Intense laser driven collision-less shock and ion acceleration in magnetized plasmas

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Abstract. The generation of strong magnetic field with a laser driven coil has been demonstrated by many experiments. It is applicable to the magnetized fast ignition (MFI), the collision-less shock in the astrophysics and the ion shock acceleration. In this paper, the longitudinal magnetic field effect on the shock wave driven by the radiation pressure of an intense short pulse laser is investigated by theory and simulations. The transition of a laminar shock (electrostatic shock) to the turbulent shock (electromagnetic shock) occurs, when the external magnetic field is applied in near relativistic cut-off density plasmas. This transition leads to the enhancement of conversion of the laser energy into high energy ions. The enhancement of the conversion efficiency is important for the ion driven fast ignition and the laser driven neutron source. It is found that the total number of ions reflected by the shock increases by six time when the magnetic field is applied.

1. Introduction

The generation of strong magnetic field with a laser driven coil has been demonstrated by many experiments [1][2][3][4]. Recently, explored are their applications to the magnetized fast ignition (MFI) [5][6], and the magnetized collision-less shock in the astrophysics[7],[8]. In this paper, we investigate the external magnetic field effect on the collision-less shock driven by intense short pulse laser and ion acceleration in near relativistic cut-off density (NRCD) laser plasmas.

In the previous paper [9], it is found that magnetic field fluctuations excited by the Weibel instability grow nonlinearly after the saturation of the linear instability, when the electron cyclotron frequency of the external magnetic field is higher than 5% of the electron plasma frequency. In the simulation, it is found that the magnetic fluctuation energy with external B field is higher by 10 times than that without B. The fluctuating magnetic fields propagate with the shock and scatter ions at the shock front. As a result of the scattering, more ions are accelerated by the shock. In this paper, the shock wave generation by the ultra-intense short pulse (UISP) laser interactions with NRCD plasmas is studied by theory and simulation in the both cases without and with external B field. It has been found that ions are reflected and accelerated by the electrostatic field when there is no external B and the laser intensity is lower than $10^{20}$W/cm$^2$ [10]. However, when the external magnetic field is applied, the magnetic fluctuation amplitude increases [9] and ions are strongly reflected and efficiently accelerated at the shock front.
The increase of total energy of accelerated ions is applicable to the ion driven fast ignition and the efficient laser driven neutron source.

2. Shock relations in magnetic fluctuations driven by the two stream and Weibel instabilities

Electromagnetic fluctuations in NRCD plasmas are high enough to cause strong scattering of electrons. So, electrons can be assumed in the local thermal equilibrium. Namely, the electron dynamics follows the thermodynamics. When the heat conduction is neglected and the adiabatic relations are assumed for electron fluid, then,

\[ \frac{p_e}{p_{e0}} = \left( \frac{n_e}{n_{e0}} \right)^{\gamma_e}, \quad e n_e \frac{d\phi}{dx} - \frac{d}{dx} p_e = 0, \quad \text{and} \quad \frac{n_e}{n_{e0}} = \left( \frac{\gamma_e - 1}{\gamma_e} \frac{\phi}{\phi_0} + 1 \right)^{\gamma_e - 1}, \tag{1} \]

where \( p_e \) is the electron pressure, \( n_e \) is the electron density, \( \gamma_e \) is the adiabatic constant of electron, which is 4/3 in the highly relativistic case, \( \phi = e n_{e0} \phi / p_{e0} \) is the normalized electrostatic potential, \( p_{e0} \) and \( n_{e0} \) are the up steam electron pressure and electron density. Here in Eq.(1), the electron inertia is neglected by assuming that the shock velocity is lower than the electron thermal velocity and the second equation of (1) were combined to obtain the last equation of (1).

When there is no external magnetic field, ions are not much scattered by magnetic fluctuations. So, the ion dynamics follows the Vlasov equation of non-magnetized plasmas which yields the ion distribution function, \( F(x,p) \), with \( n_i = \int F(x,p) dp \). In the moving frame of the shock, the distribution function satisfies, \( p_x \frac{\partial F}{\partial x} - e \frac{\partial \phi \partial F}{\partial \phi} = 0 \), when the shock is stationary. Here, \( p_x \) is the ion momentum in x (which is the direction of the shock propagation) and \( \phi \) is the electrostatic potential.

The solution of the Vlasov equation is given by \( F = F_0 \left( \frac{p_x^2}{2M} + e\phi \right) + F_+ \left( \frac{p_x^2}{2M} + e\phi_0 \right) \). Here, \( F_+ \) and \( F_- \) are the distributions functions for ions moving from left to right and from right to left in Fig.1, respectively. For simplicity, we will consider the water bag model [11] in which the distribution is assumed constant in the area between two particle orbits as shown in Fig.1.

The distribution function in the up-stream (\( x > x_s \)) is given as follows.

\[ F_- = F_0 \left( p_x < 0 \right), \]

\[ W_{\text{max}} - e\phi > \frac{p_x^2}{2M} > 0, \quad \text{or} \quad W_{\text{min}} - e\phi \]

where \( \frac{n_{i0}}{n_i} = \frac{1}{\sqrt{2M \left( W_{\text{max}} - W_{\text{min}} \right)}} \). Note that \( n_{i0} \) is the incoming ion density from the up stream. In the down stream: Here, \( W_{\text{max}} \) and \( W_{\text{min}} \) are the maximum and the minimum ion energy at \( x = \infty \), respectively. \( F_+ = F_0 \left( p_x > 0 \right) \),

for \( W_{\text{min}} - e\phi_0 + \Delta \) or \( 0 < \frac{p_x^2}{2M} < e\phi_0 + \Delta, \)

where \( e\phi_0 + \Delta \) is the maximum energy of accelerated ion. The distribution function in the down stream (\( x < x_s \) is given as, \( F_+ = F_0 \left( p_x < 0 \right) \) for \( e\phi_0 - \phi < \frac{p_x^2}{2M} < W_{\text{max}} - e\phi \). When \( n_i(\phi) \) is given as the function of \( \phi \) by integrating the above distribution function, we get the equation for the electrostatic potential by combining the Poisson equation, Eqs.(1), and ion density, as follows.

\[ \frac{d^2\phi}{dx^2} = \left( \frac{\gamma_e - 1}{\gamma_e} \frac{\phi}{\phi_0} + 1 \right)^{\gamma_e - 1} - \frac{1}{\phi_0^2} \frac{1}{\gamma_e} \frac{\phi}{\phi_0} = - \frac{dU(\phi)}{d\phi}, \tag{2} \]

where \( x \) is normalized by the Debye length: \( \lambda_D \). Here, \( U(\phi) \) is the Sagdeev potential. Note that \( \phi \) is assumed the coordinate of a virtual particle which is moving in the potential of \( U(\phi) \). When a part of
ions are reflected in front of the potential hump at \( x_s \), then the \( U(\phi) \) have different values in the region: \( x > x_s \) : up-stream and in the region: \( x < x_s \) : down stream. Actually, it is possible that in the up-stream: \( x > x_s \), \( U(\phi) = 0 \) at \( \phi = 0 \), and \( \phi \phi_0 \) and in the down-stream: \( x < x_s \), \( U(\phi) = 0 \) at \( \phi = \phi_0 \) and \( \phi \phi_0 \). Here, \( \phi_0 \) is the minimum of the potential ripple in the down-stream. Namely, the equation (2) could have asymmetric solution as shown in the Fig.1. This asymmetry is due to the ion reflection and the solution is called the electro static collision-less shock wave.

The potential and ion phase space distributions in the simulation of the intense short pulse laser interaction with NRCD plasmas as shown in the Fig.2.

![Fig.2](image)

**Fig.2.** PIC simulations for the laser intensity of \( 10^{19} \text{ W/cm}^2 \) and the plasma density of \( 7n_c \) with and without external magnetic field. A) Potential profiles, solid line: without B field, broken line: with external B field (4kT), B) Ion phase space \((x,P_x)\), (a) without B field and (b) with B field. I, II, and III in (B) indicate reflected ion, up-stream ion and down-stream ion, respectively.

The phase space distribution of ions and potential profile in the simulation without external magnetic field as shown in Fig.2B) agree with the theoretically prediction as shown in Fig.1. In the simulation without B, the reflected ion minimum velocity is \( v = 0.045c \) in the shock wave frame which is equal to the ion velocity accelerated by the potential energy \( e\phi_0 = 0.8 \text{ MeV} \).

When the external magnetic field is applied, the magnetic field fluctuations are enhanced to scatter and reflect ions. In this case, the ion dynamics is described as a fluid. Then the ion stationary flows are described the following three conservation laws. Ion mass conservation:

\[ \rho = F(\text{const}) \]

Ion momentum conservation:

\[ \rho v^2 + P_i + P_e - \frac{4}{3} \mu \frac{d}{dx} v = \rho_0 v_0^2 + P_{i0} + P_{e0}. \]

Here, \( P_i \) and \( P_e \) are the ion and electron pressure. Energy conservation:

\[ \frac{d}{dx} \left[ \frac{1}{2} \rho v^2 (v_i + 1) + \frac{1}{2} \rho_e v_e^2 \right] + \frac{d}{dx} P_{i0} = \frac{\rho_0 v_0^2}{\gamma_i - 1} \frac{d}{dx} (v_i v) \frac{d}{dx} (v_e^2 v) \]

Here, \( \gamma_i \) and \( \mu \) represent the ion adiabatic constant and the ion viscosity. The electron density given by Eq.(1), the above conservation laws, and the charge neutrality condition yield the following equation for the flow velocity: \( v \),

\[ \frac{d}{dx} 
\begin{align*}
\left[ \gamma_i P_{i0} - \frac{1}{2} F \left( (\gamma_i + 1) v - (\gamma_i - 1) v_0 \right) \right] (v - v_0) + \gamma_e P_{e0} v \left[ 1 - \frac{v_0}{v} \right] \frac{d}{dx} (v_e - v_0) = \\
- \frac{2}{3} \mu \frac{dv^2}{dx}
\end{align*}
\]

For \( x \rightarrow \pm \infty \), the right hand side of the equation (3) is zero for a shock wave. Then, we obtain \( v = v_0 \) for the up-stream \((x \rightarrow +\infty)\) and \( v = v_1 \) in the down stream \((x \rightarrow -\infty)\). The jump condition satisfies,
\begin{equation}
\frac{v_1}{v_0} = \frac{\gamma - 1}{\gamma + 1} + \frac{2m}{\gamma + 1} \frac{v_e}{v_1} \left( \frac{v_0}{v_1} \right) \frac{1}{\gamma e^{\gamma - 1}} \left[ 1 - \left( \frac{v_0}{v_1} \right)^\gamma + \frac{\gamma e^{\gamma - 1} - 1}{\gamma e^{\gamma - 1}} \left( \frac{v_0}{v_1} \right)^\gamma - \frac{v_0}{v_1} \right]. \tag{4}
\end{equation}

The meaning of Eq.(4) is clear when \( \gamma = \gamma_e = \gamma_l \). Defining the Mach number by \( M_s^2 = \frac{\rho v^2}{\gamma(P_{\text{eq}} + P_{\text{ion}})}, \)
then, Eq.(4) is reduced to
\begin{equation}
\frac{\rho_1}{\rho_0} = \frac{v_1}{v_0} \frac{1}{3} + \frac{4}{9M_s^2} \frac{v_1}{v_0} \left( 1 + \frac{\rho_0}{\rho_1} \right)^4 - 2 \frac{v_0}{v_1}. \tag{5}
\end{equation}

The Eq. (5) indicates that the shock compression ratio: \( \rho_1/\rho_0 \) is 3 in the strong shock limit (large Mach number) in the two dimensional space.

According to the simulation results of Fig.2, the potential jumps were about 0.8 MeV for both with and without B field and the up stream electron temperatures were about 0.5 MeV without B field and 0.3 MeV with B field, respectively. The density jumps evaluated by Eq.(1) with those electron temperatures and the potential jumps are 2.0 without B field and 2.8 with B field, which are consistent with those obtained in the simulations. Furthermore, the Mach number with B field is calculated by Eq.(5) is 2.7 for the density jump of 2.8. The simulation of Fig.2-B(b) indicates the shock velocity is 0.06c which corresponds to the Mach number of 2.6 since the ion acoustic velocity is 0.018c near the shock front of the up stream. This reasonably agrees with the jump condition: Eq.(5)

3. Discussions and summary
The case (I): kinetic ion dynamics with adiabatic electron compression and the case (II): viscous ion flow with adiabatic electron compression are proposed to interpret the 2-D PIC simulation results on the shock wave driven by laser radiation pressure in the NRCD plasmas. We found that the case (I) corresponds to the collision-less shock wave of the non-magnetized ions and the case (II) corresponds to the magnetized ions, where the enhanced magnetic field fluctuations scatter ions strongly. Namely, the fluid model with finite viscosity is applicable for the ion dynamics. The scale length of the shock structure in the case (I) is the electron Debye length. On the other hand, the scale length of the case (II) is \( \frac{\mu}{\rho_0} \sim \frac{c_s^2}{v_0 \omega_{ci}} \sim \frac{\lambda_{DPL}}{\omega_{ci}} \sim 5 \lambda_0 \), which reasonably agrees with the broken line of the Fig.2 (A), where the magnetic fluctuation amplitude is about 20kT near the shock front.

Finally, as it is seen by comparing Fig.2-B(a) and (b), the number of accelerated ion is larger by about 5 times in the magnetized plasma. This means the coupling efficiency from laser to ion energy will be enhanced by 5 times and it will improve the performance of the ion driven fast ignition.

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