Estimation of position and time-varying intensity of a heat source using reduced models built with the Modal Identification Method

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Abstract. This numerical study deals with the estimation of both unknown position and intensity of a heat source in diffusive problems, from the knowledge of local temperature data. The source is assumed to be fixed but its intensity varies with time. The originality of this paper lies in the use of reduced models to solve the inverse problem. The source position being unknown, a specific approach is proposed, involving the Modal Identification Method (MIM) allowing us to obtain a RM relative to a set of output temperatures. The source is modelled as \( f(r,s)u(t) \), where \( f \) covers the whole domain and mimics a source centred in \( r_s \). Starting with initial guess for \( r_s \), RMs relative to outputs and their first derivatives with respect to \( r_s \), are identified. A Quasi-Newton algorithm is used for searching \( r_s \), and according to a Taylor expansion, new RMs are built for current \( r_s \) to estimate \( u(t) \) and compute sensitivities. When \( r_s \) cannot be modified anymore by the iterative algorithm, the Detailed Model is called to update the RMs series. The approach is first described in detail for a 1D case, then expressions for 2D and 3D cases are given. An academic 3D heat diffusion problem illustrates the method.

1. Introduction

Inverse problems for the estimation of both location and strength of heat sources have already been addressed. For instance, in [1] and [2], problems involve static sources as well as moving ones and use BEM formulation. In the present work, linear heat diffusion with constant thermophysical properties is considered. In addition, the source is assumed to be fixed but its intensity is varying with time. Even if there is a linear relationship between temperature and the time-varying source intensity, the inverse problem with unknown source position is a nonlinear inverse problem because temperature depends on the source position in a nonlinear way.

The originality of this paper lies in the use of reduced models (RMs) to solve the inverse problem. Compared to a detailed model (DM) of size \( N \), RMs involve a number of equations \( n \ll N \) and are designed to reproduce the DM behavior with short computing time while preserving a good accuracy. Among reduction methods, one can cite the well known Proper Orthogonal Decomposition (POD) with a Galerkin projection. It has been used in [3] to build a reduced model for the estimation of the
time-varying intensity of a heat source in 2D nonlinear heat diffusion (position was known). In the present paper, the Modal Identification Method (MIM) is used. It allows getting a RM relative to a small set of output temperatures. MIM provides a way to obtain RMs of dynamical systems through the use of identification methods, especially for heat or mass transfer problems, when locations of boundary conditions and sources are known. These RMs can then be used to solve efficiently inverse problems with transient thermal loads. This approach has been used in [4] for an inverse heat conduction problem, and in [5] and [6] for inverse forced convection problems involving turbulent flows. Note also that experimental data have been used in [4] and [6].

A new approach is developed in this paper where the source position is unknown. The aim is:
- Firstly, to formulate a RM linking up the source intensity to the data outputs and taking into account the current position, through a Taylor expansion around a nominal source position.
- Secondly, to embed this RM in an optimisation procedure with either a reference detailed model outputs [5] or measurements at the same locations [6], for the estimation of both position and intensity of the source.

For convenience, the approach is described in detail for 1D problems in section 2. Extensions for 2D and 3D cases are given in sections 3 and 4 respectively. The general algorithm is given in section 5 and an academic 3D test case is presented in section 6. Section 7 is devoted to conclusions.

2. The proposed approach for 1D problems

2.1. Energy equation, source modelling and Detailed Model
Let us consider the 1D energy equation along with initial and Fourier boundary conditions with a surrounding temperature equal to zero, and a heat source modelled as \( g(x,x_s) \) \( u(t) \) where \( g \) covers the whole spatial domain \([0, L]\) and mimics a source centred in \( x_s \):

\[
\rho C_p \frac{\partial T}{\partial t}(x,x_s,t) = \lambda \frac{\partial^2 T}{\partial x^2}(x,x_s,t) + g(x,x_s) u(t) \quad (1 \text{ a})
\]

\[
\lambda \frac{\partial T}{\partial x}(0,x_s,t) = h_1 T(0,x_s,t) \quad \text{and} \quad -\lambda \frac{\partial T}{\partial x}(L,x_s,t) = h_2 T(L,x_s,t) \quad (1 \text{ b,c})
\]

The initial condition is \( T(x,x_s,0) = 0 \). \( T \) is the local temperature, \( \lambda \) the thermal conductivity, \( \rho \) the density and \( C_p \) the specific heat. As in [3], the function \( g(x,x_s) \) has been chosen as:

\[
g(x,x_s) = \frac{r_n}{2} \left[ 1 - \tanh^2 \left( r_n (x - x_s) \right) \right] \quad (2)
\]

This function \( g \) tends to the Dirac \( \delta \) function as \( r_n \) approaches infinity.

Whatever the chosen discretisation method (finite differences, finite volumes, finite elements, …), equation (1 a) and associated boundary conditions (1 b,c) can be written under the matrix-vector form called state space representation, which is a Detailed Model (DM) of the problem:

\[
\begin{align*}
\dot{\mathbf{T}}(x_s,t) &= \mathbf{A} \mathbf{T}(x_s,t) + \mathbf{B}(x_s) \mathbf{U}(t) \\
\mathbf{Y}(x_s,t) &= \mathbf{C} \mathbf{T}(x_s,t)
\end{align*} \quad (3 \text{ a,b})
\]

where \( \mathbf{U}(t) \) is the command vector function (here restricted to the scalar function \( u(t) \), but we prefer keeping the more general vector formulation). \( \mathbf{T} \) is the vector of temperatures at the \( N \) discretisation nodes. \( \mathbf{A} \) is the state matrix linking nodes and \( \mathbf{B} \) is the command matrix (a vector here) applying \( \mathbf{U} \). \( \mathbf{C} \) is an output matrix allowing to select \( q \leq N \) observation points stored in vector function \( \mathbf{Y} \).

2.2. Inverse problem procedure: global overview
Since inverse problems with unknown source position are nonlinear, an iterative procedure is needed. Order 0 optimisation methods such as simplex (deterministic), genetic algorithms or particle swarm optimization (stochastic) could be used, but since through RMs, we are able to quickly compute sensitivities of observed outputs with respect to the unknown position, an order 1 deterministic method can be used, such as a conjugate-gradient, Levenberg-Marquardt or Quasi-Newton method. The latter
has been used in the present work. Starting from an initial guess $x_r^*$ for $x_r$, the source position is modified during the iterative procedure. Therefore we need a RM taking into account any current position $x_r^* + \Delta x_r$. Let us assume that such a RM is available, then at each iteration, the position being fixed, the RM is linear with respect to $U(t)$ which can be estimated using linear least squares. Hence there is no need for an initial guess for $U(t)$. The sequential method that is used is described in detail in [5-6]. Once $U(t)$ estimated, sensitivities of outputs with respect to $x_r$ are computed for $x_r^* + \Delta x_r$ and a new position is found. When the position cannot be modified anymore by the iterative algorithm, the Detailed Model is called to update the RMs series for the last position. Let us now see how to obtain a RM depending on the source position.

2.3. Reduced Model relative to outputs for $x_s = x_r^*$

As shown in [4-6], the MIM is a tool able to perform the identification of a RM of equations (3 a,b) for $x_s = x_r^*$. This RM is written as:

$$
\begin{align*}
X^{(0)}(t) &= F^{(0)} X^{(0)}(t) + G^{(0)} U(t) \\
Y(x_s^*, t) &= H^{(0)} X^{(0)}(t)
\end{align*}
$$

Let $m^{(0)}$ be the RM order, i.e. the size of vector $X^{(0)}(t)$. $F^{(0)}$ is a diagonal matrix of size $(m^{(0)}, m^{(0)})$, $G^{(0)}$ a matrix (a vector here) of size $(m^{(0)}, 1)$ and $H^{(0)}$ a matrix of size $(q, m^{(0)})$. In the following, superscript (i) indicates that matrices or vectors are part of RMs relative to DMs corresponding to the $i^{th}$ differentiation of equations (3 a,b) with respect to $x_r$. However, it should be noted that such RM matrices and vectors are identified for $x_r$ fixed equal to $x_r^*$ and therefore cannot be differentiated with respect to $x_r$. It should also be underlined that the RM is not just a computation of governing equations on a coarser grid than the one used for DM. It is not also a simple interpolation between some DM solutions. In the MIM, the RM is identified through an optimization procedure, described in [4-6], which consists of minimizing the quadratic residuals between DM solutions (or in-situ measurements) and RM solutions, when a known applied signal $U(t)$ is applied to both models (or to the real system and to the RM). This means that the $F^{(0)}_i$, $i=1,.., m^{(0)}$ are not the calculated eigenvalues of state matrix $A$ but are identified through the MIM. $G^{(0)}$ and $H^{(0)}$ are also identified.

2.4. Reduced Model relative to outputs first derivatives with respect to $x_r$, for $x_s = x_r^*$

Let us differentiate equation (1 a) and associated BC (1 b,c) with respect to $x_r$:

$$
\rho C_p \frac{\partial}{\partial t} \left( \frac{\partial T}{\partial x_s} (x_s, x_r, t) \right) = \lambda \frac{\partial^2}{\partial x_s^2} \left( \frac{\partial T}{\partial x_s} (x_s, x_r, t) \right) + \frac{\partial g}{\partial x_s} (x_s, x_r) u(t)
$$

(5 a)

$$
\lambda \frac{\partial}{\partial x_s} \left( \frac{\partial T}{\partial x_s} (0, x_r, t) \right) = h \left( \frac{\partial T}{\partial x_s} (0, x_r, t) \right) \quad \text{and} \quad -\lambda \frac{\partial}{\partial x_s} \left( \frac{\partial T}{\partial x_s} (L, x_r, t) \right) = h \left( \frac{\partial T}{\partial x_s} (L, x_r, t) \right)
$$

(5 b,c)

The discretisation of equation (5 a) and associated BC (5 b,c) can be written for $x_s = x_r^*$:

$$
\frac{d}{dt} \left( \frac{\partial T}{\partial x_s} (x_r^*, t) \right) = A \frac{\partial T}{\partial x_s} (x_r^*, t) + B (x_r^*) U(t)
$$

(6 a,b)

$$
\frac{\partial Y}{\partial x_s} (x_r^*, t) = C \frac{\partial T}{\partial x_s} (x_r^*, t)
$$

Remarks:

1. Matrix $B(x_r)$ in (3 a) also depends on the chosen discretisation, on $h_1$, $h_2$ and on $r_n$. When all these parameters are fixed, $B$ depends only on $x_r$ through the discretisation of $g(x, x_r)$. As $g(x, x_r)$ is analytically differentiable with respect to $x_r$, matrix $[dG/dx_s] (x_r^*)$ is easily obtained.

2. State matrix $A$ is independent of $x_r$ and therefore is the same in equations (3 a) and (6 a). It means that a single linear system has to be solved with 2 different right hand sides. Of course, this will be also true for second order differentiation.
A RM of equations (6 a,b) can be written as:

\[
\begin{align*}
\dot{X}^{(1)}(t) &= F^{(1)}X^{(1)}(t) + G^{(1)}U(t) \\
\frac{\partial Y}{\partial x_x}(x_x^*, t) &= H^{(1)}X^{(1)}(t)
\end{align*}
\]

(7 a,b)

Let \(m^{(1)}\) be the RM order, i.e. the size of vector \(X^{(1)}(t)\). \(F^{(1)}\) is a diagonal matrix.

2.5. Reduced Model relative to outputs for \(x_x = x_x^*\) for \(x_x^*\)

In a similar way as in section 2.4, differentiating two times equation (1 a) and associated BC (1 b,c) with respect to \(x_x\) leads to the following RM:

\[
\begin{align*}
\dot{X}^{(2)}(t) &= F^{(2)}X^{(2)}(t) + G^{(2)}U(t) \\
\frac{\partial^2 Y}{\partial x_x^2}(x_x^*, t) &= H^{(2)}X^{(2)}(t)
\end{align*}
\]

(8 a,b)

Let \(m^{(2)}\) be the RM order, i.e. the size of vector \(X^{(2)}(t)\). \(F^{(2)}\) is a diagonal matrix.

2.6. Reduced Model relative to outputs for \(x_x = x_x^* + \Delta x_x\): Taylor expansion

The second order Taylor expansion of vector function \(Y(t)\) around \(x_x = x_x^*\) gives:

\[
Y(x_x^* + \Delta x_x, t) = Y(x_x^*, t) + \Delta x_x \frac{\partial Y}{\partial x_x}(x_x^*, t) + \frac{\Delta x_x^2}{2} \frac{\partial^2 Y}{\partial x_x^2}(x_x^*, t) + o(\Delta x_x^3)
\]

(9)

Inserting equations (4 b), (7 b), (8 b) and neglecting terms of order \(\geq 3\), equation (9) becomes:

\[
Y(x_x^* + \Delta x_x, t) = \begin{bmatrix} H^{(0)} & H^{(1)} \Delta x_x & H^{(2)} \frac{\Delta x_x^2}{2} \end{bmatrix} \begin{bmatrix} X^{(0)}(t) \\ X^{(1)}(t) \\ X^{(2)}(t) \end{bmatrix}
\]

(10)

Let \(H = \begin{bmatrix} H^{(0)} & H^{(1)} \Delta x_x & H^{(2)} \frac{\Delta x_x^2}{2} \end{bmatrix}\), \(X(t) = \begin{bmatrix} X^{(0)}(t) \\ X^{(1)}(t) \\ X^{(2)}(t) \end{bmatrix}\), \(F = \begin{bmatrix} F^{(0)} & 0 & 0 \\ 0 & F^{(1)} & 0 \\ 0 & 0 & F^{(2)} \end{bmatrix}\), \(G^{(1)} = \begin{bmatrix} G^{(1)} \\ G^{(1)} \\ G^{(1)} \end{bmatrix}\), \(G^{(2)} = \begin{bmatrix} G^{(2)} \\ G^{(2)} \\ G^{(2)} \end{bmatrix}\), and \(U(t) = \begin{bmatrix} G^{(1)} \\ G^{(1)} \\ G^{(1)} \end{bmatrix}\).

Then assembling (4 a), (7 a) and (8 a) and adding (10) gives the RM linking up output vector \(Y(x_x^* + \Delta x_x, t)\) evaluated for position \(x_x^* + \Delta x_x\), of the source centre, to input vector \(U(t)\):

\[
\begin{align*}
\dot{X}(t) &= F(t)X(t) + GU(t) \\
Y(x_x^* + \Delta x_x, t) &= H(t)X(t)
\end{align*}
\]

(11)

The order of RM (11), i.e. the size of vector \(X(t)\), is \(m = m^{(0)} + m^{(1)} + m^{(2)}\). \(F\) is diagonal.

2.7. RM relative to outputs first derivatives with respect to \(x_x\) for \(x_x = x_x^* + \Delta x_x\): Taylor expansion

The first order Taylor expansion of vector function \(\frac{\partial Y}{\partial x_x}(x_x, t)\) around \(x_x = x_x^*\) gives:

\[
\frac{\partial Y}{\partial x_x}(x_x^* + \Delta x_x, t) = \frac{\partial Y}{\partial x_x}(x_x^*, t) + \Delta x_x \frac{\partial^2 Y}{\partial x_x^2}(x_x^*, t) + o(\Delta x_x^2)
\]

(12)

Inserting equations (7 b), (8 b) and neglecting terms of order \(\geq 2\), equation (12) becomes:

\[
\frac{\partial Y}{\partial x_x}(x_x^* + \Delta x_x, t) = \begin{bmatrix} H^{(1)} & H^{(2)} \Delta x_x \end{bmatrix} \begin{bmatrix} X^{(1)}(t) \\ X^{(2)}(t) \end{bmatrix}
\]

(13)

Let \(H = \begin{bmatrix} H^{(1)} & H^{(2)} \Delta x_x \end{bmatrix}\), \(X(t) = \begin{bmatrix} X^{(1)}(t) \\ X^{(2)}(t) \end{bmatrix}\), \(F = \begin{bmatrix} F^{(1)} & 0 \\ 0 & F^{(2)} \end{bmatrix}\), and \(G = \begin{bmatrix} G^{(1)} & 0 \\ 0 & G^{(2)} \end{bmatrix}\).
Then assembling (7 a) and (8 a) and adding (13) gives the RM linking up output vector
\[ \frac{\partial Y}{\partial x_s}(x_s^* + \Delta x_s,t) \]
evaluated for position \( x_s^* + \Delta x_s \), of the source centre, to input vector \( U(t) \):

\[
\begin{align*}
\dot{X}(t) &= F'X(t) + G'U(t) \\
\frac{\partial Y}{\partial x_s}(x_s^* + \Delta x_s,t) &= H'X(t)
\end{align*}
\] (14)

The order of RM (14), i.e. the size of vector \( X'(t) \), is \( m' = m(1) + m(2) \). \( F' \) is diagonal.

3. Extension to 2D problems

Similarly as in 2.1, the 2D unsteady energy equation with a heat source modelled as \( f(x,y) \) \( u(t) \) where \( f \) covers the whole spatial domain and mimics a source centred in \( r_s = (x_s,y_s) \) is written:

\[
\rho C_p \frac{\partial T}{\partial t} = \lambda \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + f(x,y) \ u(t)
\] (15)

with \( f(x,x_s,y,y_s) = g(x,y) \) \( g(y,y_s) \), where \( g \) is defined by equation (2).

Initial and boundary conditions are similar as those of subsection 2.1.

Inserting RMs for \( Y(r_s^*,t) \) \( \frac{\partial Y}{\partial x_s}(r_s^*,t) \) \( \frac{\partial^2 Y}{\partial y_s^2}(r_s^*,t) \) \( \frac{\partial^2 Y}{\partial x_s \partial y_s}(r_s^*,t) \) into 2nd order Taylor expansions for \( Y(r_s^* + \Delta r_s,t) \) \( \frac{\partial Y}{\partial x_s}(r_s^* + \Delta r_s,t) \) \( \frac{\partial^2 Y}{\partial y_s^2}(r_s^* + \Delta r_s,t) \) \( \frac{\partial^2 Y}{\partial x_s \partial y_s}(r_s^* + \Delta r_s,t) \) leads to following RMs:

\[
\begin{align*}
\dot{X}(t) &= FX(t) + GU(t) \\
Y(r_s^* + \Delta r_s,t) &= HX(t)
\end{align*}
\] (16)

with \( H = H^{(0)} + H^{(1)} \Delta x_s \left( H^{(2)} \right)^2 + \left( H^{(2)} \right)^2 \Delta y_s \).

\( H^{(0)} \) being relative to \( Y(r_s^*,t) \) \( H^{(1)} \) to \( \frac{\partial Y}{\partial x_s}(r_s^*,t) \) \( H^{(1)} \) to \( \frac{\partial^2 Y}{\partial y_s^2}(r_s^*,t) \), etc.

\[
\begin{align*}
\dot{X}^{(1)}(t) &= F^{(1)} X^{(1)}(t) + G^{(1)} U(t) \\
\frac{\partial Y}{\partial x_s}(r_s^* + \Delta r_s,t) &= H^{(1)} X^{(1)}(t) \\
\frac{\partial^2 Y}{\partial x_s \partial y_s}(r_s^* + \Delta r_s,t) &= H^{(1)} \left( H^{(1)} \right) X^{(1)}(t)
\end{align*}
\] (17)

\[
\begin{align*}
\dot{X}^{(2)}(t) &= F^{(2)} X^{(2)}(t) + G^{(2)} U(t) \\
\frac{\partial Y}{\partial x_s}(r_s^* + \Delta r_s,t) &= H^{(2)} X^{(2)}(t) \\
\frac{\partial^2 Y}{\partial x_s \partial y_s}(r_s^* + \Delta r_s,t) &= H^{(2)} \left( H^{(2)} \right) X^{(2)}(t)
\end{align*}
\] (18)

In equation (17), superscript \( ^{(1)} \) indicates that matrices or vectors are part of a RM relative to outputs first derivative with respect to \( x_s \) when \( r_s = r_s^* + \Delta r_s \). The meaning of \( ^{(2)} \) in (18) is similar.

4. Extension to 3D problems

The source is now modelled as \( f(x,x_s,y,y_s,z,z_s) \) \( u(t) \) where \( f(x,x_s,y,y_s,z,z_s) = g(x,y) \) \( g(y,y_s) \) \( g(z,z_s) \) mimics a source centred in \( r_s = (x_s,y_s,z_s) \) with \( g \) defined by equation (2). As in section 3, we can formulate RMs for outputs and their first and second derivatives with respect to \( x_s, y_s, z_s \). By inserting these RMs into 2nd order Taylor expansions of outputs and their first derivatives for \( r_s = r_s^* + \Delta r_s \), we get:

\[
\begin{align*}
\dot{X}(t) &= FX(t) + GU(t) \\
Y(r_s^* + \Delta r_s,t) &= HX(t)
\end{align*}
\] (19)
with the estimated signal and position (multiplied by 1000). Figure 4 summarizes convergence results of order 32. Figure 3 shows 6 temperature recordings (each one at the centre of a boundary face) used to estimate insulated. A Finite Volumes DM has been built, with a 21

\[ \frac{\partial Y}{\partial \sigma_i} = H^{(z_i)}_{\sigma_i} \frac{\partial \sigma_i}{\partial \sigma_i} \]

\[ \frac{\partial Y}{\partial \sigma_j} = H^{(z_j)}_{\sigma_j} \frac{\partial \sigma_j}{\partial \sigma_j} \]

\[ \frac{\partial Y}{\partial \sigma_k} = H^{(z_k)}_{\sigma_k} \frac{\partial \sigma_k}{\partial \sigma_k} \]

5. General algorithm for 3D problems

The global algorithm for 3D problems is summarized in figure 1. The external loop, associated with index \( i \), corresponds to major sequences, that is when the DM is called to update the RMs series; during a major sequence \( i \), all Taylor expansions are obtained using the RMs series built for \( r_i \). The internal loop, associated with index \( j \), corresponds to iterations of the Quasi-Newton algorithm used to estimate \( r_j = (x_j, y_j, z_j) \). At a given iteration \( j \), \( r_j \) is fixed. The RM is linear with respect to \( U(t) \) which can therefore be estimated using linear least squares. Hence there is no need for an initial guess for \( U(t) \). The sequential method described in detail in [5-6] is used for minimizing:

\[ J_k(U(t_k)) = \frac{1}{2} \sum_{i=1}^{d} \sum_{j=0}^{n} \left( Y_i^{me}(t_{k+j}) - Y_i^{calc}(t_{k+j}) \right)^2, k = 1, ..., nt - nf \] with \( nf \) the number of future time steps.

Once \( U(t) \) estimated, sensitivities of outputs with respect to \( r_i \) are computed and a new position is found by minimizing

\[ J(r_k) = \frac{1}{2} \sum_{i=1}^{d} \sum_{j=0}^{n} \left( Y_i^{me}(t_j) - Y_i^{calc}(t_j) \right)^2 \] with the Quasi-Newton algorithm.

Note that the value of \( nf \) is optimized for each major sequence: the inversion is performed using \( nf = 0, nf = 1, \) etc, until the mean quadratic discrepancy \( \sigma_y = (2J(r)/q(x(nt - nf)))^{1/2} \) between temperature data and computed temperatures reaches the data standard deviation \( \sigma \) (if known). Of course, such a procedure is made possible thanks to the use of RMs and would have been very long with a DM.

6. A 3D example

Figure 2 shows an academic 3D linear heat diffusion problem with a parallelepiped \((0.27m \times 0.23m \times 0.19m)\) heated by a source whose position is \((0.21; 0.08; 0.05)\). Parameter \( r \) in equation (2) has been set to 50. The material thermophysical properties are \( \lambda = 16 \text{ W.m}^{-1}\text{K}^{-1} \) and \( \rho c_p = 4.029 \times 10^6 \text{ J.m}^{-3}\text{K}^{-1} \). East, south and bottom boundary faces are submitted to convective exchanges \((h_{s}=50, h_{t}=10 \text{ and } h_{b}=30 \text{ W.m}^{-2}\text{K}^{-1}) \) with surrounding reference temperature \( Ta=0 \). West, north and top faces are insulated. A Finite Volumes DM has been built, with a 21x17x13 nodes mesh, for a total of \( N = 4641 \) equations. In comparison, RMs of order 8 are identified at each major sequence. Therefore, the RM defined by equation (19) is of order 80 and RMs defined by equations (20), (21) and (22) are of order 32. Figure 3 shows 6 temperature recordings (each one at the centre of a boundary face) used as simulated data for inversion, as well as discrepancies between data and temperatures calculated with the estimated signal and position (multiplied by 1000). Figure 4 summarizes convergence results
for the source position with respect to major sequences. Convergence is obtained with only 4 major sequences. Mean quadratic errors on data \(s_y\) and signal \(u(t)\) are also shown (sY/40 and sU/400 in fact). Figure 5 shows the exact test signal \(u(t)\) and the corresponding estimated signal (superimposed), as well as the discrepancy between exact and estimated signal (multiplied by 100).

Figure 1: general algorithm for 3D problems
7. Conclusions and prospects

The problem of estimating both position and intensity of a heat source has been addressed in this paper. It has been shown how to formulate a RM linking up the source intensity to data outputs and taking into account the current position, through a Taylor expansion of RMs identified with the MIM at a nominal source position. A global algorithm for 3D problems has been described. For a few major sequences, RMs need to be updated by using the DM. Results show good convergence. Finally, note that as for fixed steady fluid flows, RMs with the same structure can be written, this approach could be also used for convection-diffusion problems. Other future works include the case of multiple sources.

References

[1] Lefèvre F and Le Niliot C 2002 Multiple transient point heat sources identification in heat diffusion: application to experimental 2D problems Int. J. Heat & Mass Transfer 45 1951-64
[2] Lefèvre F and Le Niliot C 2002 The BEM for point heat source estimation: application to multiple static sources and moving sources Int. J. Thermal Sciences 41 536-45
[3] Park H M, Chung O Y and Lee J H 1999 On the solution of inverse heat transfer problem using the Karhunen-Loeve Galerkin method Int. J. Heat & Mass Transfer 42 127-42
[4] Videcoq E, Petit D and Piteau A 2003 Experimental modelling and estimation of time varying thermal sources Int. J. Thermal Sciences 42 255-65
[5] Girault M, Maillet D, Fontaine J R, Braconnier R and Bonthoux F 2006 Estimation of time-varying gaseous contaminant sources in ventilated enclosures through inversion of a reduced model Int. J. Ventilation 4 365-80
[6] Girault M, Maillet D, Bonthoux F , Galland B, Martin P, Braconnier R and Fontaine J R 2008 Estimation of time-varying pollutant emission rates in a ventilated enclosure: inversion of a reduced model obtained by experimental application of the modal identification method Inverse Problems 24