Letter

Time Reversal Linearly Constrained Minimum Power Algorithm for Direction of Arrival Estimation in Diffuse Multipath Environments

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Abstract: Direction of arrival (DOA) estimation in diffuse multipath environments is a challenge for ground-based radar remote sensing applications, which has significant value in military fields, such as air defense surveillance. However, radar received echo usually contains various multipath signals caused by the reflection of complex ground or sea surface. With the introduction of multipath signals, traditional algorithms’ performance on angle estimation decreases severely. In response to this problem, the letter proposes a new time reversal (TR) algorithm used for multiple-input multiple-output (MIMO) radar angle estimation. First, the algorithm reconstructs a TR covariance matrix by multiplexing the data’s rows and columns, increasing the estimation accuracy of the TR covariance matrix. Besides, the letter applies a linearly constrained minimum power (LCMP) constraint to suppress diffuse multipath signals according to the prior knowledge of environments. Simulation results examine the improvement of estimation accuracy by the proposed algorithm, also verify the superiority of the proposed algorithm in different multipath scenarios. What’s more, the algorithm has broader applicability due to avoiding the difficulties of removing the coherence and estimating multipath number in practice.

Keywords: diffuse multipath; direction of arrival; linearly constrained minimum power; multiple-input multiple-output; time reversal

1. Introduction

Multipath is a common phenomenon in electromagnetic wave propagation, and corresponding low-altitude multipath reflection model has attracted many scholars’ attention [1–3]. Direction of arrival (DOA) tracking in multipath environments is an essential application for radar remote sensing detection. However, traditional angle estimation algorithms perform poorly or even fail under multipath scenes, because multipath interfere with target signal in space, time, and Doppler domain, which decreasing the detection performance and estimation accuracy. Moreover, most classical multipath models consider mirror multipath and ignore diffuse multipath. Because the diffuse multipath model is hard to model accurately due to diffuse multipath’s randomness and uncertainty.

As for multipath, traditional strategy treats the multipath as interference and suppresses it as entirely as possible [4]. This method limits the detection performance and estimation accuracy due to without exploiting multipath. Therefore, scholars try to find a novel technique to utilize multipath; luckily, time reversal (TR) is one such advanced tool. Fink presented the time reversal concept and time-reversal mirrors as new approaches for focusing ultrasonic field in both theory and experiments [5,6]. They verified that TR could accomplish super-resolution focusing. Then, there are some references on acoustic [7,8]
and ultrasound non-destructive testing [9,10]. With the development of TR, researchers showed the spatial resolution of TR in electromagnetic using antennas [11,12]. Rosny et al. further demonstrated the theory of electromagnetic TR mirrors in math [13]. Nowadays, TR radio applications has expended to many aspects, radiotherapy medicines [14,15], radar systems [16–18], and wireless networks [19,20]. These researches suggest TR can improve detection performance, imaging capabilities, or channel capacity. Recently, multiple-input multiple-output (MIMO) radars have attracted considerable attention from scholars [21–23]. MIMO radars, which adapt orthogonal waveforms technique to improve spatial freedom, have an excellent performance in target detection and parameter estimation [24–26]. Therefore, applying TR MIMO radars in low-angle estimation has excellent potential.

In TR MIMO DOA estimation, Tan proposes an adaptive TR multiple signal classification (MUSIC) algorithm by matching the mirror multipath channel [27]. Without referring to the precise model of the multipath channel used in [27], Liu applies forward-backward spatial smoothing (FBSS) technology to remove coherence, which accomplishes multiple target parameters estimation [28]. However, this algorithm losses an array aperture for using the FBSS technique. What’s worse, these TR algorithms suffer from various diffuse multipath, which is difficult to be modeled precisely in real low angle scenes. Avoiding modeling the multipath, Niu applies the TR minimum norm like (MNL) algorithm to solve the diffuse parameter estimation problem in complex terrains [29]. But the algorithm ignores MIMO radars’ advantage of the orthogonal waveform in parameter estimation, limiting the improvement of estimation accuracy. When it comes to TR MIMO radars in rich multipath environments, there are two methods to deal with it. Foroozan adapts space-time adaptive processing algorithm to estimate angle and Doppler jointly in urban environments [30]. This means causes severe errors due to the ambiguity between the angle and Doppler, especially for little incident angles. In [31], Sajjadieh combines compressing sensing (CS) and TR to estimate potential targets in an environment with numerous clutter. Unfortunately, its CS data dictionary is tough to be set up in low-angle settings due to the uncertainty of the low impinging angles.

In this letter, we propose a TR linearly constrained minimum power (LCMP) algorithm that addresses the angle estimation problem in diffuse multipath environments. The arrangement of this letter is organized as follows: Section 2 formulates the TR MIMO multipath model; Section 3 derives the TR LCMP algorithm; Section 4 presents the simulation results; Section 5 concludes the letter.

2. Multipath System Model

2.1. MIMO Radar

As shown in Figure 1, consider a monostatic MIMO radar consists of two colocated arrays. They are equipped with N elements and P elements at transmitter and receiver, respectively. For each array, element inner distance is half a wavelength. Suppose there is only one target considered in the scene, and also exist M pieces of transmission paths, including one direct way and M – 1 reflection paths. The number of multipath is unknown due to its randomness in practice. As for i-th path, the attenuation factor, delay of time, and DOA are $\alpha_i$, $\tau_i$, $\theta_i$ respectively. The angle of direct-path is above $0^\circ$ while others are below $0^\circ$.

The first probing signal emitted by each transmit element is $f_n(t) = e^{j2\pi f_c t} \mathbf{f}_n(t)$, where $f_c$ is the carrier frequency, $f_n(t)$ is the baseband envelope of the probing signal. All N transmitting signal can be written in the vector form: $\mathbf{f} = [f_1, f_2, \ldots, f_N]^T$. Suppose that the baseband signal is orthogonal to each other, that is, $\mathbf{f} \mathbf{f}^H = 0$, $\mathbf{f}^H = \mathbf{I}_N$. For static targets or slow-moving targets, the Doppler shift can be ignored. Thus the receiving signal of p-th ($1 \leq p \leq P$) element is [30]:

$$r_p(t) = \sum_{m_j=1}^{M} \sum_{m_f=1}^{M} \sum_{n=1}^{N} \alpha_{m_f} \alpha_{m_j} f_n(t - \tau_{nm_f}(t) - \tau_{pm_j}(t)) \times e^{j2\pi f_c (t-\tau_{nm_f}(t)-\tau_{pm_j}(t))} + n_p(t), \quad (1)$$
where \(m_b\) and \(m_f\) represent the backward scattering path and forward scattering path, \(l = (m_f, m_b)\) represents a round trip. \(\tau_{nmf}\) and \(\tau_{bmp}\) are the propagation delays via forward and backward multipath between the \(n\)-th transmit element to the target and from the target to the \(p\)-th receive element, respectively. \(n_p(t)\) is the observation noise of the \(p\)-th element in MIMO stage. In the vector form, \(r = [r_1, r_2, \ldots, r_P]^T\) is recorded as:

\[
r(t) = \sum_{l=1}^{L} \tilde{a}_l e^{-j\omega_c \tau_l(0)} A(\Theta_l) f(t - \tau_l(0)) + n(t),
\]

where the carrier angular frequency \(\omega_c = 2\pi f_c\), \(\tau_l(0) = \tau_{nmf}(0) + \tau_{bmp}(0)\) is the delay between the reference transmit and receive element (1,1), \(n(t)\) is the observation noise matrix of all the receive elements in MIMO stage. \(A(\Theta_l) = \alpha_R(\theta_{m_b})\alpha^T_T(\theta_{m_f})\) is the transmit-receive steering matrix, where \(\alpha_R\) and \(\alpha_T\) are given by:

\[
\alpha_R(\theta_{m_b}) = [1, e^{-j\omega_c \tau_{T1}(\theta_{m_b})}, \ldots, e^{-j\omega_c \tau_{TN}(\theta_{m_b})}]^T,
\]

\[
\alpha_T(\theta_{m_f}) = [1, e^{-j\omega_c \tau_{T1}(\theta_{m_f})}, \ldots, e^{-j\omega_c \tau_{TP}(\theta_{m_f})}]^T.
\]

![Schematic diagram of multipath system model.](image)

**Figure 1.** Schematic diagram of multipath system model.

### 2.2. TR MIMO Radar

According to the principle of TR, \(r(t)\) is time-reversed, conjugated, energy normalized by \(c\) and retransmitted. The second transmitting signal is \(cr^*(t)\), where \((\cdot)^*\) represents the conjugation operator. The normalization coefficient \(c\) is:

\[
c = \sqrt{||f||_2 / ||r||_2}.
\]
Following the derivation of Equation (2), the TR receiving signal can be written as:

\[ x(t) = c \sum_{l'=1}^{L} \bar{\alpha}_{l'} A^T(\Theta_{l'}) r^* (-t + \tau_{l'}(0)) + v(t) \]

\[ \approx c \sum_{l=1}^{L} |\bar{\alpha}_l|^2 A^T(\Theta_l) A^*(\Theta_l) f^* (-t) + q(t), \]

where \( v(t) \) is the observation noise for the TR stage, \( q(t) \) is the accumulated Gaussian white noise, which takes \( n(t) \) and \( v(t) \) into account. In line with the super-resolution focusing property of TR [32], the approximation in Equation (6) is valid. The normalization coefficient \( c \) is a constant value, which is set to 1 for simplicity.

3. TR LCMP Algorithm

This section formulates the novel algorithm based on the multipath system model in Section 2. Following the procedure used in conventional matched filtering, the letter applies the matched filter to the TR observation in Equation (6) by multiplying \( f f^H = f^* f^T = I_N \), the new signal \( Y(t) \) is [28]:

\[ Y(t) = E[x(t)f^T(-t)] \]

\[ = \sum_{l=1}^{L} |\bar{\alpha}_l|^2 A^T(\Theta_l) A^*(\Theta_l) + u(t), \]

where \( u(t) = E[q(t)f^T(-t)] \) is a new noise matrix whose elements obey the Gaussian distribution, \( E[\cdot] \) is the expectation operator.

Take the \( l \)-th round trip as an example, the matrix \( A_{TR}(\Theta_l) \) can be written as:

\[ A_{TR}(\Theta_l) = A^T(\Theta_l) A^*(\Theta_l) \]

\[ = (a_R(\theta_{m_1}) a_f^T(\theta_{m_1}))^T (a_R(\theta_{m_N}) a_f^T(\theta_{m_N}))^* \]

\[ = P\alpha_T(\theta_{m_1}) a_f^H(\theta_{m_1}), \]

where \((\cdot)^H\) is the conjugate transpose operator, and the matrix \( A_{TR} \) is related to the forward scattering angle \( \theta_{m_1} \) for one round trip. According to this characteristic, the \( Y(t) \) in Equation (7) can be rewritten as:

\[ Y(t) = c_1 \sum_{m_f=1}^{M} |\bar{\alpha}_{m_f}|^2 a_f^T(\theta_{m_f}) a_f^H(\theta_{m_f}) + u(t) \]

\[ = c_1 A_M A_M^H + u(t), \]

where \( A_M = [\bar{\alpha}_{m_1}, a_f^T(\theta_{m_1}), \bar{\alpha}_{m_2}, a_f^T(\theta_{m_2}), \ldots, \bar{\alpha}_{m_M}, a_f^T(\theta_{m_M})], \bar{\alpha}_{m_i} = \alpha_{m_i} e^{-j\omega\tau_{m_i}(0)} (1 \leq i \leq M) \) and \( c_1 = p(M \sum_{m_f=1}^{M} |\bar{\alpha}_{m_f}|^2) \).

The new algorithm first respectively multiplexes the rows and columns to reconstruct the TR covariance matrix. For the \( i \)-th column of the \( Y(t) \) \((i = 1, 2, \ldots, N)\), the signal covariance matrix \( R_{y_i} \) is:

\[ R_{y_i} = E[Y_i Y_i^H] \]

\[ = E[(c_1 A_M A_M^H e_i)(c_1 A_M A_M^H e_i)^H] + E[u_i u_i^H], \]

where \( e_i \) is a column vector with all elements are 0 except for 1 at \( i \)-th element, \( u_i \) is the \( i \)-th column of the noise matrix \( u(t) \).
Similarly, for the $j$-th row of $Y(t)$ ($j = 1, 2, \ldots, N$), the signal covariance matrix $R_{y,j}$ is:

$$R_{y,j} = E[Y_j^H Y_j] = E[(c_1 e_j^T A M A M^H)^H (c_1 e_j^T A M A M^H)] + E[u_j^H u_j],$$

(11)

where $e_j$ is a column vector with all elements $0$ except for $1$ at $j$-th element, $u_j$ is the $j$-th row of the noise matrix $u(t)$.

After multiplexing operation, the new TR covariance matrix $R_y'$ is:

$$R_y' = \frac{1}{2N} \left( \sum_{j=1}^N R_{y,j} + \sum_{i=1}^N R_{y,i} \right).$$

(12)

Suppose the noise covariance matrix $N_i = u_i u_i^H$, $N_{ij} = u_i^H u_j$. $N_i$ and $N_{ij}$ have the only one equal element for the $i$-th element of $u_i$ and $j$-th of the $u_j$. Thus, the accuracy of the noise covariance matrix is proposed by using the multiplexing method, which equally means the improvement of the TR covariance matrix.

According to the difference of forwarding scattering angle, the $L$ pieces of round trips can be divided into $M$ types of paths, i.e., those round trips with the same forwarding scattering angle are the same kind of paths. With the prior knowledge that the angle of the direct-path is above $0^\circ$, the optimization problem of utilizing the LCMP method to constrain the response of diffuse multipath is as follows:

$$\min_{\mathbf{w}} \quad \mathbf{w}^H R_y \mathbf{w}$$

s.t. $\quad \mathbf{w}^H \mathbf{a}_T(\theta_i) = 0, \quad \theta_i \leq 0, i = 1, 2, \ldots, N_i$

$$\mathbf{w}^H \mathbf{a}_T(\phi_j) = 1, \quad \phi_j > 0, j = 1, 2, \ldots, N_j.$$  

(13)

The analytical solution of $\mathbf{w}$ is given by:

$$\mathbf{w}_{opt} = R_y^{-1} C (C^H R_y^{-1} C)^{-1} F,$$

(14)

where $\mathbf{C}$ is an $N \times (N_i + N_j)$ matrix of array steering vectors in all constrained directions and $\mathbf{F}$ is a $1 \times (N_i + N_j)$ vector specifying the desired response in each constrained direction, i.e., $\mathbf{C} = [\mathbf{a}_T(\phi_1), \ldots, \mathbf{a}_T(\phi_{N_j}), \mathbf{a}_T(\theta_1), \ldots, \mathbf{a}_T(\theta_{N_i})], \mathbf{F} = [1_{1 \times N_j}, 0_{1 \times N_i}].$

The spatial spectrum of the angle estimation is given by:

$$P(\theta) = \frac{1}{|\mathbf{w}_{opt}^H \mathbf{a}_T(\theta)|^2},$$

(15)

where $\mathbf{a}_T(\theta)$ is the steering vector of the angle $\theta$, $| \cdot |$ is the absolute value operator.

4. Simulation Results

In this section, we verify the performance of the proposed algorithm by comparing it with the following two benchmarks: TR FBSS [28] and TR MNL [29] and formulate it from three aspects: (1) root mean square errors (RMSE) versus signal-to-noise ratios (SNRs); (2) RMSE histograms of three algorithms under two SNRs; (3) RMSE of different incident angles at a fixed SNR.

The basic parameters in the experiments are as follows: the number of array elements $N = P = 10$, the number of paths $M = 4$, the carrier frequency $f_c$ is 200 MHz, the snapshots are 256. Define the root mean square errors (RMSE) as: $\sqrt{\frac{1}{K} \sum_{k=1}^K (\hat{\theta}_1 - \theta_1)^2}$, where $K$ is the amount of Monte Carlo trials, $\hat{\theta}_1$ is the estimated angle of the angle $\theta_1$. 

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4.1. RMSE vs. SNRs

The first experiment verifies the validity of the proposed algorithm in a diffuse multipath environment. Suppose the DOAs of each path are $1.5^\circ, -1.5^\circ, -8^\circ, -20^\circ$, and corresponding delays and attenuation factors are randomly set as are $0 \text{ ns}$, $3 \text{ ns}$, $8 \text{ ns}$, $15 \text{ ns}$ and $1, 0.6120 - 0.5584i, 0.6081 - 0.3880i, 0.5878 - 0.0352i$, respectively. In this experiment, the first of every kind of parameter is corresponding to that of the real target. During the experiment, SNRs vary uniformly from $-10 \text{ dB}$ to $10 \text{ dB}$ with $2 \text{ dB}$ interval each time and 1000 Monte Carlo trials are executed at each SNR.

Figure 2 presents the performance of RMSE versus SNRs. Since TR FBSS is a kind of MUSIC algorithm, which needs the correct amounts of multipath, it is unsatisfied in terms of RMSE in diffuse environments. Our method and TR MNL have enhanced performances with the increase of SNR, where our approach outperforms the TR MNL when SNR is higher than $-6 \text{ dB}$, and even achieves an RMSE smaller than $0.01^\circ$ at SNR = $10 \text{ dB}$. The proposed method is worse under low SNR (such as SNR = $-10 \text{ dB}$), as the optimal weight obtained at low SNR fails to constrain the diffuse multipath signals.

![Figure 2. RMSE of the three algorithms versus SNRs.](image)

4.2. RMSE Histograms

The second experiment verifies the validity of the TR LCMP algorithm in different multipath environments. In this simulation, the angle of the direct-path is $1.5^\circ$, corresponding delay and attenuation factor are $0 \text{ ns}$ and $1$. These parameters are fixed during the whole simulation, and the other three paths’ parameters are randomly generated. We simulated 400 independent and different multipath scenes with 500 Monte Carlo trials in each multipath scenario. The SNR is set to $0 \text{ dB}$ and $10 \text{ dB}$, respectively.

As the SNR increases, the RMSE’s distribution range obtained by TR FBSS is almost unchanged (see Figure 3a) and is far away from the actual DOA in most cases. The result suggests that TR FBSS is not suitable for diffuse multipath environments. On the contrary, the obtained RMSE distribution intervals by the other two algorithms are significantly reduced with the increase of the SNR, which means the algorithm performance is improved consequently (see Figure 3b,c). Besides, the maximum RMSE values of the proposed algorithms under the two SNRs are less than the maximum RMSE values of the TR MNL algorithm. The proposed algorithm has apparent advantages comparing with the other two algorithms.
4.3. RMSE of Different DOAs

The third experiment examines the validity of the three algorithms for different incident angles. We set two different situations as a comparison. In the whole experiments, the incident angle varies from 0.5° to 2° with the step angle of 0.1°. For the first situation, the parameters are all the same except for the direct-path angle and its corresponding mirror angle, while the other is with different multipath parameters. To be specific, the multipath angles are −5.2° and −19.2°. Take the incident angle of 1° as an example, the DOAs of all paths are 1°, −1°, −5.2°, −19.2°, the delays and attenuation factors are fixed set as 0 ns, 8 ns, 11 ns, 12 ns and 1.0000 + 0.0000i, 0.9102 − 0.3320i, 0.6508 + 0.1378i, 0.5021 − 0.3540i during this experiment, respectively. For the other simulation, each incident angle has two different diffuse multipath angles. Similarly, we set the attenuation factor, delay of direct-path are 1 and 0 ns, while the parameters of the other paths are randomly generated. For each incident angle, the multipath environment is independent and not related to each other. 1000 Monte Carlo trials are performed for each angle under SNR = 10 dB.

As shown in Figure 4a, the proposed algorithm can achieve the smallest RMSE within the entire range of low incidence angles under the same diffuse reflection conditions. Besides, as the increase of incident angle, the angle estimation’s RMSE of the TR MNL and the TR LCMP decreases gradually, which may be related to the angle difference between the direct angle and the mirror angle. Figure 4b describes the RMSE curve for different incident angles in different multipath environments. Similarly, the TR LCMP algorithm can achieve the smallest RMSE for each incident angle. This result further shows the broader applicability in multipath environments. By comparing Figure 4a,b, it is clear that multipath parameters may enhance the algorithm performance (such as 1.5°) as well as weaken the algorithm performance (such as 1.7°), which shows that multipath parameters are related to angle estimation accuracy.
Figure 4. RMSE of the three algorithms for different DOAs under SNR = 10 dB. (a) Different DOAs with the same diffuse multipath; (b) Different DOAs with different diffuse multipath.

5. Conclusions

In this letter, we propose a novel TR LCMP algorithm for MIMO radars, which addresses the problem of low-angle estimation in diffuse multipath environments. The algorithm first reconstructs a TR covariance by utilizing the data multiplexing technique, increasing the estimation accuracy of the TR covariance matrix. Then it performs LCMP optimization to estimate DOA according to the prior knowledge. Simulation results illustrate the superior performance of the algorithm from three aspects. Moreover, the algorithm is convenient in practice due to avoiding removing coherence and estimating the multipath number. In the future, we will conduct a further study of multiple target parameter estimation problems based on the algorithm.

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