Quark propagator, instantons and gluon propagator

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Abstract

The Schwinger-Dyson formalism is used to check the consistency of instanton model solutions for the quark propagator with recent models of confining gluon propagators. We find that the models are not consistent. A major discrepancy is the absence of a vector condensate in the instanton model that is present in the solutions with nonperturbative confining gluons.

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1 Introduction

The quark propagator contains valuable information about nonperturbative quantum chromodynamics (NPQCD). In the Dyson-Schwinger formalism for QCD the integral equation for the quark propagator involves the dressed gluon propagator and quark-gluon vertex function, which leads to a set of coupled integral equations. Recently this formalism has been used to study the quark and gluon propagators. By using theoretical arguments to express the quark-gluon vertex in terms of the quark propagator \[1, 2\] or other approximations or models for the dressed vertex function it has been possible to constrain the nonperturbative gluon propagator by the quark condensates. The quark condensates, which arise from chiral symmetry breaking \[3\], have been determined by extensive fits to hadronic properties in the QCD sum rule method \[4\]. It was recently shown that space-time structure of the condensates \[5\] as well as the mixed quark condensate \[6\] can be consistently determined in the Dyson-Schwinger form with a confining gluon propagator.

The nonperturbative quark propagator has also been modeled by treating quarks propagating in the instanton medium. Instanton models were used to attempt to understand the various condensates from the very early days of the QCD sum rule method \[4\]. Based on the known instanton zero modes \[7\], an instanton medium picture was shown \[8\] to be consistent with the known quark and gluon condensates. Subsequently, the propagation of light quarks in the instanton medium was formulated quantitatively \[9\] and refined in its treatment of non-zero-mode propagation by Pobylitsa \[10\]. Although it does not give rise to a long-range confining force between quarks, the instanton vacuum has been shown to provide a good phenomenological description of many hadronic properties \[11\], as do QCD sum rules without instantons. Instanton effects have been explicitly included in the nucleon correlator in QCD sum rule calculations of the nucleon mass \[12, 13\] and the nucleon magnetic moments \[14\].

In the present work we use the forms for the nonperturbative quark propagator derived in the instanton models in the Dyson-Schwinger formalism and check the consistency of explicit gluonic compared to the instanton model treatments of NPQCD. We avoid the discussion of including instanton along with other nonperturbative QCD effects, such as condensates and Goldstone bosons, without double-counting, by simply seeing if the instanton model for the quark propagator gives a consistent solution if one uses the dressed
gluon propagators found in the previous Dyson-Schwinger models within the Dyson-Schwinger framework.

2 Dyson-Schwinger Form for Quark Propagator

The quark propagator is defined by

\[ S_q(x) = \langle 0 | T[q(x)\bar{q}(0)]|0 \rangle, \tag{1} \]

where \( q(x) \) is the quark field and \( T \) the time-ordering operator. The quark self-energy, \( \Sigma \), is defined by

\[ S_q(p)^{-1} = i\hat{p} + m_q + \Sigma(p), \tag{2} \]

with \( m_q \) the current quark mass and the notation \( \hat{p} = \sum_\alpha \gamma_\alpha p^\alpha \), where the \( \gamma_\alpha \) are Dirac matrices.

Starting with the QCD Lagrangian,

\[ L(x) = \bar{q}_f(x)(i\hat{D} - m_f)q_f(x) + L_{\text{glue}}(x), \tag{3} \]

with \( D_\alpha = \partial_\alpha - ig_s A_\alpha(x) \) and \( A(x) \) is the QCD color field, one can derive the Dyson-Schwinger equation for the quark propagator \[15\]. The self-energy of the quark is given by

\[ \Sigma(p) = \int \frac{d^4q}{(2\pi)^4} g_s^2 D_{\mu\nu}^{ab}(p - q) \gamma_\mu \frac{\lambda^a}{2} S_q(q)\Gamma_\nu^b(q,p), \tag{4} \]

with \( D_{\mu\nu}^{ab}(q) \) the gluon propagator, \( \lambda^a \) the color \( SU(3) \) matrix and \( \Gamma_\nu^b(q,p) \) the quark-gluon vertex. In the present work we use the approximation \( \Gamma_\nu^b(q,p) = \gamma_\nu \frac{\lambda^b}{2} \), the so-called “rainbow approximation”, to be consistent with Ref[5]. It is quite straightforward for us to go beyond the rainbow approximation.

Eq. (2) expresses the inverse quark propagator as a perturbative and a non-perturbative part. Alternatively one can write the propagator as

\[ S_q(x) = S_q^{PT}(x) + S_q^{NP}(x). \tag{5} \]
For short distances, the operator product expansion for the scalar part of $S_{NP}^q(x)$ gives

$$S_{NP}^q(x) \simeq \langle \bar{q}(x)q(0) : >$$

$$= \langle \bar{q}(0)q(0) : > - \frac{x^2}{4} \langle \bar{q}(0)\sigma \cdot G(0)q(0) : 0 > + \ldots ,$$

in which the local operators of the expansion are the quark condensate, the mixed condensate, and so forth. $G_{\mu\nu}^c$ is the field tensor associated with the color field.

We use the Feynman-like gauge and write the gluon propagator as

$$D_{\mu\nu}^{ab}(q) = \delta^{ab}\delta_{\mu\nu}D(q).$$

In the present work we use the results of Ref.[5] for our model of the function $D(q)$.

An important observation is that the inverse quark propagator can be written in Euclidean space as

$$S_q(p)^{-1} = i\hat{p}A(p^2) + B(p^2)$$

Except for the current quark mass and perturbative corrections, the functions $A(p^2)$ and $B(p^2)$ are nonperturbative quantities which we refer to as the vector and scalar propagator condensates, respectively. The Dyson-Schwinger equations (in the Feynman gauge) for $A$ and $B$ are the coupled set

$$[A(p^2) - 1]p^2 = \frac{8}{3}g_s^2 \int \frac{d^4q}{(2\pi)^4} D(p - q) \frac{A(q^2)}{q^2A^2(q^2) + B^2(q^2)}p \cdot q$$

$$B(p^2) = \frac{16}{3}g_s^2 \int \frac{d^4q}{(2\pi)^4} D(p - q) \frac{B(q^2)}{q^2A^2(q^2) + B^2(q^2)}.$$

The quark and mixed quark condensates are given by

$$\langle \bar{q}(0)q(0) : > = -\frac{3}{4\pi^2} \int ds sB(s)$$

$$\langle 0 | \bar{q}(0)g\sigma \cdot G(0)q(0) : 0 > = \frac{9}{4\pi^2} \int ds [s \frac{B(s)(2 - A(s))}{sA^2(s) + B^2(s)} +$$

$$81B(s)[2sA(s)(A(s) - 1) + B^2(s)]}.$$
These equations are all obtained in the rainbow approximation. The expression for the mixed quark condensate was derived in a $1/N_c$ expansion of the gluon two-point function, making use of the self-consistency relations. In Ref. the effective quark mass had reached the current quark mass by 1 GeV, which was taken as the renormalization point and cutoff for the condensate integrals. We do not use such a cutoff, consistent with Ref. [10]. We use the notation $a = -(2\pi)^2 < \bar{q}(0)q(0) : >$ for the quark condensate and $m_o^2a = (2\pi)^2 < 0| \bar{q}(0)g\sigma \cdot G(0)q(0) : |0>$ for the mixed quark condensate. The values of these condensates have been determined by fits to experiment in QCD sum rule calculations to be $a \simeq 0.55 \text{ GeV}^3$ and $m_o^2 \simeq 0.8 \text{ GeV}^2$, or $m_o^2a \simeq 0.44 \text{ GeV}^5$.

The coupled integral equations Eq.(9) have been solved for with various forms and the parameters fit to the quark condensate. It was shown in Ref. that this gave a satisfactory result for the mixed quark condensate and in Ref. that the nonlocal scale of the nonperturbative quark propagator as determined from fits to deep inelastic scattering was also fit satisfactorily. The range of the confining infra-red gluon propagator, which is taken as a Gaussian in this work, is very short. In other calculations the confining part of the gluon propagator had a longer range, but the fit to the quark condensate was not very good. We also use these models for $D(s)$ in our present study.

3 Instanton Model for Quark Propagator

In the instanton calculation of Ref. the inverse propagator is of the form (for the current quark mass $m_q = 0$)

$$ S_q(p)^{-1} = i\hat{p} + B_1(p^2) $$

$$ B_1(p) = B_0^I + O(N/VN_c). \quad (11) $$

Using the leading term of a series expansion, the author found the following instanton solution for the inverse quark propagator, Eq.(8):

$$ A_I(p) - 1.0 = 0.0 $$

$$ B_1(p) = Kp^2f^2(\frac{5}{6}p) $$

$$ f(p) = \frac{2}{p} - (3I_o(p) + I_2(p)) \times K_1(p), \quad (12) $$

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where $K = 0.29$ GeV$^{-1}$, and $p$ is in units of GeV. We refer to Eq. (12) as the Pobylitsa solution.

An important feature is that the vector propagator condensate vanishes: $A(s) - 1 = 0$. This follows from the symmetries of the model used by Pobylitsa.

This solution gives an effective quark mass which falls as $1/p^6$ for $p$ larger than a few hundred MeV. Other treatments of the effective quark mass show different behavior. For example a light-cone model for the pion form factor [18] gives $B(s)$ as almost constant and equal to the constituent quark mass for $p$ less that about 1 GeV and dropping rapidly to zero at about 1 GeV. In this model $A \approx 1.0$. Let us consider such a model, which we call the lc model:

$$A_{lc}(p) - 1.0 = 0.0$$
$$B_{lc} = \frac{M_N}{3} \text{ for } p^2 < s_o$$
$$B_{lc} = 0.0 \text{ for } p^2 > s_o,$$

where $s_o$ is a constant to be determined. This lc model can be considered as a simple alternative to the Pobylitsa model in the sense that the vector propagator condensate vanishes.

The mixed quark condensate for the instanton model of Ref [9] was estimated [19] to be $m_0^2 = 1.4$ GeV$^2$, about a factor of two larger than the empirical QCD sum rule value.

### 4 Results and discussion

We use for the form of the gluon propagator [16] $D(s)$

$$g_s^2 D(s) = 3\pi^2 X^2 e^{-s/\Delta} + c_u \frac{4\pi^2 d}{sln(\frac{s}{X} + c)},$$

with the parameter $c_u = (1.0,0.0)$ to (include,neglect) the perturbative ultraviolet behavior. The strength parameter $X$ and range parameter $\Delta$ are determined by solving the coupled Dyson-Schwinger equations, Eq.(9), and fitting $f_\pi$, the pion decay constant, and the quark condensate through Eq.(10). For the Feynman gauge with $c_u = 0.0$ these parameters are $[5] X = 1.4$ GeV and $\Delta = 2.0 \times 10^{-3}$ GeV$^2$. 
For the $lc$ model, using $a = -(2\pi)^2 <: \bar{q}(0)q(0): > = .55 \text{ GeV}^3$, we find $s_o = 0.8 \text{ GeV}^2$

$$m_o^2a = 0.831 \text{ GeV}^5$$

$$B(0) = 4.64 \text{ GeV}$$

$$A(0) - 1.0 = 7.28 \quad (15)$$

This solution clearly gives inconsistent values for $A$ and $B$ and the mixed gluon condensate is about a factor of 2 larger than that generally found in QCD sum rules. Note that this value for the mixed condensate is about the same as that obtained $[19]$ with an instanton model $[9]$.

For the instanton model, Eq.(12), the value of $K$ found by Pobylitsa gives the value of $B_I(0) \simeq 420$ MeV, rather than the constituent quark mass. By modifying the factor of $K$ in Eq.(12) by a factor of $M_N/(3 \times .420)$ so that $B_I(0) = M_N/3$, we obtain a value of quark condensate in agreement with the phenomenological one, and find for the instanton model

$$a = 0.60\text{GeV}^3$$

$$m_o^2a = 2.04 \text{ GeV}^5$$

$$B(0) = 4.74 \text{ GeV}$$

$$A(0) - 1.0 = 7.71 \quad (16)$$

Therefore we find that the $lc$ and Pobylitsa solutions give similar results for the Dyson-Schwinger formalism for the quark propagator. Although the value for the quark condensate agrees with the empirical one, the mixed condensate is almost a factor of 5 larger than the empirical value of $.44 \text{ GeV}^5$, and about 2.6 times larger than the estimate of Ref. $[19]$.

The range of the infra-red part of the gluon propagator is very small in these calculations in which the gluon propagator is constrained by the phenomenological value of the quark condensate, $a = .55 \text{ GeV}^3$. If we relax this constraint we can use the models of Ref. $[16]$ in which longer-range infra-red parts of $D(s)$ were used. The results for the Pobylitsa model for $X=1.53 \text{ GeV}$ and $\Delta = 0.02 \text{ GeV}^2$ with $c_u = 1.0$ and $\Lambda = 0.2 \text{ GeV}$ are

$$B(0) = 5.92 \text{ GeV}$$

$$A(0) - 1.0 = 9.89 \quad (17)$$
and with $X = 1.65$ GeV and $\Delta = 0.2$ GeV$^2$

\[
B(0) = 1.83 \text{ GeV} \\
A(0) - 1.0 = 3.14. \tag{18}
\]

Recall that for all of these quark propagator solutions the value of $B(0)$ is the value of the constituent quark mass, taken as .313 GeV. From Eq. (18) we note that for the longest range gluonic confining propagator, with a range of about 0.8 GeV, the instanton solution is most nearly consistent, but the value of the quark condensate is quite small in that model [16].

Our results show that the instanton model is not consistent with the Dyson-Schwinger model based on the rainbow approximation using a confining gluon propagator chosen to fit the condensates and other hadronic properties. One major difference between the NPQCD calculation solving the model with a confining gluon propagator and the simple substitution of the instanton model of the quark propagator Ref. [10] in the Dyson-Schwinger integrals is the vanishing of the vector propagator condensate in the instanton model. In future work we plan to investigate the inclusion of instanton effects along with modified confining gluon propagators.

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References

[1] C.D. Roberts and A.G. Williams, Prog. Part. Nucl. Phys. 33 (1994) 477, and references therein.

[2] P. Tandy, Prog. Part. Nucl. Phys. 39 (1997) 117, and references therein.

[3] M. Gell-Mann, R.J. Oakes and B. Renner, Phys.Rev. 175, (1968) 2195).

[4] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl.Phys. B 147 (1979) 385; 448.

[5] L.S. Kisslinger and T. Meissner, Phys. Rev. C57 (1998) 1528.

[6]  T. Meissner, Phys.Lett. B 405,(1997) 8.

[7] G. ’t Hooft, Phys. Rev. Lett 37 (1976) 8; Phys. Rev. D14 (1976) 3432.
[8] E.V. Shuryak, Nucl. Phys. B203 (1982) 93, 116, 140; Phys. Reports 115 (1985) 152.

[9] D.I. Dyakonov and V.Yu Petrov, Nucl. Phys. B245 (1984) 259; B272 (1986) 457; Sov. Phys. JETP 62 (1985) 204.

[10] P.V. Pobylitsa, Phys. Lett. B226 (1989) 387.

[11] T. Schafer and E.V. Shuryak, Rev. Mod. Phys. 70 (1998) 323.

[12] A.E. Dorokhov and N.I. Kochelev, Z. Phys. C46 (1990) 281.

[13] H. Forkel and M.K. Banerjee, Phys. Rev. Lett. 71 (1993) 484.

[14] M. Aw, M.K. Banerjee and H. Forkel, hep-ph/9902458, to be published in Phys. Lett. B (1999).

[15] C. Itzykson and J-B. Zuber, "Quantum Field Theory" (McGraw-Hill Book Co., 1985).

[16] M.R. Frank and T. Meissner, Phys. Rev. C53 (1996) 2410.

[17] H. Jung and L.S. Kisslinger, Nucl.Phys. A 586, 682 (1995).

[18] L.S Kisslinger and S.W. Wang, Nucl. Phys. B399 (1993) 63.

[19] M.V. Polyakov and C. Weiss, Phys. Lett. B387 (1996) 841.