Strong coupling analysis of diquark condensation∗†

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The phenomenon of diquark condensation at non-zero baryon density and zero temperature is analyzed in the strong coupling limit of lattice QCD. The results indicate that there is attraction in the quark-quark channel also at strong coupling, and that the attraction is more effective at high baryon density, but for infinite coupling it is not enough to produce diquark condensation. It is argued that the absence of diquark condensation is not a peculiarity of the strong coupling limit, but persists at sufficiently large finite couplings.

A weak coupling analysis of QCD, both at the level of one gluon exchange as well as using the instanton induced interaction, reveals that the quark-quark scattering amplitude is attractive in the color anti-triplet channel and repulsive in the symmetric color sextet channel. The existence of such attractive force between quarks in the color anti-triplet channel is also supported by experiments, since hadron phenomenology is best explained by models that assume baryons consisting of a quark-quark anti-triplet color bound state whose color is neutralized by the remaining quark [1].

In the deconfined phase, at high baryon densities and low temperatures, the effect may be more dramatic. If the weakly interacting quarks tend to form a Fermi surface, the quark-quark attraction will render it unstable via the Cooper mechanism. Hence, the phenomenon of color superconductivity will take place: a diquark condensate will appear in the ground state, a gap will be opened in the spectrum and the Fermi surface will be destroyed. Many exotic phenomena at high baryon density may happen as a consequence of diquark condensation, as color flavor locking and unlocking, crystalline color superconductivity, etc. [2].

The diquark operator is not gauge invariant, and therefore general arguments forbid diquark condensation in the naive way: it must be understood not as the spontaneous breaking of the gauge symmetry, but as a kind of Higgs mechanism whose physical consequences are the phenomena we name color superconductivity.

The arguments leading to the existence of diquark condensation at high baryon density are based on weak coupling analysis –although some non perturbative effects are taken into account via instantons–, and thus are only valid at asymptotically large densities. In addition, the study is performed using effective four quark theories which are not gauge invariance, and then it is difficult to trace the fate of gauge invariance. It is of utmost importance to study diquark condensation beyond the weak coupling region, because of the lack of reliable tools, since Monte Carlo simulations are not feasible at high density and low temperature due to the sign problem – however, see [3]. Hence, it is interesting to turn to the opposite regime, the strong coupling domain. Recent attempts to study finite density QCD in the strong coupling limit using hamiltonian techniques have been developed in [4]. Here, we present the results concerning diquark con-

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gously, it can be seen that the field is proportional to the chiral condensate. Analogous to chiral symmetry and its expectation value $\phi_c$ is a constant. Thus, the effective bosonic action which depends only on the canonical partition function is given by an effective action evaluated at constant fields $\varphi(x) = \bar{\varphi}$ and $|\phi(x)|^2 = v^2$. Its absolute minimum describes the equilibrium state at a given temperature and chemical potential. Other local minima, if present, are associated with metastable states.

To study the phase diagram of the theory we used the semi-classical approximation. The effective potential is given by the bosonic effective action for $\phi^a(x)$ has been obtained exactly from QCD, albeit in the strong coupling regime, and there is no gauge field coupled to it.

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$$v = 0, \quad \bar{\varphi} = \pm \left(\sqrt{33} - 5\right)^{1/2} \quad \forall \mu$$
$$v = \sqrt{5}, \quad \bar{\varphi} = 0 \quad \forall \mu$$
$$v = 0, \quad \bar{\varphi} = 0 \quad \forall \mu > 0.4416$$

Note that the positions of the minima are independent of $\mu$.

There is a first order phase transition at $\mu_c \approx 1.557$. For $\mu < \mu_c$ the -degenerate owing to chiral symmetry- absolute minima are at $v = 0$ and $\bar{\varphi} = \pm (\sqrt{33} - 5)^{1/2}$, corresponding to a phase with chiral symmetry spontaneously broken. For $\mu > \mu_c$ the absolute minimum is at $v = 0$ and $\bar{\varphi} = 0$; chiral symmetry is restored in this phase. Obviously, the transition is first order. For $\mu < \mu_c$ the baryon density is zero, and there for $\mu > \mu_c$. Three quarks per lattice site is the maximum allowed by Pauli’s principle, so that the transition separates a phase of zero baryon density from a phase saturated of quarks. The system at any intermediate density is thermodynamically unstable and splits into domains of zero and saturated baryon density. This behavior is an artifact of the strong coupling limit and has been observed in other strong coupling analysis at zero temperature and finite chemical potential.
The minimum at $v = \sqrt{5}$ and $\bar{\varphi} = 0$ is metastable in both phases. It describes a chiral symmetric state with a diquark condensate. The baryon density that correspond to this metastable state is a smooth function of the chiral condensate (see figure 1) that interpolates between zero density and saturation. This means that at any baryon density a state with diquark condensation can be formed and have some short life until the system splits into its zero density and saturated domains. The presence of the metastable state signals the attraction in the quark-quark channel. At strong coupling such attraction is not strong enough to form a stable diquark condensate. The energy difference between this metastable state and the equilibrium state decreases with $\mu$, indicating that the quark-quark attraction becomes the more effective the higher the baryon density (see figure 2). However, the energy difference is nonzero even at high $\mu$ (in the saturated phase).

The crucial question is whether this metastable state becomes stable at sufficiently large density at finite coupling. A partial answer is the following: assuming that the strong coupling expansion gives meaningful results, diquark condensation cannot take place at sufficiently large couplings, $g$, whatever the chemical potential. The reason is that, as we have seen, the energy difference between the state with diquark condensation and the stable state remains positive for any value of the chemical potential in the strong coupling limit. A correction to the effective potential of order $1/g$ cannot remove such energy difference if $g$ is sufficiently large. Hence, diquark condensation cannot take place in the strong coupling region. If, on the other hand, diquarks condense at some finite chemical potential in the weak coupling regime, there must be a critical value of the coupling, $g_c$, such that diquark condensation takes place in some interval of the baryon density for $g < g_c$, but diquarks do not condense at any density for $g > g_c$. This means that the interval of baryon densities (in lattice units) at which diquark condensation occurs as a stable state shrinks as $g$ increases, and vanishes at $g_c$. For $g > g_c$ the state with diquark condensate survives as a metastable state.

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