Quasinormal modes of semiclassical electrically charged black holes

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Abstract

We report the results concerning the influence of vacuum polarization due to quantum massive vector, scalar and spinor fields on the scalar sector of quasinormal modes in spherically symmetric charged black holes. The vacuum polarization from quantized fields produces a shift in the values of the quasinormal frequencies, and correspondingly the semiclassical system becomes a better oscillator with respect to the classical Reissner–Nordström black hole.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

Quantum theory and general relativity are two cornerstones of modern physics that for more than a century have contributed to increasing our knowledge of the Universe as never before in the human history. With the help of the quantum theory we can explain micro-world phenomena, and the general theory of relativity allows us to have a deep understanding of the Universe at cosmological scales. Unfortunately, these two beautiful theories resist all attempts to bring them together. A unified theory of gravity and the quantum world would be very important to describe, for example, the origin of the Universe and its later development.

There are other simple phenomena that can be very interesting to be described by the future quantum gravity. Among other things, from the classical side, it is well known that the response of a black hole to small perturbations at intermediate times is characterized, under suitable boundary conditions, by a discrete set of complex frequencies called quasinormal frequencies, which depend only upon the parameters of the black hole [1–4]. From the quantum side, it would be interesting to see what changes appear in the evolution of quantum black holes.
under perturbations. Particularly interesting is the behaviour at intermediate times dominated by quasinormal response, because apart from allowing us to gain some valuable information about these objects, the quasinormal spectrum permits the investigation of the black hole stability against small perturbations. Several numerical methods have been developed to study such an interesting problem [5–7].

Quasinormal modes appear to be important in other contexts, such as for example, the AdS/CFT correspondence, where the inverse of the imaginary part of quasinormal frequencies of AdS black holes can be interpreted as the dual CFT relaxation time [8, 9].

In a previous paper, we considered the influence of vacuum polarization effects due to the backreaction of a quantum massive scalar field of large mass upon the quasinormal modes of electrically charged black hole solutions obtained solving the semiclassical Einstein field equations, with the quantum renormalized stress–tensor of the quantized matter field as a source [10]. Such an influence appears essentially as an appreciable shift in the quasinormal frequencies that decrease as the bare black hole mass increases, and that do not have a strong dependence upon the quantum field parameters, leading to the conclusion that the quantum corrected black holes are less oscillatory with respect to their classical counterparts. Another previous work on similar lines was done by Konoplya [11], for the BTZ black hole dressed by a massless scalar field, but in this case he considered the influence of particle creation around the event horizon, an effect that dominates the vacuum polarization effect for massless fields.

To solve the backreaction problem in semiclassical gravity, we need to know the functional dependence of the renormalized stress–energy tensor of the quantum field surrounding the classical compact object on a wide class of metrics [12]. Unfortunately, this is a very difficult problem, and up to now, there exist only approximate methods to develop a tractable expression for this quantity [12–19]. Since the pioneering work of York [20], who solved the semiclassical Einstein equations for a Schwarzschild black hole dressed by a massless conformally coupled scalar field, using for the quantum stress–energy tensor the results given earlier by Page [13], there are some related works in the literature, both for massless and massive quantum fields of different values of the spin parameter. To see the effects of the backreaction upon the black hole response to small perturbations, quantum massless fields as sources of the quantum corrections are not the most suitable candidates, because the semiclassical metric components diverge as \(r \to \infty\) and to obtain the correct solutions to the backreaction problem we need to impose some sort of boundary to the system under study, a feature that causes a change in the quasinormal spectrum. A different situation arises in the case of very massive fields, for which the vacuum polarization effects are not difficult to compute constructing the quantum stress–energy tensor by means of the Schwinger–DeWitt expansion of the quantum effective action, whenever Compton’s wavelength of the field is less than the characteristic radius of curvature [15, 16, 18, 21–23].

It is important to mention that in semiclassical gravity the unavoidable effects due to the metric fluctuations and the associated graviton contributions to the complete quantum stress–energy tensor are ignored, a fact that is usually justified considering that there exists a regime in which the gravitational field can be regarded as a classical entity, and the effects of the remaining matter fields after quantization can be taken as quantum corrections to the bare metric. Using the quantum stress–energy tensor of the matter fields as a perturbation on the right-hand side of semiclassical Einstein equations, we can obtain a perturbative solution to the backreaction problem up to first order, and determine what changes appear in some important quantities such as the mass, the location of the event horizon and the Hawking temperature of the quantum corrected solution.

In this paper, we study the effects that vacuum polarization of very massive scalar, vector and spinor fields has on quasinormal modes of quantum corrected Reissner–Nordström black
holes in four dimensions. This is the sequel of our previous work [10] in which we focus on the quantum scalar field case. In the first section, we review the Schwinger–DeWitt technique to obtain the one-loop approximation for the effective action for massive fields in the large mass limit, and present the particular results obtained for a classical Reissner–Nordström black hole background. In section 2, we solve the backreaction problem to obtain the metric that describes the spacetime geometry of an electrically charged semiclassical black hole. Section 3 is devoted to the calculation of the massless test scalar quasinormal frequencies in this semiclassical background, by the sixth-order WKB method. Finally in section 4, we give the concluding remarks and comment on related problems to be studied.

In the following we use for the Riemann tensor, its contractions, and the covariant derivatives the sign conventions of Misner et al [24]. Our units are such that \( \hbar = c = G = 1 \).

2. Renormalized stress–energy tensor for quantum massive fields

In the following we consider the quantization of massive scalar, vector and spinor fields in the large mass limit. The results for the massive scalar field can be found in our previous works [10, 15], and for this reason we will be concerned only with the vector and spinor cases. The action for a single massive vector field \( A_\mu \) with mass \( m_v \) in some generic curved spacetime in four dimensions is

\[
S_v = - \int d^4 x \sqrt{-g} \left( \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_v^2 A_\mu A^\mu \right). \tag{1}
\]

The equation of motion for the field has the form

\[
\hat{V}_\mu (\nabla) A_\mu = 0, \tag{2}
\]

where the second-order operator \( \hat{V}_\mu (\nabla) \) is given by

\[
\hat{V}_\mu (\nabla) = \delta_\mu^\nu \Box - \nabla_\nu \nabla_\mu - R_\mu^\nu - m_v^2 \delta_\mu^\nu, \tag{3}
\]

where \( \Box = g^{\mu\nu} \nabla_\mu \nabla_\nu \) is the covariant D'Alembert operator and \( \nabla_\mu \) is the covariant derivative.

For a single massive neutral spinor field the action is

\[
S_f = i \frac{2}{\hbar} \int d^4 x \sqrt{-g} \bar{\phi} \left[ \gamma^\mu \nabla_\mu \phi + m_f \phi \right]. \tag{4}
\]

In the above expression, \( \phi \) provides a spin representation of the vierbein group and \( \bar{\phi} = \phi^* \gamma \), where \( ^* \) means transpose. The Dirac matrices \( \gamma \) and \( \gamma^\mu \) satisfy the usual relation \([\gamma^\mu, \gamma^\nu] = 2g^{\mu\nu} \hat{I} \), where \( \hat{I} \) is the \( 4 \times 4 \) unit matrix.

The covariant derivative of any spinor \( \zeta \) obeys the commutation relations [23, 25]

\[
\nabla_\mu \nabla_\nu \zeta - \nabla_\nu \nabla_\mu \zeta = \frac{1}{2} \delta_{[\alpha,\beta]} R_\mu^\alpha R_\nu^\beta, \tag{5}
\]

\[
\nabla_\mu \nabla_\nu \zeta - \nabla_\nu \nabla_\mu \zeta = \frac{1}{2} \delta_{[\alpha,\beta]} R_\mu^\alpha R_\nu^\beta, \tag{6}
\]

\[
\nabla_\mu \nabla_\nu \zeta - \nabla_\nu \nabla_\mu \zeta = \frac{1}{2} \delta_{[\alpha,\beta]} R_\mu^\alpha R_\nu^\beta, \tag{7}
\]

and so forth, where \( \delta_{[\alpha,\beta]} = \frac{1}{2} [\gamma_\alpha, \gamma_\beta] \) are the generators of the vierbein group, \([,]\) is the commutator bracket, and \( R_\mu^\alpha \) the vierbein which satisfies \( h_{\mu\nu} h_\nu^\alpha = g_{\alpha\nu} \). The covariant derivatives of \( \gamma, \gamma^\mu \) and \( \delta_{[\alpha,\beta]} \) vanishes. The equation of motion for the field \( \phi \) derived from action (4) reads

\[
(\gamma^\mu \nabla_\mu + m_f) \phi = 0. \tag{8}
\]
The operator $\hat{D}_f$ that gives the evolution of the spinor function in (8) is

$$\hat{D}_f = \gamma^\mu \nabla_\mu + m_f. \tag{9}$$

The usual formalism of quantum field theory gives an expression for the effective action of the quantum fields $A_\beta, \phi$ as a perturbative expansion,

$$\Gamma(A_\beta, \phi) = S(A_\beta, \phi) + \sum_{k \geq 1} \Gamma_{(k)}(A_\beta, \phi), \tag{10}$$

where $S(A_\beta, \phi)$ is the classical action of the free fields. The one loop contribution of the fields $A_\beta, \phi$ to the effective action is expressed in terms of operators (3) and (9) as

$$\Gamma_{(1)} = \frac{i}{2} \ln(\text{Det} \hat{V}) + \frac{i}{2} \ln(\text{Det} \hat{D}), \tag{11}$$

where $\text{Det} \hat{F} = \exp(\text{Tr} \ln \hat{F})$ is the functional Berezin superdeterminant of the operator $\hat{F}$, and $\text{Tr} \hat{F} = (-1)^{F} F = \int d^4x (-1)^A F_A^A(x)$ is the functional supertrace [21]. If Compton’s wavelength of the field is less than the characteristic radius of spacetime curvature [15, 17–19, 21–23, 26], we can develop an expansion of the above effective action in powers of the inverse square mass of the field. This is known as the Schwinger–DeWitt approximation, and can be applied to a ‘minimal’ second-order differential operator of the general form

$$K^\mu_\nu(\nabla) = \delta^\mu_\nu \Box - m^2 \delta^\mu_\nu + Q^\mu_\nu, \tag{12}$$

where $Q^\mu_\nu(x)$ is some arbitrary matrix playing the role of the potential.

Unfortunately, this is not the case of operators (3) and (9). In the case of (3), the presence of the nondiagonal term turns it to be a nonminimal operator.

By fortune we can put (3) as a function of some minimal operators if we note that it satisfies the identity $v^\mu_\nu(\nabla)(m^2 \delta^\mu_\nu - \nabla_\mu \nabla_\nu) = m^2 (\delta^\mu_\nu \Box - R^\mu_\nu - m^2 \delta^\mu_\nu).$ Then the one-loop effective action for the nonminimal operator (3) omitting an inessential constant can be written as [19]

$$\frac{i}{2} \text{Tr} \ln \hat{V}^\mu_\nu(\nabla) = \frac{i}{2} \text{Tr}(\delta^\mu_\nu \Box - R^\mu_\nu - m^2 \delta^\mu_\nu) - \frac{i}{2} \text{Tr}(m^2 \delta^\mu_\nu - \nabla_\mu \nabla_\nu). \tag{13}$$

We can see in (13) that the first term is the effective action of a minimal second-order operator $K^\mu_\nu(\nabla)$ with potential $-R^\mu_\nu$. The second term can be transformed as $\text{Tr}[\frac{1}{m^2} \nabla^\mu \nabla_\mu] = \text{Tr}[\frac{1}{m^2} \nabla^\mu \Box^{m^{-1}} \nabla_\mu] = \text{Tr}[\frac{1}{m^2} \Box^{m^{-1}}]$ and

$$\frac{i}{2} \text{Tr}(m^2 \delta^\mu_\nu - \nabla_\mu \nabla_\nu) = \frac{i}{2} \text{Tr}(m^2 \Box - \Box). \tag{14}$$

Then, the effective action for the massive vector field is equal to the effective action of the minimal second-order operator $K^\mu_\nu(\nabla)$ minus the effective action of a minimal operator $S^\mu_\nu(\nabla)$ corresponding to a massive scalar field minimally coupled to gravity.

The problem with the Dirac nonminimal operator $\hat{D}_f$ is solved introducing a new spinor variable $\psi$ connected with $\phi$ by the relation $\phi = \gamma^\alpha \nabla_\alpha \psi - m_f \psi$ so that (8) takes the form $\gamma^\mu \gamma^\alpha \nabla_\alpha \nabla_\mu \psi - m_f^2 \psi = 0$. Making use of the identity $\gamma^\mu \gamma^\alpha \nabla_\alpha \nabla_\mu = \hat{I}(\Box - \frac{1}{2} R)$ equation (8) becomes of the form

$$\hat{D}_f^{\text{min}} \psi \equiv (\Box - \frac{1}{2} R - m_f^2) \psi = 0, \tag{15}$$

where the potential matrix can be easily identified as $Q = -\frac{1}{2} R \hat{I}$.

Now using the Schwinger–DeWitt representation for the Green functions of the minimal operators, we can obtain for the renormalized one-loop effective action of the quantum massive
vector and spinor fields the expression \( \Gamma_{\text{ren}} = \int d^4x \sqrt{-g} \mathcal{L}_{\text{ren}} \) where the renormalized effective Lagrangian reads

\[
\mathcal{L}_{\text{ren}} = \frac{1}{2(4\pi)^2} \sum_{k=3}^{\infty} \frac{1}{k(k-1)(k-2)} \left[ \frac{1}{m_k^{2(k-2)}} \left( \frac{\text{Tr} a_k^{(1)}(x, x) - \text{Tr} a_k^{(0)}(x, x)}{m_k^{2(k-2)}} + \frac{\text{Tr} a_k^{(2)}(x, x)}{m_k^{2(k-2)}} \right) \right],
\]

where \( a_k^{(1)} = a_k^{(1)}(x, x'), \ [a_k^{(0)} = a_k^{(0)}(x, x') \text{ and } a_k^{(2)} = a_k^{(2)}(x, x') \), whose coincidence limit appears under the supertrace operation in (16) are the HMDS coefficients for the minimal operators \( \hat{K}, \hat{S} \) and \( D_{\mu
u}^{\text{min}} \) respectively. As usual, the first three coefficients of the DeWitt–Schwinger expansion, \( a_0, a_1, \) and \( a_2, \) contribute to the divergent part of the action and can be absorbed in the classical gravitational action by renormalization of the bare gravitational and cosmological constants.

Restricting ourselves here to the terms proportional to \( m^{-2} \), using integration by parts and the elementary properties of the Riemann tensor [15, 17–19, 21, 26], we obtain for the renormalized effective Lagrangian in the case of the massive vector field considered in this work

\[
\mathcal{L}_{\text{ren}} = \frac{1}{192\pi^2 m_c^2} \left[ \frac{27}{280} R_{\mu
u} R^\mu{}\nu - \frac{5}{6} R^2 + \frac{31}{60} R R_{\mu
u} R^{\mu\nu} - \frac{52}{63} R^\rho{}\sigma R^\sigma{}\rho R^\mu{}\nu R_{\mu
u} \right.
\]
\[
+ \frac{61}{140} R_{\mu\nu} R^\mu{}_{\gamma\rho} R^{\gamma\rho\nu} - \frac{19}{105} R_{\mu\nu} R_{\gamma\rho} R^\nu{}_{\mu\rho} - \frac{67}{2520} R_{\mu\nu} R_{\gamma\rho} R_{\sigma\tau} - \frac{1}{10} R_{\mu\nu\rho\sigma} R^\rho{}_{\nu\sigma} R^\nu{}_{\rho\mu} \left. \right] \\ (17)
\]

The interested reader can find the general result for the spinor field case, for example, in [17]. As we can see, this final expression of the one loop effective for the massive vector field only differs from that of the massive scalar and spinor fields in the numerical coefficients in front of the purely geometric terms. For \( (T_{\mu\nu})_{\text{ren}} \) we obtain a very cumbersome expression that, as in the case of (17), is different from that obtained for scalar and spinor fields only in the numerical coefficients that appear in front of the purely geometrical terms. For this reason we do not put this very long expression for the stress tensor here and refer the readers to our previous papers [15, 17, 18, 26].

For this work we deal with the Reissner–Nordström spacetime. The obtained results for the components of the stress–tensor are very simple and can be found in [18].

For a quantum scalar field \( \phi(x) \) with mass \( m \) interacting with gravity with non-minimal coupling constant \( \xi \) we find

\[
(T_{\mu\nu})^\mu_{\nu} = C^\mu_{\nu} \left( \xi - \frac{1}{6} \right) D^\mu_{\nu},
\]

where

\[
C^\mu_{\nu} = -\Upsilon_1 (1248 Q^6 - 810 r^4 Q^2 + 855 M^2 r^4 + 202 r^2 Q^4 - 1878 M^3 r^3 + 1152 M r^3 Q^2 + 2307 M^2 r^2 Q^2 - 3084 M Q^4 r),
\]
\[
D^\mu_{\nu} = \Xi (-792 M^3 r^3 + 360 M^2 r^4 + 2604 M^2 Q^2 r^2 - 1008 M Q^2 r^3 - 2712 M Q^4 + 819 Q^6 + 728 Q^4 r^2),
\]
\[
C^\mu_{\nu} = \Upsilon_1 (444 Q^6 - 1488 M Q^2 r^3 + 162 Q^4 r^4 + 842 Q^4 r^2 - 1932 M Q^4 r + 315 M^2 r^4 + 2127 M^2 Q^2 r^2 - 462 M^3 r^3),
\]

5
In the above expressions, we have
\[
\langle \theta \rangle = -104.5\text{M}^2Q^3 - 693\text{M}^2r^4 + 129.07\text{M}^2Q^2r^2 + 5365Q^6 - 169.96\text{M}^4Q^r + 1050\text{M}^3r^3,
\]
(25)
\[
\langle r^2 \rangle = -31.57Q^6 - 121.50r^4Q^2 - 1665M^2r^4 - 418.54r^2Q^4 - 935.37M^2r^3Q^2 + 3666M^3r^3 + 690.24M^3r^2Q^2 + 107.51M^4Q^4r.
\]
(26)
\[
\langle r^4 \rangle = -209.08Q^6 + 4854M^3r^3 - 440.68M^4Q^r - 317.08M^2Q^2r^3 + 7290Q^2r^4 + 308.81M^2Q^2r^2 - 2079M^2r^4 + 139.79Q^6).
\]
(27)

and for the spinor field the resulting components are given by
\[
\langle T_v \rangle^1_i = -Y_v (-214.96\text{rM}Q^4 + 4917Q^6 + 105.44r^2Q^4 - 224.64\text{Mr}^3Q^2 + 218.32M^2r^3Q^2 - 1080M^2r^4 + 2384M^3r^3 + 5400r^4Q^2),
\]
(28)
\[
\langle T_v \rangle^0_i = -Y_v (-6336M^rQ^2 + 8440M^2r^2Q^2 + 2253Q^6 + 3560r^2Q^4 - 8680rM^Q^4 + 504M^2r^4 - 784M^3r^3 + 1080r^4Q^2),
\]
(29)
\[
\langle T_v \rangle^0_i = -Y_v (120.80r^2Q^4 - 339.84rMe^4 + 9933Q^6 + 308.08M^2r^2Q^2 - 200.16\text{Mr}^3Q^2 + 1512M^2r^4 - 3536M^3r^3 + 3240r^4Q^2).
\]

In the above expressions, we have \(Y_v = \left(302.40\pi^2m^2\bar{r}^{12}\right)^{-1}\), \(Y_v = \left(100.80\pi^2m^2\bar{r}^{12}\right)^{-1}\), \(Y_v = \left(403.20\pi^2m^2\bar{r}^{12}\right)^{-1}\) and \(\Xi = \left(720\pi^2m^2r^{12}\right)^{-1}\). \(Q\) and \(M\) denote the charge and bare mass of the black hole.

The above quantum stress–energy tensors are regular at the event horizon, as is to be expected due to the local nature of the Schwinger–DeWitt approximation and the regular nature of the horizon. Also from the general form of the geometric terms conforming the general expression for the constructed stress tensor, we see that it is covariantly conserved, thus indicating that it is a good candidate for the expected exact one in our large mass approximation.

From the above expressions it is not difficult to show that, in the near horizon region of the classical black hole, the minimally coupled scalar, vector and spinor fields violate the weak energy condition (see [14, 18]). This is an interesting consequence of the quantization of fields in black hole backgrounds, that can produce some observable effects, including a change in the quasinormal spectrum of the black hole solutions obtained by considering the backreaction of the quantum fields upon the classical background spacetime, as we show in the rest of the paper.
In the specific case of a Reissner–Nordström spacetimes, Anderson et al showed, using detailed numerical results for the scalar field case, that for $m_s M \gtrsim 2$ the deviation of the approximate stress–energy tensor from the exact one lies within a few per cent [14]. The more general condition for the validity of the Schwinger–DeWitt approximation in the case of a spin-$j$ field can be written as $m_j M \gtrsim 1$, where $m_j$ and $M$ are respectively the field and black hole masses. In the following we carefully take into account the above condition in numerical calculations.

3. Semiclassical solution

In our previous paper we have shown how to find the general solution to the backreaction problem in spacetimes with spherical symmetry [10]. In the following we solve general semiclassical Einstein equations assuming that there are an electromagnetic field as a classical source, and multiple quantized massive free fields as a perturbative quantum source, so the solution to the backreaction problem gives a quantum corrected Reissner–Nordström black hole.

In this limit in which we deal with free fields on a background spacetime, the treatment of the backreaction due to a collection of fields with different spins is easy. This is due to the fact that the quantum stress tensors in this limit only depends quadratically on the fields and are flavour diagonal. For example, for a set of $N_s$ real scalars, upon renormalization we have

\[
\langle T^\mu_\nu \rangle_{\text{ren}} = \sum_{k=1}^{N_s} \langle (T^\mu_\nu)_k \rangle_{\text{ren}} = N_s \langle (T^\mu_\nu) \rangle_{\text{ren}},
\]  

(30)

where $(T^\mu_\nu)_k$ is the classical stress tensor for a single scalar. The last equality above follows from the fact that the renormalization procedure is independent of the species label $k$. In a similar manner we can arrive at the same conclusion for the renormalization of spinor and vector field stress tensors.

The above statements permit us to obtain a good approximation to the multiple-field backreaction using as the source term in the semiclassical Einstein equations an appropriate weighted combination (with weights $N_s$, $N_v$ and $N_f$) of the single-species renormalized stress–energy tensors.

In the case of our interest the general form for the line element that solves the backreaction problem, considering only terms that are linear in the perturbation parameter $\varepsilon = 1/M^2$, is given by

\[
ds^2 = -A(r) \, dt^2 + B(r) \, dr^2 + r^2 (d\theta^2 + \sin^2 \theta \, d\phi^2),
\]  

(31)

with

\[
\frac{1}{B(r)} = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{8\pi}{r} \sum_j N_j \int_\infty^r \xi^2 \langle T^t_t \rangle_j \, d\xi,
\]  

(32)

and

\[
A(r) = \frac{1}{B(r)} \prod_j \exp\{\lambda_j(r)\},
\]  

(33)

where

\[
\lambda_j(r) = \frac{8\pi N_j}{r} \int_\infty^r \xi B(\xi) \left( \langle T^t_t \rangle_j - \langle T^t_t \rangle_j \right) \, d\xi.
\]  

(34)

where the subindex $j$ denotes different single spin species considered (scalar, vector and spinor field).
As we can see from expression (32), a nonzero temporal component of the stress–energy tensor produces a quantum correction to the black hole mass, as measured by a distant observer. The principal contribution to this quantum corrected mass comes from the horizon values of the renormalized stress–energy tensor. As a consequence of the violation of the weak energy condition for quantum scalar, vector and spinor fields near the event horizon in static charged black hole backgrounds, we can see that the vacuum polarization produces a decreasing in the mass of the semiclassical black hole, as measured by a distant observer.

Inserting the corresponding expressions for the temporal component of the quantum stress tensor for the different fields considered in this work, we obtain

\[
\frac{1}{B(r)} = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{\epsilon}{\pi} \sum_j \frac{N_j}{m_j^2} F_j(r),
\]

(35)

where, for the scalar case

\[
F_s(r) = E(r) + \xi H(r),
\]

(36)

with

\[
E(r) = \frac{-613 M^3 Q^4}{840 r^9} + \frac{2327 M^2 Q^6}{1134 r^{10}} - \frac{3M^2 Q^2}{28 r^6} + \frac{5M^4}{3780 r^7} + \frac{883 M^2 Q^4}{4410 r^8}
\]

\[
H(r) = \frac{28 M^3 Q^2}{15 r^7} + \frac{113 M^3 Q^4}{30 r^9} - \frac{91 M^2 Q^2}{90 r^{10}} - \frac{52 M^2 Q^4}{45 r^8} - \frac{4M^4}{5 r^6} + \frac{22 M^5}{15 r^7} - \frac{62 M^4 Q^2}{15 r^8},
\]

\[
\lambda_s(r) = \frac{\epsilon}{\pi m^2} \left( \frac{184 M^3 Q^2}{441 r^7} - \frac{29 M^4}{140 r^6} - \frac{229 M^2 Q^4}{840 r^8} + \frac{M^2 Q^2}{35 r^6} \right)
\]

\[+ \frac{\epsilon \xi}{\pi m^2} \left( \frac{14 M^4}{15 r^6} + \frac{13 M^2 Q^4}{10 r^8} - \frac{32 M^3 Q^2}{15 r^7} \right),
\]

and for vector and spinor fields

\[
F_v(r) = \frac{26879}{5040} \frac{M^3 Q^4}{r^9} + \frac{2876}{315} \frac{M^3 Q^2}{r^7} + \frac{611}{980} \frac{M^5}{r^5} + \frac{37}{140} \frac{M^4}{r^6} - \frac{20927}{4410} \frac{M^2 Q^4}{r^8}
\]

\[
- \frac{10393}{14} \frac{M^4 Q^2}{r^5} - \frac{27}{11340} \frac{M^2 Q^4}{r^6} - \frac{31057}{30} \frac{M^2 Q^2}{r^7} - \frac{2}{11340} \frac{M^4}{r^10},
\]

\[
F_s(r) = \frac{26879}{5040} \frac{M^3 Q^4}{r^9} + \frac{1639}{15120} \frac{M^3 Q^6}{r^{10}} - \frac{149}{1890} \frac{M^5}{r^7}
\]

\[
- \frac{3}{14} \frac{M^2 Q^2}{r^6} + \frac{3}{70} \frac{M^4}{r^6} + \frac{26}{35} \frac{M^2 Q^2}{r^7} - \frac{2729}{4410} \frac{M^4 Q^2}{r^8} - \frac{659}{2205} \frac{M^2 Q^4}{r^8}.
\]

\[
\lambda_v(r) = \frac{\epsilon}{\pi m_f^2} \left( \frac{131}{420} \frac{M^4}{r^6} + \frac{9}{7} \frac{M^2 Q^2}{r^6} - \frac{9784}{2205} \frac{M^4 Q^2}{r^7} + \frac{2141}{840} \frac{M^2 Q^4}{r^8} \right)
\]

\[
\lambda_s(r) = \frac{\epsilon}{\pi m_f^2} \left( \frac{37}{560} \frac{M^2 Q^4}{r^8} - \frac{11}{210} \frac{M^4}{r^6} + \frac{1}{7} \frac{M^2 Q^2}{r^6} - \frac{52}{245} \frac{M^4 Q^2}{r^7} \right).
\]
The horizon for the quantum corrected solution will be, up to first order in the perturbation parameter, at position \( r_+ \) given by

\[
r_+ = r_H \left( 1 + \sum_j N_j \Lambda_j \right),
\]

(38)

where

\[
\Lambda_j = -\frac{4\pi}{(M - Q^2/r_H)} \int_{\infty}^{\infty} \xi^2 \langle T_{ij}(\xi) \rangle_j d\xi,
\]

(39)

and \( r_H \) is the position of the event horizon of the bare classical static charged solution.

Due to the positive sign of the temporal component of the renormalized stress tensor in this background for all the quantum fields considered in this paper, we see a decreasing of the horizon radius for the semiclassical charged solution.

Upon substitution of the required quantities in the above expression we find

\[
\Lambda_j = -\frac{\epsilon \Gamma_j}{\pi m_j^2 (M - Q^2/r_H)},
\]

(40)

with

\[
\Gamma_j = \Theta + \xi \Omega,
\]

(41)

with

\[
\Theta = \frac{613 M^3 Q^4}{1680 r_H^6} - \frac{2327 M^2 Q^6}{22680 r_H^8} + \frac{3 M^2 Q^2}{140 r_H^4} - \frac{5 M^4}{50 r_H^6} + \frac{1237 M^5}{7560 r_H^8} - \frac{883 M^2 Q^4}{8820 r_H^8}
\]

\[
+ \frac{41 M^3 Q^2}{315 r_H^4} - \frac{1369 M^4 Q^2}{3528 r_H^6},
\]

\[
\Omega = -\frac{14 M^3 Q^2}{15 r_H^4} - \frac{113 M^3 Q^4}{60 r_H^6} + \frac{91 M^2 Q^6}{180 r_H^8} + \frac{26 M^2 Q^4}{45 r_H^6} + \frac{2 M^4}{5 r_H^8} - \frac{11 M^5}{15 r_H^8} + \frac{31 M^4 Q^2}{15 r_H^8},
\]

and for the vector and fermion components we find

\[
\Gamma_v = \frac{27 M^2 Q^2}{28 r_H^6} + \frac{37 M^4}{280 r_H^8} - \frac{26879}{5040 r_H^8} - \frac{1438 M^3 Q^4}{r_H^8} + \frac{31057 M^2 Q^6}{22680 r_H^8}
\]

\[
- \frac{611 M^3}{2520 r_H^8} + \frac{20927 M^2 Q^4}{8820 r_H^8} + \frac{10393 M^4 Q^2}{1960 r_H^8},
\]

(42)

\[
\Gamma_f = -\frac{2687}{10080} \frac{M^3 Q^4}{r_H^8} + \frac{149 M^5}{3780 r_H^8} - \frac{3 M}{140 r_H^4} + \frac{3 M^4}{28 r_H^6} + \frac{2729 M^2 Q^2}{8820 r_H^8}
\]

\[
- \frac{13 M^3 Q^2}{35 r_H^6} + \frac{659 M^2 Q^4}{4410 r_H^8} + \frac{1639 M^2 Q^6}{30240 r_H^8}.
\]

(43)

### 4. Scalar perturbations and quasinormal modes

In the following we consider the evolution of a test massless scalar field \( \Phi(x^\mu) \) with \( x^\mu = (t, r, \theta, \phi) \), in the quantum corrected gravitational background studied above. The dynamics of for this test field is governed by the Klein–Gordon equation

\[
\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} \left( \sqrt{-g} g^{\mu \nu} \frac{\partial \Phi}{\partial x^\nu} \right) = 0,
\]

(44)
with $g_{\mu\nu}$ being the metric tensor of semiclassical solution and $g$ its determinant. Upon separation of the angular and radial part in the above equation and the introduction of the radial tortoise coordinate

$$\frac{d^2}{dr_+^2} Z_L - [\omega^2 - V] Z_L = 0,$$

(45)

where $Z_L(r)$ denotes the radial component of the wavefunction, $\omega$ is the quasinormal frequency and $V$ is the effective potential. The potential $V$ is a function of the metric components $g_{\mu\nu}$ and the multipolar number $L$, and for the test massless scalar field considered in this work, is given by the general expression

$$V[r(r_+)] = A(r) \frac{L(L + 1)}{r^2} + \frac{A(r)}{2r B(r)} [(\ln A(r))'' - (\ln B(r))'],$$

(46)

where the prime refers to the derivative with respect to the radial coordinate $r$. For semiclassical black holes we have in general the following expression for the effective potential:

$$V(r) = V^c(r) + \frac{\epsilon}{\pi} U(r) + O(\epsilon^2),$$

(47)

where $V^c(r)$ is the scalar effective potential of the bare black hole solution and $U(r)$ is a complicated function of the contributions $\frac{\epsilon}{\pi} U_j(r)$ due to the vacuum polarization effect related to the multiple field backreaction, which we will not write here. In the case of a classical Reissner–Nordström black hole $V^c(r)$ is given by

$$V^c(r) = \left(\frac{r_+^2 - 2 M r + Q^2}{r^6}\right)(-2 Q^2 + \beta r^2 + 2 M r),$$

(48)

where $\beta = L(L + 1)$. For the semiclassical black hole solution considered in this paper, where the vacuum polarization effects comes from the quantization of massive scalar, vector and spinor fields in the large mass limit, the particular expression for $U_j(r)$ results

$$U_j(r) = W_1(r) + \xi W_2(r),$$

(49)

where

$$W_1(r) = -\left[\frac{1751}{4410} \frac{M^2 Q^4}{r^{10}} - \frac{9}{20} \frac{M^4}{r^8} + \frac{1021}{540} \frac{M^5}{r^9} - \frac{1816}{945} \frac{M^6}{r^{10}} + \frac{6}{35} \frac{M^2 Q^2}{r^8}\right] + \left[\frac{674641}{158760} \frac{M^3 Q^6}{r^{13}} + \frac{17}{105} \frac{M^4 Q^6}{r^9} - \frac{13271}{1764} \frac{M^4 Q^4}{r^{12}} - \frac{625}{756} \frac{M^5 Q^8}{r^{14}}\right] + \left[\frac{2335}{15876} \frac{M^2 Q^8}{r^{12}} + \frac{8559}{1960} \frac{M^3 Q^4}{r^{11}} - \frac{962}{245} \frac{M^4 Q^2}{r^{10}} + \frac{16687}{2940} \frac{M^5 Q^2}{r^{11}}\right] + \frac{L(L + 1)}{22680} \left[\frac{1529}{35} \frac{M^2 Q^6}{r^{12}} - \frac{1}{70} \frac{M^4}{r^8}\right] + \frac{47}{540} \frac{M^5}{r^9} - \frac{773}{17640} \frac{M^2 Q^4}{r^{10}} + \frac{44}{441} \frac{M^3 Q^3}{r^9} + \frac{821}{3528} \frac{M^3 Q^3}{r^{11}} - \frac{1171}{4410} \frac{M^4 Q^2}{r^{10}}\right],$$

(50)

and

$$W_2(r) = L(L + 1) \left[\frac{4}{15} \frac{M^3 Q^2}{r^9} + \frac{16}{15} \frac{M^4 Q^2}{r^{10}} - \frac{29}{30} \frac{M^3 Q^4}{r^{11}} - \frac{2}{5} \frac{M^5}{r^9} + \frac{13}{90} \frac{M^2 Q^4}{r^{10}} + \frac{13}{45} \frac{M^3 Q^2}{r^{12}} + \frac{2}{15} \frac{M^4}{r^9}\right] + \frac{128}{15} \frac{M^6}{r^{10}} + \frac{13}{3} \frac{M^2 Q^2}{r^8} + \frac{182}{45} \frac{M^4 Q^4}{r^{10}} - \frac{289}{10} \frac{M^3 Q^2}{r^{11}} - \frac{28}{5} \frac{M^5 Q^2}{r^9} - \frac{42}{5} \frac{M^5}{r^9} + \frac{416}{15} \frac{M^4 Q^2}{r^{10}} + \frac{130}{3} \frac{M^4 Q^4}{r^{12}}\right].$$
Figure 1. The effective potential of $L = 0$ scalar modes for a semiclassical black hole with $M = 100$ (top curve) and $M = 200$ (bottom curve), $Q/M = 0.95$, $N_s = N_v = N_f = 10$ and the mass parameter of all quantum fields are chosen to be $m = 1/10$.

for the scalar case and

$$U_s(r) = -\frac{107 577}{980} \frac{M^3 Q^2}{r^{14}} + \frac{306 442}{2205} \frac{M^3 Q^2}{r^{10}} + \frac{349 07}{196} \frac{M^4 Q^4}{r^{12}} + \frac{13}{20} \frac{M^4}{r^8} + \frac{54}{7} \frac{M^2 Q^2}{r^8}$$

$$-\frac{491}{180} \frac{M^5}{r^9} + \frac{872}{315} \frac{M^6}{r^{10}} + \frac{1205}{21} \frac{M^3 Q^2}{r^9} - \frac{303 071}{187} \frac{M^4 Q^4}{r^{13}} - \frac{154 15961}{158 760} \frac{M^3 Q^6}{r^{14}}$$

$$+ \frac{681 461}{158 76} \frac{M^2 Q^6}{r^{12}} + \frac{290 27}{882} \frac{M^2 Q^6}{r^{10}} + \frac{664 21}{3780} \frac{M^2 Q^8}{r^{14}} + L(L+1) \left( -\frac{5}{36} \frac{M^5}{r^9} \right)$$

$$+ \frac{4678}{2205} \frac{M^3 Q^2}{r^9} - \frac{6653}{5880} \frac{M^3 Q^4}{r^{11}} - \frac{160 67}{176 40} \frac{M^2 Q^4}{r^{10}} - \frac{4307}{226 80} \frac{M^2 Q^6}{r^{12}} + \frac{1}{21} \frac{M^4}{r^8}$$

$$- \frac{6257}{4410} \frac{M^4 Q^2}{r^{10}} - \frac{9}{14} \frac{M^2 Q^2}{r^8} \right) \right). \quad (50)$$

and

$$U_f(r) = -\frac{2279}{2520} \frac{M^2 Q^3}{r^{14}} - \frac{111 01}{1470} \frac{M^5 Q^2}{r^{11}} + \frac{247 64}{2205} \frac{M^4 Q^2}{r^{10}} + \frac{5092}{441} \frac{M^4 Q^4}{r^{12}} - \frac{191}{35} \frac{M^3 Q^2}{r^9}$$

$$- \frac{1}{10} \frac{M^4}{r^8} - \frac{119 141}{211 68} \frac{M^6 Q^6}{r^{13}} + \frac{113}{7} \frac{M^5}{r^8} + \frac{80}{189} \frac{M^6}{r^{10}} - \frac{8775}{784} \frac{M^3 Q^4}{r^{11}}$$

$$+ \frac{237 97}{8820} \frac{M^2 Q^4}{r^{10}} + \frac{3569}{1323} \frac{M^2 Q^6}{r^{12}} + L(L+1) \left( \frac{7}{270} \frac{M^5}{r^9} - \frac{1}{105} \frac{M^4}{r^8} + \frac{12}{49} \frac{M^3 Q^2}{r^9} \right)$$

$$- \frac{1}{14} \frac{M^2 Q^2}{r^8} - \frac{3173}{352 80} \frac{M^2 Q^4}{r^{10}} + \frac{544}{2205} \frac{M^2 Q^6}{r^{12}} - \frac{8}{189} \frac{M^2 Q^8}{r^{14}} + \frac{6659}{352 80} \frac{M^3 Q^4}{r^{11}} \right) \right). \quad (51)$$

for the vector and spinor cases.

In figure 1 is presented the effective potential $V$ taking into account, as an example, the backreaction of multiple fields with weights $N_s$, $N_v$ and $N_f$ all equal to 10.

As it is observed, the figure shows a definite positive potential barrier, i.e. a well-behaved function that goes to 0 at spatial infinity and gets a maximum value near the event horizon. The quasinormal modes are solutions of the wave equation (45) with specific boundary conditions requiring pure outgoing waves at spatial infinity and pure incoming waves on the event horizon.
Table 1. Rescaled scalar quasinormal frequencies $\frac{\omega}{\omega_0} = 10^4 \omega$ for the classical and semiclassical $M = 100$ Reissner–Nordström black hole, with $Q/M = 0.95$, $N_s = N_c = N_f = 10$, $M = 100$ and $m = 1/10$.

| L | n | Re($\omega$) | Im($\omega$) | L | n | Re($\omega$) | Im($\omega$) |
|---|---|-------------|-------------|---|---|-------------|-------------|
| 0 | 0 | 10.2421 | 6.6426 | 0 | 0 | 5.5213 | 29.4487 |
| 1 | 0 | 28.7831 | 6.8083 | 1 | 1 | 27.1215 | 7.5690 |
| 1 | 1 | 26.5433 | 21.0202 | 2 | 0 | 45.3444 | 33.6626 |
| 2 | 0 | 47.7481 | 6.8632 | 2 | 1 | 43.6519 | 33.6050 |
| 3 | 0 | 66.7647 | 6.9466 | 3 | 0 | 63.9899 | 7.8001 |
| 3 | 1 | 65.7536 | 20.3592 | 3 | 1 | 61.9974 | 28.6634 |
| 3 | 2 | 63.7674 | 34.3047 | 3 | 2 | 60.5648 | 37.3333 |
| 4 | 0 | 85.7969 | 6.9987 | 4 | 0 | 84.0022 | 7.3011 |
| 4 | 1 | 85.0075 | 20.2988 | 4 | 1 | 83.6741 | 24.7819 |
| 4 | 2 | 83.4477 | 34.0553 | 4 | 2 | 80.9929 | 36.8832 |
| 4 | 3 | 81.1448 | 48.1531 | 4 | 3 | 80.0211 | 50.0112 |

The quasinormal frequencies are in general complex numbers, whose real part determines the real oscillation frequency and the imaginary part determines the damping rate of the quasinormal mode. In order to evaluate the quasinormal modes for the situation considered in this report field, we use the well-known WKB technique at sixth order, which can give accurate values of the lowest (that is longer lived) quasinormal frequencies, and was used in several papers for the determination of quasinormal frequencies in a variety of systems [27]. The first-order WKB technique was applied to finding quasinormal modes for the first time by Shutz and Will [28]. Later this approach was extended to the third order beyond the eikonal approximation by Iyer and Will [29] and to the sixth order by Konoplya [30].

The sixth-order WKB expansion gives a relative error which is about two orders less than the third WKB order, and allows us to determine the quasinormal frequencies through the formula

$$i\left(\omega^2 - V_0\right) = \sqrt{-2V_0} \sum_{j=2}^{6} \prod_j = n + \frac{1}{2}, \quad n = 0, 1, 2, \ldots$$

(52)

where $V_0$ is the value of the potential at its maximum as a function of the tortoise coordinate, and $V_0''$ represents the second derivative of the potential with respect to the tortoise coordinate at its peak. The correction terms $\prod_j$ depend on the value of the effective potential and its derivatives (up to the 2nd order) in the maximum, see [31] and references therein.

For all the results reported in this paper, we perform a consistency check by calculating the values of the quasinormal frequencies using one by one, all orders of the WKB approximation, and find that the results indeed converge to the sixth-order values reported in this work. Then, we expect that the results presented here can serve as a valid estimation very close to the ones that could be obtained in the future by more elaborated and accurate numerical methods.

The results of the numerical evaluation of the first four quasinormal frequencies in the case of black holes with bare mass $M = 100$ are shown in table 1. Table 2 also shows the
Table 2. Rescaled scalar quasinormal frequencies $\sigma = 10^4\omega$ for the classical and semiclassical Reissner–Nordström black hole, with $Q/M = 0.95, N_v = N_f = 10$ and $m = 1/10$.

|       | Semiclassical solution | Classical solution |
|-------|------------------------|--------------------|
|       | $M = 120$              |                    |
| $L$   | $n$                    | $\text{Re}(\sigma)$ | $-\text{Im}(\sigma)$ | $L$ | $n$ | $\text{Re}(\sigma)$ | $-\text{Im}(\sigma)$ |
| 0     | 0                      | 8.535 09           | 5.535 53              | 0   | 0  | 4.6011               | 24.5404              |
| 1     | 0                      | 23.9859            | 5.6736                | 0   | 1  | 22.2679              | 6.30758              |
| 1     | 1                      | 22.1194            | 17.0889               | 1   | 1  | 19.3661              | 28.0521              |
|       | $M = 150$              |                    |
| 0     | 0                      | 6.828 09           | 4.428 43              | 0   | 0  | 3.680 89             | 19.6324              |
| 1     | 0                      | 19.1887            | 4.5389                | 0   | 1  | 19.0144              | 5.04608              |
| 1     | 1                      | 17.6956            | 14.0135               | 1   | 1  | 15.4929              | 22.4418              |

results for the first two fundamental modes for black holes with bare masses $M = 120$ and $M = 150$.

As we can see from the tables, and from figures 2 and 3, which show the results of all our calculations concerning the fundamental mode, the backreaction of the quantized multiple fields upon the classical charged black hole gives rise to an increase in the real oscillation frequencies and to a decrease in the damping rate, for physically interesting values of the black hole mass. Then, as a consequence of the vacuum polarization effect due to the multiple massive fields, we have an effective increase in the quality factor proportional to the ratio $|\text{Re}(\omega)|/|\text{Im}(\omega)|$. As expected, the differences in the quasinormal frequencies when the black hole mass increases tend to become small. It is interesting to note that the above results show significant differences with those obtained previously considering only the backreaction of a quantized massive scalar field upon the classical charged black hole solution. In that case the quality factor is small for the semiclassical black hole with respect to the classical solution, and the shift in the quasinormal frequencies is less pronounced. As the numerical calculations show, this is not the case if the separate backreaction due to massive vector and spinor fields are considered. In each of these cases, we obtain an increase in the quality factor more relevant than the decrease shown in the scalar field case. As a result, the combination of all types of fields gives a higher quality factor with respect to the bare black hole. Thus, we arrive at a conclusion that the semiclassical Reissner–Nordström black holes have better oscillators than their classical partners. Then, in this case, there is a qualitative correspondence with the results obtained by Konoplya in [11] for the BTZ black hole dressed by a quantum conformal massless scalar field, where he studied the backreaction due to Hawking radiation upon the classical spacetime.

The above effect can be understood as a consequence of a decrease of the mass of the semiclassical black hole with respect to the bare one, due to the exotic character of the quantum fields (that violates the weak energy condition) near the event horizon. A decrease in the mass as measured by a distant observer leads to an increase in the real part of the quasinormal frequency. If we consider the semiclassical solution, as some kind of new classical one, but with a reduced value of the mass, then for the same charge (that is not affected by the presence of massive quantum fields) we obtain a larger value of the quality factor of the semiclassical oscillator.
Figure 2. Dependence of $\text{Re}(\omega)$ on $M$ for classical and semiclassical black holes. The parameters are chosen to be $Q/M = 0.95$, $m = 1/10$, $L = 0$, $N_s = N_v = N_f = 10$ and $n = 0$.

Figure 3. Dependence of $\text{Im}(\omega)$ on $M$ for classical and semiclassical black holes. The parameters are chosen to be $Q/M = 0.95$, $m = 1/10$, $L = 0$, $N_s = N_v = N_f = 10$ and $n = 0$. 
We also studied the dependence of the quasinormal frequencies for a fixed black hole bare mass and different values of the quantum field mass $m_J$, obtaining similar results with respect to the massive scalar field case: a little dependence of the quasinormal frequencies on these parameters. As the quantum field mass increases, we found a very small increment in the real part of frequencies for semiclassical black holes, and a very small decrease in the imaginary part. The similar condition occurs if we consider the variation of the coupling constant between the massive scalar field component and the gravitational background: the quasinormal frequencies are insensitive to the variation of this parameter. Therefore, the shift in the quasinormal frequencies with respect to the classical bare black hole appears to be the same for the given range of the quantum field masses. With respect to the multiplicity number $N_J$ of a given quantum field we find shift values of the same order (showing a little increment) as this parameters increases, an effect that is more pronounced for very large $N_J$. It is important to take care of the fact that very large values of these numbers can imply that the total quantum stress–tensor becomes a large quantity, such that the perturbative treatment of the backreaction problem used here becomes inadequate. By fortune, this is not the case for physically interesting values of $N_J$. We present in figures 1–3 the results for the particular case $N_J = 10$, as a representative of the general behaviour of the shift in the quasinormal spectrum.

This nontrivial behaviour of the shift in the quasinormal frequencies could be due to the complicated dependence of the complete effective potential given by expression (47) upon the quantum field parameters (mass, multiplicity factors and scalar coupling constant). Unfortunately, the general expressions are very cumbersome, and to see at its structure is of little help. The same occurs for the expression of the effective potential in the large multipole limit (given by the first-order WKB result, which is exact in this limit).

5. Concluding remarks

We have studied the influence of the backreaction due to the vacuum polarization of multiple species of large mass quantum massive fields, belonging to the standard model, upon the structure of scalar quasinormal frequencies for semiclassical charged black holes. The effect observed is a shift in the quasinormal frequencies of the semiclassical solution, such that quantum corrected black holes become better oscillators than classical ones. The observed effect appears to be related to the violation of the weak energy condition by quantum fields near the event horizon of this background, a behaviour that produces an effective decrease in the mass of the semiclassical solution as seen by a distant observer. It is important to verify that the above effects are also true for the quasinormal ringing phase in the evolution of spinor and electromagnetic test fields. We are currently investigating such problems and the results will be presented in future reports.

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