Elliptic Flow Based on a Relativistic Hydrodynamic Model

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November 5, 2017

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Abstract

Based on the (3+1)-dimensional hydrodynamic model, the space-time evolution of hot and dense nuclear matter produced in non-central relativistic heavy-ion collisions is discussed. The elliptic flow parameter $v_2$ is obtained by Fourier analysis of the azimuthal distribution of pions and protons which are emitted from the freeze-out hypersurface. As a function of rapidity, the pion and proton elliptic flow parameters both have a peak at midrapidity.

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One of the main goals in relativistic heavy-ion physics is the creation of a quark-gluon plasma (QGP) and the determination of its equation of state (EoS) \[1\]. It is therefore very important to study collective flow in non-central collisions, such as directed or elliptic flow \[2\]. Recently experimental data concerning collective flow in semi-central collisions at SPS energies has been reported \[3, 4, 5\]. This data should be analysed using various models. Some groups \[6, 7, 8\] have used their microscopic transport models to analyse the collective flow obtained by the NA49 Collaboration \[3\]. In this paper we investigate collective flow, especially elliptic flow, in terms of a relativistic hydrodynamic model.

In non-central collisions elliptic flow arises due to the fact that the spatial overlap region of two colliding nuclei in the transverse plane has an “almond shape”. That is, the hydrodynamical flow becomes larger along the short axis than along the long axis because the pressure gradient is larger in that direction. Therefore this spatial anisotropy causes the nuclear matter to also have momentum anisotropy. Consequently, the azimuthal distribution may carry information about the pressure of the nuclear matter produced in the early stage of the heavy-ion collisions \[9\].

The relativistic hydrodynamical equations for a perfect fluid represent energy-momentum conservation

\[
\partial_\mu T^{\mu\nu} = 0, \quad T^{\mu\nu} = (E + P)u^\mu u^\nu - Pg^{\mu\nu} \tag{1}
\]

and baryon density conservation

\[
\partial_\mu n_B^\mu = 0, \quad n_B^\mu = n_B u^\mu, \tag{3}
\]

where \(E, P, n_B\) and \(u^\mu\) are, respectively, the energy density, pressure, baryon density and local four velocity. We numerically solve these equations without assuming cylindrical symmetry \[10, 11\] by specifying the model EoS and we obtain the space-time dependent thermodynamical variables and the four velocity.

We use the following models of the EoS with a phase transition. Hagedorn’s statistical bootstrap model \[12\] with Hagedorn temperature \(T_H = 155\) MeV is employed for the hadronic phase. We directly use the integral representation of the solution of the bootstrap equation \[13\] instead of using the very famous hadronic mass spectrum, \(\exp(m/T_H)\), which is the asymptotic solution of this equation. It is well known that this model has a limited temperature range, i.e., the energy density and pressure diverge at \(T_H\). This singularity, however,
disappears when an exclude volume approximation [14] (with a Bag constant \(B_{14} = 230 \text{ MeV}\)) is associated with the Hagedorn model. In the QGP phase, we use massless free u, d and s-quarks and the gluon gas model for simplicity. The two equations of state are matched by imposing Gibbs’ condition for phase equilibrium. Consequently we obtain a first order phase transition model which has a critical temperature \(T_C = 159 \text{ MeV}\) and a mixed phase pressure of \(P_{\text{mix}} = 70.9 \text{ MeV/fm}^3\) at zero baryon density.

We mention our numerical algorithm for the relativistic hydrodynamic model. It is known that the Piecewise Parabolic Method (PPM) [15] is very robust scheme for the non-relativistic gas equation with a shock wave. We have extended the PPM scheme of Eulerian hydrodynamics to the relativistic hydrodynamical equation. Note that this is a higher order extension of the piecewise linear method [16].

Assuming non-central Pb+Pb collisions at SPS energy, we choose very simple formulas for the initial condition at the initial (or passage) time \(t_0 = 2r_0/(\gamma v) \sim 1.4 \text{ fm}\) (\(r_0, \gamma\) and \(v\) are, respectively, the nuclear radius, Lorentz factor and the velocity of a spectator in the center of mass system)

\[
E(x, y, z) = E_1(z) \theta(\tilde{z}_0 - z) \theta(z + \tilde{z}_0) \rho(r_p) \rho(r_t),
\]

\[
n_B(x, y, z) = n_{B1}(z) \theta(\tilde{z}_0 - z) \theta(z + \tilde{z}_0) \rho(r_p) \rho(r_t),
\]

\[
v_z(x, y, z) = v_0 \tanh(z/z_0)
\]

\[
\times \theta(\tilde{z}_0 - z) \theta(z + \tilde{z}_0) \rho(r_p) \rho(r_t),
\]

where \(\theta(z)\) is the step function, \(\rho(r)\) is the Woods-Saxon parameterization in the transverse direction,

\[
\rho(r) = \frac{1}{\exp\left(\frac{r - r_0}{\delta_r}\right) + 1},
\]

\(E_1(z)\) is Bjorken’s solution [14] and the \(z\) dependence of the baryon density \(n_{B1}(z)\) is taken from Ref. [18]

\[
E_1(z) = E_0 \times \left(\frac{\sqrt{t_0^2 - z^2}}{t_0}\right)^{-\frac{3}{2}},
\]

\[
n_{B1}(z) = \kappa \times 0.17 \sqrt{\frac{t_0^2 - z^2}{t_0}}.
\]

See also Fig. 1. We have employed Bjorken’s longitudinal solution just as an initial condition. This is in contrast to Ref. [9, 19], in which Bjorken’s boost-invariant solution was used as an assumption.
and the hydrodynamical equation was numerically solved only in the transverse plane.

At relativistic energies the Lorentz-contracted spectators leave the interaction region after ~ 1 fm, we therefore assume the hydrodynamical description is valid only in the overlap region and neglect the interaction between the spectators and the fluid. Therefore we can say that our model gives a good description only in the vicinity of the midrapidity region and fails to reproduce directed flow at present. It may be possible to treat this problem if we use a hadronic cascade model for both spectators and particles emitted from the freeze-out hypersurface, together with the hydrodynamic model.

There are four initial (and adjustable) parameters in our hydrodynamic model: 1) the energy density at $z = 0$, $E_0 = 2500 \text{ MeV/fm}^3$, 2) the factor in the baryon density distribution $\kappa = 2.5$, 3) the initial longitudinal factor $\varepsilon = 0.9$ and 4) the “diffuseness parameter” $\delta_r = 0.3 \text{ fm}$. In the present analysis we select these values ‘by hand’, i.e., we guess them. These parameters, however, should be chosen so as to reproduce the experimental data for the (pseudo-)rapidity and the transverse momentum distribution. To make our analysis more quantitative, we need this experimental data. We would like the experimental group to analyze the centrality dependence of the hadron
(To be continued on next page.)
Figure 2: Time evolution of pressure and baryon flow in the transverse plane. Left: The pressure contours. Right: The baryon flow velocity vector \((n_B v_x, n_B v_y)\).
spectra, especially, the (pseudo-)rapidity distribution. For this reason we wish to emphasize that our numerical results presented below are only preliminary.

Figure 2 shows our numerical results for the temporal behavior of the pressure (left column) and the baryonic flow (right column) at $z = 0$ in the non-central Pb+Pb collision with impact parameter $b = 7$ fm at SPS energy. Initially almost all matter in this plane is in the QGP phase and there is no transverse flow anywhere by definition. At $t = t_0 + 0.5$ fm we see the shell structure corresponding to the mixed phase with the same pressure $\sim 70$ MeV/fm$^3$, and the initial pressure gradient gives the baryons transverse flow. The QGP phase disappears at $t = t_0 + 1.0$ fm and after that the mixed phase occupies the central region. There is still no transverse flow near the origin due to the absence of a pressure gradient. At about $t = t_0 + 5.0$ fm all the nuclear matter initially in the QGP phase has gone through the phase transition and is in the hadronic phase. We can see from these figures that the shape of the nuclear matter is changing from almond (top figure on page 5) to round (bottom figure on page 6), and the elliptic flow reduces the initial geometric deformation.

The numerical results of the hydrodynamical simulation give us the momentum distribution through the Cooper-Frye formula \[ v_2(y) = \left\langle \frac{(p_x)}{p_t^2} - \left\langle \frac{(p_y)}{p_t^2} \right\rangle \right\rangle = \frac{\int_{0}^{2\pi} d\phi \cos(2\phi) \int_{p_{t+}}^{p_{t-}} p_t dp_t E \frac{d^3N}{dp^3}}{\int_{0}^{2\pi} d\phi \int_{p_{t+}}^{p_{t-}} p_t dp_t E \frac{d^3N}{dp^3}}. \] (11)

Before calculating $v_2$ in non-central collisions with impact parameter $b = 7$ fm, we checked the numerical error in our hydrodynamic model in central collisions. Since there is no special direction in the transverse plane for head-on collisions, ideally the elliptic flow vanishes in the infinite particle limit. Performing the numerical simulation with $b = 0$ fm, we obtain the value of $v_2$ as less than $10^{-1}$ percent, therefore we can safely neglect the numerical error. Note that the numerical error in the energy and baryon density conservation of the fluid is less than one percent in our analysis.

Figure 3 shows our results for the rapidity dependence of elliptic flow for pions in different transverse momentum regions. These results show that elliptic flow rises with transverse momentum $p_t$ \[ [21] \] and has a peak at midrapidity. This seems to be in contrast with the
Figure 3: Rapidity dependence of elliptic flow for pion. Four curves correspond to the different transverse momentum regions. The midrapidity is 2.92.

Figure 4: Rapidity dependence of elliptic flow for proton. Three curves correspond to the different transverse momentum regions. Note that the integral region of transverse momentum is larger than for pions.
experimental data obtained by the NA49 Collaboration \cite{3}. Their data appears to be slightly peaked at medium-high rapidity.

Our results for $v_2$ for protons are shown in Fig. 4. We see the same behavior as for the pion case. We obtain a larger $v_2$ for protons than for pions because we are integrating over a larger transverse momentum region. Since the initial parameters in our hydrodynamic model have been chosen by hand, we would like readers to not take these results quantitatively.

In summary, we reported our preliminary analysis of elliptic flow in non-central heavy-ion collisions using the hydrodynamic model. We numerically simulated the hydrodynamic model without assuming cylindrical symmetry or Bjorken’s boost-invariant solution, using the extended version of the Piecewise Parabolic Method which is known as a robust scheme for the non-relativistic gas equation with a shock wave. We presented the temporal behavior of high temperature and high density nuclear matter produced in Pb+Pb collisions with $b = 7$ fm at SPS energy. Our preliminary results showed that the elliptic flow parameter $v_2$ has a peak at midrapidity for both pions and protons and increases with transverse momentum. Since there are some ambiguities in the initial parameters of our hydrodynamical model, we should fix these parameter using experimental data for the rapidity distribution in non-central collisions. If we regard the hydrodynamical model as a predictive one, we can choose initial parameters using results from a parton cascade model, such as VNI \cite{22}. The study of these issues is a future work.

The author is much indebted to Prof. I. Ohba, Prof. H. Nakazato, Dr. Y. Yamanaka and Prof. S. Muroya for their helpful comments, and to Dr. H. Nakamura, Dr. C. Nonaka and Dr. S. Nishimura for many interesting discussions. The numerical calculations were performed on workstations of the Waseda Univ. high-energy physics group.

References

[1] See, for example, Quark Matter ’97, Nucl. Phys. A638 (1998).
[2] For a review, see J.-Y. Ollitrault, Nucl. Phys. A638 (1998) 195c.
[3] H. Appelshäuser et al. (NA49 Collaboration), Phys. Rev. Lett. 80 (1998) 4136.
[4] S. Nishimura et al. (WA98 Collaboration), Nucl. Phys. A638 (1998) 459c.
[5] F. Ceretto et al. (CERES Collaboration), Nucl. Phys. A638 (1998) 467c.
[6] H. Liu, S. Panitkin and N. Xu, Phys. Rev. C59 (1999) 348.
[7] H. Heiselberg and A.-M. Levy, nucl-th/9812034.
[8] S. Soff et al., nucl-th/9903061.
[9] J.-Y. Ollitrault, Phys. Rev. D46 (1992) 229.
[10] D. H. Rischke et al., Nucl. Phys. A595 (1995) 346.

[11] Note that to my knowledge there is only one scheme to simulate the non-central heavy-ion collisions which uses Lagrangian hydrodynamics: C. Nonaka et al., these proceedings.

[12] R. Hagedorn, Suppl. Nuovo. Cim. 3 (1965) 147; see also R. Hagedorn and J. Rafelski, in Statistical Mechanics of Quarks and Hadrons, edited by H. Satz (1981) p. 237, North Holland, Amsterdam; R. Hagedorn, in Hot Hadronic Matter, Theory and Experiment, edited by J. Letessier, H. H. Gutbrod and J. Rafelski (1995) p. 13, Plenum Press, New York.

[13] R. Hagedorn and J. Rafelski, Commun. Math. Phys. 83 (1982) 563.

[14] J. I. Kapusta and K. A. Olive, Nucl. Phys. A408 (1983) 478.

[15] P. Colella and P. R. Woodward, J. Comput. Phys. 54 (1984) 174.

[16] See, for example, V. Schneider et al., J. Comput. Phys. 105 (1993) 92.

[17] J. D. Bjorken, Phys. Rev. D27 (1983) 140.

[18] J. Sollfrank et al., Phys. Rev. C55 (1997) 392.

[19] D. Teaney and E. V. Shuryak, nucl-th/9904006.

[20] F. Cooper and G. Frye, Phys. Rev. D10 (1974) 186. Although there is a well-known problem in this formula when it is applied to the space-like freeze-out hypersurface, we use this formula for simplicity.

[21] P. Danielewicz, Phys. Rev. C51 (1995) 716.

[22] See, for example, K. Geiger, Phys. Rev. D46 (1992) 4965.