Cooperative MIMO Communications in Wireless Ad Hoc Rayleigh Fading Networks

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Abstract - In this paper addresses Rayleigh fading networks, and in particular, wireless ad-hoc and sensor networks over Rayleigh fading channels. First, we will model Rayleigh fading networks and show how to map the wireless fading channel to the upper layer parameters for cross-layer design. Based on the developed fading network model, we will consider two scarce resources of wireless networks, namely energy and medium, and develop a cross-layer way to improve their efficiency. In particular, we will first study the energy-efficiency and introduce a new parameter, Energy Cost Factor, as the counterpart of Transport Capacity in wireless transmission. We investigate the issue of cooperative node selection in MIMO communications for wireless ad hoc/sensor networks, where a source node is surrounded by multiple neighbors and all of them are equipped with a single antenna. In order to optimize system performance, we jointly consider the optimization of all these parameters, given the aforementioned system constraints. We assume that the source node either has channel state interference (CSI), or has no CSI. Heuristic algorithms, such as maximal channel gain (MCG) and least channel correlation (LCC) algorithms are proposed in order to exploit available system information and to solve the constrained optimization problem. Finally, we will give a general discussion on the cross-layer design and show how power control and route selection jointly contribute to improving the resource efficiency. A few particular routing algorithms will also be studied in detail.

Keywords - Wireless ad-hoc and sensor networks, Rayleigh fading, energy, constellation size, delay, optimization, channel state information (CSI), Maximal channel gain (MCG).

I. INTRODUCTION

The modern wireless communications technology has an enhanced spectral efficiency can be achieved by the use of multiple-input-multiple-output (MIMO) systems. Recently, MIMO has attracted extensive attention and various techniques have been proposed for both cellular systems and ad hoc networks [1],[2] to achieve improved system performance. However, in wireless ad hoc/sensor networks, direct employment of MIMO to each node might not be feasible, since MIMO might require complex transceiver and signal processing modules, which result in high power consumption. Furthermore, nodes in wireless ad hoc networks/sensor networks are often powered by batteries with limited energy. Wireless networks, such as ad-hoc and sensor networks, are composed of a set $N$ of nodes sharing a common wireless medium. Such networks have been attracting intense attention for their huge potential in the application. However, there are still many open problems at present, which hamper the effective design of high-quality wireless networks. In this article, we will deal with some of them, giving both theoretical results and practical design strategies.

Energy is widely recognized as a scarce resource in wireless networks [3] and various energy-aware routing [4] and topology control algorithms [5] were proposed. However, most of the previous works are based on the simple “Disc Model”. Owing to the lack of well-matched physical models, the energy consumption functions adopted by previous works are usually inaccurate. In this paper, we will study the energy consumption problem based on the Rayleigh fading network model, which will lead to more practical results. On the other hand, in many energy aware routing algorithms, the total energy consumption of a whole multi-hop route is used as the criterion, which typically requires central control [3]. In contrast, if a node can decide its most “economical” step based on local information, some greedy algorithms can be implemented in a distributed way while maintaining a comparable performance with their centralized counterparts. In this context, energy is the resource being consumed, and a step is said to be the most “economical” if it completes the largest amount of information transport with the same energy. How to measure the information transport quantity is of paramount importance here. Obviously, the number of packets, as in wired networks, is not a complete answer. In fact, information transmission in wireless networks is greatly different from that in wired networks.
However, the focus of the previous work (with a noticeable exception in [6]) is just one part of the entire cooperation procedure. More specifically, in order to achieve cooperative MIMO, a source node should first distribute data information to other cooperative nodes; this is the first stage or the “local distribution” stage. After each cooperative node receives information from the source node, the second stage is carried out by using a particular cooperative protocol, where the source node and the cooperative nodes collaborate together to form a virtual MIMO system and transmit to the destination node. The second stage is sometimes referred to as “long haul” transmission. Most previous work, such as [7-12], only focused on the second stage, without considering the effects in the first stage. In order to have a complete view of cooperative MIMO in wireless networks, both stages should be jointly considered.

It is widely acknowledges that the design of wireless fading networks should follow a cross-layer way. For instance, as will be shown in this paper, the route selection on the network layer plays an important role in determining the energy and medium consumption. Meanwhile, wireless networks are constrained by different factors, which leads to different design goals. As discussed above, energy and medium are two scarce resources in wireless networks, which should be exploited with caution. In light of these, we will give a general discussion on the crosslayer design of wireless fading networks, and show how power control and route selection jointly contribute to improving the resource efficiency. Though, the energy-efficiency and the medium-efficiency are not compatible all the time, both of them can be improved by the same procedure, which demonstrates a cross-layer way. Finally, we investigate how to select the cooperative nodes and how to solve the optimization problem where the source node either has perfect instantaneous channel state information (CSI), or the source node only knows the channel correlation information. It is worth noting that the problem of cooperative node selection is similar to the problem of antenna selection in MIMO [13] and [15], but in this paper, it is applied with distinct application scenarios and different system constraints.

II. SYSTEM MODEL

A.1 Rayleigh fading network model

Consider a wireless network, which consists of a set $N$ of nodes sharing a common wireless medium. The links between the node pairs are assumed to be independent narrow-band Rayleigh block fading channels. That is, each link is assumed to be constant during a packet transmission and vary from packet to packet independently according to a Rayleigh distribution. A block transmission from node $i$ to node $j$ is successful if and only if the signal-to-interference and noise ratio (SINR), $y_{ij}$, at the receiver is above a threshold $\Theta$, which is determined by the receiver design. Haenggi found that, under the Rayleigh fading assumption, the successful transmission probability from node $i$ to node $j$, $p_{ij} = P[y_{ij} > \Theta]$, can be factorized into that of a zero-noise network and that of a zero-interference network. That is, $p_{ij} = p^N_{ij}p^I_{ij}$, where

$$p^N_{ij} = \exp\left( -\frac{\Theta N_0}{P d_{ij}^\alpha} \right) < 1 \quad (1)$$

$$p^I_{ij} = \prod_{k \neq N, k \neq i} \frac{1}{1 + \Theta \frac{P_k d_{ij}^\alpha}{P_i d_{ij}^\alpha}} < 1 \quad (2)$$

In the above equations, $P_i$ denotes the transmit power of node $i$, $\alpha$ is the path loss factor, $d_{ij}$ is the distance between $i$ and $j$, $N_0$ is the noise power, and $P_k d_{ij}^\alpha$ is the average receive power at node $j$ from $i$. $p^N_{ij}$ and $p^I_{ij}$ represent the effect of noise and interference alone, respectively.

![Fig. 1: An illustration of line networks design.](image)

In addition, only $p^N_{ij}$ is affected by the absolute power level of the network, and $p^I_{ij}$ remains constant when scaling the transmit powers of all nodes by the same factor. With these in mind, we will focus on zero-interference networks, in which

$$p_{ij} = P[y_{ij} > \Theta] \approx p^N_{ij} = \exp\left( -\frac{\Theta N_0}{P d_{ij}^\alpha} \right) \quad (3)$$

With respect to the upper layer operations, we shall assume that ARQ is utilized. That is, whenever a block or packet transmission from node $i$ to node $j$ fails, which is determined by node $j$ after reception and decoding, node $i$ retransmits the same packet until its successful reception.

Thus, under the assumption that all packets have unitary transmission time, the time $T_j$ required for the successful transmission of a packet from $i$ to $j$, which includes the initial transmission and possible
retransmissions, is a Geometrically distributed random variable with probability distribution function

\[ P[T_{ij} = n] = p_{ij}(1 - p_{ij})^{n-1} \]  

(4)

Where \( p_{ij} \) is the probability of successful transmission from \( i \) to \( j \) as defined above. Note that, only the time for transmission and retransmission is counted in \( T_{ij} \). That is, the time for feedback from \( j \) to \( i \) is neglected, as it is not spent on transmitting the data packet under study at node \( i \). We have now specified the basic characteristics and operations of a Rayleigh fading network. Such model covers a large category of practical wireless networks. More assumptions on node locations will be added for particular networks in the following sections.

A.2 Energy-Efficiency and Energy Cost Factor

Information transmission in wireless networks differs from wired networks significantly. Among the differences, the geographical distance between the source and destination plays an important role. While in wired networks, the high-quality of wired links actually conceals the impact of geographical distance between nodes, the distance is crucial in determining the quality of a wireless link. In their milestone work, Gupta and Kumar brought forward a new concept, Transport Capacity, which takes the distance between the source and destination into account in measuring the capacity of wireless networks. Further studied the transport capacity of wireless networks without or with channel fading, respectively. These works inspire our defining the new parameter Energy Cost Factor. In wireless networks, Energy Cost Factor, denoted by \( \eta \), is defined as the expected energy consumption per information transmission unit. Here, information transmission is measured by the product of the number of packets and the distance they travel toward their destination, which is similar to the idea in Transport Capacity.

\[ \eta_{ij} = \frac{E[P_{ij}]}{d_{ij} - d_{jg}} = \frac{P_i}{d_{ij} - d_{jg}} \exp\left( \frac{\Theta N_0}{P_i d_{ij}^{-\alpha}} \right) \]  

(5)

As to a give route from node \( s \) to \( g \), the expected overall energy consumed by a packet.

In particular, for linear networks in which packets are transmitted along a straight line and thus \( d_{ij} = d_{ig} - d_{jg} \), can be reduced to,

\[ d_{ij} = d_{ig} - d_{jg} \]  

(6)

Obviously, for high energy-efficiency, a low Energy Cost Factor is desirable. Actually, the inverse of the Energy Cost Factor is as an indicator of energy-efficiency, in the sense that it shows how much information transmission per unit of energy can support. According to (6), both route selection and power control can contribute to minimizing \( \eta \).

III. POWER CONTROL FOR FIXED NETWORK TOPOLOGY

In many wireless sensor networks, the locations of network nodes are determined by the object that needs to be observed, and thus the network topology and the route are fixed. In such cases, the problem of power control for fixed network topology arises. Assume node \( i \) and \( j \) are two successive nodes along a route from \( s \) to \( g \), and the distance between them is \( d_{ij} \). The problem of finding the optimum transmit power \( P_i \) that minimizes the Energy Cost Factor can be formulated as follows,

\[ P_{ij}^{(i)} = \Theta N_0 d_{ij}^{-\alpha} \]  

(9)

A.1 Medium Resource And Interference Factor

In this section, we will deal with the medium resource and its consumption in Rayleigh fading networks. First, we will discuss the Medium Resource Space, which is shown in Fig.2. It not only organizes various medium resources in a systematic way but also considers a third dimension that is related to “space reuse” and “intermode interference” in multuser networks.

![Fig. 2: The evolution of Medium Resource Space.](image-url)
This leads to a relatively low throughput (1/e packet/unit time) and large end-to-end delay. Obviously, a trade-off between the energy-efficiency and the throughput, as well as the end-to-end delay, should be addressed based on particular scenarios.

A.2 Medium Resource Space

Let us begin with the point-to-point packet transmission. Generally, packets are carried out by signals that span a band of frequency and last for a period of time. The number of packets delivered by the system is linearly proportional to the product of the spectrum bandwidth and the time duration. Thus, the frequency axis and the time axis come up as the first two dimensions of the Medium Resource Space. In multiple access networks and broadcast networks, the two dimensional resource space spanned by time and frequency an be shared by multiple users, by means of FDMA, TDMA or more generally CDMA. Each user occupies a unit combination of frequency and time, which will be referred to as a channel hereafter. However, the total number of packets that can be transmitted is the same as the point-to-point case. This naturally leads to the notion that the total amount of medium resource also keeps the same as before.

IV. TRANSMISSION OPTIMIZATION AND COOPERATIVE NODE SELECTION

A.1 Long Haul Transmission Optimization

Given that \(N\) nodes have been chosen for the long haul transmission, the target is to find the power/bit allocations for each of these selected nodes under the transmit power constraint, \(P_T\), and the total bit rate constraint, \(b_T\). Therefore, the optimization problem for the long haul transmission stage is given by

\[
\max_{P, b_T} d_0^2
\]

subject to

\[
\begin{align*}
(1) & \quad \sum_{i=1}^{N} p_i = P_T \\
(2) & \quad \sum_{i=1}^{N} \log_2(B_i) = b_T
\end{align*}
\]

(10)

Since \(d_0 = \frac{d_i}{d_0}\), from (8), we have

\[
B_i = \frac{6P_iK}{d_0^2} + 1
\]

(11)

Plugging (10) into the second constraint in (11), we obtain the following equation:

\[
\sum_{i=1}^{N} \log_2 \left( \frac{6P_iK}{d_0^2} + 1 \right) = b_T
\]

\[
\Rightarrow \log_2 \left( \prod_{i=1}^{N} B_i = \frac{6P_iK}{d_0^2} + 1 \right) = b_T
\]

(12)

\[
\max_{P_i} \left\{ \prod_{i=1}^{N} 6P_iK \right\}^2 \quad s.t. \quad 1 \sum_{i=1}^{N} P_i = P_T
\]

(13)

It is clear that in order to maximize (13), we need to maximize the product of \(P_i\), i.e., \(P_1, P_2... P_N\), since the channel gain is \(P_i\). Each user occupies a unit combination of frequency and time, which will be referred to as a channel hereafter. However, the total number of packets that can be transmitted is the same as the point-to-point case. This naturally leads to the notion that the total amount of medium resource also keeps the same as before.

A.2 Cooperative Node Selection

In what follows, we describe the heuristic algorithms in two different scenarios, i.e., perfect instantaneous CSI is available at the source node or only the channel correlation information is available at the source node. As shown below, each of these two algorithms will only need to search a subspace with \(K\) possible cooperative node combinations, which is much less than that required by an exhaustive search. Among the \(K\) combinations, only the one achieving the best end-to-end performance while meeting the specified total end-to-end delay and energy constraints will be used for the transmission. It is worth noting that, compared to the exhaustive search, the proposed heuristic algorithms result in only marginal performance degradation as shown later. Perfect Full CSI is Available. In this case, the source node knows the instantaneous CSI between all the \(K\) cooperative nodes and the destination node, i.e., the channel gain matrix \(H\) with dimension \(R\times K\), and the correlation information among all the nodes.

\[
\prod_{i=1}^{N} |R_{ii}|^2 = \prod_{i=1}^{N} |\lambda_i(R)|^2 = \det(H^H H)
\]

(14)

To accomplish this, consider the use of a maximal channel gain (MCG) algorithm as follows: at the \((k+1)th\) step, where \(k\) nodes have already been chosen, and the corresponding channel matrix \(H(\hat{k})\) are known, where \(H(\hat{k})\) is the channel matrix when \(k\) nodes are chosen, we want to select one additional node \(S^*\) from the set \(S\) containing the remaining \(K-k\) nodes such that

\[
S^* = \arg \max_{S^* \subset S} \left\{ \det \left( H(\hat{k+1})^H H(\hat{k+1}) \right) \right\}
\]

(15)
We repeat this until all the $K$ nodes are chosen. Therefore, at each step, we obtain a selected combination of nodes, $\phi$, with an increasing number of nodes in it. In total, the algorithm runs $K$ steps, thus the search space for the previous optimization problem has only $K$ combinations. Finally, we choose the optimal subset $\phi^*$ which results in the largest $\Delta_0^*$ for the cooperative transmission while meeting the specified total end-to-end delay and energy constraints. Since the MCG algorithm only searches a small subset of possible combinations instead of searching all combinations, it would induce performance drop, however, the performance degradation as shown later is only marginal, which demonstrates the effectiveness of the proposed MCG algorithm.

V. SIMULATION RESULTS

Fig. 3: The probability that Algorithm I and Algorithm II make the same decision

Fig. 4: The average cost function of Algorithm II, $k=2, c=1$

Fig. 5: The effective hop length of Algorithm I

Fig. 6: Effective of channel correlation MCG with perfect CSI, $T_0=0.41s$ and $E_0=0.8j$

Fig. 7: Performance comparison of MCG with distinct channel estimation error, $T_0=0.41s$ and $E_0=0.8j$

In order to verify our theoretical analysis above and compare the performance of the routing algorithms, we carry out simulations. A Poisson Planar Network is set up in a $40 \times 40$ square region with node density $\rho = 1$. To
eliminate the edge effect, we focus on the route selection between the nodes in the central $20 \times 20$ square region. Let us begin with the cost function of a whole route, i.e. $h_c$. We search route and calculate its $h_c$ for each node pair in the central square region, under the assumption that $c = 1$. Then, $h_c$ is averaged over all node pairs. The average $h_c$ of different algorithms. As expected, the performance of the Centralized Algorithm is the best. The three distributed algorithms are just slightly inferior, especially when $h_c$ is small. When $h_c$ gets large, the gap between the centralized and distributed algorithms increases. This is because, for large $h_c$, the optimal route given by the centralized algorithm tends to have short steps and does not necessarily move the packets nearer to its destination in each step, even if this results in a longer route than the distributed algorithms, which requires to move the packets toward the destination in each step.

Fig. 6 shows the effect of $\phi$ and $n$ in Algorithm II on the simplified cost function of each hop, i.e. $f_\phi(i,j)$, and the cost function of the whole route, in the case of $\kappa = 2$. First of all, it is shown that the two curves at the bottom are close to each other. Fig. 7 shows the effective hop length of Algorithm II. First, the three curves at the bottom are close to one another, which verifies the theoretical result. Besides, as expected, it is shown that as $\phi$ decreases or $n$ increases, the effective hop length increases. However, according to Fig. 6, this also leads to larger cost function values, which is undesirable. That is, there is a trade-off between the effective hop length and the cost function, or equivalently between the end-to-end delay and the resource-efficiency. These should be addressed based on practical scenarios.

On the other hand, when no CSI is available, we can implement the proposed node selection algorithm by making use of the channel correlation information, and this can be achieved by the proposed modified MCG algorithm with no CSI and the LCC algorithm. Since the two algorithms exploit the same channel correlation information, they result in similar performance which is not distinguishable from the figure. However, as discussed previously, we also observe that when the channel correlation increases, system performance is degraded for all the algorithms. On the other hand, it is worth noting that when the correlation level increases, the performance gap between MCG with perfect instantaneous CSI and either the MCG without CSI or the LCC decreases. That means, the proposed MCG without CSI and the LCC algorithms can result in more performance gain in scenarios with high channel correlation. We present the effect of correlation on the system performance. As can be seen, when channel correlation increases, the system performance degrades accordingly. In CG algorithm with channel estimation error. The channel estimation error is modeled as a complex Gaussian random variable with zero mean and variance $\sigma^2$. The estimation error can be caused by various factors, such as feedback delay and node mobility. The estimated CSI is assumed to be the sum of the true CSI plus the random variable with zero mean and variance $\sigma^2$. The MCG algorithm with perfect CSI achieves the best performance, and as estimation error increases, system performance degrades. It is worth noting that the dotted curve is achieved with the MCG algorithm with no CSI, but with channel correlation information which is not affected by the channel estimation error. It is clear that when the channel estimation error gets large, it is desirable to use the channel correlation information instead of the erroneous CSI.

**VI. CONCLUSION**

In this paper, we modeled the Rayleigh Fading Network, from the physical layer to the network layer, which facilitates the cross-layer network design. We analyzed two scarce resources of MIMO wireless networks, namely energy and medium resource, and discussed how to improve their efficiency by power control and route selection. In addition, the Medium Resource Space was constructed, which not only organizes various resources in a systematic way but also formulate “space reuse” and “internode interference” in terms of medium resources. More specifically, we have taken into account a complete view of the node cooperation procedure, under the specified system constraints, such as the energy and delay constraints. Finally, the subset of cooperative nodes participating in the virtual MIMO communication is chosen by considering the overall system constraints, and the power level and data rate for each selected cooperative node are adaptively assigned in order to optimize the system performance.

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