One-electron spectral functions of the attractive Hubbard model at intermediate coupling.

M.Yu.Kagan\textsuperscript{a,1}, R.Fr´esard\textsuperscript{b}, M.Capezzali\textsuperscript{c}, H.Beck\textsuperscript{b},
\textsuperscript{a} P.L.Kapitza Institute for Physical Problems, Moscow 117334, Russia
\textsuperscript{b} Institut de Physique, Universit´e de Neuchˆatel, 2000 Neuchˆatel, Switzerland
\textsuperscript{c} Department of Physics, Queen’s University, Kingston, ON, K7L 3N6, Canada

Abstract

We calculate the one-electron spectral function of the attractive-$U$ Hubbard model in two dimensions. We work in the intermediate coupling and low density regime and evaluate analytically the self-energy. The results are obtained in a framework based on the self-consistent $T$-matrix approximation. We also calculate the chemical potential of the bound pairs as a function of temperature. On the basis of this calculation we analyze the low-temperature resistivity and specific heat in the normal state of this system. We compare our results with recent beautiful tunneling experiments in the underdoped regime of HTSC-materials.

Keywords: Strongly-correlated electrons, Hubbard model, superconductivity

1. Introduction

The attractive Hubbard model is very challenging for theorists since its physics is bearing quite some resemblance to the underdoped regime in High Temperature Superconductors [1], [2].

In the present paper we treat analytically the intermediate coupling regime of this model. There are two characteristic temperatures in this regime. The first one is a crossover temperature $T_\star$, where the density of electrons in bound pairs $2n_B$ is equal to the density of unpaired electrons $n_F$. Of course, the total density of charge carriers in the system satisfies the condition $n = n_F + 2n_B$. The second temperature $T_{KT}$ is a Kosterlitz-Thouless critical temperature [3] of a superfluid transition in 2D. We consider the low-temperature normal regime of the model $T_{KT} < T < T_\star$.

2. Theoretical model and results

We consider an attractive-$U$ Hubbard model [4] ($U < 0$) in the intermediate coupling regime $|U| \lesssim W$. We restrict our calculations to low density limit In this limit the crossover temperature $T_\star$ is given by [4]:

$$T_\star = \frac{|E_b|}{2\ln(|E_b|/Wn)}.$$

(1)
where $|E_b|$ is the binding energy of a local pair on the empty lattice [4]. At the same time, the Kosterlitz-Thouless transition temperature reads [5], [6]:

$$T_{KT} = \frac{nW}{4 \ln \ln (|E_b|/Wn)}. \quad (2)$$

Low density means that $T_{KT} < T_c$, or, in another words: $|E_b| > \frac{nW}{2}$.

We first calculate the two-particle $T$-matrix for small $\omega$ and $q$. This yields:

$$T(\omega, q) = \frac{|E_b|W}{(\omega - \frac{q^2}{4m} + \mu_B + i0)}. \quad (3)$$

It is important to note that the pole-structure of the $T$-matrix reflects the creation of a bound pair with a mass $m_B = 2m = 1/\hbar$ and a bosonic chemical potential $\mu_B = 2\mu + |E_b|$, where $\mu$ is the one-particle chemical potential. As a result, we obtain the following expression for the one-particle spectral function:

$$\text{Im}G(\omega, q) = \left[1 - \frac{|E_b|Wn}{(\varepsilon_q - \mu)^2}\right] \delta(\omega - \varepsilon_q + \mu)$$

$$+ \frac{|E_b|Wn}{(\varepsilon_q - \mu)^2} \delta(\omega + \varepsilon_q - \mu + \mu_B). \quad (4)$$

This expression describes an asymmetric two-band structure, consisting of a fermionic band and a bosonic band separated by a correlation gap $\Delta = |E_b|$. For $T_{KT} < T < T_c$: $\eta \sim \exp\left\{-\frac{|E_b|}{2T}\right\}$, hence $n \approx 2n_B$ and we have a new type of metal: a normal bosonic metal. Let us study the resistivity and specific heat of this metal. At very low densities, when not only the parameter $|E_b|/Wn \gg 1$, but also [5], [6] $\ln \ln (|E_b|/Wn) \gg 1$, the Kosterlitz-Thouless critical temperature is smaller than the degeneracy temperature $T_0 = \frac{nW}{4}$ of a 2D gas of bound pairs. In this case the bosonic chemical potential at temperatures $T < T_0$ becomes exponentially small and reads: $\mu_B = -T \exp(-T_0/T)$. As a result, the specific heat of the system becomes linear [7]: $C_v = nT/T_0$ for $T < T_0$. Note that since $C_v = n$ for $T > T_0$ there is no $\lambda$-point in our system.

Finally, the resistivity changes its behavior [7] from $R \sim \sqrt{T}$ for $T > T_0$ on $R \sim \frac{V}{\sqrt{T}} \exp(-\frac{T_0}{T})$ for $T < T_0$.

3. Conclusions

We would like to emphasize that already such a simple theory, as presented above, brings a lot of similarities with the recent results of beautiful tunneling experiments [8] in underdoped regime of HTSC-materials. These similarities include a large value of the ratio of $\frac{2\Delta}{T_c} \sim 8 \div 10$ and a strong anisotropy of the density of states both in theory and in experiment. Note that in our theory $2\Delta/T_c = 2|E_b|/T_{KT}$.

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