Pressure controlled transition into a self-induced topological superconducting surface state

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Ab-initio calculations show a pressure induced trivial-nontrivial-trivial topological phase transition in the normal state of 1T-TiSe2. The pressure range in which the nontrivial phase emerges overlaps with that of the superconducting ground state. Thus, topological superconductivity can be induced in protected surface states by the proximity effect of superconducting bulk states. This kind of self-induced topological surface superconductivity is promising for a realization of Majorana fermions due to the absence of lattice and chemical potential mismatches. For appropriate electron doping, the formation of the topological superconducting surface state in 1T-TiSe2 becomes accessible to experiments as it can be controlled by pressure.

Majorana fermions are half integer spin particles that are their own antiparticles1. In a two-dimensional superconductor, a vortex with an odd number of Majorana quasiparticle excitations obeys non-Abelian rather than the Bose or Fermi statistics3–5. The states are robust against local noise and, hence, are considered as strong candidates for a fault tolerant quantum information processing4. Initially, Majorana quasiparticle excitations have been predicted to occur in topological superconducting phases in which the Cooper pairs have a spin-triplet $p_x + ip_y$ pairing symmetry5. This case is very rare in nature. The most promising system is the exotic $p$-wave superconductor Sr$_2$RuO$_4$6 which for the experimental situation, however, is not clear8,9.

Later, they have also been proposed to appear in topological superconducting interface states. This would enable induction by the conventional $s$-wave superconducting proximity effect on the Dirac-like boundary states of topological insulators7,10,11, of half-metallic materials12,13, and of semiconductors with substantial Rashba spin-orbit coupling14,15. Recently, superconductivity has been reported for the prototypical three-dimensional topological insulators Bi$_2$Se$_3$ and Bi$_2$Te$_3$16–19 under doping20–24 and under pressure25. Accordingly, topological superconductivity is predicted to be induced by the superconducting proximity effect on topologically protected Dirac-like surface states of the superconductor itself20,22. Such a self-induced topological surface superconductor is very promising for the realization of Majorana quasiparticle excitations due to the absence of lattice and chemical potential mismatches25.

1T-TiSe$_2$ is one of the early discovered materials with a charge density wave (CDW)26, and has seen a revival of interest due to recently discovered superconductivity27–35. It is a layered compound, consisting of slabs in which Ti is octahedrally coordinated by six Se atoms. Se-Ti-Se slabs with ionic bonding are separated by a van der Waals gap, leading to a quasi two-dimensional nature. A commensurate CDW with a $2 \times 2 \times 2$ superstructure develops below 200 K26. No sign of superconductivity is found by lowering the temperature only. However, under controlled Cu-intercalation into the van der Waals gap the CDW is progressively suppressed and a superconducting state emerges with a maximum transition temperature of $T_c = 4.15 \text{ K}^{27}$. Suppression of the CDW and emergence of superconductivity have been discovered in pressurized 1T-TiSe$_2$ with a maximum $T_c$ of 1.8 K34.

In the following, we apply fully-relativistic all-electron ab-initio calculations to reveal a pressure induced trivial-nontrivial-trivial topological phase transition in the normal state of 1T-TiSe$_2$. We argue that the pressure range of the topologically nontrivial phase overlaps with that of the superconductivity. Our findings suggest pressurized 1T-TiSe$_2$ as another candidate for self-induced topological surface superconductivity for appropriate doping.

Results

Experimental values of the structure parameters have not been determined for pressurized 1T-TiSe$_2$ and theoretical calculations encounter serious deficiencies in predicting the parameters for 1T-layered compounds under
pressure. Our calculations confirm this observation. The structure parameters of 1T-TiSe\textsubscript{2} at ambient pressure show errors of $-1\%$, $-3\%$, and $4\%$ for the values of $a$, $c$, and $z$, respectively, employing the local density (generalized gradient) approximation. These errors are problematic because of a high sensitivity of the electronic states to the structure, see also the text below. The generalized gradient approximation with explicitly treated van der Waals interaction leads to even worse errors of $5\%$, $8\%$, and $5\%$ for the three parameters. On the other hand, the experimental structure is available for 1T-TiS\textsubscript{2} under pressure. The similarity of the crystal structure and chemical composition justifies to adopt the values of $a_{0}$, $c_{0}$, and $z$ of 1T-TiSe\textsubscript{2} for the normal state of pressurized 1T-TiSe\textsubscript{2}, which we make use of in the following. Note that the crystal symmetry of the high-temperature normal phase is adopted for the entire pressure range, although a CDW transition with lattice distortion appears at low temperature for a pressure below 2 GPa. We will argue that this assumption does not affect the conclusions of our work.

The fully-relativistic electronic band structure of the normal state of 1T-TiSe\textsubscript{2} at ambient pressure is depicted in Fig. 1(b). The observed energy overlap of the valence band maximum (VBM) and conduction band minimum (CBM) indicates a semimetallic nature. The almost full valence and almost empty conduction bands show mainly Se 4$p$ and Ti 3$d$ characters, respectively, indicating an overall ionic bonding with charge transfer from Ti to Se. Near $E_{F}$ the electronic structure of 1T-TiSe\textsubscript{2} is governed by two bands, giving rise to the hole-like Se 4$p$ derived VBM at the \textGamma-point and the electron-like Ti 3$d$ derived CBM at the L-point. The space inversion symmetry of 1T-TiSe\textsubscript{2} allows us to calculate the band topological invariants by the parity check method proposed by Fu and Kane. Due to the semimetallic nature, however, one needs to consider the valence states instead of the occupied states below $E_{F}$. The $Z_{2}$ topological invariant can be calculated directly from knowledge about the parity of each pair of Kramers degenerate bands up to the local VBM's at the eight time-reversal momenta (1\Gamma, 3M, 1A, and 3L). A value of zero is obtained for all four $Z_{2}$ topological invariants, reflecting a topologically trivial nature of the normal state of 1T-TiSe\textsubscript{2} at ambient pressure.

Due to the smaller energy gap around the \textGamma-point, as compared to the other seven time-reversal momenta, the band topology is mainly defined by the band order at the \textGamma-point, on which we focus in the following. A zoom of the VBM is displayed in Fig. 1(c) for ambient pressure. Below $E_{VBM}$, we see two spin-orbit split Se 4$p_{c,v}$ states: the lower doublet $\Gamma_{4}^{-}$ and the upper doublet $\Gamma_{56}^{-}$, where the latter forms the VBM. Above $E_{VBM}$, the spin-orbit coupling lifts the degeneracy of the Ti 3$d_{x^{2}-y^{2}}$ states and yields the lower doublet $\Gamma_{4}^{+}$ and the upper doublet $\Gamma_{56}^{+}$. Under sufficient pressure, the odd parity Se 4$p$ $\Gamma_{56}^{+}$ state is pushed on top of the Ti 3$d$ $\Gamma_{56}^{+}$ state with even parity. As an example, Fig. 1(d) illustrates the band structure of the normal state of 1T-TiSe\textsubscript{2} under a pressure of 3 GPa. We find that the band order is not changed at the other seven time-reversal invariant momenta.

According to the parity check method for calculating topological invariants, inversion of bands with a different parity at the \textGamma-point transforms 1T-TiSe\textsubscript{2} into a topologically nontrivial state of (1;000) type. The same topological phase transition due to pressure induced band inversion has been observed in layered GaS and GaSe. For higher pressure, the odd parity Se 4$p$ $\Gamma_{56}^{+}$ state is pushed above the Ti 3$d$ $\Gamma_{56}^{+}$ state with even parity, see the band structure in Fig. 1(e) for a pressure of 5 GPa. As a result, the band topology switches back to trivial. Therefore, we are confronted with a pressure induced trivial-nontrivial-topological phase transition in the normal state of 1T-TiSe\textsubscript{2}.

The phase transition may be viewed as a result of the small energy gap at the VBM and, hence, the sensitivity of the band hierarchy against modifications of the structure. In addition, spin-orbit coupling is crucial for its occurrence and the appearance of a topologically nontrivial phase. Without spin-orbit coupling, the Se 4$p_{c,v}$ $\Gamma_{4}^{-}$ and $\Gamma_{56}^{-}$ states would form a degenerate pair. Under pressure these odd parity states would simultaneously be pushed above the even parity Ti 3$d$ states and no phase transition would be observed. Therefore, an energy difference between the $\Gamma_{4}^{-}$ and $\Gamma_{4}^{+}$ states is necessary for the emergence of a pressure induced topological phase transition. In the present case, the energy difference is caused by the spin-orbit coupling. It determines the pressure range of the topologically nontrivial phase. Specifically, the critical pressures of the trivial-nontrivial ($p_{c}$) and nontrivial-trivial ($p_{c}$) transitions, are related to the energy differences between $\Gamma_{56}^{-}$ and $\Gamma_{4}^{+}$ ($\Delta_{c}^{-}$) and $\Gamma_{4}^{-}$ and $\Gamma_{56}^{+}$ ($\Delta_{c}^{+}$), respectively. Accordingly, the topological phase diagram of the normal state of 1T-TiSe\textsubscript{2} under pressure can be derived from the band order. The result is shown in Fig. 2(a) for pressures between 1.5 and 4.5 GPa.

Figure 2(a) includes the experimental pressure phase diagram for the ground state. We obtain $p_{c}^{-} = 2.7$ GPa and $p_{c}^{+} = 3.2$ GPa. On the other hand, experiments for sub-Kelvin temperature point to pressure induced phase transitions from CDW to superconductor to the normal state4. With increasing pressure, the superconductivity appears around 2 GPa and disappears beyond 4 GPa. The pressure range $p_{c}^{-}$ to $p_{c}^{+}$ overlaps with the range in which the superconducting ground state develops. Since the band hierarchy is sensitive to the structure parameters, the pressure range $p_{c}^{-}$ to $p_{c}^{+}$ could depend on uncertainties in the structure determination. Using the data set reported in Ref. 41 for $a_{0}$, $c_{0}$, and $z$, we obtain another topological phase diagram, which is depicted in Fig. 2(b). The pressure range $p_{c}^{-}$ to $p_{c}^{+}$ changes substantially with respect to Fig. 2(a), but still overlaps with the range of the superconducting ground state. As mentioned above, a hypothetical crystal symmetry without CDW distortion is used for pressures lower than 2 GPa, which, however, is in the range where our assumption on the crystal symmetry does not influence the conclusions. The discrepancy between the two experimental sets of lattice constants concerns mainly the value of $c_{0}$. For a pressure of 3 GPa, for example, the difference is 0.1% for $a_{0}$ but 0.7% for $c_{0}$. This leads to a 0.1 GPa shift for $p_{c}^{-}$ and a much larger shift of 1 GPa for $p_{c}^{+}$. Therefore, as long as $p_{c}^{-}$ is in the pressure range from 2 GPa to 4 GPa, the range of the nontrivial phase will overlap with

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**Figure 1** | Normal state of 1T-TiSe\textsubscript{2}. (a) Crystal structure. (b) Electronic band structure at ambient pressure. (c–e) Detailed band structures near the VBM under ambient pressure, 3 GPa, and 5 GPa, respectively. It should be noted that the Fermi energy is located around 0.5 eV below the VBM.
the superconducting ground state. This means that our conclusions stay valid even when the discrepancy in the experimental lattice constant is 10 to 20 times larger than that of Refs. 37 and 41. One of the most important consequences of a topologically nontrivial state is the existence of a gapless surface state\(^{39,42-44}\), which is protected by the combination of spin-orbit coupling and time-reversal symmetry and shows a spin-textured Dirac-like behavior\(^{46-49,50}\). Therefore, our results indicate that a superconducting ground state coexists with a topologically protected spin-textured Dirac-like surface state near the VBM in pressurized 1T-TiSe\(_2\).

If bulk superconductivity develops in pressurized 1T-TiSe\(_2\) at low temperature, Cooper pairs can tunnel from the bulk states into the surface states. An s-wave pairing symmetry is to be expected, because the bulk superconducting phase is governed by electron-phonon coupling\(^{51}\). This leads to the induction of a s-type superconducting energy gap in the Dirac-like surface states\(^{10}\). Note that the superconducting ground state persists for slight Cu-intercalation in pressurized 1T-TiSe\(_2\)\(^{31,34}\). By Cu-intercalation, the tunneling process can be enhanced because \(E_F\) is shifted close to the VBM, hence to the Dirac-like surface states. The resulting two-dimensional superconducting surface state resembles the spinless \(p_x + ip_y\) topological superconductor, except that the time-reversal symmetry is not violated\(^{52}\). Like the \(p_x + ip_y\) superconductor, the induced surface superconductor will have a Majorana quasiparticle excitation bound to a vortex\(^{10}\). We thus propose pressurized 1T-TiSe\(_2\) as a material in which topological superconductivity is induced by the superconducting proximity effect on topologically protected spin-textured Dirac-like surface states of the superconductor itself. Such a self-induced surface topological superconductivity is promising as it is free of lattice and chemical potential mismatches. Self-induced topological surface superconductivity before has been predicted for Bi\(_2\)Se\(_3\) and Bi\(_2\)Te\(_3\) under both doping and pressure\(^{20,22}\). The semimetallic nature and formation of a hole pocket at the VBM found in our calculations indicate that the surface Dirac-like states are not located at \(E_F\), making electron doping necessary. Assuming a rigid band model, a doping of 0.7 electrons per formula unit is required to shift \(E_F\) by 0.5 eV. A shift of more than 0.1 eV has been demonstrated for a formal doping of 0.065 electrons without losing the superconductivity\(^{53}\). Preservation for higher doping calls for experimental confirmation.

**Discussion**

In conclusion, we have applied density functional theory to demonstrate a pressure induced trivial-nontrivial-trivial topological phase transition for the normal state of 1T-TiSe\(_2\). The transition is ascribed to inversion of Se 4p and Ti 3d bands with different parity at the \(\Gamma\) point. It is very likely that the pressure range in which the topologically nontrivial phase overlaps with the one of the superconductivity, leading to self-induced topological surface superconductivity in pressurized 1T-TiSe\(_2\). As a consequence, the formation of topological superconducting surface states becomes accessible to an experimental investigation in the present material for appropriate electron doping. The surface state is promising for the realization of Majorana quasiparticle excitations because of the absence of lattice and chemical potential mismatches. We emphasize that the existence of the topological phase transition is guaranteed by spin-orbit coupling. For obtaining the pressure range of the self-induced topological surface superconductivity more exactly further experiments are required to determine the structure parameters under pressure with high accuracy. As s-type pairing is needed for topological superconducting surface states to occur, a more careful study is required to verify the conclusions of Ref. 35.

**Methods**

The high temperature normal state structure of pristine 1T-TiSe\(_2\), shows the space group \(P\overline{3}m1\) (No. 164) where the Ti and Se atoms occupy \(1a\) \([0, 0, 0]\) and \(2d\) \([1/3, 2/3, z]\) sites, respectively, see Fig. 1a). Experiments under ambient pressure yield structure parameters of \(a = 3.54\ \text{Å},\ c = 6.008\ \text{Å},\) and \(z = 0.25504\)^{54}. Our full-potential linearized augmented plane wave calculations have been performed using the WIEN2k package\(^{55}\). We employ a threshold energy of \(-6\ \text{eV}\) for separating the valence and the core states, a muffin-tin radius of \(R_{\text{MT}} = 2.2\ a_0\), high values of \(R_{\text{MT}}/R_{\text{CUT}} = 10\) and \(\eta_{\text{EFL}} = 10\), as well as a \(20 \times 20 \times 10\)-k-mesh. Moreover, a combination of the modified Becke-Johnson exchange potential and the local density approximation of the correlation potential is used in order to predict band gaps and band orders with an accuracy similar to hybrid functionals and Gutzwiller calculations\(^{56}\). Spin-orbit coupling is treated applying a second-variational method with a scalar relativistic treatment of the \(\alpha\) band model, a doping of 0.7 electrons per formula unit is used in the basis expansion.

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Author contributions
Z.Z. and U.S. conceived the study and wrote the manuscript. Z.Z. and Y.C. performed the calculations.

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