Evolution of black hole shadow in the presence of ultralight bosons

Rittick Roy† and Urjit A. Yajnik†

Department of Physics, Indian Institute of Technology Bombay, Mumbai 400076, India

(Dated: June 10, 2019)

The possible occurrence of ultralight boson clouds around Kerr black holes has attracted a lot of interest. In this work, we determine the signatures of boson cloud evolution in the shadow of a black hole. We assess the detectability of such variations in current and future imaging techniques of black hole shadow observations. We look at several black hole candidates, both intragalactic and extragalactic, and suggest SgrA∗ as an optimal candidate for observation of such signatures. Thus black hole shadow observations could be instrumental in searching for ultralight bosons or axion like particles.

Introduction.—Black holes have remained a laboratory of theoretical curiosities for the 100 years after their prediction. Their formation, stability and evolution has been extensively studied and observational evidence in their favor has been mounting. The acceleration of extra-galactic cosmic rays [2–4], production of highly relativistic jets in Blazars [2] and the observed characteristics of Gamma-Ray Bursts (GRBs) [6] all show signatures of an inner core powered by a black hole. The detection of gravitational waves by the LIGO collaboration from the merging of black hole binaries [7] in 2015 and subsequent events provided even stronger observational evidence in favor of black holes. A more recent observational signature came from the imaging of the shadow of a supermassive black hole by the Event Horizon Telescope (EHT) collaboration [8] which used a Very Long Baseline Interferometry (VLBI) array to observe the black hole at the centre of M87 (M87∗ from here onwards).

For particles in the black hole environment, there exists a least radius below which stable circular orbits are not sustainable resulting in collapse of the particles into the interior. This minimum radius is the called the Innermost Stable Circular Orbit (ISCO) and for a Schwarzschild black hole lies at \( r = 3r_g \) for photons, where \( r_g = GM/c^2 \) is the gravitational radius of the black hole. It can be shown [9], that an incoming photon with an impact parameter \( b < r_c = \sqrt{2}r_g \) would end up inside the ISCO and thus eventually inside the black hole, but the photons with \( b > r_c \) will escape to infinity and create an ‘image' of the black hole that features a bright ring with a dark interior. The dark interior is evidence of the existence of an ISCO and this image is commonly referred to as the ‘Shadow of a Black Hole'.

The black hole shadow is subject to evolution due to the interaction of the black hole with its environment. The strong gravitational field of a Kerr black hole has been studied for its superradiant scattering effects. Specifically, when the frequency \( \omega \) of an incoming wave satisfies [10]

\[
0 < \omega < m\omega_+ \tag{1}
\]

where \( \omega_+ \) is the angular velocity of the black hole horizon and \( m \) is the azimuthal separation constant, there is superradiant scattering of the wave from the black hole. These cases correspond to outgoing fluxes being larger than ingoing fluxes, at the cost of energy and angular momentum to the black hole. Hawking radiation on the other hand is analogous to Schwinger effect in quantum field theory, where quantum fluctuations can give rise to virtual particle pairs, of which one falls into the black hole with the other emerging as a net outgoing flux at infinity. Kerr geometry in presence of ultralight bosons gives rise to an intermediate effect, which has received attention in recent literature. In this, particles created by quantum fluctuations, and satisfying the superradiance condition do not emerge at infinity, but get bound in quasi-stationary states around the black hole. A precursor of this idea is the “black hole bomb” [11] wherein photons, whether introduced externally or created spontaneously, destabilize the black hole when trapped in the black hole environment due to the presence of externally provided mirrors. As observed in [12] the occurrence of Hydrogen atom like bound states in Kerr geometry for ultralight bosons substitutes for the mirror like condition. Thus such an effect is best understood as quasi-Hawking effect.

Axions are a class of ultralight bosons proposed to resolve the \( CP \) problem of strong interactions [13], but occurring more generically in string theory, with possible values of rest mass ranging from \( 10^{-9} \) – \( 10^{-21} \) eV [12]. The bound states of these axions to black holes can result in the quasi-Hawking effect leading to a reduction of the spin parameter \( a_\ast \) of the black hole which in turn could lead to substantial variation in the observed black hole shadow. In the current work we develop this idea to determine whether variation in the features of the black hole shadow is significant enough for detection and if so, whether it takes place over observational time scales. Based on this analysis, we identify the optimal candidate black hole for observation of this phenomenon. Throughout the rest of this work we will use the \( G = c = \hbar = 1 \) system of units unless mentioned otherwise.

Boson cloud formation.—Consider a Kerr black hole with mass \( M \), angular momentum \( J \) and scalar bosons of mass \( m_s \). It is convenient to use the rescaled mass parameter \( \mu_s = m_sGc/\hbar \), which in the system of units
defined above expresses the mass of the scalar field. It has been shown [14] that in the limit $\mu_a M \ll 1$ and $\omega M \ll 1$, the Klein-Gordon equation satisfied by the bosons in the Kerr geometry takes a relatively simple form of the Schrödinger equation for Hydrogen atom with the fine structure constant replaced by the gravitational fine structure constant $\alpha = \mu_a M$. The energy eigenvalues in this case develop imaginary parts [15], whereas the real part could be expressed in terms of the quantum numbers $n$ of the usual quantum numbers $nlm$ by

$$\omega = \mu_a \left(1 - \frac{\alpha^2}{2n^2}\right) \tag{2}$$

On the other hand, the small imaginary part $\Gamma_{sr}$, gives rise to either damped or growing modes. In single particle quantum mechanics these modes may be interpreted as trapped particles whose bound states get stabilized, or whose amplitude grows, suggesting instability of the black hole [16]. However if we have ultra-light bosons, whose Compton wavelength is comparable to the black hole gravitational radius, the solutions may be interpreted to lead to spontaneous creation of particles and accumulation of the same in quasi-stationary modes around the black hole [15]. We refer to this as the quasi-Hawking effect.

During this process, bosons extract mass and angular momentum from the black hole and begin to populate the quasi-stationary states. This phenomenon leads to the formation of large boson clouds in the black hole environment. This black hole-boson cloud system is often referred to as the “Gravitational Atom”. The population of the cloud increases exponentially with time and the growth stops when enough spin has been extracted from the black hole such that the superradiance condition Eq. (1) is no longer satisfied. The occupation number $N$ of a quantum level, denoted by $nlm$, that satisfies the superradiance condition will grow exponentially with time at a rate $\Gamma_{sr}$ [17]

$$\frac{dN}{dt} \bigg|_{sr} = \Gamma_{sr} N \tag{3}$$

After the process is complete, the maximum occupancy of a level with azimuthal quantum number $m$ could be expressed as [12]

$$N_{max} \approx 10^{76} \times \left(\frac{\Delta a_s}{0.1}\right)^2 \left(\frac{M}{10M_\odot}\right)^2 \tag{4}$$

where $\Delta a_s$ is the change in black hole spin. The entire process occurs over characteristic time scale $\tau_{sr}$ which depends on the mass of the black hole $M$, mass of the scalar field $\mu_a$ and the spin parameter $a_s$.

**Superradiance time scale.**—We define the parameter $\eta = \sqrt{l(l+1)}$ which denotes the angular momentum per boson in the boson-cloud. Within the approximation $\alpha/l \ll 1$, it can be shown that for black hole masses $M_\odot - 10^{10} M_\odot$, the permitted range of values for the scalar field mass is $10^{-9}$ eV $\geq \mu_a \geq 10^{-19}$ eV. The spin parameter of the black hole, at any point in time, could then be expressed in terms of $j$, $\mu_a$ and the occupation number $N$ as

$$a_s(N) = \frac{J - Nj}{(M - N\mu_a)^2} \tag{5}$$

where $J$ and $M$ are the initial angular momentum and mass of the black hole. Substituting this back in Eq. (5) we get the superradiant time scale

$$\tau_{sr} = \int_{a_i}^{a_{\min}} \frac{1}{\Gamma_{sr}(a_s, \alpha)N} \, da_s \tag{6}$$

where $a_{\min}$ denotes the lowest spin of black hole before violating the superradiance condition, $a_i$ is the initial black hole spin and $\frac{dN}{da_s}$ and $N(a_i)$ can be found by inverting Eq. (5). In the $\alpha/l \ll 1$ limit, the superradiance rate $\Gamma_{sr}$ is well approximated by [17]

$$\Gamma_{sr} \approx \Gamma_{nlm} = 2\mu_a \alpha l^4 + 4r_+(m\omega_+ - \mu_a)C_{nlm} \tag{7}$$

where

$$C_{nlm} = \frac{2^4 l^4 (l + n)!}{n^{2l+4}(n - l - 1)!} \left(\frac{ll}{(2l)!}(2l+1)!\right)^2 \times \prod_{i=1}^{l} \left(l^2 - 1 + a^2\right) + 4r_+^2(m\omega_+ - \mu_a)^2 \tag{8}$$

where $r_+ = M + \sqrt{M^2 - a^2}$ represents the event horizon of the black hole. The sign of $\Gamma_{sr}$ is determined by the sign of the term $(m\omega_+ - \mu_a)$, in agreement with Eq. (4) in the low $\alpha/l$ limit. As shown in [12], the function $\Gamma_{sr}$ drops exponentially with an increase in the quantum number $l$. Hence, the lowest level which satisfies the superradiance condition would be the fastest growing level, which in our case turns out to be the $2p$ level. We will be concerned with this level from here onwards, unless mentioned otherwise, because the fastest growing level would be the most efficient in extracting energy and angular momentum from the black hole. The minimum spin parameter could be obtained from the superradiance condition Eq. (4)

$$a_{\min} = \frac{2k_{nlm}}{1 + k^2_{nlm}} \tag{9}$$

where $k_{nlm}(\alpha) = \left(2\alpha/m\right)(1 - \alpha^2/2n^2)$. If we fix the initial spin of the black hole $a_i$, we can put an upper bound on the $\alpha$ parameter by arguing that the minimum spin should be less than the initial spin of the black hole i.e. $a_{\min}(\alpha) < a_i$. For an initial spin of $a_i = 0.95$ and the condition $\alpha \ll 1$, we get the upper bound on $\alpha$ to be
The recent imaging of the shadow of the black hole at the centre of M87 shows a bright ring (\(~42 \pm 3\) \textmu as) and a dark region at the centre with a contrast depression of about \(10:1\) with the ring \[8\]. This is evidence of the presence of an extremely compact object that lenses background light around it to create a shadow. The angular diameter of the ring is determined by the formula

\[
d = 42 \times \left( \frac{\delta x}{11} \right) \left( \frac{M}{6.5 \times 10^9 M_\odot} \right) \left( \frac{16.8 \text{ Mpc}}{D} \right) \text{\mu as} \tag{13}
\]

where \(D\) is the distance from Earth to the black hole. It is important to note here that the inclination for M87* is \(\theta_0 \sim 17^\circ\), while in our simulation of Table I we have considered \(\theta_0 = 90^\circ\) as the best case scenario. For a change in the spin parameter \(\Delta a_* = 0.1\) (from 0.95 to 0.85 in Table I) the angular diameter of the ring of SgrA*, with a mass of \(\sim 6.2 \times 10^9 M_\odot\) and a distance of \(\sim 16.8 \text{ Mpc}\) \[8\], changes by \(\Delta d \sim 1.14\) \textmu as. For observations at a wavelength of 1.3 mm, the EHT collaboration has a theoretical diffraction-limit resolution of about \(\sim 25\) \textmu as \[8\]. Thus, the variation in the shadow features falls far below the resolution of EHT. Further, for a supermassive black holes like M87*, this variation takes place over a time period of \(6.4 \times 10^4\) years. Hence, the observation of shadow evolution of M87* and similar higher mass black holes will remain unrealistic in this scenario.

Next we turn our attention to the supermassive black hole at the centre of our galaxy Sagittarius A* with \(M \sim 4.1 \times 10^6 M_\odot\) \[20, 21\] that the EHT collaboration is trying to capture. In this case, using Eq. (6) we find \(\tau_{sr} \sim 40\) years (\(\alpha = 0.28\) and \(m_\gamma 10^{-16}\text{eV}\)) . With a distance from earth of about \(\sim 7.9 \times 10^{-3}\) Mpc \[22\], the variation in the angular diameter of the ring of SgrA*, for \(\Delta a_* = 0.1\) (from 0.95 to 0.85 in Table I), is expected to be \(\Delta d = 1.6\) \textmu as. Yet again, this value falls far short of the resolution of the EHT but since the time scale for the evolution is much smaller, this is a more viable candidate for observation than M87*. Future imaging techniques at lower wavelengths \(\sim 0.8\) mm and with space-based interferometers aims to achieve a resolution of \(\sim 3\) \textmu as \[23\]. With a smaller \(\alpha = 0.24\), the variation in the ring diameter of SgrA* would be \(\Delta d = 3.03\) \textmu as, well within the precision of the future techniques. The trade off would

---

**Table I.** Shift parameter and shadow parameter for an observer at infinity, with \(\theta_0 = 90^\circ\), has been given for four values of the spin parameter \(a_*\) corresponding to four values of the \(\alpha\) parameter: 0.36, 0.28, 0.24, 0.2. The parameters are given in units of the gravitational radius \(r_g\).

| \(a_*\)   | \(\delta x\) | \(\delta y\) | \(\gamma\) |
|----------|--------------|--------------|----------|
| 0.95     | 9.16         | 10.39        | 1.90     |
| 0.85     | 9.47         | 10.40        | 1.68     |
| 0.78     | 9.75         | 10.39        | 1.55     |
| 0.69     | 10.05        | 10.39        | 1.41     |
be that for this value of $\alpha = 0.24$, the time scale over which this evolution takes place is longer, $\sim 97$ years.

The ratio of $M$ and $D$ must satisfy $M/D \geq 1.64 \times 10^{16}$ kg/m for the ring diameter $d$ to be large enough for observation in the current EHT resolution. If we focus our attention to black holes with mass $M \leq 10^7M_\odot$ which have favourable superradiant time scales, the bound on $D$ becomes $D \leq 3.95 \times 10^4$ pc. This is the intra-galactic scale and hence we only need to focus on black holes inside the Milky Way galaxy. Other than SgrA*, all other detected black hole candidates inside the Milky Way are of masses $1-100$ solar mass. For a black hole with $M = 10M_\odot$, $D$ must be $3.95 \times 10^{-2}$ pc for its ring to be observable. The nearest such black hole candidate is the V616 Monocerotis which has $D \sim 10^3$ pc [23] from Earth thus violating the requirement on $M/D$. However the discovery of $\sim 10^3M_\odot$ black holes inside the Milky Way would improve the chance of this effect being observed substantially. For the present the only candidates which satisfy the $M/D$ constraint are M87* and SgrA* and since the time duration of the effect for M87* is way too large for observations, the optimal candidate for observation of shadow evolution is SgrA*.

Conclusion.–We have searched for signatures of Ultralight Boson cloud evolution in the features of a black hole shadow. Our analysis shows that SgrA* is an optimal candidate for observation with a humanly reasonable time scale for evolution. The signals from this source require higher resolution in observational techniques which are being projected in [23]. Further, the discoverability of such an effect would improve significantly if a population of $\sim 100M_\odot$ black holes were to be found.

Another interesting signature of Ultralight Boson Cloud from black hole shadow can be obtained if the backreaction of the boson cloud becomes significant enough on the background metric. The photon geodesics in the black hole environment would then get perturbed and hence the ISCO will change. This change will be reflected in the features of the shadow of the black hole. Detailed numerical study is required to establish this effect and the results of such study would be presented elsewhere.

In conclusion we have shown that the shadow of a black hole could be used to detect the evolution ultralight boson clouds in the black hole environment, in turn providing a signal for the existence of ultralight bosons or axion like particles. Though current imaging technique is not capable of detecting these evolutions, future imaging techniques proposed with higher resolution limits might be able to confirm such variations in the shadow of a black hole.

---

* rittickrr@gmail.com

1. R. M. Wald, “General Relativity”
2. E. Fermi, Phys. Rev. 75, 1169 (1949). doi:10.1103/PhysRev.75.1169
3. P. Bhattacharjee and G. Sigl, Phys. Rept. 327, 109 (2000)
4. L. A. Anchordoqui, Phys. Rep. 801, 1 (2019)
5. F. Halzen and D. Hooper, Rept. Prog. Phys. 65, 1025 (2002)
6. T. Piran, Phys. Rept. 314, 575 (1999)
7. B. P. Abbott et al. [LIGO Scientific and Virgo Collaborations], Phys. Rev. Lett. 116, no. 6, 061102 (2016)
8. K. Akiyama et al. [Event Horizon Telescope Collaboration], Astrophys. J. 875, no. 1, L1 (2019).
9. S. Weinberg, “Gravitation and Cosmology : Principles and Applications of the General Theory of Relativity”
10. S. Endlich and R. Penco, JHEP 1705, 052 (2017)
11. W. H. Press and S. A. Teukolsky, Nature 238, 211 (1972).
12. A. Arvanitaki and S. Dubovsky, Phys. Rev. D 83, 044026 (2011)
13. R. D. Peccei, Lect. Notes Phys. 741, 3 (2008)
14. A. A. Starobinsky, Sov. Phys. JETP 37, no. 1, 28 (1973) [Zh. Eksp. Teor. Fiz. 64, 48 (1973)].
15. I. M. Ternov, V. R. Khalilov, G. A. Chizhov and A. B. Gaina, Sov. Phys. J. 21, 1200 (1978) [Izv. Vuz. Fiz. 21N9, 109 (1978)].
16. S. L. Detweiler, Phys. Rev. D 22, 2323 (1980).
17. A. Arvanitaki, M. Baryakhtar and X. Huang, Phys. Rev. D 91, no. 8, 084011 (2015)
18. J. M. Bardeen, “Timelike and null geodesics in the Kerr metric”
19. P. V. P. Cunha and C. A. R. Herdeiro, Gen. Rel. Grav. 50, no. 4, 42 (2018)
20. A. M. Ghez et al., Astrophys. J. 689, 1044 (2008)
21. S. Gillessen, F. Eisenhauer, S. Trippe, T. Alexander, R. Genzel, F. Martins and T. Ott, Astrophys. J. 692, 1075 (2009)
22. A. Boehle, A. M. Ghez, R. Schödel, L. Meyer S. Yelda, S. Albers, G. D. Martinez E. E. Becklin, T. Do, J. R. Lu, K. Matthews, M. R. Morris, B. Sitarski and G. Witzel, Astrophys. J. 830, 17 (2016)
23. V. L. Fish, M. Shea and K. Akiyama, J. Phys. 1903.09539
24. A. G. Cantrell et al., Astrophys. J. 710, 1127 (2010)