A scalar gluonium contribution to $K \to \pi\pi$ decay

A.A. Penin

Institute for Nuclear Research of the Russian Academy of Sciences,
Moscow 117312, Russia

and

A.A. Pivovarov

National Laboratory for High Energy Physics (KEK),
Tsukuba, Ibaraki 305, Japan

Abstract

We study a new $K \to \pi\pi$ decay channel with gluons in intermediate state which is normally neglected within the factorization framework. Both short-distance and long-distance parts of the amplitude are calculated. The chiral Lagrangian approach is used for obtaining the long-distance contribution. The nonperturbative contribution gives an additional enhancement to $K \to \pi\pi$ decay amplitude with $\Delta I = 1/2$. A sizable violation of factorization in the $p^4$ order of chiral perturbation theory is found.

PACS number(s): 12.15.Ji, 13.25.+m, 11.50.Li, 11.30.Rd.

---

1On leave from Institute for Nuclear Research, Moscow, Russia
The origin of considerable enhancement of $\Delta I = 1/2$ parts of non-leptonic kaon decay amplitudes remains one of subtle points within the standard model (SM) [1]. Though explained qualitatively by strong interaction effects it still escapes the reliable quantitative description. Numerous attempts have been recently made to achieve a sufficient accuracy for non-leptonic kaon decays in SM. The efforts were directed to improving the perturbative QCD analysis [2] and to accounting for long-distance effects using more advanced models of strong interactions at low energy, for example, the chiral Lagrangians [3], $1/N_c$ expansion of QCD [4,5] or lattice simulations [6]. The present results however do not fit experimental data.

An effective $\Delta S = 1$ Hamiltonian reads [7-9]

$$H_{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{us} \sum_{i=1,i\neq 4}^6 [z_i(\mu) + \tau y_i(\mu)] \tilde{Q}_i$$

(1)

where $G_F$ is a Fermi constant, $V$ is the quark flavor mixing matrix, $\tau = -V_{td}^* V_{ts}^* / V_{ud} V_{us}$; $z_i(\mu)$ and $y_i(\mu)$ are the coefficients of Wilson expansion, \{\tilde{Q}_i\} is a full basis of dimension six local operators containing light quark fields (u, d, s) only.

The renormalization group improved perturbation theory does not account for strong interaction of soft light quarks and gluons. The information about this interaction is entirely contained in the hadronic matrix elements of local quark operators. Factorization procedure for evaluation of these matrix elements [10], i.e. the procedure of replacing the four-quark operators by a product of two non-interacting quark currents accounts only for the ”factorizable” part of this interaction [11]. But there are also ”non-factorizable” contributions, for example, ones corresponding to annihilation of a quark pair from the four-quark operator into soft gluons, which are omitted within the factorization procedure. The calculation of these contributions and the generalization of matrix element estimates beyond the factorization framework can be systematically done within QCD sum rules technique combined with the chiral Lagrangian approach. This possibility is related to studying a new $K \rightarrow \pi\pi$ decay channel generated by annihilation of a quark pair from the four-quark operator into gluons with the subsequent formation of the final pion pair by the soft gluon cloud, i.e. the decay channel with gluons playing the role of an intermediate state. Being non-factorizable, this decay mode does not appear as a correction to some
leading order contribution and can be studied by its own. This feature makes obtained results more accurate.

In the present paper we study a new $K \to \pi \pi$ decay channel with the simplest scalar colorless gluon configuration forming an intermediate state. We calculate both short-distance (perturbative) and long-distance (nonperturbative) parts of the corresponding amplitude. To obtain the long-distance contribution, the chiral effective Lagrangian is used as a low energy model of strong interactions and the relevant factor of proportionality between quark and mesonic operators is derived via QCD sum rules.

Non-factorizable contributions reveal themselves in two different ways: first, they appear as corrections to the couplings characterizing the "factorizable" weak chiral Lagrangian in $O(p^4)$ and higher orders, second, some new non-factorizable terms emerge. The latter is the case for $K \to \pi \pi$ decay mode with gluons forming an intermediate state.

Before turning to the long-distance effects of meson-gluon transitions it is useful to consider the similar phenomenon arising already in perturbative QCD as a leading correction in the inverse mass of charmed quark. The effective low energy tree-level Hamiltonian for $\Delta S = 1$ transitions before decoupling of the $c$-quark reads

$$H_{\Delta S = 1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* (Q^u_2 - (1 - \tau) Q^c_2)$$  \hspace{1cm} (2)

where $Q^q_2 = 4(\bar{s}_L \gamma_\mu q_L)(\bar{q}_L \gamma_\mu d_L)$, $q = u, c$ and $q_L(R)$ stands for the left(right) handed quark. Performing the OPE to the first order terms in $\alpha_s$ and $m_c^{-1}$ one can write down a representation for the effective Hamiltonian in the form

$$H_{\Delta S = 1} = H_0 + H_1.$$ \hspace{1cm} (3)

The first addendum $H_0$ in the rhs of eq. (3) corresponds to the leading contributions in $m_c^{-1}$ and coincides with the rhs of eq. (1) while the second addendum $H_1$ is the leading order $m_c^{-1}$ correction that comes from an annihilation diagram with the charmed quark running inside the loop. If one restricts the analysis to a scalar colorless gluon configuration $G^a_{\mu \nu} G^a_{\mu \nu}$ the additional contribution reads \cite{12}

$$H_1 = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* (1 - \tau) \frac{1}{120 m_c^2} m_s \bar{s}_R d_L \frac{\alpha_s}{\pi} G_{\mu \nu}^a G_{\mu \nu}^a.$$ \hspace{1cm} (8)
To derive an effective Lagrangian involving Goldstone bosons only, which can be used for the calculation of decay amplitudes, one has to replace a QCD operator

$$m_s \bar{s}d_L \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu} \equiv m_s \bar{s}d_L G^2$$

in eq. (8) with its mesonic counterpart. The following representation holds

$$m_s \bar{s}d_L G^2 = Af_\pi^6(U_2^\dagger \chi)_{23} + Bf_\pi^4(U_2^\dagger \chi)_{23} \text{tr} \left( \partial_\mu U^\dagger \partial^\mu U \right) + \text{other } O(p^4) \text{ terms}$$

where $\chi$ stands for the meson mass matrix, $U = e^{-i\sqrt{2}/f_\pi}$ is the unitary matrix describing the octet of pseudoscalar mesons, $A$ and $B$ are dimensionless parameters to be computed. The second term on the rhs of eq. (10) is separated from other $O(p^4)$ structures for the reason that will be clarified below.

The $O(p^2)$ term in eq. (10) is a tadpole. The appearance of such a term is the consequence of working with a wrong solution for the ground state. This term merely renormalizes the effective Lagrangian of strong interactions and can be absorbed into the meson mass matrix by a suitable $SU(3)_L \otimes SU(3)_R$ rotation [3,14]; it does not affect any observables. A contribution to the physical amplitude is determined by the $O(p^4)$ part of the chiral representation (10). The most transparent way to obtain this contribution is to consider the quark-gluon operator (9) as a product of the (pseudo)scalar quark current and the scalar colorless gluon operator. Then one can replace the quark operator by its mesonic realization according to the PCAC hypothesis [13]

$$m_s \bar{s}d_L \rightarrow - \frac{f_\pi^2}{8}(U^\dagger \chi)_{23}.$$

On the other hand there is a low energy theorem based on fundamental properties of the energy-momentum tensor that gives the chiral representation of the gluon operator [15]

$$\frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu} = - \frac{2}{b} f_\pi^2 \text{tr} \left( \partial_\mu U^\dagger \partial^\mu U \right) + O(p^4)$$

where $b = 9$ for three light quark flavors. Eqs. (11,12) give for the $B$ parameter

$$B = \frac{1}{4b}.$$
This approximation corresponds to the simplest physical picture where the kaon is annihilated by the pseudoscalar quark current while the pion pair is born by the gluon operator. Eqs. (8,10-13) lead to an effective chiral Lagrangian of the form
\[ L_{\text{eff}}^G = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left(1 - \tau\right) \frac{f_\pi^4}{480 b m_c^2} \left(U^\dagger \chi \right)_{23} \text{tr}(\partial_\mu U^\dagger \partial^\mu U). \] (14)

Since the pion pair is born by the gluon operator this Lagrangian describes the decay channel with gluons forming an intermediate state.

Now the corresponding \( K \to \pi\pi \) decay amplitude becomes explicitly calculable. We use the standard parametrization of the amplitude \( A_0 \) with the isospin transfer \( \Delta I = 1/2 \)
\[ R\text{e} A_0 = \frac{G_F}{\sqrt{2}} \sin \theta_c \cos \theta_c f_K m_K^2 g_{1/2} \] (15)
where \( \theta_c \) stands for Cabibbo angle and \( g_{1/2} \) is a dimensionless parameter. The new contribution reads
\[ \Delta g_{1/2} = \frac{m_K^2}{30 b m_c^2} \sim 10^{-3} \] (16)
while experiment gives [16]
\[ g_{1/2}^{\text{exp}} = 3.9, \] (17a)
and the most recent theoretical estimate is [2]
\[ g_{1/2} \sim 2.6. \] (17b)

Thus the local (perturbative) part of new decay mode is negligible according to the general estimate of a scale of the leading order charmed quark mass corrections [12].

The local effective Hamiltonian (8) does not exhaust the whole physics of the meson-gluon transitions. It can not account for the long-distance contribution connected with the propagation of a soft \( u \)-quark round the loop of the annihilation diagram. Because of the lightness of the \( u \)-quark this contribution can not be represented as a local vertex and ultimately depends on the infrared properties of QCD. Its investigation requires some nonperturbative approach.

In so doing we start with a tree level Hamiltonian which after decoupling of the \( c \)-quark has the form
\[ H_{\Delta S=1}^{tr} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* Q_2^u. \] (18)
The quantity of interest is an effective theory realization of that part of the operator $Q_2$ which is responsible for a transfer of the kaon into gluons. Invoking the results of our previous consideration we can write down this part in the following form

$$Q_G^2 = g_G f_\pi^2 (U^\dagger \chi)_{23} tr(\partial_\mu U^\dagger \partial^\mu U) \quad (19)$$

where $g_G$ is a dimensionless parameter. We should note that the chiral representation of the whole operator $Q_2$ contains a large number of structures but we are interested only in the part corresponding to the transition with the gluons forming an intermediate state. Thus the problem is reduced to computing the chiral coupling constant $g_G$ that can be done by studying the appropriate Green’s function (GF) via QCD sum rules technique. For the technical reason working with a two point GF is preferable. In the given decay channel the pions are born by a gluon cloud therefore the gluon operator $G^a_{\mu\nu}G^a_{\mu\nu}$ can play the role of an interpolating operator of the pion pair. Thus, it is natural to choose GF in the following form

$$G(p) = \int \langle 0| T \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu}(x) Q_2(0) | K^0(q) \rangle e^{ipx} dx \bigg|_{q=0}. \quad (20)$$

A remark about the chiral limit for the kaon in eq. (20) is necessary. The representation (19) fixes the correct $O(p^4)$ chiral behavior of the considered decay amplitude and does not depend explicitly on the kaon momentum. Keeping a non-vanishing kaon momentum leads to a shift of the decay amplitude that lies beyond the accuracy of the present approach. Thus we can put $q = 0$ in eq. (20) and work with GF depending on one argument only.

Saturating GF in eq. (20) with $\pi^+\pi^-$ and $\pi^0\pi^0$ states (the lowest states with proper quantum numbers), substituting the $Q_2$ operator and using the low energy theorem (12) one obtains the following physical representation

$$G^{ph}(p) = g_G \frac{32m_K^2}{\pi^2 b f_\pi} p^4 \ln(\frac{-p^2}{\mu^2}) + O(p^6). \quad (21)$$

The theoretical side reads after making use of OPE

$$G(p) = \frac{i}{2\pi^2} \ln(\frac{-p^2}{\mu^2}) \frac{\alpha_s}{\pi} \langle 0 | m_s \bar{s}_R g_s G^a_{\mu\nu} t^a \sigma_{\mu\nu} d_L | K^0(q) \rangle \bigg|_{q=0} + O(p^{-2}). \quad (22)$$
Factor \( m_s \) in eq. (22) provides the correct chiral property of GF and justifies the representation (19) for the operator \( Q_2 \). By reducing the kaon state, one can transform eq. (22) into the following expression

\[
G(p) = \frac{1}{4\pi^2} \ln\left(\frac{-p^2}{\mu^2}\right) \frac{\alpha_s}{\pi} f_K m_K^2 m_0^2.
\]  

(23)

where \( m_0 \) determines a scale of nonlocality of the quark condensate, \( \langle \bar{q}q \rangle = m_0^2(1 GeV) = 0.8 \pm 0.2 GeV^2 [17] \). For extracting information about the chiral coupling constant \( g^G \) we use finite energy sum rules [18] with the result

\[
g^G = 3b \frac{f^2 m_0^2 \alpha_s(s_0)}{128 s_0^2 \pi}.
\]  

(24)

To take into account strong interactions at short distances the operator \( Q_2^G \) in the effective Hamiltonian has to be multiplied by the corresponding Wilson coefficient \( z_2(s_0) \). Finally, the new contribution to the theoretical estimate of the \( \Delta I = 1/2 \) amplitude in terms of the parameter \( g_{1/2} \) takes the form

\[
\Delta g_{1/2} = z_2(s_0) \frac{\alpha_s(s_0) 3bm_0^3 m_K^2}{8s_0^2}.
\]  

(25)

This result needs some comments:

1. This next-to-leading in \( 1/N_c \) expansion contribution is missed within the factorization framework and also within any approach where quark currents in four-quark operators are replaced by their mesonic counterparts separately.

2. The gluon cloud in the intermediate state does not form a resonance state and, therefore, the contribution (25) is not suppressed by a large scalar meson mass.

3. In general, some more complicated scalar colorless gluon configurations, for example, \( f^{abc} G^a_{\mu\nu} G^b_{\nu\lambda} G^c_{\lambda\mu} \), could appear as intermediate states in this channel as well. However the theorem (12) shows that the two pion form factor of such configurations could be of the \( O(p^4) \) or higher order in chiral expansion that leads to a negligible \( O(p^6) \) shift of the decay amplitude.

The question now is what numerical value for the duality interval \( s_0 \) has to be used. Actually, the allowed value of the duality threshold is quite restricted by the form of the physical spectrum and by the requirement of absence of uncontrollable \( \alpha_s \) corrections. To
suppress contributions of higher mass states, for example, a scalar meson $\sigma(0.9 \text{ GeV})$, to the considered channel one has to take $s_0 < (0.9 \text{ GeV})^2$. At the same time the physical representation (20) is obtained in the leading order in chiral perturbation theory and the whole procedure is justified until the chiral expansion parameter $s_0(8\pi^2 f_\pi^2)^{-1}$ remains small [13,19]. On the other hand at the scale $\mu$ less than 0.8 GeV the perturbative $\alpha_s$ corrections to Wilson coefficients become uncontrollable [2] and for consistency of the approach one has to set the low limit of the duality interval to be $s_0 > (0.8 \text{ GeV})^2$. The reasonable choice for the duality interval now reads $s_0 = (0.8 \text{ GeV})^2$.

Let us estimate the uncertainty of our result. On the physical side of sum rules the errors related to higher order terms in chiral expansion, which have been omitted in eq. (21), are, in general, unknown. But one can hope that in the spirit of chiral perturbation theory they are about 25% [13,19]. On the theoretical side of the sum rules the errors come from two sources. The first one is the perturbative part of $OPE$ (the unit operator) that is suppressed by an extra loop factor $\alpha_s/4\pi \sim 10^{-3}$ and can not lead to a sizable change of our result. Next nonperturbative corrections due to operators with higher dimensionality seem to be more important. They start with the dimension eight operators which have already been discussed. Numerical estimates would require knowing the matrix elements of those operators between the kaon and the vacuum state which are not available now. But as a first approximation the relative weight of these corrections can be represented by the ratio

$$\frac{\langle g_s^2 G_{\mu\nu}^a G_{\mu\nu}^a \rangle}{s_0 m_0^2} \frac{\Gamma(3)}{\Gamma(5)} \sim 10^{-1} \quad (26)$$

where $\langle \alpha_s G_{\mu\nu}^a G_{\mu\nu}^a \rangle \sim (330 \text{ MeV})^4$ and the factor $\Gamma(n)$ comes from a quark loop with $n - 1$ gluon field or mass insertions. The situation here is quite similar to that of the analysis of $1/m_c^2$ corrections [12] where a contribution of dimension eight operators is suppressed rather numerically than parametrically. Taking into account the uncertainty of determination of the parameter $m_0^2$ we estimate the error bound to be about 40%. Numerically one obtains

$$\Delta g_{1/2} = 0.56 \pm 0.22 \quad (27)$$

at the point $s_0 = \mu^2 = (0.8 \text{ GeV})^2$, $\Lambda_{QCD} = 300 \text{ MeV}$, $z_2(s_0) = 1.49$ [9].
Thus, the new contribution provides about 15% of the observed amplitude (17a). At the same time it is comparable with the leading order result for the decay amplitude obtained by the naive factorization of the four-quark operator $Q_2$ when all strong interaction corrections are neglected

$$g_{1/2}^{fac} = 5/9.$$ \hfill (28)

This implies the strong violation of factorization in the $O(p^4)$ order in chiral expansion that leads to additional enhancement of the theoretical estimate of the $K \rightarrow \pi \pi$ decay amplitude with the isospin transfer $\Delta I = 1/2$.

To conclude, in the present paper we consider a new $K \rightarrow \pi \pi$ decay channel with the gluons forming an intermediate state. Both short-distance (perturbative) and long-distance (non-perturbative) parts of the corresponding amplitude have been calculated. The latter being dominant gives an additional enhancement of the $K \rightarrow \pi \pi$ decay with the isospin transfer $\Delta I = 1/2$ and provides about 15% of the of the experimentally observed amplitude. New contribution is of the $O(p^4)$ order and is lost within the factorization framework. It allows us to conclude that there is a sizable violation of the factorization in the $O(p^4)$ order in chiral expansion. Taking into account corrections of this order along with the usually considered $\alpha_s$ corrections to coefficient functions of Wilson expansion can help to resolve the $\Delta I = 1/2$ problem within the SM.

This work was supported in part by Russian Fund for Fundamental Research under Contract No. 93-02-14428, by Soros Foundation and by Japan Society for the Promotion of Science (JSPS).

References

[1] S.L.Glashow, Nucl.Phys. 2(1961)579;
S.Weinberg, Phys.Rev.Lett. 19(1967)1264;
A.Salam, in: Elementary Particle Theory,
Ed.N.Svartholm (Almqvist and Wiksel, 1968).
[2] A.J. Buras, M. Jamin, M.E. Lautenbacher and P.H. Weisz,
   Nucl.Phys. B370(1992)69, B375(1992)501.
   A.J. Buras, M. Jamin, M.E. Lautenbacher, Preprint CERN-TH-6821/93.

[3] J.Kambor, J.Missimer and D.Wyller, Nucl.Phys. B346(1990)17.

[4] W.A.Bardeen, A.J.Buras and J.M.Gerard, Phys.Lett. 192B(1987)138.

[5] A.Pich and E.Rafael, Nucl.Phys. B358(1991)311.

[6] G.Martinelli, Nucl.Phys. B(Proc.Suppl.) 17(1990)523.

[7] A.I.Vainshtein, V.I.Zakharov and M.A.Shifman,
   JETP 45(1977)670, Nucl.Phys. B120(1977)316.

[8] F.J.Gilman and M.B.Wise, Phys.Rev. D20(1979)2392.

[9] G.Buchalla, A.J.Buras and M.K.Harlander, Nucl.Phys. B337(1990)313;
   E.A.Paschos and Y.L.Wu, Mod.Phys.Lett A6(1991)93.

[10] M.K.Gaillard and B.W.Lee, Phys.Rev. D10(1974)897.

[11] K.G.Chetyrkin et al., Phys.Lett. B174(1986)104;
    A.A.Ovchinnikov and A.A.Pivovarov, Phys.Lett. B207(1988)333.

[12] A.A.Penin and A.A.Pivovarov, Phys.Rev. D49(1994)265.

[13] J.Gasser and H.Leutwyler, Nucl.Phys. B250(1985)465.

[14] R.J.Crewther, Nucl.Phys. B264(1986)277.

[15] M.Voloshin and V.Zakharov, Phys.Rev.Lett. 45(1980)688;
    V.Novikov and M.Shifman, Z.Phys. C8(1981)43.

[16] K.Hikasa et al. [Particle Data Group], Phys.Rev. D45(1992) No. 11-II.

[17] V.M.Belyaev and B.L.Ioffe, Zh.Eksp.Teor.Fiz. 83(1982)876;
    A.A.Ovchinnikov and A.A.Pivovarov, Yad.Fiz. 48(1988)1135.
[18] N.V.Krasnikov, A.A.Pivovarov and N.N.Tavkhelidze, Z.Phys. C19(1983)301.

[19] A.Manohar and H.Georgi, Nucl.Phys. B234(1984)189;
    J.F.Donoghue, E.Golowich and B.R.Holstein, Phys.Rev. D30(1984)587.