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Abstract

This paper aims to give an alternative interpretation of the measurement process in quantum physics. This interpretation is based on conjecturing the existence of a certain “thickness” of time. This conjecture is falsifiable, fits the observed results and maximizes simplicity for explaining measurement without contradicting the established principles and laws of quantum mechanics.

Key words: time, evolution, measurement.

1. Introduction

The problem of “measurement” in quantum mechanics can be illustrated by the double-slit experiment. When particles such as electrons are sent one at a time through a double-slit plate (hereafter called slit A and slit B), single random impacts are observed on a screen behind the plate as expected out of individual particles. However, when the electrons are allowed to build up one by one, the cumulative effect of a great number of impacts on the screen reveals an interference pattern of light and dark bands characteristic of waves arriving at the screen from the two slits. Meanwhile, the interference pattern is made up of individual and sequential impacts and although these sequential impacts are separate and independent, yet it seems as if the electrons work together to produce the interference pattern on the screen. This phenomenon seems to entail that the electrons embody a wave-like feature in addition to their particle nature hence illustrating a particle-wave duality structure.

When the electrons are made to build up one by one while detectors D_A and D_B are placed at slits A and B respectively to find out through which slit each electron went, the interference pattern disappears, and the electrons behave solely as particles. It seems thus impossible to observe interference and to simultaneously know through which slit the particle has passed. The best explanation that can be made from these strange features is that the same electron seems to pass simultaneously through both slits when no detectors are present and through only one slit when detectors are present [1, 2]. This seemingly paradoxical statement is in conformity with the experimental data.

The state vector of an electron passing through slit A may be denoted |A⟩, similarly, the state vector of an electron passing through slit B may be denoted |B⟩. An electron passing through both slits A and B at the same time is said to be in a superposition state and its state-vector is denoted |ψ⟩ = a|A⟩ + b|B⟩, where “a” and “b” are called the probability amplitudes. The mod-
square of “a” represents the probability of the particle to be measured by the D_A detector at the slit A and likewise the mod-square of “b” represents the probability of the particle to be measured by the D_B detector at the slit B.

Conventionally, when no detectors are present, the state-vector $|\psi\rangle = a|A\rangle + b|B\rangle$ of the electron is said to evolve per a deterministic continuous unitary evolution U whereas, when detectors D_A and D_B measure from which slit the electron passes, the deterministic evolution of the state-vector $|\psi\rangle$ is transformed into a probabilistic discontinuous and non-linear state reduction R as explained by Penrose [2]. The two processes U and R create a conflict in the formalism of quantum mechanics. Different ontologies have been proposed to interpret the strange combination of the deterministic continuous U process with the probabilistic discontinuous R process.

According to the Copenhagen interpretation [3, 4], the state-vector $|\psi\rangle$ and the U and R processes should be regarded as a description of the experimenter’s knowledge. There exist several other interpretations amongst which the Everett interpretation or what is more commonly known as the many-world interpretation [5], according to which there is no wave function collapse and all measurement results exist but in different worlds. In line with this interpretation, it is claimed [6] that when a measurement is conducted on an electron in the superposition state $a|A\rangle + b|B\rangle$, a deterministic branching takes place where on one branch detector A detects the electron while detector B doesn’t and at the same time but on the other branch (i.e. another world), detector A doesn’t detect the electron while detector B does detect it. However, this interpretation pauses some probabilistic as well as ontological problems. In particular, the axioms of quantum mechanics say nothing about the existence of multiple physical worlds [7].

Another interpretation is the De Broglie-Bohm deterministic theory according to which particles interact via a quantum potential and are assumed to have existing trajectories at all times. This model seems to make more sense of quantum mechanics than the other interpretations as discussed in detail by Jean Bricmont in his book “Making Sense of Quantum Mechanics [8].

Conventionally, a quantum system (e.g. spin of a particle) can be defined by a state-vector in a Hilbert space. For any observable Q, the state-vector $|\psi\rangle$ is defined by a superposition of vector projections in an eigenbasis $\{|\psi_k\rangle\}$. In other words, the state-vector $|\psi\rangle$ is defined as a linear combination of the different possible states $|\psi_k\rangle$. The normalized conventional state-vector of the quantum system is expressed as follows:

$$|\psi(t)\rangle = \sum_k c_k |\psi_k\rangle \quad (1)$$

where $\sum_k (c_k)^2 = 1 \quad (2)$

The coefficients $c_k$ are complex numbers ($c_k \in \mathbb{C}$) that define the “probability amplitudes” in the specific orthonormal eigenvector basis $\{|\psi_k\rangle\}$ and $|\psi_k\rangle$ are orthonormal states of the quantum system verifying $\langle \psi_i | \psi_j \rangle = \delta_{ij}$ (Kronecker delta).

In this paper, it is intended to introduce an alternative explanatory hypothesis that makes sense of quantum measurement.
2. Introducing a time-thickness into the state-vector

It is learned from the above that a particle can be at different positions at the same instant of time and that the state of the particle can be completely described by a state-vector as a linear combination of possible states with complex coefficients according to the above equations (1, 2).

It does not seem absurd to suppose that a straightforward implication of the above observations would be the fact that an instant of time $t$ is not a geometrical point but rather a plurality of elementary instants composed of a plurality of complex indices $s$ (hereinafter referred to as state-time-indices) associated with the same index $t$ (hereinafter referred to as a physical-time-index). According to this model, time may be imagined as having a thread-like-form, hereinafter referred to as an “elementary-time-thread” having an infinitely small but non-zero thickness.

Let the radius of the “elementary-time-thread” be less than an infinitely small number $\delta$, then each physical-time-index $t$ is associated with a unique section $D_\delta(t)$ of the “elementary-time-thread” defined by a set of elementary time-instants $(t, s)$ as follows:

$$D_\delta(t) = \{(t, s); s \in C, |s| < \delta\} \quad (3)$$

The fact that a section $D_\delta(t)$, associated to a unique index $t$, is made up of an infinite number of elementary instants $(t, s)$ enables a particle to be at different positions at the same physical-time-index $t$. On the other hand, as the coefficients of the state-vector of equation (1) are complex numbers, then the state-time-indices $s$ are chosen to be complex numbers out of which the coefficients are constructed as shown below.

Indeed, let a curve $C(t)$ formed of the elementary instants $(t, s)$ belonging to the section $D_\delta(t)$ that are “visited” by the states (positions) of the particle at the physical-time-index $t$. Subdivide $C(t)$ into a set of elementary instants $\{(t, s_1), (t, s_2), \ldots, (t, s_{k-1}), (t, s_k), \ldots\}$ chosen in such a manner that each arc or interval $\Delta s_k$ joining $s_{k-1}$ to $s_k$ represents a “state-period” during which the particle is at the state $|\psi_k\rangle$. Thus, the curve $C(t)$ is the made up of all the intervals $\Delta s_k$ :

$$C(t) = \bigcup_k \Delta s_k. \quad (4)$$

On the other hand, let $\Delta s_{kj}$ ($j=1, 2, \ldots$) all the different intervals belonging to $C(t)$ that are associated to the same state $|\psi_k\rangle$, therefore, the “domain of state-time-indices” (or sub-curve) during which the particle is found in the state $|\psi_k\rangle$ is given by :

$$C_{s_k}(t) = \bigcup_j \Delta s_{kj} \quad (5)$$

Moreover, “the total state-time-period” or “state-life-time” during which the particle is in the state $|\psi_k\rangle$ is:

$$a_k = \sum_j \Delta s_{kj} \quad (6)$$

Out of the above definitions, the state of a quantum system can be characterised by two representations: an implicit representation $|\psi(t)\rangle$ and an explicit one $|\psi(t, s)\rangle$. 
2.1 Implicit Representation of the state-vector $|\psi(t)\rangle$

By considering each “state-life-time” $a_k$ as a “weight” of its corresponding state $|\psi_k\rangle$, then a weighted sum $|S(t)\rangle$ of the different states may be constructed as follows:

$$|S(t)\rangle = \sum_k a_k |\psi_k\rangle \quad (7)$$

Let $L$ be a normalizing parameter called hereafter the “total-state-life-time” at a given $t$, such that:

$$L^2 = \sum_k a_k^2 \quad (8)$$

By dividing the weighted sum $|S(t)\rangle$ by the “total-state-life-time” $L$ at a given $t$, we get:

$$\frac{|S(t)\rangle}{L} = \sum_k \frac{a_k}{L} |\psi_k\rangle \quad (9)$$

where $\sum_k \left(\frac{a_k}{L}\right)^2 = 1 \quad (10)$

Thus, by comparing equations (9) and (10) to equations (1) and (2), one can easily deduce that

$$|\psi(t)\rangle = \frac{|S(t)\rangle}{L} = \sum_k \frac{a_k}{L} |\psi_k\rangle = \sum_k c_k |\psi_k\rangle \quad (11)$$

where $c_k = \frac{a_k}{L} = \frac{|a_k|e^{i\phi}}{L} \quad (13)$

It should be noted that the phase term $e^{i\phi}$ in equation (13) is dependent on the physical-time-index $t$ and its associated state-time-indices $s$.

The state-vector $|\psi(t)\rangle$ can thus be considered as a “normalised weighted sum” or almost as a sort of a “weighted arithmetic mean” where each “weight” $c_k = \frac{a_k}{L}$ (i.e. probability amplitude) represents the ratio of the intervals $a_k$ visited by the state $|\psi_k\rangle$ at the physical-time-index $t$.

The probability $P_k$ associated to the state $|\psi_k\rangle$ is:

$$P_k = \left(\frac{a_k}{L}\right)^2 \quad (14)$$

It seems logical that for a given physical-time-index $t$, the more the “state-life-time” $a_k$ is high the greater is the probability to find the quantum system in the state $|\psi_k\rangle$.

It should be noted that equation (11) takes account of the state-time-indices $s_k$ in an implicit or “hidden” manner via the coefficients $c_k$. Thus, when a measuring apparatus is used to measure the state of a quantum system, it cannot be explicitly known at which elementary-time-instant $(t_m, s_m)$ the measure has been undertaken and thus, the measured outcome would seem as if it has resulted out of a wave-function collapse.
2.2 Explicit Representation of the stat-vector $|\psi(t,s)\rangle$

According to this second representation, the states of a quantum system are described explicitly in function of the elementary-time-instants $(t,s_k)$ by means of a “fundamental state-vector” $|\psi(t,s)\rangle$:

$$|\psi(t,s)\rangle = \begin{cases} |\psi_k\rangle & \text{if } s \in C_{s_k}(t) \\ 0 & \text{otherwise} \end{cases} \quad (15)$$

where according to equation (5), $C_{s_k}(t)$ is the “domain of state-time-intervals” associated to the state $|\psi_k\rangle$.

Equation (15) simply represents the state under which the quantum system is present at a given elementary-time-instant $(t,s)$ and can be expressed as follows:

$$|\psi(t,s)\rangle = \sum_k \delta_s(C_{s_k}(t))|\psi_k\rangle \quad (16)$$

where $\delta_s(C_{s_k}(t))$ is a Dirac measure (or indicator function) defined as:

$$\delta_s(C_{s_k}(t)) = \begin{cases} 1 & \text{if } s \in C_{s_k} \\ 0 & \text{otherwise} \end{cases} \quad (17)$$

Thus, in case it is possible to know the elementary-time-instant $(t_m, s_m)$ at which the measurement is to be undertaken, then it is theoretically possible to deterministically find out the theoretical outcome state $|\psi_m\rangle$ at that instant.

It is to be noted that the explicit representation $|\psi(t,s)\rangle$ is better adapted for describing the quantum act of measurement while the implicit representation $|\psi(t)\rangle$ may be continued to be used for describing the evolution of the quantum system in function of the physical-time-index $t$ according to the Schrödinger equation.

2.3 Evolution and Measurement

The two representations may be introduced into the Schrödinger equation as follows:

$$\left[1 - \delta_{t,t_m}\right] \left[i\hbar \frac{\partial}{\partial t} - H\right]|\psi(t)\rangle = \delta_{t,t_m} \delta_{s,s_m}[|\psi(t)\rangle - |\psi(t,s)\rangle] \quad (18)$$

where

$$\delta_{t,t_m} = \begin{cases} 0 & \text{si } t \neq t_m \\ 1 & \text{si } t = t_m \end{cases} \quad (19)$$

$$\delta_{s,s_m} = \begin{cases} 0 & \text{si } s \neq s_m \\ 1 & \text{si } s = s_m \end{cases} \quad (20)$$

The term $\delta_{t,t_m}$ gives a partial information on the act of measurement and indicates only the physical-time-index $t$ with respect to which measurement is conducted: $\delta_{t,t_m} = 0$ indicates that
no measurement has been conducted yet, while \( \delta_{t,t_m} = 1 \) indicates that a measurement has been realised at the physical-time-index \( t = t_m \).

The product-term \( \delta_{t,t_m} \delta_{s,s_m} \) gives a complete information on the act of measurement. It indicates the precise elementary-time-instant \((t_m, s_m)\) at which measurement is conducted. In other words, \( \delta_{s,s_m} \) indicates the state-time-index \( s_m \) of the measurement knowing that \( \delta_{t,t_m} \) indicates the physical-time-index \( t_m \) of the measurement process. In particular, when the product-term \( \delta_{t,t_m} \delta_{s,s_m} = 1 \) it means that the interaction of measurement took place at the elementary-time-instant \((t_m, s_m)\).

Thus, equation (18) expresses the fact that in absence of any measurement (i.e. \( t \neq t_m \)), the right-hand side of the equation is equal to zero and one gets the traditional Schrödinger equation. In contrast, when a process of measurement is conducted at the physical-time-index \( t = t_m \), then there exists a unique state-time-index \( s_m \), and hence a unique elementary-time-instant \((t_m, s_m)\) at which the precise interaction between the measuring device and the measured system took place with a probability \( P_m = \left( \frac{a_m}{L} \right)^2 \) and resulting into the unique outcome \( \psi(t_m, s_m) = |\psi_m\rangle \) without any collapse.

It is clear that even though we “ignore” the state-time-indices, the evolution and measurement of a system according to equation (18) is deterministic without any reduction.

Meanwhile, the act of measurement seems to have a double effect: on the one hand, it reveals one of the substantially pre-existing states of the system with a corresponding probability and on the other hand, “creates” a new “state” by freezing the outcome state for a “certain” physical-time-period that may depend on the “intensity” of interaction between the measured system and the measuring apparatus. The outcome of a measurement can be considered as “trapped” for a certain physical-time-period due to the interaction between the measured system and the measuring device.

Thus, just after a measurement at the physical-time-index \( t_m \) and during a “definite physical-time-period” \( \Delta t_N = \left[ t_M, t_{M+N} \right] \), the state-vector \(|\psi(t, s)\rangle\) relative to the measured state becomes stationary with respect to the state-time-indices \( s \) (i.e. \( \delta_s \left( C_{sM}(t) \right) = 1, \forall s \in \mathcal{C}(t) \) and \( t \in \Delta t_N \)) and can be expressed as follows:

\[
|\psi(t, s)\rangle = \sum_k \delta_s \left( C_{sk}(t) \right) |\psi_k\rangle = |\psi_M\rangle \quad (21)
\]

In other terms, immediately after the measurement the module of the “state-life-time” \(|a_M|\) corresponding to the measured state \(|\psi_M\rangle\) becomes equal to the “total-state-time-life” \( L \) (i.e. \(|a_M| = L \) and \( a_k = 0, k \neq M \)) entailing a stationary state-vector \(|\psi(t)\rangle\) that can be expressed as follows:

\[
|\psi(t)\rangle = \sum_k \frac{|a_k| e^{i\phi}}{L} |\psi_k\rangle = e^{i\phi} |\psi_M(x)\rangle \quad (22)
\]
3 Conclusion

If we take the conventional interpretation of equation (1) where the state-time-indices are completely ignored, and their existence is not even suspected, the different states $|\psi_k\rangle$ would appear to occur at once and thus, a measurement at any physical-time instant $t$, would seem to impose a collapse of the state-vector $|\psi(t)\rangle$ into an arbitrary outcome.

However, according to the present model even if we don’t have any knowledge about the state-time-indices, the outcome of a measurement taking place at any physical-time-index $t$ could be any state $|\psi_k\rangle$ with a corresponding probability $\left(\frac{a_k}{L}\right)^2$ (in the classical sense of probability) without any collapse whatsoever because the different states $|\psi_k\rangle$ as shown by equation (6) do not occur at once with respect to the state-time-indices.

References

[1] C. Cohen-Tannoudji, B. Diu and F. Laloë *Mécanique quantique I* (Hermann 1998).
[2] D. Rickles *The Philosophy of Physics* (Polity Press 2016).
[3] R. Penrose *The Road to Reality* (Vintage Books 2007).
[4] H. Wimmel *Quantum physics & observed reality* (World Scientific 1992).
[5] H. Everett, Rev. Mod. Phys., 29, 454 (1957).
[6] D. Wallace *Emergent Multiverse* (Oxford 2012).
[7] A. Kent, arXiv: quant-ph/0905.0624v3 (2013).
[8] Jean Bricmont, Making Sense of Quantum Mechanics, Springer (2016).