Probing two-photon decay widths of mesons at energies available at the CERN Large Hadron Collider (LHC)

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Meson production cross sections in ultra-peripheral relativistic heavy ion collisions at the LHC are revisited. The relevance of meson models and of exotic QCD states is discussed. This study includes states that have not been considered before in the literature.

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In quantum chromodynamics (QCD), meson spectroscopy is an exciting possibility to search for states which cannot be explained within the $q\bar{q}$ model. A few possibilities are multiquark states such as molecules ($q\bar{q}$)$^2$, hybrid mesons ($qg$) and glueballs ($gg$). Despite the intense and considerable experimental effort such states have yet to be established [1]. The masses predicted for these states are typically in the range of 1.5 to 2.5 GeV. Therefore, a crucial part of the search for these “abnormal” states is to establish the spectrum of “ordinary” $q\bar{q}$-mesons in the 1.5-2.5 GeV region. The “abnormal” states may only stand out clearly when the ordinary $q\bar{q}$ states are well understood and classified background [2]. This has been the subject of intense investigation for a long time (see, e.g. [2, 3, 4, 5, 6, 7, 8, 9, 10]).

Two-photon physics can contribute to the search for non-$q\bar{q}$ resonances both by establishing the spectrum of $q\bar{q}$ levels and by identifying states with anomalous $\gamma\gamma$ couplings. $\gamma\gamma$ collisions are useful for determining the spectrum of $\gamma\gamma$ states because $q\bar{q}$ mesons have a characteristic pattern of $\gamma\gamma$ couplings, for example the relative flavor factor within a flavor-SU(3) multiplet of $q\bar{q}$ states:

$$\Gamma_{\gamma\gamma}(f) : \Gamma_{\gamma\gamma}(a) : \Gamma_{\gamma\gamma}(f') = 25 : 9 : 2,$$

which, except for minor relativistic corrections, reproduce the experimental data quite well. If there were any doubt about the identity of one of the light $^3P_2$ states $f_2(1270)$, $a_2(1320)$ and $f_2'(1525)$, for example, their relative, $\gamma\gamma$ widths would be a strong argument that these are all $q\bar{q}$ states if any one had been established as $q\bar{q}$. The $\Gamma_{\gamma\gamma}$ partial widths of resonances would be even more useful in the identification of $q\bar{q}$ states if the absolute scale of these widths can be reliably calculated. But this proves to be rather sensitive to the assumptions made in calculation, as has been noted in several theoretical investigations.

There is a true motivation to observe if predictions of “abnormal” states within the quark model is verified experimentally and complies with the $\Gamma_{\gamma\gamma}$ partial widths calculated for “normal states”. This fact can only be assessed experimentally. Photon-photon (or “two-photon”) processes have long been studied at $e^+e^-$ colliders [11]. They are an excellent tool for investigating many aspects of meson spectroscopy, and allow intensity tests of QED. At the highest energy colliders, reactions such as $\gamma\gamma \rightarrow X$ may be used to probe the quark content and spin structure of meson resonances. Production of meson or baryon pairs can also probe the internal structure of hadrons.

A wonderful new possibility for similar experimental studies is the Large Hadron Collider (LHC) at CERN. At the LHC, photon-proton collisions occur at center of mass energies an order of magnitude higher than were available at previously existing accelerators, and photon-heavy ion collisions reach 30 times the energies available at fixed target accelerators. The Lorentz gamma factor $\gamma = (1 - v^2/c^2)^{-1/2}$ in the laboratory frame is 7000 for p-p, 3000 for Pb-Pb collisions. Due to the ions large charges (e.g., $Z = 82$) and their short-interaction time ($\Delta t \approx 20\gamma$ MeV), the interacting electromagnetic fields generated by these ions are typically much stronger ($\propto Z^2$) than the Schwinger critical field $E_{Sch} = m^2/\hbar e = 1.3 \times 10^{16}$ V cm. Light particles (e.g., $e^+e^-$-pairs are produced copiously by such fields [12]. The electromagnetic fields of heavy-ions are of sufficient intensity to allow the study of multi-photon reactions (for an early review, see ref. [12]). At the LHC, electroweak processes such as $\gamma\gamma \rightarrow W^+W^-$ may also be studied. Even the production of the Higgs boson is not negligible with such a mechanism [13, 16, 17]. The physics of ultra-peripheral heavy ion collisions has become object of intensive studies in recent years [18, 19, 20, 21, 22, 23, 24].

In the equivalent photon approximation, for ion beams with squared center of mass energy $s$, one can write the cross section for photon-photon fusion to a neutral state $X$ in the form [12]

$$\sigma_X = \int dx_1 dx_2 N_\gamma(x_1) N_\gamma(x_2) \sigma^X_{\gamma\gamma}(x_1 x_2 s),$$

where $N_\gamma(x)$ is the distribution function (equivalent photon numbers) for finding a quantum $\gamma$ with energy fraction $x$ and $\sigma^X_{\gamma\gamma}(x_1 x_2 s)$ is the two-photon cross section. It is given by [27]

$$\sigma^X_{\gamma\gamma}(x_1 x_2 s) = 8\pi^2 (2J + 1) \frac{\Gamma_{\gamma\gamma}}{m_X} \delta(x_1 x_2 s - m_X^2).$$
where $J$, $m_X$, and $\Gamma_{m_X\rightarrow\gamma\gamma}$ are the spin, mass, and the two-photon partial decay width. The delta-function enforces energy conservation.

The $\gamma$-nucleus vertex is given by $ZeF(t)$, where $F(t)$ is the elastic nuclear form factor and $t$ is the invariant four-momentum exchanged. Then the distribution function $N_\gamma(x)$ for a fast moving nucleus of charge $Z$ is given by \[ N_\gamma(x) = \frac{Z^2\alpha}{\pi x} \int_0^\infty dk^2 \frac{|F(x^2M^2 + k^2)|^2}{(x^2M^2 + k^2)^2}. \] (4)

If one approximates the form factor by a gaussian, $F(k^2) = \exp(-k^2/2K^2)$, one sees that the form factor imposes a cutoff $xM/K \sim xMR \sim < 1$, where $R \sim 1/K$ is the nuclear radius. Hence, an state of invariant mass $m_X$ can be produced as long as $m_X^2 = x_1x_2s < s/M^2R^2 = (2\gamma/R)^2$. This is essentially the condition for coherence: the photon wavelength must be larger than the Lorentz-contracted nuclear radius. Coherence leads to the factor of $Z^2$ in $N_\gamma$ (yielding a factor $Z^4$ in the cross section), rendering electromagnetic interactions of high-Z ions an effective tool for the production of heavy neutral particles.

The above formulation does not account for the effects of inelastic nuclear scattering, which is more easily taken into account in the impact parameter semiclassical method. The majority of the inelastic events occur at small values of the impact parameter $b$. The elastic nature of the interaction is maintained only in those collisions in which the two nuclei pass far from each other. The impact parameter representation of $N_\gamma(x)$ given by \[ N_\gamma(\omega, b) = \frac{Z\alpha\omega}{\pi^2\gamma^2} \left[K_1^2(y) + \frac{1}{\gamma^2}K_0^2(y)\right]. \] (5)

where $\omega = xE$ is the energy of the scattered quanta ($E$ is the ion energy), $y = \omega b/\gamma$, $\alpha = 1/137$, and $K_0$ and $K_1$ are modified Bessel functions. The first term, $K_1^2(y)$, gives the flux of photons transversely polarized to the ion direction and the second is the flux for longitudinally polarized photons. The photon flux is exponentially suppressed when $\omega > \gamma/b$. These photons are almost real, with virtuality $-q^2 < 1/R^2$. The usable photon flux depends on the geometry. Most peripheral reactions lead to final states with a handful of particles. These final states will be overwhelmed by any hadronic interactions between the fast moving ion and the target. Thus, the useful photon flux is that for which the ions do not overlap, i.e. when the impact parameter $b = |b_1 - b_2|$ is greater than twice the nuclear radius ($2R$). Usually, we can take $R = 1.2A^{1/3}$ fm, where $A$ is the atomic number. The photons can interact with a target nucleus in a one-photon process, (when $b_1 < R$) or with its electromagnetic field in a two-photon process when $b_1 > R$ and $b_2 > R$.

For two-photon exchange processes, the equivalent photon numbers, in equation 2 must account for the electric field orientation of the photon fluxes with respect to each ion \[ \sigma_X = \int d\omega_1d\omega_2\int_{b_1>R}\int_{b_2>R} d^2b_1d^2b_2N_\gamma(\omega_1, b_1)N_\gamma(\omega_2, b_2)\sigma_X^\gamma(\omega_1, \omega_2), \] (6)

with the condition that $|b_1 - b_2| \geq 2R$.

We use the formalism described above together with the $\Gamma_{\gamma\gamma}$ widths either taken from experiment, or from theory, to generate tables 1-4. For many cases, there is no major difference between our values and the ones obtained in previous calculations for the same mesons \[ 2, 3, 4, 5, 6, 7, 8, 9, 10]. \]

However, many of our values are predictive and have not been considered before. In these tables the properties of some $q\bar{q}$ states are given, and their production cross sections are predicted for Pb-Pb collisions at LHC. Mass values and known widths are taken from experiment when possible. Ion luminosities of $10 \times 26$ cm$^{-2}$ s$^{-1}$ for Pb-Pb collisions at LHC lead up to million of events (e.g. charmonium states) per second for the largest cross sections \[ 14]. \] The two-photon width is a probe of the charge of its constituents, so the two-photon coupling can serve to distinguish quark dominated resonances from glue-dominated resonances ("glueballs"). The absence of meson production via $\gamma\gamma$ fusion is a signal of great interest for glueball search. In ion-ion collisions, a glueball can only be produced via the annihilation of a $q\bar{q}$ pair into gluons pairs, whereas a normal $q\bar{q}$ meson can be produced directly. Due to the copious production of such states in peripheral collisions at the LHC, such studies will be viable depending on the detection setup \[ 23]. \] It requires measuring events characterized by relatively small multiplicities and a small background (especially when compared with the central collisions).

In conclusion, in this brief report, the cross sections for meson production in ultra-peripheral collisions of heavy-ions at LHC have been reviewed and additional cases have also been considered. Due to the very strong electromagnetic fields of short duration, new possibilities for interesting physics arise at the LHC. The method of equivalent photons
is a well established tool to describe this kind of reactions. But, unlike electrons and positrons, heavy ions and protons are particles with internal structure. Thus, effects arising from this structure have to be controlled and minor uncertainties coming from the exclusion of central collisions and triggering must be eliminated, e.g., by using a luminosity monitor from $\mu^-$ or $e^-$ pairs. Ultra-peripheral heavy ion collisions at the LHC is an excellent tool to produce and study “abnormal” $q\bar{q}$ states and/or glueballs. This encompasses invariant masses up to 10 GeV. The production via photon-photon fusion complements the production from single timelike photons in $e^+e^-$ collider and also in hadronic collisions via other partonic processes. The extent to which the measured production cross sections agrees with results presented here may serve as a measure of the status of several “anomalous” $q\bar{q}$ and glueballs candidacies. One must reiterate that it is in general a good strategy to study $\gamma\gamma$ meson decays through the inverse process, i.e. $\gamma\gamma$-production in heavy-ion colliders. The rate of production of these mesons can probe the relative couplings of different decay modes which are usually quite distinct for hybrid versus quarkonium assignments.

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### I. TABLES

| Mesons $a$ and $f$ | $J^{PC}$ | $\Gamma^{th}_{\gamma\gamma}$ [keV] | $\Gamma^{exp}_{\gamma\gamma}$ [keV] | Obs. | $\sigma^X_{\gamma\gamma}$ |
|-------------------|----------|---------------------------------|---------------------------------|------|--------------------------|
| $a_5^{\pi K}$ (980) | $(0^{+})$ | $0.6$ | $0.30 \pm 0.10$ | $\rightarrow K\bar{K} \rightarrow \gamma\gamma$ | $3.1$ mb |
| $a_0^{\pi}$ (980) | | $1.5$ | | hypothetical, NR q-model | $8.6$ mb |
| $a_0^{\pi}$ (980) | | $1.0$ | | hypothetical, R q-model | $5.5$ mb |
| $f_0^{\phi K}$ (980) | $(0^{++})$ | $0.6$ | $0.29^{+0.07}_{-0.09}$ | $\rightarrow K\bar{K} \rightarrow \gamma\gamma$ | $3.1$ mb |
| $f_0^{\phi}$ (980) | | $4.5$ | | hypothetical, NR q-model | $25.8$ mb |
| $f_0^{\phi}$ (980) | | $2.5$ | | hypothetical, R q-model | $14.3$ mb |
| $f_0$ (1200) | | $3.25 - 6.46$ | | unknown | $9.6 - 21$ mb |
| $f_2$ (1274) | $(2^{++})$ | $1.75 - 4.04$ | $2.6 \pm 0.24$ | $\Gamma_{\gamma\gamma} (f_0)/\Gamma_{\gamma\gamma} (f_2) = 1.86 - 1.60$ | $21 - 49$ mb |
| $f_2$ (1274) | | $1.71 - 3.93$ | | $(\lambda = 0) / (\lambda = 2) = 0.022 - 0.029$ | $20 - 44$ mb |
| $f_2$ (1274) | | $0.04 - 0.11$ | | | |
| $f_0$ (1800) | | $2.16 - 2.52$ | | unknown | $2^{3}P_0$ radial excitation | $2.5 - 3.1$ mb |
| $f_2$ (1800) | $(\lambda = 2)$ | $1.53 - 2.44$ | | $2^{3}P_2$ radial excitation | $1.7 - 2.9$ mb |
| $f_2$ (1800) | $(\lambda = 0)$ | $0.08 - 0.16$ | | | $0.08 - 14$ mb |
| $f_2$ (1525) | $(2^{++})$ | $0.17$ | $0.081 \pm 0.009$ | $s\bar{c}$, $m_s = 0.55$ GeV fixed | $0.86$ mb |
| $f_2$ (1525) | $(\lambda = 0)$ | $0.065$ | | | $0.21$ mb |
| $f_2$ (1525) | $(\lambda = 2)$ | $0.9 \times 10^{-3}$ | | | $0.42$ µb |
| $f_4$ (2050) | | $0.36 - 1.76$ | | unknown | $^{3}F_4$ | $0.03 - 0.14$ mb |
| $f_4$ (2050) | $(\lambda = 2)$ | $0.33 - 1.56$ | | | $0.02 - 0.13$ mb |
| $f_4$ (2050) | $(\lambda = 0)$ | $0.03 - 0.20$ | | | $2 - 12$ µb |
| $f_5$ (2050) | | $0.50 - 2.49$ | | unknown | $^{3}F_3$ | $0.03 - 0.13$ mb |
| $f_5$ (2050) | | $2.48 - 11.11$ | | unknown | $^{3}F_2$ | $0.12 - 0.53$ mb |
| $f_5$ (2050) | $(\lambda = 2)$ | $1.85 - 8.49$ | | | $0.09 - 0.46$ mb |
| $f_5$ (2050) | $(\lambda = 0)$ | $0.63 - 2.62$ | | | $0.01 - 0.07$ mb |
| $f_5^{\pi} K^+$ ($\approx 1750$) | | | | unknown | vector-vector molecule | $0.19$ mb |

Table 1. $\gamma - \gamma$ widths for mesons $a$ and $f$ calculated with the models described in refs. [3, 4, 5, 6, 7, 8, 9]. Masses above 1250 MeV are assumed within parenthesis. Experimental values of the $\gamma - \gamma$ widths are extracted from the Particle Data Properties Web site.

| Mesons $\eta$, $\chi$ and $h$ ($c\bar{c}$) | $J^{PC}$ | $\Gamma^{th}_{\gamma\gamma}$ [keV] | $\Gamma^{exp}_{\gamma\gamma}$ [keV] | Obs. | $\sigma^X_{\gamma\gamma}$ |
|------------------------------------------|----------|---------------------------------|---------------------------------|------|--------------------------|
| $\eta_c$ | $(0^{-+})$ | $3.4 - 4.8$ | $6.7^{+0.9}_{-0.8}$ | $m_c = 1.4 - 1.6$ GeV | $0.26 - 0.34$ mb |
| $\eta_c$ (3790) | | $1.85 - 8.49$ | $1.3 \pm 0.6$ | $m_c = 1.4$ GeV | $0.06 - 0.1$ mb |
| $\eta_c$ (3790) | | $3.7$ | unknown | $m_c = 1.4$ GeV | $0.11$ mb |
| $\eta_c$ (4060) | | $3.3$ | unknown | | $0.09$ mb |
| $\eta_c^{1D}(3840)$ | | $20 \times 10^{-3}$ | unknown | | $0.15$ µb |
| $\eta_c^{1D}(4210)$ | | $35 \times 10^{-3}$ | unknown | | $0.14$ µb |
| $\eta_c^{1G}(3550)$ | | $0.92 \times 10^{-3}$ | unknown | | $0.08$ µb |
| $\chi_2$ | $(2^{++})$ | $0.56$ | $0.258 \pm 0.019$ | $(\lambda = 2) / (\lambda = 0) = 0.005$ | $82$ µb |
| $\chi_0$ | $(0^{++})$ | $1.56$ | $0.276 \pm 0.033$ | $\Gamma_{\gamma\gamma} (\chi_0) / \Gamma_{\gamma\gamma} (\chi_2) = 2.79$ | $0.05$ mb |
| $\chi_2$ | $(2^{++})$ | $0.64$ | unknown | | $0.09$ mb |
| $h_{c2}$ (3840) | | $20 \times 10^{-3}$ | unknown | | $1^D_2$ | $82$ µb |
| $\chi_2$ (4100) | | $30 \times 10^{-3}$ | unknown | | $3F_2$ | $0.11$ µb |

Table 2. $\gamma - \gamma$ widths for $c\bar{c}$-mesons $\eta$, $\chi$ and $h$ calculated with the models described in refs. [3, 4, 5, 6, 7, 8, 9]. Masses above 1250 MeV are assumed within parenthesis. Experimental values of the $\gamma - \gamma$ widths are extracted from the Particle Data Properties Web site.
| Mesons $\eta$, $\chi$ and $h$ ($b\bar{b}$) | $J^{PC}$ | $\Gamma_{\gamma\gamma}^{th}$ [keV] | $\Gamma_{\gamma\gamma}^{exp}$ [keV] | Obs. | $\sigma_{\gamma\gamma}^X$ |
|------------------------------------------|----------|-------------------------------|-------------------------------|------|-----------------|
| $\eta_b^{1S}(9400)$                     |          | $0.17 \times 10^{-3}$         | unknown                       | 19   | nb              |
| $\eta_b^{2S}(9400)$                     |          | $0.13 \times 10^{-3}$         | unknown                       | 16   | nb              |
| $\eta_b^{3S}(9480)$                     |          | $0.11 \times 10^{-3}$         | unknown                       | 14   | nb              |
| $\eta_b^{1D}(10150)$                   |          | $33. \times 10^{-6}$          | unknown                       | 0.4  | nb              |
| $\eta_b^{2D}(10450)$                   |          | $69. \times 10^{-6}$          | unknown                       | 0.8  | nb              |
| $\eta_b^{4D}(10150)$                   |          | $59. \times 10^{-6}$          | unknown                       | 0.7  | nb              |
| $\eta_b(9366)$                         |          | $(0^{-+})$ 0.17               | unknown                       | 0.12 | $\mu$b         |
| $\eta_b'$                              |          | 0.13                          | unknown                       | 0.17 | $\mu$b         |
| $\eta_b^{''}$                          |          | 0.11                          | unknown $s\bar{s}$, $m_s = 0.55$ GeV fixed | 0.15 | $\mu$b         |
| $\chi_{62}(9913)$                      |          | $(2^{++})$ 3.7 $\times 10^{-3}$ | unknown                       | 0.09 | $\mu$b         |
| $\chi_{10}(9860)$                      |          | $(0^{++})$ 13. $\times 10^{-3}$ | unknown                       | 0.08 | $\mu$b         |

Table 3. $\gamma - \gamma$ widths for $b\bar{b}$-mesons $\eta$ and $\chi$ calculated with the models described in refs. [2, 3, 4, 5, 6, 7, 8, 9]. Masses above 1250 MeV are assumed within parenthesis. Experimental values of the $\gamma - \gamma$ widths are extracted form the Particle Data Properties Web site.

| Mesons $\pi$ | $J^{PC}$ | $\Gamma_{\gamma\gamma}^{th}$ [keV] | $\Gamma_{\gamma\gamma}^{exp}$ [keV] | Obs. | $\sigma_{\gamma\gamma}^X$ |
|--------------|----------|-------------------------------|-------------------------------|------|-----------------|
| $\pi_0$      | (0$^{++}$) | $3.4 - 6.4 \times 10^{-3}$ 8.4 $\pm 0.6 \times 10^{-3}$ | $m_q = 0.220 - .33$ GeV $27 - 52$ m b |      |                 |
| $\pi(1300)$  | (0$^{++}$) | 0.43 - 0.49                  | unknown                       | 0.69 | 0.71 mb         |
| $\pi(1880)$  |          | 0.74 - 1.0                   | unknown                       | 3$^1S_0$ | 0.8 - 1.1 mb     |
| $\pi_2(1670)$| (2$^{--}$) | 0.11 - 0.27                  | < 0.072                       | 0.41 | 1.1 mb          |
| $\pi_2(2130)$| (2$^{++}$) | 0.10 - 0.16                  | unknown                       | 2$^1D_2$ | 0.36 - 0.54 mb   |
| $\pi_4(2330)$|          | 0.21 - 1.6                   | unknown                       | 1$^1G_4$ | 0.04 - 0.31 mb   |

Table 4. $\gamma - \gamma$ widths for $\pi$-mesons calculated with the models described in refs. [2, 3, 4, 5, 6, 7, 8, 9]. Masses above 1250 MeV are assumed within parenthesis. Experimental values of the $\gamma - \gamma$ widths are extracted form the Particle Data Properties Web site.