New Magic Numbers and New Islands of Stability in Drip-Line Regions

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Abstract

The systematics of the local energies and the two-neutron separation energies obtained in the mass predictions of infinite nuclear matter model of atomic nuclei show strong evidence of new neutron magic numbers 100, 152, 164, new proton magic number 78 and new islands of stability around N=100, Z ≃ 62; N=152, Z=78; and N=164, Z≃90 in the drip line regions of nuclear chart, where the usual magic numbers are found to be no more valid.

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Nuclear matter which constitutes about 99.9% of the universe exists in many conditions in nature characterised by density, neutron-proton ratio and temperature. Our knowledge of nuclear dynamics has been mostly gathered during the last half a century or so, from the study of nuclei in the valley of stability. Obviously this knowledge cannot be considered as complete. Its adequacy and self-sufficiency has to be tested in newer and newer domains, and possibly improved upon, for its evolution towards completeness. Breakthroughs [1] achieved in nuclear techniques in recent years in the field of precision mass measurements, and production of radioactive ion beams have opened up the possibilities of study of exotic nuclear species in the drip-line regions of the nuclear "terra incognita". An era of renaissance in nuclear structure physics has commenced with many opportunities and challenges paving the ways to perfect our knowledge of nuclear dynamics. Out of many questions of momentous nature, the most important one is whether the classical magic numbers seen in the valley of stability remain valid in the exotic drip-line regions or new magic numbers and new islands of stability are awaiting to be discovered in such normally inaccessible nuclear terrain. The nuclei inhabiting such regions will be precariously balanced in regard to their stability by the disruptive dripping force and the stabilising shell effect arising out of the possible magicity. Such nuclei are produced in r-process nucleosynthesis. There abundances play important role in stellar evolution[2]. If such islands exist, they will undoubtedly provide crucial new testing grounds for nuclear dynamics. These are new nuclear terrains surrounded by relatively short lived exotic species unlike the superheavy elements which are true islands in the midst of sea of instability. This will provide new instance of the triumph of the individualistic properties of the nucleus over its liquid like behaviour in an exotic ambience like drip-line regions. In this note we present strong evidence of the existence of, new magic numbers and new islands of stability found in our mass predictions[3] in Infinite Nuclear Matter(INM) model.[4,5]

The INM model of the nucleus is based on a many-body theoretic foundation whose main elements are the infinite nuclear matter at ground state and the generalised Hugenholtz-Van Hove(HVH) theorem[6]. A good account of this can be seen in [5]. For easy reading and
completeness, a brief account of INM model is presented here. In this model the ground state energy \( E^F(A, Z) \) of a nucleus \((A, N, Z)\) with assymetry \( \beta \) is considered equivalent to the energy \( E^s \) of a perfect sphere made up of infinite nuclear matter at ground atate with the same assymetry \( \beta \) plus the residual characteristics energy \( \eta \) called the local energy. So,

\[
E^F(A, Z) = E^s_{INM}(A, Z) + \eta(A, Z) \tag{1}
\]

with

\[
E^s_{INM}(A, Z) = E(A, Z) + f(A, Z) \tag{2}
\]

where \( f(A, Z) \) characterises the finite size effects given by

\[
f(A, Z) = a^I_sA^{2/3} + a^I_c(Z^2 - 5(3/16\pi)^{2/3}Z^{4/3})A^{-1/3} - \delta(A, Z) \tag{3}
\]

The superscript \( I \) refers to the INM characteristics of the surface and Coulomb coefficients \( a^I_s \) and \( a^I_c \) respectively, and \( \delta(A, Z) \) is the usual pairing term. Hereafter the superscript ‘\( F \)’ denotes the quantities corresponding to finite nuclei.

Eq. (1) now becomes

\[
E^F(A, Z) = E(A, Z) + f(A, Z) + \eta(A, Z) \tag{4}
\]

The mass formula given by Eq.(4) consists of three distinct parts: an infinite part \( E(A, Z) \), a finite size part \( f(A, Z) \) and local energy part \( \eta(A, Z) \). We have to determine these three functions. The term \( E(A, Z) \) being the property of INM at the ground state, will satisfy the generalized HVH theorem\[6\] of many-body theory,

\[
\frac{E}{A} = [(1 + \beta)\epsilon_n + (1 - \beta)\epsilon_p] \tag{5}
\]

where, \( \epsilon_n = \left( \frac{\partial E}{\partial N} \right)_Z \) and \( \epsilon_p = \left( \frac{\partial E}{\partial Z} \right)_N \) are neutron and proton Fermi energies respectively. The solution of Eq.(5) is of the form,

\[
E = -a^I_vA + a^I_\beta\beta^2A \tag{6}
\]
where, $a_v^I$ and $a_\beta^I$ are identified as the volume and assymetry co-efficients corresponding to INM.

Using Eqs.(4), (5), and (6) we arrive at three essential equations of the model,

$$\frac{f}{A} - \frac{N}{A} \left( \frac{\partial f}{\partial N} \right) \bigg|_Z - \frac{Z}{A} \left( \frac{\partial f}{\partial Z} \right) \bigg|_N = \frac{E^F}{A} - \frac{1}{2} [(1 + \beta)\epsilon^F_n + (1 - \beta)\epsilon^F_p],$$  \hspace{1cm} (7)

$$- a_v^I + a_\beta^I \beta^2 = \frac{1}{2} [(1 + \beta)\epsilon^F_n + (1 - \beta)\epsilon^F_p] - \frac{N}{A} \left( \frac{\partial f}{\partial N} \right) \bigg|_Z - \frac{Z}{A} \left( \frac{\partial f}{\partial Z} \right) \bigg|_N,$$ \hspace{1cm} (8)

and

$$\frac{\eta(A,Z)}{A} = \frac{1}{2} [(1 + \beta)\left( \frac{\partial \eta}{\partial N} \right) \bigg|_Z + \frac{1}{2} [(1 - \beta)\left( \frac{\partial \eta}{\partial Z} \right) \bigg|_N]$$  \hspace{1cm} (9)

Eq.(7) determines the finite size coefficients characterising $f$ through its fit to the combination of the data given by the r.h.s. of the equation, consisting of total energy and proton and neutron separation energies. These values so determined are used in the fit of the Eq.(8) to determine $a_v^I$ and $a_\beta^I$, the properties of INM. Thus the coefficients $a_v^I$, $a_\beta^I$, $a_c^I$ and $a_s^I$ determined through theses two fits are called global parameters , being the charactereristics of INM and common to all nuclei. $\eta$ denotes the characteristic properties of the nucleus which comprises shell, deformation and diffuseness etc. and can be considered as its fingerprint. Once $E$ and $f$ are known, the empirical values of $\eta$ can be determined using experimental binding energies in Eq.(4). Using the empirical values of $\eta$ of all known nuclei, the $\eta$ of all unknown nuclei are determined by using Eq.(9) for extrapolation. The details of the calculation can be found in [3].

Over the years, the success of this mass formula has been well demonstrated [3-12]. Being built over a many-body theoretic foundation, it has been shown to be particularly useful in the extraction of the saturation properties and incompressibility of nuclear matter from nuclear masses leading to nuclear equation of state [5]. Recently it has been developed [3] to its full potential in the prediction of masses of 7208 nuclei with r.m.s. and mean deviations of 401 keV and 9 keV respectively. The unique feature of this prediction is that it shows shell quenching[10] for the N=82, 126 shells in agreement with the results of the
astrophysical studies[2] of the abundances of heavy elements. The systematics of the local energy $\eta$ determined in this prediction have been shown by Nayak[12] to exhibit vanishing of the shells in the drip-line region. All these successes are essentially due to the inherent long range extrapolation properties of $\eta$ demonstrated explicitly for fifteen steps in [3]. This mass formula is quite special due to its strong many-body theoretic foundation and its use of three times the data normally used in other mass formulas. It uses the binding energies and additionally, the neutron and proton separation energies. Thus it uses 5652 data for 1884 nuclei in the fitting procedure to determine its five parameters. Therefore it is endowed with better predictive power with potential for validity in the regions far from stability. This is evident in its unique success in shell-quenching[10] for higher shells $N = 82, 126$ where other mass formulas fail. This has naturally enhanced our confidence in it. Here we make a thorough search of the systematics of $\eta$ and two-neutron separation energies in our mass predictions to see if any exotic features like new magic numbers and/or new islands of stability are existing in the drip-line regions of nuclear landscape.

The local energy $\eta$ is a relatively new entity in nuclear physics. It comprises all the characteristic features of a nucleus, which includes predominantly shell effect and all other local effects like deformation, diffuseness etc. and possibly unknown ones also by its very construction. We would like to point out that the shell effect/energy depends upon the mean field one assigns to a nucleus. Since the nature of the mean field varies from region to region, its uniqueness is not global. In case of $\eta$, the INM sphere which constitutes the bulk part of the total energy is made up of the infinite nuclear matter for all nuclei giving rise to its uniquely defined status. Therefore it is expected that, if nuclear landscape contains any specific local feature, it should be reflected in $\eta$ systematics.

Before we present our results, the general features of $\eta$ distributions as function of neutron and proton number are to be discussed. As shown by Nayak[12] and also will be shown here aposteriori, the plot of the values of $\eta(N,Z)$ as a function of $N$ for a given $Z$ called the $\eta$ isoline, shows a Gaussian structure with the peak lying at a magic number. The Gaussian peak on either side is flanked by the U-shape distribution corresponding to the isotopes of
well deformed nuclei lying in the mid-shell region. The Gaussian peak gradually widens as one moves away from the magic number and finally disappears showing the shell-quenching effect. The U shapes also flatten up showing monotonic variation which implies uniform shell structure and disappearance of nuclear magicity. The width of the Gaussian peak may be taken as a measure of the degree of magicity/shell closure. Thus it has been shown by Nayak recently that $\eta$ carries strong signature of nuclear shell structure.

Another physical quantity namely the two-neutron separation energy $S_{2n}$ is known[13] to carry the signature of shell closure in the valley of stability. The $S_{2n}$ isolines show characteristic sharp bending just above and below the magic neutron number. Away from the magic number, these lines show only monotonic variations. It is interesting that $S_{2n}$ shell closure bending and $\eta$ Gaussian peak occur at the same magic number. Since, away from the magic number $S_{2n}$ shows only monotonic variation, while $\eta$ distribution shows U shape structure which sensitively varies with N and Z, $\eta$ distribution has emerged as a prominent and decisive signature of shell structure.

Here we have made a thorough search of the $\eta$ and $S_{2n}$ systematics in our mass predictions and obtained evidence of new shell closures and new islands of stability. The results are presented in Figs.1-4 for even values of Z only to avoid clumsiness. The isolines for the intervening odd-Z ones lie between the neighbouring even-Z ones at appropriate places. It may be noted that the specific elements and the range of their isotopes chosen for presentation here enables one to see in one sweep how the magicity evolves as one moves from the valley of stability to the drip-line. In Fig.1, $S_{2n}$ isolines for all the elements with even charge numbers $Z=50 \sim 62$ are plotted for neutron numbers $N=60 \sim 110$. In Fig.2, $\eta$ distributions are shown for the same isotopes. This domain includes the well known magic numbers $N=82$ and $Z=50$, and their neighbourhoods for which the behaviour of $\eta$ and $S_{2n}$ distributions can be seen to be in accord with the discussions above. The characteristic features of $\eta$ Gaussians peaking around the magic numbers $Z=50$, $N=82$ and their gradual widening as one moves away is clearly evident. This peak is flanked on either side by U shape distribution. The sharp shell closure bending in $S_{2n}$ distribution at $N=82$ in Fig.1 clearly correlates with the
peak in the $\eta$ distribution in Fig.2. It is interesting to find in Fig.2 that the $\eta$ distributions for $Z=58, 60$ and 62 show clear Gaussian peaks around $N=100$. The corresponding $S_{2n}$ isolines in Fig.1 show shell-closure type bending for these three elements at the same neutron number 100, though not that prominently as at $N=82$, but nevertheless quite conspicuously. Hence we take neutron number 100 as a shell closure. As for the proton, none of the three Gaussians are strongly peaked to qualify for a good shell closure, however they represent somewhat weak magicity which may be reminiscent of their proximity to $Z=64$ shell closure seen in the valley of stability. Hence we identify a new island of stability around $N=100$, $Z\simeq 62$. It is interesting to observe in Fig.2 how the magicity of the elements $Z=50$ varies with neutron number starting from $N=60$ to $N=110$ manifesting its strongest magicity for $N=82$. However in the drip-line regions it does not remain a valid magic number. This is the manifestation of shell quenching.

In Figs.3 and 4, the $S_{2n}$ and $\eta$ isolines are depicted for the domain $Z=78 \sim 90$ and $N=100 \simeq 170$. This domain includes the well known magic numbers $Z=82$ and $N=126$ around which clear Gaussian distribution in Fig.4, and sharp bending in Fig.3 are seen as expected. It is indeed pleasing to find two more Gaussian distributions around $N=152$ and $N=164$. The $\eta$ isolines for the four elements $Z=84, 82, 80, 78$ in Fig.4 show well defined Gaussian peaks with increasing sharpness and heights around $N=152$ as one moves down from 84 to 78. It is interesting to note that the quality of peak for $Z=78$ around $N=152$ is as good and even better than the peak for $Z=82$ around $N=126$ in the valley of stability. The $S_{2n}$ isolines in Fig.3 for these four elements show shell-closure type bending at $N=152$. Hence without reservations we assign magic number to $N=152$ and $Z=78$ and identify a new island of stability around them.

It is interesting to find in Fig.4 a Gaussian peak for the element $Z=90$ lying in the fringe of the drip region around $N=164$. Although it is of small height, it is quite conspicuous. We find the element $Z=88, 89$ exhibiting peak structure also around the same neutron number($Z=99$ case not shown in the Fig.). The $S_{2n}$ isolines in Fig.3 show clear shell-closure type bending for $Z=90, 88$ at $N=164$. The neutron number 164 has been anticipated before
to be a magic number on the basis of theoretical studies [14,15]. The present study which can be considered as an empirical one supports it. We will consider 90 as a poor proton magic number. Thus a new island of stability around N= 164 and Z≃ 90 is evident.

It may be observed in Fig.4 how the the proton number Z= 82 exhibits strongest magicity for neutron number N= 126 in the valley of stability, and looses this property in the drip-line region. On the otherhand, the element Z= 78 which does not show magicity in the valley of stability, emerges as a strong magic number in the drip line region. Thus the classical magic numbers no longer retain their magicity in the drip-line regions, and are replaced by new magic numbers.

In conclusion the present study provides strong evidence for new neutron magic numbers 100, 152 and 164, proton magic number 78 and new islands of stability around N= 100, Z≃ 62; N= 152, Z= 78, and N= 164, Z≃ 90 in the drip-line regions. This has been possible due to the uniqueness of the local energy η in the INM model and its long range extrapolation property. The classical magic numbers do not remain valid in the exotic drip-line regions where new magic numbers make their appearance. The study gives the empirical evidence of the preponderance of the individualistic properties like shell, deformation and diffuseness etc. in the drip-line regions governing the nuclear property in a decisive manner.

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FIG. 1. The $S_{2n}$ isolines obtained in the INM model for 7 elements as function of neutron number $N$ for a series of isotopes. The vertical lines represent the neutron magic numbers 82, and 100.
FIG. 2. Same as Fig 1 but for η isolines.
FIG. 3. The $S_{2n}$ isolines obtained in the INM model for 7 elements as function of neutron number $N$ for a series of isotopes. The vertical lines represent the neutron magic numbers 126, 152, and 164.
FIG. 4. Same as Fig 3 but for $\eta$ isolines.