Hair mass bound in the black hole with non-zero cosmological constants

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Abstract

We study mass bounds of Maxwell fields in RN black holes and genuine hair in Einstein-Born-Infeld black holes with non-zero cosmological constants. It shows that the Maxwell field serves as a good probe to disclose distribution of black hole hair. And we find that the Hod’s lower bound obtained in asymptotically flat space also holds in the asymptotically dS Einstein-Born-Infeld hairy black holes. In contrast, the Hod’s lower bound can be invaded in the asymptotically AdS Einstein-Born-Infeld hairy black holes. It implies that the AdS boundary could confine the field and make the hair easier to condense around the near horizon area. We also conjecture that effects of cosmological constants on hair distribution are qualitatively the same in other hairy black holes.

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I. INTRODUCTION

The famous no hair conjecture of Wheeler [1–3] was motivated by researches on uniqueness theorems that Einstein-Maxwell black holes are described by only three conserved parameters: mass, electric charge and angular momentum [4–10]. The belief in the no hair conjecture was based on a simple physical picture that matter fields outside black holes would eventually be radiated away to infinity or be swallowed by the black hole horizon except when those fields were associated with the three conserved parameters. In accordance with this logic, stationary black holes indeed exclude the existence of scalar fields, massive vector fields and spinor fields in the exterior spacetime of black holes [11–18].

However, nowadays we are faced with the surprising discovery of various types of hairy black holes. The first of which were Einstein-Yang-Mills black holes [19–21]. After that, other static hairy black hole solutions were also discovered in theories like Einstein-Skyrme, Einstein-non-Abelian-Proca, Einstein-Yang-Mills-Higgs and Einstein-Yang-Mills-Dilaton and hair formation in non-static kerr black holes was investigated, for references see [22–39] and reviews [40, 41]. The discovery of front hairy black holes provides a challenge to the validity of the classical no hair theorem. Now, it is clear that the formation of hair is due to the fact that self-interaction can bind together the hair in a region very close to the horizon and another region relatively distant from the horizon [42]. In accordance with this physical picture, a no short hair theorem was proposed as an alternative to the no hair conjecture based on the fact that the black hole hair of Einstein-Yang-Mills fields must extend above the photonsphere [42]. Shahar Hod also proved a no short scalar hair theorem that linearized massive scalar fields have no short hair behaviors in non-spherically symmetric non-static kerr black holes [43].

Along this line, it is interesting to study the distribution of hair mass. For the limit case of the linear Maxwell field, Hod showed that the region above the photosphere contains at least half of the total mass of Maxwell fields and also found that this lower bound holds for various genuine hairy black holes in Einstein-Yang-Mills, Einstein-Skyrme, Einstein-non Abelian-Proca, Einstein-Yang-Mills-Higgs and Einstein-Yang-Mills-Dilaton systems [44]. And it was found that the non-linear Einstein-Born-Infeld black holes also satisfy this lower bound that half of the Born-Infeld hair is above the photosphere [45]. In fact, it is reasonable to use Maxwell fields to study density distribution of genuine hair since the Maxwell field case is a linear limit of the non-linear Einstein-Born-Infeld theory. The front studies of hair mass bounds were carried out in asymptotically flat backgrounds. As a further step, it is meaningful to extend the discussion in asymptotically
flat black holes to spacetimes with non-zero cosmological constants.

In the following, we introduce black holes with non-zero cosmological constants and obtain bounds for linear hair mass ratio. We also disclose properties of genuine hair in Einstein-Born-Infeld black holes. We will summarize our main results at the last section.

II. ANALYTICAL STUDIES OF LINEAR HAIR MASS BOUNDS

In this paper, we use the Maxwell field as a linear limit to disclose properties of genuine hair similar to approaches in [44]. And the four dimensional Einstein-Maxwell black hole geometries with non-zero cosmological constant $\Lambda$ are described by [44, 46]:

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2),$$

where metric functions $f(r) = 1 - \frac{2m(r)}{r} - \Lambda r^2$ [46] satisfies $f(r_H) = 0$ with $r_H$ as the black hole horizon. From $f(r_H) = 1 - \frac{2m(r_H)}{r_H} - \Lambda r_H^2 = 0$, we have $m(r_H) = \frac{1}{2}r_H(1 - \Lambda r_H^2)$.

The mass $m(r)$ contained within a sphere of radius $r$ is given by

$$m(r) = \frac{1}{2}r_H(1 - \Lambda r_H^2) + \int_{r_H}^r 4\pi r'^2 \rho(r') dr'.$$

For the Maxwell field, one has the energy density $\rho(r) = -T^t_t = \frac{Q^2}{8\pi r^4}$ [44]. It yields $m(r) = M - \frac{Q^2}{2\pi}$ for the mass function, where $M$ is the total ADM mass of the spacetime.

It was found that the photonsphere can be conveniently used to describe spatial distribution of the matter field [42, 44]. According to the approach in [44], the radius $r_\gamma$ of the null circular geodesic (photonsphere) in the RN black hole is determined by the relation

$$2f(r_\gamma) - r_\gamma f'(r_\gamma) = 0.$$  

From (3), one obtains the radius $r_\gamma$ independent of the cosmological constants as

$$r_\gamma = \frac{1}{2}(3M + \sqrt{9M^2 - 8Q^2}).$$

An interesting quantity which characterizes the spatial distribution of the hair is given by the dimensionless hair mass ratio $\frac{m^+_{\text{hair}}}{m_{\text{hair}}}$, where

$$m^+_{\text{hair}} = M - m(r_\gamma)$$

is the mass of the hair above the photonsphere and

$$m^-_{\text{hair}} = m(r_\gamma) - m(r_H)$$
is the mass of the hair contained between the horizon and the photosphere. For the linear hair of Maxwell field outside the Reissner-Nordstr"om (RN) black hole, Hod obtained bounds on the ratio $\frac{m^{+}_{\text{hair}}}{m_{\text{hair}}} \geq 1$. The ratio can be expressed as

$$\frac{m^{+}_{\text{hair}}}{m_{\text{hair}}} = \frac{M - (M - \frac{Q^2}{2r_H})}{(M - \frac{Q^2}{2r_H}) - (M - \frac{Q^2}{2r_H})} = \frac{1}{r_H - 1}. \quad (7)$$

We have mentioned that $r_{\gamma} = \frac{1}{2}(3M + \sqrt{9M^2 - 8Q^2})$ is independent of the cosmological constant $\Lambda$. In order to study the ratio $\frac{m^{+}_{\text{hair}}}{m_{\text{hair}}}$, we try to research on how the cosmological constant can affect the horizon $r_H$. The horizon $r_H$ can be obtained from the equation $1 - \frac{2M}{r_H} + \frac{Q^2}{r_H^2} - \Lambda r_H^2 = \frac{1}{r_H}(r_H^2 - 2Mr_H + Q^2 - \Lambda r_H^4) = 0$. So we have

$$r_H^2 - 2Mr_H + Q^2 - \Lambda r_H^4 = 0. \quad (8)$$

For simplicity, we introduce functions $y_1 = r^2 - 2Mr + Q^2$ and $y_2 = \Lambda r^4$. The points $r$ satisfying $y_1 = y_2$ correspond to the horizon $r = r_H$. In the following, we are only interested in the outmost horizon and divide our discussion into two cases: $\Lambda > 0$ and $\Lambda < 0$.

Caes I: $\Lambda > 0$

For the case of $\Lambda > 0$ or asymptotically dS black hole spacetime, we have $r_H > M + \sqrt{M^2 - Q^2}$ with diagram of $y_1$ and $y_2$. So the mass ratio bound can be expressed with $x = \frac{Q}{M} \in [0, 1]$ as

$$\frac{m^{+}_{\text{hair}}}{m_{\text{hair}}} = \frac{1}{r_H - 1} > \frac{1}{\frac{1}{2}(3M + \sqrt{9M^2 - 8Q^2}) - 1} = \frac{1}{\frac{1}{2}(3M + \sqrt{9M^2 - 8Q^2}) - 1}. \quad (9)$$

According to the fact that

$$\left(\frac{1}{\frac{1}{2}(3M + \sqrt{9M^2 - 8Q^2}) - 1}\right)_x' = \frac{2x}{\left(1 + \frac{3M}{\sqrt{9M^2 - 8Q^2}} \right)^2} \frac{17 - 18x^2 + 3\sqrt{9 - 8x^2 + 8\sqrt{1 - x^2}}}{3\sqrt{9 - 8x^2 + 8\sqrt{1 - x^2}}} \quad (10)$$

and

$$17 - 18x^2 + 3\sqrt{9 - 8x^2 + 8\sqrt{1 - x^2}} \geq 17 - 18 + 3 + 0 = 2 > 0, \quad (11)$$

we have $\left(\frac{1}{\frac{1}{2}(3M + \sqrt{9M^2 - 8Q^2}) - 1}\right)_x' < 0$ for all $x \in [0, 1]$. So we have

$$\frac{m^{+}_{\text{hair}}}{m_{\text{hair}}} > 1 \quad (12)$$

and the lower bound is with $x = 1$ and $\Lambda \to 0$.

When we increase the value of $\Lambda$, the horizon $r_H$ increase. For $r_H = r_\gamma$ or $\Lambda = \frac{r_\gamma^2 - 2Mr_\gamma + Q^2}{r_\gamma^2} > 0$ with $r_\gamma = \frac{1}{2}(3M + \sqrt{9M^2 - 8Q^2})$, we have

$$\frac{m^{+}_{\text{hair}}}{m_{\text{hair}}} = \frac{1}{r_H - 1} = \frac{1}{r_H - 1} = +\infty. \quad (13)$$
In all, the linear hair mass ratio satisfies the Hod’s mass bound. We further conjecture that asymptotically dS genuine hairy black holes may also obey the Hod’s hair mass bound.

Caes II: $\Lambda < 0$

For another case of $\Lambda < 0$ or asymptotically AdS black hole spacetime, we have $r_H < M + \sqrt{M^2 - Q^2}$ with diagram of $y_1$ and $y_2$. So the mass ratio satisfies the upper bound

$$\frac{m^+_\text{hair}}{m^-_\text{hair}} = \frac{1}{r_H} - 1 < \frac{1}{\frac{1}{M + \sqrt{M^2 - Q^2}} - 1} \leq 2.$$  \hfill (14)

The upper bound corresponds to the case of $Q \to 0$ and $\Lambda \to 0$. The existence of this upper bound is natural since the negative cosmological constant usually serves as a potential to confine the matter field around the horizon. Since we study the case of $0 < r_H \leq r_{\gamma} < +\infty$, there is $\frac{m^+_\text{hair}}{m^-_\text{hair}} = \frac{1}{\frac{1}{r_H} - 1} > 0$. In all, we obtain bounds of the mass ratio

$$0 < \frac{m^+_\text{hair}}{m^-_\text{hair}} < 2.$$ \hfill (15)

Now we show that this lower bound can be approached as $\Lambda \to -\infty$. After choosing $Q \ll 1$, we solve $r^2 - 2Mr - \Lambda r^4 = 0$ to find the horizon $r_H$. Since $-2Mr$ is the leading term in $r^2 - 2Mr - \Lambda r^4$ around $r \approx 0$, we have $r^2 - 2Mr - \Lambda r^4 < 0$ for $r$ a little larger than 0. Also considering $r^2 - 2Mr - \Lambda r^4 \to +\infty$ as $r \to +\infty$, a horizon $r = r_H$ satisfying the equation $r^2 - 2Mr - \Lambda r^4 = 0$ must exist. In the procedure of $\Lambda \to -\infty$, we divide the horizon into three cases: $r_H \to 0$, $r_H \to +\infty$ and $r_H \to C$, where $C$ is a non-zero constant.

In the cases of $r_H \to +\infty$ and $\Lambda \to -\infty$, we have

$$r^2 - 2Mr \to +\infty$$ \hfill (16)

and

$$r^2 - 2Mr - \Lambda r^4 \to +\infty$$ \hfill (17)

in contradiction with the equation $r^2 - 2Mr - \Lambda r^4 = 0$.

In another case of $r_H \to C \neq 0$ and $\Lambda \to -\infty$, we have

$$r^2 - 2Mr - \Lambda r^4 \to C^2 - 2MC - \Lambda C^4 \to +\infty$$ \hfill (18)

in contradiction with the equation $r^2 - 2Mr - \Lambda r^4 = 0$. 

In a word, we have $r_H \to 0$ as $\Lambda \to -\infty$ and $Q \to 0$. For the case of $Q \ll 1$ and $M$ fixed, the lower bound can be approached as

$$m^+_{\text{hair}} = \frac{1}{r_H} - 1 = \frac{1}{\frac{3M}{r_H} - 1} \to 0 \quad \text{as} \quad \Lambda \to -\infty. \quad (19)$$

Here the relation (19) shows that the linear hair of Maxwell field can invade the Hod’s lower bound $m^+_{\text{hair}} \geq 1$. It implies that the Hod’s bound may be invaded in the asymptotically AdS Einstein-Born-Infeld hairy black holes according to the fact that Born-Infeld field hair can be reduced to Maxwell field in the linear limit. We will further check this in the following part.

## III. HAIR MASS BOUNDS OF EINSTEIN-BORN-INFELD BLACK HOLES

We should emphasize that the RN-(A)dS black hole is not hairy since the Maxwell field is associated with a Gauss law. In this part, we extend the discussion to Einstein-Born-Infeld hairy black holes with the Born-Infeld factor associated with no conserved charge $[45, 47]$. The Lagrangian density for Born-Infeld theory is in the form

$$L_{BI} = \frac{1}{b^2} (1 - \sqrt{1 + \frac{b^2}{r^2} F_{\mu\nu} F^{\mu\nu}}). \quad (20)$$

Here $b$ is the Born-Infeld factor parameter. Mention that in the limit $b \to 0$, this Lagrangian reduces to the Maxwell case and properties of the RN black holes may also hold in Born-Infeld hairy black holes at least for very small $b$.

Now we introduce the line element of Born-Infeld black holes with non-zero cosmological constant $\Lambda$ as follows:

$$ds^2 = -f_{EBI}(r) dt^2 + f(r)^{-1} E_{BI} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2), \quad (21)$$

where metric functions $f_{EBI}(r) = 1 - \frac{2M}{r} - \Lambda r^2 + \frac{2b^2}{3} (1 - \sqrt{1 + \frac{Q^2}{r^2} b^2}) + \frac{4Q^2}{b^2} F[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}; -\frac{Q^2}{b^2 r^4}]$, where $Q$ is the charge and $F$ is the hypergeometric function satisfying $(\frac{1}{4} F[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}; -\frac{Q^2}{b^2 r^4}])' = -\frac{1}{\sqrt{r^2 + 2b^2}}$ [48, 50]. The black hole horizon $r_H$ is defined by $f(r_H) = 0$ and the photonsphere radius $r_\gamma$ is determined by the relation $2f_{EBI}(r_\gamma) - r_\gamma f'_{EBI}(r_\gamma) = 0$ [44, 45].

The mass $m_{EBI}(r)$ contained within a sphere of radius $r$ is given by

$$m_{EBI}(r) = M - \frac{b^2 r^3}{3} (1 - \sqrt{1 + \frac{Q^2}{b^2 r^4}}) - \frac{2Q^2}{3r} F[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}; -\frac{Q^2}{b^2 r^4}]. \quad (22)$$
The hair mass ratio is
\[ \frac{m_{EBI}^+}{m_{EBI}^-} = \frac{M - m_{EBI}(r_\gamma)}{m_{EBI}(r_\gamma) - m_{EBI}(r_H)} = \frac{1}{b^2 r^3_\gamma \left(1 - \frac{Q^2}{b^2 r^4_\gamma + \frac{2 Q r_\gamma F}{r_H} \left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4} \right]} \right) - 1}. \] (23)

In the following, we calculate the hair mass ratio in the Einstein-Born-Infeld genuine hairy black holes. In the case of asymptotically flat space with \( M = 1.5, Q = 1, b = 2 \) and \( \Lambda = 0 \), the hair mass ratio is
\[ \frac{m_{EBI}^+}{m_{EBI}^-} \approx 1.896 \geq 1. \] (24)

With \( M = 1.5, Q = 1, \Lambda = 0 \) and various \( b \) from 0.001 to 1000, we find that \( \frac{m_{EBI}^+}{m_{EBI}^-} \) decreases as a function of \( b \) and the ratio approaches the limit value 1.894 for large \( b \). So we conclude that the Hod’s bound \( \frac{m_{EBI}^+}{m_{EBI}} \geq 1 \) holds in the asymptotically flat Einstein-Born-Infeld hairy black holes in accordance with results in [45].

Setting \( M = 1.5, Q = 1, b = 2 \) and \( \Lambda = 0.01 \) in the dS spacetime, we get
\[ \frac{m_{EBI}^+}{m_{EBI}^-} \approx 2.633 \geq 1. \] (25)

Fixing \( M = 1.5, Q = 1 \) and \( b = 2 \), we find that \( \frac{m_{EBI}^+}{m_{EBI}^-} \) increases with respect to \( \Lambda \). It leads to the conclusion that the Hod’s bound also holds in the asymptotically dS Einstein-Born-Infeld hairy black holes.

Now we show that the Hod’s bound can be invaded in the AdS background. With \( M = 1.5, Q = 1, b = 2 \) and \( \Lambda = -0.1 \), we find
\[ \frac{m_{EBI}^+}{m_{EBI}^-} \approx 0.850 < 1. \] (26)

In summary, we show that the Hod’s bound holds in the asymptotically dS Einstein-Born-Infeld hairy black holes. And the Hod’s bound can be invaded in the asymptotically AdS Einstein-Born-Infeld hairy black holes. Due to the confinement of the AdS boundary, these results are natural and we further conjecture that the properties are qualitatively the same for other types of black hole hairs. Since there is also no scalar hair theorem in regular neutral reflecting stars [51, 52] and static scalar fields can condense around charged reflecting stars [53–60], it is also very interesting to extend the discussion to the reflecting star background.

**IV. CONCLUSIONS**

We studied mass distribution of linear hair in RN black holes and genuine hair in Einstein-Born-Infeld theory with non-zero cosmological constants. We used the photonsphere to divide the matter into two parts and obtained lower bounds for the mass ratio. We found that the Hod’s lower bound obtained in asymptotically
flat gravity also holds in the asymptotically dS Einstein-Born-Infeld hairy black holes. In contrast, the Hod’s lower bound can be invade in the asymptotically AdS Einstein-Born-Infeld hairy black holes. This result is natural due to the physical picture that the AdS boundary can provide the confinement and make the hair easier to condense in the near horizon area. And we further conjectured that effects of cosmological constants on hair distribution are qualitatively the same in other hairy black holes.

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