Radiative transitions $\gamma \Delta(1232) \rightarrow N^*$ are discussed in the nonrelativistic quark model with spin-orbit corrections for the 70-plet $L^p = 1^-$ nucleon resonances $N^*$. The reaction $\gamma N \rightarrow \gamma \Delta$ is considered as a tool to measure some of these transitions. A particular sensitivity to photoexcitations of $S_{11}(1535)$, $D_{13}(1700)$, and $D_{15}(1675)$ is predicted.

1 Introduction: the GDH sum rule for the $\Delta(1232)$

My motivation to analyze photocouplings of the $\Delta(1232)$ was initially related to a problem of saturation of the Gerasimov–Drell–Hearn sum rule for the $\Delta$. Generally, the GDH integral

$$I = \int_{\text{thr}}^{\infty} \frac{d\omega}{\omega} \left[ \sigma_{1+s}(\omega) - \sigma_{1-s}(\omega) \right] = \pi s \left( \frac{\mu}{M} - \frac{eZ}{M} \right)^2 \quad (1)$$

$$= \sum_{N^* \neq B} \frac{4\pi M^* M}{(M^* - M)^2} \left\{ |A(1, s)|^2 - |A(1, -s)|^2 \right\} \quad (2)$$

must give the magnetic moment $\mu$ of the baryon $B$ of the mass $M$, spin $s$ and the electric charge $eZ$ through the total photoabsorption cross sections $\sigma_{1 \pm s}$ for the target spin parallel or anti-parallel to the photon helicity $\lambda = +1$. The second line re-expresses the integral (1) through the electromagnetic transition amplitudes $A(\lambda, \sigma)$ of $\gamma B \rightarrow N^*$ in the zero-width approximation for the resonances $N^*$. Usually photocouplings $A_\Lambda$ are defined instead of $A(\lambda, \sigma)$,

$$A(\lambda, \sigma) \equiv \langle N^*(p + \vec{k}), \lambda + \sigma | -\vec{\epsilon}_{\lambda} \cdot \vec{J}_{\text{e.m.}}(\vec{k}) | B(p), \sigma \rangle = \sqrt{\frac{M^* - M}{M^*}} \xi A_\Lambda \quad (3)$$

where $\Lambda = \lambda + \sigma$ is the total helicity, $\lambda = +1$, and $\xi = \pm 1$ is a phase factor determined by the pion decay amplitude of the resonance $N^*$. Collinear kinematics $p \parallel \vec{k}$ is assumed in (3).

In the framework of the nonrelativistic quark model (NQM), there is a fundamental difference between the GDH sum rule for the nucleon and for the $\Delta$. The radiative $M1$ transitions $N \rightarrow \Delta$ with the helicities $1 + \frac{1}{2} \rightarrow \frac{3}{2}$ and $1 + (- \frac{1}{2}) \rightarrow \frac{1}{2}$ nearly saturate the sum rule for the nucleon giving $I = 216 \mu b$. 


in close agreement with the experimental values $I_p = 205 \, \mu b$ and $I_n = 233 \, \mu b$ inferred from the nucleon anomalous magnetic moments. Meanwhile the same $N \rightarrow \Delta$ transition gives a big negative contribution $I = -323 \, \mu b$ to the GDH sum rule for the $\Delta$ [note the helicities $1 + (\overrightarrow{\Delta}) \rightarrow \frac{-1}{2}$] in drastic disagreement with the manifestly positive r.h.s. of Eq. (1). Assuming that the GDH sum rule is valid for all hadrons including the $\Delta$, we conclude that the $\Delta$ must have strong photocouplings to nucleon resonances of spin $J \geq \frac{5}{2}$. These resonances must compensate $\Delta \rightarrow N, N^*$ contributions from spin $J \leq \frac{3}{2}$ states, all of which are negative.

2 Photocouplings in NQM: the role of spin-orbit corrections

It was found long ago \cite{4} that nonrelativistic constituent models do not obey the GDH constraint \cite{1}. Relativistic corrections are needed in order to restore the validity of the GDH sum rule. They are especially important for high-lying states. For a target having all constituents (quarks) on the s-shell and thus having the angular and magnetic moment completely composed of constituent’s spins, it is sufficient to include only spin-orbit corrections,

$$H_{\text{c.m.}}^{\text{SO+NA}} = \sum_i \frac{e_i \vec{\sigma}_i}{8m_i^2} \cdot (\vec{p}_i \times \vec{E}_i - \vec{E}_i \times \vec{p}_i) + \sum_{i \neq j} \frac{e_i}{4M_{\text{tot}}} \left( \frac{\vec{\sigma}_i}{m_i} - \frac{\vec{\sigma}_j}{m_j} \right) \cdot \vec{p}_j \times \vec{E}_i. \quad (4)$$

Apart from ordinary additive spin-orbit terms (SO) this equation contains important non-additive (NA) two-body pieces which appear owing to the Wigner rotation of quark spins into the center-of-mass frame of the baryon \cite{4,5}.

One can illustrate the role of the SO+NA terms taking the radiative transition $\gamma N \rightarrow \Delta$ as example. Equation (3) defines the photocoupling $A_\Lambda$ of the nucleon as a Lorentz-invariant quantity. However, when nonrelativistic wave functions and the nonrelativistic electromagnetic current are used in (3), the result does depend on the frame used. For example, the photocoupling $A_{3/2}$ found in the c.m. frame ($\vec{p} = -\vec{k}$), in the Breit frame ($\vec{p} = -\frac{1}{2}\vec{k}$) and in the lab frame ($\vec{p} = 0$) is equal to $-175, -194$ and $-219 \times 10^{-3}$ GeV$^{-1/2}$, respectively [oscillator wave functions were used in this calculation with the oscillator parameter $a = 0.41$ GeV and $m_q = 336$ MeV]. When the spin-orbit terms \cite{4} are included, the result turns out be nearly frame-independent: $-198, -194$ and $-182 \times 10^{-3}$ GeV$^{-1/2}$, respectively. In this example SO and NA corrections are equally important and they are seen to nearly restore the Lorentz invariance of the transition amplitude.

Using the Karl–Isgur quark model (for a recent review see \cite{3}) with the spin-orbit correction \cite{4}, we (re)calculated photocouplings of the nucleon to
negative-parity baryons \([70, \, L^P = 1^-]\). With three exceptions, we found a full agreement with the previous work (those exceptions are the SO contribution to the amplitude \(A_{3/2}\) for \(\gamma N \rightarrow D_{33}(1700)\) and NA contributions to the amplitudes \(A_{1/2}, \, A_{3/2}\) for \(\gamma N \rightarrow S_{13}(1620)\)).

Then we calculated photocouplings of the \(\Delta\). Our results for \(\Delta^+\) are given in the Table 1 as example, separately for the cases when a pure nonrelativistic approximation is used (columns NR) and when the spin-orbit corrections (SO) are included too (columns SO). Also (some of) predictions of Carlson–Carone are shown (columns CC1) which have been obtained through a fit to experimental data on nucleon photocouplings using one-body quark operators dominating in the large \(N_c\) limit of QCD. For large amplitudes, there is a qualitative agreement between our results (when SO is included) and the results of Carlson–Carone with the except for a few signs which are probably related with different phase conventions used for the baryon wave functions; our convention follows Ref. 3.

Our calculation suggests a strong photocoupling of the \(\Delta(1232)\) to the lowest \(J = \frac{5}{2}^+\) state which is \(D_{15}(1675)\). The \(\Delta \rightarrow D_{15}\) transition contributes much to the GDH sum rule for the \(\Delta\) and it helps to reduce a gap between the negative Eq. (2) evaluated through the lowest \(56\) and \(70\)-plet baryons \(N^*\) and the manifestly positive r.h.s. of Eq. (1). See 2 for further detail.

Table 1. \(\Delta^+(1232)\) photocouplings to \([70, \, L^P = 1^-]\) baryons in units of \(10^{-3}\) GeV\(^{-1/2}\).
The phase factor of \(\xi\), Eq. (3), is included into \(A_\Lambda\).

| \(N^*\)     | \(A_{-1/2}\) | \(A_{1/2}\) | \(A_{3/2}\) |
|-------------|--------------|--------------|--------------|
|             | NR SO CC1    | NR SO CC1    | NR SO CC1    |
| \(S_{11}(1535)\) | -88 -75 108 | -94 -86 4  | -94 -86 4  |
| \(D_{13}(1520)\) | 61 20 62     | 48 1 -16     | 22 -19 -90  |
| \(S_{11}(1650)\) | -109-115 63  | -107 -60 -128 | -107 -60 -128 |
| \(D_{13}(1700)\) | -87 -64 43   | -164 -138 123 | -198 -175 169 |
| \(D_{13}(1675)\) | 9 -12 24     | -26 -62 -19  | -97 -147 -113  |
| \(S_{31}(1620)\) | 46 46 -42    | -27 -27 73   | -201 -267 -258 |
| \(D_{33}(1700)\) | 81 81 -54    | 47 47 0      | 0 0 55       |

3 Differential cross section of \(\gamma N \rightarrow \gamma \Delta\)

Using the above photoamplitudes as functions of the photon energy, we estimated differential cross section of inelastic Compton scattering \(\gamma N \rightarrow \gamma \Delta\) in the resonance region. Both \(s\) and \(u\) channel resonances, with experimental masses and widths, were included as well as seagull contributions, cf.
Ref. Dominating contributions to the reaction off the proton come from the $S_{11}(1535)$ and $D_{13}(1700)$ resonances, see Fig. For the neutron target, the $D_{11}(1675)$ resonance dominates. Note that $D_{11}(1675)$ is decoupled from the proton [i.e., the Moorhouse selection rule remains valid for $D_{15}$ even after the NA corrections are included], so that the reaction off the neutron, via $\gamma d \rightarrow \gamma p \Delta^0$, turns out the only efficient way to study the $\gamma \Delta D_{15}$ transition.

Figure 1. Differential cross sections (c.m.) of inelastic Compton scattering in the NQM including spin-orbit corrections. Contributions of $s$-channel resonances (without $u$-channel and seagull contributions) are shown separately.

The differential cross section of inelastic Compton scattering is seen to be compatible with that of elastic $\gamma N$ scattering. Therefore we may conclude that the original idea of Carlson–Carone to study photocouplings of $\Delta$ in the reaction $\gamma N \rightarrow \gamma \Delta$ can probably be realized in practice.

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