Vortex nucleation in Bose-Einstein condensates in an oblate, purely magnetic potential

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We have investigated the formation of vortices by rotating the purely magnetic potential confining a Bose-Einstein condensate. We modified the bias field of an axially symmetric TOP trap to create an elliptical potential that rotates in the radial plane. This enabled us to study the conditions for vortex nucleation over a wide range of eccentricities and rotation rates.

Present the threshold conditions for vortex nucleation in our geometry and finally discuss the role of the thermal cloud in both the nucleation and stabilization processes. A detailed description of the rotating elliptical trap is given in \[\text{(3)}\] and only a summary of the technique is presented here. Our TOP trap consists of a spherical quadrupole field and a rapidly rotating bias field \[\text{(4)}\]. The zero of the magnetic field describes a circle in the radial (XY) plane. The atoms experience a time-averaged, axially symmetric harmonic potential. If the X and Y components of the bias field \((B_x, B_y)\) are not of equal amplitude, then the axial symmetry is broken and the potential becomes elliptical in the XY plane. The ratio of the radial trap frequencies \(\omega_x/\omega_y\), can be calculated numerically from \(B_y/B_x\). For small eccentricities \(\omega_x/\omega_y - 1 \approx (B_y/B_x - 1)/4\). To rotate the elliptical potential we modulated \(B_x\) and \(B_y\) at \(\Omega\), a frequency much lower than the TOP frequency \(\omega_0\). The final bias field has the form of a rotation matrix through angle \(\Omega t\) multiplying the X and Y components of an ellipse:

\[
\begin{pmatrix}
B_x \\
B_y
\end{pmatrix} = \begin{pmatrix}
\cos \Omega t & \sin \Omega t \\
-\sin \Omega t & \cos \Omega t
\end{pmatrix} \begin{pmatrix}
EB_t \cos \omega_0 t \\
B_t \sin \omega_0 t
\end{pmatrix}
\]

During any particular evaporative cooling run the value of \(\Omega\), the angular frequency at which the elliptical trap rotates, was fixed (for technical reasons) and so the trap was always spinning. To create a ‘static’ trap for efficient evaporative cooling, the bias field ratio, \(E\), is set to 1 so that the trap is symmetric around the rotation axis. This symmetry was checked to be accurate to \(\epsilon = 0 \pm 0.005\) using measurements of the frequency of dipole oscillations in the X and Y directions. Evaporative cooling followed by an adiabatic expansion gave a condensate of \(2 \times 10^4\) atoms, at a temperature of \(0.5T/T_c\). At this stage the trap is axially symmetric, \(\omega_x = \omega_y\), with trap frequencies typically \(\omega_x/2\pi = 62\) Hz and \(\omega_x/2\pi = 175\) Hz.

To create vortices, the value of \(E\) was ramped linearly over 200ms from 1 to its final value, to give a trap that was elliptical in the rotating frame. If one creates an eccentric trap by increasing (decreasing) the TOP bias field then all three trap frequencies are reduced (increased). Since vortex lifetimes depend on the mean trap frequency \(\omega_\perp\), we adjusted the quadrupole field in the adiabatic expansion stage, to ensure that all traps had the same average radial trap frequency \(\omega_\perp\), defined as

\[
\omega_\perp = \sqrt{\frac{\omega_x^2 + \omega_y^2}{2}}.
\]
The condensate was then held in the rotating anisotropic trap for a further 800 ms before being released. After 12 ms of free expansion the cloud was imaged along the axis of rotation using an absorption imaging system with 3 μm resolution. Figure 2 shows images of the expanded condensate at different stages during the nucleation process. Initially the cloud elongates, confirming that nucleation is being mediated by excitation of a quadrupole mode (fig. 2(a)) [10]. Then finger-like structures appear on the outside edge of the condensate which eventually close round and produce vortices, ~ 800 ms after rotation began (fig. 2(b)). Approximately 200 ms later, these have moved to equilibrium positions within the bulk of the condensate. Figures 2(c) and (d) show typical, single-shot images of stable vortex arrangements. The depth of each vortex (in the integrated absorption profile) is approximately 95% of the surrounding condensate. The core diameters of ~ 5 μm after 12 ms of free expansion are consistent with the predictions in [10]. The maximum number of vortices we observed was seven, limited by the number of atoms in our condensate.

Our first study of the nucleation process involved counting the number of vortices as a function of the normalised trap rotation rate, Ω = Ω/ω⊥, for a fixed eccentricity. Results for trap deformations ϵ = 0.084 and 0.041 are given in fig. 3. These graphs show a maximum and minimum value of Ω for nucleation at a given eccentricity. Increasing ϵ increases the range of Ω over which vortices may be nucleated, both by lowering Ωmin and raising Ωmax. In the limit of small eccentricities, √2ω⊥ is the frequency of the m=2 quadrupole mode, which has been shown elsewhere to play a critical role in the nucleation process [10]. Rotation of the ellipsoidal trap at half this frequency, i.e. Ωc = 1/√2 ≈ 0.71, excites this mode. The plots in fig. 3 confirm that the nucleation depends on resonant excitation of the quadrupole mode as seen previously [4,10]. The resonance is broader at higher eccentricity, as intuitively expected for stronger driving.

Our second study involved holding Ω constant and counting the number of vortices as a function of the trap deformation ϵ. Figure 3 shows our results for two cases: (a) Ω > Ωc and (b) Ω < Ωc. Interestingly we were able to nucleate vortices under adiabatic conditions when Ω < Ωc. We employ the adiabaticity criterion ˙Ω/Ω ≪ ω⊥. As a further check of the adiabaticity we varied the time for the eccentricity ramp between 200 ms and 1 s, and detected no difference in the number of vortices formed.

The critical values of Ω and ϵ for nucleation were extracted from plots such as fig. 2 and fig. 3 for many values of Ω and compiled on fig. 4. The data points show the minimum eccentricity required for a given rotation rate and map out region 2, within which vortices nucleate. Ωc appears to be a critical rotation frequency at which vortices can be nucleated with minimum eccentricity, as predicted in [13]. Changing Ω in either direction requires a more elliptical trap for nucleation, although different physical processes control the upper and lower limits as explained below.

At Ω > Ωc, the condensate follows a particular quadrupole mode at small eccentricity, region 3 in fig. 4. This mode is referred to as the ‘overcritical branch’ in [13] and has an elliptical density distribution which is orthogonal to the trap potential. It then nucleates vortices when the eccentricity is too large for the quadrupole mode to be a solution of the hydrodynamic equations. The boundary of the region in the ϵ versus Ω plot where this quadrupole mode exists is given by

\[ \epsilon = \frac{2}{\Omega} \left( \frac{2\Omega^2 - 1}{3} \right)^{3/2}. \]

We determined this relation from the solutions of the hydrodynamic equations for superfluids and it is plotted as a solid line in fig. 4. This line agrees well with the experimental data for the critical conditions for nucleation for Ω > Ωc and a wide range of ϵ.

Below Ωc, the deformation needed to nucleate vortices appears to increase linearly with Ω. This boundary cannot be explained in terms of the stability limit of a quadrupole mode - the ‘normal branch’ is stable on both regions 1 and 2 and the ‘overcritical branch’ is stable in neither. Our data appears to be at variance with the results in [10], where no vortices were seen when the eccentricity was increased adiabatically and Ω < Ωc.

A mechanism for the creation of vortices at a frequency below Ωc has been proposed in [12]. They have shown that there are regions in the plot of ϵ versus Ω, both above and below Ωc, where the quadrupole solutions of the hydrodynamic equations become dynamically unstable. For frequencies above Ωc, the predicted instability domains coincide with the experimentally observed vortex domains in [10], and this work, thus indicating a link between their instability analysis and vortices. To make a quantitative prediction for the boundary between regions 1 and 2 shown in fig. 4 will require further detailed work for our specific case. We also note that it is possible that there exist non-negligible terms of higher order than quadratic in the rotating magnetic potential. These have been demonstrated to excite high order rotating surface modes, e.g. hexapole, which lead to vortex nucleation at frequencies less than Ωc [12].

Another possible mechanism for observation of vortices below Ωc is that the thermal cloud plays an important role. Transfer of angular momentum to the condensate from the spinning thermal cloud may provide a mechanism for vortices to form at Ω < Ωc. However the transfer rate of angular momentum must be greater than any loss rate due to residual trap anisotropy [4]. In [10], gravity produces a small static eccentricity in the trap in the
plane of rotation. The ‘spin down’ time for a rotating thermal cloud in the presence of a deformation parameter $\epsilon$ of only 0.01 is very short, 0.5 s, compared to the spin up time of 15 s and hence the cloud may never gain significant angular momentum. However, in our experiment, gravity acts along the rotation axis and hence the trap is symmetric in the plane of rotation, giving a more favourable ratio of spin-up to spin-down times.

With this hypothesis in mind, we tested our nucleation curve at lower temperature to see if there was any change when the amount of thermal cloud was reduced. When acquiring the data of fig. 3 the rf was turned off after condensation and some heating was observed during the nucleation procedure, resulting in a temperature around 0.8$T_c$. To achieve a lower temperature we left on the so-called ‘rf shield’ so as to give an effective trap depth of 800 nK during the nucleation process. This resulted in a temperature of 0.5$T_c$. No significant change was observed in the nucleation curve at this lower temperature. However, this does not totally rule out a role for the thermal cloud in the nucleation process since even at 0.5$T_c$, there was still a significant 20% of atoms in the thermal cloud.

Although the exact amount of thermal cloud seemed to have little effect on the nucleation conditions, it had a striking effect on the behaviour of vortices after nucleation. Without the rf shield during the nucleation procedure, vortices were only occasionally found in an equilibrium configuration (i.e., 1 vortex in the centre, 3 vortices in an equilateral triangle as in fig. 1) and $\sim 400$ms after forming they had already moved to the edge of the condensate before disappearing. However with the rf shield present, the vortices were normally found in equilibrium positions $\sim 200$ ms after formation and had a lifetime of $\sim 4s$, only limited by the decay of the condensate itself.

In summary, we have used a purely magnetic rotating trap to investigate conditions for vortex nucleation (after the formation of the condensate) over a wide range of trap eccentricities. For a given eccentricity, we observe both an upper and lower limit to the rotation rate for nucleation. The upper limit confirms the predictions in [13], but over a much wider range of parameters. However the lower limit to the rotation rate is different to that reported elsewhere. Further theoretical work is required to explain the linear dependence of $\epsilon$ on $T_c$ shown in fig. 4.

The thermal cloud is shown to destabilise vortex arrays, thus measurements of the vortex lifetime in an oblate geometry were made with an rf shield to prevent heating.

The rotating anisotropic magnetic trap is well suited to experiments that require a large trap eccentricity. It has recently been used to investigate the irrotational behaviour of vortex-free condensates at low rotation rates [14]. A large eccentricity is also needed to spin up a thermal cloud [3], hence this trap may be useful in attempts to condense directly into a vortex state from a spinning thermal cloud in thermodynamic equilibrium.

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FIG. 1. Images of the condensate at different stages during the vortex nucleation process. (All taken with $\Omega = 0.70$ and $\epsilon = 0.05$ and after 12ms of free expansion). (a) After the 200ms spinning eccentricity ramp, the condensate is elongated, indicating that a quadrupole mode has been excited. (b) After a further 600ms in the spinning trap one vortex has just formed near the edge. After 800ms, the vortices have reached their equilibrium positions and appear in symmetrical configurations. (c) shows one centred vortex, typical under these conditions, whilst (d) shows a triangular array of three vortices.

FIG. 2. The mean number of vortices as a function of the normalised trap rotation rate $\Omega = \Omega/\omega_\perp$. Two different trap eccentricities were used, $\epsilon = 0.041$ (open circles) and $\epsilon = 0.084$ (solid circles). Each data point is the mean of 4 runs.

FIG. 3. The mean number of vortices as a function of trap deformation at 4 different trap rotation rates: (a) above and (b) below the critical value $\Omega_c = 0.71$. In (a), $\Omega = 0.74$ (solid circles) and 0.81 (open circles). In (b) $\Omega = 0.61$ (solid circles) and 0.70 (open circles). Positive (negative) $\epsilon$ corresponds to $\omega_x < \omega_y$ ($\omega_x > \omega_y$).
FIG. 4. The critical conditions for vortex nucleation. The data points mark the minimum trap deformation for nucleation at a particular $\Omega$. Vortices may be formed in region 2. The solid line shows the theoretical limit of stability of the quadrupole mode, which is stable in region 3. This line is in good agreement with the extreme conditions for vortex formation at $\Omega > \Omega_c$. 