The $\Omega^-$ and the strange quark mass

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$\Omega^-$ correlators have been calculated on the MILC collaboration’s archive of three flavor improved staggered quark lattices. The $\Omega^-$ is stable under strong interactions (140 MeV below threshold). It provides a valuable consistency check on a combination of strange quark mass and lattice scale determination from other quantities. Alternatively, the $\Omega^-$ mass could be used to fix the strange quark mass, which gives a check on computations of the strange quark mass based on the kaon mass.

Although the most timely results of lattice QCD are otherwise unknown hadronic matrix elements needed for determination of CKM matrix elements, precise computations of experimentally well known quantities are essential tests of our methods. A small number of such quantities have been computed using three flavors of improved staggered sea quarks, showing agreement with experiment at the few percent level[1]. The mass of the $\Omega^-$ baryon is another such quantity, since the particle is well below the threshold for strong decays and its mass is precisely known. From the theoretical side, the extrapolation to the physical light quark mass is much better controlled for this particle than for baryons containing light valence quarks. The one complication is that the $\Omega^-$ mass is very sensitive to the mass of the strange quark — indeed, we could use it as a way of fixing the strange quark mass. Recently the HPQCD, MILC, and UKQCD collaborations have determined the physical strange quark mass in these lattice simulations from a study of pseudoscalar meson properties, which, roughly speaking, amounts to fixing the strange quark mass by tuning the kaon mass[2]. With the strange quark mass fixed, the computation of the $\Omega^-$ mass becomes another test of our lattice methods.

The NLO chiral corrections to the decuplet baryon masses come from loop diagrams where the baryon splits into a pseudoscalar meson and either a decuplet or octet baryon. Isospin and strangeness conservation and the absence of an octet baryon with $S = -3$ insure that the diagrams containing a pion do not contribute to the $\Omega^-$ mass. Diagrams with a kaon are present, but the kaon mass does not vanish as $m_{u,d} \rightarrow 0$. Higher order corrections will of course be present, but a linear (or polynomial) extrapolation in light quark mass is much better for the $\Omega^-$ than for other baryons.

To isolate the decuplet baryons, a nonlocal operator is needed with Kogut-Susskind quarks. We use an operator from Ref. [3], following the implementation of Ref.[4], with code written by C. DeTar[5]. While the $\Omega^-$ correlator is statistically significant to larger distances than the $\Delta$ correlator because of its heavier valence quarks, the decuplet baryons’ larger masses make them relatively noisier and more difficult to fit than the octet baryons.

Correlators for the $\Omega^-$ were calculated on the MILC collaboration three flavor improved staggered quark lattices, with light quark masses ranging from $m_s$ to $m_s/9$ and lattice spacings of about 0.12 fm (“coarse”) and 0.09 fm (“fine”). Table 1 contains a summary of the parameter values used and the $\Omega^-$ mass fits, in units of the lattice spacing. Figure 1 shows the fitted masses as a function of minimum distance included in the fit for one of the data sets. Two valence strange quark masses were used, and the results interpolated or extrapolated to the strange quark masses determined from pseudoscalar meson properties, $a m_s = 0.039$ on the coarse lattices and 0.0272 on
Table 1

Ω⁻ masses in lattice units. Valence masses are the dynamical strange quark mass and a partially quenched Ω⁻ with a valence mass close to the strange quark mass determined from $M_K$. The remaining columns are the sea quark masses, the number of configurations used, the fitted mass in units of the lattice spacing, the distance range used in the fit, the $\chi^2$ and number of degrees of freedom, and the confidence level of the fit.

| $am_{\text{valence}}$ | $am_{\text{sea}}$ | $N_{\text{conf.}}$ | $aM_{\Omega^-}$ range | $\chi^2/D$ conf. |
|------------------------|-------------------|---------------------|-----------------------|------------------|
| 0.05 (Ω) | 0.03/0.05 | 572 | 1.168(11) | 7–15 | 2.8/5 | 0.74 |
| 0.04 (Ω) | 0.02/0.05 | 485 | 1.169(9) | 7–14 | 0.9/4 | 0.93 |
| 0.04 (Ω) | 0.02/0.05 | 485 | 1.125(14) | 7–14 | 1.7/4 | 0.80 |
| 0.05 (Ω) | 0.01/0.05 | 659 | 1.175(16) | 8–15 | 2.8/4 | 0.58 |
| 0.04 (Ω) | 0.01/0.05 | 659 | 1.130(21) | 8–14 | 1.3/3 | 0.72 |
| 0.05 (Ω) | 0.007/0.05 | 498 | 1.155(14) | 8–14 | 1.1/3 | 0.79 |
| 0.04 (Ω) | 0.007/0.05 | 498 | 1.099(22) | 8–14 | 2.1/3 | 0.55 |
| 0.05 (Ω) | 0.005/0.05 | 397 | 1.178(11) | 9–16 | 0.8/4 | 0.93 |
| 0.05 (Ω) | 0.005/0.05 | 397 | 1.128(18) | 9–14 | 0.1/2 | 0.93 |
| $a \approx 0.9$ fm. | | | | |
| 0.031 (Ω) | 0.0124/0.031 | 535 | 0.791(6) | 11–22 | 2.9/8 | 0.94 |
| 0.025 (Ω) | 0.0124/0.031 | 535 | 0.761(9) | 11–20 | 1.0/6 | 0.98 |
| 0.031 (Ω) | 0.0062/0.031 | 575 | 0.785(5) | 11–23 | 8.8/9 | 0.46 |
| 0.025 (Ω) | 0.0062/0.031 | 575 | 0.752(7) | 11–21 | 7.1/7 | 0.42 |
| 0.031 (Ω) | 0.0031/0.031 | 106 | 0.773(5) | 12–23 | 6.0/8 | 0.65 |
| 0.025 (Ω) | 0.0031/0.031 | 106 | 0.736(8) | 12–22 | 5.6/7 | 0.59 |

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Figure 1. Omega mass fits versus minimum distance. This is the partially quenched $\Omega^-$, with valence mass close to the correct strange quark mass. The red octagons are fits with one state of each parity; blue squares include an (unconstrained) excited $\Omega$ state. The green diamonds are $\Delta$ fits. Symbol sizes are proportional to the confidence level, with the size of the legend corresponding to 50%.

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Figure 2. $\Omega^-$ mass in units of $r_1$, with the valence strange quark mass interpolated to the strange quark mass determined from $M_K$, with the continuum/chiral extrapolation and the experimental value.

Figure 3. Ratios of lattice results to physical values. From Ref. [1], with preliminary $\Omega^-$ point added.