Optimal Control Strategies for Multipath Routing: From Load Balancing to Bottleneck Link Management

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1. Introduction

In this work we face the Routing problem defined as an optimal control problem, with control variables representing the percentages of each flow routed along the available paths, and with a cost function which accounts for the distribution of traffic flows across the network resources (multipath routing). In particular, the scenario includes the load balancing problem already dealt with in a previous work (Bruni et al., 2010) as well as the bottleneck minimax control problem. The proposed approaches are then compared by evaluating the performances of a sample network.

In a given network, the resource management problem consists in taking decisions about handling the traffic amount which is carried by the network, while respecting a set of Quality of Service (QoS) constraints.

As stated in Bruni et al., 2009a, b, the resource management problem is hardly tackled by a single procedure. Rather, it is currently decomposed in a number of subproblems (Connection Admission Control (CAC), traffic policing, routing, dynamic capacity assignment, congestion control, scheduling), each one coping with a specific aspect of such problem. In this respect, the present work is embedded within the general approach already proposed by the authors in Bruni et al., 2009a, b, according to which each of the various sub-problems is given a separate formulation and solution procedure, which strives to make the other sub-problems easier to be solved. More specifically, the above mentioned approach consists in charging the CAC with the task of deciding, on the basis of the network congestion state, new connection admission/blocking and possible forced dropping of the in-progress connections with the aim of maximizing the number of accepted connections, whilst satisfying the QoS requirements.

According to the proposed approach, the role of the other resource management procedures is the one of keeping the network as far as possible far from the congestion state. Indeed, the more the network is kept far from congestion, the higher is the number of new connection set-up attempts that can be accepted by the CAC without infringing the QoS constraints,
and hence the traffic carried by the network increases. By so doing, the CAC and the other resource management procedures can work in a consistent way, while being kept independent.

This work deals with the multipath routing problem. Multipath routing is a widespread topic in the literature. For example, Cidon et al., 1999, and Banner and Orda, 2007, demonstrate the advantages of multipath routing with respect to single-path routing in terms of network performances; Chen et al., 2004, considers the multipath routing problem under bandwidth and delay constraints; Lin and Shroff, 2006, formulate the multipath routing problem as a utility maximization problem with bandwidth constraints; Guven et al., 2008, extend the multipath routing to multicast flows; Jaffe, 1981, Tsai et al., 2006, Tsai and Kim, 1999 deal with the multipath routing as a minimax optimization problem.

In this work we face the multipath routing problem formulated as an optimal control problem, with control variables representing the percentages of each flow routed along the available paths. As a matter of fact, in the most advanced networks each flow can be simultaneously routed over more than one path: the routing procedure has to decide the percentages of the traffic belonging to the considered flow which have to be routed over the paths associated to the flow in question. According to the above mentioned vision, we assume that other resource management control units (specifically the CAC) already dealt with and decided about issues such as how many, which ones, when and for how long connections have to be admitted in the network, with specific QoS constraints (related to losses and delays) to be satisfied. Therefore, the routing control unit has to deal with an already defined offered traffic. Thus, the admissible set for the routing control variables turns out to be closed, bounded and non-empty, and the existence of (at least) an optimal solution of the routing problem is guaranteed.

The goal of an optimal routing policy aims the routing problem solution towards a network traffic pattern which should make QoS requirements and consequently the CAC task (implicitly) easier to be satisfied. The quality of the routing solution will be evaluated by different performance indices, which take a nominal capacity for each link into account.

As far as the dynamical aspects of a routing problem, we first note that explicitly accounting for them would call for a reliable and sufficiently general dynamical model for the offered traffic. However it is widely acknowledged that such a model is not available and hard to design, due to unpredictable features of Internet traffic. And, in any case, the requested dynamical characters are committed to the CAC procedures, where the more reliable connection dynamics model along with the feedback structure may properly handle the issue.

In addition, a non-dynamical set up for the routing problem makes it much easier to be dealt with. Moreover, this approach could be justified by assuming that the time scale for changes in the routing policy is surely slower than the bit rate fluctuations in the in-progress connections, but it is reasonably faster than the evolution of traffic statistical features. Thus, the routing policy has to be periodically computed to fit the most likely traffic pattern at each given period of time.
In this work, we consider the possibility/opportunity of splitting the given network into sub-networks as detailed in Bruni et al., 2010 each one controlled by a separate subset of variables.

This work is organized as follows. In Section 2, a definition for a reference communication network and its decomposition is given, which is useful for the routing problem; in Sections 3, we in depth study the optimal routing control problem with reference to a number of different cost functions; Section 4 shows some results in order to evaluate the performance and to compare the found optimal solutions for traffic balancing and bottleneck link management; finally, concluding remarks in Section 5 end the work.

2. Reference telecommunication network definition and decomposition

At any fixed time, the telecommunication network can be defined in terms of its topological description as well as in terms of its traffic pattern. As far as network topology is concerned, we consider the network nodes \( n \in \mathbb{N} = \{n_1, n_2, \ldots, n_N\} \) and the network links defined as ordered pairs of nodes \( l \in \Lambda = \{l_1, l_2, \ldots, l_L\} \). To describe the network traffic request we first define a path \( v \in \Omega = \{v_1, v_2, \ldots, v_V\} \) as a collection of consecutive links, denoted by \( \Lambda_v \), from an ingoing node \( i \) to an outgoing node \( j \) (where \( i, j \in \mathbb{N} \)). Moreover a certain set of different Service Classes \( k \in \mathbb{K} = \{k_1, k_2, \ldots, k_K\} \), is defined, each one characterized by a set of Quality of Service (QoS) parameters. According to the most recent trends, the QoS control is performed on a per flow basis, where a flow \( f \in \Phi = \{f_1, f_2, \ldots, f_F\} \) is defined as the triple \( f = (n_i, n_j, k_p) \), with \( n_i \) denoting the ingoing node, \( n_j \) denoting the outgoing node and \( k_p \) denoting the service class. The traffic associated with a given flow \( f \) may possibly be routed on a set \( \Omega_f \) of one or more paths. We further introduce the set of indices \( \{a(l,v), l \in \Lambda, v \in \Omega\} \), defined as follows:

\[
a(l,v) = \begin{cases} 1, & \text{if} \ l \in v \\ 0, & \text{otherwise} \end{cases}
\]

For each link \( l \in \Lambda \), at the given time, we may consider its occupancy level \( c(l) \) defined as the sum of all contributions to the occupancy due to the flows routed on the link itself. Each contribution of this type will be quantified by the bit rate \( R(l,f) \) which, in turn, is the sum of bit rates of all in-progress connections going through the link \( l \) and relevant to the flow \( f \), possibly weighted by a coefficient \( \alpha(l,f) \) which accounts for the specific need of the flow itself. Therefore we have:

\[
c(l) = \sum_{f \in \Phi} \alpha(l,f)R(l,f)
\]

where \( \alpha(l,f) \) are positive known coefficients which take into account the fact that some technologies differentiate the classes of service by varying modulation, coding, and so on. For each link \( l \), we consider the so-called nominal capacity \( c_{\text{NOM}}(l) \), that is the value of the occupancy level suggested for a proper behaviour of the link (typically in terms of QoS)\(^1\).

\(^1\) \( c(l) \) and \( c_{\text{NOM}}(l) \) can be interpreted as generalizations of “load factor” and “Noise Rise” in UMTS (see Holma and Toskala, 2002).
that indicates the fraction of $R(f)$ to be routed on path $v \in \Omega_f$. Then, due to the bit conservation law, we have:

$$R(l, f) = \sum_{v \in \Omega_f} \alpha(l, f) R(f) u(f, v)$$  \hspace{1cm} (3)$$

where obviously:

$$u(f, v) \in [0, 1], \ \forall f \in \Phi, \ \forall v \in \Omega_f$$

$$\sum_{v \in \Omega_f} u(f, v) = 1, \ \forall f \in \Phi$$  \hspace{1cm} (4)$$

As shown in Bruni et al., 2010, with reference to the routing control problem, the link set $\Lambda$ might be decomposed into separated subclasses $\Lambda^j$, $j = 0, 1, 2, \ldots, P$, each of them involving separate subsets of control variables, where $\Lambda^0$ is the set, possibly empty, of links that cannot be controlled by any control variable and which therefore they are not involved in any routing control problem.

For every communicating class of links $\Lambda^j \subset \Lambda$, there exists the (uniquely) corresponding communicating class of flows $\Phi^j \subset \Phi$ defined as the set of flows such that, for each $f \in \Phi^j$, there exists (at least) a link $l \in \Lambda^0$, and therefore a pair of links (generally depending on $f$ itself), which are controllable with respect to $f$. Clearly, the set $\Phi^0$ coincides with the empty set. We now observe that the set $\{\Phi^j, j \geq 0\}$ of flow communicating classes forms a partition of $\Phi$, corresponding to the fact that the set $\{\Lambda^j, j \geq 0\}$ of link communicating classes forms a partition of $\Lambda$. This partition for $\Lambda$ and $\Phi$ immediately induces a partition of the network. Note that each $j$-th part of the network is controlled by a corresponding subvector of control variables, later defined as $u^j$ independently of the other parts; the components of the vector $u(f)$ are the variables $u(f, v)$, $f \in \Phi^j, v \in \Omega_f$. In the following $\{\Lambda^0, \Phi^0\}$ will denote a sub-network. We will use the detailed network decomposition procedure described in Bruni et al., 2010, facing the routing control problem in each sub-network (but in $\Lambda^0$).

3. A rationale for the network loading

In the following, we will focus attention on the routing problem for any given sub-network $\{\Lambda^j, \Phi^0\}$. As mentioned above, any such problem is characterized by a set $u^0$ of control variables, which may be (optimally) selected independently of the other ones. As stated in Bruni et al., 2010, the admissible set for $u^0$ is defined by the constraints:

$$u(f, v) \in [0, 1], \ \forall f \in \Phi^j, \ \forall v \in \Omega_f$$  \hspace{1cm} (5)$$

$$\sum_{v \in \Omega_f} u(f, v) = 1, \ \forall f \in \Phi^j$$  \hspace{1cm} (6)$$

so that the set itself turns out to be convex. From here on, for sake of simplicity the apices $j$ will be dropped.
The optimal choice for \( u \) within its (convex) admissible set may be performed according to a cost function which assesses the network loading. In a previous work Bruni et al., 2010, the control goal was the normalized load balancing in the sub-network, evaluated by the function:

\[
J(u) = \sum_{l \in \Lambda} \left( \frac{c(l)}{c_{NOM}(l)} - k \right)^2
\]  

with \( k \) a given constant. If, for any given \( u \), we optimize (7) with respect to \( k \), we get:

\[
k = \frac{1}{L} \sum_{l \in \Lambda} c(l)
\]

with \( L \) denoting the cardinality of \( \Lambda \). In Bruni et al., 2010, and Bruni et al., 2010 (to appear), a shortcoming of (7) was enlightened, which is due to the partial controllability property (therein defined) of some of the links. These links, in the following referred to as “ballast”, are such that they are bound to accept traffic flows not controlled by the components of the control vector \( u \). Thus other choices of the cost function might be considered which more explicitly account for the network overloading.

One first possibility is to assess the link overflow setting \( k = 0 \) in (7), thus more generally arriving at the functions:

\[
J(u) = \sum_{l \in \Lambda} \left( \frac{c(l)}{c_{NOM}(l)} \right)^m
\]

for some integer \( m \geq 1 \). If the target is to give more importance to the links belonging to several paths the function (9) can be rewritten as follows:

\[
J(u) = \sum_{v \in \Omega} \sum_{l \in \Lambda_v} \left( \frac{c(l)}{c_{NOM}(l)} \right)^m
\]

According to (9), (10) we try to distribute the total load in the network in such a way that the higher the normalized load for a link is, the stronger is the effort in reducing it. This selective attention to the most heavily loaded links progressively increases with \( m \). As \( m \) keeps increasing, then function (10) is approximated by:

\[
J(u) = \sum_{v \in \Omega} (G_v)^m
\]

where:

\[
G_v = \max_{l \in \Lambda_v} \frac{c(l)}{c_{NOM}(l)}
\]

Thus for each path \( v \) the optimization attention is just focused on the most heavily loaded link of the path itself (bottleneck). Eventually we can consider the worst bottleneck load over the whole sub-network:
Remark. Some methods are proposed in the literature to solve the above minimax optimization problem (see Warren et al., 1967, Osborne and Wetson, 1969, Blander et al., 1972, Blander and Charambous, 1972). The original minimax problem (11) is equivalent to the following:

$$J(u) = \max_{v \in \Omega} G_v$$

(13)

where $$g$$ is the vector of auxiliary variables $$g(v)$$, $$v \in \Omega$$. This is a nonlinear (linear if $$m = 1$$, quadratic if $$m = 2$$) programming problem that can be solved by well-established methods.

We observe that the equivalence lies in the fact that, once (14) (15) (16) is solved, the optimal value assumed by $$g(v)$$ coincides with $$G_v$$ in (12), for $$v \in \Omega$$, i.e., it represents the normalized bottleneck link load of path $$v$$.

The load balancing problem (7) (8), with constraints (5) (6) and the bottleneck load management problem (14) (15) (16) are easily seen to be convex. This allows standard minimization routines to be used for its solution, such as MatLab simulation tools.

Remark. The cost function (13) enlightens a further advantage of network decomposition. Indeed, in case the decomposition had not been performed, then (13) would describe an ill-posed optimal control problem whenever the worst bottleneck over the whole network happens to be an uncontrollable link. Similar considerations hold for cost function (11).

4. Evaluation and comparison of optimal routing procedures

4.1 Network structure and decomposition

The considered scenario is composed by 16 nodes and 19 links (see Fig. 1 a)). The traffic pattern involves 4 traffic flows of the same service class $$k$$, from 4 source nodes $$n_i$$, $$i = 1, \ldots, 4$$, to 4 different destination nodes $$n_j$$, $$j = 11, \ldots, 14$$. The traffic pattern is described by the set of traffic flows $$\Phi = \{f_1, f_2, f_3, f_4\}$$, where each traffic flow is identified by the following triples: $$f_1 = (n_1, n_{11}, k), f_2 = (n_2, n_{12}, k), f_3 = (n_3, n_{13}, k), f_4 = (n_4, n_{14}, k)$$. After performing the network decomposition as in Bruni et al., 2010, we recognize three sub-networks (see, Fig. 1 b), c) and d)). The network topology is summarized in Table 1, where the network decomposition is reported as well.
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Table 1. Network Topology and Decomposition; the first row shows the nominal link capacities in [Mbps]; the generic entry \((l_{i}, v_{j})\) is denoted by ‘x’ if \(l_{i} \in \Lambda^{(0)}\); the generic entry \((f_{i}, v_{j})\) is denoted by ‘x’ if it is possible to route \(f_{i}\) on path \(v_{j}\); the generic entry \((l_{i}, \Lambda^{(0)})\) is denoted by ‘x’ if \(l_{i} \in \Lambda^{(0)}\), or by ‘◊’ if \(l_{i} \in \Lambda^{(0)}\) and \(l_{i}\) is a ballast link; the generic entry \((f_{i}, \Lambda^{(0)})\) is denoted by ‘x’ if \(f_{i} \in \Phi^{(0)}\).

The considered scenario has been simulated with MatLab. In particular we have tested two simulation sets reported in subsection 4.2 and 4.3 respectively. In subsection 4.2 we considered the Bottleneck Link Management by varying the weights of the bottleneck loads, while in subsection 4.3 we made comparisons between Load Balancing and Bottleneck Link Management.

![Diagram](https://www.intechopen.com)
Fig. 1. a) Global Network, b) Sub-network 0 ($\Lambda^{(0)}$), c) Sub-network 1 ($\Lambda^{(1)}$), d) Sub-network 2 ($\Lambda^{(2)}$).

4.2 Optimal routing for different weights of bottleneck loads

In this simulation set we consider that the bit rate of traffic flows $f_1, f_3, f_4$ is equal to 5 Mbps whilst the bit rate of traffic flow $f_2$ is equal to 5.4 Mbps. Fig. 2 and 3 show the dependence of the optimal solutions on index $m$ of the Bottleneck Link Management problem.
Fig. 2. Sub-network 1: a) optimal control variables, b) bottleneck link loads, c) link loads.
Fig. 3. Sub-network 2: a) optimal control variables, b) bottleneck link loads, c) link loads.
4.3 Comparisons between load balancing and bottleneck link management

In this simulation set we consider that all the traffic sources transmit with an increasing trend from $4.5 \text{ Mbps}$ to $8.5 \text{ Mbps}$. Tables 2-5 show the network load as the sources bit rate increase, and compares the optimal bottleneck control solutions for $m = 1, 2, 3$ with the load balancing optimal solution.

In Tables 2-5, we denote by bold characters the normalized link loads exceeding 1; hereinafter, the corresponding links will be denoted as overloaded links.

The bottleneck control for $m \geq 2$ manages a higher network load than the load balancing approach. In fact, the tables show that the solutions of the bottleneck control problem are such that no link is overloaded until the flow rates exceed 5 Mbps, 6.5 Mbps and 6.5 Mbps for $m = 1, 2, 3$, respectively; on the other hand, the load balancing solutions are such that no link is overloaded until the flow rates exceed 5 Mbps. Similar results are obtained for sub-network 2.

| Rate [Mbps] | 4   | 4.5 | 5   | 5.5 | 6   | 6.5 | 7   | 7.5 | 8   | 8.5 |
|-------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $u(f_1,v_1)$ |     |     |     |     |     |     |     |     | 1.00|     |
| $u(f_1,v_2)$ |     |     |     |     |     |     |     |     | 0.00|     |
| $u(f_2,v_7)$ |     |     |     |     |     |     |     |     | 0.00|     |
| $u(f_4,v_8)$ |     |     |     |     |     |     |     |     | 1.00|     |

$g(v_1) = 0.74$, $0.83$, $0.93$, $1.02$, $1.11$, $1.20$, $1.30$, $1.39$, $1.48$, $1.57$

$g(v_2) = 0.40$, $0.45$, $0.50$, $0.55$, $0.60$, $0.65$, $0.70$, $0.75$, $0.80$, $0.85$

$g(v_7) = 0.40$, $0.45$, $0.50$, $0.55$, $0.60$, $0.65$, $0.70$, $0.75$, $0.80$, $0.85$

$g(v_8) = 0.74$, $0.83$, $0.93$, $1.02$, $1.11$, $1.20$, $1.30$, $1.39$, $1.48$, $1.57$

$l_1 = 0.40$, $0.45$, $0.50$, $0.55$, $0.60$, $0.65$, $0.70$, $0.75$, $0.80$, $0.85$

$l_2 = 0.00$, $0.00$, $0.00$, $0.00$, $0.00$, $0.00$, $0.00$, $0.00$, $0.00$, $0.00$

$l_5 = 0.00$, $0.00$, $0.00$, $0.00$, $0.00$, $0.00$, $0.00$, $0.00$, $0.00$, $0.00$

$l_6 = 0.74$, $0.83$, $0.93$, $1.02$, $1.11$, $1.20$, $1.30$, $1.39$, $1.48$, $1.57$

$l_7 = 0.40$, $0.45$, $0.50$, $0.55$, $0.60$, $0.65$, $0.70$, $0.75$, $0.80$, $0.85$

$l_{10} = 0.74$, $0.83$, $0.93$, $1.02$, $1.11$, $1.20$, $1.30$, $1.39$, $1.48$, $1.57$

$l_{11} = 0.00$, $0.00$, $0.00$, $0.00$, $0.00$, $0.00$, $0.00$, $0.00$, $0.00$, $0.00$

$l_{14} = 0.00$, $0.00$, $0.00$, $0.00$, $0.00$, $0.00$, $0.00$, $0.00$, $0.00$, $0.00$

$l_{16} = 0.74$, $0.83$, $0.93$, $1.02$, $1.11$, $1.20$, $1.30$, $1.39$, $1.48$, $1.57$

$l_{19} = 0.74$, $0.83$, $0.93$, $1.02$, $1.11$, $1.20$, $1.30$, $1.39$, $1.48$, $1.57$

Table 2. Sub-network 1: Optimal Solutions under bottleneck control, $m = 1$. 

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Table 3. Sub-network 1: Optimal Solutions under bottleneck control, $m = 2$.

| Rate [Mbps] | 4  | 4.5 | 5  | 5.5 | 6  | 6.5 | 7  | 7.5 | 8  | 8.5 |
|-------------|----|-----|----|-----|----|-----|----|-----|----|-----|
| $u(f_1, v_1)$ |    |     |    |     |    |     |    |     |    |     |
| $u(f_1, v_2)$ |    |     |    |     |    |     |    |     |    |     |
| $u(f_4, v_7)$ |    |     |    |     |    |     |    |     |    |     |
| $u(f_4, v_8)$ |    |     |    |     |    |     |    |     |    |     |
| $g(v_1)$   | 0.60 | 0.67 | 0.75 | 0.82 | 0.90 | 0.97 | 1.05 | 1.12 | 1.20 | 1.27 |
| $g(v_2)$   | 0.55 | 0.62 | 0.69 | 0.76 | 0.83 | 0.90 | 0.97 | 1.04 | 1.11 | 1.18 |
| $g(v_7)$   | 0.55 | 0.62 | 0.69 | 0.76 | 0.83 | 0.90 | 0.97 | 1.04 | 1.11 | 1.18 |
| $g(v_8)$   | 0.60 | 0.67 | 0.75 | 0.82 | 0.90 | 0.97 | 1.05 | 1.12 | 1.20 | 1.27 |
| $l_1$      | 0.32 | 0.36 | 0.40 | 0.44 | 0.48 | 0.52 | 0.57 | 0.61 | 0.65 | 0.69 |
| $l_2$      | 0.08 | 0.09 | 0.10 | 0.11 | 0.12 | 0.13 | 0.14 | 0.15 | 0.16 |     |
| $l_5$      | 0.14 | 0.16 | 0.18 | 0.20 | 0.21 | 0.23 | 0.25 | 0.27 | 0.29 | 0.30 |
| $l_6$      | 0.60 | 0.67 | 0.75 | 0.82 | 0.90 | 0.97 | 1.05 | 1.12 | 1.20 | 1.27 |
| $l_7$      | 0.55 | 0.62 | 0.69 | 0.76 | 0.83 | 0.90 | 0.97 | 1.04 | 1.11 | 1.18 |
| $l_{10}$   | 0.60 | 0.67 | 0.75 | 0.82 | 0.90 | 0.97 | 1.05 | 1.12 | 1.20 | 1.27 |
| $l_{11}$   | 0.14 | 0.16 | 0.18 | 0.20 | 0.21 | 0.23 | 0.25 | 0.27 | 0.29 | 0.30 |
| $l_{14}$   | 0.14 | 0.16 | 0.18 | 0.20 | 0.21 | 0.23 | 0.25 | 0.27 | 0.29 | 0.30 |
| $l_{16}$   | 0.60 | 0.67 | 0.75 | 0.82 | 0.90 | 0.97 | 1.05 | 1.12 | 1.20 | 1.27 |
| $l_{19}$   | 0.60 | 0.67 | 0.75 | 0.82 | 0.90 | 0.97 | 1.05 | 1.12 | 1.20 | 1.27 |

Table 4. Sub-network 1: Optimal Solutions under bottleneck control, $m = 3$.

| Rate [Mbps] | 4  | 4.5 | 5  | 5.5 | 6  | 6.5 | 7  | 7.5 | 8  | 8.5 |
|-------------|----|-----|----|-----|----|-----|----|-----|----|-----|
| $u(f_1, v_1)$ |    |     |    |     |    |     |    |     |    |     |
| $u(f_1, v_2)$ |    |     |    |     |    |     |    |     |    |     |
| $u(f_4, v_7)$ |    |     |    |     |    |     |    |     |    |     |
| $u(f_4, v_8)$ |    |     |    |     |    |     |    |     |    |     |
| $g(v_1)$   | 0.59 | 0.66 | 0.73 | 0.81 | 0.88 | 0.95 | 1.03 | 1.10 | 1.18 | 1.25 |
| $g(v_2)$   | 0.57 | 0.64 | 0.71 | 0.78 | 0.85 | 0.92 | 0.99 | 1.06 | 1.13 | 1.20 |
| $g(v_7)$   | 0.57 | 0.64 | 0.71 | 0.78 | 0.85 | 0.92 | 0.99 | 1.06 | 1.13 | 1.20 |
| $g(v_8)$   | 0.59 | 0.66 | 0.73 | 0.81 | 0.88 | 0.95 | 1.03 | 1.10 | 1.18 | 1.25 |
| $l_1$      | 0.32 | 0.36 | 0.40 | 0.44 | 0.48 | 0.52 | 0.56 | 0.59 | 0.63 | 0.67 |
| $l_2$      | 0.08 | 0.09 | 0.10 | 0.11 | 0.12 | 0.13 | 0.14 | 0.16 | 0.17 | 0.18 |
| $l_5$      | 0.15 | 0.17 | 0.19 | 0.21 | 0.23 | 0.25 | 0.27 | 0.29 | 0.31 | 0.33 |
| $l_6$      | 0.59 | 0.66 | 0.73 | 0.81 | 0.88 | 0.95 | 1.03 | 1.10 | 1.18 | 1.25 |
| $l_7$      | 0.57 | 0.64 | 0.71 | 0.78 | 0.85 | 0.92 | 0.99 | 1.06 | 1.13 | 1.20 |
| $l_{10}$   | 0.59 | 0.66 | 0.73 | 0.81 | 0.88 | 0.95 | 1.03 | 1.10 | 1.18 | 1.25 |
| $l_{11}$   | 0.15 | 0.17 | 0.19 | 0.21 | 0.23 | 0.25 | 0.27 | 0.29 | 0.31 | 0.33 |
| $l_{14}$   | 0.15 | 0.17 | 0.19 | 0.21 | 0.23 | 0.25 | 0.27 | 0.29 | 0.31 | 0.33 |
| $l_{16}$   | 0.59 | 0.66 | 0.73 | 0.81 | 0.88 | 0.95 | 1.03 | 1.10 | 1.18 | 1.25 |
| $l_{19}$   | 0.59 | 0.66 | 0.73 | 0.81 | 0.88 | 0.95 | 1.03 | 1.10 | 1.18 | 1.25 |
4.4 Decomposition evaluation

With the purpose of evaluating the decomposition strategy, in this simulation set we consider randomly generated networks, flows and paths, and use the decomposition algorithm to partition the network in sub-networks. The networks were generated starting from a grid of nodes; in particular, the considered network width is 10 nodes. Each column of the grid can be assigned a number of nodes; in the considered network, the number of nodes per column is [18, 18, 18, 16, 10, 10, 16, 18, 18, 18]. 30 flows were considered, starting from a random node of the first column of the network and directed to a random node of the last column. Similarly, each network path is directed from a node of the first column of the network and directed to a node of the last column Fig. 4 a) shows an example of randomly generated network, whereas Fig. 4 a) shows an example of sub-network. The results were obtained by averaging 20 simulations. The average number of variables of the original problem (i.e., the non-decomposed one) is 1984.8, whereas the decomposition manages to decompose the network in 10.2 sub-network (in the average): each sub-network optimization problem has therefore 194.6 variables, i.e., each sub-network problem is reduced by about one order of magnitude.
Fig. 4. a) example of a network (width=10, height=18), b) one of the sub-networks resulting from the decomposition of the network in Fig. 4 a).
5. Conclusion

In this work we formulate the multipath routing problem as an optimal control problem considering various performance indices. In particular, the scenario includes the load balancing problem already dealt with in a previous work Bruni et al., 2010, as well as the bottleneck minimax control problem, in which the traffic load of the bottleneck (raised to a given power $m$) is minimized. The mathematical structure of the problem might easily suggest some issues which are evidenced by the results of Section 4, simply intended to provide a numerical example of more general behaviours. On one side, the load balancing performance index obviously allows to achieve a higher uniformity in the loading of the various links, but it cannot prevent overloading of possible ballast links (apart from ad hoc modifications suggested in Bruni et al., 2010).

On the other side, the minimax (bottleneck) approach succeeds in keeping the bottleneck loads (including the ones of the ballast links), as low as possible, with an effort which happens to be more successful the higher the value of $m$ is. This allows accommodating for a higher traffic flow.

Moreover, we stress the fact that the choice of the proper performance index is a matter left to the network manager in charge of the routing control problem, who will have to take into account at the same time the network structure and capacity, as well as the admitted traffic flow and the possible presence of ballast links.

As a final conclusion, we have considered several cost functions for the multipath routing which are suitable for a certain network load situation. Those cost functions can be properly switched during the operations according to the network needs. In that way our approach is strongly oriented with the most innovative vision of the Future Internet perspective (see Delli Priscoli, 2010), in which the core idea is to take consistent and coordinated decisions according to the present contest.

6. References

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This book guides readers through the basics of rapidly emerging networks to more advanced concepts and future expectations of Telecommunications Networks. It identifies and examines the most pressing research issues in Telecommunications and it contains chapters written by leading researchers, academics and industry professionals. Telecommunications Networks - Current Status and Future Trends covers surveys of recent publications that investigate key areas of interest such as: IMS, eTOM, 3G/4G, optimization problems, modeling, simulation, quality of service, etc. This book, that is suitable for both PhD and master students, is organized into six sections: New Generation Networks, Quality of Services, Sensor Networks, Telecommunications, Traffic Engineering and Routing.

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