INEQUALITIES FOR WEIGHTED SPACES WITH VARIABLE EXPONENTS

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Abstract. In this article we obtain an “off-diagonal” version of the Fefferman-Stein vector-valued maximal inequality on weighted Lebesgue spaces with variable exponents. As an application of this result and the atomic decomposition developed in [16] we prove, for certain exponents $q(\cdot) \in \mathcal{P}_{\log} (\mathbb{R}_n)$ and certain weights $\omega$, that the Riesz potential $I_\alpha$, with $0 < \alpha < n$, can be extended to a bounded operator from $H^p_\alpha (\mathbb{R}_n)$ into $L^q_\omega (\mathbb{R}_n)$, for $\frac{1}{p(\cdot)} := \frac{1}{q(\cdot)} + \frac{\alpha}{n}$.

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