Solar neutrino oscillations in the quasi-vacuum regime

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Motivated by recent experimental data, we study solar neutrino oscillations in the range $\delta m^2/E \in [10^{-10}, 10^{-7}]$ eV$^2$/MeV. In this range vacuum oscillations become increasingly affected by (solar and terrestrial) matter effects for increasing $\delta m^2$, smoothly reaching the MSW regime. A numerical study of matter effects in such “quasi-vacuum” regime is performed. The results are applied to the analysis of the recent solar neutrino phenomenology.

1. Introduction

Flavor oscillations either in vacuum [1] or in matter [2] provide a viable explanation to the solar neutrino problem [3]. The corresponding survival probability $P_{\nu_e \rightarrow \nu_e}$ is determined by the mass-mixing parameters $\delta m^2$ and $\omega$, as well as by the electron density profile $N_e(x)$ along the $\nu$ path.

For solar neutrino oscillations three characteristic lengths can be identified: the astronomical unit $L = 1.496 \times 10^8$ km, the oscillation wavelength in vacuum:

$$L_{\text{osc}} = \frac{4\pi E}{\delta m^2} = 2.48 \times 10^{-3} \frac{\delta m^2}{E} \text{ km},$$

and the refraction length in matter:

$$L_{\text{mat}} = \frac{2\pi}{\sqrt{2G_F N_e}} = 1.62 \times 10^4 \frac{1}{N_e} \text{ km},$$

where $\delta m^2$ is expressed in eV$^2$, the neutrino energy $E$ in MeV and the electron density $N_e$ in mol/cm$^3$. Usually, two asymptotic regimes can be identified: 1) the “Mykheyev-Smirnov-Wolfenstein” (MSW) oscillation regime [2] ($\delta m^2/E \geq 10^{-7}$ eV$^2$/MeV) characterized by $L_{\text{osc}} \sim L_{\text{mat}} \ll L$, and the “vacuum” (VAC) oscillation regime [2] ($\delta m^2/E \sim O(10^{-11})$ eV$^2$/MeV), characterized by $L_{\text{osc}} \sim L \gg L_{\text{mat}}$.

In the MSW regime, the many oscillation cycles in vacuum ($L_{\text{osc}} \ll L$) are responsible for complete decoherence of oscillations, and the survival probability depends on the detailed density profile in the Sun (and, during nighttime, in the Earth). Conversely, in the VAC regime the matter effects suppress oscillations in the Sun and in the Earth, so that (coherent) flavor oscillations take place only in vacuum, starting approximately from the Sun surface [2].

However, there is a transition regime [that might be called of “quasi-vacuum” (QV) oscillations], corresponding to the range $10^{-10} \leq \delta m^2/E \leq 10^{-7}$ eV$^2$/MeV, in which oscillations become increasingly affected by matter effects, and decreasingly decoherent, for increasing values of $\delta m^2/E$. In this regime, characterized by $L_{\text{mat}} \leq L_{\text{QV osc}} \leq L$, none of the approximations used in the MSW or in the vacuum regimes can be applied. In the past, QV oscillations have been considered only in a few papers (see [2] and reference therein) since typical fits to solar $\nu$ rates allowed only marginal solutions in the range where QV effects are relevant. However, more recent analyses appear to extend the former ranges of the VAC solutions upwards and of the MSW solutions downwards in $\delta m^2/E$, making them eventually merge in the QV range (see, e.g., [3,4]).

The QVO regime has been recently studied in detail by Friedland in [3]. Other numerical [4] and analytical [5] studies of QVO have also been performed. Here we discuss the results of [3] in the context of the most recent solar neutrino data.

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2. Calculation of the survival probability

From general quantum-mechanical arguments we can derive the probability that a $\nu_e$ produced in the interior of the Sun is detected as a $\nu_e$ on the Earth \[8,11\]:

$$P_{\nu_e \rightarrow \nu_e} = P_\odot P_\oplus + (1 - P_\odot)(1 - P_\oplus)$$

$$+ 2\sqrt{P_\odot(1 - P_\odot)P_\oplus(1 - P_\oplus)} \cos \xi,$$ \(3\)

where $P_\odot$ and $P_\oplus$ are the transition probability $P_{\nu_{e+\nu_1}}$ along the two partial paths inside the Sun (up to its surface) and inside the Earth (up to the detector), and the total phase $\xi$ can be written as

$$\xi = \frac{\delta m^2}{2E}(L - R_\odot) + \xi_\odot + \xi_\oplus,$$ \(4\)

where $R_\odot$ is the solar radius and $\xi_\odot$ ($\xi_\oplus$) is the phase acquired in the Sun (Earth) matter.

Equation (3) is a general formula that smoothly interpolates from the vacuum to MSW regime. In particular, in the vacuum limit $P_\oplus \approx \xi_\oplus \approx P_\oplus$ and $\xi_\odot$, $\xi_\oplus \approx 0$, we obtain the standard vacuum formula. Conversely, in the MSW limit the oscillating term is averaged away and $P_\odot \approx \sin^2\theta_m P_\odot + \cos^2\theta_m (1 - P_\odot)$, where $\theta_m$ is the mixing angle at the production point and $P_\odot$ is the crossing probability $P_{\nu_{e+\nu_1}}$, where $\nu_{e+\nu_1}$ is the heavier eigenstate in matter; we obtain the standard Parkes' formula \[2,3\].

The evolution inside the Sun has been performed by direct numerical integration of the MSW equation from solar center to the solar surface. The electron density was taken from \[13\]. Figure 2 in \[3\] shows, in the mass-mixing plane, isolines of the difference $\xi_\odot - \xi_\oplus$ (solid curves), which becomes zero in the vacuum oscillation limit of very small $\delta m^2/E$. The variable $\tan^2\theta$ can be substituted in order to chart the first two octants of the mixing angle range. In \[10\] a semianalytical approximation of $P_\odot(\approx P_\oplus)$ has been find by modifying the standard resonance prescription \[1\] with the Maximum Violation of Adiabaticity (MVA) prescription \[1\]. This approximation gives $P_\odot$ with a percent accuracy.

For increasing values of $\delta m^2/E$, the oscillating term in Eq. (3) become increasingly suppressed by energy smearing. The onset of the corresponding decoherence effect is best studied for almost monochromatic neutrinos, namely for the $^7$Be and pep lines, having a width $\Delta E \sim 0(keV)$ \[17\]. When $L_{osc}/L$ become greater of $\Delta E/\langle E \rangle$ (where $\langle E \rangle$ is the average neutrino energy) the damping effect is important. This happen for $\delta m^2/E \geq 10^{-8}$ eV$^2$/MeV (and before for continuous spectra).

For narrow line spectra, the energy-averaged oscillating factor can be written as $(\cos \xi) \approx D(\delta m^2 L/2\langle E \rangle^2) \cos \xi$, where $D$ is the modulus of the Fourier-transform of the spectrum \[8\]. Figure 8 in \[10\] shows the function $D$ as function of $\delta m^2/2\langle E \rangle$ for the $^7$Be and pep lines. As anticipated, damping factor is important for $\delta m^2/E \geq 2 \times 10^{-8}$ eV$^2$/MeV. For higher values of $\delta m^2/E$, the oscillating term can be completely neglected.

The survival probability $P_{\nu_\oplus \rightarrow \nu_\oplus}$ has to be averaged in time, since it depends from $t$, both through $P_\oplus$ via $\eta$ and through the phase $\xi$ via the earth orbit eccentricity. While $P_\oplus(t)$ and $\cos \xi(t)$ can be be easily averaged separately \[8,11\], the calculation of the time average of the term $\sqrt{P_\oplus(1 - P_\oplus)} \cos \xi$ is in principle tedious.

The evolution inside the Earth is done by evolving analytically the MSW equations at any given nadir angle $\eta$, using the technique described in \[13\], which is based on a five-step biquadratic approximation of the density profile from the Preliminary Reference Earth Model (PREM) \[11\] and on a first-order perturbative expansion of the neutrino evolution operator. Figure 4 in \[3\] shows isolines of the quantity $\xi_\odot - \xi_\oplus$, which becomes zero in the vacuum oscillation limit of very small $\delta m^2/E$ for two representative values of the nadir angle.

Regarding the phases $\xi_\odot$ and $\xi_\oplus$ in Eq. (4), it has been shown in \[3\] that they are negligible for current fits. Only $\xi_\odot$ could have non-negligible effects in future high-statistics experiments. A semianalytic formula for the calculation of $\xi_\odot$ has been recently derived in \[10\].

3. Decoherence of the oscillating term

For increasing values of $\delta m^2/E$, the oscillating term in Eq. (3) become increasingly suppressed by energy smearing. The onset of the corresponding decoherence effect is best studied for almost monochromatic neutrinos, namely for the $^7$Be and pep lines, having a width $\Delta E \sim 0(keV)$ \[17\]. When $L_{osc}/L$ become greater of $\Delta E/\langle E \rangle$ (where $\langle E \rangle$ is the average neutrino energy) the damping effect is important. This happen for $\delta m^2/E \geq 10^{-8}$ eV$^2$/MeV (and before for continuous spectra).

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The survival probability $P_{\nu_\oplus \rightarrow \nu_\oplus}$ has to be averaged in time, since it depends from $t$, both through $P_\oplus$ via $\eta$ and through the phase $\xi$ via the earth orbit eccentricity. While $P_\oplus(t)$ and $\cos \xi(t)$ can be be easily averaged separately \[8,11\], the calculation of the time average of the term $\sqrt{P_\oplus(1 - P_\oplus)} \cos \xi$ is in principle tedious.
Fortunately, the Earth effect is non negligible only when the oscillating term is doubly suppressed ($P_\odot \sim 0$ and $D \sim 0$). Therefore, in Eq. (3) one can safely take $P_\oplus \simeq c^2_\omega$ as far as the oscillatory term is non negligible.

4. 2ν solutions to the solar neutrino problem

In this Section we apply the previous results to the analysis of the more recent solar neutrino phenomenology. Usually, the analysis is performed in the first octant in $\omega$ and splitting the $\delta m^2$ range in the vacuum (few $\cdot 10^{-11} \leq \delta m^2 \leq$ few $\cdot 10^{-10}$ eV$^2$) and MSW ($\delta m^2 \geq 10^{-8}$ eV$^2$) subranges.

The lack of observation of a distortion in the recoil electron spectrum in SuperKamiokande, together with a rate observed in SuperKamiokande and Gallium experiments compatible with about 1/2 suppression of the $\nu_e$ flux, strongly favor large mixing solutions. The LOW and VAC solutions tend to merge in the QV part of the plane, and acceptable solutions appear also for $\omega > \pi/4$. Consequently, the correct way to show the solutions is to use the entire mass-mixing plane without the artificial splitting in $\delta m^2$ and for $0 \leq \omega \leq \pi/2$.

Figure 1 shows the combined analysis of the total rate information coming from the SuperKamiokane (SK), Homestake, SAGE, and GALLEX-GNO experiments. Figure 2 shows the combination between rate information and day and night spectral information from SK. The technical details of the analysis are explained in the references therein. The value of $\chi^2_{\text{MIN}}$ is 35.1 for the LMA, 37.8 for the LOW and 40.7 for the SMA solution, for 36 degrees of freedom.

The LMA and LOW solutions appear to be fa-
vored over the SMA solution in the global fit, although it is rather premature to think that the latter is ruled out. The vacuum solutions appear strongly disfavored by the nonobservation of SK spectral distortions. From Fig. 2 we see that a solution to the solar neutrino problem emerges at 99% C.L. in the QV range. The matter effects in QV solutions break the symmetry $\omega \rightarrow \pi/2 - \omega$ valid in the vacuum range (and that can be well observed in the vacuum solutions in Fig. 1). From Fig. 2 it appears that large mixing angle solutions are favored. For this reason, several experimental tests have been proposed to check them [23,24].

5. Conclusions

We have studied solar neutrino oscillations in the hybrid regime $\delta m^2/E \in [10^{-10}, 10^{-7}]$ eV$^2$/MeV (“quasi-vacuum” regime), in which the familiar approximations suitable in the vacuum and MSW regime are not valid. We have discussed a general formula that interpolates smoothly between vacuum and MSW regimes. The effects of energy smearing on the oscillating term and the Earth matter effect have been taken in account. The results have been applied to the analysis of the most recent solar neutrino data, and suitable solutions to the solar $\nu$ problem have been found in the QV range.

6. Acknowledgments

The author is grateful to G.L. Fogli, E. Lisi, A. Marrone, and A. Palazzo for interesting discussions.

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