Superspeckles: a new application of optical superresolution

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Abstract: The effect of optical superresolution on speckle correlations is studied. Simulations reveal that using a lateral superresolution pupil filter more than twice the out of plane correlation length of the clear pupil can be achieved. This means that the measurement range in speckle correlation measurements doubles. To verify the correlation length an experiment is performed using a liquid crystal (LCD) spatial light modulator as a programmable superresolution filter. The results corroborate the simulation.

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1. Introduction

The extent of a laser speckle in three dimensions is of primary importance in the measurement of deformations or displacement of rough objects. It determines the measurement range within which one can ascertain the deformations in the form of fringe pattern in the case of Digital Speckle Pattern Interferometry (DSPI) or speckle correlation and Young’s fringes in case of speckle photography. Speckle correlation length limits the measurement range in DSPI or speckle photography. The latest investigations on the three dimensional size of a speckle was done in Ref. 1-3 by considering the autocorrelation function of the speckle intensity. Previously, image formation along the axial direction was studied by Frieden [4]. The average length of the speckle along the longitudinal direction is evaluated to be $8\lambda (L/D)^2$, where $L$ is the observation distance from the pupil and $D$ is the diameter of the illuminated area or the pupil diameter. The decorrelation in longitudinal direction was experimentally demonstrated by considering the reduction of fringe visibility of Young’s fringes from double exposure photographs with successive images taken at defocus positions with constant in-plane displacement.

Therefore, an increase in the average longitudinal length of the speckle without changing the lateral resolution of the imaging system is of great significance in the areas such as DSPI, speckle photography, or even blood flow imaging [5]. Here we examine the increase in the average speckle size along the longitudinal direction (optical axis) using transverse superresolution pupil filters. Simulations show that more than 100% increase in speckle correlation length is easily achievable. This is due to the fact that a reduction in the central spot size of the diffraction pattern results in a decrease of resolution in the axial direction [6]. An increase in the longitudinal speckle size would also be obtained by just reducing the diameter of the pupil which in turn increases the $L/D$ ratio. However, this would reduce the lateral resolution of the image. With the incorporation of a superresolution filter the speckle length is increased together with an increase in spatial resolution. The introduction of superresolution filters is thus destined to extend its applications, which are presently limited to microscopy [7], astronomy [8] and dense DVD writing [9], to areas such as DSPI or speckle photography.

2. Theory and simulation

To evaluate the degree of longitudinal correlation, speckle fields are first calculated for different object positions obtained by moving the object from its focal position along the optical axis. This is followed by comparing the speckle fields so obtained with respect to the speckle field for the object at the focal position. The image plane and defocused speckle fields are generated using Fresnel approximation of the diffraction integral.

The image formed by an optical system can be considered as the two dimensional convolution of the object function with the impulse response function of the system, expressed as

$$U_i(x,y) = \iint U_o(\xi,\eta)h(x-\xi,y-\eta)d\xi d\eta$$  

(1)

where $U_i$, $U_o$ and $h$ are the image field, object field and the impulse response functions, respectively; $(x, y)$ are the image plane coordinates and $(\xi, \eta)$ are the object plane coordinates. Let us consider a 4F architecture wherein a rough object is imaged by a lens with focal length $f$ onto the image plane at unit magnification. Because of this simplification we can consider image plane defocus rather than object plane defocus which produces the same result. The pupil stop is located at the center of the setup. The impulse response function can be expressed as the diffraction integral in the Fresnel approximation [10]

$$h(x,y,z) = \frac{\lambda}{z} \int_{\nu} R(u,v)e^{i \frac{k}{2f} \left( \frac{1}{2f} - \frac{1}{z} \right) u^2 + v^2} e^{i \frac{k}{f} (ux + vy)} du dv$$

(2)
where $P$ is the pupil function, $(u, v)$ are the pupil plane coordinates, $K$ is a scaling factor, $z$ is the image distance which is the sum of focal length $f$ and defocus, $k = 2\pi\lambda$ is the wave number and $\lambda$ is the wavelength of light.

The integral in Eq. (2) is the Fourier transform of the product of pupil function and an exponential factor, $E = \exp\left[\frac{k}{2} \left(\frac{1}{z - f} \right) \left(u^2 + v^2\right)\right]$. This factor is unity when $z$ is equal to the focal length. For defocused locations this factor accounts for the broadening effect of the point spread function (PSF). This is the primary reason for the decorrelation of the speckle patterns obtained at the defocus locations of the object. The energy as well as the phase are spilled into the adjacent pixels.

The object function $U_0$ of the rough object can be expressed as

$$U_0(\xi, \eta) = a(\xi, \eta) \exp\left[i\phi(\xi, \eta)\right]$$

To perform a simulation the amplitude, $a$, and the phase, $\phi$, are generated randomly at every point of the pupil plane with values in the interval $[0, 1]$ and $[0, 2\pi)$, respectively. To simulate a real situation the following values are chosen: $f = 250$ mm, $F/\# = 5$, and $\lambda = 532$ nm.

To understand the effect of transverse super-resolution, we compare the unobstructed pupil to continuous amplitude filters [11] that generate relative spot sizes $G = 90, 85$ and $81\%$. The details of the design and properties of these continuous amplitude filters are given in Refs. 11 and 12. The amplitude function is represented as

$$P(p) = \sum_{n=0}^{\infty} b_{2n} p^{2n}$$

where $p = (u^2 + v^2)^{1/2}$ is the radial distance in the pupil plane, $b_{2n}$'s are the coefficients of the even terms and $k = 6$. The coefficients $b_{2n}$'s for amplitude filters are given in Table 1. For the unobstructed pupil function, we consider $P(p) = 1$.

| $G$  | $b_0$ | $b_2$ | $b_4$ | $b_6$ | $b_8$ | $b_{10}$ | $b_{12}$ |
|------|-------|-------|-------|-------|-------|---------|---------|
| 100% | -1.00 | -     |       |       | -     | -       | -       |
| 90%  | 0.40  | -0.84 | 12.12 | -28.28| 18.87 | 7.04    | -8.30   |
| 85%  | 0.32  | -0.35 | 3.37  | -4.84 | 3.16  | 2.53    | -3.34   |
| 81%  | 0.57  | -4.30 | 10.00 | -0.95 | -3.12 | -7.11   | 5.92    |

Initially, Eq. (2) is evaluated using two dimensional FFT of the product of pupil function $P$ and exponential function $E$ at defocus positions. The lateral resolution in the pupil plane is adjusted to obtain the central spot of the PSF to fit into 4 X 4 pixels of the image. In order to reduce computation time, only the relevant central part of the FFT of the product of $P$ and $E$ is taken to calculate $U_i$ according to Eq. (1). This part also includes the side lobes of the PSF which extends over several pixels.

Before comparing the speckle correlations, it is interesting to study the axial behavior of the PSF at various relative spot size values. The amplitude distribution $U$ around the focal region of the optical system can be written as [8]

$$U(\zeta) = 2 \int_0^1 P(p) \exp(\imath n\zeta^2 / 2) p dp$$

where $\zeta = 4kz \sin^2(\alpha / 2)$ is the axial optical coordinate, and $\sin \alpha$ is the numerical aperture. Figure 1 shows the PSFs along the axial direction for the four pupil functions given in Table 1. The gradual increase in the intensity of the PSFs of the optical system with pupil

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filters tailored for $G = 90\%$, 85\% and 81\%, respectively at the first zero of the PSF of the unobstructed pupil is the main reason that has led us to explore the implication of superresolution in the speckle field.

Now Eq. (1) is evaluated at defocus positions in steps of 0.01 mm. The intensity correlation coefficient of the speckle pattern at image focus position, $I_{fi}$ and the speckle patterns at successive defocus positions, $I_{\text{def}}$ is calculated from

$$r = \frac{\sum_{m} \sum_{n} (I_f(m, n) - \langle I_f \rangle) (I_{\text{def}}(m, n) - \langle I_{\text{def}} \rangle)}{\left( \sum_{m} \sum_{n} (I_f(m, n) - \langle I_f \rangle)^2 \right)^{1/2} \left( \sum_{m} \sum_{n} (I_{\text{def}}(m, n) - \langle I_{\text{def}} \rangle)^2 \right)^{1/2}}$$

(6)

![Fig. 1. PSFs along axial direction for various pupil functions.](image1.png)

![Fig. 2. Variation of speckle intensity correlation coefficient with defocus for various pupil functions.](image2.png)
where $I_f(m,n)$ and $I_{df}(m,n)$ are the intensities of the speckle images at point $(m,n)$ at focus and defocus positions, and $<>$ represents the average intensity of the pixels under consideration, which in our case corresponds to 200 X 200 pixels. The speckle image generated was 800 X 600 pixel size. The $r$ value depends on the number of pixels under consideration and their location. We have chosen the central 200 X 200 pixels around the optical axis. Figure 2 represents the variation of the correlation coefficient, $r$, of the speckle images at focus and defocus locations for the unobstructed pupil and amplitude pupil filters.

The correlation coefficient for the unobstructed pupil drops to zero at about 0.11 mm which is in good agreement with the speckle correlation distance $8\lambda (L/D)^2 = 106$ $\mu$m. Further and in line with our discussion relative to Fig. 1, we also note that the correlation distance increases with the decrease in the $G$ value. For $G = 81\%$ the correlation distance doubles as compared to the unobstructed pupil. We term this speckle elongated in the axial direction as 'Superspeckle'. To visualize the increase in longitudinal speckle correlation length, we consider DSPI fringes along the axis at various defocus positions. First, an interferogram is obtained by simulating the interference of the speckle field at focus with a plane wave making an angle with respect to the image plane. A second interferogram is simulated with the speckle field at defocus position and a plane wave propagating along the optical axis. The pixel by pixel intensity difference between these two interferograms produces DSPI correlation fringes. Figure 3 shows the DSPI fringes for the unobstructed pupil obtained at image plane positions of 0 mm, 0.04 mm, 0.08 mm and 0.10 mm from the focus, respectively, along with their respective line intensity profiles. The line intensity profile is taken along the central rows by averaging 30 adjacent column pixels. The fringe modulation drops as the defocus increases having a minimum around 0.1 mm.

![Fig 3.](image)
Fig. 4. (a), (b), (c), (d), (e) and (f) are the DSPI fringes obtained using an amplitude filter with \( G = 81\% \) at locations 0 mm, 0.04 mm, 0.08 mm, 0.12 mm, 0.16 mm and 0.18 mm from focus, respectively. (g-l) are line intensity profiles of (a-f), respectively.

Similarly, Fig. 4 shows the DSPI fringes and their respective line intensity profiles obtained using the amplitude filter with \( G = 81\% \). The modulation drops to almost the same level as in Fig. 3d at around 0.18 mm defocus distance.

3. Experiment

We used an LCD Spatial Light Modulator (Sony Corp., having 832 x 624 pixels with pixel pitch of 32 \( \mu \)m and 1.3 inch diagonal) to implement the amplitude pupil filter. The characterization is done using two linear polarizers sandwiching the LCD. The procedure for characterization is detailed in Refs. 13-16. For speckle correlation experiments we do not need...
amplitude-only behavior, because any addition of phase values to the random phases of the speckle field cancels out during subtraction correlation.

The setup for speckle correlation measurement is shown in Fig. 5. The laser beam ($\lambda = 532 \text{ nm}$) is expanded and collimated to illuminate a rough object. A mirror and a beam splitter are used for normal incidence on the object. A doublet (L2) with 25.4 mm diameter and 400 mm focal length is used to image the speckled wavefront scattered from the rough object at unit magnification. Therefore the object and image plane distances are equal to 800 mm. The LCD is placed close to L2 in order to minimize the distance to the pupil plane. We have chosen a large focal length for two reasons. First, a large ($L/D$) value means low angle light incidence on the LCD which reduces the error in amplitude modulation from the parallel beam behavior. Second, a linear stage with a manually operated micrometer can be used to move the rough object in the axial direction since the average length of the speckle is about 6.8 mm given by $8\lambda (L/D)^2$. A CCD-camera (IMAC™) with 11 $\mu$m pitch is used for recording the speckle patterns. The average size of the speckle in transverse direction covers 4x4 pixels of the CCD. The amplitude filter for $G = 81\%$ shown in Figs. 1 and 2 is programmed onto the LCD. The speckle pattern correlations for defocus positions are obtained using Eq. (5). Figure 6 shows the correlation coefficients obtained from 200x200 pixels.

![Fig. 5. Setup for measuring speckle correlations.](image)

![Fig. 6. Variation of speckle correlation with defocus using setup shown in Fig. 5](image)
The speckle images are grabbed for each 0.5 mm step along the longitudinal axis. Without amplitude filter, i.e., the unobstructed pupil case in Fig. 6, the correlation drops around 6 mm which agrees with the theoretical value. With amplitude filter, the correction extended more than twice the distance and starts falling around 14 mm. This behavior confirms our simulation.

4. Conclusion

We have applied superresolution filters to produce 'superspeckles' to increase the correlation distance along axial direction. An experiment verifies the simulation. Therefore, we expect to increase the measurement range of DSPI to more than twice the range given by the optical system without superresolution pupil filter, with an increase in the spatial resolution of the image.