Heisenberg uncertainty of spatially gated electromagnetic fields

Vladimir Y. Chernyak, and Shaul Mukamel

Paper published as part of the special topic on Quantum Light

ARTICLES YOU MAY BE INTERESTED IN

Progress and challenges in ab initio simulations of quantum nuclei in weakly bonded systems
The Journal of Chemical Physics 154, 170902 (2021); https://doi.org/10.1063/5.0042572

Frontiers of stochastic electronic structure calculations
The Journal of Chemical Physics 154, 170401 (2021); https://doi.org/10.1063/5.0053674

Reflections on electron transfer theory
The Journal of Chemical Physics 153, 210401 (2020); https://doi.org/10.1063/5.0035434
Heisenberg uncertainty of spatially gated electromagnetic fields

Cite as: J. Chem. Phys. 154, 174110 (2021); doi: 10.1063/5.0045352
Submitted: 25 January 2021 • Accepted: 18 April 2021 •
Published Online: 7 May 2021

Vladimir Y. Chernyak\(^{1,2,a}\) and Shaul Mukamel\(^{3,4,a}\)

AFFILIATIONS
1 Department of Chemistry, Wayne State University, 5101 Cass Ave., Detroit, Michigan 48202, USA
2 Department of Mathematics, Wayne State University, 656 W. Kirby, Detroit, Michigan 48202, USA
3 Department of Chemistry, University of California, Irvine, California 92614, USA
4 Department of Physics and Astronomy, University of California, Irvine, California 92614, USA

Note: This paper is part of the JCP Special Topic on Quantum Light.

Authors to whom correspondence should be addressed: chernyak@chem.wayne.edu and smukamel@uci.edu

ABSTRACT

A Heisenberg uncertainty relation is derived for spatially-gated electric \(\Delta E\) and magnetic \(\Delta H\) field fluctuations. The uncertainty increases for small gating sizes, which implies that in confined spaces, the quantum nature of the electromagnetic field must be taken into account. Optimizing the state of light to minimize \(\Delta E\) at the expense of \(\Delta H\) and vice versa should be possible. Spatial confinements and quantum fields may alternatively be realized without gating by interaction of the field with a nanostructure. Possible applications include nonlinear spectroscopy of nanostructures and optical cavities and chiral signals.

Published under license by AIP Publishing. https://doi.org/10.1063/5.0045352

I. INTRODUCTION

The electric and the magnetic field operators \(\hat{E}(r)\) and \(\hat{H}(r')\) at two different points in space do not commute. This implies the existence of a Heisenberg uncertainty relation between them. In his 1927 Chicago lecture notes,\(^1\) Heisenberg had calculated this uncertainty for fields averaged over a box of size \(l\) and obtained \(\langle \Delta E \rangle \langle \Delta H \rangle > \hbar c l^2\). This implies that for a sufficiently small box, electromagnetic field fluctuations are strong enough so that the quantum nature of the fields may not be ignored. Heisenberg did not have a particular application in mind but was rather interested in clarifying a fundamental issue: the corpuscular vs wave nature of photons. Thanks to recent advances in nano-optics,\(^7\) this uncertainty may be tested experimentally in nanostructures. Here, we examine it for spatially gated electric and magnetic fields. An important consequence of our study is that spectroscopy of nanostructures may not be fully described by classical fields since this uncertainty may not be neglected. Spatially gated fields thus have an intrinsic quantum nature that should have experimental signatures.

To derive the Heisenberg uncertainty relation for spatially gated electric and magnetic fields, we introduce two vector gate functions, \(\eta\) and \(\gamma\), associated with the electric and magnetic fields that give rise to two gate function dependent gauge invariant Hermitian gated electric and magnetic field operators, defined by

\[
\hat{E}(\eta) = \int \, d\eta r \cdot \hat{E}(r), \quad \hat{H}(\gamma) = \int \, d\gamma r \cdot \hat{H}(r). \tag{1}
\]

In Eq. (2), we have used a standard notation \(\langle \Delta \hat{Q} \rangle = \langle \Delta \hat{Q} \rangle_{\psi}\) for the uncertainty of a Hermitian operator acting in Hilbert space (and thus defining an observable) in a quantum state \(\psi\).

\[
\langle \Delta \hat{Q} \rangle_{\psi} = \sqrt{\langle \psi | (\hat{Q} - \langle \hat{Q} \rangle_\psi)^2 | \psi \rangle}. \tag{3}
\]

Details of the derivation of Eq. (2), based on standard theory of light–matter interactions,\(^{5,6}\) are presented in Appendixes A and B.

The connection between the uncertainty relation and commutator depends on the particle statistics and holds for any system of bosons. It applies in our case since the EM field is a system of free...
bosons (photons). By applying a proper quantization procedure, we can compute the commutator in Eq. (2), resulting in

$$h(\eta, \gamma) = 4\pi c \int dr (\nabla \times \mathbf{H}(r)) \cdot \mathbf{y}(r) = -4\pi c \int dr (\nabla \times \mathbf{y}(r)) \cdot \eta(r).$$

(4)

In deriving Eq. (4), one should note the fact that the electromagnetic field is a system with first class constraints (using Dirac’s terminology) that give rise to gauge invariance or, stated differently, that photons have transverse polarization.

II. COMMUTATION RULES AND THE HEISENBERG UNCERTAINTY OF GAUSSIAN-GATED FIELDS

The contribution to the measurement of the electric or the magnetic field at point \( r\) in space comes from some small region around \( r\) due to the spatial error bar of a measurement device. Here, we describe this by using Gaussian gates for both electric and magnetic fields. To this end, we introduce a family of Gaussian approximations for the Dirac \( \delta \)-function,

$$\delta(r,l) \approx \frac{1}{\sqrt{2\pi}} \exp\left(-\left(\frac{r^2}{2l^2}\right)\right),$$

and the corresponding Gaussian-gated electric and magnetic fields,

$$E(r,l) = \int dr' \delta(r-r') E(r'), \quad H(r,l) = \int dr' \delta(r-r') H(r').$$

The Heisenberg uncertainty relation for \((\Delta E_l)(\Delta H_r)\) is completely determined by the commutator of the corresponding gated operators,

$$[\hat{E}_l(r,l),\hat{H}_r(r',l)] = 4\pi c \hbar \frac{\partial}{\partial r_m} \delta(r-r'; \sqrt{2}l),$$

$$\frac{\partial}{\partial r_m} \delta(r-r'; \sqrt{2}l) = -\frac{1}{\sqrt{2}l} \frac{1}{\sqrt{2}l} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(r-r')^2}{2l^2}\right),$$

(7)

and these immediately lead to the Heisenberg uncertainty relation

$$(\Delta E_l)(\Delta H_r) \geq 4\pi c \hbar \frac{1}{\sqrt{2\pi}} \frac{1}{l} \frac{1}{\sqrt{2l}} \left|r_m - r_m'\right| \exp\left(-\frac{(r-r')^2}{4l^2}\right).$$

(8)

To derive Eq. (7), we applied Eq. (A15) and computed \( h(\eta, \gamma) \) for Gaussian gate functions making use of Eq. (4). The Gaussian integrations are easily performed. A similar calculation shows that in the case of not necessarily identical Gaussian gates, the Heisenberg uncertainty relation for \((\Delta E_l)(\Delta H_r)\) still has a form of Eq. (8), with the gate size \( l \) in the rhs replaced by

$$l_{\text{eff}} = \sqrt{l^2 + (r')^2}. \quad (9)$$

Equation (8) provides a clear physical interpretation for the 1/l parameter in the Heisenberg’s uncertainty formula. Equation (8) depends on a single parameter: the gate size \( l \). The \((\hbar c)/(\sqrt{2l}) \) factor represents the 1/l Heisenberg’s parameter in energy units; the \(\sqrt{2} \) factor is specific to the Gaussian form of the gate. More precisely, the \((\hbar c)/(\sqrt{2l}) \) factor is the energy of a photon, whose wavelength is given by the gate size. The second factor \(\sqrt{\frac{2\pi}{\hbar}}\) is of the order of the inverse volume of the gate region. Their product thus represents the energy density of a photon, restricted to the gate region (according to the Heisenberg uncertainty principle for a photon, considered as a quantum particle). It is worth noting that \((\Delta E)(\Delta H)\) has units of energy density. The dimensionless product of the last two factors in Eq. (8) characterizes the overlap of the gates so that when \( |r-r'| \gg l \), the uncertainty vanishes. It is also interesting to note that at \( r=r' \), the uncertainty vanishes as well, provided that the gate profiles are identical. One way to look at this dependence is that even if the original field is classical and contains many photons, the nano-gated field may contain only a few photons, and for a sufficiently short gate, it will, therefore, show quantum effects.

To get a sense of the fluctuation magnitudes, we compare \((\Delta E)(\Delta H)\) for a Gaussian gate with \( l = 1 \) nm = \( 10^{-9} \) m with the \( EH \) product in a pulse with the intensity \( I = 10^{15} \) J s\(^{-1}\) cm\(^{-2}\) \( \approx 10^9 \) J s\(^{-1}\) m\(^{-2}\). The maximal value of the dimensionless function [the product of the last two factors in the rhs of Eq. (8)] is \( 1/\sqrt{c} \approx 0.6 \) so that we have for \( l = 1 \) nm = \( 10^{-9} \) m and \( hc \approx 2 \times 10^{-25} \) J m,

$$\frac{(\Delta E)(\Delta H)}{EH} \approx \frac{1}{\sqrt{2\pi}} \frac{1}{l} \frac{1}{\sqrt{2l}} \frac{1}{\sqrt{2\pi}} \frac{1}{\hbar c} \frac{1}{\sqrt{2l}} \approx \frac{1}{4\pi} \frac{2 \times 10^{-25} \text{ J} \cdot \text{m}}{10^{-26} \text{ J} \cdot \text{m}^2} \approx \frac{1}{4\pi} \frac{2 \times 10^{11} \text{ J} \cdot \text{m}^3}{2 \times 10^{11} \text{ J} \cdot \text{m}^3}. \quad (10)$$

Making use of the Poynting vector,

$$S = -\frac{c}{4\pi} [E,H]$$

so that \( I = S = \frac{c}{4\pi} EH \),

we have

$$EH = \frac{4\pi}{c} \frac{1}{3} \frac{10^{10} \text{ J} \cdot \text{s}^{-1} \cdot \text{m}^{-2}}{3 \times 10^6 \text{ m} \cdot \text{s}^{-1}} \approx 4\pi \frac{1}{3} \frac{10^{11} \text{ J} \cdot \text{m}^{-3}}{10^{10} \text{ J} \cdot \text{m}^{-3}} \approx 4\pi \frac{1}{3} \frac{10^{11} \text{ J} \cdot \text{m}^{-3}}{10^{10} \text{ J} \cdot \text{m}^{-3}} \approx \frac{1}{3} \frac{10^{11} \text{ J} \cdot \text{m}^{-3}}{10^{10} \text{ J} \cdot \text{m}^{-3}} \approx 6 \times 10^{-2}. \quad (12)$$

finally setting \( E = H \) for a plane wave and \( \Delta E = \Delta H \) for a coherent state of the electromagnetic field, we obtain

$$\frac{(\Delta E)(\Delta H)}{EH} \approx \frac{1}{4\pi} \frac{2 \times 10^{-25} \text{ J} \cdot \text{m}}{10^{-26} \text{ J} \cdot \text{m}^2} \approx 6 \approx 4 \times 10^{-2}. \quad (13)$$

Finally setting \( E = H \) for a plane wave and \( \Delta E = \Delta H \) for a coherent state of the electromagnetic field, we obtain

$$\frac{(\Delta E)(\Delta H)}{EH} \approx 0.2. \quad (14)$$

implying that for a nanometer gate, quantum effects are non-negligible even for a very strong pulse, which is usually thought of as classical.
III. UNCERTAINTIES OF THE GATED ELECTRIC AND MAGNETIC FIELDS FOR A COHERENT STATE

We now compute the uncertainties of the gated and electric and magnetic fields prepared in a coherent state. Since, in the coordinate representation, this state is simply a displaced ground state, the problem is reduced to computing the variance of the quantum fluctuations of the gated fields in vacuum, i.e., we have

$$\langle \Delta E(\eta)^2 \rangle = \langle \Omega | (\hat{E}\eta)^2 | \Omega \rangle, \quad \langle \Delta H(\gamma)^2 \rangle = \langle \Omega | (\hat{H}\gamma)^2 | \Omega \rangle. \quad (15)$$

The computation is carried out by introducing the photon creation/annihilation operators, see Appendix B for details.

Applying the continuum limit to Eqs. (B12) and (B13), we obtain

$$\langle \Delta E(\eta)^2 \rangle = \int \frac{dk}{(2\pi)^2} 2\pi \hbar c [(\hat{\eta}^*(k) \cdot \hat{\eta}(k)) $$

$$- k^2 (\hat{\eta}^*(k)) \cdot (\hat{\eta}(k))] , \quad (\Delta H(\gamma)^2) = \int \frac{dk}{(2\pi)^2} 2\pi \hbar c [(\hat{\gamma}^*(k) \cdot \hat{\gamma}(k)) $$

$$- k^2 (\hat{\gamma}^*(k)) \cdot (\hat{\gamma}(k))] . \quad (16) $$

Hereafter, we assume a Gaussian gate, i.e., $\eta(r) = u \delta(r; l)$ and $\gamma(r) = v \delta(r - r_0; l)$, with $\delta(r; l)$ given by Eq. (5), and $u$ and $v$ are unit vectors. A simple inspection of Eq. (16) shows that the uncertainties do not depend on a particular choice of $u$, $v$, and $r_0$ (reflecting the Poincare symmetry of the vacuum state), and, for our case, we have $\Delta H(\gamma) = \Delta E(\eta)$ so that we need only compute $\Delta E(\eta)$. Choosing the $z$-axis along $u$ and applying the spherical coordinates, we obtain upon substitution of the Fourier transform

$$\hat{\eta}(k) = e^{-i\hat{k}^2/2} \quad (17)$$

of $\delta(r; l)$ into Eq. (16)

$$\langle \Delta E(\eta)^2 \rangle = \frac{2\pi \hbar c}{(2\pi)^3} \int_0^\infty k^2 dk \int_{-\infty}^{2\pi} d\phi \int_0^\pi \sin \theta d\theta dk $$

$$\times (1 - \sin^2 \theta) e^{-\hat{\theta}^2/2} \Sigma^2 \frac{\hbar c}{2\pi} \int_{-\infty}^{2\pi} d\tau \int_0^\infty dk \tanh \frac{k L}{\hbar c} $$

$$= \frac{\hbar c}{3\pi \hbar c L^2} $$

so that

$$\langle \Delta E(\eta)^2 \rangle = \langle \Delta H(\eta)^2 \rangle = \frac{\hbar c}{3\pi L^2} . \quad (18)$$

In Sec. II, we identified the maximal value of the rhs of the Heisenberg uncertainty principle for the EM field for Gaussian gates of identical shape, with respect to the shift between the gates [see Eq. (10)], so that the uncertainty Heisenberg principle is, indeed, satisfied,

$$\langle \Delta E(\eta) \cdot \Delta H(\gamma) \rangle = \frac{\hbar c}{3\pi L^2} > \frac{\hbar c}{2\sqrt{2} \pi \sqrt{\epsilon} \theta^2 L^2} . \quad (19)$$

The commutator of $\hat{E}$ and $\hat{H}$ is a number that determines the lower bound of their uncertainty. This bound does not depend on the quantum state of the field and exists also in the vacuum state. We can then associate it with vacuum fluctuations. The $\Delta E$ and $\Delta H$ uncertainties do depend on the quantum state of light. We thus found that the vacuum state (and, hence, the coherent state) fluctuations are larger than the minimal uncertainty. This is similar to the results, reported in Ref. 7, that the minimal product of precision in intensity and degree of spatial localization is achieved not at coherent states but rather at the Epanechnikov distributions. Points of future interest include (i) finding a quantum state that satisfies the minimum uncertainty; (ii) investigating the connection of the presented uncertainty relations to the intensity vs spatial localization counterparts, explicitly considered in Ref. 7; and (iii) minimizing $\Delta E$ or $\Delta H$ to create squeezed fluctuations. Another future goal is to identify an experimental signal that is sensitive to this uncertainty, e.g., chirality in a nanostructure.

An alternative experimental realization of the present ideas is possible using nano-structures rather than spatial gating. Let us consider a setup whereby a nanosample interacts with a small part of a beam. The entire beam has many photons and is classical, but the relevant part of the beam that interacts with matter in this measurement is only a nanoslice that contains very few photons and should, therefore, be described by a quantum light. The measurement should thus show quantum light effects even though it employs a classical field! A simple analogy of this state of affairs exits in the time/frequency (rather than space) domain. When a short broadband pulse interacts with a system that has a narrow spectral line, only a few of its spectral components participate, and the experiment can be described by an effective narrow band (and long) pulse. The temporal resolution is eroded and not given by the pulse duration. The experiment may be then described by an effective pulse, which is different from the incoming pulse. This is the temporal analog of the spatial gating discussed here.

ACKNOWLEDGMENTS

V.Y.C. gratefully acknowledges the support of the U.S. Department of Energy, Office of Science, Materials Sciences and Engineering Division, Condensed Matter Theory Program. S.M. gratefully acknowledges the support of the National Science Foundation (Grant No. CHE-1953045).

APPENDIX A: QUANTIZATION OF THE ELECTROMAGNETIC FIELD AND THE HEISENBERG UNCERTAINTY RELATION

In this appendix, we derive the commutation relation [Eq. (4)] between the gated electric and magnetic field operators. We start with quantization of the electromagnetic field in vacuum, considered as a dynamical system. Its classical action is given by

$$S[A(r), A_0(r); \tilde{A}(r), \tilde{A}_0(r)] = \int dt L, \quad L = \frac{1}{8\pi} \int dr (E^2 - \tilde{H}^2), \quad (A1)$$

$$E = e^{-iA - \tilde{A}} A_0, \quad H_i = e^{-iA} \partial_i A_0, \quad \partial_j = \partial / \partial r_j . \quad (A2)$$
To switch to the (classical) Hamilton formalism, we identify the conjugate momenta as variational derivatives,

$$\pi_n(r) = \frac{\delta L}{\delta A_n(r)} = \frac{1}{4\pi\epsilon_0} E_n(r), \quad \pi_0(r) = \frac{\delta L}{\delta A_0(r)} = \frac{1}{4\pi\epsilon_0} E_0(r) = 0,$$

(A2)

and the second relation \(\pi_0(r) = 0\) indicates that we are dealing with a dynamical system with constraints. The classical Hamiltonian is obtained in a standard way,

$$\mathcal{H} = \int dr (\pi_n(r) A_n(r) + \pi_0(r) \cdot \dot{A}_0(r) + L)$$

$$= \frac{1}{8\pi} \int dr (\mathbf{E}^2 + \mathbf{H}^2) + \frac{1}{4\pi} \int dr E \cdot \partial A_0$$

$$= \frac{1}{8\pi} \int dr (\mathbf{E}^2 + \mathbf{H}^2) - \frac{1}{4\pi} \int dr A_0 \text{div} E,$$

(A3)

where we have made use of Eq. (A2). The Poisson bracket has a standard form

$$\{E_n(r), A_k(r')\} = 4\pi c\delta_{jk}\delta(r-r'), \quad \{E_0(r), A_0(r')\} = 4\pi c\delta(r-r').$$

(A4)

For the constraint \(E_0(r)\) to be preserved by the dynamics, we need to have \(E_0(r) = 0\), which combined with the Hamilton equation \(\dot{E}_0(r) = \{\mathcal{H}, E_0(r)\}\) and upon a direct computation of the rhs of the latter [making use of Eqs. (A3) and (A4)], yields \(\text{div} E(r) = 0\), referred to as the secondary constraints. Computation of the time derivative of the secondary constraints yields 0, meaning that there are no higher-order constraints so that the complete set of them is given by

$$E_0(r) = 0, \quad \text{div} E(r) = 0.$$

(A5)

Obviously, the Poisson bracket between constraint is zero so that all constraints are type-1 in Dirac’s classification.

Gauge invariant quantization is performed using a canonical quantization of a dynamical system with type-2 (weak) constraints. A state in the extended Hilbert space is represented by a wavefunction \(\Psi[A(r), A_0(r)]\) with the electric field operators naturally defined as variational derivatives,

$$\dot{E}_0(r) = -4\pi ci \hbar \frac{\delta}{\delta A_0(r)}, \quad \dot{E}_j(r) = -4\pi ci \hbar \frac{\delta}{\delta A_j(r)}.$$

(A6)

The constraints are applied in a weak sense, i.e., we introduce a physical subspace of the wavefunctions that satisfy the conditions \(E_0(r)\Psi = 0\) and \(\text{div} E(r)\Psi = 0\), or explicitly

$$\frac{\delta}{\delta A_0(r)} \Psi = 0, \quad \frac{\delta}{\delta A_j(r)} \Psi = 0,$$

(A7)

which means that a physical wavefunction is the one that does not depend on the scalar potential and the longitudinal component of the vector counterpart. A physical, or equivalently a gauge invariant, operator is the one that acts closely in the physical subspace.

To simplify the application of constraints, we switch to the momentum domain. To this end, we consider the space as a box of size \(L\) (with volume \(V\)) with periodic boundary conditions so that the quantization conditions for the photon wavevector are \(kL = 2\pi n\), allowing us to represent

$$A(r) = \frac{1}{\sqrt{V}} \sum_k A_k e^{ikr}, \quad A_k = \frac{1}{\sqrt{V}} \int dr A(r) e^{-ikr}.$$

(A8)

We further associate with each allowed \(k\) an orthonormal basis set \((e_k^{(\alpha)})_{\alpha = 1, 2, 3}\) with \(e_k^{(3)} = n = |k|^2 k\) and represent

$$A_k = \sum_{\alpha=1}^3 A_{\alpha,k} e_k^{(\alpha)}, \quad E_k = \sum_{\alpha=1}^3 E_{\alpha,k} e_k^{(\alpha)},$$

(A9)

resulting in

$$\{E_{\alpha,k}, A_{\beta,k'}\} = \frac{4\pi c}{V} \int dr dr' e^{ikr+ik' r'}$$

$$\times \{e_k^{(\alpha)} \cdot E(r), e_k^{(\beta)} \cdot A(r')\}$$

$$= \frac{4\pi c}{V} \int dr dr' e^{ikr+ik' r'} e_k^{(\alpha)} \cdot e_k^{(\beta)} \delta(r-r')$$

$$= 4\pi c e_k^{(\alpha)} \cdot e_k^{(\beta)} \delta_{k,k'},$$

(A10)

Applying the constraints, we have \(E_k^{(3)} = 0\) so that

$$E(r) = \frac{1}{\sqrt{V}} \sum_k \sum_{\alpha=1}^3 E_{\alpha,k} e_k^{(\alpha)},$$

$$H(r') = \frac{i}{\sqrt{V}} \sum_k \sum_{\beta=1}^3 [e_k^{(\beta)}] \cdot e_k^{(\alpha)} A_{\beta,k'},$$

(A11)

and denoting with \((u_j | j = 1, 2, 3)\) a lab frame, we compute

$$\{E_j(r), H_i(r')\} = \{u_j \cdot E(r), u_i \cdot H(r')\}$$

$$= \frac{i}{V} \sum_{k'} \sum_{\alpha\beta} [e_k^{(\alpha)}] \cdot u_j \{[e_k^{(\beta)}] \cdot u_i\}$$

$$\times e^{ikr+ik' r'} \{E_{\alpha,k}, A_{\beta,k'}\}$$

$$= -\frac{4\pi ci}{V} \sum_k \sum_{\alpha\beta} [e_k^{(\alpha)}] \cdot u_j \{[e_k^{(\beta)}] \cdot u_i\}$$

$$\times [e_k^{(\alpha)} \cdot e_k^{(\beta)}] e^{ikr-ik' r'},$$

(A12)

where the summation in Eq. (A12) runs over \(\alpha, \beta = 1, 2\). We further compute

$$\sum_{\alpha\beta} [e_k^{(\alpha)}] \cdot u_j \{[e_k^{(\beta)}] \cdot u_i\}$$

$$= \sum_{\alpha\beta} (e_k^{(\alpha)} \cdot u_j) ([u_i, e_k^{(\beta)}] \cdot e_k^{(\alpha)} - e_k^{(\alpha)} \cdot e_k^{(\beta)})$$

$$= (\sum_{\alpha} (u_i \cdot e_k^{(\alpha)} e_k^{(\alpha)}) \cdot \sum_{\beta} ([u_i, e_k^{(\beta)}] \cdot e_k^{(\beta)}))$$

$$= (u_i \cdot [u_i, k]) = [u_i, u_i] = k = \epsilon_{ijmn}(u_m \cdot k) = \epsilon_{ijmn}k_m.$$

(A13)
Note that in the first expression in the second line in Eq. (A13), the summation runs over \( \alpha, \beta = 1, 2 \), whereas to obtain the next equality in the chain, the summation has to be extended to its full range \( \alpha, \beta = 1, 2, 3 \). This is, indeed, allowed since the summand with \( \beta = 3 \) turns out to 0, which also implies that the scalar product of the summand with \( \alpha = 3 \) with the sum over \( \beta \) also turns out to zero so that the summations can be safely extended to their full ranges. Upon the substitution of Eq. (A13) into Eq. (A12), we obtain

\[
\langle E(r) H_i(r') \rangle = -4\pi c \epsilon_{ijm} \frac{\partial}{\partial r_m} \delta(r - r') \\
= -4\pi c \epsilon_{ijm} \frac{\partial}{\partial r_m} \delta(r - r'). \tag{A14}
\]

Finally, we make use of Eq. (A14) to compute the Poisson bracket of the gated variables,

\[
\{E(\eta), H(\gamma)\} = -4\pi c \epsilon_{ijm} \int d^3r d^3r' \eta_i(r') \gamma_j(r) \frac{\partial}{\partial r_m} \delta(r - r') \\
= -4\pi c \epsilon_{ijm} \int d^3r d^3r' \delta(r - r') \frac{\partial \eta_i(r)}{\partial r_m} \gamma_j(r) \\
= 4\pi c \int d^3r \frac{\partial \eta_i(r)}{\partial r_m} \gamma_j(r) \\
= 4\pi c \int d^3r \text{curl} \eta(r) \cdot \gamma(r) = h(\eta, \gamma). \tag{A15}
\]

with \( h(\eta, \gamma) \) given by Eq. (4).

Upon canonical gauge invariant quantization of the electromagnetic field, Eq. (A15) provides the expression for the commutator of the gated operators, given by Eq. (2).

**APPENDIX B: UNCERTAINTIES OF THE GATED ELECTRIC AND MAGNETIC FIELDS IN A COHERENT STATE: DETAILS**

We start with representing the EM field operators in terms of the creation/annihilation operators,

\[
\hat{A}_{a,k} = u_k a_{a,k} + u_k^* a_{a,-k}^\dagger, \quad \hat{E}_{a,k} = v_k a_{a,k} + v_k^* a_{a,-k}^\dagger, \tag{B1}
\]

together with a choice \( \epsilon^{(a)}_k = \epsilon^{(a)}_k, \quad u_{a,k} = u_k \), and \( u_{a,-k} = u_k \) so that the conditions \( \hat{A}_{a,-k} = \hat{A}_{a,k}^\dagger \) and \( \hat{E}_{a,-k} = \hat{E}_{a,k}^\dagger \) that reflect the real nature of the electromagnetic field are satisfied. Upon quantizing the expression for the Poisson bracket [Eq. (A10)] and postulating the commutation relations for the photon operators, we obtain

\[
[E_{a,k}, \hat{A}_{\beta,k'}] = -4\pi \hbar c \epsilon^{(a)}_k \epsilon^{(b)}_k \delta_{k,k'} \delta_{\alpha,\beta}, \quad [a_{a,k}, a_{\beta,k'}^\dagger] = \delta_{\alpha,\beta} \delta_{k,k'}. \tag{B2}
\]

and by substituting Eq. (B1) into Eq. (B2), we obtain

\[
\{\hat{E}_{a,k}, \hat{A}_{\beta,k'}\} = -4\pi \hbar c \epsilon^{(a)}_k \epsilon^{(b)}_k \delta_{k,k'} \delta_{\alpha,\beta}. \tag{B3}
\]

We further have

\[
\hat{E}_{a,k} \hat{E}_{a,-k} = v_k^* v_k (a_{a,k} a_{a,-k} + a_{a,-k}^\dagger a_{a,k}^\dagger) + v_k^* v_k + (v_k^* a_{a,k} a_{a,-k} + (v_k^* a_{a,-k}^\dagger a_{a,k}^\dagger)),
\]

\[
\hat{A}_{a,k} \hat{A}_{a,-k} = u_k^* u_k (a_{a,k} a_{a,-k} + a_{a,-k}^\dagger a_{a,k}^\dagger) + u_k^* u_k + (u_k^* a_{a,k} a_{a,-k} + (u_k^* a_{a,-k}^\dagger a_{a,k}^\dagger)). \tag{B4}
\]

and the Hamiltonian \( \hat{H} \) of the EM field in vacuum reads

\[
\hat{H} = \sum_k \sum_a \hat{H}_{a,k}, \tag{B5}
\]

with

\[
\hat{H}_{a,k} = \frac{1}{8\pi} (v_k^* v_k + k^2 u_k^* u_k) (a_{a,k} a_{a,-k} + a_{a,-k}^\dagger a_{a,k}^\dagger) + \frac{1}{8\pi} (v_k^* v_k + k^2 u_k^* u_k) + \frac{1}{8\pi} (v_k^* v_k + k^2 u_k^* u_k) a_{a,k} a_{a,-k}^\dagger + \frac{1}{8\pi} (v_k^* v_k + k^2 u_k^* u_k) a_{a,-k}^\dagger a_{a,k}. \tag{B6}
\]

Combining the condition that the unwanted terms [the last two terms in Eq. (B6)] disappear with Eq. (B3), we obtain a system of two equations,

\[
v_k^2 + k^2 u_k^2 = 0, \quad v_k^2 u_k^* - v_k u_k^* = -i\hbar c, \tag{B7}
\]

whose solution

\[
u_k = \sqrt{\frac{2\pi \hbar c}{k}}, \quad v_k = -i\sqrt{2\pi \hbar c k} \tag{B8}
\]

results in the explicit expressions,

\[
A_{a,k} = \sqrt{\frac{2\pi \hbar c}{k}} (a_{a,k} + a_{a,-k}^\dagger), \quad E_{a,k} = -i\sqrt{2\pi \hbar c k} (a_{a,k} - a_{a,-k}^\dagger). \tag{B9}
\]

Upon substitution of Eq. (B8) into Eqs. (B5) and (B6), we obtain the standard expression for the Hamiltonian \( \hat{H} \) of the EM field, recast in terms of the photon creation/annihilation operators,

\[
\hat{H} = \hat{E}_0 + \sum_k \sum_a \hbar c a_{a,k} a_{a,k}, \quad \hat{E}_0 = \frac{1}{2} \sum_k \sum_a \hbar c k. \tag{B10}
\]

Upon the substitution of Eqs. (A11) and (B9) into Eq. (1), we obtain

\[
\hat{H} = \frac{1}{2} \sum_k \sum_a \hbar c k + \frac{1}{2} \sum_k \sum_a \hbar c k + \frac{1}{2} \sum_k \sum_a \hbar c k. \tag{1}
\]
\[ \hat{E}(\eta) = \int d\eta \cdot E(r) = \frac{1}{\sqrt{V}} \sum_{k \alpha} \int d\eta \cdot a^{(\alpha)} E_{\alpha k} \]
\[ = -i \sum_{k \alpha} \langle \eta_{-k} \cdot a^{(\alpha)} \rangle i \sqrt{2\pi} \hbar c \kappa (a_{\alpha k} - a_{\alpha k}^\dagger) \],
\[ (\text{B11}) \]
and further, making use of Eq. (B11) and applying abbreviated notation \((\bullet) = \langle \Omega| \bullet |\Omega \rangle\), we obtain
\[ (\Delta E(\eta))^2 = \langle (\hat{E}(\eta))^2 \rangle \]
\[ = - \sum_{k \alpha k' \alpha'} \sum_{k \alpha \alpha'} (\eta_{-k} \cdot a^{(\alpha)})(\eta_{-k} \cdot a^{(\alpha')})(\bar{\eta}_{k'} \cdot a^{(\alpha)})(\bar{\eta}_{k'} \cdot a^{(\alpha')}) \sqrt{2\pi} \hbar c \kappa \sqrt{2\pi} \hbar c \kappa' \delta_{kk'} \delta_{\alpha \alpha'} \]
\[ = \sum_{k} 2\pi \hbar c \kappa (\sum_{k} (\eta_{-k} \cdot a^{(\alpha)})(\eta_{k} \cdot a^{(\alpha)}) \]
\[ = \sum_{k} 2\pi \hbar c \kappa (\eta_{-k} \cdot k^{-2} (\eta_{-k} \cdot k))(\eta_{k} \cdot k^{-2} (\eta_{k} \cdot k)) \]
\[ = \sum_{k} 2\pi \hbar c \kappa (\eta_{-k} \cdot k^{-2} (\eta_{-k} \cdot k))(\eta_{k} \cdot k) \]
\[ = \sum_{k} 2\pi \hbar c \kappa (\eta_{-k} \cdot k^{-2} (\eta_{-k} \cdot k))(\eta_{k} \cdot k). \]
\[ (\text{B12}) \]
A similar computation for the uncertainty of the magnetic field yields
\[ (\Delta H(\eta))^2 = \sum_{k} 2\pi \hbar c \kappa ((\eta_{k}^* \cdot k) - k^{-2} (\eta_{k}^* \cdot k)(\eta_{k} \cdot k)). \]
\[ (\text{B13}) \]
These result in Eq. (16) in the main text.

**DATA AVAILABILITY**

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

**REFERENCES**

1. W. Heisenberg, *The Physical Principles of the Quantum Theory* (University of Chicago Press, Dover, 1930); N. Bohr and L. Rosenfeld, *Phys. Rev.* 78, 794 (1950).
2. L. Novotny, *Principles of Nano-Optics* (Cambridge University Press, 2006).
3. L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, 1995).
4. D. L. Andrews et al., *I. Chem. Phys.* 148, 040901 (2018).
5. P. A. M. Dirac, *Lectures on Quantum Mechanics* (Dover Publications, 2001).
6. A. Salam, *Non-Relativistic QED Theory of the Van der Waals Dispersion Interactions* (Springer, 2016).
7. T. E. Gureyev et al., *Sci. Rep.* 7, 4542 (2017); T. E. Gureyev et al., *Sci. Rep.* 10, 7890 (2020).
8. A. L. Kuznetsov et al., *Science* 354, 2472 (2016).
9. J. Zeng et al., *ACS Nano* 12(12), 12159–12168 (2018).
10. A. Fast and E. O. Potma, *Nanophotonics* 8(6), 991–1021 (2019).
11. J. Langer et al., *ACS Nano* 14, 28–117 (2020).
12. E. A. Pozzi et al., *Chem. Rev.* 117, 4961–4982 (2017).
13. D. Polli et al., *Phys. Rev. A* 82, 053809 (2010).