Large extra dimensions and deep-inelastic scattering at HERA

Prakash Mathews\textsuperscript{1}, Sreerup Raychaudhuri\textsuperscript{2}, K. Sridhar\textsuperscript{1}

1) Department of Theoretical Physics, Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay 400 005, India.
2) Department of High Energy Physics, Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay 400 005, India.

ABSTRACT

In scenarios motivated by string theories, it is possible to have extra Kaluza-Klein dimensions compactified to rather large magnitudes, leading to large effects of gravity at scales down to a TeV. The effect of the spin-2 Kaluza-Klein modes on the deep-inelastic cross-section at HERA is investigated. We find that the data can be used to obtain bounds on the effective low energy scale, $M_S$. 

\textsuperscript{*}prakash@theory.tifr.res.in
\textsuperscript{†}sreerup@iris.hecr.tifr.res.in
\textsuperscript{‡}sridhar@theory.tifr.res.in
The Standard Model (SM) has proved enormously successful in providing a description of particle physics up to energy scales probed by current experiments, which is in the region of several hundred GeV. In the SM, however, one assumes that effects of gravity can be neglected, because the scale where the effects of gravity become large i.e. the Planck scale ($M_P = 1.2 \times 10^{19}$ GeV) is vastly different from the TeV scale. The separation between the TeV scale and the Planck scale is what manifests itself as the hierarchy problem, whose solution has become one of the foci of the search for the correct physics beyond the SM. This problem is exacerbated in traditional unification scenarios: the scale of grand-unification is of the order of $10^{16}$ GeV and again implies a huge desert. Further, in spite of the unification scale being so close to the Planck scale, traditional unification models make no reference whatsoever to gravity.

Recent advances in string theory provide indications for a major paradigm shift — in particular, unification of gravity with other interactions now seems possible in the strongly-coupled limit of string theory called $M$-theory [1, 2]. But other interesting effects may also manifest at lower energies, for example, it was pointed out that the fundamental scale of string theory can be as low as a TeV [3]. Of tremendous interest to phenomenology is the possibility that the effects of gravity could become large at very low scales ($\sim$ TeV), because of the effects of large extra Kaluza-Klein dimensions where gravity can propagate [4]. The starting point for such a scenario is a higher-dimensional theory of open and closed strings [3, 4]. The extra dimensions of this theory are then compactified to obtain the effective low-energy theory in 3+1 dimensions, and it is assumed that $n$ of these extra dimensions are compactified to a common scale $R$ which is relatively large, while the remaining dimensions are compactified to much smaller length scales which are of the order of the inverse Planck scale. In such a scenario, the SM particles correspond to open strings, which end on a 3-brane. It implies that SM particles are localised on this 3-brane, and are, therefore, confined to the 3 + 1-dimensional spacetime. On the other hand, the gravitons (corresponding to closed strings) propagate in the 4 + $n$-dimensional bulk. The relation between the scales in 4 + $n$ dimensions and in 4 dimensions is given by [4]

$$M_P^2 = M_S^{n+2} R^n,$$

where $M_S$ is the low-energy effective string scale. This equation has the interesting consequence that we can choose $M_S$ to be of the order of a TeV and thus get around the hierarchy problem. For such a value of $M_S$, it follows that $R = 10^{32/n - 19}$ m, and so we find that $M_S$ can be arranged to be a TeV for any value $n > 1$. Effects of non-Newtonian gravity can become apparent at these surprisingly low values of energy. For example, for $n = 2$ the compactified dimensions are of the order of 1 mm, just below the experimentally tested region for the validity of Newton’s law of gravitation and within the possible reach of ongoing experiments [4]. In fact, it has been shown [4] that it is possible to construct a phenomenologically viable scenario with large extra dimensions, which can survive the existing astrophysical and cosmological constraints.
While the lowering of the string scale leads to the nullification of the hierarchy problem, the residual problem is that of stabilising the extra large dimensions. This problem has been recently addressed in some papers \([9]\). Moreover, the effect of the Kaluza-Klein states on the running of the gauge couplings i.e. the effect of these states on the beta functions of the theory have been studied \([9, 10]\) and it has been shown that the unification scale can be also lowered down to scales close to the electroweak scale \([1]\). For recent investigations on different aspects of the TeV scale quantum gravity scenario and related ideas, see Ref. \([14]\).

Below the scale \(M_S\) the following effective picture emerges \([15, 16, 17]\): there are the Kaluza-Klein states, in addition to the usual SM particles. The graviton corresponds to a tower of Kaluza-Klein states which contain spin-2, spin-1 and spin-0 excitations. The spin-1 modes do not couple to the energy-momentum tensor and their couplings to the SM particles in the low-energy effective theory are not important. The scalar modes couple to the trace of the energy-momentum tensor, so they do not couple to massless particles. Other particles related to brane dynamics (for example, the \(Y\) modes which are related to the deformation of the brane) have effects which are subleading, compared to those of the graviton. The only states, then, that contribute are the spin-2 Kaluza-Klein states. These correspond to a massless graviton in the \(4 + n\) dimensional theory, but manifest as an infinite tower of massive gravitons in the low-energy effective theory. For graviton momenta smaller than the scale \(M_S\), the effective description reduces to one where the gravitons in the bulk propagate in the flat background and couple to the SM fields which live on the brane via a (four-dimensional) induced metric \(g_{\mu\nu}\). Starting from a linearized gravity Lagrangian in \(n\) dimensions, the four-dimensional interactions can be derived after a Kaluza-Klein reduction has been performed. The interaction of the SM particles with the graviton, \(G_{\mu\nu}\), can be derived from the following Lagrangian:

\[
\mathcal{L} = -\frac{1}{\bar{M}_P} G^{(j)\mu\nu} T_{\mu\nu},
\]

where \(j\) labels the Kaluza-Klein mode and \(\bar{M}_P = M_P/\sqrt{8\pi}\), and \(T_{\mu\nu}\) is the energy-momentum tensor. Using the above interaction Lagrangian the couplings of the graviton modes to the SM particles can be calculated \([16, 17]\), and used to study the consequences at colliders of this TeV scale effective theory of gravity. In particular, direct searches for graviton production at \(e^+e^-\), \(p\bar{p}\) and \(pp\) colliders, leading to spectacular single photon + missing energy or monojet + missing energy signatures, have been suggested \([16, 18, 17]\). The virtual effects of graviton exchange in \(e^+e^- \rightarrow f\bar{f}\) and in high-mass dilepton production \([19]\), and in \(tt\) production \([20]\) at the Tevatron and the LHC have been studied. The bounds on \(M_S\) obtained from direct searches depend on the number of extra dimensions. Non-observation of the Kaluza-Klein modes yield

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1Efforts to lower the compactification scale have been made earlier in Ref. \([12]\). The effects of Kaluza-Klein states on the running of couplings was first investigated in Ref. \([13]\).
bounds which are around 500 GeV to 1.2 TeV at LEP2 and around 600 GeV to 750 GeV at Tevatron (for \( n \) between 2 and 6) [18]. Indirect bounds from virtual graviton exchange in dilepton production at Tevatron yields a bound of around 950 GeV [19]. Virtual effects in \( t\bar{t} \) production at Tevatron yields a bound of about 650 GeV [20].

In view of the fact that the effective Lagrangian given in Eq. 2 is suppressed by \( 1/M_p \), it may seem that the effects at colliders will be hopelessly suppressed. However, in the case of real graviton production, the phase space for the Kaluza-Klein modes cancels the dependence on \( M_p \) and, instead, provides a suppression of the order of \( M_S \). For the case of virtual production, we have to sum over the whole tower of Kaluza-Klein states and this sum when properly evaluated [17, 16] provides the correct sum [17] and this shows that the low-energy effective theories for the \( s \) and \( t \)-channels are equivalent.

In the present work, we study the effect of the virtual graviton exchange on the \( e^+p \) deep-inelastic scattering cross-section at HERA. The presence of the new couplings from the low-energy effective theory of gravity, lead to new \( t \)-channel diagrams in the \( e^+q(\bar{q}) \) or \( e^+g \) initial state. We use the couplings as given in Refs. [16, 17], and summing over all the graviton modes, we find the following expressions for the cross-sections involving the virtual graviton exchange (in the following we use the notation \( d\hat{\sigma}^{a(i)}/d\hat{t} \), for the process \( e^+i \rightarrow e^+i \), where \( i = q, g \) is the parton in the initial state, and the process is mediated by the exchange of \( a \); and the notation \( d\hat{\sigma}^{ab(i)}/d\hat{t} \), for the interference of processes \( e^+i \rightarrow e^+i \) mediated by the exchanges of the virtual particles \( a \) and \( b \), respectively):

\[
\frac{d\hat{\sigma}(e^+q \rightarrow e^+q)}{dt} = \frac{d\hat{\sigma}^{SM}}{dt} + \frac{d\hat{\sigma}^{G(q)}}{dt} + \frac{d\hat{\sigma}^{\gamma G(q)}}{dt} + \frac{d\hat{\sigma}^{ZG(q)}}{dt},
\]

\[
\frac{d\hat{\sigma}^{G(q)}}{dt} = \frac{\pi\lambda^2}{32M_S^2} \frac{1}{\hat{s}^2} \left[ 32\hat{u}^4 + 64\hat{u}^3\hat{t} + 42\hat{u}^2\hat{t}^2 + 10\hat{u}\hat{t}^3 + \hat{t}^4 \right],
\]

\[
\frac{d\hat{\sigma}^{\gamma G(q)}}{dt} = \frac{\pi\alpha e_q\lambda}{2M_S^4} \frac{1}{\hat{s}^2\hat{t}} (2\hat{u} + \hat{t})^3,
\]

\[
\frac{d\hat{\sigma}^{ZG(q)}}{dt} = \frac{\pi\alpha\lambda}{2\sin^2\theta_W M_S^2} \frac{[C_V^C C_V^q (2\hat{u} + \hat{t})^3 + C_A^C C_A^q (6\hat{u}^2 + 6\hat{u}\hat{t} + \hat{t}^2)]}{\hat{s}^2(\hat{t} - m_Z^2)},
\]

\[
\frac{d\hat{\sigma}(e^+g \rightarrow e^+g)}{dt} = \frac{d\hat{\sigma}^{G(q)}}{dt} = \frac{\pi\lambda^2}{2M_S^2} \frac{\hat{s}}{\hat{s}^2} \left[ 2\hat{u}^3 + 4\hat{u}^2\hat{t} + 3\hat{u}\hat{t}^2 + \hat{t}^3 \right],
\]

\( \lambda \) is the coupling at the effective scale \( M_S \) and is expected to be of \( \mathcal{O}(1) \), but its sign is not known \( \textit{a priori} \) and \( C_V = T^3 - 2\sin^2\theta_W Q_f \) and \( C_A = T_3 \) are the usual vector and axial-vector couplings of the fermions to the \( Z \). In our work we will explore the sensitivity of our results to the choice of the sign of \( \lambda \). The \( e^+g \) subprocess is, of course,
absent in the SM, and is completely a result of introducing the new interactions. The new interactions also contribute in the $e^+q$ channel, where there is an interference between the SM amplitude and the amplitude due to the new physics.

Figure 1: Illustrating the effects of extra dimensions on positron scattering at HERA for $\lambda = \pm 1$. The ordinate represents the ratio $R \equiv \frac{d\sigma/dQ^2_e}{d\sigma/dQ^2_e|_{SM}}$. The values of $M_S$ correspond to 543 (436) GeV for $\lambda = +1 (-1)$ for the H1 data and to 567 (485) GeV for $\lambda = +1 (-1)$ for the ZEUS data, these being the respective bounds at 95 % C.L. from a $\chi^2$-fit to the data exhibited in the figure.

The H1 [21] and ZEUS [22] collaborations at HERA have presented their results from their combined 1994-97 runs in terms of the quantity

$$ R = \frac{d\sigma/dQ^2_e|_{exp}}{d\sigma/dQ^2_e|_{SM}} $$

where $Q^2_e$ is the momentum-transfer squared constructed from the $e^+$ track. In our parton-level simulation, $Q^2_e \sim -\hat{t}$. We use this observable to compare against

$$ R = \frac{d\sigma/dQ^2_e|_{SM+NSM}}{d\sigma/dQ^2_e|_{SM}} $$

where NSM denotes non-Standard Model, and use the data to put bounds on the value of $M_S$. The cross-section $d\sigma/dQ^2_e$ is given as

$$ \frac{d\sigma(e^+p \rightarrow e^+jet)}{dQ^2_e} = \sum \int dx f_{i/p}(x) \frac{d\hat{\sigma}}{d\hat{t}}, $$

4
\( f_{i/p} \) denotes the probability of finding a parton \( i \) in the proton. The sum in Eq. 10 runs over the contributing subprocesses.

The results of our numerical evaluation of the cross-section are shown in Figure 1. We have plotted \( R \) as a function of \( Q_e^2 \), as obtained from our calculation and compared it with each experiment separately. For our computations, we use the cuts as used in the two experiments and we have used CTEQ4 parton densities [23] taken from PDFLIB [24]. The curves represent the 95% C.L. bounds from a \( \chi^2 \) fit to each set of data. These fits yield the bound on \( M_S \) to be 543 (436) GeV for \( \lambda = +1(-1) \) for the H1 data and 567 (485) GeV for \( \lambda = +1(-1) \) for the ZEUS data. For higher values of \( M_S \), the non-Standard contribution decreases, so that the curves move closer to \( R = 1 \).

The data on \( R \) from H1 and ZEUS collaborations, in fact, show a deviation from the SM for \( Q_e^2 \) values beyond \( 10^4 \) GeV\(^2 \). The errors on the ratio \( R \) at these large values of \( Q_e^2 \) are large, so this discrepancy with the SM prediction is not very significant statistically. The result of including the non-Standard Model contribution is to improve the \( \chi^2 \) of our fit to the data. We find that for \( \lambda = -1 \), the theoretical prediction, with the non-Standard Model contribution, fits the data rather well, even accounting for the modest dip in \( R \) at a value of \( Q_e^2 \) just below \( 10^4 \) GeV\(^2 \)! This behaviour follows from the fact that the interference term in the quark-initiated sector dominates at relatively low \( Q_e^2 \), and this gives a negative contribution for \( \lambda = -1 \). As one moves to larger \( Q_e^2 \), the gluon-initiated contribution starts to dominate and gives the increase at large \( Q_e^2 \). Given that a discrepancy with the SM seen in the experiments exists, though it is not statistically compelling, the bounds we derive from the data are not as strong as compared to those derived from Tevatron data on dilepton production [19] and \( t\bar{t} \) production [20]. In the event that the HERA experiments improve their data in the large \( Q_e^2 \) region and find good agreement with the SM, the bounds presented in this paper are likely to improve considerably. For example, with the 20 fold increase in luminosity planned in HERA experiments in the next few years, assuming that the data are centred around the Standard Model prediction we estimate that the bounds on \( M_S \) would go up to around 600 (925) GeV for \( \lambda = +1(-1) \).

We have studied the effect of large extra dimensions and a TeV scale gravity on the deep inelastic scattering cross-section at HERA. The fits of the theoretical curves to the data yield the value of the effective string scale, \( M_S \), to be \( > 543 \) (436) GeV for \( \lambda = +1(-1) \) for the H1 data and \( > 567 \) (485) GeV for \( \lambda = +1(-1) \) for the ZEUS data. The bounds are likely to increase with any improvement in the data, especially at large \( Q_e^2 \).
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