Radiation and Ionization Energy Loss Simulation for the GDH Sum Rule Experiment in Hall-A at Jefferson Lab

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Abstract The radiation and ionization energy loss are presented for single arm Monte Carlo simulation for the GDH sum rule experiment in Hall-A at Jefferson Lab. Radiation length and ionization energy loss calculation methods are discussed in detail for $^{12}$C elastic scattering simulation. The relative momentum ratio $\Delta p/p$ and $^{12}$C elastic cross section are compared without and with Radiation Energy Loss and a reasonable shape is obtained by the simulation. The total energy loss distribution is obtained, showing a Landau shape for $^{12}$C elastic scattering. This simulation work will give good support for radiation correction analysis of the GDH sum rule experiment.

Key words GDH Sum Rule, Radiation Thickness, Ionization, SAMC

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1 Introduction

The Gerasimov-Drell-Hearn(GDH) sum rule applied to nuclei relates the total cross section of circularly polarized photons on a longitudinally polarized nucleus to the anomalous magnetic moment of the nucleus:

$$\int_{\text{thr}}^{\infty} (\sigma_{A}(\nu,Q^2) - \sigma_{p}(\nu,Q^2)) \frac{d\nu}{\nu} = -4\pi^2 \frac{\mu_A^2}{J}$$

where $Q^2 = -(p-p')^2$ is the negative four-momentum squared of the exchanged photon; $p$ and $p'$ are the four-momenta of the incoming and scattering electrons, respectively; $\sigma_p$ and $\sigma_A$ are the total photo-absorption cross sections of the nucleus with nuclear spin $J$ parallel and antiparallel, respectively, to the photon polarization; and $\mu_A = \mu - Jq\hbar/M$ is the anomalous magnetic moment of the nucleus, where $q$ and $M$ are the charge and mass of the nucleus. The lower limit is the photo-nuclear disintegration threshold.

In order to obtain precise cross sections from GDH experiments, radiation correction analysis is important. In this article, we will discuss a radiation energy loss simulation based on the Single Arm Monte Carlo (SAMC) package for the GDH experiment in Hall-A at Jefferson Lab.

2 Radiation and Ionization Energy Loss Simulation by SAMC

SAMC is a Monte Carlo package which simulates one of the two Hall-A HRS (High Resolution Spectrometers) at Jefferson Lab. In this article, we focus on the Hadron arm (i.e. the left arm in Hall-A). SAMC works by the following procedure. Firstly, the kinematic domain illuminated and the region of interest for the analysis are defined in the input files. Secondly, the relevant variables are randomly drawn with uniform distribution. All these variables define an event. The event undergoes different checks to see if it reaches the HRS focal plane without being stopped by the various components within the spectrometer. If it passes, the event is reconstructed at the target and stored in the output file. Meanwhile, radiation and ionization energy losses are applied each time the electron goes through some material. Before storing...
the event, a weight corresponding to the cross section of the event and an asymmetry can be assigned. This option is set on or off using the input file. Physics can be added into the Monte Carlo results using this weighting factor (cross section effect) or asymmetry. They are both computed for each event according to its target reconstructed kinematic quantities. Some physical procedures such as $^{12}C$ elastic cross sections, radiative corrections and Landau tail for elastic peaks, $^3He$ quasi-elastic cross sections, asymmetries and external radiation corrections can be processed in this simulation package. The radiation lengths and ionization energy loss calculation method in the simulation are discussed in detail below [4].

2.1 Radiation Length Calculations in the Simulation

The distribution of incident electron energy loss due to bremsstrahlung depends on a frequency in the Coulombic fields of an atom [3–7]. The relation between the energy loss and frequency is expressed as follows.

$$d^2E = -\hbar \nu |N| \frac{d\sigma(\nu, E)}{d\nu} d\nu$$

(2)

where $\nu$ is the frequency of radiated photons; $|N|$ is the number density of atoms. After the following integrating over the entire frequency spectrum of radiated photon, the energy loss per thickness of material which incident electron going through can be obtained as follows.

$$\int \frac{d^2E}{dx} = \frac{dE}{dx} = -|N| \int_0^E \hbar \nu \frac{d\sigma(\nu, E)}{d\nu} d\nu$$

(3)

The variable of integration can be changed to $u = \hbar \nu / E$. Where the $\hbar \nu$ is energy of radiated photons and the $E$ is the energy of incident electron.

$$\frac{dE}{dx} = -|N| E \int_0^1 u \frac{d\sigma(u, E)}{du} du$$

(4)

If incident electron energies are larger than 50 MeV, the above integral is nearly independent of the $E$ [8]:

$$\int_0^1 u \frac{d\sigma(u, E)}{du} du \approx \int_0^1 u \frac{d\sigma(u)}{du} du \equiv \sigma_{rad}$$

(5)

The above $\sigma_{rad}$ nearly depends only on the charge of the nucleus $Z$ and can be expressed as follows [9–11].

$$\sigma_{rad}(Z) = 4\alpha r_e^2[Z^2(L_{rad}(Z) - f(Z\alpha)) + ZL'_{rad}(Z)]$$

(6)

where $r_e$ is the radius of electron and $\alpha$ is the fine structure constant in nucleon. The first and second terms are correlated with the nucleus field. The third term is correlated with the field of atomic electrons. The $L_{rad}$ and $L'_{rad}$ can be defined as follows when considering the entire screening limits.

$$4\alpha r_e^2 = 2.31787 \text{ millibarns}$$

(7)

$$L_{rad}(Z) = \begin{cases} 5.31 & Z = 1 \\ 4.79 & Z = 2 \\ 4.74 & Z = 3 \\ 4.71 & Z = 4 \\ \log(184.15Z^{-1/3}) & Z \geq 5 \end{cases}$$

(8)

$$L'_{rad}(Z) = \begin{cases} 6.144 & Z = 1 \\ 5.621 & Z = 2 \\ 5.805 & Z = 3 \\ 5.924 & Z = 4 \\ \log(1194Z^{-2/3}) & Z \geq 5 \end{cases}$$

(9)

where $f(z)$ is known as the “Coulomb correction” [12]. The $L_{rad}$ is nearly equal to $\log(183Z^{-1/3})$ when neglecting Coulomb correction. If only considering the lowest terms of Coulomb correction, the $L_{rad}$ is nearly equal to $\log(191Z^{-1/3})$.

Due to the $\sigma_{rad}$ is nearly depends only on the charge of the nucleus, the energy distribution of the incident electron can be expressed as follows when it goes through the material.

$$\frac{dE}{dx} = -|N| \sigma_{rad} dx \rightarrow E(x) = E(0) exp(-[N] \sigma_{rad}(Z)x)$$

(10)

We can make a assumption that the incident electron can only scatter with one atom at a moment in the uniform material. Under the assumption, the entire energy loss is expressed as follows.

$$E(x) = E(0) \prod_k exp(-[N]_k \sigma_{rad}(Z_k)x)$$

(11)

$$E(x) = E(0) exp(-\sum_k [N]_k \sigma_{rad}(Z_k)x)$$

where $k$ represents different types of atomic isotope. When the energy loss of electron is $1−1/e$ of its initial energy, the variable called the radiation length is related with that and defined as follows:

$$\bar{X}_0 = \left( \sum_k [N]_k \sigma_{rad}(Z_k) \right)^{-1}$$

(12)
After using the mass density $\rho$ instead of number density $[N]$, we get the normal expression formula as follows.

$$X_0 = \frac{1}{[N] \sigma_{rad}(Z)} = \frac{A}{\rho N_A \sigma_{rad}(Z)} = \frac{X_0}{\rho}$$

(13)

where $\rho$ is mass density; $A$ is the molecular weight and $N_A$ is the Avogadro constant. $X_0$ and $X_0$ are both called the radiation length in the journals [1][13]. The $X_0$ represents the radiation length in mass per unit “area” as follows.

$$X_0 = \frac{A}{N_A \sigma_{rad}(Z)}$$

(14)

Consequently, the radiation thickness with unitless is defined as follows:

$$t = \frac{\rho l}{X_0}$$

(15)

where $l$ is thickness of material.

### 2.2 Ionization Energy Loss Calculation in the Simulation

When incident electrons scattered by atomic electrons in the material, the struck atom could be ionized. This process is called “ionization”. The mean ionization energy loss per unit mass density per unit thickness is defined as follows [13]:

$$\left[ \frac{\Delta}{\rho x} \right] = \left[ \frac{\xi}{\rho x} \right] \left[ 2 \log\left( \frac{pc}{T} \right) - \delta(X) + g \right]$$

(16)

$$\left[ \frac{\xi}{\rho x} \right] = \frac{Za}{A^{3/2}}$$

(17)

$$a = 2\pi N_A r_e^2 m_e c^2 = 0.15353747 \text{ MeV} \cdot \text{cm}^2/\text{mol}$$

(18)

where $\Delta$ is the mean ionization energy loss; $Z$ is atomic number; $A$ is molecular weight of the material; $E$ and $p$ are the electron’s energy and momentum; $I$ is mean excitation potential of the material; $\delta(X)$ is the density correction [14]; the $\xi$ is the “collisional” thickness. Since the mean energy loss is given by Bethe-Bloch equation and the most probable energy loss is given by Landau, the formula of $g$ is expressed by different type as follows [13]:

$$g = \log(\gamma - 1) - F(\gamma) \text{ energy loss (Bethe–Bloch)}$$

(19)

$$g_{mp} = \log \left[ \frac{2\xi}{m_e c^2} \right] - \beta^2 + 0.198 \text{ most probable (Landau)}$$

(20)

where the $F(\gamma)$, $\beta$ and $\gamma$ are defined as follows:

$$F(\gamma) = \left[ 1 + \frac{2}{\gamma} - \frac{1}{\gamma^2} \right] \log(2) - \frac{1}{8} \left[ 1 - \frac{1}{\gamma} \right]^2 - \frac{1}{\gamma^2}$$

$$\approx \log(2) - \frac{1}{8} = 0.56 \text{ for } \gamma \gg 1$$

(21)

$$\beta = \frac{v}{c} = \frac{pe}{E}$$

(22)

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}} = \frac{E}{m_e c^2}$$

(23)

The energy loss per unit thickness can be also given by:

$$\frac{dE}{dx} = -(m_e c^2)[N] \sigma_{coll}$$

(24)

$$\sigma_{coll} = 2\pi r_e^2 \left( \frac{Z}{\beta^2} \right) B = \frac{A}{N_A m_e c^2} \left[ \frac{\xi}{\rho x} \right] B$$

(25)

$$B = \left[ 2 \log\left( \frac{pc}{T} \right) - \delta(X) + g \right]$$

(26)

where $\sigma_{coll}$ is the collisional cross section and B means the stopping number with $g = \bar{g}$. The density correction $\delta$ is defined by [14]:

$$\delta(X) = \begin{cases} \delta(X_0^k) \times 10^2(X - X_0^k) & X \leq X_0^k \\ 2\log(10)(X - X_0^k) + a_3(X_0^k - X)^{m_3} & X_0^k < X < X_1^k \\ 2\log(10)(X - X_1^k) & X \geq X_1^k \end{cases}$$

(27)

where $X_0^k$, $X_1^k$, $m_3$, and $\delta(X_0^k)$ depend on the different material. If the density correction is less than $X_0^k$, it depends on whether the material is an insulator or conductor [17].

If the above $X \geq X_1^k$, the mean energy loss has only a logarithmic relation with energy and the most probable energy loss has no relationship with energy:

$$\frac{p}{m_e c} \geq \frac{p_1}{m_e c} = 10^{X_1^k}$$

(28)

$$\delta(p > p_1) = 2\log(10)(X - X_0^k) = 2\log\left( \frac{p}{p_1} \right)$$

(29)

$$\frac{p_1}{m_e c} = 10^{X_1^k} = \exp\left( \frac{C_s}{2} \right) = \left( \frac{I}{\hbar \omega_p} \right)^2 \sqrt{e}$$

(30)

$$\left[ \frac{\Delta}{\rho x} \right] = \left[ \frac{\xi}{\rho x} \right] \left[ 2 \log\left( \frac{p}{p_1} \right) + g \right]$$

(31)

The density correction parameters from [16] and [18][19] are used. We will use the above methods to calculate the radiation and ionization energy loss in simulation below.
3 SAMC Simulation Results and Discussion

In order to run the SAMC simulation, we need one physical input file, “C12.inp” which contains the physics parameters and kinematic domain (i.e. illumination area). In this article, we only study $^{12}$C elastic scattering before and after Radiation Energy Loss. For convenience, the description of Radiation Energy Loss will include radiation and ionization energy loss. The main parameters in “C12.inp” are shown in Table 1.

Table 1. SAMC Simulation Parameters.

| Parameters | Value | Definition |
|------------|-------|------------|
| $N_{\text{trail}}$ | 2000000 | Number of events for the kinematic domain |
| $E_i$ | 1.14876 GeV | Beam energy |
| $E_p$ | 1.14875 GeV/c | HRS central momentum setting |
| $\theta_{\text{spec}}$ | $-5.99^\circ$ | HRS angle |
| $dp_{ac}$ | 5% | Relative momentum $\Delta p/p$ |
| $d\theta_{ac}$ | 110 mR | Vertical angle range |
| $d\phi_{ac}$ | 50 mR | Horizontal angle range |
| $\text{spot}_x$ | 0.00004 m | Total rastering size in horizontal direction |
| $\text{spot}_y$ | 0.00004 m | Total rastering size in vertical direction |
| tgt | 0.5 cm | Target length in the z direction |
| aspin | $0^\circ$ | Target polarization |
| inl | 0.00247 $g/cm^2$ | Thickness of the matter crossed by incoming $e^-$ |
| outl | 0.0199 $g/cm^2$ | Thickness of the matter crossed by scattered $e^-$ |
| xdi | 0.105 $g/cm^2$ | Thickness of ionization for incoming $e^-$ |
| xdo | 0.798 $g/cm^2$ | Thickness of ionization for scattered $e^-$ |

The beam profile is shown in Fig. 1 with circular raster pattern. The beam sizes in x and y direction are both 0.004 cm. The energy of incoming electrons is spread due to the beam energy dispersion. The external bremsstrahlung, ionization and internal bremsstrahlung are then applied. The beam energy dispersion is taken as $3 \times 10^{-5}$.

Fig. 1. Beam position and size distribution in simulation.

In this simulation, the code treats the elastic scattering as different from the other physics process such as quasi-elastic scattering, because of the correlation between the scattering angle and the outgoing electron momentum. In this case, only the scattering angle is chosen randomly. The momentum of the scattering particle is then computed according to the angle, the mass of the target $^{12}$C and the incoming beam energy.

Fig. 2. Beam relative momentum ratio $\Delta p/p$ (%) without (solid line) and with (dashed line) Radiation Energy Loss at target.

Fig. 2 shows the beam relative momentum ratio distributions when the beam central momentum is equal to 1.14876 GeV/c. The relative momentum ratio depends on the different momentum settings, and can be tuned by the accelerator. Adjusting this momentum setting, the solid line shows the relative momentum ratio without Radiation Energy Loss at the target, with the peak at $-0.615 \pm 0.023\%$; the dashed line shows the relative momentum ratio with Radiation Energy Loss at the target, with the peak...
at $-0.749 \pm 0.027\%$. Obviously, we can see that the $\Delta p$ distribution gets wider after adding the Radiation Energy Loss procedure.

Fig. 3. $^{12}C$ elastic cross section without (solid line) and with (dashed line) Radiation Energy Loss.

Fig. 4 shows the $^{12}C$ elastic cross section distribution. The solid and dashed lines show the distribution without and with Radiation Energy Loss, respectively. From the plot, we can see that the two peaks are both at around 2869 $\mu$ barn. The dashed line is a little lower than the solid line, but goes a little higher than the solid line when the cross section value increases due to the Radiation Energy Loss.

Fig. 4. Total energy loss distribution including radiation and ionization energy loss.

Fig. 5. $^{12}C$ elastic cross section vs total energy loss with Radiation Energy Loss.

Fig. 5 shows a two dimensional distribution of $^{12}C$ elastic cross section versus total energy loss after the Radiation Energy Loss procedure. From this plot, we can see that the cross section value increases when the total energy loss of particles gets higher, but most of the events are at lower total energy loss and cross section value.

The above results are from SAMC simulation for $^{12}C$ elastic scattering without and with Radiation Energy Loss. We have included internal radiation (vacuum polarization, vertex corrections) energy loss and external radiation (ionization and bremsstrahlung) energy loss. These simulation results can be compared with the $^{12}C$ elastic data of the GDH sum rule experiment in future. If the simulation results match well with the elastic data, it will prove the good data quality state. The radiative corrections take the general form $\sigma_{exp} = (1 + \delta)\sigma_{Born}$ where the $\delta$ represents the sum of the internal and external radiative corrections. We can make a comparison between the data and the physical theory after finishing cross section analysis. This simulation work will be helpful for the above data analysis.

4 Summary

This article studied the radiation and ionization energy loss based on single arm Monte Carlo simulation for the GDH sum rule experiment in Hall-A at Jefferson Lab. The radiation length and ionization energy loss calculation methods are discussed in detail for $^{12}C$ elastic scattering simulation. The relative momentum ratio $\frac{\Delta p}{p}$ and $^{12}C$ elastic cross section were compared without and with Radiation Energy Loss and a reasonable shape was obtained by the simulation. The total energy loss (including radiation and
ionization) distribution was obtained, giving a reasonable distribution with Landau shape for $^{12}$C elastic scattering. This simulation work will provide good support for radiation correction analysis of the GDH sum rule experiment.

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References

1. Gerasimov S. Yaz. Phys. Rev. Lett., 1965, 2:598
2. Drell S D., Hearn A C. Phys. Rev. Lett., 1966, 16:908
3. Fermi E. Phys. Rev., 1940, 57:458
4. J. Singh, V. Sulkosky, Radiation Thickness, Collisional Thickness and Most Probable Collisional Energy Loss for E97110: The GDH Sum Rule, the Spin Structure of $^3$He and the Neutron using Nearly Real Photons, JLab Hall-A Technique Note, 2007
5. H. Bethe and W. Heitler. Containing Papers of a Mathematical and Physical Character, 1934, 146(856):83-112
6. John A. Wheeler and Willis E. Lamb. Phys. Rev., 1939, 55(9):858-862
7. John A. Wheeler and Willis E. Lamb. Phys. Rev., 1956, 101(6):1836
8. Hans A. Bethe and Julius Askin. Experimental Nuclear Physics, 1953, Volume1:166-357
9. Yung-Su Tsai. Rev. Mod. Phys., 1974, 46(4):815-851
10. Yung-Su Tsai. Rev. Mod. Phys., 1977, 49(2):421-423
11. Particle Data Group. Physics Letters B, 2004, 592:1-1109
12. Handel Davies, H.A. Bethe, and L.C. Maximon. Phys. Rev., 1954, 93(4):788-795
13. Willliam R.Leo. A How-to Approach. Springer-Verlag, Berlin,Second revised edition, 1994
14. Enrico Fermi. Phys. Rev., 1940, 57(6):485-493
15. H.D. Maccabee and D. G. Papworth. Physics Letters A, 1969, 30:241-242
16. R. M. Sternheimer, M.J.Berger, and S.M.Seltzer. Atomic Data and Nuclear Data Tables, 1984, 30(2):261-271
17. R. M. Sternheimer. Phys. Rev. B, 1981, 24(11):6288-6291
18. R. M. Sternheimer and R.F. Peierls. Phys. Rev. B, 1971, 3(11):3681-3692
19. Stephen M. Seltzer and Martin J. Berger. Int. J. Appl. Radiat. Isot., 1982, 33:1189-1218