Abstract

The propagation of the excess of quasiparticles and phonons produced by a nuclear recoil inside Sn and Zn superheated superconducting granules will be discussed. The decay towards equilibrium of the initial disturbance is assumed to be a thermal diffusion process described by a set of coupled heat flow equations for the effective quasiparticle and phonon temperatures. The solution is carried out analytically for a point source located anywhere inside the superconducting granule with the initial energy distributed in both quasiparticle and phonon systems. The calculated time delay between the neutron interaction and the nucleation of the phase transition will be compared to the time delay distributions obtained by irradiating Zn and Sn SSG detectors with a 70MeV neutron beam.

1 Introduction

Irradiation tests of SSG detectors with a 70MeV neutron beam have been used to study the delay in time between the neutron elastic scattering and the phase transition of a single granule inside the detector. Typical delay times of $\sim 100\text{ns}$ and $\sim 500\text{ns}$ have been measured in Sn and Zn SSG respectively, as it is discussed elsewhere in these proceedings\(^1\).

After a neutron elastic scattering, the deposited energy is transferred to electrons and to the lattice by the recoiling nucleus. The initial disturbance is localized at the interaction point and produces an excess of quasiparticles and phonons in a state very far from equilibrium where the kinetic energy of the quasiparticles is larger than $kT_c$. After a few $\text{ps}$ this hot electron gas exhibits a local equilibrium with the phonons and a slower relaxation process dominated by heat diffusion takes place\(^2\).

The heat propagation in superconducting spheres is discussed in Ref. 3 for an ionizing particle depositing energy only in the quasiparticle system as a primary excitation. These results can not be applied to the case of nuclear recoils where a large fraction of the energy is transferred to the phonons as a primary excitation.
In the present work we will consider the heat diffusion after a nuclear recoil interaction located anywhere inside the superconducting granule and with the initial energy distributed in both quasiparticle and phonon systems.

2 Thermal Diffusion

The decay towards equilibrium of the initial disturbance is described by a set of coupled heat flow equations for the effective quasiparticle ($T_e$) and phonon temperatures ($T_p$):

\[
\begin{align*}
\frac{\partial T_e}{\partial t} &= D_e \nabla^2 T_e - \frac{1}{\tau_e} (T_e - T_p) + A_e \delta_e(r_o, t_o) \\
\frac{\partial T_p}{\partial t} &= D_p \nabla^2 T_p - \frac{1}{\tau_p} (T_p - T_e) - \frac{1}{\tau_s} (T_p - T_b) + A_p \delta_p(r_o, t_o)
\end{align*}
\]

where $D_e$ and $D_p$ are the thermal diffusivities, $\tau_e$ and $\tau_p$ are the quasiparticle and phonon lifetimes and $T_b$ the bath temperature. Since SSG detectors are made of granules embedded in a dielectric material (plasticine or $Al_2O_3$ powder) the heat transfer to the bath is due only to phonons crossing the granule surface with a rate $1/\tau_s$. The initial disturbance is assumed to be a point source in $r_o$ at the time $t_o$ described by the delta functions $\delta_e(r_o, t_o)$ and $\delta_p(r_o, t_o)$ with $A_e$ and $A_p$ accounting for the initial energy share between the two systems.

Equation (1) can be solved analytically using Fourier transforms and the eigenvalues method with the thermal diffusivities and the carrier lifetimes being constant in temperature. Due to the spherical symmetry of the problem, the spatial eigenfunctions of the solution are the even and odd spherical harmonic functions $\Psi_{m,n,l}(\vec{r})$ with the indexes $n$ and $m$ referring to the order of the spherical Bessel functions and Legendre polynomials respectively. The boundary conditions on the granule surface are used to evaluate the positive zeros of the first derivative of the Bessel functions (index $l$). The final expression for the quasiparticle temperature is:

\[
T_e(\vec{r}, t) = T_b + F_e(t) + V \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{l=0}^{\infty} [\Psi_{m,n,l}(\vec{r})\Psi_{m,n,l}(\vec{r}_o) + \Psi_{m,n,l}^o(\vec{r})\Psi_{m,n,l}^o(\vec{r}_o)] F_e(l, t)
\]

with $V$ the granule volume. A similar expression holds for $T_p(\vec{r}, t)$ replacing $F_e$ with $F_p$. The time dependent terms $F_e$ and $F_p$ are linear combinations of the initial quasiparticle ($T_e^i$) and phonon ($T_p^i$) temperatures and exhibit a mixture of relaxation and diffusion terms. The terms $F_e(t)$ and $F_p(t)$ are calculated from $F_e(t, l)$ and $F_p(t, l)$ with $l=0$ and do not depend on the spatial coordinates. The derivation of Eq. (2) is discussed in Ref. 5.

At the beginning of the diffusion process the rise in temperature is localized at the interaction point with $T_e$ and $T_p$ proportional to the partition of the initial energy.
between the two systems given by the conditions \( F_e(0, l) = F_e(0) = T_e^i \) and \( F_p(0, l) = F_p(0) = T_p^i \). The terms \( F_e(l, t) \) and \( F_p(l, t) \) decay exponentially to zero with time and at the end of the diffusion process \( (t = \infty) \) the equilibrium temperatures are \( T_e = T_p = T_\infty \) with \( F_e(\infty) = F_p(\infty) \). The energy conservation inside the granule, in the case of no heat transfer to the bath \( (1/\tau_s = 0) \), impose the conditions:

\[
E_r = V \int_{T_b}^{T_\infty} (C_{es} + C_{ph}) \, dT, \quad \frac{\tau_p}{\tau_e} = \frac{T_e^i - T_\infty}{T_\infty - T_p^i} \quad (3)
\]

The initial quasiparticle and phonon temperatures \( T_e^i \) and \( T_p^i \) are evaluated from the integral of the specific heat in the superconducting state:

\[
E_r f = V \int_{T_b}^{T_e^i} C_{es} \, dT, \quad E_r (1 - f) = V \int_{T_b}^{T_p^i} C_{ph} \, dT \quad (4)
\]

where \( f \) is the fraction of the recoil energy \( E_r \) transferred to electrons by neutron elastic scatterings and can be evaluated from Ref. 6. Typical values of \( f \) in Sn and Zn absorbers are \(~0.22\) and \(~0.4\) for recoil energies of 5keV and 100keV respectively.

The lifetimes ratio and the initial temperatures defined in Eq. \( 3 \) and Eq. \( 4 \) differ from the expression used in previous works\(^3\) where it was assumed that \( \tau_p/\tau_e = C_{ph}/C_{es} \) and \( T_e^i = E_r/(VC_{es}) \) with the specific heats constant in temperature. This approximation holds only at high bath temperatures where the relative temperature rise is small. At low temperatures where \( C_{es} \ll C_{ph} \), the values of \( \tau_p \) evaluated from the ratio of the specific heats are an order of magnitude bigger than \( \tau_e \). The carrier lifetime ratio defined in Eq. \( 3 \) depends on the recoil energy and on the partition of the initial energy.

### 3 Rise in temperature after a nuclear recoil

In order to compare the calculations with the neutron irradiation tests, we considered Sn and Zn granules with diameters 15µm and 30µm respectively. The quasiparticle diffusivity was extrapolated from reference values\(^7\) using the normal state electrical resistivity measured\(^8\) on granules similar to the ones used in the irradiation tests. The phonon thermal diffusivity is \( D_p \ll D_e \) and can not be extrapolated from measurements. We performed computations of the quasiparticle and phonon temperatures either neglecting \( D_p \) or with \( D_p = D_e/100 \) without obtaining substantial differences in the thermal diffusion process. The lifetimes \( \tau_e \) were evaluated from\(^9\) at the temperature \( T_\infty \) for quasiparticle energies two times the superconducting gap. Typical values for \( \tau_e \) are \(~300\)ns in Zn and \(~1\)ns in Sn. The phonon lifetimes \( \tau_p \) were obtained from Eq. \( 3 \). The rate \( 1/\tau_s \) at which phonons cross the granule surface was neglected since \( \tau_s \) is of the order of few \( \mu\)s. Nuclear recoils inside the superconducting granule were simulated considering a three dimensional grid of 72 equally spaced interaction points located in the half sphere with the coordinate \( \phi \) in the interval \( 0-\pi \). For each
interaction point, the quasiparticle and phonon temperatures were calculated for a point on the granule surface located on the equatorial plane at \( \phi = \pi / 2 \).

The calculated temperatures are plotted in Fig. 1 for a recoil energy of 5keV deposited in a Zn granule at the bath temperature of 50mK. Due to the differences between the lifetimes in Zn, \( \tau_e = 300\text{ns} \) and \( \tau_p = 50\text{ns} \), the initial perturbation is transferred to the quasiparticles system in a shorter time scale. At the beginning of the diffusion process, the temperature rise depends on the location of the interaction point inside the granule and temperatures higher than the final value \( T_\infty \) can be reached for interaction points close to the surface. At the end of the diffusion, all the temperatures converge to \( T_\infty \). In Sn granules, the time scale for the relaxation process is shorter because \( \tau_e \sim \tau_p \sim 1\text{ns} \) and the quasiparticle and phonon temperatures exhibit the same time dependence.

4 Time delay

The nuclear recoil interactions were simulated considering a three dimensional grid of 122 equally spaced points located in the whole granule volume. For each interaction point the temperatures were computed with time steps of 4ns from 0 up to 400ns in Sn and 1.5\( \mu \text{s} \) in Zn.

The energy threshold of a single granule is related to the magnetic threshold \( h = 1 - B_a / B_{sh}(T_b) \) where \( B_{sh} \) is the granule superheating field and \( B_a \) is the strength of the applied magnetic field. From the phase diagram it is possible to relate the magnetic threshold \( h \) to the temperature \( T^*_c \) needed to fulfill the condition \( B_a = B_{sh}(T^*_c) \).

Previous irradiation experiments on single granules with radioactive sources\(^{10}\) have shown that the phase transition tends to nucleate in a small portion of the granule surface (nucleation center) and spread afterwards into the full granule volume. This effect was found to be less evident in Zn than in Sn granules. Measurements have also shown that there is a dependence of the phase transition field on the crystallographic orientation of the granule with respect to the applied magnetic field\(^{11}\). Since SSG are made of a collection of granules, it is quite difficult to define a criteria for the occurrence of the phase transition valid for all the granules inside the detector.

To evaluate the time elapsing from the interaction to the nucleation of the phase transition, we monitored the quasiparticle temperature on 5 equidistant points on the granule equator and recorded the time for which the condition \( T_e \geq T^*_c \) was simultaneously satisfied on the 5 points. This is a simplified assumption, because the nucleation of the phase transition is assumed to be along the full equatorial plane neglecting the possibility of having a more localized nucleation center or a combination of the two mechanisms. The measured SSG superheating field distributions\(^{12}\) were used to evaluate the probability distribution of the transition temperatures \( T^*_c \).

To compare with the measurements, the calculated time distributions were weighted with the neutron elastic scattering cross section and with the detection efficiency of the neutron counter used in the experiments\(^{12}\). Despite the simplified assumption
used to define the occurrence of the phase transition, the calculation well reproduce the time scale of the experimental results as shown in Fig. 2. It is important to note that the measured distribution have a constant time offset due to the electronic delays\(^1\).

In the case of Zn granules, where the diffusion time is longer, there is a difference in the shape of the calculated distributions for recoil energies in the ranges 5-15keV and 40-60keV. At low recoil energies the theoretical distribution is shifted toward longer times in agreement with the measurements. In Sn, the quasiparticle relaxation times are shorter and both the calculated and measured time delay distributions are within 200ns.

5 Conclusions

An analytical expression to describe the heat propagation inside superconducting granules after nuclear recoils was derived. The calculations differ from previous works because the initial energy is shared in both quasiparticle and phonon systems. The elapsed time from the nuclear recoil interaction to the nucleation of the phase transition was calculated for Sn and Zn granules. The calculations are in good agreement with the distributions measured in irradiation tests of SSG with a 70MeV neutron beam.

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**Figure Captions**

Fig. 1 Quasiparticle (q.p.) and phonon temperatures versus time for 5keV nuclear recoils in Zn. The interactions are distributed in a three dimensional grid of 72 equally spaced points located inside the half sphere $\phi = 0-\pi$. The temperatures are monitored on the granule equator at the point $\phi = \pi/2$.

Fig. 2 Calculated (upper) and measured$^1$ (lower) distributions of the time delay between nuclear recoils and phase transitions in Zn and Sn granules with magnetic thresholds $h=2\%$ and $h=1\%$ respectively. The measured distributions have a constant time offset due to the electronic readout.