Digital Image Compression using Graph Coloring Quantization Based on Wavelet-SVD

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Abstract. JPEG2000 is an example of standard of compression technique. Wavelet and scalar quantization are used in the JPEG2000 compression scheme. Despite the advantages of JPEG2000 based on wavelet has low computation, the wavelet has poor performance in low-correlated image. In this paper, we propose a graph coloring technique in the quantization process on image compression scheme based on Wavelet- Singular Value Decomposition. We show that the system provides good performance based on compression ratio and PSNR. The average compression ratio generated by the system stands between 50-60%, while the average PSNR stands between 40-80 dB.

1. Introduction

The general principal used in image processing is to reduce data duplication inside the image so the memory needed to reduce and represent data duplication inside the image can be smaller [6]. Compression based on the output given by this system is a lossy compression. Lossy compression is a type of compression where some information might be missing from output data but still tolerated by user. Compression model is divided in two forms which is compression and decompression. Forward transformation, quantization and entropy encoding are parts of compression process while backward transformation, dequantization and entropy decoding are parts of decompression process.

JPEG2000 is a standard compression technique with .jp2 format as its basic core system [3,7]. JPEG2000 using Wavelet as the transformation method. However, despite the advantage of wavelet transformation which has low computation, wavelet has poor performance in low-correlated image [4]. In this Paper, Wavelet - Singular Value Decomposition (SVD) domain will be used as the transformation method. SVD will be used to achieve a better performance for transformation process in low-correlated image, while Wavelet domain is used in high-correlated image [4]. Therefore, the transformation process is supposed to give better performance for both low-correlated image and high-correlated image. In the quantization process, graph-based quantization is used to cluster image component in frequency domain. The implementation of graph based quantization is based on coloring graph method. Coloring graph is a coloring dot from a graph method where adjacent dots have different colors while using colors as minimum as possible [1,2]. On previous research, it is known that graph-based quantization provides better result than vector quantization that is widely used.
2. The Proposed Method

The input image is a color image of 512x512 pixel with RGB color components. Then RGB components are transformed to a 512x512 pixel YCbCr color components. Down-sampling process is done to the chrominance components (Cb, Cr) until the current chrominance component pixel is half of before. The down-sampled color components size is 256x256, then each of color components is divided into 8x8 sub-blocks where each of sub-block consists of 64x64 pixel.

![Flowchart of Proposed Scheme](image)

Figure 1. Flowchart of Proposed Scheme

Calculation of average of standard deviation (STDA) and standard deviation of standard deviation of each sub-block (SSTD) is necessary to determine which transformation method should be applied to related sub-block. The sub-block whose size is 64x64 is divided into another sub-block size of 8x8 pixel. For each of 8x8 size of sub-block from a 64x64 size of sub-block standard deviation is calculated. Then for each 64x64 size of sub-block the average of all standard deviation from 8x8 size sub-block from 64x64 size of sub-block related is calculated to get the STDA value and the standard deviation of all standard deviation from 8x8 size sub-block from 64x64 size of sub-block related is calculated to get the SSTD value.

High STDA values indicated that sub-block contains edges, low-correlated pixel, edges and low-correlated pixel or edges and high-correlated pixel (these can be sharp edges) [7]. Sharp edges should not be transformed using SVD because it needs larger codebook to reduce the given error [5]. That is why both STDA and SSTD threshold are needed to determine which transformation method should be used. SSTD value of sharp edges is usually much higher than low-correlated pixel or high-correlated pixel [5]. Every sub-block which has higher STDA value than STDA threshold and lower SSTD value than SSTD threshold should be transformed using SVD. And then, every sub-block which has lower STDA value than STDA threshold and higher SSTD value than SSTD threshold should be transformed using DWT.

We used 7 sample images in Table 1, which have different range of STDA to ensure that threshold
value used in the system will be valid for all images. The DWT and SVD are computationally expensive, but the SVD is even more expensive [5]. Considering the fact that SVD is too much time consuming and running time in compression image is one of important matters, the sub-block transformed using SVD is a candidate sub-block for SVD which has higher STDA value than STDA threshold and lower SSTD value than SSTD threshold. Candidate sub-block for SVD is limited to sub-block which has STDA value in range 2/3 from average of all STDA and maximum STDA for each image. This should keep the running time of compression image system stay low. The calculation of STDA and SSTD is applied for luminance component (Y) only.

Table 1 Sample images

| No | Image  | Range STDA |
|----|--------|------------|
| 1  | Brick  | 24 – 48    |
| 2  | Flowers| 8 – 37     |
| 3  | Barbara| 5 – 32.4   |
| 4  | Peppers| 3.6 – 18.97|
| 5  | Crayon | 0 – 21.9   |
| 6  | Airplane| 0.7 – 27.8 |
| 7  | Sailboat| 3.1 – 25.9 |

Testing with STDA threshold in range 15 – 50 is done for all sample images. From 7 sample images, there are 5 images which has sub-block that transformed using SVD. We provide them in Table 2.

Table 2 STDA and SSTD of sub-block that transformed using SVD

| No | Image  | STDA  | SSTD  |
|----|--------|-------|-------|
| 1  | Barbara| 32,03 | 13,17 |
|    |        | 32,41 | 12,19 |
|    |        | 29,34 | 9,78  |
|    |        | 28,52 | 9,15  |
|    |        | 29,89 | 2,25  |
|    |        | 28,72 | 8,48  |
| 2  | Airplane| 26,81 | 19,68 |
|    |        | 25,61 | 24,06 |
|    |        | 27,78 | 14,19 |
|    |        | 25,72 | 11,47 |
| 3  | Sailboat| 23,18 | 10,41 |
|    |        | 22,46 | 11,97 |
|    |        | 25,86 | 9,02  |
|    |        | 24,09 | 15,29 |
| 4  | Brick  | 45,16 | 19,21 |
|    |        | 45,19 | 14,46 |
|    |        | 44,92 | 16,96 |
|    |        | 47,96 | 15,32 |
| 5  | Flowers| 33,69 | 15,56 |
|    |        | 37,73 | 17,09 |

Based on test of 20 as STDA threshold and range 15 - 17 as SSTD threshold by considering given Peak Signal to Noise Ratio (PSNR) and compression ratio value, the threshold value applied in this compression image system is 20 as STDA threshold and 15 as SSTD threshold.

Every sub-block transformed using SVD is divided into 32x32 pixel and current amount of sub-block is twice than before. And then the results of SVD transformation are 3 matrices which is matrix
left singular vector $U$, matrix singular value $\Sigma$ and right singular vector $V^T$ for each sub-block. Two values from the lowest values of matrix singular value $\Sigma$ are assigned with 0. Because a good image representation can be achieved by taking only a few largest eigenvalues and corresponding eigenvectors [5].

Every luminance components of sub-block transformed using DWT is encoded using Daubechies 9/7 filter bank into 3 levels. The chrominance components are encoded into maximum 2 levels. Transformed sub-block using DWT results are 4 coefficient which is LL, LH, HL, and HH. For each sub-block, half of lowest level of HH coefficient is assigned with 0.

Figure 2. Flowchart of Quantization by Graph Coloring

Quantization process for transformation outputs from DWT and SVD is done separately. In Figure 2, euclidean distance is used to get distance value from all input vectors. All input vectors from transformation process are treated as dots. A distance threshold is used to determine adjacent relationship between dots. A pair of dots whose distance is larger than distance threshold has 1 as adjacent matrix value. A pair of dots whose distance is less than distance threshold has 0 as adjacent matrix value. After all values of adjacent matrix are assigned, coloring dots from a graph process is done using Welch Powell algorithm [2]. The amount of clusters representation as a graph quantization results is the amount of colors given by Welch Powell algorithm. Codeword values of each clusters is average distance value of all members of related cluster. Codebook is a group of codeword.

Outputs of DWT transformation are divided into two groups which is LL coefficient and group of other coefficients because LL coefficient is the most important coefficient which is related the most with image visual so the given error will be reduced by dividing group this way. Outputs of quantization of DWT coefficients are two codebooks. In Table 3, mean distance value of transformed sub-block using DWT based on chrominance and luminance components.
Based on test range of 1000 - 3000 as luminance distance threshold and range of 100 - 300 as chrominance distance threshold with considering given Peak Signal to Noise Ratio (PSNR), compression ratio value and the amount of colors needed, the distance threshold value applied in this compression image system is 1000 as luminance distance threshold and 300 as chrominance distance threshold for DWT transformation output.

Outputs of SVD transformation are also divided into two groups which is singular matrix value $\Sigma$ and group of $U$ and $V$ matrix because singular matrix value $\Sigma$ is image base which is related the most with image visual so the given error will be reduced by dividing group this way. Outputs of quantization of SVD matrix are two codebooks. In Table 4, mean distance value of transformed sub-block using SVD based on chrominance and luminance components. Distance threshold used for SVD transformation output chrominance and luminance components is grouped into one distance threshold because of its less difference value. Based on test range of 0.6 – 1.1 as distance threshold value by considering given Peak Signal to Noise Ratio (PSNR), compression ratio value and the amount of colors needed, the distance threshold value applied in this compression image system is 0.9 for SVD transformation output.

### Table 3 Mean distance value of transformed sub-block using DWT

| No | Image | Mean distance value | Luminance | Chrominance |
|----|-------|---------------------|-----------|-------------|
| 1  | Brick | 6291.2              | 401.8     |             |
| 2  | Flowers | 4845               | 1009      |             |
| 3  | Barbara | 4457.8             | 516.6     |             |
| 4  | Peppers | 4393.4             | 907.8     |             |
| 5  | Crayon | 6575.9             | 1335.5    |             |
| 6  | Airplane | 3783.6             | 396.7     |             |
| 7  | Sailboat | 5266.8             | 665.8     |             |

All of quantization results are arranged serially based on sub-block into a vector which will become the input of arithmetic coding or huffman coding. Transformed sub-block using SVD is marked by 0 as bit value and 1 as bit value if sub-block transformed using DWT. Codeword result of arithmetic coding has value in range 0 until 1. This range is given based on frequency probability of each symbols.

The compressed bit value from encoder output is decoded using arithmetic decoding so the result value is same as input value of arithmetic encoder. Dequantization process is one on one mapping process between codevector index as result of arithmetic decoding and codevector from codebook. The output vectors are arranged based on sub-block as input of backward transformation method.
grounded on bit mark of each sub-block after mapping process. If the bit mark is 0 the backward transformation process uses SVD. Then the eigenvectors and eigenvalues matrix are reconstructed into one matrix.

3. System Evaluation

In this section, we present the performance of system evaluation and compare with JPEG2000. We use compression ratio and PSNR as parameters of system evaluation. Compression ratio is the decreased percentage of current compressed size than original size \([8]\).

\[
\text{Ratio} = \left(1 - \frac{\text{size of compressed image}}{\text{size of original image}}\right) \times 100\%
\]

(1)

PSNR is a comparison value between maximum value of reconstructed image and the original image (before compressed) in decibel unit (dB). Let \(A\) be an original image, \(B\) as the reconstructed image, \(mn (m \times n)\) as image resolution of both of images and \(\text{max}\) as most possible maximum value of image pixel:

\[
\text{PSNR} = 10 \log_{10} \left(\frac{\text{max}^2}{\text{mean}(B-A)^2}\right)
\]

(2)

In this paper, we calculated two types of PSNR which is luminance PSNR and average PSNR. Luminance PSNR is a calculated PSNR for luminance component only. Average PSNR is average of calculated PSNR for luminance and chrominance PSNR. Luminance component is a color component which is most sensitive to human eyes.

| No | Image       | Ratio (%) | STDA | T    | Luminance PSNR (dB) | Average PSNR (dB) | Time (Second) |
|----|-------------|-----------|------|------|---------------------|-------------------|---------------|
|    |             |           | Range | mean | D | S | PA | PH | J | PA | PH | J | PA | PH | J |
| 1  | Brick       | 55        | 24-48 | 37.32 | 63 | 1 | 80.6 | 80.8 | 56.8 | 55.0 | 54.8 | 57.2 | 7  | 5  | 3 |
| 2  | Flowers     | 54        | 8-38  | 18.4  | 64 | - | 87.5 | 87.1 | 58.4 | 57.5 | 57.2 | 58.0 | 7  | 4  | 3 |
| 3  | Barbara     | 57        | 5-32  | 16.58 | 58 | 6 | 55.1 | 91.4 | 61.1 | 48.4 | 59.8 | 60.5 | 15 | 4  | 3 |
| 4  | Peppers     | 53        | 4-19  | 10.9  | 64 | - | 48.7 | 104.1 | 59.4 | 45.1 | 63.6 | 58.8 | 7  | 4  | 3 |
| 5  | Sailboat    | 57        | 3-26  | 15.17 | 61 | 3 | 50.9 | 97.1 | 55.6 | 44.9 | 59.9 | 56.4 | 9  | 4  | 3 |
| 6  | Lena        | 57        | 2-23  | 9.99  | 62 | 2 | 46.6 | 107.1 | 59.2 | 45.5 | 65.4 | 58.9 | 7  | 4  | 3 |
| 7  | Baboon      | 55        | 7-30  | 17.66 | 57 | 7 | 56.2 | 102.3 | 54.5 | 46.4 | 61.4 | 55.3 | 18 | 4  | 3 |
| 8  | Pink Flowers| 55        | 12-29 | 22.45 | 60 | 4 | 62.3 | 93.2 | 63.2 | 51.5 | 60.7 | 63.1 | 10 | 5  | 3 |
| 9  | Flower      | 59        | 0-20  | 9.4   | 64 | - | 45.6 | 89.5 | 58.6 | 44.6 | 59.2 | 58.3 | 7  | 4  | 3 |
| 10 | Airplane    | 68        | 1-28  | 11.64 | 61 | 3 | 46.4 | 69.8 | 59.1 | 46.1 | 52.9 | 59.1 | 8  | 4  | 3 |

| Mean value | 58.0 | 92.2 | 58.6 | 48.5 | 59.5 | 58.6 | 9.6 | 4.2 | 3 |

STDA = Average standard deviation of image sub-block
T = Transformation method used by system
D = Amount of sub-block transformed using DWT
S = Amount of sub-block transformed using SVD
PA = Proposed Scheme with Arithmetic Coding
PH = Proposed Scheme with Huffman Coding
J = JPEG2000

Based on conducted experiments, we have some conclusions as follow. The Luminance PSNR of reconstructed image quality of image compression scheme using proposed scheme is 48.5 dB (by Arithmetic coding) and 59.5 dB (by Huffman coding), while using JPEG2000 is 58.64 dB. The
The proposed scheme has higher PSNR than JPEG2000 because error generated by SVD for transforming low-correlated sub-block is smaller than the error generated by DWT to transform the sub-block.

### Table 6 Performance of Image Reconstructed

| Source | Recontructed with Arithmetic Coding | Recontructed with Huffman Coding | Recontructed with JPEG2000 |
|--------|------------------------------------|----------------------------------|-----------------------------|
|        | PSNR : 48.4                        | PSNR :59.8                       | PSNR :60.5                  |

### 4. Conclusion

A graph coloring technique in the quantization process on image compression scheme based on Wavelet-SVD (Singular Value Decomposition) has a good performance. Based on compression ratio and PSNR, the system provides good performance compared with JPEG2000. The average compression ratio generated by the system stands between 50-60%, while the average PSNR stands between 40-80 dB. The proposed scheme has an advantage over the image with high average standard deviation value or image that have low degree of similarity between its sub-block because it produces a smaller error rate on dequantization process thus providing a higher PSNR value. Higher level of decomposition will yield a higher value of PSNR on reconstructed image to a certain threshold where the trade off with the decreasing compression ratio still felt balanced.

### 5. References

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