Optimization problem for a portfolio with an illiquid asset: Lie group analysis

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Abstract

Management of a portfolio that includes an illiquid asset is an important problem of modern mathematical finance. One of the ways to model illiquidity among others is to build an optimization problem and assume that one of the assets in a portfolio can not be sold until a certain finite, infinite or random moment of time. This approach arises a certain amount of models that are actively studied at the moment.

Working in the Merton’s optimal consumption framework with continuous time we consider an optimization problem for a portfolio with an illiquid, a risky and a risk-free asset. Our goal in this paper is to carry out a complete Lie group analysis of PDEs describing value function and investment and consumption strategies for a portfolio with an illiquid asset that is sold in a random moment of time with a prescribed liquidation time distribution. Study of optimization problems with an illiquid asset leads to three dimensional nonlinear Hamilton-Jacobi-Bellman (HJB) equations. Such equations are not only tedious for analytical methods but are also quite challenging form a numeric point of view. To reduce the three-dimensional problem to a two-dimensional one or even to an ODE one uses some substitutions, yet the methods used to find such substitutions are rarely discussed by the authors.

We find the admitted Lie algebra for a certain class of liquidation time distributions in cases of HARA and log utility functions and formulated theorems for these cases. We use them to obtain corresponding reductions. Several of these substitutions were used in other papers before and other ones are new to our knowledge. This method gives us the possibility to provide a complete set of non-equivalent substitutions and reduced equations.