Research on AC/DC Power Flow Optimization by Using Interior Point Method

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Abstract: At present, China's power system is developing in the direction of AC-DC parallel connection. AC-DC hybrid transmission has become the inevitable pattern of China's future power grid. In the operation of AC/DC hybrid system, the optimization of multiple indexes is often considered. One of the economic problems of system operation is how to minimize the operation cost of the system. This requires the optimal solution of a system of nonlinear equations with constraints. Among the existing algorithms, the interior point method has excellent convergence performance and has been widely used in power system. In this paper, the basic principle of interior point method is introduced, and the interior point method is used to optimize the AC and DC power flow obtained. With the network operation cost as the objective optimization index. The Matlab software is used for programming and simulation calculation. On the basis of verifying the effectiveness of the algorithm, the algorithm is transplanted into the large network for calculation and analysis, and the adaptability of the test algorithm.

1. Introduction

China's energy distribution has the characteristics that the concentrated area of power and the concentrated area of load are far away. Therefore, how to reduce the long-distance transmission loss has become an urgent problem to be solved in China's power network. Due to its pure resistance characteristics, DC transmission does not have reactive power loss in the circuit, which can ensure better power quality to a large extent[1]. Therefore, in the context of the interconnection of AC and DC systems, the optimization of power flow in the overall system has become an important research topic.

Power system power flow optimization is mainly for system operation cost or system total network loss. It is designed to meet the constraints of voltage and power of each node to minimize the overall operating cost or total network loss[2][3]. Power flow optimization is essentially a complex nonlinear programming problem with constraints in engineering mathematics. The algorithms for solving such problems fall into two categories: classical algorithms and artificial intelligence algorithms[4]. Classical algorithms mainly refer to Newton method, projection gradient method, interior point method, etc. Classical algorithms generally search in the feasible domain of the problem according to the derivative and gradient information. The computational efficiency is high, the
algorithm research has a certain history, and the calculation result is higher credibility. Among them, the interior point method is highly praised by researchers because of its good robustness and excellent initial value requirements, and has become the mainstream algorithm in power system optimization. The artificial intelligence algorithm mainly refers to genetic algorithm and simulated annealing method. The selection of control parameters of such algorithms is lack of basis, and the application is given by experience, so the algorithm is unstable and the reliability of the result is poor [5][6].

The principle of the interior point method is discussed in detail in this paper, and it is applied to the power flow optimization of AC/DC hybrid power grid. The effectiveness of the algorithm is further verified by an example. At the same time, the adaptability test of the algorithm is carried out for the larger network, which proves that the algorithm has strong applicability.

2. Mathematical description of AC/DC hybrid power system

Generally, the AC-DC system has a DC part added to the pure AC power system, so it is necessary to set up a converter station to realize mutual conversion between AC and DC. The converter station includes a converter transformer and an inverter [7]. Here, in order to facilitate the subsequent analysis to divide the converter station on the DC side, then the general AC-DC hybrid system can be summarized by the network shown in figure 1.

![Figure 1. Schematic diagram of AC/DC system.](image)

The following is a mathematical analysis of the AC-DC system model. Compared with the AC system, the DC system mainly adds a rectifier and an inverter. Only the basic equations of the rectifier are analyzed here. The inverter is similar to the basic equation of the rectifier.

The rectifier often uses a three-phase bridge rectifier circuit with a semi-controlled device thyristor as the main component. Combined with the characteristics of the circuit and quantitative analysis of the circuit by power electronic technology, the following equations can be obtained:

\[
\begin{align*}
U_d &= k_T U_t \cos \theta_d - X_c I_d \quad & (1) \\
U_d &= k_y k_T U_t \cos \varphi \quad & (2) \\
I_t &= k_y k_T I_d \quad & (3)
\end{align*}
\]

The meanings of the physical quantities in the formula are as follows:

- \(U_d\) and \(I_d\) are the DC voltage and current average respectively; \(k_T\) is the converter transformer ratio; \(U_t\) is the primary side line voltage of the converter transformer; \(X_c\) is the equivalent reactance of the converter transformer; \(\theta_d\) is the converter control angle; \(\varphi\) is the commutation. The power factor of the device; \(k_y\) is a constant, and the actual problem is usually 0.995; \(I_t\) is the current flowing into the converter transformer by the AC system.

The rear side of the rectifier is connected to a DC transmission line. The basic equation of the DC transmission line is:

\[
U_{d1} - U_{d2} - I_d R = 0 \quad (4)
\]

\(U_{d1}\) and \(U_{d2}\) are the rectification side and the inverter side voltage, respectively, and \(R\) is the line resistance.

It can be known from the converter equation and the DC line equation that the DC system has more unknowns than the AC system, which brings great inconvenience to the power flow solution. Therefore, the unknown system is limited by the DC system control characteristic designation control mode. This control feature is also a significant advantage of DC systems. Commonly used control methods include constant current control, constant voltage control, constant power control, fixed control angle control, and constant ratio control. In practical applications, the rectifier usually adopts a
constant current and constant ratio control method; and the inverter usually adopts a fixed control angle and a constant ratio control method[8].

The above analysis of the DC system, and the subsequent optimization work for the trend has been obtained, so the following analysis of the power flow calculation of the AC-DC hybrid system.

The system is divided into an AC node and a DC node. The power equations of the nodes are different and are introduced separately:

For a pure AC node, the power balance equation is:

\[
\begin{align*}
\Delta P_i &= P_{is} - U_i \sum_{j=1}^{n} U_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) = 0 \\
\Delta Q_i &= Q_{is} - U_i \sum_{j=1}^{n} U_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) = 0
\end{align*}
\]

(5)

For DC nodes, the power balance equation is:

\[
\begin{align*}
\Delta P_i &= P_{is} - U_i \sum_{j=1}^{n} U_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \pm U_d l_d = 0 \\
\Delta Q_i &= Q_{is} - U_i \sum_{j=1}^{n} U_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) \pm U_d l_d \tan \varphi = 0
\end{align*}
\]

(6)

A part of the DC node is the power drawn by the DC system from the AC system, and the positive and negative signs correspond to the rectification side and the inverter side[9][10], respectively.

For the converter, write its basic equation as follows:

\[
\begin{align*}
\Delta d_1 &= U_d - k_T U_t \cos \theta_d + X_c l_d = 0 \\
\Delta d_2 &= U_d - k_p k_T U_t \cos \theta_d = 0
\end{align*}
\]

(7)

For DC lines, the DC network equation is as follows:

\[
\Delta d_3 = \pm l_d - \sum_{j=1}^{n_e} g_{dj} U_{dj} = 0
\]

(8)

For different control methods, the following control equation forms are given:

\[
\begin{align*}
\Delta d_4 &= d_4 (l_d, U_d, \cos \theta_d, k_T) = 0 \\
\Delta d_5 &= d_5 (l_d, U_d, \cos \theta_d, k_T) = 0
\end{align*}
\]

(9)

The above equations constitute the tidal current equation for AC/DC networks, and are also the constraint conditions for the subsequent power flow optimization process.

3. Basic principle of interior point method

3.1 Mathematical model of interior point method

As a nonlinear optimization calculation method, the basic principle of the interior point method is to use a "penalty function" strategy, so that the iteration points in the iterative calculation process are all within the feasible domain. Its optimized nonlinear problem can be simplified to the following model:

\[
\text{Obj. min } f(x) \quad \text{S.t. } h(x) = 0 \quad g \leq g(x) \leq \bar{g}
\]

(10)

(11)

(12)

In the above formula (10), minf(x) is a minimum value for the objective function f(x), equation (11) is an equality constraint, a total of m, and equation (12) is an inequality constraint, for a total of r. The above problem is referred to as problem A. When using the interior point method for optimization, we first introduce the relaxation factor to change the inequality constraint into an equality constraint, and then use the equality constraint to construct the augmented obstacle objective function. This function should be close to infinity at the feasible domain boundary of the constraint problem, thus defining the
convergence point within the feasible domain. At the same time, the original objective function \( f(x) \) should be approximated in the feasible domain. Using this augmentation obstacle objective function to regain the nonlinear optimization problem \( P \) is as follows:

\[
\text{Obj. } \min f(x) - \mu \sum_{i=1}^{r} \ln(u_i) - \mu \sum_{i=1}^{r} \ln(l_r)
\]

(13)

\[
\text{S.t. } \begin{cases}
  h(x) = 0 \\
  g(x) + u = \bar{g} \\
  g(x) - l = \bar{g} \\
  u \geq 0, l \geq 0
\end{cases}
\]

(14)

The relaxation variable \( u = [u_1, u_2, ..., u_r], l = [l_1, l_2, ..., l_r] \), and satisfies the slack variable non-negative, where \( r \) is the number of inequality constraints. Equation (13) is the form of the augmentation obstacle objective function commonly used in the interior point method, where \( \mu \) is the disturbance factor and is a positive number. Since the problem \( P \) contains only equality constraints, it can be used to find the optimal solution using the Lagrangian multiplier algorithm:

Construct a Lagrangian objective function:

\[
L = f(x) - y^T h(x) - z^T [g(x) - l - \bar{g}] - w^T [g(x) + u - \bar{g}] - \mu \sum_{i=1}^{r} \ln(u_i) - \mu \sum_{i=1}^{r} \ln(l_r)
\]

(15)

Where \( y = [y_1, ..., y_m], z = [z_1, ..., z_r], w = [w_1, ..., w_r] \) is a Lagrangian multiplier if and only if the Lagrangian function is all. When the partial derivative of the variable and the multiplier is zero, the objective function has a minimum value, that is, the following conditions are met:

\[
\begin{align*}
L_x &= \frac{\partial L}{\partial x} = \nabla_x f(x) - \nabla_x h(x)y - \nabla_x f(x)(z + w) = 0 \\
L_y &= \frac{\partial L}{\partial y} = h(x) = 0 \\
L_z &= \frac{\partial L}{\partial z} = g(x) - l - \bar{g} = 0 \\
L_w &= \frac{\partial L}{\partial w} = g(x) + u - \bar{g} = 0 \\
L_i &= \frac{\partial L}{\partial i} = z - \mu L^{-1} e \Rightarrow l_i^\mu = LZ e - \mu e = 0 \\
L_u &= \frac{\partial L}{\partial u} = -w - \mu U^{-1} e \Rightarrow l_u^\mu = U W e + \mu e = 0
\end{align*}
\]

(16)

Where \( L = \text{diag}[l_1, ..., l_r], U = \text{diag}[u_1, ..., u_r], Z = \text{diag}[z_1, ..., z_r], W = \text{diag}[w_1, ..., w_r] \), can be solved:

\[
\mu = \frac{l^T z - u^T w}{2r}
\]

(17)

Here, \( \text{Gap} = l^T z - u^T w \) is called a complementary gap, and equation (17) is rewritten as follows:

\[
\mu = \frac{\text{Gap}}{2r}
\]

(18)

It can be seen that when the complementary gap \( \text{Gap} \) tends to zero, \( \mu \) also tends to zero, that is, the equation (13) and the equation (10) are infinitely approximated, and the iterative sequence \( x \) generated in the process must converge to the exact analytical solution of the optimization problem \( P \). Therefore, the complementary gap \( \text{Gap} \) can be used as a criterion for judging the end of the iterative calculation. However, in the actual problem, the selection of the parameter \( \mu \) value often affects the convergence performance of the algorithm, so the parameter is modified to increase the parameter (18):

\[
\mu = \sigma \frac{\text{Gap}}{2r}
\]

(19)
Where $\sigma \in (0,1)$ is called the central parameter, and usually 0.1 is used to achieve better convergence performance.

Linearization of equation (16) can be used to obtain a modified equation for each variable, which is rewritten into a matrix form and appropriately simplified. The simplified matrix form of the modified equation is as follows:

$$
\begin{bmatrix}
I & L^{-1}Z & 0 & 0 & 0 & 0 & 0 & \Delta Z \\
0 & I & 0 & 0 & -\nabla_x f(x) & 0 & 0 & \Delta l \\
0 & 0 & I & U^{-1}W & 0 & 0 & 0 & \Delta w \\
0 & 0 & 0 & 0 & \nabla_x g(x) & 0 & 0 & \Delta u \\
0 & 0 & 0 & 0 & 0 & \nabla_x h(x) & 0 & \Delta x \\
0 & 0 & 0 & 0 & 0 & 0 & \nabla_x \phi(x) & \Delta y \\
\end{bmatrix} =
\begin{bmatrix}
-L^{-1}L_1^\mu \\
L_z \\
-U^{-1}L_1^\mu \\
-L_w \\
L_x \\
-L_y \\
\end{bmatrix}
$$

(20)

Solving equation (20), we can get the correction amount of the $k$th iteration, and use this correction amount to correct the variable, and get a new set of approximate solutions closer to the optimal solution as follows:

$$
\begin{align*}
&x^{(k+1)} = x^{(k)} + a_p \Delta x \\
l^{(k+1)} = l^{(k)} + a_p \Delta l \\
u^{(k+1)} = u^{(k)} + a_p \Delta u \\
y^{(k+1)} = y^{(k)} + a_d \Delta y \\
z^{(k+1)} = z^{(k)} + a_d \Delta z \\
w^{(k+1)} = w^{(k)} + a_d \Delta w
\end{align*}
$$

(21)

In the formula, $a_p,a_d$ is the step size, which is usually taken during the calculation:

$$
\begin{align*}
a_p &= 0.9995 \min \left\{ \min \left( \frac{-l_i}{\Delta l_i}, \Delta l_i < 0; \frac{-u_i}{\Delta u_i}, \Delta u_i < 0 \right), 1 \right\} \\
a_d &= 0.9995 \min \left\{ \min \left( \frac{-z_i}{\Delta z_i}, \Delta z_i < 0; \frac{-w_i}{\Delta w_i}, \Delta w_i < 0 \right), 1 \right\}
\end{align*}
$$

(22)

### 3.2 Basic process of interior point method

The above iterative process is repeated as described above, and the variables are continually corrected by equation (21) until the iterative precision is satisfied. The specific calculation flow chart is shown in Figure 2:
4. Example analysis

4.1 Example Mathematical Model Description

For the power flow optimization problem of AC and DC systems, the power flow optimization model of the system should be established first, that is, the form of problem A. The whole system power flow optimization model includes the objective function and its corresponding equality constraints and inequality constraints [11].

4.1.1 Objective function. In this paper, the minimum operating cost of the system is the objective function, and the form is as follows:

\[ f(x) = \sum_{i=1}^{n_g} (a_{2i} P^2_{Gi} + a_{1i} P_{Gi} + a_{0i}) \]  

(23)

Where \( a_{2i}, a_{1i}, a_{0i} \) are the secondary, primary, and constant term coefficients of the power generation coal consumption curve of the i-th generator, respectively; \( n_g \) is the total number of system generators; \( P_{Gi} \) is the working power of the i-th generator.

4.1.2 equality constraints. The AC equation constraint is the power equation of each node. The DC equation constraint is the converter equation and the DC network equation, and has the same form as equations (5) to (9).

4.1.3 Inequality constraints. Due to the complexity of the AC-DC hybrid system, the inequality constraints for different systems are different, but they can be roughly divided into AC and DC.

For the communication part, the main constraints are the upper and lower limits of the active and reactive power of the power plant, the upper and lower limits of the active and reactive power of the branch, and the upper and lower limits of the AC voltage of the node. The form is as follows:

\[
\begin{align*}
P_{Gi} & \leq \bar{P}_{Gi} \leq \bar{P}_{Gi} \\
Q_{Gi} & \leq \bar{Q}_{Gi} \leq \bar{Q}_{Gi} \\
U_i & \leq \bar{U}_i \leq \bar{U}_i \\
-P_{ij} & \leq P_{ij} \leq P_{ij}
\end{align*}
\]  

(24)

Where \( P_{Gi} \) and \( Q_{Gi} \) are the active power and reactive power of the generator respectively; \( U_i \) is the voltage amplitude of node i; \( P_{ij} \) is the line power between nodes i and j; The lower and upper limits of the variable's subscript and superscript corresponding variable.

For the DC part, it mainly includes DC voltage constraint, ratio ratio constraint, control angle constraint, DC current constraint and DC power constraint:

\[
\begin{align*}
P_{si} & \leq \bar{P}_{si} \leq \bar{P}_{si} \\
U_{di} & \leq \bar{U}_{di} \leq \bar{U}_{di} \\
I_{di} & \leq \bar{I}_{di} \leq \bar{I}_{di} \\
k_{rdi} & \leq k_{rdi} \leq k_{rdi} \\
\delta_{di} & \leq \bar{\delta}_{di} \leq \bar{\delta}_{di}
\end{align*}
\]  

(25)

4.2 Analysis of specific examples

In order to verify the effectiveness of the interior point optimization algorithm, the modified classic IEEE-5 node system is analyzed for power flow optimization. The modified system is shown in figure 3.
On the basis of the original system, the line between node 2 and node 3 is replaced with a DC line. The parameters of the replaced DC line and the initial setting parameters of the converter are shown in Table 1. The rectifier adopts constant current and constant ratio control mode; and the inverter adopts fixed control angle and constant ratio control mode. AC system data can be found in the literature [4].

According to the above parameters, the system power flow data is obtained, and the power flow optimization is performed according to the line and generator related constraint parameters. The line and generator related parameters are shown in Table 2 and Table 3, respectively.

At the same time, for the node voltage, the converter related constraints are characterized by the fluctuation range, as shown in Table 4.

In the calculation, the slack variable $l_i = 1$, $u_i = 1$, the initial value of the Lagrangian multiplier takes: $z_i = 1$, $w_i = -0.5$, and the convergence condition $\epsilon < 10^{-6}$. The number of complementary gaps in the calculation process varies with the number of iterations as shown in figure 4.

---

**Table 1.** DC system converter parameters and initial values.

| Bus number | Parameter value |
|------------|-----------------|
|            | $X_C$ | $U_d$ | $I_d$ | $k_T$ | $\cos \theta_d$ | $\varphi f^\circ$ | R |
| 2          | 0.013 | 1     | 1.5   | 1.1167 | 0.913 | 25.84 | 0.0388 |
| 3          | 0.013 | 0.9384 | 1.5   | 1.0479 | 0.9141 | 25.84 |

**Table 2.** Transmission Power Boundary of Lines.

| Branch number | Branch type | Branch end node number | Line transmission power boundary |
|---------------|-------------|------------------------|----------------------------------|
| 1             | AC          | 1-2                    | 2                               |
| 2             | AC          | 1-3                    | 0.65                            |
| 3             | DC          | 2-3                    | 1.6                             |
| 4             | AC          | 2-4                    | 6                               |
| 5             | AC          | 3-5                    | 5                               |

**Table 3.** Generator parameters.

| Generator bus number | Output upper bound | Output lower bound | Fuel consumption parameter |
|----------------------|--------------------|--------------------|----------------------------|
|                      | Active power       | Reactive power     | Active power               | Reactive power | Quadratic coefficient | Primary coefficient | Constant |
| 4                    | 8                  | 3                  | 1                           | -3            | 50.4395               | 200.4335          | 1200.6485 |
| 5                    | 8                  | 5                  | 1                           | -2.1          | 200.55                | 500.746           | 1857.201 |

**Table 4.** Node Voltage and Converter Related Parameters.

|                        | Fluctuation range |
|------------------------|-------------------|
| Node voltage           | ±0.5              |
| Inverter control angle | ±π/6              |
| Converter transformer ratio | ±0.1          |

---

**Figure 3.** Revised IEEE-5 Node AC/DC System. **Figure 4.** Optimal Convergence Curve.
As can be seen from the figure, after 18 iterations of calculation, the results of the convergence are shown in table 5.

**Table 5.** Generator Output and Operation Cost.

| Generator bus | Powerful output | Fuel cost /$  |
|---------------|-----------------|--------------|
|               | Before optimization | After optimization | Before optimization | After optimization |
| 4             | 5.0             | 5.61         | 3463.8          | 3912.51            |
| 5             | 2.69            | 2.18         | 4655.40         | 3901.92            |
| total         | 7.69            | 7.79         | 8119.2          | 7814.43            |

It can be seen from the above calculation that after optimization, the active output of the No. 5 bus generator with higher power generation cost is reduced, and the active output of the No. 4 bus generator with low power generation cost increases. The goal of reducing operating costs was achieved, and overall operating costs were reduced by $304.77.

4.3. Algorithm adaptability test

In order to further verify the credibility of the algorithm, after optimizing the power flow of the 5-node AC-DC network, the power flow optimization of the IEEE30-node AC-DC system is carried out. It is found that the results converge after 21 iterations, and the total power generation cost of the system is reduced after optimization. Network diagram and calculation results are detailed in the appendix. This example further illustrates the superior performance of the interior point method in the AC and DC power flow optimization calculation process. However, if the initial value is improperly selected, the calculation result that does not satisfy the calculation accuracy may occur, so further in-depth work is needed.

5. Conclusion

In this paper, the AC/DC hybrid power network is described by mathematical model, and the steady-state mathematical model of AC/DC system is established. The basic principle and calculation flow of the interior point algorithm are analyzed in detail. Based on the power flow optimization model of AC/DC hybrid system, the interior point algorithm is applied to optimize the network power flow. Through the calculation results of the specific example analysis, it can be found that the interior point method has strong searching ability and fast convergence speed. Under normal circumstances, the optimal solution can be obtained within 30 times of iteration. At the same time, this good convergence performance and optimization ability can make the interior point method applicable to the large-scale AC-DC grid power flow optimization problem, and provide technical support and theoretical basis for other AC-DC grid problems.

6. Appendices

Appendix A. IEEE-30 BUS SYSTEM
Appendix B. GENERATOR OUTPUT AND OPERATION COST OF 30 BUS SYSTEM

| Generator bus | Powerful output | Fuel cost ($) |
|---------------|-----------------|---------------|
|               | Before optimization | After optimization | Before optimization | After optimization |
| 1             | 3.00            | 3.21          | 2255.9            | 2363.77          |
| 2             | 2.67            | 2.25          | 4623.89           | 3999.16          |
| 5             | 2.02            | 2.31          | 2067.86           | 2211.15          |
| 8             | 2.89            | 3.01          | 3224.40           | 3327.76          |
| 11            | 1.25            | 1.10          | 2237.92           | 2143.15          |
| 13            | 1.24            | 1.18          | 2395.97           | 2349.63          |
| total         | 13.07           | 13.06         | 16805.58          | 16394.6          |

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