On the Orbital Period and Models of V Sge

J. Smak

N. Copernicus Astronomical Center, Polish Academy of Sciences,
Bartycka 18, 00-716 Warsaw, Poland
e-mail: jis@camk.edu.pl

Received

To George Preston on his 92nd Birthday

ABSTRACT

The orbital period of V Sge is decreasing at a rate which increased from \( \frac{dP}{dt} = -(4.11 \pm 0.33) \times 10^{-10} \) in 1962 to \( -(5.44 \pm 0.61) \times 10^{-10} \) in 2022. This implies that the mass transfer from the secondary component is accelerating.

From the evidence based on the orbital period variations, combined with estimates of the mass loss from the system based on radio observations, it follows that (1) the mass transfer rate from the secondary component is larger than \( \dot{M}_2 = -5 \times 10^{-6} \, M_\odot / \text{yr} \), possibly as large as \( \dot{M}_2 = -2.5 \times 10^{-5} \, M_\odot / \text{yr} \), and (2) the mass loss rate from the primary component is \( \dot{M}_1 = -4 \times 10^{-7} \, M_\odot / \text{yr} \) or larger.

Close similarity of V Sge to binary Wolf-Rayet stars supports the model with primary component being a hot, evolved star losing its mass. Several arguments are presented which exclude the alternative model with primary component being a white dwarf with an accretion disk.

Key words:
binaries: close – stars: winds, outflows – X-rays: stars – stars: mass loss – stars: individual: V Sge

1. Introduction

Sixty years ago Herbig et al. (1965) discovered V Sge to be the double-line spectroscopic and eclipsing binary. Using results of their extensive spectroscopic and photometric observations they presented a model of an interacting binary system with two hot components and an extended gaseous envelope. Shortly afterwards it was found that the orbital period is decreasing (Smak 1967). In the following years V Sge was observed by numerous authors in different spectral regions and the original model by Herbig et al. was significantly improved and/or modified (see, e.g., Smak, Belczyński and Zola 2001, for an earlier review).
The purpose of the present paper is twofold: (1) to re-discuss the variations of the orbital period (Sections 2 and 3) and (2) to critically review the two competing models of V Sge (Section 4).

2. The (O-C) Diagram

Moments of minima of V Sge observed prior to the year 1997 were listed by Lockley et al. (1997) in their Table 2. Those observed after that year have been collected from the literature and are listed in Table 1.

| JDhel. 2400000+ | ref | JDhel. 2400000+ | ref | JDhel. 2400000+ | ref |
|-----------------|-----|-----------------|-----|-----------------|-----|
| 50969.4725 (1)  |     | 53900.3619 (5)  |     | 57220.4758 (9)  |     |
| 51781.3817 (2)  |     | 53902.4117 (5)  |     | 57235.3855 (10) |     |
| 52817.4834 (3)  |     | 53940.4648 (5)  |     | 57237.4388 (10) |     |
| 53217.5102 (4)  |     | 53972.3476 (5)  |     | 57272.3977 (10) |     |
| 53233.4622 (4)  |     | 53975.4324 (5)  |     | 57273.4345 (10) |     |
| 53246.3112 (4)  |     | 53991.3636 (5)  |     | 57589.6577 (10) |     |
| 53265.3322 (4)  |     | 53992.4066 (5)  |     | 57590.6892 (10) |     |
| 53266.3649 (4)  |     | 53993.4375 (5)  |     | 57615.3703 (10) |     |
| 53267.3896 (4)  |     | 53993.4427 (5)  |     | 57975.8248 (10) |     |
| 53282.3009 (4)  |     | 54007.3107 (5)  |     | 57983.5202 (10) |     |
| 53283.321 (4)   |     | 54023.2561 (5)  |     | 57988.6675 (10) |     |
| 53284.3587 (4)  |     | 54024.2779 (5)  |     | 57989.6933 (10) |     |
| 53285.3871 (4)  |     | 54026.3407 (5)  |     | 57990.7243 (10) |     |
| 53579.5023 (5)  |     | 54388.3306 (6)  |     | 58006.6587 (10) |     |
| 53580.5293 (5)  |     | 55028.4893 (7)  |     | 58007.6907 (10) |     |
| 53581.5574 (5)  |     | 55097.3965 (7)  |     | 59067.4345 (10) |     |
| 53596.4716 (5)  |     | 56891.3994 (8)  |     | 59068.4595 (10) |     |
| 53615.5040 (5)  |     | 57219.4382 (9)  |     |                   |     |

Notes to Table 1: (1) Ogloza et al. (2000). (2) Zejda (2002). (3) Höbscher (2005). (4) Pribulla et al. (2005). (5) Parimucha (2007). (6) Höbscher et al. (2008). (7) Höbscher et al. (2010). (8) Höbscher and Lehmann (2015). (9) Höbscher (2016). (10) determined from AAVSO light curves.

The values of (O-C), calculated from the original elements given by Herbig et al. (1965).
\[ \text{Pri.Min} = JDhel. \ 2437889.9154 + 0.514195 \times E, \]  

are plotted in Fig.1.

![Diagram](image)

Fig. 1. The (O-C) diagram calculated from the original elements given by Herbig et al. (1965). The line represents Eq.(4).

Fitting a parabola to the points in Fig.1 we get

\[ (O - C) = 0.0008(8) + 0.00000241(10)E - 1.270(26) \times 10^{-10}E^2, \]  

and the corresponding period decrease

\[ \frac{dP}{dt} = (-4.94 \pm 0.10) \times 10^{-10}. \]  

Fitting – for the first time(!) – a third degree polynomial gives

\[ (O - C) = 0.0001(8) + 0.00000218(13)E - 1.057(86) \times 10^{-10}E^2 \\
- 0.41(16) \times 10^{-15}E^3, \]  

and the corresponding period derivatives

\[ \left( \frac{dP}{dt} \right)_{1962} = (-4.11 \pm 0.33) \times 10^{-10}, \quad \frac{d^2P}{dt^2} = (-6.1 \pm 2.3) \times 10^{-15} \ [d^{-1}]. \]  

(5)
The value of $dP/dt$ given above refers to $E = 0$, i.e. to the year 1962. For completeness we give its value referring to the year 2022

$$\left(\frac{dP}{dt}\right)_{2022} = (-5.44 \pm 0.61) \times 10^{-10}. \quad (6)$$

This shows that the rate at which the orbital period is decreasing is increasing and implies that the process of mass transfer from the secondary component is accelerating.

### 3. The Mass Transfer and Mass Loss Rates

In what follows we use system parameters obtained earlier by us (Smak et al. 2001). And, in particular, we refer to the mean brightness of V Sge ("MS-low" in our notation). To begin with, using formulae given by Paczyński (1971, Eqs. 10 and 11), we obtain the following estimates of the thermal time scale and the mass transfer rate from the secondary component:

$$\tau_{K-H,2} = 1.3 \times 10^5 \text{ yrs}, \quad \frac{dM_2}{dt} = \dot{M}_2 = -2.6 \times 10^{-5} M_\odot/\text{yr}. \quad (7)$$

Turning to the primary component we find (see Appendix) that even in the faintest state ("HPSP-faint 1") its bolometric luminosity is larger than the corresponding Eddington luminosity. This implies substantial mass loss from this component.

We also have an independent estimate of the mass loss from the system based on radio observations made by Lockley et al. (1997, 1999). From the flux at $\lambda = 3.6\text{ cm}$, measured during two seasons: $S = 0.07$ and 0.45mJy, assuming distance $d = 2.75\text{ kpc}$, they obtained $\dot{M}_{1+2} = -(0.7 - 2.8) \times 10^{-5} M_\odot/\text{yr}$. Using our distance $d = 4\text{ kpc}$ we get $\dot{M}_{1+2} = -(1.2 - 4.9) \times 10^{-5} M_\odot/\text{yr}$ or $\log \dot{M}_{1+2} = -(4.9 - 4.3)$; we adopt

$$\log \dot{M}_{1+2} = -4.6 \pm 0.3. \quad (8)$$

We now turn to the evidence from the observed period variations. In general they can be described by

$$\frac{d \ln P}{dt} = 3 \frac{d \ln J}{dt} + \frac{d \ln (M_1 + M_2)}{dt} - 3 \frac{d \ln M_1}{dt} - 3 \frac{d \ln M_2}{dt}, \quad (9)$$

where $J$ is the total angular momentum of the system.

Our goal is to obtain a relation between the two mass transfer/mass loss rates: $dM_1/dt = \dot{M}_1$ and $dM_2/dt = \dot{M}_2$. We begin with the angular momentum which is removed from the system together with the escaping material and consider two cases:

**Case 1.** The escaping material has the specific angular momentum identical with that of component 1
\[ j = j_1 = q < j > \quad \text{and} \quad \frac{d\ln J}{dt} = q \frac{d\ln (M_1 + M_2)}{dt}. \]  

(10)

The resulting \( \dot{M}_1 - \dot{M}_2 \) relation becomes:

\[
\frac{d\ln P}{dt} = -\frac{2}{1 + q} \frac{d\ln M_1}{dt} + \frac{3q^2 - 2q - 3}{1 + q} \frac{d\ln M_2}{dt}.
\]

(11)

**Case 2.** The escaping material has the specific angular momentum identical with the mean angular momentum of the system

\[ j = < j >, \quad \text{and} \quad \frac{d\ln J}{dt} = \frac{d\ln (M_1 + M_2)}{dt}. \]

(12)

The resulting \( \dot{M}_1 - \dot{M}_2 \) relation becomes:

\[
\frac{d\ln P}{dt} = -\frac{3q - 1}{1 + q} \frac{d\ln M_1}{dt} + \frac{q - 3}{1 + q} \frac{d\ln M_2}{dt}.
\]

(13)

The results are presented in Fig. 2. From the intersections of the two \( M_1 - M_2 \) relations with the \( M_{1+2} \) line we get crude estimates:

\[ \dot{M}_1 = -2 \times 10^{-5} \, M_{\odot}/yr \quad \text{and} \quad \dot{M}_2 = -5 \times 10^{-6} \, M_{\odot}/yr, \]

(14)

for Case 1 and

\[ \dot{M}_1 = -4 \times 10^{-7} \, M_{\odot}/yr \quad \text{and} \quad \dot{M}_2 = -2.5 \times 10^{-5} \, M_{\odot}/yr, \]

(15)

for Case 2. Worth noting is that \( \dot{M}_2 \) obtained for this case agrees with the value predicted above (Eq.7). This may indicate that Case 2 – better than Case 1 – describes the loss of angular momentum from the system. If so, we can only conclude...
that (1) the mass transfer rate from the secondary component is larger than \( \dot{M}_2 = -5 \times 10^{-6} \, M_\odot/\text{yr} \), possibly as large as \( \dot{M}_2 = -2.5 \times 10^{-5} \, M_\odot/\text{yr} \), and (2) the mass loss rate from the primary component is larger than \( \dot{M}_1 = -4 \times 10^{-7} \, M_\odot/\text{yr} \).

4. The Two Competing Models

Two models were proposed to explain the observed properties and peculiarities of V Sge: Model I – with two hot stars, forming contact or near-contact configuration (Herbig et al. 1965, Mader and Shafter 1997, Lockley et al. 1999, Smak et al. 2001) and Model II – with primary component being the white dwarf with an accretion disk (Greiner and van Teeseling 1998, Patterson et al. 1998, Hachisu and Kato 2003). In both models the more massive secondary component is a main sequence star transferring mass to the primary.

4.1. Model I

Our model (Smak et al. 2001), based on a simple analysis of spectroscopic (radial velocity) and photometric data, resulted in determination of system parameters. Its hypothetical part dealt with the hot, expanding gaseous envelope (or stellar wind), being the source of the ultraviolet and soft X-ray radiation. This requires further support.

Herbig et al. (1965) were the first to note that the emission spectrum of V Sge is similar to that of Wolf-Rayet stars. In what follows we will show that the same is true with respect to the mass outflow rate, the X-ray luminosity and the bolometric luminosity of the primary component.

As discussed in Section 3, the rate of mass loss from the system was obtained by Lockley et al. (1997,1999) from radio observations. We adopted (cf. Eq.8): \( \log M_{1+2} = -4.6 \pm 0.3 \). Both radio observations were made when the observed brightness of V Sge was \( V = 10.9 \pm 0.2 \) (cf. Lockley et al. 1999, Fig.9); the corresponding luminosity of the primary component (see Appendix) was \( \log L_1 = 39.1 \pm 0.3 \). The X-ray luminosity, as obtained by Eracleous et al. (1991) at an assumed distance of 2.7 kpc, was \( L_x = 3^{+2}_{-1} \times 10^{32} \). Using \( d = 4 \) kpc we adopt \( \log L_x = 32.8 \pm 0.2 \). At the time when the X-ray flux was measured the observed brightness of V Sge was \( V = 11.0 \pm 0.1 \) (Šimon and Mattei 1999, Fig.3); the corresponding luminosity of the primary component (see Appendix) was \( \log L_1 = 39.0 \pm 0.2 \).

We now compare V Sge with binary Wolf-Rayet stars using data from Tables 1 and 3 of Nazé et al. (2021). Results, presented in Fig.3, show that – in all three plots – V Sge is indistinguishable from binary Wolf-Rayet stars. This implies that – like in those binaries – the primary component must be a hot, evolved star loosing its mass.
Fig. 3. Comparison of V Sge (red symbols) with binary Wolf-Rayet stars involving the luminosity of the primary component $L_1$, the rate of mass outflow from the system $M_{1+2}$ and the X-ray luminosity $L_x$.

4.2. Model II

This model fails to explain the following facts:

(1) The accretion rate needed to explain the large depth of the primary eclipse in the faint state would be very high and – consequently – the secondary should be strongly heated by the boundary layer. No significant irradiation effect, however, is seen around the secondary eclipse (cf. Fig.5 in Smak et al. 2001).

(2) In this model the soft X-ray flux must come from the boundary layer between the disk and the white dwarf. It is obvious that the X-ray light curve should be dominated by a deep primary eclipse. Hoard et al. (1996) found however that the X-ray flux from V Sge, measured by ROSAT, is modulated with periods equal to 1/2, 1/3, and 1/4 of the orbital period, but not with the orbital period itself.

(3) The double fluorescent OIII emission lines are narrow and define regular, sinusoidal radial velocity curves with nearly identical values of $\gamma_1$ and $\gamma_2$ (Herbig et al. 1965). This implies that they must come from the surfaces of the two components. The hypothetical white dwarf would be totally eclipsed between phases $\phi = 0.94$ and 0.06. Consequently its OIII component should also be eclipsed. Noting that around phase zero the two OIII components cannot be separated we analyze their combined equivalent widths $W_{0.65}(1+2)$ (in units of continuum at phase $\phi = 0.65$), listed in Herbig et al. (1965, Table III). Their mean intensity outside the primary eclipse is $< W_{0.65}(1+2) > = 2.20 \pm 0.08$. The mean intensity of component 1 outside eclipse is $< W_{0.65}(1) > = 0.70 \pm 0.04$. Should this component be eclipsed we would expect – during eclipse – $W_{0.65}(1+2) = 1.50 \pm 0.09$. Instead
the observed mean value is $< W_{0.65}(1 + 2) >= 2.21 \pm 0.19$, which is practically the same as outside eclipse.

(4) From our analysis of the orbital period variations it follows that the primary component is loosing mass at a rate $\dot{M}_1 = -4 \times 10^{-7} M_\odot/\text{yr}$ or – possibly – much higher. Such a significant mass loss from the white dwarf is practically impossible.

Hachisu and Kato (2003) made an attempt to overcome some of those problems by presenting a model based on a number of arbitrary, unrealistic assumptions and involving several extra free parameters. In their model the accretion disk (see their Figs 3, 4 and 6) not only has a very peculiar shape of the outer edge and extends beyond its tidal radius, but penetrates through the secondary component(!)... Such a model cannot be taken seriously.

5. Discussion

One of the results presented above, namely that concerning the accelerated mass transfer from the more massive secondary component (Section 3), requires further comments. Situation here is similar to that during the first stage of the evolution of a close binary leading to the Algol phase (cf. Paczyński 1971): the process of mass exchange is unstable and occurs on a thermal time scale. It is obvious that the mass transfer rate must initially increase (up to a certain maximum) and then decrease. The observed acceleration of the mass transfer in V Sge is therefore not unusual.

6. Appendix: Luminosities of the Primary Component

To estimate the bolometric luminosity of the primary component at various levels of brightness we use the data and results contained in Tables 1 and 2 of Smak et al. (2001). Results corresponding to the black body and Kurucz fluxes are presented in Table 2.

| Description     | V   | log $L_B^B$ | log $L_A^A$ |
|-----------------|-----|------------|------------|
| HPSP-faint1     | 12.51 | 37.96 | 38.16 |
| HPSP-faint2     | 12.12 | 38.21 | 38.38 |
| MS-low          | 11.70 | 38.18 | 38.32 |
| HPSP-bright     | 10.87 | 39.12 | 39.28 |
| MS-high         | 10.76 | 39.25 | 39.34 |

Formal fit to the points gives the following relation between the luminosity of
the primary component and the observed brightness of the system

\[ \log L_1 = 38.323(0.068) - 0.727(0.086) \times (V - 12). \]  

(16)

REFERENCES

Eracleous, M., Halpern, J. and Patterson, J. 1991, \textit{ApJ}, \textbf{382}, 290.
Greiner, J. and van Teeseling, A. 1998, \textit{A&A}, \textbf{339}, 121.
Hachisu, I. and Kato, M. 2003, \textit{ApJ}, \textbf{598}, 527.
Herbig, G.H., Preston, G.W., Smak, J. and Paczyński, B. 1965, \textit{ApJ}, \textbf{141}, 617.
Hoard, D.W., Wallerstein, G. and Willson, L.A. 1996, \textit{PASP}, \textbf{108}, 81.
Höbscher, J. 2005, \textit{IBVS}, 5643.
Höbscher, J. 2016, \textit{IBVS}, 6157.
Höbscher, J. and Lehmann, P.B. 2015, \textit{IBVS}, 6149.
Höbscher, J., Lehmann, P.B., Monninger, G. and Steinbech, H.-F. 2010, \textit{IBVS}, 5941.
Höbscher, J., Steinbech, H.-M. and Walter, F. 2008, \textit{IBVS}, 5830.
Koch, R.H., Corcoran, F.M., Holenstein, B.D. and McCluskey, G.E. 1986, \textit{ApJ}, \textbf{306}, 618.
Lockley, J.J., Eyres, S.P.S. and Wood, J.H. 1997, \textit{MNRAS}, \textbf{287}, L14.
Lockley, J.J., Wood, J.H., Eyres, S.P.S., Naylor, T. and Shugarov, S. 1999, \textit{MNRAS}, \textbf{310}, 963.
Mader, J.A. and Shafer, A.W. 1997, \textit{PASP}, \textbf{109}, 1351.
Nazé, Y., Gosset, E. and Quentin, M. 2021, \textit{MNRAS}, \textbf{501}, 4214.
Ogloza, W., Drózdź, M. and Zola, S. 2000, \textit{IBVS}, 4847.
Paczyński, B. 1971, \textit{Ann.Rev.Astron.Astrophys.}, \textbf{9}, 183.
Parimucha, S. et al. 2007, \textit{IBVS}, 5777.
Patterson, J. et al. 1998, \textit{PASP}, \textbf{110}, 380.
Pribulla, T. et al. 2005, \textit{IBVS}, 5668.
Šimon, V. and Mattei, J.A. 1999, \textit{Astron.Astrophys.Suppl.Ser.}, \textbf{139}, 75.
Smak, J. 1967, \textit{Acta Astron.}, \textbf{17}, 55.
Smak, J.J., Belczyński, K. and Zola, S. 2001, \textit{Acta Astron.}, \textbf{51}, 117.
Zejda, M. 2002, \textit{IBVS}, 5287.