Scalar field “mini–MACHOs”: a new explanation for galactic dark matter

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We examine the possibility that galactic halos are collisionless ensembles of scalar field “massive compact halo objects” (MACHOs). Using mass constraints from MACHO microlensing and from theoretical arguments on halos made up of massive black holes, as well as demanding also that scalar MACHO ensembles of all scales do not exhibit gravothermal instability (as required by consistency with observations of LSB galaxies), we obtain the range: \( m \lesssim 10^{-7} M_\odot \) or \( 30 M_\odot \lesssim m \lesssim 100 M_\odot \). The rather narrow mass range of large MACHOs seems to indicate that the ensembles we are suggesting should be probably made up of scalar MACHOs in the low mass range (“mini–MACHOs”). The proposed model allows one to consider a non–baryonic and non–thermal fundamental nature of dark matter, while at the same time keeping the same phenomenology of the CDM paradigm.

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I. INTRODUCTION

The problem of the missing mass on galactic and galactic cluster scales is one of the most interesting open issues in present day cosmology and astrophysics [1]. At cosmological scales, recent WMAP results [2] have confirmed that the global dynamics of the universe imply a far larger non relativistic ”matter” component than what is allowed by light density and nucleosynthesis estimates of baryonic matter, further strengthening the conclusion of a significant ”cold dark matter” contribution at all scales.

The dominant approach to this problem has been the so-called ”cold dark matter” (CDM) paradigm in which the missing mass-energy in galactic halos is made of relic gases of new and yet undetected types of elementary non–baryonic (possibly supersymmetric) particles, collectively known as weakly interacting massive particles (WIMPs).

Since all theories unifying gravity with other interactions involve scalar fields, more “exotic” scenarios consider dark matter in the form of a scalar field coherent on a very large scale, similar to those associated with quintessence sources. Kaluza–Klein, Super strings theories and super-gravity [3], all contain scalar fields as reminiscent of extra dimension of spacetime. Even if, until now, these remnants of primordial scalar fields have not been directly detected, their use as models for dark energy [4] or dark matter in galactic halo structures [5] has become widespread.

So far, most of the attempts to model galactic dark matter halos out of real or complex scalar fields assume that each galactic halo is a spherical Bose-Einstein condensate of scalar particles. This was first suggested [6] assuming free ultra light (\( \sim 10^{-24} \text{eV} \)) scalar particles described by a coherent complex scalar field forming a “boson star” of a galactic scale. Subsequent studies [7, 8] added self-interaction and generalized the previous Newtonian analysis to be fully general relativistic. Other authors contributed more detailed studies which used two kinds of coupling of the scalar field to gravity, i.e. either minimal [14, 15, 16] or non-minimal [12, 13]. Static solutions (“boson stars”) are possible with a complex field, but not for a real valued field. The latter do allow for stable oscillating objects called “oscillatons” that can be used to model galactic structures of all known scales (see [12, 17, 18]).

II. SCALAR FIELDS HALO MODELS

Both, oscillatons and boson stars, are described by the field theoretical action

\[
S = \int d^4x \sqrt{|g|} \left( \frac{1}{2} \left( \nabla \Phi \right)^* \left( \nabla \Phi \right) - U(|\Phi|) + \frac{\mathcal{R}}{16\pi G} \right)
\]

where \( g \) is the metric determinant and Ricci scalar and \( \Phi \) is a real or complex scalar field, which will give rise, respectively, to oscillatons and boson stars. We look at each case separately below.

An example of stable oscillatons [14] is examined in [12, 17, 18, 19], corresponding to scalar field with mass \( m_\Phi \) and potential \( U = m_\Phi^2 \Phi^2 / 2 \), which forms stable objects with a critical mass given by

\[
m_{\text{crit}} = 0.607 \frac{m_{\text{Pl}}^2}{m_\Phi}
\]

where \( m_{\text{Pl}} \) is Planck’s mass. Since we do not have any reason for choosing the scalar field mass, this parameter

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must be selected by demanding that $m \simeq m_{\text{crit}}$ complies with appropriate ranges. The formation of the oscillaton depends on initial conditions set after inflation, when the scalar field quantum fluctuations grow very fast and form seed fluctuations with different sizes. If the seed fluctuation is small, the oscillations virialize and form a compact object in a short time after inflation. If the seed fluctuation is large but smaller than the critical mass, this virialization process (not to confuse with that of the MACHO ensemble) takes longer, while if the seed fluctuation is larger than the critical mass, the oscillaton is no longer stable. However, for masses below the critical mass the oscillations are very stable objects even long time after their virialization. The size of the oscillaton depends on the central value of the scalar field, so that using (2) and the numerical results of [13], we find that if the scalar field mass is $m_\Phi = 1/n \times 8.11 \times 10^{-11}\text{eV}$, where $n$ is a constant factor, the associated oscillations will have a critcal mass and maximal radius

$$m_{\text{crit}} = n M_\odot, \quad R_{\text{max}} \sim 10 \times n \text{ km}, \quad (3)$$

Thus, for an oscillaton of $50 M_\odot$, the scalar field mass is $m_\Phi = 1.6 \times 10^{-12}\text{eV}$ and its size is $r_* \sim R_{\text{max}} \sim 500\text{ km}$, while an oscillaton with the earth’s mass will be just a few meters across.

Boson stars, the other type of scalar MACHOS, are static rather than oscillating and exist in systems of more than one scalar field [21, 22]. In their simplest form they are described by a massive complex scalar field with a possible self-interaction: $U = m_\Phi^2 |\Phi|^2/2 + \lambda |\Phi|^4/4$. There are two quantitatively different cases. When there is no self-interaction ($\lambda = 0$), boson stars are formed with a mass of the order of $m_\Phi^2/m_\Phi$ and radius which is a little larger than the corresponding Schwarzschild radius, as for oscillatons. The critical mass is given by a relation similar to equation (2):

$$m_{\text{crit}} = 0.633 \frac{m_\text{pl}^2}{m_\Phi}. \quad (4)$$

Self-interaction changes the situation dramatically since this term dominates as long as $\lambda \gg m_\Phi^2/m_\text{pl}^2$ which holds almost for any value of $\lambda$. In that case the boson star masses are much larger - roughly by a factor of $\sqrt{\lambda} m_\text{pl}/m_\Phi$. The critical mass is given now by

$$m_{\text{crit}} \approx 0.06 \sqrt{\lambda} \frac{m_\text{pl}^3}{m_\Phi}. \quad (5)$$

Consequently, for a similar solar mass MACHO the scalar particle mass should be taken now to be of the order of 1 GeV (for $\lambda \sim 1$). As in the case $\lambda = 0$, their radii are still of the order of the corresponding Schwarzschild radius. Thus, the relation between critical mass and maximal radius given in (4) is valid (to a good approximation) also for boson stars. If we assume a boson star formation in the early universe, starting after the temperature drops below $m_\Phi$, there is a strong preference for boson star formation by the self-interacting type of bosons over formation by free bosons [20, 21]. Most of the bosons of the latter type tend to condensate into black holes rather to form boson stars while the opposite is true for the former.

In either case, boson stars or oscillatons, the idea of each halo being a single solitonic configuration runs into some problems: once fundamental scalar field parameters are chosen, the size of the “boson star halo” is fixed so that halos of only one unique size would exist, in stark disagreement with observations. Regarding the case of “single oscillaton halos”, their oscillations would probably lead to halo-scale astronomical effects that should have been detected.

It has also been suggested [23, 24] that “axitons”, i.e. boson star MACHOS in the form of sub-planet-size solitons ($m < 10^{-9} M_\odot$) of the axion field possibly produced around the QCD epoch of the universe, might account for a large proportion (but not all) of non–baryonic dark matter in galactic halos. This type of “partial” approach (see also [25]) can not be ruled out, but does not significantly add to the discussion beyond further introducing an extra free parameter, as the remaining fraction of the halo mass still has to be accounted for through a separate type of dark matter. As long as a unique type of dark matter at all galactic and cosmological scales remains feasible, we believe it is this the framework within which one must work, on grounds of conceptual economy.

III. HALOS MADE UP OF SCALAR MACHOS

In this paper we propose an alternative approach in which each galactic dark matter halo is not a single halo–sized oscillaton or boson star, but is a collisionless ensemble of such objects: i.e. “scalar field MACHOS”. In other words, we consider a scenario in which the scalar field evolves by forming a large number of stable star–sized or planet–sized scalar condensations which end up clustering into structures similar to standard CDM halos (but made of scalar field MACHOS instead of WIMPs). We assume further that these scalar MACHOS constitute the totality of non–baryonic dark matter.

Under the proposed scheme, these dark matter halos would follow very similar dynamics to the usual CDM halos, but with a different particle mass granularity given by the microlensing and dynamical constraints on the MACHO’s masses. Note that numerical N-body simulations at ultra high resolution have shown results converging for particle numbers higher than $4 \times 10^7$ for Milky Way type halos [24], i.e. they are insensitive to particle mass granularity smaller than around $10^5 M_\odot$. Therefore, from a dynamical and phenomenological point of view, cosmological N-body simulations modeling CDM particles would be indistinguishable from those modeling scalar MACHOs with $m \lesssim 10^5 M_\odot$.

In order to be consistent with the MACHO detection constraints from microlensing [27], the masses of the scalar MACHOs making up the galactic halos must lie
in the ranges: \( m \gtrsim 30 \, M_\odot \) or \( m \lesssim 10^{-7} \, M_\odot \). An upper bound on the scalar MACHO mass also arises from various considerations. First, even the smallest dwarf galactic halo with \( M \sim 10^8 \, M_\odot \) must contain at least \( \sim 1000 \) scalar MACHOs so that it can be modelled as a stable collisionless ensemble of these objects, hence we should have \( m \lesssim 10^5 \, M_\odot \). Secondly, since the scalar MACHOs we are considering are very compact objects, we can apply to them the arguments used in various articles that examine the proposal galactic halos are made up of super-massive black holes (BH) \(^2\). These papers argue that BH’s larger than \( 10^9 \, M_\odot \) would lead to a central BH that is too large \(^24\) or would destroy the observed globular cluster halo population through dynamical interactions \(^32\) \& \(^33\). Considering these arguments plus the microlensing constraints, the allowed mass range for the MACHO scalars must be

\[
m \lesssim 10^{-7} \, M_\odot \quad \text{or} \quad 30 \, M_\odot \lesssim m \lesssim 1000 \, M_\odot. \quad (6)
\]

Notice that in the lower end of this range there is no inherent minimal bound on the MACHO mass. We can think of planetary or asteroid size “mini–MACHOs” or, in principle, even much smaller ones, as long as they can be described as an ensemble of classical particles. Extremely small scalar mini–MACHOs can also be described as some sort of very large scalar field WIMPs.

### IV. GRAVOTHERMAL INSTABILITY

Observations from LSB and dwarf galaxies, overwhelmingly dominated by dark matter, seem to exhibit flat constant density cores \(^35\) \& \(^36\), instead of the high density core surrounded by a lower density halo characteristic of the gravothermal instability. Hence, halos made up of ensembles of scalar MACHOs must be characterized by two–body relaxation timescales larger than 13.7 Gyr, the estimated age of the universe (according to latest WMAP estimates \(^2\)). This relaxation timescale is \(^37\).

\[
t_{\text{rel}} = \frac{1.8 \times 10^{10} \, \text{yr}}{\ln (0.4 \, M/m)} \frac{M_\odot \, M_\odot \, \text{pc}^{-3}}{m} \frac{\sigma}{\rho} \left( \frac{\text{km s}^{-1}}{\text{yr}} \right)^3, \quad (7)
\]

where \( \sigma \) is the velocity dispersion, \( \rho \) is mass density and \( M \) is the halo mass (the virial mass), so that \( M/m = (1/n) (M/M_\odot) \) for a scalar MACHO mass \( m = n \, M_\odot \). Notice that \( t_{\text{rel}} \) varies from point to point along the halo, up to several orders of magnitude between the center and outlying regions. As it happens with globular clusters and in numerical simulations, the highest density central region might be older than \( t_{\text{rel}} \), but not the outlying lower density regions. However, if the center region is younger than \( t_{\text{rel}} \), so will the rest. Therefore, it is sufficient to evaluate \(^4\) for the central values \( \rho = \rho_c \) and \( \sigma = \sigma_c \) in order to provide a criterion for a gas of scalar MACHOS not to be older than \( t_{\text{rel}} \).

\[
t_{\text{rel}} |_c > 13.7 \, \text{Gyr} \quad (8)
\]

where \( t_{\text{rel}} \) is evaluated for \( \rho = \rho_c \) and \( \sigma = \sigma_c \). Since \( \sigma_c \) is related to the maximal rotation velocity, it can be thought of as the characteristic scale parameter of different halo structures, hence for same sized halos the relaxation state depends mostly on \( \rho \) and \( m \).

We examine in figure \(^\text{Fig. 1}\) the fulfilment of \(^\text{Eq. (8)}\) for both types of scalar MACHO candidates, by plotting \( t_{\text{rel}} \) vs the scalar MACHO mass \( m \) for various typical halo structures characterized by specific values of \( \sigma_c \), that is: 10 km s\(^{-1}\), 200 km s\(^{-1}\) and 1000 km s\(^{-1}\), for a dwarf galaxy, a large galaxy and a galaxy cluster, with \( M = 10^8, 10^{12}, 10^{15} \, M_\odot \) respectively. We use \( \rho_c = 1 \, M_\odot \, \text{pc}^{-3} \), the largest in the range of estimated values \(^38\).

\[
0.001 \, M_\odot \, \text{pc}^{-3} \lesssim \rho_c \lesssim 1 \, M_\odot \, \text{pc}^{-3}, \quad (9)
\]

since, if the halo has not undergone core collapse for this value, it will not for smaller values in the range \(^9\). The shaded regions are the mass ranges excluded in \((6)\) and the horizontal dashed line gives the current age of the universe, a good estimate of the lifetimes of the oldest of these systems. Hence, the most stringent test comes from verifying \(^\text{Eq. (8)}\) for the smallest/densest halos. As shown by figure \(\text{Fig. 1}\) this condition is not fulfilled for scalar MACHOs with mass \( m \gtrsim 100 \, M_\odot \). Since we would expect all galactic halos to be made of the same type of scalar MACHOs (same type of dark matter), figure \(\text{Fig. 1}\) would be indicating that for scalar MACHOs with \( m \gtrsim 10^2 \, M_\odot \) core–collapse would happen in small halos but not in larger structures. However, this scenario is at odds with observations in LSB galaxies of various sizes \(^35\) \& \(^36\), all of which exhibit approximately isothermal density profiles.

### V. CONCLUSION

From the discussion above, the allowed masses must lie in the mini–MACHO range: \( m \lesssim 10^{-7} \, M_\odot \), and...
within $30M_{\odot} \lesssim m \lesssim 100M_{\odot}$. These values roughly agree with the findings of Yoo et al [39], who rule out massive BH’s with $m > 43M_{\odot}$ making up our galactic halo, as they would result in a depletion of halo binaries that contradicts observations. Since the window of accept-able masses for the large scalar MACHOs is rather narrow, our results and those of [39] do not suggest “the end of the MACHO era” (as claimed in [39]), but that MACHOs making up dark matter halos are, most probably, scalar field mini–MACHOs. This proposal not only solves the problems of scalar field dark matter, but does not violate the constraints arising from Big Bang Nucleosynthesis because the scalar field is not baryonic. Furthermore, at cosmological scales the proposed ensembles of scalar MACHOs behave just as CDM, though a non-supersymmetric type of CDM. We believe that this alternative modeling can describe the early universe origin of dark matter by means of new fundamental physics, while at the same time keeping the advantages of the phenomenology of the thermal CDM paradigm. A more detailed and less idealized study of the proposed dark matter scenario is presently under consideration.

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