On the Periods and Nature of Superhumps

J. Smak
N. Copernicus Astronomical Center, Polish Academy of Sciences, Bartycka 18, 00-716 Warsaw, Poland
e-mail: jis@camk.edu.pl

ABSTRACT

It is commonly accepted that the periods of superhumps can be satisfactorily explained within a model involving apsidal motion of the accretion disk provided the frequency of the apsidal motion in addition to the dynamical term includes also the pressure effects. Using a larger sample of systems with reliable mass ratios it is shown, however, that this view is not true and the model requires further modifications.

Key words: accretion, accretion disks – binaries: cataclysmic variables, stars: dwarf novae

1. Introduction

Superhumps are periodic light variations, with periods slightly longer than the orbital period, observed in dwarf novae during their superoutbursts and in nova-like cataclysmic variables – the so-called permanent superhumpers.

There are two, competing models for superhumps: the tidal-resonance model and the irradiation modulated mass transfer model. According to the tidal-resonance (TR) model, first proposed by Whitehurst (1988) and Hirose and Osaki (1990), they are due to periodic enhancement of tidal stresses in an eccentric accretion disk undergoing apsidal motion (often incorrectly called "precession"). An important ingredient of this model is the 3:1 resonance between the orbital frequency of the binary system and the orbital frequency of the outer parts of the disk which is essential for the disk to become eccentric. This model, however, fails to explain many crucial facts (cf. Smak 2017 and references therein) but in spite of that is commonly accepted. The irradiation modulated mass transfer (IMMT) model (Smak 2009,2017), based on purely observational evidence, explains superhumps as being due to the periodically variable dissipation of the kinetic energy of the stream resulting from variations in the mass transfer rate which are produced by the modulated irradiation of the secondary component.
The periods of superhumps and their interpretation have been the subject of numerous investigations. According to the most recent ones (Murray 2000, Montgomery 2001, Pearson 2006, Smith et al. 2007) they cannot be explained within purely dynamical theory but satisfactory agreement between theoretical predictions and observations is obtained when pressure effects are included.

The main purpose of the present paper is to verify this conclusion by using a much larger sample of superhumpers with reliably determined mass ratios.

2. The Data

The data set to be analyzed below consists of 26 cataclysmic variables, with independently determined mass ratios, taken from the recent compilation by McAlister et al. (2019, Tables 2 and B2) and supplemented from the compilation by Smith et al. (2007, Table 5). Included in our sample are also two helium CV's: AM CVn with $q = 0.18 \pm 0.01$ (Roelofs et al. 2006) and YZ LMi with $q = 0.04 \pm 0.002$ (Copperwheat et al. 2011).

It is worth noting that the number of objects with $P_{orb} < 3$ hours in our sample (21) is much larger than in previous investigations (e.g. Pearson 2006, Table 3, or Smith et al. 2007, Table 3). On the other hand, however, the number of objects with $P_{orb} > 3$ hours (5) remains very small. This is due to the fact that most of them belong to the SW Sex type (cf. Dhillon et al 2013) showing effects of the stream overflow which - regretfully - make determination of the mass ratio from radial velocities or from hot spot eclipses difficult and/or unreliable.

3. The Basic Equations and Relations

We begin by listing equations and relations which will be used in further analysis. The superhump period $P_{SH}$ and the period of apsidal motion $P_{aps}$ are related by

$$\frac{1}{P_{aps}} = \frac{1}{P_{orb}} - \frac{1}{P_{SH}},$$

or, in terms of the corresponding frequencies $\omega = 2\pi/P$,

$$\omega_{aps} = \omega_{orb} - \omega_{SH}.$$

The superhump period excess defined as

$$\varepsilon_{SH} = \frac{P_{SH} - P_{orb}}{P_{orb}},$$

is related to the apsidal frequency by

$$\varepsilon_{SH} = \frac{\omega_{aps}}{\omega_{orb} - \omega_{aps}}.$$
It is commonly assumed (e.g. Pearson 2006, Smith et al. 2007 and references therein) that the apsidal frequency is the sum of the dynamical and pressure terms

\[ \omega_{\text{aps}} = \omega_{\text{dyn}} + \omega_{\text{press}}. \]  

(5)

The ratio of the dynamical part of the apsidal frequency to the orbital frequency as a function of the mass ratio and the effective radius of the disk is given by (Hirose and Osaki 1990, Pearson 2006, Eqs.6 and 7)

\[ \frac{\omega_{\text{dyn}}}{\omega_{\text{orb}}} = \frac{3}{4} \frac{q}{(1 + q)^{1/2}} r^{3/2} \sum_{n=1}^{\infty} a_n r^{2(n-1)}. \]  

(6)

where

\[ a_n = \frac{2}{3} (2n) (2n + 1) \prod_{m=1}^{n} \left( \frac{2m - 1}{2m} \right)^2. \]  

(7)

The effective radius of the disk is commonly assumed to be equal to the 3:1 resonance radius which is given by

\[ r_{3:1} = \frac{1}{3^{2/3} (1 + q)^{1/3}}. \]  

(8)

4. The Pressure Term?

Following earlier papers (Murray 2000, Montgomery 2001, Pearson 2006, Smith et al. 2007) we now determine the residuals \( \Delta \omega \) between the observed apsidal frequency \( \omega_{\text{aps}} \) and the dynamical term \( \omega_{\text{dyn}} \), which, according to earlier authors, are expected to represent the pressure term \( \omega_{\text{press}} \).

First, using the observed values of \( P_{\text{orb}}, \epsilon_{\text{SH}} \) and \( q = M_2/M_1 \) we determine the observed apsidal frequency \( \omega_{\text{aps}} \) (Eq.4). Then, assuming – as is commonly done – that the effective radius of the disk is equal to the 3:1 resonance radius (Eq.8), we calculate the dynamical contribution \( \omega_{\text{dyn}} \) (Eqs.6 and 7) and, finally, the difference: \( \Delta \omega = \omega_{\text{aps}} - \omega_{\text{dyn}} \).

Results, presented in Fig.1, can be summarized as follows:

1. For systems with \( P_{\text{orb}} < 3 \) hours the residuals \( \Delta \omega \) show large scatter and/or appear to be correlated with the mass ratio.
2. Systems with \( P_{\text{orb}} > 3 \) hours form a separate group.
3. The residuals for helium CV’s are very large: \( \Delta \omega = -5.0 \pm 0.3 \) for YZ LMi and \( \Delta \omega = -21.9 \pm 0.9 \) (!) for AM CVn.
4. The theoretical \( \omega_{\text{press}} = f(q) \) relations (Montgomery 2001, Pearson (2006) fail to represent the real data.

\(^1\)replacing \( r_{3:1} \) with \( r_{\text{tid}} \) for systems with orbital periods above the period gap change the results only slightly.

\(^2\)the problem with the AM CVn systems was already noted by Pearson (2007).
4

5. Discussion

Until now, as mentioned in the Introduction, it has been believed that the periods of superhumps can be satisfactorily explained when the apsidal frequency is assumed to be the sum of the dynamical and pressure terms. Results presented above imply that this is not true. Therefore the basic model involving the apsidal motion of the disk requires substantial modifications.

Fig. 2. The effective radii $r_{\text{eff}}$ (see text for details) are plotted against the mass ratio. Red symbols represent the two helium CV’s.

Searching for possible clues we follow Pearson (2006) and determine the effective radii $r_{\text{eff}}$ at which the apsidal frequency calculated using only the dynamical term (Eqs. 6 and 7) would be equal to the observed frequency. Results, presented in
Fig. 2, are remarkable: they show that all values of $r_{\text{eff}}$, including those representing the two AM CVn systems, fall between $\approx 0.3$ and $\approx 0.4$. The significance of this result is, however, not yet clear.

REFERENCES

Copperwheat, C.M. et al. 2011, MNRAS, 410, 1113.
Dhillon, V.S., Smith, D.A., and Marsh, T.R. 2013, MNRAS, 428, 3559.
Hirose, M., and Osaki, Y. 1990, Publ.Astr.Soc.Japan, 42, 135.
McAllister, M. et al. 2019, MNRAS, 486, 5535.
Montgomery, M.M. 2001, MNRAS, 325, 761.
Murray, J.R. 2000, MNRAS, 314, L1.
Pearson, K.J. 2006, MNRAS, 371, 235.
Pearson, K.J. 2007, MNRAS, 379, 183.
Roelofs, G.H.A., Groot, P.J., Nelemans, G., Marsh, T.R., Steeghs, D. 2006, MNRAS, 371, 1231.
Smak, J. 2009, Acta Astron., 59, 121.
Smak, J. 2017, Acta Astron., 67, 273.
Smith, A.J., Haswell, C.A., Murray, J.R., Truss, M.R., and Foulkes, S.B. 2007, MNRAS, 378, 785.
Whitehurst, R. 1988, MNRAS, 232, 35.