Speed dependent polarization correlations in QED and entanglement* †

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Abstract

Exact computations of polarizations correlations probabilities are carried out in QED, to the leading order, for initially polarized as well as unpolarized particles. Quite generally they are found to be speed dependent and are in clear violation of Bell’s inequality of Local Hidden Variables (LHV) theories. This dynamical analysis shows how speed dependent entangled states are generated. These computations, based on QED are expected to lead to new experiments on polarization correlations monitoring speed in the light of Bell’s theorem. The paper provides a full QED treatment of the dynamics of entanglement.

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1 INTRODUCTION

We carry out exact computations of joint probabilities of particle polarizations correlations in QED, to the leading order, for initially polarized and unpolarized particles. The interesting lesson we have learnt from such studies is that the mere fact that particles emerging from a process have non-zero speeds to reach detectors implies, in general, that their polarizations correlations probabilities depend on speed [1]. The present extended, and needless to say, dynamical analysis shows that this is true, in general. This is unlike formal arguments based simply on combining angular momenta. As a byproduct of this work, we obtain clear violations with Bell’s inequality (cf. [2]–[4]) of LHV theories. We will also see how QED generates speed dependent entangled states.
Several experiments have been performed in recent years (cf. [4]–[8]) on particles’ polarizations correlations. And, it is expected that the novel properties recorded here by explicit calculations following directly from field theory, which is based on the principle of relativity and quantum theory, will lead to new experiments on polarization correlations monitoring speed in the light of Bell’s Theorem. We hope that these computations will be also useful in such areas of physics as quantum teleportation and quantum information in general.

The relevant quantity of interest here in testing Bell’s inequality of LHV [2] theories is, in a standard notation,

\[
S = \frac{p_{12}(a_1, a_2)}{p_{12}(\infty, \infty)} - \frac{p_{12}(a_1, a'_2)}{p_{12}(\infty, \infty)} + \frac{p_{12}(a'_1, a_2)}{p_{12}(\infty, \infty)} + \frac{p_{12}(a'_1, a'_2)}{p_{12}(\infty, \infty)} - \frac{p_{12}(a'_1, \infty)}{p_{12}(\infty, \infty)} - \frac{p_{12}(\infty, a_2)}{p_{12}(\infty, \infty)}
\]

(1.1)
as is computed from QED. Here \(a_1, a_2\) \((a'_1, a'_2)\) specify directions along which the polarizations of two particles are measured, with \(p_{12}(a_1, a_2)/p_{12}(\infty, \infty)\) denoting the joint probability, and \(p_{12}(a_1, \infty)/p_{12}(\infty, \infty)\), \(p_{12}(\infty, a_2)/p_{12}(\infty, \infty)\) denoting the probabilities when the polarization of only one of the particles is measured. \(p_{12}(\infty, \infty)\) is normalization factor. The corresponding probabilities as computed from QED will be denoted by \(P[\chi_1, \chi_2]\), \(P[\chi_1, -]\), \(P[-, \chi_2]\) with \(\chi_1, \chi_2\) denoting angles the polarization vectors make with certain axes spelled out in the bulk of the paper. To show that QED is in violation with Bell’s inequality of LHV, it is sufficient to find one set of angles \(\chi_1, \chi_2, \chi'_1, \chi'_2\) and speed \(\beta\), such that \(S\), as computed in QED, leads to a value of \(S\) with \(S > 0\) or \(S < -1\). In this work, it is implicitly assumed that the polarization parameters in the particle states are directly observable and may be used for Bell-type measurements as discussed.

The need of a relativistic treatment based on explicit quantum field dynamical calculations in testing Bell-like inequalities is critically important. An intriguing and very recent reference [9], which appeared after the submission of our paper for publication, discusses the role of relativity in quantum information, in general, and traces the historical development of its role, and most importantly, in the light of our present investigations, emphasizes the need of quantum field theory as necessary for a consistent description of interactions. Most earlier analyses dealing with relativistic aspects, relevant to information theory and Bell-like tests are kinematical of nature or deal with basic general properties of local operators associated with bounded regions of spacetime setting limits on measurements and localizability of quantum systems. These probabilities are well documented in some of the recent monographs [10]–[12] on the subject. Notable important other recent references on such general aspects which are, however, non-dynamical of nature are [13]–[17], and a paper by Czachor [18] indicating how a possible decrease in violation of Bell’s inequalities may occur. In the present work, we are interested in dynamical aspects and related uniquely determined probabilities (intensities) of correlations based on QED, as a fully relativistic quantum field theory (i.e., encompassing quantum theory and relativity) that meet the verdict of experiments. QED is a non-speculative theory and as Feynman [19] puts it, it is the most precise theory we
have in fundamental physics. The closest investigation to our own is that of Ref. [20], a reference we encountered after the submission of our paper for publication, which considers spin-spin interactions, in a QED setting, for non-relativistic electrons and, unfortunately, does not compute their polarizations correlations which are much relevant experimentally. In the present paper, exact fully relativistic QED, computations, to the leading order, of polarizations correlations are explicitly carried out for initially polarized and unpolarized particles. The importance of also considering unpolarized spin stems from the fact that we discover the existence of non-trivial correlations, in the outcome of the processes, even for such mixed states (since one averages over spin) and not only for pure states arising from polarized spins, leading, in particular, in both cases to speed dependent probabilities. The main results of our paper are given in (2.20), (2.22), (2.23), (2.41)–(2.43), (3.10), (3.12)–(3.19). All of these probabilities lead to a violation of Bell’s inequality of LHV theories. As the computations are based on the fully relativistic QED, it is of some urgency that relevant experiments are carried out by monitoring speed.

2 POLARIZATIONS CORRELATIONS: INITIALLY POLARIZED PARTICLES

We consider the process $e^-e^- \rightarrow e^-e^-$, in the c.m., with initially polarized electrons with one spin up, along the $z$-axis, and one spin down. With $\mathbf{p}_1 = \gamma m \beta (0, 1, 0) = -\mathbf{p}_2$ denoting the momenta of the initial electrons, $\gamma = 1/\sqrt{1-\beta^2}$, we consider momenta of the emerging electrons with

$$\mathbf{p}_1' = \gamma m \beta (\sin \theta, 0, \cos \theta) = -\mathbf{p}_2'$$

where $\theta$ is measured from the $z$-axis.

For the four-spinors of the initial electrons, we have ($p^0 = \gamma m$)

$$u(p_1) = \left( \frac{p^0 + m}{2m} \right)^{1/2} \begin{pmatrix} 1 \\ 0 \\ i\rho \\ 0 \end{pmatrix}$$

$$u(p_2) = \left( \frac{p^0 + m}{2m} \right)^{1/2} \begin{pmatrix} 0 \\ 1 \\ i\rho \\ 0 \end{pmatrix}$$

$$\rho = \frac{\gamma \beta}{\gamma + 1} = \frac{\beta}{1 + \sqrt{1-\beta^2}}$$
and for the final ones

\[ u(p'_1) = \left( \frac{p^0 + m}{2m} \right)^{1/2} \left( \begin{array}{c} \xi_1 \\ \frac{\sigma p'_1}{p^0 + m} \xi_1 \end{array} \right) \] (2.5)

\[ u(p'_2) = \left( \frac{p^0 + m}{2m} \right)^{1/2} \left( \begin{array}{c} \xi_2 \\ \frac{\sigma p'_2}{p^0 + m} \xi_2 \end{array} \right) \] (2.6)

where the two-spinors \( \xi_1, \xi_2 \) will be specified later.

The expression for the amplitude of the process is well known (cf. [21])

\[ A \propto \frac{\bar{u}(p'_1)\gamma^\mu u(p_1)\bar{u}(p'_2)\gamma^\mu u(p_2)}{(p'_1 - p_1)^2} - \frac{\bar{u}(p'_2)\gamma^\mu u(p'_1)\bar{u}(p'_1)\gamma^\mu u(p_2)}{(p'_2 - p_1)^2} \] (2.7)

The following matrix elements are needed and are readily calculated

\[ \bar{u}(p'_1)\gamma^0 u(p_1) = \frac{p^0 + m}{2m} \xi_1^\dagger \left( \begin{array}{c} 1 + i\rho^2 \sin \theta \\ -i\rho^2 \cos \theta \end{array} \right) \] (2.8)

\[ \bar{u}(p'_2)\gamma^0 u(p_2) = \frac{p^0 + m}{2m} \xi_2^\dagger \left( \begin{array}{c} -i\rho^2 \cos \theta \\ 1 - i\rho^2 \sin \theta \end{array} \right) \] (2.9)

\[ \bar{u}(p'_1)\gamma^0 u(p_2) = \frac{p^0 + m}{2m} \xi_1^\dagger \left( \begin{array}{c} i\rho^2 \cos \theta \\ 1 + i\rho^2 \sin \theta \end{array} \right) \] (2.10)

\[ \bar{u}(p'_2)\gamma^0 u(p_1) = \frac{p^0 + m}{2m} \xi_2^\dagger \left( \begin{array}{c} 1 - i\rho^2 \sin \theta \\ i\rho^2 \cos \theta \end{array} \right) \] (2.11)

\[ \bar{u}(p'_1)\gamma^j u(p_1) = \frac{p^0 + m}{2m} \rho^{j1}_{\xi_1} \left( \begin{array}{c} i\sin \theta \\ -\cos \theta \end{array} \right) \delta^{j1} + i \left( \begin{array}{c} -i\sin \theta \\ -\cos \theta \end{array} \right) \delta^{j2} + \left( \begin{array}{c} -\cos \theta \\ -i\sin \theta \end{array} \right) \delta^{j3} \] (2.12)

\[ \bar{u}(p'_2)\gamma^j u(p_2) = \frac{p^0 + m}{2m} \rho^{j2}_{\xi_2} \left( \begin{array}{c} -\cos \theta \\ i\sin \theta \end{array} \right) \delta^{j1} + i \left( \begin{array}{c} \cos \theta \\ i\sin \theta \end{array} \right) \delta^{j2} + \left( \begin{array}{c} \cos \theta \\ -i\sin \theta \end{array} \right) \delta^{j3} \] (2.13)

\[ \bar{u}(p'_1)\gamma^j u(p_2) = \frac{p^0 + m}{2m} \rho^{j3}_{\xi_1} \left( \begin{array}{c} \cos \theta \\ i\sin \theta \end{array} \right) \delta^{j1} + i \left( \begin{array}{c} -\cos \theta \\ i\sin \theta \end{array} \right) \delta^{j2} + \left( \begin{array}{c} \cos \theta \\ \cos \theta \end{array} \right) \delta^{j3} \] (2.14)

\[ \bar{u}(p'_2)\gamma^j u(p_1) = \frac{p^0 + m}{2m} \rho^{j3}_{\xi_2} \left( \begin{array}{c} i\sin \theta \\ \cos \theta \end{array} \right) \delta^{j1} - i \left( \begin{array}{c} i\sin \theta \\ -\cos \theta \end{array} \right) \delta^{j2} - \left( \begin{array}{c} \cos \theta \\ i\sin \theta \end{array} \right) \delta^{j3} \] (2.15)

For \( \theta = 0 \), (see FIG. 1), we obtain from (2.7)–(2.15)
\[ A \propto \xi_1^\dagger \xi_2^\dagger \left\{ (1 + 6\rho^2 + \rho^4) \left[ \left( \begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right)_1 - \left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right)_2 \right] \\
+ 4i\rho^2 \left[ \left( \begin{array}{cc} 0 & 0 \\ 1 & 0 \end{array} \right)_1 + \left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right)_2 \right] \right\} \] (2.16)

generating the speed dependent (normalized) entangled state of the emerging electrons

\[ |\psi\rangle = \frac{1}{N} \left\{ \frac{(1 + 6\rho^2 + \rho^4)}{\sqrt{2}} \left[ \left( \begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right)_1 - \left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right)_2 \right] \\
+ \frac{4i\rho^2}{\sqrt{2}} \left[ \left( \begin{array}{cc} 0 & 0 \\ 1 & 0 \end{array} \right)_1 + \left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right)_2 \right] \right\} \] (2.17)

where

\[ N = \left[ (1 + 6\rho^2 + \rho^4)^2 + 16\rho^4 \right]^{1/2} \] (2.18)

\[ \xi_j = \frac{1}{\sqrt{2}} \left( e^{-i\chi_j/2}, e^{i\chi_j/2} \right), \quad j = 1, 2 \] (2.19)

\( \rho \) is defined in (2.4), and the angles are measured relative to the \( x \)-axis (see FIG. 1).

The joint probability of the electrons polarizations correlations is then given by

\[ P[\chi_1, \chi_2] = \left\| \xi_1^\dagger \xi_2^\dagger |\psi\rangle \right\|^2 \]

\[ = \frac{\left[ (1 + 6\rho^2 + \rho^4) \sin \left( \frac{\chi_1 - \chi_2}{2} \right) - 4\rho^2 \cos \left( \frac{\chi_1 + \chi_2}{2} \right) \right]^2}{2 \left[ (1 + 6\rho^2 + \rho^4)^2 + 16\rho^4 \right]} \] (2.20)

[For \( \beta \to 0 \), one obtains a rather familiar expression \( P[\chi_1, \chi_2] = \sin^2 [ (\chi_1 - \chi_2)/2 \] /2.]

If only one of the spins is measured, say, corresponding to \( \chi_1 \), we then have to form the state

\[ \xi_1^\dagger |\psi\rangle = \frac{[1 + 6\rho^2 + \rho^4]}{2N} \left( e^{-i\chi_1/2} \right)_1 + \frac{4i\rho^2}{2N} \left( e^{i\chi_1/2} \right)_2 \] (2.21)

from which we obtain the corresponding probability
FIG. 1: The figure depicts $e^- e^-$ scattering, with the electrons initially moving along the $y$-axis, while the emerging electrons moving along the $z$-axis. The angle $\chi_1$, measured relative to the $x$-axis, denotes the orientation of spin of one of the emerging electrons may make.

$$P[\chi_1, -] = \left\| \xi_1^\dagger |\psi\rangle \right\|^2$$

$$= \frac{1}{2} - \frac{4\rho^2(1 + 6\rho^2 + \rho^4)}{(1 + 6\rho^2 + \rho^4)^2 + 16\rho^4} \sin \chi_1$$

(2.22)

and similarly
\[ P[\chi_1, \chi_2] = \| \xi_2' |\psi\rangle \|^2 = \frac{1}{2} + \frac{4\rho^2(1 + 6\rho^2 + \rho^4)}{(1 + 6\rho^2 + \rho^4)^2 + 16\rho^4} \sin \chi_2 \]  

(2.23)

The probability \( P[\chi_1, -] \) may be equivalently obtained by summing \( P[\chi_1, \chi_2] \) over the two angles

\[ \chi_2, \quad \chi_2 + \pi \]  

(2.24)

for any arbitrarily chosen fixed \( \chi_2 \), i.e.,

\[ P[\chi_1, \chi_2] + P[\chi_1, \chi_2 + \pi] = P[\chi_1, -] \]  

(2.25)

as is easily checked, and similarly for \( P[-, \chi_2] \).

For all \( 0 \leq \beta \leq 1 \), angles \( \chi_1, \chi_2, \chi_1', \chi_2' \) are readily found leading to a violation of Bell’s inequality of LHV theories. For example, for \( \beta = 0.3, \chi_1 = 0^\circ, \chi_2 = 137^\circ, \chi_1' = 12^\circ, \chi_2' = 45^\circ, S = -1.79 \) violating the inequality from below.

The speed dependence of \( P[\chi_1, \chi_2] \) generally holds true for other angles as well. For \( \theta = \pi/2 \), however, it is readily verified that \( (2.26) \) leads to the entangled state

\[ |\psi\rangle_0 = \frac{1}{\sqrt{2}} \left[ \begin{pmatrix} 0 \\ 1 \end{pmatrix}_1 (1)_{12} - (1)_{01} \begin{pmatrix} 0 \\ 1 \end{pmatrix}_2 \right] \]  

(2.26)

for all \( 0 \leq \beta \leq 1 \), leading to a rather familiar expression \( P[\chi_1, \chi_2] = \sin^2 [(\chi_1 - \chi_2)/2]/2. \)

Now we consider the process \( e^+e^- \rightarrow 2\gamma \), in the c.m. of \( e^- \), \( e^+ \) with spins up, along the \( z \)-axis, and down, respectively. With \( \mathbf{p}_1 = \mathbf{p}(e^-) = \gamma m \beta (0, 1, 0) = -\mathbf{p}(e^+) = -\mathbf{p}_2 \), we have for \( e^-, e^+ \) the spinors given by

\[ u = \left( \frac{p^0 + m}{2m} \right)^{1/2} \begin{pmatrix} 1 \\ 0 \\ i\rho \end{pmatrix} \]  

(2.27)

\[ v = \left( \frac{p^0 + m}{2m} \right)^{1/2} \begin{pmatrix} i\rho \\ 0 \\ 1 \end{pmatrix} \]  

(2.28)
with $\rho$ defined in (2.4), and we consider momenta of the photons

$$k_1 = \gamma m (\sin \theta, 0, \cos \theta) = -k_2$$

(2.29)

where we have used the facts that

$$|k_1| = |k_2| = k_1^0 = k_2^0 = p^0(e^\pm) \equiv p^0 = \gamma m$$

(2.30)

The amplitude for the process is given by (cf. [21])

$$A \propto \bar{v} \left[ \frac{\gamma^\mu k_1^\mu}{2p_1 k_1} + \frac{\gamma^\nu k_2^\nu}{2p_1 k_2} + \frac{\gamma^\mu p_1^\nu}{p_1 k_1} + \frac{\gamma^\nu p_1^\mu}{p_1 k_2} \right] u \, e_1^\nu e_2^\mu$$

(2.31)

where $e_1^\mu = (0, e_1), e_2^\nu = (0, e_2)$ are the polarizations of the photons with $(j = 1, 2)$

$$e_j = (-\cos \theta \cos \chi_j, \sin \chi_j, \sin \theta \cos \chi_j) \equiv (e_j^{(1)}, e_j^{(2)}, e_j^{(3)})$$

(2.32)

The following matrix elements are readily derived

$$\bar{v} \left( \gamma^i \gamma^0 \right) u = \frac{p^0 + m}{2m} 2i\varepsilon_{ij23}$$

(2.33)

$$\bar{v} \gamma^i u = \frac{p^0 + m}{2m} (1 - \rho^2) \delta^{i3}$$

(2.34)

$$\bar{v} \left( \gamma^i \gamma^m \gamma^j \right) u = \frac{p^0 + m}{2m} \left( -\delta^{mj} \delta^{i3} - \delta^{mi} \delta^{j3} + \delta^{ij} \delta^{m3} \right) (1 - \rho^2)$$

$$-i \frac{p^0 + m}{2m} (1 + \rho^2) \varepsilon_{imj}$$

(2.35)

Upon setting,$$
\frac{k_i}{|k_i|} = \mathbf{n}
$$

(2.36)

the amplitude $A$ is then given by

$$A \propto -i(1 + \rho^2) \mathbf{n} \cdot (\mathbf{e}_1 \times \mathbf{e}_2) + \beta(1 - \rho^2) \left( e_1^{(2)} e_2^{(3)} + e_1^{(3)} e_2^{(2)} \right)$$

(2.37)

For $\theta = \pi/2$, this gives
\[ A = -(0, \sin \chi_1, \sin \chi_1) \times (0, \sin \chi_2, \sin \chi_2) \]

\[ \times \left\{ i(1 + \rho^2) \left[ \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}_{11} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}_{12} - \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}_{11} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}_{12} \right] - \beta(1 - \rho^2) \left[ 0 \\ 1 \\ 0 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}_{12} + 0 \\ 0 \\ 1 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}_{12} \right] \right\} \] (2.38)

(see FIG. 2), generating a speed dependent (normalized) entangled state for the photons given by

\[ |\phi\rangle = \frac{1}{N} \left\{ i(1 + \rho^2) \left[ \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}_{11} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}_{12} - \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}_{11} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}_{12} \right] - \beta(1 - \rho^2) \left[ 0 \\ 1 \\ 0 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}_{12} + 0 \\ 0 \\ 1 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}_{12} \right] \right\} \] (2.39)

with

\[ N = \left[ (1 + \rho^2)^2 + \beta^2(1 - \rho^2)^2 \right]^{1/2} \] (2.40)

Therefore the joint probability of photons polarizations correlations is given by

\[ P[\chi_1, \chi_2] = \| (0, \sin \chi_1, \cos \chi_1)_1 (0, \sin \chi_2, \cos \chi_2)_2 |\phi\rangle \|^2 \]

\[ = \frac{(1 + \rho^2)^2 \sin^2(\chi_1 - \chi_2) + \beta^2(1 - \rho^2)^2 \cos^2(\chi_1 + \chi_2)}{2[(1 + \rho^2)^2 + \beta^2(1 - \rho^2)^2]} \] (2.41)

and

\[ P[\chi_1, -] = \| (0, \sin \chi_1, \cos \chi_1)_1 |\phi\rangle \|^2 = \frac{1}{2} \] (2.42)

\[ P[-, \chi_2] = \| (0, \sin \chi_2, \cos \chi_2)_2 |\phi\rangle \|^2 = \frac{1}{2} \] (2.43)

\[ P[\chi_1, -] \text{ is also equivalently obtained by summing } P[\chi_1, \chi_2] \text{ over } \chi_2, \quad \chi_2 + \frac{\pi}{2} \] (2.44)
FIG. 2: The figure depicts $e^+e^-$ annihilation into $2\gamma$, with $e^+, e^-$ moving along the $y$-axis, and the emerging photons moving along the $x$-axis. $\chi_1$ denotes the angle the polarization vector of one of the photons may make with the $z$-axis.

For any arbitrarily chosen $\chi_2$, i.e.,

$$P[\chi_1, \chi_2] + P[\chi_1, \chi_2 + \frac{\pi}{2}] = P[\chi_1, -]$$

and similarly for $P[-, \chi_2]$.

For all $0 \leq \beta \leq 1$, angles $\chi_1$, $\chi_2$, $\chi'_1$, $\chi'_2$ are readily found leading to a violation of Bell’s inequality of LHV theories. For example, for $\beta = 0.2$, $\chi_1 = 0^\circ$, $\chi_2 = 23^\circ$, $\chi'_1 = 45^\circ$, $\chi'_2 = 67^\circ$, $S = -1.187$ violating the inequality from below.

Again the speed dependence of $P[\chi_1, \chi_2]$ generally holds true for other angles as well. For $\theta = 0$, however, it is readily checked that (2.37) leads to the entangled state

$$\langle \phi_0 \rangle = \frac{1}{\sqrt{2}} \left[ \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}_1 - \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}_2 \right]$$

for all $0 \leq \beta \leq 1$ giving a rather familiar expression $P[\chi_1, \chi_2] = [\sin^2(\chi_1 - \chi_2)] / 2$.

3 POLARIZATIONS CORRELATIONS: INITIALLY UNPOLARIZED PARTICLES

For the process $e^-e^- \rightarrow e^-e^-$, in the c.m., with initially unpolarized spins, with momenta $\mathbf{p}_1 = \gamma m \beta (0, 1, 0) = -\mathbf{p}_2$, we take for the final electrons
\[ p_1' = \gamma m \beta (1, 0, 0) = -p_2' \]  

(3.1)

and for the four-spinors

\[
\begin{align*}
    u(p_1') &= \left( \frac{p^0 + m}{2m} \right)^{1/2} \left( \frac{\xi_1}{\gamma} \sigma \cdot p' \right) 
    \xi_1 = \begin{pmatrix} -i \cos \chi_1/2 \\ \sin \chi_1/2 \end{pmatrix} 
    
    (3.2)
\end{align*}
\]

\[
\begin{align*}
    u(p_2') &= \left( \frac{p^0 + m}{2m} \right)^{1/2} \left( -\frac{\xi_2}{\gamma} \sigma \cdot p' \right) 
    \xi_2 = \begin{pmatrix} -i \cos \chi_2/2 \\ \sin \chi_2/2 \end{pmatrix} 
    
    (3.3)
\end{align*}
\]

A straightforward but tedious computation of the corresponding probability of occurrence with initially unpolarized electrons, (2.7) leads to

\[
\text{Prob} \propto \left[ u(p_1') \gamma_\mu (\gamma p_1 + m) \gamma_\sigma u(p'_1) \right] \left[ u(p_2') \gamma_\mu (\gamma p_2 + m) \gamma_\sigma u(p'_2) \right] 
- \left[ u(p_1') \gamma_\mu (\gamma p_1 + m) \gamma_\sigma u(p'_2) \right] \left[ u(p_2') \gamma_\mu (\gamma p_2 + m) \gamma_\sigma u(p'_1) \right] 
- \left[ u(p_2') \gamma_\mu (\gamma p_1 + m) \gamma_\sigma u(p'_1) \right] \left[ u(p_1') \gamma_\mu (\gamma p_2 + m) \gamma_\sigma u(p'_2) \right] 
+ \left[ u(p_2') \gamma_\mu (\gamma p_1 + m) \gamma_\sigma u(p'_2) \right] \left[ u(p_1') \gamma_\mu (\gamma p_2 + m) \gamma_\sigma u(p'_1) \right] 
\]

(3.4)

which after simplification and of collecting terms reduces to

\[
\text{Prob} \propto (1 - \beta^2)(1 + 3\beta^2) \sin^2 \left( \frac{\chi_1 - \chi_2}{2} \right) + \beta^4 \cos^2 \left( \frac{\chi_1 + \chi_2}{2} \right) + 4\beta^4
\]

\[
\equiv F[\chi_1, \chi_2]
\]

(3.5)

where we have used the expressions for the spinors in (3.2), (3.3).

Given that the process has occurred, the conditional probability that the spins of the emerging electrons make angles \( \chi_1, \chi_2 \) with the z-axis, is directly obtained from (3.5) to be

\[
P[\chi_1, \chi_2] = \frac{F[\chi_1, \chi_2]}{C}
\]

(3.6)

The normalization constant \( C \) is obtained by summing over the polarizations of the emerging electrons. This is equivalent to summing of \( F[\chi_1, \chi_2] \) over the pairs of angles

\[
(\chi_1, \chi_2), (\chi_1 + \pi, \chi_2), (\chi_1, \chi_2 + \pi), (\chi_1 + \pi, \chi_2 + \pi)
\]

(3.7)

for any arbitrarily chosen fixed \( \chi_1, \chi_2 \), corresponding to the orthonormal spinors.
\[
\begin{align*}
\left( -i \cos \chi_j / 2 \right) & \left( -i \cos (\chi_j + \pi) / 2 \right) = \frac{i \sin \chi_j / 2}{\sin (\chi_j + \pi) / 2} \\
\sin \chi_j / 2 & \cos (\chi_j + \pi) / 2
\end{align*}
\] (3.8)

providing a complete set, for each \( j = 1, 2 \), in reference to (3.2), (3.3). This is,

\[
C = F[\chi_1, \chi_2] + F[\chi_1 + \pi, \chi_2] + F[\chi_1, \chi_2 + \pi] + F[\chi_1 + \pi, \chi_2 + \pi] = 2(1 + 2\beta^2 + 6\beta^4) (3.9)
\]

which as expected is independent of \( \chi_1, \chi_2 \), giving

\[
P[\chi_1, \chi_2] = \frac{(1 - \beta^2)(1 + 3\beta^2) \sin^2 \left( \frac{\chi_1 - \chi_2}{2} \right) + \beta^4 \cos^2 \left( \frac{\chi_1 + \chi_2}{2} \right) + 4\beta^4}{2(1 + 2\beta^2 + 6\beta^4)} (3.10)
\]

By summing over

\[
\chi_2, \quad \chi_2 + \pi
\] (3.11)

for any arbitrarily fixed \( \chi_2 \), we obtain

\[
P[\chi_1, -] = \frac{1}{2} (3.12)
\]

and similarly,

\[
P[-, \chi_2] = \frac{1}{2} (3.13)
\]

for the probabilities when only one of the photons polarizations is measured.

A clear violation of Bell’s inequality of LHV theories was obtained for all \( 0 \leq \beta \leq 0.45 \). For example, for \( \beta = 0.3 \), with \( \chi_1 = 0^\circ \), \( \chi_2 = 45^\circ \), \( \chi_1' = 90^\circ \), \( \chi_2' = 135^\circ \) give \( S = -1.165 \) violating the inequality from below. For larger \( \beta \) values, alone, one cannot discriminate between LHV theories and quantum theory for this process. A violation of Bell’s inequality for at least some \( \beta \) values, as seen, however, automatically violates LHV theories.

The probability of photon polarizations correlations in \( e^+ e^- \rightarrow 2\gamma \) with initially unpolared \( e^+, e^- \), has been given in [1] to be

\[
P[\chi_1, \chi_2] = \frac{1 - \left[ \cos(\chi_1 - \chi_2) - 2\beta^2 \cos \chi_1 \cos \chi_2 \right]^2}{2[1 + 2\beta^2(1 - \beta^2)]} (3.14)
\]

\[
P[\chi_1, -] = \frac{1 + 4\beta^2(1 - \beta^2) \cos^2 \chi_1}{2[1 + 2\beta^2(1 - \beta^2)]} (3.15)
\]

\[
P[-, \chi_2] = \frac{1 + 4\beta^2(1 - \beta^2) \cos^2 \chi_2}{2[1 + 2\beta^2(1 - \beta^2)]} (3.16)
\]
and a clear violation of Bell’s inequality of LHV theories was obtained for all $0 \leq \beta \leq 0.2$. Again, for larger values of $\beta$, alone, one cannot discriminate between LHV theories and quantum theory for this process. A violation of Bell’s inequality for at least some $\beta$ values, as seen, however, automatically occurs violating LHV theories.

For completeness, we mention that for the annihilation of the spin 0 pair into $2\gamma$ the following probabilities are similarly worked out:

\[
P[\chi_1, \chi_2] = \frac{(\cos(\chi_1 - \chi_2) - 2\beta^2 \cos \chi_1 \cos \chi_2)^2}{2[1 - 2\beta^2(1 - \beta^2)]}
\] (3.17)

\[
P[\chi_1, -] = \frac{1 - 4\beta^2(1 - \beta^2) \cos^2 \chi_1}{2[1 - 2\beta^2(1 - \beta^2)]}
\] (3.18)

\[
P[-, \chi_2] = \frac{1 - 4\beta^2(1 - \beta^2) \cos^2 \chi_2}{2[1 - 2\beta^2(1 - \beta^2)]}
\] (3.19)

and violates Bell’s inequality of LHV theories for all $0 \leq \beta \leq 1$.

4 CONCLUSION

We have seen by explicit dynamical computations based on QED, that the polarizations correlations probabilities of particles emerging in processes depend on speed, for initially polarized as well as unpolarized particles, in general. We have also seen how QED leads directly to speed dependent entangled states. For processes with initially polarized particles (as well as for spin 0 pairs annihilation into $2\gamma$), a clear violation of Bell’s inequality of LHV theories was obtained for all speeds. This clear violation was also true for several speeds for processes with initially unpolarized particles, but the tests are more sensitive on the speed for such processes. The main results of the paper are given in (2.20), (2.22), (2.23), (2.41)–(2.43), (3.10), (3.12)–(3.17). We feel that it is a matter of some urgency that the relevant experiments are carried out by monitoring speed.

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