String Percolation and the Glasma

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Abstract

We compare string percolation phenomenology to Glasma results on particle rapidity densities, effective string or flux tube intrinsic correlations, the ridge phenomena and long range forward-backward correlations. Effective strings may be a tool to extend the Glasma to the low density QCD regime. A good example is given by the minimum of the negative binomial distribution parameter $k$ expected to occur at low energy/centrality.

The mechanism of parton saturation [1] and of string fusion and percolation [2] have been quite successful in describing the basic facts, obtained mostly at RHIC, of the physics of QCD matter at higher density. Here, we would like to discuss the results from string percolation [3,4] in comparison with what has been obtained in the Color Glass Condensate (CGC) and in the Glasma [5,6,7].

Strings are supposed to describe confined QCD interactions in an effective way [8,9]. They carry color charges at the ends and an extended force field between the charges. They emit particles by string breaking and pair creation. Projected in the impact parameter plane they look like disks and two-dimensional percolation theory can be applied [3,10]. Interaction between strings occurs when they overlap and the general result, due to the SU(3) random summation of charges, is that there is a reduction in the final color charge, which means a reduction in multiplicity, and an increase in the

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string tension or an increase in the average transverse momentum squared, $\langle p_T^2 \rangle$ [3].

Saturation phenomena result from the overcrowding in impact parameter of low $x$ partons of a boosted hadron or nucleous, leading to the appearance of a scale, $Q_s^2$, related to the transverse momentum of the partons, $Q_s^2 \sim \langle k_T^2 \rangle$. This is the basic idea of CGC (Color Glass Condensate). The saturation scale naturally increases with $N_{part}$, the number of participating nucleons and with the beam rapidity $Y$. Hadronic and nuclear collisions are described in terms of collisions of two CGC sheets, generating extended in rapidity longitudinal electric and magnetic fields: the flux tubes. In string percolation the strings, resulting from partonic interactions may overlap and fuse, and 2-D percolation theory is applied.

The relevant parameter is the transverse string density $\eta$, which increases with $N_{part}$ and rapidity $Y$. String fusion leads to reduction of particle density at mid rapidity, and because of energy-momentum conservation, to an increase of the rapidity length of the effective strings. We shall show that for $Q_s^2 > \Lambda_{QCD}^2$ and $\eta > \eta_c \simeq 1.2$, where $\eta_c$ is the critical density for percolation, we have $Q_s^2 \sim \sqrt{\eta}$. Note that randomness in the summation of color fields is a feature common to CGC and string percolation.

The basic formulae are, for particle density,

$$\frac{dn}{dy} = F(\eta) N_s \mu ,$$

(1)

and, for $\langle p_T^2 \rangle$,

$$\langle p_T^2 \rangle = \langle p_T^2 \rangle_1 / F(\eta) ,$$

(2)

where $F(\eta)$ is the color reduction factor

$$F(\eta) \equiv \sqrt{\frac{1 - e^{-\eta}}{\eta}} ,$$

(3)

with $F(\eta) \to 1$ as $\eta \to 0$ and $F(\eta) \to 0$ as $\eta \to \infty$, where $\eta$ is the 2-dimensional transverse density of strings,

$$\eta \equiv \left( \frac{r_0}{R} \right)^2 \bar{N}_s .$$

(4)

Note that the ratio Eq(2)/Eq(1) gives

$$\frac{\langle 2 \rangle}{\langle 1 \rangle} = \frac{\langle p_T^2 \rangle_1}{\langle p_T^2 \rangle} \left( \frac{R}{r_0} \right)^2 / \mu(1 - e^{-\eta})$$

(5)
which for large $\eta$, becomes constant, as seen in LHC [24]. The quantities $\mu$, $\langle p_T^2 \rangle_1$ and $r_0$ are the particle density, the average transverse momentum squared and the radial size of the single string respectively, and $R$ is the radial size of the overlapping region of interaction. $N_s$ is the (average) number of strings. $1/F(\eta)$, or more specifically the large $\eta$ limit $\sqrt{\eta}$, plays the role of the saturation scale $Q_s^2$ of CGC.

It should be noticed that the quantities characteristic of independent single strings, $r_0^2$ and $\langle p_T^2 \rangle_1$, are conjugate variables and we expect in general, from (2),

$$r_0^2 \langle p_T^2 \rangle_1 \equiv (F(\eta)r_0^2)(\langle p_T^2 \rangle_1/F(\eta)) \simeq 1/4,$$  \hspace{1cm} (6)  

where $F(\eta)r_0^2$ (or $r_0^2/\sqrt{\eta}$, for large $\eta$) plays the role of the area of the effective string in a medium.

As far as electric field is concerned the effective strings can be identified with the flux tubes of the Glasma picture [5,6]. The area occupied by the strings divided by the area of the effective string gives the average number $<N>$ of effective strings,

$$<N> = \frac{(1-e^{-\eta})R^2}{F(\eta)r_0^2},$$  \hspace{1cm} (7)  

or

$$<N> = (1-e^{-\eta})^{1/2}\sqrt{\eta}\left(\frac{R}{r_0}\right)^2.$$  \hspace{1cm} (8)  

Note that the average number of effective strings divided by the number of strings,

$$<N> / \bar{N}_s = F(\eta)$$  \hspace{1cm} (9)  

goes to zero as $\eta$ (energy/number of participants) increases and goes to one as $\eta$ goes to zero. Our formulae, from (1) to (9), are valid both in the low density and in the high density regimes.

In the process of fusion of strings one has to take care of energy-momentum conservation, which implies an increase in the length in rapidity of the string, [3,11], with

$$\Delta y_{\bar{N}_s} = \Delta y_1 + 2 \ln \bar{N}_s.$$  \hspace{1cm} (10)  

One further notes that overall conservation of energy/momentum requires for the number of strings to behave as

$$\bar{N}_s \sim s^\lambda \sim e^{2\lambda Y}$$  \hspace{1cm} (11)
where $Y$ is the beam rapidity, $Y = \ln(\sqrt{s}/m)$, and $\lambda \approx 2/7$.

From (10) and (11) above we conclude

$$
\Delta y_{N_s} \simeq 2\lambda \Delta Y \simeq 1/2\Delta Y .
$$

As in the CGC the length in rapidity of the classical fields is $1/\alpha_s \left( Q_s^2 \right)$ and as the saturation scale is power behaved in $Y$, we end up in a formula of the kind of (12).

Turning back to Eq. (8), we note that it clearly shows the presence of two regimes, a high density one, for $\eta \gg 1$, and a low density one for $\eta \ll 1$:

i) High density regime, $\eta \gg 1$

$$
<N> \simeq \sqrt{\frac{R}{r_0}} \eta^2 , \quad <N> \sim N_{part.} e^{\lambda Y} ,
$$

(13)

ii) Low density regime, $\eta \ll 1$

$$
<N> = \eta \left( \frac{R}{r_0} \right)^2 , \quad <N> \sim N_{part.}^{4/3} e^{2\lambda Y} .
$$

(14)

In (13) and (14) we have made the reasonable assumptions that $R$ (going like) $N_{part.}^{1/3}$ and $N_s$ (going like) $N_{part.}^{4/3}$. Note that the high density regime (13), $\sqrt{\eta} \left( \frac{R}{r_0} \right)^2 \sim N_{part.} \times e^{\lambda Y}$ is equivalent to the high density regime of CGC: $Q_s^2 R^2 \sim N_{part.} \times e^{\lambda Y}$. However, there is no equivalent for the low density regime (14).

If we write the particle density normalized to $2/N_{part.}$ we obtain, from (8),

$$
\frac{2}{N_{part.}} \frac{dn}{dy} \sim \left( 1 - e^{-\eta} \right)^{1/2} .
$$

(15)

In the CGC, as $dn/dy \sim \frac{1}{\alpha_s(Q_s^2)} Q_s^2 R^2$, one obtains

$$
\frac{2}{N_{part.}} \frac{dn}{dy} \sim \frac{1}{\alpha_s(Q_s^2)} \sim \ln(Q_s^2) .
$$

(16)

Note that as the number of participants increases (15) becomes flatter as $N_{part.}$ increases, and (16) shows a slow increase: $\sim 1/3 \ln \left( N_{part.}/2 \right)$. For data and fits, see [13,14,15].
If one wants to go from particle density to multiplicity distributions or correlations, one has to take into account fluctuations in the number of effective strings/flux tubes. Emission from free strings will be considered of Poisson type, as suggested from $e^+e^-$ annihilations at low energy. We work in the "two step scenario" [16] and write:

$$\Re \equiv \frac{\langle n^2 \rangle - \langle n \rangle^2}{\langle n \rangle^2} = \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle^2} \equiv 1/K ,$$

(17)

where $\Re$ is the normalized 2-particle correlation, $n$ stands for the number of produced particles, $N$ for the number of effective strings and $1/K$ is the normalized fluctuation of the $N$-distribution. If the particle distribution is negative binomial with a $NB$ parameter $k_{NB}$, then $K \equiv k_{NB}$.

In the low density regime the particle density, as mentioned above, is essentially Poisson and we have

$$\eta \to 0 , \quad K \to \infty .$$

(18)

In the large $\eta$, large $\langle N \rangle$ limit, if one assumes that the $N$-effective strings behave like a single string, with $\langle N^2 \rangle - \langle N \rangle^2 \simeq \langle N \rangle$, one obtains

$$\eta \to \infty , \quad K \to \langle N \rangle \to \infty .$$

(19)

Such possibility, (18) and (19), was previously discussed in [17] with a somewhat different definition for $K$. See also the work of [18].

An important consequence of (17) and (19), if one assumes that the effective strings in the high density limit emit particles as independent sources, is that $K_1$ for the single effective string is given by

$$K_1 \equiv \frac{K}{\langle N \rangle} \simeq 1 ,$$

(20)

corresponding to Bose-Einstein distribution. That behavior was pointed out before in the Glasma [7], as an amplification of the intensity of multiple emitted gluons.

A parametrization for $K$ satisfying (18) and (19) is

$$K \simeq \frac{\sqrt{\eta (R/r_0)^2}}{(1 - e^{-\eta})} \simeq \frac{\langle N \rangle}{(1 - e^{-\eta})^{3/2}} .$$

(21)

This curve shows a minimum at $\eta \simeq 1.2$. As the number of strings as a function of energy can be estimated from particle density, (1), and $\eta$ can
be constructed as a function of $\sqrt{s}$ (see [17]), in the $pp$ case the minimum of (20), is not reached yet, as it corresponds to an energy of the order of 3-4 TeV. The rule to go from $\eta_{pp}$ to $\eta_{AA}$ is: $\eta_{AA} = \eta_{pp}N_A^{2/3}$, where $N_A$ is the number of participants per nucleus $A$. In [19] a study was carried out at RHIC and it was shown the $k$ increases with centrality, the less dense situation corresponding to $Cu - Cu$ at 22.5 GeV, and $N_A = 29$. Using our rule: $\eta_{CuCu} \simeq 0.2 \times 9 \simeq 1.8$, is above the minimum of (20). In conclusion: at low density we may have $k$ decreasing with increasing $\eta$ ($pp$ case) and at larger density we may have $k$ increasing with increasing $\eta$ ([19] data).

Recent results from Alice show that the negative parameter $k$ from 0.9 to 2.38 TeV slightly decreases with energy to become at 7 TeV, constant or even slightly increasing with energy, in agreement with fig(3) of [17].

In the CGC, one takes $k \simeq \langle N \rangle$ [6], [7], being implicit that the relation only works for the high density regime.

The correlations introduced in (17) are relevant in the discussion of the ridge phenomenon, discovered at RHIC: a correlated broad peak of particles, occurring with and without jet trigger, extended in rapidity and localized in the azimuthal angle $\phi$ [20,21]. The quantity plotted, $\Delta \rho/\sqrt{\rho_{ref.}}$, is the density of particles correlated with a particle emitted at zero rapidity. The quantity $\Delta \rho$ is the difference in densities between single events and mixed events, $\rho_{ref.}$ coming from mixed examples. Correlations due to azimuthally asymmetric flow have to be included.

As the basic formula is the same, (17), we conclude that the CGC calculation [5-7]

$$\Delta \rho/\sqrt{\rho_{ref.}} = R_{dn/dy}F(\phi)$$

(22)

where $F(\phi)$ describes the azimuthal dependence taken from an independent model, is equivalent to a string percolation calculation. In CGC we have

$$R_{dn/dy} \simeq 1/ \propto_s (Q_s^2)$$

(23)

and in string percolation, see (20),

$$R_{dn/dy} \simeq \langle N \rangle/K \simeq (1 - e^{-\eta})^{3/2}.$$  

(24)

Probably, the most interesting problem is the problem of forward-backward rapidity correlations. In strings and flux tubes there is a uniform field between the colour charges. In the single string ($e^+e^-$ at low energy) there
are no intrinsic forward-backward correlations, \( p(n_F, n_B) = p(n_F)p(n_B) \) and the correlation parameter \( b \equiv \frac{\text{Covariance}}{\text{Variance}} \) is zero. The forward-backward correlation arises exclusively from fluctuations in the number of strings, and fluctuations in production from a single string. With flux tubes or effective strings in dense matter the situation may be different, and particles from the same flux tube may be correlated (independent of fluctuations in the number of flux-tubes).

In string percolation we shall use the traditional formula [22,16] for a window \( \delta y = 1 \), include the small correction due to the use of Bose-Einstein distribution instead of Poisson,

\[
b = \frac{1}{1 + A} \approx \frac{1}{1 + \langle N \rangle}.
\]  

(25)

In the Glasma approach one considers correlations along the flux-tube, neglects fluctuations in the number of flux-tubes to obtain [23]

\[
b = \frac{1}{1 + A} = \frac{1}{1 + \infty_s^2 (Q_s^2) c}.
\]  

(26)

In both cases, percolation and CGC, \( b \) increases with both energy and centrality and will remain constant as a function of the length of the rapidity interval between forward and backward windows, for lengths increasing logarithmically with energy and centrality.

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