A simplified approach in determining the c and \( \phi \) for asphalt mixtures

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Abstract. In this study, using the Mohr-Coulomb failure theory, a simplified method was proposed to determine the cohesion (c) and internal friction angle (\( \phi \)) for asphalt mixtures based on direct tension (DT) strength and indirect tension (IDT) strength or unconfined compressive strength (UCS) and IDT strength. The direct tension (DT) strength, superpave indirect tension (IDT) strength and unconfined compressive strength (UCS) were conducted to verify this method using three asphalt mixtures. It was found that the determination of c and \( \phi \) values have mean absolute errors of 5.2% and 1.8%, respectively in comparing two methods indicating that the c and \( \phi \) can be estimated from DT strength and IDT strength or IDT strength and UCS. Further study is recommended to verify using different binder grades and mixture types.

1. Introduction

In geotechnical engineering, the shear-strength of materials often determined Mohr-Coulomb failure theory which can be calculated in terms of cohesion (c) and internal friction angle (\( \phi \)). It is the most widely satisfied the failure criterion of the strength characteristics for unbound granular materials or fine-grained soils subjected to various confined stress conditions. For asphalt concrete (AC) mixtures, the idea of c - \( \phi \) concept was applied to predict the rut depth of asphalt pavement by some researchers [1,2,3]. The triaxial tests were applied commonly to determine the c and \( \phi \) properties. Although using triaxial tests in determining the shear strength is similar to field conditions considering the confining stress, the complexity of the laboratory triaxial tests which requires very expensive equipment and time consuming for sample preparation and testing. Therefore, the alternative methods need to be proposed in determining the c - \( \phi \) characteristics of AC mixtures.

Christensen and Bonaquist [4] was proposed a method to calculate the c - \( \phi \) based on unconfined compressive (UCS) and IDT tests. The results indicated that the IDT and unconfined compressive strength can be related in rational ways to mixture cohesion and internal friction. However, this method has not yet verified by other methods. Li et al. [5] was validated this method using triaxial tests. Li et al. [5] concluded that the determination of c and \( \phi \) values between the two methods are similar. Recently, There were still no studies to verify the c and \( \phi \) values based on the (DT) strength and indirect tension (IDT) strength.
This study proposed simple method to determine the cohesion (c) and internal friction angle (φ) for asphalt mixtures based on DT and IDT strength or UCS and IDT strength. The direct tension DT, IDT and UCS test were conducted to verify these methods using three asphalt mixtures.

2. Background theory
The Mohr-Coulomb failure theory, the strength of a material depends on both the cohesion (c) and the internal angle of friction (φ) by the following equation (1):

\[ \tau_{\text{max}} = c + \sigma \tan \phi \]  

Where c is the cohesion; \( \phi \) is the angle of internal friction, and \( \sigma \) is the normal stress.

According to the Mohr-Coulomb theory, the stress states incipient failure was used to plot a series of Mohr circles, which represents the Mohr-Coulomb failure envelope. This method can be used to determine c and \( \phi \) of AC mixtures. The Mohr circles for the stress state at failure using DT and IDT tests was shown as figure 1.

![Figure 1. Mohr circles using DT and IDT tests](image)

From figure 1,

\[ R_1 = R_2 = R_3 \]

\[ MO_1 = MO_2 = MO_1 + R_1 + 0.5R_2 \]  

(2)

\[ MO_1 = \frac{R_1^2 + 0.5R_1R_2}{R_2 - R_1} \]  

(3)

\[ \sin \phi = \frac{R_1}{MO_1} = \frac{R_2 - R_1}{R_1 + 0.5R_2} = \frac{2|\sigma_{IDT}| - 0.5|\sigma_{DR}|}{|\sigma_{IDT}| + 0.5|\sigma_{DR}|} = \frac{4|\sigma_{IDT}| - |\sigma_{DR}|}{2|\sigma_{IDT}| + |\sigma_{DR}|} \]  

(4)

\[ c = \frac{MO}{NO_2} = \frac{c}{\tan \phi} = \frac{c}{\tan \phi + 0.5R_2} \]  

(5)

\[ (5) \Rightarrow c = 0.5R_2 (2 - \sin \phi) = \frac{(2 - \sin \phi) |\sigma_{IDT}|}{\cos \phi} \]  

(6)

Similarly, the Mohr circles for the stress state at failure using IDT and UCS tests was shown as figure 2. The c and \( \phi \) were determined by the following equation (7) and (8).
From figure 2,

\[ \sin \varphi = \frac{O_z N}{O_z M} = \frac{\sigma_{UCS}}{\frac{2}{\tan \varphi}} = \frac{\sigma_{UCS}}{\sigma_{UCS} + 2c \cot \varphi} \] (7)

\[ \sin \varphi = \frac{O_1 K}{O_1 M} = \frac{2\sigma_{IDT}}{\sigma_{IDT} + \frac{c}{\tan \varphi}} = \frac{2\sigma_{IDT}}{\sigma_{IDT} + c \cot \varphi} \] (8)

From (7) and (8):

\[ \sin \varphi = \left| \frac{\sigma_{UCS}}{\sigma_{UCS} - 2|\sigma_{IDT}|} \right| \] (9)

\[ c = \frac{(2 - \sin \varphi)}{\cos \varphi} |\sigma_{IDT}| \] (10)

3. Experimental Program

3.1. Materials and sample preparation

Three different asphalt mixtures were used in the laboratory tests. Two of them, hot mix asphalt (HMA) and PHMA, were dense graded mixtures with nominal maximum aggregate size of 19 mm and PG 64-22 and PG 76-22 binders, respectively. The other one was a stone matrix asphalt (SMA) mixture with the nominal maximum aggregate size of 13 mm and PG 64-22 binder. The gradation curves in this study are shown in Figure 3. Information on the three mixes is provided in Table 1.

| Mix Type | Binder Grade | Asphalt Content (%) | Air-Void (%) | VMA (%) | VFA (%) |
|----------|--------------|---------------------|--------------|---------|---------|
| HMA      | PG64-22      | 5.0                 | 4.2          | 20.2    | 79.3    |
| PHMA     | PG76-22      | 5.0                 | 4.4          | 17.5    | 74.8    |
| SMA      | PG64-22      | 6.5                 | 2.8          | 20.1    | 86.1    |

All asphalt mixtures were mixed using a mechanical drum mixer and compacted using a Superpave Gyratory Compactor (SGC) after four hours of short term aging. The specimens were made in 150mm
diameter and 175mm height. The gyratory compacted samples were then cored and sawed to obtain the required 100mm diameter by 150mm tall specimens for the UCS and DT testing. Other cylindrical compacted samples were cut into two parts to obtain the required 50mm high specimens for the IDT test.

![Aggregate gradation used for study](image)

**Figure 3.** Aggregate gradation used for study

### 3.2. Test method

The DT, IDT and UCS tests were carried out at a displacement rate of 50mm/min at temperature conditions of 60 °C using a MTS servo-hydraulic testing system. The peak compressive and tensile load were measured to calculate the strength of all the samples.

### 4. Results and discussion

Analysis of the DT, IDT and UCS testing data were used to calculate the c and φ values using equation (4), (6), (9) and (10). Figure 4 provide the results of shear properties c and φ values of three typical mixtures (HMA, PHMA and SMA) at the temperature of 60°C determined from the both methods. It is observed from figure 4a that the PHMA mixtures using modified binder shows much higher c values than the other two types of mixtures using unmodified binder. This clearly showed that the main benefit of the modified binder is the increase cohesion (c). Meanwhile, the SMA mixtures generally are higher φ values than the other two mixtures with dense gradation as expected.

Moreover, it was found that the c and φ values were determined between the two methods which is statistically insignificant differences. The determination of c and φ have mean absolute errors of 5.2% and 1.8%, respectively in comparing two methods.
Figure 4. The $c$ and $\phi$ values calculated from proposed two methods (a): Cohesion and (b) Angle of internal friction.
5. Conclusion
This study presents an alternative approach to determine the c and φ properties for asphalt concrete mixtures using the Mohr-Coulomb failure theory which is based on the DT, IDT strength and UCS. A comparison study in determining the c and φ properties between the DT and IDT strength, and IDT strength and UCS. It was found that mean absolute errors provide around 5.2% and 1.8% for c and φ determination, respectively in comparing two methods. Therefore, it was concluded that the c and φ values can be estimated based DT and IDT strength values or UCS and IDT strength values for asphalt mixtures. But the verify was done only with limited data, further studies are recommended to is recommended to verify using different binder grades and mixture types.

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