Fast Radio Bursts from Axion Stars

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(Dated: Dec. 25, 2014)

Fast Radio Bursts have recently been discovered[1–3] at around 1.4 GHz frequency. The durations of the bursts are typically a few milliseconds. The origin of the bursts has been suggested to be extra-galactic owing to their large dispersion measures. This suggests that the large amount of the energies (∼10^{45}GeV ) of the bursts. We discuss that the radiations are circularly polarized when they pass through the magnetospheres of neutron stars.

PACS numbers: 98.70.-f, 98.70.Dk, 14.80.Va, 11.27.+d
Axion, Neutron Star, Fast Radio Burst

Axions are one of the most promising candidates of dark matter. The axions have been shown to form miniclusters with masses ∼10^{-12}M_{⊙} and to become dominant component of dark matter. These axion miniclusters condense to form axion stars. We show a possible origin of fast radio bursts (FRBs) by assuming the axion stars being dark matter: FRBs arise from the collisions between the axion stars and neutron stars. The FRBs are caused by the rapid conversion of the axions into electromagnetic fields under strong magnetic fields. Electric fields are induced on the axion stars under strong magnetic fields of neutron stars. The electric fields parallel to the magnetic fields oscillate with a frequency and make electrons in atmospheres of neutron stars coherently oscillate. Thus, the coherent radiations are emitted. The observed frequencies (∼1.4GHz) of the bursts are given by the axion mass \( m_a \) such as \( m_a/2\pi \simeq 2.3GHz (m_a/10^{-5}eV) \). We show that the radiations can go through the atmospheres since they are transparent for the radiations with the frequencies. The frequencies are affected by cosmological and gravitational red shifts. Owing to the change of relative velocities in the course of neutron stars going through the axion stars, the observed frequencies have finite bandwidth \( \simeq 0.16m_a/2\pi \). It is remarkable that the masses of the bursts are \( \sim 10^{-12}M_{⊙} \) of the axion stars obtained by the comparison of theoretical with observed event rates \( \sim 10^{-3} \) per year in a galaxy is coincident with the masses estimated previously as those of the axion miniclusters. Using these values we can explain observed short durations (∼ms) and large amount of the energies (∼10^{45}GeV) of the bursts. We discuss that the radiations are circularly polarized when they pass through the magnetospheres of neutron stars.

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Fast Radio Bursts have recently been discovered[1–3] at around 1.4 GHz frequency. The durations of the bursts are typically a few milliseconds. The origin of the bursts has been suggested to be extra-galactic owing to their large dispersion measures. This suggests that the large amount of the energies ∼10^{45}GeV/s is produced at the radio frequencies. The event rate of the burst is estimated to be ∼10^{-3} per year in a galaxy. Furthermore, no gamma or X ray bursts associated with the bursts have been detected. To find progenitors of the bursts, several models[4] have been proposed. They ascribe FRBs to traditional sources such as neutron star-neutron star mergers, magnetors, black holes, et al..

A model[5, 6] we consider ascribes FRBs to axions[7], which are one of most promising candidates of dark matter. A prominent feature of axions is that they are converted to radiations under strong magnetic fields. The axions form axion stars known as oscillaton[8] made of axions bounded gravitationally. The axion stars are condensed objects of axion miniclusters[9], which have been shown to be produced after the QCD phase transition and to form the dominant component of dark matter in the Universe. Furthermore, the axion miniclusters have been shown to form the axion stars by gravitationally losing their kinetic energies. Thus, the axion stars are the dominant component of dark matter.

In the paper, by assuming that dark matter is composed of the axion stars, we show that FRBs arise from the collisions between neutron stars and axion stars[5, 6, 10]. Since the axion field \( a \) is real, the field configurations of the axion stars are not static but oscillating. As we show later, the axion stars have small masses such as ∼10^{-12}M_{⊙} with radii ∼10^{2}km. The axion field representing such axion stars has the frequency given by the mass of the axions; \( a(t) \propto \cos(m_a t) \). The axion stars are coherent objects of axions so that oscillating electric fields \( \vec{E}_a(t) \propto a(t)\vec{B} \) are generated on the axion stars under strong magnetic fields \( \vec{B} \). When they collide with neutron stars with strong magnetic fields, the electric fields make electrons in atmospheres[11] of neutron stars coherently oscillate. Thus, the electrons emit coherent dipole radiations with the frequency given by the axion mass. Since the electrons are much dense in the atmospheres, the large amount of radiations with the frequency \( m_a/2\pi \simeq 2.4GHz (m_a/10^{-5}eV) \) can be produced in the collisions. This is our production mechanism of FRBs.

The depth of the hydrogen atmospheres of old neutron stars is of the order of 0.1cm, since the temperatures of the atmospheres are supposed to be of the order of 10^{9}K. The radiations can pass through the atmospheres because they are shown to be optically thin for the radiations with frequency (\( m_a/2\pi \)) ∼2.4GHz (m_a/10^{-5}eV). They can
also pass through magnetospheres of neutron stars. After they pass the magnetospheres, the radiations show circular polarization owing to the absorption of radiations with either right or left handed polarization in the magnetospheres. Our production mechanism of FRBs claims that the radiations of FRBs are monochromatic, but observed frequencies have finite bandwidth 1.4GHz±200MHz. The bandwidth is caused by Doppler effects associated with the change of the relative velocities in the course of neutron stars going through axion stars. According to our production mechanism, there are no radiations with other frequencies associated with the FRBs.

In order to find intrinsic frequencies of the radiations from the axion stars, we need to take into account the effects of cosmological and gravitational red shifts. The observed FRBs arise from surfaces of neutron stars located far from our galaxy. We assume for simplicity that both cosmological and gravitational red shifts of the observed FRBs is given by \( z = 0.3 \) where the mass \( 1.4M_\odot \) and the radius \( R = 10km \) of neutron stars is assumed.

The observations of FRBs constraint the parameters of the axion stars, that is, the mass of the axion and the mass of the axion stars. The observed frequency ( \( \simeq 1.4 \) GHz which corresponds to \( 2.4 \simeq 1.4 \times (1 + z)^2 \) GHz in the rest frame of neutron stars) of FRBs, gives the mass ( \( \simeq 10^{-5} \) eV ) of the axion, while the observed rate of the bursts ( \( \sim 10^{-3} \) per year in a galaxy ) gives the mass ( \( \sim 10^{-12} M_\odot \) ) of the axion stars under the assumption that halo of galaxy is composed of the axion stars. We should stress a remarkable finding that the masses \( 10^{-12} M_\odot \) of the axion stars obtained is coincident with those estimated previously [9] as the masses of the axion miniclusters. With the use of the theoretical formula [10, 13] relating radius \( R_a \) to mass \( M_a \) of the axion stars, we can find the radius \( R_a \sim 10^2 km \) of neutron stars at the period of equal axion-matter radiation energy density. The neutron stars pass through inside of the axion stars. Since the relative velocity \( v_c \) at the collisions is estimated to be a hundred thousand km/s, we find that the durations of FRBs are given by \( 2R_a/v_c \sim \) milliseconds. Furthermore, we show that the axion stars rapidly evaporate into radiations when they touch the atmospheres of neutron stars. Thus, the total amount of the energy of the radiations emitted in the collisions is given by \( 10^{-12} M_\odot (10km/10^2 km)^2 \simeq 10^{43} \) GeV. Therefore, we can explain the observed values of the FRBs such as the short durations and the large amount of the radiation energies.

In the next section we show approximate solutions of axion stars with small masses. We can see that the axion field oscillates with the frequency given by the axion mass. In the section (3), we determine masses of axion stars by comparison of theoretical with observed rates of FRBs. The masses obtained is found to be coincident with those estimated previously as the masses of the axion miniclusters. In the section (4), we discuss the properties of the atmospheres of neutron stars and show how fast axion stars evaporate into radiations when they touch the atmospheres. We summarize our results in the final section (5).

I. AXION STARS

First we would like to make a brief review of axions and axion stars. Axions described by a real scalar field \( a \) are Nambu-Goldstone boson associated with Pecci-Quinn global U(1) symmetry [12]. The symmetry was introduced to cure strong CP problems in QCD. After the breakdown of the Pecci-Quinn symmetry at the period of much higher temperature than 1GeV, axions are thermally produced as massless particles in the early Universe. They are however only minor components of dark matter. Since the axions interact with instanton density in QCD, the potential term \( -f_a^2 m_a^2 \cos(a/f_a) \) develops owing to instanton effects at the temperature below 1GeV; \( f_a \) denotes the decay constant of the axions. Thus, the axion field oscillates around the minimum \( a = 0 \) of the potential. But, the initial value of the field \( a \) at the temperature 1GeV is unknown. It can take a different value in a region from those in other regions causally disconnected. Thus, there are many regions causally disconnected at the epoch around the temperature 1GeV, in each of which the axion field takes a different initial value; energy density is also different. With the expansion of the Universe, the regions with different energy densities are causally connected. Thus, there arises spatial fluctuations of the axion energy density. The nonlinear effects of the axion potential cause the fluctuations with over densities in some regions grow to form axion miniclusters [9] at the period of equal axion-matter radiation energy density in the regions. Their masses have been estimated to be of the order of \( 10^{-12} M_\odot \). Furthermore, these miniclusters condense to form axion stars with gravitationally losing their kinetic energies [10]. Therefore, masses of axion stars are expected to be of the order of \( 10^{-12} M_\odot \).

Now we explain the classical solutions of the axion stars obtained in previous papers [10, 13, 15]. The solutions are found by solving classical equations of axion field \( a(\vec{x}, t) \) coupled with gravity. In particular we would like to obtain spherical symmetric solutions of axion stars with much smaller masses than the critical mass \( M_{\text{max}} \) of the axion stars; axion stars are stable when their masses are smaller than the critical mass \( M_{\text{max}} \sim 0.6 m_{\text{pl}}/m_a \), where \( m_a \sim 5 \times 10^{-5} M_\odot (10^{-5} \text{eV}/m_a) \). The mass is obtained only for free axion field without self-interaction. The gravity of the axion stars with much small masses is much weak so that we may take the space-time metrics given by

\[
d s^2 = (1 + h_t) dt^2 - (1 + h_r) dr^2 - r^2 (d\theta^2 + \sin \theta d\phi^2)
\]  
(1)
Furthermore, as we will see later, the radius $R \ll 1$. It is easy to derive the equations of motion of the axion field and gravity,

\[
(1-h_t)\partial_t^2 a = \frac{\partial_0 h_t - \partial_0 h_r}{2} \partial_0 a + (1-h_r)(\partial_r^2 + \frac{2}{r} \partial_r) a + \frac{\partial_r h_t - \partial_r h_r}{2} \partial_r a - m_a^2 a
\]

\[
\partial_r h_t = \frac{h_r}{r} + 4\pi Gr \left( (\partial_t a)^2 - m_a^2 a^2 + (\partial_0 a)^2 \right)
\]

\[
\partial_r h_r = -\frac{h_r}{r} + 4\pi Gr \left( (\partial_t a)^2 + m_a^2 a^2 + (\partial_0 a)^2 \right)
\]

with the gravitational constant $G$, where we assume the axion potential such that $V_a = -f_a^2 m_a^2 \cos(a/f_a) \approx -f_a^2 m_a^2 + m_a^2 a^2/2$ for $a/f_a \ll 1$. As we will see later, the assumption holds for the axion stars with small masses, e.g. $10^{-12} M_\odot$.

It is well known that there are no static solutions of real scalar fields coupled with gravity. The fields and the metrics oscillate with time. We expand the axion field and the metric such that

\[
a(t,r) = \sum_{n=0,1,2} a_n(r) \cos((2n+1)\omega t) = a(r)\cos(\omega t) + a_1(r)\cos(3\omega t), \ , \ ,
\]

\[
h_{t,r}(t,r) = \sum_{n=0,1,2} h_{t,r}^n(r) \cos(2n\omega t) = h_{t,r}^0(r) + h_{t,r}^1(r) \cos(2\omega t), \ , \ ,
\]

with $a(r) \equiv a_0(r)$. Then, only by taking the terms proportional to $\cos^0(\omega t) = 1$ and $\cos(\omega t)$ in the equations (2), we obtain

\[
-\omega^2 (1-h_t^0 + \frac{h_t^1}{2}) a(r) = -\omega^2 \frac{h_t^1}{2} a(r) + (1-h_r^0)(\partial_r^2 + \frac{2\partial_r}{r}) a(r) - m_a^2 a(r)
\]

\[
\partial_r h_t^0 = \frac{h_0^0}{r} + 2\pi Gr \left( (\partial_t a(r))^2 - m_a^2 a^2(r) + \omega^2 a^2(r) \right)
\]

\[
\partial_r h_r^0 = -\frac{h_0^0}{r} + 2\pi Gr \left( (\partial_r a(r))^2 + m_a^2 a^2(r) + \omega^2 a^2(r) \right)
\]

\[
\partial_r h_t^1 = \frac{h_1^1}{r} + 2\pi Gr \left( (\partial_t a(r))^2 - m_a^2 a^2(r) - \omega^2 a^2(r) \right)
\]

\[
\partial_r h_r^1 = -\frac{h_1^1}{r} + 2\pi Gr \left( (\partial_r a(r))^2 + m_a^2 a^2(r) - \omega^2 a^2(r) \right).
\]

We note that when the gravitational effects vanish i.e. $G \to 0$, there is a solution $a = \tilde{a}_0 \cos(m_a t)$ with $\omega = m_a$ as well as $h_{t,r}^{0,1} = 0$. Since we consider axion stars with small masses, $\omega$ is almost equal to $m_a$; $\omega^2 - m_a^2 \ll m_a^2$. Furthermore, as we will see later, the radius $R_a$ of the axion stars with small masses is very large; $R_a \gg m_a^{-1}$. Thus, the term $\partial_r a(r)$ is much smaller than the term $m_a a(r)$, i.e. $(\partial_r a(r))^2 \ll (m_a a(r))^2$. We may approximate the above equations in the following,

\[
(m_a^2 - \omega^2)a(r) + m_a^2 h_0^0 a(r) = (\partial_r^2 + \frac{2\partial_r}{r}) a(r)
\]

\[
\partial_r h_t^0 \approx \frac{h_0^0}{r}
\]

\[
\partial_r h_r^0 \approx -\frac{h_0^0}{r} + 4\pi Gr m_a^2 a^2(r)
\]

\[
\partial_r h_t^1 \approx \frac{h_1^1}{r} - 4\pi Gr m_a^2 a^2(r)
\]

with $h_t^1 \ll h_{t,r}^0 \ll 1$ and $h_r^1 \ll h_t^1 \ll 1$, since $h_r^1 \sim O(Gr^2((\partial_r a)^2 + (m_a^2 - \omega^2) a^2)$ and the other metrics $h \sim O(Gr^2 m_a^2 a^2)$. 
Therefore, we obtain the equation of the axion field,
\[ -\frac{k^2}{2m_a}a(r) = -\frac{1}{2m_a}(\partial^2_r + \frac{2\partial_r}{r})a(r) + m_a\phi a(r) \] (14)
with \( k^2 \equiv m_a^2 - \omega^2 \), where “gravitational potential” \( \phi \equiv \hbar^2/2 \) satisfies
\[ (\partial^2_r + \frac{2\partial_r}{r})\phi = 2\pi G m_a^2 a^2(r). \] (15)

Obviously, \( k^2/2m_a \) represents a binding energy of the axion bounded to an axion star whose mass \( M_a \) is given by \( M_a = \int d^3x((\partial_\alpha a)^2 + (\partial_r a)^2 + m_a^2 a^2)/2 \simeq \int d^3x m_a^2 a(t, r)^2 = \int d^3x m_a^2 a(r)^2/2 \) with the average taken in time; \( \omega^2 \simeq m_a^2 \). The equation (14) can be rewritten in the limit \( r \to \infty \) as
\[ -\frac{k^2}{2m_a}a(r) = -\frac{1}{2m_a}(\partial^2_r + \frac{2\partial_r}{r})a(r) - \frac{G m_a M_a}{r}a(r). \] (16)

A solution in eq(16) is given by \( a(r) = \tilde{a}_0 \exp(-kr) \) with \( k = G m_a^2 M_a \). Thus, we find that the radius \( R_a = k^{-1} = (G m_a^2 M_a)^{-1} \) of the axion star is much larger than \( m_a^{-1} \) for small mass \( M_a \). We can confirm numerically that the solution in eq(16) represents approximate solutions of the equations (14) and (15). In this way we approximately obtain spherical symmetric solutions,
\[ a(\vec{x}, t) = a_0 f_a \exp(-r/R_a) \cos(m_a t), \] (17)
with \( r = |\vec{x}| \). The solutions represent boson stars made of the axions bounded gravitationally, named as axion stars. The solutions are valid for the axion stars with small masses \( M_a \ll 10^{-5} M_\odot \). The radius \( R_a \) of the axion stars is numerically given in terms of the mass \( M_a \) by
\[ R_a = \frac{m_a^2}{m_{pl}^2} \simeq 260 \text{ km} \left( \frac{10^{-5} \text{eV}}{m_a} \right)^2 \left( \frac{10^{-12} M_\odot}{M_a} \right), \] (18)
with the Planck mass \( m_{pl} \). The coefficient \( a_0 \) can be obtained by using the relations \( M_a \simeq \int d^3x m_a^2 a(r)^2/2 = \pi m_a^2 a_0^2 f_a^2 R_a^2/4 \) and \( m_a \simeq 6 \times 10^{-6} \text{eV} \times (10^{12} \text{GeV}/f_a) \), where the average is taken in time,
\[ a_0 \simeq 0.9 \times 10^{-6}(\frac{10^2 \text{km}}{R_a})^2 \left( \frac{10^{-5} \text{eV}}{m_a} \right). \] (19)

Thus, the condition \( a/f_a \ll 1 \) is satisfied for the axion stars with small mass \( M_a \sim 10^{-12} M_\odot \). Although we have used the mass \( 10^{-12} M_\odot \) for reference, the mass is the one we obtain in the present paper. The mass is much smaller than the critical mass \( M_{\text{max}} \sim 10^{-5} M_\odot \). Thus, the solutions represent stable axion stars. In this way the solutions along with the parameters \( R_a \) and \( a_0 \) can be approximately obtained. Obviously, the axion stars are composed of axions with much small momenta \( \sim 1/R_a \).

Here we should make a comment on the quartic term \( -(m_a^2/f_a^2)a^4/24 \) of the potential \( V_a = -f_a^2 m_a^2 \cos(a/f_a) = -f_a^2 m_a^2 + m_a^2 a^2/2 - (m_a^2/f_a^2)a^4/24 \). We have neglected the term in the above discussion. Although the term is much smaller than the mass term \( m_a^2 a^2/2 \), the term gives a comparable contribution \( -m_a^2 a(r)^3/(12f_a^2) \) with \( (\omega^2 - m_a^2) a(r) \) in the eq(15) of the axion field when \( R_a \simeq 130 \text{ km} \) or less. Since the quartic term is negative in the potential, the masses of the axion stars decrease with the increase of the field amplitude \( a_0 \). That is, the axion stars become unstable when the term is comparable with the mass term. Thus, our solutions are only stable for the axion stars with larger radii than 130 km (smaller masses than \( 0.2 \times 10^{-11} M_\odot \)). This indicates that the critical mass becomes much smaller than \( M_{\text{max}} \) when the quartic term is taken into account. We do not yet know real critical mass when we take account of the full potential \( V_a \) of the axions. But the axion stars at least with masses smaller than \( 0.2 \times 10^{-11} M_\odot \) are stable since the approximation of neglecting the quartic term is valid. (Much small critical masses \( \sim 10^{-21} M_\odot \) have been previously pointed out using the procedure in the previous work[17]. But the procedure is only valid for free fields coupled with gravity. Especially, it is not applicable for the real scalar fields with nonlinear interactions such as the}
axions. The approximations such as \( \langle \dot{a}^4 \rangle = \langle \dot{a}^2 \rangle \sim \langle \dot{a}^2 \rangle \) used in the reference is too rough for obtaining the axion stars where \( \dot{a} \) represents axion field operator. Actually, the critical mass shown in the paper is of the order of \( 10^{-26} M_\odot \). As we have shown, the axion stars even with much larger masses, e.g. \( 10^{-12} M_\odot \) than the critical mass is stable. Thus, the critical masses obtained previously is not reliable.

In our analysis, the radius 130km of the axion stars with the mass \( 0.2 \times 10^{-11} M_\odot \) was roughly derived only as a guide for a critical radius. We may use the radius \( 10^2 \)km as a reference of the stable axion stars in the discussion below.

### II. AXION STARS IN MAGNETIC FIELDS

We proceed to discuss electric field \( \vec{E}_a \) generated on the axion stars under magnetic field \( \vec{B} \). It is well-known that the axion couples with both electric \( \vec{E} \) and magnetic fields \( \vec{B} \) in the following,

\[
L_{\alpha EB} = k_\alpha \frac{a(x, t) \vec{E} \cdot \vec{B}}{f_\alpha \pi} + \frac{\vec{E}^2 - \vec{B}^2}{2}
\]

with the fine structure constant \( \alpha \simeq 1/137 \), where the numerical constant \( k \) depends on axion models; typically it is of the order of one. Hereafter we set \( k = 1 \). From the Lagrangian, we derive the Gauss law, \( \vec{\partial} \cdot \vec{E} = -\alpha \partial (a \vec{B}) / f_\alpha \pi \). Thus, the electric field generated on the axion stars under the magnetic field \( \vec{B} \) is given by

\[
\vec{E}_a(r, t) = -\alpha \frac{a(x, t) \vec{B}}{f_\alpha \pi} = -\alpha \frac{a_0 \exp(-r/R_a) \cos(m_a t) \vec{B} (\vec{r})}{\pi} \\
\simeq 0.4 \times 10^{-23} \text{eV}^2 (2 \times 10^4 \text{eV/cm}) \cos(m_a t) \left( \frac{10^2 \text{km}}{R_a} \right)^2 \frac{10^{-5} \text{eV}}{m_a} \frac{B}{10^{10} \text{G}}.
\]

We find that the electric field is very strong at neutron stars with magnetic fields \( \sim 10^{10} \text{G} \), while it is much weak at the sun with magnetic field \( \sim 1 \text{G} \). The electric field \( \vec{E}_a \) is parallel to the magnetic field \( \vec{B} \) and oscillates coherently over the whole of the axion stars. When the axion stars are in magnetized ionized gases, the field induces coherently oscillating electric currents with large length scale \( R_a \) of the axion stars. Thus the large amount of dipole radiations can be emitted. We should mention that the motions of charged particles accelerated by the electric field \( \vec{E}_a \) are not affected by the magnetic field \( \vec{B} \) since \( \vec{B} \) is parallel to \( \vec{E} \). Thus, they emit dipole radiations.

Here we make a comment that the electric fields in eq.(21) are the ones generated at the rest frame of the axion stars. When the axion stars collide with neutron stars, the magnetic field \( \vec{b} = \vec{v}_c \times \vec{E}_a \) with \( \vec{v}_c \ll 1 \) is induced at the rest frame of the neutron stars, where \( \vec{v}_c \) represents a relative velocity between the axion stars and the neutron stars. Since \( v_c \sim 0.1 \), the magnetic field \( \vec{b} \) is much smaller than \( \vec{B} \). Thus, the effect can be neglected. Similarly, we can neglect the effects of the rotations of the neutron stars, whose velocities are much smaller than the relative velocities \( v_c \).

As will be shown later, all the energy of a part of the axion star touching the atmospheres of neutron stars is released into radiations, since the atmospheres of neutron stars are composed of highly dense electrons and ions. Namely, the neutron stars make the axion stars evaporate into the radiations. The frequency of the radiations is given by \( m_a/2\pi \simeq 2.4 \times (10^{-5} \text{eV}/m_a) \text{GHz} \) at the rest frame of the axion stars. (The frequency varies with time at the rest frame of neutron stars owing to the change of the relative velocities between axion stars and neutron stars in the course of the neutron stars going through the axion stars.) Therefore, when the axion stars collide with neutron stars, the large amount of the radiations is produced within a short period \( R_a/v_c \) being of the order of milli seconds; \( v_c \) is of the order of \( 10^3 \text{km/s} \), see later. These radiations can escape the atmospheres and magnetosphere of neutron stars, because they are optically thin for the radiations as we show below. Thus, it is reasonable to identify the radiations as the FRBs observed.

### III. EVENT RATE OF FAST RADIO BURSTS

We calculate the rate of the collisions between axion stars and neutron stars in a galaxy. The collisions generate FRBs so that the rate is the event rate of the FRBs. By the comparison of theoretical with observed rate of the bursts, we can determine the mass of the axion stars. We assume that halo of a galaxy is composed of the axion
stars whose velocities \( v \) relative to neutron stars is supposed to be \( 3 \times 10^3 \) km/s. Since the local density of the halo is supposed to be \( 0.5 \times 10^{-22} \text{g cm}^{-3} \), the number density \( n_a \) of the axion stars is given by \( n_a = 0.5 \times 10^{-24} \text{g cm}^{-3} / M_a \). The event rate \( R_{\text{burst}} \) can be obtained in the following,

\[
R_{\text{burst}} = n_a \times N_{\text{ns}} \times S v \times 1 \text{ year},
\]

where \( N_{\text{ns}} \) represents the number of neutron stars in a galaxy; it is supposed to be \( 10^9 \). The cross section \( S \) for the collision is given by \( S = \pi (R_a + R)^2 (1 + 2G (1.4 M_\odot) / v^2 (R_a + R)) \approx 2.8 \pi (R_a + R) G M_\odot / v^2 \) where \( R = 10 \text{km} \) denotes the radius of neutron star with mass \( 1.4 M_\odot \). It follows that the observed event rate is given by

\[
R_{\text{burst}} \approx 5 \times 10^{-24} \text{g cm}^{-3} \times 10^9 \times 2.8 \pi (10 \text{km} + R_a) G M_\odot / 10^{-6} \times 1 \text{ year} \\
\approx 10^{-3} (10^{-12} M_\odot / M_a) ^{10 \text{km} + 260 \text{km}} / 10 \text{ km} + 260 \text{ km}. 
\]

Therefore, we can determine the masses \( M_a \) of the axion stars by the comparison of \( R_{\text{burst}} \) in eq (23) with the observed event rate \( \sim 10^{-3} \) per year in a galaxy. We obtain \( M_a \approx 10^{-12} M_\odot \) when \( m_a = 10^{-5} \text{eV} \). The parameters used above still involves large ambiguities. Furthermore, the rate becomes larger than that in eq (24) when we take into account the cosmological evolution of the Universe. Thus, the observed rate only constrains the masses of the axion stars in a range such that \( M_a = 10^{-12} M_\odot \approx 10^{-11} M_\odot \). It is remarkable that the mass \( M_a \approx 10^{-12} M_\odot \) of the axion stars obtained is coincident with the masses of axion miniclusters\(^{18} \) estimated previously.

Using the formula eq (13) we find the radius \( \sim 10^2 \text{km} \) of the axion stars with the mass \( \sim 10^{-12} M_\odot \), which is larger than those of neutron stars. Then, when the collisions take place, the neutron stars pass through the insides of the axion stars. As we will show later, when the axion stars touch the atmospheres of the neutron stars, the large amount of radiations is emitted instantaneously and the stars lose their energies. The amount of the radiation energy released in the collision is given by \( 10^{-12} M_\odot (10 \text{ km} / 10^3 \text{ km})^2 \approx 10^{43} \text{ GeV} \). Thus, our production mechanism of FRBs can explain the observed energies of the FRBs.

We would like to point out the frequency of the electric field on the axion stars observed at the rest frame of the neutron stars. In the collisions between the axion stars and the neutron stars, the relative velocity is given by \( v_c \approx \sqrt{2 G \times 1.4 M_\odot / (R + R_a)} \approx 2 \times 10^{-1} \) when the neutron stars touch the edge of the axion stars, while it is given by \( v_c \approx \sqrt{2 G \times 1.4 M_\odot / R} \approx 6 \times 10^{-1} \) when the neutron stars go through the centers of the axion stars. Thus, the frequency of the electric field at the rest frame of the neutron stars can take different values depending on the relative velocity. It follows that the frequencies of the radiations emitted in the collisions vary from \( (m_a / 2 \pi) \sqrt{1 - v_c^2} \approx (m_a / 2 \pi) (1-0.02) \) to \( (m_a / 2 \pi) \sqrt{1 - v_c^2} \approx (m_a / 2 \pi) (1-0.18) \) at the rest frame of the neutron stars. It leads to the observed finite bandwidth \( \delta \omega \) of the radiations; \( \delta \omega \approx 0.16 (m_a / 2 \pi) \sqrt{1-2 G \times 1.4 M_\odot / R} (1+z)^{-1} \) \sim 230 \text{ MHz} (10^{-5} \text{ eV} / m_a) \) if \( z = 0.3 \). The value \( \delta \omega \) is roughly coincident with the observed ones.

**IV. RADIATIONS FROM AXION STARS IN ATMOSPHERES OF NEUTRON STARS**

Now, we estimate how rapidly the axion stars emit radiations in the collisions with neutron stars. In particular, we show that they rapidly lose their energies in the atmospheres\(^{11} \) of neutron stars. We consider old neutron stars which are dominant components of neutron stars in the Universe. Their temperatures (magnetic fields) are assumed to be of the order of \( 10^9 \text{K} \) (\( 10^{10} \text{G} \)).

First, we show how an electron emits radiations in the electric fields of the axion stars. The electric field \( \vec{E}_a \) on the axion stars generated under magnetic fields makes an electron oscillate according to the equation of motion \( \dot{\vec{p}} = (-e) \vec{E}_a + (-e) \vec{v} \times \vec{B} + m_e \ddot{\vec{g}} \) with electron mass \( m_e \), where \( \vec{p} \) and \( \vec{v} \) denote momentum and velocity of the electron, respectively and \( \ddot{\vec{g}} \) does surface gravity of neutron stars; \( |\ddot{\vec{g}}| = 1.4 M_\odot G / R^2 \). We note that the electric field \( \vec{E}_a \) is parallel to the magnetic field \( \vec{B} \). Thus, the direction of the oscillation is parallel to \( \vec{B} \). The magnetic field does not affect the oscillation.

Similarly, the gravitational forces does not affect it since they are much weaker than the electric fields. Then, the equation of motion of the electron parallel to \( \vec{E}_a \) is given by \( \dot{\vec{p}} = -e \vec{E}_a \). Since the electric fields oscillate such as \( E_a \propto \cos(m_a t) \), the electron oscillates with the frequency \( m_a / 2 \pi \) emitting a dipole radiation. The dipole radiation is linearly polarized. The amplitude of the oscillator is given by \( c m_a B / (m_a^2 m_e \pi) \approx 0.05 \text{ cm} \) which is smaller than the
wave length $\lambda \sim 10\text{cm}(10^{-5}\text{eV/}m_a)$ of the radiations. Thus, the emission rate of the radiation energy produced by a single electron with the mass $m_e$ is given by

$$\dot{w} \equiv \frac{2e^2p^2}{3m_e^2} = \frac{2e^2(ev_0B)}{3m_e^2} \simeq 0.7 \times 10^{-9}\text{GeV/s}\left(\frac{10^2\text{km}}{R_a}\right)^4\left(\frac{10^{-5}\text{eV}}{m_a}\right)^2\left(\frac{B}{10^{10}\text{G}}\right)^2.$$  \hspace{1cm} (25)

Electrons coherently oscillate in the volume $\lambda^3$, in which there exist a number of the electrons with their number $N_e$ given by $n_e\lambda^3$ with $n_e$ denotes the number density of electrons. Then, the total emission rate $\dot{W}$ from the gas is given such that $\dot{W} = \dot{w}(n_e\lambda)^2 = 2(n_e\lambda^3)p^2/(3m_e^2)$. On the other hand, if the depth $d$ of the atmosphere of neutron stars is less than the wave length of the radiations, the number of electrons coherently oscillating is given by $n_e\lambda^2$. Actually, the depth $d$ of the hydrogen atmosphere with temperature of the order of $10^5\text{K}$ is about 0.1cm, which is much smaller than the wave length $\sim 10\text{cm}(10^{-5}\text{eV/}m_a)$. We make a comment that the thermal effects of electron gas under consideration do not affect the oscillation by the electric field. Since the temperatures of the atmospheres are supposed to be $10^5\text{K}$, the thermal energy $\sim 10\text{eV}$ of an electron is much smaller than the kinetic energy of the oscillation, $p^2/2m_e = (eE)^2/2m_em_a^2 \sim 10^2\text{eV}(B/10^{10}\text{G})^2$ with $m_a = 10^{-7}\text{eV}$. We also make a comment about the depth of atmospheres of neutron stars. The density distribution $\rho(r) = \rho_0\exp(-r/d)$ with the depth $d = k_BT/mg$ in the atmosphere of a star can be obtained in the following ($m$ denotes average mass of the atoms composing the atmospheres, $T$ does temperature of the atmospheres and $k_B$ does Boltzmann constant). Namely, we solve the equation of the dynamical valance $\partial_t\nu(r) = -\rho(r)\eta$ between pressure $P$ and surface gravity $g \equiv GM/R^2$, using the equation of state $P(r) = n(r)k_BT$ of ideal gas under the assumption of the constant temperature, where $M$ ($R$) and $n$ denote mass (radius) of the star and number density ($n = \rho/m$) of atoms composing the atmosphere. For example, $d \sim 10\text{km}$ for $T = 300\text{K}$, $g_e = 9.8\text{m/s}^2$ and $m = 28\text{GeV}$ in the case of the earth, while $d \sim 0.1\text{cm}$ for $T = 10^5\text{K}$, $g_n = 10^{-11}\times g_e$ and $m = 1\text{GeV}$ in the case of hydrogen atmosphere of neutron stars. Although the estimation is very rough, we can grip on the depth of the atmosphere of neutron stars, which is given by 0.1cm when the temperature is of the order of $10^5\text{K}$. We can see that the number density of electrons $n_e(r) = n_0\exp(-r/0.1\text{cm})$ decreases rapidly with the distance $r$ from the bottom of the atmospheres. The radiations emitted in the atmospheres can pass through the atmospheres without absorption because the atmosphere is transparent for the radiations with transverse polarizations.

Actually, we may roughly estimate optical depth for the radiations. The optical depth $\tau(r) = \int_r^{\infty}d'r'C_e(r')$ may be calculated by using the free-free absorption coefficient $C_e(r)$ of fully ionized hydrogen atmosphere\cite{19} with magnetic field $B = 10^{10}\text{G},$

$$C_e(r) = \frac{n_e(r)}{(\omega + \epsilon\omega_e)^2 + \nu_e(r)^2} \frac{4\pi^2\nu_e(r)}{m_e} \frac{\omega^2}{(\omega - \epsilon\omega_p)^2}$$ \hspace{1cm} (26)

with $\omega = m_a/2\pi$, $\omega_e = eB/m_e$ and $\omega_p = eB/m_p$, where $m_p$ denotes proton mass and $\nu_e$ is given by

$$\nu_e(r) = \frac{2e^2\omega^2}{3m_e} + \frac{4n_e(r)e^4\Lambda_0(T,B,m_a)}{3T} \sqrt{\frac{2\pi}{m_eT}}$$  \hspace{1cm} (27)

for $T = 10^5\text{K}$ and $m_a = 10^{-5}\text{eV}$. The parameter $\epsilon = 0, \pm 1$ denotes three types of polarizations; circular polarizations $\epsilon = \pm 1$ (oscillation transverse to $B$) and longitudinal polarization $\epsilon = 0$ (oscillation longitudinal to $B$). Here the explicit formula of $\Lambda_\epsilon(T,B,m_a)$ is given in the reference\cite{19} and can be numerically estimated such that $\Lambda_0 \sim O(10)$ and $\Lambda_{\pm 1} = \Lambda_{-1} \sim O(10^{-7})$ for the parameters used in our paper. Using these values we rewrite the formula $C_{\epsilon \pm 1}(r),$

$$C_{\epsilon \pm 1}(r) \simeq \frac{n_e(r)4\pi^2\nu_{\pm 1}(r)\omega^2}{\omega_e^2 m_e} \text{ with } \nu_{\pm 1}(r) \simeq \frac{4n_e(r)e^4\Lambda_{\pm 1}(T,B,m_a)}{3T} \sqrt{\frac{2\pi}{m_eT}}.$$  \hspace{1cm} (28)

We can see the optical depth $\tau_{\pm 1}(r_c) < 1$ even at the location $r_c$ in which the number density $n_e(r_c)$ is $10^{22}/\text{cm}^3$. Here, we used, for instance, the formula $n_e = n_0\exp(-r/0.1\text{cm})$ with $n_0 = 10^{24}/\text{cm}^3$. Therefore, we find that the atmospheres are transparent for the radiations with the circular polarizations. The transparency comes from the fact that the frequency $\omega = m_a/2\pi$ is much smaller than the cyclotron frequencies $\omega_e$ and $\omega_p$, and that the electric fields of the radiations hardly make electrons move transversely to the direction of the magnetic fields $B$. On the other hand, we can easily see that the atmospheres are opaque for the radiations with the longitudinal polarization since the radiations easily make electrons oscillate longitudinally; they are absorbed by the electrons.
We would like to mention that although the radiations emitted from the atmospheres are linearly polarized, they are circularly polarized when they pass through the magnetospheres of neutron stars. The magnetospheres are composed of electrons or positrons, which are produced by the Schwinger pair production mechanism under electric field associated with the rotation of the magnetic field $B$. The charged particles screen the electric field. The number density of electrons (positrons) in the magnetospheres is given by the Goldreich-Julian density $\Omega B/2\pi \sim 10^{13} \text{cm}^{-3} (\Omega/(2\pi/s))(B(r)/10^{10} \text{G})$ with angular velocity $\Omega$ of neutron stars. These electrons (positrons) absorb right (left) handed circularly polarized radiations when the cyclotron frequency $\omega = eB(r)/m_e$ becomes equal to $m_e/2\pi$, respectively. Since $B(r)$ decreases such that $B(r) \propto 1/r^3$, the absorption arise around the location at the height $r_{ab} \sim 10^3 \text{km}$ above the surface of neutron stars. It implies that the absorption coefficient $C_{\pm 1}(r_{ab})$ is much larger for a type of circularly polarized radiations than for the other type of circularly polarized radiations, for instance, $C_{+1}(r_{ab}) \gg C_{-1}(r_{ab})$. The spatial distribution of the electrons is different from the distribution of the positrons. Therefore, the radiations passing through the magnetospheres are circularly polarized. Such a polarization has been observed in FRB 140514.

Now we show that the axion stars rapidly evaporate into the radiations when they touch the atmospheres of neutron stars. We assume that the atmospheres are composed of fully ionized hydrogen gas with temperature of the order of $10^5 \text{K}$, whose depth $d$ is about $\sim 0.1 \text{cm}$. Thus, we have the density distribution $n_e(r) = n_0 \exp(-r/0.1 \text{cm})$ where the density $n_e(r = 0) = n_0$ at the bottom is much larger than $10^{24} \text{cm}^{-3}$. It approximately corresponds to the density $1 \text{g/cm}^3$. In the paper we take $n_0 = 10^{24} \text{cm}^{-3}$. We consider the radiations arising from a region with volume $d\lambda \sim 10 \text{cm}^3$ in the atmospheres. The emission rate $\dot{W}$ of the radiations from the region is given by,

$$\dot{W} \sim 10^{-9}(d\lambda^2 n_e)^2 \text{GeV/s} \left(\frac{B}{10^{10} \text{G}}\right)^2 \sim 10^{37} \text{GeV/s} \left(\frac{n_e}{10^{22} \text{cm}^{-3}}\right)^2 \left(\frac{10^2 \text{km}}{R_a}\right)^4 \left(\frac{10^{-5} \text{eV}}{m_a}\right)^6 \left(\frac{B}{10^{10} \text{G}}\right)^2,$$

where we have taken, for instance, the number density $n_e = 10^{22} \text{cm}^{-3}$ of electrons in the region with the density $\rho \sim 10^{-2} \text{g/cm}^3$, which is located roughly at the height $r = 0.5 \text{cm}$. (If we take larger $n_e$, $\dot{W}$ becomes larger.) On the other hand, the energy of the axion stars contained in the volume $d\lambda^2 = 10 \text{cm}^3$ is given by $10^{-12} M_\odot 10 \text{cm}^3/(4\pi R_a^3)/3 \sim 10^{24} \text{GeV}$. This energy is smaller than the energy of the radiations $\dot{W} \times 10^{-11} s \sim 10^{26} \text{GeV}$ emitted within a time $0.1 \text{cm}/v_e \sim 10^{-11} \text{s}$ in which the axion stars pass the depth $d = 0.1 \text{cm}$. It should be noted that the relative velocity $v_e$ of the axion stars when they collide with the neutron stars, is given by $v_e = \sqrt{2G(1.4 M_\odot)/(R + R_a)} \sim 2 \times 10^{-1} \sim 6 \times 10^4 \text{km/s}$.

The purpose using the specific values $n_e = 10^{22} \text{cm}^{-3}$ or the depth $d = 0.1 \text{cm}$ in the estimation is simply to show that the whole energies of the axion stars passed through by neutron stars are converted into radiations. The use of different values similar to these ones does not change our results. Furthermore, the atmospheres are optically thin for the radiations produced to pass through the atmospheres. Obviously, our results do not depend on the specific values. Therefore, we conclude that the energies of the axion stars are immediately transformed into the radiation energies and the radiations can pass through the atmospheres.

There are ambiguities about the parameters (temperature, density, composition, e.t.c.) of neutron star atmospheres. Only what we need to derive our results is the fact that the atmospheres are very transparent for the radiations and that the average number density of electrons is much large such as of the order of $10^{22} \text{cm}^3$ in the atmosphere. We also need strong magnetic fields $\sim 10^{10} \text{G}$ of neutron stars. It seems that these assumptions are generally acceptable. Thus, our production mechanism of the FRBs is fairly promising.

As we mentioned above, the velocity $v_e$ or $v_e$ of the axion stars with which they collides against neutron stars is of the order of $10^3 \text{km/s}$. Thus, the time within which the neutron stars go through the axion stars is of the order of $10^3 \text{km}/v_e \sim 1 \text{ms}$. FRBs are emitted during the period. This can explain the observed durations of FRBs.

V. SUMMARY AND DISCUSSIONS

We have shown a possible production mechanism of FRBs; FRBs arise from the collisions between axion stars and neutron stars. The axion stars are rapidly converted into radiations under strong magnetic fields of the neutron stars. The radiations are emitted in the atmospheres of the neutron stars. We have shown that the atmospheres are transparent for the radiations, which are circularly polarized after passing the magnetospheres.

It apparently seems that the radiations is monochromatic with the frequency given by the axion mass. But, the observed frequencies have finite bandwidth. The bandwidth is caused by the Doppler effects associated with the relative velocities when the neutron stars go through the axion stars. Additionally, the observed frequencies are affected by the cosmological and gravitational red shifts.
As have been shown\[9\] previously, the axion miniclusters have been formed in the early Universe and become the dominant component of dark matter. Furthermore, the axion miniclusters condense to form the axion stars. Thus, we may consider that the dominant component of dark matter is the axion stars. Under the assumption that dark matter is composed of the axion stars, we have determined the masses of the axion stars by comparing the theoretical with the observed event rates of the FRBs. It is remarkable that the masses $\sim 10^{-12}M_\odot$ of the axion stars obtained in this way is coincident with the masses of the axion miniclusters estimated previously. The coincidence supports the validity of our production mechanism of FRBs. Not only the masses of the axion stars but also the short duration $\sim$ms and large amount of energies $\sim 10^{43}$GeV of the FRBs can be explained by our production mechanism. Therefore, our production mechanism is very promising. It predicts that no radiations with any frequencies are produced after the bursts. This is consistent with the results of follow-up observations\[20\]. It also predicts that FRBs contain circular polarizations. The circular polarizations arise owing to the absorption of a right ( left ) handed polarized radiations by electrons ( positrons ) in the magnetospheres. Circular polarizations have recently been observed\[20\] in a FRB.

Similar radio bursts may arise when the axion stars collide with white dwarfs. The rate of the collisions is of the order of $10^6 R_{burst} \sim 10^3$ per year in a galaxy, since the number of the white dwarfs is of the order of $10^{12}$ and their radii are $10^4$km. On the other hand, the amount of radiation energies emitted is much smaller than those in the case of neutron stars. This is owing to the weak magnetic fields $\sim 10^6$G and small electron densities $\sim 10^{10}$ cm$^{-3}$ in atmospheres of the white dwarfs. Although the electron density increases more as we go to deeper inside of the atmospheres, the radiations emitted in such regions may be absorbed in the atmospheres because plasma frequencies become larger than the frequency of the radiations. We note that the depth of the hydrogen atmospheres with temperature of the order of $10^4$K is approximately given by $\sim 10^4$cm, which is much larger than the wave length of the radiations. Since the emissions arise only when the axion stars pass through the atmospheres of the white dwarfs, the emissions last for a period $10^5$cm/$v \sim 10^6$ms with relative velocities $v \sim 10^6$cm/s at the collisions. This is ten times longer than the duration of FRBs. Furthermore, the bandwidths of the radiation frequencies are narrower than those of FRBs, because there is a little change in relative velocities in the course of axion stars going through the atmospheres of the white dwarfs. Although the luminosities of the radiations are much smaller than those of FRBs, we may observe such radiations when they are emitted near the sun.

If the our production mechanism of FRBs is true, we can reach a significant conclusion that the axions are the dominant component of dark matter and their mass is about $10^{-5}$eV, which is in the window allowed by observational and cosmological constraints\[21\].

The author expresses thanks to Prof. J. Arafune for useful comments and discussions.

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