Analysis of cycles of air-cooling machines with multi-step compression and expansion

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Abstract. The technique has been developed for estimating and analysing the thermodynamic efficiency of reversible cycles of air-cooling machines of multi-step compression and expansion using the exergy method. Analytical dependences have been obtained, and the influence of initial parameters on the main energy characteristics and thermodynamic efficiency of the cycles, expressed by exergy efficiency, has been analyzed. Numerical studies were carried out with number step compression and expansion from \( n = 1 \) to \( \infty \). Shown that with \( n = \infty \) the cycle of the air cooling machines with multi-step compression and expansion were transformed in to Carnot cycle, and with \( n = 1 \) it goes into a cycle with isobars supply and removal of heat to the working body.

1. Introduction
In thermodynamic analysis of inverse cycles, the representation of a reversible cycle, i.e. ideal for these conditions, is of great importance. In the ideal cycle of the vapor-liquid refrigerating machine, the supply and removal of heat to the working body are implemented at \( P = \text{const} \) and \( T = \text{const} \), since in the area of wet steam the isotherms and isobars coincide. For gases, isothermal and isobaric processes are significantly different. Therefore, in the reverse gas cycles, there may be two types of processes, in which both isothermal and isobaric heat exchange is implemented [1-3]. Isobaric supply or removal of heat is inevitably accompanied by a change in temperature, and isothermal ones go along with pressure changes. The expansion and compression of gas may occur over isentropic lines. Therefore, in the limit, two ideal cycles are possible: the first takes place at heat exchange with a "hot" source over an isotherm and the second takes place over isobars.

The simplest ideal cycle, in which the heat exchange between the heat sources is isothermal, is the Carnot cycle. Figure 1,a shows the Carnot cycle by processes 1-2-3-4-1 [1-3].

![Carnot Cycle](image)

**Figure 1.** Reversible cycles in \( T-s \) coordinates
The work spent on the implementation of the Carnot cycle can be estimated by the expression:

$$ l^{Carnot} = (T_{hot} - T_{cool}) \Delta s = c_p T_{hot} \frac{\theta - 1}{\theta} \ln \left[ \left( 1 - \left( \frac{P_c}{P_E} \right)^\gamma \right)^{-1} \right], $$

(1)

where $c_p$ is the heat capacity at constant air pressure; $T_{hot} = 293.15K$ is the atmospheric air temperature; $\Delta s = T_{cool} / \theta$ is the temperature at the outlet of the refrigerating chamber; $\theta$ is the ratio of the temperatures of heat sources; $P_c, P_E$ are the pressures behind the compressor and expander, respectively; and $\gamma = (k - 1) / k$; $k = 1.4$ is the ratio of specific heats.

Specific cooling capacity of Carnot cycle:

$$ q^{Carnot} = T_{cool} \Delta s = c_p T_{hot} \frac{1}{\theta} \ln \left[ \left( \frac{P_c}{P_E} \right)^\gamma \right], $$

(2)

Thermodynamic efficiency of the Carnot cycle, expressed by the exergy efficiency, is

$$ \eta_{\text{ex}}^{Carnot} = \frac{q^{Carnot}}{l^{Carnot}} = 1, $$

(3)

that is, it has the maximum value; where $q^{Carnot} = T_{hot} / T_{cool} - 1 = \theta - 1$ is the exergy Carnot factor [1–3].

Another ideal cycle with efficiency also equal to one ($\eta_{\text{ex}}^{ICC} = 1$) is the ideal cooling cycle (ICC), which consists of processes 1-2-3-5-1 (Fig. 1.a). A distinctive feature of ICC is the isobaric supply of heat from a cold source. Comparing the above cycles, we can conclude that the implementation of the ICC cycle requires a lot of work, defined by the expression:

$$ l^{ICC} = c_p T_{hot} \ln \left[ \left( \frac{P_c}{P_E} \right)^\gamma \right] - \left( \frac{P_c}{P_E} \right)^\gamma, $$

(4)

The expression for determining the specific cooling capacity of the ICC cycle has the form:

$$ q^{ICC} = c_p (T_{cool} - T_s) = c_p T_{hot} \left[ \left( \frac{P_c}{P_E} \right)^\gamma \right] - \left( \frac{P_c}{P_E} \right)^\gamma, $$

(5)

The cycle with the supply and removal of heat over isobars is presented in Fig. 1,b by the processes 1-6-3-5. This reversible cycle of the air-cooling machine (ACM) is a classical example of a reverse cycle, considered in a large number of articles and textbooks [1-3].

Specific work, cooling capacity and exergy efficiency of ACM cycle can be estimated by expressions:

$$ l^{ACM} = c_p T_{hot} \left[ \left( \frac{P_c}{P_E} \right)^\gamma \right] - \left( \frac{P_c}{P_E} \right)^\gamma, $$

(6)

$$ q^{ACM} = c_p (T_{hot} - T_s) = q^{ICC}, $$

(7)

$$ \eta_{\text{ex}}^{ACM} = \ln \left[ \left( \frac{P_c}{P_E} \right)^\gamma \right] / \left[ \left( \frac{P_c}{P_E} \right)^\gamma \right] - \left( \frac{P_c}{P_E} \right)^\gamma, $$

(8)

Specific work of the ACM cycle $l^{ACM} > l^{ICC}$, since the average temperature of the working fluid in the process 6-3 in Fig 1.b is higher than $T_{hot}$. As a result, there are losses due to irreversibility during heat exchange with a greater temperature difference, the compensation of which requires additional work. At that, the exergy efficiency $\eta_{\text{ex}}^{ACM} < \eta_{\text{ex}}^{ICC}$. Thus, the cycle 1-6-3-5 in Fig. 1,b, being internally reversible, has losses due to external irreversibility. These losses and the expended work can be reduced if a circuit with intermediate compression, expansion and cooling of the working fluid is applied.
2. Multi-step compression and expansion

In practice, it is extremely difficult to implement the processes of compression and expansion with the removal of heat during pressure changes, close to isothermal. Therefore, in the technical implementation of systems, multi-stage compression and expansion with intermediate heat exchange are used in many cases to approach the isothermal process. The principle of such processes is shown in Fig. 2 (cycle \(C+E\)), where dotted lines represent the ACM cycle. The working air is compressed in series in the compressors; and after each compression stage the isobaric cooling is implemented with the removal of heat to the environment. Heat is supplied to the cold source in a similar way [1–3]. With this implementation, there is a "narrowing" of the ACM cycle, and with an increase in the number of compression and expansion stages, it approaches the Carnot cycle. At that, there is an increase in specific cooling capacity and a decrease in the work spent on the implementation of the cycle.

![Figure 2. Multi-step compression and expansion cycle.](image)

Expressions for the evaluation of the main energy characteristics of the cycle with \(n\)-step compression and expansion can be obtained in the form of:

- specific work:
  \[
  I^{C+E} = c_p T_{hot} \left\{ \theta^{-1} \left[ \left( P_{IC,1} / P_E \right)^\gamma - 1 \right] + \sum_{i=1}^{n} \left[ \left( P_{IC,i} / P_{IC,i+1} \right)^\gamma - 1 \right] \right\} - \left[ 1 - \left( P_c / P_{E,i} \right)^\gamma \right] - \sum_{i=1}^{n} \theta^{-1} \left[ 1 - \left( P_{IE,i+1} / P_{IE,i} \right)^\gamma \right],
  \]
  (9)

  where \( P_{IC} \) and \( P_{IE} \) are the pressures of intermediate compression and expansion stages;

- specific cooling capacity:
  \[
  q^{C+E} = q_1 + \sum_{i=1}^{n-1} q_i,
  \]
  \(q_1 = c_p T_{hot} \left[ \theta^{-1} - \left( P_c / P_{IC,1} \right)^\gamma \right]\) is the specific cooling capacity after the first stage of expansion;

  \(q_{n-1} = c_p T_{hot} \left[ \theta^{-1} \left[ 1 - \left( P_{IE,n-1} / P_{IE,n} \right)^\gamma \right] \right]\);

- exergy efficiency of the cycle:
  \[
  \eta_{ex}^{C+E} = \left( q_{n-1} \tau_{ex,1} + \sum_{i=1}^{n-1} q_i \tau_{ex,i} \right) / I^{C+E},
  \]
  (11)

  where \( \tau_{ex,1} = 1 - \ln \left[ \theta^{-1} \left( P_c / P_{IE,1} \right)^\gamma \right] / \theta^{-1} \left[ \theta^{-1} - \left( P_c / P_{IC,1} \right)^\gamma \right] \), \( \tau_{ex,n-1} = 1 - \ln \left[ \left( P_{IE,n-1} / P_{IE,n} \right)^\gamma \right] / \theta^{-1} \left[ 1 - \left( P_{IE,n-1} / P_{IE,n} \right)^\gamma \right] \).
At $n = 1$ expressions (9)–(11) change to (6)–(8), respectively, which confirms the reliability of the dependences.

Figure 3 presents the graphic dependences of the influence of the number of compression and expansion stages on the main characteristics of the cycle ($C+E$). Whence it follows that the increase in $n$ makes the cooling capacity and the expended work of the cycle of multi-stage compression and expansion tend to the corresponding values of the Carnot cycle.

![Figure 3. Characteristics of the multi-stage compression and expansion cycle.](image)

Thus, the cycle of multistage compression and expansion (Fig. 2) is a cross between the Carnot cycle and $ACM$. When $n \to \infty$ it turns into an ideal gas cycle with isothermal supply and removal of heat (Carnot cycle), and with a decrease in the number of compression and expansion stages to $n = 1$ it goes into a cycle with two isobars ($ACM$ cycle).

**Conclusion**

The paper has considered the reversible (idealized) cycles of refrigerating machines, the thermodynamic efficiency of which takes the maximum possible value in the studied initial conditions. This representation of the cycles allows performing a complex thermodynamic analysis and establishing the basic laws. In addition, the considered cycle models are basic for presenting more complex regenerative and real cycles. Representation of models of real thermodynamic cycles allows performing a comprehensive analysis of exergy losses in the units of the system, assessing their impact on the exergy efficiency of cycles and proposing ways to reduce irreversible losses.

**References**

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