Damage detection of circular cylindrical shells by Ritz method

L Sarker¹, Y Xiang, B Uy and X Zhu
School of Engineering, University of Western Sydney
Locked Bag 1797, Penrith South DC NSW 1797, Australia
E-mail: l.sarker@uws.edu.au

Abstract. In this study, a new technique based on Ritz method is developed for damage detection of the circular cylindrical shell structures. Sander’s thin shell theory in association with the Ritz method is used to analyze the dynamic behaviour of circular cylindrical shells. By equating the strain and kinetic energy mathematically, the eigenvalue problem is solved which will give the natural frequencies of the circular cylindrical shell. The crack damage on the shell surface is modelled by a line spring along the circumference of the shell. Different damage scenarios are simulated by the changes of the crack locations and spring stiffness. The location and extent of the damage are predicted by the changes of modal parameters. Numerical simulations show that the method is effective and accurate to determine the crack damage in the cylindrical shell structures.

1. Introduction

Cylindrical shells are widely used in engineering in the form of structural components for pressure vessels, storage tanks, pipes, water ducts and process equipments. Loading and environmental attack all lead to the accumulation of damage in structures. In order to avoid the disastrous structural failures due to damages, it is very important to detect the damage in the very early stage of damage progression. In general, damage will result in a local reduction in stiffness which usually changes the dynamic behaviour of the structure, such as natural frequencies, mode shapes and damping ratios. Many research studies have been conducted to use the change of structural vibration properties for damage detection. Doebling et al (1998), Sohn et al (2000) and Brownjohn (2007) presented the literature review on the damage detection based on parameters such as natural frequencies, mode shapes, curvature, flexibility matrix and stiffness matrix. However, most of the researchers are working on the beam or plate structures, and only a few studies are on the shell structures.

Generally, a crack in a structure introduces local flexibility which usually changes the dynamic behaviour of the structure, and the usage of such changes could be a possible way to detect the crack. Rice and Levy (1972) modelled the crack as a line spring model. The spring stiffness is related to a specific crack depth and severity and determined by a localised flexibility matrix based on fracture mechanics. This model is widely used to analyse the dynamic behaviour of cracked beam and plate structures. There are limited studies on shell structures. Roytman and Titova (2002) developed an analytical mathematical model of the elastic oscillations of a cylindrical shell with surface closing cracks. Relay’s energy conservation method was employed in the derivation of the governing equation of motion of the system. Analytical solutions were obtained by application of Fourier transformation method. The effect of different crack models such as square, triangular, or spherical was discussed. Javidruzi et al. (2004) conducted finite element analysis on the vibration, buckling and dynamic stability of cracked cylindrical shell. The effect of crack length and orientation upon the vibration frequency of the cracked cylindrical shell was analysed. A numerical analysis of partially cracked cylindrical shell system was conducted by Mohan (1998) using the finite element analysis software ABAQUS. The line spring model was used to simulate the partial crack in the shell structure. The line-spring crack model essentially reduced a complex three dimensional cracked shell system problem into a two dimensional system by transforming the crack mechanism into a series of line spring
connections. It was also observed in the study that the results were much closer to the experimental results than that by other models. Recently Lee et al. (2006) presented a structural damage identification method based on frequency response function (FRF) data. The proposed method was very promising that it only required FRF data at damaged state. Li et al. (2007) analysed the vibrational power flow of an infinite cylindrical shell with a circumferential surface crack. The surface crack was modeled using the equivalent distributed line spring. The result showed that due to the presence of the crack the vibrational power flow changed substantially and the change is strongly related to the depth and position of the crack.

In this study the dynamic behaviour of circular cylindrical shell with circumferential surface crack is analysed using the Ritz method. The surface crack is modeled as the line spring that provides the continuity among the internal forces. Different damage scenarios are simulated by the changes of the crack locations and spring stiffness. The location and extent of the damage is predicted by the changes of modal parameters. Numerical simulations show that the method is effective and accurate to determine the crack damage in the cylindrical shell structures. This research has potential applications in damages detection for oil pipelines and other cylindrical shell-type structures.

2. Vibration analysis of shells by Ritz method

Consider a circular cylindrical thin shell of uniform thickness \( h \), radius \( R \), length \( L \), mass density \( \rho \), modulus of elasticity \( E \), Poisson’s ratio \( \nu \), and shear modulus \( G = E/[2(1 + \nu)] \). The geometry and coordinate system of the shell is given in Figure 1. Adopting Sander’s (1959) thin shell theory, the strain energy \( U \) of bending and stretching of the aforementioned cylindrical shell is given by (Leissa, 1973)

\[
U = \int_0^{2\pi} \int_0^L \left\{ \frac{Eh^3}{2(1-\nu^2)} \left[ \frac{\partial u}{\partial x} \right]^2 + \frac{1}{R^2} \left[ \frac{\partial v}{\partial \theta} \right]^2 + \frac{1}{R^2} \left[ \frac{\partial w}{\partial \theta} \right]^2 + \frac{2v}{R} \left[ \frac{\partial u}{\partial x} \right] \left[ \frac{\partial v}{\partial \theta} \right] + \frac{1-\nu}{2} \left[ \frac{\partial w}{\partial \theta} \right]^2 \right\} + \frac{Eh^3}{24(1-\nu^2)} \left( \frac{\partial^2 u}{\partial x^2} \right)^2 + \frac{1}{R^4} \left[ \frac{\partial^2 v}{\partial \theta^2} - \frac{\partial v}{\partial \theta} \right]^2 + \frac{2\nu}{R^2} \left( \frac{\partial^2 w}{\partial \theta^2} \right) \left( \frac{\partial^2 w}{\partial \theta^2} \right) \right\} R \theta dx
\]

where \( u, v, w \) = displacements in the longitudinal, tangential, and radial directions; \( x \) and \( \theta \) = longitudinal and circumferential coordinates, respectively. Neglecting the effect of rotary inertia since the shell under consideration is thin; the kinetic energy \( T \) of the cylindrical shell is given by

\[
T = \int_0^{2\pi} \int_0^L \left( \frac{1}{2} \rho h \left[ \frac{\partial u}{\partial \theta} \right]^2 + \left( \frac{\partial v}{\partial \theta} \right)^2 + \left( \frac{\partial w}{\partial \theta} \right)^2 \right) R \theta dx
\]

The Lagrangian functional \( F \) is the sum of the strain and the kinetic energy of the shell is

\[
F = U + T
\]
Assuming harmonic vibration, the following functions are adopted to separate the spatial variables \( x \), \( \theta \) and the time variable \( t \):

\[
\begin{align*}
    u(x, \theta, t) &= U(x) \sin n \theta e^{i\omega t} \\
    v(x, \theta, t) &= V(x) \cos n \theta e^{i\omega t} \\
    w(x, \theta, t) &= W(x) \sin n \theta e^{i\omega t}
\end{align*}
\]  

(4)

where \( n \) = number of circumferential waves; and \( \omega = \) circular frequency of vibration.

The Lagrangian function can be expressed as

\[
F = \frac{EhRe^{2i\omega}k^2}{2(1-v^2)} \int_0^1 \left( \left( \frac{\partial u}{\partial x} \right)^2 + \frac{1}{R^2} (nV-W)^2 - \frac{2v}{R} \left( \frac{\partial u}{\partial x} \right) (nV-W) + \frac{1}{2} \left( \frac{\partial V}{\partial x} + \frac{nU}{R} \right)^2 \right) + \frac{k^2}{12} \left( \frac{\partial^2 W}{\partial x^2} \right)^2 + \frac{1}{R^2} (n^2W-nV)^2 + \frac{2v}{R} \left( \frac{\partial^2 W}{\partial x^2} \right) (n^2W-nV) + \frac{2(1-v)}{R^2} \left( -n \frac{\partial W}{\partial x} + \frac{3 \partial V}{4 R} - \frac{nU}{4R} \right)^2 \right) dx
\]  

(5)

2.1. Geometric boundary conditions

For simply supported cylindrical shells, Sobel (1964) identified four kinds of boundary conditions that are designated as follows:

\[
S_1: W = V = 0; \quad S_2: W = 0; \quad S_3: W = U = 0; \quad S_4: W = U = V = 0
\]  

(6)

Similarly for clamped shell

\[
C_1: W = \frac{dw}{dx} = V = 0; \quad C_2: W = \frac{dw}{dx} = 0; \quad C_3: W = \frac{dw}{dx} = U = 0; \quad C_4: W = \frac{dw}{dx} = U = V = 0
\]  

(7)

2.2. Ritz functions

In view of satisfying the foregoing geometric boundary conditions, the applied Ritz functions for approximating the displacements are given by (Wang et al. 1994)

\[
\begin{align*}
    u &= \left( \sum_{i=1}^m p_i x^{i-1} \right) (x)^6 (L-x)^2 = \sum_{i=1}^m p_i u_i \\
    v &= \left( \sum_{i=1}^m q_i x^{i-1} \right) (x)^6 (L-x)^2 = \sum_{i=1}^m q_i v_i \\
    w &= \left( \sum_{i=1}^m r_i x^{i-1} \right) (x)^6 (L-x)^2 = \sum_{i=1}^m r_i w_i
\end{align*}
\]  

(8)

where \( m \) is the number of terms in the Ritz trial functions, \( p_i \), \( q_i \) and \( r_i \) are the unknown Ritz coefficients to be determined and the powers \( P_u, P_v \) and \( P_w \) may choose the values as shown in Table 1 to satisfy the simply supported and clamped edge support condition.

| Boundary Condition | Free End | \( S_1 \) | \( S_2 \) | \( S_3 \) | \( S_4 \) | \( C_1 \) | \( C_2 \) | \( C_3 \) | \( C_4 \) |
|-------------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| \( P_u \) | 0        | 0        | 0        | 1        | 1        | 0        | 0        | 1        | 1        |
| \( P_v \) | 0        | 1        | 0        | 0        | 1        | 1        | 0        | 0        | 1        |
| \( P_w \) | 0        | 1        | 1        | 1        | 1        | 2        | 2        | 2        | 2        |

The superscripts of \( P \), i.e., 0 and 1, denote the cylindrical shell ends at \( x=0 \) or \( x=1 \), respectively. Ritz polynomial functions are mathematically complete to ensure easy exact differentiation and integration.
for high accuracy. Even though the Ritz program developed in this study can obtain results for all simply supported and clamped edge support conditions as given in Eqs. (6) and (7), only the $S_1$ and $C_4$ support conditions are considered in this study.

2.3. Ritz method of analysis

To apply the Ritz method, the Lagrangian function needs to be differentiated with respect to each of the unknown functions $p_i$, $q_i$, and $r_i$ as shown below in Eq.(9).

\[
\frac{\partial F}{\partial p_i} = 0, \quad \frac{\partial F}{\partial q_i} = 0, \quad \frac{\partial F}{\partial r_i} = 0, \quad i = 1, 2, \ldots, m
\]  

(9)

Substituting Eq. (8) into Eq. (5) and then into Eq. (9) yields the following eigenvalue equation:

\[
[\mathbf{K}] - \omega^2[\mathbf{M}]\mathbf{C} = 0
\]  

(10)

where $[\mathbf{K}]$ and $[\mathbf{M}]$ = stiffness and the mass matrices of the cylindrical shell, respectively, and $\mathbf{C} = [p_1, q_1, \ldots, p_m, q_1, \ldots, q_m, r_1, \ldots, r_m]^T$ = the column vector of Ritz coefficients.

The matrix $[\mathbf{K}]$ may be expressed as

\[
[\mathbf{K}] = \begin{bmatrix}
K_{pp} & K_{pq} & K_{pr} \\
K_{qp} & K_{qq} & K_{qr} \\
K_{rp} & K_{rq} & K_{rr}
\end{bmatrix}
\]  

(11)

where

\[
(K_{pp})_{ij} = \frac{Eh}{(1 - \nu^2)} \int_0^L \frac{dU_i dU_j}{dx} dx + \frac{Eh}{(1 - \nu^2)} \left( \frac{1 - \nu}{2} \right) \frac{n^2}{R^2} \int_0^L U_i U_j dx + D \left( \frac{1 - \nu}{B} \right) \frac{n^2}{R^2} \int_0^L U_i dx
\]  

(12)

\[
(K_{pq})_{ij} = -\frac{Eh \nu}{(1 - \nu^2)} \int_0^L \frac{dU_i V_j}{dx} dx + \frac{Eh}{(1 - \nu^2)} \left( \frac{1 - \nu}{2} \right) \frac{n}{R} \int_0^L U_i V_j dx - 3 \frac{nD}{8} \int_0^L V_i dx
\]  

(13)

\[
(K_{pr})_{ij} = \frac{Eh \nu}{(1 - \nu^2)} \int_0^L \frac{dU_i W_j}{dx} dx + \frac{Eh \nu^2}{2} \int_0^L U_i \frac{dW_j}{dx} dx
\]  

(14)

\[
(K_{qq})_{ij} = \frac{Eh}{(1 - \nu^2)} \int_0^L V_i V_j dx + \frac{Eh}{2(1 - \nu^2)} \left( \frac{1 - \nu}{2} \right) \frac{n^2}{R^2} \int_0^L V_i V_j dx + D \left( \frac{1 - \nu}{B} \right) \frac{n^2}{R^2} \int_0^L dV_i dV_j dx
\]  

(15)

\[
(K_{qr})_{ij} = -\frac{Eh \nu}{(1 - \nu^2)} \int_0^L \frac{dU_i W_j}{dx} dx + D \frac{n^3}{R^3} \int_0^L \frac{dW_i}{dx} dx
\]  

(16)

\[
(K_{rr})_{ij} = \frac{Eh}{(1 - \nu^2)} \int_0^L W_i W_j dx + D \frac{n^2}{R^2} \left( \frac{d^2 W_i}{dx^2} \right) \int_0^L W_j W_j dx - D \frac{n^2}{R^2} \int_0^L \frac{d^2 W_i}{dx^2} \frac{d^2 W_j}{dx^2} dx
\]  

\[
+ 2D(1 - \nu) \frac{n^2}{R^2} \int_0^L \frac{dW_i}{dx} \frac{dW_j}{dx} dx
\]  

(17)

\[D = \frac{Eh^2}{12(1 - \nu^2)}\]
The matrix $[M]$ is given by

$$[M] = \begin{bmatrix} M_{pp} & 0 & 0 \\ 0 & M_{qq} & 0 \\ 0 & 0 & M_{rr} \end{bmatrix}$$  \hspace{1cm} (18)$$

where the elements of the $[M]$ matrix are

$$(M_{pp})_{ij} = \rho h \int_{0}^{L} U_{i} U_{j} dx$$  \hspace{1cm} (19)$$

$$(M_{qq})_{ij} = \rho h \int_{0}^{L} V_{i} V_{j} dx$$  \hspace{1cm} (20)$$

$$(M_{rr})_{ij} = \rho h \int_{0}^{L} W_{i} W_{j} dx$$  \hspace{1cm} (21)$$

3. Ritz formulation for cracked shells

The Ritz method is applied in the similar way described in the previous section, for the vibration analysis of the cracked cylindrical shell. The circumferential surface crack is modeled as a rotational spring and two parameters are used to describe the crack damage: spring stiffness and location. The stiffness parameter is related to the crack damage extent and the location is the crack position. Due to the introduction of circumferential crack on the shell surface, the shell could be divided into two segments for analysis. The strain and kinetic energy in both segments has been considered separately along with the strain energy of the spring. By equating the strain and kinetic energy in both segments along with the strain energy of the spring the eigenvalue problem is solved to get the modal parameters (natural frequency and mode shape) of the circular cylindrical shell.

3.1. Development of spring connection model

For developing the spring connection model the shell is divided into two segment connected by line spring as discussed earlier. The strain energy of the cracked shell is given by

$$U = U_{1} + U_{2} + U_{3}$$  \hspace{1cm} (22)$$

where $U_{1}$ = Total strain energy of the shell, $U_{2}$ = Strain energy of shell segment 1, $U_{3}$ = Strain energy of shell segment 2 and $U_{3}$ = Strain energy of connecting springs.

Kinetic energy of the shell is given by

$$T = T_{1} + T_{2}$$  \hspace{1cm} (23)$$

where, $T_{1}$ = Total kinetic energy of the shell, $T_{1}$ = Kinetic energy of shell segment 1 and $T_{2}$ = Kinetic energy of shell segment 2. The Lagrangian functional $F$ is the sum of the strain and the kinetic energy of the cracked shell i.e.

$$F = U + T = U_{1} + U_{2} + U_{3} + T_{1} + T_{2}$$  \hspace{1cm} (24)$$

The strain energy equation for the connecting spring is

$$U_{3} = \frac{1}{2h} \int_{x=x_{0}}^{x=x_{2}} \int_{0}^{2\pi} S_{uu\psi}(u_{1} - u_{2})^{2} d\theta \left|_{x=x_{0}}^{x=x_{2}} \right. + \int_{0}^{2\pi} \frac{1}{2} S_{uu\psi}(v_{1} - v_{2})^{2} d\theta \left|_{x=x_{0}}^{x=x_{2}} \right. + \int_{0}^{2\pi} \frac{1}{2} S_{uu\psi}(w_{1} - w_{2})^{2} d\theta \left|_{x=x_{0}}^{x=x_{2}} \right. + \frac{1}{2} \int_{x=x_{0}}^{x=x_{2}} \left( \frac{d\psi_{1}}{dx} - \frac{d\psi_{2}}{dx} \right) \frac{d\psi_{1}}{dx} \left|_{x=x_{0}}^{x=x_{2}} \right.$$

$$+ \frac{1}{2} \int_{x=x_{0}}^{x=x_{2}} \left( \frac{d\psi_{1}}{dx} - \frac{d\psi_{2}}{dx} \right) \frac{d\psi_{2}}{dx} \left|_{x=x_{0}}^{x=x_{2}} \right.$$

$$+ \frac{1}{2} \int_{x=x_{0}}^{x=x_{2}} \left( \frac{d\psi_{1}}{dx} - \frac{d\psi_{2}}{dx} \right)^{2} d\theta \left|_{x=x_{0}}^{x=x_{2}} \right.$$  \hspace{1cm} (25)$$
where \( u_1, v_1, w_1 \) = displacements in the longitudinal, tangential, and radial directions of the shell segment 1, \( u_2, v_2, w_2 \) = displacements in the longitudinal, tangential, and radial directions of the shell segment 2, \( s_{uuw} = c_{uuw}Eh^3/[12(1−v^2)R^3] \) = spring stiffness coefficient to enforce the continuity of \( u, v \) and \( w \) at the crack location, \( c_{uuw} \) = non-dimensional spring stiffness parameter, \( c_s Eh^3/[12(1−v^2)R] \) = rotational spring stiffness coefficient used at the crack location, \( c_s \) = non-dimensional spring stiffness parameter that varies the stiffness of the rotational spring connection, \( x_0 \) = position of the spring along the length of the shell.

Applying the Ritz method of analysis again discussed previously, we get the following eigenvalue problem.

\[
[[K_{cr}] - \omega_{cr}^2[M_{cr}]](C_{cr}) = [0]
\]

where, \([K_{cr}]\) and \([M_{cr}]\) = stiffness and mass matrix of the cracked cylindrical shell, \( \omega_{cr} \) = natural frequency of the cracked cylindrical shell, and

\[
[C_{cr}] = [p_{1r}, \ldots, \ldots, p_{m_1}, q_{1r}, \ldots, \ldots, q_{m_2}, r_{1r}, \ldots, \ldots, r_{m_2}]^T
\]

=the column vector of Ritz coefficients.

4. Results and discussions

The accuracy of the proposed method has been confirmed by direct comparison with exact solutions (Xiang and Zhang, 2006), as shown in Table 2. The shell model is analysed with shell length \( L=2m \), shell thickness \( h=0.005m \), shell radius \( R=0.1m \), Poisson’s ratio \( v=0.3 \), modulus of elasticity \( E=206832.4 \text{ MPa} \) and material density \( \rho=7826.4 \text{ kg/m}^3 \). The results show that natural frequencies of uncracked shells by the proposed method are quite close to the values obtained by Xiang and Zhang (2006). A program has been developed in MATLAB for vibration analysis and to determine the fundamental circumferential mode frequencies of different shell configurations. The data furnished by this model in both uncracked and cracked shell models to determine the effect of the circumferential surface crack upon the fundamental circumferential mode frequencies of specific shell system. The results generated from the uncracked and crack models are presented in Table 3. Different combination of boundary conditions have been analysed with the combination of the different circumferential wave number \( n \) to show the gradual changes in natural frequency in the shell system. The boundary conditions that have been analysed are Simply Support (S-S), Clamped-Clamped (C-C), Simply Support-Clamped (S-C) and Clamped-Free (C-F).

| Frequency Parameter, \( \Omega \) | Circumferential Wave | Exact Solution Xiang and Zhang (2006) | Proposed Method Polynomial terms |
|---|---|---|---|
| \( n \) | | | 4 | 6 |
| 1 | 0.016102 | 0.016107 | 0.016102 |
| 2 | 0.039271 | 0.039271 | 0.039271 |
| 3 | 0.109811 | 0.109811 | 0.109811 |
| 4 | 0.210277 | 0.210277 | 0.210277 |
| 5 | 0.339877 | 0.339877 | 0.339877 |

To determine a meaningful scale for the rotational spring stiffness parameter \( C_r \), an analytical analysis was conducted for the \( C_r \) parameter; the results are shown in Table 4. Considering different
boundary conditions the natural frequency parameter was found with a gradual change in the rotational spring stiffness parameter. The natural frequency parameter was analysed from $10^{-3}$ up to a rotational spring stiffness parameter value of $10^7$. As seen from Table 4, the natural frequency parameter remains unchanged when the rotational spring stiffness parameter value is in the range of $10^2$ and $10^7$. It is indicated that any value from $10^2$ to $10^7$ can be selected for the rotational spring stiffness parameter $C_r$. In this study the rotational spring stiffness parameter $C_r$ has been taken as $10^7$.

The two primary criteria for modal analysis is the natural frequency and mode shape. In the proposed method both these criteria has been used to detect the damage. The purpose of finding the natural frequency of a shell system is to understand the change in a single output value. In Table 2 the vibration analysis model has been validated as to ensure the accuracy of the natural frequency which also validates the effectiveness of the Ritz method and the program developed for the analysis.

**Table 3.** Uncracked ($\Omega$) and cracked frequencies ($\Omega_{cr}$) of shells subject to different support conditions. For cracked shell the crack is simulated at the middle (0.5xL) of the shell $[L=2m, h=0.005m, R=0.1m, \nu=0.3, E=206832.4 MPa, \rho=7826.4 kg/m^3, C_{uvw}=10^7, C_s = 10^7$ and polynomial terms=6].

| Circumferential Wave Number, n | Boundary Conditions | 1   | 2   | 3   | 4   | 5   |
|-------------------------------|--------------------|-----|-----|-----|-----|-----|
|                               | Uncracked frequencies ($\Omega$) |     |     |     |     |     |
|                               | S-S                | 0.016103 | 0.039271 | 0.109812 | 0.210277 | 0.339877 |
|                               | C-C                | 0.033425 | 0.040674 | 0.109982 | 0.210341 | 0.339920 |
|                               | S-C                | 0.024160 | 0.039790 | 0.109884 | 0.210306 | 0.339896 |
|                               | C-F                | 0.005869 | 0.038822 | 0.109563 | 0.209990 | 0.339556 |
|                               | Cracked frequencies ($\Omega_{cr}$) |     |     |     |     |     |
|                               | S-S                | 0.016103 | 0.039271 | 0.109812 | 0.210277 | 0.339877 |
|                               | C-C                | 0.033151 | 0.040665 | 0.109980 | 0.210353 | 0.339909 |
|                               | S-C                | 0.024067 | 0.039787 | 0.109883 | 0.210304 | 0.339893 |
|                               | C-F                | 0.005855 | 0.038820 | 0.109555 | 0.209969 | 0.339515 |

**Table 4.** Natural frequencies $\Omega$ of shell subjected to different boundary conditions and with different spring stiffness parameter $[n = 1, L=2m, h =0.005m, R=0.1m, \nu=0.3, E=206832.4 MPa, \rho=7826.4 kg/m^3, C_{uvw}=10^7$, polynomial terms=6 and spring position=0.5xL].

| Rotational Spring Stiffness Parameter, $C_s$ | Boundary Conditions | $10^{-3}$ | $10^{-2}$ | $10^{-1}$ | 1 | 10 | $10^2$ | $10^3$ | $10^4$ | $10^5$ | $10^6$ | $10^7$ |
|---------------------------------------------|---------------------|-----------|-----------|-----------|---|----|--------|--------|--------|--------|--------|--------|
| S-S                                         | 0.016103            | 0.01603   | 0.01603   | 0.01603   | 0.01603 | 0.01603 | 0.01603 | 0.01603 | 0.01603 | 0.01603 | 0.01603 |
| C-C                                         | 0.033141            | 0.033141  | 0.033140  | 0.033142  | 0.033148 | 0.033151 | 0.033151 | 0.033151 | 0.033151 | 0.033151 | 0.033151 |
| S-C                                         | 0.024066            | 0.024066  | 0.024066  | 0.024066  | 0.024067 | 0.024067 | 0.024067 | 0.024067 | 0.024067 | 0.024067 | 0.024067 |
| C-F                                         | 0.005855            | 0.005855  | 0.005855  | 0.005855  | 0.005855 | 0.005855 | 0.005855 | 0.005855 | 0.005855 | 0.005855 | 0.005855 |

One of the significant criteria for increasing the accuracy in the analysis by Ritz method is to increase the number of polynomial terms. From Table 2 it is found that increasing the number of polynomial terms leads to the accuracy of the results. In Table 3 the natural frequencies of the uncracked and cracked shells have been shown with a different combination of boundary conditions and circumferential wave number. From this table it is observed that the natural frequencies increase with the increase of the circumferential wave number. The data also has a close agreement between the cracked and uncracked shells. In Table 5 the effect of the crack on the uncracked vibration frequency has been shown when considering different position of the crack along the length of the shell. It shows the changes of the natural frequency as a percentage of the uncracked frequency. From Table 5 it can be seen that the changes are really very small to use it as a tool for detecting the damage in the shell.
But it still provides an understanding about the presence of an imperfection in the system. Due to the insensitiveness of natural frequency to the crack damage; a special focus has been given on mode shape analysis for damage detection of cylindrical shell.

A mode shape is a specific pattern of vibration executed by a structural system at a specific frequency. In this analysis the eigenvectors associated with the first vibration mode was used to determine the first mode shape. The mode shape and its derivatives have been used for locating the damage position. The parameters used in the analysis are circumferential wave number $n=1$, shell length $L=2m$, shell thickness $h=0.005m$, shell radius $R=0.1m$, Poisson’s ratio $\nu=0.3$, modulus of elasticity $E=206832.4$ MPa, material density $\rho=7826.4$ kg/m$^3$, rotational spring stiffness parameter $C_s=10^{-3}$ and spring stiffness parameter $C_{uvw}=10^7$. The reason for taking the rotational spring stiffness parameter $C_s=10^{-3}$ is to create a great difference in the stiffness of the continuity condition and rotation of the circular cylindrical shell.

Table 5. Effect of crack upon the uncracked fundamental mode frequency of various shell system \[n=1, L=2m, h=0.005m, R=0.1m, \nu=0.3, E=206832.4 \text{ MPa}, \rho=7826.4 \text{ kg/m}^3, C_{uvw}=10^7, C=10^7 \text{ and polynomial terms}=6].

| Boundary Conditions | Uncracked Frequency(Ω) | Crack Location |
|---------------------|-------------------------|-----------------|
|                     | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
| S-S                 | 0.016102 | 0.016103 | 0.016103 | 0.016103 | 0.016103 |
|                     | (-0.005%) | (-0.005%) | (-0.005%) | (-0.005%) | (-0.005%) |
| C-C                 | 0.033425 | 0.033179 | 0.033134 | 0.033148 | 0.033151 |
|                     | (-0.7351%) | (-0.8712%) | (-0.8518%) | (-0.8278%) | (-0.8188%) |
| S-C                 | 0.02416 | 0.024152 | 0.024135 | 0.024115 | 0.024093 |
|                     | (-0.0348%) | (-0.1035%) | (-0.1854%) | (-0.2790%) | (-0.3837%) |
| C-F                 | 0.005868 | 0.005842 | 0.005845 | 0.005845 | 0.005855 |
|                     | (-0.4465%) | (-0.4789%) | (-0.3903%) | (-0.2999%) | (-0.2215%) |

Note: Frequency parameter (frequency change in percentage)

The second derivative of mode shape \(\left(\frac{d^2W}{dx^2}\right)\) of simply supported and clamped shells have been plotted to detect the damage position. The second derivative is calculated numerically from the mode shape data which is actually the curvature of the mode shape. In Figure 2 the second derivative of mode shape of a simply supported shell is shown with the spring positioned at 0.1XL from the left support simulating the crack. From Figure 2 it is also seen that the peak along the curve (i.e. the second derivative of mode shape) has originated at point 0.2 along the X-axis and it is the only peak along the curve. The length \(L\) of the shell has been taken as 2m in the analysis; hence the damage position should be located at 0.1X2=0.2 along the X-axis. So the peak on the curve of Figure 2 confirms the position of the crack. Similar observation can be made from Figure 3 where the crack in the simply supported shell is simulated at the middle of the shell (0.5XL). A very faint peak occurs in this case but still indicating the damage position successfully.

In Figure 4 the second derivative of mode shape \(\left(\frac{d^2W}{dx^2}\right)\) of a clamped shell is shown with the spring positioned at 0.1XL from the left support simulating the crack. Again Figure 4 confirms that the peak along the curve has originated at point 0.2 along the X-axis and it is the only peak on the curve. The length \(L\) of the shell has been taken as 2m in the analysis; hence the damage position should be located at 0.1X2=0.2 along the X-axis. So the peak on the curve of Figure 4 confirms the position of the crack.
Similar observation can be made from Figure 5 where the crack in the clamped shell is simulated at the middle of the shell (0.5XL). But in this case the peak easily shows the damage position in the middle of the shell. As the peak is only generated on the curve at the position of the simulated crack on the shell surface, it can be used as a tool for locating the damage in a circular cylindrical shell.

5. Conclusions
A damage detection method based on modal analysis of circular cylindrical shells has been analysed with the application of Ritz method. The crack on the shell surface has been modeled by the application of the line spring method. Natural frequency and mode shape have been analysed with different combination of boundary condition to assess the effect of circumferential surface crack on these modal parameters. The natural frequency changes with different position of the circumferential crack along the shell length are very minimal to be used for damage detection. The second derivative of mode shape has been plotted for different types of shell systems which show that the presence of the crack produces a peak on the curve indicating the damage location. Further research for incorporating crack models based on fracture mechanics and wavelet analysis for damage detections in cylindrical shells will be carried out.

References
[1] Doebling, S.W., Farrar, C.R. and Prime, M.B. (1998) A summary review of vibration-based damage identification methods. Shock and Vibration Digest, 30:2, 91-105.
[2] Sohn, H., Czarnecki, J.A. and Farrar, C.R. (2000) Structural health monitoring using statistical process control. Journal of Structural Engineering ASCE, 126:11, 1356-1363.
[3] Brownjohn J.M.W. (2007) Structural health monitoring of civil infrastructure. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 365:1851, 589-622.

[4] Rice, J. R. & Levy, N. (1972) The Part-through Surface Crack in an Elastic Plate. *Journal of Applied Mechanics*, 39:1, 185-194.

[5] Roytman, A. & Titova, O. (2002) An Analytical Approach to Determining the Dynamic Characteristics of a Cylindrical Shell with Closing Cracks. *Journal of Sound and Vibration*, 254:2, 379-386.

[6] Javidruz, M., Vafai, A., Chen, J. F. & Chilton, J. C. (2004) Vibration, Buckling and Dynamic Stability of Cracked Cylindrical Shells. *Thin-Walled Structures*, 42:1, 79-99.

[7] Mohan, R. (1998) Fracture Analyses of Surface-Cracked Pipes and Elbows Using the Line-Spring/Shell Model. *Engineering Fracture Mechanics*, 59:4, 425-438.

[8] Lee, U. & Kim, S. (2006) Identification of Multiple Directional Damages in a Thin Cylindrical Shell. *International Journal of Solids and Structures*, 43:9, 2723-2743.

[9] Zhu, X., Li, T. Y., Zhao, Y. & Yan, J. (2007) Vibrational Power Flow Analysis of Thin Cylindrical Shell with a Circumferential Surface Crack. *Journal of Sound and Vibration*, 302:1-2, 332-349.

[10] Chondros, T. G., Dimarogonas, A. D. & Yao, J. (1998) A Continuous Cracked Beam Vibration Theory. *Journal of Sound and Vibration*, 215:1, 17-34.

[11] Dimarogonas, A. & Massourous, G. (1981) Torsional Vibration of a Shaft with a Circumferential Crack. *Engineering Fracture Mechanics*, 15:3-4, 439-444.

[12] Fernandez-Saez, J., Rubio, L. & Navarro, C. (1999) Approximate Calculation of the Fundamental Frequency for Bending Vibrations of Cracked Beams. *Journal of Sound and Vibration*, 225:2, 345-352.

[13] Leissa, A. W. (1973) *Vibration of Shells*, Scientific and Technical Information Office, National Aeronautics and Space Administration; [for sale by the Supt. of Docs., U.S. Govt. Print. Off.], Washington.

[14] Swaddiwudhipong, S., Tian, J. & Wang, C. M. (1995) Vibrations of Cylindrical Shells with Intermediate Supports. *Journal of Sound and Vibration*, 187:1, 69-93.

[15] Yokoyama, T. & Chen, M. C. (1998) Vibration Analysis of Edge-Cracked Beams Using a Line-Spring Model. *Engineering Fracture Mechanics*, 59:3, 403-409.

[16] Zhang, L., Xiang, Y. & Wei, G. W. (2006) Local Adaptive Differential Quadrature for Free Vibration Analysis of Cylindrical Shells with Various Boundary Conditions. *International Journal of Mechanical Sciences*, 48:10, 1126-1138.