The fundamental point about the familiar perturbative QCD (pQCD) factorization theorems is that the hard scattering cross sections are linear functionals (convolutions) of the appropriate parton densities in the projectile and target. A consistent analysis of forward hard dijet production in deep inelastic scattering (DIS) off nuclei revealed a striking breaking of $k_\perp$-factorization \[1\] confirmed later on in the related analysis of single-jet spectra in hadron-nucleus collisions \[2\]. Namely, following the pQCD treatment of diffractive dijet production \[3, 4\] one can define the collective nuclear unintegrated gluon density such that the familiar linear $k_\perp$-factorization (see e.g. the recent review \[5\]) would hold for the nuclear structure function $F_{2A}(x, Q^2)$ and the forward single-quark spectrum in DIS off nuclei because of its special abelian features. However, the dijet spectra and single-jet spectra in hadron-nucleus collisions prove to be highly nonlinear functionals of the collective nuclear gluon density. Furthermore, the pattern of nonlinearity for single-jet spectra was shown to depend strongly on the relevant partonic subprocess \[2\]. Our conclusions on the breaking of linear $k_\perp$-factorization for hard scattering off nuclei have recently been corroborated by other authors \[6, 7\]. In this communication we investigate the nonuniversality aspects of nonlinear nuclear $k_\perp$-factorization for different dijet excitation processes. We also comment on the process-dependence of the significance of diffractive final states. Our results shed certain light on to which extent hard processes in a nuclear environment can be described entirely in terms of the classical gluon field of a nucleus \[8\].

Here we extend the analysis \[1, 8\] of the excitation of heavy flavor and leading quark jets in DIS, $\gamma^* \rightarrow QQ$, to the excitation of open charm (or hard quark-antiquark dijet) and gluon jets in subprocesses $g^*g \rightarrow QQ$, $q^*g \rightarrow qg$ which are of direct relevance to the the large (pseudo)rapidity region of proton-proton and proton-nucleus collisions at RHIC. Our treatment is applicable when the beam and final state partons interact coherently over the whole nucleus, which at RHIC amounts to the proton fragmentation region of $x = M_{JJ}^2/2m_pE_b \lesssim x_A = 1/R_A m_p \approx 0.1 A^{-1/3}$, where $R_A$ is the radius of the target nucleus of mass number $A$, $E_b$ is energy of the beam parton $a$ in the target rest frame and $m_p$ is the proton mass \[10, 11\].

To the lowest order in pQCD, all the above processes are of the general form $ag \rightarrow bc$ and, from the laboratory frame standpoint, can be viewed as an excitation of the perturbative $[bc]$ Fock state of the physical projectile $[a]$ by one-gluon exchange with the target nucleon. In the case of a nuclear target one has to deal with the non-abelian intranuclear evolution due to multiple gluon exchanges which are enhanced by a large thickness of the target nucleus. The derivation of the master formula for dijet spectrum, based on the technique developed in \[1, 12, 13\], is found in \[2\]; here we only reproduce the main result:

$$
\frac{d\sigma(a^* \rightarrow bc)}{dz_2 d^2p_b d^2p_c} = \frac{1}{(2\pi)^4} \int d^2p_0 d^2b_0 d^2b_0' d^2b_c' \left[ \times \exp[-ip_0(b_0 - b_0') - ip_c(b_c - b_c')] \times \Psi^{*}(z_{b_0} b_0 - b_{c'}) \times \left\{ S_{abc}(b_0', b_c') + S_{abc}^{(2)}(b', b) - S_{abc}^{(3)}(b', b_c') \right\} \right].
$$

If $b_0 = b + z_0 r$, $b_c = b - z_c r$, where $z_{b,c}$ stand for the fraction the lightcone momentum of the projectile $a$ carried by the partons $b$ and $c$. All $S^{(n)}$ describe a scattering on a target of color-singlet systems of partons, as

\[a\]

\[b\]

\[c\]
indicated in Fig. II and all the dijet spectra are infrared finite observables. The $S^{(2)}$ and $S^{(3)}$ are readily calculated in terms of the 2-parton and 3-parton dipole cross sections \[13\] \[14\] \[15\]. General rules for the multiple scattering theory calculation of the coupled-channel $S^{(4)}$ are found in \[12\] and need not be repeated here.

The fundamental quantity of the color dipole formalism is the $q\bar{q}$ dipole cross section $S^{(2)}$:

$$\sigma_{q\bar{q}}(x, r) = \int d^2 k f(x, k)[1 - \exp(ikr)],$$

$$f(x, k) = \frac{4\pi \alpha_s(r)}{N_c} \cdot \frac{1}{k^2} \cdot F(x, k^2), \quad (2)$$

where $F(x, k^2) = \partial G(x, k^2)/\partial \log k^2$ is the unintegrated gluon density in the target nucleon. It furnishes a universal description of the proton structure function $F_2(x, Q^2)$ and of the final states. For instance, the linear $k_1$-factorization for forward dijet cross section reads (for applications, see \[17\] and references therein)

$$\frac{2(2\pi)^2 d\sigma_{q\bar{q}}(\gamma^* \rightarrow Q\bar{Q})}{dzd^2 p_\perp d^2 \Delta} = \int f(x, \Delta) |\Psi(z, p_-) - \Psi(z, p_- - \Delta)|^2, \quad (3)$$

where $\Psi(z, p)$ is the qg wave function of the photon and $\Delta = p_+ + p_-$ is the jet-jet correlation momentum, and $p_-, z \equiv z_\perp$ refer to the Q-jet.

The principal issue is the pQCD description of the dijet cross section for nuclear targets. First place, one needs the collective nuclear gluon density $\phi(b, x, A, \kappa)$ per unit area in the impact parameter plane. As a starting point, one can define it \[1\] \[14\] in terms of the $q\bar{q}$ nuclear profile function:

$$\Gamma_{2A}(b, \sigma_{q\bar{q}}(x, r)) = 1 - \exp \left[ -\frac{1}{2} \sigma(x, r) T(b) \right] = \int d^2 k \phi(b, x, k) \left[ 1 - \exp(ikr) \right], \quad \Gamma_{A}(b, \sigma_{q\bar{q}}(x, r)) = 1 - \sigma_0(x) T(b)$$

It satisfies the sum rule $\int d^2 k \phi(b, x, k) = 1 - S_{abs}(b)$, where $S_{abs}(b) = \exp(-\frac{1}{2} \sigma_0(x) T(b))$ and $\sigma_0(x) = \int d^2 k f(x, k)$ is the dipole cross section for large dipoles and $T(b)$ is the optical thickness of a nucleus. The explicit expansion for $\phi(b, x, A, \kappa)$ in terms of the collective gluon density for $j$ overlapping nucleons in the Lorentz-contracted nucleus \[10\] \[14\] \[15\], its saturation properties at small $\kappa$ and the saturation scale $Q_A(b, x)$ are found in \[12\] \[14\]. Here we only emphasize that the inclusive spectrum of leading quarks from the excitation $\gamma^* \rightarrow Q\bar{Q}$ off nuclei satisfies the same linear $k_1$-factorization in terms of $\phi(b, x, A, \kappa)$ as its counterpart for free-nucleon target in terms of $f(x, k)$, see Eq. \[3\]. I.e., all Initial and Final State distortions of the spectrum of leading quarks are reabsorbed into the collective nuclear gluon density; such an abelianization only holds for the color singlet projectile. We shall also make use of $\Phi(b, x, \kappa) = S_{abs}(b)\delta^{(2)}(\kappa) + \phi(b, x, A, \kappa)$.

The $t$-channel pQCD gluon exchange leaves the target nucleon debris in the color excited state. In the case of nuclear targets one must distinguish the coherent diffractive and truly inelastic processes. In the former the bc color dipole is in the same color state as the projectile $a$ and coherent diffraction $aA \rightarrow (bc)A$ with retention of the target nucleus in the ground state is possible. It gives rise to exactly back-to-back dijets, i.e., the diffractive contribution is $\propto \delta^{(2)}(\Delta)$ (for finite-size nuclei the $\Delta$-dependence is controlled by a slightly modified nuclear form factor with the width $\Delta^2 \lesssim R_A^2$, see Ref. \[1\]).

At large-$N_c$ considered here, excitation from the color-singlet to color-octet dipoles in truly inelastic DIS is suppressed $\propto 1/N_c$, see the matrix element $\sigma_{18}$ in Ref. \[1\], but this smallness is compensated for by a large number of the octet dipole states (at arbitrary $N_c$ one must speak of the adjoint and fundamental representations, referring to them as octet and triplet states must not cause any confusion). That is a reason behind the unitarity-induced coherent diffraction making $\sim 50\%$ of the total DIS off heavy absorbing nuclei \[18\]. In DIS off nuclei, the dipoles first propagate as color-singlets, then at depth $\beta$ in a nucleus excite into the octet state and further color exchanges at the remaining depth $\beta, [1]$ only rotate the dipole within the the octet state. With inclusion of the diffractive component the dijet spectrum from excitation $\gamma^* \rightarrow Q\bar{Q}$ in DIS off nuclei equals \[1\]

$$\frac{\delta(2\pi)^2 d\sigma_{A}(\gamma^* \rightarrow Q\bar{Q})}{d^2\beta d z d^2 p_\perp d^2 \Delta} = \frac{1}{2} T(b) \int_0^1 d\beta \int d^2 \kappa_1 d^2 \kappa \times f(x_A, \kappa) \Phi(1 - \beta, b, x_A, \Delta - \kappa_1 - \kappa) \Phi(1 - \beta, b, x_A, \kappa_1) \times |\Psi(\beta, z, p_- - \kappa_1) - \Psi(\beta, z, p_- - \kappa_1 - \kappa)|^2 + \delta^{(2)}(\Delta) |\Psi(1 - z, p_-) - \Psi(z, p_-)|^2, \quad (5)$$

where $\Phi(b, x, A, \kappa)$ is the collective nuclear glue for the slice $\beta$ of a nucleus defined by

$$\exp \left[ -\frac{1}{2} \beta \sigma(x, r) T(b) \right] = \int d^2 k \Phi(b, x, \kappa) \exp(ikr)$$

and

$$\Psi(\beta; z, p_-) = \int d^2 k \Phi(b, x, A, \kappa) \Psi(z, p_- + \kappa)$$

is the wave function of the incident color-singlet dipole distorted by the coherent Initial State Interaction (ISI) in the slice $\beta$ of a nucleus. The diffractive component is a quadratic functional of the collective nuclear glue. The first component in \[5\] describes truly inelastic DIS. Here the slice $(1 - \beta)$ in which the dipole is in the color-octet state gives the Final State Interactions (FSI). The singlet-to-octet transition is described by the free-nucleon gluon density $f(x_A, \kappa)$. In contrast to the free-nucleon
result, the free-nucleon jet spectrum is of fifth order in gluon field densities: a quartic functional of the collective nuclear glue for two slices of a nucleus and a linear one of the free-nucleon glue; it can not be described by the classical gluon field of the whole nucleus. In DIS the FSI looks as an independent broadening of the quark and antiquark jets. The nonlinear $k_{T}$-factorization result must be contrasted to the free-nucleon spectrum; it entails nuclear enhancement of the decorrelation of dijets from truly inelastic DIS, the semihard dijets, $|p_{\perp}|^{2} \lesssim Q_{N}^{2}(b, x_{A})$, are completely decorrelated.

The large-$N_{c}$ properties of excitation $q^* \to qg$ are similar to those of excitation $\gamma^* \to qg$ in DIS. The free-nucleon dijet cross section equals

$$
\frac{2(2\pi)^{2}d\sigma_{N}(q^* \to qg)}{dz_{g}d^{2}p_{g}d^{2}\Delta} =
\int_{0}^{1} d\beta \int d^{2}\kappa d^{2}\kappa f(x_{A}, \kappa)
\times \Phi(1 - \beta, b, x_{A}, \Delta - \kappa_{1} - \kappa)\Phi(2 - \beta, b, x_{A}, \kappa)
\times |\Psi(\beta; z_{g}, p_{g} - \kappa_{1} - \kappa) - \Psi(\beta; z_{g}, p_{g} - \kappa_{1} - \kappa)|^{2}
+ \phi(b, x_{A}, \Delta)\Psi(1; z_{g}, p_{g} - \Delta) - \Psi(z_{g}, p_{g} - z\Delta)|^{2}
+ \delta^{(2)}(\Delta)S_{abs}(b)|\Psi(1; z_{g}, p_{g}) - \Psi(z, p_{g})|^{2}.
$$

Here the third component in (4) is the contribution from the coherent diffractive excitation of color-triplet $qg$ dipoles, $q^* A \to (qg) A$. It is suppressed by the nuclear attenuation factor which is a consequence of the initial parton $q^*$ being a colored one. The second term in (4) can be associated with excitation of the color-triplet $qq$ states. It looks like satisfying the linear $k_{T}$-factorization in terms of $\phi(b, x_{A}, \Delta)$ but it does not: one of the wave functions, $\Psi(1; z_{g}, p_{g})$, is coherently distorted over the whole thickness of the nucleus. Finally, the first component of the nuclear spectrum (4) describes excitation of the color sextet and 15-plet $qg$ states. The free-nucleon result (4) is recovered to the impulse approximation.

The latter contribution to the nuclear dijet spectrum is a fifth order functional of gluon densities and resembles strongly the truly inelastic dijet spectrum of (4) for DIS. As it was the case for DIS, the free-nucleon gluon density $f(x_{A}, \kappa)$ describes the excitation of the $qq$ color dipole from the lower (triplet) to higher (sextet and 15-plet) color states. The ISI distortions are similar too, the principal difference is in the nuclear thickness dependence of the distortion factors in the second line of Eqs. (5) and (7): the asymmetric one, $\Phi(1 - \beta, b, x_{A}, \Delta - \kappa_{1} - \kappa)\Phi(2 - \beta, b, x_{A}, \kappa)$ for the fragmentation of colored quark $q^*$ vs. the symmetric one, $\Phi(1 - \beta, b, x_{A}, \Delta - \kappa_{1} - \kappa)\Phi(1 - \beta, b, x_{A}, \kappa)$ in DIS. In DIS it describes equal distortion of the both outgoing parton waves by pure FSI, for the incident quarks $q^*$ in $pA$ collisions it includes the ISI distortion of the incoming wave of the colored quark $q^*$. For the latter reason the input nonlinearity of this contribution is of still higher, sixth, order: the factor $\Phi(2 - \beta, b, x_{A}, \kappa)$ which looks as if defined for the slice of nuclear matter of thickness $(2 - \beta)$ is, as a matter of fact, a convolution $\Phi(2 - \beta, b, x_{A}, \Delta - \kappa)$, which combines the effects of distortion of the incident quark wave over the whole thickness of the nucleus and of the produced quark in the slice $(1 - \beta)$ of the nucleus.

Because one starts with the color-octet $Q\bar{Q}$ dipole, at large-$N_{c}$ the intranuclear interactions are color rotations in the space of octet $Q\bar{Q}$ states. Transitions to the color-singlet $c\bar{c}$ dipoles are suppressed and the non-abelian evolution of the $Q\bar{Q}Q'\bar{Q}'$ state becomes the single channel problem. The coherent diffraction excitation, in which the initial and final color states must be identical, is likewise suppressed. The resulting nuclear dijet cross section equals

$$
\frac{2(2\pi)^{2}d\sigma_{N}(q^* \to Q\bar{Q})}{dz_{g}d^{2}p_{g}d^{2}\Delta} =
\int d^{2}\kappa \Phi(1; b, x_{A}, \kappa)
\times \Phi(1; b, x_{A}, \Delta - \kappa)|\Psi(z, p_{-} - \kappa) - \Psi(z, p_{-} - z\Delta)|^{2}
+ S_{abs}(b)|\phi(b, x_{A}, \Delta)\Psi(1; z, p_{g}) - \Psi(z, p_{g} - z\Delta)|^{2}.
$$

Because one starts with the color-octet $Q\bar{Q}$ dipole, at large-$N_{c}$ the intranuclear interactions are color rotations in the space of octet $Q\bar{Q}$ states. Transitions to the color-singlet $c\bar{c}$ dipoles are suppressed and the non-abelian evolution of the $Q\bar{Q}Q'\bar{Q}'$ state becomes the single channel problem. The coherent diffraction excitation, in which the initial and final color states must be identical, is likewise suppressed. The resulting nuclear dijet cross section equals
\[ + \int d^2 \kappa \phi(b, x_A, \kappa) \phi(b, x_A, \Delta - \kappa) \times (|\Psi(z, p_\perp - \kappa) - \Psi(z, p_\perp - z\Delta)|)^2. \tag{9} \]

This result for the dijet spectrum \([9]\) is precisely the differential version of the single-quark spectrum, Eq. (31) of Ref. \([1]\), if in the nonlinear term one makes an identification \(\Delta = \kappa_1 + \kappa_2\). It satisfies the quadratic-nonlinear \(k_\perp\)-factorization in contrast to the quartic-nonlinear one for the leading quark-antiquark dijets in DIS and the \(qq\) dijets from \(q^* \rightarrow qg\). Although only the collective glue for the whole thickness of the nucleus enters the nonlinear term in \([9]\), and kinematically it looks as if corresponding to the subprocess \(g^* g_1 A g_2 A \rightarrow Q\overline{Q}\) with two uncorrelated collective nuclear gluons \(g_A\), the emerging combination of wave functions can not readily be associated with specific Feynman diagrams in terms of collective nuclear gluons \(g_A\). Different collinear contributions corresponding to poles of the wave functions \(\Psi(p_\perp + \kappa)\) can readily be identified \([1, 2]\), for the related discussion see also \([20]\); such an analysis goes beyond the scope of the present communication and will be addressed elsewhere.

Our findings can be summarized as follows: The nonlinear \(k_\perp\)-factorization relations \([7, 9]\) and \([9]\) for the dijet spectrum with two extreme color excitation properties, are our main new results. Such explicit representation for the dijet spectra is not contained in previous works on the subject \([7, 9]\). These examples show clearly that the nonlinear \(k_\perp\)-factorization is a generic feature of the pQCD description of the dijet production in a nuclear environment. We established how at large \(N_c\) the pattern of nonlinearity depends on color properties of the relevant QCD subprocess: (A) excitation of dijets in higher color representations from partons in a lower representation typically gives rise to the fifth, or even sixth, order nonlinearity of the dijet spectrum in gluon fields, while (B) in the processes starting from already higher-representation partons inelastic interactions can be viewed as color rotations within the same representation and the nonlinearity will be quadratic one. A part of the nonlinearity comes from the free-nucleon gluon density which emerges in all instances of excitation of higher color representations (see also the related discussion of the \(1/(N_c^2 - 1)\) expansion in Ref. \([1]\)). The coherent diffraction is not suppressed by large \(N_c\) in the class-A reactions and is color-suppressed in the class-B reactions; this is also a new observation. Still another feature of the class-A reactions is a contribution from dijets in the same color representation as the incident parton. Within our nonlinear \(k_\perp\)-factorization, all the dijet spectra are explicitly calculable in terms of the collective nuclear glue of Eq. \([4]\). However, in the class-A reactions this collective glue must be evaluated for different slices of a nucleus and enters ISI and FSI effects in quite a distinct way. The condition, \(x \lesssim x_A \approx 0.1 \cdot A^{-1/3}\), restricts the applicability domain of our formalism to the proton hemisphere of \(pA\) collisions at RHIC; the required coherency condition does not hold for the mid-rapidity dijets studied so far at RHIC \([21]\).

This work was partly supported by the grant DFG 436 RUS 17/101/04.

\[ \text{* Electronic address: N.N. Nikolaev@fz-juelich.de} \]
\[ \text{‡ Electronic address: Wo.Schaefer@fz-juelich.de} \]
\[ \text{∗ Electronic address: B.Zakharov@fz-juelich.de} \]

[1] N. N. Nikolaev, W. Schäfer, B. G. Zakharov and V. R. Zoller, J. Exp. Theor. Phys. 97, 441 (2003).
[2] N. N. Nikolaev and W. Schäfer, Phys. Rev. D 71, 14023 (2005).
[3] N. N. Nikolaev and B. G. Zakharov, Phys. Lett. B 332, 177 (1994).
[4] N. N. Nikolaev, W. Schäfer and G. Schwiete, Phys. Rev. D 63, 014020 (2001); JETP Lett. 72, 405 (2000).
[5] J. R. Andersen et al. [Small x Collaboration], Eur. Phys. J. C 35, 67 (2004).
[6] J. P. Blaizot, F. Gelis and R. Venugopalan, Nucl. Phys. A 743, 57 (2004).
[7] J. Jalilian-Marian and Y. V. Kovchegov, Phys. Rev. D 70, 114017 (2004).
[8] L. D. McLerran and R. Venugopalan, Phys. Rev. D 49, 2233 (1994); R. Venugopalan, arXiv:hep-ph/0412396 and references therein.
[9] N. N. Nikolaev, W. Schäfer, B. G. Zakharov and V. R. Zoller, Phys. At. Nucl. 68, n.4 (2005), in print.
[10] N.N. Nikolaev and V.I. Zakharov, Sov. J. Nucl. Phys. 21, 227 (1975); Phys. Lett. B 55, 397 (1975).
[11] V. Barone, M. Genovese, N. N. Nikolaev, E. Predazzi and B. G. Zakharov, Z. Phys. C 58, 541 (1993).
[12] B. G. Zakharov, Sov. J. Nucl. Phys. 46, 92 (1987).
[13] N. N. Nikolaev, G. Piller and B. G. Zakharov, J. Exp. Theor. Phys. 81, 851 (1995); Z. Phys. A 354, 99 (1996).
[14] N. N. Nikolaev and B. G. Zakharov, Z. Phys. C 49, 607 (1991).
[15] N. N. Nikolaev and B. G. Zakharov, J. Exp. Theor. Phys. 78, 598 (1994); Z. Phys. C 64, 631 (1994).
[16] N. N. Nikolaev and B. G. Zakharov, Phys. Lett. B 332, 184 (1994).
[17] A. Szcurek, N. N. Nikolaev, W. Schäfer and J. Speth, Phys. Lett. B 500, 254 (2001).
[18] N. N. Nikolaev, B. G. Zakharov and V. R. Zoller, Z. Phys. A 351, 435 (1995).
[19] N. N. Nikolaev, W. Schäfer, B. G. Zakharov and V. R. Zoller, paper in preparation.
[20] D. Kharzeev, Y. V. Kovchegov and K. Tuchin, Phys. Rev. D 68, 094013 (2003).
[21] C. Adler et al. [STAR Collaboration], Phys. Rev. Lett. 90, 082302 (2003).