Holographic compactifications of (1,0) theories from massive IIA supergravity

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We describe three analytic classes of infinitely many AdS4 BPS solutions of massive IIA supergravity, for $d = 7, 5, 4$. The three classes are related by simple universal maps. For example, the AdS7 × M3 solutions (where $M_3$ is topologically $S^3$) are mapped to AdS5 × $\Sigma_2$ × M3, where $\Sigma_2$ is a Riemann surface of genus $g \geq 2$ and the metric on $M_3$ is obtained by distorting $M_3$ in a certain way. The solutions can have localized D6 or O6 sources, as well as an arbitrary number of D8-branes. The AdS7 case (previously known only numerically) is conjecturally dual to an NS5–D6–D8 system. The field theories in three and four dimensions are not known, but their number of degrees of freedom can be computed at leading order. The AdS4 solutions have numerical “attractor” generalizations that might be useful for flux compactification purposes.

Recently, AdS7 solutions in type II theories were classified [1]. In presence of the so-called Romans mass parameter $F_0$, infinitely many new solutions were obtained numerically. These were later argued [2] to be near-horizon limits of NS5-D6-D8 brane interactions, considered long ago in [3,4]. This in turn gives some information about the holographically dual (1,0)-superconformal theories (SCFTs) in six dimensions.

The more supersymmetric (2,0) SCFT living on coincident M5s can be compactified to obtain interesting SCFTs in four and three dimensions. This can be demonstrated holographically by replacing AdS7 with either AdS5 × $\Sigma_2$ [5] or AdS4 × $\Sigma_3$ [6,7], where $\Sigma_2$ is a Riemann surface and $\Sigma_3$ is a maximally symmetric space.

It is natural to wonder whether the (1,0) SCFTs described above can also be compactified in this fashion. In recent work [8,9] we found that this can indeed be done. In the process of doing so, we were able to find analytic expressions for the AdS7 solutions of [1] themselves, and analytic maps $\psi_5, \psi_4$ from those to the AdS5 × $\Sigma_2$ and AdS4 × $\Sigma_3$ solutions. These maps are invertible and they can of course be composed:

\[
\begin{align*}
\text{AdS}_7 & \quad \xrightarrow{\psi_5} \quad \text{AdS}_5 \times \Sigma_2 \\
\text{AdS}_4 \times \Sigma_3 & \quad \xrightarrow{\psi_4} \quad \text{AdS}_5 \times \Sigma_2 .
\end{align*}
\]

So in the end we have three new classes of infinitely many new backgrounds with analytic expressions, holographically dual to SCFTs in six, four, and three dimensions, with respectively eight, four, and two $Q$-supercharges. The AdS4 vacua also have potential applications to flux compactifications.

The AdS7 solutions have the following general form. The internal space $M_3$ is an $S^2$-fibration over an interval, parameterized by a coordinate $r \in [r_-, r_+]$: $e^{2A} ds^2_{\text{AdS}_7} + dr^2 + e^{2A} v^2 ds^2_{\Sigma_2}$. $A$ (the “warping”) and $v$ are functions of $r$; so is the dilaton $\phi$. We will see below how these three functions are fixed by the equations of motion and preserved supersymmetry. The $S^2$ has a round metric, and its isometry group is the SU(2) R-symmetry of the (1,0) SCFT5. It shrinks at the endpoints $r_{\pm}$ of the interval. The fluxes now acquire all the components compatible with the R-symmetry:

\[
F_0, \quad F_2 \propto \text{vol}_{S^2}, \quad H \propto dr \wedge \text{vol}_{S^2}.
\]

The map $\psi_4$ takes the metric [2] to

\[
\sqrt{\frac{5}{8}} \left[ \frac{5}{8} e^{2A} \left( 5 ds^2_{\text{AdS}_4} + \frac{4}{5} ds^2_{\Sigma_3} \right) + dr^2 + e^{\frac{2A}{5}} v^2 \frac{ds^2_{S^2}}{1 - 6 v^2} \right] \quad (3)
\]

with $\Sigma_3$ a compact quotient of hyperbolic space, normalized so that its scalar curvature is $-6$. $S^2$ is now fibered over $\Sigma_3$, in a way associated to its tangent bundle; even though the $S^2$ is still round, the total internal space has no isometries. The solution now has four supercharges; it is dual to an $\mathcal{N} = 1$ SCFT4. The fluxes now acquire also components along $\Sigma_3$. The dilaton $\phi_7$ of the AdS7 solutions is taken to $\phi_4$ given by

\[
e^{\phi_4} = \left( \frac{5}{8} \right)^{1/4} e^{\phi_7} \sqrt{1 - 6v^2} .
\]

Similarly, the map $\psi_5$ takes the metric [2] to

\[
\sqrt{\frac{3}{4}} \left[ \frac{3}{4} e^{2A} \left( ds^2_{\text{AdS}_5} + ds^2_{\Sigma_2} \right) + dr^2 + e^{\frac{2A}{5}} v^2 \frac{ds^2_{S^2}}{1 - 4v^2} \right] \quad (5)
\]

with $\Sigma_2$ a Riemann surface, again normalized so that its scalar curvature is $-6$. $S^2$ is fibered over $\Sigma_2$ via one of its U(1) isometries; in other words, it can be written as $\mathbb{P}(\mathcal{K} \oplus \mathcal{O})$, and actually $\mathcal{K}$ is the canonical bundle of $\Sigma_2$. The isometry group is now this U(1), which is the R-symmetry of the $\mathcal{N} = 1$ superalgebra of the SCFT4. Again the fluxes acquire components along $\Sigma_2$. The dila-
ton \(\phi_7\) of the AdS\(_7\) solutions is taken to \(\phi_7\) given by
\[
\phi_7 = \left(\frac{3}{4}\right)^{1/4} \frac{e^{\phi_7}}{\sqrt{1 - 4y^2}}.
\]

So far we have described how the AdS\(_7\) solutions get mapped to AdS\(_4\) and AdS\(_5\) solutions. Remarkably, the AdS\(_5\) system of equations turned out much simpler than the ones in AdS\(_7\) and AdS\(_4\); so much so that we were able to integrate it analytically. (This simplicity ultimately results from a more general classification effort in [8], where existence of AdS\(_5\) solutions is reduced to a set of PDEs; these simplify and become solvable with an inspired Ansatz.) We can then use the maps [3] and [5] to produce AdS\(_7\) and AdS\(_5\) solutions as well. In what follows, we will describe the AdS\(_7\) solutions.

Let us first give the simplest example. The metric can be written as
\[
\frac{n_{\text{D6}}}{F_0} \sqrt{\tilde{y} + 2} \left(\frac{4}{3} ds^2_{\text{AdS}_7} + \frac{1}{4(1 - \tilde{y})(\tilde{y} + 2)} + \frac{1}{3} - \frac{8}{4 - 4\tilde{y}} ds^2_{S^2}\right)
\]
where \(\tilde{y} \in [-2, 1]\). The flux can be found in [8]; the dilaton is given by \(e^{\phi_7} = \frac{2}{\sqrt{\text{AdS}_5}} (\frac{\tilde{y} + 2}{\sqrt{8 - 4\tilde{y}}} - \frac{1}{\sqrt{12}})\). Around \(\tilde{y} = 1\), the internal metric is \(\sim \frac{\tilde{y}^2}{4(\tilde{y} - 1)} + (\tilde{y} - 1) ds^2_{S^2}\), which turns into flat space \(dp^2 + \rho^2 ds^2_{S^2}\) by the change of coordinates \(\tilde{y} = \sqrt{y - 1}\). So \(\tilde{y} = 1\) is a regular point.

On the other hand, around \(\tilde{y} = -2\) the metric behaves as \(\sim 16\sqrt{\text{AdS}_5} + \frac{1}{\sqrt{\beta}}(dp^2 + \rho^2 ds^2_{S^2})\), with \(\rho = \tilde{y} + 2\), which is the correct behavior near a stack of D6-branes wrapping AdS\(_7\).

![FIG. 1. In (a), a sketch of the internal M3 in (f); the cusp represents the single D6 stack. In (b), the brane configuration whose near-horizon should originate (f). The dot represents a stack of N NS5-branes; the horizontal lines represent \(n_{\text{D6}}\) D6-branes ending on them.]

A stack of electric charges of the same sign in a compact space would be in contradiction with Gauss’ law, but in type IIA this reads \(dF_2 - H F_0 = \delta\). Integrating this, we get \(N n_0 = n_{\text{D6}}\), where \(N = \frac{2\pi}{\sqrt{\beta}}\int H\) and \(n_0 = 2\pi F_0\) are integer by flux quantization. The corresponding brane configuration should consist of \(n_{\text{D6}}\) D6-branes ending on \(N\) NS5-branes. Notice that the net number of D6-branes ending on an NS5 is \(n_0\); this again gives \(N n_0 = n_{\text{D6}}\).

More general solutions can be described as follows. Given a solution \(\beta\) to the ODE
\[
\partial_y(q^2) = \frac{2}{9} F_0, \quad q \equiv -\frac{4y\sqrt{\beta}}{\partial_y \beta},
\]
the warping \(A\), dilaton \(\phi\), and volume function \(v\) can be described by
\[
e^A = \frac{2}{3} \left( -\frac{\partial_y \beta}{y} \right)^{1/4}, \quad e^\phi = \frac{(-\partial_y \beta/y)^{5/4}}{12\sqrt{4\beta - y\partial_y \beta}},
\]
\(v^2 = \frac{\beta/4}{4\beta - y\partial_y \beta}\).

Moreover, \(dr = \left(\frac{3}{2}\right)^2 \frac{e^A}{\sqrt{\beta}} dy\).

The local behavior of a solution can be read off from \(\beta\) as follows:

- At a single zero of \(\beta\), the \(S^2\) shrinks in a regular way, so that one has a regular point (as was \(\tilde{y} = 1\) in [7]).
- At a double zero of \(\beta\), there is a stack of D6 branes (as at \(\tilde{y} = -2\) in [7]).
- At a square root point, where \(\beta \sim \beta_0 + \sqrt{y - y_0}\), there is an O6 singularity.

The equation (8) can be easily integrated by writing \(16y^2 \beta^{-3/2} = \frac{1}{3} F_0(y - y_0)\), for \(y_0\) a constant; this can now be integrated by quadrature. For \(F_0 = 0\), the solution can be written as
\[
\beta = c(y^2 - y_0^2)^2.
\]

Using [9], this gives rise to the solution discussed in [11, Sec. 5.1], which is obtained by reducing AdS\(_7\) \(\times S^4\) to IIA. (In that paper \(c = \frac{4}{\sqrt{\beta}}, y_0 = \frac{9}{\sqrt{\beta}} R^4\).) [10] has two double zeros: hence the corresponding solution has two D6 stacks, one at \(y = y_0\) with D6-branes and one at \(y = -y_0\) with an equal number of anti-D6’s. These originate from loci where the reduction from M-theory degenerates.

For \(F_0 \neq 0\), the simplest solution reads
\[
\beta = \frac{8}{F_0}(y - y_0)(y + 2y_0)^2.
\]

This has a single zero in \(y = y_0\) and a double zero in \(-2y_0\); so it has only one stack of D6’s. Flux quantization requires \(\int_{S^2} F_2 = n_{\text{D6}}\), where \(F_2 = F_2 - B F_0\) is the closed RR form; in this case it fixes
\[
y_0 = \frac{-3 n_{\text{D6}}^2}{8 F_0}.
\]

Substituting [11] in (9), defining \(\tilde{y} \equiv \frac{y}{y_0}\), and using [12], we reproduce [11], which indeed has only one D6 stack.

We will now describe more general solutions. The first generalization will introduce either a second D6 stack, or
an O6 singularity. The second generalization will involve D8-branes. These were described numerically in [1, 2], but we will now be able to give analytic expressions. It would also be possible to combine D6, D8, and O6 into even more general solutions.

The first generalization involves finding a more general β that solves (8) for \( F_0 \neq 0 \). This can be written as

\[
\beta = \frac{y^3}{b_2 F_0} \left( \sqrt{y} - 6 \right)^2 \left( \hat{y} + 6 \sqrt{y} + 6b_2 - 72 \right)^2 ,
\]

where

\[
\hat{y} = 2b_2 \left( \frac{y}{y_0} - 1 \right) + 36 .
\]

The parameter \( b_2 \) is also equal to \( \frac{F_0}{y_0} \beta_2 \), where \( \beta_2 \) is half the second derivative of \( \beta \) in \( y = y_0 \).

- If \( b_2 < 12 \), \( \beta \) has two double zeros, so the solution corresponds to two D6 stacks, one at \( \hat{y} = -\sqrt{3 + \sqrt{81 - 6b_2}} \), one at \( \hat{y} = 36 \).
- If \( b_2 > 12 \), the solution corresponds to a D6 stack at one pole \( \hat{y} = 0 \) and an O6 singularity at \( \hat{y} = 36 \).
- If \( b_2 = 12 \), \( \beta \) simplifies to \( \frac{y^3}{1728F_0} \hat{y}(\hat{y} - 36)^2 \), which is (11) up to coordinate change; so this case corresponds to a single D6 stack at \( \hat{y} = 36 \).

The second generalization consists in introducing D8-branes. These manifest themselves as loci across which \( F_0 \) (and hence \( \beta \)) can jump. Supersymmetry requires them to wrap the round \( S^2 \) in (2) at a fixed \( r = r_{DS} \); this is indeed the only way they can preserve the SU(2) R-symmetry. The supergravity solutions consist in gluing together solutions of the type (11), (13), or (10); the only non-trivial task is fixing the parameters of those solutions, and the positions of the D8’s, using flux quantization. We will do so for an example with one D8 and one example with two D8’s; here (13) will not be needed, but we expect it to become relevant for higher numbers of D8’s.

A D8 can also have D6 charge \( \mu \) smeared on its world-volume; this is the Chern class of a gauge bundle, and as such it is an integer. D8’s with the same \( \mu \) will be stabilized by supersymmetry on top of each other. In such a situation, the flux integers of \( F_0 \) and \( F \) before and after the D8 stack, \( (n_0, n_2) \) and \( (n'_0, n'_2) \), are related to the number of branes in the stack and their charge by \( n_{DS} = n'_0 - n_0 \) and \( \mu = \frac{n'_2 - n_2}{n'_0 - n_0} \). The position is then fixed by the formula (11, 2)

\[
q|_{r=r_{DS}} = \frac{n'_0 n_0 - n_2 n'_0}{2(n'_0 - n_0)} = \frac{1}{2}(-n_2 + \mu n_0) = \frac{1}{2}(-n'_2 + \mu' n'_0) ,
\]

where \( q \) was given in (8). So we see that in the \( y \) coordinate the position of the D8-branes goes quadratically with \( \mu \). In fact, \( q \) itself has a nice interpretation: from its definition (8), and from (9), (2) we see

\[
q = \frac{1}{4} e^{A - \phi} e^{-\phi} \text{radius}(S^2) .
\]

The simplest possibility is to have one D8 stack, of charge \( \mu \). This is done by gluing two copies of (7). Concretely, the function \( \beta \) reads

\[
\beta = \begin{cases} 
\frac{8}{F_0}(y - y_0)(y + 2y_0)^2 , & y_0 < y < y_{DS} ; \\
\frac{8}{F_0}(y - y'_0)(y + 2y'_0)^2 , & y_{DS} < y < y'_0 ; \\
\end{cases}
\]

with \( y_0 < 0, y'_0 > 0 \). We need to impose flux quantization, (15), and continuity of \( \beta \) and its derivative (which, via (9), guarantees continuity of \( A, \phi \), and of the metric). This leads to

\[
F_0' = F_0 \left( 1 - \frac{N}{\mu} \right) , \quad y_{DS} = 3F_0\pi^2(N - 2\mu)(N - \mu) ,
\]

\[
y_0 = \frac{3}{2} F_0 \pi^2(N - \mu)^2 , \quad y'_0 = \frac{3}{2} F_0 \pi^2(N - \mu)(2N - \mu) .
\]

We see now that \( \beta \) has a single zero at both endpoints \( y_0 \) and \( y'_0 \). So this solution is regular, except of course for the effect of the D8 backreaction; this causes discontinuities in the first derivatives of \( A, \phi \) and the metric, as any domain wall in general relativity should do.

The next possibility is to have two D8 stacks. As in (11, 2), we assume for simplicity that the solution is symmetric under \( y \to -y \), so that the two endpoints are at \( y_0 < 0 \) and \( -y_0 \), and the two D8 stacks, of D6 charge \( \mu \) and \( -\mu \), are located at \( y_{DS} < 0 \) and \( -y_{DS} \). There are three regions: i) For \( y_0 < y < y_{DS} \), \( F_0 > 0 \); \( \beta \) is as in (11, ii) For \( y_{DS} < y < -y_{DS} \), \( F_0 = 0 \), and \( \beta \) is as in (10), namely \( \beta = \frac{1}{4} F_0 \left( y^2 - \left( \frac{9}{32} R^2 \right)^2 \right)^2 \); iii) For \( -y_{DS} < y < -y_0 \), the Romans mass is \( F_0' = -F_0 < 0 \); \( \beta \) is again as in (11), but now with \( y_0 \to -y_0 \), \( F_0 \to -F_0 \). Again in this way we avoid singularities, except for the discontinuities in the first derivatives induced by the two D8 stacks. This solution, and its brane interpretation, is showed in figure 2.

Using flux quantization and (15) we can fix the parameters as

\[
y_0 = -\frac{9}{4} k\pi(N - \mu) , \quad y_{DS} = -\frac{9}{4} k\pi(N - 2\mu) , \\
R^6 = \frac{64}{3} k^2 \pi^2(3N^2 - 4\mu^2) .
\]

This solution only exists for \( N \geq 2\mu \), in agreement with a bound in (2).

All these analytic solutions now allow us to obtain some information about the field theory duals. As we mentioned above, the six-dimensional \((1, 0)\) field theories should be dual to the theories described by NS5–D6–D8
configurations such as the ones in figures 1(b) and 2(b). These theories are a bit mysterious because of the physics of coincident NS5-branes. Separating them leads to a quiver description [3, 4], but this corresponds to moving along a “tensor branch” departing from its origin, the conformal point. Thus most degrees of freedom are not captured by the quiver description, also in four and three dimensions.

A possible measure is given by the coefficient $F_{0,d}$ in the free energy as a function of temperature $T$ and volume $V$: $F_d = F_{0,d} V T^d$. This can be estimated by taking a large black hole in $AdS_{d+1}$: this leads to

$$F_{0,d} = \frac{\lambda_{AdS_{d+1}}^4}{G_{N,d+1}}$$

where $G_{N,d+1}$ is Newton’s constant. For constant dilaton this is $\sim N_0 \sqrt{\pi} V$, where Vol is the volume of the internal manifold; for us the dilaton is not constant, and we should integrate $e^{-2\phi}$ over the internal space. All in all, $F_{0,6} = \int M_5 \text{Vol}_5 e^{5A - 2\phi}$ for the SCFT$_6$, and $F_{0,d} = \int M_5 \times \Sigma_{d-4} \text{Vol}_5 \wedge \text{Vol}_{d-4} e^{5A - 2\phi}$ for the SCFT$_d$, $d = 3, 4$. For $d = 4$, this is also related to $a$ and $c$ (which are equal up to stringy corrections, which we did not compute). Using the maps (5), (21), we find the universal relations

$$F_{0,4} = \left(\frac{3}{4}\right)^3 \pi (g - 1) F_{0,6}, \quad F_{0,3} = \left(\frac{5}{8}\right)^4 \text{Vol}(\Sigma_3) F_{0,6}.$$  

(20)

$F_{0,6}$ can be computed explicitly. As a reference point, in our normalization the (2, 0) theory gives $F_{0,6} = \frac{128}{3} \pi^4 N^3$; for its $\mathbb{Z}_k$ orbifold, this number is multiplied by $k^2$. We have computed $F_{0,6}$ for the two cases of figure 1 and 2. For the first case, we simply use the solution (7), and we get $F_{0,6} = \frac{128}{3} \pi^4 N^3$. For the case of figure 2 we have to integrate over the three different regions, recalling (19). This gives

$$F_{0,6} = \frac{128}{3} k^2 \pi^4 \left( N^3 - 4N \mu^2 + \frac{16}{5} \mu^3 \right)$$

(21)

(in agreement with the approximate result in [21]). For the corresponding SCFT$_4$, $a = c = \frac{2}{9} \pi (N^3 - 4N \mu^2 + \frac{16}{5} \mu^3)$.

We conclude by expanding a bit on our earlier comment about the possible uses of the AdS$_d$ solutions for flux compactifications. For many applications (such as to achieve de Sitter vacua, or a hierarchy between the KK and cosmological constant scales), it is useful to introduce orientifold planes. Yet they are usually artificially “smeared”, i.e. replaced with a continuous charge distribution. Flux compactifications with localized sources are not many (but see [10]), and the AdS$_d$ solutions that one obtains by applying the map (3) to (13) is the first case to our knowledge of solution with a localized O6-plane. Moreover, in [9, Sec. 5] another wide class of AdS$_d$ solutions is uncovered. Although these are so far only known numerically, they appear to be more general; for example, there are solutions with an O6-plane and no further singularity. An attractor mechanism allows to achieve regularity without need for fine-tuning.

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