$B \rightarrow f_0(980)K$ Decays and Subleading Corrections

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Abstract

The decay $B \rightarrow f_0(980)K$ is studied within the framework of QCD factorization and the two-quark scenario for $f_0(980)$. There are two distinct penguin contributions and their interference depends on the unknown mixing angle $\theta$ of strange and nonstrange quark contents of $f_0(980)$: destructive for $0 < \theta < \pi/2$ and constructive for $\pi/2 < \theta < \pi$. The QCD sum rule method is applied to evaluate the leading-twist light-cone distribution amplitudes and the scalar decay constant of $f_0$. We conclude that the short-distance approach is not adequate to explain the observed large rates of $f_0K^-$ and $f_0K^0$. Among many possible subleading corrections, we study and estimate the contributions from the three-parton Fock states of the $f_0$ and from the intrinsic gluon inside the $B$ meson. It is found that the spectator gluon of the $B$ meson may play an eminent role for the enhancement of $f_0(980)K$. We point out that if $f_0(980)$ is a four-quark state as widely perceived, there will exist extra diagrams contributing to $B \rightarrow f_0(980)K$. However, in practice it is difficult to make quantitative predictions based on the four-quark picture for $f_0(980)$ as it involves additional nonfactorizable contributions that are difficult to estimate and the calculations of the decay constant and form factors of $f_0(980)$ are beyond the conventional quark model.
I. INTRODUCTION

The decay of the $B$ meson into a scalar meson $f_0(980)$ was first measured by Belle \[1\] in the charged $B$ decays to $K^\pm\pi^\mp\pi^\pm$ and a large branching fraction product for the $f_0(980)K^\pm$ final states was found. A recent updated result by Belle yields \[2\]

$$B(B^+ \to f_0(980)K^+ \to \pi^+\pi^-K^+) = (7.55 \pm 1.24^{+1.63}_{-1.18}) \times 10^{-6}. \quad (1.1)$$

The Belle result is subsequently confirmed by the BaBar measurement \[3\]:

$$B(B^+ \to f_0(980)K^+ \to \pi^+\pi^-K^+) = (9.2 \pm 1.2^{+2.1}_{-2.0}) \times 10^{-6}. \quad (1.2)$$

A recent BaBar analysis of $B^\pm \to K^\pm\pi^\mp\pi^\pm$ gives a very similar result: $(9.2 \pm 1.5 \pm 0.8) \times 10^{-6}$ \[4\].

The world average is then given by \[5\]

$$B(B^+ \to f_0(980)K^+ \to \pi^+\pi^-K^+) = (8.49^{+1.35}_{-1.26}) \times 10^{-6}. \quad (1.3)$$

BaBar has also measured the neutral mode $B^0 \to f_0(980)K^0$ with the result \[6\]

$$B(B^0 \to f_0(980)K^0 \to \pi^+\pi^-K^0) = (6.0 \pm 0.9 \pm 1.3) \times 10^{-6}. \quad (1.4)$$

This channel is of special interest as possible indications of New Physics beyond the Standard Model (SM) may be observed in the time-dependent $CP$ asymmetries in the penguin-dominated $B$ decays such as $B^0 \to f_0(980)K_S$. The mixing-induced $CP$-violation parameter $S$ is expected to be very close to $-\sin \beta$ in the SM. The most recent measurements by BaBar and Belle yield

$$\sin \beta(f_0K_S) = \begin{cases} 0.95^{+0.23}_{-0.32} \pm 0.10 & \text{BaBar} \[7]\, \\ -0.47 \pm 0.41 \pm 0.08 & \text{Belle} \[8]. \end{cases} \quad (1.5)$$

The deviation from $\sin 2\beta = 0.726 \pm 0.037$ \[9\] measured from $B \to J/\psi K_S$ may hint at a possible New Physics.

The absolute branching ratios for $B \to f_0K$ depends critically on the branching fraction of $f_0(980) \to \pi\pi$. For this purpose, we use the results from the most recent analysis of \[10\], namely, $\Gamma_{\pi\pi} = 64 \pm 8$ MeV and $\Gamma_{tot} = 80 \pm 10$ MeV for $f_0(980)$, to obtain $B(f_0(980) \to \pi\pi) = 0.80 \pm 0.14$. \[1\]

This leads to

$$B(B^+ \to f_0(980)K^+) = (15.9^{+3.8}_{-3.7}) \times 10^{-6}, \quad (1.6)$$

Comparing with the averaged branching ratios, $(12.1 \pm 0.8) \times 10^{-6}$ for $B^+ \to \pi^0K^+$ and $(11.5 \pm 1.0) \times 10^{-6}$ for $B^0 \to \pi^0K^0$, we see that $f_0(980)K^+ \geq \pi^0K^+$ and $f_0(980)K^0 \approx \pi^0K^0$.

Theoretically, the decay $B \to f_0K$ has been studied in \[12\] and \[13\] within the framework of the pQCD approach based on the $k_T$ factorization theorem. It is found that the branching ratio is

\[1\] The ratio $R \equiv B(f_0 \to \pi^+\pi^-)/B(f_0 \to K^+K^-) \approx 7.1$ is consistent with the result of $R > 2.6^{+0.7}_{-0.6}$ inferred from the Belle measurements of $B(B^+ \to f_0(980)K^+ \to \pi^+\pi^-K^+)$ [see Eq. \[1.1\]] and $B(B^+ \to f_0(980)K^+ \to K^+K^-K^+) < 2.9 \times 10^{-6}$ \[11\].
of order $5 \times 10^{-6}$ (see Fig. 2 of [13]), which is smaller than the measured value of $\mathcal{B}(B^+ \to f_0K^+)$ by a factor of 3. In the present paper we shall re-examine this decay within the QCD factorization approach [14, 15, 16].

The underlying structure of the parity-even meson $f_0(980)$ is still controversial and not clear: It can be a conventional 2-quark $P$-wave state or a $S$-wave meson made of four quarks $qq\bar{q}\bar{q}$. It will be interesting to see if these two different scenarios for $f_0(980)$ can be tested in $B \to f_0K$ decays, an issue which we will address briefly in this work.

There are two distinct penguin contributions to $B \to f_0K$: one is related to the $u$ quark component of $f_0$ and the other to the strange quark content of $f_0$. Owing to the cancellation between two penguin terms, the former penguin contribution is severely suppressed, while the latter depends on the unknown scalar decay constant of $f_0$. Based on the sum rule approach, we shall show that the magnitude of the $f_0$ scalar decay constant is substantially larger than the corresponding decay constant of pseudoscalar mesons and hence the $f_0K$ rate can be comparable to the $\pi^0K$ one. However, the predicted branching ratio of $B \to f_0K$ is still lower than experiment by $\sim 45\%$ for the $f_0K^-$ mode and $\sim 30\%$ for $f_0K^0$. There are several possible mechanisms such as final-state interactions, flavor-singlet contributions, large annihilation contributions that may account for the enhancement of $f_0K$. In the present work, we will focus on the subleading effects arising from the three-parton Fock states of the $f_0$ and from the spectator gluon inside the $B$ meson.

It is commonly assumed that only the valence quarks of the initial and final state hadrons participate in the decays. Nevertheless, a real hadron in QCD language should be described by a set of Fock states for which each state has the same quantum number as the hadron. For instance,

$$
|B^-\rangle = \psi^B_{b\bar{u}}|b\bar{u}\rangle + \psi^B_{b\bar{d}}|b\bar{d}\rangle + \psi^B_{b\bar{q}}|b\bar{q}\rangle + \psi^B_{\bar{b}d}|\bar{b}d\rangle + \ldots,
$$

$$
|f_0\rangle = \psi^{f_0}_{nn}|n\bar{n}\rangle + \psi^{f_0}_{sn}|s\bar{n}\rangle + \psi^{f_0}_{n\bar{g}}|n\bar{n}\bar{g}\rangle + \psi^{f_0}_{sq}|s\bar{s}\rangle + \sum_q(\psi^{f_0}_{n\bar{q}g}|n\bar{n}q\bar{g}\rangle + \psi^{f_0}_{s\bar{q}g}|s\bar{s}q\bar{g}\rangle) + \ldots,
$$

$$
|K^-\rangle = \psi^K_{s\bar{u}}|s\bar{u}\rangle + \psi^K_{s\bar{s}}|s\bar{s}\rangle + \psi^K_{s\bar{n}g}|s\bar{n}\bar{g}\rangle + \psi^K_{s\bar{q}g}|s\bar{s}q\bar{g}\rangle + \ldots,
$$

where $n\bar{n} \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$. The extra gluon(s) or quark pair(s) appearing in higher Fock states are the results of QCD interactions. The $c\bar{c}$ pairs due to a single gluon splitting $g \to c\bar{c}$, described by the Gribov-Lipatov-Altarelli-Parisi (GLAP) evolution equation, $^2$ are basically extrinsic to the bound-state nature of the hadron. In contrast, the $c\bar{c}$ pairs, which are entangled through multiply gluonic interactions to the valence quarks, at least via $g^*g^* \to c\bar{c}$, are intrinsic to the hadronic structure [19, 20, 21]. It has been estimated that the intrinsic charm probability in the proton is $\lesssim 1\%$ [22]. The intrinsic charm may explain the $\rho\pi$ puzzle in $J/\psi(\psi')$ decays [23].

The study of the intrinsic charm component in the $B$ meson is interesting and has attracted a great deal of attentions [19, 20, 21]. Although the extrinsic charm pairs carry only small momentum

$^2$ We should remind the readers that the light-cone wave functions in Eq. (1.7) are Lorentz invariant using the light-cone quantization in the light-cone gauge $A^+ = 0$ [17]. However, here we adopt the equal-time quantization and choose the $B$ rest frame. Each Fock-state wave function of the $B$ meson is no longer boost invariant. In contrast, the energetic $f_0$ and $K$ produced in $B$ decays are represented in terms of light-cone distribution amplitudes (LCDAs) in conformal expansion [18]. Strictly speaking, the GLAP equation can be applied only to the partons in the infinite momentum frame or to the light-cone quantization framework.
fraction of hadrons, it has been argued that the intrinsic $c\bar{c}$ pair could share 20% momentum fraction of the parent $B$ meson \cite{20}. As discussed in \cite{19}, the intrinsic charm may give rise to 20% corrections to $B \to K\pi$ amplitudes. Consistently, $|\psi_{b\bar{c}}^B|/|\psi_{ba}^B|^2$ integrating over the phase space is estimated to be roughly of order $(20\%)^2 = 4\%$ \cite{21}, compared to the charm content $\lesssim 1\%$ in the proton. Similar to the intrinsic charm case, one may ask how large is the intrinsic gluon within the $B$ meson. The gluon content of the $B$ meson is intimately related to the energy of light degrees of freedom (or the so-called “brown muck”), namely, $\bar{\Lambda} = m_B - m_b$. Even though $\bar{\Lambda} \gtrsim 450$ MeV is larger than the constituent light quark mass, it may suggest that the intrinsic gluon within the $B$ meson cannot be neglected. To estimate the possible contributions originated from the intrinsic gluon within the $B$ meson, we introduce the three-particle operator $\bar{b}\gamma^\alpha g_s\tilde{G}^\alpha_{\mu\nu}u$, which can couple to the $B$ meson. Similar definitions for $\pi$ and $\rho$ can be found in the literature \cite{24,25}. Hence, the intrinsic gluon effects can be estimated. The detailed calculation is shown in Sec. II C. As a result, we find that the intrinsic gluon may give rise to 20-40% corrections to the decay amplitudes. In contrast, we shall show in Sec. II B that the extrinsic gluon effect due to the splitting $b \to bg$ is negligible, which is described by the GLAP evolution equation in the infinite momentum frame.

The layout of the present paper is organized as follows. In Sec. II we shall first study $B \to f_0K$ decays within the framework of QCD factorization and discuss the internal structure of the $f_0(980)$. We then continue to explore the subleading effects arising from the three-parton Fock states of the $f_0$ and from the spectator gluon inside the $B$ meson. We present numerical results and discussions in Sec. III and give conclusions in Sec. IV. Appendices A and B are devoted to the determination of the scalar decay constant and distribution amplitudes of $f_0$, respectively, Appendix C outlines the sum-rule calculation for the decay constant $\delta_{f_0}^2$ and Appendix D for the intrinsic gluon content of the $B$ meson.

II. $B \to f_0(980)K$ DECAY AMPLITUDES

A. $B \to f_0(980)K$ decay amplitudes in QCD factorization of leading Fock states

1. Framework

To proceed we first discuss the decay constants and form factors. The decay constants are defined by

$$\langle K(p)|A_\mu|0\rangle = -if_{KP}\mu, \quad \langle f_0|V_\mu|0\rangle = 0, \quad \langle f_0|\bar{q}q|0\rangle = m_{f_0}\bar{f}_q.$$  \hspace{1cm} (2.1)

The scalar meson $f_0(980)$ cannot be produced via the vector current owing to charge conjugation invariance or conservation of vector current. The scalar decay constant $f_q$ will be discussed later. Form factors for $B \to P$ and $B \to S$ transitions ($P$: pseudoscalar meson, $S$: scalar meson) are defined by \cite{20}

$$\langle P(pP)|V_\mu|B(p_B)\rangle = \left(p_{B\mu} + p_{P\mu} - \frac{m_B^2 - m_P^2}{q^2} q_\mu\right) F_{1 PB}^P(q^2) + \frac{m_B^2 - m_P^2}{q^2} q_\mu F_{0 PB}^P(q^2), \hspace{1cm} (2.2)$$
where \( q_\mu = (p_B - p_P)_\mu \), and

\[
(S(p_S)|A_\mu|B(p_B)) = -i \left[ (p_{B\mu} + p_{S\mu} - \frac{m_B^2 - m_S^2}{q^2} q_\mu) F_1^{BS}(q^2) + \frac{m_B^2 - m_S^2}{q^2} q_\mu F_0^{BS}(q^2) \right].
\]

(2.3)

The penguin-dominated \( B^- \to f_0 K^- \) receive two different types of penguin contributions as depicted in Fig. 1. Within the framework of QCD factorization \[14\], the \( B \to f_0 K \) decay amplitudes read\(^4\)

\[
A(B^- \to f_0 K^-) = -\frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left\{ \left[ a_1(f_0 K^-) \delta_{0p}^p + (a_{10}^p - r_\chi^{K+} a_0^p)(f_0 K^-) + (a_{10}^p - r_\chi^{K+} a_0^p)(f_0 K^-) \right] 
\times f_K F_0 B f_0 \left( m_K^2 + m_B^2 - m_f^2 \right) + (2a_{10}^p - a_0^p)(K^- f_0) \tilde{f}_s \frac{m_f K}{m_b} F_0 B K \left( m_f^2 + m_B^2 - m_K^2 \right) 
\times \sum_{q=u,s} f_b f_K f_q \left[ b_3^q(f_0 K^-) + b_3^{3EW}(f_0 K^-) \right] \right\},
\]

\[
A(B^0 \to f_0 K^0) = -\frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left\{ \left[ (a_{10}^p - r_\chi^{K+} a_0^p)(f_0 K^0) - \frac{1}{2}(a_{10}^p - r_\chi^{K+} a_0^p)(f_0 K^0) \right] 
\times f_K F_0 B f_0 \left( m_K^2 + m_B^2 - m_f^2 \right) + (2a_{10}^p - a_0^p)(K^0 f_0) \tilde{f}_s \frac{m_f K}{m_b} F_0 B K \left( m_f^2 + m_B^2 - m_K^2 \right) 
\times \sum_{q=u,s} f_b f_K f_q \left[ b_3^q(f_0 K^0) - \frac{1}{2} b_3^{3EW}(f_0 K^0) \right] \right\},
\]

(2.4)

where \( \lambda_p \equiv V_{pb} V_{ps}^* r_\chi^K(\mu) = 2 m_K^2 /[m_b(\mu)(m_u(\mu) + m_s(\mu))] \), and weak annihilation contributions described by \( b_3 \) and \( b_{3EW} \) terms will be discussed shortly. In Eq. (2.4), the superscript \( u \) of the form factor \( F_0 B f_0^u \) reminds us that it is the \( u \) quark component of \( f_0 \) involved in the form factor transition [see Fig. 1(a)]. In contrast, the subscript \( s \) of the decay constant \( \tilde{f}_s \) indicates that it is the strange quark component of \( f_0 \) responsible for the penguin contribution of Fig. 1(b).

\(^3\) As shown in [27], a factor of \((-i)\) is needed in Eq. (2.3) in order for the \( B \to S \) form factors to be positive. This also can be checked from heavy quark symmetry [27].

\(^4\) The relative sign between \( a_0^s(f_0 K) f_K \) and \( a_0^p(K f_0) \tilde{f}_s \) terms in the decay amplitude of \( B \to f_0 K \) obtained in [28] is opposite to ours.
The effective parameters $a_i^p$ with $p = u, c$ in Eq. (2.3) can be calculated in the QCD factorization approach \cite{14}. They are basically the Wilson coefficients in conjunction with short-distance nonfactorizable corrections such as vertex corrections and hard spectator interactions. In general, they have the expressions \cite{14, 16}

$$a_i^p(M_1M_2) = c_i + \frac{c_{i\pm 1}}{N_c} + \frac{c_{i\pm 1}}{N_c} \frac{C_F\alpha_s}{4\pi} \left[ V_i(M_2) + \frac{4\pi^2}{N_c} H_i(M_1M_2) \right] + P_i^p(M_2),$$

(2.5)

where $i = 1, \ldots, 10$, the upper (lower) signs apply when $i$ is odd (even), $c_i$ are the Wilson coefficients, $C_F = (N_c^2 - 1)/(2N_c)$ with $N_c = 3$, $M_2$ is the emitted meson and $M_1$ shares the same spectator quark with the $B$ meson. The quantities $V_i(M_2)$ account for vertex corrections, $H_i(M_1M_2)$ for hard spectator interactions with a hard gluon exchange between the emitted meson and the spectator quark of the $B$ meson and $P_i^p(M_2)$ for penguin contractions. The explicit expressions of these quantities can be found in \cite{14, 16}. The hard spectator function $H$ reads

$$H_i(f_0K) = \frac{\bar{f}_a f_B}{F_{B^0(0)m_B^2}} \int_0^{1} \frac{d\rho}{\rho} \Phi_B(\rho) \int_0^{1} \frac{d\xi}{\xi} \Phi_K(\xi) \int_0^{1} \frac{d\eta}{\eta} \left[ \Phi_{f_0}(\eta) + \frac{2m_{f_0}}{m_B} \bar{\xi} \Phi_{f_0}(\eta) \right],$$

(2.6)

for $i = 1, 4, 10$ and $H_i = 0$ for $i = 6, 8$. where $\bar{\xi} = 1 - \xi$ and $\bar{\eta} = 1 - \eta$. As for the parameters $a_{6,8}^{u,c}(f_0K)$ appearing in Eq. (2.3), they have the same expressions as $a_{6,8}(f_0K)$ except that the penguin function $\hat{G}_K$ (see Eq. (55) of \cite{14}) is replaced by $\hat{G}_{f_0}$ and $\Phi_K$ by $\Phi_{f_0}$.

Weak annihilation contributions to $B \to f_0K$ are described by the terms $b_3$ and $b_{3\text{EW}}$ in Eq. (2.3) which have the expressions

$$b_3^q = \frac{C_F}{N_c^2} \left[ c_3 A_1^{i(q)} + c_5 (A_3^{i(q)} + A_3^{f(q)}) + N_c c_6 A_3^{f(q)} \right],$$

$$b_{3\text{EW}}^q = \frac{C_F}{N_c^2} \left[ c_9 A_1^{i(q)} + c_7 (A_3^{i(q)} + A_3^{f(q)}) + N_c c_8 A_3^{i(q)} \right],$$

(2.7)

where $A_3^{f}$ is the factorizable annihilation amplitude induced from $(S-P)(S+P)$ operator and $A_3^{i}$ are nonfactorizable ones induced from $(V-A)(V-A)$ and $(S-P)(S+P)$ operators, respectively. It is evident that the dominant annihilation contribution arises from the factorizable penguin-induced annihilation characterized by $A_3^{i}$. Their explicit expressions are given by (see also \cite{16})

$$A_1^{i(q)} = \pi \alpha_s \int_0^1 \frac{dxdy}{dy} \left\{ \Phi_K(x) \Phi_{f_0}(y) \left[ \frac{1}{y(1-x\bar{y})} + \frac{1}{x^2y} \right] - 2\kappa x m_{f_0} \Phi_0(x) \Phi_{f_0}(y) \frac{2y}{x y(1-x\bar{y})} \right\},$$

$$A_3^{i(q)} = \pi \alpha_s \int_0^1 \frac{dxdy}{dy} \left\{ \frac{2m_{f_0}}{m_B} \Phi_K(x) \Phi_{f_0}(y) \frac{2y}{x y(1-x\bar{y})} + \frac{1}{y(1-x\bar{y})} + \frac{1}{x^2y} \right\},$$

$$A_3^{f(q)} = \pi \alpha_s \int_0^1 \frac{dxdy}{dy} \left\{ \frac{2m_{f_0}}{m_B} \Phi_K(x) \Phi_{f_0}(y) \frac{2(1+\bar{x})}{x^2y} - \frac{1}{y(1-x\bar{y})} - \frac{1}{x^2y} \right\},$$

(2.8)

where $q = u, s$, $\bar{x} = 1 - x$, $\bar{y} = 1 - y$, $\Phi_M$ ($\Phi_M^*$) is the twist-2 (twist-3) light-cone distribution amplitude of the meson $M$.

Although the parameters $a_i(i \neq 6, 8)$ and $a_{6,8}x_{\chi}$ are formally renormalization scale and $\gamma_5$ scheme independent, in practice there exists some residual scale dependence in $a_i(\mu)$ to finite order. To be specific, we shall evaluate the vertex corrections to the decay amplitude at the scale $\mu = m_B/2$. In contrast, as stressed in \cite{14}, the hard spectator and annihilation contributions should be evaluated
at the hard-collinear scale $\mu_h = \sqrt{\mu_0 \Lambda_h}$ with $\Lambda_h \approx 500$ MeV. There is one more serious complication about these contributions; that is, while QCD factorization predictions are model independent in the $m_b \to \infty$ limit, power corrections always involve troublesome endpoint divergences. For example, the annihilation amplitude has endpoint divergences even at twist-2 level and the hard spectator scattering diagram at twist-3 order is power suppressed and posses soft and collinear divergences arising from the soft spectator quark. Since the treatment of endpoint divergences is model dependent, subleading power corrections generally can be studied only in a phenomenological way. We shall follow [14] to parametrize the endpoint divergence $X_A \equiv \int_0^1 dx/\bar{x}$ in the annihilation diagram as

$$X_A = \ln \left( \frac{m_B}{\Lambda_h} \right) (1 + \rho_A e^{i\phi_A}),$$

where $\rho_A$ is a complex parameter $0 \leq |\rho_A| \leq 1$. Likewise, the endpoint divergence $X_H$ in the hard spectator contributions can be parametrized in a similar way.

Note that $a_4$ and $a_6$ penguin terms contribute constructively to $\pi^0 K^-$ but destructively to $f_0 K^-$. Therefore, the contribution to $B \to f_0 K$ from Fig. 1(a) will be severely suppressed. The contribution from Fig. 1(b) is suppressed by $m_{f_0}/m_b$. Hence, it is naively expected that the $f_0 K$ rate is much smaller than the $\pi^0 K$ one. However, as we shall see below, the scale dependent decay constant $\tilde{f}_s$ is much larger than $f_\pi$. As a consequence, the branching ratio of $B \to f_0 K$ turns out to be comparable to $B \to \pi^0 K$.

2. Quark structure of $f_0(980)$ and input parameters

It is known that the underlying structure of scalar mesons is not well established theoretically (for a review, see e.g. [29, 30, 31]). It has been suggested that the light scalars below or near 1 GeV–the isoscalars $f_0(600)$ (or $\sigma$), $f_0(980)$, the isodoublet $K^*_0(800)$ (or $\kappa$) and the isovector $a_0(980)$–form an SU(3) flavor nonet, while scalar mesons above 1 GeV, namely, $f_0(1370)$, $a_0(1450)$, $K^*_0(1430)$ and $f_0(1500)/f_0(1710)$, form another nonet. A consistent picture [31] provided by the data suggests that the scalar meson states above 1 GeV can be identified as a conventional $q\bar{q}$ nonet with some possible glue content, whereas the light scalar mesons below or near 1 GeV form predominately a $qq\bar{q}\bar{q}$ nonet with a possible mixing with $0^+ q\bar{q}$ and glueball states. This is understandable because in the $q\bar{q}$ quark model, the $0^+$ meson has a unit of orbital angular momentum and hence it should have a higher mass above 1 GeV. On the contrary, four quarks $q^2\bar{q}^2$ can form a $0^+$ meson without introducing a unit of orbital angular momentum. Moreover, color and spin dependent interactions favor a flavor nonet configuration with attraction between the $qq$ and $\bar{q}\bar{q}$ pairs. Therefore, the $0^+ q^2\bar{q}^2$ nonet has a mass near or below 1 GeV. This four-quark scenario explains naturally the mass degeneracy of $f_0(980)$ and $a_0(980)$, the broader decay widths of $\sigma(600)$ and $\kappa(800)$ than $f_0(980)$ and $a_0(980)$, and the large coupling of $f_0(980)$ and $a_0(980)$ to $K\bar{K}$. While the above-mentioned four-quark assignment of $f_0(980)$ is certainly plausible when the light scalar meson is produced in low-energy reactions, one may wonder if the energetic $f_0(980)$ produced in $B$ decays is dominated by the four-quark configuration as it requires to pick up two energetic quark-antiquark pairs to form a fast-moving light four-quark scalar meson. The Fock states of
(a) FIG. 2: Penguin contributions to $B^- \to f_0(980)K^-$ in the 4-quark picture for $f_0(980)$. $f_0(980)$ consists of $q\bar{q}$, $q^2\bar{q}^2$, $q\bar{q}g$ etc. [see Eq. (1.7)]. It is thus expected that the distribution amplitude of $f_0$ would be smaller in the four-quark model than in the two-quark picture. Then one will not be able to explain the observed $B \to f_0(980)K$ decays.

Nevertheless, as pointed out in [34], the number of the quark diagrams for the penguin contributions to $B \to f_0(980)K$ (see Fig. 2) in the four-quark scheme for $f_0(980)$ is two times as many as that in the usual 2-quark picture (see Fig. 1). That is, besides the factorizable diagrams in Fig. 2(a), there exist two more nonfactorizable contributions depicted in Fig. 2(b). Therefore, a priori there is no reason that the $B \to f_0(980)K$ rate will be suppressed if $f_0$ is a four-quark state. However, in practice, it is difficult to give quantitative predictions based on this scenario as the nonfactorizable diagrams are usually not amenable. Moreover, even for the factorizable contributions, the calculation of the $f_0(980)$ decay constant and its form factors is beyond the conventional quark model, though attempt has been made in [34]. In order to make quantitative calculations for $B \to f_0(980)K$, we shall assume the conventional 2-quark description of the light scalar mesons.

In the naive 2-quark picture, $f_0(980)$ is purely an $s\bar{s}$ state and this is supported by the data of $D_s^+ \to f_0\pi^+$ and $\phi \to f_0\gamma$ implying the copious $f_0(980)$ production via its $s\bar{s}$ component. However, there also exist some experimental evidences indicating that $f_0(980)$ is not purely an $s\bar{s}$ state. First, the observation of $\Gamma(J/\psi \to f_0\omega) \approx \frac{1}{2}\Gamma(J/\psi \to f_0\phi)$ [35] clearly indicates the existence of the non-strange and strange quark content in $f_0(980)$. Second, the fact that $f_0(980)$ and $a_0(980)$ have similar widths and that the $f_0$ width is dominated by $\pi\pi$ also suggests the composition of $u\bar{u}$ and $d\bar{d}$ pairs in $f_0(980)$; that is, $f_0(980) \to \pi\pi$ should not be OZI suppressed relative to $a_0(980) \to \pi\eta$. Therefore, isoscalars $\sigma(600)$ and $f_0$ must have a mixing

$$|f_0(980)\rangle = |s\bar{s}\rangle \cos \theta + |n\bar{n}\rangle \sin \theta, \quad |\sigma_0(600)\rangle = -|s\bar{s}\rangle \sin \theta + |n\bar{n}\rangle \cos \theta,$$

(2.10) with $n\bar{n} \equiv (\bar{u}u + \bar{d}d)/\sqrt{2}$. The distribution amplitudes $\Phi_s \equiv \Phi_{f_0}^{(s)}$ and $\Phi_n \equiv \Phi_{f_0}^{(n)}$ corresponding to
The asymptotic forms for kaon twist-2 and twist-3 distribution amplitudes are expansion \[18\]. Hence, we take

\[
\Phi_p^B \text{ analysis in } [36]. \text{ As we shall see below, the negative sign of } = 1 \text{ GeV. Our results are consistent with the empirical result of } \mu = 5. \text{ In particular, we obtain } F_{n,s}^{0}(1-x) = B_1 = B_1^{p} = 0.8B_1^{n} \text{ for } \Phi_s \text{ at } \mu = 1 \text{ GeV. Our results are consistent with the empirical result of } |B_1| = 1.1 \text{ inferred from the analysis in } [36]. \text{ As we shall see below, the negative sign of } B_1 \text{ is indeed strongly favored when the predicted } f_0 K \text{ rates are confronted with the data. As for the twist-3 distribution amplitude } \Phi_{p}^{0}(x), \text{ its asymptotic form is the same as the light pseudoscalar meson to the leading conformal expansion } [18]. \text{ Hence, we take }

\[
\Phi_{n,s}^{p}(x) = 1. \tag{2.15}
\]

The asymptotic forms for kaon twist-2 and twist-3 distribution amplitudes are

\[
\Phi_K(x) = 6x(1-x), \quad \Phi_K^p(x) = 1. \tag{2.16}
\]

In the \(q\bar{q}\) description of \(f_0(980)\), it follows from that

\[
F_0^{p} f_0 = \frac{1}{\sqrt{2}} \sin \theta F_0^{B_0 f_0^{q\bar{q}}}, \quad F_0^{p} f_0 = \frac{1}{\sqrt{2}} \sin \theta F_0^{B_0 f_0^{q\bar{q}}}, \tag{2.17}
\]

where the superscript \(q\bar{q}\) denotes the quark content of \(f_0\) involved in the transition. The form factor for \(B\) to the scalar meson transition has been calculated in the covariant light-front model \[27\]. From Table VI of \[27\], it is clear that \(F_0^{B_0 f_0^{q\bar{q}}}(0)\) with \(q\bar{q} = u\bar{u}\) or \(d\bar{d}\) is of order 0.25 which is...
very similar to $F_0^{B\phi}(0)$. As we shall see below, a precise estimate of the $B \rightarrow f_0(980)$ form factor is not important because the contribution from Fig. 1(a) is largely suppressed owing to the large cancellation between the $a_4$ and $a_6$ penguin terms. Based on the QCD sum-rule method, the scalar decay constant $\tilde{f}_s$ defined in Eq. (2.12) has been estimated in [37] and [38] with similar results, namely, $\tilde{f}_s \approx 0.18$ GeV at a typical hadronic scale. Notice that, in contrast to the pseudoscalar meson decay constants, the scalar decay constant $\tilde{f}_s$ is scale dependent. Taking into account the scale dependence of $\tilde{f}_s$ and radiative corrections to the quark loops in the OPE series, in Appendix A we have made a careful evaluation of the scalar decay constant using the sum rule approach and found $\tilde{f}_s(\mu = 1 \text{ GeV}) = 0.33 \text{ GeV}$ and $\tilde{f}_s(\mu = 2.1 \text{ GeV}) = 0.39 \text{ GeV}$ [see Eqs. (A9) and (A10)] and similar results for $\tilde{f}_n$.

$\tilde{f}_s = \frac{m^{(s)}_f}{m_f} \tilde{f}_s \cos \theta, \quad \tilde{f}_n = \frac{m^{(n)}_f}{m_f} \tilde{f}_n \sin \theta,$

where use has been made of Eqs. (2.1), (2.10) and (2.12).

Experimental implications for the $f_0 - \sigma$ mixing angle have been discussed in detail in [39]:

$J/\psi \rightarrow f_0 \phi, \ f_0 \omega$ [39] \Rightarrow \ \theta = (34 \pm 6)^\circ \ or \ \theta = (146 \pm 6)^\circ, \ \ 
R = 4.03 \pm 0.14$ [39] \Rightarrow \ \theta = (25.1 \pm 0.5)^\circ \ or \ \theta = (164.3 \pm 0.2)^\circ, \ \ 
R = 1.63 \pm 0.46$ [39] \Rightarrow \ \theta = (42.3^{+8.3}_{-5.5})^\circ \ or \ \theta = (158 \pm 2)^\circ, \ \ 
\phi \rightarrow f_0 \gamma, \ f_0 \rightarrow \gamma \gamma$ [40] \Rightarrow \ \theta = (5 \pm 5)^\circ \ or \ \theta = (138 \pm 6)^\circ, \ \ 
QCD sum rules and $f_0$ data [41] \Rightarrow \ \theta = (27 \pm 13)^\circ \ or \ \theta = (153 \pm 13)^\circ, \ \ 
QCD sum rules and $a_0$ data [41] \Rightarrow \ \theta = (41 \pm 11)^\circ \ or \ \theta = (139 \pm 11)^\circ,$

where $R \equiv g^2_{f_0 K^+ K^-} / g^2_{f_0 \pi^+ \pi^-}$ measures the ratio of the $f_0(980)$ coupling to $K^+ K^-$ and $\pi^+ \pi^-$. In short, $\theta$ lies in the ranges of $25^\circ < \theta < 40^\circ$ and $140^\circ < \theta < 165^\circ$. Note that the phenomenological analysis of the radiative decays $\phi \rightarrow f_0(980) \gamma$ and $f_0(980) \rightarrow \gamma \gamma$ favors the second solution, namely, $\theta = (138 \pm 6)^\circ$.\(^5\)

B. Subleading QCD factorization amplitudes arising from 3-parton Fock states of the final state mesons

We shall see in Sec. III that the leading QCD factorization contributions to $B \rightarrow f_0 K$ are not adequate to explain the observed large rate of $f_0 K^-$ and $f_0 K^0$ rates. This leads us to contemplate some possible mechanisms for enhancement. In this subsection we study the subleading QCD factorization amplitudes arising from three-parton Fock states of the $f_0$ or kaon. As will be shown later, these corrections are of order $1/m_b^2$ and turn out to be negligible. The relevant diagrams are depicted in Fig. 3. The study of these corrections is motivated by the following observations: (i) As

\(^5\) In the four-quark scenario for light scalar mesons, one can also define a similar $f_0 - \sigma$ mixing angle. It has been shown that $\theta = 174.6^\circ$.\(^{42}\)
FIG. 3: The subleading QCD factorization amplitudes originating from the $\bar{q}qg$ Fock states of the final state mesons in the $B \to f_0 K$ decays.

shown in [43], Fig. 3(b) could give significant corrections to $B \to \omega K$ and $\omega \pi$ decays because of the enhancement by the softer gluon effect accompanied by the relatively large Wilson coefficients $c_4$ and $c_6$. (ii) Consider Fig. 3(a) where the emitted meson is a pseudoscalar [14] or a vector meson [43]. Its total amplitude vanishes due to the mismatch of the $G$-parity between the emitted meson and the relevant 3-parton operators. In contrast, the $G$-parities of 3-parton operators do match with the $f_0$ meson. Moreover, the contributions can be further enhanced by the large Wilson coefficients $c_4$ and $c_6$.

In the following calculation, we adopt the conventions $D_\alpha = \partial_\alpha + i g_s T^a A^a_\alpha$, $\tilde{G}_{\alpha\beta} = (1/2) \epsilon_{\alpha\beta\mu\nu} G^{\mu\nu}$ and $\epsilon^{0123} = -1$. We also use the Fock-Schwinger gauge to express the gluon field $A_\mu^a$ in terms of $G_{\mu\nu}$ for ensuring the gauge-invariant nature of the results [43, 44].

The three-parton LCDAs of the $f_0$ meson are defined by

$$\langle f_0(p)|\bar{q}(x)g_\sigma G_{\mu\nu}(vx)\sigma_{\alpha\beta} q(0)|0\rangle = -\int_0^1 dv \nu v G_{\mu\nu}^a(vx)x'^\nu. \tag{2.20}$$

The first term is evaluated in Appendix A.

$$\langle f_0(p)|\bar{q}(x)g_\gamma g_\delta G_{\alpha\beta}(vx)q(0)|0\rangle = \bar{f}_q \left[ p_\beta \left( g_\alpha - \frac{x_\alpha P_\mu}{px} \right) - p_\alpha \left( g_\beta - \frac{x_\beta P_\mu}{px} \right) \right] \int D\alpha \phi^{\alpha q}_1 e^{ipx(\alpha u + v\alpha_3)}, \tag{2.21}$$

$$\langle f_0(p)|\bar{q}(x)g_5 g_\gamma G_{\alpha\beta}(vx)q(0)|0\rangle = \bar{f}_q \left( p_\alpha g_\beta - p_\beta g_\alpha \right) \int D\alpha \phi^{\alpha q}_1 e^{ipx(\alpha u + v\alpha_3)},$$

$$\langle f_0(p)|\bar{q}(x)g_5 g_5 G_{\alpha\beta}(vx)q(0)|0\rangle = \bar{f}_q \left( p_\alpha g_\beta - p_\beta g_\alpha \right) \int D\alpha \phi^{\alpha q}_1 e^{ipx(\alpha u + v\alpha_3)}, \tag{2.22}$$

11
where \( D\alpha = d\alpha_q d\alpha_g d\delta (1 - \alpha_q - \alpha_g) \) with \( \alpha_q, \alpha_g \) being the fractions of the \( f_0 \) momentum carried by the \( \bar{q} \)-quark, \( q \)-quark and gluon, respectively. Here \( \phi^q_{3f_0} \) is a twist-3 LCDA, and \( \phi^q_\perp, \phi^q_\parallel, \widetilde{\phi}^q_\perp, \widetilde{\phi}^q_\parallel \) are twist-4 ones given by

\[
\phi^q_{3f_0}(\alpha_i) = 360\alpha_q\alpha_g\alpha_g^2 \left[ 1 + \omega_{1,0} \left( \frac{1}{2}(7\alpha_g - 3) + \omega_{2,0}(2 - 4\alpha_q\alpha_g - 8\alpha_g + 8\alpha_g^2) \right) + \omega_{1,1}(3\alpha_q\alpha_g - 2\alpha_g + 3\alpha_g^2) \right],
\]

\[
\phi^q_\parallel(\alpha_i) = 30\delta^2_{f_0}\alpha^2_g(1 - \alpha_g) \left[ \frac{1}{3} + 2\varepsilon(1 - 2\alpha_g) \right],
\]

\[
\phi^q_\perp(\alpha_i) = -120\delta^2_{f_0}\alpha_q\alpha_q\alpha_g \left[ 1 + \varepsilon(1 - 3\alpha_g) \right],
\]

\[
\widetilde{\phi}^q_\perp(\alpha_i) = 30\delta^2_{f_0}(\alpha_q - \alpha_g)\alpha_g^2 \left[ \frac{1}{3} + 2\varepsilon(1 - 2\alpha_g) \right],
\]

\[
\widetilde{\phi}^q_\parallel(\alpha_i) = 120\delta^2_{f_0}\varepsilon(\alpha_q - \alpha_g)\alpha_q\alpha_g \alpha_g,
\]

in the conformal expansion (for a further investigation, see [18]). We obtain \( \varepsilon \approx 0.30 \), similar to the case of the pion, and

\[
\delta^2_{f_0}(\mu = 2.1 \text{ GeV}) = (0.08 \pm 0.01) \text{ GeV}^2,
\]

\[
\delta^2_{f_0}(\mu = 2.1 \text{ GeV}) = (0.09 \pm 0.01) \text{ GeV}^2,
\]

for \( \bar{q}q = \bar{s}s \) or \( \bar{q}q = \bar{n}n \). A detailed calculation of \( \delta^2_{f_0} \) is shown in Appendix C. Consider the four-quark operator \( \bar{s}_\alpha\gamma_\mu(1 - \gamma_5)b_\beta \bar{q}_\beta\gamma_\mu(1 + \gamma_5)q_\alpha \) in Fig. 3 where \( \alpha \) and \( \beta \) are color indices. The result of Fig. 3(a.1) is found to be

\[
\text{Fig. 3(a.1)} = -\frac{1}{3} \int_0^1 dv \, \nu \int d^4x \, \int \frac{d^4k}{(2\pi)^4} \langle f_0|\bar{q}(0)\gamma_\mu(1 + \gamma_5)G_{\alpha\nu}(v\nu)q(0)|0, x'' \rangle \times \langle K|\bar{s}(0)\gamma_\nu\gamma_\alpha\gamma^\nu\gamma^\alpha(1 + \gamma_5)g_s(1 - \gamma_5)\gamma_\mu(1 - \gamma_5)e^{i(p_\nu - k_\nu)\nu}b(0)|\bar{B} \rangle
\]

\[
= \frac{2f_0}{3} \langle K|\bar{s}\gamma_\nu(1 - \gamma_5)b|\bar{B} \rangle \int D\alpha \frac{1}{\alpha_g m_B^2} [(\phi^q_\parallel + 2\phi^q_\perp) + 2(\widetilde{\phi}^q_\parallel - 2\widetilde{\phi}^q_\perp)],
\]

where all the components of the coordinate \( x \) should be taken into account in the calculation before the collinear approximation is applied. Hence, in Eq. (225), the exponent arising from \( G_{\alpha\nu}(v\nu) \) is actually \( e^{i(k^\nu - x^\nu)} \), where \( k^\nu \) is the gluon’s momentum, and the resultant calculation can be easily performed in the momentum space with the substitution of \( x'' \rightarrow -(i/v)(\partial/\partial k_\nu^\nu) \). Following the same lines, the result for Fig. 3(a.2) reads

\[
\text{Fig. 3(a.2)} = -\frac{1}{3} \int_0^1 dv \, \nu \int d^4x \, \int \frac{d^4k}{(2\pi)^4} \langle f_0|\bar{q}(0)\gamma_\mu g_s(1 + \gamma_5)G_{\alpha\nu}(\alpha x)q(0)|0, x'' \rangle \times \langle K^-|\bar{s}(0)\gamma_\nu(1 - \gamma_5)i(k^\nu + m_b)\gamma_\alpha e^{-i(p_\nu - k_\nu)\nu}b|\bar{B}^- \rangle
\]

\[
= \frac{2f_0}{3} \langle K^-|\bar{s}\gamma_\nu(1 - \gamma_5)b|\bar{B}^- \rangle \int D\alpha \frac{1}{\alpha_g m_B^2} [(\phi^q_\parallel + 2\phi^q_\perp) + 2(\widetilde{\phi}^q_\parallel - 2\widetilde{\phi}^q_\perp)],
\]

12
where we have used the following trick

\[
\frac{\not{p} f_0 x^2}{p f_0 x} \simeq x_+ \gamma_- \to 2i \gamma_- \frac{\partial}{\partial p_B^-} \tag{2.27}
\]

during the course of calculation and introduced two light-like vectors: \( n_-^\mu (n_-^2 = 0) \), parallel to the momentum of \( f_0 \), and \( n_+^\mu (n_+^2 = 0, n_+ \cdot n_- = 2) \), so that \( p f_0^\mu \simeq (p f_0 \cdot n_+) n_+^\mu = p f_0 + n_-^\mu, x^2 \simeq x_+ - x_-, p f_0 \cdot \gamma = p f_0 + \gamma_- \).

In Fig. 3(b), the emitted gluon becomes a parton of the kaon. To proceed, we first take \( G_{\mu \nu}(vx) \simeq G_{\mu \nu}(0) e^{iv k_g^\perp x} \) and then adopt the collinear approximation \( k_g^\perp = (\alpha_g) p_K \) in the final stage, where \( \langle \alpha_g \rangle \) is the averaged fraction of the kaon’s momentum carried by the gluon. The calculation is straightforward and leads to

\[
\begin{align*}
\text{Fig. 3(b)} = \frac{1}{4 N_c} \int_0^1 du \int_0^1 dv \left( \sqrt{2f_n \Phi_n(u) + \tilde{f}_s \Phi_s(u)} \right) |K^-| d\gamma_\mu (1 - \gamma_5) g_s G_{\nu \beta} b |B^-\rangle \\
\times i \frac{\partial}{\partial k_g^{\beta}} \left\{ \text{Tr} \left[ \not{p} f_0 \left( \gamma^\nu (v k_g^\perp + u \not{p} f_0) \gamma^\mu - \gamma^\mu (v k_g^\perp + u \not{p} f_0) \gamma^\nu \right) \right] \right\} \\
\simeq 0, \tag{2.28}
\end{align*}
\]

where we have used the fact that \( \Phi_n(\bar{u}) = -\Phi_n(u), \Phi_s(\bar{u}) = -\Phi_s(u) \) with \( \bar{u} = 1 - u \).

In short, the perturbative contributions coming from higher Fock states of the final state mesons to the decay amplitudes are (in units of \( G_F / \sqrt{2} \))

\[
A(B^- \to f_0 K^-)_{\text{Fig. 3}} = A(B^0 \to f_0 K^0)_{\text{Fig. 3}} = \frac{4}{3} (m_B^2 - m_K^2) \frac{1}{m_B^2} \int D\alpha \left\{ V_{ub} V_{us}^* c_1 \frac{\tilde{f}_n}{\sqrt{2}} (2\phi_n^r + \phi_n^s) \right. \\
\left. - V_{ub} V_{ts}^* \left( c_4 + c_6 \right) \left( \sqrt{2} \tilde{f}_n (2\phi_n^r + \phi_n^s) + \tilde{f}_s (2\phi_s^r + \phi_s^s) \right) + \frac{1}{2} (c_8 + c_{10}) \left( \tilde{f}_n (2\phi_n^r + \phi_n^s) - \tilde{f}_s (2\phi_s^r + \phi_s^s) \right) \right\}. \tag{2.29}
\]

However, the above result is numerically negligible. Note that here we do not consider the contributions from the operators \( O_{5,7,8} \); they are not only color suppressed but also of order \( 1/m_b^3 \).

### C. Soft non-factorizable contributions arising from the intrinsic gluon inside the \( B \) meson

The marginal result of Fig. 3(a.2) with the \( b \) splitting, namely, \( b \to bg \) which is governed by the GLAP evolution equation if \( B \) is boosted to the infinite momentum frame, implies that the extrinsic gluon effect is negligible. In this subsection, we shall study the intrinsic gluon contributions to the decay amplitudes. The leading diagram is displayed in Fig. 4(a). Here the intrinsic gluon plays the spectator role so that this contribution cannot be perturbatively calculated. Before proceeding, four remarks are in order. (i) The effects for intrinsic gluon being collinear with the spectator quark have already been considered in the transition form factor. (ii) The possible effect that the
(a) The contribution to the $B \rightarrow f_0 K$ decays arising from the intrinsic gluon within the $B$ meson and (b) the diagrammatic illustration to the correlation function given in Eq. (2.32).

The gluon content of $f_0$ is produced from the spectator quark is included in the present study since the spectator quark is a soft object and therefore the gluon could be one of the constituents of the $B$ meson. (iii) As illustrated in the previous section, the extrinsic gluon effect is quite small and hence there is no double counting problem. (iv) The amplitude of finding an intrinsic gluon within the $B$ meson due to the quantum fluctuations are suppressed by $\bar{\Lambda}/m_b$. It should be stressed that the idea of twist expansion for wave functions is not suitable in the present case [see Fig. 4(a)]. Since there is no hard part ready for a perturbative calculation, the argument for the suppression by higher twist distribution amplitudes is no longer justified. Although Fig. 4(a) cannot be calculated directly, we can turn to Fig. 4(b) where the $B$ meson state is replaced by a current operator with large $-p^2$ flowing through. In this way, Fig. 4(b) can be perturbatively calculated and is related to Fig. 4(a) via the reduction formula and quark-hadron duality.

Let us embark on the calculation. Consider the four-quark operator $\bar{s}_\alpha \gamma_\mu (1-\gamma_5) b_\beta \bar{q}_\beta \gamma_\mu (1+\gamma_5) q_\alpha$ in Fig. 4 where $\alpha$ and $\beta$ are color indices. The result of Fig. 4(a) can be represented as

$$\langle f_0 K^- | \bar{s}_\alpha \gamma_\mu (1-\gamma_5) b_\beta \bar{q}_\beta \gamma_\mu (1+\gamma_5) q_\alpha | B^- \rangle_{\text{Fig. 4(a)}} = \langle f_0 K^- | 2O^8(g) | B^- \rangle_{\text{Fig. 4(a)}},$$

where

$$O^8_{\pm} = \bar{s} \gamma_\mu (1-\gamma_5) T^a b \bar{q} \gamma_\mu (1+\gamma_5) T^a q,$$

and we have added a subscript “g” to $B^-$ to emphasize the intrinsic gluon for this case. This matrix element can be calculated by considering the following correlation function:

$$\Pi^\text{IG}_\mu (p,q) = i \int d^4x e^{-ipx} \langle K(p_K), f_0(p_{f_0}) | T(2O^8_{\pm}(0), \bar{i}b(x) g_s G_{\alpha\mu}(x) \gamma^\alpha u(x)) | 0 \rangle$$

$$= \Pi^\text{IG}(p^2,p'^2)p_\mu + \cdots ,$$

where the ellipses (and the following one) denote terms with the structures $(p+p_K)^\mu$ and $(p+p_{f_0})^\mu$, respectively, which are not relevant to the present study, and the superscript “IG” stands for intrinsic gluon.
In the deep Euclidean region of $p^2$, as depicted in Fig. 4(b), the correlation function can be perturbatively calculated in QCD and expressed in terms of LCDAs of the kaon and $f_0$,

$$
\Pi_\mu^{IG(QCD)} = -\frac{1}{6} \epsilon_{\alpha \beta \rho \mu} \int d^4x e^{-ipx} \langle f_0(p_{f_0})|\bar{q}(0)\gamma^\mu (1 \mp \gamma_5)G^{\alpha \beta}(x)q(0)|0\rangle \\
\times \langle K(p_K)\bar{s}(0)\gamma_\nu (1 \mp \gamma_5) \int \frac{d^4k}{(2\pi)^4} e^{ikx} \frac{i(k - m_b)}{k^2 - m_b^2}\gamma_\rho u(x)|0\rangle
$$

$$
= \frac{1}{3} \tilde{f}_B \hat{f}_K \int du \phi_K(u) \int D\alpha \frac{m_b^2}{m_b^2 - (p - up_K - \alpha_0 p_{f_0})^2} \\
\times \left[ \frac{1}{2}(\phi_\perp + \phi_\parallel) - (1 - \alpha_0)\phi_\parallel \right] p_\mu + \ldots
$$

(2.33)

To estimate $\langle f_0 K|2O^8_s(q)|B_g\rangle$, we apply the quark-hadron duality to the correlation function. Then the ground state contribution to the correlation function can be written in the dispersive representation as

$$
\int_{m_B^2}^{s_0} ds \frac{\text{Im}\Pi^{IG(phys)}(s,0)}{s - p^2} = \int_{m_B^2}^{s_0} ds \frac{\text{Im}\Pi^{IG(QCD)}(s,0)}{s - p^2},
$$

(2.34)

where $s_0$ is the threshold of higher resonances. On the left hand side of above equation, the spectral density $\text{Im}\Pi^{IG(phys)}(s,0)$ is given by hadronic contributions and reads

$$
\text{Im}\Pi^{IG(phys)}(s,0) = f_B \delta_B^2 \langle f_0 K|2O^8_s(q)|B_g\rangle \delta(s - m_B^2) + \text{higher resonance states},
$$

(2.35)

where $\langle B^- (p_B)|i\bar{b}\gamma_\alpha g_\mu \gamma_\nu \bar{c}\alpha_\nu u|0\rangle = f_B \delta_B^2 \rho_B^\mu$. On the right hand side of Eq. (2.34), the spectral density $\text{Im}\Pi^{IG(QCD)}(s,0)$ can be easily obtained from Eq. (2.33).

Equating Eqs. (2.32) and (2.33) and performing the Borel transformation yield

$$
\mathcal{B}\left[ \frac{1}{m_B^2 - p^2} \right] = \exp \left( -\frac{m_B^2}{M^2} \right),
$$

$$
\mathcal{B}\left[ \frac{1}{m_B^2 - (p - up_K - \alpha_0 p_{f_0})^2} \right] = \frac{1}{\alpha_g} \exp \left( -\frac{1 + u\alpha_0 m_B^2}{\alpha_g M^2} \right),
$$

(2.36)

It leads to the light-cone sum rule

$$
\langle f_0 K|2O^8_s(q)|B_g\rangle = \frac{f_K \tilde{f}_B m_B^2}{3f_B \delta_B^2} \int_0^1 du \phi_K(u) \int_0^{\tilde{\alpha}_g} d\bar{\alpha}_g \Theta(\tilde{\alpha}_g - \frac{m_B^2}{s_0 - u m_B^2}) \int_0^{\delta_B} dv \frac{1}{\alpha_g} e^{(m_B^2 - \frac{1 + u\alpha_0 m_B^2}{\alpha_g m_B^2})/M^2} \\
\times \left[ \frac{1}{2}(\phi_\perp + \phi_\parallel) - \tilde{\alpha}_g \phi_\parallel \right].
$$

(2.37)

In Fig. 4 we plot $\langle f_0 K|2O^8_s(q)|B\rangle$ as a function of the Borel mass squared. The relevant normalization scale for the present process is of order $\sqrt{m_B^2 - m_b^2} \approx 2.1$ GeV. Using the parameters $\tilde{f}_q, \delta_B^2, \delta_B^2$ given in Eqs. (A10), (C6), (D4), $f_K = 160$ MeV, $f_B = 185$ MeV, $m_b = 4.85$ GeV for the pole mass of the $b$ quark, and $s_0 = 34 \pm 1$ GeV, we obtain

$$
\langle f_0 K|2O^8_s(q)|B_g\rangle = (-0.20 \pm 0.06) \cos \theta,
$$

$$
\langle f_0 K|2O^8_s(q)|B_g\rangle = (-0.23 \pm 0.06) \sin \theta,
$$

(2.38)
FIG. 5: $\langle f_0 K|2O_8^{(q)}|B_g \rangle$ as a function of the Borel mass squared. The solid curve stands for $\langle f_0 K|2O_8^{(s)}|B_g \rangle$, while the dashed curve for $\langle f_0 K|2O_8^{(n)}|B_g \rangle$.

in the Borel window $9 \text{ GeV}^2 < M^2 < 12 \text{ GeV}^2$. Note that since $\langle f_0 K|2O_8^{(q)}|B_g \rangle = \langle f_0 K|2O_8^{(s)}|B_g \rangle$, we abbreviate $O_8^{(q)}$ as $O_8$ in the ensuring study. In summary, the total contributions arising from the intrinsic gluon in the $B$ meson to the decay amplitudes are (in units of $G_F/\sqrt{2}$)

$$A(B^- \to f_0 K^-)_{\text{Fig. 4}} = A(B^0 \to f_0 K^0)_{\text{Fig. 4}}$$

$$= -V_{tb}V_{ts}^* \left[ (c_4 + c_6) \left( \sqrt{2} \langle f_0 K|2O_8^{(n)}|B_g \rangle + \langle f_0 K|2O_8^{(s)}|B_g \rangle \right) 
+ \frac{1}{2}(c_8 + c_{10}) \left( \frac{1}{\sqrt{2}} \langle f_0 K|2O_8^{(n)}|B_g \rangle - \langle f_0 K|2O_8^{(s)}|B_g \rangle \right) \right]. \quad (2.39)$$

III. RESULTS AND DISCUSSIONS

To proceed the numerical calculations, we shall follow [14, 16] for the choices of the relevant parameters needed for QCDF calculations except for the form factors and CKM matrix elements. For form factors we shall use those derived in the covariant light-front quark model [27]. For CKM matrix elements we use the Wolfenstein parameters $A = 0.801$, $\lambda = 0.2265$, $\tilde{\rho} = 0.189$ and $\tilde{\eta} = 0.358$ [46]. For endpoint divergences encountered in hard spectator and annihilation contributions we take the default values $\rho_A = \rho_H = 0$ [see Eq. (2.9)]. For the running current quark masses we employ

$$m_b(m_b) = 4.2 \text{ GeV}, \quad m_b(2.1 \text{ GeV}) = 4.95 \text{ GeV}, \quad m_b(1 \text{ GeV}) = 6.89 \text{ GeV},$$

$$m_c(m_b) = 1.3 \text{ GeV}, \quad m_c(2.1 \text{ GeV}) = 1.51 \text{ GeV},$$

$$m_s(2.1 \text{ GeV}) = 90 \text{ MeV}, \quad m_s(1 \text{ GeV}) = 119 \text{ MeV}, \quad (3.1)$$

and $m_q(\mu)/m_s(\mu) = 0.0413$ [16]. The strong coupling constants are given by [47]

$$\alpha_s(2.1 \text{ GeV}) = 0.293, \quad \alpha_s(1 \text{ GeV}) = 0.489. \quad (3.2)$$

It is ready to perform numerical calculations. At the scale $\mu = 2.1 \text{ GeV}$, the numerical results for the relevant $a_i^u(f_0 K)$ and $a_i^v(K f_0)$ are

$$a_4^u = -0.0533 - i0.0183, \quad a_4^v = -0.0610 - i0.0064,$$
\[ a_6^u = -0.0568 - i0.0163, \quad a_5^u = -0.0612 - i0.0039, \]
\[ a_8^u = (74.4 - i4.5) \times 10^{-5}, \quad a_5^u = (73.6 - i2.3) \times 10^{-5}, \]
\[ a_{10}^u = (561 + i90) \times 10^{-5}, \quad a_{10}^c = (560 + i92) \times 10^{-5}, \]
\[ a_1 = 1.337 + i0.0288, \quad a_{6,8}^p(f_0^0) = a_{6,8}^p(f_0^0 K). \] (3.3)

Note that the effective Wilson coefficients \( a_1, a_4 \) and \( a_{10} \) receive large contributions from the hard spectator interactions \( H(f_0 K) \). Consequently, \( a_4 \) and \( a_6 \) are similar. Using the distribution amplitudes of the kaon and \( f_0(980) \) given in Eqs. (2.16) and (2.14), respectively, the annihilation contributions shown in Eq. (2.8) can be simplified to

\[ A_1^{(q)} \approx \pi \alpha_s \left[ -18 B_1^{(q)}(X_A + 32 - \frac{3}{2} \pi^2) - 4r^K \frac{m_{f_0}}{m_b} X_A^2 \right], \]
\[ A_3^{(q)} \approx 6 \pi \alpha_s \left[ \frac{m_{f_0}}{m_b} (X_A - 2 X_A + \frac{1}{3} \pi^2) - 3r^K B_1^{(q)}(X_A - 4 X_A + 4) \right], \]
\[ A_4^{(q)} \approx 6 \pi \alpha_s X_A \left[ \frac{m_{f_0}}{m_b} (2 X_A - 1) + r^K B_1^{(q)}(6 X_A - 11) \right], \] (3.4)

where the endpoint divergence \( X_A \) is defined in Eq. (2.4) and only the first Gegenbauer polynomial in the distribution amplitudes is kept for simplicity.

Shown in Fig. 3 are the branching ratios of \( B^- \to f_0(980) K^- \) and \( B^0 \to f_0(980) K^0 \) versus the strange-nonstrange mixing angle \( \theta \) of \( f_0(980) \). It is evident that the coefficient \( B_1^{(q)} \) appearing in the distribution amplitude of \( f_0 \) [see Eq. (2.14)] is preferred to be negative so that the annihilation terms make constructive contributions. This indicates the reliability of the sum-rule calculation of the \( f_0 \) distribution amplitudes (see Appendix B). When \( \theta = 0 \), \( f_0 \) is a pure \( s \bar{s} \) state and hence the penguin diagram Fig. 1(a) does not contribute (i.e. the form factor \( F_1^{Bf_0} \) vanishes). On the other extreme with \( \theta = 90^\circ \), \( f_0 \) is purely a \( n \bar{n} \) state and the penguin diagram Fig. 1(b) vanishes (i.e. \( f_\pi = 0 \)). Since \( (a_4^p - r^K a_5^p) \) is positive, it follows from Eq. (2.4) that, for a finite mixing angle, the interference between \( a_6^p(f_0 K) \) and \( a_5^p(K f_0) \) penguin terms arising from Figs. 1(a) and 1(b), respectively, is destructive for \( 0 < \theta < \pi/2 \) and constructive for \( \pi/2 < \theta < \pi \). As stated before, the \( f_0 - \pi \) mixing angle is slightly favored to be in the second quadrant, i.e. \( \pi/2 < \theta < \pi \). It is evident from Fig. 3 that this mixing angle solution is also preferable by the measurement of \( B \to f_0 K \). More precisely, \( B(B^- \to f_0 K^-) \) is obtained to be \((5.5 - 8.2) \times 10^{-6} \) for \( 25^\circ < \theta < 40^\circ \) and \((7.8 - 10.9) \times 10^{-6} \) for \( 140^\circ < \theta < 165^\circ \). However, even the maximal branching ratio \( 11.1 \times 10^{-6} \) occurring at \( \theta \approx 0 \) is still too small by around 40% compared to experiment. It should be remarked that our results for \( B(B \to f_0 K) \) are larger than the previous calculations [12, 13] owing to the large scalar decay constants \( \bar{f}_{s,n} \) we have derived (see Appendix A).

The fact that the observed \( f_0(980) K \) rate is higher than the naive model prediction calls for some mechanisms beyond the conventional short-distance approach. Some possibilities are:

- Final state interactions. The predicted \( B \to \pi K \) rates in the short-distance approach are in general smaller than the data by around 30% (see e.g. [14]). It is also known that the QCDF predictions for penguin-dominated modes such as \( B \to K^* \pi, K \phi, K^* \phi \) are consistently lower than the data by a factor of 2 to 3 [16]. Long-distance rescattering via charm intermediate states (or the so-called charming penguins) will not only enhance the
FIG. 6: Branching ratios of $B \to f_0(980)K$ versus the mixing angle $\theta$ of strange and nonstrange components of $f_0(980)$, where the solid curve corresponds to the Gegenbauer moments $B_{n,s}$ given in Eq. (B12) and the dashed curve is for the same Gegenbauer moments but with opposite signs. Calculations are based on QCD factorization.

aforementioned penguin-dominated decays but also drive sizable direct $CP$ violation observed recently in $B^0 \to K^+\pi^-$ mode [48]. The same rescattering effects may be expected to enhance $f_0(980)K$ rates. Unfortunately, we are not able to estimate the long-distance rescattering contributions to the $f_0K_S$ rate from intermediate charm states owing to the absence of information on $f_0DD$ and $f_0D^*_sD^*_s$ couplings.

- Gluonic coupling of the scalar meson. It is known that a possible explanation of the enormous production of $B \to \eta'K$ and $B \to \eta'X_s$ may be ascribed to the process $b \to s + g + g$ and the two gluons fragment into $\eta'$ [49]. The same mechanism may be also responsible for the enhancement of $f_0(980)K$ [50].

- Large weak annihilation contributions. Just like the pQCD approach [51] where the annihilation topology plays an essential role for producing sizable strong phases and for explaining the penguin-dominated $B \to VP$ modes, it has been suggested in [16] (see also [52]) that a favorable scenario (denoted as S4) for accommodating the observed penguin-dominated $B \to PV$ decays and the measured sign of direct $CP$ asymmetry in $B^0 \to K^-\pi^+$ is to have a large annihilation contribution by choosing $\rho_A = 1$, $\phi_A = -55^\circ$ for $PP$, $\phi_A = -20^\circ$ for $PV$ and $\phi_A = -70^\circ$ for $VP$ modes. The sign of $\phi_A$ is chosen so that the direct $CP$ violation $A_{K^-\pi^+}$ agrees with the data. Using the same set of $\rho_A$ and $\phi_A$ as the $PP$ modes, we found $\mathcal{B}(B^- \to f_0K^-) = 17.5 \times 10^{-6}$ in agreement with the data. However, the origin of these phases is unknown and their signs are not predicted. Moreover, since both annihilation and hard spectator scattering encounter endpoint divergences, there is no reason that soft gluon effects will affect only $\rho_A$ but not $\rho_H$.

- Subleading corrections arising from the three-parton Fock states of the $f_0$ and/or from the intrinsic gluon inside the $B$ meson. It has been shown that the subleading corrections to
Fig. 7: Branching ratios of $B \to f_0(980)K$ versus the mixing angle $\theta$ of strange and nonstrange components of $f_0(980)$, where the solid curves are for full decay amplitudes, dashed curves for short-distance QCD factorization amplitudes, and only the effects from the intrinsic gluon within the $B$ meson are evaluated in the dash-dotted curve. The bands correspond to the errors in Eq. (2.38).

QCD factorization due to the extrinsic softer gluon effect could enhance the branching ratio of $K\eta'$ to the level above $50 \times 10^{-6}$.

In the present work, we have examined in Secs. II B and II C the subleading effects mentioned in the last item. For corrections due to the three-parton Fock states of the $f_0$ or kaon, we find that they are of order $1/m_b^2$ and numerically negligible in $B \to f_0(980)K$ owing to the fact that $\langle f_0(980)|V_\mu|0\rangle = 0$. The detailed calculations can be found in Sec. II B and are summarized in Eq. (2.29). As for the corrections originating from the intrinsic gluon within the $B$ meson, they belong to the higher Fock components arising from quantum fluctuations and are suppressed by $\Lambda/m_b$. Since this mechanism is nonperturbative, the twist expansion for the $f_0$ (or $K$) distribution amplitudes is no longer suitable in this case. Therefore, the contribution shown in Fig. 4(a) is expected to be suppressed by $1/(m_b)^n$ with $1 \leq n < 2$. Based on the reduction formula and quark-hadron duality, we are able to estimate the nonperturbative effects in Fig. 4(a). A detailed study of intrinsic gluon effects has been shown in Sec. II C and is summarized in Eq. (2.39).

Taking into account the aforementioned subleading corrections, we show in Fig. 7 the branching ratios of $B \to f_0(980)K$ versus the mixing angle $\theta$ of strange and nonstrange components of $f_0(980)$. At $\theta \sim 20^\circ$, the intrinsic gluon effects can make 25-40% corrections to the decay amplitude so that the resulting branching ratio is of order $(12 \sim 20) \times 10^{-6}$, in agreement with experiment. These corrections are constructive for $0^\circ \lesssim \theta \lesssim 80^\circ$ as well as $160^\circ \lesssim \theta \lesssim 180^\circ$ and destructive for $80^\circ \lesssim \theta \lesssim 150^\circ$. In short, when the effects due to the intrinsic spectator gluon within the $B$ meson are included, $0^\circ \lesssim \theta \lesssim 55^\circ$ and $155^\circ \lesssim \theta \lesssim 180^\circ$ are preferable by the $B \to f_0K$ data.
IV. CONCLUSIONS

In the present work we have studied the decay $B \rightarrow f_0(980)K$ within the framework of QCD factorization. Our conclusions are as follows:

1. While it is widely believed that $f_0(980)$ is predominately a four-quark state, in practice it is difficult to make quantitative predictions on $B \rightarrow f_0K$ based on the four-quark picture for $f_0(980)$ as it involves not only the $f_0$ form factors and decays constants that are beyond the conventional quark model but also additional nonfactorizable contributions that are difficult to estimate. Hence, we shall assume the two-quark scenario for $f_0(980)$.

2. There are two distinct penguin contributions to $B \rightarrow f_0K$ and their interference depends on the unknown mixing angle $\theta$ of strange and nonstrange quark contents of $f_0(980)$: destructive for $0 < \theta < \pi/2$ and constructive for $\pi/2 < \theta < \pi$. A mixing angle in the second quadrant is preferable by the measurement of $B \rightarrow f_0(980)K$.

3. Based on the QCD sum rule method, we have derived the leading-twist light-cone distribution amplitudes of $f_0(980)$ and the scalar decay constant $\tilde{f}_q$. It is found that $\tilde{f}_s$ is much larger than the previous estimate owing to its scale dependence and the large radiative corrections to the quark loops in the OPE series. The measured $B \rightarrow f_0K$ rates clearly favor the sign of the $f_0$ distribution amplitudes predicted by the sum rule.

4. Based on the QCD factorization approach, we obtain $B(B^- \rightarrow f_0K^-) = (5.5 - 8.2) \times 10^{-6}$ for $25^\circ < \theta < 40^\circ$ and $(7.8 - 10.9) \times 10^{-6}$ for $140^\circ < \theta < 165^\circ$. Hence, the short-distance contributions are not adequate to explain the observed large rates of $f_0K^-$ and $f_0K^0$.

5. Possible subleading corrections from the three-parton Fock states of the $f_0$ and from the intrinsic (spectator) gluon inside the $B$ meson are estimated. It is shown that while the extrinsic gluon contribution to $B \rightarrow f_0K$ is negligible, the intrinsic gluon within the $B$ meson may play an eminent role for the enhancement of $f_0(980)K$.

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APPENDIX A: DETERMINATION OF THE SCALAR COUPLING OF $f_0$

To determine the scalar decay constant $\tilde{f}_q$ of $f_0(980)$ defined by $\langle 0|\bar{q}q|f_0^0 \rangle = m_{f_0}^{(q)} \tilde{f}_q$ with $f_0^0 = \bar{n}n \equiv (\bar{u}u + \bar{d}d)/\sqrt{2}$ and $f_0^0 = \bar{s}s$, we consider the two-point correlation function

$$\Pi(q^2) = i \int d^4xe^{iqx}\langle 0|T(j^q(x)j^{q\dagger}(0))|0 \rangle,$$  \hspace{1cm} (A1)

where $j^q = \bar{q}q$. The above correlation function can be calculated from the hadron and quark-gluon dynamical points of view, respectively. Therefore, the correlation function arising from the lowest-lying meson $f_0^0$ can be approximately written as

$$\frac{m_{f_0}^{(q)2}\tilde{f}_q}{m_{f_0}^{(q)2} - q^2} = \frac{1}{\pi} \int_{s_0}^{s} ds \frac{\text{Im}\Pi_{\text{OPE}}}{s - q^2},$$  \hspace{1cm} (A2)

where $\Pi_{\text{OPE}}$ is the QCD operator-product-expansion (OPE) result at the quark-gluon level, $s_0$ is the threshold of the higher resonant states and the contributions originating from higher resonances are approximated by

$$\frac{1}{\pi} \int_{s_0}^{\infty} ds \frac{\text{Im}\Pi_{\text{OPE}}}{s - q^2}.$$  \hspace{1cm} (A3)

We apply the Borel transformation to both sides of Eq. (A2) to improve the convergence of the OPE series and suppress the contributions from higher resonances. Consequently, the sum rule with OPE series up to dimension 6 and $O(\alpha_s)$ corrections reads [53]

$$m_{f_0}^{(q)2}\tilde{f}_q e^{-m_{f_0}^{(q)2}/M^2} \left(\frac{\alpha_s(\mu)}{\alpha_s(M)}\right)^{8/b} = \frac{3}{8\pi^2} M^4 \left[1 + \frac{\alpha_s(M)}{\pi} \left(\frac{17}{3} + \frac{2}{f(1)} - \ln \frac{M^2}{\mu^2}\right) f(1)\right]$$

$$+ \frac{1}{8} \left(\frac{\alpha_s G^2}{\pi}\right) + 3(m_q\bar{q}q) - \frac{\pi\alpha_s}{M^2} \left(\langle \bar{q}\gamma_\mu T^a q \bar{q}\gamma^\mu T^a q\rangle + \frac{2}{3} \langle \bar{q}\gamma_\mu T^a \bar{q}\gamma^\mu T^a q\rangle\right),$$  \hspace{1cm} (A4)

where $f(1) = 1 - e^{-s_0/M^2}(1 + s_0/M^2)$, $I(1) = \int_{e^{-s_0/M^2}}^{1} (-\ln t) dt$, and we have taken into account the scale dependence of $\tilde{f}_q$:

$$\tilde{f}_q(M) = \tilde{f}_q(\mu) \left(\frac{\alpha_s(\mu)}{\alpha_s(M)}\right)^{4/b},$$  \hspace{1cm} (A5)

with $b = (11N_c - 2n_f)/3$. Here, the anomalous dimensions of $\alpha_s G^2$ and $m_q\bar{q}q$ are equal to zero, while the anomalous dimensions of the 4-quark operators have been neglected. In the numerical analysis, we shall use the following values for vacuum condensates and quark masses at the scale of $\mu = 1$ GeV [44]:

$$\langle \alpha_s G^a G^{a \mu \nu} \rangle = 0.474 \text{ GeV}^4/(4\pi),$$

$$\langle \bar{u}u \rangle \cong \langle \bar{d}d \rangle = -0.24 \text{ GeV}^3,$$

$$\langle \bar{s}s \rangle = 0.8 \langle \bar{u}u \rangle,$$

$$(m_u + m_d)/2 = 5 \text{ MeV},$$

$$m_s = 119 \text{ MeV},$$

and adopt the vacuum saturation approximation for describing the four-quark condensates, i.e.,

$$\langle 0|q\Gamma_i T^a \bar{q}q\Gamma_i T^a q|0 \rangle = -\frac{1}{16N_c^2} \text{Tr}(\Gamma_i \Gamma_i) \text{Tr}(T^a T^a) \langle \bar{q}q \rangle^2.$$  \hspace{1cm} (A7)
FIG. 8: $m^{(q)}_0$ and $\tilde{f}_q$ as functions of the Borel mass squared $M^2$. The solid curve is obtained for $j^s = \bar{s}s$ and the dashed curve for $j^n = \bar{n}n$.

Taking the logarithm of both sides of Eq. (A4) and then applying the differential operator $M^4\partial/\partial M^2$ to them, we obtain the mass sum rule for $f^{(q)}_0$. In Fig. 8 we explore two scenarios for (i) $\bar{q}q = \bar{s}s$ and (ii) $\bar{q}q = \bar{n}n$. Numerically, we get

$$m^{(s)}_0 \simeq (1.02 \pm 0.05) \text{ GeV} \quad \text{for } s_0 = 2.6 \text{ GeV}^2,$$

$$m^{(n)}_0 \simeq (0.99 \pm 0.05) \text{ GeV} \quad \text{for } s_0 = 2.6 \text{ GeV}^2,$$

(A8)

where the value of $s_0$ is determined when the maximum stability for the mass sum rule is reached.

Substituting the above results for the threshold $s_0$ and the masses obtained in the mass sum rule into Eq. (A4), we obtain the sum rule for $\tilde{f}_q$ at $\mu = 1 \text{ GeV}$ as a function of the Borel mass squared $M^2$ (see Fig. 8). The results are

$$\tilde{f}_s(\mu = 1 \text{ GeV}) = 0.33 \text{ GeV}, \quad \tilde{f}_n(\mu = 1 \text{ GeV}) \simeq 0.35 \text{ GeV},$$

(A9)

and

$$\tilde{f}_s(\mu = 2.1 \text{ GeV}) \simeq 0.39 \text{ GeV}, \quad \tilde{f}_n(\mu = 2.1 \text{ GeV}) \simeq 0.41 \text{ GeV}.$$  

(A10)

Evidently, they are much larger than the typical decay constant of pseudoscalar mesons.

Two remarks are in order. First, the scale dependence of $\tilde{f}_q$ was not considered in [37, 38]. If the scale dependence is neglected, we will obtain $s_0 \simeq 1.6 \text{ GeV}^2$, in accordance with the results in [37, 38]. Second, the large radiative correction to the quark loop in the OPE series arises mainly from one-gluon exchange and is likely the effect of the color Coulomb interaction, as in the case for the $B$ meson [54]. The values of $\tilde{f}_q$ and $s_0$, rather than $m^{(q)}_0$, are very sensitive to the radiative corrections to the quark loop and to the scale dependence of $\tilde{f}_q$. However, these effects were not considered in [37, 38]. If neglecting $\alpha_s$ corrections to the mass sum rule while keeping $s_0 = 2.6 \text{ GeV}^2$, the resulting $\tilde{f}_s(\mu=1 \text{ GeV})$ will be 0.26 GeV. Moreover, if further setting $s_0 = 1.6 \text{ GeV}^2$, the sum rule yields $\tilde{f}_s(\mu=1 \text{ GeV})=0.19 \text{ GeV}$, in agreement with [37, 38]. The validity of our results is thus realized. It is interesting to note that a larger scalar decay constant for $K_0^*(1430)$ has been obtained in [55, 56] based on the sum-rule method.
APPENDIX B: LEADING TWIST LCDA FOR $f_0$

The LCDA's $\Phi_q(x, \mu)$ corresponding to the ideal states $f_0^q = \bar{q}q$ are defined by

$$\langle f_0^q(p)|\bar{q}(z)\gamma_\mu q(0)|0\rangle = p_\mu \tilde{f}_q \int_0^1 dx e^{ixp\cdot z}\Phi_q(x, \mu),$$  \hfill (B1)

where $x$ (or $\bar{x} = 1 - x$) is the $f_0$ momentum fraction carried by the quark $q$ (or antiquark $\bar{q}$) and $\mu$ is the normalization scale of the LCDA. $\Phi_q(x, \mu)$ can be expanded in a series of Gegenbauer polynomials \cite{18, 57}

$$\Phi_q(x, \mu) = 6x(1-x) \sum_{l=1,3,5,...} \phi_l^q(\mu)C_l^{3/2}(2x-1),$$  \hfill (B2)

with multiplicatively renormalizable coefficients (or the so-called Gegenbauer moments):

$$\phi_l^q(\mu) = \frac{2(2l + 3)}{3(l + 1)(l + 2)} \int_0^1 C_l^{3/2}(2x-1)\Phi_q(x, \mu).$$  \hfill (B3)

Consider the following two-point correlation function

$$\Pi_l(q) = i \int d^4x e^{ix\sigma} \langle 0|T(O_l(x) O^\dagger(0)|0\rangle = (zq)^{l+1}l_I(q^2),$$  \hfill (B4)

where

$$\langle 0|O_l|f_0^q(p)\rangle \equiv \langle 0|\bar{q}(iz D)^l q|f_0^q(p)\rangle = (zp)\tilde{f}_q \int_0^1 (2x-1)^2\Phi_q(x)dx \equiv (zp)\tilde{f}_q \langle \xi_l^q \rangle,$$

$$\langle 0|O_l^\dagger|f_0^q(p)\rangle \equiv \langle 0|\bar{q}\bar{q}|f_0^q(p)\rangle = m_f^{(q)} \tilde{f}_q,$$  \hfill (B5)

with $z^2 = 0$. The Gegenbauer moments $\phi_l^q(x, \mu)$ can be easily expressed in terms of the above defined moments $\langle \xi_l^q \rangle$, for which the sum rule reads

$$\langle \xi_l^q \rangle = \frac{1}{m_f^{(q)} \tilde{f}_q} e^{\frac{m_f^{(q)} / M}{2}} \left((-1)^{l+1} + 1\right) \left(\frac{\langle \bar{q}q \rangle}{24} + \frac{3l - 1}{M^2} \frac{\langle \bar{q}q \sigma \cdot Gq \rangle}{48} + \frac{l(l-1)}{M^4} \langle g_s^2 G^2 \rangle \langle \bar{q}q \rangle \right).$$  \hfill (B7)

The conformal invariance in QCD exhibits that partial waves, in the expansion of $\Phi_q(x, \mu)$ in Eq. (B2), with different conformal spin cannot mix under renormalization to the leading-order accuracy. As a consequence, the Gegenbauer moments $\phi_l$ in (B2) renormalize multiplicatively:

$$\phi_l(\mu) = \phi_l(\mu_0) \left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)}\right)^{-\gamma_l(b)},$$  \hfill (B8)

where the one-loop anomalous dimensions are \cite{58}

$$\gamma_l = C_F \left(1 - \frac{2}{(l + 1)(l + 2)} + 4 \sum_{j=2}^{l+1} \frac{1}{j} \right),$$  \hfill (B9)

with $C_F = (N_c^2 - 1)/(2N_c)$. We consider the renormalization-improved sum rule of Eq. (B7), where the anomalous dimensions of relevant operators can be found in \cite{44}, such that

$$\langle \bar{q}q \rangle_\mu = \langle \bar{q}q \rangle_{\mu_0} \left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)}\right)^{\frac{\gamma}{2}},$$

$$\langle g_s \bar{q}q \sigma \cdot Gq \rangle_\mu = \langle g_s \bar{q}q \sigma \cdot Gq \rangle_{\mu_0} \left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)}\right)^{-\frac{\gamma}{2}},$$

$$\langle g_s^2 G^2 \rangle_\mu = \langle g_s^2 G^2 \rangle_{\mu_0}.$$  \hfill (B10)
are illustrated in Fig. 9. The leading-twist LCDAs for the $f_0$ meson vs the momentum fraction carried by the quark, where the solid curves are evaluated with considering the $B_1, B_3, B_5$ central values given in Eq. (B12), while the dash-dotted and dashed lines are obtained by setting $B_5 = 0$ and $B_3 = B_5 = 0$, respectively. The darker lines correspond to the normalization scale $\mu = 2.1$ GeV, while the lighter lines to $\mu = 1$ GeV.

In the numerical analysis, we choose the Borel window $0.9$ GeV$^2 < M^2 < 1.2$ GeV$^2$, consistent with the previous case, where the contributions originating from higher resonances and the highest OPE terms are well under control. We also use the input parameters given in Eq. (A6) and [44]

$$\langle g_s \bar{u} \sigma G u \rangle \cong \langle g_s \bar{d} \sigma G d \rangle = -0.8 \langle \bar{u}u \rangle,$$
$$\langle g_s \bar{s} \sigma G s \rangle = 0.8 \langle g_s \bar{u} \sigma G u \rangle.$$  \hfill (B11)

It should be noted that in the large $l$ limit for the moment $\langle \xi^l \rangle$ sum rule, the actual expansion parameter is $M^2/l$. As a result, for $l \geq 7$ and fixed $M^2$, the OPE series are convergent slowly or even divergent, i.e. the resulting sum-rule result becomes less reliable. We thus obtain the first three non-zero moments, at the normalization scale $\mu = 1$ GeV,

$$\langle \xi_1^1 \rangle = -0.55 \pm 0.05 \Rightarrow B_1^{(n)} \equiv \phi_1^n = 5\langle \xi_1^1 \rangle/3 = -0.92 \pm 0.08,$$
$$\langle \xi_3^3 \rangle = -0.43 \pm 0.02 \Rightarrow B_3^{(n)} \equiv \phi_3^n = \frac{21}{4}\langle \xi_3^3 \rangle - \frac{9}{4}\langle \xi_1^1 \rangle = -1.00 \pm 0.05,$$
$$\langle \xi_5^5 \rangle = -0.33 \pm 0.01 \Rightarrow B_5^{(n)} \equiv \phi_5^n = \frac{3}{4}(7\langle \xi_3^3 \rangle - 3\langle \xi_1^1 \rangle) = -0.40 \pm 0.05,$$  \hfill (B12)

where the sub-(super-)script $n$ denotes $\bar{m}n$. Rescaling to the normalization point $\mu = 2.1$ GeV, we find

$$\langle \xi_1^1 \rangle = -0.44 \pm 0.04 \Rightarrow B_1^{(n)} \equiv \phi_1^n = -0.73 \pm 0.07,$$
$$\langle \xi_3^3 \rangle = -0.33 \pm 0.02 \Rightarrow B_3^{(n)} \equiv \phi_3^n = -0.74 \pm 0.04,$$
$$\langle \xi_5^5 \rangle = -0.24 \pm 0.01 \Rightarrow B_5^{(n)} \equiv \phi_5^n = -0.017 \pm 0.005.$$  \hfill (B13)

As for $\bar{q}q = \bar{s}s$, the analysis yields $\langle \xi_1^{1,3,5} \rangle \cong (\langle \bar{s}s \rangle/\langle \bar{u}u \rangle)\langle \xi_1^{1,3,5} \rangle \cong 0.8\langle \xi_1^{1,3,5} \rangle$ and $B_{1,3,5}^{(n)} \cong 0.8B_{1,3,5}^{(n)}$. The leading-twist LCDAs $\Phi_s(x, \mu)$ and $\Phi_n(x, \mu)$ for the $f_0$ meson, which vanish in the large $\mu$ limit, are illustrated in Fig. 9.
APPENDIX C: DECAY CONSTANT $\delta_{f_0}$

The parameter $\delta_{f_0}$ appearing in Eq. (2.23) can be defined through the matrix element of the local current operator $\bar{q}q_{\alpha}G_{\alpha\beta}q$, 

$$\langle 0|\bar{q}_{\beta}(0)|f_0(p)\rangle = i\bar{f}_0 q_\beta \delta_{f_0}^2.$$  \hfill (C1)

To calculate the $\delta_{f_0}$ parameter, we consider the following non-diagonal two-point correlation function

$$\Pi_{\beta}(q^2) = i\int d^4xe^{iqx} \langle 0|T(\bar{j}_q(0)x)j^\dagger(0)|0\rangle = q_\beta \Pi(q^2),$$ \hfill (C2)

where $j_q = \bar{q}q_{\alpha}G_{\alpha\beta}q$ as in Eq. (A1). In the deep Euclidean region of $q^2$, the above correlation function can be perturbatively calculated in QCD and the result is given by

$$\Pi^{QCD}(q^2) = -i\frac{2\alpha_s}{3\pi} \langle \bar{q}q \rangle \left( \ln\left(-\frac{q^2}{\mu^2}\right) + \frac{7}{6} \right) - i\frac{2}{q^2} \langle \bar{q}g_s\sigma \cdot Gq \rangle.$$ \hfill (C3)

After approximating the higher resonances as given in Eq. (A2) and performing the Borel transformation, the resultant sum rule with OPE series up to the order of dimension 7 and radiative corrections of order $O(\alpha_s)$ reads

$$e^{-m_{f_0}(q)/M^2} f_0^2 \delta_{f_0} G_0 \left( \frac{\alpha_s(\mu)}{\alpha_s(M)} \right)^{\frac{2}{3}} M^2 (1 - e^{-s_0/M^2}) + \frac{1}{2} \langle \bar{q}g_s\sigma \cdot Gq \rangle \left( \frac{\alpha_s(\mu)}{\alpha_s(M)} \right)^{\frac{2}{3}},$$ \hfill (C4)

where the anomalous dimensions of some operators have been listed in Eq. (B10) and that of $\bar{q}q_{\alpha}G_{\alpha\beta}q$ is $-32/(9b)$. \hfill (59)

In the Borel window $0.9 \, \text{GeV}^2 < M^2 < 1.2 \, \text{GeV}^2$ where the contributions originating from higher resonances and the highest OPE term are well under control, using input parameters in Eqs. (A6) and (B11), we obtain

$$\delta_{f_0}^2(\mu = 1 \, \text{GeV}) = (0.09 \pm 0.01) \, \text{GeV}^2,$$

$$\delta_{f_0}^2(\mu = 1 \, \text{GeV}) = (0.10 \pm 0.01) \, \text{GeV}^2,$$ \hfill (C5)

and, as rescaled to $\mu = 2.1 \, \text{GeV}$,

$$\delta_{f_0}^2(\mu = 2.1 \, \text{GeV}) = (0.08 \pm 0.01) \, \text{GeV}^2,$$

$$\delta_{f_0}^2(\mu = 2.1 \, \text{GeV}) = (0.09 \pm 0.01) \, \text{GeV}^2,$$ \hfill (C6)

for $j^q = \bar{s}s$ and $\bar{n}n$. The results are depicted in Fig. 10.

APPENDIX D: DETERMINATION OF $\delta_B^2$ 

The parameter related the intrinsic gluon content of the $B$ meson is defined by

$$\langle 0|\bar{u}\gamma_{\alpha}g_sG_{\alpha\mu}|B\rangle = if_B \delta_B^2 p_\mu.$$ \hfill (D1)
FIG. 10: $\delta^2_{j_0}(\mu = 1 \text{ GeV})$ as a function of the Borel mass squared $M^2$ with the same notation as Fig. 8. We have used $m_{j_0}^{(s)} = 1.02 \text{ GeV}$ and $\tilde{f}_s = 0.33 \text{ GeV}$ for the solid curve, while $m_{j_0}^{(n)} = 0.98 \text{ GeV}$ and $\tilde{f}_n = 0.35 \text{ GeV}$ for the dashed curve.

To estimate the parameter $\delta^2_B$, we consider the non-diagonal two-point propagator

$$\Pi^B_\mu = i \int d^4xe^{iqx}\langle 0 | T(\bar{b}(x)g_s\vec{G}_{\alpha\mu}(x)\gamma^\alpha u(x) \bar{u}(0)\gamma_5 b(0)|0 \rangle.$$  (D2)

Following the same line as in Appendix C, we arrive at the $\delta^2_B$ sum rule

$$\delta^2_B \simeq \frac{m_b + m_u}{4m_B^2} e^{\frac{m_b^2 - m_u^2}{M^2}} \langle \bar{q}q \sigma \cdot Gq \rangle,$$  (D3)

where the contribution arising from $\alpha_s \langle \bar{q}q \rangle$ is much smaller than the quark-gluon mixed condensate shown in Eq. (C4) and hence can be neglected. Using the value for quark-gluon mixing condensate in Eq. (B11), $f_B = 185 \text{ MeV}$, and $m_b = 4.85 \text{ GeV}$ for the pole mass of the $b$ quark, we obtain

$$\delta^2_B = 0.022 \pm 0.002 \text{ GeV}^2,$$  (D4)

within the Borel window $9 \text{ GeV}^2 < M^2 < 12 \text{ GeV}^2$. Note that the scale dependence of $\delta^2_B$ is weak.
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