Lattice Boltzmann Model based 3D Image Segmentation

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Abstract. Image segmentation plays a very important role in three-dimensional volume data processing. In three-dimensional segmentation, the curve evolution method based on geometric active contour model and level set method is one of the most widely concerned methods. Among these methods, the CV model is a hot research topic. Based on the Mumford-Shah model, CV model is proposed by Chan and Vese. Because of the huge amount of data in three-dimensional images, the computation of solving CV model is very large, which consumes a lot of time. In this study, a 3D image segmentation algorithm based on Lattice Boltzmann model is proposed. The experiment shows our method is effective. The algorithm is simple and efficient. More importantly, the LB model has natural parallelism. Especially, our method can be implemented on massively parallel image processing platform, for example, FPGAs, DSPs and GPUs.

1. Introduction

Image segmentation plays a very important role in three-dimensional volume data processing. It is the basis of volume data analysis and visualization. Moreover, three-dimensional image segmentation has always been a classic challenge in the research of image processing[1][2][3]. Because of its importance and difficulty, since the 1970s, image segmentation has attracted a lot of researchers' efforts. In three-dimensional segmentation, the curve evolution method based on geometric active contour model[4][5] and level set method[6][7] is one of the most widely concerned methods. Among these methods, Among these methods, the CV model[8] is a hot research topic. Based on the Mumford-Shah model[9], CV model is proposed by Chan and Vese. However, because of the huge amount of data in three-dimensional images, the computation of solving C-V model is very large, which consumes a lot of time. To overcome this problem, a 3D image segmentation algorithm based on Lattice Boltzmann model[10] is proposed in our study. The algorithm is simple and efficient. More importantly, the LB model has natural parallelism. Especially, our method can be implemented on massive parallel image processing platform, for example, embedded FPGAs, DSPs and GPUs.

2. Basic Lattice Boltzmann Model

LB model is actually a “Bottom-up” numerical tool for partial differential equation by simulating the particles’ streaming and collision on lattice nodes. Starting from the system’s micro-behavior, the LBM model realizes the simulation of the macro-equation of the system which traditionally are described by partial differential equations. The simulation by LB model can be divided into 2 steps: the first is a streaming step. The particles moves between each others on the lattice in variant directions. The second step is collision. The particles collide inside the nodes and the particle densities
are redistributed. Combine the two steps, we can represent the total procedure by lattice Boltzmann equation (LBE). In this section, a D2Q9 model \[12\] is introduced, where "D2" means "2 dimensions", while "Q9" means "9 speeds". The discrete LBE is written as:

\[
f_i \left( r + \sigma_i e_i, t + \tau \right) - f_i \left( r, t \right) = \Omega_i \left( f \right) = -\frac{1}{\tau} \left( f_i \left( r, t \right) - f_i^{eq} \left( r, t \right) \right) \tag{1}
\]

The direction \( e_i \) is defined as:

\[
e_i = \begin{cases} (0,0), & i = 0 \\ (1,0), & i = 1,3,5,7 \\ (1,1), & i = 2,4,6,8 \\ \end{cases}, \quad \sigma_i = v \sigma_i \tag{2}
\]

where \( v \) is the velocity of the particles.

The right side of the equation (1) describes the collision step. Parameter \( \tau \) is the relaxation factor, which determines the time of reaching to equilibrium. The equilibrium distribution functions \( f_i^{eq} \left( r, t \right) \) in direction \( e_i \) is only determined by the total density \( \rho \) which is defined as follows in our study:

\[
f_i^{eq} = \begin{cases} \rho / (9c), & i = 1,2,3,...,8 \\ 1 - 8\rho / (9c), & i = 0 \end{cases} \tag{4}
\]

where

\[
\sum_{i=0}^{8} f_i = \sum_{i=0}^{8} f_i^{eq} = \rho \tag{5}
\]

\( f_i^{eq} \) and \( f_i \) must be positive, and \( c \) is the regulator for the diffusion speed. Both sides of equation (1) can be Tylor expanded, and base on Chapman-Enskog expansion, we can obtain the global description of the LB model in the form of PDE as follows:

\[
\frac{\partial \rho}{\partial t} = \frac{\sigma_i^2}{18\sigma_i} \text{div} \left[ \nabla \left( \left( \tau - 0.5 \right) \rho \right) \right]. \tag{6}
\]

3. LB method based Three-Dimensional C-V model

The model in section 2 can easily extend to three-dimensional model. A D3Q15 model is used in our study. The evolution equation is:

\[
f \left( r + \sigma_i e_i, t + \tau \right) - f \left( r, t \right) = -\frac{1}{\tau} \left( f_i \left( r, t \right) - f_i^{eq} \left( r, t \right) \right) \tag{7}
\]

where \( F = \left[ \left( I - c_i \right)^2 - \left( I - c_2 \right)^2 \right] \). \( c_i \) is the average gray level inside the surface and \( c_2 \) outside.

The equilibrium distribution function is as follows:

\[
f_i^{eq} = \begin{cases} \rho / \left( 15c \right), & i = 1,2,3,...,14 \\ 1 - 14\rho / \left( 15c \right), & i = 0 \end{cases} \tag{8}
\]

Direction \( \vec{e}_i \) (as shown in figure 1) is defined as:

\[
\vec{e} = \left[ e_0, e_1, e_2, ..., e_{14} \right] = \begin{bmatrix} 0 & 1 & -1 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 \end{bmatrix}. \tag{9}
\]

Similar as section 2, we can obtain get the macroscopic equation:
\[ \frac{\partial \rho}{\partial t} = \frac{2\sigma}{3c} \text{Div} \left( \frac{\nabla \rho}{\rho} \right) + \sigma \left[ (I - c_1)^2 - (I - c_2)^2 \right]. \] \tag{9}

4. Experiment and Discussion

The algorithm consists of following steps:

a. Initialize the distance function \( \rho \);

b. Initialize \( f_i = \begin{cases} \rho / (15c), & i = 1, 2, 3, \ldots, 14 \\ 1 - 14\rho / (15c), & i = 0 \end{cases} \)

c. Initialize \( f_i^{eq} = \begin{cases} \rho / (15c), & i = 1, 2, 3, \ldots, 14 \\ 1 - 14\rho / (15c), & i = 0 \end{cases} \)

d. Collision and streaming according to formula (7);

e. Update distance function \( \rho(x, y, z) = \sum_i f_i \).

f. Goto step c.

It should be noted that the value of the distance function of the model must be greater than 0 to ensure the stability of the LB model. That is because the LB will get unstable if the particle density is negative. Because of this, we initialize the distance function as:

\[ \rho(x, y, z) = B + \sqrt{x^2 + y^2 + z^2} - \text{radius} \] \tag{10}

where, \( \sqrt{x^2 + y^2 + z^2} - \text{radius} \) initialize the contour as a circle. The distance function is positive outside the contour and negative inside. Parameter B should satisfy \( B > \text{radius} \). Therefore distance function \( \rho > 0 \). According to equation (6), relaxation factor \( \tau \) should be larger than 0.5, otherwise equation (6) will get ill-posed. It should be highlighted that there are many 3D LB models except for D3Q15, e.g. D3Q7, D3Q19 and D3Q27. A LB model with more velocities can achieve higher calculation accuracy but more time consuming. In the experiment, we used the D3Q15 for considering the compromise between the calculation accuracy and time consuming.

Figure 1. The 3D image (181×217×181) and the segmentation
To test the performance of our LB model for 3D image segmentation, we used the BrainWeb MRI Simulated Normal Brain Database[12]. All the experiments are run on the PC with Intel(R) Core(TM) i7-4702MQ CPU, 8G RAM, with Matlab 2016. Figure 1 shows the 3D image (181×217×181) and the segmentation. Figure 2 gives the segmentation of the slices of original MR volume data. Figure 3 shows our segmentation in a 3D view.

Figure 2. segmentation of the slices(#60,#90 and # 120) of original MR volume data

Figure 3. Segmentation in 3D view (Displayed by ParaView) a. The contour of the MR volume data; b. one slice of segmentation
5. Conclusion
In this study, a 3D image segmentation algorithm based on Lattice Boltzmann model is proposed. The experiment shows our method is effective. The algorithm is simple and efficient. More importantly, the LB model has natural parallelism. Especially, our method can be implemented on massively parallel image processing platform, for example, FPGAs, DSPs and GPUs.

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