Either neutralino dark matter or cuspy dark halos

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Abstract

We show that if the neutralino in the minimal supersymmetric standard model is the dark matter in our galaxy, there cannot be a dark matter cusp extending to the galactic center. Conversely, if a dark matter cusp extends to the galactic center, the neutralino cannot be the dark matter in our galaxy. We obtain these results considering the synchrotron emission from neutralino annihilations around the black hole at the galactic center.

The composition of dark matter is one of the major issues in cosmology. A popular candidate for non-baryonic cold dark matter is the lightest neutralino appearing in a large class of supersymmetric models [1]. In a wide range of supersymmetric parameter space, relic neutralinos from the Big Bang are in principle abundant enough to account for the dark matter in our galactic halo [2].

A generic prediction of cold dark matter models is that dark matter halos should be have steep central cusps, meaning that their density rises as $r^{-\gamma}$ to the center. Semi-analytical calculations find a cusp slope $\gamma$ between $\sim 1$ [3] and 2 [4]. Simulations find a slope $\gamma$ ranging from 0.3 [5] to 1 [6] to 1.5 [7]. It is unclear if dark matter profiles in real galaxies and galaxy clusters have a central cusp or a constant density core.

There is mounting evidence that the non-thermal radio source Sgr A* at the galactic center is a black hole of mass $M \sim 3 \times 10^6 M_\odot$. This inference is based on the large proper
motion of nearby stars [8], the spectrum of Sgr A* (e.g. [9,10]), and its low proper motion [11]. It is difficult to explain these data without a black hole [12].

The black hole at the galactic center modifies the distribution of dark matter in its surroundings [13], creating a high density dark matter region called the spike – to distinguish it from the above mentioned cusp. Signals from particle dark matter annihilation in the spike may be used to discriminate between a central cusp and a central core. With a central cusp, the annihilation signals from the galactic center increase by many orders of magnitude. With a central core, the annihilation signals do not increase significantly.

Stellar winds are observed to pervade the inner parsec of the galaxy [9], and are supposed to feed the central black hole (e.g. [10,14]). These winds carry a magnetic field whose measured intensity is a few milligauss at a distance of \( \sim 5\text{pc} \) from the galactic center [15]. The magnetic field intensity can rise to a few kilogauss at the Schwarzschild radius of the black hole in some accretion models for Sgr A* [16].

In this letter we examine the radio emission from neutralino dark matter annihilation in the central spike. (Previous studies of radio emission from neutralino annihilation at the galactic center have considered an \( r^{-1.8} \) cusp but no spike [17].) Radio emission is due to synchrotron radiation from annihilation electrons and positrons in the magnetic field around Sgr A*. Comparing the radio emission from the neutralino spike with the measured Sgr A* spectrum, we find that neutralino dark matter in the minimal supersymmetric standard model is incompatible with a dark matter cusp extending to the galactic center.

There are two ways to interpret our results. If we believe that there is a dark matter cusp extending to the center of our galaxy, we can exclude the neutralino as a dark matter candidate. Conversely, if we believe that dark matter is the lightest neutralino, we can exclude that a dark matter cusp extends to the center of the galaxy.

Dark matter candidate. We examine the lightest neutralino in the minimal supersymmetric standard model. This model provides a well-defined calculational framework, but contains at least 106 yet-unmeasured parameters [18]. Most of them control details of the squark and slepton sectors, and are usually disregarded in neutralino dark matter studies.
(cfr. [1]). So, following Bergström and Gondolo [19], we restrict the number of parameters to 7. Out of the database of points in parameter space built in refs. [2,13,20], we use the 35121 points in which the neutralino is a good cold dark matter candidate [2], in the sense that its relic density satisfies $0.025 < \Omega_{\chi} h^2 < 1$. The upper limit comes from the age of the Universe, the lower one from requiring that neutralinos are a major fraction of galactic dark halos. Present understanding of the matter density in the universe (e.g. [21]) suggests a narrower range $0.08 < \Omega_{\chi} h^2 < 0.18$, but we conservatively use the broader range.

**Spike profile.** We summarize the results of ref. [13] for the spike profile. We assume the cusp has density profile

$$\rho_{\text{cusp}} = \rho_D \left(\frac{r}{D}\right)^{-\gamma},$$

with $\rho_D = 0.24\text{GeV}/c^2/\text{cm}^3$ the density at the reference point $D = 8.5\text{kpc}$, the Sun location (this is a conservative value for $\rho_D$, see [13]). Then within a central region of radius $R_{\text{sp}} = \alpha_{\gamma} D (M/\rho_D D^3)^{1/(3-\gamma)}$, where $\alpha_{\gamma}$ is given in ref. [13] and $M = (2.6 \pm 0.2) \times 10^6 M_\odot$ is the mass of the central black hole, the dark matter density is modified to

$$\rho_{\text{sp}} = \frac{\rho'(r)\rho_c}{\rho'(r) + \rho_c},$$

Here $\rho_c = m_\chi / (\sigma v t_{bh})$, where $t_{bh}$ is the age of the black hole (conservatively $10^{10} \text{yr}$), $m_\chi$ is the mass of the neutralino, and $\sigma v$ is the neutralino–neutralino annihilation cross section times relative velocity (notice that for neutralinos at the galactic center $\sigma v$ is independent of $v$). Furthermore,

$$\rho'(r) = \rho_R g(r) \left(\frac{R_{\text{sp}}}{r}\right)^{\gamma_{\text{sp}}},$$

with $g(r) = [1 - (8GM)/(rc^2)]^3$ accounting for dark matter capture into the black hole, $\gamma_{\text{sp}} = (9 - 2\gamma)/(4 - \gamma)$, and $\rho_R = \rho_D (R_{\text{sp}}/D)^{-\gamma}$.

**Annihilation rate.** The total number of neutralino annihilations per second in the spike follows from the density profile as

$$\Gamma = \frac{\sigma v}{m^2} \int \rho_{\text{sp}}^2 4\pi r^2 dr = \frac{4\pi \sigma v \rho_{\text{sp}}^2 R_{\text{in}}^3}{m^2},$$

(4)
with $\rho_{\text{in}} = \rho_{\text{sp}}(R_{\text{in}})$ and $R_{\text{in}} = 1.5 [(20R_S)^2 + R_c^2]^{1/2}$. The latter expression is a good approximation (6\%) to the numerical integration of the annihilation profile.

Most of the annihilations occur either close to the black hole at $\sim 13R_S \sim 3 \times 10^{-6}$pc (where $R_S \equiv 2GM/c^2$ is the Schwarzschild radius) or around the spike core radius $R_c = R_{\text{sp}}(\rho_R/\rho_c)^{1/\gamma_{\text{sp}}}$, whichever is larger.

*Radio signals.* The electrons and positrons produced by neutralino annihilation in the spike are expected to emit synchrotron radiation in the magnetic field around the galactic center.

The strength and structure of this magnetic field is known to some extent. A magnetic field of few milligauss has been detected [15] few parsecs from the center. Models of Sgr A$^*$ contain accretion flows, either spherical [16] or moderately flattened [10], which carry a magnetic field towards the black hole. The strength of this magnetic field is assumed to increase inwards according to magnetic flux conservation or equipartition.

Including the gas and the radial dependence of the magnetic field in the synchrotron emission from neutralino annihilations is a complicated problem. Electrons and positrons in the regions where the magnetic field is strong may lose their energy almost in place, while those at the outskirts of the spike may have time to diffuse to very different radii. Moreover, the plasma may affect the shape of the synchrotron spectrum. We postpone this complicated analysis, and consider three simple but relevant models for the magnetic field and the electron/positron propagation.

In model A, we assume that the magnetic field is uniform across the spike, with strength $B = 1$ mG, and that the electrons and positrons lose all their energy into synchrotron radiation without moving significantly from their production point.

In model B, we also assume that the magnetic field is uniform across the spike with strength $B = 1$ mG, but that the electrons and positrons diffuse efficiently and are redistributed according to a gaussian encompassing the spike (we take the gaussian width $\lambda = 1$ pc).
In model C, we assume that the magnetic field follows the equipartition value \( B = 1 \mu G (r/\text{pc})^{-5/4} \) (from ref. [10]) and that the electrons and positrons lose all their energy into synchrotron radiation without moving significantly from their production point. In addition, in this model, we neglect synchrotron self-absorption.

Under these assumptions, the electron plus positron spectrum follows from the equation of energy loss \(-dE/dt = P(E) \equiv (2e^4B^2E^2)/(3m_e^4c^7)\) as

\[
\frac{dn_e}{dE} = \frac{Y_e(>E)}{P(E)} \Gamma f_e(r),
\]

where

\[
f_e(r) = \frac{\rho_{sp}^2}{\int \rho_{sp}^2 4\pi r^2 dr}
\]

in models A and C, and

\[
f_e(r) = \frac{1}{(2\pi \lambda^2)^{3/2}} e^{-r^2/2\lambda^2}
\]

in model B.

\(Y_e(>E)\) is the number of annihilation electrons and positrons with energy above \(E\). We obtain \(Y_e(>E)\) with the DarkSUSY code [22], which includes a Pythia simulation of the \(e^\pm\) continuum and the \(e^\pm\) lines at the neutralino mass [23].

The synchrotron luminosity is given by

\[
L_\nu = \frac{A_\nu \Gamma}{\nu} \int dr 4\pi r^2 f_e(r) \int_{m_e}^{m} \frac{Y_e(>E)}{\nu_e(E)} F \left( \frac{\nu}{\nu_e(E)} \right) dE,
\]

where

\[
\nu_e(E) = \frac{3eB}{4\pi m_e c} \left( \frac{E}{m_e c^2} \right)^2
\]

and

\[
F(x) = \frac{9\sqrt{3}}{8\pi} x \int_x^\infty K_{5/3}(y)dy.
\]

The factor \( A_\nu \) accounts for synchrotron self-absorption. In models A and B, we write
\[ A_\nu = \frac{1}{a_\nu} \int_0^\infty \left[ 1 - e^{-\tau(b)} \right] \pi b \, db, \quad (11) \]

where \((b, z)\) are cylindrical coordinates,

\[ \tau = a_\nu \int_{-\infty}^{+\infty} f_e(b, z) \, dz, \quad (12) \]

and

\[ a_\nu = \frac{e^3 B \Gamma}{9 m_e v^2} \int_{m_e}^{m} \frac{d}{dE} \left( \frac{Y_e(>E)}{E^2 P(E)} \right) F \left( \frac{\nu}{\nu_e(E)} \right) \, dE. \quad (13) \]

In model C, we neglect self-absorption \((A_\nu = 1)\).

We have evaluated equation (8) numerically for each point in supersymmetric parameter space. In model C, we use the approximation \(F(x) \simeq \delta(x - 0.29)\), which selects the peak of the synchrotron emission from each electron or positron (profuse thanks to Pasquale Blasi for suggesting this approximation).

Figure 1 shows a comparison of typical synchrotron spectra from neutralino annihilation in the spike with the measured spectrum of Sgr A* (the latter is taken from the compilation in ref. [10]). Four spectra are plotted, corresponding to two points in supersymmetric parameter space (thick and thin lines) and two assumptions for the magnetic field (solid and dashed lines; for models A and C, respectively). The spectra are normalized to their maximal intensity, which is fixed by the upper bound at 408 MHz [24]. This upper bound limits the synchrotron intensity for all points in supersymmetric parameter space.

**Results.** If a dark matter cusp extends to the galactic center, the neutralino cannot be the dark matter in our galaxy. For example, let us assume that the halo profile is of the Navarro-Frenk-White form [6], namely \(\rho \propto r^{-1}\) in the central region. Figure 2 shows the expected radio fluxes \(S_\nu = L_\nu / 4\pi D^2\) at 408 MHz and the upper limit from [24]. The upper panel is for model A, the lower panel for model C. Results of model B are similar to those of model A. Irrespective of the assumption on the magnetic field or the \(e^\pm\) propagation, all points in supersymmetric parameter space where the neutralino would be a good dark matter candidate are excluded by several orders of magnitude.
Conversely, if the neutralino is the dark matter, there is no steep dark matter cusp extending to the galactic center. We see this by lowering the cusp slope $\gamma$ until the expected flux at 408 MHz decreases below the upper limit. We obtain a different maximum value $\gamma_{\text{max}}$ for each point in supersymmetric parameter space. These values are plotted in figure 3 together with the range $0.3 \lesssim \gamma \lesssim 1.5$ obtained in cold dark matter simulations. The upper bounds $\gamma_{\text{max}}$ are generally orders of magnitude smaller than the simulation results.

We conclude that neutralino dark matter in the minimal supersymmetric standard model is incompatible with a dark matter cusp extending to the galactic center. If there is a dark matter cusp extending to the center, we can exclude the neutralino in the minimal supersymmetric standard model as a dark matter candidate. Conversely, if the dark matter of the galactic halo is the lightest neutralino in the minimal supersymmetric standard model, we can exclude that a dark matter cusp extends to the center of the galaxy.

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REFERENCES

[1] G. Jungman, M. Kamionkowski, and K. Griest, Phys. Rep. 267, 195 (1996).

[2] J. Edsjö and P. Gondolo, Phys. Rev. D56, 1879 (1997).

[3] D. Syer and S.D.M. White, MNRAS 293, 337 (1998).

[4] Y. Hoffman and J. Shaham, Ap. J. 297, 16 (1985).

[5] A.V. Kravtsov, A.A. Klypin, J.S. Bullock, and J.R. Primack, Ap. J. 502, 48 (1998); J. S. Bullock et al., astro-ph/9908152.

[6] J.F. Navarro, C.S. Frenk, and S.D.M. White, Ap. J. 462, 563 (1996); ibid. 490, 493 (1997).

[7] T. Fukushige and J. Makino, Ap. J. 477, L9 (1997); B. Moore, F. Governato, T. Quinn, J. Stadel, and G. Lake, Ap. J. 499, L5 (1998); S. Ghigna et al., astro-ph/9910160.

[8] A. Eckart and R. Genzel, Nature 383, 415 (1996); MNRAS 284, 576 (1997); A.L. Ghez, B.M. Klein, M. Morris, and E.E. Becklin, Ap. J. 509, 678 (1998); R. Genzel, C. Pichon, A. Eckart, O.E. Gerhard, and T. Ott, astro-ph/0001428.

[9] P.G. Mezger, W.J. Duschl, and R. Zylka, Astron. Astrophys. Rev. 7, 289 (1996); F. Yusef-Zadeh, F. Melia, and M. Wardle, Science 287, 85 (2000).

[10] R. Narayan, R. Mahadevan, J.E. Grindlay, R.G. Popham, and C. Gammie, Ap. J. 492, 554 (1998).

[11] D.C. Backer and R.A. Sramek, Ap. J. 524, 805 (1999).

[12] E. Maoz, Ap. J. 494, L131 (1998).

[13] P. Gondolo and J. Silk, Phys. Rev. Lett. 83, 1719 (1999).

[14] R. Coker, F. Melia, and H. Falcke, Ap. J. 523, 642 (1999).
[15] F. Yusef-Zadeh, D.A. Roberts, W.M. Goss, D. Frail, and A. Green, Ap. J. 466, L25 (1996); Ap. J. 512, 230 (1999).

[16] F. Melia, Ap. J. 387, L25 (1992).

[17] V. Berezinsky, A. Bottino, and G. Mignola, Phys. Lett. B325, 136 (1994).

[18] S. Dimopoulos and D. Sutter, Nucl. Phys. B465, 23 (1995).

[19] L. Bergström and P. Gondolo, Astropart. Phys. 5, 183 (1996).

[20] L. Bergström, P. Ullio, and J. H. Buckley, Astropart. Phys. 9, 137 (1998).

[21] N. Bahcall, J.P. Ostriker, S. Perlmutter, and P.J. Steinhardt, Science 284, 1481 (1999).

[22] P. Gondolo et al., unpublished.

[23] E.A. Baltz and J. Edsjö, Phys. Rev. D59, 023511 (1999).

[24] R.D. Davies, D. Walsh, and R. Booth, MNRAS 177, 319 (1976).
FIG. 1. Comparison of the Sgr A* spectrum with the synchrotron emission from neutralino annihilation in the spike. The figure shows four typical synchrotron spectra: two points in supersymmetric parameter space (thick and thin lines), and two models for the magnetic field (solid and dashed lines). The spectra are normalized to the upper bound at 408 MHz.
FIG. 2. Expected radio emission from the galactic center at 408 MHz from neutralino annihilations in the dark matter spike, assuming a Navarro-Frenk-White profile and (a) a uniform magnetic field of 1 mG, (b) a magnetic field at the equipartition value. All models exceed the present upper bound by several orders of magnitude.
FIG. 3. Upper bound on the inner halo slope $\gamma$ imposed by the constraint on the radio emission from the galactic center at 408 MHz, assuming (a) a uniform magnetic field of 1 mG, and (b) a magnetic field at the equipartition value. Each dot corresponds to a point in supersymmetric parameter space. The results of cold dark matter simulations are much higher than the upper bounds.