Thermodynamics of Apparent Horizon and Friedmann Equations in Big Bounce Universe

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In this paper, the thermodynamics of apparent horizon and Friedmann equations are studied in a big bounce universe typified by a non-singular big bounce, as opposed to a singular big bang. This cosmological model can describe radiation dominated early universe and matter dominated late universe in FRW model. Our calculational results show that Einstein gravitational field equations could be derived by the first law of thermodynamics and the fluid’s continuity equation. The connections between thermodynamics and gravity are observed in big bounce universe. In the late stages of cold and hot universes, the apparent horizons are convergent and the time when apparent horizons begin to bounce essentially in agreement with that of universe’s scalar factor. In the early stage of both cold and hot universes, we find there is only one geometry containing a 4D de Sitter universe with general state parameter. Furthermore, we also find the form of apparent horizon in early universe is strongly dependent on the extra dimension which suggests that the effect of extra dimension could be found in early universe.

PACS numbers: 04.50.-h; 98.80.Cq; 11.10.Kk
Keywords: high dimensional gravity; cosmology; apparent horizon

\textbf{I. INTRODUCTION}

Recently, the holographic principle points that the N dimensional gravity theory could be equivalent to one (N - 1) dimensional theory without gravity, such as the AdS/CFT correspondence \cite{1}. The holographic principle implies that one deep connection could exist between thermodynamics and gravity. One pioneer work of this subject was presented by Jacobson in Ref.\textsuperscript{2}, in which the Einstein field equation was obtained by using the Clausius relation \(\delta Q = T \delta S\) where \(\delta Q\) is the energy flux, \(T\) is Unruh temperature and \(S\) is the entropy of thermodynamic system. Then, Brustein and Hadad proved that the motion equations of generalized gravity theories were equivalent to above thermodynamic Clausius relation by using a more general definition of the Noether charge entropy \cite{3}. Soon after that, the gravitational equations following from thermodynamics is generalized to diffeomorphism invariant theories of gravity by Padmanabhan in Ref.\textsuperscript{4}, and Parikh and Sarkar in Ref.\textsuperscript{5}. Another inverse procedure put forward is to build one connection that Einstein equation could be treated as a thermodynamic identity. Padmanabhan first noticed that the gravitational field equation of spherically symmetric spacetime can be rewritten as a form of the ordinary first law of thermodynamics at a black hole horizon \cite{6}. Then, Cai and Kim employed a given entropy related horizon area to obtain the Friedmann equations from the first law of thermodynamics in corresponding gravity including \((n + 1)\) dimensional FRW universe with general spatial curvature, the higher derivative Gauss-Bonnet gravity and
Lovelock gravity \[7\]. Under FRW cosmological setup, the relationships between thermodynamics and gravitational field equation have been intensively investigated by various gravity models, such as \( f(r) \) gravity \[8\], scalar tensor gravity \[9\], braneworld cosmology \[10\]. About the reviews of this aspect, one can refer to Refs.\[11, 12\].

In the cosmology, the question of whether the time exists before the so-called “big bang” has always been the interesting topic \[13–17\]. Tolman proposed an oscillating cosmological model by using general relativity and pointed that the universe had to face the cosmological singularity and enormous inhomogeneities in every evolutional cycle \[14\]. Early this century, under the brane world scenario the so-called “ekpyrotic” cosmological model was presented by Khoury et.al. in Ref.\[15, 16\]. Then, Steinhardt and Turok presented a cyclic model where universe is dynamical from the big bang to the big crunch in every cyclic \[13\]. Most cosmological problems such as horizon problem and flatness problem could be solved well without inflation in above various cosmological models \[13–16\].

On the other hand, based on the considering about the problems of cosmological constant and singular big bang, Liu and Wesson proposed a big bounce cosmological model with a variable cosmological “constant”, in which there is only one transition from contraction to expansion \[17\]. Before the bounce, it is contracted from an empty de Sitter vacuum. This cosmological model was derived from a class of exact solutions of the 5D field equations were obtained by Liu and Mashhoon in Ref.\[18\]. This type of big bounce universe satisfies the 5D field equations \( R_{AB} = 0 \). Because two functions \( \eta \) and \( \zeta \) are contained, it has rich qualities mathematically. The most interesting is that the big bang appeared in standard 4D cosmological model is replaced by a big bounce in this model. At the big bounce point, universe arrives at a finite minimum scale. Our universe is contracted before bounce and is expanded after bounce. In usual 4D bounce universe of standard general relativity, there is a so-called inhomogeneity problem that the enormous inhomogeneity is generated in the collapsing phase \[19\]. However, in this 5D bounce universe the spacetime is not symmetric before and after the bounce point. The contraction is dominated by the new matter created because the universe could be empty at the beginning of contraction. Hence, this situation could avoid the inhomogeneity problem appeared in former 4D big bounce universe. It should be noticed that the function adopted as \( \mu(t) = t^n \) is based on the fact of that in the spatially flat FRW universe the scale factor has the form of power law. Considered two different components in early and later universe, the radiation dominated and matter dominated standard FRW universes are obtained respectively. By employing the first law of thermodynamics and the fluid’s continuity equation to the apparent horizon, we could derive the Friedmann equations in the 5D big bounce universe. Considering recently astronomical observations made in the same direction of flat universe, we also study its universal spatially flat model and its two kinds of extreme cases: the cold 3D flat case and the hot 3D flat case. Considering above situations, the motivation of this paper is trying to study the thermodynamics of apparent horizon and Friedmann equations and observe the connections between the thermodynamics and gravity about this big bounce universe.

This paper is organized as follows: In Section II, the 5D big bounce cosmological model is presented and the general Friedmann equations are derived through the thermodynamics of the apparent horizon. In Section III, thermodynamics of the apparent horizon is discussed in the general spatially flat model. In Section IV, thermodynamics of the apparent horizon is discussed in the late cold 3D flat model. In Section V, thermodynamics of the apparent horizon is discussed in the late hot 3D flat model. In Section VI, we discuss the early stage of general spatially flat universe. Section VII is conclusion. We adopt the signature \((+, -, -, -, -)\) and put \( h \) and \( c \) equal to unity. Greek indices \( \mu, \nu, \cdots \) will be taken to run over 0, 1, 2, 3 as usual, while capital indices \( A, B, C, \cdots \) run over all five coordinates \((0, 1, 2, 3, 5)\).
II. 5D BIG BOUNCE SOLUTIONS AND THEIR THERMODYNAMICS OF THE APPARENT HORIZON

A class of exact solutions of the 5D field equations $R_{AB} = 0$ are obtained by Liu and Mashhoon in Ref. [18],

$$dS^2 = \zeta^2(t, y)dt^2 - \eta^2(t, y) \left[ \frac{dr^2}{1 - kr^2} + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \right] - dy^2,$$

which represents the big bounce universe. Here, $k$ is the 3D curvature index ($k = \pm 1$ or 0) and the scale factors $\zeta$ and $\eta$ are listed as

$$\eta^2 = (\mu^2 + k) y^2 + 2\nu y + \frac{\nu^2 + K}{\mu^2 + k},$$

$$\zeta = \frac{1}{\mu} \frac{\partial \eta}{\partial t} = \dot{\eta} / \mu,$$

where $\mu = \mu(t)$ and $\nu = \nu(t)$ are two arbitrary functions varied with time. $K$ is a 5D curvature constant related to the Riemann-Christoffel tensor via

$$R_{ABCD}R^{ABCD} = \frac{72K^2}{A^8},$$

which agrees with the case of canonical coordinates models [18]. In this 5D universe, there is a freedom to fix $\mu = \mu(t)$ or $\nu = \nu(t)$ without changing the form of solutions (1) because $\zeta(t, y)dt$ is invariant under arbitrary transformation.

The 4D component of above 5D metric (1) could be written in the Robertson-Walker form under the standard FRW model as

$$ds^2 = \zeta^2(t, y)dt^2 - \eta^2(t, y) \left[ \frac{dr^2}{1 - kr^2} + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \right].$$

The non-vanishing components of the 4D Ricci tensor are given by

$$^{(4)}R_0^0 = -\frac{3}{\zeta^2} \left( \frac{\dot{\eta}}{\eta} - \frac{\dot{\eta}}{\eta^2} \right),$$

$$^{(4)}R_1^1 = ^{(4)}R_2^2 = ^{(4)}R_3^3 = -\frac{1}{\zeta^2} \left[ \frac{\ddot{\eta}}{\eta} + \dot{\eta} \left( \frac{2\ddot{\eta}}{\eta} - \frac{\dot{\zeta}}{\zeta} \right) + 2k \frac{\zeta^2}{\eta^2} \right].$$

So the 4D Ricci scalar is

$$^{(4)}R = -6 \left( \frac{\mu \dot{\mu}}{\eta \dot{\eta}} + \frac{\mu^2 + k}{\eta^2} \right).$$

Hence, according to the 4D Einstein tensor

$$^{(4)}G^\alpha_\beta \equiv ^{(4)}R^\alpha_\beta - \delta^\alpha_\beta ^{(4)}R/2,$$

the non-vanishing components are obtained as

$$^{(4)}G_0^0 = \frac{3(\mu^2 + k)}{\eta^2},$$

$$^{(4)}G_1^1 = ^{(4)}G_2^2 = ^{(4)}G_3^3 = \frac{2\mu \dot{\mu}}{\eta \dot{\eta}} + \frac{\mu^2 + k}{\eta^2}.$$

According to the proper time defined as $d\tau = \zeta(t, y)dt$, we can get the Hubble parameters written as,

$$H(t, y) = \frac{1}{\zeta \eta} \frac{\dot{\eta}}{\eta} = \frac{\mu}{\eta}. $$
Then in this 5D big bounce universe model, we can get the 4D Friedmann equations shown by

\[
\frac{1}{\zeta} \dot{H} - \frac{k}{\eta^2} = -4\pi G (p + \rho), \tag{13}
\]

\[
H^2 + \frac{k}{\eta^2} = \frac{8\pi}{3} G \rho, \tag{14}
\]

where the energy-momentum tensor of a perfect fluid with density \(\rho\) and pressure \(p\) has the following form

\[
T^{\alpha\beta} = (p + \rho) u^\alpha u^\beta - p g^{\alpha\beta} \tag{15}
\]

and the 4-velocity is \(u^\alpha \equiv dx^\alpha/ds\). Interestingly, Eqs. (13) and (14) reduce to the usual Friedmann equations in GR gravity under the limits of \(\zeta \to 1\) and \(\eta \to R\),

\[
\dot{H} - \frac{k}{R^2} = -4\pi G (p + \rho), \tag{16}
\]

\[
H^2 + \frac{k}{R^2} = \frac{8\pi}{3} G \rho. \tag{17}
\]

It is known that the apparent horizon \(\tilde{r}_A\) is a marginally trapped surface with vanishing expansion. In order to obtain the apparent horizon \(\tilde{r}_A\) conveniently, the 4D metric of this 5D big bounce universe is rewritten as,

\[
ds^2 = h_{ab} dx^a dx^b + \tilde{r} d\Omega^2, \tag{18}
\]

where \(d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2\), \(\tilde{r} = \eta r\), \(x^0 = t\), \(x^1 = r\), \(h_{ab} = \text{diag}(\zeta^2, -\eta^2/(1 - kr^2))\). Hence the radius of apparent horizon \(\tilde{r}_A\) could be given out by the null hypersurface, i.e. \(h^{ab} \partial_a \tilde{r} \partial_b \tilde{r} = 0\). By using 4D component metric shown above, we can get

\[
\tilde{r}_A = \frac{1}{\sqrt{H^2 + k/\eta^2}}, \tag{19}
\]

which also could be rewritten as

\[
\frac{\dot{\tilde{r}}_A}{\tilde{r}_A} = H \left( \frac{k}{\eta^2} - \dot{H} \right). \tag{20}
\]

Then we need to give out the thermodynamics of apparent horizon \(\tilde{r}_A\) following. Considering the spherical structure of spacetime, we assume the entropy associated with the apparent horizon written as the area formula, i.e. \(S = A_0/4G\) where \(A_0\) is the area of apparent horizon, i.e. \(A_0 = 4\pi \tilde{r}_A^2\). Like black hole physics, the temperature of apparent horizon is defined by \(T = \kappa/2\pi\) where \(\kappa\) is surface gravity \(\kappa = \frac{1}{2\sqrt{-h}} \partial_{\mu}(\sqrt{-h} h^{\mu\nu} \partial_{\nu} \tilde{r})\).

According to the perfect fluid’s energy-momentum tensor \(T_{\mu\nu}\), the energy-supply vector is given by

\[
\Psi_a = T^b_a \partial_b \tilde{r} + W \partial_a \tilde{r}, \tag{21}
\]

where \(W\) is the work density defined by

\[
W = -\frac{1}{2} T^{ab} h_{ab}. \tag{22}
\]

By using above energy-supply vector, the amount of energy crossing the apparent horizon is given by

\[
- dE = A_0 (p + \rho) \zeta H \tilde{r}_A dt. \tag{23}
\]
Then the first law of thermodynamics shown below is applied to the apparent horizon

\[-dE = TdS.\]  

(24)

Based on the apparent horizon’s temperature $T$, the entropy $S$ and the crossing amount energy $dE$, the first law of thermodynamics Eq. (24) can give the space-space component of Friedmann equations shown by

\[4\pi G(\rho + p) = - \left( \frac{1}{\zeta} \dot{H} - \frac{k}{\eta^2} \right),\]  

(25)

which describes the 4D FRW universe with spatial curvature $k$ in this big bounce universe. Then based on above Friedmann equation (25), the perfect fluid’s continuity equation is shown by

\[\dot{\rho} + 3\zeta H (\rho + p) = 0,\]  

(26)

can help us give another time-time component of Friedmann equations

\[\frac{8\pi G}{3} \rho = H^2 + \frac{k}{\eta^2}.\]  

(27)

Hence, the Friedmann equations (25) and (27) for the 4D FRW universe in this 5D big bounce universe are obtained by the first law of thermodynamics and the perfect fluid’s continuity equation near the apparent horizon $\tilde{r}_A$. The connections between the gravity of 5D big bounce universe and the thermodynamics of its apparent horizon are observed. On each hypersurface the first law of thermodynamics of apparent horizon is successfully corresponding to the spatial component of Friedmann equations of gravity. The fluid’s continuity equation is corresponding to the time component of Friedmann equations of gravity.

III. THE UNIVERSAL SPATIALLY FLAT MODEL

It is generally known that the results of recent CMB’s astronomical observations incline our universe towards a spatially flat case. In fact if the universe is not flat the geodesics of massless particles are that photons starting out parallel to each other slowly diverge [20]. Meanwhile many other evidences also support the flat fact, such as the researching of anisotropy spectrum based on three small scale experiments: DASI [21], Maxima [22], Boomerang [23]. The anisotropy spectrum in flat versus open universe where the pattern of peaks and troughs persists in the open universe but is shifted to smaller scales, clearly favoring the flat case. Else, the early astrophysical data such as the age of the universe [24, 25] also points that the space sections of FRW universe probably is flat. Just as what said in Ref. [17] this 5D big bounce universe can describe many universe models under proper scalar factor function, especially in the universal spatially flat case. Hence, considering these situations, the scale factors of this big bounce are given by [17]

\[\eta(t, y)^2 = \mu^2 (y - Ct)^2 + \frac{K}{\mu^2},\]  

(28)

\[\zeta(t, y) = \frac{\dot{\eta}}{\mu},\]  

(29)

\[\mu(t) = t^n, n = -\frac{1 + 3\gamma}{3(1 + \gamma)},\]  

(30)

where the equation of state is taken to be the isothermal one i.e. $p = \gamma \rho$. The ordinary matter is represented by the state with $\gamma$ in the range $(0, 1/3)$. Two extreme cases the dust and radiation or ultrarelativistic particles are
represented by the states of $\gamma = 0$ and $\gamma = 1/3$, respectively. The constant $C$ in above formula depends on the state parameter $\gamma$,

$$C = \begin{cases} \frac{2}{3}(1 + \gamma), & 0 \leq \gamma < \frac{4}{3}, \\ \sqrt{\frac{4}{3} - K}, & \gamma = \frac{4}{3}. \end{cases} \quad (31)$$

The density of matter and the cosmological term are listed by

$$\rho = \left(\frac{2}{1 + \gamma}\right) \frac{\mu}{\eta} \left[\frac{\mu}{\eta} - \frac{\dot{\mu}}{\dot{\eta}}\right],$$

$$\Lambda = \left(\frac{2}{1 + \gamma}\right) \frac{\mu}{\eta} \left[\left(\frac{1 + 3\gamma}{2}\right) \frac{\mu}{\eta} + \frac{\dot{\mu}}{\dot{\eta}}\right]. \quad (33)$$

The apparent horizon is

$$\tilde{r}_A = \frac{\zeta \eta}{\dot{\eta}} = \frac{1}{H} = \sqrt{\frac{t^{4n}(y - C t)^2 + K}{t^{2n}}}. \quad (34)$$

It is shown that the apparent horizon identically equals the Hubble horizon. This point is also justified by Ref.[7].

This equation gives a relationship between $\dot{H}$ and $\tilde{r}_A$

$$\frac{\dot{r}_A}{r_A^3} = -H \dot{H}. \quad (35)$$

The entropy of black hole is assumed as the normal form $S = A/4G$ and the temperature is $T = 1/2\pi \tilde{r}_A = H/2\pi$. Hence, according to the amount of energy crossing the apparent horizon, i.e. Eq.(23) and the first law of thermodynamics, the Friedmann equation is given by

$$4\pi G(\rho + p) = -\frac{1}{\zeta} \dot{H}. \quad (36)$$

Substituting the continuity equation (26) into Eq.(36) and integrating the equation, we can obtain another component of Friedmann equations in this spatially flat model,

$$\frac{8}{3} \pi G \rho = H^2. \quad (37)$$

According to the Hubble constant $H$ (34) and the spatial Friedmann equation (37), the density of big bounce universe with general 3D flat is given out by

$$\rho = \frac{3t^{4n}}{8\pi G \left[t^{4n}(y - Ct)^2 + K\right]} \quad (38)$$

**IV. THE LATE STAGE OF COLD 3D FLAT UNIVERSE**

Here, we will discuss the cold 3D flat case. The energy of component comes chiefly from the rest energy which is greater than its thermodynamic kinetic energy. The work of pressure changes only the trivial thermodynamic kinetic energy, and hence the modification of total mass of $\rho_m R^3$ could be ignored safely. In the natural unit, the pressure of dust is equal to its density of thermal kinetic energy, the dust satisfies $p_{dust} \ll \rho_{dust}$ with $\gamma = 0$ which could give us the equation of state, i.e. $p_{dust} = 0$, to represent the matter content of the late universe. Hence, in this 5D big bounce universe, the cold 3D flat case is given by

$$\eta^2 = \frac{9}{4} t^{4/3} + K t^{2/3} - 3yt^{1/3} + y^2 t^{-2/3}, \quad (39)$$
where \( \mu(t) = t^{-1/3} \) and the parameters are listed as \( \gamma = 0, \ p = 0, \ n = -1/3, \ C = 3/2 \). About the detail discussion of this case, one can refer to Ref. [17]. It’s interesting that there is a finite minimum of scalar factor \( \eta \) at \( t = t_{m1} \), before which our universe contracts and after which it expands. In usual FRW model, when the time limits zero the scalar factor vanishes. This situation leads directly a singularity geometry and the divergent matter, which is called the big bang. However, in this big bounce universe, the 5D curvature invariant i.e. Eq.(4) is finite on the hypersurface of \( y = \text{constant} \). Furthermore, because that \( \frac{d\eta}{dt} = 0 \) and \( \nu(t) \neq 0 \) are satisfied at the point of \( t_{m1} \), all components of this big bounce universe are diverge such as \( \rho, \Lambda, \ (4)^{R} \) and so on, except for the 5D curvature invariant Eq.(4) keeps finite.

According to the definition of Hubble parameter \( H(t, y) \) [12], we have

\[
H^{-1} = \sqrt{\frac{9}{4}t^2 + Kt^{4/3} - 3yt + y^2} = \tilde{r}_A. \tag{40}
\]

After calculating the amount of energy crossing the apparent horizon, we can get the Friedmann equation in this cold 3D-flat model as

\[
4\pi G\rho \zeta = -\dot{H}. \tag{41}
\]

According to the continuity equation [26], the density could be rewritten as

\[
\rho = \frac{3}{8\pi G} H^2. \tag{42}
\]

The integral former of Hubble parameter in this cold case could be written as

\[
H^{-1} = \frac{3}{2} \int \zeta(y, t) dt. \tag{43}
\]

For the case of \( t \gg t_{m} \), the scalar factor is

\[
\eta = \frac{3}{2} t^{2/3} \left[ 1 + \frac{2}{9} K t^{-2/3} - \frac{2}{3} y t^{-1} + \mathcal{O}(t^{-4/3}) \right], \tag{44}
\]

\[
\zeta = \frac{\dot{\eta}}{\mu} = 1 + \frac{y}{3t} + \mathcal{O}(t^{-4/3}). \tag{45}
\]

The 5D line element reads

\[
dS^2 = \left[ 1 + \mathcal{O}(t^{-1}) \right] dt^2 - \left[ \frac{3}{2} t^{2/3} + \frac{1}{3} K + \mathcal{O}(t^{-1/3}) \right]^2 (dr^2 + r^2 d\Omega^2) - dy^2, \tag{46}
\]

which is an Einstein de Sitter like spacetime. When \( t \) approaches proper time, the scale factor varies as \( t^{2/3} \).

According to the definition of Hubble parameter \( H(t, y) \) [12], we have

\[
H^{-1} = \frac{2}{3} \left( t + \frac{1}{3} y \ln t \right) + C_1, \tag{47}
\]

where \( C_1 \) is integral constant. Hence, the density of this case is shown by

\[
\rho = \frac{243}{8\pi G (6t + 2y \ln t + 9C_1)^2}. \tag{48}
\]
The apparent horizons of cold 3D flat model are illustrated in Fig.1. Similar with the scalar factor $\eta$, there is also a finite minimum of $\tilde{r}_A$ at $t = t_{m1}$, before $t_{m2}$ the apparent horizon contracts and after $t_{m2}$ it expands. The apparent horizons coincide with each other at the time $t = 1$ where intersection point has nothing to do with extra dimension which could be seen easily by apparent horizon’s formula Eq.(48). Before the time of intersection $\tilde{r}_A$ decreases with bigger extra dimension. Otherwise, after this intersection $\tilde{r}_A$ increases with bigger extra dimension.

Then we need to discuss two bounce points: $t_{m1}$ and $t_{m2}$. Comparing the late cold universe’s scalar factor $\eta$ with the Hubble parameter ($H$), we can find the bounce point $t_{m2}$ of apparent horizon lags behind the big bounce $t_{m1}$. Because that the big bounce $t_{m1}$ could be obtained through the derivative of $\eta$ with respect to time,

$$\left. \frac{d\eta(t)}{dt} \right|_{t=t_{m1}}^{\text{cold}} = 0,$$

which leads following formula as

$$t_{m1}^{-1/3} \left(1 + \frac{y}{3t_{m1}} \right) = 0.$$

On the other hand, the apparent horizon’s bounce point $t_{m2}$ could be obtained by the derivative of $H^{-1}$ with respect to time,

$$\left. \frac{d(H(t)^{-1})}{dt} \right|_{t=t_{m2}}^{\text{cold}} = 0,$$

which also leads a simple result as

$$3 + \frac{y}{t_{m2}} = 0.$$

Then comparing Eq.(50) with Eq.(52), one can easily find that the bounce point of apparent horizon $t_{m2}$ is the same to universe’s big bounce point $t_{m1}$ exactly.
V. THE LATE STAGE OF HOT 3D FLAT UNIVERSE

Here, we will discuss the early universe dominated by the radiation or ultrarelativistic particles, such as photon radiation. Because the rest mass of photon is null and the thermal kinematic velocity is speed of light, the whole density of photon comes from its kinematic mass completely. Based on the Planck’s distribution law, the equation of state is \( P_\gamma = \rho_\gamma / 3 \). Hence, in this 5D big bounce universe, the hot 3D flat case is given by

\[
\eta^2 = 4t - 2Cy + \frac{y^2}{t},
\]

where \( \gamma = 1/3 \), \( n = -1/2 \), \( C = \sqrt{4 - K} \) and \( \mu = t^{-1/2} \). The behavior of this kind hot universe is similar with cold case, which contracts before the time of big bounce \( t_{m1} \) and expands after \( t_{m1} \). About the further discussions, one can refer to Ref. [17].

The Hubble parameter of this hot 3D flat model is given by

\[
H^{-1} = \sqrt{4t^2 - 2yt \sqrt{4 - K} + y^2}.
\]

The Friedmann equations are given by former apparent horizon method,

\[
\rho = -\frac{3}{16\pi G} \frac{\dot{H}}{\zeta}.
\]

By using the continuity equation, we can get the Hubble parameter

\[
H^{-1} = 2 \int \eta(y,t) dt.
\]

When \( t \gg t_m \), the first term of \( \eta^2 \) is dominant. We have following approximate expressions

\[
\eta = 2t^{1/2} \left( 1 - \frac{\sqrt{4 - K} y}{4t} + O(t^{-2}) \right),
\]

\[
\zeta = 1 + \frac{y}{4t} \sqrt{4 - K} + O(t^{-2}).
\]

The 5D radiation metric is obtained as

\[
dS^2 = \left[ 1 + \frac{Cy}{2t} + O(t^{-2}) \right] dt^2 - 4t \left[ 1 - \frac{Cy}{2t} + O(t^{-2}) \right] (dr^2 + r^2 d\Omega^2) - dy^2,
\]

which are analogous to the 4D radiation metric. Unlike the cold case, when \( t \) approaches proper time, the scale factor varies as \( t^{1/2} \).

The Hubble parameter is

\[
H^{-1} = 2t + \frac{y}{2} \sqrt{4 - K} \ln t + C_3
\]

where \( C_3 \) is integral constant. The density of this case is shown by

\[
\rho = \frac{3}{2\pi G \left( 4t + y\sqrt{4 - K} \ln t + 2C_3 \right)^2}.
\]

It is illustrated in Fig. 2 that in this late hot case the apparent horizons are convergent at the point of \( t = 1 \) which point has nothing to do with extra dimension. With increasing extra dimension \( y \), the apparent horizons decrease
FIG. 2: the apparent horizons vs time in the $t \gg t_m$ case in the hot 3D flat model with $K = 1$, $C_3 = 0$ and $y = -1/4$(dashed line), $-2/4$(dotted line), $-3/4$(dash-dot line), $-1$(solid line). accordingly. Furthermore, according to the scalar factor $\eta$ and Hubble parameter $H$ we can easily find the bounce point $t_{m_2}$ of apparent horizons is also the same to the big bounce of universe $t_{m_1}$ exactly. Because that the big bounce $t_{m_1}$ could be obtained through the derivative of $\eta$ with respect to time,

$$
\left. \frac{d\eta(t)}{dt} \right|_{t=t_{m_1}}^{hot} = 0,
$$

which leads following formula as

$$
t_{m_1}^{-1/2} \left( 4 + \frac{y\sqrt{4-K}}{t_{m_1}} \right) = 0.
$$

On the other hand, the apparent horizon's bounce point $t_{m_2}$ could be obtained by the derivative of $H^{-1}$ with respect to time,

$$
\left. \frac{d(H(t)^{-1})}{dt} \right|_{t=t_{m_2}}^{hot} = 0,
$$

which also leads a simple result as

$$
4 + \frac{y\sqrt{4-K}}{t_{m_2}} = 0.
$$

Comparing formula Eq. (63) derived by scalar factor of universe with formula Eq. (65) obtained by apparent horizon, one can easily find these two bounce points have the same magnitude exactly.
VI. THE EARLY STAGE OF COLD AND HOT 3D FLAT UNIVERSE

For the early stage of cold 3D flat universe, the last term in $\eta^2$ is dominant with $t \ll t_m$. So we have the approximate expressions of $\eta$ and $\zeta$ shown by

$$\eta = yt^{-1/2} \left(1 - \frac{3}{2y} t + \frac{K}{2y^2} t^{4/3} + \mathcal{O}(t^2) \right),$$
(66)

$$\zeta = -\frac{y}{3t} - 1 + \frac{K}{2y^{1/3}} + \mathcal{O}(t^2),$$
(67)

The 5D line element has the canonical form as

$$dS^2 = \frac{y^2}{L^2} \left[ dT^2 - e^{-2T/L} \left(dr^2 + r^2 d\Omega^2 \right) \right] - dy^2,$$
(68)

where the coordinate transformation $t = L^3 e^{3T/L}$ is adopted. This metric is a canonical form given by Ref.[18].

The Hubble parameter is

$$H^{-1} = \frac{y}{2} \ln \frac{1}{t} - \frac{3}{2} t + \frac{9}{16} y^{4/3} + C_2,$$
(69)

where $C_2$ is integral constant. Hence the density of this case is shown by

$$\rho = \frac{96}{\pi G \left(8y \ln 1/t - 24t + 9K/y^{4/3} + 16C_2\right)^2}.$$  
(70)

For the early stage of hot 3D flat universe, the last term of $\eta^2$ is dominant with $t \ll t_m$, which gives us following formulas

$$\eta = yt^{-1/2} \left(1 - \sqrt{4 - K}t + \mathcal{O}(t^2) \right),$$
(71)

$$\zeta = -\frac{y}{2t} - \sqrt{1 - \frac{K}{4}} + \mathcal{O}(t^2),$$
(72)

The canonical form metric could be obtained as

$$dS^2 = \frac{y^2}{L^2} \left[ dT^2 - e^{-2T/L} \left(dr^2 + r^2 d\Omega^2 \right) \right] - dy^2,$$
(73)

where the coordinate transformation $t = L^2 e^{2T/L}$ is used. Comparing with the cold case, one can easily find that this canonical metric solution is the same to former case.

The Hubble parameter is

$$H^{-1} = y \ln \frac{1}{t} - \sqrt{4 - Kt} + C_4,$$
(74)

where $C_4$ is integral constant. The density of this case is shown by

$$\rho = \frac{3}{8\pi G \left(y \ln 1/t - \sqrt{4 - Kt} + C_4\right)^2}.$$  
(75)

Until now, it is surprise slightly that whether in the cold case or in the hot case, the geometries have the same format exactly. In order to discuss this point, we need to shift our vision to the start point, i.e. the universal formulas Eqs.(28), (29) and (30) related the 3D flat model no matter in cold case or in hot case. Then considering the situation of the early universe stage $t \ll t_m$, the scalar factor is rewritten approximately as

$$\eta(t,y)^2 = y^2 t^{2n} + K t^{-2n}.$$  
(76)
Calculating the partial derivative of $\eta(t, y)^2$ with respect to $t$ directly, one can get the expression of $\zeta(t, y)$

$$\frac{\dot{\eta}}{\mu} = (y^2 - \mu^{-4}K) \frac{\mu}{\eta^2} = \zeta(t, y),$$

which gives us $\zeta(t, y)^2$ appeared in the metric,

$$\zeta(t, y)^2 = \frac{n^2}{t^2} \left( \frac{y^2 - Kt^{-4n}}{y^2 + Kt^{-4n}} \right)^2.$$  \hspace{1cm} (78)

Then in this big bounce universe, the parameter $n$ is negative if the state parameter $\gamma$ is positive. So the negative $n$ ensures the term $t^{-2n} K$ contained in Eqs. (76) and (78) could be safely ignored in the early stage of 3D flat universe. Finally, the expressions of $\eta^2$ and $\zeta^2$ can be rewritten in the simple formulas shown as

$$\eta(t, y)^2 = y^2 t^{2n},$$

$$\zeta(t, y)^2 = y^2 \frac{n^2}{t^2},$$

Then we carry out the coordinate transformation $t = L^{-1/n}e^{-\tau/nL}$ where $L$ is a constant, the big bounce metric becomes

$$dS^2 = \frac{y^2}{L^2} \left[ d\tau^2 - e^{-2\tau/L} \left( dr^2 + r^2 d\Omega^2 \right) \right] - dy^2,$$  \hspace{1cm} (81)

which is a canonical form metric shown by Refs. 18, 26. The brackets of metric Eq. (81) is the 4D de Sitter metric which could be interpreted as $\rho = 0$ and $\Lambda = 3/L^3$. Substituting Eq. (79) into the density (32) and cosmological term (33), with general state parameter $\gamma$ we can get $\rho = 0$ and $\Lambda = 3/y^2$ which is justified as the limit of $t \to 0$ in the early stage of cold and hot 3D flat universe shown by Ref. 17. Else, according to Eq. (79) the apparent horizon has $\tilde{r}_A = y$ which proves again that the extra dimension effects the early universe more deeply. Then according to the coordinate transformation $t = L^{-1/n}e^{-\tau/nL}$, one can see that $t \to 0$ corresponds to $\tau \to -\infty$, in which $\tau$ is the real proper time of 4D. So the universe existed forever before big bounce in $\tau$ time. So in the early stage of 3D flat universe, the geometries of spacetime, which contain the 4D de Sitter metric, have the same forms exactly no matter in cold case or in hot case. This result does not depend on the EOS and has nothing to do with the specific form of matter.

VII. CONCLUSION

In this paper, we have studied the thermodynamic properties of cosmological apparent horizon in the 5D big bounce universe. We summarize what has been achieved.

Firstly, the connections between the thermodynamics and gravity are observed in big bounce universe. The Friedmann equations of universe can be obtained by using the first law of thermodynamics and the perfect fluid’s continuity equation to the apparent horizon with general spatial curvature. Meanwhile, we also check the result in spatially flat universe as well as its two extreme cases: one is the dust case described the matter content of the late universe with $\gamma = 0$ and $p = 0$, another is the case of radiation or ultrarelativistic particles described the early universe with $\gamma = 1/3$ and $p = \rho/3$. The continuity equation used here is differen from that of normal 4D case, i.e. $\dot{\rho} + 3H(\rho + p) = 0$. The reason is that Hubble parameters Eq. (12) of big bounce universe are determined both by
the scale factor of 3D space $\eta$, i.e. Eq. (2), and the scale factor of the time $\zeta$, i.e. Eq. (3). This situation is unlike the Hubble parameter of normal 4D universe which is determined only by the scale factor of 3D space.

Secondly, in the late time of cold universe, there is a bounce time $t_{m2}$ about the apparent horizon, which is physically different from the bounce time of universe $t_{m1}$. The apparent horizons of cold and hot 3D flat model are illustrated in Fig. 1 and Fig. 2 respectively. In general, the evolution of apparent horizon $\tilde{r}_A(y, t)$ is essentially in agreement with universe’s scalar factor $\eta(y, t)$. Interestingly, the bounce times of $\tilde{r}_A(y, t)$ and $\eta(y, t)$ have the same magnitude, i.e. $t_{m2} = t_{m1}$. However, one different place is that $\tilde{r}_A(y, t)$ is convergent at $t = 1$. Before convergent point $\tilde{r}_A(y, t)$ decreases with increasing extra dimension. It is the opposite way that $\tilde{r}_A(y, t)$ increases with bigger extra dimension after convergent point. In the late time of hot universe, $\tilde{r}_A(y, t)$ and $\eta(y, t)$ also have the same bounce time, i.e. $t_{m2} = t_{m1}$. Similar with the cold case, the apparent horizons are convergent at $t = 1$ which is unaffected by the extra dimension. $\tilde{r}_A(y, t)$ and $\eta(y, t)$ are both increasing with decreasing extra dimension.

Thirdly, in the early stage of universe, it is interesting that under a general state parameter $\gamma$ the original 5D big bounce metric (1) reduces to a canonical form metric (81) which contains a 4D de Sitter metric. This result is independent on the EOS. The apparent horizon of early universe, $\tilde{r}_A = y$, is dependent on the extra dimension sensitively. It means that the extra dimension effects the early universe strongly.

Acknowledgments

Liu’s work is supported in part by NSFC (No.11475143), HASTIT (No.14HASTIT043) and Foundation for University Key Teacher by He’nan Educational Committee. Xu’s work is supported in part by NSFC(No.11275035). Lv’s work is supported in part by NSFC(No.11205078).

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