Mathematical Models for the Design of Electrical Machines

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Electrical machines are used in many electrical engineering applications, viz, transports (e.g., electric, hybrid, and fuel cell vehicles, railway traction, and aerospace), energy harvesting (e.g., flywheels), renewable energy (e.g., wind power turbines and hydroelectric power plants) and magnetic refrigeration devices, among others. For decades, numerical methods (e.g., finite element, finite difference or boundary-element analysis) were widely used in research and development (R&D) departments for their accuracy as compared to measurements. Nevertheless, mainly in three-dimensional (3-D) applications, these approaches are time-consuming and not suitable for optimization problems. Nowadays, in order to reduce the computation time, R&D engineers must develop full computer-aided design for electrical machines with accurate and fast models in simulations. Hence, the main objective of this special issue is to bring the latest advances and developments in the mathematical modeling and design of electrical machines to different applications. The main models discussed will be based on the following:

- Equivalent circuits (e.g., electrical, thermal and magnetic);
- The Schwarz–Christoffel mapping method;
- The Maxwell–Fourier method (e.g., multilayer models, eigenvalues models and the subdomain technique).

The interest topics in the mathematical models include, but are not restricted to the following:

- Two-dimensional (2-D), quasi 3-D and 3-D;
- Global and local saturation, slotting and eddy current effects;
- Adaptive generic models;
- Multiphysics modeling with new materials;
- Hybrid models.

The numerical method, as well as the experimental tests, will be used as comparisons or validations. In this special issue, the authors of selected works contributed to the topics listed above, since contents of their works can be synthesized as follows:

- **Maxwell–Fourier Method** (i.e., the formal resolution of Maxwell’s equations by using the separation of variables method and the Fourier’s series) in 2-D or 3-D with a quasi-Cartesian or polar coordinate system:

  ○ Jabbari [1]. In this research, an analytical model was proposed to calculate the magnetic vector potential in surface-mounted permanent magnet (PM) machines. It was based on the subdomain technique and applied a hyperbolic function. The saturation effect was
neglected. A mathematical expression was also derived for optimizing the PM shape to reduce the cogging torque and electromagnetic torque components. The analytical results were validated through finite element analysis (FEA);

- Ben Yahia et al. [2]. In this contribution, the authors proposed a 2-D exact subdomain technique in switched reluctance machines, supplied by a sinusoidal current waveform (i.e., variable flux reluctance machines) by applying the Dubas’ superposition technique [3]. The global saturation effect was considered with a constant magnetic permeability corresponding to the linear zone of the nonlinear $B(H)$ curve. The comparisons with FEA showed good results for the proposed approach;

- Mendonça et al. [4]. Here, a novel solution for magnetic field calculation in 2-D problems based on the multilayer model, in which one region is defined with a space-varying magnetic parameter was proposed. This contribution was effective at evaluating more realistic magnetic parameters, where measurements of a high-speed PM generator prototype indicated saturation in the retaining sleeve due to pole-to-pole leakage flux. The saturation profile is a function of the mechanical angle and can be modeled with the aid of a space-varying relative permeability, expressed in terms of a Fourier’s series. The analytical solution was confronted with FEA, which confirmed the validity of the proposed methodology;

- Custers et al. [5]. This work describes the scattering matrix approach to obtain the solution to electromagnetic field quantities in harmonic multilayer models. The method is more memory efficient than classical methods used to solve boundary conditions. The method has been applied to a 3-D electromagnetic configuration for verification and compared to numeric results;

- Vahaj et al. [6]. In this research, a 2-D semi-analytical model based on the subdomain technique was proposed to calculate the magnetic vector potential in outer rotor PM machines. The saturation effect was neglected. The electromagnetic performances were verified by comparing them with those obtained from FEA;

- Electrical, Thermal or Magnetic Equivalent Circuit (EEC, TEC or MEC)

- Asfirane et al. [7]. The authors have been developing a 3-D nonlinear MEC for a hybrid excitation synchronous machine. The semi-analytical results have been compared with those obtained from 3-D numeric results, as well as experimental data. The 3-D nonlinear MEC exhibited fairly accurate results when compared to the 3-D FEA, with a significant gain in computation time (viz, 4.7 s and 1560 s for one position, respectively).

- Petitgirard et al. [8]. This original study concerned the winding head thermal design of electrical machines in difficult thermal environments. Based on geometrical assumptions (viz, Delaunay triangulation and Voronoï tessellation), an adaptive generic tool of a 2-D TEC in a steady state was developed, which could be adapted for all basic shapes and solved the thermal behavior of a random wire layout. The network set-up, adaptation, matrix writing and resolution were detailed. The model has been compared with the finite volume method, and several experiments have been planned;

- Mekahlia et al. [9]. In this significant research, a harmonic EEC was presented for multi-phase squirrel-cage induction machines. In order to predict the torque pulsations, the reduced-order model of the rotor was applied. The proposed analysis allowed for avoiding incorrect design with non-sinusoidal magnetomotive forces. The semi-analytical approach has been confirmed by FEA for a three- and five-phase induction machine;

- Hybrid Models

- Aleksandrov et al. [10]. In this contribution, the authors developed a 2-D hybrid steady-state magnetic field model, capable of accurately modeling the electromagnetic behavior in
a linear induction motor. This model, in a Cartesian coordinate system, integrated a complex harmonic modeling technique (or the Maxwell–Fourier method) with a discretized MEC without the saturation effect. The analytical solution was applied to regions with homogeneous material properties, while the linear MEC approach was used for the regions containing non-homogeneous material properties. The resulting thrust and normal forces showed excellent agreement with respect to FEA and the measurement data;

Benmessaoud et al. [11,12]. In [11], the authors developed a 2-D hybrid model in Cartesian coordinates, combining an MEC with the Maxwell–Fourier method for eddy current loss calculation. The model coupling was applied to a U-cored static electromagnetic device. Experimental tests and 3-D FEA were compared with the proposed approach on massive conductive parts in aluminum. In [12], the developed hybrid model was extended in polar coordinates to multi-phase synchronous machines for the volumic PM eddy current losses. A global revision on the calculation and analysis of PM eddy current losses can be found in [13].

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