Deep Incomplete Multi-View Clustering via Mining Cluster Complementarity

Jie Xu¹, Chao Li¹, Yazhou Ren¹*, Liang Peng¹, Yujie Mo¹, Xiaoshuang Shi¹*, Xiaofeng Zhu¹,²
¹School of Computer Science and Engineering, University of Electronic Science and Technology of China, Chengdu 611731, China
²Shenzhen Institute for Advanced Study, University of Electronic Science and Technology of China, Shenzhen 518000, China
jiexuwork@outlook.com, lichao.cfm@gmail.com, yazhou.ren@uestc.edu.cn, larrypengliang@gmail.com, moyujie2017@gmail.com, xsshi2013@gmail.com, seanzhuxf@gmail.com

Abstract

Incomplete multi-view clustering (IMVC) is an important unsupervised approach to group the multi-view data containing missing data in some views. Previous IMVC methods suffer from the following issues: (1) the inaccurate imputation or padding for missing data negatively affects the clustering performance, (2) the quality of features after fusion might be interfered by the low-quality views, especially the inaccurate imputed views. To avoid these issues, this work presents an imputation-free and fusion-free deep IMVC framework. First, the proposed method builds a deep embedding feature learning and clustering model for each view individually. Our method then nonlinearly maps the embedding features of complete data into a high-dimensional space to discover linear separability. Concretely, this paper provides an implementation of the high-dimensional mapping as well as shows the mechanism to mine the multi-view cluster complementarity. This complementary information is then transformed to the supervised information with high confidence, aiming to achieve the multi-view clustering consistency for the complete data and incomplete data. Furthermore, we design an EM-like optimization strategy to alternately promote feature learning and clustering. Extensive experiments on real-world multi-view datasets demonstrate that our method achieves superior clustering performance over state-of-the-art methods.

Introduction

Data often has multiple views collected from diverse sources in practical applications, such as classification (Liu, Li, and Zhang 2016), community detection (Cao et al. 2019), dimensionality reduction (Liu et al. 2017), and cross-modal retrieval (Xu et al. 2020; Wei et al. 2021). Multi-view clustering is an important unsupervised approach, aiming to improve the model effectiveness by mining the complementary information hidden in multi-view data. Recently, many multi-view clustering methods have been proposed (Tao et al. 2017; Li et al. 2019; Peng et al. 2019; Tang et al. 2020; Chen et al. 2020; Huang et al. 2021; Xu et al. 2021b). These methods mainly deal with the complete multi-view data, where the information of all views is observed.

However, the information of multi-view data might be incomplete in some views, as known as incomplete multi-view data (Li, Jiang, and Zhou 2014; Xu, Tao, and Xu 2015). For example, different medical tests of a patient can be treated as different views, where cheap medical tests are easily available and expensive ones are often missing due to price. Images and texts are two views to describe scenes, but only some images have textual captions. The incomplete multi-view data inevitably makes existing multi-view clustering methods limited and inapplicable. To this end, an increasing attention is paid to partial multi-view clustering or incomplete multi-view clustering (IMVC) problems (Rai et al. 2016; Wang et al. 2018; Ye et al. 2018; Huang et al. 2020).

In the literature, existing IMVC methods can be categorized into two groups, i.e., traditional methods and deep methods. Traditional IMVC methods usually adopt zero or mean values to pad missing data (Wen et al. 2021) firstly, and then design specific machine learning techniques to conduct multi-view clustering, such as non-negative matrix factorization methods (Li, Jiang, and Zhou 2014; Hu and Chen 2019), subspace learning methods (Kang et al. 2020; Liu et al. 2021a), kernel methods (Guo and Ye 2019; Liu et al. 2021b), and graph methods (Rai et al. 2016; Li, Wan, and He 2021). Nevertheless, traditional IMVC methods are limited in their representation capability and high complexity (Guo and Ye 2019). Recently, deep IMVC methods have gradually been attracting attentions due to their powerful generalization capability and scalability. Deep IMVC methods often utilize the imputation strategies to infer the possible values for missing data before conducting multi-view clustering. For instance, (Xu et al. 2019a; Wang et al. 2021) proposed to take advantage of generative adversarial networks to generate desired data for the missing data. (Lin et al. 2021) designed to recover the missing data with contrastive learning.

Although existing IMVC methods achieve important progress by padding missing data with imputation strategies, they have at least two issues. On the one hand, the effectiveness of imputation strategies depends on the quality of imputed data. It is difficult to correctly estimate the missing data based on the observed data, especially when the number of missing data is large. Moreover, it is also knotty to measure the quality of the imputation as the ground truth of the missing data is unknown. On the other hand, existing IMVC methods usually explore the complementary information among multi-view data by the fusion process. For instance, (Guo and Ye 2019) excavated the complementary in-
formation by fusing multiple similarity matrices. (Wen et al. 2020; Wang et al. 2021) pointed out to utilize fusion layers to mine the complementary information. As some views might be inherently of low quality or inaccurately imputed, they will negatively affect the fusion process.

In this paper, we propose an imputation-free and fusion-free deep IMVC framework (named DIMVC) to address the aforementioned issues. That is, the missing data does not need to be imputed or padded and the cluster assignments do not depend on the fusion process of multiple views. To fulfill this, there are two crucial challenges to be solved, i.e., (i) a new strategy needs to be developed to explore complementary information in the fusion-free model, and (ii) it is difficult to obtain consistent cluster assignments for both the complete data and incomplete data without imputation.

We illustrate the framework of our proposed DIMVC in Figure 1. More specifically, we build an individual model on all observed data for each view. Each model consists of an autoencoder and a clustering mapping. Based on our observation that the complementary information across multiple views can be described by nonlinear mappings, we tackle the challenge (i) by a high-dimensional mapping. Concretely, the embedding features of the complete multi-view data are non-linearly projected into a concatenated weighted feature space, where the high-separability view is assigned with a high weight. Intuitively, the high-separability view means that there are well-separated cluster structures in the features. Moreover, we show that the linearly separable cluster information can be transferred to the high-dimensional features, called multi-view cluster complementarity. This complementary cluster information is then transferred to supervised information with high confidence, aiming to have consistent cluster assignments for all views, i.e., solving the challenge (ii). In addition, we present an EM-like optimization strategy, including P-step and Z-step, to alternately promote feature learning and clustering.

Different from existing traditional and deep IMVC methods, our contributions can be summarized as follows:

- We propose DIMVC, a novel deep IMVC method with an imputation-free and fusion-free framework, which can avoid the noise caused by inaccurate imputation and alleviate the disturbance from the views with low quality.
- We propose to mine the complementary information in the high-dimensional feature space via a nonlinear mapping of multiple views. Moreover, we show the mechanisms to achieve the multi-view cluster complementarity and the multi-view clustering consistency.
- We design an alternate (EM-like) optimization strategy to effectively optimize the proposed deep IMVC framework. Extensive experiments demonstrate that our method achieves superior clustering performance, compared to state-of-the-art IMVC methods.

**Method**

**Notations.** Given an incomplete multi-view dataset of \( N \) samples \( \{X^v \in \mathbb{R}^{N_v \times D_v}\}_{v=1}^V \), \( V \) is the number of views. For the \( v \)-th view, \( D_v \) is the dimensionality of samples and \( N_v \) represents the number of samples, where \( N_v \leq N \) due to missing data. \( K \) is the number of categories to be clustered. We denote the samples with complete data as a set \( \mathcal{X} \).

**Deep Model of Feature Learning and Clustering**

Firstly, we introduce the feature learning and clustering model of each view, i.e., Model #1, #2, \ldots, #V in Figure 1.

Deep autoencoder can capture salient features of the data and has been applied in many unsupervised fields (Feng, Wang, and Li 2014; Song et al. 2018; Xu et al. 2019b; Zhang...
reconstruction loss between
are the learnable parameters of autoencoder network. The
assignments
we utilize a parameterized mapping to obtain soft cluster
d
th view, the embedding features denoted as 

\( X^v \)

et al. 2020; Cao et al. 2020; Lin et al. 2021). Therefore, we
employ autoencoders to convert heterogeneous multi-view
data into clustering-friendly embedding features. For the
v-th view, the embedding features denoted as 

\( Z^v \)

is the dimensionality of embedding features, \( \theta^v \) and \( \phi^v \)
are the learnable parameters of autoencoder network. The
reconstruction loss between 

\( X^v \)

and 

\( \hat{X}^v \)

of all views is

\[
L_{\text{rec}} = \sum_{v=1}^{V} L_{\text{rec}}^v = \sum_{v=1}^{V} \left\| X^v - f_{o}^{-1}(Z^v) \right\|_F^2
\]

In order to obtain clustering predictions, for each view,
we utilize a parameterized mapping to obtain soft cluster
assignments \( Q^v \), i.e., \( M_v(Z^v; U^v) : Z^v \in \mathbb{R}^{N_v \times d_v} \mapsto Q^v \in \mathbb{R}^{N_v \times K} \), where \( U^v = [u^v_1; u^v_2; \ldots; u^v_K] \in \mathbb{R}^{K \times d_v} \) represent the learnable parameters. Concretely,

\[
g_{ij}^v = \frac{(1 + \| z^v_i - u^v_j \|_2^2)^{-1}}{\sum_{j'=1}^{K} (1 + \| z^v_i - u^v_{j'} \|_2^2)^{-1}} \in Q^v, \tag{2}
\]

which is a commonly used manner to perform end-to-end
clustering (Xie, Girshick, and Farhadi 2016; Guo et al. 2017;
Xu et al. 2021a). In the v-th view, \( u^v_j \) is the j-th view centroid and \( g_{ij}^v \) is considered as the probability that the embedding feature \( z^v_i \) is assigned to the j-th cluster.

There is no connection among the different views so far,
and the complete and incomplete data of each view can be
learned without imputation for the missing data. Subse-
sequently, we present our strategy to explore complementary
information among all views for multi-view clustering.

Multi-View Cluster Complementarity

Since multiple views share common semantic information,
every view can be regarded as the mappings of the other
views, e.g., \( Z^2 = \mathcal{F}_2(Z^1) \) is a mapping of \( Z^1 \). As shown in
Figure 2(a), if one view is a linear mapping of the other
views, there is no complementary information among them.
If there exists a complementary relationship among multiple
views, e.g., the inseparable clusters in one view are separable
in other views, this complementarity can be described by
nonlinear mappings as shown in Figure 2(b).

Based on the above observations, we treat the clustering
problem as a classification problem by considering the cluster
assignments as the pseudo labels of samples.

**Assumption 1.** Cover’s theorem (Cover 1965): a complex
classification problem is more likely to be linearly separable
when it is nonlinearly projected to high-dimensional spaces.

Considering Assumption 1, we propose to map multi-view
embeddings into a high-dimensional space by a nonlinear
mapping \( \mathcal{H} \). Many functions can lead to such \( \mathcal{H} \). In this
paper, we provide a simple manner, i.e., mapping the embed-
dings into a concatenated weighted feature space (CWFS):

\[
H = \mathcal{H}(\{Z^v\})_{v=1}^{V} = (w_1Z^1, w_2Z^2, \ldots, w_VZ^V), \tag{3}
\]

where \( H \in \mathbb{R}^{|X| \times \sum_{v=1}^{V} d_v} \) denotes the obtained high-
dimensional features. \( w_v \) is the weight calculated by

\[
w_v = 1 + \log \left( \frac{1 + \sigma(U^v)}{\sum_{v=1}^{V} \sigma(U^v)} \right), \tag{4}
\]

where \( \sigma(U^v) \) is the variance on cluster centroids \( U^v \) of
the v-th view. Intuitively, the high-separability view means
that the features have well-separated clusters, whose cen-
troids have large variance. Therefore, in CWFS, the weights
\( \{w_v\}_{v=1}^{V} \) are proposed to increase the influence of the high-
separability view as well as to reduce the influence of other
views with unclear cluster structures. In this way, the mapping
\( \mathcal{H} \) can push the cluster assignment of a sample in
CWFS in agreement with that in the high-separability view.

Additionally, Theorem 1 indicates that the linearly separ-
able probability of \( H \) is improved compared with the embed-
ding features \( \{Z^v\}_{v=1}^{V} \) of any single view.

**Theorem 1.** \( H \) is a high-dimensional nonlinear mapping of
\( \{Z^v\}_{v=1}^{V} \). The high-dimensional features \( H \) are more likely to be linearly separable than any \( Z^v \) for \( v \in \{1, 2, \ldots, V\} \).

**Proof.** Considering the embedding feature \( z^v_i \) of any i-th sample in any v-th view, we assume that there are unknown mappings that make \( z^v_i = \mathcal{F}_v(z^v_i) \in \mathbb{R}^{d_v} \) for \( t \in \{1, 2, \ldots, V\} \). If some of the mappings \( \{\mathcal{F}_v : Z^v \mapsto Z^v\}_{v \neq v} \) are nonlinear, the concatenation of all embedding features,

\[
(\{z^1_1, z^2_1, \ldots, z^V_1\}, \mathcal{F}_1(z^1_1), \mathcal{F}_2(z^2_1), \ldots, \mathcal{F}_V(z^V_1)) \in \mathbb{R}^{\sum_{v=1}^{V} d_v}
\]

where \( \sum_{v=1}^{V} d_v > d_v \), is a high-dimensional
nonlinear mapping of \( z^v_1 \). Additionally, \( w_v \) is a nonlinear weight-
ing function, and thus the proposed \( \mathcal{H} : \{Z^v\}_{v=1}^{V} \mapsto H \) is also a high-dimensional nonlinear mapping for the embed-
ding features of each view. Therefore, given Assumption 1, Theorem 1 holds.

Based on Theorem 1, the proposed mapping \( \mathcal{H} \) ensures
that the obtained \( H \) contains more separable cluster patterns
than that of single-view embedding features, which is achieved by utilizing the information of the clusters in the
high-separability views to avoid the incorrect information
from the views with unclear cluster structures (verified in
Table 3). This is called multi-view cluster complementarity
in this paper. Consequently, we can obtain the new cluster

\[Z^2 = \mathcal{F}_2(Z^1)\]

(a) \( \mathcal{F}_2 \) is linear mapping

\[Z^2 = \mathcal{F}_2(Z^1)\]

(b) \( \mathcal{F}_2 \) is nonlinear mapping

Figure 2: The illustration of the mapping between two views.
centroids $C \in \mathbb{R}^{K \times \sum_{v=1}^{V} d_v}$ in CWFS by the following objective $L_{com}$ to explore complementary cluster information:

$$
L_{com} = \min_{C} \frac{1}{v} \sum_{v=1}^{V} \sum_{j=1}^{K} \sum_{i=1}^{I} \| w_i z_i^v - c_{ij}^v \|^2_2,
$$

where $h_i \in H$ and $c_{ij} = (c_{ij}^1, c_{ij}^2, \ldots, c_{ij}^V) \in \mathbb{R}^{\sum_{v=1}^{V} d_v}$. The multi-view cluster complementarity is learned from the complete data $X$. We further discuss the details to obtain consistent cluster assignments for all data, i.e., achieving clustering consistency for incomplete multi-view data.

### Multi-View Clustering Consistency

In CWFS, the linearly separable cluster information of all views is transferred to the high-dimensional features $H$. The centroids $C$ calculated on $H$ reflect more accurate cluster structures due to the multi-view cluster complementarity. Motivated by (Xu et al. 2021c), we can generate supervised information for all views by a mapping $M(H; C, A) : H \in \mathbb{R}^{K \times \sum_{v=1}^{V} d_v} \rightarrow P \in \mathbb{R}^{K \times K}$, which is formulated by

$$
P = M(H; C, A) = \mathcal{E}(S(H, C))A,
$$

where the function $S$ is leveraged to measure the confidence $s_{ij}$ of the $i$-th sample being assigned to the $j$-th centroid:

$$
s_{ij} = S(h_i, c_j) = \frac{1}{1 + \| h_i - c_j \|^2_2} \in \mathbb{S}.
$$

In this way, the confidence $s_{ij}$ is high when $h_i$ is closer to $c_j$. The function $\mathcal{E}(S)$ scales the confidence of each sample to $[0, 1]$, and meanwhile, enhances the confidence (such as $s_{ij}$) when it is the largest in $\{s_{i1}, s_{i2}, \ldots, s_{iK}\}$. Specifically,

$$
s_{ij} = \mathcal{E}(s_i) = \frac{(s_{ij}/\sum_j s_{ij})^2}{(\sum_j (s_{ij}/\sum_j s_{ij})^2)},
$$

by which the mined complementary cluster information is transformed to the supervised information with high confidence. $A$ satisfies $AA^T = I_K$, which is a boolean matrix to adjust the arrangement of $S$. Furthermore, the cross-entropy loss between $P$ and $Q^v$ of all views is optimized:

$$
L_{com} = \sum_{v=1}^{V} H(P, Q^v) = -\sum_{v=1}^{V} \sum_{i \in X} p_i \log q_i^v.
$$

As the same $P$ is shared by all views, the optimization of $L_{com}$ can achieve the consistency of multi-view clustering, i.e., learning the consistent $\{Q^v\}_{v=1}^{V}$. Moreover, the consistency from the complete data can be generalized to the incomplete data through deep models (verified in Figure 3).

After training the deep models, the obtained models of all views are fusion-free, because each view has its own model to perform the feature extraction and clustering, significantly, which does not depend on the feature fusion upon other views. Then, we could obtain robust results by averaging the cluster assignments of the observed data. Concretely, the clustering prediction of the $i$-th sample is inferred by

$$
y_i = \arg \max_j \sum_w q^v_{ij}.
$$

### Optimization

In conclusion, the loss function of our proposed framework consists of three parts:

$$
L = L_{rec} + L_{com} + L_{con}
$$

$$
= \min_{C, A, \{Q^v\}_{v=1}^{V}} \sum_{v=1}^{V} \| X^v - f_{v}^{-1}(Z^v) \|^2_F + \sum_{v=1}^{V} L(P, Q^v),
$$

$$
s.t. \ P = M(H; C, A), \ A A^T = I_K, \ Q^v = M_v(Z^v; U^v),
$$

where $Z^v = f_{v}(X^v)$, $H = \mathcal{H}(\{Z^v\}_{v=1}^{V})$, $L_{rec}$ is the reconstruction loss of autoencoders, $L_{com}$ and $L_{con}$ achieve the multi-view complementarity and consistency, respectively.

To optimize the above non-differentiable objective function, we present an alternate optimization strategy which is similar to Expectation Maximization algorithm, as follows:

**Initialization**: Firstly, the deep autoencoders are initialized by Eq. (1) to obtain meaningful embedding features. As thus, the cluster centroids $\{U^0, U^2, \ldots, U^V\}$ can be initialized by $K$-means (MacQueen 1967). $A$ is initialized as $I_K$.

**P-step**: Update $\{P, C, A\}$ with fixed $\{Z^v, U^v\}_{v=1}^{V}$.

In the first place, $C$ is obtained by optimizing Eq. (5), which can be efficiently calculated with $K$-means.

Letting $i^{(t)}_i = \arg \max_j s_{ij}^{(t)}$ denote the cluster label of $h_i$ in the $t$-th iteration, in unsupervised context, the clusters represented by $i^{(t+1)}_i$ and $i^{(t)}_i$ might be not consistent. Letting $\tilde{m}_{ij} = \sum_{n \in X} I(s_{ni}^{(t+1)} = i) I(l_n^{(t)} = j)$, we define a cost matrix $M \in \mathbb{R}^{K \times K}$, where $m_{ij} = \max_{i'} s_{ij'} - \tilde{m}_{ij'}$, and solve a maximum matching problem as follows:

$$
\min_{A} \sum_{i=1}^{K} \sum_{j=1}^{K} m_{ij} a_{ij}
$$

$$
s.t. \ A A^T = I_K,
$$

where $a_{ij} \in A$ and $A \in \{0, 1\}^{K \times K}$ is a boolean matrix. Eq. (12) is optimized with Hungarian algorithm (Jonker and Volgenant 1986) to obtain the maximum match (denoted by $A$) between the clustering results of two iterations.

Subsequently, $P$ can be computed directly by the mapping $M(H; C, A)$ with the inputs of $H$, $C$, and $A$.

**Z-step**: Update $\{Z^v, U^v\}_{v=1}^{V}$ with fixed $\{P, C, A\}$. Given fixed $C$ and $A$, $P$ is treated as constant pseudo labels for all views. Then, Eq. (11) can be divided into $\{L^1, L^2, \ldots, L^V\}$, where $L^v = L^v_{rec} + L^v_{con} = \| X^v - f_{v}^{-1}(Z^v) \|^2_F + H(P, Q^v)$. In this way, the model of each view can be learned independently. Letting $\lambda$ denote the learning rate and $n$ be the batch size, we train the deep model via the mini-batch gradient descent algorithm:

$$
U^v = U^v - \frac{\lambda}{n} \sum_{i=1}^{n} \frac{\partial L^v}{\partial U^v},
$$

$$
Z^v = Z^v - \frac{\lambda}{n} \sum_{i=1}^{n} \frac{\partial L^v}{\partial Z^v},
$$

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where \( Z^v \) is optimized by updating the neural network parameters (i.e., \( \theta^v \) and \( \phi^v \)) of the autoencoder.

Furthermore, the alternate (EM-like) strategy can make the feature learning and the clustering promote each other (verified in Table 3). Concretely, the P-step produces more precise supervised information by mining cluster complementarity from the embedding features of all views. The Z-step makes the model of each view learn better clustered embedding features with the supervised information.

**Complexity analysis.** Letting \( V, K, N \) denote the number of views, clusters, and samples, respectively, \( D \) represent the maximum number of neurons in deep autoencoders’ hidden layers, and \( M = \sum_{v=1}^{V} d_v \) denote the dimensionality of high-dimensional features, \( N \gg V, K, M \) generally holds. In Algorithm 1, the complexity to optimize Eq. (5), Eq. (12), and Eq. (6) in the P-step is \( O(NMK), O(K^3 + NK), \) and \( O(NK) \), respectively, while the complexity to optimize Eq. (13) in the Z-step is \( O(NVD^2) \). In conclusion, the total complexity of our algorithm is \( O(K^3 + NMK + NVD^2) \) in each iteration, which is linear to the data size \( N \).

**Experiments**

We evaluate the effectiveness of our proposed DIMVC by comparing it with seven state-of-the-art IMVC methods on real-world multi-view datasets, in terms of three clustering metrics, including clustering accuracy (ACC), normalized mutual information (NMI), and adjusted rand index (ARI).

**Settings**

**Comparison methods.** The comparison methods include four traditional methods (i.e., SRLC (Zhuge et al. 2019), APMC (Guo and Ye 2019), TMBSD (Li et al. 2021), and IMVTSC-MVI (Wen et al. 2021)) and three deep methods (i.e., DiMVMC (Wei et al. 2020), CDIMC-net (Wen et al. 2020), and COMPLETER (Lin et al. 2021)).

**Datasets.** We use four datasets in our experiments, i.e., BDGP (Cai et al. 2012), Caltech (Fei-Fei, Fergus, and Perona 2004), RGB-D (Kong et al. 2014), and Scene (Fei-Fei and Perona 2005). Table 1 presents the description of the used datasets. We construct incomplete multi-view datasets with varying missing rates (0.1, 0.3, 0.5, 0.7). When the missing rate is 0.5, for example, we randomly select 50% samples and randomly drop partial views of these samples.

**Implementation details.** For our DIMVC, the following settings are adopted for all datasets. Concretely, the autoencoders of all views are implemented by fully connected neural networks with the same structure. For the \( v \)-th view, the network structure can be denoted as \( X^v \rightarrow FC_500 \rightarrow FC_500 \rightarrow FC_2000 \rightarrow Z^v \rightarrow FC_500 \rightarrow FC_500 \rightarrow FC_500 \rightarrow X^v \), where \( FC_500 \) represents the fully connected neural network with 500 neurons. The dimensionality of embeddings \( Z^v \) is reduced to 10. The activation function is ReLU (Glorot, Bordes, and Bengio 2011). We adopt Adam (Kingma and Ba 2014) to optimize the deep models with a learning rate of 0.001. In the initialization phase, the autoencoders are pre-trained for 500 epochs. The batch size is set to 256. In every iteration of the proposed alternate (EM-like) optimization strategy, the Z-step will train the deep models for 1000 batches after the P-step updates the learning targets. The number of iterations is set to 10. The code is provided in the website\(^1\).

**Experimental Results and Analysis**

The clustering performance of all methods on four datasets is listed in Table 2, from which we have the following observations: (1) Our DIMVC obtains the best performance on all datasets. Compared with the second-best methods, DIMVC has considerable improvements especially on BDGP, Caltech, and Scene. (2) It is obvious that the clustering performance of all methods is reduced when the missing rate varies from 0.1 to 0.7. Nevertheless, our DIMVC still achieves superior clustering performance in most cases.

The reasons for the above observations can be explained as follows: (1) If the missing rate of multi-view data becomes high, the complementary information among multiple views becomes rare. This results in the reduction of clustering quality of all methods. (2) Imputation methods depend on the estimation of data distribution. Hence, the cumulative error increases when the missing rate is high, e.g., the performance of certain methods (such as TMBSD and COMPLETER) is unsatisfied when the missing rate is 0.5 or 0.7. (3) Fusion methods might be influenced by the views with low quality, especially for incomplete multi-view learning, e.g., even on BDGP with low missing rates, the performance of certain methods (like DiMVMC and CDIMC-net) is poor.

Different from existing traditional and deep IMVC methods, our DIMVC is imputation-free and fusion-free. It explores the cluster complementarity by the proposed high-dimensional mapping, and obtains promising results through the EM-like optimization strategy. In addition to clustering

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\(^1\)https://github.com/SumissionsIn/DIMVC.
### Table 2: Clustering results of all methods on four datasets. The best and second-best results are highlighted with bold and underline, respectively. The symbol ‘-’ denotes unknown results as COMPLETER mainly focuses on two-view clustering.

| Missing rates | 0.1 | 0.3 | 0.5 | 0.7 |
|---------------|-----|-----|-----|-----|
| **Evaluation metrics** | ACC | NMI | ARI | ACC | NMI | ARI | ACC | NMI | ARI |
| **BDGP** | | | | | | | | | |
| SRLC (Zhuge et al. 2019) | 0.691 | 0.514 | 0.470 | 0.697 | 0.458 | 0.430 | 0.626 | 0.366 | 0.320 |
| APMC (Guo and Ye 2019) | 0.859 | 0.662 | 0.583 | 0.814 | 0.589 | 0.594 | 0.770 | 0.672 | 0.626 |
| TMBSD (Li et al. 2021) | 0.739 | 0.620 | 0.582 | 0.714 | 0.597 | 0.546 | 0.605 | 0.273 | 0.275 |
| IMVTSC-MVI (Wen et al. 2021) | 0.962 | 0.889 | 0.908 | 0.934 | 0.816 | 0.844 | 0.923 | 0.813 | 0.836 |
| DiMVMC (Wei et al. 2020) | 0.730 | 0.673 | 0.560 | 0.730 | 0.677 | 0.565 | 0.595 | 0.511 | 0.282 |
| CDIMC-net (Wen et al. 2020) | 0.875 | 0.755 | 0.640 | 0.757 | 0.692 | 0.467 | 0.657 | 0.530 | 0.271 |
| COMPLETER (Lin et al. 2021) | 0.596 | 0.528 | 0.254 | 0.552 | 0.511 | 0.255 | 0.541 | 0.504 | 0.228 |
| DIMVC (ours) | 0.964 | 0.892 | 0.912 | 0.954 | 0.866 | 0.889 | 0.947 | 0.845 | 0.873 |
| **Caltech** | | | | | | | | | |
| SRLC (Zhuge et al. 2019) | 0.478 | 0.588 | 0.349 | 0.453 | 0.566 | 0.311 | 0.433 | 0.570 | 0.304 |
| APMC (Guo and Ye 2019) | 0.523 | 0.625 | 0.421 | 0.437 | 0.600 | 0.302 | 0.429 | 0.597 | 0.295 |
| TMBSD (Li et al. 2021) | 0.404 | 0.608 | 0.274 | 0.415 | 0.615 | 0.281 | 0.407 | 0.620 | 0.286 |
| IMVTSC-MVI (Wen et al. 2021) | 0.625 | 0.642 | 0.507 | 0.590 | 0.668 | 0.445 | 0.549 | 0.520 | 0.395 |
| DiMVMC (Wei et al. 2020) | 0.373 | 0.551 | 0.315 | 0.358 | 0.521 | 0.301 | 0.322 | 0.454 | 0.285 |
| CDIMC-net (Wen et al. 2020) | 0.452 | 0.589 | 0.325 | 0.443 | 0.439 | 0.265 | 0.362 | 0.400 | 0.184 |
| COMPLETER (Lin et al. 2021) | 0.742 | 0.712 | 0.843 | 0.741 | 0.690 | 0.835 | 0.716 | 0.681 | 0.814 |
| DIMVC (ours) | 0.772 | 0.726 | 0.870 | 0.761 | 0.697 | 0.842 | 0.758 | 0.685 | 0.835 |
| **RGB-D** | | | | | | | | | |
| SRLC (Zhuge et al. 2019) | 0.383 | 0.237 | 0.152 | 0.352 | 0.214 | 0.126 | 0.322 | 0.184 | 0.104 |
| APMC (Guo and Ye 2019) | 0.412 | 0.319 | 0.216 | 0.373 | 0.271 | 0.166 | 0.345 | 0.254 | 0.149 |
| TMBSD (Li et al. 2021) | 0.377 | 0.323 | 0.263 | 0.317 | 0.210 | 0.132 | 0.299 | 0.182 | 0.107 |
| IMVTSC-MVI (Wen et al. 2021) | 0.422 | 0.322 | 0.229 | 0.401 | 0.311 | 0.193 | 0.362 | 0.267 | 0.151 |
| DiMVMC (Wei et al. 2020) | 0.371 | 0.323 | 0.299 | 0.355 | 0.269 | 0.178 | 0.251 | 0.222 | 0.116 |
| CDIMC-net (Wen et al. 2020) | 0.358 | 0.334 | 0.249 | 0.393 | 0.368 | 0.161 | 0.302 | 0.270 | 0.149 |
| COMPLETER (Lin et al. 2021) | 0.418 | 0.265 | 0.237 | 0.398 | 0.236 | 0.213 | 0.370 | 0.236 | 0.127 |
| DIMVC (ours) | 0.436 | 0.353 | 0.258 | 0.405 | 0.316 | 0.215 | 0.391 | 0.288 | 0.193 |
| **Scene** | | | | | | | | | |
| SRLC (Zhuge et al. 2019) | 0.366 | 0.351 | 0.189 | 0.332 | 0.307 | 0.163 | 0.333 | 0.292 | 0.148 |
| APMC (Guo and Ye 2019) | 0.433 | 0.434 | 0.269 | 0.414 | 0.401 | 0.248 | 0.408 | 0.383 | 0.236 |
| TMBSD (Li et al. 2021) | 0.437 | 0.398 | 0.271 | 0.364 | 0.325 | 0.185 | 0.344 | 0.293 | 0.166 |
| IMVTSC-MVI (Wen et al. 2021) | 0.330 | 0.302 | 0.161 | 0.277 | 0.245 | 0.117 | 0.265 | 0.211 | 0.101 |
| DiMVMC (Wei et al. 2020) | 0.315 | 0.291 | 0.156 | 0.241 | 0.206 | 0.083 | 0.183 | 0.136 | 0.045 |
| CDIMC-net (Wen et al. 2020) | 0.346 | 0.374 | 0.143 | 0.246 | 0.219 | 0.112 | 0.309 | 0.288 | 0.136 |
| COMPLETER (Lin et al. 2021) | - | - | - | - | - | - | - | - | - |
| DIMVC (ours) | 0.474 | 0.465 | 0.306 | 0.440 | 0.403 | 0.254 | 0.428 | 0.394 | 0.252 |

Figure 3: Visualization of the embedding features and centroids on BDGP (textual view) with 4 missing rates via t-SNE (Maaten and Hinton 2008). Dots denote the samples with complete data and digits represent the samples with incomplete data.

performance, the learned embedding features and centroids are visualized in Figure 3. We find that the incomplete data points have the similar cluster structures in accord with that of the complete data points. This indicates that our method achieves the consistency for the complete data and incomplete data, and has good feature generalization capability.

### Ablation Study

We conduct the ablation study to demonstrate the importance of each component of our method. As shown in Table 3, View-ν denotes the K-means clustering performance on the ν-th view’s embedding features obtained by the autoencoder (AE). Item-1 obtains better performance than any single view, which experimentally validates our Theorem 1, i.e., the multi-view complementary information mined by the mapping 𝒇 improves clustering performance. Item-2 does not apply the proposed EM-like strategy to optimize the framework. The results indicate that Item-2 cannot effectively leverage the mined complementary information. The improvement of Item-3 is limited as it does not apply the function 𝔉(𝒮) to enhance the confidence of samples. The
In this paper, we presented an imputation-free and fusion-free deep incomplete multi-view clustering framework. Firstly, we developed a novel strategy to mine the complementary information among multiple views, i.e., mapping embedding features of all views into the concatenated weighted feature space (CWFS). As a result, the new features in CWFS are more likely to be linearly separable due to the multi-view cluster complementarity. Furthermore, the feature learning and clustering were conducted with an alternate (EM-like) optimization strategy, where the complementary information contained in embedding features of autoencoders as well as achieving the consistency of multi-view clustering, respectively. In conclusion, the hyper-parameter is insensitive in the range of $[10^{-1}, 10^1]$ and we let $\alpha = 1.0$ for all datasets.

**Conclusion**

In this paper, we presented an imputation-free and fusion-free deep incomplete multi-view clustering framework. Firstly, we developed a novel strategy to mine the complementary information among multiple views, i.e., mapping embedding features of all views into the concatenated weighted feature space (CWFS). As a result, the new features in CWFS are more likely to be linearly separable due to the multi-view cluster complementarity. Furthermore, the feature learning and clustering were conducted with an alternate (EM-like) optimization strategy, where the complementary information was transformed to the supervised information to achieve the consistency of multiple views. In conclusion, our method can effectively explore the multi-view complementary information from the complete data, and has robust generalization capability to handle the incomplete data. Extensive experiments demonstrated that our method can achieve the state-of-the-art clustering performance.
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