Abstract

We consider a cosmic ray spectrum that is a power law in momentum down to a cutoff and derive a lower cutoff corresponding to $E_{\text{kin}} \sim (30 - 60)$ MeV from the observed ionization rates in nearby diffuse clouds. While the real spectra of cosmic rays may not be so simple, we argue that one expects a substantial change in the spectra at such energies and that, at first approximation, a power law spectra with a lower cutoff is appropriate. Such a description of the cosmic rays in the interstellar medium is not only theoretically more attractive than the spectra used in the literature, but is also supported by recent observations.

Keywords: Interstellar medium : cosmic rays.

Introduction

The importance of cosmic rays in heating the interstellar medium (ISM) has been recognized since long. Hayakawa, Nishimura and Takayanagi (1961) calculated the flux of low-energy cosmic rays required to account for a kinetic temperature of 125 K, employing a power law in energy that is observed at high energies and extrapolating it down to $\sim 10$ MeV. Spitzer and Tomasko (1968) reconsidered the problem in the light of observations of lesser temperatures ($\sim 60$ K) and calculated the ionization rate for the demodulated
spectrum of cosmic rays to be $6.8 \times 10^{-18}$ sec$^{-1}$. They determined a heating rate of $\sim 5.7$ eV per ionization event. Field \textit{et al.} (1969) assumed a cosmic ray energy density that corresponds to an ionization rate of $4 \times 10^{-16}$ sec$^{-1}$.

However, on one hand, recent observations of abundances in the ISM point to an ionization rate much different from the numbers above. Black \textit{et al.} (1990) and van Dishoeck and Black (1991) recently inferred an ionization rate of $(3 - 7) \times 10^{-17}$ sec$^{-1}$ from observations of nearby diffuse clouds. The dominant contributions to the ionization, especially for the abundances of species such as H$_3^+$, are expected to come from low energy cosmic rays. On the other hand, theoretical advances in the field of cosmic ray acceleration have shown that the demodulated cosmic ray spectrum, that pervades the ISM, is most probably a power law in momentum ($p$) (Krymsky 1977; Axford \textit{et al.} 1977; Bell 1978; Blandford and Ostriker 1978). This is very different from the spectrum used by Spitzer and Tomasko (1968) or Field \textit{et al.} (1969). We assume here that the cosmic ray spectrum is a power law in momentum and that any substantial change at the lower end can be approximated as a lower cutoff. Then, the ionization and heating will depend on (a) the spectral index, (b) the amplitude and (c) the lower cutoff. Furthermore, if one uses the spectral index and the amplitude as one observes for the high energy tail, then the only parameter that remains to be determined is the lower cutoff. The lower cutoff for different ions will be different, depending on the energy losses as they traverse the ISM.

The description of the low energy cosmic ray spectrum in terms of a single lower cutoff is ad hoc but is the simplest. In reality, due to various energy loss processes, the spectrum will deviate from the power law at some energy scale. However we still lack any detail knowledge of the spectrum at energies below a few hundred MeV. We therefore assume a lower cutoff to describe the characteristic energy scale at which the spectrum changes from the power law in momentum.

In this paper we propose to describe the ionization and the heating of the ISM with a simple but theoretically plausible cosmic ray spectrum. We infer the lower cutoff by calculating the ionization rate and comparing with the observations. The lower cutoff that we infer could be important in understanding and predicting possible heating of gas in other galaxies, and the intergalactic gas in the vicinity of our Galaxy (Nath and Biermann 1993).

**Ionization by energetic particles**

\textit{(a) Ionization cross-section}

The cross-section for ionization ($\sigma$) of hydrogen atom in the ground state by an energetic particle with velocity $v = c\beta$ and with atomic number $Z$ is given by (Bethe 1933)

$$\sigma = \frac{1.23 \times 10^{-20} Z^2}{\beta^2} \left(6.2 + \log_{10} \frac{\beta^2}{1 - \beta^2} - 0.43\beta^2\right) \text{cm}^2.$$  \hspace{1cm} (1)
The effect of secondary electrons that are released by the initial ionization events has been dealt with by Dalgarno and Griffing (1958) and Spitzer and Scott (1969). The first generation electrons give rise to additional ionization and more electrons. For nearly neutral gas or whose fractional ionization \( f \) is small, the total number of secondary electrons produced in \( \frac{5}{3} \) times the number of electrons produced by ‘first generation’ ionization. This number is a function of \( f \) and drops down to 1.12 for \( f \sim 0.3 \). However, since \( f \) is small in diffuse clouds, we will use the value \( \frac{5}{3} \).

The ionization rate of H atoms due to the cosmic rays then could be written as

\[
\zeta = \frac{5}{3} \int 4\pi n(p)\sigma dp, \tag{2}
\]

where \( n(p)dp \) is the intensity of cosmic rays per unit area per second per unit of solid angle with momentum within the range \( dp \). This is essentially Spitzer and Tomasko’s (1968) eq. (4), however, with a differential spectrum in momentum and not in energy as they used.

(b) Cosmic rays

As mentioned in the Introduction, theoretical attempts to explain the acceleration of cosmic rays in shocks have shown that the resultant spectrum is one in momentum, \( p \) (Krymsky 1977; Axford et al.1977; Bell 1978; Blandford and Ostriker 1978). Biermann a, b), Biermann and Cassinelli (1993), Biermann and Strom (1993), Stanev et al. (1993), Rachen and Biermann (1993), and Rachen et al. (1993) has discussed the theoretical spectrum and compared it with airshower data. They calculate an injection spectrum of \( p^{-2.42\pm0.04} \) from the idea that cosmic rays up to about 10 TeV (particle energy for protons) are dominated by supernova explosions in the ISM and beyond that by supernova explosions into stellar winds (till EeV) and radio galaxies (beyond EeV). The observed \( p^{-2.75\pm0.04} \) spectrum below 10 TeV is explained by arguing that the leakage time due to diffusion in the ISM, which has a Kolmogorov spectrum of turbulence, changes the index.

It is possible that there are other sources of low energy cosmic rays in the ISM. However, the available data seems to be adequately explained by the supernova explosions (see the references above, especially, Biermann and Strom 1993). In the present work, we adopt a minimalist approach and assume the supernova explosions to be the underlying mechanism for low energy cosmic rays, which suggest a spectrum with a power law in momentum.
The observed amplitudes of the cosmic rays of different elements at high energy have been reported as (Weibel 1992),

\[
\phi_H = (9.81 \pm 0.32) \times 10^{-2} \left( \frac{E}{\text{TeV}} \right)^{-2.74 \pm 0.02}
\]
\[
\phi_{He} = (6.03 \pm 0.19) \times 10^{-2} \left( \frac{E}{\text{TeV}} \right)^{-2.61 \pm 0.03}
\]
\[
\phi_C = (1.09 \pm 0.15) \times 10^{-2} \left( \frac{E}{\text{TeV}} \right)^{-2.69 \pm 0.03}
\]
\[
\phi_O = (1.87 \pm 0.08) \times 10^{-2} \left( \frac{E}{\text{TeV}} \right)^{-2.64 \pm 0.04}
\]
\[
\phi_{Fe} = (1.82 \pm 0.016) \times 10^{-2} \left( \frac{E}{\text{TeV}} \right)^{-2.63 \pm 0.05}
\]

(3)

where the unit of \( \phi \) is \((\text{m}^2 \text{s sr TeV/nucleus})^{-1}\). These spectral data are consistent with the possibility that at TeV energies He-Fe spectra are all dominated by wind supernovae, with spectra like \( E^{-8/3} \), (Biermann 1993a). The subscripts H, He, C, O and Fe refer to the ions of hydrogen, helium, carbon, oxygen and iron respectively.

(c) Lower cutoff

As the cosmic ray particles travel through the IGM to us (i) the spectrum index is changed as mentioned above and (ii) the low energy particles are deleted from the spectra due to various losses. At energies below \( \sim 100 \) MeV, the important mechanisms for energy loss are due to ionization and spallation of the nuclei. The total fragmentation cross-section of a nucleus (atomic weight \( A_T > 1 \)) by a beam of protons for can be approximated as (Berezinsky et al. 1990),

\[
\sigma_{sp} = 2 \times 10^{-25} (A_T^{3/4} - 0.7)^2 \text{ cm}^2.
\]

(6)

We compare the energy losses of He nuclei due to ionization and spallation in fig. 1, in the units of per second per \( n_{HI,ISM} \), the hydrogen atom density in the ISM (for ionization), and per \( n_{p,CR} \), cosmic ray proton density (for spallation). In reality, of course, \( n_{HI,ISM} \gg n_{p,cr} \). It is clear that for \( E_{kin} < 100 \) MeV, ionization losses outweigh the losses due to spallation.

Since the energy loss depends on the atomic number, we expect the lower cutoffs of various ions to be different. We can estimate the dependence of the cutoff on the atomic number \( Z \) by considering the underlying physical mechanism of diffusion. Consider an ISM with a spatial spectrum of turbulence given by \( I(k) \). Here, \( I(k)k \) is the energy density of the turbulence and \( k \) is the wavenumber. Assuming that the most effective scattering takes place when the turbulence has lengthscale comparable to \( r_g \), the radius of gyration of the
particle in a magnetic field of strength $B$, one readily finds that the mean free path $\lambda$ is given by (Drury 1983),

$$\lambda = r_g \frac{B^2}{8\pi I(k) k}.$$  \hspace{1cm} (4)

For a Kolmogorov spectrum ($I(k) \propto k^{-5/3}$), the mean free path $\lambda \propto r_g^{1/3}$. Since the radius of gyration $r_g \propto p/Z \propto \left(\frac{E_{\text{kin}} A}{Z}\right)^{1/2}$ for the non-relativistic particles (where $A \sim 2Z$ is the atomic weight). The diffusion coefficient of the particle is, therefore, $\kappa \propto Z^{-1/3} A^{1/3} E_{\text{kin}}^{2/3}$.

Assuming that the particles come from the same distance, the diffusion time $t \propto \frac{1}{\kappa} \propto Z^{1/3} A^{-1/3} E_{\text{kin}}^{-2/3}$. The lower cutoff is proportional to the total energy loss ($t \times \frac{dE}{dt}$). Since energy loss due to ionization (neglecting the slow logarithmic term) $\frac{dE}{dt} \propto \frac{Z^2}{v}$, we can write

$$E_{\text{kin, cutoff}} \propto Z^{7/3} A^{5/6} E_{\text{kin}}^{-7/6}, \text{ i.e.,}$$

$$E_{\text{kin, cutoff}} \propto Z^{14/13} A^{5/13} \propto Z^{19/13}.$$  \hspace{1cm} (7)

With this dependence of the lower cutoffs of different ions, we can express the ionization of the cosmic rays in terms of the lower cutoff of a single ion, say, proton. In the regimes where spallation dominates over ionization energy losses, the cutoff $E \propto Z^{29/14}$ ($dE/dt$ in this case scales differently).

Note that, in reality the spectra will not have a sharp low energy cutoff, mainly because the observed particles come from various sources at different distances. However, here we attempt to approximate any possible change or turnover in the spectra with a single parameter, i.e., a lower cutoff.

\textit{(d) Rate of ionization}

Using eq. (2) to calculate the ionization rate of cosmic rays with the spectra as in (3) (in momentum space) and with the lower cutoffs obeying the proportionality rule of (8), we show the results in fig. 2. The observations of Black (1990) are plotted along with the timescale for ionization ($1/\zeta$) from the eq. (2) as a function of the lower cutoff in kinetic energy of cosmic ray protons. The figure shows that a cutoff in the range of $(30 - 60)$ MeV is implied by the observations.

\textit{(e) Heating by the cosmic rays}

Ionization produces energetic secondary electrons in the ISM. These electrons, in a weakly ionized gas, lose their energy mainly by excitation and further ionizations until the energy falls below $10.2$ eV. The average energy left to heat the gas depends on these low energy electrons and their spectrum. Spitzer and Tomasko (1968) calculated the average energy from the proton spectrum they used, viz.,

$$j(E_{\text{kin}}) \text{ (particles cm}^{-2}\text{s}^{-1}\text{sr}^{-1}\text{GeV/nucleon}^{-1}) = \frac{0.9}{(0.85 + E_{\text{GeV}})^{2.6}} \frac{1}{(1 + 0.01/E_{\text{GeV}})}.$$

\hspace{1cm} (7)
This spectrum was folded with the spectrum one gets for a single proton (of momentum $p$), $n_e(E)dE \propto E^{-2}p^{-1}dE$. Since the spectrum that we propose to use has a lower cutoff, at $\sim 30$ MeV as shown in the previous section, the spectrum of the first generation electrons is not very clear. However, it is known that the mean energy of the secondary electrons is a very slow function of the incident proton energy and can be taken as 36 eV (Dalgarno and Griffing 1958). If we, for simplicity, assume that all the secondary electrons have this energy, then it is easy to calculate the average energy following the prescription by Spitzer and Tomasko. We find that a heat source of 6.3 eV is associated with each primary ionizing event. For comparison, Spitzer and Tomasko, whose spectrum had no lower cutoff, found a value of 5.7 eV, and Field et al., using a different spectrum, 8.5 eV.

Comparing with the total energy 36 eV lost per free electron produced, the average energy (6.3 eV) implies an efficiency of $\epsilon = 0.175$ for heating. However one must note that in the case of high fractional ionization ($n_e/n_H > 0.01$), the secondary electrons will lose energy more efficiently due to interactions with abundant free electrons.

Discussion

It is worth noting that the lower cutoff derived above is also suggested by creation of light elements from spallation by cosmic ray particles. Gilmore et al. (1992) noted that the production ratio $N_{10B+11B}/N_{9Be}$ increases rapidly for particle energies below $\sim 30$ MeV as the cross-section is dominated by resonances and so that the particle spectrum ought to have a cutoff about $\sim 30$ MeV.

The model of the heating and ionization of the ISM sketched above is probably too simplistic. But we believe that such a picture is closer to reality than the spectra used by previous workers. Seo et al. (1991) analyzed the data obtained during a solar minimum and concluded that the spectra of cosmic ray ions are almost certainly power laws in rigidity outside the solar system, at least down to $\sim 700$ MeV. They found that below this energy (for protons), the flux of particles after solar modulation is lesser than what a single power law would predict. They suggested that this may be the hint of a break in the power law in below $\sim 200$ MeV. We have tried to describe the spectrum in terms of a minimum number of parameters, that is, a single power law with a lower cutoff. While the result of Seo et al. is highly suggestive of such a picture, it is difficult to confirm it without working out the complete mechanism of solar modulation and comparing with their data.

It is possible that low energy cosmic rays are accelerated inside the molecular clouds. Dogiel and Sharov (1990) showed that the energy density inside the clouds could be 10–100 times higher than what is observed on the earth and this could imply a larger low energy cutoff for the cosmic rays inside these clouds for the same ionization rate. However, the uncertainties involved motivated us to use the observed fluxes only and derive a low energy
cutoff for the ISM. One must note that such a description in terms of a single value of the low energy cutoff everywhere in the Galaxy is probably unrealistic, but is the simplest one.

Conclusion

We have used the observed ionization rates in the ISM to derive a low cutoff in the cosmic ray spectrum which is a power law in momentum. We have derived the scaling relations for the low cutoff for the ions as a function of $Z$ from the consideration of energy loss in traversing the ISM. We find that a cutoff in the kinetic energy of protons in the range of $30 - 60$ MeV tallies with the observations. We have noted that such a cutoff is also supported by arguments that spallations by cosmic rays should not produce too much light elements than are observed.

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**Figure Captions**

Figure 1: The rate of loss of energy for an ion (Helium is taken as an example here, since it is the lightest ion that loses energy by spallation) is plotted against the kinetic energy of the ion. For spallation, the rate (per second) corresponds to per cosmic ray proton, and for ionization, per neutral hydrogen atom in the ISM. The curves show that ionization losses dominate over those from spallation for the energy range we consider here for the lower cutoff.

Figure 2: Ionization timescale (second) is plotted as a function of the lower cutoff of the proton kinetic energy in the cosmic ray spectrum. The horizontal lines correspond to the observed limits on the ionization rate $(3 - 7) \times 10^{-17} \text{ s}^{-1}$. 