Field strength correlations in the QCD vacuum at short distance

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Abstract

We determine by numerical simulations on a lattice the gauge–invariant two–point correlation function of the gauge field strengths in the QCD vacuum, down to a distance of 0.1 fm.

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1. Introduction

The gauge–invariant two–point correlators of the field strengths in the QCD vacuum are defined as

\[ D_{\mu\rho,\nu\sigma}(x) = \langle 0 | \text{Tr} \left\{ G_{\mu\rho}(x) S(x, 0) G_{\nu\sigma}(0) S^\dagger(x, 0) \right\} | 0 \rangle , \]  

(1.1)

where \( G_{\mu\rho} = g T^a G^a_{\mu\rho} \) and \( T^a \) are the generators of the colour gauge group in the fundamental representation. Moreover, in Eq. (1.1),

\[ S(x, 0) = \text{P exp} \left( i \int_0^1 dt \, x^\mu A_\mu(xt) \right) , \]  

(1.2)

with \( A_\mu = g T^a A^a_\mu \), is the Schwinger phase operator needed to parallel–transport the tensor \( G_{\nu\sigma}(0) \) to the point \( x \).

These field–strength correlators play an important role in hadron physics. In the spectrum of heavy \( Q\bar{Q} \) bound states, they govern the effect of the gluon condensate on the level splittings [1, 2, 3]. They are the basic quantities in models of stochastic confinement of colour [4, 5, 6] and in the description of high–energy hadron scattering [7, 8, 9, 10].

A numerical determination of the correlators on lattice (with gauge group \( SU(3) \)) already exists, in the range of physical distances between 0.4 and 1 fm [11]. In that range \( D_{\mu\rho,\nu\sigma} \) falls off exponentially

\[ D_{\mu\rho,\nu\sigma}(x) \sim \exp(-|x|/\lambda) , \]  

(1.3)

with a correlation length \( \lambda \simeq 0.22 \text{ fm} \) [11].

What makes the determination of the correlators possible on the lattice, with a reasonable computing power, is the idea [12, 13] of removing the effects of short–range fluctuations on large distance correlators by a local cooling procedure. Freezing the links of QCD configurations one after the other, damps very rapidly the modes of short wavelength, but requires a number \( n \) of cooling steps proportional to the square of the distance \( d \) in lattice units to affect modes of wavelength \( d \):

\[ n \simeq kd^2 . \]  

(1.4)
Cooling is a kind of diffusion process. If $d$ is sufficiently large, there will be a range of values of $n$ in which lattice artefacts due to short-range fluctuations have been removed, without touching the physics at distance $d$; by lattice artefacts we mean statistical fluctuations and renormalization effects from lattice to continuum. This removal will show up as a plateau in the dependence of the correlators on $n$. This was the technique successfully used in Ref. [11]. There, the range of distances explored was from from 3–4 up to 7–8 lattice spacings at $\beta \simeq 6$, which means approximately from 0.4 up to 1 fm in physical distance. The lattice size was $16^4$.

We have now new results on a $32^4$ lattice, at $\beta$ between 6.6 and 7.2: at these values of $\beta$ the lattice size is still bigger than 1 fm, and therefore safe from infrared artefacts, but $d = 3, 4$ lattice spacings now correspond to physical distances of about 0.1 fm. Since what matters to our cooling procedure is the distance in lattice units, we obtain in this way a determination of the correlators at distances down to 0.1 fm.

2. Computations and results

The most general form of the correlator compatible with the invariances of the system is

$$D_{\mu\nu,\rho\sigma}(x) = (g_{\mu\nu}g_{\rho\sigma} - g_{\mu\sigma}g_{\rho\nu}) \left[ D(x^2) + D_1(x^2) \right]$$

$$+ \left( x_{\mu}x_{\nu}g_{\rho\sigma} - x_{\mu}x_{\sigma}g_{\rho\nu} + x_{\rho}x_{\sigma}g_{\mu\nu} - x_{\rho}x_{\nu}g_{\mu\sigma} \right) \frac{\partial D_1(x^2)}{\partial x^2}.$$  \(2.1\)

$D$ and $D_1$ are invariant functions of $x^2$. We work in the Euclidean region.

We can define a $D_{\parallel}(x^2)$ and a $D_{\perp}(x^2)$ as follows. We go to a reference frame in which $x^\mu$ is parallel to one of the coordinate axes, say $\mu = 0$. Then

$$D_{\parallel} \equiv \frac{1}{3} \sum_{i=1}^{3} D_{0i,0i}(x) = D + D_1 + x^2 \frac{\partial D_1}{\partial x^2},$$

$$D_{\perp} \equiv \frac{1}{3} \sum_{i<j=1}^{3} D_{ij,ij}(x) = D + D_1.$$  \(2.2\)

On the lattice we can define a lattice operator $D_{\mu\nu,\rho\sigma}^L$, which is proportional to $D_{\mu\nu,\rho\sigma}$ in the continuum limit, i.e., when the lattice spacing $a \to 0$. Since the lattice analogue of the
field strength is the plaquette $\Pi_{\mu\rho}(n)$ (the parallel transport along an elementary square of the lattice, lying on the $\mu\rho$-plane), $D_{\mu\rho,\nu\sigma}^L$ will be defined as

$$D_{\mu\rho,\nu\sigma}^L(\hat{da}) = \frac{1}{2} \Re \left\{ \langle \mbox{Tr}[\Pi_{\mu\rho}(n + \hat{da})S(n + \hat{da}, n)\Omega_{\nu\sigma}(n)S^\dagger(n + \hat{da}, n)] \rangle \right\} ,$$

(2.3)

where $\Re$ stands for real part and the lattice operator $\Omega_{\nu\sigma}(n)$ is given by

$$\Omega_{\nu\sigma}(n) = \Pi_{\nu\sigma}^\dagger(n) - \Pi_{\nu\sigma}(n) - \frac{1}{3}\mbox{Tr}[\Pi_{\nu\sigma}^\dagger(n) - \Pi_{\nu\sigma}(n)] .$$

(2.4)

$\hat{da}$ is a line parallel to one of the coordinate axes, with integer length $d$ in units of the lattice spacing $a$, joining the two sites to be correlated; $S$ is the parallel transport along this line. The inclusion of the operator $\frac{1}{2}\Omega_{\nu\sigma}$ (in place of simply putting $\Pi_{\nu\sigma}^\dagger$) on the right-hand side of Eq. (2.3) ensures that the disconnected part and the singlet part of the correlator are left out. In particular, the subtraction of the singlet part (which is anyway a small contribution of order $O(a^4)$) means that we are indeed taking the correlation of two operators with the quantum numbers of a colour octet.

In the naïve continuum limit ($a \to 0$) we have that

$$D_{\mu\rho,\nu\sigma}^L(\hat{da}) \sim a^4 D_{\mu\rho,\nu\sigma}(\hat{da}) + O(a^6) .$$

(2.5)

Making use of the definition (2.1) we can also write, in the same limit,

$$D_{\parallel}^L(\hat{da}) \sim a^4 D_{\parallel}(d^2a^2) + O(a^6) ,$$

$$D_{\perp}^L(\hat{da}) \sim a^4 D_{\perp}(d^2a^2) + O(a^6) .$$

(2.6)

Equations (2.5) and (2.6) come from a formal expansion of the operator, and are expected to be modified, when the expectation value is computed, by lattice artefacts, i.e., by effects due to the ultraviolet cutoff. These effects can be estimated in perturbation theory and subtracted [14]. Instead we remove them by freezing the quantum fluctuations at the scale of the lattice spacing. After cooling, $D_{\parallel}^L$ and $D_{\perp}^L$ are expected to obey Eq. (2.6).

We shall omit the details of our cooling procedure, which have been described in several papers (see Refs. [12, 13] and references therein). We only remark that it is a local procedure, which affects correlations at distances that grow with the number of
cooling steps as in a diffusion process. We then expect that, if the distance at which we observe the correlation is sufficiently large, lattice artefacts are frozen by cooling long before the correlation is affected: this produces a plateau in the correlation versus cooling step. Our data are the values of the correlation at the plateau; the error is the typical statistical error at the plateau, plus a systematic error which is estimated as the difference between neighbouring points at the plateau (whenever the plateau is not long enough, looking more like an extremum). Typically the global error is three times larger than the statistical error. The typical behaviour of $D_\parallel$ and of $D_\perp$ along cooling is shown in Fig. 1.

We have measured the correlations on a $32^4$ lattice at distances ranging from 3 to 14 lattice spacings and at $\beta = 6.6$, $6.8$, $7.0$, $7.2$. From renormalization group arguments,

$$ a = \frac{1}{\Lambda_L} f(\beta) . \quad (2.7) $$

As $\beta \to \infty$, $f(\beta)$ is given by

$$ f(\beta) = \left( \frac{8}{33} \pi^2 \beta \right)^{51/121} \exp \left( -\frac{4}{33} \pi^2 \beta \right) \left[ 1 + O(1/\beta) \right] , \quad (2.8) $$

for gauge group $SU(3)$ and in the absence of quarks. At sufficiently large $\beta$ one expects that

$$ D_\parallel f(\beta)^{-4} = \frac{1}{\Lambda_L^4} D_\parallel \left( \frac{d^2}{\Lambda_L^2} f^2(\beta) \right) , $$

$$ D_\perp f(\beta)^{-4} = \frac{1}{\Lambda_L^4} D_\perp \left( \frac{d^2}{\Lambda_L^2} f^2(\beta) \right) , \quad (2.9) $$

where $f(\beta)$ is given by Eq. (2.8) and terms of higher order in $a$ are negligible.

In Figs. 2 and 3 we plot respectively $D_\parallel f(\beta)^{-4}$ and $D_\perp f(\beta)^{-4}$ versus $d_{\text{phys}} = (d/\Lambda_L) f(\beta)$. In these figures we have also plotted the values of the correlators obtained in Ref. [11], corresponding to physical distances $d_{\text{phys}} \geq 0.4$ fm. We have applied a best fit to all of these data with the functions

$$ D(x^2) = A \exp \left( -\frac{|x|}{\Lambda_A} \right) + \frac{a}{|x|^4} \exp \left( -\frac{|x|}{\Lambda_a} \right) , $$

$$ D_1(x^2) = B \exp \left( -\frac{|x|}{\Lambda_A} \right) + \frac{b}{|x|^4} \exp \left( -\frac{|x|}{\Lambda_a} \right) . \quad (2.10) $$
We have obtained the following results:

\[
\frac{A}{\Lambda^4_L} \simeq 3.3 \times 10^8, \quad \frac{B}{\Lambda^4_L} \simeq 0.7 \times 10^8, \\
a \simeq 0.69, \quad b \simeq 0.46, \\
\lambda_A \simeq \frac{1}{\Lambda L 182}, \quad \lambda_a \simeq \frac{1}{\Lambda L 94},
\]

(2.11)

with \(\chi^2/N_{\text{d.o.f.}} \simeq 1.7\). The continuum lines in Figs. 2 and 3 have been obtained using the parameters of this best fit. With the value of \(\Lambda_L\) determined from the string tension \([15]\) we obtain

\[
\lambda_A \simeq 0.22 \text{ fm}, \quad \lambda_a \simeq 0.43 \text{ fm}.
\]

(2.12)

The correlation length \(\lambda_A\), which enters the non–perturbative exponential terms of \(\mathcal{D}\) and \(\mathcal{D}_1\), as well as the magnitude of the coefficients \(A\) and \(B\), are compatible with the values obtained in Ref. \([11]\). We have also tried a different fit, in which \(\mathcal{D}_1\) is described by a purely perturbative–like term, while \(\mathcal{D}\) is still the sum of a non–perturbative exponential term plus a purely perturbative–like term. In other words, we have fixed \(B = 0\) and \(\lambda_a = 0\) in the parametrization (2.10) for \(\mathcal{D}\) and \(\mathcal{D}_1\):

\[
\mathcal{D}(x^2) = A \exp \left( -\frac{|x|}{\lambda_A} \right) + \frac{a}{|x|^4}, \\
\mathcal{D}_1(x^2) = \frac{b}{|x|^4}.
\]

(2.13)

We have thus obtained:

\[
\frac{A}{\Lambda^4_L} \simeq 2.7 \times 10^8, \quad \lambda_A \simeq \frac{1}{\Lambda L 183}, \\
a \simeq 0.4, \quad b \simeq 0.3.
\]

(2.14)

The value of \(\chi^2/N_{\text{d.o.f.}}\) is again acceptable (about 2); the slope of the non–perturbative exponential term is practically unchanged. The behaviour for \(\mathcal{D}_\parallel\) and \(\mathcal{D}_\perp\) obtained using the parameters from this best fit is close to the continuum lines reported in Figs. 2 and 3.

As a final comment we notice that we have been able to observe terms proportional to \(1/|x|^4\) in the correlations because we have worked at larger values of \(\beta\), where the distance between two points (far enough in lattice units so that the correlation is not modified by cooling before lattice artefacts are eliminated) is small compared with 1 fm in physical units. A larger lattice (32\(^4\)) has been necessary to avoid infrared artefacts.
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References

[1] D. Gromes, Phys. Lett. 115B (1982) 482.

[2] M. Campostrini, A. Di Giacomo and S. Olejnik, Z. Phys. C31 (1986) 577.

[3] Yu.A. Simonov, S. Titard and F.J. Yndurain, Phys. Lett. B354 (1995) 435.

[4] H.G. Dosch, Phys. Lett. 190B (1987) 177.

[5] H.G. Dosch and Yu.A. Simonov, Phys. Lett. 205B (1988) 339.

[6] Yu.A. Simonov, Nucl. Phys. B324 (1989) 67.

[7] O. Nachtmann and A. Reiter, Z. Phys. C24 (1984) 283.

[8] P.V. Landshoff and O. Nachtmann, Z. Phys. C35 (1987) 405.

[9] A. Krämer and H.G. Dosch, Phys. Lett. 252B (1990) 669.

[10] H.G. Dosch, E. Ferreira and A. Krämer, Phys. Rev. D 50 (1994) 1992.

[11] A. Di Giacomo and H. Panagopoulos, Phys. Lett. B285 (1992) 133.

[12] M. Campostrini, A. Di Giacomo, M. Maggiore, H. Panagopoulos and E. Vicari, Phys. Lett. B225B (1989) 403.

[13] A. Di Giacomo, M. Maggiore and S. Olejnik, Phys. Lett 236B (1990) 199; Nucl. Phys. B347 (1990) 441.

[14] M. Campostrini, A. Di Giacomo and G. Mussardo, Z. Phys. C25 (1984) 173.

[15] C. Michael and M. Teper, Nucl. Phys. B305 (1988) 453.
**FIGURE CAPTIONS**

**Fig. 1.** A typical behaviour of $D_L^\parallel$ (crosses; $d = 6, \beta = 6.6$, lattice $32^4$) and of $D_L^\perp$ (triangles; $d = 12, \beta = 6.6$, lattice $32^4$) during cooling.

**Fig. 2.** The function $D_L^\parallel f(\beta)^{-4}$ versus physical distance (in fermi units). Crosses correspond to $\beta = 6.6$, triangles to $\beta = 6.8$, hexagons to $\beta = 7.0$, diamonds to $\beta = 7.2$; crossed-circles correspond to the data of Ref. [11]. The line is the curve for $D_\parallel$ obtained from the best fit of Eqs. (2.10) and (2.11).

**Fig. 3.** The function $D_L^\perp f(\beta)^{-4}$ versus physical distance (in fermi units). The symbols are the same as in Fig. 2. The line is the curve for $D_\perp$ obtained from the best fit of Eqs. (2.10) and (2.11).
