Core-Stability in Assignment Markets with Financially Constrained Buyers

ELENI BATZIOU, Technical University of Munich
MARTIN BICHLER, Technical University of Munich
MAXIMILIAN FICHTL, Technical University of Munich

CCS Concepts: • Theory of computation → Computational pricing and auctions; Market equilibria; • Mathematics of computing → Integer programming.

Additional Key Words and Phrases: core stability, budget constraints, unit demand, welfare maximization

ACM Reference Format:
Eleni Batziou, Martin Bichler, and Maximilian Fichtl. 2022. Core-Stability in Assignment Markets with Financially Constrained Buyers. In Proceedings of the 23rd ACM Conference on Economics and Computation (EC ’22), July 11–15, 2022, Boulder, CO, USA. ACM, New York, NY, USA, 2 pages. https://doi.org/10.1145/3490486.3538262

1 OVERVIEW
We study markets where a set of indivisible items is auctioned to bidders with unit-demand valuations and hard budget limits. Without financial constraints and pure quasi-linear bidders, it is well-known that there exists a competitive equilibrium where the outcome is at the same time welfare-maximizing, core-stable and envy-free. Moreover, a simple truthful ascending auction for finding such an outcome exists [2]. If bidders have hard budget limits, however, an outcome with these preferable properties does in general not exist, so one must be satisfied with mechanisms fulfilling a subset of them. We consider the problem of finding welfare-maximizing core-outcomes in assignment markets with budget constraints. Our main contributions are twofold: On the one hand, we provide a simple ascending auction that provably terminates in a core-allocation, given that bidders are truthful. If an additional easy-to-check ex-post condition is satisfied, this mechanism also maximizes welfare among all core-outcomes. This auction neither requires prior knowledge of the bidders’ valuations nor of their budget. On the other hand, we prove that the problem of finding welfare-maximizing core-allocations is in general NP-complete.

2 ECONOMIC SETTING
A bidder’s unit-demand preferences are determined by a valuation function \( v: S \to \mathbb{Z}_{\geq 0} \), where \( S \) is the set of all available items, identified with their respective sellers, and the budget limit \( b \). The utility for receiving item \( j \) at price \( p(j) \) is \( v(j) - p(j) \) if \( p(j) \leq b \), and \( -\infty \) if \( p(j) > b \). Given a set \( B \) of bidders, indexed by \( i \), an outcome is a tuple \((\mu, p)\), where \( \mu: B \to S \) is an assignment of bidders to items (where \( S \) contains a dummy item), and \( p: S \to \mathbb{R}_{\geq 0} \) determines the anonymous and linear prices for each of the items. A core-outcome is an outcome without a blocking buyer-seller pair, i.e., there are no \( i \in B \) and \( j \in S \) such that \( i \) would prefer \( j \) to \( \mu(i) \) and could improve \( j \)'s utility by paying strictly more than \( j \) currently receives, i.e., \( v_i(j) - p(j) > v_i(\mu(i)) - p(\mu(i)) \) and...
Our central problem is to solve \[ \max \{ \sum_i u_i(\mu(i)) : (\mu, p) \text{ is a core-outcome} \}, \] i.e., to find welfare-maximizing core-outcomes.

3 AN ASCENDING AUCTION

We provide a simple ascending auction mechanism for assignment markets with budget constraints. Similar to the well-known ascending auction mechanism by Demange et al. [2] for unconstrained unit-demand buyers, the basic idea is to raise prices of overdemanded items by 1 in every iteration. However, due to budget constraints, we may observe sudden drops in demand by doing so. Whenever this occurs, we blacklist some items for one bidder with tight budget constraints and undo the latest price increment. We prove that this mechanism always terminates in a core-outcome. If during the auction process multiple bidders have tight budgets in the same iteration, the auctioneer must decide which of the buyers to blacklist. Depending on the auctioneer’s decision, the observed welfare may vary. However, there always exists a sequence of decisions leading to maximum welfare.

**Proposition.** The auction terminates after a finite number of iterations in a core-outcome. There is a sequence of decisions by the auctioneer, such that the resulting outcome maximizes welfare. If at no point in the auction, bidders simultaneously have tight budgets, the outcome is unique and thus maximizes welfare.

4 NP-COMPLETENESS

In our iterative auction, we cannot expect to find welfare-maximizing core-outcomes in general, since the auctioneer does not have enough information to decide which buyer to blacklist. A natural next question is whether such a decision could be made with reasonable computational effort, given that complete information about the bidders’ preferences is available to the auctioneer. More generally, we ask whether it is computationally feasible to determine a welfare-maximizing core-allocation by any algorithm given the valuations and budgets of all bidders. We call the decision version of this problem the **Maximum Welfare Budget Constrained Stable Bipartite Matching (MBSBM)** problem. We show that MBSBM is NP-complete, rendering the problem intractable for larger instances. The proof is based on an elaborate reduction from the Maximum Independent Set problem.

**Theorem.** The MBSBM problem is NP-complete.

5 CONCLUSIONS

In this work, we thoroughly study the problem of computing welfare-maximizing core-outcomes in assignment markets with budget-constrained bidders. While a core-outcome can be determined efficiently via a simple ascending auction, we prove that computing a welfare-maximizing one is computationally intractable, even if complete information about the bidders’ preferences is available. The result complements the analysis of markets that allow for general preferences (beyond unit-demand) and hard budget constraints as it has been analyzed recently [1]. As a result, claims about the efficiency of simple (polynomial-time) market designs need to be considered with care in markets where financial constraints of bidders play a role.

REFERENCES

[1] Martin Bichler and Stefan Waldherr. 2022. Core-Pricing in Combinatorial Exchanges with Financially Constrained Buyers: Computational Hardness and Algorithmic Solutions. *Operations Research* 70, 1 (2022), 241 – 264.

[2] Gabrielle Demange, David Gale, and Marilda Sotomayor. 1986. Multi-item auctions. *Journal of Political Economy* 94, 4 (1986), 863–872.

A full version of this paper is available at https://arxiv.org/abs/2205.06132.