Research Article

Aeroacoustic Attenuation Performance of a Helmholtz Resonator with a Rigid Baffle Implemented in the Presence of a Grazing Flow

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1. Introduction

Helmholtz resonators are widely applied in automobile [1] and aerospace industries as an effective acoustics noise damper [2, 3]. Compared with other dampers, such as half- and quarter-wave resonators, Helmholtz resonators are found to be associated with a higher noise damping capacity [4]. They are typically coupled to an engine in the presence of a mean flow, which is also known as a grazing flow. To achieve its maximum noise attenuation performance, these resonators need to be well-designed so that they are tuned to the natural resonant frequencies [5]. Poor noise damping performances are generally expected, when the noise frequency is quite different from the near-resonant one, i.e., off-resonance conditions. The resonant frequencies can be predicted by using $\omega^2 = c^2S/VL_{\text{eff}}$. Here, $c$ denotes the speed of sound. $V$ is the resonator cavity volume. $L_{\text{eff}}$ and $S$ denote the effective length and the cross-sectional...
area of the resonator neck. $L_{\text{eff}}$ is greater than the neck physical length due to the end correction effect [6, 7]. Chanaud [6] developed a theoretical equation to determine the end correction.

The classical theoretical formula $\omega^2 = c^2S/VL_{\text{eff}}$ to predict the resonant frequency [8] fails in capturing the geometry shape of the resonator cavity and its neck, since it is derived by using a lumped mass model. Neither is the mean flow effect considered. The neck asymmetry is found to affect the resonant frequencies for both rectangular and cylindrical resonators [9]. Mercier et al. [10] confirmed the critical role of the neck shape on affecting noise damping capacity of Helmholtz resonators by conducting theoretical analysis via homogenization means. The noise damping mechanism is mainly due to a thermoviscous effect [11] and/or vortex shedding. Nonlinear damping [12] could be involved, depending on the amplitude of the incident sound to the resonator. To achieve optimum noise damping, acoustic resonance is expected, at which a large volume of the working fluid in the cavity periodically expands and compresses. This means that if the resonator cavity is structurally modified, the resonant frequency and the effective bandwidth are then varied. This is confirmed by extending the neck into the cavity [13] or filling porous materials in the resonator cavity [14]. However, alternative structural modification is expected. This partially motivated the present work.

Two or more Helmholtz resonators could be applied to broaden the effective frequency range in practice. Tang and Sirignano [15] derive a generalized theoretical model to predict a conventional Helmholtz resonator’s damping performance. It is found that long damping resonator should not be applied in practical designs. Li and Cheng [16] develop a generalized model to study the acoustic response of a Helmholtz resonator array coupled with an enclosure. A frequency formula is derived, which can be used to determine the variation at off-designed and designed conditions. Griffen et al. [17] experimentally study the noise attenuation performance of mechanically coupled Helmholtz resonators. It is found that a wider bandwidth is achievable. Similar coupled Helmholtz resonators are theoretically and experimentally studied by Johansson and Kleiner [18]. Xu et al. [19] propose and test two coupled Helmholtz resonators in the configuration of neck-cavity-neck-cavity. It is found that the geometric shape of the 2nd cavity affects little on the resonant frequencies. However, increasing the length of the 2nd resonator neck leads to the resonant frequencies being decreased. Similar configuration of coupled Helmholtz resonators is proposed by Cai and Mak [20]. A broader bandwidth is achieved. Slaton and Nishikawa [21] propose to mount two Helmholtz resonators coaxially on a cylindrical due in a presence of a low Mach number mean flow. It is found that 90-degree bending of the resonator necks has a little effect on changing the aeroacoustic damping response of the resonator network.

The aeroacoustic damping performance of a conventional Helmholtz resonator may be improved by implementing a vibrating sidewall or back-wall of its cavity. Nudehi et al. [22] design and test a Helmholtz resonator with a flexible backplate. Multiple resonant peaks are found in comparison with a single peak of the conventional resonator with a rigid backplate. The idea of applying a flexible plate is adopted by Zhao [23] to study the transmission loss performance of a parallel-coupled Helmholtz resonator network. Further applying tunable backplate is evaluated by Zhang et al. [24] to minimize combustion-driven periodic noises in a Rijke tube combustor. 50 dB sound pressure level is achieved. The tunable resonator works well even at off-design conditions [23, 24]. In industrial engine applications, the mean grazing flow is hot. It affects the acoustic damping performances of Helmholtz resonators. Ćosić et al. [25] experimentally study the acoustic noise damping performance of Helmholtz resonators, when the grazing and cooling flows have a temperature difference between them. The temperature difference is found to affect strongly on the resonant frequency of the resonator and its noise damping performance.

Extensive theoretical, experimental [26, 27], and numerical studies [28] are conducted on the noise attenuating performances of Helmholtz resonators in different geometric shapes or by introducing additional components [13, 14, 26–28]. Yang et al. [29] conducted experimental measurements of the acoustic impedance of a Helmholtz resonator. Its neck is filled with perforated ceramics. A higher power absorption coefficient and a broader effective frequency range are found. In general, foregoing studies are conducted to improve the acoustic noise damping performance and/or broaden the effective frequency ranges of Helmholtz resonators. Extensive researches are ongoing to achieve an optimum or a better design of a Helmholtz resonator. This motivated partially the current study.

In this work, a rectangular Helmholtz resonator with a rigid baffle attached inside its cavity is proposed and systematically studied. The resonator is flushed mounted on a rectangular duct with a mean grazing flow. For this, 2D numerical investigations are performed in frequency domain via determining the solutions of linearized NS (Navier-Stokes) equations. This is done by using COMSOL 5.4. In Section 2.1, the system equations and the 2D model are described. The dominant variables are evaluated. These parameters include (1) the geometric width $L_p$ of the rigid baffle, (2) its relative height/location $H_p$, attached in the cavity, (3) the implementation configuration of the rigid baffle (i.e., attached to the left or the right sideway), (4) the grazing flow Mach number $M_a$, and (5) the neck shape. In Section 2.2, the numerical 2D model is validated first. This is achieved by benchmarking with available experimental data. In Section 3, the 2D model is then modified and used to study the acoustic attenuating performances of the designed Helmholtz resonator. These impacts of the identified 5 design parameters are compared and discussed. Key findings are summarized in Section 4.

2. Description of the Model and Validation Studies

2.1. Description of the System Equations. In the present work, a 2D rectangular pipe with a rectangular Helmholtz resonator (HR) implemented is considered. It is illustrated
schematically in Figure 1. The HR is axially implemented at $x = L_u$. There is a mean duct flow [28] with a Mach number of $M_u$. It is also known as the grazing flow. The air flow is assumed to be compressible and viscous. Thus, the NS equations could be linearized and be the governing one in frequency domain. The dimensions of the modelled rectangular pipe with a HR [1] used are summarized in Table 1. The physical dimensions and geometric shape are exactly the same as the experimental one in Ref. [1].

The system equations consist of the mass, momentum, and energy conservation ones [1, 8]. The working air is assumed to behave like a perfect gas. Molecular diffusion of heat and gravity forces are neglected (i.e., $\lambda = 0$ and $g_j = 0$). These governing equations are then given in time domain as

\[
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j} (\rho u_j) = 0, \tag{1}
\]

\[
\frac{\partial}{\partial t} (\rho u_j) + \frac{\partial}{\partial x_j} (\rho u_j u_k) + \frac{\partial p}{\partial x_j} = \frac{\partial \tau_{ij}(u_k)}{\partial x_j}, \tag{2}
\]
\[
\frac{\partial p}{\partial t} + u_i \frac{\partial p}{\partial x_i} + c^2 \rho \frac{\partial u_i}{\partial x_i} = (\kappa - 1) \left( \frac{\partial \bar{u_i}}{\partial x_i} + u_i \frac{\partial \bar{p}}{\partial x_i} \right),
\]

where the viscous stress is defined as \( \tau_{ij}(u_k) = \mu(\partial u_i/\partial x_j) + (\partial u_j/\partial x_i) - (2/3)(\partial u_i/\partial x_i)\delta_{ij}, \) \( \rho, \ u_i, \) and \( p \) denote the instantaneous density, velocity vector, and pressure variables; \( c \) denotes the speed of sound. \( \kappa \) is the ratio of specific heats of the air.

The thermodynamic state equation also holds as \( p = \rho RT. \)

The compressible Navier-Stokes equations, i.e., Equations (1)–(3), can be linearized to obtain the acoustic governing equations. The instantaneous variables are assumed to consist of a mean and a fluctuating part. In physics, it means that a small-amplitude fluctuation (denoted by a prime) is superimposed on a mean flow filed (denoted by an overbar). In mathematics, it means that

\[
\rho = \bar{\rho} + \rho', \quad u_i = \bar{u}_i + u_i', \quad p = \bar{p} + p'.
\]

The fluctuating part of the air density, flow velocity, and pressure can be expressed in terms of Fourier series expansion as

\[
\rho' = \bar{\rho}(\omega)e^{i\omega t}, \quad u_i' = \bar{u}_i(\omega)e^{i\omega t}, \quad p' = \bar{p}(\omega)e^{i\omega t}.
\]

These quantities \( \rho'/\bar{\rho}, \ u_i'/\bar{u}_i, \) and \( T'/\bar{T} \) are assumed to be a small order \( \epsilon, \) where \( \epsilon < 1 \) and \( c^2 = kRT \) is the mean speed of sound. Note that the mean part of the flow variables satisfies continuity of mass, momentum, and energy. An overbar in Equation (5) denotes the flow parameter in frequency domain.

Substituting Equation (4) into Equations (1)–(3) gives

\[
\frac{\partial \rho'}{\partial t} + \frac{\partial}{\partial x_j}\left( \bar{u}_j\rho' + \bar{\rho}u_j' \right) = 0,
\]

\[
\frac{\partial \rho u_i'}{\partial t} + \frac{\partial}{\partial x_j}\left( \bar{p}u_i' + \bar{u}_i\rho' \right) + \frac{\partial \bar{\rho}u_j'}{\partial x_j} = \frac{\partial \tau_{ij}(u_k)}{\partial x_i},
\]

\[
\frac{\partial p'}{\partial t} + \frac{\partial}{\partial x_j}\left( \bar{u}_j p' + \bar{p}u_j' \right) + (\kappa - 1) \left( \bar{p} \frac{\partial \bar{u}_i}{\partial x_i} + \bar{u}_i \frac{\partial \bar{p}}{\partial x_i} \right) = (\kappa - 1) \left[ \frac{\partial \tau_{ij}(u_k)}{\partial x_i} \right].
\]

Equations (6)–(8) describe the spatiotemporal evolution of fluctuating quantities \( \rho', \ u_i', \) and \( p'. \) Note that the pressure fluctuation is selected as a primitive variable in the energy equation (Equation (8)). Furthermore, the source term of Equation (8) is attributed to the molecular stresses. The linearized mass conservation (Equation (6)) is needed to further simplify the linearized momentum (Equation (7)). By conducting linearization, any second-order terms are negligible. These include the term \( u_ju_i \) characterizing the turbulence.

| Parameters | Values | Parameters | Values |
|------------|--------|------------|--------|
| \( L_0 \) | 0.65 cm | \( H_p \) | 0.3 cm |
| \( L_d \) | 65.0 cm | \( L_p/D_t \) | 0.25 —— 0.75 |
| \( D_d \) | 4.86 cm | \( M_u \) | 0 —— 0.1 |
| \( L_\alpha \) | 8.05 cm | \( \bar{p} \) | 1.2 kg/m³ |
| \( D_n \) | 0.0404 m | \( \bar{p} \) | 101325 Pa |
| \( D_t \) | 0.1532 m | \( \omega/2\pi \) | 50 —— 500 Hz |
| \( H_t \) | 24.42 cm | \( T \) | 297 K |
| \( H_g/H_t \) | 0.25 —— 0.75 | \( D_{nc} \) | 5.04 cm |
| \( Re (M_u) \geq 0.0029 \) | ≥3200 |

Substituting Equation (5) into Equations (6)–(8) and eliminating the common factor of \( e^{i\omega t} \) lead to the linearized Navier-Stokes equations in frequency domain as

\[
i\omega \bar{\rho} + \frac{\partial}{\partial x_j}\left( \bar{u}_j \bar{\rho} + \bar{\rho} \bar{u}_j \right) = 0,
\]

\[
i\omega \bar{p}u_i + \frac{\partial}{\partial x_j}\left( \bar{p}u_i \bar{u}_j + \bar{u}_i \bar{p} \right) + \frac{\partial \bar{\rho}u_i}{\partial x_j} = \frac{\partial \tau_{ij}(u_k)}{\partial x_i},
\]

\[
i\omega \bar{p} + \frac{\partial}{\partial x_j}\left( \bar{u}_j \bar{p} + \bar{p} \bar{u}_j \right) + (\kappa - 1) \left( \bar{p} \frac{\partial \bar{u}_i}{\partial x_i} + \bar{u}_i \frac{\partial \bar{p}}{\partial x_i} \right) = (\kappa - 1) \left[ \frac{\partial \tau_{ij}(u_k)}{\partial x_i} \right].
\]

These frequency domain governing Equations (9)–(11) are iteratively determined via COMSOL 5.4. Here, UMFPACK (a set of routines for solving unsymmetric sparse systems) direct solver and Menter’s Shear Stress Transport (SST) turbulence model are applicable due to the nonnegligible duct/grazing flow, since it introduces much less numerical diffusion and leads to a better prediction of the eddy viscosity. The SST model is a combination of the \( k-\epsilon \) model in the free stream and the \( k-\omega \) model near the solid surface of the duct in order to capture the turbulence attenuation in details.

The unstructured meshes are illustrated in Figure 2. Mesh independence investigation is conducted first. In order to better capture the vortices and the shear layer at the duct-neck joint sections, a finer mesh with 1,260,360 cells is chosen. The selected mesh is enough by benchmarking the present predictions with the available (experimental) data in Refs. [26–28]. The benchmarking (i.e., validation) investigation is described in Section 2.2. The frequency step is set to 5 Hz and remains the same in all simulation cases. It will be confirmed that the frequency step is fine enough to produce good solutions. Note that the step could be decreased. However, a small frequency step gives rise to a significantly increased computational time.
and cost. Meanwhile, a smaller step is found to negligibly affect the transmission loss variation [21, 22]. Therefore, the frequency step is set to 5 Hz.

Plane travelling waves [23–26] are propagating along the duct and then passing through the HR. The boundary conditions of both duct ends are set to be PML (perfectly matched
layer). To characterize the noise damping of the HR [30, 31], TL (transmission loss) is defined as

$$ TL \equiv 20 \log_{10} \left( \frac{\left| \hat{P}_i(\omega) \right|}{\left| \hat{P}_t(\omega) \right|} \right), $$

where $\hat{P}_i$ denotes the incident plane waves, and $\hat{P}_t$ is the transmitted wave in the upstream and downstream, respectively. It is worth noting that in practice, TL is experimentally determined via the classical TMT (two-microphone technique). It can be related to sound absorption coefficient, which is an alternative parameter widely used to indicate the noise damping performance of Helmholtz resonators [32, 33].

2.2. Validation Studies. In order to benchmark the developed model, comparison investigations are conducted on a 2D rectangular duct, as a grazing mean flow is present and a conventional Helmholtz resonator is implemented. Our numerical predictions are then compared with the experimental data available in Ref. [1] and 3D simulation results of the cylindrical duct and the cylindrical resonator. Note that all variables in the current numerical studies are chosen to be the same as the experimental investigations, for example, the geometry and physical dimensions of the HR and the duct, and the grazing flow conditions. To be consistent with the experimental measurements, the studied frequency range is set to be between 50 and 250 Hz. Figure 3 shows the comparison of transmission loss varied with acoustic frequency between the present numerical, 3D time-domain simulation (STAR-CD Ref. [1]) and experimental results, as the grazing flow Mach number $M_u$ is set to 3 different values. It can be seen that for a given $M_u$, a good agreement between the

![Graphs showing transmission loss comparison](image.png)
numerical, 3D simulation and the experimental results is obtained in general. As the grazing flow Mach number is increased, the maximum transmission loss is found to be reduced dramatically. The corresponding resonant frequency is found to be shifted from 88 Hz to about 100 Hz, as $M_u$ is increased from 0 to 0.1. This investigation reveals that the developed numerical tool is applicable to evaluate the acoustic attenuation performances of a HR (Helmholtz resonator) [34, 35], as there is a grazing mean flow.

Implementing a rigid baffle with a fixed thickness is proposed to improve the aeroacoustic damping performance of the Helmholtz resonator [36, 37]. There are 2 implementation configurations:

(i) Config. 1: the rigid baffle is attached to the right sidewall of the resonator

(ii) Config. 2: the rigid baffle is attached to the left sidewall of the resonator

With the conventional design (i.e., $L_p/D_r = 0$), there are 3 different configurations. Figure 4 shows the comparison of the transmission loss with the same dimension rigid baffle implemented in these different configurations. It is revealed that for a given $M_u$ (i.e., the grazing flow Mach number), transmission loss is generally independent on the implementation configuration. Further observation shows that implementing the rigid baffle with $H_g/H_r = 0.5$ and $L_p/D_r = 0.5$ does not improve the transmission loss performance in comparison with the conventional Helmholtz resonator. This reveals that further studies are needed to shed light on the dimensions and geometric implementing location of the rigid baffle. This is summarized and discussed in details later.
3. Results and Discussing Remarks

3.1. Effect of Implementing the Rigid Baffle

With the model being validated, the Helmholtz resonator with a rigid baffle attached to the right sidewall is numerically studied, as the grazing flow Mach number $M_u$ is set to 4 different values. Figure 5 shows the transmission loss varied with forcing frequency for a given $M_u$, as $L_p/D_t = 0.25$, $L_p/D_t = 0.5$, and $L_p/D_t = 0.75$. (a) $M_u = 0$, (b) $M_u = 0.05$, (c) $M_u = 0.07$, and (d) $M_u = 0.10$.

Figure 6: Variation of the transmission loss with forcing frequency, as the rigid baffle is attached at $H_g = 0.5H_t$ but 3 different widths, i.e., $L_p/D_t = 0.25$, $L_p/D_t = 0.5$, and $L_p/D_t = 0.75$. (a) $M_u = 0$, (b) $M_u = 0.05$, (c) $M_u = 0.07$, and (d) $M_u = 0.10$.

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The effect of the rigid baffle length $L_p$ on the transmission loss is evaluated and summarized in Figure 6. It shows the variation of the transmission loss with forcing frequency $f$ for a given $M_u$, as the rigid baffle is attached at $H_b = 0.5H_t$ but 3 different widths, i.e., $L_p/D_t = 0.25$, $L_p/D_t = 0.5$, and $L_p/D_t = 0.75$. Comparison with the resonant frequency of the conventional Helmholtz resonator (i.e., $f_{res}(M_u = 0, L_p = 0)$) denoted by the green dash line is also made. It can be seen that there are two local peaks of transmission loss, as the length $L_p/D_t$ of the rigid baffle is increased to 0.75. It reveals that implementing the rigid baffle with a length of $L_p/D_t = 0.75$ broadens the effective frequency range of the resonator by introducing one additional transmission loss peak at around 400 Hz. The corresponding secondary maximum transmission loss is approximately 12 dB. As $M_u$ is increased from 0 to 0.10, the global maximum TL (transmission loss) is reduced significantly. In addition, a closer observation of Figures 6(c) and 6(d) shows that increasing $L_p/D_t$ from 0.25 to 0.75 gives rise to the global maximum transmission loss being increased slightly. However, the dominant peak at around 100 Hz is more “flattened” at $L_p/D_t \leq 0.5$. This is especially true at $M_u = 0.1$ as shown in Figure 6(d).

In order to shed lights on the physics behind the 2 transmission loss peaks, vorticity contours and velocity contours are captured, as the rigid baffle is attached at $H_b/H_t = 0.5$ but with 3 different $L_p$. These are illustrated in Figure 7, as $M_u = 0.1$. It is apparent that as $L_p/D_t \leq 0.5$, a large clockwise vortex is locked in the resonator cavity, as shown in Figures 7(a), 7(b), 7(d), and 7(e). However, when $L_p/D_t = 0.75$, dual counterrotating vortexes are locked in the cavity below the rigid baffle, which is part of the original resonator’s cavity. In other words, the rigid baffle acts like the “backplate” of the “smaller resonant cavity.” In the cavity above the rigid baffle, there is no clear observation of any lock-in vortex, as shown in Figures 7(c) and 7(f). In general, the 2nd transmission loss peak is generated due to the structure modification, i.e., introducing the rigid baffle. The interaction between the modified structure and the incident sound leads to the generation of multiple resonance peaks in the transmission loss spectrum, as shown in Figure 6.

3.2. Effect of the Grazing Flow Mach Number. The grazing flow Mach number $M_u$ effect on the aeroacoustic damping performance [3, 38–40] of the modified Helmholtz resonator is evaluated and discussed now. Figure 8 illustrates the modified resonator’s damping behaviours over the interested frequency range, as the rigid baffle is attached at $H_b = 0.5H_t$, but its width $L_p$ is set to 3 different values. It can be seen that when $L_p/D_t \leq 0.5$, there is only one TL (transmission loss) peak over the studied frequency range. However,
one additional peak is observed, as $L_p/D_r = 0.75$. As $M_u$ is increased from 0 to 0.1, the global TL$_{\text{max}}$ is decreased for a given $L_p$. This means that the nonnegligible grazing flow deteriorates the acoustic damping effect of the modified structure Helmholtz resonator. In addition, one additional transmission loss peak is obtained at around 400 Hz, when $L_p/D_r = 0.75$. This additional peak in terms of the maximum value and the corresponding resonant frequency is independent on $M_u$.

Figure 9 summarizes the variation trend of the global maximum TL with the implementing location $H_g$ and $M_u$, as the rigid baffle width is set to $L_p = 0.5D_r$. It illustrates that in the presence of a low Mach number grazing flow, i.e., $M_u \leq 0.05$, $H_g$ does affect the acoustic attenuation performance of the modified Helmholtz resonator [33, 41]. However, as $M_u$ is increased above 0.05, the attached location is found to play a negligible role on affecting the resonator’s damping performance. This is most likely due to the fact that the vortex damping mechanism is based on the resonance of the entire resonator cavity [41, 42], since the grazing flow is nonnegligible ($M_u \geq 0.05$). This is clearly visualized in Figures 7(b) and 7(e). As $L_p/D_r = 0.5$, the rigid baffle does not contribute significantly to the acoustic damping performance in the presence of a larger Mach number grazing flow.

3.3. Effect of the Implementation Configurations. The rigid baffle could be implemented in 2 different configurations,
i.e., attaching to the left or right sidewall of the resonator. These configurations are schematically shown in Figure 1 and described in Section 2.2. The effects of these implementation configurations are evaluated and compared, as $M_u$ is set to 4 different values. Figure 10 illustrates the TL (transmission loss) performances of the modified Helmholtz resonator with frequency, as $H_b/H_r = 0.5$, $L_b/D_r = 0.75$, and the rigid baffle is implemented in 2 different configurations. It can be seen that the implementation configurations play a negligible role on affecting the transmission loss, especially when $M_u \leq 0.07$. When the grazing flow Mach number is increased to $M_u = 0.1$, the dominant resonant frequency corresponding to Config. 2 is approximately 10 Hz larger than that of Config. 1. However, the secondary resonant frequency is almost unchanged, no matter to which sidewall the rigid baffle is attached. Furthermore, the difference of the local maximum transmission losses (corresponding to the 2 local peaks) between the 2 implementation configurations is less than 2.0 dB.

To shed lights on the dominant and the secondary resonant frequencies and the corresponding maximum transmission loss difference (see Figure 11(d)), the acoustic pressure contours of the Helmholtz resonator with the rigid baffle implemented are calculated. The acoustic pressure contours at the dominant resonant frequency are shown in Figures 12(a) and 12(b) as the rigid baffle is attached to the right and left sidewall, respectively, while the secondary resonant frequency contours are illustrated in Figures 12(c) and 12(d).

It can be seen that at the dominant resonant frequency, see Figures 12(a) and 12(b), vortex shedding is generated at the resonant neck. However, no vortex is observed in the cavity and the cavity is “resonant” as the baffle were not present. This is most likely due to the lower frequency and the longer wavelength. However, as the resonant frequency is increased to approximately 410 Hz as shown in Figures 12(c) and 12(d), the resonator is “resonant,” like two connected [43–45] but separated cavities with a smaller volume due to the presence of the baffle. This could be the main physics behind the 1-2 dB improved transmission loss peak performance as shown in Figure 11(d) and the generation of the high-frequency transmission loss peak.

3.4. Effect of the Neck Shape of the Modified Helmholtz Resonator. Further design of the modified resonator is proposed by changing its neck shape. Here, we propose and test 1 additional shape, with the cross-sectional view of arc-shape “(+)”, in addition to the conventional resonator neck shape, i.e., “[ ]”. Figure 13 illustrates the computational unstructured mesh of the 2D-modelled Helmholtz resonator [28] with an arc-shaped neck and a rigid baffle implemented. The total number of cells is 960219.

Figure 10 shows the transmission loss performance of the arc-shaped Helmholtz resonator with a rigid baffle implemented. Its performances at different Mach numbers are evaluated and compared with those of the conventional [ ]-shaped resonator. It can be seen that the resonant frequency in Hz corresponding to the dominant TL (transmission loss) peak is increased in general, as shown in Figures 10(a)–10(c). For example, 20% increase of the resonant frequency is observed at $M_u = 0.07$, as shown in Figure 10(c). In addition, the local maximum transmission loss at the secondary peak is increased by approximately 2-5 dB. This means that modifying the structure of the Helmholtz resonator, such as the neck shape, or implementing a rigid baffle leads to the shift of the resonant frequency and the improvement on the secondary transmission loss peaks. The improvement is most likely due to the increased equivalent cross-sectional area of the resonator neck with the arc shape. These are beneficial in the design of an effective Helmholtz resonator with a broader frequency range and a larger acoustic damping effect [46, 47].
4. Conclusions

Experimental and 2D numerical investigations are conducted on a cold-flow duct with a Helmholtz resonator (HR) implemented. A mean flow is forced to pass tangentially over the HR. It is also known as a grazing flow. The Mach number of the grazing flow is variable (0 ≤ \( M_u \) ≤ 0.1). Unlike the conventional one, the modified resonator is involved with an attached rigid baffle. Here, the rigid baffle could be attached in 2 different configurations, i.e., attaching to the left sidewall or right sidewall with reference to the duct flow direction. Five main variables are evaluated. These identified variables include (1) the width of the rigid baffle \( L_p \), (2) the implementation location/height of the baffle \( H_p \), (3) the grazing flow Mach number \( M_u \), (4) the implementation configurations, and (5) the neck shape. Before applying the model to predict the transmission loss performances of the modified resonators, it is benchmarked first with the available experimental and 3D numerical data. Good agreement is achieved, as \( M_u \) is varied from 0 to 0.1. The model is then used to evaluate the acoustic attenuating performance of the modified resonator with the rigid baffle attached, as a nonnegligible duct flow is present.

The key findings obtained from the current investigation are summarized as follows:

(i) When the grazing flow Mach number is small (\( M_u \leq 0.05 \)), the implementation location of the rigid baffle is found to affect the transmission loss of the modified Helmholtz resonator.

![Figure 10: Comparison of the transmission loss performances, as the resonator neck shape is changed from the conventional \(|-|\) to arc (\( . \)) configuration. Here, the rigid baffle dimensions are set to be \( L_p/D_t = 0.75 \), \( H_p/H_t = 0.5 \).](image-url)
(ii) In the presence of a nonnegligible grazing flow or the rigid baffle, the theoretical formula $\omega^2 = \pi c^2 D_n^2/4V_r L_{eff}$ fails in determining the resonant frequency of the modified Helmholtz resonator

(iii) As the duct flow Mach number is higher than 0.07 and $L_p/D_r \leq 0.5$, the implementation location of the rigid baffle is found to play a negligible role on affecting the transmission loss of the modified Helmholtz resonator

(iv) Increasing the Mach number of the duct flow leads to a deteriorated acoustic attenuation effect of the modified resonator in general

(v) When the width of the rigid baffle is increased to $L_p = 0.75D_r$, two transmission loss peaks at two different frequencies are observed. This is most likely due to the incident sound and structure interaction. The grazing flow Mach number is shown to affect strongly the dominant transmission loss peak

(vi) The secondary transmission loss peak at a higher frequency is found to be unchanged, as the grazing flow is varied. The flow characteristic contours reveal that the higher frequency peak is generated due to the "dual" vortex that is locked in the smaller cavity built by the rigid baffle and the neck

(vii) As the width of the rigid baffle is less than $0.5D_r$, there is an optimum grazing flow Mach number, which gives rise to a larger global maximum transmission loss

Figure 11: Comparison of the transmission loss performances of the modified Helmholtz resonator, as $H_g = 0.5H_r$, $L_p = 0.75D_r$, and the rigid baffle is implemented in 2 different configurations. (a) $M_u = 0$, (b) $M_u = 0.05$, (c) $M_u = 0.07$, and (d) $M_u = 0.1$. 
(viii) Changing the neck shape from the conventional \( \cdot \) to an arc (\( \cdot \)) one results in the dominant resonant frequency being increased by approximately 20%. The secondary transmission loss peak is increased by 2-5 dB. This is due to the increased equivalent neck cross-sectional area of the arc-shaped resonator.

In general, the present work systematically studies an improved design of a Helmholtz resonator with an additional transmission loss peak at a high frequency, besides the dominant peak at a low frequency. How to improve the transmission loss peak at the high frequency is proposed. It is found that 5-10 dB more transmission loss could be achieved by changing the resonator neck shape from the conventional shape of \( \cdot \) into the angle-bracket-“(\( \cdot \)” one.

**Data Availability**

Data will be available on reasonable request.

**Conflicts of Interest**

The authors declare that there is no conflict of interest.

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