New Beam Tracking Technique for Millimeter Wave-band Communications

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Abstract—In this paper, we propose an efficient beam tracking method for mobility scenario in mmWave-band communications. When the position of the mobile changes in mobility scenario, the base-station needs to perform beam training frequently to track the time-varying channel, thereby spending significant resources for training beams. In order to reduce the training overhead, we propose a new beam training approach called “beam tracking” which exploits the continuous nature of time varying angle of departure (AoD) for beam selection. We show that transmission of only two training beams is enough to track the time-varying AoD at good accuracy. We derive the optimal selection of beam pair which minimizes Cramer-Rao Lower Bound (CRLB) for AoD estimation averaged over statistical distribution of the AoD. Our numerical results demonstrate that the proposed beam tracking scheme produces better AoD estimation than the conventional beam training protocol with less training overhead.

I. INTRODUCTION

The next generation wireless communication systems aim to achieve Giga bit/s throughput to support high speed multimedia data service [1], [2]. Since there exist ample amount of unutilized frequency spectrum in millimeter Wave (mmWave) band (30 GHz-300 GHz), wireless communication over mmWave band is considered as a promising solution to achieve significant leap in spectral efficiency [3]. However, one major limitation of mmWave communications is significant free space path loss, which causes large attenuation of signal power at the receiver. Furthermore, the overall path loss gets worse when the signal goes through obstacles, rain, foliage, and any blockage to mobile devices. Recently, active research on mmWave communication has been conducted in order to overcome these limitations [4]-[5]. In mmWave band, many antenna elements can be integrated in a small form factor and hence, we can employ high directional beamforming using a large number of antennas to compensate high path loss.

In order to perform high directional beamforming, it is necessary to estimate channels for all transmitter and receiver antenna pair. While this step requires high computational complexity due to large number of antennas, channel estimation can be performed efficiently by using the angular domain representation of channels [6]. In angular domain, only a few angular bins contain the most of the received energy. Hence, if we identify the dominant angular bins (which correspond to the angle of arrival (AoA) and the angle of departure (AoD)), we can obtain the channel estimate without incurring computational complexity.

Basically, both AoD and AoA can be estimated using so called “beam training” procedure. The base-station sends the training beams at the designated direction and the receiver estimates the AoD/AoA based on the received signals. Widely used beam training method (called “beam cycling method”) is to allow the base-station to transmit $N$ training beams one by one at the equally spaced directions. However, to ensure good estimate of AoD/AoA, $N$ should be large, leading to significant training overhead. This problem becomes even more serious for the mobility scenario in mmWave communications. Since the location of mobiles keeps changing, the base-station should transmit training beams more frequently to update AoD/AoA estimates, causing significant drop in data throughput [7]. Recently, several adaptive beam training schemes have been proposed to improve the conventional beam training method [10]-[13].

In this paper, we introduce a novel beam training method for mobility scenario in mmWave communications. Our idea is based on the observation that for mobility scenario, the AoD of the particular user does not change drastically so that continuous nature of the AoD change can be accounted to improve the efficacy of the beam training. Since this approach exploits temporal dynamics of AoD, we call such beam training scheme “beam tracking”. While the conventional method makes no assumption on the state of AoD, we use statistical distribution of the AoD given the previously state of AoD. Using the probabilistic model on AoD change, we derive effective beam tracking strategy which employs transmission of two training beams from the base-station. Optimal placement of two training beams in angular domain is sought by minimizing (the lower bound of) variance of the estimation error for AoD. As a result, we choose the best beam pair from the beam codebook for the given prior knowledge on AoD. Our simulation results show that the proposed beam tracking method offers the channel estimation performance comparable to the conventional beam training methods with significantly reduced training overhead.

The rest of this paper is organized as follows: In section II we introduce the system and channel models for mmWave communications and in section III we describe the proposed beam tracking method and the simulation results are provided in section IV. Finally, the paper is concluded in section V.

II. SYSTEM MODEL

In this section, we describe the system model for mmWave communications. First, we describe the angular domain representation of the mmWave channel and then we introduce the
procedure for beam training and channel estimation.

A. Channel Model

Consider single user mmWave MIMO systems with the base-station with \( N_b \) antennas and the mobile with \( N_m \) antennas. The MIMO channel model with \( L \) paths at time \( t \) is described by \[ (1) \]

\[
H(t) = \sqrt{N_b N_m} \sum_{l=1}^{L} \alpha_l(t) a_m(\theta_l^m(t)) a_b^H(\theta_l^b(t))
\]

where \( \alpha_l(t) \) is the \( l \)-th path gain at time \( t \), \( \theta_l^b(t) \) and \( \theta_l^m(t) \) are the \( l \)-th path AoD and the \( l \)-th path AoA, respectively, the beam steering vectors \( a_b(\theta_l^b) \) and \( a_m(\theta_l^m) \) for the base-station and the mobile are given by \[ (2) \]

\[
a_b(\theta_l^b) = \frac{1}{\sqrt{N_b}} [1, e^{j2\pi d\theta_l^b/\lambda}, \ldots, e^{j2\pi (N_b-1)d\theta_l^b/\lambda}]^T
\]

\[
a_m(\theta_l^m) = \frac{1}{\sqrt{N_m}} [1, e^{j2\pi d\theta_l^m/\lambda}, \ldots, e^{j2\pi (N_m-1)d\theta_l^m/\lambda}]^T
\]

where \( d \) is a distance between the adjacent antennas and \( \lambda \) is wavelength. Note that \( \theta_l^b \) is a normalized angle defined as

\[
\theta_l^b = \sin(\phi)
\]

where \( \phi \in [-\pi, \pi] \) is a physical angle for AoD. The AoA \( \theta_l^m \) is defined similarly. The canonical representation of channels in angular domain can be obtained using \[ (3) \]

\[
H(t) = A_m H_v(t) A_b^H
\]

where the columns of \( A_m \) and \( A_b \) are the beam steering vectors obtained at the \( M \)-point uniformly quantized angular grid, i.e.,

\[
A_b = \frac{1}{\sqrt{N_b}} [a_b(-1 + 2\frac{0}{M}), \ldots, a_b(-1 + 2\frac{(M-1)}{M})]
\]

\[
A_m = \frac{1}{\sqrt{N_m}} [a_m(-1 + 2\frac{0}{M}), \ldots, a_m(-1 + 2\frac{(M-1)}{M})]
\]

Note that the \((i, j)\)th element of \( H_v(t) \) is the channel gain corresponding to the \( i \)-th angular bin for the AoA and the \( j \)-th angular bin for the AoD. With channel exhibiting \( L \) multi-paths, \( H_v(t) \) has dominant value only in the \( L \) elements and almost zero value for the rest.

B. Beam Training and Channel Estimation

For channel estimation, the standard mmWave systems employ “beam training method” where the base-station transmits the known symbols using the \( N \) training beams and the mobile estimates the channel using the received signals. Each beam training cycle consist of transmission of the \( N \) training beams. It repeats periodically for update of the channel estimate. From now on, we use the index \( t \) to denote the \( t \)th beam training opportunity. At the \( t \)th beam transmission in the \( t \)th beam training cycle, the base-station selects the beamforming vector \( f_i \in \mathbb{C}^{N_b \times 1} \) from the beam codebook \( D \) and send the known symbol \( s_i = 1 \). The receiver applies the combining vector \( w_i \in \mathbb{C}^{N_m \times 1} \) to the received signal \( y_i(t) \), which is expressed as

\[
y_i(t) = w_i^H H(t) f_i s_i + n_i(t),
\]

where \( n(t) \) is the i.i.d. Gaussian noise vector. The \( N \) received signal vectors are collected during the beam training and we have the matrix \( Y(t) = [y_1(1), \ldots, y_1(N)] \) \[ (4) \]

\[
Y(t) = W^H H(t) F + N(t)
\]

\[
= W^H A_m H_v(t) A_b^H F + N(t),
\]

where \( N(t) = [n_1(1), \ldots, n_1(N)] \) contains the i.i.d. Gaussian noise, \( F = [f_1, \ldots, f_N] \), and \( W = [w_1, \ldots, w_N] \). If we vectorize \( Y(t) \), we have \[ (5) \]

\[
y(t) = \text{vec}(W^H H(t) F) + \text{vec}(N(t))
\]

\[
= (F^T \otimes W^H) \text{vec}(H(t)) + n(t)
\]

\[
= (F^T \otimes W^H) (\text{conj}(A_b) \circ A_m) h(t) + n(t)
\]

\[
= (F^T \text{conj}(A_b) \otimes W^H A_m) h(t) + n(t)
\]

where \( \text{vec}(\cdot) \) and \( \text{conj}(\cdot) \) are the vectorization and the conjugation operations, respectively, and \( h(t) = \text{vec}(H_v(t)) \). Here \( (F^T \otimes W^H) \) is Kronecker product of beamforming vector \( F \) and combining vector \( W \) and each column of the matrix \( (\text{conj}(A_b) \circ A_m) \) consists of \( (\text{conj}(a_b(\theta_l^b)) \otimes a_m(\theta_l^m)) \). Note that the channel estimation is equivalent to estimation of \( h(t) \) from the received signal vector \( y(t) \) in \[ (6) \]

III. PROPOSED BEAM TRACKING TECHNIQUE FOR MOBILITY SCENARIO

One widely used beam training strategy is “beam cycling” which transmits \( N \) training beams at the uniformly spaced directions. Since this approach does not exploit the knowledge on the location of the mobile, the value of \( N \) required for the receiver to achieve good channel estimation quality should be large. While adaptive beam training approaches have been proposed to improve the overhead of beam cycling \[ (10)–(14) \], they require the feedback from the mobile during the same beam training cycle. In this section, we introduce the efficient beam training scheme which exploits the temporal dynamics of AoD to reduce the training overhead of the conventional beam training methods. Since the proposed scheme exploits the tracking of the time-varying AoD for beam training, we will refer to our scheme as “beam tracking method”.

A. Overall system description

Fig. \[ 1 \] depicts the proposed beam training protocol in comparison with conventional beam cycling scheme. While the conventional beam cycling transmits each beam one at a time towards all directions, the proposed scheme transmits only two beams toward the directions optimized by the proposed beam selection method. In the beginning, the proposed scheme does not have knowledge of the AoD and hence it employs the conventional beam cycling. Once the mobile obtains the estimate of the AoD from the received signal, the AoD estimate is fed back to the base-station. Then, using the feedback from the mobile, the base-station select the best beam
B. Statistical Model for Channel Dynamics

In order to design the proposed beam tracking scheme, we employ the statistical model capturing the smooth characteristics of channels under mobility. Specifically, we model the temporal dynamics of the AoD using the Markov random process. Note that the distribution of the current value of AoD depends only through the previous state of the AoD. For example, the AoD at the $t$th beam training cycle, $\theta^b_{l}(t)$ is distributed by

$$\theta^b_{l}(t) \sim \text{Pr}(\theta^b_{l}(t) | \theta^b_{l}(t-1); \sigma^2_p)$$

$$= N(\theta^b_{l}(t-1), \sigma^2_p),$$

where $\theta^b_{l}(t-1)$ is the AoD at the previous beam training cycle and $\sigma^2_p$ is the variance of Gaussian distribution. Note that various distribution (such as Laplacian) can be used instead of Gaussian. The parameter $\sigma^2_p$ indicates the extent of the mobility for the mobile. The stronger the mobility is, the larger $\sigma^2_p$ gets. Hence, in practice, we can find one dimensional mapping of the average speed of the mobile to the appropriate value of $\sigma^2_p$. As the AoD is discretized in our model in (3), we can easily transform the distribution in (7) into that of discrete random variable.

C. Signal Model for Single Path Scenario

For now, we assume that strong line of sight (LOS) exists, i.e., $L = 1$. Hence, we will omit the path index $l$ for the time being. As mentioned above, at the $t$th beam transmission cycle, the base-station transmits the two beamforming vectors $f_i(t)$ and $f_j(t)$ in a row. The beamforming matrix $F_{i,j}(t) \in \mathbb{C}^{N_b \times 2}$ is obtained by choosing the two beam pair from the codebook $D$, i.e.,

$$F_{i,j}(t) = [f_i(t) \ f_j(t)]$$

where $i$ and $j$ are the selected indices of beamforming vectors in the codebook. Note that the codebook we generate includes the beamforming vectors with different beam-widths and with different steering directions at uniformly quantized angular bin. Once the optimal beamforming vectors are selected, we can modify them accounting for the hardware limitation of mmWave systems [9].

$$y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} w^H(t)a_m(\theta^m(t)) \beta(t)a^H_b(\theta^b(t))f_i(t) \\ w^H(t)a_m(\theta^m(t)) \beta(t)a^H_b(\theta^b(t))f_j(t) \end{bmatrix} + \begin{bmatrix} n_1(t) \\ n_2(t) \end{bmatrix}$$

where $n_1$ and $n_2$ are i.i.d. Gaussian noise vectors $\mathcal{CN}(0, 2\sigma^2 I)$ and $\beta(t)$ is the channel gain for LOS path. Though the selection of the combining vector $w(t)$ should be considered for the optimal beamforming design, we exclude the combining matrix from our design parameters for the sake of convenience. Hence, we assume that the receiver obtains the correct estimate of the AoA and hence we can let $w(t) = a_m(\theta^m(t))$. Using $W^H(t)a_m(\theta^m(t)) = 1$, we get

$$y(t) = \beta(t) \begin{bmatrix} a^H_b(\theta^b(t))f_i(t) \\ a^H_b(\theta^b(t))f_j(t) \end{bmatrix} + \begin{bmatrix} n_1(t) \\ n_2(t) \end{bmatrix}$$

$$= \beta(t) \begin{bmatrix} f_i^T(t) \\ f_j^T(t) \end{bmatrix} \text{conj}(a_b(\theta^b(t))) + \begin{bmatrix} n_1(t) \\ n_2(t) \end{bmatrix}. \quad (12)$$

Note that in (12), the channel estimation boils down to estimating both the AoD $\theta^b(t)$ and the channel gain $\beta(t)$ based on the model for $y(t)$.

D. AoD Estimation

The joint estimation of the AoD $\theta^b(t)$ and the channel gain $\beta(t)$ can be obtained from maximum likelihood (ML) criterion. The log-likelihood function is given by

$$\ln P(y(t)|\theta^b(t), \beta(t))$$

$$= -\frac{1}{2\sigma^2} \left\| y(t) - \beta(t) \begin{bmatrix} f_i^T(t) \\ f_j^T(t) \end{bmatrix} \text{conj}(a_b(\theta^b(t))) \right\|^2 + C. \quad (13)$$
Then, the ML estimate is given by

\[
(\hat{\theta}^b(t), \hat{\beta}(t)) = \arg\min_{\theta^b(t), \beta(t)} \left\| y(t) - \beta(t) \begin{pmatrix} f_1^T(t) \\ f_2^T(t) \end{pmatrix} \text{conj}(a_b(\theta^b(t))) \right\|^2
\]

\[
= \arg\min_{\theta^b(t)} \left( \min_{\beta(t)} \left\| y(t) - \beta(t) \begin{pmatrix} f_1^T(t) \\ f_2^T(t) \end{pmatrix} \text{conj}(a_b(\theta^b(t))) \right\|^2 \right)
\]

\[
= \arg\min_{\theta^b(t)} \left\| (I - Q_{\theta^b(t)}(\theta^b(t))) y(t) \right\|^2
\]

(14)

where

\[
Q_{\theta^b(t)}(\theta^b(t)) = \begin{pmatrix} f_1^T(t) \\ f_2^T(t) \end{pmatrix} \text{conj}(a_b(\theta^b(t))) \cdot a_b^H(\theta^b(t)) \begin{pmatrix} f_1^H(t) \\ f_2^H(t) \end{pmatrix}
\]

\[
= \frac{|a_b(\theta^b(t))| \cdot |a_b^H(\theta^b(t))|}{|a_b(\theta^b(t))f_i(t)|^2 + |a_b^H(\theta^b(t))f_j(t)|^2}
\]

(15)

Note that the optimization in (14) is performed by searching for the candidate of \(\theta^b(t)\) minimizing the cost metric \(\left\| (I - Q_{\theta^b(t)}(\theta^b(t))) y(t) \right\|^2\) over uniformly quantized angular grid for representing the AoD. In order to reduce the search complexity, we can restrict the search range within the angle formed by the two training beams \(f_i(t)\) and \(f_j(t)\). (see Fig. [2]) This allows for significant reduction in computational complexity required for estimation of the AoD. Alternatively, we can increase the resolution of AoD estimation without incurring additional computational complexity.

E. Beam Selection

Now, we present the proposed beam selection algorithm which selects the best beamforming vectors \(f_i(t)\) and \(f_j(t)\) from the codebook \(\mathcal{D}\) that yield the best performance in AoD estimation. Note that we use the statistical distribution of the AoD \(\theta^b(t)\) in the derivation of optimal beam selection. Since it is not straightforward to derive the analytical expression for the mean square error (MSE), \(E[\|\hat{\theta}^b(t) - \theta^b(t)\|^2]\), we use the Cramer Rao lower bound (CRLB) averaged over the distribution of \(\theta^b(t)\) as a performance metric. The Fisher information matrix for joint estimation of \(\theta^b(t)\) and \(\beta(t)\) is expressed as

\[
I(\xi) = E(\frac{\partial \ln P(y(t)|\theta^b(t), \beta(t))}{\partial \xi^*} \cdot \frac{\partial \ln P(y(t)|\theta^b(t), \beta(t))^H}{\partial \xi^*})
\]

\[
= E \begin{bmatrix} \frac{\partial \ln P(y(t)|\theta^b(t), \beta(t))}{\partial \beta(t)} \\ \frac{\partial \ln P(y(t)|\theta^b(t), \beta(t))}{\partial \theta^b(t)} \end{bmatrix} \begin{bmatrix} \frac{\partial \ln P(y(t)|\theta^b(t), \beta(t))^H}{\partial \beta^*} \\ \frac{\partial \ln P(y(t)|\theta^b(t), \beta(t))^H}{\partial \theta^b(t)} \end{bmatrix}
\]

(16)

where \(\xi = \begin{bmatrix} \beta^*(t) \\ \theta^b(t) \end{bmatrix}\). When \(\theta^b(t)\) is given, the CRLB of the AoD \(\theta^b(t)\) is given by [16].

\[
CRLB_{i,j}(\theta^b(t)) = \left[ I(\xi) \right]_{3,3} = |Q - 2Re\{PCP^H\}|^{-1}
\]

(17)

(18)

where

\[
Q = \frac{1}{\sigma^2} \left\| \beta(t) \begin{pmatrix} f_i^T(t) \\ f_j^T(t) \end{pmatrix} \partial\text{conj}(a_b(\theta^b(t))) \right\|^2;
\]

\[
P = \frac{1}{2\sigma^2} \beta(t) \begin{pmatrix} f_i^T(t) \\ f_j^T(t) \end{pmatrix} \partial a_b(\theta^b(t)) \partial\text{conj}(a_b(\theta^b(t))),
\]

\[
C = \left( \frac{1}{2\sigma^2} \right) \left| \beta(t) \begin{pmatrix} f_i^T(t) \\ f_j^T(t) \end{pmatrix} \partial\text{conj}(a_b(\theta^b(t))) \right|^2^{-1}
\]

Now, we average the CRLB over the distribution of \(\theta^b(t)\) when \(\theta^b(t)\) is given. The average CRLB is given by

\[
CRLB_{\text{avg}}(f_i(t), f_j(t)|\theta^b(t) - 1, \sigma_p^2) = \int CRLB(\theta) \cdot Pr(\theta|\theta^b(t) - 1, \sigma_p^2) d\theta
\]

(19)

where \(Pr(\theta|\theta^b(t) - 1, \sigma_p^2)\) is drawn from [8]. In case when the distribution is discretized, we can replace the integration by the summation in (19). Note that we choose the best beamforming vectors \(f_i(t)\) and \(f_j(t)\) i.e.,

\[
F_{i,j}(t) = \arg\min_{k,l \in \text{index}(\mathcal{D})} CRLB_{\text{avg}}(f_k(t), f_l(t)|\theta^b(t) - 1, \sigma_p^2).
\]

(20)

Note that the optimization in (20) requires two dimensional search over all beam indices in the code book. Fortunately, we observe that the directions for the optimized beam pair are symmetric with each other around the previous AoD estimate \(\theta^b(t) - 1\). This allows us to conduct one dimensional search over the angle made between two beamforming vectors. In practical applications, we conduct the optimization for beam selection in offline and generate the look-up table which maps \(\sigma_p^2\) to the optimal beam indices directly.

Though our derivation is based on the assumption that the previous AoD \(\theta^b(t - 1)\) is known, the assumption is not strict since we use the estimate of the previous AoD fed back from the mobile. In order to compensate this mismatch, we refine the AoD model introducing the perturbation error \(\epsilon\) in \(\theta^b(t - 1) = \theta^b(t - 1) + \epsilon\) and derive the CRLB given the estimate of the previous AoD \(\theta^b(t - 1)\) in an iterative fashion.
F. Proposed Beam Tracking for Multi Path Scenarios

So far, we have presented the new beam tracking strategy for single path scenarios. We can easily extend the proposed scheme for the scenario where there exist $L$ multi paths in mmWave channels. If the AoDs associated with each path are well separated in angular domain, it is possible to apply the proposed tracking scheme derived for single path for each individual path while ignoring the existence of other paths. In this scenario, the base-station transmits two training beams for each of $L$ path, requiring $2L$ beam transmissions in total. Since we search for the AoD estimate within the restricted range, we can separate each path from each other without negligible performance loss. When the different paths are clustered in angular domain, we have to find joint estimate of AoDs based on the received signals generated from $2L$ beam transmissions. The optimization for designing $2L$ beamforming vectors can be performed for each path. The estimation of $L$ values of AoD can be performed via compressed sensing techniques such as orthogonal matching pursuit (OMP) [15].
the proposed scheme achieves significant performance gain in AoD estimation over the conventional methods over all range of SNR of interest. Note that the large performance gain is maintained for different values of $\sigma_p$.

V. CONCLUSIONS

In this paper, we have presented the novel beam training protocol which exploits the dynamic model for the AoA and AoD for the mobility scenario in mmWave band communications. We demonstrate that by exploiting the property of smooth variation in the AoD, the good channel estimation performance can be achieved only with transmission of two training beams. The simulation results corroborates that the proposed scheme achieves significant reduction in training overhead over the existing beam training methods while maintaining good channel estimation performance.

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