Constraints on the sum of neutrino masses using cosmological data including the latest extended Baryon Oscillation Spectroscopic Survey DR14 quasar sample

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Abstract: We investigate the constraints on the sum of neutrino masses (\(\Sigma m_\nu\)) using the most recent cosmological data, which combines the distance measurement from baryonic acoustic oscillation in the extended Baryon Oscillation Spectroscopic Survey DR14 quasar sample with the power spectra of temperature and polarization anisotropies in the cosmic microwave background from the Planck 2015 data release. We also use other low-redshift observations, including the baryonic acoustic oscillation at relatively low redshifts, Type Ia supernovae, and the local measurement of the Hubble constant. In the standard cosmological constant \(\Lambda\) cold dark matter plus massive neutrino model, we obtain the 95% upper limit to be \(\Sigma m_\nu < 0.129\) eV for the degenerate mass hierarchy, \(\Sigma m_\nu < 0.159\) eV for the normal mass hierarchy, and \(\Sigma m_\nu < 0.189\) eV for the inverted mass hierarchy. Based on Bayesian evidence, we find that the degenerate hierarchy is positively supported, and the current data combination cannot distinguish between normal and inverted hierarchies. Assuming the degenerate mass hierarchy, we extend our study to non-standard cosmological models including generic dark energy, spatial curvature, and extra relativistic degrees of freedom, but find these models are not favored by the data.

Keywords: neutrino mass, mass hierarchy, cosmological constraints

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1 Introduction

The phenomenon of neutrino oscillation has provided convincing evidence for non-zero neutrino masses and mass splittings (see Ref. [1] for a review). However, the current experimental results cannot decisively tell if the third neutrino is heavier than the other two or not. There are thus two potential mass hierarchies for three active neutrinos, namely, the normal hierarchy (NH), in which the third neutrino is the heaviest, and the inverted hierarchy (IH), in which the third neutrino is the lightest. The sum of neutrino masses (\(\Sigma m_\nu\)) also remains unknown, and different neutrino hierarchies would have different total masses. Taking into account the experimental results of the squared mass differences [1], the lower bound on \(\Sigma m_\nu\) is estimated to be 0.06 eV for NH, and 0.10 eV for IH. The experimental upper limit on \(\Sigma m_\nu\) is much looser. For instance, the most sensitive neutrino mass measurement to date, involving the kinematics of tritium beta decay, provides a 95% confidence level (CL) upper bound of 2.05 eV on the electron anti-neutrino mass [2].

Cosmology plays a significant role in exploring the neutrino masses (see Ref. [3] for a review), since it can place stringent upper limits on \(\Sigma m_\nu\), which is a key to resolving the neutrino masses by combining with the squared mass differences measured. At the beginning of the Universe, massive neutrinos would initially be relativistic, and become non-relativistic after a transition when their rest masses begin to dominate. Imprints would have been left on the cosmic microwave background (CMB) and the large scale structure (LSS). The CMB is affected through the early-time integrated Sachs-Wolfe effect [4], which shifts the amplitude and location of the CMB acoustic peaks due to a change of the redshift of matter-radiation equality. The LSS is modi-
fied through suppressing the clustering of matter, due to the large free-streaming velocity of neutrinos. Therefore, massive neutrinos can be weighed using measurements of the CMB and LSS [5–8].

Assuming the Λ cold dark matter (ΛCDM) model and that the three active neutrinos are in a degenerate hierarchy (DH), namely with equal mass, the Planck Collaboration recently reported the 95% CL upper limit on Σmν to be 0.49eV using the CMB temperature and polarization anisotropies from the Planck 2015 data [5]. Adding the gravitational lensing of the CMB from Planck 2015 data relaxes the upper limit to be 0.59 eV, which is a less stringent bound. Given the accuracy of the Planck 2015 data, we assume the neutrinos being NH and IH would have a negligible impact on the constraints on the sum of neutrino masses.

Combining baryon acoustic oscillation (BAO) and other low redshift data such as the local Hubble constant H0 and Type Ia supernovae can further tighten the constraints [9–18]. There are also efforts to tighten the constraints by combining experimental data from particle physics [19–21]. Especially, since the BAO data can significantly break the acoustic scale degeneracy, the sum of neutrino masses is tightly constrained to 0.15 eV by Ref. [22] after adding the BAO data from LSS surveys [23–25]. This result is close to the lower bound of neutrino masses in the IH, namely 0.10 eV. Current tightest constraints on the neutrino total mass [26, 27] from cosmological data, especially from Planck high-l CMB polarization data, have already reached ~0.10 eV given certain combinations of the data set, implying the NH is favoured. Nevertheless, it is necessary to take the mass prior set by the neutrino mass hierarchy into account to get consistent constraints. After taking the squared mass differences into account, we [22, 28] found that the upper limit of the neutrino total mass becomes 0.18 eV for NH, and 0.20 eV for IH. It was also shown in Refs. [28–36] that extensions of the standard cosmology model, e.g., dynamical dark energy and non-zero spatial curvature, would have subtle impacts on constraining the sum of neutrino masses. In this work we investigate the mass prior effect on constraining the neutrino total mass in the standard cosmological model.

Most recently, the SDSS-IV extended Baryon Oscillation Spectroscopic Survey (eBOSS) [37] measured a BAO scale in redshift space in the redshift interval 1<z<2 for the first time, using the clustering of 147,000 quasars with redshift 0.8<z<2.2. A spherically averaged BAO distance to z = 1.52 is obtained, namely, D_V (z = 1.52) = 3843±147(r_s/r_d, fid) [Mpc], which is of 3.8% precision. In this paper, this data point is denoted by eBOSS DR14 for simplicity. The SDSS-IV eBOSS measurement of the BAO scale is expected to break the degeneracy between the NH and IH scenarios of the three active neutrinos at 2σ confidence level [38].

In this work, we constrain the sum of neutrino masses in the ΛCDM model by adding the recently released eBOSS DR14 data. Besides the maximum likelihood analysis, we also employ Bayesian statistics to infer the parameters, and especially to perform model selections. We expect to show that the current observations can improve the previous constraints on Σmν significantly, and that the neutrino mass hierarchies have to be considered to analyze the current observational data. Due to possible degeneracy between the sum of neutrino masses and a few extended cosmological parameters, similar studies have been done under the framework of extended cosmological models by introducing generic dark energy (w), non-zero spatial curvature (Ω_k), and extra relativistic degree of freedom (N_elec), respectively.

The rest of this paper is arranged as follows. Section 2 introduces the cosmological models used, the cosmological observations, and the statistical analysis methods. Section 3 gives the result of the constraints on Σmν in the ΛCDM model, while Section 4 gives the effect of a few extended cosmological parameters on the constraints on Σmν. In Section 5, the conclusions are summarized.

2 Models, dataset and methodology

2.1 Cosmological models

In the ΛCDM plus massive neutrino model (hereafter, νΛCDM for short), we put constraints on the sum of neutrino masses with and without taking into account the neutrino mass hierarchies. Specifically, we consider the DH, NH, and IH of the neutrinos. Based on neutrino oscillation, the squared mass differences between the three active neutrinos have been measured to be Δm^2_21 = 7.5×10^{-5}eV^2 and Δm^2_31 = 2.5×10^{-3}eV^2 [1]. Therefore, there is a lower bound, i.e. Σmν ≳ 0.06 eV for the NH, and Σmν ≳ 0.10 eV for the IH. There is not such a lower bound for the DH, but Σmν should have a positive value.

In the νΛCDM model, there are six base parameters denoted by \{ω_b, ω_c, 100θ_MC, τ, n_s, ln(10^10 A_s)\} plus a seventh independent parameter denoted by Σmν for the sum of neutrino masses. Here ω_b and ω_c are, respectively, the physical densities of baryons and cold dark matter today. θ_MC is the ratio between the sound horizon and angular diameter distance at the decoupling epoch. τ is the Thomson scatter optical depth due to reionization. n_s and A_s are, respectively, the spectral index and amplitude of the power spectrum of primordial curvature perturbations. The pivot scale is set to be k_p = 0.05 Mpc^{-1}.

When generic dark energy is taken into account, an eighth independent parameter is introduced, which describes the equation of state (EoS) of the dark energy.
This parameter is denoted by $w$, and the corresponding cosmological model is the $\nu w$CDM model. When spatial curvature is considered, the eighth independent parameter is denoted by $\Omega_k$, and the corresponding model is the $\nu \Omega_k$CDM model. When an extra relativistic degree of freedom is considered, the eighth independent parameter is denoted by $N_{\nu eff}$, and the corresponding model is the $\nu N_{\nu eff}$CDM model. For these three extended models, we only consider the upper limits on $\Sigma m_{\nu}$ in the DH scenario of three massive neutrinos, because the neutrino mass hierarchy would have negligible effects on the constraints given the current cosmological observations.

2.2 Cosmological data

The cosmological observations used in this work include CMB, BAO, and other low-redshift surveys. To be specific, the CMB data are composed of temperature anisotropies, polarizations, and gravitational lensing of the CMB reported by the Planck 2015 data release [5]. The CMB lensing is used here since it is very sensitive to the neutrino masses [39]. Specifically, we utilize the angular power spectra of TT, TE, EE, lowTEB, and gravitational lensing of the CMB. The BAO data points come from the 6dF galaxy survey [23], SDSS DR7 main galaxy sample [24], SDSS-III BOSS DR12 LOWZ and CMASS galaxy samples [25], and the SDSS-IV eBOSS DR14 quasar sample [37]. The supernovae dataset is the “joint light-curve analysis” (JLA) compilation of the supernovae of type Ia (SNe Ia) [40]. The local measurement of Hubble constant ($H_0$) comes from the Hubble Space Telescope [41]. The full data combination combines together all the cosmological observational data mentioned above. In fact, one can further add other astrophysical data, such as galaxy weak lensing [42, 43], redshift space distortion [44], and Planck cluster counts [45], to improve the constraints on the neutrino masses. Though these datasets are directly related to the neutrino masses, the amplitudes of power spectra of the cosmological perturbations obtained from these observations are in tension with that obtained from the Planck CMB data. It is believed that these observations have underlying uncontrolled systematics, which may bias the global fitting. Hence, we do not take them into account in this paper.

2.3 Statistical methods

Given the observational dataset and the corresponding likelihood functions, we use the Markov-Chain Monte-Carlo (MCMC) sampler in CosmoMC [46] to estimate across the parameter space, and the PolyChord [47, 48] plug-in of CosmoMC to calculate the Bayesian evidence for model selection.

The Bayesian evidence ($E$) is defined as an integral of posterior probability distribution function (PDF), i.e., $P(\theta)$, over the parameter space \{\theta\}, i.e. $E = \int d\theta P(\theta)$ [49]. Given two different models $M_1$ and $M_2$, the logarithmic Bayesian factor is evaluated as $\Delta \ln E = \ln E_{M_1} - \ln E_{M_2}$. When $0 < \Delta \ln E < 1$, the given dataset indicates no significant support for either model. When $1 < \Delta \ln E < 3$, there is positive support for $M_1$. When $3 < \Delta \ln E < 5$, there is strong support for $M_1$. When $\Delta \ln E > 5$, there is very strong support for $M_1$. Conversely, negative values mean that the dataset supports $M_2$, rather than $M_1$.

In this work, $M_2$ usually denotes $\nu_{DH}$CDM while $M_1$ denotes one of other models. An exception is that $M_1$ denotes the NH while $M_2$ denotes the IH, when we compare the NH and the IH in $\nu$CDM. The parameter space \{\theta\} consists of the six base parameters plus the sum of neutrino masses for $\nu$CDM, while an eighth parameter is further added when an extended model is considered. For each model, the independent parameters have been shown explicitly in Section 2.1.

We also evaluate the best-fit $\chi^2$. For any scenario, a smaller value of the best-fit $\chi^2$ implies that this scenario fits the dataset better. For both Bayesian and maximum likelihood methods, the prior ranges for all the independent parameters are set to be sufficiently wide to avoid affecting the results of the data analysis.

3 Results for $\nu$CDM model

For the $\nu$CDM model, we present the results of parameter inference and model comparison in Table 1 and in Fig. 1. To be specific, we show the 68% CL constraints on the six base parameters of the $\Lambda$CDM, and the 95% CL upper limits on the sum of neutrino masses in Table 1. For the three mass hierarchies of massive neutrinos, we find that the constraints on the six base parameters of the $\Lambda$CDM are compatible within 68% CL. We also list in Table 1 the best-fit values of $\chi^2$ and the logarithmic Bayesian evidence. In Fig. 1, we show the posterior PDFs of the sum of neutrino masses. The red, green, and blue solid curves, respectively, denote the posterior PDFs of $\Sigma m_\nu$ for DH, NH, and IH of massive neutrinos. In addition, we wonder if the difference in neutrino mass constraints are due to the different priors in the parameter $\Sigma m_\nu$. Therefore, we also show in Fig. 1 the neutrino mass constraints for the $\nu_{DH}$CDM model with two non-vanishing lower bounds, i.e., 0.06 eV (red dashed curve) and 0.10 eV (red dot-dashed curve).

For three neutrinos with degenerate mass, the upper limit on the sum of neutrino masses is obtained to be

$$\Sigma m_\nu < 0.129 \text{ eV} \quad (95\% \text{ CL}).$$

This upper limit is close to the lower bound of 0.10 eV required by the IH scenario. In fact, the upper limit even becomes $\Sigma m_\nu < 0.10 \text{ eV}$ if the gravitational lensing of the CMB is discarded in the global fitting. As ex-
expected, the upper limit $\Sigma m_\nu < 0.129$ eV is 3.7% tighter than the existing one, e.g. $\Sigma m_\nu < 0.134$ eV in Wang et al. [28], which did not include the eBOSS DR14 data, and $\Sigma m_\nu < 0.197$ eV in Zhang [31], which used a different BAO dataset. However, this constraint is slightly looser than that of $\Sigma m_\nu < 0.12$ eV, which was obtained by combining BOSS Lyman-α with Planck CMB [15].

Table 1. The 68% CL constraints on the six base parameters of the ΛCDM, and the 95% CL upper limits on the sum of neutrino masses, as well as the best-fit values of $\chi^2$ and the logarithmic Bayesian evidence, i.e. $\ln E$ (68% CL).

|         | $\nu_{\text{DH}}$ACDM | $\nu_{\text{NH}}$ACDM | $\nu_{\text{IH}}$ACDM |
|---------|-----------------------|-----------------------|-----------------------|
| $\omega_b$ | 0.02232±0.00014       | 0.02240±0.00014       | 0.02242±0.00014       |
| $\omega_c$ | 0.1178±0.0010        | 0.1174±0.0010        | 0.1171±0.0010        |
| $10^9\theta_{\text{MC}}$ | 1.04105±0.00030     | 1.04107±0.00029     | 1.04108±0.00030     |
| $\tau$ | 0.0708±0.0133          | 0.0775±0.0131          | 0.0825±0.0128          |
| $n_s$ | 0.9692±0.0040         | 0.9704±0.0041         | 0.9711±0.0040         |
| $\ln(10^{10} A_s)$ | 3.071±0.025       | 3.084±0.024       | 3.093±0.024       |
| $\Sigma m_\nu$ [eV] | <0.129             | <0.159             | <0.189             |
| $\chi^2_{\text{min}}/2$ | 6832.461       | 6833.353       | 6833.577       |
| $\ln E$ | -6890.50±0.23         | -6892.61±0.23         | -6892.54±0.23         |

![Fig. 1](image_url)

Fig. 1. (color online) The posterior probability distribution functions of the sum of neutrino masses for three mass hierarchies of massive neutrinos. The red, green, and blue solid curves denote DH, NH, and IH, respectively. The red dashed (dot-dashed) curve denotes DH with a lower bound of 0.06 eV (0.10 eV) on $\Sigma m_\nu$. From left to right, the vertical dashed lines denote $\Sigma m_\nu = 0.06$ eV and 0.10 eV, respectively.

The above results reveal the necessity of taking into account the squared mass differences between the three massive neutrinos when one constrains the neutrino masses with current cosmological observations. For three neutrinos with NH, the upper limit on the sum of neutrino masses is given by

$$\Sigma m_\nu < 0.159 \text{ eV} \quad (95\% \text{ CL}),$$  

while for IH, the result is

$$\Sigma m_\nu < 0.189 \text{ eV} \quad (95\% \text{ CL}).$$  

Here we further consider the Bayesian model selection. The logarithmic Bayesian factor between the neutrino NH and IH scenarios is compatible with zero within one standard deviation, while the difference of the best-fit $\chi^2$ is given by $\chi^2_{\text{NH},\text{min}} - \chi^2_{\text{IH},\text{min}} = -0.448$. The adopted dataset is fitted nearly equally well by both NH and IH, but it cannot distinguish the two scenarios. Therefore, more experiments with higher precision are needed in the future to decisively distinguish the neutrino mass hierarchies [32, 50]. In addition, comparing the Bayesian evidences of the two scenarios with that of $\nu_{\text{DH}}$CDM, we find that the $\nu_{\text{DH}}$CDM is positively supported by the adopted data combination. Based on the best-fit $\chi^2$, we find that the $\nu_{\text{DH}}$CDM fits the data combination better, since $\chi^2_{\text{min}}$ in this scenario is smaller by a factor of around 2 than those in the other scenarios. In addition, the constraints in Eqs. (2) and (3) are consistent with those in Ref. [28], which did not include the eBOSS DR14 data. Another existing work [29] has shown that the 95% CL upper bound is $\Sigma m_\nu < 0.118$ eV for the NH, and $\Sigma m_\nu < 0.135$ eV for the IH. These constraints appear to be tighter than those obtained by this work. However, the neutrino mass hierarchy was parameterized in a different way from this work, and the data combination discarded the CMB lensing and the eBOSS DR14 BAO but included the redshift space distortion data.

From Fig. 1, we can confirm that the difference in neutrino mass constraints is mainly due to the different priors in the parameter $\Sigma m_\nu$. Given the current data combination, the posterior PDF of $\Sigma m_\nu$ in the $\nu_{\text{NH}}$CDM ($\nu_{\text{IH}}$ACDM) is approximately overlapped with that in the $\nu_{\text{DH}}$CDM with a lower bound of 0.06 eV (0.10 eV) on $\Sigma m_\nu$. In the $\nu_{\text{DH}}$ACDM model, the 95% CL constraint on $\Sigma m_\nu$ is 0.164 eV for a lower bound of 0.06 eV, while it is 0.186 eV for a lower bound of 0.10 eV. These constraints are consistent with those in Eqs. (2) and (3), respectively. Therefore, the priors in $\Sigma m_\nu$ have significant influence on the constraints on $\Sigma m_\nu$, given the current data. However, $\nu_{\text{NH}}$ACDM and $\nu_{\text{IH}}$ACDM can fit the data slightly better than $\nu_{\text{DH}}$ACDM with non-zero priors in $\Sigma m_\nu$, since the best-fit $\chi^2$ in the former two scenarios are smaller by factors of 3−4 than those in the latter two. Comparing $\nu_{\text{NH}}$ACDM with the $\nu_{\text{DH}}$ACDM+0.06 eV prior, we find negative support for the former scenario due to $\Delta \ln E = -1.29$. Comparing $\nu_{\text{IH}}$ACDM with the $\nu_{\text{DH}}$ACDM+0.1 eV prior, we find no significant support for either scenario due to $\Delta \ln E = -0.11$. 


4 Results for extended cosmological models

Based on the precision of current cosmological observations, the neutrino mass hierarchies have negligible effects on the constraints on $\Sigma m_\nu$ in the extended cosmological models explored here. We thus explore the parameter space by assuming the degenerate mass hierarchy in the following. For the extended cosmological models, we present the results of our data analysis in Table 2 and in Fig. 2. Specifically, we show the 95% CL upper limits on the sum of neutrino masses, and the 68% CL constraints on the remaining seven parameters in Table 2. For each extended model, we depict the 1$\sigma$ and 2$\sigma$ CL contours in the two-dimensional plane spanned by the sum of neutrino masses and the extended parameter in Fig. 2.

Table 2. The 68% CL constraints on the six base parameters of the $\Lambda$CDM, the 95% CL upper limits on the sum of neutrino masses, and the best-fit values of $\chi^2$ and the logarithmic Bayesian evidence, i.e. $\ln E$ (68% CL).

| Parameter          | $\nu_{\text{CDM}}$ | $\nu_{\text{NH}}$ | $\nu_{\text{AH}}$ |
|--------------------|---------------------|--------------------|--------------------|
| $\omega_b$         | 0.02230$\pm$0.00015 | 0.02222$\pm$0.00016 | 0.02252$\pm$0.00018 |
| $\omega_c$         | 0.11860$\pm$0.0012  | 0.11980$\pm$0.0015  | 0.12120$\pm$0.0027  |
| 100$\theta_{\text{MC}}$ | 1.04093$\pm$0.00030 | 1.04074$\pm$0.00034 | 1.04067$\pm$0.00041 |
| $\tau$             | 0.06590$\pm$0.0146  | 0.07360$\pm$0.0159  | 0.07340$\pm$0.0145  |
| $n_s$              | 0.96700$\pm$0.0043  | 0.96430$\pm$0.0049  | 0.97650$\pm$0.0009  |
| $\ln(10^{10} A_s)$ | 3.0630$\pm$0.027   | 3.0820$\pm$0.031   | 3.0840$\pm$0.029   |
| $w$                | $-1.06^0.05_{-0.04}$ | $-1.00^0.02_{-0.02}$ | $-0.98^0.03_{-0.02}$ |
| $\Omega_k$         | $0.0043^0.0024_{-0.0028}$ | $-0.0043^0.0024_{-0.0028}$ | $-0.0043^0.0024_{-0.0028}$ |
| $N_{\text{eff}}$   | $3.264^0.166_{-0.161}$ | $3.264^0.166_{-0.161}$ | $3.264^0.166_{-0.161}$ |
| $\Sigma m_\nu$ [eV] | $<0.214$ | $<0.294$ | $<0.174$ |
| $\chi^2_{\text{min}}/2$ | 6829.089 | 6831.799 | 6832.353 |
| $\ln E$            | $-6892.81\pm0.24$ | $-6892.47\pm0.24$ | $-6893.50\pm0.24$ |

For the $\nu w$CDM model, we obtain the upper limit on the sum of neutrino masses to be $\Sigma m_\nu<0.214$ eV at 95% CL. This upper limit on $\Sigma m_\nu$ is indeed improved compared with the existing ones, e.g. $\Sigma m_\nu<0.268$ eV [28], $\Sigma m_\nu<0.304$ eV [31], and $\Sigma m_\nu<0.25$ eV [34]. The constraint on $w$ is $w=-1.06^{0.05}_{-0.04}$ at 68% CL, deviating from $w=-1$ with a significance of 1.2$\sigma$. From the top panel of Fig. 2, $\Sigma m_\nu$ is found to be anti-correlated with $w$. Compared with the $\nu_{\text{DH}}$CDM model, we find that the logarithmic Bayesian factor is $\ln E_{\nu wCDM}-\ln E_{\nu_{\text{DH}}}=2.31$, and the difference of the best-fit $\chi^2$ is $\chi^2_{\nu wCDM,\text{min}}-\chi^2_{\nu_{\text{DH}},\text{min}}=-6.744$. There is thus negative support for the $\nu w$CDM model, but this extended model fits the adopted dataset better than the $\nu_{\text{DH}}$CDM model.

For the $\nu \Omega_k$CDM model, we obtain the upper limit on the sum of neutrino masses to be $\Sigma m_\nu<0.294$ eV at 95% CL, and the constraint on $\Omega_k$ is $\Omega_k=0.0043^{0.0024}_{-0.0028}$ at 68% CL. The significance of a non-zero value of $\Omega_k$ is found to be around 1.5 standard deviations. From the middle panel of Fig. 2, $\Sigma m_\nu$ is found to be positively correlated with $\Omega_k$. Therefore, adding the parameter $\Omega_k$ worsens the constraints on the neutrino masses. This constraint on $\Sigma m_\nu$ is compatible with the existing one in Ref. [51], which studied two different scenarios of neutrino mass hierarchy. Compared with the $\nu_{\text{DH}}$CDM model, we find that the logarithmic Bayesian factor is $\ln E_{\nu \Omega_kCDM}-\ln E_{\nu_{\text{DH}}}=-1.97$, and the difference of the best-fit $\chi^2$ is given by $\chi^2_{\nu \Omega_kCDM,\text{min}}-\chi^2_{\nu_{\text{DH}},\text{min}}=-1.324$. There is thus negative support for the $\nu \Omega_k$CDM model, even though this extended model fits the adopted dataset slightly better than the $\nu_{\text{DH}}$CDM model.

For the $\nu N_{\text{eff}}$CDM model, we obtain the upper limit on the sum of neutrino masses to be $\Sigma m_\nu<0.17$ eV at 95% CL, and the constraint on $N_{\text{eff}}$ is $N_{\text{eff}}=3.265^{0.159}_{-0.157}$ at 68% CL. Since $N_{\text{eff}}=3.046$ in standard $\Lambda$CDM model, the significance of extra relativistic degree is 1.4$\sigma$. From the bottom panel of Fig. 2, $\Sigma m_\nu$ is found to be positively
correlated with $N_{\text{eff}}$. This constraint on $\Sigma m_\nu$ is looser than the existing one $\Sigma m_\nu<0.14eV$ in Ref. [14], which used the high-$\ell$ CMB data and the Lyman-$\alpha$ data. Compared with the $\nu_{\text{CDM}}$ model, we find that the logarithmic Bayesian factor is $\ln E_{\nu_{\text{CDM}}} - \ln E_{\text{DH}} = -3.00$, and the difference of the best-fit $\chi^2$ is given by $\chi^2_{\nu_{\text{CDM}},\text{min}} - \chi^2_{\text{DH},\text{min}} = -0.216$. Therefore, there is negative support for the $\nu_{\text{CDM}}$ model, even though this extended model fits the adopted dataset nearly as well as the $\nu_{\text{DH}}$ model.

5 Discussion and conclusion

In this paper, we updated the cosmological constraints on the sum of neutrino masses $\Sigma m_\nu$, using the most up-to-date observational data. Two more realistic mass hierarchies of massive neutrinos, namely, the normal mass hierarchy and the inverted mass hierarchy, are employed in addition to the degenerate mass hierarchy. In the $\nu_{\text{CDM}}$ model, for the DH, we obtained an improved upper limit $\Sigma m_\nu<0.129eV$ at 95% CL. Taking into account the squared mass differences between three massive neutrinos, we obtained the 95% CL upper bound to be $\Sigma m_\nu<0.159eV$ for the NH, and $\Sigma m_\nu<0.198eV$ for the IH. Based on the Bayesian evidence, the adopted dataset cannot distinguish the two mass orderings. In addition, we found that the priors in $\Sigma m_\nu$ can significantly impact the cosmological constraints on $\Sigma m_\nu$, given the adopted data combination. Future cosmological observations with higher precision are needed to get more decisive conclusions. For example, BAO [38, 52], CMB [53–56], and galaxy shear surveys [57, 58] might reach the sensitivity needed to measure the neutrino masses and to determine the mass hierarchy in the future.

Since the extended cosmology model can have degeneracy with the neutrino mass, we extended our studies to include generic dark energy, spatial curvature, and extra relativistic degrees of freedom, respectively, by assuming the degenerate mass hierarchy. Compared with the $\nu_{\text{DH}}$ model, we found negative support for these extended cosmological models based on Bayesian model selection, due to the introduction of an additional independent parameter in each model.

We compared the results of this work with existing results. Comparing with our previous works [22, 28], which used the same data sets except for eBOSS DR14, we found that the eBOSS DR14 data brings about at most a few percent correction to the neutrino mass constraints. However, it is challenging to compare this work with others, since different combinations of cosmological data are usually used, or even different models. For example, adding the CMB lensing to the data combination can worsen the constraints on the neutrino masses [5]. When the CMB lensing was discarded for the $\nu_{\text{DH}}$ model, the 95% CL upper limit on $\Sigma m_\nu$ even became $0.10eV$, as found by this work. It is much tighter than that in Eq. (1), and has reached the minimal mass expected in the IH scenario. For a second example, adding a prior on the reionization optical depth to the data combination could tighten the constraints on the neutrino masses, see for example Refs. [16, 27, 32, 59]. In addition, the degeneracy between the hot dark matter model and the massive neutrinos has been considered in Ref. [30], while the impact of the dynamical dark energy model on weighing massive neutrinos has been studied in Refs. [28, 31, 34, 36]. We have compared the results of this work with several existing results in the last two sections.

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