An Optimization Approach to the Ordering Phase of an Attended Home Delivery Service

P. Hungerländer * and K. Maier † and J. Pöcher ‡ and A. Rendl § and C. Truden ¶

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Abstract

Attended Home Delivery (AHD) systems are used whenever a supplying company offers online shopping services that require that customers must be present when their deliveries arrive. Therefore, the supplying company and the customer must both agree on a time window, which ideally is rather short, during which delivery is guaranteed. Typically, a capacitated Vehicle Routing Problem with Time Windows forms the underlying optimization problem of the AHD system. In this work, we consider an AHD system that runs the online grocery shopping service of an international grocery retailer.

The ordering phase, during which customers place their orders through the web service, is the computationally most challenging part of the AHD system. The delivery schedule must be built dynamically as new orders are placed. We propose a solution approach that allows to (non-stochastically) determine which delivery time windows can be offered to potential customers. We split the computations of the ordering phase into four key steps. For performing these basic steps we suggest both a heuristic approach and a hybrid approach employing mixed-integer linear programs. In an experimental evaluation we demonstrate the efficiency of our approaches.

Keywords: Attended Home Delivery; Logistics; Heuristics.

1 Introduction

In recent years, online grocery shopping has gained increased popularity in several countries, such as the United Kingdom where about 6.3% [16] of all grocery shopping is bought online. Nowadays, all major supermarket chains provide online shopping services, where customers select groceries as well as a delivery time window on the supermarket’s website. This provides several benefits to the customers, such as 24-hour opening hours of the online store, quicker shopping times, the avoidance of traveling times, no carrying of heavy or bulky items and facilitated access for citizens with reduced mobility. Despite of the benefits for the customers, e-grocery shopping services pose several interrelated logistic and optimization challenges to the supplying companies. Especially the Ordering Phase, during which customers place their orders, imposes a computationally challenging problem.

In this paper, we tackle this challenge in the context of a large international supermarket chain that offers online grocery shopping. E-grocery services are a paradigm for Attended Home Delivery Problems (AHD) [14] where the customers must be present for their deliveries. In order to ensure customer satisfaction and to minimize undeliverable orders, it is crucial that the supplying company provides a wide selection of rather narrow delivery time windows. Hence, in this work, we aim to

* Laboratory for Information & Decision Systems, Massachusetts Institute of Technology Cambridge, MA 02139, USA, philipp.hungerlaender@aau.at
† Department of Mathematics, Alpen-Adria Universität Klagenfurt, Austria, kerstin.maier@aau.at
‡ Department of Mathematics, Alpen-Adria Universität Klagenfurt, Austria, joerg.poecher@aau.at
§ Satalia, London, UK, andrea@satalia.com
¶ Department of Mathematics, Alpen-Adria Universität Klagenfurt, Austria, christian.truden@aau.at
provide a framework that (non-stochastically) determines the available time windows and dynamically builds the delivery schedule during the Ordering Phase.

This paper is organized as follows. First we provide an overview of the logistic process behind the considered AHD system and discuss the Ordering Phase in detail. In Section 2 we introduce the related optimization problem and suggest algorithmic strategies for solving it. In Section 3 we demonstrate the efficiency of our solution approaches on benchmark instances related to an online grocery shopping system. Finally, Section 4 concludes the paper.

1.1 The Attended Home Delivery Process

Let us start with giving a short overview of the overall planning and fulfillment process behind the Attended Home Delivery service, where we describe the actions taken by the supplying e-grocery retailer in order to fulfill the deliveries of a single day.

Tactical Planning Phase - (several months or weeks before delivery):
- A fleet of vehicles is set up and operation times of those vehicles are defined.
- Drivers are assigned to the vehicles in accordance to the legal regulations concerning drive and rest times.
- The supplying company defines the set of possible delivery time windows that will be offered to the customers through the web service.

Ordering Phase - (several weeks up to days/hours before delivery): This phase begins once the web service starts to allow booking of delivery time windows for the specific day of delivery. Hence, the system must handle the following tasks:
- Customers use the web service through a web site or a mobile app to place their orders.
- The system must decide which delivery time windows can be offered to a specific customer such that the delivery can be fulfilled within the time window. Only the resources that have been assigned during the tactical planning phase are available during the ordering phase.
- Once a customer has booked a delivery time window, the system must adapt the existing delivery schedule to accommodate the respective order. Furthermore, the system periodically tries to improve the current schedule.

During this phase, the objective is to accept as many customers as possible, while offering as many time windows as possible to each potential customer in order to achieve a high degree of customer satisfaction and to also ensure good resource utilization which translates to the overall logistics operations being cost efficient.

Preparation Phase - (days/hours before delivery): This phase is triggered once the system does no longer accept new orders through the web service. The objective function is now changed to minimization of the transportation costs (overall fulfillment costs). Another relevant aspect to be considered is the traffic flow at the depot and the vehicle loading bays. Hence, the system must handle the following tasks:
- The delivery schedule is improved regarding the new objective function.
- Meanwhile, at the depot, the ordered goods are fetched from storage and consolidated accordingly to the customer orders.

Delivery Phase: In this phase the vehicles are first packed with the consolidated orders and prepared to leave the depot. Then the vehicles visit the customers according to the delivery schedule, which was generated by the system, such that the customers receive their orders within the selected time windows.
1.2 Related work

First, let us give a short overview of related work by the authors:

- In [8] parts of the Ordering Phase are tackled using two Mixed-Integer Linear Programs (MILPs).
- In [9] we introduce the Slot Optimization Problem. It describes the problem of determining the maximal number of available delivery time windows for a new customer.
- Reference [10] focuses on providing competitive MILP formulations for the Traveling Salesperson Problem with Time Windows.

Next, we want to give a brief overview of related literature that is concerned with the overall planning and fulfillment process described above:

- For an excellent literature review of the online grocery shopping process and the corresponding logistics operations in the context of AHD systems we refer to reference [14].
- Reference [2] describes a Home Delivery system that decides if a customer order is accepted. Furthermore, the system assigns accepted orders to a time window under consideration of the opportunity cost of the orders. In contrast to that, in our setup the customer takes the decision to which delivery time window their order is assigned to. The authors describe the fulfillment process by three phases, (1) order capture and promise, (2) order sourcing and assembly, and (3) order delivery. From an algorithmic point of view the authors propose a two-step insertion heuristic to tackle the order capture and promise phase.

1.3 Challenges & Key Steps of the Ordering Phase

In this paper we focus on the Ordering Phase and suggest solution approaches to deal with the computational challenges arising during this phase. In particular the run time requirements for the optimization problems applied during this phase are much more severe than in any other phase.

All decisions taken in the foregoing Tactical Planning Phase are considered as input variables. During the Ordering Phase customers can book their grocery deliveries through a web service. Figure 1 illustrates a generic example website and its main features. Clearly, the web service should respond to the customer requests with as little delay as possible. Fetching and providing the input data for the booking process requires communication across several services and many database queries. As this already requires a significant amount of time, there is even less time to solve the actual optimization problem.

![Figure 1: Illustration of a generic example website of an AHD service for grocery online shopping. Based on the customer’s address, the system determines the availability of the predefined delivery time windows. Non-available delivery time windows, e.g., 09:00-10:00, are crossed out.]
During the *Ordering Phase*, the web service must repeatedly solve an online variant of a *capacitated Vehicle Routing Problem with Time Windows* (cVRPTW). As the cVRPTW is known to be NP-hard [13], the naive approach of solving a new cVRPTW instance from scratch for each new customer order is far from being applicable in an online environment, even when using fast Meta-heuristics, see [5, 15].

For clarity of exposition, we further split the *Ordering Phase* into the following *four key steps*:

**Initialization step:** The system sets up an empty delivery schedule, i.e., a cVRPTW instance with a fixed number of vehicles and corresponding operation times that were determined during the *Tactical Planning Phase*, but not having any customers assigned to yet.

**Get TWs step - The system determines available delivery time windows:** Based on the current delivery schedule the system determines which delivery time windows are available to a new customer. During times with high customer request rates this step has to be performed within milliseconds. The available time windows are then presented to the customer through the web service. Note that the customer has to provide a delivery address such that a routing system can estimate the travel times between all pairs of customers.

Optionally, for reasons of profit maximizing, some available time windows can be hidden from the customer or be offered at different rates. However, we do not consider any kind of slot pricing in this work. For related and recent work on pricing in the context of AHD systems we refer the reader to [11, 12, 17].

**Set TW step - Customer books a delivery time window:** Using the website or app, a new customer selects her/his preferred delivery time window. As other customers might have booked deliveries since the *Get TWs step*, the system must double-check if this delivery time window is still available. If the insertion is still feasible, the system adds the new order into the working schedule.

In order to avoid queuing issues during the critical *Set TW* step, the system does not allow any other manipulations of the schedule. In case that the requested delivery time window is not available anymore, the *Get TWs* step is triggered again, and an updated list of available delivery time windows is presented to the customer.

**Improvement step:** In this step the system tries to improve the working schedule such that as many delivery time windows as possible can be offered to potential future customers and therefore more customers can place their orders. Choosing the total travel time as objective function has proven to be a reasonable choice to achieve this goal. While the fleet and the assignment of customers to time windows is fixed, the assignment of customers to delivery vehicles as well as the sequences in which the vehicles visit the customers can be altered. Typically the *Improvement* step may take several seconds, but during times with high customer request rates the step can be omitted or triggered after, e.g., every 10 *Set TW* steps.

Note that at any time there is exactly one working schedule in the system. Figure 2 depicts a flow chart of the four steps in the wider context of an AHD service. In the following section we formally introduce the cVRPTW as the underlying optimization problem of the *Ordering Phase* and propose algorithmic strategies for dealing with the cVRPTW during each of the four steps described above.

## 2 Algorithms

### 2.1 Formal Definition of the cVRPTW

In this section we now formally introduce the cVRPTW and some further required notations.

**Basic Definitions:** A cVRPTW instance is typically defined by the following input data:
A set of time windows $W = \{w_1, \ldots, w_q\}$, where each time window $w \in W$ is defined through its start time $s_w$ and its end time $e_w$. We assume that the time windows are unique. Hence, there do not exist time windows $w_a, w_b, \in W, w_a \neq w_b$ with $s_{w_a} = s_{w_b}$ and $e_{w_a} = e_{w_b}$.

A set of customers $C$, $|C| = p$, with corresponding order weight function $c : C \rightarrow \mathbb{R}^>0$, a service time function $s : C \rightarrow \mathbb{R}^>0$, and a travel time function $t : C \times C \rightarrow \mathbb{R}^>0$ where we set the travel time from a customer $a$ to itself to 0, i.e. $t(a, a) = 0, a \in C$.

A function $w : C \rightarrow W$ that assigns to each customer a time window, during which the delivery vehicle has to arrive at the customer.

A schedule $S = \{A, B, \ldots\}$, consisting of $|S| = m$ tours with assigned capacities $C_k, k \in S$, where $C_k$ corresponds to the capacity of the vehicle that operates tour $k$.

A tour $A = \{a_1, a_2, \ldots, a_n\}$ contains $n$ customers, where the indices of the customers display the sequence in which the customers are visited. To improve clarity of exposition, we sometimes
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Additionally use upper indices, i.e., $A = \{a_1^{w(a_1)}, a_2^{w(a_2)}, \ldots, a_n^{w(a_n)}\}$, which indicate the time windows assigned to the customers. Furthermore, each tour $A$ has assigned start and end times that we denote as $\text{start}_A$ and $\text{end}_A$ respectively. Hence, the vehicle executing tour $A$ can leave from the start depot no earlier than $\text{start}_A$ and must return to the end depot no later than $\text{end}_A$.

**Structured Time Windows:** Two time windows $w_a$ and $w_b$ are non-overlapping if and only if $e_{w_a} \leq s_{w_b}$ or $e_{w_b} \leq s_{w_a}$. Therefore, $w_a$ and $w_b$ do overlap if and only if $s_{w_a} < e_{w_b}$ and $s_{w_a} < e_{w_b}$. We speak of structured time windows, if all time windows in $W$ are pair-wise non-overlapping and if the number of customers $|C| = p$ is much larger than the number of time windows $|W| = q$, i.e., $n \gg q$, and therefore typically several customers are assigned to the same time window. We denote the corresponding variant of the cVRPPTW as the capacitated Vehicle Routing Problem with structured Time Windows (cVRPstTW). Structured time windows are a specialty that arises in the Attended Home Delivery use case, as well as some other modern routing applications. Note that the corresponding assumptions do not impose severe restrictions to the supplier nor the customers, but allow for a more efficient optimization of the corresponding logistics operations.

### 2.2 Arrival Times & Feasibility

Next let us give formal, recursive definitions of the earliest and latest arrival times that are needed to define feasibility of a schedule and of the insertion of a new customer.

**Earliest and Latest Arrival Times:** We consider a fixed tour $A = \{a_0, a_1, \ldots, a_n, a_{n+1}\}$, where $a_0$ is the start depot, $a_{n+1}$ is the end depot and $\{a_1, \ldots, a_n\}$ is the set of customers assigned to tour $A$. Note that all our approaches do not move the depots. Hence, customers can only be inserted after the start depot and before the end depot.

The earliest (latest) arrival time $\alpha_{a_i}$ ($\beta_{a_i}$) gives the earliest (latest) time at which the vehicle may arrive at $a_i$, who is the $i^{th}$ customer on the tour, while not violating time window and travel time constraints on the preceding (subsequent) tour:

$$\alpha_{a_0} = \text{start}_A, \quad \alpha_{a_{j+1}} = \max \{s_{w(a_{j+1})}, \alpha_{a_j} + s(a_j) + t(a_j, a_{j+1})\}, j \in [n],$$

$$\beta_{a_{n+1}} = \text{end}_A, \quad \beta_{a_{j-1}} = \min \{e_{w(a_{j-1})}, \beta_{a_j} - s(a_{j-1}) - t(a_{j-1}, a_j)\}, j \in [n].$$

**Feasibility of a Schedule:** Now we can concisely define the feasibility of a tour and a schedule with the help of the earliest arrival times. A schedule $S$ is feasible, if all its tours are feasible. A tour $A$ is feasible, if it satisfies both of the following conditions:

$$s_{w(a_i)} \leq \alpha_{a_i} \leq e_{w(a_i)}, \quad i \in [n], \quad \sum_{i \in [n]} c(a_i) \leq C_A,$$

(TEAS),

(CFEAS).

While TFEAS ensures that the arrival times at each customer are within their assigned time windows, CFEAS ensures that the capacity of $A$ is not exceeded.

**Feasibility of an Insertion:** Now we further use the concepts of earliest and latest arrival time to facilitate and algorithmically speed up feasibility checks of tours after inserting an additional customer. A new customer $\tilde{a}^w$ can be feasibly inserted with respect to time between customers $a_i$ and $a_{i+1}$, $i \in [n_0]$, into a feasible tour $A$ if the following condition holds:

$$\alpha_{\tilde{a}^w} \leq \beta_{\tilde{a}^w}, \quad \text{TFEAS}(\tilde{a}^w, i+1, A),$$

where,

$$\alpha_{\tilde{a}^w} = \max \{s_{w}, \alpha_{a_i} + s(a_i) + t(a_i, \tilde{a}^w)\},$$

$$\beta_{\tilde{a}^w} = \min \{e_{w}, \beta_{a_{i+1}} - s(\tilde{a}^w) - t(\tilde{a}^w, a_{i+1})\}.$$
Condition \[1\] ensures that we arrive at customer \(\tilde{a}^{w}\) early enough, such that we can leave from \(\tilde{a}^{w}\) early enough, to handle all subsequent customers of \(A\) within their assigned time windows. We refer to Figure 3 for an illustration of the above condition.

Additionally, we have to check, if the sum of the weights of the customer orders assigned to tour \(A\), does not exceed the capacity \(C_A\). The insertion of \(\tilde{a}^{w}\) into tour \(A\) is feasible with respect to capacity, if the following condition holds:

\[
\sum_{i \in [n]} c(a_i) + c(\tilde{a}^{w}) \leq C_A, \quad \text{CFEAS}(\tilde{a}, A). \tag{2}
\]

Assuming that all earliest and latest arrival times and the sum of capacities have already been calculated, Conditions \[1\] and \[2\] allow to check the feasibility of an insertion of a new customer into a given time window in \(O(1)\).

If we conduct an insertion that is feasible with respect to time and capacity and decide to insert \(\tilde{a}^{w}\) we receive a new tour \(A = \{a_0, a_1, \ldots, a_i, \tilde{a}^{w}, a_{i+1}, \ldots, a_n, a_{n+1}\}\). Customer \(\tilde{a}^{w}\) is then assigned index \(i+1\) and the indices of all succeeding customers are increased by one. Clearly, earliest and latest arrival times and the sum of capacities of the modified tour must be updated, which can be done in \(O(n)\).

Note that, in the context of an offline Traveling Salesperson Problem with Time Windows, the Generalized Insertion Heuristic proposed in [6] uses concepts analog to our earliest and latest arrival times and the sum of capacities of the modified tour must be updated, which can be done in \(O(n)\).

2.3 Local Search Heuristic

We consider a Local Search heuristic that uses two neighborhoods for exchanging customer orders between two tours:

1. The 1-move neighborhood moves a customer from one tour to another tour.
2. The 1-swap neighborhood swaps two customers between two different tours.

Accordingly we define the 1move(\(\tilde{a}^{w}, A, B\)) operation as the procedure where we remove customer \(\tilde{a}^{w}\) from tour \(A \in S\) and try to feasible insert it into tour \(B \in S, A \neq B\), within time window \(w\). If at least one feasible insertion position for \(\tilde{a}\) in \(B\) is found that additionally decreases the total travel time of the delivery schedule, we denote the 1-move as improving.

As a 1swap(\(\tilde{a}^{w}, A, B\)) operation we define the procedure where we try to exchange customer \(\tilde{a}^{w}\) with any customer within assigned time window \(w\) from a different tour \(B\). If at least one such exchange decreases the total travel time of the schedule, we denote the 1-swap as improving. In general we always select the exchange of an improving 1-swap that results in the largest decrease of the total travel time of the delivery schedule. In Figure 4 we provide an illustration of an improving 1swap(\(a_{i}^{w+1}, A, B\)).
2.4 Algorithmic Strategies

In this subsection we describe how to combine the Local Search heuristics presented in the previous subsection in order to conduct sufficiently fast Get TWs, Set TW and Improvement steps.

The Get TWs step: In this step we aim to quickly identify all time windows during which a new customer \( \tilde{a} \) can be inserted into (at least one of) the current tours. We suggest to use the following procedure:

- **Simple Insertion:** For each time window \( w \in W \) iterate over all tours \( A \in S \) and all possible insertion points within \( w \) and check Conditions 1 and 2. A time window \( w \) is considered as being available, if both conditions hold for at least one insertion point. In this case we add \( w \) to the set of available time windows \( T_{\tilde{a}} \subseteq W \). This procedure is computationally very cheap and runs within 1 millisecond (ms) for all benchmark instances considered in our computational study.

Then the time windows \( T_{\tilde{a}} \) are offered to the customers through the web service. Note that [2] proposes a similar procedure for the VRP, i.e., without considering time windows.

Additionally to this Simple Insertion heuristic we introduced an Adaptive Neighborhood Search (ANS) in [9] that is especially tailored to the Get TWs step. The ANS applies 1-move and 1-swap operations to free up time during a specific time window on a selected tour in order to enable the insertion of the new customer. ANS has proven to find more available time windows than Simple Insertion, while still being fast enough for most applications, as long as the customer request rate is moderate.

The Set TW step: Once the customer has selected a time window \( \tilde{w} \) from the set \( T_{\tilde{a}} \), we double check its availability in the same manner as in the Get TWs step, and then we immediately insert \( \tilde{a} \) into \( \tilde{w} \) at the insertion point found.

The Improvement step: In this step we aim to reduce the total travel time of the delivery schedule by using one of the following two procedures:

- **Local-improvement:** Our computationally cheap, yet quite effective Local Search heuristic builds the foundation of the Improvement step. We combine 1-move and 1-swap operations, where we focus on the 1-move operations when possible, as they are computationally cheaper and in general more effective than the 1-swap operations. We stop our Local Search heuristic once we reach a local minimum of our objective function with respect to our neighborhoods.

Figure 4: Reduction of the total travel time of a schedule induced by an improving 1-swap operation.
• **Local+TSPTW-improvement**: After the Local Search heuristic we additionally use MILPs proposed in our previous paper [10] for optimizing all single tours that have changed since the last improvement step. In [10] we motivated and analyzed the *Traveling Salesperson Problem with Time Windows* (TSPTW) that is a subproblem of the cVRPTW as each tour of the delivery schedule corresponds to a TSPTW instance. Optimizing the single tours of a schedule to optimality has been proven to be critical to ensure driver satisfaction. Hence, it ensures that drivers not encounter any obvious loops on their routes [7]. Also note that we use the current tours of our delivery schedule for warm starting the TSPTW MILPs.

During the **Ordering Phase** our Local Search heuristic only performs improving operations. However, the algorithms can be simply altered into a **Simulated Annealing** approach by allowing also non-improving operations, which is more appropriate for the **Preparation Phase** when there is more time available for optimization.

### 3 Computational Experiments

In this section, we present computational results on a set of benchmark instances that are motivated by an online grocery shopping service of an international grocery retailer. We restrict our experiments to a setup with structured time windows as it has proven to be much more computationally efficient than using arbitrary, overlapping time windows and therefore is better suited for the use in an AHD system. Accordingly we consider the *Traveling Salesperson Problem with structured Time Windows* (TSPstW) [10], which is a special case of the cVRPstW as each tour of the delivery schedule corresponds to a TSPstW instance.

In this section we first describe our benchmark instances in Subsection 3.1. In Subsection 3.2, we describe our experimental setup and in Subsection 3.3 we present the results of our experiments and interpret them.

#### 3.1 Benchmark Instances

We design benchmark instances similar to those which arise in online grocery shopping services. All instances can be downloaded from [http://tinyurl.com/vrpstw](http://tinyurl.com/vrpstw). In more detail our benchmark instances have the following characteristics:

- **Travel times**: The customers are randomly placed on a square-grid. Their coordinates are sampled from a two-dimensional uniform distribution and the travel times are calculated as the Euclidean distance between customers rounded to integers.

- **Customer weights**: The order weights of customers are sampled from a truncated normal distribution with mean of 7 and standard deviation of 2, where the lower bound is 1 and the upper bound is 15.

- **Time window preferences**: The customer preferences for particular time windows are simulated by randomly selecting a time window for each customer, following a uniform distribution. Note that this is in contrast to real-world applications, where usually certain time windows are more prominent among customers than others. However, we chose a uniform distribution to obtain unbiased results that allow for an easier identification and clearer interpretation of the key findings.

- **Shift patterns**: All tours have the same start and end times.

For our computational experiments we consider two benchmark sets that contain 1000 customers each:

1. A benchmark set with many short (45–55) tours with a capacity of 150 and 5 time windows each.
2. A benchmark set with fewer long (18–23) tours with a capacity of 450 and 15 time windows each.
3.2 Experimental Setup

All experiments were performed on an Ubuntu 14.04 machine equipped with an Intel Xeon E5-2630V3 @ 2.4 GHz 8 core processor and 132 GB RAM. We implemented all algorithms in Java version 8 and use Gurobi 6.5.1 as an IP-solver in single thread mode. We compare the algorithmic strategies presented in the previous section for both the Get TWs and the Improvement step.

In all our experiments we iteratively insert new customers into the schedule, simulating customers placing orders online, where the preferred time window of each customer is set beforehand in the benchmark instance. If the preferred time window is not offered to the customer, we assume that the customer does not place the order and hence the customer is not inserted into the schedule. Due to the iterative setup results we can omit the Set TW step and insert the new order without double-checking the availability of the selected delivery time slot.

We determine the following metrics:

- **Get TWs** step:
  - Number of feasible time windows determined for each customer: corresponds to the number of time windows in which the order can be inserted.
  - Runtime of the insertion step.

- **Improvement** step:
  - Reduction of the sum of travel times over all tours.
  - Number of TSPsTW - MILPs solved.
  - Runtime of each improvement step.

3.3 Results

Now let us present the results of our computational evaluation. We examine the performance of our approaches on large instances with 1000 customers. The results for both benchmark sets are summarized in Table 1 for the Get TWs step, and in Table 2 for the Improvement step.

| Get TWs step | 1000 customers | 1000 customers |
|--------------|----------------|----------------|
| Simple-insertion | 150 capacity units | 450 capacity units |
| Tours: | 5 time windows | 15 time windows |
| Average runtime (sec:ms) | 45 | 50 | 55 | 18 | 20 | 23 |
| Number of time windows offered (avg.) | 4.29 | 4.75 | 4.97 | 12.91 | 14.18 | 14.97 |
| Total customers inserted (avg.) | 865.2 | 952.8 | 995.0 | 877.2 | 953.4 | 998.8 |

Table 1: Results for the Get TWs step for our benchmark scenario.

The first benchmark set with many short tours corresponds to the left column, and the results for the second benchmark set with few long tours are displayed in the right column. First, we observe that the runtimes for both the Get TWs and the Improvement step are very low despite of the large instances, which demonstrates that our solving approaches scale very well. It is worth pointing out that the Get TWs step stays below 1ms even for a large number of customers. This is crucial in order to deal with high customer request rates at peak times. Considering that between two Improvement steps the schedule is altered only by insertion of one customer, a reduction of our objective function by 0.27% to 0.74% per step is remarkable. Furthermore, we see a moderate improvement of the hybrid heuristics over the simple heuristics.

When comparing the two different benchmark sets, we notice that the average runtimes for both the Get TWs and the Improvement step, are higher in the second setup with fewer long tours. The reason for this lies in the larger number of customers and time windows per tour which makes these
instances more difficult to solve. We also observe that the reduction of the objective function achieved by the hybrid heuristics, compared to the simple heuristics, is similar on both benchmark sets.

In summary, our suggested algorithms perform very good on both benchmark sets as they are able to produce delivery schedules on large scale instances within the tight runtime restrictions imposed by the considered application.

### 4 Conclusion

In this work, we considered an Attended Home Delivery (AHD) system in the context of an online grocery shopping service offered by an international grocery retailer. AHD systems are used whenever a supplying company offers online shopping services that require that customers must be present when their deliveries arrive. Therefore, the supplying company and the customer must both agree on a time window, which ideally is rather short, during which delivery is guaranteed.

In this paper we considered the overall fulfillment process of the AHD system that can be described by four consecutive phases: (1) Tactical Planning, (2) Ordering, (3) Preparation, and (4) Delivery. We focused on the ordering phase, during which customers place their orders through the web service. Generally, this phase is the most challenging phase of an AHD system from a computational point of view. As for most AHD approaches in the literature, we considered a capacitated Vehicle Routing Problem with Time Windows as the underlying optimization problem of the ordering phase. The online characteristic of this phase requires that the delivery schedule is built dynamically as new orders are placed. We split the computations of the ordering phase into four key steps and proposed a solution approach that allows to (non-stochastically) determine which delivery time windows can be offered to potential customers. Furthermore we employed a Local Search heuristic to improve the delivery schedule and we also suggested a hybrid approach that additionally to the Local Search heuristic employs MILPs, which optimize single tours.

Finally, in an experimental evaluation, we demonstrated the efficiency of our approaches on benchmark sets that are motivated by an online grocery shopping service. We considered the capacitated Vehicle Routing Problem with structured Time Windows (cVRPsTW) for our benchmarking experiments. The special feature of the cVRPsTW is the additional structure of the time windows which does not impose severe restrictions neither to the supplying company nor to the customers. Our computational study showed that the suggested algorithms can solve the considered cVRPsTW instances fast enough to comply with the very strict runtime restrictions as they arise in AHD systems with high customer request rates.
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