Shedding light on boundaries: re-sequencing Snell’s law instruction to first build conceptual understanding

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Abstract
Refraction is a foundational concept within introductory physics. Physics students need a deep understanding of refraction, including Snell’s Law, in order to progress towards more complex optics topics such as lenses and images. Unfortunately, many physics students obtain only a superficial understanding of refraction. Although many students can use Snell’s Law to perform basic calculations, the mathematical relationship is often divorced from students’ conceptual knowledge, which often harbours misconceptions. In this article, we describe a sequence of instructional activities that we have used in an introductory optics course that aims to address common issues of students’ learning of optics. Instead of leading with Snell’s Law and emphasising calculations, the instructional sequence places conceptual understanding in the foreground. Mathematical representations are introduced only after students have developed a conceptual foundation; in this way, mathematics becomes integrated with students’ conceptual understanding rather than existing apart from it. After describing the

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1. Introduction/Rationale

The concept of refraction, along with its mathematical description in Snell’s Law, lies at the core of geometric optics and is the foundation upon which more complex concepts are built (e.g. behaviour of lenses) [1]. Given its centrality, Snell’s Law is typically presented to students early in their optics instruction, often accompanied by mathematical exercises that develop students’ capacities to use Snell’s Law for calculations.

Unfortunately, for many students, mathematical relationships such as Snell’s Law rarely carry the same conceptual meaning as they do for experts. Experts use mathematical relationships that are deeply integrated with conceptual ideas [2–4] as cognitive tools to make sense of a variety of situations [5, 6]. In contrast, many students view equations as simply ‘formulae’ to be recalled, and mathematics is often divorced from conceptual principles [3, 4, 7–11]. Students’ abilities to carry out calculations can therefore mask conceptual misconceptions that they might hold [5, 14]. These issues have been documented not only for science in general but optics concepts in particular [10, 12, 13].

An over-emphasis on calculation is potentially unproductive in terms of promoting conceptual understanding, and it may also exacerbate mathematical anxieties that are common among students. Math anxieties often lead students to disengage from their STEM coursework and exit STEM majors [15–17]. Yet mathematics is an integral part of physics, and avoiding mathematics during physics instruction is neither desirable nor feasible. The goal, then, is to foreground conceptual understanding so that students can integrate the mathematical representations with those concepts, thus developing more ‘expert-like’ knowledge [3, 4, 8, 9, 11].

In this paper, we present an instructional sequence that we designed to help students develop an expert-like, well-integrated understanding of Snell’s Law and refraction. This sequence was originally designed for a general education introductory optics class (2 h time block) at a liberal arts school, but the methodology is easily adaptable to other educational settings. Instead of leading with the mathematics, the sequence begins with a set of rich experiences with phenomena. Canonical explanations, including Snell’s Law, are then strategically introduced when students need them to make sense of the situations at hand [14]. In this way, the instructional sequence supports the use of mathematics as a conceptual resource [6] rather than as formulae-to-be-memorised. Our instructional approach is innovative in its sequencing rather than its use of revolutionary new classroom experiences; in fact, many of its individual components will likely be familiar to most readers. To better establish how our approach is indeed different, we begin with a brief description of a more ‘typical’ treatment of refraction and Snell’s Law.

1.1. Typical approach to teaching refraction and Snell’s law

In introductory physics courses, refraction is often first presented using a ray diagram representation (see figure 1) that shows the difference in the angle of incidence ($\theta_1$) and refraction ($\theta_2$).

The standard explanation that is then provided explains refraction as due to the unequal speed of light in the two media. Students are presented with Snell’s Law, expressed as $n_1 \sin \theta_1 = n_2 \sin \theta_2$. The index of refraction ($n$) is defined as the ratio of the speed of light in a vacuum to the speed of light through that medium: $n = \frac{c}{v}$. In addition to the basic presentation of Snell’s Law and refraction, students are often provided a justification related to the change in speeds across the boundaries (see figure 2) or more complex explanations like Huygens’ Principle [19] or Fermat’s Principle [18].
Resequencing Snell’s Law Instruction

After establishing the canonical equations and explanations, students might then be shown a variety of demonstrations and applications of the principles. Often this is followed by a set of mathematical exercises and perhaps a laboratory investigation where students experimentally verify Snell’s Law by measuring incident and refracted angles.

This common presentation represents how experts understand refraction, but the instructional problem is that expert-like knowledge cannot be simply transmitted from the teacher to the student [5]. The foregrounding of the mathematics causes students to focus on the equations rather than on the underlying concepts, and students are unlikely to integrate the two [10, 12].

2. An alternative way to establish Snell’s Law

In contrast to the one described above, our instructional sequence first provides students with a set of experiences with optical phenomena, helps students explain those phenomena in qualitative terms, and then guides students through a derivation of the mathematical formulation of Snell’s Law. Placing the mathematics at the end of the instructional sequence is a critical feature. Because students will have already experienced and made sense of optical phenomena, they will be better equipped to see Snell’s Law not merely as a calculation tool, but as a way to quantitatively represent relationships that they already comprehend.

We designed and implemented the sequence in a general education, introductory optics course, taught by the first author at a liberal arts institution. The course was taught in a space where lab and lecture components could be interwoven. The course enrolls a relatively small group of students (approximately 20), 1 or 2 of which are typically physics majors, a handful are science majors (biology or chemistry) and the rest range in majors from social sciences to product design to the arts and humanities. Although our description of the learning activities aligns with that particular course context, the activities could be easily modified for different audiences and educational settings, such as high school physics classrooms, introductory physics courses or labs, or larger lecture settings that incorporate demonstrations and interactive teaching techniques [20].

2.1. Prior optics instruction

Prior to the instructional sequence, we had addressed several key ideas about light in the course, including: the wave model of light, the movement of light in straight lines, reflection, and key ideas about how humans perceive light. Students had not yet received any formal instruction on refraction in the course, and we did not assume any prior knowledge on the topic. Although some
students might have learned about refraction in high school, at most we found that their prior knowledge was superficial.

2.2. Investigation 1: water pearls

We begin our sequence with an investigation of ‘water pearls’ (see [21] for a description). These small plastic spheres have the ability to absorb a large quantity of water, whereupon they swell to become gelatinous and transparent. When they do, their index of refraction becomes extremely close to that of water. They come in a variety of colours, but for this activity we use the colourless ones because when they are submerged in water, they are very difficult to see.

We begin the activity by showing students a tank of water that is filled with colourless water pearls. We ask students what they think is in the tank, and most state that it simply looks like it is full of water. We then reach into the tank and pull out a handful of the water pearls, much to the students’ surprise! The driving question therefore is, ‘Why are the water pearls visible when they are in the air but not when they are in the water?’

In groups of 2–4, we give students their own beakers filled with water and colourless water pearls so that they can make more extensive observations. We give them about 5 min to explore the materials and record any observations that they think are important, then share them with the class. Students often comment on the physical properties of the pearls (e.g. they are squishy, bouncy, etc), but the most crucial observations are those that highlight how the pearls behave differently in air versus water. We indicate that our goal over the next few activities will be to explain those observations.

2.3. Investigation 2: beakers and numbers

Students are now given the materials described in figure 3. Students are first told to shine the laser pointer through the large square plastic container to confirm that they can see a straight beam of light. We then direct students’ attention to the labels on the three beakers. We do not tell students what the numbers on the containers mean, only that they represent something about what is inside the containers. For the case of the beaker labelled 1.00, we point out that the substance inside the container is air and that the weight is there to keep it submerged. In groups of 2–4, students work through the following:

(a) Place one of the beakers in the middle of the large plastic container (it will be partially submerged). Make note of the numbers on the containers. Shine the laser so that it passes through the liquid in the large container and the beaker. Try shining the laser at different angles. (See figure 4).
(b) Create a sketch of your observations (figure 4). In your sketch, show the path of the light through the container and the beaker and address the following:
   1. When will the light go straight through?
   2. When will the light change directions?
   3. Where does the light change directions?
(c) Repeat for the other two beakers.

After students make observations, we gather them together for discussion. During the ensuing conversations, we encourage students to share their sketches. If there are uncertainties or disagreements about the phenomena, we send students back to their materials to double-check and reach consensus. Our goal is to establish the following:

- The light will go straight through either when the light is hitting the beaker at a 90° angle of incidence or when the number on the beaker matches the number on the large plastic container.
- The light changes direction at the boundary between two materials.
- Not all of the light passes through the objects. Some is reflected, which is an important part of why we are able to see them.

2.3.1. Application of findings to water pearls

We now draw students’ attention back to the water pearls and ask, ‘Given that we cannot easily
Figure 3. Materials used for Investigation 2. A large clear plastic container with flat sides (labelled with ‘1.33’) is filled with a water solution that consists of just enough yellow dye to approximate the colour of vegetable oil and a pinch of cornstarch to help with laser beam scattering. There are three beakers: one is filled with the yellow cornstarch water solution and labelled ‘1.33’; one is filled with vegetable oil and labelled ‘1.47’; the third is filled with air (and a weight to hold it down) and marked with ‘1.00’. Students are provided with a laser pointer.

Figure 4. Example of student sketch (right) based on observations (left) the laser beam as it passes through the beaker (containing the \( n = 1.47 \) solution) and the surrounding \( n = 1.33 \) solution from Investigation 2. Students readily make the connection that the two numbers must be equal, and we ask several of them to explain why this makes sense to them.

2.3.2. Extension of findings to oil, water, and pyrex. We now show students a beaker that is filled with vegetable oil and the same yellow-cornstarch-water solution used in the previous activity. Two layers have been given time to form in the beaker, and each layer has been labelled with the appropriate ‘number’ on the beaker (see figure 5). We place a pyrex stirring rod in the beaker, as shown, and ask students what they observe about the stirring rod. We then ask how they might explain those observations, as well as how the situation is similar to, but different from, the water pearls.
2.4. Introducing the formal physics principles

Having explored key phenomena and developed several fundamental principles, we now introduce students to formal science ideas about refraction. We first label the ‘mysterious number’ as the index of refraction. We define this term using \( n = c/v \), emphasising that higher indices imply slower speeds of light through the medium. To explain how this concept relates to refraction, we provide the standard account referenced in figure 2. In short, the change in velocity means a change in wavelength, which in turn requires a change in direction. We also use these ideas to explain why light perpendicular to a surface does not change direction, and why refraction only occurs at the boundary between two media.

2.5. Investigation 3: how much bending?

We now launch an investigation to develop students’ quantitative understanding of refraction.

In groups of 2–4, students are given the materials in figure 6 and the following instructions:

(a) Place two half moon dishes together on the grid so that they form a circle. Fill with marked liquids and record the index of refraction of each dish.
(b) Shine the laser light so that it goes through the centre of the circle (see figure 7. Record the angle of incidence and refraction that occurs at the interface between the two dishes.
(c) Change your angle of incidence by moving your laser pointer to different locations around your half-moon dishes. For each new location, record the angle of incidence and refraction.
(d) Use your data to answer the following questions:
   1. Is the angle of incidence \( \theta_i \) the same as the angle of refraction \( \theta_r \)?
   2. In terms of what you see when you look at the light bending, does it matter which direction the light is going?
   3. When you compare the angle of incidence to the angle of refraction, what patterns do you see in terms of which one is larger and which is smaller?
(e) Collect similar data for each pair of dishes (see table 1. Do the relationships you found above still hold true?)

2.5.1. Interpreting data and formalising the physics principles. After students have gathered data for each of the three setups and addressed the questions posed, they are gathered together to
make sense of their data. As a first step, students share their responses to the investigation questions and make the following observations:

- Unlike the law of reflection, for refraction across boundaries $\theta_1 \neq \theta_2$.
- Whichever material has the smaller index of refraction has the larger angle regardless of which is the incident angle.

We now direct students to determine a relationship between the two angles and two indices of refraction. Students typically try out different candidate relationships to see if they fit the data, often beginning with simple relationships of the form $n_1 \theta_1 = n_2 \theta_2$, and are encouraged to check whether that relationship holds. If they are comfortable, we encourage them to use spreadsheet software to do this.

After a short time, students realize that $n_1 \theta_1 = n_2 \theta_2$ and other simple relationships do not work. If needed, we step in to discuss other mathematical alternatives with them. We remind students of the drawing of wave crests incident on a boundary that was introduced previously (see figure 8) and draw students’ attention to the angles present in the drawing. Using triangles, we remind students of the trigonometric relationships (sin, cos, tan) that might be relevant, and students brainstorm candidate relationships that include trigonometry functions (e.g. $n_1 \cos \theta_1 = n_2 \cos \theta_2$, $\frac{n_1}{\cos \theta_1} = \frac{n_2}{\cos \theta_2}$, $n_1 \sin \theta_1 = n_2 \sin \theta_2$, $n_1 \tan \theta_1 = n_2 \tan \theta_2$ etc). We then give students the time and freedom to work
in their groups and try to eliminate or confirm the various options.

Relatively quickly, students find that the relationship \( n_1 \sin \theta_1 = n_2 \sin \theta_2 \) seems to fit the data. We prompt them to further confirm that idea by using the equation to predict the outcome of new trials. After doing this for several different angles and combinations of materials, students tend to be both confident and excited that ‘their equation’ works. Only at this point do we formally label that relationship as Snell’s Law. Importantly, the students have ownership over the equation, because it was not simply presented to them as a ‘truth of physics’.

**Figure 7.** Figure on the left shows an example view of Investigation 3. In this image the laser beam is entering on the bottom left (the Oil side) and exiting through the water on the top right. The sketch on the right shows a schematic of this setup.

**Table 1.** Typical observation tables created by students for each of the setups in Investigation 3.

| Setup 1  | Setup 2  | Setup 3  |
|----------|----------|----------|
| Half Moon Dish \( n = 1.33 \) | Half Moon Dish \( n = 1.47 \) | Half Moon Dish \( n = 1 \) |
| 20       | 18       | 20       |
| 30       | 27       | 30       |
| 40       | 36       | 40       |
| 50       | 44       | 50       |
| 60       | 52       | 60       |
| 70       | 58       | 70       |
| 80       | 63       | 80       |
| Half Moon Dish \( n = 1.47 \) | Half Moon Dish \( n = 1 \) | Half Moon Dish \( n = 1.33 \) |
| 20       | 13       | 20       |
| 30       | 20       | 30       |
| 40       | 26       | 40       |
| 50       | 31       | 50       |
| 60       | 36       | 60       |
| 70       | 40       | 70       |
| 80       | 42       | 80       |
3. Summary and discussion of student learning

Throughout the instructional sequence, our primary aim is for students to develop a deep and expert-like understanding of refraction. Students with that level of understanding have extensive connections between key concepts and can apply those concepts to a range of relevant situations [5, 8, 14]. Their knowledge includes mathematical representations that are integrated with related concepts and can be readily used as cognitive resources during applications [3, 4, 6].

Although we have not conducted a formal research study with our students, we have evidence that our instructional sequence achieved its aims. Most of our students were non-science majors, and many freely expressed mathematics anxiety. It was common to hear students say things like ‘I have never been able to do math’ and ‘I cannot do math.’ Despite those anxieties, many students were genuinely excited by the final segments of the instructional sequence, where they used mathematics not to simply grind through calculations but to further develop their burgeoning conceptual ideas. They saw mathematics not as an obstacle or barrier to their learning of science, but as a valuable part of their understanding.

The depth of students’ understanding was made evident by their ability to apply their knowledge of refraction to novel situations. With little prompting from the instructor, we saw students leverage their understanding of refraction to make sense of the separation of colours in a prism, the focal lengths of lenses, and phenomena related to virtual images. We even saw students reach for Snell’s law when they needed to reason quantitatively about those situations. For instance, they used mathematical reasoning to explain why a
fisher will perceive a fish to be higher in the water than it actually is. In this way, students were not merely using Snell’s Law as a calculation tool but as a cognitive resource [6] to facilitate their reasoning.

Our experiences with students support our contention that undergraduate physics instruction ought to be designed with the learner’s perspective, rather than the expert’s, in the foreground. Perhaps the most important shift in perspective lies in the positioning of mathematics. Formal mathematical relationships might occupy central locations within the expert’s understanding of physics, but that does not imply that a novice should begin by trying to grasp those formal mathematical ideas. Layering mathematics on top of a robust foundation of conceptual knowledge is a far more fruitful path toward deep learning.

Data availability statement
No new data were created or analysed in this study.

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