IR RENORMALON EFFECTS AND LIGHT MESONS’ ELM FORM FACTOR

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Abstract

The pion and kaon electromagnetic form factors $F_M(Q^2)$ are calculated at the leading order of pQCD using the running coupling constant method. In calculations a dependence of the mesons distribution amplitudes on the hard scale $Q^2$ is taken into account. The Borel transform and resummed expression for $F_M(Q^2)$ are found.

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Investigation of the infrared (ir) renormalon effects in various inclusive and exclusive processes is one of the important and interesting problems in the perturbative QCD (pQCD) [1]. It is known that ir renormalons are responsible for factorial growth of coefficients in perturbative series for the physical quantities. But these divergent series can be resummed by means of the Borel transformation [2] and the principal value prescription [3] and effects of ir renormalons can be taken into account by scale-setting procedure \( \alpha_S(Q^2) \to \alpha_S(\exp(f(Q^2))Q^2) \) at the one-loop order results. Technically, all-order resummation of ir renormalons corresponds to the calculation of the one-loop Feynman diagrams with the running coupling constant \( \alpha_S(-k^2) \) at the vertices or, alternatively, to calculation of the same diagrams with non-zero gluon mass. Both these approaches are generalization of the Brodsky, Lepage and Mackenzie (BLM) scale-setting method [4] and amount to absorbing certain vacuum polarization corrections appearing at higher-order calculations into the one-loop QCD coupling constant. Studies of ir renormalon problems have opened also new prospects for evaluation of power suppressed (higher twist) corrections to processes’ characteristics [5].

Unlike inclusive processes exclusive ones have additional source of ir renormalon contributions [6-9]. Thus, integration over longitudinal fractional momenta of hadron constituents in the expression of the electromagnetic (elm) form factor [6-8] or, in the amplitude of exclusive subprocesses [9] generates ir renormalon corrections.

In this work we calculate the pion and kaon elm form factors using the running coupling constant method and by taking into account dependence of the mesons distribution amplitudes on the hard scale \( Q^2 \).

2. In the context of pQCD the meson elm form factor has the form,

\[
F_M(Q^2) = \int_0^1 \int_0^1 dx dy \phi_M^*(y, Q^2) T_H(x, y; Q^2, \alpha_S(\hat{Q}^2)) \phi_M(x, Q^2),
\]

where \( Q^2 = -q^2 \) is the square of the virtual photon’s four-momentum. In Eq.(1) \( \phi_M(x, Q^2) \) is the meson distribution amplitude, containing all non-perturbative hadronic binding effects, whereas \( T_H(x, y; Q^2, \alpha_S(\hat{Q}^2)) \) is the hard-scattering amplitude of the subprocess \( q\bar{q} + \gamma^* \to q\bar{q} \) and can be found using pQCD. At the leading order \( T_H \) is given by the following expression

\[
T_H(x, y; Q^2, \alpha_S(\hat{Q}^2)) = \frac{16\pi C_F}{Q^2} \left[ \frac{2\alpha_S(Q^2(1-x)(1-y))}{3(1-x)(1-y)} + \frac{1}{3} \frac{\alpha_S(Q^2xy)}{xy} \right].
\]

In Eq.(2) \( \hat{Q}^2 \) is taken as the square of the momentum transfer of the exchanged hard gluon in corresponding Feynman diagrams, \( C_F = 4/3 \) is the color factor.

One of the important ingredients of our study is the choice of the meson distribution amplitude \( \phi_M(x, Q^2) \). In the framework of pQCD it is possible to predict the dependence of \( \phi_M(x, Q^2) \) on \( Q^2 \) using evolution equation, but not its shape (its
dependence on \(x\). An information about the shape of the distribution amplitude \(\phi_M(x, Q^2)\) can be obtained by means of the QCD sum rules method or other non-perturbative approaches and as a result some model functions for \(\phi_M(x, Q^2)\) may be proposed. Model distribution amplitudes for the mesons are, therefore, objects of contradictory statements and conclusions made in the literature. Such discussions is particularly intensive in the case of the pion amplitude \(\phi_\pi(x, Q^2)\) [10,11]. Nevertheless, in this article for the pion we take ”traditional” model distribution amplitudes proposed in the literature [12-14]. They have the following form

\[
\phi_\pi(x, \mu_0^2) = \phi_{\text{asy}}^\pi(x) \left[ a + b(2x - 1)^2 + c(2x - 1)^4 \right],
\]

where \(\phi_{\text{asy}}^\pi(x)\) is the pion asymptotic distribution amplitude,

\[
\phi_{\text{asy}}^\pi(x) = \sqrt{3} f_\pi x (1 - x),
\]

and \(f_\pi\) is the pion decay constant; \(f_\pi = 0.093\ GeV\).

The constants \(a, b, c\) in Eq.(3) were found by means of the QCD sum rules method at the normalization point \(\mu_0 = 0.5\ GeV\) and have the values

\[
a = 0,\ b = 5,\ c = 0,\ CZ\ amplitude\ [12],
\]

\[
a = -0.1821,\ b = 5.91,\ c = 0,\ amplitude\ from\ Ref.[13],
\]

\[
a = 0.6016,\ b = -4.659,\ c = 15.52,\ amplitude\ from\ Ref.[14].
\]

The meson distribution amplitude evolves in accordance with the expression [12,15],

\[
\phi_M(x, Q^2) = \phi_{\text{asy}}(x) \sum_{n=0}^{\infty} r_n C_n^{3/2}(2x - 1) \frac{\alpha_s(Q^2)}{\alpha_s(\mu_0^2)} \frac{\gamma_n}{\beta_0}.
\]

In Eq.(6) \(\left\{C_n^{3/2}(2x - 1)\right\}\) are the Gegenbauer polynomials, \(\beta_0 = 11 - 2n_f/3\) is the QCD beta-function’s first coefficient and \(\gamma_n\) is the anomalous dimension,

\[
\gamma_n = \frac{4}{3} \left[ 1 - \frac{2}{(n + 1)(n + 2)} + 4 \sum_{j=2}^{n+1} \frac{1}{j} \right].
\]

The pion distribution amplitude (Eq.(3)) can be rewritten in the form (6). But after defining of the coefficients \(r_n\) and taking into account the evolution of \(\phi_\pi(x, Q^2)\) on \(Q^2\), for our purposes it is instructive to give to the distribution amplitude its old form, namely

\[
\phi_\pi(x, Q^2) = \phi_{\text{asy}}^\pi(x) \left[ a + \beta(2x - 1)^2 + \gamma(2x - 1)^4 \right].
\]
In Eq.(7) the new coefficients \( \alpha, \beta, \gamma \) are functions of \( Q^2 \)

\[
\alpha = a + \left( \frac{b}{5} + \frac{14c}{105} \right) (1 - A_2) - \frac{c}{21} (1 - A_4) , \\
\beta = bA_2 + \frac{14c}{21} (A_2 - A_4) , \\
\gamma = cA_4 .
\]

(8)

Here \( A_n \) is

\[
A_n = \left[ \frac{\alpha_s (Q^2)}{\alpha_s (\mu_0^2)} \right]^{\gamma_n/\beta_0}.
\]

Using the same method for the kaon we find

\[
\phi_K(x, Q^2) = \phi^K_{asy}(x) \left[ \alpha + \gamma (2x - 1) + \beta (2x - 1)^2 + \delta (2x - 1)^3 \right] ,
\]

(9)

\[
\alpha = a + \frac{b}{5} (1 - A_2) , \beta = bA_2 , \gamma = \frac{3c}{4} (A_1 - A_3) , \delta = cA_3 .
\]

In Eq.(9) \( \phi^K_{asy}(x) \) has the same form as in Eq.(4), but with \( f_\pi \) replaced by \( f_K = 0.122 \) GeV and constants \( a, b, c \) take values [12]

\[ a = 0.4 , \ b = 3 , \ c = 1.25 . \]

The QCD running coupling constant \( \alpha_s(\hat{Q}^2) \) in Eq.(2) suffers from ir singular-

\[ \text{ities associated with the behaviour of the } \alpha_s(\hat{Q}^2) \text{ in the soft regions } x \to 0 , \ y \to 0 ; \ x \to 1 , y \to 1 . \] Therefore, \( F_M(Q^2) \) can be found after proper regularization of \( \alpha_s(Q^2) \) in these soft end-point regions. For these purposes let us relate the run-

\[ \text{ning coupling } \alpha_s(\lambda Q^2) \text{ in terms of } \alpha_s(Q^2) \text{ by means of the renormalization group equation} \]

\[
\alpha_s(\lambda Q^2) \simeq \frac{\alpha_s(Q^2)}{1 + (\alpha_s(Q^2)\beta_0/4\pi) \ln \lambda} ,
\]

(10)

where \( \alpha_s(Q^2) \) is the one-loop QCD coupling constant.

3. As was demonstrated in our previous works [6-8], integration in Eq.(1) using

\[ \text{Eqs.}(2),(10) \text{ generates ir divergences and as a result for } F_M(Q^2) \text{ we get a perturba-
}

\[ \text{tive series with factorially growing coefficients. This series can be resummed using the Borel transformation } [2] , \]

\[
\left[ Q^2 F_M(Q^2) \right]^{res} = \frac{(16\pi f_M)^2}{\beta_0} \int_0^\infty \! du \exp \left( - \frac{4\pi u}{\beta_0 \alpha_s(Q^2)} \right) B \left[ Q^2 F_M \right](u) ,
\]

(11)

where \( B[Q^2 F_M](u) \) is the Borel transform of the corresponding perturbative series

[7,8].
But simple trick allow us from Eq.(1) directly obtain Eq.(11). In fact, after changing in Eq.(1) the variables $x$ and $y$ to $z = \ln(1 - x)$ an $w = \ln(1 - y)$ (or $z = \ln x$, $w = \ln y$) and using formula

$$\int_0^\infty e^{-u(t+w+z)} du = \frac{1}{t+w+z}, \quad t = \frac{4\pi}{\beta_0 \alpha_s(Q^2)},$$

and after integration over $z, w$ we find for the meson elm form factor the expression (11), where the Borel transform has the form

$$B \left[ Q^2 F_M \right](u) = \sum_{n=1}^N \left( \frac{m_n(Q^2)}{(n-u)^2} + \frac{l_n(Q^2)}{n-u} \right). \quad (12)$$

The exact expressions for $m_n(Q^2), l_n(Q^2)$ are given in the Appendix.

The exact expression (12) has double and single poles at $u = n$. Then the resummed expression (11) can be calculated with the help of the principal value prescription [3],

$$\left[ Q^2 F_M(Q^2) \right]^{res} = \frac{(16\pi f_M)^2}{\beta_0} \sum_{n=1}^N \left[ \frac{m_n}{n} + (l_n + m_n \ln \lambda) \frac{Li(\lambda^n)}{\lambda^n} \right], \quad (13)$$

where $Li(\lambda)$ is the logarithmic integral [16],

$$Li(\lambda) = P.V. \int_0^\lambda dx \ln x, \quad \lambda = Q^2/\Lambda^2. \quad (14)$$

In Eq.(13) we have taken into account the dependence of the distribution amplitude $\phi_M(x, Q^2)$ on the scale $Q^2$ and it is the generalization of our results for the pion and kaon elm form factor. Indeed, if we switch off this dependence ($A_n \equiv 1$), then Eq.(13) coincides for $a = 0, b = 5, c = 0$ and $N = 4$ with our result for the pion Ref.[7] and for $a = 0.4, b = 3, c = 1.25$ and $N = 5$ with the kaon elm form factor obtained in Ref.[8].

4. For studying phenomenological consequences of the ir renormalon contributions to $Q^2 F_M(Q^2)$, it is instructive to introduce the ratio $R_M = [Q^2 F_M(Q^2)]^{res}/[Q^2 F_M(Q^2)]^0$, where $[Q^2 F_M(Q^2)]^0$ is the meson elm form factor calculated in the context of the frozen coupling approximation.

In the frozen coupling approximation, for example, the pion form factor is

$$Q^2 F_\pi = 64\pi f_\pi^2 \left\{ \frac{a}{2} + \frac{b}{10} + \frac{3c}{70} + \frac{1}{15} \left( b + \frac{2c}{3} \right) A_2 + \frac{4c}{15 \cdot 21} A_4 \right\}^2. \quad (15)$$

In Fig.1 the dependence of the ratio $R_\pi$ on $Q^2$ (CZ distribution amplitude) is shown. As is seen the ir renormalon contributions enhance the perturbative result and are considerable for all values of $Q^2$, where the running coupling constant
method is applicable \((Q^2/\Lambda^2 >> 1, Q^2 \geq 2 \text{GeV}^2)\). The dependence of the distribution amplitude on the scale \(Q^2\) changes only the numerical results for the ratio \(R_\pi\), particularly in the region \(Q^2 \geq 5 \text{GeV}^2\), preserving at the same time the shape of the curve. The \(\text{ir}\) corrections can be transferred into the scale of \(\alpha_S(Q^2)\) in Eq.(15)

\[
Q^2 \rightarrow e^{f(Q^2)}Q^2, \quad f(Q^2) = c_1 + c_2\alpha_S(Q^2).
\]

Numerical fitting allow us to get (for \(n_f = 3\)) the values of \(c_1, c_2\);

\[
c_1 \simeq -8.528, \quad c_2 \simeq 26.28
\]

The similar numerical results can be obtained also for the \(R_\pi\) using other pion distribution amplitudes. For the kaon a more reliable form of \(f(Q^2)\) is \([8]\)

\[
f(Q^2) = c_1 + c_2\alpha_S(Q^2) + c_3\alpha_S^2(Q^2)
\]

where the coefficients found after numerical fittings are

\[
c_1 \simeq -1.304, \quad c_2 \simeq -35.604, \quad c_3 \simeq 127.25
\]

The results of numerical calculation for the kaon are depicted in Fig.2.

The obtained results have important consequences for a fate of the hard- scattering model based on pQCD at intermediate (moderate) energies \((2 \text{GeV}^2 \leq Q^2 \leq 10 \text{GeV}^2)\). It is well known that Eq.(1) was derived in the framework of pQCD by neglecting transverse momenta of meson’s constituents and higher Fock components of the distribution amplitude \([15]\). In Ref.[17] a modified perturbative QCD approach was proposed, which takes into account the partonic transverse momenta, as well as the Sudakov corrections. In the context of this approach the pion electromagnetic form factor was reexamined in works \([18,19]\). In these papers the authors used model light-cone wave functions and included also the unconventional helicity components \(h_1 + h_2 = \pm 1\) of the pion wave function (see, Ref.[19]). The authors of Ref.[18] made conclusion that the perturbative predictions are smaller than the experimental data, or that adding the \(h_1 + h_2 = \pm 1\) components suppresses the hard-scattering significantly and non-perturbative contributions will dominate in the present experimentally-accessible energy region \([19]\). But in these works in calculations the frozen coupling approximation was applied. We think that the running coupling constant method may improve the situation with the modified pQCD in the region of moderate \(Q^2\), because the \(\text{ir}\) renormalon effects work in opposite direction than factors considered in Refs.[18,19]. Therefore, we do not agree with Ref.[19], where the authors rule out the hard-scattering model for describing the light mesons form factors at moderate \(Q^2\).

Our results do not allow us also to exclude the CZ distribution amplitude as the reliable model function, because in deriving of Eqs.(12),(13) we have neglected the second term in Eq.(10) (Contopanagos, Sterman, Ref.[3]), as well as the second order corrections to \(T_{HH}\) \([20]\); before comparison with the experimental data \([21]\), these points must be clarified.
**APPENDIX**

The functions $m_n(Q^2)$, $l_n(Q^2)$ for the pion have the forms (N=6),

$$m_1(Q^2) = (\alpha + \beta + \gamma)^2, \quad m_2(Q^2) = (\alpha + 5\beta + 9\gamma)^2, \quad m_3(Q^2) = 64(\beta + 9\gamma)^2,$$

$$m_4(Q^2) = 16(\beta + 14\gamma)^2, \quad m_5(Q^2) = 2304\gamma^2, \quad m_6(Q^2) = 256\gamma^2,$$

and

$$l_1(Q^2) = -2(\alpha + \beta)\left(\alpha + \frac{7}{3}\beta\right) - \frac{2}{15}\gamma(58\alpha + 90\beta + 43\gamma),$$

$$l_2(Q^2) = 2\left(\alpha^2 - 25\beta^2\right) - 2\gamma(6\alpha + 120\beta + 135\gamma),$$

$$l_3(Q^2) = 8\beta(\alpha + \beta) + \frac{8}{3}\gamma(12\alpha - 161\beta - 692\gamma),$$

$$l_4(Q^2) = \frac{4}{3}\beta(35\beta - \alpha) - \frac{4}{3}\gamma(14\alpha - 417\beta + 1022\gamma),$$

$$l_5(Q^2) = 8\gamma(\alpha + 17\beta + 321\gamma), \quad l_6(Q^2) = \frac{8}{15}\gamma(1717\gamma - 3\alpha - 20\beta).$$

For the kaon we get (N=5),

$$m_1(Q^2) = (\alpha + \beta)^2 + (\gamma + \delta)^2 + \frac{2}{3}(\alpha + \beta)(\gamma + \delta),$$

$$m_2(Q^2) = (\alpha + 5\beta)^2 + (3\gamma + 7\delta)^2 + \frac{2}{3}(\alpha + 5\beta)(3\gamma + 7\delta),$$

$$m_3(Q^2) = 64\beta^2 + 4(\gamma + 9\delta)^2 + \frac{32}{3}\beta(\gamma + 9\delta),$$

$$m_4(Q^2) = 16\beta^2 + 400\delta^2 + \frac{160}{3}\beta\delta, \quad m_5(Q^2) = 64\delta^2,$$

and

$$l_1(Q^2) = -2(\alpha + \beta)\left(\alpha + \frac{7}{3}\beta\right) - 4(\gamma + \delta)\left(\gamma + \frac{4}{3}\delta\right) - \frac{2}{9}(9\alpha\gamma + 11\alpha\delta + 13\beta\gamma + 15\beta\delta),$$

$$l_2(Q^2) = 2\left(\alpha^2 - 25\beta^2\right) - 2(3\gamma + 7\delta)\left(\gamma + \frac{29}{3}\delta\right) + \frac{2}{9}(6\alpha\gamma - 8\alpha\delta - 60\beta\gamma - 250\beta\delta),$$

$$l_3(Q^2) = 8\beta(\alpha + \beta) + 2(\gamma + 9\delta)(5\gamma - 19\delta) + \frac{2}{3}(\alpha\gamma + 9\alpha\delta + 21\beta\gamma - 67\beta\delta),$$

$$l_4(Q^2) = \frac{4}{3}\beta(35\beta - \alpha) + \frac{20}{3}\delta(5\gamma + 41\delta) + \frac{2}{9}(-10\alpha\delta + 10\beta\gamma + 432\beta\delta),$$

$$l_5(Q^2) = 4\delta\left(-\gamma + \frac{157}{3}\delta\right) + \frac{4}{9}\delta(\alpha + 17\beta).$$
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FIGURE CAPTION

Fig.1 The ratio $R_\pi$ for the pion (CZ amplitude) is shown as a function of $Q^2$. The solid curves 1 and 2 correspond to $R_\pi$ with $[Q^2 F_\pi(Q^2)]^0$ in the frozen coupling approximation and after the scale-setting procedure, respectively. The dashed curve is the ratio $R_\pi$ from Ref.[7], where the dependence of the pion distribution amplitude on the hard scale $Q^2$ was neglected. In calculations the QCD parameter $\Lambda$ is taken to be equal to 0.1 $GeV$.

Fig.2 The same, but for the kaon. The dashed curve is taken from Ref.[8].
