The impact of changing the type of understeer on vehicle handling

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Abstract. The transient response of a truck with a step control action on the steering studies in the paper. The car is represented by a one-mass flat design model. The equations of the machine motion are written in the form of stability derivatives. Calculation of direct estimates of the quality of the transition process on the angular velocity of the car at a stepwise (step) control action is made. The influence of the of the rotation type change on the car handling is presented.

1. Introduction

Studying the car handling, it can be noted that a number of authors indicate that when designing a car it is necessary to ensure its understeer throughout the range of mass from curb to maximum allowed. However, often, in the operation of trucks those cases are allowed where the cargo weight exceeding the established capacity. At the same time, it is possible to change the rotation type from insufficient to excessive. In this paper, the case of such a transition is considered. In this case, the vehicle handling will be evaluated on the basis of direct assessments of the quality of the vehicle transient response for different initial traffic conditions.

In this paper, we take the linear one-mass plane model of the car as the basis (the motion equations are presented in the form of stability derivatives [1]). Let us recall briefly the main assumptions typical for this calculation model. The drag coefficient of the side wheel drive is a constant. There is no redistribution of vertical loads on the wheels of one axis, and the position of the car gravity center when it moves is not changed. The angles of the wheels slip and the steering angles of the steered wheels have small values. Applying the classical traditional approach of the automatic and control regulation theory, the car transient response is determined by an instant change in the angle of rotation of the driven wheels. When calculating, we also take the following: the translational speed of the car remains constant for each movement case, only disturbances from control are taken into account, as for aerodynamic disturbances and road effects, they are not taken into account. On the basis of direct assessments of the quality of the transition process, under different conditions, theoretical conclusions about the impact on the quality of the transition reaction of the car on the angular speed of rotation changes in the type of rotation will be made.

2. Main part

In this article the following notations are accepted:

- \(a, b\) – distance (horizontal) from the centre of gravity of the vehicle to the front (rear) axle [m];
- \(\theta\) – angle of rotation of operated wheels [rad];
- \(\omega\) – angular velocity of the car rotation \([1/s]\);
- \(s\) – Laplace’ operator;
- \(C_1, C_2\) – coefficient of resistance to the tire of the front (rear) wheels \([N/\text{rad}]\);
\( J_z \) – moment of inertia of the vehicle, relative to the vertical axis passing through its center of gravity \([\text{kg} \cdot \text{m}^2]\); 
\( M \) – vehicle weight \([\text{kg}]\); 
\( V \) – forward speed of the car \([\text{m}/\text{s}]\); 
\( Y_\theta = C_1 + C_2 \) - derivative of the resistance (for lateral force), defined by the presence of the slip of the car wheels \([\text{N}/\text{rad}]\); 
\( Y_\omega = \frac{1}{V} (aC_1 - bC_2) \) - derivative of the resistance (for lateral force), defined by the rotation of the vehicle about a vertical axis passing through its center of gravity \([\text{N} \cdot \text{s}]\); 
\( Y_\phi = -C_1 \) - the derivative of stability (for lateral force), determined by the rotation of the driven wheels \([\text{N}/\text{rad}]\); 
\( N_\beta = aC_1 - bC_2 \) - derivative of stability (by turning moment), caused by the presence of the slip of the car wheels \([\text{N} \cdot \text{m}/\text{rad}]\); 
\( N_\omega = \frac{1}{V} (a^2 C_1 + b^2 C_2) \) - derivative of stability (by turning moment), caused by the rotation of the car, with respect to the vertical axis passing through its center of gravity \([\text{N} \cdot \text{m} \cdot \text{s}]\); 
\( N_\phi = -aC_1 \) - the derivative of stability (by turning moment), caused by the rotation of the driven wheels \([\text{N} \cdot \text{m}/\text{rad}]\).

For studying, we considered a two-axle truck with a rear driving axle, which has the technical characteristics specified in table 1.

**Table 1.** General technical characteristics of the truck.

| Wheelbase                  | 2.9 m |
|----------------------------|-------|
| The coefficient of cornering resistance | -40 \( \text{kN/} \text{rad} \) |
| Gross vehicle weight / Curb weight               | 3500 kg / 1850 kg |
| Curb weight distribution (front axle \([M_1]\) / rear axle \([M_2]\)) | 1050 kg / 800 kg |
| Total mass distribution (front axle \([M_1]\) / rear axle \([M_2]\)) | 1200 kg / 2300 kg |

Having the value of the coefficient of resistance to the lateral lead of one tire, taking into account that all the wheels are equipped with the same tires with the same parameters, we obtain for the axes:

\[
C_1 = 2C = -80 \left[ \frac{\text{kN}}{\text{rad}} \right]; \quad C_2 = 4C = -160 \left[ \frac{\text{kN}}{\text{rad}} \right]
\]

The necessary values for the position of the centre of gravity of the vehicle at different values of its mass and moment of inertia are presented in table 2.

**Table 2.** Individual parameter values.

| Vehicle weight \((M), \text{kg}\) | Position of the car centre of gravity | Vehicle moment of iner \((J_z), \text{kg} \cdot \text{m}^2\) |
|-------------------------------|--------------------------------------|----------------------------------|
|                               | \(a, \text{m}\) | \(b, \text{m}\) |                                     |
| 1850                          | 1.256               | 1.644               | 3820                                      |
| 3500                          | 1.91                | 0.99                | 6620                                      |
| 3700                          | 1.96                | 0.94                | 6820                                      |
Let's make a characteristic equation of a truck and not take its roots. Using the work of [2] we have:

\[ s^2 + B_1 s + B_2 = 0 \]  

(1)

where \( B_1 = \left( \frac{N_\omega}{J_z} + \frac{Y_\beta}{MV} \right) \) \( B_2 = \frac{N_\beta - \frac{N_\beta Y_\omega}{J_z} + \frac{Y_\beta N_\omega}{MVJ_z}}{MVJ_z} \)

Consider the case when the roots of characteristic equation (1) are real, then the equality is secured:

\[ \omega(t) = E_1 + E_2 e^{s_1 t} + E_3 e^{s_2 t} \]  

(2)

where \( A_1 = \frac{N_\beta \theta_{\text{vc}}}{J_z} \); \( A_2 = \frac{\theta_{\text{vc}} (Y_\beta N_\omega - N_\beta Y_\beta)}{MVJ_z} \);

\[ s_{1,2} = \frac{-B_1 \pm \sqrt{B_1^2 - 4B_2}}{2} \]

\[ E_1 = \frac{A_2}{s_1 s_2}; \quad E_2 = \frac{-E_1 s_2 + A_1}{s_2 - s_1}; \quad E_3 = -(E_1 + E_2). \]

For this case the quality of the transition process is evaluated using only control time and by employing the traditional approach of the automatic control and regulation theory we will define the time interval from the moment of application of the impact to the moment when the value becomes less than the value of 0.1 \( \omega_{\text{ns}} \) at all subsequent moments of time. The analysis of the expression (2) shows that if the roots of the characteristic equation of the truck are valid and negative, then with increase in time the value of the angular velocity of rotation of the machine will tend to the value of \( E_1 \) which equals to the magnitude of the steady-state angular velocity \( (\omega_{\text{ns}}) \) after the completion of the transition process.

Now let us consider the situation when the roots of equation (1) are imaginary. In this case, the transient response of the vehicle is oscillatory and is determined by:

\[ \omega(t) = T_1 + Z e^{\frac{-B_1 t}{2}} \sin(\omega_k t) + T_2 e^{\frac{-B_1 t}{2}} \cos(\omega_k t) \]  

(3)

where \( \omega_k^2 = B_2 - \left( \frac{B_1}{2} \right)^2 \); \( T_1 = \frac{A_2}{B_2}; \quad T_2 = -T_1; \quad T_3 = A_1 - B_1 T_2; \quad Z = -\frac{T_2}{\omega_k} \left( \frac{B_1}{2} - \frac{T_3}{T_2} \right) \).

Here we evaluate the quality of the transition process by the previously mentioned control time and the magnitude of overshoot \( (\sigma) \) which is numerically equal to the ratio of the difference of the maximum value of the angular velocity of rotation of the vehicle rotation during the period of the transition process and the steady-state value of angular speed of after the end of the transition process to the steady-state value of angular speed.

Using expressions (2) or (3), for the corresponding roots of the characteristic equation of the truck, using the necessary data from tables 1 and 2, we counted the values of stability derivatives and constructed the car transient responses to the instantaneous change in the angle of rotation of the driven wheels from 0 to 0.17 radians at different values of the truck mass. Figure 1 shows the transient response for curb weight at three different translational speeds of the vehicle.
In figures 2 and 3 the dependence of the regulation time and over-regulation on different values of the translational speed is presented.

In the same sequence in Fig. 4-Fig. 6 the corresponding dependences for the maximum permissible mass of the vehicle are given.
Next, we consider the case for mass of 3700 kg. Here the character of the transition reaction for different speeds will have the following form shown in Fig. 7.
3. Result of work.
The analysis of the relationships of characteristic quantities and for different masses of the car shows that within the permissible mass [1850...3500 kg] there is understeer, in the case of exceeding the maximum allowed masses we have oversteer. The conducted direct evaluations of the quality of the transient response of the angular velocity of the car showed that at curb weight the control time determining the speed of the linear dynamic system (truck) is in the range from 0 to 0.52 seconds for speeds from 0 to 32 m/s, and the value of overshoot characterizing the oscillation of the system reaches 29% at a speed of 32 m/s at the maximum permitted mass, the vibration of the transient reaction is weakly expressed, and the maximum control time increases to 0.76 seconds. At the onset of oversteer, the law of change of the transient reaction is monotonic aperiodic in nature, with the system speed significantly reduced, and the control time begins to increase to the value of 1.86 seconds.

In conclusion, it should be noted that the change in the type of rotation from insufficient to excessive leads to decreasing in the operating speed of the truck as a linear dynamic (mechanical) system and inevitably has a negative impact on its handling.

4. References.
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