Spin-plasmons in topological insulator

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1. Introduction

In recent years, topological insulators with a non-trivial topological order, intrinsic to their band structure, were predicted theoretically and observed experimentally (see [1] and references therein). Three-dimensional (3D) realizations of “strong” topological insulators (such as Bi2Se3, Bi2Te3, and Sb2Te3) are insulating in the bulk, but have gapless topologically protected surface states with a number of unusual properties [2]. These states obey two-dimensional Dirac equation for massless particles, similar to that for electrons in graphene [3], but related to real spin of electrons, instead of sublattice pseudospin in graphene.

The consequence of that is a spin-momentum locking for electrons on the surface of strong 3D topological insulators, i.e., spin of each electron is always directed in the surface plane and perpendicularly to its momentum [1,4]. The surface of topological insulator can be chemically doped, forming a charged “helical liquid”.

Collective excitation of electrons in such helical liquid were considered in [5], where relationships between charge and spin responses to electromagnetic field were derived. It was shown that charge-density wave in this system is accompanied by spin-density wave. Application of spin-plasmons to create “spin accumulator” was proposed in [6]. Also the surface plasmon-polaritons under conditions of magnetoelectric effect in 3D accumulator [6] was proposed in [6]. Also the surface plasmon-spin-density wave. Application of spin-plasmons to create “spin that charge-density wave in this system is accompanied by responses to electromagnetic field were derived. It was shown that charge-density wave in this system is accompanied by spin-density wave. Application of spin-plasmons to create “spin accumulator” was proposed in [6]. Also the surface plasmon-polaritons under conditions of magnetoelectric effect in 3D accumulator [6].

In the present article we consider the properties and internal structure of spin-plasmons in a helical liquid. Within the random-phase approximation, we derive plasmon wave function and calculate amplitudes of charge- and spin-density waves in the plasmon state.

2. Wave function of spin-plasmon

Low-energy effective Hamiltonian of the surface states of Bi2Se3, in the representation of spin states $|\uparrow\rangle$ or $|\downarrow\rangle$ is $H_0=\nu_F(p_x\sigma_x−p_y\sigma_y)$ for a surface in the $xy$ plane, where the Fermi velocity $\nu_F\approx 6.2 \times 10^5$ m/s [2]. Its eigenfunctions can be written as $\psi_p = (e^{−i\text{bp}/\hbar})/\sqrt{S}$, where $S$ is the system area and $|f_{pq}|^2 = (e^{−i\text{bf}/\hbar})/\sqrt{2}$ is the spinor part of the eigenfunction, corresponding to electron with momentum $p$ (its azimuthal angle in the $xy$ plane is $\phi_p$) from conduction ($\gamma=+1$) or valence ($\gamma=−1$) band. Many-body Hamiltonian of electrons populating the surface of topological insulator is $H = \sum_{p}\langle f_{pq}\rangle f_{pq}+(1/2S)\sum_{q}\langle \text{a}_{q}\text{a}^\dagger_{q}\rangle$. Where $a_p$ is the destruction operator for electron with momentum $p$ from the band $\gamma$, $\xi_p = |\nu_F|p−\mu$ is its energy measured from the chemical potential $\mu$, $V_q = 2ne^2/\hbar q$; $\rho_{pq} = \sum_{p}f_{pq}\langle f_{p}\rangle \text{a}_{p}\text{a}^\dagger_{q}$. is the charge density operator for helical liquid.

The creation operator for spin-plasmon wave with vector $q$ can be presented in the form:

$$Q_q = \sum_{p\gamma}C_{pq}^\gamma a^\dagger_{q}\text{a}_{p\gamma},$$

This operator should obey the equation of motion $\{H,Q_q^\dagger\} = \Omega_q Q_q^\dagger$, where $\Omega_q$ is the plasmon frequency. We can get solution of this equation in the random phase approximation at $T=0$ (similarly to [8]):

$$C_{pq}^\gamma = \frac{\nu_F−\nu_p−\xi_p−\xi_q}{\sqrt{\Omega_q+\xi_p−\xi_q+i\hbar}}|\langle f_{p\gamma}\rangle f_{f\gamma}\rangle N_q,$$

where $\nu_p = \Theta(p_F−|p|)$ and $n_p$ are occupation numbers for electron-doped helical liquid ($p_F=\mu/\nu_F$ is the Fermi momentum).
The plasmon frequency is determined in this approach from the equation $1 - V^2 \Pi(q, \omega) = 0$, where

$$I(q, \omega) = \sum_{\Omega} \langle f_{p+q, \gamma} | f_{p} \rangle ^2 \frac{n_{p+q} - n_{p}}{\omega + i n_{p+q} + i 0}$$  \hspace{1cm} (3)$$

is the polarization operator of the helical liquid, different from that for graphene [3] only by degeneracy factor. The factor $N_q$ in (2) can be determined from the normalization condition

$$\langle 0 \big| (Q_{q}, Q_{-q}^+ ) \big| 0 \rangle = \delta_{qq} \sum_{\gamma} D_{\gamma} = \delta_{qq},$$

$$D_{\gamma} = \sum_{p} \left| C_{pq}^\gamma \right|^2 (n_{p} - n_{p+q})$$  \hspace{1cm} (4)$$

($\langle 0 \rangle$ is the ground state), so that $|N_q|^2 = -S\Pi(q, \omega)/\omega|_{\omega=\omega_q}$. The quantities $D_{\gamma}$ in (4) can be considered as total weights of intraband ($D_{\omega}$) and interband ($D_{\omega+} + D_{\omega-} = 1 - D_{\omega}$) electron transitions, contributing to the plasmon wave function (1). Note that all these formulas are also applicable to the case of graphene.

Spin-plasmon dispersion $\Omega_q$ and contribution of intraband transitions into its wave function are plotted in Fig. 1 at various $r_s = e^2/\epsilon \nu_F$, where $\epsilon$ is the dielectric susceptibility of surrounding 3D medium. For Bi$_2$Se$_3$, $r_s \approx 0.09$ with $e \approx 40$ for dielectric half-space [5] (for such small $r_s$, the corresponding dispersion curve approaches very closely to the upper bound $\omega = \nu_F q$ of the intraband continuum). The results for suspended graphene with rather large $r_s = 8.8$ (for $\nu_F \approx 10^6$ m/s, $\epsilon = 1$ and with the degeneracy factor 4 incorporated into $r_s$) are also presented for comparison. It is seen that the undamped spin-plasmon consists mainly of intraband transitions. When the dispersion curve enters the interband continuum, the spin plasmon becomes damped and inter- and intraband transitions contribute almost equally to its wave function.

3. Charge- and spin-waves

The helical liquid in the state $|1_q\rangle = Q_{q}^+ |0\rangle$ with one spin-plasmon of wave vector $q$ has a distribution of electron–hole excitations (2), shifted towards $q$. Due to the spin-momentum locking, the system acquires a total nonzero spin polarization, perpendicular to $q$. A similar situation occurs in the current-carrying state of the helical liquid, which turns out to be spin-polarized [4].

Introducing one-particle spin operator as $s = \sigma/2$, we can calculate its average value in the one-plasmon state $\langle s \rangle = \langle 1_q | s | 1_q \rangle$ as

$$\langle s \rangle = \sum_{p+q} \langle f_{p+q, \gamma} | f_{p, \gamma} \rangle C_{pq}^\gamma$$

$$+ C_{pq}^{\gamma*} \langle f_{p, \gamma} | f_{p+q, \gamma} \rangle (n_{p} - n_{p+q})$$  \hspace{1cm} (5)$$

If $q$ is parallel to $\epsilon_x$, only the $y$-component of $\langle s \rangle$ is nonzero. Its dependence on $q$ at various $r_s$ is plotted in Fig. 2(a).

Charge- and spin-density waves, accompanying spin-plasmon with the wave vector $q$, can be characterized by corresponding spatial harmonics of charge- and spin-density operators: $\rho_{\epsilon}^q$ and $s_q^\gamma = \sum_{p+q} \langle f_{p+q, \gamma} | f_{p, \gamma} \rangle (n_{p} - n_{p+q})$. Using, similarly to [9], the
unitary transformation, inverse with respect to (1), we can write:
\[
\rho_q^+ = S N_q^+ \Pi(q, \Omega_q) Q_q^+ + \beta_q^+ ,
\]
where the operators \( \beta_q^+ \) and \( s_q^+ \) are the contributions of single-particle excitations and are dynamically independent on plasmons. Here the crossed spin-density susceptibility of the helical liquid [5] has been introduced:
\[
\Pi(q, \omega) = \sum_{p, \gamma} \langle f_p | \gamma, f_p \rangle | s f_p \rangle | s f_p \rangle = \frac{n_p - n_p \delta_{q, \omega \gamma}}{(\omega + s_p \gamma)} - \frac{s_p \gamma}{(\omega + s_p \gamma) + i \delta}
\]
(8)

The average values of \( \rho_q^+ \) and \( s_q^+ \) in the \( n_q \) plasmon state \( |n_q\rangle = [Q_q^+]^{s_q \gamma} / \sqrt{n_q!} |0\rangle \) vanish, therefore we consider their mean squares in \( |n_q\rangle \) after subtracting their background values in \( |0\rangle \), i.e.,
\[
\langle \rho_q^+ \rangle = \langle n_q | \rho_q^+ | n_q \rangle - \langle 0 | \rho_q^+ | 0 \rangle = n_q s^2 \langle N_q^+ \Pi(q, \Omega_q) \rangle^2
\]
(9)
\[
\langle s_q^+ s_q^\gamma \rangle = \langle n_q | s_q^+ s_q^\gamma | n_q \rangle - \langle 0 | s_q^+ s_q^\gamma | 0 \rangle = n_q s^2 \langle N_q^+ \Pi(q, \Omega_q) \rangle^2
\]
(10)

(only the in-plane transverse component \( s^\gamma \) of the spin \( s \) is nonzero in these averages). The normalized amplitudes \( A_i(q) = \langle \rho_q^+ \rangle / \sqrt{n_q} \) and \( A_i(q) = \langle s_q^+ s_q^\gamma \rangle / \sqrt{n_q} \) are the charge- and spin-density waves are plotted in Fig. 2(b) (\( \rho = p^2 / 4 \pi \) is the average electron density). The “continuity equation” for density and transverse wave, following from the spin-momentum locking [5], requires that \( \Omega A_i(q) = 2V q A_i(q) \), in agreement with our results.

4. Conclusions

We have considered microscopically spin-plasmons in helical liquid in the random phase approximation. The developed quantum-mechanical formalism can be applied for a number of problems in spin-plasmon optics.

We calculated the average spin polarization, acquired by the helical liquid in a spin-plasmon state, as well as mean-square amplitudes of charge- and spin-density waves, arising in this state. Coupling between these amplitudes, caused by spin-momentum locking, was demonstrated. The interconnection between charge- and spin-density waves can be applied for constructing various spin-plasmonic and spintronic devices.

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