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REVEALING THE PATTERNS OF CHANGE IN THE TECHNICAL CONDITION OF REFRACTORY ELEMENTS IN THERMAL UNITS DURING OPERATION

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Operating conditions of thermal units for processing raw materials predetermine defects in refractory elements resulting in their gradual accumulation, which leads to a change in technical condition. A large number of defects, their development, and the achievement of critical values lead to difficulties in modeling the physical processes of changing the technical condition of refractory elements.

This study has investigated the mechanism of the occurrence, development, and accumulation of defects in refractory elements, as well as the processes of cumulative accumulation of damages; a probability model of their degradation has been constructed. The model was built using Markov chains; it describes the sequences of change in the states of refractory element damage and the probability of transitions between these states. Based on the statistical data about a change in the state of damage, the model makes it possible to assess the probability of a defect reaching the critical condition following the predefined number of load cycles. A special feature of the model is the possibility of its application to individual defects, as well as to refractory elements on which defects occur and develop, as well as to assemblies where such refractory elements are installed.

The main patterns of change in the technical condition of refractory elements of coke ovens have been established: the distribution of cracks of a certain length according to the number of coke oven output cycles; the probability of the occurrence of a crack of a critical length at a certain point during operation; the dependence of the probability of a refractory element failure on the predefined number of coke oven output cycles.

Based on the modeling results, it has been proposed, in order to prevent the degradation of refractory elements, to strengthen the structure of the surface layer of the refractory element by cold gas-dynamic spraying, to arrange laying elements that would stop the evolution of defects, and to make up schedules of hot repairs based on the time when the defects may reach critical values, determined during modeling.

Keywords: refractory element, crack, change in technical condition, probability model, Markov chains

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1. Introduction

The service life of existing industrial thermal units spans quite a large period and may reach 25 years or longer. Over this time, they have significantly changed their technical condition and their resource is almost exhausted [1–5].

One of the main elements of such units, which most intensively change their technical condition, are refractory
elements – heating partitions, heating elements, thermal insulation, lining, etc. The main purpose of refractory elements is:

- to transfer heat from heating structural elements to the processed raw materials (for example, heating partitions of coke ovens and peko coke ovens);
- to form a volume for processing raw materials under the influence of high temperatures (for example, coking chamber, coke dry extinguishing chamber);
- to thermally insulate the structural elements of thermal units from the influence of high temperatures, aggressive environments (for example lining);
- to accumulate heat from one heat carrier to transmit it to another heat carrier.

The main materials that the refractory elements of thermal units are made of are fireclay and Dinas rock. Fireclay products include materials containing Al₂O₃ + TiO₂ in the amount of 28...45 % [6, 7]. Dinas rocks include materials containing silicon dioxide SiO₂ in the amount of over 93 % [6, 7]. Fireclay refractory materials have been used in the lining of metallurgical assemblies in the production of coke, as well as an insulation material [6–9]. Dinas refractory materials, in contrast to fireclay, ensure masonry resistance to deformations at high temperatures and have a higher coefficient of thermal conductivity [8–10].

In terms of changing the technical condition of refractory elements, it is advisable to tackle one of the types of thermal technological units – coke ovens and peko coke ovens [6–8]. Refractory elements often operate under very difficult conditions and are exposed to the damaging effect of the following external factors:

- mechanical loads exerted by structural elements (working bodies, mechanical clamps, anchorage, etc.);
- contact with disparate working environments (air, coke oven gas, combustion products of the gas mixture, raw material mass, which often undergoes phase transitions);
- deep temperature changes (50...500 °C) at high heating values of 1,000...1,350 °C);
- uneven heating for the height of some structural elements, which causes temperature drops.

Such difficult conditions lead to that the resource of such units is exhausted, their reliability and efficiency are reduced. Thus, at present, it is a relevant task to prolong the service life of coke and peko coke batteries, taking into consideration the processes of changing the technical condition of the main structural elements made of refractory materials.

### 2. Literature review and problem statement

Methods and measures aimed at extending the service life of coke ovens and peko coke ovens can be divided into the following areas:

- the improvement of masonry structure and optimization design [11–17];
- the improvement of operational properties and design of refractories [5, 10, 17–19];
- the use of refractory protective coatings [20–23];
- the implementation of refractory masonry state monitoring systems [24–27];
- the introduction of effective repair technologies to eliminate defects and restore the condition of refractory elements [6, 7, 18, 28–34].

The area of masonry structure improvement and optimization design includes the issues of structural solutions, as well as the physical-mathematical modeling of important processes. Work [11] reports a design change to increase the usable volume of coking chambers to 51.0 m³ (chamber height, 7.0 m), 63.4 m³ (chamber height, 7.4 m). This improvement of the design made it possible to ensure the operational stability of the unit’s heating system and the quality of coke, improve the specific performance, and reduce the specific load on the environment. However, the issue of destruction of refractory structural elements remains relevant for such structures.

Solving optimization design tasks is based on the results from modeling the main processes related to coal coking (mass- and heat-exchange, hydrodynamic). Thus, the authors of work [12] developed a two-dimensional model of the non-stationary coking process, which makes it possible to predict the process parameters taking into consideration the basic influence factors and determine the optimal geometric dimensions. In addition, the constructed mathematical model of the gas-air flow movement and stepped combustion allowed the researchers to determine the optimal size and configuration of the heating system’s elements.

The authors of paper [13] derived a mathematical model linking internal gas pressure and pressure on the walls of the coking chamber. The model makes it possible, based on the known properties of charge, mode parameters of the coking process, and geometric parameters of the chamber, to predict the value of coking pressure on heating partitions. Thus, it has become possible to determine the mode parameters of the process at which the values of coking pressure on the heating partitions would not exceed the maximum allowed value of 10 kPa.

Work [14] considers the hydrodynamics of the coke oven battery and shows that the pressure values in the gas lines should be reduced in order to avoid the outflow of raw coke oven gas into the heating system. In addition, the authors of the cited work demonstrated the impact of defects in refractory elements on violation of the hydrodynamic mode of unit operation.

Study [15] addresses the task of modeling a change in the mechanical properties of refractory elements and the design of the heating partition under the influence of a cyclic change in the temperature and coking pressure. The devised model makes it possible to predict creep curves at the dangerous points of a heating partition, taking into consideration the simultaneous change in the stresses and temperature during the operation of a coke oven battery.

Structural solutions also apply to certain elements of the coke oven battery design: an anchor crimp system [16], or the configuration and composition of refractory elements [17]. The authors of work [16] propose to replace the springs of the anchorage system with unilateral hydraulic cylinders, which would lead to the balance of loads on the refractory part of the structure and the extension of its resource. The task of prolonging service life was also set by the authors of work [17]; it is resolved by changing the elements of the lining of the coke oven battery.

However, as a result of the complexity of the units’ design, and the multifactor nature of the destroying influence, the issue of defect occurrence in refractory elements, as well as their destruction, has remained relevant up to now. Extensive studies into the causes and mechanisms of destruction of the refractory masonry of coke oven batteries were reported in [3, 4, 29, 32, 35–37].

A preliminary investigation of the microstructure of the refractories’ surface was conducted in [38]. Photographs acquired at an SEM microscope show the characteristic porous structure of the refractory (Fig. 1). The characteristic
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size of cavities is in the range of 50...100 μm; that of the pores is about 10 μm.

An analysis of the occurrence and development of defects on the refractory elements of coke ovens and peko coke ovens [4, 38] reveals the same pattern of their evolution. At the micro-level, there is a certain initial number of defects that occurred during the manufacture, installation, commissioning of coke ovens and peko coke ovens.

During operation, micro defects accumulate; a gradual accumulation is accompanied by a transition from a micro-level to a macro level. The cumulative effect of defect accumulation is present at the micro and macro levels.

It is known [38] that a defect passes several stages in its development from the micro defect to the through macro-effect, which leads to the failure of both a separate refractory element and a heating partition in general.

The modification of refractories [5, 10, 17–19], as well as the formation of protective coatings on their surfaces [20–23], also does not make it possible to solve the task of ensuring the durability and tightness of refractory masonry throughout the entire life cycle of units.

Therefore, the operation of such units is typically accompanied by the use of hot (heat unit-specific) repairs [6, 7], carried out during operation without disconnecting individual furnaces from the heating system. The issue of maintenance and repair of the refractory masonry of coke oven batteries has been addressed in many studies. Generalized information on the types of repairs and their characteristics is given in works [28, 29, 32]. Paper [30] reveals the experience of using hot repairs to prolong the life cycle of coke oven batteries under conditions of actual coke production. In addition, generalized recommendations were given to coke production enterprises on the frequency of repairs, the cases of execution of a certain type of repair [33]. The most progressive is the technology of ceramic surfacing (welding), which makes it possible to ensure an average life cycle of the repaired section of 2...3 years [31, 34]. To improve the effectiveness of such measures, various approaches and means of monitoring the state of refractory masonry [24, 26, 27], and the heating systems in general, are developed and used [25–27].

However, in practice, hot repairs are used in cases when refractory elements are already critically damaged and their repair would require significant costs. Therefore, in order to timely prevent the occurrence of a critical state of refractory elements, it is advisable to generalize patterns of defect evolution and to forecast their technical condition taking into consideration the operating conditions.

3. The aim and objectives of the study

The aim of this study is to identify a change in the technical condition of refractory elements in thermal units during operation due to the occurrence, development, and accumulation of defects, which would make it possible to devise measures to prolong their resource.

To accomplish the aim, the following tasks have been set:

- to build a mathematical model of the development, accumulation, and transitions to the critical state of defects on refractory elements;
- to define the main patterns of changes in the technical condition of refractory elements;
- to propose directions for the organizational and technical measures to prolong the resource of refractory elements in thermal units.

4. A mathematical model of the development, accumulation, and transitions to the critical condition of defects on refractory elements

4.1. Justification of the type of mathematical model

In terms of the defect development process essence, the most appropriate is the theory of the finite Markov chains [39, 40] in addition to the process of damage accumulation [39]. We shall term the defect, the refractory element, and the partition, whose states change, the object of destruction. The process will be considered as follows: the object of destruction is cyclically exposed to high temperatures and mechanical stress. During load cycles (LC), its parameters undergo irreversible changes. Increasing their number and parameter values proceeds until the critical values are reached or the object of destruction fails. An analysis of the schemes of the development of refractory element defects [38], their damage, and the damage to a structure made from refractories, allows us to conclude that these processes are degrading and obey general patterns. Therefore, a generalized mathematical model is required, which makes it possible, based on a change in the state of these elements, to assess the probability of failure of the refractory element at any time during the operation of the object. Based on known results from the statistical analysis of the processes of the occurrence and development of defects [4, 36, 37], we shall summarize the main features in the processes of defect growth and the destruction of a refractory element in order to define mathematical models:

1) the occurrence and development of defects happen as a result of periodic influences from the set of loads during the periodic load cycles;
2) the process of defect evolution is irreversible, that is, there are no self-recovery processes;
3) repairs lead to a disruption of the process of defect development and to the restoration of the processes of destruction of the refractory element from that moment;
4) the initial value of the number and parameters of defects are random values;
5) at different stages of operation of refractory elements, the processes of their development proceed with different intensity.

An important point in predicting the state of any object is the choice of the predictive function. Given the nature of degradation processes

Fig. 1. The surface of the refractory showing micro defects [38]
and the available results of statistical analysis [37], we identify the predictive function in the presence of uncertainties similar to random processes. We also take into consideration the jump-like nature of the process progress. As the most suitable and appropriate to the essence of the defect development process, we shall use the theory of the finite Markov chains [40] in addition to the process of damage accumulation [39].

4.2. Model assumptions and limitations

The process studied will be considered as follows: a refractory element is cyclically exposed to high temperatures and mechanical stress. Irreversible changes in defect parameters occur during cycles. Increasing their number and parameter values proceeds until the critical values are reached or a refractory element fails. We introduce the following assumptions.

The operation process consists of repetitive load cycles (LC) that have constant parameters.

We assume that the time factor \( \tau \) is associated with load cyclicity and is a discrete quantity, that is, \( \tau = 0, 1, 2, \ldots, n \). Accordingly, the factor of operation time is represented by the number of load cycles.

During operation, a refractory element may enter one of the damage states, which are characterized by the degree of damage \( a \) (the quantity and/or magnitude of defects). The damage states are also discrete: \( D = 1, 2, \ldots, j, \ldots, Z \). Accordingly, the quantitative parameter of the degree of damage is \( a = a_1, a_2, \ldots, a_j, \ldots, a \). When the state \( Z \) is reached, there is a failure of the refractory element, that is, the state \( Z \) is the final event [8, 29].

The state of damage to the refractory element is considered at the limits of LC, that is, a change in the quantitative parameters of defects, which occurs during the LC, is not considered.

Since the process of defect evolution is irreversible, the state of damage to the refractory element cannot enter any previous state.

The state of damage to the refractory element over the LC may either not change or enter the next state.

4.3. Mathematical notation of the development, accumulation, and transitions of defects to the critical state

We assume that at the initial point in time, which corresponds to the first inspection of the refractory element state, and is denoted through \( \tau = 0 \), the damage state corresponds to state \( 1 \). During LC, a refractory element perceives loads caused by temperature changes, the mechanical and chemical influences. If the set of impacts is below some critical level, the damage state does not change, otherwise, the element enters the next damage state.

We shall denote the probability that a measure of devastating impacts is below the critical level by \( p_1 \). Accordingly, the probability that the measure of devastating impacts exceeds the critical value, provided that the damage was initially in a state \( 1 \), is denoted by \( q_1 = 1 - p_1 \). In this case, the refractory element enters the next state of damage, number \( 2 \).

Then, in a general case, if a refractory element before the next LC is in a state of damage, number \( j (j = 1, 2, \ldots, j, \ldots, Z - 1) \), the probability that it remains in its current state is \( p_j \), and the probability that the element enters the next state \( (j + 1) \), after exceeding the permissible impact value, is the value of \( q_j = 1 - p_j \). Next, the state of the refractory element changes up to the \( Z \) state when the failure occurs.

Denote the state of damage in which the refractory element is at time \( \tau = 0 \) by a random value (RV) \( X_0 \). The initial distribution of probabilities \( p_0 \) based on damage states at \( \tau = 0 \) is assigned by vector string (1):

\[
p_0 = \{x_1, x_2, \ldots, x_{Z-1}, 0\}, \quad (1)
\]

\[
P \{X_0 = j\} = x_j \geq 0,
\]

\[
\sum_{j=1}^{Z-1} x_j \geq 0. \quad (3)
\]

No refractory element is to be used in a failure state, so it is accepted that \( x_Z = 0 \). The \( x_j \) values form a probable weight function (PWF) for \( X_0 \).

Each LC with constant parameters is associated with a matrix of transitional probabilities (MTP) \( P \). According to assumption 5, the matrix takes the following form:

\[
P = \begin{bmatrix}
    p_{11} & q_{12} & q_{13} & \cdots & q_{1Z-1} & q_{1Z} \\
    0 & p_{21} & q_{23} & \cdots & q_{2Z-1} & q_{2Z} \\
    0 & 0 & p_{31} & q_{34} & \cdots & q_{3Z-1} & q_{3Z} \\
    \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
    0 & 0 & 0 & \cdots & p_{Z-1, Z-1} & q_{Z-1, Z} \\
    0 & 0 & 0 & \cdots & 0 & 1
\end{bmatrix} \quad (4)
\]

where \( p_{jk} \) is the probability that the refractory element remains in state \( j \) in one step, \( 0 < p_{jk} < 1 \); \( q_{jk} \) is the probability of transitioning to the damage state in one step to one of the following states \( (j+1, \ldots, Z) \).

\[
p_j + q_{j+1} + \cdots + q_{Z-1} = 1. \quad (5)
\]

Damage state \( 1, \ldots, Z - 1 \) – transitional states; \( Z \) is the final state (a state of failure).

The graphical interpretation of a change in the damage state of a refractory element corresponding to matrix (4) is shown in Fig. 2.

![Fig. 2. Change in the damage state of a refractory element according to matrix (4)](image-url)
According to assumption 6, MTP takes the following form:

\[
P = \begin{pmatrix}
p_{11} & q_{12} & 0 & 0 & \cdots & 0 & 0 \\
0 & p_{22} & q_{23} & 0 & \cdots & 0 & 0 \\
0 & 0 & p_{33} & q_{34} & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & p_{Z-1,Z-1} & \cdots & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & 1
\end{pmatrix}.
\]

(6)

The graphical interpretation of change in the damage state of a refractory element corresponding to matrix (4) is shown in Fig. 3.

To assess the probability of a refractory element’s failure after any predefined number of load cycles \( p_0 \), it is necessary to know the distribution of the probability that a refractory element may enter possible damage states at the moment \( p_0 \) and MTP, starting from the predefined point \( P^* \).

It is known from the theory of Markov chains [39, 40] that:

\[
p_i = p_i P^* = p_i., \quad \tau = 0, 1, 2, \ldots, \tau_Z.
\]

(7)

Denote the damage state of a refractory element at the time \( \tau \) by a random value \( X_{\tau} \), then:

\[
P\{X_{\tau} = j\} = p_i (j), \quad j = 1, 2, \ldots, Z.
\]

(8)

Here:

\[
p_i (j) \geq 0, \quad \sum_{j=1}^{Z} p_i (j) = 1.
\]

(9)

Thus, \( p_i(j) \) form MTP at the time \( \tau \) for the damage states of a refractory element 1, \ldots, Z:

\[
p_i = \{p_i (1), \ldots, p_i (Z)\}.
\]

(10)

Graphically, ratio (7) is shown in Fig. 4, which shows that as the number of load cycles \( \tau \) grows, the probability value of changing the state of the refractory element at the moment \( \tau \) and the number of LC \( \tau \) upon reaching the state of failure Z, equation (7) is transformed using generating and characteristic functions \[40, 41\]. The differentiation of the characteristic functions obtained as a result of the transformation of equation (7) makes it possible to derive equations for the statistical moments (mathematical expectation \( M \), variance \( \sigma \), etc.) of the number of LCs over which the damage to the object reaches the state of failure.

\[
\Phi_y (y; 1, 2) = \frac{q_{1,2} q_{2,3} \cdots q_{Z-1, Z}}{(1 - p_{1, 2}, p_{2, 3}) \cdots (1 - p_{Z-1, Z})} y^{Z-1} + \frac{q_{1,2} q_{2,3} \cdots q_{Z-1, Z}}{(1 - p_{1, 2}, p_{2, 3}) \cdots (1 - p_{Z-1, Z})} y^{Z-2} + \cdots
\]

\[
\quad + \text{Similar terms in quantity } (Z-2, 1)\]

(11)

Mathematical expectation \( M \{W_{1, Z}\} \) of the number of LCs over which a refractory element reaches a failure state \( W_{1, Z} \) is the first derivative from the characteristic function \[40, 41\]:

\[
M \{W_{1, Z}\} = \frac{d\Phi_y (y; 1, 2)}{dy} \bigg|_{y=1}.
\]

(12)
The variance \( \sigma(W_{I, Z}) \) in the number of LCs over which a refractory element reaches a failure state \( W_{I, Z} \) is determined as follows [40, 41]:

\[
\sigma W_{I, Z}^2 = \frac{d^2 \Phi_W}{dy^2}igg|_{y=1} + \frac{d \Phi_W}{dy}igg|_{y=1} - \left[\frac{d^2 \Phi_W}{dy^2}\right]_{y=1},
\]

(13)

The differentiation of characteristic function (11) according to formula (12), the mathematical expectation and variance:

\[
M(W_{I, Z}) - \sigma(W_{I, Z}) = \frac{d \Phi_W}{dy}igg|_{y=1},
\]

(14)

\[
\sigma^2 W_{I, Z} = \sum_{j=1}^{n_j} (1+r_j),
\]

(15)

where \( r_j \) is the ratio of the probability that over the determined number of LCs a refractory element remains in the current state to the probability that the refractory element enters the next state:

\[
r_j = p_{j, r_{j, 1}}.
\]

(16)

The value of a damage extent to a refractory element \( a \) corresponds to the state of the model \( n_j \). That is, each state of the model \( n_j \) corresponds to the average value of the number of LCs to reach this state.

Given that each damage state \( j \) corresponds to the interval of the states of the model \( [n_{j-1}+1; n_j] \) and the form of MTP (6), expressions (14) and (15) take the following form:

\[
M(W_{I, Z}) = (n_j-1)(1+r_{j,1}) + \sum_{j=2}^{n_j} (n_j-n_{j-1})(1+r_{j,1}),
\]

(17)

\[
\sigma W_{I, Z} = \sum_{j=1}^{n_j} (1+r_j).
\]

(18)

To determine the parameters of the model \( r_j \) and \( n_j \), we shall use the method of moments [42], whereby the values of mathematical expectation \( M(W_{I, Z}) \) and variance \( \sigma(W_{I, Z}) \) in the number of LCs over which a refractory element reaches the specified state of damage \( j \) from its original state, which is equal to unity from the left-hand part of expressions (17) and (18), are determined from the statistical analysis of the change in damage to the refractory element during operation.

The correspondence between each state of damage \( j \) of the extent of damage to a refractory element \( a \), and the mathematical expectation \( M(W_{I, Z}) \) and variance \( \sigma(W_{I, Z}) \) in the number of LCs over which a refractory element reaches this state is shown in a tabular form (Table 1).

| Damage state number, \( j \) | Refractory element damage extent, \( a \) | Mathematical expectation, \( M(W_{I, Z}) \) | Variance, \( \sigma(W_{I, Z}) \) |
|---|---|---|---|
| 1 | \( a_1 \) | \( M(W_{1, Z}) \) | \( \sigma(W_{1, Z}) \) |
| 2 | \( a_2 \) | \( M(W_{2, Z}) \) | \( \sigma(W_{2, Z}) \) |
| 3 | \( a_3 \) | \( M(W_{3, Z}) \) | \( \sigma(W_{3, Z}) \) |
| ... | ... | ... | ... |
| \( j \) | \( a_j \) | \( M(W_{j, Z}) \) | \( \sigma(W_{j, Z}) \) |
| ... | ... | ... | ... |
| \( Z \) | \( a_Z \) | \( M(W_{Z, Z}) \) | \( \sigma(W_{Z, Z}) \) |

Table 1

| Damage state number, \( j \) | Refractory element damage extent, \( a \) | Interval of the model state numbers, \( [n_{j-1}+1; n_j] \) | Parameter \( r_j \) | Probability of being in a current state \( p_{j, r_{j, 1}} \) | Probability of entering the next state \( q_{j, r_{j, 1}} \) |
|---|---|---|---|---|---|
| 1 | \( a_1 \) | \( 1 \) | \( n_1 \) | \( r_{1,1} \) | \( p_{1,1} \) | \( q_{1,2} \) |
| 2 | \( a_2 \) | \( n_{1+1} \) | \( n_2 \) | \( r_{2,2} \) | \( p_{2,2} \) | \( q_{3,3} \) |
| 3 | \( a_3 \) | \( n_{2+1} \) | \( n_3 \) | \( r_{3,3} \) | \( p_{3,3} \) | \( q_{3,4} \) |
| ... | ... | ... | ... | ... | ... | ... |
| \( j \) | \( a_j \) | \( n_{j-1}+1 \) | \( n_j \) | \( r_{j,1} \) | \( p_{j,1} \) | \( q_{j,1} \) |
| ... | ... | ... | ... | ... | ... | ... |
| \( Z \) | \( a_Z \) | \( n_{Z-1}+1 \) | \( n_Z \) | \( r_{2Z} \) | \( p_{2Z,1} \) | ... |

Table 2

Linking the accumulation model parameters to MTP
Thus, the developed model makes it possible, based on the statistical data \(M(W_{j-1},j), \sigma(W_{j-1},j)\) on a change in the state of damage, to assess the probability of a failure \(p_n\) of the refractory element at any time during its operation.

5. An analysis of patterns in the development, accumulation, and transitions of defects on refractory elements to critical condition

5.1. Source data and model parameters

We shall consider a model of the degradation of a refractory element using an example of the development of one of the most common types of defects – cracks in the refractory masonry of heating partitions of coke oven batteries.

We shall use data from the periodic inspections of laying the partitions in coke oven battery No. 9 at Kryvyi Rih Coke Plant with a volume of coking chambers of 30.2 m³. Information on the development of cracks in the head area of the partitions was selected from a database on the masonry state for coke oven battery No. 9, by the type of defect (crack), as well as inspection data [4, 36, 37]. The sample size is 214 cases, 100 of which refer to the machine side, and 114 cases refer to the coke side (Table 3).

The main parameter of a crack is its length. We assume that the crack states correspond to the value of its length, \(m\): 0.1; 0.3; 0.5; 0.7; 1.0; 1.2; 1.4; 1.6; 1.8. Cracks up to 0.1 m long are not taken into consideration. Accordingly, the moment of crack origination and its initial condition is its length of 0.1 m. The critical crack length is considered a value of 1.8 m because after that there is a sharp increase in the rate of its growth and the probability of forming a through crack. The value of the crack length in the given row is multiple of the height of the refractory brick masonry of a heating partition.

The link between a coke oven output cycle and calendar duration is as follows: 1 coke oven output cycle may last over 18…21 hours under normal operating conditions. The time of the coking period depends on the properties of raw materials, the type of coke oven battery, and a technological mode. Therefore, the indicator of operating time, in this case, is the number of coke oven output cycles.

The values of mathematical expectation and variance in the number of coke oven output cycles after a crack reaches a certain length are derived from the statistical analysis of data in Table 3 and summarized in Table 4. In Table 4, the state \(j=1\) is the original one and corresponds to the moment of the first inspection of the state of refractory elements, that is, we consider it a moment of crack origination and the original state of damage to the refractory element.

The resulting system of equations (21) consists of 16 equations and has 16 unknowns: \(n_1, n_2, ..., n_6\) and \(r_1, r_2, ..., r_8\).

5.2. Modeling results

Solving a system of equations (21) produces the intervals of the model state numbers \(\{n_{j-1} + 1; n_j\}\), the parameter \(r_{ij}\) values, and the corresponding \(q_{ij}\) and \(q_{i,j+1}\), which are given in Table 5.

Change in a crack length \((\sigma, m)\) during operation depending on the number of coke oven output cycles \((N)\)

| Defect No | The number of coke oven output cycles, \(\tau\) |
|----------|----------------------------------|
|          | 250    | 500    | 750    | 1,000  | 1,250  | 1,500  | 1,750  | 2,000  | 2,250  | 2,500  | 2,750  | 3,000  |
| 1        | 0.1    | 0.1    | 0.1    | 0.3    | 0.4    | 0.8    | 0.8    | 1.0    | 1.0    | 2.2    | 2.2    |
| 2        | 0.1    | 0.3    | 0.4    | 0.4    | 0.7    | 0.7    | 0.7    | 0.7    | 0.7    | 2.3    | 2.3    |
| 3        | 0.1    | 0.1    | 0.2    | 0.3    | 0.6    | 0.9    | 0.9    | 0.9    | 0.9    | 1.4    | 2.1    | 2.2    |
| ...      | ...    | ...    | ...    | ...    | ...    | ...    | ...    | ...    | ...    | ...    | ...    |
| 213      | 0.1    | 0.2    | 0.3    | 0.3    | 0.3    | 0.3    | 0.6    | 1.0    | 1.0    | 1.7    | 2.9    |
| 214      | 0.1    | 0.1    | 0.2    | 0.3    | 0.3    | 0.3    | 0.6    | 0.6    | 0.7    | 1.0    | 1.4    | 1.7    |
Thus, the matrix of transitional probabilities (MTP) (6) takes the following form:

\[
P = \begin{pmatrix}
0.394 & 0.606 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.636 & 0.364 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.705 & 0.295 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.898 & 0.102 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.875 & 0.125 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.926 & 0.074 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.967 & 0.033 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.928 & 0.072 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{pmatrix}
\tag{22}
\]

The resulting MTP \( P \) (22) makes it possible to assess the probability of failure of a refractory element after any given number of load cycles \( p \) according to expression (7) and to obtain the integrated laws of the distribution of a probability that a crack reaches a certain length according to the number of coke oven output cycles (Fig. 5).

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0 & 0 & 0 & 0 & 0 & 0 & 0.967 & 0.033 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.928 & 0.072 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{pmatrix}
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The resulting MTP \( P \) (22) makes it possible to assess the probability of failure of a refractory element after any given number of load cycles \( p \) according to expression (7) and to obtain the integrated laws of the distribution of a probability that a crack reaches a certain length according to the number of coke oven output cycles (Fig. 5).

Our study has made it possible to establish a series of patterns in defect development. This is a connection between the crack length and the number of a model state (Fig. 6); a connection between the model state number and the number of coke oven output cycles (Fig. 7); a connection between a crack length and the number of coke oven output cycles (Fig. 8); the integrated function of the distribution of the number of coke oven output cycles based on a crack reaching a critical length (Fig. 9). By using the diagram in Fig. 6, one can determine, for any state, the corresponding value of the crack length. The integrated function of the distribution of the number of coke oven output cycles \( \tau \) over which a defect reaches a critical size (Fig. 9) or a refractory element reaches a damaged state makes it possible to determine the probability of such an event for a certain number of coke oven output cycles. For example, for the given result, at the unit’s operating time of \( \tau = 2,500 \) coke oven output cycles, the probability of the occurrence of a crack of the critical length of 1.8 m is 0.85.
When one knows the preconditions for the occurrence of defects of different types and the established patterns in their development, it is possible to formulate the main directions of combating the defects and comprehensive counteraction to the degradation of refractory elements.

First, we propose that measures to counteract the emergence of defects should be taken at the stage of refractories manufacture. To this end, it is proposed to change the surface structure of the refractory element. Changing the structure is to eliminate the pores of the surface layer with a filler. The filler does not make it possible to penetrate the array of the refractory for carbon microparticles and other destruction products of the processed raw materials. This could create conditions for stopping the process of destruction of the refractory under conditions of cyclicity for its operation. One of the methods of such a change in the structure of the surface layer of a refractory element is the method of cold gas-dynamic spraying [43]. Spraying can be carried out in several layers, with different properties (Fig. 10). This would counteract the development of defects at the micro-level and reduce the probability of transition of the refractory element to a state of greater damage for the first or second states (Fig. 3).

Second, it is proposed that the laying elements of different sizes [44, 45] should be arranged (Fig. 11) throughout the entire volume of a refractory element. When a defect (a crack and a microcrack of different types) reaches such laid elements, their further development stops, the laying element is destroyed, its components are mixed, interact, and seal the edges of the crack. This would counteract the development of defects at the micro and macro levels and reduce the probability of the refractory element entering a state of greater damage for the third and further states (Fig. 3).

Third, traditional hot repairs are proposed for the final damage states [6, 7, 18, 34]. Our results could improve the efficiency of repair and restoration measures for technological units, which include refractory elements. The improved efficiency is achieved by taking into consideration the necessary criteria during the planning of repairs: the probability of failures; the predefined level of safety; the permissible number of defects that would reach critical parameters over a certain time of operation.

Our study has made it possible to establish the main patterns of change in the technical condition of refractory elements at the stage of operation. The preconditions for defects are the peculiarities of the technology of manufacturing refractory elements and their structure.

The resulting system of equations (21) makes it possible to calculate the probability of changes in the damage states of refractory elements and derive a matrix of transitional probabilities (22). It should be noted that the dimensionality of the...
matrix and the number of calculated parameters (20) depend on the accepted sequence of damage extent by a refractory element (Table 1).

The developed model makes it possible, based on the statistical data (Table 1) on a change in the state of damage of the refractory element, to assess the probability of failure (7) of the refractory element at any time during its operation and to obtain an integrated law of the distribution of the probability that a crack reaches a certain length in accordance with a certain number of coke oven output cycles (Fig. 5). The integrated function of the distribution of the number of coke oven output cycles over which a crack reaches a critical length (Fig. 9) has made it possible to establish that the probability that a crack reaches the critical value (1.8 m) is almost 0.87 at 2,600 coke oven output cycles. This makes it possible to determine the operating time of the unit, which requires the elimination of defects using known hot repair technologies. This approach of adjusting the repair time could help avoid the occurrence of critical defects due to the preventive repair measures.

Based on the established connection between a crack length and the state number and the number of coke oven output cycles (Fig. 6), one can see that after 975 coke oven output cycles cracks begin to develop intensively and their average length quickly approaches a critical value of 1.8 m. Identifying such data makes it possible to recommend hot repairs during this operation time.

The proposed model could be used to analyze the technical condition of objects that operate under cyclical loads, as a result of which defects occur and develop. This study could be further advanced by investigating the development of other types of defects in refractory elements (chips, caverns, etc.) and taking into consideration the repair and recovery effects on defects, that is, when the state of damage to the refractory element enters one of the previous states.

8. Conclusions

1. Based on Markov chains, we have built a mathematical model of change in the technical condition of refractory elements, on which defects occur and develop, which makes it possible to predict the time when a defect reaches critical condition and to assess the probability of a failure. The model makes it possible, based on the statistical data on changes in the state of damage, to assess the probability of a refractory element failure after any predefined number of load cycles. A special feature of the model is the possibility of its application not only to describe the development of a defect but also to describe the technical condition of a refractory element on which defects develop, as well as of the thermal unit that includes refractory elements.

2. By using the developed mathematical model to study the partitions in coke ovens, we have established the main patterns of changes in their technical condition: the distribution of cracks of a certain length (0.3...1.6 m) according to the operating time (the number of coke oven output cycles is 0...3,000). It was found that the critical length of a crack of ~1.8 m is achieved through ~2,800 coke oven output cycles with a probability of 97 %. In addition, we derived the dependences of the probability of failure of the refractory element after the predefined number of coke oven output cycles. For example, at ~1,800 coke oven output cycles, the probability of failure (at the critical length of the crack of ~1.8 m) is <1 %; at ~2,500 coke oven output cycles, it is already 19 %; and at ~2,800 coke oven output cycles, it is already 97 %. All this explains the rapid growth in the number of failures of heating partitions after 9–10 years of operation in the absence of repair and restoration measures.

3. Based on the established patterns of changes in the technical condition of refractory elements during operation, we have proposed the directions of organizational and technical measures to combat defects and comprehensively counteract the degradation of refractory elements. Namely, strengthening the structure of the surface layer of a refractory element by a cold gas-dynamic spraying method, the arrangement of the laying elements that could stop the development of defects, as well as conventional methods of hot repairs on a schedule drawn up on the basis of the deadlines when defects may reach critical values specified in this work.

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