Theory of $\tau$ mesonic decays

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Abstract

Studies of $\tau$ mesonic decays are presented. A mechanism for axial-vector current at low energies is proposed. The VMD is used to treat the vector current. All the meson vertices of both normal parity and abnormal parity (Wess-Zumino-Witten anomaly) are obtained from an effective chiral theory of mesons. $a_1$ dominance is found in the decay modes of $\tau$ lepton: $3\pi$, $f(1285)\pi$. Both the $\rho$ and the $a_1$ meson contribute to the decay $\tau \to K^*K\nu$, it is found that the vector current is dominant. CVC is tested by studying $e^+e^- \to \pi^+\pi^-$. The branching ratios of $\tau \to \omega\pi\nu$ and $K\bar{K}\nu$ are calculated. In terms of similar mechanism the $\Delta s = 1$ decay modes of $\tau$ lepton are studied and $K_a$ dominance is found in $\tau \to K^*\pi\nu$ and $K^*\eta\nu$. The suppression of $\tau \to K\rho\nu$ is revealed. The branching ratio of $\tau \to \eta\bar{K}\nu$ is computed. As a test of this theory, the
form factors of $\pi \to e\gamma\nu$ and $K \to e\gamma\nu$ are determined. Theoretical results agree with data reasonably well.
1 Introduction

The $\tau$ mesonic decays have been studied by Tsai[1] before the discovery of the $\tau$ lepton. All the hadrons in $\tau$ hadronic decays are mesons, therefore the $\tau$ mesonic decays provide a test ground for all meson theories. The mesons produced in $\tau$ hadronic decays are made of the light quarks. Therefore, chiral symmetry play an important role in studying $\tau$ mesonic decays. In Ref.[2] the $\tau$ mesonic decays are associated with the chiral dynamics. It is pointed out[2] that $\rho$-dominance is necessary to be introduced and the chiral limits of the hadronic matrix elements at low energies are set up. In Ref.[3] the $\tau$ mesonic decays are studied by using $SU(3) \times SU(3)$ chiral dynamics with the resonances phenomenologically introduced. In Ref.[4] a Lagrangian for pseudoscalar and vector mesons has been constructed to investigate $\tau$ physics. In Refs.[5] in studying three pseudoscalar mesons decays of $\tau$ lepton the form factors of these decays are constructed by chiral symmetry and dominated by the lowest resonances. The abnormal decays have also been studied[6,7,8,9,10]. The vector meson dominance(VMD)[12] has been applied to study the $\tau$ decays in which the meson states have even G-parity[2-10].

In Ref.[11] an effective Lagrangian of three nonets of pseudoscalar, vector, and axial-vector mesons with $U(3)_L \times U(3)_R$ is obtained. The chiral symmetry breaking scale $\Lambda$ determined in Ref.[11] is $1.6\text{GeV}$, therefore, this theory is suitable to be applied to study $\tau$ mesonic
decays. The VMD is a natural result of this theory. Wess-Zumino-Witten action [WZW][15] is obtained from the leading terms of the imaginary part of the effective Lagrangian. This theory has been applied to study the form factors of $K_{l3}$, $\tau \to \rho \nu$, $\tau \to K^* \nu$ and theoretical results are in good agreements with data[11]. Based on this effective chiral theory of mesons[11], a theory of $\tau$ mesonic decays is developed and a unified study of $\tau$ mesonic decays is presented in this paper.

In the standard model the W bosons are coupled to both the vector and the axial-vector currents of the ordinary quarks. The hadronization of the quark currents is a problem of nonperturbative QCD. In the dynamics of ordinary quarks both chiral symmetry and chiral symmetry breaking are important. The VMD and CVC are successful in studying the matrix elements of vector current[16]. VMD is a natural result of the effective chiral theory of mesons[11]. In this paper the VMD is exploited to treat the matrix elements of the vector currents of $\tau$ decays. Before this paper the VMD has been already exploited to study $\tau$ mesonic decays[2-10]. The difference between this paper and others in the case of two flavors is that the coupling of $\rho \pi \pi$, $f_{\rho \pi \pi}$, derived in Ref.[11] is no longer a constant, but a function of $q^2$ ($q$ is the momentum of $\rho$ meson). It means that the vertex $\rho \pi \pi$ has a form factor. Detailed discussion is presented in section 7.

It is well known that in the chiral limit, the axial-vector currents and the vector currents of ordinary quarks form a $SU(2)_L \times SU(2)_R$ algebra which leads to Weinberg’s sum rules[13].

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On the other hand, $a_1$ meson is the chiral partner of $\rho$ meson[14]. However, $a_1$ meson is much heavier than $\rho$ meson. In Ref.[11] this mass difference refers to the spontaneous chiral symmetry breaking and this effect should be taken into account in bosonizing the axial-vector currents. The effect of the spontaneous chiral symmetry breaking on the bosonization of the axial-vector currents at low energies is studied in this paper. The axial-vector currents contribute to the meson states of $\tau$ decays, which have negative G-parity. The hadronic matrix elements of the axial-vector currents derived in this paper are different from other studies. The Breit-Wigner formula for $a_1$ resonance takes (see Eq.(35))

$$\frac{-g^2 f_a^2 m_\rho^2 + i\sqrt{q^2} \Gamma_a(q^2)}{q^2 - m_a^2 + i\sqrt{q^2} \Gamma_a(q^2)}.$$ 

The difference originates in the spontaneous chiral symmetry breaking which is responsible for the mass difference of the $\rho$ and the $a_1$ mesons. Due to the chiral symmetry breaking, at $q^2 = 0$ this formula is equal to $g^2 f_a^2 m_\rho^2 m_a^2$ and the mass of the $a_1$ meson is still there.

There are two kinds of meson vertices in $\tau$ mesonic decays: the vertices of normal parity and the ones of abnormal parity. The later are from the Wess-Zumino-Witten Lagrangian. In Ref.[11] it shows that the WZW Lagrangian is the leading term of the imaginary part of the effective meson Lagrangian and the fields in WZW Lagrangian are normalized to physical meson fields. The normalization of the fields of the WZW action is very important. The normalization constants for the vector and axial-vector fields are different[11] and due
to the mixing effects the axial-vector fields are always associated with \( \partial_\mu P \), where P is the corresponding pseudoscalar field.

All the meson vertices are obtained from Ref.[11] and they are fixed completely and most of them have been tested already[11]. It is necessary to point out that the vertices of VPP obtained in ref.[11] are functions of momentum (see Eq.(37) \( f_{\rho\pi\pi} \) and see section 7 for details) and the vertices of AVP depend on momentum strongly (see Eq.(27), for example) and due to the cancellation in the vertices AVP the dependence of momentum is very important in understanding \( \tau \to \nu(mesons)_{G=-} \).

In the chiral limit, the theory used to study \( \tau \) mesonic decays consists of three parts: VMD for vector currents, a new expression of axial-vector currents, and vertices of mesons. All these three parts are determined by the effective chiral theory of mesons[11]. All parameters have been fixed.

In many studies[5,6,8,17] besides the \( \rho \) meson the excited \( \rho (\rho' \text{ and } \rho'') \) are taken part in. In this paper only the \( \rho \) meson is included. In the region of higher \( q^2 \) the effects of the form factor of VPP vertex and other decay channels of \( \rho (\text{such as } K\bar{K}, KK^*,... \) are taken into account in calculating the decay widths. So far, theoretical results agree with data reasonably well. In this paper only the lowest resonances are taken into consideration.

It has been shown in Ref.[11] that the diagrams at tree level are at order of \( N_C \) and the loop diagrams of mesons are at higher order in large \( N_C \) expansion. In Ref.[11] large
$N_C$ expansion is implored to argue the success of the effective theory. Following the same argument, all calculations are done at tree level in this paper.

The paper is organized as 1.introduction; 2.general expression of the axial-vector currents; 3.determination of $\mathcal{L}^{V,A}$; 4.$a_1$ dominance in $\tau \to \pi\pi\nu$ decay; 5.$a_1$ dominance in $\tau \to f_1(1285)\pi\nu$ decay; 6.$\tau \to K^*\bar{K}\nu$; 7.CVC and $e^+e^- \to \pi^+\pi^-$; 8.$\tau \to \omega\pi\nu$; 9.$\tau \to K\bar{K}\nu$; 10.the form factors of $\pi \to e\gamma\nu$; 11.effective Lagrangian of $\Delta s = 1$ weak interactions; 12.$K_a$ dominance in $\tau \to K^*(892)\pi\nu$; 13.$\tau \to K^*\eta\nu$; 14.$\tau \to \eta\bar{K}\nu$; 15.the form factors of $K \to e\gamma\nu$; 16. conclusions.

2 General expression of the axial-vector currents

In the case of two flavors the expression of VMD[12] is written as

$$e \{ -\frac{1}{2} F^{\mu\nu} (\partial_\mu v_\nu - \partial_\nu v_\mu) + A_\mu j^{\mu}_v \} ,$$

where $v = \rho^0, \omega, \phi$, $f_v$ is the decay constant of these vector mesons respectively, and $j^{\mu}_v$ are the appropriate currents determined by the substitution

$$v_\mu \to \frac{e}{2f_v} A_\mu$$

in the vertices involving neutral vector mesons. CVC works very well in the weak interactions of hadrons and in $\tau$ mesonic decays[16]. In the chiral limit, the vector part of the weak
interaction of ordinary quarks is determined by CVC

\[ \mathcal{L}^V = \frac{g_W}{4} \cos \theta_C \frac{1}{f_\rho} \left\{ -\frac{1}{2} \left( \partial_\mu A^i_\nu - \partial_\nu A^i_\mu \right) \left( \partial_\mu \rho^i_\nu - \partial_\nu \rho^i_\mu \right) + A^i_\mu j^{i\mu} \right\}, \]

(3)

where \( i = 1, 2 \) and \( A^i_\mu \) are W boson fields. In the vector part of the weak interaction there is \( \rho \) dominance (two flavor case). \( j^i_\mu \) is derived by the substitution

\[ \rho^i_\mu \rightarrow \frac{g_W}{4f_\rho} \cos \theta_C A^i_\mu \]

(4)
in the vertices involving \( \rho \) mesons. At low energies the matrix elements of the vector currents go back to the chiral limit[2].

Chiral symmetry is one of major features of QCD. It is known for a long time that the \( a_1 \) meson is the chiral partner of \( \rho \) meson[14] and both are treated as nonabelian chiral gauge fields[14]. On the other hand, it is well known that in the chiral limit, the vector and axial-vector currents form a \( SU(2)_L \times SU(2)_R \) algebra which leads to Weinberg’s sum rules[13]. Based on the chiral symmetry it is reasonable to think that in the axial-vector part of weak interaction of ordinary quarks there is a term which is similar to VMD

\[ -\frac{g_W}{4} \cos \theta_C A^i_\mu \frac{1}{f_a} \left\{ -\frac{1}{2} \left( \partial_\mu A^i_\nu - \partial_\nu A^i_\mu \right) \left( \partial_\mu a^i_\nu - \partial_\nu a^i_\mu \right) + A^{i\mu} j^{iW}_\mu \right\}; \]

(5)

where \( a^i_\mu \) is the \( a_1 \) meson field, \( f_a \) is a constant, and \( j^{iW}_\mu \) is the appropriate current obtained by substituting

\[ a^i_\mu \rightarrow -\frac{g_W}{4f_a} \cos \theta_C A^i_\mu \]

(6)
into the Lagrangian in which $a_1$ meson is involved. On the other hand, pion can couple to W boson directly. The second term in the axial-vector part of weak interaction of the ordinary quarks is

$$- \frac{g_W}{4} \cos \theta_C f_{a} A_{\mu}^i \partial^\mu \pi^i. \quad (7)$$

As a matter of fact, $a_1$ meson is much heavier than $\rho$ meson. The spontaneous chiral symmetry breaking is responsible for the mass difference. Therefore, due to the effect of spontaneous chiral symmetry breaking in the Lagrangian of meson theory there should be an additional mass term for $a_1$ meson

$$\frac{1}{2} \Delta m^2 f_a^2 a_{\mu}^i a^{i\mu}. \quad (8)$$

Using the substitution $(6)$, a new coupling between W bosons and $a_1$ mesons is revealed

$$- \frac{g_W}{4} \cos \theta_C \Delta m^2 f_a A_{\mu}^i a^{i\mu}. \quad (9)$$

Adding these three terms $(5, 7, 9)$ together, the axial-vector part of the effective Lagrangian of weak interaction of ordinary quarks is obtained

$$\mathcal{L}^A = - \frac{g_W}{4} \cos \theta_C \frac{1}{f_a} \{ - \frac{1}{2} (\partial_{\mu} A_{\nu}^i - \partial_{\nu} A_{\mu}^i) (\partial_{\mu} a_{\nu}^i - \partial_{\nu} a_{\mu}^i) + A^{i\mu} j^{W}_i \}$$

$$- \frac{g_W}{4} \cos \theta_C \Delta m^2 f_a A_{\mu}^i a^{i\mu} - \frac{g_W}{4} \cos \theta_C f_{\pi} A_{\mu}^i \partial^\mu \pi^i. \quad (10)$$

In Eq. $(10)$ there are two parameters $f_a$ and $\Delta m^2$ which are necessary to be determined.
Weinberg’s first sum rule

\[ \frac{g_\rho^2}{m_\rho^2} - \frac{g_a^2}{m_a^2} = f_\pi^2 \]  \hspace{1cm} (11)

is derived by using \( SU(2)_L \times SU(2)_R \) chiral symmetry, current algebra, and VMD, where \( g_\rho \) and \( g_a \) are defined by following formulas

\[ < 0 | \bar{\psi} \gamma_\mu \psi | \rho^\lambda_j > = g_\rho \delta_{ij} \epsilon_\mu^\lambda, \quad < 0 | \bar{\psi} \gamma_\mu \gamma_5 \psi | a^\lambda_j > = g_a \delta_{ij} \epsilon_\mu^\lambda. \]  \hspace{1cm} (12)

Using Eqs.(3,10), we obtain

\[ g_\rho = - \frac{m_\rho^2}{f_\rho}, \quad g_a = - \frac{m_a^2}{f_a} + \Delta m^2 f_a. \]  \hspace{1cm} (13)

It can be seen from Eqs.(4,6) that the \( \rho \) fields are associated with \( f_\rho \) and \( a_1 \) with \( f_a \). After spontaneous chiral symmetry breaking the effective mass terms of \( \rho \) and \( a_1 \) mesons are written as

\[ \frac{1}{2}(\Delta m^2 + m_0^2) f_a^2 a^i_\mu a^{i\mu} + \frac{1}{2} m_0^2 f_\rho^2 \rho^i_\mu \rho^{i\mu} \]  \hspace{1cm} (14)

and

\[ m_\rho^2 = m_0^2 f_\rho^2, \quad m_a^2 = f_a^2 (\Delta m^2 + m_0^2). \]  \hspace{1cm} (15)

Eq.(15) leads to

\[ \Delta m^2 = \frac{m_a^2}{f_a^2} - \frac{m_\rho^2}{f_\rho^2}. \]  \hspace{1cm} (16)

From Eqs.(13,16) we obtain

\[ g_a = - \frac{f_a}{f_\rho^2} m_\rho^2. \]  \hspace{1cm} (17)
Substituting Eqs. (13, 17) into Eq. (11), we determine

\[ f_a^2 = f_\rho^2 \left( 1 - \frac{f_\pi^2 f_\rho^2}{m_\rho^2} \right) \frac{m_a^2}{m_\rho^2}, \quad \Delta m^2 = f_\pi^2 \left( 1 - \frac{f_\pi^2 f_\rho^2}{m_\rho^2} \right)^{-1}. \]  

(18)

The values of \( f_a \) and \( \Delta m^2 \) are determined by \( f_\pi, f_\rho, m_\rho, \) and \( m_a \). In general, \( f_a \neq f_\rho \). Therefore, \( \mathcal{L}^A \) is fixed. The vector current is conserved in the limit of \( m_q = 0 \). The axial-vector current derived from Eq. (10) must satisfy PCAC. It will be shown that it is necessary to have all the terms in Eq. (10) to satisfy PCAC.

3 Determination of \( \mathcal{L}^{V,A} \)

An effective chiral theory of pseudoscalar, vector, and axial-vector mesons has been proposed[11]. In this theory both the physical processes of normal parity and abnormal parity are studied by one Lagrangian. Theoretical results agree with data well. In the limit of \( m_q = 0 \), the Lagrangian is

\[
\mathcal{L} = \bar{\psi}(x)(i\gamma \cdot \partial + \gamma \cdot v + \gamma \cdot a\gamma_5 - mu(x))\psi(x) \\
+ \frac{1}{2}m_0^2(\rho_i^\mu \rho_i^{\mu i} + \omega^\mu \omega_\mu + a_i^\mu a_{i i}^\mu + f^\mu f_\mu)
\]

(19)

where \( a_\mu = \tau_i a_\mu^i + f_\mu, \quad v_\mu = \tau_i \rho_\mu^i + \omega_\mu, \) and \( u = \exp\{i\gamma_5(\tau_i \pi_i + \eta)\} \). \( m \) is a parameter. The fields in the Lagrangian (19) are not physical and the physical meson fields have been defined
in Ref.[11]. This Lagrangian is global $SU(2)_L \times SU(2)_R$ chiral symmetric. On the other hand, the spontaneous chiral symmetry breaking is revealed in this theory when $\pi^i, \eta = 0$ are taken. This theory has both explicit chiral symmetry breaking (PCAC) by adding the current quark masses $-\bar{\psi}M\psi$ (M is the quark matrix) to the Lagrangian and dynamical chiral symmetry breaking (quark condensate)[11].

The explicit expression of VMD(1) has been found in Ref.[11]. The expressions of $\mathcal{L}^{VA}(3,10)$ have been derived by this effective theory (see Eqs.(76,77,78) of Ref.[11]) too. Following expressions are revealed from this effective chiral theory

$$g_\rho = -gm^2,$$

$$g_a = -g(1 - \frac{1}{2\pi^2g^2})^{-\frac{1}{2}}m^2_{\rho},$$

$$f_\rho = g^{-1},$$

$$f_a = g^{-1}(1 - \frac{1}{2\pi^2g^2})^{-\frac{1}{2}},$$

$$(1 - \frac{1}{2\pi^2g^2})m^2_a = 6m^2 + m^2_{\rho},$$

$$\Delta m^2 = 6m^2g^2 = f^2_\pi(1 - \frac{f^2_\pi}{g^2m^2_{\rho}})^{-1},$$

where $g$ is a universal coupling constant and $m$ is a parameter related to quark condensate. The term $6m^2$ in Eq.(24) and the factor $\left(1 - \frac{1}{2\pi^2g^2}\right)$ in Eqs.(23,24) are from spontaneous chiral symmetry breaking of this theory. In this paper we choose $g = 0.39$. Using this value, the theoretical results obtained in Ref.[11] agree with data well. For example, we
obtain $\Gamma_\rho = 142 MeV$ and $m_a = 1.20 GeV$. It is necessary to point out that Weinberg’s first sum rule is satisfied analytically in this effective chiral theory. Eqs.(18) are satisfied too. Therefore, all the parameters in the Lagrangians($\mathcal{L}^{V,A}$) are fixed.

Besides the Lagrangians(3,10), appropriate meson(pseudoscalar, vector, and axial-vector)vertices are needed in studying the $\tau$ mesonic decays and all these vertices can be derived from the Lagrangian(19) and they are fixed[11]. The most of these vertices have been tested by calculating appropriate decay widths and the results agree with data well. Therefore, the Lagrangians of the weak interactions of mesons are completely determined. There are no other undetermined parameters in studying $\tau$ mesonic decays. This effective theory makes definite predictions for $\tau$ mesonic decays.

4. $a_1$ dominance in $\tau \to \pi \pi \pi \nu$

The $a_1$ meson has a long history. In determining the parameters of this meson the process of $a_1$ production in $\tau$ decays play an important role[18,19]. In Ref.[19] the flux tube quark model has been exploited to study $\tau \to \pi \pi \pi \nu$. The decay rate of $\tau \to a_1 \nu$ has been calculated by the effective chiral theory[11]. However, the effect of wide resonance should be taken into account. On the other hand, the experimental observations[20-23,25,27] have reported the $a_1$ dominance in $\tau \to 3\pi \nu$. 

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Only the axial-vector part of the Lagrangian, $\mathcal{L}^A$, takes part in this process. There are five diagrams contributing to this decay: $a_1$ couples to $\rho\pi$, W-boson couples to $\rho\pi$ directly, $\pi$ couples to $\rho\pi$, $a_1$ directly couples to three pions, and $\pi$ directly couples to three pions. The study done in Ref.[11] indicates that the contribution of the four $\pi$ coupling to $\pi\pi$ scattering is smaller than the contribution of $\rho$ exchange by two orders of magnitude. From Ref.[11] it is learned that the contribution of the vertex of $a_1\pi\pi\pi$ to $a_1$ decay is very small and this result agrees with data. Therefore, we omit the contributions of both the vertices of the four $\pi$ and the $a_1\pi\pi\pi$. The remaining three diagrams indicate the existence of $\rho$ resonances in the final states of $\tau \to 3\pi\nu$ and this result is in agreement with data[26,27]. From these three vertices: $a_1\rho\pi$, $W\rho\pi$, and $\pi\rho\pi$ ($\rho\pi\pi$ is included too) it is not obvious why $a_1$ dominates this decay. This is a crucial test on this theory. The vertices derived in Ref.[11] contribute to $\tau \to \pi\pi\pi\nu$

$$\mathcal{L}^{a_1\rho\pi} = \epsilon_{ijk} \{ A a^{i}_{\mu} \rho^{j\mu} \pi^{k} - B a^{i}_{\mu} \rho^{j}(\rho^{\nu} \pi^{k}) + D a^{i}_{\mu} \rho^{j}(\rho^{\nu} \pi^{k}) \}$$  \hspace{1cm} (26)

$$A = \frac{2}{f_\pi} g f_a \left\{ \frac{m_A^2}{g^2 f_a^2} - m_\rho^2 + p^2 \left[ \frac{2c}{g} + \frac{3}{4\pi^2 g^2} \left(1 - \frac{2c}{g}\right) \right] \right\}$$

$$+ \frac{q^2}{g} \left[ \frac{1}{2\pi^2 g^2} - \frac{2c}{g} - \frac{3}{4\pi^2 g^2} \left(1 - \frac{2c}{g}\right) \right],$$

$$c = \frac{f_\pi^2}{2gm_\rho^2},$$

$$B = -\frac{2}{f_\pi} g f_a \frac{1}{2\pi^2 g^2} \left(1 - \frac{2c}{g}\right),$$

$$D = -\frac{2}{f_\pi} g f_a \left\{ 2c + \frac{3}{2\pi^2 g} \left(1 - \frac{2c}{g}\right) \right\}.$$

$$14$$
\[
\mathcal{L}^{\rho\pi\pi} = \frac{2}{g} \epsilon_{ijk} \rho_{\mu i} \pi^j \partial^\mu \pi^k - \frac{2}{\pi^2 f_\pi^2 g} \{(1 - \frac{2c}{g})^2 - 4\pi^2 c^2\} \epsilon_{ijk} \rho_{\mu i} \partial_{\nu} \pi^j \partial^{\mu \nu} \pi^k \\
- \frac{1}{\pi^2 f_\pi^2 g} \{3(1 - \frac{2c}{g})^2 + 1 - \frac{2c}{g} \} \epsilon_{ijk} \rho_{\mu i} \partial^2 \partial_{\mu} \pi^k,
\]

(31)

where p is the momentum of \(\rho\) meson and q is the momentum of \(a_1\). Because the mesons of the vertices are not necessary to be on mass-shell, hence, in Eqs.(26,31) the divergence of \(a_\mu\) and \(\partial^2 \pi_k\) are kept. In the chiral limit, these new terms do not contribute to the decays of \(\rho\) or \(a_1\), however they are important in keeping the axial-vector current conserved in \(\tau \rightarrow 3\pi\nu\) in the chiral limit. The vertex \(\mathcal{L}^{W\rho\pi}\) is derived by using the substitution (6) in Eq.(26).

The two pions in \(\tau \rightarrow 3\pi\nu\) are from the decays of \(\rho\) meson, therefore, we only need to show that the axial-vector current is conserved (in the limit of \(m_q \rightarrow 0\)) in \(\tau \rightarrow \rho\pi\nu\), then this conservation is satisfied in \(\tau \rightarrow 3\pi\nu\).

Using \(\mathcal{L}^A(10)\) and three vertices \(\mathcal{L}^{a_1\rho\pi}(26), \mathcal{L}^{W\rho\pi}, \) and \(\mathcal{L}^{\rho\pi\pi}(31)\), the matrix element of the axial-vector current of \(\tau^- \rightarrow \rho^0\pi^-\) is obtained as

\[
<\rho^0\pi^-|\bar{\psi}\tau_+\gamma_\mu\gamma_5\psi|0> = \frac{i}{\sqrt{4\omega E}} \left\{ \frac{1}{f_a(q^2 - m_a^2)}(q_\mu q_\nu - q^2 g_{\mu\nu})(A g_{\lambda\nu} + B k_\lambda k_\nu)\epsilon_\sigma^{*\lambda} \right. \\
- \frac{\Delta m^2 f_a}{q^2 - m_a^2} \left( q_\mu q_\nu - g_{\mu\nu} \right)(A g_{\lambda\nu} + B k_\lambda k_\nu)\epsilon_\sigma^{*\lambda} - \frac{\Delta m^2 f_a}{m_a^2} q_\mu q_\nu (A + k \cdot q B) k \cdot \epsilon_\sigma^{*} \\
+ \frac{1}{f_a}(A g_{\mu\nu} + B k_\mu k_\nu)\epsilon_\sigma^{*\nu} - \left( \frac{1}{f_a} - \frac{\Delta m^2 f_a}{m_a^2} \right)Dk \cdot \epsilon_\sigma^{*} q_\mu \\
- \frac{4f_\pi q_\mu}{g} \{ 1 + \frac{p^2}{2\pi^2 f_\pi^2} [(1 - \frac{2c}{g})^2 - 4\pi^2 c^2] + \frac{q^2}{2\pi^2 f_\pi^2} (1 - \frac{2c}{g})(1 - \frac{c}{g}) \} k \cdot \epsilon_\sigma^{*} \},
\]

(32)

where k, p, and q are the momenta of pion, \(\rho\), and \(a_1\) respectively. In the effective Lagrangian
of mesons[11] derived from the Lagrangian (19) there is mass term of the $a_1$ meson, therefore, the propagator of $a_1$ field is taken to be

$$\frac{i}{(2\pi)^4} \frac{1}{q^2 - m_a^2} (-g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{m_a^2}).$$

(33)

There are cancellations in Eq.(32). Using Eqs.(24,25,27,28,29,30), it is proved

$$-\frac{\Delta m^2 f_a}{m_a^2} (A + k \cdot qB) + \frac{1}{f_a} (A + B k \cdot q) - \left(\frac{1}{f_a} - \frac{\Delta m^2 f_a}{m_a^2}\right) Dq^2$$

$$-\frac{4f_\pi}{g} \left\{1 + \frac{p^2}{2\pi^2 f_\pi^2}[(1 - \frac{2c}{g})^2 - 4\pi^2 c^2] + \frac{q^2}{2\pi^2 f_\pi^2}(1 - \frac{2c}{g})(1 - \frac{c}{g})\right\} = 0. \quad (34)$$

Eq.(34) leads to the conservation of axial-vector current in the limit of $m_q = 0$

$$q^\mu < \rho^0 \pi^- |\bar{\psi}\tau + \gamma_\mu \gamma_5 \psi|0 >= 0.$$

From this discussion it is learned that in order to have the axial-vector conserved (in the limit of $m_q = 0$) the new term (9) is necessary to be included in Eq.(10). Using Eq.(34), the matrix element (32) is rewritten as

$$< \rho^0 \pi^- |\bar{\psi}\tau - \gamma_\mu \gamma_5 \psi|0 >= \frac{i}{\sqrt{4\omega E}} (q_{\mu}q_{\nu} - g_{\mu\nu})(Ag_{\nu\lambda} + Bk_{\nu}k_{\lambda})\epsilon^{\nu\sigma} - \frac{\Delta m^2 f_a + q^2 f_a^{-1}}{q^2 - m_a^2 + \sqrt{q^2}\Gamma_a(q^2)}$$

$$+ \frac{i}{\sqrt{4\omega E}} (q_{\mu}q_{\nu} - g_{\mu\nu})(Ag_{\nu\lambda} + Bk_{\nu}k_{\lambda})\epsilon^{\nu\sigma}$$

$$= \frac{i}{\sqrt{4\omega E}} (q_{\mu}q_{\nu} - g_{\mu\nu})(Ag_{\nu\lambda} + Bk_{\nu}k_{\lambda})\epsilon^{\nu\sigma}$$

$$= \frac{g^2 f_a m_a^2 - if_a^{-1}\sqrt{q^2}\Gamma_a(q^2)}{q^2 - m_a^2 + \sqrt{q^2}\Gamma_a(q^2)} \quad (35)$$

It is necessary to point out that the Breit-Wigner formula of the axial-vector meson, $a_1$, is new and is different from the one of the vector meson (see section 7). This difference is caused
by the dynamical chiral symmetry breaking. It is also important to notice that the amplitude \( A(27) \) derived in Ref.[11] strongly depends on the momentum. Due to the cancellation in Eq.(27) this dependence is significant.

The decay width of \( a_1 \) meson has been introduced. The \( a_1 \) dominance in \( \tau \to \rho \pi \nu \) is revealed and the dominance is caused by the cancellation(34) which leads to the axial-vector current conservation. Using the vertex \( \mathcal{L}^{\rho \pi \pi}(31) \), the matrix element is derived

\[
<\pi^+ \pi^- \pi^- | \bar{\psi} \gamma_\mu \gamma_5 \psi | 0 > = \frac{i}{\sqrt{8\omega_1 \omega_2 \omega_3}} (-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}) \frac{g^2 f_a m_\rho^2 - i \sqrt{q^2 f_a^{-1} \Gamma_a(q^2)}}{q^2 - m_a^2 + i \sqrt{q^2 \Gamma_a(q^2)}} \frac{A(k^2)}{k^2 - m_\rho^2 + ik \Gamma_\rho(k^2)} \frac{A(k'^2)}{k'^2 - m_\rho^2 + ik' \Gamma_\rho(k'^2)} [A(k^2)(k_2^2 - k_3^2) + Bk_1^2 k_1 \cdot (k_2^2 - k_3^2)] \]

\[
\frac{f_{\rho\pi\pi}(k^2)}{k^2 - m_\rho^2 + ik \Gamma_\rho(k^2)} [A(k'^2)(k_1^2 - k_3^2) + Bk_2^2 k_2 \cdot (k_1^2 - k_3^2)] \], \quad (36)
\]

\[
f_{\rho\pi\pi}(k^2) = \frac{2}{g} \left( 1 + \frac{k^2}{2\pi^2 f_\pi^2} \right) \left[ (1 - \frac{2c}{g})^2 - 4\pi^2 c^2 \right],
\]

\[
\Gamma_\rho(k^2) = \frac{f_{\rho\pi\pi}^2 k^2}{48\pi m_\rho^4} (1 - \frac{4m_\rho^2}{k^2})^\frac{3}{2}, \quad (37)
\]

where \( k_i (i = 1, 2, 3) \) are the momenta of \( \pi^- \), \( \pi^- \), and \( \pi^+ \), \( k = k_2^2 + k_3^2 \), \( k' = k_1^2 + k_3^2 \), \( q = k_1 + k_2 + k_3 \). \( A(k^2)(A(k'^2)) \) are obtained by taking \( p^2 = k^2(k'^2) \) and \( p_a^2 = q^2 \) in Eq.(27) , and \( \Gamma_a \) is defined in Eq.(39). The distribution of the decay width of \( \tau^- \to \pi^- \pi^- \pi^+ \nu \) is derived

\[
\frac{d\Gamma}{dq^2dk^2dk'^2} = \frac{G^2 \cos^2 \theta_C}{(2\pi)^3} \frac{3072m_\tau^3q^6(m_\tau^2 - q^2)^2(m_\tau^2 + 2q^2)(q^2 - k^2)^2}{g^4 f_a^2 m_\rho^4 + q^2 f_a^{-2} \Gamma_a(q^2)} \frac{1}{(q^2 - m_a^2)^2 + q^2 \Gamma_a(q^2)} F(k^2, k'^2),
\]

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equal branching ratio. The branching ratios are computed to be

\[ \pi \text{ the three pions respectively. There are two decay modes:} \]

\[ q \text{ where} \]

\[ \Gamma \text{ a} \]

\[ G(k^2, k'^2) = \left\{ \frac{f_{\rho \pi \pi}(k^2)}{(k^2 - m_{\rho}^2)^2 + k^2 \Gamma_{\rho}^2(k^2)}(k^2 - m_{\rho}^2)[A(k^2) + k_1 \cdot (k_2 - k_3)B] \right. \]

\[ + \left\{ \frac{f_{\rho \pi \pi}(k^2)}{(k'^2 - m_{\rho}^2)^2 + k'^2 \Gamma_{\rho}^2(k'^2)}(k'^2 - m_{\rho}^2)2A(k'^2) \right\}^2 + \left\{ \frac{f_{\rho \pi \pi}(k^2)}{(k'^2 - m_{\rho}^2)^2 + k'^2 \Gamma_{\rho}^2(k'^2)} \sqrt{k'^2 \Gamma_{\rho}(k'^2)}[A(k'^2) + k_1 \cdot (k_2 - k_3)B] \right\}^2 \]

\[ + \frac{f_{\rho \pi \pi}(k^2)}{(k'^2 - m_{\rho}^2)^2 + k'^2 \Gamma_{\rho}^2(k'^2)} \sqrt{k'^2 \Gamma_{\rho}(k'^2)}2A(k'^2) \right\}^2. \]

\[ F(k^2, k'^2) = \frac{1}{2} \{ G(k^2, k'^2) + G(k'^2, k^2) \}. \]

\[ (38) \]

\[ \Gamma_{a} \text{ is derived from the vertices}(26,31) \]

\[ \Gamma_{a}(q^2) = \frac{1}{192(2\pi)^3 m_{\rho} q^4} \int dq_1^2 dq_2^2 (q^2 - q_1^2)^2 \left\{ \frac{f_{\rho \pi \pi}(q_1^2)}{(q_1^2 - m_{\rho}^2)^2 + q_1^2 \Gamma_{\rho}(q_1^2)}(q_1^2 - m_{\rho}^2) \right\}^2 \]

\[ + \left\{ \frac{f_{\rho \pi \pi}(q_2^2)}{(q_2^2 - m_{\rho}^2)^2 + q_2^2 \Gamma_{\rho}(q_2^2)} \sqrt{q_2^2 \Gamma_{\rho}(q_2^2)}[A(q_1^2) + \frac{1}{2} \right\}^2 \]

\[ (q_2^2 - q_2^2)B] + \frac{f_{\rho \pi \pi}(q_2^2)}{(q_2^2 - m_{\rho}^2)^2 + q_2^2 \Gamma_{\rho}(q_2^2)} \sqrt{q_2^2 \Gamma_{\rho}(q_2^2)}2A(q_2^2) \}}^2, \]

\[ (39) \]

where \( q_1^2 = (q - k_1)^2, q_2^2 = (q - k_2)^2, q_3^2 = (q - k_3)^2, \) and \( k_1, k_2, \) and \( k_3 \) are momentum of the three pions respectively. There are two decay modes: \( \pi^+ \pi^- \pi^- \) and \( \pi^- \pi^0 \pi^0 \) which have equal branching ratio. The branching ratios are computed to be

\[ B(\tau \to \pi^+ \pi^- \pi^-) = B(\tau \to \pi^- \pi^0 \pi^0) = 6.3\%. \]

\[ (40) \]

The comparison with experiments is presented in Table I. The distribution of \( d\Gamma(\tau \to \)
$\pi^+\pi^-\pi^-\nu)/dq$ is shown in Fig.1. From Fig.(1) the decay width is determined to be

$$\Gamma_a = 386\, MeV.$$  \hfill (41)

The data is $\sim 400\, MeV$\cite{28}. The comparison with experiments using the model\cite{19} is presented in Table II. The starred results are taken from\cite{19}.
5 $a_1$ dominance in $\tau \to f_1(1285)\pi\nu$

$f_1$ meson is the chiral partner of $\omega$ meson[11] and the mass formula of $f_1$ meson is derived in Ref.[11]

\[(1 - \frac{1}{2\pi^2 g^2})m_{f_1}^2 = 6m^2 + m_\omega^2, \quad m_{f_1} = 1.21\text{GeV}.\] (42)

The vertex of $f_1(1285)a_1\pi$ is presented in Ref.[11] (a factor of -4 has been lost),

\[L_{f_1a_1\pi} = \frac{1}{\pi^2 f_\pi g^2} (1 - \frac{1}{2\pi^2 g^2})^{-1} \varepsilon^{\mu\nu\alpha\beta} f_\mu \partial_\nu \pi^i \partial_\alpha a^i.\] (43)

The narrow width of the decay $f_1 \to \rho\pi\pi$ is revealed from this vertex[11]. Using the substitution (6), the vertex $L_{Wf_1\pi}$ is derived. The vertex $L_{f_1a_1\pi}$ has abnormal parity, hence it belongs to WZW anomaly. Therefore, the WZW anomaly can be tested in $\tau$ mesonic decay.

Only the axial-vector part of the weak interaction contributes to the decay $\tau \to f_1\pi\nu$. Using the Lagrangian $L^A(10)$, it is obtained

\[<f_1\pi|\bar{\psi}\tau^+\gamma_\mu\gamma_5\psi|0> = -\frac{1}{\sqrt{4\omega E}} \frac{1}{\pi^2 f_\pi g^2} (1 - \frac{1}{2\pi^2 g^2})^{-1} \frac{g^2 f_a m_\rho - iqf_a^{-1} \Gamma_a(q^2)}{q^2 - m_a^2 + iq\Gamma_a(q^2)} \varepsilon^{\mu\nu\alpha\beta} k_\nu q_\alpha \epsilon^\sigma_\beta,\] (44)

where $k$ is the momentum of the pion and $q = p + k$, $p$ is the momentum of $f_1$ meson.

The decay width is derived

\[\Gamma = \frac{G^2 \cos^2\theta_C}{(2\pi)^3 128 m_\tau^3} \int dq^2 \frac{1}{q^4} (m_\tau^2 - q^2)^2 (m_\tau^2 + 2q^2)(q^2 - m_{f_1}^2)^3 \]

\[\frac{f_a^4 g^4 f_a^2 m_\rho^4 + q^2 f_a^{-2} \Gamma_a^2(q^2)}{\pi^4 f_\pi^2 (q^2 - m_a^2)^2 + q^2 \Gamma_a^2(q^2)}.\] (45)
\(a_1\) is dominant in this decay. The theoretical prediction of the branching ratio of this decay is

\[
B(\tau \to f_1\pi\nu) = 2.91 \times 10^{-4}.
\] (46)

The data is \((6.7 \pm 1.4 \pm 2.2) \times 10^{-4}\)\([29]\). Two factors result in the small branching ratio. Small phase space is the first factor and the second factor is the anomalous coupling. The effective theory proposed in Ref.[11] is a theory at low energies, therefore, derivative expansion is exploited. In this theory the anomalous couplings are at the fourth order in derivatives.

Comparing with the couplings at the second order in derivatives, the anomalous couplings are weaker. This is the reason why the widths of \(\rho\) and \(a_1\) are broader (the two vertices are at the second order in derivative expansion) and \(\omega\) and \(f_1 \to \rho\pi\pi\) are narrower.

The distribution of the invariant mass of \(f_1\pi\) is shown in Fig.2. The peak of the distribution is resulted by both the effects of the threshold and the \(a_1\) resonance.

The experimental measurement of this decay is a test of the Wess-Zumino-Witten anomaly and the mechanism proposed in this paper.

6 \(\tau \to K^*(892)K\nu\)

The processes \(\tau \to KK\pi\nu\) have been studied by many authors. The earlist study is done by using a chiral Lagrangian \([3]\). In Ref.[17] a chiral Lagrangian and three resonances \(\rho\),
\( \rho' \) and \( \rho'' \) are used. In Ref. [4] the \( \rho \) meson field is introduced to a chiral Lagrangian. In Refs. [5] a comprehensive resonance model including vector and axial-vector resonances has been exploited. In this paper the process \( \tau \to K^*K\nu \) is studied in terms of the same formulas (5,10) used to study \( \tau \to 3\pi\nu \) and \( f_1\pi\nu \).

Both the vector and the axial-vector currents of the weak interactions contribute to the decay \( \tau^- \to (K^*K)^-\nu \). The vector part comes from the anomaly and the vertex is presented in Ref. [11]

\[
\mathcal{L}^{K^*K\rho} = -\frac{N_C}{\pi^2 g^2 f_\pi} \varepsilon^{\mu\alpha\beta} d_{a\mu} K^a_\nu \partial_{\nu} \rho_\alpha^i \partial_{\beta} K^b, \tag{47}
\]

\( \mathcal{L}^{K^*K\text{W}} \) can be derived by using the substitution (4). There are three vertices in the axial-
vector part: $\pi K^* K$, $a_1 K^* K$, and $W K^* K$. As mentioned above, the later can be derived by using the substitution (6) in the vertex $a_1 K^* K$. The vertex $\mathcal{L}^{\pi K^* K}$ is given in Ref. [11]

$$
\mathcal{L}^{\pi K^* K} = i f_{K^* K} (q^2) \left\{ -\frac{1}{\sqrt{2}} K_\mu^0 (\pi^+ \partial^\mu K^- - K^- \partial^\mu \pi^+) - \frac{1}{\sqrt{2}} K_\mu^+ (\pi^- \partial^\mu K^- - K^- \partial^\mu \pi^-) 
+ \frac{1}{2} K_\mu^0 (\pi^0 \partial^\mu K^0 - K^0 \partial^\mu \pi^0) - \frac{1}{2} K_\mu^+ (\pi^0 \partial^\mu K^- - K^- \partial^\mu \pi^0) \right\} + h.c.,
$$

where $q^2$ is the momentum squared of $K^*$. In the limit of $m_q = 0$, $f_{K^* K} (q^2)$ is the same as $f_{\rho \pi \pi} (q^2)$. The vertex $a_1 K^* K$ is derived from Ref. [11]

$$
\mathcal{L}^{a_1 K^* K} = f_{abi} K_\mu^a K_\nu^b a_\mu \{ A(q^2) K^0 g_{\mu \nu} + B k_\mu k_\nu \} - f_{abi} D K_\mu^a \partial^\nu K^b \partial^\mu a_\nu,
$$

where $A(q^2) K^*$ is defined in Eq. (76). In the limit of $m_q = 0$, A, B, and D are the same as (27,29,30). The vector matrix element is obtained from $\mathcal{L}^V (3)$ and the vertex (47)

$$
< K^- K^0 | \bar{\psi} \tau_+ \gamma_\mu \psi | 0 > = \frac{1}{\sqrt{4 \omega E \sqrt{2} \pi^2 g_f \, q^2 - m_\rho^2 + i \sqrt{q^2 \Gamma_\rho (q^2)} \xi^\mu \nu \sigma \tau_+ \partial^\mu a_\nu >},
$$

where $k$ is the momentum of kaon and $q = p + k$, $p$ is the momentum of $K^*$. Because of $q^2 > 4 m_K^2$, the decay mode of $\rho \to K \bar{K}$ is open and we have

$$
\Gamma (\rho \to \pi \pi) = \frac{f_{\rho \pi \pi}^2 (q^2)}{48 \pi} \frac{q^2}{m_\rho^2} (1 - \frac{4 m_\pi^2}{q^2})^2, \\
\Gamma (\rho \to K \bar{K}) = \frac{f_{\rho \pi \pi}^2 (q^2)}{96 \pi} \frac{q^2}{m_\rho^2} (1 - \frac{4 m_K^2}{q^2})^2, \\
\Gamma_\rho (q^2) = \Gamma (\rho \to \pi \pi) + \Gamma (\rho \to K \bar{K}).
$$
The axial-vector matrix element is derived by using $\mathcal{L}^{K^*K\pi}(48)$, $\mathcal{L}^{a_1K^*K}(49)$, $\mathcal{L}^{WK^*K}$, and $\mathcal{L}^A(10)$

$$< K^- K^{*0} | \bar{\psi}_\tau \gamma_\mu \gamma_5 \psi | 0 > = -\frac{i}{\sqrt{4\omega E}} \frac{g^2 f_a m^2_\rho - i \sqrt{q^2} f^- a_1 \Gamma_a(q^2)}{q^2 - m^2_a + i m_a \Gamma_a(q^2)} \left( \frac{q_\mu q_\nu}{q^2} - g_{\mu\nu} \right) (A(q^2) K^* g_{\nu\lambda} + B k_\nu k_\lambda - D k^\nu p^\lambda) e^{*\lambda}_\sigma. $$

(52)

The decay width is derived

$$\frac{d\Gamma}{dq^2}(\tau^- \rightarrow K^{*0} K^- \nu) = \frac{G^2 \cos^2 \theta_C}{64 m^2 q^4} \frac{1}{(2\pi)^3} \frac{1}{\sqrt{q^2}} \frac{1}{(q^2 + m^2_{K^*} - m^2_K)^2 - 4 q^2 m^2_{K^*}} \left( \frac{3}{\pi^4} g^2 f^2_a \frac{m^4_\rho + q^2 \Gamma^2_\rho(q^2)}{(q^2 - m^2_\rho)^2 + q^2 \Gamma^2_\rho(q^2)} [(p \cdot q)^2 - q^2 m^2_{K^*}] \right)$$

$$+ \frac{1}{2} \frac{g^4 f^2_a m^4_\rho + f^- a_2^2 \Gamma^2_{a_2}(q^2)}{(q^2 - m^2_\rho)^2 + q^2 \Gamma^2_{a_2}(q^2)} [A^2(q^2) K^* - \frac{1}{3} \left( \frac{q \cdot k}{q^2} \right)^2 (2 A(q^2) m^2_{K^*} D^2)] ,$$

(53)

The distribution of $\frac{d\Gamma}{dq^2}$ is shown in Fig.3. There is a peak located at 1.51GeV which is caused by both the threshold and resonance effects. The branching ratio is computed to be

$$B(\tau \rightarrow K^{*0} K^- \nu) = 0.392\%,$$

The calculation shows that the vector current is the dominant contributor and the contribution of the axial-vector current is only 7.5% of the decay rate. The data are: CLEO[30] 0.32 ± 0.08 ± 0.12%; ARGUS[31] 0.20 ± 0.05 ± 0.04%. The branching ratio of $\tau^- \rightarrow K^{*-} K^0 \nu$ is the same as $\tau^- \rightarrow K^{*0} K^- \nu$.  

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7  CVC and $e^+e^- \rightarrow \pi\pi$

As discussed in Ref.[16] that CVC works very well in both meson productions in $e^+e^-$ annihilation and $\tau$ decays. As a test of the theory explored in this paper, the same theory[11] is used to study $e^+e^- \rightarrow \pi^+\pi^-$. 

The expression of the VMD[11] is 

$$e^2g\{-\frac{1}{2}F^\mu\nu(\partial_\mu\rho^0_\nu - \partial_\nu\rho^0_\mu) + A^\mu j_\nu\}.$$  \hspace{1cm} (54)

Using the substitution 

$$\rho_\mu \rightarrow \frac{e}{2}gA_\mu,$$
the current \( j_\mu \) is obtained from \( \mathcal{L}^{\rho \pi \pi}(31) \). There are two diagrams: the photon is coupled to \( \pi \pi \) directly and the photon is via the \( \rho \) meson coupled to \( \pi \pi \). The matrix element is derived

\[
<\pi^+ \pi^- | \bar{\psi} \gamma_\mu \gamma_5 \psi | 0 > = \frac{1}{\sqrt{4\omega_1 \omega_2}} g f_{\rho \pi \pi}(q^2) \{1 - \frac{q^2}{q^2 - m_\rho^2 + iq \Gamma_\rho (q^2)} \}(k_1 - k_2)_\mu
\]

\[
= \frac{1}{\sqrt{4\omega_1 \omega_2}} g f_{\rho \pi \pi}(q^2) \frac{-m_\rho^2 + iq \Gamma_\rho (q^2)}{q^2 - m_\rho^2 + iq \Gamma_\rho (q^2)} (k_1 - k_2)_\mu, \tag{55}
\]

where \( k_1 \) and \( k_2 \) are momentum of \( \pi^+ \) and \( \pi^- \) respectively, \( q^2 = (k_1 + k_2)^2 \). The cross section is found to be

\[
\sigma = \frac{\pi \alpha^2}{12} \frac{1}{q^2} (1 - \frac{4m_\pi^2}{q^2})^3 g^2 f_{\rho \pi \pi}^2 (q^2) \frac{m_\rho^4 + q^2 \Gamma_\rho^2(q^2)}{(q^2 - m_\rho^2)^2 + q^2 \Gamma_\rho^2(q^2)}. \tag{56}
\]

The numerical results are shown in Fig.(4). Theoretical results are in good agreement with data[32]. Systematic study of meson production in \( e^+e^- \) collisions will be presented somewhere else.

The pion form factor is found from Eq.(56)

\[
|F(q^2)|^2 = (\frac{g}{2} f_{\rho \pi \pi}(q^2))^2 \frac{m_\rho^4 + q^2 \Gamma_\rho^2(q^2)}{(q^2 - m_\rho^2)^2 + q^2 \Gamma_\rho^2(q^2)}. \]

The new point in this study is that the coupling \( f_{\rho \pi \pi}(37) \) is a function of \( q^2 \). As a matter of fact, \( f_{\rho \pi \pi}(q^2) \) is the form factor of the vertex \( \rho \pi \pi \). The chiral theory of mesons presented in Ref.[11] is a theory at low energies (the energy scale \( \Lambda \) is determined to be 1.6GeV[11]) and covariant derivative expansion is exploited. In Eq.(37) part of the \( q^2 \) dependence of \( f_{\rho \pi \pi}, \)
\( \frac{q^2}{2\pi f_\pi^2} (1 - \frac{2c}{g})^2 \) comes from the fourth order in derivatives and \( \frac{q^2}{2\pi f_\pi^2} (-4\pi^2c^2) \) comes from \( -\frac{1}{8} Tr \rho^\mu\nu \rho_{\mu\nu}. \)

where \( \rho_{\mu\nu} \) is the strength of the nonabelian \( \rho \) field[11]. The radius of the charged pion is derived from the pion form factor in the space-like region of \( q^2 \)

\[ <r^2>_\pi = \frac{6}{m_\rho^2} + \frac{3}{\pi^2 f_\pi^2} \{(1 - \frac{2c}{g})^2 - 4\pi^2c^2\}. \]

In Ref.[11] \( g = 0.35 \) is chosen and the last two terms are cancelled out. In this paper we choose \( g = 0.39 \) to have better fits and

\[ <r^2>_\pi = (0.393 + 0.0549) fm^2 = 0.447 fm^2. \]

The first number is from the \( \rho \) pole and the second comes from the form factor \( f_{\rho\pi\pi} \) and it is 12.2% of the total value.

The data[33] is \( (0.44 \pm 0.01) fm^2. \)

It is necessary to point out that the new expression of the pion form factor is still resulted in the VMD and the \( \rho\pi\pi \) coupling constant is substituted by the form factor of \( \rho\pi\pi \). On the other hand, the \( \rho\pi\pi \) form factor increases the value of the pion form factor at higher \( q^2 \).

In the chiral limit, the form factors of the vertices \( \rho K \bar{K}, \rho K^* K, K^* K \pi, \) and \( K^* K \eta \) are the same. These form factors result physical effects in corresponding \( \tau \) decays.
8 $\tau \to \omega \pi \nu$

This process has been studied in Ref.[8] by using the abnormal vertex $\rho \omega \pi$. The effects of excited $\rho$ mesons have been taken into account. In this paper we only take the contribution of the $\rho$ meson. The $\omega \rho \pi$ vertex is presented in Ref.[11]

$$\mathcal{L}^{\omega \rho \pi} = -\frac{N_C}{\pi^2 g^2 f_\pi} \varepsilon^{\mu \nu \alpha \beta} \partial_\mu \omega_\nu \rho_\alpha \partial_\beta \pi^i.$$ 

The vertices of $\pi^0 \gamma \gamma$, $\omega \pi \gamma$, $\rho \pi \gamma$, and $\omega 3\pi$ are via the VMD derived from this vertex and theoretical results agree with data well. The coupling constant of this $\omega \rho \pi$ vertex is different
with the one presented in Ref.[8]. The decay width of \( \tau \to \omega \pi \nu \) is derived

\[
\Gamma = \frac{G^2}{128m_\tau^3} \frac{\cos^2 \theta_C}{(2\pi)^3} \int dq^2 \frac{1}{q^4} \left( m_\tau^2 - q^2 \right)^2 (m_\tau^2 + 2q^2)(q^2 - m_\omega^2)^3 \left( m_\rho^4 + q^2\Gamma_\rho^2(q^2) \right) \frac{3}{\pi^4 g^2 f_\pi^2 (q^2 - m_\rho^2)^2 + q^2\Gamma_\rho^2(q^2)}.
\]

The numerical result of the branching ratio is

\[
B = 1.2\%,
\]

and the experiment is 1.6 ± 0.5\%[28]. This result is the same as the one obtained in Ref.[8] when only \( \rho \) meson is taken. The distribution of the invariant mass of \( \omega \) and \( \pi \) is shown in Fig.(5).
\[ \tau \rightarrow K \bar{K} \nu \]

This process has been studied by several groups\[34\]. The \( a_1 \) field doesn’t couple to \( K \bar{K} \). The vertex \( a_1 K \bar{K} \) in which the tensor \( \varepsilon^{\mu\nu\alpha\beta} \) must be involved cannot be constructed. Therefore, only the vector current contributes to this decay. The vertex \( \mathcal{L}^{aK\bar{K}} \) has been used to calculate the electric form factors of charged kaon and neutral kaon and theoretical predictions are in good agreements with data\[11\]. We use the same vertex (only the isovector part) to calculate the decay rate of \( \tau \rightarrow K \bar{K} \nu \). This is a test of CVC. The related vertex is presented in Ref.\[11\]

\[ \mathcal{L}^{\rho KK} = \frac{i}{\sqrt{2}} f_{\rho KK} \{ \rho_\mu (K^+ \partial^\mu \bar{K}^0 - \bar{K}^0 \partial^\mu K^+) - \rho_\mu (K^- \partial^\mu K^0 - K^0 \partial^\mu K^-) \}, \quad (58) \]

where \( f_{\rho KK} \) is the same as the \( f_{\rho \pi \pi} \) in the limit of \( m_q = 0 \). By using VMD(3) and the vertex\[58\], the matrix element is derived

\[ \langle K^- K^0 | \bar{\psi}_\tau + \gamma_\mu \psi | 0 \rangle = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} (k_1 - k_2)_\mu g f_{\rho \pi \pi}(q^2) \frac{-m_\rho^2 + i q \Gamma_\rho(q^2)}{q^2 - m_\rho^2 + i q \Gamma_\rho(q^2)}, \quad (59) \]

where \( k_1 \) and \( k_2 \) are momentum of two kaons respectively and \( q = k_1 + k_2 \). Using this matrix element, the decay width is obtained

\[ \frac{d\Gamma}{dq^2}(\tau \rightarrow K^0 K^- \nu) = \frac{G^2 \cos^2 \theta \varphi}{(2\pi)^3 384m^3_r} (m^2_r - q^2)^2 (m^2_r + 2q^2)(1 - \frac{4m^2_r}{q^2})^2 g^2 f_{\rho \pi \pi}(q^2) \frac{m^4_\rho + q^2 \Gamma^2_\rho(q^2)}{(q^2 - m^2_\rho)^2 + q^2 \Gamma^2_\rho(q^2)}. \quad (60) \]
The branching ratio is computed to be

\[ B = 0.27\% \]

and the data are TPC/2γ[35]: < 0.26, ALEPH[36]: 0.26 ± 0.09 ± 0.02, CLEO[37]: 0.151 ± 0.021 ± 0.022. The distribution of the decay rate is shown in Fig.6.

Due to the effects of the threshold and the \( \rho \) resonance there is a peak in the distribution and it is positioned at 1.17GeV.
10 The form factors of $\pi^- \to e\gamma\nu$

As a test of $\mathcal{L}^{V,A}(3,10)$, we study the vector and the axial-vector form factors of $\pi^- \to e\gamma\nu$. In $\pi^- \to \gamma e\nu$ there are inner bremsstrahlung from the lepton and meson and structure-dependent term[28]. The form factors of the structure-dependent term have been studied[28].

The the structure-dependent form factors are defined as[28]

$$M^V_{SD} + M^A_{SD} = e \frac{G}{\sqrt{2}} \cos \theta \epsilon \cdot m_\pi \epsilon^* \bar{\nu} \gamma_\mu (1 - \gamma_5) e$$

$$\{ F^V \varepsilon^{\mu \nu \alpha \beta} p_{\pi \alpha} k_{\beta} + i F^A [g^{\mu \nu} k \cdot p_\pi - k^\mu p_\pi^\nu] + i R t g^{\mu \nu} \},$$  \hspace{1cm} (61)

where $k$ and $p_\pi$ are momentum of photon and pion respectively, $t = k^2$. It is known that $F^V$ is via CVC determined by the amplitude of $\pi^0 \to 2\gamma$. In this theory the amplitude of $\pi^0 \to 2\gamma$ obtained by triangle anomaly is obtained by combining the vertex $\mathcal{L}^{\omega\rho\pi}$ and VMD. Using $L^V(3)$, $\mathcal{L}^{\omega\rho\pi}$, and VMD, it is obtained

$$F^V = \frac{m_\pi}{2\sqrt{2} \pi^2 f_\pi} = 0.0268.$$ \hspace{1cm} (62)

The experiments[28] are $0.014 \pm 0.009$, $0.023^{+0.015}_{-0.013}$.

The form factors $F^A$ and $R$ are determined by calculating the matrix element $< \gamma | \bar{\psi}_\tau - \gamma_\mu \gamma_5 \psi | \pi^+ >$ which is via VMD found from $< \rho^0 | \bar{\psi}_\tau - \gamma_\mu \gamma_5 \psi | \pi^+ >$ (35). However, for pion weak radiative decay besides the axial-vector current conservation(in the limit $m_q = 0$) the electric current is conserved too. In order to satisfy the electric current conservation, the divergence of $\rho$
field which is ignored in the vertex $\mathcal{L}^{a\rho\pi}(26)$ must be kept and the term derived from the effective Lagrangian of Ref.[11] is

$$- D\epsilon_{ijk} a^i_{\mu} \partial^{\mu} \pi^j \partial^{\nu} \rho^k_{\nu}, \quad (63)$$

where $D$ is given in Eq.(30) Adding the term(63) to the matrix element(35) we obtain

$$<\gamma|\bar{\psi}_\tau - \gamma_\mu \gamma_5 \psi|^\pi^+> = \frac{ie}{\sqrt{4\omega_\pi\omega_\gamma}} (q_\mu q_\nu - g_{\mu\nu}) \{A g_{\lambda\nu} + B p_\pi p_\tau + D k_\nu k_\lambda\} \epsilon_\sigma^{\lambda\rho} \frac{1}{2} g^3 f_a \frac{1}{q^2 - m_a^2} \frac{m_\rho^2}{m_\rho^2 - k^2}, \quad (64)$$

where $q = p_\pi - k$. Using the expressions of $A(27)$, $B(29)$, and $D(30)$, it is proved that

$$A + k \cdot p_\pi B + D k^2 = 0. \quad (65)$$

Eq.(65) guarantees the electric current conservation. Ignoring $q^2$ and $k^2$, the two form factors are found

$$F^A = \frac{1}{2\sqrt{2} \pi^2} \frac{m_\pi^2 m_\rho^2}{f_\pi m_a^2} (1 - \frac{2c}{g})(1 - \frac{1}{2\pi^2 g^2})^{-1} = 0.0102, \quad (66)$$

$$R = \frac{g^2}{2\sqrt{2}} \frac{m_\pi^2 m_\rho^2}{f_\pi m_a^2} \{\frac{2c}{g} + \frac{1}{\pi^2 g^2} (1 - \frac{2c}{g})\} (1 - \frac{1}{2\pi^2 g^2})^{-1}. \quad (67)$$

The experimental values[28] of $F^A$ are $0.0106 \pm 0.006, 0.0135 \pm 0.0016, 0.011 \pm 0.003$.

11 Effective Lagrangian of $\Delta s = 1$ weak interactions

It is natural to generalize the expressions of $\mathcal{L}^{V,A}(3,10)$ to the case of three flavors. The vector part of the weak interaction, instead $\rho$ meson in Eq.(3) the $K^*(892)$ meson takes part
\begin{equation}
\mathcal{L}^{Vs} = \frac{g_W}{4} \sin\theta_C \frac{1}{f_{K^*}} \left\{ - \frac{1}{2} \left( \partial_\mu W^\mu_\nu - \partial_\nu W^\mu_\mu \right) \left( \partial^\mu K^{-\nu} - \partial^\nu K^{-\mu} \right) \\
+ \left( \partial_\mu W^-_\nu - \partial_\nu W^-_\mu \right) \left( \partial^\mu K^{++\nu} - \partial^\nu K^{++\mu} \right) + W^+_\mu j^\mu - W^-_\mu j^\mu \right\},
\end{equation}

(68)

Where $j_\mu^\pm$ are obtained by substituting $K_\mu^\pm \rightarrow \frac{g_W}{4} \frac{1}{f_{K^*}} \sin\theta_C W^\pm_\mu$ into the vertex in which $K_\mu$ fields are involved. In the chiral limit, $f_{K^*}$ is determined to be $g^{-1}[11]$. This Lagrangian has been used to calculate the form factors of $K_{3l}[11]$ and the results are in good agreement with data.

For axial-vector part $\mathcal{L}^{A}$ there are two $1^+$ K-mesons: $K_1(1400)$ and $K_1(1275)$. In Ref.[11] the chiral partner of $K^*(892)$ meson, the $K_1$ meson, is coupled to

$$\bar{\psi} \lambda^a \gamma_\mu \gamma_5 \psi.$$  

(69)

The mass of this $K_1$ meson is derived as

$$(1 - \frac{1}{2\pi^2 g^2}) m^2_{K_1} = 6m^2 + m^2_{K^{*+}}, \quad m_{K_1} = 1.32 GeV. \quad (70)$$

Theoretical value of $m_{K_1}$ is lower than the mass of $K_1(1400)$ and greater than $K_1(1270)$’s mass. The widths of three decay modes ($K_1 \rightarrow K^*\pi$, $K\rho$, $K\omega$) are calculated[11]. It is found that $K^*\pi$ channel is dominant, however, $B(K\rho)$ is about 11%. The data[28] show that the branching ratio of $K_1(1400)$ decaying into $K\rho$ is very small. Therefore, the meson coupled
to the quark axial-vector current is not a pure $K_1(1400)$ state, instead, it is a mixture of the two $K_1$ mesons. This state is coupled to the quark axial-vector current(69) and it is $K_a$.

$$K_a = \cos \theta K_1(1400) + \sin \theta K_1(1270),$$

$$K_b = -\sin \theta K_1(1400) + \cos \theta K_1(1270).$$

(71)

In this theory $K_a$ is coupled to the quark axial-vector current and the amplitudes of $\tau \to K_a \nu$ is from the tree diagrams and at $O(N_C)$ [11]. The production of $K_b$ in $\tau$ decay is through loop diagrams of mesons which is at $O(1)$ in large $N_c$ expansion[11]. This theory predicts a small branching ratio for $K_b$ production in $\tau$ decays.

In the limit of $m_q = 0$, the currents $\bar{\psi} \lambda_a \gamma_\mu \gamma_5 \psi$ and $\bar{\psi} \lambda_a \gamma_\mu \gamma_\nu \gamma_5 \psi$ form an algebra of $SU(3)_L \times SU(3)_R$. In this theory $K_a$ is taken as the chiral partner of $K^*$ meson. The axial-vector part of the weak interaction $\mathcal{L}^{As}(10)$ is generalized to the case of $\Delta s = 1$

$$\mathcal{L}^{As} = -\frac{g_\nu}{4 f_a} \sin \theta_C \left\{ -\frac{1}{2} (\partial_\mu W^\pm_\nu - \partial_\nu W^\pm_\mu)(\partial^\mu K^\mp_\nu - \partial^\nu K^\pm_\mu) + W^\pm_\mu j^\mp_\mu \right\} - \frac{g_\nu}{4} \sin \theta_C \Delta m^2 f_a W^\pm_\mu K^\mp_\mu - \frac{1}{4} \sin \theta_C f_K W^\pm_\mu \partial^\mu K^\mp,$$

(72)

where $j^\pm_\mu$ are obtained by substituting $K^\pm_\mu \rightarrow -\frac{g_\nu}{4 f_a} \sin \theta_C W^\pm_\mu$ into the vertex in which $K_a$ fields are involved. In the limit of $m_q = 0$, $f_a$, $\Delta m^2$ are the same as Eqs.(23,25) and $f_K = f_\pi$. $\mathcal{L}^{As}(72)$ can be exploited to study $\tau$ mesonic decays. On the other hand, $\tau$ mesonic decays of $\Delta s = 1$ provide crucial test on $\mathcal{L}^{As}$.
12 \( K_a \) dominance in \( \tau \to K\pi\pi\nu \) decay

The processes \( \tau \to K\pi\pi\nu \) have been studied by many authors. In Ref.[3] a chiral Lagrangian of the pseudoscalars with introduction of vector resonances(\( \rho \) and \( K^* \)) has been used to calculate the branching ratios of \( \tau \to K\pi\pi \). In Ref.[4] a chiral Lagrangian of pseudoscalars and \( \rho \) mesons is exploited. In Refs.[38] the the mixture of the two \( K_1 \) resonances are phenomenologically taken into account in studying the decay \( \tau \to K_1(1400)(K_1(1270))\nu \). In Refs.[5] meson vertices(independent of momentum) and normalized Breit-Wigner propergators of the resonances are exploited.

In this theory, like \( \tau \to 3\pi\nu \), the contribution of the contact terms containing more than three mesons to the processes \( \tau \to K\pi\pi\nu \) is too small and the processes are dominated by \( \tau \to K^*\pi\nu \) and \( K\rho\nu \).

12.1 \( \tau \to K^*\pi\nu \)

We study the decay \( \tau \to K^*\pi\nu \) first. Both the vector and axial-vector currents contribute to the decay \( \tau^- \to K^{*0}\pi^-\nu \). The vertex \( \mathcal{L}^{\pi K^* K^*} \) contributes to the vector part and has abnormal parity. It is from anomaly. This vertex is derived from

\[-i\frac{2m}{f}\pi^i < \bar{\psi} \gamma_i \gamma_5 \psi > .\]
The method obtaining the vertex $\mathcal{L}^{\pi K^* K^*}$ from this quantity is the same as the one used to
derive the vertices of $\eta v v (v = \rho, \omega, \phi)$ in Ref.[11].

$$\mathcal{L}^{\pi K^* K^*} = -\frac{N_C}{\sqrt{2 \pi^2 g^2 f_\pi}} \varepsilon^{\mu \nu \rho \sigma} \{ \partial_\mu K^*_\nu \partial_\rho K_\sigma^0 \pi^- + \partial_\mu K^*_\nu \partial_\rho K_\sigma^0 \pi^+ \\
+ \frac{1}{\sqrt{2}} \pi^0 (\partial_\mu K^*_\nu \partial_\rho K_\sigma^0 - \partial_\mu K^*_\nu \partial_\rho K_\sigma^0) \}.$$ (73)

The vertex $\mathcal{L}^{W K^* \pi^-}$ is derived by using the substitution. Using $\mathcal{L}^V s (68)$ and the vertex (73),
the vector matrix element is obtained

$$<\bar{K}^0 \pi^- | \bar{\psi} \gamma_\lambda + \gamma_\mu \psi | 0 > = \frac{-1}{\sqrt{4 \omega E}} \frac{N_C}{\sqrt{2 \pi^2 g^2 f_\pi}} \frac{m_{\pi}^2}{q^2} \sqrt{2 \pi^2 g^2 f_\pi} \frac{(q^2 - m_{K^*}^2)}{q^2} + i \sqrt{q^2 \Gamma_{K^*} (q^2)} \varepsilon^{\mu \nu \rho \sigma} k_\nu p_\mu \varepsilon^\sigma_\beta,$$ (74)

where $p$ and $k$ are momentum of $K^*$ and pion respectively, $q = k + p$.

The axial-vector matrix element is obtained by using the vertices: $K_a K^* \pi$, $K K^* \pi$ which
are presented in Ref.[11]. In the chiral limit, the expression of the matrix element of the
axial-vector current is similar to Eq.(35)

$$<\bar{K}^0 \pi^- | \bar{\psi} \gamma_\lambda + \gamma_\mu \gamma_5 \psi | 0 > = \frac{i}{\sqrt{4 \omega E}} \frac{1}{\sqrt{2}} \frac{(q^2 - m_{K^*}^2)}{q^2} \frac{m_{K^*}^2}{q^2} \frac{\sqrt{q^2 \Gamma_{K^*} (q^2)} \varepsilon^{\mu \nu \rho \sigma} k_\nu p_\mu \varepsilon^\sigma_\beta}{q^2} \cos \theta (A_{K_{1}(1400)} (q^2) K^* \gamma^\nu k^\lambda + B_{K_{1}(1400)} k^\mu k^\lambda)$$

$$+ \frac{g^2 f_a m_{K^*}^2}{q^2} - i g f_a^{-1} \frac{\sqrt{q^2 \Gamma_{K_{1}(1400)} (q^2)} \varepsilon^{\mu \nu \rho \sigma} k_\nu p_\mu \varepsilon^\sigma_\beta}{q^2} \sin \theta (A_{K_{1}(1270)} (q^2) K^* \gamma^\nu k^\lambda + B_{K_{1}(1270)} k^\mu k^\lambda) \}$$ (75)

Let’s determine the amplitudes $A_{K_{1}(1400)}$, $B_{K_{1}(1400)}$, $A_{K_{1}(1270)}$, and $B_{K_{1}(1270)}$. Eqs.(71) are
written as

$$K_{1}(1400) = \cos \theta K_a - \sin \theta K_b,$$

37
\[ K_1(1270) = \sin \theta K_a + \cos \theta K_b. \]  

(76)

The vertex of \( K_1 VP \) is presented in Ref.[11]

\[ L^{K_1 VP} = f_{abc} \{ AK_{1\mu}^a V^{b\mu} P_c - BK_{2\mu}^a V^{b\mu} \partial_\mu \partial_\nu P_c \}. \]  

(77)

It is similar to Eq.(27) the amplitude \( A_{K_a}^{K^*} \) is determined to be

\[
A(q^2)_{K_a}^{K^*} = \frac{2}{f_\pi} g f_a \left( \frac{m_{K_a}^2}{g^2 f_a^2} - m_{K^*}^2 + m_{K^*}^2 \left[ \frac{2c}{g} + \frac{3}{4\pi^2 g^2} (1 - \frac{2c}{g}) \right] \right)
+ q^2 \left[ \frac{1}{2\pi^2 g^2} \frac{2c}{g} - \frac{3}{4\pi^2 g^2} (1 - \frac{2c}{g}) \right].
\]  

(78)

\( B_{K_a} \) is the same as Eq.(29). The amplitudes \( A_{K_b}^{K^*} \) and \( B_{K_b}^{K^*} \) are unknown and we take them as parameters. Both \( K_1(1400) \) and \( K_1(1270) \) decay to \( K\rho \) and \( K\omega \). Using the \( SU(3) \) coefficients, for both \( K_1 \), it is determined

\[ B(K\omega) = \frac{1}{3} B(K\rho). \]

This relation agrees with data[28] reasonably well. For the \( K\rho \) decay mode \( A_{K_b}^{\rho} \) and \( B_{K_b}^{\rho} \) are other two parameters. In the decays of the two \( K_1 \) mesons the momentum of pion or kaon is low, therefore, the decay widths are insensitive to the amplitude B. We take

\[ B_{K_b}^{K^*} = B_{K_b}^{\rho} \equiv B_b. \]

The decay width of the \( K_1 \) meson is derived from Eq.(77)

\[
\Gamma_{K_1} = \frac{k}{32\pi} \frac{1}{\sqrt{q^2 m_{K_1}}} \left\{ (3 + \frac{k^2}{m_v^2}) A^2(q^2) - A(q^2) B(q^2 + m_v^2) \frac{k^2}{m_v^2} + \frac{q^2}{m_v^2} k^4 B^2 \right\},
\]  

(79)
where \( q^2 = m_{K_1}^2 \), \( v = K^* \), \( \rho \), \( k \) is the momentum of pion or kaon

\[
k = \left( \frac{1}{4m_{K_1}^2} (m_{K_1}^2 + m_{\nu}^2 - m_p^2)^2 - 4m_{\nu}^2 \right)^{\frac{1}{2}},
\]

\( m_p \) is the mass of pion or kaon.

we choose the parameters as

\[
\theta = 30^0, \quad A_b^{K^*} = -4.5GeV, \quad A_b^\rho = 5.0GeV, \quad B_b = 0.8GeV^{-1},
\]

(80)

from which the decay widths are obtained

\[
\Gamma(K_1(1400) \to K^*\pi) = 159MeV, \quad \Gamma(K_1(1400) \to K\rho) = 10.5MeV,
\]

\[
\Gamma(K_1(1270) \to K^*\pi) = 12.4MeV, \quad \Gamma(K_1(1270) \to K\rho) = 26.8MeV.
\]

(81)

The value of \( \theta \) is about the same as the one determined in Ref.[38]. The data[28] are 163.4(1±0.13)MeV, 5.22±5.22 MeV, 37.8(1±0.28)MeV, and 14.4(1±0.27)MeV respectively.

Using the two matrix elements(74,75), the distribution of the decay rate is derived

\[
\frac{d\Gamma}{dq^2} (\tau^- \to \bar{K}^{*0}\pi^-\nu) = \frac{G^2}{(2\pi)^3} \frac{\sin^2\theta_C}{128m_{\tau}^2q^4} (m_{\tau}^2 - q^2)^2 (m_{\tau}^2 + 2q^2) \{(m_{K^*}^2 - m_{K^*}^2) - 2q^2m_{K^*}^2\}^{\frac{1}{2}}
\]

\[
- \frac{6}{\pi^4g^2f_{\pi}^2} \frac{m_{K^*}^2 + q^2\Gamma_{K^*}}{(q^2 - m_{K^*}^2)^2 + q^2\Gamma_{K^*}^2(q^2)^2} [(p \cdot q)^2 - q^2m_{K^*}^2]
\]

\[
+ |A|^2 \left[ 1 + \frac{1}{12m_{K^*}^2 q^2} (q^2 - m_{K^*}^2)^2 \right] - (BA^* + B^*A) \frac{1}{24m_{K^*}^2 q^2} (q^2 + m_{K^*}^2)(q^2 - m_{K^*}^2)^2
\]

\[
+ \frac{|B|^2}{48m_{K^*}^2 q^2} (q^2 - m_{K^*}^2)^4 \}.
\]

(82)
where $p$ is the momentum of $K^*$, $q^2$ is the invariant mass squared of $K^*\pi$, and

$$
A = \frac{g^2 f_a m_{K^*}}{q^2 - m_{K_1(1400)}^2} - i\sqrt{q^2} f_a^1 \Gamma_{K_1(1400)} \cos \theta A_{K_1(1400)} + \frac{g^2 f_a m_{K^*}}{q^2 - m_{K_1(1270)}^2} - i\sqrt{q^2} f_a^1 \Gamma_{K_1(1270)} \sin \theta A_{K_1(1270)},
$$

$$
B = \frac{g^2 f_a m_{K^*}^2 - i\sqrt{q^2} f_a^1 \Gamma_{K_1(1400)} \cos \theta B_{K_1(1400)} + \frac{g^2 f_a m_{K^*}^2 - i\sqrt{q^2} f_a^1 \Gamma_{K_1(1270)} \sin \theta B_{K_1(1270)}}{q^2 - m_{K_1(1400)}^2 + i\sqrt{q^2} \Gamma_{K_1(1400)}}}
$$

where

$$
A_{K_1(1400)} = \cos \theta A_a^* - \sin \theta A_b^*, \quad B_{K_1(1400)} = \cos \theta B_a - \sin \theta B_b,
$$

$$
A_{K_1(1270)} = \sin \theta A_a^* + \cos \theta A_b^*, \quad B_{K_1(1270)} = \sin \theta B_a + \cos \theta B_b,
$$

(84)

In the range of $q^2$ the main decay channels of $K^*$ are $K\pi$ and $K\eta$(the vertex of $K^*K\eta$ is shown in (88)). The decay width of $K^*$ is derived

$$
\Gamma(q^2)_{K^*} = \frac{f_{\rho\pi\pi}(q^2)}{8\pi} \frac{k^3}{\sqrt{q^2 m_{K^*}}} + \cos^2 2\theta A_{\rho\pi\pi}(q^2) \frac{k^2}{8\pi} \frac{\sqrt{q^2 m_{K^*}}}{m_{K^*}},
$$

$$
k = \{\frac{1}{4q^2} (q^2 + m_{K^*}^2 - m_{\pi}^2)^2 - m_{K^*}^2\}^{\frac{1}{2}},
$$

$$
k' = \{\frac{1}{4q^2} (q^2 + m_{K^*}^2 - m_{\eta}^2)^2 - m_{K^*}^2\}^{\frac{1}{2}}.
$$

(85)

In $\Gamma_{K_1}(q^2)$ the decay modes $K^*\pi$, $K\rho$ and $K\omega$ are included.

$$
\Gamma(q^2)_{K_1} = \frac{k}{32\pi \sqrt{q^2 m_{K_1}}} \{(3 + \frac{k^2}{m_{K^*}^2}) A^2(q^2)_{K^*} - A(q^2)_{K^*} B(q^2 + m_{K^*}^2) \frac{k^2}{m_{K^*}^2} + \frac{q^2}{m_{K^*}^2} k^4 B^2\}
$$

$$
+ \frac{4}{32\pi \sqrt{q^2 m_{K_1}}} \{(3 + \frac{k^2}{m_{K^*}^2}) A^2(q^2) - A(q^2) B(q^2 + m_{\rho}^2) \frac{k^2}{m_{\rho}^2} + \frac{q^2}{m_{\rho}^2} k^4 B^2\},
$$

$$
k = \{\frac{1}{4q^2} (q^2 + m_{K^*}^2 - m_{\pi}^2)^2 - m_{K^*}^2\}^{\frac{1}{2}},
$$

$$
k' = \{\frac{1}{4q^2} (q^2 + m_{K^*}^2 - m_{\rho}^2)^2 - m_{K^*}^2\}^{\frac{1}{2}}.
$$

(86)
For $K_1(1270)$ $\Gamma(K_1(1270) \rightarrow K_0^*(1430)\pi) = 25.2\,MeV$ is included. The distribution is shown in Fig.7 and the branching ratio is calculated

$$B(\tau^- \rightarrow \bar{K}^0\pi^-\nu) = 0.23\%,$$

The contribution of the vector current is about 7.4%. Therefore, $K_a$ is dominant in this decay. The data are

$0.38 \pm 0.11 \pm 0.13\%$(CLEO[39])

$0.25 \pm 0.10 \pm 0.05\%$(ARGUS[40])

There is another decay channel $\tau^- \rightarrow K^{*-}\pi^0\nu$ whose branching ratio is one half of $B(\tau^- \rightarrow$
$K^{*0}\pi^-\nu$. The total branching ratio is

$$B(\tau^- \to \bar{K}\pi\nu) = 0.35\%,$$

The narrow peak in Fig.7 is from $K_1(1270)$ and the wider peak comes from $K_1(1400)$. The width is about 230MeV.

### 12.2 $\tau \to K\rho\nu$ and $K\omega\nu$

It is the same as $\tau \to K^*\pi\nu$, $K_a$ dominates the decay $\tau \to K\rho\nu$. Both the vector and axial-vector currents contribute to this decay mode. The matrix element of the vector current,

$$<\bar{K}\rho^-|\bar{\psi}\lambda_+\gamma_{\mu}\gamma_5\psi|0>$$

is determined by the vertex $\mathcal{L}^{K^*K\rho}(47)$ and is the same as Eq.(50). The axial-vector matrix element

$$<\bar{K}\rho^-|\bar{\psi}\lambda_+\gamma_{\mu}\gamma_5\psi|0>$$

is obtained by substituting $K^* \to \rho$, $K \to \pi$ in Eq.(75). Using the same substitutions in Eq.(83), the distribution of the decay rate of $\tau \to K\rho\nu$ is found. The branching ratio of $\tau \to K\rho\nu$ (two modes $\bar{K}_0^0\rho^-$ and $K^-\rho^0$) is computed to be

$$B = 0.75 \times 10^{-3}. \quad (87)$$

It is about 18% of $\tau \to K\pi\pi\nu$. The vector current makes 8% contribution. The DELPHI[41] has reported that $\tau \to K^*\pi\nu$ is dominant the decay $\tau \to K\pi\pi\nu$ and $K\rho\nu$ decay mode has
not been observed. The ALEPH\cite{42} has reported the $K^*\pi$ dominance and a branching ratio of $30\pm 11\%$ for the $K\rho$ mode.

Due to the $SU(3)$ coefficient we expect

$$B(\tau \to K\omega\nu) = \frac{1}{3} B(\tau \to K\rho\nu).$$

The theoretical results are in reasonably agreement with data. In this paper the spontaneous chiral symmetry breaking effect(for the mass difference between $K^*$ and $K_a$) is taken into account and the resonance formula is(Eq.(83))

$$BW_{K_1}[s] \equiv \frac{-g^2f_a^2m_{K^*}^2 + i\sqrt{q^2}\Gamma_{K_1}}{q^2 - m_{K_1}^2 + i\sqrt{q^2}\Gamma_{K_1}}.$$

Because the spontaneous chiral symmetry breaking effect doesn’t disappear in the limit of $q^2 \to 0$, we have a different low energy limit which is $g^2f_a^2m_{K^*}^2/m_{K_1}^2$. On the other hand, in this theory the amplitude A strongly depends on $q^2$ and this dependence plays important role in understanding the large branching ratio of $K^*\pi$ mode and the smaller one for $K\rho$ mode.

13 $\tau \to K^*\eta$

There are vector and axial-vector parts in this decay. The calculation of the decay rate is similar to the decay of $\tau \to K^*\pi\nu$. The vertices $L^{K^*\bar{K}^*\eta}$ and $L^{WK^*\eta}$ via the Lagrangian $L^{V(68)}$ contribute to the vector part and the vertices $L^{K_1K^*\eta}$, $L^{KK^*\eta}$, and $L^{WK^*\eta}$ via $L^{A(43)}$
take the responsibility for the axial-vector part. The vertex $L^{K^*\bar{K}^*\eta}$ comes from anomaly. Using the same method deriving the vertices $\eta\nu\nu$ (in Ref. [11]), it is found

$$L^{K^*\bar{K}^*\eta} = -\frac{3a}{2\pi^2g^2f_\pi}d_{a88}^\nu\varepsilon_{\mu\alpha\beta}^\eta\partial_\mu K_\nu^a\partial_\alpha K_\beta^b - \frac{3b}{2\pi^2g^2f_\pi}d_{ab}^\nu\varepsilon_{\mu\alpha\beta}^\eta\partial_\mu K_\nu^a\partial_\alpha K_\beta^b,$$

(88)

where $a$ and $b$ are the octet and singlet component of $\eta$ respectively, $a = \cos\theta$, $b = \sqrt{2/3}\cos\theta$, and $\theta = -20^\circ$. Due to the cancellation between the two components the vector matrix element is very small and can be ignored.

The vertices $L^{K_1K^*\eta}$ and $L^{KK^*\eta}$ contribute to the axial-vector matrix element and they are derived from the effective Lagrangian presented in Ref. [11].

$$L^{K_1K^*\eta} = af_{ab}(A(q^2)_{K_1\mu}K_\mu^aK_\beta^b\eta - BK_\mu^aK_\beta^b\partial^\mu\eta),$$

(89)

$$L^{KK^*\eta} = af_{K^*K\eta}d_{a88}K_\mu^a(K_\mu^b\partial^\mu\eta - \eta\partial^\mu K_\mu^b),$$

(90)

where $f_{K^*K\eta}$ is the same as $f_{\rho\pi\pi}(37)$ in the limit of $m_q = 0$. The decay width is similar to the one of $\tau \rightarrow K^*\pi\nu$

$$\frac{d\Gamma}{dq^2}(\tau^- \rightarrow K^{*-}\eta\nu) = \frac{G^2}{(2\pi)^3}\cos^2\theta\frac{\sin^2\theta_C}{4\pi m_\pi^4}20^0\frac{\sin^2\theta_C}{64m_\pi^2q^4}(m_\pi^2 - q^2)^2(m_\pi^2 + 2q^2)^2$$

$$\times \left\{\frac{3}{4}\left\{|A|^2[1 + \frac{1}{12m_\pi^2q^2}(q^2 - m_\pi^2)^2] - (BA^* + B^*A)\frac{1}{24m_\pi^2q^2}(q^2 + m_\pi^2)^2(q^2 - m_\pi^2)^2 \right\} + \frac{|B|^2}{48m_\pi^2q^2}(q^2 - m_\pi^2)^4\right\}. \right\}$$

(91)

The distribution is shown in Fig. 8. The branching ratio is computed to be

$$B = 1.01 \times 10^{-4}.$$
The axial-vector current is dominant.

\[ \tau \rightarrow K\eta\nu \]

The decay \( \tau \rightarrow \eta K\nu \) has been studied in terms a chiral Lagrangian\([3,7]\) and only the vector current contributes. The prediction is \( 1.2 \times 10^{-4} \). The experiments are

CLEO\([43]\): \( (2.6 \pm 0.5) \times 10^{-4} \),

ALEPH\([44]\): \( (2.9^{+1.3}_{-1.2} \pm 0.7) \times 10^{-4} \).

In the effective chiral theory the vertex \( K_1 K\eta \) doesn’t exist. The reason is that if it exists it has abnormal parity and comes from the anomaly in which there is antisymmetric tensor.
It is impossible to construct a vertex with a antisymmetric tensor by using $K_1$, $K$, and $\eta$ fields. Therefore, only the vector current contributes to this process and $K^*$ is dominant in this process. The vertex $\mathcal{L}^{K^*K\eta}$ is shown in Eq.(90). The decay width is found

$$\frac{d\Gamma}{dq^2} = \frac{3}{4(2\pi)^3}\sin^2\theta_C\cos^22\theta_0\frac{1}{384m^2} \frac{1}{q^2} (m^2_\tau - q^2)^2 (m^2_\tau + 2q^2) \frac{m^4_{K^*} + q^2\Gamma^2_{K^*}(q^2)}{(q^2 - m^2_{K^*})^2 + q^2\Gamma^2_{K^*}(q^2)}.$$ (92)

The branching ratio is computed to be

$$B(\tau^- \rightarrow \eta K^- \nu) = 2.22 \times 10^{-4}.$$ 

The distribution of the invariant mass of $\eta$ and $K$ is shown in Fig.(9). The figure indicates a peak at 1.086GeV which is slightly above the threshold. The peak is resulted by the effects of the threshold and the resonance.

The branching ratio of $\tau \rightarrow \eta' K \nu$ is 200 times smaller.

## 15 Form factors of $K^+ \rightarrow \gamma l\nu$

As a test of $\mathcal{L}^{V_A}(68,72)$ we study the form factors of $K^- \rightarrow e\gamma\nu$. The form factors of $K^+ \rightarrow \gamma l\nu$ are calculated in the chiral limit. The vector form factor is determined by the vertex which comes from the anomaly[11]

$$\mathcal{L}^{K^+K^-\gamma} = -\frac{e}{2\pi^2 g_{\pi}} \varepsilon^{\mu \nu \alpha \beta} K^+_{\mu} \partial_{\beta} K^- \partial_{\nu} A_{\alpha}.$$ (93)
The vector form factor is determined

\[ F^V = \frac{1}{2\sqrt{2\pi^2}} \frac{m_K}{f_\pi} = 0.095. \]  

(94)

The vertices \( \mathcal{L}_{K^1 K^\gamma} \) can be found from Ref.[11].

\[ \mathcal{L}_{K^1 K^\gamma} = \frac{i}{2} e g (A g_{\nu\lambda} + B p_{\nu} p_{\lambda} + D k_{\nu} k_{\lambda}) K_1^{-\nu} K^+ A^\lambda. \]  

(95)

\( \mathcal{L}_{K^K K^\gamma} \) is[11]

\[ \mathcal{L}_{K^K K^\gamma} = i e (K^+ \partial_\mu K^- - K^- \partial_\mu K^+) A^\mu. \]  

(96)

The axial-vector form factors are derived

\[ F^A = \frac{1}{2\sqrt{2\pi^2}} \frac{m_K m_{K^*}}{f_\pi} \frac{m_{K^*}}{m_{K_1}} (1 - \frac{2c}{g})(1 - \frac{1}{2\pi^2 g^2})^{-1} = 0.04, \quad m_{K_1}^2 = (1.32 GeV)^2 \]  

(97)
\[ R = \frac{g^2}{2\sqrt{2}} \frac{m_K}{f} \frac{m_{K^*}^2}{m_{K^0}^2} \left\{ \frac{2c}{g} + \frac{1}{\pi^2 g^2} \left(1 - \frac{2c}{g}\right) \right\} (1 - \frac{1}{2\pi^2 g^2})^{-1} = 0.078. \]  

We obtain \( F^A + F^V = 0.135, \) \( F^A - F^V = -0.055. \) The data[28] are \( F^A + F^V = 0.147 \pm 0.011, \) \( 0.150^{+0.018}_{-0.023}, \) \( F^A - F^V = < 0.49. \) In the calculation of \( F_A \) the \( K_a \) is used, we obtain the \( F_A = 0.032. \) It is necessary to point out that the factor \( m_K \) in Eqs.(88,91,92) comes from the definitions of the form factors.

16 Conclusions

The Lagrangian of weak interaction of mesons consists of vector part and axial-vector part. In the chiral limit, the VMD takes responsibility for the vector part. Based on chiral symmetry and spontaneous chiral symmetry breaking the Lagrangian of the axial-vector part of weak interactions of mesons is determined. The whole Lagrangian is derived from the effective chiral theory of mesons. All the vertices of mesons are obtained from the same theory. This theory provides a unified study for \( \tau \) mesonic decays. The \( a_1 \) dominance in the matrix elements of the \( \Delta s = 0 \) axial-vector currents and \( K_a \) dominance in the ones of \( \Delta s = 1 \) axial-vector currents in \( \tau \) decays are found. All theoretical studies are done in the limit of \( m_q = 0 \) and the results are in reasonable agreements with data. There are many other \( \tau \) mesonic decay modes can be studied by this theory.
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References

[1] Y.S.Tsai, Phys.Rev. D4, 2821(1971).

[2] R.Fischer, J.Wess, and F.Wagner, Z.Phys. C3,313(1980).

[3] G.Aubrecht II, N.Chahrouri, and K.Slanec, Phys.Rev.D24, 1318(1981).

[4] E.Braaten, R.J.Oakes, and S.M.Tse, Inter.Jour.Modern Phys., 5,2737(1990).

[5] R.Decker, E.Mirkes, R.Sauer and Z.Was, Z.Phys., C58,445(1993); M.Finkemeier and E.Mirkes, Z.Phys., C69, 243(1996).

[6] G.Kramer and W.F.Palmer, Z.Phys., C25,195(1984) and ibid, 39,423(1988).

[7] A.Pich, Phys.Lett., 196,561(1987).

[8] R.Decker, Z.Phys.,C36,487(1987).
[9] E.Braaten, R.Oakes, and S.Z.Tse, Phys.Rev., D36, 2187 (1987).

[10] R.Decker and E.Mirkes, Phys.Rev., D47, 4012 (1993).

[11] B.A.Li, Phys.Rev., D52, 5165(1995), 5184(1995).

[12] J.J.Sakurai, Currents and Mesons, Univ. of Chicago Press, 1969.

[13] S.Weinberg, Phys.Rev.Lett., 18, 507(1967).

[14] J. Schwinger, Phys. Lett., B24, 473(1967); J. Wess and B. Zumino, Phys. Rev., 163, 1727 (1967); S. Weinberg, Phys. Rev., 166, 1568(1968); B. W. Lee and H. T. Nieh, Phys. Rev., 166, 1507(1968).

[15] J.Wess and B.Zumino, Phys.Lett., B37, 95(1971), E.Witten, Nucl.Phys., B223, 422(1983).

[16] S.I.Eidelman and V.N.Ivanchenko, Proc. of the Third Workshop on Tau Lepton Physics, p.131, Montreux, Switzerland, 19-22 September 1994, ed. by L.Rolandii.

[17] J.J.Gomez-Cadennas, M.C.Gonzalez-Garcia and A.Pich, Phys.Rev., D42, 3093(1990).

[18] M.G.Bowler, Phy.Lett., 182B, 400(1986), N.A.Tornquist, Z.Phys., C36, 695(1987), J.H.Kuhn and A.Santamaria, Z.Phys., C48, 445(1990), M.K.Volkov, Yu.P.Ivanov, A.A.Osipov, Z.Phys., C49, 563(1991).
[19] N.Isgur, C.Morningstar, and C.Reader, Phys.Rev., D39, 1357(1989).

[20] H.J.Behrend et al., CELLO Collaboration, Z.Phys., C46, 537(1990).

[21] D.Decamp et al., ALEPH Collaboration, Z.Phys., C54, 211 (1992).

[22] P.Abreu et al., DELPHI Collaboration, Z.Phys., C55, 555 (1992).

[23] G.Crawford CLEO Collaboration, Proc. of the Second Workshop on Tau Lepton Physics, Columbus, Ohio, ed. by K.K.Ga, World Scientific, p.183, 1992.

[24] D.Antreasyan et al., Crystalball Collaboration, Phys.Lett., B259,216(1991).

[25] H.Evans, “Charged Current Measurements a Tau96 Overview“, talk presented in Fourth International Workshop on Tau Lepton Physics, Estes Park, Colorado, Sept.15-19,1996.

[26] BES Collaboration, High Energy Phys. and Nuclear Phys., 19,385(1994).

[27] H.Albrecht et al., ARGUS Collaboration, Z.Phys., C58 , 61(1993).

[28] Particle data group, Phys.Rev., D50 No.3(1994).

[29] T.E.Coan et al., CLEO Collaboration, CLEO CONF 96-15, ICHEP96 PA01-085.

[30] M.Goldberg et al., CLEO Collaboration, Phys.Lett., B251, 223(1990).

[31] H.Albrecht et al., ARGUS Collaboration, Z.Phys., C68, 215(1995).
[32] see paper by M.Benayoun et al., Z.Phys., C58,31(1993).

[33] E.B.Dally et al., Phys.Rev.Lett., 48,375(1982).

[34] S.I.Eidenlman and V.N.Ivanchenko, Phys.Let., B257,437(1991), S.Nelson and A.Pich, Phys.Lett., B304,359(1993).

[35] H.Aihara et al., TPC/2γ Collaboration, Phys.Rev.Lett., 59,751(1987).

[36] D.Buskulic et al., ALEPH Collaboration, CERN-PPE/95-140.

[37] J.Barlet et al., CLEO Collaboration, CLNS 96/1391, CLEO 96-3.

[38] H.Lipkin, Phys Lett., B303,119(1993), M.Suzuki, Phys.Rev., D47,1252(1993).

[39] M.Goldberg et al., CLEO collaboration, Phys.Lett., B251 223(1992).

[40] H.Albrecht et al., ARGUS Collaboration, Z.Phys., C68, 215(1995).

[41] W.Hao(DELPHI Collaboration), ”Study of charged kaon production in three Prong tau decays”, Doctoral thesis.

[42] M.Davier(ALEPH Collaboration), talk presented in Fourth Inter. Workshop on Tau-Lepton Physics, Estes Park, Colorado, Sept. 15-19, 96.

[43] J.Barlet et al., CLEO Collaboration, CLNS 96/1395, CLEO 96-5.
[44] D. Buskulic et al., ALEPH Collaboration, CERN-PDE/96-103.
Table 1: Table I Branching Ratios

| Experiment [25] | $B(2h^-h^+\nu)\%$ | $B(h^-2\pi^0\nu)\%$ |
|-----------------|-------------------|-------------------|
| New W.A.        | 9.26 ± 0.26       | 9.21 ± 0.14       |
| DELPHI (92-95)  | 8.69 ± 0.12 ± 0.16| 9.22 ± 0.43 ± 0.20|
| ALEPH (89-93)   | 9.46 ± 0.10 ± 0.11| 9.32 ± 0.13 ± 0.10|
| CELLO (90)      |                   | 9.1 ± 1.3 ± 0.9   |
| OPAL (91-94)    | 9.83 ± 0.10 ± 0.24|                   |
| L3 (92)         |                   | 8.88 ± 0.37 ± 0.42|
| CLEO (93)       | 8.7 ± 0.8         | 8.96 ± 0.16 ± 0.449|
| CLEO (95)       | 9.47 ± 0.07 ± 0.20|                   |
| CBALL (91)      |                   | 5.7 ± 0.5 ± 1.4   |
| ARGUS (93)      | 7.3 ± 0.1 ± 0.5   |                   |
| MAC (87)        |                   | 8.7 ± 0.4 ± 0.11  |
| BES [16]        | 7.3 ± 0.5($\pi^+\pi^-\pi^-$) |              |
| Taula 2.4       | 7.0 ± 2.8         | 6.4 ± 2.8         |
| This study      | 6.3($\pi^+\pi^-\pi^-$) | 6.3($\pi^-\pi^0\pi^0$) |
Table 2: Table II Parameters of $a_1$ meson

| Experiment   | $m_{a_1}$ (GeV) | $\Gamma_{a_1}$ (GeV) |
|--------------|-----------------|----------------------|
| ARGUS[27]    | 1.211 ± 0.007   | 0.446 ± 0.021        |
| DELCO*       | 1.180 ± 0.060   | 0.430 ± 0.190        |
| MARKII*      | 1.250 ± 0.050   | 0.580 ± 0.100        |
| ARGUS*       | 1.213 ± 0.011   | 0.434 ± 0.030        |
| This study   | 1.20            | 0.386                |