High-Frequency Electromagnetic Inversion for a Dispersive Layered Earth

Hee Joon Kim¹, Yoonho Song², and Ki Ha Lee³

¹Pugyong National University, Pusan 608-737, Korea
²Korea Institute of Geology, Mining and Materials, Taejon 305-350, Korea
³Lawrence Berkeley National Laboratory, Berkeley CA 94720, U.S.A.

(Received October 25, 1996; Revised February 12, 1997; Accepted May 26, 1997)

Electrical properties in most geologic materials have been known to be frequency dependent, and resulting dispersion relationship can be a useful diagnostic tool for investigating the shallow subsurface. In this paper we investigate the determination of dispersive electrical properties of the shallow subsurface with inversion of high-frequency electromagnetic (EM) fields. We have limited the dispersive characteristics to the electrical permittivity and used the Cole-Cole model to describe the frequency dependence of the permittivity. For horizontally layered earth models high-frequency EM fields are successfully inverted via Marquardt-Levenberg least-squares method and simulated annealing method. Inversion experiments show that the simulated annealing yields slightly better parameter resolution than the least-squares inversion.

1. Introduction

An important application of electromagnetic (EM) methods is shallow subsurface investigation for engineering and environmental problems. EM methods have been successfully applied to exploration of mineral and energy resources, reservoir characterization for oil and geothermal production and management, and crustal soundings for earthquake and general geologic studies. There is an increasing need for developing reliable methods for characterizing and monitoring subsurface processes critically important in environmental remediation and containment operations. Many environmental sites require non-invasive monitoring techniques in order to minimize the risk of further contamination. High-frequency EM methods are ideal for this purpose since EM induction and wave propagation mechanisms do not require physical contacts with the medium under investigation.

Ground penetrating radar (GPR), for example, is an excellent tool to obtain shallow subsurface stratigraphic information using reflection surveys, and an overview of the utility of this tool is given by Annan (1996). The application of GPR, however, is limited to areas where the electrical conductivity is low because of severe attenuation of transmitted EM energy with increasing conductivity. When electrical properties of geologic materials are frequency dependent at high frequencies, GPR is not suitable for handling data severely affected by dispersion. In cross borehole configurations, borehole radar can be used to image electrical properties of the medium between boreholes using raytracing. Scattering by spurious electrical heterogeneities including clay minerals often causes difficulties in picking travel times. In addition, since raytracing can handle models with moderate contrasts in electrical properties, application of this configuration may also be limited in terms of its practicality.

Another limitation of GPR is that it is difficult to get the information of conductivity distribution. In interpreting GPR data, pulse transit time is normally used to estimate the velocity distribution that is dependent on the electrical permittivity. Instead, the high-frequency EM method presented here involves full waveform EM fields in layered-earth environments, which makes it possible to resolve the electrical conductivity as well as the electrical permittivity even if they are frequency dependent. Frequency-dependent material properties can be fully incorporated in generating the frequency responses, and the time-domain results may be obtained by inverse Fourier transform of the frequency responses. For this
purpose we have used a numerical code EM1D (Pellerin et al., 1995) to generate high frequency responses of dispersive layered earth. The focus of this paper concerns the inversion of electrical parameters of the layered earth at shallow depths. The inversion parameters consist of layer thickness, electrical conductivity, and electrical permittivity. The electrical permittivity can be either constant or frequency dependent. The magnetic permeability has been fixed as its free space value throughout this study but could, in principle, be handled in the same way as the electrical permittivity is.

2. Model Description

Electrical properties of most geologic materials often show significant frequency-dependent characteristics, while magnetic properties tend to be of less important concern (Keller, 1987). The Cole-Cole model (Cole and Cole, 1941) has been widely used as a dispersion or relaxation model in describing the frequency dependence of materials. In this study we used the Cole-Cole model for describing the dispersion of layered earth, but the application has been limited to the dielectric permittivity. Electrical conductivity of certain minerals and rocks may also be dispersive and can be represented by a Cole-Cole type model, but the dispersion takes place at a much lower frequency range. Spectral induced polarization method (Seigel, 1959; Swift, 1973; Pelton et al., 1978; Johnson, 1984; to list a few) is a useful exploration tool that takes advantage of the low-frequency dispersion in the electrical conductivity. Magnetic permeability may also show frequency-dependent characteristics, but in most geophysical applications its free space value is used throughout, so the dispersion is not considered.

In the frequency domain the generalized Cole-Cole expression of the dielectric permittivity, $\varepsilon(\omega)$, is given by

$$\varepsilon(\omega) = \varepsilon(\omega) + \frac{\varepsilon(0) - \varepsilon(\omega)}{1 + (i\omega\tau)^{\alpha}},$$

where $\omega$ is the angular frequency and $i = \sqrt{-1}$. Symbols $\varepsilon(\omega)$, $\varepsilon(0)$, $\tau$, and $\alpha$ are the high-frequency limit of electrical permittivity, the low-frequency limit, the time constant of relaxation, and the Cole-Cole relaxation distribution parameter, respectively. Figure 1 shows typical amplitude and phase spectra described by Eq. (1). Two Cole-Cole models are shown in this figure. Both are homogeneous half space models whose values for relaxation parameters ($\tau$ and $\alpha$) are identical to those for Layer 1 of Table 1. The low-frequency limit value used is $\varepsilon(0)/\varepsilon_0 = 10$ for both models, while the high-frequency limit values are $\varepsilon(\omega)/\varepsilon_0 = 1$ for the first model and $\varepsilon(\omega)/\varepsilon_0 = 5$ for the second model. The relaxation mechanism causes effective permittivity to decrease with increasing frequency. Also, there can be seen a negative peak of phase data, the frequency of which is mainly dependent on $\tau$ and $\alpha$, which in turn are functions of intrinsic electrochemical as well as physical properties of material.

| Parameter    | Layer 1 | Layer 2 |
|--------------|---------|---------|
| $\sigma$ (S/m) | 0.01    | 0.1     |
| $\varepsilon(0)/\varepsilon_0$ | 10      | 30      |
| $\varepsilon(\omega)/\varepsilon_0$ | 5       | 10      |
| $\tau$ (sec) | $5 \times 10^{-7}$ | $7 \times 10^{-7}$ |
| $\alpha$    | 0.5     | 0.7     |
| $h$ (m)     | 2       | $\infty$ |
To investigate the effect of dispersive permittivity on EM fields, we consider a simple two-layered earth model (Fig. 2). The thickness of the first layer is fixed to 2 m. The model is energized by a vertical magnetic dipole source, and three components of EM fields ($E_y$, $H_x$, and $H_z$) are computed at a position 2 m away from the dipole source. Other types of EM source may be considered but the vertical magnetic dipole source generates only one mode of excitation (TE mode), and as a result the successive derivation of response may be simplified and the interpretation focused. Electrical properties of each layer are given in Table 1. The range of frequencies considered is 100 kHz to 50 MHz, which is just below the typical GPR frequency range and above standard EM systems.

3. Frequency-Domain Responses

We compute frequency-domain EM fields due to a vertical magnetic dipole source over a horizontally layered earth using the code EM1D. The code is capable of handling Cole-Cole type earth models with the source and receiver arbitrarily located anywhere in the layered earth. Kernel functions of required Hankel transforms are found to undergo a rapid change at high frequencies, rendering the convolution method of
evaluating the Hankel transform inappropriate. Even an accurate Gaussian quadrature scheme (Chave, 1983) fails to work properly at high frequencies if contributions across the branch point in the kernel function are not handled carefully.

EM fields in the frequency range from 100 kHz to 50 MHz have been computed for a family of several earth models. These models correspond to the parameter variations in the first layer, and unless otherwise specified, all other parameters take values given in Table 1. In this manner we demonstrate the effect of model variation on the observed synthetic EM field data. The results are shown in Figs. 3 through 10.

---

Fig. 3. Amplitude and phase spectra of $E_y$ fields for four high-frequency limits of permittivities in the first layer; $\varepsilon_r(\omega)/\varepsilon_0 = 1, 4, 7, \text{and } 10$.

Fig. 4. Amplitude and phase spectra of $H_x$ fields for four high-frequency limits of permittivities in the first layer; $\varepsilon_r(\omega)/\varepsilon_0 = 1, 4, 7, \text{and } 10$. 
Figures 3 to 5 show the computed amplitude and phase spectra of $E_y$, $H_z$, and $H_z$ fields, respectively, for four high-frequency limits of permittivity in the first layer: $\varepsilon_r(\infty)/\varepsilon_0 = 1$, 4, 7, and 10. Up to about 10 MHz, the amplitude curves for all components show little difference for different high-frequency limits of permittivity and do not reveal any frequency dependence. Beyond 10 MHz, however, EM field amplitudes generally increase with increasing frequency. The greater the value of $\varepsilon_r(\infty)/\varepsilon_0$, the steeper the slope of amplitude curve. However, $|E_y|$ for $\varepsilon_r(\infty)/\varepsilon_0 = 10$ shows a peak at about 47 MHz. Whether this change of slope could be viewed as the transition from diffusion to wave propagation is not clear, since this trend is not present in other components.

Fig. 5. Amplitude and phase spectra of $H_z$ fields for four high-frequency limits of permittivities in the first layer; $\varepsilon_r(\infty)/\varepsilon_0 = 1$, 4, 7, and 10.

Fig. 6. Amplitude and phase spectra of $H_z$ fields for four conductivities of the first layer; $\sigma_1 = 0.001, 0.003, 0.01$, and 0.03 S/m.
Phase curves also exhibit great changes in response to the high-frequency limit of permittivity in the frequency range of interest. EM field phases generally decrease with increasing frequency. The greater the value of $\varepsilon_1(\infty)/\varepsilon_0$, the steeper the slope of phase curve.

Figures 6 to 10 show effects of the other model parameters on observed $H_z$ fields. Electrical conductivity is the most significant model parameter affecting EM fields (see Fig. 6). At low frequencies the curves are essentially flat, and this is reasonable because at these frequencies EM responses are more sensitive to the lower half space conductivity. At high frequencies, however, each curve exhibits dramatic changes of slope in response to the electrical conductivity of the top layer. With decreasing electrical

![Fig. 7. Amplitude and phase spectra of $H_z$ fields for two layer thicknesses of the first layer; $h_1 = 1$ and 2 m.](image)

![Fig. 8. Amplitude and phase spectra of $H_z$ fields for three low-frequency limits of permittivities in the first layer; $\varepsilon(0)/\varepsilon_0 = 5, 20,$ and 40.](image)
conductivity of the first layer, wave propagation mode is more likely to dominate diffusion so that both amplitude and phase spectra undergo severe changes with increasing frequency.

Figure 7 shows the computed $H_z$ amplitude and phase spectra for two different values of top layer thickness: $h_1 = 1$ and 2 m. Further increase of $h_1$ has little effect to altering the shape of spectra; the curves for $h_1 = 2$ and $h_1 = 4$ (not shown) show almost no difference from each other. The reason for this behavior may be explained with skin depth consideration. At the frequency of 10 MHz, if we only take account of diffusion regime, the skin depth in the first layer of 0.01 S/m is about 1.58 m. So the top layer thickness

![Figure 9](image_url)

**Fig. 9.** Amplitude and phase spectra of $H_z$ fields for two Cole-Cole relaxation time constants of the first layer; $\tau_1 = 5 \times 10^{-3}$ and $5 \times 10^{-5}$.

![Figure 10](image_url)

**Fig. 10.** Amplitude and phase spectra of $H_z$ fields for two Cole-Cole relaxation distribution parameters of the first layer; $\alpha_1 = 0.25$ and 0.5.
with more than 2 m has little effect on the response curve. Note that if we neglect dispersion, the dielectric permittivity has no effect on the attenuation of EM energy.

Figures 8 to 10 show the frequency responses of EM fields to the variation of other Cole-Cole parameters in the first layer; $\varepsilon_1(0)/\varepsilon_0$, $\tau_1$, and $\alpha_1$. From these illustrations one can see that these parameters have little effects on EM fields. In other words, the sensitivity of EM fields to these parameters is very low, implying that a good resolution of these parameters in the inversion analysis may be difficult to achieve.

4. Inversion Experiments

In this section we investigate the ability of high-frequency EM methods in determining electrical parameters of shallow layered earth. We assess the information content of high-frequency EM data through sensitivity analysis and inversion experiments. Sensitivity or Fréchet derivative is a partial derivative of object function, usually the data, with respect to the independent variable which is to be determined. So sensitivity analysis itself can be a good indicator for predicting the result of inversion process.

Figure 11 shows the sensitivities of $H_z$ fields to seven parameters of the two-layered model shown in Fig. 2 and Table 1. In this case $\sigma_1$ and $\varepsilon_2(0)$ show the maximum and minimum sensitivities, respectively, and $\sigma_2$ and $h_1$ have similar sensitivities. Because $|dH_z/d\varepsilon_2(0)|$ is about $10^{-7}$ times smaller than $|dH_z/d\sigma_1|$, $\varepsilon_2(0)$ would be an irrelevant parameter in the data inversion. As expected from the skin depth consideration, while $|dH_z/d\sigma_1|$ increases with increasing frequency, $|dH_z/d\sigma_2|$ does not do so. The sensitivity to $\varepsilon(\infty)$ is always higher than that to corresponding $\varepsilon(0)$. This suggests that $\varepsilon(\infty)$ is easier to determine through an inversion of EM data than $\varepsilon(0)$. Because $|dH_z/d\varepsilon_2(\infty)|$ increases with increasing frequency, although the value itself is not so high, high-frequency data are essential for determining $\varepsilon(\infty)$ in the inversion (Tables 3 and 4). If the noise level is higher than the parameter sensitivity, however, it may be difficult to resolve this parameter through data inversion.

![Fig. 11. Sensitivity of $H_z$ fields to seven parameters of the two-layered Cole-Cole model: $\sigma_1$, $\varepsilon_1(\infty)/\varepsilon_0$, $\varepsilon_1(0)/\varepsilon_0$, $h_1$, $\sigma_2$, $\varepsilon_2(\infty)/\varepsilon_0$, and $\varepsilon_2(0)/\varepsilon_0$.](image-url)
Figure 12 shows the sensitivity of $H_z$ to the electrical permittivity when this parameter is not dispersive; i.e., $\varepsilon_i = \varepsilon_i(0)$, $i = 1$ and 2. In this case the sensitivities of EM fields to $\sigma_2$, $h_1$, and $\varepsilon_1$ are similar to those of the dispersive medium (Fig. 11) at low frequencies, but are very close each other at high frequencies with slow but monotonic increase.

We have used inversion schemes to determine the extent to which these high-frequency EM fields could be used to resolve the model parameters. Data used for this purpose are $E_y$, $H_x$, and $H_z$ fields, and they have been sampled at 15 frequencies ranging from 100 kHz to 50 MHz. The frequencies were chosen logarithmically at low frequencies and linearly at high frequencies. Inversion schemes used here are Marquardt-Levenberg least-squares (LS) method and simulated annealing (SA) method (Kim, 1995).

Table 2 shows the result of LS inversion for a two-layered model with non-dispersive dielectric permittivities. The layer thickness, the electrical conductivities, and the first layer permittivity have been almost completely resolved through the inversion. However, the uncertainty for $\varepsilon_2$ is relatively large and this result has been expected from the poor sensitivity of EM fields to this parameter shown in Fig. 12.

![Fig. 12. Sensitivity of $H_z$ fields to five parameters of the two-layered non-dispersive model: $\sigma_1$, $\varepsilon_1/\varepsilon_0$, $h_1$, $\sigma_2$, and $\varepsilon_2/\varepsilon_0$.](image)

Table 2. Least-squares inversion for two-layered model with non-dispersive dielectric permittivities. Three-component EM fields ($E_y$, $H_x$, and $H_z$) at 15 frequencies ranging from 100 kHz to 50 MHz are used for the inversion.

| Parameter          | Given | Initial | Inversion        |
|--------------------|-------|---------|------------------|
| $\sigma_1$         | 0.01  | 0.02    | 0.01000 ± 0.04%  |
| $\varepsilon_1/\varepsilon_0$ | 10    | 5       | 10.00 ± 0.04%    |
| $h_1$              | 2     | 3       | 1.998 ± 0.23%    |
| $\sigma_2$         | 0.1   | 0.05    | 0.1000 ± 0.43%   |
| $\varepsilon_2/\varepsilon_0$ | 30    | 50      | 28.96 ± 6.87%    |
| RMS error          | $3.052 \times 10^{-3}$ | 1.758 | $2.994 \times 10^{-3}$ ± 0.12% |
From this result one can expect that it is possible to reconstruct layer parameters involving constant permittivities using high-frequency EM data.

If we extend the model to include Cole-Cole parameters, the problem of resolving these parameters becomes difficult. Table 3 shows the result of LS inversion for a two-layered Cole-Cole model. In this case only two Cole-Cole parameters, $\varepsilon(\infty)$ and $\varepsilon(0)$, are considered as inversion parameters in each layer. The other relaxation Cole-Cole parameters, $\tau$ and $\alpha$, are fixed to their values given in Table 1, and thus are not considered as inversion parameters. While the first-layer Cole-Cole parameters are fully resolved, the second-layer ones can not be resolved at all. In particular, the uncertainty involving $\varepsilon_2(0)$ is very large as expected from Fig. 11.

The inversion method SA is a technique that has attracted significant attention as suitable for optimization problems where a desired global minimum is hidden among many local minima (Sen and Stoffa, 1995). Table 4 shows the result of SA inversion for the same two-layered Cole-Cole model tested with the LS inversion scheme as shown by Table 3. From this table one can see that the uncertainties for the second-layer Cole-Cole parameters are substantially reduced when compared with Table 3. Note that the uncertainties listed in the table are obtained from posterior variances of corresponding parameters (Kim, 1995; Sen and Stoffa, 1995).

Next example is a full Cole-Cole model. In this example we try to reconstruct all Cole-Cole parameters in the top layer, while those of bottom layer are fixed and thus excluded from inversion exercise. Since the electrical conductivity of the bottom layer is not considered dispersive, it remains as

| Parameter  | Given | Initial | Inversion                  |
|------------|-------|---------|----------------------------|
| $\sigma_1$ | 0.01  | 0.02    | 0.01001 ± 0.04%            |
| $\varepsilon_1(0)/\varepsilon_0$ | 10    | 20      | 9.964 ± 0.77%              |
| $\varepsilon_1(\infty)/\varepsilon_0$ | 5     | 3       | 5.000 ± 0.12%              |
| $h_1$      | 2     | 3       | 1.999 ± 0.94%              |
| $\sigma_2$ | 0.1   | 0.05    | 0.1000 ± 1.50%             |
| $\varepsilon_2(0)/\varepsilon_0$ | 30    | 40      | 36.47 ± 369%               |
| $\varepsilon_2(\infty)/\varepsilon_0$ | 10    | 15      | 9.746 ± 61.8%              |
| RMS error  | $3.045 \times 10^{-3}$ | 0.772 | $2.960 \times 10^{-3} ± 0.48\%$ |

Table 4. Simulated annealing inversion for two-layered Cole-Cole model. The relaxation time constant $\tau$ and the relaxation distribution parameter $\alpha$ in each layer are fixed to their correct values in the inversion.

| Parameter  | Given | Initial | Inversion                  |
|------------|-------|---------|----------------------------|
| $\sigma_1$ | 0.01  | 0.02    | 0.01001 ± 0.04%            |
| $\varepsilon_1(0)/\varepsilon_0$ | 10    | 20      | 9.926 ± 0.23%              |
| $\varepsilon_1(\infty)/\varepsilon_0$ | 5     | 3       | 5.005 ± 0.03%              |
| $h_1$      | 2     | 3       | 2.004 ± 0.12%              |
| $\sigma_2$ | 0.1   | 0.05    | 0.1004 ± 0.20%             |
| $\varepsilon_2(0)/\varepsilon_0$ | 30    | 40      | 30.86 ± 47.9%              |
| $\varepsilon_2(\infty)/\varepsilon_0$ | 10    | 15      | 11.29 ± 9.71%              |
| RMS error  | $3.045 \times 10^{-3}$ | 0.772 | $2.994 \times 10^{-3} ± 0.87\%$ |
High-Frequency Electromagnetic Inversion for a Dispersive Layered Earth

Table 5. Simulated annealing inversion for two-layered Cole-Cole model. The Cole-Cole parameters of the second layer are fixed to their correct values in the inversion.

| Parameter | Given | Initial | Inversion |
|-----------|-------|---------|-----------|
| $\sigma_1$ | 0.01  | 0.02    | 0.009974 ± 0.79% |
| $\varepsilon_r(0)/\varepsilon_0$ | 10    | 20      | 9.561 ± 3.48% |
| $\varepsilon_r(\infty)/\varepsilon_0$ | 5     | 3       | 5.007 ± 1.50% |
| $\tau_1$ | $5 \times 10^{-7}$ | $2 \times 10^{-7}$ | $5.304 \times 10^{-6} ± 167\%$ |
| $\omega_1$ | 0.5   | 0.8     | 0.5195 ± 23.5% |
| $h_1$ | 2     | 3       | 2.000 ± 0.45% |
| $\sigma_2$ | 0.1   | 0.05    | 0.1004 ± 1.53% |

RMS error $3.045 \times 10^{-3}$ 1.195 3.032 $\times 10^{-3} ± 3.14\%$

Table 6. Least-squares inversion for three-layered non-dispersive model.

| Parameter | Given | Initial | Inversion |
|-----------|-------|---------|-----------|
| $\sigma_1$ | 0.01  | 0.02    | 0.1000 ± 0.02% |
| $\varepsilon_r(0)/\varepsilon_0$ | 10    | 20      | 9.997 ± 0.03% |
| $h_1$ | 1     | 2       | 1.001 ± 0.14% |
| $\sigma_2$ | 0.001 | 0.0005  | 0.000993 ± 2.22% |
| $\varepsilon_r(\infty)/\varepsilon_0$ | 30    | 15      | 30.00 ± 0.13% |
| $h_2$ | 1     | 1.5     | 0.9999 ± 0.14% |
| $\sigma_3$ | 0.1   | 0.05    | 0.1004 ± 0.24% |
| $\varepsilon_r(\infty)/\varepsilon_0$ | 10    | 20      | 9.817 ± 1.83% |

RMS error $2.722 \times 10^{-3}$ 1.549 2.673 $\times 10^{-3} ± 1.13\%$

an inversion parameter. Table 5 shows the result of SA inversion for the two-layered Cole-Cole model. For this model the LS inversion with the same initial guess listed in the table failed to provide a reasonable solution. Although the time constant $\tau_1$ is poorly resolved, all other parameters seem to have been well resolved with the SA inversion. Clearly, the estimation of $\tau$ from data inversion is difficult, and this outcome is consistent with the expectation from the field response curve of this parameter shown in Fig. 10.

Final example involves a non-dispersive three-layered model. Table 6 shows the result of LS inversion for the model. All parameters appear to have been completely resolved. For this model electrical conductivities of the upper two layers are low enough for the high-frequency EM fields to have good penetrations. A test was made to see what happens if the middle resistive layer (0.001 S/m) is replaced by conductive one (0.1 S/m). The result (not shown) is poorly resolved parameters. This test suggests that good parameter resolution of multi-layered earth may be possible if all layer conductivities are sufficiently low. Nevertheless, the result shown in Table 6 is encouraging, and it shows that the high-frequency EM method can be a useful tool for shallow subsurface imaging.

5. Conclusions

Broad band EM fields obtained from a magnetic dipole source over a wide range of transmitting frequencies have been analyzed for their sensitivities to each electrical parameter including the Cole-Cole
This analysis has been very useful in developing an efficient inversion strategy, which resulted in a conclusion that the high-frequency EM method could be a powerful geophysical tool for imaging shallow subsurface electrical parameters. With a configuration of fixed source-receiver separation, the broad spectrum of EM fields provides data for frequency sounding analysis. For a horizontally layered earth model the electrical conductivity, layer thickness, and dielectric permittivity of each layer can be obtained from the high-frequency EM data using an ordinary least-squares or simulated annealing inversion scheme.

The effect of frequency on parameter resolution was investigated through sensitivity analysis and inversion experiments. Generally speaking, the sensitivities of dielectric Cole-Cole parameters are not so high except for the high-frequency limit of permittivity, and therefore these parameters are difficult to resolve through data inversion. Inversion experiments show that the simulated annealing yields slightly better parameter resolution than the least-squares inversion method. The simulated annealing method is generally more powerful in searching for the global minimum, and this may have been the reason why the simulated annealing method showed better performance than the least-squares method. However, the simulated annealing requires excessive computer time, and for this reason it is uncertain whether or not this method could be still more effective if the inversion involves many parameters to be resolved.

As indicated in the three-layered model inversion analysis, high-frequency EM fields have difficulties in penetrating the subsurface strata when there is a conductive layer. The result is poorly resolved parameters because the resulting EM fields are strongly affected by the high conductivity and are not sensitive to other minor electrical parameters. We may overcome this difficulty if we adapt a two-step inversion scheme where both the low- and high-frequency EM fields are used. The first step involves solving for the electrical conductivity and layer thickness parameters using only the low-frequency EM fields. At low frequencies the effect of the electrical permittivity may be negligible. EM fields may penetrate deeper at this frequency range and the conductivity and thickness parameters could be correctly and easily resolved purely on the basis of EM diffusion. Once the conductivity and thickness parameters have been resolved, then one can proceed to resolving the dispersive dielectric permittivity using high-frequency EM fields. Even with the help of this improved scheme, however, the dispersion parameters may often be difficult to resolve in the presence of typical noise.

This study was partially supported by the Korea Science and Engineering Foundation, the Center for Mineral Resources Research, and the Assistant Secretary for Environmental Restoration and Waste Management, Office of Technology Development, of the U.S. Department of Energy under contract No. DE-AC03-76SF00098.

REFERENCES

Annan, A. P., Transmission dispersion and GPR, J. Eng. Environ. Geophys., 0, 125–136, 1996.
Chave, A. D., Numerical integration of related Hankel transforms by quadrature and continued fraction expansion, Geophysics, 48, 1671–1686, 1983.
Cole, K. S. and R. S. Cole, Dispersion and absorption in dielectrics, I, alternating current characteristics, J. Chem. Phys., 9, 341–351, 1941.
Johnson, I. M., Spectral induced polarization parameters as determined through time-domain measurements, Geophysics, 49, 1993–2003, 1984.
Keller, G. V., Rock and mineral properties, in Electromagnetic Methods in Applied Geophysics, Volume 1, edited by M. N. Nabighian, pp. 13–51, Theory. Soc. Expl. Geophys., 1987.
Kim, H. J., Nonlinear inversion of resistivity sounding data using simulated annealing, Butsuri-Tansa, 48, 214–220, 1995 (in Japanese with English abstract).
Pellerin, L., V. F. Labson, and M. C. Pfeifer, VETEM—A very early time electromagnetic system, Proc. of the Symp. on the Application of Geophysics to Engineering and Environmental Problems, SAGEEP ’95, April 23–26, 725–731, 1995.
Pelton, W. H., S. H. Ward, P. G. Hallof, W. P. Sill, and P. H. Nelson, Mineral discrimination and removal of inductive coupling with multifrequency IP, Geophysics, 43, 588–609, 1978.
Seigel, H. O., Mathematical formulation and type curves for induced polarization, Geophysics, 24, 547–565, 1959.
Sen, M. and P. L. Stoffa, Global Optimization Methods in Geophysical Inversion, 281 pp., Elsevier. 1995.
Swift, C. M., The L/M parameter of time domain IP measurements—A computational analysis, Geophysics, 38, 61–67, 1973.