IMPACT OF A PRECISE TOP MASS MEASUREMENT

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The physics impact of a precise determination of the top-quark mass, \( m_t \), at the Linear Collider (LC) is discussed, and the results are compared with the prospective accuracy at the LHC. The importance of a precise knowledge of \( m_t \) for electroweak precision observables and for Higgs physics in the MSSM is pointed out in particular. We find that going from hadron collider to LC accuracy in \( m_t \) leads to an improvement of the investigated quantities by up to an order of magnitude.

1 Introduction

The mass of the top quark, \( m_t \), is a fundamental parameter of the electroweak theory. It is by far the heaviest of all quark masses and it is also larger than the masses of all other known fundamental particles. The large value of \( m_t \) gives rise to a large coupling between the top quark and the Higgs boson and is furthermore important for flavour physics. It could therefore provide a window to new physics. The correct prediction of \( m_t \) will be a crucial test for any fundamental theory. The top-quark mass also plays an important role in electroweak precision physics, as a consequence in particular of non-decoupling effects being proportional to powers of \( m_t \). A precise knowledge of \( m_t \) is therefore indispensable in order to have sensitivity to possible effects of new physics in electroweak precision tests.

The current world average for the top-quark mass is \( m_t = 178.0 \pm 4.3 \text{ GeV} \). The prospective accuracy at the Tevatron and the LHC is \( \delta m_t = 1-2 \text{ GeV} \), while at the LC a very precise determination of \( m_t \) with an accuracy of \( \delta m_t \lesssim 100 \text{ MeV} \) will be possible. This error contains both the experimental error of the mass parameter extracted from the \( t\bar{t} \) threshold measurements at the LC and the envisaged theoretical uncertainty from its transition into a suitable short-distance mass (like the \( \overline{\text{MS}} \) mass).

In the following some examples of the impact of a precise determination
2 Electroweak Precision Observables

Electroweak precision observables (EWPO) can be used to perform internal consistency checks of the model under consideration and to obtain indirect constraints on unknown model parameters. This is done by comparing experimental results of the EWPO with their theory prediction within, for example, the Standard Model (SM) or its minimal supersymmetric extension (MSSM).

There are two sources of theoretical uncertainties: those from unknown higher-order corrections (“intrinsic” theoretical uncertainties), and those from experimental errors of the input parameters (“parametric” theoretical uncertainties). The intrinsic uncertainties within the SM are \( \Delta M_W^{\text{intr, today}} \approx 4 \text{ MeV} \), \( \Delta \sin^2 \theta_{\text{eff, intr, today}} \approx 5 \times 10^{-5} \) at present. The parametric uncertainties induced by the current experimental error of \( m_t \) are \( \Delta M_W^{\text{para, today}} \approx 26 \text{ MeV} \) and \( \Delta \sin^2 \theta_{\text{eff, para, today}} \approx 14 \times 10^{-5} \). They are larger than the uncertainties induced by the experimental errors of all other input parameters and are almost as large as the current experimental errors of \( M_W \) and \( \sin^2 \theta_{\text{eff}} \). A future experimental error of \( \delta m_t \approx 1.5 \text{ GeV} \) at the LHC will give rise to parametric uncertainties of \( \Delta M_W^{\text{para, LHC}} \approx 9 \text{ MeV} \), \( \Delta \sin^2 \theta_{\text{eff, LHC}} \approx 4.5 \times 10^{-5} \), while the LC precision of \( \delta m_t \approx 0.1 \text{ GeV} \) will reduce the parametric uncertainties to \( \Delta M_W^{\text{para, LC}} \approx 1 \text{ MeV} \), \( \Delta \sin^2 \theta_{\text{eff, LC}} \approx 0.3 \times 10^{-5} \). A comparison with the parametric uncertainties induced by the other input parameters shows that the LC accuracy on \( m_t \) will be necessary in order to keep the parametric error induced by \( m_t \) at or below the level of the other uncertainties. With the LHC accuracy on \( m_t \), on the other hand, \( \delta m_t \) will be the dominant source of uncertainty.

In Fig. 1 the predictions for \( M_W \) and \( \sin^2 \theta_{\text{eff}} \) in the SM and the MSSM are shown in comparison with the prospective experimental accuracy obtainable at the LHC and a LC with GigaZ option (low-energy running at the \( Z \)-boson resonance and the \( WW \)-threshold). The MSSM parameters have been chosen in this example according to the reference point SPS 1b, and all SUSY parameters have been varied within realistic error intervals. The figure shows that the improvement in \( \delta m_t \) from \( \delta m_t = 2 \text{ GeV} \) to \( \delta m_t = 0.1 \text{ GeV} \) strongly reduces the parametric uncertainty in the prediction for the EWPO. In the SM case it leads to a reduction by about a factor of 10 in the allowed parameter space of the \( M_W - \sin^2 \theta_{\text{eff}} \) plane. In the MSSM case, where many additional parametric uncertainties enter, a reduction by a factor of more than 2 is obtained in this example. This precision will be crucial to establish effects of new physics via EWPO.
80.30 80.35 80.40 80.45

M

W

[GeV]

0.2312

0.2314

0.2316

0.2318

sin

2

θ

eff

δ

m

t

exp

= 2.0 GeV

δ

m

t

exp

= 0.1 GeV

m

h

= 115 GeV,

δ∆α

had

= 7 \times 10^{-5}

MSSM

(SPS1b)

SM

prospective exp. errors 68\% CL:

LHC/LC

GigaZ

Figure 1: Predictions for \( M_W \) and \( \sin^2 \theta_{\text{eff}} \) in the SM and the MSSM (SPS 1b). The inner (blue) areas correspond to \( \delta m_t = 0.1 \text{ GeV} \) (LC), while the outer (green) areas arise from \( \delta m_t = 2 \text{ GeV} \) (LHC). The anticipated experimental errors on \( M_W \) and \( \sin^2 \theta_{\text{eff}} \) at the LHC/LC and at a LC with GigaZ option are indicated.

3 Implications For The MSSM

In contrast to the SM, where the Higgs-boson mass is a free input parameter, the mass of the lightest CP-even Higgs boson in the MSSM, \( m_h \), can be predicted in terms of other parameters of the model. While the tree-level prediction for \( m_h \) arises from the gauge sector of the theory, large Yukawa corrections from the top and scalar top sector (for large values of \( \tan \beta \), the ratio of the vacuum expectation values of the two Higgs doublets, also from the bottom and scalar bottom sector) enter at the loop level. The leading one-loop correction is proportional to \( m_t^4 \). The one-loop corrections can shift \( m_h \) by 50–100%.

Since these very large corrections are proportional to the fourth power of the top-quark mass, the predictions for \( m_h \) and many other observables in the MSSM Higgs sector strongly depend on the precise value of \( m_t \). As a rule of thumb, \(^9\) a shift of \( \delta m_t = 1 \text{ GeV} \) induces a parametric theoretical uncertainty of \( m_h \) of also about 1 GeV, i.e. \( \Delta m_h^{\delta m_t} \approx \delta m_t \).

In Fig. 2 the impact of the experimental error of \( m_t \) on the prediction for \( m_h \) in the MSSM is shown. The parameters are chosen according to the \( m_h^{\text{max}} \) benchmark scenario\(^{10}\). The band in the left plot\(^{11}\) corresponds to the present experimental error of \( m_t \),\(^{1,2}\) while in the right plot the situation at the LHC (\( \delta m_t = 1, 2 \text{ GeV} \)) is compared to the LC (\( \delta m_t = 0.1 \text{ GeV} \)). The figure shows that the LC precision on \( m_t \) will be necessary in order to match the experimental precision of the \( m_h \) determination with the accuracy of the theory prediction (assuming that the intrinsic theoretical uncertainty can be reduced to the same level, see Ref.\(^ {12}\)).
Figure 2: Prediction for $m_h$ in the $m_h^{\text{max}}$ scenario of the MSSM as a function of $\tan \beta$ (left) and the mass of the CP-odd Higgs boson, $M_A$ (right). In the left plot the impact of the present experimental error of $m_t$ on the $m_h$ prediction is shown. The three bands in the right plot correspond to $\delta m_t = 1.2$ GeV (LHC) and $\delta m_t = 0.1$ GeV (LC). The anticipated experimental error on $m_h$ at the LC is also indicated.

Further examples of the importance of a precise determination of $m_t$ in the MSSM are the prediction of sparticle masses, parameter determinations, and the reconstruction of the supersymmetric high scale theory.

References

1. P. Azzi et al., hep-ex/0404010.
2. V. M. Abazov et al., Nature 429 (2004) 638.
3. M. Beneke et al., hep-ph/0003033.
4. J. A. Aguilar-Saavedra et al., hep-ph/0106315; T. Abe et al. hep-ex/0106055; K. Abe et al. hep-ph/0109166.
5. A. Hoang et al., Eur. Phys. Jour. C 3 (2000) 1; M. Martinez, R. Miquel, Eur. Phys. Jour. C 27 (2003) 49; S. Boogert, these proceedings.
6. S. Heinemeyer, S. Kraml, W. Porod, G. Weiglein, JHEP 0309 (2003) 075.
7. M. Awramik, M. Czakon, A. Freitas, G. Weiglein, Phys. Rev. D 69 (2004) 053006; hep-ph/0407317.
8. B. Allanach et al., Eur. Phys. Jour. C 25 (2002) 113.
9. S. Heinemeyer, W. Hollik, G. Weiglein, JHEP 0006 (2000) 009.
10. M. Carena, S. Heinemeyer, C. Wagner, G. Weiglein, hep-ph/9912223; Eur. Phys. Jour. C 26 (2003) 601.
11. G. Weiglein, Nature 429 (2004) 613.
12. G. Degrassi, S. Heinemeyer, W. Hollik, P. Slavich, G. Weiglein, Eur. Phys. Jour. C 28 (2003) 133.