Collapse of a hot vapor bubble in subcooled liquid

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Abstract. Numerical simulations are performed for a spherical bubble containing hot vapor in subcooled water. Unlike conventional cavitation problems, the initial pressure in the bubble and surrounding liquid are equal to each other. The physical mechanism for the appearance of pressure difference driving the bubble collapse is rapid cooling and condensation of vapor in the bubble. A model for the fluid dynamics and thermal processes in the bubble and in the liquid is presented. The governing equations are solved numerically in the transformed coordinates where the bubble size remains constant, with the Rayleigh-Plesset equation describing the time evolution of bubble radius. The effect of the initial superheat of vapor in the bubble with respect to the saturation temperature is studied. The bubble collapse of hot vapor is compared with an idealized case, resembling collapse of a cavitation bubble. Estimates for the kinetic energy of liquid gained in the course of bubble collapse are obtained. The results are applicable to the evaluation of the perturbations to spreading melt surface due to interaction with the unstable water-vapor interface, and to the evaluation of the premixing zone in the stratified steam explosions.

1. Introduction
Condensation of saturated steam bubble in subcooled liquid, having high importance in many applications, has been studied quite thoroughly in numerous experimental and theoretical works [1, 2]. In most of the studies, the bubble was first formed in the surrounding liquid at the equilibrium (saturated) conditions, after which the liquid pressure was increased abruptly, making the liquid subcooled with respect to its new saturation state. The pressure drop between the liquid and vapor caused contraction of the bubble during which the internal vapor pressure increased, so that vapor condensation on the bubble surface occurred.

Two limiting cases of condensing bubble dynamics are distinguished: thermal and inertial. In the thermal regime, the bubble collapse speed is limited by heat conduction in liquid, with the bubble pressure practically equal to that in the surrounding liquid. The rate of bubble shrinking is governed exclusively by the rate at which heat is transferred by thermal conduction from the interface to colder ambient liquid. The inertial regime occurs at high thermal conductivity of liquid, when heat is transferred in the liquid so fast that the bubble pressure remains constant and equal to the saturation pressure at the temperature of the ambient subcooled liquid.

However, if the vapor in the bubble is initially superheated significantly (by hundreds of degrees Kelvin), new aspects appear in the process of bubble shrinking which need special consideration. Steam bubbles with high initial superheat are encountered, for example, in nuclear reactor severe accidents...
with core degradation and meltdown, when a layer of high-temperature melt is spreading over a pool bottom under a layer of water [3, 4]. As a result of melt-coolant interaction, powerful steam explosion can occur, posing significant threat to nuclear reactor containment integrity [5, 6]. For example, in [6] experiments on the thermal interaction of high-temperature (above 1000 K) melt with water were carried out in the stratified configuration (melt was spreading under a shallow water layer). Strong steam explosions with the peak pressure reaching up to 10 MPa were registered, which indicates existence of a zone where melt was mixed with water prior to the explosion.

A hypothesis put forward in [6] on the mechanisms for the formation of this premixed zone was that significant perturbations to the spreading melt could be caused by collapse of hot vapor bubbles captured by subcooled water due to development of instability and buoyancy effects. The downward cumulative jets resulting from collapse of such bubbles could impinge on the melt surface, causing melt splashes and promoting the development of a premixed zone containing intermittent water, melt and vapor phases.

Since the vapor film separating the melt from water above it is heated to high temperature by direct contact with the melt, the initial state of the bubble captured by subcooled (i.e., having temperature below the saturation) water is different from that encountered in cavitation, as well as in the above-mentioned cases of saturated vapor bubbles. In this work, the principal phenomena related to superheated bubble rapid shrinking and collapse in subcooled water are analyzed.

2. Mathematical model

Condensation of a significantly superheated steam bubble in subcooled water is considered on the basis of a one-dimensional model for spherically symmetric compressible conductive gas flow:

$$\frac{\partial \rho_g}{\partial t} + \frac{1}{r^2} \frac{\partial r^2 \rho_g w_g}{\partial r} = 0$$

$$\frac{\partial w_g}{\partial t} + w_g \frac{\partial w_g}{\partial r} = - \frac{1}{\rho_g} \frac{\partial P_g}{\partial r}$$

$$\frac{\partial \rho_g e_g}{\partial t} + \frac{1}{r^2} \frac{\partial r^2 \rho_g e_g w_g}{\partial r} + \frac{P_g}{r^2} \frac{\partial w_g}{\partial r} = \frac{1}{r^2} \frac{\partial \left( r^2 \lambda_g \frac{\partial T_g}{\partial r} \right)}{\partial r}$$

Here, $r$ is the radial coordinate, $t$ is the time, $\rho_g$, $w_g$, $P_g$, $e_g$, $T_g$, and $\lambda_g$ are the vapor density, velocity, pressure, specific internal energy, temperature, and thermal conductivity, respectively. The thermodynamic and transport properties of vapor are determined from the IAPWS steam tables [7].

Spherically symmetric water flow outside the vapor bubble is described by the continuity and momentum equations. Under the assumption that water is incompressible, these equations are reduced to the Rayleigh-Plesset equation, an ordinary differential equation that describes the evolution of bubble interface (see [1]):

$$\frac{dw_a}{dt} + \frac{3 w_a^2}{2 a} = \frac{P_a - P_0}{\rho_i}$$

Here, $a$ is the current bubble radius, $w_a = da/dt$ is the interface velocity, $\rho_i$ is the water density (assumed to be constant), $P_a$ is the pressure on the bubble surface, $P_0$ is the water pressure far from the bubble, assumed to be constant during the whole process. The formulation (4) of the Rayleigh-Plesset equation implies that water viscosity, as well as surface tension are neglected, which is well-grounded for the problem in question [2, 8]. The one-dimensional formulation is applicable provided that the bubble is located far enough from any walls; gravity is not taken into account because the processes considered are too fast for its effects to be any significant.
In the surrounding water \((r \geq a)\), velocity distribution is found from the incompressibility condition: 
\[ w_r = \frac{a^2}{r^2}. \]
Pressure distribution in water is obtained by integrating the momentum equation outside the bubble, taking into account Eq. (4), see, e.g., [1]:
\[
P_r = P_0 - \frac{\rho_r w_r^2}{2} \left( \frac{a}{r} \right)^4 + \frac{a}{r} \left( \frac{\rho_r w_r^2}{2} + P_u - P_0 \right)
\]  
(5)

The temperature distribution in water is described by the energy equation accounting for convective and conductive heat transfer:
\[
\rho_r C_r \left( \frac{\partial T_r}{\partial t} + w_r \frac{\partial T_r}{\partial r} \right) = \frac{1}{r^2} \frac{\partial}{\partial r} \left( \lambda_r r^2 \frac{\partial T_r}{\partial r} \right)
\]  
(6)

Here, \(T_r, C_r,\) and \(\lambda_r\) are the temperature, specific heat, and thermal conductivity of liquid water.

On the interface (bubble surface), the well-known quasi-equilibrium scheme is applied which assumes that the phase properties on the interface obey the thermodynamic equilibrium condition, while the heat fluxes incident on the interface from the vapor and liquid are related to the phase change rate by the following balance condition [1]:
\[
\lambda_{\text{vapor}} \frac{\partial T_{\text{sat}}}{\partial r} \Bigg|_{r=a} - \lambda_{\text{water}} \frac{\partial T_r}{\partial r} \Bigg|_{r=a} = \dot{m}(i_{\text{v}} - i_{\text{w}})
\]  
(7)

Here, \(i_{\text{v}}\) and \(i_{\text{w}}\) are specific enthalpies of vapor and water on the interface (their difference is equal to the specific heat of evaporation), \(\dot{m}\) is the phase change rate per unit area of the interface.

At the bubble center, symmetry conditions are posed for vapor velocity and temperature: \(r = 0:\ w_r = 0, \ \frac{\partial T_r}{\partial r} = 0\); at infinity, stagnant liquid at the ambient temperature is assumed: \(r \to \infty:\ w_r = 0, \overline{T_r} = T_0\).

On the interface \((r = a)\), the phase pressure and temperatures are continuous: \(\overline{P_r} \big|_{r=a} = \overline{P} \big|_{r=a} = P_a, \overline{T_r} \big|_{r=a} = T_{\text{sat}} (P_a),\) where \(T_{\text{sat}} (P_a)\) is the saturation temperature at the interface pressure \(P_a.\) The phase velocities, on the contrary, are discontinuous on the interface due to the phase change: the water velocity is equal to the interface velocity \(w_r \big|_{r=a} = w_a,\) while the vapor velocity is \(w_r \big|_{r=a} = w_a - \dot{m}/\rho_{\text{vapor}}\), where \(\rho_{\text{vapor}}\) is the vapor density on the bubble surface, \(\dot{m}\) is determined by (7).

The problem was solved numerically by the finite-difference method [9]. In order to maintain the solution accuracy for a significantly shrinking bubble, new independent variables were introduced, with the spatial coordinate dependent on time in such a way that the current bubble radius remained constant (equal to unity).

3. Model validation

The model was validated against the experiments [10] where condensation of vapor bubbles in subcooled water was studied. To exclude the gravity effects and ensure spherically symmetric bubble shrinking and collapse, the working unit was dropped to free fall for about 0.6 s. During this time, high-speed video recording at a rate from 700 to 3000 frames per second was performed. The cylindrical stainless steel vessel, 5 in. in diameter and 7 in. high, had three 3 in. glass ports for lighting, photography, and viewing. The vessel was mounted on a 36×10.5 in. platform supported by two electromagnets, the drop distance was 6 ft.

In the experiments [10], water was electrically heated to boiling at sub-atmospheric pressure, some amount of vapor was trapped in a tube to be released later as a bubble. After the heating was stopped...
and convective flows died out, the experimental sequence was followed: i) the cover on top of the pipe was opened by an electromagnet to release the bubble; ii) the platform was released to free fall; iii) pressure relief valve was opened in the vessel, increasing the pressure to atmospheric and initiating the bubble collapse. The time history of the bubble radius was obtained by processing the video recordings. Under the experimental conditions, the initial bubble radius was in the range 3.2–9 mm, while water subcooling was 5 to 45 K.

In figure 1, results of a typical validation simulation performed for the initial bubble radius of 3.66 mm and water subcooling 12.2 K are presented. Reasonable agreement of the results obtained with the measured data is achieved, the local deviation is within 12%. Similar results were obtained for other experimental regimes.

![Figure 1. Time history of the radius for a saturated vapor bubble collapsing in subcooled water (dots: experiment [10], solid line: current calculation).](image)

4. **Numerical study on superheated vapor bubble collapse**

Consider the results obtained for collapse of a vapor bubble in the case where the initial temperature of steam exceeds the saturation temperature. Such bubbles can be formed in the melt-water interactions which can potentially result in the stratified steam explosions [6].

Simulations were performed at the following initial conditions: the vapor pressure in the bubble was equal to the pressure in the ambient water $P_0$, the initial gas temperature was $T_g^0 > T_{sat}(P_0)$ (superheated vapor), whereas the water temperature was $T_w^0 < T_{sat}(P_0)$ (subcooled water). Here, $T_{sat}(P_0)$ is the saturation temperature at the ambient pressure.

Consider the main features of the bubble shrinking and collapse obtained in the simulations. On the initial (“wave”) stage, vapor condensation causes propagation of rarefaction waves in the bubble, travelling from the bubble boundary towards its center and back. The time of wave passage is determined by the bubble radius and speed of sound in steam. In figure 2, pressure profiles are shown at several instants, with the abscissa corresponding to non-dimensional radial coordinate $\xi = r / a$ (note that during this stage the bubble radius changes only insignificantly).

Two physical processes proceed on the bubble surface: i) intensive heat transfer from the vapor to the interface due to high initial superheat of vapor, and ii) vapor condensation due to heat transfer to the ambient subcooled water. As a result, vapor pressure is dropping near the surface, and a rarefaction wave is propagating into the bubble, seen in figure 2. Starting from time 0.01 ms, the pressure at the bubble center is decreasing, it drops below the pressure at the surface by 0.0125 ms, marking the reversal of the rarefaction wave propagation direction. At this time interval, the vapor velocity in the bubble starts to decrease. Due to its high inertia, the surrounding liquid at this time interval remains practically stagnant.
Further on, the wave processes in the bubble are gradually damping, the pressure and temperature distributions in the bubble tend to uniform ones, whereas the radial velocity distribution in the bubble is determined by the condensation rate at its surface. This new stage of bubble collapse can be termed “inertial” one. At this stage, the pressure in liquid far from the bubble exceeds the internal bubble pressure, and due to this pressure drop the liquid accelerates, gaining noticeable velocity and affecting the gas state in the bubble.

In figure 3, the time histories are shown for the bubble radius at different initial superheats of vapor in the bubble with respect to the saturation temperature, equal to 300, 400, and 650 K, respectively; the initial bubble size is $a_0 = 3.4$ mm. The dashed line shows the time dependence of bubble radius in an idealized case where the bubble pressure is assumed to be constant and equal to the saturation pressure at the initial temperature of the surrounding liquid. This case can be interpreted as the one where the and the initial vapor temperature is equal to that of the surrounding liquid, and heat conductivity of both vapor and liquid is so high that the vapor temperature and pressure remain constant during the collapse.

The dependence plotted by the dashed line is obtained by integrating (4), it describes the limiting inertial regime of bubble collapse. It follows from figure 3 that, with the increase in the initial vapor superheat, the time histories of bubble radius tend to that obtained in the idealized case. It can be concluded that, the higher the vapor superheat, the closer the dynamics of bubble shrinking and collapse to that corresponding to the limiting inertial regime.

**Figure 2.** Pressure distributions on the initial stage of bubble collapse. Initial vapor superheat 50 K, initial bubble radius 3.4 mm.

**Figure 3.** Time histories of bubble radius at the initial vapor superheat 300, 400, and 650 K ($a_0 = 3.4$ mm). The dashed line corresponds to the inertial regime.
Condensation of vapor in the collapsing bubble accelerates the flow of surrounding water which gains kinetic energy as the bubble is shrinking. The kinetic energy of water reaches its peak value at the time close to the bubble collapse instant. As was shown in calculations, practically all the kinetic energy is concentrated in a thin spherical layer surrounding the collapsing bubble.

In Table 1, the calculated maximum kinetic energy of water obtained for a bubble with the initial vapor superheat 650 K are presented for four values of the initial bubble radius relevant to the experiments [6].

| Bubble radius $a_0$ (mm) | Kinetic energy $E_{\text{max}}$ (J) |
|--------------------------|-----------------------------------|
| 3.4                      | 0.00917                           |
| 5                        | 0.02949                           |
| 10                       | 0.21725                           |
| 15                       | 0.66638                           |

For estimates, assume that all the kinetic energy just before the collapse is concentrated in the volume occupied by the bubble at the initial instant. Then the corresponding velocity of this water mass, in the case $a_0 = 15$ mm, will be about 10 m/s. Although we considered only the spherically symmetric problem where any directional flow of water after collapse (which is of interest for the interaction of bubble collapse-induced water-melt interaction) is not possible, one can assume, for estimates, that the asymmetric flow possesses the same (by the order of magnitude) velocity.

Assume that all the momentum of water jet directed towards the melt, is completely transferred to the melt, which, after reflection from the pool bottom, is splashed upwards. The estimates based on the momentum conservation show that collapse of a vapor bubble of the initial radius 15 mm can lead to throwing upwards to the height of 5 cm of melt droplets of about the same size as the initial bubble (for the melt/water density ratio of 8, as in the experiments [6]). This sequence of event, occurring repeatedly, can lead to the formation of a dynamic melt/water mixing layer, capable of generating steam explosions.

Thus, the estimates of the kinetic energy of water generated by significantly superheated steam bubble collapse near the water-melt interface have confirmed that this process can indeed result in splashing of melt to water and formation of a potentially explosive mixture. These estimates substantiate the validity of condensation mechanism of underwater melt-coolant mixing leading to powerful steam explosions.

The results obtained provide the basis for the development of a macroscopic mixing model in the stratified configuration, suitable for implementation in the computer codes intended for simulation of thermal fuel-coolant interactions. Calculation of such regimes are indispensable in the severe accident design of nuclear power reactors where design measures must be undertaken to exclude or minimize the possibility of steam explosions.

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