Population trapping of a two-level atom via interaction with CEP-locked laser pulse

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Abstract

The trapping states of a quantum system occupy a very special place in quantum optics. Here, we propose a method to realize the population trapping (PT) in a two-level atom driven by a carrier envelope phase (CEP) locked laser pulse. We present detailed numerical results on the influence of CEP in different detuned atoms beyond the rotating wave approximation and find the population is strongly dependent on CEP, the detuning and the Rabi frequency. This allows us to choose proper parameters to realize three specific trapping states: upper trapping state, half trapping state and lower trapping state. To explain the fantastic influence of CEP, we present an analytical solution under weak-field approximation, which is consistent with the numerical solution. We further define the action quantity of the pulse acting on the atom as the integral for the interaction Hamiltonian and find the population varies approximatively linearly with the action quantity. This investigation provides directive significance for achieving PT at arbitrary quantum states and insightful schemes to regulate and control the quantum dynamics of atoms via accurately controlled optical field, and the results could be beneficial for applications to laser cooling, isotope separation and so on.

Introduction

Population trapping (PT) has been regarded as a significant quantum phenomenon as it provides a method to prepare an atomic system staying in a certain state indefinitely. In the fields of isotope separation and single atom (molecule) detection, where the most crucial issue is pumping the atom from the ground state into an ionization state effectively, PT helps to obtain high ionization probability [1]. In addition, the importance of PT is well illustrated by its numerous applications in laser cooling [2], stimulated Raman adiabatic passage [3], electromagnetically induced transparency [4] and lasing without inversion [5].

In general, PT is realized in a Λ-type atomic system, where an atom is lying in a coherent superposition of two lower atomic states, both of which are coupled by lasers at the right frequency to the upper state. With the atom lying in a particular superposition of the two states, it can be made transparent to both lasers as the probability of absorbing a photon goes to 0 [6]. So far, a plenty of works have been done on the PT phenomenon in the multiple level atomic system [7–9], while much less efforts have been made to the two-level atomic system since Meystre firstly investigated the PT phenomenon of a two-level atom in 1988 [10]. Agarwal demonstrated the existence of PT states in a two-level atom driven by a frequency modulated field in 1994 [11]. However, Rabi oscillation occurs when a two-level atom interacts with a single-mode optical field. The probabilities of the atom being in the upper or lower state rely on time and the two-level atom undergoes optical Rabi oscillations at the Rabi frequency between the two levels under the action of the driving electromagnetic field [12, 13]. The population indeed can be trapped on a time scale that is shorter than the scale of Rabi period under certain conditions as demonstrated by Agarwal, while the atom still transits between the two states of atom. The population cannot be trapped in a state for a longer time than the Rabi period.
The emergence of ultrashort pulse laser technology [14, 15] has provided a powerful tool for probing physical problems in an unprecedented fast timescale. With the progress of ultrashort pulse technology, it is possible to generate optical pulses that are only a few cycles in duration [16–20]. The few-cycle laser pulses have induced enormous interests on nonlinear optical phenomena, such as above-threshold ionization [21, 22], high harmonic generation [23, 24], isolated attosecond generation [25–27], and so on. At such a short pulse duration, the relative phase between the carrier and the envelope of the pulse, which is named carrier envelope phase (CEP), becomes a very crucial parameter which can dramatically affect almost all dynamical processes [28–31] in the laser–matter interaction, such as atomic and molecular excitation, ionization, and emission in laser pulses. Rabi oscillation will not occur when a two-level atom is driven by an ultrashort pulse. This feature hopefully may be considered for preparing an atomic system staying in a certain state indefinitely.

Most of the existing theoretical investigations that discuss the trapping states in two-level atoms are made by adopting the rotating wave approximation (RWA). The rotating wave approximation [32, 33] is a basic and important approximation used in atomic physics, quantum optics, and nuclear magnetic resonance, and has been well discussed in many books [12, 13] and articles [32–35]. But the approximation is only valid when the two levels are resonant or nearly resonant with the applied electromagnetic radiation, according to our recent studies [36]. When the system is far away from the resonance, in other words, in the highly detuned system, the RWA might not work accurately.

In this work, we demonstrate the realization of PT in a two-level atom driven by an ultrashort CEP-locked pulse. Firstly, we investigate the influence of CEP on the dynamical evolutions of different detuned atoms beyond the RWA. Then we propose proper parameters to realize three trapping states including the upper trapping state, the half trapping state where half of the population is trapped in the upper state and half in the lower state, and the lower trapping state. Finally we present an analytical solution under the weak-field approximation and analyze the action quantity of pulse acting on a two-level atom, which reveals the physical mechanism of the PT phenomenon.

Model and analysis

In the traditional scheme that realizes PT states in a two-level atom, the atom is driven by a frequency-modulated field on resonance with the frequency of the atomic transition. The modulated field can be written as

\[ E(t) = E_0 \cos(\omega t + M \sin(\omega_m t + \phi)), \]

where \( E_0 \) is the amplitude, \( \omega \) is the frequency of the laser field, \( M \) is the index of modulation, \( \omega_m \) and \( \phi \) are the frequency of modulation and the initial phase of modulation, respectively. We firstly consider a system where the two levels of atom are resonant with a single mode optical field expressed as

\[ E(t) = E_0 \cos(\omega t + \phi), \]

in which situation the RWA is valid. Let \( |a\rangle \) and \( |b\rangle \) represent the lower and upper level states of the atom. They are eigenstates of the unperturbed part of the Hamiltonian \( H_0 \) with the eigenvalues \( E_a = \hbar \omega_a \) and \( E_b = \hbar \omega_b \), respectively, and \( \omega_{ba} = \omega_b - \omega_a = -\omega_a > 0 \) is the transition frequency of the two levels. The probabilities of the atom being in states \( |a\rangle \) and \( |b\rangle \) at time \( t \) are then given by \( |\psi_a(t)\rangle^2 \) and \( |\psi_b(t)\rangle^2 \). Assume the frequency of the field \( \omega \) is the dominant frequency of Ti: Sapphire pulse laser, corresponding to a wavelength of 800 nm, namely \( \omega = 2.33 \times 10^{15} \) Hz. To describe the parameters for simplicity, \( \omega \) is set as a non-dimensional value of 10. All other frequency and time parameters are normalized similarly as non-dimensional quantities. Therefore, if the Rabi frequency \( \Omega_{ab} \) is set as 1, according to [36], we find that the laser intensity is about \( I = 1 \times 10^{13} \) W cm\(^{-2}\) equivalently. Taking \( \omega_{ba} = 10, \omega = 10, \Omega_{ab} = 6 \) for example, they correspond to a transition wavelength of the two levels as 800 nm and a laser intensity of about \( I = 6 \times 10^{13} \) W cm\(^{-2}\). Under the resonant condition, the system oscillates with the Rabi frequency of \( \Omega_{ab} \) between the two atomic levels as plotted in figure 1(a).

When the frequency-modulated field is introduced to interact with the atom, PT can be realized when the system satisfies the adiabatic condition and slowly-varying condition in the time interval

\[ (\omega_{2a} j + 1, \frac{\omega_{2a}}{2}\text{integer}) \]

In figure 1(b), the population is trapped in the upper state during a time range of about (2.09, 6.28) with the parameters of \( M = 30.63 \) and \( \omega_m = 0.75 \), whereas the system without the frequency modulation oscillates 3.5 times during the same time range as shown in figure 1(a). However, this scheme to realize PT has two disadvantages: one is that there exist small oscillations at fast time scales during the PT period; the other is that the system can only remain trapped at a special range of modulation periods. A question naturally arises: Can one trap the atom in the upper state indefinitely without small oscillations as assumed in figure 1(c) (black curve)? Or one step further, can one manage the trapping of atom in any level with controllable probability? For example, as shown in figure 1(c), can a system with half of the population trapped in the upper level and the other half trapped in the lower state (red curve) or with all the population trapped in the lower state (blue curve) be prepared?

To truly realize PT, we employ an alternative scheme and consider the interaction of a single two-level atom with an ultrashort pulse. As illustrated in figure 2, the full width at half maximum (FWHM) duration of the laser
pulse is one optical cycle. The electric field of the laser pulse is

\[ E(t) = E_0 \exp(-\alpha^2 t^2) \cos(\omega t + \varphi) \]  

(1)

where \( \alpha = \sqrt{2 \ln(2)} / \tau \), \( \tau \) is the FWHM, and \( \varphi \) is the CEP.

According to [36, 38], the corresponding Schrödinger equation is

\[ i\dot{\psi}(t) = -i(H_0 + H_1)|\psi(t)\rangle / \hbar. \]  

(2)

In the above, the unperturbed Hamiltonian is

\[ H_0 = \hbar \omega_0 |a\rangle \langle a| + \hbar \omega_1 |b\rangle \langle b|, \]  

(3)

and the interaction Hamiltonian is

\[ H_1 = -(D_{ab}|a\rangle \langle b| + D_{ba}|b\rangle \langle a|)E(t), \]  

(4)

respectively, and \( D_{ab} = D_{ab}^* \) is the dipole matrix element. Here dipolar approximation describing light-atom interaction has been adopted.

The equations of motion for the slowly varying probability amplitudes of the two levels, \( c_a \) and \( c_b \) may be written as

\[ \dot{c}_a(t) = i\Omega_{ab} c_b \exp(-\alpha^2 t^2)(e^{-i(\omega_0 - \omega)t + i\varphi} + e^{-i(\omega_0 + \omega)t - i\varphi}) / 2, \]  

(5.1)

\[ \dot{c}_b(t) = i\Omega_{ab} c_a \exp(-\alpha^2 t^2)(e^{i(\omega_0 - \omega)t - i\varphi} + e^{i(\omega_0 + \omega)t + i\varphi}) / 2, \]  

(5.2)

where the Rabi frequency \( \Omega_{ab} \) is defined as

\[ \Omega_{ab} = |D_{ab}| E_0 / \hbar. \]  

(6)

To analyze the dynamical evolutions of a two-level atom driven by an ultrashort pulse, we directly solve the ordinary differential equations (5.1) and (5.2) with initial conditions \( c_a(0) = 1, c_b(0) = 0 \). The laser pulse has a single optical cycle duration, which means \( \alpha = 1.87 \), and the pulse only lasts about \( t = 0.94 \), which corresponds to 4 fs according to equation (1).

To achieve PT at arbitrary quantum states as designated in the above, we first investigate the parameters which can greatly influence the dynamical evolutions of different detuned atoms. CEP is a critical parameter which can dramatically affect the oscillation of the atom before the end of a pulse, and this directly determines

Figure 1. The comparison of the probability of the atom being in upper state \(|b\rangle \rangle \) at time \( t \) of a two-level system driven by (a) a single mode optical field and (b) a frequency-modulated field. (c) The assumed population trapping states that trap the system in the upper state (black curve), lower state (blue curve) or half in the upper and half in the lower state (red curve).

Figure 2. Schematic of the interaction of a single two-level atom with an ultrashort pulse.
the population trapped in a specific level. For a highly detuned atom, RWA is no longer applicable as the counter-rotating terms cannot be ignored [36]. Figures 3(a)–(c) show the influence of CEP on the population trapping of different detuned atoms beyond the RWA. The Rabi frequency $\Omega_{ab} = 8$ in figures 3(a)–(c) stands for a laser intensity of $I = 8 \times 10^{13}$ W cm$^{-2}$. In panels (a)–(c), the parameter is (a) $\omega_{ba} = 20$, which means a positive detuned atom with the detuning $\Delta = \omega_{ba} - \omega = 10$, (b) $\omega_{ba} = 10$, which means a resonant atom, and (c) $\omega_{ba} = 1$, which means a negative detuned atom with $\Delta = -9$. As assumed above, the transition wavelength of the two levels are 400 nm, 800 nm and 8 $\mu$m, respectively for these three atomic systems. We can see that the population trapped in the upper state has an evident CEP dependence for all these three different detuned systems.

After the pulse, we calculate the steady population of the upper state, which is a constant over time. Figures 3(d)–(f) present the population as a function of the CEP for three systems corresponding to the systems in figures 3(a)–(c), respectively (blue curves). It clearly shows that CEP plays a modulating role to the population trapping in the term of a sine function and the modulation period is $\pi$. However, the modulation is strongly dependent on the detuning, which results in a phase shift of the sine items. The population of the upper state in the resonant atom changes with CEP as a function of $\sin(2 \varphi + 0.1 \pi)$, whereas the population in the positive detuned atom and negative detuned atom conform to the function of $\sin(2 \varphi + 0.5 \pi)$ and $\sin(2 \varphi - 0.5 \pi)$, respectively. Another critical parameter, which also plays an important role in affecting the oscillation period and the maximum probability, is the Rabi frequency. We compare the population trapping of atoms with $\Omega_{ab} = 1$ (black curve) and $\Omega_{ab} = 8$ (blue curve) in figures 3(d)–(f), which correspond to the laser intensity of about $I = 1 \times 10^{13}$ W cm$^{-2}$ and $I = 8 \times 10^{13}$ W cm$^{-2}$, respectively. For highly detuned atoms, the Rabi frequency only affects the quantity of population and has nothing to do with the modulation phase shift of population. For resonant atoms, the different Rabi frequency will induce a large phase shift of the modulation. When the Rabi frequency changes from $\Omega_{ab} = 8$ to $\Omega_{ab} = 1$, the modulation function of CEP in the resonant atom transforms from $\sin(2 \varphi + 0.1 \pi)$ to $\sin(2 \varphi + \pi)$, whereas it remains the same in the highly detuned atoms. The modulation in the positive detuned atom has 0.25$\pi$ phase shift while the negative detuned atom has $-0.25\pi$ compared to that in the resonant atom when $\Omega_{ab} = 1$.

Until now, we have analyzed the influence of CEP, detuning and Rabi frequency on the population trapping and found that the population transfer is small in the positive detuned atom, whereas large in the resonant atom. For a negative detuned atom, the population has a large scope change. As a consequence, the resonant atom is a good choice to realize the upper trapping state and the positive detuned atom is applicable for the lower trapping state, whereas the half trapping state needs a negative detuned atom. The analysis provides directive significance.
for producing trapping in any level with controllable probability. For example, to produce an atomic system staying in the upper state, we choose a pulse on resonance with the frequency of the atomic transition. As shown in Figure 3(e), the Rabi frequency greatly influences the modulation law and at $\omega_{ab} = 8$ the laser pulse can pump the majority of the resonant atom to the upper level, although it is still less than 1. To find out the very Rabi frequency that can indeed realize the perfect upper trapping state, we calculate the population as a function of Rabi frequency in a resonant atom with $\varphi = 0$ and $\alpha = 1.87$ as plotted in Figure 4(a). The population reaches maximum when $\varphi = 0.94\pi$, which means the laser intensity of about $I = 6.5 \times 10^{13}$ W cm$^{-2}$. As different Rabi frequency will induce a large phase shift of the modulation, we plot the population as a function of CEP with parameters as $\omega_{ab} = 10$, $\omega = 10$, $\Omega_{ab} = 6.5$ and $\alpha = 1.87$ in Figure 4(b). It is clearly seen that when $\varphi = 0.94\pi$, the maximum of the upper-level population appears with its value being 100%. The analysis above thus provides a set of precise parameters that can trap the population in the upper level; (b) $\omega_{ab} = 1$, $\omega = 10$, $\Omega_{ab} = 8$, $\varphi = 0.35\pi$ and $\alpha = 1.87$ to pump half of the population to the upper state; (c) $\omega_{ab} = 20$, $\omega = 10$, $\Omega_{ab} = 8$, $\varphi = 0.5\pi$ and $\alpha = 1.87$ to trap the population in the lower state.

Figure 5. Three set of parameters as (a) $\omega_{ab} = 10$, $\omega = 10$, $\Omega_{ab} = 6.5$, $\varphi = 0.94\pi$ and $\alpha = 1.87$ to trap the population in the upper level; (b) $\omega_{ab} = 1$, $\omega = 10$, $\Omega_{ab} = 8$, $\varphi = 0.35\pi$ and $\alpha = 1.87$ to pump half of the population to the upper state; (c) $\omega_{ab} = 20$, $\omega = 10$, $\Omega_{ab} = 8$, $\varphi = 0.5\pi$ and $\alpha = 1.87$ to trap the population in the lower state. (d)-(f) The comparison of trapping states with different CEP.

To realize trapping in the lower state or half trapping in the upper state, a highly detuned system is a best choice. For example, we take the parameters as $\omega_{ab} = 1$, $\omega = 10$, $\Omega_{ab} = 8$, $\varphi = 0.35\pi$ and $\alpha = 1.87$ for a half
trapping state system in which half of the population is trapped in the upper level and the other is half trapped in the lower state, as shown in figure 5(b). In figure 5(c), we show a positive detuned atom which traps the population in the lower state with the parameters as $\omega_{ba} = 20$, $\omega = 10$, $\Omega_{ab} = 8$, $\varphi = 0.5\pi$ and $\alpha = 1.87$. The selection of CEP is based upon the modulating law we have discussed in figure 3, which shows that the CEP greatly influences the arrangement of the three atomic systems. As shown in figures 5(d)–(f), the changes of CEP result in the transformation of the trapping states. In figure 5(d), when CEP is changed from $\varphi = 0.94\pi$ to $\varphi = 0.5\pi$, the population can no longer be perfectly trapped in the upper state. In figure 5(e), the system drastically changes from the half trapping states to the lower trapping state as CEP is changed for 0.35$\pi$. The atom no longer lies in the lower trapping state when the CEP changes from $\varphi = 0.5\pi$ to $\varphi = 0$ as shown in figure 5(f).

Up to now, we have found the great influence of the CEP on the population trapping in different detuned atoms via numerical calculations. For better physical insights, we proceed to explore an analytical solution to explain the fantastic modulation. Because equations (5.1) and (5.2) are non-integrable, an approximate solution is necessary. Here, we solve the ordinary differential equations (5.1) and (5.2) analytically with initial condition $c_{a}(0) = 1$, $c_{b}(0) = 0$ under the weak-field approximation. Under the approximation, the probability in the lower state remains approximately constant during the interaction time and the population of upper level remains small compared with that of lower level for times $t \ll T$. The equations are then reduced to a form that can be solved exactly with an iterative procedure starting with $c_{a}(0) = 1$, $c_{b}(0) = 0$. With these assumptions the first approximation of equation (5) gives

$$c_{b}(t) = i \Omega_{ab} \exp \left(-\alpha^{2}t^{2}\right) \left(e^{i\left[\left(\omega_{ba} - \omega\right)^{2}t - i\varphi\right]} + e^{i\left[\left(\omega_{ba} + \omega\right)^{2}t + i\varphi\right]}\right) / 2.$$

The integration of equation (7) from $t = 0$ to $\infty$ yields

$$c_{b}(t) = i \Omega_{ab} \sqrt{\pi} \left[\exp \left(-\left(\omega_{ba} - \omega\right)^{2}/4\alpha^{2}\right) \operatorname{erfc}\left(-i\left(\omega_{ba} - \omega\right)/2\alpha\right)ight.
\left.\exp \left(-\left(\omega_{ba} + \omega\right)^{2}/4\alpha^{2}\right) \operatorname{erfc}\left(-i\left(\omega_{ba} + \omega\right)/2\alpha\right)\right] / 2\alpha.
\tag{8}$$

The probability of finding the atom in the upper level state is

$$|c_{a}(t)|^{2} = c_{a}c_{a}^{*} = \Omega_{ab} \sqrt{\pi} \left[\exp \left(-\left(\omega_{ba} - \omega\right)^{2}/2\alpha^{2}\right) (A^{2} + B^{2}) + \exp \left(-\left(\omega_{ba} + \omega\right)^{2}/2\alpha^{2}\right) (C^{2} + D^{2})ight.
\left.\exp \left(-\left(\omega_{ba} - \omega\right)^{2}/4\alpha^{2}\right) (E \cos 2\varphi + F \sin 2\varphi)\right] / 4\alpha^{2}.
\tag{9}$$

where

$$\operatorname{erfc}\left(-i\left(\omega_{ba} - \omega\right)/2\alpha\right) = A + Bi,$$
$$\operatorname{erfc}\left(-i\left(\omega_{ba} + \omega\right)/2\alpha\right) = C + Di,$$
$$E = AC + BD,$$  
$$F = -AD + BC.$$

In the above formulae ‘erfc’ denotes a standard error function. The interference between the approximate co-rotating terms proportional to $\exp \left[\pm i\left(\omega_{ba} - \omega\right)\right]$ and counter-rotating terms proportional to $\exp \left[\pm i\left(\omega_{ba} + \omega\right)^{2}/4\alpha^{2}\right]$ generates a cross term written as

$$2 \exp \left(-\left(\omega_{ba} - \omega\right)^{2}/4\alpha^{2}\right) \exp \left(-\left(\omega_{ba} + \omega\right)^{2}/4\alpha^{2}\right) (E \cos 2\varphi + F \sin 2\varphi)$$
$$= 2 \exp \left(-\left(\omega_{ba} - \omega\right)^{2}/4\alpha^{2}\right) \exp \left(-\left(\omega_{ba} + \omega\right)^{2}/4\alpha^{2}\right) \sqrt{E^{2} + F^{2}} \sin (2\varphi + \theta),
\tag{11}$$

where $\sin \theta = E / \sqrt{E^{2} + F^{2}}$ and $\cos \theta = F / \sqrt{E^{2} + F^{2}}$. The cross term plays a modulation role in the term of sine function to the probability evolution and it is influenced by the detuning. For a resonant atom, as $|E| \ll |F|$, $E > 0$ and $F < 0$, we find $\sin \theta = 0$, $\cos \theta = -1$ and the modulation law is $\sin (2\varphi + \pi)$. In comparison, in the positive and negative detuned atom, the modulation is in form of $\sin (2\varphi + 0.5\pi)$ and $\sin (2\varphi - 0.5\pi)$, respectively, which is consistent with the numerical calculations discussed in figures 3(e)–(f) (black curves) and supports the effectiveness of the weak-field approximation. Therefore, the developed analytical model in some extent can explain the modulation law of the CEP in different detuned atoms. The analytical and numerical solutions suggest potential ways to regulate and control the quantum dynamics of two-level atoms via interaction with CEP-locked laser pulse. Finely tuning the interaction allows for achieving PT at arbitrary quantum states.

In order to have an in-depth understanding of the physical mechanism underlying the PT phenomenon of a two-level atom interacting with a CEP locked ultrashort laser pulse, we calculate the action quantity of the pulse acting on the atom by computing the integral for the interaction Hamiltonian. According to equation (4), the action quantity $Q$ is defined as
It is clearly seen that the action quantity is determined by the incident pulse, the dipole matrix element and the slowly varying probability amplitude of the atom being in states $|a\rangle$ and $|b\rangle$, which is greatly influenced by CEP, the values of Rabi frequencies, and detuning.

Using equation (1) for the incident pulse, equation (8) for the slowly varying probability amplitude of the atom being in states $|a\rangle$ and $|b\rangle$, and the dipole matrix element $D_{ab} = D_{ba} = \Omega_{ab} \hbar / E_0$, the action quantity (12) reduces to

$$Q = \int_0^\infty H_1(t) dt = \int_0^\infty - (D_{ab}|a\rangle \langle b| + D_{ba}|b\rangle \langle a|) E(t) dt. \quad (12)$$

As plotted in figures 6(a)–(c), the variations of action quantity $Q'$ in the positive detuned atom, resonant atom, and negative detuned atom perform as functions of $\sin(2\varphi + \pi)$, $\sin(2\varphi - 0.5\pi)$, and $\sin(2\varphi)$, respectively, which all have a 0.25$\pi$ phase shift compared to the modulation law of the population as plotted in figures 6(d)–(f) (blue curves). The 0.25$\pi$ phase shift might come from the neglect of the variation of the probability of finding the atom in the lower level state, which in fact oscillates with the CEP. In figures 6(d)–(f), we compare the modulation of CEP on the population (blue curves) with the modulation on the action quantity (black curves) where the phase shift is neglected. In three different detuned atoms, the modulation functions of CEP on the population are exactly consistent with that on the action quantity excepting a constant coefficient. In other words, the population varies approximately linearly with the action quantity. The relationship between the action quantity and the population can be written as

$$Q' = aP + b, \quad (14)$$

where $Q'$ and $P$ correspond to the action quantity and the population, respectively. The constant $a$ and $b$ are determined by the detuning.

From the equation (14), we can draw a conclusion that the action quantity of laser pulse acting on a two-level atom is the crucial key to determine the trapping state. When the quantity is large, it can pump the majority of the atom to the upper state. When the action quantity reaches the maximum, it happens to be large enough to pump the atom from the lower state totally to the upper state and realizes the upper trapping state. In some cases, the quantity is too small to pump the atom to the upper state, which leads to a trapping state in the lower state. The success of achieving PT at arbitrary quantum states lies in the choice of proper incident CEP-locked
ultrashort pulse to provide suitable action quantity. Of course, this action quantity is strongly dependent on the CEP of ultrashort laser pulse.

Another issue concerns about the validity of the two-level model in describing the interaction of atoms with ultrashort laser pulses, which cover a broadband of spectrum. It can be calculated that for a few-cycle (e.g., two-cycle, three-cycle, or more) optical pulse, the spectral bandwidth is about 13.6 eV, 9.1 eV or less, respectively. This bandwidth is smaller than the maximum energy level gap and (thus the maximum transition energy) of many atoms, including He, H₂ and so on. So, if we choose proper parameters, such as duration of few-cycle optical pulses, the transition frequency of atom energy levels, the central frequency of optical field, and the CEP, the classical two-level atomic system can be a good approximate model to handle the interaction of atoms with ultrashort laser pulses, and our theoretical analyses and results are reasonable and trustful.

Conclusion

In conclusion, we have shown numerically that population trapping can be realized in a two-level system under both resonant and off-resonant conditions by applying a CEP-locked ultrashort pulse to pump the atoms. We investigate the influence of CEP on the population evolutions of different detuned atoms beyond the RWA and find that the population variation is strongly dependent on CEP, the values of Rabi frequencies and detuning. The CEP plays a modulating role to the population trapping in the term of sine function. According to the modulation rule, we can choose proper parameters to realize PT at three specific quantum states: one is trapping population in the upper state; another is trapping half of the population in the upper state and half in the lower state; the third is trapping in the lower state.

To explain the fantastic modulation of the CEP, we present an analytical solution under weak-field approximation. The analytical solution is consistent with the numerical solution under weak field incident and illustrates the different modulation of CEP in different detuned atoms. Then we calculate the action quantity of the pulse acting on the atom by computing the integral for the interaction Hamiltonian and find the modulation functions of CEP on the population are exactly consistent with that on the action quantity excepting a coefficient and phase shift. The population varies approximately linearly with the action quantity.

The investigation provides directive significance for achieving PT at arbitrary quantum states via laser technologies with unique features of CEP, which can be viewed as a new key to manipulate a two-level system trapping with a higher pumping efficiency and lower laser power compared to the traditional schemes. The issues discussed in this paper could be extended to three-level atomic systems as well, which have potential applications in the concepts of lasing without inversion, electromagnetically induced transparency and enhancement of the index of refraction accompanied by vanishing absorption. This work suggests a way to regulate and control the quantum dynamics of atom and how to achieve population trapping states, which is the key issue of improving pumping efficiency.

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