Disentangling the hadronic molecule nature of the $P_c(4380)$
pentaquark-like structure

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Abstract

We demonstrate that the relative ratio of the decays of hidden charm pentaquark-like structure $P_c^+(4380)$ to $D^*\Lambda_c^+$ and $J/\psi p$ are very different for $P_c^+$ being $D\Sigma_0^*(2520)$ or $D^*\Sigma_c(2455)$ molecule states. While the partial width of the $D\Sigma_0^*(2520)$ molecule to the $D^*\Lambda_c^+$ is much larger, by one order of magnitude, than that to the $J/\psi p$, the $D^*\Sigma_c(2455)$ molecule shows a different pattern. Our analysis shows that the $D\Sigma_0^*$ bound state ansatz is more reasonable than the $D^*\Sigma_c$ one to explain the broad $P_c(4380)$ structure. We suggest to search for the $P_c(4380)$ in the $D^*\Lambda_c^+$ system, which can be used to disentangle the nature of the $P_c^+(4380)$ structure.
1 Introduction

Exploration of the exotic baryons that have more than three constituent quarks is an important issue in hadron physics. Recently, observation of two hidden-charm pentaquark-like structures $P_c^+(4380)$ and $P_c^+(4450)$ in the $J/\psi p$ invariant mass distribution in the process of $\Lambda_c^0 \rightarrow J/\psi pK^-$ decay was reported by the LHCb Collaboration [1]. The values of the masses and widths from the fit with the Breit–Wigner parameterization are $M_{P_c(4380)} = (4380 \pm 8 \pm 29)$ MeV, $\Gamma_{P_c(4380)} = (205 \pm 18 \pm 86)$ MeV, $M_{P_c(4450)} = (4449.8 \pm 1.7 \pm 2.5)$ MeV, and $\Gamma_{P_c(4450)} = (39 \pm 5 \pm 19)$ MeV, with spin-parity $J^P$ being either 3/2$^+$ or 5/2$. Possible existence of such pentaquark states with hidden charm has already been predicted [2, 3, 4, 5, 6] prior to the experimental observation. In the earliest prediction [2], a $D^*\Sigma_c(2455)$ S-wave bound state with $J^P = 3/2^-$ was predicted to be around 4412 MeV with $J/\psi N$ as its largest decay mode, in the framework of the meson-baryon coupled channel unitary approach with the local hidden gauge formalism. In this approach, the $t$-channel vector meson exchange dominance is assumed for the $D^*\Sigma_c$ interaction. Taking into account of other meson exchanges, the mass of the predicted $D^*\Sigma_c$ S-wave bound state could be shifted by ±40 MeV [4]. Considering coupled channel effects with $D\Sigma_c^*$ and $D^*\Sigma_c^*$ channels, three $J^P = 3/2^-$ pentaquark states were predicted to be around 4334 MeV, 4417 MeV and 4481 MeV, mainly coupled to $D\Sigma_c^*$, $D^*\Sigma_c$ and $D^*\Sigma_c^*$, respectively [5]. Therefore both $P_c^+(4380)$ and $P_c^+(4450)$ could be the predicted $D\Sigma_c^*$ and $D^*\Sigma_c$ states. The predicted masses for genuine pentaquark states with both negative and positive parity [3] suffer large model dependence, but also cover the observed masses of the two $P_c^+$ structures.

After the observation of the two $P_c^+$ structures, many theoretical works have been triggered, see for example, Refs. [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21], proposing various explanations for these structures. Among them, it was suggested that the observed structures could be due to kinematical triangle singularities [14, 16], the possibility of which needs to be examined by future experiments. If these two $P_c^+$ structures correspond to two particle states, since they sit close to the mass thresholds of the $D\Sigma_c^*$ and $D^*\Sigma_c$ at 4387 MeV and 4461 MeV, respectively, a popular explanation for them is still either S-wave $D\Sigma_c^*(2520)$ or $D^*\Sigma_c(2455)$ molecular states [7, 8, 9, 10, 11, 12, 13] with $J^P = \frac{3}{2}^-$, roughly consistent with previous predictions [2, 4, 5] but with parameters tuned to reproduce the observed masses of $P_c^+$ structures. However, the observed decay width of the $P_c^+(4380)$ state is about a few times larger than the predicted one [2, 5]. Also the LHCb experiment claims that the two states have opposite parity, which is against that both states are S-wave molecules of $D^*(\Sigma_c^*)$ to have spin-parity of 3/2$^-$. 

In this work, we want to make an estimate of the partial decay widths of the $P_c(4380)$ into the $D^*\Lambda_c^+$ and $J/\psi p$ assuming it be a hadronic molecular state. We will point out that the previous prediction [5] for the decay width of $D\Sigma_c^*(2520)$ bound state underestimate the contribution of the $D^*\Lambda_c^+$ decay by more than an order of magnitude due to its assumption of vector meson exchange dominance. Rather than the $J/\psi p$ mode, the dominant decay mode for a $D\Sigma_c^*(2520)$ bound state should be $D^*\Lambda_c^+$ due to t-channel pion exchange. We demonstrate that the relative ratio of the decays of hidden charm $P_c^+$ pentaquark states to $D^*\Lambda_c^+$ and $J/\psi p$ are very different for $P_c^+$ to be $D\Sigma_c^*(2520)$ or $D^*\Sigma_c(2455)$ molecular states. While the $D\Sigma_c^*(2520)$ molecule decays dominantly into the $D^*\Lambda_c^+$, the $D^*\Sigma_c(2455)$ molecule has a larger branching fraction for the decay into the $J/\psi p$. Therefore, were the $P_c$ structures hadronic molecular states, future measurement of this ratio can help us to pin down their nature. The unexpected large decay width of the $P_c^+(4380)$ can get a natural explanation if it is a $D\Sigma_c^*(2520)$ molecule.

This article is arranged as follows. In the next section, we present the theoretical framework of our calculation. In Sect. 3, the numerical results and some discussions are presented.

2 Theoretical framework

Among the two observed structures, the existence of the narrow $P_c(4450)$, no matter what it is, is affirmative from the data for the $J/\psi p$ invariant mass distribution. However, introducing the second structure, the $P_c(4380)$, does not seem that necessary since there is still a discrepancy
between the best fit with two $P_c$ structures and the data in the right shoulder of the peak in the $J/\psi p$ invariant mass distribution. Yet, a recent phenomenological analysis of the data affirms the necessity of introducing the $P_c(4380)$ [22]. The nominal mass of the $P_c(4380)$ is just 7 MeV below the $D\Sigma_c^*$ threshold and 81 MeV below $D^*\Lambda_c$ threshold. It seems more natural to be a $D\Sigma_c^*$ dominant molecule [5, 11, 21]. However the possibility to be a deeply bounded $D^*\Sigma_c$ state cannot be excluded [7]. Here, we assume the $P_c^+(4380)$ exists with the properties reported by the LHCb Collaboration, and study its decays to the two final states $D^*\Lambda_c^0$ and $J/\psi p$ with the assumption that it is a bound state of $D\Sigma_c^*(2520)$ (type I) or $D^*\Sigma_c(2455)$ (type II). These two decays can proceed through triangular diagrams as shown in Fig. 1. Since we only aim at making a rough estimate, which is sufficient for the conclusion, we only consider the exchange of lightest possible mesons. This means that we will consider the one-pion-exchange, as well as the one-rho-exchange in previous work [2], between the charmed baryons and anti-charmed mesons, and the exchange of ground state pseudoscalar and vector charmed mesons, which are related to each other via heavy quark spin symmetry, for the decays into the $J/\psi p$.

Figure 1: Diagrams representing the decays of the $P_c^+(4380)$ state to $D^*\Lambda_c^0$ and $J/\psi p$ as $D\Sigma_c^*(2520)$ molecule (a-d) or $D^*\Sigma_c$ molecule (e-h).

In order to evaluate the decay amplitudes of the diagrams shown in Fig. 1, we need the structure of the involved interaction vertices which can be described by means of the following effective Lagrangian [23, 24],

\[
\mathcal{L}_{PPV} = g_{PPV} \phi_P(x) \partial_{\mu} \phi_P(x) \phi_V^\mu(x),
\]

\[
\mathcal{L}_{VVV} = g_{VVV} \partial_{\mu} \phi_V(x) \partial^\mu \phi_V(x) \phi_P(x),
\]

\[
\mathcal{L}_{VVV} = g_{VVV} i \left[ \partial_{\mu} \partial_{\nu} \phi_V(x) - \partial_{\mu} \phi_V(x) \partial_{\nu} \phi_V(x) \right] \phi_V^\mu(x),
\]

\[
\mathcal{L}_{BBP} = g_{BBP} \left[ \tilde{\psi}_B(x) \gamma_{\mu} \psi_B(x) + \tilde{\psi}_B(x) \psi_B(x) \right] \partial_{\mu} \phi_P(x),
\]

\[
\mathcal{L}_{BBV} = g_{BBV} \left[ \tilde{\psi}_B(x) \gamma_{\mu} \psi_B(x) \phi_V^\mu(x) + 2 f_{BBV} \tilde{\psi}_B(x) \sigma_{\mu \nu} \psi_B(x) \left( \partial_{\mu} \phi_V^\nu(x) - \partial_{\nu} \phi_V^\mu(x) \right) \right],
\]

where $P, V, B, B^*$ denote pseudoscalar, vector meson, octet and decuplet baryon, respectively. The coupling constants $g_{DD^*\pi}, g_{\Sigma_c\Lambda_c\pi}$ and $g_{\Sigma_c^*\Lambda_c\pi}$ can be determined from the experimental data of the
decay widths of the \( D^* \), \( \Sigma_c \) and \( \Sigma_{c}^* \), respectively. The extracted values of \( g_{\Sigma_c \Lambda_c \pi} \) and \( g_{\Sigma_c^* \Lambda_c \pi} \) fulfill very well the relation predicted by heavy quark spin symmetry (HQSS). The coupling constant \( g_{D^* D^* \pi} \) can be related to the value of \( g_{DD \pi} \) by heavy quark spin symmetry. The other coupling constants cannot be measured directly. Since we only aim at making a rough estimate of the partial decay widths, we take model values \([25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37]\) for them which are listed in Table 1.

Table 1: The values of coupling constants involved in the calculation \([25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37]\).

| Coupling constant | Value | Coupling constant | Value |
|-------------------|-------|-------------------|-------|
| \( g_{DD^* \pi} \) | 6.3   | \( g_{DDJ/\psi} \) | 7.4   |
| \( g_{DD^* \rho} \) | 2.8 GeV\(^{-1}\) | \( g_{DD^* J/\psi} \) | 2.5 GeV\(^{-1}\) |
| \( g_{D^* D^* \rho} \) | 6.3 GeV\(^{-1}\) | \( g_{D^* D^* J/\psi} \) | 8.0 GeV\(^{-1}\) |
| \( g_{\Sigma_c \Lambda_c \pi} \) | 7.4 GeV\(^{-1}\) | \( g_{\Sigma_c^* \Sigma_c \pi} \) | 6.5 GeV\(^{-1}\) |
| \( g_{\Sigma_c \Lambda_c \rho} \) | 10.0 GeV\(^{-1}\) | \( g_{\Sigma_c^* \Sigma_c \rho} \) | 2.9 GeV\(^{-1}\) |
| \( g_{\Sigma_c \Lambda_c \pi} \) | 9.3 | \( g_{\Sigma_c \Sigma_c \pi} \) | 2.7 |
| \( g_{\Sigma_c \Lambda_c \rho} \) | 0.4 | \( g_{\Sigma_c^* \Sigma_c \rho} \) | 3.0 |
| \( f_{\Sigma_c \Lambda_c} \) | 8.1 GeV\(^{-1}\) | \( f_{\Sigma_c^* \Sigma_c} \) | 6.0 GeV\(^{-1}\) |

While for those interaction vertices including the spin-3/2 \( P_c(4380) \) state, we use the Lorentz covariant orbital-spin \((L-S)\) scheme as illustrated in Ref. \([38]\). With this scheme, we can easily write down the effective Lagrangians as

\[
\mathcal{L}_{P_c((\frac{3}{2}^-)_{\Sigma_c \bar{D}^*})} = g_{P_c \Sigma_c \bar{D}^*} \bar{\Sigma}_c P_{\mu} \bar{D}^{*\mu} + \text{H.c.}, \tag{8}
\]

\[
\mathcal{L}_{P_c((\frac{1}{2}^-)_{\Sigma_c^* \bar{D}^*})} = g_{P_c \Sigma_c^* \bar{D}} \bar{\Sigma}_{c}^{*\mu} P_{\mu} \bar{D} + \text{H.c.}, \tag{9}
\]

where \( P_c \) is the pentaquark fields with \( J^P = 3/2^- \). Here we have assumed that the \( P_c \) is an \( S\)-wave hadronic molecular state of either \( \bar{D}^* \Sigma_c \) or \( \bar{D} \Sigma_{c}^* \). When we are only interested in the ratio between the partial widths to \( \bar{D} \Lambda_c \) and \( J/\psi \) of a given hadronic molecule structure, either \( \bar{D}^* \Sigma_c \) or \( \bar{D} \Sigma_{c}^* \), the coupling constant gets cancelled.

Combining the Lagrangians and propagators given above together, we can easily get the decay amplitudes for the process shown in Fig. 1, and the expressions are given in Appendix A.

The loop integrals in the amplitudes are ultraviolet (UV) divergent, which means that we need counterterms to absorb the divergence. Here, in order to be able to make an estimate we will neglect the counterterms and simply use a Gaussian regulator with the cutoff taking values in a large range. For the explicit form of the regulator, we take the one used in Refs. \([39, 40, 41]\):

\[
\Phi_{P_c}(q_E^2 / \Lambda^2) \equiv \exp(-q_E^2 / \Lambda^2), \tag{10}
\]

where \( q_E \) is the Euclidean Jacobi momentum.

The partial decay width of the two-body decay of the \( P_c(4380) \) state in its rest frame is given by

\[
d\Gamma = \frac{1}{32\pi^2} |\mathcal{M}|^2 |\mathbf{p}_2| M^{-2} d\Omega, \tag{11}\]

where \( M \) is the mass of the \( P_c(4380) \), while \( \mathbf{p}_2 \) is the \( \Lambda_c^* \) (or \( p \)) three-momentum in the rest frame of the \( P_c(4380) \). The averaged squared amplitude \( |\mathcal{M}|^2 \) can be obtained from

\[
\overline{|\mathcal{M}|^2} = \frac{1}{4} \sum_{s_{\Lambda_c^*}, s_{D^*}} |\mathcal{M}|^2, \tag{12}\]
for the $P_c(4380) \to \bar{D}^* \Lambda_c$ decay, with

$$\mathcal{M} = \mathcal{M}_a + \mathcal{M}_c, \quad \text{for type I,}$$

$$\mathcal{M} = \mathcal{M}_c + \mathcal{M}_g, \quad \text{for type II,}$$

and

$$|\mathcal{M}|^2 = \frac{1}{4} \sum_{s_P, s_p, s_{J/\psi}} |\mathcal{M}|^2,$$

for $P_c(4380) \to J/\psi p$ decay, with

$$\mathcal{M} = \mathcal{M}_b + \mathcal{M}_d, \quad \text{for type I,}$$

$$\mathcal{M} = \mathcal{M}_f + \mathcal{M}_h, \quad \text{for type II,}$$

where $\mathcal{M}_{a,b,c,d,e,f,g,h}$ are given in Appendix A.

### 3 Results and discussions

The partial decay width is proportional to $g_{P_c \Sigma_c \bar{D}}^2$ and $g_{P_c \Sigma_c \bar{D}^*}$ for the cases of type I and type II, respectively, which are canceled in the calculation of the ratio $R$ defined as

$$R_I = \frac{\Gamma(P_c(4380) \to \bar{D} \Sigma_c^* \to \bar{D}^* \Lambda_c)}{\Gamma(P_c(4380) \to \bar{D} \Sigma_c^* \to J/\psi p)},$$

for the case of type I and

$$R_{II} = \frac{\Gamma(P_c(4380) \to \bar{D}^* \Sigma_c \to \bar{D}^* \Lambda_c)}{\Gamma(P_c(4380) \to \bar{D}^* \Sigma_c \to J/\psi p)},$$

for the case of type II. Using the values of the coupling constants given in Table 1, we show diagrammatically the $\Lambda$ dependence of $R$ for the case of type I and type II in Figs. 2.

![Figure 2: The $\Lambda$ dependence of $R$ for the cases of type I and II.](image)

One sees that the dependence of both ratios on the cutoff is rather weak. The partial decay width of the $P_c$ into the $\Lambda_c \bar{D}^*$ is much larger than that into the $J/\psi p$ for the type I hadronic molecule, while the situation is different for type II. Because the $P_c$ structures were observed by the LHCb Collaboration in the $J/\psi p$ invariant mass distribution, our results show that the $P_c$ should be copiously produced in $\Lambda_c \bar{D}^*$ and thus can be easily searched for by reconstructing events.
for $\Lambda_c$ and $\bar{D}^*$ if it is a type I hadronic molecule. Therefore, this ratio can be employed to tell the nature of the $P_c$ resonances in the future experiments, such as experiments at LHCB, the $\gamma p$ experiments at JLab [42], or the $\pi p$ experiments at JPARC [43].

It is a firm conclusion that the partial width of $P_c(4380) \to \bar{D}^*\Lambda_c$ for the $P_c(4380)$ being a $\bar{D}\Sigma_c^*$ hadronic molecule is much larger than the $\bar{D}^*\Sigma_c$ hadronic molecular case. This conclusion does not depend on any unknown coupling constant, and is analyzed in details using the nonrelativistic formalism taking heavy quark spin symmetry into account in Appendix B.

We also find that the ratio $R$ is insensitive to the mass of the $P_c$ in the range between 4.36 GeV and 4.50 GeV which covers the locations of both LHCB $P_c$ structures. Yet, we need to notice that because of the mass, the $P_c(4450)$, located 10 MeV below the $\bar{D}^*\Sigma_c$ threshold, cannot be a $\bar{D}\Sigma_c^*$ bound state.

\[
P_c \to \bar{D}\pi \Lambda_c
\]

Figure 3: Diagram representing the decay $P_c^+(4380) \to \bar{D}\pi \Lambda_c^+$ for the $P_c(4380)$ being a $\bar{D}\Sigma_c^*$ hadronic molecule.

A hadronic molecule with unstable constituents can decay naturally through the decays of its constituents. However, the widths of $\Sigma_c^{(*)}$ are small, which leads to small three-body decay widths for the $P_c$. For instance, the three-body decay $P_c \to \bar{D}\pi \Lambda_c$ shown in Fig. 3 leads to a width of only 7.3 MeV, much smaller than the reported width of the $P_c(4380)$. Here we evaluated the value of the coupling $g_{P_c \Sigma_c^* \bar{D}}$ using [44, 45]

\[
g^2 = \frac{4\pi}{4Mm_2} \frac{(m_1 + m_2)^{5/2}}{(m_1 m_2)^{1/2}} \sqrt{32\epsilon}, \tag{20}
\]

where $M$, $m_1$ and $m_2$ are the masses of $P_c$, $\bar{D}(\bar{D}^*)$ and $\Sigma_c^*(\Sigma_c)$, respectively, and $\epsilon$ is the binding energy, which is valid for an $S$-wave shallow bound state. Here we have introduced the factor $1/(4Mm_2)$ to account for the normalization of fermion fields in comparison with the formula used in, e.g. Ref. [46]. If we take the mass of the $P_c$ as 4.38 GeV, then $g_{P_c \Sigma_c^* \bar{D}} = 1.3$. The large value of $R_I$ makes possible that the $\bar{D}\Sigma_c^*$ molecule decays dominantly into the $\bar{D}^*\Lambda_c$ rather than the three-body tree-level decay mode. We can make an order-of-magnitude estimate of $\Gamma(P_c(4380) \to \bar{D}^*\Lambda_c)$ for type I hadronic molecule. Taking the cutoff to be in the range between 0.7 GeV and 1.2 GeV, which reflects the intrinsic model dependence because of the UV divergence of the loop integrals, the partial width in question could be as large as $O(100 \text{ MeV})$. The nonrelativistic formalism with a Gaussian form factor as described in Appendix B leads to the same conclusion.

It is worthy to mention that our results also depend on the values of those coupling constants shown in Table 1, and some of them are obtained from flavor $SU(4)$. Fortunately, as shown in Figs. 2, the magnitude of $R$ for type I and type II are different by more than an order of magnitude, hence even if these values only present a rough estimate of the real values of the coupling constants, our main conclusion should still be valid. The large decay branching ratio of the $\bar{D}\Sigma_c^*$ molecule to $\bar{D}^*\Lambda_c$ results in a much larger decay width than that of the $\bar{D}^*\Sigma_c$ molecule. Furthermore, the nominal mass of the $P_c(4380)$ is just a few MeV below the $\bar{D}\Sigma_c^*$ threshold. These properties makes more plausible to explain the $P_c(4380)$ as a $\bar{D}\Sigma_c^*$ hadronic molecule than a $\bar{D}^*\Sigma_c$ one.

In summary, we have studied the decays of hidden charm pentaquark $P_c^+(4380)$ state to $\bar{D}^*\Lambda_c^+$ and $J/\psi p$, assuming that its quantum numbers are $J^P = 3/2^-$, under the hadronic molecular assumption of either $\bar{D}\Sigma_c^*$ or $\bar{D}^*\Sigma_c$. The two decays can be described by means of the triangle diagrams where the two constituents of the $P_c^+$ can exchange the pseudoscalar and vector mesons leading to the $J/\psi p$ or $\bar{D}^*\Lambda_c^+$ final states. We estimate the ratio of these two decay modes. The
results show that its value is sensitive to the ansatz of whether the \( P_c^+ \) is a \( \bar{D}\Sigma_c^*(2520) \) bound state or a \( \bar{D}^*\Sigma_c(2455) \) bound state. According to our calculation, if the \( P_c(4380) \) is a \( D\Sigma_c^* \) bound state, it would have a much larger branching ratio to the \( \bar{D}^*\Lambda_c \) than that to the \( J/\psi p \). And the situation is different if the \( P_c(4380) \) is a \( \bar{D}^*\Sigma_c \) bound state. As a result, the \( D\Sigma_c^* \) bound state ansatz is more reasonable than the \( \bar{D}^*\Sigma_c \) one to explain the broad \( P_c(4380) \) structure. We suggest to search for the \( P_c(4380) \) in the \( \bar{D}^*\Lambda_c^+ \) system, which can be used to disentangle the nature of the \( P_c^+(4380) \) structure.

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A  Decay amplitudes

The amplitudes involved in the calculation are

\[ \mathcal{M}_a = g_{\rho \Sigma_c D D^* \pi g_\Lambda g_\Sigma_c} \int_{-\infty}^{\infty} \frac{d^4q}{(2\pi)^4} \Phi_{P_\rho}(q_\rho^2/\Lambda^2) \bar{u}(p_2, s_{\Lambda_c}(p_2 - q)_\mu G_{\Sigma_c}^{\mu\nu}(q) u_\nu(p_1, s_{P_\rho}) \times G_D(p_1 - q) G_\pi(q - p_2)(p_1 + p_2 - 2q) \epsilon^*_\lambda(p_1 - p_2, s_{\tilde{D}^*}) \] (21)

\[ \mathcal{M}_b = g_{\rho \Sigma_c D D^* \pi g_\Lambda g_\Sigma_c} \int_{-\infty}^{\infty} \frac{d^4q}{(2\pi)^4} \Phi_{P_\rho}(q_\rho^2/\Lambda^2) \bar{u}(p_2, s_{P_\rho}) (p_2 - q)_\mu G_{\Sigma_c}^{\mu\nu}(q) u_\nu(p_1, s_{P_\rho}) \times G_D(p_1 - q) G_\pi(q - p_2)(p_1 + p_2 - 2q) \epsilon^*_\lambda(p_1 - p_2, s_{\tilde{D}^*}) \] (22)

\[ \mathcal{M}_c = g_{\rho \Sigma_c D D^* \pi g_\Lambda g_\Sigma_c} \int_{-\infty}^{\infty} \frac{d^4q}{(2\pi)^4} \Phi_{P_\rho}(q_\rho^2/\Lambda^2) \bar{u}(p_2, s_{\Lambda_c}) \gamma_\gamma((p_2 - q) g_{\mu\nu} - \gamma_\mu(p_2 - q)_\nu) \times G_{\Sigma_c}^{\mu\nu}(q) u_\alpha(p_1, s_{P_\rho}) G_D(p_1 - q) G_{\pi}^{\mu\nu}(q - p_2) \epsilon_{\alpha\beta\lambda}(p_2 - q)^\beta(p_1 - p_2)^\lambda \epsilon^*\lambda(p_1 - p_2, s_{\tilde{D}^*}) \] (23)

\[ \mathcal{M}_d = g_{\rho \Sigma_c D D^* \pi g_\Lambda g_\Sigma_c} \int_{-\infty}^{\infty} \frac{d^4q}{(2\pi)^4} \Phi_{P_\rho}(q_\rho^2/\Lambda^2) \bar{u}(p_2, s_{P_\rho}) \gamma_\gamma((p_2 - q) g_{\mu\nu} - \gamma_\mu(p_2 - q)_\nu) \times G_{\Sigma_c}^{\mu\nu}(q) u_\alpha(p_1, s_{P_\rho}) G_D(p_1 - q) G_{\pi}^{\mu\nu}(q - p_2) \epsilon_{\alpha\beta\lambda}(p_2 - q)^\beta(p_1 - p_2)^\lambda \epsilon^*\lambda(p_1 - p_2, s_{\tilde{D}^*}) \] (24)

\[ \mathcal{M}_e = g_{\rho \Sigma_c D D^* \pi g_\Lambda g_\Sigma_c} \int_{-\infty}^{\infty} \frac{d^4q}{(2\pi)^4} \Phi_{P_\rho}(q_\rho^2/\Lambda^2) \bar{u}(p_2, s_{\Lambda_c}) \gamma_\gamma G_{\Sigma_c}(q) u_\mu(p_1, s_{P_\rho}) \times G_{\mu\nu}(p_1 - q) \epsilon_{\alpha\beta\lambda}(p_1 - q)^\alpha(p_1 - p_2)^\beta G_\pi(q - p_2) \epsilon^*\lambda(p_1 - p_2, s_{\tilde{D}^*}) \] (25)

\[ \mathcal{M}_f = g_{\rho \Sigma_c D D^* \pi g_\Lambda g_\Sigma_c} \int_{-\infty}^{\infty} \frac{d^4q}{(2\pi)^4} \Phi_{P_\rho}(q_\rho^2/\Lambda^2) \bar{u}(p_2, s_{P_\rho}) \gamma_\gamma G_{\Sigma_c}(q) u_\mu(p_1, s_{P_\rho}) \times G_{\mu\nu}(p_1 - q) \epsilon_{\alpha\beta\lambda}(p_1 - q)^\alpha(p_1 - p_2)^\beta G_\pi(q - p_2) \epsilon^*\lambda(p_1 - p_2, s_{\tilde{D}^*}) \] (26)

\[ \mathcal{M}_g = g_{\rho \Sigma_c D D^* \pi g_\Lambda g_\Sigma_c} \int_{-\infty}^{\infty} \frac{d^4q}{(2\pi)^4} \Phi_{P_\rho}(q_\rho^2/\Lambda^2) \bar{u}(p_2, s_{\Lambda_c}) \gamma_\gamma - f_{\rho\alpha\beta\lambda}(\gamma_\beta(p_2 - q) \gamma_\beta G_{\Sigma_c}(q) u_\mu(p_1, s_{P_\rho}) G_{\mu\nu}(p_1 - q) G_{\nu\alpha}(2q - p_1 - p_2) \epsilon_{\alpha\beta\lambda}(p_1 - p_2, s_{\tilde{D}^*}) \epsilon_{\alpha\beta\lambda}(p_1 - p_2, s_{\tilde{D}^*}) \] (27)

\[ \mathcal{M}_h = g_{\rho \Sigma_c D D^* \pi g_\Lambda g_\Sigma_c} \int_{-\infty}^{\infty} \frac{d^4q}{(2\pi)^4} \Phi_{P_\rho}(q_\rho^2/\Lambda^2) \bar{u}(p_2, s_{P_\rho}) \gamma_\gamma - f_{\rho\alpha\beta\lambda}(\gamma_\beta(p_2 - q) \gamma_\beta G_{\Sigma_c}(q) u_\mu(p_1, s_{P_\rho}) G_{\mu\nu}(p_1 - q) G_{\nu\alpha}(2q - p_1 - p_2) \epsilon_{\alpha\beta\lambda}(p_1 - p_2, s_{\tilde{D}^*}) \epsilon_{\alpha\beta\lambda}(p_1 - p_2, s_{\tilde{D}^*}) \] (28)

where \( u_\mu \) and \( u \) are dimensionless Rarita-Schwinger and Dirac spinors, respectively, while \( \epsilon^*_\lambda(p_1 - p_2, s_{\tilde{D}^*}) \) (\( \epsilon^*_\lambda(p_1 - p_2, s_{\tilde{D}^*}) \)) is the \( D^* \) (\( J/\psi \)) polarization vector. Here \( p_1, p_2 \) and \( q \) are the momentums of \( P_c(4380) \), \( \Lambda_c \) (or \( p \)) and \( \Sigma_c \) (or \( s \)), respectively. Besides, the \( s_{P_\rho}, s_{\tilde{D}^*}, s_{\Lambda_c}, s_{\tilde{D}^*}, \) and \( s_{P_\rho} \) are polarization variables for \( P_c(4380) \), \( \tilde{D}^*, \Lambda_c, J/\psi, \) and \( p \), respectively. \( G_{\pi/D}(q), G_{\mu\nu}(q), G_{\Sigma_c}(q), \) and \( G_{\Sigma_c}^{\mu\nu}(q) \) are the propagators for the \( \pi, (D \text{ or } \tilde{D}^*), D^* \text{ or } \tilde{D}^*, \Sigma_c, \) and \( \Sigma_c^* \), respectively, which are

\[ G_{\pi/D}(q) = \frac{1}{q^2 - m_{\pi/D}^2} \] (29)

\[ G_{\mu\nu}(q) = \frac{-q^{\mu\nu} + q^\mu q^\nu / q^2}{q^2 - m_{\tilde{D}^*}^2} \] (30)

\[ G_{\Sigma_c}(q) = \frac{1}{q^2 - m_{\Sigma_c}^2} \] (31)

\[ G_{\Sigma_c}^{\mu\nu}(q) = \frac{1}{q^2 - m_{\Sigma_c}^2} + \frac{1}{m_{\Sigma_c}^2} \left( -g^{\mu\nu} + \frac{\gamma^\mu \gamma^\nu}{3} + \frac{g^{\mu\nu} - \gamma^\nu \gamma^\mu}{3q^2/m_{\Sigma_c}^2} + \frac{2q^\mu q^\nu}{3q^2} \right) \] (32)
B Nonrelativistic formalism

In this appendix, we will describe a nonrelativistic formalism which can be used to calculate the one-pion-exchange loop diagrams for the decays of the $D^* \Sigma_c$ and $\bar{D} \Sigma^*_{c+}$ into the $D^* \Lambda_c$. The reason is that for these two decays, all the involved particles except for the pion can be treated nonrelativistically: the $P_c$ is near the $D^* \Sigma_c$ and $\bar{D} \Sigma^*_{c+}$ thresholds, and the center-of-mass momentum in the $D^* \Lambda_c$ system is only 0.43 GeV for $M_{P_c} = 4.38$ GeV.

We will take the two-component notation for fields containing heavy quarks [47]. Then the field for charmed mesons is given by $H_a = \bar{D}_a^* \cdot \bar{\sigma} + D_a$, where $D_a$ and $D_a^*$ annihilate the pseudoscalar and vector charmed mesons, respectively, $a$ is the light flavor index, and $\bar{\sigma}$ are the Pauli matrices acting in the spinor space. The field for anti-charmed mesons reads $\bar{\Sigma}_a = \bar{\Sigma}^\dagger_a \cdot \bar{\sigma} a_{\bar{b}} H_b$ [47, 48]. The axial coupling of the pions to the heavy mesons is contained in the following leading order chiral Lagrangian [47, 48]

$\mathcal{L}_{HH\pi} = -\frac{g}{2} \langle H^\dagger_a H_b \bar{\sigma} \cdot \bar{u}_{ab} \rangle + \frac{g}{2} \langle \bar{H}^\dagger_a \bar{\sigma} \cdot \bar{u}_{ab} H_b \rangle$

(33)

where $\bar{u}_{ab} = -\sqrt{2} \partial_\phi \phi_{ab} / F + \mathcal{O}(\phi^3)$ contains the pion fields

$\phi = \begin{pmatrix} \sqrt{2} \pi^0 + \frac{1}{\sqrt{6}} \eta \\ \pi^- \\ -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta \\ K^+ \\ -\frac{1}{\sqrt{6}} \eta \\ -\frac{1}{\sqrt{6}} \eta \end{pmatrix}$

(34)

when we consider three light flavors, and $F$ is the pion decay constant in the chiral limit and we will take 92.2 MeV. The value of the coupling constant $g = 0.57$ can be extracted from the measured decay width of $D^{*+}$ [49]. We can also write down the trace formalism for the sextet heavy baryon fields introduced in Ref. [50] in the two-component notation as

$S^i_{ab} = B^i_{6,ab} + \frac{1}{\sqrt{3}} \sigma^i B_{6,ab}$

(35)

where $B^i_{6,ab}$ and $B_{6,ab}$ annihilate the $J^P = \frac{3}{2}^+$ and $\frac{1}{2}^+$ sextet charmed baryons, which degenerate in the heavy quark limit, respectively. The leading order chiral Lagrangian for the axial pionic coupling between the sextet and anti-triplet charmed baryons [51] can be written as

$\mathcal{L}_{SB \pi} = -\frac{\sqrt{3}}{2} g_2 B^i_{3,ab} \bar{u}_{bc} \cdot \bar{S}^i_{ca} + \text{h.c..}$

(36)

The charmed baryon matrices in SU(3) flavor space are given by

$B_3 = \begin{pmatrix} 0 & \Lambda_+^c & \Xi_+^c \\ -\Lambda_+^c & 0 & \Xi_0^c \\ -\Xi_+^c & -\Xi_0^c & 0 \end{pmatrix}$, $B_6 = \begin{pmatrix} \Sigma^*_c & \sqrt{\frac{1}{12}} \Sigma^0_c & \sqrt{\frac{1}{12}} \Xi^0_c \\ \sqrt{\frac{1}{12}} \Sigma^0_c & \Sigma^0_c & \Xi^0_c \\ \sqrt{\frac{1}{12}} \Xi^0_c & \Xi^0_c & \Xi^0_c \end{pmatrix}$

(37)

The value of $g_2$ extracted from the decays $\Sigma^*_c \to \Lambda_+^c \pi^+$ and $\Sigma^*_c \to \Lambda_+^c \pi^+$ are 0.56 and 0.55, respectively. At leading order of the nonrelativistic expansion, summing over the polarizations of the vector meson is given by

$\sum_{\alpha} \epsilon^i (\vec{p}, \lambda) \epsilon^j (\vec{p}, \lambda) = \delta^{ij}$

(38)

and summing over the polarizations for spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ spinors results in

$\sum_{\alpha} u(\vec{p}, \alpha) u^\dagger(\vec{p}, \alpha) = 2m,$

$\sum_{\alpha} u^i(\vec{p}, \alpha) u^{i\dagger}(\vec{p}, \alpha) = \frac{2m}{3} (2\delta^{ij} - i\varepsilon^{ijk} \sigma^k).$

(39)
We define the $S$-wave couplings of the $J^P = \frac{3}{2}^-$ $P_c$ state to the $\bar{D}^*\Sigma_c$ and $\bar{D}\Sigma_c^*$ by

$$L_{P_c} = -\sqrt{\frac{3}{2}} \left( g_{P_c} \bar{D}_{a}^{\ast}\Sigma_{c, ab}^{\ast} \cdot \bar{P}_{c, b} + g'_{P_c} \bar{D}_{a}^{\ast}\Sigma_{c, ab}^{\ast} P_{c, b}^{i} \right).$$

(40)

If we assume that the $P_c$ is a $\bar{D}\Sigma_c^*$ ($\bar{D}^*\Sigma_c$) bound state, then $g_{P_c}^2$ ($g'_{P_c}^2$) is given by Eq. (20) and $g_{P_c}^2 = 0$ ($g'_{P_c} = 0$).

Using the above Lagrangians, we can calculate the amplitudes for the one-pion-exchange diagrams shown in Fig. 1. The amplitude of $P_c^0 (p) \rightarrow \Lambda_c^+(k)D^{*0}(q)$ for $P_c$ being the $\bar{D}\Sigma_c^*$ molecule and the $\bar{D}^*\Sigma_c$ molecule are

$$A_{\bar{D}^*\Sigma_c}^{\text{OPE}} = -3Ng_{P_c} m_{\Sigma_c} u^i(\vec{k}, \omega)(2\delta^{ij} - i\varepsilon^{ijk}\sigma^k) u^l(\vec{p}, \alpha) e_n^j(\vec{q}, \lambda) F^{in}(m_D, m_{\Sigma_c}, m_\pi, \vec{q}),$$

$$A_{\bar{D}\Sigma_c^*}^{\text{OPE}} = -i3\sqrt{3}Ng_{P_c} m_{\Sigma_c} e^{ij} e^j(\vec{q}, \lambda) u^i(\vec{k}, \omega) (\sigma^a u^k(\vec{p}, \alpha) F^{in}(m_{D^*}, m_{\Sigma_c}, m_\pi, \vec{q}),$$

(41)

respectively. Here the factor 3 takes into account the contributions from the isospin multiplets of the intermediate states in the triangle diagrams, $N = 2gg_2g_{P_c}/(3\sqrt{2}F^2)$, $\omega$, $\alpha$ and $\lambda$ denote the polarization of the relevant particles, and the tensor loop integral is defined in the $P_c$ rest frame as

$$P^{ij}(m_1, m_2, m_3, \vec{q}) = \frac{i}{2m_1m_2} \int \frac{d^4l}{(2\pi)^4} \frac{[l^{ij}]}{(q^0 + l^0 - \omega_1 + i\epsilon)(k^0 + l^0 - \omega_2 + i\epsilon)(l^2 - m_3^2 + i\epsilon)},$$

(42)

where $\omega_1 = \sqrt{m_3^2 + (\vec{l} - \vec{q})^2}$, $\omega_2 = \sqrt{m_2^2 + (\vec{l} - \vec{q})^2}$, and the propagators of the charmed meson and baryon have been treated nonrelativistically. The $J^P = \frac{3}{2}^-$ $P_c$ can decay into the $\Lambda, D^*$ in both $S$ wave and $D$ wave. It is reflected by the fact that the tensor loop integral defined in Eq. (42) can be decomposed into an $S$-wave part and a $D$-wave part which will be denoted by $I_S$ and $I_D$, respectively. The decomposition can be easily done by defining the $S$-wave and $D$-wave projectors

$$P^{ij}_S = \frac{1}{3} \delta^{ij}, \quad P^{ij}_D = \frac{g_{ij}}{q^2} - \frac{1}{3} \delta^{ij},$$

(43)

which satisfy $P^{ij}_S P^{ij}_S = 1/3$, $P^{ij}_D P^{ij}_D = 2/3$, and $P^{ij}_S P^{ij}_D = 0$. We get

$$I_S(m_1, m_2, m_3, q^2) = I^{ij}(m_1, m_2, m_3, \vec{q}),$$

(44)

and

$$I_D(m_1, m_2, m_3, q^2) = \frac{3}{2} I^{ij}(m_1, m_2, m_3, \vec{q}) P^{ij}_D.$$  

(45)

It turns out that the $D$-wave part $I_D$ is UV convergent as discussed in Ref. [52], while the $S$-wave part $I_S$ is UV divergent. The divergence might be regularized by introducing a Gaussian form factor $\exp \left(-\frac{(q - l)^2}{\Lambda^2}\right)$, which is the nonrelativistic analogue of the form factor in Eq. (10).

For the purpose of qualitatively comparing the relative size of the partial widths calculated from the one-pion-exchange diagrams for the two hadronic molecular assignments, we do not need to specify a value for $\Lambda$ as can be seen in the following. Using Eqs. (38) and (39), we have

$$\sum_{\omega, \alpha, \lambda} \left| A_{\bar{D}^*\Sigma_c}^{\text{OPE}} \right|^2 = 144N^2 g_{P_c}^2 m_{\Lambda_c} m_{P_c} m_{\Sigma_c}^2 \left[ |2I_D(m_D, m_{\Sigma_c}, m_\pi, \vec{q}^2)|^2 + |I_S(m_D, m_{\Sigma_c}, m_\pi, \vec{q}^2)|^2 \right],$$

(46)

$$\sum_{\omega, \alpha, \lambda} \left| A_{\bar{D}\Sigma_c^*}^{\text{OPE}} \right|^2 = 48N^2 g_{P_c}^2 m_{\Lambda_c} m_{P_c} m_{\Sigma_c}^2 \left[ 5|I_D(m_{D^*}, m_{\Sigma_c}, m_\pi, \vec{q}^2)|^2 + |I_S(m_{D^*}, m_{\Sigma_c}, m_\pi, \vec{q}^2)|^2 \right].$$

(47)
One sees that the partial widths for both the $\bar{D}\Sigma^*_c$ and $\bar{D}^*\Sigma_c$ molecules decays into $\Lambda_c\bar{D}^*$ contains both $S$-wave and $D$-wave components, but the $S$-wave component for the decay of the $\bar{D}^*\Sigma_c$ molecule is parametrically three times larger than that for the decay of the $D^*\Sigma_c$. Numerically, the difference is a factor of $O(10)$, for $\Lambda$ taking a value in the range of $[0.5, 2]$ GeV, after we have included the kinematic effects that the $P_c(4380)$ mass is closer to the $\bar{D}\Sigma^*_c$ threshold than to the $\bar{D}^*\Sigma_c$ one and that the $D\pi\Lambda_c$ can be on shell simultaneously, if we take the mass of the $P_c$ to be 4.38 GeV.

Therefore, it is a firm conclusion that if the $P_c(4380)$ is a $\bar{D}\Sigma^*_c$ molecule, its partial width of the decay into $\Lambda_c\bar{D}^*$ is much larger than that for the case if it is a $\bar{D}^*\Sigma_c$ molecule.

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