MASS GENERATION VIA GRAVITATIONAL CONFINEMENT
OF RELATIVISTIC NEUTRINOS

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Abstract

We analyze the energy dependence of the gravitational attraction between ultrarelativistic elementary particles and we show how this attraction leads to the spontaneous generation of mass, i.e. of hadrons and bosons, the masses of which can be computed within an astonishing 1% precision.

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1 Introduction

Both special relativity (SR)\textsuperscript{[1,2]} coupled with the equivalence principle\textsuperscript{[3,4]}, and also the Schwarzschild geodesics of General Relativity (GR)\textsuperscript{[4,5]} have shown that the gravitational attraction between ultrarelativistic neutrinos with energies above 150 MeV is stronger than the electrostatic attraction of positron-electron pairs at the same distance and reaches the value of the strong force for neutrino energies above 300 MeV. This very strong force confines these particles to circular orbits with extremely high speeds corresponding to Lorentz factor $\gamma$ values above $10^{10}\textsuperscript{[3,4,6,7]}$.

The energy, $\gamma m_\nu c^2$, of these rotating particles is of the order of 300 MeV, i.e. on the order of quark energies and the mass of the rotating triads is of the order of 1 GeV/$c^2$, i.e. of the order of the mass of hadrons.

The resulting simple equations can be used to either compute hadron masses from neutrino masses and compare them with experiment or to compute neutrino masses from hadron masses and again compare them with experiment\textsuperscript{[3,4,5,6,7]}.

This simple procedure produces the mass of composite particles (e.g. protons and neutrons) which equals $K/c^2$ where $K$ is the kinetic energy of the rotating ultrarelativistic neutrinos which form the composite particle. Agreement with experiment regarding hadron and neutrino masses is typically 1% and 5% respectively without the use of any adjustable parameters\textsuperscript{[3,4,5,6,7]}.

2 Comparison of the gravitational and electrostatic forces.

The Newton-Einstein gravitational equation

Although gravitational forces between two neutrinos at rest are very weak in relation with the electrostatic attraction of a positron-electron pair at the same distance, the situation changes dramatically when the two neutrinos have a large velocity, $v$, with respect to a laboratory observer.
Figure 1: Comparison of the gravitational force, $F_{g,\nu\nu}$, between two relativistic neutrinos with the electrostatic force, $F_e$, of a positron-electron pair at the same distance $d$.

This is demonstrated in Figure 1 where the gravitational force, $F_{g,\nu\nu}$, between two neutrinos with a speed $v$ is compared with the electastatic force, $F_e$, of a positron-electron pair at the same distance. We thus consider the ratio

$$\rho_{ge} = \frac{F_{g,\nu\nu}}{F_e}$$

(1)

and we employ Coulomb’s law to express $F_e$ and Newton’s Universal gravitational law, using the gravitational masses, $m_g$, of the two neutrinos to express $F_{g,\nu\nu}$, i.e.

$$\rho_{ge} = \frac{Gm_g^2}{\varepsilon d^2} = \frac{\varepsilon Gm_g^2}{e^2}$$

(2)

It is worth noting that the equality

$$F_g = \frac{Gm_g^2}{d^2}$$

(3)

used in (2) is equivalent with the definition of the gravitational mass, $m_g$, of a particle, which is defined as the mass value which when used in Newton’s Universal gravitational law, gives the correct force $F$, value, i.e.

$$m_g^2 = \frac{F_g d^2}{G}$$

(4)

According to the equivalence principle the gravitational mass, $m_g$, is equal to the inertial mass, $m_i$, and for linear motion the latter can be computed using the special relativistic definitions of force and of momentum i.e. $p = \gamma m_\nu v$ where $\gamma$ is the neutrino Lorentz factor and $m_\nu$ is the neutrino rest
mass $m_{\nu}$, to obtain

$$ F = \frac{dp}{dt} = \frac{d(\gamma m_{\nu})}{dt} = m_{\nu} \left[ \gamma + v \frac{d\gamma}{dv} \right] \frac{dv}{dt} = $$

$$ = m_{\nu} \left[ \gamma + \frac{d}{dv} \left( 1 - \frac{v^2}{c^2} \right)^{-1/2} \right] \frac{dv}{dt} = m_{\nu} \left[ \gamma + \frac{v^2 \gamma^3}{c^2} \right] \frac{dv}{dt} = $$

$$ = m_{\nu} \left[ \gamma + \left( 1 - \frac{1}{\gamma^2} \right) \gamma^3 \right] \frac{dv}{dt} = m_{\nu} \gamma^3 \frac{dv}{dt} \quad (5) $$

Since for linear motion it is

$$ m_{i} = F/\left( dv/dt \right) \quad (6) $$

it follows

$$ m_{i} = \gamma^3 m_{\nu} \quad (7) $$

and therefore from the equivalence principle [3] one obtains

$$ m_{g} = m_{i} = \gamma^3 m_{\nu} \quad (8) $$

Using instantaneous reference frames [2, 3] one can show that equation (8) remains valid for arbitrary particle motion.

Thus using equation (8) in equation (2) one obtains

$$ \rho_{ge} = \frac{eG m_{\nu}^2 \gamma^6}{c^2} \quad (9) $$

Upon recalling the Einstein equation

$$ E = \gamma m_{\nu} c^2 \quad (10) $$

to express the total (kinetic plus rest) energy of the neutrino and denoting $m_o = m_{\nu}$ one obtains from (9)

$$ \rho_{ge} = \frac{F_{g,\nu\nu}}{F_e} = \frac{eG E^6}{c^2 m_{\nu}^4 c^{12}} \quad (11) $$

By introducing $\epsilon = 1.112 \cdot 10^{-10} \ C^2/Nm^2$, $G = 6.676 \cdot 10^{-11} \ m^3/ks^2$, $e = 1.602 \cdot 10^{-19} \ C$, $m_{\nu} \approx 0.0437 \ eV/c^2=7.79 \cdot 10^{-38} \ kg$ for the heaviest neutrino [3] [8], one obtains

$$ \rho_{ge} = \frac{eG}{c^2 c^{12}} \left( \frac{E^6}{m_{\nu}^4} \right) \quad (12) $$

As shown in Figures (1) and (2), for $m_{\nu} \approx 0.0437 \ eV/c^2$ [3] [8], the ratio $\rho_{ge}$ exceeds unity for $E > 147 \ MeV$, which is of the order of the quark effective energy.

As also shown by equation (12) the ratio $\rho_{ge}$ increases dramatically with decreasing particle mass $m_{\nu}$. Thus for $E=147 \ MeV$, the ratio $\rho_{ge}$ exceeds unity for $m_{\nu} < 0.0437 \ eV$. This is why, due to their small mass, neutrinos have been chosen by nature as building stones of composite particles and thus, as building stones of our Universe.

Figure 2 presents plots of the following forces as a function of the neutrino energy $E$: 
Figure 2: Dependence on neutrino energy $E$, of the experimentally measured cross section for $\bar{\nu}_e e^- \rightarrow \bar{\nu}_e e^-$ scattering [10] (top) and comparison (bottom) of the dependence on $E$ at any fixed distance, of the ratios of Coulombic force $F_{ee}$ of a positron electron pair and of the gravitational forces between two neutrinos, $F_{g,\nu\nu}$, and between an electron and a neutrino, $F_{g,e\nu}$, all divided by the strong force $F_S = \frac{\hbar c}{d^2}$, computed from equations [13], [14] and [15]: $\alpha = e^2/\epsilon \hbar c = 1/137.035$; Points p and W correspond to the energy of formation (baryogenesis) of protons (i.e. of quarks with effective mass $m_p/3 = 313$ MeV$/c^2$) and of W bosons (from cosmic neutrinos and terrestrial $e^\pm$, the latter at rest with the observers on earth).
1. The gravitational force, \( F_{g,\nu\nu} \) between 2 neutrinos, where

\[
F_{g,\nu\nu} = G m_\nu^2 \gamma^6 / d^2 = \frac{GE^6}{m_\nu^2 c^{12} d^2} \tag{13}
\]

2. The gravitational force, \( F_{g,\nu e} \), between an electron/positron and a neutrino, where

\[
F_{g,\nu e} = \frac{G m_e m_\nu \gamma^3}{d^2} = \frac{GE^3 m_e}{m_p^2 c^6 d^2} \tag{14}
\]

In figure 2 these forces are plotted vs the neutrino energy \( E = \gamma m_\nu c^2 \) and are compared with

A. The electrostatic force \( F_e \) of a positron-electron pair \( e^2 / \epsilon d^2 \)

B. The strong force expression \[ [3, 9] \].

\[
F_S = \frac{\hbar c}{d^2} \tag{15}
\]

The key features of Figure 2 are the following:

a. The neutrino-neutrino gravitational force, \( F_{g,\nu\nu} \), reaches the strong force, \( F_S \), at point p (proton), i.e. at \( E = 3^{12/13} m_p c^2 (= 938 \text{ MeV/c}^2) \), where \( m_p \) is the proton mass. Consequently, point p corresponds to proton formation, i.e. to hadronization (or baryogenesis).

b. The neutrino-electron (positron) gravitational force \( F_{g,\nu e} \) reaches the strong force value, \( F_S \), at point W, (Fig. 2 bottom).

This point W is in the range of the Glashow resonance \[ [10] \] (Figure 2 top) and also in the range of the energy

\[
E = \frac{(m_{pl}/m_e)^{1/3}(m_{pl} m_e^2/2)^{1/3}} = 6.5 \cdot 10^{15} eV = 6.5\text{PeV} \tag{16}
\]

which is the Rotating Lepton Model (RLM) computed energy \[ [7] \] of formation of a W boson from a terrestrial \( e^\pm \) and a cosmic neutrino as also shown in Fig. 2 top.

3 Computation of the masses of hadrons generated via gravitational confinement of neutrinos

In the case of neutrons and protons one considers three gravitating rotating neutrinos (Figure 3) and combines the special relativistic equation for circular motion \[ [3, 4] \]

\[
F = \gamma m_\nu v^2 / r \tag{17}
\]

with equation \[ [3] \], also using equation \[ [8] \] to obtain

\[
F = \frac{G m_\nu^2 \gamma^6}{\sqrt{3} r^2} \tag{18}
\]

where the \( \sqrt{3} \) factor comes from the equilateral triangle geometry (Fig. 3).

Combining \[ (17) \] and \[ (18) \] one obtains

\[
2\sqrt{3} \frac{r}{r_s} = \frac{\gamma^7}{\gamma^2 - 1} \tag{19}
\]
with
\[ r_s = \frac{2Gm_\nu}{c^2} \]  
which is the Schwarzschild radius of a particle with rest mass \( m_\nu \).

Equation (20) contains two unknowns, i.e. \( r \) and \( \gamma \). A second equation is obtained using the de Broglie wavelength expression, as in the H Bohr model [3], i.e.
\[ \lambda = r = \frac{\hbar}{\gamma m_\nu v} \]  
(21)

Solution of equations (19) and (21) gives
\[ \gamma = 3^{1/12}(m_{Pl}/m_\nu)^{1/3} = 7.163 \times 10^9 \]  
(22)
\[ m_n = 3\gamma m_\nu = 3^{13/12}(m_{Pl}m_\nu^2)^{1/3} = 939.56\text{MeV}/c^2 \]  
(23)
\[ r = \frac{3\hbar}{m_pc^2} = 0.630\text{fm} \]  
(24)

where \( m_{Pl} \) is the Planck mass \((\hbar G)^{1/2} = 1.221 \times 10^{19}\text{GeV}/c^2\).

For \( m_\nu \approx 0.0437\text{eV}/c^2 \), the heaviest neutrino mass [5], these results are in quantitative agreement with experiment [3, 7].

4 Mechanism of mass generation

Equations (22) and (23) imply that the mass of the composite (neutron) state formed by the three rotating neutrinos is a factor of \( 7.163 \times 10^9 \) larger than the initial three neutrino mass of \( 3 \times (0.0437)\text{eV}/c^2 \).

Figure 3 provides a direct explanation for this impressive phenomenon. Gravity maintains the three rotating particles at a highly relativistic speed \((v \sim c, \gamma = 7.163 \times 10^9)\), so that the composite particle mass \( 3\gamma m_\nu \) is a factor of \( \gamma \) larger than that of the three neutrinos at rest. Notice the spontaneous generation of neutrino kinetic energy, which constitutes the rest energy of the neutron formed and the corresponding negative potential energy \(-5\gamma m_\nu c^2 \) [3, 4, 7] which leads to a negative
Hamiltonian and a thus to stable structure composite particle.

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