Indeterministic Quantum Gravity and Cosmology

VIII. Gravilon: Gravitational Autolocalization

Vladimir S. MASHKEVICH

Institute of Physics, National academy of sciences of Ukraine
252028 Kiev, Ukraine

Abstract

This paper is a sequel to the series of papers [1-7]. Gravitational autolocalization of a body is considered. A self-consistent problem is solved: A quantum state of the center of mass of the body gives rise to a classical gravitational field, and the state, on the other hand, is an eigenstate in the field. We call a resulting solution gravilon. Gravilons are classified, and their properties are studied. Gravitational autolocalization is predominantly a macroscopic effect. The motion of a gravilon as a whole is classical.

1E-mail: mashkevich@gluk.apc.org
Watching the rolling ball, Mr. Tompkins noticed to his great surprise that the ball began to 'spread out’. This was the only expression he could find for the strange behavior of the ball which, moving across the green field, seemed to become more and more washed out, losing its sharp contours.

George Gamow, Mr. Tompkins in Wonderland

Introduction

The matter of the universe is governed by quantum laws. Yet macroscopic bodies exhibit a pronounced classical behavior. This leads to the problem of classicality (see, e.g., [8]). The most essential aspect of the problem is the well-marked localization of macroscopic objects. We quote Penrose [9]: "Why, then, do we not experience macroscopic bodies, say cricket balls, or even people, having two completely different locations at once? This is a profound question, and present-day quantum theory does not really provide us with a satisfying answer”. Indeed, in quantum mechanics, for a stationary state of a free body, the center of mass of the body is not localized whatsoever.

We argue that the classicality problem, specifically the localization problem, cannot be solved unless classical elements have been incorporated into a theory from the outset. But it is only spacetime, not matter, that allows a classical description, so that we argue for semiclassical gravity and its using for solving the localization problem.

Indeterministic quantum gravity and cosmology (IQGC)—the theory being developed in this series of papers—is based on a variant of semiclassical gravity, so that IQGC provides a natural groundwork for handling the localization problem.

The idea is to consider a self-consistent problem: A quantum state of the center of mass of a body gives rise to a classical gravitational field, and the state, on the other hand, is an eigenstate in the field. A model solution to the problem is very simple.

We call a gravitationally autolocalized body gravilon: gravilon=gravity+localization. The main results are as follows.

Let $a_0$ be the radius of a body with a mass $M$, $r_0$ be the radius of the wave function of the center of mass of the body, $a = a_0 + r_0$, and $m_P$ be the Planck mass. A light (respectively heavy) gravilon is that with $M \lesssim m_P$ (respectively $M \gg m_P$); a fuzzy (respectively quasiclassical) gravilon is that with $r_0 \gtrsim a_0$ (respectively $r_0 \ll a_0$).

A gravilon is quasiclassical iff $(a_0/l_P)(M/m_P)^3 \gg 10$ where $l_P$ is the Planck length. For the quasiclassical gravilon with a constant density, $r_0 \propto 1/a_0^{3/2}$.

A heavy gravilon is quasiclassical, a fuzzy one is light. A light gravilon may be both fuzzy and quasiclassical.

For any gravilon the conditions $a \gg l_P(M/m_P)$, $a \gtrsim l_P(m_P/M)^3$ should be fulfilled; it follows $a \gg l_P$.

Gravitational autolocalization is predominantly a macroscopic effect.

The motion of a gravilon as a whole is classical.
1 Model

Consider a ball with a radius \( a_0 \), a mass \( M \), and a constant density \( \rho \), so that

\[
M = \frac{4\pi}{3} a_0^3 \rho. \tag{1.1}
\]

Let \( r_0 \) be the radius of the wave function of the center of mass of the ball, and

\[
a = a_0 + r_0. \tag{1.2}
\]

We use the Newtonian approximation for gravitational potential \( \Phi(r) \) and put

\[
\Phi(r) = \kappa M \begin{cases} 
\frac{(r^2 - 3a^2)/2a^3}{2a^3} & \text{for } r \leq a \\
(-1/r) & \text{for } r \geq a,
\end{cases} \tag{1.3}
\]

where \( \kappa \) is the gravitational constant.

Now we consider a quantum particle with the mass \( M \) in a potential well

\[
U(r) = \frac{\kappa M^2}{2a^3} r^2, \tag{1.4}
\]

i.e., a harmonic oscillator. From the relation

\[
U(r) = \frac{M \omega^2 r^2}{2} \tag{1.5}
\]

it follows for the frequency

\[
\omega = \left( \frac{\kappa M}{a^3/2} \right)^{1/2}. \tag{1.6}
\]

We have (from here on \( \hbar = 1, c = 1 \))

\[
r_0 = \sqrt{\frac{2}{M \omega}} = \frac{\sqrt{2}a^{3/4}}{\kappa^{1/4}M^{3/4}}, \tag{1.7}
\]

which, in view of eq.(1.2), is an equation for \( r_0 \).

2 Conditions

The condition for the Newtonian approximation is

\[
2|\Phi(0)| \ll 1. \tag{2.1}
\]

The condition that a black hole does not form is

\[
a > 2\kappa M. \tag{2.2}
\]

The condition that there is no creation of particle-antiparticle pairs is

\[
|\Phi(0)| < \Phi_{\text{critical}}, \quad \Phi_{\text{critical}} > 1. \tag{2.3}
\]
It is seen that the condition (2.1) implies the conditions (2.2), (2.3). Thus we assume that

$$a \gg l_p \frac{M}{m_p}$$  \hspace{1cm} (2.4)$$

holds where \( l_p = t_p \) is the Planck length and/or time and \( m_p \) is the Planck mass \((\kappa = t_p^2, m_p = 1/t_p)\). 

Furthermore, the condition

$$\frac{\omega}{2} \ll |\Phi(0)|M$$  \hspace{1cm} (2.5)$$

should hold, which results in

$$a \gg \frac{1}{9} l_p \left(\frac{m_p}{M}\right)^3.$$  \hspace{1cm} (2.6)$$

Thus we have obtained the conditions

$$a \gg l_p \frac{M}{m_p}, \quad a \gtrsim l_p \left(\frac{m_p}{M}\right)^3 \equiv a_1.$$  \hspace{1cm} (2.7)$$

It follows from those

$$a \gg l_p.$$  \hspace{1cm} (2.8)$$

3 Solution

For the sake of simplicity, we drop the factor \( \sqrt{2} \) in eq.(1.7) and obtain

$$r_0^4 = l_p \left(\frac{m_p}{M}\right)^3 (r_0 + a_0)^3.$$  \hspace{1cm} (3.1)$$

Let us introduce quantities

$$x = \frac{r_0}{a_0}, \quad \alpha = \frac{a_0}{a_1},$$  \hspace{1cm} (3.2)$$

then eq.(3.1) takes the form of

$$\frac{(1 + x)^3}{x^4} = \alpha.$$  \hspace{1cm} (3.3)$$

We call the quantity \( \alpha \) characteristic parameter.

Let

I. \hspace{1cm} \alpha \ll 1.$$  \hspace{1cm} (3.4)$$

Then

$$x \gg 1, \quad x \approx \frac{1}{\alpha},$$  \hspace{1cm} (3.5)$$

so that

$$r_0 \approx a_1, \quad a_0 \ll a_1, \quad r_0 \gg a_0 \quad a \approx a_1.$$  \hspace{1cm} (3.6)$$

The conditions (2.7) reduce to

$$M \ll m_p.$$  \hspace{1cm} (3.7)$$

Let

II. \hspace{1cm} \alpha = 1.$$  \hspace{1cm} (3.8)$$
Then
\[ x = 2.63, \] (3.9)
so that
\[ r_0 = 2.6 a_0, \quad a_0 = a_1, \quad a = 3.6 a_1. \] (3.10)
The conditions (2.7) reduce to
\[ M < m_P. \] (3.11)

Let
\[ \text{I iii. } \alpha = 8. \] (3.12)
Then
\[ x = 1, \] (3.13)
so that
\[ r_0 = a_0 = 8 a_1, \quad a = 9 a_1. \] (3.14)
The conditions (2.7) reduce to
\[ M \lesssim m_P. \] (3.15)

Piecing I i, ii, iii together, we obtain
\[ \text{I. } \alpha \lesssim 10, \] (3.16)
\[ r_0 \gtrsim a_0, \quad a_0 \lesssim 10 a_1. \] (3.17)
The conditions (2.7) reduce to
\[ M \lesssim m_P. \] (3.18)

Now let
\[ \text{II. } \alpha \gg 10. \] (3.19)
Then
\[ x \ll 1, \quad x \approx \frac{1}{\alpha^{1/4}}, \quad \frac{r_0}{a_1} = x\alpha = a^{3/4} \gg 1, \] (3.20)
so that
\[ a_1 \ll r_0 \ll a_0, \quad a \cong a_0. \] (3.21)
The conditions (2.7) reduce to
\[ a_0 \gg l_P \frac{M}{m_P}. \] (3.22)

4 Gravilons

We introduce the following terminology. A gravilon is a gravitationally autolocalized system:
\[ \text{gravilon} = \text{gravity} + \text{localization}. \] (4.1)
Light gravilon:
\[ M \lesssim m_P, \] (4.2)
heavy gravilon: \[ M \gg m_P. \] (4.3)

Fuzzy gravilon: \[ r_0 \gtrsim a_0, \] (4.4)

quasiclassical gravilon: \[ r_0 \ll a_0. \] (4.5)

From eqs.(3.17),(3.18) we obtain the following results: a fuzzy gravilon is light, a heavy gravilon is quasiclassical. A light gravilon may be both fuzzy and quasiclassical. A gravilon is quasiclassical iff eqs.(3.19),(3.22) hold.

For a quasiclassical gravilon, we obtain from eqs.(3.2),(3.20),(1.1)
\[ r_0 = \left( \frac{3}{4\pi} \right)^{3/4} l_p^{1/4} \left( \frac{m_P}{\rho} \right)^{3/4} \frac{1}{a_0^{3/2}}, \] (4.6)

so that
\[ r_0 \propto \frac{1}{a_0^{3/2}} \quad \text{for} \quad \rho = \text{const.} \] (4.7)

The condition (3.22) reduces to
\[ a_0 \ll 0.5 \cdot 10^{14} \cdot \frac{1}{\rho^{1/2}}, \quad [a_0] = \text{cm}, \quad [\rho] = \text{g/cm}^3. \] (4.8)

For the characteristic parameter \( \alpha \), eq.(3.2), we have
\[ \alpha = \left( \frac{4\pi}{3} \right)^{3/2} \frac{1}{l_p} \left( \frac{\rho}{m_P} \right)^{3/4} a_0^{10} \approx 0.05 \rho^{3} (10^5 a_0)^{10}, \quad [a_0] = \text{cm}, \quad [\rho] = \text{g/cm}^3. \] (4.9)

Thus for \( \rho \approx 1 \text{ g/cm}^3 \) and \( a_0 \gtrsim 2 \cdot 10^{-5} \text{cm} \)
\[ \alpha \gg 10, \] (4.10)
the gravilon is quasiclassical.

Let
\[ \rho = 1 \text{ g/cm}^3, \quad a_0 = 1 \text{ cm}; \] (4.11)
then
\[ \alpha \approx 5 \cdot 10^{48}, \quad r_0 \approx 10^{-12} \text{ cm}. \] (4.12)

Let
\[ a_0 = 0, \quad M = 10^{-24} \text{ g}; \] (4.13)
then
\[ \alpha = 0, \quad r_0 = a_1 \approx 10^{24} \text{ cm}. \] (4.14)

Thus, gravilon formation, i.e., gravitational autolocalization is predominantly a macroscopic effect.

In conclusion, it should be pointed out that the motion of a gravilon as a whole, i.e., the motion of the gravitational well, is classical. In the Newtonian approximation, the internal (quantum) degree of freedom of the center of mass is frozen: the state of the center of mass is fixed. Thus we should not consider transitions of the center of mass in the well.
Acknowledgment

I would like to thank Stefan V. Mashkevich for helpful discussions.

References

[1] Vladimir S. Mashkevich, *Indeterministic Quantum Gravity* (gr-qc/9409010, 1994).
[2] Vladimir S. Mashkevich, *Indeterministic Quantum Gravity II. Refinements and Developments* (gr-qc/9505034, 1995).
[3] Vladimir S. Mashkevich, *Indeterministic Quantum Gravity III. Gravidynamics versus Geometrodynamics: Revision of the Einstein Equation* (gr-qc/9603022, 1996).
[4] Vladimir S. Mashkevich, *Indeterministic Quantum Gravity IV. The Cosmic-length Universe and the Problem of the Missing Dark Matter* (gr-qc/9609033, 1996).
[5] Vladimir S. Mashkevich, *Indeterministic Quantum Gravity V. Dynamics and Arrow of Time* (gr-qc/9609046, 1996).
[6] Vladimir S. Mashkevich, *Indeterministic Quantum Gravity and Cosmology VI. Predynamical Geometry of Spacetime Manifold, Supplementary Conditions for Metric, and CPT* (gr-qc/9704033, 1997).
[7] Vladimir S. Mashkevich, *Indeterministic Quantum Gravity and Cosmology VII. Dynamical Passage through Singularities: Black Hole and Naked Singularity, Big Crunch and Big Bang* (gr-qc/9704038, 1997).
[8] Murrey Gell-Mann and James Hartle, *Strong Decoherence* (gr-qc/9509054, 1995).
[9] Roger Penrose, *The Emperor’s New Mind* (Vintage, 1990).