A Phenomenological Analysis of Higher Fock State Contributions to the $\chi_{cJ}$ Decays

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Abstract

We present a phenomenological analysis of higher Fock state contributions to the $\chi_{cJ}$ decays by using the recent BES experimental data. It is found that the higher Fock state $|(c\bar{c})_8g>$ makes an important contributions to the inclusive and exclusive processes with respect to that from the valence Fock state $|c\bar{c}>$ of the $\chi_{cJ}$ and some constraints of these contributions are obtained for the $\chi_{c0}$ and $\chi_{c2}$ states in order to fit the experimental data.

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1. Introduction

Recently, BES has reported about 30 channels of $\chi_{cJ}$ hadronic decay$^{[1]}$ and the decays to $\pi\pi, KK$ among them have improved greatly with high precision$^{[2]}$. For example, some branching ratios are shown in the following table.

| Decay channels | BES ($\times 10^{-3}$) |
|----------------|------------------------|
| $Br(\chi_{c0} \rightarrow \pi^+\pi^-)$ | 4.68 ± 0.26 ± 0.65 |
| $Br(\chi_{c2} \rightarrow \pi^+\pi^-)$ | 1.49 ± 0.14 ± 0.22 |
| $Br(\chi_{c0} \rightarrow K^+K^-)$ | 5.68 ± 0.35 ± 0.85 |
| $Br(\chi_{c2} \rightarrow K^+K^-)$ | 0.79 ± 0.14 ± 0.13 |

$\chi_{cJ}$ are the P-wave bound state of $c\bar{c}$ and can be treated approximately as a non-relativistic system due to the large mass of the charm quark. In such a system, the charm quark and anti-charm quark are coupled to light quarks by two-gluon exchange which has large momentum flow. Then these light quarks will build up the products of the decay, i.e. light hadrons. The total hadronic widths of $\chi_{cJ}$ are approximately equal to the widths of the decays into two gluons$^{[3]}$, 

$$
\Gamma^{(1)}_{tot}(\chi_{c0}) = \frac{6 \alpha_s^2(M_{cJ}^2)}{M_c^4} \left|R'_p(0)\right|^2
$$

$$
\Gamma^{(1)}_{tot}(\chi_{c2}) = \frac{8 \alpha_s^2(M_{cJ}^2)}{5M_c^4} \left|R'_p(0)\right|^2
$$

where $R'_p(0)$ is the derivative of the radial wavefunction of the heavy quarkonium at origin. Here we only consider the valance Fock state contribution. Therefore the ratio of the total decay widths

$$
R \equiv \frac{\Gamma_{tot}(\chi_{c0})}{\Gamma_{tot}(\chi_{c2})} \cong \frac{\Gamma^{(1)}_{tot}(\chi_{c0})}{\Gamma^{(1)}_{tot}(\chi_{c2})} = \frac{15}{4}
$$

after neglecting the $v^2$ corrections. It was assumed that the single non-perturbative quantity $R'_p(0)$ includes the long-distance effects. However, Barbieri et al.$^{[4]}$ found...
that the coefficients of $| R'_p(0) |^2$ depend logarithmically on an infrared cutoff on the energies of the final-state gluons. Bodwin et. al.\cite{5} pointed out that to calculate the coefficients of $| R'_p(0) |^2$ to relative order $\alpha_s^n(\alpha_s(M v))$ should include all operators whose matrix elements are of relative order $v^n$ or less. Thus the annihilation rate of the $\chi_{cJ}$ into light hadrons becomes\cite{6}

$$\Gamma(\chi_{cJ}) = \sum_n \frac{2 \text{Im} f_n(\Lambda)}{M_{d_n-4}} < \chi_{cJ} | O_n(\Lambda) | \chi_{cJ} > \quad (4)$$

and the dependence on the arbitrary factorization scale $\Lambda$ in Eq.(4) cancels between the coefficients and the operators. The P-wave $\chi_{cJ}$ state can be expressed as

$$| \chi_{cJ} > = O(1) | (c\bar{c})_1(3P_J) > + O(v) | (c\bar{c})_8(3S_1)g > + O(v^2) \quad (5)$$

As keeping the first term in Eq.(5), Eq.(4) becomes Eqs.(1) and (2). They also show that the inclusion of the higher Fock states in the factorization formulas removes the dependence of the decay rate on an arbitrary infrared cutoff. Eq.(4) gives the corrections to the leading contribution Eqs.(1) and (2).

Now we consider the case of exclusive decays of $\chi_{cJ}$, such as $\chi_{cJ} \rightarrow \pi\pi, KK, \ldots$. Following the framework of calculations of exclusive processes at large momentum transfers\cite{7}, the decay amplitude of $\chi_{cJ}$ can be factorized into two parts: a hard amplitude $T_H$ calculable in perturbative QCD, and the distribute amplitudes $\phi_H$ for each hadron $H$. For example, taking the valence Fock state in Eq.(5), the decay width of $\chi_{cJ} \rightarrow \pi\pi$ is given by

$$\Gamma^{(1)}(\chi_{c0} \rightarrow \pi\pi) = \frac{C^2[4\pi \alpha_s(M^2)]^4}{2\pi^2 \cdot 32M^8} | R'_p(0) |^2 | I_0^\pi |^2 \quad (6)$$

and

$$\Gamma^{(1)}(\chi_{c2} \rightarrow \pi\pi) = \frac{C^2[4\pi \alpha_s(M^2)]^4}{5\pi^2 \cdot 32M^8} | R'_p(0) |^2 | I_2^\pi |^2 \quad (7)$$

where

$$I_0^\pi = \int_0^1 dx \int_0^1 dy \phi_\pi(x, Q^2) \frac{2 + \frac{(x-y)^2}{x + y - 2xy}}{x(1-x)(x+y-2xy)y(1-y)} \phi_\pi(y, Q^2) \quad (8)$$

and

$$I_2^\pi = \int_0^1 dx \int_0^1 dy \phi_\pi(x, Q^2) \frac{1 - \frac{(x-y)^2}{x + y - 2xy}}{x(1-x)(x+y-2xy)y(1-y)} \phi_\pi(y, Q^2) \quad (9)$$

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and

\[ C = \left( \frac{1}{\sqrt{n_c}} \right)^3 \sum_{AB} |Tr(T^AT^B)|^2 = \frac{2}{3\sqrt{3}} \]  

(10)

is the color factor. The distribution amplitude \( \phi_\pi(x,Q^2) \) in Eqs.(8) and (9) is defined by the valence wave function \( \psi_{qq/\pi}(x,k_\perp) \) in the light-cone\[^8\],

\[ \phi_\pi(x,Q^2) = \int_0^{Q^2} d^2k_\perp \psi_{qq/\pi}(x,k_\perp). \]  

(11)

Usually, one assumes that higher Fock state contributions are suppressed by \( v \) with respect to that from the valence Fock state \(|(c\bar{c})_1>\). It was shown\[^6\] this suppression for inclusive decays of \( \chi_{cJ} \) does not hold and the contribution from the color-octet \(|(c\bar{c})_8(3S_1)g>\) (i.e. the one arising from the higher Fock state \(|c\bar{c}g>\) with \( c\bar{c} \) in a color-octet state) in Eq.(5) is not suppressed by \( v \) at all. Furthermore, Ref.[9] argue that for exclusive \( \chi_{cJ} \) decays the color-octet contribution is not suppressed by powers of either \( v \) or \( 1/M_c \) too.

After taking into account for the higher Fock state contribution we can write down the decay width of the process \( \chi_{cJ} \to \pi\pi \),

\[ \Gamma(\chi_{cJ} \to \pi\pi) = \Gamma^{(1)}(\chi_{cJ} \to \pi\pi) + \Gamma^{(8)}(\chi_{cJ} \to \pi\pi), \]  

(12)

where \( \Gamma^{(8)}(\chi_{cJ} \to \pi\pi) \) is determined by the color-octet contribution. It will be estimated by the wave function of the color-octet \(|(c\bar{c})_8g>\) and there are more uncertainties to calculate the color-octet contribution.

2. Phenomenological analysis to the decay widths

It is shown that the color octet of \( \chi_{cJ} \) will make contributions to the decay width of them. However we haven’t a good framework to calculate their contributions precisely yet at present. In order to estimate them phenomenologically we rewrite the decay widths of \( \chi_{cJ} \) as

\[ \Gamma(\chi_{cJ}) = \Gamma^{(1)}(\chi_{cJ})[1 + \Delta_J] \]  

(13)

\[ \Gamma(\chi_{cJ} \to \pi\pi) = \Gamma^{(1)}(\chi_{cJ} \to \pi\pi)[1 + a^\pi] \]  

(14)
\[ \Gamma(\chi_{cJ} \rightarrow KK) = \Gamma^{(1)}(\chi_{cJ} \rightarrow KK) [1 + a^K_J] \] (15)

where \( \Gamma^{(1)}(\chi_{cJ}) \), \( \Gamma^{(1)}(\chi_{cJ} \rightarrow \pi\pi) \) and \( \Gamma^{(1)}(\chi_{cJ} \rightarrow KK) \) are determined by the color-singlet state of \( \chi_{cJ} \). All of contributions from the color-octet state of \( \chi_{cJ} \) are included in the parameters \( \Delta_J, a^\pi_J \) and \( a^K_J \). \( \Gamma^{(1)}(\chi_{c0}) \) and \( \Gamma^{(1)}(\chi_{c2}) \) are determined by Eqs.(1) and (2). Then we have the ratio \( \Gamma \)

\[ R \equiv \frac{\Gamma_{tot}(\chi_{c0})}{\Gamma_{tot}(\chi_{c2})} = \frac{\Gamma^{(1)}_{tot}(\chi_{c0})(1 + \Delta_0)}{\Gamma^{(1)}_{tot}(\chi_{c2})(1 + \Delta_2)} \]

(16)

and the branching ratios

\[ Br(\chi_{c0} \rightarrow \pi^+\pi^-) = \frac{8\pi^2}{81} \frac{\alpha_s^2(M_c^2)}{M_c^4} |I^\pi_0|^2 \frac{1 + a_0^\pi}{1 + \Delta_0} \]

(17)

\[ Br(\chi_{c2} \rightarrow \pi^+\pi^-) = \frac{4\pi^2}{27} \frac{\alpha_s^2(M_c^2)}{M_c^4} |I^\pi_2|^2 \frac{1 + a_2^\pi}{1 + \Delta_2} \]

(18)

\[ Br(\chi_{c0} \rightarrow K^+K^-) = \frac{8\pi^2}{81} \frac{\alpha_s^2(M_c^2)}{M_c^4} |I^K_0|^2 \frac{1 + a_0^K}{1 + \Delta_0} \]

(19)

\[ Br(\chi_{c2} \rightarrow K^+K^-) = \frac{4\pi^2}{27} \frac{\alpha_s^2(M_c^2)}{M_c^4} |I^K_2|^2 \frac{1 + a_2^K}{1 + \Delta_2} \]

(20)

Thus we eliminate the uncertainty of \( R'(0) \) in Eqs.(16-20). In fact, the parameters \( \Delta_J, a^\pi_J \) and \( a^K_J \) may include all of contributions from the higher Fock state and the higher order terms. From the experiments

\[ \Gamma_{tot}(\chi_{c0}) = 14.3 \pm 2.0 \pm 3.0 \text{ MeV} \]
\[ \Gamma_{tot}(\chi_{c2}) = 2.00 \pm 0.18 \text{ MeV} \]

follows

\[ \frac{1 + \Delta_0}{1 + \Delta_2} \sim 1 - 3 \] (21)
which leads an inequality

$$\Delta_0 > \Delta_2$$  \hspace{1cm} (22)

Recent BES measurement gets the ratios of the branching fractions,

$$\frac{Br(\chi_{c0} \to \pi^+\pi^-)}{Br(\chi_{c0} \to K^+K^-)} = 0.82 \pm 0.15$$

and

$$\frac{Br(\chi_{c2} \to \pi^+\pi^-)}{Br(\chi_{c2} \to K^+K^-)} = 1.88 \pm 0.51$$

Comparing the experimental data with Eqs.(17-20), we can obtain some constraints on the parameters $a_\pi^J$ and $a^K_J$,

$$\frac{|I_0^\pi|^2 (1 + a_0^\pi)}{|I_0^K|^2 (1 + a_0^K)} = 0.82 \pm 0.15 \hspace{1cm} (23)$$

$$\frac{|I_2^\pi|^2 (1 + a_2^\pi)}{|I_2^K|^2 (1 + a_2^K)} = 1.88 \pm 0.51 \hspace{1cm} (24)$$

If we ignore the SU(3) symmetry breaking of the $\pi$ and $K$ wave functions, Eqs.(23) and (24) tell us one constraint

$$a_0^\pi \simeq a_0^K \hspace{1cm} (25)$$

for the $\chi_{c0}$ state and another constraint

$$a_2^\pi > a_2^K \hspace{1cm} (26)$$

for the $\chi_{c2}$ state. Therefore the present experimental data have put some constraint on the color-octet contributions. In order to fit the BES data it requires that (1) the contributions to the $\chi_{c0}$ inclusive decay are larger than the contributions to $\chi_2$ from the higher Fock states i.e. $\Delta_0 > \Delta_2$; (2) For the $\chi_{c0}$ state, the corrections to the process $\chi_{c0} \to \pi\pi$ from the higher Fock states is approximately equal to the corrections to the process $\chi_{c0} \to KK$; (3) For the $\chi_{c2}$ states, the corrections to the process $\chi_{c2} \to \pi\pi$ is larger than the corrections to the process $\chi_2 \to KK$ from the higher Fock states.
3. Numerical estimate to the valence and higher Fock state contribution

Now let us calculate the valence Fock state contribution, such as $\Gamma^{(1)}(\chi_{cJ})$, $\Gamma^{(1)}(\chi_{cJ} \rightarrow \pi\pi)$ and $\Gamma^{(1)}(\chi_{cJ} \rightarrow KK)$ in the leading order. The results, according to the Eqs.(7-9), are all dependent on the distribution amplitude $\phi(x, Q^2)$ of the pion and kaon which are independent of the processes apart from the energy scale $Q^2$. The $\phi_M(x, Q^2)$ of the meson is defined by Eq.(11) and $\psi_{q\bar{q}/M}(x, k_\perp)$ is the valance wavefunction of the meson in the light-cone framework. The general solution of the QCD evolution equation\[8\]

$$\phi_M(x, Q^2) = x(1-x) \sum_{n=0}^{\infty} a_n(Q_0^2) C_n^{\alpha_s/2}(2x-1)(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)})^n$$  \hspace{1cm} (27)$$

and the coefficients $a_n(Q_0^2)$ are dependent upon the initial distribution amplitude $\phi_M(x, Q_0^2)$ through its definition

$$a_n(Q_0^2) = \frac{2(2n+3)}{(2+n)(1+n)} \int_{-1}^{1} d(2x-1) C_n^{\alpha_s/2}(2x-1) \phi_{\pi}(x, Q_0^2)$$  \hspace{1cm} (28)$$

with $a_0 = \sqrt{3} f_M$. $f_M$ is the decay constant of the meson. As $Q^2 \rightarrow \infty$, $\phi_M(x, Q^2)$ goes to the asymptotic form $\phi_M^{as}(x)$

$$\phi_M^{as}(x) = \sqrt{3} f_M x(1-x).$$  \hspace{1cm} (29)$$

If employing the asymptotic form, one finds the asymptotic values of the branching ratios of the $\chi_{cJ}$ are much smaller than the experimental values. In order to fit the data one possible solution is to take a much wider distribution amplitude$^{[10]}$. For example, from the Chernyak-Zhitnisky (CZ) form$^{[11]}$

$$\phi_M(x) = \frac{f_M}{\sqrt{3}} 15(2x-1)^2 x(1-x)$$  \hspace{1cm} (30)$$

the valence Fock state contribution can be consistant with experimental values very well without any color-octet contribution. However the recent studies on the processes involving the pion disfavor the CZ distribution amplitude$^{[12-15]}$. It follows that the pion distribution amplitude is close to the asymptotic form. Following Refs.$^{[12,16]}$, we take the model wavefunction

$$\psi_{\pi}(x, k_\perp) = A_{\pi} \exp\left[-\frac{k_\perp^2 + m^2}{8\beta^2 x(1-x)}\right]$$  \hspace{1cm} (31)$$

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\[ \psi_K(x, k_\perp) = A_K \exp[-(\frac{k_\perp^2 + m^2}{8\beta^2x} + \frac{k_\perp^2 + m_s^2}{8\beta^2(1-x)})] \]  \hspace{1cm} (32)

with \( A_\pi = 32 GeV^{-1}, A_K = 52.6 GeV^{-2}, \beta = 385 MeV, m = 298 MeV \) and \( m_s = 550 MeV \). It leads the distribution amplitudes

\[ \phi_\pi(x) = \frac{2\beta^2 A_\pi}{(2\pi)^2} x(1-x) \exp(-\frac{m^2}{8\beta^2 x(1-x)}) \]  \hspace{1cm} (33)

and

\[ \phi_K(x) = \frac{2\beta^2 A_K}{(2\pi)^2} x(1-x) \exp(-\frac{1}{8\beta^2}\left(\frac{m^2}{x} + \frac{m_s^2}{1-x}\right)) \]  \hspace{1cm} (34)

Substituting Eqs.(33-34) into (7-9, 17-20) we can get the following numerical results

\[ Br(\chi_{c0} \rightarrow \pi^+ \pi^-) = B_0(0.1109)^2 \frac{1 + a_0^\pi}{1 + \Delta_0} \]  \hspace{1cm} (35)

\[ Br(\chi_{c2} \rightarrow \pi^+ \pi^-) = B_2(0.4538 \times 10^{-1})^2 \frac{1 + a_2^\pi}{1 + \Delta_2} \]  \hspace{1cm} (36)

\[ Br(\chi_{c0} \rightarrow K^+ K^-) = B_0(0.1256)^2 \frac{1 + a_0^K}{1 + \Delta_0} \]  \hspace{1cm} (37)

\[ Br(\chi_{c2} \rightarrow K^+ K^-) = B_2(0.5329 \times 10^{-1})^2 \frac{1 + a_2^K}{1 + \Delta_2} \]  \hspace{1cm} (38)

where

\[ B_0 = \frac{8\pi^2}{81} \frac{\alpha_s^2(M_c^2)}{M_c^4(GeV)^{-4}}, \quad B_2 = \frac{4\pi^2}{27} \frac{\alpha_s^2(M_c^2)}{M_c^4(GeV)^{-4}} \]  \hspace{1cm} (39)

From Eqs.(35-38) follows

\[ \frac{Br(\chi_{c0} \rightarrow \pi^+ \pi^-)}{Br(\chi_{c0} \rightarrow K^+ K^-)} = 0.78 \frac{1 + a_0^\pi}{1 + a_0^K} \]  \hspace{1cm} (40)

and

\[ \frac{Br(\chi_{c2} \rightarrow \pi^+ \pi^-)}{Br(\chi_{c2} \rightarrow K^+ K^-)} = 0.73 \frac{1 + a_2^\pi}{1 + a_2^K} \]  \hspace{1cm} (41)

These ratios are independent of the parameters \( \alpha_s(M_c) \) and \( M_c \). Comparing Eqs.(40-41) with BES experimental values one obtains

\[ a_0^\pi \simeq a_0^K \]  \hspace{1cm} (42)
\[ a_2^\pi \cong 2.58 \quad a_2^K + 1.58 \]  

which means that the higher Fock state contribution depends on the total spin J of the p-wave state \( \chi_{cJ} \). Eqs. (42-43) show that \( a_2^\pi \) is rather larger than \( a_2^K \).

The precise prediction will be determined by the parameters \( \alpha_s(M_c), R'_p(0), M_c, \Delta_J, a^\pi_J \) and \( a^K_J \). In this paper we only apply the phenomenological analysis to the branching ratios to eliminate the parameters \( \alpha_s(M_c), R'_p(0) \) and \( M_c \) for getting some constraints on the parameters \( \Delta_J, a^\pi_J \) and \( a^K_J \).

4. Conclusion

We have presented a phenomenological analysis of higher Fock state contributions to the \( \chi_{cJ} \) decays by using the recent BES experimental data. In this paper we include the higher Fock state contributions to the decays of \( \chi_{cJ} \) beyond the valence Fock state contribution. We parameterize the higher Fock state and higher order contributions as \( \Delta_J \) for inclusive processes and \( a^\pi_J \) and \( a^K_J \) for exclusive processes of the \( \chi_{cJ} \). The recent experimental data requires that (i) For the inclusive process of the \( \chi_{cJ} \), the higher Fock state \( |(c\bar{c})_8g> \) makes an important contribution with respect to that from the valance Fock state \( |c\bar{c}> \) and they are different for the \( \chi_{c0} \) and \( \chi_{c2} \) state, i.e. \( \Delta_0 > \Delta_2 \). (ii) The similar results are obtained for the exclusive processes and we find \( a^\pi_0 \simeq a^K_0 \) for the \( \chi_{c0} \) state and \( a^\pi_2 > a^K_2 \) for the \( \chi_{c2} \) state.
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