Distributed Estimation of Oscillations in Power Systems: an Extended Kalman Filtering Approach

Zhe Yu†, Di Shi†, Zhiwei Wang†, Qibing Zhang‡, Junhui Huang‡, and Sen Pan‡†

Abstract—Online estimation of electromechanical oscillation parameters provides essential information to prevent system instability and blackout and helps to identify event categories and locations. We formulate the problem as a state space model and employ the extended Kalman filter to estimate oscillation frequencies and damping factors directly based on data from phasor measurement units. Due to considerations of communication burdens and privacy concerns, a fully distributed algorithm is proposed using diffusion extended Kalman filter. The effectiveness of proposed algorithms is confirmed by both simulated and real data collected during events in State Grid Jiangsu Electric Power Company.

Index Terms—Oscillation detection and estimation; extended Kalman filter; distributed estimation.

I. INTRODUCTION

ELECTROMECHANICAL oscillations are observed in interconnected power systems after large disturbances. Poorly damped oscillations reduce margins of power systems and could cause system instability or blackout. Wide-area measurement system (WAMS) technology using phasor measurement units (PMUs) makes it possible to observe the phenomenon of oscillations and estimate parameters such as frequencies, damping factors and magnitudes. These parameters contain vital information about modes of the power system and help operators to identify event categories and locations.

Compared to conventional supervisory control and data acquisition (SCADA) systems, PMUs have higher sampling rate and are able to measure phase angles, which attract great interests and investments in the past decade. Department of Energy (DoE) has spent over $328 million in aggregate on synchrophasor technology and related communications networks [1]. By 2015, there were over 1,700 PMUs on the North American power grid, covering the entire U.S. high-voltage transmission network [2]. In Jiangsu province of China, more than 160 PMUs have been deployed, covering all 500kV and majority of 220kV substations.

On one hand, abundant time stamped data from PMUs allow system operators to monitor the system in detail and better understand its dynamics. On the other hand, high sampling rate of PMU creates huge burden to the communication infrastructure, in which all data are uploaded to and processed in the control center. This mechanism limits many potential applications of WAMS which require PMU to have higher reporting rates. In addition, a centralized mechanism is vulnerable to cyber attacks. An improved framework is needed which can help better fulfill PMU’s capabilities given limited network transit and computation capacity.

A. Summary of Results

We develop a fully distributed algorithm to monitor system oscillations. In this framework, information is exchanged locally and computation is distributed to each PMUs or PDC.

We first extend the formulation from [3] to a multi-measurements framework. A nonlinear system is formulated, whose state includes frequencies, damping factors and magnitudes of various oscillation modes from each PMU. The frequencies and damping factors are assumed consistent across the entire system while the amplitudes and phasors of each PMU may be different. The observation of the nonlinear system is measurements from PMUs plus white noises.

Then we propose a centralized EKF which can directly estimate oscillation frequencies and damping factors of multiple modes with online implementation. The proposed method requires much lower computational resource compared to conventional methods because of the recursive nature of EKF.

Furthermore, a fully distributed EKF framework is developed in which no central coordinator is needed, and PMUs communicate information only with neighbour(s). The EKF computation is carried out at each PMU based on local information to estimate oscillation parameters. A consensus is achieved by a diffusion process.

B. Related Work and Organization

There is expanding literature on oscillation detection and estimation, most of which is focusing on centralized mechanisms. Some of the well known methods include matrix-pencil method (MP) [4], [5], eigenvalue realization algorithm (ERA) [6], [7], Hankel total least-squares (HTLS) [8], Hilbert-Huang transform (HHT) [9], Prony methods [10], [11], and extended Kalman filter (EKF) [3], [12], [13]. Most of these methods are not designed for online implementation and do not scale up well. An attempt of applying online estimators of Prony method can be found in [14].

Due to the explosion of data volume, distributed computing and data processing algorithms started to obtain increasing attention in recent years [15], [16], while few works has been reported on distributed oscillation monitoring. Authors in [17] model the Prony method as a consensus optimization and

Z. Yu†, D. Shi† and Z. Wang† are with GEIRI North America, San Jose, CA 95134, USA, Q. Zhang‡ and J. Huang‡ are with State Grid Jiangsu Electric Power Company, 215 Shanghai Road, Gulou, Nanjing, 210024, China. Sen Pan‡† is with GEIRI, Nanjing, 210024, China. Email: {zhe.yu,di.shi,zhiwei.wang}@geirina.net, zqp_z12@163.com, xshhj@126.com, pansen@geirii.sgcc.com.cn. This work is funded by SGCC Science and Technology Program under contract No. 5455HJ160007.
applied alternating direction methods of multipliers (ADMM)
and exam the algorithm’s performance with different commu-
ication environments. However, the proposed ADMM method
still requires a central coordinator to implement.

The work closest to this paper is [13], in which the authors
presented a consensus extended Kalman filter to indirectly
estimate oscillation modes. In this work, however, oscillation
frequencies and damping factors are directly estimated.
Furthermore, measurement diffusion and state reduction are
applied to enhance the performance of EKF.

The remainder of the paper is organized as follows. In
section II we formulate a nonlinear state space model, whose
states include oscillation frequencies, damping factors and
magnitudes. A centralized extended Kalman filter is applied
in section III. Considering the data volume and communi-
tion burden, we present a fully distributed EKF framework in
section IV. Numerical results based on simulated and real data
in section VI confirm the desirable performance of proposed
algorithms, and section VII concludes the paper.

II. PROBLEM FORMULATION

As discussed in [10] and [13], electromechanical oscillations
in power systems can be represented as a sum of some
exponentially damped sinusoids. In a discrete framework,
a system measurement \( y[k] = [y_1[k], \cdots, y_M[k]]^T \) can be
expressed as follows:

\[
y_m[k] = \sum_{l=1}^{M} A_{l,m} \exp(-\frac{\sigma_l}{f_s} k) \cos(\frac{\omega_l}{f_s} k + \phi_{l,m}) + \epsilon_m[k],
\]

where \( y_m[k] \) is the measurement of the \( m \)th PMU at the \( k \)th
time instant \( T \), the transpose operator, \( M \) the number of
PMUs, \( A_{l,m} \in \mathbb{R} \) the amplitude, \( L \) the number of oscillation
modes, \( \sigma_l \) the damping factor, \( \omega_l \) the frequency, \( \phi_{l,m} \)
the phase angle, \( f_s \) the sampling rate, and \( \epsilon_m[k] \) the measurement
error. The measurement noise is assumed to be a white
Gaussian noise with zero mean and a diagonal covariance
matrix \( \boldsymbol{R}_0 = \text{diag}(R_{1,1}, \ldots, R_{M,M}). \) Measurements from
different PMUs may have various amplitudes and phase angles,
while frequencies and damping factors are assumed to be
consistent across the system.

Inspired by [3], we formulate a nonlinear system whose
states contain frequencies and damping factors of the oscilla-
tion modes. Consider a sinusoidal signal as follows.

\[
s_{l,m}[k] \triangleq A_{l,m} \exp(-\frac{\sigma_l}{f_s} k) \cos(\frac{\omega_l}{f_s} k + \phi_{l,m}) + \epsilon_m[k],
\]

where \( B_{l,m} \) and \( B^*_{l,m} \) represent the sinusoidal signals.
Consider the evolution of the sinusoid signal as follows.

\[
s_{l,m}[k+1] = \exp(-\frac{\sigma_l}{f_s} k) B_{l,m} \cos(\frac{\omega_l}{f_s} k + \phi_{l,m}) + \epsilon_m[k],
\]

Define system states as signal magnitudes, frequencies, and
damping factors as follows.

\[
x_{l,m}[k] = \begin{bmatrix} x_{l,m}^T[k] \\ y_{l,m}^T[k] \end{bmatrix} = \begin{bmatrix} B_{l,m} \exp(-\sigma_l k/f_s) \cos(\omega_l k/f_s) \\ B_{l,m} \exp(-\sigma_l k/f_s) \sin(\omega_l k/f_s) \end{bmatrix},
\]

The state transition is presented as follows.

\[
x_{l,m}[k+1] = \begin{bmatrix} x_{l,m}^T[k+1] \\ y_{l,m}^T[k+1] \end{bmatrix} = \begin{bmatrix} B_{l,m} \exp(-\sigma_l k/f_s) \cos(\omega_l k/f_s) \\ B_{l,m} \exp(-\sigma_l k/f_s) \sin(\omega_l k/f_s) \end{bmatrix},
\]

where \( \epsilon \) is the system noise. The observation equation
is then written as

\[
y_m[k] = \sum_{l=1}^{L} (x_{l,m}^T[k] + x_{l,m}^T[k]) + \epsilon_m[k].
\]

Denote the amplitude vector of modes measured by the
\( m \)th PMU by \( \alpha_m[k] = [x_{1,m}[k], \cdots, x_{L,m}[k]]^T. \)
Define the state of the system as \( x[k] = \alpha_m[k], \cdots, \omega_m[k], \sigma_1[k], \cdots, \omega_M[k], \sigma_M[k]^T, \)
which has a dimension of \( (2ML + 2L) \)-by-1 and is composed
of two parts. The first part, \( \alpha_1[k], \cdots, \omega_M[k], \)
is the magnitude of oscillation modes measured at different
PMUs. The second part, \( \omega_1 \) and \( \sigma_1 \), is the frequency and damping factor of each
mode and this part is the consensus across different PMUs.

We can write the transition in a general form as follows.

\[
x[k+1] = f(x[k]) + \epsilon[k]
\]

where the transition function \( f(\cdot) \) is nonlinear and can be
derived from equation (2). We assume that \( \epsilon[k] \) is a white
Gaussian noise with zero mean and covariance matrix \( \boldsymbol{Q}_k. \)

Given the system states and equation (1), we obtain the
observation function as follows.

\[
y[k] = \boldsymbol{H} \alpha[k] + \epsilon[k]
\]

where

\[
\boldsymbol{H} = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_M \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_M \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & \cdots & 0 & 0 \end{bmatrix}
\]

In practice, each PMU may contain multiple channels. For notation
simplicity, we assume each PMU contains single channel measurements.
Results can be easily extended to the case of multi-channel measurements.
Here \( H_i \in \mathbb{R}^{1 \times (2ML + 2L)} \) is the \( i \)th row of the observation matrix \( H \). The \((2(i-1)L + 1)\)th to \(2iL\)th elements in \( H_i \) are 1s and others are zeros. Thus, the constructed system is summarized as follows.

\[
x[k+1] = f(x[k]) + \varepsilon[k] \\
y[k] = Hx[k] + \epsilon[k]
\]

(3)

### III. Centralized Extended Kalman Filter

In this section, a centralized framework of EKF is considered as shown in Figure [1]. At each time instant, new measurements are collected and sent from PMUs to a control center. The control center carries out a centralized extended Kalman filter to estimate the system state \( x[k] \) based on all data across the system.

Given the system equations (3), we apply an extended Kalman filter to estimate the system state. Kalman filter (KF) is a recursive algorithm to estimate the state of a linear dynamic system based on a series of noisy measurements. Based on the dynamic model, the KF predicts the priori state into the future and computes the difference between the predictions and the measurements. Then KF updates the posteriori estimation using the optimal Kalman gain and repeats the process. With white noises, Kalman filter minimizes the mean squared estimation error.

When the dynamic system is nonlinear, the extended Kalman filter can be applied. Around the current estimated state, the EKF approximates the nonlinear system by a first order linearization and applies the KF to the linearized system to find the optimal Kalman gain. The nonlinear system model and new measurements are used to calculate new state predictions. This process iterates and the state space model is re-linearized around updated state estimates.

#### A. Centralized Extended Kalman Filter

Let \( \hat{x}[k|j] \) denote the minimum mean squared error estimate of \( x[k] \) given measurements up to and including time \( j \) and \( P[k|j] \) the covariance matrix of the estimation error. Starting from the initial estimate \( \hat{x}[0|-1] \) and \( P[0|-1] \), the iteration of the extended Kalman filter for the system equation (3) is summarized in Algorithm 1

**Algorithm 1 Centralized Extended Kalman Filter (CEKF)**

1. Initialize \( \hat{x}[0|-1] \) and \( P[0|-1] \).
2. for \( k = 0 : N - 1 \)

\[
S = R_k + HP[k|k-1]HT \\
K = P[k|k-1]HTS^{-1} \\
\hat{x}[k|k] = \hat{x}[k|k-1] + K(y[k] - H\hat{x}[k|k-1]) \\
P[k|k] = P[k|k-1] - KHHP[k|k-1] \\
\hat{x}[k+1|k] = f(\hat{x}[k|k]) \\
P[k+1|k] = F_k P[k|k] F_k^T + Q_k
\]

end

Here \( F_k = \frac{\partial f(x)}{\partial x}|_{x = \hat{x}[k|k]} \) is the linearization of the system, and \( N \) is the time length of measurements. The prediction process \( f(\hat{x}[k|k]) \) is stated as follows.

\[
f(\hat{x}[k|k]) = \begin{bmatrix}
\hat{\alpha}_1[k+1|k] \\
\vdots \\
\hat{\alpha}_M[k+1|k] \\
\end{bmatrix} = \begin{bmatrix}
\hat{x}_{1,1}[k+1|k] \\
\vdots \\
\hat{x}_{L,M}[k+1|k] \\
\end{bmatrix}
\]

where

\[
\hat{x}_{l,m}[k+1|k] = \begin{bmatrix}
\hat{x}_{l,m}[k+1|k] \\
\hat{x}_{l,m}[k+1|k] \\
\end{bmatrix} \\
= \begin{bmatrix}
\exp(-\hat{\sigma}_L[k|k])\hat{x}_{l,m}[k|k]\cos(\hat{\omega}_l[k|k]) - \hat{x}^e_{l,m}[k|k]\sin(\hat{\omega}_l[k|k]) \\
\exp(-\hat{\sigma}_L[k|k])\hat{x}_{l,m}[k|k]\sin(\hat{\omega}_l[k|k]) + \hat{x}^e_{l,m}[k|k]\cos(\hat{\omega}_l[k|k]) \\
\end{bmatrix} \\
\hat{\omega}_l[k+1|k] = \hat{\omega}_l[k|k] \\
\hat{\sigma}_l[k+1|k] = \hat{\sigma}_l[k|k]
\]

#### B. Initial Point and Coefficient Choice

The accuracy and convergence of EKF rely heavily on the choice of initial points. In the context of oscillation estimation, a Fast Fourier Transform (FFT) or other similar technology can be employed as a trigger and the result can be used as a choice of initial points. FFT can estimate the spectral of sinusoids with limited measurements and alarm the operator with potential oscillations if the spectral of some frequency differs from noises significantly. Theses results can be used as inputs to Algorithm 1 and EKF will estimate the accurate frequency and damping factors. Other approaches such as singular value decomposition (SVD) [12] can be applied to increase the confidence of the initial values.

Another possible choice is to use a look-up table, which can be built according to system operators’ knowledge of the system and its typical oscillation modes. These modes can serve as the initial estimates which are fed into the EKF algorithm.

The proposed EKF algorithm is a model-based method and its performance relies on the proper choice of coefficients. Tuning of the covariance matrix of noise, \( Q_k \) and \( R_k \), is the major approach to adjust the performance of EKF. A large \( Q_k \) or a small \( R_k \) usually causes fluctuation around the actual
value, while a small $Q_k$ or a large $R_k$ normally results in poor tracking. In this work, the tuning of coefficients is based on heuristic.

IV. DISTRIBUTED EXTENDED KALMAN FILTER

In this section, we consider a distributed framework of EKF. Note that, the term “PMU” in this section is in a much broader sense. It refers to any agent that can collect measurement data, communicate with other “PMUs”, and carry out algorithms. A PMU in this work can be an actual PMU device, a PDC, a super PDC, or a data center.

As shown in Figure 2, no control center is needed in the fully distributed framework. At each time step, PMUs communicate with their neighbor(s), and EKF is carried out at each PMU. Under this framework, each PMU has its estimation, $\hat{x}_m = [\hat{a}_{1,m}, \cdots, \hat{a}_{M,m}, \hat{\omega}_{1,m}, \hat{\sigma}_{1,m}, \cdots, \hat{\omega}_{M,m}, \hat{\sigma}_{M,m}]^T$, of the system state. The objective is to design algorithms to make the estimation converge to the actual value.

![Figure 2](image)

Fig. 2: A fully distributed framework. PMUs communicate with their neighbour(s). EKF is carried out based on local information at each PMU.

We formulate the topology of PMUs as a graph. Consider an undirected graph $G = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N} = \{1, \ldots, M\}$ represents the PMU set and $\mathcal{E}$ the edge set. Each edge $(i, j) \in \mathcal{E}$ represents that PMU $i$ and PMU $j$ can communicate with each other. We define the set of nodes connected to a certain PMU $i$ as the neighbors of $i$, denoted by $\mathcal{N}_i = \{j \in \mathcal{N} : (i, j) \in \mathcal{E}\}$. A PMU is always a neighbor of itself. The number of neighbors of PMU $m$ is referred to as degree, denoted by $|\mathcal{N}_m|$. Here we assume the graph is connected.

We extend a distributed Kalman filter framework proposed by [19], referred to as diffusion Kalman filter. The diffusion Kalman filter attempts to approximate the global KF estimation by local information. As shown in Figure 3(a) each PMU carries out EKF to obtain a pre-estimation of the system state, $\varphi_m$. Then PMUs broadcast its pre-estimation to its neighbour(s) and updates its estimate $\hat{x}_m$ by diffusion of all the pre-estimation collected from its neighbour(s). The diffusion EKF algorithm is described in Algorithm 2.

![Figure 3](image)

Fig. 3: Information exchanges at PMU 4.

PMU takes new measurements and collects new information from its neighbour(s). Based on this local information, each PMU updates its state estimate $\hat{x}_m[k]$ and covariance $P_m[k]$. The diffusion EKF algorithm is described in Algorithm 2.

Algorithm 2 Diffusion EKF (DEKF)

1. Initialize $\hat{x}_m[0] = \mathbf{0}$ and $P_m[0] = \mathbf{1}$ for each PMU $m$.
2. for $k = 0 : N - 1$
   Each PMU sends measurements to its neighbour(s).
   Incremental Update
   for $m = 1 : M$
   $\varphi_m = \hat{x}_m[k | k - 1]$
   $P_m = P_m[k | k - 1]$
   $S = R_j + H_j P_m H_j^T$
   $K = P_m H_j^T S^{-1}$
   $\varphi_m = \varphi_m + K(y_j[k] - H_j \varphi_m)$
   $P_m = P_m - K H_j P_m$
   end
   Each PMU sends pre-estimation to its neighbour(s).
   Diffusion Update
   for $m = 1 : M$
   $\hat{x}_m[k | k] = \sum_{j \in \mathcal{N}_m} c_{m,j} \varphi_j$
   $P_m[k | k] = F_{m,k} P_m[k | k] F_{m,k}^T + Q_k$
   end

Here $F_{m,k} = \frac{\partial f(x_k)}{\partial x_k} |_{x = \hat{x}_m[k | k]}$, and $c_{m,j}$ is a diffusion factor and satisfies the following properties.

$$\sum_{j \in \mathcal{N}_m} c_{m,j} = 1$$
$$c_{m,j} \geq 0 \quad (4)$$

The diffusion of the pre-estimation is a weighted average. Note that, the diffusion update is not taken into account in the recursion of the matrices $P_m[k | k - 1]$ and $P_m[k | k]$, and they are no longer the covariance matrix of the state estimation.

A. State Reduction

In Algorithm 2 each PMU estimates the entire states of the system, including amplitudes of signals from PMUs that are not its neighbour(s). In the incremental update, for each PMU, only its neighbors’ states are updated while the rest stay unchanged, which makes the process slow and the communication inefficient. Here we propose a state reduction framework of the DEKF to enhance the performance. Define the reduced estimate made by the $n$th PMU as $\hat{x}_n = [\hat{a}_{m,j}, \cdots, \hat{a}_{m,j} | j \in \mathcal{N}_m, \hat{\omega}_n, \hat{\sigma}_n]^T$, where $\mathcal{N}_m = \{j_1, \cdots, j_{|\mathcal{N}_m|}\}$ is the neighbor set of the $n$th PMU, $\hat{a}_{m,j}$ is the estimate of the $j$th PMU’s amplitudes by the $n$th PMU, and $\hat{\omega}_n$ is the estimate of the frequency and damping.
factor part. The reduced estimate is a $(2|\mathcal{N}_m|L+2L)$-by-
1 dimension vector. Frequencies and damping factors of all 
modes and amplitudes of all neighboring PMUs are included 
while those of the non-neighboring ones are excluded.

In this case, the observation matrix $H$, estimation covariance 
matrix $P_m[k|i]$, process noise covariance $Q_k$, system functions 
$f(x)$ and Jacobian matrix $F_{m,k}$ are modified to $H_m$, $P_m[k|i]$, 
$Q_{m,k}$, $R_{m,k}$, $f_m(x)$ and $F_{m,k}$ accordingly. For the $n$th PMU, 
define the observation matrix $H_m \in \mathbb{R}^{|\mathcal{N}_m| \times (2|\mathcal{N}_m|L+2L)}$ as

$$
H_m = \begin{bmatrix}
H_{m,j1} & H_{m,j2} \\
\vdots & \\
1 1 \cdots 0 0 \cdots \cdots 0 0 \cdots 0 0 \\
0 0 \cdots 1 1 \cdots \cdots 0 0 \cdots 0 0 \\
\vdots & \\
0 0 \cdots 0 0 \cdots \cdots 1 1 \cdots 0 0
\end{bmatrix},
$$

where $H_{m,j1}$ is the $i$th row of $H_m$ with the $(2(i-1)+L+1)$th 
to $2iL$th elements being 1s and the rest being zeros. The formulas 
of other matrices are derived accordingly and omitted here.

The diffusion EKF under the reduced state framework is 
similar to Algorithm 3. Each PMU receives measurements from 
its neighbour(s) and estimates accordingly. After obtaining 
the pre-estimates, PMUs comminute this information and 
make a diffusion to update its estimate. However, each PMU 
only maintains amplitude estimates of its neighbour(s) and the 
diffusion is carried out across neighbour(s) who estimates 
the same amplitudes.

Recall that the reduced estimation $\hat{x}_m$ by the $n$th PMU is 
comprised of the amplitudes of its neighbors and the frequency 
and damping factor part. Denote $\varphi_m$ as the pre-estimate of 
$\hat{x}_m$, and $\varphi_m$ as the one of $\hat{x}_m$. The reduced state diffusion EKF is summarized in Algorithm 3

Here $d_{m,j,i}$ and $c_{m,j}$ are diffusion factors, where $c_{m,j}$ 
satisfies properties stated in equation (4) and $d_{m,j,i}$ satisfies 
the following properties.

$$
\sum_{i \in \mathcal{N}_m \cap \mathcal{N}_j} d_{m,j,i} = 1
$$

V. NUMERICAL RESULTS

In this section, we present numerical results using both 
simulated and real PMU data collected from real-world system 
oscillation events. We first apply the proposed algorithms on 
a noisy ring down sinusoid signal and compare the accuracy of 
the proposed algorithms with PRONY [11] and ADMM-
PRONY [17], a decentralized extension of PRONY. Then we 
test EKF and DEKF-R using a test case library [20]. After that, 
real oscillation data from Jiangsu Electric Power Company in 
China are examined.

Algorithm 3: Diffusion EKF with reduced state (DEKF-R)

1. Initialize $\hat{x}_m[0|0]$ and $P_m[0|0]$ for each PMU $m$.
2. for $k = 0 : N - 1$
   Each PMU sends measurements to its neighbour(s).
   Incremental Update
   for $m = 1 : M$
   
   $$
   \varphi_m = \hat{x}_m[k|k-1],
   P_m = P_m[k|k-1];
   $$
   for $j \in \mathcal{N}_m$
   
   $$
   S = R_{m,j} + H_m P_m (H_m)^T,
   K = P_m (H_m)^T S^{-1},
   \varphi_m = \varphi_m + K (y_j[k] - H_m \varphi_m),
   P_m = P_m - K H_m P_m.
   $$
   end
   end
   
   Each PMU sends pre-estimation to its neighbour(s).
   Diffusion Update
   for $m = 1 : M$
   for $j \in \mathcal{N}_m$
   
   $$
   \hat{a}_{m,j}[k|k] = \sum_{i \in \mathcal{N}_m \cap \mathcal{N}_j} d_{m,j,i} \varphi_{i,j}^a,
   $$
   end
   
   $$
   \hat{x}_m[k|k] = \sum_{i \in \mathcal{N}_m \cap \mathcal{N}_j} c_{m,j} \hat{a}_{m,j}[k|k],
   \hat{x}_m[k|k] = \begin{bmatrix} \hat{a}_{m,j_1}[k|k], \cdots, \hat{a}_{m,j_{|\mathcal{N}_m|}}[k|k], \hat{x}_m[k|k] \end{bmatrix}^T,
   P_m[k|k] = P_m,
   \hat{x}_m[k+1|k] = f_m(\hat{x}_m[k|k]),
   P_m[k+1|k] = F_{m,k} P_m[k|k] (F_{m,k})^T + Q_{m,k},
   $$
   end
   end
   
A. Ring Down Sinusoids with Different Noise Levels

In this case, the measurement is an exponentially damped 
sinusoid with a zero-mean white Gaussian noise stated as 
follows.

$$
y_m[k] = m \left[ \exp(-\sigma k / f_s) \cos(\omega / f_s k + \phi_m) + \varepsilon_m[k] \right],
$$

where the frequency is $\omega = 4\pi$ rad/s, the damping factor 
$\sigma = 0.0126$, $\phi_m$ the phase angle, and $\varepsilon_m[k]$. The 
corresponding damping ratio is $\zeta = \sigma / \sqrt{\sigma^2 + \omega^2} = 0.1\%$. The phase angle $\phi_m$ is assumed to be uniformly distributed within $[-\pi/2, \pi/2]$. The total number of PMUs is set to be $M = 5$.
The amplitudes of PMUs are made different to model real-world 
signals from power systems. The sample rate $f_s$ is 
selected to be 30Hz and the length of the time window is 
set as 10 seconds.

For the centralized EKF algorithm, the initial point is 
assumed to be uniformly distributed within a $[-70\%, 130\%]$ ange of the real value. The filter parameters are selected as 
$R_k = 10^{-3} I$ where $I$ is an identity matrix with a proper 
dimension. $Q_k$ is a diagonal matrix whose first $2ML$ diagonal
TABLE I: Single sinusoid with different noise levels

| Error   | Freq | Damping | Freq | Damping |
|---------|------|---------|------|---------|
| Mean (PRONY) | 0.00% | 4.85%   | 0.00% | 1.51%   |
| Std (PRONY)  | 0.00% | 3.84%   | 0.00% | 1.17%   |
| Mean (ADMM)  | 0.00% | 0.84%   | 0.00% | 2.67%   |
| Std (ADMM)   | 0.00% | 0.74%   | 0.00% | 2.03%   |
| Mean (EKF)   | 0.00% | 1.54%   | 0.00% | 1.71%   |
| Std (EKF)    | 0.00% | 1.42%   | 0.00% | 1.24%   |
| Mean (DEKF)  | 0.02% | 2.01%   | 0.07% | 7.25%   |
| Std (DEKF)   | 0.02% | 2.73%   | 0.02% | 5.63%   |
| Mean (DEKF-R)| 0.00% | 3.33%   | 0.05% | 6.95%   |
| Std (DEKF-R)| 0.00% | 2.41%   | 0.03% | 4.05%   |

For the distributed EKF algorithm, the communication topology is illustrated in Figure 2 where the communication path of five PMUs forms a connected graph with each PMU communicating with its neighbor(s) only. The initial estimate at each PMU is assumed to be uniformly distributed within a [−70%, 130%] range of its corresponding true value. Diagonal elements in the measurement noise covariance matrix $R_{m,k}$ are around $10^{-4}$ and the process noise covariance $Q_{m,k}$ is a diagonal matrix whose first $2|N_m|L$ diagonal elements are zeros and last $2L$ diagonal elements are around $10^{-8}$.

The authors in [17] extended PRONY algorithm to a decentralized version, referred as ADMM-PRONY. Under this framework, a system coordinator is still needed who collects information across the entire system. Each PMU first carries out a linear estimation locally based on private measurements and sends the coordinator estimation results. The coordinator collects all estimates, averages them and broadcasts the diffusion back to PMUs. Given this global diffusion, each PMU solves a quadratic optimization to trade off the estimation accuracy and the tracking error of the diffusion, weighted by $\rho$, and sends the updated estimate to the coordinator. The iteration continues till the diffusion converges. Compared to the centralized PRONY, ADMM algorithms require exchange of the estimates rather than the PMU measurements between PMUs and the coordinator, which relieves greatly the communication burden. However, the requirement of a system-level coordinator makes it not fully distributed, and the system will be vulnerable and subject to a single point of failure at the coordinator. In this simulation, the weight of tracking error is set $\rho = 0.01$ and the convergent tolerance is set as 0.01.

One thousand Monte Carlo runs for each level of noise are carried out. Mean and standard deviations of estimation error are summarized in Table II. It is shown that, performance of distributed EKF is close to PRONY. The accuracy of distributed EKF with reduced state is comparable with the one of ADMM, while DEKF-R requires no coordinator and no global information exchange. With a low level of noise, both EKF and PRONY work well. DEKF-R gives slightly bigger error as compared to ADMM due to less information exchange, but the errors are all within the acceptable range. As the level of noise increases, EKF starts to outperform PRONY, both in the accuracy and stability of the estimates. ADMM is based on PRONY and also sensitive to noise and performs worse than DEKF when SNR is as large as 20db. Another observation to note is that DEKF-R dominates DEKF which indicates the effectiveness of state reduction.

B. WECC Test Case Library

Authors of [20] established a test case library for oscillation detection and forced oscillation source location in power systems. A reduced WECC 179-bus 29-machine system is simulated in TSA with integration a step size around 0.004s and sampled at rate 30Hz. All generators are presented by a classical second-order differential model with damping parameter equals to 4.

In each test case, damping parameters of some generators are set such that they are poorly or negatively damped. Taking the first test case as an example, the damping factor of generator 45 and 159 are set to be $-2$ and 1, respectively. At 0.5 second, a three-phase short circuit is added at bus 159 and cleared by 0.55 second to trigger oscillations in the system. As can be found in Figure 4(a) before 0.5 second, the speed of rotors remains at 60Hz. After the fault at 0.5 second, all measurements begin to oscillate with different magnitudes and shift phase angles.

In this case, we first apply FFT to determine the number of modes and generate initial estimates for EKF methods. Feed FFT with speed data of rotor 30 from 0.7 second to 10.7 second, and the spectral shows there are two modes of frequency 0.7Hz and 1.4Hz, respectively. Set the initial estimates at $\omega_1 = 0.7$, $\omega_2 = 1.4$ and $\sigma_1 = \sigma_2 = 0$ and run centralized EKF with parameters $Q_k = 10^{-9}I$ and $R_k = 10^{-3}I$. The estimation results are summarized in Table II. It can be found that EKF successfully identify the poorly damped frequency at 1.4Hz, and the difference between the measurements and fitted signal is small. The measurements and fitted speed curve of rotor 159 are plotted in Figure 4(c) as an example.

TABLE II: Test case 1: 2 oscillation modes

| Estimate   | $\omega$ | $\sigma$ | $\omega$ | $\sigma$ |
|------------|----------|----------|----------|----------|
| CEKF       | 1.4016   | 0.0016   | 0.6927   | 0.4715   |
| DEKF-R     | 1.3999   | 0.0011   | 0.6892   | 0.2761   |

Substitute the same initial points to the proposed Algorithm 3 assuming that the communication of PMUs forms a circle like the one shown in Figure 2 and each PMU communicates with its two neighbours. As shown in Figure 4(c), the tracking error of rotor 159 is larger than the one of centralized EKF due to the lack of global information. But the difference is still small and within the acceptable range.

C. Real PMU Data from Jiangsu Electric Power Company

Jiangsu Electric Power Company, one of the largest provincial power company in China, has installed generation capacity...
of 100GW and peak load of 92GW. Over 160 PMUs, with thousands of measurement channels, have been installed in the Jiangsu system. These PMUs cover all 500kV substations, a majority of the 220kV substations, major power plants, and all renewable power plants. In this subsection, PMU data collected from real system oscillation events are used to validate the proposed oscillation monitoring algorithms.

1) Case 1: In this case, real and reactive power, voltage magnitude and angle, current magnitude and angle, and rotor speed and angle measurements from more than 160 PMUs are collected, at a reporting rate of 25Hz. As shown in Fig. 5(a), an oscillation mode with frequency around 1Hz lasts for 25 seconds. We remove the DC component in the data, carry out the normalization and feed the data from 60 to 80 seconds to FFT. As shown in Figure 5(b), FFT results suggest a dominant mode around 0.67Hz. Using this result as initial estimates, the proposed EKF algorithms are carried out, and estimation results can be found in Table IV and Figure 6. The estimated frequencies are around 1.14Hz and 4.97Hz, with negative damping. Figure 6(c) and 6(d) show that the proposed methods perform well with multiple modes.

2) Case 2: In this case, the voltage signal oscillates more than 60 seconds. Use a 10 seconds window of data and repeat the same process. FFT suggests that there are two oscillation modes around 1Hz and 5Hz, respectively. Given this initial estimates, the proposed EKF algorithms are carried out, and estimation results can be found in Table IV and Figure 6. The estimated frequencies are around 1.14Hz and 4.97Hz, with negative damping. Figure 6(c) and 6(d) show that the proposed methods perform well with multiple modes.

| TABLE III: Jiangsu data case 1: 1 oscillation mode |
|-----------------------------------------------|
| Estimate | ω   | σ  |
| CEKF     | 0.6513 | 0.017 |
| DEKF-R   | 0.6438 | 0.0163 |

| TABLE IV: Jiangsu Data case 2: 2 oscillation modes |
|-----------------------------------------------|
| Estimate | ω   | σ  | ω   | σ  |
| CEKF     | 1.1461 | -0.0215 | 4.9749 | -0.0222 |
| DEKF-R   | 1.1450 | -0.0203 | 4.9494 | -0.037 |
Oscillation monitoring is essential in power systems to detect events and help system operators to identify the causes and locations of events. Conventional centralized algorithms put heavy burdens on data communications infrastructure and suffer from single point of failure and data privacy problem. A novel distributed EKF based algorithm is proposed to estimate oscillation frequency and damping ratios directly. The fully distributed framework makes it possible to estimate at a fast reporting rate without information disclosure concerns. The effectiveness of the proposed algorithm is demonstrated using both simulated and real data.

VI. Conclusion

Oscillation monitoring is essential in power systems to detect events and help system operators to identify the causes and locations of events. Conventional centralized algorithms put heavy burdens on data communications infrastructure and suffer from single point of failure and data privacy problem. A novel distributed EKF based algorithm is proposed to estimate oscillation frequency and damping ratios directly. The fully distributed framework makes it possible to estimate at a fast reporting rate without information disclosure concerns. The effectiveness of the proposed algorithm is demonstrated using both simulated and real data.

REFERENCES

[1] North American SynchroPhasor Initiative. “Use of IEC 61850-90-5 to Transmit Synchrophasor Information According to IEEE 37.118,” NASPI, Tech. Rep., Aug. 2014. [Online]. Available: https://www.naspi.org/sites/default/files/reference_documents/48.pdf

[2] D. of Energy, “Advancement of SynchroPhasor Technology in Projects Funded by the American Recovery and Reinvestment Act of 2009,” DoE, Tech. Rep., Mar. 2016. [Online]. Available: https://www.smartgrid.gov/document/Synchrophasor_Report_201603.html

[3] M. Yazdianian, A. Mehrizi-Sani, and M. Mojiri, “Estimation of electromechanical oscillation parameters using an extended kalman filter,” IEEE Transactions on Power Systems, vol. 30, no. 6, pp. 2994–3002, 2015.

[4] T. K. Sarkar and O. Pereira, “Using the matrix pencil method to estimate the parameters of a sum of complex exponentials,” IEEE Antennas and Propagation Magazine, vol. 37, no. 1, pp. 48–55, 1995.

[5] M. Bouzou, S. Lefebvre, and R. Malhame, “A spectral algorithm for extracting power system modes from time recordings,” IEEE Transactions on Power Systems, vol. 7, no. 2, pp. 665–683, 1992.

[6] J.-N. Juang and R. S. Pappa, “An eigensystem realization algorithm for modal parameter identification and model reduction,” Journal of Guidance, Control, and Dynamics, vol. 8, no. 5, pp. 620–627, 1985.

[7] L. D. Peterson, “Efficient computation of the eigensystem realization algorithm,” Journal of Guidance, Control, and Dynamics, vol. 18, no. 3, pp. 395–403, 1995.

[8] J. Sanchez-Gasca and J. Chow, “Computation of power system low-order models from time domain simulations using a hankel matrix,” IEEE Transactions on Power Systems, vol. 12, no. 4, pp. 1461–1467, 1997.

[9] D. Ruiz-Vega, A. R. Messina, and G. Enríquez-Harper, “Analysis of interarea oscillations via non-linear time series analysis techniques,” in Proc. of 15th Power Systems Computation Conf, 2005.

[10] J. F. Hauer, C. Demeure, and L. Scharf, “Initial results in proxy analysis of power system response signals,” IEEE Transactions on Power Systems, vol. 5, no. 1, pp. 80–89, 1990.

[11] D. Tradnowski, J. Johnson, and J. Hauer, “Making proxy analysis more accurate using multiple signals,” IEEE Transactions on Power Systems, vol. 14, no. 1, pp. 226–231, 1999.

[12] J. C.-H. Peng and N.-K. C. Nair, “Enhancing kalman filter for tracking ringdown electromechanical oscillations,” IEEE Transactions on Power Systems, vol. 27, no. 2, pp. 1042–1050, 2012.

[13] T. Jiang, I. Matei, and J. Baras, “A trust based distributed kalman filtering approach for mode estimation in power systems,” in Proc. of the First Workshop on Secure Control Systems, 2010.

[14] N. Zhou, Z. Huang, F. Tufnfer, J. Pierre, and S. Jin, “Automatic implementation of proxy analysis for electromechanical mode identification from phasor measurements,” in Proc. of Power and Energy Society General Meeting, Providence, RI, USA, 2010, pp. 1–8.

[15] S. Kar and G. Hug, “Distributed robust economic dispatch in power systems: A consensus + innovations approach,” in Proc. of Power and Energy Society General Meeting, San Diego, CA, USA, 2012, pp. 1–8.

[16] Z. Zhang and M.-Y. Chow, “Convergence analysis of the incremental cost consensus algorithm under different communication network topologies in a smart grid,” IEEE Transactions on Power Systems, vol. 27, no. 4, pp. 1761–1768, 2012.

[17] S. Nabavi, J. Zhang, and A. Chakrabortty, “Distributed optimization algorithms for wide-area oscillation monitoring in power systems using interregional pmu-pdc architectures,” IEEE Transactions on Smart Grid, vol. 6, no. 5, pp. 2529–2538, 2015.

[18] P. Korba, “Real-time monitoring of electromechanical oscillations in power systems: first findings,” IET Generation, Transmission & Distribution, vol. 1, no. 1, pp. 80–88, 2007.

[19] F. S. Cattivelli and A. H. Sayed, “Diffusion strategies for distributed kalman filtering and smoothing,” IEEE Transactions on Automatic Control, vol. 55, no. 9, pp. 2069–2084, 2010.

[20] S. Maslennikov, B. Wang, Q. Zhang, E. Litvinov et al., “A test cases library for methods locating the sources of sustained oscillations,” in Proc. of Power and Energy Society General Meeting, Boston, MA, USA, 2016, pp. 1–5.