Contribution to the Modeling of the Robust Control, of the Longitudinal and Lateral Helicopter Motion Analysis

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Abstract- The purpose of this article is to present the state of the art which concerns the application of robust synthesis techniques of linear systems and mu-analysis of longitudinal and lateral flight of the helicopter.

Index Terms -- Helicopter, state feedback control, state representation, robust control, mu-analysis.

1 INTRODUCTION
The robust control of the helicopter is a type of control that aims to guarantee the performance and stability of a system during flight in the face of environmental disturbances and uncertainties of the model and a difficult task since the dynamics of the system is no linear, unstable on certain flight ranges and has a strongly coupled dynamic. Indeed, the mathematical model that models a real system is a representation that aims to best approximate, with simplifying assumptions, the system we want to control.

2 METHODOLOGY
In practice, the modeling is not precise enough to faithfully reflect the behavior of a system. Often designers use models with unstructured uncertainties that generally affect poorly known or intentionally neglected dynamics.

2.1 Simulation of the synthesis and analysis model
Robustness analysis is performed by incorporating all model uncertainties into a single transfer matrix without imposing any particular structure on it. In this study, assuming that several sources of uncertainties of different natures coexist, we adopt a system with model errors of direct additive form. Two other input signals, applied in two different places of the servo-control, are then taken into account, and the evolution of the performance of the looped system is monitored. The problem therefore arises as the search for a compromise between the objective sought and the means necessary to correct the system by using different methods of synthesis.

2.2 Robust control in longitudinal and lateral flight of the helicopter
The parameters of the system are marred by parametric uncertainty. Each uncertain coefficient \( \bar{p}_i \) is modeled as:

\[
\bar{p}_i = p_i (1 + \omega_i \delta_i), |\delta_i| < 1
\]  (1)

Or :

\( p_i \) :is the nominal value of the parameter considered;
\( \omega_i \) : the corresponding weighting coefficient.

We have chosen \( \omega_i = 0.2 \) for all the parameters.

By higher fractional linear transformation, we have:

\[
\bar{p}_i = [Q_{22} + Q_{21} \Delta_u (I - Q_{11} \Delta_u)^{-1} Q_{12}]
\]  (2)

Is :

\[
Q_i = \begin{bmatrix}
0 & p_i \\
\omega_i & p_i
\end{bmatrix}
\]  (3)

A direct additive form model error is adopted for the synthesis of the corrector for this system (Figure 1). In our case, we will take as scaling function the scalar function:

\[
W_u = 10^{-7} \frac{(s + 10)}{10(s + 500)}
\]  (4)

\[
W_p = 10 \frac{(s + 5)}{10(5s + 1000)}
\]

To achieve the desired performance, it is necessary to satisfy the inequality \( \|W_u(I + GK)^{-1}\|_\infty < 1 \). Here, the weighting function is a scalar function, so the singular values of the sensitivity function \( (I + GK)^{-1} \) over all frequency ranges must be obtained by the curve \( \frac{1}{W_u} \). It means that
\[ \|W_u(I+GK)\|_\infty < 1 \text{ if and only if all frequencies:} \]

\[ \sigma[I+GK]^{-1}(j\omega) < \frac{1}{\|W_u(j\omega)\|}. \]

By isolating the corrector \( K \) to determine the matrix of the open-loop interconnection \( P \) and the closed-loop transfer matrix \( M \), FIG. 2 below:

\[ M = \begin{bmatrix}
1.094 \times 10^{-4} s^2 - 1.163 \times 10^5 s^2 \\
+ 3.637 \times 10^7 s + 6.02 \times 10^{-7} \\
s^4 + 8.804 \times 10^4 s^3 + 3.47 \times 10^{11}s + 4.22 \times 10^9 \\
4.888 \times 10^{-5}s^2 - 3.64 \times 10^4 s^2 \\
- 8.122 \times 10^4 s - 1.998 \times 10^7 \\
s^3 + 8.804 \times 10^4 s^2 + 3.47 \times 10^{11}s + 4.22 \times 10^9 \\
1.15 \times 10^{-5}s^2 + 7.81 \times 10^7 s^2 \\
+ 1.49 \times 10^4 s + 1.206 \times 10^7 \\
s^2 + 8.804 \times 10^4 s^2 + 3.47 \times 10^{11}s + 4.22 \times 10^9 \\
1.051 \times 10^{-5}s^2 - 8.552 \times 10^4 s^2 \\
- 1.636 \times 10^4 s + 3.319 \times 10^7 \\
s^3 + 8.804 \times 10^4 s^2 + 3.47 \times 10^{11}s + 4.22 \times 10^9 \\
\end{bmatrix}. \]

\[ K_a = \begin{bmatrix}
1298 s^2 + 1.77 \times 10^{14} s \\
+ 1.838 \times 10^{14} \\
s^2 + 1.255 \times 10^5 s^2 \\
+ 4.526 \times 10^{14}s + 6.02 \times 10^{14} \\
27.2 s^4 + 3.79 \times 10^4 s^2 \\
- 2.155 \times 10^2 \\
s^3 + 1.255 \times 10^4 s^2 \\
+ 4.526 \times 10^{14}s + 6.02 \times 10^{14} \\
218.1 s^2 + 2.974 \times 10^3 s^2 \\
+ 2.111 \times 10^{13} \\
s^2 + 1.255 \times 10^4 s^2 \\
+ 4.526 \times 10^{14}s + 6.02 \times 10^{14} \\
-361.2 s^2 - 4.926 \times 10^4 s^2 \\
- 4.592 \times 10^2 \\
s^3 + 1.255 \times 10^4 s^2 \\
+ 4.526 \times 10^{14}s + 6.02 \times 10^{14} \\
\end{bmatrix}. \]

3.3 State equation of the flight subsystem of the helicopter

The equation of state of the longitudinal subsystem of the helicopter can be written in matrix form:

\[
\begin{bmatrix}
\dot{u} \\
\dot{w} \\
\dot{q} \\
\dot{\theta}
\end{bmatrix} = \begin{bmatrix}
X_u & X_w & X_q & X_{\theta_0} \\
Z_u & Z_w & Z_q & Z_{\theta_0} \\
M_u & M_w & M_q & M_{\theta_0} \\
0 & 0 & 1 & 0
\end{bmatrix} \begin{bmatrix}
\dot{u} \\
\dot{w} \\
\dot{q} \\
\dot{\theta}
\end{bmatrix} + \begin{bmatrix}
X_{\theta_1} \\
Z_{\theta_1} \\
M_{\theta_1}
\end{bmatrix} \begin{bmatrix}
\theta_0(t) \\
\theta_1(t)
\end{bmatrix}
\]
With:

\[
A_{\theta} = \begin{bmatrix}
X_{\theta} & X_{W} & X_{q} & X_{\theta} \\
Z_{\theta} & Z_{W} & Z_{q} & Z_{\theta} \\
M_{\theta} & M_{W} & M_{q} & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

\[
B_{\theta} = \begin{bmatrix}
X_{\theta0} & X_{\theta1} \\
Z_{\theta0} & Z_{\theta1} \\
M_{\theta0} & M_{\theta1} \\
0 & 0
\end{bmatrix}
\]

\[
C_{\theta} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
D_{\theta} = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix}
\]

One can write in matrix form the state equation of the lateral subsystem of the helicopter:

\[
A_{\theta} = \begin{bmatrix}
Y_{p} & Y_{q} & Y_{\phi} & Y_{r} \\
L_{p} & L_{q} & L_{\phi} & L_{r} \\
\cos \phi & \tan \theta & 0 & 0 \\
0 & 1 & 0 & N_{r}
\end{bmatrix}
\]

\[
B_{\theta} = \begin{bmatrix}
Y_{\theta1c} & Y_{\theta \phi T} \\
\theta_{\phi,c} & \theta_{\phi,T} \\
0 & 0 \\
N_{\theta1c} & N_{\theta \phi T}
\end{bmatrix}
\]

\[
C_{\theta} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
D_{\theta} = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix}
\]

### 4 RESULTS

#### 4.1 Mu-analysis in longitudinal flight

The frequency response of the lower and upper bounds of the structured and unstructured singular value of the matrix \( M_{11} \), the nominal performance analysis matrix, for the corrector is shown in Fig. 4.

| Looping | \( \omega_{\text{ny}} \) (rad/s) | \( \max[\mu(M_{11})] \) |
|---------|-------------------------------|------------------|
| \( K_{\theta} \) | \( 10^3 \) | 0.2000 |

Table 1: Analysis of the nominal performance.

The system is considered efficient because \( \max[\mu(M_{11})] < 1 \) for this loopback (Table 1).

The frequency response of the lower and upper bounds of the structured and unstructured singular value of the matrix \( M_{22} \), stability stability analysis matrix, for the corrector is shown in Fig. 4.

The system is considered robust in performance because \( \max[\mu(M)] < 1 \) for this loopback (Table 2). Yet there is a large guarantee of performance for the corrector obtained by the synthesis \( H_{\text{op}} \).

Table 2: Analysis of robustness in stability.

#### 4.2 Robustness in System Performance

The frequency response of the lower and upper bounds of the structured and unstructured singular value of the matrix \( M \), the system for analyzing the robustness of the system, for each type of corrector is shown in Fig. 5.

| Looping | \( \omega_{\text{ny}} \) (rad/s) | \( \max[\mu(M)] \) | Garantie de la performance |
|---------|-------------------------------|------------------|-----------------------------|
| \( K_{\theta} \) | 0.2341 | 0.7725 | \( \| \Delta \|_{\infty} < \frac{1}{0.7725} \) |

Table 3: Analysis of the robustness in performance.
4.2 Mu-analysis of systems in lateral flight

The block diagram for analyzing the system with unstructured uncertainties is shown in Fig. 6.

Fig. 7. Block diagram of the disturbed system with unstructured uncertainties in lateral flight.

The frequency response of the lower and upper bounds of the unstructured singular value of matrix $M_{22}$, the nominal performance analysis matrix, for the corrector is shown in the Fig. 8 below.

| Looping | $\omega_m$ (rad/s) | $\max[\mu(M_{11})]$ |
|---------|------------------|-------------------|
| $K_{la}$ | 0.7053           | 0.7741            |

Table 4: Analysis of the nominal performance.

The system is considered efficient because $\max[\mu(M_{12})] < 1$ for this loopback.

The frequency response of the lower and upper bounds of the unstructured singular value of the matrix $M_{22}$, stability analysis matrix, for the corrector is shown in Fig. 9.

| Looping | $\omega_m$ (rad/s) | $\max[\mu(M_{12})]$ | Stability guarantee            |
|---------|------------------|-------------------|--------------------------------|
| $K_{la}$ | $10^7$           | $6 \times 10^{-7}$ | $\|\Delta\|_\infty < \frac{1}{6 \times 10^{-7}}$ |

Table 5: Analysis of robustness in stability.

The system is considered stable in robustness because $\max[\mu(M_{11})] < 1$ for this loopback.

The frequency response of the lower and upper bounds of the unstructured singular value of the matrix $M$, the system of analysis of the robustness of the system, for each type of corrector is shown in the figure below.

Table 6: Analysis of the robustness in performance.

5 CONCLUSION

In this article, we have been able to develop robustness synthesis analysis tools for a linear system. Structured singular value is one of the very powerful tools for analyzing the robustness of a linear system tainted by uncertainty. It has been explained how the μ-analysis method can analyze the robustness of stability and performance of a system. The standard problem explains the synthesis of a controller by minimizing the H-infinite standard of the LFT $F(P, K)$. The resolution of this optimization leads to the determination of K. The use of μ-analysis and the problem of H-infinity has established another synthesis tool that is μ-synthesis.

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