Effect of critical speed in machining of the main shaft of cone crushers on accuracy of treated surfaces

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Abstract: One of the industry development factors is the advancement of design and technological solutions of production applications. Mining industry makes heavy use of cone crushers, with the main shaft being one of the main components under serious loads, thus requiring repair or replacement. The paper suggests the method to determine the critical speed of the main shaft of cone crushers in machining and the effect, which the type of fixing on metal-cutting machines has on the accuracy of its surfaces.

1. Introduction
Ore preparation in mining industries predetermines considerably the final technological and economic indicators of an enterprise. Cone crushers ensure the most efficient crushing of hard and abrasive ores, which include copper and iron ores, and gold-containing quartzites [1-3].

Cone crushers are widely used in mining enterprises for fine, medium and primary crushing (Figure 1).

![Cone Crushe Diagram](image)

**Figure 1.** Cone crusher diagram: 1 - base; 2 - main shaft; 3 - mantle; 4 - drive shaft.

Main shaft 1 of any type of the cone crusher is a large-sized turned part with a solid loading, located in the center of the shaft. Two main processing methods are used to produce such parts with a given accuracy – turning and circular grinding [4-9].

Object of the paper – is to offer the method to determine the critical speed of the main shaft, designed for cone crushers, during its cutting in turning and circular machines.

2. Main Part
The specificity of the process turning or circular-grinding operation is that the main shaft, based in the chuck jaws and the center of the tailstock of the machine (Figure 2) [10, 11] performs not only rotational movement during machining. The shaft axis bends in rotary mode and performs a precessional motion. When the angular velocity of rotation reaches a certain value, called the critical...
speed, shaft deflections become quite substantial. Hence, it is recommended that the operating speed fall within the range of (stiff shaft) (flexible shaft). Not counting the gyroscopic effect, the critical speed of the shaft is equal to the natural frequency of transverse (bending) vibrations of the system [12-18].

**Figure 2.** View of the main shaft mounting on the turning machine: 1 – chuck; 2 – main shaft; 3 – movable tailstock center.

Let us review the design model of the main shaft with the lumped mass M, which is in the center of span (Figure 3).

**Figure 3.** Main shaft design model: 1 – simple; 2 – complex; 3 – precise.

To define the natural frequency of system vibration, let us consider several design model options:
1. Simple, with no account of the shaft mass.
2. Complex, considering the shaft mass as lumped in the center (m – mass of the length unit).
3. Precise, accounting for shaft mass, distributed along its length.

Design models 1 and 2 are the one-degree-of-freedom systems. [19-27]. Their natural frequencies and critical speeds consequently equal to, $s^{1}$:

$$ p_1 = \omega_1 = \sqrt{\frac{C_f}{M}}, $$

(1)

$$ p_2 = \omega_2 = \sqrt{\frac{C_f}{M + m \cdot l}}, $$

(2)

where:
- $M$ – lumped mass, kg;
- $m$ – mass of the shaft length unit, kg/m;
- $l$ – shaft length, m;
- $C_f$ – shaft flexural rigidity. The shaft inverse compliance, which is equal to the displacement of the shaft centre by the unit force, is determined by Mohr integral, calculated by Vereshchagin method [2].
In the following, we make the bending-moment diagram from the external force $F$ (Figure 3, 1), and the bending-moment diagram from the unit force, in the main statically determinate system of force [28-36] method (Figure 3, 2).

![Figure 4. Bending-moment diagrams: a – from external force $F$; b – from unit force $F$.](image)

Vertical movement of point $K$ (see Figure 4) equals to Mohr integral, calculated by Vereshchagin method, m:

$$
\gamma_V = \frac{6}{E \cdot J} \int \frac{M F \cdot M}{y \cdot dz} = \frac{1}{E \cdot J} \left[ 2 \cdot \frac{3}{16} F \cdot l \cdot l - \frac{5}{32} F \cdot l \cdot l - \frac{1}{12} \right] \frac{l}{768} = \frac{F \cdot l^3}{768 \cdot E \cdot J} .
$$

External force is determined from the formula, $N$:

$$
F = C_f \cdot \gamma_V .
$$

The resulting shaft inverse compliance is, $N/m$ :

$$
C_f = \frac{768 \cdot E \cdot J}{7 \cdot l^3} ,
$$

where:

$E$ – Young's modulus of steel $2 \cdot 10^3$ MPa $= 2 \cdot 10^{11}$ N/m²

$J = \pi D^4/64$ – Shaft axial moment of inertia, m⁴;

Substituting expression (3) in formulas (1) and (2), we get the value of shaft critical speeds for the first (see Figure 3, 1) and the second design models (see Figure 3, 2) respectively:

$$
\omega_{c1} = \sqrt{\frac{768 \cdot E \cdot J}{7 \cdot l^3 \cdot M}} ,
$$

$$
\omega_{c2} = \sqrt{\frac{768 \cdot E \cdot J}{7 \cdot l^3 \cdot (M + ml)}} .
$$

Pursuant to the first design model (see Figure 3, 1), which disregards the shaft mass, formula (1) disregards the shaft mass respectively. The second design model and formula (5) account for inertia characteristics of the shaft mass, as a lumped mass. Evidently, the value of the natural frequency of the third design model (see Figure 3, 3), which considers the inertia characteristics of the mass, distributed along the shaft length, should be halfway of the interval between and, that is:

$$
\omega_{c3} = \frac{\omega_{c1} + \omega_{c2}}{2} .
$$

Validity of this statement is illustrated with the example of calculating the natural frequencies of transverse vibrations or critical speeds of the shaft.
Using the data proposed in Table 1, we calculate the values of critical speeds using the approach formulated in the article [14].

Moment of inertia and area of section of the hollow shaft are calculated from the formulas:

\[ J = \frac{\pi D^4}{64} \left( 1 - \left( \frac{d}{D} \right)^4 \right) \]

\[ A = \frac{\pi D^2}{64} \left( 1 - \left( \frac{d}{D} \right)^2 \right) \]

(7)

(8)

### Table 1. Parameter points for critical speeds calculation.

| Shaft type     | \( l \), m | \( M \), kg | \( D \), m | \( d \), m | \( J \), m\(^4\) | \( A \), m\(^2\) | \( m \), kg/m |
|----------------|-----------|------------|----------|---------|--------------|--------------|------------|
| Solid shaft    | 2         | 5          | 0.02     | -       | 0.75 \times 10^{-8} | 3 \times 10^{-4} | 2.4        |
| Hollow shaft   | 2         | 5          | 0.02     | 0.016   | 0.44 \times 10^{-8} | 1.13 \times 10^{-4} | 0.864      |

Values of solid \( \omega_c \) and hollow \( \omega_c \) shafts, calculated from the formulas (4) and (5) are specified in Table 2.

### Table 2. Critical speeds values for design models (see Figure 3).

| Shaft type     | \( \omega_{c1} \), s\(^{-1}\) | \( \omega_{c2} \), s\(^{-1}\) | \( \omega_{c3} \), s\(^{-1}\) |
|----------------|-------------------------------|-------------------------------|-------------------------------|
| Solid shaft    | \( \omega_{c1} = 255 \) | \( \omega_{c2} = 178 \) | \( \omega_{c3} \approx 216 \) |
| Hollow shaft   | \( \omega_{c1} = 192 \) | \( \omega_{c2} = 166 \) | \( \omega_{c3} \approx 179 \) |

Results of calculation using the suggested method, evidenced validity of the statement, set out in the formula (6).

### 3. Conclusions

The proposed method of calculation facilitates largely the complexity of determining the critical speed of the main shaft of the cone crusher.

The following result was obtained upon comparative analysis of calculations made in the present paper and the article [14]: the average critical speed for machining the main shaft of cone crushers, fixed in the chuck and the center of the tailstock, exceeds six times that of the part with the same dimensions, fixed in the centers. Based on these data, we conclude that in order to reach a specified surface accuracy while increasing the processing performance, it is more efficient to fix the parts, which design type is that of the main shaft of the cone crushers, in the chuck and the center of the tailstock.

### 4. Results of the study

When the carbide band is placed on a steel roll with the tension between the contacting surfaces, as a rule, a plastic unsaturated or saturated contact occurs.

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