A novel image reconstruction method applied to deep Hubble Space Telescope images

A. S. Fruchter\textsuperscript{a} and R. N. Hook\textsuperscript{b}
\textsuperscript{a}Space Telescope Science Institute
3700 San Martin Drive
Baltimore, MD 21210 USA
\textsuperscript{b}Space Telescope European Coordinating Facility
D-85748 Garching Germany

ABSTRACT
We have developed a method for the linear reconstruction of an image from undersampled, dithered data, which has been used to create the distributed, combined Hubble Deep Field images – the deepest optical images yet taken of the universe. The algorithm, known as Variable-Pixel Linear Reconstruction (or informally as “drizzling”), preserves photometry and resolution, can weight input images according to the statistical significance of each pixel, and removes the effects of geometric distortion both on image shape and photometry. In this paper, the algorithm and its implementation are described, and measurements of the photometric accuracy and image fidelity are presented. In addition, we describe the use of drizzling to combine dithered images in the presence of cosmic rays.

Keywords: image reconstruction, image restoration, undersampled images, Hubble Space Telescope

1. INTRODUCTION
The Hubble Space Telescope (HST) is now capable of providing the superb images for which it was designed. However, the detectors on HST are only able to take full advantage of the full resolving power of the telescope over a limited field of view. In particular, the primary optical imaging camera on the HST, the Wide Field and Planetary Camera\textsuperscript{2},\textsuperscript{3} is composed of four separate 800x800 pixel CCD cameras, one of which, the planetary camera (PC) has a scale of 0\textquoteleft 046 per pixel, while the other three, arranged in a chevron around the PC, have a scale of 0\textquoteleft 097 per pixel. These latter three cameras, referred to as the wide field cameras (WFs), are the primary workhorse for deep imaging surveys on HST. However, these cameras greatly undersample the HST image. The width of a WF pixel equals the full-width a half maximum of the optics in the the near-infrared, and greatly exceeds it in the blue. The effect of undersampling on WF images is illustrated by the “Great Eye Chart in the Sky” in Figure 1.

Fortunately, much of the information lost to undersampling can be restored. In the lower right of Figure 1 we display a restoration made using one of the family of techniques we refer to as “linear reconstruction.” The most commonly used of these techniques are shift-and-add and interlacing. The image in the lower right corner has been restored by interlacing dithered images. However, due to the occasional small positioning errors of telescope and the non-uniform shifts in pixel space caused by the geometric distortion of the optics, true interlacing of HST images is often infeasible. The other standard linear reconstruction technique, shift-and-add can easily handle arbitrary dither postions, but it convolves the image yet again with the orginal pixel, adding to the blurring of the image and the correlation of the noise. The importance of avoiding unnecessarily convolving the image with the pixel is emphasized by comparing the upper and lower right hand images in Figure 1. The deterioration in image quality is due entirely to convolution of the image by the WF pixel. Here we present a new method which has the versatility of shift-and-add yet largely maintains the resolution and independent noise statistics of interlacing.

2. THE METHOD
The Drizzle algorithm is conceptually straightforward. Pixels in the original input images are mapped into pixels in the subsampled output image, taking into account shifts and rotations between images and the optical distortion of the camera. However, in order to avoid convolving the image with the large pixel “footprint” of the camera, we allow the user to shrink the pixel before it is averaged into the output image, as shown in Figure 2.
Figure 1. In the upper left corner of this figure, we present the “true image”, i.e. the image one would see with an infinitely large telescope. The upper right shows the image after convolution with the optics of the Hubble Space Telescope and the WFPC2 camera – the primary wide-field imaging instrument presently installed on the HST. The lower left shows the image after sampling by the WFPC2 CCD, and the lower right shows a linear reconstruction of dithered CCD images.
**Figure 2.** A schematic representation of Drizzling. The input pixel grid (shown on the left) is mapped onto a finer output grid (shown on right), taking into accounts shift, rotation and geometric distortion. The user is allowed to "shrink" the input pixels to smaller pixels, which we refer to as drops (faint inner squares). A given input image only affects output image pixels under drops. In this particular case, the central output pixel receives no information from the input image.

The new shrunken pixels, or "drops", rain down upon the subsampled output. In the case of the HDF, the drops used had linear dimensions one-half that of the input pixel — slightly larger than the dimensions of the output subsampled pixels. The value of an input pixel is averaged into an output pixel with a weight proportional to the area of overlap between the “drop” and the output pixel. Note that if the drop size is sufficiently small not all output pixels have data added to them from each input image. One must therefore choose a drop size that is small enough to avoid degrading the image, but large enough so that after all images are “dripped” the coverage is fairly uniform.

The drop size is controlled by a user-adjustable parameter called \( \text{pixfrac} \), which is simply the ratio of the linear size of the drop to the input pixel (before any adjustment due to the geometric distortion of the camera). Thus interlacing is equivalent to setting \( \text{pixfrac} = 0.0 \), while shift-and-add is equivalent to \( \text{pixfrac} = 1.0 \).

When a drop with value \( i_{xy} \) and user defined weight \( w_{xy} \) is added to an image with pixel value \( I_{xy} \), weight \( W_{xy} \), and fractional pixel overlap \( 0 < a_{xy} < 1 \), the resulting value of the image \( I'_{xy} \) and weight \( W'_{xy} \) is

\[
W'_{xy} = a_{xy} w_{xy} + W_{xy} \tag{1}
\]

\[
I'_{xy} = \frac{a_{xy} i_{xy} w_{xy} + I_{xy} W_{xy}}{W'_{xy}} \tag{2}
\]

This algorithm has a number of advantages over the more standard linear reconstruction methods presently used. Since the area of the pixels scales with the Jacobian of the geometric distortion, this algorithm preserves both surface and absolute photometry. Therefore flux can be measured using an aperture whose size is independent of position on the chip. As the method anticipates that a given output pixel may receive no information from a given input
pixel, missing data (due for instance to cosmic rays or detector defects) does not cause a substantial problem, so long as there are enough dithered images to fill in the gaps caused by these zero-weight input pixels. Finally, the linear weighting scheme is statistically optimum when inverse variance maps are used as weights.

3. Image Fidelity

The drizzling algorithm was designed to obtain optimal signal-to-noise on faint objects while preserving image resolution. These goals are unfortunately not fully compatible. For example non-linear image restoration procedures which attempt to remove the blurring of the PSF and the pixel by enhancing the high frequencies in the image (such as such as the Richardson-Lucy[3,4,5] and maximum entropy methods[6,7]) directly exchange signal-to-noise for resolution. In the drizzling algorithm no compromises on signal-to-noise have been made; the weight of an input pixel in the final output image is entirely independent of its position on the chip. Therefore, if the dithered images do not uniformly sample the field, the “center of light” in an output pixel may be offset from the center of the pixel, and that offset may vary between adjacent pixels. The large dithering offsets which may be used in WFPC2 imaging combined with geometric distortion can produce a sampling pattern that varies across the field. The output PSFs produced by the combination of such irregularly dithered datasets may, on occasion, show significant variations about the best fit Gaussian. Fortunately this does not noticeably affect aperture photometry performed with typical aperture sizes. In practice the variability about the Gaussian appears larger in WFPC2 data than our simulations would lead us to expect. Examination of recent dithered stellar fields leads us to suspect that this excess variability results from a problem with the original data, possibly caused by charge transfer errors in the CCD.

4. Photometry

The WFPC2 optics geometrically distort the images: pixels at the corner of each CCD subtend less area on the sky than those near the center. However, after application of the flat field, a source of uniform surface brightness on the sky produces uniform counts across the CCD. Therefore point sources near the corners of the chip are artificially brightened compared to those in the center.

In order to study the ability of Drizzle to remove the photometric effects of geometric distortion, we have created a sub-sampled grid of $19 \times 19$ artificial stellar PSFs and adjusted their counts to reflect the effect of geometric distortion – the stars in the corners are up to $\sim 4\%$ brighter than those in the center of the chip. This image was then shifted and down-sampled onto four simulated WF frames and the results combined using drizzling with typical parameters. Aperture photometry on the $19 \times 19$ grid after drizzling reveals that the effect of geometric distortion on the photometry has been dramatically reduced: the RMS photometric variation in the drizzled image is 0.004 mags.

5. Cosmic Ray Detection

Few HST observing proposals have sufficient time to take a number of exposures at each of several dither positions. Therefore, if dithering is to be of wide-spread use, one must be able to remove cosmic rays from data where few, if any, images are taken at the same position on the sky. We have therefore begun to adapt Drizzle to the removal of cosmic rays.

A technique which appears quite promising is described below:

1. Drizzle each image onto a separate sub-sampled output image using $\text{pixfrac} = 1.0$
2. Take the median of the output drizzled images.
3. Map the median image back to the input plane of each of the individual images, taking into account the image shifts and geometric distortion. The is now done by interpolating the values of the median image using a program we call “Blot”.
4. Take the spatial derivative of each of the blotted output images.
Figure 3. The image on the left shows a region of one of twelve 2400s archival images taken with a wide near-infrared filter on WFPC2. Numerous cosmic rays are visible. On the right is the drizzled combination of the twelve images, no two of which shared a dither position.

5. Compare each original image with the corresponding blotted image. Where the difference is larger than can be explained by noise statistics, or the flattening effect of taking the median or perhaps an error in the shift (the magnitudes of the latter two effects are estimated using the image derivative), the suspect pixel is masked.

6. Repeat the previous step on pixels adjacent to pixels already masked, using a more stringent comparison.

7. Finally, drizzle the input images onto a single output image using the pixel masks created in the previous steps.

Figure 3 shows the result of applying this method to data originally taken by Cowie and colleagues, which we have reprocessed using Drizzle.

6. The Hubble Deep Field Images

Drizzle was originally developed for the Hubble Deep Field, a project whose purpose was to image an otherwise unexceptional region of the sky to depths far beyond those of previous astronomical images. Exposures were taken in four color bands from the near ultraviolet to the near infra-red. The resulting images are available in the published astronomical literature as well as from the Space Telescope Science Institute via the World Wide Web at http://www.stsci.edu/ftp/science/hdf/hdf.html.

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