Language-dependent I-Vectors for LRE15

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Abstract

A standard recipe for spoken language recognition is to apply a Gaussian back-end to i-vectors. This ignores the uncertainty in the i-vector extraction, which could be important especially for short utterances. A recent paper by Cumani, Plchot and Férr proposes a solution to propagate that uncertainty into the backend. We propose an alternative method of propagating the uncertainty.

1 Introduction

A standard recipe for language recognition via i-vectors [1], is to extract the i-vectors—i.e. point estimates of the hidden variables—and then score them using a linear Gaussian back-end (LGBE).1

In this document, we combine the LGBE and the i-vector model into one monolithic model. In training and test, we can now integrate out the hidden i-vectors to directly produce language recognition scores, without having to go via explicit point estimates of the i-vectors.

Since the i-vector model is intractable in closed form, we resort to mean-field VB, using an approximate posterior where the GMM state path and i-vector posterior are independent. We compare our recipe to a similar one by Cumani, Plchot and Férr [2], which makes use of a language-independent i-vector posterior approximation. In our recipe, the i-vector posterior approximation is instead language-dependent and can be expected to more closely approximate the true posterior.

1The LGBE has a common, within-class covariance, shared by all languages and language-dependent means. The score is linear (affine, from i-vector to score vector), because the quadratic term in the Gaussian log-likelihood is language-independent and cancels.
On a practical note, if we already have extracted i-vectors, we can still apply our scoring recipe, as long as we have available the zero-order stats associated with each i-vector.

2 The model

The model is shown in figure 1. It is almost a standard i-vector extractor, except that we have allowed the i-vector prior to be non-standard, with a language dependent mean, $m_\ell$ and a within-class precision, $W$.

![Figure 1: The i-vector model.](image)

This model incorporates the i-vector extractor and a linear Gaussian back-end into one and will allow joint training of both and will allow language scores to be directly extracted, without having to go via intermediate i-vectors.

3 Plugin model parameters

Here we derive a plugin ML recipe for the model parameters $W$ and the $\mu_\ell$. (The i-vectors, $x_s$, are not plugged in, but instead integrated out.)

As mentioned in the introduction, we make the model tractable by a mean-field VB approach. More details of this approach can be found in [3].
3.1 The VB lower bound

We handle this model via mean-field VB, where the approximate posterior for the GMM path is fixed and given by the UBM responsibilities, $q_{st}$, which sum to unity over states, $i$. The i-vector posterior, $Q_{ts}(x)$ is language-dependent. The VB lower bound thus obtained is:

$$L_{ts} = \langle \log \frac{N(x \mid m_{t}, W^{-1})}{Q_{ts}(x)} + \sum_{t=1}^{T_s} \sum_{i=1}^{N} q_{st}^i \log \frac{w_i N(\phi_{st}^i \mid T_i x, I)}{q_{st}^i} \rangle_{Q_{ts}}$$

(1)

where $\phi_{st}^i$ denotes a feature vector, centred and whitened w.r.t. the parameters of UBM component $i$, so that we can further ignore the UBM parameters.

3.2 The i-vector posterior

The approximate i-vector posterior is:

$$\log Q_{ts}(x) = \log N(x \mid m_{t}, W^{-1}) + \sum_{t=1}^{T_s} \sum_{i=1}^{N} q_{st}^i \log N(\phi_{st}^i \mid T_i x, I) + \text{const}$$

$$= x' \left[ W_{m_{t}} + \sum_i T_{i} \left( \sum q_{st}^i \phi_{st}^i \right) \right] - \frac{1}{2} x' \left[ W + \sum_i T_{i} T_{i} \left( \sum q_{st}^i \right) \right] x + \text{const}$$

$$= x' \left[ W_{m_{t}} + a_{s} \right] - \frac{1}{2} x' \left[ W + B_{s} \right] x + \text{const}$$

(2)

This is a multivariate Gaussian. The factors in square brackets are the natural parameters of the Gaussian: the natural mean (precision times mean); and the precision (inverse covariance). For convenience, we have defined zero and first-order stats, $n_{is}$ and $f_{is}^i$, as well as $a_{s}$ and $B_{s}$, which represent the data-dependent parts of the natural mean and precision.

The language-independent posterior covariance is:

$$C_{s} = (W + B_{s})^{-1}$$

(3)

The language-dependent posterior mean is:

$$\mu_{ts} = C_{s}(W_{m_{t}} + a_{s})$$

(4)
We shall later need the posterior expectations:
\[ \langle x \rangle_{Q_{\ell s}} = \mu_{\ell s} \]  
(5)

and for some symmetric matrix \( M \):
\[ \langle x'Mx \rangle_{Q_{\ell s}} = \langle \text{tr}(xx'M) \rangle_{Q_{\ell s}} = \text{tr} \left( \langle xx' \rangle M \right) = \text{tr} \left( (C_s + \mu_{\ell s}\mu_{\ell s}')M \right) \]
(6)

### 3.3 Parameter updates

To learn the model parameters, we can alternate E and M steps. The E-step is computing the i-vector posterior. The M-step follows. To update \( m_\ell \), we need to maximize:

\[
\sum_{s \in S_\ell} L_{\ell s} = \sum_{s \in S_\ell} \langle \log \mathcal{N}(x | m_\ell, W^{-1}) \rangle_{Q_{\ell s}} + \text{const}
\]

\[ = \sum_{s \in S_\ell} -\frac{1}{2} m'_\ell W m_\ell + m'_\ell W \mu_{\ell s} + \text{const} \]

which is maximized, independently of \( W \), at:
\[ m_\ell = \bar{\mu}_\ell = \frac{1}{|S_\ell|} \sum_{s \in S_\ell} \mu_{\ell s} \]
(8)

To update \( W \), given the \( \bar{\mu}_\ell \), we need to maximize:
\[
W^{-1} = \frac{1}{N} \sum_{\ell, s} \bar{\mu}_\ell \bar{\mu}'_s - \bar{\mu}_\ell \mu_{\ell s}' - \mu_{\ell s} \bar{\mu}'_\ell + C_s + \mu_{\ell s}\mu_{\ell s}'
\]
\[ = \frac{1}{N} \sum_{\ell, s} C_s + \bar{\mu}_\ell (\bar{\mu}_s - \mu_{\ell s})' + \mu_{\ell s} (\mu_{\ell s} - \bar{\mu}_\ell)' \]
\[ = \frac{1}{N} \sum_{s=1}^{N} C_s + \frac{1}{N} \sum_{\ell} \sum_{s \in S_\ell} (\mu_{\ell s} - \bar{\mu}_\ell)(\mu_{\ell s} - \bar{\mu}_\ell)' \]
(10)
where $N$ is the total number of segments.\textsuperscript{2}

### 3.4 Language scores

We can form language scores (approximate log-likelihoods) by evaluating lower bound for each $\ell$, while omitting any terms independent of $\ell$. For a to-be-scored speech segment $s$, we compute separately for each language $\ell$, the lower bound:

$$L_{\ell s} = \left\langle \log \frac{N(x | m_{\ell}, W^{-1})}{Q_{\ell s}(x)} + \sum_{t=1}^{T_s} \sum_{i=1}^{N} q_{st}^i \log \frac{N(\delta_{st}^i | T_s, x, I)}{q_{st}^i} \right\rangle_{Q_{\ell s}}$$

We can simplify this expression by omitting any terms independent of $\ell$.\textsuperscript{3}

$$L_{\ell s} = \left\langle \log N(x | m_{\ell}, W^{-1}) + \sum_{t,i} q_{st}^i \log N(\delta_{st}^i | T_s, x, I) \right\rangle_{Q_{\ell s}} + \text{const}$$

So let’s drop the constant terms and define the language score as:

$$\sigma_{\ell s} = -\frac{1}{2}m_{\ell}'(W - WC_s W)m_{\ell} + m_{\ell}'WC_s a_s$$

To examine the behaviour of this score, keep in mind $C_s = (W + B_s)^{-1}$; and that $a_s$ and $B_s$ are zero at $T_s = 0$ and keep increasing with $T_s$. At $T_s = 0$, we get the nice effect $\sigma_{\ell s} = 0$. Conversely, for large $T_s$, we find $WC_s W$ eventually vanishes, while $C_s a_s$ converges.

\textsuperscript{2}The middle line simplifies to the last because the second term is zero and the last term can be symmetrized by viewing mean subtraction as multiplication by the idempotent centering matrix.

\textsuperscript{3}Note in particular, that the entropy term for $Q_{\ell s}$ is language-independent, because the entropy depends only on the covariance, not the mean.
to \( \tilde{\mu}_s \), the classical i-vector and the score reduces to that given by the stand-alone linear Gaussian back-end:

\[
\sigma_{\ell s} |_{T_s \gg 1} \approx -\frac{1}{2} m'_{\ell} W m_{\ell} + m'_{\ell} W \tilde{\mu}_s
\]

(14)

### 3.4.1 Practical scoring

The above scoring recipe is expressed in terms of \( a_s = \sum_i T_i f_s^i \) and \( B_s = \sum_i T_i T_i n_s^i \), which are in turn obtained from the first and zero-order stats, \( f_s^i \) and \( n_s^i \). We can therefore apply this recipe without explicitly going via i-vectors.

It is however possible (and perhaps preferable) to instead apply this recipe using already extracted i-vectors. The i-vectors are much smaller than the first-order stats and therefore much easier to work with on disk and in memory. The classical i-vector is:

\[
\tilde{\mu}_s = (I + B_s)^{-1} a_s
\]

(15)

We can therefore recover \( a_s \) from the i-vector:

\[
a_s = (I + B_s) \tilde{\mu}_s
\]

(16)

Of course, we also need \( B_s \). As long as we have \( T_i \) available, then \( B_s \) can be computed via the zero-order stats, \( n_s^i \), which are only moderately larger than the i-vectors and can be conveniently stored alongside them.

### 3.5 CPF Scoring

We can compare the above scoring recipe to Cumani-Pichot-Fér (CPF) scoring [2]. In the CPF recipe, the i-vector is also integrated out, but the classical i-vector posterior is used instead of the \( Q_{\ell s} \) of section 3.2.

The difference between the classical posterior and \( Q_{\ell s} \) is that the classical one uses a language-independent, standard normal prior. We can therefore expect \( Q_{\ell s} \) to be closer to the true posterior.

Denoting the classical posterior precision as \( E_s = I + B_s \), the classical, language independent i-vector (posterior mean) is \( \tilde{\mu}_s = E_s^{-1} a_s \).
The CPF score is:

\[
-\frac{1}{2}(\mu_s - m_\ell)'(W^{-1} + E_s^{-1})^{-1}(\mu_s - m_\ell) \\
= -\frac{1}{2}m_\ell'(W^{-1} + E_s^{-1})^{-1}m_\ell + m_\ell'(W^{-1} + E_s^{-1})^{-1}\mu_s + \text{const} \\
= -\frac{1}{2}m_\ell'W(W + E_s)^{-1}E_s m_\ell + m_\ell'W(W + E_s)^{-1}E_s \mu_s + \text{const} \\
= -\frac{1}{2}m_\ell'W(W + E_s)^{-1}E_s m_\ell + m_\ell'W(W + E_s)^{-1}a_s + \text{const}
\]

(17)

Defining \( \hat{C}_s = (W + E_s)^{-1} = (W + I + B_s)^{-1} \) in analogy to \( C_s = (W + B_s)^{-1} \) and dumping constant terms, the CPF score is:

\[
\tilde{\sigma}_\ell_s = -\frac{1}{2}m_\ell'W\hat{C}_sE_s m_\ell + m_\ell'W\hat{C}_sa_s
\]

(18)

This score behaves like \( \sigma_\ell_s \) for large \( T_s \) and also converges to the standalone linear Gaussian back-end. But for very small \( T_s \), the behaviour is different—in particular this score does not become independent of language at \( T_s = 0 \):

\[
\tilde{\sigma}_{\ell s}|_{T_s=0} = -\frac{1}{2}m_\ell'W(W + 1)^{-1}m_\ell
\]

(19)

References

[1] D. Martinez, O. Plchot, L. Burget and P. Matejka, “Language recognition in i-vectors space”, Interspeech, Florence Italy, 2011.

[2] S. Cumani, O. Plchot and R. Fér, “Exploiting i-Vector Posterior Covariances for Short-Duration Language Recognition”, Interspeech 2015, available: http://www.isca-speech.org/archive/interspeech_2015/i15_1002.html.

[3] Niko Brummer, “VB calibration to improve the interface between phone recognizer and i-vector extractor”, arXiv 2015, available: arxiv.org/abs/1510.03203.

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\(^4\)We use the identity: \( (W^{-1} + E_s^{-1})^{-1} = W(W + E_s)^{-1}E_s \). If this seems surprising, just look at the scalar case: \( \frac{1}{w^{-1} + e^{-1}} = \frac{we}{w+e} \).