1. Introduction
Saddle-shaped cable roofs with a compliant supporting contour are rational type of structures from the point of view of materials consumption (Serdjuks, Rocens 2003). But at the same time, saddle-shaped cable roofs possess an increased compliance and vertical displacements, which complicate using this type of structures.

Basing on the results obtained in (Pakrastinsh et al. 2001) and the above-mentioned information, we can suppose that the best method to decrease the displacements of the saddle-shaped roof is to use the cable trusses made of the materials with increased moduli of elasticity as structures of several groups of the cables. Effectiveness of cable trusses used as tension cables construction was considered in (Serdjuks, Rocens 2004).

High-strength materials, such as FRCC and FRP possess potential for their application as structural materials in combination with steel (Serdjuks, Rocens 2003). Carbon fibre reinforced plastic (CFRP), glass fibre reinforced plastics (GFRP) and Vectran are examples of such materials. As structural materials, they have following advantages:
- high specific strength;
- good durability in aggressive surroundings;
- CFRP is adaptable to be used in structures not allowed to be magnetic or electric conductive;
- low density.

However, CFRP, GFRP and Vectran have a number of disadvantages, which limit their application as structural materials. Relatively small elongation at break (Serdjuks, Rocens 2003), probability of surface damages and increased cost (Pakrastinsh et al. 2001) are most significant disadvantages of CFRP, GFRP and Vectran in comparison with steel cables.

Small elongation at break significantly decreases safety of construction due to probability of brittle failure during a short-time growing of the load. This disadvantage could be improved by adding some steel component, which enables to increase the cable reliability. The hybrid composite tensioned elements can be created on the base of joined together separate bands (Fig. 1a) or strands (Fig. 1b).

An addition of distribution layer, which could be made of GFRP, significantly decreases the possibility of surface damages of CFRP in hybrid composite cable. Yet the volume fractions of the components should be evaluated.

Thus, the purpose of the study is evaluation of rational volume fractions for hybrid composite cable components. Effectiveness of cable truss application for the main diagonal cable structure, as a method to decrease the dis-
placements of the saddle-shaped cable roof, also should be evaluated. Rational geometrical characteristics of the cable truss shall be estimated.

The behaviour of hybrid composite cable should be investigated analytically and experimentally.

2. Choice of rational components for hybrid composite cable with an increased specific strength

2.1. Material combinations for hybrid composite cable

The main directions of the considered hybrid composite cables application are prestressed nets of saddle-shaped roofs. Two types of hybrid composite cables with the increased specific strength should be investigated. First of them is a hybrid composite cable with an increased, in comparison with the CFRP ultimate elongation, and decreased, in comparison with that of steel dead weight. These types of cables could be used for the tension and suspension cables of the prestressed net. The second one is a hybrid composite cable with a relatively high ultimate elongation of stressing cables of the prestressed nets. Combination of a high strength and increased ultimate elongation is the main requirement for the first hybrid composite cable type. But the second type, unlike the first one, should possess, first of all, an increased ultimate elongation.

Thus, the first cable type should obligatorily contain two types of materials: one material with a large limit of strength and the other with an increased ultimate elongation. The third type of materials should be added to the transfer perpendicular to the direction of axial force action pressure of the external layer at the surface of the internal one.

Steel wire strands can be treated as a material with an increased up to 10% ultimate elongation for the first type of the cable. Properties of GFRP (E-glass and epoxy matrix at 60% fibre content), CFRP (AS4/3501-6 graphite fibre and epoxy matrix at a 60% fibre content), Vectran HS 1500 and strands of steel wire are taken in accordance with the sources (Beers, Ramirez 1990; Bengtson 1994; Berger 2005; Blum 2000; Costello 1997; Houtman 2003; Kumar, Cochran 1997). Moduli of elasticity of steel wire strands, GFRP and Vectran are equal to $2 \times 10^5$, $0.75 \times 10^5$, $1.37 \times 10^5$ and $0.65 \times 10^5$ MPa, respectively. Limits of strength are equal to 1900, 760, 2100 and 2850 MPa, respectively (Beers, Ramirez 1990; Bengtson 1994; Berger 2005; Blum 2000; Costello 1997; Houtman 2003). Ultimate elongations are equal to 10, 2.64, 1.6 and 3.3%, respectively.

Basing on the above-mentioned materials properties, two following materials combinations can be considered for the first type of hybrid composite cable: steel, GFRP, CFRP and steel, Vectran, CFRP (Fig. 2).
Vectran and GFRP, probably, enables to obtain hybrid composite cables with such properties.

2.2. Prediction of behaviour of hybrid composite cable

The behaviour of hybrid composite cables is determined by the dependence of proportional components summing on the base of behaviour of separate components. A generally known dependence of proportional components summing was used for the engineering evaluation of elasticity modulus of the hybrid composite cable.

The axial tension force, acting in the cable, was evaluated as a sum of axial forces in separate components, which corresponds to certain level of strains.

The mentioned model for predicting the behaviour of hybrid cables was checked by the experiment. Three groups of specimens were tested. The specimens of the first group consist of two steel and one CFRP tapes, which were joined together by epoxy glue. The specimens of the second and third groups were single CFRP and steel tapes, respectively.

The dimensions of the tapes cross-sections were 50×1.2 mm. The specimens length was determined taking into account the width of the tape and base of gauge in accordance with the ASTM D3039/D3039M-95a. The length of base of the strain-measuring device was 50 mm. The specimens were loaded until the failure by the testing machine with capacity of 300 kN.

The explosive type of failure was stated for all CFRP specimens, which occurs in the middle of the specimen (Fig. 3a). The mode of failure of hybrid composite specimens is shown in Fig. 3b).

The results of the test show, that the failure of carbon fibres in hybrid composite specimen occurs in the several sections. This is a reason, why the carbon fibres are visible in zone of failure of hybrid composite specimen (Fig. 3b).

The type of failure of hybrid composite specimens was characterised by the types of failure of separate components. The failure of middle CFRP layer occurs when the strains exceed 1.8%, what corresponds to the axial force of 204 kN. Then the axial force fall down until 36.6 kN and yielding of steel component occurs, until the strains exceed by 43%, and the steel component disrupt, too.

The mean curves for all 3 groups of specimens together with the analytically obtained curve are shown in Fig. 4.

The comparison of results, which were obtained by the experiment and analytically, indicates that the max difference does not exceed the experiment precision. Thus, the model for prediction of behaviours of hybrid composite cables basing on the behaviour of the separate component can be used for coarse evaluations.
2.3. Evaluation of mechanical properties of hybrid composite cables

The behaviour of hybrid composite cable were evaluated basing on the behaviour of separate component.

Generally known dependence of proportional components summing was used for the engineering evaluation of modulus of elasticity of the hybrid composite cable. The dependences are given for the variant of the hybrid composite cable on the base of steel, GFRP and CFRP.

\[ E = \Omega_c E_c + (1 - \Omega_c - \Omega_S) E_G + \Omega_S E_S, \]  
(1)

where

\[ \Omega_c = \frac{A_c}{A}, \quad \Omega_S = \frac{A_S}{A}, \]  
(2)

\[ E_S = \frac{\sum_{i=1}^{n} m_i A_i E_i \sin(90 - \alpha_i) \left[ 1 - (1 + \nu) p_i \cos^2 \alpha_i \right]}{A_S}, \]  
(3)

\[ A_i = \pi r_i^2, \]  
(4)

\[ p_i = \left( 1 - \frac{R}{r_i} \cos^2 (90 - \alpha_i) \right) \times \left[ 1 - \frac{R^2}{4r_i^2} \frac{1 - \frac{1}{1 + \nu} \cos^2 (90 - \alpha_i)}{\cos^2 (90 - \alpha_i)} \right], \]  
(5)

where \( E \) – modulus of elasticity of hybrid composite cable; \( A \) – cross-sectional area of hybrid composite cable; \( A_c, A_S \) – cross-sectional areas of CFRP and steel components, respectively; \( E_c, E_G, E_S \) – moduli of elasticity of CFRP, GFRP and steel components, respectively; \( m_i \) – amount of steel wires in the \( i \)th strand; \( A_i \) – cross-sectional area of the separate steel wire in the \( i \)th strand; \( r_i \) – radius of separate steel wire; \( E_i \) – modulus of elasticity of steel wire; \( \alpha_i \) – angle of \( i \)th steel wire strands twisting; \( \nu \) – Poisson’s ratio of steel wire; \( R \) – radius of \( i \)th steel wire strand; \( r_i \) – distance between the centres of \( i \)th steel wire strand and cable.

Modulus of elasticity of the steel component of the hybrid composite cable was evaluated by the method of Kumar and Cochran (1997).

Specific strength and load-bearing capacity were obtained for 3 variants of composite cables with the area of cross-sections equal to 0.001 m². Empty space was not taken into account. Angle of steel wire strands twisting was equal to 12 degrees for all the variants of hybrid composite cables. Specific strength of the cables was determined as a relation of ultimate strength of the cable and its density.

\[ Y = \frac{R}{p}, \]  
(6)

where \( Y \) – specific strength of the cable; \( R \) – ultimate strength of the cable; \( p \) – density of the cable.

Ultimate strength of the cable was found as a relation of max axial force, which can be taken by the cable and area of it cross-section. Volume fraction of steel changes was within the limits of 0.1 to 0.7.

Increase of volume fraction of steel from 0.1 to 0.7 causes growing of moduli of elasticity from \( 1.25 \times 10^5 \) to \( 1.57 \times 10^5 \) MPa for the cable on the base of steel, GFRP and CFRP. For the cables on the base of steel, GFRP and Vectran and steel, Vectran and CFRP the increase of volume fraction of steel from 0.1 to 0.7 causes growing of moduli of elasticity from \( 0.94 \times 10^5 \) to \( 1.53 \times 10^5 \) and \( 1.27 \times 10^5 \) to \( 1.6 \times 10^5 \) MPa, correspondingly.

Three variants of the hybrid composite cables next will be mentioned as the first, second and third variant, correspondingly.

The values of max axial forces, which can be taken up by the cables, are given in Table 1 for 3 variants of hybrid composite cables.

| Variant of hybrid composite cable | Volume fraction of steel |
|---|---|
| 0.1 | 0.4 | 0.7 |
| 1. Variant (steel; GFRP; CFRP) | 1 191 | 1 343 | 1 496 |
| 2. Variant (steel; Vectran; CFRP) | 1 212 | 1 365 | 1 518 |
| 3. Variant (steel; GFRP; Vectran) | 892 | 1 173 | 1 454 |

It is shown, that increase of volume fraction of steel from 0.1 to 0.7 enables to increase by 20.4%, 20% and 1.63 times max axial forces, which can be taken up by the hybrid composite cables for first, second and third variant, respectively.

The dependences of specific strengths of the cables on the volume fractions of steel are given in Fig. 6.

The dependence illustrates, that the increase of volume fraction of steel from 0.1 to 0.7 causes the decrease of specific strengths of the cables by 2.40, 2.17 and 1.75 times for the first, second and third variants of the cables, respectively.
2.4. Evaluation of rational volume fractions of steel

Thus increase of volume fraction of steel causes the increase of load bearing capacity of the cable and the decrease of its specific strength from other side. Yet there is a min volume fraction of steel, which enables to prevent failure of single cable or cable net in case of emergency, when all the components of hybrid composite cable, excluding the steel, are disrupted. The min volume fraction of steel was considered as a rational.

Diagonal suspension cable of saddle-shaped cable roof with dimension 50×50 m was considered as an object for evaluating rational volume fraction of steel for 3 variants of hybrid composite cables. Design scheme of diagonal suspension cable was a prestressed simple cable with the supports at one level, which is loaded by the uniformly distributed load.

Initial deflection and span of the cable were equal to 20 and 70.71 m, accordingly. Intensity of uniformly distributed load was equal to 1.97 kN/m. The value of prestressing is assumed as a 20% of the tension force, which acts in the cable due to the vertical design load. Three above-mentioned variants of hybrid composite cable were considered. Volume fraction of steel changes within the limits of 0.1 to 0.7 from the initial area of cross-section. The dependences of stresses, acting in the steel component of hybrid composite cables after other components disruption on the volume fraction of steel, are given in Fig. 7.

The dependence illustrates, that the min volume fraction of steel, which prevents failure of the cable in the case, when other components are disrupted, is equal to 0.23, 0.28 and 0.29 for the first, second and third variants of hybrid composite cable, respectively. So, the hybrid composite cables with a rational volume fraction of steel possess the following mechanical properties.

- Moduli of elasticity are equal to 1.32 × 10⁵, 1.35 × 10⁵ and 1.36 × 10⁵ MPa, for the first, second and third variant of the hybrid composite cable, respectively.
- Max axial forces, which can be taken up by the cables, are equal to 1242, 1313 and 1294 kN, for the first, second and third variant of the hybrid composite cable, respectively.
- Specific strengths are equal to 43.00 × 10³, 41.00 × 10³ and 40.50 × 10³ m, for the first, second and third variant of the hybrid composite cable, respectively.

However, a single cable cannot characterise behaviour of the cable roof in the full scale. So, hybrid composite cable on the basis of steel, GFRP and CFRP was considered as a material of tension and diagonal suspension cables of saddle-shaped cable roof with dimensions 50×50 m. The behaviour of the cable roof was evaluated for the diagonal suspension and tension cables in cases, when all the components of hybrid composite cables, excluding the steel, were disrupted. Parameters of cable roof and methodology of numerical experiment are explained in Chapter 3.

It was stated, that the max vertical displacements of the cable roof grows by 3 mm in case of GFRP and CFRP components disruption of diagonal suspension cable. Max growing of the stresses from 899 to 1110 MPa took place in the suspension cables, which are neighbouring the diagonal suspension cables.

The max vertical displacements of the cable roof grow by 1.37 m in case of GFRP and CFRP components disruption of tension cables. Max growing of stresses from 897 to 1050 MPa took place in the diagonal suspension cable. Still the growing stresses and max vertical displacements did not cause failure of any more cables.
3. Using composite cable for increasing the saddle-shaped cable roof rigidity

Let us consider how the usage of hybrid composite cable in combination with the cable truss enables to increase the rigidity of saddle-shaped cable roof.

Such a roof of 50×50 m in plan was investigated. The existence of two symmetry planes allows us to regard, as a design scheme, a quarter of the cable net of a roof with a main stressing cable as the shape of the cable truss, subjected to the prestressing and vertical design load (Fig. 8).

Fig. 8. Design scheme of cable roof: 1 – cable truss; 2 – cable net; $f_{n1}$ – initial deflection of cable truss top chord; $f_{s}$ – initial deflection of main stressing cable; $a_{1}$ – distance between the support points of tee-bars

Three quarters of the cable roof are replaced by the bonds imposed on its one-quarter part. Hybrid composite cables on the basis of steel, GFRP and CFRP with an elastic modulus of 1.32 × 10^5 MPa were assumed as a material of cable truss elements. Steel cables with an elastic modulus of 1.3 × 10^5 MPa were assumed as a material for the suspension, stressing (excluding main diagonal) and tension cables.

From the viewpoint of material consumption, the saddle-shaped cable roof has rational geometrical characteristics: the initial deflection of the contour cables was 8.6 m, the initial deflection of suspension and stressing cables 20 m, and the step in plan of the latter ones was 8.6 m, the initial deflection of suspension cables fixation were considered:

- the pointwise forces applied to the nodes of these cables, were divided by two. The pointwise force applied to the intersection node of the main diagonal cables was divided by four.

The relations between the initial deflection of the cable truss top chord, the distance between the support points of tie-bars and effectiveness of cable net materials used for max vertical displacements decrease were determined in the form of second power polynomial functions applying the method of experimental design (Спиридонов 1981). The effectiveness of cable net materials used for max vertical displacements decrease was determined by the formula:

\[
\vartheta = \frac{\Delta \delta}{V/A},
\]

where $\Delta \delta$ – max vertical displacements decrease; $V/A$ – the volume of the material of the cable net per unit of the covered area (relative volume).

The coefficients of the 2nd power polynomial functions were found from the results of a numerical experiment, which was joined with determining forces in the net cables, necessary to select the cable cross-section, calculate the relative volume of the material of the cable net, and max vertical displacements of the cable net. The numerical experiment was conducted with the values of initial deflection of cable truss top chord, changing from 4 to 16 m and values of the distance between the nodes of cable truss changing from 2.8 to 8.4 m.

The cross-sectional areas of the cables were found according to the recommendations in (Трущев 1983), from the formula:

\[
F \geq \frac{1.6N}{kR},
\]

where $F$ – the cross-sectional area of the cable; $N$ – the design force in the cable; $k$ – a coefficient, taking into account the drop in the breaking force of the cable caused by the inhomogeneity of stress distribution; $R$ – the ultimate strength of the cable material, and 1.6 is the reliability index of the material.

The area, covered by the roof, was found with regard to the initial deflections of tension cables.

Using a computer program “ANSYS/ED 5.3” for WINDOWS the numerical experiment was carried out.
The program enables to calculate values of the tension forces acting in the cables of the net and max vertical displacements of the cable net. In calculating a cable net, the program uses the iteration method, which consists of the division of the applied vertical design load into several parts in an ascending order. The cable net was modelled by finite elements of LINK10 type, with three degrees of freedom for each node. Each finite element was divided into two parts of the same length. The judicious values of the basic geometrical characteristics of the cable truss were found from the system of Eqs (9). The 1st and 2nd Eq of the system was obtained by taking of partial derivations from the 2nd power polinomial functions, by the initial deflection of cable truss top chord and distance between support points of tie-bars, respectively.

\[
\begin{align*}
\frac{\partial \mathcal{E}}{\partial a_{f_1}} &= \theta_1 + \theta_1 a_{f_1} + \theta_1 f_{f_1} = 0, \\
\frac{\partial \mathcal{E}}{\partial f_{f_1}} &= \psi_1 + \psi_1 a_{f_1} + \psi_1 f_{f_1} = 0, \\
\end{align*}
\]

where \( \mathcal{E} \) – effectiveness of cable net materials using for max vertical displacements decrease; \( f_{f_1} \) – initial deflection of cable truss top chord; \( a_1 \) – distance between the support points of tie-bars.

Coefficients of the Eq (9) are given in Table 2.

| Coefficients | *Values of coefficients |
|--------------|-------------------------|
| \( \theta_0 \) | 0.0090/0.013 |
| \( \theta_1 \) | 0.000041/0.00023 |
| \( \theta_2 \) | -0.000066/-0.00093 |
| \( \psi_0 \) | 0.012/0.0087 |
| \( \psi_1 \) | 0.000041/0.00023 |
| \( \psi_2 \) | -0.0032/-0.0029 |

Note: *Values, given in the numerator, are obtained for the case, when the cable net displacements were restricted by the guys deformations.

The values of the initial deflection of cable truss top chord and distance between support points of tie-bars are equal to 15 and 4 m, respectively.

4. Investigation of displacements decrease

Max vertical displacements of the cable roof for all combinations of the main geometrical characteristics of the cable truss were determined as a max difference in the vertical coordinate of the cable net nodes before and after application of design vertical load. The dependence of the max vertical displacements of cable net on the initial deflection of cable truss top chord and distance between the support points of tie-bars is shown in Fig. 9.

![Fig. 9. Max vertical displacements \( \delta \) vs the initial deflection \( f_{f_1} \) of cable truss top chord and distance \( a_1 \) between the support points of tie-bars: 1 – displacements of the cable net at the support points are restricted by the guys deformations; 2 – the support points of the cable net are fixed](image)

The dependence shows, that the min values of vertical displacements of cable net were obtained, when the initial deflection of cable truss top chord was equal to 16 m and the distance between the nodes of the cable truss was equal to 2.8 m for both variants of support points fixation.

The dependence of the effectiveness of cable net materials using for max vertical displacements decrease on the initial deflection of cable truss top chord and distance between the cable truss nodes is shown in Fig. 10.

![Fig. 10. Effectiveness of the cable net materials use for max vertical displacements decrease \( \mathcal{E} \) vs the initial deflection \( f_{f_1} \) of cable truss top chord and distance \( a_1 \) between the support points of tie-bars: 1 – displacements of the cable net at the support points are restricted by the guys deformations; 2 – the support points of the cable net are fixed](image)
The dependence shows, that decrease by 31% of max vertical displacements values is connected with the growing by 24% of relative volume of cable net materials expenditure for the variant, when the cable net displacements at the support points are fixed. Max vertical displacements decrease by 38% in case, when displacements of cable net were restricted by the guys deformations. Relative volume of the cable net materials consumption grows by 27% in the case.

The max value of the cable net materials consumption was obtained, when the initial deflection of cable truss top chord was equal to 16 m and distance between the cable truss nodes was equal to 2.8 m.

5. Conclusions

Model of prediction of behaviour of hybrid composite element was suggested and checked by the experiment. It was shown, that the max difference between the results, which were obtained by the experiment and analytically, does not exceed the experiment precision.

Rational components for hybrid composite cable with the increased specific strength have been chosen. It was shown, that the min volume fraction of steel component is within the limits of 0.23–0.29 for the cables on the base of steel, CFRP, GFRP and Vectran.

The opportunity to decrease the displacements of composite saddle-shaped cable roof by using the cable trusses as the main stressing diagonal cable structure was investigated.

It has been shown by the numerical experiment, that the rational initial deflections of cable truss top chord and distance between the support points of tie-bars for the cable roof with dimensions in plan 50×50 m was equal to 15 and 4 m, respectively.

It was shown that using a cable truss as a structure of main stressing diagonal cable enables to decrease by 31–38% the max vertical displacements of the cable net and to increase by 24–27% the relative volume of the cable net materials consumption in case, when the main stressing diagonal cable is strengthened by the truss, made of hybrid composite cable, but suspension and tension cables are of steel.

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