Transmission spectrum simulation of long period fiber grating

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Abstract. According to the three-layer structure model of fiber and the theory of fiber coupling mode, the mode coupling characteristics of LPFG are deduced and simulated by MATLAB. The LPFG transmission spectrum obtained by simulation is basically consistent with the existing experimental results. In addition, by changing the input parameters, the transmission spectra of different LPFG can be simulated, which is of great value for theoretical research and parameter design in practical application of LPFG.

1. Introduction

Long period fiber grating (LPFG) is a kind of can achieve some of a particular wavelength prior to transmission mode and cladding mode coupling between optical fiber devices, for the first time since Vengsarkar \cite{1} of AT&T bell laboratories in hydrogen loaded silicon germanium photosensitive fiber successfully made the first LPFG using amplitude mask template, due to the excellent performance of LPFG, the applied research has attracted wide attention of scholars both at home and abroad. The analysis of LPFG's transmission characteristics is very helpful for the application of LPFG.

In this paper, based on the coupled mode theory and the cladding mode theory of fiber three-layer model, numerical simulation of the transmission spectrum characteristics of the cosine refractive index modulated LPFG written by the ultraviolet laser is conducted by MATLAB, which lays a foundation for the analysis and production of LPFG.

2. Theoretical analysis of LPG

2.1. Three-layer model of single-mode fiber

The fiber used in the experiment is generally single-mode fiber with step index, so only step index fiber is considered in the theoretical analysis. The three-layer model of single-mode fiber \cite{2} with step index is shown in figure 1: the innermost part is the fiber core, the cladding layer is the outer layer of the fiber core, and the air layer is outside the cladding layer. For the LPFG written by uv laser, only the refractive index (dielectric constant) of the fiber core is disturbed, while the refractive index of the cladding is not changed. The refractive index of the fiber grating changes periodically with the z direction. In the whole fiber exposure region, the refractive index distribution of the fiber grating can generally be written as \cite{3}:
$$n_i (r, z) = \begin{cases} n_1 \left( 1 + \sigma(z) \left[ 1 + m \cos \left( \frac{2\pi}{A} \cdot z + \varphi(z) \right) \right] , |r| \leq a_i \right) \\ n_2, a_i \leq |r| \leq a_2 \\ n_1, |r| > a_2 \end{cases}$$

In formula 1, $\sigma(z)$ is the envelope of slow refractive index variable. For uniform LPG, $\sigma(z)$ is a constant and $m$ is the modulation coefficient ($0 \leq m \leq 1$), which can be generally taken as 1. $\varphi(z)$ is the additional phase related to the phase shift or chirp of the grating.

**Fig. 1** Three-layer model of single-mode fiber

2.2. **Coupling mode theory analysis**

Coupling mode theory states that no energy exchange occurs between the modes that transmit light. But when there is perturbation of the waveguide structure of the fiber, such as optical fiber drawing process results in uneven fiber core roundness or late or artificially uneven distribution of refractive index fiber refractive index change, etc.) destroyed the consistency of the optical fiber waveguide optical properties, the fiber along the longitudinal periodic modulation (disturbance), makes the energy exchange occurred between different patterns.

The variation of amplitudes $A_j$ and $B_j$ of any mode propagating in the positive and negative directions along the $z$-axis is subject to the equations as follows [4]:

$$\frac{dA_j}{dz} = i \sum_k A_k (K'_{ij} + K''_{ij}) \exp[i(\beta_k - \beta_j)z] + i \sum_k B_k (K'_{ij} - K''_{ij}) \exp[-i(\beta_k + \beta_j)z] \quad \text{(2-a)}$$

$$\frac{dB_j}{dz} = -i \sum_k A_k (K'_{ij} - K''_{ij}) \exp[i(\beta_k + \beta_j)z] - i \sum_k B_k (K'_{ij} + K''_{ij}) \exp[-i(\beta_k - \beta_j)z] \quad \text{(2-b)}$$

In formula 2, $A_k$ and $B_k$ are the amplitudes of the forward and reverse k-order modes, respectively. $\beta_j$ and $\beta_k$ represent the transmission constants of the j-th order and k-th order modes, respectively. $K'_{ij}$ and $K''_{ij}$ respectively represent the coupling coefficients of transverse mode and
longitudinal mode between mode j and mode k. As \( K^j_k \) is usually small, it can be ignored. The expression of \( K^j_k \) is:

\[
K^j_k (z) = \frac{c}{d} \int \Delta \varepsilon(x, y, z) \vec{e}_\nu(x, y) \vec{e}_\rho^*(x, y) dx dy
\]  

(3)

In formula 3, \( \Delta \varepsilon(x, y, z) \) represents the change of dielectric constant, \( \vec{e}_\nu(x, y) \) and \( \vec{e}_\rho^*(x, y) \) represent the transverse mode field components of order j and order k modes respectively, and * represents conjugate.

2.3. Mode coupling of LPFG

Mode coupling mainly occurs between modes of forward propagation in LPFG. The coupling between longitudinal coupling coefficient and cladding mode is ignored, and the circular symmetry of refractive index perturbation is taken into account to study the coupling between a fiber core fundamental mode and multiple cladding modes. The coupling mode equation can be simplified as follows [5]:

\[
\frac{dA^v}{dz} = \imath \kappa_{\nu-1-0}^{\nu-0-0} A^\nu + \sum_j \frac{m}{2} \kappa_{\nu-1-0}^{\nu-0-0} A^j \exp(-\imath 2 \delta_{\nu-1-0}^{\nu-1-0}
\]

4-a

\[
\sum_j \left[ \frac{dA^j}{dz} = \frac{m}{2} \kappa_{\nu-1-0}^{\nu-0-0} A^j \exp(\imath 2 \delta_{\nu-1-0}^{\nu-1-0}) \right]
\]

4-b

In formula 4, \( A^\nu \) is the amplitude of the core fundamental mode, \( A^j \) is the amplitude of the \( \nu \)-th cladding mode, \( \kappa_{\nu-1-0}^{\nu-0-0} \) is the self-coupling coefficient between the core guide modes, \( \kappa_{\nu-1-0}^{\nu-0-0} \) is the coupling coefficient between the core fundamental mode and the first-order cladding mode, \( \kappa_{\nu-1-0}^{\nu-0-0} \) and \( \kappa_{\nu-1-0}^{\nu-0-0} \) are given in detail by the formula that defines them [6], and it will not be repeated here. \( \delta_{\nu-1-0}^{\nu-1-0} \) is the detuning parameter between the fundamental mode and the \( \nu \)-th cladding mode of the first order, indicating the deviation degree of the phase matching condition.

\[
\delta_{\nu-1-0}^{\nu-1-0} = \frac{1}{2} \left( \beta_{\nu-0}^{\nu-0} - \beta_{\nu-1}^{\nu-1} - \frac{2\pi}{\Lambda} \right)
\]

(5)

In formula 5, \( \beta_{\nu-0}^{\nu-0} \) and \( \beta_{\nu-1}^{\nu-1} \) are the propagation constants of the core fundamental mode and the \( \nu \)-th cladding mode of the first order, respectively, which can be obtained from the characteristic equation of the core fundamental mode and cladding mode. \( \beta_{\nu-0}^{\nu-0} \) and \( \beta_{\nu-1}^{\nu-1} \) can be obtained from the characteristic equation of core base mode and cladding mode.

Combining with the boundary conditions, the transmittance of LPFG can be obtained as follows:

\[
t = \cos^2 \left( \sqrt{\kappa_{\nu-1-0}^{\nu-0-0} + \delta^2} z \right) + \frac{\delta^2}{\sigma^2 + \kappa_{\nu-1-0}^{\nu-0-0}} \sin^2 \left( \sqrt{\kappa_{\nu-1-0}^{\nu-0-0} + \delta^2} z \right)
\]

(6)
In formula 6: $\hat{\sigma}^2$ represents the dc self-coupling coefficient, which does not involve energy exchange between modes, but only affects the propagation constant of the fiber core guide mode. For uniform LPG, $\hat{\sigma} = \delta_{\text{cl-co}} + k_{\text{co-co}} / 2$.

According to formula 6, if the dc coupling coefficient $a$ and the cross-coupling coefficient $b$ of LPG at a certain wavelength can be calculated, then the transmittance of LPFG at this wavelength can be calculated. The transmission spectrum in a range can be obtained by sampling each wavelength and calculating its transmittance respectively. Finally, the transmission spectrum simulation of LPFG can be obtained.

3. Simulation results and analysis

The transmission equation of LPFG is obtained by deducing the coupling equation of LPFG mode. During MATLAB simulation, the parameters of the fiber were set as follows: core radius $a_1 = 4.15\mu m$, core refractive index $n_1 = 1.4681$, cladding radius $a_2 = 62.5\mu m$, cladding refractive index $n_2 = 1.4628$, environmental refractive index $n_3 = 1.0$ and, $\sigma(z) = 0.783 \times 10^{-4}$. The necessary parameters of formula 7 were obtained successively, and the transmittance $t$ at each wavelength was obtained. The final LPFG transmission spectrum was drawn as shown in figure 2. The positions of the four loss peaks were 1340.7nm, 1.369.3nm, 1428.1nm and 1533.7nm and the corresponding peak loss values were -1.0dB, -3.9db, -10.1dB and -30dB.

![Fig. 2 Transmission spectrum simulation of LPFG](image)

By comparing the LPFG transmission spectra in the literature [7], it is found that the size of the third loss peak is in good agreement with the position of the resonant wave length. The simulated peak loss value of the second peak is slightly smaller than the experimental value, and the resonant wavelength position is slightly larger than the experimental value. The positions of peak loss and resonant wavelength of the first peak and the fourth peak are different from the experimental values. The reasons for these differences can be summarized as follows: (1) the refractive index distribution of LPFG is not strictly sinusoidal, which can be seen from the inconsistency of the values of $\sigma(z)$ calculated by the loss peaks; (2) for LPFG, the coupling of higher order mode is more influenced by other parameters; (3) the reason why the peak loss of the fourth peak is much larger than the experimental value is that the spectrometer itself is unable to measure the lowest point of the transmission rate.
4. Conclusion
The mode coupling characteristics of LPFG are studied according to the three-layer structure model and the coupling mode theory of optical fiber. The transmission spectrum of LPFG obtained by MATLAB simulation is basically consistent with the existing experimental results. In addition, the transmission spectrum of different LPFG can be simulated by changing the input parameters, which is of great value for theoretical research and parameter design of practical application of LPFG.

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