The Calculation Model of Reinforced Concrete Constructions complex Resistance in Buildings and Structures under the Action Torsion with Bending

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Abstract. It is proposed a complex resistance computational model of reinforced concrete constructions in buildings and structures under the action torsion with bending. It consists of from the block near the support (formed by a spatial crack and a compressed concrete zone closed by it – a spatial section \( k \)) and a second block, which is formed by a vertical cross section \( I - I \) passing perpendicularly to the longitudinal axis of the reinforced concrete element along the edge of the compressed zone, which closes the spatial spiral-shaped crack. In this case, as the calculated forces are taken into account in the spatial section: normal and tangential forces in the concrete of the compressed zone; components of axial and “dowel” efforts in the working reinforcement, intersected by a spiral spatial crack. The formation of a spatial crack occurs perpendicular to the direction of the main deformations of the elongation of concrete at the lower edge of the reinforced concrete element. The location of the compressed fiber is close to the direction of the main deformation of the concrete. It forms a compressed zone together with the vertical cross-section and having the form of a rectangle with a height \( x_{b,k} \), formed by a spatial crack. The vertical cross-section is refracted and passes along the lateral planes of the compressed zone, coinciding with the axes of the angular longitudinal reinforcement. Destruction of the concrete compressed zone occurs in a certain volume adjacent to the middle section by the size \( l_2 \) of its broken section when the intensity of deformations \( \varepsilon \) reaches its limiting values on octahedral sites. Thus, the spatial crack has a spiral shape closing on a broken vertical section of the compressed zone. The resolving equations are constructed that form a closed system and the Lagrange function is unified. Using the partial derivatives of the constructed function with respect to all the variables entering into it and equating them to zero, an additional system of equations is constructed. The dependence is obtained after the corresponding algebraic transformations, that allows us to search for the projection of a dangerous spatial crack \( c_{\text{inc}} \).

1. Introduction
The construction of computational models of complex resistance – torsion with bending becomes more and more urgent [1, 2], firstly, because there are comparatively few such studies [3–9], and secondly, it is caused by the urgent need to take into account the spatial work of the vast majority reinforced concrete structures of more and more original buildings and structures that significantly
alter the architectural appearance of modern cities [10–12]; thirdly, it becomes a universally accepted postulate that there is nothing more practical than a good theory of their calculation [13–16].

2. Purpose and objectives

Therefore, the purpose of the present studies is the development of a computational model of reinforced concrete structures resistance under the action torsion with bending of any cross-sections, which most fully reflects the features of their actual work [2, 12, 17, 18].

The solving equations of equilibrium and deformation are composed for computational effort. In this case, the projection of a dangerous spatial crack is defined as a function of several variables using Lagrange multipliers [13, 15].

3. The main part. Defining equations

From the equation of the moments equilibrium of internal and external forces in the cross-section I–I with respect to the z axis, passing through the point of application of the resultant forces in the compressed zone (ΣMz = 0), we obtain:

\[
\phi_{0, \text{st}} \sigma_{h, x, I} A_{h} [h_{0} - \phi_{x} \cdot x] + m \cdot R_{s, \text{up}, \text{up}} \cdot \omega_{0} A_{s, \text{up}} x_{h} - a_{0} + \sum R_{s, \text{le}, \text{le}} \cdot \omega_{0} A_{s, \text{le}} h_{0} - a_{0}^{' -} + \sum R_{s, \text{le}, \text{le}} \cdot \omega_{0} A_{s, \text{le}} h_{0} - x - a_{s, 1} - \sum R_{s, \text{le}, \text{le}} \cdot \omega_{0} A_{s, \text{le}} h_{0} - x - a_{s, 1} - \sum R_{s, \text{le}, \text{le}} \cdot \omega_{0} A_{s, \text{le}} h_{0} - x - a_{s, 1} = 0 \text{.} \tag{1}
\]

Here \( \phi_{0, \text{st}} \) is a parameter that takes into account the projection of the stress components in the plane \( k \) onto the plane I–I, perpendicular to the longitudinal axis of the reinforced concrete element; * – means an inverse transition from the section \( k \), in the upper fiber of which the deformation criterion of strength is triggered, to the section I–I through the transitional relations of the projection of the diagram \( \sigma_{1} - \varepsilon_{1} \) onto the direction perpendicular to the plane \( k \) (figure 1) – the value known; \( K_{M} \) is a numerical coefficient that takes into account the static loading scheme from the point of additional bending moments view of along the length of the rod (if necessary, the local stress field \( \Delta_{M} \) is taken into account according to the proposals of S.P. Timoshenko, will be considered \( \Delta_{M} \) a known quantity); \( K_{pr, M} \) is the coefficient, the ratio (it is known, – is given) between \( R_{\text{sup}} \) and \( M \); \( \phi_{0, x} \cdot x_{h, x} = \text{const} \) is a static-geometric parameter, taking into account the location of the center of gravity of the compressed concrete zone in the cross-section I–I (in the section \( x_{B} \) the diagram of the compressive stresses is rectangular, in the section \( x - x_{B} \), it is triangular); \( R_{\text{sup}} \) – the support reaction of the first block (figure 1) at the moment of exhaustion of the load-bearing capacity of the reinforced concrete element; \( a \) is the horizontal distance from the support to the cross-section I–I.

The unknown value of \( R_{\text{sup}} \) is determined from this equation (1).

The height of the concrete compressed zone of \( x \) is determined from the equilibrium equation of the projections of all forces acting in the cross-section I–I on the \( x \) axis \( \sum X = 0 \):

\[
\phi_{0, y} m x_{h, x} = \sum \sigma_{s, 1, d} A_{s, 1, d} m A_{s, 1, d} + m \cdot R_{s, \text{up}, \text{up}} \cdot \omega_{0} A_{s, \text{up}} - \sum R_{s, \text{le}, \text{le}} \cdot \omega_{0} A_{s, \text{le}} - \sum R_{s, \text{le}, \text{le}} \cdot \omega_{0} A_{s, \text{le}} + \sum R_{s, \text{le}, \text{le}} \cdot \omega_{0} A_{s, \text{le}} = 0 \text{.} \tag{2}
\]

Here \( \phi_{0, y} \cdot x_{h, x} \) is a parameter equal, up to a numerical coefficient to the parameter \( \phi_{0, x} \cdot x_{h, x} \).

From the equation of equilibrium of the projections of internal and external forces, acting in cross-section I–I on the \( Y \)-axis, \( \sum Y = 0 \) the “dowel” efforts in the working reinforcement in the middle cross-section I–I are zero): 

\[
-\tau_{\text{pl}, x} \cdot x \cdot b - \gamma_{\text{Q}, x} \cdot \tau_{\text{pl}, x} \cdot \psi_{\text{R}, Q} \cdot h_{0} - x \cdot b + K_{M} \cdot R_{\text{sup}} = 0 \text{.} \tag{3}
\]
From (3) it is determined

\[ \gamma_{Q,t} = \frac{K_M \cdot R_{sup} - \tau_{pl,x} \cdot x \cdot b}{\tau_{pl,x} \cdot \psi_{R,Q} \cdot h_0 - x \cdot b}, \]  

(4)

where \( K_M \) is the same parameter such as in formula (1).

In this case, the transverse force perceived by the concrete of the compressed zone will be equal to:

\[ Q_{I,b} = \tau_{pl,x} \cdot x \cdot b. \]

(5)

In turn, the transverse force perceived by the concrete of the stretched zone will be equal to:

\[ Q_{II,T} = \gamma_{Q,t} \cdot \tau_{pl,x} \cdot \psi_{R,Q} \cdot h_0 - x \cdot b. \]

(6)

On the other hand:

\[ Q_{II,T} = Q - Q_{I,b}. \]

(7)

The last equality can be used to determine the parameter \( \psi_{R,Q} \) that takes into account the presence of adjacent spatial cracks on the stress-strain state of the stretched zone of the middle cross-section I–I:

\[ \psi_{R,Q} = \frac{Q - Q_{I,b}}{\gamma_{Q,t} \cdot \tau_{pl,x} \cdot h_0 - x \cdot b}. \]

(8)

Figure 1. The design scheme of reinforced concrete structure resistance under the joint action of a bending moment, torque and transverse force (case 1):

- \( \mathcal{C} \) – compressed area of spatial cross-section; \( \mathcal{O} \) – compressed zone of cross-section area I–I

In this case, the transverse force perceived by the concrete of the compressed zone will be equal to:

\[ Q_{I,b} = \tau_{pl,x} \cdot x \cdot b. \]

In turn, the transverse force perceived by the concrete of the stretched zone will be equal to:

\[ Q_{II,T} = \gamma_{Q,t} \cdot \tau_{pl,x} \cdot \psi_{R,Q} \cdot h_0 - x \cdot b. \]

On the other hand:

\[ Q_{II,T} = Q - Q_{I,b}. \]

The last equality can be used to determine the parameter \( \psi_{R,Q} \) that takes into account the presence of adjacent spatial cracks on the stress-strain state of the stretched zone of the middle cross-section I–I:

\[ \psi_{R,Q} = \frac{Q - Q_{I,b}}{\gamma_{Q,t} \cdot \tau_{pl,x} \cdot h_0 - x \cdot b}. \]

Drawing up the following equations requires some explanation (figure 2). Upper, lower and lateral longitudinal reinforcement (in the presence of multi-tier), in figure 1 are not shown conditionally to exclude cumbersome images. The arising stresses are taken into account in the marked reinforcement under the conditions of equilibrium. The only exception is the equilibrium equation for the moments of internal and external forces acting in cross-section I–I with respect to the axis perpendicular to this
section and passing through the point of application of the resultant forces in the compressed zone ($T_{b,r}=0$) ("dowel" efforts in the entire reinforcement in the middle section $I-I$ is assumed to be zero).

In the spatial section $k$, for the block 2, the entire reinforcement is taken into account \[12, 19\]. It passes through this cross-section along the spiral spatial crack and along the broken section of the compressed zone (figure 1). In this case, the compressed upper longitudinal reinforcement cut off in cross-sections I–I and III–III (the "dowel" effect is not taken into account in the compressed reinforcement), and in all the remaining longitudinal and transverse reinforcement, the "dowel" effects are taken into account. They are determined by the use of a special second level model \[1, 2, 13, 15\].

The need for the use of a complex broken section of the compressed zone of concrete is due to the fact that its destruction occurs (as shown by experimental studies \[3–9\]) in a certain volume located not along the entire length between points $A$ and $B$ (figure 1), but only in a certain volume located in the middle part.

The lateral surfaces of the broken section in the compressed concrete coincide with the planes of the location of the axis (or the "smeared" plane) of the working longitudinal reinforcement. In this case, the angular reinforcement is considered to be located on the left for section $I-I$ and on the right for section III–III at the intersection with a broken section. Thus, it is intersected by planes I–I, III–III, respectively, at the end sections of a complex broken section (figure 4).

The distribution of the torques in a compressed and stretched zone in the middle section $I-I$ is shown in figure 3. In evaluating the resistance of reinforced concrete constructions of rectangular and complex cross-sections (consisting of a set of rectangles), the method proposed by the authors is used. It is built on the fact that the rectangular section is divided into a series of squares, which are subsequently replaced by the circles inscribed in them (figure 3).

The equations for determining tangential torsion stresses $\tau_t$ in the corresponding circle of the cross-section, located at a distance $x$ from the support, are written in a cylindrical and Cartesian coordinate system in accordance with figure 3:

$$
\tau_t = \tau_{ij} = \frac{M_{t,ij}}{I_{t,j}} \cdot \rho = \frac{M_{t,ij}}{I_{t,j}} \cdot \zeta y^2 + z^2 \sqrt{2} \leq \tau_{t,ut},
$$

where $\rho$ is the distance from the center of the $j$-th circle to the point at which tangential torsion stresses $\tau_t$ are determined; $\zeta$ is a coefficient of transition to local axes; $\tau_{t,ut}$ are the limiting values of tangential torsion stresses.

Here, the moment of inertia in torsion in the general case of a complex cross-section consisting of rectangles is equal to the sum of the inertia moments of the squares into which the rectangles are
divided, with their subsequent approximation by circles inscribed in these squares (one of the overlapping parts of the intersecting sections enters with the sign "minus", and the angular sections, because of their insignificant influence on the tangential stresses, are not taken into account, figures 1, b and 3),

\[ I_t = I_{t,1} + I_{t,2} + \ldots + I_{t,j} = \sum I_{t,j}, \]  

(10)  

and each of the torsion moments incident on the inscribed circles are respectively determined,

\[ M_{t,1} = M_t \frac{I_{t,1}}{I_t}; \quad M_{t,2} = M_t \frac{I_{t,2}}{I_t}; \quad \ldots \quad M_{t,j} = M_t \frac{I_{t,j}}{I_t}, \]  

(11)  

where \( I_{t,j} \) – the moment of inertia of the circle inscribed in the corresponding square used in the formula (10) (the lower circle is used, as a rule, for cracks of the first type, the middle circle is used for the second and third types).

It is important to note that all geometric characteristics are considered with respect to the geometric center of the complex section.

With respect to the middle cross-section I–I, which is under conditions of complex resistance – torsion with bending (figure 3), it is advisable to take into account the fact that a considerable part of this section is subject to stretching. It is known [1, 2, 15, 20] that in a stretched concrete there are a number of spatial adjacent cracks that affect the stress-strain state of the middle cross-section I–I.

We will take into account such an effect of adjacent cracks with the help of the parameter \( \psi_{RT} \). In this case, \( \tau_{t,j} \) is sought by formula (9), we will consider as a resultant of two components \( \tau_{t,xy} \) and \( \tau_{t,xz} \), which are determined from the following dependences:

\[ \tau_{t,xy} = \tau_{t,xy,j} = \tau_{t,j} \cdot \sin \alpha = \frac{M_{t,j}}{I_{t,j}} \frac{\zeta y^2 + z^2}{I_{t,j}} \frac{\zeta y}{I_{t,j}} \frac{\zeta y}{I_{t,j}} = \frac{M_{t,j}}{I_{t,j}} \zeta y \leq \tau_{t,xy,ul}, \]  

(12)  

\[ \tau_{t,xz} = \tau_{t,xz,j} = \tau_{t,j} \cdot \cos \alpha = \frac{M_{t,j}}{I_{t,j}} \frac{\zeta y^2 + z^2}{I_{t,j}} \frac{\zeta y}{I_{t,j}} \frac{\zeta y}{I_{t,j}} = \frac{M_{t,j}}{I_{t,j}} \zeta \leq \tau_{t,xz,ul}, \]  

(13)  

where \( \tau_{t,xy,ul}, \tau_{t,xy,ul} \) are the components of the limiting values of tangential torsion stresses.

In addition to the resultant \( \tau_{t,j} \), calculated by the formula (9) for the corresponding circles, it is necessary to take into account the components associated with the deplanation of the rectangular section [1, 2, 20]. In this case, the displacement caused by the deplanation of the cross-section is written in the form:

\[ w = \frac{M_t}{G \cdot I_t} \cdot f \cdot y \cdot z \cdot f_2 \cdot x = \frac{M_t}{G \cdot I_t} \cdot \frac{a^2 - b^2}{a^2 + b^2} \cdot y \cdot z \cdot l \left( 1 - \frac{x}{l} \right). \]  

(14)
Thus, the displacement $W$ is a complex function, depending on the coordinates $y, z, x$. When finding the relative angular (shear) deformations of deplanation $\gamma_{d,yx}$ and $\gamma_{d,zx}$ with using the Cauchy curves, take such form:

$$\gamma_{d,yx} = \frac{\partial w}{\partial y} + \frac{\partial w}{\partial x} = \frac{\partial w}{\partial y} + 0 = \frac{\partial w}{\partial y},$$  

$$\gamma_{d,zx} = \frac{\partial w}{\partial z} + \frac{\partial w}{\partial x} = \frac{\partial w}{\partial z} + 0 = \frac{\partial w}{\partial z},$$

where, $w, v, \omega$ – the displacement in the direction of the axes $x, y$ and $z$, respectively. With respect to the deplanation model described by formula (14), the displacements $v=\omega=0$. In the end we will have:

$$\gamma_{dep,yx} = \frac{M_t}{G \cdot I_t} \cdot \frac{a^2 - b^2_x}{a^2 + b^2_x} \cdot l \cdot \left(1 - \frac{x}{l}\right) \cdot z \leq \gamma_{dep,yx,ul},$$

$$\gamma_{dep,zx} = \frac{M_t}{G \cdot I_t} \cdot \frac{a^2 - b^2_y}{a^2 + b^2_y} \cdot l \cdot \left(1 - \frac{x}{l}\right) \cdot y \leq \gamma_{dep,zx,ul},$$

where $\gamma_{dep,yx,ul}$ and $\gamma_{dep,zx,ul}$ are the components of the limiting relative angular deformations of deplanation.

The components of tangential stresses caused by the deplanation are determined from next dependencies:

$$\tau_{dep,yx} = \gamma_{dep,yx} \cdot G = \frac{M_t}{G \cdot I_t} \cdot \frac{a^2 - b^2_x}{a^2 + b^2_x} \cdot l \cdot \left(1 - \frac{x}{l}\right) \cdot z \cdot G \leq \tau_{dep,yx,ul},$$

$$\tau_{dep,zx} = \gamma_{dep,zx} \cdot G = \frac{M_t}{G \cdot I_t} \cdot \frac{a^2 - b^2_y}{a^2 + b^2_y} \cdot l \cdot \left(1 - \frac{x}{l}\right) \cdot y \cdot G \leq \tau_{dep,zx,ul},$$

where $\tau_{dep,yx,ul}$ and $\tau_{dep,zx,ul}$ are the components of the limiting tangential stresses of the deplanation.

If the torque changes along the longitudinal axis of the reinforced concrete structure, an additional relationship is introduced, a proportional relationship between the torques in the section $k$ and in the cross-section $l-I$.

$$\frac{K_T \cdot K_{pc,T} \cdot M_{T,j}}{M_{T,k}} = \frac{a}{c - 0.5b \cdot \sin \alpha}, \quad M_{T,j} = \frac{a \cdot M_{T,k}}{K_T \cdot K_{pc,T} \cdot c - 0.5b \cdot \sin \alpha}.$$  

Here $K_T$ – a numerical coefficient that takes into account the static loading scheme from the position of additional twisting moments along the length of the rod; $K_{pc,T}$ – the coefficient, the ratio (it is known, – is given) between $R_{sup}$ and $T$. Here $a$ is the horizontal distance from the center of the support to the cross-section $l-I$.

Adding the components of tangential stresses with torsion $\tau_{t,xy}, \tau_{t,xz}, \tau_{dep,yx}$ and $\tau_{dep,zx}$ and we get the resultant stress $\tau_{sum}$:

$$\tau_{sum,j} = \left[ \tau_{t,xy,j} + \tau_{dep,yx,j} \right]^2 + \left[ \tau_{t,xz,j} + \tau_{dep,zx,j} \right]^2 \right]^{1/2},$$

where $\tau_{dep,yx,j}$ and $\tau_{dep,zx,j}$ are the components of the tangential stresses of the deplanation averaged in the $j$-th circle; their signs are determined automatically, using the second factor in formulas (18)–(20), which controls quadrants of rectangular section. If within the $j$-th circle the signs of the tangential stresses of the deplanation are different and when they are averaged, some complications arise, then the radii of the approximating circles need to be reduced.
Knowing $\tau_u$ (or $\tau_{sum,j}$) from equation (9) one can find the torque that falls on the $j$-th circle of the compressed zone in cross-section I–I by the formula:

$$T_c = M_{i,c} = \frac{\tau_u \cdot I_{i,j}}{\zeta y^2 + z^2 \psi_2}.$$  

(23)

In the case, when along the longitudinal axis of the reinforced concrete structure the torque changes and has a smaller value than in the section $k$, then in (23) instead of $\tau_u$, it is necessary to insert $\tau_{sum,j}$.

Performing the summation of all $M_{i,j}$ for all $j$-th circles $m$ of the cross-sections I–I located in the compressed zone, we will have the total torque perceived by the compressed concrete zone:

$$M_{i,c} = \sum_{j=1}^{m} M_{i,j}.$$  

(24)

In turn, the torque perceived by the concrete of the stretched zone will be equal to:

$$T_R = M_{i,R} = \frac{\tau_u \cdot \psi_{R,T} \cdot I_{i,j}}{\zeta y^2 + z^2 \psi_2},$$  

(25)

where $\psi_{R,T}$ is the parameter taking into account the presence of adjacent spatial cracks on the stress-strain state due to the torsion of the stretched zone of the middle cross-section I–I.

On the other hand, again returning to the construction of common resolving equations (Figure 1), the equation of equilibrium of the moments of internal and external forces acting in section I–I relative to the axis perpendicular to this section and passing through the point of application of the resultant forces in the compressed zone can be used here $(T_b=0)$:

$$M_{i,R} = M_{i,c} - M_{i,c}.$$  

(26)

From this equation, a parameter $\psi_{R,T}$ is determined that takes into account the presence of adjacent spatial cracks on the stress-strain state due to torsion of the stretched zone of the middle cross section I–I:

$$\psi_{R,T} = \frac{M_{i} - M_{i,c}}{\tau_u \cdot I_{i,j}}.$$  

(27)

Here $\tau_u$ is the tangential torsion stress in the compressed concrete (the compressed concrete zone has the form of a rectangle), obtained in the third stage of the stress-strain state by projecting the diagram $\sigma_l - \varepsilon_l$ onto the plane I–I for the dependence $\tau - \gamma$, taking into account the proportional ratio $Q:T$ or $M:T$ (one of them, as rule is given). In the necessary cases, it is necessary to take into account the additional dependence resulting from the ratio of the torques in the cross-section I–I and in the cross-section $k$.

Note: It should be noted that the stresses $\tau_{1,u}$, $\tau_{1,xy,ul}$, $\tau_{1,zx,ul}$, $\tau_{dep,xy,ul}$, $\tau_{dep,zx,ul}$, $\sigma_{p,ul}$ are known (located on the horizontal sections of the "strain–strain" connection diagrams), since the plastic state occurs simultaneously for tangential and normal stresses.

From the hypothesis of proportionality of longitudinal deformations, we find:

$$\sigma_{s,I} = \frac{\varphi_{0,0,s} \sigma_{h,0,x,I} \cdot E_{s}(\lambda) \cdot b_0 - x}{x} + \sigma_0 \leq R_{s,I}.$$  

(28)

Here $\sigma_0$ are the preliminary stresses in the prestressed reinforcement at the moment when the value of prestressing in the concrete decreases to zero when the structure is loaded by external forces, taking into account the loss of prestress in the prestressed reinforcement corresponding to the stage of
construction in question. If condition (28) is not satisfied, then we set \( \sigma_{x,l} \) it equal to \( R_{a,i} \). In equation (28), we use the notation \( \varphi_{i,0} \) from formula (1).

The second prism block is separated from the reinforced concrete element by the spatial section formed by a spiral-shaped crack and a vertical section passing through the compressed zone of concrete through the end of the front of the spatial crack [19].

The equilibrium of this block is ensured by the following conditions.

![Figure 4. The approximation by a broken section of a compressed zone formed by a spiral spatial crack](image)

The sum of the moments of all internal and external forces acting in the vertical longitudinal plane with respect to the z axis passing through the point of application of the resultant forces in the compressed zone is zero (\( \sum M_{b,k}=0 \), block II):

\[
\sigma_{s,k} \cdot (h_0-0.5x_b) + \sum \omega_s \cdot A_{s,i} + \phi_{r} \cdot R_{s} \cdot \eta_{hor,s} \cdot c_{i-1} \sum \omega_s \cdot A_{s,i,rig} - \phi_{r} \cdot R_{s} \cdot \sum \omega_s \cdot A_{s,lef} +
+ \sum R_{s,i,rig} \cdot \omega_{s,rig} \cdot A_{s,i,rig} \left[ h_0 - x_B - a_{s,i,rig} \right] + \sum R_{s,i,lef} \cdot \omega_{s,lef} \cdot A_{s,i,lef} \left[ h_0 - x_B - a_{s,i,lef} \right] +
+ \phi_{r} \cdot R_{s} \cdot \sum \omega_s \cdot A_{s,i,rig} - \frac{1}{2} - q_{sw,lef} \cdot c_{s,rig} c_0 \left[ c_s - 0.5 \cdot h \cdot \cos \alpha_0 + l_2 - z_{sw} \right] -
- K_M \cdot K_{pr,M} \cdot R_{sup} - R_{sup} \cdot a_{m,b} + \eta_{hor,b} \cdot l_2 - \tau_{sc} \cdot \omega_s \cdot 0.5 \cdot (x_{B,3} + x_{B,3}) \cdot l_1 \cdot 0.5l_1 +
+ \tau_{sc} \cdot \omega_s \cdot 0.5 \cdot (x_{B,3} + x_{B,3}) \cdot l_1(c) \cdot 0.5l_1(c) = 0,
\]

where \( K_M, K_{pr,M} \) — numerical coefficients, deciphered in the formula (1); \( a_{m,b} \) — the horizontal distance from the support to the center of gravity of the compressed concrete zone in cross-section \( k \); \( c_{i-1} = const \) at each step of the iteration; lateral compressed reinforcement in this equation is not taken into account in view of the smallness of its arms relative to the point \( b_k \) (because of the smallness of the parameter \( x_B \)); \( \varphi_5, \varphi_5, \varphi_5, \varphi_5, \varphi_5, \varphi_8 \) are the parameters that take into account the components of the “dowel” effect in the reinforcement (at each step of the iteration are taken into account as constants, and not as functions and are determined based on the second-level model); other geometric parameters given in formula (29) are shown in figure 1.

The unknown \( \sigma_{s,k} \) is determined from equation (29). The sum of the projections of all forces, acting in the spatial section \( k \) on the x axis is zero (\( \sum X=0 \), block II):

\[
\sigma_{s,k} \cdot \sum \omega_s \cdot A_{s,i} - \sum R_{scapi} \cdot \omega_{s,api} + \sum R_{s,i,rig} \cdot \omega_{s,rig} \cdot A_{s,i,rig} + \sum R_{s,i,lef} \cdot \omega_{s,lef} \cdot A_{s,i,lef} +
+ \sum R_{s,i,rig} \cdot \omega_{s,rig} \cdot A_{s,i,rig} \left[ x_B - z_{sw} \right] - \phi_{r} \cdot \sum \omega_s \cdot A_{s,rig} + \phi_{r} \cdot \sum \omega_s \cdot A_{s,rig} -
- \tau_{sc} \cdot \omega_s \cdot 0.5 \cdot (x_{B,3} + x_{B,3}) \cdot l_1(c) + \sigma_{s,ang} \cdot A_{s,ang} + \sigma_{s,ang} \cdot A_{s,ang} = 0
\]

* — the same parameter as in formula (1); \( \omega_{th} \) is taken at each iteration in the form of constants.

The unknown \( x_B \) is sought from this equation.
The sum of the projections of all forces, acting in the spatial section \( k \) on the \( y \) axis is zero (\( \sum Y = 0 \), block II):

\[
- q_{sw,rig} \cdot c_{inc} - q_{sw,lef} \cdot h \cdot \cos \alpha_0 - \phi_{z,r} R_s \sum \omega_3 A_{z,r} - \phi_{z,l} R_s \sum \omega_{3,rig} A_{3,rig} + \phi_{z,lef} R_s \sum \omega_{3,lef} A_{3,lef} + R_{sup} + \tau_{yz} \cdot \omega_4 \cdot 0.5 \cdot (x_{b,1} + x_{b,2}) \cdot l_1 + \tau_{yz} \cdot \omega_5 \cdot 0.5 \cdot (x_{b,1} + x_{b,3}) \cdot l_3 (c) = 0,
\]

(31)

where \( \phi_{z,lef} \), \( \phi_{z,rig} \) are the parameters that take into account the components of the “dowel” effect in the reinforcement (at each step of the iteration are taken into account as constants, and not as functions and are determined on the basis of the second-level model).

From equation (31) an unknown \( q_{sw,rig} \) is sought.

The sum of the moments of internal and external forces in the vertical transverse plane relative to the \( x \) axis passing through the point of application of the resultant forces in the compressed zone is zero (\( \sum T_{b,k} = 0 \), block II):

\[
- q_{sw,\sigma} l_1^2 + b^2 1/2 \cdot h_0 - 0.5 x_b - q_{sw,rig} \cdot 0.5b - \eta_{hor,b} b + q_{sw,lef} \cdot 0.5b + \eta_{hor,b} b + M_{T,K,11} + M_{T,K,12} \cdot \sin \alpha_0 + M_{T,K,13} +
\]

\[
+ l_1^2 + b^2 1/2 \cdot \eta_{hor,b} b \cdot \phi_{z,r} R_s \sum \omega_3 A_{3,r} - \sqrt{l_1^2 + b^2} \cdot h_0 - 0.5 x_b \cdot \phi_{z,r} R_s \sum \omega_3 A_{3,r} -
\]

\[
- 0.5b - \eta_{hor,b} b \cdot \phi_{z,rig} R_s \sum \omega_{3,rig} A_{3,rig} - h_0 - x_b - a_{j,rig} \cdot \phi_{z,rig} R_s \sum \omega_{3,rig} A_{3,rig} -
\]

\[
- 0.5b + \eta_{hor,b} b \cdot \phi_{z,lef} R_s \sum \omega_{3,lef} A_{3,lef} - h_0 - x_b - a_{j,lef} \cdot \phi_{z,lef} R_s \sum \omega_{3,lef} A_{3,lef} +
\]

\[
+ K_{pr} \cdot K_{pr,M} \cdot R_{sup} - R_{sup} \cdot \eta_{hor,b},
\]

(32)

where \( K_{M} \), \( K_{pr,M} \) are numerical coefficients, deciphered in the formula (1).

From equation (32) an unknown \( q_{sw,lef} \) is sought; \( \phi_{z,lef} \), \( \phi_{z,rig} \), \( \phi_{8,lef} \), \( \phi_{8,rig} \) are parameters that take into account the components of “dowel” effect in the reinforcement (at each iteration step are carried as a constant and not as functions and are based on the model of the second level), \( \omega_0 \) is the angle of inclination of the middle portion of the compressed area of concrete to the horizontal (close to 45º); \( M_{T,K,11} \), \( M_{T,K,12} \), \( M_{T,K,13} \) are torques sensed by the concrete compression zone \( k \) in the spatial section in the first, second and third portions sections of a broken cross-section, respectively, defined by relationships similar to formula (19). It also \( M_{T,K,13} \) depends on the variable \( c \).

Here \( \tau_{rig} \) and \( \tau_{ef} \) the tangential stress, caused by torsion and transverse force in the compressed concrete acting in the right and left parts respectively (it is expedient to average them within the limits of the height of the compressed zone (i.e. \( \tau_T = \tau_{pl_z} \cdot \omega \), \( \tau_Q = \tau_{pl_y} \cdot \omega \)), but more accurately it was decided to divide them into left and right parts, relative to the point \( b_k \) (figure 1, respectively).

\[
\frac{l_2}{b} = \frac{z_{sw} - b}{0.5b - \eta_{hor,b} b}.
\]

(33)

It follows,

\[
z_{sw} = \frac{l_2}{b} \cdot 0.5b - \eta_{hor,b} b = \frac{l_2}{b} \cdot 0.5 - \eta_{hor,b}.
\]

(34)

Note: when differentiating, geometric parameters \( z_{sw} \) are taken as constants at each iteration step.

\( \omega \) is the coefficient of filling the diagrams of tangential torsion stresses in compressed concrete, taking into account the elastic-plastic work; \( q_{sw,T} \) is the linear force in clamps, which appears on the side faces of the reinforced concrete element from the torque \( T \) (figure 1); \( q_{sw,\sigma} \) is the linear force in
the clamps, which appears on the lower face of the reinforced concrete element from the torque T (figure 1).

The sum of the projections of all forces acting in the spatial section $k$ on the $0z$ axis is zero ($\sum Z = 0$, block II):

$$-q_{sw,\sigma} l_2^2 + b^2 \frac{v^2}{2} - \phi_{b,rig} R_s \sum \omega_{sig} A_{s,rig} - \phi_{b,lef} R_s \sum \omega_{def} A_{s,lef} - l_2^2 + b^2 \frac{v^2}{2} \cdot \phi_{b} \cdot R_s \sum \omega A_s = 0,$$

(35)

where $\phi_{b}$ is the parameter that takes into account the components of the “dowel” effect of the reinforcement (at each step of the iteration they are taken into account as constants, and not as functions and are determined on the basis of the second-level model).

From the equation (35) an unknown $q_{sw,\sigma}$ is calculated.

Note: The “dowel” effect in the working longitudinal and transverse rods of the reinforcement is taken into account by a special second-level model and is calculated iteratively [2, 14, 15], i.e. when substituted into equations they are discrete constants. When compiling the function of many variables, it is taken into account that $A_s = \phi \cdot x$, $a_{m,s} = \phi \cdot c$, $a_m = \phi \cdot c$, $a = const = c_{\alpha} = const$; $\sigma_{b,rig} = const$; $\sigma_{b,lef}$ is unknown value and $M_k = const = M = f \cdot c$. Transient coefficients $\phi_{10}$, etc. are calculated iteratively, they are discrete constants.

Constructing a function $F = R_{sup}, x, x_R, \gamma_{Q,t}, \gamma_{T,k}, \sigma_{1,1}, \sigma_{2,2}, \sigma_{3,3}, q_{sw,rig}, q_{sw,lef}, q_{sw,\sigma}, c, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8, \lambda_9$ of several variables with Lagrange multipliers $\lambda_i$, using equations (1) – (35), and equating the partial derivatives with respect to all variables entering into it, to zero, we obtain an additional system of equations [13, 15]:

$$\begin{align*}
\frac{\partial f}{\partial x_1} + \lambda_1 \frac{\partial \phi_1}{\partial x_1} + \lambda_2 \frac{\partial \phi_2}{\partial x_1} + \ldots + \lambda_m \frac{\partial \phi_m}{\partial x_1} = 0, \\
\frac{\partial f}{\partial x_2} + \lambda_1 \frac{\partial \phi_1}{\partial x_2} + \lambda_2 \frac{\partial \phi_2}{\partial x_2} + \ldots + \lambda_m \frac{\partial \phi_m}{\partial x_2} = 0, \\
\vdots \\
\frac{\partial f}{\partial x_n} + \lambda_1 \frac{\partial \phi_1}{\partial x_n} + \lambda_2 \frac{\partial \phi_2}{\partial x_n} + \ldots + \lambda_m \frac{\partial \phi_m}{\partial x_n} = 0.
\end{align*}$$

(36)

From the system (36) after the corresponding algebraic transformations, we have an equation for the unknown dangerous spatial crack $c_{inc}$ on the horizontal:

$$a_{33} \cdot c_{inc}^2 + c + a_{34} \cdot c_{inc} + a_{35} = 0.$$

(37)

The coefficients in equation (37) include practically all the calculated parameters of the proposed design model (figure 1).

4. Conclusion

As a result, a design model has been constructed that makes it possible to approximate the complex limiting resistance of reinforced concrete structures under the action torsion with bending to the actual, which will contribute to their more efficient design.

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