CAN THE DENSITY DISTRIBUTION AND SHAPE OF THE GALACTIC DARK HALO BE DETERMINED BY LOW-FREQUENCY GRAVITATIONAL WAVES?

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ABSTRACT

Under the assumption that the Milky Way's dark halo consists of primordial black hole MACHOs (PBH MACHOs), the mass density of the halo can be measured by the low-frequency gravitational waves \( (10^{-3} \leq f_{\text{gw}} \leq 10^{-1}) \) Hz from PBH MACHO binaries whose fraction is \( \sim 10^{-6} \). We find that 10 yr of observations by the Laser Interferometer Space Antenna (LISA) should detect \( \sim 700 \) PBH MACHO binaries and enable us to determine the power index of the density profile within 10% (20%) and the core radius within 25% (50%) at about a 90% (99%) confidence level, respectively. The axial ratios of the halo may also be determined within \( \sim 10% \). LISA and the Orbiting Medium Explorer for Gravitational Astrophysics (OMEGA) may give us a unique observational method to determine the density profile and the shape of the dark halo to open a new field of observational astronomy.

Subject headings: black hole physics — dark matter — Galaxy: halo — Galaxy: structure — gravitation — gravitational lensing

1. INTRODUCTION

It is important to determine the density profile of the Milky Way's dark halo observationally in order to gain insight into the Galaxy's formation and evolution. Unfortunately, little is known about the halo density profile (HDP), since the dark halo emits little light. The H I rotation curve will tell us about the radial distribution of dark matter (e.g., Fich & Tremaine 1991). However, this has not been accurately measured for \( r > D_{0} \sim 8 \) kpc, and recently there has also been an argument that the Galactic rotation curve may deviate from that of the standard halo model (Honma & Sofue 1996, 1997; Olling & Merrifield 1998b). The dynamics of satellite galaxies and globular clusters can provide the mass inside \( r \sim 50 \) kpc, with some biases (e.g., Lin, Jones, & Klemola 1995; Kochanek 1996; Zaritsky et al. 1989; Einasto & Lynden-Bell 1982; Peebles 1995). The HDP can also be recovered by using the tidal streams from Galactic satellites (e.g., Johnston et al. 1999). All methods so far are, however, indirect methods. In this paper, we investigate the possibility of a direct measurement of the density distribution of the Galactic dark halo.

Recently, Ioka, Tanaka, & Nakamura (1998b; hereafter ITN) proposed the possibility of determining a map of the HDP using low-frequency gravitational waves \( (10^{-4} \leq f_{\text{gw}} \leq 10^{-1}) \) Hz from PBH MACHO binaries, which can be detected by the planned Laser Interferometer Space Antenna (LISA) and Orbiting Medium Explorer for Gravitational Astrophysics (OMEGA). ITN were motivated by observations of gravitational microlensing toward the Large Magellanic Cloud (LMC). Analysis of the first 2.1 yr of photometry of 8.5 \( \times 10^{6} \) stars in the LMC by the MACHO Collaboration (Alcock et al. 1997) suggests that a fraction \( (0.62 \pm 0.2) \) of the halo consists of massive compact halo objects (MACHOs) of mass \( 0.5^{+0.3}_{-0.2} \) \( M_{\odot} \), assuming the standard spherical flat rotation halo model.

At present, we do not know what MACHOs are. There are several candidates proposed to explain MACHOs, such as brown dwarfs, red dwarfs, white dwarfs, etc. (Chabrier 1999; Gould, Flynn, & Bahcall 1998; see also references in ITN). Any objects clustered somewhere between the LMC and the Sun with a column density larger than \( 25 M_{\odot} \) pc\(^{-2} \) could also explain the data (Nakamura, Kan-ya, & Nishi 1996). These include the possibilities of LMC-LMC self-lensing, a thick disk, warps, tidal debris, etc. (Sahu 1994; Zhao 1998a, 1998b; Evans et al. 1998; Gates et al. 1998; Bennett 1998; for discussions of SMC events, see also Alfonso et al. 1998; Albrow et al. 1999; Alcock et al. 1999; Honma 1999) However, none of them convincingly explain the microlensing events toward the LMC and SMC.

Freese, Fields, & Graff (1999) claimed that on theoretical grounds, one is pushed to either exotic explanations or a non-MACHO halo. We here simply adopt the suggestion by the MACHO Collaboration (Alcock et al. 1997) and consider an example of exotic explanations: a primordial black hole MACHO (Nakamura et al. 1997). This possibility is free from observational constraints at present (Fujita et al. 1998), and PBH MACHOs may be identified by LIGO, VIRGO, TAMA, and GEO within next 5 yr (Nakamura et al. 1997; Ioka et al. 1998a), if they exist as dark matter.

If primordial black holes (PBHs) were formed in the early universe at \( t \sim 10^{-5} \) s (Yokoyama 1997; Kawasaki & Yanagida 1999; Jedamzik 1997), some of them evolved into binaries through three-body interactions (Nakamura et al. 1997; Ioka et al. 1998a). Some of these binaries emit gravitational waves (GWs) at low frequencies at present. ITN found that 1 yr of observations by LISA should be able to identify at least several hundred PBH MACHO binaries. Since LISA can measure distances and positions of PBH MACHO binaries (Cutler 1998; see also LISA Pre--Phase A Report), it may be possible to obtain HDP from the distribution map of PBH MACHO binaries.

In this paper, we will quantitatively investigate how well HDP can be determined by the observation of low-
frequency GWs from PBH MACHO binaries and show that LISA and OMEGA will serve as excellent instruments for determining the shape of our dark halo.

2. PBH MACHO MODEL

For simplicity, we assume that PBHs dominate the dark matter, i.e., that \( \Omega = \Omega_{\text{BH}} \), where \( \Omega_{\text{BH}} \) is the density parameter of PBHs at present, and that all PBHs have the same mass, \( M_{\text{BH}} \). Throughout this paper, we will set \( M_{\text{BH}} = 0.5 M_\odot \) and \( \Omega h^2 = 0.1 \), where \( h \) is the present Hubble parameter \( H_0 \) in units of 100 km s\(^{-1}\) Mpc\(^{-1}\).

Assuming that PBHs are distributed randomly at their formation, we can obtain the probability distribution function (PDF) for the orbital frequency \( v_g \) and eccentricity \( e \) of the binary, \( f_r(v_g, e) dv_g de \) (Nakamura et al. 1997; Ioka et al. 1998a). For \( e \ll 1 \), an approximate PDF is given by

\[
f_r(v_g; t_0) dv_g \sim \frac{425}{355} \left( \frac{v_g}{f} \right)^{3/2} \left( \frac{a}{a_0} \right)^3 \left( \frac{58}{37} \right) \frac{dv_g}{v_g},
\]

where \( a = (GM_{\text{BH}}/2\pi^2 v_g^2)^{1/3} \) is the semimajor axis, \( t_0 = 10^{10} \) yr is the age of the universe, \( a_0 = 2.0 \times 10^{11} (M_{\text{BH}}/M_\odot)^{3/4} \) cm is the semimajor axis of a binary in a circular orbit that coalesces in \( t_0 \), \( \bar{x} = 1.2 \times 10^{10} (M_{\text{BH}}/M_\odot)^{1/2} (\Omega h^2)^{-4/3} \) cm is the mean separation of black holes at the time of matter-radiation equality, and \( \bar{t} = \bar{B}'(\bar{x}/a_0)^{1/2} t_0 \) (ITN). The terms \( \alpha \) and \( \beta \) are constants of order unity. In this paper we adopt \( \alpha = 0.5 \) and \( \beta = 0.7 \) (ITN). Note that a circular binary with orbital frequency \( v_g \) emits GWs at GW frequency \( v_{gw} = pv_g \), with the second harmonic \( p = 2 \) (Peter & Mathews 1963; Hils 1991).

3. INDIVIDUALLY OBSERVABLE SOURCES

To be observed as individual sources, the amplitudes of the GWs from the binaries must exceed the threshold amplitude, \( h_{\text{th}} = 5h_{\Lambda}(\Delta v)^{1/2} \), which is determined by the GW background, \( h_{\Lambda} \), and the frequency resolution, \( \Delta v = 1/T \) (Schutz 1986; Thorne 1987). Here \( T \) is the observation time, and we set the signal-to-noise ratio (S/N) at \( 5 \).

4. DENSITY PROFILE RECONSTRUCTION

In this section we show one of our simulations of real observations. For simplicity, we assume that the distribution of the number density of PBH MACHOs obeys the law

\[
n(r) = \frac{n_2}{[1 + (r/D_{\text{pc}}^2)^{1/4}]^4},
\]

where \( n, n_2, D_{\text{pc}}, \) and \( \lambda \) are the Galactocentric radius, the number density of PBH MACHOs at the Galactic center, the core radius, and the power index, respectively. As the “real” parameters for a simulation, we set \( n_2 = 2.60 \times 10^{-2} \) pc\(^{-3}\), \( D_{\text{pc}} = 5 \) kpc, and \( \lambda = 1 \). The total number of PBH MACHOs within \( r < D_{\text{halo}} \) is given by \( N_{\text{total}} = \int_{r < D_{\text{halo}}} n(r) d^3x \), where \( D_{\text{halo}} \) is the size of the halo. Since the fraction of PBH MACHO binaries with \( (10^{-3} \text{ Hz} < v_{gw} < v_{gw}^\text{max}) \) is increased correspondingly.

5 For simplicity, we do not consider the effects of the inclination of binaries and a reduction factor due to the antenna pattern of the detector in detail. These effects can be absorbed in the observation time \( T \), and the conclusion of this paper will hold if the effective \( T \) is increased correspondingly.

6 The results of Cutler (1998) are only valid for large S/N, and the angular resolution may be worse in a realistic detection with S/N = 5 (Balasubramanian, Sathyaprakash, & Dhurandhar 1999), although here we simply adopt Cutler’s results. Note also that the relative velocities of the sources to the solar system are not taken into account in Cutler (1998).
in one experimental realization. We adopt distances from the Galactic center are within (i = 1, 2, ..., in one experimental realization, we adopt $\delta r = 1$ kpc. Here we set $T = 10$ yr and $D_{\text{obs}} = 50$ kpc, which corresponds to $v_{\text{obs}} = 9.63 \times 10^{-3}$ Hz. The fitted curve (solid line) and the “real” curve (dashed line) are also shown. The fitted parameters are $n_i/n_i^{\text{real}} = 0.779$, $D_a = 6.20$ kpc, and $\lambda = 1.04$, respectively, where $n_i^{\text{real}} = 2.60 \times 10^{-2}$ pc$^{-3}$ is the “real” value. The reduced $\chi^2$ is 0.913, with 47($=D_{\text{obs}}/\delta r - 3$) degrees of freedom.

Fig. 1.—Number $N_i$ of observed PBH MACHO binaries whose distances from the Galactic center are within $(i - 1)\delta r \leq r < i\delta r$ ($i = 1, 2, \ldots$) in one experimental realization. We adopt $\delta r = 1$ kpc. Here we set $T = 10$ yr and $D_{\text{obs}} = 50$ kpc, which corresponds to $v_{\text{obs}} = 9.63 \times 10^{-3}$ Hz. The fitted curve (solid line) and the “real” curve (dashed line) are also shown. The fitted parameters are $n_i/n_i^{\text{real}} = 0.779$, $D_a = 6.20$ kpc, and $\lambda = 1.04$, respectively, in the plane. The contours of constant $s_i$ and $\lambda$ are in the range $(i - 1)\delta r \leq r < i\delta r$ ($i = 1, 2, \ldots$). We adopt $\delta r = 1$ kpc to determine the structure within a few kpc from the Galactic center. Here we set $T = 10$ yr and $D_{\text{obs}} = 50$ kpc, which corresponds to $v_{\text{obs}} = 9.63 \times 10^{-3}$ Hz. In this realization, $N_i$ is obtained from $n_i = n_i/\lambda d_i/\delta r$, where $d_i = (\Delta x^2/r_\Omega)^{1/2}$, since the distribution of $N_i$ will follow the Poisson distribution with mean $n_i(n_i, D_a, \lambda)$, assuming that

\begin{align*}
\text{fit: } & D_a = 6.20 \text{ kpc}, \lambda = 1.04 \\
\text{real: } & D_a = 5 \text{ kpc}, \lambda = 1
\end{align*}

Fig. 3.—“Real” density profile, with $D_a = 5$ kpc and $\lambda = 1$ (dashed line), and the density profile fitted from Fig. 1, with $n_i/n_i^{\text{real}} = 0.779$, $D_a = 6.20$ kpc, and $\lambda = 1.04$ (solid line). These density profiles are normalized by $n_i^{\text{real}} = 2.60 \times 10^{-2}$ pc$^{-3}$.

The variances of $s_i$ and $\lambda$ may be estimated by $\sigma_i = \left[\sigma_i^{\text{fit}} - \sigma_i^{\text{real}}(n_i, D_a, \lambda)\right]^{1/2}$, where $\sigma_i^{\text{fit}} = \sigma_i^{\text{real}}(n_i, D_a, \lambda)$, and $\sigma_i^{\text{real}}(n_i, D_a, \lambda)$ is the observed variance. The variance, $\sigma_i$, of $N_i$ can be estimated by $\sigma_i = \left[\sigma_i^{\text{fit}} - \sigma_i^{\text{real}}(n_i, D_a, \lambda)\right]^{1/2}$, since the distribution of $N_i$ will follow the Poisson distribution with mean $n_i(n_i, D_a, \lambda)$, assuming that

Strictly speaking, we may have to maximize the probability for observing $N_i$ PBH MACHO binaries in the $i$th bin from the Poisson distribution, $P(n_i, D_a, \lambda) = \left[\prod_i \left(n_i(n_i, D_a, \lambda)\right)^{N_i}e^{-n_i(n_i, D_a, \lambda)}\right]/\prod_i N_i$, instead of minimizing $\chi^2$. However, since almost all $N_i$ are larger than 10, it will be a reasonable assumption that the shape of the Poisson distributions governing the fluctuations is nearly Gaussian.
the statistical uncertainty dominates the instrumental uncertainty due to the observational errors. For this realization, the fitted parameters turn out to be $n_j/n_j^\text{real} = 0.779$, $D_\lambda = 6.20$ kpc, and $\lambda = 1.04$, where $n_j^\text{real} = 2.60 \times 10^{-2}$ pc$^{-3}$ is the “real” value. The reduced $\chi^2$ is 0.913, with $47(=D_{\text{obs}}/\delta r - 3)$ degrees of freedom. In Figure 2, the “real” parameter and the fitted parameter are marked with a filled square and a cross, respectively, in the $D_\lambda$-$\lambda$ plane. The contours of constant $\Delta \chi^2$ are also plotted with $\Delta \chi^2 = 1.00$, 2.30, 4.00, and 6.17. Figure 3 shows the “real” HDP and the reconstructed HDP normalized by $n_j^\text{real}$. It seems that in our method, the HDP is reconstructed quite well, except for the central region.

We performed $10^4$ simulations of observations with and without the instrumental error to obtain the probability distributions of the core radius, $D_\lambda$, and the power index, $\lambda$. The mean values, $\langle w \rangle$, and the dispersions, $\Delta w = (\langle w^2 \rangle - \langle w \rangle^2)^{1/2}$, of these parameters $w$ with (or without, values given in parentheses) the instrumental error are shown in Table 1. From Table 1, we find that the instrumental error does not affect the results very much. The probabilities that these parameters $w$ are within $|w - \langle w \rangle| < \Delta w$ and $2\Delta w$ turn out to be about 70% and 95%, respectively, from these realizations. Although the power index $\lambda$ is determined within 10% (20%) error at the 89% (99.7%) confidence level (CL), respectively, by 10 yr of observation, the dispersion of the core radius $D_\lambda$ is somewhat large, 25% (50%) error in the 63% (93%) CL.

After we have found the power index $\lambda$ accurately by this global observation, we can analyze the HDP for shorter distances, $r < D_{\text{obs}} < D_{\text{obs}}$, using the PBH MACHO binaries with lower frequencies, $v_{\text{gw}} > v_{\text{obs}}$, where $v_{\text{obs}}(<v_{\text{obs}})$ is determined by $D_{\text{obs}}^{(2)}(v_{\text{obs}}) = D_{\text{obs}} + 5D_\lambda$, from equation (2). For example, for $T = 10$ yr and $D_{\text{obs}} = 10$ kpc, which corresponds to $v_{\text{obs}} = 5.15 \times 10^{-3}$ Hz, the mean value and the dispersion of $D_\lambda$ are found to be $\langle D_\lambda \rangle = 4.81$ kpc and $\Delta D_\lambda = 0.710$ kpc from $10^4$ realizations with $\delta r = 0.5$ kpc and $\lambda = 1$. The dispersion, $\Delta D_\lambda$, is reduced by a factor of 0.5. Then the core radius $D_\lambda$ is determined within 25% (50%) error in the 91% (99.8%) CL.

5. DISCUSSION

In this paper we have quantitatively investigated how well the HDP consisting of PBH MACHOs can be determined by observation of the low-frequency GWs, assuming the spherical HDP given in equation (3). We have found that 10 yr of observation by LISA should be able to determine $\lambda$, the power index of the HDP, within 10% (20%) error, and $D_\lambda$, the core radius, within 25% (50%) error in about 90% (99%) CL, respectively.

The halo of our Galaxy may be nonspherical (e.g., Olling & Merrifield 1998a). For a nonspherical halo, if we calculate quadrupole moments of positions of PBH MACHO binaries, we can determine axial ratios ($c/a$ and $b/a$) of the dark halo (for details, see Dubinski &Carlberg 1991). Since about 700 PBH MACHO binaries can be provided by 10 yr of observation, errors in the axial ratios are estimated to be less than 10% if the axial ratios are less than 0.8.

We have assumed that MACHOs are PBHs. However, they may be white dwarfs, or some other compact objects (e.g., Freese et al. 1999). For such cases, it might not be so strange to expect that some of them are also binaries. If a fraction ($\sim 10^{-6}$) of them are in binary systems emitting GWs in the frequency range of $10^{-3} \lesssim v_{\text{gw}} \lesssim 10^{-1}$ Hz, arguments similar to the one in this paper will hold even for non–black hole MACHOs (see also Bond & Carr 1984).

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| $T$ (yr) | $v_{\text{gw}}$ (mHz) | $v_{\text{obs}}$ (mHz) | $\langle N_{\text{map}} \rangle^*$ | $\langle D_\lambda \rangle \pm \Delta D_\lambda$ (kpc) | $\langle \lambda \rangle \pm \Delta \lambda$ |
|---------|----------------|----------------|----------------|---------------------------------|-----------------|
| 2 ...... | 4.08          | 15.1           | 217 (214)      | $4.60 \pm 2.44 (4.68 \pm 2.43)$ | $1.00 \pm 0.114 (1.01 \pm 0.116)$ |
| 4 ...... | 2.80          | 12.4           | 367 (363)      | $4.57 \pm 1.90 (4.67 \pm 1.90)$ | $0.992 \pm 0.0886 (1.00 \pm 0.0894)$ |
| 6 ...... | 2.24          | 11.1           | 496 (492)      | $4.62 \pm 1.64 (4.71 \pm 1.64)$ | $0.992 \pm 0.0763 (1.00 \pm 0.0771)$ |
| 8 ...... | 1.92          | 10.2           | 614 (608)      | $4.65 \pm 1.47 (4.74 \pm 1.46)$ | $0.991 \pm 0.0684 (0.999 \pm 0.0689)$ |
| 10 ...... | 1.70          | 9.63           | 723 (716)      | $4.67 \pm 1.35 (4.76 \pm 1.34)$ | $0.990 \pm 0.0626 (0.999 \pm 0.0632)$ |

Note.—Numbers in parentheses indicate values without instrumental error.

* Minimum frequency of the binaries to which LISA can measure distances.

* Minimum frequency of the binaries that we can use to determine the density profile within $r < D_{\text{obs}} = 50$ kpc.

* Mean number of PBH MACHO binaries that can be used to determine the density profile.
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