Deeply Virtual Compton Scattering to the twist-four accuracy: Impact of finite-$t$ and target mass corrections

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based on

V. Braun, A. Manashov, D. Müller, B. Pirnay, Phys.Rev. D89 (2014) 074022

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Hard exclusive processes involve off-forward matrix elements

**DVCS:** $\gamma^* P \rightarrow \gamma P'$

Form factors: $\gamma^* \pi \rightarrow \gamma, B \rightarrow \rho \ell \bar{\nu}_\ell$, ...

Operator Product Expansion

$$J(x) J(0) \sim \sum_N C_N(x^2, \mu^2) O_N(\mu^2)$$

involves

$$\langle P' | O_N(\mu^2) | P \rangle \quad \langle \rho(p) | O_N(\mu^2) | 0 \rangle$$

Kinematic variables: hadron mass $m^2$, momentum transfer $t = (P - P')^2$

**How to calculate effects** \( \sim m^2/Q^2 \) and \( t/Q^2 \)?
Nucleon Tomography?

access to three-dimensional picture of the nucleon (M. Burkardt)

\[ \rightarrow \text{first two moments of transverse spin parton density} \]

\[ \text{computer simulations: M. Göckeler et al., Phys. Rev. Lett. 98 (2007) 222001} \]

• paradigm shift: finite \( t \) a “nuisance” \( \rightarrow \) important tool
How to calculate effects $\sim \frac{m^2}{Q^2}$ and $\frac{t}{Q^2}$ in DVCS?

Early work:

- **DVCS:**
  - Extension of Nachtmann's approach to target mass corrections in DIS
  - Spin-rotation (Wandzura-Wilczek)
    - Blümlein, Robaschik: NPB581 (2000) 449
    - Radyushkin, Weiss: PRD63 (2001) 114012
    - Belitsky, Müller: NPB589 (2000) 611
  - Results not gauge invariant
  - Results not translation invariant

- **B-decays:**
  - Ball, Braun: NPB543 (1999) 201
  - Problem localized but not solved
Contributions of different twist are intertwined by symmetries:

- Conservation of the electromagnetic current and translation invariance

\[
\partial^{\mu} T\left\{ j_{\mu}^{\text{em}}(x)j_{\nu}^{\text{em}}(0) \right\} = 0
\]

\[
T\left\{ j_{\mu}^{\text{em}}(2x)j_{\nu}^{\text{em}}(0) \right\} = e^{-i\mathbf{P} \cdot x} T\left\{ j_{\mu}^{\text{em}}(x)j_{\nu}^{\text{em}}(-x) \right\} e^{i\mathbf{P} \cdot x}
\]

are valid in the sum of all twists but not for each twist separately.

- Higher-twist contributions that restore gauge/translation invariance are due to descendants of leading-twist operators obtained by adding total derivatives

\[
T\left\{ j_{\mu}^{\text{em}}(x)j_{\nu}^{\text{em}}(0) \right\} = \sum_{N} a_{N} O_{N} + \sum_{N} \left( b_{N} \partial^{2} O_{N} + c_{N} (\partial O)_{N} \right) + \text{other operators}
\]

leading-twist

- “Kinematic” and “Dynamic” contributions must have autonomous scale-dependence

- Explicit diagonalization of the mixing matrix for twist-4 operators not feasible, but, conformal symmetry implies that this matrix is hermitian w.r.t. to a certain scalar product.

\[V.B., A. Manashov: PRL 107 (2011) 202001; JHEP 1201 (2012) 085\]
BMP amplitudes

**Introduction**

**DVCS observables**

**Summary**

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**BMP reference frame**

Braun, Manashov, Pirnay: PRD 86 (2012) 014003

**longitudinal plane** \((q, q')\)

\[ n = q', \quad \tilde{n} = -q + \frac{Q^2}{Q^2 + t} q' \]

with this choice \(\Delta = q - q'\) is longitudinal and

\[ |P_\perp|^2 = -m^2 - \frac{t}{4} \frac{1 - \xi^2}{\xi^2} \sim t_{\text{min}} - t \]

where

\[ P = \frac{1}{2} (p + p'), \quad \xi_{\text{BMP}} = -\frac{(\Delta \cdot q')}{2(P \cdot q')} = \frac{x_B(1 + t/Q^2)}{2 - x_B(1 - t/Q^2)} \]

**photon polarization vectors**

\[ \varepsilon_\mu^0 = - \left( q_\mu - q'_\mu \frac{q^2}{(qq')} \right) / \sqrt{-q^2}, \]

\[ \varepsilon_\mu^\pm = (P_\mu^\perp \pm i \tilde{P}_\mu^\perp) / (\sqrt{2} |P_\perp|), \quad \tilde{P}_\mu^\perp = \epsilon_{\mu \nu}^\perp P^\nu \]
BMP helicity amplitudes

\[
A_{\mu\nu}(q, q', p) = i \int d^4 x \, e^{-i(z_1 \cdot q - z_2 \cdot q')} x \langle p', s' | T\{J_\mu(z_1 x) J_\nu(z_2 x)\} | p, s \rangle
\]

\[
= \varepsilon^+_\mu \varepsilon^-_\nu A^{++} + \varepsilon^-_\mu \varepsilon^+_\nu A^{--} + \varepsilon^0_\mu \varepsilon^-_\nu A^{0+} + \varepsilon^0_\mu \varepsilon^+_\nu A^{0-} + \varepsilon^0_\mu \varepsilon^+_\nu A^{+-} + \varepsilon^-_\mu \varepsilon^-_\nu A^{-+} + q'_\nu A^{(3)}
\]

for the calculation to the twist-4 accuracy one needs

- \(A^{++}, A^{--}\): \(1 + \frac{1}{Q^2}\)
- \(A^{0+}, A^{0-}\): \(\frac{1}{Q}\) \(\leftarrow\) agree with existing results
- \(A^{-+}, A^{+-}\): \(\frac{1}{Q^2}\) \(\leftarrow\) straightforward
BMP Compton form factors (CFFs)

- Photon helicity amplitudes can be expanded in a given set of spinor bilinears

\[
\mathcal{A}_q^{a\pm} = \mathbb{H}_q^{a\pm} h + \mathbb{E}_q^{a\pm} e \mp \mathbb{H}_q^{a\pm} \tilde{h} \mp \mathbb{E}_q^{a\pm} \tilde{e}
\]

with, e.g.

\[
h = \frac{\bar{u}(p') (\bar{q} + \bar{q}') u(p)}{P \cdot (\bar{q} + \bar{q}')} \quad \ldots
\]

- The results read

Belitsky, Müller, Ji: NPB 878 (2014) 214

Braun, Manashov, Pirnay: PRL109 (2012) 242001

\[
\begin{align*}
\mathbb{H}_{++} &= T_0 \otimes H + \frac{t}{Q^2} \left[ -\frac{1}{2} T_0 + T_1 + 2\xi \mathbf{D}_\xi T_2 \right] \otimes H + \frac{2t}{Q^2} \xi^2 \partial_\xi \xi T_2 \otimes (H+E) \\
\mathbb{H}_{0+} &= -\frac{4|\xi P_\perp|}{\sqrt{2}Q} \left[ \xi \partial_\xi T_1 \otimes H + \frac{t}{Q^2} \partial_\xi \xi T_1 \otimes (H+E) \right] - \frac{t}{\sqrt{2}Q|\xi P_\perp|} \xi T_1 \otimes \left[ \xi (H+E) - \tilde{H} \right] \\
\mathbb{H}_{-+} &= \frac{4|\xi P_\perp|^2}{Q^2} \left[ \xi \partial_\xi^2 \xi T_1^{(+)} \otimes H + \frac{t}{Q^2} \partial_\xi^2 \xi^2 T_1^{(+)} \otimes (H+E) \right] \\
&\quad + \frac{2t}{Q^2} \xi \left[ \xi \partial_\xi \xi T_1^{(+)} \otimes (H+E) + \partial_\xi \xi T_1 \otimes \tilde{H} \right]
\end{align*}
\]
BMP Compton form factors (CFFs)

- Photon helicity amplitudes can be expanded in a given set of spinor bilinears

\[ \mathcal{A}_q^{a\pm} = \mathbb{H}_a^{q\pm} h + \mathbb{E}_a^{q\pm} e \mp \mathbb{H}_a^{q\pm} \tilde{h} \pm \mathbb{E}_a^{q\pm} \tilde{e} \]

with, e.g.

Belitsky, Müller, Ji: NPB 878 (2014) 214

\[ h = \frac{\bar{u}(p')(\slashed{q} + \slashed{q}')u(p)}{P \cdot (\slashed{q} + \slashed{q}')} \]

... etc.

- The results read

Braun, Manashov, Pirnay: PRL109 (2012) 242001

\[
\begin{align*}
\mathbb{E}_{++} & = T_0 \otimes E + \frac{t}{Q^2} \left[ -\frac{1}{2} T_0 + T_1 + 2\xi D_\xi T_2 \right] \otimes E - \frac{8m^2}{Q^2} \xi^2 \partial_\xi \xi T_2 \otimes (H + E) \\
\mathbb{E}_{0+} & = -\frac{4|\xi P_\perp|^2}{\sqrt{2}Q} \left[ \xi \partial_\xi T_1 \otimes E \right] + \frac{4m^2}{\sqrt{2}Q|\xi P_\perp|} \xi T_1 \otimes \left[ \xi (H + E) - \tilde{H} \right] \\
\mathbb{E}_{--} & = \frac{4|\xi P_\perp|^2}{Q^2} \left[ \xi \partial_\xi^2 \xi T_1^{(+)} \otimes E \right] - \frac{8m^2}{Q^2} \xi \left[ \xi \partial_\xi \xi T_1^{(+)} \otimes (H + E) + \partial_\xi \xi T_1 \otimes \tilde{H} \right]
\end{align*}
\]
where \( F = H, E, \tilde{H}, \tilde{E} \) are \( C \)-even GPDs

\[
T \otimes F = \sum_q e_q^2 \int_{-1}^{1} \frac{dx}{2\xi} \frac{1}{2(\xi - i\epsilon)} F(x, \xi, t)
\]

the coefficient functions \( T_{k-}^{T} \) are given by the following expressions:

\[
T_0(u) = \frac{1}{1 - u}
\]

\[
T_1(u) = \frac{\ln(1 - u)}{u}
\]

\[
T_1^{(+)}(u) = \frac{(1 - 2u)\ln(1 - u)}{u}
\]

\[
T_2(u) = \frac{\text{Li}_2(1) - \text{Li}_2(u)}{1 - u} + \frac{\ln(1 - u)}{2u}
\]

and

\[
D_\xi = \partial_\xi + 2|\xi P_\perp|^2 \partial_\xi^2 \xi = \partial_\xi - \frac{t - t_{\text{min}}}{2t} (1 - \xi^2) \partial_\xi^2 \xi
\]
Main features:

- Two expansion parameters

\[
\frac{t}{Q^2}, \quad \frac{t - t_{\text{min}}}{Q^2} \sim \frac{|\xi P_\perp|^2}{Q^2}
\]

- All mass corrections for scalar targets absorbed in \( t_{\text{min}} = -4m^2\xi^2/(1 - \xi^2) \); always overcompensated by finite-\( t \) corrections in the physical region

- Some extra \( m^2/Q^2 \) corrections for nucleon due to spinor algebra; disappear in certain CFF combinations

- Factorization checked to \( 1/Q^2 \) accuracy
- Gauge and translation invariance checked to \( 1/Q^2 \) accuracy
- Correct threshold behavior \( t \to t_{\text{min}}, \xi \to 1 \)
From CFFs to DVCS observables

- The only existing calculation to the required accuracy: BMJ

Belitsky, Müller, Ji: NPB 878 (2014) 214

!!! Subtlety: BMJ use a different reference frame to define photon helicity amplitudes; hence a different set of CFFs (calligraphic) related to BMP CFFs (blackboard bold) by a kinematic trafo

\[
\mathcal{F}_{\pm} = F_{\pm} + \frac{\kappa}{2} \left[ F_{++} + F_{--} \right] - \kappa_0 F_{0+}, \\
\mathcal{F}_{0+} = -(1 + \kappa) F_{0+} + \kappa_0 \left[ F_{++} + F_{--} \right]
\]

where

\[
\kappa_0 \sim \sqrt{\left( t_{\text{min}} - t \right) / Q^2}, \quad \kappa \sim \left( t_{\text{min}} - t \right) / Q^2
\]

Adopted strategy is, thus,

BMP CFFs $\xrightarrow{\text{exact}}$ BMJ CFFs $\xrightarrow{\text{exact}}$ observables

$\mathcal{O}(1/Q^2)$
Defining the Leading Twist approximation

**Kumerički-Müller convention (KM)**

\[
\mathbf{LT}_{\text{KM}}: \begin{cases} 
\mathcal{F}_{++} = T_0 \otimes F, & \mathcal{F}_{0+} = 0, \\
\mathcal{F}_{--} = 0, & \xi = \xi_{\text{KM}}
\end{cases}
\]

**Braun-Manashov-Pirnay convention (BMP)**

\[
\mathbf{LT}_{\text{BMP}}: \begin{cases} 
\mathcal{F}_{++} = T_0 \otimes F, & \mathcal{F}_{0+} = 0, \\
\mathcal{F}_{--} = 0, & \xi = \xi_{\text{BMP}}
\end{cases}
\]

\[
\Downarrow
\]

\[
\mathbf{LT}_{\text{BMP}}: \begin{cases} 
\mathcal{F}_{++} = (1 + \frac{\xi}{2}) F_{++}, & \mathcal{F}_{0+} = \kappa_0 F_{++}, \\
\mathcal{F}_{--} = \frac{\kappa}{2} F_{++}, & \xi = \xi_{\text{BMP}},
\end{cases}
\]

Changing frame of reference results in

- Different skewedness parameter
  \[
  \xi_{\text{KM}} = \frac{x_B}{2 - x_B}
  \]
  vs.
  \[
  \xi_{\text{BMP}} = \frac{x_B(1 + t/Q^2)}{2 - x_B(1 - t/Q^2)}
  \]

- Numerically significant excitation of helicity-flip CFFs \(\mathcal{F}_{0+}, \mathcal{F}_{--}\)
Unpolarized target

GPD model: GK12

Braun, Manashov, Müller, Pirnay: arXiv:1401.7621

Figure: Unpolarized cross section [upper panels] and electron helicity dependent cross section difference [lower panels] from HALL A
(new) Transversely polarized target

Figure: Transverse target spin asymmetries by HERMES collaboration

GPD model: GK12 (Kroll, Moutarde, Sabatie, Eur.Phys.J. C73, 2278)
Summary and conclusions

- Target mass and finite-\( t \) corrections to DVCS are known to twist-4 accuracy. They are relatively simple and can be implemented with moderate effort.

- **Premium:**
  
  - Gauge and translation invariance of the Compton tensor is restored to \( 1/Q^2 \) accuracy.
  - Convention-dependence of the common leading-twist calculations is removed.
  - Theoretically motivated limits \(-t/Q^2 \lesssim 1/4\).

- For several key observables, the lion share of the twist-4 effects is captured by going over to the BMP frame.

- Standardization badly needed for all steps, starting from the Compton tensor:

\[
A_{\mu\nu}(q, q', p) = \varepsilon_+^\mu \varepsilon_-^\nu A^{++} + \varepsilon_-^\mu \varepsilon_+^\nu A^{--} + \varepsilon_0^\mu \varepsilon_-^\nu A^{0+} + \varepsilon_-^\mu \varepsilon_0^\nu A^{0-} + \varepsilon_+^\mu \varepsilon_0^\nu A^{+-} + \varepsilon_-^\mu \varepsilon_+^\nu A^{-+} + q'_\nu B_\mu
\]
Backup slides
Unpolarized target (2)

Braun, Manashov, Müller, Pirnay: arXiv:1401.7621

Figure: Single electron beam spin asymmetry by CLAS collaboration

GPD model: GK12 (Kroll, Moutarde, Sabatie, Eur.Phys.J. C73, 2278)
Unpolarized target (3)

Braun, Manashov, Müller, Pirnay: arXiv:1401.7621

Figure: The single electron beam spin asymmetry [left panel] in the charge-odd sector and the unpolarized beam charge asymmetry [right panel] measured by the HERMES collaboration

GPD model: GK12 (Kroll, Moutarde, Sabatie, Eur.Phys.J. C73, 2278)
Longitudinally polarized targets

Braun, Manashov, Müller, Pirnay: arXiv:1401.7621

Figure: Longitudinal proton spin asymmetry from CLAS [left panel], measured with an electron beam, and HERMES [right panel], measured with a positron beam

GPD model: GK12 (Kroll, Moutarde, Sabatie, Eur.Phys.J. C73, 2278)
Collider kinematics

Braun, Manashov, Müller, Pirnay: arXiv:1401.7621

Figure: The DVCS cross section from H1 (squares, diamonds, triangles) and ZEUS (circles)

GPD model: GK12 (Kroll, Moutarde, Sabatie, Eur.Phys.J. C73, 2278)