MEDIUM EFFECTS ON CHARGED PION RATIO IN HEAVY ION COLLISIONS

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We have recently studied in the delta-resonance–nucleon-hole model the dependence of the pion spectral function in hot dense asymmetric nuclear matter on the charge of the pion due to the pion $p$-wave interaction in nuclear medium. In a thermal model, this isospin-dependent effect enhances the ratio of negatively charged to positively charged pions in neutron-rich nuclear matter, and the effect is comparable to that due to the uncertainties in the theoretically predicted stiffness of nuclear symmetry energy at high densities. This effect is, however, reversed if we also take into account the $s$-wave interaction of the pion in nuclear medium as given by chiral perturbation theory, resulting instead in a slightly reduced ratio of negatively charged to positively charged pions. Relevance of our results to the determination of the nuclear symmetry energy from the ratio of negatively to positively charged pions produced in heavy ion collisions is discussed.

1. Introduction

The nuclear symmetry energy is the energy needed per nucleon to convert all protons in a symmetric nuclear matter to neutrons. Knowledge on the density dependence of nuclear symmetry energy is important for understanding the dynamics of heavy ion collisions induced by radioactive beams, the structure of exotic nuclei with large neutron or proton excess, and many important issues in nuclear astrophysics. At normal nuclear matter density, the nuclear symmetry energy has long been known to have a value of about 30 MeV from fitting the binding energies of atomic nuclei with the liquid-drop mass formula. Somewhat stringent constraints on the nuclear symmetry energy below the normal nuclear density have
also been obtained during past few years from studies of the isospin diffusion and isoscaling in heavy-ion reactions, the size of neutron skins in heavy nuclei, and the isotope dependence of giant monopole resonances in even-\(A\) Sn isotopes.

For nuclear symmetry energy at high densities, transport model studies have shown that the ratio of negatively to positively charged pions produced in heavy ion collisions with neutron-rich nuclei is sensitive to its stiffness. Comparison of this ratio from an isospin-dependent Boltzmann-Uehling-Uhlenbeck (IBUU) transport model based on the non-relativistic momentum-dependent (MDI) nuclear effective interactions with measured data from heavy ion collisions by the FOPI Collaboration at GSI seems to indicate that the nuclear symmetry energy at high density might be very soft. Although this study does not include the relativistic effects, which may affect the charged pion ratio as shown in Ref, it provides an important step in the determination of the nuclear symmetry energy at high densities.

The transport model used in Ref. neglects, however, medium effects on pions, although it includes those on nucleons and produced \(\Delta\) resonances through their isospin-dependent mean-field potentials and scattering cross sections. It is well-known that pions interact strongly in nuclear medium as a result of their \(p\)-wave couplings to the nucleon-particle–nucleon-hole and delta-particle–nucleon-hole (\(\Delta\)-hole) excitations, leading to the softening of their dispersion relations or an increased strength of their spectral functions at low energies. Including pion medium effects in the transport model has previously been shown to enhance the production of low energy pions in high energy heavy ion collisions, although it does not affect the total pion yield. Since pions of different charges are modified differently in asymmetric nuclear matter that has unequal proton and neutron fractions, including such isospin-dependent medium effects is expected to affect the ratio of negatively to positively charged pions produced in heavy ion collisions.

2. Pion \(p\)-wave interactions in nuclear medium

Considering only the dominant \(\Delta\)-hole excitations as in Ref, as the contribution from the nucleon particle-hole excitations is known to be small, the self-energy of a pion of isospin state \(m_t\), energy \(\omega\), and momentum \(k\) in a hot nuclear medium due to its \(p\)-wave interaction is given by

\[
\Pi_0^{m_t} \approx \frac{4}{3} \left( \frac{f_\Delta}{m_\pi} \right)^2 k^2 F_\pi^2(k) \sum_{m_r,m_T} |\langle \frac{3}{2} m_T | 1 m_t \frac{1}{2} m_r \rangle|^2 \\
\times \int \frac{d^3p}{(2\pi)^3} \frac{1}{e^{(m_N+p^2/2m_N+U_N^{m_T}-\mu_B-2m_r\mu_Q)/T}+1} \left( \frac{1}{\omega-\omega_{m_T}} + \frac{1}{-\omega-\omega_{m_T}} \right),
\]

(1)
with $\omega^{m_{\tau}}_{T} \approx m_{\Delta} + U^{m_{\tau}}_{\Delta} + (\vec{k} \pm \vec{p})^2/2m_{\Delta} - i\Gamma^{m_{\tau}}_{\Delta}/2 - m_{N} - U_{N}^{m_{\tau}} - p^2/2m_{N}$. In the above, $m_{\pi} \approx 138$ MeV, $m_{N} \approx 939$ MeV, and $m_{\Delta} \approx 1232$ MeV are the masses of pion, nucleon, and $\Delta$ resonance, respectively; $f_{\Delta} \approx 3.5$ is the $\pi N \Delta$ coupling constant and $F_{\pi}(k) = [1 + 0.6(k^2/m_{\pi}^2)^{-1/2}]$ is the $\pi N \Delta$ form factor determined by fitting the decay width $\Gamma_{\Delta} \approx 118$ MeV of $\Delta$ resonance in free space. The summation in Eq. (1) is over the nucleon isospin state $m_{T}$, and the $\Delta$ resonance isospin state $m_{T}$; and the factor $(\frac{1}{2} m_{T}|1 m_{t} \frac{1}{2} m_{r})$ is the Clebsch-Gordan coefficient from the isospin coupling of pion with nucleon and $\Delta$ resonance. The momentum integration is over that of nucleons in the nuclear matter given by a Fermi-Dirac distribution with $\mu_{B}$ and $\mu_{Q}$ being, respectively, the baryon and charge potentials determined by charge and baryon number conservations; $\rho_{N}^{m_{\tau}}$ and $U_{N}^{m_{\tau}}$ are, respectively, the density and mean-field potential of nucleons of isospin state $m_{T}$ in asymmetric nuclear matter; and $\Gamma^{m_{\tau}}_{\Delta}$ and $U^{m_{\tau}}_{\Delta}$ are, respectively, the width and mean-field potential of $\Delta$ resonance of isospin state $m_{T}$.

For the nucleon mean-field potential $U_{N}^{m_{\tau}}$, we have used the one obtained from the momentum-independent (MID) interaction, i.e., $U_{N}^{m_{\tau}}(\rho_{B}, \delta_{\text{like}}) = \alpha(\rho_{B}/\rho_{0}) + \beta(\rho_{B}/\rho_{0})^{\gamma} + U^{m_{\tau}}_{\text{asy}}(\rho_{B}, \delta_{\text{like}})$, with $U^{m_{\tau}}_{\text{asy}}(\rho_{B}, \delta_{\text{like}}) = -\{4F(x)(\rho_{B}/\rho_{0}) + [18.6 - F(x)][(\rho_{B}/\rho_{0})^{G(x)}]m_{\text{like}} + [18.6 - F(x)][G(x) - 1](\rho_{B}/\rho_{0})^{G(x)}\delta_{\text{like}}^2 \}$ being the nucleon symmetry potential. The parameters $\alpha = -293.4$ MeV, $\beta = 240.1$ MeV, and $\gamma = 1.216$ are chosen to give a compressibility of 212 MeV and a binding energy per nucleon of −16 MeV for symmetric nuclear matter at the saturation or normal nuclear density $\rho_{0} = 0.16$ fm$^{-3}$. The nucleon symmetry potential $U^{m_{\tau}}_{\text{asy}}(\rho_{B}, \delta_{\text{like}})$ depends on the baryon density $\rho_{B} = \rho_{n} + \rho_{p} + \rho_{\Delta^{0}} + \rho_{\Delta^{+}} + \rho_{\Delta^{-}}$ and the isospin asymmetry $\delta_{\text{like}} = (\rho_{n} - \rho_{p}/\rho_{B})$ of the asymmetric hadronic matter, which is a generalization of the isospin asymmetry $\delta = (\rho_{n} - \rho_{p})/\rho_{B}$ usually defined for asymmetric nuclear matter without $\Delta$ resonances. The nucleon mean-field potential also depends on the stiffness of nuclear symmetry energy through the parameter $x$ via the functions $F(x)$ and $G(x)$. We consider the three cases of $x = 0$, $x = 0.5$, and $x = 1$ with corresponding values $F(x = 0) = 129.98$ and $G(x = 0) = 1.059$, $F(x = 0.5) = 85.54$ and $G(x = 0.5) = 1.212$, and $F(x = 1) = 107.23$ and $G(x = 1) = 1.246$. The resulting nuclear symmetry energy becomes increasingly softer as the value of $x$ increases, with $x = 1$ giving a nuclear symmetry energy that becomes negative at about 3 times the normal nuclear matter density. These symmetry energies reflect the uncertainties in the theoretical predictions on the stiffness of nuclear symmetry energy at high densities. For the mean-field potentials of $\Delta$ resonances, their isoscalar potentials are assumed to be the same as those of nucleons, and their symmetry potentials are taken to be the average of those for neutrons and protons with weighting factors depending on the charge state of $\Delta$ resonance, i.e., $U^{\Delta^{+}}_{\text{asy}} = U^{p}_{\text{asy}}$, $U^{\Delta^{0}}_{\text{asy}} = \frac{3}{2}U^{p}_{\text{asy}} + \frac{1}{2}U^{n}_{\text{asy}}$, $U^{\Delta^{-}}_{\text{asy}} = \frac{1}{2}U^{p}_{\text{asy}} + \frac{3}{2}U^{n}_{\text{asy}}$, and $U^{\Delta^{+}}_{\text{asy}} = U^{n}_{\text{asy}}$.

Including the short-range $\Delta$-hole repulsive interaction via the Migdal parameter $g'$, which has values $1/3 \leq g' \leq 0.6$, modifies the pion self-energy...
to $\Pi^{m_\pi} = \Pi_0^{m_\pi}/(1 - g\Pi_0^{m_\pi}/k^2)$. The pion spectral function $S_\pi^{m_\pi}(\omega, k)$ is then related to the imaginary part of its in-medium propagator $D^{m_\pi}(\omega, k) = 1/[\omega^2 - k^2 - m_\pi^2 - \Pi^{m_\pi}(\omega, k)]$ via $S_\pi^{m_\pi}(\omega, k) = -(1/\pi)\text{Im}D^{m_\pi}(\omega, k)$.

The modification of the pion properties in nuclear medium affects the decay width and mass distribution of $\Delta$ resonance. For a $\Delta$ resonance of isospin state $m_T$ and mass $M$ and at rest in nuclear matter, its decay width is then given by $^{[25]}$

$$\Gamma^{m_\pi}_\Delta(M) \approx -2 \sum_{m_r, m_\pi} |(\frac{4}{3} m_T|1 m_r \frac{1}{2} m_\pi)|^2 \int \frac{d^3k}{(2\pi)^3} \left( \frac{f_{\Delta}}{m_\pi} \right)^2 F^2_\pi(k)$$

$$\times \left[ \frac{1}{z_\pi} \frac{1}{e^{(\omega - m_r \mu_Q)/T} - 1} + 1 \right] \left[ 1 - \frac{1}{e^{(m_N + k^2/2m_N + U^{m_\pi}_N - \mu_B - 2m_r \mu_Q)/T} + 1} \right]$$

$$\times \text{Im} \left[ \frac{k^2}{3} \left( 1 - g_\pi^{m_\pi} (\omega, k)/k^2 \right)^2 + g_\pi^{m_\pi} (\omega, k) \right].$$

In the above, the first term in the last line is due to the decay of the $\Delta$ resonance to pion but corrected by the contact interaction at the $\pi N \Delta$ vertex, while the second term contains the contribution from its decay to the $\Delta$-hole state without coupling to pion. The first two factors in the momentum integral take into account, respectively, the Bose enhancement for the pion and the Pauli blocking of the nucleon. To include possible chemical non-equilibrium effect, a fugacity parameter $z_\pi$ is introduced for pions. The pion energy $\omega$ is determined from energy conservation, i.e., $M + U^{m_\pi}_\Delta = \omega + m_N + k^2/2m_N + U^{m_\pi}_N$. The resulting mass distribution of $\Delta$ resonances is then given by $P_\Delta(M) = A[\Gamma^{m_\pi}_\Delta(M)/2]/[(M - m_\Delta)^2 + \Gamma^{m_\pi}_\Delta^2(M)/4]$, where $A$ is a normalization constant to ensure the integration of $P_\Delta(M)$ over $M$ is one.

We have solved Eqs. (1) and (2) self-consistently to obtain the pion spectral functions and the mass distributions of $\Delta$ resonances in asymmetric nuclear matter. The results obtained with the Migdal parameter $g^\prime = 1/3$ are illustrated in Fig. 1 and Fig. 2 for an asymmetric nuclear matter of isospin asymmetry $\delta_{\text{like}} \approx 0.133$, twice the normal nuclear matter density $\rho_B = 2\rho_0$, temperature $T \simeq 43.6$ MeV, and chemical potentials $\mu_B \simeq 941.89$ MeV and $\mu_Q \simeq -18.26$ MeV, corresponding to those to be used in our thermal model and also similar to those reached in the transport model with the nuclear symmetry energy $x = 1$ for central Au+Au collisions at the beam energy of 0.4 AGeV. $^{[13]}$ Shown in Fig. 1 are the pion spectral functions as functions of pion energy for different values of pion momentum. It is seen that for low pion momenta the spectral function at low energies has a larger strength for $\pi^-$ (dotted line) than for $\pi^0$ (solid line), which has a strength larger than that for $\pi^+$ (dashed line). This behavior is reversed for high pion energies. Fig. 2 shows the mass distributions of $\Delta$ resonances at rest in asymmetric nuclear matter as functions of mass. One sees that they are similar to that in free space (solid line) as a result of the cancelation between the pion in-medium effects, which enhance the strength at low masses, and the Pauli-blocking of the nucleon from delta decay, which reduces the strength at low masses. This is consistent with the observed
similar energy dependence of the photo-proton and photo-nucleus absorption cross sections around the $\Delta$ resonance mass. Furthermore, the strength around the peak and near the threshold of the $\Delta$ resonance mass distribution slightly decreases with increasing charge of the $\Delta$ resonance due to nonzero isospin asymmetry of the nuclear medium.

3. Charged pion ratio in hot dense asymmetric nuclear matter

To see the above isospin-dependent pion in-medium effects on the $\pi^-/\pi^+$ ratio in heavy ion collisions, we have used a thermal model which assumes that pions are in thermal equilibrium with nucleons and $\Delta$ resonances. In terms of the spectral

Fig. 1. (Color online) Spectral functions of pions in asymmetric nuclear matter of density $2\rho_0$ and isospin asymmetry $\delta_{\text{like}} = 0.133$ as functions of pion energy for different pion momenta of (a) $m_\pi$, (b) $2m_\pi$, (c) $3m_\pi$, and (d) $4m_\pi$. All are calculated with the Migdal parameter $g' = 1/3$. 

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function $S_i(\omega, k)$, the density of a particle species $i$ is then given by

$$\rho_i \approx g_i \int \frac{d^3k}{(2\pi)^3} d\omega n_i S_i(\omega, k) \frac{1}{z_i^{-1} e^{(\omega - B_i \mu_B - Q_i \mu_Q)/T} \pm 1}. \quad (3)$$

In the above, $g_i$, $B_i$, and $Q_i$ are the degeneracy, baryon number, and charge of the particle. The fugacity parameter $z_i$ is introduced to take into account possible chemical non-equilibrium effect. The exponent $n_i$ is 2 for pions and 1 for nucleons and $\Delta$ resonances. For the spectral functions of $\Delta$ resonances, we neglect their momentum dependence and replace the integration over the energy $\omega$ by that over mass. The $\omega$ in the Fermi-Dirac distribution for $\Delta$ resonances is then simply $\omega = M + k^2/2M + U^\Delta_m$. For nucleons, their spectral functions are taken to be delta functions if we neglect their imaginary part of their self-energies, i.e., $S^m_N(\omega, k) = \delta(\omega - m_N - k^2/2m_N - U^m_N)$.

According to studies based on the transport model\cite{12,13,23}, the total number of pions and $\Delta$ resonances in heavy ion collisions reaches a maximum value when the colliding matter achieves the maximum density, and remains essentially constant during the expansion of the matter. For Au+Au collisions at the beam energy of 0.4 AGeV, for which the $\pi^-/\pi^+$ ratio has been measured by the FOPI Collaboration at GSI\cite{15}, the IBUU transport model gives a maximum density that is about

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**Fig. 2.** (Color online) Mass distributions of $\Delta$ resonances at rest in asymmetric nuclear matter of density $2\rho_0$ and isospin asymmetry $\delta_{\text{like}} = 0.133$. The solid line corresponds to that in free space. The distributions near the threshold and at the peak are enlarged in the insets.
twice the normal nuclear matter density and is insensitive to the stiffness of the nuclear symmetry energy, as it is mainly determined by the isoscalar part of the nuclear equation of state \cite{14}. This density is thus used in the thermal model. The temperature in the thermal model is determined by fitting the measured pion to nucleon ratio, which is about 0.014 including pions and nucleons from the decay of \Delta resonances \cite{15}, without medium effects and with unity fugacity parameters for all particles, and the value is $T \approx 43.6$ MeV. The assumption that pions and \Delta resonances are in chemical equilibrium is consistent with the short chemical equilibration times estimated from the pion and \Delta resonance production rates. The isospin asymmetry of the hadronic matter is then taken to be $\delta_{\text{like}} \approx 0.080, 0.106,$ and 0.143, corresponding to net charge densities of $0.920\rho_0, 0.894\rho_0$ and $0.857\rho_0,$ for the three symmetry energies given by $x = 0, 0.5,$ and 1, respectively, in order to reproduce the $\pi^-/\pi^+$ ratios of 2.20, 2.40, and 2.60 predicted by the IBUU transport model of Ref. \cite{14} using corresponding symmetry energy parameters without pion in-medium effects. Since the medium effects enhance the pion and \Delta resonance densities, to maintain the same pion to nucleon ratio as the measured one requires the fugacity parameters for pions and \Delta resonances to be less than one. Also, the pion in-medium effects have been shown to affect only slightly the pion and \Delta resonance abundance \cite{23}, indicating that both pions and \Delta resonances are out of chemical equilibrium with nucleons when medium effects are included, as expected from the estimated increasing pion and \Delta resonance chemical equilibration times as a result of the medium effects. Because of the small number of pions (about 0.3\%) and \Delta resonances (about 1.1\%) in the matter, the density, temperature, and net charge density of the hadronic matter are expected to remain unchanged when the pion in-medium effects are introduced. They lead to, however, a slight reduction of the isospin asymmetry to $\delta_{\text{like}} \approx 0.073, 0.097,$ and 0.133 for the three symmetry energies, respectively. We note that with the fugacity of nucleons kept at $z_N = 1,$ the fugacity parameters of about $z_\pi = 0.061$ and $z_\Delta = 0.373$ are needed to maintain same total number of pions and \Delta resonances as in the case without pion in-medium effects, and that the required values for the fugacity parameters increase only slightly for the other two symmetry energies considered here.

Results on the $\pi^-/\pi^+$ ratio in Au+Au collisions at the beam energy of 0.4 AGeV are shown in Fig. \ref{fig:results}. With the value $g' = 1/3$ for the Migdal parameter, values for the $\pi^-/\pi^+$ ratio are 2.32, 2.60, and 2.94 for the three symmetry energy parameters $x = 0, 0.5,$ and 1, respectively, which are larger than corresponding values for the case without including the pion in-medium effects as shown by those for $g' = \infty$ in Fig. \ref{fig:results}. These results indicate that the isospin-dependent pion in-medium effects on the charged pion ratio are comparable to those due to the uncertainties in the theoretically predicted stiffness of the nuclear symmetry energy. The measured $\pi^-/\pi^+$ ratio of about 3 by the FOPI Collaboration, shown in Fig. \ref{fig:results} by the dash-dotted line together with the error bar, which without the pion in-medium effects favors a nuclear symmetry energy softer than the one given by $x = 1,$ is now best described by a less softer one.
Fig. 3. (Color online) The $\pi^-/\pi^+$ ratio in Au+Au collisions at the beam energy of 0.4 AGeV for different values of nuclear symmetry energy ($x = 0$, $0.5$, and $1$) and the Migdal parameter $g' = 1/3$, $0.4$, $0.5$, and $0.6$ in the $\Delta$-hole model for the pion $p$-wave interaction. Results for $g' = \infty$ correspond to the case without the pion in-medium effects.

Fig. 3 further shows the results obtained with larger values of $g' = 0.4$, $0.5$ and $0.6$ for the Migdal parameter. It is seen that the isospin-dependent pion in-medium effects are reduced in these cases compared to the case of $g' = 1/3$ as the repulsive interaction between $\Delta$-hole states becomes stronger, thus reducing the pion in-medium effects. With these larger values of $g'$, symmetry energies softer than that given by $x = 1$ are then needed to describe the measured $\pi^-/\pi^+$ ratio.

4. Pion $s$-wave interactions in nuclear medium

The above study does not include the $s$-wave interactions of pions with nucleons. Calculations based on the chiral perturbation theory have shown that the pion $s$-wave interaction modifies the mass of a pion in nuclear medium, and for asymmetric nuclear matter this effect depends on the charge of the pion.\cite{20} Up to the two-loop approximation in chiral perturbation theory,\cite{20} the self energies of $\pi^-$, $\pi^+$, and $\pi^0$ in asymmetric nuclear matter of proton density $\rho_p$ and neutron density $\rho_n$ are
given, respectively, by
\[
\Pi^{-}(\rho_p, \rho_n) = \rho_n[T_{\pi N}^- - T_{\pi N}^+] - \rho_p[T_{\pi N}^- + T_{\pi N}^+] + \Pi^{\text{rel}}_{\Pi}(\rho_p, \rho_n) + \Pi^{\Pi}_{\text{cor}}(\rho_p, \rho_n)
\]
\[
\Pi^{+}(\rho_p, \rho_n) = \Pi^{-}(\rho_n, \rho_p)
\]
\[
\Pi^{0}(\rho_p, \rho_n) = -(\rho_p + \rho_n)T_{\pi N}^+ + \Pi^{0}_{\text{cor}}(\rho_p, \rho_n).
\]

(4)

In the above, \(T^{\pm}\) are the isospin-even and isospin-odd \(\pi N\)-scattering \(T\)-matrices which have the empirical values \(T_{\pi N}^- \approx 1.847\) fm and \(T_{\pi N}^+ \approx -0.045\) fm extracted from the energy shift and width of the 1s level in pionic hydrogen atom. The term \(\Pi^{\text{rel}}_{\Pi}\) is due to the relativistic correction, whereas the terms \(\Pi^{\Pi}_{\text{cor}}\) and \(\Pi^{0}_{\text{cor}}\) are the contributions from the two-loop order in chiral perturbation theory. Numerically, it was found in Ref. [29] that changes of pion masses in asymmetric nuclear matter of density \(\rho = 0.165\) fm\(^{-3}\) and isospin asymmetry \(\delta = 0.2\) are \(\Delta m_{\pi^-} = 13.8\) MeV, \(\Delta m_{\pi^+} = -1.2\) MeV, and \(\Delta m_{\pi^0} = 6.1\) MeV.
although that around the peak still decreases with increasing $\Delta$ resonance charge. As a result, the $\pi^-/\pi^+$ ratio in Au+Au collisions at the beam energy of 0.4 AGeV is slightly reduced after the inclusion of both pion s-wave and p-wave interactions in asymmetric nuclear matter as shown in Fig. 4.

5. Summary

The pion spectral function in asymmetric nuclear matter becomes dependent on the charge of a pion. For the p-wave interaction of the pion, modeled by its couplings to the $\Delta$-hole excitations in nuclear medium, it leads to an increased strength of the $\pi^-$ spectral function at low energies relative to that of the $\pi^+$ spectral function in dense asymmetric nuclear matter. In a thermal model, this isospin-dependent effect increases the $\pi^-/\pi^+$ ratio from heavy ion collisions, and the effect is comparable to that due to the uncertainties in the theoretically predicted stiffness of the nuclear symmetry energy at high densities. However, including also the pion s-wave interaction based on results from the chiral perturbation theory reverses the isospin-dependent pion in-medium effects, leading instead to a slightly reduced $\pi^-/\pi^+$ ratio in neutron-rich nuclear matter. Taking into consideration of the isospin-dependent pion in-medium effects in the transport model thus would have some influence on the extraction of the nuclear symmetry energy from measured $\pi^-/\pi^+$ ratio.

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