Time-dependent Numerical Model for Studying the Very-high-energy Emissions of Distant Gamma-Ray Burst GRB 201216C

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Abstract

Recently, the MAGIC Collaboration reported a ~5σ statistical significance of the very-high-energy (VHE) emission from a distant gamma-ray burst (GRB), GRB 201216C. Such distant GRB may be effectively absorbed by the extragalactic background light (EBL). The origin of the VHE emission from such distant objects is still unknown. Here, we propose a numerical model for studying the afterglow emission of this distant GRB. The model solves the continuity equation governing the temporal evolution of electron distribution, and the broadband observed data can be explained by the synchrotron plus synchrotron self-Compton (SSC) radiation of the forward shock. The predicted observed 0.1 TeV flux can reach ~10^{-9}–10^{-10} erg cm^{-2} s^{-1} at t ~ 10^{3}–10^{4} s, even with strong EBL absorption, such strong sub-teraelectronvolt (sub-TeV) emissions still can be observed by the MAGIC telescope. Using this numerical model, the shock parameters in the modeling are similar to two other sub-TeV GRBs (i.e., GRB 190114C and GRB 180720B), implying that the sub-TeV GRBs have some commonalities: they have energetic burst energy, low circumburst medium density, and a low magnetic equipartition factor. We regard GRB 201216C as a typical GRB, and estimate the maximum redshift of GRB that can be detected by the MAGIC telescope, i.e., z ~ 1.6. We also find that the VHE photon energy of such distant GRB can only reach ~0.1 TeV. Improving the low energy sensitivity of the VHE telescope is very important to detect the sub-TeV emissions of these distant GRBs.

Unified Astronomy Thesaurus concepts: Gamma-ray bursts (629); Shocks (2086)

1. Introduction

The origin of very-high-energy (VHE) emissions of gamma-ray bursts (GRBs) has been an open issue for a long time. There are several mechanisms that can produce the VHE gamma-ray during the afterglow phase (MAGIC Collaboration et al. 2019b; Zhang 2019): (1) synchrotron radiation from the accelerated electrons in the forward shock; however, the radiation has maximum energy, due to the Lorentz factor of the outflow, i.e., \( \varepsilon_{\text{max}} \sim 100 \text{ MeV} \times \Gamma_{b}(t)/(1 + z) \). The Lorentz factor of GRB outflow \( \Gamma_{b}(t) \) is usually on the order of 10 during the afterglow phase, so the maximum energy of the synchrotron radiation is only approximately a gigaelectronvolt (GeV); (2) synchrotron radiation from the accelerated protons in the forward shock; however, the protons are much less efficient emitters than the electrons and the requirements to reproduce the observed teraelectronvolt (TeV) flux and spectrum are more severe; (3) inverse Compton (IC) emission by the accelerated electrons of the forward shock, where the seed photons may be X-ray flares (Murase et al. 2011; Zhang et al. 2021a, 2021b), shock breakout emission (Wang & Mészáros 2006), hypernova envelope emission (He et al. 2009), or the synchrotron photons (which are emitted by the same accelerated electrons) (Sari & Esin 2001), et al.

Recently, the VHE gamma-rays from some GRBs (e.g., GRB 190114C, GRB 180720B, and GRB 190829A) have been detected by the Ground-based Imaging Atmospheric Cherenkov Telescope (MAGIC Collaboration et al. 2019a, 2021; Abdalla et al. 2019; H.E.S.S. Collaboration et al. 2021), i.e., the Major Atmospheric Gamma Imaging Cherenkov (MAGIC) telescope, and the High Energy Stereoscopic System (H.E.S.S.). The observed VHE photons of these GRBs are more likely to be explained by the IC mechanism of the forward shock in the afterglow phase (Wang et al. 2019; Zhang 2019; Zhang et al. 2021a).

GRB 201216C is a distant GRB that has been recently announced to be detected VHE emission by the MAGIC telescope. GRB 201216C is particularly bright and hard, with its GBM light curve showing a broad, structured peak. The duration (\( T_{90} \)) is about 29.9 s (50–300 keV), the time-averaged spectrum from \( T_{0} -0.03 \) to +49.665 s (\( T_{0} \) is the trigger time of GBM) is best fitted by a band function with \( E_{\text{peak}} \sim 326 \pm 7 \text{ keV} \), \( \alpha = -1.06 \pm 0.01 \), \( \beta = -2.25 \pm 0.03 \), and the event fluence (10–1000 keV) in this interval is (1.41 ± 0.06) × 10^{-4} \text{ erg cm}^{-2} (Malacaria et al. 2020). VLT X-shooter identifies the redshift of this GRB is \( z = 1.1 \), making this object the most distant known VHE source (Vielvaure et al. 2020). Using the Fermi-GBM parameters, the isotropic energy release of this GRB is \( E_{\text{iso}} = (4.71 \pm 0.16) \times 10^{53} \text{ erg} \).

The MAGIC telescope was designed to perform gamma-ray astronomy in the energy range from 50 GeV to greater than 50 TeV. The MAGIC telescope observed GRB 201216C following the trigger by Swift-BAT and Fermi-GBM under good conditions for about 57 s. The preliminary of the duration of prompt emission, we prefer to believe that the VHE emission originates from the afterglow phase.

The GRB position was not in the Fermi-LAT field of view until \( T_{0} +3500 \text{ s} \), and no significant high-energy gamma-ray emission is associated with this GRB in the time interval from...
corresponding upper limit for energy flux is \(3 \times 10^{-10}\) erg cm\(^{-2}\) s\(^{-1}\) (95\% confidence level, 100 MeV–1 GeV) (Bissaldi et al. 2020). Swift-XRT began to observe from 3–1938 ks after the BAT trigger. The light curve can be modeled with an initial power-law decay with its decay index of \(\alpha_x = 2.09^{+0.16}_{-0.15}\), followed by a break at \(T_0 + 9078\) s and the decay index after the break is \(\alpha_x = 1.07^{+0.15}_{-0.16}\). The spectrum can be fitted with an absorbed power law with a photon spectral index of \(\sim 2.35\) (Evans et al. 2021).

At 177 s after the BAT trigger, the Liverpool Telescope (LT) observed the field of Swift GRB 201216C and confirmed the optical counterpart of this GRB. They measured \(r = 18.38\), and the \(r\)-band data was calibrated with respect to nearby APASS secondary standard stars (Shrestha et al. 2020). The \(r\)-band light curve made along with the VLT data point and inferred data from FRAM-ORM show a power-law decay in flux versus time with \(\alpha_r \sim 1.07\) (Jelisiek et al. 2020). From the light curve, LT observations seem to be around the peak of afterglow.

We are interested in the origin of VHE emissions from such distant GRB. In this work, we base on the available broadband data of GRB 201216C and propose a paradigm to explain the VHE emission of GRB 201216C. In Section 2, we perform a numerical model of the afterglow emissions. We consider the details of the dynamics of shock evolution, apply the time-dependent treatment to calculate the temporal electron distributions, then derive the synchrotron and synchrotron self-Compton (SSC) emissions. The Klein–Nishina (KN) suppression and the absorption are also considered in this work. In Section 3, we present the details of the numerical approach, and compare the numerical results with the analytical tests to confirm the validity of our code. Our numerical model is applied to the afterglow of GRB 201216C in Section 4. Finally, we present a discussion and our conclusions in Section 5.

2. Model Description

2.1. Hydrodynamic Evolution

For the hydrodynamic evolution of GRB external shock, we refer to Huang et al. (1999). We consider an impulsive outflow with kinetic energy \(E_k\) and the initial Lorentz factor \(\Gamma_0\), propagating into an external medium of constant density \(n\), typical for interstellar medium (ISM). The differential dynamic equation of the afterglow shock can be derived as

\[
\frac{d\Gamma}{dm} = -\frac{\Gamma^2 - 1}{m_{ej} + 2\Gamma m},
\]

where \(\Gamma\) is the bulk Lorentz factor of the shocked shell and \(m_{ej} = E_k/\Gamma_0 c^2\) is the mass of the electron, \(m = (4\pi/3) R^3 n m_p\) is the mass of the swept-up external matter, and \(\Gamma\) is the bulk Lorentz factor of the shocked shell. In order to obtain the time dependence of \(\Gamma\), we make use of

\[
\frac{dm}{dR} = 4\pi R^2 n m_p
\]

and

\[
\frac{dR}{dt} = \beta c (\Gamma + \sqrt{\Gamma^2 - 1}),
\]

where \(t\) is the observer time, \(R\) and \(\beta = \sqrt{1 - \Gamma^{-2}}\) are the radius and velocity of the shell, respectively.

Assuming 10\% of the kinetic energy is converted into the isotropic energy of prompt emission, the kinetic energy of the outflow is about \(E_k \sim 5 \times 10^{54}\) erg. For ultrarelativistic outflow, the deceleration timescale of the shock is

\[
t_{\text{dec}} = \left(\frac{17E_k}{1024\pi n m_p c^5 \Gamma_0^8}\right)^{1/3},
\]

then we have \(t_{\text{dec}} \approx 235 s E_k^{1/3} \Gamma_0^{-2/3} n^{-1/3}\), where \(E_k = 10^{54}\) erg, \(\Gamma_0 = 10^2 \Gamma_{0,2}\), and \(n\) is normalized to 1 cm\(^{-3}\). The observed peak time of the \(r\)-band light curve is around \(\sim 177\) s, very close to this deceleration timescale.

For the nonrelativistic outflow, the deceleration timescale of the shock is on the order of years (Nakar & Piran 2011), much larger than 100 s and seriously inconsistent with the peak position of the \(r\)-band light curve. Hence, we only consider the ultrarelativistic outflow. Then the comoving time \(t'\) can be related to the observer’s time \(t\) for the ultrarelativistic shock by

\[
dt = (1 + z)(1 - \beta) dt' \approx \frac{1 + z}{2\Gamma} dt',
\]

and the position of the jet head is \(dr = \beta c dt'\). Hereafter, the superscript prime (\('\)) is used to denote the quantities in the comoving frame of the shock fluid.

2.2. Distribution of Shock-accelerated Electrons

The distribution of the shock-accelerated electrons behind the shock is usually obtained by solving the continuity equation of nonthermal electrons and it is best to work in the comoving frame of the shock fluid while we solve the continuity equation. First, we assume that the distribution of injected electrons is taken as a power law between \(\gamma_m'\) and \(\gamma_{\text{max}}' (\gamma_{\text{max}}' > \gamma_m')\),

\[
dN_{e,0}' = N_0' \frac{p - 1}{\gamma_m'} \left(\frac{\gamma_m'}{\gamma_m'}\right)^{-p} d\gamma_e',
\]

with

\[
\dot{N}_e' = \Gamma \left(\beta + \frac{1}{3}\right) 4\pi n R^2 c
\]

is the number of electrons swept and accelerated by the shock wave per unit of time. \(\gamma_e'\) is the random Lorentz factor of an electron. \(\gamma_m'\) is the minimum Lorentz factor, which can be determined by the shock jump conditions (Blandford & McKee 1976; Sari et al. 1998),

\[
\gamma_m' = \frac{p - 2}{p - 1} \epsilon_c (\Gamma - 1) \frac{m_p}{m_e},
\]

and \(\gamma_{\text{max}}'\) is the maximum Lorentz factor, which can be determined by comparing the acceleration timescale \(t_{\text{acc}}'\) with the cooling timescale \(t_{\text{cool}}'\). The acceleration timescale is to be \(t_{\text{acc}}' \approx k_B' \gamma_e' m_e c / (e B')\), where \(k_B'\) is the correction factor, which accounts for the downstream electrons not returning to the downstream region once they are deflected by an angle of \(1/\Gamma\); therefore, \(k_B'\) should not be smaller than 1, i.e., \(k_B' \gtrsim 1\). We take \(k_B' = 1\) here. The cooling timescale due to synchrotron and IC cooling is estimated to be \(t_{\text{cool}}' = (\sigma_T m_e c^3 / (\gamma_e' \sigma_T B'^2)) (1 + Y)\), where \(\sigma_T\) is the Thompson scattering cross section, and
where

\[ F(q, g) = 2q \ln q + (1 + 2q)(1 - q) + \frac{1}{2} \left( \frac{4gq^2}{\nu^2} - (1 - q) \right). \]

\( g = \frac{\gamma_{\text{esc}} h\nu}{m_e c^2}, \quad w = \frac{h\nu_{\text{esc}}}{m_e c}, \quad \text{and} \quad q = \frac{w}{4g(1 - w)} \nu^2 / \nu^2 \text{IC} \),

in Equation (15) are the frequencies of the seed photons and the IC photons, respectively. According to the dynamics of the collision between a relativistic electron and a photon, i.e., \( 1 < h \nu / \gamma_{\text{esc}} m_e c^2 \leq h \nu_{\text{IC}} / \gamma_{\text{esc}} m_e c^2 \leq 4g(1 + 4g) \), and

\( 1 / 4 \gamma_{\text{esc}}^2 \leq q \leq 1 \), the upper limit of the second integral can be derived as \( h\nu_{\text{IC, max}} = \gamma_{\text{esc}} m_e c^2 \frac{4g}{4g + 1} \), and the lower limit is

\( \nu_{\text{IC, min}} = \frac{1}{\gamma_{\text{esc}}} \nu^2 \).

GRB 201216C was not accompanied by the obvious X-ray flare, shock breakout, and hypernova envelope emission; hence, it has been proposed that the VHE \( \gamma \)-ray photons may be produced by the SSC scenario, and then the quantity \( n_{\nu} \)

needed in Equation (15) is the synchrotron seed photon spectra, which is calculated by

\[ n_{\nu} \approx \frac{T'}{h\nu_{\nu} m_e c^2} \int_{\nu_{\text{min}}^{\nu}} \gamma_{\text{esc}}^2 \nu_{\text{min}}^{\nu} R(\nu / \nu_{\text{esc}}) d\nu_{\text{esc}}. \]

After obtaining the solved temporal evolution of electron distribution 2N_{\nu} / d\gamma_{\text{esc}}', the total optically thin synchrotron radiation power at frequency \( \nu / \nu_{\text{esc}} \) in the comoving frame is

\[ P_{\nu}(\nu / \nu_{\text{esc}}') = \int d\nu_{\text{esc}}' \frac{dN_{\nu}'}{d\gamma_{\text{esc}}'} R(\nu / \nu_{\text{esc}}'). \]

In the same way, the total spectral energy distribution of the SSC radiation is

\[ P_{\nu, \text{SSC}}(\nu / \nu_{\text{esc}}') = \int \frac{dN_{\nu}'}{d\gamma_{\text{esc}}'} h\nu_{\text{esc}} \frac{dN_{\nu}'}{dt' d\nu_{\text{esc}}} d\gamma_{\text{esc}}'. \]

The quantity \( dN_{\nu} / dt' d\nu_{\text{esc}}' \) in Equation (19) is the scattered photon spectrum per electron (Blumenthal & Gould 1970), and we can express \( dN_{\nu} / dt' d\nu_{\text{esc}}' \) in terms of the frequency distribution of the seed photons \( n_{\nu} \),

\[ \frac{dN_{\nu}}{dt' d\nu_{\text{esc}}} = \frac{3\gamma_{\text{esc}}^2 c n_{\nu} d\nu}{4\gamma_{\text{esc}}^2 \nu^2} F(q, g). \]

Given the synchrotron spectra \( P_{\nu, \text{SSC}}(\nu / \nu_{\text{esc}}') \) in the comoving frame, assumed the luminosity distance of the source from the observer is \( DL \), if we ignore the effect of the equal-arrival-time surface (EATS; Waxman 1997; Granot et al. 1999), the intrinsic spectral flux at the observer
frame can be expressed as
\[ F_{\nu, \omega, \text{EATS}} = \frac{(1 + z)P' (\nu / \nu_0) \Gamma}{4 \pi D_L^2}, \quad (21) \]
where \( \nu = \frac{4 \pi^2}{3(1 + z)} \) is the observed frequency. But if we take the EATS effect into account, the respective intrinsic synchrotron and SSC fluxes at the observer frame are given by Geng et al. (2018) as
\[ F_{\nu, \omega, \text{EATS}} = \frac{1 + z}{4 \pi D_L^2} \int_0^{\theta_0} P' (\nu' / \nu_{\text{obs}}) D_{\text{Dopp}}^3 \sin \theta \, d\theta, \quad (22) \]
where \( \theta_0 \) is the half-opening angle of the jet, \( \nu_{\text{obs}} = D_{\text{Dopp}} \nu / (1 + z) \) is the observed frequency when considering the EATS effect, and \( D_{\text{Dopp}} = 1 / [\Gamma (1 - \beta \cos \theta)] \) is the Doppler factor.

In this work, the luminosity distance \( D_L \) is obtained by adopting a flat \( \Lambda \) cold dark matter universe, in which \( H_0 = 71 \) km s\(^{-1}\), \( \Omega_m = 0.27 \), and \( \Omega_\Lambda = 0.73 \).

### 2.4. Absorption

High-energy gamma-rays may suffer internal absorption by ambient photons inside the source and external absorption by extragalactic background light (EBL) while they travel through the ISM. When we consider the internal absorption by the source, the high-energy gamma-rays can be attenuated due to interaction with synchrotron photons of the afterglow phase through the pair production effect. The optical depth for a gamma-ray with energy \( \varepsilon' = \frac{4 \pi^2}{3(1 + z)} \gamma^3 \) is expressed as (Vernetto & Lipari, 2016; Murase et al., 2011)
\[ \tau_\infty = \frac{\Delta'}{2} \int_{-1}^{1} d\mu (1 - \mu) \int d\nu' \sigma_{\gamma\gamma}, \quad (23) \]
where
\[ \sigma_{\gamma\gamma} = \frac{3}{16} \sigma_T (1 - \beta_{\text{cm}}^2) \times \left[ 2 \beta_{\text{cm}} (\beta_{\text{cm}}^2 - 2) + (3 - \beta_{\text{cm}}^4) \ln \frac{1 + \beta_{\text{cm}}}{1 - \beta_{\text{cm}}} \right], \quad (24) \]
is the pair production cross section, \( \beta_{\text{cm}} \) is the velocity of the e\(^\pm\) in the center of mass frame, i.e.,
\[ \beta_{\text{cm}} = \frac{v}{c} = \sqrt{1 - \frac{2 (mc^2)^2}{\varepsilon'^2 \gamma' (1 - \mu)}}, \quad (25) \]
\( \mu = \cos \theta_r \), \( \theta_r \) is the angle between the directions of the interacting photons, and \( \varepsilon' = h \nu' \) is the energy of a target photon (synchrotron photon). \( \Delta' \) cannot be canceled out in Equation (23) and we take \( \Delta' \approx R / 10 \Gamma \); we assume that the emitting electrons are homogeneously distributed in the shell.

If the intrinsic flux of a source is \( F_\nu \), and the flux after the internal \( \gamma \gamma \) absorption is \( F'_\nu \), then we have
\[ F'_\nu = F_\nu (1 - \exp(-\tau_\infty)) / \tau_\infty. \quad (26) \]

When we consider the absorption by EBL, the optical depth \( \tau_{\text{EBL}} \) versus the observed energy of gamma-ray photons at different redshifts \( z \) are given by Domínguez et al. (2011), after absorption by EBL, the observed flux \( F_{\nu, \text{obs}} \) is
\[ F_{\nu, \text{obs}} = F'_\nu \exp(-\tau_{\text{EBL}}). \quad (27) \]

### 3. Numerical Approach

In order to calculate the time-dependent spectra of high-energy afterglow emission, it is necessary to solve the hydrodynamic evolution of GRB shock. The parameters are set as \( \varepsilon_e = 3 \times 10^{24} \text{erg} \), \( \Gamma_0 = 200 \), and \( n = 0.1 \text{ cm}^{-3} \). Numerically solving Equations (1)--(3), we obtain three dynamic phases of the shock: (1) coasting phase: at \( t \lesssim t_{\text{dec}} \), the shock propagates at a constant velocity \( \Gamma_0 \) and the shock radius is \( R \propto t \); (2) Blandford–McKee self-similar phase: at \( t \lesssim t_{\text{dec}} \), the shock dynamics transits into the self-similar Blandford–McKee solution, the shock Lorentz factor is \( \Gamma \propto t^{-3/5} \), and the shock radius is \( R \propto t^{1/5} \) (Blandford & McKee, 1976; Sari et al., 1998). The deceleration timescale is close to the peak time of the r-band light curve, i.e., \( t_{\text{dec}} \approx 10^3 \text{s} \); (3) Sedov–Taylor self-similar phase: at \( t \gg t_{\text{dec}} \), the shock dynamics transits into the nonrelativistic outflow, then we can approximate that \( \beta \propto t^{-3/5} \) and \( R \propto t^{2/5} \) (Sedov, 1959; Taylor, 1950). It should be emphasized that the hydrodynamic evolution in this work is available for strong shocks not only the relativistic, but also the nonrelativistic, or mildly relativistic regime.

In general, the display difference scheme is usually used to solve the continuous equation of electron, but when we discretize the time and energy space, the time step and the energy step have to follow the Courant–Friedrichs–Lewy condition, otherwise we cannot ensure the convergence of the solution for the arbitrary values of time steps. To avoid this problem, we must adopt an unconditional stable scheme: the fully implicit difference scheme (FIDS), which is proposed by Chang & Cooper (1970) and Chiaberge & Ghisellini (1999). The FIDS can find a more stable, non-negative, and particle number-conserving solution, reduce the number of mesh points and obtain accurate solutions. For more details on this numerical method, see Chiaberge & Ghisellini (1999). Here, we simply present the discretization procedures to solve Equation (11). The Lorentz factor \( \gamma' \) of electron distribution ranges from 10\(^{-10}\) to 10\(^{9}\), so it is necessary to use an energy grid with equal logarithmic resolution. Then, the observed time step for every evolution ranges from 10\(^{-10}\) to 10\(^{3}\), also with equal logarithmic resolution. After obtaining the solved temporal evolution of electron distribution \( N'_\gamma / d\nu' \), the total synchrotron and SSC radiation power in the comoving frame are calculated by Equations (18) and (19). In the performed simulations, a grid of 500 points has been used both for the electron Lorentz factor and photon frequency, and a grid of 121 points has been used for the observed time.

In order to test the validity of our code, we have compared our simulation results with those obtained by Wang et al. (2010), who have developed a similar analytical method, but adiabatic cooling was not considered. These electron distributions (in the comoving frame) and the radiation spectra (in the observed frame, without considering the EATS effect and absorption) at different observed times (\( t = 10^2, 10^3, 10^4, \) and \( 10^5 \) s) are shown in Figure 1. The results are obtained with the same set of parameters. We can see that the two methods produce electron distributions and radiation spectra with very similar shapes. The high-energy electrons are cooled above \( \gamma' \) (\( \gamma' \) is the cooling Lorentz factor), and the electron distributions are as \( dN'_\gamma / d\nu' \propto \gamma'^{-p-1} \). The low energy electrons are uncooled between \( \gamma'_m \) and \( \gamma'_c \), and the electron distributions gradually approach the slope of the injected function, i.e., \( dN'_\gamma / d\nu' \propto \gamma'^{-p} \). The results indicate that the electron distributions should be expressed with the slow cooling.
approximation for $t \gtrsim 10^2 \text{s}$. The analytical solution obtains a pure broken-power-law segment with a sharp break that deviates from the numerical model. With time evolution, we can see that the normalization of the numerical method is slightly different from that of Wang et al. (2010), and the corresponding cooling Lorentz factor $\gamma_i$ is also slightly different, but the deviations are within half an order of magnitude. The simple broken-power-law function is not appropriate for the numerical results. We suspect this difference arises from the fact that our numerical model considers a time-dependent electron distribution, but the analytical model considers the electron distribution at a certain moment and ignores the details of the intermediate evolution. To study whether the adiabatic cooling affects the electron distribution, we turned off this process for the test simulation, but there has no visible impact on the electron distribution and radiation spectra. Based on the consistency of the results, we can conclude that the FIDS is a good method to solve the continuity equation to ensure the convergence of the solution in the intermediate steps.

4. Application to GRB 201216C

In this section, we perform modelings of the available multiwavelength data for GRB 201216C. The modelings are based on Section 2 and the numerical methods are based on Section 3. Since there are only a few measurements and the model has six (without considering the EATS effect) or seven (with considering the EATS effect) parameters plus very detailed physics. We consider several sets of parameters for the following reasons:

1. The Fermi-GBM observations imply that the isotropic energy release of this GRB is $E_{\text{iso}} \sim 5 \times 10^{53} \text{erg}$. Assuming that the radiative efficiency for producing prompt gamma-ray emission is 1%–100%, then we can obtain the kinetic energy of the outflow $E_k \sim 5 \times 10^{53}$ to $5 \times 10^{55} \text{erg}$.

2. Assuming an ultrarelativistic outflow, we take the initial Lorentz factor $\Gamma_0 \sim 100$.

3. $n$ is the external medium density of GRBs, and for the long GRB scenario, $n$ is taken as $\sim 1 \text{ cm}^{-3}$.

4. The observed peak time of the $r$-band light curve is $t_p \sim 177 \text{s}$. Assuming that $\Gamma_0 \sim 100$ and the peak time equals the deceleration timescale of the shock, we can obtain a rough relationship between $n$ and $E_k$. The smaller $n$ is, the larger $E_k$ is.

5. The observed $r$-band light curve shows a power-law decay, i.e., $F_r \propto t^{-1.1}$. Assuming an ISM scenario, the $r$-band (low frequency) light-curve evolution for a standard GRB afterglow is $F_r(t) \propto t^{3(1-p)/4}$ (late time) or $F_r(t) \propto f^{2-3p/4}$ (very late time) (Sari et al. 1998), we obtain the corresponding electron distribution index $p \sim 2.5$ or $\sim 2.1$.

6. The values of $\epsilon_e$ and $\epsilon_B$ are set by the microphysics of relativistic shocks. From the literature, $\epsilon_e$ ranges from 0.02–0.6 and $\epsilon_B$ ranges from $10^{-6}$–$10^{-3}$ (Santana et al. 2014).

7. Since no jet breaks are observed in the X-ray and $r$-band light curves until $t \sim 10^3 \text{s}$, the afterglow radiation model described so far is based on the assumption of an on-axis viewing angle and the ejecta is almost isotropic. Therefore, we do not consider the EATS effect in model A and model B.

8. However, it is also possible that the jet breaks are not obvious or not observed. In this case, we should consider the EATS effect in our calculation, and add one more parameter $\theta_j$ in model C.

On the basis of the above discussions, we manually adjust the six (or seven) parameters of our numerical model to explain the observed multiwavelength light-curve data by eye. We have tested three models, whose parameters are summarized in Table 1 and the modeling results are shown in Figures 2 and 3.

In model A, we consider the large $\epsilon_e$ case and the results are shown in Figure 2. In the upper panel of Figure 2, the solid lines show the theoretical multiwavelength light curves, which we then compare with the observations. We can see that the observed X-ray and $r$-band light curves data are well explained by model A. The predicted light curve of the $r$ band shows a power-law decay in flux with its decay index of about $-1.1$ and the peak time of the $r$-band light curve is about a few hundred seconds, consistent with the observations of LT, VLT, and FRAM-ORM (Jelinek et al. 2020; Shrestha et al. 2020). The predicted flux of 1 GeV at $10^3$–$10^4 \text{s}$ is $F_{\gamma} \sim 10^{-8} \text{ mJy}$, and...
then \( \nu F_{\nu} \sim 10^{-11} \text{ erg cm}^{-2} \text{s}^{-1} \) is too low to be detected by Fermi-LAT. But if Fermi-LAT is sensitive enough and this GRB was on the Fermi-LAT field of view, we would see the emission mechanism is switching from synchrotron to SSC at \( t \sim 10^3 \text{ s} \), i.e., the early light curve of 1 GeV is contributed by the synchrotron radiation, while the late time light curve of 1 GeV is contributed by the SSC radiation. The light curve of 1 GeV cannot be a smooth power law. We are more concerned about the VHE emissions, but the detailed observations of the MAGIC telescope have not been published yet. We only know that the VHE emissions of this burst have a \( \sim 5 \sigma \) statistical significance of excess. If we do not consider any absorption and just consider the intrinsic flux of this GRB, the predicted light curve of 0.1 TeV is represented by the cyan solid line. We can see that the light curve of 0.1 TeV has a peak flux \( F_{\nu} \sim 10^{-9} \text{ mJy} \), then \( \nu F_{\nu} \sim 10^{-3} \text{ erg cm}^{-2} \text{s}^{-1} \), and the peak time of 0.1 TeV light curve is \( \sim 10^4 \text{ s} \). After the peak flux, the light curve decays as the power law with \( F_{\nu} \propto t^{-1.5} \).

In the lower panel of Figure 2, we show the spectral energy distributions (SEDs) of synchrotron, SSC, and total emissions with or without absorption at \( t \sim 10^2, \sim 10^3 \), and \( \sim 10^4 \text{ s} \) for

### Table 1

| Model | \( E_k \) [erg] | \( \Gamma_0 \) | \( \epsilon_e \) | \( \epsilon_\nu \) | \( n \) [cm\(^{-3}\)] | \( p \) | EATS |
|-------|-----------------|----------------|-----------------|-----------------|-----------------|------|------|
| A     | \( 3 \times 10^{54} \) | 200            | 0.7             | \( 6 \times 10^{-6} \) | 0.1             | 2.6  | no   |
| B     | \( 6 \times 10^{55} \) | 100            | 0.02            | \( 2 \times 10^{-5} \) | 1               | 2.6  | no   |
| C     | \( 6 \times 10^{53} \) | 100            | 0.6             | \( 8 \times 10^{-5} \) | 0.5             | 2.1  | yes, \( \theta_j = 0.05 \) |

Figure 2. Light curves and spectra of model A for GRB 201216C. Upper panel: the theoretical intrinsic light curves at the optical (\( r \)-band), X-ray (1 keV), and gamma-ray (1 GeV) to sub-TeV (0.1 TeV) band; we then compare with the observations. The optical data is taken from Shrestha et al. (2020), the X-ray data are taken from Evans et al. (2021) by assuming a power-law spectrum with photon index \( \Gamma_X = 2.35 \), and the Fermi-LAT (1 GeV) data are taken from Bissaldi et al. (2020) by assuming a power-law spectrum with photon index \( \Gamma_{LAT} = 2 \). The purple dotted line shows a 1 GeV light curve, which is only obtained by switching on the SSC emission artificially. Lower panel: spectra of the synchrotron, SSC, and total emission with or without absorption at \( t \sim 10^2 \text{ s} \) (red lines), \( t \sim 10^3 \text{ s} \) (blue lines), and \( t \sim 10^4 \text{ s} \) (green lines). The pink shaded region is the energy range of the MAGIC telescope (50 GeV–10 TeV).

Figure 3. Light curves of model B (upper panel) and model C (lower panel) for GRB 201216C. The data points are the same as in Figure 2 and the model light curves are plotted with solid lines. The purple dotted line represents the 1 GeV light curve, which switches on the SSC emission artificially.
model A. The synchrotron component contributes dominantly to GeV emissions at \( t \approx 10^2 \) s and the flux of 1 GeV is \( \gtrsim 10^{-9} \) erg cm\(^{-2}\) s\(^{-1}\). Unfortunately, this GRB was not in the field of view of Fermi-LAT at \( t \approx 10^2 \) s. The maximum energy of synchrotron radiation can reach \( \sim 10 \) GeV at \( t \approx 10^2 \) s and the flux decays exponentially with energy above 10 GeV. The SED at \( t = 10^3 \) s shows an obvious transition from the synchrotron component to the SSC component around approximately a GeV at \( t = 10^3 \) s. Then, the SSC component contributes dominantly to the GeV emissions at \( t = 10^3 \) s and the flux keeps rising with the energy above a GeV. However, the flux of the GeV is only \( \sim 10^{-10} \) erg cm\(^{-2}\) s\(^{-1}\) at \( t = 10^3 \) s and 10\(^3\) s and the current sensitivity of Fermi-LAT is not good enough to detect this evolution of SEDs. What is exciting is that the expected sub-TeV flux of this GRB is comparable to the GeV flux, which can be explained by the detection of MAGIC. For the VHE energy range, we can see that the internal absorption hardly changes the energy spectra of the SSC component, but the EBL absorbs most of the SSC emissions, especially above 0.3 TeV. However, the emissions below 0.1 TeV are not completely absorbed by EBL. Therefore, as long as the emissions below 0.1 TeV are strong enough, the VHE telescope can still observe a low energy VHE signal.

The small value of \( \epsilon_e \) leads to a large value of \( E_k \). In model B, we consider the small value of \( \epsilon_e \), and the fitting results are shown in the upper panel of Figure 3. We can see that model B can explain the observed X-ray and optical data points, as well as the decay slope of the \( r \)-band and 0.1 TeV light curves. However, the predicted flux of the 0.1 GeV light curve is higher and has already reached the upper limit values of Fermi-LAT at \( 10^3 \) s.

If we consider the EATS effect, the fitting results are shown in the lower panel of Figure 3. We can see that model C can explain the observed optical data point, the Fermi-LAT upper limit value, and the decay slope of the 0.1 TeV light curve. However, the predicted X-ray light curve is slightly lower than the observed data, and the power-law decay index of the \( r \)-band light curve is steeper than \(-1.1\). In addition, we obtain a relatively small jet half-opening angle, i.e., \( \theta_j = 0.05 \) rad, indicating that the kinetic energy of the shock is smaller than those of model A and model B.

### 5. Discussion and Conclusions

In this paper we performed self-consistent numerical modeling to calculate the temporal evolutions of the energy distributions of electrons and photons in the GRB shock, and then we applied our numerical model to explain the available multiwavelength observations of distant GRB 201216C. The observed optical and X-ray afterglow emissions can be explained by the synchrotron radiation, while the sub-TeV emissions are dominated by the SSC radiation. The physical parameters of this GRB are still uncertain, due to the limited observations and the parameter values used in Table 1 are optimized to explain the optical and X-ray light curves, and the Fermi-LAT upper limit. However, we offer the possibility of explaining the available observations for this GRB.

The afterglow model described so far is based on the assumption of an ISM model. We found that all the models in Table 1 have a very small value of \( \epsilon_B \); the low value of \( \epsilon_B \) leads to a slow cooling regime for the shocked electrons, i.e., \( \gamma_m' < \gamma_v' \), and the observed optical frequency is away between \( \nu_m' \) and \( \nu_c' \) \((\nu_m' \) and \( \nu_c' \) are the characteristic frequencies of \( \gamma_m' \) and \( \gamma_v' \)). If we consider the wind-like scenario, the late time afterglow light-curve evolution of the \( r \) band is \( F_r(t) \propto t^{p-3/2)/4} \) (Chevalier & Li 2000). Assuming \( p \sim (-3 \rightarrow -2) \), the decay rate of the \( r \)-band light curve is \( \alpha_r \sim (-2 \rightarrow -1.25) \), more steeper than the observed decay rate \( \alpha_r \sim -1.1 \). But the ISM model can explain this decay slope (shown in Figures 2 and 3). Also, our model predicts that the density of the external medium is relatively low, and such low medium density is not consistent with the wind-like scenario.

It must be noted that when we dealing with the continuity equation of electrons, the creation and annihilation of pairs were not considered in our numerical calculation. This is because the cooling effects of the pairs’ creation and annihilation have no observable impact on the afterglow emissions. We can obtain this conclusion simply from the internal absorption of VHE emissions. The effects of EBL absorption together with that of internal absorption derived from model A are illustrated in Figure 4. The upper panel of Figure 4 shows the internal absorption at different times. The results suggest that the internal absorption is minimal in the VHE energy range. Even at \( t = 10^3 \) s, the internal absorption only absorbs \( \lesssim 20\% \) emissions at 1 TeV. The internal absorption is even less at lower energy and earlier time. The lower panel of Figure 4 shows the EBL absorption at different redshifts. The results suggest that with the increase of redshift, the EBL absorption becomes stronger. The EBL absorption is a major effect for energies up to 30 GeV. For energies up to 0.1 TeV, the observed fluxes after EBL absorption are 0.8, 0.4, and 0.2 times the original flux for \( z = 0.5 \), 1.1, and 1.6, respectively. For energies up to 0.3 TeV, the observed fluxes...
after EBL absorption are only 0.2, $10^{-2}$, and $10^{-3}$ times the original flux for $z = 0.5, 1.1$, and $1.6$, implying that the emissions of distant GRBs ($z \gtrsim 1$) are almost completely absorbed by EBL. Hence, for distant GRBs, we do not expect to observe emissions above 0.3 TeV.

We can estimate the maximum distance of GRBs that can be detected by MAGIC. The specific approaches are as follows: First, assume that GRB 201216C is a typical VHE GRB and the parameters of model A are the typical intrinsic parameters; Second, place GRB 201216C at different cosmological distances artificially and calculate the corresponding observed flux; Third, compare the observed flux with the differential sensitivity of MAGIC for different duration times. If the observed flux is higher than the sensitivity, the VHE emissions can be observed by MAGIC; otherwise, this GRB cannot be observed. The results are shown in Figure 5. For the case of low redshift ($z \sim 0.5$), the GRB has a higher flux and less EBL absorption. The spectral flux observed at different times is higher than the corresponding sensitivity, so the early and late time ($\sim 10^{2} – 10^{4} s$) VHE emissions can be observed by MAGIC, and the maximum energy of VHE photons can reach the order of several TeV. In the case of medium redshift (GRB 201216C, or $z \sim 1.1$), due to the good sensitivity of MAGIC, it is likely that the $\lesssim 0.3$ TeV emissions of a GRB can be observed, especially for the observation at $10^{3} – 10^{4} s$. But the early ($t \lesssim 1000$ s) sub-TeV emissions may be invisible to MAGIC. The maximum energy of VHE photons can only reach a few hundred GeV. In the case of high redshift ($z \sim 1.6$), the GRB flux is weaker and the EBL absorption is even stronger. We can see that the observed VHE flux exactly equals the sensitivity of MAGIC. We believe that if the distance of this GRB is farther, it cannot be observed by MAGIC. Therefore, we concluded that the GRB falls below MAGIC sensitivity at $z \sim 1.6$ ($D_L \sim 10$ Gpc). The local event rate of long GRBs is $\rho_0 \sim 1$ Gpc$^{-3}$ yr$^{-1}$ (Wanderman & Piran 2010); we can then estimate that $\sim 1000$ GRBs occurred every year for $z < 1.6$. However, only a few VHE GRBs have been detected at present, suggesting that the generation and observation of VHE GRBs require extremely strict conditions.

Compared with the two other normal GRBs in which sub-TeV emissions were also detected, such as GRB 190114C, GRB 180720B, GRB 201216C has a high redshift and we cannot avoid the EBL absorption, but they share some common features (Huang et al. 2020). (1) Both GRBs are energetic enough, the kinetic energy of these three GRBs can reach $E_k = 10^{53} – 10^{54}$ erg, and it is 1–2 orders of magnitude higher than most normal long GRBs. (2) The external medium density of both shocks is relatively low, $n \sim 0.1 – 1$ cm$^{-3}$. It is likely a uniform density ISM, rather than a stratified stellar wind, and such low medium density can reduce the internal absorption of the source and make the sub-TeV emissions easier to detect. (3) The outflows of both GRBs are ultrarelativistic with the initial bulk Lorentz factor of 100–600, and both GRBs have a relativistic electron power-law index with $p = 2.2 – 2.6$. (4) The modeling gives a small magnetic equipartition factor of $\epsilon_B \sim 10^{-4} – 10^{-6}$ and a relatively large value of the electron energy density fraction of $\epsilon_e = 0.1 – 0.7$. The case of $\epsilon_B \ll \epsilon_e$ suggests that a significant SSC component is expected (Zhang & Mészáros 2001). Such a small value of $\epsilon_B$ also implies that the magnetic field may decay with distance downstream of the shock front (Lemoine 2013, 2015; Lemoine et al. 2013).

High redshift is a unique feature of GRB 201216C. Even with strong EBL absorption, its sub-TeV emissions can still be observed. The observable VHE GRBs may be required the following special conditions: (1) The internal absorption can be avoided. The EBL absorption is not very important at $\sim 50$ GeV, if the internal absorption can be avoided, then it is possible to observe the VHE emissions by MAGIC around 50–100 GeV. (2) The GRBs are energetic enough and the SSC.

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1. In the medium range of duration times, the sensitivity of the MAGIC telescope follows the usual $\propto 1/\sqrt{t}$ dependence (Aleksić et al. 2016).
2. GRB 190829A also observed the VHE emissions by H.E.S.S., but several indications implied that GRB 190829A is a low luminosity GRB (Chand et al. 2020; Zhang et al. 2021a; H.E.S.S. Collaboration et al. 2021; Lu-Lu et al. 2021), different from GRB 201216C.

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Figure 5. We take the parameters of model A as the typical values and compare the observed VHE spectral distribution at different times (red solid lines for $t \sim 10^2$ s, blue solid lines for $t \sim 10^3$ s, and green solid lines for $t \sim 10^4$ s) and at different redshifts (upper panel for $z = 0.5$, middle panel for $z = 1.1$, and lower panel for $z = 1.6$). The differential sensitivity of MAGIC scaled to the duration times of the observation, i.e., $10^2$ s (red data points), $10^3$ s (blue data points), and $10^4$ s (green data points) are also shown. The differential sensitivity of MAGIC is adopted from https://magic.mpp.mpg.de/uploads/pics/info.txt.
component is significant enough, even with strong EBL absorption, and such sub-TeV emissions can still be observed.

(3) The sensitivity of the VHE telescope is good enough at lower energy, and improving the low energy sensitivity of the VHE telescope is very important to detect the sub-TeV emissions of these high redshift GRBs.

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