Toward The Classification of
The Realistic Free Fermionic Models

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Abstract

The realistic free fermionic models have had remarkable success in providing plausible explanations for various properties of the Standard Model which include the natural appearance of three generations, the explanation of the heavy top quark mass and the qualitative structure of the fermion mass spectrum in general, the stability of the proton and more. These intriguing achievements makes evident the need to understand the general space of these models. While the number of possibilities is large, general patterns can be extracted. In this paper I present a detailed discussion on the construction of the realistic free fermionic models with the aim of providing some insight into the basic structures and building blocks that enter the construction. The role of free phases in the determination of the phenomenology of the models is discussed in detail. I discuss the connection between the free phases and mirror symmetry in (2,2) models and the corresponding symmetries in the case of the (2,0) models. The importance of the free phases in determining the effective low energy phenomenology is illustrated in several examples. The classification of the models in terms of boundary condition selection rules, real world–sheet fermion pairings, exotic matter states and the hidden sector is discussed.

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1 Introduction

Despite interesting progress over the last few years in understanding non perturbative aspects of superstring theory, trying to connect string theory to experimental low energy physics still relies on the analysis of perturbative heterotic string vacua\(^*\). The study of perturbative string vacua has progressed significantly over the past decade and we now have several distinct, but perhaps related, ways to construct perturbative string models in four dimensions. Among those are the geometrical formulations that include the Calabi–Yau compactifications \[^1\] and related potential constructions \[^2\], and the orbifold compactifications \[^3\]. On the other side we have algebraic constructions that include for example the free fermionic formulation \[^4\] and more general constructions obtained by tensoring higher level conformal field theories \[^5, 6\]. Similarly we have broaden our view on the possible open avenues for trying to connect between string vacua and low energy physics. Among those are the string GUT models \[^7\], and in particular the three generation string GUT models \[^8, 9\], the semi–simple string GUT models \[^10, 11, 12, 13\] and the superstring standard–like models \[^14, 15, 16\], in which the non Abelian content of the Standard Model gauge group is obtained directly at the string level.

Among the semi–realistic superstring models constructed to date the models in the free fermionic formulation have gone the furthest in trying to recapture the physics of the Standard Model. Some general properties of this class of models suggest that their success is not accidental. First is the fact that the free fermionic construction is formulated at a highly symmetric point in the string moduli space. Second, the realistic free fermionic models correspond to \(Z_2 \times Z_2\) orbifold compactifications at an enhanced symmetry point in the Narain moduli space \[^20\]. The existence of three generations in the free fermionic models is correlated with the underlying orbifold structure. This is exhibited in the fact that the free fermionic models produce a large class of three generation models with varied phenomenological properties. The emergence of three generations in these models is not ad hoc and motivates the hypothesis that the true string vacuum is a \(Z_2 \times Z_2\) orbifold at the vicinity of the free fermionic point in the string moduli space. Another important property of the three generation free fermionic models is the fact that the weak hypercharge has the standard \(SO(10)\) embedding. Thus, although the \(SO(10)\) symmetry is broken at the string level rather than in the effective low energy field theory, some of the appealing features of standard \(SO(10)\) unification are retained while some of the generic difficulties associated with grand unification, like doublet–triplet splitting, may be resolved.

The realistic superstring models \[^13, 17, 18, 20, 19\] in the free fermionic formulation \[^7\] have had intriguing success in describing the real world. Some of the important properties of these models are:

\(^*\)For discussions of phenomenological aspects related to M and F theory see e.g. ref. \[^2, 3\]
The existence of three generations arises due to the underlying \( Z_2 \times Z_2 \) orbifold compactification \[21\]. Qualitatively realistic fermion mass spectrum can be envisioned \[22, 23, 24, 25\]. The generation mass hierarchy and the general texture of the fermion mass matrices are also seen to arise from the symmetries of the underlying \( Z_2 \times Z_2 \) orbifold compactification \[24\].

The heavy generation Yukawa couplings have been calculated explicitly in specific models and found to be in agreement with the observed masses. The heaviness of the top quark arises because only the top quark gets a cubic level mass term while the lighter fermion obtain their mass terms from nonrenormalizable terms \[26\].

Dimension four and five operators that may mediate rapid proton decay are suppressed due to stringy symmetries and due to a superstring doublet–triplet splitting mechanism in which the color triplets are projected out by the GSO projections \[27\]. Thus, the superstring derived standard–like models may naturally give rise to a stable proton \[27, 28\].

Exotic matter states arise in these models due to the breaking of the non–Abelian grand unified gauge symmetries at the string level by Wilson lines. These exotic matter states may yield new dark matter candidates and new exotic leptoquark states that in general do not arise in Grand Unified theories \[29\]. Such states are referred to generically as Wilsonian matter states.

A see–saw mechanism which suppresses left–handed neutrino masses was proposed \[30, 31\]. The superstring see–saw mechanism makes use of the exotic Wilsonian matter states. The neutrino–Higgsino mixing problem, which is expected to arise generically in Gravity Unified Theories can also be resolved due to the “non–standard” charges of the exotic Wilsonian matter states \[32\].

An important property of the realistic free fermionic models is the standard \( SO(10) \) embedding of the weak hypercharge. Consequently, it was suggested that in these models string gauge coupling unification may be in agreement with \( \sin^2 \theta_W(M_Z) \) and \( \alpha_s(M_Z) \), provided that there exist additional matter beyond the Minimal Supersymmetric Standard Model spectrum at intermediate energy thresholds \[33, 34, 35\].

The list of remarkable accomplishments of the realistic free fermionic models makes evident the need to better understand their general structure. To date the study of these models has mainly focused on several isolated examples and a systematic classification is still lacking. The large space of three generation models offers on the one hand the intriguing possibility that there exist a model in this space which satisfies all of the experimental constraints. On the other hand this richness makes
a systematic classification a seemingly impossible challenge. The aim of this paper is to provide some of the insight that has been developed into the basic structures and building blocks that enter the construction, thus taking the first steps toward a classification of the realistic free fermionic models. From a different view such a classification is required if we are to uncover whether a fully realistic model can, or cannot, be obtained in this class of models.

Models in the free fermionic formulation are constructed by specifying a set of boundary condition basis vectors and a choice of GSO projection coefficients, consistent with the string consistency constraint. The low energy spectrum and allowed interactions are then determined. The classification of the models can then proceed by developing general rules of how the low energy properties of the models are determined by the fermionic boundary conditions and the GSO phases. To date, some rules based on the world-sheet fermion boundary conditions have been obtained. However, a general discussion of the basic building blocks and of the role of the GSO phases is still lacking. In this paper I make a small step in trying to fill this gap. Due to the large number of possibilities the discussion is necessarily incomplete. Several general properties, such as mirror symmetry for (2,2) as well as (2,0) are explained in terms of free phases. Furthermore, the notion of duality symmetries due to the discrete choices of free phases can be extended to the set of basis vectors and GSO phases which span the realistic models, thus providing one mean of classifying the models. The choices of GSO phases also play an important role in the determination of the low energy spectrum and the effective low energy phenomenology. For example, as will be shown, discrete choices of phases affect the massless matter states, in a way which affects the string gauge coupling unification problem. The classification of the models in terms of the real world–sheet fermion pairings, the exotic matter states and the hidden sector is discussed. In several examples it will be shown how the different choices of real fermion pairing relate to the phenomenological properties of the models. The classification of exotics is done by classifying all the possible types of exotic states that may appear in the models as well as by the type of exotic states that actually appear in specific models. Some phenomenological implications of the classification by the exotic states and hidden sector are examined.

The paper is organized as follows. Section 2 contains a review of the realistic free fermionic construction. Section 3 contains a discussion of the classification of the models by the boundary condition rules. In section 4 the role of the free phases is examined. The relation between the GSO phases and mirror symmetry is shown for (2,2) models. It is then shown that mirror symmetry, as a result of the free phases also exists in the (2,0) models, interchanging 16 of SO(10) with the 16. The connection between the free phases and other properties of the models, like space–time supersymmetry and the presence of exotics is further discussed. In section 6, 7 and 8 the classification of the models in terms of, the real world–sheet fermion pairings; the types of exotics; and the hidden sector; is discussed. Finally, section 9 contains the conclusions.
2 Realistic free fermionic models

In the free fermionic formulation of the heterotic string in four dimensions all the world–sheet degrees of freedom required to cancel the conformal anomaly are represented in terms of free fermions propagating on the string world–sheet. In the light–cone gauge the world–sheet field content consists of two transverse left– and right–moving space–time coordinate bosons, \(X_{1,2}^\mu\) and \(\bar{X}_{1,2}^\mu\), and their left–moving fermionic superpartners \(\psi_{1,2}^\mu\), and additional 62 purely internal Majorana–Weyl fermions, of which 18 are left–moving, \(\chi^I\), and 44 are right–moving, \(\phi^a\). In the supersymmetric sector the world–sheet supersymmetry is realized non–linearly and the world–sheet supercurrent is given by

\[
T_F = \psi^\mu \partial X_\mu + f_{IJK} \chi^I \chi^J \chi^K ,
\]

(2.1)

where \(f_{IJK}\) are the structure constants of a semi–simple Lie group of dimension 18. The \(\chi^I\) \((I = 1, \ldots, 18)\) world–sheet fermions transform in the adjoint representation of the Lie group. In the realistic free fermionic models the Lie group is \(SU(2)^6\). The \(\chi^I\) \(I = 1, \ldots, 18\) transform in the adjoint representation of \(SU(2)^6\), and are denoted by \(\chi^I\), \(y^I\), \(\omega^I\) \((I = 1, \ldots, 6)\). Under parallel transport around a noncontractible loop on the toroidal world–sheet the fermionic fields pick up a phase

\[
f \rightarrow - e^{i\pi\alpha(f)} f .
\]

(2.2)

The minus sign is conventional and \(\alpha(f) \in (-1, +1]\). Each set of specified phases for all world–sheet fermions, around all the non–contractible loops is called the spin structure of the model. Such spin structures are usually given is the form of 64 dimensional boundary condition vectors, with each element of the vector specifying the phase of the corresponding world–sheet fermion. The partition function is a sum over all such spin structures:

\[
Z(\tau) = \sum_\alpha (-1)^F \text{Tr} (\alpha) .
\]

(2.3)

where the sum is over all the sectors (spin structures) \(\alpha\) of the theory, \((-1)^F\) is a space–time fermion number operator, and \(\text{Tr} (\alpha)\) indicate a trace over the Fock space of mode excitations of the world–sheet fields. In string models this trace is generically realized as GSO projections between subsets of the theory,

\[
\text{Tr} (\alpha) = \frac{1}{g} \sum_\beta c^{(\alpha)}_{(\beta)} \text{Tr} \left( \begin{array}{c} \alpha \\ \beta \end{array} \right) .
\]

(2.4)

Here the \(\beta\)-sum implements the GSO projection, \(c^{(\alpha)}_{(\beta)}\) are the chosen GSO phases, \(g\) is a normalization factor, and \(\text{Tr} \left( \begin{array}{c} \alpha \\ \beta \end{array} \right)\) indicates a restricted trace over the appropriate \(\left( \begin{array}{c} \alpha \\ \beta \end{array} \right)\) subsector.
The partition function is required to be invariant under modular transformation. Modular transformation, in general, mix between the different spin–structures. Therefore, requiring invariance under modular transformation results in a set of rules which constrains the allowed spin–structures and their amplitudes \( c(\alpha, \beta) \). It turns out that a string model can be specified in terms of boundary condition basis vectors and the GSO projection coefficients for these basis vectors. The modular invariance constraints are in turn translated to a set of rules which constrain the allowed boundary condition basis vectors and their GSO projection coefficients. These rules are given in ref. [7]. In general the partition function is summed over all the genus–\( g \) string world–sheet. Due to factorization theorem the genus–\( g \) amplitude factorizes into a product of genus one amplitudes. One–loop modular invariance is therefore sufficient to constrain the consistent spin structures. Thus, a model in this construction is defined by a set of boundary conditions basis vectors and by a choice of generalized GSO projection coefficients, which satisfy the one–loop modular invariance constraints. The boundary conditions basis vectors \( b_k \) span a finite additive group

\[ \Xi = \sum_k n_k b_k \]  

(2.5)

where \( n_i = 0, \cdots, N_{\zeta} - 1 \). The physical massless states in the Hilbert space of a given sector \( \alpha \in \Xi \) are then obtained by acting on the vacuum state of that sector with the world-sheet bosonic and fermionic mode operators, with frequencies \( \nu_f, \nu_f^* \), and by subsequently applying the generalized GSO projections,

\[ \left\{ e^{i\pi(b_i F_\alpha)} - \delta_\alpha c^* \left( \frac{\alpha}{b_i} \right) \right\} |s\rangle = 0 \]  

(2.6)

with

\[ (b_i F_\alpha) \equiv \left\{ \sum_{\text{real}+\text{complex}} - \sum_{\text{real}+\text{complex}} \right\} (b_i(f) F_\alpha(f)), \]  

(2.7)

where \( F_\alpha(f) \) is a fermion number operator counting each mode of \( f \) once (and if \( f \) is complex, \( f^* \) minus once). For periodic complex fermions \( [i.e. \ for \ \alpha(f) = 1] \) the vacuum is a spinor in order to represent the Clifford algebra of the corresponding zero modes. For each periodic complex fermion \( f \), there are two degenerate vacua \( |+\rangle \), \(|-\rangle \), annihilated by the zero modes \( f_0 \) and \( f_0^* \) and with fermion number \( F(f) = 0, -1 \) respectively. In Eq. (2.6), \( \delta_\alpha = -1 \) if \( \psi^\mu \) is periodic in the sector \( \alpha \), and \( \delta_\alpha = +1 \) if \( \psi^\mu \) is antiperiodic in the sector \( \alpha \). The states satisfy the Virasoro condition:

\[ M_L^2 = -\frac{1}{2} + \frac{\alpha_L \cdot \alpha_L}{8} + N_L = -1 + \frac{\alpha_R \cdot \alpha_R}{8} + N_R = M_R^2 \]  

(2.8)

where \( \alpha = (\alpha_L, \alpha_R) \in \Xi \) is a sector in the additive group, and

\[ N_L = \sum_f (\nu_L); \quad N_R = \sum_f (\nu_R) \]  

(2.9)
\[ \nu_f = \frac{1 + \alpha(f)}{2}; \quad \nu_{f^*} = \frac{1 - \alpha(f)}{2}. \]  

(2.10)

The \( U(1) \) charges with respect to the unbroken Cartan generators of the four dimensional gauge group are in one to one correspondence with the \( U(1) \) \( ff^* \) currents. For each complex fermion \( f \):

\[ Q(f) = \frac{1}{2} \alpha(f) + F(f). \]  

(2.11)

The representation (2.11) shows that \( Q(f) \) is identical with the world–sheet fermion numbers \( F(f) \) for states in the Neveu–Schwarz sector \( (\alpha(f) = 0) \) and is \( (F(f) + \frac{1}{2}) \) for states in the Ramond sector \( (\alpha(f) = 1) \). The charges for the \( |\pm\rangle \) spinor vacua are \( \pm \frac{1}{2} \).

### 2.1 The NAHE set

The boundary condition basis vectors which generate the realistic free fermionic models are, in general, divided into two major subsets. The first set consist of the NAHE set \[37, 19\], which is a set of five boundary condition basis vectors denoted \( \{1, S, b_1, b_2, b_3\} \). With ‘0’ indicating Neveu-Schwarz boundary conditions and ‘1’ indicating Ramond boundary conditions, these vectors are as follows:

\[
\begin{array}{cccccccc}
\psi_\mu & \chi^{12} & \chi^{34} & \chi^{56} & \bar{\psi}_{1,\ldots,5} & \bar{\eta}^1 & \bar{\eta}^2 & \bar{\eta}^3 & \bar{\phi}_{1,\ldots,8} \\
1 & 1 & 1 & 1 & 1 & 1,\ldots,1 & 1 & 1 & 1,\ldots,1 \\
S & 1 & 1 & 1 & 1 & 0,\ldots,0 & 0 & 0 & 0,\ldots,0 \\
b_1 & 1 & 1 & 0 & 0 & 1,\ldots,1 & 1 & 0 & 0,\ldots,0 \\
b_2 & 1 & 0 & 1 & 0 & 1,\ldots,1 & 0 & 1 & 0,\ldots,0 \\
b_3 & 1 & 0 & 0 & 1 & 1,\ldots,1 & 0 & 0 & 1,\ldots,0 \\
\end{array}
\]

(2.12)

\[
\begin{array}{cccccccc}
y^{3,\ldots,6} & \bar{y}^{3,\ldots,6} & y^{1,2,5,6} & \bar{y}^{1,2,5,6} & \omega^{1,\ldots,4} & \bar{\omega}^{1,\ldots,4} \\
1 & 1,\ldots,1 & 1,\ldots,1 & 1,\ldots,1 & 1,\ldots,1 & 1,\ldots,1 \\
S & 0,\ldots,0 & 0,\ldots,0 & 0,\ldots,0 & 0,\ldots,0 & 0,\ldots,0 \\
b_1 & 1,\ldots,1 & 1,\ldots,1 & 0,\ldots,0 & 0,\ldots,0 & 0,\ldots,0 \\
b_2 & 0,\ldots,0 & 0,\ldots,0 & 1,\ldots,1 & 1,\ldots,1 & 0,\ldots,0 & 0,\ldots,0 \\
b_3 & 0,\ldots,0 & 0,\ldots,0 & 0,\ldots,0 & 0,\ldots,0 & 1,\ldots,1 & 1,\ldots,1 \\
\end{array}
\]

(2.12)

with the following choice of phases which define how the generalized GSO projections are to be performed in each sector of the theory:

\[ C \left( \begin{array}{c} b_i \\ b_j \end{array} \right) = C \left( \begin{array}{c} b_i \\ S \end{array} \right) = C \left( \begin{array}{c} 1 \\ 1 \end{array} \right) = -1. \]  

(2.13)

The remaining projection phases can be determined from those above through the self-consistency constraints. The precise rules governing the choices of such vectors
and phases, as well as the procedures for generating the corresponding space-time particle spectrum, are given in Refs. \[7\].

The basis vector \(S\) generated the space–time supersymmetry. The set of basis vectors \(\{1, S\}\) generates a model with \(N = 4\) space–time supersymmetry and \(SO(44)\) gauge group in the right–moving sector. Imposing the GSO projections of the basis vectors \(b_1, b_2\) and \(b_3\) reduces the \(N = 4\) supersymmetry to \(N = 1\) and brakes the gauge group to \(SO(10) \times SO(6)^3 \times E_8\). The space-time vector bosons that generate the gauge group arise from the Neveu-Schwarz sector and from the sector \(I \equiv 1 + b_1 + b_2 + b_3\). The Neveu-Schwarz sector produces the generators of \(SO(10) \times SO(6)^3 \times SO(16)\). The sector \(1 + b_1 + b_2 + b_3\) produces the spinorial \(128\) of \(SO(16)\) and completes the hidden gauge group to \(E_8\). The three basis vectors \(b_1, b_2\) and \(b_3\) correspond to the three twisted sectors of the \(Z_2 \times Z_2\) orbifold. Each one of these sectors produces 16 multiplets in the \(16\) representation of \(SO(10)\).

The correspondence of the NAHE set with \(Z_2 \times Z_2\) orbifold is illustrated by adding to the NAHE the boundary condition basis vector \(X\) with periodic boundary conditions for the world–sheet fermions \(\{\overline{\psi}_1, \cdots, \overline{\psi}_5, \overline{\eta}_1, \overline{\eta}_2, \overline{\eta}_3\}\) and all the other world–sheet fermions have antiperiodic boundary conditions,

\[
X = (0, \cdots, 0 | 1, \cdots, 1, 0, \cdots, 0).
\tag{2.14}
\]

The choice of generalized GSO projection coefficients is

\[
C\left(\begin{array}{c}X \\ b_j\end{array}\right) = -C\left(\begin{array}{c}X \\ S\end{array}\right) = C\left(\begin{array}{c}X \\ 1\end{array}\right) = +1.
\tag{2.15}
\]

With the set \(\{1, S, b_1, b_2, b_3, X\}\) the gauge group is \(E_6 \times SO(4)^3 \times E_8\) with 24 generations in the 27 representation of \(E_6\). The same model is constructed in the orbifold formulation by first constructing the Narain lattice with \(SO(12) \times E_8 \times E_8\) gauge group \(N = 4\) supersymmetry. The gauge group is broken to \(E_6 \times SO(4)^3 \times E_8\) after applying the \(Z_2 \times Z_2\) twisting on the \(SO(12)\) lattice. The three twisted sectors produce 48 fixed points, which correspond to the 24 generations in the fermionic construction.

The focus in this paper is on models that contain the full NAHE set of boundary condition basis vectors. Three generation free fermionic models that do not use the full NAHE set can also be constructed \[38, 39\]. Such models correspond to \(Z_2 \times Z_2\) orbifold compactifications without the standard embedding of the gauge connection. However, the NAHE set facilitates the illustration of the basic ingredients that enter the construction of the realistic free fermionic models. These basic ingredients can then also be extended to models that do not use the full NAHE set.

Note that the NAHE set has an \(S_3\) permutation symmetry obtained by permuting the three basis vectors \(b_j\) \((j = 1, 2, 3)\). This permutation symmetry if again a reflection of the underlying \(\tilde{Z}_2 \times Z_2\) orbifold compactification. The basis vectors which extend the NAHE set will typically break the permutation symmetry.
2.2 Beyond the NAHE set

At the level of the NAHE set the observable gauge group is $SO(10) \times SO(6)^3$ and the number of generations is 48, sixteen from each sector $b_1$, $b_2$ and $b_3$. The $SO(6)^3$ symmetries are horizontal flavor dependent symmetries. To break the gauge group and to reduce the number of generations, we must add additional boundary condition basis vectors to the NAHE set. These additional vectors break the $SO(10)$ and the flavor $SO(6)$ gauge symmetries and in turn reduce the number of generations to three generations. The $SO(10)$ gauge group is broken to one of its subgroups $SU(5) \times U(1)$, $SO(6) \times SO(4)$ or $SU(3) \times SU(2) \times U(1)^2$ by the assignment of boundary conditions to the set $\bar{\psi}_{1\cdots 5}^1$:

1. $b\{\bar{\psi}_{1\cdots 5}^1\} = \{11111\} \Rightarrow SU(5) \times U(1)$,
2. $b\{\bar{\psi}_{1\cdots 5}^2\} = \{11000\} \Rightarrow SO(6) \times SO(4)$.

To break the $SO(10)$ symmetry to $SU(3) \times SU(2) \times U(1)_C \times U(1)_L$ both steps, 1 and 2, are used, in two separate basis vectors.

The breaking of the gauge group and the reduction to three generations are done simultaneously. In fact the reduction to three generations is correlated with the breaking of the flavor $SO(6)^3$ symmetries to a product of horizontal $U(1)$ symmetries. The appealing property of the realistic free fermionic models is that the emergence of three generations is correlated with the underlying $Z_2 \times Z_2$ orbifold structure. In the superstring standard–like models each generation is obtained from one of the twisted sectors of the $Z_2 \times Z_2$ orbifold. At the level of the NAHE set each sector $b_1$, $b_2$ and $b_3$ produces sixteen generations, transforming as $2 \otimes 4 \oplus \bar{2} \otimes \bar{4}$ under the left and right–moving flavor symmetries $SO(4)_L \otimes SO(6)_R$. The reduction to three generation is achieved by breaking the flavor symmetries to $U(1)$ flavor symmetries and consequently the multiplicity of generations is reduced. It is clear therefore that it is also possible to construct models in which the flavor symmetry is not fully broken and in that case one of the sectors $b_{1,2,3}$ can give more than one generations while by a suitable choice of GSO projection coefficient all the generations from the orthogonal sectors are projected out. Realizing this possibility it is also possible to construct models with non–Abelian flavor symmetry.

In the models that utilize the full NAHE set and in which each generation is obtained from one of the sectors $b_1$, $b_2$ and $b_3$ the flavor symmetries are broken to $U(1)^n$, with $3 \leq n \leq 9$. Three $U(1)$ symmetries arise from the complex right–moving fermions $\bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3$. Additional horizontal $U(1)$ symmetries arise by pairing two of the right–moving real internal fermions $\{\bar{y}, \bar{\omega}\}$. For every right–moving $U(1)$ symmetry, there is a corresponding left–moving global $U(1)$ symmetry that is obtained by pairing two of the left–moving real fermions $\{y, \omega\}$. Each of the remaining world–sheet left–moving real fermions from the set $\{y, \omega\}$ is paired with a right–moving real fermion from the set $\{\bar{y}, \bar{\omega}\}$ to form a Ising model operator. The rank of the final gauge group

$^aU(1)_C = \frac{1}{2}U(1)_{B-L}; U(1)_L = 2U(1)_{T_{3R}}$. 

9
depends on the number of such pairings. If all right–moving (and hence left–moving) fermions were complex, then the gauge group would have rank 22. Each complexified right–moving pair of real world–sheet fermions generates a $U(1)$ subgroup of the rank 22 gauge group. The rank is reduced by the combinations left– and right–moving real fermions which form the Ising model operators. We can form twelve such combinations and therefore the minimal gauge group has rank sixteen. In addition to fixing the rank of the final gauge group, this pairing of left– and right–moving fermions, through the assignment of boundary conditions, plays an important role in fixing some of the low energy properties of the physical spectrum.

To study the construction of the basis vectors beyond the NAHE set it is convenient to use a notation which emphasizes the division of the world–sheet fermions by the NAHE set. This division of the world–sheet fermions is a reflection of the equivalent underlying $Z_2 \times Z_2$ orbifold compactification. Each one of the sectors $b_1$, $b_2$ and $b_3$ has periodic boundary conditions with respect to $\{\psi^{1,2}, \bar{\psi}^{1,\ldots,5}\}$ and one of the sets,

$$\{\chi^{12}, y^{3,\ldots,6}, \bar{y}^{1}\}, \quad \{\chi^{34}, y^{1,\ldots,2}, \omega^{5,6}, \bar{y}^{1,2}, \bar{\omega}^{5,6}, \bar{\eta}^{2}\} \quad (2.16)$$

$$\{\chi^{56}, \omega^{1,\ldots,4}, \bar{\omega}^{1,\ldots,4}, \bar{\eta}^{3}\} \quad (2.17)$$

$$\{\chi^{56}, \omega^{1,\ldots,4}, \bar{\omega}^{1,\ldots,4}, \bar{\eta}^{3}\} \quad (2.18)$$

The $\psi^{\mu}_{1,2}$ are the space–time fermions and $\bar{\psi}^{1,\ldots,5}$ produce the observable $SO(10)$ symmetry. The three complex fermions $\chi^{12}$, $\chi^{34}$ and $\chi^{56}$ correspond to the fermionic superpartners of the compactified dimensions and carry the space–time supersymmetry charges. The set of internal fermions $\{y, \omega|\bar{y}, \bar{\omega}\}^{1,\ldots,6}$ corresponds to the left–right symmetric conformal field theory of the heterotic string, or equivalently to the six–dimensional compactified manifold in a bosonic formulation. This set of left/right symmetric internal fermions plays a fundamental role in the determination of the low energy properties of the realistic free fermionic models. The analysis of models beyond the NAHE set is reduced almost entirely to the study of the boundary conditions of these internal real fermions. Below I employ a table notation which emphasizes the division of the internal fermionic states according to their division by the NAHE set. In the tables, the real fermionic states $\{y, w|\bar{y}, \bar{w}\}$ are divided according to their division by the NAHE set. The pairing of real fermions into complex fermions or into Ising model operators is noted in the table. The entries in the table represent the boundary conditions in a basis vector for all the fermionic states. The basis vectors in a given table are the three basis vectors which extend the NAHE set. The notation used is exemplified in table $[2.19]$ below. In the first table the boundary condition of: the space–time world–sheet fermions, the left–moving complex fermions $\chi^{12}$, $\chi^{34}$ and $\chi^{56}$ and of the sixteen complex right–moving world–sheet fermions $\psi^{1,\ldots,5}, \bar{\eta}^{1}, \bar{\eta}^{2}, \bar{\eta}^{3}, \bar{\psi}^{1,\ldots,8}$ are shown. In the second table the boundary conditions under the world–sheet fermions which correspond to the compactified space, $\{y, \omega|\bar{y}, \bar{\omega}\}^{1,\ldots,6}$ are given. The boundary conditions in the first table therefore fix the
final observable and hidden gauge groups. The boundary conditions in the second table fix many of the properties of the low energy observable spectrum, like for example the number of generations, the rank of the final gauge group, the presence of Higgs doublets and the projection of Higgs color triplets, and the non vanishing Yukawa couplings. The discussion below will therefore mostly focus on the assignment of boundary conditions to these set of world–sheet fermions and the boundary conditions of the remaining world–sheet fermions will sometimes not be shown explicitly.

Note that in the notation used here, basis vectors which preserve the $SO(10)$ symmetry are denoted by $b_j$ ($j = 4, 5, ...$) while basis vectors which break the $SO(10)$ symmetry are denoted by small Greek letters. The model of table [2.19] is an example of a three generation $SU(5) \times U(1)$ model. The twelve real left–moving fermions \{\(y, \omega\}\} are combined with twelve real right–moving fermions \{\(\bar{y}, \bar{\omega}\}\} to form twelve Ising model operators. Therefore, the rank of the final right–moving gauge group in this model is sixteen.

The boundary condition basis vectors span a finite additive group. Then any subset of the independent vectors of the additive group will span the same model, up to the choice of GSO projection coefficients. Given that the additive group contains typically $2^7 \times 4$ sectors the question then is how can we avoid reproducing the same models. In computerized searches this problem is addressed in [40]. However, when trying to develop an insight how the boundary conditions fix the physical properties of the models a computerized search is not suitable. The problem is avoided by requiring that the basis vectors of a new model cannot all be obtained by a linear combination of the basis vectors of a previous model. Each new model must contain at least one basis vector which cannot be realized as a linear combination of the basis vectors of a previous model. The range of allowed basis vectors is still however very large. In addition each choice of basis vectors can span a set of distinct models by the discrete choices of free phases, as will be illustrated below.
3 Classification by boundary condition rules

In this section I discuss the role of the boundary conditions in the determination of the low energy properties of the superstring models. The purpose here is to illustrate how several of the properties of the low energy spectrum are determined by general boundary condition rules. As some of these results have already appeared previously in the literature, the discussion will be concise.

3.1 Higgs doublet–triplet splitting

The Higgs doublet–triplet splitting operates as follows. The Neveu–Schwarz sector gives rise to three fields in the 10 representation of $SO(10)$. These contain the Higgs electroweak doublets and color triplets. Each of those is charged with respect to one of the horizontal $U(1)$ symmetries $U(1)_{1,2,3}$. Each one of these multiplets is associated, by the horizontal symmetries, with one of the twisted sectors, $b_1$, $b_2$ and $b_3$. The doublet–triplet splitting results from the boundary condition basis vectors which breaks the $SO(10)$ symmetry to $SO(6) \times SO(4)$. We can define a quantity $\Delta_i$ in these basis vectors which measures the difference between the boundary conditions assigned to the internal fermions from the set $\{ y, w \}$ and which are periodic in the vector $b_i$:

$$\Delta_i = |\alpha_L(\text{internal}) - \alpha_R(\text{internal})| = 0, 1 \quad (i = 1, 2, 3) \quad (3.1)$$

If $\Delta_i = 0$ then the Higgs triplets, $D_i$ and $\bar{D}_i$, remain in the massless spectrum while the Higgs doublets, $h_i$ and $\bar{h}_i$, are projected out and the opposite occurs for $\Delta_i = 1$.

Thus, the rule in Eq. (3.1) is a generic rule that can be used in the construction of the free fermionic models, with the NAHE set. The model of table [3.2] illustrates this rule. In this model $\Delta_1 = \Delta_2 = 0$ while $\Delta_3 = 1$. Therefore, this model produces two pairs of color triplets and one pair of Higgs doublets from the Neveu–Schwarz sector, $D_1$, $\bar{D}_1$, $D_2$, $\bar{D}_2$ and $h_3$, $\bar{h}_3$.

| $\psi^\mu$ | $\chi^{12}$ | $\chi^{34}$ | $\chi^{56}$ | $\bar{\psi}^{1\ldots5}$ | $\bar{\eta}^1$ | $\bar{\eta}^2$ | $\bar{\eta}^3$ | $\bar{\varphi}^{1\ldots8}$ |
|------------|------------|------------|------------|----------------|------------|------------|------------|----------------|
| $\alpha$   | 1 1 0 0 0 0 | 1 1 1 0 0 0 | 1 0 1       | 1 1 1 1 0 0 0 | 1 1 1 0 0 0 | 1 1 1 0 0 0 | 1 1 1 0 0 0 | 1 1 1 0 0 0 0 |
| $\beta$    | 1 0 1 0 0 0 | 1 1 1 0 0 0 | 0 1 1       | 1 1 1 1 0 0 0 | 1 1 1 0 0 0 | 1 1 1 0 0 0 | 1 1 1 0 0 0 | 1 1 1 0 0 0 0 |
| $\gamma$   | 1 0 0 1 0 1 | 1 1 1 1 1 1 | 1 1 1 1 1 1 | 1 1 1 1 1 1 | 1 1 1 1 1 1 | 1 1 1 1 1 1 | 1 1 1 1 1 1 | 1 1 1 1 1 1 |

With the choice of generalized GSO coefficients:

$$c \left( \begin{array}{c} b_1, b_3, \alpha, \beta, \gamma \\ \alpha \end{array} \right) = -c \left( \begin{array}{c} b_2 \\ \alpha \end{array} \right) = c \left( \begin{array}{c} 1, b_j, \gamma \\ \beta \end{array} \right) =$$
c \left( \gamma \right) = -c \left( \frac{\gamma}{b_3} \right) = -1

(j=1,2,3), with the others specified by modular invariance and space–time supersymmetry.

Another relevant question with regard to the Higgs doublet–triplet splitting mechanism is whether it is possible to construct models in which both the Higgs color triplets and electroweak doublets from the Neveu–Schwarz sector are projected out by the GSO projections. This is a viable possibility as we can choose for example

$$\Delta_j^{(\alpha)} = 1 \text{ and } \Delta_j^{(\beta)} = 0,$$

where $\Delta_j^{(\alpha,\beta)}$ are the projections due to the basis vectors $\alpha$ and $\beta$ respectively. This is a relevant question as the number of Higgs representations, which generically appear in the massless spectrum, is larger than what is allowed by the low energy phenomenology. Consider for example the model in table \[3.3\]

\[
\begin{array}{c|cccc|cccc|cccc}
\psi^\mu & \chi^{12} & \chi^{34} & \chi^{56} & \bar{\psi}^{1,...,5} & \bar{\eta}^1 & \bar{\eta}^2 & \bar{\eta}^3 & \bar{\phi}^{1,...,8} \\
\hline
\alpha & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
\beta & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\gamma & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

(3.3)

With the choice of generalized GSO coefficients:

$$c \left( s, b_j, \alpha, \beta, \gamma \right) = c \left( \alpha, \beta, \gamma \right) =$$

$$c \left( \frac{\beta}{\beta, \gamma} \right) = c \left( \frac{\gamma}{1} \right) = -1$$

(j=1,2,3), with the others specified by modular invariance and space–time supersymmetry. In this model $\Delta_1^{(\alpha)} = \Delta_2^{(\alpha)} = \Delta_3^{(\alpha)} = 1$, and $\Delta_1^{(\beta)} = \Delta_3^{(\beta)} = 0$, Therefore, In this model irrespective of the choice of the generalized GSO projection coefficients, both the Higgs color triplets and electroweak doublets associated with $b_1$ and $b_3$ are projected out by the GSO projections. However, it is found that the combination of these projections also results in the projection of some of the representations from the corresponding sectors $b_1$ and $b_3$ and therefore these sectors do not produce the full chiral 16 of $SO(10)$. Therefore, realization of this mutual projection of both Higgs triplets and doublets from the Neveu–Schwarz sector requires that the chiral generations be obtained from non–NAHE set basis vectors.
3.2 Cubic level Yukawa couplings

As a second example I discuss how the boundary condition basis vectors fix the cubic level Yukawa couplings for the quarks and leptons. These Yukawa couplings are fixed by the vector $\gamma$ which breaks the $SO(10)$ symmetry to $SU(5) \times U(1)$. Each sector $b_i$ gives rise to an up–like or down–like cubic level Yukawa coupling. We can define a similar quantity $\Delta$ in the vector $\gamma$ which breaks the $SO(10)$ symmetry to $SU(5) \times U(1)$. The quantity $\Delta$ again measures the difference between the left– and right–moving boundary conditions assigned to the internal fermions from the set \{y, w, $\bar{y}$, $\bar{w}$\} and which are periodic in the vector $b_i$,

$$\Delta_i = |\gamma_L(\text{internal}) - \gamma_R(\text{internal})| = 0, 1 \quad (i = 1, 2, 3)$$

If $\Delta_i = 0$ then the sector $b_i$ gives rise to a down–like Yukawa coupling while the up–type Yukawa coupling vanishes. The opposite occurs if $\Delta_i = 1$. The model of table [3.5] illustrates how this rule works. In this model $\Delta_1 = \Delta_3 = 1$ and $\Delta_2 = 0$. Therefore in this model the sectors $b_1$ and $b_3$ produce up type quark Yukawa couplings while the sector $b_2$ produces Yukawa couplings for the down type quark and for the charged lepton.

$$\begin{array}{cccccccc}
\alpha & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\
\beta & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\
\gamma & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 1 & 1 & 1 & 1 \\
\end{array}$$

(3.4)

With the choice of generalized GSO coefficients:

$$c(b_1, b_3) = -c(\gamma) = -c(\alpha, \beta) =$$

$$c(\gamma) = c(\beta) = -c(\alpha, \beta) = -1$$

($j = 1, 2, 3$), with the others specified by modular invariance and space–time super-symmetry. In this model therefore we obtain at the cubic level $Q_1u_1h_1$, $L_1N_1\bar{h}_1$, $Q_2d_2h_2$, $L_2e_2h_2$ and $Q_3u_3\bar{h}_3$, $L_3N_3\bar{h}_3$, irrespective of the choice of GSO projection coefficients.

Another point that should be noted is that there is a $Z_2$ ambiguity in the definition of the weak hypercharge. We can define the weak hypercharge by

$$U(1)_Y = \frac{1}{3}U(1)_C \pm \frac{1}{2}U(1)_L$$

(3.6)
where the + choice is the one typically chosen in the literature. The alternative choice corresponds to the flipping of the representations

\begin{align*}
e^c_L &\leftrightarrow N^c_L \\
u^c_L &\leftrightarrow d^c_L
\end{align*}

This flip is equivalent to the flip between the straight and flipped $SU(5)$ representations. In the case of $SU(5)$ only the later choice is allowed as there are no adjoint representations to break the non–Abelian gauge symmetry in the former. In the case of the standard–like models, as the GUT non–Abelian symmetry is broken directly at the string level, this flip can be consistent with the low energy constraints. We note however that under this $Z_2$ flip the Higgs representations are also flipped $h \leftrightarrow \bar{h}$ and therefore the Yukawa coupling rule is invariant under this ambiguity in the definition of the weak hypercharge.

The above two examples illustrate how general rules can be extracted which determine how the boundary conditions fix the low energy phenomenological properties of the string models. These results can be proven by using the definition of the GSO projection and the modular invariance rules [41, 27].

4 Classification by the role of the free phases

The focus in this section will be on the effect of the one–loop GSO phases

\[ c(a_i a_j) \]

on the properties of the massless spectrum of the realistic free fermionic models. Here \( a_i, a_j \) represent the NAHE set, and the additional, boundary conditions basis vectors which generate the realistic free fermionic models.

4.1 General remarks

Like with the boundary conditions it is convenient to split the free phases into the NAHE and beyond the NAHE sectors. In Eq. (4.1) I employ a matrix notation for the free phases between all the boundary condition basis vectors. For example,
in the model of ref. [17] the following choice of free phases has been made,

\[
\begin{pmatrix}
1 & S & b_1 & b_2 & b_3 & \alpha & \beta & \gamma \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & i \\
S & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
b_1 & -1 & -1 & -1 & -1 & -1 & -1 & i \\
b_2 & 1 & -1 & -1 & -1 & -1 & -1 & i \\
b_3 & -1 & -1 & -1 & -1 & -1 & -1 & 1 \\
\alpha & 1 & -1 & -1 & -1 & 1 & -1 & i \\
\beta & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\
\gamma & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\
\end{pmatrix}
\]

(4.1)

Only the entries above the diagonal are independent and those below and on the diagonal are fixed by the modular invariance constraints. In the matrix above blank lines are inserted to emphasize the division of the free phases between the different sectors of the realistic free fermionic models. Thus, the first two lines involve only the GSO phases of \(c^{\left({1, S}\right)}\). The set \(\{1, S\}\) generates the \(N = 4\) model with \(S\) being the space–time supersymmetry generator and therefore the phases \(c^{\left(S\right)}\) are those that control the space–time supersymmetry in the superstring models. Similarly, in the free fermionic models, sectors with periodic and anti–periodic boundary conditions, of the form of \(b_i\), produce the chiral generations. The phases \(c^{\left(b_i\right)}\) determine the chirality of the states from these sectors, and as will be shown below give rise to the mirror symmetry. Likewise, in the free fermionic models the basis vector \(b_i\) are those that respect the \(SO(10)\) symmetry while the vectors denoted by Greek letters are those that break the \(SO(10)\) symmetry. The phenomenology of the Standard Model sector including the texture of fermion mass matrices is obtained exclusively from the untwisted sector, the sectors \(b_i\) and \(b_i + 2\gamma\) and the sector \((b_1 + b_2 + \alpha + \beta)\). These are the sectors which preserve the \(SO(10)\) symmetry. Thus, the phases that play a role in the phenomenology of the Standard Model sector are, in general, the phases \(c^{\left(b_i\right)}\). On the other hand the basis vectors of the form \(\{\alpha, \beta, \gamma\}\) break the \(SO(10)\) symmetry. The phases associated with these basis vectors in general are associated with the exotic physics sectors that go beyond the Standard Model. These sectors for example control the number of additional exotic color triplets, beyond the spectrum of the Minimal Supersymmetric Standard Model. Therefore the phases associated with these basis vectors will be important, for example, with regard to the problem of string scale gauge coupling unification.

### 4.2 \(E_6 \rightarrow SO(10)\) breaking

The NAHE set plus the vector \(X\) produce a model with \(SO(4)^3 \times E_6 \times U(1)^2 \times E_8\) gauge group and with (2,2) world–sheet supersymmetry. The realistic free fermionic
models have at the level of the NAHE set an $SO(10)$ symmetry and only $(2, 0)$ world-sheet supersymmetry. However, the realistic free fermionic models have some underlying $(2,2)$ structure which is why we can identify the internal fermions $\{y, \omega|\bar{y}, \bar{\omega}\}$ with the compactified dimensions in the bosonic formulation. In the realistic free fermionic models the basis vector $2\gamma$ replaces the vector $X$. The set $\{1, S, I = 1+b_1+b_2+b_3, 2\gamma\}$ produces a model with $N = 4$ supersymmetry and $SO(12) \times SO(16) \times SO(16)$ gauge group. Alternatively, we can construct the same model by using the set $\{1, S, X, I\}$. Using this set we can construct two $N = 4$ models which differ by the discrete choice of the free phase

$$c \left( \begin{array}{c} X \\ I \end{array} \right) = \pm 1$$

(4.2)

For one choice of this phase the gauge group is $SO(12) \times E_8 \times E_8$ which is the standard toroidal compactification. However, for the second choice of this phase the states in the spinorial representation of $SO(16)$ which make up the adjoint of $E_8$ are projected out by the GSO projections. Thus, we are left with $SO(12) \times SO(16) \times SO(16)$ gauge group. Applying the orbifold twisting on the first choice, with $E_8 \times E_8$ gauge group, and with standard embedding, breaks one of the $E_8$ to $E_6 \times U(1)^2$ and gives rise to the models with $(2,2)$ world–sheet supersymmetry. Applying the same orbifold twisting to the second discrete choice results in the models with $SO(10) \times U(1)^3$ gauge group. Alternatively, we can start with the basis vectors that produce the $E_6 \times U(1)^2$ model and then turn on the GSO projection which projects out the spinorial $16 + \overline{16}$ in the adjoint of $E_6$. We end with the same model, as the spectrum is invariant under the reordering of the GSO projections. Thus, the breaking of the right–moving $N = 2$ world–sheet supersymmetry can be seen to be a result of a discrete choice of the free phases. Further, analysis of this breaking in connection with the anomalous $U(1)_A$ which appears in these models, will be reported in ref. [36]. This is an important observation because many of the useful simple relations that are obtained for $(2,2)$ models can be used for the realistic free fermionic models. It is precisely for this reason that the set of real fermions $\{y, \omega|\bar{y}, \bar{\omega}\}$ can be identified with the six left– and right–moving compactified dimensions.

### 4.3 free phases and mirror symmetry

Mirror symmetry in the free fermionic models is a result of the choices of free phases. To see how this works I consider for example the $(2,2)$ model generated by the set

$$\{1, S, X, I, b_1, b_2\}.$$ 

with the choice of free phases

$$c \left( \begin{array}{c} b_i \\ b_j \end{array} \right) = c \left( \begin{array}{c} b_i \\ S \end{array} \right) = c \left( \begin{array}{c} b_i \\ I, X \end{array} \right) = -c \left( \begin{array}{c} I \\ S \end{array} \right) = -c \left( \begin{array}{c} I \\ X \end{array} \right) = -c \left( \begin{array}{c} 1 \\ 1 \end{array} \right) = -1,$$ 

(4.3)

In this model the gauge group is $SO(4)^3 \times E_6 \times U(1)^2 \times E_8$ with $N = 1$ space–time supersymmetry. There are 24 generations in the $27$ representation of $E_6$, eight from
each twisted sector. In the fermionic construction these are the sectors \((b_1; b_1 + X)\), \((b_2; b_2 + X)\) and \((b_3; b_3 + X)\), where the sectors \(b_j\) produce the 16 and the sectors \(b_j + X\) produce the \(10 \oplus 1\) representations in the decomposition of the \(E_6\) representations under \(SO(10) \times U(1)\).

In this model the only internal fermionic states which count the multiplets of \(E_6\) are the real internal fermions \(\{y, w|\bar{y}, \bar{w}\}\). This is observed by writing the degenerate vacuum of the sectors \(b_j\) in a combinatorial notation. The vacuum of the sectors \(b_j\) contains twelve periodic fermions. Each periodic fermion gives rise to a two dimensional degenerate vacuum \(|\pm\rangle\) and \(|\mp\rangle\) with fermion numbers 0 and \(-1\), respectively. The GSO operator, is a generalized parity, operator which selects states with definite parity. After applying the GSO projections, we can write the degenerate vacuum of the sector \(b_1\) in combinatorial form

\[
\left[\begin{array}{c}
\binom{4}{0} + \binom{4}{2} + \binom{4}{4} \\
\binom{5}{0} + \binom{5}{2} + \binom{5}{4}
\end{array}\right] \left[\begin{array}{c}
\binom{2}{0} + \binom{2}{1} + \binom{2}{3} + \binom{2}{5} \\
\binom{1}{1} + \binom{1}{3}
\end{array}\right]
\]

(4.4)

where \(4 = \{y^3 y^4, y^3 y^6, \bar{y}^4 \bar{y}^6, \bar{y}^5 \bar{y}^6\}\), \(2 = \{(\psi^\mu, \chi^{12})\}, \) \(5 = \{\bar{\psi}^{1, \cdots, 5}\}\) and \(1 = \{\bar{\eta}^1\}\). The combinatorial factor counts the number of \(|\mp\rangle\) in the degenerate vacuum of a given state. The two terms in the curly brackets correspond to the two components of a Weyl spinor. The \(10 + 1\) in the 27 of \(E_6\) are obtained from the sector \(b_j + X\). From Eq. (4.3) it is observed that the states which count the multiplicities of \(E_6\) are the internal fermionic states \(\{y^3, \cdots, 6|\bar{y}^3, \cdots, 6\}\). A similar result is obtained for the sectors \(b_2\) and \(b_3\) with \(\{y^{1,2}, \omega^{5,6}|\bar{y}^{1,2}, \bar{\omega}^{5,6}\}\) and \(\{\omega^{1, \cdots, 4}|\bar{\omega}^{1, \cdots, 4}\}\) respectively, which indicates that these twelve states correspond to a six dimensional compactified orbifold with Euler characteristic equal to 48.

The chirality of the states from a twisted sector \(b_j\) is fixed by the GSO projection induced by another twisted sector \(b_i\), with \(i \neq j\). The chirality under the space–time spinor group is determined by the sign of the vacuum of the space–time fermion \(\psi^\mu\). To obtain the chiral representation of the \(E_6\) gauge group, i.e. to project the 27 from the spectrum a “chirality condition” is obeyed \([37]\). This “chirality condition” states that in the sector \(b_i\), which induces the chirality projection, the world–sheet fermions \(\{\psi^\mu, \bar{\psi}^{1, \cdots, 5}\}\) are periodic. The remaining world–sheet fermions which are periodic in the vector \(b_j\) are antiperiodic in the vector \(b_i\). Thus, the chirality of the states from a twisted sector \(b_j\) is determined by the free phase \(c(b_j/b_i)\). Since we have a freedom in the choice of the sign of this free phase, we can get from the sector \((b_j; b_j + X)\) either the 27 or the \(\bar{27}\). Which of those we obtain in the physical spectrum depends on the sign of the free phase. Thus, mirror symmetry in the free fermionic construction is the statement

\[
c\left(\frac{b_j}{b_i}\right) \rightarrow - c\left(\frac{b_j}{b_i}\right)
\]

(4.5)
The free phases \( c^{(b_j)}_{b_i} \) also fix the total number of chiral generations. Since there are two \( b_i \) projections for each sector \( b_j \), \( i \neq j \) we can use one projections to project out the states with one chirality and the other projection to project out the states with the other chirality. Thus, the total number of generations with this set of basis vectors is given by

\[
8\left( \frac{c^{(b_j)}_{b_2} + c^{(b_i)}_{b_1}}{2} \right) + 8\left( \frac{c^{(b_2)}_{b_1} + c^{(b_3)}_{b_2}}{2} \right) + 8\left( \frac{c^{(b_1)}_{b_2} + c^{(b_3)}_{b_1}}{2} \right)
\]

Since the modular invariance rules fix \( c^{(b_j)}_{b_i} = c^{(b_i)}_{b_j} \) we get that the total number of generations is either 24 or 8.

We can now see that the same mirror phenomena also holds in the case of the \((2,0)\) models. As the first example consider the case in which we start with the \((2,2)\) model and then project the \(16 \oplus 16\) generators in the adjoint of \(E_6\). As explained above, this is achieved by the choice of the free phase or by replacing the vector \(X\) with the vector \(2\gamma\). The gauge group now is \(SO(10) \times U(1)^3 \times SO(4)^3 \times SO(16)\) In this case the \(10 \oplus 1\) states from the sectors \(b_j + X\) are projected out from the massless spectrum and instead we obtain the states in the vectorial 16 representations of the hidden \(SO(16)\) gauge group. The sectors \(b_j\) still produce the chiral 16 representation of \(SO(10)\), with the same vacuum degeneracy as in Eq. (4.4). The chirality is obtained in the same way and therefore the mirror models are obtained by taking \(c^{(b_j)}_{b_i} \rightarrow -c^{(b_j)}_{b_i}\) as in the \((2,2)\) model. Thus, in this case the \((2,2)\) structure is still preserved after the projection. The world–sheet fermions \(\{y, \omega | \bar{y}, \bar{\omega}\}\) are identified with the compactified dimensions.

The mirror symmetry as a consequence of the free phases is also observed in models that do not preserve the \((2,2)\) structure. Consider for example the model generated by the NAHE set, \(\{1, S, b_1, b_2, b_3\}\). This model has a gauge group \(SO(10) \times SO(6)^3 \times E_8\) gauge symmetry. The world–sheet fermions \(\{\bar{y}, \bar{\omega}\}\) and \(\bar{\eta}^{1,2,3}\) are mixed and produce the horizontal \(SO(6)^3\) symmetries. However, also in this case the same “chirality condition” operates and the chiral spectrum is produced by the projection of \(b_j\) on \(b_i\), \(i \neq j\). In this case we obtain twice the number of generations, namely 48 (16 from each sector \(b_j\)). In addition to the states in Eq.(4.4) we have the states with the spin \(-1/2\) vacuum for \(\bar{\eta}^j\) and with parity \(-1\) for the fermions from the set \(\{y, \omega | \bar{y}, \bar{\omega}\}\). In this model as well the mirror families are obtained by the transformation \(c^{(b_j)}_{b_i} \rightarrow -c^{(b_j)}_{b_i}\). Thus, depending on the choice of the free phases we can construct with this choice of boundary condition basis vectors, models with 48 or 16 generations and their mirrors. The mirror symmetry operates in this model in the same way that it does in the \((2,2)\) model and in the previous \((2,0)\) model which preserves the \((2,2)\) structure.

The mirror symmetry in the free fermionic models with \(Z_2\) twists is therefore seen to be a result of the freedom to choose the free GSO phases. In general, for any sector of the form of the sectors \(b_j\) we can get the opposite chirality by flipping the sign of
the phase $c^{(b_i)}_{(b_j)}$ provided that the sectors $b_j$ and $b_i$ satisfy the “chirality condition”. Provided that all the vectors which satisfy the “chirality condition” with the sector $b_j$ have the same sign for $c^{(b_i)}_{(b_j)}$, insures that the states from the sector $b_j$ are not completely projected out. The mirror model of a given model is obtained by taking $c^{(b_i)}_{(b_j)} \rightarrow -c^{(b_i)}_{(b_j)}$ for all $i$ and $j$.

It is important to note that the mirror symmetry in terms of the free phases corresponds to the same phenomena in terms of discrete torsion in the orbifold models [42, 43]. However, we note that the description in terms of the free fermions seems to be richer than the corresponding orbifold construction. In the orbifold model we can get a model and its mirror by turning on the discrete torsion. However, in the fermionic models, in addition to a model and its mirror obtained by a global GSO phase change $c^{(b_i)}_{(b_j)} \rightarrow -c^{(b_i)}_{(b_j)}$ for all $i$ and $j$, we have the models which are not related by the change of the sign of the Euler characteristic, but in which the Euler characteristic is modified discretely. It is intriguing also to note that similar ambiguity in terms of discrete–like torsion may also play a role in string strong–weak coupling duality [44].

4.4 Free phases and space–time supersymmetry

In this subsection I briefly discuss the relation between free phases and space–time supersymmetry. While supersymmetry is not yet an observed symmetry of nature it does have many attractive features which motivates the search for superparticles at the electroweak scale. Supersymmetry for example explains the origin of the electroweak symmetry breaking scale in terms of the higher unification scale and the evolution of the couplings to the low scale. This is an important property and the recent observation of the top quark mass is in good agreement with the expectations from supersymmetry. Another important aspect of supersymmetry is that it naturally arises in superstring theory which at present is the leading candidate for a theory of quantum gravity. However as the breaking of supersymmetry, in this context, is still obscured, it is important to search for alternative models where supersymmetry is broken at a high scale. For example in ref. [45] this option has been explored and it has been shown that even if supersymmetry is broken near the Planck scale there still exist a “misaligned supersymmetry” that mixes between states across mass levels.

In the realistic free fermionic models the space–time supersymmetry is obtained from the basis vector $S$. Thus, the relevant phases for the space–time supersymmetry are the phases $c^{(S_a)}_{(a_i)}$ where $a_i$ is any basis vector. The sector $S$ produces $N = 4$ space–time supersymmetry and it is broken to $N = 1$ by the basis vectors $b_1$ and $b_2$. This breaking to $N = 1$ is irrespective of the choice of free phases $c^{(S)}_{(b_1)}$ and $c^{(S)}_{(b_2)}$. However, once we fix those phases, if we want to preserve $N = 1$ space–time supersymmetry, all the remaining $c^{(S)}_{(a_i)}$ phases are fixed. For example, it is found that a sufficient way to insure $N = 1$ space–time supersymmetry is to impose $c^{(S)}_{(a_i)} = \delta_{a_i}$ (or $c^{(S)}_{(a_i)} = -\delta_{a_i}$), where $\delta_{a_i} = -1(+1)$ if $\psi^\mu$ periodic (antiperiodic) in the vector $a_i$, respectively. It is
noted that the first choice is the same as the phases for the Neveu–Schwarz sector with any other vector, \( c_{\alpha i}^{(NS)} = \delta_{\alpha i} \).

Therefore, with the first choice to break supersymmetry at the Planck scale requires that we take \( c_{\alpha}^{(S)} \neq \delta_{\alpha} \) for some basis vector beyond the NAHE set. We could also choose \( c_{b_3}^{(S)} \neq \delta_{b_3} \) but this is not a good choice as it may also project out the fermions from the sector \( b_3 \) rather than the bosons. Non-supersymmetric tachyon free models can then be constructed. As an example we can take the model of ref. [18]. Imposing \( c_{\alpha,\beta,\gamma}^{(S)} \neq \delta_{\alpha,\beta,\gamma} \) produces a tachyon free non-supersymmetric model. These models have in general a non-vanishing cosmological constant and are therefore unstable due to the presence of dilaton tadpole diagrams.

### 4.5 Free phases beyond the NAHE set

The GSO phases discussed in the previous sections fix the spectrum that arises from the NAHE set basis vectors. These are the free phases \( c_{b_i}^{(S)} \) \( i, j = 1, 2, 3 \) and \( c_{\alpha i}^{(S)}, a_i = \{S, b_1, b_2, b_3, \alpha, \beta, \gamma\} \). In addition we have the freedom to choose, up to the modular invariance constraints, the discrete phases

\[
c_{b_j}^{(\alpha,\beta,\gamma)} \quad \text{and} \quad c_{\alpha,\beta,\gamma}^{(\alpha,\beta,\gamma)}
\]

These discrete choices of free phases affect as well the physical spectrum and consequently the low energy phenomenology for a given choice of boundary condition basis vectors. The phases in Eq. (4.6) affect mostly the final charges of the states which are obtained from the sectors \( b_1, b_2 \) and \( b_3 \) and therefore fix the charges of three generations under the flavor \( U(1) \) symmetries. The phases in Eq. (4.7) on the other hand fix the spectrum which arises from the exotic sectors and give rise to the exotic Wilsonian matter states. Therefore, these phases are those which determine the spectrum in the models which goes beyond the spectrum of the Standard Model. As a first example the free phases will be shown to affect the number of color triplets in the massless string spectrum and therefore to play an important role in resolving the string coupling unification problem.

It is well known that perturbative string unification predicts that the gauge couplings unify at a scale which is about a factor of twenty larger than the scale at which the couplings are seen to intersect if one assumes only the MSSM spectrum below the unification scale. This discrepancy seemingly should have many possible resolutions remembering that it involves the extrapolation of the gauge parameters over fifteen orders of magnitude. However, surprisingly the problem is not easily resolved. In ref. [35] it was shown that heavy string threshold corrections, light SUSY thresholds, intermediate enhanced non-Abelian gauge symmetry or modified weak hypercharge
normalization do not resolve the problem. It was suggested that the only way to resolve the problem is by having additional color triplets and electroweak doublets beyond the spectrum of the minimal supersymmetric standard model. Alternative proposals based on non-perturbative effects [2, 46] and product of moduli [47] were proposed. However, it should be remarked that these proposals were made in abstract setting and their realization in the context of concrete, viable, string models remains an open question.

In some superstring models the extra needed representations appear from sectors which arise due to the Wilson line breaking of the non-Abelian gauge symmetries. The free phases play again a crucial role in the determination of the physical spectrum and therefore perform a very important function in fixing the physical properties of the string models.

This essential role of the free phases is exemplified with regard to the massless spectrum from the Wilsonian sectors. As indicated above the Wilsonian sectors produce additional massless states. These states can be color triplets, electroweak doublets, color-weak singlets with fractional electric charge, or they can be Standard Model singlets with fractional charge under the $U(1)_Z$. For example in the model of ref. [26], we obtain several additional electroweak doublets from the sectors $b_1 + b_3 + \alpha \pm \gamma + (I)$ and $b_2 + b_3 + \beta \pm \gamma + (I)$, while in the model of ref. [34] we obtain from the same sectors the color triplets while the electroweak doublets are projected out. The two models differ by the choice of one and only one phase, with

$$c^\gamma_1 = -1 \quad \text{(ref. [26])} \quad \rightarrow \quad c^\gamma_1 = +1 \quad \text{(ref. [34])} \quad (4.8)$$

As a second example consider the model of ref. [18]. In this model in addition to the Neveu–Schwarz sector, the sector $X = b_1 + b_2 + b_3 + \alpha + \beta + \gamma + (I)$, where $I = 1 + b_1 + b_2 + b_3$, have $X_L \cdot X_L = 0$ and $X_R \cdot X_R = 8$. Therefore, this sector may give rise to additional space–time vectors bosons that would modify the space–time gauge group. With the choice of GSO phases in ref. [18] all the extra gauge bosons are projected out by the GSO projections. However, with the modified GSO phases

$$c^\gamma_1 \rightarrow -c^\gamma_1, \quad \alpha^\gamma_\beta \rightarrow -c^\alpha_\beta, \quad \gamma^\gamma_\beta \rightarrow -c^\gamma_\beta \quad (4.9)$$

additional space–time vector bosons are obtained from the sector $b_1 + b_2 + b_3 + \alpha + \beta + \gamma + (I)$. The sector $b_1 + b_2 + b_3 + \alpha + \beta + \gamma + (I)$ produces the representations $3_1 + 3_{-1}$ of the hidden $SU(3)_H$ gauge group, where one of the $U(1)$ combinations is the $U(1)$ in the decomposition of $SU(4)$ under $SU(3) \times U(1)$. In this case the hidden $SU(3)_H$ gauge group is extended to $SU(4)_H$ and the hidden sector contains two nonabelian factors $SU(5) \times SU(4)$. The possibility of extending the hidden sector gauge group from twisted sectors may be instrumental in trying to implement the dilaton stabilization mechanism of ref. [18].
As a further example of the role of the free phases, beyond the NA HE set, in the determination of the phenomenology of the free fermionic models consider the model in table [4.10]

\[
\begin{array}{c|cccc|cccc|c}
\psi^\mu & \chi^{12} & \chi^{34} & \chi^{56} & \bar{\psi}^{1\ldots,5} & \bar{\eta}^1 & \bar{\eta}^2 & \bar{\eta}^3 & \bar{\phi}^{1\ldots,8} \\
\hline
b_4 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\beta & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
\gamma & 1 & 0 & 1 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\
\end{array}
\]

With the choice of generalized GSO coefficients:

\[c\left(\frac{b_4}{b_j, \beta}\right) = -c\left(\frac{b_4}{1}\right) = -c\left(\frac{\beta}{1}\right) = c\left(\frac{\beta}{b_j}\right) = -c\left(\frac{\beta}{\gamma}\right) = c\left(\frac{\gamma}{b_2}\right) = -c\left(\frac{\gamma}{b_1, b_3, \alpha, \gamma}\right) = -1\]

\((j = 1, 2, 3),\) with the others specified by modular invariance and space–time supersymmetry. With this choice of GSO projection coefficients the following \(U(1)\) symmetries are anomalous: \(\text{Tr} U_1 = -24, \text{Tr} U_2 = -30, \text{Tr} U_3 = 18, \text{Tr} U_5 = 6, \text{Tr} U_6 = 6\) and \(\text{Tr} U_8 = 12.\) With the anomalous \(U(1)\) symmetry being

\[U_A = -4U_1 - 5U_2 + 3U_3 + U_5 + U_6 + 2U_8.\]

Changing

\[c\left(\frac{b_4}{1}\right) = +1 \rightarrow c\left(\frac{b_4}{1}\right) = -1,\]

changes the anomalous \(U(1)\)s to: \(\text{Tr} U_C = -18, \text{Tr} U_L = 12, \text{Tr} U_1 = -18, \text{Tr} U_2 = -24, \text{Tr} U_3 = 24, \text{Tr} U_4 = -12, \text{Tr} U_5 = 6, \text{Tr} U_6 = 6, \text{Tr} U_7 = -6, \text{Tr} U_8 = 12\) and \(\text{Tr} U_9 = 18,\) and the anomalous \(U(1)\) combination is

\[U_A = -3U_C + 2U_L - 3U_1 - 4U_2 + 4U_3 - 2U_4 + U_5 + U_6 - U_7 + 2U_8 + 3U_9.\]

This modification has an important phenomenological implication. In the model with the phase modification, Eq. (4.12), while the weak hypercharge combination, \(U_Y = 1/3U_C + 1/2U_L,\) is anomaly free, the orthogonal combination, which is embedded in \(SO(10), U_{Z'} = U_C - U_L\) is anomalous. This implies that there exist models in which this \(U_{Z'}\) must be broken near the Planck scale. In such models therefore the universal part of the observable gauge group, arising from the \(SO(10)\) gauge group
of the NAHE set, must be the Standard Model gauge group. A $U(1)$ combination of the flavor dependent $U(1)$s, however, may still remain unbroken.

The examples above illustrate the importance of the discrete choice of the free phases in the phenomenology of the free fermionic models. Depending on the choices of discrete phases we can construct models with different phenomenological properties. Another question of interest is how does the “mirror symmetry”, exhibited by the discrete change of phases in section (4.3) extend to the models that include basis vectors and GSO phases beyond the NAHE set. The basis vectors beyond the NAHE set correspond to Wilson lines in the orbifold language. The duality symmetries in the extended fermionic models then correspond to duality symmetries in orbifold constructions that include Wilson lines. Naturally a complete classification of all such duality symmetries is beyond the scope of this paper, and in general it is difficult to assess what general, model independent, patterns would persist. The simplest examples that can be considered are the models that utilize solely periodic and antiperiodic boundary conditions. These are the Pati–Salam $SO(6) \times SO(4)$ type models. As an example we can examine the model of the first reference in [15].

The basis vectors and GSO phases are shown in table (4.14)

| $S$ | $b_4$ | $b_5$ | $\alpha$ | $\gamma$ |
|-----|------|------|---------|------|
| $\psi_\mu$ | $\chi^{12}$ | $\chi^{34}$ | $\chi^{56}$ | $\bar{\psi}^{1...5}$ | $\bar{\eta}^1$ | $\bar{\eta}^2$ | $\bar{\eta}^3$ | $\bar{\phi}^{1...8}$ |
| $b_4$ | 1   | 0   | 0   | 1   | 1   | 0   | 0   | 0   |
| $b_5$ | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
| $\alpha$ | 1   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
| $\gamma$ | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |

With the choice of generalized GSO coefficients:

$$c \left( S, b_j, b_4, b_5 \right) = c \left( b_2, b_3, b_4 \right) = c \left( b_1, b_3, b_5 \right) = c \left( b_4, b_5, \gamma \right) = c \left( b_1, b_2, \alpha \right) = -c \left( b_i, b_j \right) = -c \left( b_1, b_4, b_5 \right) = -c \left( b_2, b_4, b_5 \right) = -c \left( S, b_1, b_4, b_5, b_6, \alpha \right) = 1$$

$(j = 1, 2, 3)$, with the others specified by modular invariance and space–time supersymmetry. We can now examine what is the effect of discrete modification of the GSO phases. Some modifications, as shown in the examples above, will result in new
models in which the physical spectrum and the phenomenology is modified. However, certain discrete changes will result in models in which the spectrum is invariant but some of the charges under the $U(1)$ world–sheet currents will be modified. This is similar to the mirror symmetry phenomena discussed in section (4.3). Consider for example a global change of the GSO phases in the [4.14] model. Under this modification, as expected, the chiral representations under $SO(10)$ from the sectors $b_j$ ($j = 1, 2, 3$) reverse their chirality. Because of the modification of the $c(S_a)$ GSO phases the space–time gravitino reverses its chirality as well. However the spectrum remains essentially identical. Similarly, we can consider the model which preserves the chirality of the space–time gravitino by keeping the phases $c(S_a)$ unchanged, but taking the opposite sign for all other phases. In this model again the chirality of the chiral representations of the $SO(10)$ gauge group is flipped, but up to this flip the models are identical. Thus, there is a larger set of duality symmetries which exist when considering the larger set of free phases beyond the NAHE set and that may not be necessarily be related to the geometrical dualities which have been discussed in the literature.

5 The $\alpha\beta$ sector

In all the three generation free fermionic models, that utilize the NAHE set and that were studied in detail to date, there exist a special combination of the basis vectors which extend the NAHE set. In the flipped $SU(5)$ and in the Pati–Salam like models this combination is the combination $S + b_4 + b_5$, while in the standard–like models of ref. [18] and ref. [28] the combination is $S + b_1 + b_2 + \alpha + \beta$. The $\alpha$, $\beta$ basis vectors in these two models can be replaced by the combinations $b_1 + \alpha$ and $b_2 + \beta$ which results, with a suitable modification of the GSO phases, in the same physical spectrum. The boundary conditions in this vector combination are, for example, in the model of ref. [18],

$$\{\psi^{1,2}, \chi^5, \omega^1, \bar{\omega}^1, y^2, \bar{y}^2, \omega^3, \bar{\omega}^3, y^4, \bar{y}^4, \bar{\eta}^1, \bar{\eta}^2\}$$

are periodic while all the remaining world–sheet fermions are antiperiodic. In this sector we have $(\alpha\beta)_R \cdot (\alpha\beta)_R = 4$ and $(\alpha\beta)_L \cdot (\alpha\beta)_L = 4$. Massless states from this sector are obtained by acting on the vacuum with one fermionic oscillator with Neveu–Schwarz boundary conditions and frequency $\pm 1/2$. This sector then produces additional vectorial representations of the observable $SO(10)$ symmetry which after applying the GSO projections gives rise to additional electroweak Higgs doublets and color triplets. This is the important property of this sector as the additional Higgs doublets from this sector are crucial for obtaining qualitatively realistic fermion mass matrices. In addition this vector combination produces additional $SO(10)$ singlet fields which are charged under the flavor $U(1)$ symmetries and are used in the cancellation of the anomalous $U(1)$ D–term equation. Therefore, it is desirable to require
that the assignment of boundary conditions beyond the NAHE set admits such a vector combination in a given model. This requirement in turn constrains further the possible distinct models.

6 Classification by real fermion pairings

One of the important constraints in the construction of the free fermionic models is the requirement of a well defined super–current. In the models that utilize only periodic and anti–periodic boundary conditions for the left–moving sector, the eighteen left–moving fermions are divided into six triplets in the adjoint representation of the automorphism group $SU(2)^6$. These triplets are typically denoted by $\{\chi_i, y_i, \omega_i\}$ $i = 1, \cdots, 6$. In this case the allowed boundary conditions of each of these six triplets depend on the boundary condition of the world–sheet fermions $\psi_{1,2}^\mu$. For sectors with periodic boundary conditions, $b(\psi_{1,2}^\mu) = 1$, i.e. those that produce space–time fermions the allowed boundary condition in each triplet are $(1, 0, 0), (0, 1, 0), (0, 0, 1)$ and $(1, 1, 1)$. For sectors with antiperiodic boundary conditions, $b(\psi_{1,2}^\mu) = 0$, i.e. those that produce space–time bosons, the allowed boundary conditions are $(1, 1, 0), (1, 0, 1) (0, 1, 1)$ and $(0, 0, 0)$. The left–moving fermions can be combined in pairs to form to form complex left–moving fermions. Alternatively, a real left–moving fermion can be combined with a real right–moving fermion to form an Ising model operator. The super current constraint and the various desirable phenomenological criteria then limit the possible complex or Ising model combinations of the left–moving fermions. Below I illustrate these constraints in several examples.

In the type of models that are considered here a pair of real fermions which are combined to form a complex fermion or an Ising model operator must have the identical boundary conditions in all sectors. In practice it is sufficient to require that a pair of such real fermions have the same boundary conditions in all the boundary basis vectors which span a given model. The NAHE set of boundary condition basis vectors already divides the eighteen left–moving real fermions into three groups

\begin{align*}
\{(\chi^1, \cdots, \chi^6), (y^1, \cdots, y^6), (\omega^1, \cdots, \omega^6)\} & \quad (6.1) \\
\{(\chi^1, \cdots, \chi^6), (y^1, \cdots, y^6), (\omega^1, \cdots, \omega^6)\} & \quad (6.2) \\
\{(\chi^1, \cdots, \chi^6), (y^1, \cdots, y^6), (\omega^1, \cdots, \omega^6)\} & \quad (6.3)
\end{align*}

where the notation emphasizes the division of the eighteen left–moving internal world–sheet fermions into the $SU(2)^6$ triplets. The $\chi^{12,34,56}$ are the complexified combinations which generate the $U(1)$ current of the $N = 2$ left–moving world–sheet supersymmetry \cite{19}. We have the freedom to complexify all, some or none of the remaining twelve left–moving world–sheet fermions. These different choices will in turn produce superstring models with substantially different phenomenological implications. The first obvious example is that if all of the left–moving fermions are
complexified then the rank of the right–moving gauge group is twenty–two, while if none are complexified then the rank of right–moving gauge group is sixteen.

A more subtle example is noted from the discussion in sections (3.1) and (3.2). The rules obtained there show that for the Higgs color triplets to be projected out in the standard–like models the condition $\Delta_j = 1$ must hold, where $\Delta_j$ is the difference between the boundary conditions of the left– and right–moving real world–sheet fermions. Similarly the up–down quark Yukawa coupling rule shows that to get a cubic level up type mass term requires again $\Delta_j = 1$. It follows that if we impose the presence of Higgs doublets from the Neveu–Schwarz sector in the massless string spectrum, and consequently the projection of the corresponding color Higgs triplets, that not all the twelve left–moving real fermions can be combined to form Ising model operators, as in this case we always have $\Delta_j = 0$. This example illustrate how the pairing of the real world–sheet fermions affects the phenomenology of the free fermionic models.

We can then ask further which desired properties of the fermionic models, which are motivated by the phenomenological constraints, can be consistent with a given choice of pairings. Expressed differently we can ask which properties are compatible with a given choice of real fermions pairings and the consistency of the world–sheet supercurrent.

As an example let us suppose that we would like to impose the existence of three generations, the existence of the type of sector discussed in section (5), and the projection of all color Higgs triplets from the Neveu–Schwarz sector and consider the pairing

$$\{(y^3 y^6, y^4 y^4, y^5 y^5, \bar{y}^3 \bar{y}^6),$$
$$\{y^1 \omega^6, y^2 \bar{y}^2, \omega^5 \bar{\omega}^5, \bar{y}^1 \bar{\omega}^6),$$
$$\{\omega^1 \omega^3, \omega^2 \omega^2, \omega^4 \bar{\omega}^4, \bar{\omega}^1 \bar{\omega}^3\} \}$$

(6.4)

(6.5)

(6.6)

Note that with this pairing the complexified left-moving pairs in Eqs. $(6.4,6.5)$ mix between the first, third and sixth $SU(2)$ triplets of the left–moving automorphism group. The boundary condition of a complexified pair then are forced to be identical in all boundary condition basis vectors. The supercurrent constraints then restrict the possible boundary conditions of the remaining real left–moving world–sheet fermions. With this pairing we can start building the phenomenological characteristics that we would like to impose. We can start by constructing directly the $\alpha\beta$ sector discussed in section (5), while at the same time reducing the number of generations by a factor of 2 from each of the sectors $b_1, b_2$ and $b_3$. This is obtained by the following assignment of boundary conditions

$$\begin{array}{cccccccc}
\alpha\beta & y^3 y^6 & y^4 y^4 & y^5 y^5 & \bar{y}^3 \bar{y}^6 & y^1 \bar{\omega}^6 & y^2 y^2 & \omega^5 \bar{\omega}^5 & \bar{y}^1 \bar{\omega}^6 & \omega^1 \omega^3 & \omega^1 \omega^2 & \omega^4 \bar{\omega}^4 & \bar{\omega}^1 \bar{\omega}^3 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1
\end{array}$$

(6.7)

with $b_{\alpha\beta}\{\psi^1_{12}, \chi^{56}, \bar{\eta}^1, \bar{\eta}^2\} = 1$ as well. Note that the modular invariance constraints on the product $b_{\alpha\beta} \cdot b_j = 0 \mod 2$ forces this form of boundary conditions for the $\alpha\beta$
sector. The next phenomenological criteria that we would like to impose is that all color triplets from the Neveu–Schwarz sector be projected by the GSO projections. This requirement forces the breaking of $SO(10)$ to $SO(6) \times SO(4)$, thus imposing the boundary conditions $b\{\psi^{1,\cdots,5}\} = \{1, 1, 1, 0, 0\}$. Let us denote the additional basis vector as $\delta$. In addition the quantity $\Delta_j$ from section (3.11) must be equal to one for $(j = 1, 2, 3)$. Without loss of generality it is sufficient to contemplate possible assignments with $b_\delta \{\psi^{12}, \chi^{12}, \chi^{34}, \chi^{56}\} = 0$ as all other allowed possibilities can be obtained with combinations of the NAHE set basis vectors. Note that the basis vector $\delta$ can only contain periodic and anti–periodic boundary conditions. If we assign $1/2$ boundary condition to $\tilde{\phi}^{1,\cdots,8}$ we then have $2\delta = 1 + b_1 + b_2 + b_3$ in contradiction with the rule that all basis vectors must be linearly independent. The modular invariance restrictions on $\delta \cdot b_j = 0 \text{ mod } 2$ then forces the vector $\delta$ to have the form

$$
\delta \begin{pmatrix}
 y^3 y^6 & y^4 y^6 & y^5 y^6 & y^5 y^6 & y^1 \omega^6 & y^2 \omega^5 & \omega^6 & \omega^6 & \omega^6 \\
 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\
 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0
\end{pmatrix}
$$

(6.8)

However, this form does not satisfy the modular invariance constraints on the product $b_\delta \cdot b_{\alpha\beta}$. It is therefore found that this two phenomenological properties, namely existence of $\alpha\beta$ sector and projection of color triplets from the Neveu–Schwarz sector cannot be mutually compatible with this choice of pairing of the left–moving real world–sheet fermions.

Next let us consider the pairings

$$
\{(y^3 y^6, y^4 y^6, y^5 y^6, y^5 y^6),
(y^1 \omega^6, y^2 \omega^5, \omega^6, \omega^6),
(\omega^2 \omega^4, \omega^1 \omega^1, \omega^3 \omega^3, \omega^2 \omega^4)\}
$$

(6.9)

Note that with this pairing the complexified left–moving pairs mix between the sixth $SU(2)$ triplet of the left–moving automorphism group. That is the boundary condition of $y^3 y^6$ fixes the boundary condition of $y^1 \omega^6$. A basis vector of the form of $b_{\alpha\beta}$ can again be constructed directly by the following assignment of boundary conditions

$$
\alpha\beta \begin{pmatrix}
 y^3 y^6 & y^4 y^6 & y^5 y^6 & y^5 y^6 & y^1 \omega^6 & y^2 \omega^5 & \omega^6 & \omega^6 & \omega^6 \\
 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1
\end{pmatrix}
$$

(6.10)

with $b\{\psi^{12}, \chi^{56}, \tilde{\eta}^1, \tilde{\eta}^2\} = 1$ as well. The projection of the color triplet Higgs from the Neveu–Schwarz sector is achieved with $b\{\tilde{\psi}^{1,\cdots,5}\} = \{1, 1, 1, 0, 0\}$. Again without loss of generality it is sufficient to contemplate possible assignments with $b\{\psi^{12}, \chi^{12}, \chi^{34}, \chi^{56}\} = 0$ as all other allowed possibilities can be obtained with combinations of the NAHE set basis vectors. The modular invariance restrictions on $\delta \cdot b_j = 0 \text{ mod } 2$ then forces the vector $\delta$ to have the form

$$
\delta \begin{pmatrix}
 y^3 y^6 & y^4 y^6 & y^5 y^6 & y^5 y^6 & y^1 \omega^6 & y^2 \omega^5 & \omega^6 & \omega^6 & \omega^6 \\
 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0
\end{pmatrix}
$$

(6.11)
and the remaining string consistency constraints are fixed by choosing appropriate boundary conditions for the world–sheet fermions which generate the hidden gauge group. Therefore, we note that both the existence of a $\alpha\beta$ combination as well as the projection of the color Higgs triplets can be compatible with this choice of pairing of the left–moving world–sheet fermions. However, the number of generations must still be reduced by breaking the degeneracy among the world–sheet fermions $\{(y^3 y^6, y^5 \bar{y}^5), (y^1 \omega^6, \omega^5 \bar{\omega}^5), (\omega^1 \bar{\omega}^1, \omega^3 \bar{\omega}^3)\}$. We must therefore assign either $b_\gamma(\omega^1) = 1$ or $b_\gamma(\omega^3) = 1$ but not both. Again I am considering assignments with $b_\gamma(\psi_{12}^\mu, \chi^{12}, \chi^{34}, \chi^{56}) = 0$. We note however that $b_\gamma(\omega^1) = 1$ and the supercurrent constraint forces $b_\gamma(y^1, \omega^6) = 1$ which forces $b_\gamma(y^3, y^6) = 1$ which forces $b_\gamma(\omega^3) = 1$.

Similarly, we can consider the other possible assignments for the complex world–sheet fermions $\{(\psi_{12}^\mu, \chi^{12}, \chi^{34}, \chi^{56})\}$ observing that the degeneracy cannot be removed in any case. Therefore, the degeneracy between $\{(\omega^1 \bar{\omega}^1, \omega^3 \bar{\omega}^3)\}$ cannot be removed and a three generation model, compatible with these phenomenological criteria and this choice of pairings cannot be obtained.

Next let us consider the pairings

\[
\begin{align*}
&\{ (y^3 y^6, y^4 \bar{y}^4, y^5 \bar{y}^5, y^3 \bar{y}^6), \\
&(y^1 \omega^6, y^2 \bar{y}^2, \omega^5 \bar{\omega}^5, y^1 \bar{\omega}^6), \\
&(\omega^1 \bar{\omega}^1, \omega^2 \bar{\omega}^2, \omega^3 \bar{\omega}^3, \omega^1 \omega^4) \},
\end{align*}
\tag{6.12}
\]

and consider the formation of a $\alpha\beta$ sector. A sector $\alpha\beta$ is symmetric with respect to two of the NAHE set basis vectors $b_j$. Because of the cyclic permutation of the NAHE set we can impose, without loss of generality, that the $\alpha\beta$ sector be symmetric with respect to $b_1$ and $b_2$. This means that in the vector $\alpha\beta$ the world–sheet fermions $\{\psi_{12}^\mu, \chi^{34}, \bar{\eta}^1, \bar{\eta}^2\}$ have periodic boundary conditions. We observe that with the choice made in Eq. (6.12) a $\alpha\beta$ vector which is symmetric with respect to $b_1$ and $b_2$ cannot be formed. This follows because each one of the first four $SU(2)$ triplets of the left–moving supercurrent must contain one and only one periodic fermion. On the other hand, to reduce the number of generations, we impose that each of the sets in Eq. (6.12) have at least one periodic fermion in the $\alpha\beta$ vector. This two requirements are observed not to be mutually compatible. A similar consideration also applies to the choice of pairings

\[
\begin{align*}
&\{ (y^3 y^6, y^4 \bar{y}^4, y^5 \bar{y}^5, y^3 \bar{y}^6), \\
&(y^1 \omega^5, y^2 \bar{y}^2, \omega^6 \bar{\omega}^6, y^1 \bar{\omega}^5), \\
&(\omega^1 \omega^4, \omega^2 \bar{\omega}^2, \omega^3 \bar{\omega}^3, \omega^1 \omega^4) \}.
\end{align*}
\tag{6.13}
\]

Next let us consider the pairing

\[
\begin{align*}
&\{ (y^3 y^6, y^4 \bar{y}^4, y^5 \bar{y}^5, y^3 \bar{y}^6), \\
&(y^1 \omega^5, y^2 \bar{y}^2, \omega^6 \bar{\omega}^6, y^1 \bar{\omega}^5), \\
&(\omega^2 \omega^4, y^1 \bar{\omega}^1, \omega^3 \bar{\omega}^3, \omega^2 \omega^4) \}.
\end{align*}
\tag{6.14}
\]
This pairing mixes all six triplets of the $SU(2)^6$ left–moving automorphism group and is the one used in the models of refs. [18] and [20]. With this pairing the following assignment of boundary conditions satisfies the desired phenomenological criteria, namely, three generations, existence of $\alpha\beta$ sector and projection of the color Higgs triplets from the Neveu–Schwarz sector by, for example, the following assignment of boundary conditions,

\[
\begin{array}{cccc}
y_3y_6 & y_4y_4 & y_5y_5 & y_6y_6 \\
y_1\omega_5 & y_2y_2 & \omega_6\omega_6 & y_1\omega_5 \\
y_2\omega_4 & \omega_1\omega_2 & \omega_3\omega_3 & \omega_2\omega_4 \\
\end{array}
\]

(6.15)

The above examples illustrate the effect the different choices of pairings of the left–moving real world–sheet fermions on the massless spectrum and phenomenological properties of the free fermionic models. Clearly the choices considered here do not exhaust all the possibilities and it is of further interest to examine which pairings can be compatible with the array of phenomenological constraints that must be imposed on a realistic superstring model.

An interesting question is what is the number of independent pairings that can be formed. Before imposing the NAHE set boundary conditions we have twelve left–moving real fermions. Any of those can be combined to form Ising model or complex fermion. After imposing the NAHE set projections the twelve left–moving real fermions are divided into three groups of four left–moving real fermions each. Thus, the number of allowed pairings is reduce significantly. From each group of four real fermions we can form ten possible combinations. For example for the set \{y_3,...,y_6\} we can form the combinations

\[
\begin{align}
(y^3y^4, y^5y^6), & \quad (y^3y^4, y^5y^5, y^6y^6), & \quad (y^3y^3, y^4y^4, y^5y^6), & \quad (6.16) \\
(y^3y^5, y^4y^6), & \quad (y^3y^5, y^4y^4, y^6y^6), & \quad (y^3y^3, y^5y^5, y^4y^6), & \quad (6.17) \\
(y^3y^6, y^4y^5), & \quad (y^3y^6, y^4y^4, y^5y^5), & \quad (y^3y^3, y^6y^6, y^4y^5), & \quad (6.18) \\
(y^3y^3, y^4y^4, y^5y^5, y^6y^6). & \quad (6.19)
\end{align}
\]

With similar number of possible pairings from the sets \{y^{1,2}, \omega^{5,6}\} and \{\omega^{1,...,4}\}. Thus, we have a total of one thousand possible pairings. Due to the permutation symmetry of the NAHE set and a $Z_2$ symmetry due to the exchange of the two triplets in each set, we can reduce this number by at least a factor of six. We are still left with a number of pairings in excess of 150 possibilities, some of which may still equivalent. However, this illustrates the large number of possible distinct models. It is expected that all possible pairings can produce three generation models with either one of the $SO(10)$ subgroups and the distinct choices differ by their other phenomenological properties as exemplified above.
7 Classification by exotics

In the free fermionic models which utilize the NAHE set the structure of the Standard Model fermion sector is obtained from the NAHE set basis vectors and possibly from one or two of the basis vectors which extend the NAHE set. Therefore the structure of the observable matter spectrum is similar, up to $U(1)$ charges, in different models. The models differ only by the basis vectors which extend the NAHE set. These basis vectors and their combinations with the NAHE set basis vectors produce additional massless spectrum. In the flipped $SU(5)$ and $SO(6) \times SO(4)$ models two of the basis vectors, denoted $b_4$ and $b_5$, do not break the $SO(10)$ symmetry and produce additional $SO(10)$ spinorial representations. However, in each class of models at least one of the basis vectors must break the $SO(10)$ symmetry. Such a basis vector, combined with its combinations with the NAHE set basis vectors, produces additional matter states which do not fall into representations of the original $SO(10)$ gauge group. These new states arise due to the breaking of the GUT gauge group at the string level rather than at the effective field theory level. They are a characteristic of the breaking of the non-Abelian gauge symmetry by Wilson lines and are a generic feature of superstring compactifications \[50, 29\]. In the realistic free fermionic models these type of exotic states can be classified first according to the sectors in which they appear. Namely in each of the sectors which produces exotic states we have different boundary conditions for the right–moving world–sheet fermions, $\bar{\psi}_1^{1, \ldots, 5}$, which generate the $SO(10)$ symmetry. Each type of boundary conditions results in different types of exotic states. Each choice of the final $SO(10)$ gauge subgroup, $SU(5) \times U(1)$, $SO(6) \times SO(4)$ or $SU(3) \times SU(2) \times U(1)^2$ allows distinct types of exotic states. The last possibility, as it contains the previous two $SO(10)$ breaking sectors, admits exotic states which appear also in the $SU(5) \times U(1)$ and $SO(6) \times SO(4)$ models, and in addition gives rise to exotic states which can appear only in the $SU(3) \times SU(2) \times U(1)^2$ models. Thus, one classification by the exotic states is by the type of final $SO(10)$ subgroup which is left unbroken. The other possible classification classifies the models by the actual exotic representations which appear in each model.

The $SO(6) \times SO(4)$ type exotic states are obtained from sectors with $X_R \cdot X_R = 8$ and with

$$X(\bar{\psi}_1^{1, \ldots, 5}) = (1, 1, 1, 0, 0) \quad (7.1)$$

or

$$X(\bar{\psi}_1^{1, \ldots, 5}) = (0, 0, 0, 1, 1). \quad (7.2)$$

The $SU(5) \times U(1)$ type exotic states may be produced from sectors with $X_R \cdot X_R = 8$, $X_R \cdot X_R = 6$ or $X_R \cdot X_R = 4$ and with

$$X(\bar{\psi}_1^{1, \ldots, 5}) = \left(\frac{1}{2}, \frac{1}{2}; \frac{1}{2}, \frac{1}{2}; \frac{1}{2}, \frac{1}{2}\right) \quad (7.3)$$
The $SU(3) \times SU(2) \times U(1)^2$ type exotic states may be produced from sectors with $X_R \cdot X_R = 8$, $X_R \cdot X_R = 6$ or $X_R \cdot X_R = 4$ and with

$$X(\bar{\psi}^{1,\cdots,5}) = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$$

(7.4)

where it should remembered that the weak hypercharge is defined by

$$U(1)_Y = \frac{1}{3}U(1)_C + \frac{1}{2}U(1)_L.$$

In the following I use the notation

$$[(SU(3)_C \times U(1)_C); (SU(2)_L \times U(1)_L)]_{(Q_Y, Q_{Z'}, Q_{e.m.})},$$

(7.5)

where $Q_Y$ is the weak hypercharge of a given state, $Q_{Z'}$ is the charge under the orthogonal combination $U(1)_{Z'} = U(1)_C - U(1)_L$, and for the electroweak doublets the electric charge of the two components is written.

The possible $SO(6) \times SO(4)$ type exotic states are

$$[(3, \frac{1}{2}); (1, 0)]_{(1/6, 1/2, 1/6)} ; \quad [(\bar{3}, -\frac{1}{2}); (1, 0)]_{(-1/6, -1/2, -1/6)} ; \
[(1, \pm 3/2); (1, 0)]_{(\pm 1/2, \pm 1/2, \pm 1/2)}$$

(7.6)

(7.7)

from sectors of the form of Eq. (7.1) and

$$[(1, 0); (2, 0)]_{(0, 0, \pm 1/2)}$$

(7.8)

$$[(1, 0); (1, \pm 1)]_{(\pm 1/2, \pm 1/2, \pm 1/2)}$$

(7.9)

from sectors of the form of Eq. (7.2).

The possible $SU(5) \times U(1)$ type exotic states are

$$[(1, \pm 3/4); (1, \pm 1/2)]_{(\pm 1/2, \pm 1/4, \pm 1/2)}$$

(7.10)

from sectors of the form of Eq. (7.3) with $X_R \cdot X_R = 8$. In the other two cases, $X_R \cdot X_R = 6$ and $X_R \cdot X_R = 4$, the massless states are obtained acting on the vacuum with one, or two, fermionic oscillators with frequency $1/4$, respectively. We can then obtain states that transform as $(5, 1/4), (\bar{5}, -1/4), (10, 1/2), (\bar{10}, -1/2)$, under $SU(5) \times U(1)$.

In $SU(3)\times SU(2)\times U(1)^2$ type models there may exist sectors of the $SO(6)\times SO(4)$ and $SU(5) \times U(1)$ types, and in addition sectors of the form of Eq. (7.4) which can appear only in the $SU(3) \times SU(2) \times U(1)^2$ models. These sectors are obtained from combinations of the basis vectors that contain both the $SO(6) \times SO(4)$ and $SU(5) \times U(1)$ breaking vectors. The possible $SU(3) \times SU(2) \times U(1)^2$ type exotic
states are

\[
[(3, 1/4); (1, 1/2)]_{(-1/3, -1/4, -1/3)} \quad ; \quad [(3, -1/4); (1, 1/2)]_{(1/3, 1/4, 1/3)} \quad ; \quad (7.11)
\]

\[
[(1, \pm 3/4); (2, \pm 1/2)]_{(1/2, 1/4, (1, 0); (0, -1))} \quad ; \quad (7.12)
\]

\[
[(1, \pm 3/4); (1, \mp 1/2)]_{(0, 5/4, 0)}, \quad (7.13)
\]

Sectors with \( X_R \cdot X_R = 8 \) give rise only to exotic states which are Standard Model singlets with the quantum numbers of Eq. (7.13). Sectors with \( X_R \cdot X_R = 6 \) give rise to all the exotic states with the quantum numbers of Eqs. (7.11, 7.12, 7.13). In addition to the states above, sectors with \( X_R \cdot X_R = 4 \) can give rise to exotic diquarks. In these sectors the massless states are obtained by acting on the vacuum with two fermionic oscillators with frequency \( 1/4 \), thus producing exotic diquarks with

\[
[(3, -1/4); (2, 1/2)]_{(1/6, -3/4, 2/3, -1/3)} \quad ; \quad [(3, 1/4), (2, 1/2)]_{(-1/6, 3/4, -2/3, 1/3)} \quad . \quad (7.14)
\]

from sectors with the boundary conditions in Eq. (7.14) while sectors with the boundary conditions in Eq. (7.13) with \( X_R \cdot X_R = 4 \) may give rise to exotic diquarks with

\[
[(3, -1/4), (2, -1/2)]_{(-1/3, 1/4, 1/6, -5/6)} \quad ; \quad [(3, 1/4), (2, 1/2)]_{(1/3, -1/4, -1/6, 5/6)} \quad , \quad (7.15)
\]

The above classification exhausts all the possible exotic states which may in principle appear in the realistic free fermionic models. The classification then proceeds by examining which type of sectors and therefore which type of exotic states actually appear in specific models. The task then is to further understand which choices of boundary condition basis vectors and generalized GSO projection coefficients produce specific classes of exotic states.

Such a complete classification and understanding is beyond the scope of the present paper. However, short of that, we can still make several important observations. The first observations is in regard to the presence of states with fractional electric charge. Both the \( SU(5) \times U(1) \) and \( SO(6) \times SO(4) \) type exotic states carry electric charge \( \pm 1/2 \). From a phenomenological point of view such states must be confined to produce integrally charged states or must become super–heavy by the breaking of the four dimensional gauge group along the flat F and D directions. In the revamped flipped \( SU(5) \) model [13] it is found that all the fractionally charged states transform in representations of the hidden non–Abelian gauge groups. Therefore, in this model indeed all the fractionally charged states are confined. As an example of the alternative solution, examining the fractionally charged states and the cubic level superpotential of the model of ref. [17]

\[
W_2 = \frac{1}{\sqrt{2}}(H_1 H_2 \phi_4 + H_3 H_4 \bar{\phi}_4 + H_5 H_6 \bar{\phi}_4 + (H_7 H_8 + H_9 H_{10}) \phi'_4
\]

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it is observed that all the fractionally charged states receive a Planck scale mass by giving a VEV to the neutral singlets \( \tilde{\phi}_4, \tilde{\phi}_4', \phi_4, \phi_4' \) which imposes the additional F flatness constraint \((\phi_4 \tilde{\phi}_4 + \phi_4 \phi_4') = 0\). The other exotic states which are Standard Model singlets do not receive mass by this choice of flat direction. Therefore, at this level of the superpotential, the fractionally charged states can decouple from the remaining light spectrum. This example indicates the existence of free fermionic models in which fractionally charged states appear only at the massive level.

The question of further interest is therefore which exotic sectors and corresponding states actually arise in given models. One example is provided by the state in Eq. (7.11). This state is a color triplet and is obtained from \( SU(3) \times SU(2) \times U(1)^2 \) type exotic sectors with \( X_R \cdot X_R = 6 \), which requires one fermionic oscillator acting on the vacuum. The states from this type of sectors are of particular phenomenological interest because they carry the standard charges under the Standard Model gauge group, but carry “fractional” charges under the \( U(1)_{\epsilon} \) embedded in \( SO(10) \). Consequently, this type of matter states may in fact remain sufficiently light and be in agreement with current experimental constraints. Such sectors and states appear generically in the free fermionic Standard–like models. The question then is whether their appearance is a necessary property of this choice of vacuum. The following model illustrates that this is not the case.

| \( y^3y^6 \) | \( y^4y^4 \) | \( y^5y^5 \) | \( y^3y^6 \) | \( y^1\omega^5 \) | \( y^2y^2 \) | \( \omega^6\omega^6 \) | \( y^1\bar{\omega}^5 \) | \( \omega^2\omega^4 \) | \( \omega^1\bar{\omega}^2 \) | \( \omega^3\omega^3 \) | \( \bar{\omega}^2\bar{\omega}^4 \) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| \( \alpha \) | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| \( \beta \) | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| \( \gamma \) | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 |

With the choice of generalized GSO projections coefficients:

\[
c \left( \begin{array}{c} b_i \\ S, b_j, \alpha, \beta, \gamma \end{array} \right) = c \left( \begin{array}{c} \alpha, \beta, \gamma \end{array} \right) = c \left( \begin{array}{c} \beta \end{array} \right) =
\]

\[
c \left( \begin{array}{c} \gamma \end{array} \right) = -c \left( \begin{array}{c} \beta \end{array} \right) = -1
\]

with \((j = 1, 2, 3)\). This model is a modification of the model of ref. [18] obtained by taking \( \alpha(\omega^2\omega^4) = 1 \rightarrow \alpha(\omega^2\omega^4) = 0 \) and \( \beta(\bar{\omega}^2\bar{\omega}^4) = 1 \rightarrow \beta(\bar{\omega}^2\bar{\omega}^4) = 0 \). With this modification and the rules of section (3.1) we see that in this model we obtain from the Neveu–Schwarz sector the Higgs color triplets \( D_3 \) and \( \bar{D}_3 \), rather than the
corresponding doublets $h_3$ and $\bar{h}_3$. Spanning the additive group, with this choice of boundary condition basis vectors, it is observed that this model does not contain any sector with $X_R \cdot X_R = 6$. Therefore, this model does not contain states from this type of exotic sectors irrespective of the choice of the generalized GSO projection coefficients. This shows the existence of standard–like models in which the states of the form of Eq. (7.11) do not appear. Another example is provided by the revamped flipped $SU(5)$ model in which sectors with $X_R \cdot X_R = 6$ do appear. In the flipped $SU(5)$ this type of sectors generate exotic 5 and 5̅ representations with $U(1)_5$ charges $\pm 1/4$. However, there the exotic 5 and 5̅ are projected out by the GSO projections.

As another example of the classification by exotics we note that none of the three generation free fermionic models, which were studied in detail to date, contains sectors that can produce diquark states. However, such a model can be constructed by building the desired vector with $X_R \cdot X_R = 4$ into the set of basis vectors. Table (7.18) provides an example of such a model.

| $\psi^\mu$ | $\chi^{12}$ | $\chi^{34}$ | $\chi^{56}$ | $\bar{\psi}^{1,...,5}$ | $\bar{\eta}^1$ | $\bar{\eta}^2$ | $\bar{\eta}^3$ | $\bar{\phi}^{1,...,8}$ |
|------------|-------------|-------------|-------------|-----------------|------------|------------|------------|-----------------|
| $\alpha$   | 1           | 1           | 0           | 0               | 1          | 1          | 1          | 1               |
| $\beta$    | 1           | 0           | 1           | 0               | 1          | 1          | 1          | 1               |
| $\gamma$   | 1           | 0           | 0           | 1               | 1          | 1          | 1          | 1               |

| $y^3\bar{y}^3$ | $y^4\bar{y}^4$ | $y^5\bar{y}^5$ | $y^6\bar{y}^6$ | $y^1\omega^6$ | $y^2\omega^2$ | $\omega^5\bar{\omega}^5$ | $\bar{y}^1\bar{\omega}^6$ | $\omega^1\omega^3$ | $\omega^2\omega^2$ | $\omega^4\omega^4$ | $\omega^1\bar{\omega}^3$ |
|---------------|---------------|---------------|---------------|-------------|------------|------------|-----------------|-------------|------------|------------|-------------|
| $\alpha$      | 1             | 0             | 0             | 0            | 0          | 1          | 0               | 0            | 1          | 1          | 1            |
| $\beta$       | 0             | 0             | 1             | 0            | 1          | 0          | 0               | 0            | 1          | 0          | 0            |
| $\gamma$      | 0             | 1             | 0             | 0            | 1          | 0          | 0               | 1            | 0          | 0          | 0            |

With the choice of generalized GSO coefficients:

$$c\left(\begin{array}{c} b_j \\ \alpha, \beta, \gamma \end{array}\right) = c\left(\begin{array}{c} \alpha \\ 1 \end{array}\right) = c\left(\begin{array}{c} \beta \\ 1 \end{array}\right) = c\left(\begin{array}{c} \gamma \\ 1 \end{array}\right) = -c\left(\begin{array}{c} \gamma \\ \alpha, \beta \end{array}\right) = 1$$

$(j = 1, 2, 3)$, with the others specified by modular invariance and space–time supersymmetry. The sector $\gamma$ has the desired form $\gamma^2_L = \gamma^2_R = 4$ and give rise to exotic diquarks.

The classification by the type of exotics which actually appear in specific models is crucial from a phenomenological point view, as can be seen from the following example. This is exemplified by examining the exotic spectra in the models of refs. [18] and [51] and considering the neutrino see–saw mechanism. Examining the exotic
spectrum in the model of ref. [18] we see that the massless spectrum contains from
the sectors $b_2 + b_3 + \alpha \pm \gamma + (I)$ and $b_1 + b_3 + \alpha \pm \gamma + (I)$ states which are Standard
Model singlets and carry $1/2$ the charge of the right–handed neutrino, and it complex
conjugate, with respect to the $U(1)_{Z'}$ which is embedded in $SO(10)$. In The model of
ref. [18] we find that there exist such representations which are 5 and $\bar{5}$ of the hidden
$SU(5)$ gauge group, and some which are singlets of the non–Abelian observable and
hidden gauge groups. The see–saw mechanism which operates in this model makes
use of a product of these representations which forms the quantum numbers of the
complex conjugate of the right–handed neutrino [31, 32]. Examining the spectrum in
the model of ref. [51] we observe that in this model all the exotic states with similar
$U(1)_{Z'}$ charge, transform in representations of the non–Abelian hidden gauge groups.
Therefore a similar combination of fields, which mimics the charge of the conjugate
of the right–handed neutrino cannot be formed in this model.

8 Classification by the hidden sector

The hidden sector in the free fermionic models is determined by the boundary
condition of the internal right-moving world–sheet fermions $\bar{\phi}^1, \ldots, 8$. Several comments
are important to note. First we should define what is meant by the hidden sector. The
term hidden sector here means that the states which are identified with the standard
Model states are singlets of the hidden sector. On the other hand, both observable and
hidden matter states may be charged with respect to the flavor $U(1)$ symmetries and
similarly the exotic states may be charged with respect to the $U(1)$ symmetries which
are embedded in $SO(10)$ and may transform under the non–Abelian gauge groups
of the hidden sector. In the models that I consider here there is no non–Abelian
mixing, i.e. there are no representations that transform simultaneously under the
non–Abelian gauge groups of the observable and hidden sector. It should be however
noted that models with such representations may be constructed as well.

At the level of the NAHE set the Hidden gauge group is $E_8$. Let us recall that the
adjoint representation of $E_8$ decomposes as $120 \oplus 128$ under $SO(16)$. There are two
sectors which produce the adjoint representation of the $E_8$ gauge group. The first is
the Neveu–Schwarz sectors which produces the 120 representation of $SO(16)$, and the
second is the sector $1 + b_1 + b_2 + b_3$, which produces the spinorial 128 representation
of $SO(16)$. In addition to these two sources some models may contain additional
space time vectors bosons which can enhance the hidden sector gauge group. These
additional space–time vector bosons are obtained from combinations of the boundary
condition basis vectors which extend the NAHE set.

The hidden sector gauge group depends on which of these sectors contributes
to the final hidden sector gauge group. In the flipped $SU(5)$ and $SO(6) \times SO(4)$
models the hidden gauge group is typically broken in two stages. The first is by the
vector $2\gamma$ which breaks the $E_8$ gauge group to $SO(16)$ and the second stage is to
an $SO(16)$ subgroup. For example, in the revamped flipped $SU(5)$ model the final
hidden gauge group is $SO(10) \times SO(6)$. In the flipped $SU(5)$ the vector $\gamma$ is the basis vector which breaks $SO(10) \rightarrow SU(5) \times U(1)$ and has $1/2$ boundary conditions for $\{\bar{\psi}^{1\cdots5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3 \bar{\phi}^{1\cdots4}\}$. Recall from section (4.2) that the vector $2\gamma$ is the basis vector that breaks $(2,2) \rightarrow (2,0)$ world–sheet supersymmetry. As we have seen in section (4.2) this breaking can also be achieved by a suitable choice of the GSO projection coefficients. Therefore, the first stage of the hidden sector gauge group breaking is the breaking due to the breaking of the right–moving $N = 2$ world–sheet supersymmetry. This stage of breaking is common to all the semi–realistic free fermionic models which utilize the NAHE set. The second stage of the breaking of the hidden sector gauge group is more model dependent. The remaining boundary conditions of the world–sheet fermions, which determine the hidden sector gauge group, are to a large degree fixed by the phenomenological requirements, already imposed on the observable sector and by the modular invariance constraints which constrain the possible boundary conditions.

The space–time vector bosons from the Neveu–Schwarz sector which contribute to the final hidden sector gauge group are always present. The presence of additional gauge multiplets from the sector $1 + b_1 + b_2 + b_3$, which enlarge the hidden sector gauge group, depends on the GSO projection coefficients. For example, in the model of ref. [17] the choice $c(b_4) = 1$ yields the projection of the states from this sector and therefore the resulting hidden sector gauge group arises solely from the Neveu–Schwarz gauge multiplets. It should be noted that the choice of GSO phases which project the space–time vector bosons from the sector $1 + b_1 + b_2 + b_3$ affect the phenomenological properties of the observable sector. The final hidden gauge group in a given model, to some degree, is fixed due to the phenomenological properties imposed on the observable sector.

An example which shows how this correlation works is provided in the free fermionic standard–like models. Suppose that we want to project all three color triplets pairs from the Neveu–Schwarz sector. It then follows that a vector $\alpha$, which breaks the $SO(10)$ symmetry to $SO(6) \times SO(4)$ has an odd number of periodic fermions from the set $\{\bar{\psi}^{1\cdots5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3 \bar{\phi}^{1\cdots4}\}$. To obey the modular invariance rule, $\alpha \cdot \gamma \equiv 0 \mod 1$, an odd number of fermions from the set $\{\bar{\phi}^{1\cdots8}\}$ must be periodic in the vector $\alpha$, and receive boundary condition of $1/2$ in the vector $\gamma$. Therefore, the hidden gauge symmetry is broken in two stages. Typically it is broken to $SU(5) \times SU(3) \times U(1)^2$. However, other possibilities do exist. For example, modifying the vector $\gamma$ of ref. [18] by $\gamma\{\bar{\phi}^3 \bar{\phi}^4\} = 1 \rightarrow \gamma\{\bar{\phi}^3 \bar{\phi}^4\} = 0$, enhances the hidden gauge group from $SU(5)_H \times SU(3)_H \times U(1)^2$ to $SU(7)_H \times U(1)^2$, for an appropriate choice of the generalized GSO projection coefficients.

As noted above in some models additional space–time vector bosons may arise from combinations of the basis vectors which extend the NAHE set. For some choices of the additional basis vectors that extend the NAHE set, there exist a combination

$$X = n_\alpha \alpha + n_\beta \beta + n_\gamma \gamma$$ (8.1)
for which $X_L \cdot X_L = 0$ and $X_R \cdot X_R \neq 0$. Such a combination may produce additional space–time vector bosons, depending on the choice of GSO phases. For example, in the model of ref. [18] the combination $X = b_1 + b_2 + b_3 + \alpha + \beta + \gamma$ has $X_L \cdot X_L = 0$ and $X_R \cdot X_R = 8$. The space–time vector bosons from this sector are projected out by the choice of GSO phases, and this vector combination produces only space–time scalar supermultiplets. However, with the modified GSO phases in Eq. (4.9) additional space–time vector bosons are obtained from the sector $b_1 + b_2 + b_3 + \alpha + \beta + \gamma + (I)$. In this case the hidden $SU(3)_H$ gauge group is extended to $SU(4)_H$.

Further classification of free fermionic models by the hidden sector can be done by the hidden matter content. In all the models the sector $b_j + 2\gamma$ produce the vectorial 16 representation of the hidden $SO(16)$ gauge group decomposed under the final hidden sector gauge group. These matter representations are always present in the three generation models that utilize the NAHE set of boundary condition basis vectors. Additional hidden sector matter states arise from other combinations of the additional basis vectors which extend the NAHE set. These additional hidden matter play an important role, for example, in generating a neutrino see saw mass matrix [31]. In general, the number of additional hidden sector matter states, transforming under the hidden sector non–Abelian gauge groups, affects the scale at which the hidden sector non–Abelian gauge groups become strongly interacting and therefore is important, for example, in the context of supersymmetry breaking.

9 Conclusion

The realistic models in the free fermionic formulation have had remarkable success in providing plausible explanations to various properties of the Standard Model. Among those we should list the natural emergence of three generations, the qualitative structure of the fermion mass spectrum and the possible resolution of the string gauge coupling problem. Furthermore, specific models can also explain the origin of proton stability in a robust way. The gross characteristics of this class of models arise because of the underlying $Z_2 \times Z_2$ orbifold compactification at the free fermionic point in the moduli space. This compactification then leads to a large number of three generation models, which differ by their detailed phenomenological properties, giving rise to the hope that a fully realistic model can be constructed. While it is likely that superstring theory is only an effective approximation to the truly fundamental Planck scale theory, a fully realistic superstring model is likely to be more than an accident. Such phenomenological string models in turn will serve as toy models in which we can learn about the fundamental Planck scale dynamics.

In this paper I discussed in detail the basic ingredients and building blocks that enter the construction of the realistic free fermionic models. The aim of this paper is to provide some of the insight into the basic structures that underly these models. The eventual goal of the program initiated here is to uncover whether a fully realistic superstring model in the free fermionic models can be constructed. The classification
of the models by boundary condition rules was discussed. These rules illustrate how some phenomenological properties of the models are fixed by the boundary conditions. The role of the free phases in the determination of the physical properties of the realistic string models was discussed in detail. Mirror symmetry is seen to arise in the fermionic models due to the discrete choices of free phases. The mirror symmetry exist in these models for the (2,2) models as well as for the more general (2,0) models with periodic–antiperiodic boundary conditions. Similar duality symmetries can also be found in the extended models, which include Wilson line breaking of the non–Abelian gauge symmetry. Such duality symmetries provide one criteria for classifying the models. The free phases also play an important role in fixing the physical properties of the string models. In this paper this role was illustrated in regard to the final gauge group and the string gauge coupling unification problem. The classification of the models by the pairing of the world–sheet real fermions was elucidated, and several examples demonstrated how the different choices of pairings relate to the phenomenological properties of the models. Further classification of the models, by the different types of exotic representations that may appear in the models and by the hidden sector was discussed. Further classification of the models by the anomalous $U(1)$ will be discussed in a separate publication [36].

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