A microscopic theory of skyrmion excitations in the fractional quantum Hall system

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We present a microscopic theory of skyrmion and antiskyrmion excitations in fractional quantum Hall systems, and calculate in an analytical fashion their excitation energies. From the calculated net spins at various fractional filling factors, we find the magnetic field dependence of the spin polarization of the skyrmion-condensated state.

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1. Introduction

Charged spin textures known as skyrmions in the quantum Hall system have been of great interest. In the limit of weak Zeeman coupling, it was predicted that the lowest energy excitations around filling factor \( \nu = 1/M \) with odd integer \( M \) are skyrmions \[1\]. Recent experiments demonstrated the existence of skyrmions for \( \nu \) near 1 in two-dimensional electron systems \[2, 3, 4\]. Since the skyrmion size is determined by relative strength of Coulomb and Zeeman interaction, the effective field theory of nonlinear sigma (NL\( \sigma \)) model has not been successful in describing correctly the observed spin polarization \[2, 3, 4\]. Despite several microscopic theoretical attempts to overcome the drawback of the field theory \[2, 3, 4\], most calculations were limited to the case of \( \nu = 1 \).

In this paper, we develop a microscopic approach to the skyrmion in the fractional quantum Hall regime, which allows for analytical calculations of the total energies and net spins of skyrmion and antiskyrmion. We perform infinite-size calculations for filling factors \( \nu = 1, 4/5, 2/3, 1/3, \) and \( 1/5 \), and predict the field dependence of the spin polarization for the skyrmion-condensated state.

2. Theory

At filling factor \( \nu = 1/M \), the ground state of a quantum Hall system is well approximated by the incompressible spin-polarized state in the symmetric gauge:

\[
\Phi_g^{\nu} = \sum_{i_1 < i_2 < \ldots < i_N} C^{\nu}(i_1, i_2, \ldots, i_N) \prod_{k=1}^{N} c_{i_k, \uparrow}^{\dagger} | 0 \rangle,
\]

where \( | 0 \rangle \) is the vacuum state and \( c_{i, s}^{\dagger} \) creates an electron of spin \( s \) and angular momentum \( m \) in the lowest Landau level. Here the coefficients \( C^{\nu} \) can be determined by expanding the known Laughlin
wave function. In analogy, the variational wave function for a charged spin-texture excitation will be of the form

$$\Psi^\nu_{\pm} = \sum_{i_1 < i_2 < \ldots < i_N} D^\nu_{\pm}(i_1, i_2, \ldots, i_N) \prod_{k=1}^N \gamma^\dagger_{i_k, \pm} \mid 0\rangle,$$

(2)

where $\gamma^\dagger_{m, \pm}$ are required to satisfy the relation $\gamma^\dagger_{m, \pm} = u_m c^\dagger_{m, \downarrow} + v_m c^\dagger_{m, \uparrow}$ and variational parameters $u_m$ and $v_m$ under the condition $|u_m|^2 + |v_m|^2 = 1$ determine the orientation of spins. Then, $\Psi^\nu_{\pm}$ are invariant under rotation by $R_0 \equiv \exp[i\theta(L_Z \pm S_Z)]$, where $L_Z$ and $S_Z$ denote the z-components of total orbital and spin angular momentums, respectively. The rotational invariance under $R_0$ is not associated with the exact symmetry of the Hamiltonian, but originated from the spin-charge relation in the lowest Landau level. The changes of the charge densities $\rho(r)$ associated with $\Psi^\nu_{\pm}$ in the core region are exactly related to the topological charges $Q = \pm 1$ via $\int d^2r (\rho(r) - \nu/(2\pi^2)) = \nu Q$, where $l$ is the magnetic length $\sqrt{\hbar c/(eB)}$. Thus, we see that $\Psi^\nu_{\nu}$ ($\Psi^\nu_{\nu}$) represents the wave function for a skyrmion (antiskyrmion). Our conjecture that the skyrmion and antiskyrmion states are eigenstates of $L_Z \pm S_Z$ is consistent with the result of the $\sigma$ model and was verified by recent numerical calculations around $\nu = 1$. The coefficients $D^\nu_{\nu, \pm}$ can be well approximated by $C^\nu_{\nu}$ in Eq. (1) because their values must be equivalent to those of $C^\nu_{\nu}$ in the case of $\{u_m = 1; m = 0, 1, 2, \ldots\}$. Moreover, considering a finite size of skyrmion, the electron correlation effects in the ground state and skyrmion excitation state will be nearly equal at sufficiently large distances from the origin. In fact, our choice of $C^\nu_{\nu}$ for $D^\nu_{\nu, \pm}$ gives the charge and spin density profiles of skyrmion and antiskyrmion similar to those obtained from the field theory. For $\nu = 1$, our wave functions are found to be consistent with those proposed by Fertig et al. It is also noted that $\Psi^\nu_{\nu}$ ($\Psi^\nu_{\nu}$) is equivalent to the quasi-hole state of MacDonald and Girvin. For the antiskyrmion state, we need to set $u_0 = -1$ and $v_0 = 0$ to project the single particle state with $m = -1$ onto the lowest Landau level.

For the wave functions $\Psi^\nu_{\nu, \pm}$, the expectation values of the Hamiltonian $H$ can be calculated analytically, using the following relations:

$$\langle \Psi^\nu_{\nu, \pm} | c_{m, \uparrow} c_{m, \downarrow} | \Psi^\nu_{\nu, \pm} \rangle = | u_m |^2 \langle \Phi^\nu_{\nu} | c_{m, \uparrow} c_{m, \downarrow} | \Phi^\nu_{\nu} \rangle$$

$$\langle \Psi^\nu_{\nu, \pm} | c_{m, \dagger} c_{m, \dagger} | \Psi^\nu_{\nu, \pm} \rangle = | v_m + 1 |^2 \langle \Phi^\nu_{\nu} | c_{m, \dagger} c_{m, \dagger} | \Phi^\nu_{\nu} \rangle$$

$$\langle \Psi^\nu_{\nu, \pm} | \gamma_{m, \uparrow} \gamma_{m, \downarrow} \gamma_{m, \uparrow} \gamma_{m, \uparrow} | \Psi^\nu_{\nu, \pm} \rangle = \langle \Phi^\nu_{\nu} | \gamma_{m, \dagger} \gamma_{m, \dagger} \gamma_{m, \dagger} \gamma_{m, \dagger} | \Phi^\nu_{\nu} \rangle.$$  

(3)

At $\nu = 1/M$, the expectation values in Eq. (3) are given in literatures and their analytical forms can be extended to the case of $\nu = 1-1/M$ by using the particle-hole symmetry between $\nu$ and $1-\nu$. Defining $\Psi^\nu_{\nu, -} = \Psi^\nu_{\nu, -}(u_m = 0; m = 0, 1, 2, \ldots)$ and $\Psi^\nu_{\nu, +} = \Psi^\nu_{\nu, -}(u_0 = -1, u_m = 0; m = 1, 2, \ldots)$ for the skyrmion and antiskyrmion excitations, respectively, we calculate the energy differences $\delta \epsilon_\nu \equiv \langle \Psi^\nu_{\nu} | H | \Psi^\nu_{\nu} \rangle - < \Psi^\nu_{\nu} | H | \Psi^\nu_{\nu} >$. At $\nu = 1$, the energy functionals of skyrmion and antiskyrmion are equal because of the particle-hole symmetry between $\nu$ and $2-\nu$, as previously noted by Fertig et al. In this case, we may consider a spin configuration of $\{u_0 = u_1 = \ldots = u_p = 1, u_m = 0; m > p\}$, however, we find this state to be energetically unstable due to large Coulomb interaction between opposite spins on the boundary of the reversed spins. This result leads us to consider skyrmions as symmetry breaking states.

### 3. Results and discussion

For $\nu = 1, 4/5, 2, 3, 1/3,$ and $1/5$, the calculated energy differences $\delta \epsilon_-$ and $\delta \epsilon_+$ are plotted in Fig. 1, as a function of the effective Zeeman coupling $g = (1/2) g^* B/[\epsilon^2/(\epsilon l)]$. We find that the skyrmion...
undergoes a transition into the spin-polarized quasihole state $\Psi_{\nu}^{-}$ at critical value $\tilde{g}_c$, while no such a transition exists for the antiskyrmion state. Since the energy of the reversed electrons increases for stronger Zeeman couplings, the skyrmion size tends to shrink to zero at $\tilde{g}_c$. For $\tilde{g} > \tilde{g}_c$, all $u_m$'s for the skyrmion state are found to be zero. Thus, skyrmions exhibit the critical behavior with the order parameter $u_m$ varying with $\tilde{g}$. Based on the Landau theory, the number of reversed spins in the limit $\tilde{g} \to \tilde{g}_c$ is expressed such as

$$\delta s_+ = \nu \sum_{m=0}^{m=\infty} |u_m|^2 \sim |\tilde{g} - \tilde{g}_c|^{\beta},$$

where $\beta$ represents the critical exponent. From the calculated net spins $\delta s_-$, we estimate $\beta$ to be about 1 [14]. If we assume that the skyrmion statistics is the same as that of the Laughlin quasiparticles, as addressed before [6], the skyrmion condensation can be considered in the framework of hierarchy construction. Then, for the daughter states of the $\nu = 1/M$ parent, the filling factor $\nu'$ is given by $\nu' = \frac{2p}{2MP-\alpha}$, where $p$ is integer number, and the spin polarization $P$ defined as $< S_Z > / (N/2)$ is written as

$$P = 1 - \frac{\delta s_+ + (\alpha + 1)/(2M)}{p},$$

where $\alpha = 1$ for antiskyrmion condensation and $\alpha = -1$ for skyrmion condensation. Thus, it is predicted that as the skyrmion-condensated quantum Hall system at $\nu = \frac{2p}{2MP+1}$ transforms to a spin-polarized state, its polarization behaves as $P - 1 \sim (\tilde{g} - \tilde{g}_c) \sim (\sqrt{B} - \sqrt{B_c})$, where $B_c$ denotes the critical field.

For filling factors near $\nu = 1/3$ and 1/5, the skyrmion excitations exist up to the critical fields of 7.2 and 1.3 T, respectively, for GaAs samples, while previous Monte Carlo calculations showed the corresponding values of 1.6 and 0.21 T. Our calculations suggest that the skyrmions around $\nu = 2/3$...
and 4/5 are reliable quasiparticle states, if the ground states at these filling factors are the particle-hole conjugate states of the Laughlin $\nu = 1/3$ and 1/5 states. There are some experimental evidences for the skyrmion excitation at $\nu = 2/3$; the magnetoabsorption spectroscopy reveals a dramatic reduction of the spin polarization near $\nu = 2/3$ [13], and the magnetotransport measurement of the activation energy indicates that relevant quasiparticles are associated with several reversed spins. It is pointed out that the skyrmion state around $\nu = 2/3$ can not be explained in the composite fermion picture [14], where the $\nu = 2/3$ state is obtained from the $\nu = 2$ singlet state by composite fermion transformation. Using the calculated energies of the polarized and unpolarized ground states at $\nu = 2/3$ [13], we find that $\tilde{g}$ should be greater than 0.0087.

4. Conclusions

We have presented the microscopic theory of skyrmion and antiskyrmion in fractional quantum Hall systems and calculated in an analytical fashion their excitation energies using variational wave functions. We find that the single-Slater-determinant state for the skyrmion near $\nu = 1$ is energetically unstable because of the large Coulomb interaction between neighboring electrons with opposite spins. The skyrmion-condensated states are predicted to show the critical behavior in the vicinity of the transition into the spin-polarized state.

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