Heavy Particle Effective Field Theories

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Abstract

Starting from a theory of heavy particles and antiparticles, the path integral formulation of an effective field theory which describes the low momentum interactions is presented. The heavy degrees of freedom are identified and explicitly integrated out from the functional integral of the full theory. Using this method, the effective Lagrangian, which may be calculated to arbitrary subleading order in an inverse mass expansion, is derived for fields of spin-0, 1/2, and 1.
1 Introduction

The application of quantum field theory to processes involving external states with a characteristic momentum scale $\Lambda$ is often complicated by the contributions from real and virtual excitations occurring at a much higher momentum of order $m \gg \Lambda$ which may preclude a straightforward perturbative treatment. The description of such phenomena with disparate scales can be greatly facilitated by the formulation of an effective field theory (EFT). In this approach, starting from a theory which describes processes at some high energy scale, heavy degrees of freedom, which are no longer dynamical at lower scales, are successively integrated out to yield an effective theory appropriate for the description of lower energy phenomena in terms of the remaining light excitations.

A familiar effective field theory occurs in the description of electroweak phenomena. In the modern $\text{SU}(2)_L \times \text{U}(1)_Y$ theory which provides a description of the electroweak interactions up to energies of order hundreds of GeV, both matter fields and the weak gauge boson fields, $W^\pm$ and $Z^0$, appear explicitly. However, there are a multitude of weak processes which occur at energies of around a GeV, far below the gauge boson masses, where such heavy particles only appear as virtual degrees of freedom. For such low-momentum phenomena, it is convenient and advantageous to integrate out the $W^\pm$ and $Z^0$ fields to give an effective field theory in which the effects of these gauge particles appear as non-renormalizable fermionic operators suppressed by a mass of order the weak scale; the dominant subleading four-fermion operators reproduce the old Fermi theory of the weak interactions.

In the following, we examine effective field theories of heavy particles which have some features that are qualitatively different from the example above. The aim of this presentation is to look at such theories in a more systematic and general context. For instance, the techniques to be illustrated here apply not just to elementary fields but also to those of composite particles as well as those of different spin.

Let us begin by considering a quantum field theory which describes a particle with a mass $m$ which is coupled strongly to a non-Abelian gauge field at low energies (below $m$) so that it is not amenable to a perturbative treatment in this kinematic region. In addition, let $\Lambda_{\text{SI}}$ characterize the typical scale of the interactions.\footnote{A concrete example is the strong interactions of heavy quarks which will be discussed later in this paper.}

When the mass $m$ of this particle is taken to be very large compared to this scale, $m \gg \Lambda_{\text{SI}}$, with $\Lambda_{\text{SI}}$ fixed, there is a kinematic range below the large mass $m$ where the heavy particle is no longer fully dynamical — for example, processes where heavy particle and antiparticle pair production occur are highly suppressed — and particle and antiparticle number are separately conserved. Furthermore, when the magnitude of the typical momen-
tum exchanged in such interactions and those involving other light degrees of freedom in the
system is of order $\Lambda_{SI}$, the heavy particle propagates at some velocity $v$ (with $v^2 = 1$), which
is unaltered by these strong interactions, and its momentum can be expressed in the form

$$p = mv + k,$$

where $k$ is a “residual momentum” with magnitude of order $\Lambda_{SI} \ll m$. Interactions
conserve the velocity and will only perturb $p$ by an amount of order $\Lambda_{SI}$. Typical momentum
transfers are also of order $\Lambda_{SI}$.

In this situation, it is convenient to integrate out the high-momentum excitations so
that in the resulting effective field theory its effects are reproduced by higher-dimensional
operators, which are accompanied by powers of $\Lambda_{SI}/m$, involving the remaining degrees of
freedom. This theory would be valid for a single heavy particle in the kinematic region where
low energy phenomena with momenta of order $\Lambda_{SI}$ take place. However, in this case, there is
an important distinction from the above example of the electroweak theory: here, the heavy
particle is real, and the heavy degrees of freedom which are removed correspond not to the
entire heavy field but to the components of this field which decouple from the physical states
in the infinite mass limit. Hence, external states in the EFT may still involve the heavy
particle unlike the above case where they would only appear as virtual excitations.

It is such an EFT, where the heavy particle may still be present as an external field, that
we would like to construct starting from a full theory in a functional integral framework for
particles of different spin. First, the heavy excitations will be identified and then explicitly
integrated out from the path integral: since the particles and antiparticles decouple in the
limit $m \to \infty$ independent of their spin, the heavy degrees of freedom correspond to either the
particle or antiparticle components of the heavy field. In the subsequent analyses presented
in this paper, the antiparticle component will be removed to give an EFT of heavy particles,
but the procedure to obtain a heavy antiparticle EFT is virtually identical as we shall see.

The result is an effective action which is usually non-local in the light fields that are left.
Then expanding out this non-local action in an operator product expansion yields an infinite
series of local operators of increasingly higher dimension suppressed by powers of the large
scale. This procedure disentangles the low-energy physics, which may be non-perturbative,
and is given by the structure of these operators from the effects of physics at high-energy
which resides in the coefficients of these operators. In doing so, one also extracts the full
dependence of physical observables on the heavy mass. In the following sections of this paper,
such an approach will be used to formulate effective field theories for heavy particles of spin
0, 1/2, and 1. Although it may be intuitively evident that such effective theories should
exist, it is nevertheless enlightening and reassuring to carry out this program explicitly.
2 An Effective Field Theory of Heavy Scalar Particles

Consider a non-Hermitian scalar field \( \phi \) coupled strongly to a non-Abelian gauge field \( A^\mu \) described by the Lagrangian

\[
L_s = (D^\mu \phi)^\dagger D^\mu \phi - m_s^2 \phi^\dagger \phi + J^\dagger \phi + \phi^\dagger J, \tag{2}
\]

where

\[
D^\mu \phi = (\partial^\mu - igA^\mu)\phi, \tag{3}
\]

\( J^\dagger \) and \( J \) are external sources for \( \phi \) and \( \phi^\dagger \), respectively, and \( m_s \) is the mass. Eq. (2) is the most general renormalizable Lagrangian in the absence of internal degrees of freedom (which may be incorporated straightforwardly) except for scalar self-interaction terms. Such terms are excluded because the object is to construct an EFT in the one-heavy-particle sector. As in the above discussion, let \( \Lambda_{SI} \) characterize the scale of the interactions, with \( \Lambda_{SI} \ll m_s \). A concrete example of such a theory is one which is described by a chiral effective Lagrangian of a heavy pseudoscalar meson such as a \( B \)-meson interacting with a pseudo-Goldstone bosons where the interaction scale is \( \Lambda_\chi \ll m_B \). However, to illustrate the methodology we shall continue to use the theory with the Lagrangian in eq. (2).

Since the conjugate momentum field of \( \phi^\dagger \) is

\[
\pi = \frac{\partial L_s}{\partial \dot{\phi}^\dagger} = D^0 \phi, \tag{4}
\]

the Hamiltonian is given by

\[
H_s = \pi^\dagger \dot{\phi}^\dagger \pi - L_s = \pi^\dagger \pi + i g A_0 \phi^\dagger \pi + \pi^\dagger i g A_0 \phi - (D_j \phi)^\dagger D_j \phi + m_s^2 \phi^\dagger \phi - J^\dagger \phi - \phi^\dagger J. \tag{5}
\]

The generating functional for the Green functions of this theory can be written as a functional integral over these fields:

\[
Z[j\phi, j^\dagger \phi, j\pi, j^\dagger \pi] = N \int e^{i \int [\pi^\dagger (D_0 \phi)^\dagger - H_s + j^\dagger \phi + \phi^\dagger j_\phi + j^\pi \pi + j^\dagger \pi j_\pi] + D_\phi D\phi^\dagger D\pi D\pi^\dagger} \cdot \cdot \cdot \tag{6}
\]

where \( N \) is a normalization constant.

This procedure may be generalized to a frame moving with velocity \( v^\mu \) where the conjugate momentum field of \( \phi^\dagger \) is now

\[
\pi_v = \frac{\partial L_s}{\partial (v \cdot \partial \phi)^\dagger} = v \cdot D\phi. \tag{7}
\]
Then the generating functional becomes

$$Z[j_\phi, j_{\phi}^\dagger, j_{\pi_v}, j_{\pi_v}^\dagger] = N \int e^{i \int [\pi_v^\dagger (v \cdot D) \phi + (v \cdot D) \phi] \cdot \pi_v - H_{s} + j_{\phi}^\dagger \phi + \phi^\dagger j_{\phi} + j_{\pi_v}^\dagger \pi_v + \pi_v^\dagger j_{\pi_v}] d^4x \ D\phi^\dagger D\phi D\pi_v^\dagger D\pi_v}$$

$$= N \int e^{i \int [L_s(\phi, \pi) + j_{\phi}^\dagger \phi + j_{\phi} + j_{\pi_v}^\dagger \pi_v + \pi_v^\dagger j_{\pi_v}] d^4x \ D\phi^\dagger D\phi D\pi_v^\dagger D\pi_v}, \quad (8)$$

where

$$L_s(\phi) = \pi_v^\dagger (v \cdot D) \phi + (v \cdot D) \phi\pi_v - \pi_v^\dagger \pi_v + (D_{\mu}^\dagger \phi)(D_{\mu} \phi) - m_s^2 \phi^\dagger \phi + J^\dagger \phi + \phi^\dagger J, \quad (9)$$

and $D_{\mu}^\dagger$ is the component of the covariant derivative orthogonal to the direction of the velocity $v^\mu$:

$$D_{\mu}^\dagger = D_{\mu} - v_{\mu} (v \cdot D) \quad (10)$$

To identify the heavy degrees of freedom, first observe that in the large mass limit, the (predominantly) heavy particle field (with positive energy) is given by the projection

$$\phi^+ = \frac{1}{2} \left( 1 + \frac{i v \cdot D}{m_s} \right) \phi, \quad (11a)$$

while the (predominantly) heavy antiparticle field (with negative energy) is given by

$$\phi^- = \frac{1}{2} \left( 1 - \frac{i v \cdot D}{m_s} \right) \phi, \quad (11b)$$

so that

$$\phi = \phi^+ + \phi^- \quad (12a)$$

$$\left( \frac{i v \cdot D}{m_s} \right) \phi = \phi^+ - \phi^- \quad (12b)$$

Making the decomposition into particle and antiparticle components yields

$$Z[j_{\phi^+}, j_{\phi^+}^\dagger, j_{\phi^-}, j_{\phi^-}^\dagger] = N \int e^{i \int [L_s(\phi^+, \phi^-) + j_{\phi^+}^\dagger \phi^+ + j_{\phi^-}^\dagger \phi^- + \text{h.c.}] d^4x \ D\phi^+ D\phi^- D(\phi^+) D(\phi^-)^\dagger}, \quad (13)$$

where

$$L_s(\phi^+, \phi^-) = 2m_s i[(\phi^+)\dagger v \cdot D\phi^+ - (\phi^-)\dagger v \cdot D\phi^-] - 2m_s^2[(\phi^+)\dagger \phi^+ + (\phi^-)\dagger \phi^-]$$

$$+ [D_{\mu}^\dagger (\phi^+ + \phi^-)]^\dagger D_{\mu} (\phi^+ + \phi^-) + J^\dagger (\phi^+ + \phi^-) + (\phi^+ + \phi^-)^\dagger J, \quad (14)$$

and quantities independent of the fields have been absorbed into $N$. Note that this function $L_s$ which appears in the generating functional is different from the original Lagrangian $L_s$. The “h.c.” denotes hermitian conjugate terms.

To arrive at an EFT of heavy scalars, the antiscalar component must be integrated out. However, it is useful to first remove from the total momentum of the heavy field the large momentum piece $m_s v$ in eq. (11) by defining a new field $\phi_v$ at a velocity $v$:

$$\phi(x) = e^{-im_s v \cdot x} \phi_v(x) \quad (15a)$$
and similarly for the component fields

\[ \phi^\pm(x) = e^{-i m_s v \cdot x} \phi^\pm_0(x). \]  

(15b)

Now, derivatives acting on \( \phi_v \) only give factors of the residual momentum \( k \) and thus facilitating a systematic derivative expansion of operators in powers of \( k/m_s \sim \Lambda_{\text{QCD}}/m_s \). To arrive at an EFT of heavy antiparticles, the factor \( e^{-i m_s v \cdot x} \) would be replaced by \( e^{+i m_s v \cdot x} \) in eq. (15a–15b); for particles with spin this is also the appropriate replacement \( (v \to -v) \) together with a suitable change in the mass. Implementing these transformations in the above generating functional gives

\[ Z[j^+, j^+_\dagger, j^-, j^-_\dagger] = N \int e^{i \int \left[L^s_\varphi(\phi^+_v, \phi^-_v) + (j^+_v \phi^+ + j^+_\dagger_v \phi^-_v + h.c.)\right] d^4 x} \]  

(16)

where

\[ L^s_\varphi(\phi^+_v, \phi^-_v) = 2 m_s [(\phi^+_v)^\dagger i v \cdot D \phi^+_v - (\phi^-_v)^\dagger (2 m_s + i v \cdot D) \phi^-_v] - (\phi^+_v + \phi^-_v)^\dagger (D_\varphi^\dagger)^2 (\phi^+_v + \phi^-_v) \]  

+ \( J^\dagger e^{-i m_s v \cdot x} (\phi^+_v + \phi^-_v) + e^{i m_s v \cdot x} (\phi^+_v + \phi^-_v)^\dagger J \). \]  

(17)

Now setting the sources for the \( \phi^-_v \) and the \( \phi^+_v \) fields to zero, \( j^+_v = j^+_\dagger_v = 0 \), and performing the functional integral over these fields yields the result

\[ Z[j_{\varphi_v}, j_{\varphi_v}^\dagger, 0, 0] = N \int e^{i \int \left[L^s_\varphi(\varphi_v) + (j_{\varphi_v} \varphi_v + h.c.)\right] d^4 x} \{ \det i [2 m_s (2 m_s + i v \cdot D) + (D_\varphi)^2] \}^{-1} D\varphi_v D\varphi_v^\dagger, \]  

(18)

where one has used the simplified notation

\[ \varphi_v = \phi^+_v. \]  

(19)

In eq. (18)

\[ L^s_\varphi(\varphi_v) = \varphi_v^\dagger [2 m_s i v \cdot D - (D_\varphi^\dagger)^2] \varphi_v + J^\dagger e^{-i m_s v \cdot x} \varphi_v + \varphi_v e^{i m_s v \cdot x} J \]  

+ \( [-\varphi_v^\dagger (D_\varphi^\dagger)^2 + J^\dagger e^{-i m_s v \cdot x}] [2 m_s (2 m_s + i v \cdot D) + (D_\varphi)^2]^{-1} \) \[ [- (D_\varphi^\dagger)^2 \varphi_v + J e^{i m_s v \cdot x}], \]  

(20)

and

\[ j_{\varphi_v} = j_{\phi_v}^\dagger = e^{i m_s v \cdot x} j_{\phi_v} \]  

(21)

is the source for \( \varphi_v^\dagger \). The determinant factor in eq. (20) is a consequence of quantum effects.

Eq. (21) clearly contains non-local terms, but now one may systematically expand in powers of derivatives over the large mass to arrive at the heavy scalar effective field theory.
(HSEFT) Lagrangian

\[ \mathcal{L}_{\text{HSEFT}}^v(\varphi_v) = \varphi_v^\dagger [2 m_s i v \cdot D - (D^\perp)^2] \varphi_v + J^\dagger e^{-i m_s v \cdot x} \varphi_v + \varphi_v^\dagger e^{i m_s v \cdot x} J \\
+ \frac{1}{4 m_s^2} [\varphi_v^\dagger (D^\perp)^4 \varphi_v - \varphi_v^\dagger (D^\perp)^2 J e^{i m_s v \cdot x} - J^\dagger e^{-i m_s v \cdot x} (D^\perp)^2 \varphi_v + J^\dagger J] \\
- \frac{1}{8 m_s^4} \left\{ [-\varphi_v^\dagger (D^\perp)^2 + J^\dagger e^{-i m_s v \cdot x}] \left[i v \cdot D + \frac{(D^\perp)^2}{2 m_s} \right] \left[-(D^\perp)^2 \varphi_v + J e^{i m_s v \cdot x} \right] \right\} \\
+ \mathcal{O} \left( \frac{1}{m_s^4} \right), \tag{22} \]

with the generating functional

\[ Z_{\text{HSEFT}}[j_{\varphi_v}, j_{\varphi_v}^\dagger, 0, 0] = \mathcal{N} \int e^{i \int \mathcal{L}_{\text{HSEFT}}^v(\varphi_v) + (j_{\varphi_v}, \varphi_v + \text{h.c.})} d^4 x \\
\{ \text{det} i [2 m_s (2 m_s + i v \cdot D) + (D^\perp)^2] \}^{-1} D \varphi_v D \varphi_v^\dagger. \tag{23} \]

When the theory was expressed in terms of velocity-dependent fields above, a particular velocity \( v \) was selected which breaks the Lorentz covariance of the theory. Furthermore, since the different velocity sectors are not coupled to one another by the “velocity superselection rule” \[3\], in order to recover Lorentz covariance all possible velocities should be included so that the complete generating functional becomes

\[ Z[j_{\varphi}, j_{\varphi}^\dagger] = \mathcal{N} \int e^{i \sum_v \int \mathcal{L}_{\text{HSEFT}}^v(\varphi_v) + (j_{\varphi_v}, \varphi_v + \text{h.c.})} d^4 x \\
\prod_v \{ \text{det} i [2 m_s (2 m_s + i v \cdot D) + (D^\perp)^2] \}^{-1} D \varphi_v D \varphi_v^\dagger. \tag{24} \]

In this HSEFT as it has been formulated here, \( \varphi_v \) only acts on scalars and not on antiscalars. For a theory with antiscalars, they would have to be included separately through the transformation indicated above. Moreover, additional flavours of heavy scalars (each with mass \( m_{s_i} \gg \Lambda_{\text{ST}} \)) are readily incorporated into the above formalism: the complete generating functional is then the product of the generating functionals for each species \( i \) and the corresponding effective Lagrangian is the sum of the individual ones. Hence if the fields were scaled as

\[ \varphi_v = \frac{\varphi_v'}{\sqrt{2 m_s}}, \tag{25} \]

then for \( N_s \) heavy scalar flavours the leading order effective Lagrangian in eq. (22) would have a SU\((N_s)\) symmetry. Operator insertions can also be readily accommodated by adding to the full theory Lagrangian, eq. (2), a term with the operator coupled to a source.

The calculation performed here yields the tree-level effective Lagrangian, eq. (22). This quantity can also be derived by using the classical equation of motion for \( \phi_v^- \) from eq. (17), namely

\[ \phi_v^- = - \left[ 2 m_s (2 m_s + i v \cdot D) + (D^\perp)^2 \right]^{-1} \left[ (D^\perp)^2 \phi_v^+ - e^{i m_s v \cdot x} J \right], \tag{26} \]
to express $\phi_v^-$ in terms of $\phi_v^+$ in that Lagrangian. However, these two approaches will differ when quantum effects are included and herein lies an advantage of the functional integral approach where such contributions can be incorporated methodically. In particular, the relation between $\phi_v^-$ and $\phi_v^+$ in eq. (26) will be altered by such effects. The equation of motion method also fails to generate the determinantal factor in eq. (23). To calculate physical quantities when radiative corrections are taken into account, it is necessary to choose a suitable regularization and renormalization scheme. However, since the choice of such schemes is the same for the theories considered in this paper, a discussion of this subject will be postponed until the following section where we examine heavy spin-$\frac{1}{2}$ particles because they occur in some theories of considerable interest and thus affords us the opportunity to treat them in a physically realized setting.

Finally, since any observable may be expressed in terms of a Green function which are, in turn, all generated by the action functional, eq. (23) or (24), these equations along with a regularization and renormalization scheme provide a complete framework for performing calculations in this theory.

3 An Effective Field Theory of Heavy Spin-$\frac{1}{2}$ Fermions

There are a number of examples in nature of spin-$\frac{1}{2}$ fermions whose masses are large compared with their characteristic interaction energies: for instance, heavy $b$ and $c$ quarks in QCD, and the chiral interactions of heavy spin-$\frac{1}{2}$ baryons with pseudo-Goldstone bosons amongst others. In this paper, we will apply the above method to the low-momentum interactions of heavy quarks in QCD [4]; applications of this methodology to the chiral interactions of heavy baryons will be presented in a subsequent publication.

The strong interactions of a given flavour of heavy quark, having a mass $m_Q \gg \Lambda_{\text{QCD}}$, with coloured gluons $A^\mu$ and coupled to an external source $\zeta$ is described by the Lagrangian

$$\mathcal{L}_{\text{HQ}} = \bar{\psi}(i\gamma - m_Q)\psi + \bar{\zeta}\psi + \bar{\psi}\zeta.$$  

(27)

$\psi$ is the heavy quark field in QCD and $D^\mu$ is the gauge-covariant derivative:

$$D^\mu \psi = (\partial^\mu - igA^\mu T^a)\psi$$  

(28)

The gluon field tensor $G^{\mu\nu}_a$ is defined by

$$[D^\mu, D^\nu] = -igG^{\mu\nu}_a T^a,$$  

(29)

where $T^a$ is the colour SU(3) generator.
The action functional in a frame moving at velocity \( v^\mu \) with sources \( \eta \) and \( \bar{\eta} \) for \( \bar{\psi} \) and \( \psi \), respectively, is

\[
Z[\eta, \bar{\eta}] = N \int e^{\int_{\mathbb{R}^4} \! \left[ \rho^\dagger v \cdot \partial \psi + v \cdot \partial \psi \rho - \mathcal{H}_{\text{HQ}} + \bar{\eta} \psi + \bar{\psi} \eta \right] dt^4} \, \mathcal{D} \psi \, \mathcal{D} \bar{\psi},
\]

where \( \rho^\dagger \) is the conjugate momentum field of \( \psi \),

\[
\rho^\dagger = \frac{\partial \mathcal{L}_{\text{HQ}}}{\partial (v \cdot \partial \psi)} = \frac{i}{2} \bar{\psi} \psi,
\]

and \( \mathcal{H}_{\text{HQ}} \) is the Hamiltonian:

\[
\mathcal{H}_{\text{HQ}} = -i \bar{\psi} D^\perp \psi + m_Q \bar{\psi} \psi - \bar{\zeta} \psi - \bar{\psi} \zeta.
\]

Eq. (30) can be simplified to read

\[
Z[\eta, \bar{\eta}] = N \int e^{\int_{\mathbb{R}^4} \! \left[ L_{\text{HQ}} + \bar{\eta} \psi + \bar{\psi} \eta \right] dt^4} \, \mathcal{D} \psi \, \mathcal{D} \bar{\psi},
\]

where

\[
L_{\text{HQ}} = \bar{\psi} [i (\not{\! v} D + \not{\! D}^\perp) - m_Q] \psi + \bar{\zeta} \psi + \bar{\psi} \zeta.
\]

Although in this case the quantity appearing in the generating functional of eq. (33) is \( L_{\text{HQ}} \) which coincides with the original Lagrangian \( \mathcal{L}_{\text{HQ}} \), this is generally not true whenever there are two or more time derivatives in kinetic terms of the Lagrangian as one may see from the above analysis of the scalar field theory.

As in the usual treatment of heavy quarks in an effective field theory, the heavy quark and heavy antiquark components at a velocity \( v^\mu \), \( \psi_v^+ \) and \( \psi_v^- \), respectively, are defined by

\[
\psi_v^\pm = P_v^\pm \psi,
\]

with

\[
P_v^\pm = \frac{1 \pm \not{\! v}}{2},
\]

\[
\psi = \psi_v^+ + \psi_v^-.
\]

Then to obtain an effective theory for heavy quarks, the kinematic dependence of the fields on the heavy mass is removed by the transformation

\[
\psi_v^\pm(x) = e^{-im_Q v^\mu x} h_v^\pm(x),
\]

with

\[
h_v = h_v^+ + h_v^-.
\]

Implementing these changes gives

\[
Z[\eta, \bar{\eta}] = N \int e^{\int_{\mathbb{R}^4} \! \left[ L_{\text{HQ}}(\psi, \bar{\psi}) + \bar{\eta} \psi + \bar{\psi} \eta \right] dt^4} \delta(h_v^+ - e^{im_Q v^\mu} P_v^+ \psi) \delta(h_v^- - e^{im_Q v^\mu} P_v^- \psi) \delta(h_v^+ - e^{-im_Q v^\mu} \bar{\psi} P_v^+) \delta(h_v^- - e^{-im_Q v^\mu} \bar{\psi} P_v^-) \mathcal{D} \psi \mathcal{D} \bar{\psi} \mathcal{D} h_v^+ \mathcal{D} h_v^- \mathcal{D} \bar{\psi} \mathcal{D} \bar{\psi}. \tag{40}
\]
Defining
\[ \eta_v^\pm = e^{imQv \cdot x} P_v^\pm \eta, \quad \zeta_v^\pm = e^{imQv \cdot x} P_v^\pm \zeta, \]
for the sources for \( h_v^\pm \), and integrating over the \( \psi \) and \( \bar{\psi} \) fields gives the action functional in terms of the new \( h_v^\pm \) fields:
\[ Z[\eta_v^+, \eta_v^-, \bar{\eta}_v^+, \bar{\eta}_v^-] = N \int e^{i \int [L_{\text{HQ}}^v(h_v^+, \bar{h}_v^+) + L_{\text{source}}(h_v^+, \bar{h}_v^+, \eta_v^+, \bar{\eta}_v^+)]d^4x} \mathcal{D}h_v^+ \mathcal{D} \bar{h}_v^+ \mathcal{D} \eta_v^+ \mathcal{D} \bar{\eta}_v^+, \]
where
\[ L_{\text{HQ}}^v(h_v^+, \bar{h}_v^+) = \bar{h}_v^+ i \psi \cdot D h_v^+ - \bar{h}_v^- (i \psi \cdot D + 2m_Q) h_v^- + \bar{h}_v^+ i \psi \cdot D h_v^- + \bar{h}_v^- i \psi \cdot D h_v^+ + (\zeta_v^+ h_v^+ + \bar{\zeta}_v^- h_v^- + \text{h.c.}), \]
and
\[ L_{\text{source}} = \bar{\eta}_v^+ h_v^+ + \bar{\eta}_v^- h_v^- + \bar{h}_v^+ \eta_v^+ + \bar{h}_v^- \bar{\eta}_v^- \]
As in the scalar case, the heavy antiquark component is the heavy degree of freedom to be removed, so setting the corresponding sources to zero, \( \eta_v^- = \bar{\eta}_v^- = 0 \), and integrating over \( h_v^- \) and \( \bar{h}_v^- \) yields the generating functional
\[ Z[\eta_v^+, \bar{\eta}_v^+] = N \int e^{i \int [L_{\text{HQ}}^v(h_v^+, \bar{h}_v^+)] + L_{\text{source}}(h_v^+, \bar{h}_v^+, \eta_v^+, \bar{\eta}_v^+)]d^4x} \det(2m_Q + i \psi \cdot D) \mathcal{D}h_v^+ \mathcal{D} \bar{h}_v^+, \]
where
\[ L_{\text{HQ}}^v = \bar{h}_v^+ i \psi \cdot D h_v^+ + (\bar{h}_v^+ i \psi \cdot D + 2m_Q + i \psi \cdot D)^{-1} (i \psi \cdot D h_v^+) + (\zeta_v^+ h_v^+ + \text{h.c.}), \]
and
\[ L_{\text{source}}(h_v^+, \bar{h}_v^+, \eta_v^+, \bar{\eta}_v^+ ) = \bar{\eta}_v^+ h_v^+ + \bar{h}_v^+ \eta_v^+ \]
The determinant in eq. (45) arises from integrating out the quantum fluctuations of the \( h_v^- \) field. This quantity may be regulated so that gauge invariance is preserved, and when it is evaluated in an axial gauge with \( v \cdot A = 0 \), it turns out to be constant [3].

As before, integrating out the heavy degrees of freedom leads to a non-local effective Lagrangian, but one which has a systematic derivative expansion in powers of \( \Lambda_{\text{QCD}}/m_Q \) and where short and long distance scales are separated; this heavy quark effective field theory (HQEFT) Lagrangian is
\[ L_{\text{HQEFT}}^v = \sum_{n=0}^{\infty} L_{\text{HQEFT}}^{v(n)}, \]
where the superscript \( n \) denotes the \( n \)th order term in the \( 1/m_Q \) expansion of \( L_{\text{HQEFT}}^v \). The first several terms (with the sources set to zero) are
\[ L_{\text{HQEFT}}^{v(0)} = \bar{Q}_v i (v \cdot D) Q_v, \]
\[ \mathcal{L}^{(1)}_{\text{HQEFT}} = \frac{1}{2m_Q} \bar{Q}_v \left[ (iD)^2 + \frac{g}{2} \sigma^{\mu \nu} G_{\mu \nu} - (v \cdot D)^2 \right] Q_v, \]
\[ \mathcal{L}^{(2)}_{\text{HQEFT}} = \frac{i}{4m_Q} \bar{Q}_v \left[ iD(v \cdot D) \not\! D - (v \cdot D)^3 \right] Q_v, \]
\[ \mathcal{L}^{(3)}_{\text{HQEFT}} = \frac{1}{8m_Q^3} \bar{Q}_v \left[ \frac{1}{2} g v^\mu [D^\nu, G_{\mu \nu}] + \frac{i g}{2} \sigma^{\alpha \mu \nu} \{ D_\alpha, G_{\mu \nu} \} \right. \]
\[ \left. + \frac{i}{2} \{ D^2 - \frac{g}{2} \sigma^{\mu \nu} G_{\mu \nu}, v \cdot D \} - (v \cdot D)^3 \right] Q_v, \]
\[ \mathcal{L}^{(4)}_{\text{HQEFT}} = \frac{1}{8m_Q^3} \bar{Q}_v \left[ iD(v \cdot D)^2 \not\! D - (v \cdot D)^4 \right] Q_v. \] (49)

And the HQEFT generating functional is given by eq. (45) with \( L^{(p)}_{\text{HQEFT}} \) replaced by \( \mathcal{L}^{(p)}_{\text{HQEFT}} \).

Just as for scalar fields, Lorentz covariance of this effective theory is recovered by analogously including the various velocity sectors leading to the effective Lagrangian

\[ \mathcal{L}_{\text{HQEFT}} = \sum_v \mathcal{L}^{(v)}_{\text{HQEFT}}, \] (50)

which at leading-order has the well-known SU(2\( N_f \)) spin-flavour symmetry for \( N_f \) flavours of heavy quarks.

The path integration over the heavy excitations gives the HQEFT Lagrangian, eq. (48) and (50), which reproduces QCD at tree-level for scales below \( m_Q \). In doing so, all of the internal heavy quark loops have been integrated out from the theory. However, as we had alluded to in the previous section, to completely specify an EFT requires, in addition, the specification of a regularization and renormalization scheme. Since dimensional regularization preserves all of the physical properties of the theory except that space-time is no longer four-dimensional, it is the most suitable choice for massive particles coupled to gauge fields. Perhaps the most convenient renormalization scheme is one involving a mass-independent subtraction (such as MS or \( \overline{\text{MS}} \)) and it is the one employed here.

Since the high energy behaviour of HQEFT is different from that of QCD, when radiative contributions are included, the HQEFT Lagrangian must be corrected from its tree-level form by introducing short distance coefficients for the operators in eq. (49) which are determined by matching physical quantities calculated in the two theories. Since the matching is generally performed at the heavy mass thresholds, the values of the coefficients at lower scales are determined by solving for their evolution as governed by the renormalization group equations.

3 Although in a mass-independent subtraction scheme the heavy particles do not decouple, this does not present a problem here because below its mass \( m_Q \) the dynamical degrees of freedom of the heavy quark have been explicitly integrated out.

4 See for instance ref. [1].
4 An Effective Field Theory of Heavy Vector Particles

In nature, there are some instances where the low-momentum behaviour of heavy spin-1 particles may be best described by an effective field theory such as the chiral interactions of $D^*$ or $B^*$ vector mesons with pseudo-Goldstone bosons. For simplicity, we shall illustrate the formulation of an effective field theory for a heavy vector field $A^\mu$ with mass $m_V$ described by the Lagrangian

$$\mathcal{L}_V = -\frac{1}{2} (D_\mu A_\nu - D_\nu A_\mu)^\dagger (D^\mu A^\nu - D^\nu A^\mu) + (m_V)^2 A_\mu^\dagger A^\mu,$$

where the covariant derivative prescribes the interaction of the massive vector with the gauge field

$$D^\mu A^\nu = (\partial^\mu - igA^\mu)A^\nu,$$

with a typical interaction scale of $\Lambda_{SI}$. The equation of motion for the field is

$$D_\mu (D^\mu A^\nu - D^\nu A^\mu) + (m_V)^2 A^\nu = 0.$$

Without internal symmetries (which may be included), eq. (51) is the most general Lagrangian when self-interaction terms for the heavy field are excluded (as they are irrelevant in the one-heavy-particle sector). The procedure given below can be used to derive a heavy particle effective field theory for more complicated and physically realized cases such as the example given above.

We shall first obtain the Hamiltonian which will be needed subsequently. In a coordinate frame with velocity $v^\mu$, the conjugate momentum field to $A_\nu^\dagger$ is

$$\Pi^\nu = \frac{\partial \mathcal{L}_V}{\partial (v^\nu \partial A_\nu^\dagger)} = -v_\mu\mathcal{G}_{\mu\nu} = -v_\mu(\mathcal{G}^{\parallel\mu\nu} + \mathcal{G}^{\perp\mu\nu}),$$

where

$$\mathcal{G}^{\parallel}_{\mu\nu} = (v_\mu v \cdot D)A_\nu - (v_\nu v \cdot D)A_\mu,$$

and

$$\mathcal{G}^{\perp}_{\mu\nu} = D^\perp_\mu A_\nu - D^\perp_\nu A_\mu,$$

so the Hamiltonian may then be written as

$$\mathcal{H}_V = -\Pi^\nu\Pi_\nu - [\Pi^\nu v^\mu \mathcal{G}^{\perp}_{\mu\nu} + \Pi^\nu v \cdot (-igA)A_\nu + \text{h.c.}] - v^\mu \mathcal{G}^{\perp\dagger}_{\mu\nu} v_\alpha \mathcal{G}^{\perp\alpha\nu} + \frac{1}{2} \mathcal{G}^{\perp\dagger}_{\mu\nu} \mathcal{G}^{\perp\mu\nu} - (m_V)^2 A_\mu^\dagger A^\mu.$$

This theory is singular as one may see from the fact that in the rest frame the $A^0$ component has no conjugate momentum field, and consequently, there are constraints. The

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5 Some instances where such a theory is used in chiral interactions are ref. [7, 8].
Hamiltonian formalism, in which such constraints are taken into account, will be employed to quantize the system [1]. In this moving frame there is a primary constraint:

$$\Phi^{(1)} = v \cdot \Pi = 0$$ (58)

Secondary constraints may be derived by requiring that this primary constraint is consistent with the equations of motion:

$$v \cdot \partial \Phi^{(1)} = \{ \Phi^{(1)}, \mathcal{H}_V \} = 0$$ (59)

Commuting the primary constraint with the Hamiltonian gives

$$\Phi^{(2)} = -v \cdot D(v \cdot \Pi) + D_\mu^\dagger \Pi^\mu + (m_V)^2 v \cdot A.$$ (60)

Since massive spin-1 particles have only three physical degrees of freedom and it is represented here as a four-component vector field, eq. (60) serves to eliminate the spurious degree of freedom. There are no other secondary constraints, and $\Phi = (\Phi^{(1)}, \Phi^{(2)})$ is the full system of constraints. The matrix consisting of the Poisson bracket of all constraints, namely

$$\{ \Phi(x), \Phi(y) \} = \begin{pmatrix} 0 & -(m_V)^2 \\ -(m_V)^2 & 0 \end{pmatrix} \delta(x - y),$$ (61)

is nonsingular here, so this is a theory with second-class constraints.

The generating functional for the heavy vector field $A^\mu$ and the conjugate momentum field $\Pi^\nu$ with the corresponding sources $j_A^\mu$ and $j_\Pi^\mu$ which implements these constraints is

$$Z[j_A^\mu, j_\Pi^\mu, j_{A}^{\dagger}, j_{\Pi}^{\dagger}] = N \int \exp \left[ i \int [L_V(A^\mu, \Pi^\nu; \text{h.c.}) + (j_A^\mu A^\mu + j_\Pi^\mu \Pi^\mu; \text{h.c.})] d^4x \right]$$

$$\det \frac{1}{2} \{ \Phi, \Phi \} \delta(v \cdot \Pi) \delta(v \cdot \Pi^\dagger)$$

$$\delta(-v \cdot D(v \cdot \Pi) + D_\mu^\dagger \Pi^\mu + (m_V)^2 v \cdot A)$$

$$\delta[(-v \cdot D(v \cdot \Pi) + D_\mu^\dagger \Pi^\mu + (m_V)^2 v \cdot A)^\dagger]$$

$$\mathcal{D}A_\mu \mathcal{D}A^{\dagger}_\mu \mathcal{D}\Pi_\mu \mathcal{D}\Pi^{\dagger}_\mu,$$ (62)

where

$$L_V(A^\nu, \Pi^\nu) = \Pi^{\dagger}(v \cdot \partial A^\nu) + (v \cdot \partial A^\nu)^\dagger \Pi^\nu - \mathcal{H}_V(A^\nu, \Pi^\nu).$$ (63)

Because the matrix $\{ \Phi, \Phi \}$ in eq. (61) is independent of the fields, the corresponding determinant factor $\det \frac{1}{2} \{ \Phi, \Phi \}$ can be factored out of the path integral and henceforth will be absorbed into the normalization.

It is convenient to change variables into quantities defined with respect to the velocity by introducing the following projectors parallel and perpendicular projectors

$$\mathcal{P}_{\mu\nu,v} = v_\mu v_\nu$$ (64a)

$$\mathcal{P}_{\mu\nu,v}^\perp = g_{\mu\nu} - v_\mu v_\nu.$$ (64b)
Hence, one defines the component of the vector field parallel to the velocity as

\[ A^\| = v \cdot A, \tag{65a} \]

and the perpendicular component to be

\[ A^\bot = P^\bot_{\mu\nu,v} A_\nu = A_\mu - v_\mu A^\|, \tag{65b} \]

with the constraint

\[ v \cdot A^\bot = 0, \tag{65c} \]

so that

\[ A_\mu = (P^\|_{\mu\nu,v} + P^\bot_{\mu\nu,v}) A_\nu = v_\mu A^\| + A^\bot_\mu. \tag{65d} \]

Similarly, the parallel and perpendicular components of the conjugate momentum field are defined to be, respectively,

\[ \Pi^\| = v \cdot \Pi, \tag{66a} \]

\[ \Pi^\bot_\mu = P^\bot_{\mu\nu,v} \Pi_\nu = \Pi_\mu - v_\mu \Pi^\|, \tag{66b} \]

with

\[ v \cdot \Pi^\bot = 0, \tag{66c} \]

\[ \Pi_\mu = (P^\|_{\mu\nu,v} + P^\bot_{\mu\nu,v}) \Pi_\nu = v_\mu \Pi^\| + \Pi^\bot_\mu. \tag{66d} \]

Making this change of variables and then integrating over the fields \( \Pi^\|, \Pi^\| \) and implementing the above delta function constraints in \( L_V \) yields

\[
Z[j^\mu^\|_A, j^\mu^\bot_A, j^\mu_\Pi, j^\mu_\Pi] = N \int e^{i \int [L_V(A^\mu_+, A^\|, \Pi^\|; \text{h.c.}) + (j^\mu_A^+) A^\mu_+ + j^\mu_\Pi \Pi^\|; \text{h.c.})] d^4x} \]
\[
\delta(v \cdot A^\bot) D A^\bot \delta(v \cdot \Pi^\bot) D \Pi^\bot \times (\text{h.c.)}), \tag{67} \]

where

\[
L_V(A^\mu_+, A^\|, \Pi^\|; \text{h.c.}) = (\Pi^\mu^\bot)^\dagger (v \cdot D A^\mu_+ - D^\mu_+ A^\|) + (v \cdot D A^\mu_- - D^\mu_- A^\|)^\dagger \Pi^\mu^\bot \]
\[
+ (\Pi^\mu^\bot)^\dagger \Pi^\mu^\bot + \frac{1}{2} (D^\mu_+ A^\| - D^\mu_- A^\|)^\dagger (D^\mu_+ A^\| - D^\mu_- A^\|) + (m_V)^2 [(A^\mu_+)^\dagger A^\mu^\bot + (A^\|)^\dagger A^\|]. \tag{68} \]

The next step is to identify the heavy degrees of freedom. In the heavy mass limit, the heavy vector \( A^\mu_+ \) and heavy antivector \( A^-_\mu \) component fields may be identified with

\[
A^\pm_\mu(x) = \frac{1}{2} \left( 1 \pm \frac{i v \cdot D}{m_V} \right) A_\mu(x), \tag{69} \]
The original field $A^\mu$ and its derivative may then be expressed in terms of these components through

$$A_\mu = A_\mu^+ + A_\mu^-,$$

$$\left(\frac{iv \cdot D}{m_V}\right)A_\mu = A_\mu^+ - A_\mu^-.$$  \hspace{1cm} (70a, 70b)

These equations can then be used to reexpress the action functional in terms of the positive and negative energy components $A_\mu^\pm$.

It is clear that to obtain an EFT which describes heavy vectors, $A^-_\mu$ should be integrated out. As before, however, it is convenient to first remove the kinematic dependence of the heavy field on $m_V$ by defining new velocity-dependent fields:

$$A_\mu(x) = e^{-im_Vv \cdot x} A_{\mu,v}(x),$$
$$A_\mu^\pm(x) = e^{-im_Vv \cdot x} A_{\mu,v}^\pm(x),$$
$$A^\parallel(x) = e^{-im_Vv \cdot x} A^\parallel_v(x).$$  \hspace{1cm} (71)

Expressing the generating functional in terms of these new quantities and integrating out $A^\parallel, A_{\mu,v}^\pm$ as well as their hermitian conjugate fields yields

$$Z[j_{A,v}^{\mu \perp}, (j_{A,v}^{\mu \perp})^\dagger] = N \int \mathcal{D}A_{\mu,v}^{\mu \perp} \mathcal{D}A_{\mu,v}^{\mu \perp} \mathcal{D}A_{\mu,v}^{\parallel} [\det B]^{-1} \delta(v \cdot A_{\mu,v}^{\perp}) \delta(v \cdot A_{\mu,v}^{\perp}) \mathcal{D}A_{\mu,v}^{\perp} \mathcal{D}(A_{\mu,v}^{\perp})^\dagger,$$  \hspace{1cm} (72)

with

$$B = iB_1 B_2 B_3,$$  \hspace{1cm} (73)
$$B_1 = (D^\perp)^2 + (m_V)^2,$$  \hspace{1cm} (74)
$$(B_2)_{\mu \nu} = [2m_V(2m_V + iv \cdot D) + (D^\perp)^2] g_{\mu \nu} - D^\perp_\nu D^\perp_\mu$$
$$- (v \cdot D - im_V) D^\perp_\mu [(D^\perp)^2 + (m_V)^2]^{-1} D^\perp_\nu (v \cdot D - im_V),$$  \hspace{1cm} (75)
$$B_3 = (B^{-1}_2)_{\mu \nu} v^{\mu} v^{\nu},$$  \hspace{1cm} (76)

and

$$L_{\text{HVEFT}}^v = (A_{\mu,v}^{\mu \perp})^\dagger \{-2m_V iv \cdot D + (D^\perp)^2\} g_{\mu \nu} - D^\perp_\nu D^\perp_\mu$$
$$- (v \cdot D - im_V) D^\perp_\mu [(D^\perp)^2 + (m_V)^2]^{-1} D^\perp_\nu (v \cdot D - im_V)\} A_{\nu,v}^{\mu \perp}$$
$$- \{(A_{\mu,v}^{\mu \perp})^\dagger (D^\perp)^2 - (A_{\alpha,v}^{\alpha \perp})^\dagger D^\perp_\mu D^\perp_\alpha$$
$$- (A_{\alpha,v}^{\alpha \perp})^\dagger (v \cdot D - im_V) D^\perp_\alpha [(D^\perp)^2 + (m_V)^2]^{-1} D^\perp_\nu (v \cdot D - im_V)\} (B_2^{-1})^\mu \nu$$
$$\{ (D^\perp)^2 A_{\nu,v}^{\mu \perp} - D^\perp_\alpha D^\perp_\nu A_{\alpha,v}^{\alpha \perp}$$
$$- (v \cdot D - im_V) D^\perp_\nu [(D^\perp)^2 + (m_V)^2]^{-1} D^\perp_\alpha (v \cdot D - im_V) A_{\alpha,v}^{\alpha \perp}\},$$  \hspace{1cm} (77)
Expanding out the non-local expressions in powers of $\Lambda_{SI}/m_V$ finally yields the heavy vector effective field theory (HVEFT) Lagrangian:

$$\mathcal{L}_{\text{HVEFT}}^v = (A_v^{\mu+})^\dagger \left\{ g_{\mu\nu} \left[ -2m_V i v \cdot D + (D^\perp)^2 \right] - D_\mu^\perp D_\nu^\perp + D_\mu^\perp D_\nu^\perp \right. + \frac{i}{m_V} \left( D_\mu^\perp D_\nu v \cdot D + v \cdot D D_\mu^\perp D_\nu^\perp \right) \right. \\
+ \frac{1}{4(m_V)^2} \left[ g_{\mu\nu} \left( -(D^\perp)^4 + \frac{(D^\perp)^2 i v \cdot D(D^\perp)^2}{2m_V} \right) + 4D_\mu^\perp(D^\perp)^2D_\nu^\perp \right. \\
- D_\mu^\perp D_\nu^\perp(D^\perp)^2 - (D^\perp)^2 D_\mu^\perp D_\nu^\perp + D_\nu^\perp D_\mu^\perp(D^\perp)^2 \\
\left. \left. + (D^\perp)^2 D_\nu^\perp D_\mu^\perp - 4(v \cdot D)D_\mu^\perp D_\nu^\perp(v \cdot D) \right] + \mathcal{O} \left( \frac{1}{(m_V)^3} \right) \right\} A_v^{\mu+}$$

Using eq. (65), $\mathcal{L}_{\text{HVEFT}}$ can be rewritten in terms of $A_v^{\mu+}$:

$$\mathcal{L}_{\text{HVEFT}}^v = (A_v^{\mu+})^\dagger \left\{ -2m_V i v \cdot D + (D^\perp)^2 - \frac{(D^\perp)^4}{4(m_V)^2} + \frac{(D^\perp)^2 i v \cdot D(D^\perp)^2}{8(m_V)^3} \right. \left( g^{\mu\nu} - \epsilon^{\mu\nu} \right) \\
+ D_\mu^\perp D_\nu^\perp - D_\nu^\perp D_\mu^\perp + \frac{i}{m_V} \left( D_\mu^\perp D_\nu^\perp v \cdot D + v \cdot D D_\mu^\perp D_\nu^\perp \right) \right. \\
+ \frac{1}{4(m_V)^2} \left[ g_{\mu\nu} \left( -(D^\perp)^4 + \frac{(D^\perp)^2 i v \cdot D(D^\perp)^2}{2m_V} \right) + 4D_\mu^\perp(D^\perp)^2D_\nu^\perp \right. \\
- D_\mu^\perp D_\nu^\perp(D^\perp)^2 - (D^\perp)^2 D_\mu^\perp D_\nu^\perp + D_\nu^\perp D_\mu^\perp(D^\perp)^2 \\
\left. \left. + (D^\perp)^2 D_\nu^\perp D_\mu^\perp - 4(v \cdot D)D_\mu^\perp D_\nu^\perp(v \cdot D) \right] + \mathcal{O} \left( \frac{1}{(m_V)^3} \right) \right\} A_v^{\mu+}$$

By scaling the field as

$$A_v^{\mu+} = \frac{A_v^{\mu+}}{\sqrt{2m_V}},$$

it can be seen from eq. (73) that a theory with $N_V$ flavours of heavy vector particles will have a SU($3N_V$) spin-flavour symmetry at leading order. And just as in the cases examined above, a Lorentz-covariant theory may be recovered by appropriately including the contributions from the different possible velocities.

The remarks made in the previous investigations regarding radiative contributions, regularization, renormalization, and matching are also appropriate here. Moreover, there is a remarkable similarity between this analysis and the previous one involving bosons namely for spin-0 particles.

5 Summary

In this paper, a functional integral method for deriving an effective field theories for heavy particles of different spin has been presented. It gives the effective Lagrangian to all orders in
an inverse heavy mass expansion. Radiative contributions can be systematically incorporated through matching and renormalization group running. These effective theories provide a convenient description of phenomena occurring below the heavy mass and in the kinematic region where all other interaction scales are much smaller. The results derived here will be utilized in the analysis of a hidden symmetry of heavy particle effective field theories [10].

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References

[1] H.D. Politzer and M.B. Wise, *Phys. Lett.* **B208**, 504 (1988).

[2] B. Grinstein, *Nucl. Phys.* **B339**, 253 (1990).

[3] H. Georgi, *Phys. Lett.* **B240**, 247 (1990).

[4] T. Mannel, W. Roberts, and Z. Ryzak, *Nucl. Phys.* **B368**, 204 (1992).

[5] N. Isgur and M.B. Wise, *Phys. Lett.* **B232**, 113 (1989); **B237**, 527 (1990).

[6] T. Applequist and J. Carazzone, *Phys. Rev.* **D11**, 2856 (1975).

[7] E. Jenkins, A.V. Manohar, and M.B. Wise, *Phys. Rev. Lett.* **75**, 2272 (1995).

[8] J. Bijnens, P. Gosdzinsky, and P. Talavera, LU TP 97/16, NORDITA-97/50 N/P, [hep-ph/9709232](https://arxiv.org/abs/hep-ph/9709232).

[9] The formalism of the quantization of constrained systems is rather involved; see for example,
P.A.M. Dirac, *Lectures on Quantum Mechanics* (Belfer Graduate School of Science, Yeshiva, New York, 1964),
P. Senjanovic, *Ann. Phys.* **100**, 227 (1976).

[10] C.L.Y. Lee, UCSD-TH-97-24.