String no-scale supergravity

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Abstract

We explore the postulates of string no-scale supergravity in the context of free-fermionic string models. The requirements of vanishing vacuum energy, flat directions of the scalar potential, and stable no-scale mechanism impose strong restrictions on possible string no-scale models, which must possess only two or three moduli, and a constrained massless spectrum. The soft-supersymmetry-breaking parameters involving all twisted and untwisted fields are given explicitly. This class of models contain no free parameters, i.e., in principle all supersymmetric particle masses and interactions are completely determined. A computerized search for free-fermionic models with the desired properties yields a candidate $SU(5) \times U(1)$ model containing extra $(10,10)$ matter representations that allow gauge coupling unification at the string scale. Our candidate model possesses a benign non-universal assignment of supersymmetry breaking scalar masses, which may have interesting low-energy experimental consequences.
1 Introduction

Experiments at LEP and the Tevatron have given the strongest to-date support to the Standard Model of the strong and electroweak interactions. Yet despite all this experimental evidence, physicists believe that this model is incomplete. One possible completion of the Standard Model is embedded in the physics of supersymmetry. In fact, the same experimental evidence confirming the Standard Model, can also be used to support its supersymmetric extension. This can be seen through the unification of the gauge couplings at very high energies in supersymmetric models, but perhaps more pervasively from the fact that supersymmetric models have built-in mechanisms that make them look almost identical to the Standard Model at presently available facilities. This extreme similarity with the Standard Model occurs at the tree-level for energies below the threshold for production of supersymmetric particles, and also at one-loop if the supersymmetric mass scales exceed the electroweak scale. This similarity is not totally devoid of predictivity, since in supersymmetric models the lightest Higgs boson is expected to be light ($m_h \sim M_Z$). In view of these facts, attention has turned strongly towards supersymmetric models, in particular those that can be understood as low energy limits of more fundamental theories, such as grand unification, supergravity, and superstrings. These models are highly predictive, with all supersymmetric physics typically dependent on four or less parameters.

Four-parameter supersymmetric models are obtained as low-energy effective supergravity models with assumed universal soft-supersymmetry-breaking terms: the scalar mass ($m_0$), the gaugino mass ($m_{1/2}$), the trilinear scalar coupling ($A$), and (at low energies) the ratio of Higgs vacuum expectation values ($\tan \beta$). Most phenomenological analyses are content with exploring such a four-dimensional parameter space. However, theoretically speaking it is not clear that all possible combinations of these parameters are consistent, since in a specific supergravity model, i.e., one specified by the Kähler function $G$ and the gauge kinetic function $f$, the parameters $m_0, m_{1/2}, A$ can be explicitly calculated in terms of the gravitino mass ($m_{3/2}$). Guidance in this matter has come from string-inspired choices for $G$ and $f$, which lead to simple values for the ratios $m_0/m_{1/2}$ and $A/m_{1/2}$, and thus to two-parameter supersymmetric models.

Despite this great reduction in the model parameters, several questions remain unanswered: (i) can one construct an explicit string-derived model where the various ratios of soft supersymmetry breaking terms are calculated, and at the same time the usual low-energy phenomenology is explained? (ii) does this model possess a sufficiently suppressed cosmological constant? (iii) how is the scale of supersymmetry breaking ($m_{3/2}$) determined in such model? This model would be a “no-parameter” model.

No-scale supergravity provides satisfactory answers to the latter two questions, i.e., vanishing cosmological constant (at the tree level) and dynamical determi-
nation of $m_{3/2}$ via the no-scale mechanism. *String no-scale supergravity* is postulated to be the subset of string models which can provide satisfactory answers to all three questions. This subset is rather restricted, since in practice it is seen that most known string models do not obey the postulates of string no-scale supergravity.

Our purpose in the present paper is to explore the postulates of string no-scale supergravity in the context of fermionic string models. Our investigations lead to a set of constraints on the spectrum and interactions in realistic string models, as well as to novel predictions for the soft-supersymmetry-breaking parameters. We are also able to provide an existence proof that realistic string no-scale supergravity models do exist. A search for models of this type with the gauge group $SU(5) \times U(1)$ turns up a rather interesting phenomenon: among the class of models which we have explored, a necessary condition to satisfy the postulates of no-scale supergravity appears to be the existence in the spectrum of extra $(10, \overline{10})$ matter representations that allow gauge coupling unification at the string scale.

This paper is organized as follows. In Sec. 2 we summarize the postulates of no-scale supergravity and the no-scale mechanism. In Sec. 3 we discuss the Kähler potential and the superpotential in free-fermionic string models, and compute the vacuum energy, and the quantity $\text{Str} \mathcal{M}^2$. We also discuss the conditions under which these quantities would vanish, as required in no-scale models. In Sec. 4 we compute the soft-supersymmetry-breaking parameters, including the Goldstino composition, the gaugino and scalar masses and the $A$ terms. We also discuss the origin of the $\mu$ term and the associated parameter $B$. In Sec. 5 we discuss the normalization of the fields and how these affect the observable Yukawa couplings. In Sec. 6 we perform a search for realistic string no-scale free-fermionic models, and present evidence for the conjecture mentioned in the previous paragraph. We also study the supersymmetry-breaking parameters in the candidate model found. Finally, in Sec. 7 we summarize our conclusions, in Appendix A we collect some details about the transformation between the string and supergravity bases, and in Appendix B we give details of the calculation of the twisted sector Kähler potential in a specific model.

## 2 No-scale supergravity

A supergravity theory is specified by two functions, the Kähler function

$$G = K + \ln |W|^2,$$

where $K$ is the Kähler potential and $W$ the superpotential, and the gauge kinetic function $f$. All masses and interactions are explicitly calculable from these inputs. In particular, the (tree-level) scalar potential is given by

$$V = e^G(G^I G_I - 3) + V_D,$$

where the sum is over all scalar fields in the spectrum, $G_I = \partial_I G$, $G^I = G^{IJ} G_J$, and $G^{IJ}$ is the inverse Kähler metric (i.e., the transpose of the inverse of $G_{IJ} = K_{IJ}$).
The second term in Eq. (2) is the contribution from the $D$-terms; we will assume in what follows that this vanishes at the minimum of the potential. The scalar potential is used to determine the vacuum, its energy, and any flat directions it may have. Also, small deviations around it determine the supersymmetry-breaking masses and couplings of the scalar fields. Finally, derivatives of the Kähler function determine the supersymmetry-breaking masses of the (non-chiral) fermions, and derivatives of the gauge kinetic function determine the supersymmetry-breaking gaugino masses.

Spontaneous breakdown of supergravity induces a mass for the gravitino
\[
m_{3/2} = \langle e^{G/2} \rangle = \langle e^{K/2} |W| \rangle .
\] (3)

This relation shows that supersymmetry breaking can only occur if $\langle W \rangle \neq 0$. All soft-supersymmetry-breaking parameters are proportional to $m_{3/2}$, with typically $O(1)$ coefficients of proportionality. Therefore, $m_{3/2}$ values are expected to be not much higher than the electroweak scale. Moreover, restoring the dimensions in Eq. (3) one sees that the right-hand-side has units of $10^{18}$ GeV and therefore a strong suppression of $e^{K/2}$ or $\langle W \rangle$ is typically necessary. Two scenarios for $\langle W \rangle \neq 0$ have received the most attention in the literature: gaugino condensation in the hidden sector [2], giving $W \sim e^{-b/\pi^2/g^2}$; and string tree-level breaking via coordinate-dependent compactifications [3], giving $W \sim 1$.

No-scale supergravity is defined by three constraints on a supergravity model:

- The vacuum energy vanishes ($V_0 = \langle V \rangle = 0$) by suitable choice of the Kähler function ($G$) [4].
- At the minimum of the potential there are flat directions (“moduli”) which leave the value of $m_{3/2}$ undetermined [4].
- The quantity $\text{Str} \mathcal{M}^2$ should vanish at the minimum. This constraint protects the potential from large one-loop corrections which would otherwise force $m_{3/2} = 0$ or $m_{3/2} = M_{Pl}$ [4].

These three constraints impose severe restrictions on the possible $G$ and $f$ functions. Particularly non-trivial is the last one, i.e., $\text{Str} \mathcal{M}^2 \equiv 2Qm_{3/2}^2$, with $\text{5, 6}$
\[
Q = N - 1 - G^I (R_{I\bar{J}} - H_{I\bar{J}})G^J ,
\] (4)
where $N$ is the total number of chiral superfields, and
\[
R_{I\bar{J}} = \partial_I \partial_J \ln \det G_{MN} ,
\] (5)
\[
H_{I\bar{J}} = \partial_I \partial_J \ln \det \text{Re} (f_{ab}) .
\] (6)

If the above three conditions are satisfied, the low-energy theory, obtained by renormalization-group evolution from the Planck scale down to the electroweak scale, will be undetermined to the extent that $m_{3/2}$ is undetermined, as it depends on the
undetermined moduli vacuum expectation values (VEVs). The low-energy one-loop effective potential ($V_{\text{eff}}$) then depends on the usual Higgs fields, as well as the moduli fields. The no-scale mechanism \[7\] consists of minimizing this potential with respect to all these fields, thus determining the Higgs and moduli vacuum expectation values. The thusly determined moduli VEVs then determine $m_{3/2}$, and therefore all of the supersymmetry breaking masses.

The no-scale mechanism has an additional unsuspected consequence: it may solve the strong CP problem \[8\]. Indeed, the equally undetermined imaginary parts of the moduli fields leave the $\theta_{\text{QCD}}$ parameter undetermined, i.e., the potential in the imaginary directions is also flat.\[3\] According to the usual argument, non-perturbative QCD dynamics at low energies determines $\theta_{\text{QCD}} = 0$, which in our language corresponds to lifting the imaginary flat directions, giving zero VEVs to the corresponding fields.

In practice, this procedure is subtle and complicated by the existence of a remnant vacuum energy term at high energies. This field-independent term ($Cm_{3/2}^4/2$) needs to be added to the low-energy effective potential to ensure its renormalization-scale independence \[10, 11, 12\], i.e.,

$$V_{\text{eff}} = V_{\text{tree}} + \frac{1}{64\pi^2} \text{Str} \mathcal{M}^4 \left( \ln \frac{\mathcal{M}^2}{Q^2} - \frac{3}{2} \right) - Cm_{3/2}^4 , \hspace{1cm} (7)$$

where $V_{\text{tree}}$ is the tree-level Higgs potential. It is worth mentioning that just as the usual radiative breaking mechanism (i.e., minimization of $V_{\text{eff}}$ with respect to the Higgs fields) does not always work, the no-scale mechanism may also not work. This happens when the effective potential does not have a good minimum in the moduli directions. In the case of a single modulus field (as we discuss below), it can be shown that a necessary condition for a good minimum is $\text{Str} \mathcal{M}^4 > 0$ \[12\]. If the explicit $m_{3/2}$-dependent contribution to $\text{Str} \mathcal{M}^4$ is negative, then \[12\]

$$\text{Str} \mathcal{M}^4 > 0 \implies \frac{m_{3/2}}{m_\tilde{q}} \lesssim \mathcal{O}(1) , \hspace{1cm} (8)$$

which imposes restrictions on the allowed low-energy parameter space.

### 3 Fermionic string models

The discussion in the previous section can be applied to any supergravity or superstring model. However, the requirement of $\text{Str} \mathcal{M}^2 = 0$ can only be investigated if the full spectrum of the model is known, as in string models. We are interested in exploring the three postulates of string no-scale supergravity in the context of string models built within the free-fermionic formulation \[13\]. Our motivation for such choice is that

\[\text{This is certainly the case at tree-level in the Kähler potential and for moduli-independent Yukawa couplings. In free-fermionic models, moduli dependence of the superpotential does not arise until the quartic order \[9\].}\]
fairely realistic models already exist in this construction \cite{4, 13, 16, 17}, and we would like to know whether this class of models satisfies the postulates, or what constraints may need to be imposed on model-building so that these postulates are satisfied.

3.1 Generalities

All of the level-one free-fermionic models built to date have been based on the simplest supersymmetry-generating basis vector $S$. This choice is not unique, but should suffice for our present purposes of investigating the viability of no-scale supergravity in free-fermionic models. In this class of models, all states in the spectrum fall into three sets, depending on the quantum numbers they carry with respect to some internal symmetries of the two-dimensional world-sheet theory \cite{18}. The spectrum further divides itself into two sectors: untwisted and twisted. More specifically, all matter fields carry charges under three world-sheet $U(1)$ currents. The sum of these three currents provides the additional current which extends the manifest $N = 1$ world-sheet supersymmetry to $N = 2$ world-sheet supersymmetry \cite{18}, as required for $N = 1$ space-time supersymmetry to exist. The scalar components of untwisted (or Neveu-Schwarz) matter superfields carry one of three possible types of charges, whereas the twisted matter superfields carry two of them, i.e.,

\[
\begin{array}{c|c|c}
\text{Untwisted} & \text{Twisted} \\
\hline
\text{First set} & \{0, 1, 0\} & \{1, \frac{1}{2}, \frac{1}{2}\} \\
\text{Second set} & \{0, 1, 0\} & \{\frac{1}{2}, 0, \frac{1}{2}\} \\
\text{Third set} & \{0, 0, 1\} & \{1, \frac{1}{2}, \frac{1}{2}, 0\}
\end{array}
\]

(9)

The charges of their fermionic partners are obtained by a uniform shift of $\{-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\}$. From these charge assignments it follows immediately what kind of cubic superpotential couplings are allowed in a model. Indeed, the sum of the charges of the three fields in question must be $\{1, 1, 1\}$; charge conservation follows from the two required shifts by $\{-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\}$, since a cubic coupling contains two fermionic fields and one scalar field. Thus, one gets five types of cubic couplings

\[
\begin{align*}
U^{(1)} U^{(2)} U^{(3)} & \quad U^{(1)} T^{(1)} T^{(1)} T^{(2)} T^{(2)} T^{(3)} \\
U^{(2)} T^{(2)} T^{(2)} & \quad U^{(3)} T^{(3)} T^{(3)}
\end{align*}
\]

(10)

where $U^{(I)} [T^{(I)}]$ denotes a generic untwisted (twisted) field belonging to the $I$-th set.

Several features of the Kähler function for the untwisted sector of such models had been known for some time \cite{19}, and have been recently further clarified, extended, and applied to specific models in Ref. \cite{20}. The corresponding contributions from the twisted sector have been known for some time in simple models \cite{21}, and have been only recently calculated in realistic models \cite{22}. Abstracting all that is known about free-fermionic models, we write

\[
G = -\ln(S + \bar{S}) + \sum_{I=1,2,3} K^{(I)} + K_{TS} + \ln |W|^2,
\]

(11)
with
\[
K_I = -\ln \left[ 1 - \sum_{i} \alpha_i \bar{\alpha}_i + \frac{1}{4} \left( \sum_{i} \alpha_i^2 \right) \left( \sum_{i} \bar{\alpha}_i^2 \right) \right],
\]  
(12)
where \( n_I \) represents the number of untwisted fields in set \( I \), and set-indices \( I = 1, 2, 3 \) on the \( \alpha_i \), i.e., \( \alpha_i^{(I)} \) are understood. (The twisted sector contribution \( K_{TS} \) will be addressed below.) The Kähler function in Eqs. (11),(12) is written in the “string basis”. For purposes of low-energy effective supergravity analyses, it is more convenient to make suitable field redefinitions of the \( \alpha_i \) to exhibit the moduli fields that may be present in the spectrum, i.e., to go from the “string basis” to the “supergravity basis”. In the class of fermionic models which we consider here, three possibilities for the untwisted moduli space of any of the sets were identified in Ref. [20]:

(i) \([SU(1, 1)/U(1)]^2\), with two moduli fields denoted by \( \tau_1, \tau_2 \) (a “\( \tau_1, \tau_2 \) set”).

(ii) \( SU(1, 1)/U(1) \) with one modulus field denoted by \( \tau \) (a “\( \tau \) set”).

(iii) No moduli at all (an “\( \alpha \) set”).

Whichever of the three possibilities may be realized for a given set depends on the choice of basis and GSO projections of the fermionic model. In what follows, if a set has any modular symmetry at all, we perform a field redefinition of the fields in that set which leaves the Kähler function \( G \) unchanged. (Details of these manipulations are given in Appendix A.) The redefined \( K_I \) are given by:

• \( \tau_1, \tau_2 \) set:

\[
K = -\ln \left[ (\tau_1 + \bar{\tau}_1)(\tau_2 + \bar{\tau}_2) - \sum_{i} (\phi_i + \bar{\phi}_i)^2 \right],
\]  
(13)
where \( n_\phi = n_I - 2 \), since two of the \( \alpha_i \) are transformed into the moduli \( \tau_1, \tau_2 \). The scalar fields parametrize the Kähler manifold \( SO(2, 2 + n_\phi)/SO(2) \times SO(2 + n_\phi) \), which has as a subspace the moduli space \( SO(2, 2)/SO(2) \times SO(2) \approx [SU(1, 1)/U(1)]^2 \).

• \( \tau \) set:

\[
K = -\ln \left[ (\tau + \bar{\tau})^2 - \sum_{i} (\psi_i + \bar{\psi}_i)^2 \right],
\]  
(14)
where \( n_\psi = n_I - 1 \), since one of the \( \alpha_i \) is transformed into the modulus \( \tau \). The scalar fields parametrize the Kähler manifold \( SO(2, 1+n_\psi)/SO(2) \times SO(1+n_\psi) \), which has as a subspace the moduli space \( SO(2, 1)/SO(2) \approx SU(1, 1)/U(1) \).

\[\text{In Ref. [20], } \tau_1, \tau_2 \text{ were denoted by } T, U. \text{ Such notation would cause confusion here.}\]

\[\text{We note that in the old no-scale models } \text{[3], the Kähler potential was assumed to be of the form } K \propto \ln(T + T - \sum_i C_i C_i), \text{ which yields the metric of the space } SU(1, n_C + 1)/U(1) \times SU(n_C + 1). \text{ The old no-scale ansatz and the string free-fermionic } \tau\text{-set result agree only for } n_\psi = n_C = 0.\]
The modular symmetries exhibited above can be extended from the Kähler potential \( K \) to the whole Kähler function \( G = K + \ln|W|^2 \) if the matter fields have suitable transformation properties under the modular symmetries \[20\]. In fact, the superpotential should transform as a modular form of weight \(-1\). For instance, if the modulus field in question belongs to the first untwisted set, then the modular weights of the matter fields are the negative of the first world-sheet \( U(1) \) charge in Eq. \( (8) \), i.e.,

\[
\begin{align*}
U^{(1)} & -1 & T^{(1)} & 0 \\
U^{(2)} & 0 & T^{(2)} & -\frac{1}{2} \\
U^{(3)} & 0 & T^{(3)} & -\frac{1}{2}
\end{align*}
\]

(15)

With this modular weight assignment one can verify that all cubic superpotential couplings in Eq. \( (10) \) have modular weight \(-1\). A similar argument holds for moduli in the other sets. When quartic or higher-order superpotential couplings are considered, the resulting modular weight imbalance has to be compensated by the insertion of suitable powers of the Dedekind eta function \([9]\).

It is important to reiterate that modular symmetries inferred from the Kähler potential must be respected by the whole Kähler function. If presumed moduli fields appear in the cubic superpotential, then the symmetry is explicitly broken and the presumed moduli are to be discarded. This phenomenon is quite common in free-fermionic models and has the beneficial effect of reducing the number of moduli in the model. Therefore, in what follows, when discussing the number and type of moduli and their impact on the vacuum energy and other properties of the models, we refer to fields which have been indentified as untwisted moduli and that have no superpotential couplings.

The twisted sector contribution to the Kähler potential \( K_{TS} \) has been obtained some time ago for simple models with a specific type and number of twisted sectors \[21\]. It turns out that the result found in Ref. \[21\] in fact applies to realistic free-fermionic models with a large number of twisted sectors \[22\] (see Appendix \[B\] for details). The result is

\[
K_{TS} = \sum_i^{n_{T_1}} \beta_i^{(1)} \bar{\beta}_i^{(1)} e^{\frac{1}{2}[K_{(2)} + K_{(3)}]} + \sum_i^{n_{T_2}} \beta_i^{(2)} \bar{\beta}_i^{(2)} e^{\frac{1}{2}[K_{(1)} + K_{(3)}]} + \sum_i^{n_{T_3}} \beta_i^{(3)} \bar{\beta}_i^{(3)} e^{\frac{1}{2}[K_{(1)} + K_{(2)}]}
\]

(16)

where the \( \beta_i^{(I)} \) are twisted sector fields that belong to the \( I \)-th set, \( n_{T_1,T_2,T_3} \) are the numbers of these fields, and \( K_{(1,2,3)} \) are given in Eq. \( (12) \). It is important to realize that this result is only valid to lowest order in the twisted matter fields.

3.2 Computation of \( V_0 \)

The computation of the scalar potential in Eq. \( (4) \) requires the knowledge of \( G^{IJ} \), i.e., the transpose of the inverse of the matrix of second derivatives of the Kähler potential. The computation is simplified by that fact that the matrix \( G_{IJ} = K_{IJ} \)
possesses a block-diagonal form once the twisted sector matter fields are set at their zero vacuum expectation values. Indeed, schematically we have

\[
K_{IJ} \sim \begin{pmatrix}
U^{(1)} & U^{(2)} & U^{(3)} & T^{(1)} & T^{(2)} & T^{(3)} \\
X_{U^{(1)}} & [T^{(3)}]^2 & [T^{(2)}]^2 & 0 & T^{(2)} & T^{(3)} \\
T^{(1)} & [T^{(2)}]^2 & [T^{(1)}]^2 & T^{(1)} & 0 & T^{(3)} \\
0 & T^{(1)} & T^{(1)} & X_{T^{(1)}} & 0 & 0 \\
0 & 0 & T^{(2)} & 0 & X_{T^{(2)}} & 0 \\
0 & 0 & 0 & 0 & X_{T^{(3)}} & 0
\end{pmatrix}.
\]  

(17)

At this point of the calculation all derivatives have been taken and we can set \[\langle T^{(1,2,3)} \rangle = 0\], which reveals the block-diagonal structure with

\[K^{IJ} = \text{diag} \{X_{U^{(1)}}, X_{U^{(2)}}, X_{U^{(3)}}, X_{T^{(1)}}, X_{T^{(2)}}, X_{T^{(3)}}\} .\]  

(18)

(There is an additional contribution to \[K^{IJ}\] from the dilaton field.) For each of these blocks one can compute \[G^I G_I = G^{IJ} G_J G_I\]. This calculation depends on the superpotential since \[G_I = K_I + \partial_I \ln W, \text{ i.e.,}\]

\[G^I G_I = K^I K_I + K^I \partial_I \ln W + K^I \partial_I \ln W + K^{IJ} \partial_I \ln W \partial_J \ln W .\]  

(19)

With the help of Mathematica \[\texttt{[23]}\] we obtain for the untwisted fields

\[
\text{Dilaton : } \quad [K^I K_I]^{(S)} \mid_{\ell, \tau_1, \tau_2} = 1 . \]  

(20)

\[
\tau_1, \tau_2 \text{ set : } \quad [K^I K_I]^{(\tau_1, \tau_2)} = 2 \quad (\forall n_\phi) . \]  

(21)

\[
\tau \text{ set : } \quad [K^I K_I]^{(\tau)} = 2 \quad (\forall n_\psi) . \]  

(22)

\[
\alpha \text{ set : } \quad [K^I K_I]^{(\alpha)} = \sum_n \alpha_i \tilde{\alpha}_i . \]  

(23)

We do not show the corresponding results for the twisted fields since \[\langle K^I K_I \rangle = 0\] in this case (i.e., \[\langle K_\beta \rangle \propto \langle \tilde{\beta} \rangle = 0\]). The scalar potential then becomes

\[
V = e^G \left\{ \left[ \frac{1}{2}, 1, \lambda_f = 1 \right] + 2n_{\tau_1 \tau_2} \right\} + 2n_\tau + n_\alpha \sum_i \alpha_i \tilde{\alpha}_i - 3 + F(\beta, \partial \ln W), \]  

(24)

where the sum of the three kinds of sets \[n_{\tau_1 \tau_2} + n_\tau + n_\alpha = 3\] is fixed. The term \[n_\alpha \sum_i \alpha_i \tilde{\alpha}_i\] is meant to represent however many \[\alpha\]-set contributions may exist in a given model. Also, \[\lambda_f = 1\] indicates that \[W\] does not depend on \[S\], whereas \[\lambda_f = 0\] indicates that \[W\] does depend on \[S\] in which case \[G_S = -1/(S + \tilde{S}) + (\partial_S W)/W\]. The
contributions which depend on $\partial_1 \ln W$, $\partial_2 \ln W$, or the twisted fields are collectively denoted by $F(\beta, \partial \ln W)$. The minimum of the potential is given by

$$V_0 = e^G [\lambda_f + 2n_{\tau_1 \tau_2} + 2n_\tau - 3],$$

with $\langle \alpha \rangle = \langle \beta \rangle = 0$, $\langle \partial_1 \ln W \rangle = 0$\footnote{Since $F(\beta, \partial \ln W)$ is generally a complicated expression, it may be possible to find special minima for particular non-zero values of $\langle \partial_1 \ln W \rangle$. In the case of the old no-scale models, $F(\beta, \partial \ln W) = \sum_i |\partial C_i \ln W|^2 > 0$, and therefore $\langle \partial C_i \ln W \rangle = 0$ is required.} and if $\lambda_f = 0$ also $\langle -1/(S + \bar{S}) + (\partial_3 W)/W \rangle = 0$.

In view of Eq. (25), there are only two choices for the untwisted modular symmetry which are consistent with $V_0 = 0$, namely

$$\begin{array}{cccc}
\lambda_f & n_{\tau_1 \tau_2} & n_\tau & n_\alpha \\
1 & 1 & 0 & 2 & S, \tau_1, \tau_2 \\
1 & 0 & 1 & 2 & S, \tau \\
\end{array}$$

(26)

Note that the dilaton is required to be a modulus field, and that only one set contributes moduli fields. If these requirements are satisfied, we would obtain a model with zero vacuum energy and flat directions, thus satisfying the first two no-scale supergravity postulates. Moreover, the gravitino mass is given by (see Eq. (3))

$$m^2_{3/2} = \left\{ \begin{array}{l}
\langle W^2 \rangle \\
\langle (S + \bar{S}) (\tau_1 + \bar{\tau}_1)(\tau_2 + \bar{\tau}_2) - \sum_i (\phi_i + \bar{\phi}_i)^2 \rangle \\
\langle (S + \bar{S}) (\tau + \bar{\tau})^2 - \sum_i (\psi_i + \bar{\psi}_i)^2 \rangle \\
\end{array} \right\}$$

(27)

for each of the two cases in Eq. (26), and is undetermined as anticipated. In these equations the values of $\langle \phi_i, \psi_i \rangle$ are determined by the flatness conditions $\langle \partial_1 \ln W \rangle = 0$ (typically $\langle \phi_i \rangle = \langle \psi_i \rangle = 0$).

### 3.3 Computation of $Q$

We now compute $Q$ using the formula in Eq. (4). The basic quantity to be computed is the determinant of $G_{M\bar{N}} = K_{M\bar{N}}$. Since we know the untwisted sector contribution to $K$ exactly, whereas we only have a first-order approximation to the twisted sector contribution (see Eq. (14)), we address the untwisted sector first. As in the computation of $G^I G_I$ above, the $K_{M\bar{N}}$ matrix is block-diagonal (three untwisted sets and the dilaton) and (with the help of Mathematica) we obtain

Dilaton : $[\det G_{M\bar{N}}]^{(S)} = (S + \bar{S})^{-2}.$

(28)

$\tau_1, \tau_2$ set : $[\det G_{M\bar{N}}]^{(\tau_1, \tau_2)} = 2^{n_\phi} \left[ (\tau_1 + \bar{\tau}_1)(\tau_2 + \bar{\tau}_2) - \sum_i (\phi_i + \bar{\phi}_i)^2 \right]^{-n_\phi - 2}$

(29)

$\tau$ set : $[\det G_{M\bar{N}}]^{(\tau)} = 2^{n_\phi + 1} \left[ (\tau + \bar{\tau})^2 - \sum_i (\psi_i + \bar{\psi}_i)^2 \right]^{-n_\phi - 1}$

(30)

$\alpha$ set : $[\det G_{M\bar{N}}]^{(\alpha)} = \left[ 1 - \sum_i \alpha_i \bar{\alpha}_i + \frac{1}{4} \left( \sum_i \alpha_i^2 \right) \left( \sum_i \bar{\alpha}_i^2 \right) \right]^{-n_i}$

(31)
From these results and Eqs. (11, 12, 13, 14), we see that $R_{I,J} = \partial_I \partial_J \ln \det G_{MN}$ for each block is just a multiple of $G_{I,J}$ for that block, i.e.,

\[
\begin{align*}
\text{Dilaton:} & \quad [R_{I,J}]^{(S)} = 2G^{(S)}_{SS}, \\
\tau_1, \tau_2 & \quad [R_{I,J}]^{(\tau_1, \tau_2)} = (n_\phi + 2)G^{(\tau_1, \tau_2)}_{I,J}, \\
\tau & \quad [R_{I,J}]^{(\tau)} = (n_\psi + 1)G^{(\tau)}_{I,J}, \\
\alpha & \quad [R_{I,J}]^{(\alpha)} = n_I G^{(\alpha)}_{I,J}.
\end{align*}
\]

With the above observation, the quantity that appears in $Q$ can be readily obtained: for each block $G^I R_{I,J} G^J \propto G^I G_{I,J} G^J = G^I G_I$, and these quantities have been given in Eqs. (20)–(23) (at the minimum $G$ for each block $f$).

The computation of $Q$ also involves evaluating $H_{I,J} = \partial_I \partial_J \ln \det \Re (f_{ab})$. In string models, $f_{ab}$ receives tree-level and one-loop contributions only [24]. Writing $f_{ab} = \delta_{ab} S + f^{1\text{-}loop}$ (suitable for level-one Kac-Moody constructions), and neglecting the one-loop contribution, we obtain $\det \Re (f_{ab}) = [\frac{1}{2} (S + \bar{S})]^{d_f}$, where $d_f$ is the dimension of the gauge group ($d_f \gg 1$). Also, $H_{SS} = -d_f G_{SS}$ and the contribution to $Q$ is $G^I H_{I,J} G^J = -d_f G^I \bar{G} G_S = -\lambda_f d_f$ (at the minimum).

The total contribution to $Q$ is then

\[
Q = \left\{ \begin{array}{l}
2n_I - n_\phi - 4 - d_f + Q_{TS} \\
2n_I - n_\psi - 3 - d_f + Q_{TS}
\end{array} \right.,
\]

where the terms are displayed in correspondence with those in Eq. (4), and $Q_{TS}$ is the twisted sector contribution to $Q$. For the two cases in Eq. (26), which give $V_0 = 0$, we obtain

\[
Q = \left\{ \begin{array}{l}
2n_I - n_\phi - 4 - d_f + Q_{TS} \\
2n_I - n_\psi - 3 - d_f + Q_{TS}
\end{array} \right.,
\]

where “$2n_I$” is meant to represent the sum of the untwisted fields in the two sets which do not contain moduli.

Now let us address the twisted sector contribution to $Q$ (i.e., $Q_{TS}$). Of the two cases in Eq. (26), consistent with $V_0 = 0$, the second case corresponds to a specific string model which will be discussed in Sec. 3 below; we focus on this case in what follows. The complete Kähler potential in this case is (see Appendix B for details)

\[
K = -\ln (S + \bar{S}) - \ln \left[ (\tau + \bar{\tau})^2 - \sum_i (\psi_i + \bar{\psi}_i)^2 \right] + \frac{1}{2} \left( \sum_i \beta^{(1)}_i \bar{\beta}^{(1)}_i \right) + \frac{1}{(\tau + \bar{\tau})^2 - \sum_i (\psi_i + \bar{\psi}_i)^2} \left( \sum_i \beta^{(2)}_i \bar{\beta}^{(2)}_i + \sum_i \beta^{(3)}_i \bar{\beta}^{(3)}_i \right)
\]

which is valid to first order in the twisted fields $\beta^{(1,2,3)}_i$, and the untwisted fields $\alpha^{(2,3)}_i$, and corresponds to $2n_I = n_{U2} + n_{U3}$ in Eq. (37). The contribution to $Q_{TS}$ from the
first twisted set comes only from the \( n_{T1} \) contribution to the total number of chiral superfields \( (N) \) in Eq. (3), since \( K_{\beta(1)\bar{\beta}(1)} = 1 \) has unit determinant and thus \( R_{\beta(1)\bar{\beta}(1)} = 0 \). The second and third twisted sets contribute to \( N(n_{T2}, n_{T3}) \), and in principle get tangled up with the fields in the first untwisted set \( (\tau, \psi) \). However, since \( \langle G_{\beta(2,3)} \rangle = 0 \), we only need to worry about their possible additional contributions to \( [R_I\bar{\beta}(\tau)] \) in Eq. (34). This means that in this enlarged determinant (involving \( \tau, \psi, \beta^{(2)}_i, \beta^{(3)}_i \)) we can set \( \beta^{(2,3)}_i = 0 \) after calculating the derivatives but before calculating the determinant. The enlarged determinant is then the one given in Eq. (30) times the factor \( \left[(\tau + \bar{\tau})^2 - \sum_i^n\psi_i(\psi_i + \bar{\psi}_i)^2\right]^{-(n_{T2}+n_{T3})/2} \). That is, the exponent in Eq. (30) and the coefficient in Eq. (34) receive a further contribution of \(-\frac{n_{T2}+n_{T3}}{2}\). The result for \( Q \) is then

\[
Q = [1 + (1 + n_\psi) + n_{U2} + n_{U3} + n_{T1} + n_{T2} + n_{T3}] - 1 - \{2 + 2(n_\psi + 1 + (n_{T2} + n_{T3})/2\} + d_f
= n_{U2} + n_{U3} + n_{T1} - n_\psi - d_f - 3 .
\] (39)

What are the prospects for obtaining \( Q = 0 \)? In typical models one observes that \( n_{U2} \sim n_{U3} \sim n_\psi \sim \mathcal{O}(10) \ll n_{T1} \). On the other hand \( d_f > d_{\text{SM}} = 12 \), although in realistic models we expect a number significantly exceeding this lower bound since, e.g., the hidden sector gauge group needs to be large enough for supersymmetry breaking via gaugino condensation to occur at a sufficiently high scale; a typical value would be \( d_f \sim \mathcal{O}(50 - 100) \). Thus, it is not inconceivable that models can be found where the various contributions to \( Q \) cancel each other out. In Sec. 6 we exhibit a model where this cancellation is almost perfect.

## 4 Soft-supersymmetry-breaking parameters

With the knowledge of the Kähler function, the scalar potential, and the gauge kinetic function one can compute the usual soft-supersymmetry-breaking parameters on which the low-energy model predictions depend so crucially.

### 4.1 Goldstino composition

Before we proceed with these calculations, it is instructive to determine the field dependence of the goldstino field, which has received considerable attention in “model-independent” approaches to this problem. The goldstino, which is eaten by the gravitino upon spontaneous breaking of supergravity, is given by

\[
\tilde{\eta} = \langle e^{G/2}G_I \rangle \chi^I ,
\] (40)

where \( \chi^I \) are the fermionic partners of the scalar fields which appear in the scalar potential (3). At the minimum of the scalar potential we have \( \langle G_{\alpha,\beta} \rangle = 0 \). In
computing the $G_I$ derivatives, for present purposes it suffices to approximate the Kähler function in Eqs. (13), (14) in the limit where the $\psi_i, \phi_i$ fields have vevs much smaller than the moduli vevs. This gives

\[
\begin{align*}
\text{Dilaton :} & \quad \langle G_I \rangle^{(S)} = -\lambda_f (S + \tilde{S})^{-1} \\
\tau_1, \tau_2 \text{ set :} & \quad \langle G_I \rangle^{(\tau_1, \tau_2)} \approx \left\{ -(\tau_1 + \tilde{\tau}_1)^{-1}, -(\tau_2 + \tilde{\tau}_2)^{-1}, \\
& \quad 2(\phi_i + \tilde{\phi}_i) [(\tau_1 + \tilde{\tau}_1)(\tau_2 + \tilde{\tau}_2)]^{-1} \right\} \quad (42) \\
\tau \text{ set :} & \quad \langle G_I \rangle^{(\tau)} \approx \left\{ -2(\tau + \tilde{\tau})^{-1}, 2(\psi_i + \tilde{\psi}_i)(\tau + \tilde{\tau})^{-2} \right\} \quad (43)
\end{align*}
\]

These results can be substituted back into Eq. (10) to obtain $\tilde{\eta}$. The final step is to express the $\chi^I$ fields in terms of the properly normalized $\tilde{\chi}^I$ fields. This operation entails a rescaling of the fields which will be discussed in detail in Sec. 5. Let us just quote the results:

\[
\begin{align*}
\tilde{S} &= \frac{S}{\langle S + \tilde{S} \rangle}, \quad \tilde{\tau}_1 = \frac{\tau_1}{\langle \tau_1 + \tilde{\tau}_1 \rangle}, \quad \tilde{\tau}_2 = \frac{\tau_2}{\langle \tau_2 + \tilde{\tau}_2 \rangle}, \quad \tilde{\tau} = \frac{\sqrt{2}\tau}{\langle \tau + \tilde{\tau} \rangle}, \quad (44) \\
\tilde{\phi}_i &= \frac{\sqrt{2}\phi_i}{\langle (\tau_1 + \tilde{\tau}_1)(\tau_2 + \tilde{\tau}_2) \rangle^{1/2}}, \quad \tilde{\psi}_i = \frac{\sqrt{2}\psi_i}{\langle \tau + \tilde{\tau} \rangle}. \quad (45)
\end{align*}
\]

The goldstino fields corresponding to the two cases in Eq. (23) are then

\[
\tilde{\eta} \propto \begin{cases} 
\tilde{S} + \tilde{\tau}_1 + \tilde{\tau}_2 + \sqrt{2} \sum_i n_i \langle \frac{\phi_i + \tilde{\phi}_i}{\langle (\tau_1 + \tilde{\tau}_1)(\tau_2 + \tilde{\tau}_2) \rangle^{1/2}} \rangle (\tilde{\phi}_i + \tilde{\phi}_i) & \rightarrow \tilde{\eta} = \left\{ \frac{1}{\sqrt{3}}(\tilde{S} + \tilde{\tau}_1 + \tilde{\tau}_2) \\
\tilde{S} + \sqrt{2} \tilde{\tau} + \sqrt{2} \sum_i \langle \frac{\psi_i + \tilde{\psi}_i}{\langle \tau + \tilde{\tau} \rangle} \rangle (\tilde{\psi}_i + \tilde{\psi}_i) & \to \tilde{\eta} = \left\{ \frac{1}{\sqrt{3}}(\tilde{S} + \sqrt{2} \tilde{\tau})
\end{cases} \quad (46)
\]

where the second form holds in the limit $\langle \psi_i, \phi_i \rangle \approx 0$. Thus, we get a goldstino field which contains substantial components of both “dilaton” and “moduli”. Note that, in principle light matter fields also appear, although their contribution is highly suppressed, by a factor $\langle \phi \rangle / \langle \text{moduli} \rangle \sim \mathcal{O}(10^2/10^{18})$.

### 4.2 Gaugino masses

The properly normalized gaugino masses are obtained from the expression

\[
M_a = \frac{e^{G/2}}{2\text{Re} f_a} \sum_I \partial_I f_a G^I, \quad (47)
\]

where the sum over $I$ runs over all matter fields which $f_a$ depends on. For $f_a$ we use the following one-loop (although correct to all orders) expression [24]

\[
f_a = k_a S - \frac{1}{16\pi^2} B_a^{(\tau_1, \tau_2)} \ln |\eta(\tau_1)\eta(\tau_2)|^4 \cdot n_{\tau_1, \tau_2} - \frac{1}{16\pi^2} B_a^{(\tau)} \ln |\eta(\tau)|^4 \cdot n_\tau, \quad (48)
\]

where the level of the Kac-Moody algebra is one ($k_a = 1$), and $\eta$ is the Dedekind eta function. Also, $B_a$ is a quantity which depends on the massless sector of the
theory and their modular weights, as well as on the coefficient $\delta_{GS}$ which arises in the
Green-Schwarz modular-anomaly cancellation. For our present purposes, a detailed
specification of $B_a$ is not required. The derivatives $\partial_I f_a$ in the expression for $M_a$
are non-zero only for $S, \tau_1, \tau_2, \tau$. One obtains, e.g., $\partial_{\tau_1} f_a = \frac{1}{16\pi^2} B_a^{(\tau_1, \tau_2)} \tilde{G}_2(\tau_1)$, where
$\tilde{G}_2(\tau_1) = G_2(\tau_1) - 2\pi/(\tau_1 + \bar{\tau}_1)$, and the Eisenstein function $G_2$
is related to the Dedekind function via $G_2(\tau_1) = -4\pi \partial_{\tau_1} \ln \eta(\tau_1)$ \([25]\). We also note that $\tilde{G}_2(\tau_1)$ has
zeroes at $\tau_1 = 1, e^{i\pi/6}$.

The other ingredient in the expression for $M_a$ is $G^I = G^{IJ} G_I = K^{IJ}(K_f + \partial_J \ln W) = K^{IJ} K_J = K^I$ at the minimum. This expression can be evaluated, with the result

$$
\text{Dilaton : } [K^I]^S(S) = - (S + \bar{S}) ,
$$

$$
\tau_1, \tau_2 \text{ set : } [K^I]^{(\tau_1, \tau_2)} = \{- (\tau_1 + \bar{\tau}_1), -(\tau_2 + \bar{\tau}_2), -(\phi_i + \bar{\phi}_i)\} ,
$$

$$
\tau \text{ set : } [K^I]^{(\tau)} = \{- (\tau + \bar{\tau}), -(\psi_i + \bar{\psi}_i)\} ,
$$

$$
\alpha \text{ set : } [K^I]^{(\alpha)} = \{ \alpha_i + O(\alpha_i^2)\} .
$$

With these results we finally obtain (only $I = S, \tau_1, \tau_2, \tau$ are relevant)

$$
M_a = \frac{m_{3/2}}{2 \text{Re } f_a} \left\{ -(S + \bar{S}) \lambda_f - \frac{1}{16\pi^2} B_a^{(\tau_1, \tau_2)} \left[ (\tau_1 + \bar{\tau}_1) \tilde{G}_2(\tau_1) + (\tau_2 + \bar{\tau}_2) \tilde{G}_2(\tau_2) \right] \cdot n_{\tau_1 \tau_2} - \frac{1}{16\pi^2} B_a^{(\tau)} (\tau + \bar{\tau}) \tilde{G}_2(\tau) \cdot n_\tau \right\} .
$$

For the two cases in Eq. (26) $\lambda_f = 1$ and thus the tree-level contribution to $M_a$
is non-zero and therefore dominant, giving nearly (up to small one-loop corrections)
universal gaugino masses, i.e.,

$$
M_a = m_{1/2} = m_{3/2} .
$$

### 4.3 Scalar masses

The scalar masses are obtained by taking second derivatives of the scalar potential. These masses have two sources: “supersymmetric” masses from the superpotential,
and supersymmetry-breaking masses from the Kähler potential. For the untwisted
fields, the latter can be deduced from the expression for $V$ given in Eq. (24) (only the
$K^I K_I$ term in Eq. (19) matters). We see that neither the $\phi_i$ nor the $\psi_i$ fields appear, thus

$$
\tilde{m}_{\phi_i} = \tilde{m}_{\psi_i} = 0 ,
$$

where $\tilde{m}_f$ represents the supersymmetry-breaking contribution to the mass of the scalar field $f$. On the other hand, the $\alpha_i$ do appear in $V$ and their mass is given by

$$
\tilde{m}_{\alpha_i} = m_{3/2} .
$$

\footnote{One has to properly normalize the fields (e.g., $\psi \rightarrow \tilde{\psi}$) to obtain the physical masses. The normalization factors are given in Eqs. \([4, 10, 88, 88]\), and can be trivial (i.e., 1) in many instances.}
In other words, untwisted sector fields in sets with moduli receive no (tree-level) supersymmetry-breaking masses, whereas those in sets with no moduli receive a universal mass equal to the gravitino mass. Obviously, it also follows that the moduli are massless (including the dilaton since $\lambda_f = 1$ is required).

Turning to the twisted field scalar masses, let us again consider the second case in Eqs. (26,37), with the Kähler potential given in Eq. (38). For the fields in the first untwisted set $[\beta_i^{(1)}]$ it is clear that their (Kähler potential) masses are equal to those of the second $[\alpha_i^{(2)}]$ and third $[\alpha_i^{(3)}]$ untwisted set fields. Indeed, in this case $K_{\beta^{(1)}\beta^{(1)}} = 1$ and $K' K_I = \sum_i \beta_i^{(1)} \beta_i^{(1)}$, and as expected

$$\tilde{m}_{\beta_i^{(1)}} = m_{3/2}. \quad (57)$$

The scalar masses of the second and third twisted set fields can be obtained by considering the following portion of the Kähler potential in Eq. (38)

$$K_0(\tau, \bar{\tau}) + K_1(\tau, \bar{\tau}) \sum_i (\psi_i + \bar{\psi}_i)^2 + K_2(\tau, \bar{\tau}) \left( \sum_i \beta_i^{(2)} \bar{\beta}_i^{(2)} + \sum_i \beta_i^{(3)} \bar{\beta}_i^{(3)} \right), \quad (58)$$

with

$$K_0 = -\ln(\tau + \bar{\tau})^2, \quad K_1 = \frac{1}{(\tau + \bar{\tau})^2}, \quad K_2 = \frac{1}{\tau + \bar{\tau}}, \quad (59)$$

where we have performed an expansion to first order in both $\psi_i$ and $\beta_i^{(2,3)}$. The approximate expression in Eq. (58) is sufficient to compute the scalar masses. After some algebra we obtain for this subset of the fields (i.e., $\tau, \psi_i, \beta_i^{(2)}, \beta_i^{(3)}$)

$$K' K_I = \frac{K_{0\tau} K_{0\bar{\tau}}}{K_{0\tau\bar{\tau}}} + 2 K_1 \left[ 1 - \frac{K_{0\tau} K_{1\tau\bar{\tau}} + K_{0\bar{\tau}} K_{1\tau\bar{\tau}}}{2 K_{0\tau\bar{\tau}}} - \frac{K_{0\tau} K_{0\bar{\tau}} \left( K_{1\tau\bar{\tau}} - 2 K_{1\tau} K_{1\bar{\tau}} \right)}{2 (K_1)^2 K_{0\tau\bar{\tau}}} \right] \sum_i (\psi_i + \bar{\psi}_i)^2$$

$$+ K_2 \left[ 1 - \frac{K_{0\tau} K_{0\bar{\tau}} (\ln K_2)_{\tau\bar{\tau}}}{K_{0\tau\bar{\tau}}} \left( \sum_i \beta_i^{(2)} \bar{\beta}_i^{(2)} + \sum_i \beta_i^{(3)} \bar{\beta}_i^{(3)} \right) \right]. \quad (60)$$

That is

$$\tilde{m}_{\psi_i}^2 = 1 - \frac{K_{0\tau} K_{1\tau\bar{\tau}} + K_{0\bar{\tau}} K_{1\tau\bar{\tau}}}{2 K_{0\tau\bar{\tau}}} - \frac{K_{0\tau} K_{0\bar{\tau}} K_{1\tau\bar{\tau}} - 2 K_{1\tau} K_{1\bar{\tau}}}{2 (K_1)^2 K_{0\tau\bar{\tau}}}, \quad (61)$$

$$\tilde{m}_{\beta_i^{(2)}}^2 = \tilde{m}_{\beta_i^{(3)}}^2 = 1 - \frac{K_{0\tau} K_{0\bar{\tau}} (\ln K_2)_{\tau\bar{\tau}}}{K_{0\tau\bar{\tau}}}, \quad (62)$$

where we have properly normalized the $\psi_i$ and $\beta_i^{(2,3)}$ fields by absorbing the overall factors (see Eqs. (60,61)). For the choices of $K_0, K_1, K_2$ in Eq. (53), the first term in Eq. (60) is $K_0 K_{0\tau}/K_{0\tau\bar{\tau}} = 2$, as expected from the vacuum energy calculation above. It also follows that $\tilde{m}_{\psi_i}^2 = 0$ (confirming the result in Eq. (55)) and the new result

$$\tilde{m}_{\beta_i^{(2)}} = \tilde{m}_{\beta_i^{(3)}} = 0. \quad (63)$$
The expression for the $\beta_{i}^{(2,3)}$ masses in Eq. (62) agrees with that given in Ref. [30]; the expression for the $\psi_{i}$ masses is new.

To summarize, for the model with Kähler potential given in Eq. (38), the scalar masses of all fields are the following multiples of $m_{3/2}$:

$$
U^{(1)}: \begin{array}{c}
\psi_{i} \\
0
\end{array}, \quad 
T^{(1)}: \begin{array}{c}
\beta_{i}^{(1)} \\
1
\end{array}, 
U^{(2)}: \begin{array}{c}
\alpha_{i}^{(2)} \\
1
\end{array}, \quad 
T^{(2)}: \begin{array}{c}
\beta_{i}^{(2)} \\
0
\end{array}, 
U^{(3)}: \begin{array}{c}
\alpha_{i}^{(2)} \\
1
\end{array}, \quad 
T^{(3)}: \begin{array}{c}
\beta_{i}^{(3)} \\
0
\end{array} \quad (64)
$$

Note the close correlation between the scalar masses and the corresponding modular weights of the matter fields given in Eq. (15) (where the same choice of moduli fields was made). The scalar mass spectrum in Eq. (64) is non-universal. This situation is likely to be an important model-building constraint, given what we know about needed near-degeneracies in certain low-energy squark and slepton masses. For example, data on $K^{0} - \bar{K}^{0}$ mixing and leptonic flavor-changing decays like $\mu \rightarrow e\gamma$ strongly constrain the mass differences for squarks and sleptons of the first two generations with the same electric charge but of different flavor [20]. The scenario which appears to emerge in string no-scale supergravity seems to explain this phenomenological requirement naturally: since all light chiral matter fields usually arise from the twisted sector, one would assign the first two generations to the second and third sets (with vanishing scalar masses); the third generation could be assigned to any of the sets.

4.4 Fermion masses

Supersymmetry breaking can also induce masses for (non-chiral) fermions in real representations of the gauge group. These (unnormalized) masses are given by the following expression [8]

$$
(M_{f})_{IJ} = m_{3/2} \left( G_{IJ} - G_{1JK}G_{K}^{*} + \frac{1}{3}G_{1J}G_{1} \right). \quad (65)
$$

As above, we focus on the model with Kähler potential given in Eq. (38). For fermions in the second and third untwisted sets $[\alpha_{i}^{(2,3)}]$ and in all of the twisted sets $[\beta_{i}^{(1,2,3)}]$ one has $G_{IJ} \equiv 0$ and $\langle G_{I} \rangle = 0$, and thus

$$
m_{\alpha_{i}^{(2)}} = m_{\alpha_{i}^{(3)}} = 0, \quad m_{\beta_{i}^{(1)}} = m_{\beta_{i}^{(2)}} = m_{\beta_{i}^{(3)}} = 0. \quad (66)
$$

The remaining fields are $S, \tau, \psi_{i}$. If we make the simplifying assumption $\langle \psi_{i} \rangle = 0$ (i.e., $\langle G_{\psi_{i}} \rangle = 0$) one can show that the normalized fermion mass matrix reduces to

$$
(M_{f})_{IJ} = m_{3/2} \begin{pmatrix}
\hat{S} & \hat{\tau} & \hat{\psi}_{j} \\
2/3 & -\sqrt{2}/3 & 0 \\
-\sqrt{2}/3 & 1/3 & 0 \\
0 & 0 & \delta_{ij}\end{pmatrix}. \quad (67)
$$
To obtain this result we have made use of the various normalization factors given in Eqs. (44,45). This matrix has zero determinant, indicating the presence of a massless eigenstate, namely the goldstino ($\tilde{\eta}$). Indeed, from Eq. (67) this eigenstate is $\tilde{\eta} = (\tilde{S} + \sqrt{2}\tilde{\tau})/\sqrt{3}$, in agreement with our previous result in Eq. (46) (for $\langle \psi_i \rangle = 0$). From Eq. (67) it also follows that the orthogonal linear combination $\tilde{\eta}_\perp = (\sqrt{2}\tilde{S} - \tilde{\tau})/\sqrt{3}$, and all of the $\hat{\psi}_i$ get masses equal to the gravitino mass, i.e.,

$$m_{\tilde{\eta}_\perp} = m_{3/2}, \quad m_{\hat{\psi}_i} = m_{3/2}.$$  

(68)

4.5 A consistency check

In the previous three subsections we have computed all of the supersymmetry breaking masses, in particular for the model with Kähler potential given in Eq. (38). One can then perform a consistency check of result for $Q$ given in Eq. (39), since we can calculate directly $\text{Str}M^2 = \sum_j (-1)^{2j}(2j + 1)M_j^2 = 2Qm_{3/2}^2$. The masses of the complex scalars ($j = 0$) are given in Eq. (64) and contribute to the supertrace (in units of $m_{3/2}^2$) in the amount of $2(n_{u2} + n_{u3} + n_{T1})$. The masses of the Majorana fermions ($j = 1/2$) are given in Eqs. (66,68) and contribute $-2(1 + n_\psi)$, whereas the Majorana gaugino masses (given in Eq. (64)) contribute $-4$. Putting it all together gives $Q = n_{u2} + n_{u3} + n_{T1} - n_\psi - d_f - 3$, which is the result found in Eq. (39) by less direct means.

4.6 $A$ terms

The supersymmetry-breaking cubic scalar couplings (or $A$ terms) are contained in the term $c^G K^I \partial_I \ln W$ (plus hermitian conjugate) of the scalar potential in Eq. (19). The main input required to evaluate these couplings is the value of $K^I$ for each of the types of untwisted and twisted states. For the untwisted states these inputs are given in Eqs. (51,52), i.e.,

$$K^{\psi_i} = -\psi_i, \quad K^{\alpha_i^{(2)}} \approx \alpha_i^{(2)}, \quad K^{\alpha_i^{(3)}} \approx \alpha_i^{(3)}.$$  

(69)

For the twisted states in the model with Kähler potential given in Eq. (38), the first twisted set fields $[\beta_i^{(1)}]$ have the same functional dependence as the second and third untwisted set fields $[\alpha_i^{(2,3)}]$, and therefore the result is as above: $K^{\beta_i^{(1)}} \approx \beta_i^{(1)}$. For the second and third twisted set fields, an intermediate step in the calculation that yields the result in Eq. (60) gives

$$K^{\beta_i^{(2,3)}} \approx \frac{K_{0r\tau}K_2 - K_{0r}\beta_i^{(2,3)}}{K_{0r\tau}K_2} K^{\beta_i^{(2,3)}}.$$  

(70)

Inserting the values for $K_0, K_2$ (Eq. (59)) one finds a zero result to first order, i.e.,

$$K^{\beta_i^{(1)}} \approx \beta_i^{(1)}, \quad K^{\beta_i^{(2)}} \approx 0, \quad K^{\beta_i^{(3)}} \approx 0.$$  

(71)
With the above results one can proceed to compute the $A$ terms for all the types of cubic couplings given in Eq. (10), with the field identifications given in Eq. (64). The expression to manipulate is $e^{G/K}I\partial I\ln W = e^{G/2}e^{K/2}K'I\partial I\ln W = m_{3/2}e^{K/2}K'I\partial I\ln W$. Since the cubic superpotential does not depend on $S$ or $\tau$, all we need to do is take derivatives with respect to the untwisted and twisted matter fields. Each time one such field is removed from a cubic coupling by the $\partial I\ln W$ operation, the corresponding $K'I$ factor puts it back in restoring the original coupling, although a coefficient $(0, 1, -1)$ is picked up in this process. After summing over all fields in a given cubic coupling, and over all cubic couplings one ends up with $m_{3/2}e^{K/2}W = m_{3/2}e^{K/2}\hat{W}$ where $\hat{W}(\hat{\phi}) = e^{K/2}W(\phi)$ is the superpotential written in terms of the properly normalized fields, as discussed in Sec. 5. The constant $c$ is common to all of the types of cubic couplings in Eq. (10), and in fact $c = 1$, i.e.,

$$
\begin{align*}
\psi\alpha^{(2)}\alpha^{(3)} & : -1 + 1 + 1 = 1 \\
\psi\beta^{(1)}\beta^{(1)} & : -1 + 1 + 1 = 1 \\
\alpha^{(2)}\beta^{(2)}\beta^{(2)} & : 1 + 0 + 0 = 1 \\
\alpha^{(3)}\beta^{(3)}\beta^{(3)} & : 1 + 0 + 0 = 1 \\
\beta^{(1)}\beta^{(2)}\beta^{(3)} & : 1 + 0 + 0 = 1
\end{align*}
$$

Thus we conclude that for all cubic couplings

$$A = m_{3/2} .$$

(In passing we note that in the old no-scale models, $K' = \{-[T + \bar{T} - C_{ij}\bar{C}_{ij}], \bar{0}\}$, and therefore $A \equiv 0$.)

### 4.7 $\mu$ and $B$

The possible origin of the low-energy Higgs mixing parameter $\mu$ (and its associated supersymmetry-breaking bilinear coupling $B$) has been discussed in the literature for some time. It is well-known that this term $(\mu h_1 h_2)$ must be present in the superpotential, and have a magnitude comparable to all other dimensional parameters of the low-energy theory. In the framework of string theory, where explicit mass parameters are not present in the superpotential, the nature of the $\mu$ term is particularly intriguing. Three scenarios have been put forward:

- The low-energy theory possesses an additional singlet field ($N$) which couples to the two Higgs doublets ($\lambda N h_1 h_2$) and gets a vacuum expectation value which effectively produces $\mu = \lambda \langle N \rangle$. Even though such couplings proliferate in fermionic string models, in all known instances the singlet fields are heavy and decouple from the low-energy spectrum.

- The quadratic $\mu$ term arises as an effective non-renormalizable fourth- (or higher) order term in the superpotential, i.e., $\frac{1}{M}\lambda_4 H \bar{H} h_1 h_2$ where $M \sim 10^{18}$ GeV
is the string scale $\mu$. In this case $\mu = \frac{1}{7} \lambda_4 \langle HH \rangle$; for $\mu \sim 1 \text{ TeV}$, one requires $\langle HH \rangle^{1/2} \sim 10^{11} \text{ GeV}$ which is typical of hidden sector matter condensates in string models.

- The quadratic $\mu$ term is built into the theory through the Kähler potential, and becomes non-zero and of $O(m_{3/2})$ upon supersymmetry breaking \cite{19, 29, 34}.

Let us first address the third scenario. From the calculation of the fermion masses in Eq. (68) ($m_{\tilde{\psi}} = m_{3/2}$) one could think that these may come from a superpotential $\mu$ term, e.g., $\mu \tilde{\psi} \tilde{\psi}$ with $\mu = \frac{1}{2} m_{3/2}$. However, Eq. (53) shows that the corresponding scalar masses vanish ($\tilde{m}_{\tilde{\psi}} = 0$), a result apparently inconsistent with the possible presence of a superpotential $\mu$ term. It turns out that things are more intricate and the interpretation of a superpotential $\mu$ term is not inconsistent. What happens is that one has to split the Kähler function into two pieces, one which is absorbed into the superpotential to provide the squared scalar masses (but not to the fermion masses). Thus, such a Kähler-induced $\mu$-term does break supersymmetry, even though it can be incorporated into the superpotential.

Let us illuminate this result by studying a simple example in detail. Consider the Kähler potential $K = -\ln[(\tau + \bar{\tau})^2 - (\psi + \bar{\psi})^2]$, which gives $\tilde{m}_{\tilde{\psi}} = 0$ and $m_{\psi} = m_{3/2}$.

Let us expand the Kähler function to first order in the $\psi$ field

$$K \approx -\ln(\tau + \bar{\tau})^2 + \frac{2}{(\tau + \bar{\tau})^2} \psi \bar{\psi} + \frac{1}{(\tau + \bar{\tau})^2} (\psi \bar{\psi} + \bar{\psi} \psi).$$

(74)

Ignoring the last term in this expression (to be absorbed into $W$) the scalar mass can be obtained from Eq. (72) with $K_0 = -\ln(\tau + \bar{\tau})^2$ and $K_2 = 2/(\tau + \bar{\tau})^2$. The result is $\tilde{m}_{\tilde{\psi}}^2 = -m_{3/2}^2$. The last term in Eq. (74) can be lumped with the superpotential $W \rightarrow W e^{K_1 \psi \bar{\psi}} \approx W + WK_1 \psi \bar{\psi}$ with $K_1 = 1/(\tau + \bar{\tau})^2$. Proper normalization entails multiplying $W$ times $e^{K_1/2}$, thus giving the new superpotential term $e^{K/2} WK_1 \psi \bar{\psi} = \frac{1}{2} m_{3/2} \tilde{\psi} \tilde{\psi}$, where we have also properly normalized the $\psi$ field. We therefore get $\mu = \frac{1}{2} m_{3/2}$, which leads to a superpotential fermion mass $m_{\psi} = 2\mu = m_{3/2}$. This also entails a superpotential scalar mass-squared $\tilde{m}_{\tilde{\psi}}^2 = 4\mu^2 = m_{3/2}^2$, which when added to the Kähler potential mass-squared found above gives the expected vanishing result.  

\footnote{This procedure can be easily generalized to what would be the case of interest with $\psi \psi \rightarrow \psi_1 \psi_2$, as discussed in Ref. \cite{31}. In $SU(5) \times U(1)$ free-fermionic models one finds $\psi_1 = \frac{1}{\sqrt{2}} (h_1 + h_2)$, $\psi_2 = \sqrt{2} (h_1 - h_2)$ with $h_1(h_2)$ a 5 (5) of $SU(5)$ \cite{21}. It follows that $\langle \psi_1 + \tilde{\psi}_1 \rangle^2 + \langle \psi_2 + \tilde{\psi}_2 \rangle^2 = 2h_1 h_2 + 2(h_1 h_2)^* + 2h_1^* h_2 + h_2^* h_2$ and $K_1 = K_2 = 2/(\tau + \bar{\tau})^2$. The new superpotential term is $\mu h_1 h_2$ with $\mu = m_{3/2}$, which gives $\tilde{m}_{\tilde{\psi}} = \tilde{m}_{\tilde{h}_2} = 0$ and $m_{\tilde{h}_1 \tilde{h}_2} = m_{3/2}$.}

\footnote{Note that in principle the coefficients $K_1$, $K_2$ could be related in a different manner, with even the $K_2$ piece leading to a vanishing scalar mass (if $K \propto 1/(\tau + \bar{\tau})$) and to the so-called “super-soft” supersymmetry breaking terms \cite{31} which do not break supersymmetry.
If this mechanism for generation of the $\mu$ term is present in realistic string models, one has to be careful with its embedding into the traditional supergravity-induced soft-supersymmetry-breaking parameters. For the squared scalar masses of the Higgs doublets one normally writes $(m_1^2 + \mu^2)|H_1|^2 + (m_2^2 + \mu^2)|H_2|^2$. The Kähler-induced $\mu$ term has the property that $m_1^2 = m_2^2 = -\mu^2$, which is what would need to be used as initial conditions in the corresponding renormalization group equations. (In practice we do not find this kind of $\mu$ term present in realistic string models, at least involving the two Higgs doublets which are to remain in the light spectrum.)

Now let us consider the second mechanism for generating a $\mu$ term, namely via a non-renormalizable coupling in the superpotential. Given such coupling we would like to know what is the associated supersymmetry breaking $B$ term. Essentially this term arises in a very similar manner as the $A$ terms discussed above, that is, from the $e^G K^I \partial_I \ln W$ term in the scalar potential. There is one crucial difference: the non-renormalizable couplings may need to be multiplied by powers of the Dedekind eta function to restore modular invariance of the superpotential, as discussed after Eq. (15). This possible new dependence on the moduli must be taken into account when taken the derivatives of $W$. Let us first list the types of quartic and quintic superpotential couplings in free-fermionic models. In the same spirit as the cubic couplings given in Eq. (10), the types of higher-order couplings can be deduced from Ref. [18].

Next we need to determine if there is a modular weight imbalance, but this can only be done in a specific class of models, such as the ones with Kähler potential given in Eq. (38). The modular weights are then given in Eq. (15). For every unit of modular weight imbalance we multiply the non-renormalizable coupling by $\eta^2(\tau)$, as indicated in Eq. (75). Note that in some instances there is no modular weight imbalance. The value of the corresponding $B$ parameters can then be determined in analogy with the procedure followed for the $A$ terms, except when a power of $\eta^p$ is present. In this case one adds to the result the quantity

$$-(\tau + \bar{\tau})(\partial_\tau \eta^p) / \eta^p = p/2 + (\tau + \bar{\tau})(p/4\pi)\hat{G}_2(\tau).$$

With the use of Eqs. (69,71) one can determine the $B$ parameters for each of the types of quartic or quintic couplings, as follows (in units of $m_{3/2}$)

\[\begin{array}{ccc}
\text{Quartic couplings} & B & \text{Quintic couplings} \\
\eta^0 \left[T^{(1)}\right]^2 \left[T^{(2)}\right]^2 & 2 & \eta^0 \left[T^{(1)}\right]^3 \left[T^{(2)}\right] \left[T^{(3)}\right] \\
\eta^0 \left[T^{(1)}\right]^2 \left[T^{(3)}\right]^2 & \eta^0 \left[T^{(1)}\right]^3 U^{(3)} & \eta^0 \left[T^{(1)}\right]^3 \left[T^{(2)}\right] \left[T^{(3)}\right] \left[T^{(3)}\right] \\
\eta^2 \left[T^{(2)}\right]^2 \left[T^{(3)}\right]^2 & \eta^2 \left[T^{(2)}\right]^3 U^{(1)} & \eta^2 \left[T^{(2)}\right]^3 \left[T^{(1)}\right] \left[T^{(1)}\right] \\
\end{array}\]
Concerning the calculated values of the $B$ terms, we should note that $\hat{G}_2(\tau) \approx \frac{\tau^2}{3}(1 - 24e^{-2\pi\tau} - 2\pi/(\tau + \bar{\tau})$ with $\hat{G}_2(\tau = 1) = 0$ [27]. Also, for values of $\tau$ in the fundamental domain (i.e., $\tau \geq 1$ if $\tau$ is real) $\hat{G}_2(\tau) \geq 0$. Therefore, the integer values of $B$ shown in Eqs. (77, 78) are actually the minimum possible values.

5 Field normalizations and Yukawa couplings

The superpotential couplings in free-fermionic models are easily calculable in the string basis. Moreover, in specific models the fermion Yukawa couplings have led to structures which bear close resemblance to the observed hierarchical fermion mass spectrum. On the other hand, results about the Yukawa couplings in the string model are not necessarily directly related to those which would be observed at low energies. The possible snag lies in the normalization of the fields in the supergravity lagrangian.

For the scalar fields the relevant term is

$$K_{ij} \partial^\mu \phi_i \partial^\mu \bar{\phi}_j .$$

If the Kähler potential ($K$) is non-trivial, then the scalar fields would need to be normalized appropriately. If this effect propagates to the Yukawa couplings, the physical ones may differ from those naively expected.

We seek a matrix $A$ such that

$$\phi = A \hat{\phi} , \quad \bar{\phi} = \bar{A} \hat{\phi} ,$$

where $\hat{\phi}, \bar{\hat{\phi}}$ are the properly normalized fields. From the condition $\partial_\mu \phi^T K \partial^\mu \bar{\phi} = \partial_\mu \hat{\phi}^T A^T K A \partial^\mu \hat{\phi} = \partial_\mu \bar{\hat{\phi}}^T \partial^\mu \bar{\hat{\phi}}$ we obtain

$$A = (K^{-1/2})^T , \quad \bar{A} = K^{-1/2} .$$

Here $K^{-1/2}$ is the matrix obtained by taking the square root of $K^{MN}$, where $K^{MN}$ has been used above in calculating, e.g., $K^T = K^{IJ} \bar{K}_J$. If $K^{-1/2}$ is obtained, one can determine the physical Yukawa couplings (which couple properly normalized fields) from the expression [30, 6]

$$\hat{\lambda}_{ij \kappa} = \epsilon^{K/2} (K^{-1/2})_{i\ell} (K^{-1/2})_{j\ell'} (K^{-1/2})_{k\ell'} \lambda_{\ell \ell' \kappa} ,$$

where $\lambda_{ij \kappa}$ are the couplings which appear in the superpotential of the string model. That is, $\hat{W}(\hat{\phi}) = \epsilon^{K/2} W(\phi)$. 

20
Calculating the square root of a matrix can be a complicated task. Before we attempt this, it is quite illuminating to consider the limit of small matter field vevs. To leading order we only keep the diagonal contributions to $K_{M\bar{N}}$, and obtain for the untwisted matter fields

$$
\tau_1, \tau_2 \text{ set: } [K^{-1/2}]^{(\tau_1, \tau_2)} \approx \text{diag} \left[ (\tau_1 + \bar{\tau}_1), (\tau_2 + \bar{\tau}_2) \right], \quad \frac{1}{\sqrt{2}}(\tau_1 + \bar{\tau}_1)^{1/2}(\tau_2 + \bar{\tau}_2)^{1/2} 1_{n_\phi}, \tag{83}
$$

$$
\tau \text{ set: } [K^{-1/2}]^{(\tau)} \approx \text{diag} \left[ \frac{1}{\sqrt{2}}(\tau + \bar{\tau}), \frac{1}{\sqrt{2}}(\tau + \bar{\tau}) 1_{n_\psi} \right], \tag{84}
$$

$$
\alpha \text{ set: } [K^{-1/2}]^{(\alpha)} \approx \text{diag} \left[ 1_{n_j} \right], \tag{85}
$$

where $1_n$ is a vector with $n$ unit entries. In the case of the model with Kähler potential given in Eq. (38), and in this same approximation, we obtain the explicit normalized fields as follows

$$
\hat{S} = \frac{S}{\langle S + \bar{S} \rangle}, \quad \hat{\tau} = \frac{\sqrt{2}\tau}{\langle \tau + \bar{\tau} \rangle}, \quad \hat{\psi}_i = \frac{\sqrt{2}\psi_i}{\langle \tau + \bar{\tau} \rangle}; \tag{86}
$$

$$
\hat{\alpha}_i^{(2)} = \alpha_i^{(2)}, \quad \hat{\alpha}_i^{(3)} = \alpha_i^{(3)}, \quad \hat{\beta}_i^{(1)} = \beta_i^{(1)}; \tag{87}
$$

$$
\hat{\beta}_i^{(2)} = \frac{\beta_i^{(2)}}{\langle \tau + \bar{\tau} \rangle^{1/2}}, \quad \hat{\beta}_i^{(3)} = \frac{\beta_i^{(3)}}{\langle \tau + \bar{\tau} \rangle^{1/2}}. \tag{88}
$$

Now let us determine the properly normalized cubic Yukawa couplings. From Eq. (38) we have $\langle e^{K/2} \rangle = 1/[(S + \bar{S})^{1/2}(\tau + \bar{\tau})]$. For the various types of cubic couplings in Eq. (10) we write generically $\hat{\lambda} = f\lambda$, with the normalization factor $f$ given by

$$
\psi \alpha^{(2)} \alpha^{(3)} : \quad e^{K/2} \frac{1}{\sqrt{2}}(\tau + \bar{\tau}) \langle 1 \rangle (1) = \frac{1}{2}g \\
\psi \beta^{(1)} \beta^{(1)} : \quad e^{K/2} \frac{1}{\sqrt{2}}(\tau + \bar{\tau}) \langle 1 \rangle (1) = \frac{1}{2}g \\
\alpha^{(2)} \beta^{(2)} \beta^{(2)} : \quad e^{K/2} \langle 1 \rangle (\tau + \bar{\tau})^{1/2} (\tau + \bar{\tau})^{1/2} = \frac{1}{\sqrt{2}}g \\
\alpha^{(3)} \beta^{(3)} \beta^{(3)} : \quad e^{K/2} \langle 1 \rangle (\tau + \bar{\tau})^{1/2} (\tau + \bar{\tau})^{1/2} = \frac{1}{\sqrt{2}}g \\
\beta^{(1)} \beta^{(2)} \beta^{(3)} : \quad e^{K/2} \langle 1 \rangle (\tau + \bar{\tau})^{1/2} (\tau + \bar{\tau})^{1/2} = \frac{1}{\sqrt{2}}g ; \tag{89}
$$

where we have used the result $g^2 = 1/\text{Re}S$. We conclude that all the normalized cubic Yukawa couplings are independent of the moduli but depend on the dilaton (or the gauge coupling).

Our assumption that the $K^{-1/2}$ matrices are nearly diagonal can be justified by studying some simple cases where the calculation can be done exactly. For instance, let us take a $\tau$ set with $n_\psi = 1$, which amounts to a $2 \times 2$ matrix. The two matrices of relevance are

$$
K_{M\bar{N}} = \frac{2}{x^2} \begin{pmatrix}
2a + b^2 & -4ab \\
-4ab & a^2 + b^2
\end{pmatrix}, \quad K^{-1} = \frac{1}{2} \begin{pmatrix}
a^2 + b^2 & 2ab \\
2ab & a^2 + b^2
\end{pmatrix}; \tag{90}
$$

\footnote{This property does not appear to allow the dynamical determination of Yukawa couplings via the no-scale mechanism, as recently advocated \cite{11, 32}.}
where \( a = \tau + \bar{\tau}, b = \psi + \bar{\psi}, \) and \( X = a^2 - b^2. \) From \( K^{-1} \) we can compute the square root,

\[
K^{-1/2} = \frac{1}{\sqrt{2}} \begin{pmatrix} a & b \\ b & a \end{pmatrix},
\]

and therefore

\[
\begin{pmatrix} \tau \\ \psi \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \tau + \bar{\tau} & \psi + \bar{\psi} \\ \psi + \bar{\psi} & \tau + \bar{\tau} \end{pmatrix} \begin{pmatrix} \hat{\tau} \\ \hat{\psi} \end{pmatrix},
\]

which agrees with Eq. (84) in the limit \( \langle \psi + \bar{\psi} \rangle / \langle \tau + \bar{\tau} \rangle \rightarrow 0. \) However, the exact expression allows us to study the possibility of moduli-matter mixing. For the all-untwisted superpotential coupling \( \lambda_{\psi\alpha\bar{\alpha}} \psi\alpha \), we find in an obvious notation

\[
\hat{\lambda}_{\hat{\psi}i\hat{\alpha}} = \frac{1}{\sqrt{2}} (\tau + \bar{\tau}) \lambda_{\psi\alpha\bar{\alpha}} \frac{1}{(S + \bar{S})^{1/2}[(\tau + \bar{\tau})^2 - (\psi + \bar{\psi})^2]^{1/2}},
\]

\[
\hat{\lambda}_{\hat{\tau}\hat{\alpha}\bar{\alpha}} = \frac{1}{\sqrt{2}} (\psi + \bar{\psi}) \lambda_{\psi\alpha\bar{\alpha}} \frac{1}{(S + \bar{S})^{1/2}[(\tau + \bar{\tau})^2 - (\psi + \bar{\psi})^2]^{1/2}}.
\]

The novelty here is a new (although small) Yukawa coupling between matter (\( \hat{\alpha} \)) and moduli (\( \hat{\tau} \)) fields, of order \( \hat{\lambda}_{\hat{\psi}i\hat{\alpha}} = O[(\psi + \bar{\psi})/(\tau + \bar{\tau})]. \) Otherwise, the results derived above in the diagonal approximation are quite accurate.

The above exact calculation for \( n_{\psi} = 1 \) does not allow quantification of possible matter-matter mixing through field normalizations. One can repeat the exercise for \( n_{\psi} = 2 \) to study the magnitude of such mixings. The square root of such 3 x 3 matrix can be readily obtained by the use of Sylvester’s formula

\[
P(U) = \sum_{r=1}^{n} P(\lambda_r) \prod_{j \neq r} \frac{\lambda_j I - U}{\lambda_j - \lambda_r},
\]

where \( U \) is the given matrix, \( P \) is the required operation (square root in our case), and \( \lambda_r \) are the eigenvalues of \( U. \) Mathematica can be programmed to calculate \( K^{-1/2} \) using Sylvester’s formula, but the result is messy. In the limit of interest we find, for example

\[
\hat{\lambda}_{\hat{\psi}1\hat{\alpha}} \approx \frac{1}{\sqrt{2}} \lambda_{\psi1\alpha} + \frac{1}{\sqrt{2}} \lambda_{\psi2\alpha} \frac{1}{(S + \bar{S})^{1/2}} \left[ \frac{1}{\sqrt{2}} (\psi_1 + \bar{\psi}_1) \frac{1}{\sqrt{2}} (\psi_2 + \bar{\psi}_2) \right]^2.
\]

The possible matter-matter mixing is therefore very small \( O[(\psi + \bar{\psi})/(\tau + \bar{\tau})]^4. \) Matter-matter mixing through the Kähler potential (as in the case of the \( \mu \) term) or through the superpotential are therefore the realistic possible sources of mixing. For \( n_{\psi} = 2 \) the matter-moduli mixing is also highly suppressed \( O[(\psi + \bar{\psi})/(\tau + \bar{\tau})^2]. \)
6 Possible realistic models

In the previous sections we have explored what consequences would string no-scale supergravity models have regarding the soft-supersymmetry-breaking terms and the low-energy Yukawa couplings. However, it remains to be shown that such models actually exist, \textit{i.e.}, that the first postulate of string no-scale supergravity is satisfied. Here we describe a search for such models, and the properties of any that may be found.

6.1 The search for models

We have performed a computerized search of free-fermionic string models with the desired properties. The two cases in Eq. (26) indicate that we should look for models which \textit{effectively} possess one untwisted set with moduli (a $\tau_1, \tau_2$ set or a $\tau$ set) and the other two untwisted sets with all moduli projected out. As discussed in Sec. 3.1, this determination should be done after the cubic superpotential of the model is calculated, since we would discard “moduli” which have superpotential couplings.

Let us consider the kind of truncations of the moduli space which could occur if moduli have superpotential couplings. If we start off with a $\tau$ set, all that can happen is that the modulus field appears in $W$, and therefore the set would effectively become an $\alpha$ set. In the case of a $\tau_1, \tau_2$ set, if both fields appear in $W$ we have an $\alpha$ set, whereas if only one appears we have a $\tau$ set. In the latter case ($\tau_1, \tau_2$ set $\rightarrow$ $\tau$ set) we would perform the field redefinition in Eq. (14), instead of that in Eq. (13).

With the restriction that we obey Eqs. (26) after moduli present in $W$ are discarded, we have performed a search for free-fermionic models following the methods described in Ref. [14]. A free-fermionic model is specified by a basis of $n$ basis vectors of boundary conditions, plus an $n \times n$ matrix of GSO projections (the “$k$-matrix”). Our search is based on the reasonable assumption that the basis vectors of the fermionic model contain five “standard” vectors which have appeared in all known models of this kind, these are denoted by 1, $S, b_1, b_2, b_3$. Since we are interested in models with $SU(5) \times U(1)$ observable gauge symmetry, we also assume the presence of two other vectors ($b_4, b_5$) which have been used in the two $SU(5) \times U(1)$ string models in the literature [2, 10]. The eighth and last vector, called $\alpha$, is decisive. In Ref. [10] this vector was allowed to take all possible values consistent with the free-fermionic model-building rules. In addition, the $k$-matrix was varied at random. This search was specifically focused on finding models which allow unification of the low-energy gauge couplings at the string scale. As such, the model had to contain five $\bf{10}$’s and two $\bf{10}$’s of $SU(5)$ (a “5/2 model”), as opposed to the original “revamped” model which is a “4/1 model”. In fact, one such 5/2 model was found, which we will call the “search” model. The “search” model is in fact an example of a class of 5/2 models, with slightly varying properties.

\footnote{Note that even if a field appears in the superpotential, it may still be a flat direction if the fields coupled to it conspire in the appropriate way. In what follows we disregard such exceptional possibilities.}
Our purpose here is somewhat different, since the search is more constrained. We seek either 4/1 or 5/2 models with a single effective untwisted $\tau_1, \tau_2$ or $\tau$ set (and two effective untwisted $\alpha$ sets). Our search procedure consists of picking representative $\alpha$ vectors and varying the $k$-matrix at random.

- $\alpha = \alpha_{\text{search}}$. In 10,000 $k$-matrices we find $\sim 6\%$ $N = 1$ supersymmetric models, of which 19 are 5/2 models and none are 4/1 models. All these models possess two $\tau_1, \tau_2$ sets and one $\alpha$ set, since untwisted (Neveu-Schwarz) states like the moduli depend only on the choice of basis vectors, and not on the $k$-matrix. Calculation of the cubic superpotential reveals that the 5/2 models divide into two “Yukawa sets”: 1/3/2 and 2/3/2 with the 2/3/2 case preferred phenomenologically [16]. Moreover, we discover that all models with the 1/3/2 Yukawa set have their $\tau_1, \tau_2$ sets broken down to $\tau$ sets, whereas all the models with the preferred 2/3/2 Yukawa set have one $\tau_1, \tau_2$ set broken to an $\alpha$ set and the other $\tau_1, \tau_2$ set broken to a $\tau$ set. Therefore, the 2/3/2 models (like the “search” model) possess precisely the desired moduli content. (The 1/3/2 models give $V_0 > 0$.) Schematically

\[
\begin{align*}
\text{5/2 models} & \\
\alpha = \alpha_{\text{search}} & \\
\{ & \\
1/3/2 & \begin{array}{c}
\tau_1 \tau_2 \rightarrow \tau \\
\tau_1 \tau_2 \rightarrow \tau \\
\alpha \rightarrow \alpha
\end{array} \\
V_0 & > 0 \\
2/3/2 & \begin{array}{c}
\tau_1 \tau_2 \rightarrow \tau \\
\tau_1 \tau_2 \rightarrow \alpha \\
\alpha \rightarrow \alpha
\end{array} \\
V_0 & = 0
\end{align*}
\]

(97)

- $\alpha = \alpha_{\text{revamped}}$. In 5,000 $k$-matrices we find 41 4/1 models and no 5/2 models. All these models contain one $\tau_1, \tau_2$ set and two $\tau$ sets. The 4/1 models come with two possible Yukawa sets (1/3/3 and 2/3/3, the latter is preferred), and in all instances we find that the $\tau$ sets are unbroken, whereas the $\tau_1, \tau_2$ set is broken to an $\alpha$ set, i.e.,

\[
\begin{align*}
\text{4/1 models} & \\
\alpha = \alpha_{\text{revamped}} & \\
\{ & \\
1/3/3 & \begin{array}{c}
\tau_1 \tau_2 \rightarrow \alpha \\
\tau \rightarrow \tau
\end{array} \\
V_0 & > 0
\end{align*}
\]

(98)

- $\alpha = \alpha_{3a}$. We choose two other $\alpha$ vectors which belong to “class 3a” in the notation of Ref. [16]. Models with $\alpha$ vectors in this class are expected to be 4/1 models ($\alpha_{\text{revamped}}$ belongs to this class). We obtain the same result as in Eq. (98).

\[\text{[11]}\]

An “$m/n/p$ Yukawa set” includes $m$ potential up-quark like Yukawa couplings, $n$ potential down-quark like Yukawa couplings, and $p$ potential charged-lepton Yukawa couplings [16].
\( \alpha = \alpha_{\text{price}} \). Unlike our previous choices for \( \alpha \), \( \alpha_{\text{price}} \) (introduced in Ref. [34]) produces both 4/1 and 5/2 models. In this case the three sets are \( \tau \) sets and the three moduli appear in the superpotential, \textit{i.e.},

\[
\begin{align*}
5/2, 4/1 \text{ models} & \quad \begin{cases} 
\text{Moduli} & V_0 \\
\tau \rightarrow \alpha & \\
\tau \rightarrow \alpha & V_0 < 0 \\
\tau \rightarrow \alpha & 
\end{cases} 
\end{align*}
\tag{99}
\]

These models are not realistic, but we consider them since we want to establish a connection between the value of \( V_0 \) and the 4/1 or 5/2 nature of a model.

- \textit{Change} \( b_4, b_5 \). We finally allow for changes in the core basis, in addition to varying \( \alpha \). In this case 4/1 and 5/2 models are found, although very unappealing ones (\textit{e.g.}, with no Yukawa couplings!). Nonetheless, a sample case yields

\[
\begin{align*}
5/2, 4/1 \text{ models} & \quad \begin{cases} 
\text{Moduli} & V_0 \\
\tau \rightarrow \tau & \\
\tau \rightarrow \tau & V_0 > 0 \\
\tau \rightarrow \alpha & 
\end{cases} 
\end{align*}
\tag{100}
\]

The above search for models, although limited in extent, provides support for the following

Conjecture : 4/1 models always give \( V_0 \neq 0 \). \tag{101}

This would imply that a \textit{necessary} condition for \( SU(5) \times U(1) \) string no-scale supergravity models is a 5/2 field content. This condition is consistent with the string-theory nature of the model which requires unification at the string scale, which can be accomplished in a 5/2 model. Moreover, a realistic model which satisfies the postulates of string no-scale supergravity already exists, namely the “search” model of Ref. [16].

### 6.2 A realistic example

As we just saw, the “search” model of Ref. [16] is a good candidate for a string no-scale supergravity model, with a single effective untwisted \( \tau \) set and a Kähler potential of the general form given in Eq. (38). With this information it should be possible to get a good idea of what the spectrum of sparticles may look like. First of all, we note that any model of this kind would be a \textit{no-parameter model} [33]. That is, the whole supersymmetric spectrum would be unambiguously determined. Indeed, with the ability to compute all soft-supersymmetry-breaking parameters (including \( \mu \) and \( B \)) in terms of \( m_{3/2} \), the high-energy theory would be determined up to the value of \( m_{3/2} \). At low energies one new parameter arises, namely \( \tan \beta \), but one also has two radiative electroweak symmetry breaking conditions which therefore allow one to
determine $m_{3/2}$ and $\tan \beta$. The no-scale mechanism can then be used to compute the quantity $C$ in Eq. (7). Thus, if the radiative breaking conditions can be solved, we would have a complete determination of the sparticle spectrum. In this exercise the top-quark mass is not an independent parameter: the top-quark Yukawa coupling at the string scale is a hallmark prediction of string models \cite{36} and the value of $\tan \beta$ would be self-consistently determined by the radiative breaking conditions.

Let us first address the question of the value of $Q$ in the “search” model of Ref. \cite{16}. The formula in Eq. (39) is

$$Q = n_{U2} + n_{U3} + n_{T1} - n_\psi - d_f - 3.$$  

The gauge group is $SU(5) \times SO(10) \times SU(4) \times U(1)^6$ which gives $d_f = 90$. The number of untwisted and twisted fields is: $n_\psi = 13$, $n_{U2} = 14$, $n_{U3} = 16$ and $n_{T1} = 80$, $n_{T2} = 80$, $n_{T3} = 68$, where we count $p$-dimensional representations of the gauge group as $p$. Putting it all together gives

$$Q = 14 + 16 + 80 - 13 - 90 - 3 = 110 - 106 = 4,$$  

which is remarkably close to the desired zero result (c.f., if all terms were to be added in magnitude, we are off by 2%). Pragmatically speaking $Q \neq 0$ and a destabilizing one-loop correction to the scalar potential is expected. However, given our incomplete knowledge of string dynamics (e.g., the role played by the anomalous $U_A(1)$) and of additional contributions to $Q$ from massive string states \cite{3} and string loop corrections, we are not ready to discard this model hastily. For instance, if the one-loop corrections to the gaugino masses (see Eq. (53)) increased them by 4.4%, we would obtain $Q = 0$. Such small string one-loop shifts on the scalar and gaugino masses are expected and quantify our statement that $Q = 4$ is a “small” number. We will therefore proceed exploring the manifold observable implications of this model, carrying the $Q = 4$ result as a warning flag.

Considering the spectrum of the “search” model of Ref. \cite{16}, the various relevant observable fields (and the sets they belong to) are as follows

| Set       | Untwisted fields | Twisted fields |
|-----------|------------------|----------------|
| First     | $\Phi_0, \Phi_1$; $h_1, \bar{h}_1$ | $F_0, F_1, F_4, \bar{F}_4$ |
| Second    | $h_2, \bar{h}_2$ | $F_2, \bar{f}_2, l^c_2; F_5, \bar{f}_5, l^c_5$ |
| Third     | $\Phi_3, \Phi_5$; $h_3, \bar{h}_3$ | $F_3, \bar{f}_3, l^c_3; h_{45}, \bar{h}_{45}$ |

The possible moduli are $\Phi_{0,1,3,5}$, of which all but $\Phi_1$ appear in the cubic superpotential (given in Eq. (6.3a) in Ref. \cite{16}). Therefore, the first set is a $\tau$ set, whereas the other two are $\alpha$ sets. In Ref. \cite{16} it was argued that $F_4 = \{Q_4, d_4^c, \nu_4^c\}$ should contain the third generation squarks, whereas $F_0, F_1, \bar{F}_4$ contain either Higgs particles or intermediate scale particles, therefore the first and second generation squarks and sleptons belong to the second and third twisted sets. Moreover, the light Higgs boson doublets

12When considering a Yukawa coupling at the string scale, care must be taken to include any normalization factors that may arise, as discussed in Sec. 5 especially in Eq. (89).

13In contrast, the second paper in Ref. \cite{3} presents a model with zero vacuum energy where $Q = -272$. Also, in the “revamped” model \cite{14}, for the value of $V_0$ closest to zero (it cannot be exactly zero), we get $Q = -83$.  

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are located inside \( h_1 \) and \( \bar{h}_{45} \), which in the usual notation correspond to \( H_1, H_2 \) respectively. Contrasting Eq. (103) with the general result for the scalar masses in this class of models in Eq. (24) we obtain the following spectrum of supersymmetry-breaking scalar masses:

- **First generation**: \( m^2_{Q_1, U_1^c, D_1^c, L_1, E_1^c} = 0 \), (104)
- **Second generation**: \( m^2_{Q_2, U_2^c, D_2^c, L_2, E_2^c} = 0 \), (105)
- **Third generation**: 
  \[
  \begin{align*}
  m^2_{Q_3, D_3^c} &= m_{3/2} \\
  m^2_{U_3^c, L_3, E_3^c} &= 0 
  \end{align*}
  \]
  (106)
- **Higgs masses**: \( m^2_{H_1} = 0, \ m^2_{H_2} = 0 \). (107)

We also know that the gaugino masses are degenerate \( m_{1/2} = m_{3/2} \) (see Eq. (54)), and that the \( A \) parameter is universal \( A = m_{3/2} \) (see Eq. (73)). The \( \mu \) parameter is expected to arise at the quintic level in the superpotential (no suitable terms exist at the quartic level). Since one of the light Higgs doublets belongs to the first untwisted set \( (h_1) \) and the other one to the third twisted set \( (\bar{h}_{45}) \), Eq. (75) singles out only one possible type of quintic term: \( [\beta^{(2)}]^2 [\beta^{(3)}]^2 \psi \). Moreover, from Eq. (78) we get \( B = \left[ 1 + \frac{\tau + \bar{\tau}}{\bar{\pi}} \hat{G}_2(\tau) \right] m_{3/2} \), which has a minimum at \( B = m_{3/2} \). In sum, without identifying the specific quintic term giving rise to \( \mu \), we can nonetheless predict the corresponding \( B \) parameter. Therefore, our no-parameter model reduces in practice to a one-parameter model until we compute such a quintic term.

An important ingredient in the viability of our candidate model is that the extra \( (10, \bar{10}) \) matter representations have suitable masses to allow gauge coupling unification at the string scale. In fact, the \( Q \) and \( D^c \) components of these representations should acquire different masses \( \sim 10^{12} \text{GeV} \) and \( \sim 10^6 \text{GeV} \) respectively [37], but this is allowed since \( SU(5) \times U(1) \) is broken at the string scale. Morever, the \( Q \) mass scale is determined by an effective superpotential mass term, whereas the \( D^c \) mass scale is obtained by working out the eigenvalues of the extended Higgs triplet mass matrix [10].

To summarize, the above candidate model has several notable properties: (i) a \( Q \) parameter tantalizingly close to zero, (ii) a benign non-universal spectrum of supersymmetry-breaking scalar masses which bears close resemblance to the old “no-scale” result, (iii) a universal trilinear term \( A = m_{3/2} \), (iv) a \( \mu \) parameter at the quintic level of superpotential interactions associated with \( B \geq m_{3/2} \), and (v) extra \( 10, \bar{10} \) representations dynamically required and likely to possess the desired mass spectrum. The low-energy consequences of this model will be explored in detail elsewhere [35].

## 7 Conclusions

In this paper we have explored the postulates of string no-scale supergravity in the context of free-fermionic string models. These postulates are not trivially satisfied, and in fact impose important new restrictions on string model building. In particular
the moduli sector should be rather minimal, and the massless matter spectrum and
gauge group should be correlated in a way such that the parameter $Q$ vanishes. We
have given plausibility arguments indicating that this condition may be possible to
satisfy in specific models, and in fact presented a model where $Q$ is very close to
zero. For all (untwisted and twisted) matter fields we have computed explicitly the
associated supersymmetry breaking parameters. Models of this kind are in fact “no-
parameter” models, with all needed masses and couplings completely determined.
A search for free-fermionic models which satisfy the minimal necessary conditions
yielded one candidate model with the $SU(5) \times U(1)$ observable gauge group, calcu-
lated novel supersymmetry breaking parameter space, and several desirable properties
regarding the stability of the no-scale mechanism. This search also appears to imply
that viable $SU(5) \times U(1)$ models always contain additional matter representations
that allow unification at the string scale.

It is interesting to remark that the models studied in this paper possess a
goldstino composition with significant admixtures of both “dilaton” and “moduli”. This hybrid scenario borrows desirable features from the two extremes, where either one or the other dominates the goldstino field. From the dilaton admixture we get a
tree-level contribution to the gaugino mass, and from the moduli admixture (or more
properly old no-scale) we get universality of scalar masses in the subset of fields where
it is desired.

We should also note that the no-scale supergravity realized in free-fermionic
models $[SO(2, n)/SO(2) \times SO(n)]$ differs in structure from that in the old no-scale
models $[SU(n, 1)/U(1) \times SU(n)]$. The effects of the different structures is most evident
in the computation of the vacuum energy and in the computation of the supersym-
metry breaking parameters.

Throughout our discussion we have said little about the supersymmetry break-
ing mechanism which creates $\langle W \rangle \neq 0$, thus implicitly assuming that its precise nature
would not affect our results. In gaugino condensation models, the non-perturbative
superpotential depends explicitly on $S$ and fixes its value, but this $S$-dependence is
likely to also affect the calculation of $V_0$. Nonetheless, it may be possible to retain
all the desirable no-scale supergravity properties. In coordinate-dependent com-
 pactifications we expect our results to hold since $W=\text{constant}$, although here the
question is: what determines $S$? The no-scale mechanism extended to the $S$ field
would appear to answer this question. A third possibility to obtain $\langle W \rangle \neq 0$ ap-
pears possible in models with an anomalous $U_A(1)$. In this case various singlet fields
would acquire vacuum expectations values $\langle \phi \rangle /M \sim 1/10$ and a cubic term in the
superpotential would give $\langle W \rangle = (\langle \phi \rangle /M)^3 \sim 10^{-3}$, if the flatness conditions can
be simultaneously satisfied. The crucial question in the phenomenologically viability
of these supersymmetry breaking mechanisms is the whether the calculated value of
$m_{3/2}$ is of electroweak scale size, since all supersymmetry-breaking parameters are
proportional to $m_{3/2}$.
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A Field redefinitions

As discussed in Sec. 3, the modular properties of the theory are evident in the supergravity basis, while the direct string calculations are in the string basis. Here we discuss how one goes from one basis to the other, or more specifically, how Eqs. (13) and (14) are obtained from Eq. (12). Our purpose is to identify a transformation which leaves the Kähler function \( G \) invariant. In this way all results which follow from it will not depend on the transformation. Some discussion of the relevant transformation has been given in Ref. [38].

The main observation is that the two forms of the untwisted Kähler potential are simply two different parametrizations of the metric for the same coset space: \( SO(2,n)/SO(2) \times SO(n) \). Following Gilmore [39], we introduce complex variables \( t_j = x_j + i y_j \) with \( 1 \leq j \leq n + 2 \), which describe the coset space of dimension \( 2n \) (two of the variables are auxiliary). The first parametrization is in terms of the variables \( \alpha_j \)

\[
\alpha_j = \frac{t_j}{t_{n+1} - it_{n+2}} , \quad 1 \leq j \leq n ,
\]

which have the following properties [39]

\[
\left| \sum_{j=1}^{n} \alpha_j \bar{\alpha}_j \right| < 1 ,
\]

\[
Y(\alpha) = 1 - 2 \sum_{j=1}^{n} \alpha_j \bar{\alpha}_j + \sum_{j=1}^{n} \alpha_j^2 = \left| t_{n+1} - it_{n+2} \right|^2 > 0 .
\]

This parametrization corresponds to that given in Ref. [20] and used in Eq. (12). In fact, \( Y(\alpha) \) is the argument of the logarithm in the Kähler potential. The second parametrization is in terms of the variables \( \beta_k \)

\[
\beta_k = \frac{t_k}{t_1 - it_{n+2}} , \quad 2 \leq k \leq n + 1 ,
\]

which have the property

\[
Y(\beta) = \sum_{k=2}^{n} (\beta_k - \bar{\beta}_k)^2 - (\beta_{n+1} - \bar{\beta}_{n+1})^2 = \left| t_{n+2} - t_1 \right|^2 > 0 .
\]
Performing the phase transformation $\beta_k \rightarrow i\beta_k$, we get

$$Y(\beta) \rightarrow Y(\beta) = (\beta_{n+1} + \bar{\beta}_{n+1})^2 - \sum_{k=2}^{n} (\beta_k + \bar{\beta}_k)^2,$$  

which reproduces Eq. (14) with the identifications $\beta_{n+1} \rightarrow \tau$, $\beta_k \rightarrow \psi_k$. Moreover, we can rewrite Eq. (113) as follows

$$Y(\beta) = (\tau_1 + \bar{\tau}_1)(\tau_2 + \bar{\tau}_2) - \sum_{i=1}^{n_\phi} (\phi_i + \bar{\phi}_i)^2$$  

(114)

where the second equality follows from the identifications $\beta_{n+1} + \beta_n \rightarrow \tau_1$ and $\beta_{n+1} - \beta_n \rightarrow \tau_2$. This result reproduces Eq. (13).

We now show that the transformation $\alpha \leftrightarrow t \leftrightarrow \beta$ leaves $G$ unchanged. Let us write $e^G$ as follows

$$e^G = \frac{|W|^2}{Y_0Y_1Y_2Y_3} = \frac{\lambda_{ij} \lambda_{ij'} \bar{\alpha}_i \bar{\alpha}_{i'} \bar{\alpha}_{j'} \bar{\alpha}_{j'}}{Y_1(\alpha) Y_2(\alpha) Y_3(\alpha)},$$  

(115)

where $Y_0 = (S + \bar{S})$, $Y_{1,2,3} = e^{-K_{1,2,3}}$ with the $K$’s in Eq. (12), and the superpotential is $W = \lambda_{ijk} \alpha_i^{(1)} \alpha_j^{(2)} \alpha_k^{(3)}$. Now we note that from Eqs. (111),(112) we can write

$$\frac{\beta_i \bar{\beta}_{i'}}{Y(\beta)} = \frac{t_i t_{i'}}{|t_1 - t_{n+2}|^2} \frac{|t_{n+2} - t_1|^2}{4} = \frac{1}{4} t_i \bar{t}_{i'},$$  

(116)

whereas from Eqs. (108),(110) we can write

$$\frac{\alpha_i \bar{\alpha}_{i'}}{Y(\alpha)} = \frac{t_i t_{i'}}{|t_{n+1} - it_{n+2}|^2} \frac{|t_{n+1} - it_{n+2}|^2}{4} = \frac{1}{4} t_i \bar{t}_{i'}.$$  

(117)

Therefore we conclude that $\beta_i \bar{\beta}_{i'}/Y(\beta) = \alpha_i \bar{\alpha}_{i'}/Y(\alpha)$, and thus Eq. (115) shows that $e^G$ remains invariant when written in terms of the $\beta$ variables. Note that the transformation involves the Kähler potential and the superpotential, and that the superpotential has the same couplings when written in terms of the $\beta$ variables. It appears unnecessary to relate the $\alpha$ to the $\beta$ variables directly (i.e., eliminating $t$), although this has apparently been done in Ref. [38].

### B Example of twisted sector Kähler potential

The twisted sector contribution to the Kähler potential in free-fermionic models was calculated in Ref. [21] for a simple model with $N = 1$ spacetime supersymmetry and
fermionic basis $\mathcal{B} = \{1, S, b_1, b_2, b_3\}$. This model has only three (massless) twisted sectors: $b_1, b_2, b_3$. The result, obtained to lowest order in the twisted fields, is

$$K_{TS} = \sum_{i}^{nT_1} \beta^{(1)}_i \beta^{(1)}_i e^{h_1(K_2 + K_3) + \sum_{i}^{nT_2} \beta^{(2)}_i \beta^{(2)}_i e^{h_2[K_2 + K_3]} + \sum_{i}^{nT_3} \beta^{(3)}_i \beta^{(3)}_i e^{h_3[K_2 + K_3]}} \quad (118)$$

where $\beta^{(1,2,3)}$ are the twisted fields (numbering $n_{T_1,T_2,T_3}$) in the $b_{1,2,3}$ sectors, and $K_{(1,2,3)}$ are the contributions to the Kähler potential from the untwisted fields as given in Eq. (12). Realistic free-fermionic models contain more basis vectors and a great deal more massless twisted sectors. For instance, the “search” model discussed above has basis $\mathcal{B} = \{1, S, b_1, b_2, b_3, b_4, b_5, \alpha\}$ and 22 massless twisted sectors. Despite this enlargement of the model, we conjecture that the structure of the twisted sector Kähler potential remains as simple as in Eq. (118) once we generalize the concept of twisted sector to “twisted set” with the meaning given in Sec. 3.1. In what follows we prove this conjecture by explicit calculation in the context of the “search” model.\footnote{For a more general discussion of twisted sector Kähler potentials in fermionic models see Ref. [22].}

We start by listing all of the massless states of the model divided into untwisted and twisted fields and by the set they belong to

| Set        | Untwisted fields | Twisted fields |
|------------|-----------------|----------------|
| First      | $\phi_0, \phi_1$| $F_0, F_1$ [b$_1$] |
|            | $\phi_{23}, \phi_{23}$| $F_4, \tilde{F}_4$ |
|            | $h_1, \tilde{h}_1$| $\tilde{F}_3, \tilde{F}_{1,2,4}, D_{1,2,5,6}$ |
| Second     | $\eta_1, \bar{\eta}_1$| $F_2, \bar{F}_2, l_2^c$ [b$_2$] |
|            | $\phi_{31}, \phi_{31}$| $\tilde{F}_5, \tilde{F}_5, l_5^c$ |
|            | $h_2, \tilde{h}_2$| $\tilde{F}_{1,5,6}, \tilde{F}_3, D_{3,7, T_1}$ |
| Third      | $\eta_2, \bar{h}_2$| $F_3, \bar{F}_3, l_3^c$ [b$_3$] |
|            | $\phi_{12}, \phi_{12}$| $h_{45}, \bar{h}_{45}$ |
|            | $\phi_{12}, \phi_{12}$| $\bar{F}_{2,4}, \bar{F}_{5,6}, F_4, T_2$ |

(The [b$_{1,2,3}$] that appear next to some states indicates that these states belong to that particular twisted sector.) With the exception of $\Phi_{0,1,3,5}$, the above fields carry charges under $SU(5) \times SU(4) \times SO(10)$ and various $U(1)$ symmetries: $\Phi_{23,31,12}, \Phi_{12,31,12}, \bar{h}_{1,2}, \bar{\eta}_{1,2}$ and $\phi_{45}^+, \bar{\phi}_{45}^+, \phi_{45}^-, \bar{\phi}_{45}^-$, $\phi_{3,4}^+, \bar{\phi}_{3,4}^-$ are gauge singlets; $F_{0,1,2,5}$ ($\tilde{F}_4$) are 10 (10) under $SU(5)$; $F_{2,3,5}$ ($l_{2,5,3}$) are 5 (1) under $SU(5)$; $\tilde{F}_{1,2,3,4,5,6}$ ($\tilde{F}_{1,2,3,4,5,6}$) are 4 (4) under $SU(4)$, and $D_{1,2,3,4,5,6,7}$ are 6 under $SU(4)$; $T_{1,2,3}$ are 10 under $SO(10)$.

To verify our conjecture (at least to lowest order) we work in the string basis and expand the exponentials in Eq. (118) to first order in the untwisted states: $K_{(1)} \approx$
\[ \sum_i \alpha_i^{(I)} \tilde{\alpha}_i^{(I)} \]. We end up with generic terms of the form
\[
\beta_i^{(I)} \tilde{\beta}_i^{(I)} + \frac{1}{2} \beta_i^{(I)} \tilde{\beta}_i^{(I)} \sum_j \left[ \alpha_j^{(J)} \tilde{\alpha}_j^{(J)} + \alpha_j^{(K)} \tilde{\alpha}_j^{(K)} \right],
\]
(120)

where \( J, K \neq I \). To verify the presence of the quartic term with the \( \frac{1}{2} \) coefficient we need to compute string scattering amplitudes of the type \( \langle \beta_i^{(I)} \beta_i^{(I)} \alpha_j^{(J)} \alpha_j^{(J)} \rangle \), which should exhibit a term proportional to \( s \) (in fact \( (g^4/4)(s/2) \)). This type of calculations have been performed in detail in Ref. [20]. Here we give the new results of interest and point out subtleties that need to be dealt with in the process.

To verify the old result in Ref. [21], we need to pick \( \beta \)'s from the twisted sectors \( b_{1,2,3} \) (see Eq. (119)). For instance, for \( \alpha^{(1)} = \Phi_{23} \) and \( \beta^{(1)} = F_{0,1} \) we find
\[
\langle F_{0,1} \Phi_{31} \Phi^\dagger_{31} F_{0,1}^\dagger \rangle = \frac{g^2}{4} \frac{s u}{t},
\]
(121)

which exhibits no term \( \propto s \) since both \( \alpha \) and \( \beta \) belong to the same set. The \( s u/t \) term is just the expected graviton exchange contribution. There are no further contributions since \( F_{0,1} \) and \( \Phi_{23} \) do not have any \( U(1) \) charges in common (\( i.e., \) no \( \text{“D terms”} \)) or appear in the same superpotential coupling (\( i.e., \) no \( \text{“F terms”} \)).\(^{15}\) Now let us consider \( \alpha^{(2)} = \Phi_{31} \), \( \bar{\Phi}_{31} \) and \( \beta^{(1)} = F_{0,1} \),
\[
\langle F_{0,1} \Phi_{31} \Phi^\dagger_{31} F_{0,1}^\dagger \rangle = \frac{g^2}{4} \left[ \frac{s}{2} + \frac{s u}{t} + 2 \ln 2 \right] - \frac{g^2}{2} \left( \frac{s - u}{t} - 1 \right),
\]
(122)
\[
\langle F_{0,1} \Phi_{31} \Phi^\dagger_{31} F_{0,1}^\dagger \rangle = \frac{g^2}{4} \left[ \frac{s}{2} + \frac{s u}{t} - 2 \ln 2 \right] + \frac{g^2}{2} \left( \frac{s - u}{t} - 1 \right).
\]
(123)

In these expressions we see the expected graviton exchange term (\( \propto s u/t \)) and also the contact term \( (g^2/4)(s/2) \), signaling the non-trivial quartic coupling in the Kähler function (see Eq. (122)). The last term is just a \( \text{“D term”} \): under a common \( U(1) \) \( F_{0,1} \) carry \(-1/2\) charge, whereas \( \Phi_{31} \) (\( \bar{\Phi}_{31} \)) carries \(+1\) charge, which give \( D = -\frac{1}{2} |F_{0,1}|^2 - |\Phi_{31}|^2 - |\bar{\Phi}_{31}|^2 \). Therefore, we expect gauge boson exchange \( \propto (s - u)/t \) and a contact term from \(-\frac{2}{s^2} D^2 \), all evidently present in Eqs. (122,123). We do not expect an \( \text{“F term”} \) since there is no superpotential coupling involving these fields. The last matter concerns the disturbing term \( \propto 2 \ln 2 s \). In Ref. [20] it was shown that the proper coordinates in which to write the untwisted sector Kähler function may be linear combinations of the string coordinates, in this case \( \chi_1 = \frac{1}{\sqrt{2}} (\Phi_{31} + \bar{\Phi}_{31}) \) and \( \chi_2 = \frac{1}{\sqrt{2}} (\Phi_{31} - \bar{\Phi}_{31}) \). The amplitudes to consider are then \( \langle F_{0,1} \chi_{1,2} \chi_{1,2} F_{0,1}^\dagger \rangle = \frac{1}{2} \langle F_{0,1} \Phi_{31} \Phi_{31}^\dagger F_{0,1}^\dagger \rangle + \frac{1}{2} \langle F_{0,1} \Phi_{31} \Phi_{31}^\dagger F_{0,1}^\dagger \rangle = \frac{g^2}{4} [s/2 + s u/t] \), which is the expected result.

Continuing in this fashion one can calculate all of the terms of the form \( \langle \_ \rangle \), where the \( \beta \)'s come exclusively from the \( b_{1,2,3} \) sectors. In each instance one can

\(^{15}\)A listing of all \( U(1) \) charges associated with the fields in Eq. (119) is given in Table 4 in Ref. [19]; the cubic and quartic superpotential is given in Eq. (6.3) of this same reference.
account for all pieces of the amplitudes, thus confirming the old result in Ref. [21].
We have conjectured that this result can be extended to all states in the spectrum. This conjecture has been verified explicitly for all twisted sectors in this model. For instance, consider \( \alpha^{(1)} = \Phi_0 \) and \( \beta^{(1)} = F_4 \)

\[ \langle F_4 \Phi_0 F_4^\dagger \rangle = \frac{g^2}{4} \frac{su}{t} - \frac{g^2}{2}. \]  

(124)

According to our conjecture, in this case we do expect a term \( \propto s \) since both fields belong to the first set. We also get the usual graviton exchange term, and a contact term \(-g^2/2\). This latter is an “F term” originating from the superpotential coupling \( \frac{1}{2}g\sqrt{2}F_4\Phi_0 \). Another example would be to take \( \alpha^{(2)} = \Phi_{31}, \Phi_{\bar{3}1} \) and \( \beta^{(1)} = F_4 \). The result in this case is identical to that in Eqs. (122,123); repeating the subsequent discussion shows the appearance of the term \( \propto s \), as conjectured. A lot more of these brute force calculations shows that the conjecture holds for all states in the massless spectrum. That is, in Eq. (118) one is to interpret \( \beta^{(I)} \) as fields belonging to the \( I \)-th set as defined in Eq. (9).

We should add that we have also verified the results of Ref. [20] for the untwisted sector of the “search” model. The novelty here is that some of the untwisted singlet fields (some of the presumed moduli) have superpotential couplings (in fact \( \Phi_{0,3,5} \)), a feature that does not alter the structure of the Kähler function derived in Ref. [20]. We have also repeated the above exercise (and therefore proven our conjecture) for the “revamped” model of Ref. [14]. For remarks regarding the generality of these results see Ref. [22].

In the main text we have used the twisted sector contribution to the Kähler potential to write down Eq. (38). In this case we have one untwisted set which is a \( \tau \) set, and the other two are \( \alpha \) sets. In Eq. (38) the two \( \alpha \) sets have been expanded to first order in \( \alpha^{(2,3)}_i \). The same expansion has been carried out in the exponents in Eq. (118), reducing it to unity for the first twisted set, and to the square-root of the argument of the \( \ln \) in the first untwisted set Kähler potential. Equation (38) then follows immediately. We should remark that these various approximations in calculating the Kähler function are immaterial as far as the observable quantities which we have calculated are concerned.

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\(^{16}\)This amplitude involves the Ising model correlator \( \langle \sigma f f \sigma \rangle = \frac{1}{2}|z_\infty|^{-1/4}[4(1-z)^{-2} + z^{-1}]^{1/2} \), which has been calculated using the methods of Ref. [18].

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