Asymptotic behavior of the Daily Increment Distribution of the IPC, the Mexican Stock Market Index

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Abstract. In this work, a statistical analysis of the distribution of daily fluctuations of the IPC, the Mexican Stock Market Index is presented. A sample of the IPC covering the 13-year period 04/19/1990 - 08/21/2003 was analyzed and the cumulative probability distribution of its daily logarithmic variations studied. Results showed that the cumulative distribution function for extreme variations, can be described by a Pareto-Levy model with shape parameters $\alpha = 3.634 \pm 0.272$ and $\alpha = 3.540 \pm 0.278$ for its positive and negative tails respectively. This result is consistent with previous studies, where it has been found that $2.5 < \alpha < 4$ for other financial markets worldwide.

Key words. Econophysics, stock market, Power-Law, stable distribution, Levy regime.

1 Introduction

The behavior of extreme variations of economics indexes, stock prices or even currencies, has been a topic of interest in finance and economics, and its study becomes relevant in the context of risk management and Financial Risk Theory. However, these analyses are usually difficult to perform due to the small number of extreme observations in the tails of the distributions of financial time series variations. Recently, the interest of the physics community in the behavior of financial markets, has strongly increased, boosted by the availability of worldwide, electronically recorded financial data, giving rise to a different approach to confront problems arising from economics. The collection of methods and techniques originally developed in the area of physics, which are currently applied in the study of financial complex systems, is now called Econophysics and it is becoming an emergent branch of physics by itself [1-3,4,5,6].

In order to describe the behavior of the distribution of financial time series variations, several models have been proposed. Some of them are:

- Gaussian distribution [7].
- Log-gaussian distribution (Geometric Brownian Motion) [8].
- Stable Levy distribution [9,10,11].
- Truncated Levy Distribution [12,13,14].
- Poisson like distribution [15,16].
- Power law Distribution with $\alpha \simeq 3$ [17] (Asymptotically).

In this paper, a statistical analysis of the distribution of daily variations of the IPC\(^1\) is presented. It is organized as follows. In the remaining of this section, we briefly review

\(^1\) The Mexican Stock Market index or Indice de Precios y Cotizaciones for its Spanish meaning.
the Pareto-Levy distribution, and some of the phenomena it describes are mentioned. In next section, a very short introduction to the variables of common use in the study of financial index and prices variations is given. In section 3, we introduce the sample data analyzed in this work and some important statistical properties of financial time series variations (fat tails, clustering volatility, etc)\(^2\) are discussed, all the above in the context of the IPC data. In section 4 we explain and justify the procedure to estimate the Pareto-Levy exponent from data and we show the results concerning the fit on the tails of the cumulative distribution of the IPC daily logarithmic variations. Finally, last section is devoted to compare our results with some other related studies of different international stock markets previously reported.

1.1 Pareto-Levy Distribution. Stable Distributions

At this point, it is convenient to make a review of the definition of the Pareto-Levy distribution: An absolutely continuous random variable \(Y\), is said to follow a Pareto-Levy distribution with parameters \(\alpha\) and \(\gamma\), if its cumulative distribution function \(F\) has the form:

\[
F(y) := P\{Y \leq y\} = 1 - \left(\frac{y_0}{y}\right)^{\alpha} = 1 - \frac{\gamma}{y^{\alpha}} \tag{1}
\]

with \(y_1 \geq y_0\), \(y_0^\alpha = \gamma\) and \(\alpha > 0\). When the condition \(\alpha > 2\) holds, the mean and variance of \(Y\) are both finite and by the central limit theorem, the sum of independent Pareto-Levy distributed random variables, converges in probability to a gaussian law. On the other hand, when \(\alpha < 2\), the Pareto-Levy distribution has infinite variance, and it is said that the distribution is stable. For a mathematical treatment of these topics, consult [19] and [20]. For a review from an econophysics point of view, references [6] and [21] are recommended.

The Pareto-Levy distribution is often known as the Power-law distribution, and its role in Physics and other areas seems ubiquitous. In particular, we can illustrate this point by the following examples taken from finance:

- \(N_{\Delta t}\), the number of trades in a given interval of time \(\Delta t\), follows a power-law distribution with exponent \(\alpha\approx \frac{3}{2}\) [24].
- Pareto-Levy tails, with \(\alpha \approx 3\) for extreme variations of individual stocks prices [24] and also for indexes of different leading stocks markets [17,24,24].
- Decay of volatility correlations follows a power law distribution [20,27].
- The tail behavior of the cumulative distribution function of volatility, is consistent with a power law distribution with exponent \(\alpha \approx 3\) [28].

\(^2\) In finance, to these statistical properties of financial time series, jointly with other such as the properties of intermittency, asymmetry in time scales, absence of autocorrelations, gain/loss asymmetry etc., are called "Stylized facts" [18].

All the above suggests universality in financial complex systems and in order to explain the above facts, new models and even theories, are currently being proposed [20,29,30,31].

2 Study of variations of financial time series

In the study of price variations of financial assets, many observables can be analyzed [8]. If \(Y(t)\) is the value of the index at time \(t\), some commonly used observables are:

- Prices or indexes changes themselves, for some interval of time \(\Delta t\):
  \[
  Z(t) := Y(t + \Delta t) - Y(t) \tag{2}
  \]
- Deflated prices and index changes:
  \[
  Z_D(t) := Z(t) \times D(t) \tag{3}
  \]
  Where \(D(t)\) is a statistical factor or index called a discount or a deflation factor, and used to adjust the time value of money, enabling the comparison of prices while accounting for inflation, devaluation, etc. in different time periods.
- Returns, defined as:
  \[
  R(t) := \frac{Y(t + \Delta t) - Y(t)}{Y(t)} \tag{4}
  \]
- Differences of the natural logarithm of prices\(^3\), defined for some interval of time \(\Delta t\) as:
  \[
  S(t) := \ln Y(t + \Delta t) - \ln Y(t) \tag{5}
  \]
  Each one having its own merits and disadvantages [6]. In this analysis, we have used the former variable \(S(t)\).

3 Data sample and IPC variations

The database containing the IPC series analyzed here is available at [32] and covers the 13-year period 04/19/1990 - 08/21/2003. Figure 1 shows the IPC evolution for this time period. We have used in our analysis the daily closure values of the IPC, that is, its recorded value at the end of each trading day

In this work, our observable is \(S(t)\) as defined in equation 5 and we studied the tail behavior of \(P(S(t)) = 1 - F(S(t))\), for \(t = 1, \ldots, N\), where \(N = 3337\) is our sample size and \(\Delta t = 1\) day.

Figure 3 shows the histogram of \(Z(t)\), the IPC daily changes. It is interesting that this strongly symmetric and leptokurtic (fat tailed) distribution does not follow any well known model, which could appropriately describe the probability of events in its central region and in its tails at once.

\(^3\) Note: Some authors call returns to the difference of the natural logarithm of prices, and call normalized returns to our returns. For the case of high frequency data, each one approaches the other.
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4 Parameter estimation from IPC empirical data

The procedure to estimate \( \alpha \), the Pareto-Levý exponent from empirical data is straightforward: Pareto-Levý tails behave as straight lines in a logarithmic plot. Then, after performing a linear fit of log \( P(S(t)) \) on log \( S(t) \), the obtained slope gives us an estimate of the exponent \( \alpha \) of the Pareto-Levý Distribution. Clearly a distribution whose tails do not behave linearly in a log-log plot, can not be properly described by the Pareto-Levý model.

4 In finance, volatility is a relative measure of price movement during a given time period. It can be modeled by the standard deviation of stock price changes. Econophysicists usually use absolute returns to model the dynamics of volatility.

5 The 2003 Nobel Price of Economy was awarded jointly to Robert F. Engle for a related topic:...for methods of analyzing economic time series with time-varying volatility (ARCH)". Engle owns an M. S. in Physics by Cornell University (1966).
The fit was carried out using data available in the tails regions \(|S(t)| > 0.035\). Both tails are marked with vertical lines in figure 4. Those regions were chosen simply by examining the set of points for which the corresponding log-log plot behaves linearly. \(P(S(t))\) was then reduced to 79 and 93 events in its negative and positive tails respectively. Note that, as is showed in figure 4, and in order to deal with the undefined logarithmic scale for the left tail of \(P(S(t))\), we have used \(-S(t)\) in our analysis. It was found, that for these regions, the tails of \(P(S(t))\) decay following a power law model. A straight line provides a good fit for them in a logarithmic plot. Both fits are shown in figure 4.

![Positive Tail](image1.png)  ![Negative Tail](image2.png)  

**Fig. 4.** Linear fitted tails in a log-log plot of the cumulative distribution function \(P(S(t))\) on \(S(t)\). Right image positive tail. Left image negative tail. Fitted parameters are shown.

Table 1 shows the estimated parameters and the 95% confidence intervals obtained from the linear regression fit for the negative and positive tails of \(P(S(t))\).

| Fitted region  | \(\alpha\)   |
|----------------|-------------|
| \(S(t) > 0.035\) | 3.634 ± 0.272 |
| \(S(t) < -0.035\) | 3.540 ± 0.278 |

**Table 1.** Fitted Parameters plus minus twice the standard error of the estimates, for positive and negative tails.

5. Discussion

Results shown in table 1 are consistent with similar studies, where the Pareto-Levéy model with \(2.5 < \alpha < 4.0\), has been found be useful for describe the behavior of extreme variations of diverse financial markets. Table 2 summarizes results of some of these studies. High frequency studies of price variations, most of them performed for stock markets belonging to developed countries show that the distribution of returns follows a Pareto-Levéy form with exponent converging to \(\alpha \approx 3\) as \(\Delta t\) decreases to time intervals of about one minute. For the case of stock markets of emergent economies, it seems they may belong to a different universality class, some studies \[36, 37\] show that the return distributions from emergent markets have fatter tails than the observed in developed markets.

In summary, it has been shown that the cumulative probability of daily extreme logarithmic changes of the Mexican IPC index, can be approximated by the Pareto-Levéy model, with exponents \(\alpha = 3.634 ± 0.272\) and \(\alpha = 3.540 ± 0.278\) for its positive and negative tails respectively. As a consequence of these values, we can affirm that the stochastic process that governs the time series \(S(t)\), is well outside the Lévy stable regime \((0 < \alpha < 2)\).

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**Table 2.** Pareto exponent for some international Stock Markets. Daily data (d). High Frequency data (*).

| Market       | \(\alpha\) (Right tail) | \(\alpha\) (Left tail) | Data Period   |
|--------------|-------------------------|------------------------|---------------|
| AMEX         | 2.84 ± 0.12             | 2.73 ± 0.14            | 01/94-12/95   |
| NASDAQ       | 3.00 ± 0.10             | 2.97 ± 0.12            | 1962-1996d    |
| NYSE (Combined) (USA) | 3.66 ± 0.011 | 3.61 ± 0.11 | 10/97-12/99* |
| (Germany)    | 3.30 ± 0.05             | 3.37 ± 0.07            | 1984-1996     |
| NIKKEI       | 2.4 (minutely)          | 3.61 ± 0.11            | 1950-2001d    |
| Hang-Seng    | 3.05 ± 0.16             | 3.98 ± 0.16            | 1984-1997     |
| (Hong Kong)  | 3.03 ± 0.16             | 3.07 ± 0.16            | 1986-1997     |

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