A Simple “Quantum Interrogation” Method

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Abstract

A simple non-interferometric “quantum interrogation” method is proposed which uses evanescent wave sensing with frustrated total internal reflection on a surface. The simple method has the advantage over the original interferometric Elitzur-Vaidman method of being able to detect objects that are neither black nor non-diffracting and that are such that they cannot be introduced into an arm of an interferometer for whatever reason (e.g. its size, sensitivity, etc.). The method is intrinsically of high efficiency.

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1 Introduction

The basic idea behind “quantum interrogation” (or “interaction-free” detection as it is often called) is to detect the presence of an ultra-sensitive object (an object that is damaged by a single quantum of the probe beam) and

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also image it without damaging it in most cases. This is of great significance for imaging photo-sensitive biological systems such as cells and "delicate" quantum systems such as Bose-Einstein Condensates, trapped atoms, etc. [1].

The best known quantum interrogation method is the Elitzur-Vaidman (EV) method [2]. In the original version of the method a black object (usually referred to as the 'bomb' which can be triggered by a single incident quantum) is placed in one arm of a balanced Mach-Zehnder interferometer, and the photons arriving at the dark port signal its presence without damaging it. The intrinsic efficiency of the method with 50-50 beam splitters is 33 % in principle (only 33% of the bombs can be detected without exploding them). However, the efficiency can be increased arbitrarily and the restriction to black objects somewhat relaxed by modifying the original method in various ways [3, 4, 5, 6, 7, 8, 9].

We present here a different quantum interrogation method which does not require an interferometer but uses a combination of the basic ideas of "evanescent wave sensing" using "frustrated total internal reflection" (FTIR) on a surface.

2 Evanescent Wave Sensing with Frustrated Total Internal Reflection

Consider two adjacent materials with refractive indices $n_i$ and $n_r$ with $n_i > n_r$. It is well-known that total internal reflection (TIR) occurs at the boundary of these materials when light is incident on the boundary from the material with the higher refractive index $n_i$ at an angle $\theta_i$ greater than the critical angle

$$\theta_c = \sin^{-1} \frac{n_r}{n_i}$$

The wave does not, however, vanish in the second medium with refractive index $n_r$ but is exponentially damped:

$$\psi_{Ev}(x, y) = \psi(x, 0)e^{-y/\xi}$$

with the ‘penetration depth’

$$\xi(n_i, n_r, \lambda) = \frac{\lambda_i}{2\pi \sqrt{n_i^2 \sin^2 \theta_i - n_r^2}} = \frac{\lambda_i}{2\pi n_i \sqrt{\sin^2 \theta_i - \sin^2 \theta_c}},$$

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taking the x-axis along the boundary and the penetration in the transverse direction y. The wave in the second medium with lower refractive index is called the ‘evanescent wave’. The electric and magnetic vectors \( \mathbf{E} \) and \( \mathbf{B} \) in an evanescent wave are in time quadrature, and so the Poynting vector \((c/4\pi)(\mathbf{E} \wedge \mathbf{B})\) vanishes.

If a material with a refractive index \( n_t \neq n_r \) comes within the ‘penetration depth’ \( \xi \) of the evanescent wave, it scatters the wave, i.e. the electric and magnetic vectors are no longer in time quadrature, a part of the energy leaks (tunnels) out across the boundary and propagates parallel to the boundary, frustrating total internal reflection. Thus, for fixed \( \theta_i \) and \( n_i \), any roughness of the surface of the material (variation in y) or inhomogeneity in its refractive index (variation of \( n_r \) along x due to the presence of the object) will be reflected as intensity variations in the beam cross-sections.

There are two significant features of evanescent waves in this context. Firstly, the component of the momentum perpendicular to the boundary surface is imaginary, which is why the wave is exponentially damped and non-radiating. This implies through momentum conservation across the boundary that the momentum components of the evanescent wave parallel to the surface are large. High momentum components imply small spatial dimensions and high resolution. Secondly, the expression (3) for the penetration depth \( \xi \) shows that for a fixed wavelength \( \lambda \), the penetration depth increases indefinitely as \( \theta_i \) approaches \( \theta_c \). Thus, it is possible to adjust the penetration depth by varying the angle of incidence and make it sufficiently large when required. This is particularly simple when prisms are used as total internal reflectors. Optical fibres may be specially modified to take advantage of this feature.

The above analysis also applies to photons which are quantum mechanical objects [10]. In this case there is tunnelling across a barrier which is a well-understood quantum mechanical phenomenon.

3 The Method

As shown in the schematic Figure, let a pair of single-photons be emitted from a source \( S \) (for example, a pulsed parametric down-conversion source PDC), and let one of the photons be incident on the detector \( D_I \) and the other photon be totally internally reflected by the surface \( TIR \) into the detector \( D_S \). The detection of a photon by \( D_I \) can be used to herald the other photon.
In the absence of the object and in ideal conditions the two detectors $D_S$ and $D_I$ will count $\bar{n}_0$ photons per second if $\bar{n}_0$ is the flux of signal and idler photons emitted by $S$. However, when an object $O$ with a refractive index different from that of its surroundings is present on the far side of TIR (as shown in the Figure) but close enough to its back surface, i.e., within the ‘penetration depth’ of the evanescent wave which is of the order of the wavelength of the photons, the evanescent wave associated with total internal reflection will interact with the object and a fraction of the photons will tunnel out, partially frustrating total internal reflection. If $|\psi\rangle$ is the normalized state of the signal photon before reflection by TIR, it splits into an evanescent state and a reflected state after TIR so that the state after TIR can be written as

$$|\phi\rangle = (a + b)|\psi\rangle$$

with $|a|^2 + |b|^2 = 1$ and

$$|b| = \exp[-d/\xi(n_i, n_t, \lambda)]$$

where $d$ is the distance of the object from the back surface of TIR and $\xi(n_i, n_t, \lambda)$ is given by Eq. (3) above. Thus there is a reduction in amplitude of the state that is totally reflected by TIR, and the count rate at $D_S$ is reduced from $\bar{n}_0$ to $|a|^2\bar{n}_0$. Since single photons are sent in one at a time, a photon arriving at $D_S$ could not have ‘touched’ the object, for then it would have been absorbed by it (assuming the object to be ultra-sensitive or black). Yet the decrease in the counting rate unambiguously indicates the presence of the object. The important point here is that although the photon wave function splits into an evanescent part and a reflected part at the total
internal reflector (for angles of incidence $\theta_i \geq \theta_c$), a single photon either interacts with the object and tunnels out, or it is entirely reflected. These are mutually exclusive alternatives. This is impossible with multi-photon states of classical light because with such light reflection and evanescent wave interaction are concurrent and not mutually exclusive.

Some important differences from the EV method must be pointed out. In the EV method every photon in the dark port signals the presence of an ultra-sensitive object. This is not the case here since the detector $D_S$ clicks whether or not an object is present. However, every photon that is totally internally reflected in the presence of the object and is detected by $D_S$ carries information of its presence through its reduced probability amplitude, resulting in a lower counting rate.

Furthermore, the original EV method works only with a black and non-diffracting object because with a semi-transparent object, for example, the wave function in the arm of the interferometer with the bomb is not fully blocked, and hence interference on the second beam-splitter is not totally destroyed. Consequently, a detection by the dark port cannot obviously be described as completely “interaction-free”. In the new method even a fully transparent object will allow a fraction of photons to pass through it (so to speak), depending on the gap between the back face of the total internal reflector and the object [13], but these photons leak out and are never detected by $D_S$. Only the photons that are totally internally reflected by TIR are detected and these can be metaphorically said not to “touch” the object because if they did, they would have tunnelled out.

However, it is equally important to bear in mind the following similarity between the original EV method and the new method being proposed here. Let us consider the case in which the object to be detected is an ultra-sensitive bomb that is triggered by a single photon. What happens in both cases is that the wave function of the incident photon is split into two parts. Only the method of splitting the wave function is different in the two cases. One part interacts with the wave function of the bomb, and the other part does not. The photons corresponding to the non-interacting part signal the presence of the bomb without exploding it. It is only in this sense that the detection can be said to be “interaction-free” in both cases. A better term to use would perhaps be “damage-free”.

It follows from Eq. (4) that the probability of absorption of a photon by the object is $P_{abs} = |b|^2$ and the probability of damage-free detection is $P_{dfd} = |a|^2 (a < 1)$. (The limit $|b| = 0$ and hence $a = 1$ corresponds to the
absence of an object within the penetration depth of the evanescent wave, and the method fails.) Hence, for a given object, $P_{df}$ can be increased by reducing $P_{abs}$ by placing the object at a suitable distance from the back surface of TIR (the dependence on the distance (Eq. (2)) is exponential!). Such flexibility in placing the object makes the method intrinsically of high efficiency, i.e., for any ensemble of identically prepared incident photons, the fraction of photons that interact with the object (triggering the bomb) can be made arbitrarily small compared to the fraction of photons that are totally reflected with a reduced amplitude.

An advantage of using a single-photon source for damage-free detection (and imaging) over continuous laser light is that the power induced damage is much less in the former case. In a typical cw laser beam the power is of the order of mW to 1 W and the flux of photons is of the order of $10^{17}$ to $10^{19}$ photons/s whereas single-photon sources have much lower power ($\sim$ pW) and much lower photon fluxes ($10^2$ to $10^6$ photons/s). Although the power in a cw laser can be reduced to levels comparable to a heralded PDC single-photon source by using filters, the statistics is different (Poissonian) from that of single-photon states (sub-Poissonian) \[11\], the light is still classical \[12\] in character and hence it cannot be used for true damage-free detection.

4 Experimental Feasibility

The feasibility of the method has already been experimentally demonstrated by Mizobuchi and Ohtaké \[13\] who used a total internal reflector (a prism face) to detect the presence of a second prism within the penetration depth of the incident single photons generated by a Nd:Yag laser, although their results suffered from insufficient statistical precision \[14\]. In the present case, a suitable configuration has to be identified to hold the object at controlled distances behind the total internal reflector TIR. One way is to follow what Mizobuchi and Ohtaké did, namely to use Langmuir-Blodgett films. One can also consider using integrated optics methods to fabricate the required TIR as an optical sensor.

To cut out photons of the original pump laser of frequency $\nu_0$ that are inevitably present as background, suitable interference filters may be used to select the single photons of the right frequency $\nu_1$ to herald the conjugated photons of frequency $\nu_2$ within a coincidence time window of the first detection, together with appropriate logic circuits.
The quantum or shot noise in a single-photon beam also places certain constraints on the detection of small absorptions by matter over short intervals of time and must be taken care of. Let $\bar{n}_0$ be the mean number of heralded photons received by the detector $D_S$ per second as determined over a sufficiently long time. This is a true population characteristic. Let a sample of $N$ counts be recorded by $D_S$ in $T$ seconds when the object is suspected to be present. By hypothesis the sample mean is then $\mu = |a|^2 \bar{n}_0$, and assuming the distribution to be sub-Poissonian, the standard deviation $\sigma = \sqrt{|a|^2 \bar{n}_0 \eta}$ where $\eta = 1 - \epsilon (\epsilon << 1)$. Then the null hypothesis (namely, that the sample mean $\mu = \bar{n}_0$) can be tested by using the T-test and calculating the statistic

$$t = \frac{\mu - \bar{n}_0}{\sigma / \sqrt{N}} = -\sqrt{\frac{N \bar{n}_0}{|a|^2 \eta}} |b|^2. \tag{6}$$

The null hypothesis can be rejected at the 99% confidence level if $|t| \geq 2.58$ for $N > 30$. If one wishes to restrict the fraction of photons that trigger the bomb to, say 1%, then $|a|^2 / |b|^2 = 10^2$, and

$$N \geq \frac{6.7243 \times 10^4 \eta}{\bar{n}_0}. \tag{7}$$

Thus, for example, if $\bar{n}_0 \geq 6.7243 \times 10^2$ photons/s, $N \geq 10^2$ would more than suffice. For small absorptions it would be reasonable to assume that $N \approx \bar{n}_0 T$, and therefore $T \approx 0.15$ s would be required to collect a sample size of 100, and only 1 bomb would be triggered.

## 5 Conclusion

The method has the advantage over the original EV method of being able to detect objects that are neither black nor non-diffracting and that are such that they cannot be introduced into an arm of an interferometer for whatever reason (e.g. its size, sensitivity, etc.). The object to be detected must, however, be present within the penetration depth of the evanescent wave generated by the sensing surface $TIR$. It is a simple method and is likely to have a wider scope of applications.
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