Improving the Quality of Response Surface Analysis of an Experiment for Coffee-supplemented Milk Beverage: II. Heterogeneous Third-order Models and Multi-response Optimization

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Abstract  This research was motivated by our encounter with the situation where an optimization was done based on statistically non-significant models having poor fits. Such a situation took place in a research to optimize manufacturing conditions for improving storage stability of coffee-supplemented milk beverage by using response surface methodology, where two responses are $Y_1$=particle size and $Y_2$=zeta-potential, two factors are $F_1$=speed of primary homogenization (rpm) and $F_2$=concentration of emulsifier (%), and the optimization objective is to simultaneously minimize $Y_1$ and maximize $Y_2$. For response surface analysis, practically, the second-order polynomial model is almost solely used. But, there exists the cases in which the second-order model fails to provide a good fit, to which remedies are seldom known to researchers. Thus, as an alternative to a failed second-order model, we present the heterogeneous third-order model, which can be used when the experimental plan is a two-factor central composite design having -1, 0, and 1 as the coded levels of factors. And, for multi-response optimization, we suggest a modified desirability function technique. Using these two methods, we have obtained statistical models with improved fits and multi-response optimization results with the predictions better than those in the previous research. Our predicted optimum combination of conditions is $(F_1, F_2)=(5,000, 0.295)$, which is different from the previous combination. This research is expected to help improve the quality of response surface analysis in experimental sciences including food science of animal resources.

Keywords  response surface methodology, central composite design, heterogeneous third-order model, multi-response optimization, desirability

Introduction

Response surface methodology (RSM) is a set of statistical techniques for modeling and optimizing responses through the design and analysis of experiments (Myers et al., 2009), which has been widely used in engineering, agriculture, life science,
microbiology and food sciences. A search by Google Scholar revealed that the number of scientific articles whose titles mentioned ‘response surface’ was 157 in 2000 but it became 1,860 in 2018, which is an 11.8-fold increase during recent 18 years. This indicates that RSM has been established as an important tool for modeling and optimization in experimental sciences including food sciences of animal resources.

In RSM, the central composite designs (CCD, Box and Wilson, 1951) have been most frequently used as experimental plans, and the second-order polynomial regression models have been usually employed for data analysis. And, the response surface model can be said to be well fitted and reliable when it satisfies the following criteria: (1) the model is significant (the model p-value $\leq 0.05$), (2) the lack of fit is non-significant (the lack-of-fit p-value $> 0.05$), (3) the $r^2 \geq 0.9$ (Giunta, 1997), and (4) the adjusted $r^2 \geq 0.8$ (Myers et al., 2009).

However, in reality, it is observed that, for some data, the analysis models, which are the second-order models in most cases, do not satisfy the above criteria. A remedy in this case is to use a third-order model that consists of linear, quadratic, cubic, and relevant interaction terms (Rheem and Rheem, 2012). For example, when there are two factors, letting $X_1$ and $X_2$ denote coded factors, the third-order model has the following terms: linear terms $X_1$ and $X_2$, quadratic terms $X_1^2$ and $X_2^2$, cubic terms $X_1^3$ and $X_2^3$, and the two-factor interaction term $X_1X_2$. There exist the cases where this method improves the model. But, this method is applicable to a CCD that uses five values, which are denoted by $-\alpha$, $-1$, 0, 1, and $\alpha$, as the levels of coded factors.

When the experimental design is a CCD in which $-1$, 0, and 1 are the levels of coded factors, as in Table 1, cubic terms cannot be added to the model, since $(-1)^{\alpha}=-1$, $(0)^{\alpha}=0$, and $(1)^{\alpha}=1$, which makes the cubic terms equal to the linear terms. For example, in Table 1B, we can see that $X_1=X_1^3$ and $X_2=X_2^3$, and thus the $X_1^3$ and $X_2^3$ terms cannot be chosen from among the candidates of additional model terms for augmenting the second-order model.

This problem can be solved by adding the terms of the interaction between the linear term of one factor and the quadratic term of another factor. For example, in Table 1B, $X_1^2X_2$ and $X_1X_2^2$ are such interaction terms. The model that contains such interaction terms can be named a heterogeneous third-order model, since the sum of the exponents in each of such interaction terms is three. Thus, a remedy in this case is to augment the second-order model to the heterogeneous third-order model by adding the $X_1^2X_2$ and $X_1X_2^2$ terms, which are chosen from among the candidates of additional model terms in Table 1B, to the second-order model.

A dataset, which is obtained through the screening of the data in Ahn et al. (2017), will be re-analyzed for the illustration of the remedy suggested in this research note. Since Ahn et al. (2017) has two responses and a purpose of it is the multi-response optimization of them, this research note, which is a continuation of Rheem and Oh (2019), will model both responses by using heterogeneous third-order models, and optimize them simultaneously by employing the desirability function technique.

**Materials and Methods**

**Dataset to be re-analyzed**

Data analysis should include data screening, which is necessary for accurate modeling. The original data to be used for re-analysis is the data described in Ahn et al. (2017), in which they tried to optimize manufacturing conditions for improving storage stability of coffee-supplemented milk beverage by using RSM. Through data screening, one outlier was deleted from their data (Rheem and Oh, 2019). The response variables, $Y_1$ and $Y_2$, and the factors in this experiment are described in Table 1A. The dataset from which an outlier is eliminated is given in Table 1B. Here, the experimental design is a CCD for two
factors with the coded levels of –1, 0, and 1. Using this data, we will fit to the data second-order models and heterogeneous third-order models.

### Statistical analysis

Data were analyzed by the use of SAS software. SAS/STAT (2013) was employed for the statistical modeling of data. Graphs were produced by SAS/GRAPH (2013).

### Results and Discussion

#### Fitting the second-order model to the data

First, for each of $Y_1$ and $Y_2$, the second-order polynomial regression model containing 2 linear, 2 quadratic, and 1 interaction terms was fitted to the data by using RSREG procedure of SAS/STAT.

For both $Y_1$ and $Y_2$, the second-order models are unsatisfactory. For $Y_1$, the model is non-significant ($p=0.5962$), the lack of fit is significant ($p=0.0131$), the $r^2=0.40<0.9$, and the adjusted $r^2=–0.11<0.8$. Also, for $Y_2$, the model is non-significant ($p=0.2924$), the lack of fit is significant ($p=0.0203$), and the $r^2=0.57<0.9$, and the adjusted $r^2=0.21<0.8$. None of the four criteria are met for both $Y_1$ and $Y_2$. Thus, we will augment the analysis models for their improvement.
Fitting the heterogeneous third-order model to the data

For each of $Y_1$ and $Y_2$, since the second-order model has a poor fit for the data, next we will fit to the data a heterogeneous third-order model that consists of the $X_1, X_2, X_1^2, X_2^2, X_1X_2, X_1^2X_2$, and $X_1X_2^2$ terms, by adding the $X_1^2X_2$ and $X_1X_2^2$ terms to the second-order model, in the anticipation of a possible improvement in modeling.

For both $Y_1$ and $Y_2$, the heterogeneous third-order models are satisfactory. For $Y_1$, the model is significant ($p=0.0243$), the lack of fit is non-significant ($p=0.1276$), and the $r^2=0.94>0.9$, and the adjusted $r^2=0.84\geq0.8$. Also, for $Y_2$, the model is significant ($p=0.0371$), the lack of fit is non-significant ($p=0.0820$), and the $r^2=0.93>0.9$, and the adjusted $r^2=0.80\geq0.8$. All of the four criteria are satisfied for both $Y_1$ and $Y_2$.

Thus, we accept these models as our final models. Letting $\hat{Y}_1$ and $\hat{Y}_2$ denote the predicted values of $Y_1$, and $Y_2$, we specify our heterogeneous third-order models as

$$\hat{Y}_1 = b_0 + b_1X_1 + b_2X_2 + b_{11}X_1^2 + b_{22}X_2^2 + b_{12}X_1X_2 + b_{112}X_1^2X_2 + b_{122}X_1X_2^2$$

and

$$\hat{Y}_2 = c_0 + c_1X_1 + c_2X_2 + c_{11}X_1^2 + c_{22}X_2^2 + c_{12}X_1X_2 + c_{112}X_1^2X_2 + c_{122}X_1X_2^2$$

where the coefficients $b_1, b_2, \ldots, b_{122}$ and $c_1, c_2, \ldots, c_{122}$ are given in Table 2A and Table 2B.

Drawing the 3D plots of the response surface

Each of the three-dimensional (3D) response surface plots was drawn with the vertical axis representing the predicted response and two horizontal axes indicating the two explanatory factors. Fig. 1A and 1B are the 3D response surface plots for the effects of the two actual factors on the two predicted responses.

Multi-response optimization of two responses

In Ahn et al. (2017), the optimization objective was to minimize $Y_1$ (particle size) and maximize $Y_2$ (zeta-potential) simultaneously. For this multi-response optimization, we modified the desirability function technique of Derringer and Suich (1980). In this modified technique, first, we define the desirability function for the minimization of $Y_1$ as

$$D_1 = [\text{Maximum}(\hat{Y}_1) - \hat{Y}_1]/[\text{Maximum}(\hat{Y}_1) - \text{Minimum}(\hat{Y}_1)],$$

and define the desirability function for the maximization of $Y_1$ as

$$D_2 = [\hat{Y}_2 - \text{Minimum}(\hat{Y}_2)]/[\text{Maximum}(\hat{Y}_2) - \text{Minimum}(\hat{Y}_2)].$$

Here, for $\hat{Y}_1$, when $\hat{Y}_1$ is minimized, $D_1$ becomes 1; otherwise $0 \leq D_1 < 1$, and for $\hat{Y}_2$, when $\hat{Y}_2$ is maximized, $D_2$ becomes 1; otherwise $0 \leq D_2 < 1$. Now, we define $CD$, which means the composite desirability, as

$$CD = (D_1D_2)^{1/2}$$
which is the geometric mean of D1 and D2. Then, we find the combination of the values of X1 and X2 that maximizes CD. This combination is the optimum point of (X1, X2). Now, by converting this optimum point to the combination of the levels of the actual factors, we achieve the multi-response optimization of minimizing Y1 and maximizing Y2 simultaneously.

For the minimization and maximization of Y1 and Y2 and the maximization of CD, we performed the searches on a grid (Oh et al., 1995). First, we obtained Minimum (Y1)=170.8131135, Maximum (Y1)=221.6698750, Minimum (Y2)=24.7334750, and Maximum (Y2)=35.2957228, and using these values, we implemented our modified desirability function technique that maximizes the composite desirability defined above. Fig. 1C shows the 3D surface plot of the composite desirability function for our multi-response optimization. Table 2C presents the results of our multi-response optimization.

In Ahn et al. (2017), at their optimum point, their Y1 value was 190.1 and their Y2 value was –25.94±0.06, whereas, at our predicted optimum point, the predicted Y1 was 183.4 and the predicted Y2 was 30.93. We can see that our predicted minimum of Y1 is smaller than their observed Y1, and our predicted maximum of Y2 is greater than their observed Y2. Their optimum conditions for their multi-response optimization were F1=speed of primary homogenization (rpm)=5,000 and F2=concentration of emulsifier (%)=0.2071, whereas our optimum conditions are F1=5,000 and F2=0.295. We can see that our predicted combination of optimum factor levels is different from theirs. A validation experiment will be needed to verify the result of multi-response optimization obtained by the method proposed in this article.

### Table 2. Results of modeling and optimization

#### A. Coefficient estimates in the heterogeneous 3rd-order model on Y1

| Term         | Parameter estimate | Standard error | t-value | p-value |
|--------------|--------------------|----------------|---------|---------|
| Intercept    | b0=182.99          | 2.76           | 66.24   | <0.0001 |
| X1           | b1=–19.75          | 4.28           | –4.62   | 0.0099  |
| X2           | b2=–1.58           | 4.28           | –0.37   | 0.7302  |
| X1^2         | b11=–11.33         | 3.71           | 3.06    | 0.0378  |
| X2^2         | b22=–3.04          | 3.71           | –0.82   | 0.4579  |
| X1X2         | b12=10.47          | 3.03           | 3.46    | 0.0258  |
| X1^2X2       | b112=11.23         | 5.24           | 2.14    | 0.0987  |
| X1X2^2       | b122=30.03         | 5.24           | 5.73    | 0.0046  |

#### B. Coefficient estimates in the heterogeneous 3rd-order model on Y2

| Term         | Parameter estimate | Standard error | t-value | p-value |
|--------------|--------------------|----------------|---------|---------|
| Intercept    | c0=30.08           | 0.58           | 51.52   | <0.0001 |
| X1           | c1=–3.92           | 0.90           | –4.33   | 0.0124  |
| X2           | c2=–0.45           | 0.90           | –0.50   | 0.6450  |
| X1^2         | c11=1.23           | 0.78           | 1.57    | 0.1904  |
| X2^2         | c22=–2.30          | 0.78           | –2.94   | 0.0426  |
| X1X2         | c12=1.45           | 0.64           | 2.27    | 0.0860  |
| X1^2X2       | c112=3.13          | 1.11           | 2.83    | 0.0474  |
| X1X2^2       | c122=3.77          | 1.11           | 3.40    | 0.0273  |

#### C. Results of multi-response optimization

| X1  | X2  | F1=Speed of primary homogenization (rpm) | F2=Concentration of emulsifier (%) | Predicted minimum of Y1=Particle size | Predicted maximum of Y2=Zeta-potential | D1   | D2   | CD= Composite desirability |
|-----|-----|----------------------------------------|-----------------------------------|---------------------------------------|----------------------------------------|------|------|---------------------------|
| –1  | 0.95| 5,000                                  | 0.295                             | 183.4                                 | 30.93                                  | 0.752| 0.587| 0.664                     |
Conclusion

This article suggests the use of the heterogeneous third-order model for better modeling and a modified desirability function technique for multi-response optimization. The heterogeneous third-order model can be used when (1) the experimental design is a two-factor central composite design having –1, 0, and 1 as the coded levels of factors, and (2) a second-order model fails to provide a good fit for the data. How to construct the heterogeneous third-order model is to use $X_1$, $X_2$, $X_1^2$, $X_2^2$, $X_1X_2$, $X_1^2X_2$, and $X_1X_2^2$ as model terms. A modified desirability function technique first defines a desirability function for each response according to each optimization objective, and then finds out the combination of factor levels that maximizes the geometric mean of the values from desirability functions for multiple responses. An illustrative new analysis of the data from a previous research has produced statistical models with better fits and optimization results with better predictions. This suggestion is expected to help enhance the quality of response surface analyses of the experiments in food science of animal resources.

**Fig. 1.** 3D surface plots of predicted responses and the composite desirability function. (A) 3D surface plot of the predicted response $Y_1$, (B) 3D surface plot of the predicted response $Y_2$, (C) 3D surface plot of the composite desirability function.
Conflict of Interest

The authors declare no potential conflict of interest.

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Author Contributions

Conceptualization: Rheem S, Rheem I, Oh S. Data curation: Rheem S. Formal analysis: Rheem S. Methodology: Rheem S, Rheem I, Oh S. Software: Rheem S. Investigation: Rheem S, Rheem I, Oh S. Writing - original draft: Rheem D, Oh S. Writing - review & editing: Rheem S, Rheem I, Oh S.

Ethics Approval

This article does not require IRB/IACUC approval because there are no human and animal participants.

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