I. INTRODUCTION

One of the most common properties of attractive fermion many-body system is the arising of superfluid state at low temperature. Depending on the strength of the attractive interaction between two fermions, however, the physical contents of the superfluid state could be distinguishably altered. When the interaction is weak, the system can be well described by the Bardeen-Cooper-Schrieffer (BCS) theory. In this case, the superfluidity is due to the condensate of loosely correlated Cooper pairs and superfluid gap is much smaller than the Fermi energy. When the interaction becomes sufficiently strong, the two-fermion bound state could form, which may behave like a boson. In this situation, the superfluidity is due to the Bose-Einstein condensation (BEC) of the tightly bound two-fermion state and the superfluid gap could be much larger than the Fermi energy. Although the BCS and BEC limits have quite different physics, it was found that there is no true phase transition (traditional symmetry breaking) happening in between. The transition from BCS state to BEC state is smooth and is often called the BCS-BEC crossover [1–4]. Such BCS-BEC crossover was recently realized in cold atomic experiments (see Refs. [5, 6] and references therein).

It was well known that cold nuclear matter can be in a superfluid state, which plays a crucial role in a variety of nuclear many-body problems, from neutron stars, low-energy heavy-ion collisions, to finite nuclei. It was argued that the BCS-BEC crossover should also occur in nuclear matter where the BCS state of neutron-proton (np) Cooper pairs at high density undergoes a smooth transition into BEC state of deuterons at low density [7–14]. At the same time, the chemical potential changes its sign at a certain density and finally approaches one-half of the deuteron binding energy at the low-density limit. Recently, a similar situation was also studied for the neutron-neutron (nn) pairs in the $^1S_0$ channel [14–25]. It was found that in certain (low) density regions the nn pairs can be strongly correlated. However, no assured BEC state was found for np pairs.

So far, most of the investigations of nuclear BCS-BEC crossover in the literatures focused on the ground state crossover described by BCS theory. Although the BCS theory succeeds in describing the BCS-BEC crossover at zero temperature, it, as a mean-field theory, is not sufficient to describe low-density nuclear matter at finite temperature where the pairing fluctuation is substantial due to the strongly correlation nature of the system. Actually, as a consequence of the strong correlation, the low-density nuclear matter exhibits “pseudogap” phenomena above the critical temperature $T_c$ of superfluid transition and has an exotic normal state that is different from the Fermi liquid normal state associated with BCS theory [26–28]. Similar situations were also found in other strongly correlated systems, such as high $T_c$ superconductors [29–34] and cold atomic Fermi gases under Feshbach resonances [33, 35–40]. To include the pairing fluctuation effects and investigate the pseudogap phenomena, we will adopt a $T$-matrix formalism based on a $G_0G$ approximation for the pair susceptibility, which was first introduced by the Chicago group [34–40]. This formalism generalizes the early works of Kadanoff and Martin [41] and Patton [42], and can be considered as a natural extension of the BCS theory since they share the same ground state. Moreover, this formalism allows quasi-analytic calculations and gives a simple physical interpretation of the pseudogap phase. It clearly shows that the pseudogap is due to the incoherent pairing fluctuation.

Our focus will be put on the np pair in symmetric nuclear matter (mainly in the low-density region), since the interaction in this case is more attractive than in the nn or pp channels and it provides a very good playground for the BCS-BEC crossover. We will extend the early studies [7–14] to include the pairing fluctuation effects and determine the magnitude of the pseudogap. Furthermore, the transition temperature for the onset of the superfluid and the thermodynamic properties will be also a concern. Such a study will be helpful to understand the strongly coupling nature of low-density nuclear matter and may give useful information on the physics of the surface of nuclei, expanding nuclear matter from heavy-ion collisions, collapsing stars, etc.

The article is organized as follows. We give a brief summary of the effective nucleon-nucleon potential in Sec. II. In Sec. III, we give a detailed theoretical scheme of how the $T$-matrix-based formalism works at finite temperature. The nu-
merical results are presented in Sec. V. We summarize our results in Sec. V. Throughout this article, we use natural units \( \hbar = k_B = c = 1 \).

II. EFFECTIVE NUCLEON-NUCLEON POTENTIAL

The aim of this article is not to determine the precise values of the pairing gap, the critical temperature, etc., but rather to perform a qualitative (or semi-quantitative) study of the effects of pairing fluctuation on BCS-BEC crossover. In order to highlight the essential physics, we will adopt a simple density dependent contact interaction (DDCI) developed in Refs. [43, 44]. The potential is of the form

\[
V(x - x') = v_0 \left\{ 1 - \eta \left[ \frac{\rho(x)}{\rho_0} \right]^\gamma \right\} \delta(x - x'),
\]

where \( v_0, \eta, \gamma \) are three adjustable parameters, \( \rho(x) = \rho_n(x) + \rho_p(x) \) is the nuclear density and \( \rho_0 = 0.17 \text{ fm}^{-3} \) is the normal nuclear density. Taking suitable values of the parameters, one can reproduce the pairing gap \( \Delta(k_F) \) as a function of Fermi momentum \( k_F = (3\pi^2 \rho/2)^{1/3} \) in the channels \( L = 0, I = 1, I_z = \pm 1, S = 0 \) and \( L = 0, I = 0, S = 1, S_z = 0 \) calculated from realistic nucleon-nucleon potentials [43, 44], where \( L \) is orbital angular momentum, \( I \) denotes isospin, and \( S \) is spin. According to Garrido et al. [43, 44], we will choose in the following numerical calculation \( \eta = 0, v_0 = -530 \text{ MeV fm}^3 \) in the \( I = 0, 3S_1 \) (np pairing) channel and a energy cutoff \( \epsilon_c = 60 \text{ MeV} \) to regularize the integration. With these parameters one must use a density-dependent effective nucleon mass \( m(p) \) corresponding to the Gogny interaction[43, 44],

\[
\left[ \frac{m(p)}{m_0} \right]^{-1} = 1 + \frac{m_0 k_F}{2} \sqrt{\pi} \sum_{c = 1}^{2} \left[ W_c + 2(B_c - H_c) - 4M_c \right] \times \mu_c^3 e^{-x_c} \left[ \frac{\cosh x_c}{x_e} - \frac{\sinh x_c}{x_e^2} \right],
\]

where \( x_c = \frac{k_F^2 \mu_c^2}{2 m_0} \), \( m_0 = 939 \text{ MeV} \) is the bare mass of nucleon, and \( \mu_c, W_c, B_c, H_c, M_c \) are parameters corresponding to the Gogny force D1[45, 46], their values are listed in Table I.

TABLE I: Parameters in the effective mass of nucleon (2.2) corresponding to the Gogny interaction D1[45, 46].

| c  | \( \mu_c [\text{fm}] \) | \( W_c [\text{MeV}] \) | \( B_c [\text{MeV}] \) | \( H_c [\text{MeV}] \) | \( M_c [\text{MeV}] \) |
|----|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1  | 0.7             | -402.4          | -100.0          | -496.2          | -23.56          |
| 2  | 1.2             | -21.30          | -11.77          | 37.27           | -68.81          |

III. T-MATRIX-BASED FORMALISM

We consider the nuclear matter as an infinite system of interacting fermions. In the low-density region, np pairing is realized mainly in the spin-triplet s-wave channel, so let us consider the following Lagrangian describing neutron and proton interaction via two-body attractive forces in \( 3S_1, S_z = 0 \) channel,

\[
\mathcal{L} = \sum_{i = n, p \sigma = \uparrow, \downarrow} \bar{\psi}_i \sigma \left( -\partial_\tau + \frac{\nabla^2}{2m} + \mu \right) \psi_i \sigma + \frac{g}{2} \left( \bar{\psi}_n \gamma^5 \psi_p \downarrow - \bar{\psi}_p \gamma^5 \psi_n \uparrow \right) \left( \bar{\psi}_p \gamma^5 \psi_n \uparrow - \bar{\psi}_n \gamma^5 \psi_p \downarrow \right),
\]

(3.1)

where \( g = -v_0 > 0 \) is the coupling strength in np channel and \( \tau = it \) is the imaginary time. Introducing auxiliary fields \( \Delta \equiv (g/2) \left( \bar{\psi}_n \gamma^5 \psi_p \downarrow - \bar{\psi}_p \gamma^5 \psi_n \uparrow \right) \) and \( \bar{\Delta} \equiv (g/2) \left( \bar{\psi}_p \gamma^5 \psi_n \uparrow - \bar{\psi}_n \gamma^5 \psi_p \downarrow \right) \), we can recast Eq. (3.1) as

\[
\mathcal{L} = \sum_{i = n, p \sigma = \uparrow, \downarrow} \bar{\psi}_i \sigma \left( -\partial_\tau + \frac{\nabla^2}{2m} + \mu \right) \psi_i \sigma + \frac{2g}{\bar{\Delta}} \Delta \bar{\Delta},
\]

(3.2)

where we have introduced the Nambu-Gorkov spinor \( \Psi = (\bar{\psi}_n \gamma^5 \psi_p \downarrow, \bar{\psi}_p \gamma^5 \psi_n \uparrow)^T \), and

\[
S^{-1} = \begin{pmatrix} S_1^{-1} & 0 \\ 0 & S_2^{-1} \end{pmatrix},
\]

(3.3)

with

\[
S_1^{-1} = \begin{pmatrix} -\partial_\tau + \nabla^2/(2m) + \mu & \Delta_1 \\ \Delta_1 & -\partial_\tau - \nabla^2/(2m) - \mu \end{pmatrix},
\]

(3.4)

\[
S_2^{-1} = \begin{pmatrix} -\partial_\tau + \nabla^2/(2m) + \mu & -\Delta \bar{\Delta} \\ -\Delta \bar{\Delta} & -\partial_\tau - \nabla^2/(2m) - \mu \end{pmatrix},
\]

(3.5)

It is seen that \( S_2 \) differs from \( S_1 \) only by minus signs in front of \( \Delta \) and \( \bar{\Delta} \). To make our formulae more compact, in the following discussions we will treat \( S_1 \) only and neglect the subscript 1 without confusion.

In the rest of this section, following the works of the Chicago group, we will introduce the basic method of the \( T \) matrix. This \( T \) matrix is defined as an infinite series of ladder-diagrams in particle-particle channel (rather than particle-hole channel) by constructing the ladder by one free nucleon propagator and one full nucleon propagator. Then, as usual, the \( T \) matrix enters the nucleon self-energy in place of the bare interaction vertex. The coupled \( T \)-matrix equation and the self-energy equation (as well as number density equation) should be solved self-consistently. One can view this approach as the simplest generalization of the BCS scheme, which formally can also be cast in a \( T \)-matrix formalism. Let us discuss this point in the following subsection.
A. BCS theory

The BCS theory is based on mean-field approximation to the (anomalous) self-energy, i.e., $\Delta$ and $\bar{\Delta}$ are chosen as their mean-field values $\Delta = \Delta_{sf}$ and $\bar{\Delta} = \Delta_{mf}$ (without loss of generality, we put $\Delta_{sf}$ and $\bar{\Delta}_{sf}$ to be constants and $\Delta_{sf} = \Delta_{sf}$), which are regarded as order parameters for superfluid phase transition. We start with the Nambu-Gorkov formalism in momentum space,

$$S_{mf}^{-1}(K) = \left( \frac{G_{mf}^{-1}(K)}{\Delta_{sf}} - g \frac{\delta_{mf}^{-1}(-K)}{\bar{\Delta}_{sf}} \right),$$

where $K = (i\omega_n, k)$ and $i\omega_n = i(2n + 1)\pi T$ is the fermion Matsubara frequency. $G_{mf}^{-1}(K) = i\omega_n - \xi_k$ is the inverse of free nucleon propagator, $\xi_k = k^2/(2m) - \mu$ is the dispersion relation of free nucleon. From Eq. (3.6) one gets,

$$S_{mf}(K) = \left( \frac{F_{mf}(K)}{\Delta_{mf}} - g \frac{\delta_{mf}(-K)}{\bar{\Delta}_{mf}} \right),$$

where $F_{mf}(K)$ is the anomalous propagator,

$$F_{mf}(K) = \Delta_{sf} G_{mf}(K) \xi_0(-K) = \frac{-\Delta_{sf}}{(i\omega_n)^2 - E_k^2},$$

and $G_{mf}(K)$ is the mean-field single nucleon propagator,

$$G_{mf}(K) = \left[ G_{mf}^{-1}(K) - \Sigma_{mf}(K) \right]^{-1} = \frac{i\omega_n + \xi_k}{(i\omega_n)^2 - E_k^2},$$

with the mean-field dispersion relation of nucleon $E_k = \sqrt{\xi_k^2 + \Delta_{mf}^2}$ and the mean-field self-energy

$$\Sigma_{mf}(K) = -\Delta_{mf}^2 \xi_0(-K).$$

The coupled gap and density equations read

$$\Delta_{sf} \frac{g}{\beta V} \sum_K F_{mf}(K) = \frac{g\Delta_{sf}}{\beta V} \sum_k \frac{1}{2E_k} \left[ 1 - 2n_F(E_k) \right],$$

$$\rho \frac{2}{\beta V} \sum_K e^{\eta n_\nu} G_{mf}(K) = \frac{2}{\beta V} \sum_k \left[ 1 - \frac{\xi_k}{E_k} (1 - 2n_F(E_k)) \right],$$

where $n_F(x) = 1/\left[ \exp (\beta x) + 1 \right]$ is the Fermi-Dirac function and $e^{\eta n_\nu}$ with $\eta \rightarrow 0$ is a convergence factor for Matsubara summation. The prefactor 2 on the right-hand side of density equation counts the degeneracy of $S_1$ and $S_2$.

In BCS theory, $np$ pairs enter into the problem below $T_c$, but only through their condensates at zero momentum. By rewriting the mean-field self-energy $\Sigma_{mf}(K)$ in a manner of

$$\Sigma_{mf}(K) = \frac{1}{\beta V} \sum_Q t_{mf}(Q) \xi_0(Q - K),$$

we are aware of that these condensed pairs can be associated with a $T$ matrix in the following form

$$t_{mf}(Q) = -\Delta_{sf}^2 \delta(Q),$$

with $Q = (q_0, \mathbf{q}), q_0 = i\omega_n = i2\nu_{\pi}T, \nu \in \mathbb{Z}$ being the boson Matsubara frequency and $\delta(Q) = \beta \delta_{q_0,0}\delta^{(3)}(\mathbf{q})$. Furthermore, if we define the mean-field pair susceptibility as

$$\chi_{mf}(Q) = \frac{1}{\beta V} \sum_K G_{mf}(K) \xi_0(Q - K),$$

we can re-write the gap equation in superfluid phase as

$$1 - g\chi_{mf}(0) = 0, \quad T \leq T_c.$$  

This suggests that one can consider the uncondensed pair propagator or $T$ matrix to be of the form

$$t_{pair} = \frac{-g}{1 - g\chi_{mf}(Q)},$$

and then the gap equation is given by $t_{pair}^{-1}(0) = 0$.

It is well known that the critical temperature $T_c$ in the BCS theory is related to the appearance of a singularity in a $T$ matrix in the form of Eq. (3.16) but with $\Delta_{sf} = 0$. This is the so-called Thouless criterion for $T_c$ [47]. But the meaning of Eq. (3.15) is more general as stressed by Kadanoff and Martin [41]. It states that under a asymmetric choice of $\chi$, the gap equation is equivalent to the requirement that the $T$ matrix associated with uncondensed pair remains singular at zero momentum and energy for all temperatures below $T_c$.

Although the construction of the uncondensed pair propagator (3.16) in BCS scheme is quite natural, the uncondensed pair has no feedback to the nucleon self-energy (3.12). When the coupling is weak, such a feedback is not important, but if the system is strongly coupled, this feedback will be significant. The simplest way to include the feedback effects is to replace $t_{mf}$ in Eq. (3.12) by $t_{mf} + t_{pair}$. But to make such an inclusion self-consistent, $t_{pair}$ should be somewhat modified which we discuss in next subsection.

B. $G_0G$ formalism at $T \leq T_c$

Physically, the BCS theory involves the contribution to nucleon self-energy below $T_c$ only from those condensed pairs, i.e., the $q = 0$ Cooper pairs. This is justified only at weak-coupling region. Generally, in superfluid phase, the self-energy consists of two distinctive contributions, one from the superfluid condensate, and the other from thermal pair excitations. Correspondingly, it is natural to decompose the self-energy into two additive terms

$$\Sigma(K) = \frac{1}{\beta V} \sum_Q t(Q) \xi_0(Q - K) = \Sigma_{mf}(K) + \Sigma_{pe}(K),$$

(3.17)
with the $T$ matrix accordingly given by
\[
t(Q) = t_{mf}(Q) + t_{pg}(Q),
\]
\[
t_{pg}(Q) = \frac{-g}{1 - g\chi(Q)},
\] (3.18)
where the subscript $pg$ indicates that this term will lead to the pseudogap in nucleon dispersion relation as will become clear soon. See Fig. 1 for the Feynman diagrams for $t_{pg}(Q)$ and $\Sigma(K)$. Comparing with the BCS scheme, $t_{mf}(Q)$ in Eq. (3.12) is replaced by $t(Q)$, and $\Sigma(K)$ now contains the feedback of uncondensed pairs. The pair susceptibility $\chi(Q)$, as inspired by Eq. (3.14), is chosen to be the following asymmetric $G_0G$ form,
\[
\chi(Q) = \frac{1}{\beta V} \sum_K G(K) G_0(Q - K). \tag{3.19}
\]

In spirit of Kadanoff and Martin, we now propose the superfluid instability condition or gap equation as [extension of Eq. (3.15)]
\[
1 - g\chi(0) = 0, \quad T \leq T_c. \tag{3.20}
\]
We stress here that this condition has quite clear physical meaning in BEC regime. The dispersion relation of the bound pair is given by $t^{-1}(Q) = 0$, hence $t^{-1}(0) \propto \mu_b$ with $\mu_b$ the effective chemical potential of the pairs. Then the BEC condition requires $\mu_b = 0$ for all $T \leq T_c$.

The gap equation (3.20) tells us that $t_{pg}(Q)$ is highly peaked around $Q = 0$, so we can approximate $\Sigma_{pg}$ as
\[
\Sigma_{pg}(K) \approx -\Delta_{pg}^2 G_0(-K), \quad T \leq T_c, \tag{3.21}
\]
where we have defined the pseudogap parameter via
\[
\Delta_{pg}^2 = -\frac{1}{\beta V} \sum_Q t_{pg}(Q). \tag{3.22}
\]
The total self-energy now is
\[
\Sigma(K) = -\Delta^2 G_0(-K), \tag{3.23}
\]
with $\Delta^2 = \Delta_{af}^2 + \Delta_{pg}^2$. It is clear that $\Delta_{pg}^2$ also contributes to the energy gap in quasi-nucleon excitation. Physically, the pseudogap $\Delta_{pg}$ below $T_c$ can be interpreted as an extra contribution to the excitation gap of nucleon quasi-particle: an additional energy is needed to overcome the residual attraction between nucleons in a thermal excited pair to produce fermion-like quasi-particles. One should note that the $\Delta_{pg}$ is associated with the fluctuation of the pairs $\Delta_{pg}^2 \sim \langle \Delta^2 \rangle - \langle \Delta \rangle^2$ [34, 37], hence it does not lead to superfluid (symmetry breaking).

With Eq. (3.21), the pair susceptibility reads,
\[
\chi(Q) = \frac{1}{\beta V} \sum_k \left[ \frac{E_k + \xi_k n_F(-\xi_{q-k}) - n_F(E_k)}{2E_k} \right] - \frac{E_k - \xi_k}{2E_k} \left[ \frac{n_F(E_k) - n_F(\xi_{q-k})}{E_k - \xi_{q-k} + q_0 + i0^+} \right] = \frac{1}{\beta V} \sum_{k,s=\pm} \frac{sE_k + \xi_k n_F(-sE_k) - n_F(\xi_{q-k})}{2sE_k} - \frac{sE_k + \xi_k}{2sE_k}(sE_k + \xi_{q-k} + q_0 + i0^+), \tag{3.24}
\]
\[
\Sigma = \Sigma_{pg} + \Sigma_{mf}
\]
\[
\text{FIG. 1: Feynman diagrams for the } T \text{ matrix of non-condensed pairs and the nucleon self-energy in the } G_0G \text{ formalism.}
\]

with $E_k = \sqrt{\xi_k^2 + \Delta^2}$. The number equation remains unchanged except the replacement of $\Delta_{mf} \rightarrow \Delta$.

Furthermore, the gap equation (3.20) suggests that we can make the following pole approximation to the pair propagator or $T$ matrix $t_{pg}(Q)$ as
\[
t_{pg}(Q) \approx \frac{Z^{-1}}{q_0 - q^2/(2m_b)}, \tag{3.25}
\]
where the residue $Z^{-1}$ and effective “boson” mass are given by
\[
Z = \frac{\partial \chi}{\partial q_0} \bigg|_{q=0},
\]
\[
Z_{mb} = -\frac{1}{3} \frac{\partial^2 \chi}{\partial q^2} \bigg|_{q=0}. \tag{3.26}
\]

We stress here that in general, the expansion of $t_{pg}^{-1}(Q)$ should also contain a term $\propto \xi_{q_k}^0$, but such term does not bring qualitative change to the crossover physics [38], hence we neglect it in Eq. (3.25).

A straightforward calculation gives
\[
Z = \frac{1}{V} \sum_k \sum_{s=\pm} \frac{s}{2E_k} \left[ n_F(E_k) - n_F(\xi_{q-k}) \right] = \frac{1}{\Delta^2} \left[ \frac{\rho}{4} - 1 - \frac{1}{V} \sum_k n_F(\xi_k) \right], \tag{3.27}
\]
and
\[
Z_{mb} = \frac{1}{V} \sum_k \sum_{s=\pm} \frac{2k^2}{3m_b^2} \left[ \frac{n_F(-sE_k) - n_F(\xi_{q-k})}{sE_k + \xi_k} \right] - \frac{2k^2}{3m_b^2} \left[ \frac{n_F(-sE_k) - n_F(\xi_{q-k})}{sE_k + \xi_k} \right] = \frac{1}{V} \sum_k \frac{2k^2}{3m_b^2} \left[ \frac{n_F(-sE_k) - n_F(\xi_{q-k})}{sE_k + \xi_k} \right] - \frac{1}{V} \sum_k \frac{k^2}{3m_b^2} \left[ \frac{n_F(\xi_{q-k})}{sE_k + \xi_k} \right]
\]
\[
\times \left\{ (E_k^2 + \xi_k^2)(1 - 2n_F(E_k)) - 2E_k \xi_{q-k}[1 - 2n_F(\xi_{q-k})] \right\}. \tag{3.28}
\]

The expression in the square bracket of right-hand-side of Eq. (3.27) is nothing but one-half the density of the pairs $\rho_b/2$, we then have $\rho_b = 2Z\Delta^2$. 

Substituting Eq. (3.25) into Eq. (3.22) leads to
\[
\Delta_{pg}^2 = \frac{1}{ZV} \sum_q n_B[q^2/(2m_b)]^2
= \frac{1}{Z} \left( \frac{T m_b}{2\pi} \right)^{3/2} \zeta \left( \frac{3}{2} \right),
\]
(3.29)
where \( n_B(x) = 1/[\exp(\beta x) - 1] \) is the Bose-Einstein function and a vacuum term was regularized out. It should be stressed that at zero temperature \( \Delta_{pg}^2 = 0 \), hence the \( G_0G \) scheme yields the BCS ground state. One should note that \( \Delta_{pg}^2 = \rho_b^{\text{uncondensed}}/2Z \), and hence \( \Delta_{st}^2 = \rho_b^{\text{condensed}}/2Z \).

Now, Eq. (3.20), Eq. (3.29), as well as number equation are coupled to determine the total excitation gap \( \Delta \), the pseudogap \( \Delta_{pg} \) and the nucleon chemical potential \( \mu \) at given density and temperature below \( T_c \). In short, they are
\[
1 = \frac{g}{V} \sum_k \frac{1}{2E_k} [1 - 2n_F(E_k)],
\]
\[
\rho = \frac{2}{V} \sum_k \left[ 1 - \frac{\xi_k}{E_k} (1 - 2n_F(E_k)) \right],
\]
\[
\Delta_{pg}^2 = \frac{1}{Z} \left( \frac{T m_b}{2\pi} \right)^{3/2} \zeta \left( \frac{3}{2} \right).
\]
(3.30)

C. \( G_0G \) formalism above \( T_c \)

Above \( T_c \), Eq. (3.20) does not apply, hence Eq. (3.21) no longer holds. To proceed, we extend our more precise \( T \leq T_c \) equations to \( T > T_c \) in the simplest fashion. We will continue to use Eq. (3.23) to parameterize the self-energy but with \( \Delta = \Delta_{pg} \), and ignore the finite lifetime effect associated with normal state pairs. It was shown that this is still a good approximation when temperature is not very high [36, 38, 40]. The \( T \) matrix \( t_{pg}(Q) \) at small \( Q \) can be approximated now as
\[
t_{pg}(Q) \approx \frac{Z^{-1}}{q_0 - \Omega_q},
\]
(3.31)
where \( \Omega_q = q^2/(2m_b) - \mu_b \). Since there is no condensation in normal state, the effective pair chemical potential \( \mu_b \) is no longer zero, instead, it should be calculated from
\[
Z\mu_b \equiv t^{-1}(0) = -\frac{1}{g} + \chi(0)
= -\frac{1}{g} + \frac{1}{V} \sum_k \frac{1 - 2n_F(E_k)}{2E_k},
\]
(3.32)
This is used as the modified gap equation. Similarly, above \( T_c \) the pseudogap \( \Delta_{pg} \) is determined by
\[
\Delta_{pg}^2 = \frac{1}{ZV} \sum_q n_B(\Omega_q)
= \frac{1}{Z} \left( \frac{T m_b}{2\pi} \right)^{3/2} \zeta \left( \frac{3}{2} \right),
\]
(3.33)
where \( \text{Li}_n(z) \) is the polylogarithm function. Then Eq. (3.32), Eq. (3.33) and the number equation which remains unchanged determine \( \Delta_{pg} \), \( \mu \) and \( \mu_b \).

In summary, at \( T > T_c \), the order parameter is zero, and \( \Delta = \Delta_{pg} \). The closed set of equations determining \( \Delta, \mu \) and \( \mu_b \) is
\[
Z\mu_b = -\frac{1}{g} + \frac{1}{V} \sum_k \frac{1 - 2n_F(E_k)}{2E_k},
\]
\[
\rho = \frac{2}{V} \sum_k \left[ 1 - \frac{\xi_k}{E_k} (1 - 2n_F(E_k)) \right],
\]
\[
\Delta_{pg}^2 = \frac{1}{Z} \left( \frac{T m_b}{2\pi} \right)^{3/2} \text{Li}_\frac{3}{2} \left( e^{\mu_b/T} \right).
\]
(3.34)

D. Thermodynamics

The thermodynamics of the matter are governed by the thermodynamic potential, which reads
\[
\Omega = \Omega_f + \Omega_b,
\]
(3.35)
where \( \Omega_f \) and \( \Omega_b \) are the contributions from nucleons and thermal excited pairs,
\[
\Omega_f = 2\Delta^2 \chi(0) - \frac{4T}{V} \sum_k \left[ \frac{E_k - \xi_k}{2} + \ln \left( 1 + e^{-\beta E_k} \right) \right],
\]
(3.36)
\[
\Omega_b = \frac{2}{\beta V} \sum_q \ln \left( 1 - e^{-\beta \Omega_q} \right).
\]
(3.37)
Other thermodynamic quantities can be derived from \( \Omega \), e.g., the entropy density is given by \( s = -\partial\Omega/\partial T \), and the specific heat \( c_V \) is given by \( c_V = T \partial s/\partial T \).

IV. NUMERICAL RESULTS

We discuss now the results obtained by numerically solving Eq. (3.30) for \( T \leq T_c \) and Eq. (3.34) for \( T > T_c \). We will mainly focus on the intermediate (the crossover region, see Fig. 2) and low density regions, since the high density region is proven to be well understood in BCS theory. We begin with the results concerning the critical temperature for superfluid transition in the BCS-BEC crossover.

A. BCS-BEC crossover and critical temperature

At zero temperature, the \( G_0G \) formalism reproduces the usual BCS theory. In order to have a quantitative examination of the BCS-BEC crossover, it is convenient to define the condensed \( np \) Cooper pair wave function at zero temperature,
\[
\psi(r) = C(BCS|a_{np}^\dagger(x)a_{np}^\dagger(x + r)|BCS)
= C' \int \frac{d^3k}{(2\pi)^3} \psi(k)e^{ikr},
\]
(4.1)
where $a_{\sigma}^{\dagger}$ ($a_{\sigma}$) is the creation operator of neutron (proton) with spin $\sigma$ and $\psi(k)$ are the anomalous density distribution function

$$
\psi(k) = \langle BCS | a^\dagger_{n,\uparrow}(k) a^\dagger_{p,\downarrow}(-k) | BCS \rangle = \frac{\Delta}{2E_k}.
$$

After substituting Eq. (4.2) into the number and gap equations, we get the following Schrödinger-like equation,

$$
\frac{k^2}{m} \psi(k) - g(1 - 2n_k) \int \frac{d^3k'}{(2\pi)^3} \psi(k') = 2\mu \psi(k).
$$

In the limit of vanishing density, $n_k \to 0$, this equation goes over into the Schrödinger equation for the $np$ bound states (the deuterons) in the center-of-mass frame, and the chemical potential $2\mu$ then plays the role of the binding energy. Hence, one expects that at sufficiently low density and low temperature, the symmetric nuclear matter should be in the BEC phase.

To have more quantitative description of the BCS-BEC crossover, we define other characteristic quantities: the mean-square-root size of the $np$ pair,

$$
\xi^2 = \frac{\int d^3x |\psi(x)|^2}{\int d^3x |\psi(x)|^2},
$$

and the $s$-wave scattering length $a$ that relates the coupling constant $g$ to the low-energy limit of the two-body $T$ matrix of $np$ scattering in vacuum,

$$
\frac{m}{4\pi a} = - \frac{1}{g} + \int \frac{d^3k}{(2\pi)^3} \frac{m}{k^2}.
$$

In BCS region, $\xi$ is expected to be larger than the average distance between neutron and proton $d \equiv (\rho/2)^{-1/3}$ and at the same time the scattering length $a$ should be negative to ensure that the interaction between neutron and proton is attractive. In the BEC region, however, $\xi/a$ should be very small reflecting the compactness of the pair, and the scattering length will be positive to guarantee the appearance of two-body bound state.

In Fig. 2, we show the nucleon chemical potential over Fermi energy ratio $\mu/\varepsilon_F$, the scaled pair size $\xi/d$, and the scaled scattering length of $np$ collision $1/(k_F a)$ as functions of density. The right and left vertical lines that separate the BCS, BEC, and crossover regions are, respectively, determined by the conditions $\xi/d = 1$, and $\mu/\varepsilon_F = 0$. The red vertical line in crossover region denotes the unitary point where $1/(k_F a) = 0$.

FIG. 2: (Color online) The nucleon chemical potential over Fermi energy ratio $\mu/\varepsilon_F$, the scaled pair size $\xi/d$, and the scaled scattering length of $np$ collision $1/(k_F a)$ as functions of density. The right and left vertical lines that separate the BCS, BEC, and crossover regions are, respectively, determined by the conditions $\xi/d = 1$, and $\mu/\varepsilon_F = 0$. The red vertical line in crossover region denotes the unitary point where $1/(k_F a) = 0$.

The numerical result for the critical temperature is shown in Fig. 3. The dashed line shows the critical temperature $T_c$ scaled by the Fermi energy $\varepsilon_F$ as a function of density. Also shown is the BCS prediction (dashed line).

FIG. 3: (Color online) The critical temperature $T_c$ scaled by the Fermi energy $\varepsilon_F$ as a function of density. Also shown is the BCS prediction (dashed line).
theory does not give correct critical temperature for superfluid/normal transition which is mainly determined by the bosonic degree of freedom in these regions. The solid line is for $T_c$, obtained from Eq. (3.30). We can see that the evolution of $T_c$ is smooth and the superfluid phase transition is second order in the whole density region. Also, it can be seen that $T_c$ is not a monotonous function of density: There is a local maximum in $T_c$ curve which is roughly located around the unitary point. One should notice that a similar local maximum also appears in the famous Nozieres–Schmitt-Rink approach for $T_c$ [3]. At low density limit, all the nucleons participate into the deuterons which are long lived at temperature lower than $T_c$, the system is essentially a deuteron gas and the superfluid is totally due to the BEC of deuterons. In this case, we have, at $T_c$, $2Z\Delta _{pg}(T = T_c) = \rho _b^{\text{uncondensed}} = \rho /2$. Solving out $T_c$, we get,

$$T_c = \frac{2\pi}{m_b} \left[ \frac{\rho}{4\zeta (3/2)} \right]^{2/3}. \quad (4.6)$$

This is just the BEC transition temperature for boson of mass $m_b$. Adopting that $m_b \approx 2m$, we arrived at the well-known result,

$$T_c \approx 0.218\varepsilon_F, \quad (4.7)$$

which coincides well with our numerical result.

It should be stressed that the BCS critical temperature $T_c^{\text{BCS}}$ was found to be a good approximation for the pair dissociation temperature $T^*$ (above which the pairs are essentially dissociated by thermal motion of the participators) [36]. So it is a pseudogap dominated region in between $T_c$ and $T_c^{\text{BCS}}$.

Corresponding to the evolution of the critical temperature, it is indicative to see how the pseudogap evolves. In Fig. 4, we plot the zero temperature excitation gap $\Delta (T = 0)$ as well as the pseudogap $\Delta _{pg}(T = T_c)$ at $T_c$. It can be seen that at low density $\rho \lesssim 0.01\rho_0$, $\Delta _{pg}(T = T_c)$ is roughly equal to $\Delta (T = 0)$ (but they do not completely coincide) reflecting the strong coupled nature in this case; however, at high density, $\Delta _{pg}(T = T_c)$ is much smaller than $\Delta (T = 0)$, indicating that the pairing fluctuation is not essential there and BCS theory can work well. In the following subsections, we will focus on the crossover and BEC regions where BCS theory is not applicable at finite temperature.

### B. Unitary matter

As shown in last subsection, the $np$ scattering length $k_Fa$ is infinite at the unitary point $\rho \approx 0.06\rho_0$. We call the nuclear matter at this point a unitary matter. It is interesting because it exhibits universal behaviors, i.e., the physical properties of unitary matter are independent of the details of the interactions [48]. Hence, the unitary nuclear matter behaves just like unitary cold atomic Fermi gas which has been realized in laboratory through the Feshbach resonance. For unitary matter, the unique characteristic scale is given by the Fermi momentum $k_F$, so we have $\mu_{T=0} = \zeta \varepsilon_F, \Delta _{T=0} = \gamma \varepsilon_F$, $\gamma = 64\pi^2\hbar^2a^2/m_F^2$, $\zeta$, $\lambda$ are scattering length, $\mu$ is the energy per particle in unitary matter is

$$E/N = \zeta (E/N)_{\text{free}}$$

$\zeta$, $\gamma$, $\alpha$, $\lambda$, etc, being universal constants. In our $G_0G$ scheme, the universal coefficients are given by $\zeta \approx 0.59, \gamma \approx 0.64, \alpha \approx 0.26$ and $\lambda \approx 0.53$. For comparison, we would like to list the values obtained by Monte-Carlo techniques: $\zeta \approx 0.42$ [49], $\gamma \approx 0.50$ [49], $\alpha \approx 0.157$ [50] or 0.25 [51]. Our results are larger than Monte-Carlo values. It is easy to show that the energy per particle in unitary matter is $E/N = \zeta (E/N)_{\text{free}}$ where $(E/N)_{\text{free}}$ is energy per particle for free fermion gas. Moreover, the equation of state of unitary matter is the same as free fermion gas, $\varepsilon = 3P/2$, with $P$ being the pressure.

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In Fig. 5, we plot $\Delta _{sf}$ (in units of $\varepsilon_F$, the same below), $\Delta _{pg}$, and $\Delta$ as functions of temperature for unitary matter. As a comparison, we also plot the BCS result $\Delta _{\text{BCS}}$ above $T_c$. As we can see from the figure, with decreasing temperature below $T_c$, $\Delta _{pg}(T)$ is a monotonically decreasing function from
its maximum value at \( T_c \) and it essentially vanishes at \( T = 0 \) roughly according to \( \Delta_{pg}(T) \propto T^{3/4} \) [see Eq. (3.29)], while \( \Delta_{d}(T) \) and \( \Delta(T) \) both increase monotonically and become coincident at \( T = 0 \). Such kinds of temperature dependence reflect the fact that the pseudogap is due to the thermally excited pairs: When \( T \) grows higher and higher, more and more pairs are excited from the condensate and at \( T_c \) all the condensed pairs are thermally excited; after that the thermal motion of nucleons begins to dissociate the pairs and hence \( \Delta_{pg} \) (more exactly, \( Z \Delta_{pg}^2 \)) begins decreasing above \( T_c \). In addition, the critical temperature \( T_c \) is smaller than the BCS prediction which shows the fact that the pairing fluctuation tends to destroy the order of the system. Although the physical picture is clear, our formalism cannot be applied to very high temperature, where the effects of finite life-time of the pairs become significant which are not included in our formalism.

One should note that \( \Delta \) and its derivative \( d\Delta/dT \) are continuous at \( T_c \). This is very different from the BCS case, where \( d\Delta_{BCS}/dT \) is discontinuous at \( T_{c}^{BCS} \). Such difference could be reflected in thermodynamic quantities such as the specific heat. In Fig. 6, we illustrate the entropy density and the specific heat \( c_V \) for unitary matter. We compute \( c_V \) through \( c_V = T\partial s/\partial T \) which involves the derivatives \( \partial\mu/\partial T, \partial\Delta/\partial T, \partial m_{b}/\partial T, \) and \( \partial\mu_{b}/\partial T \), so it is a nontrivial calculation. As is well-known, for the weak-coupling BCS case, the specific heat has a jump \( \Delta_{cV} \propto d\Delta^2/dT \) at \( T_{c}^{BCS} \) which reflects the sudden opening of the excitation gap. For unitary matter, however, we found a continuous \( c_{V} \) at \( T_c \). This continuity of \( c_{V} \) reflects the previous existence of the excitation gap above \( T_c \) due to the pairing fluctuation. It may serve as an experimentally accessible signal for the existence of the pseudogap in the normal phase.

C. Deuteron gas

When density is very low, say \( \rho < 0.002\rho_0 \) from Fig. 2, the symmetric nuclear matter is effectively a Bose gas of deuteron. We in this subsection study the properties of deuteron gas based on our \( G_{0}G \) formalism. First, we observed from Fig. 2 and Fig. 3 that when \( \rho \rightarrow 0, -\mu \gg T_c \). In this case, \( \Delta, \mu \) and \( Z \) are almost temperature independent below \( T_c \) reflecting the strong \( np \) attraction. Then for \( T < T_c \), the governing equations become (expanding in powers of \( a^{2}\rho \) and \( \Delta^2/\mu^2 \))[38],

\[
m = \frac{1}{4\pi a} \sum_{k} \left( m \left( k^2 - \frac{1}{2} E_k \right) \right) \approx \frac{m e^{-2m|\mu|}}{4\pi} \left( 1 + \frac{\Delta^2}{16\mu^2} \right),
\]

\[
\rho = \frac{2}{4\pi} \sum_{k} \left( 1 - \xi_k \right) E_k \approx \frac{m^2 \Delta^2}{2\pi \sqrt{2m|\mu|}},
\]

\[
\Delta_{pg}^2 \approx \frac{4\Delta^2}{\rho} \left( \frac{T m}{\pi} \right)^{3/2} \xi \left( \frac{3}{2} \right).
\]

Hence at low temperature, we have

\[
\Delta^2 \approx \frac{2\pi \rho}{m^2 a} \left( 1 - \frac{\pi}{2} a^3 \rho \right),
\]

\[
\Delta_{pg}^2 \approx \frac{8\pi}{m^2 a} \xi \left( \frac{3}{2} \right) \left( \frac{T m}{\pi} \right)^{3/2} \left( 1 - \frac{\pi}{2} a^3 \rho \right),
\]

\[
\mu \approx -\frac{1}{2ma^2} \left( 1 - \frac{\pi}{2} a^3 \rho \right).
\]

These relations give how \( \Delta(\rho), \Delta_{pg}(\rho), \mu(\rho), \) and \( m_{b}(\rho) \) evolve with \( \rho \) at \( T < T_c \) in the deep BEC region.

Next, let us study the temperature dependence of these characteristic quantities. To specify the problem, we fix the density as \( \rho = 0.001\rho_0 \). By solving the coupled equations (3.30), we get the transition temperature \( T_c \approx 0.22 \varepsilon_F \approx 0.08 \) MeV.

In Fig. 7, we show the superfluid gap \( \Delta_{sf} \), the pseudogap \( \Delta_{pg} \), the total excitation gap \( \Delta \) as functions of temperature. Due to the stronger attraction, unlike for the unitary matter, now \( \Delta(T) \) is almost a constant below \( T_c \), and at \( T = 0, \Delta/\varepsilon_F \) is even larger than 1. But near zero temperature, \( \Delta_{pg}(T) \) still behaves as \( \Delta_{pg} \propto T^{3/4} \) as shown in Eq. (4.9).

In Fig. 8, we give the temperature dependence of nucleon chemical potential \( \mu(T) \) and effective deuteron chemical potential \( \mu_{b}(T) \). Below \( T_c \), \( \mu_{b} \) is zero meaning that BEC superfluid is formed. Above \( T_c \), both \( \mu \) and \( \mu_{b} \) decrease, corresponding to the thermal dissociating effect. Just above \( T_c \), simple calculation leads to that \( \mu(T) - \mu(T_c) \propto \mu_{b} \propto (T - T_c)^{2} \). Since \( \mu_{b} \) is related to \( \mu \) and the deuteron binding energy \( E_{b} \) through \( \mu_{b} = 2\mu + E_{b} \), we get that the binding energy \( E_{b} \) at zero temperature is roughly 0.6 \( \varepsilon_F \) at \( \rho = 0.001\rho_0 \) from Fig. 8. The effective deuteron mass \( m_{b}(T) \) is shown in Fig. 9. It is seen that at low temperature \( m_{b} \) is almost a constant, but it drops when temperature becomes higher. We note here that \( m_{b} \) can be regarded as the medium renormalized deuteron mass only in deep BEC region [52, 53]. It is renormalized because it contains indirectly the deuteron-deuteron interaction through the nucleon-deuteron coupling in the nucleon self-energy. Hence, this effective mass is not equal to
2m – E_k, as one may intuitively expect. It is actually a parameter measuring the effective size of the non-condensed pairs, hence it is also defined in intermediate coupling and even weak coupling regions. The drop of m_b at high temperature simply indicates that the effective size of the non-condensed pair is enlarged by the thermal motion of participate nucleons.

Finally, we depict the entropy density and the specific heat for deuteron gas in Fig. 10. One should note that at low temperature, the behavior of c_V is very different from the prediction of BCS theory: It shows T^{3/2} dependence at low T rather than an exponential suppression. Actually, since the condensate does not contribute to entropy, c_V at T ≪ T_c contains contributions from quasi-nucleons and from thermal excited pairs. The former contribution is just the BCS theory result,

$$c_V^{BCS} \propto e^{-\Delta_0/T}, \ T \ll T_c,$$

with \Delta_0 = \Delta_{T=0}. The latter one is dominated by \[\partial \Delta_{pg}^2 / \partial T\] for other quantities are almost independent of T [see Eq. (4.9)], hence

$$c_V^{pr} \propto T^{3/2}, \ T \ll T_c,$$

which dominates c_V at low temperature. At the phase transition point, similarly with unitary matter, due to the continuity of the temperature derivative of excitation gap, c_V does not get a discontinuity, but a \lambda-type behavior. Now, both the low temperature and the critical behaviors of c_V are quite similar with the situation found in ideal BEC superfluid. It indicates that the symmetric nuclear matter at very low density is a nearly ideal deuteron gas.

Figures 6 and 10 inspire us that the specific heat jump at T_c may serve as a possible thermodynamic signal for BCS-BEC crossover. We hence draw in Fig. 11 the specific heat jump \[\Delta c_V \equiv c_V(T_c - 0^+) - c_V(T_c + 0^+)\] at T_c over density ratio as a function of density. As density decreases, \[\Delta c_V / \rho\] monotonously decreases and vanishes when density is smaller than 0.06\rho_0 which is just the unitary point. The physical reason for such kind of behavior is clear: as density decreases the system becomes more and more bosonic and the finite jump of the specific heat at T_c which is a typical BCS feature gets suppressed.

V. SUMMARY AND DISCUSSION

As is well known, BCS theory is only applicable to weak coupling system or at zero temperature since it does not contain any pairing fluctuation effects. To study the BCS-BEC crossover at finite temperature, it is necessary to go beyond BCS description. We, in this article, studied the effects of pseudogap due to pairing fluctuation on the BCS-BEC
crossover problem in symmetric nuclear matter. For this purpose, we adopted a $T$-matrix method based on a $G_0G$ approximation for the pair susceptibility. This method is a natural extension of BCS theory and has been widely used in theoretical studies of high $T_c$ superconductor and BCS-BEC crossover problems in cold fermion atoms.

The pseudogap is determined by the density of thermally excited $np$ pairs and vanishes at zero temperature. We found that its effects are substantial for intermediate and strong coupling regions (corresponding to intermediate and low-density regions) in the BCS-BEC crossover when temperature is not zero. At high-density region, the pseudogap is essentially small, and the $G_0G$ theory recovers the BCS theory. Taking into account the pseudogap effects, we calculated the critical temperature for superfluid phase transition shown in Fig. 3. At high density, $T_c$ follows the BCS result, but at intermediate and low densities, it deviates from the BCS prediction remarkably. At dilute limit, $T_c$ coincides with the BEC transition temperature of dilute deuterons.

The pseudogap persists in both $T > T_c$ and $T < T_c$ regions. We investigated how the pseudogap affects the properties of unitary matter and dilute deuteron matter. At intermediate and low densities, the significant result is that due to the pseudogap, the specific heat is continuous at $T_c$. At low density, $c_V \propto T^{3/2}$ at low temperature and has a continuous $\lambda$-type behavior at $T_c$ just like an ideal BEC superfluid. The qualitative change of the temperature dependence of specific heat from high density to low density also indicates the BCS-BEC crossover. Moreover, as density decreases, the jump of specific heat at $T_c$ decreases and eventually disappears when $\rho \lesssim 0.06\rho_0$ as shown in Fig. 11. This may serve as a thermodynamic signal for BCS-BEC crossover.

We stress that the $G_0G$ approximation is, in principle, not applicable at temperature much higher than $T_c$, since then the thermal dissociation effect will result in a finite width of the pairs which we did not take into account. Besides, symmetric nuclear matter may favor the formation of $\alpha$ cluster [54], but we did not consider this situation in present article. Finally, since most of the nuclear systems in nature do not contain equal numbers of neutrons and protons, it will be significant to extend present formalism to asymmetric nuclear matter [14]. We leave all these challenges for future studies.

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