Geometric phase of Wannier–Stark ladders in alkaline-earth(-like) atoms

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Abstract. We discuss the geometric phase of Wannier–Stark ladders generated by periodically driven clock states in alkaline-earth(-like) atoms. Using \textsuperscript{\textit{171}}Yb atoms as a concrete example, we show that clock states coupled by a set of detuned clock lasers can be mapped to a pair of two-band Wannier–Stark ladders, where dynamics of the system along each ladder is mapped to Bloch oscillations in a one-dimensional topological lattice. When the adiabatic condition is satisfied, the geometric phase accumulated in one period of the oscillation is quantized, and reveals the change of band topology as the laser parameters are tuned. We show how the difference in geometric phase between different ladders can be experimentally detected through interference between different nuclear spin states, revealing the inherent topological phase transition. Our study sheds light on the engineering of exotic band structures in Floquet dynamics.

1 Introduction

An increasingly important tool for coherent quantum control, periodic driving is not only useful for the manipulation of atoms and spins through atom-photon couplings, but has also found applications in quantum simulation across a wide range of synthetic systems such as cold atoms [1–6], trapped ions [7], solid-spin systems [8–10], photons [11–13], acoustics [14], and superconducting qubits [15,16]. Periodic driving gives rise to a wealth of interesting phenomena, such as topological charge pumping [17–23]. Floquet topological phases [24–32], and dynamic topological constructions [33–36], which have no counterparts in static systems. The synthetic degrees of freedom afforded by Floquet dressed states further offer intriguing possibilities of quantum control and quantum engineering, where these synthetic dimensions facilitate the observation of Floquet Raman transitions [9] or the design of exotic lattice models [37–39].

In this work, we show that Floquet dynamics in a periodically driven two-level system can simulate the Bloch oscillation in a one-dimensional lattice with topologically non-trivial bands. For concreteness, we use the clock states \textsuperscript{1}S\textsubscript{0} and \textsuperscript{3}P\textsubscript{0} of \textsuperscript{\textit{171}}Yb atoms as an example, where the ground \textsuperscript{1}S\textsubscript{0} and the metastable \textsuperscript{3}P\textsubscript{0} manifolds are separated by an optical wavelength of 578nm, and the electronic- and nuclear-spin degrees of freedom are decoupled [40–43]. For periodic driving, we consider a cross coupling of the clock states \( |\textsuperscript{1}S\textsubscript{0}, m_F = \pm \frac{1}{2}\rangle\) (labelled as \( |g, \pm \frac{1}{2}\rangle\)) and \( |\textsuperscript{3}P\textsubscript{0}, m_F = \mp \frac{1}{2}\rangle\) (labelled as \( |e, \mp \frac{1}{2}\rangle\)) with circularly polarized lasers, which divides the four clock states into two decoupled groups of two-level systems, as illustrated in Fig. 1. Under the rotating frame of the clock transition, the two-level system in each group is dictated by a periodically driven Hamiltonian, with the driving frequency \( \omega \) given by the detuning between the coupling lasers. We map the resulting Floquet dynamics for each two-level system to a two-band Wannier–Stark ladder [44], which can be described as a tilted one-dimensional lattice in the synthetic Floquet dimension. Remarkably, these lattices possess topologically non-trivial bands, such that the Floquet dynamics can be understood as Bloch oscillations along topological lattices. Under the adiabatic condition, which corresponds to a small laser detuning \( \omega \), the non-trivial band topology is reflected in the quantized geometric phase the system acquires after one period of Bloch oscillation, which equals the Zak phase of the corresponding band [45,46]. In particular, as the driving parameters are tuned, the geometric phase undergoes an abrupt change, indicating a transition in the band topology. Away from the adiabatic regime with large laser detunings, the geometric phase is no longer quantized, with its behavior well-captured by time-dependent perturbations. We propose to detect the difference in geometric phase between the two sets of ladders, through interference measurements between the two groups of clock states, which reveals the location of the underlying topological phase transition. Our work explicitly demonstrates the potential of simulating exotic band structures and interesting topological phe-

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nomina in the synthetic dimensions available to Floquet dynamics.

The paper is organized as follows. In Sect. 2, we discuss the system setup and model Hamiltonian. We show how the Floquet dynamics is mapped to Wannier–Stark ladders and further to Bloch oscillations along topological lattices in Sect. 3. In Sect. 4, we perform numerical calculations of the Zak-phase difference in both the adiabatic and non-adiabatic regimes. We then discuss the experimental detection of our scheme in Sect. 5, before summarizing in Sect. 6.

2 Wannier–Stark Ladder as a tilted topological lattice

We consider a cross-coupling scheme which divides the hyperfine states in the clock-state manifolds of $^{171}$Yb atoms into two groups of two-level systems. As illustrated in Fig. 1a, in the group labelled by $\alpha = \pm$, hyperfine states $|g, \pm \frac{1}{2}\rangle$ and $|e, \mp \frac{1}{2}\rangle$ are coupled by a pair of $\sigma^{\pm}$-polarized lasers. The general time-dependent Hamiltonian for each group is written as

$$H_{0}^{\alpha} = \frac{\omega_{0}}{2} \sigma_{z} + \Omega_{-\alpha} \cos(\omega_{0}t) \sigma_{x} + \Omega_{\alpha} \cos((\omega_{0} + \omega)t) \sigma_{y}, \tag{1}$$

where $\omega_{r}$ is the frequency of the clock transition, $\Omega_{-\alpha}$ and $\Omega_{\alpha}$ are the Rabi frequencies of the coupling lasers, $\omega_{0}$ and $\omega_{0} + \omega$ are the frequencies of the coupling lasers with $\omega_{0} \gg \omega$, and $\sigma_{i}$ $(i = x, y, z)$ are the Pauli operators associated with the two-level system in either group, with $\sigma_{z} = |e\rangle\langle e| - |g\rangle\langle g|$. Applying the unitary transformation $U = \exp(i \frac{\omega_{0}t}{2} \sigma_{z})$ and the rotating-wave approximation, we arrive at an effective Hamiltonian with periodically driven parameters

$$H_{\text{eff}}^{\alpha} = \frac{\Delta_{z}}{2} \sigma_{z} + \left[ \frac{\Omega_{-\alpha}}{2} + \frac{\Omega_{\alpha}}{2} \cos(\omega t) \right] \sigma_{x} \pm \frac{\Omega_{\alpha}}{2} \sin(\omega t) \sigma_{y}, \tag{2}$$

where $\Delta_{z} = \omega_{r} - \omega_{0}$. Following the standard derivation, the Floquet Hamiltonian $H_{F}^{\alpha} = H_{\text{eff}}^{\alpha} - i \frac{\partial}{\partial t}$ can be written, in the second-quantized form, as

$$H_{F}^{\alpha} = - \sum_{n} n\omega (\hat{a}_{n,\alpha}^{\dagger} \hat{a}_{n,\alpha} + \hat{b}_{n,\alpha}^{\dagger} \hat{b}_{n,\alpha}) + \frac{\Delta_{z}}{2} \sum_{n} (\hat{a}_{n,\alpha}^{\dagger} \hat{b}_{n,\alpha} + \hat{b}_{n,\alpha}^{\dagger} \hat{a}_{n,\alpha}) + \frac{\Omega_{-\alpha}}{2} \sum_{n} (\hat{a}_{n,\alpha}^{\dagger} \hat{b}_{n,\alpha} - \hat{b}_{n,\alpha}^{\dagger} \hat{a}_{n,\alpha}) + \frac{\Omega_{\alpha}}{2} \sum_{n} (\hat{a}_{n,\alpha}^{\dagger} \hat{b}_{n+1,\alpha} + \hat{b}_{n-1,\alpha}^{\dagger} \hat{a}_{n,\alpha}), \tag{3}$$

where $n$ is the Floquet-band index, and $\hat{a}_{n,\alpha}^{\dagger}$ $(\hat{b}_{n,\alpha}^{\dagger})$ creates an atom in the state $e^{-i\omega_{0}t}|g, \pm \frac{1}{2}\rangle \ (e^{-i\omega_{0}t}|e, \mp \frac{1}{2}\rangle)$ for $\alpha = \pm$, respectively. As shown in Fig. 1b, for each group of two-level system, Hamiltonian (3) is essentially a two-band Wannier–Stark ladder, with couplings indicated by blue and black arrows for states in group $\alpha = +$; and by red and black arrows for those in group $\alpha = -$.

From Eq. (3), we define $H_{L}^{\alpha} := H_{F}^{\alpha} + \sum_{n} n\omega (\hat{a}_{n,\alpha}^{\dagger} \hat{a}_{n,\alpha} + \hat{b}_{n,\alpha}^{\dagger} \hat{b}_{n,\alpha})$, which is essentially a Rice–Mele model in the synthetic Floquet dimension. The Hamiltonian (3) can thus be identified as a tilted Rice–Mele model in the same synthetic dimension, and Floquet dynamics of the system can be mapped to Bloch oscillations along the tilted lattice. In particular, for $\Delta_{z} = 0$, where $H_{F}^{\alpha}$ reduces to a tilted Su–Schrieffer–Heeger (SSH) model, geometric phases accumulated in the Floquet dynamics reflect the quantized Zak phases of the underlying topological bands when the adiabatic condition is satisfied.

3 Geometric phase in Floquet dynamics

To characterize geometric phases accumulated in the Floquet dynamics, we define creation operators in the synthetic-momentum space ($k$-space) by performing the Fourier transformation $\hat{a}_{k,\alpha}^{\dagger} = \frac{1}{\sqrt{N}} \sum_{n} e^{ink} \hat{a}_{n,\alpha}$ and $\hat{b}_{k,\alpha}^{\dagger} = \frac{1}{\sqrt{N}} \sum_{n} e^{ink} \hat{b}_{n,\alpha}$, where $k \in [0, 2\pi]$. The effective lattice Hamiltonian $H_{L}^{\alpha}$ can be written in the $k$-space as
with
\[
H_k^\alpha = \frac{1}{2} \left[ \Omega_{\alpha} - \Omega_\alpha e^{i\theta_k} e^{i\omega t} + \Omega_\alpha e^{i\omega t} - \Omega_{\alpha} e^{i\theta_k} \right].
\]

The quasienergy bands are the same for both \( \alpha = \pm \), and are given by
\[
\epsilon_{k,i}^\alpha = \pm \sqrt{\Omega_\alpha^2 + \Omega_\alpha^2 + 2\Omega_\alpha \Omega_{-\alpha} \cos k},
\]
where \( i = 1, 2 \) is the band index. For later reference, we write the creation operators for eigenstates of the quasienergy bands as
\[
c_i^\alpha = [\mu_i^\alpha]^T \left( \hat{a}_{k,\alpha} \right)^\dagger \left( \hat{b}_{k,\alpha}^\dagger \right),
\]
where
\[
\mu_{k,1}^\alpha = \left( \begin{array}{c} \cos \frac{\gamma_k^\alpha}{2} e^{i\theta_k^\alpha} \\ \sin \frac{\gamma_k^\alpha}{2} e^{i\theta_k^\alpha} \end{array} \right),
\]
\[
\mu_{k,2}^\alpha = \left( \begin{array}{c} -\sin \frac{\gamma_k^\alpha}{2} e^{i\theta_k^\alpha} \\ \cos \frac{\gamma_k^\alpha}{2} e^{i\theta_k^\alpha} \end{array} \right),
\]
with \( \tan \theta_k^\alpha = \Omega_\alpha \sin k/(\Omega_{-\alpha} + \Omega_\alpha \cos k) \), \( \tan \gamma_k^\alpha = 2|\epsilon_k^\alpha|/\Delta_z \) and \( |\epsilon_k^\alpha| = \frac{1}{2} \sqrt{\Omega_\alpha^2 + \Omega_\alpha^2 + 2\Omega_\alpha \Omega_{-\alpha} \cos k} \).

The tilting of the lattice leads to an effective driving force, which gives rise to a Bloch oscillation along the lattice. For atoms initialized in the upper or lower band, they would adiabatically follow the corresponding quasienergy band, when \( \omega \), or effectively, the driving force is small. In contrast, when \( \omega \) becomes comparable with other parameters such as \( \Omega_\alpha \) or \( \Omega_{-\alpha} \), Landau-Zener tunneling dominates, giving rise to the population of the other band. We note that while Bloch bands in the quasimomentum space are destroyed in a tilted lattice such as the Wannier–Stark ladder, understanding the dynamics on Bloch bands provides a clear physical picture even beyond the adiabatic limit. In the following, let us first focus on the adiabatic dynamics with small \( \omega \).

When the adiabatic condition is satisfied, the time-dependent single-particle field operator can be written as
\[
\hat{\Psi}_\alpha(t) = P_0 e^{i\varphi_\alpha(t)} c_{k,1}^\alpha + Q_0 e^{i\varphi_\alpha(t)} c_{k,2}^\alpha,
\]
where the dynamics is encoded in the evolution of the phase factors \( \varphi_\alpha(t) \), while the initial condition is given by \( P_0 \) and \( Q_0 \). Applying the Heisenberg equation
\[
\frac{d}{dt} \hat{\Psi}_\alpha(t) = \frac{i}{\hbar} [H_F, \hat{\Psi}_\alpha(t)],
\]
and considering an adiabatic sweep of the synthetic momentum \( k(t) = k_0 - \omega t \) (\( k_0 \) is the initial synthetic momentum), we have
\[
\varphi_\alpha(t) = \epsilon_k^\alpha(t) + i\omega (\mu_k^\alpha(t)|\partial_k \mu_k^\alpha(t), i \rangle),
\]
where \( |\mu_k^\alpha(t)\rangle := \hat{c}_{k,1}^\alpha(t)|0\rangle \).

The key message here is that for atoms initialized within the same band but within different groups of clock states (with different \( \alpha \) index), after one period of Bloch oscillation \( T = 2\pi/\omega \), they would accumulate the same dynamical phase, but different Zak phases. Here the dynamic phase and the Zak phase associated with the \( i \)th band are respectively defined as
\[
\varphi_{d,i}^\alpha = -i \int_0^T \epsilon_k^\alpha(t)|\partial_k \mu_k^\alpha(t), i \rangle dt,
\]
\[
\varphi_{z,i}^\alpha = -i \int_0^T \omega (\mu_k^\alpha(t)|\partial_k \mu_k^\alpha(t), i \rangle dt.
\]
Our results thus suggest that, by probing the relative phase between clock states in different groups, we can extract the Zak-phase difference between the corresponding topological bands of the two groups, which helps to identify the topological phase transition.

### 4 Numerical simulation

To confirm the conclusion above, we perform numerical simulations under typical experimental parameters of \(^{171}\)Yb. We initialize the system in the state \( \hat{c}_{k=0,1}^\alpha + \hat{c}_{k=0,2}^{-\dagger} |0\rangle \), which corresponds to a superposition of eigenstates at \( k = 0 \) of the lower quasienergy bands of the two groups of clock states. For our numerical simulation, we first set \( \Delta_z = 0 \), and evolve the system under the Hamiltonian (2) with \( \omega \ll \Omega_+ \), \( \Omega_- \) for one period of the Bloch oscillation \( T \).

In Fig. 2a, b, we show, respectively, the Zak phases \( \varphi_{z,2} \) and the Zak-phase difference \( \delta \varphi = \varphi_{z,2} - \varphi_{z,1} = \varphi_2 - \varphi_1 \) at the end of the time evolution. While we fix the Zak phases under the gauge given by the unitary transformation \( U = \exp(\frac{i\omega T}{2}\sigma_z) \), the Zak-phase difference \( \delta \varphi \) is gauge independent and detectable. We also adopt the convention that the Zak phases and the Zak-phase difference are within the range of \([\pi, \pi]\). As shown in Fig. 2a, the Zak phases are quantized except for a narrow region close to the topological phase transition \( \Omega_- = \Omega_+ \), where the gap between the upper and lower quasienergy bands becomes vanishingly small and the adiabatic condition breaks down. Further, under our configuration, \( \delta \varphi \) changes from \( \pi \) to \( -\pi \) as \( \Omega_- \) is tuned across the transition point. Whereas in the adiabatic limit, \( \delta \varphi \) should be the same (modular \( 2\pi \)) before and after the topological transition, non-adiabaticity makes \( \delta \varphi \) \( < \pi \) (\( \delta \varphi > -\pi \)) to the left (right) of the topological phase transition, thus allowing us to differentiate the two regimes. Under larger \( \omega \), the non-adiabatic region becomes wider, see Fig. 2d, e, which makes it
under the unitary transformation $U$ for $\omega = 0.02$ kHz. The Zak-phase difference $\delta \varphi$ and fidelity $F$ are defined in the main text. d-f Numerically calculated $\varphi_{z,2}^\pm$ (red line for $\varphi_{z,2}^-$ and green for $\varphi_{z,2}^+$), $\delta \varphi$ and fidelity $F$ for $\omega = 1$ kHz. For our calculations, we take $\Omega_z = 100$ kHz and $\Delta_z = 0$. For both cases, we initialize the atom at $z = 0$ in the lower band. Correspondingly, $\delta \varphi$ approaches the Zak-phase difference of the lower band, which shows a topological transition near $\Omega_z = \Omega_+$. While the transition is reflected as a precipitous drop of the fidelity due to the closing of the band gap, with increasing $\omega$, the adiabatic condition begins to breakdown, as is clearly seen in (e) and (f). Note the Zak phases shown in (a), (b), (d) and (e) are taken within the range $[-\pi, \pi]$ (modular $2\pi$), and are fixed by the gauge under the unitary transformation $U$. We also restrict the Zak-phase difference $\delta \varphi$ within the range $[-\pi, \pi]$ (modular $2\pi$).

harder to pin down the exact location of the transition point. However, it is easier to differentiate the two regimes due to a larger deviation of $\delta \varphi$ from $\pi$ ($-\pi$) to the left (right) of the transition.

The general picture above is also reflected in the fidelity of the final state relative to the initial state, defined as $F = |\langle \Psi_+(\omega) \Psi_0^0(t) |0 \rangle|^2$, which should approach unity under the adiabatic condition. As shown in Fig. 2c, f, the fidelity rapidly drops to zero near the location where the band topology changes, suggesting the dominance of Landau-Zener tunneling close to the transition point.

The break down of adiabatic condition can be systematically studied by plotting the Zak-phase difference and the fidelity with increasing $\omega$. As shown in Fig. 3, both $\delta \varphi$ and $F$ deviate from their adiabatic values under larger $\omega$. While the oscillatory behavior in $\delta \varphi$ is due to Zitterbewegung-type interference [47, 48], the overall decay is caused by the Landau-Zener tunneling, which can be well-explained using time-dependent perturbation. Specifically, under the time-dependent perturbation, the time-evolved state at time $T$ can be written as

$$
\begin{align*}
|\Psi_+(T)\rangle &= e^{i(\varphi_{z,2}^- + \varphi_{z,2}^+)} |\mu_{0,2}^+\rangle \\
&+ i \langle \mu_{k(t),1}^\pm | \partial_t | \mu_{k(t),2}^\pm \rangle / 2 | \mu_{0,1}^+ \rangle \\
&= \frac{1}{\sqrt{2}} e^{i(\varphi_{z,2}^- + \pi)} \left( -1 e^{2i\pi} - f(\omega, T) \left( 1 e^{2i\pi} \right) \right),
\end{align*}
$$

where $f(\omega, t) = \frac{\Omega_+ \Omega_- \cos(\omega t) + \Omega_0^2}{2(\Omega_+^2 + \Omega_-^2 \cos(\omega t) + \Omega_0^2)} e^{i\pi/2}$. As shown in Fig. 3b, Eq. (14) fits well with the overall decay profile of the fidelity.

When $\omega$ increases further, the perturbative calculation is no longer valid, while the fidelity increases again and approaches unity in the large-$\omega$ limit. This is because under the condition $\omega \gg \Omega_+, \Omega_-$, different Floquet bands are effectively decoupled from one another, such that the initial state is close to the eigenstate of the Floquet Hamiltonian $H_0^\alpha$.

In previous discussions, we show that both the Zak phases and the Zak-phase difference are quantized under small $\omega$ and a vanishing $\Delta_z$. When $\Delta_z$ becomes finite, the dynamics is mapped to the Bloch oscillation along a Rice–Mele lattice. The Zak phases of the corresponding bands are then no longer quantized, but continuously changes as $\Delta_z$ increases. This is reflected in Fig. 4: the Zak-phase difference $\delta \varphi(\Delta_z)$ continuously changes from $-\pi$ to 0 with increasing $\Delta_z$. We note that a similar behavior has been observed in the Bloch oscillation of cold atoms in a superlattice [46].
5 Detection

The Zak-phase difference $\delta \varphi$ can be detected through interference measurements between different nuclear spin states in the clock-state manifold. As we have illustrated previously, due to the opposite sign of detunings in the laser coupling of different groups of clock states ($\alpha = \pm$), Zak phases accumulated in the same period of Bloch oscillation for different groups are different, whereas dynamic phases are the same. The phase difference between states in different groups after one period of Bloch oscillation thus reveal the accumulated Zak phase. In practice, one can first initialize the atoms in an equal superposition of the clock states $\frac{1}{2}(-|g, -\frac{1}{2}\rangle - |g, \frac{1}{2}\rangle + |e, -\frac{1}{2}\rangle + |e, \frac{1}{2}\rangle)$, before switching on the cross-coupling lasers for a duration $2\pi/\omega$. A standard interference measurement can then be applied to the states $|g, \pm \frac{1}{2}\rangle$ or $|e, \pm \frac{1}{2}\rangle$ to extract the Zak-phase difference.

6 Conclusion

We show how periodic driving of the clock states in alkaline-earth(-like) atoms can simulate Bloch oscillations along topological Wannier–Stark ladders in the synthetic Floquet dimension. Making use of the nuclear spin degrees of freedom, we demonstrate how the Zak-phase difference between bands of distinct Wannier–Stark ladders can be extracted from the Floquet dynamics, which is experimentally detectable through interference measurements. While our scheme can also be applied to generic two-level quantum systems such as vacancy centers in solids and superconducting qubits, the optical clock transition and the narrow line width of $^3P_0$ clock states make the system particularly suitable for the simulation of Floquet dynamics in the adiabatic regime. Specifically, since the line width of $^3P_0$ states is on the order of 10 mHz, it is convenient to make $\omega$ much smaller than $\Omega_-$ and $\Omega_+$. Based on the current scheme, one may further consider spatially periodic coupling lasers, which offers the intriguing possibility of simulating complex band structures in higher dimensions.

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Author contributions

D.-H. Cai performed the theoretical analysis; W.Y. conceived the idea and supervised the project; both authors contributed to the writing of this work.

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