TOPICS IN M-THEORY

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We give a brief history of the passage from strings to branes and we review some aspects of the following topics in M-theory: (a) an extended brane scan, (b) superembedding approach to the dynamics of superbranes and (c) supermembranes in anti de Sitter space, singletons and massless higher spin field theories.

1 Tribute to Abdus Salam

Abdus Salam was a truly unique man with great achievements not only in physics but also in promoting science in developing countries. His place in the annals of science as one of the finest physicists in this century is assured. He is certainly immortalized with his work on the unification of electromagnetic and weak interactions. His achievements in physics extend beyond this monumental work. He contributed many important ideas in particle physics, covering important aspects of renormalization theory, spontaneous symmetry breaking, grand unified theories, superspace, string theory and supermembrane theory.

Abdus Salam’s odyssey in physics began in earnest in 1950 when, after having realized that he “saidly lacked the sublime quality of patience” needed for conducting research in experimental particle physics, he started to work under the guidance of Nicholas Kemmer, who advised him to collaborate with Paul Matthews (who was completing his PhD work at Cambridge University at the time) on renormalization of meson theories. This marked the beginning of an amazing journey from the pion-nucleon theory in 1950 to the marvelous discovery of the Standard Model 17 years later. Abdus Salam has given a wonderful account of “the story of the short-lived rise of the pion-nucleon theory as the standard model of 1950-51” and the “story of the rise of chiral symmetry, of spontaneous symmetry breaking and of electroweak unification”, including the story of his interactions with Pauli, Peierls, Ward, Weinberg, Glashow and others, in his Nobel Lecture of 1979. Reading the account of the twists and turns encountered in the remarkable odyssey which lead to the unification of electroweak interactions, one feels the excitement of it and

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appreciates all the more what the research in our discipline is really all about.

I first met Abdus Salam in 1981 in Trieste, when I joined the Abdus Salam International Center for Theoretical Physics as a postdoctoral fellow. This marked the beginning of a very enjoyable and fruitful collaboration. I was privileged to have interacted with Abdus Salam for more than a decade. I will always cherish this experience. It was amazing how Abdus Salam treated a young post-doc that I was with so much humility. When we completed our first paper, he insisted that I would put my name first. I had to argue vigorously to convince him to put our names in alphabetical order.

During the 80’s, we wrote a series of papers [6-19] and we edited a reprint collection with commentaries on supergravities in diverse dimensions [20]. Our work span topics in higher dimensional Poincaré, anti de Sitter and conformal supergravities, their anomalies and compactifications, string theory and supermembrane theory. Abdus Salam was legendary in being open minded to new ideas. He embraced the developments in the subject of supermembranes when not many others did. He gave his full support for the supermembrane conference which was held in Trieste in 1989. As far as I know, this was the first conference ever to be held on membranes. The last papers we collaborated on dealt with connections between membranes, singletons and massless higher spin fields, which are among my favorites. I believe that the full significance of the ideas put forward in those papers will someday be better appreciated, in the process of discovering what M-theory is. It was a joy to speculate about the tantalizing brane-singleton-higher spin gauge theory connections in collaboration with him.

Abdus Salam expressed his motivation in his research very humbly when he said: “I have spent my life working on two problems: first, to discover the basic building blocks of matter; and secondly to discover the basic forces among them” [21]. He was deeply religious man who realized the limitations of science. He wrote: “my own faith was predicated by the timeless spiritual message of Islam, on matters on which physics is silent, and will remain so” [22]. In the same article, he wrote: “the scientist of today knows when and where he is speculating; he would claim no finality for the associated modes of thought”. Such was the humility and wisdom of the man.

Abdus Salam was a man with boundless energy and many creative ideas, not only in physics but also in the process of promoting science in the political domain. He travelled frequently all over the world and in addition to his extremely productive research activities that resulted with over 250 publications, he gave many speeches and he wrote several articles on subjects other than
physics. He had an amazing ability to focus on the heart of matters at hand.
He was always able to bear in mind the big picture. He was very eloquent in his
speeches and his writing. Among many topics, ranging from the importance
of transferring science to developing nations to the interaction of science with
religion and society, he wrote with passion about the glorious era which some
of today's developing countries once had in science. He lamented the decline
of science in those countries and he was deeply disappointed with the existing
and ever widening gap between the developed and developing countries. He
wrote passionately about why most of the developing countries need help in
building up scientific infrastructure at all levels and why science transfer must
accompany technology transfer if the latter is to take root in those countries.
He wrote: “of the two passions of my life, the second has been to stress the
importance of “science transfer” for developing countries. After building up
the Theoretical Physics Department at Imperial College, London, I have spent
20 years fighting the battle of stressing the necessity of science transfer for
developing countries.” He highlighted many important aspects of this and
many other related problems in brilliant speeches and essays which have been
collected in a book entitled “Ideals and Realities”.

Abdus Salam made it a mission to himself to work towards the advancement
of science in developing countries. The immensely successful Abdus Salam
International Center for Theoretical Physics in Trieste (AS-ICTP) which he
founded in 1964 is a monument to his extraordinary achievement in this sphere.
The story of how AS-ICTP came to existence and how it transformed to what
it is today is an amazing one which can be glimpsed from various essays that
appeared in [1]. Abdus Salam was the primary driving force in this process
from the very beginning. He always had brilliant ideas for how to expand the
functions of the Center and he saw to it that those ideas were actually put into
action [1]. Among many functions of the Center aimed at promoting scientific
activities in the developing countries perhaps the most important one was to
make it possible for the physicists from those countries to visit the Center,
and as he put it, to “recharge their intellectual batteries, work on research
problems while at the Center, and then return to their countries carrying a
new line of work, refreshed with new ideas and new interactions”.

It should be pointed out that this success did not come easy. Abdus Salam
had to fight critical battles at times to ensure the continuation and expansion
of the Center. He worked incredibly hard to this end. He had to make sac-
rifices, among which was the amount of time he could devote to his beloved
research activities. I remember once hopping into a car which was taking him
to the Venice airport for one of his frequent trips, so that we could continue
our physics discussion en route to the airport. He was constantly at work in trying to ensure the success of the Center and at the same time trying to carry out his research in physics which he loved so much. With his relentless efforts, he contributed to the advancement of science in the Third World in many ways. Indeed, the legacy of Abdus Salam is not only his epoch-making contribution to physics but also his brilliant contributions in building up the scientific manpower in the developing nations.

Abdus Salam, a great man, brilliant, humanitarian, idealist, visionary, articulate, eloquent and passionate, the man never at rest, is no longer with us but he will always be remembered.

2 From Strings to Branes

A Brief History

The notion of extended objects in the context of elementary particle physics arose several years before the discovery of supersymmetry. The most notable introduction of the idea is due to Dirac who, in 1962, envisaged the possibility of the muon being an excited state of a membrane in four dimension whose ground state corresponds to an electron. With the discovery of supersymmetry in early 70’s, the physics of the extended objects took a remarkable turn, though for nearly twenty years the focus was on the simplest extended object, namely the string.

It should be pointed out, however, that prior to the proposal of Yoneya, Scherk and Schwarz in 1974 to interpret the dual models as theories of elementary particles rather than hadrons, there were attempts to generalize the dual models to exhibit the four dimensional conformal SO(4, 2) symmetry. Inspired by the connection between the dual resonance models and strings, a further step was taken in [25] (see also [26]) to associate the dual models possessing extended conformal symmetry with extended objects (see [27] for a review). In [26], the \((N-1)\) extra spatial dimensions were associated with a globally \(SO(N, 2)\) invariant theory, and these dimensions were interpreted as the orbital degrees of freedom of \((N-1)\)-dimensionally extended objects, related to the “dimension” of the hadronic matter. In [26], the study of the asymptotic behaviour of generalized dual model amplitudes led to the consideration of \(2k\) dimensional extended objects. In [27], the intrinsic nonlinearity of Nambu-Goto type actions for 3-dimensional extended objects was recognized and the compactification of a four dimensional worldvolume to a two...
dimensional worldsheet with continuous internal symmetry was considered.

Turning to the story of super extended objects, to begin with, a manifestly worldsheet supersymmetric Neveu-Schwarz-Ramond formulation of string theory was discovered in 1971 (see [21] for a review). The considerations of anomalies led to the critical target space dimension of $D = 10$. This result, which can be derived in many different ways, is one of the most amazing discoveries in physics. The target space supersymmetry was not manifest in this formulation, but this was remedied by Green and Schwarz who discovered just such a formulation, though they had to sacrifice manifest worldsheet supersymmetry. The Green-Schwarz superstring has a local fermionic symmetry on the worldsheet known as $\kappa$-symmetry, which is necessary for theory to make sense for several reasons. The Lorentz covariant quantization of the Green-Schwarz string proved to be a difficult problem, however, and consequently most of the work done in string theory has been based on the Neveu-Schwarz-Ramond formulation.

It turned out that the extension of the superstring construction to higher extended objects heavily favors the Green-Schwarz formalism, where one constructs an action for the map from a bosonic $(p + 1)$ dimensional worldvolume to a target superspace. Significant progress towards the generalization of the Green-Schwarz action to higher branes came after a better understanding of its interpretation. A particularly useful such understanding was achieved in a paper by Hughes and Polchinski where the classical Green-Schwarz superstring action in $D = 4$ was understood as the effective low energy action for a Nielsen-Olesen vortex solution of an $N = 1, D = 4$ supersymmetric Abelian Higgs model such that the $N = 1$ supersymmetry is broken down to $(2,0)$ supersymmetry in the $(1+1)$ dimensional worldsheet. Soon after, the analog of this phenomenon was shown to arise in the context of a three-brane solution of $(1,0)$ supersymmetric Yang-Mill theory in $D = 6$, such that, this time the $(1,0)$ supersymmetry is broken down to $N = 1$ supersymmetry on the worldvolume of the three-brane. The action was constructed for the collective coordinates which form an $N = 1$ scalar supermultiplet on the three-brane worldvolume.

In 1987, inspired by these results, Bergshoeff, Townsend and the author constructed an eleven dimensional supermembrane action. The target space was taken to be a curved superspace, and the requirement of $\kappa$-symmetry was shown to require the equations of motion of the eleven dimensional supergravity! Thus, connection was made between the eleven dimensional supergravity which was invented in its own right nearly a decade before and supersymmetric extended objects. The action was constructed directly without the
knowledge of any membrane solution, which was to be found years later. It seemed to be a very natural extension of the Green-Schwarz superstring action in $D = 10$ to a supermembrane action in $D = 11$. Thus, it was very tempting to consider it as a candidate for the description of a fundamental supermembrane theory that went beyond string theory in a natural way. While it was hoped that this passage from string to membrane theory might have welcome consequences in solving some of the outstanding unsolved problems of string theory, the theory was put forward essentially because “it was there”. In other words, it was considered as a logical possibility, primarily on the basis of symmetry considerations. It was not invented out of pressing needs in physics based on paradoxes or anomalies, with the possible exception of the desire to “explain” the existence of $D = 11$ supergravity. In fact, once one comes to term with the basic idea of transition from elementary particles to strings, then the passage from strings to membranes is very natural. Indeed, the higher than two dimensional extended objects were also considered and their actions were constructed in a proper classification (with certain assumptions) was made soon afterwards and it was found that the maximum target space dimension allowed was $D = 11$ and the maximum possible extension, $p$, of the object was $p = 5$. One of the important assumptions made was that the worldvolume fields always form scalar supermultiplets. It was a number of years later that branes which support other supermultiplets, most notably the Maxwell supermultiplets in $(p + 1)$ dimensions with $p = 0, 1, \ldots, 9$ and the 2-form supermultiplets in $(5 + 1)$ dimensions were discovered.

The idea of a fundamental supermembrane in $D = 11$ was pursued for couple years after 1987 intensely by a number of authors. Primarily the following issues were addressed: (a) spectrum and stability, (b) anomalies, (c) perturbation theory, (d) covariant quantization, (e) chirality/non-Abelian internal symmetries (f) renormalizability (g) supermembrane in anti de Sitter space and its relation to singleton field theory on the boundary of AdS. We shall come back to these points later.

Considerably intense activities on supermembrane culminated in a Trieste conference in 1989, devoted to the subject. I believe this was the first conference ever on supermembranes. The spectrum problem was emphasized considerably, though other aspects of the supermembranes were also covered. The spectrum issue appeared to be problematic due to the indications that “the supermembrane can grow hair” without cost in energy, which seemed to imply a continuum in the spectrum. No dent could be made in the quantization problem. Despite the lack of covariant quantization scheme, and the lack of any information about the consequences of the $\kappa$-symmetry at the quantum
level, the theory was widely considered to be nonrenormalizable. Moreover, it
appeared to be hopeless that the theory could ever produce a chiral spectrum
by any compactification scheme, for it seemed to be intrinsically nonchiral. It
also appeared that a realistic internal symmetry gauge groups could not be
obtained.

There were, however, some results of promising nature by 1989 (see [138] for
an extensive list of references on (super)membranes covering the period by mid
1990). To begin with, the particle, and string limits of the supermembrane
were obtained. These limits were sensible and they were suggestive of an
important role for the supermembrane to play in the larger scheme of things.
The study the zero modes of the supermembrane in the superparticle limit
showed that the zero-mode oscillators gave exactly the spectrum of massless
states in $D = 11$ supergravity. Even more interesting was the result that
a double dimensional reduction of the $D = 11$ supermembrane action yielded
the type IIA superstring action. The significance of this result was not
appreciated at the time, partially due to the fact that the type II strings were
considered as academic cases since they could not give rise to any promising
physical picture in $D = 4$ by contrast with the heterotic string. It would have
been nearly impossible at the time to imagine that one day (in less than a
decade, to be more specific) the heterotic string would be related to eleven
dimensional supergravity and that all strings would be unified in an eleven
dimensional theory! See below.

In a related development, the $D = 11$ supermembrane was quantized in the
limit in which the membrane was wrapped around a circle or torus. This
procedure brought into focus the study of the Kaluza-Klein modes of the type
IIA string as well. Again, the utility of the procedure of wrapping membrane
around compact spaces so as to examine the physics of the branes in regions
where they look like particles or strings, amenable to perturbation theory was
not fully appreciated until much later.

Another interesting development was the emergence of the area-preserving dif-
feormorphisms, SDiff, of the supermembrane as a useful tool in the study of
the quantum theory (for a generalization to volume preserving diffeomo-
orphisms of super $p$-branes, see [40]). The interesting fact that the superme-
brane Hamiltonian in a light-cone gauge turned into a gauge Yang-Mills
theory in $(0 + 1)$ dimensions (i.e. supersymmetric quantum mechanics) with
SDiff $\sim SU(\infty)$ as a gauge group was discovered. This story too was to be

$^5$Interestingly, there exists a generalized Virasoro algebra for the type IIA string which can
be deduced from the algebra of worldvolume diffeomorphisms and $\kappa$-symmetry of the $D = 11$
supermembrane.
appreciated later more fully, in the context of the matrix model approach to M-theory 66.

Last but not least, soon after the eleven dimensional supermembrane was discovered, it was speculated 106, 108, 109 that singletons, which are special representations of the anti de Sitter group 98, 100, may play a role in its description. Soon after, it was conjectured 107, 108, 109 that a whole class of AdS compactifications of supergravity theories may be closely related to various singleton field theories. The singleton representations of the AdS group are special in that they are ultra short and, strikingly, they admit no Poincaré limit. Indeed, they can be realized in terms of fields that propagate on the boundary of AdS 100. The kinds of singleton field theories studied back then were free field theories, and that raised the hope that while supermembrane theory may seem to be non-renormalizable in general, it might miraculously be renormalizable in special backgrounds involving AdS space, where it may be treated as a free singleton field theory. The recent exciting developments in AdS/CFT correspondence 133, 134, 135 is reminiscent of these hopes, though the exact fashion in which this connection has emerged certainly goes beyond what was known and imagined previously. One thing that was imagined before, and has not been materialized yet, is a byproduct of the conjectured brane-singleton connection, namely the possible occurrence of infinitely many massless higher spin fields in AdS 4 as two supersingleton bound states in supermembrane theory! This conjecture was put forward in 17, 18, 109. We will discuss this topic further in section 3.

Despite these interesting developments, the quantization of the supermembrane and higher superbranes remained to be an unsolved problem. As early as 1988, however, aspects of branes as solitons or topological defects which break the target space symmetries were revisited 48 and this proved to be a very fruitful move. It was suggested 48 that all p–branes known at the time should correspond to solitonic solutions of certain supersymmetric field theories (just as in the case of 3-brane of 32) or in the case of 10D strings and 11D supermembrane, they should arise as solutions of appropriate supergravities. Few years later, interesting and significant results started to appear in this direction. In 1990, a string solution of D = 10 supergravity was found 49. Soon after, a fivebrane solution of D = 10 supergravity coupled to Yang-Mills was discovered 50. These results meant that string theory contained solitonic objects in its spectrum which were nonperturbative in nature. This implied a simple yet very important fact that supersymmetric extended objects could not possibly be ignored any longer; they were simply there and inevitable! The soliton fever carried over to eleven dimensions as well. In 1991, the supermembrane soliton 51 and in 1992 the superfivebrane soliton of D = 11 supergravity were discov-
The discovery of the superfivbrane was somewhat surprising for it was not in the original brane scan. Fascinating aspects of the superfivbranes have been discovered since then and they, of course, occupy an important place in the big scheme of things.

In the early 90’s, a whole class of brane solutions in type II string theories were also found. The study of type II branes eventually led to a remarkable discovery by Polchinski in 1995 that the type II $p$-branes carrying Ramond-Ramond charges can be understood as Dirichlet branes, or D-branes for short. These are $p$ dimensional surfaces on which an open string can end. Thus, it became possible to study the dynamics of at least special kinds of $p$-branes (at weak coupling limit) by studying the (perturbative) dynamics of an open string! The “D-brane technology” developed rapidly (see [142] for a review) and it provided an important tool for studying the dynamics of intricate brane configurations, leading to discoveries of novel physical phenomena and to breakthroughs in the study of long standing problems in black hole physics.

Concomitant to these developments, and closely related to them, were the important discoveries in the arena of duality symmetries of string theory. Remarkable results on T-duality symmetries relating strings in backgrounds involving a circle of radius $R$ and $1/R$ and $S$-duality transformations interchanging strong and weak coupling limits started to accumulate with increasing rate. This is a vast subject, even a brief review of which would take us beyond the scope of this brief introduction. We refer the reader to the excellent reviews of this subject existing in the literature; see for example, [129-138].

Further studies in strong-weak coupling dualities led to major developments in late 94 and early 95 which finally put the string-membrane connection on a firm footing. Firstly, it was observed that the soliton spectrum of the compactified $D = 11$ supergravity agreed with that of compactified type IIA string by the inclusion of the wrapping modes of the supermembrane and superfivbrane and by taking into account the wrapping modes of the type IIA D-branes carrying Ramond-Ramond charges. Next, it was argued that the $D0$–branes of type IIA string correspond to the Kaluza-Klein modes of $D = 11$ supergravity. These were tantalizing new hints for a deep connection between type IIA string and the eleven dimensional supermembrane that went beyond the connection based on double dimensional reduction, because it was not necessary to consider only the zero modes. Finally, Witten in his celebrated paper argued convincingly that the strong coupling limit of type IIA string theory is the $D = 11$ supergravity! Furthermore, the ensuing developments showed that all string that all string theories in $D = 10$ were unified via duality relations involving an eleven dimensional origin in one way or another! The view
started to develop that there exists an intrinsically nonperturbative and quantum consistent eleven dimensional theory with the properties that (a) it can be approximated by $D = 11$ supergravity at low energies, (b) it contains the supermembrane and superfivebrane in its spectrum and (c) when expanded perturbatively in different coupling constants, it gives different perturbative theories, which can be superstrings or superparticles. This mysterious theory was named (upon a suggestion by Witten) the M-theory. Work on M-theory continues with rapid pace (see, for example, [141, 144, 145] for technically detailed reviews) and striking new results have emerged from the studies of M-theory in anti de Sitter background.

In the light of these developments, it is interesting now to revisit the questions that arose in the late eighties in the context of the eleven dimensional supermembrane, which were mentioned above. The problem of chirality and non-Abelian internal symmetries found a remarkable answer with the discovery made by Horava and Witten that M-theory compactified on an interval leads to the $E_8 \times E_8$ heterotic string! This phenomenon provides a surprisingly simple and elegant answer to the question of how to obtain a chiral theory starting from eleven dimensions. As for the problems associated with the quantization of the supermembrane, there is no solution in sight (not yet!)

**M Theory-Supermembrane Connection**

Notwithstanding the presently unsolved problem of how to quantize the supermembrane (perturbatively or nonperturbatively) in eleven dimensions, it is tempting to pose the following question: Is it conceivable that M-theory is nothing but the eleven dimensional supermembrane theory? According to [147], “most experts now believe that M-theory cannot be defined as a supermembrane theory”. While we are not aware of all the arguments leading to this assertion, some of them go as follows: (a) The supermembrane action is only meant to describe a macroscopic object in M-theory and therefore one should not even attempt to quantize the supermembrane. (b) The fundamental and solitonic supermembranes should be identified. The latter has a finite core due to its horizon. Since the known supermembrane action does not take into account this classical structure of the membrane, it is not an appropriate starting point for quantization. (c) As far as the perturbative formulation goes, there is no suitable perturbative expansion parameter (assuming that the theory is not compactified) to order the spacetime amplitudes and moreover the worldvolume perturbation theory is non-renormalizable.

Despite all these considerations, the eleven dimensional supermembrane the-
ory seems to be the only theory that goes beyond $D = 11$ supergravity and which does incorporate supergravity equations of motion. Indeed, it does so already at the classical level, thanks to the power of $\kappa$–symmetry. The theory goes beyond $D = 11$ supergravity because we know that the quantization of the supermembrane collapsed to a point yields the $D = 11$ supergravity spectrum, and that the wrapping the supermembrane around a circle gives type IIA string theory, a perturbative treatment of which yields an infinite set of higher derivative corrections to type IIA supergravity. It is natural to expect a supermembrane origin of these corrections.

In this context, let us recall that while the $\kappa$–symmetry of the supermembrane uniquely leads to the $D = 11$ supergravity equations of motion \[33\], there is one correction to these equations \[68\] that has been motivated by the considerations of sigma model anomalies in $M5$–brane and one-loop effects in type IIA string \[67\], and takes the form $C_3 \wedge X_8$, where $C_3$ is the 3-form potential in $D = 11$ and $X_8$ is an 8-form made out of the Riemann curvature. Supersymmetrization of this term implies an infinite number of terms in the action, just as in the type IIA theory in $D = 10$. Some aspects of these terms have been deduced from one-loop effects in $D = 11$ supergravity, but this cannot be the full story \[c\].

What then is the principle which governs the corrections to $D = 11$ supergravity? Since we no longer have the benefit of worldsheet superconformal invariance in curved background that helps in answering a similar question in $D = 10$, we have to find a new principle here. Local supersymmetry is very helpful, but we need more than that, based on the lessons learnt from string theory. We mentioned above that the $\kappa$–symmetry of the supermembrane was very restrictive by giving the usual $D = 11$ supergravity equations. Perhaps one should re-examine the issue of $\kappa$-symmetry by allowing more general superspace than the standard $D = 11$ Poincaré superspace.

To have a control over the higher derivative corrections in $D = 11$, it is very useful to work in superspace. One possible approach is to modify the superspace Bianchi identity as follows

$$dH_7 = G_4 \wedge G_4 + X_8,$$

where the $G_4 = dC_3$ and $H_7$ is a 7-form whose purely bosonic components are Hodge dual to those of $G_4$. The $\theta = 0$ component of this equation has been extensively discussed in \[68\], but not much is known about the superspace \[c\].

\[c\] It is nonetheless an amazing fact that these one-loop effects are capable of producing nonperturbative effects in type II string theory \[c\].
consequences of the full equation, essentially due to its complexity. It is possible that the solution requires the modification of the standard supergeometry by introducing the 2nd and/or 5th rank Γ–matrix terms in the constraint for the dimension zero supertorsion components $T_{\alpha\beta}^{a\theta_{1} \theta_{2}}$. It would be very interesting to derive the corrections to the minimal superspace geometry from the $\kappa$–symmetry considerations, or better yet, from the formalism of superembeddings, which will be summarized in section 4. It is clear that any modification of the standard $D = 11$ supermembrane action, or equations of motion, would be very interesting and it would effect the discussion of just what is the role of the supermembrane in M-theory.

In the above discussion we omitted the superfivebrane. It is natural to expect a sort of duality relationship between supermembranes and superfivebranes (see [143] for a discussion of the membrane/fivebrane duality in $D = 11$). Moreover, it is well known that an open supermembrane can end on a superfivebrane in $D = 11$. Thus, the connection between $M2$–branes and $M5$–branes in $D = 11$ is similar (in some respects) to the connection between open strings and $Dp$–branes in $D = 10$. In fact, it has been shown that the superfivebrane equations of motion follow from the $\kappa$–symmetry of an open supermembrane ending on a superfivebrane!

We conclude this introduction by emphasizing the importance of exploring the consequences of the M-theory unification at the level of interactions. Much of the work done so far understandably has dealt with spectral issues and this has been very beneficial. However, at some point several problems about interactions need to be probed more deeply. Some encouraging results have already emerged. Interesting connections between the string interactions in $D = 10$ and membrane interactions in $D = 11$ have been noted. We already mentioned the one-loop effects in $D = 11$ supergravity giving rise to nonperturbative information about type IIA string amplitudes. Another example is the deduction of the quantum Seiberg-Witten effective action for $N = 2$ supersymmetric Yang-Mills theory from the classical M-fivebrane equations of motion with $N$ three branes moving in its worldvolume.

In the next section we will discuss an extended brane scan covering various kinds of branes that have emerged until now. The emphasis will be on the amount of supersymmetry breaking by the branes. The purpose of this section is to give a feel for the panorama of branes in M-theory and their properties. The superembedding theory plays a significant role not only in their classifi-

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$d$ It is rather amusing to see that the Planck constant emerges as a combination of certain integration constants arising in the course of solving the three-brane equations.

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cations but also in the description of their dynamics. With this motivation in mind, section 4 is devoted to a summary of the basic ideas behind the superembedding theory. When the target space is taken to be a supercoset involving anti de Sitter space, remarkable things happen, as it has become abundantly clear with recent exciting developments. In section 5, we summarize some aspects of the connections between supermembranes, singletons and higher spin gauge theory.

3 Types of Branes

A Minimal Brane Scan (The Scalar Branes)

Until the discovery of the superfivebrane solution of the $D = 11$ supergravity in 1992, the types of branes that were known were rather limited in number. Some of them were already anticipated in [33] and classified properly in [35]. The result is reproduced in Table 1. For uniformity in the nature of the scan, we have left out the type IIB, type I and heterotic strings. The main characteristic of all the branes occurring in this scan is that they all support a scalar multiplet with $1, 2, 4, 8$ scalars and matching spin $1/2$ fermionic degrees of freedom. The branes in this scan fall into four series: The octonionic, quaternionic, complex and real series with co-dimensions $8, 4, 2, 1$ embeddings, respectively. All the members of a given series can be obtained from the one that occupies the highest dimension by the process of double dimensional reduction. All branes in this scan for $p > 1$ have minimal possible target space supersymmetry.

The brane scan shown in Figure 1 was originally derived from the requirement of $\kappa$-symmetry of their actions. This requirement amounted to the existence of suitable Wess-Zumino terms which was possible whenever a closed super $(p + 2)$ in target superspace existed. This in turn meant finding the values of the pairs $(p, D)$ for which the following $\Gamma$-matrix identity is satisfied

$$\Gamma_{\mu(\alpha\beta} \Gamma^{\mu_{\nu_1}...\nu_{p-1}}_{\gamma\delta)} = 0 ,$$

where $\mu = 0, 1, ..., D - 1$ and $\alpha$ labels a minimal dimensional spinor in $D$-dimensions. There is a simple alternative way to deduce the same brane scan. Indeed, using the rule

$$D = (p + 1) + n_S ,$$

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where $D$ is the bosonic dimension of the target space, $p$ is the spatial brane dimension and $n_S$ is the number of real scalars in the scalar multiplet, together with the knowledge of which supermultiplets exist in diverse dimensions, the brane scan shown in Table 1 can easily be reproduced.

Of the branes for $p > 1$, only the supermembrane in $D = 11$ attracted the most interest for some time. There was a modest attempt to try to rule out the “other branes” on various grounds though, primarily on the basis Lorentz anomaly considerations. At the time, intersecting branes were not known and all the scalar branes were considered in their own right. The constraints implied by the $\kappa$-symmetry of their actions have been determined long ago. The consequences of these constraints have not been studied so far except for the cases of strings in $D = 3, 4, 6, 10$ and the supermembrane in $D = 11$. In the case of fivebrane in $D = 10$, one can check that the dual formulation of (1,0) supergravity is consistent with the $\kappa$-symmetry constraints, though it is considerably more difficult to show that it is uniquely implied by these constraints. As for the solution of the $\kappa$-symmetry constraints for the remaining branes that occur in the old brane scan (see Figure 1), we expect that the dimensional reduction of the dual (1,0) supergravity in $D = 10$, which contains a 6-form potential, followed by a truncation of the resulting vector multiplets, provides a solution.

While the old $p$–branes we have been discussing may emerge in the physics of intersecting branes, it is still interesting to determine if they can describe consistent brane theories formulated in subcritical $D < 10$ dimensional spacetimes. If so, these branes might correspond to an interesting class of M-theory limits in which the $(10 - D)$ or $(11 - D)$ extra dimensions decouple in a rather drastic way. In fact, this is somewhat reminiscent of the situation arising in the so called “little m” theories. However, in the latter case the target space theory typically involves the Yang-Mills supermultiplet but not the supergravity fields. For example, the little $m$-theory in $D = 7$ involves only the super Yang-Mills theory. This is to be contrasted with the supermembrane in $D = 7$ which arises in the old brane scan (see Figure 1), where the target space theory naturally involves the $N = 1, D = 7$ supergravity but not super Yang-Mills. In fact, this raises the interesting question of whether one can couple supergravity plus Yang-Mills system to the supermembrane in $D = 7$ that arises in the old brane scan. If such a coupling exists, it would be reasonable to expect a limit in which supergravity is decoupled.

Another aspect of the branes listed in Figure 1 which might be worthwhile to study is their anomalies; namely the gravitational anomalies in the target field theories as well as the $\kappa$–symmetry and global anomalies on the worldvolume.
In doing so, one immediately rules out the fivebrane in $D = 10$ on the basis of its incurable gravitational anomaly. However, many of the lower dimensional branes, in particular those for which the target space is odd-dimensional such anomalies do not arise. For example, the supermembrane in $D = 7$ has odd dimensional target as well as odd dimensional worldvolume, and consequently, the pertinent anomalies are the global ones. Indeed, such anomalies have
been studied by Witten \cite{Witten} in the case of the $D = 11$ supermembrane, and it was found that a mild restriction arises on the topology of spacetime and the possible configuration which the membrane Kalb-Ramond field may assume. With similar conditions satisfied, we expect that the supermembrane in $D = 7$ is anomaly free, though we do not know at present how this brane might possibly arise in M theory.

**M-Branes and D-Branes**

With the discovery of the superfivebrane, the novel situation in which the worldvolume supported a multiplet other than scalar multiplet emerged. Indeed, it was found that the $5 + 1$ dimensional worldvolume theory was that of $(2,0)$ tensor multiplet, containing a two-form potential with self-dual field strength, giving rise to 3 on-shell degrees of freedom, five scalars and 8 on-shell fermi degrees of freedom. The rule (3) still holds. It was speculated later that there should be an analog of this brane with $(1,0)$ tensor multiplet that contains a single scalar, in addition to the a self-dual tensor and 4 on-shell fermions. The single scalar suggests a seven dimensional target space. This model has been analyzed in considerable detail in \cite{97}.

In 1985, the $D$-branes were discovered \cite{61}. These are branes on which open strings can end. The worldvolume dynamics of these branes is described by vector multiplets. Considering the maximally symmetric Maxwell multiplets in various dimensions, i.e. those with 8 bose and 8 fermi on-shell degrees of freedom and using the rule (3), one finds they all imply $D = 10$ target space with type II supersymmetry. Allowing nonmaximal vector supermultiplets on the worldvolume gives rise to a revised brane scan \cite{53} and applying (3) one finds $D = 3, 4, 6$ dimensional target spaces.

At this point, it is tempting to contemplate a classification of all possible branes by classifying all possible globally supersymmetric multiplets in dimensions $p + 1 \leq 10$. There are some complicating factors, however.

Firstly, there is the possibility of dualizing a member of the worldvolume supermultiplet, say a $q$-form potential, to a $p - q - 1$ form potential. Indeed, one may consider the dualization of a number of forms at the same time. While this may be a simple matter at the free field theory level, it is considerably more difficult for branes where the worldvolume multiplets are self-interacting in a highly nonlinear fashion. In fact, the dualization of forms on the worldvolume is intimately connected with the $S, T$ or more general duality transformations. Consequently, there is the additional question of which branes should be considered as the fundamental ones from which all others can be derived by one
duality transformation or by Kaluza-Klein type reductions of the target space and/or the worldvolume.

Secondly, it is possible that the description of the worldvolume theory involves more than one supermultiplet. The simplest example of this is the heterotic string where the worldsheet contains scalar multiplets in the left-moving sector and heterotic fermions or bosons, which are supersymmetry singlets, in the right-moving sector. The fact that worldsheet supersymmetry allows supersymmetric singlets is special to 1 and 2 dimensions, and it cannot occur in higher than two dimensions. Nonetheless, focusing our attention on the fact that the heterotic string is described by two distinct multiplets on the worldsheet, we tentatively refer all branes which support more than one irreducible supermultiplet as heterotic branes.

Given the complications just described, the classification of branes becomes a rather nontrivial task. However, one may consider an alternative scheme in which one focuses on the amount of supersymmetry breaking due to the embedding of superbranes in a given target superspace rather than emphasizing the worldvolume supermultiplet. This approach especially becomes powerful if one considers both the target space as well as the worldvolume to be superspaces. Thus, one considers the embedding theory of supersurfaces, which turns out to be a geometrical and natural framework, not only for classifying the superbranes, but also for providing the manifestly worldvolume and target space supersymmetric dynamical equations. Indeed, the problem of describing the superfivebrane equations of motion was solved for the first time by using this formalism\[87,88\]. The main criteria in the theory superembedding theory from the physical point of view is that the basic equations which describe the superembedding lead to sensible equations of motion for the worldvolume fields. This can be typically checked in a reasonably straightforward way at the linearized level, at least for a large class of superbranes.

The classification of all possible branes is still a formidable task despite the universal nature of the superembedding approach. A further complicating factor is that the geometry and topology of the spaces involved can mathematically become rather complicated. For example, the full actions for intersecting branes is yet to be constructed in any approach. Nonetheless the existence of a large class of branes has been deduced from the study of brane solutions to supergravity theories, or from the study of the possible brane charges in supersymmetry algebras, or from the analysis of the linearized embedding equations. Putting together a variety of information available on the nature of existing and predicted types of branes, they can be classified according to the amount of supersymmetry they preserve. For the purposes of the proposed scan, we
will assume that the maximum real dimension of supersymmetry is 32 and we shall furthermore consider flat target superspaces with Lorentzian signature.

32 → 16 Branes

Assuming Lorentzian signature, the maximum dimension in which real 32 symmetries can occur is \( D = 11 \). In \( D = 11 \) the target superspace has (even—odd) dimensions \((11|32)\). Embedding a \((3|16)\) dimensional super worldsurface gives the supermembrane, and embedding a \((6|16,0)\) dimensional super worldsurface gives the superfivebrane. The former is a co-dimension 8 embedding, and the later is a co-dimension 4 embedding. In the latter case, the notation \((16,0)\) means that there are 16 real left-handed spinors and no right-handed spinors. This means \((2,0)\) supersymmetry, since the minimum real dimension of a spinor in \( 5+1 \) dimensions is 8. Such hybrid superspaces can occur in \( 2 \mod 4 \) dimensions (assuming Lorentzian signature).

The supermembrane worldvolume multiplet is an \( N = 8 \) scalar multiplet, which has 8 real scalars and the superfivebrane worldvolume multiplet is the tensor multiplet of \((2,0)\) supersymmetry which has 5 real scalars and a 2-form potential with self-dual field strength. Thus, the relation (3) holds in both cases.

In \( D = 11 \) there are two other “superbranes” which preserve half supersymmetry, but for which the relation (3) does not hold. These are the pp-waves, which can be considered in some sense as 1-branes and the Kaluza-Klein monopole, which can be viewed in a certain sense as 6-branes. The existence of 9-branes has also been speculated briefly in \([87]\) and in some more detail in \([75]\). Sometimes the boundaries of the \( D = 11 \) spacetime arising in the Horava-Witten configuration \([80]\) is also referred to as a M9-brane. See \([80]\) for a recent discussion of various aspects of M9 branes.

In all these cases, the formula (3) breaks down, and one finds 8 scalars for the pp-wave, 3 scalars for the KK monopole and no scalars for the 9-brane, as opposed to 9, 4, 1 scalars, respectively. This is essentially due to the fact that the transverse space lacks the isometries of \( R^9, R^4, R^1 \), respectively; for example, in the case of KK monopole, the transverse space is a Taub-Nut space. A detailed discussion of these branes can be found, for example, in \([75]\) where they have been called the G-branes (G standing for gravitational).

In classifying the superbranes in \( D = 10 \), one should take into account the ordinary (vertical) dimensional reduction, or (diagonal) double dimensional of the branes in \( D = 11 \). Moreover, one may take the eleventh dimension to be \( S^1 \) or \( S^1/Z^2 \). This would generate a set of branes known to exist in \( D = 10 \).
In $D = 10$ we can embed the $(p+1|16)$ dimensional $Dp$-brane worldsheets for $p = 1, 3, 5, 7, 9$ in $(10|16, 16)$ superspaces, where $(16, 16)$ denotes the 16 left-handed and 16 right-handed Majorana-Weyl spinor coordinates, associated with type IIA $(1, 1)$ supersymmetry. Similarly, the $(p+1|16)$ dimensional $Dp$-brane worldsheets for $p = 0, 2, 4, 6, 8$ can be embedded in the $(10|32, 0)$ superspace associated with type IIB $(1, 0)$ supersymmetry. The worldvolume supermultiplets for the $Dp$-branes are the maximally supersymmetric vector multiplets in $p+1$ dimensions, which have a single vector and $(9-p)$ scalars in the bosonic sector. Thus, the relation (3) holds for all these branes.

In $D = 10$, there also exist the fundamental strings, NS5-branes, pp-waves and Kaluza-Klein monopoles (which may be viewed as 5-branes), both in type IIA and type IIB superspaces. These are denoted by $1_F$, $5_S$, $1_W$ and $5_{KK}$, respectively, in Figure 2. The $Dp$ branes for $p = -1, 7, 8, 9$ are somewhat special cases which are not shown in this Figure. The type IIA/B branes are related to each other by T- and S-dualities as shown in Figure 2. The type IIA theory compactified on a circle of radius $R$ is $T$-dual to the type IIB theory compactified on a circle of radius $1/R$. The $SL(2, Z)$ $S$-duality transformation, on the other hand, is a strong-weak coupling symmetry operative in type IIB
theory: the $(1_F, 1_D)$ and $(5_S, 5_D)$ form a doublet and $3_D$ is singlet under this symmetry. The action of the S-duality on the type IIB $D7$– and $D9$–branes is more involved [76, 75, 77]. For a more detailed version of Figure 2 which summarizes almost all the known dualities between the type II branes, including the $D7$– and $D9$–branes, see [77]. For the S-duality involving type IIB KK-monopoles, see [78].

The worldvolume field content of all these branes have been obtained in some cases speculated in a number of references. We refer to [75] where an extensive discussion and references to earlier work can be found. For reader’s convenience and following [75] and [79], the $D = 11$ and type II brane worldvolume contents are listed in Table 1.

| Branes          | Worldvolume Fields         |
|-----------------|----------------------------|
| **D=11 M-Branes** |                            |
| pp-wave         | $A_1, 8 \times \phi$       |
| M2-brane        | $8 \times \phi$            |
| M5-brane        | $A_2, 5 \times \phi$       |
| KK monopole     | $A_1, 3 \times \phi$       |
| **D=10 Type IIA Branes** |                       |
| $Dp$-branes     | $A_1, (9 - p) \times \phi \ (p = 0, 2, 4, 6, 8)$ |
| Fundamental string | $8 \times \phi$           |
| pp-wave         | $A_1, 8 \times \phi$       |
| KK monopole     | $A_1, 3 \times \phi, S \sim A_4$ |
| NS 5-brane      | $A_2^+, 4 \times \phi, S \sim A_4$ |
| **D=10 Type IIB Branes** |                       |
| $Dp$-branes     | $A_1, (9 - p) \times \phi \ (p = 1, 3, 5, 7, 9)$ |
| Fundamental string | $8 \times \phi$           |
| pp-wave         | $8 \times \phi$            |
| NS 5-brane      | $A_3, 4 \times \phi$       |
| KK monopole     | $A_2^+, 3 \times \phi, S \sim A_4, S' \sim A'_4$ |

Table 1: Worldvolume bosonic fields. $A_q$ denotes a $q$-form potential, $A_2^+$ is 2-form potential with self-dual field strength, $S, S'$ are scalars which can be dualized to appropriate $q$ forms.
Branes/ Doubly Intersecting Branes

The maximum dimensional target space admitting 16 real component spinor is $D = 10$. There is only one case to consider here, which is the $(1, 0)$ supersymmetric $(10|16, 0)$ superspace (or any of its dimensional reductions). We can embed a super 5-brane with $(6|8, 0)$ world supersurface. This gives a hypermultiplet of $(1, 0)$ supersymmetry on the worldvolume and this is the old super 5-brane which was considered long ago in its own right, prior to the discovery of intersecting branes. The target supergravity theory is the dual formulation of $(1, 0)$ supergravity, which has fatal gravitational anomalies (see [19] for a review). There is an alternative way to interpret this embedding, however. It can be viewed as a $D5$-brane within a $D9$-brane of type IIB theory. This is shown as the point $5 \cap 9 = 5$ in Figure 3. It is important to note that both branes are embedded in type IIB superspace which has 32 real supersymmetry, though the 5-brane residing inside the 9-brane possesses only 8 real supersymmetries. This is known as 1/4 supersymmetry preserving brane.

We also emphasize that the supergravity theory in the target of this 5-brane is induced supergravity where all the members of the supergravity multiplet are composites of the type IIB supergravity theory. Since type IIB theory is free from anomalies, the $D5$-brane residing inside the $D9$-brane is presumably anomaly free as well. Starting from the fivebrane inside the ninebrane, one can perform two kinds of $T$-duality transformation which generates all the intersecting branes shown in Fig. 3. Starting from an intersecting pair $p \cap q$, a vertical move corresponds to a $T$ duality transformation over one of the transverse directions of the $q$ brane. An oblique move corresponds to $T$-duality over one of $q$-brane the worldvolume directions. The details of such transformations to obtain one brane solution from the other is treated in a number of papers; see for example, [73, 74].

Intersecting branes involving $NS$ branes can be obtained by utilizing combined $S/T$-duality transformations. Not including the cases involving the pp-waves and KK monopoles one thus finds

\[
\begin{align*}
Dp \cap 1_F &= 0, & p &= 1, 2, \ldots, 9, \\
Dp \cap 5_S &= (p - 1), & p &= 1, 2, \ldots, 6, \\
1_F \cap 5_S &= 1, \\
5_S \cap 5_S &= 3.
\end{align*}
\]

For completeness, we also list the by now well known double intersections of
Figure 3: Intersecting D-branes. A $D_{q_1}$ brane intersecting a $D_{q_2}$ brane over a $p$-brane is denoted by $q_1 \cap q_2 = p$. The cases $q_1 - q_2 = 4, 2, 0$ correspond to $q_1$ brane within $q_2$ brane, ending on $q_2$ brane or intersecting with $q_2$ brane, respectively. In all these cases, $p$-brane is viewed as moving in $D = q_2 + 2$ dimensional target space. Worldvolume and target supersymmetries for these $p$-branes are shown along the $p$- and $D$-axis, respectively. All the points shown in this Figure are related to each other by $T$-duality transformations described briefly in the text.
The relative and overall transverse dimensions for these intersection are $(4, 3)$, $(5, 4)$ and $(4, 6)$, respectively.

$8 \to 4$ Branes/ Triply Intersecting Branes

The maximum dimensional target space admitting 8 real component spinor is the $(1, 0)$ superspace in $D = 6$ (or any of its dimensional reductions). We can embed a superstring with $(2|4, 0)$ super worldsheet, or a super 3-brane with $(4|4)$ world supersurface in this target superspace. This gives the $(4, 0)$ supersymmetric hypermultiplet on the string worldsheet and the $N = 2$ scalar multiplet on the 3-brane worldvolume. Considered as scalar branes, these are shown in Table 1. An alternative way to interpret the above embeddings is to view them as triple intersections of suitable M- or D-branes. The multi-intersections of all known branes have been extensively studied in a number of papers. The most extensive classification known to us can be found in [73]. Here, we shall be content with the reproduction of the list for triply intersecting M- and D-branes.

To begin with, the complete list of triple intersections of D-branes is given in Figure 4. The arrows indicate various T-duality transformations whose precise nature is spelled out in [73]. The common property of these intersections is that the relative transverse dimension for any pair is always 4. The overall transverse dimension is $(3, 2, 1)$ for the triple intersections over $(0, 1, 2)$–branes, respectively.

Finally, the triple intersections of M-branes are given by

$$5 \cap 5 = 3, \quad 5 \cap 5 = 2, \quad 5 \cap 5 = 0,$$

$$5 \cap 5 = 1, \quad 2 \cap 5 = 1, \quad 2 \cap 5 = 0,$$

$$2 \cap 2 = 0.$$  

The relative transverse dimension for any pair in all of these intersections is always 4 or 5 (see, for example, [73]).
| 6 ∩ 2 ∩ 2 = 0 | → | 7 ∩ 3 ∩ 3 = 1 | → | 8 ∩ 4 ∩ 4 = 2 |
| --- | --- | --- | --- |
| ↓ | ↓ | ↓ | ↓ |
| 5 ∩ 3 ∩ 3 = 0 | → | 6 ∩ 4 ∩ 4 = 1 | → | 7 ∩ 5 ∩ 5 = 2 |
| ↓ | ↓ | ↓ | ↓ |
| 4 ∩ 4 ∩ 2 = 0 | → | 5 ∩ 5 ∩ 3 = 1 | → | 6 ∩ 6 ∩ 4 = 2 |
| ↓ | ↓ | ↓ | ↓ |
| 3 ∩ 3 ∩ 3 = 0 | → | 4 ∩ 4 ∩ 4 = 1 | → | 5 ∩ 5 ∩ 5 = 2 |
| ↓ | ↓ | ↓ | ↓ |
| 4 ∩ 2 ∩ 2 = 0 | → | 5 ∩ 3 ∩ 3 = 1 | → | 6 ∩ 4 ∩ 4 = 2 |
| ↓ | ↓ | ↓ | ↓ |
| 5 ∩ 3 ∩ 1 = 0 | → | 6 ∩ 4 ∩ 2 = 1 | → | 7 ∩ 5 ∩ 3 = 2 |
| ↓ | ↓ | ↓ | ↓ |
| 4 ∩ 4 ∩ 0 = 0 | → | 5 ∩ 5 ∩ 1 = 1 | → | 6 ∩ 6 ∩ 2 = 2 |
| ↓ | ↓ | ↓ | ↓ |
| 3 ∩ 3 ∩ 1 = 0 | → | 4 ∩ 4 ∩ 2 = 1 | → | 5 ∩ 5 ∩ 3 = 2 |
| ↓ | ↓ | ↓ | ↓ |
| 2 ∩ 2 ∩ 2 = 0 | → | 3 ∩ 3 ∩ 3 = 1 | → | 4 ∩ 4 ∩ 4 = 2 |

Figure 4: The triple intersections of D-branes. The arrows indicate various T-duality transformations.

## 4 Superbrane Dynamics from Superembedding

*Background*

Superembedding approach to supersymmetric extended objects provides a natural and powerful geometrical framework for describing the dynamics of superbranes. One of the most attractive aspects of this approach is its universality; it seems to apply to any kind of branes, regardless of whether their worldvolume multiplets are scalar, vector, tensor or, indeed, any other supermultiplet. Another virtue of this approach is that the target space and worldvolume supersymmetry are both made manifest.

In the superembedding approach to branes, the brane under consideration is described mathematically as a sub-supersurface (the worldsurface) of the target superspace. The coordinates transverse to the worldsurface are then the Goldstone superfields which encode the information about the worldsurface supermultiplets. The key point is then to impose a natural geometrical constraint on the embedding which can translate into a constraint on the Goldstone superfield. Indeed, a constraint of this nature does exist, and it will be explained below.
The superembedding approach has its origin in what is known as the twistor-like approach to superparticles/strings/branes. This approach was initiated some time ago in the context of superparticles. The formalism has been extended to branes and it has been developed by several authors over the years. In particular we refer to [85] and [87] for an extensive list of references.

Starting with [87], in a series of papers, the superembedding formalism has been developed further. In particular, in [87], the nature of the worldsurface supermultiplets emerging from the embedding formalism was spelled out. As applications, the full covariant field equations of the M theory fivebranes were obtained for the first time by using this formalism. Moreover, the existence of new types of superbranes were deduced and/or conjectured within this framework. Later, this formalism was used to describe open superbranes. The superembedding formalism yields naturally the field equations rather than an action. However, it is possible to obtain an action as well, and in a recent work the approach of has been extended to cover essentially any superbrane that does not involve worldsurface self-dual field strengths. We will come back to these points briefly later. First, let us begin with the description of the basics of superembedding formalism, followed by some examples.

**Basics of the Superembedding Formalism**

We consider superembeddings \( f : M \rightarrow M \), where the worldsurface \( M \) has (even|odd) dimension \( (d|D') \) and the target space has dimension \( (D|D') \). In local coordinates \( M \) is given as \( z^M(z^M) \), where \( z^M = (x^m, \theta^\mu) \) and \( z^M = (x^m, \theta^\mu) \) (if no indices are used we shall distinguish target space coordinates from worldsurface ones by underlining the former). The embedding matrix \( E_\Lambda^\Delta \) is defined to be

\[
E_\Lambda^\Delta = E_\Lambda^M \left( \partial_M z^\Delta \right) E^\Delta_M,
\]

in other words, the embedding matrix is the differential of the embedding map referred to standard bases on both spaces. Our index conventions are as follows: latin (greek) indices are even (odd) while capital indices run over both types; letters from the beginning of the alphabet are used to refer to a preferred basis while letters from the middle of the alphabet refer to a coordinate basis, the two types of basis being related to each other by means of the vielbein matrix \( E_\Lambda^M \) and its inverse \( E^\Delta_M \); exactly the same conventions are used for the target space and the worldsurface with the difference that the target space indices are underlined. Primed indices are used to denote directions normal to
the worldsurface. We shall also use a two-step notation for worldsurface spinor indices where appropriate: in general discussions, a worldsurface spinor index such as \( \alpha \) runs from 1 to \( \frac{1}{2} D' \), but it may often be the case that the group acting on this index includes an internal factor as well as the spin group of the worldsurface; in this case we replace the single index \( \alpha \) with the pair \( \alpha i \) where \( i \) refers to the internal symmetry group. A similar convention is used for normal spinor indices.

![Superembedding of a world supersurface](image)

**Figure 5**: Superembedding of a world supersurface \( M \) in a target superspace \( \mathcal{M} \). \( X^{a'} \) indicates the transverse coordinates which are the Goldstone superfields associated with the breaking of translations in \( \mathcal{M} \) to translations in \( M \).

The basic embedding condition is

\[
E_{a}^{\alpha} = 0.
\]  

(8)

It implies that the odd tangent space of the worldsurface is a subspace of the odd tangent space to \( \mathcal{M} \) at each point in \( M \subset \mathcal{M} \). In many cases, equation
\[ \nabla_A E_B E^C - (-1)^{AB} \nabla_B E_A E^C + T_{AB} E^C E^D = (-1)^{A(B+1)} E_B E_A T_{AB} E^C \quad (9) \]

which involves the super torsion tensors of both, the world and target super-spaces. Thus, it is clear that feeding the basic embedding constraint (8) into this integrability equation will have consequences for the worldvolume and target space supergeometries, and hence the dynamics.

Remarkably, the basic embedding equation (8) turns out to be sufficient to determine the full covariant equations of motion for the collective coordinates of the superbrane for most cases. For example, this is the case for both the \( M_2 \) and \( M_5 \) branes. A qualitative aspect of this is that the larger the codimension of the embedding is the stronger is the constraint (8). In fact, it was found in [87] that three types of multiplet can arise as a consequence of (8): on-shell, off-shell or underconstrained. In the on-shell case, there can be no superspace actions of the heterotic string type since such actions would necessarily involve the propagation of the Lagrange multipliers that are used in this construction. Nevertheless, on-shell embeddings are useful for deriving equations of motion; for example, the full equations of motion of the \( M \)-theory fivebrane were first obtained this way [88]. In the off-shell case, by which it is meant that the worldsurface multiplet is a recognisable off-shell multiplet, it is possible to write down actions of the heterotic string type. The third case that arises, and which we call underconstrained here, typically occurs for branes with low codimension. For example, in codimension one the basic embedding condition gives rise to an unconstrained scalar superfield. In order to get a recognisable multiplet further constraints must be imposed. An example of this is given by IIA D-branes where the basic embedding condition yields an on-shell multiplet for \( p = 0, 2, 4 \), but an underconstrained one for \( p = 6, 8 \). Imposing a further constraint which states that there is a worldsurface vector field with the usual modified Bianchi identity whose superspace field strength vanishes unless all indices are bosonic, one recovers on-shell vector multiplets [85]. (For \( p = 0, 2, 4 \) one can show that the vector Bianchi identity follows from the basic embedding condition.)

The description of the superembedding formalism given above may understand-
ably give the impression that it provides only the superbrane field equations but not an action from which they can be derived. In fact, a rather universal method has been proposed recently by which a superspace action can be obtained for a large class of superbranes. See [94] for a detailed description of the method and comparison with the work of [86].

The power of superembedding formalism has also been put to use in (a) the derivation of the M5–brane equations of motion from those of an open M2–brane ending on the M5–brane and the dynamics of $Dp$–branes ending on $D(p + 2)$–branes and (b) the derivation of a Born-Infeld type action for branes involving a higher than 2-form potentials in their worldvolumes. These branes have been called the L-branes because their worldvolume fields form supermultiplets known as the linear multiplets. For example, the L5–brane in $D = 9$ has the linear multiplet on the worldvolume which contains a 4-form potential, and action has been obtained for this object in [96].

**Deriving the Field Equations from the Superembedding Constraint**

In order to get a feel for how the embedding condition (8) really determines the worldvolume supermultiplet field equations, it suffices to study the linearised version of the constraints resulting from the embedding condition in flat target space limit. The supervielbein for the flat target superspace is,

\[
E^\alpha = dx^\alpha - \frac{i}{2} d\theta^\alpha (\Gamma^\alpha)_{\beta\gamma} \theta^\beta \theta^\gamma,
\]

\[
E^\alpha = d\theta^\alpha.
\]  

(10)

Let us choose the physical gauge,

\[
x^\alpha = \{x^\alpha, x^\alpha'(x, \theta)\}
\]

\[
\theta^\alpha = \{\theta^\alpha, \theta^\alpha'(x, \theta)\}
\]  

(11)

and take the embedding to be infinitesimal so that $E_A^M \partial_M$ can be replaced by $D_A = (\partial_a, D_\alpha)$ where $D_\alpha$ is the flat superspace covariant derivative on the worldsurface, provided that the embedding constraint holds. In this limit the embedding matrix is:
\[ E_{a}^{b} = (\delta_{a}^{b}, \partial_{a}X^{b'}) \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad E_{a}^{\beta} = (0, \partial_{a}\theta^{\beta'}) \]  

\[ E_{a}^{b} = (0, D_{a}X^{a'} - i(\Gamma^{a'})_{\alpha\beta'}\theta^{\beta'}) \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad E_{a}^{\beta} = (\delta_{a}^{\beta}, D_{a}\theta^{\beta'}) \]  

(12) \]

where

\[ X^{a'} := x^{a'} + \frac{i}{2} \theta^{\alpha}(\Gamma^{a'})_{\alpha\beta'}\theta^{\beta'} . \]  

(13)

Using the expressions given in (12) in the embedding condition (8), we find, at the linearized level,

\[ D_{a}X^{a'} = i(\Gamma^{a'})_{\alpha\beta'}\theta^{\beta'} . \]  

(14)

This is a general formula. Next, we consider the examples of M2 and M5 branes. Interestingly, the same embedding constraint (14) yields the on-shell field equations of \( N = 8 \) worldvolume scalar supermultiplet in the M2 brane case, consisting of 8 Bose and 8 Fermi on-shell physical degrees of freedom, and the (2, 0) worldvolume tensor multiplet in the case of M5 brane, consisting of 5 real scalars, a two-form potential with self-dual field strength describing 3 on-shell degrees of freedom and 8 fermionic on-shell degrees of freedom. Let us show how this works, starting with the case of M2 brane.

**The M2 Brane**

The M2 brane worldvolume is an (3|16) dimensional supermanifold embedded in the target superspace of dimension (11|32). The index \( \alpha = 1, \ldots, 16 \) which labels worldsurface fermionic coordinates carry a spinor representation of \( SO(2, 1) \times SO(8) \), which will be denoted by a pair of indices \( \alpha A \) where \( \alpha = 1, 2 \) labels two component Majorana spinor of \( SO(2, 1) \) and \( A = 1, \ldots, 8 \) labels the chiral spinor of \( SO(8) \). The index \( \alpha' = 1, \ldots, 16 \) labels the fermionic directions that are normal to the worldsurface which will be denoted by \( \alpha A \), where \( \bar{A} = 1, \ldots, 8 \) labels the anti-chiral representation of \( SO(8) \). The master constraint (8) can then be written as

\[ D_{\alpha A}X^{a'} = i(\sigma^{a'})_{AB} \theta_{\alpha B} \]  

(15)
where $\sigma^a$ are the chirally projected Dirac $\Gamma$-matrices of $SO(8)$ (the van der Wardeen symbols). Differentiating both sides with $D_{\beta B}$ and using the algebra of supercovariant derivatives

$$\{D_\alpha, D_\beta\} = i (\gamma^a)_{\alpha\beta} \partial_a , \quad a = 0, 1, 2 ,$$

(16)
after straightforward manipulations one finds the result

$$D_\alpha A^\theta_{\beta} = (\sigma^a)_{A}^{\cdot C} (\gamma^a)_{\alpha\beta} \partial_a X_{a'} .$$

(17)

The equations of motion now arise as follows. Differentiating (17) with $D_\gamma B$, using (14), symmetrizing the equation in $\gamma\alpha$ indices, using the algebra (16) of supercovariant derivatives and multiplying with the $SO(2, 1)$ charge conjugation matrix $\epsilon^{\alpha\beta}$, we obtain the Dirac equation

$$(\gamma^a)_{\beta\gamma} \partial_a X_{a'} = 0 .$$

(18)

Acting on (17) with $(\gamma^b)^{\alpha\beta} \partial_b$, and using the Dirac equation (18), on the other hand gives the Klein-Gordon equation

$$\partial^a \partial_a X_{a'} = 0 .$$

(19)

Continuing in this manner, it can be shows that no new components arise in the Goldstone superfields $X_{a'}$ and $\theta_{\beta}^{\cdot C}$. Thus, what we have found is an $N = 8$ on-shell scalar supermultiplet with 8 real scalars obeying the Klein-Gordon equation and 8 two-component Majorana spinors obeying the Dirac equation, altogether representing the 8 fermi and 8 bose on-shell degrees of freedom on the $M2$ brane worldvolume.

The $M5$ Brane

The procedure for analysing the constraint (14) for the case of $M5$ brane is parallel to the case of $M2$ brane just described in detail. Here, the $M5$ brane worldvolume is an (6|16) dimensional supermanifold embedded in the target superspace of dimension (11|32). The index $\alpha = 1, ..., 16$ which labels worldsurface fermionic coordinates carry a spinor representation of $SO(5, 1) \times SO(5)$, which will be denoted by $\theta_{\alpha i}$ where $\alpha = 1, ..., 4$ is the chirally projected...
spinor index of $SO(5,1)$ and $i = 1, ..., 4$ labels the spinor of $SO(5)$. The index $\alpha' = 1, ..., 16$ labels the fermionic coordinates that are normal to the worldsurface which will be denoted by $\theta_{\alpha i}$. We are using the well established chiral notation in which the lower $\alpha$ index denotes a chiral spinor, the upper $\alpha$ index denotes an anti-chiral spinor, and these indices are never to be raised and lowered by a charge conjugation matrix. Furthermore, all the spinors in question are symplectic Majorana-Weyl. See [87] for further notation and conventions.

To analyze the constraint (14) for the case of $M^5$ brane, we begin by writing it more explicitly as

$$D_{\alpha i} X^{a'} = i (\gamma^{a'})_{ij} \theta^j_{\alpha} , \quad a' = 1, ..., 5 , \quad \alpha = 1, ..., 4 , \quad (20)$$

where $(\gamma^{a'})_{ij}$ are the Dirac $\gamma$-matrices of $SO(5)$ (which are antisymmetric). The raising and lowering of the $SO(5)$ spinor index is with the antisymmetric charge conjugation matrix $\Omega_{ij}$.

Starting from (20), manipulations parallel to those described above for the case of $M^2$ brane now lead to the result

$$D_{\alpha i} \theta_{\beta j} = -\frac{1}{2} (\gamma^{a'})_{ij} (\gamma^a)_{\alpha\beta} \partial_a X^{a'} + \Omega_{ij} h_{\alpha\beta} , \quad (21)$$

where $(\gamma)_{\alpha\beta}$ are the chirally projected Dirac $\gamma$-matrices of $SO(5,1)$, and the symmetric bispinor $h_{\alpha\beta}$ defines a self-dual third-rank antisymmetric tensor

$$h_{\alpha\beta} \equiv (\gamma^{abc})_{\alpha\beta} h_{abc} , \quad h_{abc} = \frac{1}{3!} \epsilon_{abcdef} h^{def} . \quad (22)$$

Comparing the result (21) with (17), reveals that the difference between the $M^2$ and $M^5$ branes is due to the occurrence of a new worldvolume field $h_{abc}$ in the latter case. Continuing in the manner described for the $M^2$ brane case earlier, one finds by applying further spinorial covariant derivatives that the fermion field satisfies the Dirac equation, the scalar fields satisfy the Klein-Gordon equation and the tensor field satisfies the Bianchi identity and field equation for a third-rank antisymmetric field strength tensor. Furthermore, there are no other spacetime components, so that equation (14) defines an on-shell tensor multiplet. It is remarkable that this result follows from the superembedding constraint which takes exactly the same form for both the $M^2$ brane as well as the $M^5$ brane. This shows the universal nature of the embedding approach; although the worldvolume supermultiplets are rather
different in nature, they both arise from one universal superembedding constraint. Unlike the Green-Schwarz type formulation of branes in which one has to search for different kinds of actions depending on the nature of the expected worldvolume supermultiplet, here one starts from a universal and geometrical embedding formula which then determines the worldvolume supermultiplet and provides their equations of motion, if the codimension of the embedding is large enough to make the constraint sufficiently strong to do so. We just saw that this is the case for the $M2$ and $M5$ branes which are described by codimension 8 and 5 embeddings.

The $L5$ Brane

As a last example to illustrate the universality of the superembedding approach, we examine the $L5$ brane in $D = 9|87$. The $(6|8)$ dimensional worldvolume superspace is embedded in $(9|16)$ dimensional target superspace. This is a codimension 3 embedding in which the 3 Goldstone superfields give rise to a linear supermultiplet with $(1,0)$ supersymmetry in the $L5$ brane worldvolume. This multiplet consists of 3 real scalars, a 4-form potential describing 5 degrees of freedom and an $Sp(1)$ symplectic Majorana spinor describing 8 real degrees of freedom. This is an example for a superembedding in which the embedding constraint is not sufficient to put the theory off-shell. As a consequence, it is easier to write down an action formula for this theory.

The calculations at the linearized level are again very similar to those explained in detail for the case of $M2$ brane earlier, so it suffices to outline briefly how the worldvolume supermultiplet arises.

The index $\alpha = 1, ..., 8$ now labels worldsurface fermionic coordinates which carry a spinor representation of $SO(5,1) \times SO(3)$, which will be denoted by $\theta_{\alpha i}$ where $\alpha = 1, ..., 4$ is the chirally projected spinor index of $SO(5,1)$ and $i = 1, 2$ labels the spinor of $SO(3)$. The index $\alpha' = 1, ..., 8$ labels the fermionic coordinates that are normal to the worldsurface which will be denoted by $\theta_{\alpha'}^i$. The chiral notation for the spinors is as explained earlier for the case of $M5$ brane. Thus, the master embedding constraint (8) again takes the form (20). The only difference is that the index $i = 1, 2$ now labels an $SO(3)$ spinor. Steps parallel to those described above then lead to the formula

$$D_{\alpha i} \theta_{\beta j} = - (\gamma^{\alpha'})_{ij} (\gamma^a)_{\alpha\beta} \partial_a X^{a'} + \epsilon_{ij} (\gamma^a)_{\alpha\beta} h_a ,$$

where $h_a$ is the conserved vector in the multiplet, $\partial^a h_a = 0$. This field, together with the 3 scalars $X^{a'}$ and the 8 spinors $\Theta_{\alpha i}$ (evaluated at $\theta = 0$) are the
components of the (off-shell) linear multiplet. At the linearized level the field equations are obtained by imposing the free Dirac equation on the spinor field. One then finds the Klein-Gordon equation \( \partial_a \partial^a X_{ij} = 0 \) for the scalars and the field equation for the antisymmetric tensor gauge field \( \partial_a h_{ij} = 0 \). The full equations of motion can be obtained either by directly imposing an additional constraint in superspace or by using the recently proposed brane action principle which has the advantage of generating the modified Born-Infeld term for the tensor gauge fields in a systematic way.

5 Supermembranes in AdS Background, Singletons and Higher Spin Gauge Theory

String theory has often been studied in Minkowski target spacetime or in a product of Minkowski spacetime with tori, orbifolds or Ricci flat spaces such as K3 or Calabi-Yau manifolds. These are spaces which allow a perturbative formulation of string theory to all orders in \( \alpha' \) as a conformal field theory on the string worldsheet. Group manifolds also allow an exact conformal field theoretic treatment and they have been studied in the context of string theory as well, though to somewhat lesser extent. An additional motivation for focusing attention on Ricci flat spaces has been the fact that phenomenologically the most promising string theory is the heterotic string theory which has natural compactifications that require Ricci-flat internal spaces.

The study of duality symmetries in the early 90’s and the discovery of D-branes in 1995 brought the type II theories under focus. The most often studied type II backgrounds continued to be Minkowski \( \times \) flat or Ricci flat spaces for sometime but that changed drastically with the discovery in 1997 of a remarkable connection between type IIB string on \( AdS_5 \times S^5 \) and \( D = 4, N = 4 \) supersymmetric \( SU(N) \) Yang-Mills theory. The AdS background has emerged as the near horizon geometry of certain brane solutions, and connections with Yang-Mills have been found by taking particular limits in the parameter space of the theory. The study of branes in AdS space is now in full swing, and it brings together nicely many aspects of brane physics, supersymmetric field theories, Kaluza-Klein supergravities, gauged and conformal supergravities. It has the further dividend of giving new handles on old problems in nonperturbative Yang-Mills gauge theory.

The study of branes in AdS is not altogether a new development though. Already back in late 80’s, the \( D = 11 \) supermembrane was studied in AdS background. Indeed, the solutions of the \( D = 11 \) supermembrane equations
were studied in a series of papers\cite{110,111,112} in $M_4 \times M^7$ background, where $M_4$ was taken to be $AdS_4$ or its suitable covering and $M_7$ to be a suitable seven dimensional Einstein space, such as $S^7$. Particularly interesting solution was found in which a static spherical membrane resided at the boundary of $AdS_4$. This was named the Membrane at the End of the Universe. As mentioned earlier, the hope was that a perturbative expansion of the supermembrane around this solution would give a free field theory at the boundary of AdS, thereby having significant consequences for the renormalizability issue. At the time, the properties the full supermembrane action in AdS background (that is, without expanding around a particular solution) were not investigated. Recent developments, however, have provided abundant motivation to do just that. It is convenient to discuss some general features of the supermembrane in AdS space before we turn to a description of the Membrane at the End of the Universe.

**Supermembrane in AdS Background and Singletons**

For definiteness, let us consider the supermembrane in $AdS_4 \times S^7$ background, which is a well known $N = 8$ supersymmetric solution of $D = 11$ supergravity \cite{136}. The $D = 11$ supermembrane action in a generic background is given by \cite{33}

$$S = - \int d^3 \xi \left( \sqrt{-g} + \epsilon^{ijk} C_{ijk} \right) ,$$

(24)

where $\xi^i$ ($i = 0, 1, 2$) are the coordinates on the membrane worldvolume, $g_{ij}$ is the induced metric on $\Sigma$ and $g = \det g_{ij}$. This metric and the components of the pulled-back 3-form $C$ are defined as

$$g_{ij} = E_i^A E_j^B \eta_{AB} , \quad C_{ijk} = E_i^A E_j^B E_k^C C_{ABC} ,$$

(25)

where $\eta_{AB}$ is the Minkowski metric in eleven dimensions, and

$$E_i^A = \partial_i z^M E_M^A ,$$

(26)

and $E_M^A$ is the target space supervielbein.

Thus, the $OSp(8|4)$ invariant supermembrane action is \cite{123} in a target superspace with isometry group $OSp(8|4)$ which supports a closed 4-form $dH = 0$, which can be locally solved as $H = dC$. The superspace we seek must have $AdS_4 \times S^7$ as a bosonic subspace and consequently it can be chosen to be \cite{134,113,135}
\[
\frac{G}{H} = \frac{OSp(8|4)}{SO(3,1) \times SO(7)}.
\]  
(27)

The generators of \(G\) and \(H\) are

\[
G : \ M_{ab}, P_c, T_{IJ}, P_J, Q_{\alpha A}
\]

\[
H : \ M_{ab}, T_{IJ}
\]

(28)

where \(Q_{\alpha A}\) are the 32 real supergenerators transforming as spinor of \(SO(3,2) \times SO(8)\) and the rest of the notation is self explanatory. The supervielbein and the 3-form \(C\) on \(G/H\) can be calculated straightforwardly from the knowledge of the structure constants of \(G\). See \([117, 118, 119]\) for further details.

The action \((24)\), with target superspace \(G/H\) specified in \((27)\), is manifestly invariant under \(OSp(8|4)\) since this is the isometry group of \(G/H\). It is also invariant under the worldvolume local diffeomorphism and local \(\kappa\)-symmetry. Fixing a physical gauge by identifying the worldvolume coordinates with three of the target space coordinates and setting half of the target space fermionic coordinates equal to zero (by means of a suitable projection), breaks the local diffeomorphisms, local \(\kappa\)-symmetry as well as the rigid isometries of \(G/H\). The requirement of maintaining the physical gauge fixes the local symmetry parameters in terms of the rigid parameters, and consequently one arrives at a gauged fixed worldvolume action which is invariant under the rigid \(G\) symmetry \([116, 118]\).

Thus one obtains an action for the 8 real scalars and 8 Majorana spinors on the worldvolume, which is invariant under the rigid superconformal group \(OSp(8|4)\) transformations some of which are linearly realized (and hence manifest) and the rest are nonlinearly realized. All this is perfectly analogous to the discussion of the lightcone gauge fixing in \(D = 11\) supermembrane theory in Poincaré superspace \([14]\).

Now, we ask the following question: Is there a vacuum solution of the supermembrane equation such that a perturbative expansion around it yields a free but still \(OSp(8|4)\) invariant action? The answer is yes. To see this, it is convenient to use horospherical \(\times\) hyperspherical coordinates to parametrize the \(AdS_4 \times S^7\) metric as
\[ ds^2 = \phi^2 \left( -d\tau^2 + d\sigma^2 + d\rho^2 \right) + a^{-2} \left( \frac{d\phi}{\phi} \right)^2 + 4a^{-2} d\Omega_7 , \]  

where \( d\Omega_7 \) is the \( SO(8) \) invariant metric on \( S^7 \). The boundary of this metric is the three dimensional Minkowski space \( M_3 \) at \( \phi \to \infty \), completed by the point \( \phi \to 0 \), so that the inversion element of the conformal group action on the boundary is well defined. The nature of this boundary has been discussed in great detail, for example, in \cite{135,116,118}.

Denoting the coordinates of the seven sphere by \( y^I \) (\( I = 1, \ldots, 7 \)), a simple class of solutions to the supermembrane equations is \cite{118}

\[ x^i = \xi^i , \quad \phi = \phi_0 , \quad y^I = y^I_0 , \]  

where \( x^i = (\tau, \sigma, \rho) \) and \( \xi \) are the worldvolume coordinates, \( \phi_0 \) and \( y^I_0 \) are arbitrary constants and the fermionic variables are set equal to zero. The singleton action is obtained by expanding around \( \phi_0 = 0 \) which corresponds to expanding around the boundary of \( AdS_4 \). For convenience, one can also set \( y^I_0 = 0 \). In using the normal coordinate expansion formulae \cite{18}, it is important to rescale the fluctuation fields appropriately. Defining the fluctuations as \cite{118}

\[ \phi = \phi_0 + \sqrt{\phi_0} \varphi , \quad y^I = \frac{1}{\sqrt{\phi_0}} \varphi^I , \quad \theta = (\phi_0)^{-3/2} \lambda , \]  

and taking the limit \( \phi_0 \to 0 \) after substituting to the normal coordinate expansion formulae \cite{18}, one finds that the zeroth and first order terms in the normal coordinate expansion vanish and the second order term yields the \( N = 8 \) singleton action for eight free scalars \( (\varphi, \varphi^I) \) and eight free fermions \( \lambda \). The action is \( OSp(8|4) \) invariant and this does not require a mass term for the bosons, since the boundary of \( AdS_4 \) is characterized as a Minkowski space in the coordinate system used here.

**Membrane at the End of the Universe**

Now, we turn to the description of an interesting set of solutions to the supermembrane equations in \( AdS_4 \times S^7 \) background which were found sometime ago \cite{110,111} and which are closely related to solution (30). We begin by considering the bosonic field equation for a configuration where the spacetime gravitino and the fermionic co-ordinates are set equal to zero:

\[ y^I = y^I_0 , \quad \theta = \theta_0 , \]
\[
\partial_i(\sqrt{-h} h^{ij} \partial_j X^N g_{MN}) - \frac{1}{2} \sqrt{-h} h^{ij} \partial_i X^N \partial_j X^P \partial_M g_{NP} \\
+ \frac{1}{3} \varepsilon^{ijk} \partial_i X^N \partial_j X^P \partial_k X^Q H_{MNPQ} = 0 ,
\]

where \( h_{ij} \) is the induced metric

\[ h_{ij} = \partial_i X^M \partial_j X^N g_{MN} , \]

\( X^M(\tau, \sigma, \rho) \) are the spacetime coordinates of the membrane, \( g_{MN} \) is the spacetime metric and \( H_{MNPQ} = 4 \partial_{[M} B_{NPQ]} \). The \( \kappa \)-symmetry of the supermembrane action requires that \( g_{MN} \) and \( H_{MNPQ} \) satisfy the usual bosonic equations of \( d = 11 \) supergravity. Let us consider the solution of these equations is which the spacetime is \( AdS_4 \times M_7 \), such that \( AdS_4 \) has inverse radius \( a \), and \( M_7 \) is an Einstein space with Ricci tensor \( R_{mn} = \frac{3}{2} a^2 g_{mn} \). In [11] we used the coordinate system in which the \( AdS_4 \) metric is given by

\[ ds^2 = -(1 + a^2 r^2) dt^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) + (1 + a^2 r^2)^{-1} dr^2 . \]

The maximal \( SO(3,2) \) symmetry of this metric requires that the period of \( t \) is an integer multiple of \( 2\pi/a \), namely \( \Delta t = 2\pi q/a \), where \( q \) is an integer [11].

The four-index field strength is taken to be proportional to the Levi-Civita tensor in \( AdS_4 \):

\[ H_{\mu \nu \rho \sigma} = \frac{3}{2} a \sqrt{-g} \epsilon_{\mu \nu \rho \sigma} . \]

The metric for the seven dimensional internal space will be taken to be an \( S^7 \) moded by \( Z^p \), which is a Lens space, \( L(1, p) \) that can also be viewed as a \( U(1) \) bundle over \( CP^3 \) with fibers having the period \( 2\pi/p \), for some integer \( p \):

\[ ds^2 = \frac{4}{a^2} [(d\psi + 2A)^2 + ds^2(CP^3)] , \]

where \( ds^2(CP^3) \) is the standard Fubini-Study metric on \( CP^3 \), with Einstein metric satisfying \( R_{ab} = 8g_{ab} \), \( \psi \) is the coordinate on the \( U(1) \) fibres and \( A \) is a one-form potential satisfying \( dA = J \), where \( J \) is the Kähler two-form on \( CP^3 \). The fibre coordinate \( \psi \) has the period \( \Delta \psi = 2\pi/p \), for some integer \( p \).
The space \( L(1, 1) \) is the round seven sphere \( S^7 \), and \( L(1, 2) \) is the projective space \( \mathbb{RP}^7 \).

To solve the supermembrane equations, we make the ansatz

\[
\begin{align*}
    t &= \tau, \\
    \theta &= \sigma, \\
    \phi &= \rho, \\
    r &= r_0, \\
    y^I &= y^I_0, \\
    \psi &= \alpha \tau / 2,
\end{align*}
\]

where \( \alpha, r_0 \) and \( y^I_0 \) are constants, and \( y^I \) are the coordinates on \( \mathbb{CP}^3 \), when we view the internal space as \( U(1) \) bundles over \( \mathbb{CP}^3 \). Substituting this ansatz into the supermembrane field equations (32), we find that all components of the equation are satisfied identically except in the \( r \) direction, which is solved by (\( \alpha = 1, \; r_0 = \text{arbitrary} \)), or by (\( \alpha > 1, \; r_0 = \sqrt{4(\alpha^2 - 1)/3a^2} \)). Furthermore we note that the periods of \( \tau \) and \( \psi \) specified above, together with the ansatz (37), imply that \( \alpha = 2/pq \). Thus we find the following solutions:

\[
\begin{align*}
    \alpha = 1 : & \quad \tilde{\text{AdS}}_4 \times S^7, \quad r_0 = \text{arbitrary}, \\
    \alpha = 2 : & \quad \text{AdS}_4 \times \mathbb{RP}^7, \quad r_0 = \text{arbitrary}, \\
    & \quad \text{AdS}_4 \times S^7, \quad r_0 = 2/a,
\end{align*}
\]

where \( \tilde{\text{AdS}}_4 \) denotes the double covering of \( \text{AdS}_4 \). As was shown in [111], the supersymmetry of the solutions presented above requires that \( r_0 \) be taken to infinity (hence the terminology of the “Membrane at the End of the Universe”). This requirement peaks the \( \alpha = 1 \) solutions above. A normal coordinate expansion of the action around these solutions should yield an \( OSp(8|4) \) invariant action formulated on the boundary of \( \text{AdS}_4 \) (or \( \tilde{\text{AdS}}_4 \)). However, it has been noted [111] that the rescalings of the fluctuation fields needed to extract a non-vanishing quadratic action (which also eliminate all the interaction terms in the limit \( r_0 \to \infty \)) had the effect of spoiling the extraction of meaningful supersymmetry transformations (which required different rescalings). Thus, a rigorous derivation of the action is still missing. Nonetheless, we expect the standard \( OSp(8|4) \) singleton field theory on the boundary of \( \text{AdS}_4 \) to arise. The action for this theory [107, 108] can be described as follows.

The \( OSp(8|4) \) singleton supermultiplet consists of 8 real scalars \( \phi^I (I = 1, \ldots, 8) \) in the \( 8_v \) representation of \( SO(8) \) and 8 four-component spinors \( \lambda_\alpha^I (\alpha = 1, \ldots, 8) \) in \( 8_s \) of \( SO(8) \). These fields live on \( S_2 \times S^1 \) boundary of \( \text{AdS}_4 \). In
addition to the four-dimensional Majorana condition $\tilde{\lambda} = \lambda^T C$, the spinor $\lambda$ satisfies the chirality condition

$$\gamma_0 \gamma_1 \gamma_2 \lambda_- = -\lambda_- ,$$

which, unlike the usual chirality condition, is compatible with the Majorana condition. The $N = 8$ supersingleton action is given by $^{107, 108}$

$$\mathcal{L} = -\frac{1}{2} \sqrt{-h} \left( h^{ij} \partial_i \phi^I \partial_j \phi_I + \frac{1}{4} a^2 \phi^I \phi_I - i \bar{\lambda} \gamma^i \nabla_i \lambda_- \right) ,$$

where $\nabla_i$ is the covariant derivative on $S^2 \times S^1$. This action is invariant under the rigid $OSp(8|4)$ transformations, the details of which can be found in $^{107, 108, 18}$. Notice the presence of the mass term for the bosonic singletons in the action. In the case of $S^p \times S^1$ boundary of $AdS_{p+2}$, this term is given by $^{14}$

$$\frac{p-1}{4p} R \phi^2 ,$$

where $R = p(p-1)a^2$ is the curvature scalar of $S^p \times S^1$ and $a$ is the inverse radius of $S^p$. In fact, the difficulty in obtaining the Lagrangian (40) from the normal coordinate expansion of the supermembrane action on $AdS_4 \times S^7$ around the Membrane at the End of the Universe solutions lies precisely in getting this mass term right, for it seems to vanish if one takes a naive $r_0 \to \infty$ limit $^{13, 116, 14, 146}$.

**Singletons and Higher Spin Gauge Theory**

The $N = 8$ singleton field theory formulated on $S^2 \times S^1$ boundary of $AdS_4$ was quantized sometime ago $^{3, 4}$, with the hope that it might play a role in the quantization of the supermembrane on $AdS_4 \times S^7$ background $^{107, 11}$. Furthermore, a remarkable group theoretical property of the singleton representation which states that the direct product of two singletons decomposes into infinitely many massless field with higher spins $^{99}$, motivated Bergshoeff, Salam, Tani and the author $^{17, 18, 109}$ to conjecture that the $D = 11$ quantum supermembrane on $AdS_4 \times S^7$ should give rise to a higher spin gauge theory which contains the usual $N = 8$ AdS supergravity as a subsector. The occurrence of the infinitely many massless higher spin fields implies the existence
of infinitely many (local) gauge symmetries analogous to the Yang-Mills, general coordinate and local supersymmetries associated with spin 1, 2 and 3/2, respectively.

Interestingly enough, in a related development Fradkin and Vasiliev were in the course of developing a higher spin gauge theory in its own right (see [120] for references to earlier work). These authors succeeded in constructing interacting field theories for higher spin fields. It was observed that the previous difficulties in constructing higher spin theories can be bypassed by formulating the theory in AdS space and to consider an infinite tower of gauge fields controlled by various higher spin algebras based on certain infinite dimensional extensions of super AdS algebras. In particular, the AdS radius could not be taken to infinity since its positive powers occurred in the higher spin interactions and therefore one could not take a naive Poincaré limit.

In a series of papers Vasiliev pursued the program of constructing the AdS higher spin gauge theory and simplified the construction considerably. In [122] the spin 0 and 1/2 fields were introduced to the system within the framework of free differential algebras. The theory was furthermore cast into an elegant geometrical form by extending the higher spin algebra to include new auxiliary spinorial variables (see [124] for a review).

Applying the formalism of Vasiliev to a suitable higher spin algebra that contains the maximally extended super AdS algebra $OSp(8|4)$, the resulting spectrum of gauge and matter fields remarkably coincide with the massless states resulting from the symmetric product of two $OSp(8|4)$ supersingletons. In a recent paper, the Vasiliev theory of higher spin fields (which is applicable to a wide class of higher spin superalgebras) was examined in the context of $N = 8$ supersymmetry, and the precise manner in which the $N = 8$ de Wit-Nicolai gauged supergravity can be described within this framework was studied.

In the rest of this section, we will outline some kinematical aspects of the higher spin gauge theory. To begin with, the $N = 8$ singleton representation of the $D = 4$ super AdS group $OSp(8|4)$, which are also known as Di’s and Rac’s, are

$$
\begin{align*}
\text{Rac} & : \quad D\left(\frac{1}{2}, 0\right) \otimes 8_s \\
\text{Di} & : \quad D\left(1, \frac{1}{2}\right) \otimes 8_c
\end{align*}
$$

where $D(E_0, s)$ denotes an UIR of $SO(3, 2)$ for which $E_0$ is the minimal energy eigenvalue of the energy operator $M_{04}$, and $s$ is the maximum eigenvalue of
the spin operator $M_{12}$ in the lowest energy sector. The decomposition of the symmetric tensor product $[(Rac \oplus Di) \otimes (Rac \oplus Di)]_S$ under the $OSp(8|4)$ leads to the UIR’s of $OSp(8|4)$ given in Table 1.

Table 2: The $SO(3, 2) \times SO(8)$ content of the symmetric tensor product of two $N = 8$ singletons. This product generates the infinite spectrum of an $shsE(8|4)$ gauge field ($s \geq 1$) and the spectrum of a finite number of additional matter fields ($s < 1$).

| $k \backslash s$ | 0 | $\frac{1}{2}$ | 1 | $\frac{3}{2}$ | 2 | $\frac{5}{2}$ | 3 | $\frac{7}{2}$ | 4 | $\cdots$ | $2s$ | $2s + \frac{1}{2}$ | $2s + 1$ | $2s + \frac{3}{2}$ | $2s + 2$ | $\cdots$ |
|-----------------|---|-------------|---|-------------|---|-------------|---|-------------|---|-------------|---|-------------|---|-------------|---|-------------|
| 0               | 35_+ + 35_- | 56 | 28 | 8 | 1 |
| 1               | 1+1 | 8 | 35_+ + 35_- | 56 | 28 | 8 | 1 |
| 2               | | | 1 | 8 | 35_+ + 35_- | $\cdots$ |
| $\vdots$        | | | | | | | | | | | | | | | | |
| $s - 1$         | | | | | | | | | | | | | | | | |
| $s + 2$         | | | | | | | | | | | | | | | | |
| $\vdots$        | | | | | | | | | | | | | | | | |

The integer $k = 0, 1, 2, \ldots$ can be considered as level number. Levels $\kappa = 0, 1$ are somewhat special. At level $k = 0$, there is the familiar $N = 8$ supergravity multiplet consisting of 128 Bose and 128 Fermi on-shell degrees of freedom. At level $k = 1$, there is a $256 + 256$ multiplet that has 2 scalars as the lowest member of the supermultiplet and a single spin 4 field as the highest member. For levels $k > 2$ the structure of all the supermultiplets is the same, namely they start with a singlet spin $(2k - 2)$ field and end with a singlet $(2k + 2)$ field, in 8 steps of spin $1/2$ increments. The associated $SO(8)$ irrep's are: $(1, 8, 28, 56, 35 + 35, 56, 28, 8, 1)$.

Among the interesting and important properties of the spectrum shown in Table 1 is that there are two distinct classes of fields; those with spin $s \geq 1$, which forms an infinite set, and few fields that have spin $s \leq 1/2$. These two classes of fields are separated with a vertical line in Table 1. The fields with spin $s \geq 1$ can be associated with generators of an infinite dimensional algebra, called $shsE(8|4)$, while the fields with spin $s \leq 1/2$, clearly cannot be associated with any generator. Note however the important fact that all fields shown in Table 1, including those with spin $s \leq 1/2$ are exactly those which arise in the symmetric tensor product of two $OSp(8|4)$ singletons. Physical consistency of gauge field theory based on $shsE(8|4)$ requires that the complete particle spectrum forms a unitary representation of the full, infinite dimensional al-
gebra \( shs^E(8|4) \). The product of the \( OSp(8|4) \) singletons which give the field content shown in Table 1, indeed does form a unitary representation of \( shs^E(8|4) \). Consequently, the matter fields (the left hand side of the vertical line shown in Table 1) must be included, in addition to the gauge fields (the right hand side of the vertical line) in a sensible (consistent and unitary) formulation of higher spin \( N = 8 \) supergravity theory.

Recently \(^{22}\), the \( N = 8 \) higher spin supergravity theory based on \( shs^E(8|4) \) was investigated in considerable detail, and the precise manner in which it contains the \( N = 8 \) de Wit-Nicolai gauged supergravity \(^{126}\) has been shown at the linearized level. In our opinion, this constitutes a positive step towards the understanding of the M-theoretic origin of the massless higher spin gauge theory. To make further progress, one has to compare the interactions of the spin \( s \leq 2 \) fields in the higher spin theory with those of de Wit-Nicolai gauged supergravity theory at the next order, namely the quadratic order in fields, in the equations of motion. It would be very interesting to find out how the \( E_7/SU(8) \) structure of the scalar fields \(^{22}\) will manifest itself and to determine how the higher spin fields interact with the spin \( s \leq 2 \) fields. Ultimately, the \( N = 8 \) higher spin supergravity should emerge from the dynamics of the \( N = 8 \) singleton field theory defined at the boundary of \( AdS_4 \). In this context, we note that the OPE's of the stress energy tensor in the \( OSp(8|4) \) singleton theory have been studied \(^{113}\), but a great deal of work remains to be done to shed more light on the issue of how to extract information about the physics in the bulk of AdS.

In summary, it is worth emphasizing the following points about the \( N = 8 \) higher spin supergravity whose properties have been outlined above: (a) the existence of the theory is highly nontrivial, (b) the theory is based on an infinite dimensional extension of the \( D = 4, N = 8 \) super AdS group \( OSp(8|4) \), (c) it fuses matter fields with the gauge fields in such a way that the full spectrum of massless states are exactly those which arise from the two \( N = 8 \) singleton states and (d) the theory contains the equations of motion of the \( D = 4, N = 8 \) AdS supergravity as a subsector. This last property is very significant in that it is in the spirit of discovering new structures that build upon what we already know. Should this theory survive further scrutiny, then the appropriate question to ask is not if this theory fits into the big picture of M-theory, but rather how it will do so.

The massive Kaluza-Klein states coming from the \( S^7 \) compactification of \( D = 11 \) supergravity must also be taken into account in a suitable extension of the \( N = 8 \) higher spin supergravity. These states are expected to arise in the product of three or more \( N = 8 \) singletons. An infinite set of new massive
states would appear in the spectrum as a byproduct. A higher spin theory taking into account massive states is yet to be constructed.

An extreme point of view would be to imagine a pure gauge theory formulation of M-theory which contains only the massless fields corresponding to an infinite dimensional symmetry. All the phases of M-theory (the known ones and those yet to be discovered) are then to emerge from the breaking of this master symmetry in various ways.

One can imagine the construction of a higher spin gauge theory directly in $D = 11$ AdS space. However, the anticommutator of two supersymmetries necessarily involves (tensorial) generators in addition to the AdS generators in $D = 11$. One can take the AdS group in $D = 11$ to be the diagonal subgroup of $OSp(1\vert 32) \oplus OSp(1\vert 32)$ and study its field theoretic realizations. Apparently, no such realizations are known at present. Nonetheless, the singleton representation for this group has been studied recently by G"unaydin who found that the product of two such irreps do not contain the $D = 11$ supergravity states, but a further product of the resulting representation does contain the $D = 11$ supergravity fields and additional fields as well. This group theoretical result suggests the construction of a 10D singleton field theory that lives on the boundary of the $D = 11$ AdS space.

It is clear that much remains to be discovered and that these are exciting times in the quest for an understanding of the mysterious and magic membrane theory.

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