Dynamical electro-weak symmetry breaking from deformed AdS: vector mesons and effective couplings.

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We study a modification of the five-dimensional description of dynamical electro-weak symmetry breaking inspired by the AdS/CFT correspondence. Conformal symmetry is broken in the low-energy region near the IR brane by a power-law departure from the pure AdS background. Such a modification—while not spoiling the identification of the IR brane with the scale of confinement—has a dramatic effect on both the coupling of the first composite states to the standard model currents and their self-couplings. Chiral symmetry breaking can take place at a scale larger than the IR cut-off. This study shows that observables, such as the precision parameter $\hat{S}$, which depend on the dynamics in the low energy region where conformal symmetry is lost and electro-weak symmetry is broken just above the scale of confinement. Therefore results of calculations of these observables in AdS/CFT inspired scenarios should be interpreted conservatively. The most important phenomenological consequence for physics at the LHC is that the bound on the mass scale of the heavy excitations (technirho mesons) in a realistic model is in general lower than in the pure AdS background with a simple hard-wall cut-off in the IR.

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I. INTRODUCTION

The mechanism responsible for electro-weak symmetry breaking in the standard model will be tested at the Large Hadron Collider (LHC). One important goal for this experimental program is to understand whether the interactions responsible for electro-weak symmetry breaking are strong or weak. It is essential to identify theoretically clean, measurable quantities that can help distinguish these two possibilities unambiguously.

One might think that this is an easy task: after all, a strongly coupled model in the spirit of technicolor predicts the existence of towers of broad, strongly coupled composite resonances, with a rich spectroscopy at the TeV scale, in analogy with what is known about QCD at the GeV scale. Yet, indirect experimental data about electro-weak symmetry breaking cannot be easily reconciled with this framework and already suggest that, if electro-weak symmetry breaking is a dynamical effect, the low-energy effective field theory description of the new strongly-coupled sector has to exhibit features that are not generic. Walking technicolor is a plausible candidate for such a strongly coupled model, based on conformal behavior and large anomalous dimensions in the IR, but calculability within this framework has been a challenge.

In recent years, based on the ideas of the AdS/CFT correspondence, and on the pioneering work of Randall and Sundrum, many models have been investigated which exhibit in the low energy region the basic properties expected in a walking theory, while being calculable. Examples now exist of models that are compatible (within the errors) with the precision data and can be discovered at the LHC. The literature on the subject is already extensive. Most of these models assume a conformal behavior of the strongly coupled sector in the energy region spanning few orders of magnitude above the electro-weak scale and the existence of a weakly-coupled effective field theory description of the low-energy dynamics of the resonances. The construction of the effective field theory is derived by writing a weakly coupled extra-dimension model with a non-trivial gravity background, and by using the dictionary of the AdS/CFT correspondence to relate back to four dimensions. A generic phenomenological feature of all these models is that, unless a clever mechanism arranging for non-trivial (often fine-tuned) cancellations is implemented, a quite severe lower bound on the mass $M_1 \gtrsim 2.5$–3 TeV of the lightest spin-1 resonance (technirho) results, in particular from the bounds on the electro-weak parameter $S$. This result, together with the assumption that the effective field theory be weakly coupled (and hence calculable), gives rise to a spectacular signature (a sharp resonance peak) at the LHC. Unfortunately, it is very difficult to distinguish it from the signature of a generic, weakly-coupled extension of the standard model with an extended gauge group, predicting a new massive $Z'$ gauge boson.

Indisputable evidence proving that a strongly-coupled sector is responsible for electro-weak symmetry break-
ing would be the discovery of at least the first two spin-1 resonances, hence proving that these new particles are not elementary, but higher energy excitations of a composite object. The major obstacle against this scenario is the unfortunate numerology emerging from the combination of precision data and LHC high-energy discovery reach. If $M_1 \gtrsim 2.5$–$3$ TeV, than it follows that the mass of the second resonance must be $M_2 \gtrsim 5$–$6$ TeV and just beyond the region where LHC data are expected to give convincing evidence\cite{14}. Yet, a pretty mild relaxation of the experimental bounds would be enough to change this situation radically, since $M_1 \sim 1.5$ TeV would imply $M_2 \approx 2.5$–$4$ TeV, well within reach even at moderate luminosity \cite{15}. It is hence timely, just before LHC starts collecting data, to question how accurate the AdS/CFT description of realistic dynamical electro-weak symmetry breaking is, and whether some of the approximations implied by this description could account for the desired softening of the bounds, without at the same time spoiling the calculability of the effective field theory.

In analogy with \cite{2}, the five-dimensional picture usually contains two hard boundaries representing the UV and IR cut-off between which the theory is conformal. This is the weakest link with the idea that electro-weak symmetry breaking be triggered by a non-abelian gauge theory with an approximate IR fixed point. Taken literally, this picture means that, both in the UV and in the IR, conformal symmetry is lost instantaneously, via a sharp transition. As for the UV cut-off, this is not a real problem from the low-energy effective field theory point of view. The details of how an asymptotically-free fundamental theory in the far UV enters a quasi-conformal phase below the UV cut-off, can always be reabsorbed (via holographic renormalization \cite{11,16}) in the definition of otherwise divergent low-energy parameters of the effective field theory, defined at a given order in the perturbative expansion of the effective field theory itself.

Rather different is the case against using a hard-wall regulator in the IR. There is no sense in which IR effects decouple and can be renormalized away, and hence the low-energy effects we are interested in, when comparing the effective field theory to the experimental data, are inherently sensitive to the choice of the IR regulator. On the one hand, the very validity of the effective field theory description based on the AdS/CFT dictionary requires that the hard-wall cut-off be at least a reasonable leading order approximation (otherwise the effective field theory itself would be strongly coupled, and not admit a controllable expansion). On the other hand, corrections are expected to be present, and estimating their size and understanding their phenomenological consequences is crucial, at the very least in order to know what to expect in experiments such as those at the LHC, which is going to test precisely the energy range close to the IR cut-off.

To be more specific. In the IR, three different phase transitions are taking place: electro-weak symmetry breaking, conformal symmetry breaking and confinement. These cannot define three parametrically separate scales, since they are all triggered by the same physical effect, namely the fact that the underlying (unknown) theory possesses an approximate fixed point in the IR. Hence the RG flow of the underlying dynamics is not going to reach the IR fixed point (which is only approximate), but will drift away from it at low energies, after spending some time (walking) in its proximity. Yet, there is no reason to expect these three effects to arise precisely at the same energy (temperature), and they might define three distinct critical scales (temperatures) that differ by $O(1)$ coefficients.

An illustration of this point can be obtained by considering an $\mathcal{N}=1$ supersymmetric QCD model with $N_c$ colors and $N_f$ quarks. At least at large-$N_c$, for $3N_c/2 < N_f < 3N_c$, the theory is asymptotically free, but has a fixed point in the IR \cite{17,18} (for recent progress towards the rigorous construction of the gravity dual see, for instance, \cite{19}). If $N_f$ is not far from the lower bound, so that the theory is strongly coupled at distances larger than a UV cut-off $1/L_0$, then the theory might be approximately described by a large-$N_c$ conformal field theory at strong ’t Hooft coupling. Suppose now that at some smaller energy, characterized by a length scale $L \gg L_0$, for some reason (for example the existence of a suppressed symmetry-breaking higher-order operator, which acquires a large anomalous dimension in the IR turning it into a relevant deformation) a symmetry-breaking condensate forms, reducing further $N_f$ to a value $N'_f$ closer to or below $3N_c/2$. Symmetry-breaking drives the theory away from the original fixed point, and induces the loss of conformal symmetry. The coupling now runs fast (because the coupling itself was already big and large anomalous dimensions are present), and (depending on $N'_f$) the theory either enters a new conformal phase at stronger coupling or confines. The breaking of the global $SU(N_f)_L \times SU(N_f)_R$, conformal symmetry-breaking and confinement take place approximately at the same scale. Yet, the energy at which the coupling reaches its upper bound defines a new scale $L_1$ which might well be some numerical factor away from $L$, the scale at which the RG-flow trajectory departed away from the fixed point.

If this is the qualitative behavior of the UV-complete dynamical model that is ultimately responsible for electro-weak symmetry breaking, describing it as a slice of AdS space between two hard walls is a good leading-order approximation. Nevertheless, we may wonder whether a factor of 3 or 4 separating the scales of conformal symmetry-breaking and confinement can be completely ignored, in the light of the phenomenological consequences at the LHC that a mere factor of two might have. In this paper, we study the effect of such a factor. We consider the simplest possible effective field theory description of dynamical electro-weak symmetry breaking as a 5D weakly-coupled system (see also \cite{11}), introduce (besides the UV brane at $L_0$ and the IR brane at $L_1$) a new discontinuity at the scale $L$, very close to the IR scale $L_1$, and assume that the background deviates from the AdS case for $L < z < L_1$. 


As for the origin and description of electro-weak symmetry-breaking, we will treat it as a completely non-dynamical effect localized in the IR, somehow in the spirit of Higgless models. The breaking could take place at $L_1$ as well as at $\bar{L}$ (or anywhere in between), as suggested by the SQCD example above. We compare the effects on the electro-weak precision parameter $\hat{S}$ in these two cases, as illustrative of two extreme possibilities, without committing ourselves to either of them. The idea that chiral symmetry breaking might, for a generic model, take place at a scale higher than confinement has been in the literature for a while \cite{20}, has been supported by lattice evidence in some special case \cite{21}, and has recently been discussed also in string-inspired models \cite{22}.

A realistic model should also implement a dynamical mechanism generating the mass of the standard model fermions. This can be done either via extended technicolor higher-order interactions between the standard model fermions and the new strong sector \cite{23,24} (represented in the 5D picture by Yukawa interactions localized at the UV, with the symmetry-breaking vacuum expectation value not localized, but exhibiting a non-trivial power-law profile in the bulk), or via the assumption that standard model fields are themselves (partially) composite, in the spirit of topcolor and related models \cite{25} (which would imply the fermions be allowed to propagate in the bulk of the 5D model). A detailed discussion of how the global family symmetry of the standard model is broken would be required in order to study how the phenomenology of flavor-changing transitions and the physics of the third generation would be affected by the proposed modification of the background. In this paper, we treat the standard model fermions as non-dynamical fields, described by a set of external currents, and do not address the problem of their mass generation.

II. PRELIMINARIES

A non-trivial departure of the dynamics of the spin-1 resonances, with respect to that on pure AdS geometry, may be either due to a modification of the gravity background or to the presence of a non-dynamical background (dilaton). Since we consider an effective field theory where only spin-1 states are dynamical, it is not possible to distinguish between these two effects at this level. We choose to describe the model in terms of a deformation of the gravity background, for simplicity.

Consider the five-dimensional space described by the metric

$$ds^2 = a(z)^2 (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

where $L_0 < z < L_1$. We will assume that the geometry approaches pure AdS in the UV region, $a(z) \to L/z$ as $z \to L_0$, and departs from it at a scale $z \sim L$. In most of the calculations we take $L_0 = L$ for simplicity.

We are interested in describing a model that at low-energy (below $1/L_1$) can be matched to the electro-weak chiral Lagrangian \cite{27}. This requires to introduce a 5-dimensional gauge group which is at least $SU(2)_L \times U(1)_Y$, but may be enlarged to accommodate custodial symmetry. Irrespectively of the details, the model contains a vectorial sector (the neutral part of which consists of the photon and its excitations) and an axial sector (containing the $Z$ boson and its excitations). In this paper we describe only the phenomenology connected with the neutral gauge bosons, hence we dispense with the details of the complete symmetry group. For concreteness, we take the vectorial sector to be described by the pure Yang-Mills $SU(2)$ theory with the following action:

$$S = \int d^4x \int_{L_0}^{L_1} dz \sqrt{G} \left[ G^{\alpha\beta} G^{\mu\nu} \left( -\frac{1}{2} \text{Tr} F_{\alpha\beta} F_{\mu\nu} \right) + 2g\delta(z-L_0)G^{MN} \text{Tr} J_M A_N \right],$$

where $F_{MN} = \partial_M A_N - \partial_N A_M + ig[A_M, A_N]$ is the field strength tensor of $A_M = A_M^a T^a$ with $T^a = \epsilon^a/2$ the generators of $SU(2)$, where $g$ is the (dimensionful) gauge coupling, and where $J_M = (J_\mu(x),0)$ is the four-dimensional external current localized on the UV-brane.

Quantization requires to add appropriate gauge-fixing terms, canceling the mixing terms between spin-0 and spin-1 fields, which in unitary gauge implies $A_5(x,z) = 0$.

After Fourier transforming in 4D, $A_\mu(x,z) = \int \frac{d^4q}{(2\pi)^4} e^{iqx} A_\mu(q) v(q,z)$, the free bulk equations read

$$\partial_5 [a(z) \partial_5 v(q,z)] = -q^2 a(z) v(q,z).$$

Substituting the solutions in the action, and canceling the boundary terms at $z = L_1$, without breaking the gauge symmetry, requires to impose Neumann boundary conditions:

$$\partial_5 v(q,L_1) = 0.$$

This set of equations admits always a constant, massless zero mode.
Finally, the action can be rewritten as a pure boundary term at the UV, from which one can read the vector two-point correlator, that for $L_0 \to L$ is

$$
\Sigma_V(q^2) = g^2 \frac{v(q, L_0)}{\partial_5 v(q, L_0)},
$$

which can be expanded as

$$
\Sigma_V(q^2) = e^2 \left( \frac{1}{q^2} + \sum_i \frac{R_i}{q^2 - M_i^2} \right),
$$

where $M_i$ ($i = 1, 2, \ldots$) are the masses of the excited states, and $R_i$ define their effective couplings to the four-dimensional currents, normalized to the coupling $e^2$ of the massless mode (to be identified with the electro-magnetic coupling of the photon).

The (dimensionful) bulk coupling $g$ controls the perturbative expansion used to extract this correlator. It is not directly related to the effective coupling $e$ of the standard model gauge boson (photon), but rather is related to the strength of the effective interactions among its heavy (composite) excitations. The relation between these two effective couplings depends on how the theory is regularized in the UV, and is not a calculable quantity, because of the divergences in the $L_0 \to 0$ limit. A rigorous treatment requires to introduce appropriate counterterms and treat the ratio $e^2L/g^2$ as a free parameter. For the purposes of this paper, which primarily require comparing identical UV settings with different IR deformations, we can simplify this procedure by assuming that $L_0 \ll L_1$ be finite and fixed, and express this ratio as a function of the scales and couplings in the model. We discuss later how good the perturbative expansion is by estimating the size of the effective self-coupling of the composite states.

In order to compute $\hat{S}$ one has to introduce also the axial-vector excitations, and a symmetry-breaking mechanism. For the purposes of this paper, we only consider the Higgsless limit, defined by the introduction of a localized, infinitely massive Higgs scalar which assumes a non-trivial symmetry-breaking vacuum expectation value.

The axial-vector modes $v_A(q, z)$ still satisfy Eq (3), but their boundary conditions (and the gauge fixing action) are modified. We consider two cases in the following. In the first, symmetry-breaking takes place on the boundary $L_1$ so that the axial-vector profiles $v_A(q, z)$ obey generalized Neumann boundary condition:

$$
\partial_5 v_A(q, L_1) + mv_A(q, L_1) = 0.
$$

The effective symmetry-breaking parameter $m$ has dimension of a mass. In the limit $m \to 0$ one recovers the symmetric case, while for $m \to +\infty$ one recovers the Dirichelet boundary conditions. The mass of the $Z$ boson depends on $m$ is such a way that it vanishes for vanishing $m$, but is determined by $L_1$ for arbitrarily large $m$. In the second case we consider a symmetry-breaking vacuum expectation value localized at a different point $L < L_1$ in the fifth dimension. The modifications to be implemented in this case will be discussed in the next sections.

All of this allows to define the axial-vector correlator $\Sigma_A(q^2)$ by replacing in Eq. (5) $v_A(q, z)$ and its derivative to $v(q, z)$. After these manipulations, the precision parameter $\hat{S}$ is given by

$$
\hat{S} = e^2 \cos^2 \theta_W \frac{d}{dq^2} \left( \frac{1}{\Sigma_V(q^2)} - \frac{1}{\Sigma_A(q^2)} \right) \bigg|_{q^2=0},
$$

where $\epsilon$ has been defined before, and corresponds to the electro-magnetic coupling, while $\theta_W$ is the effective weak-mixing angle. We recall here that an approximate extrapolation to large Higgs masses yields the experimental limit $\hat{S} \lesssim 0.003$ at the 3σ level [3].

### III. PURE ADS BACKGROUND

We summarize here the results of the case in which the background is purely AdS with $a(z) = L/z$, and assume for simplicity that $L_0 = L$. The vector correlator is

$$
\Sigma_V^{(0)}(q^2) = g^2 \frac{(J_0(L_1q)Y_1(Lq) - J_1(Lq)Y_0(L_1q))}{q(J_0(L_1q)Y_0(Lq) - J_0(Lq)Y_0(L_1q))}.
$$

In order to discuss the spectrum and couplings, the following approximations can be used:

$$
\Sigma_V^{(0)}(q^2) \simeq \frac{g^2 J_0(L_1q)}{Lq^2 \left( \frac{\gamma_E}{2} Y_0(L_1q) - J_0(qL_1) \left( \gamma_E + \log \frac{L_1}{2} \right) \right)},
$$

where $\gamma_E$ is the Euler-Mascheroni constant.
the first of which is valid for $L \ll L_1$, and the second for $qL_1 > 1$.

From [10] one can read the coupling of the zero mode:

$$e^2 = \frac{g^2}{L \log L_1/L}. \quad (12)$$

From [11] one can look for the poles and the residues $R_i$. The poles (besides the pole at zero) are in the vicinity of those of $\tan(L_1q - \pi/4)$:

$$M_i \simeq \frac{\pi}{4L_1} \left( (4i - 1) - \frac{2}{\gamma_E + \log(4i - 1)\pi(8L_1)} \right), \quad (13)$$

while the residues are approximately given by:

$$R_i \simeq \frac{4 \log(L_1/L)}{-2 + \frac{2\pi L_1 M_i}{1 + \sin(2L_1 M_i)}}, \quad (14)$$

with $i = 1, 2, \ldots$. These approximations are acceptably accurate as long as $L \ll L_1$. A numerical calculation will be performed later on, when discussing the phenomenology for some relevant choice of parameters.

The axial correlator can be computed exactly:

$$\Sigma_A^{(0)} (q^2) = \frac{g^2 ((qJ_0(L_1q) + mJ_1(L_1q)) Y_1(Lq) - J_1(Lq) (qY_0(L_1q) + mY_1(L_1q))))}{q (qJ_0(L_1q)Y_0(Lq) + mJ_1(L_1q)Y_0(Lq) - J_0(Lq) (qY_0(L_1q) + mY_1(L_1q))))}, \quad (15)$$

and, for $L_0 \ll L_1$, yields

$$\hat{S} = \frac{\cos^2 \theta_W L_1 m (3L_1 m + 8)}{4(L_1 m + 2)^2 \log \left( \frac{L_1}{L} \right)}. \quad (16)$$

In the limit $m \to +\infty$ we have

$$\hat{S} = \frac{3 \cos^2 \theta_W}{4 \log L_1/L}. \quad (17)$$

Imposing the (3$\sigma$-level) experimental limit we find that

$$\frac{g^2}{L} = e^2 \log \frac{L_1}{L} = e^2 \frac{3 \cos^2 \theta_W}{4 \hat{S}} \gtrsim 20, \quad (18)$$

where $e^2 \simeq 0.1$ is the effective coupling of the electro-magnetic $U(1)_Q$ in the standard model. Since, as discussed later, $g^2/L$ gives a measure of the effective strength of the self interactions between resonances (and the dimensionful coupling $g$ is the expansion parameter in the 5D action) the experimental bounds are satisfied only at the price of loosing calculability, as is the unfortunate case also when trying to build QCD-like technicolor models in 4D, either using the large-$N$ expansion, hidden local symmetry, or deconstruction (see for instance [20]). We do not discuss further this limit.

In the more interesting and realistic case in which $mL_1 \ll 1$, the axial-vector spectrum and couplings are approximately the same as the vectorial sector. In this framework $m$ is just a free parameter, and we treat it as such. With finite $mL_1 \ll 1$, the mass of the lightest axial-vector state is approximately $M_2^2 \simeq m/(L_1 \log(L_1/L))$, and hence

$$\hat{S} \simeq \frac{\cos^2 \theta_W}{2 \log L_1/L} mL_1 \simeq \frac{\cos^2 \theta_W}{2} M_2^2 L_1^2 \quad \text{(19)}$$

satisfies the bounds on $\hat{S}$ for $1/L_1 \gtrsim 1$ TeV, which depending on the value of $L_0/L_1$ translates into a bound $M_1 \gtrsim 2.5-4$ TeV. For instance, for $g^2/L < 1/2$ it requires $M_1 \gtrsim 2.8$ TeV, and consequently $M_2 \gtrsim 6$ TeV, which is beyond the projected reach of the LHC searches.
IV. DEPARTURE FROM ADS

We now consider the possibility that conformal invariance be violated at some energy regime above the confinement scale and suppose there exists a hierarchy of scales \( L_0 = L < \bar{L} < L_1 \) such that the space is the usual AdS for \( L_0 < z < \bar{L} \), but departs from it in the IR region \( \bar{L} < z < L_1 \). Our aim is to model this behavior without affecting the approximate description of confinement provided by the IR brane (different motivations lead the authors of [10] to other parameterizations). The simplest form one can choose in order to achieve this goal is a power-law warp factor

\[
a(z) = \begin{cases} \frac{L}{z} & z < \bar{L} \\ \frac{L}{z} (\frac{L}{z})^{n-1} & z > \bar{L} \end{cases}
\]

This parameterization may be viewed as a leading order approximation of a smooth background describing the appearance of some relevant deformation in the conformal field theory before the underlying fundamental theory confines.

We will see later that a power-law avoids generating an explicit mass gap from the bulk equations, so that the quantity \( 1/L_1 \) can still be interpreted as the scale of confinement. Moreover, with our parameterization we can solve the equations exactly and in a very straightforward way, which is in itself a welcome property when modeling an otherwise untreatable dynamical system.

Most of the algebraic manipulations can be performed for generic \( n \). Yet, we discuss explicitly only the \( n > 1 \) case. A variety of arguments, all ultimately descending from unitarity, suggest that we should limit ourselves to \( n \geq 1 \). An extra-dimensional argument can be derived along the lines of [27], in which it is shown how the weaker energy condition leads to a c-theorem controlling the behavior of the curvature in crossing a phase transition towards the IR. This is related to the fact that, in the context of strongly-coupled four-dimensional models, in going through a phase transition it is reasonable to expect the effective number of light degrees of freedom to decrease [28]. Hence the effective coupling of the effective field theory description, which is related to the \( 1/N \) expansion, is expected to increase. We show later in the paper that the effective self-coupling of the heavy resonances is enhanced for \( n > 1 \), in agreement with the four-dimensional intuitive expectation, and that this enhancement is controlled by a power of the ratio of relevant scales, in agreement with naive expectations for a theory with a generic deformation due to a relevant operator. The fact that all of our results agree with the intuitive interpretation gives an indication in support both of the power-law parameterization chosen here and of the \( n \geq 1 \) restriction.

The solutions to the bulk equations in the IR region \( z > \bar{L} \) are of the form

\[
v^{IR}(q, z) = z^{\frac{n+1}{2}} \left( c_1^{IR}(q) J_{\frac{n+1}{2}}(qz) + c_2^{IR}(q) Y_{\frac{n+1}{2}}(qz) \right),
\]

while in the UV region

\[
v^{UV}(q, z) = z \left( c_1^{UV}(q) J_1(qz) + c_2^{UV}(q) Y_1(qz) \right).
\]

The bulk profile is obtained by applying the IR boundary conditions to \( v^{IR} \), and then by requiring that the junction of the two solutions be smooth, so that no boundary action localized at \( \bar{L} \) is left:

\[
\partial_5 v^{IR}(q, L_1) = 0, \quad v^{IR}(q, \bar{L}) = v^{UV}(q, \bar{L}),
\]

\[
\partial_5 v^{IR}(q, L) = \partial_5 v^{UV}(q, L).
\]

The correlator is then obtained from Eq. (21) by using \( v^{UV} \). From all of this, one can extract the masses and couplings of the resonances. In particular, the coupling of the zero-mode (photon) is

\[
e^2 = \frac{(n-1)\bar{L}^2}{(n-1)\log(\bar{L}/L_0) + (1 - (\frac{\bar{L}}{L_0})^{n-1})}.
\]

For \( n = 1 \), or for \( \bar{L} = L_1 \), one recovers the AdS result [12]. For \( n > 1 \) and \( \bar{L} < L_1 \) this estimate is enhanced (for fixed \( g^2/L \)). In order to understand how significant this effect is, one needs to compare this coupling to the effective self-coupling, which is discussed in the next section.

Analytical expressions for the couplings and masses of the vector-like resonances are rather involved, and numerical results will be plotted later on. In order to gain a semi-quantitative understanding of how these quantities are modified
with respect to the pure AdS case, we discuss the (unrealistic) extreme case in which $\bar{L} = L_0 \ll L_1$. For $qz \gg 1$:

$$J_{\bar{L}L}(qz) \approx \sqrt{\frac{2}{\pi qz}} \cos \left( qz - \frac{n\pi}{4} \right),$$

$$Y_{\bar{L}L}(qz) \approx \sqrt{\frac{2}{\pi qz}} \sin \left( qz - \frac{n\pi}{4} \right)$$

and the masses of $i$-th resonances, for $n \gg 1$, are approximately given by the zeros of $J_{\bar{L}L}(qL_1)$,

$$M_i(n) \approx \frac{2i - 1}{2} \frac{\pi}{L_1} + \frac{n\pi}{4L_1},$$

with $i = 1, 2, \ldots$. This agrees with the pure AdS case ($n = 1$) at least for $L/L_1 \ll 1$. For the more realistic case in which $L_0 \ll \bar{L} < L_1$, the spectrum of heavy modes is going to be shifted, with masses heavier by approximately $(n - 1)\pi/(4L_1)$ with respect to the AdS case, for those resonances whose masses are comparable with the new scale $1/L$. The spectrum connects back to the pure AdS case for higher excitation number $i$.

As for the residues, the couplings to the currents of the heavy modes are approximately going to be suppressed with a power-law dependence $\approx \left(\frac{\bar{L}_{uv}}{L_1}\right)^{(n-1)}$ with respect to the AdS case. Again, this suppression applies only to the lightest resonances, those for which the mass is shifted to higher values.

For the axial-vector case, if the symmetry breaking takes place at $L_1$ the only change is the IR boundary condition:

$$\partial_5 v_{A}^{IR}(q, L_1) + n v_{A}(q, L_1) = 0,$$

$$v_{A}^{IR}(q, \bar{L}) = v_{A}^{UV}(q, \bar{L}),$$

$$\partial_5 v_{A}^{IR}(q, \bar{L}) = \partial_5 v_{A}^{UV}(q, \bar{L}).$$

For generic $n > 1$, we find the following approximations for the mass of $Z$ boson and for $\hat{S}$:

$$M_Z^2 \approx \left(\frac{\bar{L}}{L_1}\right)^{n-1} \frac{(n-1)m}{L_1 \left(1 - (L/L_1)^{n-1} + (n-1) \log L/L_1\right)},$$

$$\hat{S} \approx \cos^2 \theta_W \left(\frac{1}{n+1} + \frac{1}{2} \left(\frac{\bar{L}}{L_1}\right)^2 - \frac{1}{n+1} \left(1 + \frac{1}{n+1}\right)\right) \bar{L}^2 M_Z^2,$$

where the last approximation is valid as long as $mL_1 \ll 1$. For small $\bar{L}/L_1$ this approximation would not hold, because of the dependence of $M_Z$ on $m$ and on $L/L_1$. We do not admit a parametric separation between $L$ and $L_1$, and hence the approximations are acceptable. We also checked this numerically, using the exact bulk profiles and correlators.

The other extreme possibility we are interested in is the one in which the symmetry-breaking condensate is localized at $\bar{L}$, for which the boundary conditions become

$$\partial_5 v_{A}^{IR}(q, L_1) = 0,$$

$$v_{A}^{IR}(q, \bar{L}) = v_{A}^{UV}(q, \bar{L}),$$

$$\partial_5 v_{A}^{IR}(q, \bar{L}) = \partial_5 v_{A}^{UV}(q, \bar{L}) + \tilde{m} v_{A}(q, \bar{L}).$$

For generic $n$:

$$M_Z^2 \approx \frac{(n-1)\bar{m}}{L_1 \left(1 - (L/L_1)^{n-1} + (n-1) \log L/L_1\right)},$$

$$\hat{S} \approx \cos^2 \theta_W \left(\frac{1}{n+1} + 2 - \frac{\bar{L}}{L_1}\right) \bar{L}^2 M_Z^2,$$

in which in the last expression only the leading order of the expansion in $M_Z$ has been kept, and all the expressions are valid as long as $\bar{m}L_1 \ll 1$.

Notice how the dependence of $M_Z$ on $\bar{m}$ is not suppressed by powers of $\bar{L}/L_1$, as in the former case, where $m$ came from a localized term at $L_1$. This result agrees with the intuitive notion that moving the symmetry-breaking towards the UV enhances its effect for the zero-mode, while suppressing the mass splitting of the heavy resonances. The result is well illustrated by $\hat{S}$, which is proportional to $M_Z^2$ through the position $\bar{L}$ or $L_1$ of the symmetry-breaking condensate in the fifth dimension.
V. ESTIMATING THE STRENGTH OF THE SELF-INTERACTIONS

The departure from conformal invariance, explicitly added via a power-law deviation from the AdS background in the IR region, might imply that the dynamics of the effective field theory itself be strongly coupled, as is the case for a QCD-like dynamical model. It has to be understood if the effective field theory treatment still admits a power-counting allowing to use a cut-off $L_0$ much larger than the electro-weak scale. A fully rigorous treatment of this problem is not possible, because it requires to extend the effective field theory Lagrangian beyond the leading order in $1/N_c$. Yet, a reasonable estimate of the effective coupling can be extracted by looking at the cubic and quartic self-couplings of the resonances, the structure of which (at the leading order) is dictated by 5D gauge-invariance.

Consider first the pure AdS background and define

$$g_{\rho}^{(i)2} \equiv \frac{g^2}{L} \frac{\int_{L_0}^{L_1} dz |v(M_i, z)|^4}{\left( \int_{L_0}^{L_1} \frac{dz}{2\pi} |v(M_i, z)|^2 \right)^2}. \quad (40)$$

The expansion parameter is related to $g_{\rho}$, which we define as the asymptotic limit of the effective self-coupling for large excitation number. As long as $L_0 \ll L_1$ and $M_i L_1 \gg 1$, the bulk profiles of the heavy modes can be approximated by

$$v(M_i, z) \propto \frac{z}{\sqrt{M_i}} J_1(M_i z) \propto \frac{z}{M_i} \cos \left( M_i z - \frac{3\pi}{4} \right). \quad (41)$$

yielding

$$g_{\rho}^2 \equiv \lim_{i \to +\infty} g_{\rho}^{(i)2} \approx \frac{3}{4} \frac{g^2}{L}. \quad (42)$$

For the smallest values of $i = 1, 2$ this is a moderate underestimate. For instance for $i = 1$, from the exact solution one obtains $g_{\rho}^{(1)2} \sim 1.2 g^2 / L$. The meaning of this definition of $g_{\rho}$ is that it gives a reasonable estimate of the strength of the self-coupling of the resonances, and hence of the expansion parameter of the effective field theory (which is related to the large-$N_c$ expansion). As expected, this turns out to be controlled by $g^2 / L$, up to $O(1)$ coefficients.

The actual value of $g^2$ is related (with the treatment of the UV cut-off used here) to the coupling of the zero mode $e^2 = g^2 / (L \log L_1 / L_0)$, so that $g_{\rho}^2 \approx e^2 \log(L_1 / L_0)$. This yields the relation between strength of the effective coupling and the effective cut-off in the UV, which as expected is logarithmic, ultimately because of conformal symmetry. The requirement that this defines a perturbative coupling $g_{\rho}^2$ implies a bound on $L_1 / L_0$. Choosing for instance $L_1 = 100 L_0$ (a value that is not justifiable by applying naive dimensional analysis to the electro-weak chiral Lagrangian), yields $g_{\rho}^2 \approx 0.3$, which means that the effective field theory admits an acceptable expansion in powers of $g_{\rho}^2 / (4\pi)$ even with large choices of the UV cut-off $1/L_0$.

Generalizing this estimate in presence of the non-trivial background [20] is somehow more difficult, largely because of the junction conditions at $\bar{L}$. This can be done numerically, but for the present purposes a semi-quantitative assessment of the size of the effective coupling suffices. We again focus on large values of $M_i L_1$ and modify the definition of the effective couplings to

$$g_{\rho}^{(i)2} \equiv \frac{g^2}{\bar{L}} \frac{\int_{L_0}^{L_1} \frac{dz}{2\pi} |v(M_i, z)|^4}{\left( \int_{L_0}^{L_1} \frac{dz}{2\pi} |v(M_i, z)|^2 \right)^2}. \quad (43)$$

The specific case we are interested in lies somewhere in between the pure AdS and the pure power-law. In the latter case an acceptable approximation would be:

$$v(M_i, z) \propto \frac{z^{\frac{n+1}{2}}}{\sqrt{M_i}} J_{\frac{n+1}{2}}(M_i z) \propto \frac{z^{\frac{n}{2}}}{M_i} \cos \left( M_i z - \frac{(n+2)\pi}{4} \right). \quad (44)$$

The effective coupling receives power-law contributions in $L_1 / \bar{L}$, plus terms that are logarithmic in $L_0 / \bar{L}$ and hence subleading $O(1)$ corrections. The power-law is the most important effect and, for largish choices of $L_1 / \bar{L}$ and in the case $n > 1$, we obtain:

$$g_{\rho}^2 \approx \frac{3}{2(n+1)} \left( \frac{L_1}{\bar{L}} \right)^{n-1} \quad (45)$$

$$\approx \frac{3e^2}{2(n^2 - 1)} \left( \frac{L_1}{\bar{L}} \right)^{n-1} \quad (46)$$
which, as in the pure AdS case, represents a defective approximation by roughly a factor of 2 for the very first resonance. We see that, for \( g_\rho \) to be acceptably small as to define an expansion parameter, \( L_1/\bar{L} \) cannot be large.

The power-law dependence on \( \bar{L}/L_1 \) in Eq. (45) is expected in a non-conformal effective theory, in presence of relevant operators, in which case there cannot be a substantial scale separation between the UV cut-off and the mass scale \( L_1 \) of the effective theory itself. This result agrees with naive dimensional analysis counting. For instance, taking \( \bar{L} = L_0 \) implies that the model is strongly coupled, unless \( (L_1/\bar{L})^{n-1} \ll 4\pi \), which implies a very low cut off, and the impossibility of describing the resonances as weakly coupled.

Notice that this result depends smoothly on \( n \gtrsim 1 \). But in trying to extend the analysis to \( n < 1 \) one immediately faces a problem. For instance, for \( L_0 \to \bar{L} \ll L_1 \), \( n < 1 \), and keeping \( g^2/L \) fixed, the effective coupling becomes vanishingly small. This behavior would imply that, in the region of the parameters space in which the theory admits an effective approach, the original conformal theory flows into a new phase that is described by a new effective field theory which has effectively a weaker coupling. This violates all possible logical expectations, according to which such a phase transition always drives the theory towards stronger coupling, such that the new effective field theory has always a smaller number of light degrees of freedom, and hence a larger expansion parameter. Though not rigorous, this argument seems to support the hypothesis that only \( n \gtrsim 1 \) is an admissible choice.

From the phenomenological point of view, one way to assess how strongly coupled is the first resonance, is to consider \( \gamma_1 \), the first excited mode with the quantum numbers of a photon, and compare its partial width into two on-shell \( W \) bosons:

\[
\frac{\Gamma(\gamma_1 \to f\bar{f})}{\Gamma(\gamma_1 \to W^+W^-)} \approx \frac{8\alpha}{3} \frac{R_1}{g_\rho} \frac{48\pi}{g_\rho^2} \approx \frac{\pi R_1}{g_\rho^2}. \tag{47}
\]

For a weakly-coupled theory this approximate estimate should be \( O(1) \) or bigger. In other words, a rough estimate of the width of the first resonance gives \( \Gamma \approx g_\rho^2 M_1/(48\pi) \), and hence the approximation of treating this resonance as infinitely narrow (as expected at large-\( N_c \)) makes sense only as long as \( g_\rho^2 \) is at most some \( O(1) \) number. A more detailed study of this quantities, and the phenomenological consequences relevant at LHC energies, will be discussed in a subsequent study.

VI. PHENOMENOLOGICAL IMPLICATIONS

A. Spectrum and couplings to the currents

We start with a numerical analysis of the spectrum and couplings of the vectorial excited states. We perform the numerical analysis because the results discussed in the previous section for these quantities give only semi-quantitative approximate expressions. Since we always consider values of \( m \) and \( \bar{m} \) that are small compared to \( 1/L_1 \), the results apply also to the axial-vector modes, irrespectively of the choice of localizing the symmetry-breaking effects at \( L_1 \) or at \( \bar{L} \).

The masses \( M_i \) depend in a complicated way on \( L_1, \bar{L}, L_0, \) and \( n \). In Figure 1 we plot the mass (in units of \( 1/L_1 \)) for the first three excited states, as a function of \( L_1/\bar{L} \). We compare four choices of the relevant parameters, characterized by \( n = 2, 3 \) and by the choice of the UV cut off \( L_0 = L_1/20 \) and \( L_0 = L_1/100 \).

As anticipated, the masses are larger than in the \( n = 1 \) case (pure AdS), which is recovered when \( \bar{L} = L_1 \). The enhancement is only moderate, it affects the heavier states only for large values of \( L_1/\bar{L} \), and is proportional to \( n \).

The coupling \( R_i \) is, in the pure AdS case, a monotonically decreasing function of the excitation number \( i \). In Figures 2 and 3 we plot the numerical results obtained for this quantity, for the same choices of parameters used for the masses. In going from \( L_1/\bar{L} = 1 \) (pure AdS) to larger values and/or to large \( n \), a suppression of the coupling is obtained for the lightest state. This suppression is a very big effect, and it becomes relevant at large values of \( L_1/\bar{L} \). As a result, for instance in the case \( n = 3 \), with \( L_1/\bar{L} \gtrsim 4 \) the third resonance has the strongest coupling, followed by the second and by the first.

B. Self-couplings and symmetry-breaking

We want the 5D action to define a reasonable effective field theory treatment of the strong dynamics and of the resulting electro-weak symmetry breaking effects, with a well-behaved perturbative expansion. We implement this requirement by imposing the bound \( g_\rho^2 \lesssim 1/2 \) (a reference value that we fix in such a way that for the choices of parameters discussed here the ratio of partial width estimated in Eq. 47 is \( \gtrsim 1 \)), where \( g_\rho \) has been defined in the
body of the previous section. In the pure AdS case we require that $L_1/L_0 \lesssim 200$, which means that the model is very modestly sensitive to the position of the UV cut-off and, unless extreme choices of $L_0 \ll L_1$ are used, we can neglect the effect of $L_0$ in driving the effective coupling strong. We can therefore impose the bound directly on the modification due to the new non-conformal energy regime:

$$\left(\frac{L_1}{L}\right)^{n-1} \lesssim \frac{(n^2 - 1)}{3e^2}. \quad (48)$$

For small values of $n \approx 1$, the bound is not relevant, unless very large values of $L_1/L_0$ are used. We do not discuss further this case. For $n \gtrsim 3$ the bound is very restrictive, and only $L_1/L \sim O(1)$ is allowed. This confirms the intuitive notion that if large power-law deviations are allowed over a large energy window, the model is strongly coupled and does not admit a perturbative and controllable effective field theory expansion. For $n = 2 - 3$, values of $L_1/L \sim 3 - 8$ are compatible with the requirement that the effective field theory be weakly coupled, and offer an interesting possibility from the phenomenological point of view. We focus on this possibility.

The effects of symmetry breaking are encoded in the estimate of $\hat{S}$. This is the quantity that ultimately sets a bound on $L_1$, and hence on the mass of the excited resonances. If the symmetry-breaking effects are localized at $L_1$, the analytical expression derived in Eq. (34) shows that, for all practical purposes, the bounds are the same as those obtained in the pure AdS case, $L_1 \lesssim 1$ TeV$^{-1}$. This is the case because the only sizable suppression factors are the $1/(n + 1)$ and the $\hat{L}/L_1$ terms, but at large values of $n$ only $\hat{L}/L_1 \sim 1$ is allowed.

Let us discuss the case in which symmetry-breaking takes place at $\tilde{L}$. In order to assess how sizable the reduction in the experimental bounds is, we require that $\hat{S} < 0.003$, and calculate the minimum value of $1/L_1$ which is compatible with this bound, using the expression in Eq. (39). We show the result in Figure 4 assuming various values of $L_1/\tilde{L}$. We plot, as a function of $n$, the lower bound for $\pi/(M_Z L_1)$ – which, up to boundary effects and model-dependent shifts, gives a reasonable estimate of the ratio $M_1/M_Z$ (see Figure 1) – starting from the pure AdS case, but without exceeding the $(n$-dependent) bound in Eq. (48).

In the pure AdS case ($L_1/\tilde{L} = 1$), the lower bound in Figure 4 implies (using the experimental value of $M_Z$) $M_1 \gtrsim 3$ TeV, and $M_2 \gtrsim 6-7$ TeV. Going to larger values of $L_1/\tilde{L}$ allows for a very significant reduction of such bounds, even when this ratio is small enough to be compatible with the requirement that the effective coupling $g_{\rho}^2$ be smaller than 1/2. As a result, the value of the scale $1/L_1$ can be greatly reduced. Values such as $M_1 \sim 1.5$ TeV, $M_2 \sim 3$ TeV and $M_3 \sim 4.5$ TeV are not excluded experimentally.
FIG. 2: Relative coupling $R_i$ to the currents of the first $i = 1, 2, 3$ excited vector modes, as a function of $L_1/L$. The curves are drawn for $n = 2$, and for $L_0 = L_1/20$ (thick line) and $L_0 = L_1/100$ (thin line).

A detailed calculation of the coupling to the currents and of the partial widths is necessary in order to draw firm quantitative conclusions, but these preliminary estimates indicate that the first three resonances have $R_i \sim 0.15 - 0.35$, while $g^{(1/2)}_0 \lesssim 0.5$. These resonances should have a sizable branching fraction in standard-model fermions, and a sizable production cross-section in Drell-Yan processes. In particular, for this range of masses and couplings, LHC has a good chance of detecting all of these states even at moderate integrated luminosity, by combining data on $\mu^+\mu^-$ and $e^+e^-$ final states.

VII. DISCUSSION

The starting point for the construction of an effective field theory description of dynamical electro-weak symmetry breaking is the assumption that some fundamental, possibly asymptotically free, field theory, defined in the far UV, flows towards an (approximate) strongly-coupled fixed-point in the IR. Accordingly, there is a regime at intermediate-to-low energies in which the (walking) theory can be described by a weakly-coupled five-dimensional model, in the spirit of the AdS/CFT correspondence. The presence of a deformation away from the AdS metric—in the form of some operator that becomes relevant and dominates the dynamics at long distances—drives the model away from the fixed point (inducing the loss of conformal behavior), produces non-trivial condensates (which trigger spontaneous electro-weak symmetry breaking), and ultimately leads the theory towards confinement (and hence introducing a mass gap in the spectrum of bound states).

This paper proposes a toy-model that allows for a quantitative study of the effects that such a relevant deformation might have on the low-energy observable quantities, in the regime at and below the LHC relevant energies. The basic idea is to parameterize the effects of such a deformation in terms of a power-law departure from the AdS background over a limited energy window just above the scale of confinement. This treatment proves to be useful thanks to its intrinsic simplicity and the lack of any more systematic (calculable) approach. It has its limitations as well. Hence we summarize and critically analyze our results, in order to draw some important model-independent conclusion and in order to highlight the areas where more work, and possibly some guidance from the experimental data to come, are necessary.

First of all, the type of modification of the background we propose has a very modest effect on the spectrum of composite resonances. The properties of such spectrum are still determined by the presence of a hard-wall in the IR, that acts both as a regulator and as a physical scale determining the mass gaps and spacings. It is inappropriate to believe that this model can describe accurately more than a handful of resonances, and one should be very careful
FIG. 3: Relative coupling $R_i$ to the currents of the first $i = 1, 2, 3$ excited vector modes, as a function of $L_1/L$. The curves are drawn for $n = 3$, and for $L_0 = L_1/20$ (thick line) and $L_0 = L_1/100$ (thin line).

when talking about resonances with large excitation number $i$. Yet, the model-independent message here is quite clear, and very important. While the spectrum is substantially independent of the possible presence, and type, of deformation that is driving the theory away from the fixed point in the IR, the effective couplings of the resonances, both to other resonances and to the standard model fermions, are very sensible to the departure from conformality that this deformation is introducing.

The calculation of the coupling to the currents and the estimate of the self-couplings show a large departure from the expectations based on the pure AdS case, in presence of the same regulators in the IR and in the UV. The coupling to the currents is suppressed, and the suppression in not a universal effect, but rather it is different for different resonances. The self couplings are enhanced with respect to the pure AdS case, following the four-dimensional intuition. This poses some important limitation on how long it is admissible to assume that it will take for the theory to flow from the region in proximity of the IR fixed point, where it is walking, to the new phase transition at which confinement takes place. It is very encouraging that our estimates indicate that this regime, though limited, might be long enough to allow for very sizable $O(2-4)$ effects to result, without spoiling the calculability of the effective field theory that the AdS/CFT language is supposed to provide.

The deformation responsible for the loss of conformal symmetry might or might not be related with electro-weak symmetry breaking. If not, then electro-weak symmetry breaking is triggered at the same scale as confinement, as is the case for QCD. In this case this model allows us to say that we do not expect any significant modification of the precision electro-weak parameters and of the coefficients of the electro-weak chiral Lagrangian with respect to the results obtained in the pure AdS background. In this case, the couplings of the excited states are the only observable quantities carrying information about the existence of an energy regime above the scale of confinement where the dynamics is not conformal.

At large-$N_c$ or in presence of a complicated fermionic field content in the fundamental theory, the chiral symmetry breaking condensates may form at a temperature larger than the scale of confinement. In this case, the formation of such condensates might itself be the deformation that drives the theory away from the fixed point, and that leads to confinement at some lower scale. The phenomenological consequences of such a scenario are relevant not only for the LHC, but even in analyzing LEP and Tevatron data. Our simple model allows us to show that it is reasonable to expect that in this case the estimates of the coefficients of the chiral Lagrangian (we focused on $S$ because best known and most model-independent) might be suppressed by large numerical factors, without entering a strongly coupled regime for the effective field theory, and with a resulting drastic reduction of the experimental bounds on the masses of the lightest new spin-1 states (techni-$\rho$). This toy-model highlights the fact that, whatever the fundamental theory is in the far UV, if the dynamics contains a mechanism leading to a separation of the scales of chiral symmetry breaking
FIG. 4: Lower bound on $\pi/(M_T L_1)$ as a function of $n$ in the case in which symmetry-breaking takes place at $\bar{L}$. The green curves are obtained using $L_1/\bar{L} = 1, 2, 3, 4, 5$. The cyan curve is obtained by using the limiting value of $L_1/\bar{L}$ such that $g_\rho$ be perturbative. We interrupt the (green) curves obtained at constant $L_1/\bar{L}$ at the value of $n$ for which Eq. (48) would not be satisfied, which is at the intersection with the cyan curve.

and confinement, then the expectations for $\hat{S}$, and for other precision parameters related with isospin breaking, can be changed drastically. At the LHC, this implies that, without requiring any additional custodial symmetry, nor any fine-tuning, the dynamics itself might be compatible with the detection of the first two or even three excited states, which would provide unmistakable evidence for a strongly-coupled origin of electro-weak symmetry breaking.

The techniques used here, and the choices of parameters we make, are affected by systematic uncertainties. The numerical results we obtain are to be taken as an indication of what is possible, rather than as robust predictions. Yet, part of the results are completely general: for any admissible choice of $L_1/\bar{L}$, of $n > 1$ and of the position in the fifth dimension at which we localize the symmetry-breaking terms, there is always a suppression of the coupling of the vector mesons to the currents, an enhancement of their self-couplings, and a suppression of $\hat{S}$. These are quantitative model-independent results, indicating that for these quantities the pure AdS case yields always a limiting, conservative estimate. And they all point in the direction of making the experimental searches at the LHC easier.

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