Radio signal of PSR B0950+08 is detected over the whole phase

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ABSTRACT

Pulsars, known as the “lighthouses” in the universe, are thought to emit periodic pulses with duty-cycle ∼ 10%. In this report, the 160 min-data of a nearby pulsar, PSR B0950+08, observed with the Five-hundred-meter Aperture Spherical radio Telescope (FAST) is analysed. Thanks to the extremely high sensitivity of FAST, it is found that the radiation of PSR B0950+08 could be detected over the entire pulse period. To investigate the radiative characteristics of the pulsar’s bridge emission, a function, Θ(n), is defined to reveal the weak radiation there. It is suggested that the narrow peaks of both the main and the interpulse could be radiated at low altitude, while other weak emission (e.g., the “bridges”) from high magnetosphere far away from the surface though its radiative mechanism is still a matter of debate. The measured mean pulse behaviors are consistent with previous results in the phase of strong emission, and both the frequency-independent separation between the interpulse and main pulse and the narrow pulse width may support a double-pole model. Nonetheless, in order to finalize the magnetospheric geometry, further polarization observation with FAST is surely required, which would only be believable in the phase of weak emission if the baseline is determined with certainty in the future.

Key words: pulsar – radio astronomy – emission mechanism – individual pulsar (PSR B0950+08)

1 INTRODUCTION

The nearby bright pulsar, PSR B0950+08, is well known, with a spin period of 253 ms and dispersion-measure (DM) of 2.97 pc/cm3. Abundant observations have already been obtained for this normal pulsar (e.g. Hankins et al. 1991; Kuzmin et al. 1998; Everett & Weisberg 2001; Shabanova & Shitov 2004; Johnston et al. 2005). For instance, the averaged pulse profile of PSR B0950+08 shows frequency-dependent properties (e.g. Hankins et al. 1991; Kuzmin et al. 1998), and Shabanova & Shitov (2004) reported that the frequency dependent sinusoidal modulation of the pulse shape may result from the Faraday rotation effect. Also, its linear polarization is shown in the literature (Everett & Weisberg 2001; Shabanova & Shitov 2004; Johnston et al. 2005), but is too low to fit the polarization position angle (PPA) at the main pulse longitude by typical ‘S’ shape in the Rotating Vector Model (RVM, Radhakrishnan & Cooke 1969). Popov et al. (2002) confirmed the characteristic timescales of micro-structure and narrow micro-structure are orders of 0.1 ms and 10 µs, respectively, while the timescale of fine structure is ∼ 1 ms (Ulyanov et al. 2016). Besides, the separation between interpulse and main pulse is frequency independent below 5 GHz, and this feature demonstrates that multi-pole magnetic field may contribute significantly in order to understand the emission “bridge” (Hankins & Cordes 1981; Nowakowski et al. 2002).

In addition to the mean pulse profile, the single pulse behaviors are also focused for PSR B0950+08. After a detailed study of intensity dependence of individual pulses, Nowakowski et al. (2002) found a possible correlation between the intensities and locations of the interpulse with the leading component of the main pulse. Additionally, PSR B0950+08 also exhibits the nulling pulse phenomenon and the sharp flux variation in a short-time scale (e.g. Cairns 2004; Shabanova & Shitov 2004; Singal & Vats 2012). Besides, the giant pulse (GP) and giant micro-pulse phenomena had also been detected from PSR B0950+08 (e.g. Cairns 2004; Singal & Vats 2012; Tsai et al. 2015), but the origin of them is still a mystery.

Nevertheless, both properties of the mean pulse and single pulse would be significantly influenced by the baseline determination, as the conventional way of baseline elimination can hardly be acceptable for a pulsar with unusually wide pulse like that of PSR B0950+08. Hankins & Cordes (1981) strongly support the conclusion that the radio emission from PSR B0950+08 occurs over at least 83% of the
rotation period. They pointed that, because of limited sensitivity of telescope, the radio emission characters of the two minimum low levels at the bridge regions between interpulse and main pulse (the remaining 17% pulse longitude) could not be defined with certainty.

The largest single-dish radio telescope in the world, China’s five-hundred-meter aperture spherical radio telescope (FAST), would be an appropriate tool to understand more about radio emission as well as the radiative mechanism of PSR B0950+08, taking advantage of its extremely high sensitivity (Jiang et al. 2019, 2020). New scientific achievements have already been obtained with FAST, including studies of pulsars (e.g. Lu et al. 2019) and fast radio bursts (e.g. Xu 2021). In this article, the emission from the “bridge” of PSR B0950+08 is determined (as shown in Fig. 1), exhibiting a concave structure. In this paper, a detailed study of the radiation properties of the entire 360°-pulse longitude is presented. The observations with FAST and the data reduction are described in Section 2. To determine the emission at the entire pulse period, a depth data analysis and the radiation characteristics of this pulsar are presented in Section 3. Finally, general conclusions and discussions are provided in Section 4.

2 OBSERVATION AND DATA REDUCTION

FAST locates in Gouzhou, a southwest province of China, at longitude 106.9° E and latitude 25.7° N. In this work, the 19 beam receiver system was adopted. The central frequency and bandwidth of the receiver are 1250 and 400 MHz, respectively. The system temperature of the 19 beam receiver is less than 24 K for central beam, and the system stability for most observation modes with the observation zenith angle less than 26.4°, is ~1% over 3.5 hours (Jiang et al. 2019, 2020).

In this work, PSR B0950+08 was observed with tracking mode on MJD 59306 (July 1st, 2021). The data were recorded as the 8 bit-sampled search mode PSRFITS format (Hotan et al. 2004) with 4096 frequencies channels, and the frequency resolution is ~0.122MHz. The entire integration time is 160 minutes, and the time resolution is ~50 μs. The DSPSR software package (van Straten & Bailes 2011) was adopted in the data reduction, and the individual pulses were generated with 1024 phase bins across the pulse period of 253 ms (Hobbs et al. 2004). The narrow-band radio frequency interference (RFI) had also been eliminated in the process.

3 DATA ANALYSIS AND RADIATION CHARACTERISTICS

After eliminated RFI, the narrow band with these frequencies of 1100, 1305, 1350, and 1395 MHz are reserved, and their bandwidth are 80, 50, 40, and 50 MHz, respectively. The averaged pulse profile at all these frequencies are shown Fig. 1. As shown in Fig. 1, the radio emission features across the entire pulse longitude are exhibited, and the observational properties of the two minimum low levels (the bridge components) are clearly revealed from zoomed-in view in the right panels. The averaged pulse profile of the full bandwidth is also shown in Fig. 2. Apparently, from the figure, there is a concave emission structure rather than flat platform at the two relative minimum levels, the first concave structure appears at pulse phase of 0.25, meanwhile, another concave structure is revealed at pulse phase of 0.90. This feature demonstrates a radio emission behaviour without off-pulse longitude, i.e., the pulsar radiates over the entire pulse period.

To reveal the emission characteristics of the two minimum levels, a mathematical method is proposed to analyze the emission characteristics of the entire pulse longitude. The function $\Theta(n)$ is defined as following,

$$\Theta(n) = \sum_{k=1}^{N_{\text{period}}} \left\{ 2 \times (I_{k,n-1} - I_{k,n+1})^2 - [(I_{k,n} - I_{k,n-1})^2 + (I_{k,n} - I_{k,n+1})^2] \right\},$$

where $I_{k,n}$ represents the signal intensity contributed by the k-th pulse and the n-th pulse phase bin, and $N_{\text{period}} = 36746$ is the pulse numbers. The results are shown as Fig. 3.

The function $\Theta(n)$ describes the statistics properties of the radio emission signal. The region of the on-pulse and off-pulse can be distinguished by this function, because the value of the function $\Theta(n)$ is positive at on-pulse regions and goes to zero at the off-pulse regions (see Appendix A). It should be noted that the fluctuation of the radio emission signals and noise may lead to the statistics error, and this error can possibly be eliminated if the number of the individual pulse ($N_{\text{period}}$) is big enough. This statistics properties implicates that the value of the function $\Theta(n)$ may be influenced by the distribution of the noise intensity. A detailed behaviour of $\Theta(n)$ at the entire pulse longitude is described as black curves in Fig. 3, and the detailed property of the blue region in the middle panel is zoomed in the bottom panel. From this figure, the result is obviously influenced by the noise at the relative minimum levels. To determine the emission properties of these regions, the values of $\Theta(n)$ at the minimum levels need to be smoothed. The red curve in the bottom panel is the smoothed result via low-pass filtering, and apparently, this curve at the minimum levels are greater than zero (dash-dot gray reference line). It is evident that, in the observed data, the values of the function $\Theta(n)$ at the two minimum low levels are positive. The behaviour of this function strongly illustrates that radio emission signals filling the minimum levels, and this result supports the whole phase radiation picture that the emission signals fill the entire 360° pulse longitude.

4 CONCLUSIONS AND DISCUSSIONS

In this work, it is shown that the averaged pulse profile of PSR B0950+08 is consistent with previously presented works in the strong emission regions (e.g. McLaughlin & Rankin 2004; Everett & Weisberg 2001; Johnston et al. 2005). From Figs. 1, 2 and 3, it is evident that the emission of the relatively minimum levels (from the pulse phase 0.65 to 0.95, the emission properties at this pulse longitude are undetected and usually regarded as off-pulse regions in published literature) had been detected for PSR B0950+08. With the detection of the emission at the bridge regions, the whole phase radiation characteristics of this pulsar are identified. However, there are some issues which need to be discussed.

The whole phase radiation behaviour of the pulsar is different from that of PSR B0826–34. Esamdin et al. (2005)
reported that the radiation extends through the whole phase of PSR B0826−34 in the strong emission mode. The radio emission signals of PSR B0826−34 covers the whole phase could naturally be understandable because the inclination angle, $\alpha$, is extremely low ($\sim 0.5^\circ$), as is evident from the large pulse width. Whereas, the pulse width of strong emission for PSR B0950+08 is relatively small ($\sim 31.4^\circ$), which implicates a large inclination angle. With the assumption of the radiation altitude as 1000 km (at this altitude, the multipole field is weak and the field is dominated by the dipole component), one can calculate the maximum half width of the radiation beam, $\theta_\mu$, (corresponding to the radiation from

\textbf{Figure 1.} The averaged pulse profile at 1100, 1305, 1350, and 1395 MHz are shown in the left panel, and the corresponding ×800 expanded scale views are plotted in the right panels. The relative minimum region is regarded as the baseline position.
Figure 2. The averaged pulse profile of the whole bandwidth is shown in the top panel, and the corresponding \times 60 and \times 900 expanded scale views are plotted in the middle and bottom panels, respectively. The relative minimum region is regarded as the baseline position.

The last open field lines), to be \( \sim 25.8^\circ \). Assuming that the directions of the magnetic axis and rotation axis are uniform and ignore the beam scale distribution, with the Eq. 2 in Lee et al. (2009), we estimate the probability density of magnetic inclination angle, \( \alpha \), which is shown in Figure 4. It is obvious that the the inclination angle can hardly be smaller than \( 10^\circ \) or larger than \( 170^\circ \). Moreover, the inclination angle measured through polarization observation of the strong emission is \( \alpha \simeq 70^\circ \) (Everett & Weisberg 2001), which implicates the radiation of PSR B0950+08 originates from both poles of an almost orthogonal rotator. Therefore, the results support double-pole models of PSR B0950+08.

However, the assumption of radiation altitude at 1000 km should be discussed carefully. In fact, the mean pulse profile at lower frequency is much wider, with pulse width \( \sim 70^\circ \) (Bilous et al. 2022), which leads to that the radiation altitude must be higher than 1500 km under the assumption of \( \alpha \simeq 70^\circ \). Furthermore, for the whole phase radiation, the maximum radiation altitude should be larger than 5000 km even after the adjustment for the effects of aberration and retardation. Considering that the radius of the light cylinder is \( \sim 12000 \) km and that the inclination angle is \( 70^\circ \), we can derive an extremely high altitude of 5000 km, already reaching

Figure 3. The \( \Theta(n) \) defined in equation (1) of the whole pulse longitude are shown in the top panel, and vertically zoomed-in view is plotted in the middle panel. Moreover, the detailed property of the blue region is zoomed in the bottom panel.

Figure 4. The probability density of magnetic inclination angle \( \alpha \) derived from the period and pulse width of PSR B0950+08.
Radio signal detected over the whole phase

Figure 5. The distribution of the signal intensity as a function of pulse phase.

the location of “outer gap” to be popular for explaining the X-/γ-ray emission of pulsars (Cheng et al. 1986). However, for the radio radiation of normal pulsars, the radiative particles are supposed to lose energy rapidly and not to keep radiating till reaching such a high altitude (Zhang et al. 1997). For PSR B0950+08, the two bridges between main pulse and inter pulse most probably originate from this height, as a challenge to the origin and the coherence mechanism of the relativistic plasma for the radio emission.

The whole pulse radiation characteristics is unveiled with the function $\Theta(n)$, which exploits the variation property of the pulsar radiation. In other words, the invariant radiation does not affect the analysis results. For example, the stable emission from pulsar wind nebula which contributes off-pulse emission (Ruan et al. 2020) is different from the whole phase radiation from pulsar. In fact, the whole phase radiation is similar to the non-100% pulsed fraction radiation of the high energy pulsar, in which the emission occurs over the whole phase while some specific phase regions are preferred.

It is worth noting that, although the whole pulse longitude exhibits radio emission, the emission properties vary with different pulse longitude. The significance of this characteristics is exhibited in Figs. 2 and 3. The difference between inter-pulse and main pulse is described in Fig. 2, and the well-resolved conal double structure of inter-pulse is different with the unresolved conal single structure of main pulse. Fig. 3 reveals that the values of $\Theta(n)$ at two minimum low levels are almost equal to each other, and this result possibility indicates that similar emission properties appear at the two minimum low levels.

To determine the emission properties of the bridge regions, the distribution of the radio emission signals at certain pulse longitude is also considered. This method reveal the emission properties through the distribution of the noise and radio emission at each pulse phase bin of the folded data, which follow different statistics laws. Apparently, the fluctuation of the noise follows the Gaussian distribution, and this statistics property is different with the log-normal distribution of the radio emission signals of the normal pulse (Cairns 2004). This property may cause that the distribution of the emission at the on-pulse phase interval is asymmetric. The statistical results are shown in Fig. 5. From this figure, it faintly shows that the distribution of the upper and lower of the maximum distribution point (the red points) at each pulse phase bin is asymmetric. Moreover, the dispersion of the upper area is greater than the lower, and this distribution characteristics supports that there is radio emission components at each pulse phase bin. Compared with this method, the function $\Theta(n)$ is better revealing the emission properties, because the conclusion of the $\Theta(n)$ is not influenced by the RFI in time domain.

Although the relatively minimum levels as the baseline position is not suitable for the whole phase radiation pulsar, the determination of the baseline is surely difficult. The baseline changing with time will affect the baseline determination, and this effect can be eliminated through the flattening of the baseline evolution in each individual pulse with considering the narrow pulse phase property of the individual pulse. Meanwhile, the radiation intensity of PSR B0950+08 is influenced by the effect of the interstellar scintillation (Smirnova & Shishov 2008), which would also influence determining the baseline.

PSR B0950+08 is a nearby and bright radio pulsar, and its whole phase radiation phenomenon is detected for the first time. Up to now, similar properties have also been discovered in PSR B1929+10 and in some millisecond pulsars, and all these pulsars have been reported that the emission covers an unusually wide range of pulse longitude (e.g. McLaughlin & Rankin 2004; Dai et al. 2015). Consequently,
the baselines for the observation of this type of pulsars need to be determined with caution.

The polarization position angle (PPA) is related to the radiation geometry, but the PPA of the whole phase radiation is hard to be calibrated because it is sensitive to the baseline determination in the weak radiation phase region. The polarization features of this pulsar are not presented in this paper, since we have not made satisfied calibration during the observations. The non-linear evolution of polarization response with time severely influenced the calibration. Therefore, real time calibration information is needed to determine the baselines of the Stokes parameters precisely, and the optimized calibration information is needed to determine the baseline determination in the weak radiation phase region. The polarization behaviors of repeating fast radio bursts have been investigated extensively with FAST too (Wang et al. 2022, 2021), a new era of studying pulsar electrodynamics and related radiative mechanisms is expected.

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DATA AVAILABILITY

The data underlying this work are available in the FAST project PT2020–0034, and can be shared on request to the FAST Data Center.

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APPENDIX A: THE PROPERTY OF THE FUNCTION $\Theta$

Detailed expression of the function $\Theta(n)$ is shown as equation(1), and here we will present that the values of $\Theta(n)$ at the emission regions are positive.

For the data, at any pulse phase interval with emission, the intensity of the noise follows the Gaussian distribution. The properties of $\Theta(n)$ can be described by the follows,

$$\Theta(n) = \sum_{k=1}^{N_{\text{period}}} \left\{ 2 (I_{k,n-1} - I_{k,n+1})^2 - \left[ (I_{k,n-1} - I_{k,n})^2 + (I_{k,n} - I_{k,n+1})^2 \right] \right\}$$

$$\approx \sum_{k=1}^{N_{\text{period}}} \left\{ 2 (S_{k,n-1} - S_{k,n+1} + \sigma_{k,n-1} - \sigma_{k,n+1})^2 - \left[ (S_{k,n-1} - S_{k,n} + \sigma_{k,n-1} - \sigma_{k,n})^2 \right] \right\}$$

$$\approx \sum_{k=1}^{N_{\text{period}}} \left\{ 2 (S_{k,n-1} - S_{k,n})^2 - \left[ (S_{k,n-1} - S_{k,n})^2 + (S_{k,n} - S_{k,n+1})^2 \right] \right\}$$

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where $S_{k,n}$ and $\sigma_{k,n}$ are the emission signal and noise intensity contributed by the $k$-th pulse and $n$-th pulse phase bin, respectively.

For the observation data, a same structure of the sub-pulse repeat many times in a large number of cycles, so that a pulse phase is ergodic in all positions of sub-pulse. In other words, the summation of all cycles is equivalent to that of all points in sub-pulses with different shape,

$$\Theta(n) \approx \sum_{k=1}^{N_{\text{period}}} \frac{1}{T} \sum_{l=1}^{L} \left\{ 2 (S_{k,n,l-1} - S_{k,n,l+1})^2 - [(S_{k,n,l} - S_{k,n,l-1})^2 + (S_{k,n,l} - S_{k,n,l+1})^2] \right\} \approx \sum_{k=1}^{N_{\text{period}}} \frac{1}{T} \sum_{l=1}^{L} \left\{ (S_{k,n,l} - S_{k,n,l-1})^2 + (S_{k,n,l+1} - S_{k,n,l})^2 + 4 (S_{k,n,l} - S_{k,n,l-1}) (S_{k,n,l+1} - S_{k,n,l}) \right\} \approx \sum_{k=1}^{N_{\text{period}}} \frac{1}{T} \sum_{l=1}^{L} (\Delta_l^2 + \Delta_{l-1}^2 + 4 \Delta_l \Delta_{l-1})$$

(A2)

where $S_{k,n,l}$ is the $l$-th bin of the sub-pulse with $L$ bins of the $k$-th pulse and $n$-th pulse phase bin, $\Delta_l = S_{k,n,l+1} - S_{k,n,l}$ is the first order difference of the sub-pulse.

In this work, the time resolution of the folded data is $\sim$0.25 ms. According to the previous work, the characteristics time-scale of the fine structure is $\sim$1 ms for PSR B0950+08 (Ulyanov et al. 2016). In other words, the second order difference can hardly change sign in succession in 1 ms ($\sim$4 phase bins) for the folded data. In the interval that the second order difference has same sign (the bin index of the start and end of the interval are $l_s$ and $l_e$, respectively), $\Delta_l$ is monotonically increasing. If the sign of $\Delta_l$ does not change, we have

$$M = \sum_{l=l_s}^{l_e} (\Delta_l^2 + \Delta_{l-1}^2 + 4 \Delta_l \Delta_{l-1}) \geq 0.$$  Otherwise, if $\Delta_l$ changes signs, without loss of generality, suppose $\Delta_{l_s} \geq ... \geq \Delta_{l_m} > 0 \geq \Delta_{m+1} \geq ... \geq \Delta_{l_e}$, it follows that

$$\sum_{l=m+1}^{l_e} (\Delta_l^2 + \Delta_{l-1}^2 + 4 \Delta_l \Delta_{l-1}) \geq (\Delta_m^2 + \Delta_{m-1}^2 + 4 \Delta_m \Delta_{m-1}) + \sum_{l=m+2}^{l_e} (\Delta_l^2 + \Delta_{l-1}^2 + 4 \Delta_l \Delta_{l-1}) \geq (\Delta_m^2 + \Delta_{m-1}^2 + 4 \Delta_m \Delta_{m-1}) + (\Delta_{m+1}^2 + \Delta_m^2 + 4 \Delta_{m+1} \Delta_m) \geq \Delta_m^2 - 2 \Delta_m^2 + 4 \Delta_m \Delta_{m-1} + (\Delta_{m+1} + 2 \Delta_m)^2 \geq \Delta_m^2 - 2 \Delta_m^2 + 4 \Delta_m \Delta_{m-1} \geq \Delta_m^2 - 2 \Delta_m^2 + 4 \Delta_m^2 = 3 \Delta_m^2 > 0.$$  (A3)

To sum up, $\Theta(n) > 0$ at the interval of the emission pulse phase.