Quantum anomalous energy effects on the nucleon mass

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Apart from the quark and gluon kinetic and potential energies, the nucleon mass includes a novel energy of pure quantum origin resulting from anomalous breaking of scale symmetry. We demonstrate the effects of this quantum anomalous energy (QAE) in QED, as well as in a toy 1+1 dimensional non-linear sigma model where it contributes non-perturbatively, in a way resembling the Higgs mechanism for the masses of matter particles in electro-weak theory. The QAE contribution to the nucleon mass can be explained using a similar mechanism, in terms of a dynamical response of the gluonic scalar field through Higgs-like couplings between the nucleon and scalar resonances. In addition, the QAE sets the scale for other energies in the nucleon through a relativistic virial theorem, and contributes a negative pressure to confine the colored quarks.

proton mass, quantum anomaly, spontaneous symmetry breaking, scale symmetry

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1 Introduction

Mass is one of the most important and fundamental properties of a physical system. For macroscopic ones, mass is simply the sum of individual parts. The additive rule works with a high degree of accuracy up to atomic nuclei, for which the masses are the sum of each individual nucleon’s mass (proton and neutron) subtracting the binding energy effects, à la Einstein. Physically, the binding energy takes into account the quantum-mechanical average of the nucleons’ kinetic energy as well as the interacting potential energies between them (see for example ref. [1]). In the electroweak theory, the origin of the masses of elementary particles has a different paradigm: They arise from these particles’ interactions with the Higgs potential, which acquires a vacuum condensate after the well-known spontaneous electroweak symmetry breaking, or the Higgs mechanism [2].

The generation of nucleon mass is a combination effect of the Higgs mechanism for the quarks and the inner workings of the fundamental theory of strong interactions, the quantum chromodynamics (QCD). In numerical simulations of QCD on the lattice, the nucleon mass along with masses of other hadrons has been routinely calculated to a very high degree of accuracy [3-6]. However, to unravel its physical content, one has to analyze the sources of QCD energy according to the famous equation $M = E/c^2$. Since the original
work by one of the authors of the current paper [7, 8], several papers that calculated the QCD Hamiltonian operator’s matrix elements on the lattice have appeared in the literature [9-12]. Alternative structures of the QCD Hamiltonian have also been proposed and analyzed [13-16]. Proposals have been made to measure the anomalous gluon matrix element through heavy-quarkonium electroproduction on the nucleon at Jefferson Lab and future Electron-Ion Collider [17-19].

The goal of this paper is to further establish the existence and physical significance of the QCD trace anomaly contribution to the nucleon mass [7]. Although the connection between the trace anomaly and the nucleon mass as well as the related low-energy theorems has been well studied in the literature [20, 21], the anomalous contribution to the QCD energy itself is a less familiar concept. We consider the role of this quantum anomalous energy (QAE) in quantum electrodynamics (QED), where perturbation theory is well-established. We argue that the QAE holds the key to the relationship between the trace anomaly and the nucleon mass as well as the role of this quantum anomalous energy (QAE) in quantum electrodynamics (QED), where perturbation theory is well-established. We argue that the QAE holds the key to the QCD nucleon mass generation in two important ways: First, it sets the scale for the quark and gluon kinetic and potential energies through a relativistic version of virial theorem. Second, its contribution to the mass is non-perturbative and bears analogy to the Higgs mechanism for the masses of elementary particles in the standard model [2, 22].

2 Scalar and tensor energy and relativistic virial theorem

In classical mechanics, the virial theorem provides an equation that relates the average kinetic and potential energies of a stable system of discrete particles, bound by potential forces, with that of the total potential energy of the system. In quantum field theories (QFTs), a similar relation exists between the matrix elements of the scalar and tensor parts of the Hamiltonian operator, \( H = \int d^4x T^\mu_\nu(x) \), where \( T^\mu_\nu \) is the symmetric energy-momentum tensor. Since any symmetric second-order tensor can be decomposed into irreducible representations \((1,1) + (0,0)\) of the Lorentz group \( T^\mu_\nu = \tilde{T}^\mu_\nu + \tilde{T}^\mu_\nu \), we can correspondingly decompose the Hamiltonian into the tensor and scalar parts:

\[
H = H_T + H_S, \tag{1}
\]

where \( H \) is a conserved charge and thus ultra-violet (UV) renormalization scale-independent. The separate parts, \( H_T \) and \( H_S \), are UV scale-independent as well, a consequence of Lorentz symmetry. The space-time symmetry further dictates a relation between average of the tensor and scalar energies in any stationary state \((\mathbf{P} = 0)\):

\[
E_T = (d - 1) E_S, \tag{2}
\]

where \( E_{T,S} = \langle H_{T,S} \rangle \) is a quantum average in the zero momentum hadron state, and \( d = 4 \) is the space-time dimension. This has been referred to as a virial theorem in QFT [7].

The above relation has to do with the virial theorem in classical physics and may be seen heuristically as follows. In classical mechanics, the virial theorem relates the average kinetic energy \((K)\) and the total potential energy \((V)\) of a system, with the exact coefficient depending on the scaling property of the potential under scale transformation \( r \rightarrow \lambda r \), where \( \lambda \) is a scale factor. In QFT, the behavior of a system under scale transformation depends on the dilatation current \( j^\alpha_\mu = x^\alpha T^\mu_\alpha \), which has a divergence \( \partial_\mu j^\alpha_\mu = T^\alpha_\alpha \sim d H_S \). Thus the scalar energy density, \( H_S \), comes entirely from scale-breaking effects. In gauge theories such as QED and QCD, \( H_T \) reads

\[
H_T = \int d^{d-1}x \left[ \bar{\psi} \left( i \gamma \cdot \mathbf{D} + \frac{d - 1}{d} m \right) \psi + \frac{1}{2} \left( \mathbf{E}^2 + \mathbf{B}^2 \right) \right], \tag{3}
\]

where \( \psi \) is a fermion field with \( m \) as its mass, \( \mathbf{E} \) and \( \mathbf{B} \) are the electric and magnetic fields, and \( D \) is a covariant derivative. Thus the tensor energy \( E_T \) includes the familiar kinetic and potential energies of particles and fields. Eq. (2) can be interpreted as the scalar energy \( E_S \) sets a scale for the tensor energy \( E_T \) that grows linearly with space dimension \( d - 1 \).

The most familiar scale-breaking effect is the mass terms such as \( m^2 \psi \) for Dirac fields \( \psi \) or \( \mu^2 \phi^2 \) for scalar field \( \phi \). In the case of QED, the electron mass is the only mechanical scale that, together with the dimensionless coupling \( \alpha \sim 1/137 \), sets the energy scale for atomic physics, chemistry, and biology. In cases like 1+1 dimensional QCD, the dimensional coupling introduces scalar energy as well [23]. An important mechanism to generate a new source of scale-breaking is through the condensates of scalar fields in the ground states of QFTs. This mechanism has been used to provide masses of elementary particles in the standard model (the Higgs mechanism) [2], and energy needed for the inflationary universe as well as a mechanism for cosmic dark energy [24].

It is easy to check that in the non-relativistic limit, the above equation for the positronium system in 3+1 QED reduces to the known virial theorem in quantum mechanics. Indeed in Coulomb gauge \( \nabla \cdot \mathbf{A} = 0 \), and in the leading component of the non-relativistic positronium state, one can show that \( E_S = \frac{1}{4} (2m - \langle T \rangle) \) and \( E_T = \frac{3}{4} (2m - \langle T \rangle) + 2\langle T \rangle + \langle V \rangle \). Therefore, eq. (2) reduces to

\[
2\langle K \rangle + \langle V \rangle = 0, \tag{4}
\]
which is nothing but the non-relativistic virial theorem. It is easy to check that this statement is actually gauge invariant.

3 Quantum anomalous energy and its perturbative role in QED

The QAE is a novel scale-breaking energy source arising from short-distance quantum fluctuations. The UV physics caused by these quantum effects cannot be completely removed but leaves a trace through running of the coupling constants and associated composite scalar fields as an anomalous contribution to Hamiltonian that represents those “residue memories” of the microscopic world.

To see this new energy emerging from scaling breaking, we consider a re-scaling in time direction, \( t \to (1 + \delta t) t \). In this case, the cutoff in the temporal direction needs to change accordingly and results in asymmetric cutoff theory. The anomalous contribution to the Hamiltonian can be derived by studying the response of the system under the re-scaling, \( S \to S + \delta t \int dtH \) [25]. For example, for the classical gauge theory, in terms of the time re-scaled field variables \( A'_\mu(t',x) = A_\mu(t,x) \), the action transforms to

\[
\frac{1}{2g_0} \int d^4 x \left( \frac{1}{1 + \delta t} (\mathbf{B'})^2 + (1 + \delta t)(\mathbf{E'})^2 \right).
\]

The contribution at order \( \delta t \) is nothing but the Hamiltonian \( H = \int d^4 x \frac{1}{2g_0} (\mathbf{B}^2 + \mathbf{B}^2) \) integrated over time, consistent with the general principle above. However, in quantum theory and in the presence of a cutoff in the temporal direction, such as a lattice cutoff, the \( g_0 \) in eq. (5) must depend on \( \delta t \) in order to be equivalent to the time translated theory [26]. Therefore, the derivative of \( g_0 \) with respect to \( \delta t \) will generate anomalous terms in addition to the canonical energy, which has been shown to be exactly the anomalous contribution [27]:

\[
H_a = \frac{1}{4} \int d^3 \mathbf{x} \left( \frac{\beta(g_0)}{2g_0} F^2 + m_0 \gamma_m \bar{\psi} \psi \right), \tag{6}
\]

where \( F^\mu_\nu \) is the gauge field strength with coupling \( g_0 \), which arises from logarithmic running of couplings \( m_0 \) and \( g_0 \) with the UV cut-off scale, as reflected in the beta function \( \beta(g_0) \) and the mass anomalous dimension \( \gamma_m \). This agrees with the original mass decomposition in ref. [7], in which the role of the trace anomaly is explicit. The mass decomposition in ref. [16], although technically correct in dimensional regularization, missed the key role that the anomalous term plays. We call the expectation value of \( H_a \) in a state, \( E_a \), the quantum anomalous energy.

There is an anomalous energy contribution to the QED Hamiltonian. However, owing to the perturbative nature of the theory, this anomalous energy does not bring in any new scale, and its contribution is embedded in covariant perturbation theory and is proportional to the electron mass. The electron pole mass \( m_e \) at the one-loop level receives a contribution from the mass anomalous dimension term, \( \langle e|H_a|e \rangle = \frac{3m_e}{\pi \alpha} \). In an external field \( \mathcal{A} \), the anomalous energy contains a mixing term between the external field and the radiative field. As such, this will contribute to Lamb shift for hydrogen atoms which is at order \( \alpha \) in radiative field. A calculation shows that to leading order in radiative correction, the anomalous part leads to the contribution

\[
\langle n, j|H_a|n, j \rangle = \frac{\alpha^2}{6\pi} \int d^3 \mathbf{x} \left( u_{n,j}(\mathbf{x}) u_{n,j}(\mathbf{x}) \right) + \frac{3\alpha^3}{8\pi} E_{n,j}, \tag{7}
\]

for the energy level \( n, j \), where \( n \) is the radial quantum number and \( j \) is the total spin. The first term is the photonic contribution, while the second term is the fermionic contribution. Here \( u_{n,j}(\mathbf{x}) \) is the quantum-mechanical wave function that solves the Dirac equation in a static Coulumb field, and \( E_{n,j} \) is the bound state energy. In the non-relativistic limit, eq. (7) can be further expanded in \( \alpha \) by using explicit form of \( E_{n,j} \) [28] and contains contributions at \( O(\alpha) \), \( O(\alpha^3) \) and \( O(\alpha^5) \). To obtain the first term in eq. (7) we need to use the Feynmann-Hellman theorem to evaluate the average of \( \frac{1}{Z} \).

The \( O(\alpha) \) and \( O(\alpha^3) \) contributions will be cancelled by other terms, while the contribution at \( O(\alpha^5) \) reads

\[
\langle n, j|H_a|n, j \rangle^{(5)} = -\frac{7m_e\alpha^5}{24\pi\alpha^3} \left( \frac{3}{8} - \frac{1}{2j + 1} \right). \tag{8}
\]

This contributes to the famous Lamb shift at \( O(\alpha^5) \).

4 Non-perturbative QAE and dynamical Higgs mechanism in 1+1 nonlinear sigma model

In more interesting cases, the QAE will generate a non-perturbative contribution characterized with a new mass scale (dimensional transmutation [29]). On the other hand, the anomalous scalar field can be considered as a dynamical one, and the QAE contribution to the mass comes from its dynamical response to the matter, in analogy to the Higgs mechanism for fermion masses in the standard model.

To see this analogy, let’s first review the Higgs mechanism for matter particles with a simplified scalar field \( \Phi \) with potential \( V(\Phi) = -\frac{\nu^2}{2} \Phi^2 + \frac{1}{4} \Phi^4 \) that couples to a massless fermion \( \Psi \) through Yukawa interaction \(-g \bar{\Psi} \Phi \). At the tree level, the scalar \( \Phi \) develops a condensate at \( \langle \Phi \rangle^2 = \frac{\nu^2}{\lambda} \), which gives the fermion mass interaction \( -g \Psi \Phi \Phi \).

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to the fermion mass through the intermediate Higgs particle:
\[
\langle \Psi | H_3 | \Psi \rangle = (-g) f_s/m_\nu^2 = (1/4) m_\nu, \quad \text{where } f_s = -\frac{1}{2} \mu^2(\Phi) \text{ is a scalar decay constant and } m_\nu = \sqrt{\beta} \mu \text{ is the Higgs mass. The } \frac{1}{m_\nu^2} \text{ is due to the zero-momentum propagator of the Higgs field. Therefore, the scalar part of the Hamiltonian contributes 1/4 of the fermion mass through the dynamical Higgs. See Figure 1 for a depiction of the mechanism. The example demonstrates that the scalar part of the mass of the fermions can also be measured by the response of the fluctuating part of the scalar field in the presence of the matter.}

In the above model, the Higgs scalar field is introduced at the Lagrangian level “by hand”. However, in case that the scalar field emerges dynamically, the scenario remains qualitatively the same. We use the example of the 1+1 dimensional nonlinear sigma model in the large N limit to demonstrate it. The model consists of an N component scalar field \( \sigma = (\sigma^1, ..., \sigma^N) \) that lives in the unit N−1 sphere \( \sum_{i=1}^N \sigma^i \sigma^i = 1 \) with the action
\[
S = \frac{1}{2g_0^2} \int d^2 x \sum_{i=1}^N (\partial_\mu \sigma^i)(\partial_\mu \sigma^i),
\]
where \( g_0 \) is a dimensionless coupling. The theory is the scale-invariant at the classical level, but the scale symmetry is broken by an UV cut-off at the quantum level and the coupling becomes scale dependent. The model can be analytically solved in the large-N limit where the coupling \( \lambda_0 = g_0^2 N \) stays finite [30, 31]. The constraint can be removed by adding an auxiliary term to the action,
\[
S_\Sigma = \frac{i}{2g_0^2} \int d^2 x \left( \sum_{i=1}^N \sigma^i \sigma^i - 1 \right).
\]

By integrating out the \( \sigma^i \) fields, an effective potential for \( \Sigma \) can be generated, which has a saddle point \( \langle \Sigma \rangle = -i \lambda_0^2 \) and introduces a new mass scale. The condensate \( \lambda_0^2 \) satisfies the gap equation
\[
\frac{1}{g_0^2 N} \int \frac{d^2 k}{(2\pi)^2} \frac{1}{k^2 + m^2} = \frac{1}{4\pi} \ln \frac{\Lambda_{\text{UV}}^2}{m^2},
\]
which determines the bare \( \lambda_0 \) as a function of \( m \) and the UV cutoff \( \Lambda_{\text{UV}} \). Therefore, the theory is asymptotically free with the beta function \( \beta(g_0) = -\frac{Ng_0^4}{4\pi} \). Furthermore, the vacuum condensate for \( \Sigma \) generates a mass \( m \) for \( \sigma \), and the spectrum consists of \( N \) massive scalars \( \sigma \) with equal mass.

The anomalous part of the Hamiltonian is
\[
H_a = -\frac{\beta(g_0)}{2g_0} \int d^4 x \sum_{i=1}^N (\partial_\mu \sigma^i)(\partial_\mu \sigma^i) + \frac{m^2}{2} \langle \sigma \rangle.
\]

Although in the limit of \( g_0 \to 0 \), \( \frac{\beta(g_0)}{2g_0} = -\frac{Ng_0^4}{4\pi} \) is proportional to \( g_0^2 \). \( H_a \) does not vanish. The operator \( (\partial_\mu \sigma^i)^2 \) receive quantum fluctuations from the loop diagram proportional to
\[
\int \frac{d^4 k}{(2\pi)^4} \frac{k^2}{(k^2 + m^2)^2},
\]
which diverges logarithmically. It can be shown that the contribution to the \( \sigma \) mass is always 1/2 independent of regularization scheme,
\[
\langle \sigma^i | H_a | \sigma^i \rangle = \frac{m^2}{2}
\]
which is consistent with the virial theorem in eq. (2).

The QAE contribution to the meson mass can be explained in terms of a dynamical Higgs mechanism as follows. Using the equation of motion, the anomalous Hamiltonian can also be re-written in terms of the auxiliary scalar
\[
H_a = -\frac{iN m^2}{8\pi} \int d^4 x, \quad (15)
\]
where the dimensionless scalar \( \sigma = (\Sigma - \langle \Sigma \rangle)/m^2 \) contains the quantum fluctuation part. This is similar to the Higgs example above, in that the scalar part of the Hamiltonian is linear in the sigma field. Its contribution to the pion mass is determined by \( \langle \sigma^i | H_a | \sigma^i \rangle \). By using the \( \tau \pi \sigma \) vertices in eq. (10), and the dominance of the zero-momentum \( \sigma \) propagator \( \langle \sigma(0) \sigma(0) \rangle = 8\pi/(N m^2) \) in the intermediate state, the response of the scalar \( \sigma \) to \( \pi \) state exactly makes \( H_a \) contributing \( 1/4 \) of the \( \pi \) mass. We shall mention that the propagator of \( \sigma \) [31] contains only a cut starting at the two-\( \pi \) threshold \( p^2 = 4m^2 \) but no poles, unlike the Higgs field \( h \) in the previous example. Nevertheless, the zero-momentum propagator of \( \sigma \) contributes to the average of the anomalous Hamiltonian in exactly the same way as the zero-momentum propagator of the Higgs field \( h \).

5 Dynamical scalar and QAE contribution to the nucleon mass and pressure

For simplicity, we consider the limiting case of massless up and down quarks. The anomalous Hamiltonian comes entirely from the gluon composite scalar, \( H_a = \int d^3 x \Phi(x), \)
where $\Phi(x) = \beta(g)/(8\pi)F^{\mu\nu}F_{\mu\nu}(x)$. As in the non-linear sigma model, its contribution to the nucleon mass can be seen as a form of dynamical Higgs mechanism, which is consistent with that the Higgs and confining phases of matter-coupled gauge theory are smoothly connected [32, 33].

It is useful to recall that for the infinite-heavy $Q\bar{Q}$ state separated by $r$ in pure gauge theory, it has been shown [13, 27] that the non-perturbative contribution of $H_\sigma$ to the static potential is $\frac{1}{2}(V(r) + rV'(r))$. At large $r$ where the confinement potential dominates $V(r) \sim \sigma r$, the anomalous contribution is exactly one half of the confinement potential.

The scalar field $\Phi(x)$ has a vacuum condensate $\Phi_0 = \langle 0|\Phi|0 \rangle$ [34, 35]. However, in the presence of the nucleon, the quantum response is measured by

$$\phi(x) = \Phi(x) - \Phi_0,$$

which is a dynamical version of the MIT bag-model constant $B$ [22]. Its contribution to the nucleon mass can be seen as the response of the scalar field to the nucleon source,

$$E_s = \langle \phi \rangle_N = \langle N|\phi(x)|N \rangle,$$

where the nucleon state is normalized as $\langle N|N \rangle = (2\pi)^3\delta^3(0)$. If $\phi(x)$ is a static constant $B$ inside the nucleon, $E_s$ will be of order $BV$, where $V$ is the effective volume in which the valence quarks are present.

The static response of the composite gluon scalar $\phi$ in the nucleon state can be measured in the electro-production of heavy quarkonium on the proton [17, 36-41] or leptoproduction of heavy quarkonium at large photon virtuality [42]. The color dipole from the quarkonium will be an effective probe of the $F^2$. This also provides a direct determination of the QAE contribution to the mass.

An interesting mechanism for its physics is to consider a dynamical response of the $\phi$ in the presence of the nucleon through a tower of scalar $0^+$ spectral states, as in the Higgs model. Assuming an effective coupling between the nucleon and scalar $g_{NN\phi}\bar{N}N\phi$, the QAE contribution to the mass can be related to the scalar field response function,

$$\langle N|\phi|N \rangle = ig_{NN\phi}\langle \phi(0)\phi(0) \rangle,$$

where $\langle \phi(0)\phi(0) \rangle$ is the zero-momentum propagator of the scalar field $\phi$. If the propagator is dominated by a series of scalar resonances, or $\langle \phi(0)\phi(0) \rangle = \sum_s \frac{f_s^2}{m_s^2}$, one has

$$\langle N|\phi|N \rangle = \sum_s \frac{g_{NN\phi}f_s}{m_s^2},$$

where $m_s$ is the mass of the scalar resonances, $f_s = \langle \phi(0) \rangle$ is the decay constant and $g_{NN\phi} = g_{NN\phi}f_s$ is the coupling of the nucleon to the scalars. See Figure 2 for a depiction.

One might assume the dominance of the lowest mass scalar glueball-like state, generically called $\sigma$, for the above equation. If the coupling constant $g_{NN\phi}$ can be extracted through experiment, one can perform a consistency check on the $\sigma$ dominance picture by combining the glueball masses and the decay constants extracted from lattice QCD calculations [9, 43]. In fact, for the lowest glueball state $\sigma$, low-energy theorem predicts [44] that $f_\sigma = m_\sigma \sqrt{|\Phi_0|}$. Given this relation and assuming the sigma dominance, we predict that $g_{NN\sigma} = \frac{m_\sigma}{4|\Phi_0|}$ in the chiral limit. This then exactly corresponds to the Higgs model mentioned earlier. Of course, the QCD reality is in between the simple Higgs and the 1+1 sigma models. With massive quarks, the proton mass will receive contributions both from the dynamical Higgs mechanism and the traditional Higgs mechanism. And the scalar glueball-like particles in QCD are resonance-like states instead of stable particles. However, we expect the coupling between the nucleon (or any other hadrons) and the scalars to be proportional to the masses which can be tested by experiments, the same as in the Higgs case which has been tested recently at LHC [45-47].

For pion state, it has been shown in ref. [7] that $H_\sigma$ contributes to $\frac{1}{2}$ of the total pion mass. Assuming the $\sigma$ dominance, the effective coupling between the pion and the scalar glueball $g_{\sigma\pi\pi}$ is again proportional to the pion mass, what consistent with a dynamical Higgs effect.

Finally, the $\phi$ contributes a negative mechanical pressure to the trace part of the energy-momentum tensor [8], just like what the cosmological constant does in Einstein’s gravity theory [24]. The physics of this has been well explored in the context of MIT bag model [22]. Its contribution confines the colored quarks and cancels the positive quarks and gluon contributions, which are measurable through deeply virtual Compton scattering [48-50].

6 Conclusion

To conclude, the mass of the nucleon contains a quantum
anomalous contribution which sets the scale for other types of contributions such as quark and gluon kinetic and potential energies. This contribution has a physical mechanism similar to the Higgs model, with a dynamical scalar field generating a response, having the characteristic feature that the coupling to the scalars is proportional to the fermion mass. Furthermore, it contributes a negative pressure to confine the colored quarks.

In preparation of the paper, there appeared another calculation of anomalous energy contribution to hydrogen atom mass [51]. Their result differs from ours by a factor of 2.

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