Angular momentum content of the $\rho(1450)$ from chiral lattice fermions

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We identify the chiral and angular momentum content of the leading quark-antiquark Fock component for the $\rho(770)$ and $\rho(1450)$ mesons using a two-flavor lattice simulation with dynamical Overlap Dirac fermions. We extract this information from the overlap factors of two interpolating fields with different chiral structure and from the unitary transformation between chiral and angular momentum basis. For the chiral content of the mesons we find that the $\rho(770)$ slightly favors the $(1,0) \oplus (0,1)$ chiral representation and the $\rho(1450)$ slightly favors the $(1/2,1/2)_b$ chiral representation. In the angular momentum basis the $\rho(770)$ is then a $^3S_1$ state, in accordance with the quark model. The $\rho(1450)$ is a $^3D_1$ state, showing that the quark model wrongly assumes the $\rho(1450)$ to be a radial excitation of the $\rho(770)$.

I. INTRODUCTION

The potential constituent quark model has been quite successful in describing the low-lying hadron spectrum \cite{1}. Being an effective classification scheme, it does not care about foundations in terms of underlying QCD dynamics. Despite its successes the non-relativistic description clearly has limitations.

In this paper we investigate the angular momentum content of the $\rho(770)$ and $\rho(1450)$ mesons. In the spectroscopic notation $n^2S^1_J$ the $\rho(770)$ is assigned to the $1^3S_1$ state by the quark model. The $\rho(1450)$ is assigned to the $2^3S_1$ state, hence being the first radial excitation of the $\rho(770)$. However, this assumption is by far not clear from the underlying QCD dynamics, and is an output of the non-relativistic potential description of a meson as a two-body system.

In principle the angular momentum content of the leading quark-antiquark Fock components of mesons can be identified by a lattice simulation \cite{2,3,4}. The crucial ingredients to such a study are the overlap factors obtained with operators that form a complete set with respect to the chiral-parity group. From these overlap factors the chiral content of a state can be identified. Then, given a unitary transformation between the chiral basis and the $2S^1_J$ basis we can reconstruct the angular momentum content. Since the chiral content is important for such a study we need a lattice fermion discretization, which respects chiral symmetry. This is why we use overlap fermions, which distinguishes the present study from the previous ones.

We find in contrast to the previous studies that the $\rho(1450)$ is practically a pure $1^3D_1$ state. We will argue that our result is correct due to a careful analysis of the signs of the overlap factors.

Further, we remove the low-lying Dirac eigenmodes of the spectrum, which has been done recently to show an emergent $SU(2N_f)$ symmetry in the QCD spectrum.

This symmetry connects all flavors and quark chiralities, which means in the two-flavor case that $u_L, u_R, d_L, d_R$ are connected with each other. Here we study the effect of the Dirac eigenmode removal on the overlap factors of vector and pseudotensor interpolators.

The outline of the article is as follows: In section II we present the method how to extract the chiral and angular momentum content of the physical states. In section III we discuss our simulation parameters and introduce a resolution scale for our measurements. In section IV we present our main findings. In section V we discuss the effect of removing the low-lying eigenmodes on the $\rho$ states. Finally, in section VI we give a short conclusion.

II. FORMALISM

The formalism how to extract chiral and angular momentum content from lattice correlators has been explained in detail in Ref. \cite{5}. We review the basic steps.

To generate states with $\rho$ quantum numbers $(1,1^{--})$ two different local interpolators can be used, which belong to two distinct chiral representations \cite{6}:

\begin{equation}
J^V_T(x) = \bar{\Psi}(x)(\tau^a \otimes \gamma^5)\Psi(x) \in (0,1) \oplus (1,0) \quad (1)
\end{equation}

\begin{equation}
J^T_T(x) = \bar{\Psi}(x)(\tau^a \otimes \gamma^0\gamma^5)\Psi(x) \in (1/2,1/2)_b. \quad (2)
\end{equation}

We denote them according to their Dirac structure as vector $(V)$ and pseudotensor $(T)$ interpolators. Both of them couple to the physical $\rho$ states. The interpolators \cite{6,7} transform differently under $SU(2)_L \times SU(2)_R$ and therefore belong to different chiral representations. If chiral symmetry would be manifest in nature, these two interpolators would generate two different particles and the index of the irreducible representation of the chiral-parity group would be an additional quantum number. In the real world, where chiral symmetry is broken, a physical $\rho$-meson is a mixture of two possible chiral representations and consequently both interpolators create the same physical $\rho$-meson.

\begin{footnotesize}
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\item \footnotemark[1] For a detailed description of the chiral-parity group we refer to Ref. \cite{8}.
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In a next step we connect the chiral basis to the angular momentum basis with quantum numbers isospin $I$ and $2S+1|J_f⟩$. For spin-1 isovector mesons there are only two allowed states $|1;^3S_1⟩$ and $|1;^3D_1⟩$, which are connected to the chiral basis by a unitary transformation \[ S \]:

\[ |ρ(0,1)⟩_{3(1,0)}⟩ = \sqrt{\frac{2}{3}} |1;^3S_1⟩ + \sqrt{\frac{1}{3}} |1;^3D_1⟩ \quad (3) \]

\[ |ρ(1/2,1/2)⟩_{3(1,0)}⟩ = \sqrt{\frac{1}{3}} |1;^3S_1⟩ - \sqrt{\frac{2}{3}} |1;^3D_1⟩ \quad (4) \]

This transformation is valid only in the rest frame.

On the lattice we evaluate the correlators ⟨$J_l(t)J_l^†(0)$⟩. We apply the variational technique, where different interpolators are used to construct the correlation matrix ⟨$J_l(t)J_l^†(0)$⟩ = $C(t)_{lm}$. By solving the generalized eigenvalue problem

\[ C(t)_{lm}u_{m}^{(n)} = λ^{(n)}(t,t_0)C(t_0)_{lm}u_{m}^{(n)} \quad (5) \]

the masses of the states can be extracted from the eigenvalues:

\[ λ^{(n)}(t,t_0) = e^{-E^{(n)}(t-t_0)} \left( 1 + O \left( e^{-\Delta E^{(n)}(t-t_0)} \right) \right) \quad (6) \]

A single entry of the correlation matrix reads

\[ ⟨J_l(t)J_l^†(0)⟩ = \sum_n a_l^{(n)} a_m^{(n)*} e^{-E^{(n)}t} \quad (7) \]

where $a_l^{(n)} = ⟨0| J_l |n⟩$ is the overlap of interpolator $J_l$ with the physical state $|n⟩$. The ratio of these overlap factors gives a relative weight of the different chiral representations in a given physical state. It can be constructed as

\[ \frac{a_l^{(n)}}{a_k^{(n)}} = \frac{C(t)_{lj}u^{(n)}_j}{C(t)_{kj}l^{(n)}_{j}} \quad (8) \]

We note that the ratio of eigenvector components $u_l$ is not suited due to the lack of a unique normalization for different operators.

We can extract the ratio $a_V/a_T$ for each state $n$. Then via the unitary transformation \[ S \], we arrive at the angular momentum content of the $ρ$ mesons. In Fig. 1 we show the dependence of the partial wave content of a state on the ratio \[ S \]. For example, if the $ρ$ state is a pure $1;^3S_1$ state, then two different chiral components have to mix in the following way:

\[ \sqrt{2} |ρ(0,1)⟩_{3(1,0)}⟩ + |ρ(1/2,1/2)⟩_{3(1,0)}⟩ = |1;^3S_1⟩ \quad (9) \]

In our lattice evaluation for the ground state $ρ$ we will not find the value $\sqrt{2}$ but a value which is close to it.

### III. LATTICE TECHNICALITIES

#### A. Simulation parameters

We use gauge configurations generously provided by the JLQCD collaboration, see Ref. [9]. The ensemble consists of 100 configurations of two-flavor dynamical Overlap fermions. The topological sector is fixed to $Q_{top} = 0$. Lattice size and spacing are $16^3×32$ at $a ≈ 0.12$ fm.

The pion mass is at $m_π = 289(2)$ MeV [9].

We calculate the isovector correlators with extended sources with different smearing widths of Gaussian type, described below.

#### B. Resolution scale via smeared sources

The vector current $J_μ^V$ is conserved, i.e. its coupling $a_V$ to the physical state should be independent of the scale. The pseudotensor current $J_μ^T$ is not conserved. Hence, $a_T$ should depend on the scale where it is measured. Consequently the ratio $a_V/a_T$ should also depend on the scale.

An intrinsic resolution scale is set by the lattice spacing $a$. If we probe the hadron structure with the point-like source then the result should display a structure of a hadron that is obtained at the scale fixed by the ultraviolet regularization $a$. In principle we could study the $a$-dependence by means of different lattices with different $a$. However, such a procedure does not allow to measure the structure close to the infrared region, i.e. at large $a$.

Instead of varying $a$, we can smear the sources of the quark propagators using different widths $σ$. Clearly, the smeared source cannot supply us with the information about the hadron structure that is sensitive to distances that are smaller than the smearing width $σ$. Consequently the smearing width $σ$ defines a scale at which we probe the structure of our hadron.

This is done using the Gaussian gauge invariant smearing of the source and sink operators. For a definition of the Gaussian (Jacobi) smearing we refer to Ref. [3] and references therein. We use four different smearing widths in this study. The corresponding profiles are given in Table [4]. The radius $σ$ of a given source $S(x;x_0)$ located
Table I. Different Jacobi-smeared sources, generating parameters $\kappa$ and $N$, widths $\sigma$ and resolution scales $R$.

| Source          | $\kappa$ | $N$ | $\sigma/a$ | $R$/fm |
|-----------------|----------|-----|-------------|--------|
| Super-Narrow    | 0.3      | 4   | 1.024       | 0.245  |
| Narrow          | 0.21     | 18  | 1.905       | 0.455  |
| Wide            | 0.191    | 41  | 2.236       | 0.530  |
| Ultra-Wide      | 0.19     | 100 | 3.748       | 0.890  |

at $x_0$ is calculated by

$$\sigma^2 = \frac{\sum_x (\vec{x} - \vec{x}_0)^2 |S(x; x_0)|^2}{\sum_x |S(x; x_0)|^2}.$$  \hspace{3cm} (10)

We define the resolution scale as $R = 2\sigma a$. The profiles are pictured in Figure 2. The Super Narrow source probes the hadron wave function at the resolution $\sim 0.25$ fm and marks the ultraviolet end of our parameter space. Narrow and Wide probe the hadron in the mid-momentum region. The Ultra Wide source does not resolve details smaller than $\sim 0.9$ fm and marks our infrared end. In this study it is not reasonable to go any further in the infrared due to the box size of $\sim 2$ fm.

Note that the gauge configurations remain untouched throughout the whole process.

IV. RESULTS

As a consistency check we first extract the masses of the $\rho$ states with our four source profiles and end up with the same results as already found in Ref. [12]. Masses of mesons, as expected, do not depend on the resolution scale $R$ and on a choice of a number of smearings used in the eigenvalue problem (5).

To study the ratio $a_V/a_T$ at different resolution scales $R$ we solve the $8 \times 8$ eigenvalue problem (5)-(7) with operators (1) and (2) and four different smearings. Then using (8) we extract the ratio $a_V/a_T$ as a function of $R$. In Fig. 3 we show the ratio $a_V/a_T$ at different resolution scales $R$. We find a clear $R$-effect on the ratio $a_V/a_T$. For both $\rho$ and $\rho'$ states we see a linear dependence on the resolution scale between 0.2 fm and 0.9 fm.

In the infrared, at the resolution scale 0.9 fm, the ratios are given by $a_V/a_T = (1.26 \pm 0.05)$ for the ground state $\rho$ meson and $a_V/a_T = -(0.88 \pm 0.18)$ for the first excited state. Here it is important to note that the ratio for the first excited state is negative. The ratio $a_V/a_T = 1.26$ for the ground state $\rho$ meson then means that the chiral representation $(0,1) \oplus (1,0)$ is slightly favouréd. For the first excited state with $a_V/a_T = 0.88$ the $(1/2,1/2)_b$ representation is slightly favourí.

Using now transformations [3], (1) we find:

$$|\rho(770)\rangle = + (0.998 \pm 0.002) |^3S_1\rangle$$ \hspace{3cm} (11)

$$- (0.05 \pm 0.025) |^3D_1\rangle \ ,$$

$$|\rho(1450)\rangle = - (0.106 \pm 0.09 ) |^3S_1\rangle$$ \hspace{3cm} (12)

$$- (0.994 \pm 0.005) |^3D_1\rangle \ .$$

The ground state $\rho$ is therefore practically a pure $^3S_1$ state, in agreement with the potential quark model assumption.

The first excited $\rho$ is, however, a $^3D_1$ state with a very small admixture of a $^3S_1$ wave. The latter result is in clear contradiction with the potential constituent quark model that attributes the first excited state of the $\rho$-meson as a radially excited $^3S_1$ state.

2 We found an error in the data which led to the conclusions in Refs. [3], [6]. Correcting this error gives the same result as presented in Fig. 3.
V. EFFECT OF LOW-MODE TRUNCATION ON THE OVERLAP FACTORS

We now study the effect of removing the low-lying modes of the Dirac operator on the ratio $a_V/a_T$. Its effect on the hadron spectrum has been studied extensively in Refs. [10]-[13]. The Banks-Casher relation connects the low-lying modes of the Dirac operator to the quark condensate. Hence by removing the lowest eigenmodes we decouple our $\rho$ states from the chiral symmetry breaking dynamics. The procedure of removing the low modes from the quark propagator ($D^{-1}$ denotes the quark propagator) is given as:

$$D_k^{-1}(x,y) = D_{FULL}^{-1}(x,y) - \sum_{i=1}^{k} \frac{1}{\lambda_i} v_i(x) v_i(y).$$

(13)

In Fig. 4 we show the value $a_V/a_T$ for the ground state $\rho$ and excited $\rho'$ at different resolutions $R$ for an increasing number of removed modes. For $k = 0$, i.e. the full theory, the mesons are a strong mixture of both chiral representations. With an increasing number of modes removed the ground state $\rho$ meson approaches a pure $(0, 1) \oplus (1, 0)$ state, whereas the first excited state $\rho'$ becomes a pure $(1/2, 1/2)_b$ state. Already at $k = 10$ the states are strongly dominated by one chiral representation: the chiral representations, which are slightly favored for $k = 0$, become dominant for $k \neq 0$.

After removal of $\sim 10-20$ lowest modes both $\rho$ and $\rho'$ get degenerate, which reflects a $SU(4)$ symmetry [14][15] of QCD in Euclidean space-time [16]. For other recent studies of this issue see Refs. [17][18].

VI. SUMMARY AND CONCLUSIONS

In this paper we addressed the issue of the angular momentum content of the leading quark-antiquark component of $\rho$ and $\rho'(1450)$. In the potential constituent quark model both states are assumed to be $^3S_1$ state.

We investigated this issue via a lattice simulation with dynamical Overlap fermions. We studied the ratio of overlap factors of vector and pseudotensor interpolators that belong to different chiral representations. We observed this ratio of overlap factors to be negative for the $\rho'(1450)$, which implies that this state is a $^3D_1$ state with only a tiny $^3S_1$ component. The $\rho(770)$ is, in agreement with the quark model, a $^3S_1$ state.

Then we studied the effect of removing the low-lying Dirac eigenmodes on the ratio of overlap factors. The state, which was identified as the ground state $\rho$ at truncation zero, becomes the $(1, 0) \oplus (0, 1)$ state, while the $\rho'$ meson becomes a pure $(1/2, 1/2)_b$ state. They are both degenerate, which is a manifestation of the previously found $SU(4)$ symmetry.

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