I. INTRODUCTION

The exquisite ability to tune interatomic interactions using magnetic Feshbach resonances lies at the heart of many ultracold-atom experiments [1]. A single resonance can provide access to the full range of contact interactions, characterized by the s-wave scattering length \( a \). Near resonance, strongly correlated phases of matter [2] can be realized, with highlights including unitary Bose [3–10] and Fermi [11–13] gases. Focusing on Bose gases, the weakly repulsive regime offers a textbook setting for exploring interacting Bose–Einstein condensates [14, 15], while tunable attractive interactions facilitate studies of condensate collapse [16–19], soliton formation [20–24], and negative absolute temperatures [25]. The zero crossing of \( a \) provides pristine conditions for simulating noninteracting quantum phenomena (see e.g. [26, 27]). Knowledge of the details of Feshbach resonances and the resulting map of scattering lengths across multiple atomic states facilitates studies of quantum mixtures [28] and many-body interferometry [7, 31, 32].

Feshbach resonances also provide a testbed for fundamental few-body physics, with the resonance positions \( B_{\text{res}} \) acting as invaluable benchmarks. Sufficienctly close to resonance, the structure of few-body bound states is particularly simple: a dimer of size \( a \) exists on the repulsive side (\( a > 0 \)) and becomes unbound as the magnetic field \( B \rightarrow B_{\text{res}} \). This provides a gateway into the realm of ultracold molecules [33–35], and a particularly accurate method for pinpointing \( B_{\text{res}} \) [1, 36–38].

In Bose gases, the Efimov effect [39–43] leads to a spectrum of three-body bound states, persisting even when the dimer is unbound. A hallmark of Efimov states is their discrete scaling symmetry, which manifests in the log-periodic modulation of few-body observables, such as the three-body recombination rate [44–51] (see also [52]). While the functional form of these modulations is predicted by Efimov theory, their phase (captured by \( a_- \), where the lowest Efimov state meets the continuum) depends on the details of the short-range interaction. Remarkably, for many Feshbach resonances, across spin states and atomic species, \( a_- \) was measured to be within 20% of \( -9r_{vdW} \) [49, 53, 61], where \( r_{vdW} \) is the van-der-Waals length [1]. This Efimov–van-der-Waals universality was traced to the universal form of the interaction potential, predicting \( a_- \approx -9.7r_{vdW} \) for broad resonances [62, 63].

The Efimov effect is poised to influence the many-body state at unitarity as well, from setting the lifetime of the unitary gas [3, 6] to predictions of a low-temperature superfluid of Efimov trimers [64, 65]. So far, three-body correlations in a thermal unitary Bose gas have been observed [7], but in the degenerate case signatures are limited to experiments that convert the strongly correlated state into an atom-molecule mixture by sweeping from unitary to weak interactions [8, 9].

Over the last 15 years \(^{39}\text{K}\) has proven to be a versatile atom for quantum-gas experiments. Numerous \(^{39}\text{K}\) Feshbach resonances were surveyed early on [66–68], with recent precision measurements of a select few [7, 38, 69]. The intermediate-strength nature of its broad resonances has rendered \(^{39}\text{K}\) ideal for tests of the Efimov universalities [38, 47, 51, 68]. Recently, a high-precision measurement on a 33.6 G resonance revealed a breakdown of Efimov–van-der-Waals universality [38], finding \( a_- \approx -14.05(17)r_{vdW} \), while the Efimov ratios were largely consistent with universal theory [51].

In this article, we use a combination of bound-state and loss spectroscopy to pinpoint and characterize eight intrastate Feshbach resonances in \(^{39}\text{K}\) and six previously unexplored interstate ones. We compare the properties of the resonances with the results of coupled-channel scattering calculations. Using the resultant accurate determination of the interaction landscape and leveraging the advantages of homogeneous samples [70, 71], we map out Efimov loss extrema across four broad Feshbach resonances. Our measurements reveal a ubiquitous breakdown of Efimov–van-der-Waals universality [53, 61], consistently finding \( a_- \) in the range \( -13(1)r_{vdW} \), which we attribute to the similar intermediate-strength resonance...
characters. The Efimov ratios between features across the Feshbach resonance are in agreement with Efimov theory. We also explore the many-body loss dynamics in unitary gases for the three broadest Feshbach resonances in different spin states. We observe universal behavior, consistent with measuring similar Efimov width parameters $\eta^*$.

II. BOUND-STATE SPECTROSCOPY

In the vicinity of an isolated Feshbach resonance, neglecting inelasticity, $a$ approximately follows [72]

$$a = \alpha_{bg} \left(1 - \frac{\Delta}{B - B_{\text{res}}}\right),$$

(1)

where $\alpha_{bg}$ is a slowly varying background scattering length and $\Delta$ is the resonance width. Close to the pole, $a \approx -\alpha_{bg}\Delta/(B - B_{\text{res}})$. If $\alpha_{bg}$ is constant across the entire resonance, $a$ crosses zero at $B_{\text{res}} + \Delta$. More generally, however, the position $B_{\text{zero}}$ of the zero crossing may differ from this value. In the presence of inelasticity, Eq. (1) breaks down close to resonance [73]. However, for the resonances considered here, this breakdown is insignificant more than 1 mG from $B_{\text{res}}$, as shown theoretically in Appendix A.

For large positive values of $a$ close to resonance, the binding energy of the least-bound molecular state is [74]

$$E_b = \frac{\hbar^2}{m(a - \bar{a})^2},$$

(2)

where $m$ is the atom mass, $\hbar$ the reduced Planck’s constant, $\bar{a} = [4\pi/\Gamma(1/4)^2]r_{\text{vdW}} \approx 61.8a_0$, and $r_{\text{vdW}} = 64.6a_0$ [75] for $^{39}$K, and $a_0$ is the Bohr radius.

For our bound-state spectroscopy we begin with a quasipure $^{39}$K Bose–Einstein condensate confined in the uniform potential of a cylindrical optical box trap [19, 70, 71] of volume $V \approx 3 \times 10^{-14}$ m$^3$. We prepare weakly interacting repulsive spin-polarized samples in a hyperfine state of choice within the $F = 1$ manifold. Following Refs. [76, 77], and as depicted in Fig. 1(a), we employ a Ramsey-like quench protocol. After tuning $B$ to a value of interest over tens of ms [78], we perform two field pulses of magnitude $\delta B$ towards $B_{\text{res}}$, separated by a time $t_0$ [80]. We use $t_1$ on the order of tens of ms, short $t_2 = 10 \mu$s, and our field changes occur in $\approx 2 \mu$s [9]. The first pulse initiates a coherent superposition of atoms and molecules [81]. During the evolution time $t_0$ a phase difference (set by $E_b$) accumulates, before the second pulse projects this difference onto a population imbalance. Finally, we record a time-of-flight absorption image of the sample.

As shown in Fig. 1(b), the observed atom number $N_{\text{obs}}$ exhibits decaying oscillations with $t_0$. Our imaging detects only free atoms, and we associate the initial increase of $N_{\text{obs}}$ with the number of atoms transferred to the molecular state. Once the oscillation has decayed, we are still able to revert a significant fraction of molecules to atoms using the second pulse, consistent with a long-lived molecular sample that eventually decays due to molecular loss processes (see e.g. [51]).

To extract the oscillation frequency $\omega_b$, which directly gives $E_b = \hbar\omega_b$, we fit $N_{\text{obs}}(t_0)$ with $N_0 - \alpha t_0 + N_0 \exp(-t_0/\tau) \sin(\omega_b t_0 + \phi)$ [82]. While most extracted fit parameters are sensitive to the details of the quench protocol, crucially, $\omega_b$ is robust [76, 77].

In Fig. 1(c) we show $\omega_b/(2\pi)$ versus $B$ for the $|1, 1$ state near the 402.7 G Feshbach resonance. The solid line shows a fit to the data using Eqs. (1,2), which yields $B_{\text{res}} = 402.76(3)$ G, $\alpha_{bg}\Delta = 1600(70) a_0 G$, and a relatively uncertain $a_{bg} = -60(30) a_0$. If we assume that $\Delta = B_{\text{zero}} - B_{\text{res}}$ and use the independently measured $B_{\text{zero}} = 350.4(1)$ G [19, 83], to constrain the fit, we obtain a refined $B_{\text{res}} = 402.74(1)$ G and $a_{bg} = -29.3(3) a_0$.

In Fig. 1(d) we plot $1/a$ versus $B - B_{\text{res}}$, and also show analogous measurements across three other resonances. We depict the constrained fits (solid lines) and their linear approximation valid near $B_{\text{res}}$ (dashed lines).
a\text{action landscape and quench protocol. We prepare thermal samples in the (1,1) state ±3G away from \(B_{\text{res}} = 402.74(1)\) G (dashed line). In both cases we quench to the same final \(B\), wait for a time \(t_1\), before quenching back and probing the sample. (b) Fractional change in both cases we quench to the same final \(B\text{res}\). We observe maximal loss near \(\delta N/N_0 = 1\), which occurs at \(B_{\text{res}}\), validating our loss spectroscopy, while the secondary feature arises due to Efimov physics.

**III. BENCHMARKING LOSS SPECTROSCOPY**

We next benchmark the more economical loss spectroscopy as an accurate albeit less precise method for measuring \(B_{\text{res}}\). We focus on the (1,1) state near \(B_{\text{res}} = 402.74(1)\) G and prepare samples of density \(n_0 = 1.3\mu\text{m}^{-3}\) at a temperature \(T_0 = 100\text{nK}\); thermal samples are used to facilitate starting both above and below resonance. As depicted in Fig. 2(a), in both cases we then quench to the same final \(B\) and wait for a time \(t_1\), before quenching back to measure \(N_{\text{obs}}\).

In Fig. 2(b) we plot the fractional change in atom number \(\delta N/N_0 = N_{\text{obs}}/N_0 - 1\) after \(t_1 = 400\mu\text{s}\) as a function of \(B - B_{\text{res}}\) for both quenches. We observe maximal loss near \(B_{\text{res}}\), and see that the quench direction does not matter (this relies on choosing a sufficiently long \(t_1\), see Appendix B). The central loss feature is asymmetric, with enhanced loss for \(a < 0\) and a pronounced secondary peak, as expected from Efimov physics. We associate the secondary peak with the lowest-lying Efimov resonance at \(a_{\text{res}}\), which occurs near but visibly closer to \(B_{\text{res}}\) than \(a_{\text{res}} = -9.7vdW\) predicted from Efimov–vander-Waals universality. Our measurements establish quench-based loss-spectroscopy as an accurate tool to pinpoint Feshbach resonances up to the width of the loss feature [84].

**IV. SUMMARY OF \(^{39}\text{K} FESHBACH RESONANCES**

In Table I we summarize our intrastate resonance measurements, including those from Fig. 1(c) and four based on loss spectroscopy. We also include the high-precision measurement from Ref. [38] and the high-field one from Ref. [67] (see also [85]). Moreover, we include our independent measurements of the associated zero-crossings [88] based on either the critical condition for collapse [16, 19] or thermalization rates [90]. Note that our measured values of \(a_{bg}\Delta\) are those that characterize the strength of the pole in \(a\), and are not necessarily consistent with independently measured values of \(B_{\text{zero}}\) and \(a_{bg}\) far from the pole.

In Table II we summarize our interstate resonance measurements, also including one from Ref. [69]. By transposing loss spectroscopy to spin mixtures we locate six previously predicted [91] but experimentally elusive interstate resonances. The resonances at 445.4 G and 526.2 G are promising candidates for studies of quantum mixtures (e.g. Bose polarons [92–94]). In particular, the magnetic moments of the two states involved \((\bar{\mu}_1, \bar{\mu}_2)\) are very similar (differing by only 3% and 0.3%, respectively), which opens the door to forming magneti-
cally levitated uniform Bose mixtures with tuneable interparticle interactions. We further characterize these two resonances using rf molecular association spectroscopy [1, 95, 96]; see Appendix D for details.

We also compare our measurements to coupled-channel scattering calculations using state-of-the-art interaction potentials [87]; see Appendix A for details. The calculated resonance positions differ from the experiment by no more than 0.25 G, even for the broadest resonances. The calculated values of \( a_{02}/\Delta \) are in good agreement with the experiment within at most two standard deviations.

V. TESTING EFIMOV UNIVERSALITIES

Equipped with a high-precision map \( a(B) \), we now turn to tests of Efimov universalities. The three-body loss rate of a thermal Bose gas is given by

\[
\dot{N}/N = -L_3 n^2, \tag{3}
\]

which upon integration yields \( 1/N^2 = 2tL_3/V^2 + 1/N_0^2 \) in a uniform system of volume \( V \). The scaling \( L_3 \propto \hbar a^{1/3}m \), expected on dimensional grounds (for \( r_{cdw} < a < \lambda_T \), where \( \lambda_T \) is the thermal wavelength) is further modulated by Efimov physics; zero-range zero-temperature theory predicts log-periodic deviations [40], captured by the dimensionless

\[
\mathcal{Z}_3 = \frac{mL_3}{3\hbar a^2}, \tag{4}
\]

with \( s_0 = 1.00624 \). For \( a < 0 \), loss resonances occur at \( a = e^{i\pi j/s_0} \), where \( j \in \{0, 1, \ldots, \} \), whereas for \( a > 0 \) the log-periodic sinusoidal modulation exhibits minima at \( a = e^{i\pi j/s_0} \) and corresponding maxima at \( a = a_s e^{i\pi j/2s_0} \), with \( a_s = a_+ e^{i\pi /2s_0} \), such that \( a_-/a_p < -1 \). Here \( \eta^* \) denotes the Efimov width parameter, which encodes the lifetime of the Efimov trimers.

To study the loss dynamics we prepare box-trapped thermal gases with initial densities \( n_0 \) ranging from 0.08\,\mu\text{m}^{-3} \text{ to } 3\,\mu\text{m}^{-3}. \) We use trap depths \( U_0/k_B \approx 1\,\mu\text{K} \text{ and restrict ourselves to temperatures } T < 100 \, \text{nK} \) to mitigate nonzero-\( T \) effects, which are especially prominent for \( a < 0 \) near \( a_+ [38, 97, 98] \). For a given \( T \) and \( n_0 \), we initiate the loss measurements either by slowly (in tens of ms) ramping \( B \) to set a final \( a \) or by performing a spin flip at the field of interest [98]; see also Appendix B for quench-based loss measurements featuring rich molecular dynamics and high-density effects.

In Fig. 3(a) we show an example of the loss dynamics for a thermal gas at \( a = -930(30)\,a_0 \) in the \( |1,1 \rangle \) state. Our uniform samples offer negligible anti-evaporation; heating occurs only for high \( a \) when the mean free path \( \ell = (8\pi n^2)^{-1} \) becomes short, such that the loss products (with energies \( \sim E_B \gg U_0 \)) undergo additional collisions and deposit some of their energy before leaving the trap. In all cases we restrict our analysis to times where \( T \) remains within \( \sim 10\% \) of its initial value. The slope of the linear \( 1/N^2(t) \) [99] yields \( L_3/V^2 \), from which we calculate \( \mathcal{Z}_3 [100] \).

In Fig. 3(b,c) we show the extracted \( \mathcal{Z}_3(a) \) on both sides of the 402.7 G Feshbach resonance. For \( a < 0 \) (b), we see an order of magnitude enhancement of \( \mathcal{Z}_3 \) signaling \( a_+ \), while for \( a > 0 \) (c) \( \mathcal{Z}_3 \) is consistent with a log-periodic oscillation. To extract the positions of the maxima of \( \mathcal{Z}_3 \) we fit each series with Eqs. (5), but also include a free amplitude \( P \) to account for systematic uncertainties in density and the approximation \( n^2 = n^2 \) made in Eq. (3); we find \( P \approx 0.5, \text{ consistent with } 1 \) within our systematic uncertainties. The solid lines display the average fit across series, while the dotted ones extend it outside of the fit range.

FIG. 3. Testing Efimov universalities in the \( |1,1 \rangle \) state (\( B_{\text{res}} = 402.74 \, \text{G} \)). (a) Loss dynamics in a box-trapped thermal gas at \( a = -930(30)\,a_0 \), plotting the evolution of \( 1/N^2 \) (top) and temperature \( T \) (bottom). As expected for three-body recombination in a uniform system, \( 1/N^2(t) \) is linear and \( T(t) \) constant. (b,c) Modulation of the three-body loss coefficient \( \mathcal{Z}_3 = mL_3/(3\hbar a^2) \) for \( a < 0 \) (b) and \( a > 0 \) (c). Different data series are grouped by their approximate \( T \) (see legend) and the symbol filling denotes the preparation protocol: \( B \)-field ramps (closed) and spin flips (open). Our error bars combine fit errors and the uncertainty in \( a \), but exclude a systematic density uncertainty \( \lesssim 30\% \). We extract the positions of the Efimov features \( a_- \) (b) and \( a_p \) (c) with fits using Eqs. (5) and a free amplitude (solid lines). While the ratio \( a_p/a_- \approx -1.08(9) \) is consistent with Efimov universality (\( a_p/a_- = -1 \), the absolute positions exclude \( a_- = -9.7r_{cdw} \) predicted from Efimov–van-der-Waals universality.
In Table III we summarize our characterization of the Efimov features for measurements across four Feshbach resonances, as well as measurements from Refs. [38, 51]. In all cases we find values of $a_{-} = -13(1)r_{vdW}$, with the Efimov ratios $a_p/a_{-} \approx 1$. We also find little variation between the Efimov width parameters $\eta^{*}$ (extracted from resonances at $a < 0$) for different states. Curiously, for the $|1, 1\rangle$ state at $B_{res} = 402.74$ G the three-body loss rate of thermal unitary gases has suggested lower values $\eta^{*} \approx 0.1$ [4, 9, 101].

Table III includes values of the effective range $r_{eff}$ near $B_{res}$ for each resonance, obtained from coupled-channel calculations as described in Appendix A. For the four stronger resonances with $s_{res} \sim 2.6(2)$, $r_{eff}$ is within a few percent of 134 $a_0$. This is about 25% less than the value of $\sim 2.8r_{vdW}$ expected for broad resonances [102] (see also [51]).

VI. LOSS DYNAMICS AT UNITARITY

Finally, we also study the loss dynamics of initially degenerate gases quenched to unitarity at different resonances. For both a thermal and a degenerate homogeneous Bose gas at unitarity one can define a dimensionless loss rate

$$\Gamma = -N_t \bar{N}/N,$$

where $t_n$ is the density-set timescale $t_n = \hbar/E_n$, with $E_n = h^2(6\pi^2n^{2/3})/(2m)$.

For a thermal unitary gas [3, 4, 6]

$$\Gamma = (1 - e^{-4\eta^*}) \frac{18\sqrt{3}}{\pi^2} \left( \frac{E_n}{E} \right)^2,$$

where $E = (3/2)k_B T$, whereas for a degenerate gas $\Gamma = A$, where $A$ depends only on the Efimov physics [103]. Previously, $A = 0.28(3)$ was measured for $B_{res} = 402.7$ G in $|1, 1\rangle$ [9] and $A \approx 0.18$ estimated from a measurement in $^{85}$Rb [8], where $\eta^* = 0.057(2)$ [50].

As in Ref. [9], we start with a quasi-pure Bose–Einstein condensate and quench it to unitarity. As the cloud decays and heats, from the time evolution of $N$ and $E$ we map out $\Gamma$ as a function of $E/E_n$, and observe a crossover from degenerate- to thermal-gas behavior.

In Fig. 4 we show $\Gamma(E/E_n)$ for the three broadest intrastate resonances [at 402.74 G, 472.33 G, and 561.14 G]. Consistent with observing similar values of $\eta^*$ from the Efimov loss resonances, we find remarkably universal behavior of $\Gamma$ across both degenerate- and thermal-gas regimes. In all cases degeneracy persists for $t_c \approx 4t_{n0}$, with a remaining fraction $N_c / N_0 \approx 0.4$ [see Fig. 4(b)].

VII. CONCLUSIONS & OUTLOOK

In conclusion, our experiments provide a high-precision characterization of few-body physics in $^{39}$K, offering invaluable input for modeling its full interaction landscape and understanding complex multi-channel effects [38, 87, 104]. Our precise determination of $a(B)$ across multiple Feshbach resonances and spin states allows a comprehensive study of Efimov physics in $^{39}$K. For the accessible Feshbach resonances, which all feature a similar intermediate-strength character (captured by $s_{res}$), we find $a_{-} = -13(1)r_{vdW}$, suggesting a universal breakdown of Efimov–van-der-Waals universality set by $s_{res}$. Moreover, the observed Efimov ratios uphold Efimov universality and we find that the loss dynamics, both at and away from unitarity, are remarkably universal.

Our work points to many avenues for further research.
It would be interesting to explore the effects of few-body physics on the many-body state at unitarity [10]. This includes surveying Feshbach resonances across atomic species to find cases where the unitary lifetime is longer, and understanding the intricate few-body loss channels (see e.g. [65]) to devise schemes that could extend the lifetime (as e.g. in molecular samples [105, 106]). Finally, the rich \( a(B) \) landscape of \(^{39}\text{K} \) also offers exciting prospects for novel experiments. The pair of overlapping Feshbach resonances at 472.3 G and 491.2 G gives access to a steep zero crossing for studies of the collapse of uniform condensates via modulational instability, while the two interstate resonances at 445.4 G and 526.2 G are particularly promising for creating uniform Bose mixtures.

VIII. ACKNOWLEDGMENTS

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APPENDIX A: COMPARISON OF FESHBACH RESONANCE PROPERTIES WITH COUPLED-CHANNEL CALCULATIONS

We have carried out coupled-channel scattering calculations for comparison with our measurements of the Feshbach resonance properties. The total wave function is expanded in a complete basis set of functions for electron and nuclear spins and end-over-end rotation, producing a set of coupled differential equations that are solved by propagation with respect to uniform condensates via modulational instability, while the two interstate resonances at 445.4 G and 526.2 G are particularly promising for creating uniform Bose mixtures.

\[
\hat{H} = \frac{\hbar^2}{2\mu} \left[ -\frac{1}{R} \frac{d^2}{dR^2} R + \frac{\hat{L}^2}{R^2} \right] + \hat{H}_A + \hat{H}_B + \hat{V}(R),
\]

where \( \mu \) is the reduced mass and \( \hat{L} \) is the two-atom rotational angular momentum operator. The single-atom Hamiltonians \( \hat{H}_i \) contain the hyperfine couplings and the Zeeman interaction with the magnetic field. The interaction operator \( \hat{V}(R) \) contains the two isotropic Born–Oppenheimer potentials, for the \( X \Sigma_g^+ \) singlet and \( a \Sigma_u^+ \) triplet states, and anisotropic spin-dependent couplings that arise from magnetic dipole–dipole and second-order spin-orbit coupling. Here we use the singlet and triplet interaction potentials of Tiemann et al. [87] (their model 3), with the second-order spin-orbit coupling function of Xie et al. [51].

The scattering calculations are carried out using the MOLSCAT package [108, 109]. The wavefunction is expanded in a fully uncoupled basis set that contains all allowed electron and nuclear spin functions, and is limited by \( L_{\text{max}} = 2 \). Solutions are propagated from \( R_{\text{in}} = 5a_0 \) to \( R_{\text{mid}} = 30a_0 \) using the fixed-step symplectic log-derivative propagator of Manolopoulos and Gray [110] with an interval size of 0.002\( a_0 \), and from \( R_{\text{mid}} \) to \( R_{\text{max}} = 8,000a_0 \) using the variable-step Airy propagator of Alexander and Manolopoulos [111].

Each scattering calculation produces the scattering matrix \( S \) for a single value of the collision energy \( E_{\text{coll}} \) and magnetic field \( B \). The complex energy-dependent s-wave scattering length \( a(k_0) \) is obtained from the diagonal element of \( S \) in the incoming channel, \( S_{00} \), using the identity [73]

\[
a(k_0) = \frac{1}{i k_0} \left( \frac{1 - S_{00}(k_0)}{1 + S_{00}(k_0)} \right),
\]

where \( k_0 \) is the incoming wavenumber, related to the collision energy by \( E_{\text{coll}} = \hbar^2 k_0^2 / (2\mu) \). Note that \( a(k_0) \) becomes constant at sufficiently low \( E_{\text{coll}} \), with limiting value \( a \). In the present work, s-wave scattering lengths are calculated at \( E_{\text{coll}} / k_0 = 100 \text{ pK} \), which is low enough to neglect the dependence on \( k_0 \).

When both atoms are in their lowest state \((1,1)\), only elastic scattering is possible. The scattering length is then real and approximately follows Eq. (2) in the vicinity of an isolated resonance. If either atom is excited, however, inelastic collisions may also occur. For most of the atomic states considered here, these are spin-relaxation collisions, mediated by the weak spin-spin and second-order spin-orbit couplings. The scattering length is then complex, \( a(B) = a(B) - i\beta(B) \), and the rate coefficient for two-body loss is approximately proportional to \( \beta \) [73]. Even if there is very little inelastic scattering away from resonance, \( \beta(B) \) shows a narrow peak of height \( a_{\text{res}} \) near \( B_{\text{res}} \) and \( a(B) \) shows an oscillation of magnitude \( \pm a_{\text{res}}/2 \) instead of a pole. A feature of MOLSCAT is that it can converge automatically on both elastic and inelastic Feshbach resonances and characterize them to obtain their parameters, as described by Frye and Hutson [112]. For elastic scattering, the parameters extracted are \( B_{\text{res}}, \Delta \), and \( \alpha_{bg} \); in the presence of weak inelastic scattering, these are supplemented by \( a_{\text{res}} \), which can be recast as \( \Gamma_{\text{inel}} = -2\alpha_{bg}\Delta/a_{\text{res}} \) to assess the size of the region around \( B_{\text{res}} \) where \( a(B) \) is not pole-like [73, 112].

We have characterized theoretically all the resonances for which experimental properties are given in Table I and Table II. The calculated properties are compared with experiment in Table IV. The agreement is generally good. The calculated resonance positions differ from experiment by up to 0.25 G, but that is comparable to the deviations between experiment and theory in Ref. [87]. Note that the position measured here for the resonance near 403 G is far closer to theory than the experimental value used in Ref. [87]. The calculated values of \( \alpha_{bg}\Delta \) are generally within one or two standard deviations of the measured ones, but the calculation breaks them down into their components \( \alpha_{bg} \) and \( \Delta \), which the present experiments cannot do without assuming that Eq. (1) holds all the way to \( B_{\text{zero}} \). For the resonances at higher thresholds, the small values of \( \Gamma_{\text{inel}} \) confirm that the scattering length is pole-like until very close to \( B_{\text{res}} \).
molscat can also converge numerically upon fields $B_{\text{zero}}$ where $a(B)$ is zero. These may differ from the value $B_{\text{res}} + \Delta$ implied by the approximate Eq. (1) if $a_{bg}$ varies significantly across the width of the resonance. These values are included in Table IV, and are close to the experimental values even in cases where $B_{\text{zero}}$ differs significantly from $B_{\text{res}} + \Delta$.

We have also calculated values of the effective range near $B_{\text{res}}$ from the expansion

$$\frac{1}{a(k_0)} = \frac{1}{a} - \frac{1}{2} r_{\text{eff}} k_0^2 + \ldots \quad (10)$$

using the methods of Ref. [113]. The results are included in Table III.

**APPENDIX B: PECULIARITIES OF QUENCH-BASED LOSS SPECTROSCOPY**

Using field quenches for loss spectroscopy circumvents substantial atom loss in the time it takes to reach the B-field of interest. This leads to more narrow spectra and avoids systematic asymmetries due to the quench direction, allowing a reliable extraction of $B_{\text{res}}$. However, such quenches can also induce additional dynamics, primarily dictated by shallow molecular bound states, which we explore below.

In Fig. 5(a) we plot $N_{\text{obs}}(t_1)$ for the quench experiments from Fig. 2 at three different fields, below ($B < B_{\text{res}}$), on ($B = B_{\text{res}}$), and above ($B > B_{\text{res}}$) resonance. The early times reveal rich dynamics, while for late times ($t_1 = 400\,\mu s$) we observe loss essentially independent of the quench direction.

For $a \approx 3000a_0$, the first quench to $B$ already creates a non-negligible superposition of atoms and molecules, leading to an oscillation in $N_{\text{obs}}(t_1)$ upon quenching back to $\approx 500\,a_0$. Meanwhile, for quenches back to $\approx -500\,a_0$, where the dimer

![FIG. 5. On the perils of quench-based loss spectroscopy [see Fig. 2(a) for protocol]. (a) Evolution of $N_{\text{obs}}(t_1)$ for quenches below (left), on, and above resonance (left to right), with initial $N_0 = 4.6(1) \times 10^4$ atoms at $T \approx 100\,\mu \text{K}$. While the early times display rich dynamics, the long-time loss is independent of the quench direction. (b) Fractional change in energy $\delta E / E_{\text{0}}$ for quench-based loss spectroscopy, measured using $t_1 = 400\,\mu \text{s}$ [as in Fig. 2(b)]. Here substantial differences persist between the quench directions even at long times. We note that $\delta E / E_{\text{0}}$ is not centered at $B_{\text{res}}$, making it unsuitable for accurate $B_{\text{res}}$ measurements.](image-url)
APPENDIX C: OVERLAPPING FESHBACH RESONANCES

For two overlapping resonances with widths of the same sign, Eq. (1) extends to the more general form [114, 115]

\[ a(B) = a_{bg} \left( 1 - \frac{\Delta_1}{B - B_{res,1}} \right) \left( 1 - \frac{\Delta_2}{B - B_{res,2}} \right) \]  

where \( a_{bg} \) is the common background scattering length, \( B_{res,1} \) the first (second) resonance position, and \( \Delta_{1(2)} = B_{zero,1(2)} - B_{res,1(2)} \). For \( \Delta_1/\Delta_2 \gg 1 \), the effective width of the narrow resonance is \( a'_{bg,2} = a_{bg} \left( 1 - \Delta_1/(B_{res,2} - B_{res,1}) \right) \Delta_2 \).

For the narrow intrastate resonance near 491 G we find \( a'_{bg,2} \Delta_2 = 140(30) a_0 G \) using our measurements (see Table I), while \( a'_{bg,2} / a_{bg} \approx 5.2 \). To calculate \( a(B) \) for our \( a_- \) measurements near the 162.4 G resonance we include the small \((\sim 10 a_0)\) contribution from the 33.6 G one.

APPENDIX D: INTERSTATE RF ASSOCIATION SPECTROSCOPY

Here we present \( E_b \) measurements for two interstate resonances based on rf molecular association spectroscopy [1, 95, 96] (see Fig. 7, plotted akin to Fig. 1(d)). To measure \( E_b \) we prepare thermal clouds spin-polarized in one of the two states, and then apply a weak rf pulse up to tens of ms to associate the interstate Feshbach dimer, which enhances atom loss.

A typical rf spectrum is shown in the inset of Fig. 7, which exhibits features corresponding to both the bare atomic transition and the dimer. To extract \( E_b \) (dashed line) from the asymmetric dimer peak, we fit a Lorentzian convolved with a Maxwell–Boltzmann distribution (solid line) [96] (see also [69]); this asymmetry arises due to the initial kinetic energy of the associated atoms. Curiously, the atomic feature near zero detuning also exhibits an asymmetric tail, which we also attribute to kinetic effects. We restrict ourselves to cases where the two peaks are well separated.
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We use a range of different-sized box traps and independently calibrate V using in situ measurements, also accounting for slight changes in V with T and Ud owing to the finite sharpness of the trap walls.

We observe additional slow essentially a-independent loss captured by a one-body lifetime ∼ 30 s, which we include in the same way.

While such quenches can lead to the formation of both dimers and trimers [8, 9], we find that our measurements are consistent with assuming a mixture of atoms and dimers.

Note that these measurements are susceptible to several systematic errors used for the low positive a measurements, we observe additional slow essentially a-independent loss captured by a one-body lifetime ∼ 30 s, which we include in the same way.

We typically exclude the first 10 μs from the fit to mitigate effects of our finite quench times. We directly relate the oscillation frequency to aω, neglecting model-dependent frequency shifts due to damping, which we assess to be negligible.

During some measurements, we also include effects of one-body losses (with lifetime ∼ 100 ns) due to collisions with background-gas particles. Note that for our highest temperatures used for the low positive a measurements, we observe additional slow essentially a-independent loss captured by a one-body lifetime ∼ 30 s, which we include in the same way.

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