Superfield formalism for 5D Lorentz violating models

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Abstract. We construct a supersymmetric higher dimensional theory which includes 5D Lorentz violation terms in a superfield formalism. We show that in order to obtain a consistent construction which incorporates the most general terms that explicitly breaks the 5D Lorentz invariance, it is only necessary to redefine the supersymmetric auxiliary fields. We also analyze the meaning of these violation terms and point out the possibility of resizing the radius of compactification to obtain an equivalent 4D Lorentz invariant effective theory.

1. Introduction

On December 2011, CMS and ATLAS groups presented their latest results on the Higgs search, where they showed the possibility of finding this particle with a mass between 119 and 130 GeV [1]. A Higgs particle with such a mass leads to the famous hierarchy problem in the Standard Model. As it is well known, supersymmetry [2] (SUSY) and Extra Dimensional theories (XD) [3] offer some suggestions to solve this problem, unfortunately we do not have any experimental evidence of their existence yet.

SUSY is introduced as a global symmetry of the Lagrangian which transforms fermions into boson and viceversa, that implies the existence at least of one superpartner for each particle of the SM. On an exact SUSY theory, a particle and its superpartner must have same charge, spin and mass. There is not any indication for the existence of superpartners, and for this reason global supersymmetry, if present, must be broken, but a convincing mechanism to obtain this rupture is yet unknown. [5]. Most supersymmetric theories can be rewritten in superspace formalism which was proposed by Salam and Strathdee in 1974 [6]. This formalism involves the consideration of superfield \( \Phi(x, \theta, \bar{\theta}) \) defined on an 8-dimensional space which is the product of ordinary space-time with a 4-dimensional space whose points are labeled by the anticommuting Grassman variables \( \theta_\alpha \).

Extra dimensional models, on the other hand, introduce additional space-like coordinates in the usual theories, trying to solve some conceptual problems of the Standard Model (SM). However, the required compactification of such dimensions leads to an infinite number of particles arranged on the so called Kaluza Klein (KK) towers, where different levels incorporate copies of SM particle representations, but with larger masses. Lowest levels corresponding to SM particle spectrum. Mass gaps and the KK spectrum characteristics do depend on the geometry of the compact space. [4]. One of the common features of such constructions, is that compactification of the extra space implies the breaking of the higher dimensional space-time symmetries. In particular, in flat extra space models, higher dimensional Lorentz invariance cannot be advocated in general as a true symmetry. Nevertheless, the way this symmetry is broken and the physical
implications of the breaking at the level of the effective theories had not yet been studied in detail so far. This is the issue we are dealing with, and here we present some technical advances on our studies. In a previous work we build a SUSY model with one extra dimension in which the 5D Lorentz symmetry is broken in a explicit way, whereas the 4D Lorentz symmetry remains after the compactification process [7]. The goal of this work is to present the model discussed above in a superfield formalism which explicitly depict the degrees of freedom that are relevant at the level of the 4D effective theory, and further discuss the physical meaning of the terms that break Lorentz invariance.

The outline for this work is as follow: In the section 2 we briefly present a review of the superfield formalism for standard SUSY theories which should serve as the reference frame for the rest of the discussion. In the section 3 we show the convention for Dirac matrices in five dimensions for the model, and rewrite the Dirac Lagrangian in terms of two Weyl spinors. In the section 4 and 5 we present the 5D SUSY model on our superfield formalism. In section 6 we discuss how a modification on compactification radius provides a mechanism to hide the terms of Lorentz violation. Finally, our conclusions are presented in section 7.

2. Superfield formalism.

The SUSY algebra is generated by the momentum operators $P_\mu$ and the Weyl spinor supersymmetry generators $Q_\alpha$ and $\bar{Q}_{\dot{\alpha}}$

\[
[Q_\alpha, P_\mu] = [\bar{Q}_{\dot{\alpha}}, P_\mu] = [P_\mu, P_\nu] = 0
\]
\[
\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0
\]
\[
\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma^{\mu}_{\alpha\beta}P_\mu ,
\]

and, a finite element of the corresponding group is given by

\[
G \left( x^\mu, \theta, \bar{\theta} \right) = \exp \left( i \left( \theta Q + \bar{\theta} \bar{Q} - x^\mu P_\mu \right) \right).
\]

Hausdorff formula leads to express the group product as

\[
G \left( a^\mu, \xi, \bar{\xi} \right) G \left( x^\mu, \theta, \bar{\theta} \right) = G \left( x^\mu + a^\mu + i\xi\sigma^\mu \bar{\theta} - i\theta\sigma^\mu \xi, \theta, \bar{\theta} + \bar{\xi} \right).
\]

Therefore, the SUSY generators induce a translation in group parameter space

\[
\exp \left( i \left( \xi Q + \bar{\xi} \bar{Q} - a^\mu P_\mu \right) \right) : (x^\mu, \theta, \bar{\theta}) \longrightarrow (x^\mu + a^\mu + i\xi\sigma^\mu \bar{\theta} - i\theta\sigma^\mu \xi, \theta, \bar{\theta} + \bar{\xi}).
\]

For a superspace function $S(x^\mu, \theta, \bar{\theta})$ (referred to as a superfield) we have

\[
S \left( x^\mu + a^\mu - i\xi\sigma^\mu \bar{\theta} + i\theta\sigma^\mu \xi, \theta, \bar{\theta} + \bar{\xi} \right) = S \left( x^\mu, \theta, \bar{\theta} \right) + (a^\mu + i\xi\sigma^\mu \bar{\theta} - i\theta\sigma^\mu \xi) \frac{\partial S}{\partial x^\mu} + \xi^\alpha \frac{\partial S}{\partial \theta^\alpha} + \bar{\xi}_{\dot{\alpha}} \frac{\partial S}{\partial \bar{\theta}_{\dot{\alpha}}} + \ldots
\]

from which it follows that the action of the SUSY algebra on superfields is generated by

\[
P_\mu = i\partial_\mu
\]
\[
iQ_\alpha = \frac{\partial}{\partial \theta^\alpha} + i\sigma^\mu_{\alpha\dot{\alpha}} \bar{\theta}_{\dot{\alpha}} \partial_\mu
\]
\[
i\bar{Q}_{\dot{\alpha}} = -\frac{\partial}{\partial \bar{\theta}^\dot{\alpha}} - i\theta^\alpha \sigma^\mu_{\alpha\dot{\alpha}} \partial_\mu.
\]
At this level we can define fermionic derivatives which anticommute with the generators of the supersymmetry algebra, an example for they is given by

\[ D_\alpha = \frac{\partial}{\partial \theta^\alpha} - i \sigma^\mu_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \partial_\mu \]

\[ \bar{D}_{\dot{\alpha}} = - \frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} + i \theta^\alpha \sigma_{\alpha\dot{\alpha}} \mu \partial_\mu \]

and these covariant derivatives obey the algebra

\[ \{ D_\alpha, \bar{D}_{\dot{\alpha}} \} = 2 \sigma^\mu_{\alpha\dot{\alpha}} P_\mu \]

\[ \{ D_\alpha, D_\beta \} = \{ \bar{D}_{\dot{\alpha}}, \bar{D}_{\dot{\beta}} \} = 0 \]  

A general superfield \( S(x^\mu, \theta, \bar{\theta}) \) may be expanded as a power series in \( \theta \) and \( \bar{\theta} \) involving not more than two powers of \( \theta \) and \( \bar{\theta} \) because they are two-component Grassmann variables. The coefficients of the various powers of \( \theta \) and \( \bar{\theta} \) in this expansion are ordinary fields (functions of \( x^\mu \)). The superfield on which has been imposed the constraint \( \bar{D}_{\dot{\alpha}} S = 0 \) is denoted by \( \Phi \) and we refer to it as a chiral superfield. Furthermore, in this work we are going to define the chiral superfield as

\[ \Phi(x^\mu, \theta, \bar{\theta}) = A(x) + \sqrt{2} \theta \chi(x) + \theta^2 F(x) - i (\theta \sigma^\mu \bar{\theta}) \partial_\mu A(x) + \frac{i}{\sqrt{2}} \theta^2 (\partial_\mu \chi(x) \sigma^\mu \bar{\theta}) - \frac{1}{4} \theta^2 \bar{\theta}^2 \partial_\mu \partial^\mu A(x) \]

here \( \mu = 0, \ldots, 3 \), \( A \) is a complex scalar field, \( \chi \) is a Weyl spinor, \( F \) is an auxiliary field and \( \sigma^\mu = (1, \vec{\sigma}) \). To obtain the supersymmetric transformations for the fields, let's consider an infinitesimal supersymmetry transformation on the chiral superfield

\[ \Phi \rightarrow \Phi + \delta \Phi \]

where

\[ \delta \Phi = i (\xi Q + \bar{\xi} \bar{Q}) \Phi \]

Thus, this leads to

\[ \delta \Phi = \xi^\alpha \left( \frac{\partial}{\partial \theta^\alpha} + i \sigma^\mu_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \partial_\mu \right) \Phi + \bar{\xi}^{\dot{\alpha}} \left( \frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} + i \theta^\alpha \sigma_{\alpha\dot{\alpha}} \mu \partial_\mu \right) \Phi \]

\[ = \sqrt{2} \xi \chi + 2 \xi \theta F - 2 i \theta \sigma^\mu \bar{\theta} \partial_\mu A + \frac{i}{\sqrt{2}} \theta^2 \partial_\mu (\chi \sigma^\mu \bar{\xi}) + \ldots \]

\[ = \delta A + \sqrt{2} \theta \delta \chi + \theta^2 \delta F + \ldots \]  

which implies

\[ \delta A = \sqrt{2} \xi \chi \]

\[ \delta \chi = \sqrt{2} \xi F - i \sqrt{2} \sigma^\mu \bar{\theta} \partial_\mu A \]

\[ \delta F = \sqrt{2} \partial_\mu \chi \sigma^\mu \bar{\xi} \]  

In order to find an object \( \mathcal{L}(\Phi) \) such that \( \delta \mathcal{L} \) is a total derivative under SUSY transformation, we can exploit that

- For a general scalar superfield \( S = \ldots + \theta^2 \bar{\theta}^2 D(x) \), the \( D \)-term transforms as a total derivative.
• For a chiral superfield $\Phi = \ldots + \theta^2 F(x)$, the $F$-term transforms as a total-derivative (see (5)).

That means, the most general Lagrangian for a chiral superfield can be written as

$$\mathcal{L} = K(\Phi^\dagger, \Phi) \bigg|_D + (W(\Phi)|_F + h.c.) .$$

The function $K$ is known as the Kähler potential. This is a real function of $\Phi$ and $\Phi^\dagger$. $W(\Phi)$ is known as the superpotential, and it is a holomorphic function of the chiral superfield $\Phi$. For example, to the Wess Zumino model Lagrangian,

$$\mathcal{L} = \partial_\mu A^\dagger \partial^\mu A + i\chi\sigma^\mu \partial_\mu \bar{\chi} + F^\dagger F ,$$

(6)

corresponds the Kähler potential

$$K = \Phi^\dagger \Phi .$$

3. Spinors in five dimensions.

Without loss of generality, a massless fermion field can be defined as the solution of the equation of motion for the Dirac Lagrangian

$$\mathcal{L} = i\bar{\Psi} \Gamma^M \partial_M \Psi ,$$

where $M = 0, \ldots, 3, 5$, Gamma matrices ($\Gamma^M$) satisfy the Clifford algebra relation

$$\{\Gamma^M, \Gamma^N\} = 2\eta^{MN}\mathbb{I}$$

where $\mathbb{I}$ is the identity matrix and $\eta^{MN}$ is the Minkowski metric. In this work we use the convention $\eta^{MN} = \text{diag}(1, -1, \ldots, -1)$.

In five dimension we use the Weyl basis

$$\Gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \quad \Gamma^5 = i \begin{pmatrix} -\mathbb{I} & 0 \\ 0 & \mathbb{I} \end{pmatrix} .$$

(7)

Therefore, a fermion on five space-time dimensions is necessarily a four component spinor, nevertheless for our proposes we can always arrange it in terms of two two-component spinors as

$$\Psi = \begin{pmatrix} \psi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix} .$$

Under this decomposition, the corresponding 5D Dirac Lagrangian can be rewritten as

$$\mathcal{L} = i\chi\sigma^\mu \partial_\mu \bar{\chi} + i\psi\sigma^\mu \partial_\mu \bar{\psi} - \psi\partial_5 \bar{\chi} + \bar{\chi}\partial_5 \bar{\psi} + \text{total derivatives} .$$

(8)

4. Superfield formalism for a 5D SUSY model.

In order to proceed with the development of our formulation, lets us consider the general 5D action

$$S = \int d^5x \left( \partial_M A^\dagger \partial^M A + \partial_M B^\dagger \partial^M B + i\bar{\Psi} \Gamma^M \partial_M \Psi + P^\dagger P + Q^\dagger Q \right) ,$$

(9)

where $A$ and $B$ are complex scalar fields; $\Psi$ is given as above, and $P$ and $Q$ are general auxiliary fields. This is a straightforward generalization of the action in Eq. (6) which contains twice the number of degrees of freedom. The goal next would be to conveniently rewrite the above action in terms of a 4D-like superfield formalism which would explicitly exhibit the degrees of freedom.
that are relevant at the 4D effective level of the theory (upon compactification). This formalism, as we will see later, will also allow us for a straightforward introduction of Lorentz violation 5D terms on the theory. Notice that this is actually possible, since the 5D construction turns out to be a \(N = 2\) SUSY model, for which the hyperepresentation can always be decomposed (at the level of 4D) as made out of a couple of \(N = 1\) chiral fields. As a matter of fact, by using the 5D fermion represented in terms of two Weyl spinor we introduced in previous section, it is easy to see that for a SUSY transformation each Weyl spinor would require its own complex scalar field and its own auxiliary field. Therefore, the field content so far would be right what it is required to have two chiral superfields.

For our proposes, let us define the required chiral superfields as

\[
\begin{align*}
\Phi &= \Phi (A(x^M), \psi(x^M), F(x^M)) , \\
\Omega &= \Omega (B(x^M), \chi(x^M), G(x^M)) ,
\end{align*}
\]

where to keep things as general as possible we named the corresponding auxiliary fields as \(F\) and \(G\). As it is clear, with these superfields one could straightforwardly write down two Wess-Zumino like actions, as

\[
S' = \int d^5x \left( \Phi^\dagger \Phi|_D + \Omega^\dagger \Omega|_D \right) ,
\]

nonetheless, these terms do not reproduce the total action as given in Eq. (9) As a matter of fact, the difference among both expressions gives

\[
S - S' = \int d^5x \left( \partial_5 A^\dagger \partial^5 A + \partial_5 B^\dagger \partial^5 B + i \bar{\Psi} \Gamma^5 \partial_5 \Psi + P^\dagger P + Q^\dagger Q - F^\dagger F - G^\dagger G \right) .
\]

(10)

Notice that last essentially contains all terms with fifth derivatives. Thus, the simple Wess-Zumino-like action misses the actual terms depicting the fact that the construction lives in five dimension, and, furthermore, it implicitly breaks 5D Lorentz invariance, although, of course, it maintains 4D Lorentz symmetry. However, considering the following interaction-like term that involves both superfields

\[
S_{int} = \int d^5x \left( \Phi \partial_5 \Omega|_F + h.c. \right)
\]

where

\[
\begin{align*}
\Phi \partial_5 \Omega & \supset \theta^2 \left( F \partial_5 B - G \partial_5 A - \psi \partial_5 \chi \right) , \\
\Phi^\dagger \partial_5 \Omega^\dagger & \supset \theta^2 \left( F^\dagger \partial_5 B^\dagger - G^\dagger \partial_5 A^\dagger + \bar{\chi} \partial_5 \bar{\psi} \right) ,
\end{align*}
\]

one could require that \(S_{int} = S - S'\), which leads to the relationship

\[
\partial_5 A^\dagger \partial^5 A + \partial_5 B^\dagger \partial^5 B + P^\dagger P + Q^\dagger Q - F^\dagger F - G^\dagger G = F \partial_5 B - G \partial_5 A + F^\dagger \partial_5 B^\dagger - G^\dagger \partial_5 A^\dagger .
\]

Last expression is easily satisfied, provided that one takes

\[
\begin{align*}
F &= P - \partial_5 B^\dagger \\
G &= Q + \partial_5 A^\dagger .
\end{align*}
\]

(12)

Therefore, the action (9) can be rewritten in terms of superfields considering two Kähler potentials, one superpotential and the redefinition of the auxiliary fields, such that \(S = S' + S_{int}\). Here, the fifth derivative components get explicitly exposed, which would be particularly useful in what follows.
5. 5D Lorentz violation terms.
Here we are interested only in those modifications to the above theory which explicitly break 5D Lorentz symmetry, but leaving 4D Lorenz invariance intact. This problem was already considered in general in a previous work, where those terms were identified (see Ref. [7]). Here we will focus on presenting the deformations that we use to incorporate 5D Lorentz violation to the scheme.

Key observation for our goal is that by introducing a single parameter deformation change in the expressions given by Eqs. (12), and in the superpotential (11), such that,

\[
\begin{align*}
F' &= P - a \partial_5 B^\dagger \\
G' &= Q + a \partial_5 A^\dagger \\
S'_{int} &= a \int d^5x \left( \Phi \partial_5 |F| + h.c. \right)
\end{align*}
\]

then, the action we obtain from \( S'' = S' + S'_{int} \) becomes

\[
S'' = \int d^5x \left( \partial_\mu A^\dagger \partial^\mu A + \partial_\mu B^\dagger \partial^\mu B + i \bar{\Psi} \Gamma^\mu \partial_\mu \Psi + P^\dagger P + Q^\dagger Q \right) \\
+ \int d^5x \left( a^2 \partial_5 A^\dagger \partial^5 A + a^2 \partial_5 B^\dagger \partial^5 B + i a \bar{\Psi} \Gamma^5 \partial_5 \Psi \right).
\]

(13)

(14)

This expression can be rewritten, by taking the reparameterization \( a = 1 + \kappa \), as

\[
S'' = \int d^5x \left( \partial_\mu A^\dagger \partial^\mu A + \partial_\mu B^\dagger \partial^\mu B + i \bar{\Psi} \Gamma^\mu \partial_\mu \Psi + P^\dagger P + Q^\dagger Q \right) \\
+ \int d^5x \left( (2 \kappa + \kappa^2) \left( \partial_5 A^\dagger \partial^5 A + \partial_5 B^\dagger \partial^5 B \right) + i \kappa \bar{\Psi} \Gamma^5 \partial_5 \Psi \right),
\]

(15)

which is the very same action discussed in our previous work [7], which was found to contain all possible terms that violate 5D Lorentz symmetry, but remained invariant under 4D Lorentz transformations and under Supersymmetry, just as desired. We should stress that this procedure of reconstructing the last action neatly suggest that 5D Lorentz violation can be expressed as a simple deformation of the fifth derivatives. This will have immediate implications at the level of the effective 4D theory, in particular on the KK mass spectrum.

6. Parameter interpretation.
To have a more clear physical idea of the effects, and the meaning of the Lorentz violation parameter we have introduced above, let us consider the aftercome of the compactification process on the theory. As an example, considering only the Lagrangian of the scalar like field,

\[
L = \partial_M A^\dagger \partial^M A + \omega^2 \partial_5 A^\dagger \partial^5 A = -A^\dagger \left( \partial_M \partial^M + \omega^2 \partial_5 \partial^5 \right) A.
\]

(16)

A solution of the equation of motion for the \( A \) field (on compact space) can be in general separated as

\[
A(x^\mu, y) = \sum_n \phi_n(x^\mu) \zeta_n(x^5),
\]

where \( \phi \) and \( \zeta \) correspond to the four and fifth wave function components, respectively. Thus, to open the mode spectrum, one can always require that

\[
\partial_\mu \partial^\mu \phi_n = -m_n^2 \phi_n,
\]

which leads to

\[
\zeta_n(x^5) = C_1 \cos \left( \alpha_n x^5 \right) + C_2 \sin \left( \alpha_n x^5 \right)
\]

6
where $\alpha_n = \left( m_n / \sqrt{1+\omega^2} \right)$. Assuming that fifth dimension has being compacted into a circle of radius $2\pi R$, last expression implies that eigen masses are given as

$$m_n = \left( \frac{n}{R} \right) \sqrt{1+\omega^2} .$$  \hspace{1cm} (17)

That means, the Lorentz violation parameter corresponds to a shift on the mass for each Kaluza-Klein mode, except for the zero mode.

Interestingly, if one rather suppose to have a usual Lorentz invariant theory (that is where $\omega = 0$), but instead consider a compactification on a circle with a radius $2\pi R'$, it is known that KK mass would simply be

$$m_n = \frac{n}{R'} .$$ \hspace{1cm} (18)

A comparison between Eq. (17) and Eq. (18) straightforwardly shows that a simple redefinition of the former compactification radius, such that

$$R = R' \sqrt{1+\omega^2},$$

will give the same spectrum as that of 5D Lorentz violating theory. Therefore, at least at this level, it is clear that a non 5D Lorentz invariant theory can be seen, at the effective level, as derived from a totally Lorentz invariant one, just by a simple redefinition on the compactification radius.

7. Conclusions

In this work we have presented a superfield formulation for a five dimensional SUSY and Lorentz invariant theory, which clearly shows how 5D Lorentz violation can be incorporated in a consistent way, respecting supersymmetry, through a simple parameter deformation on the fifth derivative involved in a superpotential-like term, and, in simple field redefinition for the SUSY auxiliary fields. The construction explicitly requires two chiral superfields, which means the possibility of studying, under certain conditions, the rupture of $N = 1$ to $N = 2$ supersymmetry. This is yet a problem we are still working on. Furthermore, we presented how the Lorentz violation terms can be hidden in a redefinition of the compactification radius of the original theory, making the effective theory equivalent to that obtained from a rather Lorentz invariant formulation. Nevertheless, it is important to remark that this redefinition do depend on the particular Lorentz violation parameter, and thus, only in the case where this were universal for all involved fields the effect of 5D Lorentz breaking could be scaled away. Otherwise, asymmetries on the KK mass gaps for different fields would be expected.

References.

[1] http://cms.web.cern.ch/org/cms-papers-and-results
[2] S. P. Martin, In *Kane, G.L. (ed.): Perspectives on supersymmetry II* 1-153 [hep-ph/9709356].
[3] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B 429 (1998) 263 [hep-ph/9803315].
[4] A. Perez-Lorenzana, AIP Conf. Proc. 562, 53 (2001) [arXiv:hep-ph/0008333].
[5] D. Bailin and A. Love, Bristol, UK: IOP (1994) 322 p. (Graduate student series in physics)
[6] A. Salam and J. A. Strathdee, Phys. Rev. D 11, 1521 (1975).
[7] J. D. Garcia-Aguilar, A. Perez-Lorenzana and O. Pedraza-Ortega, AIP Conf. Proc. 1396, 140 (2011).