Gravitational Wave Signal for Quark Matter with Realistic Phase Transition

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Abstract

The cores of neutron stars (NSs) near the maximum mass realize the most highly compressed matter in the universe where quark degrees of freedom may be liberated. Such a state of dense matter is hypothesized as quark matter (QM) and its presence has awaited to be confirmed for decades in nuclear physics. Gravitational waves from binary NS mergers are expected to convey useful information called the equation of state (EOS). However, the signature for QM with realistic EOS is not yet established. Here, we show that the gravitational wave in the post-merger stage can distinguish the theory scenarios with and without a transition to QM. Instead of adopting specific EOSs as studied previously, we compile reliable EOS constraints from the ab initio approaches. We demonstrate that early collapse to a black hole after NS merger signifies softening of the EOS associated with the onset of QM in accord with ab initio constraints. Nature of hadron-quark phase transition can be further constrained by the condition that electromagnetic counterparts need to be energized by the material left outside the remnant black hole.
Heavy nuclei have a property called saturation and the baryon density in central regions inside nuclei remains almost constant around \( n_0 \sim 0.16 \text{fm}^{-3} \). The corresponding rest-mass density is \( \rho_0 = m_N n_0 \sim 2.7 \times 10^{14} \text{g/cm}^3 \) with \( m_N \) being the nucleon mass. This is already a density of unimaginable order, but in the universe we can find even denser matter compressed with the help of gravitational force. Among various compact stellar objects neutron stars (NSs) are unique systems with complicated structures having its typical mass around \( \sim 1.4M_\odot \), where \( M_\odot \) denotes the solar mass, and its estimated size of \( \sim 10 \text{km} \).

In the NS environment the gravitational force is so strong that the saturation property of nuclear matter (NM) can be overwhelmed and the maximum rest-mass density can reach \( \sim 5\rho_0 \) or even higher. The quantitative determination of the maximum rest-mass density in the NS requires a relation of the pressure in response to gravitational compression. There have been a huge number of theoretical and experimental attempts to extract information about the equation of state (EOS); that is, the pressure \( p \) as a function of the rest-mass density \( \rho \) of NS matter [1–3].

Since seminal works [4, 5] on the hypothetical existence of quark matter (QM) in NSs, there have been disputes on the interpretation of NS observational data. For example, it was argued [6] that EXO 0748-676 would rule out the soft EOS and thus the presence of QM in the NS cores. Soon later, however, a counter argument [7] appeared to claim that EXO 0748-676 can be consistent with some EOS variants in the presence of QM. The discovery of a two-solar-mass NS (PSR J1614-2230) in 2010 [8] gave more decisive impacts to the nuclear physics community. The possible EOS candidates are severely constrained and a strong first-order phase transition up to a certain density has been excluded without doubt [9]. Now, there are similar radio measurements that have confirmed massive NSs: PSR J0348+0432 [10] and PSR J0740+6620 [11]. A more comprehensive test of the EOS candidates with available data including collective properties measured in the terrestrial experiment of heavy-ion collision has also been proposed [12].

In principle, the EOS is to be identified from the first-principles theory of the Strong Interaction, i.e., quantum chromodynamics (QCD) in terms of quarks and gluons [13]. The leading order (LO) in perturbative QCD (pQCD) expansion leads to thermodynamic quantities of a non-interacting quark gas. From the next-to-next-to-LO (N^2LO), a logarithmic term involving the renormalization scale emerges, and the EOS suffers theoretical uncertainty in resumming logarithmic singularities [14]. The possible EOS window including the strange
quark mass effect has been carefully quantified [15], and the resultant EOS can be reliable at high enough density (see Ref. [16] for the state-of-the-art pQCD EOS). Alternative to QCD at low density in the vicinity of $\sim \rho_0$ is the effective theory approach with pion and nucleon degrees of freedom. The chiral effective theory ($\chi$EFT) is based on the derivative expansion of QCD and it is regarded as an *ab initio* approach [17]. So far, the next-to-next-to-next-to-LO ($N^3\text{LO}$) calculation from the $\chi$EFT has been applied for the nuclear properties [18] and the analysis of NS matter [19].

In this way, the $\chi$EFT at low density and the pQCD at high density tighten the favored region of the EOS variations [20] and the NS observation can further constrain possible parameters. The recent analysis [21] suggests EOS softening at a density around $(5\text{--}6)\rho_0$, which may well be interpreted as the onset of QM in the NS cores. The evidence for QM is, however, not quite conclusive yet and the multimessenger analysis is indispensable [22]. Among various NS observations, gravitational waves from binary NS mergers would be a promising probe into the state of dense matter beyond the presumable QM onset.

In 2017 the LIGO-Virgo collaboration reported the first observation of the gravitational wave from the binary NS merger in GW170817 [23], which was two years later from the first detection of the merger of the binary black holes (BHs) by LIGO. For GW170817, the gravitational wave from the inspiral stage was identified, and the total mass of the system is found to be $2.74^{+0.04}_{-0.01}M_\odot$ (90\% credibility). The tidal deformability was extracted, and the EOS in the intermediate density region has been constrained accordingly [24, 25]. In the future, it is expected that third-generation detectors can find several tens of binary NS mergers per year. They will also enable us to achieve high-signal-to-noise-ratio detection, so that the uncertainty window should be narrowed and the information on the phase transition may eventually be extracted [26]. While it is more challenging to detect the post-merger signals, they are more sensitive to the phase transition to QM. It is, therefore, of utmost importance to make theoretical predictions for the gravitational wave signals at the post-merger stage to seek for a hint of QM. Along these lines, numerical simulations have found that significant effects on the gravitational wave should result from an assumed strong first-order phase transition to QM [27–29]. The multimessenger signals associated with supernova explosion and NS mergers are also discussed [30].

At the same time, a continuous transition (called “crossover” in the QCD context) toward QM is theoretically supported. In the presence of color superconducting states, NM and
QM can have identical global symmetry, implying duality between the confining phase with baryons and mesons and the Higgs phase with quarks and gluons [31] (see also Refs. [32, 33] for discussions on topological phase transition). This idea of continuity can be generalized to a more realistic situation with finite strange quark mass [34]. Moreover, it has been known that NM and QM cannot be distinguished in an idealized world with the infinitely large number of gluon species (i.e., large-$N_c$ limit), and this dual nature of dense matter is referred to as Quarkyonic Matter [35]. It is notable that Quarkyonic Matter can give a natural account [36] for a possible peak in the speed of sound with increasing density as seen from the NS data inference [37, 38]. From this point of view, the merger simulations with preferable EOS candidates with continuous crossover to QM should be important. Recently, numerical simulations with quark-hadron crossover (QHC) EOSs have been reported [39, 40].

We would remind that the \textit{ab initio} constraints have not been properly taken into account for any gravitational wave simulations (see Ref. [44] for the latest analysis of the \textit{ab initio} constraints). Because QCD is the most fundamental theory, if the density is high enough to justify the perturbative expansion, the genuine EOS must eventually meet the quark branch from pQCD. If the phase transition onto the quark branch takes place at higher density, a discontinuous jump is inevitable, leading to a stronger first-order transition. For lower transition densities the observational data can constrain the EOS better. In this way we can classify theory possibilities into several distinct scenarios according to the critical density of the phase transition. Figure 1 summarizes two representative scenarios.

Now, we shall explain the common ingredients for the EOS construction. First of all, we note that the terms, “soft” and “stiff”, refer to the EOS with $p$ relatively low and high, respectively, for a given density, $\rho$, or the energy density, $\varepsilon$. At low density ($\varepsilon \lesssim 0.25$ GeV/fm$^3$ in Figure 1), a blue shaded region labeled as “Nuclear Matter” represents the nuclear branch, that is, a theoretical window predicted from N$^3$LO $\chi$EFT [19]. At high density ($\varepsilon \gtrsim 1$ GeV/fm$^3$ in Figure 1), on the other hand, the pQCD prediction is shown by an orange shaded region (quark branch) labeled as “Quark Matter” on the figure. Although these are reliable constraints on the genuine EOS, we still need to introduce (at least) two parameters corresponding to the validity limits of two branches. Here, one parameter, $\rho_{\text{stiff}}$, represents the upper limit of the $\chi$EFT prediction, above which the EOS must become stiff to support massive NSs. The unknown intermediate region between the nuclear and the quark branches may be parametrized by piecewise polytrope: for our purpose of demonstrating
the distinguishability of crossover transition the intermediate EOS is represented by a single polytrope, \( p(\rho) = K \rho^\Gamma \) with the adiabatic index, \( \Gamma \). Detailed shapes will be constrained by the future NS observations \([45, 46]\). Now, let us look into representative scenarios in theory.
Crossover

Strictly speaking, the category of smooth crossover would not exclude a second-order nor a weak first-order phase transition. The EOS is continued to QM at the density when the intermediate EOS overshoots the quark branch. The quark branch has uncertainty from renormalization scale, but the stiffest edge of the uncertainty band is the most favored; The smoothly connected EOS over the quark branch should support massive NSs and it is very hard to satisfy this condition unless the stiffest one is adopted. This choice is also consistent with the more convergent EOS from the resummed perturbation theory [47]. We have performed extensive analyses of $\rho_{\text{stiff}}$ and $\Gamma$ allowed by two conditions of the causality and the two-solar-mass bound (see Method). For a realistic set of parameters, we have chosen $\rho_{\text{stiff}} = 1.6\rho_0$ and $\Gamma = 3.5$ to draw the solid line in the top panel of Figure 1. It is noteworthy that our choice of the intermediate EOS looks consistent with observational data-driven EOSs based on the Bayesian analysis [26, 42, 43, 48, 49] as well as the deep learning [37, 38].

Strong First-order Phase Transition at High Density

In the first-order phase transition case, one more parameter, $p_{\text{1st}}$, is necessary to specify the critical pressure. The bottom panel in Figure 1 shows an EOS with the strong first-order phase transition for $p_{\text{1st}} = 0.5$ GeV/fm$^3$ with $\rho_{\text{stiff}} = 1.6\rho_0$ and $\Gamma = 3.5$ unchanged. We have numerically confirmed that the NS structure is insensitive to the phase transition up to the maximum-mass configuration for $p_{\text{1st}} \gtrsim 0.4$ GeV/fm$^3$. This is because $p_{\text{1st}} \gtrsim 0.4$ GeV/fm$^3$ is higher than the pressure achievable in the NS cores. The EOS on the quark branch is softer than most of the empirical EOSs that are phenomenologically accepted, and a strong first-order phase transition is unavoidable in order to match the quark branch. Such a scenario of the strong first-order phase transition is logically possible, though it is unlikely that supposedly valid pQCD could describe two competing vacua there.

As quantified in Method, other parameter sets are still possible, but we must first confirm detectable differences for the representative cases with and without crossover; only after this confirmation we are demonstrating in the present work, more systematic survey would make sense. Moreover, we note that a first-order transition in the intermediate density region is disfavored; see Method for more details and also discussions in Ref. [50]. Here, our terminology, “without crossover”, specifically means a possibility of stiff EOSs (as the most
FIG. 2. Plus-mode gravitational waveforms $h_+$ with the EOSs without crossover (left) and with crossover (right). The signals are normalized as dimensionless $Dh_+/m_0$ by the luminosity distance to the source, $D$, divided by the total mass of the system, $m_0$, in unit of $G = c = 1$. The corresponding spectrograms from the short-time Fourier transform are displayed below.

empirical EOSs) followed by a strong first-order phase transition at too high density to affect the gravitational wave signal. Then, for this comparison $p_{1st}$ is no longer a relevant parameter. For actual simulations for the case without crossover, we implicitly assume a large $p_{1st}$ and used the extrapolated stiff EOS with slight modification not to violate the causality; $\Gamma = 3.5 \rightarrow 2.9$ in the high density region corresponding to the quark branch in the crossover scenario.

Now, we come to discussions of our central results in Figure 2, where a merger simulation is performed with a numerical relativity code SACRA [51]. The upper and lower panels show
the gravitational waves from binary NS mergers with equal masses, 1.375\(M_\odot\)-1.375\(M_\odot\), and unequal masses, 1.55\(M_\odot\)-1.2\(M_\odot\), respectively, chosen to be consistent with GW170817 [23].

The left panels in Figure 2 show the gravitational wave expected with the EOS without crossover, i.e., a conventional EOS without rapid softening at detectable density, while the right panels are results in the crossover scenario. In both scenarios, a remnant massive NS is transiently formed after merger. Because the EOS in the intermediate density is characterized by a large value of \(\Gamma\), the remnant exhibits a non-axisymmetric, ellipsoidal structure [52]. Thus, the maximum density of the system increases only moderately right after merger, and the gravitational collapse does not occur immediately irrespective of the crossover. When the density eventually grows up above the QM onset due to the hydrodynamical angular momentum transport and gravitational-wave emission, the EOS softening associated with the quark branch makes the transient NS collapse into a BH at \(\approx 7-8\) ms after the first bounce of merging NSs in our models. The time scale to the gravitational collapse depends primarily on the total mass of the binary, which can be extracted from the inspiral waveform with reasonable accuracy as in the case of GW170817 [23], but only weakly on the mass ratio of the system, the strength of the thermal effect (characterized by the thermal index \(\Gamma_\text{th}\); see Method), and the grid resolution. By contrast, if the transition to QM does not set in, the remnant collapses (if possible) only after the long time scale of magnetic dipole radiation (see, e.g., Ref. [53]) thanks to the large maximum mass of the NS. Accordingly, in this case, the BH formation cannot be identified by the gravitational-wave signal.

In the lower panels of Figure 2, the corresponding spectrograms from the short-time Fourier transform are shown. It is evident that the high frequency components are enhanced when the gravitational collapse occurs, which signifies EOS softening induced by the QM onset. From this comparison of the gravitational-wave patterns we conclude that the future measurement of the lifetime of the remnant NS after merger and the comparison to systematic simulations can constrain the presence/location of the softening point in the EOS (QM onset).

We finally emphasize that the crossover scenario is consistent with multimessenger observations of GW170817. Specifically, it is widely recognized that the associated kilonova,

\footnote{One may think that the softening could be caused by hadronic effects such as hyperons, but according to Hagedorn’s picture of deconfinement, the crossover is approached by liberation of more and more massive degrees of freedom; see Ref. [54] and references therein.}
FIG. 3. Remnant mass on the material outside the apparent horizon of the BH as a function of the time after the gravitational collapse, $t_{\text{collapse}}$.

AT 2017gfo, requires ejection of $\approx 0.05M_\odot$ because of its luminosity [55, 56]. For the total mass of $2.75M_\odot$, such a substantial material can be ejected by unequal-mass systems even if the crossover-induced collapse sets in. Figure 3 shows the mass of material left outside the apparent horizon (one variant of BH horizons), $M_{r>r_{\text{AH}}}$. As the mass of the dynamical ejecta is $< 0.01M_\odot$ for the current models, $M_{r>r_{\text{AH}}}$ needs to be larger than $\approx 0.05M_\odot$ as a minimal requirement unless the mass ejection is extremely efficient for $\approx 7$–$8$ ms between the first bounce and the gravitational collapse. While the equal-mass binary violates this criterion at $\approx 15$ ms after the gravitational collapse, the unequal-mass binary may sustain $\gtrsim 0.05M_\odot$ much longer, giving greater potential for ejecting material responsible for AT 2017gfo. The precise mass and other characteristics of ejected material may be clarified by high-resolution simulations incorporating magnetic fields and neutrinos (see, e.g., Ref. [57]), which we leave as a topic for future studies along with calculations of nucleosynthesis and electromagnetic emission. In the coming future, the precise measurements of binary masses with the gravitational-wave signals together with the characterization with electromagnetic counterparts for various mergers with different masses will enable us to constrain the EOS at high density. Thus, multimessenger observations will further delineate properties of QM and the nature of transition.
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ACKNOWLEDGMENTS

We thank Mark Alford, Aleski Kurkela, Sanjay Reddy, and Wolfram Weise for discussions. This work was partially supported by Japan Society for the Promotion of Science (JSPS) KAKENHI Grant No. 20J10506 (YF), 18H01211, 22H01216 (KF), 20K14513, 20H05639, 20H00158 (KH), 18H05236, 20H00158, 22K03617 (KK), and JST FOREST Program Grant No. JPMJFR2136 (KH).

AUTHOR CONTRIBUTIONS

Y.F. made crucial suggestions on the EOS parametrization and checked the EOS calculation consistency. K.F. designed the EOS parametrization and performed the numerical calculations of the EOSs. K.H. gave discussions on the gravitational wave signals at the inspiral stage and reformatted the spectrograms. K.K. performed the numerical simulation for the gravitational wave signals and analyzed the ejected and the remnant masses. All authors equally contributed to writing the manuscript.

AUTHOR INFORMATION

The authors declare no competing financial interests. Interested readers can send comments to K.F. (fuku@nt.phys.s.u-tokyo.ac.jp) or Y.F. (yfuji@uw.edu) for the EOS construction and to K.H. (kentah@g.ecc.u-tokyo.ac.jp) or K.K. (kyutoku@tap.scphys.kyoto-u.ac.jp) for the simulation of numerical relativity.
TABLE I. Parameters in the polytropic EOS; the pressure unit is dyne/cm² and the rest-mass density unit is g/cm³. The stiffening density is fixed as \( \rho_{\text{stiff}} = 1.6 \rho_0 = 10^{14.632255} \text{ g/cm}^3 \). The crust parameters are adopted from Ref. [58].

| i          | \( \rho_i \)     | \( \Gamma_i \) | \( K_i \)  |
|------------|------------------|----------------|------------|
| Crust      | 1.02148E+14      | 1.35692        | 3.9987E−8 |
| \( \chi \)EFT | \( \rho_{\text{stiff}} = 4.28800E+14 \) | 2.64258 | 3.50290E−5 |
| Interpolated| 8.55829E+14      | \( \Gamma = 3.5 \) | 9.96414E−18 |
| pQCD       | —                | 1.45303        | 3.66934E+13 |

METHODS

Piecewise Polytropic EOS:

The pressure in a piecewise density window \([\rho_{i-1}, \rho_i]\) is given by

\[
p(\rho) = K_i \rho^{\Gamma_i},
\]

and the continuity of the pressure to the adjacent segment requires,

\[
K_{i+1} = \frac{p(\rho_i)}{\rho_{i+1}} = K_i \frac{\rho_i^{\Gamma_i-1}}{\Gamma_i - 1}.
\]

The energy density, \( \varepsilon = -p + \mu n \), involves an integration constant, \( a_i \), and is written as

\[
\frac{\varepsilon}{\rho} = \frac{K_i}{\Gamma_i - 1} \rho^{\Gamma_i-1} + 1 + a_i.
\]

Following the prescription in Ref. [58], the integration constant, \( a_i \), is fixed by the condition to make \( \varepsilon \) continuous, i.e.,

\[
a_{i+1} = a_i + \frac{\Gamma_i+1 - \Gamma_i}{(\Gamma_{i+1} - 1)(\Gamma_i - 1)} K_i \rho_i^{\Gamma_i-1}.
\]

We note that the polytropic EOS is a function of \( \rho \), not of the chemical potential \( \mu \), and this is why the integration constant is needed; see Ref. [50] for related discussions.

Constraining the Realistic EOS:

The EOS from N³LO \( \chi \)EFT can be well fitted by the polytropic form with the parameters in Table I. Actually the uncertainty band allows for the soft and the stiff edges, respectively, with \( \Gamma \simeq 2.378 \) and 2.832, and we adopt the middle value as a representative. In the same
Figure S1 displays the allowed region of parameters. The upper region (i.e., large-\(\Gamma\) region) shaded with gray color is excluded from the causality bound that the sound speed, \(c_s\), must not exceed the speed of light up to the maximum-mass configuration. Two dashed lines represent the uncertainty band from the \(\chi\)EFT EOS (and the quark branch is fixed at the stiffest edge). The lower region with small \(\Gamma\) is bounded by the maximum mass of the NS, which has a width associated with the uncertainty band from the \(\chi\)EFT EOS as well. The blue shaded region represents the condition that the maximum NS mass is \(2M_\odot\). The thick line in the middle of the shaded region corresponds to our choice of the parameters for the \(\chi\)EFT EOS and the cross symbol to our choice of \(\rho_{\text{stiff}}\) and \(\Gamma\) as given in Table I. The NS mass from our EOS is consistent with pulsar observations within observational errors. More systematic studies on the choice of parameters are left for future studies. If the maximum
mass is further raised to $2.05M_\odot$ and $2.1M_\odot$, the EOS should be stiffer and $\Gamma$ should be larger accordingly as shown by the purple dashed line and the red dotted line, respectively.

In conventional EOSs, the reasonable values of $\Gamma$ ranges around 2–4. For example, the comprehensive EOS list is found in Ref. [58], from which we took $\Gamma$’s of several representative EOSs at the second density segment, $\rho = (1.85–3.7)\rho_0$; $\Gamma = 3.791$ for WFF1, $\Gamma = 3.445$ for AP4, and $\Gamma = 2.988$ for SLy are shown in Figure S1 for reference. The present analysis aims to quantify visible impacts from the EOS softening and we choose $\Gamma = 3.5$ at $\rho_{\text{stiff}} = 1.6\rho_0$ for our demonstration purpose.

We can perform allowed parameter search in a similar fashion for the EOS with a first-order phase transition. In this case the first-order phase transition point, $p_{\text{1st}}$, is involved in addition to $\rho_{\text{stiff}}$ and $\Gamma$. To draw Figure S2, $\rho_{\text{stiff}} = 1.6\rho_0$ is chosen and the allowed region of $p_{\text{1st}}$ and $\Gamma$ is plotted. If $p_{\text{1st}}$ is higher than $\sim 0.4\text{GeV/fm}^3$, the lower bound of $\Gamma$ is almost insensitive to $p_{\text{1st}}$ because the maximum pressure of the NS would no longer reach $p_{\text{1st}}$. Also, if the stiff EOS from the nuclear branch is extended to the higher density, the causality bound is violated at smaller $\Gamma$ and the allowed region becomes narrower as seen in Figure S2. Therefore, the allowed window is already very limited. The Bayesian analysis can estimate the likelihood and the first-order phase transition is disfavored in the detectable
region [50]. If $p_{1st}$ is large, in contrast, even the gravitational wave from the NS merger is not influenced by phase transitions and the parameter search for $p_{1st} \gtrsim 0.5 \text{GeV}/\text{fm}^3$ is not crucial in practice.

**Thermal Index:**

After the collision of NSs, shock interactions increase the temperature to a few tens of MeV. Because the cooling time scale due to neutrino emission is as long as $\sim 1 \text{s}$, it is necessary to incorporate thermal effects at least approximately. The thermal correction is included through

$$p = p_{\text{cold}} + p_{\text{th}}, \quad \varepsilon = \varepsilon_{\text{cold}} + \varepsilon_{\text{th}},$$

where the first term is the zero-temperature part and the second term represents the thermal effect. In numerical simulations, hydrodynamic evolution equations determine the values of $\varepsilon$ and $\rho$. Using the zero-temperature EOS, we may readily derive $p_{\text{cold}}$ and $\varepsilon_{\text{cold}}$ from $\rho$. Then, the thermal index is introduced from an ideal-gas Ansatz:

$$p_{\text{th}} = (\Gamma_{\text{th}} - 1) \rho \varepsilon_{\text{th}}.$$  

The conventional choice of $\Gamma_{\text{th}}$ is within the range of 1.5–1.8, where a larger value typically gives a larger thermal effect and a longer lifetime of the remnant NS. The $\chi$EFT gives a theory estimate of $\Gamma_{\text{th}}$ depending on the density. According to Ref. [59] a peak is located around $\Gamma_{\text{th}} \sim 1.75$ around the density $\sim 0.5 \rho_0$ and $\Gamma_{\text{th}}$ decreases toward $\sim 1$ around the density $\sim 2 \rho_0$ (see also Ref. [60] for model calculations of $\Gamma_{\text{th}}(\rho)$). In the present work the gravitational collapse makes a sharp contrast to signify QM, and to strengthen our argument, we choose the most conservative value of $\Gamma_{\text{th}} = 1.75$ for our simulation presented here. This choice is compatible with the preceding studies [27, 29]. We have numerically checked that the qualitative behavior in the post-merger dynamics is not changed for $\Gamma_{\text{th}} = 1.5$. In reality density-dependent $\Gamma_{\text{th}}$ could be even smaller in some density range. Besides, the effects of magnetic fields and neutrinos are also likely to decrease the survival time of the remnant after merger.

**Simulation:**

2 If $\Gamma_{\text{th}} = 2$ is used, the thermal pressure can sustain the massive NS after the merger as presented in Ref. [39] and the BH formation may not be seen shortly after merger. However, this choice of $\Gamma_{\text{th}} = 2$ in Ref. [39] is motivated for a specific purpose to enhance the $f_2$ signal.
Numerical simulations of binary NS mergers are performed with a numerical-relativity code, SACRA [51]. This code has been well-tested and used for simulating compact object binaries (see, e.g., Ref. [53]). Specifically, we solve the Einstein equation for gravity and ideal hydrodynamics equations for matter. Formulation adopted in this work is described in Ref. [61], in which the method for constructing initial data is also presented. Because we focus on the short time scale of $\sim 20\text{ ms}$ after the first bounce, magnetic fields or neutrino radiation transfer are not incorporated.

We mainly simulate two binary models with the total mass being fixed to $2.75M_\odot$ motivated by GW170817 [23]: an equal-mass binary with $1.375M_\odot-1.375M_\odot$ and an unequal-mass binary with $1.55M_\odot-1.2M_\odot$. To check that our numerical results are insensitive to the grid resolution, we performed two simulations for each model. The finest grid spacing in our adaptive-mesh-refinement structure is $\approx 190\text{ m}$ and $\approx 280\text{ m}$ for high- and low-resolution runs, respectively. In all the models, the outer boundary is located at $\approx 3000\text{ km}$ from the center of mass with the aid of the adaptive-mesh-refinement algorithm. Gravitational waves are extracted at $\approx 150\text{ km}$ from the center of mass and extrapolated to null infinity as also described in Ref. [61].