Deep uGMRT observations of the ELAIS-North 1 field: statistical properties of radio–infrared relations up to $z \sim 2$

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ABSTRACT

Comprehending the radio–infrared (IR) relations of the faint extragalactic radio sources is important for using radio emission as a tracer of star-formation in high redshift ($z$) star-forming galaxies (SFGs). Using deep uGMRT observations of the ELAIS-N1 field in the 0.3–0.5 GHz range, we study the statistical properties of the radio–IR relations and the variation of the ‘$q$-parameter’ up to $z = 2$ after broadly classifying the faint sources as SFGs and AGN. We find the dust temperature ($T_{\text{dust}}$) to increase with $z$. This gives rise to $q_{24\mu m}$, measured at 24 $\mu$m, to increase with $z$ as the peak of IR emission shifts towards shorter wavelengths, resulting in the largest scatter among different measures of $q$-parameters. $q_{70\mu m}$ measured at 70 $\mu$m, and $q_{\text{TIR}}$ using total-IR (TIR) emission are largely unaffected by $T_{\text{dust}}$. We observe strong, non-linear correlations between the radio luminosities at 0.4 and 1.4 GHz with 70 $\mu$m luminosity and TIR luminosity ($L_{\text{TIR}}$). To assess the possible role of the radio-continuum spectrum in making the relations non-linear, for the first time we study them at high $z$ using integrated radio luminosity ($L_{\text{RC}}$) in the range 0.1–2 GHz. In SFGs, the $L_{\text{RC}}–L_{\text{TIR}}$ Relation remains non-linear with a slope of 1.07 ± 0.02, has a factor of 2 lower scatter compared to monochromatic radio luminosities, and $q_{\text{TIR}}$ decreases with $z$ as $q_{\text{TIR}}^{L_{\text{RC}}} = (2.27 ± 0.03) (1+z)^{-0.12±0.03}$. A redshift variation of $q$ is a natural consequence of non-linearity. We suggest that a redshift evolution of magnetic field strengths and/or cosmic ray acceleration efficiency in high-$z$ SFGs could give rise to non-linear radio–IR relations.

Key words:
radio continuum: galaxies, infrared: galaxies, galaxies: active, galaxies: ISM

1 INTRODUCTION

Deep radio-continuum observations at micro-Jansky (mJy) level below about 10 GHz provide an unobscured view of the extragalactic sky up to a very early Universe (e.g., Condon 1992; Williams et al. 2016; Novak et al. 2017). Of late, deep observations have enabled the study of the properties of a diverse population of sources, ranging from star-forming galaxies (SFGs), typically at the faint flux density end, through radio-quiet and Fanaroff–Riley (FR) class 0-type active galactic nuclei (AGN) with intermediate flux densities (few tens of mJy), to powerful FRI- and FR-II-type AGN at the high flux density regime (≥ 100 mJy). These observations not only facilitate finding and determining the relative properties of these sources, but when combined with multi-waveband information, they provide an excellent means to statistically study their intrinsic properties, impact of their environment, and constrain their evolution over the history of the Universe (see, e.g., Padovani et al. 2009, 2011; Padovani et al. 2015; Novak et al. 2018; Baldi et al. 2019; Tisanić et al. 2019; Mingo et al. 2019; Hardcastle et al. 2019; Hardcastle & Croston 2020).

Broadly speaking, in terms of the origin of emission, extragalactic sources can be classified into AGN and SFGs. Although the emission mechanism in both classes of sources is dominated by synchrotron radiation, AGN are powered by relativistic jets launched by the central black hole, while for SFGs, the emission originates from cosmic ray electrons (CREs) accelerated in the shock fronts of supernovae explosions. Deep radio-continuum surveys with sensitivity ≤ 100 mJy are opening up a new window on what is usually considered the ‘normal’ galaxy population at high redshift ($z$). Since radio observations are free from dust obscuration, unlike ultraviolet (UV) and Hα tracers, and the source-photometry are generally not confused, unlike mid- to far-infrared tracers (e.g., Madau & Dickinson 2014; Jarvis et al. 2015b); radio continuum emission is being used to trace the star-formation history of the Universe and the evolution in the star forming main sequence (Daddi et al. 2007; Seymour et al. 2008; Novak et al. 2017; Ocran et al. 2020; Leslie et al. 2020). Lately, the scenario of co-evolution is emerging, wherein feedback from star formation and jet launching play an important role in the evolution of the AGN and SFG populations (Jurlin et al. 2020; Webster et al. 2021). Therefore, it is crucial to carefully characterize the properties of the sources in order to determine the dominant component, i.e., star-formation or the AGN activity, which is contributing to the radio continuum emission. This in turn, enables a contamination-free
estimation of the star formation rate (SFR) from radio continuum measurement alone.

Using radio continuum emission as a tracer of SFR relies on one of the tightest, near-linear correlations in astrophysics observed between the radio and infrared (IR) luminosities, the radio–IR relation (e.g., Helou et al. 1985; Condon 1992; Yun et al. 2001). This relation spans over five orders of magnitude in luminosity with dispersion less than a factor of two, and holds good from dwarf (Chyży et al. 2011; Roychowdhury & Chengalur 2012; Jurusik et al. 2014) to ultraluminous infrared galaxies (ULIRGs) on galaxy-integrated scales (e.g., Appleton et al. 2004; Sargent et al. 2010a; Mao et al. 2011; Basu et al. 2015b). It is believed that the UV photons from massive (≥ 10 M☉) OB-type stars, that are absorbed by the dust, is re-radiated in the infrared wavebands and the same OB-type stars provide the synchrotron emitting CREs when they end their short lives (up to a few Myr) as supernovae explosions, giving rise to the correlation.

However, a number of seemingly independent physical parameters of the interstellar medium (ISM) are responsible for the emission processes in the radio and infrared wavebands, such as, the number density of CREs; energy losses and escape of CREs; magnetic field amplification mechanism; star formation history; dust absorption efficiency; and densities of dust and gas. The tightness and the slope of the radio–IR relation depends on the interplay between these physical parameters, and on whether or not energy equipartition between magnetic fields and CREs are valid (Voelk 1989; Helou & Bicay 1993; Niklas & Beck 1997; Bell 2003; Lacki et al. 2010; Berkhuijsen et al. 2013; Basu et al. 2017b). Based on theoretical and empirical results, a framework based on efficient amplification of the magnetic fields via supernovae-driven fluctuation dynamo in galaxies has been put forward that connects magnetic field strengths and cosmic ray, and gas densities, to explain the tightness and cosmic evolution of the radio–IR relation (see, e.g., Lacki & Thompson 2010; Schleicher & Beck 2013; Schober et al. 2016). These studies bring to light that the relation is expected to evolve with redshift due to a combination of evolution of the properties of the ISM, or confinement of CREs by the magnetic fields, manifesting as either a change in the slope, making it significantly non-linear, or a change in the ratio of infrared to radio luminosities. Therefore, it is essential to study the properties of radio–IR relations at high redshifts.

Radio continuum emission in star-forming galaxies mostly originates from the synchrotron and the free–free mechanisms. Owing to the steep spectrum of the synchrotron emission, it dominates at frequencies ≲ 2 GHz (Basu et al. 2012). Since the free–free emission with flat spectrum directly originates as a consequence of star formation, high frequency (≥ 20 GHz) radio observations are well suited to constrain the cosmic star formation history (Murphy et al. 2011, 2015). However, performing deep, large sky-area surveys at these frequencies is time expensive due to the relatively small field-of-view, and the emission is contaminated by anomalous microwave emission (Leitch et al. 1997; Murphy et al. 2010). Low frequency radio surveys significantly below 1 GHz are important as the rest-frame emission is dominated by the synchrotron emission, and are relatively less contaminated by the free–free emission as compared to observations near or above 1 GHz. Furthermore, the emission from the AGN is typically optically thick and less variable at frequencies below 1 GHz (Condon & Ransom 2016). This give rise to less biases in radio photometry when compared to high radio frequency observations, making low-frequency observations well suited for identifying steady AGN emission and thereby studying the star formation history via the radio–IR relations.

Ongoing and future sensitive large sky-area radio continuum surveys, such as, the LoTSS using the LOFAR (Shimwell et al. 2017), the VLASS using the Karl G. Jansky Very Large Array (VLA; Lacy et al. 2020), and the MIGHTEE using the MeerKAT (Jarvis et al. 2016), and surveys using the Square Kilometre Array (SKA; Jarvis et al. 2015a) and next-generation VLA (ngVLA; Di Francesco et al. 2019) later in the decade, are going to detect several tens of millions of radio sources. Robust characterization of all the sources, primarily based on optical and/or infrared spectroscopy, for example, using the James Webb Space Telescope (JWST; Kalirai 2018) is going to be a challenging proposition, and much of the initial source classification is expected to rely on existing, ancillary multi-waveband data. Therefore, it is crucial to investigate the efficacy of source classification based on existing photometric surveys in the optical and infrared wavebands, and using relatively shallow but large sky-area spectroscopy from the Sloan Digital Sky Survey (SDSS) data and investigate their impact on the radio–IR relations.

In order to prepare for these large sky-area surveys, it is important to first investigate smaller sky-areas which are prototypical examples of future surveys. To this end, we have performed deep observations of the European Large Area ISO Survey–North 1 (ELAIS-N1) field covering an area of 1.8 deg² with a root mean square (rms) sensitivity of ~ 15 μJy beam⁻¹ centered at 400 MHz using the upgraded Giant Metrewave Radio Telescope (uGMRT) presented in Chakraborty et al. (2019). In this paper, we broadly classify the radio sources into AGN and SFGs using publicly available ancillary multi-waveband data to investigate the radio–IR relations up to z ~ 2. This paper is organised as follows: Section 2 describes the radio and multi-wavelength data used in this work. The different methods of identifying SFGs and AGN are described in Section 3. The radio and infrared spectral energy distribution (SED) fitting for k-correcting to the rest-frame are described in Section 4. In Section 5, we present our results on the statistical properties of the radio–IR relations, and discuss them in Section 6. A summary of our work is presented in Section 7. In this work, we have used the best-fit cosmological parameters from the Planck 2018 results (Planck Collaboration et al. 2020): ΩΛ = 0.68, Ωm = 0.31 and H0 = 67.36 km s⁻¹ Mpc⁻¹.

2 DATA

In this section, we present our observations of the ELAIS-N1 field using the uGMRT between 300–500 MHz, and discuss the salient features of radio continuum data at other radio frequencies. In addition, we also discuss in brief the assorted multi-wavelength survey data used for further analyses in this paper. The salient features of the different surveys used in this work are summarized in Table 1. In this work, we have identified the counterparts of our uGMRT sample by cross-matching them to their nearest neighbour in various multi-wavelength catalogues. We have used a search radius of 3 arcsec for all datasets except at 1.4 GHz where a search radius of 5 arcsec was used.

2.1 Radio continuum data

2.1.1 uGMRT observations at 400 MHz

Observations of the ELAIS-N1 field, centered at RA = 16h 10m 1s, Dec = 54d 30m 36s (J2000), were carried out in May–June 2017 using the uGMRT for a total of 25 hrs (including calibration overheads) spanning over four nights (proposal code: 32_120). These observations were performed in Band 3 covering the frequency range 300 to 500 MHz centered at 400 MHz using the new GMRT wideband (GWB) correlator with a frequency resolution of 24 kHz. A


Table 1. Salient features of various multi-waveband surveys of the ELAIS-N1 field. The columns represent the multi-wavelength catalogues with total area covered, resolution, and, corresponding 5σ sensitivity in mJy. The column ‘Size’ represents the number of sources in a catalogue with 400-MHz uGMRT counterpart, and their corresponding percentage are listed in the last column.

| Catalogue               | Total Area (deg²) | Resolution (" × ") | 5σ sensitivity (mJy) | Size     | Percentage |
|-------------------------|-------------------|--------------------|----------------------|----------|------------|
| uGMRT 400 MHz           | 1.8               | 4.6 × 4.3          | 0.075                | 2528¹    | 100        |
| LoTSS                   | 64                | 6 × 6              | 0.1                  | 2225     | 88         |
| GMRT 612 MHz            | 1.13              | 6 × 6              | 0.04                 | 1518     | 60         |
| FIRST                   | Large area survey | 5 × 5              | 0.75                 | 144      | 6          |
| BOSS                    | Large area survey |                    |                      | 597      | 24         |
| SWIRE all IRAC bands    | 2.0               |                    |                      | 1470     | 58         |
| SWIRE 24 μm             | 8.72              | 5.6 × 5.6          | 0.45                 | 1201     | 48         |
| SWIRE 70 μm             | 8.72              | 16.7 × 16.7        | 2.75                 | 388      | 15         |
| HerMES 250 μm           | 3.25              | 18.2 × 18.2        | 25.8                 | 702      | 28         |
| HerMES 350 μm           | 3.25              | 25 × 25            | 21.2                 | 686      | 27         |
| HerMES 500 μm           | 3.25              | 36.3 × 36.3        | 30.8                 | 557      | 22         |
| Redshifts               |                    |                    |                      | 2319     | 92         |

¹ Represents the number of sources compiled above ~ 6 σ with point source sensitivity ≥ 100 μJy. All other catalogues are matched to these 2528 sources.

2.1.4 FIRST data at 1.4 GHz

We have also utilized the Faint Images of the Radio Sky at Twenty centimetres (FIRST) survey (White et al. 1997) that has used the Very Large Array to compile a catalogue of 946,432 sources covering a sky-area of 10,575 deg² (Helfand et al. 2015). FIRST covers the ELAIS-N1 region with a relatively shallow 5σ sensitivity limit of 0.75 mJy at an angular resolution of 5 ″. On cross-matching with our 400-MHz uGMRT catalogue, we find 144 sources that has also been detected in the FIRST survey.

2.2 Ancillary multi-wavelength data

In order to classify the detected radio sources as AGN and SFGs, and to perform k-correction, we have used publicly available data in the mid- to far-infrared (MIR and FIR, at 24 and 70 μm, respectively) wavelength regime, and optical spectroscopy from the Sloan Digital Sky Survey (SDSS). Here we discuss the salient features of these data.

2.2.1 BOSS/SDSS Spectroscopy

The Baryon Oscillation Spectroscopic Survey (BOSS) refers to the dark time survey of the third phase of the SDSS (SDSS-III; York et al. 2000). BOSS consists of two spectroscopic surveys over an area of 10,000 deg² (see Eisenstein et al. 2005, 2011, for details). To observe ancillary science programs, a series of plates were added to the SDSS-III survey beyond 2012. Four plates were granted to observe and obtain spectra for the radio sources in the ELAIS-N1 field, and are publicly available as a part of the SDSS DR12.¹ The SDSS catalogue provides spectroscopic redshift (zspec) along with object-type classification of the spectra (see Bolton et al. 2012, for details). The radio counterparts of the sources in the BOSS catalogue were, in part, classified based on this information. Furthermore, the sources with reliable zspec are identified with the flag WARNING = 0, and 597 sources in the uGMRT catalogue at 400 MHz were found to have a reliable zspec. These sources were classified into AGN and SFGs, where SFGs also contain the starburst galaxies.

¹ https://www.sdss.org/dr12/algorithms/ancillary/boss/ sdslofar/
2.2.2 Infrared Data

The Spitzer Wide-Area Infrared Extragalactic (SWIRE) survey covers a sky area of 49 deg$^2$ using the Infrared Array Camera (IRAC) at 3.6, 4.5, 5.8, 8 μm, and using the Multi-Band Imaging Photometer (MIPS) at 24, 70 and 160 μm of the Spitzer space telescope (Lonsdale et al. 2003; Mauduit et al. 2012). As a part of the SWIRE survey, six extragalactic deep-fields were observed, including 8.72 deg$^2$ on the ELAIS-N1 field (Rowan-Robinson et al. 2008, 2013). This also includes another five bands (U', g', r', i', Z') from the Wide Field Survey using the 2.5-m Isaac Newton Telescope (McMahon et al. 2001). The revised SWIRE catalogue includes the J, H and K$_s$ bands from the Two Micron All Sky Survey (2MASS) and the UKIRT Infrared Deep Sky Survey (UKIDSS) in the near-infrared (Lawrence et al. 2007). The availability of a large number of photometric data reduces the fraction of catastrophic outliers making the photometry in the SWIRE catalogue one of the most reliable.

In addition, we have also made use of the Herschel Multi-tiered Extra-galactic Survey (HerMES) performed using the Herschel Space Telescope that mapped a set of nested fields covering a total area of ~380 deg$^2$ using the Herschel-Spectral and Photometric Imaging Receiver (SPIRE) at 250, 350 and 500 μm (Roseboom et al. 2010, 2012). SPIRE has covered 3.25 deg$^2$ area of the ELAIS-N1 field (Oliver et al. 2012) at these bands. We have utilized these far-infrared photometry in our study.

2.2.3 Redshifts

We obtained $z_{\text{spec}}$ for 597 sources (23 per cent) in the uGMRT catalogue from the BOSS catalogue discussed earlier. For 2216 sources (87 per cent) in the uGMRT catalogue, we have used the redshifts provided as a part of the LoTSS catalogue (Duncan et al. 2021), of which 555 sources have $z_{\text{spec}}$ from BOSS. For the remaining 1661 sources, 66 sources (4 per cent) have $z_{\text{spec}}$ from various other spectroscopic data, and 1595 sources (95 per cent) have photometric redshifts ($z_{\text{phot}}$). We also found 63 additional sources in the uGMRT catalogue that have no redshift information either from BOSS or LOFAR redshift catalogue but have $z_{\text{phot}}$ from the SPIRE photometric redshift catalogue (Rowan-Robinson et al. 2013). Overall, 2321 (92 per cent) of the radio sources in the uGMRT catalogue have redshift information, and these sources form the core sample of our analysis of the radio–IR relations.

3 AGN/SFG CLASSIFICATION

Since the majority of the radio sources in the 400-MHz uGMRT data do not have spectroscopic identification, it is challenging to robustly identify the AGN and the SFGs in a radio source catalogue. The complication mostly arises due to the fact that, although continuum and/or emission lines at other wavebands, e.g., in the infrared, optical and X-ray bands can discern AGN activity, whether the AGN-component of the emission significantly dominates over the emission due to star formation activity at radio frequencies remain unclear. This is especially the case for the ‘radio-quiet’ AGN (RQ AGN) population whose radio emission has been suggested to be dominated by star formation in the host galaxy (e.g., Sopp & Alexander 1991; Hodge et al. 2008; Retana-Montenegro 2022). To compare the properties of the radio–IR relations with SFGs, we have also studied the AGN population as well.

Following Bonzini et al. (2013), we have used four different source classification criteria, these are based on radio luminosity, colours in the IRAC bands, the flux density ratio at 24 μm in the MIR to that at 1.4 GHz in the radio ($q_{\mu m}$ parameter), and spectroscopy. In addition, we have also used source classification available from the LoTSS catalogue. We denote the number of AGN and SFGs identified using these methods as $N_{\text{AGN}}^+$ and $N_{\text{SFG}}^+$, respectively, where ‘+’ denotes the classification scheme, ‘m’ IR, ‘LoTSS’ ‘q’, and ‘spec’, for classification based on radio luminosity, IRAC colours, LoTSS catalogue, $q$-parameter, and spectroscopy, respectively. Table 2 summarizes the number of sources identified in both categories using these source classification schemes.

(i) **Radio luminosity based classification:** We computed the rest-frame luminosity at 1.4 GHz ($L_{1.4\text{GHz}}$) for the 2321 sources as,

$$L_{1.4\text{GHz}} = 4\pi d_L^2 \frac{S_{1.4\text{GHz}}}{(1+z)^{\alpha}}$$

where $S_{1.4\text{GHz}}$ is the observed flux density at 1.4 GHz, $d_L$ is the luminosity distance, and $\alpha$ is the spectral index (defined as $S \propto \nu^\alpha$) of a given source (see Section 4.1 for details). The sources with $L_{1.4\text{GHz}} > 10^{25} \text{W Hz}^{-1}$ are classified as ‘radio-loud’ AGN (RL AGN) (Sajina et al. 2007, 2008; Jiang et al. 2007), as such luminosities from star formation related synchrotron emission are highly unlikely in a large population of galaxies. In Fig. 1a, we show the variation of $L_{1.4\text{GHz}}$ as a function of $z$, and we found 190 sources ($N_{\text{AGN}}^+$), shown as the red points, meeting the above criterion that were classified as RL AGN.

(ii) **IRAC colour based classification:** The thermal radiation from dust, predominantly heated by the AGN, emits with a characteristic power-law spectrum at MIR wavelengths. We have used the criterion described by Donley et al. (2012) in the IRAC colour-colour plot for 1470 sources with redshift information shown in Fig. 1b to identify AGN using a black wedge. Using the IRAC colours, we found a total of 103 AGN ($N_{\text{IR}}^+$) in the 400-MHz uGMRT catalogue, all of which have redshifts.

(iii) **Spectroscopic classification:** As discussed in Section 2.2.3, the 597-sources with $z_{\text{spec}}$ from the BOSS catalogue also contain classified sources based on their spectra. We used the CLASS and SUBCLASS keywords in the BOSS catalogue for identifying AGN and SFGs which include starburst galaxies (see Bolton et al. 2012, for details). From BOSS spectroscopy, we found 533 sources that were classified as GALAXY and 62 as QSOs. The remaining two sources were classified as STAR and were not included in our further analysis. Of the 533 sources with CLASS GALAXY, 119 were sub-classified as STARFORMING (includes one STARFORMING BROADLINE source), 102 as STARBURST, and 20 as AGN (includes SUBCLASSes BROADLINE and AGN BROADLINE). The remaining 292 sources had no sub-classification. In summary, we identify a total of 82 AGN ($N_{\text{spec}}^+$), and 513 SFGs ($N_{\text{spec}}^+$) from BOSS spectroscopy (see Table 2).

(iv) **LOFAR based classification:** The LOFAR photometric redshift catalogue (Duncan et al. 2021) also contains sources classified based on various multi-wavelength information found in the literature. This catalogue includes AGN identified from the Half Million Quasars (HMQ) catalogue (Flesch 2015), IRAC color–color using Donley et al. (2012), and X-ray catalogue using Boller et al. (2016). We used the flag AGN in the LOFAR redshift catalogue for selecting AGN. In this way, we identify 219 AGN ($N_{\text{LoTSS AGN}}^+$) in our uGMRT sample.

(v) **$q_{24\mu m}$ based classification:** In addition to the above classification schemes, for the 1201 sources detected at 24 μm, we have also used the logarithmic ratio of the observed mid-IR at 24 μm and radio flux densities, $q_{24\mu m} = \log_{10}(S_{24\mu m}/S_{1.4\text{GHz}})$, to identify AGN. Here, $S_{24\mu m}$ is the observed flux density at 24 μm. In Fig. 1d we show the plot of observed $q_{24\mu m}$ versus redshift for the uGMRT-
Figure 1. Source classification schemes used in this work. (a) Variation of rest-frame luminosity at 1.4 GHz ($L_{1.4\,\text{GHz}}$) with redshift ($z$). The dashed black line indicates the threshold $L_{1.4\,\text{GHz}} = 10^{25}\,\text{WHz}^{-1}$ above which the sources were classified as RL AGN, and are shown as the red dots (see Section (i)). (b) IRAC color-color diagram (see Section (ii)). The black lines are from Donley et al. (2012) and the sources within these lines represents AGN while those outside are SFGs. The red dots represent AGN, and the blue triangles represent SFGs. (c) The redshift distribution of the sources in ELAIS-N1 field. The open histograms are spectroscopic redshift ($z_{\text{spec}}$) from BOSS which provides sources classified based on their spectra. The red and the dashed blue histograms are for AGN and SFGs, respectively. (d) IRAC colour-colour distribution. All histograms are normalized to unit area.

(d) Plot of observed $q_{24\,\mu m}$ values with $z$

Table 2. Summary of the number of sources identified using the four classification schemes used in this work (see Section 3.1). We find the following matches of AGN identified from radio luminosity with other classification schemes: $N_{\text{AGN}}^{\text{lum}} \cap N_{\text{IR}}^{\text{AGN}} = 8$, $N_{\text{AGN}}^{\text{lum}} \cap N_{\text{spec}}^{\text{AGN}} = 14$, $N_{\text{AGN}}^{\text{lum}} \cap N_{\text{LoTSS}}^{\text{AGN}} = 7$, $N_{\text{AGN}}^{\text{lum}} \cap N_{\text{LoTSS}}^{\text{SFG}} = 22$. Also note that we have dropped 2 sources that were classified as stars in the BOSS catalogue. Hence, we are left with 2319 sources with redshift measurements. $^\dagger$ These represents the number of non-AGN sources from the respective criterion, and does not identify SFGs adequately. In our study, all those sources are considered as SFGs that remained identified as non-AGN after combining all the selection criteria (see text for detail).
selected sources. We find a median $q_{24\mu m} = 0.62 \pm 0.20$ using the observer’s frame flux densities with median $z = 0.61$. Our value of median $q_{24\mu m}$ in the observer’s frame obtained by extrapolating the 400 MHz flux densities to 1.4 GHz matches excellently with those reported in the literature by using observations at 1.4 GHz (e.g., Appleton et al. 2004; Huyhn et al. 2010; Ibar et al. 2008). We have utilized the redshifted $q_{24\mu m}$ values from the SED template of the nearby starburst galaxy M 82 to differentiate SFGs and RL AGN as described in Bonzini et al. (2013). The M 82 template was normalized to the local average value of $q_{24\mu m}$ by Sargent et al. (2010b) which is shown as the solid black curve in Fig. 1d. The SFG locus is defined as the region with $\pm 2\sigma$ of the M 82 template shown by the black dashed curves in Fig. 1d, where $\sigma$ is the typical scatter of 0.35 dex. In summary, we classified a source as RL AGN if it has $L_{1.4\,GHz} > 10^{25}$ W Hz$^{-1}$ or if the $q_{24\mu m}$ value lies $2\sigma$ below the M 82 template curve. A source is classified as a radio-quiet AGN (RQ AGN) if an AGN identified using any of the other criteria lies within the SFG locus. Besides the SFGs that have spectroscopic identification in the BOSS catalogue, we consider the remaining sources within the SFG locus as SFGs for this work. Using the observer’s frame $q_{24\mu m}$ values, we identify 166 sources as RL AGN, 146 sources as RQ AGN, and the remaining 889 as SFGs, as described above. Thus, based on observed $q_{24\mu m}$ values, we identify 312 sources ($N_{\text{AGN}}$) as AGN that comprise both RQ and RL AGN obtained using this criterion. The black stars, purple dots, and green diamonds in Fig. 1d represent SFGs, RL AGN, and RQ AGN, respectively. We emphasize that we have not made any distinction between RL and RQ AGN in the rest of the paper, and both of them are referred to as AGN. For the remaining 283 sources with redshift measurements from SWIRE catalogue but having no counterpart at 24 $\mu$m, we have used the upper limits on their $q_{24\mu m}$, and are shown as the downward pointing arrows in Fig. 1d. It is clear that a bulk of these sources lie in the RL AGN regime (~ 80 per cent of the undetected sources at 24 $\mu$m) as per the redshifted SED template of M 82.

In Table 3, we summarize the number and fraction of AGN and SFGs with respect to the total of 2319 sources that were classified. The redshift distributions of AGN and SFGs identified from different methods are shown in Fig. 1c. The distribution of $z_{\text{spec}}$ from BOSS for AGN and SFGs are shown by the open histograms, and the distributions for $z_{\text{spec}}$ and $z_{\text{ph}}$ combined are shown by the shaded histograms, where the median redshift of AGN and SFGs in our sample are found to be 1.45 and 0.94, respectively.

### Table 3. Total unique number of SFGs and AGN identified from the selection criterion discussed in Section 3.1. We point out that we have not made any distinction between RL and RQ AGN, and commonly refer to them as AGN in this paper.

| Class       | Number | Percentage |
|-------------|--------|------------|
| AGN         | 556    | 23.9       |
| RQ AGN      | 146    | 6.3        |
| RL AGN      | 333    | 14.3       |

$$N_{\text{Total}}^{\text{AGN}} = N_{\text{lum}}^{\text{AGN}} \cup N_{\text{IR}}^{\text{AGN}} \cup N_{\text{LoTSS}}^{\text{AGN}} \cup N_{\text{spec}}^{\text{AGN}}$$

$$N_{\text{Total}}^{\text{RL AGN}} = (N_{\text{lum}}^{\text{AGN}} \cap N_{\text{SFG}}^{\text{RL}}) \cup (N_{\text{IR}}^{\text{AGN}} \cap N_{\text{SFG}}^{\text{RL}}) \cup (N_{\text{spec}}^{\text{AGN}} \cap N_{\text{SFG}}^{\text{RL}}) \cup (N_{\text{LoTSS}}^{\text{AGN}} \cap N_{\text{SFG}}^{\text{RL}})$$

$$N_{\text{Total}}^{\text{SFGs}} = 2319 - N_{\text{Total}}^{\text{AGN}}$$

3.1 Salient features of the classification schemes

In Table 2, we summarize the various methods we have used for classifying the radio sources in the ELAIS-N1 field into AGN and SFGs. Note that the number of SFGs ($N_{\text{SFG}}^{\#}$) represent those sources that were not identified as AGN from the different criteria discussed in the previous section, except for the ones that were identified as SFGs from BOSS spectroscopy. We would like to emphasize that the statistical selection of SFGs in this way could possibly misclassify AGN in composite systems as SFGs or vice versa. Larger gas content in star-forming low-excitation radio galaxies (LERGs) marks an increase in the population at high redshifts (Williams et al. 2018). Hence, the population of SFGs in our sample could be contaminated by LERGs at $z \geq 0.7$. As described in the previous section, we identify 146 sources as RQ AGN that indicate emission from AGN identified from the other four selection criteria (lum, IR, LoTSS, and spec), and lie within the M 82 SFG locus. Further, a source is considered as RL AGN if it was classified as AGN either from radio luminosity or from the observer’s frame $q_{24\mu m}$ values. In this way, we classify an overall of 333 sources as RL AGN. There are 77 AGN that were neither classified as RQ or RL AGN. Combining all the five source classification criteria, we find a total of 580 (~ 25 per cent) AGN (146 RQ AGN + 333 RL AGN + 77 AGN), while the rest 1763 (76 per cent) sources that have redshift measurements but do not show AGN characteristics are considered as SFGs in our study.

Interestingly, only 10 out of 190 RL AGN classified based on $L_{1.4\,GHz}$ has been detected in BOSS, of which 7 are identified as AGN from BOSS spectroscopic classification. Out of these, 4 sources were identified as AGN from IRAC colors. Additionally, 8 sources were identified as AGN using the $L_{1.4\,GHz}$ criteria and the IRAC colours. On the other hand, 14 of the 190 RL AGN identified using radio luminosity are also identified as RL AGN from the $q_{24\mu m}$ criteria. In general, from Fig. 1a we notice that RL AGN are largely missing in our radio observations, especially at $z \leq 0.2$. This is partly due to the relatively small co-moving volume probed by our uGMRT observations and flattening of the radio luminosity function of AGN at low redshifts (Mauch & Sadler 2007). Furthermore, the AGN population is dominated by LERGs at low redshifts (Heckman & Best 2014; Hardcastle & Croston 2020) making their detection challenging in large sky-area spectroscopic surveys, such as the BOSS. In contrast, spectroscopy tends to identify high-excitation radio galaxies (HERGs) via their strong high-excitation emission lines at high redshifts. Furthermore, from Fig. 1d it is clear that several sources that are undetected at 24 $\mu$m, falls under the RL AGN category. A combination of relative faintness and dust obscuration at near- and mid-infrared wavebands perhaps makes them undetectable in the SWIRE catalogue and explains the relatively low number of RL AGN that are identified by the classification schemes we have used. These RL AGN are hence missed when we study the radio–IR relations in later sections. We again emphasize that no distinction is made between RL and RQ AGN henceforth, and both types are considered as AGN in our study.
Counts

Figure 2. Left: Typical rest-frame radio SED fitted for one of the sources J161041+5410.54 at z = 0.227. The errorbars are the 3σ errors on the flux densities. The light blue lines show the power-law fit for 1000 samples randomly drawn with the 3σ flux density error at each frequency (see section 4.1 for details). The red dashed line represents the best-fit SED for this source. Right: The spectral index (α) distribution of the radio sources in the ELAIS-N1 field obtained by fitting the power-law shown on left for 1278 sources (see section 4.1). The shaded histograms represent the distribution for SFGs, AGN for and the combined population (SFG+AGN). The dashed line represents the median alpha of −0.58 for all the sources. The open histograms show the distribution by accounting for the error in the estimated α for each source. The median α then measured for SFGs and AGN are −0.58 and −0.57, respectively.

4 CORRECTION TO REST-FRAME

In this section, we present the k-correction methods we have applied to the observed flux densities at the radio and infrared wavebands for determining the respective rest-frame emissions. For this, we use only those sources that have redshift information.

4.1 k-correction at radio frequencies

To investigate the radio–IR relations, we determined the luminosity of the radio sources at 1.4 GHz and 400 MHz in the rest-frame. For this, we constructed the radio SED by taking advantage of the 200 MHz wide bandwidth of the uGMRT data, and by using flux densities measured at 146 MHz, 612 MHz and near 1.4 GHz, wherever available. To obtain a reliable spectral index (α), we model the radio SED between the frequency range 100 MHz and 1.4 GHz as a power-law of the form,

\[ S(ν) = S_{1.4\,\text{GHz}} \left( \frac{ν}{1.4\,\text{GHz}} \right)^{α}, \]

where ν is expressed in GHz.

To achieve a robust radio SED modeling, we first divided the uGMRT data, covering the 300–500 MHz frequency range, into narrower 50 MHz bandwidth centered at 325, 375, 425, and 475 MHz. Each of these narrow bandwidth data were imaged using the task tclean available as a part of the CASA package.\(^2\) Using P-BDSF on each image individually, we compiled four catalogues at 325, 375, 425, and 475 MHz, wherein we obtained 1513, 1584, 2199, and 1366 sources, respectively, above 5σ.

Out of the 2321 sources in the 400-MHz uGMRT catalogue that have redshift information, we have obtained α for 1278 sources, measured using power-law SED fitting. In the right-hand panel of Fig. 2, we show the distribution of α of these 1278 sources as the grey shaded histogram. The distributions of 351 AGN and 927 SFGs are shown as the red and blue shaded histograms. For the overall sample of 1278 sources, we find that the median α = −0.57 ± 0.005 having a dispersion of 0.12. For the SFGs, we find that the median α = −0.57 ± 0.006 with a dispersion of 0.12, and for AGN, the median α = −0.57 ± 0.008 that has a dispersion of 0.14. In order to accurately account for the error on individual estimates of α for determining its statistical properties, we have drawn 1000 random samples of α within their respective error for each of the 1278 sources. The distribution of α for these 1000 random samples is shown as the open histograms in Fig. 2 (right-panel). From this, for all the sources, we find the median α = −0.58 ± 0.15, wherein for SFGs the median α = −0.58 ± 0.15, and for AGN the median α = −0.57 ± 0.16. Here, the errors on the median values are the MAD measured from the distribution. For the remaining 1043 sources that have redshift measurements but do not have enough data points to perform SED fitting, we have assumed a constant spectral index of \( α = −0.7 \) to estimate \( L_{1.4\,\text{GHz}} \) for classification in Section 3 only. We would like to point out that for our further analyses, we have used only those sources that have α measured from the SED fits to keep the statistical population the same throughout the paper.

4.2 k-correction at infrared wavelengths

In this study we examine the radio–IR correlation between the rest-frame IR emission (both monochromatic and bolometric) and the radio emission. We match our radio catalogue with the SWIRE and

\(^2\) http://casa.nrao.edu/ (McMullin et al. 2007).
the HerMES catalogues to obtain a total of 634 sources for which we fit the infrared SED as described below.

For estimating the monochromatic and bolometric luminosities at infrared wavelength, we model the infrared SED using a composition of single temperature modified-blackbody (graybody) and a truncated mid-IR power-law (Casey 2012). We chose this composite model of the infrared SED because a single temperature graybody spectrum does not fit the mid-IR observations well, while multi-temperature graybody SED model introduces several free parameters. The infrared SED is modeled as (Casey 2012),

\[ S(λ) = A_{\text{GB}} \left( \frac{1 - e^{-\tau_1}}{\text{e}^{(λ/λ_1) - 1}} \right) + A_{\text{PL}} \left( \frac{λ}{λ_0} \right)^{α_{\text{IR}}} e^{-λ/(κλ)} , \]  

(3)

Here, \( A_{\text{GB}} \) and \( A_{\text{PL}} \) are the amplitude normalization for graybody and mid-IR power-law, respectively; \( λ_1 \) is the mid-IR turnover wavelength; \( α_{\text{IR}} \) is the mid-IR powerlaw index; \( τ_1 = (A_0/λ)^β \) is the dust optical depth and has a value of unity at \( A_0 = 200 \, μm \); \( β \) is the dust emissivity index assumed to be constant with \( β = 1.5 \) (Casey 2012; Magnelli et al. 2014); \( T \) is the temperature; and \( c, h, κ, κ_0 \) are Planck constant, speed of light, and Boltzmann constant, respectively. The parameters \( A_{\text{PL}} \) and \( λ_0 \) coupled to the rest (see Casey 2012), and for the assumed value of \( β = 1.5 \), the number of free parameters reduces to three, namely, \( A_{\text{GB}}, T, \) and \( α_{\text{IR}} \). Therefore, at least four data points in the infrared waveband were used to constrain the SED in the mid- to far-infrared wavelengths. Further, in order to constrain the mid-IR power-law, we ensured that all sources be detected at 24 μm in the SWIRE catalogue. In Fig. 3, we show a typical SED in the rest-frame infrared waveband for the same source, J161041+5410.54, shown in Fig. 2 (left-hand panel). The graybody and the mid-IR power-law components are shown as the red dashed and the blue dot-dashed curves, respectively, and the solid black curve shows the total SED model. Note that, the effective dust temperature (\( T_{\text{dust}} \)) was obtained by using the Wien’s displacement law, \( T_{\text{dust}} = b/λ_{\text{peak}} \), where \( b = 2.898 \times 10^3 \, μm \, K \), and \( λ_{\text{peak}} \) (in μm) is the wavelength of the peak of the SED.

In order to determine the \( k \)-corrected bolometric luminosity for a source, we integrated the fitted SED between the rest-frame wavelength range 8 and 1000 μm to obtain the total infrared (TIR) flux, and converted them to luminosity \( L_{\text{TIR}} \). On the other hand, monochromatic flux densities were obtained at 24 and 70 μm from the best-fit SED in the rest-frame, and were converted to \( L_{24μm} \) and \( L_{70μm} \), respectively.

In the left-hand panel of Fig. 4, we show the variation of \( T_{\text{dust}} \) with \((1+z)\), and find \( T_{\text{dust}} \) to increase with redshift for both SFGs and AGN in our sample. A linear increase of \( T_{\text{dust}} \) with redshift was reported by Kovács et al. (2006) for sub-millimetre galaxies in the redshift range 1.5–3.5, and by Basu et al. (2015b) for ‘blue cloud’ galaxies up to \( z = 1.2 \) in the XMM-LSS field. This increasing trend could be caused due to \( T_{\text{dust}} \) being correlated with the total infrared luminosity, as shown in the right-hand panel of Fig. 4, and which, in turn, is correlated with redshift due to flux-limited surveys. Overall, there are 634 sources (520 SFGs and 114 AGN) for which we have obtained the rest-frame infrared luminosities. For 450 of these sources (349 SFGs and 101 AGN), we have measured values of \( α \) using multiple radio frequencies, and these sources form the core sample in our study of the radio–IR relations in the next sections. We would like to emphasize that, although the parent sample is drawn from the 400-MHz uGMRT data, these 450 sources used for further analysis have a complicated selection function due to the requirement imposed for the radio and the infrared SED fitting.

5 RESULTS

Here, we focus on the properties of the radio–IR relations using the rest-frame emission for the SFGs and AGN detected in the deep uGMRT observations of the ELAIS-N1 field at 400 MHz. In the radio waveband, we use the monochromatic rest-frame emission at 1.4 GHz and 400 MHz; and, to our knowledge, for the first time we will use the bolometric radio emission in high-redshift sources. In the infrared, we have used different measures of luminosity, i.e., monochromatic luminosities in the MIR at 24 μm \( (L_{24μm}) \) and in the FIR at 70 μm \( (L_{70μm}) \); and the total infrared luminosity integrated between 8 and 1000 μm \( (L_{\text{TIR}}) \). We first present our results obtained for the ‘\( q \)’ parameter, defined as the logarithmic ratio of the \( k \)-corrected luminosity at infrared \( (L_{\text{IR}}) \) to that at radio frequencies \( (L_{ν}) \), and is given as,

\[ q_{\text{IR}} = \log_{10} \left( \frac{L_{\text{IR}}}{L_{ν}} \right) \].

(4)

Here, \( IR = 24μm, 70μm \) and \( ν \) is the radio frequency. In this analysis, we will present the statistical properties of the different types of \( q_{\text{IR}} \) to study the radio–IR relations. We then study the slope \( b \) of the radio–IR relations given as \( L_{ν} \propto L_{\text{IR}}^b \) in the log–log space.

5.1 Variation of monochromatic ‘\( q \)’ parameter

The ‘\( q \)’ parameter is often used in the literature to study the evolution of the radio–IR relations. At MIR wavelengths, \( q_{24μm} \) is used for discerning dominant emission from AGN which gives rise to an excess in the radio emission (e.g., Padovani et al. 2011; Bonzini et al. 2013), and thereby, lower \( q_{24μm} \) as compared to the emission from the star-forming counterparts. Once the SFG population in a sample is identified, the radio–MIR relation can be used to calibrate the radio emission to trace star formation rates in high redshift galaxies (Madau & Dickinson 2014; Murphy et al. 2011). At MIR \((~ 10–30μm)\), however, the emission could be contaminated by cirrus dust heated by old stars and/or an obscured AGN, or emission from polycyclic aromatic hydrocarbons (PAHs). In that case, \( q_{70μm} \) or \( q_{\text{TIR}} \) are alternatively used. For a modified graybody-type SED, the IR emission in these bands are dominated by cold \( ~ 20 \, K \) dust emission in star forming...
In the left panel of Fig. 5, we show the variation of $T_{\text{dust}}$ with redshift $(1+z)$. In this section, we will investigate the statistical properties of $q_{24\mu m}$ and $q_{70\mu m}$, and their variation with redshift.

5.1.1 $q_{24\mu m}$

In the left panel of Fig. 5, we show the variation of $q_{24\mu m}$, computed using $k$-corrected emission at $24\mu m$ and at $1.4\times10^4$ Hz, with $z$ for the 450 sources. The SFGs and AGN are shown as the star and diamond symbols respectively, and the colours represent their $T_{\text{dust}}$. For these sources, we find $(q_{24\mu m}) = 1.03 \pm 0.01$ with 1 $\sigma$ dispersion of 0.37. Hereafter, we will present the corresponding 1 $\sigma$ standard deviation in parenthesis. For comparison, we show the mean and 1 $\sigma$ dispersion of $q_{24\mu m}$ from Appleton et al. (2004) as the solid and dashed lines, respectively. At an average, we find $(q_{24\mu m})$ for AGN to be slightly lower: $(q_{24\mu m})_{\text{AGN}} = 0.83 \pm 0.01(0.47)$ than that of SFGs: $(q_{24\mu m})_{\text{SFG}} = 1.09 \pm 0.01(0.32)$. However, within errors, this difference is insignificant. The values of $q_{24\mu m}$ are listed in the top three rows of Table 4.

The values of $(q_{24\mu m})$ and its dispersion we have obtained for the sources in the ELAIS-N1 field are consistent with those reported in the literature for cosmologically distant sources. For example, Ibar et al. (2008) find $(q_{24\mu m}) = 1.03 \pm 0.31$ which remains roughly constant in the range $0 < z < 1$, and up to $z \sim 1.4$. Sargent et al. (2010a) find $(q_{24\mu m}) = 1.26 \pm 0.13$ for SFGs in the COSMOS field. However, on a cursory look, in contrast to previous studies, we find $q_{24\mu m}$ to generally increase with $z$ in Fig. 5, especially at $z \gtrsim 1.5$ for our radio selected sample of SFGs and AGN. This is a consequence of increasing $T_{\text{dust}}$ with both $z$ and luminosity as seen in Fig. 4, and can be gleaned from the left-hand panel of Fig. 5 which indicates that at a given redshift, the higher values of $q_{24\mu m}$ correspond to higher $T_{\text{dust}}$.

Often in the literature, $k$-correction at MIR wavelengths is performed, or $q_{24\mu m}$ in the observer’s frame is compared, by assuming a M82-like SED template (e.g., Appleton et al. 2004; Ibar et al. 2008; Sargent et al. 2010a; Bonzini et al. 2013; Ocran et al. 2017). Therefore, to compare our $k$-corrected $q_{24\mu m}$ with that of a M82-like template, we compute the normalized $q_{24\mu m}(q_{24\mu m,\text{norm}})$ for a source as the ratio of the observed $q_{24\mu m}$ (points in Fig. 1d) to the mean value of $q_{24\mu m}$ at the redshift of the source obtained from the M82 template, i.e., the solid black curve shown in Fig. 1d. In the right panel of Fig. 5, we show the variation of $q_{24\mu m,\text{norm}}$ from M82-like $k$-correction with $z$ in black, and of $k$-corrected $q_{24\mu m}$ from our SED fitting, divided by $(q_{24\mu m})$, in blue. From the figure, it is clear that both the methods indicate an increasing trend in $q_{24\mu m}$ with $z$.

Furthermore, it also indicates that $k$-correction of $q_{24\mu m}$ obtained either by using a M82-like template, or by directly fitting the mid-to-far-infrared SED using equation 3, do not show any strong systematic differences for the sample of radio selected sources from the uGMRT observations, apart from a constant offset of the absolute values of $q_{24\mu m}$. The estimated $q_{24\mu m}$ lies within 0.36 dex of each other.

5.1.2 $q_{70\mu m}$

The variation of $q_{70\mu m}$ with $z$ is shown in the left-hand panel of Fig. 6 where we have used the $k$-corrected flux densities at $70\mu m$ and $1.4\times10^4$ Hz. The symbols and color scheme are the same as used in the left-hand panel of Fig. 5. The solid black and the dashed lines show the mean $q_{70\mu m}$ value of 2.15 and $1\sigma$ dispersion of 0.16 from Appleton et al. (2004). Table 4 tabulates the $q_{70\mu m}$ values for both the classes of sources. The $(q_{70\mu m}) = 1.84 \pm 0.01(0.42)$ for AGN is found to be significantly lower than the $(q_{70\mu m}) = 2.18 \pm 0.01(0.26)$ for SFGs implying that, in the FIR wavelengths, an excess radio emission from AGN is better captured compared to $q_{24\mu m}$. For the combined population, we find $(q_{70\mu m}) = 2.10 \pm 0.01(0.34)$.

Our measured value of $(q_{70\mu m})$ is close to that reported in the literature (e.g., Appleton et al. 2004; Sargent et al. 2010a; Mao et al. 2011; Basu et al. 2015b). Unlike $q_{24\mu m}$ increasing with $z$, we find $q_{70\mu m}$ to remain roughly constant up to $z = 1.5$. This is caused due to the increasing dust temperature with both, the redshift and the luminosity of the sources (see Fig. 4).

Further, we have also determined the values of $(q_{24\mu m})$ and $(q_{70\mu m})$ for SFGs and AGN using rest-frame luminosity at 400 MHz ($L_{400\text{MHz}}$). These are listed in the Table 4 (rows four–six). Overall, the mean $q$ obtained using $L_{400\text{MHz}}$ are lower than that for $L_{1.4\text{GHz}}$.

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Note that, henceforth, we will present $k$-corrected $q_{24\mu m}$ values unless mentioned otherwise. This is different from the $q_{24\mu m}$ at observer’s frame discussed in Section (v).
due to the nature of the radio continuum spectrum. We find the dispersion of $q_{24\mu m}$ and $q_{70\mu m}$ with respect to their respective mean values to be similar for both $L_{1.4\,\text{GHz}}$ and $L_{400\,\text{MHz}}$ in SFGs, however, these are slightly higher for AGN at 400 MHz.

5.2 Variation of bolometric $q_{\text{TIR}}$ parameter

5.2.1 Using monochromatic radio luminosities

The statistical properties of monochromatic $q$, i.e., $q_{24\mu m}$ and $q_{70\mu m}$ are usually affected by the variation of $T_{\text{dust}}$ of the sources in a sample which gives rise to larger scatter, and/or systematic variation as seen in Fig. 5. Therefore, total IR luminosity ($L_{\text{TIR}}$) integrated between 8 and 1000 μm is used (Helou et al. 1985; Bourne et al. 2011), and the corresponding $q_{\text{TIR}}$ is defined as,

$$q_{\text{TIR}} = \log_{10} \left( \frac{L_{\text{TIR}}}{3.75 \times 10^{12} \, \text{W}} \right) - \log_{10} \left( \frac{L_{1.4\,\text{GHz}}}{4 \, \text{kHz}} \right).$$

(5)

In the right panel of Fig. 6, we show the variation of $q_{\text{TIR}}$ with redshift. The SFGs and AGN are marked as star and diamond symbols, and their colours represent the $T_{\text{dust}}$). We measure a mean $\langle q_{\text{TIR}} \rangle$ of 2.39 ± 0.01 (0.30) for the whole sample in the entire redshift range, which is in excellent agreement with previous estimates in the literature (e.g., Bell 2003; Ivison et al. 2010; Thomson et al. 2014; Basu et al. 2015b; Ocran et al. 2017). The $q_{\text{TIR}}$ value for our uGMRT sample remains roughly constant upto $z \sim 1.5$ with $\langle q_{\text{TIR}} \rangle = 2.46 \pm 0.01 (0.23)$ for SFGs across the full redshift range, whereas, AGN have a slightly lower $\langle q_{\text{TIR}} \rangle = 2.15 \pm 0.01 (0.38)$ and larger scatter compared to the SFGs. The values of $q_{\text{TIR}}$ are tabulated in Table 4.

In the top panel of Fig. 7, the distribution of $q_{\text{TIR}}$ for SFGs and AGN are respectively shown as the open and the filled histograms. From the distributions of $q_{\text{TIR}}$ it is clear that for the SFGs, $q_{\text{TIR}}$ has a symmetric distribution with smaller dispersion compared to AGN which shows a broader tail especially towards lower values of $q_{\text{TIR}}$. This is a manifestation of the fact that there is an excess of radio emission in AGN as compared to that in the SFGs. We note that the tail towards lower values of $q_{\text{TIR}}$ is likely to extend further.

However, as discussed in Section 3.1, the RL AGN population are missed in our sample due to the flux limits applied to our sample and perhaps obscuration at 24 μm. Because of this, in our sample, a large fraction of the AGN are found to be overlapping with the SFGs in the radio–IR relations. This may indicate that the radio emission in these AGN, that are likely to be RQ AGN, is dominated by star formation.

5.2.2 Using bolometric radio luminosity

Here we extend the radio–IR relation by using the bolometric radio continuum (RC) emission in our sample of sources. We computed the bolometric radio luminosity ($L_{\text{RC}}$) for the 450 sources by integrating their radio SEDs between the frequencies $v_1 = 0.1 \times 10^9 \, \text{Hz}$ and $v_2 = 2 \times 10^9 \, \text{Hz}$ in the rest-frame as,

$$L_{\text{RC}} = \int_{v_1}^{v_2} \left( \frac{L_v}{\text{W} \, \text{Hz}^{-1}} \right) \left( \frac{\text{d}v}{\text{Hz}} \right).$$

(6)

We note that, for a sample of nearby galaxies, Tabatabaei et al. (2017) computed the radio emission integrated between 1 and 10 GHz to study the distribution of $q$-parameters after separating the relatively high contribution of the thermal free–free emission in this frequency range.

To our knowledge, there is no standard definition of bolometric radio luminosity in the literature. Due to the relatively higher contribution of free–free emission above 2 GHz (e.g., Tabatabaei et al. 2017; Algera et al. 2021), higher radio frequency emission would be contaminated. On the other hand, at frequencies below ~ 0.1 GHz, free–free absorption and/or ionization losses could affect the radio continuum emission in star forming galaxies (Basu et al. 2015a). Both these effects tend to modify the power-law synchrotron spectrum. Therefore, we have avoided frequencies above 2 GHz and below 0.1 GHz for computing $L_{\text{RC}}$. Compared to $L_{1.4\,\text{GHz}}$ or $L_{400\,\text{MHz}}$, $L_{\text{RC}}$ has the advantage of being dominated by synchrotron emission and is less susceptible to systematic and statistical fluctuations in the estimated values of $\alpha$. In Appendix A, we discuss the various advantages of using $L_{\text{RC}}$.

Figure 5. Left: Variation of k-corrected $q_{24\mu m}$ with redshift ($z$). The symbols are color-coded on the basis of their dust temperature ($T_{\text{dust}}$). Right: Variation of normalised $q_{24\mu m}$ ($q_{24\mu m,\text{norm}}$) with $z$. The black symbols are for $q_{24\mu m}$ in the observer’s frame divided by the redshifted value of $q_{24\mu m}$ from M82-type SED template, and the cyan symbols represents the k-corrected $q_{24\mu m}$ using SED fitting divided by the mean value. The star and diamond symbols represent SFGs and AGN, respectively, in both the panels.
Using $L_{\text{RC}}$, we define the different $q$-parameters as,

$$q_{\lambda}^\text{RC} = \log_{10} \left( \frac{L_{\lambda}}{\text{W Hz}^{-1}} \right) - \log_{10} \left( \frac{L_{\text{RC}}}{1.4 \times 10^9 \text{[W]}} \right),$$

for monochromatic infrared luminosities ($L_{\lambda}$), where $\lambda = 24$ and 70 $\mu$m, and,

$$q_{\text{TIR}}^\text{RC} = \log_{10} \left( \frac{L_{\text{TIR}}}{3.75 \times 10^{12} \text{[W]}} \right) - \log_{10} \left( \frac{L_{\text{RC}}}{1.4 \times 10^9 \text{[W]}} \right),$$

for total infrared luminosity. The $1.4 \times 10^9$ factor is for normalizing $L_{\text{RC}}$ at 1.4 GHz.

The mean values of $q_{24\mu m}^\text{RC}$, $q_{70\mu m}^\text{RC}$ and $q_{\text{TIR}}^\text{RC}$ are listed in the bottom three rows of Table 4. The mean values of $q_{\text{TIR}}^\text{RC}$ for SFGs and AGN are found to be $2.16 \pm 0.01(0.23)$ and $1.87 \pm 0.01(0.39)$, respectively, and the AGN have larger relative scatter compared to the SFGs. For $q_{\text{TIR}}^\text{RC}$, the difference in the mean values for SFGs and AGN are more discernible compared to that for $q_{\text{TIR}}$. This can be gleaned from the bottom panel of Fig. 7 which shows indication that the peak of the distribution of $q_{\text{TIR}}^\text{RC}$ for AGN is shifted towards lower values compared to SFGs. Although there is significant overlap in the distribution of $q_{\text{TIR}}^\text{RC}$ between SFGs and AGN, $L_{\text{RC}}$ is perhaps a better measure to distinguish low luminosity AGN.

5.3 Radio–infrared relations

5.3.1 Using monochromatic radio luminosity

Here, we study the variation of the radio luminosity $L_{1.4\text{GHz}}$ with different rest-frame infrared luminosities, namely, monochromatic $L_{24\mu m}$ and $L_{70\mu m}$ at 24 $\mu$m and 70 $\mu$m, and bolometric $L_{\text{TIR}}$ of the total infrared emission. We fit these relations using orthogonal distance regression (ODR) in the log–log space with the form $L_{1.4\text{GHz}} = a L_{\text{TIR}}^b$. Here, $a$ is the normalization, $b$ is the slope (in log–log space), and IR = 24 $\mu$m, 70 $\mu$m and TIR. Further, to avoid any biases that could arise from inadequate source identification and severe incompleteness in our flux-limited sample beyond $z \sim 2$, we have fitted using the data up to $z = 2$. The values of $b$ obtained for the different radio–IR relations are listed in Table 4. In the table, for completeness, we also present the slopes obtained for AGN, and for AGN and SFGs together.

For all the three types of radio–IR relations, we find the radio and infrared luminosities to be strongly correlated with Spearman’s rank correlation $r > 0.9$. Of them, the correlation between $L_{1.4\text{GHz}}$ and $L_{\text{TIR}}$ is found to be the strongest with $r = 0.95$. For our sample,

Figure 6. Left: Variation of $k$-corrected $q_{70\mu m}$ values with redshift ($z$). The solid line represent the mean $q_{70\mu m}$ value of 2.15 with 1 $\sigma$ dispersion of 0.16 shown in dashed lines from Appleton et al. (2004). Right: Variation of $q_{\text{TIR}}$ with $z$. The black open pentagon denotes the median $q_{\text{TIR}}$ values with each redshift bin for SFGs up to $z = 2$ and the power-law fit for the same is shown in dashed black line. Other lines represent the evolution of $q_{\text{TIR}}$ with $z$ from previous works reported in the literature. The star and diamond symbols represent the SFGs and AGN, respectively, in both the panels, and are colour coded based on dust temperature ($T_{\text{dust}}$).

Figure 7. Distributions of $q_{\text{TIR}}$ (top panel) using monochromatic radio luminosity at 1.4 GHz and of $q_{\text{TIR}}^\text{RC}$ (bottom panel) using bolometric radio luminosity. The open green and the shaded gray histograms are for SFGs and AGN, respectively.
we find the slope $b$ for all the relations to be non-linear with high statistical significance ($> 3\sigma$). For SFGs, we find the $L_{1.4\,\text{GHz}}-L_{24\,\mu\text{m}}$ relation to be sub-linear with slope $b = 0.93 \pm 0.02$, and the $L_{1.4\,\text{GHz}}-L_{70\,\mu\text{m}}$ and $L_{1.4\,\text{GHz}}-L_{\text{TIR}}$ relations to be super-linear with slopes $1.05 \pm 0.02$ and $1.07 \pm 0.02$, respectively. For the AGN, the radio–IR relations are slightly weaker compared to the SFGs with $r \sim 0.9$, and also have super-linear slopes of $1.10 \pm 0.06$ and $1.08 \pm 0.07$ for the $L_{1.4\,\text{GHz}}-L_{\text{TIR}}$ and $L_{1.4\,\text{GHz}}-L_{70\,\mu\text{m}}$ relations, respectively. Similar to SFGs, AGN also show a sub-linear slope of $b = 0.87 \pm 0.05$ for the $L_{1.4\,\text{GHz}}-L_{24\,\mu\text{m}}$ relation. When both the SFGs and AGN are combined, the slopes for the three radio–IR relations do not change significantly (see Table 4). This is perhaps due to the fact that MIR based sample-selection misses the radio bright AGN, while the RQ AGN in our sample follow the relations in a same way as the SFGs. Furthermore, within the uncertainties, the normalization $a$ of the relations are similar for both SFGs and AGN (see Table 4). This further reiterates the fact that the radio emission in our sample of AGN, $\sim 50$ per cent of which are RQ AGN, are dominated by star-formation in the host galaxies as have been suggested previously (e.g., Kimball et al. 2011; Padovani et al. 2011; Ocran et al. 2017).

The slope for both the $L_{1.4\,\text{GHz}}-L_{\text{TIR}}$ and $L_{1.4\,\text{GHz}}-L_{70\,\mu\text{m}}$ relations we obtained using our sample are in good agreement with Bell (2003) who also reported a super-linear slope of $1.10 \pm 0.04$ for their sample of normal star-forming galaxies. A similar non-linear slope of 1.12 $\pm 0.06$ was found by Basu et al. (2015b) by stacking blue-cloud galaxies up to $z = 1.2$ in the XMM-LSS field. In fact, for a carefully selected sample of about 2000 SFGs at relatively low-$z$ ($< 0.2$), a similar non-linear slope of $b = 1.11 \pm 0.01$ for the $L_{1.4\,\text{GHz}}-L_{\text{TIR}}$ relation has been found (Molnár et al. 2021). This suggests that the slope of the $L_{1.4\,\text{GHz}}-L_{70\,\mu\text{m}}$ relation remains the same up to at least $z = 2$ and that the non-linearity is likely to be intrinsic for SFGs irrespective of the selection criterion.

We have also studied the radio–IR relations at 400 MHz using the rest-frame luminosity $L_{400\,\text{MHz}}$. In rows four to six of Table 4, we present the values of the slopes for all the three measures of infrared luminosity using $L_{400\,\text{MHz}}$. The slopes of the $L_{400\,\text{MHz}}-L_{24\,\mu\text{m}}$, $L_{400\,\text{MHz}}-L_{70\,\mu\text{m}}$, and $L_{400\,\text{MHz}}-L_{\text{TIR}}$ relations are similar to those obtained using $L_{1.4\,\text{GHz}}$. We do not find any systematic differences in the statistical properties of the relations when $L_{400\,\text{MHz}}$ is used instead of $L_{1.4\,\text{GHz}}$.

Figure 8. Variation of bolometric radio luminosity (from 0.1–2 GHz) with rest-frame luminosity at 24 $\mu$m ($L_{24\,\mu\text{m}}$) in the left and at 70 $\mu$m ($L_{70\,\mu\text{m}}$) in the right. The star and the diamond symbols represents SFGs and AGN, respectively, and are colored based on their redshifts. The solid lines in both the panels show the best-fit straight line in log–log space using only SFGs up to $z = 2$.

Figure 9. The variation of bolometric radio luminosity ($L_{\text{RC}}$) with total infrared luminosity ($L_{\text{TIR}}$). The best-fit straight line to these luminosities in log–log for SFGs are shown by the solid black line up to $z = 2$. The star and diamond symbols represent SFGs and AGN, respectively, and are color coded on their redshifts.

5.3.2 Using constant $\alpha = -0.7$ for all sources

In order to check whether $L_{1.4\,\text{GHz}}$ when computed using measured values of $\alpha$ has any systematic impact on the radio–IR relation as compared to when $L_{1.4\,\text{GHz}}$ is computed by assuming the same value of $\alpha$ for all the sources, we also studied the radio–IR relation using the latter. For this, we computed $L_{1.4\,\text{GHz}}$ by assuming $\alpha = -0.7$ for the same population of 450 sources and $k$-corrected the flux densities measured at 400 MHz. The results of the radio–IR relations thus obtained are presented in Table 4 (rows seven–nine). Except for a slight decrease in the scatter of the radio–IR relations, we do not find any systematic differences in the mean values of the $q$ parameters and the slopes for this assumption on $\alpha$. This suggests that the method of $k$-correcting the radio flux densities does not systematically affect the radio–IR relations.

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5.3.3 Using bolometric radio luminosity

We also study the variation of the bolometric radio luminosity integrated in the frequency range 0.1–2 GHz with monochromatic infrared luminosities $L_{24\mu m}$ and $L_{70\mu m}$, and, total infrared luminosity $L_{\text{TIR}}$ at high $z$. Fig. 8 shows the variation $L_{\text{RC}}$ with $L_{24\mu m}$ (left-hand panel) and with $L_{70\mu m}$ (right-hand panel). In Fig. 9, we show the variation of $L_{\text{RC}}$ with $L_{\text{TIR}}$. The solid lines show the best-fit using ODR as discussed in Sec. 5.3.1.

For our uGMRT sample, we find that all the three bolometric radio–IR relations, $L_{\text{RC}}$–$L_{24\mu m}$, $L_{\text{RC}}$–$L_{70\mu m}$ and, $L_{\text{RC}}$–$L_{\text{TIR}}$ are strongly correlated with $r > 0.93$ (marginally stronger than the relations with $L_{1.4\text{GHz}}$ for SFGs and have super-linear slopes (rows ten–twelve in Table 4). The $L_{\text{RC}}$–$L_{\text{TIR}}$ relation for SFGs show the strongest correlation with $r = 0.96$ and slope $b = 1.07 \pm 0.02$. In fact, AGN also show a strong $L_{\text{RC}}$–$L_{\text{TIR}}$ correlation with $r = 0.92$ (slightly weaker than SFGs) and a slope of $b = 1.10 \pm 0.06$. Interestingly, within the errors, the slopes of the $L_{\text{RC}}$–$L_{\text{TIR}}$ relations for SFGs and AGN are found to be similar to that with monochromatic radio luminosities, however, the slopes are found to be significantly ($>4.5\sigma$) larger for $L_{\text{RC}}$–$L_{24\mu m}$ and $L_{\text{RC}}$–$L_{70\mu m}$ relations compared to the corresponding relations with $L_{1.4\text{GHz}}$ and $L_{400\text{MHz}}$ for the SFGs (see Table 4). For AGN, this increase in the slopes is at $>2\sigma$ level. Furthermore, in contrast to the sub-linear slopes we have found between monochromatic radio and $24\mu m$ luminosities, the slope is super-linear for $L_{\text{RC}}$. It is unclear what gives rise to this significant change in slopes of the bolometric radio–IR relations, and this trend requires to be investigated using multi-frequency radio continuum data in other well-known deep fields.

### 6 DISCUSSIONS

Deep observations at radio frequencies in combination with multi-wavelength information, makes the statistical study of SFGs and AGN feasible. After broadly classifying the sources in the ELAIS-N1 field in our uGMRT observations at 400 MHz into AGN and SFGs, we have studied the properties of the radio–IR relations. In this section, we discuss our results on these relations and their evolutionary properties in the context of SFGs in our sample. We will primarily focus on the properties of the radio–IR relation for monochromatic radio emission, and later compare with them what we observe for the bolometric radio emission in the context of their variation with redshift.

#### 6.1 Dispersion in q parameters

In Section 5.1, we studied the variation of the monochromatic $q$ parameters at 24 and 70 $\mu m$ with $z$, and in Section 5.2 that of the TIR using the radio emission at 1.4 GHz. In general, we find the relative dispersion of up to $\sim 40$ per cent in $q_{24\mu m}$ to be significantly larger than those of $q_{70\mu m}$ and $q_{\text{TIR}}$ which have $\leq 16$ per cent dispersion for the whole sample. This behaviour is also true for the SFGs and AGN in our sample, however the dispersions of $q$ values in SFGs are lower than the AGN (see Table 4). To assess the impact on $q$ parameters for $k$-correcting the radio emission using the standard method of assuming a constant value of $\alpha$ for all the sources, we have also presented the values of $q_{24\mu m}$, $q_{70\mu m}$ and ($q_{\text{TIR}}$) obtained by assuming a typical $\alpha = -0.7$ for all the sources in Table 4.

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### Table 4: Results of the $k$-corrected radio–IR relations for MER emission at 24 $\mu m$, FIR emission at 70 $\mu m$, and total IR emission (TIR) ($\ldots$) of the $L_{\text{RC}}$–$L_{\text{IR}}$ relations.

| Frequency | $L_{\text{RC}}$ ($q_{24\mu m}$) | $L_{\text{RC}}$ ($q_{70\mu m}$) | $L_{\text{RC}}$ ($q_{\text{TIR}}$) |
|-----------|-----------------------------|-----------------------------|-----------------------------|
| 1.4 GHz   | 0.95 $\pm$ 0.00 (32)        | 0.93 $\pm$ 0.00 (47)        | 0.92 $\pm$ 0.00 (42)        |
| 2.4 GHz   | 0.95 $\pm$ 0.00 (30)        | 0.95 $\pm$ 0.00 (31)        | 0.95 $\pm$ 0.00 (30)        |
| 6.7 GHz   | 0.95 $\pm$ 0.00 (27)        | 0.95 $\pm$ 0.00 (27)        | 0.95 $\pm$ 0.00 (27)        |

---

4 Defined as the ratio of 1 $\sigma$ standard deviation of the $q$ parameters to their corresponding mean values expressed in percentages.
Within error, we do not find any significant difference in the statistical properties of the \( q \) parameters when the radio emission is \( k \)-corrected by fitting the radio SED or by assuming a constant \( \alpha \). Note that, from the radio SED fitting we find \( \alpha \) to have a substantial scatter of \( \sim 26 \) per cent between sources (see right-hand panel of Fig. 2). This is expected to give rise to a larger scatter in the values of \( q \) as compared to assuming a constant \( \alpha \). Since both the \( k \)-correction methods yield similar scatter in \( q \), it implies that the fluctuations in the physical parameters which determine the infrared luminosity, e.g., dust emissivity, \( T_{\text{dust}} \) and/or density of dust, and comparatively larger contamination from dust heated by AGN activity at MIR wavelengths introduces significant scatter in \( q \) compared to fluctuations in \( \alpha \) within the sample.

From the left-hand panel of Fig. 5, we find the \( q_{24\mu m} \) values to increase with \( z \) for our sample of sources in the ELAIS-N1 field, especially at \( z \gtrsim 1 \). Depending on the type of MIR SED template used for \( k \)-corrections, the value of \( q_{24\mu m} \) may vary strongly, especially towards higher redshifts (Ibar et al. 2008; Bourne et al. 2011). Thus, in the right-hand panel of Fig. 5 we present the variation of \( q_{24\mu m,\text{norm}} \) with \( z \) for \( k \)-corrections using our SED fitting discussed in Section 4.2 and by using the M82-like SED template. We do not find any systematic difference between the two methods of MIR \( k \)-correction, demonstrating that \( k \)-correction using a M82-like template does not introduce any systematic variation in the values of \( q_{24\mu m} \) as compared to direct fitting of the mid- to far-infrared SED. This is perhaps not surprising because the high star-forming galaxy M82 in the nearby Universe is likely to be a prototypical example of the main sequence SFGs at high redshifts (Magnelli et al. 2009; Madau & Dickinson 2014; Béthermin et al. 2017). As \( q_{24\mu m} \) is relatively more sensitive to emission by hot-dust heated by the AGN activity, it shows the largest scatter among the three \( q \) parameters (see Table 4).

The trend of increasing \( q_{24\mu m} \) with \( z \) seen in Fig. 5 is caused by the flux-limitation of our sample in the MIR where higher luminosity sources with higher \( T_{\text{dust}} \) are preferentially being detected at higher redshifts. This can be gleaned from Fig. 4. As the detected sources at \( z \gtrsim 1 \) have higher \( T_{\text{dust}} \), the peak of the dust emission shifts towards shorter wavelengths resulting in an increase of the monochromatic emission at 24 \( \mu m \). As a consequence \( q_{24\mu m} \) increases along with its dispersion. This brings out the critical fact about flux-limited study of the radio–IR relation at MIR wavelengths that \( T_{\text{dust}} \) variation in the sample of sources introduces systematic and statistical biases. Furthermore, the PAHs are one of the major constituents of the interstellar dust that show broad emission features. These PAH emission features at 7.7, 8.6, 11.3 and 12.7 \( \mu m \) (e.g., Roche et al. 1991; Genzel et al. 1998) in SFGs when redshifted beyond \( z \approx 1 \) falls in the 24 \( \mu m \) band and therefore could also give rise to further scatter and biases.

In contrast to the \( q_{24\mu m} \), we observe \( q_{70\mu m} \) to remain nearly constant up to \( z \sim 1 \) and \( q_{\text{IR}} \) shows a mild decrease with \( z \). Both \( q_{70\mu m} \) and \( q_{\text{IR}} \) have significantly lower dispersion compared to that of \( q_{24\mu m} \). This is because the dust emission near the peak of the infrared spectrum around \( \sim 70–80\mu m \) is mainly unaffected by the fluctuations in \( T_{\text{dust}} \), and the total infrared emission is independent of \( T_{\text{dust}} \). In previous studies, \( q_{70\mu m} \) has been observed to remain constant at \( z \lesssim 1.5 \) (see, e.g., Sargent et al. 2010a; Basu et al. 2015b), and is consistent with the observations of Smith et al. (2014) who found the monochromatic \( q \) values at wavelengths near the peak of the infrared emission to remain constant with \( T_{\text{dust}} \). The increase in \( T_{\text{dust}} \) with both the luminosity and \( z \) interplay in a way such that the decrease in monochromatic emission at 70 \( \mu m \) due to the shift in the peak of the infrared spectrum towards shorter wavelengths is largely compensated by an increase of the radio luminosity towards higher redshift. This delicate balance results in \( q_{70\mu m} \) to remain roughly constant over a large redshift range.

Interestingly, we do not find any discernible trend with \( T_{\text{dust}} \) in the variation of \( q_{\text{IR}} \) as a function of \( z \) in Fig. 6 (right-hand panel). This is because \( L_{\text{IR}} \) integrated over the infrared spectrum is independent of \( T_{\text{dust}} \). Thus, for the radio and MIR flux-limited sample used by us, \( q_{\text{IR}} \) is a better indicator of the intrinsic evolution of the radio–IR relation. In Section 6.3, we will discuss about the mild decrease in \( q_{\text{IR}} \) with \( z \) in detail.

### 6.2 Scatter in the radio–infrared relations

In Section 5.3, we studied the monochromatic radio–IR relations between \( L_{1.4\,\text{GHz}} \) and \( L_{24\mu m} \), \( L_{70\mu m} \), and \( L_{\text{IR}} \). In our flux-limited sample of SFGs and AGN, we find the radio–IR relations, modelled as \( L_{1.4\,\text{GHz}} = a L_{\text{IR}} \) to be significantly non-linear (see Table 4), wherein, for SFGs up to \( z = 2 \), the \( L_{1.4\,\text{GHz}} \) versus \( L_{24\mu m} \) relation is sub-linear with a slope \( b = 0.93 \pm 0.02 \) and \( L_{1.4\,\text{GHz}} \) versus \( L_{70\mu m} \) and, versus \( L_{\text{IR}} \) are super-linear with \( b = 1.05 \pm 0.02 \), and \( 1.07 \pm 0.02 \), respectively. The sub-linear slope of the \( L_{1.4\,\text{GHz}} \)–\( L_{24\mu m} \) relation can be explained by the increase in luminosity at 24 \( \mu m \) with \( T_{\text{dust}} \), that also manifests as an increase in \( q_{24\mu m} \) with \( z \). In general, the slopes for AGN are larger than the SFGs, and for our sample as a whole, i.e., including both SFGs and AGN, the slopes of the radio–IR relations do not change significantly. This is due to the fact that, in deep radio observations, the sample is dominated by fainter SFGs, and contamination by AGN that are mostly RQ does not affect the radio–IR relations statistically. Except for the handful of radio luminosity-selected (\( L_{1.4\,\text{GHz}} > 10^{25} \, \text{W Hz}^{-1} \)) RL AGN in our sample (Section 1), MIR- and spectroscopically-selected AGN mostly follow the radio–IR relations over the entire redshift range, up to \( z \sim 4 \), probed in this study. This makes it a challenge to identify AGN using the radio–IR relations alone. Nonetheless, this also implies that AGN contamination for a photometrically-selected sample of SFGs is unlikely to affect the estimation of the cosmic star formation history when the radio emission is calibrated using...
the radio–IR relations. This will form the basis of our forthcoming paper (Sinha A. et al., in preparation).

The non-linearity of the radio–IR relations implies that the q parameters are expected to show a variation with redshift. For general expression of the radio–IR relations as \( L_v = a L_{\text{IR}}^q \), where \( L_v \) is the luminosity at a radio frequency \( v \). \( q \) can also be written as,

\[
q = -\left( \frac{\log_{10} a}{b} \right) + \left( \frac{1 - b}{b} \right) \log_{10} L_v
\]

\[
= -\left( \frac{\log_{10} a}{b} \right) + \left( \frac{1 - b}{b} \right) \log_{10} L_{\text{IR}, \text{obs}} - \alpha \log_{10}(1 + z) \quad (9)
\]

Here, \( L_{\text{IR}, \text{obs}} \) is the radio luminosity in the observer’s frame. This relation is true for both the monochromatic \( q_{24\mu m} \) and \( q_{70\mu m} \), and for bolometric \( q_{\text{TIR}} \). It is clear that for non-linear slopes, the quantity \( q \) depends on the radio luminosity, the slope, the spectral index, and the redshift. Hence, \( q \) is expected to systematically vary with redshift. We therefore quantify the scatter \( (\sigma_q) \) directly from the radio–IR relation as, the 1\( \sigma \) dispersion of the quantity \( \Delta \) defined as,

\[
\Delta = \left( \frac{L_v - a L_{\text{IR}}^b}{a L_{\text{IR}}^b} \right). \quad (10)
\]

The normalization \( a \) and slope \( b \) are obtained by fitting the corresponding radio–IR relation.

The scatter in the radio–IR relations for SFGs, AGN, and the combined population up to \( z = 2 \) are listed in Table 4. Firstly, the scatter of less than a factor of two for SFGs is significantly lower than the scatter of more than factor of ~ 3 for AGN. Secondly, in general, the scatter in the \( L_{\text{bol}} - L_{\text{TIR}} \) relation of SFGs, \( \sigma_{\text{TIR}} \), is lower when compared to the \( L_{\text{bol}} - L_{24\mu m} \) relations \( (\sigma_{24\mu m} = 1.10) \) (for \( 24\mu m < 1.45 \)). This implies that the radio–IR relations are tighter when studied for the bolometric luminosity \( L_{\text{TIR}} \) than the monochromatic \( L_{24\mu m} \) or \( L_{70\mu m} \) luminosities. This is expected because the infrared luminosity integrated over the infrared spectrum is independent of \( T_{\text{dust}} \) while the monochromatic luminosities vary with \( T_{\text{dust}} \). In Table 4, we also present the scatter in the radio–IR relations measured by assuming the same \( a = -0.7 \) for all the sources. Except for AGN, the slopes and scatter of the radio–IR relations for SFGs and the entire sample lies within the error when \( L_{\text{bol}} \) is determined by assuming a constant \( a = -0.7 \) or when \( a \) is measured for each source. However, the scatter \( \sigma_{\text{TIR}} \) is slightly lower when measured using a constant \( a \). This was not readily evident from the scatter of \( q_{\text{TIR}} \) and \( q_{70\mu m} \). It is evident that for studying the intrinsic evolutionary properties of the radio–IR relations, the impact of the radio continuum spectrum needs to be investigated in detail when large surveys at multiple radio frequencies become available. This is because the nature of the radio continuum spectrum is determined by the mechanisms of cosmic ray particle injection and their subsequent energy loss by synchrotron and inverse-Compton cooling. These respectively depend on the star formation activity and magnetic field strengths in the SFGs. Otherwise, by assuming a constant \( a \), scatter introduced in the radio–IR relations due to fluctuations in the radio continuum spectra between sources are missed.

The scatter in the bolometric radio–TIR (\( L_{\text{bol}} - L_{\text{TIR}} \)) relation for SFGs is significantly lower \( (\sigma_{\text{bol}} = 0.45) \) when compared to that for the monochromatic radio–TIR relation \( (\sigma_{\text{TIR}} = 0.81 \); see Table 4). This is because of the fact that \( L_{\text{bol}} \) is largely insensitive to fluctuations of the synchrotron spectrum caused by energy loss/gain of the CREs. Interestingly, for monochromatic infrared emission, i.e., \( L_{\text{RC}} - L_{24\mu m} \) and \( L_{\text{RC}} - L_{70\mu m} \) relations, the scatter is comparable to those of \( L_{1.4 \text{GHz}} - L_{24\mu m} \) and \( L_{1.4 \text{GHz}} - L_{70\mu m} \) (also for \( L_{400 \text{MHz}} \)).

Suggesting that fluctuations in physical parameters that determine the monochromatic infrared emission, e.g., \( T_{\text{dust}} \), dominates over fluctuations in synchrotron emission. An interesting feature of the bolometric radio–IR relations for AGN is that, although they have slopes and normalization similar to that of the SFGs (Sections 5.3.1 and 5.3.3), the scatter is significantly higher and remains unaffected for all the \( L_{\text{RC}} - L_{\text{TIR}} \) relations.

As discussed before, it should be noted that, due to the flux limitations of the currently available near- and mid-infrared surveys, RL AGN populations are largely missing, and the so-called SFGs in our sample could be contaminated by LERGs, especially at redshifts above 0.5. It is interesting to notice that such contamination does not affect the radio–IR relations as all the non-AGN sources identified as SFGs in our study lie within the typical dispersion of the relations expected for SFGs. This suggests that the radio and far-infrared emission from mid-infrared selected sample of SFGs are likely to be dominated by the star formation activity.

6.3 Apparent evolution of radio–IR relations with redshift

The evolution of the radio–IR relation is typically studied by modelling the variation of the \( q_{\text{TIR}} \) with \( z \) as \( q_{\text{TIR}} = q_0 (1 + z)^\gamma \) (e.g., Ivison et al. 2010; Bourne et al. 2011; Basu et al. 2015b; Calistro Rivera et al. 2017; Delvecchio et al. 2021). Here, \( q_0 \) is the value at \( z = 0 \) and \( \gamma \) is the exponent. A weak but significant decrease in \( q_{\text{TIR}} \) with \( z \) has been observed in these studies. While it is clear from equation 9 that non-linearity of the radio–IR relations could in fact give rise to a variation in \( q \) parameters with \( z \), however, it remains unclear what gives rise to such a variation physically. In order to perform a similar modelling using our data, we binned the SFGs in our sample into nine redshift bins up to \( z = 2 \) in a way that each bin contains an equal number of sources (≈ 40 in our case) to avoid statistical biases introduced because of the binning, especially at higher \( z \) where the number of sources are less. The median \( q_{\text{TIR}} \) values were computed for each of the redshift bins and was fitted using the form described above. These median \( z \) and the corresponding median \( q_{\text{TIR}} \) values are listed in Table 5. We present the errors on the median \( q_{\text{TIR}} \) in each bin along with their 1\( \sigma \) dispersion in parenthesis. To reflect the robustness in the variation of \( q_{\text{TIR}} \) with \( z \), we used the dispersions in each redshift bin as the errors while fitting. Here, we will discuss about the variation of \( q_{\text{TIR}} \) because they are less prone to variations as compared to variations in \( T_{\text{dust}} \) (see Section 6.1). We find a mild variation of \( q_{\text{TIR}} \) with \( z \) given by \( q_{\text{TIR}} = (2.58 \pm 0.04) (1 + z)^{-0.12 \pm 0.03} \), as shown as the dashed black line in the right-hand panel of Fig. 6. The index \( \gamma = -0.12 \pm 0.03 \) for the SFGs in our sample is slightly lower than those reported previously. For example, Basu et al. (2015b) found \( \gamma = -0.16 \pm 0.03 \) for star-forming blue-cloud galaxies in the XMM-LESS field by extrapolating from data at 325 MHz; Delhaize et al. (2017) found \( \gamma = -0.19 \pm 0.01 \) for their sample in the COSMOS field; Calistro Rivera et al. (2017) found \( \gamma = -0.15 \pm 0.03 \) for SFGs in the Boötes field by extrapolating from data at 150 MHz; and Ocran et al. (2020) reported \( \gamma = -0.20 \pm 0.02 \) for their sample of SFGs up to \( z = 1.8 \) in the ELAIS-N1 field.

A major difference of estimating \( q_{\text{TIR}} \) in these previous studies compared to ours is the way rest-frame luminosity at 1.4 GHz is obtained. Most of the previous studies either used a constant spectral index value (e.g., Basu et al. 2015b; Ocran et al. 2020), or relied on spectral index measured between two frequencies with relatively shallower flux density cut-off compared to our study (e.g., Calistro Rivera et al. 2017; Delhaize et al. 2017). Thus, to verify whether the measured spectral indices have any impact on the variation of \( q_{\text{TIR}} \) with \( z \), we also estimated \( q_{\text{TIR}} \) values assuming a constant spectral
index of $-0.7$ for our sample of SFGs. Following the same binning procedure described above, we obtain $q_{TIR} = (2.57 \pm 0.03)(1 + z)^{-0.14\pm0.02}$. This suggests that, for our sample, the method of k-correction plays little role in significantly affecting the variation of $q_{TIR}$ with $z$.

In fact, for bolometric radio luminosity, although the dispersion of the $L_{RC} - L_{TIR}$ is significantly lower, the relation remains super-linear, and as a consequence $q_{TIR}$ is also observed to decrease with $z$. Similar to $q_{TIR}$ estimated using $L_{1.4\,GHz}$, we find $q_{TIR}^{RC}$ to vary as $q_{TIR}^{RC} = (2.27 \pm 0.03)(1 + z)^{-0.12\pm0.03}$ and is shown as the blue dashed line in Fig. 10. This clearly indicates that the decrease in $q_{TIR}$ or $q_{TIR}^{RC}$ with $z$ is an intrinsic feature of the radio–IR relation.

Note that the variations of both $q_{TIR}$ with $z$ depends on several physical factors that give rise to emission in the radio and infrared wavebands (see Section 6.4), how they vary with $z$, and therefore in a way, on the sample selection. The slope of the radio–IR relations is also essential in determining the variation of $q$ with $z$ (see equation 9). For linear–radio–IR relations, corresponding $q$ should remain constant with $z$. However, in Section 5.3 we find the radio–IR relations to be significantly non-linear, and thus, the corresponding $q$ values are expected to vary with $z$. Following equation 9, a super-linear slope for the radio–IR relations in the context of SFGs at high redshifts.

Table 5. The median values of $q_{TIR}$ and $q_{TIR}^{RC}$ of SFGs in different redshift bins. The standard deviation for each bin are shown in parentheses.

| Range of $z$ | Median $z$ | Median $q_{TIR}$ | Median $q_{TIR}^{RC}$ |
|-------------|-------------|------------------|-----------------------|
| 0.030 – 0.137 | 0.08       | 2.51 \pm 0.01 (0.12) | 2.21 \pm 0.01 (0.11) |
| 0.137 – 0.241 | 0.21       | 2.50 \pm 0.01 (0.07) | 2.20 \pm 0.01 (0.09) |
| 0.242 – 0.336 | 0.27       | 2.49 \pm 0.01 (0.10) | 2.19 \pm 0.01 (0.13) |
| 0.337 – 0.439 | 0.37       | 2.44 \pm 0.01 (0.12) | 2.15 \pm 0.01 (0.12) |
| 0.448 – 0.565 | 0.50       | 2.46 \pm 0.01 (0.08) | 2.20 \pm 0.01 (0.08) |
| 0.567 – 0.760 | 0.63       | 2.53 \pm 0.01 (0.13) | 2.19 \pm 0.01 (0.10) |
| 0.760 – 0.913 | 0.82       | 2.41 \pm 0.01 (0.19) | 2.08 \pm 0.01 (0.17) |
| 0.914 – 1.148 | 0.99       | 2.38 \pm 0.01 (0.15) | 2.12 \pm 0.01 (0.16) |
| 1.154 – 1.920 | 1.25       | 2.34 \pm 0.01 (0.13) | 2.05 \pm 0.01 (0.14) |

To mitigate this galaxy-selection bias, deep mid- to far-infrared observations are necessary to capture gas-rich, dynamically settled, star-forming galaxy populations at high redshifts. However, unfortunately, a super-Spitzer or super-Herschel space telescope is nowhere in the horizon. Therefore, spectroscopically confirmed normal galaxies up to a moderate redshift of $z \sim 0.5$ needs to be explored with deep radio observations using sensitive telescopes, such as the MeerKAT and upcoming SKA and ngVLA, combined with existing infrared surveys. For higher redshifts, stacking the existing infrared survey data at the location of the normal galaxies expected to be detected in deeper optical surveys with the JWST and the LSST is perhaps the only promising way forward to unravel the intrinsic redshift evolution of the radio–IR relations. The upcoming WHT Enhanced Area Velocity Explorer (WEAVE) survey\(^5\) (Dalton et al. 2014) will be important in advancing toward these directions.

6.4 On the efficacy of using $q_{TIR}$ and $q_{TIR}^{RC}$ to study ISM evolution

In a well selected sample of SFGs based on stellar mass and/or star formation activity, the $q$ parameter can perhaps be used to study the cosmic evolution of ISM. Besides equation 9, assuming that a single-temperature dust emission is a good representation of the infrared SED, and the radio emission contains negligible contribution from the free–free emission, $q$ for monochromatic radio emission can also be expressed in terms of the physical properties of the ISM as (Basu et al. 2017b),

$$q_{TIR} = \log_{10} \left( \frac{n_{UV}}{n_{CRE}} \left( \frac{B_{1} (T_{dust}) Q (\lambda, \alpha)}{B_{tot} \alpha \gamma} \right) \right) + C. \quad (11)$$

Here, $n_{UV}$ and $n_{CRE}$ are the number densities of dust-heating UV photons and synchrotron emitting CREs; $B_{1}$ is the Planck function; $Q (\lambda, \alpha) \propto \lambda^{\beta}$ is the absorption coefficient for dust grains with radius $\alpha$; $B_{tot}$ is the total magnetic field strength; and $C$ is a normalization comprised of standard constants. All these parameters, namely, the ratio $n_{UV}/n_{CRE}$, $T_{dust}$, $B_{tot}$, $\alpha$, and $Q$, can vary with redshift. Their interplay can therefore result in the variation of $q$ as a function of $z$. For galaxy-integrated emission, and for bolometric $q_{TIR}$, $T_{dust}$ variation can be neglected, as indicated by our study. Therefore, by neglecting negligible variation in dust properties with $z$, a decrease in $q_{TIR}$ can be caused due to a combination of reasons, such as, flattening of the radio continuum spectrum, decrease in $n_{UV}/n_{CRE}$, and an increase in $B_{tot}$ with $z$. It has been suggested in recent studies that the decrease in $q_{TIR}$ is possibly caused due to increasing stellar mass at higher $z$ (Delvecchio et al. 2021) since at higher $z$, flux limited surveys are biased by more massive galaxies, or a consequence of selection bias based on star formation rate (Molnár et al. 2021).

Our result on the variation of $q_{TIR}$ brings out an important fact about the decrease in the values of $q_{TIR}$. Note that, $L_{RC}$ is mostly immune to CRE energy gain/loss mechanisms which affect monochromatic radio emission at different frequencies differently. As indicated by the data, $q_{TIR}$ and $q_{TIR}^{RC}$ are largely independent of $T_{dust}$ (see Figs. 6 and 10),\(^6\) and $L_{RC}$ is independent of the radio continuum spectrum.

\(^5\) https://www.ing.iac.es/confluence/display/WEAVE/The+WEAVE+Project

\(^6\) Since $L_{TIR}$ is obtained by integrating over the dust emission, it is expected to be independent of $T_{dust}$. The variation seen in Fig. 4 is likely to be a consequence of flux limitation, especially above 200 \(\mu\)m. For SFGs up to $z = 2$, the sample of our interest, $T_{dust}$ and $L_{TIR}$ are weakly correlated with $r = 0.58$. 

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equation 11 simplifies as,\(^7\)

\[
\log_{10} q_{\text{RC}} = \log_{10} \left( \frac{n_{\text{UV}}}{n_{\text{CRE}}} \right) + \log_{10} \left( \frac{1}{B_{\text{tot}}^{1-\alpha}} \right) + \tilde{C}.
\]  

(12)

The flattening of radio continuum spectrum in high mass galaxies at high redshifts, where it becomes flatter, having more massive stellar masses (IMF) where it becomes flatter having more massive redshifts, the acceleration efficiency of CREs in supernovae remnants (SFR). This implies that at high redshifts, the radio luminosity should increase with redshift, and from equation 12, this can be caused due to an increase in magnetic field strengths with redshift. An increase of \(B_{\text{tot}}\) with redshift is possible because the small-scale (~ 100 pc) turbulent dynamo action (Cho & Vishniac 2000; Brandenburg & Subramanian 2005; Gent et al. 2013; Schleicher & Beck 2013; Schober et al. 2016) generates stronger magnetic fields in high redshift massive galaxies that have higher gas (Chowdhury et al. 2020, 2021) and star-formation density (Madau & Dickinson 2014; Pillepich et al. 2017; Gruppioni et al. 2020; Jo et al. 2021). The magnetic fields amplified by the action of turbulent dynamo can lead to a coupling between the magnetic fields and the gas densities, and therefore with star formation rate which is perhaps the cause of the non-linear radio–IR relations (Niklas & Beck 1997; Schleicher & Beck 2013). Furthermore, in addition to turbulence driven by star-formation, magnetic fields can also be amplified by galaxy merger-driven turbulent in the luminous galaxies at high redshifts (Veilleux et al. 2002; Kiferči Eser et al. 2014; Whitingham et al. 2021). However, except for a handful of studies, a robust observational constraint on the evolution of magnetic fields with redshift remains unclear (Oren & Wolfe 1995; Bernet et al. 2008; Joshi & Chand 2013; Kim et al. 2016; Mao et al. 2017). On the other hand, since the mean-free path of dust-heating UV photons is \(O(50\text{pc})\), i.e., the size of the Strömgen sphere ionized by OB-type stars, UV photons are expected to remain trapped within the dense environment of massive galaxies at high redshifts, except perhaps in Lyman-\(\alpha\) emitters and in low metallicity galaxies near the epoch of reionization. That means, a decreasing \(n_{\text{UV}}/n_{\text{CRE}}\) implies an increase in \(n_{\text{CRE}}\), which indicates an increased CRE injection at high \(z\), likely due to the increase in cosmic star formation rate density up to \(z \sim 2\) (Magnelli et al. 2011; Madau & Dickinson 2014; Leslie et al. 2020). Additional data are required to pin down the cosmic evolution of magnetic fields and the escape fraction of UV photons in galaxies to unravel the cause of non-linearity in the radio–IR relations.

(ii) **Evolving cosmic ray acceleration efficiency:** Another interesting possibility for the decrease of \(q_{\text{TIR}}\) with \(z\) can be a super-linear dependence of \(n_{\text{CRE}}\) with the star formation rate (SFR). To reproduce the variation of \(q_{\text{TIR}}\), \(n_{\text{UV}}/n_{\text{CRE}}\) should vary with \(z\) as \(n_{\text{UV}}/n_{\text{CRE}} = n_0 (1+z)^{\beta} \propto (1+z)^{\gamma}\), where \(\beta > 0\). Since both \(n_{\text{UV}}\) and \(n_{\text{CRE}}\) are related to SFR, and \(n_{\text{UV}} \propto \text{SFR}\), and say, \(n_{\text{CRE}} \propto \text{SFR}^\alpha\), then for \(q_{\text{TIR}} \propto (1+z)^{\gamma}\) implies \(\text{SFR}^{\beta-\alpha} \propto (1+z)^{\gamma}\). Thus, for a negative value of \(\gamma, \delta > 1\). This implies that at high redshifts, the acceleration efficiency of CREs in supernova remnants changes with SFR and/or there is a significant change in the initial mass function (IMF) where it becomes flatter having more massive stars. In order to establish the scenario of changing IMF, optical to infrared SED fitting in stellar-mass-selected galaxies in bins of redshift needs to be performed by including its variation. On the other hand, the scenario of changing CRE acceleration efficiency is not entirely unfeasible. Numerical simulations have suggested that the acceleration efficiency increases substantially with the Mach number (\(M\)), Caprioli & Spitkovsky 2014; van Marle et al. 2022), where \(M = v_{sh}/c_s = \sqrt{\rho_{gas}}\) for similar \(v_{sh}\). Here, \(v_{sh}\) is the shock velocity of supernovae explosions and \(c_s\) is the sound speed which depends on the gas density (\(\rho_{gas}\)). Observations suggest that the average Hi-to-stellar mass ratio in star forming main-sequence galaxies increases from \(\approx 0.4\) in the local Universe to \(\approx 2.5\) at \(z = 1.3\) (Santonge et al. 2017; Chowdhury et al. 2021). This increase in the relative Hi mass hints at a possible increase in the gas density, and therefore an increase in the average \(M\) in high-z galaxies which can give rise to an increased CRE acceleration efficiency and thus a super-linear dependence of \(n_{\text{CRE}}\) on SFR. Detailed semi-analytical calculations and/or numerical simulations are needed to quantitatively confirm this scenario.

These scenarios, however, can be further complicated depending on—(i) whether or not energy equipartition between magnetic fields and CREs are valid (Niklas & Beck 1997; Basu & Roy 2013; Basu et al. 2017b); (ii) whether magnetic fields and gas densities are coupled; and (iii) whether high redshift SFRs are CRE calorimeters (e.g., Werhahn et al. 2021) or they lose CREs via winds on galactic scales (e.g., Wiener et al. 2017; Heald et al. 2022). In the first case, a breakdown in the energy equipartition due to CRE energy losses could also lead to an evolution of the radio–IR relation (e.g., Schleicher & Beck 2013). In the second case, magnetic fields amplified by the action of fluctuation dynamo can lead to a coupling between the magnetic fields and the gas densities, and therefore with the star formation rate. While in the third case, depending of the efficiency of cosmic ray escape from galaxies at high redshifts, the form of the radio–IR relations could be affected. Hence, in order to infer the cosmic star formation rate density evolution at \(z \geq 2\) using radio continuum emission as a tracer via the radio–IR relations, the evolution of magnetic fields, gas densities and the rate of CRE escape also needs to be considered appropriately.

### 7 SUMMARY

We have performed deep observations of the ELAIS-N1 field using the uGMRT at 400 MHz achieving a RMS noise of 15 \(\mu\)Jy beam\(^{-1}\) which provides 6\(\sigma\) point-source sensitivity of \(~ 100 \mu\)Jy beam\(^{-1}\). A total of 2528 extragalactic sources were detected, of which 2321 sources having redshift information were broadly classified into SFGs and AGN using a host of publicly available multi-waveband data at infrared wavelengths and spectroscopy at optical wavelengths. About 24 and 76 per cent of the sources were identified as AGN and SFGs, respectively, suggesting that at faint flux density end, SFGs dominate the population of extragalactic sources. Using these sources, we studied the statistical properties of the radio–IR relations, and our key findings are summarized below.

(i) The median spectral index (\(\alpha\)) of the sources in the ELAIS-N1 field detected at 400 MHz is found to be \(\alpha = 0.58 \pm 0.15\). While, the median \(\alpha\) for SFGs and AGN, when measured separately, are \(-0.58 \pm 0.15\) and \(-0.57 \pm 0.16\), respectively. Here the errors represent the median absolute deviation of the sample.

(ii) The value of \(T_{\text{dust}}\) for the sources in the ELAIS-N1 field is found to increase with \(z\) and the total infrared (between

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\(^7\) Here, \(\tilde{C}\) is a different normalization constant compared to equation 11.
8–1000 μm) luminosity ($L_{\text{TIR}}$), implying that the three quantities are correlated, perhaps due to the flux limitations. As a result, the monochromatic luminosity at 24 μm increases with $z$, which in turn results in $q_{24\mu m}$ to increase with $z$. This implies that $q_{24\mu m}$ is of limited value while investigating the radio–IR relations.

(iii) The value of $q_{70\mu m}$ mostly remain constant up to $z \sim 1$. Since the emission at 70 μm lies near the peak of the infrared spectrum, $q_{70\mu m}$ is less prone to variations in $T_{\text{dust}}$. However, $q_{\text{TIR}}$ mildly decreases with $z$. Since the total infrared luminosity is independent of $T_{\text{dust}}$. $T_{\text{dust}}$ variation with $z$ does not affect the variation of $q_{\text{TIR}}$. AGN shows lower $q_{\text{TIR}}$ values than SFGs, signifying an excess radio emissions in AGN.

(iv) We observe the $L_{1.4\text{GHz}}$–$L_{70\mu m}$ and $L_{1.4\text{GHz}}$–$L_{\text{TIR}}$ relations to have super-linear slopes while the $L_{1.4\text{GHz}}$–$L_{24\mu m}$ relation is sub-linear for both SFGs and AGN.

(v) The statistical properties of $q$ and the slope of the radio–IR relations do not significantly depend on the method of how spectral indices are estimated using radio continuum observations between 0.1 and 1.4 GHz for correcting the radio emission to rest-frame, i.e., directly from SED of each source or assuming the same value for all sources.

(vi) For the first time, we investigated the radio–IR relations at high redshifts using the radio luminosity integrated over the low (0.1 and 2 GHz ($L_{\text{RC}}$) that also exhibit super-linear slopes with various measures of infrared luminosity. The $L_{\text{RC}}$–$L_{\text{TIR}}$ relation for SFGs is the tightest of all correlations with a scatter lower by a factor of $\approx 2$ when compared to monochromatic $L_{1.4\text{GHz}}$–$L_{\text{TIR}}$ and $L_{400\text{MHz}}$–$L_{\text{TIR}}$ relations. This is because, in $L_{1.4\text{GHz}}$–$L_{\text{TIR}}$ and monochromatic radio luminosities, $L_{\text{RC}}$ is independent of the fluctuations of the synchrotron spectrum caused due to CRE energy loss/gain mechanisms and/or contamination due to free–free emission in the sample of SFGs.

(vii) We present the variation of $q$ parameters derived using bolo metric radio and IR luminosities as, $q_{\text{RC}} = (2.27 \pm 0.03) (1 + z)^{-0.12 \pm 0.03}$. This moderate evolution could be attributed to the non-linearity of the $L_{\text{RC}}$–$L_{\text{TIR}}$ relation, and suggests that non-linearity of the relations and variation of $q$ parameters have a common physical origin (see equation 12).

(viii) The $q$ parameters depend on various physical parameters (see equation 11). From our results on the non-linear $L_{\text{RC}}$–$L_{\text{TIR}}$ relation and decrease of $q_{\text{RC}}$ with $z$, we suggest that an increase in magnetic field strength and/or an increase in CRE acceleration efficiency with redshift are plausible reasons that could give rise to non-linearity in the radio–IR relations. More data are needed to investigate these scenarios.

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This research also made use of Astropy,8 a community-developed core Python package for Astronomy (Astropy Collaboration et al. 2013; Price-Whelan et al. 2018), NumPy (van der Walt et al. 2011), and Matplotlib (Hunter 2007).

DATA AVAILABILITY

The raw interferometric data from the uGMRT in Band3 are publicly available at the GMRT online archive (https://naps.ncra.tifr.res.in/goa) under project 32_120. The source classified catalogue at 400 MHz including the spectral indices will be shared on a reasonable request to the corresponding authors.

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**APPENDIX A: COMPARISON BETWEEN BOLOMETRIC AND MONOCROMATIC RADIO LUMINOSITY**

In Sec. 5.2.2, we have introduced the quantity \( L_{RC} \), the integrated radio continuum luminosity in the range 0.1–2 GHz. Here we discuss the various advantages of using \( L_{RC} \) over \( L_{1.4\,\text{GHz}} \).

(i) *Dominated by synchrotron emission:* As the synchrotron emission typically has steeper spectrum (\( \alpha \approx -0.5 \)) compared to the thermal free–free emission (\( \alpha \approx -0.1 \)), it dominates at frequencies below \( \approx 2 \) GHz. For example, for a typical thermal fraction of \( \approx 10(20) \) per cent at 1.4 GHz (Basu & Roy 2013), the contribution of free–free emission to \( L_{RC} \) is \( \approx 7(13) \) per cent for synchrotron spectral index \( \approx -0.5 \).

(ii) *Less susceptible to systematic variations of\( \alpha \):* Beside being dominated by synchrotron emission, \( L_{RC} \) has another advantage over \( L_{1.4\,\text{GHz}} \) or \( L_{400\,\text{MHz}} \). The bolometric \( L_{RC} \) is largely unaffected by possible systematic fluctuations of \( \alpha \) in a sample of SFGs, either due to calibration offsets between data from different telescopes or due to flux measurement methods or due to \( \alpha \) being affected by an increased contribution from free–free emission at the larger rest-frame frequencies. A large offset in the estimated value of \( \alpha \) by up to \( \approx 50 \) per cent (with respect to the true value of \( \alpha \)) affects \( L_{RC} \) by less than \( \approx 10 \) per cent. In contrast, the error in \( L_{1.4\,\text{GHz}} \) or \( L_{400\,\text{MHz}} \) can be significantly more, up to 40 per cent, when \( k \)-corrected using the offset value of the measured \( \alpha \).

(iii) *Less susceptible to errors in\( \alpha \):* \( L_{RC} \) is mildly affected by statistical errors in the measured values of \( \alpha \) and is therefore expected to capture the scatter in the radio–IR relations better compared to \( L_{1.4\,\text{GHz}} \). To demonstrate the advantage of \( L_{RC} \) over \( L_{1.4\,\text{GHz}} \) for our sample, in Fig. A1 we show the variation of the standard deviation of the quantity \( L_{MC}/L_{\text{best-fit}} \), \( \sigma(L_{MC}/L_{\text{best-fit}}) \), obtained from the Monte-Carlo simulations for each source discussed in Sec. 4.1, as a function of the fractional error of \( \alpha \), \( \Delta \alpha/|\alpha| \), for \( L_{1.4\,\text{GHz}} \) (black dots) and \( L_{RC} \) (hexagons). For a source, \( L_{MC} \) is the corresponding luminosity for a Monte-Carlo realization, and \( L_{\text{best-fit}} \) is the corresponding best-fit luminosity used in this work. It is clear that the scatter in the values of \( L_{1.4\,\text{GHz}} \) is significantly larger than the scatter in the values of \( L_{RC} \). The red stars and the blue diamonds are the median values of the scatter in \( L_{1.4\,\text{GHz}} \) in bins of \( \Delta \alpha/|\alpha| \). In our sample, the scatter in \( L_{1.4\,\text{GHz}} \) could be up to 90 per cent larger than that of \( L_{RC} \).

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9 Here we consider a systematic offset in the estimated values of \( \alpha \).
10 This depends on the \( \alpha \) and slightly on the redshift of a source. Here the larger numbers correspond to a typical value of \( \alpha \) in the range \(-0.7 \) to \(-1 \).