Meson cloud contributions to baryon axial form factors

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Abstract. The axial form factor as well as the axial charge of octet baryons are studied in the perturbative chiral quark model (PCQM) with the quark wave functions predetermined by fitting the theoretical results of the proton charge form factor to experimental data. The theoretical results are found, based on the predetermined quark wave functions, in good agreement with experimental data and lattice values. This may indicate that the electric charge and axial charge distributions of the constituent quarks are the same. The study reveals that the meson cloud plays an important role in the axial charge of octet baryons, contributing 30\%-40\% to the total values, and strange sea quarks have a considerable contribution to the axial charge of the Σ and Ξ.

1. Introduction

The axial form factors play an extremely important role in hadron physics since they supply necessary information on the internal structure and weak interaction properties. Recently, the $Q^{2}$-dependence of the axial form factor of nucleon has been studied in Lattice-QCD and other approaches. The hyperon axial charges, which are the axial form factors in zero-recoil, have been predicted in Lattice-QCD, chiral perturbation theory and relativistic quark model (RQM). However, there is few theoretical works on the $Q^{2}$-dependence of the axial form factor of hyperons, especially in the chiral quark model. This inspires us to study the axial form factors of octet baryons in perturbative chiral quark model (PCQM).

The PCQM is a powerful tool to study the baryon structure and properties in the low energy particle physics. However, our previous work on axial form factor of nucleon shows that the PCQM theoretical result of the nucleon axial form factor is consistent well with the experimental data only at very low momentum transfer $Q^{2}$, descending quickly with the momentum transfer $Q^{2}$ increasing. It is noted that a variational Gaussian ansatz has been employed for the quark wave functions.
As we argue in [20], the Gaussian-type quark wave functions of baryons lead to the theoretical predictions for the form factors of baryons consistent well with experimental data only at very low momentum transfer $Q^2$. Furthermore, the more reasonable quark wave functions have been determined in [20] by fitting the PCQM theoretical result of the proton charge form factor to the experimental data, as shown in figure 1. And the $Q^2$-dependence of the theoretical electromagnetic form factors with the determined wave functions in the region $Q^2 \leq 1 \text{ GeV}^2$ is consistent with experimental data. More details could be found in [20]. In this work, we attempt to study the axial form factors of octet baryons in the PCQM with the determined wave functions in SU(3) and analyze the strangeness contributions to the axial form factors. We also predict the axial charges of light hyperons (Λ and Ξ). There are no further parameters to be adjusted in the present work.

The paper is organized as follows. In section 2 we present the theoretical expressions of octet baryon axial form factors in the PCQM. The numerical results based on the predetermined quark wave functions and discussion are given in section 3.
2. Axial form factors in the PCQM

In the framework of the PCQM, the axial form factors of octet baryons up to one-loop corrections are defined by

\[
\chi_{B_{\sigma}}\bar{\sigma}_B\chi_{B_s}G_A^B(Q^2) = B\langle\phi_0\lfloor|n\rfloor\rceil e^{-iq\cdot x}\times T[\mathcal{L}_I^W(x_1)\cdots\mathcal{L}_I^W(x_n)|\bar{A}_3(x)]\rangle|\phi_0\rangle_B,
\]

where the state vector $|\phi_0\rangle_B$ corresponds to the unperturbed three-quark states projected onto the respective baryon states, which are constructed in the framework of the $SU(6)$ spin-flavor and $SU(3)$ color symmetry. The subscript $c$ in (1) refers to contributions from connected graphs only. $G_A^B(Q^2)$ are the axial form factors of octet baryons with the squared momentum transfer $Q^2$. $\chi_{B_s}$ and $\chi_{B_{\sigma}}$ are the baryon spin wave functions in the initial and final states, $\bar{\sigma}$ is the baryon spin matrix.

The quark-meson interaction Lagrangian $\mathcal{L}_I^W(x)$ in (1) is taken the form

\[
\mathcal{L}_I^W(x) = \frac{1}{2F}\partial_\mu\Phi_i(x)\bar{\psi}(x)\gamma^\mu\gamma^5\lambda_i\psi(x) + \frac{f_{ijk}}{4F^2}\Phi_i(x)\partial_\mu\Phi_j(x)\bar{\psi}(x)\gamma^\mu\lambda_k\psi(x).
\]

where $\psi$ is the triplet of u, d and s quark fields, and $\Phi_i$ are the octet meson fields.

The axial-vector current $A_\mu^i$ in (1) is given by

\[
A_\mu^i = F\partial_\mu\Phi_i + \bar{\psi}\gamma^\mu\gamma^5\lambda_i\frac{1}{2}\psi - \frac{f_{ijk}}{2F}\bar{\psi}\gamma^\mu\lambda_j\psi\Phi_k + \bar{\psi}(\hat{Z} - 1)\gamma^\mu\gamma^5\lambda_i\frac{1}{2}\gamma^\nu\lambda^\nu\psi + o(\Phi_i^2).
\]

where the renormalization constants $\hat{Z}$ is defined as

\[
\hat{Z} = 1 - \frac{3}{4(2\pi F)^2} \int_0^\infty dk k^4 \frac{1}{\omega^3(k^2)} + \frac{2}{3\omega^3(k^2)} + \frac{1}{9\omega^3(k^2)},
\]

with $\omega_k(k^2) = \sqrt{M^2_\pi + k^2}$ and the vertex function $F_1(k)$ for the $qq\Phi$ system taking the form

\[
F_1(k) = 2\pi \int_0^\infty dr \int_0^\pi d\theta r^2 \sin \theta e^{ikr \cos \theta} [g(r)^2 + f(r)^2 \cos 2\theta].
\]

The ground state quark wave function $u_0(\vec{x})$ may, in general, be expressed as

\[
u_0(\vec{x}) = \begin{pmatrix} g(r) \\ i\hat{\sigma} \cdot \hat{r} f(r) \end{pmatrix} \chi_s\chi_f\chi_c.
\]

where $\chi_s$, $\chi_f$ and $\chi_c$ are the spin, flavor and color quark wave functions, respectively. In the numerical analysis, we employ the radial quark wave functions $g(r)$ and $f(r)$ which have been extracted in [20] by fitting the theoretical results of the proton charge form factor to the experimental data. More information on the PCQM and quark wave functions can be found in [20].

The Feynman diagrams contributing to the axial form factor of octet baryons in accordance with the $\mathcal{L}_I^W(x)$ in (2) and the $A_\mu^i$ in (3) are shown in figure 2. The corresponding analytical expressions for the relevant diagrams are derived as follows:
Figure 2. Diagrams contributing to the axial form factor of octet baryons: 3q-core leading order (a), 3q-core counterterm (b), self-energy I (c) self-energy II (d), meson exchange (e), and vertex correction (f).

(a) Three-quark core leading-order (LO) diagram

\[ G^B_A(Q^2)|_{LO} = c^B_1 2\pi \int_0^\infty dr \int_0^\pi d\theta r^2 \sin \theta [g(r)^2 + f(r)^2 \cos(2\theta)] e^{iQr\cos \theta}. \] (7)

(b) Three-quark core counterterm (CT) diagram

\[ G^B_A(Q^2)|_{CT} = (\hat{Z} - 1)G^B_A(Q^2)|_{LO}, \] (8)

(c) Self-energy I (SE:1) diagram

\[ G^B_A(Q^2)|_{SE:1} = \frac{1}{2(2\pi F)^2} \int_0^\infty dk \int_{-1}^1 dx k^4 (1 - x^2) F_I(k) F_{II}(k-) \times \frac{1}{\sqrt{k^2} \left[ \frac{c^B_1}{\omega^2(k^2)} + \frac{c^B_2}{\omega^R(k^2)} \right]} \] (9)

where the vertex function for the quark-pion-axial vector current \( F_{II}(k) \) is given by

\[ F_{II}(k) = -2i\pi \int_0^\infty dr \int_0^\pi d\theta r^2 g(r) f(r) \sin 2\theta e^{ikr\cos \theta}. \] (10)
Table 1. The constants $c_i^B$ for the octet baryons axial form factors $G_A^B(Q^2)$.

|   | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ |
|---|-------|-------|-------|-------|-------|
| $N$ | 5/3   | 5/6   | 8     | 0     | $-5/9$|
| $\Sigma$ | 4/3   | 2/3   | 0     | 4     | $-4/9$|
| $\Xi$  | $-1/3$| $-1/6$| 0     | $-4$  | 1/9   |

(d) Self-energy II (SE:II) diagram

$$G_A^B(Q^2)_{SE:II} = \frac{1}{2(2\pi F)^2} \int_0^\infty \int_{-1}^1 dk \int_0^1 dx k^4 (1 - x^2) F_I(k) F_{II}(k_-)$$

$$\times \frac{1}{\sqrt{k^2}} \left[ \frac{c_1^B}{\omega_\pi^2(k^2)} + \frac{c_2^B}{\omega_K^2(k^2)} \right]. \quad (11)$$

(e) Exchange (EX) diagram

$$G_A^B(Q^2)_{EX} = \frac{1}{4(2\pi F)^2} \int_0^\infty \int_{-1}^1 dk \int_0^1 dx k^4 (1 - x^2) F_I(k) F_{II}(k_-)$$

$$\times \frac{1}{\sqrt{k^2}} \left[ \frac{c_3^B}{\omega_\pi^2(k^2)} + \frac{c_4^B}{\omega_K^2(k^2)} \right]. \quad (12)$$

(f) Vertex-correction (VC) diagram

$$G_A^B(Q^2)_{VC} = \frac{1}{20(2\pi F)^2} \int_0^\infty dkk^2 F_I^2(k) \left[ \frac{c_1^B}{\omega_\pi^2(k^2)} + \frac{c_5^B}{\omega_\eta^2(k^2)} \right] \cdot G_A^N(Q^2)_{LO}. \quad (13)$$

The constants $c_i^B$, which depend on the spin and flavor of baryons, are given in table 1.

3. Numerical results and discussion

In this section, we present the axial charges and form factors of octet baryons with the determined quark wave functions [20]. The calculations are extended to the SU(3) flavor symmetry, including $\pi$, kaon and $\eta$-meson cloud contributions. Note that there is no further parameters in the following numerical calculations on the axial form factors of octet baryons.

The numerical results for the axial charges, which are the axial form factors in zero-recoil, are listed in table 2. The uncertainties in the total values of the axial charges caused by the fitting errors of the quark wave functions [20] are estimated around 15%. As shown in table 2, the theoretical results reveal that the meson cloud plays an important role in the axial charge of octet baryons, contributing 30%-40% to the total values. Except for the $N$, there is no direct experimental data for the axial charge of the $\Sigma$ and $\Xi$, thus we have the lattice-QCD results [7] compiled in the table for comparison. It is found that the theoretical $N$ axial charge is in good agreement with the experimental value [21], and the work predictions on $\Sigma$ and $\Xi$ axial charges are consistent well with the Lattice-QCD values [7] and the RQM results [9].
Table 2. Numerical results for the octet baryon axial charges $g_A^B$, where the uncertainties are from the errors of the quark wave functions. The experimental data are taken from [21], while the Lattice-QCD results are taken from [7] with $m_q = 35$ MeV.

| 3q Meson loops | Total | Lattice [7] | Exp. [21] |
|----------------|-------|-------------|-----------|
| LO CT+SE+EX+VC |       |             |           |
| $g_N^A$        | 0.883 | 1.301±0.230 | 1.314±0.024 | 1.269±0.003 |
| $g_Σ^A$        | 0.707 | 0.927±0.132 | 0.970±0.029 | —           |
| $g_Ξ^A$        | −0.177| −0.283±0.033| −0.299±0.014| —           |

Table 3. Contribution of $\pi$, $K$ and $\eta$ mesons to the axial charges $g_A^B$.

| Meson loops | $\pi$ | $K$ | $\eta$ |
|-------------|-------|-----|--------|
| $g_N^A$     | 0.375 | 0.045| −0.002 |
| $g_Σ^A$     | 0.118 | 0.104| −0.002 |
| $g_Ξ^A$     | −0.030| −0.077| −0.001 |

Furthermore, we have studied the separate contribution of $\pi$, $K$ and $\eta$ mesons to the axial charges. As shown in table 3, the $\pi$ meson contribution to the $N$ axial charge dominates over the ones from the $K$ and $\eta$ mesons, but the $K$ meson contributions to the $\Sigma$ and $\Xi$ axial charges are in the same order as the $\pi$ ones. It is noticed that the contribution from the $\eta$ meson is negligible.

We show the $Q^2$-dependence of the axial form factors of octet baryons in figure 3, which are normalized to one at zero-recoil, with the experimental data on the nucleon axial form factor [22–29] plotted as well. As expected, the theoretical axial form factors fall off smoothly as the momentum transfer $Q^2$ increases. It is also found that the theoretical result for the $N$ axial form factor is in good agreement with the experimental data [22–29], and the axial form factors for $\Sigma$ and $\Xi$ show a similar $Q^2$ dependence based on the SU(3) symmetry.

We present in figure 4 the individual contributions of various processes shown in figure 2 to the axial form factors of octet baryons. As shown in figure 4, the 3q-core leading order (LO) diagram dominates the axial form factors of octet baryons while the self-energy (SE) and exchange (EX) diagrams contribute considerably.

In summary, one may conclude that the fact that the theoretical results of the axial form factors and axial charges agree well with experimental data and lattice-QCD values, with the predetermined quark core wave functions in the electromagnetic sector, may indicate that the electric charge and axial charge distributions of the constituent quarks are the same. The study reveals that the meson cloud plays an important role in the axial charge of octet baryons, contributing 30%-40% to the total values, and strange
Figure 3. Normalized axial form factors \( G_A^B(Q^2)/g_A^B \) of octet baryons. The experimental data on nucleon axial form factor are taken from [22–29].

Figure 4. The individual contributions of the different diagrams of figure 2 to the axial form factors of octet baryons (left panel for \( N \), middle panel for \( \Sigma \) and right panel for \( \Xi \)).

sea quarks have a considerable contribution to the axial charge of the \( \Sigma \) and \( \Xi \).

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References

[1] Liu K F, Dong S J, Draper T, Wu J M and Wilcox W 1994 Phys. Rev. D 49 4755
[2] Alexandrou C, Brinet M, Carbonell J, Constantinou M, Harraud P A, Guichon P, Jansen K, Korzec T and Papinutto M 2011 Phys. Rev. D 83 045010
[3] Schindler M R and Scherer S 2007 Eur. Phys. J. A 32 429
[4] Erkol G and Ozpineci A 2011 Phys. Rev. D 83 114022
[5] Eichmann G and Fischer C S 2012 Eur. Phys. J. A 48 1434
[6] Lin H W and Orginos K 2009 Phys. Rev. D 79 034507
[7] Erkola G, Oka M and Takahashi T T 2010 Phys. Lett. B 686 36
[8] Jiang F J and Tiburzi B C 2008 Phys. Rev. D 78 017504
Jiang F J and Tiburzi B C 2009 Phys. Rev. D 80 077501
[9] Choi K, Plessas W and Wagenbrunn R F 2010 Phys. Rev. D 82 014007
Choi K, Plessas W and Wagenbrunn R F 2010 Phys. Rev. D 82 039901(E)
[10] Lyubovitskij V E, Gutsche Th, Faessler A and Drukarev E G 2001 Phys. Rev. D 63 054026
[11] Lyubovitskij V E, Gutsche Th and Faessler A 2001 Phys. Rev. C 64 065203
[12] Lyubovitskij V E, Gutsche Th, Faessler A and Vinh M R 2001 Phys. Lett. B 520 204
[13] Lyubovitskij V E, Gutsche Th, Faessler A and Vinh M R 2002 Phys. Rev. C 65 025202
[14] Pumsa-ard K, Lyubovitskij V E, Gutsche Th, Faessler A and Cheedket S 2003 Phys. Rev. C 68 015205
[15] Cheedket S, Lyubovitskij V E, Gutsche Th, Faessler A, Pumsa-ard K and Yan Y 2004 Eur. Phys. J. A 20 317
[16] Khosonthongkee K, Lyubovitskij V E, Gutsche Th, Faessler A, Pumsa-ard K, Cheedket S and Yan Y 2004 J. Phys. G: Nucl. Part. Phys. 30 793
[17] Dong Y, Faessler A, Gutsche Th, Kuckei J, Lyubovitskij V E, Pumsa-ard K and Shen P 2006 J. Phys. G: Nucl. Part. Phys. 32 203
[18] Dib C, Faessler A, Gutsche Th, Kovalenko S, Kuckei J, Lyubovitskij V E and Pumsa-ard K 2006 J. Phys. G: Nucl. Part. Phys. 32 547
[19] Faessler A, Gutsche Th, Lyubovitskij V E and Oonariya C 2008 J. Phys. G: Nucl. Part. Phys. 35 025005
[20] Liu X Y, Khosonthongkee K, Limphirat A and Yan Y 2014 J. Phys. G: Nucl. Part. Phys. 41 055008
[21] Beringer J et al (Particle Data Group) 2012 Phys. Rev. D 1 010001
[22] Amaldi E, Borgia B, Pistilli P, Balla M, Di Giorgio G V, Giazotto A, Serbassi S and Stoppini G 1970 Nuovo Cimento A 65 377
[23] Nambu Y and Yoshimura M 1970 Phys. Rev. Lett. 24 25
[24] Amaldi E, Benevantino M, Borgia B, De Notaristefani F, Frondaroli A, Pistilli P and Severi M 1972 Phys. Lett. B 41 216
[25] Bloom E D, Cottrell P L A, DeStaebler H, Jordan C L, Piel H G, Prescott C Y, Siemann R, Stein S and Taylor R E 1973 Phys. Rev. Lett. 30 1186
[26] Brauel P et al 1973 Phys. Lett. B 45 389
[27] Read B J 1974 Nucl. Phys. B 74 482
[28] Guerra A D, Giazotto A, Giorgi M A, Stefanini A, Botterili D R, Montgomery H E Norton P R and Matone G 1976 Nucl. Phys. B 107 65
[29] Esaulov A S, Pilipenko A M and Titov Y I 1978 Nucl. Phys. B 136 511