Charged and Neutral Vortex Excitations in a Mean Field Theory
for the Fractional Quantum Hall Effect

Nobuki MAEDA
Department of Physics, Hokkaido University, Sapporo 060

(Received June 9, 1994)

Applying a bi-local mean field approximation to the fractional quantum Hall state of \( \nu=1/3 \), we obtain charged and neutral vortex mean field solutions numerically. We calculate the mean field energy and the fluctuation corrections. The charged vortex has a fractional charge and a fractional angular momentum. The neutral vortex has a zero charge and a zero angular momentum and vorticity two. The creation energy of the neutral vortex is about a half of the pair creation energy of two charged vortices. The magnetic field dependence of the gap energy agrees with Laughlin's quasiparticle gap energy.

§ 1. Introduction

We consider the two-dimensional interacting spinless electron system in a perpendicular magnetic field. The fractional quantum Hall effect (FQHE)\(^1\) can be characterized by the existence of an energy gap in a partially filled Landau level. Strong correlations among electrons with the Coulomb interaction are responsible for the energy gap. Since a partially filled many-electron state in the Landau level has an enormous degeneracy, systematic calculations based on a microscopic theory are very difficult. Laughlin proposed a phenomenological many-body wave function for a ground state of an incompressible quantum liquid.\(^2\) In Laughlin's theory the excited state includes quasiholes and quasiparticles which have fractional charges and obey the fractional statistics. The pair creation energy of a quasihole and a quasiparticle gives a finite energy gap and is in good agreement with experimental values at strong magnetic fields. Although Laughlin's theory explains the FQHE phenomenologically, it may be desirable to have more systematic calculational method.

Recently a bi-local mean field theory\(^3\),\(^4\) was applied to the many-body system for the FQHE and obtained a uniform mean field solution for a ground state\(^3\) and charged vortex solutions for a topological excited state.\(^5\) It was found that the vortex solutions have a fractional charge and a fractional angular momentum and have vorticity one. In this theory the correlation function \( \langle \phi^*(y)\phi(x) \rangle \) is treated as a mean field. At the stationary point of the effective action, the mean field solution has to satisfy the self-consistency condition which includes the interaction effect as a Fock term. It was found that the gap energy is rather smaller in a mean field theory than Laughlin's one. In the present paper we calculate the fluctuation corrections to the vortex energy and find that the gap energy becomes close to Laughlin's one. We also calculate charged and neutral vortex solutions of vorticity two. The neutral vortex solution has a zero charge and zero angular momentum. The neutral vortex is considered to be a bound state of two charged vortices which have charges of opposite sign. The neutral vortex energy is about a half of the pair creation energy of two
charged vortices.

In § 2, we review a bi-local mean field theory for the FQHE. We point out that we need to change the self-consistent condition for a systematic expansion around the vortex mean field solutions slightly. Nevertheless, it is shown in the Appendix that our results obtained in this paper is not changed. In § 3, we consider the ansatz forms of charged vortex solutions of vorticity \( n_v \) and get the exact relation between the fractional charge and fractional angular momentum of vortex in the large \( B \) limit. We also consider a neutral vortex of vorticity two. The fluctuation corrections to the vortex energy are calculated in § 4. The results of numerical calculations are presented in § 5. In § 6, we summarize our results.

§ 2. Mean field approximation for the FQHE

We consider the two-dimensional electron system in a perpendicular uniform magnetic field \( B \) which is described by a Hamiltonian

\[
H(\psi^*, \psi) = \int d^2x \left[ \phi^+(x) \left( \frac{\mathbf{P} + e\mathbf{A}}{2m} \right) \phi(x) + \frac{1}{2} \int d^2y \phi^+(x) \left( x - y \right) V(x-y) \phi(y) \phi(x) \right],
\]

where \( \mathbf{P} = -i \partial / \partial x^i \), \( \mathbf{A} = (B/2)(-x_2, x_1) \) and \( V(r) = e^2/(kr) \). We use the unit \( \hbar = c = 1 \) and fix \( \kappa = 13 \), \( m = 0.07 \) \( m_e \) for GaAs. We supposed that the magnetic field \( B \) is strong enough to make electrons fully polarized and neglected the spin effect of electrons.

In the functional integral formalism, the partition function is written as

\[
Z = \text{Tr} \left( e^{-\beta H(\psi^*, \psi)} \right) = \int \mathcal{D} \phi^* \mathcal{D} \phi \exp \left[ - \int_0^\beta dt \left( \int d^2x \phi^* \partial_t \phi + H(\psi^*, \psi) \right) \right],
\]

where \( \phi^* \) and \( \phi \) are anti-commuting field variables, \( \hat{\phi}^* \) and \( \hat{\phi} \) are anti-commuting field operators and \( \beta = 1/k_B T \). We introduce a bi-local auxiliary field \( U(x, y; t) \) and convert the interaction term into a quadratic form of \( \phi^* \) and \( \phi \) as

\[
H_v(\psi^*, \psi) = \int d^2x \phi^* \left( x, t \right) \left( \frac{\mathbf{P} + e\mathbf{A}}{2m} \right) \phi \left( x, t \right)
\]

\[
+ \frac{1}{2} \int d^2x d^2y V(x-y) \left[ U(x, y; t) U(y, x; t) - U(x, y; t) \psi^* (x; t) \psi (y; t) - U(y, x; t) \psi^* (y; t) \psi (x; t) \right],
\]

\[
Z = \mathcal{N}^{-1} \int \mathcal{D} \phi^* \mathcal{D} \phi \mathcal{D} U \exp \left[ - \int_0^\beta dt \left( \int d^2x \phi^* \partial_t \phi + H_v(\psi^*, \psi) \right) \right]
\]

\[
= \mathcal{N}^{-1} \int \mathcal{D} U \exp \left[ - S_{\text{eff}}(U) \right].
\]

In the mean field approximation, the partition function is calculated at the stationary point of \( S_{\text{eff}} \) as

\[
Z_0 = e^{-S_{\text{eff}}(U_0)} = \text{Tr} \left[ e^{-\beta H_{u_0}(\hat{\phi}^*, \hat{\phi})} \right],
\]
Using a correlation function, Eq. (2·6) can be written as
\[ U_0(x, y) = \langle \phi^*(y) \phi(x) \rangle_{U_0}, \]
\[ \langle \phi^*(y) \phi(x) \rangle_{U_0} = Z_0^{-1} \text{Tr}(\phi^*(y) \phi(x) \exp[-\beta H_{U_0}(\phi^*, \phi)]) \rightarrow \langle E_0 | \phi^*(y) \phi(x) | E_0 \rangle, \]
in the limit \( \beta \rightarrow \infty \). The state \( |E_0) \) is the lowest energy eigenstate of \( H_{U_0}(\phi^*, \phi) \). We solve the self-consistency condition (2·7) under the following ansatz form:

\[ U_0(x, y) = U_0 \rho(x, y) e^{-\gamma(x-y)^2} \exp[i \int_x^y a_\xi(\xi) d\xi], \]

where \( U_0 = \nu/\pi R_0^3 \), \( \gamma = 1/2 R_0^5 \), \( R_0 = \sqrt{2/eB} \) and \( \rho(x, y) = \rho(y, x) \). The line integral is along a straight line between \( x \) and \( y \). \( \rho(x, y) \) and \( a_\xi(\xi) \) are unknown real functions and we will determine these functions under appropriate assumptions. We assume that the mean field \( U_0(x, y) \) coincides with a uniform self-consistent solution of the filling factor \( \nu \) at infinity. By substituting Eq. (2·9) into Eq. (2·3), we get a mean field Hamiltonian:

\[ H_{U_0} = \int d^2 x \phi^*(x) h_0 \phi(x) + \frac{U_0^2}{2} \int d^2 x d^2 y V(x-y) \rho^2(x, y) e^{-2\gamma(x-y)^2}, \]

\[ h_0 = \left[ \frac{(P + eA)^2}{2m} - F((P + eA)^2) \rho(\bar{x}, x) \right]_{\bar{x} \rightarrow x}. \]

Explicit form of \( F(p^2) \) is given in Ref. 5). \( h_0 \) is a single-particle Hamiltonian and \( [\ ]_{\bar{x} \rightarrow x} \) represents to take a limit \( \bar{x} \rightarrow x \) after \( P \) operates on a function of \( x \). The second term of \( h_0 \) is called a Fock term in the Hartree-Fock approximation.

We assume that the system has a rotational invariance and the eigenfunctions are classified by the angular momentum quantum number, \( l \), as

\[ h_0 U_{l,n}(x) = E_{l,n} U_{l,n}(x), \quad E_{l,0} < E_{l,1} < E_{l,2} < \cdots, \]

and expand the electron field operator as \( \phi(x) = \sum_{l,n} U_{l,n}(x) \bar{a}_{l,n} \). \( \bar{a}_{l,n} \) and \( \bar{a}_{l,n}' \) are anti-commuting creation and annihilation operators. The \( N \)-electron state is defined by a totally anti-symmetric tensor \( F_{l_1, \cdots, l_N} \) as

\[ |\Psi\rangle = \sum_{l_1, l_2, \cdots, l_N=0}^M F_{l_1, \cdots, l_N} \bar{a}_{l_1,0} \bar{a}_{l_2,0} \cdots \bar{a}_{l_N,0} |0\rangle, \]

\[ \bar{a}_{l,n} |0\rangle = 0 \quad \text{for any } l, n, \]

where \( M \) is a maximum value of the angular momentum of a single-particle. Since we concentrate upon the case \( \nu < 1 \) (especially \( \nu = 1/3 \)), we restrict \( |\Psi\rangle \) within the \( n = 0 \) eigenstates in Eq. (2·12). For a free electron system the \( n = 0 \) eigenstates belong to the lowest Landau level. \( F_{l_1, \cdots, l_N} \) is zero unless \( l_1 + l_2 + \cdots + l_N = L_T \) for a fixed total angular momentum \( L_T \). From this property one can prove the following relations:
We do not need to know the explicit form of $F_{l_1 \cdots l_N}$ but need to know the value of $\nu_l$ for the following calculations. By Eqs. (2.12) and (2.13) we can calculate the correlation function as

$$
\langle \Psi | \tilde{\phi}(y) \tilde{\phi}(x) | \Psi \rangle = \sum_{l=0}^{M} \nu_l U_{l,0}(y) U_{l,0}(x).
$$

The condition that the electron density becomes a uniform density of filling factor $\nu$ means $\lim_{l \to \infty} \nu_l = \nu = N/(M+1)$ for $N \to \infty$. Using Eqs. (2.9) and (2.14), we solve the self-consistency condition (2.7).

Generally the $N$-electron state $|\Psi\rangle$ in Eq. (2.12) is a superposition of different $M+1$ states. Hence the different states in $|\Psi\rangle$ have to be degenerate in the energy eigenvalue because the lowest energy eigenstate survives in the self-consistency condition (2.7) as Eq. (2.8). We will show later that the energy eigenvalue $E_{l,0}$ is degenerate for a large $l$, but is not degenerate for a small $l$ in the vortex mean field solution. Therefore we need to modify the self-consistency condition (2.7) so as to make the eigenvalues degenerate. In fact this is possible by using an arbitrariness in the limiting procedure of discretized time slices.$^6$ The details are summarized in the Appendix. Since the results obtained by using Eq. (2.7) become the same results as obtained by using a modified one, we continue to use Eq. (2.7) in the following.

For a non-uniform state, energy eigenvalues become non-degenerate by the Coulomb interaction generally. Therefore the usual Hartree-Fock approximation does not work in this case for the above reason.

§ 3. Vortex mean field solutions

Following Ref. 5, we work on the ansatz form of a vortex as

$$
a_i(x) = eA_i(x) + n_i(x), \quad n(x) = \frac{n(r)}{r^2} (-x_2, x_1)
$$

with the boundary conditions:

$$
n(0) = n_\nu = 1, 2, 3, \ldots,
\lim_{r \to \infty} n(r) = 0,
\rho(x, 0) = 0, \quad \rho(x, x) = \rho(r).
$$

In Eqs. (3.1) and (3.2), the form of $n_i(x)$ is a reminiscent of the vortex solution in the Abelian Higgs theory.$^7$ It is seen by separating the non-singular part $\tilde{n}_i$ in $n_i$ as

$$
\int_{x}^{x'} n_i(\xi) d\xi' = n_\nu(\theta_y - \theta_x) + \int_{x}^{x'} \tilde{n}_i(\xi) d\xi',
$$
Charged and Neutral Vortex Excitations in a Mean Field Theory

\[ \tilde{n}_i(x) = \frac{n(r) - n(0)}{r^2} (-x_2, x_1), \]

\[ \int_{\mathbb{R}} \tilde{n}_i(\xi) d\xi^i = -2\pi n_\nu. \]  

(3.3)

The singular part of \( n_i \) gives a phase dependence \( e^{i\nu \theta} \) to the expectation value of the field operator and the non-singular part gives a dynamical magnetic field, \( |\mathbf{v} \times \tilde{n}| = (1/r)(dn/dr) \), localized around a vortex. \( n_\nu \) is called vorticity and characterizes the topological excitations.

The uniform density mean field solution was already obtained in Ref. 3). We denote the uniform density state as

\[ |\Psi_0\rangle = \sum_{l_1, \ldots, l_N=0}^{N} F_{l_1, \ldots, l_N}^0 \hat{a}_{l_1,0} \hat{a}_{l_2,0} \cdots \hat{a}_{l_N,0} |0\rangle, \]

\[ (N! \cdot N!) \sum_{l_1, \ldots, l_N=0}^{N} F_{l_1, \ldots, l_N}^0 \ast F_{l_1, \ldots, l_N}^0 = \nu \delta_{n,m}. \]  

(3.4)

The eigenfunction of \( h_0 \) for the uniform density mean field is written as

\[ u_i^0(x) = (\pi R_0^2 l!)^{-1/2} z^i e^{-z^2/2}, \quad l = 0, 1, 2, \ldots, \]

\[ z = (x_1 + ix_2)/R_0, \]  

(3.5)

and the energy eigenvalue is degenerate as

\[ E_0^0 = \frac{eB}{2m} - F(eB). \]  

(3.6)

(A) Charged vortex

Using the uniform density state we construct a quasihole state of a charge \(-n_\nu \nu\) as

\[ |\Psi_{-n_\nu \nu}\rangle = \sum_{l_1, l_2, \ldots, l_N=0}^{N} F_{l_1, \ldots, l_N}^{(0)} \hat{a}_{l_1,0}+n_\nu \hat{a}_{l_2,0}+n_\nu \cdots \hat{a}_{l_N,0}+n_\nu |0\rangle. \]  

(3.7)

Note that \( \hat{a}_i^\dagger \) in Eq. (3.4) and \( \hat{a}_i^\dagger \) in Eq. (3.7) are not the same because \( \hat{a}_i^\dagger \) is dependent upon the set of eigenfunctions of \( h_0, \{u_i, n(x)\} \), and \( h_0 \) is dependent upon the state of electrons. The state \( |\Psi_{-n_\nu \nu}\rangle \) gives \( \nu = \ldots = \nu_{n_\nu-1} = 0 \) and \( \nu_{n_\nu} = \nu_{n_\nu+1} = \cdots = \nu \). In fact the charge difference from the uniform density state is

\[ \langle \Psi_{-n_\nu \nu} | \int d^2 x e^i \hat{\phi}^\dagger \hat{\phi} | \Psi_{-n_\nu \nu} \rangle - \langle \Psi_0 | \int d^2 x e^i \hat{\phi}^\dagger \hat{\phi} | \Psi_0 \rangle = e \sum_{l=0}^{n_\nu-1} (-\nu) = -n_\nu \nu. \]  

(3.8)

In the large \( B \) limit, the single-particle state is restricted in the lowest Landau level and the eigenstate becomes \( u_i^{(0)}(x) \) of Eq. (3.5). Thus the correlation function for a quasihole state becomes

\[ \pi R_0^2 \langle \Psi_{-n_\nu \nu} | \hat{\phi}^\dagger(y) \hat{\phi}(x) | \Psi_{-n_\nu \nu} \rangle \]

\[ \rightarrow \nu \sum_{l=n_\nu}^{\infty} \frac{(Z_2^* Z_2)^l}{l!} e^{-(x_2 z_2 + z_2 x z_2)/2} \]

Downloaded from https://academic.oup.com/ptp/article-abstract/93/3/503/1894624 on 26 July 2018
From Eq. (3·9), \( \rho^2(x, y) \) reads
\[
\rho^2(x, y) \to 1 - 2 \frac{e^{-\frac{x^2+y^2}{4}}}{\pi^2} \sum_{l=0}^{\infty} \frac{(\xi_x \xi_y l)!}{l!} \cos(l\theta - \xi_x \xi_y \sin \theta)
\]
\[
+ \frac{e^{-\frac{x^2+y^2}{4}}}{\pi^2} \sum_{l,m=0}^{\infty} \frac{(\xi_x \xi_y l+m)!}{(l-m)!m!} \cos(l-m)\theta,
\]

as \( B \to \infty \), where \( \xi_x = \xi_x e^{i\theta} \), \( \theta = \theta_x - \theta_y \). For \( |z_x - z_y| < 1 \), \( n(r) \) reads
\[
n(r) = \frac{\xi_x^{2n_v} (n_v - 1)!}{e^{\xi_x^2} - \sum_{l=0}^{n_v-1} \xi_x^{2l}/l!}, \quad \text{as } B \to \infty,
\]
\[
n(0) = n_v.
\]

Using Eqs. (3·10) and (3·11), the angular momentum of the vortex is given by
\[
\langle \Psi_{-n_v} | \int d^2x \hat{L}(x) | \Psi_{-n_v} \rangle = -U_0 \int d^2x \rho(r) n(r) \to -n_v \nu,
\]

as \( B \to \infty \), where \( \hat{L}(x) \) is an angular momentum density operator.

Similarly we construct a quasiparticle state of a charge \( +n_v \nu \) for \( \nu = 1/3 \) as
\[
| \Psi_{+n_v} \rangle = \tilde{a}_{1+2n_v,0} \cdots \tilde{a}_{1,0} \sum_{l_1, \ldots, l_M} F_{M,1-n_v}^{(0)} \tilde{a}_{2l+2n_v,0} \tilde{a}_{1+2n_v,0} \cdots \tilde{a}_{1+2n_v,0} | 0 \rangle.
\]

The state \( | \Psi_{+n_v} \rangle \) gives \( \nu_0 = \cdots = \nu_{-n_v} = 0 \) and \( \nu_{-n_v+1} = 1 \) and \( \nu_{2n_v} = \nu_{2n_v+1} = \cdots = \nu = 1/3 \). In fact the charge difference from the uniform density state is
\[
\langle \Psi_{+n_v} | \int d^2x \tilde{\phi}^* \tilde{\phi} | \Psi_{-n_v} \rangle - \langle \Psi_0 | \int d^2x \tilde{\phi}^* \tilde{\phi} | \Psi_0 \rangle = e \left( \sum_{l=0}^{n_v-1} (-\nu) + \sum_{l=n_v}^{2n_v-1} (1-\nu) \right)
\]
\[
= + n_v \nu \tilde{\phi}.
\]

In Eq. (3·14), it is essential that \( \nu \) is equal to 1/3. In the large \( B \) limit the correlation function for a quasiparticle state becomes
\[
\pi R_0^2 \langle \Psi_{+n_v} | \tilde{\phi}^* (y) \tilde{\phi} (x) | \Psi_{+n_v} \rangle
\]
\[
= \nu \sum_{l=0}^{n_v} (1 - \delta_l) \frac{(\xi_x \xi_y l)!}{l!} e^{-\frac{(x^2+y^2+z_x z_y)}{2}}
\]
\[
= \nu \left( 1 - \sum_{l=0}^{2n_v-1} \delta_l \frac{(\xi_x \xi_y l)!}{l!} e^{-\frac{(x^2+y^2+z_x z_y)}{2}} \right) \exp \left[ -|z_x - z_y|^2/2 + i \int_x e^{A_x \xi_x^2} \right],
\]

as \( B \to \infty \), where
\[
\delta_l = \begin{cases} 
1 & \text{for } 0 \leq l \leq n_v - 1, \\
-2 & \text{for } n_v \leq l \leq 2n_v - 1, \\
0 & \text{otherwise}.
\end{cases}
\]

From Eq. (3·15), \( \rho^2(x, y) \) reads
\[
\rho^2(x, y) \rightarrow 1 - 2e^{-2txy\cos \theta} \sum_{l=0}^{2n-1} \frac{(-\xi_x \xi_y)^l}{l!} \cos(l \theta - \xi_x \xi_y \sin \theta) \\
+ e^{-2txy\cos \theta} \sum_{l,m=0}^{2n-1} \delta_{l,l} \delta_{m,m} \frac{(-\xi_x \xi_y)^{l+m}}{l!m!} \cos(l - m) \theta ,
\]
\[ (3.16) \]

as \( B \to \infty \). For \(|z_x - z_y| \ll 1 \), \( n(r) \) reads
\[
n(r) \rightarrow \frac{3e^{2n\nu}/(n\nu - 1)! - 2e^{4n\nu}/(2n\nu - 1)!}{e^{y} - \sum_{l=0}^{2n-1} \delta_{l,l} \xi^l/!} , \quad \text{as } B \to \infty ,
\]
\[ (3.17) \]
\[ n(0) = n_\nu . \]

Using Eqs. (3.16) and (3.17), the angular momentum of the vortex is given by
\[
\langle \Psi_{\nu\nu}| \int d^2x \hat{L}(x)|\Psi_{\nu\nu}\rangle = -U_0 \int dx^2 \rho(r) n(r) \rightarrow +n_\nu ,
\]
\[ (3.18) \]
as \( B \to \infty \).

(B) **Neutral vortex**

We also construct a neutral vortex state for \( \nu = 1/3 \) as
\[
|\Psi_{\text{neu}}\rangle = \hat{a}_{l=1} \sum_{l_1 l_2 l_3 l_4} \hat{F}_{l_1, l_2, l_3, l_4} \hat{a}_{l_1} \hat{a}_{l_2} \hat{a}_{l_3} \hat{a}_{l_4} |0\rangle .
\]
\[ (3.19) \]
The state \( |\Psi_{\text{neu}}\rangle \) gives \( \nu_0 = \nu_1 = 0, \nu_2 = 1, \nu_3 = \nu_4 = \cdots = \nu = 1/3 \). The charge difference from the uniform density state is zero. In the large \( B \) limit, \( \rho^2(x, y) \) reads
\[
\rho^2(x, y) \rightarrow 1 - 2e^{-2txy\cos \theta} (\cos(\xi_x \xi_y \sin \theta) + \xi_x \xi_y \cos(\theta - \xi_x \xi_y \sin \theta)) \\
- (\xi_x \xi_y)^3 \cos(2\theta - \xi_x \xi_y \sin \theta)) + e^{-2txy\cos \theta} \left( 1 + (\xi_x \xi_y)^2 \right) \\
+ (\xi_x \xi_y)^4 + 2\xi_x \xi_y \cos \theta - 2(\xi_x \xi_y)^3 \cos \theta - 2(\xi_x \xi_y)^2 \cos 2\theta \right) .
\]
\[ (3.20) \]
For \(|z_x - z_y| \ll 1 \), \( n(r) \) reads
\[
n(r) \rightarrow \frac{\xi^4(3 - \xi^2)}{e^y - 1 - \xi^2} , \quad \text{as } B \to \infty ,
\]
\[ n(0) = 2 . \]
\[ (3.21) \]

From Eqs. (3.20) and (3.21), one can see that the angular momentum of the neutral vortex is zero.

We see from Eqs. (3.8), (3.12), (3.14) and (3.18) that the charge \( Q_v \) and the angular momentum \( L_v \) of our vortex have a relation:
\[
eL_v = Q_v ,
\]
\[ (3.22) \]
in the large \( B \) limit.

\section*{§ 4. Fluctuation corrections}

In this section we study the fluctuation corrections to the mean field solutions.
We approximate the integration over $U$ in Eq. (2.4) into a sum over the mean field solutions of Eq. (2.7) and the quadratic fluctuations around the mean field solutions. That is

$$Z = e^{-\frac{\delta S_{\text{eff}}(U_0^{(n)})}{\mathcal{N}}} \int \mathcal{D}U \exp\left[-\frac{1}{2} \left( \frac{\delta^2 S_{\text{eff}}}{\delta U^2} \right)_n \delta U \delta U + (\delta U^3) \right]$$

$$\sim \sum_n e^{-\frac{\delta S_{\text{eff}}(U_0^{(n)})}{\mathcal{N}}} \int \mathcal{D}U \exp\left[-\frac{1}{2} \left( \frac{\delta^2 S_{\text{eff}}}{\delta U^2} \right)_n \delta U \delta U \right], \quad (4.1)$$

where the $U_0^{(n)}$'s are solutions of Eq. (2.7) and $(\delta^2 S_{\text{eff}}/\delta U^2)_n$ is given by

$$\left( \frac{\delta^2 S_{\text{eff}}}{\delta U^2} \right)_n = \frac{\delta^2 S_{\text{eff}}}{\delta U(y', x'; t'; t'' \delta U(x, y; t)} |_{U=U_0^{(n)}} = \delta(x'-x) \delta(y'-y) \delta(t'-t) V(x-y) - V(y'-x') D_n(y', x', x, y, t) V(x-y) = V(1-D_n V), \quad (4.2)$$

$$D_n(y', x', x, y, t) = \langle \phi'(y'; t') \psi'(x'; t') \psi'(x; t) \psi(y; t) \rangle_{U_0^{(n)}} - \langle \phi'(y'; t') \phi'(x'; t') \rangle_{U_0^{(n)}} \langle \phi'(x; t) \psi(y; t) \rangle_{U_0^{(n)}} \psi'(x; t) \psi(x; t) \rangle_{U_0^{(n)}}. \quad (4.3)$$

The functional Gaussian integration in Eq. (4.1) is calculated as

$$\int \mathcal{D}U \exp\left[-\frac{1}{2} \left( \frac{\delta^2 S_{\text{eff}}}{\delta U^2} \right)_n \delta U \delta U \right] = \left[ \text{Det}(1-D_n V) \right]^{-1/2}$$

$$= \exp\left[-\frac{1}{2} \text{Tr} \ln(1-D_n V) \right]$$

$$= \exp\left[\frac{1}{2} \sum_{j=1}^{\infty} \frac{1}{j} \text{Tr}(D_n V)^j \right], \quad (4.4)$$

where

$$\frac{1}{2j} \text{Tr}(D_n V)^j = \frac{1}{2j} \int dt_1 d^2 x_1 d^2 y_1 \cdots dt_j d^2 x_j d^2 y_j V(x_1 - y_1) D_n(y_j, x_j, t_j, x_2, y_2, t_2) \cdots V(x_j - y_j) D_n(y_j, x_j, t_j, x_1, y_1, t_1). \quad (4.5)$$

Especially the $j=1$ term is the first order of $V$ and only includes the equal time propagator as

$$\frac{1}{2} \text{Tr}(D_n V) = \frac{1}{2} \int dt d^2 x d^2 y \langle \phi'(y; t) \psi(y; t) \rangle_{U_0^{(n)}} V(x-y) \langle \phi(x; t) \psi'(x; t) \rangle_{U_0^{(n)}}$$

$$= -\frac{\beta}{2} \int d^2 x d^2 y \langle U_0^{(n)}(x, x) V(x-y) U_0^{(n)}(y, y) \rangle. \quad (4.6)$$

Thus the $j=1$ term is a direct interaction term. Graphical representations of Eqs. (4.5) and (4.6) are given in Fig. 1. The energy eigenvalue appears in the partition function as $e^{-\beta E_{\text{tot}}}$ for $\beta \to \infty$. Thus
\(E_n^{\text{tot}} = E_n^{\text{mean}} + E_n^{\text{ex}} + E_n^{\text{dir}} + O(V^2),\)

\[E_n^{\text{mean}} = \langle E_n | \int d^2x \bar{\phi}^{\dagger} \hbar \phi E_n \rangle = \sum_{\ell=0}^{N} \nu_{\ell} E_{\ell,0},\]

\[E_n^{\text{ex}} = \frac{1}{2} \int d^2x d^2y U_{0}^{(n)}(x, y) \times V(x-y) U_{0}^{(n)}(y, x),\]

\[E_n^{\text{dir}} = \frac{1}{2} \int d^2x d^2y (U_{0}^{(n)}(x, x) - U_0) V(x-y) (U_{0}^{(n)}(y, y) - U_0).\]

\(E_n^{\text{ex}}\) is an exchange energy and \(E_n^{\text{dir}}\) is a direct energy. In \(E_n^{\text{dir}}\), we replace \(U_{0}^{(n)}(x, x)\) by \(U_{0}^{(n)}(x, x) - U_0\), because the electron density \(U_{0}^{(n)}(x, x)\) becomes a uniform density \(U_0\) at infinity and \(V(r)\) does not couple the zero mode of the Fourier component.

From a dimensional analysis, \(V^j\) term behaves as \(B^{1-j/2}\) and \(j=1\) term is a leading term of the perturbation expansion in the large \(B\) limit.

\section{5. Numerical results}

We will solve the self-consistency condition \((2\cdot7)\) numerically for the charged and neutral vortex states of Eqs. \((3\cdot7)\), \((3\cdot13)\) and \((3\cdot19)\). At the start we expand \(F(p^2)\) in Eq. \((2\cdot10)\) around \(p^2 = eB\) which is the energy of the lowest Landau level and approximate as

\[F(p^2) = F(eB) + F'(eB)(p^2 - eB) = F_0 - \frac{p^2}{2m'}.\]

\(F_0\) and \(m'\) are given in Ref. 5). In this approximation, the eigenvalue equation in Eq. \((2\cdot11)\) becomes

\[\left[ \frac{(P+eA)^2}{2m} + \frac{(P+eA)^2}{2m'} \rho(\bar{x}, x) + F_0(1 - \rho(x)) \right]_{\bar{x}=x} u_{i,0}(x) = \tilde{E}_i u_{i,0}(x),\]

\[u_{i,0}(x) = v_{i,0}(r)e^{-i\omega}; \quad l = \text{integer},\]

where we add a constant \(F_0\) to the energy \(\hbar_0\) for a simplicity and put \(\tilde{E}_i = E_{i,0} + F_0\). For the energy eigenvalue of the uniform density state, we define \(\tilde{E}_{i}^{(0)}\) as

\[\tilde{E}_{i}^{(0)} = E_{i,0} + F_0 - \frac{1}{2} \left( \frac{eB}{M} \right),\]

where \(M = mm'/(m + m')\). We use the analytic form of Eqs. \((3\cdot11)\), \((3\cdot17)\) and \((3\cdot21)\) for \(n_i(x)\) and solve Eq. \((5\cdot2)\) self-consistently with respect to \(\rho\). We restrict the form of \(\rho(x, y)\) as
Fig. 2. $\rho_{-n/v}$ for quasiholes of $n_v=1, 2$ at $B=10[T]$.

Fig. 3. $\rho_{+n/v} / 3$ for quasiparticles of $n_v=1, 2$ at $B=10[T]$.

$\rho(x, y) = f(\sqrt{r_x r_y}, \theta^2)$,

$f(r, 0) = \rho(r), \quad (5\cdot4)$

and calculate $\rho$ by a numerical self-consistent method.$^3$

(A) Charged vortex for $n_v=1, 2$

Using Eqs. (3·11) and (3·17), we obtain the numerical solutions of $\rho$. See Figs. 2 and 3. The electron densities for the quasihole and quasiparticle vanish at the vortex core. Figures 5 and 6 show the energy eigenvalues $\tilde{E}_l$. For large $l$, $\tilde{E}_l$ becomes degenerate to $\tilde{E}_l^{(0)}$. For $l \leq -1$, $\tilde{E}_l$ belongs to higher energy levels and is larger than $\tilde{E}_l$ for $l \geq 0$ by $O(eB/M)$. The energy differences from the uniform density state, $\Delta E_n = E_n - E^{(0)}$

Fig. 4. $\rho_{\text{wac}}$ for a neutral vortex of $n_v=2$ at $B=10[T]$.

Fig. 5. $\tilde{E}_l$ for quasiholes of $n_v=1, 2$ at $B=10[T]$. ○ represents an eigenvalue for $n_v=1$ and ● for $n_v=2$.

Fig. 6. $\tilde{E}_l$ for quasiparticles of $n_v=1, 2$ at $B=10[T]$. ○ represents an eigenvalue for $n_v=1$ and ● for $n_v=2$.
Charged and Neutral Vortex Excitations in a Mean Field Theory

Table I. Energy differences from the uniform density state at $B=10\text{[T]}$. The unit is $eB/M$.

| $\rho_n$ | $\rho_{-1/3}$ | $\rho_{1/3}$ | $\rho_{-2/3}$ | $\rho_{3/3}$ | $\rho_{\text{neu}}$ |
|---------|---------------|--------------|---------------|--------------|-----------------|
| $\Delta E_n^{\text{mean}}$ | 0.03286 | -0.15412 | 0.04899 | -0.39914 | -0.08970 |
| $\Delta E_n^{\text{ex}}$ | -0.06075 | 0.18320 | -0.11480 | 0.39538 | 0.10189 |
| $\Delta E_n^{\text{dir}}$ | 0.03391 | 0.02580 | 0.10696 | 0.09780 | 0.02356 |
| $\Delta E_n^{\text{tot}}$ | 0.00682 | 0.05488 | 0.04115 | 0.09224 | 0.03575 |

Fig. 7. $E_{1/3}$ for a neutral vortex of $n_v=2$ at $B=10\text{[T]}$.

$=\bar{E}_n-\bar{E}^{(0)}$, are listed in Table I for $B=10\text{[T]}$. The gap energy $\Delta$ is a half of the pair creation energy of a quasihole and quasiparticle which are separated infinitely as

$$\Delta n_{1/3} = \frac{1}{2} (\Delta E_{n_{1/3}}^{\text{tot}} + \Delta E_{n_{1/3}}^{\text{tot}})^{\text{tot}}.$$  \hspace{1cm} (5.5)

Experimentally $\Delta$ is measured as an activation energy by the temperature dependence of the diagonal resistivity as $\rho_{xx} \propto e^{-\Delta T}$. We calculate $\Delta n_{1/3}$ (the unit is kelvin) at $B=5$, 10, 15, 20 [T]. See Fig. 8. The gap energy $\Delta_{2/3}$ is about two times as large as $\Delta_{1/3}$.

(B) Neutral vortex for $n_v=2$

Using Eq. (3.21) we obtain the numerical solution of $\rho$. See Fig. 4. Figure 7 shows the energy eigenvalue $\bar{E}_i$. The energy differences from the uniform density state are listed in Table I for $B=10\text{[T]}$. The gap energy $\Delta_{\text{neu}}$ for a neutral vortex is equal to $\Delta E_{\text{tot}}^{\text{neu}}$. See Fig. 8. It seems that our neutral vortex (charge=0, $n_v=2$) is a bound state of two charged vortices, $|\Psi_{-1/3}\rangle$ (charge = $-e/3$, $n_v=1$) and $|\Psi_{1/3}\rangle$ (charge = $e/3$, $n_v=1$). Since both of the two charged vortices have vorticity one, their bound state has vorticity two. The gap energy $\Delta_{\text{neu}}$ is about a half of the pair creation.
energy $2\Delta_{1/3}$. Since the sum of energies for two charged vortices is larger than the energy of one neutral vortex ($\Delta E_{1/3}^{\text{tot}} + \Delta E_{1/3}^{\text{vac}} > \Delta E_{\text{neu}}^{\text{tot}}$), it is possible to make a bound state.

§ 6. Summary and discussion

In this paper we applied a mean field theory to the FQHE for $\nu = 1/3$ and obtained the charged and neutral vortex solutions numerically. These solutions have fractional charges and fractional angular momenta and have a relation (3·22). We calculated the gap energies for these vortices. The smallest one among the gap energies is $\Delta_{1/3}$ for the pair of vortices which have charges $\pm e/3$. The $B$ dependence of $\Delta_{1/3}$ is written as

$$\Delta_{1/3} = C \cdot \frac{e^2}{2k} \sqrt{eB}, \quad (6\cdot1)$$

and $C \sim 0.09$. This result is consistent with other theoretical calculations ($C \sim 0.1$) based on Laughlin's theory $^{8,9}$ and on the exact diagonalization of the finite system $^{10,11}$. The experimental values for the activation energy are about half of these theoretical values at $B = 10 \sim 20[T]$. This discrepancy is explained by the effect of a thickness of 2D layer and effect of disorder $^{12}$. Below $B \sim 5[T]$ the experimental gap energy becomes very small and the discrepancy still remains. In Ref. 5), the gap energy vanished at $B = 5.5[T]$ without the thickness effect and disorder effect. However we obtained negative results in the present paper by calculating fluctuation corrections. The gap energy did not vanish at $B = 5 \sim 20[T]$.

A similar construction of a quasihole was performed by MacDonald and Girvin $^{13}$. Using the Laughlin wave function, they calculated the energy expectation value for the quasihole state. The effect of the interaction is involved in the Laughlin wave function implicitly. In our case, on the other hand, the effect of the interaction appears in the one-particle Hamiltonian $h_0$ in Eq. (2·10) explicitly and one-particle eigenstates is modified by the interaction effect. They also constructed a quasiparticle and the charge density of their quasiparticle state does not vanish at origin. Therefore their quasiparticle is different from ours and is not a vortex solution.

In addition to charged vortices we obtained a neutral vortex which has a zero charge and a zero angular momentum. The $B$ dependence of $\Delta_{\text{neu}}$ is written as Eq. (6·1) and $C_{\text{neu}} \sim 0.11$. The neutral excitation in the FQHE was studied by means of the single-mode approximation and the exact diagonalization of the finite system $^{10,11}$. These studies show that the excitation energy has a magnetoroton minimum at finite wave vector ($C_{\text{roton}} \sim 0.15$). We speculate that our neutral vortex may be a wave packet made out of the collective mode around the roton minimum. Recently the $q = 0$ collective mode in the FQHE was observed by inelastic light scattering $^{14}$. In Ref. 9) it is speculated that the $q = 0$ excitation may be a two-roton bound state. We hope that the magnetoroton will be observed by inelastic light scattering or other methods.

As a summary, starting from microscopic Hamiltonian for the FQHE, we have derived the self-consistency condition for the correlation function, which includes a...
charged and neutral vortex excitations in a mean field theory

Fock term, and shown that it is possible to solve this condition for the topological excited states and to calculate the fluctuation corrections systematically.

Acknowledgements

The author is grateful to Professor K. Ishikawa for valuable discussions.

Appendix

We see in §5 that the energy eigenvalues become non-degenerate for small \( l \). These cause a difficulty that many-electron state \( |\Psi\rangle \) is not the eigenstate of \( H_{\text{ne}} \) in Eq. (2.10). We can overcome this difficulty by using an arbitrariness in the limiting procedure of discretized time slices. In this appendix we mainly use the notation of Ref. 6).

The partition function can be written as

\[
Z = \text{Tr}(e^{-\beta \hat{H}}) = \lim_{\varepsilon \to 0} \text{Tr}\left[1 - \epsilon \hat{\phi}^\dagger \hat{K} \hat{\phi} - \frac{\epsilon}{2} \hat{\phi}^\dagger \hat{V} \hat{\phi} \hat{\phi} \right]^N,
\]

where \( \beta = \frac{\alpha}{e} \) and we use the shorthand notations:

\[
\hat{\phi}^\dagger \hat{K} \hat{\phi} = \int d^2 x \phi^\dagger(x) \left(\frac{P + eA}{2m}\right) \phi(x),
\]

\[
\hat{\phi}^\dagger \hat{V} \hat{\phi} = \int d^2 x d^2 y \phi^\dagger(x) \phi^\dagger(y) V(x - y) \phi(y) \phi(x).
\]

Using an auxiliary field \( \sigma \), \( Z \) becomes

\[
Z = \lim_{\varepsilon \to 0} \int_{\mathcal{D}} \sigma e^{-(\varepsilon/2)\sum_k \sigma_k \sigma_k} \text{Tr}\left[1 - \epsilon \hat{\phi}^\dagger \hat{K} \hat{\phi} + \epsilon \hat{\phi}^\dagger \sigma_k \phi - \frac{\varepsilon^2}{2} \sigma_k \sigma_k^\dagger \hat{V} \hat{\phi} \hat{\phi} \right],
\]

\[
\mathcal{N} = \int_{\mathcal{D}} \sigma e^{-(\varepsilon/2)\sum_k \sigma_k \sigma_k},
\]

where

\[
\sigma_k \sigma_k = \int d^2 x d^2 y \sigma_k(x, y) \sigma_k(y, x),
\]

\[
\hat{\phi}^\dagger \sigma_k \phi \phi = \int d^2 x d^2 y d^2 x' d^2 y' \phi^\dagger(x') \sigma_k(x, y) \nu(x, y; x', y') \phi(y'),
\]

\[
\sigma_k \phi^\dagger \phi^\dagger \phi \phi \sigma = \int d^2 x d^2 y d^2 x' d^2 y' \sigma_k(x', y') \phi^\dagger(x') \phi^\dagger(y) V(x - y) \phi(y') \phi(x) \sigma_k(y, x).
\]

The suffix \( k \) specifies a discrete time slice. We can choose a function \( \nu(x, y; x', y') \) in Eq. (A·3) arbitrarily because the term \( \hat{\phi}^\dagger \sigma_k \phi \phi \) is linear in \( \sigma \) and vanishes after integrating over \( \sigma \). For a general \( \nu \) we cannot exchange the order of the limit \( \varepsilon \to 0 \) and integration \( \int \mathcal{D} \sigma \). Therefore we have to integrate over \( \sigma \) for a finite \( \varepsilon \) and take a limit \( \varepsilon \to 0 \) in the end. With a fixed \( \sigma \) the \( \varepsilon^2 \)-term seems not to contribute to Eq.
(A·3). However the integration with a Gaussian factor \( e^{-\varepsilon^2/2} \) makes \( \sigma \) become of order \( \varepsilon^{-1/2} \). Thus the \( \varepsilon^2 \)-term becomes of order \( \varepsilon \) by integration \( \int \mathcal{D} \sigma \) and is not negligible. We change the integral variable as

\[
\sigma_k(x, y) = \sigma^{(0)}(x, y) + \xi_k(x, y),
\]

where \( \sigma^{(0)} \) is a time-independent mean field solution which is determined in the following.

The mean field \( \sigma^{(0)} \) is of order \( \varepsilon^0 \) and \( \sigma_k \) is of order \( \varepsilon^{-1/2} \), then \( \xi_k \) is also of order \( \varepsilon^{-1/2} \). Hence, \( Z \) becomes

\[
Z = \lim_{\varepsilon \to 0} \int \frac{D \xi}{\mathcal{H}} e^{-\varepsilon/2} \mathcal{H}_k(\sigma^{(0)} + \xi) \mathcal{H}_k(\sigma^{(0)} + \xi) \mathcal{H} \mathcal{H}_k(1 - \varepsilon \delta \xi) \mathcal{H} \mathcal{H}_k(e^{\delta \xi} \sigma^{(0)} + \xi)
\]

\[
= \lim_{\varepsilon \to 0} \int \frac{D \xi}{\mathcal{H}} e^{-\varepsilon \mathcal{H}(\sigma^{(0)}, x)}.
\]

\( \sigma^{(0)} \) is fixed by the stationary condition:

\[
\frac{\partial S_{\text{eff}}(\sigma^{(0)}, \xi)}{\partial \xi} \bigg|_{\xi = 0} = 0.
\]

In the limit \( \varepsilon \to 0 \), \( \sigma^{(0)} \) is given by

\[
\sigma^{(0)}(x, y) = \int dx' dy' \phi(x, y; x', y') \phi(x', y') \phi(x, y') \phi(x'y') \sigma^{(0)},
\]

\[
\langle \phi^*(y) \phi(x) \rangle_{\sigma^{(0)}} = \text{Tr} \left[ \phi^*(y) \phi(x) e^{-\mathcal{H}_{\sigma^{(0)}}} \right] / \text{Tr} \left[ e^{-\mathcal{H}_{\sigma^{(0)}}} \right].
\]

\[
\mathcal{H}_{\sigma^{(0)}} = \int dx \phi^* \phi \mathcal{H} - \int dx dx' dy dy' \phi^*(x, y') \phi^*(x', y') \phi(x, y') \phi(x', y')
\]

\[
\times \left\{ \phi^*(x') \phi^*(y') \phi(x') \phi(y') \phi(x') \phi(y') \phi(x') \phi(y') \right\}.
\]

We choose \( \nu(x, y; x', y') \) as

\[
\int dx dx' dy dx' dy' \phi(x', y') \phi(x, y') \phi(x', y') \phi(x, y')
\]

\[
= V(x' - y') \delta(x' - y') \delta(y' - x')
\]

\[
+ \delta(x' - x') \delta(y' - y') \sum_{i=1}^{N} \Delta E_{i,0} \nu_{i,0}(y') \nu_{i,0}(x') \phi(x') \phi(y') \phi(x') \phi(y') \sigma^{(0)}.
\]

Using the ansatz form (2.9) for \( \phi^*(y) \phi(x) \), the energy eigenvalue equation for a single-particle becomes

\[
[K - F((\mathbf{P} + \mathbf{a})^2) \rho(\mathbf{x}, x) - \Delta E_{i,0}] \mathcal{H}_{\sigma^{(0)}} = E_{i,0} \mathcal{H}_{\sigma^{(0)}} \mathcal{H}_{\sigma^{(0)}},
\]

where we choose \( \Delta E_{i,0} \) as
\[ \Delta E_{\ell,0} = E_{\ell,0} - E_{\ell,0}^{(0)} \]  
\[ \text{(A·12)} \]

\( E_{\ell,0}^{(0)} \) is the eigenvalue for a uniform density state and is independent of \( \ell \). Thus the energy eigenvalues become degenerate in \( \ell \) and the state \( |\Psi\rangle \) of Eq. (2·12) becomes eigenstate of \( \tilde{H}_{\sigma^{(0)}} \) of Eq. (A·9). Since a difference between the eigenvalue equation in Eqs. (2·11) and (A·11) is only a constant \( \Delta E_{\ell,0} \), the eigenfunction \( \nu_{\ell,0}(x) \) is common in Eqs. (2·11) and (A·11). From Eqs. (A·10) and (A·11), the mean field energy and the exchange energy become

\[ E_{\ell,0}^{\text{mean}} = \sum_{\ell=0}^{M} \nu_{\ell} E_{\ell,0}^{(0)}, \]

\[ E_{\ell,0}^{\text{ex}} = \frac{1}{2} \int d^{2}x d^{2}y d^{2}x' d^{2}y' d^{2}z d^{2}z' v(x, y; x', y') v(y, x; z', z) \]

\[ \times \langle \phi^{\dagger}(x') \phi^{\dagger}(y') \rangle_{\sigma^{(0)}} \langle \phi(x') \phi(y') \rangle_{\sigma^{(0)}} \]

\[ = \frac{1}{2} \int d^{2}x d^{2}y V(x - y) U_{0}^{(n)}(x, y) U_{0}^{(n)}(y, x) + \frac{1}{2} \sum_{\ell=0}^{M} \nu_{\ell} \Delta E_{\ell,0}. \]  
\[ \text{(A·13)} \]

The quadratic fluctuation around \( \sigma^{(0)} \) is calculated as

\[ \frac{\partial^{2} S_{\text{eff}}}{\partial \xi_{kk} \partial \xi_{kk}'} |_{\sigma^{(0)}} = \delta_{kk} \delta(x' - x) \delta(y' - y) + \epsilon \delta_{kk} S(y', x'; x, y) \]

\[ - \epsilon(1 - \delta_{kk}) \int d^{2}x d^{2}y d^{2}x' d^{2}y' d^{2}z d^{2}z' v(y', x'; x, y) \]

\[ \times D_{n}(x'', y'', t_k; x', y', t_k) v(x, y; x'', y''), \]

\[ S(y', x'; x, y) = \langle \phi^{\dagger}(y') \phi^{\dagger}(x) V(x - y) \phi(x') \phi(y) \rangle_{\sigma^{(0)}} \]

\[ + \int d^{2}x d^{2}y d^{2}x' d^{2}y' d^{2}z d^{2}z' v(y', x'; x, y) \]

\[ \times \langle \phi^{\dagger}(x') \phi^{\dagger}(y') \rangle_{\sigma^{(0)}} \langle \phi(x') \phi(y') \rangle_{\sigma^{(0)}}. \]  
\[ \text{(A·14)} \]

From Eq. (A·14), the direct energy included in the fluctuation correction is written as

\[ \beta E_{\ell,0}^{\text{dir}} = \frac{1}{2} \text{Tr} \epsilon \delta_{kk} S \]

\[ = \beta \int d^{2}x d^{2}y (U_{0}^{(n)}(x, x) - U_{0}) V(x - y)(U_{0}^{(n)}(y, y) - U_{0}) + \frac{\beta}{2} \sum_{\ell=0}^{M} \nu_{\ell} \Delta E_{\ell,0}. \]  
\[ \text{(A·15)} \]

From Eqs. (A·13) and (A·15), the total energy becomes

\[ E_{\ell,0}^{\text{tot}} = E_{\ell,0}^{\text{mean}} + E_{\ell,0}^{\text{ex}} + E_{\ell,0}^{\text{dir}} + O(V^{2}) = E_{\ell,0}^{\text{tot}}. \]  
\[ \text{(A·16)} \]

Thus the results obtained in this paper are correct for the total energy \( E_{\ell,0}^{\text{tot}} \).

References

1) *The Quantum Hall Effect*, ed. R. E. Prange and S. M. Girvin (Springer, New York, 1990).
2) R. B. Laughlin, Phys. Rev. Lett. 50 (1983), 1395.
3) K. Ishikawa, Prog. Theor. Phys. Suppl. No. 107 (1992), 167; Prog. Theor. Phys. 88 (1992), 881.
N. Maeda

4) K. Ishikawa and N. Maeda, Phys. Lett. A186 (1994), 65.
5) K. Ishikawa and N. Maeda, Prog. Theor. Phys. 91 (1994), 237.
6) A. K. Kerman, S. Levit and T. Troudet, Ann. of Phys. 148 (1983), 436.
7) A. Abrikosov, ZhETF(USSR) 22 (1957), 1442 [JETP-Sov. Phys. 5 (1957), 1174].
   H. Nielsen and P. Olesen, Nucl. Phys. B61 (1973), 45.
8) R. Morf and B. I. Halperin, Phys. Rev. B33 (1986), 2221.
9) S. M. Girvin, A. H. MacDonald and P. M. Platzman, Phys. Rev. Lett. 54 (1985), 581; Phys. Rev. B33 (1986), 2481.
10) D. Yoshioka, B. I. Halperin and P. A. Lee, Phys. Rev. Lett. 50 (1983), 1219.
11) F. D. M. Haldane and E. H. Rezayi, Phys. Rev. Lett. 54 (1985), 237.
12) R. L. Willet, H. L. Stormer, D. C. Tsui, A. C. Gossard and J. H. English, Phys. Rev. B37 (1988), 8476.
13) A. H. MacDonald and S. M. Girvin, Phys. Rev. B34 (1986), 5639.
14) A. Pinczuk, B. S. Dennis, L. N. Pfeiffer and K. West, Phys. Rev. Lett. 70 (1993), 3983.