Theory of Neutrinos

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Abstract

After a brief overview of the present knowledge of neutrino masses and mixing, we summarize what can be learned about physics beyond the standard model from the various proposed neutrino experiments. We also comment on the impact of the experiments on our understanding of the origin of the matter–antimatter asymmetry of the Universe as well as what can be learned from some experiments outside the domain of neutrinos.

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I. INTRODUCTION

Our understanding of neutrinos has changed tremendously in the past six years. Thanks to the efforts of several neutrino oscillation studies of solar, atmospheric and reactor (anti)neutrinos, we learned that neutrinos produced in a well defined flavor eigenstate can be detected, after propagating a macroscopic distance, as a different flavor eigenstate. The simplest interpretation of this phenomenon is that, like all charged fermions the neutrinos have mass and that, similar to quarks, the neutrino weak, or flavor, eigenstates are different from neutrino mass eigenstates, i.e., neutrinos mix \[1\]. This new state of affairs has also raised many other issues which did not exist for massless neutrinos: For example, (i) massive neutrinos can have nonzero magnetic moments, like the electron and the quarks; (ii) the heavier neutrinos may decay into lighter ones, like charged leptons and quarks, and (iii) (most importantly) the neutrinos can be either Majorana or Dirac fermions \[2\].

Learning about all these possibilities can not only bring our knowledge of neutrinos to the same level as that of charged leptons and quarks, but may also lead to a plethora of laboratory as well as astrophysical and cosmological consequences with far reaching implications. Most importantly, knowing neutrino properties in detail may also play a crucial role in clarifying the blueprint of new physical laws beyond those embodied in the
Standard Model.

One may also consider the possibility that there could be new neutrino species beyond the three known ones ($\nu_e, \nu_\mu, \nu_\tau$). In addition to being a question whose answer would be a revolutionary milestone pointing to unexpected new physics, it may also become a necessity if the LSND results are confirmed by the MiniBooNE experiment, now in progress at Fermilab. This would, undoubtedly, be a second revolution in our thinking about neutrinos and the nature of unification.

The existence of neutrino masses qualifies as the first evidence of new physics beyond the Standard Model. The answers to the neutrino-questions mentioned above will add substantially to our knowledge about the precise nature of this new physics, and in turn about the nature of new forces beyond the Standard Model. They also have the potential to unravel some of the deepest and long-standing mysteries of cosmology and astrophysics, such as the origin of matter, the origin of the heavy elements, and, perhaps, even the nature of dark energy.

Active endeavors are under way to launch the era of precision neutrino measurement science (PNMS), that will surely broaden the horizon of our knowledge about neutrinos. We undertake this survey to pin down how different experimental results expected in the coming decades can elucidate the nature of neutrinos and our quest for new physics. In particular, we would like to know (i) the implications of neutrinos for such long standing ideas as grand unification, supersymmetry, extra dimensions, etc; (ii) the implications of the possible existence of additional neutrino species for physics and cosmology, and (iii) whether neutrinos have anything to do with the origin of the observed matter-antimatter asymmetry in the universe and, if so, whether there is any way to determine this via low-energy experiments. Once the answers to these questions are at hand, we will have considerably narrowed the choices of new physics, providing a giant leap in our understanding of the physical Universe.

The purpose of this document is to briefly summarize what we know about neutrino masses and mixings, their context in overall physics, their connection to theoretical models, and open questions. There is a companion document ("Theory White Paper") where further technical details are presented.
II. WHY NEUTRINOS?

Neutrinos are elementary particles with spin 1/2, electrically neutral, and obey Fermi-Dirac statistics. Even though their existence has been known since the 1950s and the existence of three types of them was experimentally confirmed in the 1990s, it has been difficult to study their intrinsic properties due to their weak interactions. Nonetheless, they have rather unique roles in the world of elementary particles. They are ubiquitous in our universe, provide a unique window to physics at very short distances, and may even be relevant to the question “Why do we exist?” Moreover, the history of neutrinos has been full of surprises, which is likely to continue in the future.

A. Ubiquitous Neutrinos

Neutrinos are the most ubiquitous matter particles in the universe. They were produced in the Big Bang, when universe was so dense that neutrinos, despite their only weak interactions, were in thermal equilibrium with all other particle species. Similarly to the cosmic microwave background photons, their number density has been diluted by the expansion of the universe. In comparison, constituents of ordinary matter, electrons, protons, and neutrons, are far rarer than photons and neutrinos, by about a factor of ten billions. It is clear that we need to understand neutrinos in order to understand our universe.

In terms of energy density, the yet unknown dark matter and dark energy dominate the universe. If neutrinos were massless, their energy density could have been completely negligible in our current universe. However, in the last several years, we learned that neutrinos have small but finite masses, implying that the neutrinos contribute to the total energy of the universe at least as much as all stars combined. We do not yet know the mass of neutrinos precisely, and they may in fact be a sizable fraction of dark matter. The precise amount of the neutrino component is relevant to the way galaxies and stars were formed during the evolution of the universe.

Neutrinos are an important part of the stellar dynamics; without them, stars would not shine. There are about $7 \times 10^9 \text{ cm}^{-2}\text{ sec}^{-1}$ neutrinos from the Sun reaching (and streaming through) the Earth. They also govern the dynamics of supernovae.
B. Special Role of Neutrino Mass

One way to characterize physics is an attempt to understand nature at its most fundamental level, namely at the shortest distance scales, or equivalently the highest energy scales, possible. There has been two approaches. One way to access physics at the highest energy scales possible is to build powerful particle accelerators and reach the energy scale directly. Another way is to look for rare effects from physics at high-energy scales that do not occur from physics at known energy scales, namely the Standard Model. Physics of neutrino mass (currently) belongs to the second category.

Rare effects from physics beyond the Standard Model are parameterized by effective operators added to the Standard Model Lagrangian,

\[ \mathcal{L} = \mathcal{L}_{SM} + \frac{1}{\Lambda} \mathcal{L}_5 + \frac{1}{\Lambda^2} \mathcal{L}_6. \]  

(1)

The effects in \( \mathcal{L}_5 \) are suppressed by a single power of the high energy scale, \( \mathcal{L}_6 \) by two powers, etc. The possible terms have been classified systematically by Weinberg, and there are many terms suppressed by two powers:

\[ \mathcal{L}_6 \supset QQQL, \bar{L}\sigma^{\mu\nu}W_{\mu\nu}He, W_\nu^\mu W_\lambda^\nu B_\mu^\lambda, \bar{s}d\bar{s}d, (H^\dagger D_\mu H)(H^\dagger D^\mu H), \cdots. \]

(2)

The examples here contribute to proton decay, \( g - 2 \), the anomalous triple gauge boson vertex, \( K^0 - \bar{K}^0 \) mixing, and the \( \rho \)-parameter, respectively. It is interesting that there is only one operator suppressed by a single power, \( \mathcal{L}_5 = (LH^\dagger)(LH) \). After substituting the expectation value of the Higgs, the Lagrangian becomes

\[ \mathcal{L} = \frac{1}{\Lambda}(LH)^\dagger(LH) \to \frac{1}{\Lambda}(L\langle H\rangle)(L\langle H\rangle) = m_\nu \nu \nu, \]

(3)

nothing but the neutrino mass.

Therefore the neutrino mass plays a very unique role. It is the lowest-order effect of physics at short distances. This is an extremely small effect. Any kinematical effects of the neutrino mass are suppressed by \( (m_\nu/E_\nu)^2 \), and for \( m_\nu \sim 1 \text{ eV} \) (which we now know is already too large) and \( E_\nu \sim 1 \text{ GeV} \) for typical accelerator-based neutrino experiments, it is as small as \( (m_\nu/E_\nu)^2 \sim 10^{-18} \). At first sight, there is no hope to probe such a small number. However, any physicist knows that interferometry is a sensitive method to probe extremely small effects. For interferometry to work, we need a coherent source.
Fortunately there are many coherent sources of neutrinos: the Sun, cosmic rays, reactors, etc. We also need interference for an interferometer to work. Fortunately, there are large mixing angles that make the interference possible. We also need long baselines to enhance the tiny effects. Again fortunately there are many long baselines available, such as the size of the Sun, the size of the Earth, etc. nature was very kind to provide all the necessary conditions for interferometry to us! Neutrino interferometry, a.k.a. neutrino oscillation, is a unique tool to study physics at very high energy scales.

At the currently accessible energy scale of about a hundred GeV in accelerators, the electromagnetic, weak, and strong forces have very different strengths. But their strengths become the same at \(2 \times 10^{16}\) GeV if there the Standard Model is extended to become supersymmetric. Given this, a natural candidate energy scale for new physics is \(\Lambda \sim 10^{16}\) GeV, which suggests \(m_\nu \sim \langle H \rangle^2 / \Lambda \sim 0.003\) eV. Curiously, the data suggest numbers quite close to this expectation. Therefore neutrino oscillation experiments may be probing physics at the energy scale of grand unification.

C. Surprises

Even though some may argue that the neutrino mass was observed with the theoretically expected order of magnitude, it is fair to say that we had not anticipated another important leptonic property: neutrinos “oscillate” from one species to another with a high probability. Their mixing angles are large. We’ve known that different species of quarks
mix, but their mixing angles are very small; the largest one is 13 degrees, and others are much smaller. In comparison, the known mixing of muon and tau neutrinos is consistent with being maximal, or 45 degrees.

Another surprise is the neutrino mass spectrum. The quarks and charged leptons have what are referred to as hierarchical mass spectra, namely that the masses of charged fermions that share the same quantum numbers are drastically different among three generations of elementary particles. For instance, the masses of the up- and top-quarks are different by almost five orders of magnitude. On the other hand, the two heavier neutrino masses differ at most by a factor of about five, and may possibly even be degenerate.

Our knowledge of neutrino masses and mixing angles is still imprecise and incomplete. Future measurements are likely to bring more surprises.

D. Why Do We Exist?

There is very little matter in our universe. That was not the case at an earlier stage. There were approximately equal amount of matter and anti-matter particles as there were photons and neutrinos. As the universe cooled, most of the matter and anti-matter annihilated into pure energy, and disappeared. The fact that there is still matter left means that there must have been a small imbalance between matter and anti-matter, at the level of one part in ten billion. If not for this excess, all matter would have annihilated with all the anti-matter and we would not exist. Why do we exist? The scientific question is rather what caused this tiny imbalance. It turns out that the finite mass of the neutrino may well have played a fundamental role.

All neutrinos detected are left-handed, namely that their spins point the opposite directions from their momenta. Likewise, all anti-neutrinos are right-handed.

Now that neutrinos were found to have finite masses, as discussed in the previous section, we have to incorporate the massive neutrinos by extending the Standard Model. Note that massive neutrinos do not travel at speed of light. In principle, an observer can go faster than the neutrino and look back at it. Then a neutrino would appear right-handed to the observer. This is a state we have not seen before. Is this a new particle? If so, we have to introduce right-handed neutrinos, which do not have any of the Standard Model
gauge interactions, into the theory. This is the possibility of the Dirac neutrino. On the other hand, we already know neutral right-handed fermions: anti-neutrinos. Could this state be an anti-neutrino? If so, we have to abandon the fundamental distinction between neutrinos and anti-neutrinos, and hence matter and anti-matter. This is the possibility of the Majorana neutrino.

If the neutrino is a Majorana fermion, the the neutrino and the anti-neutrino are the same object. This being the case, it becomes possible for matter to transform into anti-matter and vice-versa. Therefore, the existence of neutrino masses makes it possible to create an imbalance between matter and anti-matter in early universe. This possibility is known as “leptogenesis.” In other words, neutrino masses may play a role in providing the environment necessary for our very existence.\footnote{It has been shown, however, that leptogenesis is possible also for Dirac neutrinos (see Subsec. \ref{sec:leptogenesis}).}

III. OUR PRESENT KNOWLEDGE ABOUT MASSES AND MIXINGS

A. Dirac versus Majorana Neutrinos

The fact that the neutrino has no electric charge endows it with certain properties not shared by the charged fermions of the Standard Model: i.e. it can be its own antiparticle without violating electric charge conservation. In that case, the neutrino is called a Majorana fermion; otherwise it is called a Dirac neutrino. This leads to a whole new class of experimental signatures, the most prominent among them being the process of neutrino-less double beta decay of heavy nuclei, ($\beta\beta_{0\nu}$). Since $\beta\beta_{0\nu}$ arises due to the presence of neutrino Majorana masses, the observation of $\beta\beta_{0\nu}$ decay, in addition to establishing the existence of lepton number violation, can also provide very precise information about neutrino masses and mixing, provided (i) one can satisfactorily eliminate other contributions to this process that may arises from other interactions in full beyond-the-Standard-Model theory, as we discuss below, (ii) one can precisely estimate the values of the nuclear matrix elements associated with the $\beta\beta_{0\nu}$ in question.
B. Neutrino masses and mixings

We will use the notation where the weak-eigenstates (defined as the neutrino that is produced in a charged-current weak interaction process associated with a well-defined charged lepton) are denoted by $\nu_\alpha$ (with $\alpha = e, \mu, \tau, \ldots$), where the ellipsis indicate yet to be discovered “sterile” states that are not produced in association with charged leptons and/or fourth-generation neutrinos.

Let us now focus on the case of only three Majorana neutrinos, with mass matrix $m^\alpha_\nu$ in the weak-eigenbasis (note that $m_\nu$ is symmetric, i.e., $m^\alpha_\nu = m^\beta_\nu$). In the weak-basis where the charged lepton mass-matrix and the charged current coupling-matrix is diagonal, the neutrino mass-matrix is

$$m^\alpha_\nu = \sum_i (U^*)_\alpha i m_i (U^\dagger)_{i\beta},$$

where $U$ is the Maki-Nakagawa-Sakata-Pontecorvo (MNSP) matrix, and $m_i, i = 1, 2, 3,$ are the neutrino mass-eigenvalues, which can be taken real and positive. We choose to write $U = V \times K$, where $V$ and $K$ are given by Eq. (4). The phases in $K$ are the so-called Majorana phases. The case for Dirac neutrinos is similar, except that $K$ can be absorbed into the phases of neutrino mass eigenstates.

The mass-eigenstate $\nu_i, i = 1, 2, 3, \ldots$, has a well-defined mass $m_i$ and we will order the mass eigenvalues such that $m_1^2 < m_2^2$ and $\Delta m^2_{i2} < |\Delta m^2_{i3}|$, where $\Delta m^2_{ij} \equiv m_j^2 - m_i^2$. Flavor eigenstates are expressed in terms of the mass eigenstates as follows: $\nu_\alpha = \sum_i U_{\alpha i} \nu_i$. $U_{\alpha i}$ are the elements of the MNSP matrix, and are related to the observable mixing angles in the basis where the charged lepton masses are diagonal.

For the case of three Majorana neutrinos, the MNSP matrix $U$ can be written as: $VK$, where $V$ will be parameterized as

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, K = \begin{pmatrix} 1 \\ e^{i\phi_1} \\ e^{i\phi_2} \end{pmatrix}.$$

Neutrino oscillation experiments have already provided measurements for the neutrino mass-squared differences, as well as the mixing angles. The allowed values for the $\theta_{ij}$ as well as the $\Delta m^2$'s are, at the $3\sigma$ level: $\sin^2 2\theta_{23} \geq 0.92; 1.2 \times 10^{-3} \text{ eV}^2 \leq |\Delta m^2_{13}| \leq 3.4 \times 10^{-3} \text{ eV}^2$.
FIG. 2: Left: Compilation of various neutrino oscillation experiments. Right: Global fit to solar, atmospheric, and reactor neutrino oscillation data.

$4.8 \times 10^{-3} \text{ eV}^2$; $0.70 \leq \sin^2 2\theta_{12} \leq 0.95$; $5.4 \times 10^{-5} \text{ eV}^2 \leq \Delta m^2_{12} \leq 9.5 \times 10^{-5} \text{ eV}^2$; $\sin \theta_{13} \leq 0.23$. There is currently no constraint on any of the CP-odd phases or on the sign of $\Delta m^2_{13}$.

Since the oscillation data are only sensitive to mass-squared differences, they allow for three possible arrangements of the different mass levels:

(i) Normal hierarchy, i.e. $m_1 \ll m_2 \ll m_3$. In this case, we can deduce the value of $m_3 \simeq \sqrt{\Delta m^2_{23}} \simeq 0.03 - 0.07 \text{ eV}$. In this case $\Delta m^2_{23} \equiv m_3^2 - m_2^2 > 0$. The solar neutrino oscillation involves the two lighter levels. The mass of the lightest neutrino is unconstrained. If $m_1 \ll m_2$, then we get the value of $m_2 \simeq 0.008 \text{ eV}$.

(ii) Inverted hierarchy, i.e. $m_1 \simeq m_2 \gg m_3$ with $m_{1,2} \simeq \sqrt{\Delta m^2_{23}} \simeq 0.03 - 0.07 \text{ eV}$. In this case, solar neutrino oscillation takes place between the heavier levels and we...
have $\Delta m_{23}^2 \equiv m_3^2 - m_2^2 < 0$. We have no information about $m_3$ except that its value is much less than the other two masses.

(iii) Degenerate neutrinos, i.e. $m_1 \simeq m_2 \simeq m_3$.

Oscillation experiments do not tell us about the overall scale of masses. It is therefore important to explore to what extent the absolute values of the masses can be determined. While discussing the question of absolute masses, it is good to keep in mind that none of the methods discussed below can provide any information about the lightest neutrino mass in the cases of a normal or inverted mass-hierarchy. They are most useful for determining absolute masses in the case of degenerate neutrinos, i.e., when all $m_i \geq 0.1$ eV.

One can directly search for the kinematical effect of nonzero neutrino masses in beta-decay by looking for structure near the end point of the electron energy spectrum. This search is sensitive to neutrino masses regardless of whether the neutrinos are Dirac or Majorana particles. One is sensitive to the quantity $m_\beta \equiv \sqrt{\sum_i |U_{ei}|^2 m_i^2}$. The Troitsk and Mainz experiments place the present upper limit on $m_\beta \leq 2.2$ eV. The proposed KATRIN experiment is projected to be sensitive to $m_\beta > 0.2$ eV, which will have important implications for the theory of neutrino masses. For instance, if the result is positive, it will imply a degenerate spectrum; on the other hand a negative result will be a very useful constraint.

If neutrinos are Majorana particles, the rate for $\beta\beta_{0\nu}$ decay Majorana mass for the
neutrino \( \nu \) depends on the combination \( m_{ee} = \sum U_{ei}^2 m_i \), provided heavy particle contributions to this process present in various theories are small. If they are not small however (which will need additional experiments to decide), observing \( \beta\beta_{0\nu} \) decay will still be of fundamental significance since it will provide the first observation of lepton number violation, for which there is strong theoretical motivation.

The present best upper bound on \( \beta\beta_{0\nu} \) decay lifetimes come from the Heidelberg-Moscow and the IGEX experiments and can be translated into an upper limit on \( m_{ee} \lesssim 0.3 \) eV. There is a claim of discovery of neutrinoless double beta decay of enriched \(^{76}Ge\) experiment by the Heidelberg-Moscow collaboration. Interpreted in terms of a Majorana mass of the neutrino, this implies \( m_{ee} \) between 0.11 eV to 0.56 eV. If confirmed, this result is of fundamental significance. For more discussions of this result, we refer the reader to the report of the double beta decay working group.

A very different way to get information on the absolute scale of neutrino masses is to study the spectrum of the cosmic microwave background radiation (CMB), as well as to study the large scale structure of the universe. This is discussed in the cosmology working group report. Observations of CMB anisotropy and surveys of large scale structure have set a limit on the sum of neutrino masses \( \sum m_i \leq 0.7 - 2 \) eV. More recent results from the Sloan Digital Sky Survey (SDSS) place the limit of \( \sum m_i \leq 1.6 \) eV. A point worth emphasizing is that the above result is valid for both Majorana and Dirac neutrinos as long as the “right-handed” neutrinos decouple before the BBN epoch and are not regenerated subsequently.

These limits already provide nontrivial information about neutrino masses: the limit \( \sum_i m_i = 0.7 \) eV, if taken at face value, implies that each individual neutrino mass is smaller than 0.23 eV, which is similar to the projected sensitivity of the proposed KATRIN experiment.

It is clear from the above discussion that there are three urgent pieces of information needed to answer the question of whether the neutrinos are Majorana or Dirac fermions. Three experiments that play a crucial role in this are (i) neutrinoless double beta decay search; (ii) determination of the sign of \( \Delta m^2_{13} \) and (iii) direct search for neutrino mass

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2 In the Dirac case the “right-handed” degrees of freedom are decoupled because of the smallness of the corresponding Yukawa couplings.
TABLE I: Different possible conclusions regarding the nature of the neutrinos and their mass hierarchy from the three complementary experiments.

| $\beta\beta_{0\nu}$ | $\Delta m^2_{13}$ | KATRIN | Conclusion |
|----------------------|------------------|--------|------------|
| yes                  | $>$ 0            | yes    | Degenerate, Majorana |
| yes                  | $>$ 0            | No     | Degenerate, Majorana or normal, Majorana with heavy particle contribution |
| yes                  | $<$ 0            | no     | Inverted, Majorana |
| yes                  | $<$ 0            | yes    | Degenerate, Majorana |
| no                   | $>$ 0            | no     | Normal, Dirac or Majorana |
| no                   | $<$ 0            | no     | Dirac |
| no                   | $<$ 0            | yes    | Dirac |
| no                   | $>$ 0            | yes    | Dirac |

in tritium decay (e.g. KATRIN) or similar decay experiments. In Table I, we present conclusions about the nature of neutrinos for different outcomes of these three types of experiments (for KATRIN a goal of 0.2 eV and for $\beta\beta_{0\nu}$ decay a goal of about 2 meV is taken and $\Delta m^2_{13} = m_3^2 - m_1^2$).

It is clear from Eq. (5) that for Majorana neutrinos, there are three CP phases that characterize neutrino mixings and our understanding of the leptonic sector will remain incomplete without knowledge of these [7, 8]. There are two possible ways to explore CP phases: (i) one way is to perform long baseline oscillation experiments and look for differences between neutrino and anti-neutrino survival probabilities [9]; (ii) another way is to use possible connections with cosmology. It has often been argued that neutrinoless double beta decay may also provide an alternative way to explore CP violation.

C. Sterile neutrinos

A question of great importance in neutrino physics is the number of neutrino species. Measurement of the invisible Z-width in LEP-SLC experiments tell us that only three types of neutrinos couple to the $W$ and $Z$ boson. They correspond to the three known neutrinos $\nu_e, \mu, \tau$. This implies that if there are other neutrino species, then they must
have little or no interaction with the \( W \) and \( Z \). They are called sterile neutrinos. So the question is: are there any sterile neutrinos and, if so, how many?

In the Los Alamos Liquid Scintillation Detector (LSND) experiment, neutrino oscillations both from a stopped muon (DAR) as well as the one accompanying the muon in pion decay have apparently been observed. The evidence from the DAR is statistically more significant and is an oscillation from \( \bar{\nu}_\mu \) to \( \bar{\nu}_e \). The mass and mixing parameter range that fits data is \( \Delta m^2 \approx 0.2 - 2 \text{ eV}^2 \), \( \sin^2 2\theta \approx 0.003 - 0.03 \). There are points at higher masses specifically at 6 eV\( ^2 \) which are also allowed by the present LSND data for small mixings. The KARMEN experiment at the Rutherford laboratory has very strongly constrained the allowed parameter range of the LSND data. Currently the MiniBooNE experiment at Fermilab is under way to probe the LSND parameter region.

Since this \( \Delta m^2_{\text{LSND}} \) is much larger than \( \Delta m^2_{12,23} \), the simplest way to explain these results is to add one \([10, 11]\) or two \([12]\) sterile neutrinos. The sterile neutrinos raise important issues of consistency with cosmology as well as physics beyond the simple three neutrino picture and will be discussed in a subsequent section.

### D. Neutrino magnetic moment and neutrino decay

A massive neutrino can have a magnetic moment. The presence of a magnetic moment allows for new electromagnetic interactions between neutrinos and other fermions of the Standard Model. In particular in neutrino-electron scattering, in addition to the usual weak interaction contribution, there will be a photon exchange contribution to the scattering cross section. The existing neutrino scattering measurements therefore provide an upper limit on the neutrino magnetic moment: \( \mu_{\nu_e} \leq (1 - 1.3) \times 10^{-10} \mu_B \) where \( \mu_B = \frac{e}{2m_e} \) is a Bohr magneton. As we discuss in detail later on, the magnetic moment is a sensitive measure of any new TeV scale physics, i.e. if all physics beyond the Standard Model is at the scale of grand unification or higher, the neutrino magnetic moments will be of order \( 10^{-19} \mu_B \left( \frac{m_e}{1 \text{ eV}} \right) \). Thus any magnetic moment above this value implies the existence of new physics at the TeV scale. A high precision search for a magnetic moment is therefore very important for learning about physics just beyond the Standard Model scale.

Neutrino magnetic moment also leads to new processes that can alter our understanding
of energy balance in astrophysical systems such as in stars and supernovae  [13]. It can also affect considerations involving the neutrinos in the early universe such as the BBN. In sec. (VII A) we discuss more details on magnetic moment and what one can learn from various proposed experiments.

The existence of a neutrino magnetic moment is also related to neutrino decays. For instance if there is a cross-generational structure to magnetic moment as will necessarily be the case if neutrinos are Majorana fermions, then heavier neutrino species can decay radiatively to the lighter ones. Such decays can be detectable in astrophysical experiments. Present upper limits coupled with the general idea about spectra of neutrinos from oscillation experiments, imply that lifetimes of active neutrinos are larger than $10^{20}$ sec., much longer than the age of the universe. Such decays do not therefore affect the evolution of the universe.

It is however possible that there are other scalar particles to which the neutrinos decay; one such example is the majoron, which is a Goldstone boson corresponding to the spontaneous breaking of a global $B - L$ symmetry [14]. The decay to these scalar bosons may occur at a faster rate than that to photons and may therefore have astrophysical and cosmological implications [13].

IV. THE QUESTIONS

The existing data on neutrinos have already raised very important questions, such as the very different mixing angles, that are blazing new trails in physics beyond that Standard Model. They are also helping to define sharp questions to be addressed by near future experiments:

- Are neutrinos Dirac or Majorana?
- What is the absolute mass scale of neutrinos?
- How small is $\theta_{13}$?
- How “maximal” is $\theta_{23}$?
- Is there CP Violation in the neutrino sector?
- Is the mass hierarchy inverted or normal?
- Is the LSND evidence for oscillation true? Are there sterile neutrino(s)?
In the near future, we hope to significantly improve the determination of the elements of the neutrino mass-matrix, although some uncertainty will still remain [17]. Through neutrino oscillation experiments, all three mixing angles $\theta_{12}, \theta_{23},$ and $\theta_{13}$ are expected to be determined with good precision (this is one of the main goals of next-generation neutrino oscillation experiments), while there is hope that the “Dirac phase” $\delta$ can be probed via long-baseline $\nu_\mu \rightarrow \nu_e$ oscillation searches. Neutrino oscillation experiments will also determine with good precision the neutrino mass-squared differences ($\Delta m^2_{12}$ at the 5%-10% level, $\Delta m^2_{13}$ [including the sign] at the few percent level). In order to complete the picture, three other quantities must also be measured, none of which is directly related to neutrino oscillations.

One is the overall scale for neutrino masses. As already briefly discussed, this will be probed, according to our current understanding, by studies of the end-point spectrum of beta-decay, searches for neutrinoless double beta decay, and cosmological observations (especially studies of large-scale structure formation). Note that neutrinoless double-beta decay experiments are sensitive to $|m^{ee}_\nu|$, i.e., they directly measure the absolute value of an element of $m_\nu$. The other two remaining observables are the “Majorana” phases.

Neutrinoless double beta decay experiments are sensitive to a particular combination of masses, mixings and phases:

$$|m^{ee}_\nu| = |\cos^2 \theta_{13} \left( |m_1| \cos^2 \theta_{12} + |m_2| e^{-2i\phi_1} \sin^2 \theta_{12} \right) + \sin^2 \theta_{13} |m_3| e^{-2i(\phi_2 - \delta)}|.$$  \hspace{1cm} (6)

In practice, however, it seems at least very challenging [18] to obtain any information regarding Majorana phases from neutrinoless double-beta decay, in part due to the fact that the relevant nuclear matrix elements need to be computed with far more precision than has been currently achieved.

It must of course be made clear that neutrinoless double-beta decay rate is related to the Majorana phases and neutrino masses only under the assumption that the neutrino masses are the only source of lepton-number violation. Second, only a combination of the two independent Majorana phases can be determined in this way. It is fair to say that there is no realistic measurement one can look forward to making in the near future that will add any information and help us disentangle the “other” Majorana phase. Third, it is curious to note that the effect the Majorana phases have on the rate for neutrinoless double-beta decay is CP-even. While Majorana phases can mediate CP violating phenomena
it seems unlikely that any of them can be realistically studied experimentally in the foreseeable future.

In spite of all the uncertainty due to our inability to measure Majorana phases, it is fair to say that we expect to correctly reconstruct several features of the neutrino mass matrix \cite{17}, especially if the overall mass-scale and the neutrino mass hierarchy are determined experimentally. This will help us uncover whether there are new fundamental organizing principles responsible for explaining in a more satisfying way the values of the neutrino masses and the leptonic mixing angles e.g. whether there are flavor (or family) symmetries, capable of dynamically distinguishing the different generations of quarks and leptons and/or whether there is quark-lepton unification at short distances etc.

Answers from experiments will have crucial impact on the development of the new Standard Model that incorporates newly discovered neutrino mass. Moreover, there are many deep theoretical questions that will be influenced by data. For example,

- We find that the value of $\theta_{13}$ is a good discriminator of models.
- Testing the seesaw hypothesis and discriminating between different types of seesaw using lepton flavor violation.
- Are there new exotic interactions, possibly flavor-changing, for neutrinos?
- What are the admixtures of sterile neutrinos both heavy and light?
- Do neutrinos have magnetic moments?
- How can we understand the neutrinos’ role in the origin of the cosmic baryon asymmetry?
- Can we use neutrinos as probes of other physics beyond the Standard Model?

This document briefly addresses many of these questions.

V. NEUTRINO MASS MODELS

To discuss neutrino masses, we have to specify if they are of Dirac or Majorana type.

Dirac neutrinos require the existence of new right-handed neutrinos that have not been observed. The neutrino mass is generated by the Yukawa coupling of left- and right-handed neutrinos to the Higgs boson, in exactly the same fashion as the charged-lepton and quark masses. Even though this is a simple extension of the Standard Model,
FIG. 4: Seesaw mechanism that explains small Majorana neutrino mass by the exchange of GUT-scale particles.

The extreme smallness of the neutrino masses requires the Yukawa couplings to be of $O(10^{-13})$ or less, which most theorists believe requires an explanation. Some models of extra dimensions or supersymmetry may offer an explanation as discussed later in this report.

The Majorana mass, on the other hand, does not require new light degrees of freedom, but rather a new higher-dimensional operator Eq. (3) $\frac{1}{\Lambda}{(LH)(LH)}$. The atmospheric neutrino data require that $\Lambda \lesssim 10^{16}$ GeV, much lower than the Planck or string scales. It mostly likely means that there are new heavy degrees of freedom whose interaction result in this operator. There are two such possibilities. One is to exchange heavy Majorana right-handed neutrinos that have (not small) Yukawa coupling to the left-handed lepton and Higgs. This mechanism is called the seesaw mechanism [16] because the heavier right-handed neutrino mass implies a lighter left-handed neutrino mass. Another is to exchange an $SU(2)_L$ triplet scalar coupled to $LL$ and $HH$, possible in some SO(10) grand-unified models. The latter possibility is called type II seesaw mechanism.

In this section, we discuss models of Majorana neutrino masses and mixings and their predictions on important quantities such as $\theta_{13}$ and neutrinoless double beta decay rates.

A. Bottom-Up Models

Here, we discuss phenomenological models inspired by the data that in turn give predictions to parameters not measured so far.

In Table I, we identify several textures for the neutrino mass matrix that lead to the currently observed mass-squared differences and mixing angles, and some of the measure-
TABLE II: Different leading-order neutrino mass-textures and their “predictions” for various observables. The fifth column indicates the “prediction” for $|\cos 2\theta_{23}|$ when there is no symmetry relating the different order one entries of the leading-order texture (‘n.s.’ stands for ‘no structure’, meaning that the entries of the matrices in the second column should all be multiplied by and order one coefficient), while the sixth column indicates the “prediction” for $|\cos 2\theta_{23}|$ [19] when the coefficients of the leading order texture are indeed related as prescribed by the matrix contained in the second column.

| Case | Texture | Hierarchy | $|U_{e3}|$ | $|\cos 2\theta_{23}|$ (n.s.) | $|\cos 2\theta_{23}|$ | Solar Angle |
|------|---------|-----------|--------|----------------|----------------|------------|
| A    | $\sqrt{\Delta m_{13}^2/2}$ $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ | Normal | $\sqrt{\Delta m_{12}^2/\Delta m_{13}^2}$ | O(1) | $\sqrt{\Delta m_{12}^2/\Delta m_{13}^2}$ | O(1) |
| B    | $\sqrt{\Delta m_{13}^2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & -1/2 \\ 0 & -1/2 & 1/2 \end{pmatrix}$ | Inverted | $\Delta m_{12}^2/|\Delta m_{13}^2|$ | – | $\Delta m_{12}^2/|\Delta m_{13}^2|$ | O(1) |
| C    | $\sqrt{\Delta m_{13}^2/\sqrt{2}} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ | Inverted | $\Delta m_{12}^2/|\Delta m_{13}^2|$ | O(1) | $\Delta m_{12}^2/|\Delta m_{13}^2|$ | $|\cos 2\theta_{12}|$ $\sim \Delta m_{12}^2/|\Delta m_{13}^2|$ |
| Anarchy | $\sqrt{\Delta m_{13}^2} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ | Normal$^a$ | > 0.1 | O(1) | – | O(1) |

$^a$One may argue that the anarchical texture prefers but does not require a normal mass hierarchy.

ments that will allow us to identify which textures best describe nature. One caveat to the usefulness of this approach is that we have made a choice of weak basis where the charged lepton masses are diagonal and fundamental theories need not manifest themselves in this basis. Nonetheless, by studying some of these textures we can determine some of the measurements (and how precise they should be) that will shed a significant amount of light on the issue of interpreting neutrino masses and mixing angles.

As is clear from Table II, there are several completely different textures that explain the current neutrino data. They differ, however, on the prediction for yet unknown parameters. In particular, one can identify that knowledge of the mass hierarchy and
TABLE III: The maximal values of $\langle m \rangle_{\text{eff}}$ (in units of meV) for the NH and IH spectra, and the minimal values of $\langle m \rangle_{\text{eff}}$ (in units of meV) for the IH and QD spectra, for the best fit values of the oscillation parameters and $\sin^2 \theta_{13} = 0.0, 0.02$ and 0.04. The results for the NH and IH spectra are obtained for $|\Delta m^2_{23}| = 2.6 \times 10^{-3} \text{ eV}^2$ (2.0 $\times$ $10^{-3} \text{ eV}^2$ – values in brackets) and $m_1 = 10^{-4} \text{ eV}$, while those for the QD spectrum correspond to $m_0 = 0.2 \text{ eV}$. (From ref. [20]).

| $\sin^2 \theta_{13}$ | $\langle m \rangle_{\text{eff}}_{\text{NH max}}$ | $\langle m \rangle_{\text{eff}}_{\text{IH min}}$ | $\langle m \rangle_{\text{eff}}_{\text{IH max}}$ | $\langle m \rangle_{\text{eff}}_{\text{QD min}}$ |
|----------------------|-----------------|-----------------|-----------------|-----------------|
| 0.0                  | 2.6 (2.6)       | 19.9 (17.3)     | 50.5 (44.2)     | 79.9            |
| 0.02                 | 3.6 (3.5)       | 19.5 (17.0)     | 49.5 (43.3)     | 74.2            |
| 0.04                 | 4.6 (4.3)       | 19.1 (16.6)     | 48.5 (42.4)     | 68.5            |

measurements of whether $|U_{e3}|^2 \gtrsim 0.01$ and/or $|\cos 2\theta_{23}| \gtrsim 0.01$ will allow us to determine the best path to follow as far as understanding neutrino masses and leptonic mixing is concerned.

As already noted, Majorana nature of massive neutrinos lead to neutrinoless double beta decay processes of type $(A, Z) \rightarrow (A, Z + 2) + e^- + e^-$. This is subject of another working group report. Therefore we do not discuss it in depth here except to note the very interesting prediction that the currently contemplated precision of the next generation of experiments will throw important light on the different pattern of masses which is yet to be experimentally determined. Given the current information on the neutrino oscillation parameters, we summarize in Table [II] some typical predictions for $\langle m \rangle_{\text{eff}}$ in different models. Especially interesting is the lower bound on $\langle m \rangle_{\text{eff}}$ for the case of inverted hierarchy, a value that the next generation experiments are supposed to be able to probe.

B. Grand Unified Models

One of the major ideas for physics beyond the Standard Model is supersymmetric grand unification (SUSY GUT). It is stimulated by a number of observations that are in accord with the general expectations from SUSY GUTs: (i) A solution to the gauge hierarchy problem i.e why $v_{\text{wk}} \ll M_{\text{Pl}}$; (ii) unification of electroweak, i.e. $SU(2)_L \times U(1)_Y$ and strong $SU(3)_c$ gauge couplings assuming supersymmetry breaking masses are in the
TABLE IV: The table lists some typical predictions for $\theta_{13}$ in different SO(10) models and shows how the next generation of experiments can narrow the field of possible SO(10) unification models.

| 126 based models | $\theta_{13}$ | 16 based models | $\theta_{13}$ |
|------------------|---------------|-----------------|---------------|
| Goh, Mohapatra, Ng | 0.18          | Albright, Barr  | 0.014         |
| Chen, Mahanthappa | 0.15          | Ross, Velasco-Sevilla | 0.07 |
|                  |               | Blazek, Raby, Tobe | 0.05 |

TeV range, as would be required by the solution to the gauge hierarchy; (iii) a natural way to understand the origin of electroweak symmetry breaking. The unification of gauge couplings points to a unification scale around $10^{16}$ GeV and simple seesaw intuition leads to a seesaw scale under $10^{16}$ GeV in order to fit atmospheric neutrino data. This suggests that the seesaw scale could be the GUT scale itself; thus the smallness of neutrino mass could go quite well with the idea of supersymmetric grand unification (although one can also get light neutrinos in, e.g., a model with a TeV scale seesaw [21]). However, in contrast with the items (i) through (iii) listed above, the abundance of information for neutrinos makes it a highly nontrivial exercise to see whether the neutrino mixings indeed fit well into SUSY GUTs. In fact, most GUT models proposed before 1998 have been ruled out by the discovery of large mixing angles. In turn, the freedom in constructing realistic GUT models allows many different ways to explain current neutrino observations. Thus, even though neutrino masses are solid evidence for physics beyond the Standard Model, the true nature of this physics still remains obscure. The hope is that the next round of the experiments will help to narrow the field of candidate theories a great deal.

While the $SU(5)$ group is enough to unify $SU(3)_c \times SU(2)_L \times U(1)_Y$ into a simple group, it does not unify the matter content. They are split into $5^*(d^c, L)$ and $10(Q, u^c, e^c)$. The right-handed neutrinos can be introduced to the model but are not required. On the other hand, the $SO(10)$ group unifies the matter content into a single $16$, which in turn requires right-handed neutrinos. Their mass is naturally of the order of the grand-unification scale, once $B - L$ is broken by a Higgs in either $16$ or $126$ representation, and their exchange produces the neutrino mass operator Eq. (3). The Higgs in $126$, however, also contains
a weak-triplet whose exchange can give rise to the type-II seesaw as well. We give a very small sample of the different predictions for $\theta_{13}$ in models with either 16 or 126 in Table IV and a very incomplete list of references in Ref. [22, 23].

Here, we briefly summarize generic consequences of SO(10) models. (i) The neutrino mass hierarchy is normal although with type II seesaw, the spectrum can be degenerate. In fact any evidence for a degenerate neutrino spectrum would be an indication for type II seesaw in general. (ii) They make definite predictions about the mixing angle $\theta_{13}$ as given in Table IV and often for the other mixing angles. (iii) Neutrinos are Majorana. The first two predictions can be tested by long-baseline neutrino oscillation experiments, while the third by neutrinoless double beta decay experiments.

C. Renormalization Effects

In the study of these top-down models, the renormalization group (RG) evolution may affect the neutrino masses and mixings significantly. Formalisms have been developed to study it model-independently. The RG equation of the effective neutrino mass operator in the SM and MSSM [24] can be translated into differential equations for the energy dependence of the mass eigenvalues, mixing angles and CP phases [25]. In the SM and in the MSSM with small $\tan \beta$, the RG evolution of the mixing angles is negligible due to the smallness of the $\tau$ Yukawa coupling. It is the stronger the more degenerate the mass spectrum is. For a strong normal mass hierarchy, it is negligible even in the MSSM with a large $\tan \beta$, but for an inverted hierarchy a significant running is possible even if the lightest neutrino is massless. Furthermore, non-zero phases tend to damp the running. Typically, $\theta_{12}$ undergoes the strongest RG evolution because the solar mass squared difference is much smaller than the atmospheric one. The RG equations for the CP phases show that whenever the mixings run sizably, the same happens for the phases.

Apart from modifying the predictions of top-down models, RG effects also open up new possibilities for model building, such as the radiative magnification of mixing angles [26]. If one restricts oneself to the running below the lowest seesaw scale, $M_1$, significant magnification can occur only if $m_1 \geq 0.1$ eV (a value observable in $\beta\beta_{0v}$ decay). With the running above $M_1$, magnification can be achieved for less degenerate light neutrino spectra, too
(see e.g. [27]). RG effects can also cause important changes of the input parameters for calculations of high-energy processes relevant for leptogenesis. Furthermore, they induce deviations of $\theta_{13}$ from zero and $\theta_{23}$ from the maximal angle that provide an additional motivation for planned oscillation experiments. Given the expected accuracy of these measurements, even relatively small RG effects are interesting in this context.

VI. LEPTOGENESIS AND LOW ENERGY CP PHASE IN SEESAW MODELS

Understanding the origin of matter is one of the fundamental questions of cosmology the answer to which is most likely going to come from particle physics. The seesaw mechanism is at the heart of one particle physics mechanism and we discuss what we can learn about neutrino physics as well as the pattern of right handed neutrino masses from the observed baryon asymmetry.

Three ingredients are required to generate the observed Baryon Asymmetry of the Universe: baryon number violation, CP violation and some out-of-thermal equilibrium dynamics. The seesaw model [16], which was introduced to give small neutrino masses, naturally satisfies these requirements, producing the baryon asymmetry by “leptogenesis” [28]. It is interesting to investigate the relation between the requirements of successful leptogenesis, and the observable neutrino masses and mixing matrix. In particular, does the CP violation that could be observed in neutrino oscillations bear any relation to leptogenesis?

The idea of leptogenesis is to use the lepton number violation of the $N_i$ Majorana masses $M_i$, in conjunction with the $B + L$ violation contained in the Standard Model, to generate the baryon asymmetry. The most cosmology-independent implementation is “thermal leptogenesis” [28, 32, 37, 39].

A. Thermal Leptogenesis

If the temperature $T_{RH}$ of the thermal bath after inflation is $\gtrsim M_1$, the lightest $N_i$, $N_1$, will be produced by scattering. If $N_1$ subsequently decays out of equilibrium, a CP
asymmetry
\[ \epsilon_1 = \frac{\Gamma(N_1 \to LH) - \Gamma(N_1 \to \bar{L}H^*)}{\Gamma(N_1 \to LH) + \Gamma(N_1 \to \bar{L}H^*)} \] (7)
in the decay produces a net asymmetry of Standard Model leptons. This asymmetry is partially transformed into a baryon asymmetry by the non-perturbative \( B + L \) violation. Thermal leptogenesis has been studied in detail [32, 37, 39]; the baryon to entropy ratio produced is
\[ Y_B \simeq C \kappa \frac{n}{s} \epsilon_1 , \] (8)
where \( \kappa \leq 1 \) is an efficiency factor to be discussed in a moment, \( n/s \sim 10^{-3} \) is the ratio of the \( N_1 \) equilibrium number density to the entropy density, and \( \epsilon_1 \) is the CP asymmetry in the \( N_1 \) decay. \( C \sim 1/3 \) tells what fraction of the produced lepton asymmetry is reprocessed into baryons by the \( B + L \) violating processes. \( Y_B \) depends largely on three parameters: the \( N_1 \) mass \( M_1 \), its decay rate \( \Gamma_1 \), and the CP asymmetry \( \epsilon_1 \) in the decay. The decay rate \( \Gamma_j \) of \( N_j \) can be conveniently parameterized as \( \Gamma_j = \frac{|h_j h_{\nu}|^2 M_2}{8 \pi} \equiv \frac{\tilde{m}_j M^2}{8 \pi v^2} \), where \( \tilde{m}_j \) is often of order of the elements of the \( \nu_L \) mass matrix, although it is a rescaled \( N_1 \) decay rate.

Eq. (8) can be of the order of the observed \( Y_B \sim 3 \times 10^{-11} \) when the following conditions are satisfied:

(i) \( M_1 \) should be \( \lesssim T_{RH} \): This temperature is unknown, but bounded from above in certain scenarios.

(ii) The \( N_1 \) decay rate \( \propto \tilde{m}_1 \) should sit in a certain range. \( \tilde{m}_1 \) must be large enough to produce an approximately thermal number density of \( N_1 \)'s, and small enough that the \( N_1 \) lifetime is of order the age of the Universe at \( T \sim M_1 \) (the out of equilibrium decay condition). These two constraints are encoded in the efficiency factor \( \kappa \).

(iii) \( \epsilon_1 \) must be \( \gtrsim 10^{-6} \).

The second requirement sets an upper bound on the mass scale of light neutrinos. The decay rate \( \tilde{m}_1 \) is usually \( \sim m_2, m_3 \); for hierarchical light neutrinos, it naturally sits in the desired range. One can show that \( m_1 \leq \tilde{m}_1 \), so \( m_1 \lesssim 0.15 \) eV [29, 30, 31, 32] is required for thermal leptogenesis in the type I seesaw.\(^4\) This is shown in Fig. 6.

\(^3\) In the so-called ‘strong washout’ regime, \( T_{RH} \) can be an order of magnitude smaller than \( M_1 \) [33].

\(^4\) Note that Ref. [33] derives a somewhat tighter bound. Also note that for type II leptogenesis there is no longer any upper bound on \( m_1 \) (with important implications for neutrinoless double beta decay). Also
FIG. 5: Contour plot of the baryon to photon ratio produced in thermal leptogenesis in the plane of parameters $M_1$ and $\tilde{m}_1$. The three (red) close-together correspond to the observed asymmetry as observed by WMAP. The plot is an updated version of a plot of Ref. [30].

FIG. 6: Upper bound on the light neutrino mass scale, assuming hierarchical $M_i$, taken from [31]. The plot shows the measured baryon asymmetry (horizontal line) compared with the maximal leptogenesis value as function of the heaviest neutrino mass $m_3$.

In the type I seesaw with hierarchical $N_i$, the third condition imposes $M_1 \gtrsim 10^8$ GeV, because $\epsilon_1 \leq 3M_1(m_3 - m_1)/(8\pi v_w^2)$ in most of parameter space [30, 31]. If the $N_i$ are degenerate, with $\Delta M_{ij} \sim \Gamma_i$, this bound on $M_1$ can be evaded [36, 37]. For three $N_i$, the value of $M_1$ has little implication on low energy neutrino observables. If $\epsilon_1$ is maximal – that is, $M_1$ close to its lower bound, – this sets one constraint on the 21 parameters of the type I seesaw. This has no observable consequences among Standard Model particles, because at most 12 masses, angles and phases are measurable, and $\epsilon$ can be maximized by choice of the nine other parameters. The situation is more promising [35] in SUSY models with universal soft terms, where some of the 9 additional parameters can contribute to

in type II leptogenesis, the lower bound on $M_1$ could be reduced by about an order of magnitude [34].
slepton RGEs and thereby to charged lepton flavour violating processes.

B. Any relation with CP Violation in neutrino oscillations?

The leptogenesis parameter $\epsilon_1$ is a $\mathcal{CP}$ asymmetry, suggesting a possible correlation with CP violation in $\nu$ oscillations (the phase $\delta$). It turns out that there is no linear connection between the MNSP phase and leptogenesis, i.e. leptogenesis can work when there is no $\mathcal{CP}$ in MNSP, and measuring low energy leptonic phases does not imply that there is CP violation available for leptogenesis \([38]\). In specific models, however, one may be able to relate the MNSP phase to the leptogenesis phase.

Turning to the type II seesaw case, the phase counting is same as in the type I case and also there is in general no connection between low and high energy CP violation either. The number of CP phases can be obtained by going to a basis in which both $m_\nu$ and $M$ are real and diagonal since they are proportional to each other. Any CP violation will then stem from the matrices $m_D$ and $m_\ell m_\ell^\dagger$ (with $m_\ell$ being the charged lepton mass matrix). Those two matrices posses in total $9 + 3 = 12$ phases. Since the type II models have $M_\nu = v_L f_L$, with $f_L$ a symmetric $3 \times 3$ coupling matrix, which represents the coupling of weak-triplet $\Delta L$ Higgs field to leptons, there are new contributions to leptogenesis. The decay asymmetry in both $N_1$ and $\Delta$ decay may arise from either $N_1$ or $\Delta$ exchange \([41, 42]\). Thus, depending on which contribution dominates, four different situations are possible \([42]\). If $M_1 \ll M_{\Delta L}$ and the conventional term $\mathcal{M}_I^I$ dominates $\mathcal{M}_I^{II}$, we recover the usual seesaw and leptogenesis mechanisms and the statements given earlier apply.

C. Resonant Leptogenesis

If the mass difference between two heavy Majorana neutrinos happens to be much smaller than their masses, the self-energy ($\varepsilon$-type) contribution to the leptonic asymmetry becomes larger than the corresponding ($\varepsilon'$-type) contribution from vertex effects \([37, 43]\). Resonant leptogenesis can occur when this mass difference of two heavy Majorana neutrinos is of the order of their decay widths, in which case the leptonic asymmetry could be even of order one \([37, 44]\). As a result, one can maintain the RH neutrino masses around the GUT scale \([36]\) or one can contemplate the possibility that the heavy neutrino mass
scale pertinent to thermal leptogenesis is significantly lower being in the TeV energies \[37\]. This of course requires a different realization of the seesaw mechanism \[45\] but it can be in complete accordance with the current solar and atmospheric neutrino data \[44\].

The magnitude of the ε-type CP violation occurring in the decay of a heavy Majorana neutrino \(N_i\) is given by \[37\],

\[
\varepsilon_{N_i} = \frac{\text{Im}(h_v^\dagger h_{\nu})_{ij}^2}{(h_v^\dagger h_{\nu})_{ii}(h_v^\dagger h_{\nu})_{jj}} \frac{(M_i^2 - M_j^2)M_i \Gamma^{(0)}_j}{(M_i^2 - M_j^2)^2 + M_i^2 \Gamma^{(0)}_j^2}, \tag{9}
\]

where \(\Gamma^{(0)}_i\) is the tree level total decay width of \(N_i\). It is apparent that the CP asymmetry will be enhanced, possibly to \(\varepsilon \sim 1\), provided if the first factor above is order unity and if \(M_2 - M_1 \sim \frac{1}{2} \Gamma^{(0)}_{i,1,2}\). It is important to note that Eq. (9) is only valid for the mixing of two heavy Majorana neutrinos. Its generalization to the three neutrino mixing case is more involved and is given in \[44\].

Successful leptogenesis requires conditions out of thermal equilibrium. To quantify this, we introduce the parameter, \(K_i = \Gamma^{(0)}_i / H(T = M_i)\) where \(H(T)\) is the Hubble parameter. \(K_i\) should be smaller than a certain value, \(K^{\text{max}}_i\) for successful leptogenesis. Using the parameter \(\tilde{M}_i\) defined in Section VI A, it can be re-expressed as \(\tilde{M}_i \lesssim 10^{-3} K^{\text{max}}_i\ eV\).

Resonant leptogenesis can be successful with values of \(K^{\text{max}}_i\) larger than 1000 \[44\]. This has implications for leptogenesis bounds on the absolute mass scale of the light neutrinos. If a large, \(\gtrsim 0.2\ eV\), Majorana mass was seen in neutrinoless double beta decay, this could be naturally accommodated with resonant leptogenesis.

The conditions for resonant leptogenesis can be met in several ways. For instance, the ‘heavy’ Majorana neutrinos can be as light as 1 TeV \[44\]. SO(10) models with a type III seesaw mechanism naturally predict pairs of nearly degenerate heavy Majorana neutrinos suitable for resonant leptogenesis \[45, 46\].

Soft SUSY breaking terms can give small mass differences between sneutrinos in soft leptogenesis \[47\]. Resonant effects allow sneutrino decay to generate the required CP asymmetry. A model of neutrino mass from SUSY breaking has also been shown to naturally lead to conditions suitable for resonant leptogenesis \[48\].
D. Dirac Leptogenesis

In passing, we would like to mention that lepton number, or, more precisely, $B - L$ has not necessarily to be violated in order to explain our existence, i.e. the observed baryon asymmetry. In the context of neutrino-based baryogenesis mechanisms, one can exploit the fact that only left-handed particles couple to the sphalerons. It has been shown that, in the case of Dirac neutrinos, lepton number can be stored in the right-handed neutrinos during the washout \[49\]. Thus, baryogenesis can work even if $B - L$ is conserved. In particular, the requirement of successful baryogenesis does not imply that neutrinos have to be Majorana particles.

VII. ISSUES BEYOND THE MINIMAL THREE NEUTRINO PICTURE

A. Neutrino magnetic moments

Once neutrinos are massive, they can have magnetic moments. Magnetic moment always connects one species of neutrino with another. When an active neutrino ($\nu_{e,\mu,\tau}$) connects with an active neutrino, we will call it a Majorana type magnetic moment. On the other hand when one of the $\nu_i$ is a sterile neutrino, we will call it Dirac moment. The two have fundamentally different physical implications.

Neutrino magnetic moments can be directly measured in terrestrial experiments using the neutrino beam from the Sun as in Super-K \[50\] or with neutrinos from close by nuclear reactors as in the MUNU \[51\] and in the Texono \[52\] experiments because the presence of magnetic moment gives additional contribution to neutrino scattering off electrons. These experiments have put upper bounds of the order of $10^{-10}\mu_B$ on the effective neutrino magnetic moment where $\mu_B$ is Bohr magneton ($= \frac{e}{2m_e c}$). It is also possible to put bounds on $\mu_{ij}$ from SNO-NC data \[53\] using the fact that neutrinos with non-zero magnetic moments can dissociate deuterium \[54\] in addition to the weak neutral currents. The bounds established from SNO-NC data do not depend upon the oscillation parameters unlike in the case of Super-K. However the bounds are poorer due to the large uncertainty in our theoretical knowledge of the theoretical $^8B$ flux from the Sun \[55\].

In minimal extensions of the Standard Model that include neutrino mass, the value of
the neutrino magnetic moment is \(10^{-19}(m_\nu/1\text{ eV})\mu_B\)\[100\]. However new physics around a TeV tends to give larger values for the magnetic moment \[21\] and therefore search for magnetic moment is a sensitive indicator of new physics near the TeV scale. The effective magnetic moment of the neutrinos can get substantially enhanced in a certain class of extra dimensions models. Searching for \(\mu_\nu\) can therefore be used to put limits on theories with extra dimensions. In particular, in a reactor experiment that searches for differential cross section for \(\nu_e - e\) scattering as a function of the electron recoil energy, the extra dimension models produce a distortion of the spectral shape, which can therefore be a crucial signature of low fundamental scale, large extra dimension models.

B. The search for other light neutrinos

A neutrino that does not participate in Standard Model interactions (sterile) might seem of little interest, but this concept includes reasonable theoretical constructs such as right-handed neutrinos themselves. Furthermore, the hypothesis of ‘sterility’ concerns the weak forces; gravity is expected to be felt anyway, and we cannot exclude that the ‘sterile’ neutrino participates in new forces, perhaps, mostly coupled to quarks; or carried by new heavy mediators; or that sterile neutrinos have preferential couplings with new particles – say, with majorons. Even putting aside these possibilities, we can probe sterile neutrinos by the search for observable effects due to their mixing with the ordinary neutrinos. In this section, we will further restrict our attention on ‘light’ sterile neutrinos (say, below 10 eV) and discuss the impact on oscillations. We make extensive reference to ref. \[101\], an updated overview on the phenomenology of one extra sterile neutrino.

Many extensions of the Standard Model incorporate particles behaving as sterile neutrinos. The main question is \[102\] why these are light. Models with mirror matter (and mirror neutrinos) offer a straightforward answer: ordinary and mirror neutrinos are light for the same reason. It is easy to arrange a ‘communication’ term between ordinary and mirror worlds, e.g., due to the operator \(\sim \nu\phi'\phi'/M_{\text{Planck}}\). This leads to long-wavelength oscillations into sterile neutrinos (see Fig. 7, from \[103\]). There are many other possibilities. Already with mirror matter, the VEV \(\langle\phi'\rangle\) could be different from \(\langle\phi\rangle = 174\text{ GeV}\), and this has important consequences for the phenomenology \[104\]. Alternatively, one
could guess on dimensional grounds the value $\frac{\text{TeV}^2}{M_{\text{Planck}}}$ as the mass (or mixing) of sterile neutrinos, and relate the TeV-value, e.g., to supersymmetry breaking \[105\]. Understanding neutrino mass in extra dimension models also requires the existence of light sterile neutrinos.

In the following discussion of phenomenology, we will be concerned mostly with oscillations. However, the implications can be also elsewhere. To see that, it is sufficient to recall that when we add 3 sterile neutrinos we can form Dirac masses, which means that there is no contribution to neutrinoless double beta decay process.

\textit{a. Terrestrial oscillation experiments} Broadly speaking, there are two types of terrestrial experiments. The first one includes several disappearance experiments and LSND; the second one includes atmospheric neutrinos and long baseline experiments. The first type is sensitive mostly to the mixing of $\nu_e$ and a sterile state, the other one also to $\nu_\mu$ or $\nu_\tau$. Both types of experiments probe only relatively large mixing angles, $\theta_s \sim 0.1$. Sterile neutrinos within the sensitivity regions are disfavored if standard cosmology (mostly BBN) applies; further important tests will be done by CMB+LSS or BBN data. None of these experiments \textit{alone} requires the existence of sterile neutrinos. A case for sterile neutrinos can be made interpreting in terms of oscillations LSND together with solar and atmospheric anomalies \[106\]. The hypothesis that LSND signal is due to a relatively heavy and mostly sterile neutrino should be regarded as conservative \[107\], even though it leads to some problems with disappearance in terrestrial experiments, and interesting predictions for cosmology (BBN and CMB+LLS spectra). In view of this situation, the test of the LSND result is of essential importance. At the same time, we should not forget that sterile neutrinos could manifest themselves in other manners.

\textit{b. Solar and KamLAND neutrinos} The solar and KamLAND data can be explained well without sterile neutrinos. Even more, the ‘LMA’ solution received significant confirmations: the sub-MeV energy regions have been probed by Gallium experiments and the super-MeV ones by SNO and Kamiokande, and LMA is in agreement with KamLAND. Thus we are led to consider minor admixtures of sterile neutrinos, presumably not more than 20 %. In many interesting cases sterile neutrinos are invisible at KamLAND but affect the survival probability of solar neutrinos. Quite generally, to test the hypothesis of oscillations into sterile states it would be important to improve on (or measure precisely)
FIG. 7: The double degeneracy between mass eigenstates of ordinary and mirror world ($\nu_i$ and $\nu'_i$) is lifted when the small mixing terms are included in the $6 \times 6$ mass matrix. The new mass eigenstates ($\nu_i^+$ and $\nu_i^-$) are in good approximation maximal superpositions of $\nu_i$ and $\nu'_i$.

the fluxes from Beryllium and pp-neutrinos.

c. Ultra-high energy neutrinos  Although there is a great deal of interest in the search for ultra-high energy neutrinos, the number of reasonable (or even, less reasonable) mechanisms that have been discussed to produce them is not large. The reason is that neutrinos are produced along with electromagnetic radiation, that can be observed in a variety of ways, even when this is reprocessed. Following this line of thought, the astrophysical mechanism that can be conceived to overcome such a stricture is the concept of a ‘hidden source’. Another escape from this constraint involves sterile neutrinos. Indeed, if there are ultra-high energy mirror neutrinos, they inevitably oscillate into neutrinos from our world on cosmic scales \cite{108}. This scenario can provide intense fluxes of ultra-high energy neutrinos, subject only to the observable electromagnetic radiation from their interaction with the relic neutrino sea.

C. Supersymmetry and neutrinos:

Neutrino masses are not the only motivation to extend the Standard Model. One also likes to extend it in order to solve the gauge hierarchy problem. Models of low-energy supersymmetry are attractive candidates for the theory of TeV scale physics. In the minimal supersymmetric extension of the Standard Model (MSSM) neutrinos are massless. Thus, we need to consider supersymmetric extensions of the Standard Model that allow for neutrino masses.
There are basically three questions we like to answer when we talk about the relations between supersymmetry and neutrinos:

(i) Can successful predictions for neutrino masses of non-supersymmetric extensions of the Standard Model be retained once these models are supersymmetrized? In particular, can supersymmetry help in making such models more motivated?

(ii) Are there models where neutrino masses arise only due to supersymmetry?

(iii) Are there interesting phenomena in the slepton sector that can shed light on the issue of neutrino masses, lepton number violation and lepton flavor violation?

In the following we briefly describe two frameworks where neutrino masses are tightly connected to supersymmetry. We also discuss two effects, that of charged lepton flavor violation and sneutrino–antisneutrino oscillation, that can help us disentangle the origin of neutrino masses using supersymmetric probes.

1. Seesaw Mechanism and Charged Lepton Flavor Violation

In the Standard Model, there is no lepton flavor violation. Neutrino oscillation experiments have revealed that flavour is much more violated in the lepton than in the quark sector. However if one simply extends the Standard Model by the addition of an appropriate neutrino mass matrix in a gauge invariant manner, the magnitude of charged lepton flavor violation is very small (the branching ratio being given by \( \left( \frac{m_\nu}{m_W} \right)^4 \)). The situation remains unchanged even when the seesaw mechanism is used to generate neutrino masses, since the seesaw scale is very high. However, if the theory is supersymmetric, the flavor mixings in either the Dirac neutrino mass matrix or the RH neutrino mass matrix (or both) can transmit flavor mixings to the slepton sector which can then lead to lepton flavor violating processes such as \( \mu \rightarrow e + \gamma \) and \( \tau \rightarrow \mu + \gamma \). Current bounds for \( B(\mu \rightarrow e + \gamma) \leq 1.2 \times 10^{-11} \) and \( B(\tau \rightarrow \mu + \gamma) \leq 2 \times 10^{-7} \). These limits are expected to be pushed down to the level of \( 10^{-14} \) and \( 10^{-8} \) level. For reasonable values for the supersymmetry parameters, seesaw models can predict these branching ratios at these levels. Combined with supersymmetry searches at the LHC, one can hope to probe the validity of the seesaw mechanism. One exception to this is the class of models with TeV scale seesaw, where even without supersymmetry, charged lepton flavor violation
could be large.

2. Neutrino masses from R-parity violation

Neutrino masses from R-parity violation have been extensively studied. Here we briefly summarize the main results [58]. Once R-parity is violated there is no conserved quantum number that would distinguish between the down-type Higgs doublet and the lepton doublets. Thus, these fields in general mix. Such mixing generates neutrino masses; in fact, they generically produce too large masses. One neutrino gets a tree level mass which depends on the mixings between the Higgs and the sneutrinos. The other two neutrinos get their masses at the one loop level, and thus their masses are smaller by, roughly, a loop factor. The most attractive feature of R-parity violation models of neutrino masses is that they naturally generate hierarchical neutrino masses with large mixing angles. This is due to the fact that only one neutrino gets a mass at tree level, while the other neutrinos only acquire loop induced masses. Numerically, however, the predicted mass hierarchy is in general somewhat too strong. The biggest puzzle posed by R-parity violation models is to understand the smallness of the neutrino masses. There must be a mechanism that generates very small R-parity violating couplings. There are several ideas of how to do it. For example, the small R-parity violation couplings can be a result of an Abelian horizontal symmetry [59] or left-right SUSY [60].

3. Neutrino masses from supersymmetry breaking

The smallness of neutrino masses can be directly related to the mechanism of supersymmetry breaking, in particular to the mechanism that ensures a weak scale $\mu$ parameter [61, 62, 63]. In general, there is no reason why the MSSM $\mu$ parameter is of the order of the weak scale. Generically, it is expected to be at the cut-off scale of the theory, say the Plank or the GUT scale. Phenomenologically, however, $\mu$ is required to be at the weak scale. One explanation, which is known as the Giudice-Masiero mechanism, is that a $\mu$ term in the superpotential is not allowed by a global symmetry. The required effective weak scale $\mu$ is generated due to supersymmetry breaking effects.

The Giudice-Masiero mechanism can be generalized to generate small neutrino masses.
It might be that the large Majorana mass term that drives the seesaw mechanism is forbidden by a global symmetry. Effective Majorana mass terms for the right handed neutrinos, of the order of the weak scale, are generated due to supersymmetry breaking. The same global symmetry can also suppress the Dirac mass between the right and left handed neutrinos. Then, the left handed neutrinos have very small Majorana or Dirac masses as desired. The emerging neutrino spectrum depends on the exact form of the global symmetry that is used to implement the Giuduce-Masiero mechanism. Nevertheless, the feature that the left-handed neutrino masses are very small is generic.

4. Sneutrino oscillation

Supersymmetric models can also lead to sneutrino–antisneutrino mixing and oscillation [64]. This phenomena is analogous to the effect of a small $\Delta S = 2$ perturbation to the leading $\Delta S = 0$ mass term in the $K$-system which results in a mass splitting between the heavy and light neutral $K$ mesons. The very small mass splitting can be measured by observing flavor oscillations. The sneutrino system can exhibit similar behavior. The lepton number is tagged in sneutrino decay using the charge of the outgoing lepton. The relevant scale is the sneutrino width. If the sneutrino mass splitting is large, namely when $x_\tilde{\nu} \equiv \Delta m_{\tilde{\nu}}/\Gamma_{\tilde{\nu}} \gtrsim 1$, and the sneutrino branching ratio into final states with a charged lepton is significant, then a measurable same sign dilepton signal is expected. Any observation of such oscillation will be an evidence for total lepton number violation, namely for Majorana neutrino masses.

D. Neutrinos in extra dimensions

The pioneering idea by Kaluza and Klein (KK) [65] that our world may have more than four dimensions has attracted renewed interest over the last ten years [66, 67]. The possible existence of extra dimensions has enriched dramatically our perspectives in searching for physics beyond the Standard Model. Obviously, extra dimensions have to be sufficiently compact to explain why they have escaped detection so far, although their allowed size is highly model-dependent. This means that the derived constraints not only depend on the number of the fields sensitive to extra dimensions but also on the geometry
and/or the shape of the new dimensions.

Models with large extra dimensions generically have a low fundamental scale, and it is often the case that the seesaw mechanism cannot be properly implemented. An alternative way to understand small neutrino masses is to introduce singlet neutrinos that propagate in a higher \([1 + (3 + \delta)]\)-dimensional space (where \(\delta\) is the number of the additional spatial compact dimensions). In this formulation, the ordinary SM particles reside in a \((1 + 3)\)-dimensional Minkowski subspace, which is called the wall. The overlap of their wave-functions with the bulk neutrinos is suppressed by the volume of the extra-dimensional space \((R M_F)^{\delta/2} \approx M_P/M_F\), where \(R\) is the common compactification radius, \(M_F\) is the fundamental gravity scale and \(M_P \approx 10^{16}\) TeV is the usual Planck mass. This volume-suppression factor gives rise to effective neutrino Yukawa couplings that are naturally very small, i.e. of order \(M_F/M_P \sim 10^{-15}\), for \(M_F = 10\) TeV, although the original higher-dimensional Yukawa couplings of the theory could be of order unity.

There are several generic consequences of these models:

(i) There is a closely spaced tower of sterile neutrinos in such models which can be emitted in any process where the final state is a sterile neutrino. A typical example is the magnetic moment contribution to \(\nu_e - e\) scattering in a reactor \([70, 71]\). Reactor searches for magnetic moment can therefore shed light on the size of extra dimensions (see Fig. 8).

(ii) When neutrinos travel through dense matter there can be MSW resonances \([69]\) that can rise to a dip pattern \([39, 72]\) in the neutrino survival probability corresponding to energies spaced by \(E \approx \Delta m^2_{\nu_FKK}/2 \sqrt{2} G_F N_e\) (i.e. \(E, 4E, 9E,\ldots\)) since typically the survival probability goes like \(e^{-\Delta m^2_E/2}\). For solar neutrinos, such dip structure is quite pronounced \([72]\). In the hierarchical pattern for neutrino masses, this would correspond to \(E \approx 10\) MeV for densities comparable to solar core. The value of the energy clearly depends on the size of the extra dimensions; therefore looking at neutrinos of different energies such as those from Sun, atmosphere and distant galaxies, one can probe different sizes of the extra dimensions.

(iii) The cumulative effect of the neutrino KK tower also leads to enhanced flavor violating effects \([73]\).

(iii) One may add lepton-number violating bilinears of the Majorana type in the Lagrangian \([68]\), e.g. operators of the form \(N^T C^{(5)-1} N\), where \(C^{(5)} = -\gamma_1 \gamma_3\) is the charge
FIG. 8: The figure shows the contribution of a neutrino magnetic moment for the case of single Dirac neutrino, for two large extra dimensions (and a comparison between the two) to differential cross section $\frac{d\sigma}{dT}$ (where $T$ is the electron recoil energy) for neutrino electron scattering and compares it to the case of one right handed neutrino (“Standard Model with one right handed neutrino”).

conjugation operator, which can then add new contributions to neutrinoless double beta decay. These models provide other sources of both lepton flavor violation as well as lepton number violation that can be experimentally interesting.

VIII. EXOTIC PHYSICS AND NEUTRINOS

A. New long range forces

The possible existence of new long range forces has always been an interesting one in particle physics. A special class of long range forces which distinguish between leptonic flavors has far reaching implications for neutrino oscillations [90, 91] which may be used as probes of such forces. Anomaly considerations leave a limited choice for such forces, i.e., the ones coupling to $L_i - L_j$ (where $i, j = e, \mu, \tau$). It is possible in this case to have long range forces with range of the order of the Earth-Sun distance. Such forces would induce
matter effects in terrestrial, solar and atmospheric neutrino oscillations. For example, the electrons inside the Sun generate a potential $V_{LR}$ at the earth surface given by

$$V_{LR} = \alpha \frac{N_e}{R_{es}} \approx (1.04 \times 10^{-11} eV) \left( \frac{\alpha}{10^{-50}} \right),$$

where $\alpha \equiv \frac{g^2}{4\pi}$ corresponds to the gauge coupling of the $L_e - L_{\mu,\tau}$ symmetry, $N_e$ is the number of electrons inside the Sun and $R_{es}$ is the Earth-Sun distance $\approx 7.6 \times 10^{26} GeV^{-1}$. The present bound on the $Z$-dependent force with range $\lambda \sim 10^{13}$ cm is given by $\alpha < 3.3 \times 10^{-50}$. Eq. (10) then shows that the potential $V_{LR}$ can introduce very significant matter-dependent effects in spite of the very strong bound on $\alpha$. One can define a parameter $\xi \equiv 2E_\nu V_{LR}/\Delta m^2$ which measures the effect of the long range force in any given neutrino oscillation experiment. Given the terrestrial bound on $\alpha$, one sees that $\xi$ is given by $\xi_{atm} \sim 27.4$ in atmospheric or typical long baseline experiments while it is given by $\xi_{solar} \sim 7.6$ in the case of the solar or KamLAND type of experiments. In either case, the long range force would change the conventional oscillation analysis. The relatively large value of $\alpha$ suppresses the oscillations of the atmospheric neutrinos. The observed oscillations then can be used to put stronger constraints on $\alpha$ which were analyzed in [90]. One finds the improved 90% CL bound: $\alpha_{e\mu} \leq 5.5 \times 10^{-52}$, $\alpha_{e\tau} \leq 6.4 \times 10^{-52}$, in case of the $L_e - L_{\mu,\tau}$ symmetries respectively.

Although these bounds represent considerable improvement over the conventional fifth force bound, they still allow interesting effects which can be used as a probe of such long range forces in future long baseline experiments with super beam or at neutrino factories. As a concrete example, the influence of the $L_e - L_\mu$ gauge interactions on the long baseline oscillations of muon neutrinos of $\mathcal{O}(GeV)$ energy. The survival probability in this case as a function of energy is given in Fig. 9.

B. Non-standard Neutrino Neutral Current Interactions

The latest results of neutrino oscillation experiments indicate that the conversion mechanism between different neutrino flavors is driven by a non-vanishing mass difference between mass eigenstates together with large mixing angles between families. However, these conclusions are achieved supposing that no non-standard neutrino interactions (NSNI) are present. The inclusion of NSNI can modify the characteristics of neutrino conversion, and
FIG. 9: The long baseline neutrino oscillation probability $P_{e\mu}$ in case of vacuum (solid), the earth matter effects (dotted) and with inclusion of the long range potential $V_{LR}$ (dashed). The plotted curves correspond to a baseline of 740 km, $\Delta m_{32}^2 = 2.5 \times 10^{-5}\text{eV}^2$, $\Delta m_{21}^2 = 7.0 \times 10^{-5}\text{eV}^2$, $(\theta_{12}, \theta_{23}) = (32^0, 45^0)$, $\alpha_{e\mu} = 5.5 \times 10^{-52}$ and $\sin \theta_{13} = 0.05$.

in general large values of NSNI parameters worsen the quality of the fit to data. We can then use neutrino oscillation experiments to set limits to NSNI parameters.

The atmospheric neutrino data are well described by the oscillation driven by one mass scale, $\Delta m_{32}^2$, and with maximal mixing between second and third families. The addition of sufficiently large NSNI on top of the masses and mixing can be expected to spoil the fit. A two-family ($\nu_\mu$, $\nu_\tau$) analysis constrains the flavor-violating NSNI to be less than a few percent of the standard weak interaction. A generalization to the three-family analysis, however, significantly relaxes this bound and in fact allows NSNI comparable in strength to the standard model interactions.

The oscillation of solar neutrinos is driven by only one mass scale, $\Delta m_{21}^2$. The upper limits on the flavor-conserving NSNI are at tens of percents of the standard weak interaction and hence are surprisingly weak. Moreover, the flavor-changing NSNI is likewise weakly constrained.

Apart from phenomena that involve neutrino oscillations, bounds on NSNI can also
come from the bounds of such non-standard interactions on the charged leptons. One should be careful in translating such bounds to the neutrino sector, since one is only possible if details regarding the model that generates the non-standard interactions are known. Recent analyses of such bounds can be found in [96, 97, 98, 99].

One can argue that here is a hint for NSNI. The NuTeV experiment [74] at Fermilab has measured the ratios of neutral to charged current events in muon (anti)neutrino – nucleon scattering and from these has obtained values of effective coupling parameters $g^{2}_{L} = 0.30005 \pm 0.00137$ and $g^{2}_{R} = 0.03076 \pm 0.00110$ [73]. Standard Model (SM) predictions of these parameters based on a global fit to non-NuTeV data, cited as $[g^{2}_{L}]_{SM} = 0.3042$ and $[g^{2}_{R}]_{SM} = 0.0301$ in Ref. [74], differ from the NuTeV result by 3$\sigma$ in $g^{2}_{L}$. The significance of the result remains controversial [76] and a critical examination of the initial analysis is ongoing, but it remains a distinct possibility that the discrepancy with the SM prediction is genuine and that its resolution lies in physics beyond the SM [77].

Neglecting $g^{2}_{R}$, the ratio of neutral to charged current events is simply $g^{2}_{L}$. Since the NuTeV value for $g^{2}_{L}$ is smaller than its SM prediction, possible new physics explanations of the NuTeV anomaly would be those that suppress the neutral current cross sections over the charged current cross sections, or enhance the charged current cross sections over the neutral current cross sections. Two classes of models have been proposed which accomplish this.

The first class comprises models which introduce new neutrino-quark interactions, mediated by leptoquarks or extra $U(1)$ gauge bosons ($Z'$s), which interfere either destructively with the $Z$-exchange amplitude, or constructively with the $W$-exchange amplitude [74, 78]. Models in this class are constrained strongly by lepton universality and predict gauge boson masses in the several 100 GeV to TeV range, within reach of LHC. Models of the second class suppress $Z\nu\nu$ and $W\mu\nu_{\mu}$ couplings by mixing the neutrino with heavy gauge singlet states (neutrissimos) [79, 80, 81, 82]. Suppressions of the neutrino-gauge couplings also affect most other electroweak observables and may violate lepton universality. These models predict new heavy particles which might be found at LHC and can be constrained by tests of lepton universality, lepton flavor violation [83, 84, 85, 86], muon $g - 2$ [87, 88], and violations of CKM unitarity [89].
C. Lorentz noninvariance, CPT violation and decoherence

CPT is a symmetry in any theory that satisfies the three assumptions that are normally taken for granted: (1) locality, (2) Lorentz invariance, and (3) hermiticity of the Hamiltonian. In particular, it predicts that the mass is common for a particle and its anti-particle. Any violation of CPT would have profound consequences on fundamental physics.

The best limit on CPT violation is in the neutral kaon system, $|m(K^0) - m(\bar{K}^0)| < 10^{-18}m_K = 0.50 \times 10^{-18}$ GeV [109]. Such a stringent bound does not seem to naively allow sizable CPT violation in the neutrino sector. However, the kinematic parameter is mass-squared instead of mass, and the constraint may naturally be considered on the CPT-violating difference in mass-squared $|m^2(K^0) - m^2(\bar{K}^0)| < 0.25$ eV$^2$. In comparison, the combination of SNO and KamLAND data leads to the constraint $|\Delta m^2_{\nu} - \Delta m^2_{\bar{\nu}}| < 1.3 \times 10^{-3}$ eV$^2$ (90% CL) and hence currently the best limit on CPT violation [110].

New motivation for considering CPT violation among neutrinos arose recently with the observation that the LSND, solar, and atmospheric neutrino data can be accommodated simultaneously without invoking a sterile neutrino provided the neutrino and anti-neutrino masses are not the same violating CPT [107, 111, 112].

The KamLAND data, however, require $\bar{\nu}_e \rightarrow \bar{\nu}_{\mu,\tau}$ oscillations with parameters consistent with the solar neutrino oscillation, and CPT-violation alone cannot explain LSND. A different proposal to explain LSND and atmospheric anti-neutrino oscillations with a single $\Delta m^2$ [113], is excluded by the atmospheric neutrino data [114]. The introduction of CPT violation improves significantly four neutrino (three active and one sterile) fits to all neutrino data (including LSND) [115]. This is due to the fact that the short-baseline experiments constraining the interpretation of the LSND data with a sterile neutrino involve mostly neutrinos but not anti-neutrinos, and the 3 + 1 spectrum (Fig. 10) is allowed if there is little mixing of the sterile state with the active ones.

Other possibilities that go beyond conventional quantum field theory have been proposed as a way to understand the LSND anomaly. Decoherence is one such possibility [116], which can be tested using neutrino oscillation studies.
FIG. 10: The revised proposal in [115] that combines CPT violation and a sterile neutrino. The neutrinos always have $2 + 2$ spectrum, while the anti-neutrinos may have either $3 + 1$ or $2 + 2$ spectrum.

IX. CONCLUSION

In this report, we presented a brief review of the present knowledge of neutrino physics and what we can learn from planned experiments in the next decade. The discussion group feels that three most important experiments (beyond the KATRIN experiment which is already under way) that will have a significant impact on clarifying the nature of neutrino mass hierarchy, the nature of the neutrino (Dirac or Majorana) as well as the search for physics beyond the Standard Model are: (i) search for $\beta\beta$ decay, (ii) determination of the sign of the $\Delta m_{13}^2$ and (iii) measurement of the value of $\theta_{13}$. The last one will not only specify the neutrino mass matrix more precisely than we know today but will considerably narrow the field of models. Next in our top priority list is the measurement of the Dirac phase, which will give a partial understanding of CP violation in the leptonic sector.

We believe that all support should be given to MiniBooNE experiment till it provides a complete resolution of the LSND result. If the MiniBooNE confirms the LSND result, we need to completely revise our current understanding of neutrinos and, perhaps, particle physics. Therefore search for properties of the sterile neutrinos will become a very high priority item at the same level as those discussed in the previous paragraph.
Within the three neutrino picture, the precise measurement of the solar and atmospheric mixing angles will significantly help discriminate among various new physics possibilities. We consider it as the next level of priority.

If MiniBooNE does not confirm LSND, the light sterile neutrinos could still be playing a subdominant role in solar neutrino physics, as has been suggested by several theoretical models. At the next level of priority, we consider items such as: (i) search for subdominant effects of light sterile neutrinos using precision measurements of pp neutrinos from the Sun; (ii) search for neutrino magnetic moment, whose values below the current astrophysical limit of $10^{-11} \mu_B$ will be a sure indication of TeV scale new physics, such as a TeV scale left-right model, horizontal models, or large extra dimensions; (iii) searches for exotic physics involving neutrinos that can test the limits of the assumptions on which the Standard Model is based e.g., the violation of Lorentz invariance, the existence of new long range forces coupled to lepton number, CPT violation, etc.

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