Geometry learning in vocational high school: Investigating the students’ difficulties and levels of thinking

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Abstract Student’s difficulties and low-level thinking in geometry topic is not a new problem in mathematics education. It is portrayed on student’s low achievement both on national and international tests. Van Hiele’s model has been used as a useful framework to justify student’s difficulties as well as their level of thinking in geometry. Therefore, the aim of this study to investigate student’s difficulties and level of thinking using Van Hiele’s theory. Participants in this study were 12th grades vocational high school students. This study is descriptive qualitative research with a phenomenology approach. Data has been collected through a test and interviews. Further, the obtained data were analyzed by looking for a specific theme, then, the researcher found the inter-theme relationship in order to gain a proper understanding of student’s difficulties in geometry and level of student’s geometrical thinking. Based on data analysis, there are some difficulties faced by the student such as, unable to properly identify the geometry shapes based on their formal definition, lack of visualization ability, failed to understand specific terms or symbols of geometry, insufficiency in providing proper reasoning related relationship within geometric shapes. Moreover, it can be concluded that most of the students are on Level 1 of geometry thinking.

1. Introduction

Mathematics is a universal knowledge and skill that must be mastered by any kind of human being in the world because it is the basic for every aspect for human’s life. For instances, in the latest global change, it has been the great contribution for development of science, engineering or technology. Undeniably, mathematics mastery is a must if someone want to success in the modern area. Because its importance, mathematics is one of the core subjects to be offered to all students in every level of their education. Through mathematic learning students were practiced to develop logical thinking, analytical, systematic, critical, creative and the ability to work together [1]. One of the important topics in mathematic that must be mastered by student in school is geometry.

Geometry is always the essential area in mathematics because its connection with daily life situation especially to solve human and natural problems like everyday life problems [2] for example to specify quantities, to measure figures, land and earth, and makes maps [3]. In mathematics curriculum. geometry usually studies in every level in school education either as a separate course or integrated with other mathematics topics [4]. The study of geometry contributes to helping students develop the skills of visualisation, critical thinking, intuition, perspective, problem-solving, conjecturing, deductive
reasoning, logical argument and proof [5-6]. Besides that, geometry also can be used to help students make sense of other areas of mathematics such as fractions and multiplication in arithmetic, the relationships between the graphs of functions (of both two and three variables) and graphical representations of data in statistics [5].

Despite its importance, geometry is still regarded as a problematic topic in mathematics [6-7]. It is shown by student’s poor performance on national and international test results. Based on Trend in International Mathematics and Science Study (TIMSS) result in 2011, Indonesia student achievement score for geometry topic was 377, that is less than the average standard achievement [9-10]. Over past few years, geometry has been known as one of the topics with lowest average score among other topic that had been tested in Indonesia national examination. This issue also experienced by vocational high school students, where the percentage of student’s with right answer for the subtopic “distance between two points within cube” was 6.25%, which is very low. With these result, that geometry can be regarded the topic which perceived difficult to learn by students including vocational high school student [6-7, 12-13].

Based on the problem stated before, there must be an attempt to identify kind of student’s difficulties in learning geometry. One of theory that can explain student’s difficulties in geometry is Van Hiele Theory [15-16]. According to the Van Hiele’s theory, there are of five levels of hierarchy thinking that children pass through in their acquisition of geometric understanding. The levels are visualization, analysis, informal deduction, formal deduction and rigor [17]. Accordingly, the purpose of the study is to apply Van Hiele’s model to identify various possible difficulties of vocational high school students in geometry especially in accomplishing geometry problems. The test used in this study were developed based on level on Van Hiele’s model. The study also was conducted because there is still not much concern from education practitioner for mathematics curriculum in vocational school level [18].

2. Theoretical Review
Geometry is considered as oldest branch of mathematics since it has become part in several ancient culture such as Indian, Babylonian, Egyptian, Chinese and Greek [5]. It used for practical measurement of land in ancient Egypt and the study of the properties of shapes in Greek geometry [19-20]. Geometry can be found in the solar system structure, in of plants and flowers geological formation, and even in animals. It is also a major part of our synthetic world such as art, architecture, cars, machines, and virtually everything humans create [21].

In mathematics curriculum, geometry usually teach as compulsory topic in every level of school. Van de Walle [14, 22-23] stated some advantages in learning geometry. First, geometry helps human to have a whole appreciation of their world. Second, the geometrical exploration can help the students' development of problem-solving skill. Third, it plays a significant role in the other mathematics fields. Fourth, geometry is used by a lot of people on their daily basis. Fifth, it is full of interesting challenges. Regardless all the importance, geometry is still considered as one of difficult topic and not preferred to learn by students.

The kind of difficulties experienced by student in geometry such as: 1) difficult in representing characteristics of and relationships between two-dimensional and three-dimensional objects, 2) problems in analysing and solving problem if geometric objects were placed in a variety of orientations and positions, 3) difficult to view objects from different angles, 4) unable to analyse a three-dimensional problem and calculate the surface area correctly includes breaking a figure up into rectangles and right-angled surfaces [24]. Asemani [21] also addressed poor reasoning skills as another area of concern among secondary school student. This is showed by student inability to extract necessary information from given data and many more are unable to interpret answers and make conclusions.

Awareness on these difficulties arise various study related to it. On describing student difficulties, many researchers have been using Van Hiele’s theory. According to Van Hiele’s model, student must pass 5 hierarchy levels in learning geometric concepts namely visualization, analysis, informal deduction, formal deduction and rigor. In the first level, children are able to recognise and name basic shapes. In the second level, children are able to describe attributes or properties of the basic shapes and
sort, classify and make them. In the third level, students begin to establish relationships between the properties of shapes. They are able to identify families of shapes, and make conjectures and simple deductions. In the fourth level, student was able recognise relationships between properties and make logical arguments about properties. Meanwhile in fifth level, students can form chains of reasoning and justify their thinking. For each of these levels, there is different skill that must be possessed by student. These differences allow different difficulties experienced by students in solving geometry problems at each Van Hiele’s level. Therefore, it seems important to analyse the difficulties experienced by students in solving geometry problems viewed from Van Hiele’s level.

3. Method
The research applied qualitative method with phenomenology approach. Phenomenology approach has been used because it suitable to study particular phenomena on particular group of people. In the context of study, the phenomena are difficulties in geometry and geometry thinking level. Meanwhile the group of people has been studied was vocational high school student on their last year of senior high school level in Yogyakarta. The participants consist of 9 male students and 14 female students.

Data were collected through test and interview. Test was develop based on Van Hiele’s theory indicator for Level 1 to Level 4. Level 5 was not included because the Indonesian mathematics curriculum for grade 12 have not include material at level 5 [25]. After test was held, students were interviewed based on interview protocol that had prepared by researcher. Moreover, interview was conduct to confirm some of student’s answer of the question on the test. Later, data were analysed using Bogdan and Biklen model which followed some step such as: 1) looking for the specific theme, then 2) the researcher found the inter-theme relationship to get proper understanding to obtain conclusion related kind of student’s difficulties and student’s geometry thinking level [26-27].

4. Results

4.1 Student’s answer for Problem 1
On the first question, students were asked to give name for three geometric shapes in Figure 1. This question has been made based on indicator of first level of Van Hiele’s theory. On this level, student supposedly could give the exact name for geometric shape based on their physical appearances.

![Figure 1. Problem 1](image)

The following figures are some samples of students’ answer of Problem 1.
Most of participant on the study can correctly answer the question. But there were some of student had difficulties to give the proper and complete name for the shapes, especially the first and last shape. Based on Figure 2, Figure 3, Figure 4, and Figure 5, students were likely named the first shape as “pyramid” (See Figure 2) or “cuboid” (See Figure 5) or just “prism” (See Figure 4) instead of “triangular prism”. Meanwhile for the third shape, students named it as “pyramid” or “triangular pyramid” instead of “rectangular pyramid”.

Furthermore, we conducted an interview to see the difficulties faced by the students. The difficulties also reflected on student’s answer of interview questions below.
4.2 Student’s answer for Problem 2
The second problem was created based on third level of student’s geometry level thinking, as presented in Figure 7. On this level student was asked to categorize space geometry shapes that has been given either as pyramid or prism or cube or cuboid.

For this problem, most of students can easily categorized the shapes on number 2,4,5,6,7,8,9,10,11, and 12, which number 2,4,7,9 are prisms, number 5,8,10,11 are pyramids, number 6 is cuboid, and number 12 is cube. Meanwhile, the geometric shapes on number 1, 3, 13, and 14 are classified as the difficult shapes to categorize by students. The difficulties represented on student’s answer sheet as presented in Figure 8, 9, 10, and 11.
For shape on number 1, most of the students did not categorize it in any kind of shapes. Some of students tend to categorize it as cube or prism (prism is the right answer), meanwhile others categorized it as cuboid. For shape 13, most of student has the right answer (prism is the right answer). Even though, student could correctly categorize the shape, but they could not give proper reason to categorize a shape on particular category not others, as shown on the piece of interview between researcher and student in Figure 12.

From the interview above, students were likely to categorize the shapes based on the physical appearances rather than the formal definition of the shapes. Student categorized shapes on number 3 as cubes because it was looked like one. Indeed, there was a student who recognized shape on number 8 as a kite and not as the solid geometry shape. This difficulty also closely related with student difficulty to recognize the shapes when it was positioned in different angles. For example, some of the students assumed the shape one the number 14 was a cuboid. They argued that the image on number 13 showed the cuboid appearances from front view.

4.3 Student’s answer for Problem 3
On Problem 3, students were asked to identify parallel lines, perpendicular lines, and skew lines on cuboid (Figure 13). This question was made based on second level of Van Hiele. In this level, student expected to identify the properties of geometric shapes.

The following figures are some of the sample students’ answer of Problem 3.
For Problem 3, most of the students correctly answered the questions on part (i) and part (ii). Students could list the pairs of parallel and perpendicular lines correctly, even though the answer had been given not quite complete (see Figure 16). After doing a depth interview, it revealed that the student failed to identify other pairs because they depended on the visual appearances of the shape shown on the picture. For example, the lines $BC$ and $CD$, not identified as perpendicular lines because the lines formed acute angles on the picture rather than the right angle.

Meanwhile for part (iii), most of the students could not write the correct pairs of skew lines. Based on the result of the interview, it revealed that the student misunderstood the word “skew” in “skew lines.” Students were likely to understand the word as the condition where two lines crossed each other like the line $AF$ and $BE$. 
4.4 Student’s answer for Problem 4

For the fourth problem, students were asked to choose picture which showed condition of the skew lines among six given pictures. The question was made based on fourth level of Van Hiele where student is expected to build logical arguments in proving the skew lines.

![Figure 17. Problem 4](image)

From the given figures above, which figure shown condition that two lines are skew lines? Give further explanation about your answer.

For the question 4, students were asked to point out which condition satisfy that skew lines between (i), (ii), (iii), (iv) or (v). The following figures are the sample answers for Problem 4.

![Figure 18. Student S5 answer for Problem 4](image)

![Figure 19. Student S6 answer for Problem 4](image)

![Figure 20. Student S4 answer for Problem 4](image)

Most of the student didn’t answer the question because they don’t know the meaning of “skew lines” or misinterpret the word “skew” like what happened on third problem. Few of them give the answer, but not all the answer was correct. Like shown above, student’s answer on Figure 19 wasn’t correct, both for choice of correct figure and the reason of the choice. But there were some of students who answer
correctly the question number for like in the Figure 19 and Figure 21. Based on the interview, student couldn’t give proper argumentation about their answer of the problem. It is likely student get used to do rote learning in geometry rather than develop deep reasoning.

5. Discussion

5.1 Student’s difficulties
There are some kinds of difficulties faced by vocational high school student shown on the study. First, students failed to identify the proper characteristics for geometry shapes. From analysis of the data it shows that student still confuse in giving names and properties of the given geometric shapes. This condition indicates that student doesn’t have enough understanding about concept in geometry especially formal definition of geometric shapes. Further, this leads to another difficulties where student could not properly categorize some geometry shapes. Bayuningsih [1] addressed this issue through the statement “many students find difficulty in geometry because their understanding concept”. Study of Alex and Mammen [28] also found many students had an inadequate understanding regarding the identification of the geometric shapes using their properties.

Second, it also was found that student troubled to visualise some specific geometric shape. Visualization is the ability to see motion or displacement on the part of a shape. Lacking of this ability makes student prefer to rely on the visual prototype rather than formal definition when classifying and identifying shapes [29-30], for example in categorizing shapes on Problem 2. Students failed to recognize the properties of solid shapes because its figure was placed on plain surface This finding has been addressed on Hock et al [31] and Gutiérrez [32]. On their studies, many students cannot visualize and imagine three-dimensional objects on a plain surface. So that, students tend to base their arguments and conclusions on the appearance of the solid on the screen.

The third type of difficulties faced by student’s related with their inadequate understanding about language in geometry especially related to specific symbol or term used in geometry. Some studies also has pointed out the geometric language as one of the contributing factor in student’s difficulties [21, 33]. On this study, students did not understand or simply misinterpret some geometric terminology like “parallel lines” or “perpendicular lines” or “skew lines”.

The last type of difficulties identified in this study is related with student’s lack of ability to construct the proper logical structure to prove geometric statement. Students unable to establish possible connection between statements [6]. This showed when students could not provide reasoning behind the choice they had been made on answering the question on this study. According to Asemani [21] this difficulty is also showed by student inability to extract necessary information from given data and many more are unable to interpret answers and make conclusions.

5.2 Student’s level of thinking
Based on the description of student difficulties above, most of the student already can recognize the shapes based on their visual appearances (Level 1). Some of student could describe the properties of the given geometric shapes (Level 2). But there were students who failed to give the correct answer because they had no enough understanding of geometry concepts especially on some particular term in geometry.

Furthermore, there are little amount of the student that can answer the question related to Level 3. Although they could answer the question, still they could not give the complete answer. This could happen on student who did not surpass Level 2. They have no enough knowledge about formal definition or properties of shapes Students tends to rely on the visual prototype of shape rather than formal definition. Whereas, none of the student achieved the level 4. It was shown when there is no student who can give proper reasoning and proof on Problem 4.

According to Luneta [20], senior high school students are expected to operate between level 3 and level. However, none of student in this study reach level 4 of Van Hiele. Less than a half of students are on level 2 and level 3. Most of the student are on level 1 of Van Hiele. Moreover, this study also confirms
findings from Hock et al study [31] that students cannot progress into higher level of thinking if they have failed in the previous level.

6. Conclusion
Based on the result and discussion above, it can be concluded that there are some specific difficulties experienced by students in geometry. The difficulties are unable to properly identify the geometry shapes based on their formal definition, lack of visualization ability, failed to understand specific term or symbol of geometry, insufficiency in provide proper reasoning related relationship within geometric shapes. Meanwhile for the student’s geometry level thinking, none of the student were identified on the fourth level of geometric thinking level. Most of them are still on the first level of geometry thinking, with some of them already achieve level 2 and level 3 of geometric thinking.

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References
[1] Bayuningsih A S, Usodo B and Subanti S 2018 Critical thinking level in geometry based on self-regulated learning J. Phys. Conf. Ser. 983 012143
[2] Retnawati H, Arlinwibowo J and Sulistyaningsih E 2017 The Student’s Difficulties In Completing Geometry Items Of National Examination Int. J. New Trends Educ. Their Implic. 8 28–41
[3] Sunzuma G, Masocha M and Zezekwa N 2013 Secondary School Students’ Attitudes towards their Learning of Geometry: A Survey of Bindura Urban Secondary Schools Greener J. Educ. Res. 3 402–10
[4] Martinovic D and Manizade A G 2018 The challenges in the assessment of knowledge for teaching geometry ZDM - Math. Educ. 50 613–29
[5] Jones K 2002 Issues in the teaching and learning of geometry Aspects of Teaching Secondary Mathematics: Perspectives on practice (New York: Routledge Falmer) pp 121–39
[6] Dimla R B 2018 Probing Students’ Levels of Geometric Thinking in Geometry and Their Enacted Example Space Function J. Educ. Black Sea Reg. 4 155–63
[7] Snyders M 1995 Geometry in senior secondary class Proceedings of the 4th Amesa Conf. (Port Elizabeth: University of Port Elizabeth) pp 5–13
[8] Alex J K and Mammen K J 2016 Lessons Learnt from Employing van Hiele Theory Based Instruction in Senior Secondary School Geometry Classrooms EURASIA J. Math. Sci. Technol. Educ. 12 2223–36
[9] Riastuti N, Mardiyan M and Pramudya I 2017 Students’ Errors in Geometry Viewed from Spatial Intelligence J. Phys. Conf. Ser. 895 012029
[10] Utami A K D, Mardiyan and Pramudya I 2017 Analysis of junior high school students’ difficulty in resolving rectangular conceptual problems AIP Conf. Proc. 1868 050008
[11] Adolphus T 2011 Problems of teaching and learning of geometry in secondary schools in Rivers State, Nigeria Int. J. Emerg. Sci. 1 143–52
[12] Rohendi D, Septian S and Sutarno H 2018 The Use of Geometry Learning Media Based on Augmented Reality for Junior High School Students, IOP Conf. Ser. Mater. Sci. Eng. 306 012029
[13] Sulistiwawati D L, Herman T and Jupri A 2019 Student difficulties in solving geometry problem based on Van Hiele thinking level J. Phys. Conf. Ser. 1157 042118
[14] Nurwijayanti A, Budiyono and Fitriana L 2018 The geometry ability of junior high school students in Karanganyar based on the Hoffer’s theory J. Phys. Conf. Ser. 983 012085
[15] Chua G L L, Tengah K A, Shahrill M, Tan A and Leong E 2017 Analysing Students’ Perspectives
on Geometry Learning from The Combination of Van Hiele Phase-Based Instructions and Geogebra Proceeding of the 3rd International Conference on Education (Kuala Lumpur: TIIKM) pp 205–13

[16] Wheatley J 2011 An Investigation Of Three-Dimensional Problem Solving And Levels Of Thinking Among High School Geometry Students (Washington : Central Washington University)

[17] Ismail Z and Rahman S N A 2017 Learning 2-Dimensional and 3-Dimensional Geometry with Geogebra: Which Would Students Do Better? Int. J. Emerg. Math. Educ. 1 121–34

[18] Zeynivandnezhad F Ismail Z and Yusof Y M 2012. Mathematics Requirements for Vocational and Technical Education in Iran Procedia - Soc. Behav. Sci. 56 410–5

[19] Cooke H 2007 Mathematics for primary and early years: Developing subject knowledge. (London: SAGE Publications Ltd)

[20] Luneta K 2014 Foundation phase teachers’ (limited) knowledge of geometry South African J. Child. Educ. 4 71–86

[21] Asemani E, Asiedu-Addo S K and Oppong R A 2017 The geometric thinking levels of senior high school students in Ghana Int. J. Math. Stat. Stud. 5 1–8

[22] Van de Walle JA 2001 Geometric Thinking and Geometric Concepts in Elementary and Middle School Mathematics: Teaching Developmentally (Boston: Allyn and Bacon)

[23] Suyitno H, Nurul E and Budhiati R 2019 The development of geometry concepts understanding based on NCTM reference in learning using discovery learning model Unnes J. of Math. Educ. 8 42–8

[24] Kotzé G 2007 Investigating shape and space in mathematics: A case study South African Journal of Education 27 19–35

[25] Razak F Sutrisno A B Immawan Z and Muchsin S B 2018 Analysis students’ thinking level with cognitive style ‘field independent’ based on van Hiele’s theory J. Phys. Conf. Ser. 1028 012161

[26] Bogdan R.and Biklen S K 1982 Qualitative research for education: An introduction to theory and methods (Boston: Allyn and Bacon, Inc)

[27] Retnawati H Hadi S and Nugraha A C 2016 Vocational high school teachers’ difficulties in implementing the assessment in curriculum 2013 in Yogyakarta Province of Indonesia Int. J. Instr. 9 33–48

[28] Alex J K and Mammen K J 2012 A survey of South African Grade 10 learners’ geometric thinking levels in terms of the van Hiele Theory The Anthropologist 14 123–9

[29] Gal H and Linchevski L 2010 To see or not to see: analyzing difficulties in geometry from the perspective of visual perception Educ. Stud. Math. 74 163–18

[30] Özerem A 2012 Misconceptions in geometry and suggested solutions Int. J. New Trends Arts, Sport. Sci. Educ. 1 23–35

[31] Hock T T, Tarmizi R A, Aida A S and Ayub A F 2015 Understanding the primary school students’ van Hiele levels of geometry thinking in learning shapes and spaces: A Q-methodology Eurasia J. Math. Sci. Technol. Educ. 11 793–802.

[32] Gutiérrez Á 1996 Visualization in 3-dimensional geometry: In search of a framework Proc. 20th PME Conf. 1 3–19

[33] Fabiyi T R 2017 Geometry concepts in mathematics perceived difficult to learn by senior secondary school students in Ekiti State, Nigeria IOSR J. Res. Method Educ. 7 83–90