\( \mathcal{N} = 2 \) AdS supergravity and supercurrents

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Abstract

We consider the minimal off-shell formulation for four-dimensional \( \mathcal{N} = 2 \) supergravity with a cosmological term, in which the second compensator is an improved tensor multiplet. We use it to derive a linearized supergravity action (and its dual versions) around the anti-de Sitter (AdS) background in terms of three \( \mathcal{N} = 2 \) off-shell multiplets: an unconstrained scalar superfield, vector and tensor multiplets. This allows us to deduce the structure of the supercurrent multiplet associated with those supersymmetric theories which naturally couple to the supergravity formulation chosen, with or without a cosmological term. Finally, our linearized \( \mathcal{N} = 2 \) AdS supergravity action is reduced to \( \mathcal{N} = 1 \) superspace. The result is a sum of two \( \mathcal{N} = 1 \) linearized actions describing (i) old minimal supergravity; and (ii) an off-shell massless gravitino multiplet. We also derive dual formulations for the massless \( \mathcal{N} = 1 \) gravitino multiplet in AdS. As a by-product of our consideration, we derive the consistent supergravity extension of the \( \mathcal{N} = 1 \) supercurrent multiplet advocated recently by Komargodski and Seiberg.

Dedicated To The 50th Anniversary
Of The First Man In Space
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1 Introduction

One of the reasons to study linearized off-shell supergravity actions around exact supergravity backgrounds is the possibility to generate consistent supercurrent multiplets, that is supermultiplets containing the energy-momentum tensor and the supersymmetry current(s) [1]. In a recent paper [2] we have found the linearized superfield action of the minimal 4D $\mathcal{N} = 2$ supergravity with a tensor compensator [3] around a Minkowski superspace background. This has allowed us to construct a new $\mathcal{N} = 2$ supercurrent multiplet, in addition to those proposed in the past [4, 5, 6]. In the present paper, we will extend the main constructions of [2] to the case of $\mathcal{N} = 2$ supergravity with a cosmological term. We will heavily use some of the results of our work [7] which in turn built on the series of papers [8, 9, 10, 11] concerning the projective-superspace formulation for general $\mathcal{N} = 2$ supergravity-matter couplings.

A natural question to ask is the following: Is there anything interesting to be learnt from an extension of the results in [2] to the anti-de Sitter (AdS) case? The answer is certainly ‘Yes’ in the sense that AdS supercurrent multiplets are usually more restrictive than those corresponding to the Poincaré supersymmetry. To clarify this point, we would like to discuss, in some detail, the situation in $\mathcal{N} = 1$ supersymmetry.

In the case of $\mathcal{N} = 1$ Poincaré supersymmetry, the most general form (see, e.g., [12, 13]) of a supercurrent multiplet is as follows:

$$\begin{align*}
\bar{D}^\dot{a} J_{\alpha \dot{a}} &= \chi_\alpha + i \eta_\alpha + D_\alpha X , \\
\bar{D}^\dot{a} \chi_\alpha &= \bar{D}^\dot{a} \eta_\alpha = \bar{D}^\dot{a} X = 0 , \\
D_\alpha \chi_\alpha - \bar{D}_\dot{a} \bar{\chi}^{\dot{a}} &= D^\alpha \eta_\alpha - \bar{D}_\dot{a} \bar{\eta}^{\dot{a}} = 0 .
\end{align*}$$

(1.1)
Here \( J_{\alpha\dot{\alpha}} = \bar{J}_{\alpha\dot{\alpha}} \) denotes the supercurrent, while the chiral superfields \( \chi_{\alpha}, \eta_{\alpha} \) and \( X \) constitute the so-called multiplet of anomalies. Some of the superfields \( \chi_{\alpha}, \eta_{\alpha} \) and \( X \) are actually absent for concrete models, and all of them vanish in the case of superconformal theories. The three terms on the right of (1.1) emphasize the fact that there exist exactly three different linearized actions for minimal \((12+12)\) supergravity, according to the classification given in [14], which are related by superfield duality transformations. The case \( \chi_{\alpha} = \eta_{\alpha} = 0 \) describes the Ferrara-Zumino multiplet [1] which corresponds to the old minimal formulation for \( \mathcal{N} = 1 \) supergravity [15, 16]. The choice \( X = \eta_{\alpha} = 0 \) corresponds to the new minimal supergravity [17] (this supercurrent was studied in [18]). Finally, the third choice \( X = \chi_{\alpha} = 0 \) corresponds to the minimal supergravity formulation proposed in [19].

If only one of the superfields \( \chi_{\alpha}, \eta_{\alpha} \) and \( X \) in (1.1) is zero, the supercurrent multiplet describes \( 16 + 16 \) components. Of the three such supercurrents studied in [13], the most interesting is the one advocated by Komargodski and Seiberg [20]. Their conservation law is

\[
\bar{D}^{\dot{\alpha}} J_{\alpha\dot{\alpha}} = \chi_{\alpha} + D_{\alpha} X , \quad \bar{D}_{\dot{\alpha}} \chi_{\alpha} = \bar{D}_{\dot{\alpha}} X = 0 , \quad D^{\alpha} \chi_{\alpha} - \bar{D}_{\dot{\alpha}} \bar{X}^{\dot{\alpha}} = 0 . \tag{1.2}
\]

Finally, the most general supercurrent multiplet with \( 20 + 20 \) components, for which all the superfields \( \chi_{\alpha}, \eta_{\alpha} \) and \( X \) in (1.1) are non-zero, is related to a linearized version of the non-minimal formulation for \( \mathcal{N} = 1 \) supergravity [21, 22]. The non-minimal supercurrent can be written in the form [23]

\[
\bar{D}^{\dot{\alpha}} J_{\alpha\dot{\alpha}} = -\frac{1}{4} D^2 \zeta_{\alpha} - \frac{1}{4} \left( n + \frac{1}{3n + 1} \right) D_{\alpha} D_{\beta} \bar{\zeta}^{\beta} , \quad D_{(\alpha\beta)} \zeta_{\beta} = 0 , \tag{1.3}
\]

where \( n \) is a real constant, \( n \neq -1/3, 0 \), parametrizing the different versions of non-minimal supergravity [22]. The constraint on \( \zeta_{\alpha} \) in (1.3) is solved by \( \zeta_{\alpha} = D_{\alpha} Z \), for some complex superfield \( Z \), and then eq. (1.3) takes the form (1.1).

Let us now turn to the case of \( \mathcal{N} = 1 \) AdS supersymmetry. To start with, we could try to generalize the conservation equation (1.1) by replacing the flat covariant derivatives \( D_A = (\partial_{\alpha}, D_{\alpha}, \bar{D}^{\dot{\alpha}}) \) in (1.1) with those corresponding to the AdS super-space, \( D_A \rightarrow \nabla_A = (\nabla_{\alpha}, \nabla_{\dot{\alpha}}, \bar{\nabla}^{\dot{\alpha}}) \). However, a simple analysis shows that the only consistent generalization obtained in this way is

\[
\bar{\nabla}^{\dot{\alpha}} J_{\alpha\dot{\alpha}} = \nabla_{\alpha} X , \quad \bar{\nabla}_{\dot{\alpha}} X = 0 . \tag{1.4}
\]

\(^1\)Unlike the old minimal and the new minimal theories, this formulation is known at the linearized level only.
It corresponds to the old minimal supergravity with a cosmological term, for which
the linearized action around the AdS background \[24\] is
\[
S_{\text{old}} = - \int d^4x \, d^2\theta \, d^2\bar{\theta} \, E \left\{ \frac{1}{16} H^{\dot{a}a} \nabla^\beta (\bar{\nabla}^2 - 4R) \nabla_\beta H_{a\dot{a}} - \frac{1}{48} ([\nabla_\alpha, \bar{\nabla}_{\dot{\alpha}}] H^{\dot{a}\alpha})^2 \\
+ \frac{1}{4} (\nabla_{a\dot{a}} H^{\dot{a}a})^2 + \frac{R\bar{R}}{4} H^{\dot{a}a} H_{a\dot{a}} + iH^{\dot{a}a} \nabla_{a\dot{a}} (\phi - \bar{\phi}) + 3(\phi\bar{\phi} - \phi^2 - \bar{\phi}^2) \right\}, \tag{1.5}
\]
with \( H_{a\dot{a}} \) the gravitational superfield, \( \phi \) the chiral compensator, \( \nabla_{\dot{a}}\phi = 0 \), and \( R \) the constant torsion of the AdS superspace. This action is invariant under the linearized
supergravity gauge transformations
\[
\delta H_{a\dot{a}} = \nabla_\alpha \bar{L}_{\dot{a}} - \bar{\nabla}_{\dot{\alpha}} L_\alpha, \quad \delta \phi = -\frac{1}{12} (\bar{\nabla}^2 - 4R) \nabla^a L_\alpha, \tag{1.6}
\]
with \( L_\alpha \) an unconstrained superfield parameter. Looking at the explicit structure of
the action (1.5), it is easy to understand why the AdS supersymmetry allows only for
one minimal supercurrent multiplet, which is given by eq. (1.4). In particular, the
Komargodski-Seiberg supercurrent (1.2) does not admit a minimal AdS extension.
The point is that the theory (1.5) does not possess, for \( R \neq 0 \), a dual formulation
in which the chiral compensator \( \phi \) and its conjugate \( \bar{\phi} \) get replaced by a real linear
superfield.

It is instructive to see how the consistency issue mentioned arises in terms of the
non-minimal supercurrent (1.3). Starting from (1.3), let us again replace the flat co-
variant derivatives \( D_A = (\partial_a, D_\alpha, \bar{D}_{\dot{\alpha}}) \) with the AdS ones, \( D_A \rightarrow \nabla_A = (\nabla_a, \nabla_\alpha, \bar{\nabla}_{\dot{\alpha}}) \).
It turns out that such a generalization is consistent only in the case \( n = -1 \),
\[
\bar{\nabla}_{\dot{\alpha}} J_{a\dot{a}} = -\frac{1}{4} \bar{\nabla}^2 \zeta_\alpha, \quad \nabla_{(\alpha} \zeta_{\beta)} = 0. \tag{1.7}
\]
This supercurrent multiplet is associated with the linearized supergravity action \[24\]
\[
S_{n=-1} = - \int d^4x \, d^2\theta \, d^2\bar{\theta} \, E \left\{ \frac{1}{16} H^{\dot{a}a} \nabla^\beta (\bar{\nabla}^2 - 4R) \nabla_\beta H_{a\dot{a}} + \frac{1}{4} R\bar{R} H^{\dot{a}a} H_{a\dot{a}} \\
+ \frac{1}{2} H^{\dot{a}a} (\nabla_\alpha \bar{\nabla}_{\dot{\alpha}} \Gamma - \bar{\nabla}_{\dot{\alpha}} \nabla_\alpha \bar{\Gamma}) + \Gamma^\alpha + \bar{\Gamma}^\alpha + \bar{\Gamma}^2 \right\}, \tag{1.8}
\]
with \( \Gamma \) the complex linear compensator obeying the constraint
\[
(\bar{\nabla}^2 - 4R) \Gamma = 0. \tag{1.9}
\]
This action is invariant under the gauge transformations
\[
\delta H_{a\dot{a}} = \nabla_\alpha \bar{L}_{\dot{a}} - \bar{\nabla}_{\dot{\alpha}} L_\alpha, \quad \delta \bar{\Gamma} = -\frac{1}{4} \bar{\nabla}^a \bar{\nabla}^2 L_\alpha, \tag{1.10}
\]
and is dual to the linearized theory (1.5). Unlike the situation in Minkowski superspace, where an infinite family of non-minimal supergravity actions exists, with the corresponding supercurrents being given by eq. (1.3), the theory (1.8) proves to be the only dual formulation of the old minimal model (1.5).

Our consideration shows that the structure of the $\mathcal{N} = 1$ AdS supercurrent multiplets is more restrictive than in the super-Poincaré case. In what follows, we will study a consistent $\mathcal{N} = 2$ AdS supercurrent.

This paper is organized as follows. In section 2, we consider $\mathcal{N} = 2$ supergravity with a cosmological constant and construct its solution, which corresponds to an AdS geometry. In section 3, we derive the linearized AdS supergravity action. Its $\mathcal{N} = 1$ reduction is the topic of section 4. Both of these results are generalizations of our previous work [2]. We address some general issues regarding $\mathcal{N} = 2$ supercurrents in section 5 and postulate the general form of the supercurrent for $\mathcal{N} = 2$ supergravity + matter theories coupled to vector and tensor compensators. There are five technical appendices. Appendices A and B review briefly the geometry of $\mathcal{N} = 1$ and $\mathcal{N} = 2$ superspace with structure group $\text{SL}(2, \mathbb{C}) \times \text{U}(\mathcal{N})_R$. Appendix C reviews the improved $\mathcal{N} = 2$ tensor multiplet in curved projective superspace. Appendix D contains further technical details of the derivation of the linearized $\mathcal{N} = 2$ supergravity action. Similarly, Appendix E provides further details about the $\mathcal{N} = 1$ reduction procedure.

2 $\mathcal{N} = 2$ supergravity with a cosmological term

In this section we discuss the $\mathcal{N} = 2$ supergravity formulation of [3] using the superspace approach of [7]. This supergravity formulation makes use of two compensators: the vector multiplet [25] and the tensor multiplet [26].

2.1 Conformal compensators

The vector multiplet can be described in curved superspace by its covariantly chiral field strength $\mathcal{W}$ subject to the Bianchi identity [25, 27]

$$\mathcal{D}_i \mathcal{W} = 0 \ , \quad \Sigma^{ij} := \frac{1}{4} \left( \mathcal{D}^{\alpha (i} \mathcal{D}^{j)}_{\alpha} + 4 S^{ij} \right) \mathcal{W} = \frac{1}{4} \left( \mathcal{D}_\alpha (i \mathcal{D}^j)_{\alpha} + 4 S^{ij} \right) \mathcal{W} , \quad (2.1)$$

Such a superfield is often called reduced chiral.
where $S^{ij}$ and $\bar{S}^{ij}$ are special dimension-1 components of the torsion. The superfield $\Sigma^{ij}$ is real, $\Sigma_{ij} := (\Sigma^{ij})^* = \varepsilon_{ik}\varepsilon_{jl}\Sigma^{kl}$, and obeys the constraints

$$D_\alpha^{(i}\Sigma^{jk)} = \bar{D}_\dot{\alpha}^{(i}\Sigma^{jk)} = 0 . \tag{2.2}$$

These constraints are characteristic of the $\mathcal{N} = 2$ linear multiplet [30, 31, 32]. In off-shell formulations of $\mathcal{N} = 2$ supergravity, one of the compensators is usually a vector multiplet such that its field strength $W$ is nowhere vanishing, $W \neq 0$.

There are several ways to realize $W$ as a gauge invariant field strength. One possibility, which we will use in what follows, is to introduce the curved-superspace extension [7] of Mezincescu’s prepotential [33] (see also [34]), $V_{ij} = V_{ji}$, which is an unconstrained real SU(2) triplet. The expression for $W$ in terms of $V_{ij}$ [7] is

$$W = \frac{1}{4}\bar{\Delta}\left(D^{ij} + 4S^{ij}\right)V_{ij} , \tag{2.3}$$

where $\bar{\Delta}$ is the chiral projection operator (A.13). Note that $V_{ij}$ is defined only up to gauge transformations of the form

$$\delta V^{ij} = D^a_k\Lambda^{kij} + \bar{D}_{ak}\bar{\Lambda}^{kij} , \quad \Lambda^{kij} = \Lambda^{(kij)} , \quad \bar{\Lambda}^{\dot{k}ij} := (\Lambda^{kij})^* , \tag{2.4}$$

with the gauge parameter $\Lambda^{kij}$ being completely arbitrary modulo the algebraic condition given.

The tensor (or linear) multiplet can be described in curved superspace by its gauge invariant field strength $G^{ij}$ which is defined to be a real SU(2) triplet (that is, $G^{ij} = G^{ji}$ and $\bar{G}_{ij} := (G^{ij})^* = G_{ij}$) subject to the covariant constraints [31, 32]

$$D_\alpha^{(i}G^{jk)} = \bar{D}_\dot{\alpha}^{(i}G^{jk)} = 0 . \tag{2.5}$$

These constraints are solved in terms of a covariantly chiral prepotential $\Psi$ [34, 35, 36, 37] as follows:

$$G^{ij} = \frac{1}{4}\left(D^{ij} + 4S^{ij}\right)\Psi + \frac{1}{4}\left(D^{ij} + 4S^{ij}\right)\bar{\Psi} , \quad D_\alpha^i\Psi = 0 . \tag{2.6}$$

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3Our curved-superspace conventions follow Ref. [9]. In particular, we use the superspace geometry of $\mathcal{N} = 2$ conformal supergravity introduced in [27] (see also [28]) in which the structure group is SL(2, $\mathbb{C}$) $\times$ U(2). The relevant information about Howe’s formulation is collected in Appendix A. In what follows, we will use the notation: $D^{ij} := D^a(iD^j_a)$ and $\bar{D}^{ij} := D_{\dot{a}}(\dot{i}\bar{D}_j^{\dot{a}})$. It should be noted that Howe’s realization of $\mathcal{N} = 2$ conformal supergravity [27] is a simple extension of Grimm’s formulation [29] with the structure group SL(2, $\mathbb{C}$) $\times$ SU(2). The precise relationship between these two formulations is spelled out in [9].

4Another realization for $W$, in terms of the weight-zero tropical prepotential in projective superspace, is briefly mentioned in Appendix C. A more extensive discussion of this realization can be found, e.g., in Appendix E of [7] where its relation to the Mezincescu prepotential is derived.
The prepotential is defined up to gauge transformations of the form
\[ \delta \Psi = i \Lambda , \quad \left( \mathcal{D}^{ij} + 4 S^{ij} \right) \Lambda = \left( \mathcal{D}^{ij} + 4 \tilde{S}^{ij} \right) \tilde{\Lambda} , \] (2.7)
with \( \Lambda \) an arbitrary reduced chiral superfield.

If the tensor multiplet is chosen as one of the two supergravity compensators, then the scalar
\[ G := \sqrt{\frac{1}{2} G^{ij} G_{ij}} \] (2.8)
must be nowhere vanishing, \( G \neq 0 \).

### 2.2 Dynamics in supergravity

In accordance with the analysis given in [7], the gauge-invariant supergravity action can be written as
\[ S = \frac{1}{\kappa^2} \int d^4x \, d^4\theta \, \mathcal{E} \left\{ \Psi \mathbb{W} - \frac{1}{4} \mathbb{W}^2 - \xi \Psi \mathbb{W} \right\} + \text{c.c.} \] (2.9a)
\[ = \frac{1}{\kappa^2} \int d^4x \, d^4\theta \, \mathcal{E} \left\{ \Psi \mathbb{W} - \frac{1}{4} \mathbb{W}^2 \right\} + \text{c.c.} - \frac{\xi}{\kappa^2} \int d^4x \, d^4\theta \, d^4\bar{\theta} \, \mathcal{E} G^{ij} \mathcal{V}_{ij} , \] (2.9b)
where
\[ \mathbb{W} := \frac{G}{8} \left( \mathcal{D}^{ij} + 4 \tilde{S}^{ij} \right) \left( \frac{G^{ij}}{G^2} \right) \] (2.10)
is a composite reduced chiral superfield [3, 7] (that is \( \mathbb{W} \) obeys the same conditions (2.1) as the field strength \( \mathbb{W} \)). Here \( \kappa \) is the gravitational constant, and \( \xi \) the cosmological constant. In what follows, we will choose \( \kappa = 1 \). The first representation for the action, eq. (2.9a), involves the integration over the chiral subspace, with \( \mathcal{E} \) the chiral density. In the second form, eq. (2.9b), the cosmological term is given as an integral over the full superspace, with \( E^{-1} = \text{Ber}(E_A^M) \). In Appendix C, we give an alternative expression for the supergravity action.

The equations of motion associated with the above action were derived in [7]. They are
\[ \mathcal{G} - \mathbb{W} \mathbb{W} = 0 , \] (2.11a)
\[ \Sigma^{ij} + \xi G^{ij} = 0 , \] (2.11b)
\[ \mathbb{W} - \xi \mathbb{W} = 0 . \] (2.11c)
Eq. (2.11a) corresponds to the Weyl multiplet \[38, 39\], i.e. the multiplet of conformal supergravity. As shown in detail in [6], modulo purely gauge degrees of freedom, the Weyl multiplet can be described by a real scalar superfield \(\mathcal{H}\) which we call the gravitation superfield.\(^5\) The remaining equations (2.11b) and (2.11c) correspond to the vector and the tensor compensators, respectively.

### 2.3 Solution to the equations of motion: AdS geometry

A simple solution to the supergravity equations (2.11a)–(2.11c) can be obtained in the case that the supersymmetric Weyl tensor \(W^\alpha{}\bar{\beta}\) is zero,

\[
W^\alpha{}\bar{\beta} = 0 ,
\]

which corresponds to a conformally flat superspace. Such a solution is easy to derive explicitly by using the super-Weyl gauge freedom to fix \(\mathcal{W}\bar{\mathcal{W}} = \mathcal{G} \) to a positive constant, which we denote \(g\). This will imply that coordinate dependence of \(\mathcal{W}\) lies only in its phase; similarly, \(\mathcal{G}^{ij}\) will vary only in the direction it points in isovector space. Both of these residual degrees of freedom can be fixed by using almost all of the local \(U(2)_R\) invariance, but we wish to examine the consequences of leaving the \(U(2)_R\) gauge freedom unfixed for now.

We consider first the consequences of super-Weyl gauging \(\mathcal{W}\bar{\mathcal{W}}\) to be constant, for some reduced chiral superfield \(\mathcal{W}\) such that \(\mathcal{W} \neq 0\). It is immediately apparent that \(\mathcal{W}\) must itself be annihilated by all the spinor covariant derivatives, \(\mathcal{D}_\alpha{}\bar{\alpha}\mathcal{W} = 0\), by applying the covariant derivative to \(\mathcal{W}\bar{\mathcal{W}} = g = \text{const}\). It follows from the algebra of covariant derivatives, eq. (A.3b), that \[27, 9\]

\[
G^{ij}_{\alpha\bar{\beta}} = 0 ,
\]

as well as \(\mathcal{D}_{\alpha\bar{\alpha}}\mathcal{W} = -2i G_{\alpha\bar{\alpha}} \mathcal{W}\). In conjunction with the properties \(\mathcal{D}_\alpha{}\bar{\alpha}\mathcal{W} = \bar{\mathcal{D}}^\alpha{}\bar{\beta}\mathcal{W} = 0\), the latter relation immediately allows us to solve for the \(U(1)_R\) connection\(^7\)

\[
\Phi_A = -\frac{i}{4} E_A \log(\mathcal{W}/\bar{\mathcal{W}}) + \delta^b_A G_b ,
\]

with \(E_A = E_A^M \partial_M\) the vielbein, see Appendix A. It also follows from the reduced chirality condition (2.1) that

\[
\mathcal{W}S^{ij} = \bar{\mathcal{W}}\bar{S}^{ij} .
\]

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\(^5\) The appearance of \(\mathcal{H}\) at the linearized supergravity level was revealed in [34, 40, 41].

\(^6\) The \(SU(2)_R\) gauge freedom is only partially fixed if we take \(\mathcal{G}^{ij}\) to a constant. A residual \(U(1) = SO(2)\) remains, which corresponds to rotations about the axis of \(\mathcal{G}^{ij}\).

\(^7\) The contribution of \(G_{\alpha\bar{\alpha}}\) to the connection \(\Phi_{\alpha\bar{\alpha}}\) is the result of a conventional constraint. One may redefine the connection to eliminate this term if one desires.
Using the local $U(1)_R$ symmetry allows us to gauge away the phase of $W$, that is to impose the gauge condition $W = w = \text{const}$, and then the $U(1)_R$ connection simplifies dramatically, as follows from (2.14).

Now we turn to the tensor multiplet and consider the consequences of enforcing the super-Weyl condition $G = g = \text{const}$. Using the constraints of the $\mathcal{N} = 2$ tensor multiplet (2.5), one may show [7] that $G^{ij}$ is annihilated by the spinor covariant derivatives, $\mathcal{D}_i G^{ij} = \mathcal{D}_{\dot{a}} G^{ij} = 0$. From (A.3a) and (A.3b), one may show that $\mathcal{D}_{\dot{a}a} G^{ij} = 4G_{\dot{a}a}^k G_k^{ij}$ as well as

\[ G_{\alpha\dot{a}} = Y_{\alpha\beta} = 0 , \quad S^{ij} \propto G^{ij} . \]  

(2.16)

The vanishing of the spinor derivatives of $G^{ij}$ is a powerful condition; it implies a set of conditions on the $SU(2)_R$ connection

\[ \Phi_A^{kl} \left( \delta^{ij}_{kl} - \frac{1}{2g^2} G_{kl} G^{ij} \right) = \frac{1}{2g^2} G^{ik} E_A G_{k}^{j} - 2\delta_A^b G_k^{ij} \left( \delta^{ij}_{kl} - \frac{1}{2g^2} G_{kl} G^{ij} \right) . \]  

(2.17)

Note that this determines the isospin connection only along directions perpendicular to $G^{ij}$; the parallel component gauges rotations about the axis of $G^{ij}$ and is completely undetermined.

Using the local $SU(2)_R$ symmetry allows us to turn the covariantly constant $G^{ij}$ into a truly constant isovector, $G^{ij} = g^{ij} = \text{const}$. When such a gauge is chosen, the first term on the right of (2.17) drops out.

It remains to enforce the equation of motion $G = \mathcal{W} \bar{\mathcal{W}}$ which allows both sets of the above conditions to be applied simultaneously. This implies, in particular, that the only torsion superfield is $S^{ij}$, which along with $W$ and $G^{ij}$ are all covariantly constant. From the other two equations of motion (2.11b) and (2.11c), one may read off the solution

\[ S^{ij} = -\xi G^{ij} / W , \quad S^{\dot{i}\dot{j}} = -\xi G^{\dot{i}\dot{j}} / \bar{W} . \]  

(2.18)

The superspace background we have found is maximally symmetric with the covariant derivatives obeying the algebra [8 42]:

\[ \{ \mathcal{D}_i^\alpha , \mathcal{D}_j^\beta \} = 4S^{ij} M_{\alpha\beta} + 2\epsilon_{\alpha\beta}^{\dot{a}\dot{b}} S^{ij} J_{k l} , \quad \{ \mathcal{D}_i^\alpha , \mathcal{D}_{\dot{a}}^{\dot{\alpha}} \} = -2i\delta^j_i \mathcal{D}_{\alpha\dot{a}} , \]  

(2.19a)

\[ [ \mathcal{D}_i^\alpha , \mathcal{D}_{\dot{a}}^{\dot{\alpha}} ] = -i\epsilon_{\beta\dot{a}}^{\dot{b}j} S^{ij} \mathcal{D}_{\dot{\alpha}j} , \quad [ \mathcal{D}_a , \mathcal{D}_b ] = -S^2 M_{ab} , \]  

(2.19b)

where we have denoted $S^2 := \frac{1}{2} S^{ij} S_{ij} = \xi^2 g$. From the explicit form of the Riemann tensor, it is clear that the space-time geometry is AdS, with the curvature scale determined by the magnitude of the constant $\xi^2 g$.

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8The contribution of $G^{ij}_{\alpha\dot{a}}$ to $\Phi^{ij}_{\alpha\dot{a}}$ is the consequence of another conventional constraint.

9
3 Linearized AdS supergravity action

Our goal is to linearize the supergravity action (2.9a) or (2.9b) around AdS superspace which has been shown to be an exact solution of the supergravity equations of motion (2.11a)–(2.11c). We represent the compensators in the form

\[ G^{ij} \rightarrow G^{ij} + G_{ij}, \quad (3.1a) \]
\[ \mathcal{W} \rightarrow \mathcal{W} + \mathcal{W}, \quad (3.1b) \]

where \( G^{ij} \) and \( \mathcal{W} \) on the right hand side correspond to the covariantly constant background compensators, while \( G^{ij} \) and \( \mathcal{W} \) are arbitrary deformations obeying their respective Bianchi identities. They may be represented in terms of linearized prepotentials via

\[ G^{ij} = \frac{1}{4}(\mathcal{D}^{ij} + 4S^{ij})\Psi + \frac{1}{4}(\bar{\mathcal{D}}^{ij} + 4\bar{S}^{ij})\bar{\Psi}, \quad \bar{\mathcal{D}}^i \Psi = 0, \quad (3.2a) \]
\[ \mathcal{W} = \frac{1}{4}\bar{\Lambda} (\mathcal{D}^{ij} + 4S^{ij}) \mathcal{V}_{ij}, \quad \mathcal{V}_{ij} = \mathcal{V}_{ji} = (\mathcal{V}^{ij})^*. \quad (3.2b) \]

We also introduce \( H \) for the linearized gravitational superfield.\(^9\)

3.1 Linearized supergravity gauge transformations

We postulate the linearized supergravity gauge transformations:

\[ \delta \Psi = 4\bar{\Lambda} (\bar{\Omega}^{ij} G_{ij}), \quad (3.3a) \]
\[ \delta \mathcal{V}^{ij} = -4\Omega^{ij} \mathcal{W} - 4\bar{\Omega}^{ij} \mathcal{W}, \quad (3.3b) \]
\[ \delta H = (\mathcal{D}^{ij} + 4S^{ij}) \Omega_{ij} + (\bar{\mathcal{D}}^{ij} + 4\bar{S}^{ij}) \bar{\Omega}_{ij}, \quad (3.3c) \]

as natural generalizations of those given in [2]. The rule for \( \mathcal{V}^{ij} \) is exactly as in the Minkowski background, while that for \( \Psi \) is the only possible generalization when we take into account that \( \delta \Psi \) must be covariantly chiral. Note that the formulae (3.3a) and (3.3b) are background super-Weyl covariant if \( \Omega_{ij} \) possesses weight \(-3\) with \( \Psi \) and \( \mathcal{V}^{ij} \) having weights 1 and \(-2\), respectively.

There remains some arbitrariness in the choice of \( \delta H \), in particular the choice of the numerical factor in front of \( S^{ij} \). The particular choice made in (3.3c) is the one respecting background super-Weyl covariance when \( H \) is assumed to transform with weight \(-2\). That the variations should be background super-Weyl covariant

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\(^9\)The background covariant derivatives depend on some background prepotentials. However, the explicit form of such a dependence is not essential for our purposes.
is reasonable since the original theory is super-Weyl invariant, but we can marshal some additional evidence for this. For instance, the supergravity equations of motion (2.11a)–(2.11c) arise from the first order action
\[
S^{(1)} = \int d^4x d^4\theta \mathcal{E} \mathbf{\Psi}(\mathbb{W} - \xi \mathbb{W}) + \int d^4x d^4\bar{\theta} \bar{\mathcal{E}} \mathbf{\bar{\Psi}}(\mathbb{\bar{W}} - \xi \mathbb{\bar{W}}) + \int d^4x d^4\theta d^4\bar{\theta} E \left\{ H(G - \mathbb{W} \mathbb{\bar{W}}) - V_{ij}(\Sigma_{ij} + \xi G_{ij}) \right\} \tag{3.4}
\]
which is gauge invariant precisely for the choice (3.3). More generally, in any super-conformal theory H couples to a supercurrent J which is itself super-Weyl covariant with weight +2; in the above example J = G - \mathbb{W} \mathbb{\bar{W}}. For this linearized coupling to be sensible, H must share this covariance property and have weight -2.

From eqs. (3.3a) and (3.3b) we read off the supergravity gauge transformations of the linearized field strengths (3.2a) and (3.2b):
\[
\delta G^{ij} = (\mathcal{D}^{ij} + 4S^{ij})\Delta(\Omega^{ij} G_{ij}) + (\mathcal{D}^{ij} + 4S^{ij})\Delta(\Omega^{ij} \mathcal{G}_{ij}) , \tag{3.5a}
\]
\[
\delta W = -\Delta(\mathcal{D}^{ij} + 4S^{ij})(\Omega^{ij} \mathbb{W} + \mathcal{\bar{\Omega}}^{ij} \mathcal{\bar{W}}) . \tag{3.5b}
\]
It should be emphasized that, in this subsection, no assumption has been made about the background fields chosen. The linearized supergravity gauge transformations (3.3a)–(3.3c) hold for an arbitrary supergravity background generated by some covariant derivatives \( \mathcal{D}_A \) and compensators \( G^{ij}, \mathcal{W} \) and \( \mathcal{\bar{W}} \).

### 3.2 Linearized supergravity action

We are now prepared to derive the linearized supergravity action around the AdS background as described in subsection 2.3. It can be uniquely constructed by including all the terms quadratic in the compensators \( \Psi, V^{ij} \) and the gravitational superfield \( H \) with the coefficients chosen in such a way as to render a gauge invariant
result. A several-day calculation\footnote{Its details are collected in Appendix D.} leads to the linearized AdS supergravity action:

\[
S^{(2)} = \int d^4x d^4\theta E \left( -\frac{1}{4} WW + \hat{\Psi} \hat{\bar{W}} - \xi \Psi \bar{W} \right) + c.c.
\]

\[
+ \int d^4x d^4\theta d^4\theta E \left\{ - V W \bar{W} - \bar{W} V \bar{W} + \frac{1}{2} G_{ij} G^{ij} H
\]

\[
- \frac{1}{2} \bar{W}^2 H \Delta H - \frac{1}{4} \bar{W}^2 H \Delta H - \frac{g}{8} H S^{ij} \bar{D}_{ij} H - \frac{g}{8} H S^{ij} \bar{D}_{ij} H
\]

\[
- \frac{1}{64} G_{ij} G^{kl} H \bar{D}^{ij} \bar{D}_{kl} H - \frac{g}{32} H \bar{D}^{ij} \bar{D}_{ij} H + \frac{g}{2} H \square H \right\},
\]

where \( \square = D^a D_a \) and

\[
\hat{\bar{W}} = - \frac{1}{24g} D_{ij} G^{ij}.
\]

One can check that \( \hat{\bar{W}} \) is a reduced chiral superfield,

\[
\bar{D}^i \hat{\bar{W}} = 0, \quad (\bar{D}^{ij} + 4S^{ij}) \hat{\bar{W}} = (\bar{D}^{ij} + 4S^{ij}) \hat{\bar{W}}.
\]

The linearized supergravity action (3.6) is one of the main results of our work. In the rigid supersymmetric limit it reduces to the action constructed in \cite{2}. It is natural to choose units so that

\[
W\bar{W} = G = g = 1
\]

which implies that \( W \) is a pure phase superfield and \( G^{ij} \) is a unit isovector superfield. We will make this assumption from section 4 on.

It is worth emphasizing that the background compensators \( W \) and \( G^{ij} \) in the above action are covariantly constant\footnote{This form for the \( \mathcal{N} = 2 \) action with covariantly constant compensators is closely related to the \( \mathcal{N} = 1 \) procedure advocated in \cite{43}.}. In particular, they can be made \textit{truly} constant by choosing a specific U(2)\(_R\) gauge. In our treatment of the Minkowski case \cite{2}, we implicitly took this gauge, with \( W = w \) and \( G^{ij} = g^{ij} \). We will frequently find it useful to refer to this gauge; it should be clear from context (i.e. the appearance of \( w \) and \( g^{ij} \) in formulae) when we are using it.

### 3.3 Dual versions of the supergravity action

Before moving on to the \( \mathcal{N} = 1 \) reduction, we will briefly discuss a duality which may be applied to the supergravity action, both in its nonlinear (2.9) and linearized (3.6) forms.
We review first the nonlinear version. We begin by writing the action (2.9) with an additional complex parameter $\alpha$ with unit real part,

$$S = \int \! d^4x \, d^4\theta \mathcal{E} \left\{ \Psi \bar{\Psi} - \frac{\alpha}{4} \mathcal{W}^2 - \xi \mathcal{W} \Psi^2 \right\} + \text{c.c.}, \quad \alpha + \bar{\alpha} = 2. \quad (3.10)$$

The imaginary part of $\alpha$ is physically irrelevant at first. Next we relax $\mathcal{W}$ to a general chiral superfield and enforce its reduced chirality using a Lagrange multiplier $\mathcal{W}_D$, which is a reduced chiral superfield:

$$S = \int \! d^4x \, d^4\theta \mathcal{E} \left\{ \Psi \bar{\Psi} - \frac{\alpha}{4} \mathcal{W}^2 - \xi \mathcal{W} \mathcal{W}_D + i \mathcal{W} \bar{\mathcal{W}}_D \right\} + \text{c.c.} \quad (3.11)$$

Performing the duality, we find

$$S = \int \! d^4x \, d^4\theta \mathcal{E} \left\{ \Psi \bar{\Psi} + \frac{\xi^2}{\alpha} \left( \Psi - \frac{i}{\xi} \mathcal{W}_D \right)^2 \right\} + \text{c.c.} \quad (3.12)$$

For nonzero $\xi$ it is clear that $\mathcal{W}_D$ is a Stueckelberg field for $\Psi$. We can absorb it into $\Psi$, by applying a finite gauge transformation (2.7), and then we end up with a massive tensor compensator

$$S = \int \! d^4x \, d^4\theta \mathcal{E} \left\{ \Psi \bar{\Psi} + \frac{\xi^2}{\alpha} \Psi^2 \right\} + \text{c.c.} \quad (3.13)$$

If we parametrize $\alpha = 1 - i e/\mu$ with $e^2 + \mu^2 = 4 \xi^2$, the mass-like term for $\Psi$ becomes $\xi^2/\alpha = \mu(\mu + ie)/4$, which can be interpreted as a combination of magnetic and electric contributions which are associated with the two possible mass terms $B \wedge * B$ and $B \wedge B$ for the component two-form $B$ (see, e.g. [44] for a pedagogical discussion). As discussed in [7], this is a formulation for $N = 2$ supergravity with a cosmological constant and a single chiral compensator $\Psi$.\[12\]

We can perform the same duality at the linearized level. The result is

$$S^{(2)} = \int \! d^4x \, d^4\theta \mathcal{E} \left( \Psi \bar{\Psi} + \frac{1}{\alpha} (\xi \Psi - i \mathcal{W}_D)^2 \right) + \text{c.c.}$$

$$+ \int \! d^4x \, d^4\theta \, d^4\bar{\theta} \mathcal{E} \left\{ \frac{2W}{\alpha} (\xi \Psi - i \mathcal{W}_D) \mathcal{H} + \frac{2W}{\alpha} (\xi \Psi + i \mathcal{W}_D) \mathcal{H} + \frac{1}{2g} G_{ij} G^{ij} \mathcal{H} \right. \nonumber \right. \right. \nonumber \right. \nonumber \right. \nonumber \right. \nonumber \right. \nonumber \right. \nonumber \right. \nonumber \right.$$

$$+ \frac{\mathcal{W}^2 \bar{\alpha}}{2} H \Delta \mathcal{H} + \frac{\mathcal{W}^2 \alpha}{2} H \Delta \mathcal{H} - \frac{g}{8} H S^{ij} \mathcal{D}_{ij} \mathcal{H} - \frac{g}{8} H \mathcal{S}^{ij} \mathcal{D}_{ij} \mathcal{H}$$

$$- \frac{1}{64g} G_{ij} G^{kl} H \mathcal{D}_{ij} \mathcal{D}_{kl} \mathcal{H} - \frac{g}{32} H D^{ij} \mathcal{D}_{ij} \mathcal{H} + \frac{g}{2} H \square \mathcal{H} \} \right. \nonumber \right. \right. \nonumber \right. \right. \right. \right. \right. \right.$$

\[12\]The vector multiplet has been eaten up by the tensor multiplet which is now massive. The vector compensator acts as a Stueckelberg field to give mass to the tensor multiplet. This is an example of the phenomenon observed originally in [43] and studied in detail in [10] [47] [44] [45] [49] [50] [10] [51].
Under the linearized supergravity gauge transformations, eqs. (3.3a) and (3.3b), the Stueckelberg field $W_D$ transforms as

$$\delta W_D = -\frac{i}{2} \bar{\Delta} (D^{ij} + 4S^{ij})(\bar{\alpha}\Omega_{ij}\bar{W} - \alpha\bar{\Omega}_{ij}W),$$  

(3.15)

compare with (3.5b). As before, we may redefine $\Psi$ to “eat” $W_D$. In the gauge $W_D = 0$ the supergravity gauge transformation of $\Psi$, eq. (3.3a), turns into

$$\delta \Psi = 2\bar{\alpha}\bar{\Delta}(G^{ij}\Omega_{ij} + G^{ij}\Omega_{ij}) + \frac{1}{2\xi}\bar{\Delta}D^{ij}(\alpha\mathcal{W}\Omega_{ij} - \bar{\alpha}\bar{\mathcal{W}}\Omega_{ij}).$$  

(3.16)

Of course, the transformation of $G^{ij}$ does not change.

It is also possible to perform a duality to a massive vector multiplet (i.e. with a polar multiplet acting as a Stueckelberg field). This formulation lives naturally in projective superspace, so we won’t attempt to describe it here. The nonlinear version was discussed in [10].

4 $\mathcal{N} = 1$ reduction

In our previous paper [2], we considered the $\mathcal{N} = 1$ reduction of the $\mathcal{N} = 2$ supergravity without a cosmological constant (i.e. $\xi = 0$). This was an easy procedure since the background was Minkowski and the reduction was quite straightforward. The situation in AdS is markedly different and there are in principle two ways we might proceed. One is to treat AdS as a special case of a general curved space and construct explicitly the $\mathcal{N} = 1$ local superspace reduction of a generic $\mathcal{N} = 2$ action. Needless to say this would be a difficult task, even given the effort already expended on $\mathcal{N} = 1$ reductions in superspace [52, 53, 54, 55].

The other approach is to exploit the fact that AdS is a conformally flat geometry. That is, we can always go to a super-Weyl gauge where we have flat superspace geometry and a non-vanishing compensator field.

This is most easily illustrated by the $\mathcal{N} = 0$ case. One can construct Einstein gravity by taking conformal gravity in the presence of a real scalar field $\phi$ of unit conformal dimension acting as a conformal compensator$^{13}$ (The metric $g_{mn}$ has conformal dimension $-2$ in this picture.) One may describe a conformally flat geometry using the set of fields

$$g_{mn} = \eta_{mn}e^{2\Omega}, \quad \phi = 1.$$  

(4.1)

$^{13}$See for example the discussions in [56, 57], but the idea goes back to Weyl.
This is the conventional picture, with the field $\phi$ essentially playing no role. (It is placed in any given action so that Weyl invariance is formally maintained.) We call this the “Einstein frame.” Alternatively, one may perform a Weyl transformation to the set of fields

$$g_{mn} = \eta_{mn}, \quad \phi = e^{\Omega}.$$  

(4.2)

In this picture, all of the curvature is contained within $\phi$. This we refer to as the “flat frame.”

Physically there is no real difference between these two pictures. The effective metric in the theory is $g_{mn}\phi^2$, which is Weyl invariant. Our $\mathcal{N} = 2$ geometry is only a little more complicated. Instead of a single compensator field $\phi$, we have $\mathcal{G}^{ij}$ and $\mathcal{W}$ which compensate not just for super-Weyl transformations but also U(2)$_R$ transformations.

For AdS geometry, there exists an explicit solution for $\Omega(x)$ in a certain coordinate chart covering only part of the AdS hyperboloid:

$$e^{\Omega} = \left(1 - \frac{1}{4} \mu^2 x^2\right)^{-1}.$$  

(4.3)

For this case, we will refer to the Einstein frame as the “AdS frame.” An analogous construction exists in superspace. The $\mathcal{N} = 2$ geometry we have described up to this point is in a conformal frame analogous to the first set of equations (4.1). We have nontrivial curvature of all types – torsion, Lorentz, and isospin – while our compensator fields $\mathcal{W}$ and $\mathcal{G}_{ij}$ are covariantly constant, and gauge equivalent to constant values $w$ and $g_{ij}$. Because we know how to do the $\mathcal{N} = 1$ reduction in the case of flat superspace background (i.e. the analogue of (4.2)), we will exploit our ability to perform super-Weyl transformations in both $\mathcal{N} = 2$ and $\mathcal{N} = 1$ AdS geometries to construct the relation between $\mathcal{N} = 2$ and $\mathcal{N} = 1$ AdS actions.

In other words, given a set of covariant derivatives $D_{\alpha}^i$ in $\mathcal{N} = 2$ AdS, we will relate them to a set of covariant derivatives $\nabla_\alpha$ in $\mathcal{N} = 1$ AdS by the chain

$$D_{\alpha}^i \xleftarrow{\text{super-Weyl}} D_{\alpha}^i \xrightarrow{\mathcal{N}=1 \text{ reduction}} D_\alpha = D_{\alpha}^1 \xrightarrow{\text{super-Weyl}} \nabla_\alpha$$  

(4.4)

with the $\mathcal{N} = 1$ reduction performed in a frame where it is straightforward. The same chain of transformations can be applied to any objects in our theory, including actions. Given a Lagrangian $\mathcal{L}$ appearing in the $\mathcal{N} = 2$ AdS action

$$S = \int d^4 x \ d^4 \theta \ d^4 \bar{\theta} \ E \mathcal{L}$$

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we may convert it to an $\mathcal{N} = 1$ AdS Lagrangian $\mathcal{L}^{(1)}$ with action

$$S = \int d^4 x \ d^2 \theta \ d^2 \bar{\theta} \ E \mathcal{L}^{(1)}$$

via the procedure

$$\mathcal{L} \xrightarrow{\text{super-Weyl}} \mathcal{L}_0 \xrightarrow{\mathcal{N} = 1 \text{ reduction}} \mathcal{L}_0^{(1)} = \frac{1}{16}(D\bar{D})^2(D\bar{D})^2 \mathcal{L}_0 \xrightarrow{\text{super-Weyl}} \mathcal{L}^{(1)}.$$ \ (4.5)

Now we need only explicitly construct the transformations taking us to the flat geometry for $\mathcal{N} = 2$ and $\mathcal{N} = 1$. We will give first the $\mathcal{N} = 2$ solution, then the $\mathcal{N} = 1$ solution, and then describe how to connect them via a simple reduction procedure.

### 4.1 From flat $\mathcal{N} = 2$ geometry to AdS

Conformally flat geometry for $\mathcal{N} = 2$ superspace was analyzed in depth in \[42\] for the case where the structure group is $\text{SL}(2, \mathbb{C}) \times \text{SU}(2)_R$. Here we modify that presentation somewhat for our choice of structure group $\text{SL}(2, \mathbb{C}) \times \text{U}(2)_R$. We have included Appendix \[A\] to briefly review the details of that $\mathcal{N} = 2$ superspace.

A conformally flat geometry is defined as any geometry related to a flat geometry by the combination of super-Weyl and $\text{U}(2)_R$ transformations.\footnote{The fact that the $\mathcal{N} = 2$ AdS superspace is locally conformally flat was also discussed in \[58\].} For our purposes, it will be important only to consider the super-Weyl transformations. These take the flat space derivatives $D_A = (\partial_a, D^i_\alpha, \bar{D}^\dot{\alpha}_i)$ to curved space ones $\mathcal{D}_A = (\mathcal{D}_a, \mathcal{D}^i_\alpha, \bar{\mathcal{D}}^{\dot{\alpha}}_i)$ by

$$\mathcal{D}^i_\alpha = e^{\mathcal{U}/2} \left( D^i_\alpha + 2D^\beta_i U M_{\beta \alpha} - \frac{1}{2} D^i_\alpha U J^j_i \right),$$ \ (4.6a)

$$\bar{\mathcal{D}}^{\dot{\alpha}}_i = e^{\mathcal{U}/2} \left( \bar{D}^{\dot{\alpha}}_i - 2\bar{D}^{\dot{\beta}}_j U M^{\beta \dot{\alpha}} + \frac{1}{2} \bar{D}^{\dot{\alpha}}_i U J^j_i \right),$$ \ (4.6b)

where $U$ is the super-Weyl parameter. The torsion superfields in the curved space are given by

$$S_{ij} = \frac{1}{4} e^{3\mathcal{U}} D_{ij} e^{-2\mathcal{U}},$$ \ (4.7a)

$$Y_{\alpha \beta} = -\frac{1}{4} e^{-\mathcal{U}} D_{\alpha \beta} e^{2\mathcal{U}},$$ \ (4.7b)

$$G_{\alpha \dot{\alpha}} = -\frac{1}{16} e^{-\mathcal{U}} [D_{\alpha k} \bar{D}_{\dot{\alpha} k}] e^{2\mathcal{U}},$$ \ (4.7c)

$$G_{\alpha \dot{\alpha}}^{ij} = \frac{i}{4} e^{\mathcal{U}} [D_{\alpha}^{i} \bar{D}_{\dot{\alpha}}^j] U.$$ \ (4.7d)
The maximally symmetric geometry is AdS, which obeys the additional constraints

\[ Y_{\alpha\beta} = G_{\alpha\dot{\alpha}} = G_{\alpha\dot{\alpha}}^{ij} = 0, \quad (4.8) \]

which in turn imply that \( S^{ij} \) is covariantly constant

\[ \mathcal{D}_A S^{ij} = 0 \quad (4.9) \]

with constant norm

\[ S^2 = \frac{1}{2} S^{ij} S_{ij} = \text{const}. \quad (4.10) \]

In general, \( S^{ij} \) is not actually constant; however, one can always make an additional \( \text{U}(2)_R \) transformation to achieve this.

The constraints (4.8) impose a number of additional conditions on the real parameter \( U \). For example, using (4.7d) the equation \( G_{\alpha}^{ij} = 0 \) is solved by

\[ U = \Sigma + \bar{\Sigma}, \quad \bar{\mathcal{D}}^i \Sigma = 0 \quad (4.11) \]

for an arbitrary chiral scalar \( \Sigma \), neutral under the group \( \text{U}(1)_R \). We temporarily may convert our AdS covariant derivatives \( \mathcal{D} \) with the structure group \( \text{SL}(2, \mathbb{C}) \times \text{U}(2)_R \) to AdS covariant derivatives \( \bar{\mathcal{D}} \) with structure group \( \text{SL}(2, \mathbb{C}) \times \text{SU}(2)_R \) via the similarity transformation

\[ \mathcal{D}_\alpha^i = e^{-\frac{1}{2}(\Sigma - \bar{\Sigma})} \mathcal{D}_\alpha^i e^{\frac{1}{2}(\Sigma - \bar{\Sigma})} = e^{\bar{\Sigma}} \left( \mathcal{D}_\alpha^i + 2 \mathcal{D}_\beta^i \Sigma M_{\beta\alpha} + 2 \mathcal{D}_\alpha^i \Sigma J^j \right), \quad (4.12a) \]

\[ \bar{\mathcal{D}}^\dot{\alpha}_i = e^{-\frac{1}{2}(\Sigma - \bar{\Sigma})} \bar{\mathcal{D}}^\dot{\alpha}_i e^{\frac{1}{2}(\Sigma - \bar{\Sigma})} = e^\Sigma \left( \bar{\mathcal{D}}^\dot{\alpha}_i - 2 \bar{\mathcal{D}}^\dot{\beta}_i \bar{\Sigma} \bar{M}^{\dot{\beta}\dot{\alpha}} - 2 \bar{\mathcal{D}}^\dot{\alpha}_j \bar{\Sigma} J^j_i \right). \quad (4.12b) \]

These AdS covariant derivatives were given in [42]. There the requirements that their torsions \( Y_{\alpha\beta} \) and \( G_{\alpha\dot{\alpha}} \) vanish were solved by requiring the chiral superfield \( \Sigma \) to obey

\[ \exp(-2\Sigma) = \left( 1 - \frac{1}{4} s^2 y^2 + s^{ij} \theta_{ij} \right)^{-1}, \quad y^m = x^m + i\theta_j \sigma^m \bar{\theta}^j, \quad (4.13) \]

where \( s^{ij} \) is a constant complex isovector, \( s^2 := \frac{1}{2} s^{ij} \bar{s}_{ij} \), and \( \theta_{ij} := \theta^{\mu}_{ij} \theta_{ij} \). We may borrow this solution and use it with our original \( \text{SL}(2, \mathbb{C}) \times \text{U}(2)_R \) AdS covariant derivatives.

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16 The same similarity transformation must be applied to all superfields, including torsion superfields.

17 Our parameter \( s^{ij} \) actually corresponds to the parameter \( b^{ij} = q s^{ij} \) given in eq (4.16) of [42]. There, \( s^{ij} \) was a real isovector and \( q \) was a complex phase, which was subsequently set to unity. We find it useful to keep the phase unfixed and consider complex \( s^{ij} \).
The complex constants $s^{ij}$ are acted on by the global $U(2)_R$. Using (4.1a), one may show that

$$S^{ij} = s^{ij} + \mathcal{O}(\theta), \quad S^2 = s^2.$$  \hspace{1cm} (4.14)

which imply that $S^{ij}$ differs from $s^{ij}$ by a $\theta$-dependent $U(2)_R$ rotation. Thus we are always free to choose the local $U(2)_R$ gauge so that

$$S^{ij} \xrightarrow{U(2)_R} s^{ij}. \hspace{1cm} (4.15)$$

The global $U(2)_R$ transformations remain unfixed.

In section 2.3, we solved the equations of motion by going to the frame where $G^{ij}$ and $\mathcal{W}$ were covariantly constant and gauge-equivalent to the constant values $g^{ij}$ and $w$. We can now apply our super-Weyl transform to work out explicit forms for $G^{ij}_0$ and $\mathcal{W}_0$ in the flat frame. Recall that $\mathcal{W}$ is a pure phase superfield; this implies that

$$1 = \mathcal{W}\bar{\mathcal{W}} = e^{2\mathcal{W}}\mathcal{W}_0\bar{\mathcal{W}}_0. \hspace{1cm} (4.16)$$

This equation can be solved by taking

$$\mathcal{W}_0 = w e^{-2\Sigma} = w \left(1 - \frac{1}{4}s^2 y^2 + s^{ij} \theta_{ij}\right)^{-1} \hspace{1cm} (4.17)$$

for a constant phase $w$. This yields an explicit solution for $\mathcal{W}_0$ in the flat frame. Note that this solution reduces to $w$ when the AdS curvature goes to zero.

The solution for $G^{ij}_0$ can be most readily found by applying the equation of motion (2.11b):

$$G^{ij}_0 = -\frac{1}{4\xi} D^{ij} \mathcal{W}_0 = -\frac{w}{4\xi} D^{ij} \left(1 - \frac{1}{4}s^2 y^2 + s^{ij} \theta_{ij}\right)^{-1}. \hspace{1cm} (4.18)$$

Consistency with the remaining equations (2.11a) and (2.11c) requires that

$$s^2 = \xi^2, \hspace{1cm} (4.19)$$

which agrees with the physical requirement that the AdS scale be set by the cosmological constant. Note that this solution for $G^{ij}_0$ tends toward constant $g^{ij} = -ws^{ij}/\xi$ as the AdS curvature tends to zero.

It is worth noting that

$$G^{ij}_0 = -\frac{e^{-\mathcal{W}}}{\xi} \mathcal{W}_0 S^{ij} \hspace{1cm} (4.20)$$
and so the same \( U(2) \) rotation which sends \( S^{ij} \) to constant \( s^{ij} \) will send
\[
G_{0}^{ij} \rightarrow e^{-2U} g^{ij}, \quad W_{0} \rightarrow e^{-U} w.
\]
Therefore the composition of this \( U(2) \) with the super-Weyl transformation does indeed take us to an AdS frame where \( G^{ij} \) and \( W \) are actually constant
\[
G_{0}^{ij} \rightarrow e^{-2U} g^{ij}, \quad W_{0} \rightarrow e^{-U} w.
\]

It turns out we will have no need for the explicit form of this \( U(2) \) transformation; it is sufficient to know it exists.

### 4.2 From flat \( \mathcal{N} = 1 \) geometry to AdS

We now consider conformally flat \( \mathcal{N} = 1 \) AdS geometry\(^{18}\) with the structure group \( \text{SL}(2, \mathbb{C}) \times U(1)_{R} \). The details will be very similar to the standard discussion with structure group \( \text{SL}(2, \mathbb{C}) \). We have included Appendix B to briefly review the relevant details of \( \mathcal{N} = 1 \) superspace with the structure group \( \text{SL}(2, \mathbb{C}) \times U(1)_{R} \).

As with \( \mathcal{N} = 2 \), a conformally flat \( \mathcal{N} = 1 \) geometry can be connected to a flat geometry via an \( \mathcal{N} = 1 \) super-Weyl + \( U(1)_{R} \) transformation\(^{19}\). We are concerned only with the super-Weyl transformation for now, which acts on the covariant derivatives,
\[
\nabla_{\alpha} = e^{U/2} \left( D_{\alpha} + 2D_{\beta} U M_{\beta\alpha} - \frac{3}{2} D_{\alpha} \hat{J} \right),
\]
\[
\bar{\nabla}^{\dot{\alpha}} = e^{U/2} \left( \bar{D}^{\dot{\alpha}} - 2\bar{D}_{\dot{\beta}} U \bar{M}^{\dot{\beta}\dot{\alpha}} + \frac{3}{2} \bar{D}^{\dot{\alpha}} \hat{J} \right),
\]
where \( U \) is an arbitrary real scalar superfield\(^{20}\). The torsion superfields are given by
\[
R = -\frac{1}{4} e^{3U} \bar{\nabla}^{2} e^{-2U},
\]
\[
G_{\alpha\dot{\alpha}} = [\nabla_{\alpha}, \bar{\nabla}^{\dot{\alpha}}] e^{U},
\]
\[
X_{\alpha} = -\frac{3}{2} (\bar{\nabla}^{2} - 4R) \nabla_{\alpha} U.
\]

\(^{18}\)See \cite{59, 60, 61} for early papers on \( \mathcal{N} = 1 \) AdS supersymmetry and superspace.

\(^{19}\)Equivalently, a conformally flat \( \mathcal{N} = 1 \) geometry is characterized by the condition that the torsion superfield \( W_{\alpha\beta\gamma} \) vanishes.

\(^{20}\)Note that we have used a different label \( \hat{J} \) for the \( U(1)_{R} \) generator than in the \( \mathcal{N} = 2 \) case. The action of \( \hat{J} \) on the \( \mathcal{N} = 1 \) covariant derivatives is defined by eq. (B.3). This operator may related to the \( \mathcal{N} = 2 \) \( U(1)_{R} \) generator as in eq. (4.41).
The maximally symmetric geometry is AdS, which is characterized by the additional constraints

\[ G_{\alpha\dot{\alpha}} = X_{\alpha} = 0 \]  \hspace{1cm} (4.25)

These in turn imply that \( R \) is covariantly constant with constant norm

\[ \nabla_\alpha R = \bar{\nabla}_{\dot{\alpha}} R = 0, \quad R\bar{R} = \text{const} \]  \hspace{1cm} (4.26)

In general \( R \) is not actually constant, but a local \( U(1)_R \) transformation can always make it so.

The constraints (4.25) impose a number of conditions on the super-Weyl parameter \( U \), which can be solved by

\[ U = \sigma + \bar{\sigma} \]  \hspace{1cm} (4.27)

for chiral \( \sigma \) parametrized by a constant complex parameter \( \mu \)

\[ \exp(-2\sigma) = \left(1 - \frac{1}{4}\mu^2 y^2 - \bar{\mu}\theta^2\right)^{-1}, \quad y^m = x^m + i\theta \sigma^m \bar{\theta} \]  \hspace{1cm} (4.28)

Using (4.24a), one may show that

\[ R = \mu + \mathcal{O}(\theta), \quad R\bar{R} = |\mu|^2 \]  \hspace{1cm} (4.29)

which imply that \( R \) differs from \( \mu \) by a \( \theta \)-dependent \( U(1)_R \) rotation; thus we are always free to choose the local \( U(1)_R \) gauge

\[ R \xrightarrow{\text{U}(1)_R} \mu. \]  \hspace{1cm} (4.30)

A residual global \( U(1)_R \) symmetry acts on \( \mu \).

It is also possible to understand \( \mathcal{N} = 1 \) AdS geometry in terms of a compensator. One couples a chiral compensator \( \Phi \) with a cubic coupling \( \xi \) to conformal supergravity with the action

\[ S = -3 \int d^4x \, d^2\theta \, d^2\bar{\theta} \, E \Phi \bar{\Phi} + \xi \int d^4x \, d^2\theta \, \mathcal{E} \Phi^3 + \xi \int d^4x \, d^2\bar{\theta} \, \bar{\mathcal{E}} \bar{\Phi}^3. \]  \hspace{1cm} (4.31)

The equation of motion in the flat frame is

\[ -\frac{1}{4} \bar{D}^2 \Phi_0 = \xi \Phi_0^2 \]  \hspace{1cm} (4.32)

and it has the solution \( \Phi_0 = \varphi e^\sigma \) for complex phase \( \varphi \) where \( \mu = \xi \varphi^3 \) \hspace{1cm} (62). In the AdS frame, it follows that \( \Phi = \varphi \); in other words

\[ \Phi_0 \xrightarrow{\text{super-Weyl+U}(1)_R} \varphi. \]  \hspace{1cm} (4.33)
4.3 Procedure for the reduction to $\mathcal{N} = 1$

The solution for $\mathcal{N} = 2$ AdS is parametrized by a constant isovector $s^{ij} = -\xi g^{ij}/w$ which is rotated by the global $U(2)_R$ action. There are two interesting possibilities to choose for $g^{ij}$. One is

$$g^{11} = g^{22} = 0 ,$$

(4.34)

while the other is

$$g^{12} = 0 .$$

(4.35)

Making a specific choice for $g^{ij}$ is equivalent to choosing which supersymmetry to leave manifest in the $\mathcal{N} = 1$ reduction. In our previous work [2] where we considered $\mathcal{N} = 2$ Minkowski superspace (i.e. with vanishing cosmological constant, $\xi = 0$), we found that both of these were sensible choices. The first of these conditions corresponded to linearized new minimal supergravity and the second to linearized old minimal supergravity, each accompanied by a massless gravitino multiplet, which are dual to each other.

AdS offers much less freedom in this choice. As discussed in section 1, the linearized $\mathcal{N} = 1$ supergravity action exists in AdS only for two cases of compensator field: a chiral compensator (corresponding to old minimal supergravity) and a complex linear compensator (corresponding to the $n = -1$ non-minimal supergravity), which are dual to each other. Therefore, we expect that only the choice $g^{12} = 0$ will yield an elegant $\mathcal{N} = 1$ reduction.

Taking $g^{12} = 0$, it is convenient to introduce the complex phase parameter $\gamma$,

$$g_{11} = \gamma , \quad g_{22} = \bar{\gamma} , \quad \gamma \bar{\gamma} = 1 ,$$

(4.36)

yielding

$$s^{11} = -\xi \bar{\gamma} \bar{w} , \quad s^{22} = -\xi \gamma w ,$$

(4.37a)

$$\bar{s}_{11} = -\xi \gamma w , \quad \bar{s}_{22} = -\xi \bar{\gamma} w .$$

(4.37b)

The $\mathcal{N} = 2$ chiral super-Weyl parameter $\Sigma$ takes the form

$$\exp(-2\Sigma) = \left(1 - \frac{1}{4} \xi^2 y^2 - \xi \bar{\gamma} \bar{w} \theta_{11} - \xi \gamma w \theta_{22}\right)^{-1} .$$

(4.38)

\[^{21}\text{In fact, it was possible in [2] to perform an } \mathcal{N} = 1 \text{ reduction for any choice of the parameter } g^{ij} .\]
The coefficients of $\theta_{11}$ and $y^2$ have the same relationship required by the coefficients of $\theta^2$ and $y^2$ in the $\mathcal{N} = 1$ parameter $\sigma$, so we may take $\sigma = \Sigma$. This implies

$$\bar{\mu} = \xi \bar{\gamma} \bar{\nu} = -s_{11}. \quad (4.39)$$

We can now explicitly relate the $\mathcal{N} = 2$ and $\mathcal{N} = 1$ AdS derivatives. Observe that the flat space derivative identification $D_{\alpha} \downarrow = D_{\alpha}$ and the relation $U \downarrow = U$ lead to

$$D_{\alpha} \downarrow = e^{U/2} \left( D_{\alpha} + 2D^\beta U M_{\beta \alpha} - \frac{1}{2} D_{\alpha} U J \downarrow + 2D_{\alpha} \frac{3}{2} U J \downarrow \right). \quad (4.40)$$

Moreover, because $g^{12} = 0$, one can show that $D_{\alpha} \downarrow = 0$. Comparing the expression for $D_{\alpha} \downarrow$ with that for the $\mathcal{N} = 1$ covariant derivative $\nabla_{\alpha}$, eq. (4.23a), we are led to identify the $\mathcal{N} = 1$ $U(1)_R$ generator with a certain diagonal subgroup of $U(2)_R$:

$$\hat{J} = \frac{1}{3} J \downarrow - \frac{4}{3} J_{\downarrow}. \quad (4.41)$$

This leads to the very simple AdS relations:

$$D_{\alpha} \downarrow = \nabla_{\alpha}, \quad D_{\dot{\alpha}} \downarrow = \nabla_{\dot{\alpha}}, \quad D_{\alpha \dot{\alpha}} \downarrow = \nabla_{\alpha \dot{\alpha}} = \frac{i}{2} \{ \nabla_{\alpha}, \nabla_{\dot{\alpha}} \}. \quad (4.42)$$

From the AdS algebra in both cases, we find that

$$R := \bar{S}_{11}, \quad \bar{R} := -S_{11} \quad (4.43)$$

are covariantly constant. Moreover, one can show that

$$S^{12} \downarrow = 0. \quad (4.44)$$

We can now perform the general reduction to $\mathcal{N} = 1$ of any action. We begin from the AdS frame where $S^{ij}$, $G^{ij}$, and $W$ are only covariantly constant, and only a super-Weyl transformation separates us from the flat frame. The generic action

$$S = \int d^4x \ d^4\theta \ d^4\bar{\theta} \ E \mathcal{L} \quad (4.45)$$

can then be super-Weyl transformed to the flat frame

$$S = \int d^4x \ d^4\theta \ d^4\bar{\theta} \mathcal{L}_0. \quad (4.46)$$

---

Note that $\mathcal{L}_0 = \mathcal{L}$ is unchanged since it has super-Weyl weight zero; this implies that the AdS $E$ is equal to unity.
The $\mathcal{N} = 1$ reduction in the flat geometry is straightforward:

$$S = \int \! d^4 x \, d^2 \theta \, d^2 \bar{\theta} \mathcal{L}_0^{(1)}, \quad \mathcal{L}_0^{(1)} = \frac{1}{32} \{(D\bar{\theta})^2, (\bar{D}_2)^2\} \mathcal{L}_0\) . \quad (4.47)$$

Performing an $\mathcal{N} = 1$ super-Weyl transformation back to the AdS geometry gives

$$S = \int \! d^4 x \, d^2 \theta \, d^2 \bar{\theta} \, E \mathcal{L}^{(1)}, \quad \mathcal{L}^{(1)} = e^{2U} \mathcal{L}_0^{(1)} . \quad (4.48)$$

Discarding $\mathcal{N} = 1$ total derivatives, one can show that

$$\mathcal{L}^{(1)} = \frac{1}{32} \left( (D\bar{\theta})^2 + 8 S_{22} \right) (\bar{D}_2)^2 \mathcal{L}_c| + \frac{1}{32} \left( (\bar{D}_2)^2 + 8 \bar{S}_{22} \right) D\bar{\theta}^2 \mathcal{L}| . \quad (4.49)$$

This formula can now be directly applied to any $\mathcal{N} = 2$ full superspace action.

This procedure may also be applied to chiral actions. Beginning with

$$S = \int \! d^4 x \, d^4 \theta \, \mathcal{E} \mathcal{L}_c, \quad (4.50)$$

we can first perform a super-Weyl transformation to the flat frame

$$S = \int \! d^4 x \, d^4 \theta \, \mathcal{L}_{c0}, \quad \mathcal{L}_{c0} = e^{-2U} \mathcal{L}_c . \quad (4.51)$$

The $\mathcal{N} = 1$ reduction in the flat geometry is simple:

$$S = \int \! d^4 x \, d^2 \theta \, \mathcal{L}_{c0}^{(1)}, \quad \mathcal{L}_{c0}^{(1)} = \frac{1}{4} (D\bar{\theta})^2 \mathcal{L}_0| . \quad (4.52)$$

Performing an $\mathcal{N} = 1$ super-Weyl transformation back to the AdS frame gives

$$S = \int \! d^4 x \, d^2 \theta \, d^2 \bar{\theta} \, \mathcal{E} \mathcal{L}_c^{(1)}, \quad \mathcal{L}_c^{(1)} = e^{3U/2} \mathcal{L}_0^{(1)} . \quad (4.53)$$

One can show that

$$\mathcal{L}_c^{(1)} = -\frac{1}{4} \left( (\bar{D}_2)^2 + 8 \bar{S}_{22} \right) \mathcal{L}_c| . \quad (4.54)$$

4.4 Background superfields and the fixing of local $U(1)_R$

In performing the reductions of the actions, we will need to identify the $\mathcal{N} = 1$ reductions of the various background superfields. As we have mentioned, the $U(2)_R$ gauge symmetry of the $\mathcal{N} = 2$ geometry must first be fixed so that only a super-Weyl transformation separates us from the flat geometry. In such a gauge, we have
covariantly constant background superfields, which differ from constant values by a \( \theta \)-dependent U(2)_R transformation,

\[
W = w + \mathcal{O}(\theta), \quad G^{ij} = g^{ij} + \mathcal{O}(\theta).
\]  

(4.55)

The requirement of covariant constancy is quite powerful since in applying the reduction formulae to the actions, we use the AdS covariant derivatives and so no derivatives of \( W \) or \( G^{ij} \) can be generated.

The background vector multiplet yields then a single \( \mathcal{N} = 1 \) background chiral superfield \( W| \). Because of the relation (4.42), one may show that \( W| \) is also covariantly constant with respect to the \( \mathcal{N} = 1 \) AdS derivatives and is a pure phase superfield. This means we are free to fix the U(1)_R gauge freedom by imposing the gauge choice

\[
W| = w = \text{const}.
\]  

(4.56)

We will make this choice for simplicity. Because we have gauged a chiral superfield to a constant, the \( U(1)_R \) connections must vanish. It follows that \( R \) is reduced to an actual constant

\[
R = \mu = \text{const}.
\]  

(4.57)

Similarly, the tensor multiplet obeys the conditions

\[
G_{ij}| = -\frac{1}{\xi} W S_{ij}| = -\frac{1}{\xi} \bar{W} \bar{S}_{ij}|
\]  

(4.58)

leading to

\[
\gamma = G_{11}| = \frac{\bar{w}\mu}{\xi}, \quad \bar{\gamma} = G_{22}| = \frac{w\bar{\mu}}{\xi}, \quad G_{12}| = 0, \quad \gamma \bar{\gamma} = 1
\]  

(4.59)

for the complex constant \( \gamma \).

For convenience, we collect the formulae relating the various constants and background superfields in this gauge:

\[
W| = w, \quad G_{11}| = \gamma, \quad G_{22}| = \bar{\gamma}, \quad G_{12}| = 0,
\]

\[
S_{11}| = -\bar{R} = -\bar{\mu}, \quad \bar{S}_{11}| = -R = -\mu,
\]

\[
S_{22}| = -\mu w^2, \quad \bar{S}_{22}| = -\bar{\mu} \bar{w}^2,
\]

\[
S_{12}| = \bar{S}_{12}| = 0,
\]

\[
\mu = \xi \gamma w, \quad \bar{\mu} = \xi \bar{\gamma} \bar{w}, \quad w \bar{w} = \gamma \bar{\gamma} = 1.
\]

\[\text{For any choice other than } g^{12} = 0, \text{ we would find a non-vanishing background } \mathcal{N} = 1 \text{ vector multiplet field strength } W_\alpha \text{ as well. This is another way to understand the simplicity of the } g^{12} = 0 \text{ choice.} \]
It is convenient to think of $w$ and $\gamma$ as free phases, which can be chosen by using the diagonal part of the global $U(2)_R$ which is preserved as a symmetry by our reduction procedure; the last line in the equations above then defines the constant $R = \mu$.

### 4.5 $\mathcal{N} = 1$ superfields and supergravity gauge transformations

We turn next to the identification of $\mathcal{N} = 1$ superfields and the supergravity gauge transformations. Recall that we explicitly showed in [2] how to identify the $\mathcal{N} = 1$ components of the $\mathcal{N} = 2$ supergravity multiplet in Wess-Zumino gauge. This correspondence was constructed in a Minkowski frame, so we may apply those rules to performing the $\mathcal{N} = 1$ reduction in the flat frame of our current AdS model. We relegate the details of how precisely to do this to Appendix [E] and give only the results.

The supergravity gauge transformations (3.3) allow us to impose the Wess-Zumino gauge

$$H| = \mathcal{D}_\alpha^2 \mathcal{H} = \bar{\mathcal{D}}_{\dot{\alpha}} \mathcal{H} = (\mathcal{D}^2)^2 \mathcal{H} = (\bar{\mathcal{D}}_2)^2 \mathcal{H} = 0 .$$

(4.60)

The remaining components of the superfield $\mathcal{H}$ may be identified as

$$H_{\alpha\dot{\alpha}} := \frac{1}{4} [\mathcal{D}_\alpha, \mathcal{D}_{\dot{\alpha}}] \mathcal{H} ,$$  \hspace{1cm} (4.61a)

$$\Psi_\alpha := \frac{1}{8} (\mathcal{D}_2)^2 \mathcal{D}_\alpha \mathcal{H} ,$$  \hspace{1cm} (4.61b)

$$\hat{U} := \frac{1}{16} \mathcal{D}_\alpha^2 (\mathcal{D}_2^2) \mathcal{D}_\alpha \mathcal{H} + \frac{1}{12} [\nabla^\alpha, \bar{\nabla}^{\dot{\alpha}}] H_{\alpha\dot{\alpha}} .$$  \hspace{1cm} (4.61c)

Here $H_{\alpha\dot{\alpha}}$ is the $\mathcal{N} = 1$ gravitational superfield, $\Psi_\alpha$ is the spinor superfield associated with the second gravitino, and $\hat{U}$ is an auxiliary real scalar superfield, all in the $\mathcal{N} = 1$ AdS frame.\footnote{In [2], we used the $\mathcal{N} = 1$ superfield $U$ which differed in its definition from $\hat{U}$. It turns out that $\hat{U}$ has a much simpler super-Weyl transformation law than $U$. This is sensible since it is the combination $\hat{U}$, rather than $U$, which appears in the $\mathcal{N} = 1$ reduction of the superconformal $\mathcal{N} = 2$ Noether coupling of $\mathcal{H}$ to its conserved current, which we constructed in Appendix B of [2].}

It is quite natural to perform the same super-Weyl transformation on the flat frame $\mathcal{N} = 1$ gauge transformations found in [2]. In the AdS frame, they are

$$\delta H_{\alpha\dot{\alpha}} = \nabla_\alpha \bar{L}_{\dot{\alpha}} - \bar{\nabla}_{\dot{\alpha}} L_\alpha ,$$  \hspace{1cm} (4.62a)

$$\delta \Psi_\alpha = \nabla_\alpha \Omega + \Lambda_\alpha , \hspace{1cm} \bar{\nabla}_{\dot{\alpha}} \Lambda_\alpha = 0 ,$$  \hspace{1cm} (4.62b)

$$\delta \hat{U} = \rho + \bar{\rho} , \hspace{1cm} \bar{\nabla}_\alpha \rho = 0 ,$$  \hspace{1cm} (4.62c)
where ρ and Λα are covariantly chiral, while Ω and Lα are unconstrained complex superfields.  

We may apply the same procedure for identifying the N = 1 components of the other N = 2 superfields in AdS. The components of the N = 2 vector multiplet W consist of a chiral scalar χ and the abelian vector field strength Wα given by

\[ \chi := W|, \quad W_\alpha := \frac{i}{2} D_\alpha \bar{\partial} W| \quad (4.63) \]

Similarly, the components of the N = 2 tensor multiplet Gij are given by a chiral scalar η and a tensor multiplet L,

\[ \eta := G_{11}|, \quad \bar{\eta} := G_{22}|, \quad L = -2i G_{12}| \quad (4.64) \]

These N = 1 superfields naturally transform under the N = 1 supergravity gauge transformations (4.62):

\[ \delta \chi = -\frac{w}{12} (\bar{\partial}^2 - 4R) \partial^\alpha L_\alpha - w \rho \quad (4.65a) \]

\[ \delta W_\alpha = \frac{i}{4} (\bar{\partial}^2 - 4R) \partial^\alpha (w \Omega - \bar{w} \bar{\Omega}) \quad (4.65b) \]

\[ \delta \eta = -\frac{\gamma}{6} (\bar{\partial}^2 - 4R) \partial^\alpha L_\alpha + \gamma \rho \quad (4.65c) \]

\[ \delta L = i\gamma \bar{\partial}^\alpha L_\alpha - i\bar{\gamma} \bar{\partial}_\alpha \bar{L}^\alpha \quad (4.65d) \]

These are natural generalizations of the Minkowski space results found in [2]. It is clear that χ and η transform as chiral compensators with conformal dimension 1 and 2, respectively, while they carry opposite charge under the U(1) transformation gauged by the N = 1 auxiliary superfield \( \hat{U} \).  

### 4.6 The N = 1 action

We now give the action corresponding to the N = 1 reduction of (3.6). It consists of four pieces, \( S = S_{WG} + S_{WH} + S_{GH} + S_{HH} \). The terms involving the compensators alone are

\[ S_{WG} = \int d^4x \, d^2\theta \, d^2\bar{\theta} \, E \left\{ \frac{1}{4} L^2 - \frac{1}{2} \eta \bar{\eta} - 2\xi L \bar{V} - \chi \bar{\chi} \right. \\
+ \left. \frac{1}{4} \gamma \eta^2 + \frac{1}{4} \gamma \bar{\eta}^2 + \gamma \bar{\omega} \eta \chi + \gamma w \bar{\eta} \bar{\chi} - \frac{1}{R} W^\alpha W_\alpha \right\} \quad (4.66) \]

---

25 In our previous paper [2], the gauge parameter ρ was denoted \( \hat{\Phi} \).

26 This U(1) may be understood as the shadow chiral rotation discussed in [63]. It corresponds to a subset of U(2)R which rotates \( \theta_2 \) while leaving \( \theta_1 \) invariant.
The terms involving mixing between the vector and the supergravity multiplets are given by

\[ S_{WH} = \int d^4x d^2\theta d^2\bar{\theta} E \left\{ -\hat{U}(w\bar{\chi} + \bar{w}\chi) + 2i\bar{w}\Psi^\alpha W_\alpha - 2iw\bar{\Psi}_\dot{\alpha}\bar{W}^{\dot{\alpha}} - \frac{i}{3}H^{\dot{\alpha}\alpha}\nabla_{\alpha\dot{\alpha}}(\bar{w}\chi - w\bar{\chi}) \right\}, \] (4.67)

while those involving the tensor and supergravity multiplets are

\[ S_{GH} = \int d^4x d^2\theta d^2\bar{\theta} E \left\{ \frac{1}{2}\hat{U}(\gamma\bar{\eta} + \bar{\gamma}\eta) - \frac{i}{2}\gamma L\nabla^\alpha \Psi_\alpha + \frac{i}{2}\bar{\gamma}L\nabla_{\dot{\alpha}}\bar{\Psi}^{\dot{\alpha}} - \frac{i}{3}H^{\dot{\alpha}\alpha}\nabla_{\alpha\dot{\alpha}}(\bar{\gamma}\eta - \gamma\bar{\eta}) \right\}. \] (4.68)

The pure supergravity sector yields

\[ S_{HH} = \int d^4x d^2\theta d^2\bar{\theta} E \left\{ -\frac{3}{4}\hat{U}^2 - \frac{1}{16}H^{\dot{\alpha}\alpha}\nabla^\beta(\nabla^2 - 4R)\nabla_\beta H_{\alpha\dot{\alpha}} + \frac{1}{48}([\nabla_\alpha, \nabla_{\dot{\alpha}}]H^{\dot{\alpha}\alpha})^2 - \frac{1}{4}(\nabla_{\alpha\dot{\alpha}}H^{\dot{\alpha}\alpha})^2 - \frac{1}{4}RRH^{\dot{\alpha}\alpha}H_{\alpha\dot{\alpha}} - \Psi^\alpha\nabla_\alpha\nabla_{\dot{\alpha}}\bar{\Psi}^{\dot{\alpha}} - \frac{1}{4}(\gamma^{\dot{\alpha}\alpha}\Psi_\alpha - \bar{\gamma}\nabla_\dot{\alpha}\bar{\Psi}^{\dot{\alpha}})^2 - \frac{\bar{w}^2}{4}\Psi^\alpha\nabla^2\Psi_\alpha - \frac{w^2}{4}\bar{\Psi}_{\dot{\alpha}}\nabla^2\bar{\Psi}^{\dot{\alpha}} \right\}. \] (4.69)

As in the Minkowski case, the superfield \( \hat{U} \) is an auxiliary and may be integrated out algebraically. Doing so, we find that only a certain combination of chiral superfields survives, which can be denoted

\[ \bar{\varphi}\phi := \frac{1}{3}\bar{w}\chi + \frac{1}{3}\bar{\gamma}\eta \] (4.70)

We interpret \( \varphi \) as a background phase, corresponding to the background value of a chiral compensator \( \Phi \), while \( \phi \) is its quantum deformation. In our previous paper \[2\], we denoted the combination \( \bar{\varphi}\phi \) by \( \sigma \); to make the analogy with \( \mathcal{N} = 2 \) as strong as possible, we have restored a background value to this compensator.

We arrive at two decoupled actions \( S = S_{\text{old}} + S_\Psi \). The first is the linearized old minimal supergravity action in AdS

\[ S_{\text{old}} = -\int d^4x d^2\theta d^2\bar{\theta} E \left\{ \frac{1}{16}H^{\dot{\alpha}\alpha}\nabla^\beta(\nabla^2 - 4R)\nabla_\beta H_{\alpha\dot{\alpha}} - \frac{1}{48}([\nabla_\alpha, \nabla_{\dot{\alpha}}]H^{\dot{\alpha}\alpha})^2 + \frac{1}{4}(\nabla_{\alpha\dot{\alpha}}H^{\dot{\alpha}\alpha})^2 + \frac{RR}{4}H^{\dot{\alpha}\alpha}H_{\alpha\dot{\alpha}} + iH^{\dot{\alpha}\alpha}\nabla_{\alpha\dot{\alpha}}(\bar{\varphi}\phi - \varphi\bar{\phi}) + 3(\varphi\bar{\phi} + \varphi^2\phi^2 - \varphi^2\bar{\phi}^2) \right\} \] (4.71)
which is invariant under the gauge transformations

\[ \delta H_{\alpha \dot{\alpha}} = \nabla_\alpha \tilde{L}_{\dot{\alpha}} - \tilde{\nabla}_{\alpha} L_{\dot{\alpha}}, \]

\[ \delta \phi = -\frac{\varphi}{12}(\nabla^2 - 4R)\nabla^\alpha L_{\alpha}. \]

The other sector is the massless gravitino action in AdS

\[ S_{\Psi} = \int d^4x d^2\theta d^2\bar{\theta} E \left\{ -\Psi^\alpha \nabla_\alpha \nabla_\alpha \bar{\Psi}^{\dot{\alpha}} - \frac{\bar{w}^2}{4} \Psi^\alpha \nabla^2 \Psi_\alpha - \frac{w^2}{4} \bar{\Psi}_\alpha \nabla^2 \bar{\Psi}^{\dot{\alpha}} - \frac{1}{4}(\gamma \nabla^\alpha \Psi_\alpha - \bar{\gamma} \nabla_\dot{\alpha} \bar{\Psi}^{\dot{\alpha}})^2 + 2i \bar{w} \Psi^\alpha W_\alpha - 2iw \bar{\Psi}_\dot{\alpha} \bar{W}^{\dot{\alpha}} + \frac{1}{2} L(\gamma \nabla^\alpha \Psi_\alpha - \bar{\gamma} \nabla_\dot{\alpha} \bar{\Psi}^{\dot{\alpha}}) + \frac{1}{4} L^2 - 2\xi LV - \frac{1}{R} W^\alpha W_\alpha \right\}, \]

where \( W_\alpha \equiv \frac{1}{4}(\nabla^2 - 4R)\nabla_\alpha V \). Its gauge symmetries are described by

\[ \delta \Psi_\alpha = \nabla_\alpha \Omega + \Lambda_\alpha, \]

\[ \delta V = -i\bar{w} \Omega + iw \bar{\Omega} + \lambda + \bar{\lambda}, \]

\[ \delta L = i\gamma \nabla^\alpha \Lambda_\alpha - i\bar{\gamma} \nabla_\dot{\alpha} \bar{\Lambda}^{\dot{\alpha}}, \]

where \( \lambda \) is the covariantly chiral gauge parameter associated with the usual gauge invariance of an abelian vector multiplet.

The supergravity action \((4.71)\) reduces to \((1.5)\) for \( \varphi = 1 \). In the rigid supersymmetric limit, the gravitino multiplet action \( S_{\Psi} \) correctly reduces to its flat superspace counterpart derived in [2] (see also [64]).

### 4.7 Dual versions of the gravitino multiplet action

The gravitino multiplet action \( S_{\Psi} \) which we have found is quite interesting since the coupling between the tensor and vector multiplets allows several duality transformations (compare with [50]). For example, dualizing the linear multiplet to a chiral multiplet gives

\[ \tilde{S}_{\Psi} = \int d^4x d^2\theta d^2\bar{\theta} E \left\{ -\Psi^\alpha \nabla_\alpha \nabla_\alpha \bar{\Psi}^{\dot{\alpha}} - \frac{\bar{w}^2}{4} \Psi^\alpha \nabla^2 \Psi_\alpha - \frac{w^2}{4} \bar{\Psi}_\alpha \nabla^2 \bar{\Psi}^{\dot{\alpha}} + 2i \bar{w} \Psi^\alpha W_\alpha - 2iw \bar{\Psi}_\dot{\alpha} \bar{W}^{\dot{\alpha}} - 2i\xi(V + \phi + \bar{\phi})(\gamma \nabla^\alpha \Psi_\alpha - \bar{\gamma} \nabla_\dot{\alpha} \bar{\Psi}^{\dot{\alpha}}) - 4\xi(V + \phi + \bar{\phi})^2 - \frac{1}{R} W^\alpha W_\alpha \right\}, \]

\((4.75)\).
where $W_\alpha \equiv \frac{1}{4}(\nabla^2 - 4R)\nabla_\alpha V$. The gauge invariances are

$$
\delta \Psi_\alpha = \nabla_\alpha \Omega + \Lambda_\alpha ,
\quad \delta V = -i\bar{w} \Omega + iw\bar{\Omega} + \lambda + \bar{\lambda} ,
\quad \delta \phi = +\frac{i\bar{\gamma}}{4}(\bar{\nabla}^2 - 4R)\bar{\Omega} - \lambda .
$$

We have written the action to make it obvious that $\phi$ is a Stueckelberg field; it may be eliminated by sacrificing the gauge invariance $\lambda$. Similarly, one may eliminate $V$ by employing the $\Omega$ gauge invariance.

We may also perform a duality on the vector multiplet to give

$$
\tilde{S}_\Psi' = \int d^8 z E \left\{ -\Psi^\alpha \nabla_\alpha \bar{\Psi}^{\dot{\alpha}} + \frac{\bar{w}^2}{4} \bar{\Psi}^\alpha \nabla^2 \Psi_\alpha + \frac{w^2}{4} \bar{\Psi}^{\dot{\alpha}} \nabla^2 \bar{\Psi}_{\dot{\alpha}} 
\right.
\left. + \frac{1}{4} (L - i\gamma \nabla^2 \Psi_\alpha + i\bar{\gamma} \nabla^2 \bar{\Psi}^{\dot{\alpha}})^2 
\right.
\left. + \frac{2\xi^2}{\alpha R} \left( \chi^\alpha + \frac{i}{\xi} W^\alpha - i\gamma \Psi^\alpha \right) \left( \chi_\alpha + \frac{i}{\xi} W_\alpha - i\gamma \bar{\Psi}_\alpha \right) 
\right.
\left. + \frac{2\xi^2}{\bar{\alpha} R} \left( \bar{\chi}^{\dot{\alpha}} - \frac{i}{\xi} \bar{W}^{\dot{\alpha}} + i\gamma \bar{\Psi}^{\dot{\alpha}} \right) \left( \bar{\chi}_{\dot{\alpha}} - \frac{i}{\xi} \bar{W}_{\dot{\alpha}} + i\gamma \bar{\Psi}_{\dot{\alpha}} \right) \right\} .
$$

The parameter $\alpha$ is complex and constrained to have unit real part. The resemblance to the dual $\mathcal{N} = 2$ action (3.14) is strong; likely one can derive the former from the latter. This action is invariant under the gauge transformations

$$
\delta \Psi_\alpha = \nabla_\alpha \Omega + \Lambda_\alpha ,
\quad \delta \chi_\alpha = i\gamma \Lambda_\alpha + i\phi_\alpha ,
\quad \delta W_\alpha = -\xi \phi_\alpha - \frac{1}{8}(\nabla^2 - 4R)\nabla_\alpha (\bar{\alpha} \bar{w} \Omega + \alpha w \bar{\Omega}) ,
$$

where $\phi_\alpha$ and $W_\alpha$ are both reduced chiral spinors. We have written the action in a way which makes clear that $W_\alpha$ is a Stueckelberg field and can be removed by fixing the $\phi_\alpha$ gauge degree of freedom. Similarly, one may eliminate $\chi_\alpha$ by employing the $\Lambda_\alpha$ gauge invariance.

## 5 Supercurrent

We are now in a position to postulate the general supercurrent conservation equation for arbitrary matter systems coupled to $\mathcal{N} = 2$ supergravity with vector and
tensor compensators. We have confirmed the structure of the linearized supergravity gauge transformations in an AdS background, eq. (3.3). Moreover, it has been argued in subsection 3.1 that the same transformation laws hold in an arbitrary supergravity background. We can therefore state the gauge transformation laws of the supergravity prepotentials:

\[ \delta \Psi = 4 \bar{\Delta}(\bar{\Omega}^{ij} G_{ij}) , \]  
\[ \delta V^{ij} = -4 \Omega^{ij} \bar{W} - 4 \bar{\Omega}^{ij} W , \]  
\[ \delta H = (D^{ij} + 4 S^{ij}) \Omega_{ij} + (\bar{D}^{ij} + 4 \bar{S}^{ij}) \bar{\Omega}_{ij} . \]  

This information is sufficient to identify the relevant supercurrent multiplet.

### 5.1 Main construction

Given a matter system coupled to \( N = 2 \) supergravity, we give small disturbances, \( H \), \( V^{ij} \) and \( \Psi \), to the gravitational superfield \( H \) and the compensators \( V^{ij} \) and \( \Psi \), respectively, the latter being defined as in eqs. (2.3) and (2.6). To first order, the matter action changes as

\[ S^{(1)} = \int d^4x d^4\theta d^4\bar{\theta} E \left\{ J H + T_{ij} V^{ij} \right\} + \left\{ \int d^4x d^4\theta E Y \Psi + \text{c.c.} \right\} , \]  

where

\[ J = \frac{\delta S}{\delta H} , \quad T_{ij} = \frac{\delta S}{\delta V^{ij}} , \quad Y = \frac{\delta S}{\delta \Psi} . \]  

The sources \( J \) and \( T_{ij} \) must be real, and \( Y \) covariantly chiral. In addition, \( Y \) and \( T_{ij} \) must obey the constraints

\[ D_\alpha^{(k) T^{ij}} = \bar{D}_\alpha^{(k) T^{ij}} = 0 , \]  
\[ (D^{ij} + 4 S^{ij}) Y = (\bar{D}^{ij} + 4 \bar{S}^{ij}) \bar{Y} , \]  

due to the gauge invariances of \( V^{ij} \) and \( \Psi \) given by eqs. (2.4) and (2.7). In the case that the disturbances \( H \), \( V^{ij} \) and \( \Psi \) correspond to a supergravity gauge transformation, eq. (5.1), the functional \( S^{(1)} \) must vanish provided the matter fields are placed on the mass shell. Since the gauge parameters \( \Omega^{ij} \) are unconstrained, we end up with the conservation equation

\[ \frac{1}{4}(\bar{D}_{ij} + 4 \bar{S}_{ij}) J = W T_{ij} - G_{ij} Y . \]
There is an alternative way to derive the conservation equation (5.5). We can start from $\mathcal{N} = 2$ supergravity without matter and choose an on-shell background, that is a solution to the supergravity equations of motion (2.11). We then linearize the supergravity action around the background chosen. The linearized supergravity action, $S^{(2)}$, must be invariant under the supergravity gauge transformations (5.1), as well as under the gauge invariances of $V^{ij}$ and $\Psi$ given by eqs. (2.4) and (2.7). The linearized supergravity multiplet can be further coupled to external sources by replacing $S^{(2)} \rightarrow S^{(2)} + S^{(1)}$, with $S^{(1)}$ given by eq. (5.2). The obtained action is gauge invariant provided the conservation equation (5.5) holds.

If the supergravity compensators obey the equation $W\bar{W} = \mathcal{G}$, which is one of the equations of motion for pure supergravity (2.11), then $W$ and $\mathcal{G}_{ij}$ may be fixed to be constant by applying appropriate super-Weyl and local $U(2)_R$ transformations. However, the presence of the matter will modify the supergravity equation of motion (even though the matter is on-shell), so we cannot assume $W\bar{W} = \mathcal{G}$. This eliminates the possibility of simultaneously gauging both compensators to be constant (or even covariantly constant). However, it is often the case that a theory couples to only one of the two compensators, and then it is quite natural to choose the gauge where $\mathcal{G}$ or $W\bar{W}$ is constant, as appropriate.

Before elaborating on the supercurrent (5.5), we should first review the $\mathcal{N} = 1$ situation, paying particular attention to the case in AdS where additional restrictions on the supercurrent arise. Then we consider some consequences of this proposed $\mathcal{N} = 2$ supercurrent in a Minkowski background. Finally, we discuss it in a more general supergravity background and present some examples of its application.

### 5.2 $\mathcal{N} = 1$ supercurrents

Textbook derivations [23, 62] of the supercurrent multiplet in the old minimal formulation for $\mathcal{N} = 1$ supergravity make use of the chiral prepotential $\Phi$ and its conjugate $\bar{\Phi}$ introduced originally by Siegel and Gates [65, 22]. These objects and the gravitational superfield $\mathcal{H}^m$ [66, 65] are the only prepotentials, modulo purely gauge degrees of freedom, in terms of which the Wess-Zumino constraints [15] are solved [65]. In this paper, we work with a super-Weyl invariant extension of old minimal supergravity, which makes use of a covariantly chiral scalar superfield $\varphi$ and its conjugate in addition to the Weyl multiplet described by the covariant derivatives $\nabla_A$. Because of the super-Weyl symmetry, which leaves the chiral combination $(\Phi \varphi)$ invariant, there is no need at all to resort to $\Phi$. However, there appear some new
technical nuances regarding the derivation of the supercurrent conservation equation. Such a derivation is similar to the \( \mathcal{N} = 2 \) construction described above.

Given a matter system coupled to old minimal supergravity, we give small disturbances, \( H^m \) and \( \phi \), to the gravitational superfield \( \mathcal{H}^m \) and the compensator \( \varphi \), respectively. To first order, the matter action changes by

\[
S^{(1)} = \int d^4x \, d^2\theta \, d^2\bar{\theta} \, E \, J^{\dot{\alpha} \alpha} \, H_{\dot{\alpha} \alpha} - \left\{ 3 \int d^4x \, d^2\theta \, E \, X \, \phi + \text{c.c.} \right\}
\]

(5.6)

where

\[
J_{\alpha \dot{\alpha}} = \frac{\delta S}{\delta H_{\alpha \dot{\alpha}}} , \quad X = -\frac{1}{3} \frac{\delta S}{\delta \varphi} .
\]

(5.7)

By construction, \( X \) is covariantly chiral. If \( H_{\alpha \dot{\alpha}} \) and \( \phi \) correspond to a supergravity gauge transformation, then \( S^{(1)} \) must vanish with the matter fields on-shell. In the case that \( \varphi \) is chosen to be annihilated by the spinor covariant derivatives, the \( \mathcal{N} = 1 \) supergravity transformations are described by eq. (4.72). In the general case, the transformation of \( \phi \) is as follows:

\[
\delta \phi = -\frac{1}{4} (\nabla^2 - 4R) (L^\alpha \nabla_\alpha \varphi) - \frac{\varphi}{12} (\nabla^2 - 4R) \nabla^\alpha L_\alpha .
\]

(5.8)

Of course, due to the super-Weyl invariance, we can always choose a super-Weyl gauge \( \varphi = \text{const} \), and then the transformation (5.8) reduces to (4.72). The condition \( S^{(1)} = 0 \) for \( H_{\alpha \dot{\alpha}} \) and \( \phi \) given by eqs. (4.72a) and (5.8), respectively, is the conservation equation

\[
\bar{\nabla}^{\dot{\alpha}} J_{\alpha \dot{\alpha}} = \varphi \nabla_\alpha X - 2X \nabla_\alpha \varphi .
\]

(5.9)

If \( \nabla_\alpha \varphi = 0 \), then this equation reduces, modulo a trivial rescaling, to eq. (1.4).

The above argument is easily generalized in the presence of other compensating multiplets. Essentially the only input needed is the generalization of the rule (5.8) for the supergravity gauge transformation of those multiplets. For example, if there is a single linear multiplet \( \mathcal{L} \) acting as a compensator, as in new minimal supergravity, one may construct its ‘quantum’ variation \( L \) out of a chiral spinor \( \phi_\alpha \),

\[
\mathcal{L} \to \mathcal{L} + L , \quad L = \nabla^\alpha \phi_\alpha + \bar{\nabla}_{\dot{\alpha}} \bar{\phi}^{\dot{\alpha}} ,
\]

(5.10)

---

\(^{27}\)The factor of 3 in (5.6) is introduced for convenience.

\(^{28}\)This can be justified in a number of ways. In analogy to our \( \mathcal{N} = 2 \) argument, this is the unique possibility (up to overall normalization) which transforms covariantly under super-Weyl transformations when \( \varphi \) has weight 1 and \( L_\alpha \) has weight -3/2. It may also be constructed by accompanying any coordinate transformation with an appropriate super-Weyl transformation to fix the chiral pre-potential \( \sigma \) hidden within the covariant derivatives. Alternatively, it may also be derived explicitly in conformal superspace \[43\] and then gauge fixed to Poincaré superspace.
with $\phi_\alpha$ transforming as

$$\delta \phi_\alpha = \frac{1}{4} (\bar{\nabla}^2 - 4R)(L_\alpha \mathcal{L})$$

(5.11)

under the supergravity gauge transformation. If $\phi_\alpha$ occurs only in the combination $L$, then it possesses the additional symmetry $\delta \phi_\alpha = iW_\alpha$, where $W_\alpha$ is a reduced chiral spinor.

Any matter action coupled to one or both of these compensators must be invariant under the supergravity gauge transformations. Defining the new trace multiplet $\chi_\alpha$ by its contribution to $S^{(1)}$,

$$\int d^4x d^2\theta \mathcal{E}_\chi \chi_\alpha \phi_\alpha , \quad \nabla^\alpha \chi_\alpha - \bar{\nabla}^\alpha \bar{\chi}_\alpha = \bar{\nabla}_\alpha \chi_\alpha = 0 ,$$

(5.12)

leads to the modified conservation equation

$$\bar{\nabla}_\alpha J_{\alpha \dot{\alpha}} = \mathcal{L}_\chi + \varphi \nabla_\alpha X - 2X \nabla_\alpha \varphi .$$

(5.13)

Suppose we have chosen the super-Weyl gauge $\varphi \bar{\varphi} = \text{const}$. Then $\varphi$ is covariantly constant and can be made truly constant by fixing the $U(1)_R$ gauge. The question remains whether it is possible for $\mathcal{L}$ to be constant as well. However, it is quite clear that this cannot be the case in general since the background linear compensator obeys the condition

$$(\bar{\nabla}^2 - 4R)\mathcal{L} = (\nabla^2 - 4\bar{R})\mathcal{L} = 0 ,$$

(5.14)

and so one cannot simultaneously choose to work in an AdS frame $R = \mu \neq 0$ and choose the background compensator $\mathcal{L}$ to be constant.

To put it another way, if we naively generalize the flat space supercurrent (1.2), ignoring the background compensators (i.e. assuming that they are unity), we find

$$\bar{\nabla}_\alpha J_{\alpha \dot{\alpha}} = \chi_\alpha + \nabla_\alpha X .$$

(5.15)

Applying $(\nabla^2 - 4R)$ to both sides, we find immediately that

$$0 = (\bar{\nabla}^2 - 4\bar{R})\chi_\alpha = -4R \chi_\alpha ,$$

(5.16)

which is consistent only if $R$ vanishes.

So the naive supergravity extension (5.15) of the Komargodski-Seiberg supercurrent, eq. (1.2), is inconsistent. The consistent extension is given by the relation (5.13). The key point is the compensator $\mathcal{L}$ must be taken seriously as a superfield with nontrivial coordinate dependence.
5.3 $\mathcal{N} = 2$ supercurrents in Minkowski background

We consider next some aspects of the $\mathcal{N} = 2$ supercurrent (5.5) in a Minkowski background, meaning that we take all of the $\mathcal{N} = 2$ torsion superfields to vanish while simultaneously taking the background compensators to be constant, $\mathcal{W} = w = \text{const}$ and $\mathcal{G}^{ij} = g^{ij} = \text{const}$. In this case, the $\mathcal{N} = 2$ supercurrent conservation equation reads

$$\frac{1}{4} \bar{D}_{ij} J = w \mathcal{T}_{ij} - g_{ij} \mathcal{Y}. \quad (5.17)$$

(In our previous work [2], we fixed $w = i$.)

One particularly interesting application of this equation, which we neglected to discuss in [2], is that it naturally leads to the $\mathcal{N} = 1$ supercurrent (1.2) discussed by Komargodski and Seiberg [20]. Defining the $\mathcal{N} = 1$ supercurrent $J_{a\dot{a}}$ by the rule [6]

$$J_{a\dot{a}} := \frac{1}{4}[D_{a\dot{2}}, \bar{D}_{\dot{a}2}]\mathcal{J} - \frac{1}{12}[D_{a\dot{1}}, \bar{D}_{\dot{a}1}]\mathcal{J}, \quad (5.18)$$

one can show it obeys

$$D^{\dot{a}} J_{a\dot{a}} = \chi_\alpha + D_a X \quad (5.19)$$

where the contributions to the trace multiplet are given by

$$\chi_\alpha := 4ig_{12} \mathcal{Y}_\alpha, \quad X := \frac{1}{3} w T_{11} + \frac{2}{3} g_{22} Y \quad (5.20)$$

$$T_{11} := T_{11}, \quad Y := \mathcal{Y}, \quad \mathcal{Y}_\alpha := \frac{i}{2} D_{a\dot{2}} \mathcal{Y}_\alpha. \quad (5.21)$$

(One should keep in mind that $ig_{12}$ is real.) Because of the constraints (5.4a), $T_{11}$ and $Y$ are both chiral superfields while $\mathcal{Y}_\alpha$ is a chiral field strength obeying the Bianchi identity $D^a \mathcal{Y}_\alpha = \bar{D}_\alpha \mathcal{Y}^\dot{\alpha}$.

Particularly illuminating is the case where $\mathcal{T}_{ij}$ vanishes (i.e. the $\mathcal{N} = 2$ theory couples only to the tensor compensator). Then we have

$$\chi_\alpha = 4ig_{12} \mathcal{Y}_\alpha, \quad X = \frac{2}{3} g_{22} Y. \quad (5.22)$$

The background values $g_{22}$ and $g_{12}$ rotate under the global SU(2)$_R$ symmetry; for a certain choice of gauge, either one (but not both) can be eliminated. In these situations, we clearly see the supercurrent associated with old minimal ($g_{12} = \chi_\alpha = 0$) or new minimal supergravity ($g_{22} = X = 0$); in the general case, the current has the form (1.2). A global SU(2)$_R$ rotation connects them all.
5.4 Supercurrent in a supergravity background: Examples

We conclude this section by giving several examples of supercurrents in curved superspace.

5.4.1 Abelian gauge theory with a Fayet-Iliopoulos term

Consider an abelian \( \mathcal{N} = 2 \) gauge theory coupled to the tensor compensator \( G^{ij} \):

\[
S = \frac{1}{2} \int d^4x \ d^4\theta \ E \ W^2 - \lambda \int d^4x \ d^4\theta \ d^4\bar{\theta} \ E \ G^{ij} V_{ij} .
\]

(5.23)

As usual, \( W \) denotes the covariantly chiral field strength of the \( \mathcal{N} = 2 \) vector multiplet, and \( V_{ij} \) the corresponding Mezincescu prepotential. If we impose the super-Weyl and SU(2)_R gauges where \( G^{ij} \) is constant, the tensor compensator coupling resembles a Fayet-Iliopoulos term. In addition to the usual supercurrent

\[
\mathcal{J} = W\bar{W} ,
\]

(5.24)

we have nontrivial coupling due to the tensor compensator

\[
\mathcal{Y} = -\lambda W ,
\]

(5.25)

while \( \mathcal{T}_{ij} = 0 \) since the model does not couple to the vector compensator. It is easy to see that the supercurrent equation (5.5) holds due to the equation of motion

\[
\frac{1}{4}(\mathcal{D}^{ij} + 4\mathcal{S}^{ij})W = \frac{1}{4}(\mathcal{D}^{ij} + 4\mathcal{S}^{ij})\bar{W} = \lambda G^{ij} .
\]

(5.26)

5.4.2 Supersymmetric Yang-Mills theory

We next consider a general \( \mathcal{N} = 2 \) supersymmetric Yang-Mills (SYM) theory with a hypermultiplet transforming in some representation of the gauge group. The simplest off-shell description of this theory in the presence of supergravity is provided by the projective-superspace formulation for \( \mathcal{N} = 2 \) supergravity-matter systems developed in [8, 9]. The action is

\[
S_{\text{YM}} = \frac{1}{2{g_{\text{YM}}}^2} \int d^4x \ d^4\theta \text{ Tr} (\mathcal{W}^2) + \frac{i}{2\pi} \oint_C v^i dv_i \int d^4x \ d^4\theta \ d^4\bar{\theta} \ E \frac{\mathcal{W}\bar{\mathcal{W}}}{(\Sigma^{++})^2} \hat{\Sigma}^+ \Sigma^+ ,
\]

(5.27)

with \( g_{\text{YM}} \) the coupling constant. Here the first term describes the pure SYM sector, with \( \mathcal{W} \) the gauge-covariantly chiral field strength,

\[
\mathcal{D}_a^i \mathcal{W} = 0 , \quad 4\Sigma^{ij} := (\mathcal{D}^{ij} + 4\mathcal{S}^{ij})\mathcal{W} = (\mathcal{D}^{ij} + 4\mathcal{S}^{ij})\mathcal{W} .
\]

(5.28)
The gauge-covariant derivatives \( \mathbf{D}_A = (\mathbf{D}_a, \mathbf{D}_i^\alpha, \mathbf{D}_i^{\dot{\alpha}}) \) differ from the supergravity covariant derivatives \( \mathbf{D}_A \), which are described in Appendix A, by the presence of (i) the Yang-Mills connection, and (ii) the U(1) connection associated with the vector multiplet compensator \( \mathbf{W} \). The spinor derivatives obey the anti-commutation relations

\[
\{ \mathbf{D}_i^\alpha, \mathbf{D}_j^\beta \} = \{ \mathbf{D}_i^\alpha, \mathbf{D}_j^\beta \} - 2i \epsilon^{ij} \epsilon_{\alpha\beta} (\mathbf{W} \mathbf{e} + \mathbf{W}) , \tag{5.29a}
\]

\[
\{ \mathbf{\bar{D}}_i^{\dot{\alpha}}, \mathbf{\bar{D}}_j^{\dot{\beta}} \} = \{ \mathbf{\bar{D}}_i^{\dot{\alpha}}, \mathbf{\bar{D}}_j^{\dot{\beta}} \} + 2i \epsilon_{ij} \epsilon^{\dot{\alpha}\dot{\beta}} (\mathbf{W} \mathbf{e} + \mathbf{W}) , \tag{5.29b}
\]

with \( \mathbf{e} \) the U(1) charge operator associated with \( \mathbf{W} \). The anticommutator \( \{ \mathbf{D}_i^\alpha, \mathbf{\bar{D}}_j^{\dot{\alpha}} \} \) has the same form (A.3b) as \( \{ \mathbf{D}_i^\alpha, \mathbf{\bar{D}}_j^{\dot{\alpha}} \} \) with the replacement of \( \mathbf{D}_c \) with \( \mathbf{D}_c' \).

The second term in (5.27) is the hypermultiplet action. It involves a closed contour in an auxiliary isotwistor variable \( v^i \in \mathbb{C}^2 \setminus \{0\} \), with respect to which the arctic hypermultiplet \( \Upsilon^+(z, v) \), its smile-conjugate \( \mathbf{\tilde{\Upsilon}}^+(z, v) \) and \( \Sigma_i^+(z) := \Sigma_i^j(z) v^i v^j \) are holomorphic homogeneous functions, with \( \Sigma_i^j \) defined in (2.21). The hypermultiplet \( \Upsilon^+ \) and its smile-conjugate \( \mathbf{\tilde{\Upsilon}}^+ \) are special covariant projective supermultiplets\(^{29}\) annihilated by half of the supercharges. Specifically, \( \Upsilon^+ \) obeys the constraints

\[
\mathbf{D}_a^+ \Upsilon^+ = \mathbf{D}_a^{\dot{\alpha}} \Upsilon^+ = 0 , \quad \mathbf{D}_i^+ := v^i \mathbf{D}_i , \quad \mathbf{D}_a^{\dot{\alpha}} := v^i \mathbf{D}_i^{\dot{\alpha}} , \tag{5.30}
\]

and similar constraints hold for \( \mathbf{\tilde{\Upsilon}}^+ \). In terms of the complex variable \( \zeta \) defined using \( v^i = (v^1, v^2) = v^1(1, \zeta) \), the explicit functional forms of \( \Upsilon^+(v) \) and \( \mathbf{\tilde{\Upsilon}}^+(v) \) are

\[
\Upsilon^+(v) = v^1 \sum_{n=0}^{\infty} \Upsilon_n \zeta^n , \quad \mathbf{\tilde{\Upsilon}}^+(v) = v^2 \sum_{n=0}^{\infty} (-1)^n \mathbf{\tilde{\Upsilon}}_n \frac{1}{\zeta^n} . \tag{5.31}
\]

The hypermultiplet action in (5.27) is invariant under arbitrary re-scalings of \( v^i \), and therefore these variables are homogeneous coordinates for \( \mathbb{C}P^1 \). The arctic hypermultiplet is assumed to be charged under the U(1) gauge group associated with the vector multiplet compensator, and we denote by \( e \) the charge of \( \Upsilon^+ \).

It can be shown that the hypermultiplet equations of motion are equivalent to

\[
\Upsilon^+(v) = \Upsilon^i v_i , \tag{5.32}
\]

where \( \Upsilon^i \) is an ordinary isospinor superfield obeying the covariant constraints

\[
\mathbf{D}_a^{(i} \Upsilon^{j)} = \mathbf{D}_a^{(i} \mathbf{\bar{\Upsilon}}^{j)} = 0 . \tag{5.33}
\]

\(^{29}\)See [9] for the general properties of the covariant projective supermultiplets and their supergravity gauge transformation laws.
These constraints prove to imply that the multiplet $\Upsilon^i$ is on-shell, and its mass $m$ is equal to $|e|$. In the rigid supersymmetric case, the constraints were introduced by Sohnius [30]. It can also be shown that the equation of motion for the SYM multiplet implies that

$$\frac{1}{g_{\text{YM}}^2} \text{Tr} (\mathcal{W} \Sigma^{ij}) = i \bar{\Upsilon}^{(i} \mathcal{W} \Upsilon^{j)} .$$  \hspace{1cm} (5.34)

Now, if we define the supercurrent as in [6],

$$J = \frac{1}{g_{\text{YM}}^2} \text{Tr} (\mathcal{W} \bar{\mathcal{W}}) - \frac{1}{2} \bar{\Upsilon}_k \Upsilon^k ,$$  \hspace{1cm} (5.35)

a simple calculation shows that $J$ indeed satisfies the conservation equation (5.3) with $\gamma = 0$ and

$$\tau_{ij} = -ie \bar{\Upsilon}_{(i} \Upsilon_{j)} .$$  \hspace{1cm} (5.36)

We have $\gamma = 0$ since the SYM action is independent of the tensor compensator.

6 Conclusion

For the minimal off-shell $N = 2$ supergravity with vector and tensor compensators [3], we have constructed the linearized action in the AdS background, eq. (3.6). A main advantage of our construction is that it has revealed the gauge transformations of the supergravity prepotentials, eq. (5.1), and uncovered the supercurrent multiplet (5.5) corresponding to those matter theories which couple to the supergravity chosen. The $N = 2$ supergravity formulation with vector and tensor compensators allows one to realize a huge class of matter couplings within the projective-superspace approach developed in [8, 9, 10].

The action (3.6), although constructed in AdS, could serve as a springboard to reconstructing the linearized supergravity action in an arbitrary on-shell background. The only contributions missing are those arising from $W_{\alpha\beta}$, the $N = 2$ analogue of the $N = 1$ superfield $W_{\alpha\beta\gamma}$, which contains the conformal Weyl tensor and which vanishes in an AdS background but not in a general on-shell background. These are, in principle, straightforward to restore by including terms involving $W_{\alpha\beta}$ in the linearized action and determining their coefficients by requiring supergravity gauge invariance. The result is just a first step toward quantizing pure $N = 2$ supergravity and then performing one-loop calculations (similar to what was done in $N = 1$...
supergravity by Grisaru and Siegel [68]. Since $\mathcal{N} = 2$ superfield supergravity is a reducible gauge theory, its covariant quantization is nontrivial.

Another interesting open problem would be to construct massive extensions of the linearized $\mathcal{N} = 2$ supergravity action in AdS. Even in the super-Poincaré case, such a problem has not been addressed.

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### A $\mathcal{N} = 2$ superconformal geometry

We give a summary of the superspace geometry for $\mathcal{N} = 2$ conformal supergravity which was originally introduced in [27], as a generalization of [29], and later elaborated in [9]. A curved four-dimensional $\mathcal{N} = 2$ superspace $\mathcal{M}^{4|8}$ is parametrized by local coordinates $z^M = (x^m, \theta^i, \bar{\theta}_\dot{i})$, where $m = 0, 1, \ldots, 3$, $\mu = 1, 2$, $\dot{\mu} = 1, 2$ and $i = \frac{1}{2}, \frac{3}{2}$. The Grassmann variables $\theta^i$ and $\bar{\theta}_{\dot{i}}$ are related to each other by complex conjugation: $\bar{\theta}_{\dot{i}} = \bar{\theta}^i$. The structure group is $\text{SL}(2, \mathbb{C}) \times \text{SU}(2)_R \times \text{U}(1)_R$, with $M_{ab} = -M_{ba}$, $J_{ij} = J_{ji}$ and $\mathbb{J}$ be the corresponding Lorentz, $\text{SU}(2)_R$ and $\text{U}(1)_R$ generators. The covariant derivatives $D_A = (D_a, D_\alpha^i, \bar{D}_{\dot{\alpha}}^i) \equiv (D_a, D_{\alpha}, \bar{D}_{\dot{\alpha}})$ have the form

\[
D_A = E_A + \frac{1}{2} \Omega^A_{\beta\gamma} M_{\beta\gamma} + \Phi_A^{kl} J_{kl} + i \Phi_A \mathbb{J} \equiv E_A + \Omega_A^{\beta\gamma} M_{\beta\gamma} + \Phi_A^{kl} J_{kl} + i \Phi_A \mathbb{J} . \tag{A.1}
\]

Here $E_A = E_A^M \partial_M$ is the supervielbein, with $\partial_M = \partial / \partial z^M$, $\Omega^A_{\beta\gamma}$ is the Lorentz connection, $\Phi_A^{kl}$ and $\Phi_A$ are the $\text{SU}(2)_R$ and $\text{U}(1)_R$ connections, respectively.

The Lorentz generators with vector indices $(M_{ab})$ and spinor indices $(M_{\alpha\beta} = M_{\beta\alpha})$ and $\bar{M}_{\dot{\alpha}\dot{\beta}} = \bar{M}_{\dot{\beta}\dot{\alpha}}$ are related to each other by the standard rule:

\[
M_{ab} = (\sigma_{ab})^{\alpha\beta} M_{\alpha\beta} - (\bar{\sigma}_{ab})^{\dot{\alpha}\dot{\beta}} \bar{M}_{\dot{\alpha}\dot{\beta}} , \quad M_{\alpha\beta} = \frac{1}{2} (\sigma_{ab})_{\alpha\beta} M_{ab} , \quad \bar{M}_{\dot{\alpha}\dot{\beta}} = -\frac{1}{2} (\bar{\sigma}_{ab})_{\dot{\alpha}\dot{\beta}} M_{ab} . \tag{A.2}
\]

The generators of the structure group act on the spinor covariant derivatives as follows:

\[
[M_{\alpha\beta}, D^j_\gamma] = \varepsilon_{\gamma(\alpha} D^j_{\beta)\alpha} , \quad [\bar{M}_{\dot{\alpha}\dot{\beta}}, \bar{D}^i_{\dot{\gamma}}] = \varepsilon_{\dot{\gamma}(\dot{\alpha}} \bar{D}^i_{\dot{\beta)\dot{\alpha}}},
\]

\[
[J_{kl}, D^j_\alpha] = -\delta^j_k D^j_{\alpha l} , \quad [J_{kl}, \bar{D}^i_{\dot{\alpha}}] = -\varepsilon_{i(k} \bar{D}^i_{\dot{\alpha}l)} ,
\]

\[
[\mathbb{J}, D^j_\alpha] = D^j_\alpha , \quad [\mathbb{J}, \bar{D}^i_{\dot{\alpha}}] = -\bar{D}^i_{\dot{\alpha}} , \tag{A.2}
\]

\[\text{The (anti)symmetrization of } n \text{ indices is defined to include a factor of } (n!)^{-1}.\]
Our notation and conventions correspond to [62].

The spinor covariant derivatives obey the algebra

\[
\{ \mathcal{D}^i_\alpha, \mathcal{D}_j^j \} = 4 S^{ij} M_{\alpha \beta} + 2 \varepsilon^{ij \rho} \varepsilon_{\alpha \beta} Y^{\gamma \delta} M_{\gamma \delta} + 2 \varepsilon^{ij \rho} \varepsilon_{\alpha \beta} W^{\gamma \delta} \bar{M}_{\gamma \delta} \\
+ 2 \varepsilon_{\alpha \beta} \varepsilon^{ij \rho} S^{\rho kl} J_{kl} + 4 Y_{\alpha \beta} J^{ij} \text{,} \quad (A.3a)
\]

\[
\{ \mathcal{D}^i_\alpha, \mathcal{D}_j^\beta \} = -2 i \delta^i_j (\sigma^c)^{\beta \alpha} \mathcal{D}_c + 4 \left( \delta^i_j G^{\alpha \beta} + i G^{\alpha \beta} \right) M_{\alpha \beta} + 4 \left( \delta^i_j G_{\alpha \gamma} + i G_{\alpha \gamma} \right) \bar{M}_{\gamma \beta} \\
+ 8 G_{\alpha \gamma} J^j \, J_{ij} - 4 i \delta^i_j G_{\alpha \beta} \, J_{kl} - 2 \left( \delta^i_j G_{\alpha \beta} + i G_{\alpha \beta} \right) \mathbb{J} \text{,} \quad (A.3b)
\]

Commutators involving the vector derivative may be worked out using the Bianchi identity and are summarized in [9]. The conventions for the Lorentz generators \( M_{\alpha \beta} \) and \( \bar{M}_{\alpha \beta} \) as well as the \( SU(2)_R \) and \( U(1)_R \) generators \( J_{ij} \) and \( \mathbb{J} \) are also given in [9].

Here the dimension-1 components of the torsion obey the symmetry properties

\[
S^{ij} = S^{ji} \, , \quad Y_{\alpha \beta} = Y_{\beta \alpha} \, , \quad W_{\alpha \beta} = W_{\beta \alpha} \, , \quad G_{\alpha \beta}^{ij} = G_{\alpha \beta}^{ji} \text{.} \quad (A.4)
\]

and the reality conditions

\[
\bar{S}^{ij} = \bar{S}^{ji} \, , \quad \bar{W}_{\alpha \beta} = \bar{W}_{\beta \alpha} \, , \quad \bar{Y}_{\alpha \beta} = \bar{Y}_{\beta \alpha} \, , \quad \bar{G}_{\alpha \beta} = \bar{G}_{\alpha \beta} \, , \quad \bar{G}_{\alpha \beta}^{ij} = G_{\alpha \beta}^{ji} \text{.} \quad (A.5)
\]

The dimension-3/2 Bianchi identities are:

\[
\mathcal{D}^{(i} S^{jk)} = 0 \, , \quad \mathcal{D}^{(i} S^{jkl)} = i \mathcal{D}^{\beta (i} G_{\beta \alpha}^{jkl)} \text{,} \quad (A.6a)
\]

\[
\mathcal{D}^{i}_\alpha \bar{W}_{\beta \gamma} = 0 \, , \quad (A.6b)
\]

\[
\mathcal{D}^{i}_\alpha Y_{\beta \gamma} = 0 \, , \quad \mathcal{D}^{i}_\alpha S_{ij} + \mathcal{D}^{\beta} Y_{\beta \alpha} = 0 \, , \quad (A.6c)
\]

\[
\mathcal{D}^{(i} G_{\beta \alpha)}^{jkl)} = 0 \, , \quad (A.6d)
\]

\[
\mathcal{D}^{i}_\alpha G_{\beta \alpha} = - \frac{1}{4} \mathcal{D}^{i}_\beta Y_{\alpha \beta} + \frac{1}{12} \varepsilon_{\alpha \beta} \mathcal{D}_{j} S^{ij} - \frac{1}{4} \varepsilon_{\alpha \beta} \bar{\mathcal{D}}^{i} \bar{W}_{\beta \gamma} - \frac{i}{3} \varepsilon_{\alpha \beta} \bar{\mathcal{D}}^{i} \bar{G}_{\gamma \beta}^{ij} \text{.} \quad (A.6e)
\]

This structure is invariant under super-Weyl transformations involving a real unconstrained parameter \( \mathcal{U} \). The spinor covariant derivatives transform as

\[
\mathcal{D}^{i}_\alpha = e^{U/2} \left( \mathcal{D}^{i}_\alpha + 2 \mathcal{D}^{\beta i} \mathcal{U} M_{\beta \alpha} - \frac{1}{2} \mathcal{D}^{i} \mathcal{U} \mathcal{J} + 2 \mathcal{D}^{i} \mathcal{J} \right) \text{,} \quad (A.7a)
\]

\[
\mathcal{D}^{\alpha i} = e^{U/2} \left( \mathcal{D}^{\alpha i} - 2 \mathcal{D}^{\beta \alpha} \mathcal{U} \bar{M}^{\beta \alpha} + \frac{1}{2} \mathcal{D}^{\alpha} \mathcal{U} \mathcal{J} - 2 \mathcal{D}^{\alpha} \mathcal{J} \right) \text{.} \quad (A.7b)
\]
The corresponding torsion superfields are given by

\begin{align}
S'_{ij} &= e^U S^{ij} + \frac{1}{4} e^{3U} D^{ij} e^{-2U} \\
Y'_{\alpha\beta} &= e^U Y_{\alpha\beta} - \frac{1}{4} e^{-U} D_{\alpha\beta} e^{2U} \\
G'_{\alpha\dot{\alpha}} &= e^U G_{\alpha\dot{\alpha}} - \frac{1}{16} e^{-U} [\bar{\mathcal{D}}_\alpha, \mathcal{D}_{\dot{\alpha}}] e^{2U} \\
G'_{\alpha\dot{\alpha}, ij} &= e^U G_{\alpha\dot{\alpha}, ij} + \frac{i}{4} e^U \{\bar{\mathcal{D}}_\alpha (i), \mathcal{D}_{\dot{\alpha}} (j)\} U \\
W'_{\alpha\beta} &= e^U W_{\alpha\beta}.
\end{align}

A conformally primary superfield $\Psi$ of weight $\Delta$ is defined to transform as

$$
\Psi' = e^{U\Delta} \Psi.
$$

Actions in $\mathcal{N} = 2$ supergravity may be constructed from integrals over the full superspace

$$
\int d^4x d^4\theta d^4\bar{\theta} E \mathcal{L}
$$

or integrals over a chiral subspace

$$
\int d^4x d^4\theta \mathcal{E} \mathcal{L}_c , \quad \bar{\mathcal{D}}^\dot{\alpha}_i \mathcal{L}_c = 0
$$

with $\mathcal{E}$ the chiral density. Just as in $\mathcal{N} = 1$ superspace, actions of the former type may be rewritten as the latter using a covariant chiral projection operator $\bar{\Delta}$ [28],

$$
\int d^4x d^4\theta d^4\bar{\theta} E \mathcal{L} = \int d^4x d^4\theta \mathcal{E} \bar{\Delta} \mathcal{L}_c.
$$

The covariant chiral projection operator is defined as

$$
\bar{\Delta} = \frac{1}{96} \left( (\bar{\mathcal{D}}^{ij} + 16\bar{S}^{ij}) \bar{\mathcal{D}}_{ij} - (\bar{\mathcal{D}}^{\dot{\alpha}\dot{\beta}} - 16\bar{Y}^{\dot{\alpha}\dot{\beta}}) \bar{\mathcal{D}}_{\dot{\alpha}\dot{\beta}} \right)
= \frac{1}{96} \left( \bar{\mathcal{D}}_{ij} (\bar{\mathcal{D}}^{ij} + 16\bar{S}^{ij}) - \bar{\mathcal{D}}_{\dot{\alpha}\dot{\beta}} (\bar{\mathcal{D}}^{\dot{\alpha}\dot{\beta}} - 16\bar{Y}^{\dot{\alpha}\dot{\beta}}) \right).
$$

Its fundamental property is that $\bar{\Delta} U$ is covariantly chiral, for any scalar, isoscalar and $U(1)_R$-neutral superfield $U(z)$,

$$
\bar{\mathcal{D}}^\dot{\alpha}_i \bar{\Delta} U = 0.
$$

A detailed derivation of the relation \((A.12)\) can be found in \([11]\).
B \ N = 1 superconformal geometry

We give here a summary of the superspace geometry of \( \mathcal{N} = 1 \) conformal supergravity with the structure group \( \text{SL}(2, \mathbb{C}) \times \text{U}(1)_{\mathbb{R}} \). This formulation appeared originally in [27] and was elaborated upon in [23]. Our conventions for generators essentially match those appearing in Appendix A.

The covariant derivatives have the form
\[
\nabla_A = (\nabla_\alpha, \nabla_\dot{\alpha}, \bar{\nabla}_\dot{\alpha}) = E_A + \frac{1}{2} \Omega_A^{bc} M_{bc} + i \Phi_A \hat{J} \tag{B.1}
\]
and obey the algebra
\[
\begin{align*}
\{\nabla_\alpha, \nabla_\beta\} &= -4 \bar{R} M_{\alpha\beta} , \quad (B.2a) \\
\{\nabla_\alpha, \bar{\nabla}_\dot{\alpha}\} &= -2i \nabla_\dot{\alpha} , \quad (B.2b) \\
[\nabla_\beta, \nabla_\alpha]\ &= -i \epsilon_{\beta\gamma} \nabla^\gamma + i \epsilon_{\beta\alpha} \bar{R} \nabla_\dot{\alpha} - i \epsilon_{\gamma\alpha} \nabla_\gamma G_{\gamma\dot{\alpha}} M^{\gamma\dot{\beta}} + i \nabla_\dot{\alpha} \bar{R} M_{\beta\alpha} \\
&+ 2i \epsilon_{\beta\alpha} \bar{W}_{\dot{\alpha}\beta\dot{\gamma}} \bar{M}^{\dot{\beta}\gamma} - i 3 \epsilon_{\beta\alpha} \bar{X}_{\dot{\alpha}} \bar{M}_{\beta\dot{\gamma}} - 2i \epsilon_{\beta\alpha} \bar{X}_{\dot{\gamma}} \hat{J} . \tag{B.2c}
\end{align*}
\]

The remaining vector commutator can be calculated using the Bianchi identity. The \( \text{U}(1)_{\mathbb{R}} \) generator \( \hat{J} \) is defined by
\[
[\hat{J}, \nabla_\alpha] = \nabla_\alpha , \quad [\hat{J}, \bar{\nabla}_\dot{\alpha}] = -\bar{\nabla}_\dot{\alpha} . \tag{B.3}
\]

The conventional geometry [23, 62] with structure group \( \text{SL}(2, \mathbb{C}) \) can be recovered by degauging the \( \text{U}(1)_{\mathbb{R}} \) and performing a super-Weyl transformation to set \( X_\alpha = 0 \).

The components of the torsion are constrained by the Bianchi identities
\[
\begin{align*}
\bar{\nabla}_\dot{\alpha} R &= \bar{\nabla}_\dot{\alpha} W_{\alpha\beta\gamma} = 0 , \quad (B.4a) \\
X_\alpha &= \nabla_\alpha R - \nabla^\dot{\alpha} G_{\alpha\dot{\alpha}} , \quad \nabla^\dot{\alpha} X_\alpha = \nabla_\dot{\alpha} X^\dot{\alpha} , \quad (B.4b) \\
\nabla^\gamma W_{\gamma\beta\alpha} &= -\frac{1}{3} \nabla_{(\beta} X_{\alpha)} + i \nabla_{(\beta} G_{\alpha)\dot{\gamma}} . \quad (B.4c)
\end{align*}
\]

This structure is invariant under super-Weyl transformations involving a real unconstrained parameter \( U \). The \( \mathcal{N} = 1 \) covariant derivatives transform as
\[
\begin{align*}
\nabla'_\alpha &= e^{U/2} \left( \nabla_\alpha + 2 \nabla^\beta U M_{\beta\alpha} - \frac{3}{2} \nabla_\alpha U \hat{J} \right) , \quad (B.5a) \\
\bar{\nabla}'_{\dot{\alpha}} &= e^{U/2} \left( \bar{\nabla}_{\dot{\alpha}} - 2 \nabla_{\beta} U M_{\beta\dot{\alpha}} + \frac{3}{2} \nabla_{\dot{\alpha}} U \hat{J} \right) , \quad (B.5b)
\end{align*}
\]
and the $\mathcal{N} = 1$ torsion superfields transform as

$$
\begin{align*}
R' &= e^U R - \frac{1}{4} e^{3U} \nabla^2 e^{-2U}, \\
G'_{a\bar{a}} &= e^U G_{a\bar{a}} + [\nabla_a, \nabla_{\bar{a}}] e^U, \\
X'_\alpha &= e^{3U/2} X_\alpha - \frac{3}{2} (\nabla^2 - 4R) \nabla_\alpha U, \\
W'_{\alpha\beta\gamma} &= e^{3U/2} W_{\alpha\beta\gamma}.
\end{align*}
$$

(A.6a, b, c, d)

A conformally primary superfield $\Psi$ of dimension $\Delta$ transforms as

$$
\Psi' = e^{U\Delta} \Psi.
$$

(B.7)

Actions in $\mathcal{N} = 1$ supergravity may be constructed by integrals over the full superspace

$$
\int d^4x \, d^2\theta \, d^2\bar{\theta} \, E \, \mathcal{L}
$$

(B.8)
or integrals over the chiral subspace

$$
\int d^4x \, d^2\theta \, d^2\bar{\theta} \, E_c \, \mathcal{L}_c, \quad \bar{\nabla}_a \mathcal{L}_c = 0.
$$

(B.9)

Actions of the former type may be rewritten as the latter via

$$
\int d^4x \, d^2\theta \, d^2\bar{\theta} \, E \, \mathcal{L} = -\frac{1}{4} \int d^4x \, d^2\theta \, d^2\bar{\theta} \, E (\nabla^2 - 4R) \mathcal{L},
$$

(B.10)

and vice versa

$$
\int d^4x \, d^2\theta \, d^2\bar{\theta} \, E_c \, \mathcal{L}_c = \int d^4x \, d^2\theta \, d^2\bar{\theta} \, \frac{E}{R} \, \mathcal{L}_c.
$$

(B.11)

## C Improved tensor compensator

Within the projective superspace formulation given in [8, 9], the $\mathcal{N} = 2$ improved tensor compensator action[31] has the form [10]

$$
S_{\text{tensor}} = \frac{1}{2\pi} \int_C v' dv_i \int d^4x \, d^4\theta \, d^4\bar{\theta} \, E \frac{\mathcal{W}\mathcal{W}}{(\Sigma^{++})^2} G^{++} \ln \frac{G^{++}}{i\Upsilon^+\Upsilon^+}.
$$

(C.1)

Here $\mathcal{W}$ is the vector compensator and $\Upsilon^+$ is an auxiliary arctic multiplet. Although the action appears to depend on the choice of both $\mathcal{W}$ and $\Upsilon^+$, one can show [9, 10, 11] that it is independent of these two fields.

---

[31] The $\mathcal{N} = 1$ improved tensor action was constructed in [69].

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Using the techniques reviewed in [7], this action can be rewritten in the form

\[ S_{\text{tensor}} = \int d^4x \, d^4\theta \, \mathcal{E} \Psi \mathbb{W} + \text{c.c.} , \]  

(C.2)

where

\[ \mathbb{W} := \frac{1}{8\pi} \oint_C v^i dv_i \left[ (\mathcal{D}^-)^2 + 4\mathcal{S}^- \right] \ln \frac{\mathcal{G}^{++}}{i\Upsilon^{++}} \]  

(C.3)

is a reduced chiral superfield by construction. This property is necessary so that the action is invariant under gauge transformations of the prepotential \( \Psi \) (2.7). The contour integral may be evaluated [7] to give

\[ \mathbb{W} = -\frac{G}{8} (\mathcal{D}_{ij} + 4\mathcal{S}_{ij}) \left( \frac{\mathcal{G}^{ij}}{\mathcal{G}} \right) . \]  

(C.4)

In this form the reduced chirality property is far from obvious and, in fact, is a tedious calculation to demonstrate. The superfield \( \mathbb{W} \) was first constructed at the component level in [3].

In addition the improved tensor action, we have also a coupling between the tensor multiplet and the vector multiplet which generates the cosmological constant. In projective superspace, this coupling is written

\[ S_{\text{cosm}} = -\frac{\xi}{2\pi} \oint_C v^i dv_i \int d^4x \, d^4\theta d^4\bar{\theta} E \frac{\mathbb{W} \mathbb{W}}{(\Sigma^{++})^2} G^{++} \mathcal{V} , \]  

(C.5)

where \( \mathcal{V} \) is the real weight-zero tropical prepotential for \( \mathbb{W} \) [12]

\[ \mathbb{W} = \frac{1}{8\pi} \oint_C v^i dv_i \left[ (\mathcal{D}^-)^2 + 4\mathcal{S}^- \right] \mathcal{V} . \]  

(C.6)

It may be reformulated as a chiral superspace integral

\[ S_{\text{cosm}} = -\xi \int d^4x \, d^4\theta \, \mathcal{E} \Psi \mathbb{W} + \text{c.c.} \]  

(C.7)

or as a full superspace integral

\[ S_{\text{cosm}} = -\xi \int d^4x \, d^4\theta d^4\bar{\theta} E \mathcal{G}^{ij} \mathcal{V}_{ij} , \]  

(C.8)

where \( \mathcal{V}^{ij} \) is the unconstrained Mezincescu prepotential (2.3). The differing forms (C.6) and (2.3) are related and one can derive the latter from the former. The procedure is described in Appendix E of [7].
D Details for derivation of $\mathcal{N} = 2$ action

We review in this appendix some details involving the derivation of the linearized $\mathcal{N} = 2$ action (3.6). It is useful to isolate it into four sets of terms:

\begin{align}
S^{(2)} &= S^{(2)}_W + S^{(2)}_G + S^{(2)}_{\text{cosm}} + S^{(2)}_{HH}, \\
S^{(2)}_W &= -\frac{1}{2} \int d^4x d^4\theta \mathcal{E} \mathbf{WW} - \int d^4x d^4\theta d^4\bar{\theta} E (\bar{w} \mathbf{WH} + w \mathbf{WH}), \\
S^{(2)}_G &= \int d^4x d^4\theta \mathbf{\bar{\Psi}} \mathbf{W} + \text{c.c.} + \frac{1}{2g} \int d^4x d^4\theta d^4\bar{\theta} E g_{ij} G^{ij}\mathbf{H}, \\
S^{(2)}_{\text{cosm}} &= -\xi \int d^4x d^4\theta \mathcal{E} \mathbf{WW} + \text{c.c.}.
\end{align}

$S^{(2)}_W$ and $S^{(2)}_G$ are those terms involving either the vector or tensor compensators and $S^{(2)}_{\text{cosm}}$ consists of the terms involving both the vector and tensor compensators which arises from the cosmological term. These are all easy to determine from generalizing the flat space result and expanding to second order the Noether coupling of $\mathbf{H}$, as discussed in [2]. The terms $S^{(2)}_{HH}$ are all terms second order in $\mathbf{H}$. These can be determined by requiring that the entire action is gauge invariant. We work with constant background values $w$ and $g^{ij}$ for the compensators for simplicity, but the result will hold for covariantly constant $\mathcal{W}$ and $G^{ij}$.

The gauge variation of $S^{(2)}_W$ is

\begin{align}
\delta S^{(2)}_W = \int d^4x d^4\theta d^4\bar{\theta} E \left\{-4 \bar{w} \mathbf{W} \bar{\Omega}^{ij} \bar{\Omega}_{ij} + \bar{w}^2 \mathbf{H} \bar{\Delta} (\bar{D}^{ij} + 4S^{ij}) \Omega_{ij} \right. \\
&\quad + \bar{w} w \mathbf{H} \bar{\Delta} (D^{ij} + 4S^{ij}) \Omega_{ij} + \text{c.c.} \right\}
\end{align}

and the variation of the cosmological term is

\begin{align}
\delta S^{(2)}_{\text{cosm}} = \xi \int d^4x d^4\theta d^4\bar{\theta} E \left\{-4 \bar{\Omega}^{ij} g_{ij} \mathbf{W} + \bar{\Psi} (D^{ij} + 4S^{ij}) (\bar{w} \Omega_{ij} + w \bar{\Omega}_{ij}) + \text{c.c.} \right\}.
\end{align}

Adding these two together and using the AdS relation $S_{ij} = -\xi g_{ij}/w$ gives

\begin{align}
\delta S^{(2)}_W + \delta S^{(2)}_{\text{cosm}} = \int d^4x d^4\theta d^4\bar{\theta} E \left\{4 \xi G^{ij} (\bar{w} \Omega_{ij} + w \bar{\Omega}_{ij}) \\
&\quad + \bar{w}^2 \mathbf{H} \bar{\Delta} (D^{ij} + 4S^{ij}) \Omega_{ij} + \bar{w} w \mathbf{H} \bar{\Delta} (D^{ij} + 4S^{ij}) \bar{\Omega}_{ij} \right\}.
\end{align}
This depends only on the tensor compensator $G^{ij}$ and the supergravity prepotential $H$, and so we should be able to cancel it by terms involving only these prepotentials.

Next we calculate the gauge variation of $S_{G}^{(2)}$. The analysis of this for the kinetic term is simplest in projective superspace, where we observe that

$$\int d^4x d^4\theta \mathcal{E} \Psi \bar{W} + c.c. = \frac{1}{2\pi} \oint v' dv, \int d^4x d^4\theta d^4\bar{\theta} E \frac{W \bar{W} G^{++} G^{++}}{(\Sigma^{++})^2}.$$ (D.5)

The contour integral variation is given by

$$\frac{1}{2\pi} \oint v' dv, \int d^4x d^4\theta d^4\bar{\theta} E \frac{W \bar{W}}{(\Sigma^{++})^2} \delta G^{++} \frac{G^{++}}{g^{++}},$$ (D.6)

which can be rewritten

$$\int d^4x d^4\theta \mathcal{E} \delta \Psi \bar{W} + c.c.$$ (D.7)

using eq (5.10) of [7]. This result follows also from its original chiral form. Using the identity

$$D_{ij}G^{kl} = \frac{1}{3} \delta_{ij}^{kl} D_{mn} G^{mn} + 4S^{kl} G_{ij} - 4 S_{ij} G^{kl}$$ (D.8)

one may show that $\delta S_{G}^{(2)}$ can be written

$$\delta S_{G}^{(2)} = \int d^4x d^4\theta d^4\bar{\theta} E \left\{ - 4 \xi \bar{w} \Omega_{ij} G^{ij} + \frac{1}{2g} H g^{ij} g_{kl} (D_{ij} + 4S_{ij}) \bar{\Delta} \Omega^{kl} + c.c. \right\}.$$ (D.9)

Adding this to the terms we had before gives

$$\delta S_{G}^{(2)} + \delta S_{W}^{(2)} + \delta S_{\cosm}^{(2)}$$

$$= \int d^4x d^4\theta d^4\bar{\theta} E \left\{ \bar{w}^2 H \bar{\Delta} (D^{ij} + 4S^{ij}) \Omega_{ij} + \bar{w} w H \bar{\Delta} (D^{ij} + 4S^{ij}) \Omega_{ij} + \frac{1}{2g} H g^{ij} g_{kl} (D_{ij} + 4S_{ij}) \bar{\Delta} \Omega^{kl} + c.c. \right\}.$$ (D.10)

This result, as required, depends only on $H$. $S_{HH}^{(2)}$ must be constructed so that its variation cancels this term.
We construct a solution for \( S^{(2)}_{HH} \) by generalizing the Minkowski result given in [2]:

\[
S_{HH} = S_{HH,1} + S_{HH,2} + S_{HH,3} + S_{HH,4}, \tag{D.11a}
\]

\[
S_{HH,1} = -\frac{1}{2} \bar{w}^2 \int d^4x \, d^4\theta \, d^4\bar{\theta} \; E \; \bar{H} \Delta H + c.c., \tag{D.11b}
\]

\[
S_{HH,2} = -\frac{1}{64g} \int d^4x \, d^4\theta \, d^4\bar{\theta} \; E \; g_{ij}g_{kl} \bar{H} D^{ij} \bar{D}^{kl} H, \tag{D.11c}
\]

\[
S_{HH,3} = -\frac{g}{32} \int d^4x \, d^4\theta \, d^4\bar{\theta} \; \left( \bar{H} D^{ij} \bar{D}^j_i H + \frac{g}{2} \bar{H} \Box H \right), \tag{D.11d}
\]

\[
S_{HH,4} = -\frac{g}{8} \int d^4x \, d^4\theta \, d^4\bar{\theta} \; E \; H \bar{S}^{ij} \bar{D}^j_i H + c.c. \tag{D.11e}
\]

The term \( S_{HH,4} \) is new, having no analogue in a Minkowski background.

It is a straightforward exercise to calculate the variations of the first two terms:

\[
\delta S_{HH,1} = \int d^4x \, d^4\theta \, d^4\bar{\theta} \; \left\{ \bar{w}^2 \; \bar{H} \Delta (D^{ij} + 4S^{ij}) \Omega_{ij} - 4g \bar{S}^{ij} \bar{H} \Delta \Omega_{ij} + c.c. \right\}, \tag{D.12}
\]

\[
\delta S_{HH,2} = \int d^4x \, d^4\theta \, d^4\bar{\theta} \; \left\{ -\frac{1}{2g} \bar{H} g^{ij}g^{kl} (\bar{D}_{ij} + 4 \bar{S}_{ij}) \Delta \Omega_{ij} \\
+ \frac{i}{16g} \bar{H} D_{\alpha \alpha} [D^{\alpha j}, \bar{D}^{\alpha j}] (D^{mn} + 4S^{mn}) \Omega_{mn} + 4g \bar{H} \bar{S}^{ij} \Delta \Omega_{ij} \\
- \frac{g}{8} \bar{H} \bar{S}^{ij} \bar{D}^j_i (D^{mn} + 4S^{mn}) \Omega_{mn} + \frac{g}{8} \bar{H} \bar{S}^{ij} \bar{D}^j_i (D^{mn} + 4S^{mn}) \Omega_{mn} + c.c. \right\}. \tag{D.13}
\]

It helps at this point to combine these with the variation of the compensator terms since numerous cancellations result:

\[
\delta S^{(2)}_G + \delta S^{(2)}_W + \delta S^{(2)}_{FI} + \delta S_{HH,1} + \delta S_{HH,2} = \\
\int d^4x \, d^4\theta \, d^4\bar{\theta} \; \left\{ \frac{i}{16g} \bar{H} D_{\alpha \alpha} [D^{\alpha j}, \bar{D}^{\alpha j}] (D^{mn} + 4S^{mn}) \Omega_{mn} \\
- \frac{g}{8} \bar{H} \bar{S}^{ij} \bar{D}^j_i (D^{mn} + 4S^{mn}) \Omega_{mn} + \frac{g}{8} \bar{H} \bar{S}^{ij} \bar{D}^j_i (D^{mn} + 4S^{mn}) \Omega_{mn} \\
+ g \bar{H} \Delta (\bar{D}^{ij} + \bar{S}^{ij}) \Omega_{ij} + c.c. \right\}. \tag{D.14}
\]
Finally we can check that

\[ \delta S_{HH,3} = \int d^4x d^4\theta d^4\bar{\theta} \mathcal{E} \left\{ -2gH \Box (D^{mn} + 4S^{mn}) \Omega_{mn} - gH \Delta (\bar{D}^{ij} + \bar{S}^{ij}) \Omega_{ij} \right. \\
- \frac{i}{4} gHD^{\dot{a}a}[D_{a}^{i}, D_{ak}](D^{kj} + 4S^{kj}) \Omega_{ij} - \frac{3i}{16} gH D^{\dot{a}a}[D_{a}^{k}, D_{ak}](D^{ij} + 4S^{ij}) \Omega_{ij} \\
+ \frac{3}{8} gH \bar{S}_{kl}D^{kl}(D^{ij} + 4S^{ij}) \Omega_{ij} + \left. \frac{1}{8} gH S_{kl} \bar{D}^{kl}(D^{ij} + 4S^{ij}) \Omega_{ij} + \text{c.c.} \right\}, \]

(D.15)

\[ \delta S_{HH,4} = \int d^4x d^4\theta d^4\bar{\theta} \mathcal{E} \left\{ - \frac{1}{4} gHS^{ij}D_{ij}(D^{kl} + 4S^{kl}) \Omega_{kl} \\
- \frac{1}{4} gHS^{ij}D_{ij}(D^{kl} + 4S^{kl}) \Omega_{kl} + \text{c.c.} \right\}. \]

(D.16)

The sum of all these terms is

\[ \delta S^{(2)} = \int d^4x d^4\theta d^4\bar{\theta} \mathcal{E} \left\{ -2gH \Box (D^{mn} + 4S^{mn}) \Omega_{mn} \\
- \frac{i}{4} gHD^{\dot{a}a}[D_{a}^{i}, D_{ak}](D^{kj} + 4S^{kj}) \Omega_{ij} \\
- \frac{i}{8} gHD^{\dot{a}a}[D_{a}^{k}, D_{ak}](D^{ij} + 4S^{ij}) \Omega_{ij} + \text{c.c.} \right\}, \]

(D.17)

which can be shown to vanish after some complicated algebra.

### E Details of \( \mathcal{N} = 1 \) reduction

The \( \mathcal{N} = 1 \) reduction in a Minkowski background was considered in [2]. Most of that work is applicable here since we can perform a super-Weyl transform to the Minkowski frame, identify the various \( \mathcal{N} = 1 \) superfields, and then transform back.

We begin by identifying all the \( \mathcal{N} = 1 \) components of the \( \mathcal{N} = 2 \) supergravity multiplet. The AdS frame superfield \( H \) is related to the flat frame \( H_0 \) via \( H = e^{-2D^{\dot{a}a}H_0} \), since \( H \) has super-Weyl weight -2. In the flat frame, the Wess-Zumino gauge conditions read

\[ H_0| = D_{a}^{\dot{a}}H_0| = D_{a}^{\dot{a}2}H_0| = (D^{\dot{a}})^2H_0| = (D_{a}^{\dot{a}})^2H_0| = 0. \]

(E.1)

It is easy to check that these imply similar-looking conditions in AdS:

\[ H| = D_{a}^{\dot{a}}H| = D_{a}^{\dot{a}2}H| = (D^{\dot{a}})^2H| = (D_{a}^{\dot{a}})^2H| = 0. \]

(E.2)
Similarly, the flat frame $\mathcal{N} = 1$ components

\begin{align}
H_{\alpha\dot{\alpha}0} &:= \frac{1}{4} [\bar{D}_\alpha \bar{\partial}\dot{\alpha} \bar{D}_{\dot{\alpha}\dot{\beta}}] H_0|,
\Psi_\alpha &:= \frac{1}{8} (\bar{D}_\dot{\alpha})^2 D_\alpha \bar{H}_0|,
\hat{U}_0 &:= \frac{1}{16} D^\alpha D^\dot{\alpha} (\bar{D}_\dot{\alpha})^2 H_0| + \frac{1}{12} [D_\alpha, \bar{D}_{\dot{\alpha}}] H_{\alpha\dot{\alpha}0} \quad (E.3a)
\end{align}

imply in the AdS frame

\begin{align}
H_{\alpha\dot{\alpha}} &:= e^{-U} H_{\alpha\dot{\alpha}0} = \frac{1}{4} [\partial_{\alpha\dot{\alpha}} \bar{H}|,
\Psi_\alpha &:= e^{-U/2} \Psi_{\alpha0} = \frac{1}{8} (\bar{D}_\dot{\alpha})^2 D_\alpha \bar{H}|,
\hat{U} &:= \hat{U}_0 = \frac{1}{16} D^\alpha D^\dot{\alpha} (\bar{D}_\dot{\alpha})^2 D_\alpha \bar{H}| + \frac{1}{12} [\nabla_\alpha, \bar{\nabla}_{\dot{\alpha}}] H_{\alpha\dot{\alpha}} \quad (E.4c)
\end{align}

We have defined the $\mathcal{N} = 1$ components $H_{\alpha\dot{\alpha}}$, $\Psi_\alpha$, and $\hat{U}$ in AdS by performing an $\mathcal{N} = 1$ super-Weyl transformation with parameter $U$. Since $U = U|$, it is straightforward to verify the right hand side of these equations.

The $\mathcal{N} = 1$ supergravity gauge transformations in the flat geometry read

\begin{align}
\delta H_{\alpha\dot{\alpha}0} &= D_\alpha L_{\dot{\alpha}0} - \bar{D}_{\dot{\alpha}} L_{\alpha0} \quad (E.5a),
\delta \Psi_{\alpha0} &= D_\alpha \Omega_0 + \Lambda_{\alpha0} \quad (E.5b),
\delta \hat{U}_0 &= \rho_0 + \bar{\rho}_0 \quad (E.5c).
\end{align}

If we choose the gauge parameters $L_\alpha$, $\Omega$, $\Lambda_\alpha$, and $\rho$ to transform covariantly under super-Weyl transformations, we recover the AdS frame conditions $\mathcal{N} = 1$.

For our compensator fields, we had in the flat frame

\begin{align}
\chi_0 &:= W_0|, \quad W_{0\alpha} := \frac{i}{2} D^\alpha W_0| \quad (E.6)
\end{align}

for the components of the vector multiplet. Similarly, the components of the $\mathcal{N} = 2$ tensor multiplet $G_{ij0}$ are given by a chiral scalar $\eta_0$ and a tensor multiplet $L_0$,

\begin{align}
\eta_0 &:= G_{110}|, \quad \bar{\eta}_0 := G_{220}|, \quad L_0 = -2i G_{120}| \quad (E.7).
\end{align}

These generalize quite easily to the corresponding equations in the AdS frame.

However, in order to derive their transformations under the $\mathcal{N} = 1$ supergravity gauge transformations, we need to first work out their transformations in the flat geometry. This requires a generalization of the results given in [2] since within the
flat geometry the background values $G_{ij0}$ and $W_0$ are no longer constants. The results are

$$
\delta \chi_0 = -\frac{1}{12} W_0 D^2 D^\alpha L_{\alpha 0} - \frac{1}{4} D^2 (L_0^\alpha D_\alpha W_0) - \rho_0 W_0 + 2i \Lambda_0^\alpha W_{\alpha 0}
$$

(E.8a)

$$
\delta W_{\alpha 0} = \frac{1}{4} \bar{D}^2 D_\alpha \left( L_0^\alpha W_{\alpha 0} + L_{\dot{\alpha} 0} \bar{W}^\dot{\alpha} + i \bar{\Omega}_0 W_0 - i \Omega_0 \bar{W}_0 \right)
$$

(E.8b)

for the components of the $\mathcal{N} = 2$ vector multiplet, where

$$
W_{\alpha 0} := \frac{i}{2} D_\alpha D^2 W_0
$$

(E.9)

is the background value of the $\mathcal{N} = 1$ abelian vector field strength. This vanishes for the choice of $W_0$ that we make in this paper, but we have included it here for full generality. For the tensor multiplet in the flat frame, we find

$$
\delta \eta_0 = -\bar{D}^2 (G_{\dot{1}\dot{2}0} \bar{\Omega}_0) + G_{110} \rho_0 - \frac{1}{6} G_{110} \bar{D}^2 D^\alpha L_{\alpha 0} - \frac{1}{4} \bar{D}^2 (L_0^\alpha D_\alpha G_{110})
$$

(E.10a)

$$
\delta L_{0} = -\frac{i}{2} D^\alpha \bar{D}^2 (L_{\alpha 0} G_{\dot{1}\dot{2}0}) - \frac{i}{2} \bar{D}_\dot{\alpha} D^2 (L_0^{\dot{\alpha}} G_{\dot{1}\dot{2}0}) + i D^\alpha (\Lambda_{\alpha 0} G_{110}) - i \bar{D}_{\dot{\alpha}} (\bar{\Lambda}_0^{\dot{\alpha}} G_{220}).
$$

(E.10b)

For the choice of $G_{ij0}$ we make in this paper, it turns out that $G_{\dot{1}\dot{2}0}$ vanishes.

Transforming these relations to the AdS frame is straightforward. It (essentially) involves turning off $G_{\dot{1}\dot{2}0}$ and $W_{\alpha 0}$, replacing $W_0$ and $G_{110}$ with their constant AdS values $w$ and $g_{11} = \gamma$, and making the obvious covariantizations of derivatives everywhere, replacing, for example, $D^2$ with $\nabla^2 - 4R$. The results are given in (4.65).

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