Vibration characteristic of moderately thick graphene reinforced composite functionally graded plate based on state-space levy method combined with higher order shear deformation theory

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Abstract. In this paper, the mechanical properties of graphene reinforced composite functional gradient (GRC-FG) plates under plane load conditions are studied in depth. The plate consists of a segmented patterned graphene reinforcement layer and the mechanical properties of the composite are simulated by extending the Halpin-Tsai micromechanical model. The governing equations are derived from Reddy's third-order shear deformation theory (HSDT) and the Hamiltonian principle. For the study of two opposite side simple supports and the solution of the coupled motion equation, we use state space eigenvectors to solve. The effects of geometric features and boundary conditions on material properties were investigated.

1. Introduction
Functionally graded composites of many different materials have excellent mechanical properties. Among them, carbon nanotube composite materials have attracted great interest from researchers. Nonlinear bending mechanics analysis of functionally graded carbon nanotube reinforced composite plates [1], three-dimensional elastic theory based on simply supported edges of heated mechanical loads [2], study of bending properties of composite rectangular plates [3], vibration characteristics of carbon nanotubes and their composites, and functional gradients The mechanical behavior of carbon nanotube reinforced composites [4], including powder metallurgy in the preparation of functionally graded carbon nanotube reinforced aluminum matrix composites [5], has been extensively studied.

With excellent thermal and electrical conductivity, high stiffness and strength, graphene and its derivatives such as graphene oxide and graphene sheets are ideal for composites. Zuxiang Lei, Qiangqiang Su et al, [6] the nano-silicon/graphene composite material can directly synthesize the resin composite material of the resin laminate and the graphene reinforced composite material [7], and the geometric linear nonlinear excitation of the rectangular plate of the three-stage sheath functionally graded graphene plate reinforced composite material under different edge conditions. Aiwen Wang et al. [8] Evenly distributed graphene sheets enhance the buckling and free vibration of the initial stress functionally graded cylindrical shell, the vibration characteristics of the graphene reinforced composite layer cylindrical plate supported by the elastic foundation in the thermal environment, and the Nsile mechanics of the graphene composite based on the molecular dynamics method Behavior [9], large free-linear vibration of graphene sheet reinforced composite laminates, high-ordering of graphene nanosheets (GNS) is achieved by vacuum filtration and then by spark plasma sintering in GNS / Cu composites [10]. A lot of results.

The Levy method solves the mechanical problems of two relatively simple supporting edge plates, determines the stability and vibration characteristics of the laminated cross-laminate [11], and analyzes the vibration and aeroelasticity of the ordered and disordered double-span plates [12]. Expanding the Kantorovich method, Levy-type support conditions, a closed form of three-dimensional free vibration solution can be used for rectangular laminates [13]. Based on Sanders' first-order impedance
deformation theory, a reliable and accurate Levy-type cylindrical panel free vibration analysis method is proposed [14]. An Exact Levy type solution for studying the mechanical properties of anisotropic plates has also been developed.

In this paper, the graphene composites studied have different carbon nanotube volume fractions in the thickness direction. The vibration analysis of the graphene-enhanced composite functional gradient (GRC-FG) plate is combined with the high-order deformation theory. The rotational inertia and lateral impedance deformation of nanocomposites were studied by the extended Halpin-Tsai micromechanical model and Reddy's third-order deformation theory (HSDT). And using the Hamilton equation to study the equation of motion of the plate, and to study the effects of geometric features and boundary conditions on the mechanical properties of the material.

2. Problem definitions
The geometric parameters of the GRC-FG board are shown in Figure 1(a). In-plane loads are applied to the edges of the GRC-FG board. There are three types of FG-GRC boards, UD, FG-O and FG-X, as shown in Figure 1(b). These boards are divided into ten layers. For the UD type GRC-FG board, we assume that the volume fraction of graphene is \((0.07) / (0.07) / (0.07) / (0.07) / (0.07) / (0.07) / (0.07) / (0.07) / (0.07) / (0.07)\). The volume fraction of graphene is assumed to be \((0.03) / (0.05) / (0.07) / (0.09) / (0.11) / (0.1) / (0.09) / (0.07) / (0.05) / (0.03) / (0.05) / (0.07) / (0.09) / (0.11)\) for FG-O GRC-FG plate. For the FG-X GRC-FG plate, the volume fraction of graphene is assumed to be \((0.11) / (0.09) / (0.07) / (0.05) / (0.03) / (0.03) / (0.05) / (0.07) / (0.09) / (0.11)\). According to the geometry and orientation of the filler, the elastic properties of the filler and the matrix are based on the Halpin-Tsai model to calculate the elastic properties of the composite. This study uses the extended Halpin-Tsai [15] The model evaluates the effective material properties of the GRC-FG plate. The Young's modulus and shear modulus are expressed as

\[
\begin{align*}
E_{11} & = \eta_1 \frac{1 + (2a_G/h_G)\gamma_{11}V_G}{1 - \gamma_{11}V_G} E^m, \\
E_{22} & = \eta_2 \frac{1 + (2b_G/h_G)\gamma_{22}V_G}{1 - \gamma_{22}V_G} E^m, \\
G_{12} & = \eta_3 \frac{1}{1 - \gamma_{12}V_G} G^m,
\end{align*}
\]

where \(a_G\), \(b_G\) and \(h_G\) are geometric parameters of the graphene sheet. \(\eta_j (j=1,2,3)\), The graphene efficiency parameter is introduced to measure the scale effect.

\[
\gamma_{11} = \frac{E_{11}^G/E^m - 1}{E_{11}^G/E^m + 2a_G/h_G},
\]

\[
\gamma_{12} = \frac{E_{12}^G/E^m - 1}{E_{12}^G/E^m + 2b_G/h_G},
\]

\[
\gamma_{12} = \frac{G_{12}^G/G^m - 1}{G_{12}^G/G^m},
\]

where \(E_{11}^G\), \(E_{22}^G\) and \(G_{12}^G\) are the Young’s moduli and shear modulus of the graphene sheet, and \(E^m\) and \(G^m\) are the corresponding Young’s moduli and shear modulus. The volume fractions of graphene and matrix are defined as \(V_G\) and \(V_m = 1 - V_G\), respectively. The Poisson’s ratio is defined as

\[
\nu_{12} = V_G\nu_{12}^G + V_m\nu_{12}^m,
\]

where \(\nu_{12}^G\) and \(\nu_{12}^m\) are the Poisson’s ratios of the graphene and matrix, respectively.

3. Formulation of motion
Using Reddy's third-order shear deformation theory to obtain the displacement field
\[ u = u_0 + z \phi_x - \frac{4}{3h^2} z^3 \left( \phi_x + \frac{\partial w}{\partial x} \right), \quad (4a) \]
\[ v = v_0 + z \phi_y - \frac{4}{3h^2} z^3 \left( \phi_y + \frac{\partial w}{\partial y} \right), \quad (4b) \]
\[ w = w, \quad (4c) \]

where \((u_0, v_0, \phi_x, \phi_y)\) are the displacement components of the neutral plane in the \(x\) and \(y\) directions, and the rotations of a transverse normal about \(y\) and \(x\) axes.

Calculate the strain component formula of the GRC-FG board as follows
\[ \varepsilon = \varepsilon_0 + z \varepsilon_1 + z^3 \varepsilon_3, \gamma = \gamma_0 + z^2 \gamma_2, \quad (5) \]

where
\[ \varepsilon_0 = \begin{bmatrix} u_{0,x} & v_{0,x} & u_{0,y} & v_{0,y} \end{bmatrix}^T, \quad (6a) \]
\[ \varepsilon_1 = \begin{bmatrix} \phi_x, \phi_y, \phi_{x,y} + \phi_{y,x}, \phi_{x,y} + \phi_{y,x} + 2w_{,xy} \end{bmatrix}^T, \quad (6b) \]
\[ \varepsilon_3 = -c_1 \begin{bmatrix} \phi_{x,x} + w_{,xx}, \phi_{y,y} + w_{,yy}, \phi_{x,x} + \phi_{y,y} + 2w_{,xy} \end{bmatrix}^T, \quad (6c) \]
\[ \gamma_0 = \begin{bmatrix} \phi_x + w_x, \phi_x + w_x \end{bmatrix}^T, \quad (6d) \]
\[ \gamma_2 = -c_2 \begin{bmatrix} \phi_y + w_y, \phi_y + w_y \end{bmatrix}^T, \quad (6e) \]

where
\[ c_2 = 3c_1 \text{ and } c_1 = 4/3h^2. \quad (7) \]

The stress resultants can be calculated as
\[ \begin{bmatrix} N_0 \\ M_0 \\ P_0 \end{bmatrix} = \begin{bmatrix} A_0 & B_0 & E_0 \\ B_0 & D_0 & F_0 \\ E_0 & F_0 & H_0 \end{bmatrix} \begin{bmatrix} \varepsilon_0 \\ \varepsilon_1 \\ \varepsilon_3 \end{bmatrix}, \quad (8) \]

where the matrices \(A_0, B_0, D_0, E_0, F_0, H_0, A_1, D_1\) and \(F_1\) are given as
\[ A_0, B_0, D_0, E_0, F_0, H_0 = \int_{h/2}^{h/2} \hat{Q}_b(z)(1, z, z^2, z^3, z^4, z^5)dz, \quad (9a) \]
\[ A_1, D_1, F_1 = \int_{h/2}^{h/2} \hat{Q}_b(z)(1, z^2, z^4)dz, \quad (9b) \]

where
\[ \hat{Q}_b(z) = \begin{bmatrix} E_{11} & \nu_{21}E_{11} & 0 \\ \nu_{21}E_{11} & E_{22} & 0 \\ 0 & 0 & G_{12} \end{bmatrix}, \quad (10a) \]
\[ \hat{Q}_b(z) = \begin{bmatrix} G_{23} & 0 \\ 0 & G_{13} \end{bmatrix}. \quad (10b) \]

The calculation of the equation of motion for the GRC-FG plate is based on the Hamilton principle.
\[ \int_T \left[ \delta(T - \Pi + W) \right] dt = 0, \quad (11) \]

where \(T, U, W\) are the kinetic energy, potential energy and work of the external loads which are given as
\[ T = \frac{1}{2} \int_V \rho(z)(\dot{u}^2 + \dot{v}^2 + \dot{w}^2) dV, \quad (12a) \]
\[ \Pi = \frac{1}{2} \int_A (N_0^t e_0 + M_0^t e_1 + P_0^t e_3 + Q_0^t y_0 + R_0^t y_1) dA, \quad (12b) \]

\[ W = - \frac{1}{2} \int_A [N_1 (\frac{\partial w}{\partial x})^2 + N_1 (\frac{\partial w}{\partial y})^2] dA. \quad (12c) \]

By substituting the Eqs. (6) and (8) into Eq. (12), and the results into Eq. (11), we can obtain the system partial differential equation of motion as follows

\[ 2m \ddot{u} + \dot{m}_w \ddot{w}_w + \dot{n}_w \ddot{w}_w - A_4 u_{,xx} - A_{66} u_{,yy}, \quad (13a) \]

\[ - (A_{12} + A_{66}) u_{,xy} + c_1 E_1 w_{,xxx} + c_1 (E_{12} + 2E_{66}) w_{,xyy}, \]

\[ - (\hat{B}_{12})^\phi_{,xx} - \hat{B}_{66} \phi_{,xy} - (\hat{B}_{12} + \hat{B}_{66}) \phi_{,xy} = 0 \]

\[ 2m \ddot{v} + \dot{m}_w \ddot{w}_w + A_4 \ddot{v}_{,xx} + A_{66} \ddot{v}_{,yy} - (\hat{B}_{12} + \hat{B}_{66}) \phi_{,xy}, \quad (13b) \]

\[ - (A_{12} + A_{66}) u_{,x} + c_1 E_2 w_{,yxy} + c_1 (E_{12} + 2E_{66}) w_{,xyy}, \]

\[ - (\hat{B}_{22})^\phi_{,xy} - \hat{B}_{66} \phi_{,xx} - (\hat{B}_{12} + \hat{B}_{66}) \phi_{,xy} = 0 \]

\[ 2m \ddot{v} - 2m \ddot{w}_w \ddot{w}_w - 2m \ddot{w}_w \ddot{w}_w - \dot{m}_w \ddot{u}_{,x} - \dot{m}_w \ddot{v}_{,y}, \]

\[ - \dot{m}_w \ddot{v}_{,x} + \dot{m}_w \ddot{w}_w + c_1^2 H_1 w_{,xxx} + c_1^2 H_2 w_{,yyy} + c_1^2 (H_{12} + 2H_{66}) w_{,xyy}, \]

\[ + c_1^2 (A_{44} + N_4 + N_5) w_{,xy} - (\hat{A}_{44} + N_5) w_{,xy}, \]

\[ - c_1 E_1 u_{,xxx} - c_1 (E_{12} + 2E_{66}) (u_{,xy} + v_{,xy}), \quad (13c) \]

\[ c_1 E_2 w_{,yxy} - (\hat{A}_{44} + N_5) w_{,xy} - c_1 (2\hat{F}_{66} + \hat{F}_{12}) (\phi_{,xy} + \phi_{,xyy}), \]

\[ - c_1 \hat{F}_{22} \phi_{,xy} - c_1 \hat{F}_{11} \phi_{,xxx} = 0 \]

where

\[ \hat{B}_j = B_j - c_j E_j, \quad (14a) \]

\[ \hat{F}_j = F_j - c_j H_j, \quad (14b) \]

\[ \hat{D}_j = D_j - c_j F_j, \quad (14c) \]

\[ \hat{A}_j = A_j - 2c_j D_j + c_2^2 F_j, \quad (14d) \]

\[ \hat{D}_j = D_j - c_j F_j, \quad (14f) \]

\[ \hat{A}_j = A_j - c_j D_j, \quad (14g) \]

\[ \hat{A}_j = A_j - 2c_j D_j + c_2^2 F_j, \quad (14h) \]

where \( i, j = 1, 2, 6 \) for Eqs (14a)-(14e) and \( i, j = 4, 5 \) for Eqs (14f)-(14h).

We assume the solution of Eq. (13) are in the form as

\[ u = \bar{U}(x, y)e^{iu}, \quad (15a) \]

\[ v = \bar{V}(x, y)e^{iv}, \quad (15b) \]
\[ w = W(x, y)e^{i\omega t}, \]  
\[ \phi_x = X(x, y)e^{i\omega t}, \]  
\[ \phi_y = Y(x, y)e^{i\omega t}, \]  

where \( \omega \) is the natural frequency of the GRC-FG plates. \( \bar{U}, \bar{V}, \bar{W}, \bar{X}, \bar{Y} \) are the mode shapes. The present GRC-FG plates are all simply supported along two opposite edges in the \( y \) direction. Then, the modes shapes can be assumed as

\[ \bar{U}(x, y) = \hat{U}(x)\sin \beta y, \]  
\[ \bar{V}(x, y) = \hat{V}(x)\cos \beta y, \]  
\[ \bar{W}(x, y) = \hat{W}(x)\sin \beta y, \]  
\[ \bar{X}(x, y) = \hat{X}(x)\sin \beta y, \]  
\[ \bar{Y}(x, y) = \hat{Y}(x)\cos \beta y, \]

in which \( \hat{U}, \hat{V}, \hat{W}, \hat{X}, \hat{Y} \) are the mode shapes of GRC-FG plates along the \( x \) axis.

The following mode equation can be obtained by substituting Eqs. (15) and (16) into Eq. (13)

\[ \begin{bmatrix} \psi_1 & 0 & 0 & 0 & \hat{\mu}_1 \\ 0 & \psi_2 & 0 & 0 & \hat{\mu}_2 \\ 0 & 0 & \psi_{31} & \psi_{32} & \hat{\mu}_3 \end{bmatrix} = \begin{bmatrix} \partial_1 & 0 \\ 0 & \partial_2 \\ 0 & \partial_3 \end{bmatrix} \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix}, \]

where

\[ \hat{\mu}_1 = [\hat{U}^\top, \hat{X}^\top]^\top, \hat{\mu}_2 = [\hat{V}^\top, \hat{Y}^\top]^\top, \hat{\mu}_3 = [\hat{W}^\top, \hat{X}^\top, \hat{Y}^{(4)}]^\top, \]

\[ \mu_1 = [\hat{U}, \hat{V}, \hat{W}, \hat{X}, \hat{Y}]^\top, \mu_2 = [\hat{U}, \hat{V}, \hat{W}, \hat{X}, \hat{Y}]^\top, \mu_3 = [\hat{U}, \hat{V}, \hat{W}, \hat{X}, \hat{Y}]^\top, \]

\[ \begin{bmatrix} A_{11} & A_{16} & A_{12} & 0 \\ B_{11} & B_{16} & 0 & 0 \\ 0 & D_{11} & D_{16} & D_{12} \end{bmatrix}, \]

\[ \begin{bmatrix} \partial_1 \\ \partial_2 \\ \partial_3 \end{bmatrix} = \begin{bmatrix} \partial_{11} & \partial_{14} & \partial_{16} & \partial_{19} & \partial_{112} \\ \partial_{14} & \partial_{16} & \partial_{18} & \partial_{19} & \partial_{112} \\ \partial_{16} & \partial_{18} & \partial_{19} & \partial_{210} & \partial_{211} \end{bmatrix}, \]

\[ \begin{bmatrix} \partial_{32} & \partial_{33} & \partial_{35} & \partial_{37} & \partial_{310} & \partial_{311} \\ \partial_{32} & \partial_{33} & \partial_{35} & \partial_{37} & \partial_{310} & \partial_{311} \end{bmatrix}, \]

where

\[ \tau_1 = c_1E_{11}, \quad \tau_2 = c_1\hat{F}_{11}, \quad \tau_3 = -c_1^2H_{11}, \]

\[ \tau_4 = -c_1\beta(2\hat{F}_{66} + \hat{F}_{11}), \quad \tau_5 = -c_1\beta(E_{12} + 2E_{66}) \]

\[ \partial_{11} = \beta^2A_{16} - 2\omega^2\hat{m}_{uw}, \quad \partial_{14} = \beta(A_{12} + A_{16}), \]

\[ \partial_{16} = -[\omega^2\hat{m}_{uw} + c_1\beta^2(E_{12} + 2E_{66})], \]

\[ \partial_{19} = c_1E_{11}, \quad \partial_{19} = \beta^2\hat{B}_{66} - \omega^2\hat{m}_{uv}, \quad \partial_{112} = \beta\hat{B}_{12} + \hat{B}_{66}, \]

\[ \partial_{22} = -\beta(A_{12} + A_{16}), \partial_{23} = \beta^2A_{22} - 2\omega^2\hat{m}_{vw}, \]

\[ \partial_{25} = -[\beta\omega^2\hat{m}_{vw} + \beta^2c_1E_{22}], \]

\[ \partial_{27} = c_1\beta(E_{12} + 2E_{66}), \partial_{210} = -\beta(\hat{B}_{12} + \hat{B}_{66}), \]

\[ \partial_{211} = \beta^2\hat{B}_{22} - \omega^2\hat{m}_{yz}, \]
\[ \begin{align*}
\vartheta_{41} &= \beta^2 \hat{B}_{66} - \omega^2 \hat{m}_{xx}, \\
\vartheta_{46} &= (\hat{A}_{55} - \omega^2 \hat{m}_{uu}) - c_i \beta^2 (2 \hat{F}_{66} + \hat{F}_{12}), \\
\vartheta_{48} &= c_i \hat{F}_{11}, \quad \vartheta_{49} = \beta^2 \hat{D}_{66} + \hat{A}_{55} - 2 \omega^2 \hat{m}_{yy}, \\
\vartheta_{412} &= \beta(\hat{D}_{12} + \hat{D}_{66}), \\
\vartheta_{32} &= -\beta(\hat{B}_{66} + \hat{B}_{12}), \quad \vartheta_{33} = \beta^2 \hat{B}_{22} - \omega^2 \hat{m}_{yy}, \\
\vartheta_{35} &= \beta(\hat{A}_{44} - \omega^2 \hat{m}_{yy} - c_i \beta^2 \hat{F}_{22}), \\
\vartheta_{39} &= c_i \beta (2 \hat{F}_{66} + \hat{F}_{12}), \quad \vartheta_{310} = -\beta(\hat{D}_{12} + \hat{D}_{66}), \\
\vartheta_{311} &= \beta^2 \hat{D}_{22} + \hat{A}_{44} - 2 \omega^2 \hat{m}_{yy}, \\
\vartheta_{32} &= c_i \beta^2 (E_{12} + 2 E_{66}) + \omega^2 \hat{m}_{uu}, \\
\vartheta_{39} &= -\beta(c_i \beta^2 E_{22} + \omega^2 \hat{m}_{uu}), \quad \vartheta_{311} = \beta(\hat{A}_{44} - c_i \beta^2 \hat{F}_{22} - \omega^2 \hat{m}_{yy}), \\
\vartheta_{35} &= c_i^2 \beta^4 H_{22} + \beta^2 (\hat{A}_{44} + N_x) - 2 \omega^2 (\hat{m}_{uu1} + \beta^2 \hat{m}_{uu2}), \\
\vartheta_{310} &= c_i \beta^2 (2 \hat{F}_{66} + \hat{F}_{12}) - \hat{A}_{55} + \omega^2 \hat{m}_{uu}, \\
\vartheta_{37} &= \left[ 2 c_i \beta^2 (H_{12} + H_{66}) + \hat{A}_{55} + N_x - 2 \omega^2 \hat{m}_{uu2} \right], \\
\end{align*} \]

The following equations are calculated to solve the mode equations

\[ \begin{align*}
\dot{U}' &= \dot{\vartheta}_{11} U + \dot{\vartheta}_{14} \dot{V} + \dot{\vartheta}_{44} \dot{W} + \dot{\vartheta}_{49} \dot{X} + \dot{\vartheta}_{412} \dot{Y}, \\
\dot{V}' &= \dot{\vartheta}_{22} \dot{U} + \dot{\vartheta}_{23} \dot{V} + \dot{\vartheta}_{25} \dot{W} + \dot{\vartheta}_{210} \dot{X} + \dot{\vartheta}_{211} \dot{Y}, \\
\dot{X}' &= \dot{\vartheta}_{42} \dot{U} + \dot{\vartheta}_{44} \dot{V} + \dot{\vartheta}_{49} \dot{W} + \dot{\vartheta}_{412} \dot{X} + \dot{\vartheta}_{411} \dot{Y}, \\
\dot{Y}' &= \dot{\vartheta}_{22} \dot{U} + \dot{\vartheta}_{23} \dot{V} + \dot{\vartheta}_{25} \dot{W} + \dot{\vartheta}_{210} \dot{X} + \dot{\vartheta}_{211} \dot{Y},
\end{align*} \]

where

\[ \begin{align*}
\dot{\vartheta}_{11} &= \frac{D_{11}(\beta^2 A_{66} - 2 \omega^2 \hat{m}_{uu}) - \hat{B}_{11}(\beta^2 \hat{B}_{66} - \omega^2 \hat{m}_{uu})}{A_{11}D_{11} - B_{11}^2}, \\
\dot{\vartheta}_{14} &= \frac{D_{11}(\beta^2 A_{44} - \hat{A}_{44}) - \hat{B}_{11}(\hat{B}_{12} + \hat{B}_{66})}{A_{11}D_{11} - B_{11}^2}, \\
\dot{\vartheta}_{16} &= \frac{D_{11}(\omega^2 \hat{m}_{uu} + \beta^2 c_i (E_{12} + 2 E_{66}) - \hat{B}_{11}(c_i \beta^2 (2 \hat{F}_{66} + \hat{F}_{12}) + \hat{A}_{55} - \omega^2 \hat{m}_{uu})}{A_{11}D_{11} - B_{11}^2}, \\
\dot{\vartheta}_{18} &= \frac{D_{11}(\beta^2 \hat{B}_{66} - \omega^2 \hat{m}_{uu}) - \hat{B}_{11}(\beta^2 \hat{D}_{66} + \hat{A}_{55} - 2 \omega^2 \hat{m}_{uu})}{A_{11}D_{11} - B_{11}^2}, \\
\dot{\vartheta}_{19} &= \frac{D_{11}(\beta^2 \hat{B}_{66} - \omega^2 \hat{m}_{uu}) - \hat{B}_{11}(\beta^2 \hat{D}_{66} + \hat{A}_{55} - 2 \omega^2 \hat{m}_{uu})}{A_{11}D_{11} - B_{11}^2}, \\
\dot{\vartheta}_{112} &= \frac{D_{11}(\beta^2 \hat{B}_{12} + \hat{B}_{66}) - \hat{B}_{11}(\hat{D}_{12} + \hat{D}_{66})}{A_{11}D_{11} - B_{11}^2},
\end{align*} \]
\[ \hat{\beta}_{41} = -\frac{\hat{B}_1(\beta^2 A_{66} - 2\omega^2 \hat{m}_{uu}) + A_1(\beta^2 \hat{B}_{66} - \omega^2 \hat{m}_{uu})}{A_1 \hat{D}_{11} - \hat{B}_{11}^2}, \]
\[ \hat{\beta}_{44} = -\frac{\hat{B}_1 \beta (A_{12} + A_{66}) + A_1 \beta (\hat{B}_{12} + \hat{B}_{66})}{A_1 \hat{D}_{11} - \hat{B}_{11}^2}, \]
\[ \hat{\beta}_{46} = \frac{\hat{B}_1(\omega^2 \hat{m}_{uu} + \beta^2 c_i (E_{12} + 2E_{66}) + A_{11} c_i \beta^2 (2\hat{F}_{66} + \hat{F}_{12}) + \hat{A}_{44} - \omega^2 \hat{m}_{uu})}{A_1 \hat{D}_{11} - \hat{B}_{11}^2}, \]
\[ \hat{\beta}_{48} = -\frac{\hat{B}_1 c_i \hat{F}_{11} + A_1 c_i \hat{F}_{11}}{A_1 \hat{D}_{11} - \hat{B}_{11}^2}, \]
\[ \hat{\beta}_{49} = -\frac{\hat{B}_1(\beta^2 \hat{B}_{66} - \omega^2 \hat{m}_{uu}) + A_1(\beta^2 \hat{D}_{66} + \hat{A}_{44} - 2\omega^2 \hat{m}_{uu})}{A_1 \hat{D}_{11} - \hat{B}_{11}^2}, \]
\[ \hat{\beta}_{412} = \frac{\hat{B}_1 \beta (\hat{B}_{12} + \hat{B}_{66}) + A_1 \beta (\hat{D}_{12} + \hat{D}_{66})}{A_1 \hat{D}_{11} - \hat{B}_{11}^2}, \]
\[ \hat{\beta}_{22} = -\frac{\hat{D}_{66}(\beta^2 A_{22} - 2\omega^2 \hat{m}_{vv}) - \hat{B}_{66}(\beta^2 \hat{B}_{22} - \omega^2 \hat{m}_{vv})}{A_6 \hat{D}_{66} - \hat{B}_{66}^2}, \]
\[ \hat{\beta}_{25} = -\frac{\hat{D}_{66}(\beta^2 \hat{B}_{12} - \omega^2 \hat{m}_{vv}) - \hat{B}_{66}(\beta^2 \hat{D}_{22} + \hat{A}_{44} - 2\omega^2 \hat{m}_{vv})}{A_6 \hat{D}_{66} - \hat{B}_{66}^2}, \]
\[ \hat{\beta}_{27} = -\frac{\hat{D}_{66} c_i (E_{12} + 2E_{66}) + \hat{B}_{66} c_i \beta (2\hat{F}_{66} + \hat{F}_{12})}{A_6 \hat{D}_{66} - \hat{B}_{66}^2}, \]
\[ \hat{\beta}_{210} = \frac{\hat{D}_{66}(\beta^2 \hat{B}_{66} - \omega^2 \hat{m}_{uu}) - \hat{B}_{66}(\beta^2 \hat{D}_{66} - \hat{B}_{66}^2)}{A_6 \hat{D}_{66} - \hat{B}_{66}^2}, \]
\[ \hat{\beta}_{211} = \frac{\hat{D}_{66}(\beta^2 \hat{B}_{12} - \omega^2 \hat{m}_{uu}) - \hat{B}_{66}(\beta^2 \hat{D}_{22} + \hat{A}_{44} - 2\omega^2 \hat{m}_{uu})}{A_6 \hat{D}_{66} - \hat{B}_{66}^2}, \]
\[ \hat{\beta}_{32} = \frac{\hat{B}_{66}(\beta^2 A_{22} - 2\omega^2 \hat{m}_{vv})}{A_6 \hat{D}_{66} - \hat{B}_{66}^2} - \frac{A_6 \beta(\hat{B}_{12} + \hat{B}_{66})}{A_6 \hat{D}_{66} - \hat{B}_{66}^2}, \]
\[ \hat{\beta}_{33} = \frac{\hat{B}_{66}(\beta^2 A_{22} - 2\omega^2 \hat{m}_{vv}) + A_6 \beta(\hat{B}_{12} - \hat{B}_{66})}{A_6 \hat{D}_{66} - \hat{B}_{66}^2}, \]
\[ \hat{\beta}_{35} = -\frac{\hat{B}_{66}(\beta^2 \hat{B}_{12} - \omega^2 \hat{m}_{vv}) + A_6 \beta(\hat{D}_{12} - \hat{D}_{66})}{A_6 \hat{D}_{66} - \hat{B}_{66}^2}, \]
\[ \hat{\beta}_{37} = -\frac{\hat{B}_{66} c_i (E_{12} + 2E_{66})}{A_6 \hat{D}_{66} - \hat{B}_{66}^2} - \frac{A_6 c_i \beta (2\hat{F}_{66} + \hat{F}_{12})}{A_6 \hat{D}_{66} - \hat{B}_{66}^2}, \]
\[ \hat{\beta}_{52} = \frac{\hat{B}_{66}(\beta^2 A_{22} - 2\omega^2 \hat{m}_{vv})}{A_6 \hat{D}_{66} - \hat{B}_{66}^2} - \frac{A_6 \beta(\hat{B}_{12} - \hat{B}_{66})}{A_6 \hat{D}_{66} - \hat{B}_{66}^2}, \]
\[ \hat{\beta}_{53} = \frac{\hat{B}_{66}(\beta^2 A_{22} - 2\omega^2 \hat{m}_{vv}) + A_6 \beta(\hat{B}_{12} + \hat{B}_{66})}{A_6 \hat{D}_{66} - \hat{B}_{66}^2}, \]
\[ \hat{\beta}_{55} = -\frac{\hat{B}_{66}(\beta^2 \hat{B}_{12} - \omega^2 \hat{m}_{vv}) + A_6 \beta(\hat{D}_{12} - \hat{D}_{66})}{A_6 \hat{D}_{66} - \hat{B}_{66}^2}, \]
\[ \hat{\beta}_{57} = -\frac{\hat{B}_{66} c_i (E_{12} + 2E_{66})}{A_6 \hat{D}_{66} - \hat{B}_{66}^2} - \frac{A_6 c_i \beta (2\hat{F}_{66} + \hat{F}_{12})}{A_6 \hat{D}_{66} - \hat{B}_{66}^2}, \]
\[ \hat{\beta}_{510} = \frac{\hat{B}_{66}(\beta^2 \hat{B}_{66} - \omega^2 \hat{m}_{uu}) - \hat{B}_{66}(\beta^2 \hat{D}_{66} - \hat{B}_{66}^2)}{A_6 \hat{D}_{66} - \hat{B}_{66}^2}, \]
\[ \hat{\beta}_{511} = \frac{\hat{B}_{66}(\beta^2 \hat{B}_{12} - \omega^2 \hat{m}_{uu}) - \hat{B}_{66}(\beta^2 \hat{D}_{22} + \hat{A}_{44} - 2\omega^2 \hat{m}_{uu})}{A_6 \hat{D}_{66} - \hat{B}_{66}^2}. \]
\[ W^{(4)} = \hat{\partial}_{32} \hat{U} + \hat{\partial}_{33} \hat{V} + \hat{\partial}_{35} \hat{W} + \hat{\partial}_{310} \hat{X} + \hat{\partial}_{311} \hat{Y}, \]  \hspace{1cm} (22)

where

\[
\begin{align*}
\hat{\partial}_{32} &= - \left( \tau_1 \hat{\partial}_{44} \hat{\partial}_{32} + \tau_3 \hat{\partial}_{22} \hat{\partial}_{34} + \tau_1 \hat{\partial}_{44} \hat{\partial}_{32} + \tau_1 \hat{\partial}_{44} \hat{\partial}_{32} + \tau_4 \hat{\partial}_{22} + \tau_4 \hat{\partial}_{41} + \tau_4 \hat{\partial}_{22} - \hat{\partial}_{32} \right) \\
\hat{\partial}_{33} &= - \left( \tau_1 \hat{\partial}_{44} \hat{\partial}_{33} + \tau_3 \hat{\partial}_{23} \hat{\partial}_{34} + \tau_1 \hat{\partial}_{44} \hat{\partial}_{33} + \tau_4 \hat{\partial}_{23} \hat{\partial}_{33} + \tau_5 \hat{\partial}_{33} - \hat{\partial}_{33} \right) \\
\hat{\partial}_{35} &= - \left( \tau_1 \hat{\partial}_{44} \hat{\partial}_{35} + \tau_3 \hat{\partial}_{25} \hat{\partial}_{34} + \tau_1 \hat{\partial}_{44} \hat{\partial}_{35} + \tau_4 \hat{\partial}_{25} \hat{\partial}_{35} + \tau_5 \hat{\partial}_{35} - \hat{\partial}_{35} \right) \\
\hat{\partial}_{310} &= - \left( \tau_1 \hat{\partial}_{44} \hat{\partial}_{310} + \tau_3 \hat{\partial}_{210} \hat{\partial}_{34} + \tau_1 \hat{\partial}_{44} \hat{\partial}_{310} + \tau_4 \hat{\partial}_{210} \hat{\partial}_{310} + \tau_5 \hat{\partial}_{310} - \hat{\partial}_{310} \right) \\
\hat{\partial}_{311} &= - \left( \tau_1 \hat{\partial}_{44} \hat{\partial}_{311} + \tau_3 \hat{\partial}_{211} \hat{\partial}_{34} + \tau_1 \hat{\partial}_{44} \hat{\partial}_{311} + \tau_4 \hat{\partial}_{211} \hat{\partial}_{311} + \tau_5 \hat{\partial}_{311} - \hat{\partial}_{311} \right). 
\end{align*}
\]  \hspace{1cm} (23)

Eq. (20) and Eq. (22) present a set of ordinary differential equations. A state-space equation is introduced to solve this coupling motion equation. The state \( Z(x) \) is given as

\[ Z(x) = \begin{bmatrix} U & \hat{U} & V & \hat{V} & W & \hat{W} & X & \hat{X} & Y & \hat{Y} \end{bmatrix}. \] \hspace{1cm} (24)

Then, the state-space motion equation can be written as

\[ Z = \Theta Z, \] \hspace{1cm} (25)

where

\[ \Theta = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \] \hspace{1cm} (26)

The solution of Eq. (25) can be presented as

\[ Z = \Gamma K \Lambda, \] \hspace{1cm} (27)

in which \( \Gamma = [\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_r] \), and \( \mathbf{r}_i \) is the eigenvector of matrix \( \Theta \). \( K = [K_1, K_2, \ldots, K_k]^T \) is the undetermined coefficient vector which can be determined by the boundary conditions. \( \Lambda \) is a diagonal matrix. The component of \( \Lambda \) is \( \exp(\lambda_i x) \) and \( \lambda_i \) is the eigenvalue of the matrix \( \Theta \).

For the present GRC-FG plates, the boundary conditions for simply supported (S), clamped (C) and free (F) at the edges \( x = 0 \) and \( x = a \) are defined as

\[ S : v = w = \phi_y = N_{as} = M_{as} = P_{as} = 0, \] \hspace{1cm} (28a)
C: \( v = w = \phi_x = \phi_y = \frac{\partial w}{\partial x} = 0 \), \( F : N_{xx} = N_{yy} = M_{xx} = P_{xx} = M_{yy} - c_1 P_{yy} = 0 \),
\[ Q_x - c_1 R_x + c_1 \left( \frac{\partial P_{ax}}{\partial x} + \frac{\partial P_{ay}}{\partial y} \right) + N_y \frac{\partial w}{\partial x} = 0, \]

where \( N_{xx}, N_{yy}, M_{xx}, M_{yy}, P_{xx}, P_{yy}, Q_x, \) and \( R_x \) are the corresponding components of \( N_0, M_0, P_0, Q_0, \) and \( R_0 \).

Substituting Eq. (27) into Eq. (28), the natural frequency \( \omega \) can be obtained by letting the determinant of coefficients \( K_i(k = 1, 2, \ldots, 12) \) be equal to zero.

4. Numerical results and discussion

In this section, the material performance parameters of the GRC-FG board under in-plane load conditions are investigated. For all cases discussed below, the zigzag graphene sheets with geometric and material properties \( a_G = 14.76 \text{ nm}, b_G = b \text{ nm}, h_G = 0.188 \text{ nm}, \rho_G = 4118 \text{ kg/m}^3, \quad E_G^{11} = 1812 \text{ GPa}, \quad E_G^{22} = 1807 \text{ GPa}, \quad G_G^{12} = 683 \text{ GPa}, \quad \nu_G^{12} = 0.11 \) are selected as reinforcements. Poly (methyl methacrylate), referred to as PMMA, is selected for the matrix with the material properties \( \rho_m = 1150 \text{ kg/m}^3, \quad E_m = 2.5 \text{ GPa}, \quad \nu_m = 0.34 \). The graphene efficiency parameters \( \eta_j(j = 1, 2, 3) \) are obtained by matching the material properties of GRCs predicted from the Halpin-Tsai model to those from the MD simulations, which are shown in Figure 2.

First, the parameters of the isotropic plates under different boundary conditions and different plate aspect ratios are compared. First three order dimensionless frequency parameters, \( \bar{\omega} = \omega (b^2 / \pi^2) \sqrt{\rho h / D} \), are listed in Table 2. In contrast to the results in Table 2, the solution obtained by Liew et al. [16] also yielded very similar results, so the method used in this study was well validated.

Set the current in-plane load \( N_x \) and \( N_y \) values equal to \( P_e \), and the critical curved surface load to \( P_{cr} \). Figure 3 shows the variation of the critical in-plane buckling load of SSSS GRC-FG plates with different \( a / b \). Figure 4 shows the variation of the \( h / b \) and critical in-plane buckling load of the SSSS GRC-FG plate. Figure 5 shows the relationship between the different \( a / b \) and boundary conditions and the plane buckling load of the GRC-FG plate. Figure 6 shows the various in-plane buckling loads. It can be seen that as the plate becomes thicker and the constraint becomes stronger, the critical in-plane buckling load increases rapidly. Figure 7 show the first order non-dimensional frequency parameters, \( \bar{\omega} = \omega (b^2 / \pi^2) \sqrt{\rho h / E_m} \), for UD GRC-FG plates with different \( a / b \) and boundary conditions for \( P_e / P_{cr} = 0.5, 0 \) and 0.5. It can be seen that the dimensionless frequency parameter value is stronger than the \( S \) edge than the \( F \) edge at the constraint \( C \) edge, and the dimensionless frequency parameter decreases as the plate aspect ratio increases from 1.0 to 1.4.

If we consider FG-O and FG-X GRC-FG plates, Figure 8 and 9 give the first order non-dimensional frequency parameter, \( \bar{\omega} = \omega (b^2 / \pi^2) \sqrt{\rho h / E_m} \), for GRC-FG plates with different \( a / b \) and boundary conditions for \( P_e / P_{cr} = 0.5, 0 \) and 0.5. It can be seen that the compressive in-plane load \( P_e / P_{cr} = 0.5 \) can reduce the dimensionless frequency parameter of the GRC-FG plate, while the elongation in-plane load \( P_e / P_{cr} = 0.5 \) will increase the frequency.

Figure 10 presents the first order non-dimensional frequency parameters, \( \bar{\omega} = \omega (b^2 / \pi^2) \sqrt{\rho h / E_m} \), for UD GRC-FG plates with different \( h / b \) and boundary conditions for \( P_e / P_{cr} = 0.5, 0 \) and 0.5. It can be seen that as the value of \( h / b \) changes from 0.1 to 0.2, the dimensionless frequency parameter decreases.

Figure 11 and 12 show the corresponding first order non-dimensional frequency parameters, \( \bar{\omega} = \omega (b^2 / \pi^2) \sqrt{\rho h / E_m} \), for FG-O and FG-X GRC-FG plates with different \( h / b \) and boundary conditions for \( P_e / P_{cr} = 0.5, 0 \) and 0.5. It can be seen that the greater the stiffness near the top and bottom surfaces of the FG-CNT composite panel, the larger the dimensionless frequency parameter of the GRC-FG structure.
5. Conclusions
In the extended Halpin-Tsai micromechanical model, the segmented pattern graphene enhancement layer with different volume fractions in the thickness direction is considered, and the motion equation of the GRC-FG plate is obtained according to Reddy's third-order impedance deformation theory. The Hamilton principle and the Levy method were used to study the effects of different graphene distributions, plate thickness ratio width, plate aspect ratio and in-plane load on the mechanical properties of the material.

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Table 1. Comparison of non-dimensional frequency parameter, $\bar{\omega} = \frac{\omega(b^2)}{\pi^2} \sqrt{\rho h / D}$, for simply supported isotropic plate.

| Boundary conditions | $h/b$ | $a/b=1.0$ Mode sequences | $a/b=1.5$ Mode sequences |
|---------------------|-------|--------------------------|--------------------------|
|                     |       | 1 | 2 | 3 | 1 | 2 | 3 |
| SSSS                | 0.1   | Present | 1.928 | 4.593 | 4.593 | 1.407 | 2.642 | 4.1178 |
|                     |       | 6 | 4 | 4 | 3 | 0 |  |
|                     |       | Liew et al. [16] | 1.931 | 4.608 | 4.608 | 1.408 | 2.649 | 4.1303 |
|                     |       | 7 | 4 | 4 | 2 | 1 |  |
|                     | 0.2   | Present | 1.759 | 3.834 | 3.834 | 1.312 | 2.345 | 3.4825 |
|                     |       | 2 | 3 | 3 | 9 | 6 |  |
|                     |       | Liew et al. [16] | 1.767 | 3.865 | 3.865 | 1.316 | 2.361 | 3.5117 |
|                     |       | 9 | 6 | 6 | 4 | 2 |  |
| SCSC                | 0.1   | Present | 2.694 | 4.958 | 5.977 | 2.281 | 3.283 | 4.6285 |
|                     |       | 1 | 3 | 9 | 0 | 8 |  |
|                     |       | Liew et al. [16] | 2.702 | 4.976 | 5.999 | 2.350 | 3.256 | 4.9762 |
|                     |       | 1 | 2 | 2 | 0 | 2 |  |
|                     | 0.2   | Present | 2.260 | 4.016 | 4.528 | 1.898 | 2.743 | 3.8444 |
|                     |       | 9 | 8 | 2 | 3 | 0 |  |

Figure 1. (a) Schematic diagram of GRC-FG plate; (b) Configurations of GRC layers in the thickness direction.
Figure 2. Efficiency parameters for different volume fractions GRC-FG plates.

Figure 3. Variations of critical in-plane buckling loads for various types SSSS GRC-FG plates with different $a/b$.

Figure 4. Variations of critical in-plane buckling loads for various types SSSS GRC-FG plates with different $h/b$. 
Figure 5. (a) for UD, (b) for FG-O, (c) for FG-X. Critical in-plane buckling loads for various types GRC-FG plates with different $a/b$ and boundary conditions.
Figure 6. (a) for UD, (b) for FG-O, (c) for FG-X. Critical in-plane buckling loads for various types GRC-FG plates with different h/b and boundary conditions.
Figure 7. (a), (b), (c) for $P_r / P_{cr} = -0.5, 0, 0.5$. Non-dimensional frequency parameter, $\bar{\omega} = \omega(b^2 / h \pi^2)\sqrt{\rho / E}$, for UD GRC-FG plates with different $a/b$ and boundary conditions.
Figure 8. (a), (b), (c) for $P_i/P_{cr} = -0.5, 0, 0.5$. Non-dimensional frequency parameter, $\bar{\omega} = \omega (b^2 / h \pi^2) \sqrt{\rho / E}$, for FG-O GRC-FG plates with different $a/b$ and boundary conditions.
Figure 9. (a), (b), (c) for \( P_e / P_{cr} = -0.5, 0, 0.5 \). Non-dimensional frequency parameter,
\( \bar{\omega} = \frac{\omega b^2}{h \pi^2} \sqrt{\frac{\rho b^2}{E \pi^4}} \), for FG-X GRC-FG plates with different \( a/b \) and boundary conditions.
Figure 10. (a), (b), (c) for \( P_x / P_y = -0.5, 0, 0.5 \). Non-dimensional frequency parameter, 
\[ \tilde{\omega} = \omega \left( b^2 / h \pi^2 \right) \sqrt{\rho \pi / E} \], for UD GRC-FG plates with different \( h/b \) and boundary conditions.
Figure 11. (a), (b), (c) for $P_r / P_{cr} = -0.5, 0, 0.5$. Non-dimensional frequency parameter, \( \bar{\omega} = \omega \left( \frac{b^2}{h \pi^2} \right) \sqrt{\rho / E^m} \), for FG-O GRC-FG plates with different $h/b$ and boundary conditions.
Figure 12. (a), (b), (c) for $P_c / P_{cr} = -0.5, 0, 0.5$. Non-dimensional frequency parameter, $ar{\omega} = \omega (b^2 / h \pi^2) \sqrt{\rho / E}$, for FG-X GRC-FG plates with different $h/b$ and boundary conditions.