Polymer Schwarzschild black hole: An effective metric

J. Ben Achour1, F. Lamy2, H. Liu2,3,4 and K. Noui2,4

1 Department of Physics, Beijing Normal University - Beijing 100875, China
2 Laboratoire Astroparticule et Cosmologie, Université Paris Diderot Paris 7, CNRS - 75013 Paris, France
3 Centre de Physique Théorique, Universités d’Aix-Marseille et de Toulon, CNRS - 13288 Marseille, France
4 Institut Denis Poisson, Université d’Orléans, Université de Tours, CNRS - 37200 Tours, France

received 7 June 2018; accepted in final form 25 July 2018
published online 20 August 2018

PACS 04.70.Dy – Quantum aspects of black holes, evaporation, thermodynamics
PACS 04.60.Kz – Lower dimensional models; minisuperspace models
PACS 04.60.Pp – Loop quantum gravity, quantum geometry, spin foams

Abstract – We consider the modified Einstein equations obtained in the framework of effective spherically symmetric polymer models inspired by loop quantum gravity. When one takes into account the anomaly free pointwise holonomy quantum corrections, the modification of Einstein equations is parametrized by a function \( f(x) \) of one phase space variable. We solve explicitly these equations for a static interior black-hole geometry and find the effective metric describing the trapped region, inside the black hole, for any \( f(x) \). This general resolution allows to take into account a standard ambiguity inherent to the polymer regularization: namely the choice of the spin \( j \) labelling the \( SU(2) \)-representation of the holonomy corrections. When \( j = 1/2 \), the function \( f(x) \) is the usual sine function used in the polymer literature. For this simple case, the effective exterior metric remains the classical Schwarzschild’s one but acquires modifications inside the hole. The interior metric describes a regular trapped region and presents strong similarities with the Reissner-Nordström metric, with a new inner horizon generated by quantum effects. We discuss the gluing of our interior solution to the exterior Schwarzschild metric and the challenge to extend the solution outside the trapped region due to covariance requirement. By starting from the anomaly free polymer regularization for inhomogeneous spherically symmetric geometry, and then reducing to the homogeneous interior problem, we provide an alternative treatment to existing polymer interior black-hole models which focus directly on the interior geometry, ignoring the covariance issue when introducing the polymer regularization.

Introduction. – The extraordinary recent detections of gravitational waves (GW) by the LIGO and the LIGO/Virgo Collaborations allowed us to “hear” black holes for the first time, a century after Schwarzschild predicted their existence from Einstein equations. These detections have opened a new window on black holes and we hope to learn much more on these fascinating astrophysical objects in a near future. So far, the observations of GW emitted by binaries of black holes or neutron stars are in total agreement with the predictions of general relativity. However, when the GW detectors become more sensitive and allow probing deeper the “very strong” gravity regime at the merger, one will possibly measure deviations from Einstein gravity.

Perhaps, the main reason to expect gravity to be modified is the existence of singularity theorems in classical gravity. The presence of such singularity is believed to be pathological and to indicate a breakdown of the classical theory which should be modified and regularized by quantum gravity effects. However, how quantum gravity regularizes precisely black-holes singularities is still unknown simply because a complete theory of quantum gravity is still missing. Faced with such an important difficulty, one has instead proposed candidates for regular metrics with the requirements that they are non-singular modifications of the classical black-hole metric and they are physically reasonable. The Hayward [1] or more recently the Planck star metrics [2] are typical examples. Hence, the regular metrics could be interpreted as effective quantum geometries. From this point of view, it is natural to think that they could be recovered from a semi-classical limit of a black-hole quantum geometry. In practice, this is an extremely difficult problem since it will require the development of suitable coarse-graining technics of the
underlying quantum geometry, a major challenge in non-perturbative approaches to quantum gravity such as loop quantum gravity.

One way to circumvent this difficulty would be to construct and classify (from first principles) effective theories of quantum gravity as one does for studying in a systematic way dark energy, for instance. See [3–5] along this line. In that way, one could write a modified gravity action (or modified Einstein equations) which takes into account quantum corrections, and then study the spherically symmetric sector and look for black-hole solutions. Of course, these solutions are expected to be regular and to predict new physical phenomena which could be in principle observable. In the framework of loop quantum cosmology [6], one knows how to construct and classify effective quantum Friedmann equations (by the choice of the spin-j representation which labels the holonomy corrections) [7]. It is well known that they lead to a regular cosmology with no more initial singularity. However, the effective description of loop quantum black holes is much less understood, the challenge being to generalize the techniques applied in LQG to the inhomogeneous black-hole background. Indeed, in this inhomogeneous case, one has to make sure that the effective corrections do not generate anomalies in the algebra of first-class constraints, and thus do not spoil covariance. Taking care of this potential covariance issue, one can obtain modified Einstein’s equation for polymer black holes [8–10]. Their resolution for the simple vacuum modified Schwarzschild interior has not been investigated yet. In this letter, we fill this gap.

In the polymer framework, the effective corrections are introduced at the phase space level, in the Hamiltonian constraint. In the treatment of interior black holes, several regularization schemes have been developed. Models such as those in [11–13] make use of the homogeneity of the interior geometry to introduce a regularization very similar to cosmological polymer models. Yet, the exterior black-hole geometry is inhomogeneous, and the modified Einstein’s equations obtained in [11–13] hold only for the interior geometry. In this letter, we adopt a different strategy. We consider the full inhomogeneous geometry, and introduce the polymer regularization satisfying the anomaly freedom conditions of [8], paying thus attention to the underlying covariance of the effective approach. Only afterwards, we reduce the problem to the interior homogeneous geometry. The advantage is that we obtain one and only one set of modified Einstein’s field equations valid for the whole black-hole geometry (both exterior and interior regions), i.e., eqs. (13), (14). The modified field’s equation for the interior region are then simply obtained by suitable gauge fixing.

Following thus the approach of [8], the quantum corrections of the effective Hamiltonian constraint, induced by the regularization, are parametrized by a single real-valued function \( f(x) \) of one phase-space variable \( x \). This is a consequence of the requirement that the deformed symmetry algebra (generated by the effective Hamiltonian and vectorial constraint) remains closed so that there are no anomalies. See eq. (9) and discussion below. However, even though there is a standard choice for \( f(x) \) in loop quantum gravity, the precise definition of the “regularization” function \( f(x) \) is in fact ambiguous. For this reason, it is important to study the effective corrected Einstein equations for an arbitrary function \( f(x) \), as initiated in [8].

In this letter, we consider the effective theory introduced in [8] and we solve explicitly the effective Einstein equations for static spherically symmetric interior space-times. More precisely, we focus on the static region inside the horizon, where quantum gravity effects are supposed to become important, and we find an explicit form of the effective metric in this region for an arbitrary deformation function \( f(x) \). Surprisingly, the effective metric can be simply expressed in terms of \( f(x) \), and then we can easily deduce the conditions for the black hole to be non-singular as one wishes. We apply our result to the case where \( f(x) \) is the standard deformation function used in loop quantum gravity (11), and we show that the black hole presents strong similarities with the Reissner-Nordström space-time. The interior effective geometry inherits an inner horizon due to the non-perturbative quantum gravity effects. Finally, we explore the possibility to extend the black-hole solution of the effective metric to the whole space-time (outside the trapped region).

**Covariant polymer phase-space regularization.**— Let us first present the effective Einstein equations obtained in loop quantum gravity for spherically symmetric black holes and justify our choice of regularization.

We start with an ADM parametrization of the metric

\[
\text{d}s^2 = -N^2 \text{d}t^2 + q_{rr} (dr + N^r \text{d}t)^2 + g_{\theta \theta} \text{d}\Omega^2,
\]

where each function \( N, N^r, q_{rr} \) and \( g_{\theta \theta} \) depends on the radial and time coordinates \((t, r)\), and \( \text{d}\Omega^2 \) is the metric on the unit two-sphere. Following the Ashtekar-Barbero construction, it is more convenient to express the metric components \( q_{rr} \) and \( g_{\theta \theta} \) in terms of the components of the electric fields \( E^r \) and \( E^\phi \) as follows:

\[
q_{rr} \equiv \frac{(E^\phi)^2}{E^r}, \quad g_{\theta \theta} \equiv E^\phi.
\]

Hence, the phase space is parametrized by two pairs of conjugate fields defined by the Poisson brackets

\[
\{ K_\phi(r), E^\phi(s) \} = \delta(r - s), \quad \{ K_r(r), E^r(s) \} = 2\delta(r - s),
\]

where we have fixed for simplicity the Newton constant and the Barbero-Immirzi parameter to 1. The variables \( K_\phi \) and \( K_r \) are \( su(2) \) connections.

As usual, the lapse function \( N \) and the shift vector \( N^r \) are Lagrange multipliers which enforce, respectively, the Hamiltonian and vectorial constraints,

\[
H = \frac{E^\phi}{2\sqrt{E^r}} (1 + K_\phi^2 - \Gamma_\phi^2) + \sqrt{E^r} (K_\phi K_r + \partial_r \Gamma_\phi),
\]

\[
V = 2E^\phi \partial_r K_\phi - K_r \partial_r E^\phi,
\]

[39x78]
where $\Gamma_\phi \equiv -\partial_r E^\phi/2E^\phi$ is linked to the Levi-Civita connection. These constraints are first class, they generate diffeomorphisms restricted to spherically symmetric spacetimes, and they satisfy the closed Poisson algebra

$$
\{H[N], V[N'_1]\} = -H[N'_1 \partial_r N], \quad (6)
$$

$$
\{V[N'_1], V[N'_2]\} = V[N'_1 \partial_r N'_2 - N'_2 \partial_r N'_1], \quad (7)
$$

$$
\{H[N_1], H[N_2]\} = V[\sqrt{g} (N_1 \partial_r N_2 - N_2 \partial_r N_1)], \quad (8)
$$

where $H[N]$ and $V[N'_1]$ are the smeared constraints.

In this letter, we focus on the effective dynamic obtained from the anomaly free loop regularization which is introduced prior quantization. Concretely, we keep the phase-space parametrization (3) unchanged and we modify the expression of the constraint (4). As we consider solely pointwise holonomy corrections of $K_\phi$ here, only the dependency of the Hamiltonian constraint on $K_\phi$ is modified according to

$$
H = \frac{E^\phi}{2\sqrt{E^r}} [1 + f(K_\phi) - \Gamma_\phi^2] + \sqrt{E^r} [g(K_\phi) K_r + \partial_r \Gamma_\phi] \quad (9)
$$

where the functions $f$ and $g$ are not fixed yet. The requirement of anomaly freedom of the Dirac’s algebra requires then that $g(x) = f'(x)/2$. In that case, the Poisson bracket between Hamiltonian constraints (8) is deformed according to

$$
\{H[N_1], H[N_2]\} = V[\beta(K_\phi) q''(N_1 N'_2 - N'_2 N_1)] \quad (10)
$$

where $\beta(x) = f''(x)/2$, as initially derived in [8,9]. Such a deformation is a generic feature of holonomy corrected symmetry reduced models of gravity [14]. The other two brackets (6) and (7) are unchanged. Moreover, $K_r$ is not modified in our regularization since it can be completely removed from the scalar constraint by a simple redefinition of the constraints, as shown in [15]. Consequently, the regularization of $K_\phi$ does not play any role in the classical regularization and can be safely ignored at this step. The holonomies of $K_r$ will nevertheless be crucial in the quantum theory when introducing the one-dimensional spin network defining the kinematic Hilbert space. See [15] for more details. Finally, our regularization is restricted to the $\mu_0$-scheme, as in [15], since introducing holonomy corrections within the $\mu$-scheme, i.e., $K_\phi \rightarrow f(K_\phi, E^\phi)$, and requiring at the same time the anomaly freedom of the effective Dirac’s algebra generates inconsistencies as shown in [10]. Therefore, the standard improved dynamics used in polymer cosmological models cannot be generalized as it stands to such inhomogeneous spherically symmetric polymer models. See [11,12] for an alternative strategy. This concludes our justifications for our classical regularization of the phase space.

**Effective Einstein’s equations.** – Hence, as was emphasized in the introduction, the regularization induced by holonomy corrections inspired from loop quantum gravity is parametrized by the sole function $f(x)$. The explicit expression of this effective correction remains ambiguous. Nonetheless, as we require naturally that $f(x)$ reproduces the classical behavior in the low curvature regime, we must have $f(x) \approx x^2$ when $x \ll 1$. In the literature, the usual choice is

$$
f(x) = \frac{\sin^2(\rho x)}{\rho^2}, \quad (11)
$$

where $\rho$ is a deformation real parameter that tends to zero at the classical limit. The presence of a trigonometric function is reminiscent from the SU(2) gauge invariance in loop quantum gravity: roughly, one replaces the “connection” variable $K_\phi$ by a pointwise “holonomy-like” variable $\sin(\rho K_\phi)/\rho$. Note that (11) is associated to the computation of the regularization of the connection (or its curvature) in terms of holonomies within the $j = 1/2$ fundamental representation of SU(2). Yet, one could obtain more complicated trigonometric functions by evaluating this regularization in another $j$-representation of SU(2), as done for polymer cosmological models in [7]. Therefore, keeping $f(x)$ general in our resolution allows to keep track of this ambiguity of the polymer regularization.

Now, we have all the ingredients to compute the effective Einstein equations for deformed spherically symmetric space-times. They are given by the Hamilton equations

$$
\dot{F} = \{F, H[N] + V[N']\}, \quad (12)
$$

for $F$ being one of the four phase-space variables (3). The time evolutions of the electric-field components simply read

$$
\dot{E}^r = N \sqrt{E^r} f'(K_\phi) + N^r \partial_r E^r, \quad (13)
$$

$$
\dot{E}^\phi = N \left[ \sqrt{E^r} K_r + \frac{E^\phi}{\sqrt{E^r}} f(K_\phi) \right] + \partial_r (N^r E^\phi). \quad (14)
$$

The expression of $\dot{K}_\phi$ is more involved and thus we do not report it here. The component $K_\phi$ can be obtained by solving the Hamiltonian constraint (9). Note that for $N' = 0$ and static geometry, eq. (13) implies that $K_\phi = n\pi/2\rho$, where $n \in \mathbb{N}$ and where we have used (11). It implies that outside the hole, the only consistent inhomogeneous static solution is the classical Schwarzschild’s one, i.e., $K_\phi = 0$ for $n = 0$. Indeed, for $n \neq 0$, the resulting geometry has a divergent extrinsic curvature $K_\phi$ in the semi-classical limit, i.e., when $\rho \rightarrow 0$. Hence, the effective corrections introduced above do not allow to have a modified Schwarzschild geometry outside the hole when looking for a static exterior solution. We can now study the potential modifications inside the black hole.

**Inside the black hole: static anstatz.** – We turn now to the interior problem. We are interested in solving these equations inside a “static” black hole. As the role of the variables $r$ and $t$ changes when one crosses the horizon,
this corresponds to considering time-dependent fields only. In that case, the effective Einstein equations dramatically simplify and read

\[ E^r = N\sqrt{E^r} f'(K_0), \quad \]  
\[ E^\phi = \frac{N}{2} \left[ \sqrt{E^r} K_r f''(K_0) + \frac{E^\phi}{\sqrt{E^r}} f'(K_0) \right], \quad \]  
\[ \dot{K}_\phi = -\frac{N}{2\sqrt{E^r}} \left[ 1 + f(K_0) \right], \quad \]

from where we easily get the dynamics of \( K_r \).

Now, we are going to solve these equations explicitly for any function \( f \). As we are going to show, it is very convenient to fix the lapse function \( N(t) \) (by a gauge fixing) such that

\[ N f'(K_0) = 2. \quad \]  
\[ \]  
In that case, eq. (15) for \( E^r \) decouples completely from the other variables and can be easily integrated to

\[ E^r(t) = (t + a)^2, \quad \]  
\[ \]  
where \( a \) is an integration constant that we fix to \( a = 0 \) (in order to recover the Schwarzschild solution at the classical limit). Another important consequence of the gauge choice (19) is that eq. (17) for \( K_\phi \) also decouples and takes the very simple form

\[ \frac{f'(K_0)}{1 + f(K_0)} \dot{K}_\phi = -\frac{1}{t}. \quad \]  
\[ \]  
It can be immediately integrated to the form

\[ f(K_0) = \frac{r_s}{t} - 1, \quad \]  
\[ \]  
where \( r_s \) is an integration constant with the dimension of a length. As we are going to see later on, \( t = r_s \) corresponds to the location of the black-hole (outer) horizon. Hence, \( K_\phi \) is easily obtained by inverting the function \( f(x) \) (see footnote 1). The expression of \( E^\phi \) follows immediately. Indeed, if one substitutes \( K_r \) from (18) into (16), one obtains the following equation for \( E^\phi \):

\[ \frac{\dot{E}^\phi}{E^\phi} = \frac{1}{t} \left( 1 - \frac{[1 + f(K_0)] f''(K_0)}{[f(K_0)]^2} \right), \quad \]  
\[ \]  
which can be easily integrated to

\[ E^\phi = b \frac{f'(K_0)}{1 + f(K_0)}, \quad \]  
\[ \]  
where \( b \) is a new integration constant that will be fixed later. The remaining variable \( K_r \) is given immediately from the Hamiltonian constraint (18) together with (20) and (24). Hence, we have integrated explicitly and completely the modified Einstein equations in the region inside a “static” spherically symmetric black hole where the effective metric is

\[ ds^2 = -\frac{1}{F(t)} dt^2 + \left( \frac{2 b}{r_s} \right)^2 F(t) dr^2 + t^2 d\Omega^2, \quad \]  
\[ \]  
with \( F(t) \) related to \( f(x) \) by

\[ F(t) = \frac{1}{4} \left[ f' f^{-1} \left( \frac{r_s}{t} - 1 \right) \right]^2 = \left[ 2 \frac{df^{-1}}{dx} \left( \frac{r_s}{t} - 1 \right) \right]^{-2}. \quad \]  
\[ \]  
In the region where \( t \approx r_s \), quantum gravity effects are negligible and the metric should reproduce the Schwarzschild metric. We see immediately in (25) that a necessary condition for this to be the case is that \( 2b = r_s \). This fixes the constant \( b \). Furthermore, in such a regime, we know that \( f(x) \approx x^2 \), then \( f^{-1}(x) \approx \sqrt{x} \), hence \( F(t) \approx [r_s/t - 1] \). As a consequence, we recover the expected classical metric with \( r_s \) being the Schwarzschild radius.

**Inverse problem.** – Before studying concrete examples, let us consider a converse situation where a deformed metric \( g_{\mu\nu} \) of the form (25) is given. Then, one asks the question whether one can find a deformation function \( f(x) \) such that the deformed metric \( g_{\mu\nu} \) is a solution of the effective Einstein equations. The answer is positive and \( f(x) \) can be obtained immediately by inverting the relation (26) between \( F(t) \) and \( f(x) \) as follows:

\[ f^{-1}(x) = \frac{1}{2} \int_0^x du \left( F \left( \frac{r_s}{1 + u} \right) \right)^{-1/2}. \quad \]  
\[ \]  
As the function \( f^{-1}(x) \) is monotonic, one can invert this relation and define the deformation function \( f(x) \) without ambiguity. This can be done for Hayward’s metric for instance, even though in that case \( f^{-1}(x) \) is defined as an integral, and thus \( f(x) \) is implicit.

**Example: the standard \( j = 1/2 \) sine correction.** – To illustrate this result, let us consider some interesting physical situations. First, the case where there is no quantum deformation corresponds to \( f(x) = x^2 \). As we have just said above, we recover immediately the Schwarzschild metric.

Then, let us study the more interesting case where \( f(x) \) is the usual function considered in polymer black-hole models (11). In that case, the reciprocal function is

\[ f^{-1}(x) = \frac{\arcsin(x/\rho \sqrt{T})}{\rho}, \quad \]  
\[ \]  
which is defined for \( x \leq 1/\rho^2 \) only. As a consequence, the effective metric for a black hole is of the form (25) with

\[ F(t) = \left( \frac{r_s}{t} - 1 \right) \left( 1 + \rho^2 - \rho^2 \frac{r_s}{t} \right), \quad \]  
\[ \]  
\footnote{When \( f \) is monotone, it admits a global reciprocal function \( f^{-1} \), otherwise the reciprocal function is defined locally.}
which is defined for $t \leq r_s$ a priori. At this point, we can make several interesting remarks. First, one recovers the Schwarzschild metric, when $t$ approaches $r_s$. Then, in addition to the usual outer horizon (located at $t = r_s$), the metric has an inner horizon located at

$$t = \frac{\rho^2 r_s}{1 + \rho^2}. \quad (30)$$

The computation of the Ricci and Kretschmann scalars shows that there is no curvature singularity inside the trapped region. One can naturally extend this solution outside the trapped region by using a generalized advanced time coordinate $v$ such that $dv = dr + dt/F(t)$: $ds^2 = F(t)dv^2 - 2dvd t + t^2 d\Omega^2$. The metric and inverse metric are regular when $F(t) = 0$. This allows to define the expansion of null radial outgoing geodesics, leading to $\theta_\pi = F(t)/t$. Hence the zeros of $F$ correspond to the locus of the horizons (inner and outer), and the region comprised between them is trapped. The Ricci scalar $R \approx -2\rho^2/t^2$ diverges at $t = 0$, which is the locus of a time-like singularity as in Reissner-Nordström’s (RN) black hole. Our metric is actually very similar to this solution, and leads to the same Penrose diagram. However, the main difference is that an outer horizon (at $t = r_s$) is always present in our geometry, while naked singularities appear for super-extreme RN black holes. In that end, this naive extension is not satisfactory since it does not allow recovering Schwarzschild’s solution in the classical region ($r > r_s$), except if the parameter $\rho$ becomes $r$-dependent and tends to zero, which would drastically modify the equations of motion [10].

Moreover, while the extension of the metric outside the trapped region is natural for a standard RN solution, it is not clear whether the extension is allowed or not in our context. The reason is that the deformation of the Hamiltonian constraint (10) modifies the invariance of the effective theory under time reparametrizations. Then, if we believe that such a deformed symmetry is the right one and it is no longer given by usual diffeomorphisms (which can be discussed), we could not perform an arbitrary time redefinition as we did to extend the metric outside the trapped region. The deformed symmetry has recently been analyzed in great detail in [16]. It was realized that the effective metric which is invariant under these deformed transformations is slightly different from (1) where $N$ has to be rescaled according to

$$N^2 \rightarrow \beta(K_\phi) N^2, \quad (31)$$

where $\beta$, which has been introduced in (10), is explicitly given, in our case, as a function of time by

$$\beta(t) \equiv \beta(K_\phi(t)) = 1 - 2\rho^2 \left( \frac{r_s}{t} - 1 \right). \quad (32)$$

With this new invariant metric, the function $g_{tt}(t)$ acquires a zero in the trapped region which corresponds to a transition between a Lorentzian and an Euclidean signature within the trapped region at

$$t = \frac{2\rho^2 r_s}{1 + 2\rho^2}. \quad (33)$$

Such a transition was studied in more detail in [16]. Starting from another gauge choice, namely $K_\phi = \pi/(2\rho)$, and solving the field eqs. (15)–(18) deep inside the black hole, it was shown that the geometry is regular. Yet, the Lorentzian to Euclidean transition raises new difficulties concerning, for instance, the fate of matter inside this trapped region, since the standard evolution equations become elliptic [17]. Similar aspects were encountered in the context of the perturbations analysis in loop quantum cosmology known as the deformed algebra approach [18].

**Discussion.** – In this letter, we have solved explicitly a large class of modified Einstein equations arising in the effective polymer approach to black holes. We have adopted a different strategy from the existing interior Schwarzschild models such as [11–13]. We first consider the full polymer regularization of the inhomogeneous geometry consistent with covariance, and only then reduce the problem to the interior homogeneous geometry. By doing this, we ensure that the regularization of the Hamiltonian constraint does not generate any anomalies and, thus, that we still have the right number of degrees of freedom at the effective level. This point is ignored in [11–13] and the regularization introduced in these models is different, since there are no anomaly freedom condition to constrain it. Consequently, the effective metric obtained in this letter and the one presented in [11–13] are very different.

Focusing on the usual deformation considered in polymer models studied by Gambini and Pullin, we have found a black-hole (interior) solution whose structure shows strong similarities with the Reissner-Nordström black hole. The main novelty due to the quantum gravity effect is the appearance of an inner horizon, while the expected Schwarzschild solution is recovered when one approaches the outer horizon, albeit not smoothly. This last point is a consequence of the lack of a proper $\mu$-scheme regularization in the Gambini-Pullin model.

Strictly speaking, we obtained a solution only inside a trapped region, valid for $t \in [t_-, t_+]$ and the question of its extension in the whole space-time deserves to be study carefully. In particular, we see that the naive extension outside the trapped region, while always present, does not allow recovering Schwarzschild’s solution in the classical region ($r > r_s$), except if the parameter $\rho$ becomes $r$-dependent and tends to zero. This underlines the limitation of the current model to have a consistent semi-classical limit. A generalization of the current regularization is required to account for a $\mathcal{P}$-scheme, as already emphasized in [10] and more recently in [19]. See also [20] for a more recent proposal including such $\mathcal{P}$-scheme in polymer black holes using self-dual variables.

Our results open interesting theoretical and phenomenological directions to follow. First, it would be interesting...
to include additional effective corrections such as the triad corrections affecting the intrinsic geometry, which are usually considered separately. An even more challenging step is to go beyond static geometries and study dynamical black holes, an open issue up to now in the polymer framework [21–23]. This is particularly important for understanding the Hawking radiation as well as quantum gravitational collapse and eventually bouncing and black hole to white hole transitions scenarios. Finally, it would be interesting to use this model to investigate possible quantum gravity modifications of the structure inside astrophysical objects as is done in modified classical gravity, such as scalar tensor theories. We plan to address these important questions in the future.

***

JBA would like to thank S. Brahma for numerous discussions and for having shared with us their results [16] prior to publication. We would like to thank E. Livine for his interesting remarks.

REFERENCES

[1] Hayward S. A., Phys. Rev. Lett., 96 (2006) 031103 (arXiv:gr-qc/0506126).
[2] De Lorenzo T., Pacilio C., Rovelli C. and Speziale S., Gen. Relativ. Gravit., 47 (2015) 41 (arXiv:1412.6015).
[3] Bojowald M., Brahma S., Buyukcam U. and D’Ambrosio F., Phys. Rev. D, 94 (2016) 104032 (arXiv:1610.08355 [gr-qc]).
[4] Carballo-Rubio R., Di Filippo F. and Liberati S., Minimally modified theories of gravity: a playground for testing the uniqueness of general relativity, arXiv:1802.02537.
[5] Ghersi J. T. G., Desrochers M. J., Protter M. and DeBenedictis A., Hamiltonian consistency of the gravitational constraint algebra under deformations, arXiv:1711.04234.
[6] Ashtekar A. and Singh P., Class. Quantum Grav., 28 (2011) 213001 (arXiv:1108.0893).
[7] Ben Achour J., Brahma S. and Geiller M., Phys. Rev. D, 95 (2017) 086015 (arXiv:1612.07615).
[8] Bojowald M., Brahma S. and Reyes J. D., Phys. Rev. D, 92 (2015) 045043 (arXiv:1507.00329).
[9] Tibrewala R., Class. Quantum Grav., 31 (2014) 055010 (arXiv:1311.1297).
[10] Tibrewala R., Class. Quantum Grav., 29 (2012) 235012 (arXiv:1207.2585).
[11] Corichi A. and Singh P., Class. Quantum Grav., 33 (2016) 055006 (arXiv:1506.07815).
[12] Olmedo J., Saini S. and Singh P., Class. Quantum Grav., 34 (2017) 225011 (arXiv:1707.07333).
[13] Protter M. and DeBenedictis A., Phys. Rev. D, 97 (2018) 106009 (arXiv:1802.09114 [gr-qc]).
[14] Bojowald M. and Brahma S., Signature change in 2-dimensional black-hole models of loop quantum gravity, arXiv:1610.08850.
[15] Gambini R. and Pullin J., Phys. Rev. Lett., 110 (2013) 211301 (arXiv:1302.5265).
[16] Bojowald M., Brahma S. and Yeom D.-H., Effective line elements and black-hole models in canonical (loop) quantum gravity, arXiv:1803.01119.
[17] Bojowald M., Front. Phys., 3 (2015) 33 (arXiv:1409.3157).
[18] Barrau A., Bojowald M., Calcagni G., Grain J. and Kagan M., JCAP, 05 (2015) 051 (arXiv:1404.1018).
[19] Ben Achour J., Lamy F., Liu H. and Noui K., JCAP, 05 (2018) 072 (arXiv:1712.03876).
[20] Ben Achour J. and Brahma S., Phys. Rev. D, 97 (2018) 126003 (arXiv:1712.03677 [gr-qc]).
[21] Alesci E. and Modesto L., Gen. Relativ. Gravit., 46 (2014) 1656 (arXiv:1101.5792).
[22] Ben Achour J., Brahma S. and Marciano A., Phys. Rev. D, 96 (2017) 026002 (arXiv:1608.07314).
[23] Campiglia M., Gambini R., Olmedo J. and Pullin J., Class. Quantum Grav., 33 (2016) 18LT01 (arXiv:1601.05688).