Progress in bottom-quark fragmentation

G. Corcella
Department of Physics, CERN
Theory Division
CH-1211 Geneva 23, Switzerland

Abstract

I review recent progress on bottom-quark fragmentation in $e^+e^-$ annihilation, top-quark decay and $H \to b\bar{b}$ processes. I discuss the implementation of collinear and soft resummation, and the inclusion of non-perturbative information taken from LEP and SLD data.

Heavy-quark physics is currently one of the major fields of investigation in experimental and theoretical particle physics. In order to investigate heavy-quark phenomenology, fixed-order calculations are reliable as long as one considers inclusive observables, such as total cross sections. For less inclusive quantities, one needs to resum large contributions, which correspond to collinear or soft parton radiation.

In this talk I will investigate bottom-quark production in $e^+e^- \to b\bar{b}$ processes, top-quark decay $t \to bW$, and Standard Model Higgs decay $H \to b\bar{b}$. In particular, I wish to study the energy distribution of $b$ quarks and $b$-flavoured hadrons in such processes. The results will be expressed in terms of the $b$ normalized energy fraction $x_b$, which is given, e.g. in Higgs decay $H \to b\bar{b}(g)$, by:

$$x_b = \frac{2p_{h\cdot p_H}}{m_H^2}.$$  

(1)

The differential distribution for the production of a massive $b$ quark at next-to-leading order (NLO) in the strong coupling constant $\alpha_S$ is given by the following general result:

$$\frac{1}{\Gamma_0} \frac{d\Gamma}{dx_b} = \delta(1-x_b) + \frac{\alpha_S(\mu)}{2\pi} \left[ P_{qq}(x_b) \ln \frac{m_H^2}{m_b^2} + A(x_b) \right] + \mathcal{O}\left( \frac{m_b^2}{m_H^2} \right)^p.$$  

(2)

In Eq. (2) $\Gamma_0$ is the width of the Born process, $\mu$ is the renormalization scale, $A(x_b)$ is a function independent of $m_b$, $P_{qq}(x_b)$ is the Altarelli–Parisi splitting function:

$$P_{qq}(x_b) = C_F \left( \frac{1 + x_b^2}{1 - x_b} \right)_+.$$  

(3)

Equation (2) is formally equal to the bottom energy spectrum in top decay or $e^+e^-$ processes: one will just have to replace $m_H$ with the top mass or the centre-of-mass energy $\sqrt{s}$ of the $e^+e^-$ collision. Function $A(x_b)$ is instead process-dependent.

Equation (2) presents a large mass logarithm $\sim \alpha_S \ln(m_H^2/m_b^2)$, which needs to be resummed to all orders to improve the prediction. To achieve this goal, we can follow the approach of perturbative fragmentation functions [2], which expresses the energy spectrum of a heavy quark as the convolution of a coefficient function, describing the emission of a massless parton, and a perturbative fragmentation function $D(m_b, \mu_F)$, associated with the transition of a massless parton into a massive quark:

$$\frac{1}{\Gamma_0} \frac{d\Gamma_b}{dx_b}(x_b, m_H, m_b) = \sum_i \int_{x_b}^1 \frac{dz}{z} \left[ \frac{1}{\Gamma_0} \frac{d\Gamma_i}{dz}(z, m_H, \mu, \mu_F) \right]^{\text{MS}} D_{i,\text{MS}}^{\text{MS}} \left( \frac{x_b}{z}, \mu_F, m_b \right) + \mathcal{O}\left( (m_b/m_H)^p \right).$$  

(4)

In Eq. (4), $d\Gamma_i/dz$ is the differential width for the production of a massless parton $i$ in Higgs decay with an energy fraction $z$; $D_i(x, \mu_F, m_b)$ is the perturbative fragmentation function for a parton $i$ to fragment...
into a massive b quark, $\mu_F$ is the factorization scale. Neglecting $g \to b\bar{b}$ splitting, $i = b$ on the right-hand side of Eq. (4). The coefficient function for $H \to b\bar{b}$ processes has been computed in [3] and reads, in the $\overline{\text{MS}}$ factorization scheme:

$$
\left[ \frac{1}{\mu_0^2} \frac{dF_b}{dz}(z, m_H, \mu, \mu_F) \right]_{\overline{\text{MS}}} = \delta(1-z) + \frac{\alpha_s(\mu)C_F}{2\pi} \left[ \ln \frac{m_H^2}{\mu_F^2} + \frac{1}{1-z} \ln \left( \frac{1+z^2}{1-z} \right) + \left( \frac{2}{3} \pi^2 + \frac{3}{2} \right) \delta(1-z) + 1 - z \frac{3}{2} \frac{z^2}{1-z} \right] - (1+z)[\ln(1-z) + 2 \ln z - 6 \frac{\ln z}{1-z}] - 2 \frac{\ln z}{1-z} - 2 \left( \ln \frac{1-z}{1-z} \right) + .
$$

(5)

As pointed out in Ref. [3], in the calculation of Eq. (5), using the $\overline{\text{MS}}$-renormalized Yukawa coupling $\bar{y}_b(m_b)$ for the $Hb\bar{b}$ vertex turned out to be essential. The $\overline{\text{MS}}$ NLO coefficient function for $e^+e^-$ annihilation was calculated in [2], for top decay in [1].

The perturbative fragmentation function follows the Dokshitzer–Gribov–Altarelli–Parisi (DGLAP) evolution equations [4, 5]. Its value at a given scale $\mu_F$ can be obtained once an initial condition is given. In [2] the initial condition $D_b^{\text{ini}}(x_b, \mu_0 \mu_F, m_b)$ was calculated and its process-independence was established on more general grounds in [6]. It is given at NLO by:

$$
D_b^{\text{ini}}(x_b, \mu_0 \mu_F, m_b) = \delta(1-x_b) + \frac{\alpha_s(\mu_0^2)C_F}{2\pi} \left[ \ln \frac{\mu_F^2}{m_b^2} - 2 \ln(1-x_b) - 1 \right] + .
$$

(6)

The NNLO initial condition was calculated in Refs. [7, 8].

As discussed in [2], solving the DGLAP equations for an evolution from $\mu_0 \mu_F$ to $\mu_F$, with a NLO kernel, allows one to resum leading (LL) $\alpha_s^n \ln^n(\mu_F^2/\mu_0^2)$ and next-to-leading (NLL) $\alpha_s^3 \ln^{-1}(\mu_F^2/\mu_0^2)$ logarithms (collinear resummation). The explicit expression for the solution of the DGLAP equations can be found, e.g., in Ref. [2].

In particular, in $H \to b\bar{b}$ processes, for an evolution from $\mu_0 \mu_F \simeq m_b$ to $\mu_F \simeq m_H$, the large logarithms $\ln(m_H^2/m_b^2)$ appearing in the massive calculation (2) are resummed with NLL accuracy. Likewise, since the perturbative fragmentation approach and the collinear resummation are process-independent, we can resum, following the same procedure, NLL logarithms $\ln(s/m_b^2)$ and $\ln(m_b^2/m_b^2)$ in the massive $b$ spectrum in $e^+e^-$ annihilation [2] and top decay [1].

Furthermore, both the coefficient functions in Eq. (5) and Refs. [1, 6] and the initial condition of the perturbative fragmentation function (6) present terms that become large once the $b$-quark energy fraction $x_b$ approaches 1, which corresponds to soft-gluon radiation. Soft contributions to the initial condition are process-independent and were resummed in [6] in the NLL approximation. NLL soft-gluon resummation in the coefficient function is instead process-dependent and it was implemented in $e^+e^-$ annihilation [6], top decay [9] and $H \to b\bar{b}$ [3] processes. For the sake of brevity, I do not report here the expression for the soft-resummed initial condition and coefficient functions.

In Fig. 1, the bottom energy distribution in Higgs decay is given; the result is qualitatively similar to what can be found in $e^+e^-$ processes or top decay. I have plotted the NLO massive calculation (dashed), including NLL collinear resummation (dotted), and with both NLL collinear and soft resumations (solid). I have set $\mu = \mu_F = m_H$, $\mu_0 = \mu_0 \mu_F = m_b$, $m_H = 120$ GeV, $m_b = 5$ GeV. The fixed-order calculation lies below the resummed ones and grows as $x_b \to 1$, since $\sim 1/(1-x_b)$; the collinear-resummed spectrum exhibits instead a sharp peak at large $x_b$. Soft resummation is relevant at $x_b > 0.6$: the distribution is further smoothed and shows the Sudakov peak at $x_b \simeq 0.97$. References [1, 3, 6] show that, after collinear and soft logarithms are resummed, the $x_b$ distribution exhibits very mild dependence on factorization and renormalization scales, which corresponds to a reduction of the theoretical uncertainty on the prediction.

It is interesting to compare the $b$ spectrum in the considered three processes. In Fig. 2 we plot the $x_b$ distributions in $H \to b\bar{b}$, $e^+e^- \to b\bar{b}$ at $\sqrt{s} = m_Z$, i.e. LEP I and SLD centre-of-mass energy, and
Figure 1: $b$ spectrum in Higgs decay according to the NLO massive calculation (dashed), including NLL collinear resummation (dotted), and both NLL collinear and soft resummations (solid). In the inset, the same curves are shown at large $x_b$, on a logarithmic scale.

Figure 2: $b$ spectrum in Higgs decay, $e^+e^-$ annihilation at $\sqrt{s} = 91.2$ GeV (dashes), top decay (dots). All curves include NLL collinear and soft resummation.
t \to bW. The b spectrum in \( H \) decay is the highest at small \( x_b \) and the lowest at large \( x_b \); in top decay it is shifted toward large \( x_b \) and sharply peaked very close to 1. The \( e^+ e^- \to b\bar{b} \) prediction is similar to the one yielded by Higgs decay, although some difference is still visible around the Sudakov peak. It was shown in [3] that setting \( \sqrt{s} = m_H \) makes the predictions in \( H \to b\bar{b} \) and \( e^+ e^- \) annihilation more similar still. The difference noticed between the b spectra in Higgs and top decay can be experimentally relevant when distinguishing jets initiated by b quarks in the process \( pp \to t\bar{t}H \), followed by \( H \to b\bar{b} \). In fact, this is one of the search channels of the Standard Model Higgs boson at the LHC, for a Higgs mass \( m_H \leq 135 \text{ GeV} \) [10].

I would like to present results for b-flavoured B hadron production in the investigated processes. The B spectrum will be given by the convolution of the b-energy distribution with a non-perturbative fragmentation function associated with the hadronization. As in [1, 3, 9], I consider a power law with two tunable parameters

\[
D^{np}(x; \alpha, \beta) = \frac{1}{B(\beta + 1, \alpha + 1)}(1 - x)^\alpha x^\beta, \tag{7}
\]

the model of Kartvelishvili et al. [11]

\[
D^{np}(x; \delta) = (1 + \delta)(2 + \delta)(1 - x)x^\delta, \tag{8}
\]

and the Peterson model [12]

\[
D^{np}(x; \epsilon) = \frac{A}{x[1 - 1/x - \epsilon/(1 - x)]^2}. \tag{9}
\]

In Eq. (7), \( B(x, y) \) is the Euler beta function; in (9) \( A \) is a normalization constant.

The parameters \( \alpha, \beta, \delta \) and \( \epsilon \) can be obtained after fitting models (7)-(9) to \( x_B \) data in \( e^+ e^- \) collisions, where \( x_B \) is the normalized B energy fraction. In [3, 15], data from ALEPH [13] and SLD [14] Collaborations were considered. In [15] it was reported that if we fit independently the non-perturbative models to ALEPH and SLD, we get pretty different best-fit parameters. Unlike ALEPH, which detected B mesons, SLD was able to reconstruct some B baryons as well; however it is a very small fraction of the whole sample and that may not be the reason for the discrepancies discussed in [15].

In [3] a combined fit to ALEPH and SLD was performed instead, as if all data came from the same sample. This approach is used here too. In the fits, the correlations among data points are neglected, and the considered data are in the range \( 0.18 \leq x_B \leq 0.94 \). The best-fit parameters, along with the \( \chi^2 \) per degree of freedom, are listed in Table 1.

| parameter | value |
|-----------|-------|
| \( \alpha \) | 0.90 \pm 0.15 |
| \( \beta \) | 16.23 \pm 1.37 |
| \( \chi^2(\alpha, \beta)/\text{dof} \) | 33.42/31 |
| \( \delta \) | 17.07 \pm 0.39 |
| \( \chi^2(\delta)/\text{dof} \) | 33.80/32 |
| \( \epsilon \) | \( (1.71 \pm 0.09) \times 10^{-3} \) |
| \( \chi^2(\epsilon)/\text{dof} \) | 166.36/32 |

Table 1: Results of combined fits to \( e^+ e^- \to b\bar{b} \) data from the ALEPH and SLD Collaborations, using NLO coefficient functions, NLL DGLAP evolution and NLL soft-gluon resummation. I have set \( \Lambda = 200 \text{ MeV} \), \( \mu_{QF} = \mu_0 = m_b = 5 \text{ GeV} \) and \( \mu_F = \mu = \sqrt{s} = 91.2 \text{ GeV} \).

The power law (7) and the Kartvelishvili model (8) yield very good fits, while the Peterson model (9) is unable to reproduce the data. In [3] it was also shown that the inclusion of NLL soft resummation in the coefficient function and in the initial condition of the perturbative fragmentation function is necessary to reproduce the data. Figure 3 shows the data samples, along with the fitting curves corresponding to the central values of \( \alpha, \beta, \delta \) and \( \epsilon \) quoted in Table 1. We see that models (7) and (8) yield approximately the same distribution: in fact, from Table 1 we learn that, within the error ranges, \( \alpha \) is consistent with 1 and \( \beta \) with \( \delta \). The Peterson non-perturbative fragmentation function is unable to fit the data.
Figure 3: ALEPH and SLD data on $B$-hadrons, along with the best-fit curves according to the power law (7) (solid line), the Kartvelishvili model (dashes) and the Peters model (dots), with $\alpha$, $\beta$, $\delta$ and $\epsilon$ given by the central values reported in Table 1. All curves include NLL collinear and soft resummation in the parton-level calculation.

Given the results of Table 1, we can predict the $B$-hadron spectrum in Higgs or top decay. In Fig. 4 we show the $x_B$ distribution in Higgs decay according to models (7) and (8). Plotted are the edges of bands at one-standard-deviation confidence level. We can see that the predictions yielded by the two models are in statistical agreement.

In Fig. 5 we compare $b$-flavoured hadron spectra in $e^+e^-$ annihilation at LEP energy, Higgs and top decay, according to the power law (7). The results are similar to the ones at parton level presented in Fig. 2: the top-decay spectrum is shifted toward larger $x_B$, the $H \to b\bar{b}$ one is the highest at low $x_B$, the $e^+e^-$-prediction lies between the two.

Finally, we can use the moments on $B$ production in $e^+e^-$ annihilation from the DELPHI Collaboration [16] to predict the same moments in top and Higgs decay. In $N$-space we have: $\sigma_N^b = \sigma_N^b D_{NP}^b$, $\Gamma_N^b = \Gamma_N^b D_{NP}^b = \Gamma_N^b \sigma_N^b / \sigma_N$, where $\sigma_N$ and $\Gamma_N$ are the moments of $e^+e^-$-annihilation cross section, Higgs or top width, at hadron or parton level. The results are shown in Table 2: the moments of the top-decay width are the largest, the ones of the Higgs width are the lowest, and the $e^+e^-$-annihilation ones are in the middle. This result is consistent with what is observed in $x_B$-space in Fig. 5.

| $e^+e^-$ data $\sigma_N^b$ | $\langle x \rangle$ | $\langle x^2 \rangle$ | $\langle x^3 \rangle$ | $\langle x^4 \rangle$ |
|---------------------------|-------------------|-------------------|-------------------|-------------------|
|                           | 0.7153±0.0052     | 0.5401±0.0064     | 0.4236±0.0065     | 0.3406±0.0064     |
| $e^+e^-$ NLL $\sigma_N^b$| 0.7801            | 0.6436            | 0.5479            | 0.4755            |
| $D_{NP}^b$ [B]            | 0.9169            | 0.8392            | 0.7731            | 0.7163            |
| $t$-decay NLL $\Gamma_N^b$| 0.7884           | 0.6617           | 0.5737           | 0.5072           |
| $t$-decay $\Gamma_N^b$   | 0.7228           | 0.5553           | 0.4435           | 0.3633           |
| $H$-decay NLL $\Gamma_N^b$| 0.7578           | 0.6162           | 0.5193           | 0.4473           |
| $H$-decay $\Gamma_N^b$   | 0.6948           | 0.5171           | 0.4015           | 0.3204           |

Table 2: Experimental data for the moments $\sigma_N^b$ from DELPHI [16], the resummed $e^+e^-$ perturbative calculations for $\sigma_N^b$ [6], the extracted non-perturbative contribution $D_{NP}^b$. Using the resummed perturbative result $\Gamma_N^b$ on top and Higgs partonic widths, a prediction for the moments $\Gamma_N^b$ $t \to bW$ and $H \to b\bar{b}$ processes is given.

In summary, I have reviewed recent results on bottom-quark fragmentation in $e^+e^-$ annihilation, Higgs and top decay. Parton-level spectra exhibit a relevant impact of the inclusion of NLL collinear and soft resummation; using information from LEP and SLD experiments, $b$-flavoured hadron energy distributions have been predicted.
Figure 4: $B$-hadron spectrum in Higgs decay: the hadronization is described according to the power law (7) (solid) and the Kartvelishvili model (8) (dashes). Plotted are one-standard-deviation bands for the non-perturbative parameters $\alpha$, $\beta$ and $\delta$.

Figure 5: One-standard-deviation prediction for $B$-hadron spectra in Higgs decay (solid), top decay (dashes) and $e^+e^-$ annihilation at $\sqrt{s} = 91.2$ GeV (dotted), according to the hadronization model (7).
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