Flow of quantum correlations in noisy two-mode squeezed microwave states

M. Renger, S. Pogorzalek, F. Fesquet, K. Honasego, F. Kronowetter, Q. Chen, Y. Nojiri, K. Inomata, Y. Nakamura, A. Marx, F. Deppe, R. Gross, and K. G. Fedorov

Walther-Meißner-Institut, Bayerische Akademie der Wissenschaften, 85748 Garching, Germany
Physik-Department, Technische Universität München, 85748 Garching, Germany
Rohde & Schwarz GmbH & Co. KG, Mühldorfer Straße 15, 81671 Munich, Germany
RIKEN Center for Quantum Computing (RQC), Wako, Saitama 351-0198, Japan
National Institute of Advanced Industrial Science and Technology, 1-1-1 Umezono, Tsukuba, Ibaraki, 305-8563, Japan
Department of Applied Physics, Graduate School of Engineering, The University of Tokyo, Bunkyo-ku, Tokyo 113-8656, Japan
Munich Center for Quantum Science and Technology (MCQST), Schellingstr. 4, 80799 Munich, Germany

(Dated: July 14, 2022)

We study nonclassical correlations in propagating two-mode squeezed microwave states in the presence of noise. We focus on two different types of correlations, namely, quantum entanglement and quantum discord. Quantum discord has various intriguing fundamental properties which require experimental verification, such as the asymptotic robustness to environmental noise. Here, we experimentally investigate quantum discord in propagating two-mode squeezed microwave states generated via superconducting Josephson parametric amplifiers. By exploiting an asymmetric noise injection into these entangled states, we demonstrate the robustness of quantum discord against thermal noise while verifying the sudden death of entanglement. Furthermore, we investigate the difference between quantum discord and entanglement of formation, which can be directly related to the flow of locally inaccessible information between the environment and the bipartite subsystem. We observe a crossover behavior between quantum discord and entanglement for low noise photon numbers, which is a result of the tripartite nature of noise injection. We demonstrate that the difference between entanglement and quantum discord can be related to the security of certain quantum key distribution protocols.

Keywords: quantum discord, quantum entanglement, squeezing, Josephson parametric amplifier

I. INTRODUCTION

Quantum communication and quantum computing protocols often employ entanglement as a nonclassical resource to provide improvement of information transfer and achieve a quantum speed-up in processing [1]. In this regard, entanglement plays a key role in realizing quantum error correction [2], efficient quantum simulation [3], as well as in achieving quantum supremacy [4]. Prominent examples of quantum communication protocols are quantum key distribution [5], dense coding [6], or quantum teleportation [7, 8], where entanglement is exploited for an efficient and unconditionally secure state transfer. However, entanglement represents only one particular quantum resource and does not capture all nonclassical correlations. In particular, quantum discord (QD) provides a more general measure of nonclassical correlations including entanglement [9]. Quantum discord may serve as a resource in multiple quantum information processing protocols such as deterministic quantum computing with one qubit (DQC1) [10, 11], quantum sensing [12], and quantum illumination [13]. Furthermore, theoretical investigations imply that QD has multiple intriguing physical properties which still lack experimental verification, such as the asymptotic robustness against environmental noise and its relation to entanglement in mixed tripartite systems [14]. These two properties are the key focus of our experimental study. In particular, our experiments reveal that the process of noise injection into a bipartite quantum system necessarily creates multipartite quantum correlations with the environment. As a result, the noise suppresses quantum correlations in the bipartite system and simultaneously increases the correlations between one of the subsystems and the environment. We measure this effect by extracting a flow of locally inaccessible information (LII) [15]. This LII flow behaves fundamentally different as a function of noise for different subsystems. The more detailed investigation of the LII flow may be relevant for testing ideas related to quantum Darwinism [16, 17], where correlations with different fragments of the environment eventually lead to an objective reality [18]. In addition, we demonstrate that LII is related to the unconditional security of a certain class of quantum key distribution (QKD) protocols.

The potential robustness of QD versus noise is extremely useful for various quantum communication protocols, where unavoidable external fluctuations cause loss of quantum correlations, thereby lowering the efficiency of these protocols. Protocols based on quantum entanglement are particularly vulnerable to noise, since entanglement cannot survive significant noise levels, leading

* michael.renger@wmi.badw.de
† rudolf.gross@wmi.badw.de
‡ kirill.fedorov@wmi.badw.de
to the so-called sudden death of entanglement [19]. This is especially important for quantum communication protocols in the microwave regime, as room temperature is associated with thousands of thermal noise photons with characteristic frequencies of several GHz [20]. Here, the expected asymptotic robustness of QD [21] against noise offers a natural path for quantum communication and sensing. An actual challenge is to find protocols capable of exploiting QD as a quantum resource. As such, the remote state preparation (RSP) protocol stands out as one of the prominent known examples. RSP aims at the generation of a desired and known quantum state at a remote location with the assistance of classical communication and complementary nonclassical correlations [22]. A finite quantum advantage provided by RSP is associated with the smaller amount of classical information required to prepare a quantum state, as compared to a fully classical communication protocol, and the unconditional security of the feedforward signal [23]. In some scenarios, the RSP protocol appears to exploit QD as its nonclassical resource [9, 24]. Finally, there is a class of quantum sensing protocols known as quantum illumination, where entangled light is used to detect the presence of a low-reflectivity object in a bright noisy background. Naturally, entanglement vanishes in this scenario, yet residual non-classical correlations, captured by QD, persist. The latter seems to be connected to the quantum advantage of these quantum sensing schemes [13, 25].

II. EXPERIMENT

In this paper, we experimentally study the effect of noise injection into one mode of a propagating two-mode squeezed (TMS) state which is distributed along two paths, A and B. A schematic illustration of this scenario is shown in Fig. 1(a). The TMS state is generated by superimposing two orthogonally squeezed microwave states at a symmetric microwave beam splitter. Then, we inject an uncorrelated broadband noise into path B. Finally, we perform a joint quantum state tomography, which allows us to extract full information about the two-mode quantum state [26, 27]. We use the entanglement of formation (EoF), $E_F$, as a measure for bipartite entanglement between parties A and B. We choose this specific entanglement measure since it exactly coincides with QD for pure states [28]. Simultaneously, EoF is directly related to QD via various monogamy relations, which eventually enables the connection of both quantities to LII [15, 29].

For continuous-variable quantum systems, EoF quantifies the minimal amount of two-mode squeezing needed to prepare an entangled state, starting from a classical one by using local operations and classical communication [30]. In addition, we extract an asymmetric bipartite QD, $D_A$ ($D_B$), between the two subsystems A and B. It is defined as the difference between their quantum mutual information $I_{AB}$ and a one-way classical correlation $J_{A|B}$ ($J_{B|A}$),

$$D_A = I_{AB} - J_{A|B}, \quad D_B = I_{AB} - J_{B|A}, \quad (1)$$

and quantifies the non-local fraction of $I_{AB}$ [31].

Figure 1(b) illustrates our experimental setup. We use two superconducting flux-driven Josephson parametric amplifiers (JPAs) operated at the frequency $\omega_0/2\pi = 5.323$ GHz for squeezed state generation, where each JPA performs a squeezing operation on the incident weak thermal state [32]. By sending the respective squeezed states to a hybrid ring (symmetric microwave beam splitter), we generate a TMS microwave state. Here, entanglement is expressed in strong correlations between two nonlocal field quadratures [33–36]. Our final state tomography is based on heterodyne measurements of paths A and B. After digital down-conversion and filtering, we extract the statistical field quadrature moments using a reference state reconstruction method [33, 37]. Under the assumption that the reconstructed states are Gaussian, a local phase space distribution is described by the resulting two-mode covariance matrix [33–35]. The photon number calibration of the experimental setup is obtained by using Planck spectroscopy [38]. In order to test the robustness of the nonclassical correlations against noise, we perform a controlled noise injection into one of the entangled paths [33–35]. The noise signal is generated using an arbitrary function generator (AFG) which pro-
produces a low-frequency white Gaussian noise with a specified bandwidth of 160 MHz. This noise signal is upconverted to the carrier frequency ω0/2π and guided into the cryogenic setup. We implement the actual noise injection in one of the entangled paths with a directional coupler with coupling β = −20 dB. By varying the noise power emitted from the AFG, we probe both EoF and QD as a function of the injected noise photon number n for different JPA squeezing levels. The latter is defined as $S = -10 \log_{10}(\nu_s/0.25)$, where $\nu_s$ is the variance of the squeezed quadrature and the chosen vacuum reference is $\nu_{\text{vac}} = 0.25$. More details about the experimental setup are provided in Appendix A.

III. ENTANGLEMENT OF FORMATION AND QUANTUM DISCORD

Under the assumption that the state is Gaussian, EoF and QD can be extracted from the reconstructed two-mode covariance matrix $V_{AB}$ [27, 39]. In Ref. 26, it has been shown that a lower bound, $E_F$, for Gaussian EoF can be expressed as

$$E_F = s_\gamma \left[ \cosh^2 \gamma \ln(\cosh^2 \gamma) - \sinh^2 \gamma \ln(\sinh^2 \gamma) \right] \leq \mathcal{E}_F,$$

where $\gamma$ represents the minimally required amount of two-mode squeezing to disentangle the respective bipartite quantum state and $s_\gamma = \text{sign}(\gamma)$. For our analysis, we use an approximation $E_F \simeq \mathcal{E}_F$, which becomes exact in the case of symmetric local states A and B. The asymmetric Gaussian QD, $D_A$, corresponds to the correlation left after we perform a local measurement on subsystem B. It can be calculated as

$$D_A = f \left( \sqrt{I_2} \right) - f(\nu_+) - f(\nu_-) + f \left( \sqrt{E_{\text{min}}^{\text{AB}}} \right),$$

where $I_2$ denotes the second symplectic invariant of $V_{AB}$ and $\nu_{\pm}$ are the corresponding symplectic eigenvalues [28].

The quantity $\sqrt{E_{\text{min}}^{\text{AB}}}$ describes the minimized conditional entropy and $f$ is defined as

$$f(x) = (2x + \frac{1}{2}) \ln(2x + \frac{1}{2}) - (2x - \frac{1}{2}) \ln(2x - \frac{1}{2}).$$

A similar expression can be written for the quantum discord $D_B$, where the measurement is performed on system A. It can be shown that for pure quantum systems, i.e., in the limit of $n \to 0$, EoF and QD coincide [28]. Nevertheless, for mixed states, these quantities behave fundamentally different, as theory predicts asymptotic stability of QD, in contrast to EoF.

Figure 2 shows theoretically expected results for an idealized experiment with zero losses and noiseless JPAs. In Fig. 2(a) [Fig. 2(b)], we plot $D_B$ ($E_F$) as a function of the average noise photon number $n$, injected to the TMS state, and the squeezing level $S$. We observe the expected asymptotic stability of QD, $D_B > 0$, and the sudden death of entanglement, $E_F < 0$, for $n > 1$. The latter effect can be understood by expressing $\gamma$ in Eq. 2 analytically as

$$\gamma(r, n) = \frac{1}{2} \ln \left[ \frac{e^{2r} + n}{1 + e^{2rn}} \right],$$

where $r$ is the squeezing factor, which can be calculated by $r = S/(20 \log_{10} e)$ for noiseless amplification by both JPAs. We observe that $\gamma(r, 1) = 0$, independent of $r$, indicating the sudden death of entanglement. In Fig. 2(c), we plot the theory values of $E_F$ and $D_B$ for low noise photon numbers and observe a crossover between EoF and QD. We denote the crossover point in terms of a corresponding noise photon number, $n_c$. This crossover point, $n_c > 0$, exists for any positive squeezing level $S$. This effect has been predicted in Refs. 40 and 41, and is a direct result of a tripartite nature of the noise injection. The latter implies that a correct quantum mechanical description of noise necessarily requires to take into account the environment $E$ as a third interacting quantum system. For the tripartite system, it can be shown that EoF and QD are monogamous, i.e., that bipartite QD can only...
be increased by the simultaneous consumption of bipar- 
tite EoF and vice versa [29]. From this monogamic con-
servation relation, it has been shown that the difference 
between EoF and QD can be expressed as [15]
\[
\Delta_A \equiv D_A - \mathcal{E}_F = \frac{1}{2} (\mathcal{L}_{B\rightarrow A\rightarrow E} - \mathcal{L}_{E\rightarrow A\rightarrow B}),
\]
\[
\Delta_B \equiv D_B - \mathcal{E}_F = \frac{1}{2} (\mathcal{L}_{A\rightarrow B\rightarrow E} - \mathcal{L}_{E\rightarrow B\rightarrow A}),
\]
\[
\Delta_{AB} \equiv \frac{D_A + D_B}{2} - \mathcal{E}_F = \frac{1}{2} (\mathcal{L}_{(A\rightarrow B)} - \mathcal{L}_{E\rightarrow (A\rightarrow B)}),
\]
where \( \mathcal{L}_{X\rightarrow Y\rightarrow Z} \) denotes the flow of LII from the system \( X \) over \( Y \) to \( Z \) and \( \mathcal{L}_{(X\rightarrow Y)} \) is the LII flow from (to) the bipartite system \( XY \) to (from) \( Z \). As a result, if \( \Delta_{AB} > 0 \), more LII flows from the bipar- 
tite system \( AB \) to the environment \( E \) than vice versa.

IV. RESULTS AND DISCUSSION

The experimentally determined QD values, \( D_A \) and 
\( D_B \), are provided in Fig. 3(a) and Fig. 3(b) and the 
quanta entanglement measure \( E_F \) is shown in Fig. 3(c). The 
line plots correspond to a fit according to Eq. 3 for the 
QD and Eq. 2 for the EoF, where we take the finite JPA 
noise into account. The gain-dependent noise, added by 
the JPAs, is modelled by a power law dependence, 
\( n_1 = \chi_1 (G-1)^\chi_2 \), where \( G \) represents the degenerate 
gain [43]. The coefficients \( \chi_1 \) and \( \chi_2 \) are treated as fit 
parameters. We find \( \chi_1 = 0.05 \) and \( \chi_2 = 0.56 \). In 
Fig. 3(a), we additionally show the experimentally deter- 
mined squeezing level \( S_c \), as well as the respective theo- 
retical squeezing level \( S_t \) and fitted JPA noise \( n_j \). We 
observe that the fits reliably reproduce the experimen- 
tal data. More information about the fitting routine is 
given in Appendix C.

Furthermore, we find that the experimentally deter- 
mined QD is always positive and converges towards zero 
for \( n \to \infty \), thereby, proving the asymptotic robust- 
ness against noise. In contrast to that, we find that \( E_F \) 
becomes zero already at a finite noise photon number 
\( n_{ad} \approx 1 \), experimentally verifying the sudden death of 
entanglement. This value is an important fundamental 
noise threshold for two-mode squeezed light. The respec- 
tive experimental values for \( n_{ad} \) as a function of squeezing 
are shown in the inset of Fig. 3(c). They have been ex- 
tracted from the experimental data using cubic Hermite 
spline interpolation. We find that \( n_{ad} \) is independent of 
the squeezing level, as expected from theory. Note that 
the experimentally determined noise level for the sudden 
death of entanglement is lower than the theoretically pre- 
dicted noise photon number of unity, which is a result from 
the finite noise added by the JPAs themselves. In 
addition to that, further deviations from ideal theory are 
called by path losses and a pump crossstalk between the 
JPAs. However, these imperfections are not taken into 
account in the current fit model.

For most of the observed states, EoF appears to be 
smaller than QD. This can be understood by the fact 
that, by definition, QD describes more general nonlocal 
correlations than EoF. However, this simple relation is 
only true in the bipartite limit. When one considers the 
environment as a third party, the relation between bi- 
partite QD and EoF may change. In order to experiment- 
tally investigate the latter we investigate the regime of 
\( n \ll 1 \) in more detail. Here, theory predicts a crossover 
between EoF and QD, according to Fig. 2(c). To experimen- 
tally study this crossover region, we replot the measured 
\( D_B \) and \( E_F \) for noise photon numbers \( n \leq 0.2 \) 
in Fig. 3(d), revealing the intersection between EoF and 
QD, especially well observable for \( S_c = 6.5 \) dB. The solid 
(dashed) lines correspond to a cubic Hermite spline 
interpolation for \( D_B \) (\( E_F \)). From this interpolation, we 
determine the crossover noise photon number \( n_c \) as a 
function of squeezing. The same procedure is repeated 
for \( D_A \) and \( \Delta_{AB} = (D_A + D_B)/2 \). The corre- 
sponding results are plotted in Fig. 4(a), and the predictions 
are plotted on an ideal (lossless and noiseless) model are 
depicted in Fig. 4(b). In the limit \( S \to \infty \), we observe that 
\( n_c \approx 0.26 \) for \( D_A \) as well as for \( D_B \). Furthermore, we 
find a qualitative agreement between experiment and 
theory for the dependence of \( n_c \) on the squeezing level 
\( S \). Nevertheless, we observe that for \( D_A \) the experimen- 
tally determined values are lower than those predicted by 
theory. This deviation can be explained by the fi- 
nite noise, losses, and crosstalk between the JPAs, since 
these effects are not taken into account in the ideal theory 
model. Furthermore, we note that we cannot experimen- 
tally investigate the whole range shown in Fig. 4(b), as 
larger squeezing levels are not experimentally achievable 
due to the gain-dependent noise added by the JPAs. As 
shown by the red line in Fig. 4(b), the corresponding \( n_c \) 
for \( (D_A + D_B)/2 \) has a minimum at \( n_{min} \approx 0.23 \), corre- 
sponding to \( S_{min} = 5.7 \) dB. Thus, when one attempts to 
maximize \( n_c \) in the bipartite system \( AB \), it is not always 
beneficial to increase the squeezing level. Experimental 
data in Fig. 4(a) qualitatively reproduces this result.

Next, we investigate the asymmetric differences be- 
tween EoF and QD, \( \Delta_A \), \( \Delta_B \), and \( \Delta_{AB} \), as a function of 
the noise photon number \( n \). Figure 4(c) [(d)] shows the 
theoretically expected noise dependence of \( \Delta_A \) (\( \Delta_B \)) 
for various squeezing levels \( S \). We observe that the quanti- 
ties \( \Delta_A \) and \( \Delta_B \) behave fundamentally different in 
the limit of low noise. In particular, the crossover noise pho- 
ton number \( n_c \) decreases monotonically with increasing 
\( S \) in the case of \( \Delta_A \), as can also be seen in Fig. 4(b). On 
the contrary, \( n_c \) shows a monotonic increase as a func- 
tion of \( S \) for increasing \( \Delta_B \). This fundamental deviation 
can be understood by the fact that noise injection in B 
is a local process and directly leads to bipartite corre- 
lations between party B and environment E and only 
indirectly correlates A and E. Thus, the bipartite corre- 
lations between B and E increase monotonically with \( n \). 
Furthermore, the correlation between A and B monoton- 
ically decreases as a function of \( n \). In contrast to that,
Figure 3. (a) Experimentally obtained values of quantum discord $D_A$ as a function of the injected noise photon number $n$ for various squeezing levels $S$. Dots indicate the measured data and lines are fits according to a realistic model, described in Appendix C, which takes a finite JPA noise into account. The quantity $S_i$ denotes the experimentally determined squeezing level and $S_t$ is the corresponding squeezing level, obtained by fitting the data by the theory prediction. The JPA noise $n_j$ is extracted from the fit and is a function of gain. Although only shown for $D_A$, the fitted values for $S_t$ and $n_j$ are the same for $D_B$ and $E_F$. (b) Experimentally obtained values of quantum discord $D_B$ as a function of the injected noise photon number, $n$, for various squeezing levels. The inset shows the same data in a log-log plot. (c) Experimental EoF (full circles) and corresponding fits (lines) for various squeezing levels. We observe the sudden death of entanglement at $n_{sd}$ ≃ 1. The inset shows that $n_{sd}$ is independent of $S$, where $n_{sd}$ is obtained from the experimental data using cubic Hermite spline interpolation. Error bars are obtained from the statistical measurement error and are only plotted if the error exceeds the symbol size. (d) Zoom-in of experimental results for $D_B$ and $E_F$ for low noise photon numbers $n$ and various squeezing levels $S_t$. Solid (dashed) lines are the result from a cubic Hermite spline interpolation between the measured values for $D_B$ (EoF). Here, we observe the crossover behavior of QD and EoF, as predicted by the theory.

correlations between $A$ and $E$ can only result from an interplay between squeezing $S$ and noise $n$, and are not necessarily required to be monotonic in these quantities. The scenario is schematically depicted in Fig. 5(a), where solid black arrows, connecting to systems $X$ and $Y$ with $X, Y \in \{A, B, E\}$, indicate a monotonic increase of correlations. The direction of the LII flow, described by $\Delta_A$ ($\Delta_B$) according to Eq. 5 (Eq. 6), is shown by the purple (green) curved arrow. Note that $\Delta_A$ describes the net LII flow $B \rightarrow A \rightarrow E$ and $\Delta_B$ describes the respective net flow $A \rightarrow B \rightarrow E$. Therefore, the fundamentally different behavior of these quantities as a function of $n$, as shown in Figs. 4(c) and (d), can be explained by the fact that in the case of $\Delta_B$ no direct bipartite correlations between $A$ and $E$ are required to establish an LII flow, in contrast to the case described by $\Delta_A$.

Furthermore, the quantities $\Delta_A, \Delta_B$, and $\Delta_{AB}$ can become of practical interest for entanglement-based quantum key distribution (QKD) protocols [44], where an eavesdropper attempts to extract LII from a bipartite quantum system. In such a scenario, the subsystems $A$ and $B$ exploit quantum correlations to securely share a common secret key and the subsystem $E$ can be related to an eavesdropper controlling the environment [45]. Then, the noise injection can be interpreted as the result of an entangling cloner attack performed by the eavesdropper [46]. It directly follows from Eq. 7 that the eavesdropper needs to add at least $n_c$ noise photons to the system.
AB to get a positive net flow of LII. Numerically, we find that in the limit $S \to \infty$, we have $n_c \to n_\ast \simeq 0.26$ for $\Delta_A$, $\Delta_B$, and $\Delta_{AB}$. In order to investigate this interrelation, we consider a Gaussian CV-QKD scheme described in Refs. 42 and 47 under the assumption of reverse reconciliation [48]. We consider a scenario, where A and B share a TMS resource state, and assume that the eavesdropper performs an entangling cloner attack via the directional coupler. The resulting secret key which quantifies the amount of exchanged secure information can be calculated as

$$K = I_s(A : B) - \chi_E,$$

where $I_s(A : B)$ denotes the Shannon mutual information between A and B, and $\chi_E$ represents the eavesdropper’s Holevo quantity [48, 49]. Figure 5(b) shows the theoretically expected $K$ as a function of the resource state squeezing level $S$ and noise photon number $n_q$ in the detected quadrature for the case when B performs a homodyne detection on his part of the entangled state. Note that, in contrast to the full noise $n$, we only consider the noise $n_q$, added to the detected quadrature $q$, since homodyne detection is equivalent to phase-sensitive amplification and measurement of a certain quadrature which inherently deemphases the other quadrature [28]. Consequently, only the noise in the amplified quadrature has an effect on the measurement result. We observe that, similarly to the quantities $\Delta_A$, $\Delta_B$, and $\Delta_{AB}$, the threshold value for $n_q$, when we obtain a positive secret key, converges towards $n_\ast \simeq 0.26$ for $S \to \infty$. We numerically find that this asymptotic value approximately coincides

Figure 4. (a) Experimentally determined crossover noise photon number, $n_c$, as a function of the squeezing level for $D_A$ (blue) and $D_B$ (orange). Red dots represent the arithmetic mean of $n_c$ for $D_A$ and $D_B$, where we observe a minimum in the region of 5 dB. The error bars are determined from the experimental uncertainties of QD and EoF by randomized error sampling. (b) Theoretically predicted $n_c$ for $D_A$ (blue), $D_B$ (orange) for the case of an idealized (lossless and noiseless, $\beta \to 0$) experiment. In the case of $D_A$ ($D_B$), $n_c$ decreases (increases) monotonically with $S$. We observe a minimum for $S_{\text{min}} \simeq 5.73$ dB for the red curve which corresponds to the arithmetic mean of both discords, $D_A$ and $D_B$. In the limit $S \to \infty$, $n_c$ converges to the same constant $n_\ast \simeq 0.26$ for $D_A$ and $D_B$. The region of experimentally obtained squeezing levels, corresponding to panel (a), is indicated in gray. (c) Theoretical difference $\Delta_A$ between $D_A$ and $E_F$ as a function of the noise photon number $n$ for various squeezing levels $S$. (d) Theoretical difference $\Delta_B$ between $D_B$ and $E_F$ as a function of the noise $n$ for various squeezing levels $S$. 

-0.02
-0.01
0.00
0.0 0.2 0.4 0.6

-0.02
-0.01
0.00
0.0 0.2 0.4 0.6

0.00
0.01
0.02
0.03
n
0.30
3 4 5 6 7
with \( n_\ast \) implying that, in the high squeezing limit, we can only obtain a positive secret key if the noise, added to the detected quadrature, is lower than the threshold \( n_\ast \), as required for a positive LII flow to the eavesdropper. As a result, the difference between QD and EoF can act as an indicator whether it is possible to obtain a positive secret key or not. More details about the calculation of \( K \) are provided in Appendix B.

In summary, we have investigated the influence of local noise injection in propagating TMS microwave states on quantum discord correlations and quantum entanglement quantified via the entanglement of formation measure. We have experimentally verified the sudden death of entanglement around theoretically predicted values of approximately one injected noise photon, independent of the squeezing level. Furthermore, we have experimentally demonstrated that in strong contrast to entanglement, QD is asymptotically robust against noise. In addition, we have measured the theoretically predicted crossover between EoF and QD for small noise photon numbers, which is a result of the tripartite nature of mixed TMS states. Since the difference between QD and EoF can be related to the net flow of LII, it may be used to assess the security of certain QKD protocols based on squeezed states. We have demonstrated that the locality of noise injection implies a fundamental difference between the LII flows A→B→E and B→A→E. Finally, the demonstrated results on the robustness of QD against noise are relevant for the DQC1 quantum computation approach and quantum illumination protocols. These applications can be viewed as a motivation to intensify the search for quantum information processing, communication, and sensing protocols exploiting QD as a quantum resource. Such protocols would be inherently resistant to noise in contrast to entanglement-based approaches which suffer from the sudden death of entanglement.

We acknowledge support by the German Research Foundation via Germany’s Excellence Strategy (EXC-2111-390814868), the Elite Network of Bavaria through the program ExQM, the EU Flagship project QMiCS (Grant No. 820505), and the German Federal Ministry of Education and Research via the project QUARATE (Grant No. 13N15380), the project QuaMToMe (Grant No. 16KISQ036), JSPS KAKENHI (Grant No. 22H04937), and JST ERATO (Grant No. JPMJER1601). This research is part of the Munich Quantum Valley, which is supported by the Bavarian state government with funds from the Hightech Agenda Bayern Plus.

**APPENDIX**

**A. Experimental setup**

The JPAs are thermally stabilized at 50 mK to guarantee steady squeezing and noise properties. We pump the JPAs using Rohde&Schwarz SGS100A microwave sources and pulse the pump signal using a data timing generator (DTG) [23, 33, 50, 51]. Here, the TMS state is generated by superimposing two orthogonally squeezed states with equal squeezing levels using a cryogenic hybrid ring, acting as a 50:50 beam splitter. The resulting TMS state at the hybrid ring output then locally looks like thermal noise but shows strong nonlocal correlations in the covariance. The photon number in the TMS state is calibrated using the Planck spectroscopy [38], which is realized by sweeping the temperature of a heatable 30 dB attenuator in the temperature range of 40 mK – 600 mK. By using this heatable attenuator as a self-calibrated cryogenic photon source, we can directly map the detected voltage...
in the output signal to the photon number in the cryogenic quantum signal [38]. One part of the TMS state is transmitted to a directional coupler (CPL-4000-8000-20-C, Miteq/Sirius) with coupling $\beta = -20 \text{ dB}$. The coupled port of the directional coupler is used to inject white broadband noise into the system, which is generated using a Keysight 8160A arbitrary waveform generator (AFG). The generated noise has a bandwidth of 160 MHz and is upconverted to the signal reconstruction frequency of $\omega_0/2\pi = 5.323 \text{ GHz}$ using a local oscillator. Due to the low signal level, the output signal needs to go through multiple amplification stages consisting of a cryogenic high-electron-mobility transistor (HEMT) amplifier and additional room temperature amplifiers, which are stabilized in temperature by a Peltier cooler. The overall noise of the detection chain is determined by the HEMT, which results from its high gain ($\sim 40 \text{ dB}$). Frequency-resolved measurements are performed using a vector network analyzer (VNA) and the reconstruction of quantum microwave states as well as quantum correlation measurements are performed using a heterodyne receiver setup. This heterodyne detection setup is similar to the setups described in Refs. 23, 34, 35, 51. The signal is down-converted to 11 MHz and digitized by an Acqiris card with a sampling frequency of 400 MHz. The digitized data is transmitted to a computer and down converted to a dc-signal. The resulting signal is filtered using a digital finite-impulse-response (FIR) filter with a full bandwidth of 400 kHz. Subsequently, the quadrature moments $\langle I^x Q^y I^z Q^y \rangle$, $n,m,k,l \in \mathbb{N}_0$ are determined and averaged. In each measurement, the data is averaged over 210 cycles, where each cycle corresponds to $5.76 \times 10^6$ averages. The squeezing angles are stabilized using a phase-locked loop, where in each measurement cycle, the squeezing angle $\gamma_i^{\text{exp}}$ is extracted from the quadrature moments corresponding to the $i^{\text{th}}$ JPA, where $i \in \{1,2\}$. Following this approach, the difference $\delta \gamma_i = \gamma_i^{\text{exp}} - \gamma_i^{\text{target}}$ from the desired target angle $\gamma_i^{\text{target}}$ is calculated and the respective phase of the JPA pump source is corrected by $2\delta \gamma_i$. The JPA pump sources are daisy-chained with a reference frequency of 1 GHz to the local oscillator (LO) source. The LO, the DTG, the VNA, the AFG, as well as the Acqiris card are synchronized with a 10 MHz rubidium frequency clock (Stanford Research Systems, FS725). In our experiment, we assume that all reconstructed states are Gaussian and can be described by signal moments up to the second order. This assumption of Gaussianity is verified by calculating the cumulants $\kappa_{mn}$ [52]. Since it is theoretically expected that $\kappa_{mn} = 0$ for $m + n > 2$ for Gaussian states, we conclude that the Gaussian approximation of our quantum states is well justified if the experimentally reconstructed cumulants of third and fourth order are much smaller than the first and second order cumulants.

B. Quantum key distribution with Gaussian states

In order to investigate the relation between the LII flow and QKD, we consider that Alice and Bob share an ideal TMS state with the squeezing factor $r$, described by the covariance matrix

$$V_{AB} = \frac{1}{4} \begin{pmatrix} \cosh 2r & \sinh 2r & 0 & 0 \\ \sinh 2r & \cosh 2r & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(9)

where $\mathbb{1}_2$ denotes the $2 \times 2$ identity matrix and $\sigma_z$ is the Pauli $z$-matrix. For the entangling cloner attack, the eavesdropper prepares a second TMS state [48]

$$V_{E_1E_2} = \frac{1}{4} \begin{pmatrix} W_{\mathbb{1}_2} & \sqrt{W^2 - 1} \sigma_z \\ \sqrt{W^2 - 1} \sigma_z & W_{\mathbb{1}_2} \end{pmatrix}.$$  

(10)

As a result, the full covariance matrix

$$V_{ABE_1E_2} = V_{AB} \oplus V_{E_1E_2}$$

(11)

describes a pure state. The eavesdropper then couples the mode $E_1$ to B by an asymmetric beam splitter operation

$$C = \begin{pmatrix} \sqrt{1 - \beta} \mathbb{1}_2 & \sqrt{\beta} \mathbb{1}_2 \\ -\sqrt{\beta} \mathbb{1}_2 & \sqrt{1 - \beta} \mathbb{1}_2 \end{pmatrix}.$$  

(12)

The resulting covariance matrix is given by

$$V'_{ABE_1E_2} = (\mathbb{1}_2 \oplus C \oplus \mathbb{1}_2)V_{ABE_1E_2}(\mathbb{1}_2 \oplus C \oplus \mathbb{1}_2)^{\dagger},$$  

(13)

which can be analytically expressed as

$$V'_{ABE_1E_2} = \frac{1}{4} \begin{pmatrix} V'_{11} & V'_{12} & V'_{13} & 0 \\ V'_{12}^{T} & V'_{22} & V'_{23} & V'_{24} \\ V'_{13}^{T} & V'_{23}^{T} & V'_{33} & V'_{34} \\ 0 & V'_{24}^{T} & V'_{34}^{T} & V'_{44} \end{pmatrix},$$  

(14)

where

$$V'_{11} = \cosh 2r \mathbb{1}_2$$
$$V'_{12} = \sqrt{1 - \beta} \sinh 2r \sigma_z$$
$$V'_{13} = -\sqrt{\beta} \cosh 2r \sigma_z$$
$$V'_{12} = (1 - \beta) \cosh 2r \mathbb{1}_2 + \beta W \mathbb{1}_2$$
$$V'_{23} = \sqrt{\beta(1 - \beta)} (W - \cosh 2r) \mathbb{1}_2$$
$$V'_{24} = \sqrt{\beta} \sqrt{W^2 - 1} \sigma_z$$
$$V'_{33} = \beta \cosh 2r \mathbb{1}_2 + (1 - \beta) W \mathbb{1}_2$$
$$V'_{34} = \sqrt{1 - \beta} \sqrt{W^2 - 1} \sigma_z$$
$$V'_{44} = W \mathbb{1}_2.$$  

(15)

Since the matrix $V'_{22}/4$ corresponds to the local noisy TMS state in system B, we demand

$$\frac{1}{4} (\cosh 2r + 2n) \mathbb{1}_2 = \frac{1}{4} [(1 - \beta) \cosh 2r \mathbb{1}_2 + \beta W \mathbb{1}_2].$$  

(16)
In the limit $\beta \ll 1$, we find the relation $\beta W = 2n$. Since we perform a homodyne detection on B in the next step, it is practical to define the number of noise photons, added to the measured quadrature, as $n_q = n/2$. For the final state of the eavesdropper, we have

$$V_E' = \frac{1}{4} \left( \begin{array}{cc} V_{33}^\prime & V_{34}^\prime \\ V_{34}^\prime & V_{44}^\prime \end{array} \right).$$

(17)

We consider reverse reconciliation and hence perform a measurement of $B$ [46]. After a homodyne measurement of the $q$ quadrature, the conditioned covariance matrix for $E$ reads

$$V_{E|B} = V_E' - \frac{1}{4\sqrt{\text{det}V_{22}}} V_C \Pi_q V_C^T,$$

(18)

where $V_C = (V_{33}^\prime, V_{34}^\prime)^T$ and $\Pi_q$ denotes the phase-space projector on the $q$-quadrature. The corresponding Holevo quantity $\chi_E$ is then obtained as

$$\chi_E = S_E - S_{E|B},$$

(19)

where $S_E$ ($S_{E|B}$) denotes the von Neumann entropy, corresponding to $V_E'$ ($V_{E|B}$) [48, 49]. For a two-mode Gaussian state, described by a covariance matrix $V$, the von Neumann entropy is given by

$$S = f(\nu_+) + f(\nu_-),$$

(20)

where $\nu_+$ and $\nu_-$ are the symplectic eigenvalues of $V$ [53]. For a codebook of input states with variance $\sigma^2$, the Shannon mutual information is obtained to be [48]

$$I_s(A : B) = \frac{1}{2} \log_2 (1 + \text{SNR})$$

$$= \frac{1}{2} \log_2 \left( 1 + \frac{4(1 - \beta)\sigma^2}{(1 - \beta)e^{-2r} + 4n_q} \right)$$

$$\approx \frac{1}{2} \log_2 \left( 1 + \frac{\sigma^2}{n_q} \right),$$

(21)

where the last expression is valid in the limit $r \gg 1$ and $\beta \ll 1$. For the calculation of the signal-to-noise ratio (SNR) in the detected quadrature, we have considered the protocol described in Ref. 42, implying a noise of $e^{-2r}/4 + n_q$ per quadrature and $\sigma^2 = \sinh(2r)/2$. To obtain Fig. 3, we have fixed $\beta = 10^{-4}$.

**C. Fitting the experimental data**

In this section, we provide details about the model used to fit the experimental data. By taking finite coupling and amplifier noise into account, the final covariance matrix can be expressed as

$$V_{AB} = \frac{1 + 2n_j(G)}{4} \begin{pmatrix} \alpha & \gamma \\ \gamma^T & \beta \end{pmatrix},$$

(22)

where

$$\alpha = \cosh 2r \ 1_2,$$

$$\beta = [(1 - \beta) \cosh 2r + (1 + 2\bar{n})] \ 1_2,$$

$$\gamma = \sqrt{1 - \beta} \sinh 2r \ 1_2.$$

(23-25)

Furthermore, $n_j(G)$ represents the noise added by the JPAs, which depends on the degenerate gain $G$. In addition, $\bar{n}$ is the number of noise photons at the input of the coupled port of the directional coupler. The environmental noise $\bar{n}$ is related to $n$ via $n = \beta \bar{n}$ under the assumption that $\bar{n} \gg 1$. The realistic squeezing factor $r$ can be extracted from the reconstructed squeezed (antisqueezed) variance $v_\nu (v_\chi)$ via $e^{4r} = v_\chi/v_\nu$. The degenerate gain can then be expressed as $G = e^{2r}$. Furthermore, we model the gain-dependent JPA noise by a power law dependence $n_j(G) = \chi_1(G - 1)^{\chi_2}$, where we treat $\chi_1$ and $\chi_2$ as the only fit parameters. For the fit, we use $\chi = (\chi_1, \chi_2)^T$ and define a corresponding weighted least-square cost function

$$T(\chi) = \sum_{S,n} \left( w_1 |D_A(S, n, \chi) - \tilde{D}_A(S, n)|^2 + w_2 |D_B(S, n, \chi) - \tilde{D}_B(S, n)|^2 + w_3 |E_F(S, n, \chi) - \tilde{E}_F(S, n)|^2 \right),$$

(26-28)

where the sum is evaluated over all experimentally chosen squeezing levels $S$ and noise photon numbers $n$. The quantities $w_i$ are the weights, accounting for the respective contribution. The quantities $\tilde{D}_A(S, n, \chi)$, $\tilde{D}_B(S, n, \chi)$, and $\tilde{E}_F(S, n, \chi)$ are the experimentally determined data points for A-discord, B-discord, and EoF, respectively, corresponding to $S$ and $n$. The functions $D_A(S, n, \chi)$, $D_B(S, n, \chi)$, and $E_F(S, n, \chi)$ are obtained by inserting Eq. 22 into the theoretical expressions for QD and EoF. The fit parameters $\chi_1 = 0.05$ and $\chi_2 = 0.56$ are then given by

$$\left( \chi_1 \chi_2 \right) = \arg \min T(\chi),$$

(29)

where we start with the initial conditions $\chi = (0, 1)^T$. To balance the contributions of QD and EoF in the cost function, we choose the weights $w_1 = w_2 = 1/2$ and $w_3 = 1$ for the fit. To extract the noise photon numbers corresponding to the sudden death of entanglement as well as the experimental crossover points between QD and EoF, we do not make use of the fit curves. Instead, we determine these values directly from the experimental data using cubic Hermite spline interpolation as we expect this method to be more accurate than the fit. We use cubic Hermite spline interpolation instead of conventional cubic splines to increase the precision by avoiding overshoots.
[1] S. L. Braunstein and P. van Loock, “Quantum information with continuous variables,” Rev. Mod. Phys. 77, 513–577 (2005).
[2] Z. Chen et al., “Exponential suppression of bit or phase errors with cyclic error correction,” Nature 595, 383–387 (2021).
[3] A. Eddins, M. Motta, T. P. Gujarati, S. Bravyi, A. Mezzacapo, C. Hadfield, and S. Sheldon, “Doubling the Size of Quantum Simulators by Entanglement Forging,” PRX Quantum 3, 010309 (2022).
[4] F. Arute et al., “Quantum supremacy using a programmable superconducting processor,” Nature 574, 505–510 (2019).
[5] X. Wang, T. Hiroshima, A. Tomita, and M. Hayashi, “Quantum information with Gaussian states,” Physics Reports 449, 1–111 (2007).
[6] S. L. Braunstein and H. J. Kimble, “Dense coding for continuous variables,” Phys. Rev. A 61, 042302 (2000).
[7] K. G. Fedorov et al., “Experimental quantum teleportation of propagating microwaves,” Sci. Adv. 7, eabk0891 (2021).
[8] A. Furusawa, J. L. Sørensen, S. L. Braunstein, C. A. Fuchs, H. J. Kimble, and E. S. Polzik, “Unconditional Quantum Teleportation,” Science 282, 706–709 (1998).
[9] H. Ollivier and W. H. Zurek, “Quantum Discord: A Measure of the Quantumness of Correlations,” Phys. Rev. Lett. 88, 017901 (2001).
[10] A. Datta, A. Shaji, and C. M. Caves, “Quantum Discord and the Power of One Qubit,” Phys. Rev. Lett. 100, 050502 (2008).
[11] B. P. Lanyon, M. Barbieri, M. P. Almeida, and A. G. White, “Experimental Quantum Computing without Entanglement,” Phys. Rev. Lett. 101, 200501 (2008).
[12] D. Girolami, A. M. Souza, V. Giovannetti, T. Tufarelli, J. G. Filgueiras, R. S. Sarthour, D. O. Soares-Pinto, I. S. Oliveira, and G. Adesso, “Quantum Discord Determines the Interferometric Power of Quantum States,” Phys. Rev. Lett. 112, 210401 (2014).
[13] C. Weedbrook, S. Pirandola, J. Thompson, V. Vedral, and M. Gu, “How discord underlies the noise resilience of quantum illumination,” New J. Phys. 18, 043027 (2016).
[14] C. Weedbrook, S. Pirandola, R. García-Pratón, N. J. Cerf, T. C. Ralph, J. H. Shapiro, and S. Lloyd, “Gaussian quantum information,” Rev. Mod. Phys. 84, 621–669 (2012).
[15] F. F. Fanchini, L. K. Castelano, M. F. Cornelio, and M. C. de Oliveira, “Locally inaccessible information as a fundamental ingredient to quantum information,” New Journal of Physics 14, 013027 (2012).
[16] D. Girolami, A. Touil, B. Yan, S. Deffner, and W. H. Zurek, (2022), arXiv:2202.09328 [quant-ph].
[17] W. H. Zurek, “Quantum Darwinism,” Nat. Phys. 5, 181–188 (2009).
[18] A. Touil, B. Yan, D. Girolami, S. Deffner, and W. H. Zurek, “Eavesdropping on the Decohering Environment: Quantum Darwinism, Amplification, and the Origin of Objective Classical Reality,” Phys. Rev. Lett. 128, 010401 (2022).
[19] T. Yu and J. H. Eberly, “Sudden death of entanglement,” Science 323, 598–601 (2009).
[20] M. Sanz, K. G. Fedorov, F. Deppe, and E. Solano, “Challenges in Open-air Microwave Quantum Communication and Sensing,” 2018 IEEE Conference on Antenna Measurements & Applications (CAMA), 1–4 (2018).
[21] T. Werlang, S. Souza, F. F. Fanchini, and C. J. Villas Boas, “Robustness of quantum discord to sudden death,” Phys. Rev. A 80, 024103 (2009).
[22] C. H. Bennett, D. P. DiVincenzo, P. W. Shor, J. A. Smolin, B. M. Terhal, and W. K. Wootters, “Remote state preparation,” Phys. Rev. Lett. 87, 077902 (2001).
[23] S. Pogorzalek et al., “Secure quantum remote state preparation of squeezed microwave states,” Nat. Commun. 10, 2604 (2019).
[24] B. Dakić et al., “Quantum discord as resource for remote state preparation,” Nat. Phys. 8, 666 (2012).
[25] S. Lloyd, “Enhanced Sensitivity of Photodetection via Quantum Illumination,” Science 321, 1463 (2008).
[26] S. Tserkis and T. C. Ralph, “Quantifying entanglement in two-mode Gaussian states,” Phys. Rev. A 96, 062338 (2017).
[27] P. Giorda and M. G. A. Paris, “Gaussian Quantum Discord,” Phys. Rev. Lett. 105, 020503 (2010).
[28] G. Adesso and A. Datta, “Quantum versus Classical Correlations in Gaussian states,” Phys. Rev. Lett. 105, 030501 (2010).
[29] F. F. Fanchini, M. F. Cornelio, M. C. de Oliveira, and A. O. Caldeira, “Conservation law for distributed entanglement of formation and quantum discord,” Phys. Rev. A 84, 012313 (2011).
[30] S. Tserkis, S. Onoe, and T. C. Ralph, “Quantifying entanglement of formation for two-mode Gaussian states: Analytical expressions for upper and lower bounds and numerical estimation of its exact value,” Phys. Rev. A 99, 052337 (2019).
[31] L. Henderson and V. Vedral, “Classical, quantum and total correlations,” J. Phys A Math Gen 34, 6899–6905 (2001).
[32] T. Yamamoto, K. Inomata, M. Watanabe, K. Matsuba, T. Miyazaki, W. D. Oliver, Y. Nakamura, and J. S. Tsai, “Flux-driven Josephson parametric amplifier,” Appl. Phys. Lett. 93, 042510 (2008).
[33] E. P. Menzel et al., “Path Entanglement of Continuous-Variable Quantum Microwaves,” Phys. Rev. Lett. 109, 250502 (2012).
[34] K. G. Fedorov et al., “Displacement of propagating squeezed microwave states,” Phys. Rev. Lett. 117, 020502 (2016).
[35] K. G. Fedorov et al., “Finite-time quantum entanglement in propagating squeezed microwaves,” Sci. Rep. 8, 6416 (2018).
[36] E. Flurin, N. Roch, F. Mallet, M. H. Devoret, and B. Huard, “Generating Entangled Microwave Radiation over Two Transmission lines,” Phys. Rev. Lett. 109, 183901 (2012).
[37] C. Eichler, D. Bozyigit, C. Lang, M. Baur, L. Steffen, E. P. Menzel, F. Deppe, M. ´A. Araque Caballero, A. Baust, T. Niemczyk, E. Hoffmann, E. Solano, A. Marx, and R. Gross, “Planck Spectroscopy
and Quantum Noise of Microwave Beam Splitters,” Phys. Rev. Lett. 105, 133601 (2010).

[39] S. Tserkis, J. Thompson, A. P. Lund, T. C. Ralph, P. K. Lam, M. Gu, and S. M. Assad, “Maximum entanglement of formation for a two-mode Gaussian state over passive operations,” Phys. Rev. A 102, 052418 (2020).

[40] S. Luo, “Quantum discord for two-qubit systems,” Phys. Rev. A 77, 042303 (2008).

[41] P. Marian, I. Ghiu, and T. A. Marian, “Decay of Gaussian correlations in local thermal reservoirs,” Phys. Scr. 90, 074041 (2015).

[42] N. J. Cerf, M. Lévy, and G. Van Assche, “Quantum distribution of Gaussian keys using squeezed states,” Phys. Rev. A 63, 052311 (2001).

[43] M. Renger et al., “Beyond the standard quantum limit for parametric amplification of broadband signals,” Npj Quantum Inf. 7, 160 (2021).

[44] V. C. Usenko, L. Ruppert, and R. Filip, “Entanglement-based continuous-variable quantum key distribution with multimode states and detectors,” Phys. Rev. A 90, 062326 (2014).

[45] O. Ahonen, M. Möttönen, and J. L. O’Brien, “Entanglement-enhanced quantum key distribution,” Phys. Rev. A 78, 032314 (2008).

[46] Frédéric Grosshans, Nicolas J. Cerf, Jérôme Wenger, Rosa Tualle-Brouri, and Philippe Grangier, “Virtual Entanglement and Reconciliation Protocols for Quantum Cryptography with Continuous Variables,” Quantum Inf Comput 3, 535–552 (2003).

[47] F. Fesquet et al., “Perspectives of microwave quantum key distribution in open-air,” https://arxiv.org/abs/2203.05530 (2022), arXiv:2203.05530 [quant-ph].

[48] F. Laudenbach, C. Pacher, C. F. Fung, A. Poppe, M. Peev, B. Schrenk, M. Hentschel, P. Walther, and H. Hübel, “Continuous-Variable Quantum Key Distribution with Gaussian Modulation—The Theory of Practical Implementations,” Adv. Quantum Technol 1, 1800011 (2018).

[49] A. S. Holevo, “Bounds for the quantity of information transmitted by a quantum communication channel,” Probl. Inf. Transm. 9, 177–183 (1973).

[50] L. Zhong et al., “Squeezing with a flux-driven Josephson parametric amplifier,” New Journal of Physics 15, 125013 (2013).

[51] S. Pogorzalek et al., “Hysteretic Flux Response and Non-degenerate Gain of Flux-Driven Josephson Parametric Amplifiers,” Phys. Rev. Applied 8, 024012 (2017).

[52] S. Xiang, W. Wen, Y. Zhao, and K. Song, “Evaluation of the non-Gaussianity of two-mode entangled states over a bosonic memory channel via cumulant theory and quadrature detection,” Phys. Rev. A 97, 042303 (2018).

[53] A Serafini, F. Illuminati, and S. De Siena, “Symplectic invariants, entropic measures and correlations of Gaussian states,” J PHYS B-AT MOL OPT 37, L21–L28 (2004).