Structure Scalars in Charged Plane Symmetry

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Abstract

We consider non-adiabatic flow of the fluid possessing dissipation in the form of shearing viscosity in electromagnetic field. The scalar functions (structure scalars) for charged plane symmetry are formulated and are related with the physical variables of the fluid. We also develop a relationship between the Weyl tensor and other physical variables by using Taub mass formalism. The role of electric charge as well as its physical significance for the evolution of the shear tensor and expansion scalar are also explored. Finally, we discuss a special case for dust with cosmological constant.

Keywords: Structure scalars; Relativistic dissipative fluids; Electromagnetic field; Plane symmetry.

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1 Introduction

Self-gravitation is a process through which different components of a large body are binded together. In spite of it, stars, stellar clusters, galaxies and clusters of galaxies would all expand and dissipate. Self-gravitating fluids have attracted many people due to their applications in general relativity,

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astrophysics and cosmology. These are usually characterized by the set of physical variables. Many structures have been identified which produce pressure anisotropy in stellar models and make the fluid imperfect. The effect of local anisotropy is usually taken into consideration under static conditions [1].

The inclusion of an electromagnetic field has shown interesting outcome in gravitational collapse. Bekenstein [2] was the first who explored the Oppenheimer-Volkoff equations of hydrostatic equilibrium [3] from the neutral to the charged case. In the discussion of charged gravitational collapse, Nath et al. [4] concluded that electromagnetic field increases the formation of naked singularity. Sharif and Abbas [5] analyzed the effect of electromagnetic field on gravitational collapse with positive cosmological constant and found two apparent horizons (cosmological and black hole) whose area decreases in the presence of electromagnetic field. Further, they [6] extended this study from four dimensions to five dimensions in the presence of electromagnetic field and examined that the electromagnetic field reduces the pressure but favors the formation of naked singularity.

The same authors [7] discussed the gravitational collapse for cylindrical symmetry with charged perfect fluid. Sharif and Siddiqa [8] carried out the consequences of charge and dissipation for plane symmetrical gravitational collapse of real fluid. The dynamical and transport equations are coupled to check the effects of dissipation over collapsing process. Sharif and Fatima [9] discussed the coupled dynamical and transport equation for cylindrically symmetric collapsing process. Moreover, they formulated a relationship between the Weyl tensor and energy-density inhomogeneity. Rosales et al. [10] pointed out that electric charge plays the same role of anisotropy in the collapse [11], when the tangential pressure is greater than radial pressure.

Herrera et al. [12] figured out a systematic study of spherically symmetric self-gravitating relativistic fluids, based on the scalar functions (structure scalars) derived from the orthogonal splitting of the Riemann tensor. Also, Herrera et al. [13] provided a study on two aspects of Lemaitre-Tolman-Bondi (LTB) spacetimes and gave an alternatives for obtaining spherically symmetric dust solutions. Moreover, Herrera et al. [14] investigated the stability of shear free condition based on the evolution equation of the shear tensor and found that the major role is played by the scalar \( Y_{TF} \). Recently, Herrera et al. [15] provided a detailed study based on the structure and evolution of self-gravitating relativistic fluids through structure scalars. These functions are based on the orthogonal splitting [16] of the Riemann tensor in general
relativity which are denoted by $X_T, X_{TF}, Y_T, Y_{TF}$. The role of electric charge and cosmological constant on structure scalars is also analyzed for spherically symmetric spacetime [17]. In a recent paper, we have investigated the structure scalars for the charged cylindrically symmetric spacetime [18].

This paper develops structure scalars for the charged plane symmetric spacetime. The format is as follows. Section 2 is devoted for the discussion of fluid distribution and formulate the Einstein-Maxwell field equations. We also find a relation between the Weyl tensor and energy density inhomogeneity. Section 3 contains the discussion of structure scalars. In section 4, we establish the structure scalars for the dust case with cosmological constant in the absence of dissipation and viscosity. Finally, we summarize the results in the last section.

2 Fluid Distribution and the Field Equations

We consider a general non-rotating plane symmetric distribution of collapsing fluid whose line element is given by

$$ds^2 = -A^2(t,z)dt^2 + B^2(t,z)\left(dx^2 + dy^2\right) + C^2(t,z)dz^2. \quad (1)$$

We assume a fluid distribution which is locally anisotropic and suffering dissipation in the form of shearing viscosity, heat flow and free streaming radiation. Its energy-momentum-tensor is

$$T_{\alpha\beta} = (\mu + P_\perp)V_\alpha V_\beta + P_\perp g_{\alpha\beta} + (P_z - P_\perp)\chi_\alpha \chi_\beta + q_\alpha V_\beta + V_\alpha q_\beta + \epsilon_\alpha l_\beta - 2\eta \sigma_{\alpha\beta}, \quad (2)$$

where $\mu, P_\perp, V_\alpha, q_\alpha, \epsilon, \eta$ and $\sigma_{\alpha\beta}$ are the energy density, tangential pressure, four velocity, heat flux, radiation density, coefficient of shear viscosity and the shear tensor, respectively. Also, $P_z, \chi_\alpha$ and $l^\alpha$ are pressure, unit four-vector and the null four-vector in the $z$-direction, respectively. We assume the fluid to be comoving so that Eq.(1) satisfies

$$V_\alpha V^\alpha = -1, \quad \chi_\alpha \chi^\alpha = 1, \quad \chi_\alpha V^\alpha = 0,$$

$$V_\alpha q_\alpha = 0, \quad l^\alpha V_\alpha = -1, \quad l^\alpha l_\alpha = 0. \quad (4)$$
The energy-momentum tensor in electromagnetic field is

\[ S_{\alpha\beta} = \frac{1}{4\pi} \left( F^\gamma_{\alpha} F_{\beta\gamma} - \frac{1}{4} F^{\gamma\delta} F_{\gamma\delta g_{\alpha\beta}} \right), \tag{5} \]

where \( F_{\alpha\beta} = \phi_{\beta,\alpha} - \phi_{\alpha,\beta} \) is the Maxwell field tensor and \( \phi_{\alpha} \) is the four potential. The Maxwell field equations are

\[ F^\alpha_{\iota\beta} = \mu_0 J^\alpha, \quad F_{[\alpha\beta;\gamma]} = 0, \tag{6} \]

here \( \mu_0 = 4\pi \) is the magnetic permeability and \( J^\alpha \) is the four current. In comoving coordinates, we have

\[ \phi_{\alpha} = \phi_0^\alpha, \quad J^\alpha = \xi V^\alpha, \]

where \( \phi, \xi \) represent the scalar potential and the charge density, respectively, both are functions of \( t \) and \( z \). The charge conservation equation, \( J^\alpha_{;\alpha} = 0 \), yields

\[ s(z) = \int_0^z \xi C B^2 dz. \]

Equation (6) for \( \alpha = 0 \) with \( \phi'(t, 0) = 0 \) gives

\[ \phi' = \frac{\mu_0 s(z) A C}{B^2}. \tag{7} \]

The Einstein field equations in electromagnetic field lead to

\[ 8\pi (\mu + \epsilon) A^2 + \left( \frac{\mu_0 s}{B^2} \right)^2 = \frac{\dot{B}}{B} \left( \frac{2\dot{C}}{C} + \frac{\dot{B}}{B} \right) - \left( \frac{A}{C} \right)^2 \left[ \frac{2B''}{B} \right], \tag{8} \]

\[ -8\pi (q + \epsilon) A C = -2 \left( \frac{\dot{B}}{B} - \frac{A' \dot{B}}{AB} - \frac{B' \dot{C}}{BC} \right), \tag{9} \]

\[ 8\pi (P_\perp + \frac{2}{3} \eta F) B^2 + \left( \frac{\mu_0 s}{B} \right)^2 = - \left( \frac{B}{A} \right)^2 \left[ \frac{\dot{B}}{B} - \frac{\dot{C}}{C} - \frac{A'}{A} \left( \frac{\dot{B}}{B} \right) + \frac{\dot{B} \dot{C}}{BC} \right] + \left( \frac{B}{C} \right)^2 \left[ \frac{A''}{A} + \frac{B''}{B} - \frac{A'}{A} \left( \frac{C'}{C} - \frac{B'}{B} \right) - \frac{B' C''}{BC} \right]. \tag{10} \]
\[ 8\pi (P_z + \epsilon - \frac{4}{3} \eta F)C^2 - \left( \frac{\mu_0 sC'}{B^2} \right)^2 = - \left( \frac{C}{A} \right)^2 \left[ \frac{2\dot{B}}{B} + \left( \frac{\dot{B}}{B} \right)^2 \right] - \frac{2\dot{A}\dot{B}}{AB} + \left( \frac{\dot{B}'}{B} \right)^2 + \frac{2A'B'}{AB}, \]  

(11)

where dot and prime stand for differentiation with respect to \( t \) and \( z \), respectively.

We can describe the non-rotating fluid by the kinematical variables, i.e., expansion, acceleration and shear as

\[ \Theta = V^\alpha_{;\alpha}, \quad a_\alpha = V_{\alpha;\beta}V^\beta, \quad \sigma_{\alpha\beta} = V_{(\alpha;\beta)} + a_{(\alpha}V_{\beta)} - \frac{1}{3} \Theta h_{\alpha\beta}, \]

where \( h_{\alpha\beta} = g_{\alpha\beta} + V_{\alpha}V_{\beta} \) is the projection tensor. Using Eq. (11), these quantities turn out to be

\[ \Theta = \frac{1}{A} \left( \frac{2\dot{B}}{B} + \frac{\dot{C}}{C} \right), \quad a_3 = \frac{A'}{A}, \quad a^\alpha = a\chi^\alpha, \quad a^2 = a^\alpha a_\alpha = (\frac{A'}{AC})^2. \]

(12)

The magnitude of the shear tensor is defined as

\[ \sigma^2 = \frac{1}{2} \sigma_{\alpha\beta} \sigma^{\alpha\beta} = \frac{1}{9} F^2, \]

where

\[ F = \frac{1}{A} \left( \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right). \]

(13)

The Weyl tensor \( (C_{\alpha\mu\beta\nu}) \) can be broken into electric and magnetic parts. However, the magnetic part vanishes for plane symmetry, while the electric part is defined as

\[ E_{\alpha\beta} = C_{\alpha\mu\beta\nu}V^\mu V^\nu, \]

whose non-vanishing components are

\[ E_{11} = \frac{1}{3} B^2 \varepsilon = E_{22}, \quad E_{33} = -\frac{2}{3} C^2 \varepsilon, \]

where

\[ \varepsilon = -\frac{1}{2A^2} \left[ \frac{\dot{C}}{C} - \frac{\dot{B}}{B} - \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}^2}{B^2} + \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} \right] - \frac{1}{2C^2} \left[ \frac{A'C'}{AC} + \frac{A'B'}{AB} - \frac{B^2}{B^2} - \frac{B'C'}{BC} + \frac{B''}{B} - \frac{A''}{A} \right]. \]

(14)
The electric part can also be written as

\[ E_{\alpha\beta} = \varepsilon(\chi_{\alpha}\chi_{\beta} - \frac{1}{3}h_{\alpha\beta}). \]

The mass function introduced by Taub [19] in the presence of electric charge is given as

\[ m(t, z) = \frac{B}{2} \left( \frac{\dot{B}^2}{A^2} - \frac{B'^2}{C^2} \right) + \frac{(s\mu_0)^2}{2B}. \]  

(15)

Using the field equations (8), (10), (11) and (15) in Eq.(14), we obtain

\[ \varepsilon = 4\tilde{\mu}\pi + 4\pi(2\eta F - \Pi) + \frac{6\mu_0^2s^2}{2B^4} - \frac{3m}{B^3}, \]

(16)

where \( \Pi = \tilde{P}_z - P_\perp, \tilde{P}_z = P_z + \epsilon, \tilde{\mu} = \mu + \epsilon. \) Differentiating Eq.(15) with respect to \( z \) and then using the field equations (8) and (9), it follows that

\[ m' = 4\pi \left( \tilde{\mu} + \frac{\tilde{q}U}{E} \right) B'B^2 + \frac{(\mu_0s)^2B'}{2B^2}, \]

(17)

where \( U = \frac{\dot{B}}{A} \) is called the areal velocity of the collapsing fluid, \( \tilde{q} = q + \epsilon \) and \( E = \frac{B'}{C}. \) Integrating this equation, after some simplification, we obtain

\[ \frac{3m}{B^3} = 4\pi\tilde{\mu} - \frac{4\pi}{B^3} \int_0^z B^3\tilde{\mu}'dz + \frac{4\pi}{B^3} \int_0^z 3B^2UC\tilde{q}dz + \frac{3\mu_0^2}{B^3} \left[ \frac{s^2}{2B} + \int_0^z \frac{B'}{2B^2}s^2d\tilde{q}dz \right]. \]

(18)

It provides a relationship between mass function and fluid properties such as energy density, heat flux and electric charge. Substituting this value in Eq.(16), it follows that

\[ \varepsilon = 4\pi(2\eta F - \Pi) + \frac{3\mu_0^2s^2}{2B^4} + \frac{4\pi}{B^3} \int_0^z B^3\tilde{\mu}'dz - \frac{4\pi}{B^3} \int_0^z 3B^2UC\tilde{q}dz \]

\[ - \frac{3\mu_0^2}{B^3} \int_0^z \frac{B'}{2B^2}s^2d\tilde{q}dz. \]

(19)

This shows that the Weyl tensor depends on energy density inhomogeneity and local anisotropy of pressure.
3 Structure Scalars for the Charged Fluid

In this section, we formulate structure scalars from the orthogonal splitting of the Riemann tensor. For this purpose, we define the following tensors

\[ Y_{\alpha\beta} = R_{\alpha\gamma\beta\delta}V^{\gamma}V^{\delta}, \quad X_{\alpha\beta} = R^*_{\alpha\gamma\beta\delta}V^{\gamma}V^{\delta} = \frac{1}{2}\eta^\varepsilon_{\alpha\gamma}R^*_{\varepsilon\rho\beta\delta}V^{\gamma}V^{\delta}, \]

where \( R^*_{\alpha\beta\gamma\delta} = \frac{1}{2}\eta^\varepsilon_{\rho\gamma}R^*_{\alpha\beta\rho\delta}. \) The tensors \( Y_{\alpha\beta} \) and \( X_{\alpha\beta} \) can be decomposed in its trace and trace free components as

\[
Y_{\alpha\beta} = \frac{1}{3}Y_{TF}h_{\alpha\beta} + \frac{1}{3}Y_{TF}h_{\alpha\beta}, \quad X_{\alpha\beta} = \frac{1}{3}X_{TF}h_{\alpha\beta} + \frac{1}{3}X_{TF}h_{\alpha\beta}.
\] (20)

Using the field equations (8), (10), (11) and (14), the above components take the form

\[
Y_{TF} = 4\pi\left(\tilde{\mu} + 3\tilde{P}_z - 2\Pi\right) + \frac{\mu_0^2 s^2}{B^4},
\]

\[
X_{TF} = 8\pi\tilde{\mu} + \frac{\mu_0^2 s^2}{B^4},
\]

\[
Y_{TF} = \varepsilon - 4\pi (\Pi - 2\eta F) + \frac{\mu_0^2 s^2}{B^4},
\]

\[
X_{TF} = -\varepsilon - 4\pi (\Pi - 2\eta F) + \frac{\mu_0^2 s^2}{B^4}.
\] (21)

Substituting Eq. (19) in (21), it follows that

\[
Y_{TF} = -8\pi\Pi + 16\eta\Pi F + \frac{5s^2\mu_0^2}{2B^4} + 4\pi\int_0^z B^3 \left(\tilde{\mu} - \frac{3qUC}{B}\right)dz
\]

\[
- \frac{3\mu_0^2}{2B^4} \int_0^z s^2B'dz,
\]

\[
X_{TF} = -\frac{s^2\mu_0^2}{2B^4} - \frac{4\pi}{B^3} \int_0^z B^3 \left(\tilde{\mu} + \frac{3qUC}{B}\right)dz + \frac{3\mu_0^2}{2B^3} \int_0^z s^2B'dz,
\] (22)

which describes the density inhomogeneity and local anisotropy of the fluid.
For the sake of convenience, we can introduce the following effective variables as

\[-(T_0^0 + S_0^0) = \mu_{\text{eff}} = \bar{\mu} + \frac{\mu_0^2 s^2}{8\pi B^4},\]

\[(T_1^1 + S_1^1) = P_z^{\text{eff}} = \left(\tilde{P}_z - \frac{4}{3}\eta F\right) - \frac{\mu_0^2 s^2}{8\pi B^4},\]

\[(T_2^2 + S_2^2) = P_\perp^{\text{eff}} = \left(P_\perp + \frac{2}{3}\eta F\right) + \frac{\mu_0^2 s^2}{8\pi B^4},\]

\[P_z^{\text{eff}} - P_\perp^{\text{eff}} = \Pi^{\text{eff}} = \left(\tilde{P}_z - P_\perp\right) - 2\eta F - \frac{\mu_0^2 s^2}{8\pi B^4},\]

\[\Pi^{\text{eff}} = \Pi - 2\eta F - \frac{\mu_0^2 s^2}{8\pi B^4}.\]

These equations show that the effective variables have the resemblance with the ordinary variables from all the contributions (viscosity and electric charge). In the light of above effective variables, the structure scalars become

\[X_T = 8\pi \tilde{\mu}_{\text{eff}},\]

\[Y_T = 4\pi \left(\tilde{\mu}_{\text{eff}} + 3\tilde{P}_z - 2\Pi^{\text{eff}}\right),\]

\[Y_{TF} = -8\pi \Pi^{\text{eff}} + \frac{4\pi}{B^3} \int_{0}^{z} B^3 \left(\mu'_{\text{eff}} - \frac{3\tilde{q}UC}{B}\right) dz,\]

\[X_{TF} = -\frac{4\pi}{B^3} \int_{0}^{z} B^3 \left(\tilde{\mu}'_{\text{eff}} - \frac{3\tilde{q}UC}{B}\right) dz.\]

We see that the charge contribution is present in the effective variables. In the absence of electric charge, the structure scalars are directly obtained from the above equations by replacing the effective variables with the ordinary ones.

In order to understand the physical significance of the electric charge in structure scalars, we use Raychaudhuri equation which gives the evolution of expansion and the shear. In the absence of dissipation, \(X_{TF}\) controls inhomogeneities in the energy density. Moreover, it provides a differential equation which yields the inhomogeneity factor and a relationship between the Weyl tensor as well as other physical variables. The evolution equation for expansion leads to

\[-Y_T = V^\alpha \Theta_{;\alpha} + \frac{1}{3} \Theta^2 + \sigma^{\alpha\beta} \sigma_{\alpha\beta} - a^\alpha_{;\alpha},\]
The evolution equation for shear becomes
\[ Y_{TF} = \chi^\alpha a_{;\alpha} + a^2 - \frac{a B'}{BC} - V^\alpha F_{;\alpha} - \frac{2}{3} F\Theta - \frac{1}{3} F^2. \] (26)

We see that the evolution equations for expansion and shear are independent of charge contribution. This implies that the electric charge does not play any role in these two equations. Moreover, these equations turn out to be the same as in spherically symmetric spacetime. Finally, the differential equation for the Weyl tensor and energy density inhomogeneity can be written as
\[ (X_{TF} + 4\pi \mu_{eff})' = -X_{TF} \frac{3B'}{B} + 4\pi \tilde{q}C(\Theta - F), \] (27)

which gives \(X_{TF}\) as the inhomogeneity factor. This corresponds to the charged spherically symmetric and also non-charged case by replacing the effective energy density with the ordinary one.

### 4 Structure Scalars for Dust with Cosmological Constant

In the dust collapse, the role of density inhomogeneities [20], especially in the formation of naked singularities has been extensively discussed in the literature. Eardley and Smarr [21] discussed the inhomogeneous generalization of the Oppenheimer-Snyder spherical dust collapse and found naked singularities if the collapse is sufficiently inhomogeneous. Waugh and Lake [22] found the necessary conditions for the formation of the naked singularities. Joshi and Dwivedi [23] investigated the occurrence and nature of a naked singularity for the inhomogeneous gravitational collapse for spherical symmetry. Moreover, they [24] discussed the structure of naked singularities. The energy-density inhomogeneity also occurs due to the presence of dissipation [25].

Here, we consider a special case of dust with non-vanishing cosmological constant. Also, we take dissipation as well as shear effects zero. In this case, the energy-momentum tensor takes the form
\[ T_{\alpha\beta} = 8\pi \mu V_\alpha V_\beta, \] (28)

and the field equation has the form
\[ G_{\alpha\beta} = T_{\alpha\beta} - \Lambda g_{\alpha\beta}, \] (29)
where $\Lambda$ is the cosmological constant. When $\Lambda$ is negative, the universe shows contraction and inhomogeneity will increase otherwise it will decrease. In comoving coordinate system, the fluid represents geodesic behavior and $A'$ becomes zero. Further, the re-scaling of the time coordinate gives $A = 1$. Consequently, the mass function reduces to

$$m = 4\pi \int_0^z \mu B' B^2 dz + \frac{\Lambda}{6} B^3. \quad (30)$$

After some manipulations, the mass function and $\varepsilon$ for dust fluid with cosmological constant become

$$\frac{3m}{B^3} = 4\pi \mu + \frac{\Lambda}{2} - \frac{4\pi}{B^3} \int_0^z \mu' dz,$$

$$\varepsilon = \frac{4\pi}{B^3} \int_0^z B^3 \mu' dz. \quad (31)$$

In this case, the scalar functions take the following form

$$Y_T = 4\pi \mu - \Lambda, \quad Y_{TF} = \varepsilon, \quad X_T = 8\pi \mu - \Lambda, \quad X_{TF} = \varepsilon. \quad (32)$$

Also, the evolution equations for the expansion and shear become

$$V^\alpha \Theta_{\alpha} + \frac{1}{3} \Theta^2 + \frac{2}{3} F^2 - a'^2 = -4\pi \mu + \Lambda = -Y_T,$$

$$-V^\alpha F_{;\alpha} - \frac{2}{3} \Theta F - \frac{1}{3} F^2 = \varepsilon = Y_{TF},$$

respectively. The differential equation for the inhomogeneity factor can be written as

$$(X_{TF} + 4\pi \mu)' = -X_{TF} \frac{3B'}{B}. \quad (33)$$

It follows from here that $\mu' = 0$ if and only if $X_{TF} = 0$, indicating $X_{TF}$ as the inhomogeneity factor.

5 Summary

We have investigated a set of scalar functions corresponding to the charged plane symmetric distribution. We have found that $X_T$ corresponds to the energy density of the fluid with the contribution of electric charge. In the
absence of dissipation, $X_{TF}$ controls the energy density inhomogeneity with the passage of time. Also, $Y_T$ turns out to be the mass density while $Y_{TF}$ have the interaction of both the energy density inhomogeneity and local anisotropy. It is noted that the evolution of the shear and expansion have the same contribution to $Y_T$ and $Y_{TF}$ as in the spherically symmetric case. Also, it has the same effect on the inhomogeneity factor. For the dust case with cosmological constant, it is found that evolution of the expansion scalar is affected by the term $\Lambda$ in $Y_T$, while evolution of the shear and inhomogeneity factor remains the same. In this case, the relevant factor for inhomogeneity is the Weyl tensor and stability of the homogeneous energy density is equivalent to the stability of the conformal flatness.

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