Extraction of $\gamma$ from three-body $B$ decays\textsuperscript{1}

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The conventional use of two-body $B$ decays to extract $\gamma$, although theoretically clean, is currently statistics-limited. On the other hand, a bulk of data on three-body $B$ decays is available from $B$ factories. Applying the flavor-SU(3)-symmetric approach proposed in Ref. \cite{2} to BABAR data, we find the highly promising result $\gamma = (81^{+4}_{-5}(\text{avg.}) \pm 5(\text{std. dev.}))^\circ$. This establishes the use of three-body $B$ decays as a viable alternative for the extraction of weak phases. In this preliminary analysis we have neglected several sources of uncertainties such as the effect of flavor-SU(3) breaking due to meson masses, and error correlations between input experimental parameters. A better understanding of these will improve the viability of this method.

The Cabibbo-Kobayashi-Maskawa (CKM) phase $\gamma$ is conventionally extracted using two-body $B$ decays \cite{1} where the goal is to use the interference between the decay amplitudes for $b \to c\bar{u}s$ and $b \to u\bar{c}s$. This technique is theoretically clean because of the absence of penguins. However, current statistics on Cabibbo-suppressed modes of two-body $B$ decays limits the precision of $\gamma$ obtained in two-body $B$ decays. On the other hand, there is a fair amount of data available on $B$ decays to charmless three-body final states from Belle and BABAR. However, extraction of weak phases from three-body decays is difficult because of two reasons. First, the charmless three-body final states do not involve distinct quark flavors: one has to deal with penguin diagrams. Secondly, even flavor-neutral three-body final states are not CP eigenstates: absence of indirect CP asymmetry makes it harder to extract the weak phase. It

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was shown in Ref. [2] that in the limit of flavor-SU(3) symmetry these difficulties can be overcome by combining Dalitz analyses of multiple three-body B decays and considering the part of the amplitudes that are fully symmetric under the exchange of any two final-state mesons. The purpose of this talk is to show an application of the method proposed in Ref. [2] using experimental data from BABAR. The theory details including the extraction technique are available in Ref. [2, 3].

We have used Dalitz analyses for the following processes available from BABAR:

- $B^0 \rightarrow K_S \pi^+ \pi^-$ [4],
- $K_S K^+ K^-$ [5],
- $K^+ \pi^- \pi^0$ [6], and 3$K_S$ [7]. In addition it is necessary to include data from $B^+ \rightarrow K^+ \pi^+ \pi^-$ to estimate SU(3) breaking as described in Ref. [2]. However, our preliminary goal is to establish the viability of the method and we have ignored effects of SU(3) breaking. In order to relate the amplitudes of the different three-body decay modes they are written in terms of flavor-SU(3) diagrammatics. In three-body B decays, neglecting power-suppressed contributions from weak-annihilation topologies, there are ten distinct flavor-SU(3) topologies, “Color-favored Tree” ($T_1, T_2$), “Color-suppressed Tree” ($C_1, C_2$), “Color-favored electroweak Penguin” ($P_{EW1}, P_{EW2}$), “Color-suppressed electroweak Penguin” ($P_{C EW1}, P_{C EW2}$) and “Gluonic Penguins” ($P_{tc}, P_{uc}$).

The amplitudes for the relevant B decay processes, under the assumption of flavor-SU(3) symmetry may be expressed in terms of these twelve flavor-topology parameters. In order to construct a CP eigenstate from the three-body final state, Ref. [2] considers the part of the amplitude that is totally symmetric under the interchange of any two final state mesons. An added advantage of using the totally-symmetric final-state amplitude is that the “electroweak Penguin” topologies can now be related to the “Tree” topologies [3].

Although it is possible to measure both the direct and indirect CP asymmetries in $B^0 \rightarrow 3K_S$, these observables are currently limited by the available statistics [7]. This can be implemented by setting $P_{uc}$ to zero. The three-body amplitudes may now be written in terms of “effective diagrams” (linear combinations of the flavor-topology amplitudes) as follows:

\[
\begin{align*}
A(B^0 \rightarrow K^0 K^0 K^0)_{sym} & = a , \\
\sqrt{2}A(B^0 \rightarrow K^+ K^0 K^-)_{sym} & = -ce^{i\gamma} - a + \kappa b , \\
2A(B^0 \rightarrow K^+ \pi^0 \pi^-)_{sym} & = be^{i\gamma} - \kappa c , \\
\sqrt{2}A(B^0 \rightarrow K^0 \pi^+ \pi^-)_{sym} & = -de^{i\gamma} - a + \kappa d ,
\end{align*}
\]

where in terms of the flavor-topology amplitudes $a, b, c$ and $d$ are:

\[
a = -P_{tc} + \kappa \left( \frac{2}{3}T_1 + \frac{1}{3}C_1 + \frac{1}{3}C_2 \right) ,
\]

\[
b = T_1 + C_2 , 
\]

\[
c = T_2 + C_1 , 
\]

\[
d = T_1 + C_1 , 
\]

\[
\kappa \sim 0.5 .
\]

The fully-symmetric three-body amplitudes are dependent on the kinematics of the final-state particles. This dependence can be conveniently represented on a Dalitz
plot. Dalitz analyses for the relevant three-body decays have been performed by BABAR. A convenient model used to represent the dynamics of the three-body final state is the Isobar model where the final state amplitude is written as follows:

\[ A_{DP} = N_{DP} \sum_j c_j e^{i\theta_j} F_j(s_{12}, s_{13}), \]  

(3)

where the index \( j \) represents an intermediate state, \( N_{DP} \) serves the purpose of normalizing the amplitudes to the measured rates for the relevant processes, \( s_{ik} \) is the invariant mass of two final state mesons labeled \( i \) and \( k \) respectively, and \( c_j \) and \( \theta_j \) represent the amplitude and phase of the \( j \)th isobar coefficient. Although there are three choices for \( s_{ik} \), the sum of all three is a constant and hence only two of them are independent.

The values of \( c_j \) and \( \theta_j \) were taken from the respective BABAR publications. Although, the error bars on these quantities are correlated, such correlations are often not available in the literature. In our preliminary analysis we have chosen to ignore correlations, noting that a more complete work would necessarily include them. We are now able to compute the totally symmetric parts of the amplitudes as follows:

\[ A_{\text{sym}} = \frac{1}{\sqrt{6}} (A(s_{12}, s_{13}) + A(s_{13}, s_{12}) + A(s_{12}, s_{23}) + A(s_{23}, s_{12}) + A(s_{23}, s_{13}) + A(s_{13}, s_{23})) . \]  

(4)

A similar exercise can be done to obtain the symmetric part of the amplitude for CP-conjugate decay process. These may then be used to construct the observable quantities for every point on a given Dalitz plot as follows:

\[ X_{DP} = |A_{\text{sym}}|^2 + |\overline{A}_{\text{sym}}|^2 \]
\[ Y_{DP} = |A_{\text{sym}}|^2 - |\overline{A}_{\text{sym}}|^2 \]
\[ Z_{DP} = \text{Im}(A^*_{\text{sym}}A_{\text{sym}}) \]  

(5)

A simple count of parameters and observables tells us that one can extract \( \gamma \) in this scenario. For every point on the Dalitz plot we have eight independent unknown parameters: the magnitudes and relative phases of \( a, b, c \) and \( d \) (one overall phase is arbitrary) and \( \gamma \). However, we have nine observables: \( X_{DP} \) from all four processes, \( Y_{DP} \) from all but \( B \to 3K_S \) (due to low statistics CP-asymmetry measurements aren’t available for this decay), \( Z_{DP} \) from \( B^0 \to K_S(\pi^+\pi^-, K^+K^-) \). Note that the \( K^+\pi^-\pi^0 \) final state is not flavor neutral, and doesn’t give us a \( Z_{DP} \). The advantage of this method is that we are able to determine \( \gamma \) independently for every point on the Dalitz plot. Therefore an average over Dalitz plot points is expected to significantly reduce the error in \( \gamma \).

In Fig. 1 we show the kinematic boundaries of the Dalitz plots for the four relevant decay processes. In addition we show the relevant axes of symmetry, which divide
each Dalitz plot into six zones. $A_{\text{sym}}$ is totally symmetric under the exchange of any two $s_{ik}$'s. Therefore to avoid overcounting we are restricted to one of the six zones. Within the chosen sixth of the Dalitz plots we pick fourteen points to use for the determination of $\gamma$, which have been displayed in Fig. 1. We have chosen the points to lie on a grid with the squared momenta separated by $2GeV^2$ on either directions. This choice is completely arbitrary. For each point we perform a $\chi^2$ fit.

![Dalitz Plot](image)

Figure 1: Dalitz boundaries and symmetry axes for the four relevant $B$ decays. The blue dots represent the points that were used for extraction of $\gamma$, while the red dots represent the points close to the boundary of the Dalitz plots that were discarded.

In Fig. 2 we plot the $\chi^2$ minimum as a function of $\gamma$ between 0 and $180^\circ$, for two different points on the Dalitz plots. The first plot shows one solution with $\gamma \sim 90^\circ$, which can be clearly distinguished from the other solutions at the $\Delta \chi^2 = 1$ level, and several other indistinguishable solutions closer to $\gamma \sim 30^\circ$. The second plot corresponds to a point close to the Dalitz boundaries and shows multiple indistinguishable $\chi^2$ minima. We find similar results for other points close to the Dalitz boundaries. These solutions require a better understanding of the underlying dynamics and have currently been left out while computing the final result for $\gamma$. We find that a clearly distinguishable solution exists for nine points that are further away from the Dalitz boundaries.

In Table 1 we present our results for $\gamma$ obtained from each Dalitz plot point where we found a distinguishable solution. A simple mean over the values of $\gamma$ quoted in Table 1 gives us the following result:

$$ \gamma = \left(81^{+5}_{-4}(\text{avg.}) \pm 5(\text{std. dev.})\right)^\circ, $$

(6)
where the first error was obtained by square averaging over the individual errors and the second error denotes the standard deviation from the mean of the central values of \( \gamma \) for the nine Dalitz plot points used.

The error bar on \( \gamma \) obtained above is significantly smaller than that currently obtainable in two-body decays. However, we have neglected a few sources of errors. For example we have ignored correlations between isobar coefficients. In practice such correlations may introduce additional error. Furthermore we have assumed flavor-SU(3) symmetry. However, the masses of the daughter pions and kaons as well as the intermediate resonances break flavor-SU(3) symmetry. A direct consequence of flavor-SU(3) breaking is that the Dalitz boundaries for a final state \( K\pi\pi \) is different from that for \( KKK \). As a consequence points very close to the Dalitz boundaries don’t yield distinguishable solutions for \( \gamma \). One way of alleviating this problem is by including many more points on the Dalitz plot for evaluating \( \gamma \). Note that a more

\[
\begin{array}{|c|c|c|c|}
\hline
(s_{12}, s_{13}) & \gamma & (s_{12}, s_{13}) & \gamma \\
\text{in GeV}^2 & \text{(in deg)} & \text{in GeV}^2 & \text{(in deg)} \\
\hline
(4, 4) & - & (8, 8) & 80^{+15}_{-14} \\
(6, 2) & - & (10, 2) & - \\
(6, 4) & 73^{+12}_{-18} & (10, 4) & 86^{+12}_{-14} \\
(6, 6) & 80^{+10}_{-15} & (10, 6) & 79^{+12}_{-15} \\
(8, 2) & - & (10, 8) & 81^{+10}_{-13} \\
(8, 4) & 80^{+12}_{-15} & (12, 2) & - \\
(8, 6) & 78^{+13}_{-14} & (12, 4) & 88^{+14}_{-19} \\
\hline
\end{array}
\]

Table 1: \( \gamma \) extracted from several points within the Dalitz plots.
complete analysis of the combined data set for the four Dalitz plots can avoid errors due to the neglect of correlations between isobar coefficients. In order to get an idea of how large the effect of SU(3) breaking is, it would be interesting to include effects of SU(3) breaking due to one extra parameter.

The work presented in this talk was aimed at motivating interest toward a new method for extracting $\gamma$ from three-body $B$ decays. In this preliminary analysis we have neglected several sources of errors, however, the value of $\gamma$ obtained is quite promising. The hope is to find an alternative viable method for extraction of $\gamma$ using the already available statistics on three-body $B$ decays from B-factories.

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