FUNDAMENTAL AND PRACTICAL PROBLEMS
OF QKD SECURITY-THE ACTUAL AND THE
PERCEIVED SITUATION*

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Abstract

It is widely believed that quantum key distribution (QKD) has been proved unconditionally secure for realistic models applicable to various current experimental schemes. Here we summarize briefly why this is not the case, from both the viewpoints of fundamental quantitative security and applicable models of security analysis, with some morals drawn.

1 Introduction

After the appearance of papers last year on fundamental QKD security [1] and the complete breach of concrete QKD systems [2-3], claims have persistently been made [4] that QKD is already proved unconditionally secure in principle in various models. However, ref [2] and its extensions [3,5] highlight in a forceful manner the precarious situation of such widespread claims, especially on concrete experimental systems. Imagine the consequence if the Norway group kept their detector “blinding attacks” secret and makes them available to selected parties after a QKD systems has been deployed upon convincing the users its unconditional or whatever imposing security terminology that has been employed. The fact of the matter is that the security proofs of the models, assuming the deductions are totally valid (but they are actually not), contain general and specific assumptions that are simply not satisfied in practice. Moreover, the security criteria themselves used in the proofs do not guarantee proper security when satisfied. This paper tries to outline the underlying reasons, and indicates some ways to deal with the situation.
2 Security is not merely a matter of definition

Thus far a single-number criterion on a single quantity has been used as the security criterion in QKD security proofs, from mutual information to trace distance. However, in accord with detection theory an attacker Eve would obtain from the measurement result on her probe a whole probability distribution \( \{q_j\} \), \( j \in 1, \ldots, M \) for the \( M = 2^m \) possible values of an \( m \)-bit data \( X \) from which an \( n \)-bit final key \( K \) is drawn after the use of an error correction code (ECC) and a privacy amplification code (PAC). Since PAC is a known hash function, this distribution \( \{q_j\} X \) leads to the distribution \( \{p_i\} \) on \( K \), \( i \in \{1, \ldots, N\} \). A bound on the mutual information or any single-number criterion on \( X \) or \( K \) merely expresses a constrain on \( \{q_j\} \) or \( \{p_j\} \).

Since one is using \( K \) as if it is uniformly distributed to Eve, i.e., with probability \( \frac{1}{N} \) for the \( N \) possible key values, one must bound the difference between the probability \( 2^{\left|\tilde{K}\right|} \) of a uniformly distributed subset of size \( \left|\tilde{K}\right| \) and Eve’s optimum probability \( p_1^E(\tilde{K}) \) of estimating \( \tilde{K} \) correctly in any attack to be below a prescribed security level,

\[
|p_1^E(\tilde{K}) - 2^{-|\tilde{K}|}| \leq \epsilon(\tilde{K}) \quad \text{for each} \quad \tilde{K} \subseteq K
\]

When \( \epsilon(\tilde{K}) \) can be made sufficiently small, condition (1) gives \( K \) the information-theoretic (IT) “security” required for meaningful “unconditional security”. Thus, good security demands semantic security that may not be obtainable quantitatively from another criterion, especially a single-number criterion that is not bounded tightly enough. Security is a quantitative issue. It is also not a mere mathematical issue for which one can adopt whatever mathematical definition that seems intuitively suitable. In particular, it must be
expressed by Eve’s success probabilities of correctly estimating various properties of $K$ which can all be derived from (1).

Eve can try to estimate $K$ from just its generation process, the resulting security of $K$ is called “raw security” to be distinguished from its “composition security” when $K$ is actually used. General composition security is a complicated matter and we will just restrict our attention to the case when $K$ is used for encryption, say in the one-time pad format often suggested. In such situation part of $K$ may be revealed to Eve in a known-plaintext attack (KPA), which may help her find the rest of $K$ and thus find the rest of the data unknown to Eve at the beginning, which is encrypted by $K$ to yield the ciphertext. Security against such leakage of $K$ will be called “KPA security”.

It is important to note that proof of general KPA security is necessary for any claim of unconditional security on $K$ to be used for encryption. This is because the use of conventional symmetric key ciphers for key expansion also gives raw IT security, and thus they, not RSA, are the appropriate conventional ciphers one should compare QKD to. This is a fair comparison because the use of a shared secret key for message authentication during key generation is necessary, though not included thus far in the security analysis of any QKD protocol. (KCQ protocols [6] use a shared secret key explicitly.) In addition, the raw security of such conventional ciphers is far better than that of concrete QKD systems that have been studied experimentally or theoretically. The superiority of concrete QKD must lie in its KPA security, which is the usual security concern because the shared secret key is typically totally hidden when the data $X$ is uniform to Eve.
3 Problems of the mutual information criterion

Eve’s accessible information on $K$ from her attack is the most commonly used security criterion, so far the only one used in all experimental schemes. It is Eve’s mutual information $I_E$ with respect to $K$ under optimal measurement on her probe. Information or entropy expresses a constraint on Eve’s estimate on the whole distribution $\{p_j\}$ she may get from the measurement result on her probe. It has been repeatedly pointed out [7,8,1] that there are distributions consistent with a given $I_E$ such that her maximum probability $p_1$ of estimating the whole key $K$ correctly is given by

$$p_1 \sim \frac{I_E}{n} \equiv 2^{-l}$$

(2)

Thus, unless $l \sim n$, the raw security of $K$ so guaranteed may be quite inferior to a uniform key. The other subsets $\tilde{K}$ of $K$ suffer similarly. The practical values of $I_E$ obtained in experimental schemes indeed gives very large $p_1$ in this sense [1,8].

Under KPA, knowing some bits of $K$ does not render the rest of $K$ more insecure if $E$ has no quantum memory [1]. If Eve does have quantum memory, possible locking information would render $K$ insecure [9] or even very insecure [10]. In fact, the latter can be understood from (2) as follows. The bits on $K$ gained in a KPA could reduce the exponent of $p_1$ in (2). Indeed, it only takes

$$l' = l + \log n$$

(3)

number of bits to change $p_1$ to the value 1 when Eve measures on her probe with this added information on $K$.  

5
The variational distance
\[ \delta_E \equiv \delta(P, U) = \frac{1}{2} \sum_i |p_i - U| \] (4)

between Eve’s probability distribution \( \{p_i\} \) on \( K \) averaged over \( K \) and the uniform distribution \( U \) of \( n \) bits has quantitative behavior similar to \( I_E \). In particular, when \( p_1 \) is large compared to \( \frac{1}{N}, \delta_E \) could give a \( p_1 \) as big as the case of \( \delta_E = \frac{I_E}{N} \), i.e. the value of (2). Thus, the same problem occurs under KPA as in the case of an \( I_E \) criterion above. When \( \delta_E \leq \varepsilon = 2^{-l} \) with \( l \sim n \), good security close to \( U \) may be obtained.

4 Problems of the trace distance criterion

The trace distance quantum criterion
\[ \frac{1}{2} \| \rho_1 - \rho_2 \|_1 \leq \varepsilon \] (5)

between two density operators \( \rho_1 \) and \( \rho_2 \) says that the variational distance between the two distributions \( P \) and \( Q \) obtained in a measurement as derived from \( \rho_1 \) and \( \rho_2 \) satisfies \( \delta(P, Q) \leq \varepsilon \). Let \( \rho_E^k \) be the state of Eve’s probe when the actual \( K \) has value \( k \). Then (5) says, with \( \rho_1 = \rho_E^k \) and \( \rho_2 = \rho_U \) the uniform mixed state with rank \( N \), that for any measurement Eve may make on her probe one has \( \delta_E \leq \varepsilon \). The problem of such a security criterion is indicated through \( \delta_E \) above.

With \( \rho_E = E_k[\rho_E^k] \), the following trace distance
\[ d = \frac{1}{2} \| \rho_{KE} - \rho_U \otimes \rho_E \|_1 \] (6)

for a joint state \( \rho_{KE} \), is used [11] with the interpretation that when \( d = \varepsilon \), it means \( K \) equals \( U \) with probability \( 1 - \varepsilon \) to Eve and the value \( k \) also becomes independent of \( \rho_E^k \). This implies
“universal composability” including security against KPA. In ref [14] it has been analyzed in detail why this interpretation cannot be true with any probability. We can describe the reason simply as follows.

The main error arises from conclusion in ref [11] that \( \delta(P, Q) = \epsilon \) implies \( P \) and \( Q \) are the same distribution with probability \( 1 - \epsilon \). This conclusion was derived from the existence of a joint distribution \( D \) that gives \( P \) and \( Q \) as marginal and yields the above interpretation. Why would this \( D \) arise in the cryptosystem? In fact, even when a joint distribution different from the product form \( PQ \) is in force, why would it be this particular (which is actually the optimal) one for the interpretation to obtain. In reality, one is simply comparing two distributions and the joint distribution should be \( PQ \). It is indeed clear directly from the definition of \( \delta(P, Q) \) that \( P \) and \( Q \) must differ when \( \delta(P, Q) = \epsilon > 0 \).

Sometimes the term “failure probability” is used [12,13] without explicitly saying what that means. In [14] it is shown that \( \epsilon \) does not itself have a probability interpretation. Thus, it is not any “failure probability”. A related source of this error, which was not discussed in ref [1], is a misinterpretation of a notion of “\( \epsilon \)-indistinguishable” measure. It is concluded [12] from (5) that \( \rho_1 \) and \( \rho_2 \) is “\( \epsilon \)-indistinguishable” and thus the protocol has “failure probability” \( \geq 1 - \epsilon \). The problem of “\( \epsilon \)-indistinguishable” for KPA security guarantee is a quantitative one similar to (2)-(3) above. A detailed explanation of the whole situation is given in [15].
5 True key generation rate and limitation of privacy amplification

It is important to observe that Eve’s maximum probability \( q_1 \) on the data \( X \) is equal to her \( p_1 \) on \( K \), and that \( p_1 \) cannot be improved by further PA. This is because a known transformation from ECC+PAC would just bring the most likely value of the data or key to a final value of \( K \) with the same probability. On the other hand, it is clear from (2) that the rate of secure key generation is limited to \( \frac{l}{n} \). Thus, unconditionally secure key generation rate cannot be given by what has been asserted in the literature, but is determined via the \( p_1 \) exponent. Moreover, this rate is determined by \( q_1 \) and we just saw it cannot be improved via PA.

In finite protocols – no real protocol operates in the asymptotic limit \( n \to \infty \) – it makes little sense to say a quantity grows exponential in \( n \) without some estimate of the actual convergence rate, because any value can be written as exponential in \( n \). It is more accurate to just say that \( l \) secure bits are generated in the round. For concrete protocols \( l \) is very small thus far and may not even cover the message authentication bits used in any normal IT-secure authentication scheme.

6 True security and asymptotic proof

Note that even with PA to extract semantically secure bits from the \( p_1 \) exponent, fundamental security has not been guaranteed. The use of Markov Inequality to convert an average guarantee to a probable individual guarantee would just reduce the quantitative value that
has been achieved. Much more significantly, Eve could launch an optimal quantum attack on specific subset $\tilde{K}$ of $K$ which, because of quantum mechanics, can be superior to what she may obtain by attacking the whole $K$ and must be bounded in a security proof.

In this connection, it is important to point out that the asymptotic limit that so many security proofs are based upon both overestimate what the users can achieve and what Eve can achieve in a finite situation. This is clear information theoretically, because all the “capacity” like statements involving mutual information are well known to be limiting capabilities. For example, the actual “random coding exponent” or channel reliability function [16] for finite $n$, similar to the $p_1$, exponent, is what controls finite system performance, not the capacities. This applies to both the users and Eve, and is overlooked also in the classical literature on key generation. Basically, cryptographers should be working with detection theory, not information theory, for ascertaining performance by any party. Probability has operational interpretation and is what matters IT wise (but IT in the broad sense), not any other theoretical quantity like mutual information that needs to be translated back to probability as done in ref [1,7].

This last point is very important. It shows the possibility of secure key generation is not determined by any capacity statement. Indeed, in KCQ (keyed communication in quantum noise) [7] one does not allow coding or indefinitely large $n$ for Eve other than her optimum decision on a finite-n system. Quantum information locking may help significantly for KCQ but it is not necessary.

Some details and further elaboration on sections II-VI can be found in [14,15].
Figure 1: Schematic way to eliminate or reduce the effect of loss by user: loss is alleviated or eliminated with favorable pre-detection outcome.

7 Grave effect of loss on security

Real optical systems have significant loss. If the transmission loss is small one can treat deleted bits as random errors. Security claim was often made with loss taken into account just on the throughput via post-detection selection of the detected events. That this is clearly not a valid inference could be seen from the situation of B92, for which security is totally breached in an intercept-resend attack when the loss is above a certain threshold determined by the two signal states, or in any coherent-state BB84 protocol [17].

Generally, the users may try to reduce loss by pre-detection as indicated in Fig. 1, with success probability itself limited by the loss. Examples include QND measurement and “herald qubit amplifier”. However, Eve also has a similar attack approach, the “probabilistic
Figure 2: Schematic way to take advantage of loss by attacker: a more favorable input state $\rho_f$ from Eve’s viewpoint is sought with possible quantum signal detection (PRS attack).
re-send (PRS) attack” indicated in Fig. 2. Sufficient loss would allow her to cover the deleted
bits in principle and often in practice also. PRS attacks include probabilistic approximate
cloning which is itself a generalization of the attack in ref [17] that is equivalent to proba-
bilistic exact cloning. Note that the possibility of bit deletion from loss violates the usual
information-disturbance tradeoff that underlines QKD security, in that information can be
gained by Eve without causing any relevant disturbance.

While PRS attacks can be covered in a sufficiently general formulation on Eve’s probe, it
is not automatically covered by merely bringing up the possible use of post-detection selection
[18], QND measurement or squashing [19-20], or heralded qubit amplifier [21]. Indeed, it
does not seem a security proof covering all possible PRS attacks in significant loss has ever
appeared. The analysis of ref [22] includes detector inefficiency but not transmission loss.
Absorbing transmission loss in the detector efficiency with \( \rho_i \) replaced by a state without
loss is just the same as post-detection selection, in addition to yielding a possibly very
small detector efficiency. Note that there is no complete security proof in loss even just
under individual attack. This grave consequence of loss on security has been pointed out
previously in [8], and further elaborated in [23].

8 Problems of modeling versus side channel

There are two kinds of mathematical modeling problems in QKD security analysis of concrete
systems:

(A) whether the model includes typical general features of a real cryptosystem;

(B) whether the operative assumptions of the security analysis are satisfied in the real
system it is applied to.

As an example of (A), the quantum signal state space in any QKD implementation is never a qubit but an infinite-dimensional boson mode. That a different dimension from two may breach security is clearly brought out in a specific example [24]. The situation of loss is discussed above. Thus, all qubit-based security proof is not directly applicable to a real system, but such security claim was often made on the basis of qubit proofs.

There are many examples of (B), such as the use of threshold detector or Poisson source model for lasers without phase randomization. Much more significant are the time-shift attack [25] based on detector efficiency mismatch [26] and the blinding attacks [2-3,5]. Detector efficiency mismatch has been dealt with in ref [18]. What is unsettling about the time-shift attack is that the detailed detection mechanism in the detector can be exploited to lead to a huge mismatch. The blinding attacks (based on detector controllability by Eve more essentially than “faked state”) is even more unsettling, because it does not lead to any common detector imperfection representation and relies on the internal detector electronic behavior. While the particular possibility of detector blinding can be added in a security analysis [27], it is not clear how one would know all the relevant internal electronics behavior have been included in any particular model. Some discussion on similar but more general modeling question can be found in ref [28]. Note that Fig. 2 can be used to represent timing and blinding attacks, when the input state itself is already “faked” in a specific way by Eve and the detector electronic behavior and total system loss may together allow an attack to succeed.
In this connection, it may be pointed out that this is not a “side channel” issue as it is the case with the RSA timing attack. Side channels can be closed once and forever, but the detector in a QKD system is an integral part of the receiver one must have, part of the ”main channel”. For example, if a detector leaks radiation of different characteristics depending on the incoming state, it can be sealed and thus the leak is a side channel. But the detection mechanism is not a side channel. Another point is that a side channel would not affect the original cryptosystem representation, surely the case also for the RSA timing attack. However, the system model has to be extended to include the time-shift attack and blinding attack [27]. To what extent must one model the internal behavior of a detector, or any system component, so that the resulting security analysis captures all the relevant features of the cryptosystem, instead of getting new surprises from time to time? If this question is not settled there will be no security proof for any concrete QKD system even just in principle, whatever else one may have achieved. The detector representation problem goes beyond (B) and squarely to (A) above, an issue of completeness of the cryptosystem model.

There are other systems that are not subject to such detector based attacks, including continuous variable QKD which is, however, not yet proved secure according to the standard view in the literature [13]. The KCQ approach [6] is also immune to such attacks, especially Y00 in any of the formats (PSK, ISK, QAM) that have been studied, because there is essentially no deleted bit. However, general IT security has not been established for any KCQ protocol.
9 Outlook

Security is a serious matter and cannot be established experimentally. We see in the above
that, just in principle, fundamental quantitative security has not been properly addressed
in QKD security analysis and the effect of loss has not been properly accounted for. The
widespread perception of proven QKD security is based on several omission or errors of
reasoning:

(i) No proven security against known-plaintext attack when the generated key $K$ is used
    for encryption;

(ii) Use of single-number constraint on the attacker’s probability distribution on $K$ when
    the number is not or cannot be bounded tightly enough;

(iii) Not including all possible attacks in the presence of significant transmission loss;

(iv) Not including relevant device characteristics.

The situation is summarized in the following Table 1:

There are two ways to deal with these problems. The first is to limit one’s claim, for
example to known-plaintext attacks with no quantum memory. One can, say, wait an hour
before using the generated key. One can ignore joint attacks that require entanglement across
modes either in the probe or in the measurement. The resulting KPA security appears prov-
able for at least KCQ protocols and would still represents major progress beyond what can be
obtained with standard ciphers. In any case it is better to avoid misleading terminology like
“unconditional”, “no signaling”, and “device independent”. Generally, a more careful and
critical attitude in making security claim would be appropriate. The second way is to look
|                              | Perceived                                                                 | Real                                                                 |
|------------------------------|---------------------------------------------------------------------------|----------------------------------------------------------------------|
| raw security of $K$ during key generation | criterion $d \leq 2^{-l}$ implies $K$ is perfect with probability $\geq 1 - \epsilon$ | probability Eve gets a subset $\tilde{K} \subseteq K$ can be as big as $2^{-l}$ |
| composition security of $K$ against known-plaintext attack when used in encryption | criterion $d \leq 2^{-l}$ implies $K$ is perfect with probability $\geq 1 - \epsilon$ | $d$ is not the proper criterion                                      |
| privacy amplification        | given Eve’s entropy can make $d$ small and hence improve security       | cannot improve $p_1$, Eve’s maximum probability of getting the whole $K$ |
| key generation rate          | $H(A|B) - H(A|E)$                                                         | exponent $r$ of $p_1 \sim 2^{-rn}$                                  |
| determination of security    | by analysis of entropy hierarchy with respect to criterion $d$           | by bounding Eve’s success probabilities                              |
| effect of transmission loss on security | reduction of key rate but not security                                   | many possible attacks by Eve not accounted for                       |
| modeling of cryptosystem photon detector | a side channel issue                                                     | part of the completeness issue in original system representation     |

Table 1: QKD Security Situation

for new approaches or major modification of existing ones. In particular, we need a general proof that device internal electronics cannot lead to security loopholes in the protocol.
Note added for v.4: For new developments on the topics of sections II–IV, see [29]-[30].

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References

[1] H. P. Yuen, Phys. Rev. A 82, 062304 (2010).

[2] L. Lydersen, C. Weichers, C. Wittman, D. Elser, J. Skaar, and V. Makarov, Nat. Photonics 4, 686 (2010).

[3] J. Gerhardt, Q. Liu, J. Skaar, A. Lamas-Linares, C. Kurtsiefer, and V. Makarov, Nature Communications, 112349, June 2011.

[4] In this paper, examples of such alleged claims in the literature would not be quoted. Also, by QKD we refer to protocols involving intrusion level estimation which excludes KCQ protocols [6].

[5] L. Lydersen, N. Jain, C. Wittman, O. Maroy, J. Skaar, C. Marquardt, V. Makarov, and G. Leuchs, quant-ph 1106.2119 (2011), to appear in Phys. Rev. A.

[6] H. P. Yuen, IEEE J. Sel. Top. Quantum Electron. 15, 1630 (2009).

[7] H. P. Yuen, quant-ph. 0311061, 2003

[8] H. P. Yuen, in Proceedings of the QCMC, O. Hirota, J. H. Shapiro, and M. Sasaki, Eds, NICT Press, pp.163-171 (2006).
[9] R. Konig, R. Renner, A. Bariska, and U. Maurer, Phys. Rev. Lett. 98, 140502 (2007).

[10] F. Dupuis, J. Florjanczyk, P. Hayden, and D. Leung, quant-ph 1011.1612 (2010).

[11] R. Renner and R. Konig, Second Theory of Cryptography Conference (TCC), Springer, New York, Lecture Notes in Computer Science, Vol. 3378, pp.407-425 (2005).

[12] J. Muller-Quade and R. Renner, New J. Phys. 11, 085006 (2009).

[13] V. Scarani, H. Bechmann-Pasquinucci, N. J. Cerf, M. Pusek, N. Lutkenhaus, and M. Peev, Rev. Mod. Phys. 81, 1301 (2009).

[14] H. P. Yuen, arXiv: 1109.2675.v.3

[15] H. P. Yuen, arXiv: 1109.1051.v.3

[16] R. G. Gallager, Information Theory and Reliable Communication, New York, Wiley (1968).

[17] H. P. Yuen, Quantum Semiclassic. Opt. 8, 939 (1996).

[18] D. Gottesman, H.-K. Lo, N. Lutkenhaus, and J. Preskill. Quantum Inf. Comput. 4, 325 (2004).

[19] N. J. Beaudry, T. Moroder, and N. Lutkenhaus, Phys. Rev. Lett. 101, 093601 (2008).

[20] T. Tsurumaru and K. Tamaki, Phys. Rev. A 78, 032302 (2008).

[21] N. Gisin, S. Pironio, and N. Sangouard, Phys. Rev. Lett. 105, 070501 (2010).

[22] M. Hayashi, Phys. Rev. A 76, 012329 (2007).
[23] H. P. Yuen, quant-ph/1109.1049.

[24] A. Acin, N. Gisin, and L. Masanes, Phys. Rev. Lett. 97, 120405 (2006).

[25] B. Qi, C. Fung, H.-K. Lo, and X. Ma, Quantum Inf. Comput. 7, 73 (2007).

[26] V. Makarov, A. Anisimov, and J. Skaar, Phys. Rev. A 74, 022313 (2006).

[27] O. Maroy, L. Lydersen, and J. Skaar, Phys. Rev. A 82, 032337 (2010).

[28] H. P. Yuen, quant-ph 0808.2040 (2008).

[29] H. P. Yuen, arXiv: 1205.3820.v.2.

[30] H. P. Yuen, arXiv: 1205.0565.