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Experimental and Numerical Study of Structural Effects of Anisotropic Frost Damage on Reinforced Concrete Beams

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Abstract

Unlike plain concrete, frost damaged reinforced concrete (RC) exhibits anisotropy because of the presence of reinforcing bars. The resultant mechanical responses are influenced strongly by the loading direction. Therefore, to ascertain the mechanical behavior up to failure of RC beams subjected to freeze-thaw action and subsequent mechanical loading, anisotropic damage models of frost-damaged RC elements were assessed in this study using three-dimensional nonlinear finite element analysis (3D-NLFEA), which revealed that the anisotropic damage affected load-deflection responses and played a key role in failure modes that differed from those of an undamaged RC beam. This paper is the English translation from the authors’ previous work [Kanazawa, T., Sato, Y. and Takahashi, R., (2019). “Frost damage of reinforced concrete beams having various cross-sectional geometries.” Experimental and Numerical Study of Structural Effects of Anisotropic Frost Damage on Structural Concrete. Journal of Japan Society of Civil Engineers. Ser. E2 (Materials and Concrete Structures), 75(4), 293-307. (in Japanese)].

1. Introduction

Frost damage develops unevenly inside concrete. For that reason, the mechanical performance of existing structures can be influenced considerably by the location and size of damaged zones. Such damage is usually not envisaged at the design stage. Therefore, structural assessment based on visual inspection or using design equations might not be valid because they do not consider the damage distribution. An urgent need exists for an efficient method for the performance assessment for such structures subjected to frost damage (Melchers 2001).

As such a method, we intend to use nonlinear finite element analysis (NLFEA) to assess the structural performance of frost-damaged RC members. Earlier studies related to frost damage (Hasan et al. 2004; Penttala and Al-Neshawy 2002; Zhou and Mihashi 2008; Bishnoi and Uomoto 2008) have specifically examined material-related issues such as mechanical properties and deformational behavior of plain concrete (PL), although the structural effects of frost damage on RC have received little attention (Diao et al. 2011; Petersen et al. 2007; Hassanzadeh and Fagerlund 2006). Hassanzadeh and Fagerlund (2006) tested freeze-thaw (FT) exposed RC beams having various cross-sectional geometries. Hayashida et al. (2014) experimentally and numerically studied the mechanical behavior of RC beams with different sizes and locations of frost-damaged zones.

The present study demonstrates that effects of frost damage on load-carrying capacities and failure modes are not well simulated by two-dimensional (2D) NLFEA when the frost damage is introduced only into mechanical properties of PL, i.e., Young’s modulus and compressive strength. Although some constitutive models with damage index have been examined in several numerical studies using NLFEA (Berto et al. 2015; Gong and Maekawa 2018; Karthik et al. 2008), a simple framework to elucidate aspects of structural frost damage must be developed for use in engineering. As Hasan et al. (2004) reported, the volumetric expansion after FT tests corresponds well with the reductions in mechanical properties of PL. The expansion strain develops uniformly in all directions. Although RC expansion caused by alkali-silica reaction (ASR) develops non-uniformly (Karthik et al. 2016), little information is available for the expansive behavior of frost-damaged RC. If anisotropic mechanical behavior results from the presence of reinforcing bars, then such behavior must be modeled properly to evaluate the mechanical responses of frost-damaged RC beams.

To predict mechanical responses of RC beams under the effect of non-uniform frost damage, this study provides a numerical solution based on a new perspective of anisotropic frost damage on structural concrete. Figure 1 shows that this paper presents a study of seven RC beams including one control specimen N, which were prepared by Hayashida et al. (2014). Symbols T and C represent the beam tension and compression zones exposed to FT action. The subsequent number denotes the depth of damage from the compression or tension fiber of the beams. To study the beams’ mechanical response, first, a constitutive model for frost-damaged RC elements under uniaxial compression with mechanical anisotropy is developed based on the experimental work.
conducted by the authors (Kanazawa and Sato 2018). Thereafter, heat and moisture transfers during the FT test are calculated using finite-difference approximation. Then a distribution of temperature and relative humidity (RH) in a section of RC beams is linked to a three-dimensional (3D) distribution of expansion strains. The obtained strain distributions are assessed using the 3D-NLFEA and are used to simulate the mechanical anisotropy by the developed constitutive model. Finally, the mechanical behaviors of RC beams with variation in frost damaged zones were examined using the developed method.

2. Anisotropic constitutive model for RC elements under uniaxial compression

This section is devoted to develop the constitutive model that is a key element in the numerical simulation using 3D-NLFEA, as well as to provide experimental evidences of anisotropic frost damage.

2.1 Experimental evidence of anisotropic expansion

Thirty-nine prismatic specimens were prepared, each with a reinforcing bar embedded at the center. The FT test was conducted except for control specimens. Subsequently, all specimens were tested for uniaxial compression. Details were presented in the earlier report (Kanazawa and Sato 2018). Some important results are presented in Figs. 2, 3 and 4.

Figure 2 shows the development of expansion strain exhibited with the number of FT cycles. It is noteworthy that expansion strains of PL specimens in Fig. 2(a) develop uniformly whereas those of RC specimens in Figs. 2(b) to 2(d) exhibit a different tendency by which expansion perpendicular to reinforcing bars develops more than that parallel to reinforcing bars.
2.2 Mechanical anisotropy

Figure 3 presents correlation between reduction in compressive strength and expansion strain along the loading direction. The vertical axis of Fig. 3 was normalized with the compressive strength of undamaged PL cylinders, 14.5 MPa (Kanazawa and Sato 2018). Series 1 and 2 show that the compression force was applied respectively perpendicular and parallel to reinforcing bars. Figure 3 demonstrates that the tendency of strength reduction derived from frost-damaged RC elements differs considerably from that derived from PL cylinders (Hasan et al. 2004). Comparing the normalized strengths corresponding to an expansion strain, Series 1 caused larger strength than that of PL cylinders; however, Series 2 caused smaller strength. This result indicates that anisotropic expansion in Fig. 2 caused the mechanical anisotropy.

2.3 Comparison between the model predictions and test results

To model the mechanical anisotropy, the authors introduced anisotropic expansion strains into the stress-strain relation of Eqs. (1a) to (1c), of which the existing model of Vecchio and Collins (1986) was the backbone equation. Eq. (1b) is for application of compression force parallel to reinforcing bars and Eq. (1c) is for the case of application of compression force perpendicular to reinforcing bars:

$$\sigma'_{c} = \beta_{c} \alpha' \left\{ 2 \left( \varepsilon'_{c} / \varepsilon'_{c0} \right) + \left( \varepsilon'_{c} / \varepsilon'_{c0} \right)^{2} \right\}$$

$$\varepsilon'_{c0} = 0.002 + 0.56 \varepsilon$$

$$\beta = \frac{1}{0.85 + 0.27 \left( \varepsilon'_{c} / \varepsilon'_{c0} \right) + \alpha \left( \varepsilon'_{c} / \varepsilon'_{c0} \right)} < 1.0$$

where $\sigma'_c$ represents the principal compressive stress (N/mm²), $\varepsilon'_c$ denotes the principal compressive strain, $\varepsilon'_{c0}$ stands for compressive strain corresponding to peak stress, $\varepsilon$ denotes principal tensile strain, $\varepsilon$ denotes expansion strain, $\rho_{FT}$ expresses reinforcement ratio, and $\alpha$, $\beta$ and $\gamma$ represent coefficients related to expansion strain to consider anisotropic frost damage.

Figure 4 shows the comparison between experimental and predicted stress-strain curves by Eqs. (1a) to (1c). For comparison, three specimens having different magnitudes of expansion strain before the loading test were selected from each reinforcement ratio ($\rho_{10}$, $\rho_{13}$, and $\rho_{19}$). Figure 4 depicts that the proposed equation exhibits satisfactory agreement with the test results, irrespective of the loading direction. Specimen No. 1 and No. 2 in Fig. 4 are underestimated by Eqs. (1a) to (1c), perhaps because both specimens showed greater strength than the other specimens corresponding to an expansion strain, as encircled by dotted lines in Fig. 3.

3. Numerical simulation of anisotropic expansion strain

The resultant expansion under the same number of FT cycles is well-known to be dependent on water-to-cement ratio (W/C) of concrete and the minimum temperature inside the concrete. Therefore, the relations between FT cycles and expansion strains of Fig. 2 are not applicable to the RC beams because of the difference in FT environment as seen in Table 1. This section presents a generalized approach to predict FT-related expansions by converting the various FT environment into the number of FT cycles based on the procedure presented in Fig. 5.

3.1 Simulation procedure

In Fig. 5, the temperature distribution in a section of the RC beams is predicted first. Then the predicted temperatures and W/C of the RC beams are substituted to calculate the standardized FT cycles (Ishii et al. 1997). Finally, the relations between the number of FT cycles and the expansion strains are formulated based on Fig. 2. Because the distribution of the number of FT cycles in the section of RC beams was obtained earlier, the corresponding expansion strain is obtainable from the relations, both parallel and perpendicular to reinforcing bars.

### Table 1 Ambient temperatures and W/C of target specimens.

| RC beams (Hayashida et al. 2014) in Fig. 1 | RC elements (Kanazawa and Sato 2018) in Fig. 2 |
|--------------------------------------------|-----------------------------------------------|
| Minimum ambient temperatures (°C)         | Minimum ambient temperatures (°C)              |
| W/C                                        | W/C                                           |
| -18                                        | 0.61                                          |
| -25                                        | 0.65                                          |

$$\alpha = 0.73 \rho_{FT}^{-0.914} \varepsilon \times 10^{3}$$

$$\gamma = 1 / \left[ 1 + \ln \left( 1 + 2.23 \varepsilon \times 10^{3} \right) \right]$$

$$\alpha = 0.48 \varepsilon \times 10^{3}$$

$$\gamma = 1 / \left[ 1 + \ln \left( 1 + 0.23 \varepsilon \times 10^{3} \right) \right]$$
3.2 Calculation of heat and moisture transfer

To obtain the temperature distributions in a section of the RC beams, heat and moisture transfers are calculated using finite-difference approximation. Although simultaneous transfer of heat and moisture requires complicated mathematical models (Matsumoto et al. 2001), this study uses the simplified calculation that Eqs. (2a) and (2b) below are solved one after the other at every time step:

\[
\frac{\partial U}{\partial T} = K \left( \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right), \quad K = \frac{\lambda}{c \rho}
\]  

(2a)

\[
\frac{\partial R}{\partial T} = D \left( \frac{\partial^2 R}{\partial x^2} + \frac{\partial^2 R}{\partial y^2} \right)
\]  

(2b)

where \( U \) stands for temperature (K), \( T \) expresses time (s), \( \kappa \) represents thermal diffusivity (m\(^2\) s\(^{-1}\)), \( c \) denotes specific heat (J kg\(^{-1}\) K\(^{-1}\)), \( \rho \) signifies the concrete density (kg m\(^{-3}\)), \( R \) represents relative humidity and \( D \) denotes diffusion coefficient (m\(^2\) s\(^{-1}\)).

The dependence of the thermal conductivity of concrete on water content is considered with Eq. (3) (Campbell-Allen and Thome 1963):

\[
\lambda = 1.3348 + 3.6063 \phi
\]  

(3)

where \( \lambda \) stands for thermal conductivity (W m\(^{-1}\) K\(^{-1}\)) and \( \phi \) denotes volumetric water content (%).

The coefficients for Eqs. (2a) and (2b) were ascertained based on those for undamaged concrete, as listed in Table 2.

| No. | \( \phi \) | Series |
|-----|------|------|
| 1   | 0    | 1    |
| 2   | 0    | 2    |
| 3   | 0.1  | 1    |
| 4   | 0.13 | 1    |
| 5   | 0.19 | 2    |
| 6   | 0.13 | 1    |
| 7   | 0.19 | 2    |
| 8   | 0.19 | 1    |
| 9   | 0.19 | 2    |

Fig. 4 Comparison between test results and model predictions using Eqs. (1a) to (1c).

![Comparison of test results and model predictions](image)

Fig. 5 Procedure to obtain expansion strain distribution in a section of RC beams.
Figure 6 shows a discretized section of the specimen C10. The simultaneous equations are solved using Crank-Nicolson method with the time step of 60 seconds. The various damaged zones of the beams are controlled by the boundary elements in Fig. 6. The damaged zones were subjected to repeated water supply during thawing processes in FT cycling (Hayashida et al. 2014). The relative humidities of boundary elements for damaged zones were assumed to be 100% during the thawing processes. Additionally, the initial temperature and RH were, respectively, 20°C and 50%. Although the initial RH was not recorded during the test, it was ineffective in the resultant expansion strain; 20% of the difference in RH caused some 10 μ of the difference in the expansion strain.

The present numerical scheme is verified by comparison with the temperature recorded at the center of the RC beams prepared only for temperature measurements. Figure 7 presents both recorded and calculated temperature transitions. The calculated temperature shows good agreement with the recorded value. Therefore, we infer that the present numerical scheme is available to obtain the temperature distributions during FT cycling.

3.3 Effects on minimum temperature and W/C
To predict expansion strain under arbitrary temperature and W/C, the experimental FT cycles of Fig. 2 is converted to the number of standardized FT cycles as in Eq. (4), which was proposed empirically by Ishii et al. (1997). The procedure to obtain expansion strains is presented in Fig. 8 and Table 3, where and N1 and N2 denote the FT cycles, respectively for RC beams (Fig. 2) and RC elements (Fig. 2) obtained from Eq. (4). N’ represents the generalized FT cycles in terms of W/C and minimum temperatures. In Eq. (4), N represents the number of standardized FT cycles when 60% of relative dynamic modulus of elasticity is reached. Also, A and B denote coefficients determined by minimum temperatures and C/W, ω expresses the minimum temperatures inside concrete.

\[
N = 10^{6A + B}
\]

\[A = 0.14 \ln(C/W) + 0.04\]

\[B = 3.15 \ln(C/W) + 1.43\]

The minimum temperatures of Fig. 8(a), obtained from the finite-difference scheme are substituted to ω in Eq. (4) to obtain the number of standardized FT cycles for the RC beams, N1, as seen in Fig. 8(b). For the RC elements in Fig. 2, N2 = 1.90 is calculated by Eq. (4) from the assumption that the minimum temperature inside the concrete was approximated as -25°C during the FT test.

![Fig. 6 Finite difference mesh of C10 specimen.](image)

![Fig. 7 Experimental and numerical temperature transitions.](image)

![Fig. 8 Procedure to obtain N’ distribution of C15 specimen.](image)

| Coefficient (unit) | Value | Reference |
|--------------------|-------|-----------|
| c ( J kg\(^{-1}\) K\(^{-1}\)) | 963   | Ogata 2008 |
| ρ ( kg m\(^{-3}\)) | 2300  | -         |
| D ( m\(^2\) s\(^{-1}\)) | 20.0 × 10\(^{-8}\) | Murata 1961 |
because of its smaller cross-section: each side of 100 mm, than that of the RC beams. Note that larger $N_1$ and $N_2$ signify a lower degree of damage in accordance with the definition of standardized FT cycles (Ishii et al. 1997).

**Figure 8(b)** shows that the inner part of the section presents larger $N_1$, meaning less degree of damage. Thereafter, we can generalize the number of FT cycles of **Fig. 2** by multiplying $N_2$ to $N_1$ ratio by the 291 times of FT cycles that the RC beams sustained as shown by Eq. (5), where $N'$ as shown in **Fig. 8(c)** is used to obtain the expansion strain in the following part.

$$N' = 291 \frac{N_2}{N_1}$$

### 3.4 Relations between expansion strains and numbers of FT cycles

The relations between the number of FT cycles and expansion strains will be regressed from **Fig. 2**. To evaluate the anisotropy, the relations parallel and perpendicular to reinforcing bars are formulated separately.

**Figure 2** shows that the expansion strain increases exponentially with the number of FT cycles. However, because the exponential form might overestimate the obtained strains, the behavior is modeled as a bilinear relation. **Figure 9** shows that the slopes of the approximated relations are presented with the reinforcement ratio. When the expansion strain is less than a threshold of 500 $\mu$, as represented by blue points, the expansion is independent of the reinforcement ratio. By contrast, when the expansion strain is greater than 500 $\mu$, as shown by red points, the opposite trend with respect to the direction is observed. Expansion along reinforcing bars is restrained, whereas that perpendicular to reinforcing bars is promoted. Such differences can be attributed to the fact that the thermal properties of reinforcing bars are independent of temperature (Dahmani et al. 2007), whereas those of concrete depend on frost damage (Sicat et al. 2013), which indicates that anisotropic expansion results from large degrees of frost damage that can impair the bond between reinforcing bars and concrete. The anisotropic expansion parallel to reinforcing bars is formulated as shown in Eq. (6a) and that perpendicular to reinforcing bars formulated by Eq. (6b):

$$\varepsilon = 3.98 \times 10^{-8} N' \quad (0 \mu < \varepsilon \leq 500 \mu)$$

$$\varepsilon = (14.3 - 2.45 \rho_{FT}) \times 10^{-8} N' \quad (500 \mu < \varepsilon)$$

where $\varepsilon$ signifies the expansion strain, $\rho_{FT}$ stands for the reinforcement ratio related to anisotropic frost damage ($\%$), and $N'$ is defined as shown in Eq. (5). Examples of expansion strain obtained from the above formulations are shown in **Fig. 10**.

Unlike the ordinary reinforcement ratio $\rho$ determined by the effective cross-sectional area, values $\rho_{FT}$ in Eqs. (6a) and (6b) are determined as $A/A_{c,FT}$ in which $A_{c,FT}$ represents the area of damaged concrete. Eqs. (6a) and (6b) show that the degrees of anisotropy depends on $A_{c,FT}$. For this study, $A_{c,FT}$ was fixed as the area exposed to FT action in the test (Hayashida et al. 2014), as shown in **Table 4**.

### 3.5 Distribution of expansion strains of frost-damaged RC beams

**Figure 11** presents an example of the resultant distribution of maximum principal expansion strain of specimen...
C15 predicted by Eqs. (6a) and (6b). To obtain the principal strains, shear strains calculated from the strain compatibility after uniform expansion was assumed along the longitudinal axis of RC beams. Figures 11(a) and 11(b) respectively display the strain distributions in which anisotropy is ignored $\rho_{FT}=0\%$ in Eqs. (6a) and (6b), and those in which it is considered $\rho_{FT}=0.85\%$ in Eqs. (6a) and (6b)]. Greater expansion was predicted as presented in Fig. 11(b) considering the anisotropy. The strain distribution is the initial strain in the 3D-NLFEA. As ASR-related expansion of RC presents, confinement by reinforcing bars might induce prestress into the surrounding concrete. Nevertheless, this study neglects the stress because little information is available about the frost-damaged RC members.

4. Numerical simulation of mechanical behavior of frost-damaged RC beam using 3D-NLFEA

4.1 Fundamental information related to numerical modeling

This study used 3D-NLFEA code developed at Hokkaido University. This code has been applied to punching-shear analysis (Takahashi et al. 2005) and shear analysis of T-beams with stirrups (Pansuk and Sato 2007).

Table 5 presents the mechanical properties used for concrete and reinforcing bars in the undamaged specimen N. As presented in Table 6, this numerical method can assess anisotropic expansion and the resultant mechanical anisotropy of the RC beams. Specifically, the frost-damaged zones are treated separately as PL and RC zones. The respective zones behave following the isotropic and anisotropic initial strains. Damaged zones of specimens C5 and C10 are conceived as PL, whereas those of the others are assumed as RC as presented in Fig. 12.

Figure 13 presents cracks initiated only to RC damaged zones before the load application. The cracks are introduced based on experimental observations (Hassanzadeh and Fagerlund 2006; Hayashida et al. 2014), probably because of the anisotropic expansion. The horizontal cracks trigger the application of constitutive models of Eqs. (1) and (12) in 3D-NLFEA. Isotropic reductions for PL damaged zones in compressive strengths and Young’s moduli are considered with Eqs. (7) and (8) before cracking and with Eq. (10) after cracking. However, mechanical anisotropy for compression and shear response of RC damaged zone is considered respectively with Eq. (1) and with Eq. (12). Because

Table 4 $A_{FT}$ and $\rho_{FT}$ for RC beams with reinforcing bars in damaged zones.

| RC beams | $A_{FT}$ (cm$^2$) | $\rho_{FT}$ (%) |
|----------|------------------|-----------------|
| C15      | 300 (15 × 20)    | 0.85            |
| T10      | 200 (10 × 20)    | 1.27            |
| T15      | 300 (15 × 20)    | 0.85            |

Table 5 Mechanical properties of concrete and reinforcing bars used for RC beams.

| Concrete (specimen N) | $f'c$ (N/mm$^2$) | $Ec$ (N/mm$^2$) | $ft$ (N/mm$^2$) | $Es$ (kN/mm$^2$) | $fy$ (N/mm$^2$) |
|-----------------------|------------------|-----------------|-----------------|-----------------|-----------------|
| Concrete              | 30.4             | 28120           | 2.24            |                 |                 |
| Reinforcing bars      | 200              | 361             |                 |                 |                 |

Fig. 10 Examples of expansion strain obtained from Eq. (6a) (left) and Eq. (6b) (right).

(a) Parallel to reinforcing bars
(b) Perpendicular to reinforcing bars

Fig. 11 Distribution of maximum principal expansion strain of C15 specimen.
the reduction in tensile strengths has a minor effect on mechanical responses of RC beams, it is decreased by Eq. (9) (Noguchi 2007), based on the relative dynamic modulus of elasticities measured before the loading test (Hayashida et al. 2014). For reinforcing bars, the stress-strain relation proposed by Kato (1979) was used:

\[
f'_{c,\theta} = \exp(-0.32\varepsilon \times 10^3)
\]

(7)  

\[
E_{c,\theta} = \exp(-0.50\varepsilon \times 10^1)
\]

(8)  

\[
f_{s,\theta} = \exp\{(-0.141(100 - \text{RDME})^{0.52})
\]

(9)

where RDME represents the relative dynamic modulus of elasticity (%).

Table 7 presents the elements used in the present analysis. The reinforcing bars are modeled by truss elements with three nodes. The relative displacement between reinforcing bars and concrete are modeled by spring elements. Additionally, two non-orthogonal and fixed cracks are formed at each Gauss point. The RC beams presented in Fig. 1 are idealized by finite elements as presented in Fig. 14. Because of the symmetric geometry and loading condition, a quarter of the beam is modeled. Enforced displacement was applied up to failure on the steel plate elements using the displacement-controlled technique. The modified Newton-Raphson method is used to solve the nonlinear equilibrium equations.

4.2 Constitutive models

For proper simulation by NLFEA, we found three constitutive models described hereinafter. All play an important role: compression for cracked concrete, shear transfer, and bond between reinforcing bars and concrete (Kanazawa et al. 2017).

4.2.1 Compression response of cracked concrete

Eqs. (1a) to (1c) used for RC damaged zone is only applicable to the two directions of compression force: parallel and perpendicular to reinforcing bars. Therefore, the parameters of \(\alpha\) and \(\gamma\) are related to \(\theta\), the angle between a crack and reinforcing bar, as presented in Fig. 15 and Table 8. Eq. (10) is used for PL damaged zone, for which the reductions in strength and stiffness are independent of the compression force direction:

Table 6 Relevant equations and crack conditions to PL and RC damaged zones.

| PL damaged zone: | RC damaged zone: |
|------------------|------------------|
| C5, C10          | C15, T10, T15    |
| Initial strain conditions | \(\rho_{ST} = 0\) in Eqs. (6a) and (6b) \(\rho_{ST} \neq 0\) in Eqs. (6a) and (6b) |
| Horizontal cracks | Not introduced   | Introduced as Fig. 13 |
| \(f_{c,\theta}\) and \(E_{c,\theta}\) before cracking | Eqs. (7) and (8) | Unnecessary |
| \(f_{s,\theta}\) | Eq. (9)          |
| Compression response after cracking | Eq. (10) developed based on the model of Vecchio and Collins (1986) | Eq. (1) developed by the authors |
| Shear stiffness | Eq. (12) developed based on the model of Maekawa and Okamura (1991) | Eq. (11) developed by the authors |

Table 7 Element types used in 3D-NLFEA.

| Material            | Element type       | No. of nodes | No. of Gauss points |
|---------------------|--------------------|--------------|---------------------|
| Concrete            | Hexahedron solid   | 20           | 8                   |
| Reinforcing bars    | Truss              | 3            | 2                   |
| Loading plate       | Hexahedron solid   | 20           | 8                   |

Fig. 12 Respective damaged zones for each beam.

Fig. 13 Initial crack because of FT action.

Fig. 14 Finite element mesh for RC beam.
is associated with reductions in compressive strength
Shima et al. (1987). The frost damage in concrete cover
damaged to damaged concrete known from experiments.

4.2.2 Bond model between reinforcing bars and concrete
Reductions in mechanical properties of concrete cover
have caused bond deterioration because of frost damage
(Petersen et al. 2007). However, we found that the bond
stress degradation resulted from temperature fluctuation
without damage in concrete cover and developed the
bond model coupled with the deterioration mechanism
due solely to temperature fluctuation as Eq. (11), which
is based on bond stress-slip-strain relation proposed by
Shima et al. (1987). The frost damage in concrete cover
is associated with reductions in compressive strength
and Young’s modulus. In addition, the deteriora-
tion mechanism associated with temperature fluctu-
tation was linked to reduction in tensile strength of the
concrete near reinforcing bars. The authors have demon-
strated that the shear transfer in the damaged zones
governed the mechanical behavior of the RC beams damaged in compression zones of C5, C10, and C15
(Kanazawa et al. 2017). Although mechanical interlocking of coarse aggregates plays a key role in the
transfer of shear stress, proper modeling is difficult even
for the undamaged concrete (Mattock and Hawkins
1972). Therefore, shear stiffness is reduced based on the
series of spring models (Maekawa and Okamura 1991) to
model the shear stress reduction corresponding to a shear
strain in damaged zones. The original model was modi-
fied with \( \xi \), indicating the reduction rate in compressive
strength, as shown in Eq. (12). Furthermore, the maxi-
num shear transfer stress, \( f_s \) is reduced by \( \xi \).

\[
\sigma' = \beta f'_c \exp \left\{ -\left( \frac{\varepsilon'_r - \varepsilon'_e}{\eta \varepsilon'_r - \varepsilon'_o} \right)^\gamma \right\}
\]

(10)

\[
\beta = \frac{1}{0.85 + 0.27(\varepsilon'_r + \varepsilon / \varepsilon'')} < 1.0
\]

(11)

where \( \xi \) denotes the compressive strength ratio of un-
damaged to damaged concrete known from experiments.

4.2.3 Shear transfer on the crack surface
The authors have demonstrated that the shear transfer in
the damaged zones governed the mechanical behavior of the
cementitious materials (Hayashida et al. 2014). For 2D-FEA
of the broken lines, the authors assumed perfect bond between reinforcing bars and concrete. The mechanical properties of intact
concrete, \( f''_c, f_s, \) and \( E_c \) are decreased isotropically. Ha-
yashida et al. (2014) did not examine specimen T5 because of the disturbance of water insulation during FT
testing. Therefore, T5 was excluded from the discussion.

4.3 Simulation of mechanical responses of
frost-damaged RC beams
The developed numerical method is verified by com-
parison with the work of Hayashida et al. (2014), in
which the same RC beams were studied using two-di-
rectional finite element analysis (2D-FEA). The 2D-FEA
assumed a perfect bond between reinforcing bars and concrete. The mechanical properties of intact
concrete, \( f''_c, f_s, \) and \( E_c \) are decreased isotropically. Ha-
yashida et al. (2014) did not examine specimen T5 because of the disturbance of water insulation during FT
testing. Therefore, T5 was excluded from the discussion.

4.3.1 Load-deflection responses
Figure 16 shows load versus midspan deflections ob-
tained from 3D-NLFEA, 2D-FEA, and the loading test
(Hayashida et al. 2014). For 2D-FEA of the broken lines,
deflections corresponding to maximum loads are much
smaller than the test results for all cases. However,
3D-NLFEA reproduces the stiffness reduction of speci-
mens C5, C10, and C15 relative to undamaged specimen
N. In addition, the tendency of C5 and C10 by which the
loads were maintained after the maximum loads, are
simulated by the 3D-NLFEA. The reason for underes-
timation of the maximum load of C5 remains unclear. For
specimen C15, the RC damaged zone is changed to the
PL damaged zone to investigate the contribution of the
anisotropy to load-deflection responses, as presented in
Fig. 16(d). Computation without anisotropy: the blue
line overestimates the test result and presents a bilinear response similarly to the undamaged specimen N. This result suggests that the anisotropic frost damage involved in 3D-NLFEA is fundamentally important to simulate the test result by which reinforcing bars are embedded at frost-damaged zones.

For specimens T10 and T15 with frost damage in tension zones, a mismatch is apparent between the test and simulated result. This mismatch could result from the numerical accuracy of bond deterioration. Therefore, the mechanical strain distributions of reinforcing bars are compared with the test result in Fig. 17. Results of 3D-NLFEA; solid lines coincide with the test result because of the bond model of Eq. (11). By contrast, the strain distributions obtained from 2D-FEA: broken lines are much smaller than the test results because a perfect bond is assumed.

The accuracy of the developed method can be improved by the determination of PL and RC zones and by the initiation of horizontal cracks. The former, the reinforcement ratio of $\rho_{FT}$, governs the degree of anisotropy and resultant stress distribution in damaged concrete, as presented in Eqs. (6a) and (6b). By the examination of an effective area of bond action, $A_{eff}$ can be found reasonably (An and Maekawa 1997). The latter influences the stress path in damaged zones. The initial cracks should not be considered based on the experimental observation, but on principal directions of the initial stress induced by freeze-thaw action.

4.3.2 Failure modes

Figure 18 presents a comparison between experimental and numerical failure modes. The numerical results of principal compressive strains were obtained when 95% of maximum load was reached. For specimens damaged in their compression zones, failure zones extend to the shear span as damage depth increases. This tendency corresponds to the test results. Note that 2D-FEA caused flexural failure, which the failure zones were limited to the elements adjacent to loading point (Hayashida et al. 2014). This result suggests that the 3D-NLFEA can predict shear-dominant failure, which differs from the undamaged specimen. Additionally, the flexural-dominant failure of specimen T10 is simulated.

5. Conclusions

This study specifically examined anisotropic frost damage both for the expansion and for mechanical responses of RC beams. A numerical method was developed for frost-damaged RC beams with variation in the frost damaged zone by 3D-NLFEA. The following conclusions can be drawn.

(1) To ensure proper evaluation of mechanical behavior of frost-damaged RC beams by NLFEA, PL and RC must be treated separately. The test results show that RC exhibits anisotropic expansion and consequent mechanical anisotropy, unlike PL.

(2) To develop a generalized approach to predict FT-related expansions, differences in minimum
temperature and W/C are converted to the number of FT cycles. The anisotropic expansion is incorporated into the relations between number of FT cycles and expansion strain. This approach can predict anisotropic expansion and temperature variation in an RC beam section.

(3) The predicted anisotropic expansion is introduced as initial strains and cracks into 3D-NLFEA. The mechanical anisotropy is regarded with constitutive models coupled with expansion strain. This method can predict failure behavior that differs from that of an undamaged beam and can predict load-deflection responses of RC beams showing variation in a frost damaged zone.

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