Can the CP asymmetries in $B \to \psi K_S$ and $B \to \psi K_L$ differ?

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Abstract

In the standard model the CP asymmetries in $B \to \psi K_S$ and $B \to \psi K_L$ are equal in magnitude and opposite in sign to very good approximation. We compute the order $\epsilon_K$ corrections to each of these CP asymmetries and find that they give a deviation from $\sin 2\beta$ at the half percent level, which may eventually be measurable. However, the correction to $a_{\text{CP}}(B \to \psi K_S) + a_{\text{CP}}(B \to \psi K_L)$ due to $\epsilon_K$ is further suppressed. The dominant corrections to this sum, at the few times $10^{-3}$ level, come from the $B$ lifetime difference, and CP violation in $B - \bar{B}$ mixing and $B \to \psi K$ decay. New physics could induce a significant difference in the $\sin(\Delta m_B t)$ time dependence in the asymmetries if and only if the “wrong-flavor” amplitudes $B \to \psi \bar{K}$ or $\bar{B} \to \psi K$ are generated. A scale of new physics that lies well below the weak scale would be required. Potential scenarios are therefore highly constrained, and do not appear feasible. A direct test is proposed to set bounds on such effects.

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I. INTRODUCTION

In the standard model (SM) the CP asymmetries in $B \to \psi K_S$ and $B \to \psi K_L$ have the same magnitudes and opposite signs,

$$a_{CP}(B \to \psi K_S) = -a_{CP}(B \to \psi K_L). \quad (1)$$

Since these two modes have the largest weight in the BABAR and BELLE measurements of CP violation quoted as $\sin 2\beta$ \textsuperscript{[1]}, it is important to understand the accuracy of Eq. (1) in the SM, and whether it could be altered by new physics. In Section II, we review the necessary formalism and explain the conditions that have to be fulfilled in order to violate Eq. (1), in the limit where the $K_S$ and $K_L$ are considered to be pure CP eigenstates. Specifically, we are interested in how different magnitudes for the $\sin(\Delta m_B t)$ terms in the asymmetries could be realized. A necessary condition is shown to be the presence of “wrong-flavor” kaon amplitudes, $B \to \psi K$ or $\bar{B} \to \psi K$, which are negligibly small in the SM.

There are corrections to both sides of Eq. (1) proportional to $\epsilon_K$, due to the fact that the $K_S$ and the $K_L$ are not pure CP eigenstates. One may also expect the $K_S$ to $K_L$ lifetime ratio to enter, since the $K_S$ is identified experimentally by two pions that are produced at a distance from the interaction point that is less than a few times the typical $K_S$ decay length. The probability that a $K_L$ decays into two pions within the same region is suppressed. However, to obtain the corrections to Eq. (1), it is necessary to fully take into account interference effects between the (unobserved) intermediate $K$ and $\bar{K}$ states. In Section III we show that $a_{CP}(B \to \psi K_S)$ and $a_{CP}(B \to \psi K_L)$ receive corrections at order $\epsilon_K$, but the correction to Eq. (1) is further suppressed.

In Section IV we investigate how new physics could yield the wrong-flavor kaon amplitudes required to obtain $a_{CP}(B \to \psi K_S) \neq -a_{CP}(B \to \psi K_L)$. Sizable effects are possible in principle, but we find that the scale of new physics would have to lie well below the weak scale. Potential scenarios are therefore tightly constrained by bounds on flavor changing neutral current processes, and a significant contribution appears rather unlikely. This is illustrated with an example that arises in supersymmetric models with an ultra-light sbottom squark. We also discuss experimentally testable predictions which can be used to set bounds on the wrong-flavor amplitudes. Section V contains our conclusions.
II. FORMALISM

The time dependent CP asymmetries in $B \to \psi K_{S,L}$ (for notation and formalism, see [2, 3, 4]) are given by

$$a_{CP}(B \to \psi K_{S,L}) = \frac{\Gamma(B(t) \to \psi K_{S,L}) - \Gamma(B(t) \to \psi K_{S,L})}{\Gamma(B(t) \to \psi K_{S,L}) + \Gamma(B(t) \to \psi K_{S,L})}$$

$$= -\frac{(1 - |\lambda_{S,L}|^2) \cos(\Delta m_B t) - 2\text{Im}\lambda_{S,L} \sin(\Delta m_B t)}{1 + |\lambda_{S,L}|^2}$$

$$\equiv S_{S,L} s_B - C_{S,L} c_B.$$  (2)

Here $s_B \equiv \sin(\Delta m_B t)$, $c_B \equiv \cos(\Delta m_B t)$, $\Delta m_B \equiv m_H - m_L$, and the last line defines $S_{S,L}$ and $C_{S,L}$. Furthermore,

$$\lambda_{S,L} \equiv \left(\frac{q_B}{p_B}\right)\left(\frac{\bar{A}_{S,L}}{A_{S,L}}\right),$$  (3)

where $\bar{A}_{S,L} \equiv A(\bar{B} \to \psi K_{S,L})$ and $A_{S,L} \equiv A(B \to \psi K_{S,L})$. The neutral $B$ and $K$ meson mass eigenstates are defined in terms of flavor eigenstates as

$$|B_{L,H}\rangle = p_B|B\rangle \pm q_B|\bar{B}\rangle, \quad |K_{S,L}\rangle = p_K|K\rangle \pm q_K|\bar{K}\rangle.$$  (4)

In the $|\lambda_{S,L}| = 1$ limit, which is usually considered, the asymmetries reduce to the simple form $a_{CP}(B \to \psi K_{S,L}) = \text{Im}\lambda_{S,L} \sin(\Delta m_B t)$, and $\text{Im}\lambda_{S,L} = S_{S,L} = \pm \sin 2\beta$ and $C_{S,L} = 0$. Our goal is to investigate possible deviations from this limit.

Since $B$ meson decays are better described in terms of flavor eigenstates at short distances, we rewrite $\lambda_{S,L}$ in terms of the right-flavor kaon decay amplitudes

$$\bar{A}_K \equiv A(\bar{B} \to \psi \bar{K}), \quad A_K \equiv A(B \to \psi K),$$  (5)

and the wrong-flavor kaon decay amplitudes

$$\bar{A}_K \equiv A(\bar{B} \to \psi K), \quad A_K \equiv A(B \to \psi \bar{K}).$$  (6)

To parameterize the contributions due to possible wrong-flavor amplitudes from new physics, we define

$$a \equiv \left(\frac{q_K}{p_K}\right)\left(\frac{\bar{A}_K}{A_K}\right), \quad b \equiv \left(\frac{p_K}{q_K}\right)\left(\frac{A_K}{\bar{A}_K}\right).$$  (7)

Then we can rewrite $\lambda_{S,L}$ defined in Eq. (3) as

$$\lambda_{S,L} = \pm \lambda_B \left(\frac{1 + a}{1 + b}\right),$$  (8)
\[ \lambda_B \equiv \left( \frac{q_B}{p_B} \right) \left( \frac{\bar{A}_K}{A_K} \right) \left( \frac{p_K}{q_K} \right). \]  

(9)

In the SM, and in any extensions of it in which the wrong-flavor kaon amplitudes are negligibly small (i.e., \(|a| \) and \(|b| \ll 1 \), Eq. (8) reduces to \( \lambda_{S,L} = \pm \lambda_B \), and so \( \lambda_S + \lambda_L = 0 \). As a result, for the two CP asymmetries in Eq. (2), \( S_S = -S_L \) and \( C_S = C_L \). However, for arbitrary \( a \) and \( b \),

\[ \lambda_S + \lambda_L = \lambda_B \frac{2(a - b)}{1 - b^2}. \]  

(10)

We learn that a necessary and sufficient condition for \( \lambda_S \neq -\lambda_L \) is the presence of non-vanishing wrong-flavor amplitudes with \( a \neq b \). Such a situation can arise either if \(|a| \neq |b| \) or if \( \text{arg}(a) \neq \text{arg}(b) \).

To get a rough idea of the size of the expected difference between the two asymmetries, note that if each right-flavor and wrong-flavor kaon amplitude is dominated by a single contribution, then \(|a| \approx |b| \) holds. We further assume that the CP violating phases are not small, namely that \( \text{Re}\lambda_{S,L} \sim \text{Im}\lambda_{S,L} \sim O(1) \) as in the SM, and that \( \text{arg}(a) \sim \text{arg}(b) \sim O(1) \). Under these assumptions,

\[ \text{Im}(\lambda_S + \lambda_L) \sim |a|. \]  

(11)

Thus, \( S_S + S_L \) is expected to be of the order of the ratio of wrong-flavor to right-flavor kaon amplitudes. If the strong phases between the wrong-flavor and right-flavor kaon amplitudes are not small, effects of similar order will also be generated for the \( C_{S,L} \) terms in the CP asymmetries in Eq. (2).

III. THE DIFFERENCE IN CP ASYMMETRIES IN THE SM

The amplitudes for \( B \) decays to wrong-flavor kaons are negligible in the SM. Any contribution would require at least two \( W \) propagators in an (exchange) annihilation graph, and would involve small CKM matrix elements. Naive estimates in the SM lead to \(|a|, |b| < 10^{-6} \). There are much larger effects which contribute to the CP asymmetries at the \( 10^{-3} \) level. Since they are all small, we can expand to linear order in each of them. The finite \( B \) meson width difference results in equal contributions to the \( S_S \) and \( S_L \) terms in Eq. (2).

The deviation of \(|\lambda_B| \) from unity due to CP violation in \( B \) mixing or in \( B \to \psi K \) decay results in non-zero \( C_{S,L} \) terms, satisfying \( C_S = C_L \). CP violation in decay also results in
corrections to the $S_{S,L}$ terms of equal magnitude, but of opposite sign. We will return to a discussion of these effects later.

CP violation in $K - \bar{K}$ mixing contributes to $a_{CP}(B \to \psi K_{S,L})$ via corrections to $\lambda_B$. If the measured final states were the $K_S$ and $K_L$ mass eigenstates, this would be the only effect of $\epsilon_K$ and the relation $\lambda_S = -\lambda_L$ would not be altered. However, there is an additional effect due to the fact that the experimentally reconstructed $\psi K_S$ final state is actually a coherent superposition of $\psi K \to \psi \pi \pi$ and $\psi \bar{K} \to \psi \pi \pi$ with some constraint on the kaon decay time. For example, at BABAR and BELLE the $K_S$ is identified by requiring two pions in the tracking system. This requirement selects kaons that decay after a short time. Thus they are mainly $K_S$, but there is a small $K_L$ admixture, since the $K_L$ can also decay to two pions. Final states reconstructed as $K_L$, on the other hand, are identified by hits in the hadronic calorimeter. This requires that the kaon decay time must be much longer than the $K_S$ lifetime, therefore such states are pure $K_L$ to very good accuracy.

To study the effect of kaon mixing, it is most convenient to use the cascade mixing formalism \cite{cascade}. In particular, to obtain the total $B \to \psi K \to \psi \pi \pi$ amplitude a coherent sum is performed over the physical $B$ and $K$ mass eigenstate contributions,

$$A(B \to \psi K \to \psi \pi \pi) = \sum_{M,N} A(K_N \to \pi \pi)e^{-i(m_{K_N}-i\Gamma_{K_N}/2)t_K} \times A(B_M \to \psi K_N)e^{-i(m_{B_M}-i\Gamma_{B_M}/2)t_K} \langle B_M|B \rangle,$$

(12)

where $M = H, L$ and $N = S, L$ are summed over, and $t_K$ is the time between the formation and decay of the $K$ meson.

We are interested in obtaining the corrections to the CP asymmetries due to $\epsilon_K$, so in the following we set $\Delta \Gamma_B = 0$ but allow for deviations of $|\lambda_B|$ from unity. The resulting decay rates can be expressed as

$$\Gamma[B(\bar{B}) \to \psi K \to \psi \pi \pi] \propto \left[ e^{-\Gamma_{st}t_K} c_{11} + e^{-\Gamma_{lt}t_K} c_{22} + 2e^{-(\Gamma_{st}+\Gamma_{lt})t_K/2} c_{12} \right] e^{-\Gamma_B t}.$$  

(13)

For the $c_{ij}$ coefficients, following Ref. \cite{cascade}, we obtain

$$c_{11} = |1 + \lambda_K|^2 \left\{ 1 + |\lambda_B|^2 \mp 2s_B \text{Im} \lambda_B \pm c_B(1 - |\lambda_B|^2) \right\},$$

$$c_{22} = |1 - \lambda_K|^2 \left\{ 1 + |\lambda_B|^2 \mp 2s_B \text{Im} \lambda_B \pm c_B(1 - |\lambda_B|^2) \right\},$$

$$c_{12} = \pm \left\{ 2 \left( 1 - |\lambda_K|^2 \right) (c_Bc_K - s_Bs_K \text{Re} \lambda_B) - 4 \text{Im} \lambda_K (c_Bs_K + s_Bc_K \text{Re} \lambda_B) \right\}$$

$$- \left( 1 - |\lambda_B|^2 \right) \left\{ (1 - |\lambda_K|^2) (\pm c_Bc_K - c_K) + 2 \text{Im} \lambda_K (s_K \mp c_Bs_K) \right\},$$

(14)
where the upper (lower) signs stand for decays of a $B$ ($\bar{B}$) meson,

$$\lambda_K \equiv \frac{q_K}{p_K} \frac{A(K \to \pi \pi)}{A(K \to \pi \pi)},$$  \hspace{1cm} (15)

and

$$s_K \equiv \sin \Delta m_K t_K, \quad c_K \equiv \cos \Delta m_K t_K.$$  \hspace{1cm} (16)

The $c_{11}$ ($c_{22}$) term corresponds to decays of $K_S$ ($K_L$), and the $c_{12}$ term is due to the interference between them.

To obtain corrections to the CP asymmetries due to $\epsilon_K$ we need the following relations, valid to leading order in $\epsilon_K$ ($\epsilon'_K$ is neglected throughout),

$$\lambda_B = -e^{-2i\beta} (1 + 2\epsilon_K),$$  \hspace{1cm} (19)

where $|\epsilon_K| \approx 2.28 \times 10^{-3}$ and

$$\text{Im} \epsilon_K = x_K \text{Re} \epsilon_K \left[ 1 + \mathcal{O} \left( \frac{\Gamma_L}{\Gamma_S} \right) \right], \quad x_K \equiv \frac{2\Delta m_K}{\Gamma_S + \Gamma_L} \approx 0.95.$$  \hspace{1cm} (18)

As can be seen from Eqs. (9) and (17), $\lambda_B$ to leading order in $\epsilon_K$ is given by

$$\lambda_B = -e^{-2i\beta} (1 + 2\epsilon_K),$$  \hspace{1cm} (19)

where $\beta$ is the usual angle of the unitarity triangle.

What is experimentally called $a_{\text{CP}}(B \to \psi K_S)$ is obtained by integrating the rates in Eq. (13) with respect to $t_K$ from (almost) zero to some cutoff $t_{\text{cut}}$ that depends on the experimental setup, and then forming the asymmetry defined in Eq. (2). Since this cutoff is much larger than the $K_S$ lifetime (by about a factor of 10 at BABAR and BELLE), it is a good approximation to perform the integrals over the terms proportional to $c_{11}$ and $c_{12}$ from zero to infinity. Using the above relations, we find to leading order in $\epsilon_K$,

$$a_{\text{CP}}(B \to \psi K_S) = \left[ \sin 2\beta - 2\text{Im} \epsilon_K \cos 2\beta \right] s_B - 2\text{Re} \epsilon_K c_B,$$  \hspace{1cm} (20)

where it is to be understood that $K_S$ stands for the state identified in the experiments as $K_S$. The corrections to $a_{\text{CP}}(B \to \psi K_L)$ are obtained from Eq. (2), taking into account the correction to $\lambda_B$ of order $\epsilon_K$ given in Eq. (19). The result is

$$a_{\text{CP}}(B \to \psi K_L) = -\left[ \sin 2\beta - 2\text{Im} \epsilon_K \cos 2\beta \right] s_B + 2\text{Re} \epsilon_K c_B.$$  \hspace{1cm} (21)
Remarkably, to leading order in $\epsilon_K$, the relation $a_{CP}(B \to \psi K_S) = -a_{CP}(B \to \psi K_L)$ is maintained. The terms with $s_B$ time dependence in Eqs. (20) and (21) originate from $\text{Im}\lambda_B$ and its small deviation from $\sin 2\beta$. The third term in Eq. (20) receives contributions from both the interference term $c_{12}$ (given by $-4\text{Re}\epsilon_K c_B$), and the correction due to $|\lambda_B| \neq 1$ in $c_{11}$ (given by $+2\text{Re}\epsilon_K c_B$). The relation $a_{CP}(B \to \psi K_S) = -a_{CP}(B \to \psi K_L)$ is maintained because the ratio of these two terms is $-2$, and the third term in Eq. (21) comes entirely from the $|\lambda_B| \neq 1$ contribution (that is $+2\text{Re}\epsilon_K c_B$).

Corrections to $a_{CP}(B \to \psi K_S) = -a_{CP}(B \to \psi K_L)$ due to $\epsilon_K \neq 0$ only occur suppressed by other factors, and are therefore not shown explicitly in Eqs. (20) and (21). There are contributions of order $\epsilon_K$ to $S_S$ from the $c_{12}$ interference term, which are suppressed by either $\text{Im}\epsilon_K - x_K \text{Re}\epsilon_K \propto \Gamma_L/\Gamma_S$ according to Eq. (18), or by $e^{-\Gamma_S t_{\text{cut}}/2}$ due to the finite experimental cut $t_K < t_{\text{cut}}$. The largest correction numerically actually comes from a contribution of the $c_{22}$ term to $S_S$, which is given by $-2|\epsilon_K|^2 (1 - e^{-\Gamma_L t_{\text{cut}}}) \Gamma_S/\Gamma_L$. For $t_{\text{cut}} \sim 10 \tau_S$, it is about $-1 \times 10^{-4}$.

To close this section, we return to discuss the relative importance of the corrections to the CP asymmetries from $\epsilon_K$, from the $B$ lifetime difference, and from CP violation in $B$ mixing and decay. The $B$ lifetime difference, $\Delta\Gamma_B \equiv \Gamma_H - \Gamma_L$, modifies the asymmetries to first order in $\Delta\Gamma_B/\Gamma_B$ as [4]

$$\delta a_{CP}(B \to \psi K_{S,L}) = \frac{1}{2} \sin 2\beta \cos 2\beta \left( \Delta\Gamma_B t \right) s_B. \quad (22)$$

Using $t \sim 1/\Gamma_B$ and the estimate $\Delta\Gamma_B/\Gamma_B \sim 3 \times 10^{-3}$ [7], these corrections are expected to be comparable to the $s_B$ terms arising at $O(\epsilon_K)$. (Note that new physics contributions to the $B$ lifetime difference are unlikely to be sufficiently large to significantly modify the size of this effect.) Corrections due to CP violation in $B - \bar{B}$ mixing ($|q_B/p_B| \neq 1$), to first order in $\Gamma_{12}/M_{12}$, only modify the $C_{S,L}$ terms in the asymmetries, and are given by

$$\delta a_{CP}(B \to \psi K_{S,L}) = -\frac{1 - |\lambda_B|^2}{2} c_B. \quad (23)$$

At this order, $1 - |\lambda_B|^2 = \text{Im}(\Gamma_{12}/M_{12})$, which is also equal to the measurable CP asymmetry in semileptonic decays, $A_{SL}$. A recent estimate gives $\text{Im}(\Gamma_{12}/M_{12}) \approx -(0.5 - 1.3) \times 10^{-3}$ [13], so this correction is somewhat smaller than the $c_B$ terms induced at $O(\epsilon_K)$. Finally, we consider the effect of direct CP violation in $B \to \psi K_{S,L}$ decays due to the CKM suppressed penguin diagrams. We denote by $T$ all contributions to the decay amplitude proportional to
CKM elements $\lambda_c$ and by $P$ all contributions proportional to $\lambda_u$, where $\lambda_q \equiv V_{qb} V_{qs}^*$. Then $A(B^0 \to \psi K^0) = \lambda_c T + \lambda_u P$, and the resulting modifications of the CP asymmetries are

$$
\delta a_{\text{CP}}(B \to \psi K_{S,L}) = \mp 2 \cos 2\beta \Im \frac{\lambda_u}{\lambda_c} \Re P \frac{T}{s_B} - 2 \Im \frac{\lambda_u}{\lambda_c} \Im P T c_B.
$$

The CKM suppression ($|\lambda_u/\lambda_c| \sim 1/50$), and the hard to estimate matrix element suppression and strong phases in $P/T$ imply that such effects are of order a few times $10^{-3}$ or below.

Note that Eqs. (22)–(24) include corrections which are of equal magnitude and sign for the two asymmetries. Therefore, when combined with Eqs. (20) and (21), they introduce a difference between the magnitudes of $a_{\text{CP}}(B \to \psi K_S)$ and $a_{\text{CP}}(B \to \psi K_L)$. In view of the fact that $a_{\text{CP}}(B \to \psi K_S)$ may be measured below the percent level during the next decade, we collect Eqs. (22)–(24) and (20) to obtain

$$
a_{\text{CP}}(B \to \psi K_S) = \left[ \sin 2\beta - 2 \cos 2\beta \Im \epsilon_K + \frac{1}{4} \sin 4\beta (\Delta \Gamma_B t) - 2 \cos 2\beta \Im \frac{\lambda_u}{\lambda_c} \Re P T \right] s_B
- \left[ 2 \Re \epsilon_K + \frac{1}{2} \Im \left( \frac{\Gamma_{12}}{M_{12}} \right) + 2 \Im \frac{\lambda_u}{\lambda_c} \Im P T \right] c_B.
$$

We conclude that in the SM the $S_S = -S_L = \sin 2\beta$ and $C_S = C_L = 0$ relations between the CP asymmetries in $B \to \psi K_S$ and $B \to \psi K_L$ hold at the 1% level, therefore it is safe to average the asymmetries in the $B \to \psi K_S$ and $B \to \psi K_L$ modes. However, below the percent level, there are several effects shown in Eq. (24) which can be calculated with varying degrees of reliability that enter the relation between $a_{\text{CP}}(B \to \psi K_S)$ and $\sin 2\beta$.

**IV. CONSTRAINTS ON NEW PHYSICS**

Consider $A_{K}$, the amplitude of the wrong-flavor decay $B^0 \to \psi K$. As the final state does not contain a $\bar{d}$ quark, the decay must proceed via annihilation of the $B$ meson. The flavor structure of the operator that mediates this decay is $(\bar{d}b)(\bar{s}s)(\bar{c}c)$. (Here, and in what follows, the color indices and Dirac structure of the operators are suppressed.) While the $(\bar{d}b)(\bar{s}s)$ part, which violates flavor, must come from new short distance physics, the $(\bar{c}c)$ pair can be generated either by gluons or by exchange of heavy particles. In the following we study both cases.

First, we consider models where the $c\bar{c}$ pair is generated from gluon exchange. The high
energy theory is assumed to produce an effective four-Fermi interaction

\[ O_{4}^{\text{NP}} = \frac{1}{M_{4}^{2}} \bar{db} \bar{ds}, \tag{26} \]

where \( M_{4} \) is the effective scale of new physics, which includes all possible dimensionless couplings. The bounds on such operators are very strong, as we find below, so we may crudely estimate their contributions to \( A_{K} \). The final state could be produced either by forming the \( \psi \) in a color octet Fock state from a hard gluon, or via an OZI suppressed graph where the \( \psi \) is formed out of three gluons. Taking into account the fact that both processes are power suppressed, and using factorization, we obtain

\[ A_{K} \lessapprox \frac{1}{M_{4}^{2} m_{B}^{2}} \frac{\Lambda_{\text{QCD}}}{m_{B}} \alpha_{s} f_{B} f_{K} f_{\psi} m_{\psi} (\epsilon_{\psi} \cdot p_{K}). \tag{27} \]

Upper bounds on such contributions to the amplitude can be obtained by considering the effect of the new operators on the rare decays \( B^{\pm} \rightarrow K^{\pm} \pi^{\pm} \pi^{\pm} \) [14] and \( B^{\pm} \rightarrow \pi^{\pm} K_{S} \) [15]. (In addition to modifying the \( B^{\pm} \rightarrow \pi^{\pm} K_{S} \) decay rate, they must increase the ratio \( \Gamma(B^{\pm} \rightarrow \pi^{\pm} K_{S})/\Gamma(B^{\pm} \rightarrow \pi^{0} K^{\pm}) \) which cannot be much larger than its value in the SM according to current data.) Assuming factorization, the latter gives the strongest bound [15]

\[ M_{4} \mathrel{\gtrsim} 3 \text{ TeV}. \tag{28} \]

For comparison, we note that in the SM the right-flavor amplitude using the factorization hypothesis is given by

\[ A_{K}^{\text{SM}} = \frac{G_{F}}{\sqrt{2}} V_{cb} V_{cs}^{\ast} a_{2} f_{\psi} m_{\psi} F_{1} (\epsilon_{\psi} \cdot p_{K}), \tag{29} \]

where \( F_{1} \) is the \( B \rightarrow K \) form factor at \( q^{2} = m_{\psi}^{2} \), and \( a_{2} \) depends on the current-current operators’ Wilson coefficients, \( C_{1} \) and \( C_{2} \). The observed \( B \rightarrow \psi K_{S} \) rate is reproduced if \( a_{2} F_{1} \approx 0.2 \) [16]. Using \( \alpha_{s}(m_{b}) = 0.2 \) and \( f_{B} \sim f_{K} \sim f_{\psi} \sim 200 \text{ MeV} \), Eqs. (27)–(29) imply \( |a| \lesssim O(10^{-4}) \). While these estimates are very crude, it is clear that large effects cannot occur.

Next, we turn to the case of \( c \bar{c} \) pair production by exchange of heavy particles. The wrong-flavor amplitudes would be due to six-quark operators, with an effective Hamiltonian of the form

\[ O_{6}^{\text{NP}} = \frac{1}{M_{6}^{2}} \bar{db} \bar{ds} \bar{cc}, \tag{30} \]
where $M_6$ is the effective scale of new physics, which includes all possible dimensionless couplings. A crude estimate of the wrong-flavor amplitude using factorization yields

$$A_K \sim \frac{1}{M_6^6} m_B f_B f_K m_{\psi} \psi \psi (\epsilon_{\psi} \cdot p_K).$$  

(31)

Comparing Eqs. (31) and (29) we find

$$|a| \sim \left( \frac{20 \text{ GeV}}{M_6} \right)^5.$$  

(32)

Thus, a difference of CP asymmetries greater than a percent for the sin $\Delta m_B t$ terms, i.e., $|a_{CP}(B \rightarrow \psi K_S)| - |a_{CP}(B \rightarrow \psi K_L)| \gtrsim 10^{-2}$, would require a new physics scale that lies well below the weak scale.

We know of only one new physics scenario which could in principle accommodate large wrong-flavor amplitudes: supersymmetric models with a light bottom squark of mass $2 - 5.5 \text{ GeV}$ and a light gluino of mass $12 - 16 \text{ GeV}$ [17]. Such models have been proposed to enhance the $b$ quark production cross section at the Tevatron. Among the new operators which can arise at tree-level, there are several of the form $\bar{d}b \bar{\tilde{b}} \tilde{\tilde{b}}$. Stringent upper bounds on their coefficients have been obtained from rare $B$ decays [18]. Interactions of the desired form in Eq. (30) would be generated from these operators if the $R$-parity violating Yukawa couplings mediating $\tilde{b} \rightarrow \bar{c} \tilde{d}$ and $\tilde{b} \rightarrow \bar{c} \tilde{s}$ decays were also present. Moreover, large wrong-flavor amplitudes could be generated if these couplings were of unit strength. However, an upper bound of order $10^{-5}$ on the product of these two couplings from box-graph contributions to $K - \bar{K}$ mixing implies that the wrong-flavor kaon amplitudes must be negligibly small, i.e., $|a| \sim |b| \lesssim 10^{-5}$. This example illustrates the difficulties any scenario with large wrong-flavor amplitudes would face due to the requirement of a low mass scale for new flavor-changing interactions. The possibility of significantly different CP asymmetries in $B \rightarrow \psi K_S$ and $B \rightarrow \psi K_L$ decays is therefore extremely unlikely.

The most direct test for such new physics effects is provided by searching for the wrong-flavor decay $B \rightarrow \psi \bar{K}^*$, by studying the time dependence of flavor tagged $B$ decays. It is very likely that the matrix elements are similar in $B$ decays to $\psi K^*$ and $\psi K$, so the ratios of the wrong-flavor to the right-flavor decay amplitudes should be similar in the two cases. Although $\psi \bar{K}^*$ is a vector-vector final state, and thus it is a mixture of CP even and odd components, this is not expected to yield a significant difference in the ratio of wrong-flavor to right-flavor decay amplitudes. In the presence of wrong-flavor amplitudes
the time dependent rate is

$$\Gamma[B(\bar{B}) \to \psi K^*] \propto 1 + |\lambda_{\psi K^*}|^2 \pm \left[(1 - |\lambda_{\psi K^*}|^2) c_B - 2 \text{Im}\lambda_{\psi K^*} s_B\right].$$  \hspace{1cm} (33)

Here $\lambda_{\psi K^*}$ is of the order of the wrong-flavor to right-flavor amplitude ratio, and the upper (lower) signs stand for decays of $B$ ($\bar{B}$). The time dependence for the $\bar{B}(B) \to \psi K^*$ decay is obtained by replacing $\lambda_{\psi K^*}$ by $\lambda_{\psi K^*}^{-1}$ and $\pm$ by $\mp$ in Eq. (33). Fitting to these time dependences, the $B$ factories should be able to bound the magnitudes of the wrong-flavor amplitudes, which constrains $|a_{CP}(B \to \psi K_S) + a_{CP}(B \to \psi K_L)|$ using Eq. (10).

V. CONCLUSIONS

The $B \to \psi K_S$ and $B \to \psi K_L$ decays are the golden modes for studying CP violation, since the hadronic uncertainties are below the 1% level. We studied possible effects within the standard model and in the presence of new physics that can make the absolute values of the CP asymmetries in these two channels different. We computed the corrections due to $\epsilon_K$, taking into account the way the $K_S$ and $K_L$ mesons are identified at the $B$ factories, and found that although $\epsilon_K$ induces corrections to each CP asymmetry at the few times $10^{-3}$ level, it only introduces a difference in their magnitudes at the $10^{-4}$ level. Nevertheless, in the SM the difference in the absolute values of the two CP asymmetries is of order $10^{-3}$ due to the $B$ lifetime difference and CP violation in $B - \bar{B}$ mixing and in $B \to \psi K$ decay. These effects modify the relation between $a_{CP}(B \to \psi K_S)$ and $\sin 2\beta$ as summarized in Eq. (25).

New physics in $B - \bar{B}$ mixing, which would modify $a_{CP}(B \to \psi K_S)$ and $a_{CP}(B \to \psi K_L)$ while leaving their magnitudes equal, has been extensively studied. Direct CP violation in $B \to \psi K$ decays, which would lead to contributions equal in magnitude and opposite [same] in sign for the $\sin(\Delta m_B t) [\cos(\Delta m_B t)]$ terms in the two asymmetries, has also been discussed previously. We investigated how new physics could violate $a_{CP}(B \to \psi K_S) = -a_{CP}(B \to \psi K_L)$ via unequal magnitudes for the $\sin(\Delta m_B t)$ terms, and found that the presence of the wrong-flavor kaon amplitudes $B \to \psi K$ or $\bar{B} \to \psi K$ are necessary to obtain significant effects, i.e., in excess of 1%. (Small effects are, in principle, possible via new physics contributions to the $B$ lifetime difference.) This would require a scale for new physics which lies well below the weak scale, therefore the existence of a viable scenario is unlikely due to bounds on flavor changing neutral currents. Using the current data sets,
it should be possible for the \( B \) factories to put tight bounds on the related wrong-flavor
\( B \to \psi K^* \) and \( B \to \psi \bar{K}^* \) decay amplitudes.

While it is important to constrain the decay amplitudes to wrong-flavor kaons experimentally, unless the results indicate large new physics contributions, it is safe to combine the \( a_{\mathrm{CP}}(B \to \psi K_S) \) and \( a_{\mathrm{CP}}(B \to \psi K_L) \) measurements. If and when \( a_{\mathrm{CP}}(B \to \psi K_S) \) will be measured at or below the one percent level, it will become important to include the various subleading effects discussed in this paper.

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