Absence of aging in the remanent magnetization in Migdal-Kadanoff spin glasses

F. Ricci-Tersenghi\(^a\) and F. Ritort\(^b\)
\(^a\) Abdus Salam International Center for Theoretical Physics, Condensed Matter Group
Strada Costiera 11, P.O. Box 586, 34100 Trieste (Italy)
\(^b\) Department of Physics, Faculty of Physics, University of Barcelona
Diagonal 647, 08028 Barcelona (Spain)
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We study the non-equilibrium behavior of three-dimensional spin glasses in the Migdal-Kadanoff approximation, that is on a hierarchical lattice. In this approximation the model has an unique ground state and equilibrium properties correctly described by the droplet model. Extensive numerical simulations show that this model lacks aging in the remanent magnetization as well as a maximum in the magnetic viscosity in disagreement with experiments as well as with numerical studies of the Edwards-Anderson model. This result strongly limits the validity of the droplet model (at least in its simplest form) as a good model for real spin glasses.

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Spin glasses are disordered magnets which for low impurity concentrations above the Kondo regime display interesting non-equilibrium phenomena. In particular, a freezing of the dynamics appears at a temperature \(T_c\) below which slow relaxation phenomena manifest through non-stationary effects in the zero-field-cooled magnetization. In this regime different non-equilibrium phenomena have been observed such as aging, remanence and several memory as well as chaotic effects. Despite the great activity devoted to understand the nature of the low temperature phase in three-dimensional spin glasses (numerical simulations, experiments and theory, see the review) still many questions regarding the ground state (e.g. its shape and its uniqueness) and the type of excitations remain unanswered.

The mean-field picture for spin glasses (i.e. the results obtained for the Sherrington-Kirkpatrick model), despite its great theoretical interest, is not able to furnish a real space picture of the type of excitations present in spin glasses. To fill this gap, and based on domain wall scaling arguments (initially proposed by McMillan and Bray and Moore), Fisher and Huse proposed what has been termed as droplet model for spin glasses. In the droplet model there are two unique ground states related by spin inversion symmetry. Thermal fluctuations activate droplets which are supposed to be compact droplets of typical size \(L\) and fractal surface of dimension \(d_s \geq d - 1\). These excitations cost a free energy which grows like \(\mathcal{Y}(T)L^\theta\) where \(\theta\) is a zero-temperature exponent and \(\mathcal{Y}(T)\) is a temperature dependent stiffness constant. The idea that excitations in spin glasses are compact droplets is the simplest description that finds its most successful application in the study of phase transitions in ordered systems. Despite its inherent simplicity, the droplet model has a severe limitation, i.e. its main assumptions remain to be proven from a correct microscopic theory. If one of its key assumptions were wrong then the whole set of predictions coming out from the model should need to be revisited.

Now, what is an appropriate microscopic model which describes the spin glass transition? The simplest proposal was put forward by Edwards and Anderson almost twenty years ago, who introduced a random bond nearest-neighbor interaction model, the so called Edwards-Anderson model (EA model). It is widely believed that the EA model is a real spin glass, i.e. it reproduces the major part of results experimentally measured in the laboratory. So the question is whether the droplet model is the appropriate theory to describe the phenomenology already contained in the EA model. Despite of the large number of numerical works devoted to this question (see the reviews) there is still no an universal agreement on it.

Our work has been motivated by recent results by the Manchester group who found that finite-size effects in the Migdal-Kadanoff approximation (MKA) of the three-dimensional EA model are mean-field like. While in the thermodynamical limit the MKA is known to be described by the droplet model with \(d_s = d - 1\) and \(\theta \approx 0.26\), the Manchester group suggested that the droplet model could also explain the vast majority of numerical simulation results for the EA model obtained during the last decade (which on the other hand, have been taken by the Rome group as evidences against the droplet picture). This is an interesting observation whose physical meaning and consequences need to be better understood and was already anticipated quite long ago in a theoretical study of the one-dimensional spin-glass chain. This controversy has been centered around the study of the spin-glass equilibrium properties. So, it is now time to check whether non-equilibrium behavior is well reproduced by the droplet model. This is of the outmost importance because experimental measurements in spin glasses in the low temperature regime are always
taken in the out-of-equilibrium regime.

In this paper we want to show that the droplet model lacks one of the key features of real spin glasses found in the laboratory, i.e. the aging in the zero-field-cooled magnetization. Consequently, the physics contained in the droplet model corresponds to a limited class of disordered systems being far from what is observed in real spin glasses.

The EA model in the presence of a field is defined by,
\[ \mathcal{H} = - \sum_{(i,j)} J_{ij} \sigma_i \sigma_j - h \sum_i \sigma_i \]
where the site indexes run on the nodes of a cubic lattice, \((i,j)\) stands for nearest neighbor pairs, the spins take values \(\sigma_i = \pm 1\) and the couplings are extracted from a Gaussian distribution of zero average and unit variance. Following \cite{8} we will consider the three dimensional EA model in the MKA which amounts to consider a hierarchical lattice constructed iteratively by replacing each bond by eight bonds as indicated in fig. 1. Denoting by \(g\) the number of generations then the total number of bonds is \(8^g\) which corresponds to the number of sites for a cubic lattice with lattice size \(L = 2^g\).

**FIG. 1.** Elementary step in the construction of the hierarchical lattice, where the MK approximation is exact.

The order parameter can be defined through the equilibrium autocorrelation function,
\[ q_{EA} = \lim_{t \to \infty} \lim_{V \to \infty} \frac{\sum_{i=1}^V x_i (\sigma_i(t) \sigma_i(0))}{\sum_{i=1}^V x_i} , \]
where \(V = 8^g = L^3\) is the volume and the averages \(\langle \cdot \rangle\) are taken over dynamical histories starting from different equilibrium initial conditions at time 0. The parameters \(x_i\) are weights which may consider the fact that a given site is connected with a different number of bonds depending on its generation level (i.e. depending on which iteration in the recursive construction of the lattice that site was generated). Our results concentrate on the choice \(x_i = 1\), i.e. all sites are identically weighted. However also the results obtained with \(x_i = c_i\), where \(c_i\) is the connectivity of site \(i\) (so all bonds are identically weighted) corroborate our conclusions \cite{12}.

We have concentrated our attention in the study of the relaxational dynamics in the low-temperature phase \(T < T_c \approx 0.88\) below which \(q_{EA}\) defined in Eq.\( (2)\) is different from zero. We have used Monte Carlo dynamics with Metropolis algorithm and random updating \cite{13}. In our runs we follow the typical aging experiment scheduling, that is: at \(t = 0\) we quench the system from infinite temperature to a finite one \(T < T_c\) without magnetic field, letting the system evolve for a time \(t_w\). At time \(t_w\) we switch the magnetic field on. For subsequent times \((t > t_w)\) the system continues to relax in a magnetic field \(h\) and then we measure the following two quantities: a) the autocorrelation function
\[ C(t, t_w) = \frac{\sum_{i=1}^V x_i (\sigma_i(t) \sigma_i(t_w))}{\sum_{i=1}^V x_i} , \]
and b) the zero-field-cooled susceptibility defined by
\[ \chi_{ZFC}(t, t_w) = \lim_{h \to 0} \frac{M_{ZFC}(t, t_w)}{h} , \]
where
\[ M_{ZFC}(t, t_w) = \frac{\sum_{i=1}^V x_i (\sigma_i(t) - \sigma_i(t_w))}{\sum_{i=1}^V x_i} . \]

The limit in eq.\( (4)\) is usually ignored, because we always work in the linear response regime. All the data we present have been obtained with a magnetic field of intensity \(h = 0.1\) and we have checked that the same susceptibility is obtained by doubling the perturbing field.

We have performed extensive numerical simulations for \(g = 5\) (\(L = 32\)) and \(g = 6\) (\(L = 64\)) at two different temperatures (\(T = 0.7, 0.5\)) and for many values of \(t_w\). We obtain the same results for both temperatures. Here we present only those for \(T = 0.7\), while the complete set of data will be reported elsewhere. Note that the ratio \(T/T_c \approx 0.86\) used here is very similar to that used in many experiments \cite{14}.

In fig. 2 we show the autocorrelation function for different values of \(t_w\). One can observe the presence of aging characteristic of many glassy systems.

**FIG. 2.** The correlation functions for \(g = 6, T = 0.7\) and different \(t_w\) clearly show aging. In the inset the same curves as a function of the scaling variable \(\log(t)/\log(t_w)\).

Following general assumptions, in the asymptotic regime \(t_w \to \infty\), the correlation function decomposes in two terms each one governing a different time regime. In the quasi-equilibrium regime \(t - t_w << t_w\) the system is in some sort of local equilibrium and correlation...
functions are time translational invariant. In the aging regime \( t-t_w \gg t_w \) the system ages and correlation functions depend on both times through non trivial scaling relations. So in general one can write \( \text{(6)} \):

\[
C(t, t_w) = C_{sl}(t-t_w) + C_{aging}(t, t_w)
\]

with \( \lim_{\tau \to \infty} C_{sl}(\tau) = q_{EA} \). In equilibrium the aging part vanishes and one recovers the previous result of eq.(3).

The very difference between the experimental data and the EA model on one hand and the MKA and the droplet model on the other is the large times scaling of dynamical functions. As can be seen in the inset of fig. 4 in the MKA we find that the aging part of the autocorrelation function is well described, in the large times limit, by a function of the ratio \( \log(t)/\log(t_w) \). On the other hand in experiments and in the EA model the scaling is far from the \( \log(t)/\log(t_w) \) and similar to \( t/t_w \). Because of the use of the scaling variable, in the inset of fig. 4 the data corresponding to the quasi-equilibrium regime collapse on the line \( \log(t)/\log(t_w) = 1 \). Thus we can estimate the value of \( q_{EA} \) as the limit of the scaling function for \( \log(t)/\log(t_w) \to 1^+ \), i.e. \( q_{EA} \approx 0.6 \) (a value compatible with \( \text{(3)} \)).

Our most striking result is found for the zero-field-cooled susceptibility \( \chi_{ZFC}(t, t_w) \) shown in fig. 3. In the MKA there is no dependence of the susceptibility on \( t_w \). We believe that such a result, which is characteristic of droplet models and kinetic growth, makes the droplet model, at least in its simplest form, inadequate for the description of the EA one. Aging in both zero-field-cooled and field-cooled magnetization is so commonly found in experiments on spin glasses that it is not clear to us how this result can be explained by the standard droplet theory. Note also that the peak in the magnetic viscosity \( S(t, t_w) = \partial \chi_{ZFC}(t, t_w)/\partial \log(t-t_w) \) (experimentally very well observed \( \text{(3)} \)) is completely absent in the MKA.

We should remind that aging in \( \chi_{ZFC}(t, t_w) \), with a peak in the \( S(t, t_w) \), is naturally found in the EA model (see inset of fig. 4) as well as in mean-field models. Then it remains to be explained why these aging effects are naturally and easily observed in the EA model and not in the MKA.

Finally we consider the analysis of the fluctuation-dissipation ratio useful to compare the results obtained in the MKA with those obtained in the EA and coarsening models \( \text{(7)} \). In the quasi-equilibrium regime \( t-t_w \ll t_w \) the system is in local equilibrium. Consequently both correlation and susceptibility are time-translational invariant and the fluctuation-dissipation theorem (FDT) is satisfied,

\[
T \chi_{ZFC}(t-t_w) = 1 - C(t-t_w) \quad . \quad \text{(7)}
\]

In the aging regime \( t-t_w \gg t_w \) the system ages and FDT is violated. Then it is useful to define the so called fluctuation-dissipation ratio \( \text{(8)} \)

\[
X(t, t_w) = \frac{TR(t, t_w)}{\partial C(t, t_w)/\partial t_w} \quad , \quad \text{(8)}
\]

which, in the asymptotic long-times limit \( t, t_w \to \infty \), may be uniquely expressed as function of the correlation \( C(t, t_w) \) yielding

\[
T \chi_{ZFC}(t, t_w) = \int_{C(t, t_w)}^1 X(C) dC \quad . \quad \text{(9)}
\]

Moreover the \( X(C) \) can be related to equilibrium quantities \( \text{(9)} \). The previous expression reduces to Eq.(9) in the quasi-equilibrium regime where \( X = 1 \). A plot of \( T \chi_{ZFC}(t, t_w) \) as a function of \( C(t, t_w) \) is expected to show two different behaviors. For \( q_{EA} < C < 1 \) we have \( X = 1 \) and so the curve \( T \chi_{ZFC} \) versus \( C \) has slope \( -1 \). For \( C < q_{EA} \) the \( X \) may be a non-vanishing function of \( C \) and we have \( T \chi_{ZFC}(t, t_w) = (1-q_{EA}) + \int_{C(t, t_w)}^q q_{EA} X(C) dC \). In coarsening models, \( X = 0 \) for \( C < q_{EA} \) and so the function \( \chi_{ZFC}(C) \) is flat for \( C < q_{EA} \). In fig. 4 we show the \( \chi_{ZFC} \) as a function of \( C \) for different values of \( g \) and \( t_w \), which show that the behavior rapidly converges to that of coarsening models and strongly differs from that observed in finite-dimensional EA spin glasses \( \text{(7)} \). The horizontal line in fig. 4 is the infinite time limit of the susceptibility, extrapolated from the data of fig. 3 and from those for the field-cooled magnetization (not shown). It is an upper bound for the plotted curves, thanks to the positiveness of \( X \) ratio. From fig. 3 we can also get an estimate for the \( q_{EA} \) order parameter, defined as the abscissa value where the curves leave the FDT line \( (T \chi_{ZFC} = 1 - C) \). Very reasonably this point is converging, in the large times limit, near to the intersection of the two lines, giving \( q_{EA} \approx 0.6 \) (as already found from the data of fig. 3).
Fig. 4 adds more evidence on the fact that spin glasses in the MKA do not capture all the key features of finite-dimensional spin glasses as we know them from the three-dimensional EA model.

To summarize, we have shown that in the MKA spin glasses do not show aging in the integrated response function. This aging is experimentally observed in real spin glasses through zero-field-cooled and field-cooled measurements being one of the key features which distinguishes spin glasses from other disordered systems. The study of the fluctuation-dissipation ratio suggests that relaxation in this model is driven by coarsening, like in conventional ferromagnets. One could argue that these results for the MKA are not extensible to the droplet model because, in the general case, the inequality $d_s \geq d - 1$ could restore aging. Despite this possibility our results unambiguously show that the MK model is not a good model for realistic spin glasses. A new class of excitations or droplets must be present in spin glasses. The droplet model in its simplest version does not capture the physics behind real spin glasses.

One possible generalization of the droplet model (which would be no longer simply droplet) is to consider two kind of basic excitations in a spin glass: on small length scale the usual droplets and, in addition, system-size scale collective rearrangements \[\text{[20]}\]. The second kind of excitation are, at present, ignored in the droplet model (they are exponentially rare), but they could be responsible for the many mean-field like features observed in finite-dimensional spin glasses. In terms of a very simplified energy landscape the two excitations would correspond, respectively, to the local movements of the system in a single “valley” and to the jumps from one valley to another one. In our opinion a new theory comprehensive of the small scale droplets and the system-size scale excitations (with a clear real space picture) would be welcome and could hopefully terminate the longstanding discussion on finite-dimensional spin glasses.

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