Acoustic Waves in the Early Universe

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Abstract. Acoustics waves in the cosmology connect the astronomically large and the astronomically small. The largest acoustic waves in the Universe are the baryon acoustic oscillations that can be seen on the sky in the Cosmic Microwave Background anisotropy and in the current distribution of galaxies with a wavelength of 8 yottameters, nearly one billion lightyears. The sound travel distance in the 400,000 years prior to the Universe becoming transparent becomes one half of this wavelength, and then the expansion of the Universe stretched all distances by a factor of 1000. Precise measurements of these signals reveal the baryon density (0.4 yoctograms per cubic meter), the dark matter density (2 yg per cubic meter), and the radius of curvature of the Universe (more than 700 yottameters).

1. Introduction

The anisotropy of the CMB was recognized as an important probe of cosmological models very shortly after the CMB was discovered. The gravitational potential wells produced by density excesses produce cold spots on the sky via the Sachs-Wolfe (Sachs & Wolfe, 1967) effect. In addition, excess density in the baryon-photon fluid is associated with a temperature excess that leads to hot spots on the sky (Silk, 1968). Sachs & Wolfe predicted $\Delta T/T = 10^{-2}$ and Silk predicted $\Delta T/T = 3 \times 10^{-4}$ on smaller scales, but observed upper limits well below these limits were quickly established. These early calculations did not include the possibility of cold dark matter (CDM), but Peebles (1982) did include the effects of “X” particles with $m_X > 1$ keV/c$^2$. The effects of the acoustic waves and their interference with the CDM density pattern were included by Bond & Efstathiou (1984) and by 1987 a graph showing the interference pattern in the angular power spectrum of the CMB was produced by Bond & Efstathiou (1987). All of these calculations were done well before the first detection of any intrinsic anisotropy of the CMB. Only the dipole anisotropy (Conklin 1969, Henry 1971, Corey & Wilkinson 1976, Smoot et al 1977) due to the motion of the Solar System through the Universe was known until 1992 (Wright et al 1992).

2. Cosmic Definitions

Distances between unbound systems and wavelengths of light all scale like $a(t)$ in the expanding Universe. The scale factor is defined to be unity at the current time $t_o$, so $a(t_o) = 1$. The redshift is defined as $z = \lambda_o/\lambda_{em} - 1$ so it can be computed for light emitted at time $t_{em}$ as $1 + z = 1/a(t_{em})$.

The Hubble constant $H$, which is a function of time but not position, is given by $H = \dot{a}/a$. The density of the Universe that makes the escape velocity from a sphere equal to the Hubble...
Figure 1. Quantum fluctuations occur through space-time. Here virtual particle-antiparticle pairs are created and then annihilated.

Flow velocity is called the critical density, is given by

$$\rho_{\text{crit}} = \frac{3H^2}{8\pi G}$$

which is 18.8 yg/m$^3$ for $H = 100$ km/sec/Mpc. 1 Mpc (megaparsec) is $3.086 \times 10^{24}$ cm. The ratio of the actual density to the critical density is called $\Omega = \rho/\rho_{\text{crit}}$. The ratio of the matter density to the critical density is given by $\Omega_m$, and similarly the radiation density gives $\Omega_r$, and the vacuum energy density gives $\Omega_v$. While these $\Omega$’s depend on the redshift, the usual shorthand is to write $\Omega_m$ for the ratio now, which could be more accurately written as $\Omega_{m,0}$.

Finally one define a “curvature” term $\Omega_k = 1 - \Omega_m - \Omega_r - \Omega_v$. With these definitions one can write the expansion rate of the Universe as

$$\dot{a} = H_0 \sqrt{\Omega_r/a^2 + \Omega_m/a + \Omega_k + \Omega_v a^2}.$$  

(2)

The Hubble constant is often given as a fraction of a fiducial value: $h = H_0/(100$ km/sec/Mpc).
Figure 2. Left: space-time diagram showing quantum fluctuations during inflation. Right: three spatial slices showing the pattern on the sky produced by quantum fluctuations at a given instant during inflation.

The combination $\Omega h^2 = \omega$ is the ratio of the density to the critical density for $H = 100$ km/sec/Mpc.

3. Primordial Fluctuations
The origin of all cosmic structures is thought to be quantum fluctuations during the inflationary epoch. The Higgs mechanism for inducing masses through spontaneous symmetry breaking in symmetric potential with a minimum at a non-zero value of the potential (such as a Mexican Hat potential $V(\phi_1, \phi_2) = \lambda (\phi_1^2 + \phi_2^2 - \sigma^2)^2$) suggested that the Universe may have been dominated by this vacuum energy density for a brief period very soon after the Big Bang. During this inflationary epoch the Universe grew exponentially with a time constant given by $1/H = \sqrt{3c^2/8\pi GV}$. Guth (1981) used this scenario to explain the observed flatness of the Universe, the nearly perfect isotropy of the CMB, and the absence of magnetic monopoles. But quantum fluctuations, as suggested by the space-time diagram in Figure 1, occur at all times on very small spatial and temporal scales. During the inflationary epoch, these small regions of space-time can be swept up in the exponential expansion of space and become the size of stars, galaxies, clusters, superclusters and even larger than the observable Universe. Figure 2 shows this process.

Quantum fluctuations start out small, grow, and then get carried along by the exponential
expansion of the Universe. Meanwhile new quantum fluctuations are formed in the middle of the previous generation of fluctuations. Once these perturbations are larger than the speed of light times the expansion time scale, the gravitational potential associated with them is conserved. This scale, $c/H$ during inflation, is quite small: $O(10^{-24})$ cm, so it is reasonable that quantum effects are seen. Since the Universe is nearly in a steady state exponential expansion during the inflationary epoch, the final result is a primordial power spectrum of density fluctuations that gives “equal power on all scales”. This means that the gravitational potential perturbations on scales of 1 Gpc are the same as those on 2, 4, 8 or 16 Gpc scales.

4. Acoustic Effects
The equal power on all scales only applies for waves with lengths that are always larger than the speed of sound times the expansion time scales of the Universe, $1/H$. The speed of sound in the dark matter is zero because the dark matter has no pressure, and $c_s^2 = \partial P/\partial \rho = 0$. The speed of sound in the baryon-photon fluid is quite high. The pressure is given by the radiation pressure of the photons, which is $1/3$ of the energy density and scales as the $-4$ power of the scale factor of the Universe $a$. The density of the baryon-photon fluid includes both the photon energy density which scales like $a^{-4}$ and the baryon density which scales like $a^{-3}$. This gives

$$c_s = \frac{c}{\sqrt{3}} \left( \frac{4\rho_\gamma}{4\rho_\gamma + 3\rho_b} \right)^{1/2}.$$  \hspace{1cm} (3)

The current value of $\rho_\gamma$ is given by

$$\rho_\gamma = \frac{4\sigma s_B T_0^4}{c^3} = 4.6 \times 10^{-34} \text{ gm/cm}^3.$$  \hspace{1cm} (4)

![Figure 3](Eisenstein et al 2005 Figure 3)

Figure 3. The two-point correlation function $\xi$ vs. separation $s$ for luminous red galaxies showing the secondary peak marked by the arrow at $L_{ac}$. Data from Eisenstein et al. (2005).
Angular Scale [Degrees]

$[\ell (\ell+1) C_{\ell}/2\pi]^{1/2}$ [uK]

- - - $\Lambda$CDM

$\omega_b = 0.0222$
$\omega_c = 0.1035$
$\Omega_v = 0.7765$
$\Omega_{\text{tot}} = 1$
$H_0 = 75$

Figure 4. CMB data from the WMAP 3 year results (Hinshaw et al 2007) and other CMB experiments: ARCHEOPS (Benoit et al 2003), BOOM01 (Netterfield et al 2002), MAXIMA (Lee et al 2001), DASI (Halverson et al 2002), ACBAR (Kuo et al 2004), VSA (Grainge et al 2003), CBI (Readhead et al 2004) & BOOM05 (Jones et al 2006). A flat $\Lambda$CDM model is a good fit to all these data.

with $T_0 = 2.725$ K (Mather et al 1999). With the best value for the current baryon density $\rho_b = 0.42 \times 10^{-30}$ gm/cm$^3$ (Spergel et al 2007) the speed of sound is given by

$$c_s = \frac{c}{\sqrt{3(1 + 680/(1 + z))}}. \quad (5)$$

Thus for redshifts less than 679 the speed of sound in the baryon-photon fluid departs significantly from its high redshift value of $c/\sqrt{3}$. But at $z = 1089$ the hydrogen plasma in the Universe recombines to make neutral hydrogen, a transparent gas. At this point, called recombination or the surface of last scattering, the baryon-photon fluid ceases to exist, and the speed of sound in atomic hydrogen is more than 10,000 times smaller than the previous speed of sound. Any wave that has a wavelength substantially longer than $c_s/H$ at recombination has not been affected by acoustic processes since the inflationary epoch and is thus part of the equal power on all scales power spectrum. Shorter waves are affected and have substantially smaller amplitudes as a result. The time at recombination is 380,000 years after the Big Bang, so a wave with a wavelength about $10^6$ lightyears underwent about one radian of phase shift before recombination. This wave now has a wavelength of about $10^9$ lightyears.
Figure 5. Combined constraints on the matter density $\Omega_M$ and the vacuum density $\Omega_v$ from measurements of the acoustic scale $L_{ac}$ in the galaxy distribution (BAO) and the scale $\ell_a$ in the CMB anisotropy angular power spectrum.

A more accurate calculation of the relevant length gives

$$L_{ac} = \int^{t_{rec}}_0 (1 + z) c_s dt = \int^{1/1090}_0 \frac{c_s da}{a \dot{a}}. \quad (6)$$

This length is mainly determined by $1/(H_0\sqrt{\Omega_m})$ coming from the equation for $\dot{a}$, since $\Omega_k$ and $\Omega_v$ have negligible effect. When measuring this length one usually determines recession velocities which are given by $H_0 L_{ac}$, so the Hubble constant cancels out and a measure of $\Omega_m$ is obtained. $L_{ac}$ gives the distance that the baryon density excess travels as a sound wave from the Big Bang until recombination. This leaves a spherical shell of excess baryon density around any initial
density peak. The dark matter does not travel as a sound wave and moves only slightly due to gravitational forces. As a result, there will be a secondary peak in the two-point correlation function of galaxies at a separation given by \( L_{ac} \). Figure 3 shows the data obtained by Eisenstein et al. (2005) which determine this length. The data give \( H_0 L_{ac} = 10,400 \text{ km/sec} \).

One can also see this acoustic scale in angular power spectrum of the CMB anisotropy. \( L_{ac} \) gives the half wavelength of the “acoustic scale” which gives the spacing between the peaks. Specifically one gets

\[
\ell_a = \frac{\pi (1 + z_{LS}) D_A(z_{LS})}{L_{ac}} \tag{7}
\]

The angular size distance \( D_A \) is given by

\[
D_A = \frac{cZ(z)J(\Omega_k Z^2)}{H_0 (1 + z)} \tag{8}
\]

with

\[
J(x) = \begin{cases} 
\sin \sqrt{-x}, & x < 0; \\
\sinh \sqrt{x}, & x > 0; \\
1 + x/6 + x^2/120 + \ldots + x^n/(2n + 1)! + \ldots, & x \approx 0.
\end{cases} \tag{9}
\]

and

\[
Z = \int_{1/(1+z)}^{1} \frac{da}{a \sqrt{X}} \tag{10}
\]

where

\[
X(a) = \Omega_m/a + \Omega_r/a^2 + \Omega_v a^2 + \Omega_k. \tag{11}
\]

The angular size distance is significantly affected by the vacuum energy density so the combination of the \( \ell_a \) and \( L_{ac} \) can determine both \( \Omega_m \) and \( \Omega_v \).

The combination of the two measured acoustic scale quantities gives a very tight constraint on the geometry of the Universe. Figure 5 shows the current data, and the intersection of the allowed regions is very small and very close to the flat Universe line. The upper limit on the curvature is \( |\Omega_k| < 0.04 \). Since the radius of curvature of the Universe is given by \( R = (c/H_0)/\sqrt{|\Omega_k|} \) this gives an lower limit on the radius of curvature, \( R > 700 \text{ Ym} \).

5. Discussion

Sound waves in the Universe have given us the most precise constraints on the density of the components of the Universe. Much more precise values for both \( L_{ac} \) and \( \ell_a \) can be expected in the near future from the Joint Dark Energy Mission (JDEM) and from Planck, the ESA CMB mission to be launched in 2008.

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