The Milan factor for jet-shape observables

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This talk discusses and explains the two-loop corrections, also known as the Milan factor, for
power corrections to jet-shape observables.

1 Introduction

This talk aims to give a qualitative discussion of the issues related to the two-loop calculations
of power-suppressed contributions to event-shape variables. For simplicity, it will concentrate
mostly on the situation in $e^+e^-$. For the one- and two-loop DIS results, the reader is referre
d to the work of Dasgupta and Webber.

The essential issue in the phenomenological study of power-suppressed contributions to QCD
observables is that of universality, namely that the leading power correction to any given ob-
servable $V$ can be expressed in terms of a (perturbatively) calculable coefficient $c_V$ multi-
plied by an unknown (non-perturbative) number $A_{2p,q}$. This latter number can be thought of as a
particular moment of the non-perturbative part of the QCD coupling:

$$A_{2p,q} = \frac{1}{2} \int_0^\infty \frac{dk^2}{k^2} k^{2p} \ln^q \frac{k^2}{\mu^2} \alpha_s^{NP} (k^2).$$

Observables can be divided into classes according to the relevant values of $p$ and $q$. For example
many event-shape variables have $2p = 1$, $q = 0$, and so are expected to have a power-suppressed
contribution $\mathcal{P}$,

$$\mathcal{P} = \frac{4C_F c_V A_{1,0}}{\pi^2} \frac{Q}{Q},$$

where $Q$ is the hard scale of the process. So in some sense universality is the statement that for
a certain class of observables, the relative coefficients of the power corrections are all calculable.
Experimentally, at the 20% level, there is already some fair evidence for universality among
event-shape observables.

The rest of this talk will discuss the calculability of the coefficients $c_V$, in particular the
problems associated with the 1-loop calculation of power corrections and their solution with a
two-loop calculation.

2 Power corrections at one loop

The methods for calculating power corrections at 1-loop accuracy have reached a point of some
maturity (and controversy). This talk will discuss calculations in the particular context of

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the dispersive approach, though a number of other approaches are available. In general one introduces a gluon mass $m$, and looks for non-analyticity in $m^2$. In reality one should understand this gluon mass as in figure 1, namely as the virtuality of a gluon which then decays into two massless partons. In a one-loop calculation, one has no information about the momenta of these decay products, so the contribution to the event shape must be taken as coming from the parent gluon. It was pointed out by Nason and Seymour that as a result of the partial non-inclusiveness of event-shape variables this can be a rather poor approximation: the decay products will not go in the same direction as the parent, and therefore the parent and its decay products give different contributions to the event shape.

Another manifestation of the same problem arises in the introduction of the mass into the event-shape variable. Since the mass is fake, one is free to introduce it into the event-shape definition as one likes. If one takes as an example the thrust variable,

$$T = \max_\vec{n} \frac{\sum_i |\vec{p}_i\cdot\vec{n}|}{\sum_i |\vec{p}_i|},$$

one finds two suggestions for the power correction in the literature. Beneke and Braun chose to include the effects of the gluon mass in the denominator (i.e. truly the sum of the modulus of the 3-momenta), whereas Dokshitzer and Webber chose not to. Beneke and Braun obtained for the coefficient of the power correction $c_T = -4G$, with $G$ being Catalan’s constant, while Dokshitzer and Webber obtained $c_T = -2$. At the one-loop level there is no way to resolve this problem and one cannot unambiguously calculate the coefficient $c_V$ of the power correction for a variable $V$: universality is lost.

3 The Milan factor

The solution to the problem is to carry out a two-loop calculation. What one finds is that the two-loop result modifies the one-loop result by a factor (known as the Milan factor). At first sight this is a somewhat surprising result for a two-loop correction. On the other hand, if the two-loop modification is to resolve the one-loop ambiguity (a factor), then it has to be a factor itself. Another argument is that a two-loop correction can always be recast as a change of scale:

$$\alpha_s(Q^2) + \alpha_s^2(Q^2) \simeq \alpha_s(A^2Q^2),$$

where $A^2 = \exp(-4\pi c/\beta_0)$. But for a power correction, a change of scale corresponds to a modification by a factor:

$$\frac{\Lambda}{Q} \Rightarrow \frac{\Lambda}{AQ}.$$

At this point one may start to wonder higher order corrections will also be factors. The above argument suggests not, since the addition of an $\alpha_s^3$ correction to (2), changes $A$ by

$$A \rightarrow A(1 + O(\alpha_s/\pi)).$$
The values obtained for the first-moment of the coupling suggest that at small scales $\alpha_s/\pi \simeq 0.2$. This leads us to hope that higher order corrections to the Milan factor will remain relatively small ($\simeq 20\%$?), rather than a factor of order one.

One obvious worry if the two-loop corrections are factors is that fits to data which worked reasonably well with old coefficients will suddenly worsen. In fact most of the coefficients change by a common factor $M \simeq 1.8$: given a suitable naive or one-loop definition of the event-shape variable, typically in terms of the Sudakov variables $\alpha$ and $\beta$, one finds that the entire variable dependence of the power correction, both at one and two loops enters through the quantity

$$\rho_V = \int_{-\infty}^{\infty} d\eta f^{(V)}(\eta),$$

with for example for thrust and the $C$-parameter

$$f^{(T)}(\eta) = -e^{-|\eta|}, \quad f^{(C)}(\eta) = \frac{3}{\cosh \eta}.$$  

The rest of the calculation is variable independent. As a result the relative coefficients of the power corrections stay the same at one and two loops.

There are a few exceptions to this. The tables below summarise the old and new results. For $e^+e^-$ one has

| Variable | $1-T$ | $C$ | $M_T^2$ | $M_H^2$ | $B_T$ | $B_W$ |
|----------|-------|-----|---------|---------|-------|-------|
| old $c_V$ | 2     | -   | 2       | 2       | 2     | 2     |
| new $c_V/M$ | 2     | 3\pi | 2       | 1       | 1     | $\frac{3}{2}$ |

and for DIS\footnote{Except for $C_c$ in DIS where the change is due to the manner in which the gluon mass was included in the one-loop treatment. Note also that the coefficients given here for $1-T_z$ differ from those presented at the conference, which were wrong.}

| Variable | $1-T_z$ | $1-T_c$ | $C_c$ | $\rho_Q$ |
|----------|---------|---------|-------|----------|
| old $c_V$ | 2       | 2       | 6\pi  | 1        |
| new $c_V/M$ | 2     | 2       | 3\pi  | 1        |

In general the changes between the relative $c_V$ of different variables can be attributed to the interplay between perturbative and non-perturbative effects\footnote{As a result the coefficient for $M_H^2$ from the dispersive approach is now in agreement with that found earlier by Akhoury and Zakharov.}.

For example in the original one-loop calculation to determine the non-perturbative correction to the heavy-jet mass one considered only a single gluon. The hemisphere which contained this gluon was automatically the heaviest, and therefore the heavy-jet mass acquired the whole non-perturbative contribution, and the light-jet mass none. On the other hand, the formulation of the two-loop calculation takes into account the presence also of perturbative radiation (though the relevant techniques were first introduced for one-loop calculations\footnote{As a result the coefficient for $M_H^2$ from the dispersive approach is now in agreement with that found earlier by Akhoury and Zakharov.}). In general it is the perturbative radiation which determines which jet is heavy, and since this is uncorrelated with the non-perturbative radiation, the non-perturbative “trigger” gluon will be present in the heavy jet only half the time, and therefore the heavy jet acquires half the correction of the total jet mass.

A similar argument applies in the comparison of the total and wide-jet broadenings. Here though, there is the additional element of quark recoil. The broadening is the sum of the transverse momenta (with respect to the thrust axis) of all particles in the event:

$$B_T = \frac{1}{2Q} \sum_i k_{T,i}$$

In the presence of only a single non-perturbative gluon, the transverse momentum of the recoiling quark is equal to that of the gluon, and so the contribution to the broadening is twice that from
the gluon alone. But in practice perturbative radiation will give a certain transverse momentum to the quark (somewhat larger than the non-perturbative gluon transverse momentum), so that the effect of the recoil from the non-perturbative gluon averages out to zero after the integration over azimuth (or rather, gives a \((\ln Q)/Q^2\) type correction). This halves one’s naive expectation for the non-perturbative contribution to the jet broadening. Analogous considerations are expected to affect the predictions for the jet-broadening in DIS.

It is to be recalled that the jet-broadenings are all expected to exhibit a \((\ln Q)/Q\) type power correction, as opposed to \(1/Q\) for the other variables.

4 Fractional power corrections

In this section we will examine the topic of fractional power corrections, in particular for the energy-energy correlation (EEC), defined as

\[
\text{EEC}(\chi) = \sum_{ab} E_a E_b \delta(\cos \chi - \cos \theta_{ab}).
\]

In the central region, namely \(\chi \sim \pi/2\), the ratio of the non-perturbative (NP) to perturbative (PT) contributions has the behaviour

\[
\frac{\text{NP}}{\text{PT}} \sim \frac{1}{Q \sin \chi}.
\]

The configurations leading to this correction are those where one triggers on a soft gluon and either the quark or anti-quark. The divergence as \(\sin \chi \to 0\) is closely connected to the collinear divergence in gluon emission from a quark.

Figure 2: Back-to-back EEC, showing the positions of the detectors and a typical configuration (only the non-perturbative gluon is shown)

The question of interest is what happens to the divergence when \(\chi \to \pi\). Figure 2 illustrates a typical situation. The choice \(\chi = \pi\) means that the triggered gluon and quark are back to back. Naively one would expect the gluon and the anti-quark to be parallel, but in practice as a consequence of all-order perturbative gluon emission, the quark and anti-quark are not quite back-to-back so that there is a small angle \(\bar{\chi}\) between the anti-quark and gluon, which when integrated over gives a power correction (relative to the perturbative result)

\[
\frac{1}{Q} \int d^2 \bar{\chi} \frac{P(\bar{\chi})}{\sin \bar{\chi}} \sim \frac{1}{Q} \left( \frac{Q}{\Lambda_{\text{QCD}}} \right)^{\beta_0/(\beta_0 + 4C_F)} \propto Q^{-0.372}.
\]

This estimate is the result of a steepest descent approach, which appears to be too naive to produce the correct exponent, however non-integer powers of \(Q\) will definitely be there. Analogous effects are expected for the energy-weighted particle sum in the photon direction in the Breit frame in DIS, and for the height of the plateau in the distribution of the transverse momentum of Drell-Yan pairs.
5 Merging

So far a number of rather glib references have been made to “non-perturbative gluons” on the one hand and to “perturbative gluons” on the other. This is connected with the use in \( \alpha_{NP}^s \), the non-perturbative part of the coupling. In practice one defines \( \alpha_{NP}^s \) as the difference between the true coupling and the perturbative expansion, \( \alpha_{PT}^s \), as used in the fixed-order perturbative calculation, both in the CMW scheme:

\[
\alpha_{NP}^s(k^2) \equiv \alpha_s(k^2) + \alpha_{PT}^s(k^2).
\]

One expects \( \alpha_{NP}^s \) to go to zero very rapidly, as the true and the perturbative couplings coincide at moderate and large scales. As a result one chooses to truncate the moments \( A_{1,0} \) at an infra-red matching scale \( \mu_I \), so that for example \( A_{1,0} \) becomes

\[
A_{1,0} \simeq \int_0^{\mu_I} dk \alpha_s(k) - \int_0^{\mu_I} dk \alpha_{PT}^s(k) \\
\simeq \mu_I \alpha_0(\mu_I) - \mu_I \left[ \alpha_s(Q) - \beta_0 \frac{\alpha_s^2}{2\pi} \left( \ln \frac{Q}{\mu_I} + 1 \right) \right] .
\]

On the second line, the second term, known as the merging part, has been shown for the case of the two-loop perturbative calculation. It is instructive to examine its form for higher orders of perturbation theory. In general, at \( n^{th} \) order the merging piece will have the form \( \alpha_{s,n!} \). This factorial divergence should cancel the corresponding factorial (or renormalon) divergence in the fixed order perturbative calculation which was the origin of the power correction in the first place, leaving in the end a renormalon-free answer.

6 Conclusions

The main point of this talk has been that to determine the coefficients for event-shape power corrections, and thus to be able to test the concept of universality, it is not sufficient to perform a one-loop determination of the power correction. This is because of the non-inclusive nature of event shapes, which leads to an unresolvable ambiguity in the result, according to one’s choice of how to include the fake dispersive gluon mass into the event-shape definition.

The solution is to perform a two-loop calculation of the power correction. As a result of the linearity of event-shape variables (at least those with \( 1/Q \) leading power-suppressed contributions) in the soft limit, it turns out that the variable dependence enters both in the one- and two-loop calculations entirely through one common simple integral; thus in one and two loops the relative coefficients for event-shape power corrections stay the same (as long as one chooses a suitable convention for the inclusion of the gluon mass at one loop).

For certain variables, the relative coefficient of the power correction has changed: in the old 1-loop calculations there was generally no simultaneous treatment of the perturbative and non-perturbative gluon radiation, whereas the interaction between the two is an essential part of the physics. One particularly interesting example of the interaction between NP and PT physics is the back-to-back energy-energy correlation where it leads to a fractional power correction.

Finally, a critical element of all these discussions is that the power correction is not simply of the form \( 1/Q^p \) but that there is an essential “merging” piece which subtracts out the renormalon divergence in the perturbative calculation, ensuring that the final answer is well defined.

\(^d\)Note that the merging term has been given for the case of a renormalisation scale \( Q \); should one choose to use a renormalisation scale \( \mu \) one should replace \( Q \) with \( \mu \). Note also that it has been given in the CMW scheme.
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