Individual popularity and activity in online social systems

Haibo Hu\textsuperscript{a}, Dingyi Han\textsuperscript{b}, Xiaofan Wang\textsuperscript{a}

\textsuperscript{a}Complex Networks and Control Lab, Shanghai Jiao Tong University, Shanghai 200240, China
\textsuperscript{b}Apex Data and Knowledge Management Lab, Shanghai Jiao Tong University, Shanghai 200240, China

Abstract

We propose a stochastic model of web user behaviors in online social systems, and study the influence of attraction kernel on statistical property of user or item occurrence. Combining the different growth patterns of new entities and attraction patterns of old ones, different heavy-tailed distributions for popularity and activity which have been observed in real life, can be obtained. From a broader perspective, we explore the underlying principle governing the statistical feature of individual popularity and activity in online social systems and point out the potential simple mechanism underlying the complex dynamics of the systems.

\textbf{Key words:} Popularity, Activity, Online social system

\textbf{PACS:} 89.65.-s, 89.75.-k, 05.45.Tp

1. Introduction

Currently the WWW is undergoing a landmark revolution from the traditional Web 1.0 to Web 2.0 characterized by social collaborative technologies, such as social networking site, blog, Wiki and folksonomy. The social Web (or more specifically, online social systems), a label that includes both social networking sites (such as MySpace and Facebook) and social media sites (such as Digg, CiteULike and Flickr), is changing the way content is created and distributed. Web-based authoring tools enable users to rapidly publish content, from stories and opinion pieces on weblogs, to photographs and videos on Flickr and YouTube, to advice on Yahoo!Answers, and to web discoveries on Del.icio.us and Furl. The availability of large-scale electronic databases has delivered us extraordinary new insights on the human behaviors and human dynamics on the web. The clear patterns and regularities in individual distributions in respect of popularity and activity in some online social systems have been revealed [1-8].

Evidently web users vary widely in their activity levels. Take Digg as example, some users casually browse the front page, voting on one or two stories. Others spend hours a day combing the web for new stories to submit, and voting on stories they found on Digg. Also different items on the web vary widely in their popularity. Some stories can attract large attention and their influence can last for a long time while most stories only can attract very little attention and their impact vanishes rapidly. In social media sites a tag-cloud is usually used to visualize the popularity of items, or more specifically, tags. Typical tag-clouds have between 30 and 150 tags. The popularity is represented using font sizes, colors or other visual clues. Fig. 1 shows a tag-cloud with terms related to Web 2.0.

Recently much attention has been devoted to investigating the statistical feature of individual popularity and activity in online social systems. Their distributions show the wide-spread believed power law or general heavy-tailed ones intermediate between exponential and power law, such as stretched exponential or log-normal [2, 5, 9]. Despite the great progress made, little work is done on the underlying mechanism governing the statistical feature of popularity and activity in online social systems, which will be explored in the work.

We can start our analysis from a time-ordered table of item assignments. For the system as a whole, we can define an intrinsic time $T$ as the index of an item assignment into such a table, so that $T$ runs from 1 to the number of total item assignments. The temporal process shown in Fig. 2 can be regarded as the process of appearance of entity $u_i$ or $m_i$. And the frequency of occurrence for some user/item in the total $T$ events can be defined as its activity/popularity.
Thus activity measures how frequently a user performs a specific action, such as listening to music, seeing films, browsing posts and sending friendship invitations to other users on the web, and popularity measures how frequently an item (such as music, films, posts and tags) is visited by web users. Note that for items we can only measure their popularity, while for users, in some cases, we can measure not only their activity but also popularity. For instance in social networks, users can invite other users to be their friends. Thus we can measure the activity of users in terms of the number of sent invitations, and can also measure the popularity of users in terms of the number of received invitations.

It is natural that the number of distinct items or users \( N \) increases with \( T \), however different growth patterns can appear. Generally \( N(T) \propto T^\gamma \), where \( \gamma < 1 \) implies sub-linear growth while \( \gamma = 1 \) linear growth, i.e. the generation probability of new individuals is a constant value (homogeneous Poisson process). Besides the more frequently an individual appears, the more possibly the individual will appear once again. Specifically, when an old individual joins the sequence, the probability that it will be a specific old individual is \( k_i \). The preference metric \( 0 \leq \beta < 1 \) implies sub-linear preference while \( \beta = 1 \) linear preference. The case where \( N(T) \propto T \) and \( \beta = 1 \) corresponds to the classic Simon model [10, 11].

When an old individual joins the sequence, the probability that individual \( i \) with frequency \( k_i \) is selected can be expressed as \( \Pi (k_i) = \frac{k_i^\beta}{\sum_j k_j^\beta} \). Thus we can compute the probability \( \Pi (k) \) that an old individual of frequency \( k \) is chosen, and it is normalized by the number of individuals of frequency \( k \) that exist just before this step [12, 13]:

\[
\Pi (k) = \frac{\sum [e_i = v \land k_i(t - 1) = k]}{\sum [\{u : k_u(t - 1) = k\}] - k^\beta, \text{ where } e_i = v \land k_i(t - 1) = k \text{ represents that at time } t \text{ the old individual whose frequency is } k \text{ at time } t-1 \text{ is chosen. We use } [\cdot] \text{ to denote a predicate (take value of 1 if expression is true, else 0). Generally } \Pi (k) \text{ has significant fluctuations, particularly for large } k. \text{ To reduce the noise level, instead of } \Pi (k) \text{ we can study the cumulative function to obtain the preference metric } \beta: k(k) = \int_0^k \Pi (k)dk \sim k^{\beta+1}.
\]

2. Model

Consider that users are listening to music. At a discrete time step \( T \), a new user may appear with probability \( \alpha \), whereas with probability \( 1 - \alpha \) an existing old user can appear. We can apply the mean field method to analytically obtain the probability distribution for individual popularity and activity. When \( 0 < \beta < 1 \), we have

\[
\frac{\partial k_i}{\partial t} = (1 - \alpha) \frac{k_i^\beta}{\sum_j k_j^\beta} \tag{1}
\]
According to
\[
\begin{align*}
\sum_j k_j^0 &= t\alpha \\
\sum_j k_j^1 &= t
\end{align*}
\]  
(2)
and
\[
\sum_j k_j^0 = \mu t \quad (\alpha < \mu < 1),
\]
We obtain
\[
\frac{\partial k_i}{\partial t} = (1 - \alpha) \frac{k_i^\beta}{\mu t}.
\]
(4)
The initial condition is \(k_i(t_i) = 1\), where \(t_i\) is the time when the individual \(i\) appeared for the first time. Thus
\[
k_i^{1-\beta}(t) = 1 + \ln \left(\frac{t}{t_i}\right)^{(1-\beta)(\frac{\mu}{1-\alpha})}
\sim (1 - \beta) \left(\frac{1 - \alpha}{\mu}\right) \ln t
\]
and
\[
P(k_i(t) < k) = P \left[ t_i > t \cdot \exp \left( -\frac{k_i^{1-\beta} - 1}{1 - \beta} \cdot \frac{\mu}{1 - \alpha} \right) \right].
\]
(6)
The probability density function for \(t_i\) is \(P_i(t) = \alpha / (1 + t)\) and thus
\[
P(k_i(t) < k) = 1 - P \left[ t_i \leq t \cdot \exp \left( -\frac{k_i^{1-\beta} - 1}{1 - \beta} \cdot \frac{\mu}{1 - \alpha} \right) \right]
\]
\[
= 1 - \frac{t\alpha}{1 + t} \exp \left( -\frac{k_i^{1-\beta} - 1}{1 - \beta} \cdot \frac{\mu}{1 - \alpha} \right).
\]
(7)
The probability distribution \(P(k)\) for individual popularity and activity is
\[
P(k) = \frac{\partial P(k_i(t) < k)}{\partial k}
\]
\[
= \frac{\alpha t}{1 + t} \cdot k^\beta \cdot \exp \left( -\frac{k^{1-\beta} - 1}{1 - \beta} \cdot \frac{\mu}{1 - \alpha} \right),
\]
(8)
which is a stretched exponential distribution. Its complementary cumulative distribution functions (CCDF) is \(P_c(k) = \exp[-(k/k_0)^c]\), where \(c = 1 - \beta\) and \(k_0\) is a constant. When \(\beta \rightarrow 1\), lim \(\beta \rightarrow 1\frac{t^{1-\beta} - 1}{1 - \beta} = \ln k, \mu = 1,\) and
\[
P(k) = \frac{t\alpha}{(1 + t)(1 - \alpha)}, k^{-1 + \frac{1}{1 - \alpha}}.
\]
(9)
which is a power law distribution. Its CCDF is \(P_c(k) \sim k^{-\nu}\), where \(\nu = 1/(1 - \alpha)\). The special situation of absent preference \(\beta = 0\) reduces Eq. (8) to an exponential distribution. Generally the stretched exponential distribution is correlative with sub-linear preference while power law distribution linear preference [14, 15].

3. Results and discussion

We ground our empirical analysis on actual log data extracted from an online media site Comic [5] and an online social network Wealink [6]. Note that our approach of investigation is also applicable to other online social systems. Comic is located in a large Chinese university with more than 40,000 undergraduate and graduate students, and only is

\[1\] http://comic.sjtu.edu.cn/music.asp
\[2\] http://www.wealink.com
Figure 3: The CCDFs for users’ activity (a) and music’s popularity (b) in Comic. A stretched exponential distribution will show a straight line if we use $\ln P_c(k)$ as $x$-axis and $k'$ as $y$-axis. The solid lines represent the fitted lines.

Figure 4: The CCDFs of users’ activity and popularity in Wealink. Both distributions have a power law tail.

accessible to the IP addresses within the university. We recorded its visiting log from October 25th, 2006 to February 6th, 2007 in the data format: time $t_i$/user ID number $u_i$/music ID number $m_i$, i.e. a user $u_i$ listened to a song $m_i$ at time $t_i$. Users were distinguished by their IP addresses. The total number of log we obtained is 2,136,149, the number of different music recorded is 98,747 (mostly popular songs), and the number of users recorded is 8472.

Wealink is a large social networking site in China whose users are mostly professionals, typically businessmen and office clerks. Each registered user has a profile, including his/her list of friends. For privacy reasons, the data, logged from May 11th, 2005 (the inception day for the Internet community) to August 22nd, 2007, include only each user’s ID and list of friends, and the time of sending and accepting friendship invitations. The final data format is time $t_i$/user ID number $u_i$/user ID number $v_i$/flag $s_i$. $s_i$ can take value of 0 or 1. $s_i=0$ indicates that at time $t_i$ a user $u_i$ invited another user $v_i$ to be his/her friend while $s_i=1$ indicates that at time $t_i$ user $u_i$ accepted user $v_i$’s invitation. During our data collection period, there are 273,395 sent invitations and more than 99.9% have been accepted. The total number of users recorded is 223,482. Like most social networking sites, in Wealink, only when the sent friendship invitations are accepted, can the inviters and receivers become online friends. We can measure users’ activity and popularity in terms of their numbers of sent and received invitations.

In Comic, individual activity and popularity can be well described by stretched exponential distribution, which is shown in Fig. 3. While in Fig. 4, the distribution of users’ activity and popularity in Wealink has a power law tail.
We can compare the distributions of individual activity and popularity in real data with the predicted by the stochastic model. Fig. 5 shows the $\kappa$ versus $k$ for music and users in Comic. For these two cases, the sub-linear preferential selection hypothesis can offer a good approximation. The values of $\beta$ for users and music are approximately 0.61 and 0.79, respectively. For the CCDF of users’ activity, our model gives $c = 1 - \beta \approx 0.39$ and the empirical distribution gives $c \approx 0.34$. Fig. 6 shows the $\kappa$ versus $k$ for users in Wealink. Approximately $\beta \approx 1$ for users’ activity and popularity, indicating linear preference. The appearance probabilities $\alpha$ of new users in the time-ordered lists of users of sending and receiving invitations are 0.53 and 0.35, respectively. For the CCDF of users’ activity, the model gives $\nu = 1/(1 - \alpha) \approx 2.13$, while for users’ popularity the model gives $\nu = 1.54$. The power law exponents achieve proper agreement with the empirical results in Fig. 4.

Fig. 7 shows the growth of the numbers of different users/music $N$ in Comic and users $N$ in Wealink with $T$. The traditional assumption, as applied in the previous deduction, is that the generation probability of new individuals is a constant value, i.e. $N(T) \propto T$. However as shown in Fig. 7, the hypothesis is unrealistic to some extent. For the Wealink users, approximately the slopes for senders and receivers are 1.09 and 0.97, respectively, however for the users/music in Comic, the growth lines show several segments with different slopes. In some cases the number of distinct items $N$ introduced by users after $T$ assignments can grow approximately as $N(T) \propto T^\gamma$ with $\gamma < 1$. When dealing with the evolution of the number of attributes pertaining to some collection of objects, this sub-linear growth is generally referred to as Heaps’ law [16]. As an example, sub-linear behavior has been observed in the
growth of vocabulary size in texts, i.e. in the number of different words in a text as a function of the total number of words observed while scanning through it. For the case of English corpora, vocabulary growth exponents in the range $0.4 < \gamma < 0.6$ have been reported [17].

The rate at which new items appear at time $T$ scales as $dN(T)/dT \sim T^{-\gamma}$. That is, new items appear less and less frequently, with the invention rate of new items monotonically decreasing towards zero. The approach to zero is however so slow that the cumulated number of items, asymptotically, does not converge to a constant value but is unbounded - assuming the observed trend stays valid.

Different users or items with distinct activity or popularity may have quite different $\gamma$. Recent research on the collaborative tagging system del.icio.us reveals that for less and less popular resources being bookmarked, the distribution of growth exponent $P(\gamma)$ of distinct tags gets broader and its peak shifts towards higher values of $\gamma$, indicating that the growth behavior is becoming more and more linear [3].

Table 1 summarizes the probability distributions of popularity and activity for different patterns of growth and preference. For sub-linear growth and linear preference, the recent research shows that when the rate at which new items appear $dN(T)/dT \ll 1$, the distribution can be approximately viewed as a power law $P(k) \sim k^{-1-\gamma}$ [18]. For sub-linear growth and sub-linear preference, unfortunately the analysis for probability distribution can lead to a rather intractable relation whose analytical solution is hard to obtain. Qualitatively in this case the distribution is still a fat-tailed one intermediate between exponential and power law. For some sub-linear growth exponent, the distribution resulted from sub-linear preference will be more homogeneous than that (power law) resulted from linear preference; while for some sub-linear preference exponent, the distribution resulted from sub-linear growth will be more heterogeneous than that (stretched exponential distribution) resulted from linear growth.

The distributions of individual popularity and activity in many online social systems can follow generic heavy-
tailed ones, unnecessarily power law [19-24]. Several aspects of the underlying intricate dynamics may be responsible for the feature. Except sub-linear preference discussed above, another possible origin is the memory effect, that is, newly appeared individuals will appear more frequently than old ones. For example web users tend to listen to recently added music or apply recently added tags more frequently than old ones, which may be equivalent to the ageing effect of individuals. The popularity or activity of an entity will inevitably undergo a decaying process. Users become less active and items become less attractive over the time [25-29].

According to growth and preference characteristic, it is possible to predict the amount that would be devoted over time to given ones by measuring the data at an early time. However the method does not consider the semantics of popularity and why some items become more popular than others [30]. That is, popularity prediction in the presence of a large table of item assignments can essentially be made based on the observed early time series, while semantic analysis of content may be more useful when no early click-through information is known. Semantic attraction can lead to the initial prevalence of items and subsequent preferential selection strengthens the popularity.

Acknowledgments

We thank the anonymous reviewers for their constructive remarks and suggestions which helped us to improve the quality of the manuscript to a great extent. This work was partly supported by the NSF of PRC under Grant No. 60674045.

References

[1] R. Lambiotte, M. Ausloos, Phys. Rev. E 72 (2005) 066107.
[2] S. Sinha, R. K. Pan, How a ‘hit’ is born: The emergence of popularity from the dynamics of collective choice, in: B. K. Chakrabarti, A. Chakrabarti, A. Chatterjee (Eds.), Econophysics and Sociophysics: Trends and Perspectives, Wiley-VCH, Berlin, 2006, pp. 417-447.
[3] C. Cattuto, A. Baldassarri, V. D. P. Servedio, V. Loreto, arXiv:0704.3316.
[4] C. Cattuto, V. Loreto, L. Pietronero, Proc. Natl. Acad. Sci. U. S. A. 104 (2007) 1461-1464.
[5] H. B. Hu, D. Y. Han, Physica A 387 (2008) 5916-5921.
[6] H. Hu, X. Wang, Physics Letters A 373 (2009) 1105-1110.
[7] A. Capocci, G. Caldarelli, J. Phys. A: Math. Gen. 41 (2008) 224016.
[8] F. Benevenuto, F. Duarte, T. Rodrigues, V. Almeida, J. Almeida, K. Ross, arXiv:0804.4865.
[9] T. Zhou, H. A. T. Kiet, B. J. Kim, B. H. Wang, P. Holme, Europhys. Lett. 82 (2008) 28002.
[10] G. U. Yule, Phil. Trans. R. Soc. Lond. B 213 (1925) 21-87.
[11] H. A. Simon, Biometrika 42 (1955) 425-440.
[12] A. L. Barabási, H. Jeong, Z. Nédé, E. Ravasz, A. Schubert, T. Vicsek, Physica A 311 (2002) 590-614.
[13] H. Jeong, Z. Nédé, A. L. Barabási, Europhys. Lett. 61 (2003) 567-572.
[14] P. L. Krapivsky, S. Redner, F. Leyvraz, Phys. Rev. Lett. 85 (2000) 4629-4632.
[15] B. Frenesleben de Blasio, A. Svensson, F. Liljeros, Proc. Natl. Acad. Sci. U. S. A. 104 (2007) 10762-10767.
[16] H. S. Heaps, Information Retrieval: Computational and Theoretical Aspects, Academic Press, New York, 1978.
[17] D. Harman, Overview of the third text retrieval conference, in: D. K. Harman (Eds.), Proc. Third Text REtrieval Conference (TREC-3), NIST Special Publication 500-226, 1995, pp. 1-20.
[18] D. H. Janette, M. A. Montemurro, J. Quant. Linguist. 12 (2005) 29-40.
[19] S. Whittaker, L. Terveen, W. Hill, L. Cherry, The dynamics of mass interaction, in: Proc. ACM Conf. Computer-Supported Cooperative Work, ACM Press, New York, 1998, pp. 257-264.
[20] P. Holme, C. R. Edling, F. Liljeros, Social Networks 26 (2004) 155-174.
[21] S. A. Goldor, D. Wilkinson, B. A. Huberman, arXiv:cs/0611137.
[22] G. Ghoshal, P. Holme, Physica A 364 (2006) 603-609.
[23] Y. Y. Ahn, S. Han, H. Kwak, S. Moon, H. Jeong, Analysis of topological characteristics of huge online social networking services, in: Proc. 16th Int. World Wide Web Conf., ACM Press, New York, 2007, pp. 835-844.
[24] J. Leskovec, E. Horvitz, Planetary-scale views on a large Instant-Messaging network, in: Proc. 17th Int. World Wide Web Conf., ACM Press, New York, 2008, pp. 915-924.
[25] S. N. Dorogovtsev, I. F. F. Mendes, Phys. Rev. E 62 (2000) 1842.
[26] F. Wu, B. A. Huberman, Proc. Natl. Acad. Sci. U. S. A. 104 (2007) 17599-17601.
[27] A. Grabowski, N. Kruszewska, R. A. Kosiński, Eur. Phys. J. B 66 (2008) 107-113.
[28] Z. K. Zhang, L. Lü, J. G. Liu, T. Zhou, Eur. Phys. J. B 66 (2008) 557-561.
[29] R. Crane, D. Sornette, Proc. Natl. Acad. Sci. U. S. A. 105 (2008) 15649-15653.
[30] G. Szabo, B. A. Huberman, arXiv:0811.0405.