REAL BOTT MANIFOLDS AND ACYCLIC DIGRAPHS

SUYYOUNG CHOI

ABSTRACT

A Small cover, defined by Davis and Januszkiewicz [2], is an $n$-dimensional closed smooth manifold $M$ with a smooth action of standard real torus $\mathbb{Z}_2^n$ such that the action is locally isomorphic to a standard action of $\mathbb{Z}_2^n$ on $\mathbb{R}^n$ and the orbit space $M/\mathbb{Z}_2^n$ can be identified with a simple (combinatorial) polytope. In this talk, we assume that the orbit polytope is a hypercube. Small covers over cubes, called the real Bott manifolds, are obtained as iterated $\mathbb{R}P^1$ bundles starting with a point, where each stage is the projectivization of a Whitney sum of two real line bundles. In this talk, we discuss about the topological classification of real Bott manifolds. In fact, by Kamishima and Masuda [3], two real Bott manifolds are diffeomorphic if their cohomology rings with $\mathbb{Z}_2$ coefficients are isomorphic. In addition, Masuda [4] has explained the diffeomorphism classes of them by 3 operations of the set of real Bott manifolds. In other words, two Bott manifolds are diffeomorphic if one is transformed to the other through a sequence of the three operations.

On the other hand, Choi [1] has found a 1-1 correspondence between the set of real Bott manifolds and the set of combinatorial objects, acyclic digraphs. So we may regard operations of real Bott manifolds as operations of digraphs. One of operations is well-known for the local complementation. We investigate many invariants under the these operations. This work is jointly with Sangil Oum.

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Department of Mathematical Sciences, KAIST, 335 Gwahangno, Yuseong-gu, Daejeon 305-701, Republic of Korea
E-mail address: choisy@kaist.ac.kr
URL: http://topology.kaist.ac.kr/schoi

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