Calculation of trajectories and the rate of growth of curvilinear fatigue cracks in isotropic and composite plates

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Abstract. The methods of calculating the trajectories and the rate of growth of curvilinear fatigue cracks in isotropic and composite plate structure elements during cyclic loading along straight or curvilinear trajectories are developed. For isotropic and anisotropic materials, the methods are developed on the basis of the force criterion of destruction with the additional application of the fatigue fracture diagrams. To find the change in the shape of the cracks in the loading process, the step-by-step method was used. At each stage, the direction of the growth of all vertices of cracks and the lengths of their arcs was found on the basis of determining the intensity coefficients of stresses by the method of singular integral equations. The results of calculations of the cracks system growth process are presented.

1. Introduction
One of the most common reasons of lamellar elements destruction in cyclic loads is the growth of small initial cracks until they reach critical dimensions. [1,2].
Investigation of the process of growth of fatigue cracks makes it possible to predict the survivability of structures, that is, to evaluate the process from the origin of cracks to complete destruction [1-10]. The criteria for destructive mechanics are used to develop such methods (to find the directions of cracks growth), Fatigue Fracture Diagram (FFD, necessary for finding the speed of peaks) and numerical algorithms for calculating Stress Intensity Factor (SIF) for cracks [11-13]. In calculations in the literature, the most commonly used step-by-step methods, in which the trajectory of cracks growth is described by segments of a straight line (polygonal chain). In this paper, the trajectory of a crack is described by splines of the third degree, which enables simultaneously to control the accuracy of calculations, increase steps and thereby reduce the volume of computations and obtain smooth trajectories of cracks, which are usually observed in practice.
For composite materials, the question of determining the directions of fatigue cracks growth is studied much less than for isotropic materials due to the complication of constructing general kinetic equations of the peaks movement. In the work to find the trajectories of growth of cracks, a force destruction criterion is used, which takes into account the anisotropy of the strength characteristics of the composite material. Algorithms based on the method of singular integral equations are used to calculate stresses at cracks. [11-14].
2. Algorithm for calculating the growth process of the system of curvilinear fatigue cracks in plates with a small degree of anisotropy

We consider a plate that at the initial moment of time is weakened by a system of cracks placed along the curves, which is loaded with cyclically variable forces at infinity (for plates of infinite size), concentrated forces, stress, which are applied in accordance with the boundary of the plate and the sides of the cracks.

For isotropic materials, when finding the direction of change in the trajectory of the selected peak of the crack (depending on the conditions) used power, displacement or mixed criteria of destruction. According to them

\[ \theta = f(K', K^*, C_p) , \]  

where \( \theta \) is the angle measured from the tangent to the crack at its peak, \( C_p \) is set of additional parameters, which take into account the property of the material and the conditions of operation of the structure, \( K', K^* \) are characteristic (for example, high or medium) value of SIF in a cycle.

In determining the length of the arc increments were used FFD, which were selected in the form

\[ V = F(K_{\text{max}}, K_{\text{min}}, C_v) , \]  

where \( V \) is the growth rate of the crack, \( K_{\text{max}}, K_{\text{min}} \) are respectively equal to the largest and smallest value of K, which characterizes SSS (Stress-Strain State) at the peak of the crack during cyclic loading, \( C_v \) are additional parameters that take into account the properties of the material and the conditions of operation of the structure. The relations (1) and (2) are used for isotropic materials. In the same form, relations can also be written for anisotropic materials, while taking into account the strength of anisotropy materials near the peak of cracks.

The calculation algorithm is as follows. Let's consider n – step. We determine the peaks of j cracks by \( A_{j,n}, B_{j,n} \). In the process of tracking the cracks change their shape. To find SIF using the method of integral equations \[12, 13, 14\] it is necessary to describe them in an analytical form. For this purpose, on each of the cracks, we will specify the coordinates of the nodal points and, in addition, the angles of the inclination of the tangents at the peaks.

We denote the selected nodes on the crack at the number j at the number of the step n – via \((x_{j,n}^{(1)}, y_{j,n}^{(1)}), (x_{j,n}^{(2)}, y_{j,n}^{(2)}),..., (x_{j,n}^{(N_j)}, y_{j,n}^{(N_j)})\), where \( N_j \) is number of nodal points. Note that the first and last points correspond to the crack tips. We will also give the values of the angles of inclination of tangents at the peaks of the crack \( \phi_{j,n}^A, \phi_{j,n}^B \) to the axis Ox, which we will denote by \( \phi_{j,n}^A, \phi_{j,n}^B \).

On the basis of the given coordinates of the nodal points and the derivatives of the equality of each crack, we describe crack form with the help of a cubic spline, which is continuous along with the first two derivatives and has predetermined inclining tangents at the vertices. Then using known methods of calculation SSS \[12, 13, 14\] of plates with cracks we find SIF for both vertices of cracks, which we denote by a \( K_{j,1}, K_{j,2} \).

Using equation (1) we find angles \( \theta_{j,n}^A, \theta_{j,n}^B \), under which two peaks will grow j crack, which are counted from the corresponding tangent. Let's assume that in the process of destruction, all the peaks of the cracks can move. Denote the coordinates of the peaks of the crack in the next step through \( A_{j,n+1}, B_{j,n+1} \) and the distances to which they moved, through \( \Delta_{j,n+1}, \Delta_{j,n+1}^B \), where

\[ \Delta_{j,n+1}^A = |A_{j,n+1} - A_{j,n}|, \Delta_{j,n+1}^B = |B_{j,n+1} - B_{j,n}| .\]

Based on the ratio (2) we have

\[ \Delta_{j,n+1}^A = F_{j,n}(A)\Delta_{n+1}, \Delta_{j,n+1}^B = F_{j,n}(B)\Delta_{n+1} , \]  

where \( F_{j,n}(A) = F(K_{\text{max}}(A_{j,n}), K_{\text{min}}(A_{j,n})), F_{j,n}(B) = F(K_{\text{max}}(B_{j,n}), K_{\text{min}}(B_{j,n})) \), \( \Delta_{n+1} \) is time of growth.
Here we assumed that the peaks of the crack grew along the straight line. When calculating, we assume that the distance to which the peak with the highest value of SIF has moved at a certain point is equal to some value, small in size. Then, based on (2) we will get
\[ \Delta r_{n+1} = \Delta / F_{j,\text{max}} \]
where \( F_{j,\text{max}} \) are highest value \( F_{j,a} (A), F_{j,a} (B), j = 1, ..., J \).

On the basis of the above relations, we find the coordinates of the peaks of the cracks at a new stage
\[
A_{j,a+1} (x_{j,a+1}^A, y_{j,a+1}^A), B_{j,a+1} (x_{j,a+1}^B, y_{j,a+1}^B),
\]
where \( x_{j,a+1}^A = x_{j,a}^A + \Delta x_{j,a+1}^A \cos \phi_{j,a}, \)
\( y_{j,a+1}^A = y_{j,a}^A + \Delta y_{j,a+1}^A \sin \phi_{j,a}, \)
\( x_{j,a+1}^B = x_{j,a}^B + \Delta x_{j,a+1}^B \cos \phi_{j,a}, \)
\( y_{j,a+1}^B = y_{j,a}^B + \Delta y_{j,a+1}^B \sin \phi_{j,a}. \)

Next we find
\( \phi_{j,a+1} = \phi_{j,a} + \theta_{j,a}, \quad \phi_{j,a+1} = \phi_{j,a} + \theta_{j,a} - \) angles of inclination tangent to a crack in its peaks to the axis Ox. The results obtained allow us to proceed to the next stage of tracking the trajectory of the crack.

For this purpose, for the description of the crack, we lead nodal points in consideration \( (x_{j1}^{(n+1)}, y_{j1}^{(n+1)}), (x_{j2}^{(n+1)}, y_{j2}^{(n+1)}), ..., (x_{jN_n}^{(n+1)}, y_{jN_n}^{(n+1)}), \) where \( N_n = N_{n+2} \);
\( x_{j1}^{(n+1)} = x_{j1}^{(n)}, \quad x_{j2}^{(n+1)} = x_{j2}^{(n)}, \quad x_{jN_n}^{(n+1)} = x_{jN_n}^{(n)}, \quad y_{j1}^{(n+1)} = y_{j1}^{(n)}, \quad y_{j2}^{(n+1)} = y_{j2}^{(n)}, \quad y_{jN_n}^{(n+1)} = y_{jN_n}^{(n)}, \)
where \( k = 2, ..., N_n). \)

In this way, we sequentially establish the shape of the crack at the following stages and moments of time, which these stages correspond to relative to the crack.

The time required for crack growth \( \tau_n \) (number of cycles), for which peaks of crack number \( j \) will reach points \( A_{j,a}, B_{j,a} \) is determined by the formula \( \tau_n = \Delta \tau_0 + \Delta \tau_1 + ... + \Delta \tau_n. \)

We divide the entire load process, which we will characterize by the parameter \( p \) on the stages. Assume that we have already found the equation of a crack at the end of stage \( n \) and denote its equation via \( x = \varphi_n (t), y = \psi_n (t), \) where \( t \) is the parameter that varies within \(-1 \leq t \leq 1.\) We will assign the number 0 to the initial crack.

3. Determination of cracks growth directions

Consider the general case where both SIF values are different from zero and in the process of loading, the crack grows along the curvilinear trajectories. In this case, it is necessary to choose a criterion for changing the direction of growth of fatigue cracks on steps. In the literature is shown [12], which in many cases can be used for this purpose specified above \( \sigma_p \) - criterion. Calculations showed, that in the process of tracking the motion of the tops of the crack SIF \( K_p \) practically is equal to zero. The results obtained in this paper also confirm this conclusion. That is, in this case, the destruction process will pass as the mechanism of normal separation, at which \( K_p = 0. \) Thus, when applied \( \sigma_p \) is the criterion for calculating the curvilinear trajectories can FFD be used, which are based on the study of the growth of rectilinear cracks by the mechanism of normal separation.

4. Determination of the curvilinear cracks system growth process by force criterion of destruction

In the literature, the development of a single fracture is sufficiently studied. Let's consider a less studied problem about the growth of the system of fatigue cracks.

To find the length of the arc of the crack, it is also necessary to use the equation for the rate of cracks growth. We use the following formula [15].

\[ L = \int_{t_0}^{t_1} \sqrt{\left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2} dt \]
\[ v = v_0 \left( \frac{\Delta K - \Delta K_{th}}{\Delta K_{th} - \Delta K_{fc}} \right)^{\eta}, \]  
where \( \Delta K_{th}, \Delta K_{fc} \) are respectively, the initial and critical ranges of SIF \( \Delta K_f \), \( v_0 \) and \( q \) are constants of the material. Note that when \( \Delta K < \Delta K_{th} \) the crack does not grow, but with \( \Delta K > \Delta K_{fc} \) there is a fragile (instantaneous) destruction.

Let's rewrite this formula in the form

\[ v_0 \Delta N = \left( \frac{1 - k \eta}{k \eta - \varepsilon} \right)^\eta \Delta l, \]

where \( k = \frac{\Delta K}{\Delta K_0}, \eta = \frac{\Delta K_0}{\Delta K_{fc}}, \varepsilon = \frac{\Delta K_{th}}{\Delta K_{fc}}, \Delta K_0 = p \sqrt{\pi L}, \Delta l \) is increase in the length of the trajectory, \( \Delta N \) is the number of cycles during which the growth takes place. Note that \( \Delta K_0 \) is range of SIF for the initial single fracture in the plate, which is under the action of tension perpendicular to it.

5. Results of calculations

Calculations are performed at \( \varepsilon = 0.12, q = 1.26, \eta = \sqrt{\pi} / 3 \approx 0.6 \)

The case of one-sided stretching of plates with straight and weakly curved cracks is studied in detail. The results of calculations of the crack growing process, located on a circle of radius \( R = 0.1 \) m at \( p / 4 < j < p / 4 \) for stretching the plates with effort \( pC \), where \( C \) – cyclic factor, which varies in each of the load cycles from 0 to 1 (ie, in the cycle, the maximum value of the effort is \( p \), and the minimum value is 0) at an angle \( \theta \) to the axis \( OX \) shown in Fig. 1. Calculations were made at \( p = 5 \) MPa (ie, in the cycle, the maximum value of the effort is \( p \), and the minimum value is 0), \( N = 60, K = 80 \), where \( K \) – number of steps. Trajectories of the growth crack are shown on the upper figure; the rate of crack growth (number of cycles), depending on the number of steps, is shown on the middle figure; the SIF value at the top of the crack is on the bottom figure. Here the numbers on charts 0, 1, 2 correspond to the numbers of the steps: 0 (initial crack), \( K/4, K/2, 3K/4, K \). On the basis of the calculations performed, it was found that rectilinear and weakly curved cracks in endless plates, which are in the form of one-piece stretch, grow, as a rule, perpendicular to the direction of the efforts.

Consider a plate with two half-length cracks, parallel to the axis \( OY \) at a distance between them \( d \). Let's assume that the plate is stretched along the axis \( OX \) with efforts \( pC \).

Calculations are performed at 40 steps in the tracking method. On figure 2 above shown the calculated trajectories of the two identical cracks movement, the distance between them \( L \), and here the coordinates are attributed to the half-length of the cracks. At the left side below shows the dependence of the value \( \tau = v_0 N / L \) from the value of the steps. Note that the characteristic points on the trajectories, near which numbers 1, 2, 3, 4 placed match the step numbers 10, 20, 30, 40. At the right side below is the value \( K_m = k \eta K_{fc} \), which characterizes the size of SIF in the process of propagation of cracks, and
accepted $K_{\phi} = 37MPa\sqrt{m}$, which takes place for steel 75KhGST (75XGCT).

To ensure accuracy the number of nodes in the quadrature formula was chosen sufficiently large - up to 30. With less accuracy of calculations, there was a loss of symmetry, which corresponds to the real instability of crack growth in practice. It follows that minor random factors can significantly change the propagation of parallel, closely placed cracks. Note that the calculated trajectories almost coincide with the data of the work [16], in which the problem was solved taking into account the symmetry of the problem, in which instability of calculations does not arise.

Consider the case when the right split has a smaller length - equal $0.9L$. The results of calculations for this case are shown in figure 3. Curves 1, 2 on the lower right figure correspond to SIF in the lower and upper peaks of the left crack, and 3, 4 - for the right cracks.

The upper figure shows that a smaller fracture grows at a slower rate than a larger one. From the bottom left figure it follows that a slight change in the length of one of the cracks leads to a significant
reduction in durability (at about two times at \( L_2/L_1 = 0.9 \)). We give relative values of SIF 
\[ F_{i,j} = K_{i,j} / (p \sqrt{\pi L}) \] for initial cracks: 0.8309, 0.0970 (left crack) and 0.7554, -0.1331 (right crack).

Now assume that in the above system the center of the right-handed crack is shifted - it is at the point \( (L,0.1L) \). The results of calculations for this case are shown in figure 4.

From the upper figure it can be seen that the upper peaks of the cracks grow at almost the same speed (This is confirmed by the fact that curves 2 and 4, which describe the SIF in these peaks are close), with the trajectories of their movement somewhat removed from each other. The maximum speed has a lower peak of a longer crack, and closer to it peak of a smaller crack is practically immobile. The durability of the crack system, considered in the last two cases, is practically the same.

5.1. System of four parallel cracks

We will initially calculate the SIF for the system parallel to the axis \( O_y \) our equidistant cracks halflength of \( L \), stretching by the forces of \( pC \) in the direction of the axis \( O_x \). The calculation results of the relative values of the SIF 
\[ F_{i,j} = K_{i,j} / (p \sqrt{\pi L_j}) \] for the average (\( j=1 \)) and for the right (\( j=2 \)) top peaks of the cracks are shown in Table 1.

| \( d/L \) | \( F_i \) | \( F_{i,i} \) | \( F_j \) | \( F_{j,i} \) |
|-------|------|------|------|------|
| 0.25  | 0.441 | -0.002 | 0.675 | 0.147 |
| 0.50  | 0.532 | 0.000 | 0.724 | 0.113 |
| 0.75  | 0.606 | 0.001 | 0.766 | 0.085 |
| 1.00  | 0.672 | 0.002 | 0.805 | 0.062 |
| 1.50  | 0.778 | 0.002 | 0.867 | 0.033 |
| 2.00  | 0.848 | 0.001 | 0.909 | 0.018 |
| 2.50  | 0.892 | 0.001 | 0.935 | 0.011 |
| 3.00  | 0.920 | 0.001 | 0.952 | 0.007 |
| 4.00  | 0.952 | 0.000 | 0.971 | 0.003 |
| 5.00  | 0.968 | 0.000 | 0.981 | 0.002 |

The table shows that the SIF values for the side cracks are much larger than for the central. In addition, for central cracks the value \( K_{ii} \) practically equal to zero. From here it is clear that side cracks will grow faster, and they will grow in the direction opposite to the central cracks.

Tables 2 and 3 show similar results for a case where the half-length of the central cracks is equal \( L_1 = 1.1L \) and \( L_2 = 1.2L \), and side cracks - \( L_2 = L \).
Table 2. Relative SIF for a system of four cracks (the half-length of the central cracks is equal $L_1 = 1.1L$ and side cracks is $L_2 = L$.)

| d/L  | $F_{i}$ | $F_{1i}$ | $F_j$ | $F_{1j}$ |
|------|---------|----------|-------|----------|
| .25  | 0.546   | 0.035    | 0.574 | 0.167    |
| .50  | 0.578   | 0.018    | 0.671 | 0.126    |
| 0.75 | 0.628   | 0.013    | 0.729 | 0.095    |
| 1.00 | 0.681   | 0.011    | 0.775 | 0.070    |
| 1.25 | 0.731   | 0.009    | 0.814 | 0.051    |
| 1.50 | 0.775   | 0.007    | 0.846 | 0.037    |
| 2.00 | 0.842   | 0.004    | 0.893 | 0.021    |
| 2.50 | 0.886   | 0.003    | 0.924 | 0.012    |

From Table 2 it can be seen that the character of the SIF distribution is the same as for the cracks of the same length. At $L_1 = 1.1L$ SIF values for central crack dominate the SIF for the side crack. It follows that in the latter case faster growth of central crack should be expected.

Let's now calculate the process of fatigue destruction of the plate. Figure 5 shows the results of calculations for four cracks of equal length $L$, the distance between which is equal to $L$.

Similar results of calculations for cases where side cracks have a half length of $L$, and average $0.9L$ and $0.8L$ shown in figure 6, 7.

Figure 5. The system of four parallel cracks of the same length

Table 3. Relative SIF for a system of four cracks (the half-length of the central cracks is equal $L_1 = 1.2L$ and side cracks is $L_2 = L$.)

| d/L  | $F_{i}$ | $F_{1i}$ | $F_j$ | $F_{1j}$ |
|------|---------|----------|-------|----------|
| 0.25 | 0.619   | 0.074    | 0.462 | 0.173    |
| 0.50 | 0.617   | 0.038    | 0.613 | 0.136    |
| 0.75 | 0.649   | 0.026    | 0.688 | 0.104    |
| 1.00 | 0.690   | 0.020    | 0.744 | 0.077    |
| 1.25 | 0.733   | 0.016    | 0.788 | 0.057    |
| 1.50 | 0.772   | 0.013    | 0.825 | 0.042    |
| 2.00 | 0.836   | 0.008    | 0.877 | 0.024    |
| 2.50 | 0.879   | 0.005    | 0.912 | 0.014    |
5.2. Consider a composite orthotropic plate with a crack. The calculation of the fatigue cracks growth process is complicated, since the direction of growth is determined not only on the basis of SIF, but also taking into account the placement of a crack relative to the orthotropic axes. Building a FFD for composite materials is also a difficult task. Therefore, we will restrict ourselves to the tasks in which the crack growth trajectory can be found without using FFD.
Consider the crack, the tangent to which at its peak $A$ inclined at an angle $\varphi$ to the axis $Ox$. Here and then the Ox axis is chosen so that along with it the module of elasticity of the material was maximal. We will use the polar coordinate system $(r, \theta)$ with center at the top of the crack so that the angular coordinate $\theta = 0$ coincided with the tangent to the crack at its peak (figure 8).

![Figure 8. Load scheme of the plate with a crack](image)

We will consider the known SIF for this peak, which we denote by $K_I$, $K_{II}$. For stresses in the neighbourhood of the peak of the crack, we have asymptotic relations [12,16]

$$\sigma_r(r, \theta) \approx \frac{K_{II}(\theta, \varphi)}{\sqrt{2\pi r}}, \quad \tau_{r\theta}(r, \theta) \approx \frac{K_{II}(\theta, \varphi)}{\sqrt{2\pi r}},$$

(6)

where

$$K_{II}(\theta, \varphi) = K_{I1}f_{11}(\theta + \varphi, \varphi) + K_{II}f_{12}(\theta + \varphi, \varphi),$$

$$K_{II}(\theta, \varphi) = K_{I1}f_{21}(\theta + \varphi, \varphi) + K_{II}f_{22}(\theta + \varphi, \varphi),$$

(7)

$$f_{11}(\gamma, \varphi) = \text{Re}\left[\left(g_1\delta_2(\gamma)\sqrt{\delta_2(\gamma)}d_2\frac{d_2 - g_2\delta_1(\gamma)}{\delta_1(\gamma)}d_1\right)/(s_2 - s_1)\right],$$

$$f_{12}(\gamma, \varphi) = -\text{Re}\left[\left(d_1\delta_2(\gamma)\sqrt{\delta_2(\gamma)}d_2\frac{d_2 - g_2\delta_1(\gamma)}{\delta_1(\gamma)}d_1\right)/(s_2 - s_1)\right],$$

$$f_{21}(\gamma, \varphi) = \text{Re}\left[\left(g_1\delta_2(\gamma)\sqrt{\delta_2(\gamma)}d_2\frac{d_2 - g_2\delta_1(\gamma)}{\delta_1(\gamma)}d_1\right)/(s_2 - s_1)\right],$$

$$f_{22}(\gamma, \varphi) = -\text{Re}\left[\left(d_1\delta_2(\gamma)\sqrt{\delta_2(\gamma)}d_2\frac{d_2 - g_2\delta_1(\gamma)}{\delta_1(\gamma)}d_1\right)/(s_2 - s_1)\right],$$

$$\delta_j = \delta_j(\gamma) = \cos \gamma + s_j \sin \gamma, \quad \gamma_j = \gamma_j(\gamma) = \sin \gamma - s_j \cos \gamma, \quad g_j = g_j(\varphi),$$

$$d_j = d_j(\varphi), \quad \gamma = \varphi + \theta, \quad s_{1,2}$$ are the roots of the characteristic Lehniks equation [11].

Assume that the fracture toughness is known $K_c(\varphi)$, which is determined on the basis of the stretching of the plate in a direction perpendicular to the crack. Consider the fiberglass $E\Phi 32-301$, which is made of fiberglass ACTT. For functions $K_c(\gamma)$ use experimental data [17] (table 4).

**Table 4.** Values $K_c$ (MPa • m$^{1/2}$) depending on the angle $\gamma$ (angle between the crack and the direction with maximum stiffness of the material)/

| $\gamma$, radian | 0 | $\pi/6$ | $\pi/4$ | $5\pi/16$ | $\pi/2$ |
|-----------------|---|---------|---------|----------|---------|
| E\Phi 32-301    | 13.0 | 16.79 | 18.66 | 22.74 | 25.55 |

Function $K_c(\gamma)$ (MPa • m$^{1/2}$) based on the data in the table for $-\pi \leq \gamma \leq \pi$ we will describe in the
The coefficients in this formula, found by the method of least squares, will be: $a_0 = 25.6074$, $a_1 = -11.2031$, $a_2 = -6.0858$, $a_3 = 4.7243$.

The crack will grow at an angle $\theta$, at which value $\sigma(r, \theta)/K_c(\gamma)$ is the maximum. Based on (7) then we get that angle $\theta = \gamma - \varphi$ is such that the value is the maximum.

$$I(\gamma) = \frac{K_1 f_1(\gamma, \varphi) + K_2 f_2(\gamma, \varphi)}{K_c(\gamma)},$$

Calculations of the trajectory growth of the initially rectilinear a half-longitudinal fracture $a_1$, inclined at an angle $\varphi = 0^\circ$ when stretching the plate with the forces acting at angles $\omega = 18^\circ$, $j = 1,...,9$ to the axis Ox. Calculated trajectories are shown in figure 9 at which angle $\omega$, is indicated at the top and here the coordinates are assigned to the half-length of the crack $a$.

**Figure 9.** Trajectories of the crack growth, inclined at an angle $\varphi = 0^\circ$ to the axis Ox

Similar data at the angle of crack inclination $\varphi = 30^\circ$ shown in figure 10.
Figure 10. Trajectories of the crack growth, inclined at an angle $\phi = 30^\circ$ to the axis Ox.

Figure 9-10 shows that the cracks mainly grow along the straight lines.

6. Conclusions
The method of calculating the durability of isotropic and composite plate elements of constructions under cyclic loading is developed on the basis of the study of the growth process of fatigue cracks along the curvilinear trajectories. Calculations are made on the basis of singular integral equations, force criterion of destruction, diagrams of fatigue failure. The characteristic features of fatigue cracks system growth in isotropic materials are defined. In particular, in the system of four identical cracks, central cracks do not propagate. For longer $1,1L$ central cracks comparatively to side cracks at the initial stage all cracks grow at almost the same speed, however, further central cracks stop their growth. Only at significantly shorter lengths of the side crack, the growth rate in the central cracks is greater than in the side one, and in the process of propagation the side cracks almost stop their growth.

The curvilinear trajectories of the growth of the initially rectilinear crack in the composite plate are investigated depending on the direction of the applied load.
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