Probing the core and envelope structure of DBV white dwarfs

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ABSTRACT
We investigate the global pulsation properties of DBV white dwarf models that include both the double-layered envelope structure expected from time-dependent diffusion calculations, as well as a non-uniform C/O core expected from prior nuclear burning. We compare these models to otherwise identical models containing a pure C core to determine whether the addition of core structure leads to any significant improvement. Our double-layered envelope model fit to GD 358 that includes an adjustable C/O core is significantly better than our pure C core fit (7σ improvement). We find a comparable improvement from fits to a second DBV star, CBS 114, though the values of the derived parameters may be more difficult to reconcile with stellar evolution theory. We find that our models are systematically cooler by 1,900 K relative to the similar models of Fontaine & Brassard (2002). Although a fit to their model reproduces the mass and envelope structure almost exactly, we are unable to reproduce the absolute quality of their fit to GD 358. Differences between the constitutive physics employed by the two models may account for both the temperature offset and the period residuals.

Key words: stars: evolution – stars: individual (GD 358, CBS 114) – stars: interiors – stars: oscillations – white dwarfs

1 INTRODUCTION
The vast majority of what we can learn about stars is derived from observations of the thin outermost skins of their surface layers. For the most part, we are left to infer the properties of the interior based upon our current best understanding of the constitutive physics. There are a number of exceptions to this general rule, including observations of supernovae explosions and interacting binary stars, where the deeper layers of the interior are either ejected from the system or are gradually exposed as mass is transferred from one star to the other, respectively. However, both of these processes are disruptive to the stellar interior, making inferences about the original structure ambiguous at some level.

Pulsating stars represent the best opportunity for probing stellar interiors while they are still intact. The most dramatic example is the Sun, where observations of light and radial velocity variations across the visible surface have led to the identification of thousands of unique pulsation modes, each sampling the solar interior in a slightly different and complementary way. These observations have led to such precise constraints on the standard solar model that the inverted radial profile of the sound speed, for example, now agrees to better than a few parts per thousand over 90 per cent of the solar radius (Christensen-Dalsgaard 2002).

If we were to move the Sun to the distance of even the nearest star, most of the pulsation modes that we now know to be present would be rendered undetectable. We would lose nearly all of our spatial resolution across the disc of the star, and only those modes of the lowest spherical degree (ℓ ≤ 3) would produce significant variations in the total integrated light or the spectral line profiles. This would reduce the number of detectable modes from thousands to merely dozens, leading to a corresponding reduction in the ability of the observations to constrain the internal structure (e.g., see Kieldse et al. 1999). Even so, such data would still allow us to determine the global properties of the star and to probe the gross internal composition and structure, providing valuable independent tests of stellar evolution theory.

Pulsations in white dwarfs were first discovered in the cooler DA stars by Landolt (1968), and later in the hotter PG 1159 stars (McGraw et al. 1979) and DB stars (Winget et al. 1983). These differ from the solar oscillations in that they are non-radial g-modes instead of p-modes; the restoring force is gravity rather than pressure, and so the most important physical quantity is the buoyancy frequency, not the acoustic frequency. White dwarf stars are the end-points of stellar evolution for all stars with masses below what is necessary to produce elements much heavier than carbon and oxygen. Their interior structure contains a record of the physical processes that operate during the later stages in the lives of most stars, so there is a potential wealth of information encoded in their pulsation frequencies.

Much of the observational data for white dwarf asteroseismology has come from an international collaboration known as the Whole Earth Telescope (WET, Nather et al. 1993). This network of small telescopes situated around the globe obtains the long time-series of photometric measurements that are often necessary to unambiguously identify and resolve dozens of unique
pulsation modes. One of the most notable successes of this collaboration was an observing campaign in 1990 on the brightest known DB variable, GD 358 (Winget et al. 1994). Among many interesting results, these observations established the simultaneous presence of 11 low-degree (\(\ell = 1, m = 0\)) modes of consecutive radial overtone (\(k = 8–18\)) with periods in the range 400–800 seconds (Bradley & Winget 1994).

The theoretical interpretation of these data has grown gradually more sophisticated as our computational capabilities have expanded over the decade following the observations. The most recent analysis produced a model that leads to a root-mean-square difference between the observed and calculated periods of only \(\sim 1\) second (Metcalfe 2003). However, a comparable match to the same observations was recently published by Fontaine & Brassard (2002), who used a completely independent model with an internal structure that was physically distinct from that assumed by Metcalfe (see Fig. 1). Essentially, Fontaine & Brassard’s model contains additional structure in the envelope, while Metcalfe’s model has extra structure in the core.

While this duality in model-fits is now understood to be due to an inherent symmetry in the way the pulsations sample the interior (Montgomery, Metcalfe, & Winget 2003), there is good reason to believe that the physical basis of each model is sound, but that neither represents a complete description of the true interior structure. In this paper, we attempt to bridge the gap between these two models by including the essential elements of each into a ‘hybrid’ model that contains both the double-layered envelope structure expected from time-dependent diffusion calculations, and an adjustable carbon/oxygen (C/O) core. In §2 we outline the physical basis of our model parametrizations, and in §3 we present the results of our model-to-model and model-to-data comparisons. We conclude in §4 with a brief discussion of the promise of asteroseismology.

2 THEORETICAL MODELS

2.1 Adjustable C/O Cores

The generic shape of a theoretical white dwarf internal oxygen profile is set by the nuclear and mixing processes that occur during its formation in the core of a red giant. The detailed shape is less certain, since it depends on the specific physical and numerical treatments utilized by the adopted model. Many models agree that the inner \(\sim 0.5\) in fractional mass should contain an approximately uniform C/O mixture (Salaris et al. 1997; Althaus et al. 2002). The precise extent of this region is determined by the maximum size of the central convective core (and by the amount of convective overshooting, if it occurs) during helium burning. The C/O ratio in this uniform region is primarily set by the rate of the \(^{12}\text{C}(\alpha, \gamma)^{16}\text{O}\) reaction. Outside of the uniform C/O core the exact manner in which the oxygen mass fraction drops to zero is uncertain, since it depends on the adopted recipe for semiconvection during core helium burning and on the physical conditions in the models during helium shell burning. However, the location and slope of the initial break from a constant C/O ratio seems to be the most important feature from an asteroseismological standpoint.

Our parametrization for the C/O core is identical to that used in the earlier model fitting of Metcalfe (2003), Metcalfe, Salaris, & Winget (2002), and Metcalfe, Winget, & Charbonneau (2001). We fix the oxygen mass fraction to its central value (\(X_0\)) out to some fractional mass (\(q\)) where it then decreases linearly in mass to zero oxygen at the 0.95 fractional mass point (see Fig. 2). This form for the profile was chosen to facilitate comparison with the earlier work of Bradley, Winget, & Wood (1993).

Despite this simplification, Metcalfe (2003) obtained fits to two different pulsating DB white dwarfs that independently led to...
find a match with root-mean-square period residuals nearly as low as the fit of [Metcalf 2003]. They also attempted similar fits using models with cores of pure O and a uniform C/O mixture, resulting in models that were only ‘marginally worse’. They speculated that the 11 periods observed in GD 358 may not contain enough information to measure the core and envelope structure simultaneously.

To investigate this possibility more thoroughly, we replaced the single-layered envelopes in our models with a parametrization of the double-layered structure used by [Fontaine & Brassard 2002]. This allows us to specify the locations in the envelope where the He mass fraction reaches 0.15 (the middle of the transition between the C/O core and the He/C mantle) and 0.65 (the middle of the transition between the mantle and the pure He surface)\(^1\). The He profiles from several different models are shown in Fig. 4. Although there are slight differences between the profile from our parameterization compared to that of [Fontaine & Brassard 2002], we have verified that modifications to the detailed shape of the profile comparable to these differences lead to only marginal improvements in the final fits. Note that the two layers are theoretically expected to merge into one at low values of \(T_\text{eff} \) [Gautschy & Althaus 2002], leading to structures that have effectively been explored by the single-layered global model-fitting of [Metcalf, Winget, & Charbonneau 2001] and [Handler, Metcalf, & Wood 2002] for GD 358 and CBS 114 respectively.

### 3 NEW MODEL FITTING

To get a sense of the relative improvement that might be possible by including an adjustable C/O core in our models, we performed two sets of fits to the pulsation data. In both cases, we used a genetic algorithm (see [Metcalf & Charbonneau 2003]) to minimize the root-mean-square residuals between the observed and calculated periods (\(\sigma_P\)) for models with effective temperatures (\(T_\text{eff}\)) between 20,000 and 30,000 K, and total stellar masses (\(M_*\)) between 0.45 and 0.95 \(M_\odot\). We allowed the location of the inner He transition (between the core and the mixed He/C mantle) to assume values of \(\log(M_\text{env}/M_*)\) between \(-2.0\) and \(-4.0\), and for the outer He transition (between the mixed He/C mantle and the pure He surface) the value of \(\log(M_\text{env}/M_*)\) was in the range \(-5.0\) to \(-7.0\). For the first set of fits we fixed the core composition to be pure C. For the second set we included an adjustable C/O core, allowing the central oxygen mass fraction (\(X_O\)) to vary between 0.0 and 1.0, with the break from a uniform C/O mixture beginning at a fractional mass (\(q\)) between 0.10 and 0.85.

We performed these fits on three sets of pulsation data. To explore the systematic differences between our own models and those of [Fontaine & Brassard 2002], we fit the 11 model pulsation periods of their fit to GD 358, which had a pure C core. For our adjustable C/O fit to these periods, we used a single-layered envelope model as part of an investigation of the symmetry between these two types of models [Montgomery, Metcalf, & Winget 2003]. The second set of data came from the 1990 WET campaign on GD 358 [Winget et al. 1994], which is the same set of 11 periods fit by [Fontaine & Brassard 2002]. To investigate whether the double-layered envelope models could produce fits to a second DBV star that were physically consistent with time-dependent diffusion theory, our third set of data came from the slightly hotter white dwarf PG 1159 that were physically consistent with time-dependent diffusion theory, our third set of data came from the slightly hotter white dwarf PG 1159.
3.1 Carbon Core Models

From our pure C core fit to the model periods of Fontaine & Brassard (2002), we retrieved the same mass and virtually identical locations for the two He transitions in the envelope, but our temperature was systematically low by 1,900 K relative to their value (24,800 K). Because the internal structure of the two models is very similar, this temperature offset is most likely due to differences between the constitutive physics (equations of state, opacities, etc.) as well as the convective prescription employed by each. In a study of the influence of various updates to the constitutive physics on the pulsational properties of their models, Fontaine & Brassard (1994) found that the OPAL radiative opacities (Iglesias & Rogers 1993) for pure He and C were systematically lower than the older LAO data (Huebner et al. 1977), and that models using the higher opacity data could mimick hotter models with otherwise identical parameters. This is consistent with the results of our fit, since our models use the more opaque LAO data, while Fontaine & Brassard use the more recent OPAL data (Fontaine, Brassard, & Bergeron 2001). However, our preliminary attempts to use the OPAL data in our own code suggest that this may only account for roughly half of the temperature difference between the models. Other possible contributions include differences between our equations of state – both of our models use tables derived from Lamb & Van Horn (1975) and Fontaine Graboske & Van Horn (1977) for parts of the interior and the envelope respectively, but Fontaine & Brassard's models include additional unpublished modifications (Fontaine, Brassard, & Bergeron 2001). More work will be required to document these differences and to evaluate their influence on the effective temperature scales of our models.

Our fit to the observed pulsation periods of GD 358 has a higher mass and a lower temperature than the fit of Fontaine & Brassard (2002), but the locations of the two He transitions are both within their 1σ uncertainties. The most striking difference between our fits is that while Fontaine & Brassard were able to match the periods observed in GD 358 with residuals of only σp = 1.30 s, our optimal fit has σp = 2.17 s (a difference nearly 30 times larger than the observational uncertainty, σobs ≈ 0.03 s). Again, differences between the constitutive physics employed by the two models may account for at least part of this difference in the absolute quality of the two fits – using the OPAL radiative opacities with our models led to a significant decrease in the residuals (e.g., from 1.63 to 1.45 s in the case of our fit to Fontaine & Brassard's model). Despite the unresolved differences between the constitutive physics, we can still use the relative quality of our own fits to determine whether or not the addition of extra free parameters in a given model lead to significant improvements.

Our double-layered envelope fit to CBS 114 is only marginally (2σobs) better than the pure C core single-layered envelope model fit of Handler, Metcalfe, & Wood (2002), compared to what is expected from the addition of an extra parameter. Our values for the mass, the temperature, and the (inner) He transition are indistinguishable from the single-layered fit. Although the derived temperature for CBS 114 is higher than for GD 358, as expected from the spectroscopic measurements of Beauchamp et al. (1999), the pure He surface layer for this fit is thicker than for GD 358 – just the opposite of what time-dependent diffusion calculations would lead us to expect for models with similar masses.

3.2 C/O Core Models

When we used an adjustable C/O core and a single-layered envelope to fit the model periods of Fontaine & Brassard (2002), it led to a significant improvement to our match even though the interior structure of our model was dramatically different from the source model. Our derived value of q corresponds exactly to the true location of the outer He transition reflected through the core/envelope symmetry described by Montgomery, Metcalfe, & Winget (2003). Essentially our fit confirms empirically that such a symmetry exists, and that it is possible to fit real structure in the envelopes by assuming structure at the corresponding symmetric location in the core.

Our fit to GD 358 leads to inferred locations for all chemical transition zones that are distinct from each other with respect to the core/envelope symmetry. Statistically, the addition of an adjustable C/O core to the double-layered envelope fit leads us to expect the residuals to decrease to 1.74 s just from the addition of the two extra parameters. In fact the residuals of our C/O fit are reduced to 1.52 s, a 7σobs improvement beyond what is expected. In this model, the C/O core is effectively fitting the same mode trapping structure attributed to the outer He transition.
in Fontaine & Brassard’s fit. Our outer He transition, centered at log($M_{\text{He}}/M_\ast$) = \textasciitilde-5.22, is distinct since no structure is expected at the corresponding symmetric location in the core, which is near $M_\ast/M_\ast \sim 0.6$ (see Montgomery, Metcalfe, & Winget 2003, their fig. 3).

The improvement in the fit to CBS 114 is even more substantial. While the derived locations of the two He transitions do not change by much between the pure C and the C/O fits, the addition of structure in the core leads to an improvement in the fit of nearly $12\sigma_{\text{obs}}$ beyond what is expected from the extra parameters. As with GD 358, the locations of the core and outer envelope structure are distinct from one another, from the standpoint of the symmetry inherent in the models. Interestingly, the conflict between the higher temperature and the thickness of the surface He layer disappears for the C/O fits. However, both fits to CBS 114 lead to an unusually small total envelope mass relative to the range expected [log($M_{\text{env}}/M_\ast$) \textasciitilde-2.0 to \textasciitilde-3.0] from simulations of carbon dredge-up in DQ stars (see Pelletier et al. 1986; Fontaine & Brassard 2002). In addition, the masses of the C/O fits are in conflict with the spectroscopic measurements of Beauchamp et al. (1999).

4 DISCUSSION

Our global exploration of models containing double-layered envelope structure make it clear that the addition of an adjustable C/O core leads to significantly better fits to the observations, relative to pure C core models. This is reassuring, since there are sound physical reasons that lead us to expect composition transition zones in both the cores and the envelopes of real white dwarf stars.

The absolute quality of our fits to GD 358 are generally worse than the fits of Fontaine & Brassard (2002) or Metcalfe (2003). If white dwarf envelopes really do contain a double-layered structure, this implies that we will need to know the detailed shapes of the composition transition zones in both the core and the envelope before better fits can be found.

The fits to CBS 114 support our general conclusion that the models are sensitive to structure in both the core and the envelope, but only the C/O fit is consistent with the expectations from time-dependent diffusion theory (thinner surface He layers for hotter white dwarfs of comparable mass). However, the relatively small derived values for the stellar mass and the total envelope mass remain a concern, implying that double-layered models may be less able to explain the observations of both GD 358 and CBS 114 in a physically self-consistent manner (cf. Metcalfe 2003). Additional calculations of the mass-dependence of time-dependent diffusion profiles in white dwarf envelopes and higher signal-to-noise observations of CBS 114 to yield a larger number of periods for fitting would both clearly be useful.

Comparison of our model to that of Fontaine & Brassard revealed a systematic difference of 1,900 K between our effective temperature scales. This may help us to understand the relatively low effective temperatures derived in previous studies using our models (Metcalfe, Winget, & Charbonneau 2001, Metcalfe, Salaris, & Winget 2002, Metcalfe 2003). The source of this temperature offset appears to be related to our use of the larger LAO radiative opacities, and possibly due to subtle differences between our convective prescriptions and the equations of state that we use for both the core and the envelope.

Our understanding of stellar interiors through white dwarf asteroseismology is evolving rapidly. It is now clear that the pulsations really do sample the star globally, and we must be careful to avoid the potential ambiguities caused by the intrinsic core/envelope symmetry. Our challenge is to find a physically self-consistent description of both the core and the envelope that will allow us to match the pulsation periods within the observational uncertainties. With persistence, and with an open-source development philosophy for our models (Metcalfe et al. 2002), the future holds great promise for unveiling the detailed composition and structure of white dwarf interiors.

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