The $R$ Axion From Dynamical Supersymmetry Breaking

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Abstract

All generic, calculable models of dynamical supersymmetry breaking have a spontaneously broken $R$ symmetry and therefore contain an $R$ axion. We show that the axion is massive in any model in which the cosmological constant is fine-tuned to zero through an explicit $R$-symmetry-breaking constant. In visible-sector models, the axion mass is in the 100 MeV range and thus evades astrophysical bounds. In nonrenormalizable hidden-sector models, the mass is of order of the weak scale and can have dangerous cosmological consequences similar to those already present from other fields. In renormalizable hidden-sector models, the axion mass is generally quite large, of order $10^7$ GeV. Typically, these axions are cosmologically safe. However, if the dominant decay mode is to gravitinos, the potentially large gravitino abundance that arises from axion decay after inflation might affect the successful predictions of big-bang nucleosynthesis. We show that the upper bound on the reheat temperature after standard inflation can be competitive with or stronger than bounds from thermal gravitino production, depending on the model and the gravitino mass.

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1 Introduction

The primary attraction of supersymmetry is that it suggests a solution to the gauge hierarchy problem. However, it does not necessarily explain the ratio $m_W/M_P$. Dynamical models of supersymmetry breaking offer a possible reason why $m_W/M_P \simeq 10^{-17}$. In such models all scales arise from the Planck scale through dimensional transmutation [1]. Supersymmetry is unbroken to all orders of perturbation theory, but nonperturbative effects, proportional to $e^{-O(1)\pi^2/g^2}$, can generate a ground state that breaks supersymmetry [2].

At present there is only a limited class of known models where dynamical supersymmetry breaking is realized in a calculable way. Some criteria for supersymmetry breaking have been proposed [3, 4]; but it is not yet clear whether this exhausts the class of viable models. Reference [5] considered the class of generic, calculable models, where all terms consistent with the symmetries are present, and the low-energy effective lagrangian does not contain strongly-interacting gauge fields. It was shown that for such models a spontaneously broken $R$ symmetry is necessary and sufficient for dynamical supersymmetry breaking. This spontaneously broken symmetry implies the existence of a Goldstone boson, called the $R$ axion.

In this paper we study the properties of the $R$ axion. We show that any generic calculable model of supersymmetry breaking always has a trivial analog for which the axion is massive, namely the same model with a constant term added to the superpotential. Although the model is technically nongeneric because it includes a single term that breaks $R$ symmetry, it is still a reasonable candidate for a supersymmetry-breaking sector. In fact, this constant is necessary in any realistic model to obtain a vanishing cosmological constant.

We also consider cosmological and astrophysical constraints on the $R$ axion. When supersymmetry is broken in a visible sector, the axion is sufficiently heavy to avoid astrophysical problems. This is important because it implies that the existence of an $R$ axion is not sufficient cause to dismiss these models.

When supersymmetry is broken in a nonrenormalizable hidden sector, the axion mass is generally of order the weak scale. Such an axion can have dangerous cosmological consequences. However, it is already known that models of this sort generally contain singlets with similar problems [6, 7]. One might hope that all the difficulties will be solved by the same mechanism. (See, for example, [8].)

Most of this paper focuses on the axion in renormalizable hidden-sector models. The axion in
these models is quite heavy, with mass of order $10^7\text{GeV}$. We point out that this axion can be a new source of gravitinos and that this leads to a new constraint on the reheat temperature of the universe after inflation. Our conclusion is that it is not necessary to eliminate the axion to build a successful model of dynamical supersymmetry breaking.

As we will see, in all models the $R$-axion mass arises because the cosmological constant must be canceled in any successful theory of dynamical supersymmetry breaking. We do not in any way attempt to solve the cosmological constant problem; our point is that until one understands better its solution, one cannot dismiss models that contain an axion. This is of particular relevance to visible-sector models.

In section 2 of this paper we discuss the relation between $R$ symmetry and the cosmological constant. We explain the origin of the axion mass, and consider three types of models: visible-sector models, nonrenormalizable hidden-sector models, and renormalizable hidden-sector models. In section 3 we consider renormalizable hidden-sector models in more detail. We study the simplest such model, based on a $\text{SU(3)} \times \text{SU(2)}$ gauge group, which we call the 3-2 model. Because supersymmetry breaking occurs in the weak-coupling regime, the spectrum and its properties can be calculated in a controlled expansion. This model provides a useful template for renormalizable hidden-sector models. In section 4 we study the relevant cosmological constraints on renormalizable hidden-sector models. We find that if the reheat temperature of the universe after inflation is too high, a large number of gravitinos may be produced by coherent oscillations of the $R$ axion field. Their subsequent decay may lead to dissociation of the light elements, in conflict with big-bang nucleosynthesis. We compare this bound on the reheat temperature with the bound from thermal production of gravitino and find it competitive (for smaller gravitino mass) or stronger (for larger gravitino mass). In section 5 we summarize our results.

## 2 The Source of Axion Mass

In this section, we discuss the axion mass in three scenarios for dynamical supersymmetry breaking: nonrenormalizable hidden-sector models (NRHS), renormalizable hidden-sector models (RHS), and visible-sector models. In the first class of models, supersymmetry is broken only when supergravity couplings are included (that is, supersymmetry is unbroken in the $M_P \to \infty$ limit). In the second class, supersymmetry is broken in the flat-space limit, and supersymmetry breaking is communicated to the visible world through Planck-mass-suppressed interactions associated with
supergravity. In the final class of models, both supersymmetry breaking and the communication of supersymmetry breaking are achieved through renormalizable couplings.

In supergravity theories, the tree-level scalar potential takes the following form [9]:

\[
V = V_D + V_F ,
\]

\[
V_D = \frac{1}{2} g^2 D^a D^a ,
\]

\[
V_F = \exp(K/M_P^2) \left( \left[ W_i + \frac{K_i}{M_P^2} W \right] K^{-1} \left[ W_j^* + \frac{K_j^*}{M_P^2} W^* \right] - \frac{3W W^*}{M_P^2} \right) ,
\]

where \( W \) and \( K \) are the superpotential and the Kähler potential, respectively, the \( D^a \) are the \( D \) terms of the various gauge groups, and we have defined \( M_P \) to absorb a factor of \( \sqrt{8\pi} \). Supersymmetry is spontaneously broken if \( \langle \exp(K/2M_P^2)[W_i + (K_i/M_P^2)W] \rangle \) is nonzero.\(^1\)

From these expressions, we see that supersymmetry can be spontaneously broken, with no cosmological constant, if the superpotential contains a constant \( W_0 \) that is adjusted to cancel the vacuum energy. If there are no Planck-scale vevs, this implies

\[
W_0 = \frac{1}{\sqrt{3}} M_S^2 M_P
\]

(2.4)
to leading order in \( 1/M_P \), where \( M_S^4 = \langle W_i K^{-1} W_j^* \rangle \) denotes the scale of supersymmetry breaking.

If the original superpotential \( W_2 \) preserves an \( R \) symmetry, it carries \( R \)-charge 2, whereas any constant term has \( R \)-charge zero. Therefore the cosmological term \( W_0 \) explicitly breaks the \( R \) symmetry, which implies that the \( R \) axion is a massive pseudo-Goldstone boson. The mass term arises from the cross terms between the \( R \)-symmetry-breaking constant and the \( R \)-preserving terms in the superpotential. The general formula for the axion mass follows from (2.3), with superpotential \( W = W_2 + W_0 \),

\[
m_a^2 = \frac{8}{f_a^2} \frac{W_0 \langle (W_2 i K^{-1} K^* j - 3W_2) \rangle}{M_P^2} .
\]

(2.5)

Here \( f_a \) is the axion coupling. (See section 3.)

\[
f_a^2 = 2 r_i r_j v_i v_j^* \langle K_{ij} \rangle ,
\]

(2.6)

where \( r_i \) and \( v_j \) are the \( R \) charges and vevs of the fields, respectively. Note that this mass arises from an \( F \) term, unlike the soft scalar masses of hidden-sector models.

\(^1\)We ignore the possibility of a Fayet-Iliopoulos \( D \)-term.
This explicit violation of $R$ symmetry does not contradict the theorem of Nelson and Seiberg [5], since the model is now technically nongeneric because we have not added all terms that violate $R$ symmetry. However, it is not inconceivable that the physics responsible for solving the cosmological constant problem communicates only through gravity, and respects different symmetries than the supersymmetry-breaking sector of the theory. Furthermore, the proof that an $R$ symmetry is both necessary and sufficient relies on possible flat directions in the potential [3]. The constant term is irrelevant to the argument, so any generic theory has a trivial nongeneric counterpart for which the cosmological constant vanishes and the axion is massive.

For models with dynamical supersymmetry breaking, the general formula (2.5) for the axion mass becomes

$$m_a^2 \simeq \frac{1}{f^2} \frac{M_S^2 \Lambda^3}{M_P},$$

(2.7)

where $\Lambda$ is related to the scale of the strong dynamics. Equation (2.7) applies for all the models we will discuss.

In renormalizable hidden-sector models, the scale of supersymmetry breaking $M_S \simeq f \simeq \Lambda$, so $m_a^2 \simeq M_S^3/M_P$. In such models $M_S^2 \simeq m_W M_P$, so $m_a \simeq \sqrt{m_W M_S}$, or about $10^7$ GeV. This mass is enhanced over the other soft masses by a factor of $(M_P/m_W)^{1/4}$. Because of the large mass, the axion decays quickly. It is cosmologically safe, except when it decays predominantly to stable heavy particles, as we will discuss.

In nonrenormalizable hidden-sector models, the energy density at the minimum of the potential is of order $M_S^4 \simeq \Lambda^6/M_P^2$, and $f \simeq M_P$. The factor of $1/M_P^2$ follows from the fact that supersymmetry is broken by nonrenormalizable Planck-mass-suppressed terms. Therefore, in NRHS models, the axion mass is of order $m_a \simeq M_S^3/M_P \simeq m_W$, where we have again used the fact that $M_S^2 \simeq m_W M_P$. This implies that the axion in NRHS models suffers from the same cosmological problems that have been identified for moduli fields [6, 7], or any other fields that have flat potentials up to nonrenormalizable terms, and whose potential is determined by the supersymmetry-breaking sector. We do not address the cosmological problems of such fields in this paper. (See, however, ref. [3].)

Finally, the axion of visible-sector models is also massive. In such models, supersymmetry breaking is typically at a much lower scale because it is communicated to the visible world through gauge interactions. Therefore the axion mass is much smaller, as can be seen from (2.7). If the axion can be produced in a supernova, its mass must exceed 10 MeV so that the supernova does not cool too quickly [10]. For visible-sector models with $M_S$ greater than $10^5$ GeV, there is no problem. For lower values of $M_S$, an alternative source of axion mass may be required. (For example, there
is a contribution to the axion mass if the associated U(1) is anomalous with respect to a gauged symmetry \[11\]. However, visible sectors seem to require a symmetry-breaking scale as high as \(10^6\) GeV because masses arise through multiloop graphs \[11\]. With such a large scale of supersymmetry breaking, the axion is sufficiently heavy to be in accord with astrophysical bounds.

3 An Explicit Calculation of the Axion Mass: the 3-2 Model

3.1 The Model

The simplest known calculable model with dynamical supersymmetry breaking is based on two-flavor supersymmetric SU(3) QCD with gauged SU(2)\(_L\) flavor symmetry \[3, 4\]. The model is remarkable because the supersymmetry-breaking ground state and the low-energy particle spectrum can be found in a controlled weak-coupling approximation.

To describe the model, we denote the left and right quark chiral superfields by \(Q\) and \(\bar{Q}\). Under the SU(3) \(\times\) SU(2) gauge symmetry, they transform as follows (\(\bar{Q}^i_\alpha \equiv (\bar{D}^i, \bar{U}^i)\)):

\[
\begin{align*}
Q^i_\alpha &\sim (3, 2), \\
\bar{U}^i &\sim (\bar{3}, 1), \\
\bar{D}^i &\sim (\bar{3}, 1),
\end{align*}
\]

(3.1)

where Greek and Roman letters denote SU(2) and SU(3) indices, respectively. Cancellation of the Witten anomaly requires another SU(2) doublet,

\[
L^\alpha \sim (1, 2).
\]

(3.2)

The particle content of the model is similar to that of the minimal supersymmetric standard model without the right-handed electron and Higgs superfields.

Apart from the gauge symmetries, the 3-2 model has two anomaly-free global symmetries: U(1)\(_Y\) hypercharge and U(1)\(_R\). The hypercharge assignments are like those in the minimal supersymmetric standard model:

\[
\begin{align*}
Y(Q) &= 1/6, \\
Y(\bar{U}) &= -2/3,
\end{align*}
\]
\[ Y(\bar{D}) = \frac{1}{3} , \quad Y(L) = -\frac{1}{2} . \]  

Under the nonanomalous \( R \) symmetry the charges of the matter superfields are given by:

\[
\begin{align*}
R(Q) & = 1 , \\
R(\bar{U}) & = R(\bar{D}) = 0 , \\
R(L) & = -3 .
\end{align*}
\]

The gauginos carry \( R \) charge \(-1\).

The Kähler potential of the model takes the usual form for a renormalizable, supersymmetric theory

\[ K = Q^\dagger Q + \bar{Q} \bar{Q}^\dagger + L^\dagger L . \]  

In (3.5) the SU(2) and SU(3) gauge superfields are not written, but are assumed to be coupled in the usual way [3].

In the absence of a superpotential, the scalar potential vanishes for a number of flat directions in field space, and the ground state is undetermined at the classical level [3]. The equations that determine the flat directions are

\[
Q_{\alpha}^\dagger Q_{\alpha} = 0 ,
\]

for the SU(3) \( D \)-terms, and

\[
Q_{\alpha}^\dagger Q_{\beta} + L_{\alpha}^\dagger L_{\beta} = \frac{1}{2} \delta_{\alpha}^{\beta} (Q^\dagger Q + L^\dagger L) ,
\]

for the SU(2) \( D \)-terms. Up to local symmetries, the solutions to these equations are parametrized by six real variables.

Let us consider the theory expanded around a solution of (3.6), (3.7), such that the scale \( v \) of the vacuum expectation values of the scalar fields obeys

\[ v \gg \Lambda_3 , \]

where

\[ \Lambda_3 = v \exp \left( -\frac{8\pi^2}{g_3(v)^2 b_0} \right) \]  

2Recall that the \( R \) charge of a fermion in a chiral multiplet \( \Phi \) of charge \( R_\Phi \) equals \( R_\Phi - 1 \) [3].
is the scale where the SU(3) gauge coupling $g_3$ becomes strong and $b_0$ is the one-loop coefficient of the beta function. For such vacua, the theory is in the weak-coupling regime. This vacuum completely breaks the SU(2) and SU(3) gauge symmetries, so the vector supermultiplets are massive. Supersymmetry is unbroken along the flat directions, and 11 out of the 14 matter chiral superfields are massive as well. The remaining 3 chiral superfields are massless.

At energies below the scale $\Lambda_3$, the low-energy effective theory can be described in terms of the following three gauge-invariant chiral superfields

$$
\begin{align*}
X_1 &= Q \bar{D} L , \\
X_2 &= Q \bar{U} L , \\
X_3 &= \det \bar{Q} Q^\alpha .
\end{align*}
$$

Their scalar components parametrize the six flat directions (3.6), (3.7) of the potential.

The superpotential is such that a global $U(1)_Y \times U(1)_R$ symmetry is preserved:

$$
W = \lambda X_1 + 2 \Lambda_3^7 X_3 .
$$

The first term is the usual renormalizable superpotential. The second is generated by nonperturbative effects. Its coefficient can be calculated in the weak coupling expansion around a constrained instanton in a vacuum that obeys (3.8) [3].

When $\lambda = 0$, the scalar potential of the model (3.5, 3.11) does not have a minimum at a finite value of the fields, so the theory does not have a ground state [3]. For the case when

$$
\lambda \ll g_2 \ll g_3 \ll 1 ,
$$

the scalar potential has almost-flat directions, given by the solutions to (3.6), (3.7). The potential now has a minimum at finite values $v$ for the fields, of order

$$
v \simeq \frac{\Lambda_3}{\lambda^{1/7}} .
$$

This value is such that the weak coupling assumption (3.8) is self-consistent, so the theory can be analyzed perturbatively.

At this minimum, the vacuum energy is nonzero and supersymmetry is spontaneously broken\footnote{For simplicity, we assume that the SU(2) gauge coupling $g_2 \ll g_3$.}

\footnote{In fact, supersymmetry is broken even when the theory is strongly-coupled.}

In this paper we restrict our attention to the weak-coupling regime, where the spectrum can be
computed using the effective low-energy theory along the almost-flat directions. This procedure is described in the next section.

3.2 The Low-Energy Sigma Model and its Spectrum

In this section we derive the gauge-invariant low-energy effective field theory, valid below the scale \( \Lambda_3 \). We find the spectrum of all particles lighter than this scale.

In the limit (3.12), the superpotential can be treated as a perturbation on the theory without a superpotential. Therefore the Kähler potential of the effective theory is given by the projection of (3.5) onto the fields \( X_1, X_2 \) and \( X_3 \) that span the flat directions of the potential. The low-energy theory is a supersymmetric nonlinear sigma model with coordinates \( X_1, X_2 \) and \( X_3 \).

From the equations for the SU(3) flat directions (3.6), it follows that

\[
Q^\dagger Q = \bar{Q} \bar{Q}^\dagger
\]  

(3.14)

along the flat directions. Using the definitions of the light fields (3.10) and the equations for the flat directions (3.6), (3.7), we find

\[
X_1^\dagger X_1 + X_2^\dagger X_2 = \frac{1}{4} (Q^\dagger Q + L^\dagger L)^2 L^\dagger L
\]

\[
\sqrt{X_3^\dagger X_3} = \frac{1}{4} (Q^\dagger Q)^2 - \frac{1}{4} (L^\dagger L)^2
\]  

(3.15)

along the flat directions. Equations (3.14), (3.15) hold for the scalar components of the superfields. The supersymmetry of the low-energy theory lifts them to superfield equations along the flat directions.

Using the notation of ref. [4],

\[
A = \frac{1}{2} \left( X_1^\dagger X_1 + X_2^\dagger X_2 \right),
\]

\[
B = \frac{1}{3} \sqrt{X_3^\dagger X_3},
\]  

(3.16)

the Kähler potential (3.5), projected onto the flat directions becomes

\[
K_{\text{flat}} = \left( Q^\dagger Q + \bar{Q} \bar{Q}^\dagger + L^\dagger L \right)_{\text{flat}} \equiv K_{\text{light}} (X_i^\dagger, X_i) = 24 \frac{A + B x}{x^2},
\]  

(3.17)
where

\[ x \equiv \left( Q^\dagger Q + L^\dagger L \right) \bigg|_{\text{flat}} = 4 \sqrt{B} \cos \left( \frac{1}{3} \arccos \frac{A}{B^{3/2}} \right). \]  \hspace{1cm} (3.18)

Note that equations (3.15), used to determine \( x \) as a function of the light superfields, have several solutions. Equation (3.18) is the only one that leads to a positive definite Kähler metric at the minimum (3.20).

The low-energy theory is therefore described by the sigma model with Kähler potential \( K_{\text{light}} \), (3.16), (3.17), (3.18) and superpotential (3.11)

\[ W_{\text{light}} = \lambda X_1 + 2 \frac{\Lambda_3^7}{X_3}. \]  \hspace{1cm} (3.19)

In the limit (3.12), (3.19) can be treated as a perturbation on the theory without a superpotential.

To find the ground state of the model, we must minimize the scalar potential of the sigma model (3.17), (3.19). Because this potential is so complicated, we found it convenient to minimize the potential of the full theory. In the limit (3.12), it suffices to minimize \( V_F \) along the minima of \( V_D \). Up to local symmetries, the space of minima of \( V_D \) is six-dimensional. The two global \( U(1)_Y \) and \( U(1)_R \) symmetries reduce this number to four. Numerically minimizing \( V_F \) with respect to these four parameters, one finds the minimum of the potential for the full theory.

At the minimum, the corresponding values of the composite light fields are

\[ X_1 = 0.50 \frac{\Lambda_3^3}{\lambda^{3/7}}, \]
\[ X_2 = 0, \]
\[ X_3 = 2.58 \frac{\Lambda_3^4}{\lambda^{4/7}}. \]  \hspace{1cm} (3.20)

We have checked that these values also minimize the scalar potential of the low-energy sigma model (3.17), (3.19).

The sigma-model approach is especially useful for finding the low-energy spectrum. The vacuum energy density is

\[ M_S^4 = 3.59 \lambda^{10/7} \Lambda_3^4. \]  \hspace{1cm} (3.21)

The scalar mass matrix is given by\(^5\)

\[ m_{\alpha \beta}^2 = \langle V_{\alpha \beta} \rangle \]  \hspace{1cm} (3.22)

\(^5\)To find the masses, one must properly normalize the kinetic terms.
where \( V = W_i K^{-1}_{ij} W_j \) and \( a, b = 1, ..., 6 \) label the six light real fields. It gives three real scalar fields of masses 3.88, 2.83 and 2.04 (in units of \( \lambda^6/\Lambda_3 \)), a complex scalar of mass 1.35 (in the same units), and a massless \( R \) axion. The fermion mass matrix is

\[
m_{ij} = \langle W_{ij} \rangle - K^{-1}_{kl} K_{ij\ell} W_{\ell} \rangle ,
\]

(3.23)

where \( i, j = 1, ..., 3 \) label the three light fermions. The fermion spectrum consists of a massless goldstino, a massless fermion of unit hypercharge, and a fermion of mass \( 3.19 \lambda^6/\Lambda_3 \).

We have also performed an expansion of the full theory around the minimum, along the lines of ref. [12]. We integrated out the heavy fields by substituting the solutions to their equations of motion into the potential \( V_D + V_F \). This gave a potential for the light fields, which we minimized in an expansion in \( \lambda^2/g_2^2 \) and \( \lambda^2/g_3^2 \).

To leading order in \( \lambda \), the light fermion mass matrix is simply

\[
m_{ij} = \langle W_{ij} \rangle ,
\]

(3.24)

where \( W_{ij} \) are the derivatives of the superpotential (3.11), evaluated in the unperturbed minimum (3.20), and \( i, j = 1, ..., 3 \) label the light fermion fields.

The light scalar mass matrix is more complicated. To leading order in \( \lambda \), it is

\[
m_{ab}^2 = \langle V_F_{ab} - V_{DA} V_{DAB} V_F_B \rangle ,
\]

(3.25)

where \( A, B = 1, ..., 11 \) and \( a, b = 1, ..., 6 \) label the eleven heavy and six light real scalars, respectively; \( V_{DAB} \) is the unperturbed heavy-scalar mass matrix; and all derivatives of \( V_F \) and \( V_D \) are evaluated at the unperturbed minimum (3.20). The second term in (3.25) is induced by the order \( \lambda^2/g_{2,3}^2 \) correction to the heavy-field vevs. We found that the light particle spectrum that follows from (3.24), (3.25) is identical to the one presented above.

### 3.3 Supergravity Couplings and R-Axion Mass

In this section we couple the model to supergravity, and compute the supergravity contribution to the \( R \)-axion mass. We also determine the \( R \)-axion couplings and calculate its decay rate into visible particles.

The supergravity coupling is straightforward and can be done either in the full theory or in the effective theory of the previous section. Since we are interested in the light sector only, we will work
with the effective theory. The results, of course, are identical to those that are obtained with the full theory.

The most important effect of the supergravity coupling is its explicit breaking of the $R$ symmetry. In models where all scales are much smaller than the Planck mass\textsuperscript{6}, the only way to cancel the cosmological constant is to add a constant term (2.4) to the superpotential,

$$W_0 = \frac{1}{\sqrt{3}} M_S^2 M_P.$$  

This constant might arise from a distinct sector of the theory; in this paper we assume that it exists, but we do not address its source\textsuperscript{6}. We also assume that there are no other contributions to the constant term in the superpotential, associated with symmetry breaking at a higher scale, because all such phase transitions preserve supersymmetry.

In this class of models, all soft breaking terms (gravitino mass, scalar masses and trilinear scalar terms) are induced by $W_0$. One finds terms in the scalar potential that break $R$ symmetry,

$$V_1 = \frac{1}{\sqrt{3}} \frac{M_S^2}{M_P} \left( W_i K_{ij}^{-1} K_j - 3 W \right) + \text{h.c.} + \ldots ,$$

where $K$ and $W$ are the Kähler potential (3.17) and superpotential (3.19) of the effective theory, and the dots denote terms suppressed by additional powers of $M_P$. The gravitino mass is

$$m_{3/2} = \frac{W_0}{M_P^2} = \frac{1}{\sqrt{3}} \frac{M_S^2}{M_P} = 1.09 \frac{\Lambda^{5/7}}{M_P}.$$  

The $R$-symmetry-breaking terms in the scalar potential also give mass to the $R$ axion. To find the mass, it is easiest to realize the $R$ axion nonlinearly in the effective theory,

$$X_k = \langle X_k \rangle \exp \left( i r_k \frac{a}{f_a} \right) ,$$  

where $X_k$ are the light fields (3.10), and $\langle X_k \rangle$ and $r_k$ are their vevs (3.20) and $R$ charges (3.4), respectively. The axion coupling constant is

$$f_a = 2.18 \frac{\Lambda_3}{\Lambda^{1/7}} = 1.58 \frac{M_S}{\sqrt{\Lambda}} ,$$

\textsuperscript{6}Models where the cosmological constant is canceled by nonrenormalizable $R$-symmetric terms in the Kähler potential typically require Planck-scale vevs \textsuperscript{[3].}

\textsuperscript{7}Actually, the constant can take either sign. The sign determines the potential of the axion field. In the stable vacuum for the axion, this is the correct sign.
while axion mass is
\[ m_a^2 = 10.0 \lambda^{11/7} \frac{\Lambda^3}{M_P} = 6.58\sqrt{\lambda} \, m_{3/2} \, M_S. \] (3.31)

For \( M_S \simeq 10^{11} \) GeV, they become
\[ f_a \simeq 10^{11} \text{ GeV}, \quad m_a \simeq 10^7 \text{ GeV}. \] (3.32)

The supergravity couplings also contribute a small correction, of order \( m_a \), to the particles of mass \( M_S \). The supergravity couplings do not give mass to the fermion of hypercharge one.

### 3.4 Axion Interactions in RHS Models

We now consider the axion interactions in renormalizable hidden-sector models. In such models the supersymmetry breaking is communicated to the visible world by gravity. As discussed above, the constant term in the superpotential induces soft supersymmetry-breaking scalar masses and trilinear terms in the visible sector, with mass parameters proportional to \( m_{3/2} \simeq 10^3 \) GeV.

Let us assume that, in the limit \( M_P \to \infty \), the visible sector has an \( R \) symmetry. Since in that limit the two sectors decouple, the model has two independent \( R \) symmetries – one for the visible and one for the hidden sector. The cosmological term in the superpotential induces terms in the scalar potential that explicitly break both \( R \) symmetries, with strength \( m_{3/2} \).

Now, the \( R \) symmetry in the hidden sector is spontaneously broken at the scale \( M_S \), much higher than the scale of explicit breaking. Therefore the hidden sector has a pseudo-Goldstone \( R \) axion, with mass \( m_a \) (3.32). In the visible sector, on the other hand, the scales of explicit and spontaneous breaking are approximately the same. This implies that there is no pseudo-Goldstone boson associated with the observable-sector \( R \) symmetry.

The interactions of the \( R \) axion with the visible fields are induced by gravity and are therefore suppressed by \( M_P \). The leading-order terms in the scalar potential that mix with the observable and hidden fields include
\[ \frac{1}{M_P^2} (W_{\text{obs}}^i K_{\text{obs}}^i W_{\text{hid}}^* + W_{\text{hid}}^i K_{\text{hid}}^i W_{\text{obs}}^* - 3 W_{\text{hid}}^* W_{\text{obs}} + \text{h.c.}). \] (3.33)

Here \( W_{\text{hid}} \) is the hidden-sector superpotential (3.19), \( K_{\text{hid}} \) is the Kähler potential (3.17), and \( W_{\text{obs}} \) and \( K_{\text{obs}} \) are the corresponding quantities for the observable sector. After substituting the nonlinearily-realized \( R \) axion (3.29), one finds that the couplings in (3.33) induce three-body decays of the \( R \) axion into observable particles, with a suppression factor of \((m_{3/2}/M_P)^2\).
In addition to the decay modes into visible particles, the $R$ axion in this model can decay into pairs of gravitinos, with a helicity suppression factor of $(m_{3/2}/m_a)^2$. If this is the only allowed two-body decay mode, an axion will decay primarily into gravitinos.

In some models there are alternative modes of axion decay. For example, the axion might have an anomalous coupling to two $U(1)_Y$ gauge bosons with a strength of order $\alpha/4\pi$. Then the partial width into this mode is of order $(\alpha/4\pi)^2 m_a^3/f_a^2 \lesssim 10^{-4} m_a^3/f_a^2$.

Another possibility is for the axion to decay into light or massless fermions. However, helicity suppression factors apply here as well. This implies that the axion cannot decay into two massless fermions. Since it is unlikely that models of dynamical symmetry breaking contain fermions with mass between $m_{3/2}$ and $m_a$, the decay rate into fermions is probably similar to that into gravitinos.

Finally, the axion might decay into light scalars. The minimum mass of scalars in these models is of order $m_{3/2}$. If the axion decays dominantly into these scalars, the cosmological problems discussed in the next section are just transferred to the scalar fields.

### 4 Cosmological Bounds

The solution to the relic gravitino problem of supergravity requires an inflationary period in the evolution of the universe. In this section, we show that the standard inflationary scenario, with Hubble constant $H_{\text{infl}} \gg m_a$ during the exponential expansion, can give rise to a coherent background axion field that decays into gravitinos before nucleosynthesis. If the axion has no alternative decay mode, this leaves a large gravitino abundance that affects the light elements abundances from standard big-bang nucleosynthesis.

In the previous section we found that in RHS models the axion typically has a mass of order $10^7$ GeV. For these models, we will derive a bound on the reheat temperature of the universe, independent from that of the thermally-produced gravitinos. For lighter gravitino masses, this bound is competitive with the standard bound on the reheat temperature, while for heavy gravitino masses, the bound is stronger. In this section, we present the bound.

According to the standard inflationary scenario, after the end of the exponential expansion, the $R$ axion field starts oscillating at a time $t_{\text{osc}}$, when the Hubble constant becomes comparable to its mass

$$t_{\text{osc}}^{-1} \simeq H \simeq m_a \simeq 10^7 \, \text{GeV} \, .$$

(4.1)
The amplitude of the oscillations is given by

\[ f \approx f_a \approx 10^{11} \text{ GeV}. \] (4.2)

The number density of R axions in this coherently oscillating wave is \( n_a \approx f^2 m_a \). Hence, at the time of reheating \( t_{rh} \), the ratio of the axion number density to the entropy density is

\[ \frac{n_a}{s} \approx \frac{2.5 f^2 m_a}{g T_{rh}^3} \left( \frac{R_{osc}}{R_{rh}} \right)^3, \] (4.3)

where \( T_{rh} \) is the reheat temperature, \( R_{osc} \) and \( R_{rh} \) are the scale factors at \( t_{osc} \) and \( t_{rh} \) respectively, and \( g \approx 300 \) is the number of effectively massless degrees of freedom. The energy density before reheating is dominated by coherent oscillations of the inflaton field [14]. Therefore the scale factor \( R \sim t^{2/3} \), which implies

\[ \frac{n_a}{s} \approx \frac{2.5 f^2 m_a}{g T_{rh}^3} \left( \frac{t_{osc}}{t_{rh}} \right)^2. \] (4.4)

Substituting

\[ t_{rh}^{-1} \approx H_{rh} \approx 1.7 \sqrt{g} \frac{T_{rh}^2}{M_P} \] (4.5)

into (4.4), we find the axion number-to-entropy ratio after reheating,

\[ \frac{n_a}{s} \approx 7.0 \frac{f^2 T_{rh}}{m_a M_P^2}. \] (4.6)

If the dominant decay mode of the axion is to gravitinos, as in the 3-2 model, its decay rate is

\[ \Gamma_{gg} \approx \frac{m_a}{8\pi} \left( \frac{m_{3/2}}{f_a} \right)^2 \approx 4 \cdot 10^{-11} \text{ GeV}. \] (4.7)

This implies that the axions will be rapidly converted into gravitinos before nucleosynthesis, at a temperature \( T_D \approx 5 \text{ TeV} \). The abundance and lifetime of the gravitinos are constrained by the successful predictions for the light-element abundance from standard big-bang nucleosynthesis.

The exact bound on the gravitino abundance is complicated, since both the abundance and the lifetime depend on the gravitino mass. It also depends on the square of the initial amplitude of the oscillating axion field. Since our assumption for \( f \) was probably low, the bound might actually be stronger (for example, in the 3-2 model, smaller \( \lambda \) means larger \( f \)). This bound also assumes that the axion mass is large, and it too will be smaller in models with smaller energy at the minimum (e.g., small \( \lambda \) in the 3-2 model).
For the purpose of comparing with the bound from ref. [15], we can use (4.6) to find the ratio of the mass density of gravitinos to the photon number density that would hold today if the gravitinos were stable:

\[ X \equiv \frac{m_{3/2} \gamma}{n_{3/2}} \quad (4.8) \]

\[ \simeq 1 \cdot 10^{-8} \text{ GeV} \ B(a \rightarrow gg) \left( \frac{10^7 \text{ GeV}}{m_a} \right) \left( \frac{m_{3/2}}{10^3 \text{ GeV}} \right) \left( \frac{T_{\text{rh}}}{10^{10} \text{ GeV}} \right) \left( \frac{f}{10^{11} \text{ GeV}} \right)^2 , \]

where \( B \) is the branching ratio for the decay of the axion into gravitino pairs. In what follows, we take \( m_a = 10^7 \) GeV and \( f = 10^{11} \) GeV.

The physics behind the bound differs depending on whether the gravitino lifetime is greater or less than about \( 10^4 \) sec. As an example of each case, we will consider gravitinos of mass 300 GeV and 3 TeV. The gravitino lifetime corresponding to a decay to a single massless gauge boson and gaugino is

\[ \tau_{3/2} \simeq 4 \cdot 10^5 \text{ sec} \left( \frac{10^3 \text{ GeV}}{m_{3/2}} \right)^3 \quad (4.9) \]

For gravitino lifetimes longer than about \( 10^4 \) sec, the strongest bounds on the gravitino density come from photodissociation of light elements by the electromagnetic showers produced by gravitino decays [13, 16]. As an example of this case, we consider \( m_{3/2} = 300 \) GeV. Then, if the gravitino decays only to photon-photino pairs, we find a lifetime of \( 1.4 \cdot 10^7 \) sec from (4.9). For this lifetime, fig. 3 of ref. [15] yields the following upper bound on \( XB_{\gamma} \),

\[ XB_{\gamma} \lesssim 3 \cdot 10^{-12} \text{ GeV} , \quad (4.10) \]

where \( B_{\gamma} = 1 \) is the branching ratio for decay into photon-photino pairs. The gravitino abundance from axion decays (4.8) is

\[ X \simeq 3 \cdot 10^{-9} \text{ GeV} \ \frac{T_{\text{rh}}}{10^{10} \text{ GeV}} . \quad (4.11) \]

Comparing (4.10) and (4.11) we see that the axion-produced gravitinos will induce dissociation of the light elements unless the reheat temperature is bounded by

\[ T_{\text{rh}} \lesssim 1 \cdot 10^7 \text{ GeV} . \quad (4.12) \]

However, axions are not the only source of gravitinos. Gravitinos can also be produced by thermal scattering after reheating. For the case of a 300 GeV gravitino, the corresponding bound
on the reheat temperature is $T_{rh} \lesssim 1 \cdot 10^7$ GeV \cite{16}. This is comparable with our bound (4.12) from axion production of gravitinos in RHS models.

If the gravitinos also have a direct hadronic decay channel into gluon-gluino pairs, so $B_\gamma = 1/9$, their lifetime is $1.6 \cdot 10^6$ sec, and the corresponding bound on the reheat temperature becomes $T_{rh} \lesssim 6 \cdot 10^9$ GeV. This bound is weaker than the bound from thermal production.

For gravitino with lifetime less than about $10^4$ sec, photodissociation of light elements does not occur and nucleosynthesis is not affected by electromagnetic showers \cite{14,17}. However, gravitino-induced hadronic showers will cause neutron-proton conversions that change the ratio of neutron-to-proton density \cite{17}. This change affects nucleosynthesis and constrains the abundance of gravitinos.

As an example of the short-lived case, we consider a gravitino of mass $3$ TeV. If the only kinematically-allowed decay of the $3$ TeV gravitino is to a photon-photino pair, hadronic showers can still be induced by quark-antiquark production from the virtual photon. The gravitino lifetime is $1.5 \cdot 10^4$ sec, and the hadronic branching ratio is estimated to be $B_h \simeq 1\%$ \cite{17}. Assuming gravitino decay into two jets of energy $m_3/2 \simeq 1$ TeV, we find the bound

$$\frac{n_{3/2}}{s} \lesssim 1 \cdot 10^{-12}, \ 4 \cdot 10^{-14}$$

from fig. 4 of ref. \cite{17}. The first and second numbers correspond to baryon-to-photon number ratio $N_B/N_\gamma \simeq 10^{-9}, \ 3 \cdot 10^{-10}$, respectively. The gravitino abundance from axion decay (4.6) is

$$\frac{n_{3/2}}{s} \simeq 1 \cdot 10^{-12} \frac{T_{rh}}{10^{10} \text{GeV}}.$$  \hspace{1cm} (4.14)

Comparing with (4.13), we find the following bound on the reheat temperature

$$T_{rh} \lesssim 1 \cdot 10^{10}, \ 3 \cdot 10^7 \text{ GeV},$$

where again the two numbers correspond to $N_B/N_\gamma \simeq 10^{-9}, \ 3 \cdot 10^{-10}$. For such a heavy gravitino, the bound on $T_{rh}$ from ref. \cite{16} comes from the present mass density of photinos. It is weaker, of order $10^{11} - 10^{12}$ GeV.

If both the hadronic and electromagnetic decay channels are open, the gravitino lifetime is $1.6 \cdot 10^3$ sec. For a hadronic branching ratio $B_h = 8/9$, and decay to two jets of energy $1.5$ TeV, the bound becomes \cite{17}

$$\frac{n_{3/2}}{s} \lesssim 1 \cdot 10^{-14}, \ 3 \cdot 10^{-16},$$

(4.16)
where again $N_B/N_\gamma \simeq 10^{-9}, \ 3 \cdot 10^{-10}$. Comparing (4.16) and (4.14), we conclude that the hadronic showers from the gravitino decay will affect nucleosynthesis unless the reheat temperature is bounded by

$$T_{\text{rh}} \lesssim 1 \cdot 10^8, \ 3 \cdot 10^6 \ \text{GeV}. \quad (4.17)$$

We see that for a heavier gravitino, the bounds on the reheat temperature from axion production can be stronger than the bounds from thermal production [16].

Of course, the most important assumption was that the axion decay to gravitinos was substantial. As discussed in the previous section, this is very model-dependent. However, for any model, we expect either a branching ratio of at least $10^4(m_{3/2}/m_a)^2$ into gravitinos, or a large decay rate into some other cosmologically dangerous species. It is interesting nonetheless that the background axion density leads to a potentially dangerous gravitino background in certain models of dynamical supersymmetry breaking.

## 5 Conclusions

It is quite difficult to construct models of dynamical supersymmetry breaking. In this paper we have shown that the existence of an axion does not further constrain these models. Our point is that the cosmological constant can (and should) be cancelled by adding a constant term to the superpotential. This constant explicitly breaks any continuous $R$ symmetry, and gives mass to the $R$ axion.

We have found that in visible-sector models with supersymmetry breaking scale greater than $10^5 \ \text{GeV}$, the axion is sufficiently heavy to evade astrophysical constraints. In nonrenormalizable hidden-sector models, the axion mass is of order the electroweak scale and can lead to cosmological difficulties of the sort already presented by other singlet fields. In renormalizable hidden-sector models, the axion mass is quite large, of order $10^7 \ \text{GeV}$.

In an inflationary scenario, the axion of renormalizable hidden-sector models can be a new source of gravitinos. If the reheat temperature after inflation is too high, the large gravitino abundance affects the successful predictions for the light elements. Our general conclusion, however, is that the axion in such models is cosmologically safe.
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