DEFINITION OF THE TOPOLOGICAL STRUCTURE OF THE AUTOMATIC CONTROL SYSTEM OF SPACECRAFTS *

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Abstract. The paper considers the problem of selection the topological structure of the automated control system of spacecrafts. The integer linear model of mathematical programming designed to define the optimal topological structure for spacecraft control is proposed. To solve the determination problem of topological structure of the control system of spacecrafts developed the procedure of the directed search of some structure variants according to the scheme “Branch and bound”. The example of the automated control system of spacecraft development included the combination of ground control stations, managing the spacecraft of three classes with a geosynchronous orbit with constant orbital periods is presented.

Introduction

Modern automated control systems (ACS) including ACS of spacecrafts are sophisticated systems, characterized by complex interaction of elements located on a wide area including the space and requiring the significant resources and time for their development.

The most important problem arisen at such systems analysis and synthesis is the structural design of the control system determining the system properties and characteristics of its functioning [1]. The development of the system structure at designing the new or improving the existing ACS requires such problems solution as the select existing or designing new points of spacecraft control stations, determination of the control system topological structure.

However, when forming the structure of the ACS of spacecrafts the uncertainty caused by the dynamics of control object in the control system functioning appears. This problem due to the presence of uncertainty relates to the problems requiring the support of decision making where the information is converted into the form simplifying the decision making.

Problem description

In the information technologies decisions making is understood to be a set of solutions in conditions of certainty allowing to determine a clear, consistent, correct solutions based on formal models of object control and their environment [2].

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To solve the problem of the structural design of ACS of spacecrafts it necessary to build model and algorithmic tools for selection places for control stations (CS) of spacecrafts that requires the development of the decision-making procedures simulation in the spacecraft control formation.

**Solution methods**

The topological structure of ACS of spacecrafts defines the position and the number of control stations of spacecrafts (SC) [3]. The problem of a topological complex of CS of spacecrafts formation is that for the given sets of spacecrafts of different classes \( E_{SC} \), a number of stations of the possible distance \( L \), movement scheme \( G_k \) and works program \( \Pi_k \) of the each class of spacecrafts it is necessary to find the lowest cost for the designing a set of CS to meet the requirements for controllability and accessibility indicator for the specified classes of spacecrafts, for loading CS (for example, accessibility on a single turn of the spacecraft not less than by two CS, minimum time spent for spacecrafts finding in the region of any CS etc.) [4], [5], [6]. In the result of this problem solution the formation is determined according to what are required in the system and what classes they will interact by.

For the formation problem formalization of the topological structures of ACS of spacecrafts let’s introduce additionally the following notations:

- \( \theta_{kl} \) – time accessibility of SC of the \( k \)-th class in the \( l \)-th station of the possible accessibility;
- \( c_j \) – costs for the designing the \( j \)-th CS;
- \( \tau_{k\min} \) – minimum duration of one session for SC of the \( k \)-th class;
- \( R_k \) – minimum number of CS necessary for SC control of the \( k \)-th class;
- \( L_{\mu k} \) – number of adjacent accessibility areas from \( L_k \) with continuous time accessibility for SC of the \( k \)-th class;
- \( H_k \) – SC accessibility indicator of the \( k \)-th class, defined as the ratio between the overall accessibility time of SC of the \( k \)-th class to the time of SC finding of the \( k \)-th class over the given territory.

Introduce the variables:

\[
z_j = \begin{cases} 
1, & \text{if the } j \text{-th CS is to be used in the system} \\
0, & \text{otherwise} 
\end{cases}
\]

\[
y_l = \begin{cases} 
1, & \text{if the } l \text{-th region of the possible accessibility is to be used in the system} \\
0, & \text{otherwise} 
\end{cases}
\]

The model of topological structure formation is as follows:

\[
\sum_{j \in j} c_j z_j \rightarrow \min 
\]

with restrictions:

on the accessibility indicator for each class of the SC:

\[
\sum_{l \in L_k} \theta_{kl} y_l / \tau_{k} \geq H_k, k \in K 
\]

on minimum number of CS necessary for each class of the SC control:

\[
\sum_{j \in j} z_j \geq R_k, k \in K 
\]
on minimum duration of one session for SC of the $k$-th class:

$$\sum_{l \in L_{\mu k}} \theta_{kl} y_t \geq \tau_{k \min}$$  \hspace{1cm} (4)

on correlation between variables:

$$\sum_{j \in J} z_j \geq y_t \geq \left( \frac{1}{|J|} \right) \sum_{j \in J} z_j, l \in L$$

$$\sum_{j \in J}$$  \hspace{1cm} (5)

The analytical part of the problem formation (expressions (1)-(5)) is a linear model of the mathematical integer programming and some structure variants of ACS of spacecraft can be obtained using the known methods [7], [8], [9]. The expressions (2)-(5) define a set of analytically given constraints $\beta'$. A set of algorithmically specified restrictions is defined with constraints on the controllability of each class of the SC and on the loading of control stations. At checking the validity under algorithmically specified attributes, as parameters of the simulation model are a version of the complex CS of SC of the $X_i$, work program of each class SC, the volume of information transferred since the implementation of controlling interaction between SC of the $k$-th class and $\xi_{kn}$, $N_k$ – number of the SC of the $k$-th class, the scheme of the SC movement of the $k$-th class.

To solve the synthesis problem of the topological structure of the ground control stations complex of ACS of spacecrafts containing analytically and algorithmically given constraints were developed and the procedure that is of the directed search of some structure variants according to the scheme “Branch and bound” [10], and further checking of their belonging to the set $\beta'$, carried out using a simulation model was implemented.

The search process of the optimal structure of the ACS of spacecrafts $X^{opt}_{k}$ can be represented as a tree of variants where a zero pea corresponds to the optimal solution $X^{opt}_{o}$, and each of the vertices with the appropriate edges corresponds to its solution with the additional condition that the variable $x^{mn}_{k}$ has the particular value $x^{mn}_{k} = \{0,1\}$.

Each of the vertices of the tree corresponds to the estimation $\gamma^{mn}_{k}$ equals to the optimal value of the objective function for the corresponding task. The need for further vertex branching occurs in case of integrity conditions non-fulfillment, so at the next step of branching a vertex with the lowest estimation is selected.

Each final vertex of the tree corresponds to the variant of the ACS of spacecrafts $X_k \in \beta''$ structure that using a simulation model examines the implementation of algorithmically specified restrictions. The process continues until the further branching becomes impossible. The optimal variant of the structure $X^{opt}_{k}$ corresponds to the final vertex satisfying algorithmically the given constraints and with the lowest estimation $\gamma^{\min}_{\min}$.

The main features of the procedure:
1. Sets subdivision (branching). The variable select for further branching is as follows. For each variable \(x_k^{m,n} \in X_n^m\) \((k = 1, m)\) receiving the no integral value, calculated the objective function values \(f_0(\tilde{X}_{n,0}^m)\) and \(f_1(\tilde{X}_{n,1}^m)\) are calculated accordingly with values \(f_1(\tilde{X}_{n,1}^m)\) and \(x_k^{m,n} = 1\). For the further subdivision a variable provided the greatest absolute difference between the received values of the objective function is selected, i.e.

\[
\max \left\{ \Delta_k^{m,n} = | \int 1(\tilde{X}_{n,1}^m) - \int 0(\tilde{X}_{n,0}^m) | \ k = 1, m \right\}
\]

and it receives the value with the lowest increasing of the objective function.

2. Estimations calculation (of the lowest bounds). For the initial set \(\tilde{\beta}_0\) the value estimation of the objective function from below \(\gamma(f(\tilde{X}_0)) = f(\tilde{X}_0)\), where \(\tilde{X}_0\) – optimal solution of the linear programming. For the set \(\tilde{\beta}_n^m\) the estimation \(\gamma(f(\tilde{X}_n^m)) = f(\tilde{X}_n^m)\), where \(\tilde{X}_n^m\) – optimal solution of the estimation problem, \(\tilde{X}_n^m \in \tilde{\beta}_n^m\), \(\gamma(f(\tilde{X}_n^m)) = \infty\).

**Application and estimation of the results**

Let’s consider an example of ACS of SC formation that includes the integrity of the ground control systems realizing SC control of three classes

\[E_{SC} = \{ E_{SC}^k / k = 1,3 \}\], with geosynchronous orbits with constant orbital period.

The following initial information is given: a set of possible CS \(E_{CS} = \{ e_j / j = 1,9 \}\), expenses for the \(j\)-th CS design; a set of participants with the possible accessibility and accessibility time of the SC of the \(k\)-th class at the \(l\)-th region \(\theta_{kl}\) (Table 1); a number of each class SC \(N_k(k = 1,3)\) and a set of possible CS for each class of SC \(m_k(k = 1,3)\); accessible duration of the \(k\)-th class SC \(j\)-th to the possible CS \(\tau_{kj}(k = 1,3, j = 1,9)\) (Table 2).

**Table 1.** Accessibility time of the spacecrafts at the regions of the possible accessibility.

| SC class | Regions of the possible accessibility | L_k |
|----------|--------------------------------------|-----|
| 1        | 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 |
| 2        | 6 2.0 5 2.5 7.5 5 7 9 3 5 6 1 9 |
| 3        | 6.5 1.5 6 1.5 5.5 |

**Table 2.** Accessible duration of spacecrafts

| SC class | Regions of controls |
|----------|---------------------|
| 1        | 8 10 5 7 9 |
| 2        | 12 6 9 7 |
| 3        | 8 7 9 |

The movement scheme of the \(k\)-th class taken into account in the imitation model is determined by bulks which elements determine the order of the SC goes through the
accessible regions of the CS, duration of the spacecraft situates in the accessibility regions of each spacecraft and out of the accessibility regions when one of CS is transferring to another CS correspondingly if there is no any gaps between neighbor CSs (Table 3).

**Table 3.** Information from the movement schemes of different classes spacecrafts

| The order of the $k$-th class SC passage | Duration of the spacecraft situates in the accessibility regions | Duration of the spacecraft situates out of the accessibility regions |
|----------------------------------------|----------------------------------------------------------------|----------------------------------------------------------------|
| $k = 1$ | $k = 2$ | $k = 3$ | $k = 1$ | $k = 2$ | $k = 3$ | $k = 1$ | $k = 2$ | $k = 3$ |
| 2 | 4 | 6 | 8 | 12 | 8 | 0 | 0 | 0 |
| 7 | 9 | 8 | 10 | 7 | 9 | 0 | 1 | 0 |
| 3 | 3 | 5 | 10 | 6 | 7 | 0 | 6 | 0 |
| 5 | 4 | 0 | 5 | 9 | 0 | 0,5 | 10 | 155 |
| 6 | 0 | 0 | 7 | 0 | 0 | 121 | 0 | 0 |

The program of the $k$-th class operation $\prod_k (k = 1,3)$ is given with the periodicity of the controllable influence of the $n$-th type realization, its duration and priority. Two types of controllable interactions are considered. The controllable interaction of the first type has the 1-st priority, the time of its realization is random value with a normal distribution law and mathematical expectation equals to 3.5 min., standard deviation is 0.4 min., maximum value is 5 min., minimum value is 2 min. The controllable influence of the second type has the 2-nd priority which duration is also distributed normally with mathematical expectation equals to 2 min., standard deviation is 0.25 min., maximum value is 3 min., minimum value is 1 min.

The periodicity of controllable interactions of the $n$-th type for all classes of SCs is given below in Table 4.

**Table 4.** Periodicity of controllable interactions

| SC class | Periodicity of controllable interactions |
|----------|----------------------------------------|
|          | Type of controllable interactions |
|          | 1 | 2 | 3 |
| 1        | 3 | 3 | 4 |
| 2        | 5 | 7 | 5 |

Moreover the restrictions for globality $H_k (k = 1,3)$ are given: the minimum number of control stations necessary for the $k$-th class SC control $M_k (k = 1,3)$, minimum duration of controllable interactions for spacecrafts of the $k$-th class $\tau_{\min} (k = 1,3)$ and for controllability of each type spacecrafts $\omega_\text{add} (k = 1,3)$ which values are given in Table 5 and also the restrictions for loading of each CS $U_j (j = 1,3)$ are given.

**Table 5.** Restrictions for classes of spacecrafts

| SC class | Restrictions |
|----------|--------------|
|          | $H_k$ | $M_k$ | $\tau_{\min}$ | $\omega_\text{add}$ |
| 1        | 0,7 | 4 | 2 | 0,95 |
| 2        | 0,8 | 2 | 1 | 0,8 |
The optimal variant of topological structure of the considered ACS of spacecrafts received with the help of imitation procedure are given in Table 6 where the optimal structure of control stations and used regions of accessibility are presented. The overall cost volume of this variant designing is 15 conventional units.

**Table 6.** Optimal variant of topological structure of ACS of spacecrafts

| SCclass | Regions of possible accessibility $L_k$ | Control stations |
|---------|----------------------------------------|------------------|
| 1       | \{P_1, P_2, P_4, P_5, P_6, P_7\}       | \{CS1, CS3, CS5, CS6\} |
| 2       | \{P_8, P_{11}, P_{13}\}               | \{CS2, CS3, CS6\}  |
| 3       | \{P_{14}, P_{17}, P_{18}\}            | \{CS4, CS5\}      |

The problem solving of the stability study of the optimal variant of the topological structure of ACS of spacecraft allowed us to determine the potential opportunities of the system development without its structure changing and requirements for the quality of its functioning.

In particular we were solving the problem of simultaneous possible increasing number of the 1-st class spacecrafts and duration of controllable interactions of the first type at realizing the requirement to spacecrafts interactions and CS loading.

For this problem solving tow functional parameters were investigated: $N_1$ – number of 1-st class spacecrafts and $T_1$ – average duration of controllable interactions of the first type, and quality of the system functioning was estimated according to the controllability of the 1-st class spacecrafts $\omega_1$ and CS loading $U_j (j = 1,3)$. As the possible values intervals of the investigated parameters $L_1 = [N_1^{\text{base}}, N_1^{\text{max}}] = [30, 45]$ and $L_2 = [T_1^{\text{base}}, T_1^{\text{max}}] = [3.5, 6.5]$ were selected.

The basis values of functional parameters $N_1^{\text{base}}$ and $T_1^{\text{base}}$, i.e. values used as the initial data at synthesis of the investigated variant structure are equal to 30 and 6/5 correspondingly. The solving results of the problem are given in Table 7 and in Figure 1.

So in the result of the structure stability we determined that the ACS of spacecraft with the given variant of the topological structure has the potential for the further development without additional costs. The number of the 1st class spacecrafts can be increased from 30 to 45; the average duration of the controllable interactions of 1-st type is from 3.5 up to 5.1 minutes taking into account the requirements for the quality of the system functioning.

**Table 7.** Solving results

| Parameters and characteristics | Number of the experiment |
|-------------------------------|--------------------------|
|                               | 1 | 2 | 3 | 4 | 5 |
| $N_1$                         | 45 | 37 | 41 | 39 | 38 |
| $T_1$                         | 6.5 | 4.9 | 5.7 | 5.3 | 5.1 |
| $\omega_1$                    | 0.843 | 0.975 | 0.880 | 0.970 | 0.960 |
| $U_1$                         | 0.710 | 0.470 | 0.600 | 0.485 | 0.775 |
| $U_2$                         | 0.670 | 0.550 | 0.550 | 0.570 | 0.560 |
| $U_3$                         | 0.720 | 0.480 | 0.550 | 0.510 | 0.490 |
Let’s consider another example of the ACS of spacecrafts structure formation. Let the topological structure of the ACS of spacecrafts to control three classes of spacecrafts be developing (Figure 1).

The initial data: number of CS – 7, number of spacecrafts classes – 3, number of accessibility – 9.

**Table 8.** Accessibility time of spacecrafts on regions of possible accessibility

| SC class | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
|----------|----|----|----|----|----|----|----|----|----|
| 1        | 10 | 5  | 6  |    |    |    |    |    |    |
| 2        |    | 5  | 6  | 5  |    |    |    |    |    |
| 3        |    |    |    |    | 3  | 4  | 4  |    |    |

**Table 9.** Restrictions and overall flying time over the trajectory.

| SC class | $N_k$ | $M_k$ | $\tau_k^{\min}$ | Overall flying time $\tau_k$ |
|----------|-------|-------|------------------|-------------------------------|
| 1        | 0.6   | 1     | 9                | 15                            |
| 2        | 0.7   | 2     | 10               | 12                            |
| 3        | 0.6   | 3     | 9                | 14                            |

**Fig. 1.** Accessibility regions and trajectory of flying.
Table 10. Cost volume of control stations building

| CS   | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|------|------|------|------|------|------|------|------|
| Cost | 30   | 50   | 40   | 45   | 50   | 55   | 55   |

Table 11. Calculation solution of the synthesis of the structure problem

| SC class | Accessibility regions $L_k$ | Control stations $m_k$ |
|----------|-----------------------------|------------------------|
| 1        | 1, 3                        | 1, 3                   |
| 2        | 5, 6                        | 3, 4                   |
| 3        | 7, 8, 9                     | 3, 6, 7                |

The overall cost volume of the received complex of control stations of spacecrafts is 205 conventional units, that is less for 100 conventional units of the initial complex.

**Conclusion**
In the work a simulation approach to the development of the automated control systems of spacecraft designed to select the optimal complex of control systems and topology system has been developed. The designed a model and algorithmic unit allows us to automate the selection process of control systems of spacecrafts identifying the minimum necessary number and rational distribution in space. The selection method of the topological structure of ACS of spacecrafts is important for a wide range of practical problems of the control systems structures formation.

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