Effect of an extrinsic curvature on a quark–hadron phase transition

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Abstract
The last phase transition predicted by the standard model of particle physics took place at the QCD scale $T \approx 200$ MeV when the universe was about $t \approx 10^{-5}$ s old and the Hubble radius was around 10 km. In this paper, we consider the quark–hadron phase transition in the context of braneworld cosmology where our universe is a 3-brane embedded in an $m$-dimensional bulk and localization of matter on the brane is achieved by means of a confining potential. We study the behavior of the physical quantities relevant to the description of the early universe such as the energy density, temperature and scale factor, before, during and after the phase transition and investigate the effects of an extrinsic curvature on the cosmological phase transition. We show that the braneworld effects reduce the effective temperature of the quark–gluon plasma and of the hadronic fluid. Finally, we discuss the case where the universe evolved through a mixed phase with a small initial supercooling and monotonically growing hadronic bubbles.

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1. Introduction
In the recent past, models with extra dimensions have been proposed in which the standard fields are confined to a four-dimensional (4D) world viewed as a hypersurface (the brane) embedded in a higher dimensional spacetime (the bulk) through which only gravity can propagate. The most well-known model in the context of braneworld theory is that proposed by Randall and Sundrum (RS). In the so-called RSI model [1], they proposed a mechanism to solve the hierarchy problem with two branes, while in the RSII model [2], they considered a single brane with a positive tension, where 4D Newtonian gravity is recovered at low energies even if the extra dimension is not compact. This mechanism provides an alternative
to compactification of extra dimensions. The cosmological evolution of such a brane universe has been extensively investigated and effects such as a quadratic density term in the Friedmann equations have been found [3, 4]. This term arises from the imposition of the Israel junction conditions which is a relationship between the extrinsic curvature and energy–momentum tensor of the brane and results from the singular behavior in the energy–momentum tensor. In brane theories, the covariant Einstein equations are also derived by projecting the bulk equations onto the brane [5, 6]. This was first done by Shiromizu et al [5] where the Gauss–Codazzi equations together with Israel junction conditions were used to obtain the Einstein field equations on the 3-brane. This method has predominantly been used in theories with one extra dimension. If the number of extra dimensions exceeds 1, no reliable method for confining matter to the brane exists. This is so since the requirement to define junction conditions is the existence of a boundary (brane) which cannot be defined if the number of extra dimensions is more than 1. For example, a boundary surface in a 3D space is a surface with one less dimension whereas a line in the same space cannot be considered as its boundary. Given such concerns, model theories have been proposed where matter is confined to the brane through the action of a confining potential, without the use of any junction condition or $Z_2$ symmetry [7]. In [8, 9] the authors used the confining potential approach to study a braneworld embedded in an $m$-dimensional bulk. The field equations so obtained on the brane contained an extra term which was identified with the X-cold dark matter (XCDM). The dynamics of test particles confined to a brane by the action of such a potential at the classical and quantum levels were studied in [10]. In [11], a Friedmann–Robertson–Walker (FRW) braneworld model was studied, offering a geometrical explanation for the accelerated expansion of the universe. The same methodology was used in [12] to find the spherically symmetric vacuum solutions of the field equations on the brane. These solutions were shown to account for the accelerated expansion of the universe and offered an explanation for the galaxy rotation curves. The classical tests stemming from a braneworld with a confining potential have been studied in [13].

During the evolution of the very early universe there have been at least two phase transitions. The electroweak theory predicts that at about 100 GeV there was a transition from a symmetric high temperature phase with massless gauge bosons to the Higgs phase, in which the $SU(2) \times U(1)$ gauge symmetry is spontaneously broken and all the masses in the model are generated. A detailed understanding of the nature and dynamics of this transition is a very difficult task, and a lot of quantitative analytical and numerical studies have been performed over the years. One of the motivations for these studies has been that non-perturbative effects might have changed the baryon asymmetry in the phase transition. Also, the quantum chromodynamics (QCD) prediction that at about 200 MeV there was a transition from a quark–gluon plasma to a plasma of light hadrons, is thought to have occurred early in the history of the universe. The physical conditions for this phase transition to take place, date it back to a few microseconds after the big bang, when the universe had a mean density of the same order as nuclear matter ($\rho \sim 10^{15}$ g cm$^{-3}$). The quark–hadron transition marks the end of the exotic physics of the very early universe and the beginning of the era of processes and phenomena which have a direct counterpart in the high energy experiments now being carried out with modern accelerators. It is also the last of the early-universe phase transitions (at least within the standard picture) and so could be relevant both as a potential filter for the relics produced by previous transitions and also as a best candidate for the production of inhomogeneities which could have survived to later epochs [14].

As is well known, phase transitions are called first or second order depending on whether the position of the vacuum state changes discontinuously or continuously as the critical temperature is reached. A first-order quark–hadron phase transition in the expanding universe
can be described generically as follows [15]. The cooling down of the color deconfined quark–gluon plasma below the critical temperature, believed to be around $T_c \approx 150$ MeV, makes it energetically favorable to form color confined hadrons (mainly pions and a tiny amount of neutrons and protons, since the net baryon number should be conserved). However, the new phase does not show up immediately. A first-order phase transition generally needs some supercooling to overcome the energy expended in forming the surface of the bubble and the new hadron phase. When a hadron bubble is nucleated, latent heat is released and a spherical shock wave expands into the surrounding supercooled quark–gluon plasma. This causes the plasma to reheat, approaching the critical temperature and preventing further nucleation in a region passed by one or more shock fronts. Bubble growth is generally described by deflagrations where a shock front precedes the actual transition front. The nucleation stops when the whole universe has reheated $T_c$. The phase transition corresponding to this phase ends promptly, in about 0.05 $\mu$s, during which the cosmic expansion is completely negligible. Afterward, the hadron bubbles grow at the expense of the quark phase and eventually percolate or coalesce. Ultimately, when all quark–gluon plasma has been converted into hadrons, neglecting possible quark nugget production, the transition ends. The physics of the quark–hadron phase transition and its cosmological implications have been extensively discussed in the framework of general relativistic cosmology in [16–25].

As was mentioned above, the Friedmann equation in braneworld scenario contains deviations from 4D cosmology. We expect this deviation from the standard 4D cosmology to have noticeable effects on the cosmological evolution, especially on cosmological phase transitions. In the context of braneworld models, the first-order phase transitions have been studied in [26] where it has been shown that due to the effects coming from higher dimensions, a phase transition requires a higher nucleation rate to complete and baryogenesis and particle abundances could be suppressed. Recently, the quark–hadron phase transition has been studied in a braneworld scenario by assuming that the phase transition is of the first order. It has been shown that the braneworld effects lead to an overall decrease of the temperature of the very early universe and accelerate the transition to the pure hadronic phase [27]. It would therefore be of interest to study the latter in a braneworld model with a constant curvature bulk without using the $Z_2$ symmetry and without postulating any form of junction conditions [8, 9]. In so doing, the gravitational field equations on the brane are modified by a local extra term, $Q_{\mu\nu}$. Using the modified Friedmann equation and the equation of state of matter, we study the effects of the extrinsic curvature on the quark–hadron phase transition.

2. The model

Let us start by presenting the model used in our calculations. We only state the results and refer the reader to [9, 11] for a detailed derivation of these results.

As was mentioned in section 1, the braneworld model we invoke here differs from the usual Randall–Sundrum type, in that no junction conditions or $Z_2$ symmetry is used. One thus starts with the usual setup in which a 4D brane is embedded in a five or, in general, $m$-dimensional bulk. Assuming that the bulk space has constant curvature, one arrives at the following equations [9]:

$$G_{\mu\nu} = \kappa_4^2 T_{\mu\nu} - \Lambda g_{\mu\nu} + Q_{\mu\nu},$$

where $\Lambda$ is the effective cosmological constant in 4D, $T_{\mu\nu}$ is the energy–momentum tensor of the matter confined to the brane and $Q_{\mu\nu}$ is a completely geometrical quantity given by

$$Q_{\mu\nu} = \frac{1}{\epsilon} \left[ (K^\rho_{\mu}K^\rho_{\nu} - K K_{\mu\nu}) - \frac{1}{2} (K_{a\beta}K^{a\beta} - K^2) g_{\mu\nu} \right].$$
We note that $Q_{\mu\nu}$ is an independently conserved quantity, that is
\[ Q_{\mu\nu}^{\mu\nu} = 0, \]  
so that equation (1) satisfies the covariant conservation law. Equation (1) is the starting point from which a class of solutions was found in [9]. Thus, starting with the metric
\[ ds^2 = -dt^2 + a(t)^2 [dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)], \]
and calculating the components of the extrinsic curvature from the Codazzi equation, the gravitational field equations on the brane become
\[ \left( \frac{\dot{a}}{a} \right)^2 = \frac{k^2}{3} \rho + \frac{\Lambda}{3} + \frac{1}{\kappa} b_0^2 a^{-3(1+w_x)}, \]  
\[ \frac{\ddot{a}}{a} = -\frac{k^2}{6} (\rho + 3p) + \frac{\Lambda}{6} + \frac{1}{\kappa} b_0^2 \frac{1}{a^3} \frac{d}{dt} a^2 \frac{d}{dt} \left( \frac{b}{a} \right), \]
\[ \dot{\rho} + 3 \frac{\dot{a}}{a} (\rho + p) = 0. \]
Now, using the geometrical energy–momentum tensor for $Q_{\mu\nu}$ (XCDM), the Friedmann equations (5) and (6) can be written as (for more details see [9])
\[ \left( \frac{\dot{a}}{a} \right)^2 = \frac{k^2}{3} \rho + \frac{\Lambda}{3} + \frac{1}{\kappa} b_0^2 \frac{1}{a^3} \frac{d}{dt} (a^2 \left( \frac{b}{a} \right)), \]
\[ \frac{\ddot{a}}{a} = -\frac{k^2}{6} (\rho + 3p) + \frac{\Lambda}{6} + \frac{1}{\kappa} b_0^2 (1 + 3w_x) a^{-3(1+w_x)}, \]
where $b_0$ is a constant of integration and $w_x$ is a constant appearing in the equation of state, $p_x = w_x \rho_x$. As can be seen, the correction term with respect to the standard Friedmann equation is given by the components of the extrinsic curvature.

For the sake of completeness, let us compare the model presented in this work to the usual braneworld models where the Israel junction conditions are used to calculate the extrinsic curvature in terms of the energy–momentum tensor and its trace on the brane. If we did that we would obtain
\[ \frac{b(t)}{a} = -\frac{1}{6} \kappa^2 \rho a^{-2}, \]  
which, upon its substitution into equation (5), gives
\[ \left( \frac{\dot{a}}{a} \right)^2 = \frac{k^2}{3} \rho + \frac{\Lambda}{3} + \frac{1}{\kappa} b_0^2 \frac{1}{3} \rho^2, \]
which is the same as equation (6) in [27]. On the other hand, for the perfect fluid with energy density $\rho = \rho_0 a^{-3(1+w_x)}$, equation (8) reduces to the Friedmann equation (11) in the usual braneworld models if we take $b_0 = -\frac{1}{6} \kappa^2 \rho_0$ and $w_x = 1 + 2w$.

3. The quark–hadron phase transition

We are going to consider phase transition in the context of the braneworld model without mirror symmetry or any form of the junction condition. In a cosmological setting, these could be modified because of the modified Friedmann equations. Let us first outline the relevant physical quantities of the quark–hadron phase transition which will be used in the following sections. In order to study the quark–hadron phase transition we should specify the equation of state of the matter, in both quark and hadron states. In this regard, a well-written and
concise review can be found in [27] and the interested reader should consult it. Here, it will suffice to mention the results relevant to our study and leave the details of the discussion to the said reference.

For a first-order phase transition in the quark phase the equation of state of the matter can generally be given by

$$\rho_q = g_q^* \left( \frac{\pi^2}{30} \right) T^4 + V(T), \quad p_q = g_q^* \left( \frac{\pi^2}{90} \right) T^4 - V(T),$$  \hspace{1cm} (12)

where $$g_q^* = 16 + 21/2(N_F = 2) + 14.25 = 51.25$$ is the effective number of degrees of freedom in the quark phase. We choose the following expression for the self-interaction potential $$V(T)$$ [28]:

$$V(T) = B + \gamma_T T^2 - \alpha_T T^4,$$  \hspace{1cm} (13)

where $$B$$ is the bag pressure constant, $$\alpha_T = 7\pi^2/20$$ and $$\gamma_T = m_s^2/4$$ with $$m_s$$ being the mass of the strange quark in the range $$m_s \in (60–200)$$ MeV. The form of the potential $$V$$ corresponds to a physical model in which the quark fields are interacting with a chiral field formed with the $$\pi$$ meson field and a scalar field. By ignoring the temperature effects, the equation of state in the quark phase takes the form of the MIT bag model equation of state, $$p_q = (\rho_q - 4B)/3$$. The results obtained in low-energy hadron spectroscopy, heavy ion collisions and phenomenological fits of light hadron properties give a range for $$B^{1/4}$$ between 100 and 200 MeV [29].

For the hadron phase we take the cosmological fluid with energy density $$\rho_h$$ and pressure $$p_h$$ as an ideal gas of massless pions and of nucleons described by the Maxwell–Boltzmann statistics. The equation of state can be approximated by

$$p_h(T) = \frac{1}{3} \rho_h(T) = g_h^* \left( \frac{\pi^2}{90} \right) T^4,$$  \hspace{1cm} (14)

where $$g_h^* = 17.25$$.

During the quark–hadron phase transition the temperature is equal to the critical temperature $$T_c$$ which is defined by the condition $$p_q(T_c) = p_h(T_c)$$ [15], and is given by

$$T_c = \left[ \frac{\gamma_T + \sqrt{\gamma_T^2 + 4B(\alpha_T + a_q - a_\pi)}}{2(\alpha_T + a_q - a_\pi)} \right]^{1/2},$$  \hspace{1cm} (15)

where $$a_q = (\pi^2/90)g_q^*$$ and $$a_\pi = (\pi^2/90)g_h^*$$. For $$m_s = 200$$ MeV and $$B^{1/4} = 200$$ MeV the transition temperature is of the order $$T_c \approx 125$$ MeV. It is worth mentioning that since the phase transition is assumed to be of first order, all the physical quantities, such as the energy density, pressure and entropy exhibit discontinuities across the critical curve. In the following section, we consider the evolution of the braneworld scenario before, during and after the phase transition era.

4. Dynamical evolution during the phase transition

To begin with, we obtain the physically important quantities in the quark–hadron phase transition in the braneworld scenario, such as the energy density $$\rho$$, temperature $$T$$ and scale factor $$a$$. These parameters are derived from the modified Friedmann equation (8), conservation equation (7) and the equations of state (12), (13) and (14).

**For $$T > T_c$$**, before the phase transition, the braneworld is in the quark phase. Use of the equations of state of the quark matter and conservation of matter on the brane, we can rewrite equation (7) as
the general solution to equation (16) becomes

\[
a(T) = a_0 T^{\frac{\gamma T}{6a_q}} \exp \left( \frac{\gamma T}{12a_q} \right),
\]

where \(a_0\) is a constant of integration.

Substituting equation (16) into the modified Friedmann equation (5), one obtains

\[
\frac{dT}{dt} = -\frac{T^3}{3a_q} \left( 3a_q - \alpha_T T \right) \frac{\gamma T T^2 + \frac{\kappa^2}{4} \frac{B}{\Lambda_1}}{(1 - \frac{\gamma T}{3a_q})T^2 + \frac{2\gamma T}{6a_q}},
\]

Now, use of equations (10) and (12) in the above equation leads to the variation of the temperature of the brane universe in the standard braneworld models, which is the same as equation (17) in [27] in the absence of nonlocal bulk effects.

Substituting equation (16) into the modified Friedmann equation (8), the basic equation describing the evolution of temperature of the brane universe in the quark phase can be written as

\[
\frac{dT}{dt} = -\frac{T^3}{\sqrt{3}} \left( \Lambda + \frac{\kappa^2}{4} \frac{B}{\Lambda_1} + \frac{\kappa^2}{4} \frac{B}{\Lambda_1} + A_0 T^2 + A_1 \right),
\]

where we have denoted

\[
A_0 = 1 - \frac{\alpha_T}{3a_q},
\]

\[
A_1 = \frac{\gamma T}{6a_q}.
\]

The behavior of the temperature as a function of time in the quark matter filled braneworld is illustrated, for \(w_x = -0.5\) and different values of the constant \(b_0\), in figure 1. The behavior of this parameter shows that the effects of the extrinsic curvature would significantly reduce the temperature of the quark–gluon plasma and accelerate the phase transition to the hadronic era. Here, the solid curve corresponds to the general relativistic limit. In figure 2, we have
illustrated the behavior of the temperature of the brane in the quark phase for $b_0 = 10^8$ and different values of $w_x$. In [11], we have discussed the role of the extrinsic curvature in dark energy. We have found that the accelerating expansion of the universe can be a consequence of the extrinsic curvature and thus a purely geometrical effect. The rate of expansion and so the age of the universe increase with decreasing $w_x$. Here, as one can see from figure 2, the temperature evolution of the early universe is also sensitive to this parameter and increases with decreasing $w_x$.

One can obtain an analytical insight into the evolution of the cosmological quark matter in braneworld scenarios by looking at the effects of the extrinsic curvature on the phase transition. Let us consider the case in which temperature corrections can be neglected in the self-interacting potential $V$, that is $V = B = \text{const}$ with the equation of state of the quark matter being given by the bag model, $p_q = (\rho_q - 4B)/3$. Thus, equation (7) leads to the following scale factor:

$$a(T) = a_0 T,$$

where $a_0$ is a constant of integration. Taking $w_x > 1/3$ in equation (8), the correction term due to the extrinsic curvature ($\propto a^{-3(1+w_x)} \propto T^{3(1+w_x)}$) is the dominant term relative to the energy density of the matter in the quark phase ($\propto T^4$). Therefore, the evolution of the quark phase of the braneworld is now described by the equation

$$\frac{\dot{a}}{a} \approx - \frac{b_0}{a_0^{3/2} (1+w_x)} T^{3/2 (1+w_x)}.$$

Now, use of equations (20) and (21) leads to the following equation for the temperature:

$$\frac{dT}{dr} \approx - \frac{b_0}{a_0^{3/2} (1+w_x)} T^{1/2 (5+3w_x)},$$

with the general solution given by

$$T(t) \approx \left[ \frac{3}{2} b_0 a_0^{-3/2 (1+w_x)} (1+w_x) t \right]^{-\frac{1}{5+3w_x}}.$$

- For $T = T_c$, during the phase transition, the temperature and the pressure are constant. In this case, the entropy $S = s a^3$ and the enthalpy $W = (\rho + p)a^3$ are conserved. For later
convenience, following [15], we replace $\rho(t)$ by $h(t)$, so that the volume fraction of matter in the hadron phase is given by

$$\rho(t) = \rho_h h(t) + \rho_q [1 - h(t)] = \rho_q [1 + n h(t)],$$

(24)

where $n = (\rho_h - \rho_q)/\rho_q$. At the beginning of the phase transition $h(t_c) = 0$, where $t_c$ is the time representing the beginning of the phase transition and $\rho(t_c) \equiv \rho_q$, while at the end of the transition $h(t_h) = 1$, where $t_h$ is the time at which the phase transition ends, representing $\rho(t_h) \equiv \rho_h$. For $t > t_h$ the universe enters the hadronic phase. Substituting equation (24) into equation (7) we have

$$\frac{\dot{a}}{a} = -\frac{1}{3} \frac{(\rho_h - \rho_q) \dot{h}}{\rho_q + p_c + (\rho_h - \rho_q) \dot{h}},$$

(25)

The above equation leads to the following scale factor:

$$a(t) = a(t_c) \left[1 + \frac{\rho_h - \rho_q}{\rho_q + p_c} h(t)\right]^{-1/3},$$

(26)

where the initial condition $h(t_c) = 0$ has been used.

Substituting equation (25) into the modified Friedmann equation (5), one obtains

$$\frac{d^2h}{dt^2} = -\sqrt{3} \left(h + \frac{\rho_q + p_c}{\rho_h - \rho_q}\right) \left[k^2 \rho_0 [1 + n h] + \Lambda + \frac{3 b^2}{\epsilon} \frac{a^2}{a^2} + \sqrt{a(t_c) \left[1 + \frac{\rho_h - \rho_q}{\rho_q + p_c} h(t)\right]^{-1/3}} - 3 \frac{a(t_c)}{a^2}\right].$$

(27)

Now, use of equations (10) and (24) in the above equation leads to the variation of the temperature of the brane universe in the standard braneworld models, which is the same as equation (29) in [27] in the absence of nonlocal bulk effects.

The evolution of the fraction of the matter for the hadron phase in our model is given by

$$\frac{d h}{d t} = -\sqrt{3} \left(h + \frac{\rho_q + p_c}{\rho_h - \rho_q}\right) \left[k^2 \rho_0 [1 + n h] + \Lambda + \frac{3 h_0^2}{\epsilon} \frac{a(t_c) \left[1 + \frac{\rho_h - \rho_q}{\rho_q + p_c} h(t)\right]^{-1/3}} - 3 \frac{a(t_c)}{a^2}\right].$$

(28)

In figure 3, the variation of the hadron fraction as a function of time is presented, for $w_x = -0.5$ and different values of $b_0$. The behavior of this parameter shows that the hadron fraction is
again strongly dependent on the brane effects so that in the presence of the term $Q_{\mu\nu}$, $h(t)$ is much larger than the standard general relativity. Figure 4 shows the variation of the hadron fraction as a function of time, for $b_0^2 = 10^6$ and different values of $w_x$.

- For $T < T_c$, after the phase transition, the energy density of the pure hadronic matter is $\rho_h = 3 p_h = 3 a_{\pi} T^4$. The conservation equation on the brane (7) gives $a(T) = a(t_h) T_c / T$.

The time variation of the temperature of the brane universe in the hadronic phase is given by

$$\frac{dT}{dr} = -\frac{T}{\sqrt{\Lambda}} \left( \Lambda + 3 a_{\pi}^2 T^4 + \frac{3 b_0^2}{\epsilon} [a(t_h) T_c]^{-3(1+w_x)} T^{3(1+w_x)} \right).$$

The behavior of the temperature of the hadron fluid filled brane universe as a function of time for $w_x = -0.5$ and different values of $b_0$ is presented in figure 5. As mentioned before the variation of the temperature of the brane universe is sensitive to both parameters $w_x$ and $b_0$.

We can obtain interesting effects on the temperature of the very early universe for the small value of $b_0$ if we take a positive value of $w_x$. Thus, the behavior of the temperature as a function of time for $w_x = 3/2$, $a(t_h) = 0.1$ and different values of $b_0$ is represented in figure 6.
The behavior of the temperature for the brane in the hadron phase for $b_0^2 = 10^8$ and different values of $w_x$ is presented in figure 7.

It is interesting to note that for $w_x = \frac{5}{3}$, $b_0 = -\frac{1}{2} \kappa_2^2 a_\pi a^4(t_h) T_c^4$ and $\kappa_4^2 = \frac{\lambda a^4}{\eta}$, equation (29) reduces to equation (34) in [27] where the variation of the temperature of the brane universe in the hadronic phase has been studied within the context of standard braneworld models.

5. Bubble nucleation in brane cosmology

In this section, we consider the process of the formation and evolution of microscopic quark nuclei in the cosmological fluid on the brane using nucleation theory. The nucleation theory computes the probability that a bubble or droplet of the A phase appears in a system initially in the B phase near the critical temperature [30].

In the previous section, we discussed the mechanisms of the phase transition and for the sake of completeness, we consider the phase transition in the context of bubble nucleation theory in this section. The phase transition can be described by an effective nucleation theory.
in the small supercooling regime. As the temperature decreases, there is a probability that a droplet of hadrons is nucleated from the quark plasma. The nucleation probability density is given by [15]

\[ p(t) = p_0 T_c^4 \exp \left( -\frac{w_0}{(1 - \hat{T}^4)^2} \right), \tag{30} \]

where \( p_0, w_0 \) are dimensionless constants and \( \hat{T} = T / T_c \), where \( T_c \) is a critical temperature of the phase transition. When a droplet of hadrons is formed, it starts to expand explosively [31], with a velocity \( v_{sh} \) smaller than the sound speed. At the same time, a much quicker shock wave is generated. During the period over which the temperature decreases, a large number of bubbles are created until the shock waves collide and re-heat the plasma to the critical temperature. The fraction of the volume which at a time \( t \) is turned into the hadronic phase in the small supercooling scenario is given by [27]

\[ f(t) = \frac{1}{\pi} \int_{t_c}^{t} \frac{4\pi}{3} v_{sh}^3 (t - t')^3, \tag{31} \]

where \( t_c \) is the time at which the critical temperature is reached and the nucleation process started. It is now appropriate to consider the effects of the extrinsic curvature in bubble nucleation in the early universe. Thus, using equations (21) and (22), we can express this integral in terms of the normalized temperature. The integral (31) may now be given by

\[ f(\hat{T}) = \frac{1}{\pi} \frac{d\hat{T}}{\hat{T}} p_0 T_c^4 v_{sh}^3 \left( \frac{\hat{T}}{c} \right)^{-3/2(1+w_s)} \exp \left[ -\frac{\omega_0}{(1 - \hat{T}^4)^2} \left( \frac{\hat{T}^{-3/2(1+w_s)} - \hat{T}^{-3/2(1+w_s)}}{3/2(1 + w_s) c^{-3/2(1+w_s)}} \right)^3 \right], \tag{32} \]

where \( c = a_0 / T_c \). To study the cosmological dynamics we use numerical methods for calculating the integral. We have plotted \( f(\hat{T}) \) against \( \hat{T} \) for different values of \( b_0^2 \) and \( w_s = 0.5 \) with \( v_{sh} = 10^{-3} \) and \( w_0 = p_0 = 1 \), in figure 8. The fraction of hadronic matter stays very close to zero until the supercooling temperature is between \( \hat{T} = 0.976 \) and \( \hat{T} = 0.979 \), then it jumps to 1 very rapidly. Here we may deduce the result that at a certain temperature below the critical value, an enormous amount of hadronic bubbles are nucleated almost
everywhere in the plasma, which grow explosively to transform all the plasma into hadrons. This result is similar to what happens in standard cosmology [15] where the small supercooling phase transition is also very rapid with respect to the simple first-order phase transition, at a temperature \( T \approx 0.979 \), which is very similar to that we have obtained in the context of braneworld scenario.

6. Conclusions

The Friedmann equation in a braneworld scenario deviates from that of the standard 4D case which imposes fundamental phenomenological consequences on the cosmological evolution, and in particular on the cosmological phase transitions. It has been found that due to the effects coming from higher dimensions, the temperature evolution of the universe in the braneworld scenario is different from the standard FRW model [27].

In the present paper, we have investigated the quark–hadron phase transition in a braneworld scenario in which the localization of matter on the brane is achieved through the action of a confining potential. We have studied the evolution of the physical quantities relevant to the physical description of the early universe such as the energy density, temperature and scale factor, before, during and after the phase transition. We have shown that for different values of parameters the phase transition occurs and results in decreasing the effective temperature of the quark–gluon plasma and of the hadronic fluid. Bubble nucleation which is one of the different simplified mechanisms used to describe the dynamics of a first-order phase transition was also studied.

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