Some examples of uses of Dirac equation and its generalizations in particle physics

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Applications of the Dirac equation with an anomalous magnetic moment are considered for description of characteristics of electrons, muons and quarks. The Dirac equation with four-dimensional scalar and vector potentials is reduced to a form suitable for a numerical integration. When a certain type of the potential is chosen, solutions can approximate quark states inside hadrons. In view of complicated behaviour of quarks in a confinement domain some generalizations are considered such as the Dirac-Gursey-Lee equation, the Dirac equation in a five-dimensional Minkowski space, the Dirac equation in a quantum phase space. Extended symmetries for the Dirac equation and its generalizations are considered, which can be used for investigation of properties of solutions of these equations and subsequent applications in particle physics.

1. Introduction

It is well known the Dirac equation for a relativistic electron is one of the main building blocks of the quantum electrodynamics (QED) and the Standard Model (SM) [1, 2]. It is used to check the validity of SM as to search for new physics beyond SM. Although the Dirac equation was offered to the description of electron it describes with success muons, tayons and moreover quarks. The fact of validity of the Dirac equation (DE) for description of quarks is astonishing because quarks do not observe in our physical spacetime. DE is used in the quantum chromodynamics (QCD) [2] and in phenomenological hadron models, for example, in the relativistic model for quasi-independent quarks (RMQIQ), which has been applied for the description of hadron properties [3, 4]. However in a domain of confinement the deviations from the Dirac equation and standard spacetime symmetry can take place. From this point of view generalizations of DE are welcome. We consider some generalized equations of Dirac type, namely, the Dirac-Gursey-Lee equation (DGLE), the Dirac equation in a five-dimensional Minkowski space (FDDE), the Dirac equation in a quantum phase space (QPSDE).
2. The Dirac equation and AMMs of electrons, muons and quarks

In 1928 Dirac presented the electron equation, which is used successfully for description of fermions at present. Let us write DE in the natural system of units: \( c = \hbar = 1 \) \((p^i = i\partial^i, x^i = \{t, \mathbf{x}\}, \gamma^i = \{\beta, \beta\alpha\})\).

\[
(\gamma^i p_i - m)\psi(x) = 0, \quad \beta = \begin{pmatrix} \sigma_0 & 0 \\ 0 & -\sigma_0 \end{pmatrix}, \quad \alpha = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix},
\]

where \( \sigma_0 = 1_2, \sigma \) are the known Pauli matrices, the metrical tensor \( \eta_{ij} = \text{diag}\{1, -1, -1, -1\} \). DE is invariant under transformations of Poincare group, which are generated by operators of momenta \( p^i \) and four-dimensional rotations

\[
J_{ij} = x_i p_j - x_j p_i + S_{ij}, \quad S_{ij} = \frac{i}{2} \sigma_{ij}, \quad \sigma_{ij} = \frac{1}{2}(\gamma_i \gamma_j - \gamma_j \gamma_i).
\]

If we define the pseudoscalar matrix \( \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 \), the left and right wave functions \( \psi_L, \psi_R \) can be written

\[
\psi_L = \frac{1 - \gamma_5}{2} \psi, \quad \psi_R = \frac{1 + \gamma_5}{2} \psi, \quad \gamma_3 \psi_{R,L} = \pm \psi_{R,L}.
\]

One can consider the charge conjugated fields \( \psi^c = C(\bar{\psi})^T \), where \( C \) is the matrix of the charge conjugation which satisfies the relations: \( C\gamma_i^T C^{-1} = -\gamma_i \), \( C^T = -C \). We can construct two Majorana fields \( \chi_1 \) and \( \chi_2 \) using \( \psi \) and \( \psi^c \):

\[
\chi_1 = (\psi + \psi^c)/\sqrt{2}, \quad \chi_2 = (\psi - \psi^c)/\sqrt{2}. \quad \text{For the Majorana fields the following relation holds} \ (\chi_{1,2})^c = \pm (\chi_{1,2}).
\]

It is known that for a particle ”a” with charge \( q \), spin \( S \), and mass \( m \) the magnetic moment arises with the absolute value \( \mu_a = g(|q|/2m)S \), where \( g \) is the gyro-magnetic factor or the g-factor. The g-factor is usually written as \( g = g_0(1 + a) \), \( a \) is the reduced anomalous magnetic moment or anomalous magnetic moment (AMM) of the particle. For electrons and muons \( g_0 = 2 \) and DE with AMM have the so-called Pauli term

\[
(\gamma^i p'_i - m - \frac{iqa}{4m}\sigma^{ij}F_{ij})\psi(x) = 0,
\]

where \( p'_i = p_i - qa_i \), \( F_{ij} = \partial_i A_j - \partial_j A_i \).

In the SM framework AMM for electrons and muons have been evaluated with high accuracy. It is interesting that the most stringent test of SM and QED is the comparison of experimental and theoretical values of AMM for electrons \( (a_e) \). For experimental determination of \( a_e \) the frequencies of radiation are measured when an electron transits between quantum levels in a cylindrical Penning trap [5]. The QED corrections entered into a value \( a_e \) are
evaluated up to 10th order (the total contribution of terms of 10th order are only estimated now) [6]. If the validity of SM is assumed, the most accurate value of the fine structure constant can be obtained at present (0.37 ppb).

\[ g/2(exp) = 1.00115965218073(28), \quad \alpha^{-1} = 137.035999084(51) \]  

The analogous calculations have been made for AMM of a muon [7, 8] and have been compared with the result of E821 experiment at Brookhaven National Laboratory (BNL) [9]: \( a_{\mu}(exp) = 11659208.9(6.3) \times 10^{-10} \). When the SM prediction \( a_{\mu}(SM) = 11659177.3(4.8) \times 10^{-10} \) have been compared to the BNL measurement result the difference is obtained \( a_{\mu}(exp) - a_{\mu}(SM) = 31.6(7.9) \times 10^{-10} \), which corresponds to the 4\( \sigma \) discrepancy! To establish physics beyond SM new \( g_{\mu} - 2 \) experiments are planned [10]. The role of electromagnetic AMMs of quarks should be made clear in the future.

3. The Dirac equation and the relativistic quasi-independent quark model

The main statement of the relativistic model for quasi-independent quarks is that hadron can be described as a system of independent constituents (or quasi-independent ones with weak residual interactions), which move in some mean self-consistent field. In the framework of RMQIQ a wave function \( \psi_i \) for any valent i-quark is a solution of DE with the mean field static potential \( U(r_i) \). Energies or mass terms \( E_i(n_i^r, j_i) \) represent the energies of constituents in a mean field \( (n_i^r \text{ and } j_i \text{ are the radial quantum number and the quantum number of the angular moment correspondingly for the } i \text{-th constituent}) \) and can be evaluated for quarks with the help of solutions of the stationary Dirac equation (for diquarks and constituent gluons with the help of solutions of the Klein-Gordon-Fock equation):

\[ E_i(n_i^r, j_i)\psi_i(r_i) = [\alpha_i p_i + \beta(m_i + V_0) + V_1]\psi_i(r_i), \]  

with \( V_0(r) = \sigma r/2 \) and \( V_1(r) = -2\alpha_s/3r \), where the model parameters \( \sigma \) and \( \alpha_s \) have meanings of the "string tension" and the strong coupling constant at small distances, correspondingly [3, 4].

Angular dependence of the single-particle wave functions in a stationary state for the spherically symmetric potential can be separated in a well-known manner, namely, the solutions of Eq.(6) with the total angular momentum \( j \) and its projection \( m \) can be represented as (the subscript \( i \) here and below is omitted)

\[ \psi(r) \propto \begin{pmatrix} f(r)\Omega_{jm}^m(n) \\ -ig(r)(\sigma n)\Omega_{jm}^m(n) \end{pmatrix}, \]  

3
where $n = r/r$. Constituent energies $E_i$ are evaluated by solving the radial equations with the model potentials for each constituent. If $k = -\omega(j + 1/2)$, where $\omega$ is connected with an eigenvalue of the space-parity operator, the system of the radial Dirac equations reads

\[
(E - V_0 - V_1 - m)f = -\frac{(1 - k)}{r}g - g',
\]

\[
(E + V_0 - V_1 + m)g = -\frac{(1 + k)}{r}f + f',
\]

(8)

One can derive the second order equation for the "large" component $f(r)$ using Eqs. (8), then making two substitutions $E = \sqrt{\lambda + m^2}$ and

\[
\phi(r) = rf(r)[V_0(r) - V_1(r) + m + \sqrt{\lambda + m^2}]^{-1/2},
\]

one comes on to the model radial equation for $\phi(r)$ in the following form [3, 4]:

\[
\phi'' + \lambda \phi = \left[V_0^2 - V_1^2 + 2(mV_0 + \sqrt{\lambda + m^2}V_1) + \frac{k(k + 1)}{r^2}\right] + \frac{3(V_0' - V_1')^2}{4(\sqrt{\lambda + m^2} - V_1 + V_0 + m)^2} + \frac{k(V_0' - V_1')}{r(\sqrt{\lambda + m^2} - V_1 + V_0 + m)^2} - \frac{3(V_0'' - V_1'')}{2(\sqrt{\lambda + m^2} - V_1 + V_0 + m)^2}\]

(9)

This equation is suitable for numerical integration and fitting of hadron mass spectra. For instance, one can use the phenomenological mass formula for $q'\bar{q}$-mesons with the quark and antiquark energies, which takes into account spin-spin interaction between quark and antiquark, in the following form [3, 4]:

\[
M(q'\bar{q}) = E_0 + E_1 + E_2 + 4s_1s_2 > V_{12}^{ss}
\]

(10)

It is interesting to relate the model of quasi-independent quarks to the constituent quark model, where the Zeldovich-Sakharov formula are frequently used [11, 12]:

\[
M(q'\bar{q}) = m_0 + m_1 + m_2 + s_1s_2 > v_{12}^{ss}
\]

(11)

where $m_1, m_2$ are masses of constituent quarks, $m_0$ is some additional phenomenological contribution. Take into account the similarity between formulae
(10) and (11) we can relate constituents’ energies $E_i$ with constituent quarks’ masses $m_i$ within uncertainties of models. Some complications arise for exited hadron states. In the model of quasi-independent quarks $E_i$ remains an energy of $i$-th exited constituent, while in the constituent quark model an additional contribution representing energy of excitation must be taken into account.

The values of parameters $\sigma$ and $\alpha_s$ entered into potentials $V_0(r)$ and $V_1(r)$ have been determined after the comparison between experimental and evaluated spectra of the vector and pseudoscalar mesons. In this manner it was found that the value of $\sigma$ is the same for the light and heavy mesons within the systematic errors of the model: $\sigma = (0.20 \pm 0.01) GeV^2$, whereas the $\alpha_s$ values run from 0.7 to 0.25 for the light and heavy mesons, correspondingly [3, 4]. The values of constituents’ energies $E_i$ for $u-, d-, s-, c-, b-$quarks and antiquarks in MeVs are: $E_U = 335 \pm 2$, $E_D = 339 \pm 2$, $E_S = 485 \pm 8$, $E_C = 1610 \pm 15$, $E_B = 4952 \pm 20$ [13, 14]. So RMQIQ may be considered as the workable generalization of the constituent quarks model. In the RMQIQ framework the large number of meson resonance masses have been evaluated, which do not contradict as the existing experimental data, as the results of other phenomenological models [4, 15, 16, 17]. In some variants of the quark model anomalous chromo-magnetic moments of quarks have been taken into account as well [18, 19, 20].

4. The DGL equation and the Dirac equation in a five-dimensional Minkowski space

It is mentioned above, the spacetime symmetry for quarks can differ from the Poincare symmetry. Let us consider some extended kinematical symmetries and generalized Dirac equations.

The structure of the algebra of Dirac $\gamma$-matrices (including the $\gamma_5$ -matrix anticommutating with other $\gamma$-matrices) shows that the extension of the Poincare group in a four-dimensional Minkowski space to a generalized Poincare group in a five-dimensional Minkowski space is possible. By this way the Dirac equation is obtained in a five-dimensional Minkowski space [21]:

$$ (\gamma^a p_a - \kappa) \psi(x) = 0, $$

where $p^a = i\partial^a$, $x^a = \{t, \mathbf{x}, x^4\}$, $\gamma^a = \{\beta, \beta\alpha, i\gamma_5\}$, the metrical tensor $\eta_{ab} = diag\{1, -1, -1, -1, -1\}$. FMSDE is invariant under transformations of Poincare group in a five-dimensional Minkowski space $P(1, 4)$, which generate by operators of momentum components $p^a$ and five-dimensional rotations

$$ J'_{ab} = x_a p_b - x_b p_a + S'_{ab}, \quad S'_{ab} = \frac{i}{2} \sigma'_{ab}, \quad \sigma'_{ab} = \frac{1}{2}(\gamma_a \gamma_b - \gamma_b \gamma_a). $$
The mass of a particle is dependent on a value of an additional momentum in a new direction $p^4$, $m^2 = \kappa^2 + (p^4)^2$, where $\kappa$ is the constant entered in Eq. (12).

If the symmetry group of a equation was restricted to the homogeneous pseudoorthogonal group $O(1, 4)$ (or $O(2, 3)$), then the equation is depended on generators of the pseudoorthogonal rotations $L_{ab}$ and have the form

$$1/2\gamma^a\gamma^bL_{ab}\psi(x) = \lambda \psi(x)$$

(14)

This equation is proposed by Dirac in 1935 [22]. However $\lambda$ has an imaginary term due to nonhermitean operators $L_{ij}$ for a finite dimensional representation. The imaginary term should be single out for a correct physical interpretation, and the equation (14) was presented by Gursey and Lee in another form [23]:

$$\left(4i + \gamma^a\gamma^bL_{ab}\right)\eta(x) = 2mR\eta(x)$$

(15)

where $\lambda = mR - 2i$, $R$ is a radius of a de Sitter space. Properties of solutions of equations of this type and its application for a description of elementary particles were studied by Fronsdal, who used the generalized momentum $p_0 + dp_0L = p_F$, where $p_0$ and $L$ have the forms of the usual generators of translations and Lorentz transformations in Minkowski spacetime, and $p_0^2 = m_0^2$, $d = \mu_s/m_0$ [24]. Take into account the results obtained in Refs. [22, 23, 24], we can apply the following modified Dirac-Gursey-Lee equation for a description of quark characteristics [25]:

$$\left[\gamma_i(p_0^i + dp_0^iL_k^i + i\mu_s\gamma^i/2) + 2i\mu_sS_{ij}(L^{ij} + S^{ij})\right]\psi = m\psi.$$  

(16)

To estimate a $\mu_s$ value with the help of a constituent quark mass $m$ and a current quark mass $m_0$ values, a quark ground state $\psi_0$ can be used in a meson so the contribution from $L\psi_0$ can be neglected. In this way we obtain from Eq.(23) the approximate relation: $m \approx m_0 + 2i\mu_s$. So the constant $\mu_s$ should be pure imaginary and $|\mu_s| \sim 0.16GeV$ [25].

The problem of quark and lepton masses is the most important problem in particle physics now. In the SM framework these masses arise due to the Yukawa coupling between fundamental fermions and Higgs field. But, it is known the Higgs meson has not been discovered [26] and LHC experiments in the near future must make clear the question of its existing. The mass problem of quarks and leptons became more complicated after discovering the neutrino masses [26]. The neutrino masses are very small, so their origin can differ from the origin of other fundamental fermion masses due to the fact that neutrinos can be Majorana particles. It is interesting that values of quark and neutrino masses can be connected with values of there mixing angles [14].
5. The Dirac equation in a quantum phase space

It is well known that the Poincare symmetry, which is the spacetime symmetry of the relativistic quantum field theory, originates from the isotropy and homogeneity of Minkowski spacetime and is based on observations of macro- and micro-phenomena concerning conventional physical bodies and particles. However, description of color particles over the whole range of interaction distances is possibly needed using extended symmetries and theories with new fundamental physical constants (other than the well known ones \( c \) and \( \hbar \)) in a noncommutative spacetime [27].

Let us consider the generalized model for a color particle motion, when coordinates and momenta are on equal terms and form an eight dimensional phase space:

\[
\begin{align*}
h &= \{ h^A | h^A = q^\mu, A = 1, 2, 3, 4, \mu = 0, 1, 2, 3, h^A = \tau p^\mu, A = 5, 6, 7, 8, \mu = 0, 1, 2, 3 \}, \\
P &= \{ P^A | P^A = p^\mu, A = 1, 2, 3, 4, \mu = 0, 1, 2, 3, P^A = \sigma q^\mu, A = 5, 6, 7, 8, \mu = 0, 1, 2, 3 \} [28].
\end{align*}
\]

The constants \( \tau \) and \( \sigma \) have dimensions of length and mass square, correspondingly. Their values can be chosen on the phenomenological ground or with the help of some functions of the quantum constants \( \mu \), \( \kappa \) and \( \lambda \). We assume the generalized length and mass squares

\[
L^2 = h_A h_A, \quad M^2 = P^A P_A
\]

are invariant under the O(2,6) transformations, where \( h_A = g_{AB} h^B, \ g_{AB} = g^{AB} = diag\{1, -1, -1, -1, 1, -1, -1, -1\} \).

Thus the generalized differential mass squared can conserve for strong interacting color particles:

\[
dM^2 = (dp_0)^2 - (dp_1)^2 - (dp_2)^2 - (dp_3)^2 + \sigma^2 (dq_0)^2 - \sigma^2 (dq_1)^2 - \sigma^2 (dq_2)^2 - \sigma^2 (dq_3)^2 = (dm)^2 + \sigma^2 (ds)^2.
\]

An important point is that the coordinates \( q^\mu \) and the momentum components \( p^\mu \) are the quantum operators satisfied the generalized Snyder-Yang algebra (GSYA) [28, 30, 31, 32]:

\[
\begin{align*}
[F_{ij}, F_{kl}] &= i(g_{jk} F_{il} - g_{ik} F_{jl} + g_{il} F_{jk} - g_{jl} F_{ik}), \\
[F_{ij}, p_k] &= i(g_{jk} p_i - g_{ik} p_j), \quad [F_{ij}, q_k] = i(g_{jk} q_i - g_{ik} q_j), \\
[F_{ij}, I] &= 0, \quad [p_i, q_j] = i(g_{ij} I + \kappa F_{ij}), \\
[p_i, I] &= i(\mu^2 q_i - \kappa p_i), \quad [q_i, I] = i(\kappa q_i - \lambda^2 p_i), \\
[p_i, p_j] &= i\mu^2 F_{ij}, \quad [q_i, q_j] = i\lambda^2 F_{ij},
\end{align*}
\]
where $F_{ij}$, $p_i$, $x_i$ are the generators of the Lorentz group and the operators of momentum components and coordinates, correspondingly, $I$ is the "identity" operator, $i$, $j$, $k$, $l = 0, 1, 2, 3$. The new quantum constants $\mu$ and $\lambda$ have dimensionality of mass and length correspondingly. The constant $\kappa$ is dimensionless in the natural system of units.

By applying the algebra (19) to the description of color particles the condition $\kappa = 0$ can be imposed. Actually it is known the nonzero $\kappa$ leads to the $CP$-violation [31, 32], but strong interactions are invariant with respect to the $P$, $C$, and $T$-transformations on the high level of precision. Moreover for color particles one can use the relation $\mu \lambda = 1$ [28]. In this case we obtain the reduction of GSYA to the special Snyder-Yang algebra (SSYA) with $\mu \lambda = 1$ and $\kappa = 0$ for strong interaction color particles. Denoting $\mu$ as $\mu_c$ and $\lambda$ as $\lambda_c$ we write the following commutation relations without the standard commutation relations with Lorentz group generators, which are shown for the GSYA above (see eqs. (19)).

\[
[p_i, q_j] = i g_{ij} I,
[p_i, I] = i \mu_c^2 q_i, [q_i, I] = -i \lambda_c^2 p_i,
[q_i, q_j] = i \lambda_c^2 F_{ij},
[p_i, p_j] = i \mu_c^2 F_{ij}.
\]  

(20)

We take into account difficulties arised when one try to prove the confinement on the basis of the QCD first principles [33], and we simulate this phenomenon with the help of an assumed high symmetry of the nonperturbative QCD interaction beyond the Poincare symmetry. So we turn from the Poincare symmetry in the Minkowski spacetime to the inhomogeneous O(2,6) symmetry in a phase space of a color particle [28].

Under these conditions the new Dirac type equation for a spinorial field $\psi$ has the following form:

\[
\gamma^A P_A \psi = M \psi, \quad \gamma^A \gamma^B + \gamma^B \gamma^A = 2 g^{AB}.
\]  

(21)

where $\gamma^A$ are the Clifford numbers for the spinorial O(2,6) representation. One can take the product of Eq. (21) with $\gamma^A P_A + M$ and apply Eqs. (19), then the following equation for $\psi$ can be obtained

\[
(p^i p_i + \sigma^0 q^i q_i + 2 \Sigma_{i<j} S^{ij} F_{ij} +
2 \sigma S^0 I) \psi = M^2 \psi, S^0 = \frac{i}{2} C^0, S^{ij} = \frac{i}{2} C^{ij},
\]  

(22)

where $C^0$, $C^{ij}$ are the combinations of the $\gamma$-matrices and the constants $\lambda$, $\mu$ and $\kappa$ [28]. Eq. (22) contains the oscillator potential, which restricts a
motion of a color quark. Besides that we broke the inhomogeneous O(2,6) symmetry with the help of the commutation relations (19). In the special case \( \mu \lambda = 1, \kappa = 0 \) and the commutation relations (20) for SSYA we will obtain more simple expressions for the \( C^0 \) and \( C^{ij} \), but the form of the Eq. (22) will remain unchanged. Note that Eq. (22) can also be applied for a description of a confinement of boson particles such as diquarks and gluons with the same confinement parameter \( \sigma \).

One can get an estimation of the \( \sigma \) value using Eq. (22). As it is seen, \( M^2 \) and \( p^2 \) entered into Eq. (22) can be considered as current and constituent quark masses squared, respectively. So Eq. (22) indicates that the conventional relation \( p_{cur}^2 = M^2 \) for a current quark should be transform to \( p^2 = M^2 + \Delta^2 \) for a constituent quark, where \( m^2 = M^2 + \Delta^2 \) is a constituent mass squared.

To estimate the \( \sigma \) value with the help of a constituent quark mass \( m \) and a current quark mass \( M \) values a ground state \( \psi_0 \) in a meson has been considered neglecting the orbital angular momentum contribution \( L\psi_0 \), as it had been done for DGLE. We can also estimate the \( \sigma \) value with the help of the value of the confinement rising potential coefficient [3]. It is seen from Eq. (22) that the coefficient of the oscillator confinement potential for a color particle is equal to \( \sigma^2 \). Within the potential approach this coefficient is connected with the string tension \( \sigma_{str} \) typically as \( \sigma^2 = \sigma_{str}^2/4 \), where \( \sigma_{str} \) varies from 0.19 GeV\(^2\) to 0.21 GeV\(^2\) [3]. Hence \( \sqrt{\sigma} \approx 0.3\)GeV and \( \lambda_{conf} \approx 0.6\)Fm. Clearly it is assumed that the conceptions of the asymptotical oscillator potential and the constituent quark are applicable in a confinement domain.

6. Conclusion

The Dirac equation is successfully employed for the description of particles with the spin 1/2. If one take into account an additional Pauli term, then the Dirac equation describes in the SM framework the modification of a magnetic moments of leptons due to radiative corrections. However, someone has cast doubt on the relevance of DE for quarks in a confinement domain, i.e. in the nonperturbative domain of QCD. It is very likely that in this case we should modify essentially the Dirac equation or to pass on to a new equation of the Dirac type. The possible generalizations of the Dirac equation have been considered in this report. The Dirac equation in a quantum phase space is the most interesting from the standpoint of description of quark motion. This equation involves an oscillatory potential, so it can play a role of a model of color particle confinement.

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