Limits on $\tan \beta$ in SU(5) GUTs with Gauge-Mediated
Supersymmetry Breaking

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Abstract

By considering the constraints from nucleon decay we obtain upper limits on $\tan \beta$ in generalized supersymmetric SU(5) grand unified theories with gauge-mediated supersymmetry breaking. We find that the predicted values of $\tan \beta$ in these models are mostly inconsistent with the constraints from nucleon decay.
Supersymmetric (SUSY) grand unified theories (GUTs) are presently considered to be among the most promising candidates for physics beyond the standard model (SM). However, for phenomenological reasons, supersymmetry cannot be exact and it is usually assumed that the theory includes a visible sector containing the observable particles, and a hidden sector where supersymmetry is broken. SUSY breaking can be communicated to the visible sector either by gravitational interactions, as in supergravity (SUGRA) inspired models, or by SM gauge interactions, as in theories with gauge-mediated SUSY breaking (GMSB).

GMSB models were initially studied in the early 1980’s [1] and have recently become the subject of much theoretical investigation. The revival of interest in these models [2] is largely due to recent dramatic improvements in our understanding of nonperturbative effects in SUSY gauge theories as a result of the pioneering works of Seiberg [3] and Seiberg and Witten [4]. Many new mechanisms for dynamical SUSY breaking (DSB) have been found since the appearance of [3,4] and new DSB models have been constructed [5]. From a phenomenological point of view, GMSB theories are interesting for a number of reasons. In these theories gauge interactions provide flavor-symmetric SUSY breaking terms and thus naturally suppress the flavor-changing neutral currents associated with soft squark and slepton masses. They also predict approximately degenerate squark and slepton masses (see below) and are specified by a relatively small number of parameters.

In GMSB theories the $SU(3) \times SU(2) \times U(1)$ gauge interactions of the “messenger” fields communicate SUSY breaking from a hidden sector to the fields of the visible world. In the simplest of such models [2], in addition to the particles in the minimal supersymmetric standard model (MSSM), there exists at least one singlet superfield $S$ which couples to vector-like messenger superfields $V + \bar{V}$ through the superpotential interaction

$$W_{\text{mess}} = \lambda_V SV\bar{V}. \quad (1)$$

$^1$Here we do not consider the modification of direct mediation models such as the one proposed, e.g., in [6].
At a scale $\Lambda \sim 10 - 100 \, \text{TeV}$, which is not much higher than the weak scale, SUSY is broken and both the lowest and $F$-component, $F_S$, of the singlet superfield $S$ acquire vacuum expectation values (VEVs) through their interactions with the hidden sector. The VEV, $\langle S \rangle$, gives masses to the vector-like supermultiplets $V + \bar{V}$, while $\langle F_S \rangle$ induces mass splittings within the supermultiplets. As a result, the gaugino and sfermion masses are generated through their gauge couplings to the messenger fields. The gauginos receive masses at one-loop, $m_\lambda \sim (\alpha/4\pi)\Lambda$, where $\Lambda = \langle F_S \rangle/\langle S \rangle$, while squarks and sleptons do so only at two-loop order, $\tilde{m}^2 \sim (\alpha/4\pi)^2 \Lambda^2$. This implies that $m_\lambda \sim \tilde{m}$, which is one of the attractive features of GMSB theories.

In the minimal version of the GMSB model [2], the messenger fields belong to the $5 + \bar{5}$ or $10 + \bar{10}$ representations of the $SU(5)$ gauge group, and the messenger Yukawa couplings, $\lambda_V$’s in (1), in any given $SU(5)$ representation are taken to be equal at the unification scale $M_{\text{GUT}}$. Consequently, the spectrum at the messenger scale consists of a set of fields in complete $SU(5)$ representations and the mass splitting among the fields in a representation is induced through the renormalization group running of the messenger Yukawa couplings from $M_{\text{GUT}}$ down to the messenger scale $\Lambda_m$.

While the phenomenological implications of the minimal GMSB model have been extensively studied [7–9], non-minimal generalizations of this class of theories have seen less investigation. In this Letter, we study generalized models of GMSB proposed by Martin [10] in which the messenger fields do not necessarily form complete $SU(5)$ GUT multiplets. It is, in fact, not difficult to see how one may be naturally led to consider messenger fields which belong to incomplete representations of the $SU(5)$ gauge group in the generalized GMSB models. This is because the unification of the messenger Yukawa couplings at the GUT scale—whose MSSM analogue is the so-called $b - \tau$ unification [11]—is not necessarily required for gauge unification. Suppose, for example, that in addition to $S$ there exist singlet superfields, $S'$, whose VEVs (but not the VEVs of their $F$-components) are just below
the GUT scale, and which couple only to some components of an $SU(5)$ multiplet. Then within the $SU(5)$ multiplet these superfields acquire masses of order $O(M_{GUT})$ and decouple from the low-energy spectrum. The other components, which get their masses only through couplings with the superfield $S$, obtain masses of order $\lambda \langle S \rangle \sim \Lambda_m$. Since $\sqrt{F_S}$ is much smaller than the masses of the heavy superfields, these (missing) particles make negligible mass contributions and play a less important role in determining the MSSM mass spectrum.

A fruitful approach for examining the phenomenological viability of SUSY GUTs has been to study processes that contribute to the nucleon decay. However, in contrast to the minimal GMSB theories, in which constraints from the nucleon decay yield quite strong results, the situation in this regard is somewhat more complicated in the generalized GMSB models. In the minimal model one begins with the renormalization group running of the SM gauge coupling constants from the electroweak scale up to the GUT scale to determine the mismatch between the SM and the $SU(5)$ gauge coupling constants at the GUT scale. The mismatch, which is expressed in terms of the sum of the contributions coming from threshold corrections at the weak scale, the messenger scale, and the GUT scale, can then be used to calculate the masses of the color-triplet Higgs bosons, $M_{H^c}$. On the other hand, in the generalized GMSB models the mass splitting within a given $SU(5)$ representation is undetermined and may result in large contributions to the mismatch at the GUT scale. This means that the masses of the color-triplet Higgs bosons cannot, in general, be reliably determined in the same way as in the minimal model.

However, it is still possible to obtain useful constraints in the generalized $SU(5)$ GMSB models. The color-triplet Higgs superfields belong to the $5 + \bar{5}$ representation of $SU(5)$ and, in general, the unification scale in $SU(5)$ GUTs can be reliably taken to be $M_{GUT} \sim 2 \times 10^{16}$ GeV. This is due to the fact that above the GUT scale, e.g., at around the reduced Planck scale, $2.4 \times 10^{18}$ GeV, other effects—such as those coming from string theory, for

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2 Horizontal symmetries, e.g., can be used to construct such models.
example—are expected to play an essential role. It is then safe to conclude that the masses of the color-triplet Higgs fields cannot be much larger than the GUT scale. As a conservative upper bound, one can take \( M_{HC} \leq 10^{17} \) GeV. [Note that in the minimal GMSB model the calculated masses of the color-triplet Higgs bosons are found to be around \( 10^{15} - 10^{16} \) GeV if the LEP result \( \alpha_3(m_Z) = 0.116 \pm 0.005 \) is used \[13\].]

Let us now describe the GMSB models that are the subject of this Letter. Following Martin \[10\], we shall consider five possible types of (chiral) superfields in the messenger sector of the generalized GMSB model

\[
\begin{align*}
\text{n}_L : & \quad L + \bar{L} = (1, 2, -\frac{1}{2}) + \text{conj.}, \\
\text{n}_D : & \quad D + \bar{D} = (3, 1, \frac{1}{3}) + \text{conj.}, \\
\text{n}_E : & \quad E + \bar{E} = (1, 1, 1) + \text{conj.}, \\
\text{n}_U : & \quad U + \bar{U} = (3, 1, -\frac{2}{3}) + \text{conj.}, \\
\text{n}_Q : & \quad Q + \bar{Q} = (3, 2, \frac{1}{6}) + \text{conj.},
\end{align*}
\]

where the multiplicities of the messenger fields are denoted by \((n_L, n_D, n_E, n_U, n_Q)\).

Requiring that the gauge couplings remain perturbative, and assuming messenger field masses that do not greatly exceed \( 10^4 \) TeV, leads (see \[10\] for further discussion) to the following set of multiplicities for the messenger fields

\[
(n_L, n_D, n_E, n_U, n_Q) \leq (1, 2, 2, 0, 1) \quad \text{or (1, 1, 1, 1)} \quad \text{or (1, 0, 0, 2, 1)} \quad \text{or (4, 4, 0, 0, 0)}.
\]

The general low energy superpotential of the messenger sector is

\[
W_{\text{mess}} = \sum_{n_L} \lambda_L^i SL^i \bar{T}^i + \sum_{n_D} \lambda_D^i SD^i \bar{D}^i + \sum_{n_E} \lambda_E^i SE^i \bar{E}^i + \sum_{n_U} \lambda_U^i SU^i \bar{U}^i + \sum_{n_Q} \lambda_Q^i SQ^i \bar{Q}^i.
\]
and the MSSM spectrum can be determined once the messenger sector is fixed. In the numerical estimates that are reported here, we shall use $\Lambda_m = 10^4$ TeV.

We can now directly calculate the nucleon decay rates for the GMSB models listed above by using the process $n \rightarrow K^0\mu\bar{\nu}$ as the characteristic mode. The short- and long-distance corrections and the hadronic matrix elements are taken at their conservative values, as e.g. in [9], giving a lower bound on the product $M_{H_u}\sin 2\beta$. We have studied all the 53 possible models which satisfy criteria (3) and contain massive gauginos. By setting the upper limit on the mass of the triplet Higgs at $10^{17}$ GeV, upper bounds on $\tan \beta$ can be obtained and some interesting configurations for $\Lambda = \langle F_S \rangle / \langle S \rangle = 100$ TeV are listed in Table I. The parameters in the columns are the ones needed in the nucleon decay formula. We find a general bound, $\tan \beta < 10$, except in cases (1,4,0,0,0), (2,4,0,0,0), and (1,3,0,0,0) for which this bound is $\tan \beta < 17$.

There is also a lower bound on $\tan \beta$ following from the nucleon decay constraints. However, this constraint is less severe than the one obtained by requiring that the Yukawa couplings should not blow up at the GUT scale, giving $\tan \beta > 0.85$ [9].

For comparison, we have also calculated the value of $\tan \beta$ with the assumption of radiatively-broken $SU(2) \times U(1)$ symmetry when trilinear and bilinear soft couplings vanish at the messenger scale—which, among other things, free these models from the supersymmetric CP problem and make them extremely predictive. For the computations we use the full one-loop effective potential [8]. The calculated values of $\tan \beta$ are plotted versus the upper limit from nucleon decay in Fig. 1. The calculations are done for $\Lambda = 100$ TeV (black circles) and $\Lambda = 200$ TeV (open rectangles). For $\Lambda = 100$ TeV, the values of $\tan \beta$ are all larger than bounds from nucleon decay, whereas for $\Lambda = 200$ TeV, the values of $\tan \beta$ from radiative symmetry breaking roughly double, and two of the models are allowed. These are the first two in Table 1. Note that less than 53 points are plotted in Fig. 1 since many of these models are on top of each and some of them (four models) turn out not to be physical.

To conclude, we have studied the phenomenological viability of generalized supersymmetric $SU(5)$ grand unified theories with gauge-mediated SUSY breaking by calculating the
upper limits on $\tan \beta$ from nucleon decay in these theories. We find that the predicted values of $\tan \beta$ are mostly inconsistent with the constraints from nucleon decay. Our results suggest that if theories with GMSB are to be taken as serious SUSY GUT candidates beyond the standard model, the bilinear and/or trilinear soft terms cannot vanish at the messenger scale, and gauge groups other than $SU(5)$—together with their associated implementation of dynamical SUSY breaking—are required for acceptable low-energy phenomenology.

ACKNOWLEDGEMENTS

HH thanks the Swedish Natural Science Research Council for financial support. The work of KP is supported by a grant from the Magnus Ehrnrooth Foundation, and the work of KH, KP and DXZ by the Academy of Finland (No. 37599).
REFERENCES

[1] M. Dine, W. Fischler, and M. Srednicki, Nucl. Phys. B189 (1981) 575; S. Dimopoulos and S. Raby, Nucl. Phys. B192 (1981) 353; M. Dine and W. Fischler, Phys. Lett. B110 (1982) 227; M. Dine and M. Srednicki, Nucl. Phys. B202 (1982) 238; L. Alvarez-Gaumé, M. Claudson, and M. Wise, Nucl. Phys. B207 (1982) 96; C. Nappi and B. Ovrut, Phys. Lett. B113 (1982) 175.

[2] M. Dine and A.E. Nelson, Phys. Rev. D48 (1993) 1277; M. Dine, A.E. Nelson, and Y. Shirman, *ibid.* D51 (1995) 1362; M. Dine, A.E. Nelson, Y. Nir, and Y. Shirman, *ibid.* D53 (1996) 2658.

[3] N. Seiberg, Phys. Lett. B318 (1993) 469; Phys. Rev. D49 (1994) 6857; Nucl. Phys. B435 (1995) 129.

[4] N. Seiberg and E. Witten, Nucl. Phys. B426 (1994) 19, Erratum-*ibid.* B430 (1994) 485; *ibid.* B431 (1994) 484.

[5] W. Skiba, Mod. Phys. Lett. A12 (1997) 737.

[6] N. Haba, N. Maru, and T. Matsuoka, Phys. Rev. D56 (1997) 4207.

[7] S. Dimopoulos, M. Dine, S. Raby, and S. Thomas, Phys. Rev. Lett. 76 (1996) 3494; S. Dimopoulos, S. Thomas, and J. D. Wells, Phys. Rev. D54 (1996) 3283; K. S. Babu, C. Kolda, and F. Wilczek, Phys. Rev. Lett. 77 (1996) 3070; J. A. Bagger, K. Matchev, D. M. Pierce, and R.-J. Zhang, Phys. Rev. D55 (1997) 3188; H. Baer, M. Brhlik, C.-H. Chen, and X. Tata, Phys. Rev. D55 (1997) 4463; N. G. Deshpande, B. Dutta, and S. Oh, Phys. Rev. D56 (1997) 519.

[8] R. Rattazzi and U. Sarid, Nucl. Phys. B501 (1997) 297; F. M. Borzumati, hep-ph/9702307.

[9] C. D. Carone and H. Murayama, Phys. Rev. D53 (1996) 1658; B. Blok, C.-D. Lü, and D.-X. Zhang, Phys. Lett. B386 (1996) 146.
[10] S. P. Martin, Phys. Rev. D56 (1997) 4207.

[11] M. S. Chanowitz, J. Ellis, and M.K. Gaillard, Nucl. Phys. B128 (1977) 506; R. Rattazzi, U. Sarid, and L.J. Hall, hep-ph/9405313.

[12] Y. Nir and N. Seiberg, Phys. Lett. B309 (1993) 337.

[13] P. Nath, A. H. Chamseddine, and R. Arnowitt, Phys. Rev. D32 (1985) 2348; J. Hisano, H. Murayama, and T. Yanagida, Nucl. Phys. B402 (1993) 46.

[14] J. Erler and P. Langacker, Phys. Rev. D52 (1995) 441.
TABLES

TABLE I. Limits on tan $\beta$ with $\Lambda = 100$ TeV.

| $(n_L, n_D, n_E, n'_U, n'_Q)^a$ | $(\tan \beta)_{\text{max}}^b$ | $m_{\text{Bino}}$ GeV | $m_{\text{Wino}}$ GeV | $m_{\tilde{Gluino}}$ GeV | $m_{\tilde{q}}$ GeV | $m_{\tilde{e}}$ GeV |
|-------------------------------|-------------------------------|-------------------|-------------------|----------------|----------------|----------------|
| (1, 4, 0, 0, 0)$^c$           | 17                            | 301               | 263               | 2565          | 2262          | 373            |
| (1, 3, 0, 0, 0)$^c$           | 13                            | 245               | 263               | 1971          | 1894          | 369            |
| (2, 4, 0, 0, 0)$^c$           | 11                            | 384               | 521               | 2565          | 2287          | 527            |
| (1, 1, 0, 0, 0)$^d$           | 5                             | 301               | 263               | 724           | 1006          | 383            |

$^a$n$_k$ specify the numbers of different types of messenger fields as defined in eq. (2).

$^b$Limit assumes $M_{HC} \leq 10^{17}$ GeV.

$^c$These three configurations are the only choices which allow tan $\beta > 10$.

$^d$5 + $\overline{5}$ model.
FIG. 1. The upper limit on $\tan \beta$ from nucleon decay vs. $\tan \beta$ calculated by assuming radiatively-broken $SU(2) \times U(1)$ symmetry and vanishing bilinear and trilinear soft couplings at the messenger scale. The black circles correspond to $\Lambda = 100$ TeV and the open rectangles to $\Lambda = 200$ TeV.