Decomposition of a complex dynamic system with a network architecture

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Abstract. The paper presents the method of decomposition of a complex dynamic network based on the diacoptic approach. The proposed method is aimed at identifying independent network segments, the removal of which from the overall structure will not affect the overall system performance. Segments are formed as simple paths in the graph model of the system with the smallest weight of incident edges and checking the condition for maintaining the balance of the product flow as applied to the transportation problem. In the general case, the presented decomposition method makes it possible to divide a complex technical object with the network architecture into subsystems, simplifying the process of analysis and control. In the case when the general task of control and the dynamics of interaction between subsystems allows the system to be divided into unrelated segments, or to limit the interaction along certain lines, the method allows reducing the load on the transmitting elements of the system.

1. Introduction
Most real technical systems are multiply connected complex dynamic objects consisting of a large number of interconnected components. For example, a network of positioned orbital satellites [1, 2], a group of mobile objects [3], networks of sensor systems [4], power system [5], distributed computer networks [6], paper machines [7], gas turbine engines [8], etc. Traditional decomposition methods are applicable to dynamical systems of small dimension. Analysis of a complex dynamic network by the traditional decomposition methods is a difficult task and often comes down to finding weak links. This can lead to the division of the system not into subsystems, but into separate components. The most preferred methods for decomposing such systems are topological methods based on diakoptics [9]. The introduction of conditions for maintaining the balance of the weights of incident edges within the subsystems will allow the network to be divided into independent segments. The functioning of these segments will be focused on the overall task of the dynamic system. Since within the subsystems (segments) the balance of the edge weights is maintained, such subsystems can be excluded from the network without affecting the system’s performance. Further analysis and control of a separated dynamic system is possible by known methods [10, 11].

The paper considers the problem of decomposition of a complex dynamic system with a network architecture. The graphical model of the dynamic system is represented as a bipartite graph, where the set of vertices of the graph M is divided into two subsets X - transmitting elements and Y - consuming elements of the system, so that $X \subseteq M$ and $Y \subseteq M$, $X \cap Y = M$, $X \cap Y = M_{XY}$, where $M_{XY}$ - a set of elements that are both transmitters and consumers depending on the state and operating conditions. The relationship between the elements of the system from X and Y is described by the product stream...
transferred from Xi to Yj. The product can be power in the power grid, gas flow in gas distribution systems, etc.

2. Decomposition method
The multi-connected model of the system is a bipartite graph, on the one hand of which are transmitting elements \( X_i \), on the other hand consuming elements \( Y_j \), each vertex of the graph has several incident edges, which reflects possible channels for transporting some product (Figure 1). The task of decomposition is to identify network sections, the allocation of which from the overall system will not affect the overall performance and the ability to achieve the overall goal of the system. This statement of the problem allows to define segments with minimal weight of incident edges. If the weight is the value of the cost of passing the resulting path, it is possible to minimize the load on the transmitting elements of the system. The most preferable channel of product transportation can be considered a way that can transfer the largest amount of product at the lowest cost. Thus, the value of losses on the way \( W_{ij} \) is put in correspondence to each edge, and the value of the produced \( P_{gi} \) or consumed \( P_{pj} \) product is put in correspondence to each vertex.

It is necessary to single out a graph in which each consumer will be associated with two transmission components capable of meeting his needs for a product with a certain supply of \( R \). To determine these transmitters is needed to meet the minimum expenditures for transportation of the product. To do this, we construct the reduced adjacency matrix of the graph for the vertices of the fraction \( Y_j \), where the adjacency indicator will be the weight of the incident edge to adjacent vertices:

\[
A = Y_j \begin{bmatrix} W_{ij} \end{bmatrix}
\]  

(1)

In case the vertices are not adjacent \( W_{ij} = 0 \). For the considered graph (Figure 1) the reduced adjacency matrix will take the form:

\[
A = \begin{bmatrix} X_1 & X_2 & X_3 & X_4 & X_5 \\ Y_1 & W_{11} & W_{21} & 0 & W_{41} & 0 \\ Y_2 & W_{12} & W_{22} & W_{32} & 0 & W_{52} \\ Y_3 & 0 & 0 & 0 & W_{43} & W_{53} \\ Y_4 & 0 & W_{24} & W_{34} & W_{44} & W_{54} \end{bmatrix}
\]  

(2)

We define an \( A_{m2} \) matrix that will contain two adjacent vertices to \( Y_j \) (each element of the set \( Y \)) with the minimum weight of incident edges, so that the condition is satisfied:

\[
A_{m2} = \begin{bmatrix} Z^*_n \end{bmatrix}_{X} + \begin{bmatrix} Z^*_n \end{bmatrix}_{X} 
\]  

(3)

where \( Z^*(n,X) = A(n,X) \rightarrow \min \), \( Z^*(n,X) = A^*(n,Y) \rightarrow \min \), \( A^* = A - Z^*_Y \times X, n = 1, \ldots, Y, Z^*_Y \times X \) – a zero matrix with a nonzero element \( Z^*(n,X) \) in each row that is equal to the minimum element in each row of the matrix \( A_{Y \times X} \).

We introduce conditions for the weights of the edges: \( Y_1: W_{11} < W_{21} < W_{41}; Y_2: W_{22} < W_{32} < W_{32} < W_{52}; Y_3: W_{43} < W_{53}; Y_4: W_{24} < W_{44} < W_{44} < W_{54} \). Based on these conditions, we define the matrix \( A_{m2} \):

\[
A_{m2} = \begin{bmatrix} W_{11} & W_{21} & 0 & 0 & 0 \\ 0 & W_{22} & W_{32} & 0 & 0 \\ 0 & 0 & 0 & W_{43} & W_{53} \\ 0 & 0 & 0 & W_{44} & W_{54} \end{bmatrix}
\]  

(4)

Thus, two unrelated graphs are obtained (Figure 2).

To isolate the resulting graphs into separate network segments, it is necessary that two conditions be met:
where $A_{n2}(Y_j,:) - \text{set of connected vertices to a vertex } Y_j \text{ from } A_{n2}$ matrix; $A'_{n2}(Y_j,X_i) - \text{a set of vertices formed from simple paths obtained from } A_{n2}$ matrix; $P_{ji}$ - product quantity required by the $j$-th user; $P_{yi}$ - the amount of product that can be produced by the $i$-th transmitting component; $R$ - the level of the required reserve of product up to a maximum transmitter performance.

**Figure 1.** Segment of a dynamic system

**Figure 2.** Decomposition of the dynamic network (1)

Check the obtained graphs (Figure 2) for compliance with the conditions (5):

- Graph 1:\n  \[
  \begin{align*}
  P_{p_1} &\leq 0.8 \cdot (P_{s_1} + P_{s_2}); \\
  P_{p_2} &\leq 0.8 \cdot (P_{s_2} + P_{s_3}); \\
  P_{p_1} + P_{p_2} &\leq 0.8 \cdot (P_{s_1} + P_{s_2} + P_{s_3}).
  \end{align*}
  \] (6.1.a)

- Graph 2:\n  \[
  \begin{align*}
  P_{p_3} &\leq 0.8 \cdot (P_{s_4} + P_{s_5}); \\
  P_{p_4} &\leq 0.8 \cdot (P_{s_4} + P_{s_5}); \\
  P_{p_3} + P_{p_4} &\leq 0.8 \cdot (P_{s_4} + P_{s_5}).
  \end{align*}
  \] (6.1.b)

In case of complete non-fulfillment of inequality (6.1), and (6.2) is not fulfilled, but in case of non-fulfillment (6.2) inequality (6.1) can be partially fulfilled (for example, inequality (6.1.a) - true, and (6.1.b) – wrong). It is possible that (6.2) is true and (6.1) is partially true. If all inequalities are satisfied, we can assume that the segments of the system are found. In the case where (6.1) is partially satisfied, we determine the vertices from the set $Y$ for which (6.1) is incorrect and select the next adjacent vertex from the set $X$ with the lowest weight of the incident edge, and form a matrix $A_{n3}$:

\[
A_{n3} = A_{n2} + \left[ Z^*_{y\times x} \right]
\] (7)

in case $P_{yi} > R \cdot \Sigma P_{gi}, \Sigma P_{ji} > R \cdot \Sigma P_{gi}$, where $Z^{*\text{a}}(Y_j,X) = A^{*\text{a}}(Y_j,X) \rightarrow \text{min}, A^{*\text{a}} = A \cdot A_{n2}$.

Suppose that the inequality (6.1.b) for graph 1 is not executed, in this case we define the matrix $Z^{*\text{b}}_{y\times x}$ and $A_{n3}$, and construct graphs 1 and 2 (Figure 3) according to the obtained $A_{n3}$ matrix:

\[
\begin{bmatrix}
0 & 0 & 0 & 0 \\
W_{11} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix};
\begin{bmatrix}
W_{11} & W_{21} & 0 & 0 \\
W_{12} & W_{22} & W_{32} & 0 \\
0 & 0 & W_{43} & W_{53} \\
0 & 0 & W_{44} & W_{54}
\end{bmatrix},
\]

If partially executed (6.1) and not executed (6.2), it is necessary to look for an adjacent vertex not belonging to the set $A^{*\text{a}}_{n2}(Y_j,X_i)$.

\[
A_{n3} = A_{n2} + \left[ Z^*_{y\times x} \right]
\] (8)

in case $P_{yi} > R \cdot \Sigma P_{gi}, \Sigma P_{ji} > R \cdot \Sigma P_{gi}$, where $Z^{*\text{c}}(Y_j,X) = A^{*\text{c}}(Y_j,X) \rightarrow \text{min}, A^{*\text{c}} = A \cdot A_{n2}$, $X^* \in (X \setminus A_{n2}(Y_j,X_i))$.

Suppose that (6.1.b) and (6.2) for graph 1 is not performed, in this case we define $Z^{*\text{d}}$ matrix and $A_{n3}$ and construct graphs 1 and 2 (Figure 4) according to the obtained matrix $A_{n3}$:
\[ Z_{Y,X}^{*} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & W_{52} \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad ; \quad A_{m3} = \begin{bmatrix} W_{11} & W_{21} & 0 & 0 \\ 0 & W_{22} & W_{32} & 0 & W_{52} \\ 0 & 0 & 0 & W_{43} & W_{53} \\ 0 & 0 & 0 & W_{44} & W_{54} \end{bmatrix} . \]

**Figure 3. Decomposition of the dynamic network (2)**

![Graph 1 and Graph 2](image1)

**Figure 4. Decomposition of the dynamic network (3)**

![Graph 1 and Graph 2](image2)

For graph 1 (Figure 3), the adjacent vertex was found from the set of vertices of graph 2, forming a single simple path combining both graphs (Figure 4). Thus, the set \( A_{n_{mk}}(Y_j,X_i) \) of graph 1 is replenished with a similar set of graph 2, and this segment becomes one and is highlighted when performing (5). If inequalities (5) are not satisfied, a return to the search for the next adjacent vertex occurs (7) or (8), depending on which inequality is not satisfied.

In the general case, the problem of finding adjacent vertices when inequalities (5) are not fulfilled is written in the form:

\[
A_{mk} = A_{m(k-1)} + \left[ Z_{Y,X}^{*} \right], \text{ in case } P_{ij} > 0.8 \cdot \sum P_{ij}, \sum P_{ji} \leq 0.8 \cdot \sum P_{ii}, k > 2, \\
A_{mk} = A_{m(k-1)} + \left[ Z_{Y,X}^{*} \right], \text{ in case } P_{ij} > 0.8 \cdot \sum P_{ij}, \sum P_{ji} > 0.8 \cdot \sum P_{ii}, k > 2, \\
Z_{Y,X}^{*} (Y_j,X_i) = A_{n_{mk}}(Y_j,X_i) \to \min, A_{n_{mk}} = A - A_{m(k-1)}. 
\]

(9)

where \( k \) – the number of iterations of the search cycle for adjacent vertices, starting with \( k=3 \) at the first access to the system of inequalities (5) (since the search for the third adjacent vertex from \( X \) for the vertex from \( Y \)).

At the end of the cycle of search for adjacent vertices and the formation of the segment will be obtained matrix \( A_{mk} \). \( A_{mk} \) – adjacency matrix for a network segment filled with interconnected vertices and edges between which the product is transferred. Let’s define an adjacency matrix for vertices connected but not involved in the product transportation process (the connection exists but will be involved in emergency situations):

\[
A_{ne} = A - A_{mk} 
\]

(10)

For the segments found in fig. 3, we define the \( A_{ne} \) matrices and construct the resulting segments taking into account the temporarily unused edges (Figure 5):

\[
A_{ne} = \begin{bmatrix} W_{11} & W_{21} & 0 & 0 \\ W_{12} & W_{22} & W_{32} & 0 \\ 0 & 0 & 0 & W_{43} & W_{53} \\ 0 & 0 & 0 & W_{44} & W_{54} \end{bmatrix} , \quad k = 3 ; \quad A_{ne} = \begin{bmatrix} 0 & 0 & 0 & W_{41} & 0 \\ 0 & 0 & 0 & 0 & W_{52} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & W_{24} & W_{34} & 0 & 0 \end{bmatrix} .
\]
3. **Modelling the decomposition method on the example of the grid**

Consider the operation of the algorithm on the example of the test scheme IEEE 118-node power system in graphical representation [12] (Figure 6). In the graphical model the black dots represent the nodes load, white dots – nodes generators. When modeling the decomposition algorithm for each edge of the graph, the value of the active resistance of the line was set in accordance. Information about lines, load, generators, mode parameters are presented in [13]. The result of the network decomposition algorithm is shown in Figure 7, the requirements for the residual power reserve were not introduced.

Based on the simulation results, the decomposition algorithm divided the original network into five segments: 1st segment consumes 458 MW and has a power reserve of 77 MW; 2nd segment consumes 462 MW and has a power reserve of 72 MW; 3rd segment consumes 1,859 MW and has a power reserve of 23 MW; 4th segment consumes 600 MW and has a power reserve of 11 MW; 5th segment consumes 800 MW and has a power reserve of 5 MW. When calculating the power balance, the maximum and total power consumption inside each segment and the maximum power of energy sources were taken into account.

![Redundant transmitters at the junction of segments](image)

**Figure 5.** Decomposition of the dynamic network (4)

![A graphical model of the grid](image)

**Figure 6.** A graphical model of the grid

![Result of the grid decomposition algorithm](image)

**Figure 7.** Result of the grid decomposition algorithm

We will analyze the energy losses in the redistribution of power flows. Suppose that the power system is operating normally, and intentional disconnection of inter-segment links will not affect the parameters of the mode, and the distribution of power flows on other lines maintains the balance of power.

Disabling the link between nodes 14-15 limits the flow of active power to 4.18 MW. Analysis of the mode parameters shows that this power is transferred from the 12th generator node, the power flow path is from node 12 to node 15: 12-14-15. If the supply of power is limited along this path, there is a shortage of power in node 15, which can be replenished from the 26th node-generator of segment 2. In the analysis of active power losses, we will operate with the active resistance of the line. For the line 12-14-15: \( R = 14 \text{ Ohm} \), for the line: 26-30-17-15: 12 Ohm. Based on the traditional ratio for calculating losses in power lines [14], we obtain that when transferring power through line 26-30-17-
15, losses will decrease by 14.29% (from 0.05 MW to 0.043 MW) compared to transfer through line 12-14-15. Further analysis of the operating modes of the power grid segments indicates that this power does not participate in the formation of the power balance within the 2nd segment and is transferred further along the line 15-33 to the 3rd segment. Taking into account that the capacity of each segment is sufficient, the power flow along the line 12-15 can be limited without damage to the power system. Thus, it is necessary to cover the resulting shortage of power in the 33rd node. On lines 15-33 transmitted power 7.22 MW, of which 4.21 and 0.74 MW was transported along the lines 12-14-15 and 10-9-8-5-4-11-13-15, respectively. The resistance of the 12-33 line will be 22.2 Ohms, and the replacement line 65-38-37-33 has a resistance of 18.6 Ohms. Thus, the loss of line 65-33 in the transmission of 4.95 MW will be 0.012 MW, and total losses along the lines 10-33 and 12-33 will be 0.0401 MW. Reduced power loss will be 70%.

Analysis of power flows along the lines connecting the segments of the grid, compared with the power losses of the replacement lines within the entire divided power system, shows a decrease in power losses of 57%. Depending on the network topology and parameters of its mode, the calculated reduction in power losses along the replacement lines connecting the segments may vary.

4. Conclusion
The article proposes a method for decomposing a complex dynamic system with a network architecture. The method belongs to the class of topological methods and is based on the diakoptic approach to the study of complex dynamic systems in parts. The paper introduces a number of conditions that reflect the preservation of the product flow balance between the components. Thus, segments are formed, the removal of which from the general structure will not affect the overall performance of the system. If it is possible to physically divide the system into segments, the method reduces the load on the transmitting elements on the destroyed communication lines to 55-60%. Otherwise, the interaction between the segments is taken into account by means of analysis of the new graph, where the segments act as elements of the system.

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Acknowledgments
This work was supported by Russia's Southern Federal University grant №ВнГр-07/2017-15: Development of theoretical foundations and creation methods of intelligent multivariable control systems for generating processes, transportation, distribution and consumption of energy. The reported study was funded by RFBR according to the research project № 16-08-00013 А.