The pion-nucleon $\sigma$ term from pionic atoms

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Abstract

Earlier work suggested that the in-medium $\pi N$ threshold isovector amplitude $b_1(\rho)$ gets renormalized in pionic atoms by $\sim 30\%$ away from its $\rho = 0$ free-space value, relating such renormalization to the leading low-density decrease of the in-medium quark condensate $\langle \bar{q} q \rangle$ and the pion decay constant $f_\pi$ in terms of the pion-nucleon $\sigma$ term $\sigma_{\pi N}$. Accepting the validity of this approach, we extracted $\sigma_{\pi N}$ from a large-scale fit of pionic-atom level shift and width data across the periodic table. Our fitted value $\sigma_{\pi N} = 57 \pm 7$ MeV is robust with respect to variation of $\pi N$ interaction terms other than the isovector $s$-wave term with which $\sigma_{\pi N}$ is associated. Higher order corrections to the leading order in density involve cancellations, suggesting thereby only a few percent overall systematic uncertainty. The value of $\sigma_{\pi N}$ derived here agrees with values obtained in several recent studies based on near-threshold $\pi N$ phenomenology, but sharply disagrees with values obtained in recent direct lattice QCD calculations.

Keywords: pion-nucleon $\sigma$ term, partial restoration of chiral symmetry, pionic atoms

1. Introduction

The $\pi N$ $\sigma$ term

$$\sigma_{\pi N} = \frac{\bar{m}_q}{2m_N} \sum_{u,d} \langle N | \bar{q} q | N \rangle, \quad \bar{m}_q = \frac{1}{2} (m_u + m_d), \quad (1)$$

records the contribution of explicit chiral symmetry breaking to the nucleon mass $m_N$ arising from the non-zero value of the $u$ and $d$ quark masses in

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QCD. A wide spectrum of evaluated $\sigma_{\pi N}$ values, from about 20 to 80 MeV, was compiled by Sainio back in 2002 [1]. Recent evaluations roughly fall into two classes: (i) pion-nucleon low-energy phenomenology, using $\pi N$ $s$-wave scattering lengths derived precisely from pionic hydrogen and deuterium, results in calculated values of $\sigma_{\pi N} \sim (50 - 60)$ MeV [2, 3, 4, 5, 6], the most recent of which is $58 \pm 5$ MeV, whereas (ii) recent lattice QCD calculations reach values of $\sigma_{\pi N} \sim (30 - 50)$ MeV [7, 8, 9, 10, 11, 12], the most recent of which is $26 \pm 7$ MeV. However, when augmented by chiral perturbation expansions such lattice calculations may lead also to values of about 50 MeV, see e.g. Refs. [13, 14, 15]. This spread of calculated $\sigma_{\pi N}$ values is discussed further in the concluding section.

Here we show that the wealth of data on pionic atoms across the periodic table provides a precise determination of $\sigma_{\pi N}$. The experimental database for pionic atoms is the most extensive of all hadronic atoms [16, 17], offering a useful test-ground for studying in-medium effects. On the theory side, the near-threshold pion-nucleus optical potential $V_{\text{opt}}$ is given by single-nucleon $\pi N$ interaction terms approximated by their free-space values, with relatively small contributions from absorption on two nucleons. Our recent analysis of pionic atoms [18] demonstrated robustness in the quality of fitting the data against details of the applied analysis methodology.

The starting point in discussing in-medium renormalization in pionic atoms is that the free-space isoscalar and isovector $\pi N$ scattering lengths derived in a chiral perturbation calculation [19] from pionic hydrogen and pionic deuterium X-ray measurements [20, 21],

$$b_0^{\text{free}} = 0.0076 \pm 0.0031 \ m_{\pi}^{-1}, \quad b_1^{\text{free}} = -0.0861 \pm 0.0009 \ m_{\pi}^{-1},$$

(2)

are well approximated by the Tomozawa-Weinberg (TW) leading-order chiral limit [22]

$$b_0^{\text{TW}} = 0, \quad b_1^{\text{TW}} = -\frac{\mu_{\pi N}}{8\pi f_\pi^2} = -0.079 \ m_{\pi}^{-1},$$

(3)

where $\mu_{\pi N}$ is the pion-nucleon reduced mass and $f_\pi = 92.4$ MeV is the free-space pion decay constant. This expression for the isovector amplitude $b_1$ suggests that its in-medium renormalization is directly connected to that of the pion decay constant $f_\pi$, given to first order in the nuclear density $\rho$ by the Gell-Mann - Oakes - Renner expression [23]

$$\frac{f_\pi^2(\rho)}{f_\pi^2} = \frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle} \simeq 1 - \frac{\sigma_{\pi N}}{m_{\pi}^2 f_\pi^2} \rho,$$

(4)
where \( \langle \bar{q}q \rangle_\rho \) stands for the in-medium quark condensate and \( \sigma_{\pi N} \) is the pion-nucleon \( \sigma \) term. The decrease of \( \langle \bar{q}q \rangle_\rho \) with density in Eq. (4) marks the leading low-density behavior of the order parameter of the spontaneously broken chiral symmetry. Recalling the \( f_\pi \) dependence of \( b_{1TW}^1 \) in Eq. (3), Eq. (4) suggests the following density dependence for the in-medium \( b_1 \):

\[
b_1 = b_1^{\text{free}} \left( 1 - \frac{\sigma_{\pi N}}{m_\pi^2 f_\pi^2 \rho} \right)^{-1}.
\]

(5)

In this model, introduced by Weise \[24, 25\], the explicitly density-dependent \( b_1(\rho) \) of Eq. (5) figures directly in the pion-nucleus s-wave near-threshold potential. Studies of pionic atoms \[26, 27, 28, 29, 30, 31, 32, 33, 34, 35\] and low-energy pion-nucleus scattering \[36, 37\] confirmed that the \( \pi N \) isovector s-wave interaction term is indeed renormalized in agreement with Eq. (5). It is this in-medium renormalization that brings in \( \sigma_{\pi N} \) to the interpretation of pionic-atom data. However, the value of \( \sigma_{\pi N} \) was held fixed around 50 MeV in these studies, with no attempt to determine its optimal value.

In the present work, we kept to the \( \pi N \) isovector s-wave amplitude \( b_1 \) renormalization given by Eq. (5), but adopted a reversed approach of fitting \( \sigma_{\pi N} \) to a comprehensive set of pionic atoms data across the periodic table. Other real \( \pi N \) interaction parameters fitted simultaneously with \( \sigma_{\pi N} \) converged at expected free-space values. Holding these parameters fixed at the converged values, except for the tiny isoscalar s-wave amplitude \( b_0 \) which is renormalized primarily by a double-scattering term (see below), we get a best-fit value of \( \sigma_{\pi N} = 57 \pm 7 \) MeV.

The paper is organized as follows. In section 2, we outline the methodology applied to fitting pionic atoms data. Results are given in section 3, followed by discussion in section 4 of estimated deviations from the linear-density expression (4) and their impact on the value derived for \( \sigma_{\pi N} \).

2. Methodology

Here we briefly review the methodology applied in our recent work \[18\] to dealing with pionic atoms data, using energy-dependent optical potentials within a suitably constructed subthreshold model. For a recent review focusing on \( K^- \) and \( \eta \) nuclear near-threshold physics, see Ref. \[38\]. The pion self-energy operator \( \Pi(E, \vec{p}, \rho) \) in nuclear matter of density \( \rho \) satisfies the Klein-Gordon equation \[17\]

\[
E^2 - \vec{p}^2 - m_\pi^2 - \Pi(E, \vec{p}, \rho) = 0,
\]

(6)
where $\vec{p}$ and $E$ are the pion momentum and energy, respectively, in nuclear matter of density $\rho$. The resulting pion-nuclear optical potential $V_{\text{opt}}$, defined by $\Pi(E, \vec{p}, \rho) = 2E V_{\text{opt}}$, satisfies the following wave equation at or near threshold:

$$\left[ \nabla^2 - 2\mu(B + V_{\text{opt}} + V_c) + (V_c + B)^2 \right] \psi = 0,$$

(7)

where $\hbar = c = 1$ was implicitly assumed in these equations. In this expression, $\mu$ is the pion-nucleus reduced mass, $B$ is the complex binding energy, $V_c$ is the finite-size Coulomb interaction of the pion with the nucleus, including vacuum-polarization terms, all added according to the minimal substitution principle $E \rightarrow E - V_c$. Interaction terms negligible with respect to $2\mu V_{\text{opt}}$, i.e. $2V_c V_{\text{opt}}$ and $2BV_{\text{opt}}$, were omitted. We use the Ericson-Ericson form [39]

$$2\mu V_{\text{opt}}(r) = q(r) + \vec{\nabla} \cdot \left( \frac{\alpha_1(r)}{1 + \frac{1}{3}\xi\alpha_1(r)} + \alpha_2(r) \right) \vec{\nabla},$$

(8)

with its $s$-wave part $q(r)$ and $p$-wave part, $\alpha_1(r)$ and $\alpha_2(r)$, given by [17]

$$q(r) = -4\pi(1 + \frac{\mu}{m_N})\{b_0[\rho_n(r) + \rho_p(r)] + b_1[\rho_n(r) - \rho_p(r)]\} - 4\pi(1 + \frac{\mu}{2m_N})4B_0\rho_n(r)\rho_p(r),$$

(9)

$$\alpha_1(r) = 4\pi(1 + \frac{\mu}{m_N})^{-1}\{c_0[\rho_n(r) + \rho_p(r)] + c_1[\rho_n(r) - \rho_p(r)]\}$$

(10)

$$\alpha_2(r) = 4\pi(1 + \frac{\mu}{2m_N})^{-1}4C_0\rho_n(r)\rho_p(r),$$

(11)

where $\rho_n$ and $\rho_p$ are the neutron and proton density distributions normalized to the number of neutrons $N$ and number of protons $Z$, respectively. The coefficients $b_0$ and $b_1$ in Eq. (9) are effective, density-dependent pion-nucleon isoscalar and isovector $s$-wave scattering amplitudes, respectively, evolving from the free-space amplitudes $b_0^{\text{free}}$ and $b_1^{\text{free}}$ of Eq. (2), and are essentially real near threshold. Similarly, the coefficients $c_0$ and $c_1$ in Eq. (10) are effective $p$-wave scattering volumes which, since the $p$-wave part of $V_{\text{opt}}$ acts mostly near the nuclear surface, are close to their free-space values provided $\xi = 1$ is applied in the Lorentz-Lorenz renormalization of $\alpha_1$ in Eq. (8). The parameters $B_0$ and $C_0$ represent multi-nucleon absorption and therefore have an imaginary part. Their real parts stand for dispersive contributions which often are absorbed into the respective single-nucleon amplitudes. Below we focus on the $s$-wave part $q(r)$ of $V_{\text{opt}}$. 


Regarding the isoscalar amplitude $b_0$, since the free-space value of $b_0^\text{free}$ in Eq. (2) is exceptionally small, it is customary in the analysis of pionic atoms to supplement it by double-scattering contributions induced by Pauli correlations which give rise to explicit density dependence of the form \[ 10 \]

\[ b_0 \rightarrow b_0 - \frac{3}{2\pi} (b_0^2 + 2b_1^2)p_F, \]

where $p_F$ is the local Fermi momentum corresponding to the local nuclear density $\rho = 2p_F^3/(3\pi^2)$.

Regarding the isovector amplitude $b_1$, it is given by the r.h.s. of Eq. (5) in terms of a free-space $b_1^\text{free}$ and $\sigma_{\pi N}$. It affects primarily level shifts in pionic atoms with $N - Z \neq 0$. However, it affects also $N = Z$ pionic atoms through the dominant quadratic $b_1$ contribution to the r.h.s. of Eq. (12).

An important ingredient in the analysis of pionic atoms are the nuclear densities that enter the potential, Eqs. (8) and (9). With proton densities determined from nuclear charge densities, we vary the neutron densities searching for a best agreement with the pionic atoms data. A linear dependence of $r_n - r_p$, the difference between the root-mean-square (rms) radii, on the neutron excess ratio $(N - Z)/A$ has been recognized to be a useful and relevant representation, parameterized across the periodic table as

\[ r_n - r_p = \gamma \frac{N - Z}{A} + \delta, \]

with $\gamma$ close to 1.0 fm and $\delta$ close to zero. Two-parameter Fermi distributions are used for $\rho_p$ and $\rho_n$ with the same diffuseness parameter for protons and neutrons, the so-called ‘skin’ shape \[ 17, 41 \] which was found to yield lower values of $\chi^2$ than other shapes do for pions. Here we used $\delta = -0.035$ fm and varied the parameter $\gamma$. With $\gamma=1$ fm, for example, the ‘neutron skin’ of $^{208}\text{Pb}$ is $r_n - r_p = 0.177$ fm which agrees well with recent values derived specifically for $^{208}\text{Pb}$ from several sources.\[ 1 \] In what follows, rather than show results as a function of the neutron-excess parameter $\gamma$ of Eq. (13), we present results as a function of the implied value of $r_n - r_p$ for $^{208}\text{Pb}$, as this quantity has been discussed extensively in recent years, e.g. Refs. \[ 46, 47 \], particularly in the context of neutron stars.

\[ ^1 \text{For example, 0.16±0.02±0.04 fm from $\bar{p}$ atoms \[ 42 \], 0.156±0.025 ±0.025 fm from $E1$ polarizability studies \[ 13 \], 0.15±0.08 fm from $\pi^-$ atoms \[ 44 \], 0.11±0.06 fm from $\pi^+$ total reaction cross sections \[ 44 \], and 0.15±0.03±0.01 fm from coherent pion photoproduction measurements at MAMI \[ 45 \].} \]
3. Results

![Graph](image)

Figure 1: Fits to pionic atoms for different radial parameters of neutron densities presented as the implied neutron skin for $^{208}$Pb. Top: $\chi^2$ for 98 data points with six adjusted parameters, including $\sigma_{\pi N}$. FR stands for finite-range folding of $\pi N$ $p$-wave interaction terms. Bottom: derived values of $\sigma_{\pi N}$.

In line with our previous studies of pionic atoms [17, 18] we performed global fits to strong interaction level shifts and widths across the periodic table, from Ne to U, including ‘deeply bound’ states in Sn isotopes and in $^{205}$Pb. This approach provides an average behavior of the $\pi N$ interaction parameters within an optical potential model, Eqs. (8,9,12). Fits were made over a wide range of radial parameters for the neutron distributions. Most of the resulting $\pi N$ interaction terms, but not the $\pi N$ $\sigma$ term $\sigma_{\pi N}$, were independent of these parameters. Extensive fits essentially displayed correlations between the average radial extent of the neutron density distribution and the resulting $\sigma_{\pi N}$, as shown in Fig. 1.
Figure 1 shows fits with six adjusted parameters, namely $b_0$, $\sigma_{\pi N}$, $\text{Im} B_0$, $c_0$, $c_1$ and $\text{Im} C_0$. The real parts of the two-nucleon terms $B_0$ and $C_0$ were found compatible with zero, hence were kept zero. The imaginary parts of $B_0$ and $C_0$ were practically independent of the variable neutron density. As in earlier work [17] a finite range (FR) folding of rms radius of 0.9 fm was applied to the $\pi N$ $p$-wave interaction terms. Note in the top part that with a $\chi^2$ per degree of freedom of 1.7 a two-points distance from the minimum implies more than one standard deviation for the adjusted parameters. The bottom part shows the derived $\sigma_{\pi N}$ values with their uncertainties. An interesting by-product of these fits is the value of the implied neutron skin of $^{208}\text{Pb}$, taken from the minimum of the $\chi^2$ curve in the top part to be $0.15\pm0.03$ fm, in agreement with the values cited at the end of Sect. 2 above.

In the fits shown in Fig. 1 the single-nucleon isoscalar $c_0$ and isovector $c_1$ parameters of the $\pi N$ $p$-wave potential $\alpha_1(r)$ turned out to agree with the corresponding values of the free $\pi N$ interaction. This is shown in Fig. 2.
Figure 3: Fits to pionic atoms for different radial parameters of neutron densities presented as the implied $^{208}$Pb neutron skin, with fixed values of $c_0$ and $c_1$. Top: $\chi^2$ values. Bottom: fitted values of $\sigma_{\pi N}$. Black (red) solid (dashed) lines correspond to FR (ZR).

With $c_0$ and $c_1$ hardly dependent on the neutron densities, one could keep these fixed during fits to reduce the uncertainties of the resulting values of $\sigma_{\pi N}$. Figure 3 shows two such fits with fixed values, both analogous to Fig. 1, one with $p$-wave finite-range folding (FR, solid lines, black), and one without folding (ZR, dashed lines, red). In both parts of Fig. 3 the red curves are shifted to the right of the corresponding black curves, but for the best fit values of $\sigma_{\pi N}$, at the minima of $\chi^2$, there is hardly any difference between the FR and ZR models, regardless of the $\sim 0.06$ fm difference between the best implied values of the $^{208}$Pb skin in these models. With fixed $c_0$ and $c_1$, the fitting errors are indeed smaller than those in Fig. 1. The average value for $\sigma_{\pi N}$ from Fig. 3 is $\sigma_{\pi N} = 57 \pm 7$ MeV.
4. Discussion and summary

The pionic atoms fits and the value of the \( \pi N \sigma \) term \( \sigma_{\pi N} \) reported in the present work are based on the in-medium renormalization of the near-threshold \( \pi N \) isovector scattering amplitude \( b_1 \) as given by Eq. (5), derived from Eq. (4) for the leading order in-medium decrease of the quark condensate \( \langle \bar{q}q \rangle \). Higher order corrections to this simple form have been proposed in the literature and are discussed briefly below to see how much they affect our fitted value of \( \sigma_{\pi N} \). Generally, one does not expect appreciable corrections simply because typical nuclear densities probed in pionic atoms are only about 0.6 [29] or even 0.5 [49] of nuclear matter density. A representative effective density of \( \rho_{\text{eff}} = 0.1 \text{ fm}^{-3} \) is used for the two types of corrections discussed below.

Kaiser et al. [50] extended Eq. (4) to

\[
\frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle} = 1 - \frac{\rho}{f_\pi^2} \left[ \frac{\sigma_{\pi N}}{m_\pi^2} \left( 1 - \frac{3p_F^2}{10m_N^2} + \frac{9p_F^4}{56m_N^4} \right) + \frac{\partial E(\rho)/A}{\partial m_\pi^2} \right],
\]

(14)

accounting for kinetic energy contributions up to order \( m_N^{-3} \) in the Fermi gas model plus \( NN \) correlation contributions from one- and two-pion interaction terms. At \( \rho_{\text{eff}} = 0.1 \text{ fm}^{-3} \) and for \( \sigma_{\pi N} = 60 \text{ MeV} \) the r.h.s. of Eq. (14) is about 0.75, higher than the purely linear density expression by about 0.03. Most of this increase is owing to the \( NN \) correlation contributions. If we wish to absorb at \( \rho_{\text{eff}} \) this departure from linearity in \( \rho \) into an effective linear density form, Eq. (4), we need to increase our fitted \( \sigma_{\pi N} \) value by about 7 MeV.

Jido and collaborators [51] argued that Eq. (4) should be extended to include the departure of the in-medium pion mass \( m_\pi(\rho) \) from its free-space value \( m_\pi \):

\[
\frac{f_\pi^2(\rho)}{f_\pi^2} = \frac{m_\pi^2}{m_\pi^2(\rho)} \frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle} \simeq \frac{m_\pi^2}{m_\pi^2(\rho)} \left( 1 - \frac{\sigma_{\pi N}}{m_\pi^2 f_\pi^2} \rho \right),
\]

(15)

and also by adding corrections of order \( \rho^{4/3} \) [52, 53] which at \( \rho_{\text{eff}} \) are negligible. The pion mass dependence in Eq. (15) leads to the following modification of Eq. (5) for the near-threshold \( \pi N \) isovector amplitude:

\[
b_1 = b_1^{\text{free}} \frac{m_\pi^2(\rho)}{m_\pi^2} \left( 1 - \frac{\sigma_{\pi N}}{m_\pi^2 f_\pi^2} \rho \right)^{-1}.
\]

(16)
The pion mass in $N = Z$ isospin-zero symmetric nuclear matter increases from its free-space value $m_\pi$ to $m_\pi(\rho)$ regardless of the pion charge state owing to the weakly repulsive $b_0$ isoscalar s-wave $\pi N$ interaction term. Identifying the in-medium pion mass with $E(\vec{p} = 0)$ in the dispersion equation (6) and using Eq. (9) with an appropriate subthreshold value $b_0 = -0.011(2) m_\pi^{-1}$ corresponding to our best fit threshold value $b_0 = -0.022(2) m_\pi^{-1}$, we obtain for $\delta m_\pi^2 = m_\pi^2(\rho) - m_\pi^2$:

$$\delta m_\pi^2 \approx -4\pi \left(1 + \frac{m_\pi}{m_N}\right) b_0 \rho_{\text{eff}} = 0.045(8) m_\pi^2,$$ (17)

or equivalently $m_\pi^2(\rho_{\text{eff}})/m_\pi^2 = 1.045(8)$, in agreement with Ref. [54]. With this increased in-medium pion mass, our best-fit central value of $\sigma_{\pi N} = 57$ MeV decreases, by just 7±1 MeV, to 50±1 MeV. Perhaps fortuitously, the two higher-order effects considered here upon deriving $\sigma_{\pi N}$ from pionic atoms, Eqs. (14) and (16), cancel perfectly each other.

It is worth recalling that the attractive isoscalar $p$-wave $\pi N$ interaction term was disregarded in this uniform nuclear matter estimate where the pion momentum vanishes. In finite-size nuclei, however, multiplying $p_F(\rho_{\text{eff}})$ by $m_\pi/(m_N + m_\pi)$ a representative pion effective momentum of $p_{\text{eff}} = 29.1$ MeV is obtained. This leads to the following $p$-wave contribution:

$$\delta m_\pi^2 \approx -\frac{4\pi(1 + \frac{m_\pi}{m_N})^{-1}c_0 \rho_{\text{eff}}}{1 + \frac{1}{4}\pi(1 + \frac{m_\pi}{m_N})^{-1}c_0 \rho_{\text{eff}}} p_{\text{eff}}^2 = -0.025 m_\pi^2,$$ (18)

using $c_0^{\text{free}} = 0.230 m_\pi^{-3}$. Adding up these s-wave and $p$-wave contributions, we get $m_\pi^2(\rho_{\text{eff}})/m_\pi^2 = 1.020(8)$, leading to a decrease of our best-fit $\sigma_{\pi N}$ central value of 57 MeV, by only 3±1 MeV, to 54±1 MeV.

To conclude the discussion, we note that unlike most determinations of $\sigma_{\pi N}$ that rely heavily on the vanishingly small and highly model dependent value of the free-space $\pi N$ isoscalar scattering length $b_0^{\text{free}}$, the present work is based on the considerably larger and nearly model independent value of the free-space $\pi N$ isovector scattering length $b_1^{\text{free}}$. The dependence of $\sigma_{\pi N}$ on the input free-space $\pi N$ scattering lengths, within any specific hadronic
model calculation, is given according to the Bonn-Jülich (BJ) group\cite{55} by

$$\sigma_{\pi N} \approx (59 \pm 3) \text{ MeV} + 1.116 \Delta b_0^{\text{free}} + 0.390 \Delta b_1^{\text{free}},$$  \hspace{1cm} (19)

where the value $\langle 59 \pm 3 \rangle$ MeV is the BJ calculated $\sigma_{\pi N}$ value\cite{4} and $\Delta b_j^{\text{free}}$, $j = 0, 1$, is the difference between the values of $b_j^{\text{free}}$ (in units of $10^{-3} m_{\pi}^{-1}$) used in that specific model and in the BJ calculation. Two sets of values were suggested by BJ for $(b_0^{\text{free}}, b_1^{\text{free}})$,

BJ: \hspace{1cm} $(-0.9, -85.3) \cdot 10^{-3} m_{\pi}^{-1}$, \hspace{0.5cm} $(+7.9, -85.4) \cdot 10^{-3} m_{\pi}^{-1}$, \hspace{1cm} (20)

depending on how charge dependence is considered. These two sets differ mostly in the $b_0^{\text{free}}$ values. To demonstrate the use of Eq. (19) we refer to the evaluation of the $\pi N \sigma$ term in Ref.\cite{56} from $\pi^\pm p$ scattering data taken by the CHAOS group at TRIUMF\cite{57}. Extrapolating from the lowest pion kinetic energy of 19.9 MeV reached in the experiment, the value used in Ref.\cite{56} was $b_0^{\text{free}} = (-9.7 \pm 0.9) \cdot 10^{-3} m_{\pi}^{-1}$. Eq. (19) ‘predicts’ then a value of $\sigma_{\pi N} = 49 \pm 3$ or $39 \pm 3$ MeV, depending on the choice made for $b_0^{\text{free}}$ in Eq. (20), in rough agreement with the value $\sigma_{\pi N} = 44 \pm 12$ MeV derived in Ref.\cite{56}.  

In conclusion, we have derived in this work a value of $\sigma_{\pi N} = 57 \pm 7$ MeV from a large scale fit to pionic atoms observables, in agreement with the relatively high values reported in recent studies based on modern hadronic $\pi N$ phenomenology\cite{6}, but in disagreement with the low $\sigma_{\pi N}$ values reached in the modern lattice QCD calculations, e.g.\cite{12}. Our derivation is based on the model introduced by Weise and collaborators\cite{24, 25, 30} for the in-medium renormalization of the $\pi N$ near-threshold isovector scattering amplitude, using its leading density dependence Eq. (5), and is robust against variation of other $\pi N$ interaction parameters that enter the low-energy pion self-energy operator.

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