Weyl-Invariant Gravity and the Nature of Dark Matter

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Abstract. The apparent missing mass in galaxies and galaxy clusters, commonly viewed as evidence for dark matter, could possibly originate from gradients in the gravitational coupling parameter, $G$, and active gravitational mass, $M_{act}$, rather than hypothetical beyond-the-standard-model particles. We argue that in (the weak field limit of) a Weyl-invariant extension of General Relativity, one can simply affect the change $\Phi_b(x) \rightarrow \Phi_b(x) + \Phi_{DM}(x)$, where $\Phi_b$ is the baryon-sourced potential and $\Phi_{DM}$ is the ‘excess’ potential. This is compensated by gradients of $GM_{act}$ and a fractional increase of $O(-4\Phi_{DM}(x))$ in the baryon density, well below current detection thresholds on all relevant scales.

1. Introduction

Two key components of the standard model (SM) of cosmology, $\Lambda$CDM, are dark energy (DE) and dark matter (DM), where evidence for DM on galactic and cosmological scales abounds. It is commonly thought that observations favor a type of ‘cold’ DM (CDM) on all relevant scales. CDM constitutes $\sim 80\%$ of the non-relativistic (NR) matter in the universe, and together with DE makes up the ‘dark sector’ of cosmology, accounting for $\sim 95\%$ of the energy budget of the universe at present.

DM has reincarnated several times over the nearly century old history of the modern era of cosmology [1-3]. The first indication for missing mass, i.e. DM, in galaxy clusters was provided by the velocity dispersion of a handful of galaxies in the Coma cluster [4, 5]. Five decades later it has been concluded based on observations of rotation curves in galaxies that DM halos dominate the mass budget on galactic scales [6]. At around the same time a $\sim 1$ Kev mass neutrino was proposed as the dominant component of the NR energy budget on cosmological scales [7].

Obvious DM candidates are massive compact halo objects (MACHOs) such as primordial black holes [8-11], but this possibility seems to be less popular at present as compared to weakly interacting massive particles (WIMPs), e.g. [12-19]. The WIMP paradigm is held by many as the most attractive microphysical explanation for DM on galactic and extragalactic scales. Yet, no definite detection had been made in any of the few dozen terrestrial DM particle detection experiments.
Cold, warm, and hot DM candidates are distinguished by the impact they have on structure formation hierarchy and are thus constrained by probes of the growth of structure. The simplest hot DM candidates are already ruled out on galactic scales. Warm DM candidates are thought to be either gravitinos or sterile neutrinos [20-22] that were non-thermally produced unlike the leading CDM particles which would account for WIMPs that are thermally produced. While CDM is the leading candidate on Hubble down to galactic scales, certain issues with CDM on dwarf galaxy scales seem to be more optimally addressed by fuzzy DM, e.g. [23]. Mimetic DM is yet another DM model that provides a geometrical (rather than material) interpretation for DM on cosmological scales, e.g. [24]. An alternative geometrical interpretation is provided by Horava gravity [25].

The allowed parameter space of DM particles ranges from ultralight to superheavy, that can be either bosons or fermions, neutral or charged, stable or unstable, interacting with DE or not, etc [26-29]. Inspired by supersymmetry, the list of candidate DM particles is large, e.g. [30]. These include: the neutralino, axion, sterile neutralino, gravitinos, axino, Q-balls, etc [31-33]. The number of model parameters in the minimal supersymmetric standard model (MSSM) exceeds 100, and under a few simplifying assumptions this number can be considerably reduced to $\sim 20$. Despite this ample freedom, there is yet no single candidate that fully explains the entire spectrum of DM phenomena, especially on the smallest cosmological scales, e.g. [34, 35]. In spite of extensive theoretical effort there is no viable unified DM model.

With dozens of direct [36-39] and indirect [40-46] detection experiments, as well as with the large hadron collider (LHC), a vigorous hunt for WIMPs, non-WIMP or Kaluza-Klein states [47, 48] is underway. Present and future indirect efforts aim at detecting either $\gamma$-ray emission from DM annihilation at the Galactic center and dwarf galaxies or via high energy neutrinos from the Sun [49-54].

Perhaps the first compelling alternative approach to explain galactic rotation curves and velocity dispersion in galaxy clusters was modified Newtonian dynamics (MOND) [55-58]. As a phenomenological model, MOND lacks a relativistic description thus preventing its application on cosmological scales. Relativistic theories of modified gravity (MOG) and mimetic gravity, also aim at explaining galactic rotation curves with no recourse to particle DM [59, 60]. These typically include both scalar, vector and tensor degrees of freedom [61, 62].

An alternative Weyl-invariant (WI) relativistic theory – fourth-order Weyl gravity – was later proposed to account for the anomalous rotation curves in galaxies with no recourse to DM [63-66]. The proposed remedy is based on an exact spherically symmetric static solution of the field equations. A similar solution based on Weyl invariant scalar tensor (WIST) gravity was found in [67]. However, it has not been demonstrated that these alternative approaches provide compelling explanations in other systems, e.g. galaxy clusters, and in more realistic (dynamical and non–symmetric) cases such as merging bullet-like galaxy clusters. Other attempts to resolve the DM enigma include, in particular, scalar-tensor (ST) theories that provide specific mechanisms with falsifiable
predictions.

The discovery of a few bullet-like merging galaxy clusters is seen by many as a serious challenge to MOND and other alternatives to the DM paradigm [68]. In these systems the center of gravity (probed by lensing) and the luminous baryonic matter (observed through its X-ray emission) are clearly separated and this is conventionally interpreted as evidence that the bulk mass of the merging clusters is made of dissipationless matter. However, the existence of DM in these systems is deduced from lensing measurements (of background quasars). The latter depends on transversal gradients of the gravitational potential $\nabla_\perp \Phi$; the equivalent DM density profile $\rho_{DM}(x)$, which is not directly measured, can be deduced from maps of $\nabla_\perp \Phi$ (integrated along the line of sight).

Crucial to this procedure in particular, and to the CDM paradigm in general, is the assumption that the gravitational coupling $G$ is a universal constant. This has been established to a high level of precision essentially only with regard to its temporal evolution, e.g. [69, 70]. However, more relevant to the current work is a possible space-dependence of the Planck mass, $m_P \propto G^{-1/2}$, and more specifically scale-dependence, if an underlying translation invariance is assumed. In comparison, the observed scale-dependence of DM phenomena is implicitly set in WIMP or fuzzy DM models by, e.g. the particle mass or de Broglie wavelength, respectively. We note in passing that even if the conservative interpretation of bullet-like clusters is accepted at face value, the existence of these systems only implies that DM cannot be fully accounted for by baryons, by no means does this prove that DM is particulate.

2. WIST theory

Our universe is highly symmetric; the electroweak and strong interactions are described by a $U(1) \otimes SU(2) \otimes SU(3)$ gauge symmetry, and the underlying symmetry of general relativity (GR) is general coordinate invariance. A hitherto hidden symmetry of GR could be WI. The ultimate test of the validity of symmetries in physical theories is experiment, as well as naturalness, falsifiability, etc.

While the assumptions that the Higgs mass scale $v \approx 246$ GeV and chiral symmetry breaking mass scale $\Lambda_{QCD} \approx 220$ MeV are universally fixed seem to work well for the SM of particle physics, a similar assumption for $G$ may fail already on galactic scales (unless DM is invoked or gravity is modified), although it is consistent with solar system observations. If $G$ is determined by a scalar field then the convention that the former is universal obviously implies constancy of the latter. In the following, we relax this assumption and allow variation of $GM_{act}$, where $M_{act}$ is the active gravitational mass, in space and time [71]. The equivalence of passive and inertial masses is essentially the ‘equivalence principle’ which is not invalidated by allowing for variations of $GM_{act}$ [72].

The WIST theory considered here is obtained by replacing the spacetime metric $g_{\mu\nu}$ with $\phi^* g_{\mu\nu}$ everywhere in the Einstein-Hilbert (EH) action $I_{EH} = \int \left[ R/(16\pi G) + L_m \right] \sqrt{-g} d^4 x$ (in units where $G \equiv \frac{3}{8\pi}$), where $\phi$ & $\phi^*$ are a scalar field and its complex
conjugate, $\mathcal{L}_m$ is the lagrangian density of matter and $g$ is the metric determinant. The resulting ST theory is

$$\mathcal{I}_{ST} = \int \left( \frac{1}{6} |\phi|^2 R + \phi_\mu \phi^{*\mu} + \mathcal{L}_m(|\phi|, \{\psi\}) \right) \sqrt{-g} d^4 x$$

$$= \int \left( \frac{1}{6} |\phi|^2 R - \phi^* \Box \phi + \mathcal{L}_m(|\phi|, \{\psi\}) \right) \sqrt{-g} d^4 x, \quad (1)$$

where $\phi_\mu \equiv \phi_\mu$, $\mathcal{L}_m$ is now allowed to explicitly depend on $|\phi|$ but not on its derivatives, and the second equality follows from integration by parts of the canonical kinetic term associated with the scalar field. All other fields are collectively denoted by $\{\psi\}$. The kinetic term associated with the scalar field $\mathcal{L}_\phi \equiv -\phi^* \Box \phi$ can be considered as a new source of the gravitational force. This term is completely ignored in GR as $|\phi|$ is a constant $\sqrt{\frac{3}{8\pi G}}$ in this case.

The model has a global U(1) symmetry but we will not need the phase of the scalar field here. The latter plays an important role in the very early universe in a cosmological model based on Eq. (1) [73]. By construction, the action is invariant under Weyl transformations, i.e. $g_{\mu\nu} \rightarrow \Omega^2 g_{\mu\nu}$, $\mathcal{L}_m \rightarrow \Omega^{-4} \mathcal{L}_m$, $\phi \rightarrow \phi/\Omega$, with $\Omega(x)$ a continuous but otherwise arbitrary spacetime-dependent function. The theory described by Eq. (1) was first considered in [74] for the case of real $\phi$ and $\mathcal{L}_m$ independent of $\phi$. This additional freedom to locally rescale fields is essential to our construction; it implies that $\Omega(x)$ can be so chosen so as to modify any single metric perturbation potential to obtain any desired value at any point in spacetime, but at the cost of corresponding modifications to $\phi$ (i.e. $G$), and $\mathcal{L}_m$ (that determine the energy density of matter and its pressure). This is exactly the case we consider below; neglecting any vector and tensor perturbation modes, and assuming vanishing stress, metric perturbations of scalar type are described by a single function $\Phi(x)$.

Variation of Eq. (1) with respect to the metric and $\phi^*$ results in the following field equations, respectively, e.g. [75]

$$\frac{|\phi|^2}{3} G_{\mu\nu} = T_{M,\mu\nu} + \Theta_{\mu\nu} \quad (2)$$

$$\frac{1}{6} \phi R - \Box \phi + \frac{\partial \mathcal{L}_m}{\partial \phi^*} = 0, \quad (3)$$

where

$$3 \Theta_{\mu\nu} = \phi^{*\alpha}_{\mu\nu} \phi - 2 \phi^*_{\mu} \phi_{\nu} - g_{\mu\nu} (\phi^* \Box \phi - \frac{1}{2} \phi^*_{\alpha} \phi^\alpha) + c.c. \quad (4)$$

Eq. (2) is a generalization of Einstein equations, and $\Theta_{\mu\nu}$ is an effective contribution to the energy-momentum tensor essentially due to gradients of $\phi$ (that determines $G$ and $M_{act}$). From Eq. (4) it then follows that the trace of $\Theta_{\mu\nu}$ is $\Theta = \mathcal{L}_\phi$.

In the weak field limit Eq. (2) results in a modified Poisson equation. Multiplying Eq. (3) by $\phi^*$ and adding the result to its complex conjugate and to the trace of Eq. (2) results in the constraint

$$\phi^* \frac{\partial \mathcal{L}_m}{\partial \phi^*} + \phi \frac{\partial \mathcal{L}_m}{\partial \phi} = T_m, \quad (5)$$
i.e. only pure radiation $T_{\text{rad}} = 0$ is consistent with WI gravity unless $\mathcal{L}_m$ (which in the case of perfect fluid equals $T_m$) explicitly depends on $|\phi|$. This fact explains the requirement in Eq. (1) that in general the matter lagrangian does depend on $|\phi|$. Recalling that $\mathcal{L}_m$ is a potential in $\phi$, it then follows that for NR matter, $\rho_{\text{NR}} \propto \rho$, i.e. $M_{\text{act}} \propto \rho$, where $\rho$ is the modulus of $\phi$. For a general equation of state (EOS) $w \equiv P_m/\rho_m$, where $P_m$ is the pressure associated with matter of energy density $\rho_m$, Eq. (5) is satisfied by $\mathcal{L}_m \propto \rho^{1-3w} = (\phi\phi^*)^{\frac{1-3w}{2}}$. We mention in passing that no violation of the equivalence principle ensues insofar inertial and passive gravitational masses are fixed, which we indeed assume here. The notion of passive gravitational masses is vacuous in the present theory and we only use the Newtonian parlance for clarity. Irrespective of that, the equivalence principle has never been directly tested beyond the solar system anyway, and obviously not directly with DM particles. 

The relevant combination for NR gravitating source is $GM_{\text{act}}$ which is $\propto \rho^{-1}$. While $GM_{\text{act}}$ is spacetime-dependent, we assume that the SM of particle physics is left unchanged (i.e. WI is not a symmetry thereof), and inertial masses are universal constants.

3. DM in gravitationally-bound structures

A key to the following considerations is the fact that 'DM' is only probed via the force its exerts, rather than $\rho_{\text{DM}}$. The latter, we recall, is absent from $\mathcal{L}_m$ in Eq. (1). In contrast, density distribution of baryonic matter, $\rho_b$, is directly observed through electromagnetic emission, absorption and scattering processes. Throughout, we assume that DM is not real matter and that its apparent attributes arise from gradients of $GM_{\text{act}}$.

Although we focus on WIST, the following arguments are equally well valid in any WI theory of gravitation. However, in general other theories do not have a Poisson-like equation as their weak field limit, and thus $G$ is only an effective notion, e.g. [76]. Although there is no scalar field in the gravitational sector of forth-order Weyl gravity such a scalar (or an effective scalar) field is expected to appear in $\mathcal{L}_m$. The latter must scale $\propto \Omega^{-4}$ in any WI theory, and since there are no mass scales in the theory, effective masses, e.g. the Planck mass or active gravitational mass (the mass that sources the gravitational field in $\mathcal{L}_m$ that need not be identical with the passive gravitational mass or the inertial mass), scale $\propto \Omega^{-1}$.

Neglecting DM effects, assuming negligible stress and vanishing vector and tensor modes, the appropriate spacetime for galaxies and galaxy clusters is described (in the weak field limit) by the line element

$$ds^2 = -(1 - 2\Phi_b)dt^2 - dtdx^i v_i + (1 + 2\Phi_b)dx_i dx^i,$$

(6)

where Latin indices (denoting spatial coordinates) are summed over, $v_i \ll 1$ is a NR velocity and $\Phi_b \ll 1$ is the linear gravitational potential that satisfies the relativistic generalization of Poisson equation with a source term $\rho_b$. Here, $v_i$ is already a linear perturbation over Minkowski spacetime. Other than the condition $\Phi_b \ll 1$ we make
no specific assumptions about the spacetime-dependence (symmetry) of $\Phi_b$ and the following results are therefore general. Linear analysis is justified by the fact that the rms internal velocities of substructure within galaxies and galaxy clusters are much lower than the speed of light, i.e. $\Phi_b \ll 1$ & $\Phi_{DM} \ll 1$.

Applying a Weyl transformation with $\Omega = 1 + \Phi_{DM}(x)$ and recalling that $g_{\mu\nu} \rightarrow \Omega^2 g_{\mu\nu}$ and $\phi \rightarrow \phi/\Omega$, then at first order, the gravitational potential in Eq. (6) is shifted, $\Phi_b \rightarrow \Phi_b + \Phi_{DM}$, at the “cost” of $G M_{act} \propto 1 + \Phi_{DM}$ and a corresponding change in $\rho_b$ that we discuss below.

We assume that $\Phi_{DM} \ll 1$ on the relevant range of scales but is a priori an arbitrary function of spacetime. It accounts for the excess gravitational potential required to explain galactic scale observations. Translation invariance further implies that $\Omega$ is a function of scale rather than location. Specifically, in the spherically symmetric static case, we could select the Navarro-Frenk-White (NFW) [77] gravitational potential profile, $\Phi_{NFW}(r) \propto r^{-1} \ln(1 + r/r_s)$ (where $r_s$ is an object-specific scale parameter), or any other profile, e.g. [78], as our $\Phi_{DM}(r)$. The underlying assumptions about the microphysics that lead to the specific r-dependence of $\Phi_{NFW}(r)$ (or any other $\Phi_{DM}(r)$ profile for that matter that fits observational data) are irrelevant to our discussion. It only matters that the combination $\Phi = \Phi_b(x) + \Phi_{DM}(x)$ provides a good fit to the data.

The excess gravitational potential $\Phi_{DM}$ is supported by gradients of $\phi$, i.e. of $G M_{act}$, via the $\Theta_{\mu\nu}$ addition to the baryonic energy-momentum tensor in Eq. (2). In this sense, “$T_{DM,\mu\nu} = \Theta_{\mu\nu}$”. Its nonvanishing components in spherical coordinates, linear in $\Phi_{DM} \ll 1$, are $\Theta_\theta = \frac{2\phi^2}{3}(\Phi_{rr} + \frac{2}{r}\Phi_r)$, $\Theta_r = \frac{2\phi^2}{3r}\Phi_r$, and $\Theta_\phi = \Theta_\varphi = \frac{\phi^2}{3}(\Phi_{rr} + \frac{1}{r^2}\Phi_r)$, where the ‘DM’ subscript was dropped and it is assumed that $\Phi = \Phi(r)$. Interestingly, the trace $\rho_{DM} - 3P_{DM} = -\Theta = 3\rho_{DM}$, which corresponds to an effective (averaged over spatial directions) EOS $-2/3$. Although $w_{DM} = -2/3$ is negative and is very nearly the EOS of the expanding Universe at the present, we believe that this is merely a coincidence. Similarly irrelevant to the present work is the fact that it is the same EOS of a ‘domain wall’ solution of the Einstein equations. The pressure in the r-direction differs in general from the pressure in the other two orthogonal directions. Therefore, the effective energy-momentum tensor associated with DM does not correspond to a perfect fluid, and in any case does not describe pressureless matter as does CDM. This is not a problem since neither DM pressure nor DM density [and consequently not its EOS $w_{DM}$] are directly observed. Again, in practice the CDM paradigm with $w_{DM} \approx 0$ is only a means to obtain an adequate $\Phi_{DM}(r)$ fit on these scales assuming the latter satisfies the Poisson, Euler and continuity equations provided that $G$ is a universal constant.

These equations are essentially obtained from Eq. (2) and its derivatives in the more general case (varying $G$ and $M_{act}$) considered here. As emphasized above, from the WI perspective any guess, or e.g. artificial intelligence informed reconstruction, of $\Phi_{DM}(r)$ is equally legitimate insofar as it provides a reasonably good fit to the data combined with $\Phi_b$, much like $|\phi|^2 = 3/(8\pi G) = constant$ across the entire universe and over Hubble time is an educated guess based on observations over the relatively small solar system scales. However, a significant difference is that in the latter convention DM is
required (to make up for the convenient $|\phi|^2 = 3/(8\pi G) = \text{constant}$ choice) which is widely believed to consist of beyond-the-SM particles.

Aside from accounting for the apparent effects of DM, the only (potentially observational) consequence of the $\Omega = 1 + \Phi_{DM}(x)$ choice in these systems is that $\rho_b$ transforms as $\rho_b \rightarrow \rho_b/(1 + \Phi_{DM})^4 \approx \rho_b(1 - 4\Phi_{DM})$. Since typically $-\Phi_{DM} = O(10^{-4})$ in galaxies or $O(10^{-5})$ in galaxy clusters, then $O(4\Phi_{DM})$ amounts to a tenth of a percent increase in $\rho_b$ at most, too weak to be measured given the typically complicated morphology and lumpiness of baryon distribution, temperature uncertainty, current observational precision, etc, on these scales. Therefore, the transformation $\Omega = 1 + \Phi_{DM}(x)$ can significantly modify the gravitational potential, e.g. outside galaxy cluster and galaxy cores, while making only a subtle change in the observable luminous matter density, thereby making the case for a ‘non-particle DM’. This procedure could clearly be applied to merging bullet-like clusters as well, where in this case $\Phi_{DM}$ is expected to depend on both space and time, with no impact on our general conclusion that DM may well be a non-WIMP, and non-MACHO-related phenomenon on galactic and galaxy cluster scales.

The approach explored here to remove the need for DM on galactic and super-galactic scales could be similarly employed on cosmological scales since typically $\Phi = O(10^{-5})$ on these scales. However, a NR DM component seems to still be required at the background level by a wealth of cosmological probes. In the concordance cosmological model the NR component accounts for $\sim 30\%$ of the total energy density at present, while baryons account for only $\sim 5\%$; the remaining $\sim 25\%$ is believed to be in CDM.

Similar to the replacement of $g_{\mu\nu}$ with $\phi^*g_{\mu\nu}$ that leads to Eq. (1), replacing the metric with $g^{\alpha\beta}\varphi_\alpha\varphi_\beta g_{\mu\nu}$ (along with the constraint $g^{\alpha\beta}\varphi_\alpha\varphi_\beta = 1$, where $\varphi$ is a real scalar field) in the EH action could be made, that leads to “mimetic gravity”. The latter theory was proposed as a solution to the DM problem on the largest cosmological scales assuming the homogeneous and isotropic Friedmann-Robertson background metric \cite{79,80}. According to this picture the additional energy provided by $\varphi_\mu$ mimics CDM with an EOS $w_{\text{mim}} = 0$. This does not contradict our proposed solution in gravitationally bound structures for which the effective matter density and pressure do not correspond with NR matter since the “missing mass” problems in these two entirely different physical environments are of completely different nature; $GM_{\text{act}}$ is indeed constant on cosmological scales as in the SM of cosmology, but spatially varies on galactic scales.

4. Summary

The perspective advocated in the present work is that WI may well be a symmetry of gravitation, and that while GR represents a convenient specific choice of units (namely constant $G$ and $M_{\text{act}}$), it may have misled us to think that some exotic form of DM is required to account for observations (either in the form of MACHOs or WIMPs). A more prosaic solution to the DM problem might be that $GM_{\text{act}}$ is actually scale-dependent.

In the context of the proposed ST theory gradients of the scalar field (essentially of
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$G M_{act}$ may provide the required additional energy and pressure for warping space to reproduce the observed gravitational pull caused by hypothesized DM without recourse to DM particles. Specifically, we show that DM manifestations could be fully accounted for through the innocuous change $\Phi_b \to \Phi_b + \Phi_{DM}$ at the ‘cost’ of $O(\Phi_{DM})$ fractional variations in $G M_{act}$ and $\rho_b$ in the range of $O(10^{-3}) - O(10^{-4})$ on the relevant galactic and super-galactic scales, levels that are too weak to be directly observed.

As the conformal factor $\Omega(\mathbf{x}) = 1 + \Phi_{DM}(\mathbf{x})$ [and consequently $\Phi_{DM}(\mathbf{x})$] is arbitrary, it could be in principle selected to better-fit observations (on all relevant scales, including dwarf galaxy scales) than would standard particle-induced CDM gravitational potential profiles do. We note that at the cosmological background level GR endowed with WI is equivalent to mimetic gravity, and thus contains DM-like energy density and energy density perturbations with no recourse to DM particles.

It may seem that since the conformal factor $\Omega(\mathbf{x}) = 1 + \Phi_{DM}(\mathbf{x})$, essentially $G M_{act}(\mathbf{x})$, can always be so chosen to fit the data then the theory is not falsifiable. However, by the same rationale the choice $|\phi|^2 = 3/(8\pi G) = constant$ employed in the SM has already been falsified by observations on galactic scales, and it is only saved by invoking an unobservable $\rho_{DM}(\mathbf{x})$ component. It is somewhat ironic that if existing DM candidates are not found, the CDM paradigm may still hold up with the only consequence that the vast WIMP parameter space is narrowed down, while in contrast, if WIMPs are found with properties that are sufficient for adequately explaining observations, our proposed solution (as is any other alternative solution) to the DM problem is essentially falsified.

We conclude that if gravitation is indeed locally scale-invariant, a symmetry hidden by universally setting $|\phi|^2 = 3/(8\pi G) = constant$ in GR (largely motivated by convenience, at least on galactic and cosmological scales), then the currently favorite identification of DM with WIMPs may just be an artifact of arbitrarily adopting a constant system of units.

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