Design of PD-type second-order ILC law for PMSM servo position control

Saleem Riaz1,2, Hui Lin1, Muhammad Bilal Anwar1 and Haider Alii

1School of Automation, Northwestern Polytechnical University, Xi'an, Shaanxi, P.R. China
2E-mail: saleemriaznwp@gmail.nwpu.edu.cn

Abstract. An iterative learning control method is better to control the high-order nonlinear strong coupling and external disturbances in the permanent magnet synchronous motor position servo system. This paper proposes a second-order P.D. type iterative learning control strategy, which can effectively achieve the optimal tracking control algorithm. By using the generalization of the Young inequality of convolution, the Lebesgue-p norm is obtained under the adequate condition that the tracking error converges monotonously. Furthermore, the convergence rate of second-order iterative learning control is compared, and it is proved by the mathematical knowledge that second-order iterative learning control is much better than a first-order iterative learning control under satisfactory conditions. Simulation results show that the effectiveness of the proposed algorithm is better than that of the traditional method, and the error of the iterative learning control strategy is smaller but with higher accuracy.

1. Introduction
As a Brushless motor, permanent magnet synchronous motor (PMSM) is widely used in aerospace, robotics, transportation, and other high-precision position servo systems due to its excessive advantages such as high efficiency, power density, and torque inertia ratio [1].

In recent years, with the increasing demand for the system performance, how to improve the accuracy of all-digital fuzzy position servo system has become a research hotspot of the traditional position servo control methods. Usually, a PID controller is used, but the PID controller itself is their control object, and it is difficult to adjust the parameters of the controller, and it is difficult to achieve satisfactory results in solving the problem of high-precision tracking [2-7].

Therefore, domestic and international numerous scholars try to apply advanced control theories and methods to improve the all-digital fuzzy position servo system using sliding mode variable structure control and precision. In this literature, an improved algorithm, the non-singular scheme of fast terminal sliding mode controller (SMC), is discussed. The adaptive law is used to estimate the uncertainty, which reduces the reaction of the SMC, but is unable to resolve the chattering problem of the sliding mode variable structure control to achieve high precision tracking [8,9].

In this literature, the backstepping control and fuzzy control are combined and applied to the servo system. The adaptive method is proposed to estimate unknown parameters of the system and improve dynamic behavior [10-13].

In reference [13], the backstepping controller of the adaptive control observer is designed for the uncertainty in the PMSM servo system, which effectively improves the precision of the control system.
However, the backstepping controller is complicated in design and requires a large amount of computation, so it is difficult to apply in practice.

In the literature [14], a robust variance controller is designed based on disturbance observer, which can ensure the dynamic performance of the system by regional pole assignment, therefore improved tracking precision achieved (ZuoFei). A kind of improved active disturbance rejection controller (ADRC) is proposed, the model compensation algorithm is combined with the extended state observer to reduce the system modeling error, and improve the system steady-state accuracy and robustness against load disturbance.

As a model-free control, iterative learning control (ILC) is simple in structure and does not require specific model parameters. Only after enough iterations can the behavior of the executed object meet the standard requirements. Because of the above attributes, this learning algorithm has been favored by many scholars in recent years [15].

At present, there are few kinds of research on iterative learning control of high precision PMSM position servo system. Such as this literature proposed an iterative learning control based on reference input, which enhanced the PMSM position servo system by learning the reference input [16].

In the literature [17], the prediction method to iterative learning control is applied, a robust iterative learning control strategy is proposed, and weight function criteria are chosen for the robustness and to improve the accuracy of the system.

In literature [18], iterative learning control algorithm and SVM modulation technology are applied to the PMSM DTC system, which improved the stable accuracy of speed and torque and the robustness of the control system. The above literature all designed iterative learning control for the position servo system, but they all lacked strict mathematical proof.

In literature [19] ILC algorithm, second-order differential of system error is proposed for the PMSM position servo system, which solves the problem that the traditional ILC law is not valid due to the particularity of the model in the position servo system. Its convergence is strictly proved by mathematical knowledge, and convergence conditions are obtained.

In this paper, the second-order PD-ILC is proposed, which realizes the fast and high-precision tracking control through the configuration of proportion and integral. Also, the convergence rate of second-order iterative learning control is compared, and the results prove that second-order iterative learning control is much better than a first-order iterative learning control. Finally, simulation analysis is carried out to prove the rationality and sufficiency of the algorithm.

2. Problem description and analysis
Consider the following PMSM position servo system model:

\[
\begin{align*}
\frac{d\theta(t)}{dt} &= \omega(t) \\
\frac{d\omega(t)}{dt} &= \frac{1}{J} T_e(t) - \frac{1}{J} T_l(t) - \frac{B_f}{J} \omega(t)
\end{align*}
\]

where \( T_e \) : the electromagnetic torque; \( T_l \) : the load torque; \( \theta(t) \) : motor mechanical angle; \( B_f \) : coefficient of viscous friction; \( \omega(t) \) : mechanical angular velocity; \( J \) : the moment of inertia of the system.

Also, the above equation shows that the control variable is \( T_e \). Equation (1) can be described in the form of state space as follows:

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t)
\end{align*}
\]
where \( x = [\theta(t), \omega(t)]^T \) denotes the system state variable, and \( u = T_i(t) = k_i i_q(t) \) is the control input and, each matrix of the system is as follows:

\[
A = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{B_f}{J} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1/J \end{bmatrix}, \quad C = [1 \ 0]
\]

(3)

The typical position servo control model for the PMSM block diagram with \( i_d = 0 \) the control strategy is shown in figure 1. The given Position \( \theta' \) is compared with the measured value \( \theta \) that can be obtained through the position sensor. Similarly, \( i_q' \) the motor axis current, then \( i_q' \) the current sensor measured current value is transformed to get the actual motor shaft current value \( i_q \). Based on the existing control system and voltage controller, the control mode of the three-phase inverter is further changed, that is, the drive motor.

3. Description of P.D. second-order ILC Control law

The typical ILC control diagram combined with the mathematical knowledge is shown in figure 2. Taking the typical ILC control representation, the controller state equations can be described as follows:

\[
\begin{align*}
\dot{x}_{k+1}(t) &= Ax_{k+1}(t) + Bu_{k+1}(t) \\
y_{k+1}(t) &= Cx_{k+1}(t) \\
x_{k+1}(0) &= 0, t \in [0, T]
\end{align*}
\]

(4)

The desired output trajectory of the system is \( y_d', t \in [0, T] \), also the second-order PD-ILC law is constructed \( L(c_1, c_2) \) as follows:
\[ u_2(t) = u_1(t) + \Gamma_p e_1(t) + \Gamma_d \dot{e}_1(t) \]  
\[ u_{k+1}(t) = c_1 \left[ (u_k(t) + \Gamma_p e_k(t) + \Gamma_d \dot{e}_k(t)) \right] + c_2 \left[ u_{k-1}(t) + \Gamma_p e_{k-1}(t) + \Gamma_d \dot{e}_{k-1}(t) \right] \]  
\[ t \in [0, T_0] \quad k = 2, 3, 4, \ldots \]

where: \( k \) denotes the iteration number, \( \Gamma_{p1}, \Gamma_{p0} \) respectively, for the first order, Second-order \( \Gamma_{d1}, \Gamma_{d0} \) of first and second-order differential, respectively gain; \( c_1, c_2 \) is the weighted plane for iterative learning components for first and second-order, respectively.

\[ e_k = y_d(t) - y_k(t) \]

Due to \( x(0) = 0 \), can be \( e_k(0) = 0, k = 1, 2, 3, \ldots \). Combining equation 1.4, 1.5, and 1.6 with \( L(e_1, c_1) \) The system tracking error under the action in the time domain is:

\[ e_{k+1} = \tilde{y}_d(t) - \tilde{y}_{k+1}(t) \]

\[ = c_1 \{ y_{d(t)} - y_{k(t)} \} + c_2 \{ y_{d(t)} - y_{k(t)} \} - \{ y_{k+1}(t) - c_1(y_k(t)) - c_2(y_{k-1}) \} \]

\[ = c_1 e_1(t) + c_2 e_{k-1}(t) - \mathbf{C} \int_0^t \exp((\sqrt{A(t-\tau)})^2) \times \mathbf{B} \left[ c_1 \Gamma_p e_1(\tau) + c_2 \Gamma_p e_{k-1}(\tau) \right] d\tau \]
\[ CB \int_0^t \exp((\sqrt{\frac{A(t-\tau)}{2}})^2)[c_1 \Gamma_{di} e_k(\tau) + c_2 \Gamma_{do} e_{k-1}(\tau)] \]

Parts integrate the last term on both sides of the right of the above equation:

\[ e_{k+1}(t) = c_1 e_k(t) + c_2 e_{k-1}(t) - C \int_0^t \exp((\sqrt{\frac{A(t-\tau)}{2}})^2)B \left[ c_1 \Gamma_{pl} e_k(\tau) + c_2 \Gamma_{p0} e_{k-1}(\tau) \right] d\tau \]

Because of \( e_k(0) = 0 \), \( y_j - y_k = 0 \) at \( t = 0 \), then the above formula can be reduced to:

\[ e_{k+1}(t) = c_1 \left( 1 - CB \Gamma_{di} \right) e_k(t) + c_2 \left( 1 - CB \Gamma_{do} \right) \times \]

\[ e_{k-1}(t) - c_1 C \int_0^t \exp((\sqrt{\frac{A(t-\tau)}{2}})^2) \left( B \Gamma_{pl} + AB \Gamma_{d1} \right) e_k(\tau) d\tau - c_2 C \int_0^t \exp(A \cdot (t-\tau)) \times \left( B \Gamma_{p0} + AB \Gamma_{d0} \right) e_{k-1}(\tau) d\tau \]

Lebesgue-p norm is taken from both sides of the above equation and should be using the generalized Young inequality, and it could obtain that:

\[ \| e_{k+1}(\cdot) \|_p \leq c_1 \left\{ 1 - CB \Gamma_{di} \right\} + \| \exp[A(\cdot)] \left( B \Gamma_{pl} + AB \Gamma_{d1} \right) \| \| e_k(\cdot) \|_p \]

\[ \| e_{k-1}(\cdot) \|_p \leq \left\{ 1 - CB \Gamma_{d0} \right\} + \| \exp[A(\cdot)] \left( B \Gamma_{p0} + AB \Gamma_{d0} \right) \| \| e_{k-1}(\cdot) \|_p \]

Results

\[ \rho_1 = \left\{ 1 - CB \Gamma_{di} \right\} + \| \exp[A(\cdot)] \left( B \Gamma_{pl} + AB \Gamma_{d1} \right) \| \]

\[ \rho_2 = \left\{ 1 - CB \Gamma_{d0} \right\} + \| \exp[A(\cdot)] \left( B \Gamma_{p0} + AB \Gamma_{d0} \right) \| \]

We can obtain

\[ \| e_{k+1}(\cdot) \|_p \leq c_1 \rho_1 \| e_k(\cdot) \|_p + c_2 \rho_2 \| e_{k-1}(\cdot) \|_p \]

(7)

When \( \lim_{k \to \infty} e_k(t) = 0 \) system convergence, so the sufficient condition of convergence is as follows: \( \rho_1 < 1, \rho_2 < 1 \).

4. First-order P.D. iterative learning strategy

In equation (6), when \( c_1 = 1 \), the control law is reduced to first-order P.D. type ILC rule \( L(1) \) as...
follows:

\[ L(t) : u_{k+1}(t) = u_k(t) + \Gamma_{pl} e_k(t) + \Gamma_{di} \dot{e}_k(t) \quad \text{for} \quad t \in [0, T_0), \quad k = 1, 2, 3, \ldots \]

The above-mentioned equation \( \Gamma_{pl}, \Gamma_{di} \) is the same, namely, \( k(t) \) is a type of the first-order component in equation 1.8 \( u_k(t) + \Gamma_{pl} e_k(t) + \Gamma_{di} \dot{e}_k(t) \). Combining equation (4) and (8) results in the error that calculated under the action of \( L(1) \) obtained as:

\[
\begin{align*}
\varepsilon_{k+1}(t) &= \tilde{y}_d(t) - \tilde{y}_{k+1}(t) \\
&= \left[ \sqrt{(y_d(t) - y_k(t))^2} \right] - \left[ \sqrt{(y_{k+1}(t) - y_k(t))^2} \right] \\
&= \varepsilon_k(t) - \left[ \sqrt{(y_{k+1}(t) - y_k(t))^2} \right]
\end{align*}
\]

\[
= \varepsilon_k(t) - C \int_0^t \exp((\sqrt{A(t-\tau)}))^2) B \Gamma_{pl} e_k(\tau) d\tau - \\
C \int_0^t \exp((\sqrt{A(t-\tau)}))^2) B \Gamma_{di} d[e_k(\tau)] \\
= \tilde{e}_k(t) - C \int_0^t \exp((\sqrt{A(t-\tau)}))^2) B \Gamma_{pl} e_k(t) d\tau - \\
C \int_0^t \exp((\sqrt{A(t-\tau)}))^2) B \Gamma_{di} e_k(t) d\tau \bigg|_{t=0}^{t=\infty}
\]

Because of \( e_k(0) = 0 \), \( y_d - y_k = 0 \) at \( t = 0 \), then the above equation can be simplified as follows:

\[
e_{k+1}(t) = (1 - CB \Gamma_{di}) e_k(t) - C \left[ \int_0^t \exp((\sqrt{A(t-\tau)}))^2) \right] \times \\
\left( B \Gamma_{pl} + A B \Gamma_{di} \right) e_k(t) d\tau
\]

Now taking Lebesgue-p norm on both sides should result in generalized Young's inequality:

\[
\|e_k(t)\|_p \leq 1 - CB \Gamma_{di} \|e_k(t)\|_p + ||C \exp[A \cdot \cdot \cdot] \times \\
\left( B \Gamma_{pl} + A B \Gamma_{di} \right) \|1 \| e_k(t) \|_p
\]

In conclusion or hence:

\[
\|e_k(t)\|_p \leq \left( 1 - CB \Gamma_{di} \right) + ||C \exp[A \cdot \cdot \cdot] \times \\
\left( B \Gamma_{pl} + A B \Gamma_{di} \right) \|1 \| e_k(t) \|_p
\]

Hence

\[
\rho_1 = ||1 - CB \Gamma_{di} \| + ||C \exp[A \cdot \cdot \cdot] \times \\
\left( B \Gamma_{pl} + A B \Gamma_{di} \right) \|1 \| e_k(t) \|_p
\]

You can get:

\[
\|e_k(t)\|_p \leq \rho_1 \|e_k(t)\|_p \quad k = 1, 2, 3, \ldots
\]
When \( \lim_{k \to \infty} e_k(t) = 0 \) the system converges, so the adequate condition for the system to converge is \( \rho_1 < 1 \).

5. Simulation results and analysis

To verify the reasonability and effectiveness of the algorithm proposed in this paper, MATLAB / Simulink was used to build a control system simulation platform for simulation analysis, in which rotational inertia \( J = 0.004 \text{kg} \cdot \text{m}^2 \) and viscosity friction coefficients \( B_f = 0.0001 \text{Nm} / \text{rad} / \text{s} \) used to set the expected reference trajectory of the system as \( y_d = 50 \sin(2\pi t) \) shown in figure 3 below. The parameters for ILC is taken as \( \epsilon_1 = c_2 = 0.5, \Gamma_p = 0.8, \Gamma_d = 0.01, \Gamma_p = 0.3, \Gamma_d = 0.006 \).

![Figure 3. desired reference position trajectory.](image)

Controller effort can be seen in the following figure that defines the system output tries to follow the desired given Position. The simulation results and analysis are shown in Figure 4.

![Figure 4. Output for different iterations versus reference.](image)
Figure 4 shows the control effect of the different iteration of the system. As we can see, the second iteration results are not better and have large errors initially, and the delay is relatively apparent, and the error is significant. The ILC control action further decreases the error and tries to reach its limit. After a little amount of time and some iterations, the error converges quickly to its limit, and the output of the system follows precisely the desired Position. Moreover, after debugging the controller parameters many times, these situations still exist. The desired and the output position of the system is shown and easily view that output concisely following the desired value, as shown in figure 5. The controller action is robust and enough adequate conditions to converge the error to its monotone convergence limit. In the second iteration of the physical system, the tracking curve of the system is shown in figure 4, and the error curve shows that the error is too large. The 4th iteration results are shown in figure 4. Error trajectory of the system has been significantly reduced but compared with the second trial, the most critical error in the tenth iteration, the results are shown in the same figure 4, has very high tracking precision. The error is very small to meet the system requirements. Finally, error convergence is shown in figure 6. The norm of the error illustrated and seen that goes to zero as the number of iteration increases.

![Figure 5](image1.png)  
**Figure 5.** Measured Position versus reference.

![Figure 6](image2.png)  
**Figure 6.** System error convergence.
The sufficient conditions and the Lebesgue-p norm of the error show that the results are satisfactory, and the system is robust to track the Position of the PMSM servo control. This method also can be applied to other dynamical speed and position servo systems for the wide variety of typical automation applications with some extra addition.

6. Conclusion
A second-order iterative learning control algorithm based on tracking error is designed for high precision position servo system of permanent magnet synchronous motor. To solve the traditional iterative learning algorithm requires full rank problem, the system tracking accuracy is significantly improved d-type iterative learning by multiple iterative learning control, and its convergence is discussed. The convergence speed of error for permanent magnet synchronous motor (PMSM) position servo system in the sense of \( L(c_1, c_2) \) is studied. According to the current research focus, the Lebesgue-p norm and the influence of different proportions and differential gains on the convergence speed are analyzed. It is proved that the second-order iterative learning control has more freedom and better robustness in gain selection. Simulation results show that the proposed control algorithm has high tracking precision and satisfies the system requirements. Robustness and more efficient controller design is the additional effort to modify for other new automation applications.

References
[1] S Chai, L Wang and E Rogers 2011 Model Predictive Control of a Permanent Magnet Synchronous Motor Conference of the IEEE Industrial Electronics Society
[2] S Ding, Z Cui, Q Wu, X Chang, Q Hu etc. 2014 Simulation study of vector control of permanent magnetic synchronous motor based on SVPWM Forgn Electronic Measurement Technology
[3] G L Demidova, D V Lukichev and K Denisov 2019 Implementation of Type-2 Fuzzy Control of PMSM Position Drive with Flexible Coupling 2019 6th International Conference on Control, Decision and Information Technologies (CoDIT)
[4] K Wang and M Yang 2019 Permanent magnet synchronous motor vector control based on improved sliding mode observer Electrical Engineering
[5] K Kysljan, V Slapak, F Durovsky, V Fedák and P Sanjeevikumar 2018 Feedforward Finite Control Set Model Predictive Position Control of PMSM 18th International Conference on Power Electronics and Motion Control PEMC 2018
[6] Q Xu, C Zhang, L Zhang and C Wang 2014 Multiobjective Optimization of PID Controller of PMSM Journal of Control ence & Engineering 2014 381-386
[7] V Kumar, P Gaur and A P Mittal 2014 ANN based self tuned PID like adaptive controller design for high performance PMSM position control Expert Systems with Applications 41 (17) 7995-8002
[8] D J Xu 2015 Based on The Direct Torque Control of Permanent Magnet Synchronous Motor Disturbances Rejection Journal of Hefei University (Natural Sciences Edition)
[9] Y Z Lei, Y P Xu and Q Zhou 2015 Direct Torque Control of Permanent Magnet Synchronous Motor Based on Improved Model Predictive Control Electric Drive
[10] B Si, S H Han and O M Alkhouli 2014 Motor control system implementing field weakening
[11] H Liu and S Li 2010 The application of predictive functional control for permanent magnet synchronous motor servo system IEEE/ASME International Conference on Mechatronics & Embedded Systems & Applications
[12] J Yu, B Chen, H Yu and J Gao 2011 Adaptive fuzzy tracking control for the chaotic permanent magnet synchronous motor drive system via backstepping Nonlinear Analysis Real World Applications 12 (1) 671-681
[13] R H Du, Y F Wu, W Chen and Q W Chen 2012 Backstepping Adaptive Fuzzy Control for
Permanent Magnet Synchronous Motor Servo Systems Advanced Materials Research 591-593 1483-1489

[14] W U Chun and Q I Rong 2013 Mixed robust variance control design of PMSM servo system Dianji Yu Kongzhi Xuebao/electric Machines & Control

[15] Yue-fei Zuo, Jie Zhang, Chuang Liu, and Tao Zhang 2017 Modified AdRC for time-varying input of permanent synchronous magnet motor Journal of Electromechanics no. 2

[16] L I Bingqiang, W U Chun and H Lin 2012 A High-precision Position Servo System of Permanent Magnet Synchronous Motors With Reference Input Iterative Learning Proceedings of the Csee 32 (3) 96-102

[17] Y B Sun, F Yan and C F Liu 2009 Robust Repetitive Control for Permanent-magnet Linear Synchronous Motor Based on μ Theory Proceedings of the Csee 29 (30) 52-57

[18] Z Zhang, J Zhao, L Zhou, X Yan and A N Bainan 2017 PMSM DTC based on ILC algorithm and SVM modulation Power System Protection & Control 45 (19) 63-70

[19] X Ruan and J Y Zhao 2012 Pulse compensated iterative learning control to nonlinear systems with initial state uncertainty Control Theory & Applications 29 (8)