Abstract

Background/Objective: Dynamic Time Warping (DTW), a similarity measure works in $O(N^2)$ complexity. Cause of this it will be used for small datasets only.

Methods/Statistical Analysis: In this work, we introduced Efficient DTW (EDTW), which works in linear time. It uses two level approaches. In the first level data reduction is performed, and in the second level warping distance and path are calculated.

Findings: While calculating the values of distance matrix, values along the warping path only considered and calculated. For time series of length $n$, maximum $n$ values of distance matrix are calculated. So it works in linear time.

Improvements/Applications: We applied this distance measure to UCR Time Series archive and calculated error rate of 1NN classification. Most of the cases it is matching, some cases it is better, and some other cases error rate is high.

Keywords: Dynamic Time Warping, Efficient DTW, 1NN Classification, Time Series Classification, UCR Time Series Data Sets

1. Introduction

Dynamic time warping (DTW) finds similarity between sequences, it is one of the similarity methods used in pattern recognition and time series data mining and other fields.

Figure 1 shows example of non-warping and warping between time series. DTW is efficient similarity method for time series datasets, but it has quadratic complexity.

Here we developed an Efficient Dynamic Time Warping (EDTW) algorithm, it takes linear time and works in two level. In first level the time series data is reduced by applying data reduction technique, i.e. the series is reduced to one by third by taking the average of three consecutive values. Second level calculates warping distance and path, while calculating, only limited values around warping path of distance matrix considered. The shape of the warping path of reduced data set and original dataset are almost similar as shown in the Figure 5.

The remaining sections are organized as follows. Section 2 outlines the details of standard DTW, relevant prior approaches to accelerate it. An in depth description of EDTW algorithm covers in Section 3. Section 4 is experimental evaluation discusses experimentation on synthetic temperature dataset and explains the experimentation results on the UCR Time Series dataset. Section 5 summarizes the EDTW.

2. Related Work

The DTW distance measure is used to find similar instances in a time series database and to overcome problems of other similarity methods. Finding similar series is base for classification and clustering and other data mining techniques.

Given two time series $X$ and $Y$, of lengths $n$ and $m$.

$$X = (x_1, x_2, x_3, ..., x_n)$$
Y = (y₁, y₂, y₃, ..., yₘ)

Construct a warping path P, P= p₁, p₂... pₖ
Where k greater than or equal to length of the larger time series and less than or equal to sum of time series.

The optimal warping path P is calculated as follows

\[
\text{Dist}(P) = \sum_{k=1}^{k} \text{Dist}(p_k, p_k)
\]

DTW distance follows boundary condition, monotonicity condition, step size condition.

Definition: A warping path is a sequence P= (p₁, p₂, pₖ) with three properties

(i) Boundary condition: p₁= (1, 1), pₖ= (N, M).
(ii) Monotonicity condition: n₁ ≤ n₂ ≤ ... ≤ nₖ and m₁ ≤ m₂ ≤ ... ≤ mₖ.
(iii) Step size condition: pₖ+ 1 – pₖ ∈ \{(0, 1), (1, 0), (1, 1)\} for L ∈ (1: L-1).

To calculate minimum-distance warping path, dynamic programming approach is used. It follows principle of optimality. Figure 2 shows dynamic time warping path, for given two series. X=(7,8,7,8,9,8,11,14,16,17,15,13), Y=(5,8,9,10,12, 12,13,17,16,16,15,14), it start with D(1,1) and ends with D(12,12).

The wrap path is w={(1,1),(2,3),(2,4),(3,5),(4,5),(4,6),(5,7),(6,7),(7,8),(8,9),(8,10),(9,11),(10,11),(11,11),(12,12)}. Initially Y₁ is warped with X₁₁, and Y₂ is warped with X₅, warped with Y₃, and so on. At the end X₁₁ is warped with Y₉, 10, 11.

Time and Space complexity of the DTW is O(NM). Because we need to fill N*M distance matrix, each value calculation of matrix takes constant time, if size of X and Y are N and M respectively. There are so many attempts in the literature to minimize the space and time complexity, some attempts succeeded but it will be applicable to some applications, may not be suitable to applications requiring full path, and merging of time series together.

2.1 Speedup Dynamic Time Warping
To speedup DTW three methods are available.

1) Constraints – It reduce the quantity of values in the cost matrix while calculating distance and warping path.
2) Data reduction – Before applying DTW try to reduce the data set size.
3) Indexing – It will decrease the DTW calculation in Data Mining functionalities.

The constraints S-C Band and the Itakura Parallelogram, which are shown in figure 3.

Applying constraints while calculating DTW will accelerate process by constant factor, still it requires quadratic time. Data reduction speeds up, even it requires O(N²) time. Figure 4 below show DTW path on reduced dataset.
Indexing can be used in time series data mining to speedup similarity and other techniques\textsuperscript{9,10}.

2.2 Fast DTW

This method uses multilevel approach by three operations Coarsening, Projection, Refinement\textsuperscript{11}.

FastDTW\textsuperscript{11} is a recursive implementation of DTW. The solution to the problem is available if time series size is less than min Size else it uses the above three operations.

The space complexity and time complexity of Fast DTW is O(N) and O(N) respectively.

Fast DTW is an approximation of DTW, the limitations of Fast DTW is it does not always find an optimal solution. The possible extensions for Fast DTW is trying for different step sizes conditions, Investigate search algorithms to help improve refinement, examining number of cells evaluated while calculating warping path and compare accuracy.

3. Efficient DTW

The EDTW follows two stages. It is motivated by data mining\textsuperscript{12} process of finding various patterns through functionalities. Where the dataset is reduced to small size suitable for existing algorithm. Then apply algorithm on reduced dataset to get the patterns.

3.1 Efficient DTW Algorithm

The EDTW works in two stages

1. Reduction – Reduce an instance into a small size by using some data reduction techniques.
2. Constraints–Reduce the number of values in the cost matrix while calculating distance and warping path

Reduction reduce an instance into a small size by replacing every three values by a single value which is the average of three values. This reduce instance size by $1/3^{rd}$ of the original instance.

Constraints limits the count of values evaluated in distance matrix, just above the warping path one line and below the warping path one line of values will be evaluated, remaining values will not be evaluated.

The EDTW algorithm first uses reduction to create the reduced dataset by taking the average of three consecutive values. The EDTW applied on reduced data, while calculating optimal warping path it follows constraints. That is only the values around the path will be calculated. The dark shaded cells is the warping path and light shaded cells are the extra cells required for calculation. This path will be approximately equivalent to original dataset optimal warping path. Figure 5 shows the optimal warping paths of reduced dataset as well as original dataset.

We will calculate maximum three values corresponding to each cell in the entire warping path using the following formula\textsuperscript{13}.

$$minDist(i,j) = \begin{cases} 
(X_i - Y_j)^2 & i = j = 1 \\
minDist(i',j') + \min & \begin{cases} 
(X_{i+1} - Y_{j+1})^2 \\
(X_{i+1} - Y_j)^2 \\
(X_i - Y_{j+1})^2
\end{cases}
\end{cases}$$

Where $(i',j')$ is the previous point on the warping path.

Whenever calculating mindist. We followed Boundary condition, monotonicity condition and step size condition.

The optimal warping paths have similar shape and the number of value of full size matrix is approximately three times the reduced matrix.

EDTW evaluated 10 cells at reduced size 4*4, and 38 cells at the original size (12*12), while DTW evaluates all...
16 cells at reduced size and 144 cells at original size. The original size is 12 and it reduced to 1/3(=4), in reduced size we calculating only 10 values. In the worst case also for time series of size N maximum N values only calculated. This increases efficiency significantly. The EDTW algorithm is shown in table 1.

**Table 1. Efficient DTW algorithm**

| Algorithm Efficient DTW (X,Y) |
|-------------------------------|
| i/p: X and Y are Time Series |
| o/p: P – warping path         |
| minDist- distance of path     |
| 1. M=|X|/3; N=|Y|/3;          |
| 2. i:=1; j:=1;                |
| 3. P[i,j]=[1,1]              |
| 4. minDist:=(X_i-Y_j)^2      |
| 5. While (i!=N and j!=M)      |
| 6. i=i+1;                     |
| 7. If (i==N) then            |
| 8. j=j+1;                    |
| 9. end                        |
| 10. If (j==M) then           |
| 11. i=i+1;                   |
| 12. if (i!=N&&j!=M) then      |
| 13. minDist=minDist+(X_i-Y_j)^2 |
| 14. P[i,j]=[i,j]             |

**Figure 6.** Some of possible warping paths in reduced data.

The number of values needed to calculate are maximum 12, means it is equivalent to original size. Original size N, reduced size N/3, for reduced size EDTW calculates N cell values in worst case. So it’s time complexity is linear. The possible wrapping paths of size 4 by 4 distance matrix is shown in figure 6. In all possible cases we computed 10 to 12 values in the distance matrix, so the space complexity of EDTW is O(N).

### 4. Experimentation

In this section we want to test the new distance measure EDTW against synthetic temperature dataset and standard UCR Time series datasets to measure the accuracy. The following subsections explain the results of EDTW:

#### 4.1 Synthetic Dataset

This section explains experimental evaluation of the EDTW algorithm using synthetic temperature dataset. In data set we took temperature for every two hours between 0 to 24 hours for 10 days. The reduced dataset is generated by taking the average of consecutive three values. The original and reduced data sets are shown in tables 2 and 3. On reduced dataset we applied EDTW and calculated neighbors and summarized in the table 4. The neighbors of original dataset using DTW(NODDTW) and reduced dataset using DTW(NRDEDTW), reduced dataset using EDTW(NRDEDTW) are mostly similar (table 4).

#### 4.2 UCR Time Series Data Set

EDTW is tested against UCR Time series datasets. The reference contains various datasets names. For
each dataset the following details available. Number of classes(NC), size of training set(STRS), size of test set(STES), time series length(TL), error rates of 1-NN classifier using Euclidean Distance(1NNED), DTW with best warping window size(DTWBWW), and DTW with no warping window(DTWNWW).

On UCR time series dataset we applied 1-NN classification with EDTW and calculated classification error rate. The error rate is compared with existing classification methods. Surprisingly its performance is good on some datasets. For example if we observe wine dataset, its error rate is 0.000. It’s a good improvement compared to rival methods. The list of data sets for which error rate is less (performance improved) is shown in the table 5.

On some data sets its performance is equivalent to existing methods. The list of datasets whose error rate is equal to existing methods is shown in the table 6. The list of datasets where error rate is high compared to rival methods is shown in the table 7. For example car dataset, Cricket_X, Face, Strawberry datasets we may observe more error rate compared to rival methods.

Table 3. Reduced dataset

|   | 7   | 11  | 15  | 15  |
|---|-----|-----|-----|-----|
| 9 | 12  | 16  | 16  |     |
| 8 | 11  | 15  | 13  |     |
| 7 | 9   | 13  | 15  |     |
| 7 | 8   | 11  | 13  |     |
| 7 | 9   | 13  | 13  |     |
| 6 | 8   | 11  | 12  |     |
| 6 | 9   | 12  | 14  |     |
| 7 | 8   | 14  | 15  |     |
| 9 | 10  | 15  | 14  |     |

Table 4. Neighbors of original/reduced dataset using DTW/EDTW distance.

| Dataset No. | NODDTW | NRDDTW | NRD EDTW |
|-------------|--------|--------|----------|
| 1           | 2,3,10 | 2,3,4,10 | 3,10,2   |
| 2           | 1,3,10 | 1,3,10 | 1,4,10   |
| 3           | 10,1,2 | 10,1,2 | 10,1,8   |
| 4           | 9,6,5  | 9,8,6 | 9,8,6    |
| 5           | 8,6,4,7,9 | 7,8,6 | 7,8,6    |
| 6           | 4,5,8,9 | 4,8,5 | 4,8,5    |
| 7           | 8,5,6 | 5,6,8 | 5,6,3    |
| 8           | 5,6,7 | 4,5,6 | 4,6,5    |
| 9           | 4,6,5,8 | 4,6,8 | 4,8,6    |
| 10          | 3,1,2 | 3,1,4 | 3,1,4    |

Table 5. Data sets with less error rate

| S.No. | Name                          | NC | STRS | STES | TL  | 1NNED | DTWBWW | DTWNWW | 1-NN Efficient DTW |
|-------|-------------------------------|----|------|------|-----|-------|--------|--------|-------------------|
| 1     | Bird Chicken                  | 2  | 20   | 20   | 512 | 0.450 | 0.300  | 0.250  | 0.020             |
| 2     | Distal Phalanx TW             | 6  | 139  | 400  | 80  | 0.273 | 0.272  | 0.290  | 0.272             |
| 3     | Ford A                        | 2  | 1320 | 3601 | 500 | 0.341 | 0.341  | 0.438  | 0.404             |
| 4     | Ham                           | 2  | 109  | 105  | 431 | 0.400 | 0.400  | 0.533  | 0.523             |
| 5     | Hand Outlines                 | 2  | 370  | 1000 | 2709| 0.199 | 0.197  | 0.202  | 0.095             |
| 6     | Haptics                       | 5  | 155  | 308  | 1092| 0.630 | 0.588  | 0.623  | 0.577             |
| 7     | Herring                       | 2  | 64   | 64   | 512 | 0.484 | 0.469  | 0.469  | 0.468             |
| 8     | Insect Wing beat Sound        | 11 | 220  | 1980 | 256 | 0.438 | 0.422  | 0.645  | 0.607             |
| 9     | Large Kitchen Appliances      | 3  | 375  | 375  | 720 | 0.507 | 0.205  | 0.205  | 0.229             |
| 10    | Middle Phalanx Outline Age Group | 3  | 154  | 400  | 80  | 0.260 | 0.253  | 0.250  | 0.242             |
| 11    | Proximal Phalanx Outline Age Group | 3  | 400  | 205  | 80  | 0.215 | 0.215  | 0.195  | 0.165             |
| 12    | Refrigeration Devices         | 3  | 375  | 375  | 720 | 0.605 | 0.560  | 0.536  | 0.538             |
| 13    | Screen Type                   | 3  | 375  | 375  | 720 | 0.640 | 0.589  | 0.603  | 0.517             |
| 14    | Wine                          | 2  | 57   | 54   | 234 | 0.389 | 0.389  | 0.426  | 0.000             |
### Table 6. The datasets with similar error rate

| S.No. | Name                                      | NC | STRS | STES | TL  | 1NNED | DTWBWW | DTWNWW | 1-NN Efficient DTW |
|-------|-------------------------------------------|----|------|------|-----|-------|--------|--------|-------------------|
| 1     | 50Words                                   | 50 | 450  | 455  | 270 | 0.369 | 0.242 (6) | 0.310  | 0.325             |
| 2     | Adiac                                     | 37 | 390  | 391  | 176 | 0.389 | 0.391 (3) | 0.396  | 0.450             |
| 3     | Distal Phalanx Outline Age Group          | 3  | 139  | 400  | 80  | 0.218 | 0.228 (1) | 0.208  | 0.227             |
| 4     | ECG 5000                                  | 5  | 500  | 4500 | 140 | 0.075 | 0.075 (1) | 0.076  | 0.079             |
| 5     | Ford B                                    | 2  | 810  | 3636 | 500 | 0.442 | 0.414 (1) | 0.406  | 0.436             |
| 6     | Medical Images                            | 10 | 381  | 760  | 99  | 0.316 | 0.253 (20)| 0.310  | 0.325             |
| 7     | Middle Phalanx Outline Correct            | 2  | 291  | 600  | 80  | 0.247 | 0.318 (1) | 0.352  | 0.371             |
| 8     | Middle Phalanx TW                         | 6  | 154  | 399  | 80  | 0.439 | 0.419 (2) | 0.416  | 0.442             |
| 9     | Plane                                     | 7  | 105  | 105  | 144 | 0.038 | 0.000 (6) | 0.000  | 0.047             |
| 10    | Proximal Phalanx Outline Correct          | 2  | 600  | 291  | 80  | 0.192 | 0.210 (1) | 0.216  | 0.240             |
| 11    | Trace                                     | 4  | 100  | 100  | 275 | 0.240 | 0.010 (3) | 0.000  | 0.020             |
| 12    | Two Patterns                              | 4  | 900  | 4000 | 128 | 0.090 | 0.002 (4) | 0.000  | 0.003             |
| 13    | Synthetic Control                         | 6  | 300  | 300  | 60  | 0.120 | 0.017 (6) | 0.007  | 0.190             |

### Table 7. The datasets with high error rate

| S.No. | Name                                      | NC | STRS | STES | TL  | 1NNED | DTWBWW | DTWNWW | 1-NN Efficient DTW |
|-------|-------------------------------------------|----|------|------|-----|-------|--------|--------|-------------------|
| 1     | Arrow Head                                | 3  | 36   | 175  | 251 | 0.200 | 0.200 (0) | 0.297  | 0.371             |
| 2     | Beef                                      | 5  | 30   | 30   | 470 | 0.333 | 0.333 (0) | 0.367  | 0.400             |
| 3     | Beetle Fly                                | 2  | 20   | 20   | 512 | 0.250 | 0.300 (7) | 0.300  | 0.450             |
| 4     | Car                                       | 4  | 60   | 60   | 577 | 0.267 | 0.233 (1) | 0.267  | 0.533             |
| 5     | CBF                                       | 3  | 30   | 900  | 128 | 0.148 | 0.004 (11)| 0.003  | 0.183             |
| 6     | Chlorine Concentration                    | 3  | 467  | 3840 | 166 | 0.35  | 0.35 (0)  | 0.352  | 0.396             |
| 7     | Cricket_X                                 | 12 | 390  | 390  | 300 | 0.423 | 0.228 (10)| 0.246  | 0.497             |
| 8     | Diatom Size Reduction                     | 4  | 16   | 306  | 345 | 0.065 | 0.065 (0) | 0.033  | 0.3               |
| 9     | Distal Phalanx Outline Correct            | 2  | 276  | 600  | 80  | 0.248 | 0.232 (2) | 0.232  | 0.495             |
| 10    | Earth quakes                              | 2  | 139  | 322  | 512 | 0.326 | 0.258 (22)| 0.258  | 0.413             |
| 11    | ECG Five Days                             | 2  | 23   | 861  | 136 | 0.203 | 0.203 (0) | 0.232  | 0.319             |
| 12    | Face (four)                               | 4  | 24   | 88   | 350 | 0.216 | 0.114 (2) | 0.170  | 0.590             |
| 13    | Face (all)                                | 14 | 560  | 1690 | 131 | 0.286 | 0.192 (3) | 0.192  | 0.384             |
| 14    | Toe Segmentation 1                        | 2  | 40   | 228  | 277 | 0.320 | 0.250 (8) | 0.228  | 0.302             |
| 15    | Toe Segmentation 2                        | 2  | 36   | 130  | 343 | 0.192 | 0.092 (5) | 0.162  | 0.492             |
| 16    | Strawberry                                | 2  | 370  | 613  | 235 | 0.062 | 0.062 (0) | 0.060  | 0.386             |
| 17    | Word Synonyms                             | 25 | 267  | 638  | 270 | 0.382 | 0.252 (8) | 0.351  | 0.426             |
5. Conclusion and Future Work

In this paper we introduced the EDTW algorithm. It has linear time and space complexity. Efficient DTW uses a two level approach, first level it reduce the dataset and second level calculates limited values in a distance matrix to find distance and wrapping path. It is efficient than DTW, and accelerate data mining functionalities. Technique applied on synthetic dataset and UCR Time series datasets. In future we will apply this technique to existing classification and clustering methods of time series data to test efficiency.

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