On the sources of the late integrated Sachs-Wolfe effect

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Abstract

In some scenarios, the peculiar gravitational potential of linear and mildly nonlinear structures depends on time and, as a result of this dependence, a late integrated Sachs-Wolfe effect appears. Here, an appropriate formalism is used which allows us to improve on the analysis of the spatial scales and locations of the main cosmological inhomogeneities producing this effect. The study is performed in the framework of the currently preferred flat model with cosmological constant, and it is also developed in an open model for comparisons. Results from this analysis are used to discuss the contribution of Great Attractor-like objects, voids, and other structures to the CMB anisotropy.

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1 INTRODUCTION

In some scenarios where linear and mildly nonlinear structures create a time varying gravitational potential, the photons of the Cosmic Microwave Background (CMB) undergo a late Integrated Sachs-Wolfe (ISW) effect. In the absence of any cosmological constant, the partial time derivative of the gravitational potential tends to zero as the universe approaches a flat one and, consequently, the ISW effect tends also to zero. This paper is devoted to the study of some aspects poorly known of the ISW effect: our goal is a detailed analysis of the locations and scales of the subhorizon structures contributing to this effect. We are particularly interested in the scales corresponding to observable objects as voids, the Great wall, et cetera and, by this reason, we will only consider scales smaller than the horizon. We choose an adequate formalism to deal with this analysis.

Two scenarios are considered: a flat universe with cold dark matter (CDM) and cosmological constant and an open universe with CDM, they are hereafter referred to as scenarios (or models) I and II, respectively. In both cases the spectrum has been first normalized by the condition $\sigma_8 = 1$, to consider other normalizations (other $\sigma_8$ values) when necessary. In case I, the spectrum corresponds to cold dark matter (CDM) with $\Omega_d = 0.25$, $\Omega_b = 0.05$, $\Omega_{\lambda} = 0.7$, $h = 0.65$ and $n = 1$, where $\Omega_b$, $\Omega_d$, and $\Omega_{\lambda}$ are the density parameters corresponding to baryonic matter, dark matter, and the cosmological constant, respectively, $h$ is the reduced Hubble constant ($h = H_0/100$, $H_0$ being the Hubble constant in units of $Km/s.Mpc$), and $n$ is the spectral index.
of the primordial scalar energy density fluctuations. The scenario II involves CDM and the relevant parameters are: \( \Omega_d = 0.25, \Omega_b = 0.05, h = 0.65 \) and \( n = 1 \). Model I is currently preferred according to recent observations of far Ia supernovae and the CMB spectrum (location of the Doppler peak), while model II can account for the abundances of rich clusters and Einstein’s rings and, here, it is mainly used for comparisons. As it is well known, the normalization \( \sigma_8 = 1 \) does not lead to a good normalization of the CMB angular power spectrum in most cases; in other words, when the \( C_\ell \) coefficients are calculated (for \( \sigma_8 = 1 \)), the resulting values do not fit well with the values observed by COBE, TENERIFE and other experiments. In each scenario, appropriate fits to the observed CMB spectrum correspond to \( \sigma_8 \) values which are, in general, different from unity. The so-called bias parameter is \( b = 1/\sigma_8 \).

Since our attention is focused on subhorizon scales, we will estimate the late ISW anisotropy in the \( \ell \)-interval \((10, 40)\). For \( \ell < 10 \), super-horizon scales would be important (see Kamionkowski and Spergel, 1994) and, then, the spatial curvature could be only neglected in model I; furthermore, the cosmic variance would lead to important uncertainties (\( \Delta C_\ell / C_\ell \) is proportional to \( [2/(2\ell+1)]^{1/2} \), see Knox, 1995) and the Sachs-Wolfe effect would be very important. For \( \ell > 40 \), the Doppler effect starts its domination hiding other effects as the ISW one. An appropriate linear approach is used in next sections to estimate the \( C_\ell \) coefficients for \( 10 \leq \ell \leq 40 \). The method used to do this estimation should facilitate the separation of the ISW effect from other contributions to the angular power spectrum and, moreover, this method should.
give information about the sizes and locations of the main subhorizon structures contributing to the late ISW effect. The numerical integration of the Boltzmann equation or the computational strategy of Hu and Sugiyama (1994) could be used to perform the analysis of this paper; nevertheless, another appropriate approach—based on a certain approximation to the sources—is described and used in next sections.

In previous papers, it was claimed that some Great Attractor-Like (GAL) objects located between redshifts 2 and 30 in open enough universes (without cosmological constant) could account for an important part of the Integrated Sachs-Wolfe (ISW) effect. Arguments in those papers were based on the Tolman-Bondi (TB) solution of Einstein’s equations, which was used to estimate both the anisotropy produced by a single GAL structure (Arnau, Fullana & Sáez, 1994; Sáez, Arnau & Fullana, 1995) and the abundance of these structures (Sáez & Fullana 1999). Unfortunately, our TB simulations have some features, as the spherical symmetry and a particular form of compensation, which could affect abundance and anisotropy estimations. By this reason, the mentioned claim should be discussed using a general formalism (not TB solution). It is done in Section 4 as a subsidiary application (in model II) of the formalism described along the paper.

In section 2, the method used to compute the angular power spectrum inside the \( \ell \) interval [10, 40] is described. Results are presented in Section 3 and, Section 4 is a general discussion and a summary of conclusions. Finally, some words about notation: whatever quantity ”A” may be, \( A_L \) and \( A_0 \) stand for the A values on the
last scattering surface and at present time, respectively. Symbols \( x^i, \phi, \bar{v}, \bar{n}, \rho_B, \delta, a, t, G \), stand for the comoving coordinates, the peculiar gravitational potential, the peculiar velocity, the unit vector in the observation direction, the background mass density, the density contrast, the scale factor, the cosmological time, and the gravitational constant, respectively. Units are chosen in such a way that the speed of light is \( C = 1 \). Quantities \( \Omega_0 \) and \( \Omega_m \) are defined as follows: \( \Omega_0 = \Omega_b + \Omega_d + \Omega_\lambda \) and \( \Omega_m = \Omega_b + \Omega_d \). The comoving wavenumber is \( k_c \), while \( k \) is the physical one.

2 \( C_\ell \) ESTIMATIONS IN THE \( \ell \) INTERVAL \([10,40]\)

We are interested in the ISW effect produced by structures much smaller than the horizon scale and, consequently, the region of the hypersurfaces \( t = \text{constant} \) where the inhomogeneities interact with the CMB can be considered as flat; namely, the spatial curvature can be neglected in the open model II. This means that, even in the open case, the spatial part of the functions defining the linear structures under consideration can be expanded in plane waves. It is not necessary the use of the complicated solutions of the Helmholtz equation, which should be used in open backgrounds to do an exact and rigorous treatment of structure evolution. Of course, in model II, the \emph{time evolution} is studied taking into account the existence of a space-time curvature distinguishing the open universe from the flat one.

The potential approximation is used in our estimates. The basic equations (Martínez-
González et al. 1994, Sanz et al. 1996) are:

\[
\frac{\Delta T}{T} = \frac{1}{3} \phi_L + \vec{n} \cdot \vec{v}_L + 2 \int_{t_e}^{t_o} dt \frac{\partial \phi}{\partial t},
\]

(1)

and

\[
\Delta \phi = 4\pi G \delta a^2 \rho_B ,
\]

(2)

where \( \frac{\Delta T}{T} \) is the relative temperature variation –with respect to the background temperature– along the direction \( \vec{n} \), and the integral is to be computed from emission \((e)\) to observation \((o)\) along the background null geodesics. Initially, this approach was designed to study the flat case (scenario I); nevertheless, the potential approximation can be also used in the open case (scenario II) provided that we are concerned with structures smaller than the horizon scale (Sanz et al. 1996). The first, second, and third terms of this equation give the Sachs-Wolfe, the Doppler and the ISW anisotropies, respectively. Hereafter, we write \( A^S \), \( A^D \), or \( A^I \) to indicate that the quantity \( A \) has been estimated using only the first, the second or the third term, respectively.

In the linear pressureless approach, the density contrast evolves as follows:

\[
\delta(x^i, t) = \frac{D_1(a)}{D_1(a_0)} \delta(x^i, t_0) ,
\]

(3)

where \( D_1(a) \) describes the evolution of the growing mode of the density contrast (Peebles, 1980). The form of the function \( D_1(a) \) is different in models I and II. Hereafter, \( D_{1q}(a) \) stands for the functions \( D_1(a) \) corresponding to our two scenarios. The same notation based on the subscript \( q \) is also used for other quantities. This
subscript $q$ is only used along the text to distinguish the model I ($q=I$) from the model II ($q=II$). Functions $D_{1q}(a)$ and all the quantities having the subscript $q$ are written in Appendix A for $q=I$ and $q=II$.

The peculiar gravitational potential can be written as follows

$$\phi(x^i, t) = \Phi(x^i) \frac{D_1(a)}{a},$$

where function $\Phi(x^i)$ satisfies the equation $\Delta \Phi = B_q \delta(x^i, t_0)$.

Let us focus our attention on the angular power spectrum of the CMB; namely, on the coefficients

$$C_\ell = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{m=\ell} \langle |a_{\ell m}|^2 \rangle.$$  \hspace{1cm} (5)

We begin with the contribution to these coefficients of the third term of the right hand side of Eq. (1), which corresponds to the ISW effect. An appropriate formula giving this contribution to the $C_\ell$’s has been derived. The most useful feature of this formula is that, given two $k$–values and two redshifts, it allows us to obtain a good measure of the ISW effect produced by the density perturbation located between the chosen redshifts and having scales between the chosen ones. A few words about the derivation–similar to the usual derivation of the $C_\ell$ coefficients of the Sachs-Wolfe effect– and characteristics of this formula are worthwhile.

Since the angular brackets in Eq. (5) stand for a mean performed from many realizations of the microwave sky, quantity $|a_{\ell m}|^2$ is first computed for an observer having comoving coordinates $x^i_\rho$ in a reference system attached to the Local Group (origin of spatial coordinates) and, then, an average over position $x^i_\rho$ is done to get
the $C_\ell$ quantities.

The equations of the null geodesics passing by the origin of spatial coordinates are

$$x^i = \lambda_q(a) e^i \ .$$

(6)

Furthermore, in flat cases (as model I), the null geodesics passing by point $x^i_p$ are:

$$x^i = x^i_p + \lambda_q(a) e^i \ ,$$

(7)

while in model II, this last equation is also valid when point P is well inside a sphere centred at the Local Group and having the size of the curvature scale. This is because—in such a case—the spatial curvature can be neglected.

Using the third term of the right hand side of Eq. (3), and Eqs. (3), (4) and (7), some Fourier expansions lead to the following relation

$$\Delta T \left( \vec{x}_p, \vec{n} \right) = \frac{2B_q}{(2\pi)^{3/2}} \int d^3k \ e^{-i\vec{k}_c \vec{x}_p} \frac{\delta_{\vec{k}_c}}{k^2} \int^Q d\vec{Q} e^{-i\lambda_q(a)\vec{k}_c \vec{n}} \frac{D_{1q}(a)}{a} \frac{d}{da} \left[ \frac{D_{1q}(a)}{a} \right] da ,$$

(8)

where the components of $\vec{x}_p$ are $x^i_p$, the observer located at P estimates $\Delta T$ in the direction $\vec{n}$, and point Q is the intersection between the last scattering surface of P and the null geodesic determined by $\vec{n}$.

Finally, from Eq. (8) plus the usual expansions in spherical harmonics and, after performing the average in $x^i_p$, the following angular power spectrum arises:

$$C_\ell^q = \Gamma^q \int \frac{P(k)}{k^2} \xi^2(k) dk ,$$

(9)

where

$$\xi^q(k) = \int^{a_L}_{a_0} j_\ell[\lambda_q(a)ka_0] \frac{d}{da} \left[ \frac{D_{1q}(a)}{a} \right] da .$$

(10)
Function $P(k) = \langle |\delta_k|^2 \rangle$ is the power spectrum of the energy density fluctuations, $j_\ell$ is the spherical Bessel function of order $\ell$, and coefficients $\Gamma_q^I$ are given in Appendix A for models I and II (all the $\Gamma$ coefficients appearing below are also listed in the same appendix).

A similar computation leads to the $C_\ell$ coefficients corresponding to the first and second terms of Eq. (1), which are usually referred to as Sachs-Wolfe and Doppler terms. In the Sachs-Wolfe case, these coefficients can be written as follows:

$$C_{\ell q}^S = \Gamma_q^S \int \frac{P(k)}{k^2} j_\ell^2 [\lambda_q(a)ka_0] dk$$

and, the coefficients of the Doppler term are

$$C_{\ell q}^D = \Gamma_q^D \int P(k) j_\ell' j_\ell^2 [\lambda_q(a)ka_0] dk ,$$

where $j_\ell'(x) = (d/dx)j_\ell(x)$.

Eqs. (11) – (12) are written in a form which is adequate to perform our numerical estimates. We define the functions $\mu_{\ell q}^I(k) = \Gamma_q^I k^{-2} P(k) \xi_{\ell q}^2(k), \mu_{\ell q}^S(k) = \Gamma_q^S k^{-2} P(k) j_\ell^2 [\lambda_q(a)ka_0],$ and $\mu_{\ell q}^D(k) = \Gamma_q^D P(k) j_\ell' j_\ell^2 [\lambda_q(a)ka_0],$ whose integrals in the variable $k$ give $C_{\ell q}^I, C_{\ell q}^S,$ and $C_{\ell q}^D,$ respectively. These definitions will be useful below.

If the three terms of the right hand side of Eq. (1) are simultaneously taken into account in order to get $\langle |a_{\ell m}|^2 \rangle$, the resulting $C_\ell$ quantities include three crossed contributions mixing the ISW, SW, and Doppler effects. We have not found fully convincing arguments to neglect these contributions in all the cases and, consequently,
they have been systematically estimated using the following formulae:

\[ C^{SD}_{\ell q} = -2 \int \left[ \mu^{S}_{\ell q}(k)\mu^{D}_{\ell q}(k) \right]^{1/2} dk , \quad (13) \]

\[ C^{SI}_{\ell q} = 2 \int \left[ \mu^{S}_{\ell q}(k)\mu^{I}_{\ell q}(k) \right]^{1/2} dk , \quad (14) \]

\[ C^{DI}_{\ell q} = -2 \int \left[ \mu^{D}_{\ell q}(k)\mu^{I}_{\ell q}(k) \right]^{1/2} dk . \quad (15) \]

Since the late ISW effect is produced by inhomogeneities evolving after decoupling, quantities \( C^{I}_{\ell q} \) can be estimated using the above pressureless approach for the sources; however, the Sachs-Wolfe and Doppler effects are produced by other inhomogeneities, which evolve in the recombination-decoupling period and, consequently, a certain radiation pressure is acting on the subdominant baryonic component. Taking into account that the importance of pressure effects increases as \( \ell \) does, we only apply our pressureless approach to calculate the Sachs-Wolfe, Doppler and crossed coefficients in the case \( \ell = 10 \). This calculation is performed with the essential aim of obtaining a rough estimate of the unknown crossed terms and, for this purpose, our approach suffices.

In the \( \ell \) interval [10,40], we can only expect significant contributions to the \( C_{\ell} \) coefficients coming from: (1) a possible background of primordial gravitational waves (this contribution would be almost independent on \( \ell \) in the interval under consideration and it is not studied here), (2) each of the three effects considered above and, (3) some crossed terms. Other effects as Sunyaev–Zel’dovich, lens anisotropy, nonlinear gravitational effects et cetera are not expected to be relevant for these angular scales, but for much smaller ones.
The sources of the term $C^I_{\ell q}$ have been identified in both scale and position using the following definitions:

$$D^I_{\ell q}(Z_{\min}, Z_{\max}) = \Gamma^I_{q} \int \frac{P(k)}{k^2} \zeta^2_{\ell q}(k; Z_{\min}, Z_{\max}) dk,$$

where

$$\zeta_{\ell q}(k; Z_{\min}, Z_{\max}) = a_0^{-1} \int Z_{\max}^{Z_{\min}} j_{\ell} \left[ \lambda_q \left( \frac{a_0}{1 + Z} \right) k a_0 \right] \frac{d}{dZ} \left[ D_{1\ell q} \left( \frac{a_0}{1 + Z} \right) (1 + Z) \right] dZ.$$  

(17)

For $Z_{\min} = 0$ and $Z_{\max} = Z_L$, where $Z_L$ is the redshift of the last scattering surface, functions $\zeta_{\ell q}(k; Z_{\min}, Z_{\max})$ and $D^I_{\ell q}(Z_{\min}, Z_{\max})$ are identical to $\xi_{\ell q}(k)$ and $C^I_{\ell q}$, respectively. Quantity $D^I_{\ell q}(Z_{\min}, Z_{\max})$ can be considered as a measure of the contribution to the ISW effect of the inhomogeneities lying between redshifts $Z_{\min}$ and $Z_{\max}$; nevertheless, it is worthwhile to emphasize that the right hand side of Eq. (16) involves the function $\zeta^2_{\ell q}(k; Z_{\min}, Z_{\max})$, which implies that the quantities $C^I_{\ell q} = D^I_{\ell q}(0, Z_L)$ are not the linear superposition of quantities of the form $D^I_{\ell q}(Z_{\min}, Z_{\max})$, even if these quantities are calculated in disjoint redshift intervals covering the total interval $(0, Z_L)$. From Eq. (16) it follows that the contribution of each scale to $D^I_{\ell q}(Z_{\min}, Z_{\max})$ is measured by the function $\nu^I_{\ell q}(k; Z_{\min}, Z_{\max}) = \Gamma^I_{q} P(k) \zeta^2_{\ell q}(k; Z_{\min}, Z_{\max})/k^2$. This function measures the contribution of the scale $k$ –for the inhomogeneities placed between redshift $Z_{\min}$ and $Z_{\max}$– to the late ISW effect. For $Z_{\min} = 0$ and $Z_{\max} = Z_L$, function $\nu^I_{\ell q}(k; Z_{\min}, Z_{\max})$ is identical to function $\mu^I_{\ell q}(k)$ and it weights the contribution of each scale –whatever the inhomogeneity location may be– to the ISW angular power spectrum.
Our calculations require a value of $Z_L$. Since the Sachs-Wolfe and Doppler effects are produced by inhomogeneities located very near the last scattering surface, estimates of $C_{\ell q}^S$ and $C_{\ell q}^D$ based on Eqs. (12) and (11) are sensitive to the value of $Z_L$; however, the late ISW effect is produced by inhomogeneities located far from this surface (see Section 3) and, consequently, it is almost independent on the assumed value of $Z_L$. In order to do the best estimate of the Doppler and SW effects for $\ell = 10$—allowed by our formalism— we have taken $Z_L = 1140$, which is the redshift corresponding to $\Omega_b = 0.05$ and $\Omega_d = 0.25$ according to the formula $Z_L \simeq 1100(\Omega_m/\Omega_b)^{0.018}$ (see Kolb & Turner 1994). Fortunately, we are focusing our attention on the late ISW effect, which is almost independent on the choice of $Z_L$.

3 RESULTS

Assuming the normalization $\sigma_8 = 1$, quantities $C_{\ell q}^I$, $C_{10q}^S$, $C_{10q}^D$, $C_{10q}^{SI}$, $C_{10q}^{SI}$, and $C_{10q}$ have been computed in models I ($q=I$) and II ($q=II$). For this first normalization, Quantity $[\ell(\ell+1)C_{\ell q}/2\pi]^{1/2}$ is shown in the left panel of Fig. 1 (for models I and II). The entries 1 and 2 of Table 1 gives $[110C_{10}/2\pi]^{1/2}$ for all the $C_{10}$ quantities. In this Table we see that: for $\ell = 10$ and model I, the ISW effect is smaller than the Doppler and SW ones, while for $\ell = 10$ and model II, the ISW and the SW effects are similar. A different normalization facilitates some comparisons of the anisotropies appearing in models I and II. We have observed that most theoretical predictions based on COBE normalization give $[\ell(\ell+1)C_{\ell}]^{1/2} \sim 28 \mu K$ for $\ell = 10$, with a small dispersion around
28 \mu K. This is true for a wide range of variation of the cosmological parameters: \( \Omega_0, \Omega_b \) et cetera. This condition is also compatible with all the observational evidences (FIRS, TENERIFE). By these reasons, the ISW effects corresponding to models I and II with the normalization \( [\ell(\ell + 1)C_\ell]^{1/2} = 28 \mu K \) for \( \ell = 10 \) are represented in the right panel of Fig. 1, where we see that: (i) the ISW effect corresponding to model I (with cosmological constant) is much smaller than that of the model II (very open universe), and (ii) the ISW effect of model I is small but it is not negligible.

Entries 3 and 4 of Table 1 correspond to the second normalization, for which, the bias parameter of model I (II) appears to be 0.93 (1.98). This means that the currently preferred model (with cosmological constant) leads to a very natural compatibility between the CMB observational data and the value \( \sigma_8 \sim 1 \) extracted from the analysis of galaxy surveys.

In scenario I, the greatest \( \ell = 10 \) crossed term is the SW–Doppler one (\( C_{10I}^{SD} \)), which is shown in the entries 1 and 3 of Table 1. The remaining crossed terms (SW-ISW and Doppler-ISW) are not given because they have appeared to be negligible. In entries 3 and 4 of Table 1, we give the SW-ISW crossed term (\( C_{10II}^{SI} \)) of the scenario II, which is not negligible; however, the terms Doppler–SW and Doppler–ISW can be neglected. As it follows from Eqs. (13) – (15), any crossed term is proportional to an integral (in the variable \( k \)), and the function to be integrated can be written as the product of two \( k \) functions corresponding to the mixed effects. For example, in the SW–Doppler (SW–ISW) case, we must integrate the product \( \left[ \mu_q^S(k) \right]^{1/2} \left[ \mu_q^D(k) \right]^{1/2} \).
\((\mu^{s}_{\ell q}(k))^{1/2}[\mu^{t}_{\ell q}(k)]^{1/2}\). In Fig. 2, we display the functions to be multiplied to get the SW-Doppler crossed term of model I (left panel) and the SW-ISW term of model II (right panel). We have assumed \(\ell = 10\) in both models, evidently, these crossed terms are not negligible as a result of the existence of a wide enough \(k\) interval where the positive functions to be multiplied take on large enough values simultaneously.

Where are located the inhomogeneities producing the ISW effect? This question can be answered using Eqs. (16) and (17) to calculate \(D_{\ell q}^{I}(0, Z_{max})\) for appropriate values of \(Z_{max}\). Results are shown in Fig. 3, where \(D_{\ell q}^{I}(0, Z_{max})\) is represented as a function of \(Z_{max}\) in models I (top) and II (bottom). The points of the horizontal straight lines of Fig. 3 have the ordinate \(C_{\ell q}^{I} = D_{\ell I}^{I}(0, 1140)\) and, consequently, the curves \(D_{\ell q}^{I}(0, Z_{max})\) must tend to the horizontal lines as \(Z_{max}\) tends to 1140. In the top panel, we see that \(D_{\ell I}^{I}(0, Z_{max})\) approaches the horizontal lines very quickly. Quantity \(D_{\ell I}^{I}(0, 2)\) is very similar to quantity \(C_{\ell q}^{I} = D_{\ell I}^{I}(0, 1140)\), which means that the most part of the late ISW is produced by inhomogeneities located at very low redshift \((Z \leq 2)\). This is because the cosmological constant is known to be significant only at very low redshifts; before, the universe can be considered as a flat one with a negligible cosmological constant, and no ISW effect is expected in this situation.

In model II (bottom panel), we see that \(D_{\ell II}^{I}(0, Z_{max})\) approaches the corresponding horizontal line more slowly than in model I. The most important part of the ISW effect is produced by inhomogeneities located at redshift \(Z < 10\), but inhomogeneities at \(Z > 10\) also produce a small but appreciable effect.
Now, let us look for the spatial scales contributing significantly to the ISW effect in the $\ell$ interval [10,40]. As stated before, the contribution of the scale $k$ to the ISW effect—for arbitrary location of the inhomogeneities—is weighted by the function $\mu_\ell^I(k)$. In Fig. 4, this function is represented with solid lines in cases I (top) and II (bottom). Left (right) panels correspond to $\ell = 10$ ($\ell = 40$). Taking into account that, for $h = 0.65$, the spatial size (diameter in the spherical case) of the structures associated to the wavenumber $k$ is $2k h^{-1} \text{ Mpc}$, Fig. 4 can be easily interpreted. In each panel, the solid lines show a $k$ value for which the function $\mu_\ell^I(k)$ reaches a maximum. The spatial scale corresponding to this $k$ value is hereafter denoted $D^*$. It is the most significant scale for ISW anisotropy production. Solid lines also show the existence of a minimum $k$ value where $\mu_\ell^I(k)$ starts to increase from negligible values. The spatial scale associated to the minimum will be denoted $D_{\text{max}}$. Scales larger than this maximum one do not contribute to the ISW effect significantly. The scales $D^*$ and $D_{\text{max}}$ corresponding to the four solid lines of Fig. 4 are given in Table 2. The meaning of these scales is discussed in next section.

Finally, among all the inhomogeneities located between redshifts $Z_{\text{min}} = 0$ and $Z_{\text{max}}$, which of them are contributing to the ISW effect? Which are the spatial scales of these inhomogeneities? In order to answer this question we have put $Z_{\text{min}} = 0$ and various values of $Z_{\text{max}}$ into Eq. (17); thus, we have found various functions $\nu_\ell^I(k,0,Z_{\text{max}})$ of the variable $k$. Only two of these functions are displayed in each panel of Fig. 4. The function corresponding to $Z_{\text{max}} = 1140$ is identical to $\mu_\ell^I(k)$.
and it is drawn with solid lines, while the dotted lines correspond to \( Z_{\text{max}} = 0.5 \) in model I (top panels) and to \( Z_{\text{max}} = 2 \) in model II (bottom panels). In Fig. 3, we see that for these \( Z_{\text{max}} \) values, a significant part of the ISW effect is produced by inhomogeneities located at \( Z < Z_{\text{max}} \). The dotted lines of Fig. 4 lead—as the solid lines—to new values of \( D^* \) and \( D_{\text{max}} \) which are presented in Table 2 and interpreted below. These scales correspond to inhomogeneities located at low redshifts smaller than the chosen values of \( Z_{\text{max}} \).

4 DISCUSSION AND CONCLUSIONS

In this paper, we have accurately estimated the scales and locations of the inhomogeneities contributing to the late ISW effect. The chosen formalism has facilitated our analysis. Now, let us focus our attention on the meaning of the resulting scales (see Table 2). They are not the scales of the inhomogeneities (density contrasts) producing the effect. According to Eq. (3), the ISW effect is produced by the scales contributing significantly to the partial time derivative of the peculiar gravitational potential and, in the linear regime under consideration, these scales are identical to those of the potential itself (see Eq. (4)).

The significant spatial scales of an overdensity and those of its peculiar gravitational potential are different, this is proved by the relation \( \phi_{\vec{k}} \propto \delta_{\vec{k}} / k^2 \) between the Fourier transforms of the density contrast \( \delta_{\vec{k}} \) and the peculiar potential \( \phi_{\vec{k}} \). The factor \( 1/k^2 \) implies that the regions where the potential is significant are more extended
than those where the density contrast is not negligible. What is the size of the regions where the potential is contributing to the late ISW effect? In order to answer this question let us consider a spherically symmetric overdensity. In such a case, Eq. (4) leads to the relation

\[ \frac{\partial \phi}{\partial r} \propto \frac{M(r)}{r^2}, \]  

(18)

where \( M(r) \) is the total mass inside a sphere of radius \( r \). This relation allows us to get various important features of the peculiar gravitational potential generated by a compensated overdensity. In fact, according to the cosmological principle, any overdensity must be compensated at some distance, \( r_c \), from its center, at which the total mass excess \( M(r_c) \) vanishes. This excess also vanishes for \( r > r_c \). Then, according to Eq. (18), the derivative \( \frac{\partial \phi}{\partial r}(r_c) \) vanishes for \( r > r_c \) and, consequently, the potential reaches a minimum constant value, which should be zero to achieve good boundary conditions at infinity. We see that the potential of a compensated structure tends to zero as \( r \) tends to \( r_c \); hence, all the shells forming a certain structure—up to compensation radius—would contribute to the ISW effect, although this contribution would be small for \( r \) values close to \( r_c \); hence, the late ISW effect produced by a given structure depends on the way in which it is compensated; namely, it depends on the size \( \sim 2r_c \) of the region where the potential is contributing to the ISW effect. This region is hereafter referred as to the pot-region associated to the inhomogeneity. Since we are considering linear scales where the peculiar velocities are proportional to the gradients of the peculiar gravitational potential, these velocities also vanish for
$r > r_c$ and, consequently, the pot-region contributing to the ISW effect is that where the peculiar velocities (potential gradients) are significant. A given overdensity would contribute to a certain $C_\ell$ coefficient, if the angular scale subtended by its pot-region (not the angular scale subtended by itself) is appropriate.

The compensation of cosmological objects is a statistical phenomenon and, consequently, structures of the same type (for example various GAL structures) could be compensated at different distances from their cores; therefore, although the compensation radius of the Great Attractor has been estimated to be $r_c^{GA} \sim 100h^{-1}\text{Mpc}$ (study of the velocity field around the GA), other GAL structures could compensate at other distances, perhaps at distances of a few times $r_c^{GA}$. Voids and Abell clusters would compensate at distances of a few tens of Mpc from the central region. Taking into account these considerations we are going to interpret the results summarized in Table 2.

For model I and $\ell = 40$ ($\ell = 10$), pot-regions with radius larger than $250h^{-1}\text{Mpc}$ ($1000h^{-1}\text{Mpc}$) are not contributing to the late ISW effect. Those having radius larger than $50h^{-1}\text{Mpc}$ ($200h^{-1}\text{Mpc}$) and located at $Z < 0.5$ do not contribute either. The maximum effect is produced by pot-regions with radius close to $100h^{-1}\text{Mpc}$ ($300h^{-1}\text{Mpc}$) and located between redshifts 0.5 and 2. and, finally, the maximum effect produced at $Z < 0.5$ is due to pot-regions having about $40h^{-1}\text{Mpc}$ ($140h^{-1}\text{Mpc}$) radius. This means that, in model I, GAL structures with $r_c \sim 100h^{-1}\text{Mpc}$ produce the maximum contribution to $C_40$ (the smallest of the $C_\ell I$ coefficients, see the top
panel of Fig. 1). GAL objects with sizes $r_c \sim 140h^{-1}$ would contribute to $C_{40}$ when located at $Z < 0.5$ and GAL objects with pot-regions of a few times $100h^{-1} \ Mpc$ would contribute to all the $C_{\ell I}$ coefficients from $\ell = 10$ to $\ell = 40$. Pot-regions with radius of $\sim 40h^{-1} \ Mpc$ and located at $Z < 0.5$ contribute to $C_{40}$.

In model II, a similar study has been developed. For $\ell = 40 \ (\ell = 10)$, the following conclusions can be obtained: (i) for arbitrary locations, the most large compensation radius contributing to the late ISW is $650h^{-1} \ Mpc \ (2500h^{-1} \ Mpc)$, (ii) for structures located at $Z < 2$, the most large radius is $125h^{-1} \ Mpc \ (500h^{-1} \ Mpc)$, (iii) for arbitrary locations, the most large contributions to the late ISW come from compensation radius of $300h^{-1} \ Mpc \ (1000h^{-1} \ Mpc)$, and (iv) for structures located at $Z < 2$, the most large contributions correspond to radius of $\sim 110h^{-1} \ Mpc \ (\sim 350h^{-1} \ Mpc)$. In model II, the scales are larger than those of model I. Scales of a few times $\sim 10h^{-1} \ Mpc$ do not play any role. GAL structures with $r_c \sim 100h^{-1} \ Mpc$ only contribute to $C_{40}$ if located at $Z < 2$. GAL objects compensated at radius around $300h^{-1} \ Mpc$ would play an important role in generating the $C_{\ell I}$ coefficients, in particular, for $\ell = 40$. This conclusion is in agreement with previous claims about the possible relevance of GAL structures in generating the late ISW effect in open universes (Arnau, Fullana & Sáez, 1994; Sáez, Arnau & Fullana, 1995). The GAL objects simulated in those studies (based on TB) undergo effective compensations at distances of a few hundred of Megaparsec from their cores and, in agreement with the results of this paper (Table 2), this type of structures would be contributing significantly to the late ISW effect.
It is worthwhile to emphasize that we have discussed the contribution to the late ISW effect of great cosmological structures (which produce peculiar velocities up to distances of tens or hundreds of Mpc). A linear approach suffices to estimate the potential (also the peculiar velocities) produced by these structures (GAL objects, voids et cetera). We have never considered the Rees-Sciama effect produced by strongly nonlinear substructures lying inside the Great Attractor and other extended inhomogeneities. Such an effect would produce CMB anisotropy on smaller angular scales and its estimate would require other nonlinear approaches.

APPENDIX A

Some quantities used in this paper have the subscript "q". Here, the explicit form of these quantities is given for $q = I$ (model I of Section 1) and $q = II$ (model II). We summarize the information as follows:

MODEL I ($q = I$)

The growing mode of the scalar energy density fluctuations is

$$ D_{1I}(a) = \frac{1}{x} \left[ \frac{2}{x} + x^2 \right]^{1/2} \int_0^x \left[ \frac{2}{y} + y^2 \right]^{-3/2} dy, $$

(19)

where

$$ x = \left[ \frac{2 \Omega_{\Lambda}}{\Omega_m} \right]^{1/3} (1 + Z)^{-1}. $$

(20)

The constant $B_q$ is

$$ B_I = -\frac{3}{2} \frac{\Omega_m H_0^2}{D_1(0)}. $$

(21)
The function $\lambda_q(a)$ can be written as follows:

$$\lambda_I(a) = \kappa(a)H_0^{-1},$$  \hspace{1cm} (22)

where

$$\kappa(a) = \int_a^1 \frac{db}{(\Omega_m b + \Omega_b b^4)^{1/2}}.$$  \hspace{1cm} (23)

Now, we give the coefficients $\Gamma^I_I$, $\Gamma^S_I$, $\Gamma^D_I$, $\Gamma^{SD}_I$, $\Gamma^{SI}_I$, and $\Gamma^{DI}_I$ defined in Section 2.

$$\Gamma^I_I = \frac{18H_0^4}{\pi} \left[ \frac{\Omega_m}{D_1(0)} \right]^2,$$  \hspace{1cm} (24)

$$\Gamma^S_I = \frac{H_0^4}{2\pi} \left[ \frac{D_1(L)\Omega_m(1 + Z_L)}{D_1(0)} \right]^2,$$  \hspace{1cm} (25)

$$\Gamma^D_I = \frac{2H_0^2}{\pi} \left[ \frac{D_1(L)}{D_1(0)} \right]^2 \Omega_m(1 + Z_L).$$  \hspace{1cm} (26)

In order to derive these formulae, the cosmological constant has been assumed to be negligible at $Z_L$ (see comments of Section 3) and, consequently, as $\Omega_m$ tends to unity, quantities $\Gamma^S_I$ and $\Gamma^D_I$ tend to the right values corresponding to a flat universe without cosmological constant ($H_0^4/2\pi$ and $2H_0^2/\pi(1 + Z_L)$).

MODEL II ($q = II$)

The same quantities as in model I are now listed:

$$D_{II}(a) = 1 + \frac{3}{\zeta} + \frac{3(1 + \zeta)^{1/2}}{\zeta^{3/2}} \ln[(1 + \zeta)^{1/2} - \zeta^{1/2}],$$  \hspace{1cm} (27)

where

$$\zeta = \frac{H_0(1 - \Omega_0)^{3/2}}{\Omega_0}a.$$  \hspace{1cm} (28)
\[ B_{II} = -\frac{3}{2} \Omega_0 \left[ H_0 D_1(a_0)(1 - \Omega_0)^{3/2} \right]^{-1}. \]  

\[ \lambda_{II}(a) = 2 \tanh(y/2), \]  

where

\[ y = \cosh^{-1}\left[ \frac{2 - \Omega_0}{\Omega_0} \right] - \cosh^{-1}\left[ \frac{2(1 - \Omega_0)^{3/2} H_0 a + 1}{\Omega_0} \right]. \]

\[ \Gamma_{II}^I = \frac{18 \Omega_0^2 H_0^2}{\pi (1 - \Omega_0) D_1^2(a_0)}, \]

\[ \Gamma_{II}^S = \frac{H_0^4 \Omega_0^2 (1 + Z_L)^2 D_1^2(a_L)}{2 \pi (1 - \Omega_0) D_1^2(a_0)}, \]

\[ \Gamma_{II}^D = \frac{2 \Omega_0}{\pi (1 - \Omega_0) D_1^2(0)(1 + Z_L)} \left( \frac{dD_1}{da} \right)_L^2. \]

As \( \Omega_0 \) tends to unity, function \( D_1(a) \) tends to the growing mode of a flat background, which is proportional to \( a \). Taking into account this fact and Eqs. (33) and (34), one easily concludes that –as the universe approaches a flat one– quantities \( \Gamma_{II}^S \) and \( \Gamma_{II}^D \) tend to \( H_0^4 / 2\pi \) and \( 2H_0^2 / \pi(1 + Z_L) \), respectively. These limit values coincide with the well known values of \( \Gamma_{II}^S \) and \( \Gamma_{II}^D \) corresponding to the flat background without cosmological constant.
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FIGURE CAPTIONS

FIG. 1.— Each panel shows the quantity \([\ell(\ell+1)C_{\ell}\pi^{1/2}]\) (in \(\mu K\)) as a function of \(\ell\). Left (right) panel corresponds to the normalization \(\sigma_8 = 1\) ([\(\ell(\ell+1)C_{\ell}\pi^{1/2} = 28\] \(\mu K\) for \(\ell = 10\)).

FIG. 2.— Left panel shows the quantity \(\mu_{10f}^{1/2} \times 10^5\) defined in the text as a function of \(k\) for the SW (solid) and Doppler (dotted) effects. In the right panel, quantity \(\mu_{10f}^{1/2} \times 10^4\) is plotted vs. \(k\) for the SW (solid) and ISW (dotted) effects. These functions must be multiplied to get the corresponding crossed contributions to the CMB angular power spectrum.

FIG. 3.— Top panel shows the quantity \(D_{\ell I}(0, Z_{\text{max}})\) defined in the text as a function of \(Z_{\text{max}}\) for \(\ell = 10\) (solid, not horizontal) and \(\ell = 40\) (dotted, not horizontal). These curves approach the horizontal lines of the same type as \(Z_{\text{max}}\) increases, crossing them at \(Z_{\text{max}} = 1140\). Bottom panel has the same structure, but it shows the quantity \(D_{\ell II}(0, Z_{\text{max}})\).

FIG. 4.— Top left: plot of \(\nu_{10f}(k, 0, Z_{\text{max}})\) (see text) vs. \(k\) for \(Z_{\text{max}} = 1140\) (solid) and \(Z_{\text{max}} = 0.5\) (dotted); top right: the same as in top left panel for the quantity \(\nu_{40f}(k, 0, Z_{\text{max}})\); bottom left: plot of \(\nu_{10f}(k, 0, Z_{\text{max}})\) vs. \(k\) for \(Z_{\text{max}} = 1140\) (solid) and \(Z_{\text{min}} = 2\) (dotted); and bottom right: the same as in bottom left for \(\nu_{40f}(k, 0, Z_{\text{max}})\).
TABLE 1

\[ \ell (\ell + 1) C_\ell / 2\pi^{1/2} \text{ (IN } \mu K) \text{ FOR } \ell = 10 \]

| MODEL | \( \sigma_8 \) | ISW | SW  | Doppler | SW–Doppler | SW–ISW | Total |
|-------|----------------|-----|-----|---------|-------------|--------|-------|
| I     | 1.00           | 6.75| 21.85| 12.63   | 1.25        | –      | 26.15 |
| II    | 1.00           | 38.18| 38.03| 12.63   | –           | 12.30  | 55.34 |
| I     | 1.07           | 7.23| 23.39| 13.52   | 1.34        | –      | 28.00 |
| II    | 0.51           | 19.32| 19.24| 6.39    | –           | 6.22   | 28.00 |
**TABLE 2**

PRESENT SIZES OF THE INHOMOGENEITIES PRODUCING THE LATE ISW EFFECT

| MODEL | $\ell$ | $Z_{max}$ | $D_{max}$ | $D^*$ |
|-------|-------|-----------|-----------|-------|
|       |       |           | $h^{-1}$ Mpc | $h^{-1}$ Mpc |
| I     | 10    | 1140      | 2000      | 600   |
| I     | 10    | 0.5       | 400       | 280   |
| I     | 40    | 1140      | 500       | 200   |
| I     | 40    | 0.5       | 100       | 85    |
| II    | 10    | 1140      | 5000      | 2000  |
| II    | 10    | 2.        | 1000      | 700   |
| II    | 40    | 1140      | 1300      | 600   |
| II    | 40    | 2.        | 250       | 220   |
