Shell-Model Description of Λ Hypernuclei

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Empirical data on the spectra of light hypernuclei, especially the data from recent γ-ray experiments, is used to constrain the parameters which govern the $p_Ns_\Lambda$ and $p_Np_\Lambda$ two-body matrix elements that enter into shell-model calculations.

1. Introduction

Three hypernuclear γ-ray experiments were run in 1998. The detector for KEK E419 and BNL E930 was the Hyperball. The primary objective of KEK E419 was to measure the spacing of the the ground-state doublet in $^7_\Lambda$Li while that of BNL E930 was to measure the spacing of the excited-state doublet in $^9_\Lambda$Be built on the $2^+$ state of the $^8$Be core. The objective of BNL E929, which used 72 NaI detectors and a live $^{13}$C target, was to measure the spacing of $p_{1/2}$ and $p_{3/2}$ Λ states at $\sim 11$ MeV excitation energy in $^{13}_\Lambda$C.

While considerable progress has been made starting from a good NN model (OBEP or OBEP + TBEP), applying symmetries, and fitting the limited YN data set, the procedure does not lead to a reliable YN interaction. Specifically, predictions for how the overall binding energy is distributed between singlet and triplet, or even and odd state, interactions vary widely. For example, six NSC97 interactions have been constructed which give equally good fits to the YN data but which exhibit a wide range of central spin-spin interaction strengths. On the other hand, one can argue that many-body effective interaction theory provides a sound connection between the free YN interaction and the effective ΛN (and ΛNN) interaction for shell-model calculations. Thus, precise hypernuclear data should strongly constrain models of the free YN interaction.

The ΛN−ΣN coupling is an important feature of YN interaction models. Recently, Akaishi et al. have calculated G-matrices for a number of YN potential models for use in the model space of $s$ orbits only. Then, the coupling between $s^3s_\Lambda$ and $s^3s_\Sigma$ configurations is simply $v = 3/2 s^3g - 1/2 s^1g$ for $0^+$ and $v = 1/2 s^3g + 1/2 s^1g$ for $1^+$, where $s^3g$ and $s^1g$ are the relative s-state g-matrix elements for triplet and singlet states. The Σ admixture and energy shift for the lower state are $(v/\Delta E)^2$ and $v^2/\Delta E$, respectively ($\Delta E \sim 80$ MeV). The $s^3g$ interaction dominates with the result that the energy shift is substantial for the $0^+$ state and very small for the $1^+$ state. The results for NSC97e and NSC97f bracket the experimental splitting of $\sim 1.1$ MeV. For NSC97f, the ΛN and ANN contributions are comparable ($v \sim 7.6$ MeV) for a total of 1.48 MeV. At this meeting, E. Hiyama showed similar results from calculations which include explicit Σ degrees of freedom in a large model space.

If one calculates the ΛN−ΣN coupling for the $1/2^+$ ground-state of $^7_\Lambda$Li by asking
for the matrix element between $p^2(1^+; 0)s_{\Lambda}$ and $p^2(0^+; 1)s_{\Sigma}$ in the LS limit, one gets

$$v = \sqrt{3}(g_{ps} - g_{ps}/2).$$

Because $g_{ps} \sim g_{ss}/2$, the energy shift should be $\sim 1/12$ of that for the $0^+$ states in the $A=4$ hypernuclei. For the $3/2^+$ state, only non-central coupling interactions contribute and the shift should be very small indeed.

The last result justifies the initial neglect of effective $\Lambda NN$ interactions for p-shell hypernuclei. Fetisov [3] has come to different conclusions using a particular form of zero-range $\Lambda NN$ interaction. In the following sections, we consider how the new $\gamma$-ray data affects the parametrizations of the $\Lambda N$ effective interaction put forward by MGDD [4] and by FMZE [5]. From this purely phenomenological point of view, the spin-dependence for a $\Lambda$ in a $0s$ orbit interacting with a p-shell core is specified by four radial integrals, conventionally denoted by $\Delta$, $S_{\Lambda}$, $S_N$, and $T$ associated with the operators $s_N.s_{\Lambda}$, $l_{NA}.s_{\Lambda}$, $l_{NA}.s_N$, and $3(\sigma_N.\vec{r})(\sigma_{\Lambda}.\vec{r}) - \sigma_N.\sigma_{\Lambda}$.

Figure 1. The bound-state spectrum of $^7\Lambda Li$. 
2. $^7\Lambda$Li

The $\gamma$-ray energies [6] and the lifetime [7] from the $^7\text{Li}(\pi^+,K^+\gamma)^7\Lambda$Li reaction used in KEK E419 are shown in Fig. [1]. The $\gamma$-ray branching ratios use the B(M1) and B(E2) values from the $^3\text{He} + N + N$ model of Hiyama et al. [8] and the $(\pi^+,K^+)$ cross sections integrated from $0^\circ - 15^\circ (\mu b)$ are also from Ref. [8]. The spectrum is from a shell-model calculation using a modified MGDD interaction with $S_N$ changed from $-0.08$ to $-0.47$ MeV to bring the energy of the $5/2^+$ state in $^7\Lambda$Li down to the measured value of 2.05 MeV,

$$\Delta = 0.50 \quad S_\Lambda = -0.04 \quad S_N = -0.47 \quad T = 0.04 . \quad (1)$$

At the same time, p-shell wave functions from a fit to 32 energy levels for $A = 6 - 9$ are used for the $^6\text{Li}$ core in place of the Cohen and Kurath wave functions used in MGDD [9]. The main effect of a stronger one-body spin-orbit component in the new p-shell interaction is to decrease the small $^3\text{S}$ admixture and increase the small $^1\text{P}$ admixture in the dominantly $^3\text{S}$ ground state of $^6\text{Li}$ [9]. Similarly, the $^3\text{P}$ component in the $0^+; 1$ state is substantially increased.

The hypernuclear wave functions are very close to the weak-coupling limit. The contribution of each component of the $\Lambda N$ interaction to the observed energy separations in $^7\Lambda$Li are given in Table [1]. As expected, the energy splitting of the ground-state doublet is dominated by the spin-spin interaction. A substantial negative value for $S_N$ is required to fit the excitation energy of the $5/2^+$ state. The excitation energy of the $1/2^+; T=1$ state provides another measure of $S_N$, the coefficient of which is controlled mainly by the $^3\text{P}$ component in the $0^+; 1$ wave function. This coefficient is very different from the original MGDD value of 0.081.

In principle, the measurement of four energy separations, the energies of two states based on excited states of the core and two doublet separations, gives the four parameters

| Spacing $\Delta$   | $\Delta$ | $S_\Lambda$ | $S_N$ | $T$  | $\Delta E_{AN}$ |
|--------------------|----------|-------------|-------|------|-----------------|
| $3/2^+ - 1/2^+$    | 1.444    | 0.054       | 0.016 | -0.271 | 712 keV         |
| $\Delta E \sim \Delta E_{AN}$ | 722      | -2          | -8    | -11   |                 |
| $5/2^+ - 1/2^+$    | 0.154    | -1.105      | 0.678 | 1.095 | 2050 keV        |
| $\Delta E \sim 2186 + \Delta E_{AN}$ | 77       | 44          | -319  | 44    |                 |
| $1/2^+ - 1/2^+$    | 0.955    | 0.045       | 0.466 | -0.057 | 3819 keV        |
| $\Delta E \sim 3563 + \Delta E_{AN}$ | 477      | -2          | -219  | -2    |                 |
| $7/2^+ - 5/2^+$    | 1.311    | 2.141       | 0.024 | -2.324 | 431 keV         |
| $\Delta E \sim \Delta E_{AN}$ | 656      | -86         | -11   | -93   |                 |
controlling the spin dependence of the $p_Ns_Λ$ interaction. In practice, $Δ$, $S_N$, and one combination of $S_Λ$ and $T$, namely $T − S_Λ$, might be determined.

3. $s_Λ$ states in $^9_Λ$Be, $^{12}_Λ$C, and $^{13}_Λ$C

There is new data on each of these hypernuclei from experiments at KEK and BNL and it is of interest to see whether there is consistency with the preceding analysis of the KEK E419 data on $^7_Λ$Li. In this analysis, the $p_Ns_Λ$ matrix elements were treated as parameters. However, the radial integrals depend on the size of the hypernucleus and to study this effect the YNG interactions of Yamamoto et al. [10], in which nuclear matter G-matrix elements for various free YN interaction models are simulated by a number of Gaussians with different ranges ($r^2$ times Gaussian for tensor interactions), are used. In particular, the parametrization [11] of the NSC97f interaction [1] for $k_F = 1$ fm$^{-1}$ is used with Woods-Saxon radial wave functions. In the first instance, the strengths of each component are scaled to reproduce the parameters of Eq. (1). To do this the singlet and triplet central interaction strengths have to be slightly reduced and increased, respectively, to reduce $Δ$ from the NSC97f value. The ALS component has to be increased by a factor of $\sim 3$ and the tensor component has to be reduced somewhat in strength. Results are shown in Table 2, where it can be seen that matrix elements are larger for more tightly bound neutron wave functions.

The value $Δ = 0.48$ MeV chosen for $^7_Λ$Li gives improved agreement with the measure ground-state doublet splitting of 692 keV. The smaller magnitudes for $S_Λ$ and $T$ give better agreement with the new data from BNL E930 on the doublet splitting in $^9_Λ$Be, to be discussed next. As can be seen from Table 2, these reductions require a small reduction in the magnitude of $S_N$ to maintain the excitation energy predicted for the $5/2^+$, which will then improve the predicted energy for the $1/2^+$; $1$ state. The spacing of the $7/2^+ − 5/2^+$ doublet would also increase, perhaps too close to 511 keV to be directly measurable.

The result from BNL E930 reported at this meeting by Tamura and by Akikawa is that the splitting of the $3/2^+$, $5/2^+$ doublet in $^9_Λ$Be is only $\sim 32$ keV, updating the limit

| Table 2 | $p_n s_Λ$ parameters as a function of $A$ for Woods-Saxon wave functions. For nucleons $r_0 = 1.25$ fm, $a = 0.6$ fm; for $Λ$’s $r_0 = 1.128 + 0.439A^{-2/3}$ fm, $a = 0.6$ fm. Energies are in MeV and lengths are in fm. |
|---------|----------------------------------------------------|
| $^9_Λ$Li | $^9_Λ$Be | $^{12}_Λ$C | $^{13}_Λ$C | $^{16}_Λ$O |
| $B_Λ(0s)$ | 5.58 | 6.73 | 10.80 | 11.67 | 13.0 |
| $B_n(0p)$ | 5.56 | 18.90 | 13.12 | 18.72 | 16.31 |
| $\langle r^2 \rangle^{1/2}(0s_Λ)$ | 2.63 | 2.55 | 2.33 | 2.32 | 2.32 |
| $\langle r^2 \rangle^{1/2}(0p_N)$ | 2.86 | 2.34 | 2.62 | 2.50 | 2.66 |
| $Δ$ | 0.480 | 0.619 | 0.550 | 0.591 | 0.521 |
| $S_N$ | −0.430 | −0.549 | −0.508 | −0.545 | −0.461 |
| $S_Λ$ | −0.010 | −0.013 | −0.012 | −0.013 | −0.011 |
| $T$ | 0.021 | 0.029 | 0.025 | 0.027 | 0.023 |
of $< 100$ keV from the work of May et al. [12] with NaI detectors. As can be seen from Table 3, the small $S = 1$ amplitudes ($\sim 4\%$ intensity) in the $^8$Be $2^+$ wave function (necessary to account for $^8$Li and $^8$B $\beta$ decay) lead to a substantial contribution from the $\Lambda N$ tensor interaction. For $S_\Lambda < 0$ and $T > 0$, the measurement implies that the magnitudes of these parameters are restricted to be very small.

There is strong evidence from both $^{12}_\Lambda$C and $^{13}_\Lambda$C that supports the substantial negative value for $S_N$ deduced from $^7$Li.

At this meeting, Sakaguchi reported an energy of $4.915(33)$ MeV from BNL E929 for the $\gamma$-ray from the first $3/2^+$ state of $^{13}_\Lambda$C. This is in agreement the value $4.89(7)$ MeV deduced from a high-statistics $^{13}$C($\pi^+, K^+$)$^{13}_\Lambda$C spectrum from KEK E336 [13], in which the ground-state and $3/2^+$ peaks are well resolved. The deviation from the $4.439$ MeV energy of the $2^+$ core can be attributed mainly to $S_N$, as can be seen from Table 4 (parameters from Table 2). For comparison, the MGD excitation energy was $4.49$ MeV.

The energies of the excited $1^-$ states in $^{12}_\Lambda$C are also raised from the unperturbed core energies. A preliminary analysis of ($\pi^+, K^+$) data from KEK E336 [13] gives $2.71$ MeV and $6.05$ MeV for the $1^-$ levels. The corresponding numbers from KEK E369 [14], which achieved an energy resolution of $1.45$ MeV, are $2.54$ MeV and $6.17$ MeV. As can be seen from Table 5, the largest contributions are again from $S_N$ with substantial contributions from $\Delta$.

A number of $\gamma$ rays in $^{12}_\Lambda$C could be measured in future runs of BNL E930 at either $0.9$ GeV/$c$ or $1.8$ GeV/$c$ depending in part on the primary beam intensity available to the D6 beamline. These include the spacing of the ground-state doublet, either directly if the spacing is large enough to preclude predominantly weak decay of the upper level or indirectly through transitions from the $1^-_\Lambda$ level. The simplest weak-coupling estimate gives $280$ ps for the partial electromagnetic lifetime of the $2^-_\Lambda$ level for the separation in Table 5. The parameters of Eq. (4) give a spacing of only $71$ keV so that weak decay would dominate.

| Table 3 |
| $^9$Be: $3/2^+ - 5/2^+$ separation |

| $\Delta$ | $S_\Lambda$ | $S_N$ | $T$ | $\Delta E$ |
|----------|-----------|-------|-----|------------|
| -0.036   | -2.463    | 0.002 | 0.985 | 40 keV     |
| -22      | 32        | -1    | 29   |            |

| Table 4 |
| $^{13}_\Lambda$C: $3/2^+ - 1/2^+$ separation $\Delta E = 4439 + \Delta E_{\Lambda N}$ |

| $\Delta$ | $S_\Lambda$ | $S_N$ | $T$ | $\Delta E$ |
|----------|-------------|-------|-----|------------|
| -0.050   | -1.450      | -0.861| -1.104 | 4823 keV   |
| -30      | 19          | 469   | -30  |            |

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Table 5
Energy separations in $^{12}_{\Lambda}C$

| Spacing          | $\Delta$| $\Delta S_N$ | $\Delta T$ | $\Delta E$ |
|------------------|---------|--------------|------------|-----------|
| $1/2^- - 1/2^+$  | 0.331   | 1.147        | -0.913     | 670 keV   |
| $\Delta E \sim 2.000 + \Delta E_{AN}$ | 182     | -14          | 464        | 17       |
| $1^- - 1^-_1$   | 0.376   | -0.388       | -1.339     | 464 keV   |
| $\Delta E \sim 4.804 + \Delta E_{AN}$ | 207     | 5            | 680        | 12       |
| $2^- - 1^-_1$   | 0.474   | 1.510        | 0.031      | -2.092    |
| $\Delta E \sim \Delta E_{AN}$ | 261     | -18          | -16        | 52       |

4. $p_\Lambda$ states in $^{13}_{\Lambda}C$

The $p_n p_\Lambda$ central interaction can be parametrized for $S = 0$ and $S = 1$ separately as:

$$V_{NN} = (F^{(0)} + F^{(2)} Q_N Q_\Lambda)(1 - \varepsilon + \varepsilon P_x)$$

where $F^{(2)}/F^{(0)}$ characterizes the range of the interaction and $\varepsilon$ characterizes the space-exchange component. The NSC97 interactions exhibit strong short-range repulsion in relative p waves, corresponding to $\varepsilon \sim 1$ in the parametrization of Eq. (2). Interactions with different space-exchange properties can be made from the YNG interactions by varying the strengths of the even and odd state interactions separately while maintaining the same overall attraction.

The essential structure of the lowest $p_\Lambda$ states in $^{13}_{\Lambda}C$, built on the lowest $0^+$ and $2^+$ states of the $^{12}C$ core, is illustrated in Fig. 12 of Ref. [15]. The $2^+ \otimes p_\Lambda$ states appear in the order $\mathcal{L} = 2,3,1$ where $\mathcal{L} = J_{core} + l_\Lambda$ under the action of the $Q_Q$ interaction and doublets form as a the result of coupling the $\Lambda$ intrinsic spin. A space-exchange interaction pushes the $0^+ \otimes p_\Lambda$ and $2^+ \otimes p_\Lambda$ states with $\mathcal{L} = 1$ apart.

The two $1/2^-\otimes p_\Lambda$ states with $\mathcal{L} = 1$ are seen strongly via $\Delta L = 0$ transitions in the $(K^-, \pi^-)$ reaction near $0^\circ$, the upper one most strongly because it tends towards the $[441]$ spatial symmetry of the $^{13}C$ target. The separation is observed to be $6.0(4)$ MeV. The NSC97f interaction, which has a very strong space-exchange component, gives a spacing of $10$ MeV when harmonic oscillator wave functions are used to reproduce the parameters of Eq. (1). From BNL E929, the lowest $1/2^-\otimes p_\Lambda$ doublet is at $E_x \sim 11$ MeV so the the $0p_\Lambda$ separation energy is only $0.67$ MeV. The Woods-Saxon parameters used in Table 2 give an rms radius for the $0p_\Lambda$ orbit of $4.53$ fm and the mismatch with the deeply bound nucleon orbits leads to a very substantial reduction in the $p_N p_\Lambda$ matrix elements. The separation of the $1/2^-\otimes p_\Lambda$ states drops to $7.7$ MeV, which is still too large. The interaction used for Table 2 has an attractive odd-state interaction of about half the strength of the even-state interaction ($\varepsilon \sim 0.25$) and reproduces the experimentally measured $1/2^-$ separation of $6.0$ MeV. The ratio of cross sections for the $1/2^-$ states increases as the strength of the space-exchange interaction increases. Experiment ($\sim 5.5$) and theory are in good agreement for the interaction which reproduces the energy separation.

The $3/2^-_1$ and $5/2^-_2$ states are seen strongly in the $(\pi^+, K^+)$ reaction or the $(K^-, \pi^-)$ reaction at larger angles and the $1/2^-_1 - 5/2^-_2$ separation has been measured to be $1.7(4)$ MeV.
The new results on $s_A$ states of $^7\Lambda$Li, $^9\Lambda$Be, $^{12}\Lambda$C, and $^{13}\Lambda$C are consistent with substantial magnitudes for $\Delta$ and $S_N$ and small values for $S_A$ and $T$.

The $p_Np_A$ matrix elements are sensitive to more features of the underlying $\Lambda N$ interaction than are the $p_Ns_A$ matrix elements. Quantities such as the space-exchange character, the $Q.Q$ component, and the even-state tensor interaction all play a role for $p_A$ states in $^{13}\Lambda$C. In particular, the magnitude of the space-exchange component of the effective $\Lambda N$ interaction is restricted to be quite small. This is in agreement with an analysis of $\Lambda$ single-particle energies by Usmani and Bodmer [18], who also find $\varepsilon \sim 0.25$. 

5. Remarks
The object of future \((K^- , \pi^- \gamma)\) runs of BNL E930 is to obtain a much more complete the set of information on the \(p_N s_\Lambda\) interaction. An important target is \(^{16}\text{O}\) because the ground-state doublet splitting in \(^{16}\text{O}\) is very sensitive to the tensor matrix element \(T\). It will probably have to be measured as the difference of energies of the \(\sim 6\) MeV \(\gamma\)-rays deexciting the \(1^-\) level. It may also be possible to observe \(\gamma\)-ray transitions in \(^{15}\text{N}\) following proton emission from higher-energy states in \(^{16}\text{O}\). As mentioned previously, studies with \(^{12}\text{C}\) and \(^{7}\text{Li}\) targets are also under consideration. Measurements for a range of nuclei throughout the \(p\) shell should both overdetermine the set of spin-dependent parameters and make it possible to test for variations with nuclear size and the empirical need for a \(\Lambda\text{NN}\) interaction.

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