1. Introduction

To reduce ground traffic pressure, urban metros have undergone rapid developments in recent years. Due to the interference of pipelines with urban underground sewage, drainage, electricity, communication, and natural gas, and the limitations on new construction due to existing buildings, underground construction space is largely limited. Under such circumstances, neighbourhood tunnels are typically adopted as urban metro tunnels (Divall 2013; Mohamad et al. 2012; Do, Dias, and Oreste 2014). In the western cities of China, there is a significant quantity of distributed loess; hence, we focus on the impact of tunnel collapse and the stability of tunnel foundations when shallow neighbourhood tunnels are constructed in collapsible loess areas, especially when it involves crossing large-thickness hydrocousolidate loess.

Hydrocousolidate loess is characterised by large pores, vertical joint development, and low water content. collapsibility and transformation after immersion is its most prominent engineering feature (Gong, Xia, and Lei 2010; Chen 1994; Chehade and Shahrou 2008). In Code for Building Construction in Collapsible Loess Regions (GB 50025–2004) (Press 2004) in China, loess...
field collapsibility types correspond to engineering measures, but they shall be classified based on the self-weight collapse (Luo 1998; Derbyshire, Meng, and Wang et al. 1995). Therefore, the self-weight collapse of loess must first be reasonably determined. The current practice is to acquire the self-weight collapse of loess through indoor compression tests, which involves calculating the self-weight collapse coefficients of different loess layers based on indoor confined compression tests and the accumulated collapse of various strata through a formula (Hou, Zhai, and Chao et al. 2016). For the collapsible indoor compression test, the Code for Building Construction in Collapsible Loess Regions (GB 50025–2004) (Press 2004) explains that vertical compressive stress functioning on sample soil is 200 kPa within a depth of 10 m for a shallow foundation and 300 kPa within a depth of 10–15 m. When the saturated self-weight pressure is greater than 300 kPa, the vertical compressive stress is determined by the saturated self-weight pressure (Assalay et al. 1997; Bishop and Blight 1963). The evaluation method for such a collapsible loess foundation has been applied to a large number of construction engineering applications, but underground engineering and construction engineering have different foundation vertical compressive stresses. Tunnel excavation is a mechanical process of discharging foundation soil, and the mutual impact between the left and right holes may affect the foundation vertical compressive stress in a neighbourhood tunnel (Bishop and Blight 1963). Thus, the foundation vertical compressive stress studied and loaded in the indoor loess collapse test appears to be especially important for shallow neighbourhood loess tunnels. Vertical pressure of twin-tunnels or shallow tunnels has been extensively studied (Wan et al. 2019; Li et al. 2019b, 2019c, 2019a, 2019d). However, a corresponding calculation method to reasonably determine the foundation vertical compressive stress of shallow neighbourhood loess tunnels is still lacking (Assalay et al. 1997; Shimin, Zhang, and Wang 2011). This study is based on the elastic foundation theory of plane strain which considers the longitudinal separation construction characteristics of a neighbourhood tunnel and deduces the foundation vertical compressive stress calculation formula at two different construction times: earlier tunnel excavation and later tunnel excavation (Wenchao, Zhong, and Wang 2003; Mingyue 2010; Chapman et al. 2006).

2. Calculation assumptions

Incremental soil stress caused by tunnel excavation is the subsidiary stress in soil. This study adopts the calculation model as shown in Figure 1 for the shallow neighbourhood loess tunnel. In Figure 1, \( W_1, W_2, \) and \( W_3 \) are the weight of rock mass \( ACEA'CE' \), \( IKOM \) and \( IKOM' \) respectively. \( T_1 \) and \( T_2 \) are frictional resistance from the soil. \( \theta \) is the friction angle (°) on both sides of earth pillar at the top of the tunnel, \( \beta_1 \) is the fracture angle (°) of the triangular rock mass on both sides of the tunnel, and \( \phi_k \) is the calculated friction angle (°). To facilitate the analysis, the assumptions below are adopted:

1. The surrounding rock has single and even lithology, and the tunnel covered ground surface is even.
2. The left and right tunnels are symmetrical in structure, and there is a certain construction distance in their longitudinal separation.
3. Tunnel foundation soil is isotropic and homogenic, the semi-space elastomer has infinite extension in the depth and vertical direction, and the foundation is deemed the strip foundation with a length ratio equal to or greater than 10. This leads to regarding the stress calculation problem in tunnel foundation soil under the load effect as a plain strain problem.
4. This study adopts the calculation model and formula of the surrounding rock in the reference (GB 50025–2004). It combines the mutual effect characteristics of successive construction of the left and right holes in the neighbourhood tunnel and constructs the failure analysis model and calculation method for a shear sliding pattern which complies with the pressure calculation of Xie (Fei, Xie, and Lai et al. 2016; Jiaxiao 1964;
Junfu, Wang, and Guo et al. 2009) for rocks surrounding a shallow tunnel.

3. Boundary depth for shallow and deep neighbour loess tunnels

Based on the formula to calculate surrounding rock pressure for a shallow neighbourhood tunnel given in this paper, when the pressure of surrounding rock reaches the maximum threshold, the broken-rock pressure does not increase with an increase in the burial depth of tunnels, and a relatively stable load-carrying arch may be generated in the surrounding rock. At this time, the embedded depth of tunnel that is acquired is the boundary depth for the shallow and deep-buried neighbourhood loess tunnels. The calculation formula of the surrounding rock pressure of the neighbourhood tunnels is as shown in Equations (1)–(4).

\[ Q_{sh} = y \left[ B(H - (H + h))^2 \left( \frac{\lambda_1 + \lambda_2}{2} \right) \tan \theta \right] \]  

\[ \lambda_1 = \frac{\tan \beta_1 - \tan \varphi_k}{\tan \beta_1[1 + \tan \beta_1(\tan \varphi_k - \tan \theta)] + \tan \varphi_k \tan \theta} \]  

\[ \lambda_2 = \frac{\frac{D}{H + h} \left( 1 - \frac{D}{H + h} \right) \tan \beta_1(\tan \beta_1 - \tan \varphi_k)}{1 + \tan \beta_1(\tan \varphi_k - \tan \theta)} + \tan \varphi_k \tan \theta \]  

\[ \tan \beta_1 = \tan \varphi_k + \sqrt{\frac{\tan^2 \varphi_k + 1}{\tan \varphi_k - \tan \theta}} \]  

where \( Q_{sh} \) is the surrounding rock pressure at the shallow-buried depth, \( \theta \) is the excavation depth (m) of the tunnel, \( h \) is the excavation height (m), \( H \) is the embedded depth (m), \( D \) is the clear distance (m) of the neighbourhood loess tunnel, \( y \) is the weight (kN/m^3) of the surrounding rock covering the tunnel, \( \lambda_1 \) is the lateral pressure coefficient outside the tunnel, \( \lambda_2 \) is the lateral pressure coefficient inside the tunnel, \( \theta \) is the friction angle (°) on both sides of earth pillar at the top of the tunnel, \( \beta_1 \) is the fracture angle (°) of the triangular rock mass on both sides of tunnel, and \( \varphi_k \) is the calculated friction angle (°).

When the surrounding rock pressure in Equation (1) reaches the maximum threshold, namely \( \partial Q_{sh} / \partial H = 0 \), the surrounding rock pressure no longer increases with an increase in embedded depth. The embedded depth of the tunnel is then the boundary depth for the shallow and deep-buried neighbourhood loess tunnels, \( H_{bd} \). After derivation, it is as shown in Equation (5):

\[ H_{bd} = \frac{B}{\lambda_1 \tan \theta} - h - \frac{D \tan \beta_1}{2} \]  

According to Equation (5), the boundary depth for shallow- and deep-buried neighbourhood loess tunnels is related to the soil parameter, excavation size, and clear distance. The boundary depth of shallow and deep-buried neighbourhood loess tunnels is inversely proportional to the clear distance of the tunnels. With an increase in clear distance, the boundary depth decreases, and when the clear distance increases to certain value, namely \( D_{max} = B / (\lambda_1 \tan \theta \tan \beta_1) \), the boundary depth of shallow and deep-buried in neighbourhood loess tunnels equals the boundary depth of the shallow and deep-buried single hold obtained from the maximum value method of Xie. As shown in Equation (6), \( D_{max} \) is the maximum applicable clear distance in the calculation formula of boundary depth of shallow and deep-buried in neighbourhood loess tunnel, namely

\[ H_{bd} = \frac{B}{\lambda_1 \tan \theta} - h - \frac{D_{max} \tan \beta_1}{2} = \frac{B}{2\lambda_1 \tan \theta} - h \]  

Therefore, the boundary depths for shallow- and deep-buried neighbourhood loess tunnels are explained by different expressions with various clear distances. The detail is as shown below:

When \( 0 \leq D \leq D_{max} \):

\[ \text{Figure 2. The distribution model for calculating vertical pressure on the vault of surrounding rock during the excavation of the first hole.} \]
\[ H_{bd} = \frac{B}{\lambda_1 \tan \theta} - h - \frac{D \tan \beta_1}{2} \]  
\begin{equation}
\text{When } D_{\text{max}} \leq D:\n H_{bd} = \frac{B}{2\lambda_1 \tan \theta} - h
\end{equation}

4. Calculation of surrounding rock pressure of shallow neighbourhood loess tunnel

4.1. Calculation of surrounding rock pressure in first hole

When we begin to excavate the first hole, similar to the excavation of a single tunnel, the arch ring bears vertical and uniform strip loads. The distribution model for calculating the pressure is shown in Figure 2, and the calculation formula for surrounding rock pressure is shown in Equations (9) – (11).

\[ q_1 = \gamma \left[ H - \frac{(H + h)^2}{B} \lambda_1 \tan \theta \right] \]  
\[ \lambda_1 = \frac{\tan \beta_1 - \tan \phi_k}{\tan \beta_1 [1 + \tan \beta_1 (\tan \phi_k - \tan \theta) + \tan \phi_k \tan \theta]} \]  
\[ \tan \beta_1 = \tan \phi_k + \sqrt{\frac{(\tan^2 \phi_k + 1) \tan \phi_k}{\tan \phi_k - \tan \theta}} \]  

4.2. Calculation of the pressure on the surrounding rock of rear hole

When we start to excavate the rear hole, according to written records, different degrees of “biasing effect” exist in vertical surrounding rock pressure on the vault of a shallow neighbourhood loess tunnel, or the pressure of the vertical surrounding rock of the outer side of the two tunnels are all less than that of the inner side (Jianwu, Caichu, and Xuewen 2010) (inner side is the side close to middle wall). In this case, the pressure of the vertical surrounding rock on the inner side of first hole is the maximum pressure because of the excavation of the rear hole, following that of the inner side of the rear hole, as shown in Figure 3. The calculation formula for surrounding rock pressure of this model is as shown in Equations (12)–(19).

\[ q_1 = \gamma \left[ H - \frac{(H + h)^2}{B} \lambda_1 \tan \theta \right] \]  
\[ \lambda_1 = \frac{\tan \beta_1 - \tan \phi_k}{\tan \beta_1 [1 + \tan \beta_1 (\tan \phi_k - \tan \theta) + \tan \phi_k \tan \theta]} \]  
\[ \tan \beta_1 = \tan \phi_k + \sqrt{\frac{(\tan^2 \phi_k + 1) \tan \phi_k}{\tan \phi_k - \tan \theta}} \]  

\[ q_2 = \gamma \left[ H - \frac{(H + h)^2}{B} \lambda_2 \tan \theta \right] \]  
\[ \lambda_2 = \frac{\frac{D}{\pi r} \left(1 - \frac{D}{4(H + h)}\right) \tan \beta_1 (\tan \beta_1 - \tan \phi_k)}{1 + \tan \beta_1 (\tan \phi_k - \tan \theta) + \tan \phi_k \tan \theta} \]  
\[ \tan \beta_1 = \tan \phi_k + \sqrt{\frac{1}{\tan \phi_k - \tan \theta} \left[ \frac{1}{\tan (\phi_k - \theta)} + \frac{4(H + h)}{D} \right]} \]  

\[ q_2' = \gamma \left[ H - \frac{(H + h)^2}{B} \lambda_2' \tan \theta \right] \]  
\[ \lambda_2' = \frac{\frac{D}{\pi r} \left(1 - \frac{D}{4(H + h)}\right) \tan \beta_1 (\tan \beta_1 - \tan \phi_k)}{1 + \tan \beta_1 (\tan \phi_k - \tan \theta) + \tan \phi_k \tan \theta} \]  

Figure 3. The distribution model for calculating vertical pressure on the vault of surrounding rock during the excavation of the rear hole.
As shown in Figure 1, \( \lambda_2 \) is the pressure coefficient for the inner side of rear hole, \( \theta \) is the friction angle (°) on both sides of the earth-pillar at the top of the tunnel, \( \beta_2 \) is the fracture angle (°) of triquetrum rock mass at the inner side of the rear hole, and \( \varphi_k \) is the calculated friction angle (°).

5. Calculation of foundation vertical compressive stress of shallow neighbourhood loess tunnel

5.1. Calculation of foundation vertical compressive stress of first hole

The distribution of the tunnel basement surface load is shown in Figure 4. We depict an X-Z coordinate system in the figure with the left arch springing off of the first hole as the centre of a circle; the horizontal direction is axis X, and vertical direction is axis Z.

In Figure 4, \( P_1 \) is the tunnel basement uniform pressure, and \( P_2 \) and \( P_3 \) are the self-weights of the soil mass on the basement on both sides of the tunnel. The calculation formula is shown as (20):

\[
P_1 = q_1 + \frac{G_z}{B}
\]

where \( q_1 \) is the pressure of the surrounding rock on the tunnel vault (kN/m\(^2\)), \( G_z \) is the gravity stress of the lining structure (kN/m), and \( B \) is the width for tunnel excavation (m).

Under the effect of uniform pressure \( P_1 \) on the tunnel foundation, we can obtain the additional compressive stress at any point of the tunnel foundation soil using Flamant’s elastic solution. As shown in Figure 5, we take
a micro-segment at $\delta$ point on axis $X$ to calculate the integral shown in Equation (21) and to obtain the additional stress value at any point $M(x, z)$ in the soil under the uniform strip load.

$$\sigma_1(z) = \frac{2\pi^2}{n} \int_0^\infty \frac{P_1}{(\delta - x)^2 + z^2} d\delta$$

$$= \frac{P_1}{\pi} \left[ \frac{(\delta - x) \cdot z}{(\delta - x)^2 + z^2} + \frac{x \cdot z}{x^2 + z^2} + \arctg \frac{\delta - x}{z} + \arctg \frac{x}{z} \right]$$

(21)

$P_1$ is the tunnel basement pressure (kPa), and $B$ is the tunnel excavation width (m).

By utilising the soil mass self-weight on the basement on both sides of the tunnel, $P_2$ and $P_3$, we can obtain the additional stresses $\sigma_2(z)$ and $\sigma_3(z)$ at any point $M(x, z)$ in the tunnel foundation soil using Flamant’s elastic solution and the same coordinate system in the calculation in Figure 5.

$$\sigma_2(z) = \frac{2\pi^2}{\pi} \int_0^\infty \frac{P_2}{(\delta - x)^2 + z^2} d\delta$$

$$= \frac{P_2}{\pi} \left[ \frac{\pi \cdot x \cdot z}{(x - B)^2 + z^2} + \arctg \frac{x - B}{z} \right]$$

(22)

$$\sigma_3(z) = \frac{2\pi^2}{n} \int_0^\infty \frac{P_3}{(\delta - x)^2 + z^2} d\delta$$

$$= \frac{P_3}{\pi} \left[ \frac{\pi \cdot z}{\pi \cdot x^2 + z^2} + \arctg \frac{\pi x}{z} \right]$$

(23)

Therefore, the vertical compressive stress at any point $M(x, z)$ in the foundation soil of a single tunnel, $\sigma_{1-3}(z)$, is as shown in Equation (24).

$$\sigma_{1-3}(z) = \sigma_1(z) + \sigma_2(z) + \sigma_3(z) + y \cdot z$$

(24)

where $y$ is the soil mass unit weight under the tunnel basement surface (kN/m$^3$).

5.2. Calculation of foundation vertical compressive stress in excavation of rear hole

Under the effect of the rear hole excavation, the surrounding rock pressure on the first hole readjusts, and the uniform pressure $P_1$ of the tunnel basement transforms into trapezoidal load $P_{1'}$. The load distribution on the tunnel basement surface is shown in Figure 6. As with the excavation of the first hole, we make a $X-Z$ coordinate system in the figure with the left arch springing off of the first hole as the centre of a circle; the horizontal direction is axis $X$, and vertical direction is axis $Z$.

In Figure 6, $P_{1'}$ is the tunnel basement trapezoidal load for the first hole, and $P_4$ is the tunnel basement load.

![Figure 6](image1.png)

**Figure 6.** Distribution of foundation vertical compressive stress during rear hole excavation.

![Figure 7](image2.png)

**Figure 7.** Stress in foundation with trapezoidal load $P_{1'}$. 
trapezoidal load for the rear hole; the calculation formulas are shown in Equations (25) and (26).

\[ P_1' = q_{ear} + \frac{G_z}{B} \]  
\[ P_4 = q_{lat} + \frac{G_z}{B} \]  

In the formula, \( q_{ear} \) and \( q_{lat} \) are the biasing effect forces on the surrounding rock of the arch ring of the first and rear holes (kN/m\(^2\)), respectively, \( G_z \) is the gravity stress of the lining structure (kN/m), and \( B \) is the excavation width (m).

(1) Calculation of first hole foundation vertical compressive stress

Under the impact of tunnel basement trapezoidal load \( P_1' \), we can also obtain the additional compressive stress at any point on the tunnel foundation soil using Flamant's elastic solution under the impact of a trapezoidal load. Taking a micro-segment at the \( \delta \) X-coordinate, we can glean \( P_1'(\delta) \), which is shown as Equations (27)–(29). Then, we complete the calculations as shown in Equation (30) to attain the additional stress value \( \sigma_1(z) \) at any point \( M(x, z) \) in the soil under the trapezoidal load of the tunnel foundation.

In Figure 7, for position \( \delta \) of the X coordinate:

\[ P_1'(\delta) = P_{min} + \frac{(P_{max} - P_{min})}{B} \cdot \delta \]  
\[ P_{min} = q_1 + \frac{G_z}{B} \]  
\[ P_{max} = q_2' + \frac{G_z}{B} \]  

where \( q_1 \) is the external pressure (kN/m\(^2\)) surrounding the rock of first hole, \( q_2' \) is the pressure (kN/m\(^2\)) from the internal surrounding rock of first hole, \( G_z \) is the gravity stress of the lining structure, and \( B \) is the tunnel excavation width (m).

\[ \sigma_1(z) = \frac{2z^2}{n} \int_0^B P_1'(\delta) \left[ \frac{(8 - x) \cdot z}{(8 - x)^2 + z^2} + \frac{x \cdot z}{x^2 + z^2} + \frac{\arctg \frac{B - x}{z}}{B} + \frac{\arctg \frac{x}{z}}{z} \right] d\delta = \frac{P_{min}}{n} \] 

\[ + \frac{(P_{max} - P_{min})}{n} \int_0^B \left[ \frac{(8 - x) \cdot z}{(8 - x)^2 + z^2} + \frac{x \cdot z}{x^2 + z^2} + \frac{\arctg \frac{B - x}{z}}{B} + \frac{\arctg \frac{x}{z}}{z} \right] d\delta \]  

where \( P_{max} \) and \( P_{min} \) are the basal stresses inside and outside of first hole, respectively, and \( B \) is the of tunnel excavation width (m).

Under the effect of \( P_1' \) and by using Flamant's elastic solution, we can obtain the additional stress \( \sigma_2(z) \) of any position \( M(x, z) \) in the foundation, as shown in Equation (31).

\[ \sigma_2(z) = \frac{2z^2}{n} \int_0^B P_2 \left[ \frac{(8 - x) \cdot z}{(8 - x)^2 + z^2} + \frac{x \cdot z}{x^2 + z^2} + \frac{\arctg \frac{B - x}{z}}{B} + \frac{\arctg \frac{x}{z}}{z} \right] d\delta \]  
\[ = \frac{P_2}{n} \int_0^B \left[ \frac{(x - B) \cdot z}{(x - B)^2 + z^2} + \frac{(B + D - x) \cdot z}{(B + D - x)^2 + z^2} + \frac{x - B}{z} + \frac{B + D - x}{z} \right] d\delta \]  

In the same coordinate system, \( \sigma_3(z) \) under the effect of upper soil gravity \( P_3 \) in the tunnel base is the same as that of Equation (23), which is not expounded here.

(5) Foundation vertical compressive expression of the rear hole

To facilitate an integral solution, according to the symmetry of the existing tunnel structure, we constructed a \( Y-Z \) coordinate system, with the right arch springing off of the rear hole as the centre; the horizontal direction is axis \( Y \), and the vertical direction is axis \( Z \) to calculate the.
foundation compressive stress under the effect of loads $P_d$ and $P_3$ on this coordinate system. Based on the calculus method, additional stresses $\sigma_4(z)$ and $\sigma_5(z)$ will be obtained.

As shown in Figure 8, under the effect of trapezoidal load $P_d$ of the base in the rear hole, we utilised Flamant’s elastic solution to calculate additional stress $\sigma_3'(z)$ for any position of the foundation soil.

According to the coordinates in Figure 8, for the $Y$ coordinate of position $\delta$:

$$P_d(\delta) = P_{\text{min}} + \left( \frac{P_{\text{max}} - P_{\text{min}}}{B} \right) \cdot \delta$$  \hspace{1cm} (32)

$$P_{\text{min}} = q_1 + \frac{G_z}{B}$$ \hspace{1cm} (33)

$$P_{\text{max}}' = q_2 + \frac{G_z}{B}$$ \hspace{1cm} (34)

where $q_1$ is the pressure (kN/m²) from the external surrounding rock of rear hole, $q_2$ is the pressure (kN/m²) from the internal surrounding rock of the rear hole, $G_z$ is the gravity stress of the lining structure, and $B$ is the tunnel excavation width (m).

Hence, the additional compressive stress in any position $M(y, z)$ in trapezoidal load of base is:

$$\sigma_4(z) = \frac{2z^2}{\pi} \int_0^B \frac{P_d(\delta)}{(\delta - y)^2 + z^2} \cdot \delta d\delta = \frac{P_{\text{min}}}{\pi} \left[ \frac{(B - y) \cdot z}{(B - y)^2 + z^2} + \frac{y \cdot z}{y^2 + z^2} + \frac{B - y \cdot \arctg \frac{y}{z}}{z} + \frac{B \cdot \arctg \frac{y}{z}}{z} \right]$$

$$+ \frac{P_{\text{max}} - P_{\text{min}}}{\pi} \left[ \frac{(B - y) \cdot z}{(B - y)^2 + z^2} + \frac{y \cdot z}{y^2 + z^2} + \frac{B - y \cdot \arctg \frac{y}{z}}{z} + \frac{B \cdot \arctg \frac{y}{z}}{z} \right]$$

$$\sigma_5'(z) = \frac{2z^2}{\pi} \int_0^B \frac{P_d(\delta)}{(\delta - y)^2 + z^2} \cdot \delta d\delta$$

$$= \frac{P_{\text{min}}}{\pi} \left[ \frac{(B - y) \cdot z}{(B - y)^2 + z^2} + \frac{y \cdot z}{y^2 + z^2} + \frac{B - y \cdot \arctg \frac{y}{z}}{z} + \frac{B \cdot \arctg \frac{y}{z}}{z} \right]$$

where $P_{\text{max}}$ and $P_{\text{min}}$ are the basal stresses (kPa) inside and outside of first hole, respectively, and $B$ is the tunnel excavation width (m).

Under the effect of upper soil gravity $P_3$ in the tunnel base, by using Flamant’s elastic solution, we can obtain the additional stress $\sigma_3'(z)$ of any position $M(x, z)$ in the foundation, as shown in Equation (37).

$$\sigma_3'(z) = \frac{2z^2}{\pi} \int_0^B \frac{P_d(\delta)}{(\delta - y)^2 + z^2} \cdot \delta d\delta = \frac{P_{\text{min}}}{\pi} \left[ \frac{(B - y) \cdot z}{(B - y)^2 + z^2} + \frac{y \cdot z}{y^2 + z^2} + \frac{B - y \cdot \arctg \frac{y}{z}}{z} + \frac{B \cdot \arctg \frac{y}{z}}{z} \right]$$

To facilitate the description, Equations (35) and (36) can be conversed by coordinate to make $y = (2B + D) - x$, by representing $\sigma_4(z)$ and $\sigma_5$ in the $X-Z$ coordinate system of Figure 7:

$$\sigma_4(z) = \frac{P_{\text{min}}}{\pi} \left[ \frac{(x - B - D) \cdot z}{(x - B - D)^2 + z^2} + \frac{(2B + D - x) \cdot z}{(2B + D - x)^2 + z^2} + \frac{x \cdot \arctg \frac{y}{z}}{z} + \frac{x \cdot \arctg \frac{y}{z}}{z} \right]$$

$$+ \frac{P_{\text{max}} - P_{\text{min}}}{\pi} \left[ \frac{(x - B - D) \cdot z}{(x - B - D)^2 + z^2} + \frac{(2B + D - x) \cdot z}{(2B + D - x)^2 + z^2} + \frac{x \cdot \arctg \frac{y}{z}}{z} + \frac{x \cdot \arctg \frac{y}{z}}{z} \right]$$

$$\sigma_5'(z) = \frac{P_{\text{min}}}{\pi} \left[ \frac{(2B + D - x) \cdot z}{(2B + D - x)^2 + z^2} + \frac{B \cdot \arctg \frac{y}{z}}{z} \right]$$

$$\sigma_5'(z) = \frac{P_{\text{min}}}{\pi} \left[ \frac{(2B + D - x) \cdot z}{(2B + D - x)^2 + z^2} + \frac{B \cdot \arctg \frac{y}{z}}{z} \right]$$  \hspace{1cm} (37)

Figure 9. Schematic diagram of the engineering case.
Hence, the vertical compressive stress of any position, \( \sigma(x, z) \), of foundation soil during excavation of the rear hole, \( \sigma_{c}(z) \), is shown as Equation (39):

\[
\sigma_{c}(z) = \sigma_{1}(z) + \sigma_{2}(z) + \sigma_{3}(z) + \sigma_{4}(z) + y \cdot z
\]

(39)

where \( y \) is the soil mass gravity (kN/m\(^3\)) below the tunnel base.

Compared with the calculation formula of vertical compressive stress of a single-hole tunnel foundation during excavation of the first hole, the calculation formula of vertical compressive stress of a double-hole tunnel foundation during excavation of the rear hole is more complicated. The stress value can be calculated by means of computer; by inputting different parameters, the vertical compressive stress of the point can be accurately calculated.

6. Case study

6.1. Project overview

In this study, we choose a typical section, as shown in Figure 9, in "East Hangtian Road to Shouzhou Avenue" of the Xi’an Metro Line (No. 4), to analyse the distribution characteristics of the foundation vertical compressive stress after metro tunnel excavation. The upper layer of the section is mainly new loess \( Q_{3} \) with collapsibility, and the lower layer is predominately old loess \( Q_{2} \). The specific soil property parameters are shown in Table 1. The circular shield is adopted to excavate the running tunnel. While the burial depth of tunnel is 10 m, and the hole diameter is 6 m, the corresponding thickness of the collapsible soil layer under the tunnel base is 6 m. The clear distance of the shallow neighbourhood loess tunnel is 6.6 m, and the construction spacing between the left and right tunnel is 19.5 m.

### Table 1. Physical and mechanical parameters of natural soil in research section of line four of Xi’an metro.

| Soil layer | Cohesive force \( c \) (kPa) | Friction angle \( \varphi \) (°) | Bulk modulus \( K \) (MPa) | Shear modulus \( G \) (MPa) | Soil weight \( \gamma \) (kN/m\(^3\)) |
|---|---|---|---|---|---|
| \( Q_{1} \) | 30 | 21 | 35.46 | 9.76 | 16.8 |
| \( Q_{2} \) | 37 | 22 | 33.04 | 12.67 | 16.9 |

6.2. Unloading rate of foundation vertical compressive stress

We calculate the downward foundation vertical compressive stress along a perpendicular line in the arch spring and inverted arch centre, contrasting and analysing the result with the gravity stress on the site. To contrast and analyse the relationship between foundation vertical compressive stress and gravity stress on the site with foundation depth, we assume that the gravity stress on certain positions of the site below the foundation before excavation is \( \sigma_{G} \), and the foundation vertical compressive stress is \( \sigma_{c} \). Then, the formula of unloading rate \( k \) of vertical compressive stress for a certain position depth below the foundation is:

\[
k = \left( 1 - \frac{\sigma_{c}}{\sigma_{G}} \right) \times 100\%
\]

(40)

6.3. Calculation result for excavation of first hole

We calculate Equation (24) with the calculation parameters and obtain the foundation vertical compressive stress \( \sigma_{c} \) and unloading rate \( k \) with the change in foundation depth \( z \) below the basal surface, as shown in Figure 10. We can see that:

1. Excavation leads to a reduction in foundation vertical compressive stress below the tunnel base. When the depth \( z \) of the tunnel foundation is
0–15 m, the reduction in vertical compressive stress is greater than that of the gravity stress on the site.

(2) When foundation depth z remains unchanged, the foundation vertical compressive stress below the inverted arch centre is the minimum value. With an increase in tunnel foundation depth z, the difference between the arch spring and foundation vertical compressive stress below the inverted arch centre reduces and eventually shows the same variation trend.

(3) The variation ranges of unloading rate k of the foundation vertical compressive stress below the tunnel arch spring and inverted arch centre are 0–23% and 0–44%, respectively. With an increase in foundation depth z, k gradually reduces in “concave type”, which means that the extent of the reduction tends to be mild. The final unloading rate k tends to approach 0, which means that at a certain depth of tunnel foundation, the vertical compressive stress tends to approach the gravity stress on the site.

The results show that the “unloading effect” below the tunnel inverted arch centre during excavation is the most significant. With an increase in tunnel foundation depth z, the “unloading effect” diminishes.

6.4. Calculation result for excavation of rear hole

Pressure from the surrounding rock in the rear hole is calculated according to Equations (12)–(19), and the foundation vertical compressive stress is calculated according to Equation (39), in which the lining parameters are: gravity \( \gamma_{\text{lining}} = 25 \text{kN/m}^3 \) and average thickness is 50 cm. Through this calculation and under natural soil parameters, the tunnel base pressure range is \( P_{\text{min}} = 142 \text{kPa} \), \( P_{\text{max}} = 159.3 \text{kPa} \), \( P_{\text{max}}' = 152 \text{kPa} \); the upper soil gravity on the two sides of tunnel is \( P_2' = P_3 = P_3' = 268 \text{.8 kPa} \).

According to different values of \( x \), we can obtain the foundation vertical compressive stresses of shallow neighbourhood tunnels at different points. The vertical compressive stresses of various control points are listed in Figure 11.

By taking \( x = 0, B/2, B, (B + D), (3B/2 + D), \) and \( (2B+D) \) into the equations and then obtaining the calculation results of vertical compressive stress \( \sigma_{d-c} \) and the unloading rate k of the foundations with varying

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**Figure 11.** The analysis diagram of the foundation vertical compressive stresses at different points of neighbourhood loess tunnel basement.

**Figure 12.** The variation curve of the unloading rate of the foundation vertical compressive stress of tunnel with the foundation depth during double-hole excavation.
tunnel foundation depth $z$ at points $a$, $b$, $c$, $d$, $e$, and $f$, as shown in Figure 12, it can be seen that:

1. During the excavation of the rear hole, the foundation vertical compressive stresses under the tunnel basement are asymmetric, and the foundation vertical compressive stress under the inverted arch centre is smaller than that of the arch foot. With an increase in foundation depth $z$, the difference in foundation vertical compressive stresses under the tunnel basement grows smaller and eventually approaches 0.

2. The foundation vertical compressive stress of the first hole is slightly greater than that of the rear hole, which is consistent with the calculation law of the vertical pressure at the vault shallow neighbourhood loess tunnel from surrounding rock.

3. When the tunnel foundation depth $z$ is in the range of 0–20 m, the vertical compressive stress of the foundation soil reduces significantly compared with the self-weight stress on the site. Compared with the case where only the first hole is excavated, the double-hole excavation causes a greater influence of the “unloading effect” due to the mutual influence between the left and right holes.

4. The unloading rate $k$ of the foundation vertical compressive stress under the arch foot and inverted arch centre vary from 0% to 22% and from 0% to 43%, respectively. Like the excavation of the first hole, the value of $k$ gradually decreases in a “concave type” trend with an increase in foundation depth $z$, and eventually the foundation vertical compressive stress approaches the self-weight stress on the site.

5. When the foundation depth $z$ is 0–5 m, the foundation unloading rate $k$ under the tunnel inner arch foot is the smallest. When the foundation depth $z$ is greater than 5 m, the unloading rate $k$ increases with an increase in the depth $z$ and gradually grows larger than that of the lateral arch foot and finally that of the inverted arch centre. The “unloading effect” on the tunnel inner arch foot tends to be the largest. The reason is that there is a “biasing effect” on the surrounding rock pressure inside the tunnel vault during the excavation of the rear hole (the surrounding rock pressure in the inner vault is greater than that of the lateral vault and inverted arch centre). Therefore, when the foundation depth $z$ ranges between 0 and 5 m, the foundation vertical compressive stress under the inner arch foot is the largest, and the unloading rate $k$ is the smallest. With the increase in the foundation depth $z$, the foundation vertical compressive stress under the

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**Figure 13.** Numerical simulation model.

**Table 2.** Elements parameters for numerical simulation.

| Element         | Cohesive force $c$ (kPa) | Friction angle $\phi$ (°) | Poisson's ratio $\mu$ | Elastic modulus $E$ (MPa) | Bulk modulus $K$ (MPa) | Shear modulus $G$ (MPa) | Soil weight $\gamma$ (kN/m$^3$) |
|-----------------|--------------------------|--------------------------|-----------------------|---------------------------|------------------------|--------------------------|---------------------------------|
| $Q_1$ Loess     | 30                       | 21                       | -                     | -                         | 25.46                  | 9.76                     | 16.8                            |
| $Q_2$ Loess     | 37                       | 22                       | -                     | -                         | 33.04                  | 12.67                    | 16.9                            |
| Equivalent zone| -                        | -                        | 0.2                   | 43.9                      | 24.40                  | 18.70                    | 20                              |
| Shied shell     | -                        | -                        | 0.27                  | $2 \times 10^3$           | -                      | -                        | 78.5                            |
| Lining          | -                        | -                        | 0.17                  | $2.8 \times 10^3$         | -                      | -                        | 25                              |
Table 5. Back tunnel foundation vertical compressive stress when dual-hole digging was applied.

| Depth (m) | Analytical results | Simulated results | Relative error |
|-----------|--------------------|-------------------|----------------|
|           | Left arch foot     | Inverted-arch centre | Right arch foot |
| 1         | 226.7              | 255.6             | 12.7%          |
| 2         | 243.3              | 260.3             | 7.0%           |
| 3         | 260.4              | 268.9             | 3.3%           |
| 4         | 277.8              | 289.8             | 4.3%           |
| 5         | 295.5              | 319.5             | 8.1%           |
| 6         | 313.5              | 336.1             | 7.2%           |
| 7         | 333.2              | 354.5             | 6.7%           |
| 10        | 387.2              | 405.7             | 4.8%           |
| 15        | 479.0              | 503.0             | 5.0%           |
| 20        | 570.0              | 578.6             | 1.5%           |
| 25        | 659.8              | 664.2             | 0.7%           |
| 35        | 836.2              | 837.1             | 0.1%           |
| 45        | 1009.9             | 1008.2            | 0.2%           |

Table 4. Front tunnel foundation vertical compressive stress when dual-hole digging was applied.

| Depth (m) | Analytical results | Simulated results | Relative error |
|-----------|--------------------|-------------------|----------------|
|           | Left arch foot     | Inverted-arch centre | Right arch foot |
| 1         | 223.2              | 254.5             | 14.0%          |
| 2         | 241.5              | 259.0             | 7.3%           |
| 3         | 260.4              | 277.4             | 6.5%           |
| 4         | 279.8              | 294.0             | 5.1%           |
| 5         | 299.4              | 319.4             | 6.7%           |
| 6         | 319.1              | 332.6             | 4.2%           |
| 7         | 339.3              | 356.1             | 5.0%           |
| 10        | 396.5              | 406.7             | 2.6%           |
| 15        | 487.6              | 505.2             | 3.6%           |
| 20        | 576.4              | 580.5             | 0.7%           |
| 25        | 664.2              | 665.3             | 0.2%           |
| 35        | 838.4              | 837.8             | 0.1%           |
| 45        | 1011.2             | 1008.6            | 0.3%           |

Table 3. Tunnel foundation vertical compressive stress when single-hole digging was applied.

| Depth (m) | Analytical results | Simulated results | Relative error |
|-----------|--------------------|-------------------|----------------|
|           | Arch foot          | Inverted-arch centre |
| 1         | 222.3              | 242.5             | 9.1%           |
| 2         | 239.9              | 264.9             | 10.4%          |
| 3         | 258.4              | 284.3             | 10.0%          |
| 4         | 277.7              | 296.2             | 6.7%           |
| 5         | 297.6              | 328.1             | 10.3%          |
| 6         | 317.7              | 340.7             | 7.2%           |
| 7         | 338.6              | 365.1             | 7.8%           |
| 10        | 398.2              | 415.4             | 4.3%           |
| 15        | 493.0              | 509.0             | 3.3%           |
| 20        | 583.9              | 595.2             | 1.9%           |
| 25        | 672.6              | 661.0             | 1.7%           |
| 35        | 846.7              | 833.6             | 1.6%           |
| 45        | 1018.7             | -1004.3           | 1.4%           |

inner arch foot is influenced less by the “biasing effect”. The foundation additional stress caused by the middle rock pillar is smaller than that of the outer soil layer above the tunnel base surface, which results in a smaller vertical compressive stress for the foundation soil under the inner arch foot near the middle rock pillar, or the gradual increase in the unloading rate $k$.

6.5. Reliability of the calculations

Due to the lack of measured data of foundation compressive stress on this site, it is necessary to test the reliability of the proposed formulas in this paper through further calculations. We use the results from the continuous medium model preformed via FLAC 3D to verify the reliability of the proposed theoretical method. The size of the model (Figure 13) is $90 \text{ m} \times 80 \text{ m} \times 30 \text{ m} (\text{Length} \times \text{Width} \times \text{Thickness})$. The pressure from the soil chamber on the tunnel face top is 0.12MPa, and the gradient from the top of the tunnel face to the bottom is 0.01MPa/m. The grouting pressure is 0.3 MPa considering the rear situation on the construction of Xi’an metro. (Fei, Yanpeng, and Changfei et al. 2013), the gap between the shield tail and rock.
mass is simulated by equivalent zone. The rock-mass and equivalent zone are simulated by solid element. The shield shell and segment lining are simulated by shell element. The parameters of different part are listed in Table 2.

The comparison results of proposed theoretical method and numerical simulation are shown in Tables 3–5.

As is shown in Tables 3–5, the vertical compressive stress of the foundation with a small clearance tunnel, compared with values obtained from our simulation model (formula), has a deviation range of 0–30 kPa. The results from the two methods show a variation of 0–15%, which is mathematically sound. Moreover, as the depth of the tunnel foundation increases, the variation decreases. In the two calculation models, the co-changing trend corresponds to several aspects, including tunnel arch on the exterior and the interior and the foundation vertical compressive stress centred below the arch. For the tunnel foundation compressive stress, the results from the analytical and modelling methods appear to have a significant variation on the surface level and a minimum variation on the deep level.

7. Conclusions

(1) The net distance of the tunnel greatly influences the depth division between deep and shallow-buried neighbourhood tunnels. There are different calculation expressions for the boundary depth with different net distances.

(2) Based on the mutual influence between the construction of the left and right holes of a neighbourhood tunnel, this study offers a different calculation method for how the boundary depth of deep and shallow-buried tunnels to neighbourhood loess tunnels vary in net distance.

(3) On the basis of the Flamant elasticity solution, this paper deduces the analytical formula of foundation vertical compressive stress of the shallow neighbourhood loess tunnel, which provides a theoretical formula for determining the vertical compressive stress of a sample in the laboratory using a loess hydrocousolidation test.

(4) The “unloading effect” under the inverted arch centre is the most significant during tunnel excavation. With an increase in the foundation depth z, the “unloading effect” caused by tunnel excavation will decrease, and the foundation vertical compressive stress of the tunnel will approach the self-weight stress on the site.

(5) During the excavation of the rear hole, due to the mutual influence between the left and right holes of the neighbourhood tunnel, the foundation vertical compressive stress under the tunnel base is asymmetric, and the foundation vertical compressive stress of the first hole is slightly larger than that of the rear hole. Compared with the single-hole tunnel, the “unloading effect” caused by the neighbourhood tunnel excavation has a greater influence.

(6) When comparing the model calculations, the formula proposed in this study has a minor variation of 15%, which proves the reliability of our proposed analytical formula and its application in the construction of shallow neighborhood loess tunnels.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

This work was supported by the National Natural Science Foundation of China [51678062,51878060]; Fundamental Research Funds for the Central Universities, CHD [300102 210124].

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