Skewon modified electrodynamics

Yakov Itin
Inst. Mathematics, The Hebrew Univ. of Jerusalem and Jerusalem College of Technology, Jerusalem, Israel
E-mail: itin@math.huji.ac.il

Abstract. Premetric electrodynamics is a representation of classical electrodynamics based on topological conservation laws. This model predicts a covariant extension of classical theory by dilaton, axion, and skewon as copartners of photon. In this paper, we report on recent results on skewon modification of classical electrodynamics. We present the skewon modified dispersion relation for electromagnetic wave propagation. It yields several kinds of the birefringence effect of topologically different types. The superluminal character of wave propagation and the Higgs-type effect for the symmetric skewon are indicated. We present the skewon modified photon propagator and discuss the corresponding modification of Coulomb’s law.

1. Introduction

Interior and exterior symmetries play an important role in classical and quantum field theory. Violations of some of these symmetries may be considered as a straightforward way of extension of field theories. Moreover, the dynamical violation of the interior symmetry (Higgs effect) is well-known as a necessary ingredient of the Standard Model.

Many modern field models in loop quantum gravity and string theory predict a modification of the light cone structure expressed by an anisotropic dispersion relation, a birefringent vacuum, and a violation of local Lorentz and CPT invariance. Such violations of the exterior symmetry, if they really exist, must yield crucial modifications of the very basis of physics. Since these models are still very far from their complete form, it is important to have a phenomenological model that predicts the indicated optics phenomena. Moreover, it is possible that even when we will already have a good quantum gravity, we will need a phenomenological model in order to give the observational predictions. It is due to a huge gap between the energies that are available in experiments and the energies for which quantum gravity effects come to be relevant.

Modification of the electrodynamics action by an 1-parametric axion was applied by Wei Tou Ni [1] already in 1977 for studying possible violation of the equivalent principle. In Carroll-Field-Jackiw electrodynamics [2], modified energy-momentum tensor and birefringent dispersion relation for axion model was derived and the astronomical data were applied for deducing the upper magnitude of the axion.

In Kostelecky’s model [3], an extension of the axion model was proposed in the form of a modified Lagrangian

\[ L = -\frac{1}{4} \eta^{ik} \eta^{jl} F_{ij} F_{kl} - \frac{1}{4} \kappa^{ijkl} F_{ij} F_{kl}. \]  

(1)

Here, the fixed set of the coefficients \( \kappa^{ijkl} \) represents the small perturbations of the standard
electrodynamics model. The symmetries of $\kappa^{ijkl}$ essentially restrict the number of independent components. Moreover, the leading term is usually absorbed by a field redefinition, thus 20 independent parameters of $\kappa^{ijkl}$ are left over. An extension of the construction (1) to the standard model, called Standard Model Extension (SME), was worked out. Theoretical work in this model has spanned a vast number of areas that include string theory, gravity theory, quantum field theory and cosmology. Several exceptionally accurate Lorentz tests have recently been conducted, and other experiments are currently underway.

In this paper, we report on an alternative construction termed premetric electrodynamics. Premetric construction can be considered as a model inspired by the ideas from solid state physics and thus it can serve as a bridge between macroscopic and microscopic physics. In this Lorentz violation model, the individual terms can be expressed via the well-known physics notations such as dielectric permittivity and magnetic permeability. Since premetric electrodynamics is based on topological conservation laws instead of a Lagrangian, it predicts an extra set of parameters that are not accounted for in (1). This set forms a pseudotensor of 15 independent components called skewon. In this paper, we discuss some recent results on the skewon modification of classical electrodynamics.

The organization of the paper is as follows: In Section 2, we present a brief account of premetric electrodynamics and introduce our basic notations. In Section 3, we discuss the wave propagation in the skewon modified electrodynamics. We show that the modified dispersion relation predicts superluminal wave motion for all types of the skewon. For the antisymmetric skewon, it describes the birefringence effect similar to one known from optics. For the symmetric skewon, the wavefront is of a topologically different type and the wave propagation is forbidden for small skewon parameters (Higgs-type behavior). In Section 4, we present the skewon modified photon propagator and describe the contribution of a skewon to the Coulomb law. Some additional information on the observational status of the skewon modified electrodynamics and its possible applications to electromagnetism technology is presented in Conclusion section.

2. Premetric electrodynamics

In this section, we give a brief account of premetric electrodynamics. For a comprehensive introduction to this subject, see the book of Hehl and Obukhov [5] and the references given therein. An account of the recent developments in this area is presented in [6].

2.1. Field equations

The basic variables of the premetric electrodynamics are the even 2-form of the field strength $F$ and the odd 3-form of the electric current $J$. Both of them are defined on a 4-dimensional differential manifold without metric and connection.

For an electric current localized on a closed hypersurface $C_3$, the electric charge conservation law is expressed in the integral and differential forms, respectively

$$\int_{C_3} J = 0, \quad dJ = 0.$$  \hfill (2)

For topologically simple spacetime regions, this equation yields existence of an odd 2-form of the excitation $H$

$$dH = J.$$  \hfill (3)

This relation is a differential form representation of the inhomogeneous Maxwell equation.

The homogeneous Maxwell equation is treated as a result of the magnetic flux conservation law. For a closed 2-dimensional surface $C$, one postulates the relations

$$\int_C F = 0, \quad dF = 0.$$  \hfill (4)
With the definitions above the tensorial form of Maxwell’s equations is given by

$$\varepsilon^{ijkl} F_{jk,l} = 0, \quad \mathcal{H}^{ij} = \mathcal{J}^j.$$  (5)

The system (5) is explicitly general relativistic covariant and does not require a metric or a connection for its formulation.

2.2. Constitutive pseudotensor

Field equations (5) describe only the formal topological structure of electromagnetism. They are endowed with a physical content only when a constitutive relation is postulated. We consider linear, local constitutive relation:

$$\mathcal{H}^{ij} = \frac{1}{2} \chi^{ijkl} F_{kl}.$$  (6)

Due to its definition, the constitutive pseudotensor $\chi^{ijkl}$ possess the symmetries

$$\chi^{ijkl} = -\chi^{jikl} = -\chi^{ijlk}.$$  (7)

Hence, in a 4-dimensional space, $\chi^{ijkl}$ has 36 independent components. A multi-component tensor, such as $\chi^{ijkl}$, can be decomposed into the sum of simpler independent sub-tensors with a fewer number of components. In group theory, such a decomposition is derived by applying the Young diagram technique. For the constitutive pseudotensor with the symmetries (7), only 3 different diagrams are relevant

$$\frac{\begin{array}{c} \emptyset \otimes \emptyset \end{array}}{=} \frac{\begin{array}{c} \emptyset \oplus \emptyset \oplus \emptyset \end{array}}{\begin{array}{c} \emptyset \end{array}}$$  (8)

It is a decomposition of the tensor space into the direct sum of invariant subspaces with the dimensions $36 = 20 + 15 + 1$, respectively. This canonical decomposition of the constitutive pseudotensor is unique and irreducible. The decomposition depicted in (8) reads

$$\chi^{ijkl} = (1) \chi^{ijkl} + (2) \chi^{ijkl} + (3) \chi^{ijkl}.$$  (9)

The principal part of 20 independent components corresponds to the first diagram

$$(1) \chi^{ijkl} = \frac{1}{6}\left(2\chi^{ijkl} + 2\chi^{klij} - \chi^{iklj} - \chi^{ijlk} - \chi^{iljk} - \chi^{jkil}\right).$$  (10)

It is a generalization of the standard Maxwell-Lorentz constitutive tensor. Two other parts do not emerge in Maxwell’s theory and present its covariant extension.

The skewon part $^{(2)}\chi^{ijkl}$ of 15 independent components corresponds to the middle diagram of (8). It reads

$$(2) \chi^{ijkl} = \frac{1}{2} \left(\chi^{ijkl} - \chi^{klij}\right).$$  (11)

The axion part $^{(3)}\chi^{ijkl}$ corresponds to the third diagram of (8). It has only 1 component

$$(3) \chi^{ijkl} = \chi^{ijkl} = \alpha \epsilon^{ijkl}.$$  (12)
2.3. Lorentz force, energy-momentum current, and action

In premetric electrodynamics, the Lorentz force density is presented by a twisted covector valued 4-form

\[ f_i = (e_i^j F) \wedge J. \]  \hfill (13)

This expression can be considered as an operational definition of the electromagnetic field strength \( F \).

Energy-momentum current in premetric electrodynamics can be treated as a pre-potential of the Lorentz force density. It is presented by a covector-valued 3-form

\[ \Sigma_i = \frac{1}{2} [F \wedge (e_i^j H) - H \wedge (e_i^j H)]. \]  \hfill (14)

When the constitutive relation (6) with the decomposition (9) is used here the axion part is canceled. The principle and the skewon part contribute to the energy-momentum current. As it was demonstrated in [8], the skewon field contributes non-trivially to the electromagnetic energy. It induces an asymmetric electromagnetic energy-momentum tensor, which can cause specific source term in the Einstein-Cartan-Maxwell system.

The electromagnetic Lagrangian 4-form of the model can be expressed in the premetric form

\[ \Lambda = -\frac{1}{2} F \wedge H. \]  \hfill (15)

Using \( F = dA \) we can derive the field equation \( dH = J \) and the current (14) from the Lagrangian (15). Observe that (15) is equivalent to the Kostelecky’s model (1). The skewon part does not give a contribution to this expression, thus it can be treated as a non-Lagrangian extension of electromagnetism. Moreover, the skewon contribution prevents the conservation of the energy-momentum tensor. Thus skewon part of the energy-momentum tensor can be treated as a dissipative term, that in general cannot be described within the Lagrangian framework, [5], [7], [8].

3. Wave propagation in skewon-modified electrodynamics

In this section, we present the results related to the wave propagation in the skewon modified electrodynamics. For the details, see [9], [11], [12], [13], [14].

3.1. Skewon field

In order to extract the pure skewon contribution to electrodynamics, we forbid the axion part \((^3/2)\chi^{ijkl}\) of the constitutive pseudo-tensor and assume its principal part to be presented in the simplest metric form

\[ ^{(1)}\chi^{ijkl} = g^{ik}g^{jl} - g^{il}g^{jk}. \]  \hfill (16)

Moreover, we assume the flat Minkowski metric \( g^{ij} = \text{diag}(1,+1,+1,+1) \). The skewon part \((^2)\chi^{ijkl}\), however, will be considered in the most general form. Since the skewon has 15 independent components, it can be expressed as a traceless tensor of the second order. We use the definition as in [5]

\[ S_i^j = \frac{1}{4} \varepsilon_{iklm} ^{(2)}\chi^{klmj}. \]  \hfill (17)

The inverse relation reads

\[ ^{(2)}\chi^{ijkl} = \varepsilon^{ijklm} [S_m^l] - \varepsilon^{klm} [S_m^i]. \]  \hfill (18)

These definitions are metric-free. On a space endowed with a metric tensor \( g_{ij} \), the mixed tensor \( S_i^j \) can be replaced by the covariant and contravariant tensors

\[ S^i_j = g^{ik}S_{k}^j, \quad \text{and} \quad S_{ij} = g_{jk}S^j_k. \]  \hfill (19)
These tensors are decomposed into a sum of symmetric and antisymmetric parts. Thus, we can define a special case of a symmetric skewon that satisfies $S_{ij} = S_{ji}$ and of an antisymmetric skewon with $S_{ij} = -S_{ji}$.

3.2. Electromagnetic waves

In order to study the electromagnetic waves in skewon modified electrodynamics, we start with the generic premetric jump conditions, [10]. These conditions are resulted from Maxwell equations and hold on an arbitrary hypersurface $\varphi(x^i) = 0$ in spacetime. In differential forms, they take the form of the wavefront conditions

$$F \wedge d\varphi = 0, \quad H \wedge d\varphi = 0.$$  \hspace{1cm} (20)

The hypersurface $\varphi(x^i) = 0$ is referred to as a wavefront. In the tensorial form, this characteristic system with the wave covector $q_i = \varphi_{,i}$ reads

$$\epsilon^{ijkl} f_{kl} q_j = 0; \quad \chi^{ijkl} f_{kl} q_j = 0$$  \hspace{1cm} (21)

The most general solution of the first equation is written by the use of an arbitrary covector $a_k$

$$f_{kl} = \frac{1}{2} (a_k q_l - a_l q_k).$$  \hspace{1cm} (22)

The covector $a_k$ is an analog of the electromagnetic potential. Substituting (22) in the second equation of (22) we remain with the characteristic equation

$$M^{ik} a_k = 0 \quad \text{where} \quad M^{ik} = \chi^{ijkl} q_k q_j.$$  \hspace{1cm} (23)

Due to the gauge invariance, this equation has a non-trivial solution if and only if its adjoint matrix is equal to zero,

$$\text{Adj}(M^{ij}) = \lambda(q) q_i q_j = 0.$$  \hspace{1cm} (24)

Thus the dispersion relation is represented as

$$\lambda(q) = 0.$$  \hspace{1cm} (25)

With the use of Eq.(9), the characteristic matrix $M^{ik}$ is irreducibly decomposed as

$$M^{ik} = P^{ik} + Q^{ik}$$  \hspace{1cm} (26)

with

$$P^{ik} = (1)\chi^{ijkl} q_j q_l; \quad Q^{ik} = (2)\chi^{ijkl} q_k q_l.$$  \hspace{1cm} (27)

Moreover, $P^{ik}$ is the symmetric part and $Q^{ik}$ is the antisymmetric part of the characteristic matrix $M^{ik}$

$$P^{ik} = M^{(ik)}; \quad Q^{ik} = M^{[ik]}.$$  \hspace{1cm} (28)

We recall that the axion part $(3)\chi^{ijkl}$ does not contribute to $M^{ik}$.

The relation

$$Q^{ij} q_j = 0.$$  \hspace{1cm} (29)

results in the expression

$$Q^{ij} = -\frac{1}{2} \epsilon^{jrs}(Y_r q_s - Y_s q_r) = \epsilon^{ijkl} q_k Y_l$$  \hspace{1cm} (30)

with an arbitrary covector $Y_i$. Consequently, the skewon contribution to the dispersion relation is completely determined by the skewon optic covector $Y_i$. Calculating the adjoint matrix $\text{Adj}(M)$ of the optic tensor $M^{ij} = P^{ij} + Q^{ij}$ we have for the most generic linear constitutive pseudo-tensor $\chi^{ijkl}$, the dispersion relation of the form

$$\Lambda(P) + P^{ij} Y_i Y_j = 0,$$  \hspace{1cm} (31)

where the scalar expression $\Lambda(P)$ is defined via the relation $\text{Adj}(P^{ij}) = \Lambda(P) q_i q_j$. 


3.3. Superluminal wave motion

Using the metric principle part (16) we obtain from Eq.(31) the dispersion relation for the most generic skewon media in the (pseudo) Riemannian vacuum

$$q^4 - Y^2 q^2 + (Y, q)^2 = 0,$$

(32)

where the scalar products are constructed with the use of the metric tensor $g^{ij}$. An analysis of this dispersion relation yields the following remarkable fact [11]:

*In the Lorentz signature metric space, the solution of the skewon modified dispersion relation (32) are spacelike or null.*

This means that the skewon generates superluminal wave motion and thus yields strong violation of Lorentz invariance. This consideration remains completely unaltered in the isotropic medium with two standard parameters $\varepsilon$ and $\mu$. In this case, however, the wave velocity for the skewon modified medium only exceeds the value $\sqrt{\varepsilon \mu}$ instead of the universal speed of light. Such behavior is certainly can not be excluded by any fundamental principle. Thus the skewon part could exist in a material medium as a phenomenological quantity. For instance, it can be applicable for a description of dissipative media. Moreover, the observation of the electromagnetic waves with velocity higher than $\sqrt{\varepsilon \mu}$ could point towards the presence of the skewon.

3.4. Symmetric skewon — Higgs-type effect

On a space endowed with a metric tensor, a special symmetric traceless skewon can be extracted. In 4-dimensional spacetime, it has 9 independent components. For a generic skewon, the dispersion relation (32) can be written in the form

$$q^2 = \frac{1}{2} \left( Y^2 \pm \sqrt{Y^4 - (q, Y)^2} \right).$$

(33)

In order to have a real Lorentz square $q^2$, the expression under the square root must be non-negative. For an antisymmetric skewon, the relation $(q, Y) = 0$ holds identically, thus there are no additional restrictions on the wave propagation. For a symmetric skewon, both terms under the square root are non-zero and the inequality

$$Y^4 - (q, Y)^2 \geq 0$$

(34)

must be imposed. Recall that the covector $Y$ is linear in the skewon matrix $S_{ij}$. If we rescale this tensor, $S_{ij} \rightarrow CS_{ij}$, Eq.(34) takes the form

$$C^2 Y^4 - (q, Y)^2 \geq 0.$$  

(35)

For small values of $C$, the first term approaches zero and the inequality is broken. Consequently, we came to the following result [13]:

*Let the skewon have a non-zero symmetric part. For sufficient small magnitudes of the skewon parameters, there are no solutions of the dispersion relation. Thus there is a gap for values of the skewon parameters near zero, where the wave propagation is forbidden.*

This way we constructed a model of an electromagnetic media with Higgs-type spontaneously broken transparency. This result is in a correspondence with an alternative description presented in [9], where the similar effect is treated as holes in wavefront surface.
4. Photon propagator in skewon electrodynamics

4.1. Propagator for linear response media

In geometrical optic approximation, Eq. (5) could be rewritten as a general system of four linear equations for four variables

$$M^m a_m = j^i.$$  (36)

The matrix $M$ is singular and thus for given functions $M^m(q)$ and $j^i(q)$, the solution of Eq. (2.8) is not unique. A formal solution of the linear system (36) can be written as

$$a_i = D_{ik} j^k.$$  (37)

The tensor $D_{ik}$ is called the propagator tensor. In standard quantum electrodynamics based on pure Minkowski constitutive tensor, it suffices to consider symmetric propagators, $D_{ik} = D_{ki}$. Such symmetry, however, does not represent any physical requirement. Indeed, in the case of a generic linear constitutive relation, the propagator tensor is asymmetric. Using the second adjoint technique [14], we derive the photon propagator in Feynman gauge

$$D_{ij} = \frac{1}{\lambda q^2} B^{jm}_{ijm}.$$  (38)

Here $B_{ijkl}$ is the second adjoint of the matrix $M^{ij}$. It is obtained by removing two rows and two columns from the initial matrix. The function $\lambda(q)$ represents the left-hand side of the dispersion relation (25).

The tensor $B^{jm}_{ijm}$ is quadratic in the entries of the matrix $M^{ij}$, hence it is a homogeneous 4-th order polynomial in the components of $q_i$. Consequently the propagator $D_{ij} \sim q^{-2}$ as the ordinary non-modified photon propagator.

4.2. Propagator for skewon modified electrodynamics

We calculate the second adjoint with the metric principle part (16) and use the representation (30) of the skewon part of the characteristic matrix $M^{ij}$ via the covector $Y_i$. The result is

$$D_{ij} = -\frac{g_{ij}q^2 + \epsilon_{ikm}q^kY^m + Y_iY_j}{q^4 - q^2Y^2 + (q,Y)^2}.$$  (39)

Let us observe some basic properties of this propagator:

1. For a vanishing skewon field, the standard vacuum form of Feynman propagator is reinstated

$$D_{ij} = -\frac{g_{ij}}{q^2}.$$  (40)

2. Skewon propagator has nontrivial symmetric and antisymmetric parts:

$$D_{(ij)} = -\frac{g_{ij}q^2 + Y_iY_j}{q^4 - q^2Y^2 + (q,Y)^2}, \quad \text{and} \quad D_{[ij]} = -\frac{\epsilon_{ikm}q^kY^m}{q^4 - q^2Y^2 + (q,Y)^2},$$  (41)

respectively.

3. In general, the propagator is singular for four roots of the denominator. Thus, there are at most two light cones at every spacetime point – birefringence.

4. In the first order approximation for a small skewon field, Eq. (41) reads

$$D_{(ij)} = -\frac{g_{ij}}{q^2}, \quad \text{and} \quad D_{[ij]} = -\frac{\epsilon_{ikm}q^kY^m}{q^4}.$$  (42)

Thus, the small skewon field contributes only to the antisymmetric part of the propagator.
4.3. Skewon and modified Coulomb law

Using the propagator expression with the skewon modifications, we are able to calculate the corresponding modification of the Coulomb law. For this, we need an expression of propagator in Coulomb gauge. Moreover, in the static case, it is enough to have only the temporal component of the propagator. It takes the form

$$ C_{D_{00}} = \frac{q^2(k^2 - Y_0^2)}{k^4(q^2 - Y^2)} , $$

(43)

where the 3-dimensional notations $q = (\omega, k_\alpha)$ and $Y = (Y_0, Y_\alpha)$ are used.

We consider now an explicit model of the antisymmetric skewon field

$$ S_0\mu = -S_{\mu 0} = \alpha_\mu . $$

(44)

Correspondingly, we are left with

$$ C_{D_{00}} = \frac{1}{k^2} \frac{k^2 - (\alpha, k)^2}{k^2 + (\alpha, k)^2} . $$

(45)

The electromagnetic potential for a point-wise charge $Q$ is calculated by the inverse Fourier transform of (45) and results in

$$ \varphi(r) = \frac{Q}{4\pi r} \left(1 - \alpha^2 + \frac{(\alpha, r)^2}{r^2}\right) . $$

(46)

The corresponding equipotential surfaces are 2-dimensional ellipsoids. They are similar to the wave-front surfaces of the skewon model [13].

5. Results and discussion

In this paper, we discussed some theoretical results related to the skewon modification of the classical electrodynamics. The vacuum birefringence effect presents in all skewon models and can be of topologically different types. The antisymmetric skewon produces two wavefronts similar to those known from anisotropic optics. Alternatively, the symmetric skewon provides very strong modification of the wavefront structure with rotation of the time axis and some forbidden directions of wave propagation - "topological holes".

Another remarkable property is the superluminal wave propagation for all types of the skewon. This fact, probably, indicates that the skewon must be forbidden in vacuum (but not in a medium). The skewon modified photon propagator shows a proper behavior, and can be used for modeling the anisotropic modifications of the Coulomb law.

An important issue in skewon electrodynamics is the estimation of the magnitude of the skewon and its comparison to the observation data. In the recent publications of Ni [16], [17], [18], the first order approximation of the skewon contributions to electrodynamics was worked out. From the dispersion relation, it is shown that the requirement of no dissipation/no amplification in propagation implies that the skewon field must antisymmetric (Type II, in the nomenclature of Ni). For the symmetric (Type I) skewon field, the dissipation/amplification in propagation comes to be proportional to the frequency and the CMB spectrum would deviate from Planck spectrum. This result is in a correspondence with our restrictions coming from the topology of the modified wavefront.

Another interesting issue is the discussion of possible applications of the skewon in engineering electromagnetism. In the publications of Lindell and Sihvola [19],[20],[21],[22], the skewon (and axion) additions to electrodynamics was used for realization the DB and SHDB boundary conditions. This approach provides a deeper understanding of the nature of metamaterials.
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