Neutrino flavour relaxation
or
neutrino oscillations?

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Abstract

We propose the new mechanism of neutrino flavour relaxation to explain the experimentally observed changes of initial neutrino flavour fluxes. The test of neutrino relaxation hypothesis is presented, using the data of modern reactor, solar and accelerator experiments. The final choice between the standard neutrino oscillations and the proposed neutrino flavour relaxation model can be done in future experiments.

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Now the phenomena of changes in initial neutrino flavour flux is observed in different neutrino experiments. The SNO experiment [1] evidently detects only 1/3 part of initial electron neutrino flux from the Sun and 2/3 part of muon and(or) tau neutrino fluxes. The Kamland reactor experiment [2] detected only 61% of expected electron antineutrino events from different reactors at mean distance of 180 km. Convincing evidence of initial neutrino flavour flux changes is observed also in Super-K [3] and MACRO [4] atmospheric neutrino data and K2K [5] accelerator experiment with muon neutrinos.

The standard common way of interpreting these results lies in neutrino oscillation hypothesis, first proposed by Bruno Pontecorvo [6] and developed in further works [7].

Here we discuss the alternative mechanism of neutrino flavour relaxation, which can also describe the observed changes of initial neutrino flavour fluxes with distance.

The proposed model is similar to the mechanism [8] of spin relaxation in random fluctuating magnetic field \( \mathbf{B} \) with zero average \(< \mathbf{B}(t) > = 0 \) and mean square fluctuating field value \(< \mathbf{B}^2(t) > \neq 0 \). The spin relaxation process is described by W.Pauli master equation [9].

Let us assume the existence of some small random fluctuating vacuum field \( \hat{V} \), causing the transitions between different lepton flavours. Such field \( \hat{V} \) can have mean zero value, with \(< \hat{V}^2(t) > \) different from 0 and \((< \hat{V}^2 > )^{1/2} > m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau} \). Interactions of neutrino flavours with the vacuum field should lead to flavour relaxation process.

The time evolution of the neutrino states are governed by the Shrodinger equation:

\[
i \frac{d}{dt} |\nu(t) > = \hat{H}(t) |\nu(t) > ,
\]

where \( |\nu(t) > \) is the neutrino vector of state and \( \hat{H}(t) \) is the time-dependent Hamiltonian of the system, which form depends on in what basis it is given.

In flavour basis the total Hamiltonian in the random fluctuating vacuum field for rest reference frame can be written as

\[
\hat{H}_f(t) = \hat{H}_m + \hat{V}_f(t) ,
\]
where

\[ \hat{H}_m = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \quad \text{and} \quad \hat{V}_f(t) = \begin{pmatrix} 0 & V_{e\mu}(t) & V_{e\tau}(t) \\ V_{\mu e}(t) & 0 & V_{\mu\tau}(t) \\ V_{\tau e}(t) & V_{\tau\mu}(t) & 0 \end{pmatrix}. \]

(3)

Here \( H_m \) is the free Hamiltonian in mass basis. Note, that in this model the neutrino flavour states are assumed to be the same as their massive states (or in other words the mixing mass matrix is diagonal).

\( V_{\alpha\alpha'}(\alpha = e, \mu, \tau) \) is the vacuum field potential with mean value \( < V_{\alpha\alpha'}(t) >= 0 \) and \( < (V_{\alpha\alpha'}(t))^2 > \neq 0 \).

The neutrino flavour evolution in time can be obtained from Eq. (1-3), using the density matrix approach and is given by Pauli master equation:

\[
\frac{d}{dt} \begin{pmatrix} n_{\nu_e} \\ n_{\nu_\mu} \\ n_{\nu_\tau} \end{pmatrix} = \begin{pmatrix} -(W_{e\mu} + W_{e\tau}) & W_{e\mu} & W_{e\tau} \\ W_{\mu e} & -(W_{e\mu} + W_{\mu\tau}) & W_{\mu\tau} \\ W_{\tau e} & W_{\tau\mu} & -(W_{e\tau} + W_{\mu\tau}) \end{pmatrix} \begin{pmatrix} n_{\nu_e} \\ n_{\nu_\mu} \\ n_{\nu_\tau} \end{pmatrix},
\]

(4)

where \( n_{\alpha}(\alpha = \nu_e, \nu_\mu, \nu_\tau) \) are the probability of observing neutrino with electron, muon or tau flavour, \( W_{\alpha\alpha'} \) corresponds to the neutrino transition rates from flavour \( \alpha' \) to flavour \( \alpha \) and

\[ \sum_{\alpha=\nu_e,\nu_\mu,\nu_\tau} n_{\alpha}(t) = 1. \]

The general solution of Eq. (4) is given by the sum of two exponents and constant:

\[
\begin{pmatrix} n_{\nu_e}(t) \\ n_{\nu_\mu}(t) \\ n_{\nu_\tau}(t) \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} e^{-t/T_1} + \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} e^{-t/T_2},
\]

(5)

with

\[ \sum_{i=1}^{3} a_i = 1, \quad \sum_{i=1}^{3} b_i = 0, \quad \sum_{i=1}^{3} c_i = 0. \]

(6)

Note, that in relaxation model the neutrino flavour lepton numbers are violated, while the total sum of the numbers are constant (also as in the case of oscillation model).
To illustrate the possibilities of the relaxation model the particular "simple" solution can be found under the following assumptions:

a) \( W_{e\mu} = W_{\mu e}, W_{e\tau} = W_{\tau e}, W_{\mu\tau} = W_{\tau\mu} \),

b) \( W_{\mu\tau} \gg W_{e\mu}, W_{e\tau} \)

Now equation (5) looks as:

\[
\begin{pmatrix}
  n_{\nu_e}(t) \\
  n_{\nu_\mu}(t) \\
  n_{\nu_\tau}(t)
\end{pmatrix} =
\begin{pmatrix}
  1/3 \\
  1/3 \\
  1/3
\end{pmatrix} + b \begin{pmatrix}
  0 \\
  1 \\
  -1
\end{pmatrix} e^{-2W_{\mu\tau}t} + c \begin{pmatrix}
  -1 \\
  1/2 \\
  1/2
\end{pmatrix} e^{-3/2(W_{e\mu}+W_{e\tau})t}.
\]

(7)

For small mass differences between different neutrino flavours \( m_{\nu_e} \approx m_{\nu_\mu} \approx m_{\nu_\tau} \) and \( \Delta m_\nu \ll m_\nu \) with \( \gamma = E_\nu/m_\nu \gg 1 \) the probability \( P \) of observing the neutrino flavour with energy \( E_\nu \) at distance \( r \) from the source is given by:

\[
\begin{pmatrix}
  P_{\nu_e}(r) \\
  P_{\nu_\mu}(r) \\
  P_{\nu_\tau}(r)
\end{pmatrix} =
\begin{pmatrix}
  1/3 \\
  1/3 \\
  1/3
\end{pmatrix} + b \begin{pmatrix}
  0 \\
  1 \\
  -1
\end{pmatrix} e^{-r/(\Lambda_0 E_\nu)} + c \begin{pmatrix}
  -1 \\
  1/2 \\
  1/2
\end{pmatrix} e^{-r/(\Lambda_1 E_\nu)}.
\]

(8)

where we use the new notation \( \Lambda_0 = (2W_{\mu\tau}m_\nu)^{-1} \)

and \( \Lambda_1 = 2/3[(W_{e\mu} + W_{e\tau})m_\nu]^{-1} \).

Equation (8) can be solved for different initial experimental conditions of neutrino flavour fluxes.

For initial pure electron neutrino or antineutrino flux (like in the case of solar or reactor experiments) \( P_{\nu_e}(0) = 1, P_{\nu_\mu}(0) = P_{\nu_\tau}(0) = 0 \) the solution is given by:

\[
\begin{align*}
  P_{\nu_e}(r) &= 1/3[1 + 2 e^{-r/(\Lambda_1 E_\nu)}], \\
  P_{\nu_\mu}(r) &= 1/3[1 - e^{-r/(\Lambda_1 E_\nu)}], \\
  P_{\nu_\tau}(r) &= 1/3[1 - e^{-r/(\Lambda_1 E_\nu)}].
\end{align*}
\]

(9)

From Kamland experiment data \[2\] it is possible to estimate the value of \( \Lambda_1 \) parameter. Taking the effective energy of reactor antineutrino flux to be equal to 4.8 MeV (here we take into account the threshold of Kamland detector \( E_{\bar{\nu}_e} > 3.4 \) MeV), mean reactor distance 180 km and \( R = P_{\bar{\nu}_e(\text{measured})}/P_{\bar{\nu}_e(\text{expected})} = 0.61 \) we obtain

\[ \Lambda_1 = 43 \text{ km/MeV}. \]
Fig.1 illustrates the distance dependence of reactor antineutrino fluxes (for $E_{\tilde{\nu}} = 4.8\, MeV$).

Figure 1: Probability to observe different flavours from reactor for $E_{\tilde{\nu}} = 4.8\, MeV$.

Note, that the final flavour survival probability $1/3$ is consistent with the SNO [1] experimental data with the ratio of measured neutrino flavour fluxes

$$\frac{CC}{NC} = 0.306 \pm 0.026\text{(stat)} \pm 0.024\text{(syst)},$$

and with the absence of distortion in the measured neutrino spectrum at low energies, while such distortion is predicted by LMA solution [11].

It agrees also with the Homestake results [12] for the ratio $R$ of observed and predicted by SSM [13] neutrino rates: $R = 0.34 \pm 0.03$.

The deviation from "simple" relaxation model exists for Gallium experiments [14] [15] with $R = 0.553 \pm 0.034$. But it is worth to mention, that the $R$ value depends on the accuracy of Standard Solar Model predictions for low energy fluxes.

For the case of pure initial muon neutrino or antineutrino flux (like in accelerator experiments) $P_{\nu_{\mu}}(0) = 1, P_{\nu_{e}}(0) = P_{\nu_{\tau}}(0) = 0$ the solution of Eq. (9) is:
\[ P_{\nu_e}(r) = \frac{1}{3}[1 - e^{-r/(\Lambda_1 E_{\nu_e})}] , \]
\[ P_{\nu_\mu}(r) = \frac{1}{3} + \frac{1}{2} e^{-r/(\Lambda_0 E_{\nu_\mu})} + \frac{1}{6} e^{-r/(\Lambda_1 E_{\nu_\mu})} , \]
\[ P_{\nu_\tau}(r) = \frac{1}{3} - \frac{1}{2} e^{-r/(\Lambda_0 E_{\nu_\tau})} + \frac{1}{6} e^{-r/(\Lambda_1 E_{\nu_\tau})} . \] (11)

Preliminary estimation of \( \Lambda_0 \) parameter can be done using K2K accelerator experiment data [5]. Taking the mean energy of muon neutrino flux \( E_{\nu} = 1.3 GeV \), distance \( r = 250 km \), \( R = P_{\nu_\mu} \text{(measured)} / P_{\nu_\mu} \text{(expected)} = 0.70 \) we obtain:
\[ \Lambda_0 = 0.21 km/MeV . \]

Fig.2 illustrates the distance dependence of neutrino fluxes from muon neutrino beam \((E_{\nu} = 1.3 GeV)\).

![Figure 2: Probability to observe different flavours from accelerator muon neutrino beam for \( E_{\nu} = 1.3 GeV \).](image)

It is necessary to note that the estimations of neutrino relaxation parameters \( \Lambda_0 \) and \( \Lambda_1 \) are very preliminary and were made mainly for illustration of relaxation model. More precise calculations can be done in future, using exact data of different experiments (taking also into account atmospheric neutrino data).
Finally we would like to emphasize the main difference between the standard oscillation theory and the proposed here relaxation model: the dependence of neutrino flavour fluxes from distance is described by the sum of constant and 2 exponents, instead of oscillation case. Fig.3 illustrates this difference for reactor antineutrino experiments.

![Graph showing the ratio of measured to expected $\bar{\nu}_e$ flux from Kamland [2] and Chooz [16] reactor experiments. Dotted curve — predictions of oscillation model with $\sin^2 2\theta = 0.91$ and $\delta m^2 = 6.9 \cdot 10^{-5} (eV)^2$ best fit parameters from [2]. Solid curve — predictions of "simple" relaxation model.](image)

Figure 3: The ratio of measured to expected $\bar{\nu}_e$ flux from Kamland [2] and Chooz [16] reactor experiments. Dotted curve — predictions of oscillation model with $\sin^2 2\theta = 0.91$ and $\delta m^2 = 6.9 \cdot 10^{-5} (eV)^2$ best fit parameters from [2]. Solid curve — predictions of "simple" relaxation model.

Future reactor and accelerator neutrino experiments can provide the necessary data to choose between the neutrino oscillations and the proposed flavour relaxation model. Of course, the possibility to have a mixture of neutrino oscillation and relaxation models also exists.

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