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Hierarchical Load Tracking Control of a Grid-connected Solid Oxide Fuel Cell for Maximum Electrical Efficiency Operation

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Abstract: Based on the benchmark solid oxide fuel cell (SOFC) dynamic model for power system studies and the analysis of the SOFC operating conditions, the nonlinear programming (NLP) optimization method was used to determine the maximum electrical efficiency of the grid-connected SOFC subject to the constraints of fuel utilization factor, stack temperature and output active power. The optimal operating conditions of the grid-connected SOFC were obtained by solving the NLP problem considering the power consumed by the air compressor. With the optimal operating conditions of the SOFC for the maximum efficiency operation obtained at different active power output levels, a hierarchical load tracking control scheme for the grid-connected SOFC was proposed to realize the maximum electrical efficiency operation with the stack temperature bounded. The hierarchical control scheme consists of a fast active power control and a slower stack temperature control. The active power control was developed by using a decentralized control method. The efficiency of the proposed hierarchical control scheme was demonstrated by case studies using the benchmark SOFC dynamic model.
Keywords: Hierarchical control scheme; maximum electrical efficiency; nonlinear programming; solid oxide fuel cell

Nomenclature

Symbols

- $a$: Tafel constant
- $b$: Tafel slope
- $c(s)$: The $i^{th}$ controller
- $C_p$: J.mol$^{-1}$ K$^{-1}$, Molar constant-pressure heat capacity
- $E_0$: V, Nernst potential at standard pressure
- $E$: V, Nernst potential
- $F$: 96485 C.mol$^{-1}$, Faraday constant
- $g(s)$: Transfer function
- $h$: J.mol$^{-1}$, Molar enthalpy
- $I_{FC}$: A, Stack current
- $I_L$: A, Limiting current
- $k$: 3.762, Mole ratio of nitrogen to oxygen in the air
- $K_i$: mol.s$^{-1}$.Pa$^{-1}$, The $i^{th}$ gas valve molar constant
- $m$: Modulation index
- $m_sC_{ps}$: J.K$^{-1}$, Stack solid mass-specific product
- $N_0$: Cell number in series
- $N$: Mole number
- $p$: MPa, Stack operating pressure
- $P$: kW, Fuel cell output power
- $P_{loss}$: kW, Power loss caused by the air compressor
- $q$: mol.s$^{-1}$, Mole flow rate
- $R$: 8.314 J K$^{-1}$ mol$^{-1}$, Universe gas constant
- $T$: K, Temperature
- $u$: Fuel utilization factor
- $V$: m$^3$, Electrode volume
- $V_{dc}$: V, Cell terminal voltage
- $V_s$: V, Grid bus voltage

Greek letters

- $\alpha$: $\Omega$, Ohmic resistant constant
- $\beta$: $K$, Ohmic resistant constant
- $\delta$: rad, Phase shift angle
- $\eta$: SOFC electrical efficiency
- $\eta_c$: Air compressor efficiency
- $\rho$: Multiplicate model factor
1. Introduction

It is a trend to replace conventional power plants by the more environmentally-friendly distributed generators (DG) in order to reduce the greenhouse gas (GHG) emission from the power sector. Among the various types of DG, the high-temperature solid oxide fuel cell (SOFC) is one of the viable options due to its relatively high electrical efficiency of 45%-65% compared to typically 30-35% efficiency in conventional power plants [1,2]. Furthermore, high temperature reaction heat produced during the energy conversion process in the fuel cell (FC) stack permits an SOFC generator to be coupled to a gas turbine to form a combined heat and power (CHP) system, which can reach higher efficiency of up to 80% [3,4].

Various cell operating variables such as output power, stack temperature and fuel utilization factor, among others, do affect the thermodynamic, mass transfer, electrochemical and electrical processes within the SOFC in complex and intricate manners [5]. In the literature, researchers studied the possible effects of operating variables on the efficiency of different types of FC [6-8]. From these studies, it is shown that it is important but difficult to determine the optimal operating condition of the FC operation in order to achieve the maximum efficiency.

Therefore, like wind turbine, photovoltaic and other kind of renewable sources [9], a controller must be carefully designed in order to ensure that the SOFC power plant operates at the maximum efficiency for tracking the external power demand. Some references have made a comprehensive review of the SOFC modeling and control [10-12]. The SOFC dynamic models range from zero-dimensional (0-D) to three-dimensional (3-D). Both 2-D and 3-D models can be used for the cell geometrical design and thermal stress analysis [13-15]. These models are able to accurately represent the behavior of the FC at the expense of a heavy computational burden and are not suitable for power system studies. The 0-D and 1-D models are for the control purposes such as steady state and transient performance prediction and optimization. Most of the 1-D models are used for the stand-alone SOFC analysis. It was reported in [16] that the SOFC stack terminal voltage and temperature will reach a steady state value after a few seconds and tens of minutes respectively under the constant fuel...
utilization factor control scheme when the stack current has a step change. In order to mitigate the temperature excursion and extend the cell material lifespan, the excess air for cooling can be adjusted by a proportional-integral (PI) controller, a variable structure controller or a neural network predictive controller [17-19]. Komatsu et al studied the transient response of the SOFC for load tracking. The PI controllers considering the constraints of temperature, fuel utilization factor and steam-to-carbon ratio were proposed based on the feedback control [20]. It is shown that the response time of the stack terminal voltage and temperature after a step change of the dc output power is very close to what have been reported in [16]. 0-D SOFC models have been widely used for load tracking studies under both stand-alone and grid-connected conditions [21-23]. Such a lumped-parameter model can emulate the FC operations with acceptable accuracy only if certain strict assumptions are met, e.g. the fuel utilization factor is constant [11]. With regard to the control of a SOFC, model predictive control (MPC) [24] and adaptive control [25] can achieve multiple objectives during the load tracking process. Sendjaja and Kariwala [26] studied the use of decentralized proportional-integral-derivative (PID) controllers on the benchmark constant temperature SOFC dynamic model given in [21]. The same benchmark model was used in [27] and [28] to study the load tracking and small-signal stability issues pertaining to a grid-connected SOFC. As the stack temperature has been recognized to have significant impacts on the cell lifespan, some studies have improved the constant temperature model by including the energy balance equation. It was reported in [29-31] that the temperature can be maintained within a safe range by regulating the air flow rate. Vijay et al showed that the response of the stack temperature is in the order of several minutes when the stack current has a step change [30]. Some control schemes examined in [30] were found to be suitable for the decentralized controller design although the maximum electrical efficiency operations of the SOFC had not been considered. Bunin et al in [31] provided the experimental validation of a strategy to achieve the optimal efficiency operation of a stand-alone SOFC. The experimental results verified the simulation studies in [16] and [20]. Without considering the possible power losses consumed by the auxiliary devices, the optimal efficiency of the SOFC reported in [31] is between 40% and 50% over the power range. When the SOFC is connected to an external ac power system through a power control unit (PCU), the active power control of the PCU shall be taken into account as well in order to achieve the maximum efficiency load tracking operation and has not been studied.

The paper presents a maximum electrical efficiency load-tracking control scheme for the grid-connected SOFC in order to improve the operation performance. In Section 2, by using an existing benchmark SOFC dynamic model specifically developed for power system studies, the maximum efficiency of the SOFC can be obtained by solving a non-linear programming problem which is subject to a set of steady-state equality and inequality constraints. Next, the locations of the open-loop poles of the dynamic model lead to the proposed structure of the hierarchical control scheme shown in Section 3. In order to achieve the optimal operating state, a decentralized power and temperature control system is proposed and described in Section 4. The performance of the maximum electrical efficiency load tracking control scheme is illustrated through the case studies in Section 5, followed by the conclusions.

2. Determination of the Optimal Operating Condition of a Grid-connected SOFC Using Nonlinear Programming
In order to facilitate the analysis, the following assumptions are made,

1. Hydrogen rich nature gas is converted to hydrogen (H\textsubscript{2}) through the external-reforming fuel processor. Like in [22], the carbon oxide (CO) shift reaction is ignored in the analysis. Only pure H\textsubscript{2} is fed to the anode;
2. Oxygen in the air is used as the oxidant. The mole ratio of nitrogen (N\textsubscript{2}) to oxygen (O\textsubscript{2}) in the air is denoted as $k_c$ which is 3.762;
3. Both the fuel and air are preheated to the same temperature before they are transmitted to the cell stack. The detailed thermal management is not studied;
4. The cell stack is well-insulated and the energy losses caused by radiation, convection and conduction are negligible.

2.1. The SOFC Dynamic Model for Power System Studies

Padullés et al [21] developed one of the earliest SOFC stack models specifically for power system studies, and the model is shown within the dash lines in Figure 1. In this model, it is assumed the stack temperature $T$ is constant. Considering the cell stack tabular structure, the channels that transport the gases along the electrodes have a fixed volume, but their lengths are small. Hence it is sufficient to define one single pressure value in the cell stack interior. The exhaust of each channel is via a single orifice. The ratio of pressures between the interior and exterior of the channel is large enough and it can be assumed that the orifice is choked and the lumped-parameter model can be derived. Therefore, the mass balance equation, expressed in terms of the partial pressures $p_i$, is given as,

$$\frac{dp_i}{dt} = \frac{RT}{V_i} \left( q_i^{\text{in}} - q_i^{\text{o}} - q_i^{\text{r}} \right) = \frac{1}{\tau_i} \left( 1 - K_i \right)$$

where the subscript ‘$i$’ denotes either H\textsubscript{2}, O\textsubscript{2} or water (H\textsubscript{2}O), the superscript ‘$\text{in}$’, ‘$\text{o}$’ and ‘$\text{r}$’ denote the input, output and reaction variable, respectively, $R$ is the ideal gas constant, $V_i$ is the anode or cathode volume, and $q_i$, $K_i$ and $\tau_i$ are the $i^{th}$ gas mole flow rate, valve molar constant and time constant, respectively. Thus, $\tau_i$ can be written as,

$$\tau_i = \frac{V_i}{(K_i RT)}$$

According to the Faraday’s Law of Electrolysis, the reaction flow rates are,

$$q_i^{\text{H}_2} = 2q_i^{\text{O}_2} = -q_i^{\text{H}_2,\text{O}} = 2K_i I_{\text{FC}}$$

where $K_i = N_0/(4F)$, $N_0$ is the number of the cells connected in series in the stack, $F$ is the Faraday constant of 96485 C.mol\textsuperscript{-1}, and $I_{\text{FC}}$ is the stack current.

In order to improve the SOFC model, Zhu and Tomsovic included the dynamics of the electrochemical reaction and the fuel processor [22]. In Figure 1, these processes are represented by two first-order equations,

$$\frac{dI_{\text{FC}}}{dt} = (I_r - I_{\text{FC}}) / \tau_e$$
$$\frac{dq_{\text{H}_2}^{\text{in}}}{dt} = (q_{\text{fuel}}^{\text{in}} - q_{\text{H}_2}^{\text{in}}) / \tau_f$$
where $I_r$ is the reference stack current, $q_{\text{fuel}}^{in}$ is the natural gas input, $\tau_e$ and $\tau_f$ are the respective process time constants.

**Figure 1.** The benchmark SOFC dynamic model.

It can be seen from (1)–(2) that the partial pressures $p_i$ are dependent on the stack temperature $T$ which has to be carefully controlled as it can affect the cell efficiency, stack reliability and lifespan [5]. In order to improve the accuracy of the stack model, the dynamic behavior of the cell according to $T$ can be derived based on the energy balance principle [30]. The relevant equation is,

$$m_i C_{ps} \frac{dT}{dt} = \sum q_{in}^{in} (\bar{h}_i - \bar{h}_i^0) + \sum q_{in}^{o} \bar{h}_i^0 - P$$  \hspace{1cm} (6)

where $P$ is the SOFC dc output power, $m_i C_{ps}$ is the mass-specific heat product of the stack, $\bar{h}_i$ is the $i^{th}$ gas per mole enthalpy and it can be written as,

$$\bar{h}_i = \bar{h}_{i,\text{std}} + C_{ps} \Delta T$$  \hspace{1cm} (7)

In (7), $\bar{h}_{i,\text{std}}$ is the $i^{th}$ gas per mole enthalpy at the standard pressure of 0.1MPa and the standard temperature $T_{\text{std}}$ of 283K, $C_{ps}$ is the $i^{th}$ gas average constant-pressure specific heat and $\Delta T$ is the temperature change.

Substituting (3) and (7) into (6), (6) can be rewritten as,

$$\frac{dT}{dt} = \left( \frac{A}{B - T - P} \right) / \tau_T$$  \hspace{1cm} (8)

where

$$A = \left[ \overline{C}_{pH_2} q_{H_2}^{in} + (\overline{C}_{pO_2} + k_v \overline{C}_{pH_2}) d_{O_2}^{in} \right] T_{in} - K_f I_{FC} T_{\text{std}} \left( 2 \overline{C}_{pH_2} q_{H_2}^{in} + \overline{C}_{pO_2} - 2 \overline{C}_{pH_2} - 2K_f I_{FC} H_{LHV} \right)$$  \hspace{1cm} (9)
\[
B = \bar{C}_{pH_2}(q_{H_2}^{in} - 2K_i I_{FC}) + \bar{C}_{pO_2}(q_{O_2}^{in} - K_i I_{FC}) + 2K_i I_{FC}\bar{C}_{pH_2O} + k_e\bar{C}_{pO_2} q_{O_2}^{in}
\]  
(10)

\[
\tau_T = m_C P_{ps} / B
\]  
(11)

In (9), \(H_{LHV}\) has the low heat value of 241.83kJ if 1 mole of \(H_2\) is fully combusted to produce gaseous \(H_2O\) at the standard state [1], and \(T_{in}\) is the stack inlet gas temperature. As \(q_{H_2}^{in} > 2K_i I_{FC}\) and \(q_{O_2}^{in} > K_i I_{FC}\), \(B\) given by (10) is positive and it will increase with \(I_{FC}\). Therefore, the stack temperature time constant \(\tau_T\) shown in (11) will be minimal when the SOFC is at the highest load condition.

The response speed of the electrochemical, mass transfer and thermodynamic processes in the SOFC are characterized by the time constants \(\tau_e, \tau_i, \tau_f\) and \(\tau_T\). This observation is used in the design of the hierarchical control scheme.

### 2.2. Power Regulation of A Grid-connected SOFC

Figure 2 schematically shows a SOFC operating under the grid-connected condition. The PCU provides the required dc/ac interface. Typically, power electronics switching devices in the PCU are controlled using the Sinusoidal Pulse Width Modulation (SPWM) technique. The modulation index \(m\) and the phase shift angle \(\delta\) are the two control variables associated with this technique. The remaining “Control and Optimization Systems” parts are described in later sections.

Figure 2. Schematic diagram of a grid-connected SOFC power plant and the overall load tracking control system for maximum electrical efficiency operation.

Denote the terminal voltage of the SOFC stack as \(V_{dc}\) and the grid voltage as \(V_s\). The turns-ratio of the transformer is \(1:k_T\), and the transformer series impedance plus the linking feeder yield the equivalent reactance \(X\). Define \(k = \sqrt{3} / (2\sqrt{2})k_T\). The injected active and reactive power \((P+jQ)\) from the SOFC to the grid system is [27],

\[
P = mkV_{dc}V_s \sin \delta / X
\]  
(12)

\[
Q = (mkV_{dc}V_s \cos \delta - V_s^2) / X
\]  
(13)
From Figures 1 and 2, it is seen that the grid-connected SOFC power plant has four control variables, namely $q_{fuel}^{in}$, $q_{O_2}^{in}$, $m$ and $\delta$. As $Q$ is strongly dependent on $m$, $m$ is therefore often manipulated to allow the SOFC power plant to operate under constant voltage, constant reactive power or constant power factor operating schemes. The present investigation, however, focuses on the active power control. Accordingly, the SOFC power plant terminal is treated as a PV bus. In addition, if the switching losses in the PCU and in the feeder are ignored, $I_{FC}$ can be regulated through $m$ and $\delta$ [27],

$$I_{FC} = mkV_s \sin \delta / X$$

### 2.3. Operating Variables and Constraints

The operating variables of the grid-connected SOFC such as $T$, $V_{dc}$, $P$, $q_{H_2}^{in}$, $q_{O_2}^{in}$ and $I_r$ ($I_{FC}$) can be calculated based on the energy balance principle, Nernst equation and Figure 1, i.e. through solving the following equations,

$$\sum q_i^{in}(\overline{H_i}^{in} - \overline{H_i}^{o}) + \sum q_i^{o}\overline{H}_i^{o} = P$$

$$V_{dc} = N_0[E_0 + \frac{RT}{2F} \ln \left(\frac{p_{H_2}^{o}}{p_{H_2}^{in}} \left(\frac{p_{O_2}^{o}}{p_{O_2}^{in}}\right)^{0.5}\right) - V_{act} - V_r - V_{con}]$$

$$P = V_{dc}I_{FC}$$

where $E_0$, the ideal standard potential, is a function of $T$ [30],

$$E_0 = 1.2856 + 0.000252T$$

In (16), $p_0$ is the standard pressure. $V_{act}$, $V_r$ and $V_{con}$, as shown in (19)–(21), are activation loss, Ohmic loss and concentration loss, respectively [1]. The detailed definitions of the parameters and their typical values are given in Nomenclature and Table 1.

$$V_{act} = a + b \log I_{FC}$$

$$V_r = \alpha \exp[\beta(1/T_{in} - 1/T)]I_{FC}$$

$$V_{con} = RT/(2F).\ln(1-I_{FC}/I_L)$$

Among the six steady-state operating variables $T$, $V_{dc}$, $P$, $q_{H_2}^{in}$, $q_{O_2}^{in}$ and $I_r$ ($I_{FC}$), if any three of them are given, the other three can be obtained by solving the nonlinear equations (15)-(17).

The cell lifespan and performance are dependent on the operating parameters. Therefore, three operating constraints must be respected for the safe operation of the cell. The most important operating constraint is the fuel utilization factor $u$, given as,

$$u = 2K_sI_{FC} / q_{H_2}^{in}$$

$$u_{min} \leq u \leq u_{max}$$

Typically $u_{min} = 0.7$ and $u_{max} = 0.9$ [21].

The other two operating constraints are $T$ and $P$,

$$T_{min} \leq T \leq T_{max}$$

$$P_{min} \leq P \leq P_{max}$$
\[ P_{\text{min}} \leq P \leq P_{\text{max}} \]  

Typically, \( T_{\text{min}} = 1173K, T_{\text{max}} = 1273K, P_{\text{min}} = 0.1\text{pu} \) and \( P_{\text{max}} = 1\text{pu} \) of the SOFC rated power [1,23].

### 2.4. Determination of the Optimal Operating Condition

The electrical efficiency \( \eta \) of the hydrogen SOFC is defined as the ratio of the net power to the total power obtainable by burning \( H_2 \) at the standard state [1],

\[ \eta = \frac{(P - P_{\text{loss}})}{(q_{\text{H}_2}^{\text{in}} H_{\text{LHV}})} \]  

However, if the stack operating pressure \( p \) is higher than 0.1MPa, not all the power generated by the SOFC will be delivered to the external circuit. The parasitic losses \( P_{\text{loss}} \) is dominated by the air compressor in the form [1],

\[ P_{\text{loss}} = C_{\text{par}} T_{\text{std}} (p^{0.286} - 1)(1 + k_c)q_{\text{O}_2}^{\text{in}} / \eta_c \]  

where \( \eta_c \) is the equivalent efficiency of the air compressor.

Under a given pressure \( p \), it can be seen from (26) and (27) that \( \eta \) is the function of \( P, q_{\text{H}_2}^{\text{in}} \) and \( q_{\text{O}_2}^{\text{in}} \).

There is one set of operating variables which enables the SOFC to operate at the maximum electrical efficiency \( \eta_{\text{max}} \). In order to optimize \( \eta \), a nonlinear programming problem (NLPP) is formulated as follows,

Objective function

Maximize \( \eta \)  

Subject to

Equality constraints (15)-(17)
Inequality constraints (23)-(25)

The optimization is obtained by treating \( P, q_{\text{H}_2}^{\text{in}} \) and \( q_{\text{O}_2}^{\text{in}} \) as the decision variables in the NLPP.

Numerical optimization software packages such as that provided by MATLAB can be used to search \( \eta_{\text{max}} \). At the end of the NLPP search, the optimal set of \( T, V_{dc}, q_{\text{in}}^{\text{in}}, q_{\text{in}}^{\text{out}}, I_r, (I_{FC}) \) as well as \( u \) for a targeted \( P \) will be obtained and pre-stored in a look-up table as the reference input signals for the SOFC load tracking control system.

### 3. Hierarchical Load Tracking Control Scheme for the Grid-connected SOFC

With the optimal operating condition of the SOFC determined by the NLPP, the load tracking control scheme for the grid-connected SOFC can be developed to track the power demand and operate at \( \eta_{\text{max}} \).

The open-loop poles of the SOFC are analyzed to study the dynamic response of the SOFC and a hierarchical control scheme for the SOFC is proposed based on the dynamic response analysis.

#### 3.1. Analysis of the Open-loop System Poles
In the load tracking control scheme for the SOFC, the internal dynamics of the PCU can be neglected as the typical response time of the PCU is a few milliseconds. The load tracking speed of the SOFC will be dominated by the dynamic response of devices on the dc side of the power plant where the typical time constants are of the order of 1s or larger.

The response characteristics of the SOFC operating at the maximum efficiency can be assessed by examining the locations of the six open-loop poles of the dynamic model shown in Figure 1. The $i^{th}$ pole can be calculated as,

$$s_i = -1/\tau_i$$  \hspace{1cm} (30)

The electrochemical reaction and fuel processor contribute to the poles $-1/\tau_e$ and $-1/\tau_f$. They are independent of $P$ and $T$. However, the location of the poles $-1/\tau_{H_2}, -1/\tau_{O_2}, -1/\tau_{H_2O}$ and $-1/\tau_T$ may change when the SOFC operates at different power levels. As shown in (2) and Figure 3, three gas time constants are the function of $T$ but independent of $I_{FC}$. Therefore, $-1/\tau_{H_2}, -1/\tau_{O_2}$ and $-1/\tau_{H_2O}$ will be away from the origin when $T$ increases. On the other hand, from (10) and (11), $\tau_T$ is seen to be inversely proportional to $I_{FC}$. Thus, the remaining pole $-1/\tau_T$ is directly proportional to $I_{FC}$ or $P$.

**Figure 3.** Illustrative of the positions of the six poles of the dynamic model

When the SOFC operates at $\eta_{\text{max}}$, the poles can be divided into two groups: plotted in Figure 3, the distance of the pole $-1/\tau_T$ to the imaginary axis is at least six times smaller than that of the other five poles. The observation on the locations of the open-loop poles shall be used to develop the structure of the hierarchical load tracking and temperature control scheme for the SOFC.

### 3.2. The Hierarchical Load Tracking Control Scheme

Figure 2 shows that $q_{\text{fuel}}$, $q_{\text{O}_2}$, $m$ and $\delta$ can be used to regulate the SOFC output active and reactive power. In the design of a control system for a multi-input-multi-output plant, it is desirable that the structure of the control system is selected in such a way that possible interactions between the control loops is minimized. The modulation index $m$ of the PCU can be used to control the bi-directional reactive power flow to the grid and this can be accomplished in a few milliseconds. Therefore, among the four control variables, $m$ can be treated as a quasi-steady state variable during the load tracking process because the electrochemical, mass transfer and thermodynamics processes usually takes a much longer period.

Based on the observation on the locations of the open-loop poles, it can be concluded that the pole $-1/\tau_T$ essentially governs the dynamics of the stack temperature $T$ whereas the load tracking process is dominated by the remaining poles. The $T$ control typically lasts for tens of seconds. Therefore, $T$ can be assumed to be constant for the load tracking operation. On the other hand, by adopting the practice of [29-31] in which $O_2$ was used as a coolant to regulate $T$, the oxygen flow rate $q_{O_2}^a$ can be
manipulated such that \( T \) is maintained at the optimal value to realize the SOFC \( \eta_{\text{max}} \) operation. It therefore results in a single-input-single-output (SISO) stack temperature control scheme, denoted as the \( T \) control system in this paper. With the \( T \) control in place, the remaining two control variables \( q_{\text{fuel}}^m \) and \( \delta \) can be utilized to perform the load tracking of \( P \) while maintaining \( u \) at the optimal value. This strategy leads to a two-input-two-output \( P \) control system.

In summary, as shown in Figure 4, a hierarchical control structure is proposed to achieve both the maximum electrical efficiency operation and stack temperature control of the SOFC when the FC tracks the power demand. The structure is based on the inherent differences in the speeds of response of \( P \), \( u \) and \( T \) of the SOFC to the demand changes. \( P \) and \( u \) can be controlled by regulating the control variables \( q_{\text{fuel}}^m \) and \( \delta \) while \( T \) is to be controlled through regulating \( d_{O_2} \), subject to the operating constraints (23)-(25). The proposed hierarchical control structure is more comprehensive, in comparison with the on-line load tracking scheme shown in [27] in which \( T \) is assumed constant.

Figure 4. Diagram of the hierarchical control scheme

4. Design of the \( P \) and \( T \) Control Systems

The detailed design procedure of the \( P \) and \( T \) control systems is described in this section.

4.1. SOFC Dynamic Model for the Design of \( P \) Controller

According to Section 3.2, \( T \) can be assumed constant during the \( P \) control process. Therefore, the nonlinear model given in Figure 1 can be linearized around the plant initial operating state. For the convenience of the analysis and controller design, the plant variables are normalized in the following way. The values of the state variables \( q_{\text{fuel, max}}, u_{\text{max}}, P_{\text{max}}, \delta_{\text{max}} \), which correspond to the operating condition when the SOFC operates at the maximum \( P \), are selected as the base for the normalization. The normalized output-control model shall be of the form
where \( y = [\Delta P, \Delta u]^T \) and \( w = [\Delta \delta, \Delta q_{fuel}^{in}]^T \) are the small deviations of the output and control variables. \( G_{r}(s) \) is the transfer function determined by taking the small signal form of (1), (4), (5), (14), (16)-(22). Algebraic manipulation shall yield the following expressions for the various elements in \( G_{r}(s) \):

\[
g_{py}(s) = \frac{RT_0 u_0}{1-u_0} \frac{1}{1+\tau_e s} \frac{q_{fuel,\text{max}}^{in}}{P_{\text{max}}} \]

\[
g_{u0}(s) = \frac{u_0 \cdot \delta_0 \delta_{\text{max}}}{1+\tau_e s} \frac{\delta_{\text{max}}}{u_{\text{max}}} \]

\[
g_{uf}(s) = \frac{-u_0^2}{2K_r I_{r,0}} \frac{1}{1+\tau_e s} \frac{q_{fuel,\text{max}}^{in}}{u_{\text{max}}} \]

where

\[
g_{E}(s) = -\frac{N_o R T_0}{2F} \left( \frac{2K_r}{q_{fuel,0}^{in} - 2K_r I_{r,0} } \frac{1}{1+\tau_{H_2} s} + \frac{1}{I_{r,0}} \frac{1}{1+\tau_{H_2} s} + \frac{K_r}{2(q_{\text{O}_2,0}^{in} - K_r I_{r,0})} \frac{1}{1+\tau_{\text{O}_2} s} \right) \]

\[
g_{\gamma}(s) = \frac{b}{I_{r,0}} + \alpha \exp[\beta(\frac{1}{T_{\text{in}}} - \frac{1}{T_0})] - \frac{RT_0}{2F} \frac{1}{I_{L} - I_{r,0}} \]

The subscript ‘0’ in (32)-(37) indicates the initial value of the respective variables when the SOFC operates at \( \eta_{\text{max}} \).

### 4.2. Selection of P Control Output-input Variables Pairs

Although there are many methods of designing a control system for a general two-input-two-output plant, the decentralized control is a widely used approach. The advantages of the decentralized control include hardware simplicity, operation flexibility, and the relative ease in the controller design and tuning. However, the dynamic performance of the resulting two SISO sub-systems may be degraded by any unaccounted interactions between the two control loops. Therefore, in order to design a feasible and robust controller, an important step is to determine the most suitable two output-input variable pairs for the two SISO sub-systems.

The relative gain array (RGA) is an established technique to measure the steady-state interactions between multiple SISO loops [32]. In the design of the P control system, there are two possible selections of output-input variable pairs: the \((P - \delta, u - q_{\text{fuel}}^{in})\) pair and the \((P - q_{\text{fuel}}^{in}, u - \delta)\) pair. The most suitable output-input variables pair shall be examined by observing the relative steady state gain between the inputs and outputs. Define the RGA matrix \( \Lambda \) of the plant (31) as the Hadamard product of \( G_{r}(0) \) and its inverse transposition,

\[
\Lambda = G_{r}(0) \odot G_{r}^{-T}(0) \]
With the typical values given in Table 1 and the SOFC operating at \( \eta_{\text{max}} \), the variations of each element of \( \Lambda \) are shown in Figure 5. It is shown that the values of the off-diagonal elements \( \lambda_{12} \) and \( \lambda_{21} \) are closer to 1 compared to that of the diagonal elements \( \lambda_{11} \) and \( \lambda_{22} \), particularly under heavy load conditions. This means the selection of the output-input pair \((P - q_{\text{fuel}}^\text{in}, u - \delta)\) will be more suitable because the interactions of the \( P - \delta \) and \( u - q_{\text{fuel}}^\text{in} \) loops are smaller and decreases as \( P \) increases. Therefore, \((P - q_{\text{fuel}}^\text{in}, u - \delta)\) were selected as the output-input variable pairs when designing the \( P \) control system.

**Figure 5.** Variations of the values of \( \Lambda \) elements with \( P \).

### 4.3. Design of the Decentralized \( P \) Controller

Figure 6(a) shows the \( P \) control block diagram where an input variable with the subscript ‘\( \text{ref} \)’ denotes its reference value. The figure has been configured to reflect the outcome of the pair selection described in the previous sub-section, i.e. the adoption of the \((P - q_{\text{fuel}}^\text{in}, u - \delta)\) output-input variable pairs. The system of Figure 6(a) is then split into two independent SISO systems, with each SISO having the structure shown in Figure 6(b). The so-called multiplicative model factor (MMF) is utilized to account for the loop interactions between the two SISO systems. In Figure 6(b), \( c_i(s) \) is the respective controller where the subscript “\( i \)” denotes either \( P \) or \( u \). The design method for \( c_i(s) \) can be summarized as follows.

**Step One:** Design the \( c_i(s) \) controllers without considering loop interactions. Suppose the controller \( c_i(s) \) in Figure 6(b) is the PID type and is tuned using the simple internal mode control (SIMC) method described in [32]. Thus for a second-order system \( g_i(s) \) with a dc-gain \( k_i \) and a time delay \( \theta_i \):

\[
g_i(s) = k_i e^{-\theta_i} [(\tau_i s + 1)(\tau_i s + 1)]^{-1}; \quad \tau_i > \tau_i; \theta_i > 0 \tag{39}
\]

\( c_i(s) \) shall be of the form,

\[
c_i(s) = k_i/(1 + 1/\tau_i s)(1 + \tau_i s) \tag{40}
\]
It can be seen from (33) and (34) that in \( g_Pf(s) \) and \( g_{ud}(s) \), \( \theta=0 \). Set the desired closed-loop crossover time constant \( \tau_i \) equals to \( \tau_{ii} \), a practice often used in the process control [32], the PID parameter settings are then given as,

\[
k_p = 1 / k_i; \tau_d = \tau_i; \tau_i = \tau_i
\]

With this set of settings, the phase margin of \( c_i(s)g_i(s) \) is 90\(^\circ\) and it meets the typically desirable phase margin of 60\(^\circ\). While it can be seen from (33) that \( g_Pf(s) \) is independent of \( P \), however, (34) shows the dc-gain of \( g_{ud}(s) \) will be the maximum when \( P = P_{min} \). Therefore, the \( c_u(s) \) controller must be designed based on the minimum SOFC output power condition.

**Figure 6.** \( P \) control as applied to: (a) the two-input-two-output SOFC plant model, (b) the individual decentralized SISO plant model.

**Step Two:** Calculate the MMF by using dynamic Relative Index (dRI) and obtain the equivalent transfer function of each SISO system. In Figure 6(b), it is shown the output of the sub-system \( i \) will be superimposed by the output \( y'_{i} \) from the neighboring system \( j \). Define the dRI between \( y'_{i} \) and the output of the subsystem \( i \) as \( \phi_y(s) \). \( \phi_y(s) \) for the \( P \) control system can then be derived using the technique described in [32] for a general process system,

\[
\phi_y(s) = -g_y(s)g_{ji}(s)g_u^{-1}(s)(c_j^{-1}(s) + g_{ji}(s))^{-1}
\]

The MMF of the \( i^{th} \) SISO system is then given as,
\[
\rho_i(s) = 1 + \phi_i(s) = |\rho_i|e^{-\Delta \theta_i s}
\]  \hspace{1cm} (43)

|\rho_i| and \(\Delta \theta_i\) are the magnitude and phase angle of the MMF, respectively. Therefore, the rectangular box formed by the delineated lines in Figure 6(b) represents the equivalent transfer function \(g'_i(s)\) where

\[
g'_i(s) = \rho_i(s)g_u(s) = |\rho_i|\left[k_i e^{-\left((\theta_i + \Delta \theta_i)\right)s}\right][(\tau_i s + 1)(\tau'_i s + 1)]^{-1}
\] \hspace{1cm} (44)

In (44), |\rho_i| and \((\theta_i + \Delta \theta_i)\) vary with the operating condition of the SOFC. \(g'_i(s)\) can be chosen under the most onerous conditions when both |\rho_i| and \((\theta_i + \Delta \theta_i)\) have the maximum values, although maximum |\rho_i| and \((\theta_i + \Delta \theta_i)\) may not necessarily occur under the same operating condition. With this practice, \(g'_i(s)\) can be chosen under the most onerous conditions when both |\rho_i| and \((\theta_i + \Delta \theta_i)\) have the maximum values, although maximum |\rho_i| and \((\theta_i + \Delta \theta_i)\) may not necessarily occur under the same operating condition. With this practice,

\[
|\rho_i| = \max(1, |\rho_i|); \theta_i + \Delta \theta_i = \max(\theta_i, \Delta \theta_i)
\] \hspace{1cm} (45)

**Step Three:** Redesign each \(c_i(s)\) based on the equivalent transfer function \(g'_i(s)\). In a manner similar to that in designing the SIMC-PID controller in Step One, if the time constant corresponding to the closed-loop cross-over frequency of \(c_i(s)g'_i(s)\) is selected to be the same as the process maximum time constant \(\tau_{ci}\), as suggested in [32], the new controller settings for \(c_i(s)\) are,

\[
k_{pi} = \tau_i/[\rho_{i,\text{max}}k_i(\tau_{ci} + \theta_{i,\text{max}})]; \tau_{ui} = \min(\tau_i, A(\tau_{ci} + \theta_{i,\text{max}})); \tau_{di} = \tau'_i
\] \hspace{1cm} (46)

### 4.4. T Control System Design

As explained in Section 3.2, the temperature control involves slower dynamics of the hierarchical control system. Since the SOFC output power \(P\) can be maintained at the targeted value through the regulation of the faster \(P^\text{-q}\) and \(u^\text{-}\delta\) control loops, \(P\) can be assumed to have reached a quasi-steady state value, even before the \(T\) control loop starts to become active. From (8)-(11) and Figure 1, selecting \(q_{O_2,\text{max}}^\text{in}\) and \(T_{\text{min}}\) as the normalization base, the small-signal perturbation equation of the temperature \(T\) is,

\[
\Delta T(s) = g_{to}(s)\Delta q_{O_2}^{\text{in}} = \frac{(C_p, O_2 + k, C_{p,N})(T_{\text{in}} - T_0) + q_{O_2,\text{max}}^{\text{in}}}{m_{f, i} C_p s + B} \frac{T_{\text{min}}}{\Delta q_{O_2}^{\text{in}}}
\] \hspace{1cm} (47)

Last equation indicates that at steady-state, an increase in \(q_{O_2}^{\text{in}}\) will lead to a decrease in \(T\) because \(T_{\text{in}} < T_0\). However, as explained in Section 2, the parameter \(B\) will increase when \(I_{FC}\) increases. As \(B\) also appears in the denominator of (47), the consequence is that the phase margin of the transfer function \(g_{to}(s)\) in (47) will be at the minimum when \(I_{FC}\) is at the minimum, i.e. when \(P = P_{\text{min}}\). Therefore, the parameters of the temperature controller \(c_T(s)\) can be determined using the same SIMC-PID tuning method as that used in the design of the \(P\) controller. \(c_T(s)\) is to be tuned under the most onerous condition when the SOFC is at the minimum load.

### 4.5. Overall Load Tracking and Temperature Control Scheme

The overall control scheme for the SOFC to achieve \(\eta_{\text{max}}\) during the load tracking process is illustrated in the “Control and Optimization System” portion of Figure 2. Based on the above analysis,
both $P$ and $u$ will be ahead of $T$ to reach the reference values. It can be seen from (8) and will be
illustrated in Section 5 that continuously varying $P$ may lead the transient $T$ to exceed the constraint
given in (24). In order to guarantee the cell lifespan, $T$ should be monitored on-line. If the measured $T$
is not within the pre-set band which is close to its operating boundaries, as shown in Figure 2, the “Pre-
filter” block will convert the error between the targeted power level $P_t$ and the SOFC output power $P$
into continuous adjustments $P_{ref}$. Since the overall control objective of the load tracking is to achieve
$\eta_{max}$, the reference signals $T_{ref}$ and $u_{ref}$ can be obtained directly from the look-up table. Hence the
SOFC shall attempt to operate at $\eta_{max}$ as it approaches $P_t$. The hierarchical control scheme will track
$P_{ref}$, $u_{ref}$ and $T_{ref}$ through the respective controllers $c_P(s)$, $c_u(s)$ and $c_T(s)$.

5. Case Studies

The benchmark SOFC power plant in [21, 22, 30] was used to carry out case studies to illustrate the
efficiency of the proposed hierarchical control scheme. The 100kW power plant is connected to a
400V ac system and the associated parameters are given in Table 1. On the 400V and 100kVA base,
the SOFC power plant ac terminal voltage is assumed to be constant at 1.05pu. It is also assumed that
the link reactance $X$ in Figure 2 is 0.05pu. The simulation tool used is MATLAB/SIMULINK.

5.1. Steady-state $\eta_{max}$ Operations

Suppose the SOFC is to operate between 10kW and 100kW. Table 2 shows part of the NLPP
calculation results. For comparison, like the steady-state operating conditions in [26] and [27], the
efficiencies ($\eta_1$) when $T=1273K$ and $u=0.8$ under different power are also given. Obviously, $\eta_{max}$ is
higher than $\eta_1$. The optimal $\eta$ can be found on the boundaries of $T$ and $u$ when $P$ is at the low level.
The highest $\eta_{max}$ is 43.4% when $P=0.3pu$. However, the power consumed by the air compressor is over
15% of the output power if the cell operating pressure is 0.15MPa. This will cause $\eta_{max}$ less than 40%
under the maximum output power condition.

| Parameters | Value                   |
|------------|-------------------------|
| $T_{in}$   | 923K                    |
| $p$        | 0.15MPa                 |
| $N_0$      | 384                     |
| $m_C_p$    | $1.1\times10^4$(J/K)    |
| $K_i$      | $9.95\times10^{-4}$mol/(s.A) |
| $K_{H2}$   | $8.32\times10^{-6}$mol/(s.Pa) |
| $K_{H2O}$  | $2.77\times10^{-6}$mol/(s.Pa) |
| $K_{O2}$   | $2.49\times10^{-7}$mol/(s.Pa) |
| $V_a$      | 2.3$m^3$                |
| $V_c$      | 0.76$m^3$               |
| $\tau_f$   | 5s                      |
| $\tau_e$   | 0.8s                    |
| $\alpha$   | 0.02$\Omega$            |
| $\beta$    | -2870K                  |
### Table 2. The maximum efficiency results by NLPP

| P (kW) | P_loss (kW) | V_{dc} (V) | q_{fuel}^{in} (mol/s) | q_{O_2}^{in} (mol/s) | T(K) | u | \eta_{\text{max}} | \eta_i |
|--------|-------------|------------|-----------------------|----------------------|------|---|-----------------|-------|
| 10     | 1.514       | 269.31     | 0.082                 | 0.208                | 1173 | 0.9 | 0.427           | 0.3904 |
| 30     | 4.438       | 272.06     | 0.244                 | 0.610                | 1173 | 0.9 | 0.434           | 0.3973 |
| 50     | 7.560       | 269.49     | 0.410                 | 1.040                | 1173 | 0.8989 | 0.428       | 0.3992 |
| 70     | 10.889      | 266.08     | 0.585                 | 1.497                | 1173 | 0.8948 | 0.418       | 0.3988 |
| 90     | 14.416      | 261.51     | 0.771                 | 1.982                | 1175 | 0.8885 | 0.406       | 0.3973 |
| 100    | 15.906      | 257.87     | 0.872                 | 2.187                | 1184 | 0.8849 | 0.399       | 0.395   |

When \( P \) is 0.1pu or 1pu, the corresponding dRI are: \( \phi_{P,0.1}(j0.2) = 1.414 \angle 26.8^\circ; \phi_{P,1}(j1.25) = 0.443 \angle 10.0^\circ; \) and \( \phi_{P,1}(j1.25) = 0.423 \angle 13.6^\circ. \) The values of \( \phi_{P,\text{MPP}(\cdot)} \) confirm the most onerous condition under which the \( P-q_{\text{fuel}}^{in} \) loop interact with the \( u-\delta \) loop is at the minimum \( P \) condition. The values of \( \phi_{G,\text{MPP}(\cdot)} \) show that this loop can contribute to the \( P-q_{\text{fuel}}^{in} \) loop with the largest gain increase, and the maximum phase lag under the minimum and the maximum \( P \) conditions, respectively. In order to guarantee satisfactory dynamic performance of the SOFC under possible loop failures for all output power conditions, the corresponding MMF can be selected to be the extreme gain and phase values simultaneously. Thus, based on the above numerical results and using (43), \( \rho_{P,\delta} = 2.391 \exp(-0.96s) \) and \( \rho_{G,\delta} = 1.442 \exp(0.04s). \) The corresponding equivalent transfer function of each SISO system can then be calculated using (44) and (45) to yield \( g_{P}(s) = 1.839 \exp(-0.96s)/(1+1/5s)(1+0.8s) \) and \( g_{u\delta}(s) = 15.147/(1+0.8s). \) From (46), the new \( P \) controllers are \( c_{P}(s) = 0.531(1+1/5s)(1+0.8s) \) and \( c_{u}(s) = 0.066(1+1/0.8s). \)

As discussed in Section 4.4, the controller for the \( T \) control system is also designed when \( P = P_{\text{min}}. \) Accordingly, the \( T \) controller is \( c_{T}(s) = -0.64(1+1/292s). \) Again, the above three controllers designed for the model shown in Figure 1 indicate the SOFC is feasible for slow load tracking application. The tracking speed is firstly limited by the \( P \) controllers. As the cross-over time constant of \( c_{P}(s) \) is around 0.2s, it will be safe for the SOFC to track the load...
within this bandwidth. On the other hand, the cross-over time constant of $c_T(s)$ is about 0.0035s. The $T$
control system is much slower than the $P$ control system. As shown in (8), the continuing output power
change will cause $T$ deviate from the acceptable value. Therefore, the load tracking speed must be slow
down until $T$ can be effectively regulated within the constraints.

5.3. SOFC Load Tracking Dynamic Performance

This section illustrates the load tracking performance of the SOFC under the proposed hierarchical
control scheme with and without considering the temperature control. The results are then compared
with that obtained from the on-line control scheme described in [27]. Suppose the power demand
increases from 0.1pu to 1pu. If the measured $T$ is within the constraints, the load demand on the SOFC
can change at the rate of 0.1 pu kW/min. Such load tracking speed is quite close to the results reported
in [20] and [31]. However, as discussed in Section 4.3 and shown in Figure 7a, the “Pre-filter” block
can generate the new power reference only the measured $T$ is below a pre-set threshold value (say 1263K). It will take about 30 minutes to achieve the targeted power due to the variable power ramp
rate. If the temperature control is not considered, the targeted power can be reached in about 12
minutes. The on-line control strategy proposed in [27] is designed such that the final load level shall be
reached within the minimum time. Indeed, the on-line method shown in Figure 7(a) has a higher speed
of response, i.e. the 0.9pu power change is reached in about 100s. However, in [27], the ratio of the
fuel flow rate to oxygen flow rate is kept constant at 1.145. It is shown in Table 2 that it is impossible
to maintain a constant $T$ with the flow rate ratio fixed. Therefore, the constant temperature assumption
made in [27] is invalid once the energy balance consideration is included in the dynamic model.

An interesting observation is that the direction of the $u$ variation based on the on-line method is
opposite to that obtained under the hierarchical control. This is shown in Figure 7(b). The reason for
this is because under the on-line scheme proposed in [27], $q_{fuel}^{in}$ is the only independent control variable
and $u$ is kept constant at a pre-set value (which, in this simulation, is 0.8). As derived in [27], $q_{fuel}^{in}$ and
$u$ will vary in the same direction following the load change. Under the hierarchical control scheme,
however, both $q_{fuel}^{in}$ and $\delta$ will affect $u$. Due to the loop interactions, $u$ will vary in a direction opposite
to that of $q_{fuel}^{in}$, as shown in (35). However, the hierarchical control scheme can achieve the optimal
value 0.8849, as can be seen in the figure.

Figure 7. Comparison of SOFC load tracking performance under hierarchical control
scheme with considering $T$ bound (──), hierarchical control scheme without considering $T$
bound (---) and on-line control scheme of [27] (−.)
From the initial optimal value 1173K, Figure 7c indicates that it will take about 90 minutes for the SOFC to reach at 1184K. $T$ can be maintained under 1273K during the transient period if $T$ bound is satisfied with the variable load tracking speed. Otherwise, $T$ will be out of the constraint due to the continuously increasing $P$. 
According to the discussion above, it can be concluded that the proposed hierarchical control scheme will be able to track the power demand in a safe manner, and the mutual loop interactions have been included in the control system design. The scheme will also lead to the maximum electrical efficiency operations of the SOFC.

6. Conclusions

By considering the power loss caused by the air compressor, the maximum electrical efficiency operating conditions of the grid-connected SOFC can be obtained by solving a nonlinear programming problem which is subject to constraints of stack temperature, fuel utilization factor and output power. In order to accommodate the inherently different dynamical processes within the SOFC, a hierarchical control scheme for the grid-connected SOFC power plant has been proposed. The scheme consists of a $P$ control system and a relatively slower $T$ control system. The case studies verify that the proposed hierarchical control scheme can achieve maximum efficiency load tracking operation for the grid-connected SOFC with the stack temperature bounded within the preset constraints.

The FC-based DG technology is still far from mature. Continuous improvements on the FC performance, durability and making it economically competitive are needed in order to realize its wide application. For power system analysis, the SOFC model and control strategies should be improved and verified through experiment in the future work.

Conflicts of Interest

The authors declare no conflict of interest.

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