Leveraging Secondary Reflections and Mitigating Interference in Multi-IRS/RIS Aided Wireless Networks

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Abstract—Reconfigurable surfaces (RS) have recently emerged as an enabler for smart radio environments where they are used to actively tailor/control the radio propagation (e.g., to support users under adverse channel conditions). If multiple RSs are deployed (e.g., coated on various buildings) to support different groups of users, it is critical to jointly optimize the phase-shifts of all RSs to mitigate interference amongst them as well as to leverage the secondary reflections amongst them. Motivated by these considerations, this paper considers the uplink transmissions of multiple users that are grouped and supported by multiple RSs to communicate with a multi-antenna base station (BS). We first formulate two optimization problems: the weighted sum-rate maximization and the minimum achievable rate (from all users) maximization. Unlike existing works that considered single user or single RS or multiple RSs without inter-RS reflections, the considered problems require the joint optimization of the phase-shifts of all RS elements and all beamformers at the multi-antenna BS. The two problems turn out to be non-convex and thus are difficult to be solved in general. Moreover, the inter-RS reflections give rise to the coupling of the phase-shifts amongst the RSs, making the optimization problems even more challenging to solve. To tackle them, we design alternating optimization algorithms that provably converge to locally optimal solutions. Simulation results reveal that by effectively mitigating interference and leveraging the secondary reflections amongst the RSs, there is a great benefit of deploying more RSs to support different groups of users so as to achieve a higher rate per user. This gain is even more significant with a larger number of elements per RS. Without properly dealing with the secondary reflections, by contrast, increasing the number of RSs can adversely impact the network throughput, especially for high transmit power.

Index Terms—Intelligent reflecting surface (IRS), reconfigurable intelligent surface (RIS), secondary reflections, multi-IRS/RIS interference, multi-user communications, cooperative beamforming.

I. INTRODUCTION

RECONFIGURABLE surface (RS) also known as Intelligent Reflecting Surface (IRS) or Reconfigurable Intelligent Surface (RIS) has been emerging as an enabler for realizing so-called smart radio environments in which the radio propagation can be deliberately controlled/tailored by deploying nearly-passive tunable metasurfaces [2]–[6]. A wireless system assisted by RSs and hence “reconfigure” the environment to combat shadowing and fading impairments or to create a well-scattered environment for spatial multiplexing.

An RS is a planar metasurface that is made up of a large number of nearly passive reflecting elements. Each of these elements can be optimized to alter the amplitude and/or the phase-shift of the reflected signal. These elements together can help to effectively “reshape” the wireless channels [2]–[6]. Such an ability to reconfigure the radio medium provides an additional dimension to design wireless systems. Conventional wireless optimization/designs, on the other hand, take the additional dimension to design wireless systems.

When multiple RSs are deployed (e.g., coated on various buildings) to support distributed users, it is critical to jointly optimize the phase-shifts and/or amplitudes of all the RSs to mitigate the interference as well as to leverage the secondary reflections amongst them. For example, consider the uplink transmission from multiple users that are grouped and supported by multiple RSs to a multi-antenna base station (BS), as shown in Fig. 1. Due to the proximity amongst these groups of users, the RSs can be deployed close to each other. The signal from a given user not only reaches the BS through a reflection from the RS dedicated to the user group (referred to as the primary reflection) but can also be reflected by nearby RSs before reaching the BS (referred to as the secondary reflection or multi-RS reflection). It is clear that...
under such a scenario, besides the primary reflections from
the users to the BS via each RS, the secondary reflections
amongst multiple RSs cannot be ignored in general. These
secondary reflections can either cause multi-user and/or multi-
RS interference or strengthen the signal reception at the
BS. Therefore, by jointly tuning the phase-shifts of all the
RSs, one can simultaneously manage the interference and
leverage the multi-RS “secondary” reflections to enhance the
signal reception at the BS. Such an observation is similar
to multi-cell cooperative multi-input multi-output (MIMO)
communications [36].

Most existing works on multiple RSs, e.g., [14]–[16],
ignored the secondary reflections amongst RSs. In [14],
the authors studied a resource allocation problem for a downlink
wireless communication network with multiple distributed RSs
to maximize the system energy efficiency with a maximum
transmit power constraint and minimum rate requirements. The
authors of [15] aimed to maximize the minimum signal-to-
interference-plus-noise ratio (SINR) amongst users by jointly
optimizing the transmit precoding vector at the BS and the
phase-shifts at the RSs for a downlink multi-RS system. In
[16], the outage probability and the average sum-rate for
a single source, single destination and multi-RS system were
studied where the best RS is selected to be active at a time.
Many other works that ignored the secondary reflections by
assuming that the strength of the signals reflected onto multiple
RSs is negligible. Such an assumption is, however, only reason-able when the RSs are far from each other or the signal
is reflected on many RSs (e.g., more than two). When the
RSs are deployed close to each other, e.g., in central business
districts (CBDs) or dense deployment scenarios, as shown in
this paper, the secondary reflections are in fact significant.
Additionally, all the aforementioned works did not study the
interference management and did not leverage the cooperation
amongst RSs via “secondary” reflections. In [17], [18], the
authors considered the beamforming design for the case with
two RSs in which one of them is placed close to the BS and
the other one is placed close to the users. The authors show
that there is a significant gain as compared to the case with
a single RS. However, in the considered scenario there are
only two RSs that aim to support a single group of users.
Therefore, the scenario in [17], [18] does not account for the
interference amongst RSs and amongst different groups of
users. In practice, especially in a dense urban environment
where shadowing and fading are severe (e.g., caused by
multiple buildings/structures), multiple RSs can be deployed
within one cell to support various groups of users whose direct
links to the BS are weak or blocked.

Motivated by these considerations, this work takes the
first step in exploring the impact of secondary reflections
amongst multiple RSs by considering the uplink transmis-
sion of multiple groups of users that are supported by
multiple RSs to communicate with a multi-antenna BS as
illustrated in Fig. 1 [1]. For this scenario, we consider
two commonly-adopted optimization problems: the weighted
sum-rate maximization and the minimum-SINR maximiza-
tion [1] in which we jointly optimize the phase-shifts of all
the RSs and the beamforming vectors at the BS for all users.

As aforementioned, besides the multi-user interference from
multiple groups of users, multi-RS reflection also induces
additional multi-user interference. The problems become more
challenging to solve due to the coupling of the phase-shifts
amongst multiple RSs. Specifically, the adjustment of the
phase-shifts at one RS impacts the reflected signals from other
RSs as well. As such, optimizing/tuning the phase-shifts at one
RS [8]–[13] or multiple RSs but for a single group of users
(e.g., [14]–[18], [38]–[49]) is insufficient in general. In fact,
we need to optimize the RS elements for all the RSs and all
users while considering the coupling among them due to the
inter-RS reflections.

Maximizing the weighted sum-rate for multi-user MIMO
systems without RSs has been well studied, for example,
in [29]–[33]. Some recent works also studied single or multiple
RSs considering the weighted sum-rate such as [37]–[45].
In [37], the authors considered the downlink of a multi-user
multi-input single-output (MISO) system with a single RS and
jointly optimized the transmit beamforming and phase-shifts
of RS elements. In [38], the authors considered multiple RSs
deployed at the cell-edge to improve the signal reception
by optimizing the precoding matrices at the BS and the
phase-shifts of the RSs. Scenarios with multiple BSs, multiple
RSs, and multiple users in downlink communications were
studied in [39] where the RS-user association was optimized
considering the co-channel interference. In [46]–[49], the
authors considered the multi-beam multi-hop routing problem
for this downlink of a multi-RS and multi-user system in which
each user communicates with the BS with the help of a set of
RSs. In these works, even though there are multiple RSs that
participate in the communications from the BS to each user,
only the primary reflections from the RSs are considered and
the signal is successively reflected on a sequence of RSs. For
the minimum-SINR maximization problem, the work in [17],
[18] is a special case of the problem investigated in this
paper, which is obtained by setting the number of RSs to two
and by assigning no user to the second BS. To the best of
our knowledge, none of the existing works in the literature
has considered the secondary reflections for multiple RSs in
general scenarios. Our major contributions in this paper are
summarized as follows:

- First, we study the impact of inter-RS or secondary
  reflections in a multi-user, multi-RS network. To this end,
  we formulate and solve two well-known optimization
  problems: the weighted sum-rate maximization and the
  minimum-SINR maximization over the users.
- Second, we tackle these two non-convex optimization
  problems by designing alternating optimization algo-
rithms that are proven to converge to locally optimal
  solutions. For the first problem, we transform each
  sub-problem to a difference of convex (DC) problem and
  use the software CVXPY/DCCP package in Python [26],
  [27] to efficiently solve it. For the second problem,
  a relaxed version of each of its sub-problems is trans-
  formed to a semidefinite program (SDP) problem which
can be efficiently solved by SDP standard solvers. Unlike
existing works that did not account for the inter-RS reflec-
tions, the two considered problems are more challenging
to solve as we need to deal with the coupling of the phase-shifts among different RS elements.

- Third, by optimizing the phase-shifts of all the RSs and the beamforming vectors for all the users, we show that the inter-RS reflections can be leveraged to contribute to improving the signal reception at the BS, leading to an improved rate per user. We observe that the gain is more significant with a larger number of elements per RS.

- Last, we perform extensive simulations to evaluate the impact of secondary reflections in various settings. For example, we observe about 6.4%, 15.6%, or 29.9% rate gains when exploiting the secondary reflections for a system with 2, 4, or 6 RSs, respectively. Also, when the number of RS elements is increased, the gain is also increased; for instance, there are about 7.2%, 15.6%, and 27.9% gains for a system with four RSs each with 32, 64 and 128 elements, respectively. However, without properly managing the secondary reflections, increasing the number of RSs may adversely impact the network throughput, especially for high transmit power. For example, the achievable rate of six RSs is worse than that of four RSs for a transmit power equal to 25.5 dBm or higher with the linear minimum mean squared error (MMSE) based receiver, or for a transmit power equal to 18.2 dBm or higher with the MMSE based non-linear receiver with successive interference cancellation (MMSE-SIC).

The rest of this paper is organized as follows. In Section II, the system model and the problem formulation for multi-RS system are presented. In Section III, a locally optimal solution for the weighted sum-rate maximization is introduced. The min-rate/SINR maximization problem is studied in Section IV. In Section V, numerical results are discussed, and conclusions are drawn in Section VI.

II. SYSTEM MODEL

We consider a system with an $N$-antenna BS and $K$ single-antenna users. Due to the presence of scattered obstacles in the network, users are partitioned/clustered into $L$ groups whose direct radio links to the BS are severely blocked. Such a scenario is often the case in dense CBD/urban areas with high-rise buildings and structures. $L$ RSs are then deployed to support the transmission of these users, as depicted in Fig. 1. We assume that the $l$-th RS is comprised of $M_l$ elements and serves $K_l$ single-antenna users in the $l$-th group, $l = 1, 2, \cdots, L$. The total number of users is $\sum_{l=1}^{L} K_l = K$.

To simplify the notation, we denote the set of users in the $l$-th group, the RS elements of the $l$-th RS, and the RSs as $K_l = \{1, 2, \cdots, K_l\}$, $M_l = \{1, 2, \cdots, M_l\}$, and $L = \{1, 2, \cdots, L\}$, respectively.

In general, each RS element can be controlled to adjust both its amplitude and phase. However, similar to many other works in literature (e.g., [2], [4], [5], [17], [18]), for simplicity and cost-effective implementations, the amplitude of all RS elements is set to one and we only tune/optimize their phases. The results and analysis in the sequel can be easily extended to optimize both amplitudes and phases.

The signal from the $k$-th user in the $l$-th group (denoted as the $k^{(l)}$-th user for short) can reach the BS by one of the following paths via at least one RS: (i) single or primary reflection from the $k^{(l)}$-th user onto any $l$-th RS before reaching the BS, (ii) double reflections (via two RSs) from the $k^{(l)}$-th user to the $l$-th RS, then to the $l$-th RS before reaching the BS (where $l \neq l'$), (iii) triple or more reflections (via three or more RSs), e.g., from the $k^{(l)}$-th user to the $l$-th RS, then to the $l$-th RS and subsequently to the $l'$-th RS before reaching the BS (where $l$, $l'$ and $l''$ are pair-wise different). As aforementioned, most of the works on multiple RSs, e.g., [14]–[18], [38]–[49], only consider the primary reflection (type (i)) and ignore the secondary and the triple/higher-order reflections amongst RSs. In triple or higher-order reflections (type (iii)), a signal traverses through more than two RSs, hence its impact is likely negligible due to the large effective pathloss. On the other hand, the secondary reflections amongst RSs may not be negligible in practice, and this is the focus of this paper. In addition, secondary reflections can cause additional interference at the BS. Therefore, if we can properly leverage the secondary reflections from the intended user and manage the interference caused by multiple RSs, the system performance can be improved. It is worth mentioning that the secondary reflections are particularly significant when the RSs are not too far from each other, e.g., in urban environments with dense deployment of RSs. In the sequel, we consider setups for which closely located RSs are still in the Fraunhofer far-field distance of each other [52], [53].

Let $u_{k^{(l)}} \in \mathbb{C}^{M_l \times 1}$, $D_{l,l'} \in \mathbb{C}^{M_l \times M_{l'}}$, and $G_l \in \mathbb{C}^{N \times M_l}$ denote the baseband equivalent channel matrix from the user $k$ in the group $l$ to the $l$-th RS, the baseband equivalent channel matrix from the $l$-th RS to the $l$-th RS (for $l \neq l'$), and the baseband equivalent channel matrix from the $l$-th RS to the BS links, respectively, with $l, l' \in L$ and $k \in K_l$. Let $\theta_k \in \mathbb{C}^{M_l \times 1}$ denote the phase-shifts vector of the $l$-th RS. We assume that the direct links from the users to the BS are weak and can be ignored. Thus, the effective channel from the $k^{(l)}$-th user can
be written as follows

\[ h_{k(l)} = \sum_{i=1}^{L} G_i \Phi_i u_{i,k(l)} + \sum_{i=1, i \neq l}^{L} G_i \Phi_i \tilde{D}_{i,l} \Phi_i u_{i,k(l)}, \tag{1} \]

for \( l \in \mathcal{L} \) and \( k(l) \in \mathcal{K}_l \), where \( \Phi_i \) denotes the diagonal matrix of the \( l \)-th RS.

Denote \( R_{i,k(l)} = G_i \text{diag}(u_{i,k(l)}) \in \mathbb{C}^{N \times M_l} \) as the cascaded channel matrix from the \( k(l) \)-th user to the BS, the cascaded channel matrices \( \tilde{D}_{i,l} \) from the primary and secondary reflections, and the thermal noise power \( \sum_{i=1}^{L} \sum_{j=1}^{M_l} \tilde{d}_{i,k(l),j} \). Thus, we can rewrite (1) as follows

\[ h_{k(l)} = \sum_{i=1}^{L} R_{i,k(l)} \theta_i + \sum_{i=1, i \neq l}^{L} G_i \Phi_i \tilde{D}_{i,l} \Phi_i u_{i,k(l)}, \tag{2} \]

where \( Q_{i,k(l),j} \) denotes the total cascaded channel matrix from the \( k(l) \)-th user to the \( j \)-th element of the \( l \)-th RS to the \( \tilde{l} \)-th RS to the BS without the phase-shifts at the \( \tilde{l} \)-th RS and the \( \tilde{l} \)-th RS, where \( l \in \mathcal{L} \), \( j \in \mathcal{M}_l \) and \( k(l) \in \mathcal{K}_l \).

From (2), we can see that it is sufficient to estimate \( \{ R_{i,k(l)} \} \) and \( \{ Q_{i,k(l),j} \} \) for jointly designing the reflection coefficients \( \{ \theta_i \} \) in the considered multi-RS cooperative system [17], [18], [21]. Channel estimation for multi-antenna systems is a well-investigated area and several mature techniques exist. In [7], [54] and [55], the authors have proposed efficient methods to estimate the effective cascaded channels for double-RS aided multi-user MIMO systems. These methods can be directly applied to estimate the CSI for both the primary reflections, i.e., \( \{ R_{i,k(l)} \} \) and the secondary reflection, i.e., \( \{ Q_{i,k(l),j} \} \) for any pair of RSs \( l \) and \( \tilde{l} \). In this paper, therefore, we assume that the cascaded channel matrices \( \{ R_{i,k(l)} \} \) and \( \{ Q_{i,k(l),j} \} \) are accurately estimated at the BS.

During the uplink data transmission, the received signal at the BS is given by

\[ y_{k(l)} = \mathbf{w}_{k(l)}^H \mathbf{y} = \mathbf{w}_{k(l)}^H \left( \sum_{i=1}^{L} \sum_{k=1}^{K_l} \left( \sum_{l'=1}^{L} \sum_{k'=1}^{K} \mathbf{R}_{l',k'} \theta_{l'} \right) + \sum_{i=1, i \neq l}^{L} \sum_{j=1}^{M_l} \mathbf{Q}_{i,k(l),j} \right) \mathbf{s}_{k(l)} + \mathbf{n}, \tag{3} \]

where \( \mathbf{n} \) is the noise vector at the BS. From (3), we can see that it is sufficient to jointly estimate \( \{ R_{i,k(l)} \} \) and \( \{ Q_{i,k(l),j} \} \) for designing the reflection coefficients \( \{ \theta_i \} \) in the considered multi-RS cooperative system [17], [18], [21]. Channel estimation for multi-antenna systems is a well-investigated area and several mature techniques exist. In [7], [54] and [55], the authors have proposed efficient methods to estimate the effective cascaded channels for double-RS aided multi-user MIMO systems. These methods can be directly applied to estimate the CSI for both the primary reflections, i.e., \( \{ R_{i,k(l)} \} \) and the secondary reflection, i.e., \( \{ Q_{i,k(l),j} \} \) for any pair of RSs \( l \) and \( \tilde{l} \). In this paper, therefore, we assume that the cascaded channel matrices \( \{ R_{i,k(l)} \} \) and \( \{ Q_{i,k(l),j} \} \) are accurately estimated at the BS.

III. WEIGHTED SUM-RATE MAXIMIZATION

First, we consider the users’ weighted sum-rate maximization problem subject to a minimum rate requirement per user. The problem is formally formulated as follows

\[ \text{(P1): max}_{\{ \mathbf{w}_{k(l)} \}, \{ \theta_i \}} \sum_{l=1}^{L} \sum_{k=1}^{K_l} \alpha_{k(l)} \log_2(1 + \gamma_{k(l)}) \]

s.t. \[ \frac{\sum_{l=1}^{L} \sum_{k=1}^{K_l} \gamma_{k(l)} - 2R_{k(l)}/B - 1}{\forall l \in \mathcal{L}, \ k \in \mathcal{K}_l, \ |\theta_{l,j}| = 1, \forall l \in \mathcal{L}, \ j \in \mathcal{M}_l}, \tag{6} \]
where \( \gamma_k(l) \) is given in (5), \( \{ \alpha_k(l) \} \) are predefined non-negative weights with \( \sum_k \alpha_k(l) = 1 \), and \( \{ R(l) \} \) specify predefined minimum rates requirement of each user \( l \).

Besides the multi-user interference term in the SINR expression given in (5), problem (P1) is non-convex due to the unitary constraints on the phase-shifts \( \{ \theta_{j,l} \} = 1 \). To tackle it, we propose an AO algorithm that can be proven to converge to a locally optimal solution of problem (P1).

### A. Optimization of \( \{ w_k(l) \} \) for Fixed \( \{ \theta_l \}_{l=1}^L \)

For fixed \( \{ \theta_l \}_{l=1}^L \), the effective channel of each user \( h_k(l) \) in (2) is fixed and thus problem (P1) reduces to \( K \) sub-problems, each of which is equivalent to maximizing the SINR of the \( k(l) \)-th user as given in (5) and can be formulated as

\[
(P2): \max_{w_k(l)} \quad \frac{P_k(l) w_k(l)^H h_k(l)^H h_k(l) w_k(l)}{w_k(l)^H \left( \sum_{(l,k) \neq (l,k')} P_{l,k'} h_k(l)^H h_k(l') + \sigma^2 I \right) w_k(l)}. \tag{7}
\]

It can be shown that (P2) is a convex optimization problem, and in fact it has a closed-form optimal solution [23]. Denoting \( H = [H_1, \ldots, H_L] \in \mathbb{C}^{N \times K} \) and \( W = [W_1, \ldots, W_L] \in \mathbb{C}^{N \times K} \), where \( H_l = [h_1(l), \ldots, h_{K_1}(l)] \in \mathbb{C}^{N \times K_1} \) and \( W_l = [w_1(l), \ldots, w_{K_1}(l)] \in \mathbb{C}^{N \times K_1} \) are the effective user-to-BS channel matrix and the receive beamforming matrix applied at the BS, respectively. The solution of problem (P2) is the MMSE based one given by [23]

\[
W_{MMSE} = (H P P H^H + \sigma^2 I)^{-1} H P, \tag{8}
\]

where \( P = \text{diag} \left( \left[ p_1, p_2, \ldots, p_L \right] \right) \in \mathbb{C}^{K \times K} \), with \( p_l \triangleq \left[ \sqrt{P_1(l)}, \sqrt{P_2(l)}, \ldots, \sqrt{P_{K_1}(l)} \right] \) is the diagonal transmit power matrix for all the \( K \) users. In practice, the equivalent noise power, \( \sigma^2 \), needs to be estimated. Also, the solution to problem (P2) can often be simplified to a suboptimal zero-forcing (ZF) precoding where the noise term \( \sigma^2 I \) in (8) is omitted and the matrix pseudo inverse is applied [23].

### B. Optimization of \( \{ \theta_l \}_{l=1}^L \) for Fixed \( \{ w_k(l) \} \)

For fixed \( \{ w_k(l) \} \), problem (P1) in (6) can be reformulated as follows

\[
(P3): \max_{\{ \theta_l \}} \quad \sum_{l=1}^L \sum_{k=1}^{M_l} \alpha_k(l) \log_2 (1 + \gamma_k(l)) \quad \text{s.t.} \quad \gamma_k(l) \geq 2^{R_k(l)}/B - 1, \quad \forall l \in L, \quad k \in K_l, \quad |\theta_{l,j}| = 1, \quad \forall l \in L, \quad j \in \mathcal{M}_l. \tag{9}
\]

Similar to (P1), problem (P3) is non-convex. We note that, because of the secondary reflections, the SINR of a given user depends on multiple RSs. This is captured by the coupling of the \( L \) phase-shift vectors \( \{ \theta_l \}_{l=1}^L \) in \( \gamma_k(l) \) for each user. Note that most of the existing works without accounting for the impact of secondary reflections do not need to deal with the coupling among the \( L \) phase-shift vectors \( \{ \theta_l \}_{l=1}^L \).

To tackle this new issue, we design an AO algorithm to solve problem (P3). Specifically, we fix \( \{ \theta_l \}_{l \neq \hat{l}} \), and solve the problem for \( \theta_{\hat{l}} \) for each \( \hat{l} \in L \) in an iterative manner. However, each resultant sub-problem is still non-convex. To solve it, we observe that the objective function is a difference of two convex (DC) functions. We thus convert it into a DC optimization problem and use the disciplined convex-concave programming (DCCP) method for solving it. This sub-problem is solved later in Section III.C. Different non-convex optimization methods, such as the alternating direction method of multipliers (ADMM) [57], [58] may be used to tackle the considered problem. However, unlike the DCCP method, the point of convergence of the ADMM algorithm and the quality of the solution are not guaranteed in general.

In the following, we first derive \( \gamma_k(l) \) as a function of \( \theta_{\hat{l}} \). There are two scenarios to be considered: i) the \( k(l) \)-th user belongs to the \( l \)-th RS (i.e., \( l = \hat{l} \)) and ii) the \( k(l) \)-th user does not belong to the \( l \)-th RS (i.e., \( l \neq \hat{l} \)).

**Theorem 1:** The SINR of the \( k(l) \)-th user at the BS can be written as a function of \( \theta_{\hat{l}} \) as follows

\[
\gamma_k(l) = \frac{\left| q_{l,k}(l) \theta_{\hat{l}} + p_{l,k}(l) \right|^2}{\sum_{(l',k') \neq (l,k)} \left| q_{l',k'}(l') \theta_{\hat{l}} + p_{l',k'}(l') \right|^2 + \sigma^2(l,k)}, \tag{10}
\]

where

\[
\sigma^2(l,k) = \sigma^2 w_{k,l}^H w_{k,l}, \tag{11}
\]

\[
q_{l,k}(l) = \begin{cases} \sqrt{P_{l,k}(l)} w_{k,l}^H R_{l,k}(l) + S_{l,k}(l) & \text{for } l = \hat{l}, \\ \sqrt{P_{l,k}(l)} w_{k,l}^H R_{l,k}(l) + T_{l,k}(l) & \text{for } l \neq \hat{l}, \end{cases} \tag{12}
\]

and

\[
p_{l,k}(l) = \begin{cases} \sqrt{P_{l,k}(l)} w_{k,l}^H U_{l,k}(l) & \text{for } l = \hat{l}, \\ \sqrt{P_{l,k}(l)} w_{k,l}^H (U_{l,k}(l) + S_{l,k}(l) \theta_{\hat{l}}) & \text{for } l \neq \hat{l}. \end{cases} \tag{13}
\]

with \( S_{l,k}(l) \), \( T_{l,k}(l) \) and \( U_{l,k}(l) \) being independent of \( \theta_{\hat{l}} \) as defined in (32) and (33), respectively.

**Proof:** See Appendix A.

Equation (10) has the same structure for all \( k(l) \) users but the constituent terms are different depending on whether the users belong to the \( l \)-th RS or not, as shown in (12) and (13). Moreover, to account for the direct links from the users to the BS, one may update the parameter \( p_{l,k}(l) \) in the SINR of the \( k(l) \)-th user, as shown in (10), which is independent of the phase-shifts of the RSs.

### C. Optimization of \( \theta_{\hat{l}} \) for Fixed \( \{ \theta_l \}_{l \neq \hat{l}} \) and Fixed \( \{ w_k(l) \} \)

For each \( \hat{l} \), we substitute \( \gamma_k(l) = \gamma_k(l)(\theta_{\hat{l}}) \) as a function of \( \theta_{\hat{l}} \) as shown in (10). Then, problem (P3) becomes

\[
(P3.1): \max_{\theta_{\hat{l}}} \quad \sum_{l=1}^L \sum_{k=1}^{M_l} \alpha_k(l) \log_2 (1 + \gamma_k(l)(\theta_{\hat{l}})) \quad \text{s.t.} \quad \gamma_k(l)(\theta_{\hat{l}}) \geq 2^{R_k(l)}/B - 1, \forall \ l, k, \quad |\theta_{\hat{l},j}| = 1, \quad \forall \ j \in \mathcal{M}_l. \tag{14}
\]

Due to the unitary constraints on \( \{ \theta_{l,j} \} \), problem (P3.1) is non-convex. However, we observe that \( \gamma_k(l)(\theta_{\hat{l}}) \) is a ratio
of two convex functions of $\theta_l$ (as shown in (10)). Hence, we can rewrite it as the difference of two convex functions. Consequently, to solve (P3.1), we first convert the problem to a DC optimization problem and use the DCCP method to solve it. For example, the CVXPY/DCCP package in Python [26], [27] can be used to efficiently solve this problem. Since current CVXPY/DCCP solvers have limited support for complex variables, we thus need to convert our problem to an equivalent one with real-valued variables. To this end, we define

$Q_{l,k}(t) \triangleq \begin{bmatrix} q_{l,k}^{(r)}(t) & -q_{l,k}^{(i)}(t) \\ q_{l,k}^{(i)}(t) & q_{l,k}^{(r)}(t) \end{bmatrix}$, \quad $\tilde{\theta}_l \triangleq \begin{bmatrix} \theta_l^{(r)}(t) \\ \theta_l^{(i)}(t) \end{bmatrix}$, and

$\tilde{p}_{l,k}(t) \triangleq \begin{bmatrix} p_{l,k}^{(r)}(t) \\ p_{l,k}^{(i)}(t) \end{bmatrix}$,

(16)

where $x^{(r)}$ and $x^{(i)}$ denote the real and imaginary part of a complex vector $x$, respectively, and $Q_{l,k}(t) \in \mathbb{R}^{2M_l \times 2}$, $\tilde{\theta}_l \in \mathbb{R}^{2M_l \times 1}$, and $\tilde{p}_{l,k}(t) \in \mathbb{R}^{2 \times 1}$. Given the definitions in (16), we can rewrite

$\|x\|_2$ denotes the $l_2$ norm of a vector $x$. To further simplify the notation, we define

$Z_{l,k}(t) \triangleq Q_{l,k}(t)Q_{l,k}^T$, \quad $\hat{Z}_{l,k}(t) \triangleq \sum_{(l',k') \neq (l,k)} Z_{l,k}(t)$,

$z_{l,k}(t) \triangleq Q_{l,k}(t)Q_{l,k}^T$, \quad $\hat{z}_{l,k}(t) \triangleq \sum_{(l',k') \neq (l,k)} z_{l,k}(t)$, and

$c_{l,k}(t) \triangleq \|\tilde{p}_{l,k}(t)\|_2^2$, \quad $\hat{c}_{l,k}(t) \triangleq \sigma_{l,k}^2 + \sum_{(l',k') \neq (l,k)} v_{l',k'}$

We then can rewrite $\gamma_k(t)$ in (14) as a function of $\hat{\theta}_l$ as follows

$\gamma_k(t) = \gamma_k(t)(\hat{\theta}_l) = \frac{\tilde{\theta}_l Z_{l,k}(t) \tilde{\theta}_l + 2z_{l,k}(t) \tilde{\theta}_l + \hat{c}_{l,k}(t)}{\|\tilde{\theta}_l Z_{l,k}(t) \tilde{\theta}_l + 2z_{l,k}(t) \tilde{\theta}_l + \hat{c}_{l,k}(t)}$.

(17)

In (17), both $Z_{l,k}(t)$ and $\hat{Z}_{l,k}(t)$ belong to $\mathbb{R}^{2M_l \times 2M_l}$ and are positive semi-definite, $c_{l,k}(t) > 0$ and $\hat{c}_{l,k}(t) > 0$. To convert problem (P3.1) in (6) to a DCCP optimization problem, we further define

$u_k(t) \triangleq \tilde{\theta}_l^T Z_{l,k}(t) \tilde{\theta}_l + 2z_{l,k}(t) \tilde{\theta}_l + c_{l,k}(t)$,

$v_k(t) \triangleq \tilde{\theta}_l^T Z_{l,k}(t) \tilde{\theta}_l + 2z_{l,k}(t) \tilde{\theta}_l + \hat{c}_{l,k}(t)$,

$s \triangleq \sum_{l=1}^{L} \sum_{k=1}^{M_l} \alpha_k(t) \log_2(v_k(t))$.

(18)

Problem (P3.1) in (14) is then equivalent to the following optimization problem.

(P3.2): \max_{\tilde{\theta}_l, u, v, s} \sum_{l=1}^{L} \sum_{k=1}^{M_l} \alpha_k(t) \log_2(u_k(t) + v_k(t)) - s

Algorithm 1 AO Algorithm to Solve the Non-Convex Optimization Problem (P1) in (6)

1. Input: Output at iteration $(t-1)$ \{\$w_k^{(t-1)}, \hat{\theta}_l^{(t-1)}\}.
2. Initialize $t = 1$, \{\$w_k^{(0)}, \hat{\theta}_l^{(0)}\}, \{\alpha_k(t)\}$, maximum number of iteration, and tolerance $\xi, \epsilon_2 > 0$.
3. Compute the weighted sum-rate $\Gamma^{(0)}(W^{(0)}, \hat{\Theta}^{(0)}) = \sum_{l=1}^{L} \sum_{k=1}^{M_l} \alpha_k(t) \log_2(1 + \gamma_k^{(0)}(w_k^{(0)}, \hat{\Theta}^{(0)}))$.
4. Repeat
5. Obtain $w_k^{(t)}$ from \{$\tilde{\theta}_l^{(t-1)}$\} by solving (8);
6. For $i = 1 \text{ to } L$:
7. Obtain $\hat{\theta}_l^{(t)}$ by iteratively solving (19) using the CVXPY/DCCP solver [26], [27] with $w_k^{(t)} = w_k^{(t)}$.
8. Compute the weighted sum-rate $\Gamma^{(t)}(W^{(t)}, \hat{\Theta}^{(t)}) = \sum_{l=1}^{L} \sum_{k=1}^{M_l} \alpha_k(t) \log_2(1 + \gamma_k^{(t)}(w_k^{(t)}, \hat{\Theta}^{(t)}))$.
9. If $\Gamma^{(t)} - \Gamma^{(t-1)} < \xi$;
10. Then
11. Set \{$w_k^{*}, \hat{\theta}_l^{*}$\} = \{$w_k^{(t)}, \hat{\theta}_l^{(t)}$\} and terminate.
12. Otherwise
13. Update $t \leftarrow t + 1$ and continue.
14. Output: A locally optimal solution $\hat{x}^{*} = \{w_k^{*}, \hat{\theta}_l^{*}\}$.

D. Proposed AO Algorithm

The proposed AO algorithm to solve the optimization problem (P1) in (6) iteratively is summarized in Algorithm 1, where $\hat{\Theta}^{(t)} = \{\hat{\Theta}_1^{(t)}, \hat{\Theta}_2^{(t)}, \ldots, \hat{\Theta}_L^{(t)}\}$ denotes the phase-shift matrix at the $t$-th iteration. The convergence of (P1) to a locally optimal solution is formally stated in the following theorem.

Theorem 2: Algorithm 1 is guaranteed to converge to a locally optimal solution of problem (P1) in (6).

Proof: See Appendix B.
Algorithm 1 (and Algorithm 2 introduced next) is solved in a centralized manner. It is generally not practical to be solved in a distributed manner as the RSs are low-cost devices with no, or very limited, computational capability and power resources [2]–[6]. They are thus not suitable to execute signal processing or optimization algorithms. In most existing works on RSs, unlike multi-cell systems, centralized solutions are widely adopted.

E. Non-Linear Receiver: MMSE-SIC

In the previous subsections, we have considered the linear MMSE based receiver. In this subsection, we consider the MMSE-SIC receiver, which is known to attain the channel capacity [56]. We also study the effect of secondary reflections on the performance of the MMSE-SIC receiver.

As described in [56], the idea of SIC receivers is to subtract the interference from decoded users during the decoding of the remaining users. In theory, by changing the order of decoding and cancellation, this nonlinear receiver structure is optimal and achieves the capacity region, but at a higher complexity cost compared with linear receivers. In this subsection, we consider a fixed decoding order for SIC, starting with the users in the first group. In each group, the users are decoded from their lower to higher indices. This choice for the decoding order is made just to simplify the mathematical presentation. In fact, the result obtained in this paper is applicable to any fixed decoding order.

We define \((l, k) > (\tilde{l}, \tilde{k})\) if and only if \(l > \tilde{l}\) or \(l = \tilde{l}\) and \(k > \tilde{k}\). Thus, the user \(k(l)\) is decoded after the user \(k(\tilde{l})\) if and only if \((l, k) > (\tilde{l}, \tilde{k})\). Consider decoding the \(k(l)\)-th user while all the \(\tilde{k}(l)\)-th users with \((\tilde{l}, \tilde{k}) < (l, k)\) have been decoded and their corresponding transmit signals or interference have been canceled. The MMSE weight vector for decoding this \(k(l)\)-th user is given by

\[
\hat{w}_{k(l)} = (H_{k(l)} P_{k(l)} P_{k(l)}^H H_{k(l)}^H + \sigma^2 I)^{-1} h_{k(l)} \sqrt{P_{k(l)}},
\]

where \(H_{k(l)} = [h_{k(l,1)}, \ldots, h_{k(l,L)}]\) is the \(H\) matrix after removing all the columns corresponding to the channels of the already decoded users, and \(P_{k(l)} = \text{diag}\left(\sqrt{P_{k(l,1)}}, \ldots, \sqrt{P_{k(l,L)}}\right)\) is defined similarly.

The SINR of the \(k(l)\)-th user given in Theorem 1 can be written as a function of \(\theta_l\) as follow

\[
\gamma_{k(l)} = \frac{\left| q_{k(l)}^H \theta_l + p_{l,k(l)} \right|^2}{\sum_{(\tilde{l}, \tilde{k}) > (l,k)} \left| q_{k(l)}^H \theta_l + p_{l,k(l)} \right|^2 + \sigma^2_{k(l)}},
\]

where we assume that the interference from the users decoded before the \(k(l)\)-th user is completely canceled, and thus the number of interfering terms in the denominator is reduced for the subsequently decoded users, and becomes zero for the last decoded user.

The remaining equations can be rewritten in a similar way for the MMSE-SIC receiver, e.g.,

\[
\hat{Z}_{k(l),k(l)} \triangleq \sum_{(\tilde{l}, \tilde{k}) > (l,k)} Z_{k(l),l}, \quad \hat{z}_{k(l),k(l)} \triangleq \sum_{(\tilde{l}, \tilde{k}) > (l,k)} z_{k(l),l},
\]

\[
\hat{c}_{k(l)} = \sigma^2_{k(l)} + \sum_{(\tilde{l}, \tilde{k}) > (l,k)} v_{k(l)},
\]

The sub-problems to find the phase-shift \(\theta_l\) in Section III-C with the MMSE-SIC receiver are the same as those for the linear MMSE receiver, with the exception that the effective interference weight matrices, \(Z_{k(l),l}, \hat{z}_{k(l),l},\) and \(\hat{c}_{k(l)}\), are now given in (22). The numerical results for the MMSE-SIC receiver will be presented in Section V-B.

F. Complexity Analysis

The complexity of solving the optimization problem in (6) is given by the combined complexity of solving the beamforming weights as described in Section III-A and the \(L\) optimization problems in (9). As shown in the previous subsection, the complexity of obtaining the weights in (8) is \(O(N^3)\). Similar to the analysis in [14], the complexity of solving the \(l\)-th optimization problem in (19), which has \(2K + 2M_l\) variables and \(3K + M_l\) constraints, is \(O((2K + 2M_l)^2 \cdot (3K + M_l) \log_2 (1/\epsilon_2)) \approx O((K^{3.5} + M_l^{3.5}) \log_2 (1/\epsilon_2))\), where \(\epsilon_2\) is the accuracy of the DCCP method for solving (19). Since the AO algorithm iteratively solves each of these \(L\) problems, the corresponding complexity is \(O((LK^{3.5} + \sum_{l=1}^L M_l^{3.5}) \log_2 (1/\epsilon_2))\). Therefore, the total complexity of Algorithm 1 is \(O(I_1 (N^3 + (LK^{3.5} + \sum_{l=1}^L M_l^{3.5}) \log_2 (1/\epsilon_2)))\). The simulation results illustrated in Section V will show that Algorithm 1 converges after about \(I_1 \sim 100\) iterations in the considered setup.

IV. MINIMUM-RATE MAXIMIZATION

In this section, we aim to maximize the minimum achievable rate among all the users by optimizing the phase-shifts of all the RSs as well as the beamforming vectors at the BS for all groups of users [1]. The problem is formally written as follows

\[
\max_{\{\theta_l\}, \{w_{l,k}\}} \min_{l, k} \log_2 (1 + \gamma_{k(l)})
\]

\[
s.t. \quad |\theta_{l,j}| = 1, \quad \forall l \in L, \quad j \in M_l,
\]

where \(\gamma_{k(l)}\) is given in (5).

It is worth noting that the authors of [17] aimed to maximize the minimum achievable rate among all users but for a different setting. Specifically, only two RSs and one group of users were considered in [17]. Given these assumptions, interference and the secondary reflections amongst more than two RSs were not considered in [17], which is, on the other hand, the focus of our paper.

Since the constraints \(|\theta_{l,j}| = 1\) are non-convex, problem (P4) is non-convex. As (P4) is a non-convex optimization problem, a typical approach to solve it is to utilize the AO method to find a suboptimal solution. Specifically, the proposed AO algorithm works as follows. First, we fix all the RSs phase-shifts \(\{\theta_l\}_{l=1}^L\) to optimize the receive beamforming vectors \(\{w_{k(l)}\}\) of all the users \(k(l)\).
In this case, the effective channel of each user \(h_{k(l)}\) in (2) is fixed and thus problem (P4) is reduced to solving \(K\) sub-problems. Each sub-problem solves the receive beamforming weight vector to maximize the SINR of the \(k(l)\)-th user. 

Then, for each \(l \in \mathcal{L}\), the corresponding \(\theta_l\) is optimized while the receive beamforming vectors \(\{w_{k(l)}\}\) and all the other phase-shift vectors \(\{\theta_j\}_{j \neq l}\) are kept fixed. Although the AO method is also used in [17] and other works, solving (P4) is not straightforward in the considered case due to the specific intricacy caused by the presence of multiple RSs, secondary reflections amongst the RSs, and multiple groups of users. In the following, we focus on optimizing the phase-shifts in (P4) given the optimal MMSE beamforming vectors.

### A. Optimization of \(\{\theta_l\}_{l=1}^L\) for Fixed \(\{w_{k(l)}\}\)

Maximizing the minimum \(\log_2(1 + \gamma_{k(l)})\) over \(\{l, k\}\) is equivalent to maximizing the minimum SINR \(\gamma_{k(l)}\) over \(\{l, k\}\). Hence, for fixed \(\{w_{k(l)}\}\), problem (P4) in (23) is equivalent to the following optimization problem

\[
\text{(P4.1): } \max_{\{\theta_l\}, \delta} \delta \quad \text{s.t.} \quad \gamma_{k(l)} \geq \delta, \quad \forall l \in \mathcal{L}, \quad k \in \mathcal{K}_l, \quad |\theta_{l,j}| = 1, \quad \forall l \in \mathcal{L}, \quad j \in \mathcal{M}_l, \quad (24)
\]

where \(\delta\) is a slack variable and \(\gamma_{k(l)}\) is given in (5).

Problem (P4.1) is non-convex because the constraints \(|\theta_{l,j}| = 1\) make the feasible region a non-convex set. We thus apply the AO algorithm to solve (P4.1) by alternately optimizing each \(\theta_l\) while keeping all the other phase-shifts fixed as detailed in the following subsection.

### B. Optimization of \(\theta_l\) for Fixed \(\{\theta_{l'}\}_{l' \neq l}\) and Fixed \(\{w_{k(l)}\}\)

For fixed \(\{\theta_{l'}\}_{l' \neq l}\) and \(\{w_{k(l)}\}\), problem (P3) simplifies to the following problem

\[
\text{(P4.2): } \max_{\theta_l, \delta} \delta \quad \text{s.t.} \quad \gamma_{k(l)} \geq \delta, \quad \forall l \in \mathcal{L}, \quad k \in \mathcal{K}_l, \quad |\theta_{l,j}| = 1, \quad \forall j \in \mathcal{M}_l, \quad (25)
\]

where \(\hat{l} \in \mathcal{L}\) is a predefined value. Problem (P4.2) is still a non-convex problem due to the unitary constraints on \(\theta_{l,j}\).

Using Theorem 1, when only \(\theta_l\) is the optimization variable, problem (P4.2) in (25) can be rewritten as follows

\[
\max_{\theta_l, \delta} \delta \quad \text{s.t.} \quad \left| q_{l,k(l)}^H \theta_l + p_{l,k(l)} \right|^2 \geq \delta \sum_{(i,k) \neq (l,k)} \left| q_{i,k(l)}^H \theta_l + p_{i,k(l)} \right|^2 + \delta \sigma_{k(l)}^2 \quad (26)
\]

Then, we can rewrite

\[
\left| q_{l,k(l)}^H \theta_l + p_{l,k(l)} \right|^2 = \theta_l^H B_{l,k(l)} \theta_l + \left| p_{l,k(l)} \right|^2 \equiv \text{Tr} \left( B_{l,k(l)} \theta_l \theta_l^H \right) + \left| p_{l,k(l)} \right|^2 \quad (27)
\]

where

\[
B_{l,k(l)} \equiv \left[ \begin{array}{c} q_{l,k(l)}^H q_{l,k(l)}^H + p_{l,k(l)} q_{l,k(l)}^H \ 0 \end{array} \right], \quad \theta_l \equiv \left[ \begin{array}{c} \theta_l \\ \theta_l^H \end{array} \right], \quad s = \text{an auxiliary variable. Define } \Psi_l \equiv \tilde{\theta}_l \tilde{\theta}_l^H. \text{ We have } \Psi_l \succeq 0 \text{ and rank}(\Psi_l) = 1. \text{ Since the rank-one constraint, rank}(\Psi_l) = 1, \text{ is non-convex, we relax this constraint. Accordingly, the problem in (26) is rewritten as}
\]

\[
\text{(P4.3): } \max_{\Psi_l, \delta} \delta \quad \text{s.t.} \quad \text{Tr}(B_{l,k(l)} \Psi_l) + \left| p_{l,k(l)} \right|^2 \geq \delta \sum_{(i,k) \neq (l,k)} \text{Tr}(B_{i,k(l)} \Psi_l) + \delta \left( \sum_{(i,k) \neq (l,k)} \left| p_{i,k(l)} \right|^2 + \sigma_{k(l)}^2 \right) \quad (28)
\]

\[
\Psi_l \succeq 0, \quad [\Psi_l]_{ij} = 1, \quad \forall j \in \mathcal{M}_l + 1, \text{ and for all } (k(l)). \quad (27)
\]

For a fixed \(\delta\), (P4.3) is a convex semidefinite program (SDP) problem, which boils down to a feasibility-check problem [17], [24]. Therefore, problem (P4.3) can be optimally solved by standard convex optimization solvers [25]. For example, (P4.3) can be efficiently solved by using the bisection method. Once a global maximum \(\delta_{\text{opt}}\) is obtained (with a certain numerical accuracy), for a given \(\Psi_l\) solution, the Gaussian randomization search is used to obtain a feasible solution for \(\theta_l\) [6]. Simulation results show that we can almost always obtain a numerical solution \(\theta_l\) such that the max-min achievable rate is within a pre-defined error, e.g., \(\epsilon = 0.1\%\), relative to the globally optimal \(\delta_{\text{opt}}\) of problem (P4.3).

Because we obtain \(\theta_l\) from \(\Psi_l\) using the Gaussian randomization method, there is a possibility that the solution at the \((t - 1)\)-th iteration is better than that at the \(t\)-th iteration. To guarantee that Algorithm 2 converges, we add a heuristic check as shown in lines 9-14. Specifically, when the solution \(\theta_l^{(t)}\) results in a strictly smaller value of the original objective function in problem (P4), it is not updated. In this case, Algorithm 2 continues to optimize the phase-shifts of the next RS, and it is terminated when the condition in line 17 is fulfilled.

It is worth noting that one can formulate (P4.2) as a DCCP problem and solve it using a DCCP-based algorithm with a binary search over \(\delta\). Then, for each fixed \(\delta\), the problem is equivalent to a feasibility check. However, for the max-min achievable rate problem, we propose to solve (P4.2) using convex SDP, since it generally performs better than the DCCP, as we can obtain the optimal solution for convex sub-problem (P4.3) using the SDP toolbox and binary search.

### C. Proposed AO Algorithm for Max-Min Achievable Rate

The proposed AO algorithm is summarized in Algorithm 2, in which \(\Theta^{(l)} = [\theta_l^{(1)}, \theta_l^{(2)}, \ldots, \theta_l^{(L)}]\) is the phase-shift matrix at the iteration \(t\). The convergence of this algorithm is provided in the following theorem [1].
Algorithm 2 AO Algorithm to Solve the Non-Convex Optimization Problem in (23)

1: **Input:** Output at iteration \((t-1)\) \(\{w^{(t-1)}_{k(l)}, \theta^{(t-1)}_{l}\}\).
2: **Initialize** \(t = 1\), \(\{w^{(0)}_{k(l)}, \theta^{(0)}_{l}\}\), tolerance \(\xi > 0\) and \(\epsilon > 0\).
3: **Compute** the (minimum) achievable rate \(\gamma^{(0)}_{\min} = \min_{l,k} \gamma_{k(l)}^{(0)}(w^{(0)}_{k(l)}, \theta^{(0)}_{l})\).
4: **Repeat**
5: Obtain \(w^{(t)}_{k(l)}\) from \(\{\theta^{(t-1)}_{l}\}\) by solving (8);
6: For \(l = 1 \) to \(L\):
7: Obtain \(\psi^{(t)}_{l}\) by solving (27) with \(w^{(t)}_{k(l)} = w^{(t)}_{k(l)}, \theta^{(t)}_{l} = \theta^{(t)}_{l}\) for \(l < i\), and \(\theta^{(t)}_{i} = \theta^{(t-1)}_{i}\) for \(l > i\), and tolerance \(\epsilon\);
8: Obtain \(\hat{\theta}^{(t)}_{l}\) from \(\psi^{(t)}_{l}\) using the Gaussian randomization method;
9: If \(\gamma^{(t)}_{\min}(\theta^{(t)}_{l}) \geq \gamma^{(t-1)}_{\min}(\theta^{(t-1)}_{l})\);
10: Update \(\theta^{(t)}_{l} ← \hat{\theta}^{(t)}_{l}\);
11: **Then**
12: Update \(\theta^{(t)}_{l} ← \hat{\theta}^{(t)}_{l}\);
13: **Otherwise**
14: Update \(\theta^{(t)}_{l} ← \theta^{(t-1)}_{l}\); (i.e., skip updating.)
15: **Compute** the (minimum) achievable rate \(\gamma^{(t)}_{\min} = \min_{l,k} \gamma_{k(l)}^{(t)}(w^{(t)}_{k(l)}, \theta^{(t)}_{l})\);
16: If \(\gamma^{(t)}_{\min} - \gamma^{(t-1)}_{\min} < \xi\);
17: **Then**
18: Set \(\{w^{*}_{k(l)}, \theta^{*}_{l}\} = \{w^{(t)}_{k(l)}, \theta^{(t)}_{l}\}\) and terminate.
19: **Otherwise**
20: Update \(t ← t + 1\) and continue.
22: **Output:** \(\chi^{t} = \{w^{*}_{k(l)}, \theta^{*}_{l}\}\).

**Theorem 3:** The objective value \(\gamma^{(t)}_{\min}\) as shown in Algorithm 2 is monotonically non-decreasing (and always bounded from the above) and thus the algorithm is guaranteed to converge.

**Proof:** See Appendix C.

**D. Complexity Analysis**

The complexity of solving the optimization problem in (23) is the combined complexity of solving the beamforming weights as shown in (8), whose complexity is mainly due to the computation of the inverse of an \(N \times N\) matrix. Hence, the complexity is \(O(N^{3})\). Similar to the analysis in [14], the complexity of solving the \(l\)-th SDP optimization problem in (27) with the bisection method, which has \(M^{2}_{L}\) variables, is \(O(M^{2.5}_{L} \log(1/\epsilon))\) where \(\epsilon\), as shown in Algorithm 2, is the accuracy of the bisection search. The algorithm alternately solves \(L\) sub-problems, hence the total complexity is \(O((M^{1.5}_{L} + \ldots + M^{1.5}_{L}) \log(1/\epsilon))\), which simplifies to \(O(L \cdot M^{4.5}_{L} \log(1/\epsilon))\) when \(M_{L} = M_{L}\) for all \(L\). Hence, the total complexity of Algorithm 2 is \(O(I_{2}(M^{1.5}_{L} + \ldots + M^{1.5}_{L}) \log(1/\epsilon) + N^{3})\), or \(O(I_{2}(L M^{4.5}_{1} \log(1/\epsilon) + N^{3})\) when \(M_{L} = M_{L}\) for all \(L\), where \(I_{2}\) is the required number of iterations for the algorithm to converge. Our simulations show that Algorithm 2 converges after about \(I_{2} \sim 100\) iterations in the considered setup.

The complexity of Algorithm 2 is higher than that of Algorithm 1 due to the SDP optimization problem. However, it has the same order of complexity as other algorithms proposed in the literature, e.g., [17], which increases linearly with the number of RSs for the multi-RS case. Furthermore, for system implementation, we can tailor the solution to tradeoff between system performance and complexity. For example, we can divide the optimization solution into multiple phases, and each phase can update only the BS beamformers and/or the phase-shifts of one or several RSs at a time (as opposed to updating all the RSs at once). We can also increase the duration of updating the phase-shifts of the RS elements in slow-fading environments.

**V. NUMERICAL RESULTS**

In this section, we consider the uplink transmission to a BS with 16 or more antennas, using orthogonal frequency-division multiplexing (OFDM) and varying the number of RSs. The number of elements per RS is varied from 32 to 128. The number of users aided by each RS is from 2 to 12 users. We assume that each user is allocated with \(B = 180\) KHz bandwidth. Thus, the noise power at the BS is assumed to be \(\sigma^{2} = -174 + 10 \log_{10}(B)\) dBm. Similar to the simulation setup and assumption in [11], [20], [39], we consider a small-cell situation as shown in Fig. 2.

In this setup, we assume that the RS locations are within \(180^\circ\) half-space reflection of each other. Particularly, the RSs are located with equal spacing in a half circle with diameter of 20 meters (unless otherwise stated) and are facing the center of the circle (see Fig. 2). The distance between the BS and the center of the circle is 50 meters. The user locations are assumed to be uniformly distributed in a half circle with diameter \(d_{rs} = 2r\) and within the \(180^\circ\) half-space reflection of each RS. We assume the heights of BS, RSs, and users are 15, 20 and 1.5 meters, respectively. The transmit power at each user is assumed to be the same. In the following, when the number of the RSs varies from 2 to 6, we specify the RS locations as \((x_{rs}, y_{rs}, 1.5)\) in meter.

We assume a non-LOS-dominant channel model between the users and the RSs with the Rician factor \(\kappa = -10\) dB. The LOS-dominant channel model between the RSs and the
BS and between any two RSs has Rician factor $\kappa = 5$ dB. Specifically, the channel model between a source, $s$, and a destination, $d$, is assumed to be as follows:

$$H_{s,d} = \sqrt{\frac{\kappa}{\kappa+1}} H_{s,\text{los}} + \sqrt{\frac{1}{\kappa+1}} \tilde{H}_{s,d},$$

where $(s, d)$ can be (User, RS), (RS, RS) or (RS, BS), $H_{s,\text{los}}$ denotes the LOS component and $\tilde{H}_{s,d}$ denotes the non-LOS component. We assume a rich scattering environment and thus the elements of the non-LOS components $\tilde{H}_{s,d}$ are modeled as independent and identically distributed complex Gaussian random variables with zero mean and unit variance, i.e., $CN(0,1)$. The LOS component is modeled as a rank-one matrix as follows [11], [49]:

$$H_{s,d} = ab^H,$$

where $a_m = \exp(j \frac{2\pi}{\lambda} d_s (m-1) \sin \theta_{s,\text{los}} \sin \theta_{d,\text{los}})$ and $b_n = \exp(j \frac{2\pi}{\lambda} d_d (m-1) \sin \phi_{d,\text{los}} \sin \theta_{d,\text{los}})$, where $\lambda$ is the carrier wavelength, $d_s, d_d$ are the inter-antenna or inter-RS-element separation at $s$ and $d$ (which can be either the BS or RS), $\phi_{s,\text{los}}, \theta_{s,\text{los}}$ are LOS azimuth and elevation AoDs at $s$ and $\phi_{d,\text{los}}, \theta_{d,\text{los}}$ are LOS azimuth and elevation AoDs at $d$, respectively. In our simulation we set $d_s = d_d = 0.5\lambda$.

Consider a three-terminal reflection link, $s \rightarrow r \rightarrow d$, where $(s, r, d)$ can be (User, RS, BS), (User, RS, RS), or (RS, RS, BS), similar to [11], [19], the combined pathloss is assumed to follow the far-field product-distance model as follows:

$$\beta_{sr,d} = \beta_{sr} \cdot \beta_{rd} = \frac{C_1}{d_{sr}^{\alpha_1}} \cdot \frac{C_2}{d_{rd}^{\alpha_2}},$$

where $C_1, C_2$ are the pathlosses at one meter for the $sr$ and $rd$ links, $d_{sr}, d_{rd}$ are the distance between $s, r$ and $r, d$, and $\alpha_1, \alpha_2$ are the pathloss exponents for the $sr$ and $rd$ links, respectively. As for the non-LOS-dominant channel between the users and the RSs, we set $\alpha_1 = 3.0$, $C_1 = 10^{-30}/m^{3.0}$. As for the LOS-dominant channel for the RS-RS or RS-BS links, we set $\alpha_2 = 2.2$, $C_2 = 10^{-30}/m^{2.2}$, where $G_s, G_r, G_d$ are the antenna/RS element gains. We assume 5 dBi gain at the BS antenna and 0 dBi at the user antenna and the RS elements. As for the secondary reflections, the signal travels from $User \rightarrow RS_1 \rightarrow RS_P \rightarrow BS$, and the total pathloss is modeled as the product of the far-field pathloss of each point-to-point link, that is, $\beta_{sr,r'}d = \beta_{sr} \cdot \beta_{rr'} \cdot \beta_{r'd}$, where $s, r, r', d$ denote $User, RS_1, RS_P, BS$, respectively. The transmit power at each user is assumed to be the same and ranges from 10 up to 40 dBm (i.e., 10 W). The numerical results averaged over 50 independent channel realizations.

### A. Sum-Rate Maximization

First, we present the simulation results for the weighted sum-rate maximization problem. We assume all the user weights are the same, i.e., $1/K$, or equivalently we maximize the sum-rate only. The minimum rate requirement (on each user achievable rate) is set to zero, unless otherwise stated. We focus on showing the performance gain by considering the additional secondary reflections amongst the RSs in our proposed design as compared to the conventional design considering the RS primary reflections only. In the following, we compare our solution (leveraging the inter-RS secondary reflections) with state-of-the-art IRS/RIS solutions e.g., [17], [37]–[40], which did not consider the secondary reflections of RSs. Also, for fair comparison with existing designs in the literature that ignore the secondary RS reflections while optimizing the phase-shifts and beamformers, we still evaluate their performance under the same secondary reflections as for the proposed scheme.

In Fig. 3(a), we plot the relative gain of the achievable sum-rate with over without considering the secondary reflections for a system with 2, 4, or 6 RSs (while the total number of users is the same, i.e., 12 users). The locations of the RSs in these scenarios are assumed to be equally spaced and the distance between two adjacent RSs is 15, 10 and 6.18 meters for the setup with 2, 4, or 6 RSs, respectively. At lower transmit power (i.e., low SNR), we observe some marginal gain, because the performance is limited by AWGN and the interference amongst the RSs can be tolerated without compromising the rate. At mid to high transmit power, we can observe significant performance gain when considering the secondary reflections. For example, at a transmit power (Ptx) of 35 dBm, we see about 6.4%, 15.6%, or 29.9% gains for the setup with 2, 4, or 6 RSs, respectively. The higher gain observed with more RSs is partially due to the stronger and more secondary reflections, which is due to the smaller distance between two adjacent RSs.

In Fig. 3(b), we study how the number of elements per RS affects the system performance with and without managing the
We observe that the more the number of RS elements, the higher the gain from managing and leveraging the secondary reflections. For example, at a transmit power of 35 dBm, we see about 7.2%, 15.6%, and 27.9% gains for 32, 64 and 128 elements per RS, respectively. The increase in gain is due to the fact that the more RS elements, the more signal energy is reflected amongst RSs, thus increasing the interference from secondary reflections.

In Fig. 4(a), we plot the achievable rate gain as a result of managing secondary reflections for a system with four RSs and three users while varying the distance between two adjacent RSs (i.e., \(d_{rs} \)) and the number of RS elements. When the distance between adjacent RSs is large, the secondary reflection effects are weak as expected, but increasing the number of RS elements, the gain can be significantly improved. For example, at a transmit power of 35 dBm, the gains for the \((5m, 32el.)\) configuration, i.e., 5 meter (m) distance and 32 RS elements per RS and \((10m, 64el.)\) are 20.6% and 15.6%, respectively, and the gains for \((15m, 96el.)\) and \((20m, 128el.)\) are 11.6% and 8.7%, respectively.

In Fig. 4(b), we plot the sum-rate based on Algorithm 1 for the system with four RSs each with 3 users versus the number of iterations without early termination (i.e., \(\xi = 0\)). We observe that the algorithm reasonably converges after about 50 iterations for \(P_{tx} = 10\) dBm. It requires a slightly higher number of iterations for higher transmit powers. A similar trend can be observed for the max-min achievable rate based on Algorithm 2; the plot is thus omitted for brevity.

B. Sum-Rate Maximization With the MMSE-SIC Receiver

In this subsection, we present the numerical results for the sum-rate maximization when the MMSE-SIC receiver presented in Section III-E is utilized. Fig. 5(a) depicts the achievable sum-rate of the MMSE-SIC receiver with versus without secondary reflections. It can be seen that the achievable sum-rate of six RSs (with two users per RS) is worse than that of four RSs (with three users per RS) when the transmit power is 18.2 dBm or higher and when the secondary reflections are not considered.

Fig. 5(b) shows the achievable sum-rate for four RSs (with three users per RS) when the MMSE and the MMSE-SIC receivers are utilized. We observe that the achievable sum-rate of the MMSE-SIC receiver is about 0.3 bps/Hz higher than that of the MMSE receiver when the secondary reflections are considered. If we do not consider the secondary reflections, i.e., the corresponding channels are not estimated and the signals from the secondary reflections are not managed, the rate provided by the MMSE-SIC receiver is even worse than that of the MMSE receiver. We observe about 0.8 bps/Hz degradation at a transmit power of 30 dBm. This is because without considering the secondary reflections, the interference from the secondary reflections is not mitigated and is accumulated from the decoded users when the MMSE-SIC
receiver is utilized. This degrades the performance of the subsequently decoded users.

Fig. 5(c) plots the achievable sum-rate gain (percentage) for both the MMSE and MMSE-SIC receivers versus the transmit power. We observe a much higher gain for the MMSE-SIC receiver as compared to the MMSE receiver. For example, at the transmit power of 30 dBm, the MMSE-SIC gains $34.7\%$ and the MMSE receiver gains $19.1\%$ if six RSs are deployed. The corresponding gains are $17.7\%$ and $8.4\%$ when four RSs are deployed.

C. Min-Rate Maximization

In Fig. 6(a), we consider the minimum achievable rate among the users under various configurations: (i) four RSs each with 3 users, (ii) two RSs each with 6 users and (iii) one RS with 12 users; that is, there are 12 users in total. The four-RS scenario is as shown in Fig. 2. When there are two RSs, their locations are $(x_{rs}, y_{rs}) = (50, \pm10)$ meters, and for the single-RS scenario, the RS is at $(50, 10)$ meters. In all these configurations, each RS has 64 elements and the BS has 16 antennas. We can see that with the same number of users and receive antennas, the more RSs, the higher the minimum achievable rate. With a transmit power of 30 dBm, for instance, we observe a $1.2$ bits-per-second (bps)/Hz or $2.4$ bps/Hz gain for a two- or four-RS scenario as compared to the single-RS system, respectively. This is because of the higher received signal power on average at the receiver antennas due to the reflections from more RSs. Note that the performance of the scenario without any RS is not shown as there is no feasible direct link between the users and the BS under the considered setup. In this figure, we also plot the average achievable rate among all users and observe that the average achievable rate has a similar trend which is not much different from the minimum achievable rate.

Fig. 6(b) compares the minimum achievable rate when both primary and secondary reflections are considered (as in this paper) and when only the primary reflections are considered (as in other works [14]–[17]). We assume that the RSs are equally spaced, i.e., the distances between two adjacent RSs for the scenarios with four and six RSs are 10 and 6.18 meters, respectively. It can be seen that, by properly managing the interference and leveraging the secondary reflections amongst RSs, the minimum achievable rate is significantly improved.

For example, at a transmit power of 30 dBm, we observe about 1.2 or 2.3 bps/Hz gain when exploiting the secondary reflections for a system with four or six RSs, respectively. The improvement becomes more pronounced with higher transmit power/SNR. This is because at high SNR, the secondary reflections amongst the RSs become more significant while the stronger interference amongst them can be mitigated/managed with the proposed design framework.

The management of the inter-RS interference becomes more critical for dense networks. Specifically, we observe that for a given area, without properly managing the secondary reflections, adding more RSs may actually degrade the overall system performance. This is due to the stronger interference caused by the secondary reflections from more RSs. For example, when the secondary reflections are not managed using our proposed method, we observe that the achievable rate under the six-RS scenario is even worse than that with four RSs at the same transmit power of 25.5 dBm or higher. In contrast, by properly managing the interference using our method, the six-RS scenario always outperforms that of the four-RS scenario.

In Fig. 7(a), we plot the achievable rate versus different number of elements per RS for a system with four RSs, three users per RS, and a BS with 16 antennas. It is seen that the larger the number of RS elements, the higher the achievable rate due to the power gain at the BS. We can see the larger the number of elements per RS, the higher the achievable rate. For example, at a transmit power of 30 dBm, we can see 1.5 bps/Hz gain for a 64-element RS system as compared to that of the 32-element RS one, and 1.4 bps/Hz gain is observed for 128 elements versus 64 elements.

In Fig. 7(b), we plot the achievable rate versus the number of users per RS for four RSs each with 64 elements and a BS with 16 antennas. It can be seen that the larger the number of users that access the same bandwidth, the lower the minimum achievable rate, whereas the sum-rate for all the users is increased (before reaching its peak at 4 users per RS). However, when the total number of users is $5 \times 4 = 20$ (or higher), which is greater than the number (16) of antennas at the BS, the achievable rate is significantly lower due to the lack of sufficient spatial channel diversity.

In Fig. 7(c), we plot the achievable rate versus the number of receive antennas at the BS for four RSs of 64 elements.
each and 3 users per RS. It shows that the larger the number of receive antennas at the BS, the higher is the minimum achievable rate. For example, at a transmit power of 30 dBm, we observe about 1.7 bps/Hz or 0.9 bps/Hz gain when there are 64 or 32 antennas as compared to 16 antennas, respectively. We also observe that the rate for 8 antennas is significantly lower than that for 12 or 16 antennas due to the lack of sufficient spatial dimension since 12 users are considered in this case.

VI. CONCLUSION

In this paper, we considered an uplink multi-IRS/RIS aided multi-user MIMO communication system, where the phase-shifts of the reflecting elements at all the RSs and the beamforming at the BS are jointly optimized. We formulated and solved two optimization problems using AO algorithms. The numerical results showed that by properly managing the interference and leveraging the secondary reflections amongst closely deployed RSs, the system throughput can be significantly improved in terms of sum-rate and minimum rate among the users, especially when a large number of RSs or reflecting elements per RS are deployed in the network.

APPENDIX A

PROOF OF THEOREM 1

First, we rewrite $h_{k(l)}$ in (2) as a function of $\theta_l$. When $l \neq \hat{l}$, we have

$$h_{k(l)} = \sum_{l'=1}^{L} R_{l',k(l)} \theta_{l'} + \sum_{l'=1}^{L} \sum_{j=1}^{M_l} Q_{l',k(l),j} \theta_{l'} \theta_{l,j}$$

$$= R_{l,k(l)} \theta_{l} + \sum_{l'=1, l' \neq \hat{l}}^{L} R_{l',k(l)} \theta_{l'} + \left( \sum_{j=1}^{M_l} Q_{l',k(l),j} \theta_{l,j} \right) \theta_{l}$$

$$= \left( R_{l,k(l)} + T_{l,k(l)} \right) \theta_{l} + \left( U_{l,k(l)} + S_{l,k(l)} \theta_{l} \right),$$

where we have defined

$$S_{l,k(l)} \triangleq \left[ s_{l,k(l),1}, \ldots, s_{l,k(l),M_l} \right],$$

with $s_{l,k(l),j} \triangleq \sum_{l' \neq l, l' \neq \hat{l}} Q_{l',k(l),j} \theta_{l'}$.

$$T_{l,k(l)} \triangleq \sum_{j=1}^{M_l} Q_{l',k(l),j} \theta_{l'} \theta_{l,j}, \text{ and } U_{l,k(l)} \triangleq \sum_{l'=1}^{L} R_{l',k(l)} \theta_{l'}.$$  

(33)

When $l = \hat{l}$, we have

$$h_{\hat{k}(l)} = R_{\hat{l},k(l)} \theta_{\hat{l}} + \sum_{l'=1}^{L} R_{l',k(l)} \theta_{l'} + \sum_{j=1}^{M_l} Q_{l',k(l),j} \theta_{l'} \theta_{\hat{l},j}$$

$$= R_{\hat{l},k(l)} \theta_{\hat{l}} + U_{\hat{l},k(l)} + S_{\hat{l},k(l)} \theta_{\hat{l}}$$

$$+ \left( R_{l,k(l)} + S_{l,k(l)} \theta_{\hat{l}} \right) \theta_{\hat{l}} + U_{\hat{l},k(l)}.$$  

(34)

By substituting (31) and (34) into (5), we obtain (10) with $q_{l,k(l)}$ and $p_{l,k(l)}$ given in (12) and (13), respectively.

APPENDIX B

PROOF OF THEOREM 2

First, we prove that the sequence $\Gamma(t)$ is non-decreasing, i.e., $\Gamma(t) \geq \Gamma(t-1)$ for all $t > 0$. In step 5 of Algorithm 1, we obtain $w_{k(l)}(t)$ from $\tilde{\theta}_{k(l)}^{(t-1)}$ by solving (8), which is the optimal MMSE-based solution. Thus, we have $\tilde{\gamma}_{k(l)}^{(t)}(w_{k(l)}(t), \tilde{\Theta}^{(t-1)}) \geq \tilde{\gamma}_{k(l)}^{(t-1)}(w_{k(l)}^{(t-1)}, \tilde{\Theta}^{(t-1)})$, for all $l,k$. It follows that the weighted sum-rate satisfies

$$\Gamma(t)\left(W^{(t)}, \tilde{\Theta}^{(t)}\right) \geq \Gamma(t-1)(W^{(t-1)}, \tilde{\Theta}^{(t-1)}).$$  

(35)

Next, in step 6 and 7, for each $\hat{l} \in L$, we obtain $\tilde{\theta}_{k(l)}^{(t)}$ by solving (19) using the CVXPY/DCCP solver with $\tilde{\theta}_{k(l)}^{(t-1)}$ as the initial value (while the other variables are kept fixed). We obtain

$$\Gamma(t)\left(W^{(t)}, \tilde{\theta}_{k(l)}^{(t)}, \ldots, \tilde{\theta}_{L}^{(t)}\right) \geq \Gamma(t)\left(W^{(t)}, \tilde{\theta}_{k(l)}^{(t)}, \ldots, \tilde{\theta}_{L}^{(t-1)}\right),$$

(36)

for all $\hat{l} \in L$. Combining the above, we have

$$\Gamma(t)\left(W^{(t)}, \tilde{\theta}_{k(l)}^{(t)}, \ldots, \tilde{\theta}_{L}^{(t)}\right) \geq \Gamma(t)\left(W^{(t)}, \tilde{\theta}_{k(l)}^{(t-1)}, \ldots, \tilde{\theta}_{L}^{(t-1)}\right),$$

(37)
and further combining (35) and (37), we have
\[ \Gamma^{(t)}(W^{(t)}, \Theta^{(t)}) \geq \Gamma^{(t-1)}(W^{(t-1)}, \Theta^{(t-1)}), \]
for all \( t > 0 \). That is, \( \Gamma^{(t)} \) is a non-decreasing sequence. Due to the limited transmit power, the SINR \( \gamma^{(t)} \) is bounded from the above by its SNR (by ignoring the interference terms); hence, \( \Gamma^{(t)} \) is bounded from the above as well. Therefore, Algorithm 1 is guaranteed to converge.

Next, we show that the algorithm converges to a local optimum in general. The following proof is similar to those in [50], [51]. When Algorithm 1 converges, denote the solution as \( \hat{x}^* = \{w_{i}^{*}, \Theta_{i}^{*}\} \). For each sub-problem (P3.2) shown in (19), if it is feasible, the local optimum is guaranteed to be achieved by the CVXPY/DCCP solver. Let \( \{\hat{\Theta}_{i}, \hat{u}^{*}, \hat{v}^{*}, s^{*}\} \) denote the locally optimal solution. Then, \( \Theta_{i}^{*} = \hat{\Theta}_{i}[1 : M_{i}/2] + j \hat{\Theta}_{i}[M_{i}/2 + 1 : M_{i}] \) is a locally optimal solution of problem (P3.1) in (14). The Karush–Kuhn–Tucker (KKT) condition for problem (P3.1) is satisfied at \( \Theta_{i}^{*} \), for each \( l \). Let \( \Gamma(\Theta_{i}) \) be the objective function of (P3.1), and \( R(\hat{x}^*) = [R_{l}(\hat{x}^*, R_{2}(\hat{x}^*)), \ldots, R_{l}(\hat{x}^*)] \) be the set of constraints of problem (P3.1). Then, we can write
\[ \nabla_{\Theta_{i}}^{T} \Gamma(\hat{x}^*) + Y^{T} \nabla_{\Theta_{i}} R(\hat{x}^*) = 0, \]
\[ y_{i} \geq 0, y_{i} R(\hat{x}^*) = 0, \forall i. \]
where \( Y = [y_{1}, y_{2}, \ldots, y_{j}] \) is the optimal Lagrangian variable set, and \( \nabla_{\Theta_{i}}^{T} \) is the gradient with respect to \( x_{i} \). We also note that \( \{w_{k}^{*}\} \) is a globally optimal solution for (8), hence its KKT condition is satisfied with respect to \( W = \{w_{k}^{*}\} \), that is
\[ \nabla_{W}^{T} \Gamma(\hat{x}^*) + Y^{T} \nabla_{W} R(\hat{x}^*) = 0, \]
\[ y_{i} \geq 0, y_{i} R(\hat{x}^*) = 0, \forall i. \]
Combining all the conditions in (39) and (39), we have
\[ \nabla_{x}^{T} \Gamma(\hat{x}^*) + Y^{T} \nabla_{x} R(\hat{x}^*) = 0, \]
\[ y_{i} \geq 0, y_{i} R(\hat{x}^*) = 0, \forall i. \]
which is the KKT condition for (P3.1) in (6), i.e., \( \hat{x}^* \) is a locally optimal solution for (6).

**APPENDIX C PROOF OF THEOREM 3**

First, we prove that the sequence \( \gamma^{(t)} \) is non-decreasing, i.e., \( \gamma^{(t)} \geq \gamma^{(t-1)} \) for all \( t > 0 \). In step 5 of Algorithm 2, we obtain \( w_{k}^{(t)} \) from \( \{w_{k}^{(t)}\} \) by solving (8), which is the optimal MMSE-based solution. Thus, we have
\[ \gamma^{(t)}(w_{k}^{(t)}(\Theta_{i}^{(t)}), \Theta_{i}^{(t-1)}) \geq \gamma^{(t-1)}(w_{k}^{(t)}(\Theta_{i}^{(t-1)}), \Theta_{i}^{(t-1)}), \]
for all \( l, k \). It follows that the minimum achievable rate among the users satisfies
\[ \gamma^{(t)}_{\min}(W^{(t)}, \Theta^{(t-1)}) \geq \gamma^{(t-1)}_{\min}(W^{(t-1)}, \Theta^{(t-1)}). \]
Next, in step 6 and 7, for each \( l \in L \), we obtain \( \Theta_{i}^{(t)} \) by solving (27) with the aid of the CVX solver and the bisection search, which is guaranteed to find the optimal value for this sub-problem. The heuristic check in step 8-14 ensures that
\[ \gamma^{(t)}_{\min}(\Theta_{i}^{(t)}) \geq \gamma^{(t-1)}_{\min}(\Theta_{i}^{(t-1)}), \]
for all \( \hat{\ell} \in L \). Therefore, we have
\[ \gamma^{(t)}_{\min}(W^{(t)}, \Theta^{(t)}) \geq \gamma^{(t-1)}_{\min}(W^{(t-1)}, \Theta^{(t-1)}). \]
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