Abstract

Systematic testing of autonomous vehicles operating in complex real-world scenarios is a difficult and expensive problem. We present Paracosm, a reactive language for writing test scenarios for autonomous driving systems. Paracosm allows users to programmatically describe complex driving situations with specific visual features, e.g., road layout in an urban environment, as well as reactive temporal behaviors of cars and pedestrians. Paracosm programs are executed on top of a game engine that provides realistic physics simulation and visual rendering. The infrastructure allows systematic exploration of the state space, both for visual features (lighting, shadows, fog) and for reactive interactions with the environment (pedestrians, other traffic).

We define a notion of test coverage for Paracosm configurations based on combinatorial testing and low dispersion sequences. Paracosm comes with an automatic test case generator that uses random sampling for discrete parameters and deterministic quasi-Monte Carlo generation for continuous parameters. Through an empirical evaluation, we demonstrate the modeling and testing capabilities of Paracosm on a suite of autonomous driving systems implemented using deep neural networks developed in research and education. We show how Paracosm can expose incorrect behaviors or degraded performance.

CCS Concepts: Software and its engineering → General programming languages; Social and professional topics → History of programming languages;

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environments for testing autonomous vehicles in realistic settings. Like several previous projects [17, 18, 43, 49, 54], PARACOSM is built on top of a high-fidelity visual environment and physics simulator provided by a game engine (Unity [55], in our case). Unlike other projects, where each virtual world needs to be pre-fabricated by a designer, PARACOSM focuses on providing language support for programmatically designing parameterized environments and test cases. The test parameters define the environment and the behaviors of the actors. As they can be programmatically adapted, PARACOSM can do systematic testing and we can describe a notion of test coverage for these test families.

PARACOSM is based on a synchronous reactive programming model [8, 28, 31, 57]. Components in PARACOSM, such as road segments or cars, receive streams of inputs and produce streams of outputs over time. In addition, components can also have graphical assets to describe their geometric properties for an underlying graphics rendering engine and physical properties relating to their dynamics for an underlying physics engine. For example, a car modelled in PARACOSM will not only have code that reads in camera feeds and outputs steering angle or braking, but also have a textured mesh representing its shape and its position and orientation in 3D space, and a physics model for its dynamical behavior. A PARACOSM configuration is a composition of a number of components. Using a set of system-defined components (road segments, cars, and pedestrians, etc.) combined together using the expressive operations from the underlying reactive programming model, users can set up complex driving scenarios where the environment’s behavior changes over time. For example, one can build an urban road network with intersections, pedestrian and vehicular traffic, and parameterize both environment conditions (lighting, fog) and behaviors (when a pedestrian crosses a street).

Streams in the world description can be left “open” and, during testing, PARACOSM will generate sequences of values for these streams. This forms the basis of an automatic testing infrastructure on which many different testing strategies from randomized to exhaustive can be tried. We formalize a notion of coverage for our setting based on \( k \)-wise combinatorial coverage [9, 30] for discrete variables and dispersion for continuous variables. Intuitively, \( k \)-wise coverage ensures that, for a programmer-specified parameter \( k \), all possible combinations of values of any \( k \) discrete parameters are covered by tests. Low dispersion [44] ensures that there are no “large empty holes” left in the continuous parameter space. We show an automatic test generation strategy that offers high coverage based on random sampling over discrete parameters and deterministic quasi-Monte Carlo methods for continuous parameters [37, 44]. We provide theoretical guarantees on the number of tests to ensure coverage with high probability. Perhaps surprisingly, quasi-Monte Carlo methods provide better guarantees than pure random exploration for dispersion.

An empirical evaluation of autonomous driving models in PARACOSM environments shows, through various case studies, how PARACOSM can be an effective testing framework for both qualitative properties (crash) and quantitative properties (distance covered in a given time).

Our main contributions are the following. (I) We provide the first programmable and expressive mocking framework for autonomous systems interacting with the physical world. (II) We define an appropriate notion of test coverage based on combinatorial \( k \)-wise coverage in discrete space and low dispersion in continuous space. We show a test generation strategy based on random generation and quasi-Monte Carlo methods that theoretically guarantees good coverage. (III) We demonstrate empirically that our system is able to construct complex scenarios to automatically test autonomous driving agents and find incorrect behaviors or degraded performance.

## 2 Language through Examples

PARACOSM is based on a synchronous reactive programming model [8, 28], where communication occurs through streams of values, and computation occurs through stream transformations. The core of PARACOSM is a reactive object. Reactive objects capture geometric and graphical features of a physical object, as well as their behavior over time. Internally, reactive objects contain graphical assets (3D meshes) for their visual representation; the assets prescribe their geometric properties to an underlying graphics engine. The behavioral interface for each reactive object has a set of input streams and a set of output streams. The evolution of the world is computed in steps of fixed duration which corresponds to events in a predefined tick stream. For streams that correspond to physical quantities updated by the physics engine, such as position and speeds of the cars, collisions detection, etc., the appropriate events are generated by the underlying physics engine.

Input streams provide input values from the environment over time; output streams represent output values computed by the object. The object’s constructor sets up the internal state of the object. An object is updated by the event triggered computations. PARACOSM provides a set of assets as base classes. While users can use any new asset, typically, users would derive new behaviors but preserve the graphical assets.

We differentiate between static environment reactive objects and dynamic actor reactive objects. Environment reactive objects represent “static” components of the world, such as road segments, intersections, houses, or trees, and a special component called the world. Actor reactive objects represent components with “dynamic” behavior: cars, bicycles, or pedestrians. The world object is used to model features of the world such as lighting or weather conditions.
Reactive objects can be composed to generate complex assemblies from simple objects. The composition process can be used to connect static components structurally—such as two road segments connecting at an intersection. Composition also connects the behavior of one object to another by binding output streams to input streams. At run time, the values on that input stream of the second object are obtained from the output values of the first. Composition must respect geometric properties—the runtime system ensures that a composition maintains invariants such as no collision.

A test configuration in Paracosm consists of a composition of reactive objects. A test configuration may contain reactive objects some of whose input streams are not connected yet. A test case replaces each such “open” input stream with a concrete input stream.

An instantiation of world and actor reactive objects gives a scene, i.e., all the visual elements present in a model of the world. A test case determines how the scene evolves over time: how the car moves, how pedestrians move, and how road conditions change over time.

It may seem surprising that we model static components as reactive objects as well. This serves two purposes. First, one can treat the number of lanes in a road segment as a constant input stream that is set by the test case. This allows writing parameterized test cases. Second, certain features of static objects can also change over time: for example, the coefficient of friction on a road segment may depend on the weather condition, and the weather condition can be a function of time.

We demonstrate the descriptive power of Paracosm through a series of examples building up on each other. We have simplified the actual API syntax for clarity.

A Simple World. The first example sets up the world, a road segment, a car on the road segment, and a pedestrian trying to cross:

```plaintext
1 World(rxinout light, rxinout fog) { ... }
2 RoadSegment(rxin friction, const rxinout len, 
   const rxinout nlanes) { ... }
3 AutonomousVehicle(value loc, 
   rxout pos, rxout vel, ...) { ... }
4 Pedestrian(value start, value target, 
   rxin dist, rxin car, rxin speed) { ... }
5 light = VarInterval(0, 1)
6 fog = VarInterval(0, 1)
7 friction = VarInterval(0.2, 0.8)
8 nlanes = VarEnum((2, 4, 6))
9 dist = VarInterval(10, 50)
10 speed = VarInterval(0.5, 10)
```

The reactive objects World, RoadSegment are pre-defined in our system, and consist of pre-defined visual assets. AutonomousVehicle and Pedestrian extend pre-defined elements. The Pedestrian also contains code for the behavior for this test (code below). The pedestrian starts at start and, when the car is closer than dist, tries to cross toward target. We use some syntactic sugar: we write `rxin` and `rxout` for input and output streams respectively. We write `rxinout` for input streams which are “passed through” and available as identical output streams as well. A `value` is a single value which can be used, for instance, to compute the initial state of the system, e.g., initial value of the output stream pos. We can connect streams explicitly, or by assigning them during creation of an object. As a shorthand, we can pass an entire object as a stream; the intended use is that any output stream of the object is available to the receiver. The keyword `const` creates a constant stream.

The World is a special object which also deals with the graphics and physics engine. It has some predefined reactive values related to the global environment conditions like the lighting. The place operation also adds objects to the scene and starts the simulation. The road r has no given position and it is placed at the origin by default. The vehicle v and pedestrian p have an initial position relative to r. The call `r.onLane(lane, at)` returns a position at a specified distance on a given lane. Negative lane numbers denote the opposite direction.

Figure 1(a) shows the representation of the world in the graphics engine. In this example, there are no open streams. Hence, the behavior is completely determined by the initial state. By setting parameters, assets, or by using derived classes, one can easily generate different worlds (see 1(b)-(d)).

A Family of Worlds. In the previous example, we used constant streams to set input streams to constant values. Instead, one can allow test cases with general streams of values but one can constrain the possible values passed into an object [47]. We do this by constraining streams to a given range. For the example above, we could define

```plaintext
1 light = VarInterval(0, 1)
2 fog = VarInterval(0, 1)
3 friction = VarInterval(0.2, 0.8)
4 nlanes = VarEnum((2, 4, 6))
5 dist = VarInterval(10, 50)
6 speed = VarInterval(0.5, 10)
```

Such constraints allow test generators to change properties of the simulation, thereby leading to different test cases (see Figure 1).

Reactive Behaviors. Reactive objects often model some physical elements in the world and, therefore, need to interface with a physics and rendering engine. There are two dedicated types for this task. First, we have Geometric objects which have a model and support collision detection. The position
of geometric object is under control of the programmer. Second, the Physical objects extends geometric objects but their motion is computed by the physics engine.

Geometric objects need to provide a textured mesh and have a pose (position and orientation in 3D space). The world evolves in fixed time interval \( dt \). We call each passing interval a *tick*. *Tick* is a stream of events with unit type. Each time the simulation engine makes a step, an event is generated on the *tick* stream. The loading and initialization of models is done at object creation and stored in the appropriate field inherited from the parent class. The models used for display and collision detection may be different. While the object’s mesh may be used for accurate collision detection, it is also possible to use simpler models, typically bounding boxes, on which detecting collision is computationally cheaper.

Not every model need have complex dynamics and a simple and precise way of controlling an object is sometime preferable. For example, we can build a simple model of a pedestrian crossing the road which simply updates the position of the pedestrian by a constant at each *tick*. In general, the reactive nature of objects enables complex scenarios to be built; for example, a pedestrian model can start crossing the road only when a car is a certain distance away. In the code below, we use ‘\_’ as shorthand for a lambda expression, i.e., “\( f(\_\)’” is the same as “\( x \mapsto f(x) \)”.

```java
Pedestrian(value start, value target, rxin dist, rxin car, rxin speed) extends Geometric {
    ... // initialization
    trigger = car . filter ( |_ - start| < dist )
    done = pos . filter ( _ == target )
    tick . skipUntil (trigger).takeUntil (done).forEach ( ... /* walk */ )
}
```

Reactive objects whose state is also modified by the physics engine have a sightly richer interface. Their internal state is restricted to an API understood by the physics engine. The state is a collection of rigid bodies connected by joints. Each body has a mesh for rendering and physical properties like mass, moments of inertia, etc., that makes it possible to simulate the effect of forces on the object. Joints connect the multiple bodies in an object and constrain their relative positions. For instance, a simple car has five different bodies: one for the frame and four for the wheels. The wheels are attached to the body and allowed to rotate around the center. The car model can be modified to change the type of drive (front, rear or four wheel drive), centre of mass, maximum steering angle, torque, maximum speed, etc.

**Invariants and Monitors.** So far, we discussed static and dynamic elements that make up the world. Additionally, Paracosm provides an API to provide qualitative and quantitative temporal specifications. For instance, in our pedestrian crossing example, we want to check that there is no collision and detect that the collision was not trivially avoided because the car did not move at all.

```java
CollisionMonitor (rxin AutonomousVehicle v) {
    assert (v.collider.isEmpty())
}

DistanceMonitor (rxin AutonomousVehicle v, value minD, rxout D) {
    pOld = v.pos.take(1).concat(v.pos)
    D = v.pos.zip(pOld).map ( |_ - _| ).sum()
    assert (D >= minD)
}
```

The ability to write monitors which read streams of system-generated events provides an expressive framework to write temporal properties. For example, one can encode monitors for metric and signal temporal logic specifications in the usual way [14, 26].

**Sensors.** Self-driving cars have a large array of sensors. In simple systems like openpilot [10], only a dash-cam is used but more complex systems mix cameras, LiDARs, radars, and GPS. Paracosm can emulate common types of sensors.
Sensors produce streams of data which can be used by the tested system. In Figure 2, we show data coming from a few sensors. Normal cameras can be mounted at specific points on the vehicle and it is possible to vary parameters like the focal length. The camera can be ideal or also include imperfections like blur or noise. During the rendering process for RGB images we can also extract depth map (Fig. 2b) to cheaply emulate LiDAR. It is also possible to use ray casting techniques to more accurately simulate a LiDAR but the computational cost is significantly higher.

In general, it is possible to provide new sensors which are not supported out-of-the-box by Paracosm. However, new types of sensors need to be implemented directly on top of the rendering engine’s API.

System Under Test (SUT). Autonomous systems naturally fit reactive programming models. They consume sensor input streams and produce actuator streams for the car model. Our default vehicle model has two control inputs: the throttle (acceleration ∈ [−1, 1]) and the steering angle (steering_angle ∈ [−1, 1]). The values are normalized. The actual maximum steering angle and acceleration/braking depends on the car.

Let us be more specific. Suppose we want to test the implementation of a controller for an autonomous vehicle. Suppose the controller is implemented using a neural network that takes a video stream, frame-by-frame, and outputs the acceleration and the steering angle. We design the tests for this controller in Paracosm in the following way. We encapsulate the controller code inside an AutonomousVehicle object. The AutonomousVehicle object subclasses Physical, and contains both geometric information about the shape and physical dimensions of the car, the position and properties of the camera, as well as a vehicle model for the physics of its dynamics. The object has an input stream for the video feed, which is provided by the rendering engine using its knowledge about the geometry of the scene. Each new image coming from the video feed is forwarded to the neural network. The neural network returns the actuation values for the acceleration and the steering angle. The values from this output stream goes to the physics engine, which calculates the new position and orientation of the vehicle.

More Complex Worlds. Until now, our tests had a very simple environment. However, constructing a world in a procedural manner allows a Paracosm’s user to quickly create large environment to run longer tests. Road segments can be composed into more complicated road networks. Paracosm provides abstractions to generate such compositions with a connect operation between road elements. Furthermore, Paracosm contains more complex road elements such as cross-intersections, T-intersections, and roundabouts. Connections can be established using a connect methods that takes physical connection identifiers and road elements as arguments. The connections are directed in order to compute the positions of the elements. One road element becomes the parent and it’s children are positioned relative its position and the specified connection points. After an object is connected a new composite road element which encapsulates all road elements along with requisite transformations (rotations and translations) is returned. The following example shows how road segments can be connected into a road network.

```paracosm
1 // Create a T-intersection and three straight road elements (east, south, west)
2 t = TIntersection(numLane = Const(4),
3     position = Const(origin))
4 e = RoadSegment(len=Const(50),nlanes=Const(4))
5 s = RoadSegment(len=Const(50),nlanes=Const(4))
6 w = RoadSegment(len=Const(20),nlanes=Const(4))
7 // connect and get new composite object
8 net = t.connect((t.ONE, e, e.TWO),
9     (t.TWO, s, s.ONE),
10     (t.THIR, w, w.ONE))
```

Connecting elements has two purposes. First, it allows Paracosm to perform sanity checks like the positions of road elements. Second, it creates an overlay graph of the road networks which can easily be followed by environment controlled cars not under test. When a road network is created, the runtime system of Paracosm checks that compositions of road elements and intersections are topologically and geometrically valid. All road elements must be connected to a matching road correctly (for example, a 2-lane road segment cannot be connected to a 6-lane road segment directly), there are no spatial overlaps between road segments, and the positions of the connection points match.

In general, Paracosm inherits all programming features of the underlying imperative programming model as well as reactive programming with streams à la functional reactive programming. Thus, one can build complex urban settings through composition and iteration. For instance, the grid world shown in Figure 3 was created by iterating a simple road network.

3 Testing Paracosm Worlds

3.1 Tests and Coverage

Worlds built using the Paracosm API directly describe a parameterized family of tests; the parameterization is provided by the reactive streams left “open” in the world. The testing framework of Paracosm allows the user to specify different strategies to generate input streams for both the static and the dynamic reactive objects in the world.

Test Cases. A test of duration T executes a configuration of reactive objects by providing inputs to every open input stream in the configuration for T ticks. The inputs for each

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3 In our implementation, we also allow the neural network to run in its own process. The processing uses remote procedure calls.
stream must satisfy const parameters and respect the range constraints from \texttt{VarInterval} and \texttt{VarEnum}. The runtime system manages the scheduling of inputs and pushing input streams to the reactive objects.

Suppose that $\ln$ denotes the set of all input streams, and let $\ln = \ln_D \cup \ln_C$ denote the partition of $\ln$ into discrete streams and continuous streams respectively. Discrete streams take their value over a finite, discrete range; for example, the color of a car, the number of lanes in a road segment, or the position of the next pedestrian (left or right) are discrete streams. Continuous streams take their values in a continuous (bounded) interval. For example, the fog density or light intensity, or the speed of an AI vehicle in the environment are examples of continuous streams.

A test case must provide discrete values to each stream in $\ln_D$ and continuous values to each stream in $\ln_C$ at each step. Thus, a test of duration $T$ can be seen as a vector over $(\mathbb{N}^{\ln_D} \times \mathbb{R}^{\ln_C})^T$; for each time index $t = 0, \ldots, T - 1$, the vector gives values to each stream in $\ln$. The test is valid if each input satisfies the constraints on the stream. For example, a constant stream will have the same value for each time step, and a stream with a constraint $[0, 2]$ will take values in that range at each time step.

**Coverage.** As usual in testing, one can use programmer-provided invariants and monitors as test oracles. In the setting of autonomous vehicles testing, however, one often wants to explore the state space of a parameterized world to check “how well” an autonomous vehicle works under various situations, both qualitatively and quantitatively. Thus, we now introduce a notion of coverage. Instead of structural coverage criteria such as line or branch coverage, our goal is to cover the parameter space “uniformly.”

In what follows, for simplicity of notation, we assume that all discrete streams take values $\{0, 1\}$, and all continuous streams take values in the real interval $[0, 1]$. Any input stream over bounded intervals—discrete or continuous—can be encoded into such streams. Further, in test vectors, we only keep a single co-ordinate for constant streams instead of a co-ordinate for every time step.

For discrete streams, there are finitely many tests, since each co-ordinate is Boolean and there is a fixed number of co-ordinates. One can define the coverage as the fraction of the number of vectors tested to the total number of vectors. Unfortunately, the total number of vectors is very high: if each stream is constant, then there are already $2^n$ tests for $n$ streams. Instead, we consider the notion of $k$-wise testing from combinatorial testing [30]. In $k$-wise testing, we fix a parameter $k$ and ask that every interaction between every $k$ elements is tested.

Let us be more precise. Suppose that a test vector has $N$ co-ordinates, where each co-ordinate $i$ can get the value $0$ or $1$. A set of tests $A$ is a $k$-wise covering family if for every subset $\{i_1, i_2, \ldots, i_k\} \subseteq \{1, \ldots, N\}$ of co-ordinates and every vector $v \in \{0, 1\}^k$, there is a test $t \in A$ whose restriction to the co-ordinates $i_1, \ldots, i_k$ is precisely $v$.

**Example 3.1.** Consider a configuration with constant input streams corresponding to the number of lanes on the road (possible values $\{2, 4\}$), the direction of $n$ vehicles along the
road (toward a test vehicle or in the same direction as the test vehicle), and the color of the vehicles (possible values red and green). There are $2 \times 2^n \times 2^n$ possible tests. For 2-wise testing, we want to ensure every pair of parameters is covered by some test. Naively, there are at most $2n(2n + 1)$ test cases, but the theory of covering arrays can be used to find much smaller test sets. For example, for $n = 3$, just 6 tests are enough: (2, 011, GGG), (2, 101, GRR), (2, 010, RRR), (4, 001, RRR), (4, 110, GGR), and (4, 100, RRG). Here, the first column denotes the number of lanes, the next denotes the direction of the three cars, and the next denotes their colors. An “*” denotes either color can be used in the test.

For continuous streams, the situation is more complex: since any continuous interval has infinitely many points, each one corresponding to a different test case, we cannot directly define coverage as a ratio (the denominator will be infinite). Instead, we define coverage using the notion of dispersion [37, 44].

Recall that we assume a (continuous) test is a vector in $[0, 1]^D$: each entry is picked from the interval $[0, 1]$ and there are $N$ co-ordinates. Dispersion over $[0, 1]^N$ can be defined relative to sets of neighborhoods, such as $N$-dimensional balls or axis-parallel rectangles. Let us define $B$ to be the family of $N$-dimensional axis-parallel rectangles in $[0, 1]^N$, our results also hold for other notions of neighborhoods such as balls or ellipsoids. For a neighborhood $B \in \mathcal{B}$, let $\text{vol}(B)$ denote the volume of $B$.

Given a set $A \subseteq [0, 1]^N$ of tests, we define the dispersion as the largest volume neighborhood in $\mathcal{B}$ without any test:

$$\text{dispersion}(A) = \sup \{\text{vol}(B) \mid B \in \mathcal{B} \text{ and } A \cap B = \emptyset\}$$

Intuitively, dispersion measures the largest empty space left by a set of tests. We use dispersion as a measure of coverage for continuous vectors: the lower the dispersion, the better the coverage for a configuration.\(^4\)

Let us summarize. Suppose that a test vector consists of $N_D$ discrete co-ordinates and $N_C$ continuous co-ordinates; that is, a test is a vector $(t_D, t_C)$ in $[0, 1]^{N_D} \times [0, 1]^{N_C}$. We say a set of tests $A$ is $(k, \epsilon)$-covering if

1. for each set of $k$ co-ordinates $\{i_1, \ldots, i_k\} \subseteq \{1, \ldots, N_D\}$ and each vector $v \in \{0, 1\}^k$ there is a test $(t_D, t_C) \in [0, 1]^{N_D} \times [0, 1]^{N_C}$ such that the restriction of $t_D$ to the co-ordinates $i_1, \ldots, i_k$ is $v$; and
2. for each $(t_D, t_C) \in A$, the set $\{t_C \mid (t_D, t_C) \in A\}$ has dispersion at most $\epsilon$.

Our goal is to automatically generate $(k, \epsilon)$-covering test sets for programmer-specified $k$ and $\epsilon$.

### 3.2 Test Generation

We now show an automatic test generation algorithm based on random sampling from discrete parameters and deterministic quasi-Monte Carlo generation for continuous parameters. Our test generation strategy has two parts. First, we construct a $k$-wise covering family for the discrete part of the test. Then, for each (discrete) vector generated in the family, we generate a set of (continuous) vectors of low dispersion.

**k-Wise Covering Family.** One can use explicit construction results from combinatorial testing to generate $k$-wise covering families [9]. However, a simple way to generate such families with high probability is through random testing. The proof is by the probabilistic method [1] (see also [34]).

**Proposition 3.2.** Let $A$ be a set of $2^k (k \log N - \log \delta)$ uniformly randomly generated $(0, 1)^N$ vectors. Then $A$ is a $k$-wise covering family with probability at least $1 - \delta$.

**Low Dispersion Sequences.** It is tempting to think that uniformly generating vectors from $[0, 1]^N$ would similarly give low dispersion sequences. Indeed, as the number of tests goes to infinity, the set of uniformly randomly generated tests has dispersion 0 almost surely. However, we do not plan to test with an infinite number of tests. When we fix the number of tests, it is well known that uniform random sampling can lead to high dispersion [37, 44]; in fact, one can show that the dispersion of $n$ uniformly randomly generated tests grows asymptotically as $O((\log \log n/n)^{1/2})$ almost surely.

Our test generation strategy will be based on deterministic quasi-Monte Carlo sequences, which have much better dispersion properties, asymptotically of the order of $O(1/n)$, than the dispersion behavior of uniformly random tests.

There are many different algorithms for generating quasi-Monte Carlo sequences deterministically (see, e.g., [37, 44]). We shall use Halton sequences in our experiments. Fix an integer $q \geq 2$. For an integer $j$, suppose the base-$q$ representation of $j$ is $j = \sum_{i \geq 0} b_i q^i$. Define the function

$$y_q(j) = \sum_{i \geq 0} b_i q^{i-1}$$

Let $q_1, \ldots, q_N \geq 2$ be co-prime integers. The Halton sequence is the set of points

$$E_n = \{ (y_{q_1}(j), y_{q_2}(j), \ldots, y_{q_N}(j)) \mid 0 \leq j \leq n \}$$

The following estimate shows that the Halton sequence has dispersion asymptotically better than uniform random sequences; in fact, the dispersion decreases linearly with the number of tests.

**Proposition 3.3 ([44]).** $\text{dispersion}(E_n) = O(1/n).$
Thus, for a given $\epsilon$, one needs to generate $O(\frac{1}{\epsilon})$ tests. In Section 4, we show an empirical comparison between uniformly random and quasi-Monte Carlo test generation.

### 3.3 Reducing the Dimension

In general, one can expect that the dimension $N$ of a test case is extremely large, and that any test generation technique is fruitless. For example, consider that a video feed can get 1000 $\times$ 600 pixels per tick, where each pixel is an RGB value (total 256$^3$ values). Thus, a single test case for $T = 100$ ticks is already 256$^{3 \times 1000 \times 600 \times 100}$ dimensional, and one cannot hope to cover even a minuscule portion of the space of tests.

However, in many testing situations, we sample from a low-dimensional feature space embedded in the high-dimensional sensor space [18, 19]: instead of covering the space of pixels, we focus on a set of features (e.g., the presence or position of an object in the environment over time). Further, one can use parameterized representations [15]: instead of modeling fog over time a 1000-dimensional vector, where each dimension is independently picked, we can pick a low-degree polynomial whose evaluation gives the fog intensity, and only test over the coefficient ranges of the polynomial.

### 4 Implementation and Tests

#### 4.1 Runtime System and Implementation

Paracosm uses the Unity game engine [55] to render scenes and to simulate physics. The physics engine in Unity is PhysX [11]. The reactive objects are built on top of UniRx [29]. Like other simulation environments [17, 49], the core reason for using a modern game engine is visually realistic rendering. In addition to this, a game engine manages global positions and orientations of 3D objects and also offers abstractions for generating realistic environmental effects. Encoding behaviours for actors, managing 3D assets, and dynamic checks (system inactivity, collisions, etc.) are implemented using the game engine interface.

Paracosm program

![Paracosm diagram](https://example.com/paracosm-diagram.png)

**Figure 4.** Architecture of a simulation in Paracosm

Figure 4 shows the architecture of Paracosm’s runtime. A simulation proceeds as follows. First, the program is read and requisite reactive components are instantiated. This involves initialization and placement of their 3D mesh (if they have one). The connection to the SUT is also established in case it runs on its own process. In our experiments, we test a number of driving systems based on neural networks. We use a modified version of Udacity’s self-driving car simulator [54] to interface with the neural networks we test.

For practical purposes, game engines typically treat physics and rendering pipelines separately. The physics engine is informed about the physical components (subclasses of Physical). Physics simulations in Paracosm work on rigid bodies, i.e., solid bodies with an assumption of no deformation. This enables accurate detection of collision, but not deformations caused by the collision. This is an acceptable limitation as a collision typically marks the end of a failure case.

### 4.2 Case studies

The previous sections discuss Paracosm’s programming interface and automatic test generation algorithm. We now demonstrate two case studies that illustrate how these can be utilized for authoring a wide array of interesting test scenarios. The first study (Section 4.2.1) covers tests on pre-trained machine-learnt models built for object segmentation, specifically, for road detection and vehicle detection. The second study (Section 4.2.2) discusses tests on an autonomous vehicle being controlled by a convolutional neural network.

#### 4.2.1 Testing Pre-Trained Networks

Most autonomous vehicles and advanced driver-assistance systems (ADAS) have on-board components for computer vision tasks like road detection, traffic light and traffic sign classification, vehicle detection and optical flow. Using Paracosm’s programming interface, we created a 6-lane road network with a cross intersection and heterogeneously spread-out traffic. We tested two object detection systems: road detection, and vehicle detection.

All the systems we test in this section are already trained. We took existing code for pre-trained networks and integrated it in Paracosm test scenarios.

**Road detection.** A road detection system takes RGB images as input and outputs those pixels that are estimated to be a part of the road. We use a network based on the fully convolutional neural network from Simonyan and Zisserman [50], trained with the KITTI road segmentation data set [23].

In this experiment, we vary the focal length of the car’s top-mounted camera to see its effect on road segmentation. Only the focal length change; the rest of the scene is kept the same. We varied the focal length between 10 and 50mm (wide to standard angle) and measured how much of the road was
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4.2.1 Road Detection

A short focal length (10mm) results in poor road detection. A longer focal length (34mm) results in better road detection. The red area depicts a falsely detected road segment.

**Figure 5.** Sample output from the road detection system.

The normalized baseline is obtained by manual annotation. A true positive indicates the percentage of the baseline that was recognized as road. A false positive shows the percentage of non-road (compared to baseline) identified as road. A correct detection and how much of the scene was falsely identified as road. For each value, we manually tag the road in the image and compare the output of the network to this. Correctly detected road has to be detected in both images; falsely detected means that the neural network marked a part of the image being part of the road while manual labeling did not mark that part. Figure 5 shows two example images and Figure 6 shows the test’s results.

**Figure 6.** Influence of focal length on the road detection. The normalized baseline is obtained by manual annotation. A true positive indicates the percentage of the baseline that was recognized as road. A false positive shows the percentage of non-road (compared to baseline) identified as road.

Vehicle detection. A vehicle detection system takes RGB images as input and outputs bounding boxes around pixels that correspond to vehicles. Figure 8 shows an example of a vehicle detection system’s output. To detect other vehicles in the vicinity of the autonomous car, we used (i) the single shot multibox detector (SSD)

![SSD code from https://github.com/ndrplz/self-driving-car/tree/master/project_5_vehicle_detection](https://github.com/ndrplz/self-driving-car/tree/master/project_5_vehicle_detection)

which frames object detection as a regression problem, and (ii) YOLO [42], a neural network based approach based on History Oriented Gradients (HOG) [12], trained on the GTI Vehicle image database [4]. HOG methods were first used for human detection and then adapted to cars.\(^7\)

In this experiment, the car is driven in a straight line at constant speed through traffic and the camera images are sent to the vehicle detection system. Using the same road network as in the example of the road segmentation experiment, the car follows the innermost lane through the traffic into the crossing, on which a white truck is passing from left to right. Every 10 meters, we save the output of these systems. We manually identify the total number of vehicles to get a baseline. Then we inspect the output of the vehicle detection systems and count the number of vehicles correctly identified and the false positives, e.g., road features identified as a car.

**Figure 8.** Sample output from the vehicle detection system.

Figure 8 summarizes the results. **Baseline** denotes the total number of cars visible, **true positive** the correctly identified vehicles, and **false positive** the erroneously detected ones.

We observe that overall, the systems we used are not able to identify vehicles that are too far away, explaining why the detection is so poor in the beginning. More worrying, no system was able to detect the truck in the middle of the crossing. As the training data consists mostly of images where all the vehicles follow the same road, our hypotheses is that the systems learned features corresponding to the front or back of the vehicles. This experiment shows the usefulness of testing pre-trained networks under different driving conditions.

4.2.2 Driving Behaviour

We now test an end-to-end system, i.e., a system which takes sensor values as input and transforms these into steering or throttle commands. **Paracosm**’s architecture enables a qualitative and quantitative analysis of such dynamic closed-loop behavior. We run tests over time to catch situations in which decisions made by a controller can have long-term consequences. In an optimistic scenario, if the output of the SUT changes abruptly due to some error in the vision system, it may be harmless if it is just for one frame as it gets smoothed out by correct behavior in surrounding frames.

\(^7\)YOLO and HOG code from https://github.com/JunshengFu/vehicle-detection.
For the experiment, we parameterize the speed of walking (1 m/s). For the experiment, we parameterize the speed of walking (1 m/s). For the experiment, we parameterize the speed of walking (1 m/s). For the experiment, we parameterize the speed of walking (1 m/s). For the experiment, we parameterize the speed of walking (1 m/s). For the experiment, we parameterize the speed of walking (1 m/s).

The light grey bar shows ground truth. The light grey bar shows ground truth. The light grey bar shows ground truth. The light grey bar shows ground truth. The light grey bar shows ground truth. The light grey bar shows ground truth. The light grey bar shows ground truth. The light grey bar shows ground truth. The light grey bar shows ground truth. The light grey bar shows ground truth. The light grey bar shows ground truth.

We use the implementation at https://github.com/naokishibuya/car-behavioral-cloning.

Figure 8. Vehicle detection rates for SSD, YOLO, and HOG. Results are summarized in Figure 9 and demonstrate the advantage of low-dispersion sampling over random sampling. This difference is more pronounced for smaller number of samples.

Behavior under Changing Environmental Settings. As discussed in Section 2, reactive variables can be used to parameterize environment settings so as to describe a large class of configurations. To demonstrate this, we train a model at a fixed light intensity and no fog. In this scenario, the car is behind a slow car and, therefore, may need to brake to avoid a collision. We analyse the car’s performance when the environment condition varies by reporting the distance covered, and whether a collision happened. Each test lasts 15 seconds. We generate test parameters using the Halton sequence. The results are aggregated in Figure 10a. The car performs best around the parameter values used during training. The distance that the car covers drops-off fairly quickly as fog density increases and perturbations to light intensity often lead to scenarios with collision.

Next, we demonstrate an experiment with test scenarios over a longer time horizon. We created a loop by connecting equally sized straight road segments with T-intersections and trained the car to drive around in the loop indefinitely (always making a right turn). The training data corresponds to around 100 seconds of driving with the front facing camera producing 10 images per second. We parameterize fog density and measure the time to failure, i.e., time until the first collision. A time-out of 200 seconds is set. We observe that the steering angle is affected by the fog. Surprisingly, even though we train the car with no fog, it understeered in low fog conditions, then oversteered properly for moderate fog. Finally, in high fog conditions, the car first understeered then oversteered.

Behavior with Different 3D Components. We now discuss some observations made when training for one of the previous experiments (Figure 10a). For these experiments, the autonomous car is trained to follow a red lead car driving in front on a two-lane road. Under tests with ideal conditions, we observe that the autonomous car is able to follow the lead car while maintaining a safe distance in-between. We do two tests to try to understand what the neural network “learns” from the training images.

For the first experiment, we have another red car, similar to the lead car come from the other direction. If the system learned to follow the lead car, this new car should have no effect on the behavior of our autonomous vehicle. However, we observe that the autonomous car picks-up on the car coming from the other direction and applies brakes when it is close enough, essentially getting confused between the lead car and this car coming from the opposite direction. The speed over time is plotted for both, the ideal and problematic case in Figure 10c. For the second experiment, we change the body color of the lead car from red to white. As the autonomous car was only trained on a red lead car, it fails to recognize the lead car and crashes into it. These two
Figure 9. Comparison of random and Halton sampling. The X axis is the walking speed of the pedestrian (0.5 to 10 meters per second). The Y axis is the distance from the car when the pedestrian starts crossing (5 to 60 metres). When the car collides with the pedestrian, we mark the scenarios with a red cross.

Figure 10. Experiences on the effect of environment conditions.

experiments point to the network learning that the throttle is related to the amount of red pixels in the image rather than recognizing a car in front.

5 Related Work

Reactive Programming Models. PARACOSM’s programming model follows a synchronous reactive style for dataflows, similar to functional reactive programming (FRP) [57] or to synchronous dataflow languages like Lustre [8]. FRP has been applied to control robotic and embedded systems [28], including car [21]. In PARACOSM, rather than using reactive programming to control the SUT, we use reactive elements to build and control the world around the system under test. Our programming model shares elements found in automata-based models like reactive modules [2]. In contrast to these languages, PARACOSM natively supports geometric and physical properties of components and integrates with a game engine. Reactive programming has gained popularity in recent years as a way to build asynchronous distributed applications [31]. Our model is synchronous because we wish to test physical systems with a global time.
Traditionally, test-driven software development paradigms [5] have advocated testing and mocking frameworks to test software early and often. Mocking frameworks and mock objects [33, 36] allow the programmer to test a piece of code against an API specification, even when the implementation of the API is not available. Unfortunately, current mocking frameworks provide rudimentary support for complex environment interactions over time. Typically, mock objects are stubs providing outputs to explicitly provided lists of inputs of simple types, with little functionality of the actual code. Thus, they fall short of providing a rich environment for autonomous driving. PARACOSM can be seen as a mocking framework for reactive, physical systems embedded in the 3D world. Our notion of constraining streams is inspired by work on declarative mocking [47].

**Testing for Cyber-Physical Systems.** There is a large body of work in automated test generation tools for cyber-physical systems through heuristic search of a high-dimensional continuous state space. While much of this work has focused on low-level controller interfaces [3, 13, 15, 16, 20, 48] rather than at the system level, specification and test generation techniques arising from this work—for example, the use of metric and signal temporal logics or search heuristics—can be adapted to our setting. Surprisingly, the notion of dispersion as a coverage criterion has not been stated before.

More recently, test generation tools have started to target autonomous systems with machine learning components, under a simulation-based semantic testing framework similar to ours [18, 19, 53]. In most of this work, an underlying visual scenario is fixed by hand, and test generation explores plausible perturbations to the nominal scenario to detect faulty system behavior. Such an adversarial analysis is shown to be preferable to the application of random noise on the input vector. Moreover, a simulation-based approach can filter benign misclassifications of the machine learning component from misclassifications that actually lead to bad system behavior. Our work extends this line of work by providing an expressive language to write parameterized tests. For example, instead of manually specifying the time and speed at which a pedestrian crosses a road, it is possible to have these variables dependent on the behavior of the autonomous car.

There is some recent work on interchange formats for test scenarios for autonomous driving [56]. The testing toolkit includes a set of visual building blocks for road elements similar to the building blocks we use to make the environment. These tools are aimed at exchanging information between humans. In contrast, PARACOSM provides parametric test scenarios readily executable in a simulated environment.

To address the problem of high time and infrastructure costs of testing autonomous systems, several robotics simulators have been developed. The most popular is Gazebo [22] for the ROS [41] robotics framework. It offers a modular and extensible architecture, however falls behind on visual realism and complexity of environments that can be generated with it. To counter this, game engines have been used. A popular example is TORCS [59] and the Grand Theft Auto V environment, modified for training or testing [46]. Simulators such as CARLA [17] and AirSim [49] use modern game engines and support creation of realistic urban environments. Though these simulators enable visually realistic simulations and enable detection of infractions such as collisions, the environments themselves are rather difficult to design. Designing a custom environment involves manual placement of road segments, buildings, and actors (as well as their properties). Performing many tests is therefore time-consuming and difficult. While these systems and PARACOSM share the same aims and much of the same infrastructure, PARACOSM focuses on procedural generation and testing of complex scenarios.

**Adversarial Examples.** Adversarial examples for neural networks [25, 51] introduce perturbations to inputs that cause a classifier to classify "perceptually identical" inputs differently. Much work has focused on finding adversarial examples in the context of autonomous driving as well as on training a network to be robust to perturbations [6, 24, 35, 38, 58]. Tools such as DeepXplore [39] and DeepTest [52] define a notion of coverage for neural networks based on the number of neurons activated during the tests over the total number of neurons in the network. They use gradient descent and random test generation to find unsafe corner cases and attain high coverage. However, the evaluations of these techniques focus mostly on individual classification tasks and apply 2D transformations on images. PARACOSM also considers the closed loop behavior of the system and our parameters directly change the world rather than applying transformations post facto. For example, we could observe, over time, that certain vehicles were never detected.

Alternately, there are recent techniques to verify controllers implemented as neural networks through constraint solving or abstract interpretation [24, 27, 45, 58]. While these tools do not focus on the problem of autonomous driving, their underlying techniques could be combined in the test generation phase for PARACOSM. For example, some of our case studies in Section 4 track the performance of the controller against errors in the perceptual system over time.

6 Future Work and Conclusion

Deploying autonomous systems like self-driving cars in urban environments raises many challenges about making sure these systems are safe. The complex software stack includes the processing of sensor data, building a semantic model of the surrounding world, decision making, trajectory planning, and control of the car. The end-to-end testing of such systems requires the creation and simulation of a whole world. Furthermore, each test may require a slightly different world. PARACOSM tackles these problems by (1) enabling
the construction of a wide range of scenarios, with precise control over elements like layout of the road, physical and visual properties of objects, and the behaviors of actors in the system, and (2) using quasi-random testing to get a good coverage over the large parametric test space.

In our evaluation, we showed that Paracosm can efficiently find erroneous behaviors in existing systems implemented using neural networks. While finding errors in sensing can be done with only a few static images, we show that Paracosm also enables the creation of longer test scenarios which exercise the controller’s feedback on the environment, e.g., physics of the car.

In the future, we plan to extend Paracosm’s testing infrastructure to also help training deep neural network based systems. The amount and quality of training data is critical in this step. For instance, we showed that a small variation in the camera model results in widely different results for road segmentation. However, generating data is a time consuming and expensive task. We plan to record data when a user manually drives within a parameterized Paracosm environment.

The data produced during this initial run can serve as the basis for generating a large amount of new data by varying the parameters which should not impact the car’s behavior. For instance, we can vary the color of other cars, positions of pedestrians who are not crossing, or even the light conditions and sensor properties (within reasonable limits). The new data can be generated completely automatically to augment the learning data set.

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