In the present study we analyse the effect of the density dependence of the symmetry energy on the hyperonic content of neutron stars within a relativistic mean field description of stellar matter. For the Λ-hyperon, we consider parametrizations calibrated to Λ-hypernuclei. For the Σ and Ξ-hyperons uncertainties that reflect the present lack of experimental information on Σ and Ξ-hypernuclei are taken into account. We perform our study considering nuclear equations of state that predict two solar mass stars, and satisfy other well settled nuclear matter properties. The effect of the presence of hyperons on the radius, the direct Urca processes, and the cooling of accreting neutron stars are discussed. We show that some star properties are affected in a similar way by the density dependence of the symmetry energy and the hyperon content of the star. To disentangle these two effects it is essential to have a good knowledge of the equation of state at supra-saturation densities. The density dependence of the symmetry energy affects the order of appearance of the different hyperons, which may have direct implications on the neutron star cooling as different hyperon neutrino processes may operate at the center of massive stars. For models which allow for the direct Urca process to operate, hyperonic and purely nucleonic ones are shown to have a similar luminosity when hyperons are included in agreement with modern experimental data. It is shown that for a density dependent hadronic model constrained by experimental, theoretical and observational data, the low-luminosity of SAX J1808.4 – 3658 can only be modelled for a hyperonic NS, suggesting that hyperons could be present in its core.

I. INTRODUCTION

The behavior of asymmetric nuclear matter is strongly influenced by the density dependence of the symmetry energy of nuclear matter, see [1] for a review. This quantity defines the properties of systems like nuclei far from the stability line or neutron stars (NS), from the neutron skin thickness to the NS radius [2]. The advancement of nuclear physics and astrophysics requires, therefore, a well-grounded knowledge of the properties of isospin-rich nuclear matter [3, 5]. In the present study, we will concentrate our attention on the effect of the density dependence of the symmetry energy on some of the properties of hyperonic stellar matter that may occur inside NSs, including the mass and radius of hyperonic stars [6-8] or their cooling evolution [9, 10].

Although the symmetry energy is quite well constrained at nuclear saturation density, see [11, 12], its density dependence at high densities is still badly known. The density dependence of the symmetry energy has been investigated in many works, see for instance [14, 21], but usually for the saturation and sub-saturation densities. Since the description of NSs requires the knowledge of the equation of state (EoS), from very low to very high densities, it is important to have a correct description of the EoS in the whole range of densities.

Hyperons may have non-zero isospin, and, therefore it is expected that the NS strangeness content and, in particular, the non-zero isospin hyperons, will be affected by the density dependence of the symmetry energy. In the present study we will analyse the interplay between the symmetry energy and the hyperon content in the framework of relativistic mean-field models, following closely the work developed in [7, 8], but with the care of choosing hyperonic models that have been calibrated to the existing experimental hypernuclei data, as developed in [22]. Besides, we will only consider unified inner crust-core EoS since a non-unified EoS may give rise to a large uncertainty on the star radius, as discussed in [23].

The possible existence of hyperons inside NSs has been questioned [6, 24] because many of the models including hyperons are not able to predict massive stars such as the pulsars PSR J1614 – 2230 [24, 25] and PSR J0348 + 0432 [26] both with a mass close to or just above two solar masses, or even the PSR J1903 + 0327 with a mass 1.67 ± 0.02 M⊙ [6, 27]. This has been designated by the “hyperon puzzle” and a review of the problem, and of the solutions that can overcome possible contradictory scenarios has been presented in [28].

We will consider that the presence of hyperons is not simply ruled out by the existence of two solar mass stars and that this problem can be controlled by either using EoSs that are hard enough at high densities [23] or by going beyond the simple SU(6) symmetry ansatz to fix the isoscalar vector meson couplings [29, 30], or even by considering that nuclear matter may undergo a phase transition to quark matter [31, 32]. Having this in mind we will explore different RMF models of nuclear matter that satisfy a set of well-established nuclear matter properties at saturation as developed in [23].

The paper will be organized in the following way: a review of the formalism and presentation of the models that will be used in the study is given in Sec. [1] In Sec. [II] and [V] we discuss, respectively, the calculation of the inner crust EoS, and the choice of the hyperon-meson couplings, including the calibration of the hyperon Λ-meson couplings for the recently proposed RMF models FSU2 [33], FSU2R and FSU2H [34]. In Sec. [V] the effect of the symmetry energy on the nucleonic direct Urca process, also in the presence of hyperons, and the effect of the still badly constrained Σ-potential in symmetric nuclear matter on the star properties, including cooling, are discussed. Finally, in Sec. [V] some conclusions are drawn.
II. THE MODEL

We will undertake the following discussion in the framework of a relativistic mean field (RMF) approach to the equation of state of nuclear and stellar matter. Many models have been proposed within this framework, see the recent publication [35] for a compilation of a large number of those models and their properties. We will restrict ourselves to a small set, both with density dependent couplings and non-linear meson terms, that we will justify later. Within this approach, we start from the following Lagrangian density

\[ \mathcal{L} = \sum_{j=1}^{8} \bar{\psi}_j \left( i \gamma_\mu \partial^\mu - m_j + g_{\sigma j} \sigma + g_{\sigma' j} \sigma' \right) \psi_j - g_{\omega j} \bar{\psi}_j \omega^\mu \psi_j - g_{\phi j} \bar{\psi}_j \phi^\mu \psi_j - g_{\rho j} \bar{\psi}_j \rho^\mu \psi_j \]

\[ + \frac{1}{2} \left( \partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2 \right) - \frac{3}{4} g_3^2 \sigma^3 - \frac{1}{4} g_3^4 \sigma^4 \]

\[ + \frac{1}{2} \left( \partial_\mu \sigma' \partial^\mu \sigma' - m_{\sigma'}^2 \sigma'^2 \right) + \frac{3}{4} c_3 \left( \omega^\mu \omega_\mu + \mathcal{L}_{nl} \right) \]

\[ - \frac{1}{4} W_{\mu \nu} W^{\mu \nu} - \frac{1}{4} P_{\mu \nu} P^{\mu \nu} - \frac{1}{4} \tilde{R}_{\mu \nu} \tilde{R}^{\mu \nu} \]

\[ + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu + \frac{1}{2} m_\phi^2 \phi^\mu \phi_\mu + \frac{1}{2} m_\rho^2 \rho^\mu \rho_\mu , \] (1)

where \( \psi_j \) stands for the field of \( j \) baryon, \( \sigma, \sigma' \) are scalar-isoscalar meson fields, coupling to all baryons \( (\sigma) \) and to strange baryons \( (\sigma') \), and \( \omega^\mu, \phi^\mu, \rho^\mu \) denote the vector isoscalar (the first two) and isovector (the last) fields, respectively. The \( \omega \) and \( \rho \) couple to all baryons and the \( \phi \) only to baryons with strangeness. \( W_{\mu \nu}, P_{\mu \nu}, \tilde{R}_{\mu \nu} \) are the vector meson field tensors \( V_{\mu \nu} = \partial_\mu V_\nu - \partial_\nu V_\mu \). The couplings \( g_{\sigma N}, g_{\omega N}, g_{\rho N}, g_{\phi N}, g_3 \), and the \( \sigma, \omega, \phi, \rho \) meson masses are fitted to different kinds of data: experimental, theoretical and observational. The function \( \mathcal{L}_{nl} \) may be very general and defines the density dependence of the symmetry energy. In the present study we will limit ourselves to

\[ \mathcal{L}_{nl}(\sigma, \omega_\mu, \phi_\mu) = \left( a_1 g_\sigma^2 \sigma^2 + b_1 g_\omega \omega_\mu \omega^\mu \right) \tilde{\bar{\mu}}^\mu \tilde{\bar{\phi}}^\mu \]

\[ = A(\sigma, \omega^\mu \omega_\mu) \tilde{\bar{\mu}}^\mu \tilde{\bar{\phi}}^\mu , \] (2)

where \( g_{\sigma N} \) and \( g_{\omega N} \) are the couplings of the nucleons to the \( \sigma \) and \( \omega \) mesons. We will only consider \( a_1 \neq 0 \) and \( b_1 = 0 \) or \( a_1 = 0 \) and \( b_1 \neq 0 \). These terms have been introduced in [24] and [56] to explicitly model the density dependence of the symmetry energy.

For the models with density-dependent couplings, all non-linear terms, including the contribution \( \mathcal{L}_{nl} \), are zero. The couplings of meson \( i \) to baryon \( j \) are written in the form

\[ g_{ij}(n) = g_{ij}(n_0) h_M(x) , \quad x = n/n_0 , \] (3)

where the density \( n_0 \) is the saturation density \( n_0 = n_{sat} \) of symmetric nuclear matter. In the present study, we consider the parametrizations DD2 [37] and DDME2 [38]. For these two parametrizations the functions \( h_M \) assumes for the isoscalar couplings the form [37],

\[ h_M(x) = a_M \frac{1 + b_M (x + d_M)^2}{1 + c_M (x + d_M)^2} \] (4)

and for the isovector couplings the form

\[ h_M(x) = \exp[-d_M (x - 1)] \] . (5)

The values of the parameters \( a_M, b_M, c_M, d_M \) can be obtained from Ref. [37] for DD2 and from [38] for DDME2.

Both types of model with constant couplings and density-dependent couplings will be considered in the mean field approximation, where the meson fields are replaced by their respective expectation values in uniform matter:

\[ m_\sigma^2 \bar{\sigma} = \sum_{j=\Lambda}^{B} g_{\sigma j} n_j - g_2 \sigma^2 - g_3 \sigma, \quad \frac{\partial A}{\partial \sigma} \bar{\sigma}^2 \] (6)

\[ m_\omega^2 \bar{\omega} = \sum_{j=\Lambda}^{B} g_{\omega j} n_j - c_3 \omega^3 - \frac{\partial A}{\partial \omega} \bar{\omega}^2 \] (7)

\[ m_\phi^2 \bar{\phi} = \sum_{j=\Lambda}^{B} g_{\phi j} n_j \] (8)

\[ m_\rho^2 \bar{\rho} = \sum_{j=\Lambda}^{B} g_{\rho j} n_j - 2A \bar{\rho} \] , (9)

where \( \bar{\rho} = \langle \rho_3 \rangle, \bar{\omega} = \langle \omega \rangle, \bar{\phi} = \langle \phi \rangle \), and \( t_{3j} \) the third component of isospin of baryon \( j \) with the convention \( t_{3p} = 1/2 \). The scalar density of baryon \( j \) is given by

\[ n_j = \langle \bar{\psi}_j \psi_j \rangle = \frac{1}{\pi^2} \int_0^{k_F j} k^2 M^2_f \frac{d k}{d k} , \] (10)

and the number density by

\[ n_j = \langle \bar{\psi}_j \gamma^0 \psi_j \rangle = \frac{k_{F j}^3}{3\pi^2} , \] (11)

where \( e_j(k) = \sqrt{k^2 + M_j^2} \), and effective chemical potential

\[ \mu_j^* = \sqrt{k_{F j}^2 + M_j^2} \]. The effective baryon mass \( M_j^* \) is expressed in terms of the scalar mesons

\[ M_j^* = M_i - g_{\sigma i} \bar{\sigma} - g_{\sigma' i} \bar{\sigma'} - g_{\omega i} t_{3i} \bar{\omega} , \] (12)

where \( M_i \) is the vacuum mass of the baryon \( i \). The chemical potentials are defined by

\[ \mu_i = \mu_j^* + g_{\omega i} \bar{\omega} + \frac{g_{\rho i}}{2} t_{3i} \bar{\rho} + g_{\phi i} \bar{\phi} + \Sigma_0^R \] , (13)

where \( \Sigma_0^R \) is the rearrangement term

\[ \Sigma_0^R = \sum_{j=\Lambda}^{B} \left( \frac{\partial g_{\omega j}}{\partial n_j} \partial n_j \partial n_j + t_{3j} \frac{\partial g_{\rho j}}{\partial n_j} \partial n_j \partial n_j + \frac{\partial g_{\phi j}}{\partial n_j} \partial n_j \partial n_j \right) \]

\[ - \frac{\partial g_{\omega j}}{\partial n_j} \partial n_j \partial n_j - \frac{\partial g_{\omega j}}{\partial n_j} \partial n_j \partial n_j - t_{3j} \frac{\partial g_{\phi j}}{\partial n_j} \partial n_j \partial n_j \) \] (4)

and, at zero temperature, the effective chemical potential \( \mu_j^* \) is given by

\[ (\mu_j^*)^2 = (M_j^*)^2 + k_{F j}^2 . \] (15)
The rearrangement term is only present in the density-dependent models and ensures thermodynamic consistency.

Besides the two models with density-dependent parameters, DD2 and DDME2, we will also consider the following set of RMF models with constant couplings (see Table I for their properties): FSU2, FSU2H and FSU2R, NL3, NL3 $\sigma \rho$ and NL3 $\omega \rho$ [2,41], TM1 [42], TM1$\omega \rho$ and TM1$\sigma \rho$ [41,43], TM1-2 and TM1-2 $\omega \rho$ [8].

| Model  | $n_0$  | $B$       | $K$  | $E_{sym}$ | $L$  | $n_t$  |
|--------|--------|-----------|------|-----------|------|--------|
| DD2    | 0.149  | -16.0     | 242.6| 31.7      | 55   | 0.067  |
| DDME2  | 0.152  | -16.1     | 250.9| 32.3      | 51   | 0.072  |
| FSU2   | 0.1505 | -16.28    | 238  | 37.6      | 113  | 0.054  |
| FSU2R  | 0.1505 | -16.28    | 238  | 30.7      | 47   | 0.083  |
| FSU2H  | 0.1505 | -16.28    | 238  | 30.5      | 44.5 | 0.087  |
| NL3    | 0.148  | -16.24    | 271  | 37.4      | 118  | 0.055  |
| NL3 $\sigma \rho$ | 0.148 | -16.24   | 271 | 31.7    | 55  | 0.080  |
| TM1    | 0.145  | -16.26    | 281  | 36.8      | 108  | 0.060  |
| TM1$\omega \rho$ | 0.145 | -16.26  | 280 | 31.6    | 55  | 0.082  |
| TM1-2  | 0.145  | -16.26    | 280  | 31.4      | 56   | 0.080  |
| TM1-2 $\omega \rho$ | 0.146 | -16.3   | 281.3| 36.9 | 111| 0.061 |
|        |        |           |      |           |      |        |

TABLE I. Nuclear matter properties of the models considered in this study: saturation density $n_0$, binding energy $B$, incompressibility $K$, symmetry energy $E_{sym}$ and its slope $L$, all defined at saturation density, and the crust-core transition density $n_t$.

III. INNER CRUST

In the present study we will only consider unified EoSs at the level of the inner crust and core, since it has been shown in [23,41] that a non-unified EoS may give rise to large uncertainties in the NS radius. The inner crust EoSs for the models we are considering have been calculated within the Thomas-Fermi approximation [44,45]. In the above approach, we assume that the inner crust is formed by non-homogeneous $npe$ matter inside a Wigner-Seitz cell of one, two or three dimensions. Besides, the fields are considered to vary slowly so that matter can be treated as locally homogeneous. Since the density of the nucleons is determined by their Fermi momenta, we can then write the energy as a functional of the density. The equations of motion for the meson fields follow from variational conditions and are integrated over the whole cell. For a given density, the equilibrium configuration is the one that minimizes the free energy. For the present study, we have calculated the inner crust EoS for the models FSU2 [33] and FSU2R, FSU2H [34]. In Table II, we give the density transitions between pasta configurations, $n_{t-r}$ from droplets to rods and $n_{t-s}$ from rods to slabs, as well as $n_t$, the crust-core transition density that defines the transition to homogeneous matter. $\beta$-equilibrium is imposed, and under these conditions, the configurations corresponding to tubes and bubbles are not present. We confirm the conclusion drawn in [47], where it was discussed that models with large values of $L$, such as FSU2, do not predict the existence of pasta phases, due to their large neutron skin thicknesses, contrary to models with a small value of $L$, such as FSU2R and FSU2H. As Supplementary Material we list the inner crust EoS, i.e. baryonic density, energy density and pressure, for the models FSU2, FSU2H and FSU2R.

TABLE II. Density transitions in the pasta phase, $n_{t-r}$ and $n_{t-s}$, for the models considered in this work. $n_{t}$ indicates the transition density to homogeneous matter. All densities are given in units of fm$^{-3}$.

| Model  | $n_{t-r}$ | $n_{t-s}$ | $n_t$   |
|--------|-----------|-----------|---------|
| FSU2   | -         | -         | 0.054   |
| FSU2R  | 0.037     | 0.060     | 0.083   |
| FSU2H  | 0.041     | 0.067     | 0.087   |

IV. CALIBRATED HYPERON COUPLINGS

In the present study, we will only consider calibrated $\Lambda$-meson couplings as obtained in [22,48] in order to reproduce experimental data of $\Lambda$-hypernuclei. The binding energies of single and double $\Lambda$-hypernuclei are calculated solving the Dirac equations for the nucleons and $\Lambda$s, following the approach described in [49,50]. For the RMF models with density-dependent couplings, we have assumed the same density dependence for hyperon- and nucleon-meson couplings.

Following the approach described in [22], we have obtained calibrated couplings for the FSU2 [33], and the FSU2R and FSU2H RMF parametrizations recently proposed in [34]. The last two parametrizations have been fitted to both properties of nuclear matter and finite nuclei and NS properties. The former one was fitted to ground-state properties of finite nuclei and their monopole response. They all describe $2M_\odot$ NSs.

The values of the coupling constant fractions $R_{\sigma \Lambda}$ and $R_{\omega \Lambda}$ to the $\sigma$ and $\omega$ mesons are given in Table III and $R_{\sigma^* \Lambda}$ and $R_{\phi \Lambda}$ to the $\sigma^*$ and $\phi$ mesons in Table IV where $R_{\sigma \Lambda} = g_{\sigma \Lambda}/g_{\Lambda N}$ and similarly for the other meson fields. For reference, we also give the $\Lambda$-potential in symmetric nuclear matter at saturation density $n_0$ in Table III and in pure $\Lambda$-matter at $n_0/5$ and $n_0/3$ in Table IV as these are quantities traditionally used to obtain hyperonic EoSs within the RMF approach.

For the coupling of the $\Lambda$ to the $\omega$ meson we consider either the $SU(6)$ quark model value: $R_{\omega \Lambda}(SU(6)) = 2/3$, the so-called models ‘-a’, or the maximum expected coupling, i.e. $R_{\omega \Lambda} = 1$, forming the models ‘-b’. For the coupling between the $\Lambda$ and the $\phi$-meson we include in the tables results obtained with the $SU(6)$ value, $R_{\phi \Lambda}(SU(6)) = -\sqrt{3}/2$ and with $3R_{\phi \Lambda}(SU(6))/2 = -\sqrt{3}/2$. We assume that the $\omega$ and $\phi$ mesons to not couple [29,51].

For a given $\phi$-meson coupling, the $\sigma^*$-meson coupling is fitted to the bond energy of the only double-$\Lambda$ hypernucleus for which it has been measured unambiguously, that is $^8\Lambda\Lambda$He. Two sets of parameters are given for each $\phi$ coupling corresponding to the lower and upper values of the bond energy of $^8\Lambda\Lambda$He: $\Delta E_{\Lambda \Lambda} = 0.50$ MeV or 0.84 MeV.

To test the new parametrizations, we have integrated the
TABLE III. Calibration to single Λ-hypernuclei: for given \( R_{\phi\Lambda} \) values of \( R_{\sigma\Lambda} \) calibrated to reproduce the binding energies \( B_{\Lambda} \) of hypernuclei in the \( s \) and \( p \) shells. The last column contains the value of the \( \Lambda \)-potential in symmetric baryonic matter at saturation in MeV, for reference.

| Model   | \( R_{\phi\Lambda} \) | \( R_{\sigma\Lambda} \) | \( U^N_\Lambda(n_0) \) | \( U^P_\Lambda(n_0) \) | \( U^S_\Lambda(n_0) \) |
|---------|-----------------|-----------------|-----------------|-----------------|-----------------|
| FSU2-a  | 2/3             | 0.619           | -30             |
| FSU2-b  | 1               | 0.894           | -32             |
| FSU2R-a | 2/3             | 0.618           | -34             |
| FSU2R-b | 1               | 0.893           | -37             |
| FSU2H-a | 2/3             | 0.620           | -35             |
| FSU2H-b | 1               | 0.893           | -38             |

TABLE IV. Calibration to double Λ-hypernuclei for models -a and -b of Table III. For a given \( R_{\phi\Lambda}, R_{\sigma\Lambda} \) is calibrated to reproduce either the upper or the lower values of bound energy of \( ^5\Lambda \) He. For reference the \( \Lambda \)-potential in pure \( \Lambda \)-matter at saturation and at \( n_0/5 \) are also given. All energies are given in MeV.

| Model   | \( R_{\phi\Lambda} \) | \( \Delta B_{\Lambda\Lambda} = 0.50 \) | \( R_{\sigma\Lambda} \) | \( \Delta B_{\Lambda\Lambda} = 0.84 \) |
|---------|-----------------|-----------------|-----------------|-----------------|
| FSU2-a  | \( -\sqrt{2}/3 \) | 0.553           | -7.98           | 0.577           |
|         | \( -\sqrt{2}/2 \) | 0.862           | -5.56           | 0.877           |
| FSU2-b  | \( -\sqrt{2}/3 \) | 0.573           | 0.48            | 0.604           |
|         | \( -\sqrt{2}/2 \) | 0.874           | 5.39            | 0.894           |
| FSU2R-a | \( -\sqrt{2}/3 \) | 0.552           | -7.52           | 0.577           |
|         | \( -\sqrt{2}/2 \) | 0.860           | -5.12           | 0.876           |
| FSU2R-b | \( -\sqrt{2}/3 \) | 0.573           | 1.31            | 0.604           |
|         | \( -\sqrt{2}/2 \) | 0.873           | 6.18            | 0.894           |
| FSU2H-a | \( -\sqrt{2}/3 \) | 0.544           | -8.62           | 0.570           |
|         | \( -\sqrt{2}/2 \) | 0.848           | -6.42           | 0.865           |
| FSU2H-b | \( -\sqrt{2}/3 \) | 0.564           | 4.20            | 0.598           |
|         | \( -\sqrt{2}/2 \) | 0.860           | 8.75            | 0.883           |

Tolman-Oppenheimer-Volkoff equations, allowing the appearance of hyperons in the core of the star. For the outer crust, we have considered the EoS proposed in Ref. [52], and the EoS of the inner crust was obtained from a Thomas Fermi calculation, see [45][46], as discussed in the previous section, consistently with the core EoS.

With the complete EoS, we have calculated the NS maximum mass \( M_{\text{max}} \) as a function of \( R_{\phi\Lambda} \) including on the \( \Lambda \) hyperons in the EoS in addition to the nucleons, for the models -a and -b, see black lines in Fig. 1. The values \( R_{\phi\Lambda}, R_{\sigma\Lambda} \) and \( R_{\sigma\Lambda} \) are adjusted to reproduce the binding energies of single \( \Lambda \)-hypernuclei and of \( ^5\Lambda \) He with \( \Delta B_{\Lambda\Lambda} = 0.50 \) MeV (solid lines) and \( 0.84 \) MeV (dashed lines).

In Fig. 1 the colored lines correspond to models that also include the \( \Xi \) and \( \Sigma \) hyperons. For these hyperons the values of hyperonic single-particle mean field potentials have been used to constrain the scalar coupling constants. The potential for a hyperon \( Y \) in symmetric nuclear matter is given by

\[
U^N_Y(n_0) = M_Y^* - M_Y + \mu_Y - \mu_Y^*,
\]

where the chemical potential \( \mu_Y \) and the effective chemical potential \( \mu_Y^* \) have been defined in Eqs. (15) and (15). For the \( \Xi \) potential we take \( U^N_\Xi(n_0) = -18 \) MeV, compatible with the analysis in [53][54] of the experimental data for the reaction \( ^{12}\text{C}(K^-, K^+)\text{Be} \), which are reproduced using a potential \( U^N_\Xi(n_0) \sim -14 \) to \(-18 \) MeV. No \( \Sigma \)-hypernucleus has been detected and this seems to indicate that the \( \Sigma \)-potential in nuclear matter is repulsive. Therefore, we have considered two values of \( U^N_\Xi(n_0) = 0 \) and \(+30 \) MeV. Since, presently no information on double \( \Xi \)- or \( \Sigma \)-hypernuclei exists, we did not include the coupling of these two hyperons to the \( \sigma^2 \) and the \( \phi \)-meson, responsible for the description of the \( YY \) interaction in RMF models. For the \( \omega \)-meson couplings we consider the \( SU(6) \) values:

\[
\mathcal{g}_{\omega\Xi} = 1, \quad \mathcal{g}_{\omega\Sigma} = 1, \quad \mathcal{g}_{\omega\Xi} = 1,
\]

(17)

In Fig. 1 the predictions obtained with the EoSs that include only the \( \Lambda \) hyperons in addition to the nucleons defining the minimal hyperonic model (black lines), may be considered as an upper limit on the maximum mass of an hyperonic NS, when compared with models including the full baryonic octet. On the other hand, including in the calculations the complete baryonic octet and not including the mesons that account for the \( YY \) interaction (colored lines), the maximal hyperonic model, gives an estimation of the lower limits for the maximum mass of hyperonic NSs. The blue stripped ar-
eas in Fig. 2 correspond, precisely, to the mass range covered when employing the minimal and maximal hyperonic models for SU(6) values of the coupling constants $R_{\rho \Lambda}$ and $R_{\omega \Lambda}$ of the $\Lambda$ hyperons, for $U_{\Sigma}(n_0) = 0$ MeV and $U_{\Sigma}(n_0) = -14$ MeV.

Under the above conditions the FSU2R model with hyperons does not describe two solar mass stars (not even 1.9 $M_\odot$ as indicated by the most recent measurements of PSR J1614 − 2230 [25]). This conclusion had already been drawn in [34]. In Fig. 2 the red curves have been obtained with the hyperon parametrization defined in [34]. It lies above the upper limit defined by the minimal hyperonic model because the $\sigma^*$ was not included, and the $\Lambda$-$\sigma$ coupling was also smaller giving rise to a potential equal to -28 MeV instead of $\sim -35$ MeV obtained with the calibrated parametrization.

![Fig. 2. $M - R$ relations for the FSU2R and FSU2H models. The grey strips correspond to the mass of the two heaviest known NSs, PSR J1614 − 2230 and PSR J0348 + 0432. The black lines are obtained for purely nucleonic models, the red ones for the models presented in [39]. The blue stripped areas correspond to the mass range covered when employing the minimal and maximal hyperonic models for SU(6) values of the coupling constants $R_{\rho \Lambda}$ and $R_{\omega \Lambda}$ of the $\Lambda$ hyperons, for $U_{\Sigma}(n_0) = 0$ MeV and $U_{\Sigma}(n_0) = -14$ MeV - see Fig. 1.](image)

![Fig. 3. Right panel: Onset density of the different hyperons ($\Lambda$, $\Sigma^-$ and $\Xi^-$) (red, yellow and green lines), and of the DU process (blue lines), and NS central density at the maximum mass (black lines) for the TM1$\omega p$ family and three different values of the $\Sigma$ potential at saturation (−10, 10, and 30 MeV) as a function of the slope of the symmetry energy $L$. Left panel: the NS masses corresponding to the different densities plotted in the right panel. The DU onset density in nucleonic matter and corresponding star mass are also shown with blue dots. All other curves were obtained with models including hyperons.](image)

V. SYMMETRY ENERGY AND HYPERONIC NEUTRON STARS

In the present section, we discuss the effect of the density dependence of the symmetry energy on the onset of the different hyperon species, and on the onset of the direct Urca process in the presence of hyperons. The study will be undertaken considering a family of models generated from the TM1 model [42]. The inclusion of the nonlinear term $Z_{\omega \Lambda}$ that couples the $\omega$ and the $\sigma$ mesons to the $p$-meson will allow the generation of a family of models with the same underlying isoscalar properties and different isovector properties [41][43]. This family is built in such a way that all the models predict the same symmetry energy, equal to the one predicted by TM1, at $n_\Sigma = 0.1$ fm$^{-3}$. It was shown in [41] that the ground-state properties of nuclei used to calibrate TM1 are still quite well reproduced when the new terms are introduced in the model. Contrary to the previous section, in the present and following sections we will consider that the $\Sigma$ and $\Xi$ hyperons couple to the $\phi$-meson with the couplings defined by the SU(6) symmetry, unless when Fig. 9 is discussed.

A. The direct Urca process: nucleonic neutron stars

The most efficient cooling mechanism of a NS by neutrino emission is the nucleonic electron direct Urca (DU) process [55] described by the equations

$$n \rightarrow p + e^- + \bar{\nu}_e \quad \text{and} \quad p + e^- \rightarrow n + \nu_e.$$  \hspace{1cm} (18)

This process operates only if momentum conservation is allowed, and this can be translated into the inequalities:

$$p_{\nu_e} \leq p_{\nu_p} + p_{\nu_e},$$  \hspace{1cm} (19)

where $p_i$ is the Fermi momentum of species $i$. As a consequence, in order for the DU process to occur the proton fraction must be equal or above a minimum proton fraction $Y_p^{\text{min}}$ [56]:

$$Y_p^{\text{min}} = \frac{1}{1 + \left(1 + x_e^{1/3}\right)^{-3}},$$  \hspace{1cm} (20)

where $x_e = n_e/(n_e + n_\mu)$, and $n_e$ and $n_\mu$ are the electron and muon densities. In the following, we will designate by $n_{\text{DU}}$ and mass $M_{\text{DU}}$, respectively, the baryonic density at which the
DU process sets in and the mass of the star where it starts operating, i.e. which has a central density equal to \( n_{DU} \).

For some models the nucleonic DU process does not operate inside NSs because the onset DU density is above the central density of the most massive star. In our study this is the case for the two models with density-dependent coupling parameters DD2 and DDME2.

In order to discuss the influence of the density dependence of the symmetry energy on the DU process, we include in Fig. 3 left panel the DU onset density as a function of the slope \( L \) of the symmetry energy at saturation density (blue curves) and the corresponding star masses on the right panels. The blue dotted line is obtained for the nucleonic EoSs from the family of TM1 models defined in section II and the other blue curves have been obtained for hyperonic EoSs and will be discussed below. It is clear that the DU process is strongly influenced by the density dependence of the symmetry energy, because this quantity defines the proton fraction in matter. A similar relation was obtained in [7, 57]. A large symmetry energy disfavors a large proton-neutron asymmetry and, therefore, favors the DU process and it sets in at low densities. On the contrary, a small symmetry energy allows for large proton-neutron asymmetries hence pushing the DU threshold to higher densities. In [58], the authors have discussed how it is possible to establish a relation between the \( ^{208}\text{Pb} \) neutron skin and the possibility of occurring the DU process. Since the nuclear neutron skin is strongly correlated with the slope \( L \), the above observation is equivalent to the one displayed in Fig. 3.

B. The direct Urca process: hyperonic neutron stars

In the presence of hyperons, other channels are opened for neutrino emission [9]:

\[
\begin{align*}
\Sigma^- &\rightarrow \Xi^0 \ell^- \bar{\nu}_\ell, \quad R = 0.61 \quad (21) \\
\Xi^- &\rightarrow \Xi^0 \ell^- \bar{\nu}_\ell, \quad R = 0.22 \quad (22) \\
\Sigma^- &\rightarrow \Lambda \ell^- \bar{\nu}_\ell, \quad R = 0.21 \quad (23) \\
\Xi^0 &\rightarrow \Sigma^+ \ell^- \bar{\nu}_\ell, \quad R = 0.06 \quad (24) \\
\Lambda &\rightarrow p \ell^- \bar{\nu}_\ell, \quad R = 0.04 \quad (25) \\
\Xi^- &\rightarrow \Sigma^0 \ell^- \bar{\nu}_\ell, \quad R = 0.03 \quad (26) \\
\Xi^- &\rightarrow \Lambda \ell^- \bar{\nu}_\ell, \quad R = 0.02 \quad (27) \\
\Sigma^- &\rightarrow n \ell^- \bar{\nu}_\ell, \quad R = 0.01. \quad (28)
\end{align*}
\]

For each process the \( R \) factor indicates the efficiency of each process with respect to the nucleonic DU process for which \( R = 1 \) (see [9]). These different hyperonic DU channels are opened as soon as the species involved set in. The most efficient processes being the ones described by Eqs. (21), (22) and (25) and, in particular, the process (21) is almost three times more efficient that the other two. This indicates that it is important to establish whether the \( \Sigma \)-hyperon occurs inside a NS. Since this hyperon has isospin equal to one, it is expected that its occurrence will be strongly influenced by the density dependence of the symmetry energy.

The occurrence of hyperons affects the neutron, proton and electron fractions. Therefore, Eq. (18) for the nucleonic DU threshold loses validity, and after hyperons set in, the minimum proton fraction for nucleonic electron DU is given by

\[
\frac{n_p}{n_p + n_n} = \frac{1}{1 + \left(1 + \chi^5\right)^{1/3}}, \quad \chi^5 = \frac{n_e}{n_p + n_n - n_{\Sigma^-}},
\]

where \( n_{\Sigma^-} = n_{\Xi^-} + n_{\Sigma^0} - n_{\Xi^0} \). The nucleonic electron DU process is not affected by the presence of hyperons in models with a large slope \( L \) because its threshold is at densities lower than the hyperon onset density. However, if \( L \leq 75 \text{ MeV} \), the presence of hyperons will affect the nucleonic electron DU process and the effect depends on the value of the \( \Sigma \) potential: if very repulsive (\( U_{\Sigma} \) of the order of couple of tens of \( \text{MeV} \)), the DU process turns on at densities larger that the hyperon DU process, the onset densities of the \( \Lambda \), \( \Sigma^- \) and \( \Xi^- \) hyperons, and the central density \( n_c \) of the NS with the maxi-
mum mass for three different values of $U_\Sigma$ at saturation: $-10$, $10$ and $30 \text{ MeV}$. Hyperons that are not included in the figure do not appear at densities below $n_c$ and hence are not present at all in NSs. The grey bands show the mass constraints set by the pulsars PSR J1614 $- 2230$ and PSR J0348 $+ 0432$. Even though the TM1 $\omega \rho$ family with hyperons and the vector meson couplings to the hyperons defined by the $SU(6)$ symmetry do not satisfy the two solar mass constraint, the main conclusions drawn with respect to the $L$ dependence of the several properties we discuss, is still valid for more massive stars.

For $L \geq 75 \text{ MeV}$ the DU process sets in at a density below the hyperon onset density and, in fact, the DU process is possible at densities of the order of $2n_0$ or below, corresponding to stars with a mass equal to $1M_\odot$ or below. Observations do not support a fast cooling for these low masses (see eg. discussion in [59]). The DU mass threshold rises monotonously as $L$ decreases below $75 \text{ MeV}$, and for $L = 50 \text{ MeV}$ attains $1.4 - 1.7M_\odot$ depending on the value of $U_\Sigma$, a large repulsive value favoring a higher threshold. Similar conclusions have been drawn in [7], although using different hyperonic models.

We finally comment on the effect of $L$ on the hyperonic species inside the star. The $\Lambda$ hyperon onset is practically not affected by the value of $U_\Sigma$, and, although its onset density increases slightly when $L$ decreases, the mass of star at the $\Lambda$-onset is essentially independent of $L$ and equal to $1.3M_\odot$. However, the other two hyperons $\Sigma^-$ and $\Xi^-$, having a nonzero isospin are strongly affected by the density dependence of the symmetry energy, the onset density decreasing as $L$ decreases. The more repulsive the $U_\Sigma$ the larger the onset density of the $\Sigma$ and the mass of the star where the hyperon sets in.

The strongest effect of the $U_\Sigma(n_0)$ is observed for $L = 56 \text{ MeV}$. In nucleonic matter the DU sets in at $n_{DU} = 0.504 \text{ fm}^{-3}$ corresponding to a star with a mass $M_{DU} = 1.81M_\odot$. The density $n_{DU}$ and mass $M_{DU}$ change to $n_{DU} = 0.411 \text{ fm}^{-3}$ and $M_{DU} = 1.39M_\odot$ if $U_\Sigma(n_0) = -10 \text{ MeV}$, and to $n_{DU} = 0.566 \text{ fm}^{-3}$ and $M_{DU} = 1.67M_\odot$ if $U_\Sigma(n_0) = +30 \text{ MeV}$. The $\Xi^-$ does not occur unless the $\Sigma$ potential is quite repulsive.

One fact that should be pointed out is that the overall effect of the value of $L$ on the star maximum mass is negligible, a conclusion that had already been drawn in [7],[8].

In Fig. 5 left panel, we show how the radius of NSs with a mass equal to $1.67M_\odot$, the mass of the pulsar PSR J1903+0327, changes with the total hyperon fraction, one third of the strangeness fraction when only hyperons with strangeness charge -1 are involved, at the maximum mass

$$N_S = \frac{1}{3} \int_0^R \frac{\sqrt{m(r)} n_s(r) \, dr}{r},$$

where $m(r)$ is the mass inside the radius $r$ and $n_s$ the strangeness density, obtained when $U_\Sigma$ varies between $-10$ and $+30 \text{ MeV}$ and the other hyperon coupling parameters are kept unchanged for parametrizations of the TM1 $\omega \rho$ family with different values of $L$.

The right panel of the same figure represents the M-R curves of the same models. For $L = 108 \text{ MeV}$ the $\Lambda$-hyperon is the responsible for almost all the strangeness content and, therefore, it is not sensitive to the $\Sigma$ potential. On the other hand, models with smaller values of $L$ are sensitive to the $\Sigma$ potential and a change of $U_\Sigma(n_0)$ between $-10$ and $+30 \text{ MeV}$ is translated into a reduction of $\sim 20\%$ of the total strangeness content and an increase of $300 - 400 \text{ m}$ of the star radius. The overall effect on the radius due to the inclusion of hyperons in the family of models considered in this section is a reduction of at most $400 - 600 \text{ m}$. Let us recall that several authors, including [7, 36, 41, 60–62], have shown that the NS radius is correlated with the nucleus neutron skin, a quantity directly related with the slope of the symmetry energy: the larger the slope of the symmetry energy the larger the radius. This behavior is clearly seen in the left panel of Fig. 5: for the non-hyperonic models, located on the vertical axis where $N_S/N_B = 0$ of the left panel, the radius of a $1.67M_\odot$ increases with the symmetry energy slope $L$, and a difference in radius of almost $1 \text{ km}$ is obtained between models with $L = 56 \text{ MeV}$ and $L = 108 \text{ MeV}$.

C. Effect of the $\Sigma$ potential

It was shown in the previous section that besides the symmetry energy the value of $\Sigma$ potential in symmetric matter at saturation, chosen to fix the value of the $\sigma$-meson coupling, could also have a strong effect on the properties of the star, in particular, if the model has a small value of $L$. In the following, we analyse this effect and, taking into account that the $\Sigma$-meson interaction is still not constrained, we allow it to vary between $-10 \text{ MeV}$ and $30 \text{ MeV}$. Experimentally no $\Sigma$-hypernucleus was detected and this seems to indicate that the $\Sigma$ interaction in nuclear matter is repulsive or, at most, slightly
1. Direct Urca process

In this section, we consider the set of models defined in Section II. Most of these models have a symmetry energy slope below 60 MeV but there are three of them with a slope above 100 MeV (NL3, TM1 and FSU2), out of the range of values $40 < L < 62$ MeV [12] and $30 < L < 86$ MeV [13] which where defined by terrestrial, theoretical, and, for the second range, also by observational constraints. In addition, these three models do not satisfy constraints obtained from microscopic calculations of neutron matter based on nuclear interactions derived from chiral effective field theory [63], or from realistic two- and three-nucleon interactions using quantum Monte Carlo techniques [64]. We keep them in the discussion because they are still frequently used and it is interesting to show how a stiff symmetry energy affects the behavior of an hyperonic EoS.

We have discussed in the previous section the effect of the density dependence of the symmetry energy on the onset of the nucleonic electron DU process, whether hyperons are included and present or not. In Fig. 6 left panel, we plot the DU onset density for the different models as a function of $U_\Sigma(n_0)$ the $\Sigma$ potential in symmetric nuclear matter at saturation. In the right panel, the corresponding NS masses are shown. Models with a large $L$, i.e. NL3, TM1 and FSU2, are not affected because $n_{DU}$ is just above saturation density and lower than any of the hyperon onset density. For all the other models the trend is similar: the more repulsive $U_\Sigma(n_0)$ is, the larger $n_{DU}$.

To conclude, let us point out that the two models with density dependent couplings do not predict the occurrence of the DU process, even in the presence of hyperons.

2. Hyperon species

In Sec. V B we have indicated the different channels that allow for hyperonic direct Urca. It is, therefore, important to determine under which conditions these processes occur, in particular, the masses of the NSs for which they are opened. In the present section we show for all models of Table I the maximum mass central density, the onset density of the differ-
ent hyperons and the onset density of the nucleonic electron DU process as a function of the $U_E$, and the corresponding NS masses.

In Fig. 7, the information above is plotted for TM1, TM1$\sigma\rho$ and TM1$\sigma\rho$ MeV (top panel), TM1-2 and TM1-2$\sigma\rho$ (middle panel), and for NL3, NL3$\sigma\rho$ and NL3$\sigma\rho$ (bottom panel). All the models with a nonlinear term in $\omega\rho$ or $\sigma\rho$ have $L \approx 55$ MeV while TM1, TM1-2 and NL3 have a slope of the symmetry energy which is twice larger: $L \approx 110 - 120$ MeV. The behavior of the TM1 and TM1-2 EoS only differ above saturation density, the TM1-2 EoS being stiffer. As a consequence, hyperons set in at lower densities in TM1-2, and the maximum masses are larger, but still below 1.9 $M_\odot$, for the set of hyperon-meson coupling chosen which considers for the vector-isoscalar mesons the SU(6) symmetry. For TM1 and TM1-2 as discussed before, the DU sets in in NSs with masses below 1$M_\odot$, independently of $U_E$. Models including the nonlinear term $\omega\rho$ or $\sigma\rho$, and having a symmetry energy slope $L \approx 55$ MeV, show a very different behavior. In this case, the magnitude of $U_E(n_0)$ has an important effect on the behavior of the system: for $U_E \lesssim 5$ MeV, the $\Sigma$ hyperon sets in at densities below the onset of $\Lambda$, and the corresponding NS masses have masses below $\sim 1.2M_\odot$, that is $\sim 0.2 - 0.3M_\odot$ smaller than the mass of the star where the nucleonic electron DU process starts operating. For $U_E \gtrsim 5$ MeV, the $\Lambda$-hyperon is the first hyperon to set in and is not affected by the magnitude of $U_E(n_0)$. This occurs for stars with a mass $\sim 1.3M_\odot$. If $U_E \gtrsim 20$ MeV, the $\Sigma^-$-hyperon sets in before $\Sigma^+$, corresponding to a star mass of $\sim 1.6M_\odot$. It is interesting to comment on the differences between models TM1$\sigma\rho$ and TM1$\sigma\rho$ which have the same symmetry energy slope at saturation, but the density dependence of the symmetry energy in TM1$\sigma\rho$ is modeled by the coupling of the $\omega$-meson to the $\rho$-meson, while in TM1$\sigma\rho$ the $\rho$-meson couples to the $\sigma$-meson. Within TM1$\sigma\rho$, the onset of the $\Lambda$ and $\Sigma$-hyperons as well as the nucleonic electron DU process occur in stars with lower masses. This is due to the fact that the softening effect on the symmetry energy, which is always very effective in TM1$\sigma\rho$ because the $\omega$-field increases with density, saturates in model TM1$\sigma\rho$ due to the behavior of the $\sigma$-meson with density. Finally, we also conclude that the overall effect of the value of $U_E(n_0)$ on the star maximum mass is negligible.

Similar conclusions may be drawn for the models NL3, NL3$\sigma\rho$ and NL3$\sigma\rho$, the main difference being that in this case much larger star masses are attained, well above $\sim 2M_\odot$, because these EoSs are harder than the EoS resulting from TM1, TM1-2 and respective families. For these models the maximum NS masses correspond to configurations where the effective nucleonic mass becomes zero, as already pointed out in [23]. As a consequence of the extra hardness, the central densities are smaller, and for NL3$\sigma\rho$ and NL3$\sigma\rho$, the different processes set in for more massive stars when compared to the TM1 like models: the $\Lambda$-hyperon appears masses at $\sim 1.5M_\odot$, the nucleonic electron DU process turns on above $\sim 1.6M_\odot$ if $U_E = -10$ MeV and $\sim 1.9M_\odot$ if $U_E = +30$ MeV. Besides the crossing between the onsets of the $\Sigma$-hyperon and the $\Lambda$-hyperon occurs for slightly smaller values of $U_E(n_0)$ than for the TM1 models.

In Fig. 8, left panel, the behavior of models FSU2, FSU2R and FSU2H is shown. Model FSU2 has a large symmetry energy slope $L = 113$ MeV, and properties similar to the ones of TM1, presenting, however, smaller star masses at the hyperon onset and smaller maximum star masses. FSU2R and FSU2H have been fitted to a different set of properties and, in particular, to a smaller symmetry energy slope ($L \approx 45$ MeV), and were built to describe a $2M_\odot$ star, even in the presence of hyperons for FSU2H. FSU2 and FSU2R in fact predict similar maximum masses taking the SU(6) symmetry to fit the vector isoscalar mesons, close to 1.75 $M_\odot$, but for FSU2H the maximum mass goes up to 2$M_\odot$. Comparing the FSU2H and FSU2R models, it is clear that because FSU2H is harder, the onset of hyperons occurs at smaller densities, which, however, corresponds to larger star masses. As an ex-
ample, the onset of As occur at $\sim 1.3M_\odot$ for FSU2R and at $\sim 1.4M_\odot$ for FSU2H. Also the nucleonic electron DU process turns on for the FSU2H model for masses $\sim 0.2M_\odot$ larger, and above $1.5M_\odot$ whichever values of $U_\Sigma$ is employed, going up to $\sim 1.7M_\odot$ for $U_\Sigma = +30$ MeV. The $\Sigma$-hyperon appears before the $\Lambda$-hyperon these two models at larger values of $U_\Sigma$ than discussed before, i.e. for $U_\Sigma \lesssim +10$ MeV. For such a slightly attractive potential hyperons appears already in stars with masses below $1.25M_\odot$. One difference with respect to the previous NL3, TM1 and TM1-2-like models is that for the FSU2 like models, the $\Xi$-meson does not set in before the $\Sigma$-hyperon for $U_\Xi \leq +30$ MeV. This is a consequence of the large isospin of $\Sigma^-$ that compensates the repulsion of the $\Sigma$ potential in symmetric nuclear matter. In order to analyze the effect of the present results on the cooling of the NSs, one would need to take into account the nucleonic and hyperonic pairing [59,65], and this will be left for a future work.

We finally consider the two models with density-dependent parameters, see Fig. 8 right panel. They have very similar behaviors, the only difference being that, since the DDME2 EoS is slightly harder, the incompressibility at saturation is $K = 251$ MeV, the onset of hyperons and of the nucleonic DU process occur at smaller densities and slightly larger star masses ($\sim 0.1M_\odot$). Just as for the FSU2-like models, for these two models the $\Xi^-$-hyperon does not set in before the $\Sigma^-$ for $U_\Xi(n_0)$ in the range $-10, +30$ MeV. The $\Lambda$-meson appears in stars with $M = 1.3 - 1.4M_\odot$ and if $U_\Sigma \sim -10$ MeV stars with $M \sim 1 - 1.1M_\odot$ already contain $\Sigma$-hyperons. The two density-dependent models do not allow for the nucleonic electron DU process to turn on. However, the hyperonic DU processes operate inside the stars, and for $U_\Sigma \leq 10$ MeV the process described in Eq. (23) is already open for stars with $M \sim 1.3M_\odot$.

Before finishing this section we would like to discuss the effect of the uncertainties introduced in the previous discussion by fixing the $U_\Sigma$ in symmetric matter to $-18$ MeV and by the unconstrained couplings of the $\Sigma$ and $\Xi$-hyperons to the $\phi$-meson.

Following [53], we could have considered $U_\Xi(n_0) = -14$ MeV. In Fig. 9 the solid (dashed) lines were obtained with $U_\Xi = -18 (-14)$ MeV. The curves corresponding to these two calculations are generally superposed, except for the ones showing the onset density of the $\Xi$-hyperon, which will occur at a density $0.05-0.1$ fm$^{-3}$ larger, if the higher value of $U_\Sigma$ is considered. All other properties, such as the onset of the DU process and of the other hyperons are insensitive to this change of $U_\Sigma$, except if the $\Sigma$ potential is so repulsive that the $\Omega$ hyperon sets in before the $\Sigma$ hyperon. If future experiments show that the $\Sigma$ potential is very repulsive in symmetric nuclear matter, models will be more sensitive to the $\Sigma$ hyperon interaction.

We discuss in the following the role of the $\phi$ meson. In Fig. 9 for the FSU2H and DDME2 models the results of switching off the coupling of the hyperons $\Sigma$ and $\Xi$ to the $\phi$ meson (as in the minimal hyperonic models defining a lower limit on the NS mass [22]) are compared with the previous calculations for which the $\phi$ couplings to $\Sigma$ and $\Xi$ hyperons are fixed to the SU(6) values. The $\phi$ meson is responsible for the description of the YY interaction and, therefore, its effect is noticeable at high densities but not on the first hyperon to appear, for which it is the YN interaction that plays a role. Once the first hyperon sets in, not including the coupling to the $\phi$ meson are results in an earlier onset (lower density) of the other hyperons. In particular, the $\Omega$ hyperon is strongly affected because, having strangeness $-2$, the coupling of the $\phi$ meson to the $\Omega$ hyperon is two times larger. An immediate consequence of this last effect is that the the maximum mass configuration is lowered and for both FSU2H and DDME2 it falls below $1.9M_\odot$, the mass of the PSR J1614–2230. Removing the $\phi$-meson also affects the DU process in the FSU2H model, bringing its onset to lower densities, because of an increased hyperon content and thus a reduction of the neutron Fermi momentum which ultimately favors the occurrence of the DU process.

![FIG. 9. Onset density and mass of the different hyperons for different values of $U_\Sigma$, $U_\Xi$ and $x_\phi \Sigma$ and for models FSU2H and DDME2.](image-url)
3. Steady thermal state of accreting NSs

We now explore how the value of the $U_C$ potential and of the symmetry energy affects the cooling of NSs. In particular, we model the thermal state of NSs in Soft X-ray transients (SXTs) and focus more specifically on SAX J1808.4-3658 (SAX J1808 in the following) [66, 67], the SXT with the lowest-observed luminosity.

In SXTs NSs accrete matter from their binary companion during short phases with a high luminosity followed by long periods of quiescence characterized by a low luminosity signaling zero or strongly reduced accretion. During the accretion phases, the accreted matter undergoes a series of nuclear reactions (electron captures and pycnonuclear fusions - see [68] and references therein) as it sinks deeper into the crust under the weight of the newly-accreted matter. These reactions release heat in the crust which propagates in the NS interior, inwardly heating the core and outwards emitted in the form of photons at the surface. This is the so-called deep crustal heating. After frequent and short periods of accretion the NS reaches a state of thermal equilibrium with a constant internal temperature throughout the star [10, 69]. This temperature is determined by the balance between the heating generated during the accretion phase which is directly proportional to the accretion rate $\dot{M}$ averaged over periods of accretion and quiescence, and the energy losses in the form of 1) photons emitted from the surface of the star and 2) of neutrinos freely escaping from the whole star (see e.g. [59] for details). Consequently the steady thermal states of accreting NS depends on three ingredients 1) the composition of the NS envelope from where the photons escape; 2) the NS core properties (EoS and composition) since the core is reponsible for most of the neutrino losses; 3) the total heat release in the accreted crust. The EoS for the crust hardly affect the thermal states, only the heat release per accreted nucleon $Q_{\text{DCH}}$ does and its values has been shown to be rather robust: $Q_{\text{DCH}} \sim 2$ MeV per accreted nucleon [68, 70]. Thus, in the following we adopt the model for the accreted crust and the deep crustal heating from Ref. [68] for lack of model consistent with the core EoSs that we employ. We use two limiting models of NS envelopes corresponding to either the absence of light elements (non-accreted envelope) or a maximum amount of them (fully accreted envelope) from Ref. [71].

In Fig. 10 for the TM1 (left) and DDME2 (right) EoSs, we show, on the left panel of each plot the luminosity in quiescence as a function of the accretion rate together with we show, on the left panel of each plot the luminosity in the following) [66, 67], the SXT with the lowest-observed luminosity. We use the TM1 and the observational data from [72] and on the right panel the quiescence as a function of the accretion rate together with we show, on the left panel of each plot the luminosity in the following) [66, 67], the SXT with the lowest-observed luminosity.

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In Fig. 10 for the TM1 (left) and DDME2 (right) EoSs, we show, on the left panel of each plot the luminosity in quiescence as a function of the accretion rate together with the observational data from [72] and on the right panel the composition for the different models. We use the TM1 and DDME2 EoSs with various hyperonic contents obtained for the different models. We use the TM1 and DDME2 EoSs with various hyperonic contents obtained for different values of the $\Sigma$ potential (dashed, dotted, and dot-dashed lines) together with their purely nucleonic versions (solid lines). TM1 is chosen as a representative model that predicts that the nucleonic DU process occurs for quite low star masses $M < 0.8 M_\odot$ while DDME2 as a model which does not allow for this process at all. For each EoS we compute 1) the upper bound on the thermal state of NSs that is obtained for NSs with a mass below the DU threshold - this defines the lowest possible neutrino losses and hence the largest luminosity, 2) the lower bound of the thermal state which is reached for maximum mass NSs with the largest neutrino emissions obtained when the DU processes operate and hence the lowest luminosity. We do not include superfluidity in the models (see discussion in [59]) as it reduces the DU emissivity. We indeed want to confront the lowest-bound on the thermal state we obtain with the observational data on SAX J1808. This object, indicated in red in the plots in Fig 10 has the lowest observed luminosity and a precisely measured accretion rate thanks to the observations of multiple type I X-ray bursts [67]. Its low-luminosity is challenging to model and suggests that very efficient neutrino processes, the most efficient of which are the nucleonic and hyperonic DU processes, are operating in its NS core. In [69], the authors could explain its luminosity only by using an hyperonic core EoS. The model they have considered for nuclear matter is GL85 [73] that predicts a quite hard EoS with an incompressibility $K = 285$ MeV and a symmetry energy at saturation $E_{\text{sym}} = 36.8$ MeV. For the hyperonic interaction the universal couplings were considered, i.e. the hyperon-meson couplings equal the nucleon-meson couplings. This choice gives rise to strongly attractive hyperon potentials in symmetric nuclear matter at saturation, of the order $-60$ to $-70$ MeV, and allows for the appearance of all six hyperons inside the maximum mass star, and, therefore, all channels defined by Eqs. (21)-(28) are opened. As a consequence in addition to the nucleonic DU process all hyperonic processes are turned on and hence the neutrino emissivity is larger and the luminosity lower for the hyperonic EoS than for purely nucleonic one. The low-luminosity of SAX J1808 could only then be modelled for a hyperonic NS, suggesting that hyperons could be present in SAX J1808.

For the hyperonic TM1 EoSs on the left plot in Fig. 10 in addition to the nucleonic DU process, for the model with a slightly attractive potential, $U_C = -10$ MeV the DU channels in Eqs. (23), (25), (28) are operating in the star with the maximum mass, for a repulsive $U_C = 10$ MeV the DU process in Eq. (27) is turned on as the $\Sigma^-$ is present. However since the $\Sigma^-$ appears at larger densities than when an attractive potential is used, the most efficient of all hyperonic DU processes turned on for such models, is the one in Eq. (23) that then operates in a smaller region of the star and the process in Eq. (27) is too weak to compensate these lesser neutrinos losses. For the model with $U_C = 30$ MeV since no $\Sigma^-$ are present only processes in Eqs. (25) and (27) set in and both are less efficient than the one in Eq. (27). Hence the model with the $U_C = -10$ MeV is the coolest of all hyperonic models. We obtain that the purely nucleonic has the lowest luminosity compared to hyperonic models but the difference is quite small. The purely nucleonic NS, in which only the nucleonic DU process, which is the most efficient process, operates is almost $\sim 0.2 M_\odot$ more massive than the hyperonic NSs. Hence for hyperonic NSs even if more DU channels are opened, these are less efficient and do not exactly compensate for the fact that the nucleonic NS has an extra region of $0.2 M_\odot$ emitting neutrinos via the most efficient channel. Thus hyperonic stars emit all in all less neutrinos and hence have a slightly larger luminosity. As in [59] we obtain that NSs with a fully accreted envelope are more luminous than with a non-accreted one. Thus we obtain
that for the TM1 EoS SAX J1808 is compatible with a NS with a small or null amount of accreted matter in the envelope, with or without hyperons.

For the DD2 parametrization (right plots of Fig. 10), as the nucleonic DU process does not operate at all for the purely nucleonic EoS, non-hyperonic NSs will have a very similar and large luminosity. Hyperonic models have, however, a small luminosity as the additional hyperonic DU processes operates and only such models can explain the low-luminosity of SAX J1808. For all hyperonic models the $\Sigma^-$, $\Sigma^0$ and $\Lambda$ are present at the maximum mass, and the latter two species in similar amount. The most efficient hyperonic DU process is then the channel in Eq. (23) between the $\Lambda$ and the $\Sigma^-$. As the model with $U_\Sigma = -10$ MeV has the largest amount of $\Sigma^-$ (it even appears before the $\Lambda$) it has the largest neutrino emissivity and hence the lowest luminosity of all models. The model with $U_\Sigma = 10$ MeV has approximately 50% less of $\Sigma^-$ and hence is slightly more luminous as it emits less neutrinos. Finally for $U_\Sigma = 30$ MeV the fraction of $\Sigma^-$ is one order of magnitude less than for the slightly attractive potential. As a consequence this model gives the largest luminosity of all hyperonic models. We conclude that for the DDME2 model, since the nucleonic DU process does not operate, SAX J1808 is only compatible with a NS with hyperons and no or a very small amount of accreted matter in the envelope.

We can see that the delicate interplay between the symmetry energy and the $\Sigma$-potential strongly affects the cooling of SXTs. These objects could potentially offer the possibility to constrain the $\Sigma$-potential and thus the properties of the $\Sigma$ hyperon, from the astrophysical observations of SXTs with a low-luminosity complementing the little experimental constraints on the properties of the $\Sigma$ hyperon currently available. A more systematic study of the thermal state of accreting NSs is beyond the scope of the present paper and will be the subject of a future work.

4. Hyperonic star radius

There are still large observational uncertainties associated with the radius of NSs including the canonical NS with a mass equal to $1.4 M_\odot$, see the discussion in Refs. [74–76], although there have been several indirect predictions from different analysis. Recently several studies have used the detection of the gravitational waves emitted from a neutron star merger GW170817 [77] to constrain the upper limit of the $1.4 M_\odot$ star radius to $\sim 13.7$ km [78,85]. Similar constraints had been obtained before from the analysis of the experimental constraints set on the symmetry energy [76,86].

Since we are interested in analysing the effect of strangeness on the radius of a NS, and as we have seen for many models, strangeness sets in inside stars with a mass above $1.4 M_\odot$, we will consider a more massive star. In the discussion of this section we calculate the radius of a star with $M = 1.67 M_\odot$, the mass of the pulsar PSR J1903+0327. Results are plotted in Fig. 11 left panel as a function of the total star hyperon fraction. On the right panel, we have plotted the hyperonic star mass-radius curves to help the discussion. The thin (thick) lines correspond to $U_\Sigma = -10 (+30)$ MeV.

The strangeness fraction increases if the $\Sigma$ potential becomes less repulsive, and simultaneously the radius decreases. The relation between the radius and the strangeness fraction is essentially linear but the slope is model dependent. For models like NL3, TM1, TM1-2 changing $U_\Sigma$ does not have a large effect on the strangeness content and on the radius. This is clearly understood looking at Fig. 7, where the star mass at the onset of the $\Sigma$ hyperon is plotted: a star with $M = 1.67 M_\odot$ has no (only a few) $\Sigma$ hyperons for $U_\Sigma = +30$ (-
10) MeV. Density-dependent models have a similar behavior, being the models that predict a larger amount of strangeness, as large as 0.075, although still satisfying the $2M_{\odot}$ constraint. For $-10 < U_\Sigma < +30$ MeV the radius increases $\sim 300$ m. Models TM1$\omega\rho$, TM1$\sigma\rho$, TM1-2$\omega\rho$, FSU2H have a similar behavior but do not predict strangeness contents above 0.05. Models FSU2 and FSU2R suffer a quite large radius change for a small increase of strangeness because, as seen in the right panel, $1.67M_{\odot}$ is very close to the maximum star mass. Contrary to [8] we do not see a linear correlation if also the $N_S/N_B=0$ radius is included. In [8] $N_S/N_B$ is the strangeness fraction and not the hyperon fraction. However, in that work the authors did not use unified crust-core EoS and different hyperon interactions, giving rise to much larger strangeness fractions inside the star, were discussed.

VI. SUMMARY AND CONCLUSIONS

In the present study, we have explored how the density dependence of the symmetry energy may affect the properties of hyperonic neutron stars. The study was undertaken within the RMF approach to nuclear matter and models that describe ground-state properties of nuclei and $\Lambda$-hypernuclei, as well as constraints from microscopic calculations of NS (except for three models) and the $2M_{\odot}$ constraint on nucleonic stars have been chosen. We have also considered a family of models based on TM1 [8, 42, 43] that has allowed us to directly discuss the effect of the density dependence of the symmetry energy on the properties of hyperonic stars. For all the models considered, we have taken an inner crust-core unified EoS. In the present work, we have calculated the FSU2, FSUR2H and FSU2H inner crust of catalyzed $\beta$-equilibrium matter, which are given as Supplementary Material.

The $\Lambda$-meson and $\Sigma$-meson couplings were constrained by the existing hypernuclei experimental data. Taking into account the present lack of knowledge concerning the properties of the $\Sigma$ hyperon in nuclear matter, we have discussed the properties of hyperonic matter considering values of the $\Sigma$ potential in symmetric nuclear matter that go from $-10$ MeV to $+30$ MeV at saturation density, having in mind that if no $\Sigma$-hypernucleus has been detected, the $\Sigma$ potential must be repulsive or only slightly attractive.

We have shown that the DU process is affected by hyperons only if the slope of the symmetry energy is $L \lesssim 70$ MeV. The nucleonic electron DU process is both sensitive to the slope of the symmetry energy and, for $L \lesssim 70$ MeV, to the value of the $\Sigma$ potential in nuclear matter. The more repulsive $U_\Sigma$ the
larger the nucleonic electron DU process. A small \(L\) shifts the DU onset to larger densities but the effect is stronger the more repulsive the \(\Sigma\) potential is. Models with density-dependent couplings simply do not allow for the nucleonic electron DU process to turn on. However, the cooling of stars within this framework is also affected when new hyperonic channels open inside the star. So, even though the density-dependent models do not predict nucleonic electron DU, when the reactions described in Eqs. (25), (23) and (28) start to operate the star is much less luminous. This occurs in stars with a mass of the order of \(1.1 - 1.3M_\odot\) models. All other models, with constant couplings, predict the occurrence of both hyperonic and nucleonic DU processes inside massive enough NSs.

We have studied how the value of the \(U_2\) potential affects the thermal state of NSs in Soft X-ray transients and focused on the lowest-observed luminosity. We have shown that the low luminosity could also be explained in the framework of a density dependent hadronic model, satisfying well established nuclear matter and nuclei properties and describing a \(2M_\odot\) star, if hyperonic degrees of freedom are allowed to occur inside the star. In this case, objects like the SAX J1808 could potentially offer the possibility to constraint the hyperonic interaction, in particular, the \(\Sigma\) potential.

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[1] B.-A. Li, A. Ramos, G. Verde, and I. Vidana, Eur. Phys. J. A50, 9 (2014).
[2] C. J. Horowitz and J. Piekarewicz, Phys. Rev. Lett. 86, 5647 (2001), arXiv:astro-ph/0009227 [astro-ph].
[3] V. Baran, M. Colonna, V. Greco, and M. Di Toro, Phys. Rept. 410, 335 (2005), arXiv:nucl-th/0412040 [nucl-th].
[4] A. W. Steiner, M. Prakash, J. M. Lattimer, and P. J. Ellis, Phys. Rept. 411, 325 (2005), arXiv:nucl-th/0410066 [nucl-th].
[5] B.-A. Li, L.-W. Chen, and C. M. Ko, Phys. Rept. 464, 113 (2008), arXiv:0804.3580 [nucl-th].
[6] I. Vidana, D. Logoteta, C. Providencia, A. Polls, and I. Bombaci, EPL 94, 11002 (2011), arXiv:1006.5660 [nucl-th].
[7] R. Cavagnoli, D. P. Menezes, and C. Providencia, Phys. Rev. C84, 065810 (2011), arXiv:1108.1735 [hep-ph].
[8] C. Providencia and A. Kabhi, Phys. Rev. C87, 055801 (2013), arXiv:1212.5911 [nucl-th].
[9] M. Prakash, M. Prakash, J. M. Lattimer, and C. J. Pethick, Astrophys. J. 390, L77 (1992).
[10] D. G. Yakovlev, K. P. Levenfish, A. Y. Potekhin, O. Y. Gnedin, and G. Chabrier, Astron. Astrophys. 417, 169 (2004), arXiv:astro-ph/0310259 [astro-ph].
[11] M. B. Tsang et al., Phys. Rev. C86, 015803 (2012), arXiv:1204.0466 [nucl-ex].
[12] J. M. Lattimer and Y. Lim, Astrophys. J. 771, 51 (2013), arXiv:1203.4286 [nucl-th].
[13] M. Oertel, M. Hempel, T. Klähn, and S. Typel, Rev. Mod. Phys. 89, 015007 (2017).
[14] A. Klimkiewicz, N. Paar, P. Adrich, M. Fallot, K. Boretzky, T. Aumann, D. Cortina-Gil, U. D. Pramanik, T. W. Elze, H. Emling, H. Geissel, M. Hellström, K. L. Jones, J. V. Kratz, R. Kulessa, C. Necifor, R. Palit, H. Simon, G. Süröoka, K. Sümmerer, D. Vretenar, and W. Walus (LAND Collaboration), Phys. Rev. C 76, 051603 (2007).
[15] M. B. Tsang, Y. Zhang, P. Danielewicz, M. Famiano, Z. Li, W. G. Lynch, and A. W. Steiner, Phys. Rev. Lett. 102, 122701 (2009).
[16] M. Centelles, X. Roça-Maza, X. Viñas, and M. Warda, Phys. Rev. Lett. 102, 122502 (2009).
[17] M. Warda, X. Viñas, X. Roça-Maza, and M. Centelles, Phys. Rev. C 80, 024516 (2009).
[18] A. Carbone, G. Cola, A. Bracco, L.-G. Cao, P. F. Bortignon, F. Camera, and O. Wieland, Phys. Rev. C 81, 041301 (2010).
[19] I. Vidana, C. Providencia, A. Polls, and A. Rios, Phys. Rev. C80, 045806 (2009), arXiv:0907.1165 [nucl-th].
[20] C. Ducoin, J. Marguereon, C. Providencia, and I. Vidana, Phys. Rev. C83, 045810 (2011), arXiv:1102.1283 [nucl-th].
[21] F. J. Fatteyev, W. G. Newton, and B.-A. Li, Phys. Rev. C 90, 022801 (2014).
[22] M. Fortin, S. S. Avancini, C. Providencia, and I. Vidana, Phys. Rev. C 95, 065803 (2017).
[23] M. Fortin, C. Providencia, A. R. Raduta, F. Gulminelli, J. L. Zdunik, P. Haensel, and M. Bejger, Phys. Rev. C 94, 035804 (2016), arXiv:1604.01944 [astro-ph.SR].
[24] P. Demorest, T. Pennucci, S. Ransom, M. Roberts, and J. Hessels, Nature 467, 1081 (2010), arXiv:1010.1078 [astro-ph.HE].
[25] Z. Arzoumanian et al. (NANOGrav), Astrophys. J. Suppl. 235, 37 (2018), arXiv:1801.01837 [astro-ph.HE].
[26] J. Antoniadis et al., Science 340, 6131 (2013), arXiv:1304.6875 [astro-ph.HE].
[27] P. Freire, C. Bassa, N. Wex, I. Stairs, D. Champion, S. Ransom, P. Lazarus, V. Kaspi, J. Hessels, M. Kramer, et al., Monthly Notices of the Royal Astronomical Society 412, 2763 (2011).
[28] D. Chatterjee and I. Vidana, Eur. Phys. J. A52, 29 (2016), arXiv:1510.06306 [nucl-th].
[29] S. Weissenborn, D. Chatterjee, and J. Schaffner-Bielich, Phys. Rev. C85, 065802 (2012), Erratum: Phys. Rev.C90,no.1,019904(2014), arXiv:1112.0234 [astro-ph.HE].
[30] S. Weissenborn, D. Chatterjee, and J. Schaffner-Bielich, Proceedings, 11th International Conference on Hypernuclear and Strange Particle Physics (HYP 2012): Barcelona, Spain, Octo-
| $n_B$ (fm$^{-3}$) | FSU2 | FSU2R | FSU2H |
|------------------|------|-------|-------|
| 0.002            | -    | -     | 0.009527397342, 1.11490624E-05 |
| 0.003            | 0.01427514106, 1.19091664E-05 |
| 0.004            | 0.01903804950, 1.55759378E-05 |
| 0.005            | 0.023801861331, 1.90398862E-05 |
| 0.006            | 0.02856343711, 2.21966583E-05 |
| 0.007            | 0.03333383816, 2.58166204E-05 |
| 0.008            | 0.03806940842, 2.78725165E-05 |
| 0.009            | 0.042862471193, 3.04570069E-05 |
| 0.01             | 0.047628492117, 3.29902718E-05 |
| 0.011            | 0.05239467636, 3.54714132E-05 |
| 0.012            | 0.057161271572, 3.81601117E-05 |
| 0.013            | 0.061920800199, 4.08965864E-05 |
| 0.014            | 0.06669452548, 4.40385373E-05 |
| 0.015            | 0.07142194847, 4.75353081E-05 |
| 0.016            | 0.076229857197, 5.14881394E-05 |
| 0.017            | 0.08097282227, 5.60997759E-05 |
| 0.018            | 0.08576527983, 6.13193705E-05 |
| 0.019            | 0.09053359930, 6.74081153E-05 |
| 0.02             | 0.09530223906, 7.4292926E-05 |
| 0.021            | 0.10007125139, 8.21985137E-05 |
| 0.022            | 0.10480473506, 9.12191389E-05 |
| 0.023            | 0.10961052039, 0.001101202575 |
| 0.024            | 0.11430883487, 0.001125542913 |
| 0.025            | 0.11915651898, 0.00125071568 |
| 0.026            | 0.12392996223, 0.001390585239 |
| 0.027            | 0.12869426923, 0.00154363064 |
| 0.028            | 0.133467406034 |
| 0.029            | 0.13824051083, 0.00189938588 |
| 0.03             | 0.143014326692, 0.002100067359 |
| 0.031            | 0.14778822651, 0.00231950025 |
| 0.032            | 0.15256063668, 0.002554136154 |
| 0.033            | 0.157340064645 |
| 0.034            | 0.162116870824 |
| 0.035            | 0.16689449548 |
| 0.036            | 0.171672984958 |
| 0.037            | 0.17652368498 |
| 0.038            | 0.18123705712 |
| 0.039            | 0.18601398197 |
| 0.04             | 0.19079262651 |
| 0.041            | 0.19557928609 |
| 0.042            | 0.200365953593 |
| 0.043            | 0.20514926314 |
| 0.044            | 0.20993756439 |
| 0.045            | 0.214723423123 |
| 0.046            | 0.21951279494 |
| 0.047            | 0.22430142859 |
| 0.048            | 0.22903321818 |
| 0.049            | 0.233854720372 |
| 0.05             | 0.238679364324 |

| $\varepsilon$ | $P$ | $\varepsilon$ | $P$ | $\varepsilon$ | $P$ |
|---------------|-----|---------------|-----|---------------|-----|
| 0.009527397342, 1.11490624E-05 | 0.009527397342, 1.11490624E-05 | 0.009527397342, 1.11490624E-05 | 0.009527397342, 1.11490624E-05 | 0.009527397342, 1.11490624E-05 | 0.009527397342, 1.11490624E-05 |

TABLE V. Equation of state of the inner crust with pasta for the FSU2, FSU2R, and FSU2H models. The energy density, $\varepsilon$, and pressure, $P$, are in units of fm$^{-4}$. 

Legend: The values in the table represent the energy density ($\varepsilon$) and pressure ($P$) for different values of $n_B$ (fm$^{-3}$) for the FSU2, FSU2R, and FSU2H models. The energy density is given in units of fm$^{-4}$.
| \( n_g \) (fm\(^{-3}\)) | FSU2 \( \epsilon \) | FSU2 \( P \) | FSU2R \( \epsilon \) | FSU2R \( P \) | FSU2H \( \epsilon \) | FSU2H \( P \) |
|-----------------|-------|-------|------|-------|------|-------|
| 0.051           | 0.24347424 | 0.00998278 | 0.24913086 | 0.00993027 | 0.24489324 | 0.00984000 |
| 0.052           | 0.24827042 | 0.00998278 | 0.24973552 | 0.00994759 | 0.24971842 | 0.00998278 |
| 0.053           | 0.25306776 | 0.01022061 | 0.25455430 | 0.01039962 | 0.25453674 | 0.01020463 |
| 0.054           | 0.25786412 | 0.01029064 | 0.25937536 | 0.01059491 | 0.25935903 | 0.01039239 |
| 0.055           | -      | -      | 0.26419672 | 0.01069659 | 0.26418164 | 0.01057128 |
| 0.056           | -      | -      | 0.26901832 | 0.01084051 | 0.26900413 | 0.01074814 |
| 0.057           | -      | -      | 0.27384021 | 0.01109828 | 0.27382791 | 0.01092479 |
| 0.058           | 0.28766235 | 0.01102228 | 0.28765105 | 0.01109427 | 0.28763990 | 0.01106772 |
| 0.059           | 0.29312992 | 0.01104930 | 0.29312408 | 0.01115955 | 0.29312079 | 0.01114580 |
| 0.060           | -      | -      | 0.29795250 | 0.01119426 | 0.29794837 | 0.01117564 |
| 0.061           | -      | -      | 0.30277532 | 0.01133649 | 0.30277383 | 0.01119782 |
| 0.062           | -      | -      | 0.30759835 | 0.01147702 | 0.30759915 | 0.01120764 |
| 0.063           | -      | -      | 0.31242162 | 0.01161357 | 0.31242173 | 0.01122337 |
| 0.064           | -      | -      | 0.31724501 | 0.01174464 | 0.31724490 | 0.01123819 |
| 0.065           | -      | -      | 0.32206863 | 0.01188599 | 0.32207649 | 0.01125301 |
| 0.066           | -      | -      | 0.32689249 | 0.01118259 | 0.32690248 | 0.01126453 |
| 0.067           | -      | -      | 0.33171657 | 0.01114748 | 0.33172548 | 0.01123573 |
| 0.068           | -      | -      | 0.33654078 | 0.01127468 | 0.33655470 | 0.01124843 |
| 0.069           | -      | -      | 0.34136188 | 0.01139935 | 0.34138119 | 0.01126415 |
| 0.070           | -      | -      | 0.34618976 | 0.01152098 | 0.34620758 | 0.01127956 |
| 0.071           | -      | -      | 0.35101455 | 0.01163956 | 0.35103482 | 0.01129406 |
| 0.072           | -      | -      | 0.35583943 | 0.01175406 | 0.35585920 | 0.01130957 |
| 0.073           | -      | -      | 0.36066454 | 0.01186609 | 0.36068925 | 0.01132475 |
| 0.074           | -      | -      | 0.36548984 | 0.01197352 | 0.36551681 | 0.01133929 |
| 0.075           | -      | -      | 0.37031516 | 0.01207387 | 0.37034451 | 0.01135390 |
| 0.076           | -      | -      | 0.37514063 | 0.01217218 | 0.37517246 | 0.01136752 |
| 0.077           | -      | -      | 0.37996290 | 0.01226286 | 0.37999228 | 0.01138159 |
| 0.078           | -      | -      | 0.38479200 | 0.01234499 | 0.38482885 | 0.01139446 |
| 0.079           | -      | -      | 0.38961771 | 0.01241948 | 0.38965229 | 0.01140725 |
| 0.080           | -      | -      | 0.39444357 | 0.01248486 | 0.39448583 | 0.01141929 |
| 0.081           | -      | -      | 0.39926946 | 0.01253909 | 0.39931452 | 0.01143279 |
| 0.082           | -      | -      | 0.40414354 | 0.01263452 | 0.40418386 | 0.01144773 |
| 0.083           | -      | -      | 0.40872501 | 0.01268242 | 0.40876651 | 0.01146204 |
| 0.084           | -      | -      | 0.41380169 | 0.01265546 | 0.41384259 | 0.01148023 |
| 0.085           | -      | -      | 0.41863068 | 0.01270194 | 0.41867270 | 0.01149920 |