Nonparametric Dark Energy Reconstruction Using the Tomographic Alcock–Paczyński Test

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Abstract

The tomographic Alcock–Paczyński (AP) method can result in tight cosmological constraints by using small and intermediate clustering scales of the large-scale structure of the galaxy distribution. By focusing on the redshift dependence, the AP distortion can be distinguished from the distortions produced by the redshift space distortions. In this work, we combine the tomographic AP method with other recent observational data sets of SN Ia+BAO+CMB+$H_0$ to reconstruct the dark energy equation-of-state $w$ in a nonparametric form. The result favors a dynamical DE at $z \lesssim 1$, and shows a mild deviation ($\lesssim 2\sigma$) from $w = -1$ at $z = 0.5$–0.7. We find the addition of the AP method improves the low-redshift ($z \lesssim 0.7$) constraint by $\sim 50\%$.

Key words: cosmological parameters – dark energy – large-scale structure of universe

1. Introduction

The late-time accelerated expansion of the universe (Riess et al. 1998; Perlmutter et al. 1999) implies either the existence of “dark energy” or the breakdown of general relativity on cosmological scales. The theoretical origin and observational measurements of cosmic acceleration, although having attracted tremendous attention, are still far from being well explained or accurately measured (Weinberg 1989; Li et al. 2011; Yoo & Watanabe 2012; Weinberg et al. 2013).

The Alcock–Paczyński (AP) test (Alcock & Paczynski 1979) enables us to probe the angular diameter distance $D_A$ and the Hubble factor $H$, which can be used to place constraints on cosmological parameters. Under a certain cosmological model, the radial and tangential sizes of some distant objects or structures take the forms of $\Delta r \sim \frac{\Delta \theta}{\theta(z)}$ and $\Delta r \sim (1+z)D_A(z)\Delta \theta$, where $\Delta z$, $\Delta \theta$ are their redshift span and angular size, respectively. Thus, if incorrect cosmological models are assumed for transforming redshifts into comoving distances, the wrongly estimated $\Delta r$ and $\Delta r$ induce a geometric distortion, known as the AP distortion. Statistical methods that probe and quantify the AP distortion have been developed and applied to a number of galaxy redshift surveys to constrain the cosmological parameters (Ryden 1995; Ballinger et al. 1996; Matsubara & Suto 1996; Outram et al. 2004; Blake et al. 2011; Lavaux & Wandelt 2012; Alam et al. 2017; Mao et al. 2016; Ramanah et al. 2019).

Recently, a novel tomographic AP method based on the redshift evolution of the AP distortion has achieved significantly strong constraints on the cosmic expansion history parameters (Park & Kim 2010; Li et al. 2014, 2015, 2016). The method focuses on the redshift dependence to differentiate the AP effect from the distortions produced by the redshift space distortions (RSDs), and has proved to be successful in dealing with galaxy clustering on relatively small scales. Li et al. (2016) first applied the method to the Sloan Digital Sky Survey (SDSS) Baryon Oscillation Spectroscopic Survey (BOSS) DR12 galaxies, and achieved $\sim 35\%$ improvements in the constraints on $\Omega_m$ and $w$ when combining the method with external data sets of the cosmic microwave background (CMB), type Ia supernovae (SN Ia), baryon acoustic oscillations (BAO), and the $H_0$.

In this work we aim to study how the tomographic AP method can be optimized to aid in measuring and characterizing dark energy. The nonparametric strategy is particularly suitable for constraining functions whose forms are not clearly known from the theoretical aspect (Marco 2019). We apply the method to reconstruct the dark energy equation-of-state $w(z)$, using the nonparametric approach developed in Crittenden et al. (2009, 2012) and Zhao et al. (2012), which has the advantage of not assuming any ad hoc form of $w$. In a recent work Zhao et al. (2017b) use this method to reconstruct $w$ from 16 observational data sets, and claim a $3.5\sigma$ significance level in preference of a dynamical dark energy. It would be interesting to see what the results would be if the tomographic AP method is used to reconstruct $w$, and whether the reconstructed $w$ is consistent with the results of Zhao et al. (2017b).

A brief outline of this paper is as follows. In Section 2 we outline the tomographic AP method and how we practically implement the nonparametric modeling of $w(z)$. In Section 3 we present the results of our analysis in combination with other data sets. We conclude in Section 4.

2. Methodology

In pursuit of reconstructing DE in a model-independent manner, we adopt the nonparametric method of $w$ (Crittenden et al. 2009; Zhao et al. 2012) without choosing any particular parameterization. To start, $w$ is parameterized in terms of its values at discrete steps in the scale factor $a$. Fitting a large number of uncorrelated bins would lead to extremely large uncertainties and, in fact, would prevent the Monte Carlo Markov Chains (MCMCs) from converging due to the large number of degenerate directions in the parameter space. On the other hand, fitting only a few bins usually lead to an unphysical discrete distribution of $w$ and significantly biases the result. The solution is to introduce a prior covariance among a large number of bins based on a
pheno-omenological two-point function,
\[ \xi_w(|a - a'|) \equiv \langle [w(a) - w_{\text{fid}}(a)][w(a') - w_{\text{fid}}(a')] \rangle, \] (1)
which is chosen as the form of (Crittenden et al. 2009)
\[ \xi_{\text{CTZ}}(\delta a) = \xi_w(a = 0) /[1 + (\delta a/\alpha_c)^2], \] (2)
where \( \delta a \equiv |a - a'| \). Clearly, \( \alpha_c \) describes the typical smoothing scale, and \( \xi_w(0) \) is the normalization factor determined by the expected variance of the mean of the \( w \)'s, \( \sigma_w^2 \). The “floating” fiducial is defined as the local average,
\[ w_i^\text{fid} = \frac{\sum_{|a_i - a| \leq \alpha_c} w_i^\text{true}}{N_i}, \] (3)
where \( N_i \) is the number of neighboring bins lying around the \( i \)th bin within the smoothing scale.

In practice, one should set the prior to conduct the analysis. A very weak prior (i.e., small \( \alpha_c \) or large \( \sigma_w^2 \)) can match the true model on average (i.e., unbiased), but will result in a noisy reconstruction. A stronger prior reduces the variance but pulls the reconstructed results toward the peak of the prior. In this paper, we use the “weak prior” \( \alpha_c = 0.06, \sigma_w = 0.04 \), the prior which was also adopted in Zhao et al. (2012). The tests performed in Crittenden et al. (2009) show that the results are largely independent of the choice of the correlation function. Also, Crittenden et al. (2012) has shown that a stronger prior \( \sigma_w = 0.02 \) is already enough for reconstructing a range of models without introducing a sizable bias.

We parameterize \( w \) in terms of its values at \( N \) points in \( a \), i.e.,
\[ w_i = w(a_i), \quad i = 1, 2, \ldots, N. \] (4)
In this analysis we choose \( N = 30 \), where the first 29 bins are uniform in \( a \in [0.286, 1] \), corresponding to \( z \in [0, 2.5] \), and the last bin covers the wide range of \( z \in [2.5, 1100] \). Given the binning scheme, together with the covariance matrix \( C \) given by Equation (2), it is straightforward to write down the prior following the Gaussian form PDF,
\[ P_{\text{prior}}(w) \propto \exp \left(-\frac{1}{2}(w - w_{\text{fid}})^T C^{-1}(w - w_{\text{fid}})\right). \] (5)

Effectively, the prior results in a new contribution to the total likelihood of the model given the data sets \( D \),
\[ P(w|D) \propto P(D|w) \times P_{\text{prior}}(w), \] (6)
thus penalizing those models that are less smooth.

The method is then applied to a joint data set of recent cosmological observations including the CMB temperature and polarization anisotropies measured by the full-mission Planck (Ade et al. 2016), the “ILAC” SN Ia sample (Betoule et al. 2014), a Hubble Space Telescope measurement of \( H_0 = 70.6 \pm 3.3 \) km s\(^{-1}\)Mpc\(^{-1}\) (Riess et al. 2011; Efstathiou 2014), and the BAO distance priors measured from 6dFGS (Beutler et al. 2011), SDSS MGS (Ross et al. 2015), and the SDSS-III BOSS DR11 anisotropic measurements (Anderson et al. 2014), as was also adopted in Li et al. (2016, 2018).

These data sets are then combined with the AP likelihood of SDSS-III BOSS DR12 galaxies (Li et al. 2016, 2018), for which we evaluate the redshift evolution of large-scale structure (LSS) distortion induced by the wrong cosmological parameters via the anisotropic correlation function,
\[ \delta \xi_{\Delta z}(z_i, \mu) = \xi_{\Delta z}(z_i, \mu) - \bar{\xi}_{\Delta z}(z_i, \mu). \] (7)
\( \xi_{\Delta z}(z_i, \mu) \) is the integrated correlation function that captures the information of LSS distortion within the clustering scales one was interested in,
\[ \xi_{\Delta z}(\mu) = \int_{z_{\text{min}}}^{z_{\text{max}}} \xi(x, \mu) \, dz. \] (8)

It was then normalized to remove the uncertainty from clustering magnitude and the galaxy bias,
\[ \hat{\xi}_{\Delta z}(\mu) = \frac{\xi_{\Delta z}(\mu)}{\int_{0}^{\mu} \xi_{\Delta z}(\mu) \, d\mu}. \] (9)
As described in Equation (7), the difference between \( \hat{\xi}_{\Delta z}(\mu) \) measured at two different redshifts \( z_i, z_j \) characterizes the amount of the redshift evolution of LSS distortion. SDSS DR12 has 361,759 LOWZ galaxies at \( 0.15 < z < 0.43 \), and 771,567 CMASS galaxies at \( 0.43 < z < 0.693 \). We split these galaxies into six, non-overlapping redshift bins of \( 0.15 < z_1 < 0.274 < z_2 < 0.351 < z_3 < 0.430 < z_4 < 0.511 < z_5 < 0.572 < z_6 < 0.693 \) (Li et al. 2016).

Li et al. (2014, 2015) demonstrated that \( \delta \xi_{\Delta z}(z, \mu) \) is dominated by the AP distortion while being rather insensitive to the RSD distortion, enabling us to avoid large contamination from the latter and probe the AP distortion information on relative small clustering scales.

The only difference in our treatment from Li et al. (2016) is that here we slightly improve the method and adopt a “full-covariance matrix” likelihood,
\[ P_{\text{AP}}(w|D) \propto \exp \left(-\frac{1}{2} \theta_{\text{AP}} C^{-1}_{\text{AP}} \theta_{\text{AP}} \right). \] (10)
where the vector
\[ \theta_{\text{AP}} = [\xi_{\Delta z}(z_2, \mu_1), \xi_{\Delta z}(z_3, \mu_2), \ldots, \xi_{\Delta z}(z_6, \mu_5)] \] (11)
summarizes the redshift evolution among the six redshift bins into its \( 5 \times n_\mu \) components (\( n_\mu \) is the number of binning in \( \xi_{\Delta z} \)). The covariance matrix \( C_{\text{AP}} \) is estimated using the 2000 MultiDark-Patchy mocks (Kitaura et al. 2016). Compared with Li et al. (2016), where the first redshift bin is taken as the reference, this current approach includes the statistical uncertainties in the system and avoids the particular dependence on which specific redshift bin is chosen as the reference.

This improved methodology was presented in Li et al. (2019), where the authors detailed how the multiredshift correlation is included, and how it affects the constraints on the various cosmological parameters.

3. Results

The derived constraints on \( w \) as a function of redshift are plotted in Figure 1. The red solid lines represent the 68.3% CL constraints based on Planck+SN Ia+BAO+H_0, while the AP-added results are plotted in the blue filled region.\(^8\) The boundaries are determined so that, for LOWZ and CMASS samples, the number of galaxies are the same in each bin, respectively.

\(^8\) The AP and BAO methods probe galaxy clustering on very different scales, so it is safe to assume they have no correlation and can be simply combined. In Zhang et al. (2018), the authors have computed the the correlation coefficient of the anisotropic information in the clustering scales of AP and BAO methods in N-body simulations, and find it as small as \(-0.054 \pm 0.034\).
Figure 1. Derived redshift evolution of $w(z)$. The mean values and 68.3\% CL regions are plotted. Adding the AP method tightens the constraints. Dynamical behavior of dark energy is mildly favored at $z \lesssim 0.7$.

likelihood distributions of $w$ at $z = 0.03, 0.33, 0.65, 1.31$ are plotted in Figure 2.

The reconstructed $w(z)$ from Planck+SN Ia+BAO+$H_0^0$ is fully consistent with the cosmological constant; the $w = -1$ line lies within the 68.3\% CL region. In the plotted redshift range ($0 < z < 2.5$), the upper bound of $w$ is constrained to $\lesssim -0.8$, while the lower bound varies from $-1.3$ at $z = 0$ to $-2.0$ at $z \gtrsim 2$, dependent on the redshift. The best constrained epoch lies around $z = 0.2$. These features are consistent with the previous results presented in the literature using a similar data set (Zhao et al. 2017a).

The constraints are much improved after adding AP to the combined data set. At $z \lesssim 0.7$, i.e., the redshift range of the SDSS galaxies analyzed by the AP method, the uncertainty of $w(z)$ is reduced by $\sim 50\%$, reaching as small as 0.2. It then increases to 0.4–1.0 at higher redshift ($0.7 < z < 2.5$). This highlights the power of the AP method in constraining the properties of dark energy, which were shown in Li et al. (2016, 2018).

Although here the AP method only probes the expansion history information at $z \in (0.1, 0.7)$, it can still affect the high-redshift constraints. At $0.7 \lesssim z \lesssim 1.0$, the constraints are tightened by the correlated prior of $w(z)$. At higher redshift, the error bars are less affected, but the values of $w$ are shifted to more negative regions. This is due to the combination of AP and CMB data. Effectively, the CMB data constrain the $w(z)$ in a manner of the “shift parameter” $R \equiv \sqrt{\Omega_m H_0 (1 + z)} D_A(z_f)$ (Bond et al. 1997), which constrains the integration of $1/H(z)$ in the range of $z \approx 0–1100$. So if the constraints on $z \lesssim 1.0$ are changed, the $z \gtrsim 1.0$ parts are also altered, correspondingly.

The most interesting phenomenon from our studies is that the result indicates a mild discrepancy with a constant $w = -1$. At $0.5 \lesssim z \lesssim 0.7$, $w > -1$ is slightly favored ($\lesssim 2\sigma$). The statistical significance of this result is not large enough to claim a detection of deviation from a cosmological constant; however, this may be readdressed in the near future as the constraining power will become much improved when combining tomographic AP with the upcoming experiments of DESI (Aghamousa 2016) or EUCLID (Laureijs et al. 2011).

The results also slightly favor a dynamical behavior of $D_E$. At $z = 0.0–0.5$, we find phantom-like dark energy $-0.7 \lesssim w \lesssim -1.0$, while at higher redshift $z > 0.5$ it becomes quintessence-like, $-1.0 \lesssim w \lesssim -0.6$. Theoretically, this is known as the quintom dark energy (Feng et al. 2005).

The advantage of the tomographic AP method is that it makes use of the clustering information in a series of redshift bins (rather than compressing the whole sample into a single effective redshift). Thus, it is able to capture the dynamical behavior of dark energy within narrow ranges of $\Delta z$.

Our results are consistent with the $w(z)$ obtained in Li et al. (2018), where the authors used the Planck+SN Ia+BAO+$H_0$+AP data set to constrain the CPL parameterization $w = w_0 + \frac{w_a(z)}{1+z}$. They found 100\% improvement in the DE figure-of-merit and a slight preference of dynamical dark energy. Benefiting from a more general form of a nonparametric $w(z)$, we are able to obtain more detailed features in the reconstruction.

To further validate the results, we did an input–output test on two MultiDark-Patchy realizations.\(^8\) We treat the mocks as the “real data” and apply the AP method to them. The constraints on $w(z)$ are plotted in Figure 3. The “true” cosmology of $w = -1$ are nicely recovered (deviation $\lesssim 1\sigma$). The size of the error bars is $\sim 1.5$ times that of the Planck+SN Ia+BAO+$H_0$+AP constraints of Figure 1. This justifies the ability of the tomographic AP method to constrain the nonparametric dark energy equation-of-state.

Finally, we note that the results with and without AP are in good consistency with each other. This implies that the information obtained from the AP effect agrees well with the other probes. As the clustering information probed by AP is independent from those probed by BAO (see the discussion in Zhang et al. 2018), to some extent, in this analysis these two different LSS probes complement and validate each other. This is also consistent with the results of Li et al. (2016), where we found the contour region constrained by AP consistently overlaps with those of SN Ia, BAO, and CMB.

4. Concluding Remarks

In this work, we consider a very general, nonparametric form for the evolution of the dark energy equation-of-state, $w(z)$. We obtain cosmological constraints by combining our tomographic AP method with other recent observational data sets of SN Ia+BAO+CMB+$H_0$. As a result, we find that the inclusion of AP improves the low-redshift ($z < 0.7$) constraint by $\sim 50\%$. Moreover, our result favors a dynamical DE at $z \lesssim 1$, and shows a mild deviation ($\lesssim 2\sigma$) from $w = -1$ at $z = 0.5–0.7$.

We did not discuss the systematics of the AP method in detail. This topic has been extensively studied in Li et al. (2016, 2018), where the authors found that for the current observations the systematical error is still much less than the statistical uncertainty.

We note that our constraint on $w(z)$ at $z \lesssim 0.7$ is the tightest within the current literature. The accuracy we achieved is as

Footnotes:

\(^7\) Note that the SDSS DR11 anisotropic BAO measurements also contain the AP information on scales of $\sim 100\, h^{-1}\, \text{Mpc}$, so it is a little inappropriate to use the abbreviations “BAO” and “AP” in the legend. Anyway, we still use them, and our “AP” only stands for the $6–40\, h^{-1}\, \text{Mpc}$ tomographic AP measurements of SDSS DR12 galaxies.

\(^8\) As a simple check we just did it on two realizations. Due to the many degrees of freedom the MCMC chains converge very slowly, making it rather difficult to perform this kind of test on a large number of mock.
good as that of Zhao et al. (2017a) in their "ALL16" combination, where they used the Planck+SN Ia+BAO+H₀ data sets, combined with the WiggleZ galaxy power spectra (Parkinson et al. 2012), the CFHTLenS weak lensing shear angular power spectra Heymans et al. (2013), the H(z) measurement using the relative age of old and passively evolving galaxies based on a cosmic chronometer approach (OHD; Moresco et al. 2016), and the Lyα BAO measurements (Delubac et al. 2015). In comparison, we use a much smaller number of data sets to achieve a similar low-redshift \( w(z) \) constraint. This highlights the great power of our tomographic AP method using anisotropic clustering on small scales.

At higher redshift \( (z \gtrsim 0.7) \) our constraint is weaker than Zhao et al. (2017a). It would be interesting to include more data sets (e.g., the ones used in their paper, the SDSS IV high-redshift results; Zhao et al. 2019) and then re-perform this analysis.

The dynamical behavior of dark energy at \( z \approx 0.5–0.7 \) has also been found in many other works (Zhao et al. 2017a; Wang et al. 2018). Due to the limitation of current observations, it is not possible to claim a detection of dynamical dark energy at \( >5 \sigma \) CL. We expect this can be achieved (or falsified) in the near future aided by more advanced LSS experiments, such as DESI (Aghamousa 2016), Euclid (Laureijs et al. 2011), and LSST (Marshall et al. 2017).

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