The Super-Radiant Mechanism and the Widths of Compound Nuclear States

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Abstract. In the introduction I will present the theory of the super-radiant mechanism as applied to various phenomena. I will then discuss the statistics of resonance widths in a many-body Fermi system with open decay channels. Depending on the strength of the coupling to the continuum such systems show deviations from the standard Porter-Thomas distribution. The deviations result from the process of increasing interaction of the intrinsic states through the common decay channels. In the limit of very strong coupling this leads to super-radiance. The results I will present are important for the understanding of recent experimental data concerning the width distribution of compound neutron resonances in nuclei.

1. Introduction
The possibility of forming a “super-radiant” (SR) state in a gas of atoms confined to a volume of a size smaller than the wave length of radiation was proposed by Dicke [1]. In the absence of a direct interaction, the atoms are coupled through their common radiation field. This indirect interaction through the continuum leads to a redistribution of lifetimes among intrinsic states. A rapidly decaying SR state is created at the expense of the rest of the states of the system that are “robbed” of their decay width and become narrow. This mechanism has a general origin and analogous phenomena should appear in many quantum systems when quasi-bound states are strongly coupled through common decay channels. The SR approach has been used in many different fields. For example, it was applied in chemistry [2], atomic physics [3], condensed matter physics [4, 5], intermediate energy nuclear physics [6, 7], in particle physics [8], in the theory of nuclear reactions [5, 9, 10]. In the next section we will develop the SR formalism for some simple cases.

2. The projection formalism.
Following the projection formalism [6,11], we divide the Hilbert space of nuclear states into two parts, the \( \{Q\} \)-subspace involving complicated many-body states \( |q\rangle \), and the subspace \( \{P\} \) of open channels \( |c\rangle \). We use the notations \( Q \) and \( P \) for the corresponding projection operators onto the above subspaces. The wave function of the total system,

\[
|\Psi\rangle = Q|\Psi\rangle + P|\Psi\rangle
\]

(1)
is the solution of the Schrödinger equation

\[
H|\Psi\rangle = E|\Psi\rangle
\]

(2)
that can be written as a set of coupled equations,

\[
\left(E - H_{QQ}\right)|\Psi\rangle = H_{QP} P|\Psi\rangle
\]

(3)
and
\[
(E - H_{pp})P\Psi = H_{pq}Q\Psi,
\]
where we use the notations \( H_{AB} = AHB \). Eliminating the part \( P\Psi \), we obtain an equation in the \( Q \)-space:

\[
(E - \tilde{H}_{QQ})Q\Psi = 0
\]
with the effective Hamiltonian

\[
\tilde{H}_{QQ} = H_{QQ} + H_{QP} \frac{1}{E^{(+)} - H_{pp}} H_{PQ}.
\]

Here, \( E^{(+)} \equiv E + i0 \) contains the infinitesimal imaginary term \( + i0 \) ensuring correct asymptotic behaviour of the continuum wave functions. The second term of the effective Hamiltonian contains a real and imaginary part of the propagator

\[
G^{(+)}(E) = \frac{1}{E^{(+)} - H_{pp}}.
\]

arising from the principal value and the delta-function \( \delta(E - H_{pp}) \) (on-shell contributions from channels \( c \) open at energy \( E \)), respectively. The imaginary part of the effective Hamiltonian is \(-i/2W\), with

\[
W = 2\pi \sum_{c} H_{QP} |c\rangle\langle c| H_{PQ}.
\]

Thus, the effective Hamiltonian (7) in \( Q \)-space is anti-Hermitian,

\[
\tilde{H}_{QQ} = \tilde{H}_{QQ} - \frac{i}{2}W,
\]

where \( \tilde{H}_{QQ} \) is a symmetric and real matrix that includes, apart from the original Hamiltonian in the \( Q \)-space- \( H_{QQ} \), the principal value contribution of the \( PQ \)-coupling. The second part is anti-Hermitian. The eigenvalues of \( \tilde{H} \), \( \tilde{E} = E - (i/2)\Gamma \) are complex poles of the scattering matrix corresponding to the resonances in the cross sections.

To demonstrate in a simple way the role of the anti-Hermitian term we assume that only one channel is open. Then the matrix \( W \), Eq. (8), has a completely separable form,

\[
\langle q|W|q'\rangle = 2\pi A^*_q A_q \delta_{qq'},
\]

where:

\[
A^*_q = \langle q|H_{QP}|c\rangle.
\]

The rank of the matrix \( W \) is 1, so that all the eigenvalues of this matrix are zero except one that has the value equal to the trace of the matrix:

\[
\Gamma_q = \sum_q \langle q|W|q\rangle = 2\pi \sum_q |A^*_q|^2 = \sum_q \Gamma^\dagger_q
\]

with \( \Gamma^\dagger_q \) denoting the escape width of the individual levels before the \( W \)-matrix is diagonalized. The special unstable state with width \( \Gamma_q \) is often referred to as the super-radiant (SR), in analogy to the Dicke coherent state [1] of a set of two-level atoms coupled through the common radiation field. Here
the coherence is generated by the common decay channel. The stable states are trapped and decoupled from the continuum.

In the more general case of $N$ intrinsic states and $N_c$ open channels with $N_c << N$ the super-radiant mechanism survives if the mean level spacing $D$ of internal states and their decay widths $\Gamma_q$ satisfy:

$$\kappa^r = \frac{\Gamma_q}{D} > 1$$

In this case we have $N_c$ broad states, while the rest of states $N - N_c$ become very narrow.

### 3. Porter-Thomas Distribution and the Super-Radiant Mechanism.

We will now discuss an example that illustrates the use of the super-radiant approach to a complicated problem in nuclear physics. One of the best examples in which the interplay between intrinsic dynamics and decay channels is manifested are the low-energy neutron resonances. These resonances are interpreted as quasi-stationary levels of the compound nucleus formed after the neutron is captured. These resonances were described, with certain degree of success, by the theory of random matrices [12]. With exceedingly complicated wave functions of the compound states their components are Gaussian distributed. This applies also to the specific component related to the channel, thus the neutron in the continuum and the residual nucleus in the ground state. The decay width of the neutron resonance is proportional to the amplitude squared of this component and the width distribution is therefore given by the $\chi^2$ function with $\nu = 1$, that is, the Porter-Thomas distribution (PTD).

Recently experiments found significant departures from the PTD [13]. Attempts to still use the $\chi^2$ distribution in order to fit the data require $\nu < 1$. This result has been interpreted as a breakdown of the random matrix theory for the case when the components of compound states are the same as for a closed system. This might be valid only in the limit of very weak continuum coupling. The standard PTD does not account for the openness of the system. The openness of the system inevitably brings in the elements of coherence through the common decay channel even in the case of intrinsic chaos. This is expected naturally to introduce deviations from the PTD.

In a paper published recently [14] it was shown that a proper description of unstable quantum states using the super-radiant mechanism can in fact produce deviations from the PTD of the same type as observed in experiments. The coupling to open channels is essential in the redistribution of the decay widths of the compound states. The details of the calculations are described in reference [14]. When the strength of the coupling between compound states and the open channel(s) (denoted by $\kappa$) is very small ($\kappa << 1$) one finds that the distribution of the calculated widths follows the PTD. With increasing coupling the deviations become more pronounced. At $\kappa \sim 1$ the super-radiant stage is reached and the distribution of widths becomes singular. In Figures 1 and 2 some of the results from reference [14] are shown.

We see in Figure 1 that as $\kappa$ increases, the fits to the numerical results involve $\chi^2$ with decreasing values of $\nu$. In Figure 2 we present the deviations from the PTD curve when $\kappa$ increases. The observed deviations from the PTD seem to exceed than the predicted by our new results for rather small values of the parameter $\kappa$. This might suggest that there are other factors related to nuclear structure that go beyond statistical considerations. The intrinsic dynamics can have some non-chaotic...
Figure 1. Distribution of widths for one open channel (M=1) and for two channels (M=2) for an intrinsic Gaussian Orthogonal Ensemble Hamiltonian and for different continuum coupling strength $\kappa$. The numerical results are given by histograms; the PTD for M=1 and the $\chi^2_{v=2}$ distribution for M=2 are shown as a smooth curve. The full squares stand for the best fit to a $\chi^2_{v=2}$ distribution. Taken from Ref. [14].

Figure 2. Distribution of widths for one open channel (M=1) and for two channels (M=2) for an intrinsic Gaussian Orthogonal Ensemble Hamiltonian and for different continuum coupling strength $\kappa$. The numerical results are given by histograms; the PTD for M=1 and the $\chi^2_{v=2}$ distribution for M=2 are shown as a smooth curve in each case indicating clearly the deviations from PTD (or $\chi^2_{v=2}$) as $\kappa$ increases.
components, the energy dependence of neutron resonances might not be the one that is usually assumed (for s states $\sqrt{E}$), especially due to single-particle resonances in the vicinity of the energies studied in the experiment [15].

In summary, the interpretation of the widths of compound resonances as a strength of the pure neutron component in the wave function fails due to the coupling to the continuum that has to be taken into account in a proper statistical description. This phenomenon is of general nature and might influence many other quantum systems.

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