Charting the landscape of supercritical string theory

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Abstract

Special solutions of string theory in supercritical dimensions can interpolate in time between theories with different numbers of spacetime dimensions (via dimension quenching) and different amounts of worldsheet supersymmetry (via c-duality). These solutions connect supercritical string theories to the more familiar string duality web in ten dimensions, and provide a precise link between supersymmetric and purely bosonic string theories. Dimension quenching and c-duality appear to be natural concepts in string theory, giving rise to large networks of interconnected theories. We describe some of these networks in detail and discuss general consistency constraints on the types of transitions that arise in this framework.

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1 Introduction

Among the major advances in string theory has been the realization that several seemingly distinct perturbative versions of the theory are linked by duality, and overwhelming evidence has accumulated that these theories collectively emerge from a more fundamental framework known as M theory. The duality web connects type I, type IIA, type IIB and heterotic SO (32) and $E_8 \times E_8$ superstring theories. This is often depicted as a pointed diagram with M theory residing in the middle, bounded by links among five vertices representing individual superstring theories (11-dimensional supergravity is sometimes included as an additional vertex). This moduli space describes only the duality network connecting supersymmetric string theories in a critical number of spacetime dimensions. A number of consistent string theories possessing either non-critical target space dimensions or exhibiting no spacetime supersymmetry (or both) can be included in this picture.

Perturbative string theory in conformal gauge is described by a 2D field theory coupled to 2D gravity. The gravity theory on the string worldsheet possesses an anomalous Weyl symmetry, and the anomaly can be made to vanish by formulating the theory in a specific, critical number of spacetime dimensions, $D_c = 10$ for the superstring. The condition $D = D_c$, however, is not the only way to cancel the Weyl anomaly. Consistent string theories are known to exist in $D > D_c$ in the presence of a linear dilaton background. Furthermore, bosonic string theory appears to be another perturbatively self-consistent physical theory, albeit with an instability. Understanding the relationship between both non-critical and bosonic string theories and the more familiar incarnations of critical superstring theory has been a longstanding problem.

The solutions we present here solve this problem conclusively. In fact, we can classify the complete set of such solutions under certain conditions (though more possibilities may be allowed when these conditions are relaxed). The dynamics of our models describe a domain wall moving to the left at the speed of light, separating two phases described by distinct string theories. The right side of the wall describes a string theory with lower potential energy. The configuration as a whole can be thought of as the late time limit of an expanding bubble of a lower-energy vacuum. Transitions to the new vacuum can be interpreted as a renormalization group flow in the worldsheet theory, dressed with an exponential of $X^+$ to make the relevant perturbation strictly scale invariant.

In some instances, the dynamics interpolate between string theories in different numbers
of spacetime dimensions: in passing across the domain wall of the expanding bubble, the number of spatial dimensions is reduced dynamically. To simplify the exposition, we collectively refer to processes in this category as dimension quenching. In the examples we study, the stable endpoint of successive stages of dimension quenching is string theory in either the critical dimension (for type II theories) or in two dimensions (for bosonic and type 0 theories).

We also describe transitions that connect type 0 superstring theory dynamically with purely bosonic string theory. These solutions constitute precise dualities, unique in that they are realized cosmologically, rather than formally, or as adiabatic motion along moduli space. As with dimension-quenching transitions, time evolution in the target space equates to an RG flow on the worldsheet. For this reason, and because they involve a transfer of central charge contribution from various sources, we dub these relations c-dualities.

The intent of this paper is to present a global atlas of connections between various consistent string theories that arise from the above processes. The resulting networks, while not exhaustive, demonstrate the rich interconnectedness of string theory. Detailed expositions of dimension quenching and c-duality can be found in references [1][3], and in forthcoming work.

2 Dimension quenching

The solutions we study describe a reduction in the matter central charge on the worldsheet as a function of light-cone time $X^+$, triggered by a non-zero tachyon expectation value. The initial matter central charge is equal to the number of spacetime dimensions $D$. There is a time-like dilaton dependence $\sim q X^0$, which compensates the matter central charge excess if we set $q = \frac{6}{26}$ for the bosonic string or $q = \frac{9}{4}$ for the superstring.

The worldsheet dynamics are exactly solvable at the quantum level, despite the fact that the underlying 2D theories are fully interacting. (Quantum corrections are either absent, or are exact at one-loop order in perturbation theory.) The on-shell condition for a generic tachyon vertex operator $T(X)$ in the linear dilaton background is

$$@^2 T - 2V @ T + \frac{4}{6} T = 0;$$

(2.1)

where $V$ is the dilaton gradient. The second term in Eqn. (2.1) signifies an anomalous dimension, while the third term represents a non-linearly realized contribution to the scaling
dimension. Non-in nitesimal vertex operators satisfying Eqn. (2.1) will generally lead to complicated theories, since multiple insertions of $T(X)$ will become singular when they approach one another.

There are special choices of $T(X)$ for which the worldsheet theory is well-de ned and conformed to all orders in perturbation theory (and nonperturbatively) in $\theta^0$. De ning the lightcone frame $X = (X^0, X^1) = P - Z$, one sees that the exponential $\exp(X^+)$ is non-singular in the vicinity of another copy of itself. The conformal properties of vertex operators involving $\exp(X^+)$ are therefore completely well-behaved. If we choose $= \frac{2}{p\eta q}$, this operator has weight $(1;1)$ and zero anomalous dimension (that is, no piece of the dimension quadratic in the exponent). The vanishing of the anomalous dimension makes it possible to construct Lagrangian perturbations that deform the free theory while preserving conformal invariance exactly.

For a product of the form $T \exp(X^+)$, the tachyon perturbation will increase exponentially into the future in a lightlike direction. The dilaton rolls to weak coupling, and we can arrange for tachyon condensation to occur long after the big bang. This means that any e ects associated with the strongly-coupled region of the cosmology (located in the far past) are washed out by the subsequent expansion of the universe.

The tachyon couples to the worldsheet according to the Lagrangian

$$L = \frac{1}{2} \theta_0 X^+ \theta_0 X \quad \theta X^+ \theta_1 X + 0^2 \exp X^+ ; \quad (2.2)$$

where $\theta_0$ are derivatives along the worldsheet. This 2D eld theory is remarkably simple and exactly conformal, even at the quantum level. The potential barrier accelerates string states to the left, and their speed rapidly approaches the speed of light. Rather than describing a transition between two conventional string theories, this simple model connects a timelike linear dilaton background to a `nothing state,' into which no excitation, including the graviton, can propagate.

We can also introduce some dependence on a third coordinate $X_2$:

$$T = 2 e^{X^+} \quad \frac{2}{k} \cos(kX_2)e^{iX^+} ; \quad (2.3)$$

where

$$k = \frac{p - 2}{q} \quad 0 \quad \frac{1}{2} k^2 \quad (2.4)$$

3
One obtains a marginal perturbation that is Gaussian in $X_2$ in the long-wavelength limit $k!0$, and the resulting theory is exactly solvable. Classically, the tachyon becomes large in the future, and the theory acquires a mass term for the coordinate $X_2$, which decreases exponentially:

$$T(X^+_2) = \frac{2}{2} \exp X^+_2 + T_0(X^+) ;$$

$$T_0(X^+) = \frac{2}{2} \exp X^+ + \exp X^+ ;$$

Like the simple case described by the Lagrangian in Eqn. (2.2), this theory describes a domain bubble expanding at the speed of light. We can consider two basic categories of string trajectories in this background. We characterize one set of trajectories as generic, in that they carry energy in the $X_2$ direction. Upon encountering the domain bubble, the $X_2$ eld becomes frozen into a state of nonzero excitation and is pushed off along the wall to $X_1$ at late times (similar to string states in the vicinity of the bubble of nothing).

In addition, there is a special set of trajectories carrying no energy in the modes of the $X_2$ eld. These states are allowed to propagate through the domain wall and into the bubble interior. (One might think of the interface as a domain wall.) The amount of dynamical matter on the worldsheet therefore decreases dynamically as a function of $X^+$. The $X_2$ dimension is quenched for large $X^+$, and the region remaining at late times inside the tachyon condensate is a $D = 1$ dimensional theory.

$$\Delta(\partial_+ \Phi) = \begin{array}{c} \text{ } \\
\end{array}$$

$$\Delta G_{++} = \begin{array}{c} \text{ } \\
\end{array}$$

**Figure 1:** Feynman diagrams contributing to the renormalizations of the dilaton and metric. The massive $X_2$ eld (solid lines) propagates in the loops, while the massless elds (dashed lines) have oriented propagators. Quantum corrections terminate at one-loop order in perturbation theory.

The contribution to the central charge from the dilaton sector of the theory is determined by the dilaton gradient and the string-frame metric. Both of these objects shift by nite
am counts during the dimension-quenching transition. From the worksheet point of view, this is a one-loop renormalization of effective couplings arising when the $X_2$ field is integrated out. From the spacetime perspective, the effect is a backreaction of the tachyon onto the dilaton and string-frame metric. The result is that the dilaton contribution to the central charge increases by one unit, compensating the loss of the $X_2$ degree of freedom.

In fact, all quantum corrections in this theory are saturated at one-loop order in perturbation theory, with $X_2$ fields propagating in the loop. (The $X_2$ fields have oriented propagators; see [2] for details.) It is therefore possible to calculate this renormalization exactly. Most corrections coming from integrating out $X_2$ vanish in the $X^+!1$ limit, apart from contributions from the effective tachyon, the dilaton and the string-frame metric. The effective tachyon can be re-tuned to zero in the future, and what remains are corrections to the metric and dilaton gradient arising from the one-loop Feynman diagrams depicted in Fig. 1. Taking these contributions into account, the dilaton central charge is shifted up by one unit, and the total central charge of the theory remains constant.

There are also generalizations of these solutions connecting type 0, type II and stable or unstable heterotic string theories in diverse dimensions. Here we will focus on connections among type 0 and type II theories. (Transitions among heterotic theories give rise to a web of connections even more intricate than the ones we describe in this article.) In mapping out the various possibilities, it is helpful to sort the allowed transitions into the following categories:

- **Stable transitions**, in which no perturbation of the solution can destroy or alter the final state qualitatively;

- **Natural transitions**, in which no instability can destabilize the solution without breaking additional symmetry;

- **Tuned transitions**, in which the initial conditions of an unstable mode must be re-tuned to preserve the qualitative nature of the final state.

Under this classification scheme, transitions among bosonic string theories in different numbers of dimensions are tuned transitions. In particular, the constant $\Omega$ in Eqn. 2.2 must be tuned to a particular value $1$ to set the bosonic string tachyon to zero in the far future of the lower-dimensional final state.

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1. In general, this value will be regulator-dependent.
Tuned transitions between type 0 theories with $D = 1$

Transitions between type 0 string theories in even numbers of dimensions are natural by virtue of a discrete R-parity $q$, that acts with $a + 1$ on $G$ and a $-1$ on $G'$ (and, possibly, with a $1$ on some number of coordinates). However, type 0 string theories in odd dimensions preserve no such symmetry (see, e.g., [4] for a discussion of this point). Transitions between even- and odd-dimensional type 0 theories are therefore tuned transitions. Here we consider the case for which the dimension of spacetime decreases by $D = 1$.

The tachyon $T$ couples to the worksheet as a superpotential:

$$L = \frac{i}{2} \bar{d} d :T(X):; \quad (2.6)$$

with

$$:T = \exp X^+ + \frac{2}{2} :X^+ + \frac{2}{2} + \frac{2}{2} :X^+ + \phi^2 : \quad (2.7)$$

The quantity $\phi^0$ is a regulator-dependent coefficient that must be tuned to make the effective superpotential vanish in the limit $X^+ \rightarrow 1$. Unlike the even-dimensional type 0 theories, which come in two varieties (0A and 0B), there is only one kind of odd-dimensional type 0 theory. Starting from odd-dimensional type 0, the sign of $\phi$ determines whether one reaches type 0A or type 0B string theory in $D = 1$ dimensions as a final state. After defining the odd-dimensional GSO projection with a particular phase ($1^w = i$) in the Ramond/Ramond sectors, altering the sign of $\phi$ changes the ground state of the $2; \bar{2}$ fermions, and thereby reverses the effective GSO projection for the $D = 1$ remaining Majorana fermions.

These tuned transitions are represented by the diagonal lines in Fig. 2, with right-pointing downward (dashed) arrows representing transitions with $\phi > 0$ and left-pointing downward (dotted) arrows representing transitions with $\phi < 0$. The tuned transitions can be defined between type 0 linear dilaton backgrounds in any number of dimensions, with any value of $D$, as long as the final state is at least two-dimensional.

Natural transitions between even-dimensional type 0 states

When the number of spacetime dimensions reduces by an even number ($D = 2K$, $K \geq 2$)
Figure 2: The dimension quenching transitions in type 0 string theory define a semi-infinite lattice of connected theories. The diagonal lines are tuned transitions that reduce the spacetime dimensionality by one (right-pointing downward arrows have $\mu > 0$, and left-pointing downward arrows have $\mu < 0$). The vertical lines are natural transitions reducing the spacetime dimensionality by two. The horizontal lines do not represent dynamical transitions; rather, they represent the standard connection between type 0A and 0B string theory by T-duality or orbifolding by left-moving spacetime fermion number $(1)^{F_L}$ (the lowest horizontal arrow represents orbifolding, or alternatively a thermal T-duality). The lowest point on the diagram represents two-dimensional string theory of the kind described by the $\mathcal{C}=1$ matrix model.
during a transition, it is possible for the \( \psi \) to be natural, in the sense that a discrete symmetry \( g_L \) can preserve the qualitative nature of the final state and prevent the generation of an effective superpotential. For instance, the tachyon profile

\[
T = \frac{2}{2}\exp X^+ X_2 X_3 \tag{2.8}
\]

reduces the number of spacetime dimensions by \( D = 2 \) while preserving the chiral R-parity \( g_L \), which acts according to

\[
g_L : X_2 ! \ X_2 ; \ X_3 ! \ X_3 ; + \ ! \ + ; ! \ : \tag{2.9}
\]

Indeed, \( g_L \) constitutes a discrete R-symmetry, which forbids the generation of a superpotential in the effective theory describing the \( D = 2 \) embedding coordinates and their superpartners. These natural transitions are represented in Fig. 2 by downward solid arrows.

More generally, the tachyon profile

\[
T = \frac{2}{2}\exp X^+ (X_3 X_5 + X_4 X_5 + 2K X_{2K +1}) \tag{2.10}
\]

gives rise to a natural transition reducing the spacetime dimension by \( 2K \), preserving the discrete global symmetry

\[
! \ ; \ (X_3 ; X_5 ; 2K ; 2K ; X_3 ; X_5 ; 2K ; X_3) : \tag{2.11}
\]

This symmetry forbids the generation of an effective worldsheet superpotential for the string propagating in \( D = 2K \) dimensions.

Stable transitions from type 0 to type II

The natural transitions described above can be converted to stable transitions if one orbifolds by a discrete R-symmetry, such as \( g_L \). Modular invariance constrains the conditions under which one can orbifold by such a symmetry. A sufficient condition is that the spacetime dimension be even and the number \( K \) of orbifolded dimensions be equal to \( \frac{1}{2}(D - 10) \). In this case, the GSO projection inherited by the theory in the future is the chiral GSO projection of the type II string worldsheet. As a result, one can condense the tachyon without fine-tuning initial conditions to recover a stable, supersymmetric background in the distant future. Since all relevant operators that could alter the nature of the final state have been projected out by the action of \( g_L \), the final state is stable against all gauge-invariant perturbations.
In the limit $X^+ \rightarrow 1$, the massive string modes of the $10D$ theory have vanishing expectation value, the dilaton is lightlike and rolling to weak coupling, and the background preserves half the supersymmetries of type II string dynamics. Starting from an initial state with $D > 10$, the final state is a half-BPS vacuum of critical superstring theory. This establishes that supercritical string theories are connected by dimension quenching to the standard duality web of critical superstring theory.

![Diagram](image.png)

**Figure 3:** Natural dimension-quenching transitions of orbifolded type $0$ string theories terminate with type II string theory in the critical dimension ($D = 10$). These hierarchies are connected laterally to the type $0$ series by Scherk-Schwarz compactification and T-duality, or by a discrete Wilson line construction [4]. The type $0$ hierarchy in the middle is labeled according to the legend in Fig. 2.
The stable transitions to 10D type II string theory are represented by the vertical arrows on the left- and right-hand ladders of Fig. 3. As noted, these transitions are related to the natural transitions by orbifolding: These connections are represented in Fig. 3 by the horizontal arrows connecting unorbifolded type 0 theories with ladders representing transitions from type 0 orbifolded down to type II in the critical dimension. The inverse connection can be formed by a particular discrete Wilson line construction of the type described in [4].

Relation to timelike tachyon condensation

Any dimension-changing exact solution of the kind described above can be deformed to make the tachyon gradient timelike rather than lightlike. We may embed both possibilities in a family of theories, where the exponential dependence of the tachyon is \( \exp (B_0 X^0 + B_1 X^1) \), with \( B_1 \) varying such that the linearized equations of motion are still satisfied.

Specifically, we can vary \( B_1 \) continuously from \( P = 2 \) to 0, taking

\[
B_0 = q + B_1^2 + q^2 + j_{\text{tach}} \quad : \quad \frac{1}{2} B_0^2 - B_1^2 : \quad (2.12)
\]

The anomalous dimension of the vertex operator is simply

\[
\frac{1}{2} B_0^2 - B_1^2 : \quad (2.13)
\]

The properties of the tree-level string theory depend on \( D \) (i.e., the number of coordinates being eliminated during the transition) and on \( D \). Fixing \( D \), there are two limits in which \( D \rightarrow 0 \) and the worldsheet CFT is solvable. The rest is the case we consider here, in which \( B_0 = B_1 \). The second is the case in which \( B_1 = 0 \) identically, and \( D \rightarrow 0 \) [5,6]. This second limit preserves the full set of symmetries of a spatially flat FRW cosmology. However, it cannot describe the return to a supersymmetric vacuum, since \( D_{\text{nal}} = D = 10 \). It has been proposed [7,8] that solutions may exist with \( B_1 = 0 \) and \( D_{\text{nal}} = D_c \), in which the theory preserves FRW symmetries and the final state lies in the critical dimension. At present, the existence of such solutions remains conjectural, and requires the absence of a phase transition between \( D \rightarrow 0 \) of order one. Another tractable limit of such models is \( B_1 = 0 \) and \( D \rightarrow 1 \) with \( D \) held fixed. In this regime, the worldsheet theory can be analyzed using the techniques of large-N vector-like models, with \( D \) playing the role of the 2D 't Hooft coupling \( g^2 N \) [9,10].
Relation to 'Dimensional Duality'

Recently, a series of papers [8,11] appeared describing dynamical transitions between certain timelike linear dilaton backgrounds in $D > D_c$ as initial states, and final-state backgrounds on negatively curved spaces in $D = D_c$. The initial states of [11] are type 0 string theories with timelike linear dilaton, running to weak coupling in the future. The models, parametrized by an integer $h$, have initial-state geometries described by 7-dimensional at spatial slices, a time direction, a 2h-dimensional torus, and 2h real fiber directions, which we will call $Y^h$. The $Y$ directions are orbifolded over the 2h-torus, as well as orbifolded by a $Z_2$ symmetry which acts as $Y^h \rightarrow Y^h$, as well as with a sign on the $G^+$ supercurrent and a + sign on the $G$ supercurrent. Indeed, the models of [8,11] are special cases of the type 0/type II transitions described in [2], with the tachyon taken to be timelike instead of lightlike. (Order one corrections contributed by the $X^0$ sector must be assumed to cause no qualitative difference to the dynamics, as in the approach of [7].)

The total number of spacetime dimensions in the initial state is $D = 4h + 6$. The number of orbifolded dimensions is $K = 2h$. As for the natural transition models described earlier in this section, the number $K$ of orbifolded dimensions is equal to $\frac{1}{2}(D - 10)$. In section [4], we will show that under broad conditions, the number $K$ of orbifolded dimensions in the initial state must be equal to $\frac{1}{2}(D - 10)$ for any model of perturbative tachyon condensation whose endpoint is the type II superstring. The models of [11] constitute a nontrivial example of the classical string theory described in sec. [4] for dimension-reducing dynamical transitions with critical type II final state.

3 c-duality

One surprising outcome arises when considering type 0 string theory with no orbifold singularities in a timelike linear dilaton background. We will focus first on cases for which the tachyon perturbation has no dependence at all on the directions transverse to the lightcone.

The type 0 tachyon couples to the string worldsheet as a $(1,1)$ superpotential. The worldsheet theory is thus deformed by the following interaction Lagrangian:

$$L_{\text{int.}} = \frac{1}{8} G \partial \bar{T} @ T + \frac{1}{4} \partial @ \bar{T} \sim ;$$

where $\partial$ and $\bar{T}$ are right- and left-moving worldsheet fermions. In addition, the supersym-
metry algebra is modified by the following $F$-term

$$F = f Q ; \quad ^M g = f Q ; \quad \sim \quad ^N g = \frac{r}{8} G_{0}^{M N} \Theta_{N} T ; \quad (3.2)$$

When the tachyon is lightlike, the worldsheet potential vanishes, but the Yukawa coupling (the second term in Eqn. (3.1)), the $F$-terms and the interaction terms all grow as $X^+ \rightarrow 1$. Despite this fact, the fermion mass matrix remains nilpotent, and the physical frequencies of the fermion modes do not grow. With no worldsheet potential, no states are expelled from the interior of the domain bubble, and the number of spacetime dimensions does not change. The presence of the $F$-term, however, indicates that the worldsheet supersymmetry is spontaneously broken as $X^+ \rightarrow 1$.

Although the worldsheet theory is strongly interacting in its original variables at large $X^+$, we can describe the physical content precisely by invoking a series of canonical transformations. The starting point is to exchange the lightcone fermions for a bc ghost system with weights $\frac{3}{2}$ and $\frac{1}{2}$, denoted by $b_1, c_1$ (and similarly for left-movers):

$$^+ = 2c_1^0 M^{-1} b_1 + 2 (\Theta_+ X^+) c_1 ;$$
$$\sim ^+ = 2\delta^2 + M^{-1} b_1 + 2 (\Theta_+ X^+) c_1 ;$$
$$\sim = M \Theta_1 ;$$

(3.3)

where $M = \exp (X^+)$. In this new set of variables the Lagrangian consists of a free theory plus a perturbation proportional to $\exp (X^+)$, which becomes vanishingly small deep inside the region of large tachyon condensate.

A subsequent series of canonical transformations can be invoked that render the stress tensor and supercurrent in the following form [3]:

$$T = \frac{3i}{2} \Theta_+ c_1 b_1 + \frac{i}{2} c_1 \Theta_+ b_2 + \frac{i}{2} \Theta_+ (c_1 \Theta_2 c_1) + T_{\text{matter}} ;$$

$$G = b_1 + i \Theta_+ c_1 b_2 c_1 \quad G_{\text{matter}} \frac{5}{2} \Theta_+^2 c_1 ; \quad (3.4)$$

At this stage, the supersymmetry is completely non-linearly realized. In this form, the $D$-dimensional theory is a free worldsheet theory with a bc ghost system, $D$ free scalars and $D-2$ free fermions (with appropriate left-or right-moving counterparts). The central charge
receives a contribution of 26 from the matter-dilaton system (the dilaton gradient is again renormalized), and 11 from the bc ghosts. The theory therefore exhibits the correct central charge for the worldsheet of an RNS superstring in conformal gauge.

In fact, the system at late times ts into a construction due to Berkovits and Vafa [12], in which the bosonic string is embedded in the solution space of the superstring (i.e., it is a formal rewriting of the bosonic string as a superstring). This is just a manifestation of the fact that to any theory one may add any amount of non-linearly realized local symmetry as a redundancy of description. From the perspective of the bosonic theory, the role of the supersymmetry is to restrict the $b_1$ and $c_1$ fields to their ground states, up to gauge transformation.

We emphasize that, in formulating the transition to bosonic string theory, we have not integrated out any fields, and no information contained in the original type 0 phase has been lost. The late-time description is achieved entirely through canonical variable redefinitions. In one set of variables (referred to as UV variables in [3]) the theory is weakly-coupled in the initial phase, while in another set (denoted IR variables in [3]) the theory is weakly-coupled in the late-time limit. Since no information is lost in moving to IR variables, in which the theory reduces to bosonic string theory at large $X^+$, this constitutes a precise duality. Because of the cosmological nature of the transition (meaning that time evolution in the target space drives RG ow on the worldsheet), and because there is again a transfer of central charge from the UV fermions to the dilaton, we refer to this transition as c-duality.

Relationship between the type 0 web and the bosonic string Bosonic string theory in various dimensions can be reached via c-duality from type 0 string theory in various dimensions. The basic transitions induced by the tachyon profile $T = \exp(X^+)$ connect $D$-dimensional type 0 string theory to the bosonic string in $D$ noncompact dimensions with an $SO(D-2)$ current algebra. The current algebra retains a memory (via the set of representations that appear) of whether the parent type 0 theory was type 0B or 0A. In both cases the current algebra contains some states in which the fermions are all simultaneously periodic. The GSO projection in the R sector of the type 0 parent theory determines the state of the fermion zero modes in the sector of the current algebra with periodic fermions. That is, the type of RR forms that appear in the type 0 parent theory
Figure 4: Transitions to bosonic string theory in two dimensions and higher can occur via c-duality, starting from points in the type 0 series. The transitions connect type 0 theories in D dimensions to bosonic string theory in D (straight, solid arrows) or D − 1 (curved, solid arrows) noncompact dimensions, with compact current algebra factors. The straight, solid arrows are c-duality transitions, and the curved solid arrows are the detuned versions of tuned dimension-changing transitions with D = 1. The detuned transitions combine c-duality and dimension quenching. At the bottom of the type 0 series, c-duality connects type 0B in 2D to two copies of bosonic string theory in 2D, while the analogous arrows connects type 0A to a single copy of bosonic string theory in 2D. Both of these endpoints are also connected by tuned transitions to type 0 string theory in 3D. The transitions to bosonic string theory in this figure are understood to be superimposed on the corresponding structure in the type 0 hierarchy depicted in Fig. 2 (which is displayed in light blue; color is available in the electronic version).
determine the \( \text{SO}(D - 2) \times \text{SO}(D - 2) \) chirality of the bispinor states of the current algebra.

A related set of transitions can be obtained by "detuning" the tuned dimension-changing type 0 transitions. For instance, we can modify the transition from type 0 in \( D \) dimensions to type 0 in \( D - 1 \) dimensions by deforming \( \theta \) away from its tuned value. This is equivalent to deforming the \( D - 1 \) dimensional nal state with a tachyon vev of the form \( T = (\theta_{\text{tuned}}^0 \exp(\lambda)) \). This induces a further transition to the bosonic string in \( D - 1 \) noncompact dimensions with an \( \text{SO}(D - 3) \times \text{SO}(D - 3) \) current algebra. We will refer to this type of solution as a detuned transition.

The detuned transitions follow the trajectories of the tuned transitions arbitrarily closely over an arbitrarily long time. Ultimately they deviate and land on a nal state described by the bosonic string. The detuned solutions can therefore be thought of as hybrid transitions combining dimension quenching and c-duality, reducing the number of noncompact dimensions and changing the type of string theory relative to the initial state.

These transitions are depicted in Fig. 4. Direct transitions to bosonic nal states are drawn with solid straight lines, while transitions that combine c-duality with dimension quenching are shown with solid curved lines. The latter are meant to suggest that, in starting from an initial type 0 phase in even dimension (\( D \)), the worksheet theory evolves along a pathway that is very similar to the tuned dimension-quenching transition that lands on type 0 string theory in odd dimension (\( D - 1 \)). In the large \( X^+ \) region, however, the ow misses the type 0 nal phase and lands instead on bosonic string theory in odd dimension (\( D - 1 \)).

c-duality in \( D = 2 \)

The c-duality transition in two dimensions involves various subtleties that depend on the GSO projection. In type 0B there is a physical Ramond ground state \( \Omega \) of weight zero that survives the GSO projection. Combining this with the NS ground state \( \Omega \), one can construct projection operators onto states of the form \( \frac{1}{2} \left( \Omega \right) \). These projectors break states of the theory into two sectors that are decoupled in the OPE \(^{13,15} \). This means that the type 0B string describes two decoupled universes in closed-string perturbation theory. However, it is known that nonperturbative effects associated with D-instantons lift the degeneracy of the projection operators and couple the two universes \(^{14,15} \).

The degeneracy of the NS and R ground states persists in the Berkovits-Vafa embedding
describing the $X^+$!1 region of the theory. In other words, there are still projection operators at $X^+$!1 on the worksheet that decompose the string Hilbert space into two sectors that are not coupled through perturbative interactions. To the extent that we can trust perturbation theory, the final state of the c-duality transition from type 0B string theory is therefore two decoupled copies of 2D bosonic string theory in a spacelike linear dilaton background. It is not clear, however, whether this description is correct nonperturbatively, or whether D-brane effects may couple the two universes as they do in the type 0B initial background.

D-instanton contributions can be suppressed, but not without altering the properties of the theory that render the transition solvable. In the presence of a Liouville wall in the initial type 0B theory, D-instantons are exponentially suppressed, and the same may be true for the final state. However, our c-duality transitions can only be constructed as exact CFTs in the strong-coupling limit, for which the Liouville wall is absent. Otherwise, there would be a nontrivial interaction in conformal perturbation theory between insertions of $\exp\left( X^1 \right)$, associated with the Liouville wall, and $\exp\left( X^+ \right)$, associated with the c-duality transition.)

In type 0A string theory in 2D, the fundamental string has no physical Ramond states whatsoever. The same is true of the Berkovits-Vafa embedding of the 2D bosonic string when the GSO projection in the R sector is $\left( 1 \right)^x = 1$. Although the Ramond sector has four degenerate ground states, it is not possible to reverse the GSO projection by acting with a fermion zero mode while satisfying the physical state conditions. Only one ground state satisfies the $G_0;G_0$ physical state conditions, and if the corresponding Ramond state is removed by the GSO projection, the Ramond sector is completely empty. In this case the CFT has only one component, and the endpoint of the transition describes a single copy of bosonic string theory in two dimensions.

4 A classification theorem

It would be useful to classify all possible supercritical vacua that can make transitions to points in the 10D supersymmetric moduli space. As we have seen in this paper, supercritical starting points above supersymmetric vacua are somewhat constrained, but still diverse. In this section we will give a partial classification of the possibilities under certain simple conditions.
4.1 Supercritical vacua above type II in 10 dimensions

The set of supercritical vacua that can relax to type II string theory in 10 dimensions by classical condensation of NS tachyons is large. We can narrow the scope of our classification by restricting to cases in which the worldsheet description of the low can be analyzed reliably with a semiclassical treatment of the worldsheet fields transverse to the X directions. (Worldsheet quantum corrections resulting from the dynamics of the X directions vanish when the tachyon gradient is lightlike [2].) In other words, we will consider cases in which the worldsheet stress tensor and supercurrent of the initial-state background are well approximated quadratic expressions in free fields. Furthermore, the tachyonic perturbation is assumed to be well approximated everywhere in the classical worldsheet vacuum manifold by a quadratic expression in free fields, dressed with an exponential of the lightlike direction $X^+$ . (For paradigmatic examples and further analysis of this type of transition, see [2].)

Let us list our technical assumptions. In addition to the requirement of semiclassical worldsheet dynamics, we will restrict to cases for which:

- The final state is a single copy of an orbifolded ten-dimensional space with a string-frame metric and lightlike linear dilaton, rolling to weak coupling in the future;

- The GSO projection of the final theory is the chiral GSO projection of the type IIA/B string;

- The initial state is described by an oriented string theory;

- The continuous worldsheet gauge symmetry of the initial theory is linearly realized $(1;1)$ superconformal invariance (i.e., the initial theory must be a type 0 or type II string);

- The $(1,1)$ worldsheet supersymmetry is unbroken in the final-state vacuum manifold (if the worldsheet supersymmetry is spontaneously broken and the vacuum energy is tuned to zero, the resulting physics is that of the bosonic string [3]).

Discrete gauge symmetry

We will first prove that the discrete worldsheet gauge symmetry in the initial state must be $\mathbb{Z}_2 \oplus \mathbb{Z}_2$ acting chirally on the supercurrents $G ; G$ . We know that the final-state discrete

\footnote{Of course, one can expand the search by relaxing some of these conditions.}
gauge symmetry is the group generated by the operators \((1^\text{LW})_k\) and \((1^\text{RW})_k\), which act trivially on the \(X^M\) and with a \(1\) on \(G;G\) respectively. The tachyon condensation is described by semi-classical worldsheet RG ow, which can only break, and not enhance, gauge symmetries. The discrete worldsheet gauge symmetry of the initial state must therefore be at least \(Z_2 \times Z_2\), which acts as a chiral \(R\)-parity on the supercurrents \(G;G\).

We can also show that the discrete gauge group is no larger than \(Z_2 \times Z_2\). The key idea is that for a larger group, the worldsheet theory at \(X^+ + 1\) must contain multiple components as direct summands of the CFT, generated by the mechanism described in [13]. If the orbifold group were larger than the minimal \(Z_2 \times Z_2\), there would have to be an element \(g_3\) that acts trivially on both supercurrents, but nontrivially on matter degrees of freedom. We now focus on the sector twisted by \(g_3\). Since we have assumed the RG ow is semi-classical, the NS tachyon describing the condensation must be untwisted. This twist is therefore a good symmetry throughout the entire ow, including in the IR limit.

Now we consider the nal-state theory and focus on the sector twisted by \(g_3\). By assumption, we have a nal state that is an unorbifolded type II theory, so \(g_3\) acts trivially on all infrared degrees of freedom. The physics of these types of discrete gauge symmetries was analyzed in [13]. The NS/NS ground state in the sector twisted by \(g_3\) has weight zero in the infrared. The Hilbert space built on the twisted vacuum gives rise to an identical copy of the ten-dimensional universe in the untwisted vacuum. Therefore, if the discrete gauge group in the initial theory is larger than the minimal \(Z_2 \times Z_2\), the nal state has at least two disconnected components. We conclude that the GSO group of the supercritical theory must be exactly \(Z_2 \times Z_2\) if the nal state is critical type II string theory on a single connected component \(\mathbb{R}^{9\vert\Phi}\).

4.2 Geometric action of the GSO group

In addition to acting on the supercurrents, the generators \((1^\text{LW})_k\) and \((1^\text{RW})_k\) of the GSO group may act on the coordinates \(X^M\) of the \(D\)-dimensional initial theory. In the general case, this geometric action must respect the symmetries of the initial state. In the limit where the initial state is at and noncompact, the geometric action of the GSO group must be a subgroup of the Euclidean group, which means it must be a combination of shifts and

\(^4\)Here we are using the fact that the gauged worldsheet supersymmetry of the string is only \(N = 1\), and not \(N = 2\) or higher.
rotations. Since the subgroup is $\mathbb{Z}_2 \times \mathbb{Z}_2$, the rotations must be 180 degree rotations at most.

We now wish to determine how the GSO group $\mathbb{Z}_2 \times \mathbb{Z}_2$ acts on the coordinates $X^0$, $P^i$.

Functions of $X^M$ can be decomposed into eigenfunctions $f^{(0)}(X)$ under the action of $\mathbb{Z}_2 \times \mathbb{Z}_2$. It is easy to see that only $f_{++}$ and $f_{--}$ are allowed by modular invariance, and not $f_{+-}$ or $f_{-+}$. If we now have a function $f_+$ that is odd under $\mathbb{1}_L^{\text{FW}}$ and even under $\mathbb{1}_R^{\text{FW}}$, acting on the normal-ordered operator $f(X)$: with the right-moving supercurrent $G$ gives an NS/NS state of half-integer spin surviving the GSO projection. This is forbidden by modular invariance. We conclude that $\mathbb{1}_L^{\text{FW}}$ must act identically to $\mathbb{1}_R^{\text{FW}}$ on the coordinates $X^M$, meaning that the geometric action of $\mathbb{1}_R^{\text{FW}}$ must be trivial on all $D$ coordinates.

Next we will show that, in addition to the total fermion parity $\mathbb{1}_R^{\text{FW}}$, the chiral fermion parity $\mathbb{1}_R^{\text{FW}}$ must act trivially on the vacuum manifold $M_R$ of the final configuration, assuming the final state is type II rather than type 0 on macroscopic scales.

Lem m a

Consider the action of $\mathbb{1}_R^{\text{FW}}$ on the zero-energy vacuum manifold $M_R$ of the worldsheet potential. By assumption, $M_R$ has ten dimensions. Let $L$ be the typical distance scale of $M_R$ (where $L = 1$ in the case for which the final state is noncompact). Our assumption that the world is semiclassical on the worldsheet implies that $L \gtrsim p_0$. It is therefore clear that $\mathbb{1}_R^{\text{FW}}$ must act trivially on $M_R$ if the local physics is to be type II, rather than type 0, on scales smaller than $L$.

For instance, there must exist an unstable mode of the tachyon $T$ if the operation $\mathbb{1}_R^{\text{FW}}$ acts nontrivially on $M_R$. To see this, we can decompose functions on $M_R$ into even modes $f_{++}$ and odd modes $f_{--}$. The GSO projection removes the $f_{++}$ modes of the tachyon and returns the $f_{--}$ modes. If $\mathbb{1}_R^{\text{FW}}$ acts nontrivially on $M_R$, then the odd modes $f_{--}$ must exist. If $\tilde{\gamma}$ is the eigenvalue of the lowest odd mode $f_{--}$, the frequency-squared of the lowest surviving tachyon mode is $\frac{2}{L}$. The eigenvalues of the Laplacian for all eigenmodes are of order $L^2$ in the semiclassical limit $L \gtrsim p_0$. In this limit, therefore, there are modes of the tachyon with imaginary frequency, which grow exponentially in time.

We conclude that the final 10-dimensional state cannot be supersymmetric type II string theory unless the chiral GSO projection acts trivially on the vacuum manifold $M_R$. We can use this fact to constrain the action of the chiral GSO projection on the $D$ coordinates of the initial supercritical state. We consider two possible cases: 1) No point of the initial theory
is xed under the action of (1\(\bar{\mathbf{f}}_{\mathbf{w}}\)) or 2) there are nonempty xed loci under the action of (1\(\bar{\mathbf{f}}_{\mathbf{w}}\)).

Case 1: Non xed loci of (1\(\bar{\mathbf{f}}_{\mathbf{w}}\))
Suppose we adopt the first case, in which (1\(\bar{\mathbf{f}}_{\mathbf{w}}\)) acts on the D-dimensional initial state without xed points. Since the vacuum manifold \(M_{\mathbb{R}}\) is a subspace of the original D-dimensional spacetime, any xed point in \(M_{\mathbb{R}}\) is necessarily a xed point in the D-dimensional space, so (1\(\bar{\mathbf{f}}_{\mathbf{w}}\)) must act on \(M_{\mathbb{R}}\) without xed points as well. The GSO projection means that the physical 10D geometry is a xed-point-free quotient of \(M_{\mathbb{R}}\), by the action of (1\(\bar{\mathbf{f}}_{\mathbf{w}}\)), with antiperiodic boundary conditions for \(T\) around the \(\mathbb{Z}_2\) one-cycle. By the lemma given above, the local physics is that of type 0, not type II, on scales smaller than \(L\).

This result is intuitively clear when one considers the necessity of having massless fermions in 10D. Spacetime fermions arise only from twisted sectors of the chiral GSO projection. If this projection has no xed points, then all such twisted sectors describe stretched strings, whose length is the typical scale \(L\) of the manifold. As long as \(P = \mathbb{Z}_2\), all spacetime fermions are much heavier than the string scale if there are non xed loci of (1\(\bar{\mathbf{f}}_{\mathbf{w}}\)). There are therefore no light fermions that could fill out supersymmetry multiplets in the \(X^+!1\) limit.

Case 2: Nonempty xed loci of (1\(\bar{\mathbf{f}}_{\mathbf{w}}\))
In the second case, we consider a xed locus of (1\(\bar{\mathbf{f}}_{\mathbf{w}}\)), whose codimension we denote by \(n_{\text{odd}}\). (If there are multiple disconnected components of the xed locus, each one may be assigned its own codimension \(n_{\text{odd}}\).) The vacuum manifold of the bosonic potential must lie entirely within the xed locus. Otherwise, the chiral GSO projection acts nontrivially on \(M_{\mathbb{R}}\), giving a background whose local physics is type 0, not type II.

Let \(Y^a\) (with \(A = 1, \ldots, n_{\text{odd}}\)) be local coordinates normal to the xed locus. These are, by definition, odd under the action of (1\(\bar{\mathbf{f}}_{\mathbf{w}}\)). Let \(X^a\) (\(a = 1, \ldots, n_{\text{even}}\)) be the coordinates longitudinal to the xed locus. The sum \(n_{\text{even}} + n_{\text{odd}}\) is equal to the total dimension \(D\) of the initial state. We will now show that \(n_{\text{even}}\) and \(n_{\text{odd}}\) is an invariant of any semiclassical RG flow.

The tachyon \(T\) couples to the worldsheet as a superpotential:

\[
L = \frac{i}{2} \int \mathcal{D}X \, T(X); \quad (4.1)
\]
where

\[ T(X) = \exp X^+ f(X^a;Y^A) \]  

We wish to expand around an arbitrary point \( x^a_{(0)} \) on the vacuum manifold, choosing coordinates such that \( x^a_{(0)} = 0 \). The condition that the ow be semiclassical means that the ow is controlled by terms that are constant, linear and quadratic in \( X;Y \) and their superpartners, with terms of cubic and higher order making no qualitative difference to the transition. The function \( f \) can therefore be approximated as a quadratic function of \( X \) and \( Y \) in an expansion around the origin.

By assumption, our vacuum manifold preserves worldsheet supersymmetry. The F-term conditions

\[ \Theta_X \cdot T(0) = \Theta_{X^a} T(0) = \Theta_{Y^A} T(0) = 0 \]  

mean that the constant and linear terms in \( f \) must be zero at \( x^a = Y^A = 0 \). In other words, the function \( f \) must vanish to second order at any point on the vacuum manifold. The form of the quadratic term is constrained by invariance under the chiral GSO projection. The superspace measure \( d^\sim d \) is odd under \( (1\hat{f}^{1\#}) \), so the function \( T(X;Y) \) must be odd as well, linking only \( X \) coordinates and \( Y \) coordinates. We thus have

\[ T = \exp X^+ q_{IA} X^a Y^A + \text{(cubic)} \]  

around any point on the vacuum manifold, for some constant matrix \( q_{IA} \). It follows that \( n_{\text{even}} \) \( n_{\text{odd}} \) is an invariant of tachyon condensation when \( 0 \) corrections are small in the dynamics of the \( X^a;Y^A \) sector. Since \( n_{\text{odd}} \) vanishes in the final, critical type II theory, we must have \( n_{\text{odd}} = \frac{1}{2}(D - 10) \) in the initial state. This reduces the set of possibilities to the models described in [2], in the limit where the initial state has all dimensions noncompact and at.

5 Other simple type 0/type II theories and connections between them

We have classified the set of supercritical vacua that can decay to type II string theory via classical solutions describing the lightlike condensation of perturbative instabilities, under
the assumption that all scales $L$ transverse to the light cone $X$ are much larger than the string scale: $L \gg P^{-0}$. The full set of supercritical type II and type 0 theories is immensely larger, and we will not attempt to classify it here. Instead, we will examine a simple subset of type 0 and type II theories in diverse dimensions, and chart the connections between them.

The sector of the supercritical landscape we will describe consists of theories with linearly-realized local $(1,1)$ superconformal symmetry on the worldsheet, with all dimensions non-compact and at, and with a possible orbifold singularity at the origin. We will assume the discrete gauge group is the minimal $\mathbb{Z}_2 \times \mathbb{Z}_2$ GSO group; larger groups can be constructed by orbifolding the theories we discuss here.

The worldsheet field content for such theories consists of D bosons $X^H$ and their worldsheet superpartners $\tilde{X}^H$. By the same argument given above, ($1^+\omega$) (which acts as a 1 on both supercurrents) must act as a +1 on all $X^H$. The chiral GSO generator ($1^+\omega$) can act as a 1 on some of the $X^H$, and the various possibilities are tightly constrained by modular invariance.

The simplest constraint comes from the level-matching condition in the R/NS and NS/R sectors. We let the number of coordinates $X^H$ that are even (resp. odd) under ($1^+\omega$) be denoted by $n_{\text{even}}$ (resp. $n_{\text{odd}}$). If we make the standard choice of GSO projection, which is NS/NS and R/NS, then level matching requires $n_{\text{even}} = 10 + 16K$. We refer to this as the standard series of type 0/II hierarchies. Each hierarchy has a particular value of $K$, and term inates in a stable type II theory with no geometric orbifold singularities. We can also consider the opposite projection on the NS side of the NS/R and R/NS sectors, namely NS/R and R/NS. These sectors are level-matched if and only if $n_{\text{even}} = 2 + 16K$. We denote the set of string theories defined this way the Seiberg series. In all cases, the GSO projection in the NS/NS sector is $+\leftrightarrow$, corresponding to a GSO projection of $=\leftrightarrow$ in the matter sector.

For any value of $K$ and $n_{\text{odd}}$, $D$ is always even in both series, and both have consistent OPEs and modular-invariant partition functions. For the stable, bottom rungs (that is,

---

5The series is so named because they are based on a generalization of the recently discovered two-dimensional type II theories [13]. Indeed, the models of [13] constitute the lowest-dimensional example of the Seiberg series.
For \( n_{\text{odd}} = 0 \) of the standard series, the partition function is

\[
Z \sum_{F} \frac{d}{2} \frac{d}{2} \left( j \right)^{2(D-2)} 4^2 0^2 \frac{\Gamma^2}{2} \left( I_{\text{NS}}^{+} \right) \left( I_{\text{R}}^{+} \right) \left( I_{\text{NS}}^{+} \right) \left( I_{\text{R}}^{+} \right) ;
\]

(5.1)

where

\[
I_{\text{NS}}^{+} \left( \right) \frac{1}{2} \left( Z_{0}^{0} \left( \right) \right)^{\frac{D-2}{2}} \left( Z_{1}^{0} \left( \right) \right)^{\frac{D-2}{2}} ;
\]

\[
I_{\text{R}}^{+} \left( \right) \frac{1}{2} \left( Z_{-1}^{0} \left( \right) \right)^{\frac{D-2}{2}} ;
\]

(5.2)

with

\[
Z \left( \right) \frac{1}{\left( \right)} \left( 0; \right) \]

(5.3)

For the stable, bottom rungs of the Seiberg hierarchies, the partition function is

\[
Z \sum_{F} \frac{d}{2} \frac{d}{2} \left( j \right)^{2(D-2)} 4^2 0^2 \frac{\Gamma^2}{2} \left( I_{\text{NS}}^{+} \right) \left( I_{\text{NS}}^{+} \right) \left( I_{\text{NS}}^{+} \right) \left( I_{\text{R}}^{+} \right) \left( I_{\text{NS}}^{+} \right) \left( I_{\text{R}}^{+} \right) ;
\]

(5.4)

For \( n_{\text{odd}} = 0 \), the theories are tachyon free, since the matter part \( V_{(\frac{1}{2} \frac{1}{2})} \) of the vertex operator must have GSO charges = , but all embedding coordinates are even under \( \left( \bar{I} \right)^{w} \). For \( n_{\text{odd}} = 0 \), there are always tachyons. If \( Y^{a} \) are the odd coordinates and \( X^{a} \) are the even coordinates, any function \( f(Y;X) \) satisfying \( f(Y;X) = f(X;Y) \) corresponding to the matter component of an allowed tachyon vertex operator: \( f(X;Y) \). Solvable classical backgrounds exist describing tachyon condensation that reduce the dimension \( D \) while keeping \( n_{\text{even}} \) \( n_{\text{odd}} \) constant. Such solutions are described by tachyon profiles of the form

\[
T = \exp X^{+} \frac{X_{\text{odd}}}{a+1} X^{a} Y^{a} ;
\]

(5.5)

By the mechanism of central charge transfer described in [2], the solution transfers \( \frac{3}{2} D = 3 n_{\text{odd}} \) units of central charge to the dilaton gradient, by an exactly calculable renormalization effect on the worksheet.

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This generates an infinite number of hierarchies of cosmological string theory, parametrized by an integer $K$. Each hierarchy is connected vertically by solvable dynamical transitions that preserve $K$ and reduce $n_{\text{odd}}$ and $n_{\text{even}}$ by the same amount. The lowest element of each hierarchy, with $n_{\text{odd}} = 0$, is tachyon-free.

These hierarchies are also connected 'laterally' to the hierarchy of unorbifolded type 0 strings. The horizontal connections can be understood either as orbifolding or as T-duality on a twisted circle. Starting from type 0 in D dimensions, one can orbifold by reflection of $n_{\text{odd}}$ directions, together with the action of $\{1\}^{\pm}$, to reach a consistent, modular-invariant theory when $D - 2n_{\text{odd}}$ is equal to $10 + 16K$ or $2 + 16K$. This orbifolding produces a theory with $n_{\text{odd}}$ odd directions in the hierarchy labeled by $K$. Starting with one of the type II theories or type 0 orbifolds, one can orbifold by spacetime fermion number modulo two ($\{1\}^z$) and reach an unorbifolded type 0 phase. Orbifolding by $\{1\}^z$ removes the geometric singularity of the type 0 orbifolds along with the twisted states that live there, since those twisted sectors are all spacetime fermions.

Instead of orbifolding, one can also move horizontally between type 0 and the chiral-GSO hierarchies by a discrete Wilson line construction, as described in [4]. We can start with type 0A/B string theory, compactifying on a circle with a Wilson line for the symmetry $g$. Taking the radius to zero and transforming to T-dual variables produces the same effect as orbifolding the type 0B/A theory by $g$, if such an orbifold is modular-invariant. (If the orbifold is not modular-invariant, the T-dual description of the small-radius limit is again an unorbifolded type 0 theory.) To avoid switching type 0A with type 0B, one may combine $g$ with $\{1\}^z$ (which reverses the signs of all R/R sectors) in the action implemented by the Wilson line. To move from the chiral-GSO hierarchies to the type 0 hierarchy, one may compactify on a circle with Scherk-Schwarz boundary conditions for the spacetime fermions and take the small-radius limit, switching to T-dual variables.

Partition functions for $n_{\text{odd}} \neq 0$

The tachyon-free case $n_{\text{odd}} = 0$ is the simplest, in that all bosons $X^M$ make the same contribution to the partition function. For $n_{\text{odd}} \neq 0$, we have contributions from odd (orbifolded) coordinates $Y^A$. To organize these contributions, we define

$$J^{-1} \sim (\cdot) \quad (5.6)$$
to be the path integral for all bosons and fermions in the sector with boundary conditions defined by the four periodicities $; \sim ; \sim$ for the two supercurrents $G; G'$ on the two-cycles. That is, a zero represents an antiperiodic supercurrent and a one represents a periodic supercurrent. By our choice of GSO projection, the periodicity of the odd bosons $Y^A$ is correlated with that of the supercurrents. In particular, an odd boson $Y$ comes back to itself with a phase of $(1)^{\sim}$ around a cycle when the supercurrents have periodicities $; \sim$. The path integrals $Y^{\sim}$ for an odd boson are

$$
Y^{\sim}_0(\; ; ) = 4 \; ^2 \; ^0 \; _2 \; \frac{1}{2} j(\; ) j^{2} ; \\
Y^{\sim}_1(\; ; ) = \frac{j(\; ) j}{j_{10}(0; \; ) j} ; \\
Y^{\sim}_0(\; ; ) = \frac{j(\; ) j}{j_{01}(0; \; ) j} ; \\
Y^{\sim}_1(\; ; ) = \frac{j(\; ) j}{j_{00}(0; \; ) j} ;
$$

(5.7)

The path integrals for the odd fermions are the same as the $Z$, except that the right-moving fermions have the periodicities of the left-moving supercurrents, and vice-versa. So the path integral for the bosons and fermions in the $Y$ multiplets is equal to

$$
J^{-\sim}_i (\; ; ) = h \; Y^{-\sim}_i (\; ; ) \; Z \; ( ) Z^{-\sim}_i (\; ) \; \frac{1}{2} \; ^{1\; i_{\text{odd}}} ;
$$

(5.8)

when $G$ has the boundary conditions $; \sim$ and $G'$ has boundary conditions $\sim; \sim$. We can include the fermionic superpartners of the even coordinates as well, which always have periodicities that match those of the supercurrents. The periodicities of the superghosts $; ; \sim; \sim$ match those of the supercurrents as well, and exactly cancel the path integral of two real fermions of each chirality.

Defining $I^{-\sim}_i (\; ; )$ to be the path integral over the $Y^A$ degrees of freedom and their superpartners, the fermions in the $X^A$ multiplets, and the superghosts, we have

$$
I^{-\sim}_i (\; ; ) = J^{-\sim}_i (\; ; ) \; Z \; ( ) Z^{-\sim}_i (\; ) \; \frac{2}{2} \; ^{2\; \text{sgn}} ;
$$

(5.9)

From these path integrals, we define partition functions over the $Y; ; \sim$ and superghost...
degrees of freedom in each sector:

\[
\begin{align*}
\mathcal{I}_{NS^+ = NS^+} &= \frac{1}{4} I_{0 \rho}^0 + I_{1 \rho}^0 + I_{0 \rho}^0 + I_{1 \rho}^0 + I_{0 \rho}^0 + I_{1 \rho}^0 ; \\
\mathcal{I}_{NS^0 = R} &= \frac{1}{4} I_{0 \rho}^0 + I_{1 \rho}^0 + I_{0 \rho}^0 ; \\
\mathcal{I}_{R^0 = NS^+} &= \frac{1}{4} I_{1 \rho}^0 + I_{0 \rho}^0 ; \\
\mathcal{I}_{R^0 = R} &= \frac{1}{4} I_{1 \rho}^0 ; \\
\end{align*}
\]

where we have suppressed the arguments ; and used the fact that \( I_{i \rho}^0 = I_{-i \rho}^0 = 0 \). The partition functions for the \( X \) and reparametrization ghosts are the same in every sector, giving a total of

\[
Z \int \frac{d^2 j}{2} \left( \cdots \right)^{2(n_{\text{even}} 2)} 4^2 0^2 \frac{\text{n_{even}} - 2}{2} 
\]

\[
\mathcal{I}_{NS^+ = NS^+} \quad \mathcal{I}_{R^0 = NS^+} \quad \mathcal{I}_{S^+ = R} \quad \mathcal{I}_{R^0 = R} 
\]

for the standard series. For the Seiberg series, we have

\[
Z \int \frac{d^2 j}{2} \left( \cdots \right)^{2(n_{\text{even}} 2)} 4^2 0^2 \frac{\text{n_{even}} - 2}{2} 
\]

\[
\mathcal{I}_{NS^+ = NS^+} \quad \mathcal{I}_{R^0 = NS} \quad \mathcal{I}_{S^+ = R} \quad \mathcal{I}_{R^0 = R} 
\]

6 Conclusions

The solutions described in this article interpolate between theories that were previously thought to be completely distinct. These solutions connect string theories in different numbers of spacetime dimensions and with different amounts of worldsheet supersymmetry. We have also found examples that connect superstring theories to purely bosonic string theory, which sheds new insight on the relationship between the two theories. These interpolations

\[6\text{For recent developments along these lines, see [11].}\]
are cosmological in nature, in that tachyon perturbations can be interpreted as the nucleation of domain walls moving at the speed of light, and the late-time physics is described by the theory deep inside the tachyon condensate. These solutions serve to connect a much wider class of string backgrounds to the supersymmetric duality web of superstring theory.

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