Rapid estimation of tsunami earthquake magnitudes at local distance

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Abstract
A tsunami earthquake is an earthquake event that generates abnormally high tsunami waves considering the amplitude of the seismic waves. These abnormally high waves relative to the seismic wave amplitude are related to the longer rupture duration of such earthquakes compared with typical events. Rapid magnitude estimation is essential for the timely issuance of effective tsunami warnings for tsunami earthquakes. For local events, event magnitude estimated from the observed displacement amplitudes of the seismic waves, which can be obtained before estimation of the seismic moment, is often used for the first tsunami warning. However, because the observed displacement amplitude is approximately proportional to the moment rate, conventional magnitudes of tsunami earthquakes estimated based on the seismic wave amplitude tend to underestimate the event size. To overcome this problem, we investigated several methods of magnitude estimation, including magnitudes based on long-period displacement, integrated displacement, and multiband amplitude distribution. We tested the methods using synthetic waveforms calculated from finite fault models of tsunami earthquakes. We found that methods based on observed amplitudes could not estimate magnitude properly, but the method based on the multiband amplitude distribution gave values close to the moment magnitude for many tsunami earthquakes. In this method, peak amplitudes of bandpass filtered waveforms are compared with those of synthetic records for an assumed source duration and fault mechanism. We applied the multiband amplitude distribution method to the records of events that occurred around the Japanese Islands and to those of tsunami earthquakes, and confirmed that this method could be used to estimate event magnitudes close to the moment magnitudes.

Keywords: Tsunami earthquake, Magnitude, Velocity-type strong-motion seismometer, Tsunami warning

Introduction
Some earthquakes generate large tsunamis even though their surface wave magnitudes are relatively small (Kanamori 1972). Such earthquakes, which have abnormally long rupture durations, are called tsunami earthquakes. The 1896 Meiji-Sanriku earthquake off northeast Japan (Kanamori 1972; Tanioka and Satake 1996), the 1946 Aleutian Island earthquake (Kanamori 1972; Johnson and Satake 1997), the 1992 Nicaragua earthquake ($M_S 7.2$, $M_w 7.6$, source duration $\sim 100$ s) (Kanamori and Kikuchi 1993; Satake 1994; Polet Kanamori 2000), the 1994 Java earthquake ($M_S 7.2$, $M_w 7.8$, source duration $80-90$ s) (Polet Kanamori 2000; Abercrombie et al. 2001; Polet and Thio 2003), the 1996 Peru earthquake ($M_S 6.6$, $M_w 7.5$, source duration 50 s) (Heinrich et al. 1998; Ihmle et al. 1998), the 2006 Java earthquake ($M_S 7.7$, $M_w 7.7$, source duration 185 s) (Ammon et al. 2006; Fan et al. 2017), and the 2010 Mentawai earthquake ($M_S 7.8$, $M_w 7.8$, source duration 90–125 s) (Newman et al. 2011; Lay et al. 2011) were all tsunami earthquakes. The moment magnitudes $M_w$ (Kanamori 1977) given here for these events are based on global CMT solutions (Dziewonski et al. 1981; Ekström et al. 2012). These tsunami earthquakes occurred at shallow interplate boundaries near trench axes, and their slow rupture speeds are considered to relate to the frictional properties around shallow plate boundaries (e.g., Polet Kanamori 2000; Bilek and Lay 2002).
Submarine landslides caused by strong ground motions produced by large earthquakes have also caused disastrous tsunamis (Okal and Synolakis 2004). Those earthquakes, although accompanied by high large tsunamis, also have relatively small seismic amplitude. Here, we consider only earthquakes with long source durations.

For tsunami warning purposes, rapid estimation of event magnitude soon after event detection is essential, because the seismic moment of tsunami earthquakes can be used to estimate the height of the expected tsunami. At present, seismic moment can be estimated within 10 min of event detection by $W$-phase analysis of regional seismic network data (Kanamori and Rivera 2008; Usui and Yamauchi 2013). However, tsunamis generated by large local events can arrive at the shoreline in less than 10 min. The Japan Meteorological Agency uses a magnitude determination method based on maximum ground displacements (Katsumata 2004) along with a few additional methods (Japan Meteorological Agency 2013; Katsumata et al. 2013) for huge earthquakes so that the first tsunami warning can be issued within about 3 min of event detection. Because far-field displacement is proportional to the seismic moment rate (e.g., Aki and Richards 2002), the longer the duration of an event, the smaller the displacement amplitude becomes. As a result, estimated magnitudes of tsunami earthquakes based on maximum ground displacement are underestimated. To overcome this problem, Tsuboi et al. (1995) proposed using the $P$-wave moment magnitude $M_{wp}$, which is obtained by integrating broadband seismometer records of $P$-waves, to estimate moment magnitude. They considered the amplitude of the second-order integral of broadband data to be proportional to seismic moment. However, application of the $M_{wp}$ method to the 1992 Nicaragua earthquake yielded an estimated magnitude of less than 7.0 (Tsuboi et al. 1995). Hara (2007) used a magnitude determination method based on the duration of high-frequency energy radiation and the maximum $P$-wave displacement amplitude and obtained values consistent with moment magnitudes, even for tsunami earthquakes. Kawamoto et al. (2017) developed the REGARD fault parameter estimation method based on real-time GNSS data for tsunami-warning purposes. The REGARD method should yield appropriate magnitudes even for tsunami earthquakes, but a tsunami earthquake with an epicenter near the Japan trench, such as the 1896 Sanriku earthquake, might not cause enough crustal deformation for analysis by real-time GNSS. For example, the crustal deformation of the shoreline given the fault model (strike 190°, dip 20°, rake 90°, length 210 km, width 50 km, seismic moment $12 \times 10^{20}$ Nm; Tanioka and Satake 1996) of the 1896 Sanriku earthquake, calculated by the method of Okada (1992) using a rigidity of $6.3 \times 10^{10}$ N/m², was several centimeters, a little smaller than the REGARD threshold (10 cm). For this calculation, we used this rigidity value, which is larger than that indicated by Tanioka and Satake (1996) because crustal deformation was evaluated at a point distant from the fault.

In this study, we investigated several magnitude estimation methods for tsunami earthquakes, based on the long-period displacement, integrated displacement, and multiband amplitude distribution recorded at local distance from the earthquake epicenter by velocity-type strong-motion (VSM) and broadband (BB) seismometers, to use for first tsunami warnings. The $P$, $S$, and surface wave seismic phases from long-duration earthquakes overlap when they arrive at a near-field station. Magnitude can be estimated at local distance more quickly by using mixed phases than by using only $P$-waves (Katsumata et al. 2013). We therefore developed magnitude estimation methods based on the amplitudes of mixed phases. Then we evaluated the methods by using finite fault models and data obtained around the Japanese Islands and other regions.

**Data**

We used data obtained by VSM and BB seismometers operated by the National Research Institute for Earth Science and Disaster Resilience (NIED) for events around the Japanese Islands (Fukuyama et al. 1996). A VSM seismometer has a similar frequency response to a BB seismometer but covers a higher amplitude range. For the tsunami earthquakes, we used data archived by the Incorporated Research Institutions for Seismology.

Real-time processing is required for our magnitude estimation methods, so time-domain processing with a recursive filter is necessary. Instrumental responses were corrected by using recursive digital filters similar to Zhu (2003) and Kanamori and Rivera (2008). If the response $H(s)$ in the Laplace transform expression of the VMS and BB seismometers on the long period side is given by two conjugate or two real poles ($s_{L1}$ and $s_{L2}$), then

$$H(s) = \frac{s^2}{(s - s_{L1})(s - s_{L2})}. \quad (1)$$

For the values of the poles, we used those given by National Research Institute for Earth Science and Disaster Resilience (2019). The recursive filter is obtained by calculating the $z$-transform of the inverse of Eq. 1.

$$H^{-1}(z) = \left((1 - cs_{L1} - cs_{L2} + c^2s_{L1}s_{L2}) + (-2 + 2c^2s_{L1}s_{L2})z^{-1} + (1 + cs_{L1} + cs_{L2} + c^2s_{L1}s_{L2})z^{-2}\right)/(1 - 2z^{-1} + z^{-2}), \quad (2)$$
where $c = \Delta t / 2$ and $\Delta t$ is the sampling interval. When the response on the long period side is given by a single real pole ($s_{L}$), the recursive filter is

$$H^{-1}(z) = \frac{(1 - cs_{L}) + (-1 - cs_{L})z^{-1}}{1 - z^{-1}}. \quad (3)$$

**Empirical method based on observed amplitude**

We derived empirical formulas to represent the relationships between moment magnitude and observed peak amplitudes using data from ordinary events around the Japanese Islands. We examined three types of amplitudes: highpass-filtered displacement (cutoff period 200 s), highpass-filtered integrated displacement (cutoff period 200 s), and bandpass-filtered displacement (passband 200–400 s). We expected integrated displacement would reflect seismic moment rather than moment rate (Tsuboi et al. 1995), and that long period displacement would be relatively insensitive to source duration. As a target, we assumed an earthquake with a rupture duration of 100 s. We set the cutoff period for the highpass-filtered displacement and the highpass-filtered integrated displacement to 200 s, twice the assumed rupture duration. A fourth-order Bessel high-pass filter (Katsumata 1993) was applied to the displacement to suppress the baseline shift of the seismic wave caused by the three time integrations (two for the instrumental response correction by Eq. 2 and one for the conversion from velocity to displacement) during continuous filtering in a real-time system. A fifth-order Bessel high-pass filter was applied to the integrated displacement.

For this analysis, we selected events that occurred near the main islands of Japan (open circles in Fig. 1, Table S1), based on hypocenter locations in the Seismological Bulletin of Japan (Japan Meteorological Agency 2019), to avoid the effect of small amplitudes attributable to the radiation pattern. Although large earthquakes have occurred around the Kuril Islands, stations on the Japanese Islands are located approximately along the null axis of the seismic wave radiation pattern from those earthquakes. We set a threshold of $M_{w} > 6.8$ as an approximate magnitude threshold of hazardous tsunami potential. Whereas these events were not tsunami earthquakes, we expected empirical analyses on the integrated displacement and long-period displacement of these events would provide formulas reflecting their seismic moments rather than their moment rates.

![Fig. 1](image-url) Map showing the locations of the stations (triangles) and epicenters (open circles) of the earthquakes used to obtain the regression relationship between earthquake magnitude and observed peak amplitudes. Open and gray-filled circles denote the events used for Figs. 5 and 7.
For example, Fig. 2 shows an original seismogram recorded at Urahoro station (URH, epicentral distance 132 km) during the 2003 Tokachi-oki earthquake (25 September UTC, $M_w$ 8.3), together with the displacement and the integrated displacement records obtained after instrumental response correction and integration.

It is possible that some amplitude data were contaminated with instrumental noise, and it is necessary to exclude those data for proper magnitude estimation. For example, Fig. 3 shows peak integrated displacement amplitudes and travel times for an earthquake ($M_w$ 6.4) that occurred on 24 January 2018 east off northeast Japan. Here, $T_0$ is the origin time, $T_P$ and $T_S$ are the calculated $P$- and $S$-wave travel times (Ueno et al. 2002), and $T_D$ is the assumed rupture duration ($2 \times 10^{(M_w−5)/2}$ s). The data from BB seismometers are distributed between $T_0 + T_S$ (solid red line in Fig. 3) and $T_0 + 2.5T_S + T_D$ (dashed red line), whereas the data from VSM seismometers are distributed mainly after $T_0 + 2.5T_S + T_D$. The data after $T_0 + 2.5T_S + T_D$ are considered to be contaminated by instrumental noise because of the scattered travel times and because the amplitudes are larger than the BB seismometer amplitudes. We divided peak amplitude data for 366 events into “Good” and “Not Good” data, where peak amplitudes between $T_0 + T_S$ and $T_0 + 2.5T_S + T_D$ were considered to be Good data, and plotted the data against the peak digital counts (BIT; Fig. 4, left panels) and the integrated displacement amplitudes (right side). The result shows that data with large integrated displacement amplitudes may not be reliable, but the digital count of the original data can be used as an index for distinguishing reliable from unreliable data. In

![Figure 2](image1.png)

**Fig. 2** Example seismic waveforms. a Original seismogram of the 2003 Tokachi-oki earthquake recorded at Urahoro station with a velocity-type strong-motion seismometer. b Displacement record (black curve) obtained after instrumental correction and integration of the original record shown in a. c Integrated displacement record (black curve) obtained from the original record shown in a. d Bandpass (200–400 s) filtered displacement record (black curve) obtained from the original record shown in a. The red curves in b–d show synthetic records for the finite fault model of U.S. Geological Survey (2020)

![Figure 3](image2.png)

**Fig. 3** Peak amplitudes of integrated displacement (upper) for an earthquake that occurred on 24 January 2018 ($M_w$ 6.4), and travel times to peak amplitude (lower) in relation to hypocentral distance for amplitudes observed with velocity-type strong-motion (VSM) seismometers (crosses) and broadband (BB) seismometers (circles). The solid blue, solid red, and dashed red curves show the calculated values of $T_0 + T_P$, $T_0 + T_S$, and $T_0 + 2.5T_S + T_D$, respectively, where $T_0$ is origin time, $T_P$ is $P$-wave travel time, $T_S$ is $S$-wave travel time, and $T_D$ is the assumed source duration. The travel times were calculated by using the velocity structure of Ueno et al. (2002)

![Table 1](image3.png)

**Table 1** Velocity structure model used to calculate the synthetic records

| Depth (km) | Velocity ($P$) (km/s) | Velocity ($S$) (km/s) | Density (g/cm³) | $Q_P$ | $Q_S$ |
|-----------|----------------------|----------------------|-----------------|-------|-------|
| 0.0       | 4.2                  | 2.42                 | 2.4             | 200   | 100   |
| 2.4       | 5.3                  | 3.06                 | 2.6             | 300   | 150   |
| 4.0       | 6.1                  | 3.52                 | 2.7             | 300   | 150   |
| 14.6      | 6.7                  | 3.87                 | 3.0             | 500   | 250   |
| 31.5      | 8.0                  | 4.62                 | 3.2             | 600   | 300   |

For example, Fig. 2 shows an original seismogram recorded at Urahoro station (URH, epicentral distance 132 km) during the 2003 Tokachi-oki earthquake (25 September UTC, $M_w$ 8.3), together with the displacement and the integrated displacement records obtained after instrumental response correction and integration.
our analysis, therefore, we used amplitude data with time
$t$, where $T_0 + T_S < t < T_0 + 2.5T_S + T_D$, and original
digital data amplitudes $> 210^2$. Because duration cannot
be set appropriately before the magnitude is estimated,
we assumed $T_D$ to be constant at 200 s.

Using the peak amplitude ($A$) of the vertical com-
ponent, we obtained the following empirical relationship:

$$M_w \simeq a \log_{10} A + b \log_{10} R + c,$$

where $a$, $b$, and $c$ are constants; $M_w$ is the moment mag-
nitude calculated with global CMT data (Dziewonski
et al. 1981; Ekström et al. 2012); and $R$ is hypocentral
distance ($\leq 1000$ km). We selected the vertical component
because we expected dip slip on the fault to result in a
large vertical displacement amplitude and because the
vertical component would be relatively insensitive to the
inclination of the crustal deformation.

### Table 2

| Year    | Mw | MD200 | MD200-400 | MD200-400 | M_BA |
|---------|----|-------|-----------|-----------|------|
| 1992 Nicaragua | 7.6 | 6.7      | 6.9      | 7.2      | 7.5  |
| 1994 Java     | 7.8 | 7.5      | 7.5      | 7.2      | 7.6  |
| 1996 Peru     | 7.5 | 7.4      | 7.2      | 7.2      | 7.4  |
| 2006 Java     | 7.7 | 6.9      | 6.9      | 7.2      | 7.3  |
| 2010 Mentawai | 7.8 | 7.3      | 7.2      | 7.3      | 7.5  |
| 2003 Tokachi  | 8.3 | 8.4      | 8.2      | 7.9      | 8.1  |
| 2011 Tohoku   | 9.1 | 8.9      | 8.8      | 8.9      | 8.9  |

**Fig. 4** Distinguishing "Good" and "Not Good" data. Good data are data with peak integrated displacement times between $T_0 + T_S$ and $T_0 + 2.5T_S + T_D$, as shown in Fig. 3. The peak digital counts of the original record (BIT, left) and the integrated displacement amplitudes (right) are compared between a velocity-type strong-motion (VSM) seismometers and b broadband (BB) seismometers. Good data are shown above the 0 line, and Not Good data (as defined in the text) are shown below the line. Ratios of Good good to the total number of data are shown by red circles (right-side scale)
We expected $b$ in Eq. 4 to be insensitive to earthquake size. By averaging the regression line slopes of 20 events (open circles in Fig. 1, Table S1), including inland events, with $M_w > 6.8$, we obtained values of $b_{D200} = 1.10$ for highpass-filtered displacements with a 200 s cutoff, $b_{ID200} = 0.857$ for highpass-filtered integrated displacements with a 200 s cutoff, and $b_{D200-400} = 0.923$ for bandpass-filtered displacement with a passband of 200-400 s. $a$ and $c$ were obtained by minimizing the expression \[ \sum_i \left( M_{wi} - (a \log_{10} A + b \log_{10} R + c) \right)^2, \] where $i$ is an index of events. The resulting magnitude estimation formulas for the three types of amplitudes are

\[
M_{D200} = 1.06 \log_{10} A_{D200} + 1.10 \log_{10} R + 6.69, \quad (5)
\]

\[
M_{ID200} = 0.919 \log_{10} A_{ID200} + 0.857 \log_{10} R + 6.31, \quad (6)
\]

\[
M_{D200-400} = 0.813 \log_{10} A_{D200-400} + 0.923 \log_{10} R + 7.63. \quad (7)
\]

We compared $M_{D200}$, $M_{ID200}$, and $M_{D200-400}$ obtained with Eqs. 5–7 with $M_w$ (Fig. 5). We included smaller events ($M_w > 6.0$) for this figure to check the results for smaller events. Underestimation is recognized for events of $M_w \sim 6.0$ in all three methods. For quick magnitude estimation, it is necessary to limit the number of stations used. Therefore, magnitudes estimated with data from the 10 closest stations (crosses in Fig. 5) are shown together with those estimated by using all available data (circles).

For events with $M_w > 6.8$, the root mean squares of $M_{D200} - M_w$, $M_{ID200} - M_w$, and $M_{D200-400} - M_w$ for magnitudes estimated by using all data within 1000 km were 0.18, 0.14, and 0.15, respectively.

**Magnitude based on multiband amplitude distribution ($M_{BA}$)**

Rupture duration is a key factor used to characterize a tsunami earthquake. To estimate the characteristic duration of an earthquake, we examined the multiband amplitude distribution obtained from the peak amplitudes of bandpass-filtered seismic records with passbands at intervals of $10^{0.2}$ from 2.51 to 631 s. The ratio of upper and lower limits of passband intervals was set to $10^{0.16}$ so that the passbands overlapped with each other. In this case, the amplitudes of 10 bands were used. To exclude noise-contaminated data, the same peak digital count limitation (described in the previous section) was applied in this analysis. Overlapping passbands were used to capture relatively short-duration seismic wave increases. If the period on the shorter side of a passband is long enough, it takes a long time for the seismic wave to reach its peak, and this long time could become a problem for rapid magnitude estimation. We compared the amplitude distributions between bandpass-filtered observed and synthetic records by assuming various rupture durations. As an example, multiband amplitude distributions of the 2003 Tokachi-oki earthquake ($M_w 8.3$) are shown in Fig. 6. The black and red bars show peak amplitudes of bandpass-filtered observed and synthetic records. The synthetic records were calculated by assuming a seismic moment value, $M_0$, and various source durations. The durations were set at $10^{0.1}$ intervals from 2.5 to 398 s. Each graph in Fig. 6 compares the observed amplitude
Fig. 6 Examples of multiband amplitude distributions of the 2003 Tokachi-oki earthquake in relation to seismic wave period. The black bars indicate observed peak displacements of the bandpass-filtered seismic record, and the red bars indicate those of the synthetic records fitted to the observation. The length of each bar shows the passband range. Each graph shows comparison for an assumed source duration. The estimate magnitude for each duration is indicated at the upper left of the graph. Duration of 25 s showed the best fit in this case.
distribution with the synthetic one for a specific source duration. The best source duration was selected based on the standard deviation of the least squares fitting, which was 25 s in the case of Fig. 6. The earthquake magnitude for a station, \( M_{BAst} \), was estimated from the assumed seismic moment for the synthetic records and logarithmic average difference between observed and synthetic amplitudes, \( \log_{10} A_{oi} - \log_{10} A_{si} \), for the selected source duration as

\[
M_{BAst} = M_{0st} + 10^{0.030 \log_{10} A_{oi} - \log_{10} A_{si}},
\]

\[
M_{BAst} = (\log_{10} M_{0st} - 9.1)/1.5,
\]

where \( M_{0st} \) is estimated seismic moment for a station, \( A_{oi} \) and \( A_{si} \) are the observed and synthetic peak amplitudes of the selected source duration for the \( i \)th bandpass filter.

For data from multiple stations, the average was calculated over station magnitudes, \( M_{BAst} \), using the duration estimated to be the mode. The minimum number of data to estimate \( M_{BA} \) was set at three so that extraordinary data would be rejected. If a dominant duration was not found, sum over three durations was evaluated to determine the dominant duration. The number of stations was limited to 10 to reduce the estimation time, because the results for 10 stations were almost the same as those for 20 stations.

The synthetic records were calculated by the method of Takeo (1985) for a point source at the hypocenter with the velocity structure shown in Table 1. Fault parameters must be assumed to calculate synthetic records. The strike and dip were assumed from the subducting plate geometry. The rake was set at 90° by assuming a pure dip slip. The assumed source time function was \( [1 - \cos(\pi t/D)]/2 \), where \( t \) and \( D \) denote time and source duration, respectively. Synthetic records to be used for rapid estimation soon after an earthquake occurrence were obtained from Green’s functions calculated in advance. We prepared Green’s functions for six seismic moment components at 2 km intervals up to 1,000 km in epicentral distance and at 2 km intervals down to 100 km depth.

We compared \( M_w \) with the magnitude estimated by observed data using the multiband amplitude distribution for events around the Japanese Islands (Fig. 7). For events with \( M_w > 6.8 \), the root mean square of \( M_{BA} - M_w \) was 0.14. Result for smaller events \( (M_w > 6.0) \) were also shown in the figure. Overestimation is recognized in Fig. 7 for events of \( M_w > 6.0 \). We tested the depth dependence of \( M_{BA} \) by changing the assumed focal depth from 2 km to 50 km for the 2003 Tokachi-oki earthquake; the resultant \( M_{BA} - M_w \) value was within about 0.3 magnitude unit.

**Discussion**

**Evaluation with finite fault models**

The proposed methods were tested by using finite fault models estimated by U. S. Geological Survey (2020). Because our main targets are tsunami earthquakes occurring around the Japanese Islands, we assumed that the past tsunami earthquakes occurred off Tohoku, Japan (Fig. 8), and that the seismic records were obtained from the current Japanese seismic network (Fig. 1). For each event, a synthetic record was calculated as the summation of seismic records from point sources on sub-faults (Fig. 8) by the method of Takeo (1985).

The estimated magnitudes are listed in Table 2 along with the results for the finite fault models of two major earthquakes around Japan, shown for comparison as examples of ordinary events. Magnitudes based on the three types of observed amplitudes (\( M_{D200} \), \( M_{ID200} \), and \( M_{P200-400} \)) underestimated the sizes of these tsunami earthquakes, especially those of the 1992 Nicaragua and the 2006 Java earthquakes, whereas \( M_{BA} \) values were closer to \( M_w \) for nearly all events.

All of the magnitudes based on the observed amplitudes tended to underestimate earthquake magnitudes. Figure 9 shows the relationship between peak amplitude normalized by the amplitude of the original model (U. S. Geological Survey 2020) and the rupture duration ratio calculated using the finite fault model for the 2003 Tokachi-oki earthquake. The amplitudes of displacement, integrated displacement, and bandpass-filtered displacement decreased quickly as the assumed rupture duration became longer. The synthetic records for the finite fault model (shown in Fig. 2) were obtained by convolution between the Green’s function and the source time function, and averaging of the Green’s function over a long rupture duration can explain the small peak amplitudes. The same cause may account for the \( M_{wp} \)
Fig. 8 (Upper) The finite fault models (U. S. Geological Survey 2020) used to test proposed magnitudes. The five tsunami earthquakes were assumed to occur off Tohoku, Japan. On each map, the circle indicates the epicenter. The various magnitudes were estimated (Table 2) for synthetic records calculated with these fault models. (Lower) Source time function of each event.
underestimation of the magnitude of the 1992 Nicaragua earthquake by Tsuboi et al. (1995).

Comparison of the source time functions of the tsunami earthquakes (Fig. 8) shows that the peak moment rates over the durations of the 1992 and 2006 events were relatively low compared with those of the 1994, 1996, and 2010 events; the source time functions of the 1994, 1996, and 2010 events show clear peaks, whereas those of the 1992 and 2006 events are relatively flat. These differences in the source time functions would have affected the degree to which the observed amplitude underestimated $M_w$ (Table 2).

One way of compensating the underestimation seen in Table 2 would be a safety factor in tsunami height estimation. The average of $M_{BA} - M_w$ in Table 2 including the 2003 and 2011 events is $-0.2$. However, $M_{BA} - M_w$ obtained with observed data for the 2003 and 2011 events (Fig. 7) are different from those in Table 2. Because number of samples is not many and results of simulation and observation are a little different, we do not show a specific value of the safety factor for $M_{BA}$ here. On the other hand, Tanioka and Seno (2001) discussed sediment effect for the 1896 Sanriku tsunami earthquakes. A safety factor for the sediment or low-rigidity effects should be added in practical use for the events near the trench.

**Application to the observed records of the tsunami earthquake**

We applied the $M_{BA}$ magnitude estimation method to the observed records of tsunami earthquakes. However, not enough data were available for the 1992 Nicaragua, 1994 Java, 1996 Peru, and 2006 Java tsunami earthquakes. Although data were available from two seismic stations within 1000 km from the epicenter of the 2006 Java event, they were insufficient for estimating $M_{BA}$. For the 2010 Mentawai earthquake ($M_w7.8$), we obtained $M_{BA}7.7$, which is close to the moment magnitude of this tsunami earthquake.

A value of $M_{BA}7.5$ for this earthquake was obtained from the finite fault model (Table 2) in which source location was assumed off Tohoku, Japan. We calculated synthetic records for this event with assuming its original source location and real station distribution, and a value of $M_{BA}7.5$ was obtained, which was the same as that in Table 2. The reason of the underestimation in simulation seems not in the relative locations of stations.

BB seismometer networks have been enhanced after the tsunami earthquakes in Table 2. It would be possible to estimate $M_{BA}$ for the all of the past tsunami earthquakes if they occur in the future. Because availability of VSM data is not enough except for in Japan, data records in the areas close to sources would be partly clipped for large earthquakes. Although data of a few stations were off the scale in the case of the 2010 Mentawai earthquake, it was possible to estimate $M_{BA}$ with the remaining data for the earthquake.

**Effect on tsunami height estimation from magnitude underestimation**

Table 2 suggested underestimation of $M_{BA}$ for an event similar to the 2006 Java earthquake. We calculated tsunami heights for the model of the event ($M_w7.7$) shown in Fig. 8 and that with reduced magnitude ($M_w7.3$) with JAGURS tsunami simulation code (Baba et al. 2017) and bathymetry data by Disaster Management Council of Japan (2003). Nonlinear longwave equation code in JAGURS and nested grids of intervals 1,350 m, 450 m, and 150 m were used for the tsunami simulation. Crustal deformation was calculated by the method of Okada (1992). A rigidity of 30 GPa was assumed for the crustal deformation calculation and calculating fault slip from seismic moment. The calculated tsunami heights along water depth of 100 m are shown in Fig. 10. The tsunami height difference was 3.6–7.5 times between the $M_w7.7$ and $M_w7.3$ models, which reflected the corresponding seismic moment difference. Figure 10 indicates that a possibility of serious underestimation could occur in
tsunami height estimation based on $M_{BA}$ for an event similar to the 2006 Java earthquake. For that case, tsunami warning upgrade after the first tsunami warning based on a moment tensor estimation and off-shore tsunami gauge data would be important to mitigate tsunami disaster.

A uniform-slip fault model is often used for the tsunami height estimation in tsunami warning systems. We calculated tsunami height from a fault model with a uniform slip. A fault of length 71 km, width 35 km, and slip 2.1 m ($M_w$ 7.3) was assumed following a uniform-slip fault model used by Nakata et al. (2019). Strike N188$^\circ$E, dip 45$^\circ$, rake 90$^\circ$, top depth 2.4 km, and a fault corner location at 38.5$^\circ$N, 143.8$^\circ$E were used as fault parameters. The result of the calculation is show in Fig. 10 with those for the finite fault models. The top tsunami height of the uniform-slip fault model was about half of that for the finite fault model with the same moment magnitude. Although peak crustal deformation of the uniform-slip fault model was larger than that of the finite fault model of $M_w$ 7.7, the resultant tsunami height was significantly smaller for the uniform-slip fault model. This tsunami height difference come from the crustal deformation distribution difference between the finite fault model and the uniform-slip fault model. Such difference should be taken into account as a safety factor of tsunami warning for events near the trench axis.

**Time required for magnitude estimation**

The time required for magnitude estimation is important for early tsunami warning. Comparison of $M_{BA}$ with the time elapsed since the origin time for the fault models shown in Fig. 8 (Fig. 11) shows that the final magnitudes of the five events were reached within about 4 min, and some of them closely approached their final values within 3 min. Variations in the estimated duration with elapsed are also shown in Fig. 11. Because the assumed source duration greatly affects the amplitude distribution (Fig. 6), $M_{BA}$ is greatly dependent on the estimated duration; thus the time needed to estimate the duration controls the time needed to estimate $M_{BA}$. To obtain the values shown in Table 2, the period ratio between the lower and upper passband was set at $10^{0.6}$. When a value of $10^{0.2}$ was used as the ratio between the lower and upper passband limits, we obtained $M_{BA}$ values closer to $M_w$ than those in the table, but 6–8 min were required for the magnitude estimation.

The estimated durations, which ranged from 20 to 50 s (Fig. 11), were much shorter than the source durations...
For the 1992 Nicaragua earthquake, we also compared the station number limit with the time required to approach the final $M_{BA}$ and $M_{BA} - M_w$.

Indicated by the source time function (Fig. 8), because the estimated durations relate the characteristic period of the seismic waves at the observation sites rather than the actual source duration.
Although the time was reduced when fewer stations were used, the time reduction was negligible when the station number limit was set to less than under about 10. When only a small number of station were used, there were intervals during which estimation of $M_{BA}$ failed. This result indicated the station number limit of 10 was appropriate from the perspective of magnitude estimation time.

Figure 12 shows variations of $M_{BA}$ and the estimated durations obtained with observed data for three large earthquakes around the Japanese Islands. In the case of the 2011 Tohoku earthquake, $M_{BA}$ approached the final value within an elapsed time of about 4 min, and in the case of the other events, $M_{BA}$ approached the final value about 2 min. The long rupture duration (> 150 s; Yoshida et al. 2011) of the 2011 event seems to have prevented the magnitude from being estimated within 3 min. We also applied the $M_{BA}$ method to events around the Kuril Islands and Taiwan, where few events have adequate azimuthal coverage of seismic stations, but we did not succeed in obtaining $M_{BA}$ for many of these events because it was not possible to estimate dominant duration.

**Conclusions**

We developed a rapid magnitude estimation method for tsunami earthquakes based on the amplitude distribution of multi-bandpass-filtered records. We expect this method to be used soon after event detection but before seismic moment estimation. The multi-band amplitude distribution was greatly affected by the source duration, which was estimated by comparison with synthetic records for various assumed source durations. The magnitude was estimated from the ratio of observed amplitudes to synthetic ones. Simulations with finite fault models demonstrated the effectiveness of this method for past tsunami earthquakes. Application to observed data also showed proper magnitude estimation for the 2010 Mentawai earthquake, a typical tsunami earthquakes. The intended use of this method is the issuance of the first tsunami warnings soon after event detection. By using the developed method, we were able to estimate the magnitudes of great earthquakes, including the 2011 Tohoku earthquake, within about 4 min.
Abbreviations
BB: Broadband seismometer, IRIS: Incorporated Research Institutions for Seismology; GNSS: Global Navigation Satellite System; NIED: The National Research Institute for Earth Science and Disaster Resilience; VSM: Velocity-type strong-motion seismometer.

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Authors’ contributions
AK proposed the estimation methods and contributed to the data analyses, the synthetic record analyses, and manuscript preparation. MT contributed to the observed data analyses and preliminary manuscript. TN contributed to the synthetic record analyses. All authors read and approved the final manuscript.

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Availability of data and materials
The seismic data analyzed in this study are available at the National Research Institute for Earth Science and Disaster Resilience (NIED) F-net repository, http://www.fnet.bosai.go.jp/top.php, and the Incorporated Research Institutions for Seismology (IRIS) repository, https://iris.edu/wibber3/find_event. The Global CMT catalog is available from the Global Centroid-Moment-Tensor Project repository, https://www.globalcmt.org/CMTsearch.html. The Seismological Bulletin of Japan is available from the Japan Meteorological Agency repository, https://www.data.jma.go.jp/svd/eqev/data/bulletin/index.html. Topographic data used for tsunami simulation is available from the Association for Promotion of Infrastructure Geospatial Information Distribution repository, https://www.geospatial.jp/ckan/dataset.

Declarations
Competing interests
The authors declare that they have no competing interests.

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Affiliation of one of authors (AK) was changed from Meteorological Research Institute to University of Toyama (akatsuma@sus.u-toyama.ac.jp) after the submission.

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