On the Capacity of Causal Cognitive Interference Channel With Delay

Mahtab Mirmohseni, Bahareh Akhbari, and Mohammad Reza Aref
Information Systems and Security Lab (ISSL)
Department of Electrical Engineering, Sharif University of Technology, Tehran, Iran
Email: mirmohseni@ee.sharif.edu, b_akhbari@ee.sharif.edu, and aref@sharif.edu

Abstract—In this paper, we introduce the Causal Cognitive Interference Channel With Delay (CC-IFC-WD) in which the cognitive user transmission can depend on \( L \) future received symbols as well as the past ones. Taking the effect of the link delays into account, CC-IFC-WD fills the gap between the genie-aided and causal cognitive radio channels. We study three special cases: 1) Classical CC-IFC \((L = 0)\), 2) CC-IFC without delay \((L = 1)\) and 3) CC-IFC with a block length delay \((L = n)\). In each case, we obtain an inner bound on the capacity region. Our coding schemes make use of cooperative strategy by generalized block Markov superposition coding, collaborative strategy by rate splitting, and Gel’fand-Pinsker coding in order to pre-cancel part of the interference. Moreover, instantaneous relaying and non-causal partial Decode-and-Forward strategies are employed in the second and third cases, respectively. The derived regions under special conditions, reduce to several previously known results. Moreover, we show that the coding strategy which we use to derive achievable rate region for the classical CC-IFC achieves capacity for a special case of this channel. Furthermore, we extend our achievable rate regions to Gaussian case. Providing a numerical example for Gaussian CC-IFC-WD, we investigate the rate gain of the cognitive link for different delay values.

I. INTRODUCTION

Cognitive Interference Channel (C-IFC) refers to a two-user interference channel in which the cognitive user (secondary user) has the ability to obtain the message being transmitted by the other user (primary user), either in a non-causal or causal manner. C-IFC was first introduced in [1], where for the non-causal C-IFC an achievable rate region is derived by combining the Gel’fand-Pinsker (GP) binning [2] and a well known simultaneous superposition coding scheme (rate splitting) applied to the Interference Channel (IFC) [3]. For the non-causal C-IFC, where the cognitive user has non-causal full or partial knowledge of the other user’s transmitted message, several achievable rate regions and capacity results in some special cases have been established [3]-[7].

In the Causal C-IFC (CC-IFC), the cognitive user can exploit knowledge of the primary user’s message from the causally received signals (information overheard by the feedback link from the channel and not sent back from the receivers). Due to the complex nature of the problem, CC-IFC which is a more realistic and appropriate model for practical applications than the non-causal C-IFC, has been far less investigated compared with the latter [8]. In [1], achievable rate regions for the CC-IFC that consist of the non-cooperative causal transmission protocols have been characterized. An improved rate region for CC-IFC employing a cooperative coding strategy based on the block Markov superposition coding (full Decode-and-Forward (DF) [9]) and GP coding was derived in [10]. A more general model in which both transmitters are causally cognitive was proposed in [11], called Interference Channel with Generalized Feedback (IFC-GF). Different achievable rate regions for IFC-GF were obtained in [11]-[13], combining the methods of rate splitting, block Markov superposition coding, and GP binning.

In this paper, we define the Causal Cognitive Interference Channel With Delay (CC-IFC-WD) as an IFC where one of the transmitters can causally overhear the channel and its transmission can depend on the \( L \) future received symbols as well as the past ones. This can equivalently be seen as the classical CC-IFC with \(-L\) unit delay on the cognitive user’s received signal (or on the link between the transmitters). This channel model fits the wireless networks where the transmitters are close. Moreover, CC-IFC-WD is a middle point between the unrealistic genie-aided (non-causal) C-IFC and complex CC-IFC. In fact, a simple strategy such as Instantaneous Relaying (IR) by itself could be beneficial, as the case in the Relay With Delay (RWD) channel [14]. Different upper and lower bounds and some capacity results have been derived for RWD in [14]-[16], where the lower bounds are achieved based on the combination of cooperative strategies such as: full or partial DF, IR (for \( L > 0 \)), and non-causal DF (for \( L = n \)). It has been shown that the capacity of the RWD is strictly larger than the classic relay channel [13].

In this paper, after introducing the general CC-IFC-WD, we focus on three special cases: 1) \( L = 0 \) which corresponds to the classical CC-IFC, 2) CC-IFC without delay \((L = 1)\) where current received symbol (at the cognitive user) could also be utilized and 3) CC-IFC with a block length delay \((L = n)\), in which cognitive user knows its entire received sequence non-causally. In each case, we obtain new inner bound on the capacity region (achievable rate region) for the general discrete memoryless case. Our coding schemes benefit the cooperative strategy by generalized block Markov coding (partial DF [9]) and superposition coding, collaborative strategy by rate splitting and GP coding in order to mitigate part of the interference. For the first case (classic CC-IFC), we use a different strategy compared to the previous results. We use partial DF instead of full DF. Therefore our achievable region improves that of [10]. Moreover, since common message should be decoded in both receivers, binning against the common message provides no improvement. Therefore, we use GP binning to pre-cancel the part of the private message. A similar result has been concluded in [17] for the Cognitive Z-IFC. In the second and third cases, besides the approach we adopt for the first case, IR and
non-causal partial DF strategies are employed, respectively. The derived achievable rate regions, under special conditions, reduce to several previously known rate regions, such as the ones in [3], [14]. Moreover, we derive the capacity region for a special case of the CC-IFC-WD, where achievability follows from our derived region. Furthermore, we consider Gaussian CC-IFC-WD and extend the achievable rate regions for $L = 0$, $L = 1$ and $L = n$, to the Gaussian case. Providing a numerical example for Gaussian CC-IFC-WD, we investigate the rate gain of the cognitive link for different delay values. Thus, we compare the strategies which are used for our coding schemes and show that IR and non-causal DF improve the rate region noticeably.

The rest of the paper is organized as follows. Section II introduces the general CC-IFC-WD channel model and the notations. In Section III we consider three different scenarios and derive new inner bound on the capacity region for each scenario. Capacity region for a special case of the CC-IFC-WD is derived in Section IV. Gaussian CC-IFC-WD is investigated.

II. CHANNEL MODELS AND PRELIMINARIES

Throughout the paper, upper case letters (e.g. $X$) are used to denote random variables (RVs) and lower case letters (e.g. $x$) show their realizations. The probability mass function (p.m.f) of a random variable (RV) $X$ with alphabet set $\mathcal{X}$, is denoted by $p_X(x)$, where occasionally subscript $X$ is omitted. $|\mathcal{X}|$ denotes the cardinality of a finite set $\mathcal{X}$. $A^n_i$ ($X,Y$) specifies the set of $i$-strongly, jointly typical sequences of length $n$, abbreviated by $A^n_i$ if it is clear. The notation $X^n_1$ indicates a sequence of RVs $(X_1, X_{1+1}, ..., X_n)$, where we use $X^i$ instead of $X^n_i$ for brevity.

Consider the CC-IFC-WD in Fig I which is denoted by $(\mathcal{X}_1 \times \mathcal{X}_2, p(y_2, y_3, y_4|x_1, x_2), \mathcal{Y}_2 \times \mathcal{Y}_3 \times \mathcal{Y}_4)$, where $X_1 \in \mathcal{X}$ and $X_2 \in \mathcal{X}_2$ are inputs of Transmitter 1 (Tx1) and Transmitter 2 (Tx2), respectively. $Y_2 \in \mathcal{Y}_2$ is the secondary user output, $Y_3 \in \mathcal{Y}_3$ and $Y_4 \in \mathcal{Y}_4$ are channel outputs at the Receiver 1 (Rx1) and Receiver 2 (Rx2), respectively. $p(y_2, y_3, y_4|x_1, x_2)$ is the channel transition probability distribution. In $n$ channel uses, each Tx sends a message $m_{1,n}$ to the Rxu where $u \in \{1, 2\}$.

Definition 1: A $(2^nR_1, 2^nR_2, n)$ code for the CC-IFC-WD consists of (i) two message sets $\mathcal{M}_1 = \{1, ..., 2^nR_1\}$ and $\mathcal{M}_2 = \{1, ..., 2^nR_2\}$ for the primary and secondary users, respectively, (ii) an encoding function at the primary user $f_1: \mathcal{M}_1 \rightarrow X^n_1$, (iii) a set of encoding functions at the secondary user $f_{2,i} = f_{2,i}(m_{2,i}, y_{2,i-1}^{i-1} + \epsilon)$, for $1 \leq i \leq n$ and $m_{2,i} \in \mathcal{M}_2$, (iv) two decoding functions at Rx1 and Rx2, $g_{1} : \mathcal{Y}^{n}_3 \rightarrow \mathcal{M}_1$ and $g_{2} : \mathcal{Y}^{n}_3 \rightarrow \mathcal{M}_2$. We assume that the channel is memoryless. Thus, for $m_{1,1} \in \mathcal{M}_1$ and $m_{2,i} \in \mathcal{M}_2$, the joint p.m.f of $\mathcal{M}_1 \times \mathcal{M}_2 \times X^n_1 \times X^n_2 \times Y^n_3 \times Y^n_4$ is given by

$$p(m_1, m_2, x_1, x_2, y_2, y_3, y_4) = p(m_1)p(m_2) \prod_{i=1}^{n} p(x_{1,i}|m_1)$$

$$\times p(x_{2,i}|m_2, y_{2,i-1} + \epsilon)p(y_{2,i}|x_{1,i})p(y_{3,i}, y_{4,i}|x_{1,i}, x_{2,i})$$

where we avoid instantaneous feedback from $X_2$ to $Y_2$, which delay may cause. The probability of error for this code is defined as $P_e = \max\{P_{e1}, P_{e2}\}$, where for $u \in \{1, 2\}$ we have:

$$P_{e,u} = \frac{1}{2^n(R_1 + R_2)} \sum_{m_{1,2}} P(y_{u,n}^n(m_{1,2}) \neq m_u(m_{1,2}))$$

Definition 2: A rate pair $(R_1, R_2)$ is achievable if there exists a sequence of $(2^nR_1, 2^nR_2, n)$ codes with $P_{e,u} \rightarrow 0$ as $n \rightarrow \infty$. The capacity region $C_L$, is closure of set of all achievable rates.
Theorem 1: For any $p(.) \in \mathcal{P}_1$ the region $\mathcal{R}_1(Z_1)$ is an achievable rate region for the discrete memoryless classical CC-IFC (CC-IFC-WD with $L = 0$), i.e., $\bigcup_{Z_1 \in \mathcal{P}_1} \mathcal{R}_1(Z_1) \subseteq \mathcal{C}_0$.

Remark 1: Consider the case where the cognitive user cannot overhear the channel, i.e. $Y_2 = 0$. If we set $T_c = T_p = U_{11} = U_{1p} = \emptyset$, $L_{20} = L_{22} = R_{10d} = R_{1id} = 0$ and $V_{1p} = X_1$, the rate region reduces to Han-Kobayashi (HK) region [3].

Remark 2: If we omit receiver 2, i.e. $Y_4 = 0$, and cognitive user does not have any message to transmit, i.e. $R_2 = 0$, the model reduces to the relay channel. By setting $T_c = T_p = U_{11} = U_{1p} = V_{1p} = \emptyset$, $L_{20} = L_{22} = R_{11} = R_{1td} = 0$ and $V_{2c} = X_2$, the rate region reduces to the partial DF rate for the relay channel [9], which includes the capacity regions of the degraded [9] and semi-deterministic relay channels [13].

Outline of the Proof: We propose the following random coding scheme, which contains regular generalized block Markov superposition coding, rate splitting and GP coding in the encoding part. For decoding at the receivers we utilize backward decoding. Messages of the primary and cognitive users are split into four and two parts, respectively, i.e.: $m_1 = (m_{10d}, m_{10n}, m_{1id}, m_{11n})$ and $m_2 = (m_{20}, m_{22})$, where subscript $d$ (or $n$) refers to the part of the primary user’s message which can (or cannot) be decoded by the cognitive user. Moreover, $(m_{10}, m_{20})$ and $(m_{11}, m_{22})$ are common and private messages as in the HK scheme [3]. $m_{10d}$ should be decoded at Rx2 (besides its intended receiver), therefore binning against $m_{10d}$ at the cognitive transmitter provides no improvement. Hence, cognitive user cooperatively with the primary user sends $m_{10d}$, while uses GP binning method against $m_{11d}$ to mitigate the effect of this known interference at Rx2. Now, consider a block Markov encoding scheme with $B$ blocks of transmission, each of $n$ symbols.

Codebook Generation: Let $q^*$ be a random sequence according to $\prod_{i=1}^{n} p(q_i)$. Generate $2^{nR_{10d}}$ independent identically distributed (i.i.d) $t_c^n$ sequences, each with probability $\prod_{i=1}^{n} p(t_{c,i}(q_i))$. Index them as $t_c^n(m_{10d})$ where $m_{10d} \in \{1, 2^{nR_{10d}}\}$. For each $t_c^n(m_{10d})$, generate $2^{nR_{11d}}$ i.i.d $t_p^n$ sequences, $2^{nR_{10d}}$ i.i.d $u_{1p}^n$ sequences and $2^{nR_{10n}}$ i.i.d $v_{1c}^n$ sequences, according to

\[ p(t_{p,i}(q_i)), p(t_{1c,i}(q_i)), p(t_{1c,i}(q_i)) \] respectively. Index them as $t_p^n(m_{11d,1}, m_{11d,2}), u_{1p}^n(m_{10d,1}, m_{10d,2})$ and $v_{1c}^n(m_{10d,1}, m_{10d,2})$ where $m_{10d} \in \{1, 2^{nR_{10d}}\}$ and $m_{10n} \in \{1, 2^{nR_{10n}}\}$. For each $(u_{1p}^n(m_{10d,1}, m_{10d,2}), t_p^n(m_{11d,1,1}, m_{11d,1,2}), t_c^n(m_{10d})$, generate $2^{nR_{11d}}$ i.i.d $u_{1p}^n$ sequences, according to

\[ \prod_{i=1}^{n} p(u_{1p,i}|u_{1c,i}, t_{p,i}, t_{c,i}, q_i) \] Index them as $u_{1p}^n(m_{11d,1}, m_{11d,2}, m_{10d,1}, m_{10d,2})$ where $m_{11d} \in \{1, 2^{nR_{11d}}\}$. From the p.m.f in (2), compute the marginals $p(u_{1p,i}|t_{c,i}, q)$ and $p(u_{2p,i}|t_{c,i})$ (drop the dependence on $t_p$). For each $t_c^n(m_{10d})$, generate $2^{nR_{10d}+2L_{20}}$ i.i.d $u_{2p}^n$ sequences, each with probability $\prod_{i=1}^{n} p(u_{2p,i}|t_{c,i}, q_i)$. Index them as $u_{2p}^n(m_{20}, m_{22}), m_{10d})$, where $m_{20} \in \{1, 2^{nR_{20}}\}$ and $m_{22} \in \{1, 2^{nR_{22}}\}$. For each $t_c^n(m_{10d})$, generate $2^{nR_{10d}+2L_{22}}$ i.i.d $u_{2p}^n$ sequences, according to

\[ \prod_{i=1}^{n} p(u_{2p,i}|u_{1c,i}, t_{p,i}, t_{c,i}, q_i) \] Index them as $u_{2p}^n(m_{20}, m_{22}), m_{10d})$, where $m_{20} \in \{1, 2^{nR_{20}}\}$ and $m_{22} \in \{1, 2^{nR_{22}}\}$. In the first block cooperative information is: $(m_{11d,1}, m_{10d,1}) = (m_{11d,0}, m_{10d,0}) = (0, 0)$ and in the last block, a previously known message $(m_{11d,1}, m_{10d,1}) = (1, 1)$ is transmitted.

Encoding (at the beginning of block b):

Primary User: In order to transmit the message $m_{11} = (m_{10d,b}, m_{10n,b}, m_{1id,b}, m_{11n,b})$, Tx1 picks codewords $v_{1c}^n(m_{11n,b}, m_{10d,b}, m_{10d,b}, m_{10d,b}), v_{1c}^n(m_{10d,b}, m_{10d,b}, m_{11d,b}, m_{11d,b}), t_{c}^n(m_{11d,b}, m_{10d,b}, m_{11d,b}, m_{10d,b}), t_{c}^n(m_{11d,b}, m_{11d,b}, m_{11d,b}, m_{11d,b})$, then, sends $x_1^n$ generated according to $\prod_{i=1}^{n} p(x_{1,i}|v_{1p,i}, v_{1c,i}, u_{1p,i}, u_{1c,i}, t_{p,i}, t_{c,i}, q_i)$. In the first block cooperative information is: $(m_{11d,b}, m_{10d,b}) = (m_{11d,0}, m_{10d,0}) = (0, 0)$ and in the last block, a previously known message $(m_{11d,b}, m_{10d,b}) = (1, 1)$ is transmitted.

Cognitive User: Tx2 at the beginning of block b, knows $m_{10d,b-1}$ and $m_{11d,b-1}$, which are estimates of the parts of the common and private messages sent by Tx1 in the previous block and can be decoded by the cognitive user. In order to send $m_{20,b} = (m_{20,b}, m_{22,b})$, encoder 2 knows codewords $t_{c}^n(m_{11d,b-1}, m_{10d,b-1})$ and $t_{c}^n(m_{10d,b-1})$, seeks an index pair $(l_{20,b}, l_{22,b})$ such that

\[ (u_{2c}^n(m_{20,b}, l_{20,b}), m_{10d,b-1}), u_{2p}^n(m_{22,b}, l_{22,b}, m_{10d,b-1}), t_{c}^n(m_{11d,b-1}, m_{10d,b-1}), t_{c}^n(m_{10d,b-1}, m_{11d,b-1}, q^*) = A^* \] (24)

If there is more than one such index pair, pick the smallest. If there are no such codewords, declare an error. There exist such indices $l_{20,b}$ and $l_{22,b}$ with enough high probability, if $n$ is sufficiently large and [3] [13] hold. Then, it sends $x_2^n$ generated according to $\prod_{i=1}^{n} p(x_{2,i}|u_{2p,i}, u_{2c,i}, t_{p,i}, t_{c,i}, q_i)$.

Decoding: Cognitive User: Tx2 at the end of block b, wants to correctly recover $(m_{11d,b}, m_{10d,b})$. Hence, it looks for a unique pair $(m_{11d,b}, m_{10d,b})$ such that

\[ (y_{2c}^n(b), u_{1p}^n(m_{11d,b}, m_{10d,b}, m_{11d,b-1}, m_{10d,b-1}), u_{1c}^n(m_{10d,b}, m_{10d,b-1}), u_{2c}^n(m_{20,b}, l_{20,b}, m_{10d,b-1}), u_{2p}^n(m_{22,b}, l_{22,b}, m_{10d,b-1}), t_{c}^n(m_{11d,b-1}, m_{10d,b-1}), t_{c}^n(m_{10d,b-1}, m_{11d,b-1}, q^*) = A^* \] (24)
\[ u_{2p}([m_{22}, l_{22}], m_{10d}, b_{1}, l_{2}, m_{10d}, b_{1}], (25) \]

\[ t_2^p(m_{10d}, b_{1}, q^n) \in A_{c}^2(Y_{2}, U_{2c}, U_{2p}, U_{1p}, U_{1c}, T_{p}, T_{c}, Q) \]

This step can be done with small enough probability of error, for sufficiently large \( n \) if \( (22), (23) \) hold.

Backward decoding is used at the receivers, hence they start decoding after all \( B \) blocks are received. \( Rx\_1 \): In block \( b \), \( Rx\_1 \) looks for a unique quadruple \( (m_{11d}, b_{1}, m_{11d}, b_{1}, m_{10d}, b_{1}) \) and some pair \( (m_{20b}, l_{20b}) \) such that

\[ (y^n(1), u_{2c}([m_{20b}, l_{20b}], m_{10d}, b_{1})), v^n_{1c}(m_{10b}, m_{10d}, b_{1}), \]

\[ v^n_{1p}(m_{10d}, m_{10d}, b_{1}, m_{10d}, b_{1}), t_2^p(m_{10d}, b_{1}, q^n) \in A_{c}^2(Y_{4}, U_{2c}, U_{2p}, U_{1c}, T_{c}, Q) \]

where \( (m_{11d}, b_{1}, m_{10d}, b_{1}) \) were decoded in the previous step. Here, for large enough \( n \), the probability of error can be made sufficiently small if \( (10), (14) \) hold.

\( Rx\_2 \): In block \( b \), \( Rx\_2 \) finds a unique pair \( (m_{20b}, m_{20b}) \) and some quadruple \( (l_{20b}, l_{20b}, m_{10n}, m_{10d}, b_{1}) \) such that

\[ (u^n_{20b}([m_{20b}, l_{20b}], m_{10d}, b_{1})), u^n_{2p}([m_{22}, l_{22}], m_{10d}, b_{1})), \]

\[ v^n_{1c}(m_{10n}, m_{10d}, b_{1}, m_{10d}, b_{1}), v^n_{1p}(m_{10d}, m_{10d}, b_{1}, m_{10d}, b_{1}), t_2^p(m_{10d}, b_{1}, q^n) \in A_{c}^2(Y_{4}, U_{2c}, U_{2p}, U_{1c}, T_{c}, Q) \]

where \( m_{10d} \) was decoded in the previous step. With arbitrary high probability, no error occurs in \( Rx\_2 \) if \( n \) is large enough and \( (15), (21) \) hold.

Now to understand the shape of the achievable region, we give a compact expression for \( R_c(Z_1) \) which is easier to compute.

**Corollary 1:** The region \( R_c(Z_1) \), after Fourier-Motzkin elimination, can be expressed as:

\[ R_1 \leq \min \left( \left\{ \begin{array}{c} I_{12} + I_{16} + I_{10} - I_{1}, \\
I_{21} + \min(I_{17} - I_{2}, I_{0}) \end{array} \right\} \right) \]

\[ R_2 \leq \min \left( \left\{ \begin{array}{c} I_{19}, I_{14} + \min(I_{10}, I_{13}), \\
I_{21} + I_{17} + \min(I_{12}, I_{13}) \end{array} \right\} \right) \]

\[ R_1 + R_2 \leq \min \left( \left\{ \begin{array}{c} I_{14} + I_{5} + \min(I_{17} - I_{8}, -I_{1}), \\
I_{21} + \min(I_{12}, I_{13}) \end{array} \right\} \right) \]

\[ 2R_1 + R_2 \leq \min \left( \left\{ \begin{array}{c} I_{10} + I_{15} + I_{2} \quad I_{21} + 2I_{4} + I_{17} + I_{6}, \\
I_{4} + I_{17} + \min(I_{12}, I_{13}) \end{array} \right\} \right) \]

\[ R_1 + 2R_2 \leq \min \left( \left\{ \begin{array}{c} I_{12} + I_{10} + I_{14} + \min(I_{14} + I_{17} - I_{8}), \\
I_{14} + I_{15} + \min(I_{20} - I_{10}, I_{8}) \end{array} \right\} \right) \]

\[ 2R_1 + 2R_2 \leq \min \left( \left\{ \begin{array}{c} I_{6} + \min(I_{14} + I_{11}, I_{17}), \\
I_{10} + I_{14} + I_{2} \quad I_{21} + I_{15} - 2I_{1} \end{array} \right\} \right) \]

\[ 2R_1 + 3R_2 \leq \min \left( \left\{ \begin{array}{c} I_{8} + \min(I_{15} - I_{1}, I_{1}), \\
I_{21} + 2I_{1} + I_{1} + 2I_{1} - 2I_{1} \end{array} \right\} \right) \]

subject to \( I_{1} \leq \min(I_{8}, I_{10}, I_{12}, I_{13}, I_{16}) \) and \( I_{2} \leq I_{17} \), where \( I_{1}, 1 \leq i \leq 21 \) were defined in \( (13), (23) \), and \( I_{1} = \max(I_{1} + I_{2}, I_{3}), I_{2} = \min(I_{10} - I_{1}, I_{4}), \) and \( I_{3} = \min(I_{6}, I_{11} - I_{1}) \).
Theorem 3: For any $p(\cdot) \in \mathcal{P}_3$ the region $\mathcal{R}_3(Z_3)$ is achievable for the discrete memoryless CC-IFC with a block length delay $(CC-IFC-WD)$ with $L = n$, i.e., $\bigcup_{\mathcal{L}_2 \in \mathcal{P}_3} \mathcal{R}_3(Z_3) \subseteq \mathcal{C}_n$.

Proof: Proof is similar to Theorem 1 except that there is no dependence on previous block messages. Hence, simultaneous joint decoding is used instead of backward decoding.

IV. CAPACITY OF DEGRADED CLASSICAL CC-IFC

In this section, we investigate classical CC-IFC (CC-IFC-WD with $L = 0$), with joint p.m.f $p^*$, given by (1) with $L = 0$. Using the achievable region in Theorem 1 we find the capacity region for a special case. We define degraded classical CC-IFC as a classical CC-IFC where degradedness condition for the $X_1$-Rx1 pair with cognitive user as a relay holds for every $p^*$:

$$p(y|x_1, x_2, y_2) = p(y|x_2, y_2) \quad (31)$$

i.e., $X_1 \rightarrow (X_2, Y_2) \rightarrow Y_3$ forms a Markov chain. Next, we impose the following strong interference conditions:

$$I(X_1; Y_3) \leq I(X_1; Y_4) \quad (32)$$

$$I(X_2; Y_4|X_1) \leq I(X_2; Y_3|X_1) \quad (33)$$

In fact, under these conditions interfering signals at Rx1 and Rx2 are strong enough to decode both messages.

Theorem 4: The capacity region of the degraded classical CC-IFC with joint p.m.f $p^*$, satisfying (32) and (33), is given by

$$C_0^* = \bigcup_{p(t)p(x_1t)p(x_2t)} \{(R_1, R_2) : R_1 \geq 0, R_2 \geq 0$$

$$R_1 \leq (X_1; Y_2|X_2T) \quad (34)$$

$$R_2 \leq I(X_2; Y_4|X_1T)$$

$$R_1 + R_2 \leq \min\{I(X_1X_2; Y_3), I(X_1X_2; Y_4)\}$$

Proof: Achievability: For this part, we use the region $\mathcal{R}_1$ in Theorem 1 or Corollary 1 and we ignore time sharing RV $Q$. Let, $T_p = U_1p = V_1p = U_2n = 0$ and $R_{11n} = R_{11d} = 0$, which cross out the private parts of both messages, making the messages common to both receivers. Furthermore, assume that cognitive user fully decode the message of the primary user ($m_1$). Hence, set $R_{10n} = 0$ and $U_{1c} = 0$. To omit the GP coding, we set $L_0 = L_2 = 0$. Redefining $T_c = T$, $U_2 = X_2$, $U_{1c} = X_1$ and applying condition (35) complete the proof for the achievability.

Converse: Consider a $(2^nR_1, 2^nR_2)$ code with $P_e \rightarrow 0$. For $1 \leq i \leq n$, define the RV $T_i = Y_2^{i-1}$. Noting joint p.m.f $p^*$, we remark that $X_1 \rightarrow T \rightarrow X_2$ forms a Markov chain. First, we provide a useful lemma which we need in the proof of converse.

Lemma 1: If (32) and (33) hold for all distribution $p^*$, then

$$I(X_1^n; Y_3^n) \leq I(X_1^n; Y_4^n) \quad (35)$$

$$I(X_2^n; Y_4^n|X_1^n) \leq I(X_2^n; Y_3^n|X_1^n) \quad (36)$$

Proof: Proof follows the same lines as in [5] Lemma 5].

Noting the independence of the messages and utilizing Fano’s inequality for the first bound, we have:

$$nR_1 - n\delta_{1n} \leq I(M_1; Y_3^n|M_2) \leq I(M_1; Y_3^nY_2^n|M_2)$$

$$\leq \sum_{i=1}^{n} I(M_1; Y_3^nY_2^n; Y_3^{i-1}Y_2^{i-1}M_2X_2,i) \quad (37)$$

where (a) and (c) due to the fact that mutual information is non-negative, (b) obtains from the chain rule and the fact that $X_2,i$ is a deterministic functions of $M_2$ and $Y_2^{i-1}$ and (d) because channel is memoryless. Using standard time-sharing argument and condition (31), we have

$$R_1 - \delta_{1n} \leq I(X_1^n; Y_3^nY_2^n|X_2) = I(X_1^n; Y_2^n|X_2, T_n)$$

Similarly, applying Fano’s inequality, we bound $R_2$ as:

$$nR_2 - n\delta_{2n} \leq I(M_2; Y_4^n|M_1) \leq I(M_2; Y_4^nY_2^n|M_1)$$

$$\leq \sum_{i=1}^{n} I(M_2; Y_4^nY_2^n; Y_4^{i-1}Y_2^{i-1}, M_1)$$

$$\leq \sum_{i=1}^{n} I(M_2, X_2,i; Y_4^nY_2^nY_3^{i-1}, T_n, M_1, X_1,i)$$

$$\leq \sum_{i=1}^{n} I(X_2,i; Y_4^nY_2^n|X_1,i, T_n) \quad (38)$$

where (a) is based on the chain rule, (b) since $X_1,i$ is a deterministic functions of $M_1$ and mutual information is non-negative and (c) because channel is memoryless with joint p.m.f $p^*$.

Now, we utilize Fano’s inequality to bound $R_1 + R_2$ as:

$$n(R_1 + R_2) - n\delta_{3n} \leq I(M_1; Y_3^n) + I(M_2; Y_4^n|M_1)$$

$$\leq I(X_1^n; Y_3^n) + I(M_2; Y_4^n|M_1)$$

$$\leq I(X_1^n; Y_3^n) + I(M_2, X_2;i; Y_4^nY_2^n|X_1^n)$$

$$\leq I(X_1^n; Y_3^n) + I(X_1^n; Y_4^n|X_1^n) \quad (39)$$

Finally, utilizing Fano’s inequality, (38)-(c) and condition (35), the last bound can be shown. Using standard time-sharing argument for these bounds, completes the proof.

V. GAUSSIAN CC-IFC-WD

We consider Gaussian CC-IFC-WD and extend the achievable rate regions $R_1(Z_1), R_2(Z_2)$ and $R_3(Z_3)$ derived for the discrete memoryless classical CC-IFC ($L = 0$), CC-IFC without delay ($L = 1$) and CC-IFC with a block length delay ($L = n$), respectively, to the Gaussian case.

Gaussian CC-IFC-WD at time $i = 1, \ldots, n$, can be modeled as

$$Y_2,i = h_2X_1,i + Z_2,i$$

$$Y_3,i = h_3X_1,i + h_3X_2,i + Z_3,i$$

$$Y_4,i = h_4X_1,i + h_4X_2,i + Z_4,i \quad (39)$$

where, $h_2$, $h_3$, $h_3$, $h_4$ and $h_2$ are known channel gains. $X_1,i$ and $X_2,i$ are input signals with average power constraints $P_1$ and
First, we consider Gaussian classical CC-IFC ($L = 0$). Using standard arguments, region $R_1$ in Theorem 1 (or Corollary 1) can be extended to the discrete-time Gaussian memoryless case with continuous alphabets ($R_1$). Hence, it is sufficient to evaluate the (23) with an appropriate choice of input distribution. We constrain all the inputs to be Gaussian and set the time sharing RV $Q = 0$. For certain $\{0 \leq \beta_1, \beta_2, s \in \{1, 2, 3\}\}$ with $0 \leq \beta_3 \leq 1$, $t \in \{1, 2, 3\}$ and $0 \leq \gamma_1, \gamma_2, \gamma_3 \leq 1$, consider the following mapping $(MAP_1)$ for the generated codebook in Theorem 1 with respect to the p.m.f. (2), which contains rate splitting, generalized block Markov superposition and GP coding:

\[
\begin{align*}
T_c & \sim N(0, \beta_3 P_1), T_p' \sim N(0, \beta_3 P_1), U_{1c} \sim N(0, \beta_2 P_1) \\
V_{1c} & \sim N(0, \beta_2 P_1), U_{1p} \sim N(0, \beta_3 P_1), V_{1p} \sim N(0, \beta_2 P_1) \\
T_p & = T_p' + T_c, U_{1c} = U_{1c}' + T_c, V_{1c} = V_{1c}' + T_c \\
U_{1p} & = U_{1p}' + U_{1c}' + T_p + T_c, V_{1p} = V_{1p}' + V_{1c}' + T_p + T_c \\
X_1 & = V_{1p}' + X_{1c}' + U_{1c}' + T_p + T_c. \\
U_{2c} & \sim N(0, \gamma_2 P_2), U_{2p} \sim N(0, \gamma_2 P_2) \\
S_1 & = h_1 T_p, S_2 = h_2 T_c + h_{42} U_{2c} \\
U_{2c} & = U_{2c}' + \alpha_1 S_1, U_{2p} = U_{2p}' + \alpha_2 S_2 \\
X_2 & = V_{2p}' + U_{2c}' + \frac{1}{\gamma_3 P_2} T_c
\end{align*}
\]

where we have $\alpha_1 = \frac{h_{42} + h_{21} P_2}{\gamma_2 + \gamma_3 P_2}$ and $\alpha_2 = \frac{h_{21} + h_{22} + h_{42} P_2}{\gamma_2 + \gamma_3 P_2}$, and $A = N_4 + h_{21} (\beta_1 + \beta_2 + \beta_3) P_1 + h_{22} \gamma_1 P_2 + (h_{41} + \gamma_3 P_2) \frac{1}{\gamma_2 + \gamma_3 P_2}$. In fact, optimal values for $\alpha_1$, $\alpha_2$, $S_1$ and $S_2$ (used for GP coding) can be found by optimizing the rate region. However, this method is cumbersome and we use the modified version of Costa’s dirty paper coding (DPC) results (21).

Next, we investigate Gaussian CC-IFC without delay ($L = 0$), i.e., $X_2 = f_2(m_2, V_{2c})$. Considering Theorem 2 $v_{2c}$ is generated according to $p(v_{2c} | u_{2c}, t, p_2, l, c, q_1)$, and $a_{2c} = f_2(v_{2c}, q_2, v_{2c}, q_1)$. In order to obtain the Gaussian counterpart of $R_2$, namely $R_2^\ast$, appropriate mapping $(MAP_2)$, consists of (40–42).

Finally, we consider the Gaussian CC-IFC with a block length delay ($L = n$). We can use $R_1^\ast$ to obtain the Gaussian counterpart of $R_3$, namely $R_3^\ast$. The only difference is that according to Theorem 3 there is no dependence on the previous block messages. Therefore, we can set $T_c = U_{1c}$, and $T_p = U_{1p}$, or equivalently $\beta_1 = \beta_2 = 0$ in $(MAP_2)$ to obtain $MAP_3$.

Fig. 2 compares $R_1^\ast$, $R_2^\ast$, $R_3^\ast$ with HK region in [3], where the overheard information is neglected. For $L = 0$, rate improvement over HK region can be seen, especially when cognitive link is good enough ($h_{21} = 1$). Due to IR, even when $h_{21} = 1$, $R_2$ outperforms both $R_1^\ast$ and HK region, significantly. Considering $R_2^\ast$ and $R_3^\ast$, it is seen that when $R_2$ is small, IR can achieve higher rates than non-causal DF. However, when cognitive user sends at higher rates, condition of the cognitive link determines better strategy. Note that, using a coding scheme based on the combination of IR and non-causal DF, convex hull of $R_2^\ast$ and $R_3^\ast$ is achievable for CC-IFC with $L = n$.

REFERENCES

[1] N. Devroye, P. Mitran, V. Tarokh, “Achievable rates in cognitive radio networks,” IEEE Trans. Inf. Theory, vol. 52, pp. 1813–1827, May 2006.
[2] S. Gelfand and M. Pinsker, “Coding for channels with random parameters,” Prob. Contr. Inf. Theory, vol. 9, no. 1, pp. 19–31, 1980.
[3] T. S. Han and K. Kobayashi, “A new achievable rate region for the interference channel,” IEEE Trans. Inf. Theory, vol. 27, pp. 49–60, 1981.
[4] A. Jovicic, P. Viswanath, “Cognitive radio: An information-theoretic perspective,” IEEE Trans. Inf. Theory, vol. 55, pp. 3945–3958, Sep. 2009.
[5] I. Maric, R. D. Yates, and G. Kramer, “Capacity of interference channels with partial transmitter cooperation,” IEEE Trans. Inf. Theory, vol. 53, pp. 3536–3548, Oct. 2007.
[6] I. Maric, A. Goldsmith, G. Kramer, and S. Shamai(Shitz), “On the capacity of interference channels with one cooperating transmitter,” European Trans. Telecomm., vol. 19, pp. 405–420, April 2008.
[7] J. Jiang and Y. Xin, “On the achievable rate regions for interference channels with degraded message sets,” IEEE Trans. Inf. Theory, vol. 54, pp. 4707–4712, Oct. 2008.
[8] Y. Cao and B. Chen, “Interference Channels with One Cognitive Transmitter,” in http://arxiv.org/abs/0910.0899v1, Oct. 2009.
[9] T. M. Cover and A. El Gamal, “Capacity theorems for relay channels,” IEEE Trans. Inf. Theory, vol. 25, pp. 572–584, 1979.
[10] S. H. Seyedi-Mehdshi, J. Jiang, Y. Xin, and X. Wang, “An Improved Achievable Rate Region for Causal Cognitive Radio,” in Proc. IEEE Int. Symp. Inf. Theory (ISIT), Kobe, Japan, 2009, pp. 611–615.
[11] D. Tuninetti, “On interference channel with generalized feedback (IFC-GF),” in Proc. ISIT, France, 2007, pp. 2861–2865.
[12] Y. Cao and B. Chen, “An achievable rate region for interference channels with conferencing,” in Proc. ISIT, France, 2007, pp. 1251–1255.
[13] S. Yang and D. Tuninetti, “A New Achievable Rate Region for Interference Channel with Generalized Feedback,” in Proc. CISS, March 2008.
[14] N. Hassanpour, Relay without delay, Ph.D. thesis, Department of Electrical Engineering, Stanford University, August 2006.
[15] E. ElGamal, N. Hassanpour, and J. Mammen, “Relay networks with delays,” IEEE Trans. Inf. Theory, vol. 53, pp. 3413–3431, 2007.
[16] A. Salimi, M. Mirmohseni, and M. R. Aref, “A new capacity upper bound for “relay-with-delay” channel,” in Proc. IEEE Int. Symp. Inf. Theory (ISIT), Korea, 2009, pp. 26–30.
[17] N. Liu, I. Marc, A. Goldsmith, and S. Shamai(Shitz), “Bounds and Capacity Results for the Cognitive x-interference Channel,” in Proc. IEEE Int. Symp. Inf. Theory (ISIT), Korea, 2009, pp. 2422–2426.
[18] A. El Gamal and M. Aref, “The capacity of the semi-deterministic relay channel,” IEEE Trans. Inf. Theory, vol. 28, pp. 536, May 1982.
[19] C. E. Shannon, “Channels with side information at the transmitter,” IBM Journal Res. Develop., pp. 289–293, 1958.
[20] M. Mirmohseni, B. Akhbari, and M. R. Aref, “Compress-and-forward strategy for the relay channel with causal state information,” in Proc. IEEE Inform. Workshop Theory (ITW), Italy, Oct. 2009, pp. 426–430.
[21] M. H. M. Costa, “Writing on dirty paper,” IEEE Trans. Inf. Theory, vol. 29, pp. 439–441, May 1983.