Characterizing the Peak in the Cosmic Microwave Background Angular Power Spectrum

Lloyd Knox\textsuperscript{1} and Lyman Page\textsuperscript{2}

\textsuperscript{1} Department of Astronomy and Astrophysics
University of Chicago, 5640 S. Ellis Ave., Chicago, IL 60637, USA
\textsuperscript{2} Department of Physics
Princeton University, Princeton, NJ, USA

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A peak has been unambiguously detected in the cosmic microwave background (CMB) angular spectrum. Here we characterize its properties with fits to phenomenological models. We find that the TOCO and BOOM/NA data determine the peak location to be in the range 175–243 and 151–259 respectively (both ranges 95\% confidence) and determine the peak amplitude to be between \(\approx 70\) and 90 \(\mu\)K. By combining all the data, we constrain the full-width at half-maximum to be between 180 and 250 at 95\% confidence. Such a peak shape is consistent with inflation-inspired flat, cold dark matter plus cosmological constant models of structure formation with adiabatic, nearly scale-invariant initial conditions. It is inconsistent with open and defect models.

\textit{Introduction.} If the adiabatic cold dark matter (CDM) models with scale-invariant initial conditions describe our cosmogony, then an analysis of the anisotropy in the CMB can reveal the cosmological parameters to unprecedented accuracy \cite{1}. A number of studies have aimed at determining, with various prior assumptions, a subset of the \(\sim 10\) free parameters that affect the statistical properties of the CMB \cite{2}. The parameter most robustly determined from current data is \(\Omega\), the ratio of the mean matter/energy density to the critical density (that for which the mean spatial curvature is zero). These investigations show that \(\Omega\) is close to one. This result, combined with other cosmological data, implies the existence of some smoothly distributed energy component with negative pressure such as a cosmological constant.

A weakness of previous approaches \cite{3,4} is that the conclusions depend on the validity of the assumed model. In this Letter we take a different tack and ask what we know independent of the details of the cosmological model. We find the peak location, amplitude and width are consistent with those expected in adiabatic CDM models. Furthermore, as \(l_{\text{peak}} \approx 200 f^{-1/2}\) in these models, the observed peak location implies \(\Omega \approx 1\). The determination of the peak location is robust; it does not depend on the parametrization of the spectrum, assumptions about the distribution of the power spectrum measurement errors, nor on the validity of any one data set. The model-dependent determinations of \(\Omega\) are further supported by the inconsistency of the data with competing models, such as topological defects, open models with \(\Omega < 0.4\), or the simplest isocurvature models.

\textit{The Data.} The last year of the 1000’s saw new results from MSAM \cite{5}, PythonV \cite{6}, MAT/TOCO \cite{7}, Viper \cite{8}, CAT \cite{9}, IAC \cite{10} and BOOM/NA \cite{11}, all of which have bearing on the properties of the peak. These results are plotted in Fig. 1. We have known for several years that there is a rise toward towards \(l = 200\) but it is now clear that the spectrum also falls significantly towards \(l = 400\).

For all the medium angular scale experiments, the largest systematic effect is the calibration error which is roughly 10\% for each. Contamination from foreground emission is also important and not yet fully accounted for in some experiments (\textit{e.g.} TOCO). A correction for this contribution, for which \(\delta T_l \sim l^{-1/2}\), will affect the amplitude of the peak though will not strongly affect its position. Thorough analyses by the MSAM \cite{12} and PYTHON \cite{6} teams show that the level of contamination in those experiments was < 3\%.

The three experiments that have taken data that span the peak are MSAM, TOCO, and BOOM/NA. All experiments exhibit a definite increase over the Sachs-Wolfe plateau though the significance of a feature based on the data alone, \textit{e.g.} a peak, differs between experiments. We may assess the detection of a feature by examining the deviation from the best fit flat line, \(\delta T\). For the three MSAM points, we find \(\delta T = 46 \pm 4.9\) \(\mu\)K with a reduced \(\chi^2\) of 0.43 (Probability to exceed, \(P_{>\chi^2} = 0.65\)). The calibration error is not included.). Thus, no feature is detected with these data alone though there is a clear increase over DMR \cite{13}. For the seven BOOM/NA points, we find \(\delta T = 55.3 \pm 4.2\) \(\mu\)K with a reduced \(\chi^2\) of 1.94 (\(P_{>\chi^2} = 0.05\), assuming the data are anti-correlated at the 0.1 level \cite{14}). For the ten TOCO points, \(\delta T = 69.3 \pm 2.7\) \(\mu\)K with a reduced \(\chi^2\) of 4.86 (\(P_{>\chi^2} < 10^{-5}\)) Calibration errors will not change \(\chi^2/\nu\), however a correction for foreground emission will have a slight effect. Though we examine all data in the following, we focus particularly on BOOM/NA and TOCO because of their detections of a feature.

\textit{Fits to Phenomenological Models.} To characterize the peak amplitude and location we fit the parameters of two different phenomenological models. For the first, we start with the best fit DK99 \cite{3} adiabatic CDM model, \(\delta T_{l_{\text{DK}}}\),
and form $\delta T_l = (\delta T^{DK}_l - \delta T^{DK}_0) \alpha + \delta T^{DK}_0$ by varying $\alpha$, and then stretching in $l \geq 10$. We characterize each stretching with the peak position and peak amplitude. This method has the virtue that the resulting spectra resemble adiabatic models and so if one assumes that these models describe Nature, then these results are the ones to which we should pay the most attention.

![Graph](image)

**Fig. 1.** Bandpowers from TOCO97 (cyan open triangles), TOCO98 (blue filled triangles), BOOM/NA (green filled squares), MSAM (red open squares), CAT (black open pentagon), IAC (black filled pentagon), PyV (black filled squares), MSAM (red open squares), CAT (black filled squares), TOCO98 (blue filled triangles), BOOM/NA (green filled circles) and Viper (green filled circles). The y-axis is the angular spectrum. The models are, peaking at left to right, the best fit models of $\Omega = 1$, $\Omega = 0.4$ and $\Omega = 0.2$. The $\Omega = 1$ model has a mean density of non-relativistic matter, $\Omega_m = 0.31$, a cosmological constant density of $\Omega_{\Lambda} = 0.69$, a baryon density of $\Omega_b = 0.019 h^{-2}$, a Hubble constant of $H_0 = 65 \text{ km/sec/Mpc}$, an optical depth to reionization of $\tau = 0.17$ and a power spectrum power-law index of $n = 1.12$, where $n = 1$ is scale-invariant. The shaded areas are the results of fitting the power in 14 bands of $l$ to all the data (from 1999 and previous years) as in [16]. Many of the bands are at low $l$ and cannot be discerned on this plot. Calibration errors are not shown though are included in the best fit.

Our second model for $\delta T^2_l$ is a Gaussian: $\delta T^2_l = A^2 \exp \left( - (l-l_c)^2 / (2\sigma^2) \right)$. Depending on the width, this spectrum can look very much like, or unlike, the spectra of adiabatic models. We view this versatility as a virtue since we are interested in a characterization of the peak which is independent of physical models.

We fit to these phenomenological models in two ways. For the stretch model, we examine the $\chi^2$ of the residuals between the published data and each model. The widths of the window functions are ignored and we assume the data are normally distributed in $\delta T_l$ with a dispersion given by the average of the published error bars (GT in Table 1). This is an admittedly crude method but it works well because the likelihoods as a function of $\delta T_l$ are moderately well approximated by a normal distribution.

For both the Gaussian shape and the stretch model, we also perform the full fit as outlined in BJK [16] (RAD in Table 1). For the Gaussian shape model, the constraints on the amplitude and location are given below after marginalization over the width $\sigma_l$. In all fitting, we ignore the experiments that are affected by $l < 30$ (DMR, FIRS [18] and Tenerife [19]) because we want the parameters of our Gaussian to be determined by behavior in the peak region.

| Data Model | Fit | $N/\nu$ | $\chi^2/\nu$ | $P_{>\chi^2}$ | $l_{peak}$ | $\delta T_{peak}$ |
|------------|-----|---------|--------------|---------------|----------------|----------------|
| All        | G   | Rad     | 58/55        | 1.25          | 0.10           | 229 ± 8.5      | 78 |
| T          | G   | Rad     | 10/7         | 0.41          | 0.89           | 206 ± 16       | 95 |
| T          | S   | GT      | 10/8         | 0.94          | 0.48           | 214 ± 14       | 88 |
| T          | S   | Rad     | 10/9         | 0.84          | 0.57           | 209 ± 17       | 92 |
| T          | G   | Rad     | 7/4          | 0.19          | 0.94           | 208 ± 21       | 69 |
| B          | G   | Rad     | 7/5          | 0.39          | 0.85           | 215 ± 24       | 69 |
| B          | S   | GT      | 7/5          | 0.23          | 0.95           | 205 ± 27       | 72 |
| B          | S   | Rad$_0$ | 7/5          | 0.39          | 0.85           | 206 ± 26       | 68 |
| P          | G   | Rad     | 33/30        | 1.13          | 0.28           | 262 ± 24       | 68 |

* ALL stands for all publically available data sets (except for VIPER which was not used because of unspecified point-to-point correlations), the T is for the TOCO data, the B for BOOM/NA and the P is for “Previous”, meaning all data prior to BOOM/NA and TOCO. $^G$ and S are for the Gaussian shape and stretch methods respectively. $^N$ is the number of data points and $\nu$ the degrees of freedom. $^\nu$ corresponds to log normal and normal distributions for the likelihood respectively.

The main thing to notice in the Table is that the position of the peak is robustly determined by either TOCO or BOOM/NA to be in the range 185 to 235, regardless of the method. For the quoted errors, we have marginalized over all parameters except the position. The peak amplitudes are subject to change as there is some dependence on the model parametrization and the foreground contamination has not been thoroughly assessed.

We account for the calibration uncertainty through a convolution of the likelihood of the fits with a normal distribution of the fractional error [20,16]. BOOM/NA, TOCO97 and TOCO98 have calibration uncertainties of 8%, 10.5% and 8% respectively. However, 5% of this is due to uncertainty in the temperature of Jupiter and therefore, assuming that these uncertainties add in quadrature, we get $\sigma_{\text{Jup}} = 0.05$, $\sigma_{\text{T97}} = 0.092$, $\sigma_{\text{T98}} = 0.062$ and $\sigma_{\text{T97}} = 0.062$. We then find, for TOCO, that the full likelihood in $\delta T_l$ and $l$ is given by...
$L(l_c, \delta T_l) = \int d\alpha_1 d\alpha_2 d\alpha_3 d\alpha_4 L_T \alpha_1 \delta T_1 \alpha_2 \delta T_2 \alpha_3 \delta T_3 \alpha_4 \delta T_4$

where $P_G(x; \sigma) = \exp \left(-x^2/(2\sigma^2)\right)/\sqrt{2\pi}\sigma^2$, $u$ is integrated from 0 to $\infty$ and, e.g., $L_T \alpha_1 \delta T_1 \sigma_1 = \exp(-\chi^2/2)$ where $\chi^2$ is evaluated on a grid of $\delta T_i^2$, $l_c$ & $\sigma_1$ using RAPACK \[2\] as discussed in BJK. We get similar results for TOCO when using a combined total calibration error of 8.5%.

For the Gaussian model we can also marginalize over $A$ and $l_c$ to place 95% confidence bounds on the width: $75 < \sigma_1 < 105$ for ALL, $50 < \sigma_1 < 105$ for TOCO and $55 < \sigma_1 < 145$ for BOOM/NA.

FIG. 2. Likelihood contours for $l_0$ vs $\delta T_0$ for the position of the peak. For BOOM/NA and TOCO, we use the stretch method using RAPACK \[2\] and include the calibration error. For Previous and ALL (tightest contours) we have not used generalizations of Eq. \[1\], but instead have fixed the calibration parameters. All contour levels correspond to 5%, 68%, and 95% enclosed, or roughly the peak, 1$\sigma$, 2$\sigma$.

Are the data in Fig 1 consistent? DK99 found that the best-fit model, given all the data at the time, had a $\chi^2$ of 79 for 63 degrees of freedom, which is exceeded 8% of the time. Here we see that the $\chi^2$ for the fit of the Gaussian model is 69 for 55 degrees of freedom, which is exceeded 10% of the time. We conclude that, although there may well be systematic error in some of these data sets, we have no compelling evidence of it. However, we take caution from the fact that we had to adjust the calibration parameters from their nominal values to their best-fit values in order to reduce the $\chi^2$ to 69. Left at their nominal values with calibration uncertainty ignored, the data are not consistent with each other. Thus we believe that the

compilation results are perhaps less reliable than those for either BOOM/NA or TOCO.

Implications for Physical Models. Flat, adiabatic, nearly scale-invariant models have similar peak properties to those of our best-fit phenomenological models. Most importantly the peak location, as determined by three independent data sets ("Previous", TOCO, BOOM/NA), is near $l \approx 210$, as expected. Depending on the data set chosen, the amplitude is higher than expected but can easily be accommodated, within the uncertainties, with a cosmological constant. Combining all the data, there is a preference for $l_{\text{peak}} > 210$ which suggests a cosmological constant \[22\] (at $\Omega = 0.65$, $l_{\text{peak}}$ goes from 200 at $\Omega = 0$ to 220 at $\Omega = 0.7$). However, this result is not seen in any individual data set.

A good approximation to the first peak in the DK99 best-fit model is given by the Gaussian model with $\sigma_l = 95$. From the $\sigma_l$ constraints quoted earlier we see that the data have no significant preference for peaks that are either narrower or broader than those in inflation-inspired CDM models.

A general perturbation is a combination of adiabatic and isocurvature perturbations. Adiabatic perturbations are such that at each point in space, the fractional fluctuations in the number density of each particle species is the same for all species. Isocurvature perturbations are initially arranged so that, despite fluctuations in individual species, the total energy density fluctuation is zero. Given multiple components, there are a number of different ways of maintaining the isocurvature condition. Below we assume the isocurvature condition is maintained by the dark matter compensating everything else.

Isocurvature initial conditions result in shifts to the CMB power spectrum peak locations. For a given wavenumber, the temporal phase of oscillations in the baryon-photon fluid depends on the initial relation between dark matter and the fluid. Those waves with oscillation frequencies such that they hit an extremum at the time of last-scattering in the adiabatic case, will hit a null in the isocurvature case \[23\]. The effect on the first peak is a shift from $l \approx 200\Omega^{-1/2}$ to $l \approx 350\Omega^{-1/2}$. Given the observation of $l_{\text{peak}} \approx 210$, simple isocurvature models require $\Omega > 2$--which is inconsistent with a number of observations \[24\].

Critical to the Doppler peak structure, in either adiabatic or isocurvature models, is the temporal phase coherence for Fourier modes of a given wavenumber \[24\]. In topological defect models, the continual generation of new perturbations by the non-linear evolution of the defect network destroys this temporal phase coherence and the acoustic peaks blend into a broad hump which is wider and peaks at higher $l$ than the observed feature.

One can make defect model power spectra with less power at $l = 400$ than at $l = 200$ with ad-hoc modifications to the standard ionization history \[25\]. But even for these models the drop is probably not fast enough \[27\].
The contrast between the power at $l = 200$ and $l = 400$ is a great challenge for these models.

There are scenarios with initially isocurvature conditions that can produce CMB power spectra that look much like those in the adiabatic case. This can be done by adding to the adiabatic fluctuations (of photons, neutrinos, baryons and cold dark mater) another component, with a non-trivial stress history, which maintains the isocurvature condition $\Delta_{i} = 400 \mu_{0}$.

Conclusions. Our phenomenological models have allowed for rapid, model-independent, investigation of the consistency of CMB datasets, and of the robustness of the properties of the peak in the CMB power spectrum. The peak has been observed by two different instruments, and can be inferred from an independent compilation of other data sets. The properties of this peak are consistent with those of the first peak in the inflation-inspired adiabatic CDM models, and inconsistent with a number of competing models, with the possible exception of the more complicated isocurvature models mentioned above. It is perhaps instructive that where the confrontation between theory and observation can be done with a minimum of theoretical uncertainty, the adiabatic CDM models have been highly successful.

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