Analysis of semi-active vehicle suspension system using airspring and MR damper

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Abstract. With the new advancements in vibration control strategies and controllable actuator manufacturing, semi-active actuators and dampers are finding their way as an essential part of vibration isolators, particularly in vehicle suspension systems. This is attributed to the fact that in a semi-active system, the damping coefficients can be adjusted to improve ride comfort and road handling performances. The currently available semi-active damper technology uses MR fluid to control the damping characteristics of the suspension system. In addition to MR dampers, combining air springs in a semi-active suspension system leads to better handling and ride performance in vehicles. Furthermore, the use of air spring in semi-active suspension system helps to ease design of variable spring stiffness. This easy design opportunity leads to independent control of stiffness and ride height of the vehicle. This paper deals with the design and modelling of variable stiffness air spring for semi-active suspension system, modelling of semi-active suspension systems with variable stiffness and MR damper, and study their

1. Introduction

The evolution of suspension is merely close to the evolution of transportation technology. For centuries, carts were not equipped with any sort of suspension at all. In the eighth century, a primitive suspension based on an iron chain system was developed \([1]\). Today’s vehicle suspension systems can be categorized as passive, active, and semi-active system. Passive suspension has design and cost related advantages, however, it has fixed parameters and limited performance on variable operating conditions. In contrast, active suspension utilizing hydraulic or pneumatic actuators to provide high control performance. Nevertheless, it requires high power and sophisticated control implementation. On the other hand, semi-active suspension system use variable dampers or variable spring stiffness with small power requirement and simpler control implementation. Today’s rapid development of semi-active suspension systems have improved ride, handling and safety of vehicles and occupants \([2]\). Fuzzy logic controllers incorporated in semi-active suspension systems gave better road handling and rid comfort \([3]\). Semi-active suspension system with modulated damping coefficient value has shown minimum percentage of overshoot and fast stabilizing time with fuzzy logic controller\([4]\). One of the effective semi-active suspension system that coupled air-springs with variable axillary volume are liable to give lower amplitude transmissibility and lower system natural frequency \([5]\).
The objective of this paper is to develop a conceptual design of variable stiffness air spring and MR model with a fuzzy logic controller. The developed model is used to analyze effects of variable stiffness in various operating frequencies, simulate semi-active suspension system using variable damper and variable air spring stiffness using Math lab Simulink and study the advantage of Fuzzy logic controller in semi-active suspension system to smooth and ease operation of MR damper.

2. Magnetorheological Damper

The semi-active suspension system considered in this paper contains variable stiffness air spring and MR damper as its suspension elements. The magnetorheological (MR) damper consists of a hydraulic cylinder containing a solution that can reversibly change from a free flowing, linear viscous fluid to a semi-solid with controllable yield strength. This solution is termed MR fluid and is composed of micron-sized magnetically polarizable particles dispersed in a carrier medium such as water, mineral or synthetic oil. Out of the many developed MR damper models, Bouc-Wen’s [6] modified model gives best results portraying the hysteretic behavior of MR dampers as shown in figure 1.

![Figure 1. Bouc-Wen modified model of MR damper.](image)

In this model, the damping force generated by the device is:

\[ F = c_1 \dot{y} + k_1(x - x_0) \]  

(1)

Where \( x \) is the total relative displacement and \( x_0 \) the initial displacement of spring \( k_1 \) associated with the force due to the accumulator; \( k_1 \) represents the accumulator stiffness; \( y \) is an internal displacement of the MR fluid damper and \( z \) is the evolutionary variable that accounts for history dependence of the response and are governed by the coupled differential equations:

\[ \dot{y} = \frac{1}{c_0 + c_1} [c_0 \dot{x} + k_0(x - y) + kz] \]  

(2)

\[ \dot{z} = -\gamma |\dot{x} - \dot{y}|z|z|^{n-2} - \nu(\dot{x} - \dot{y})|z|^n + A(\dot{x} - \dot{y}) \]  

(3)

Where \( k \) is a stiffness coefficient associated with the displacement \( z \), \( c_0 \) represents the viscous damping observed at large velocities; and \( k_0 \) is to control the stiffness at large velocities. The parameters \( n, \gamma, \nu, A, \) and \( k_1 \) are ‘loop parameters’ which control the shape and the magnitude of the hysteretic loops [6]. The parameters \( c_0, c_1, k \) and \( k_0 \) are assumed to be functions of the delayed current \( i = i(t) \) applied to the MR damper: \( c_0 = c_0(i), c_1 = c_1(i), k = k(i), k_0 = k_0(i) \). For numerical simulations the following values of the fixed coefficients were chosen: \( n = 2, \gamma = 500 \text{ N/m}, \nu = 613000 \text{ m}^{-2}, A = 30.56, k_1 = 540 \text{ N/m} \) and \( k_0 = 1050 \text{ N/m} \).
The functional approximations (equation (4)) obtained for the remaining parameters \( c_0, c_1, k \), are:

\[
\begin{align*}
  c_0 &= 26.134i + 5.164 \\
  c_1 &= 1150 \tanh 1.95i \\
  k &= 1297.2 \tanh 1.3i
\end{align*}
\]

3. Conceptual design of variable stiffness air spring

The motivation to design an automotive suspension system with independent control of stiffness, damping and ride height comes from the trade-offs involved in conflicting requirements of ride and handling. Hrishikesh Deo and Nam P Suh [7] have proposed an electromechanical suspension system capable of independent control of stiffness, damping and ride height and have discussed its application to improved vehicle dynamics [8]. The application to improved vehicle dynamics requires the stiffness change to be instantaneous and no change in ride-height during stiffness change. In the electromechanical suspension system [7], stiffness change requires power-input and is not instantaneous. This paper focuses on design of pneumatic automotive suspension system capable of instantaneous stiffness change with no power input, and no ride-height change due to stiffness change. In this section we discuss how to format the title, authors and affiliations. Please follow these instructions as carefully as possible so all articles within a conference have the same style to the title page. This paragraph follows a section title so it should not be indented.

3.1. Design for Variable Stiffness Suspension

Assuming adiabatic compression and expansion of air, the nominal stiffness \( K \) of an air-spring is:

\[
K = \gamma A^2 \frac{P_0}{V_1}
\]

Where \( P_0, T_0, \) and \( V_1 \) are the equilibrium values of pressure, temperature and volume of air in the air-spring, \( A \) is the effective area and \( \gamma = C_p / C_v = \) heat capacity ratio of air. In some existing adaptive suspensions [8], air-springs are employed to achieve variable stiffness and variable ride-height by pumping air into (or out of) the air bags. But this leads to coupling as the design parameter (DP): amount of air affects both, functional requirements (FR), FR1: controlled stiffness and FR2: controlled ride-height.

\[
\begin{align*}
  \text{FR}_1 : \text{Controlled Ride Height} \\
  \text{FR}_2 : \text{Controlled Stiffness}
\end{align*}
\]

\[
\begin{bmatrix}
  X \\
  X
\end{bmatrix} = \begin{bmatrix}
  X \\
  X
\end{bmatrix} \begin{bmatrix}
  \text{DP: Amount of air in } V_1
\end{bmatrix}
\]

Coupling due to insufficient number of DPs is a common mistake made by designers [8]. Suh proposed decoupling of such a coupled system by addition of new DPs to make the number of FRs and DPs equal. Here additional DPs have been introduced through the following proposed design modification.
The proposed modification lets us to connect the airspring volume $V_1$ to auxiliary volumes, represented by $V_2$ to $V_4$ in figure 2, through on-off valves. When all of the three valves are closed, the effective volume is $V_1$ and the stiffness is:

$$K_{\text{max}} = \gamma A^2 \frac{P_0}{V_1} \quad (9)$$

When all of the three valves are open and communicate with the airspring volume, the effective volume is $(V_1+V_2+V_3+V_4)$ and the stiffness as given by:

$$K_{\text{eff}} = \gamma A^2 \frac{P_0}{(V_1+V_2+V_3+V_4)} \quad (10)$$

By efficient choices of non-equal volumes $V_2 \neq V_3 \neq V_4$, we can get $2^3=8$ stiffness band settings between the $K_{\text{min}}$ and $K_{\text{max}}$, through different combinations of valves open and closed. In general, by adequately choosing $N$ different volumes, $V_2 \neq V_3 \neq \ldots \neq V_{N+1}$, we can get $2^N$ different stiffness bands. Note that all the volumes are at the same pressure. As a result, there will not be any change in pressure or ride-height. When the valves are opened or closed, to adjust the stiffness, they do not use any effort and they are instantaneous. Damping control is achieved using magneto-rheological damper [8] that is mounted in parallel with the air spring arrangement. The coupled design matrix in equation (11).

$$\begin{bmatrix}
FR_1 : \text{Controlled Rideheight} \\
FR_2 : \text{Controlled Stiffness} \\
FR_3 : \text{Controlled Damping}
\end{bmatrix} = \begin{bmatrix}
X & 0 & 0 \\
0 & X & 0 \\
0 & 0 & X
\end{bmatrix} \begin{bmatrix}
DP_1 : \text{Compressor Control} (V_1) \\
DP_2 : \text{Valve Control} (\sum V_i) \\
DP_3 : \text{MR damping}
\end{bmatrix} \quad (11)$$

The set-up in figure 2 with only one auxiliary volume connected to the main volume through an on-off valve can be schematically represented as shown in figure 3, where $\beta_2 = \infty$ (for valve closed) results in stiffness $K = K_{\text{max}}$, and $\beta_2 = 0$ (for valve open) results in stiffness $K = K_{\text{min}}$. $\beta$ denotes the variable damper in parallel with the air spring.
3.2. Thermodynamic Modelling to Estimate Damping due to the Valve

With the valve closed, $\beta_2 = \infty$ is an accurate assumption. But for the valve open, $\beta_2 = 0$ is not accurate. There is always restriction to the flow across the valve. Pressure drop in most valves is proportional to the square of the volume flow through the valve. This introduces damping in the system and in this section it has been tried to determine the damping through thermodynamic modelling of the system. The system is characterized by the $P_1$, $V_1$, $m_1$, $T_1$ for CV1 and $P_2$, $V_2$, $m_2$, $T_2$ for CV2. We have made an assumption that all the gas in CV1 is in the same state and all the gas in CV2 is in the same state. It is reasonable to assume that the gas properties are not a function of position within the chamber for modelling the air spring, as the corner frequency for the suspension system is in order of magnitude lower than the frequency at which pressure wave propagates with in the chamber to equalize the pressure at different points in the chamber. The damping in the system is due to the pressure drop “effort” across the valve which is proportional to the square of the volume flow through the valve. There is an additional source of irreversibility in the system, namely the mixing of air at the outlet of the valve as it flows from one chamber to another. There are eight parameters and we need a relation of the form $\Delta P = f(\Delta V, \omega)$, i.e., pressure variation in response to a sinusoidal forcing of magnitude $\Delta V$ and frequency $\omega$.

$$V_1 = V_0 + \Delta V \sin(\omega t), \quad \Delta V = A_0 \cdot x_2$$

$$V_2 = \text{Constant}$$

3.3. Conservation of mass

The total mass of a control volume is conserved and net rate of change of mass of the system is zero since there is no mass flowing out of the system boundary. The relation is given by:

$$m_1 + m_2 = m$$
$$\dot{m}_1 + \dot{m}_2 = 0$$

i.e., $\dot{m}_1 = -\dot{m}_2$

3.4. Constitutive relation for gas in control volume

For isentropic processes where $K$ is a constant, When this relationship is combined with the equation of state for a perfect gas given by, where $R$ is the gas constant, we can write the following relationships linking the variables at two different states (or stations) of an isentropic flow.
\[ \frac{\rho}{\rho_1} = \frac{P}{P_1}, \quad \text{and} \quad \frac{\rho}{\rho_2} = \frac{P}{P_2} \]  \hspace{1cm} (15)

From these relations it follows that:

\[ \frac{T_1}{T_2} = \left( \frac{P_1}{P_2} \right)^{\frac{m}{m-1}} \]  \hspace{1cm} (16)

From the equations of state for quasi-one dimensional adiabatic flow, constitutive relation for gas in CV1 in algebraic and differential form becomes:

\[ \frac{d(P/\rho)}{dx} = 0 \]  \hspace{1cm} (17)

Expanding equation (15) and rearranging gives

\[ \frac{dP}{\rho} = \frac{d\rho}{\rho} - \frac{dT}{T} = 0, \]  \hspace{1cm} further simplification and rearrangement gives the following relation:

\[ P_i = \frac{m_iRT_i}{V_i} \quad \text{and} \quad \frac{dP}{\rho} = \frac{d(mV^{-1})}{mV^{-2}} \]

\[ \frac{\dot{P}_2}{\dot{P}_1} = \frac{\dot{m}_1 + \dot{t}_1}{\dot{m}_2} \Rightarrow \dot{P}_1 = \dot{P}_2 \left( \frac{\dot{m}_1 + \dot{t}_1}{\dot{m}_2} \right) \]

In a similar fashion the derivation for constitutive relation for gas in CV2 in algebraic and differential form is given in by:

\[ P_2 = \frac{m_2RT_2}{V_2}, \quad \frac{\dot{P}_2}{\dot{P}_1} = \frac{\dot{m}_1 + \dot{t}_1}{\dot{m}_2} - \frac{\dot{t}_2}{\dot{m}_2} \]

The mass/volume flow rate is related to the pressure drop across the valve by the valve flow equation. Mass flow rate can be calculated from the density of the substance, the cross sectional area through which the substance is flowing, and its velocity relative to the area of interest.

\[ \dot{m} = \rho \cdot v \cdot A \]  \hspace{1cm} (20)

This is equivalent to multiplying the volume flow rate by the density.

\[ \dot{M} = \rho \cdot q \]  \hspace{1cm} (21)

The symbol for mass flow rate is Newton’s notation for a derivative:

\[ \dot{m} = \frac{dm}{dt} \]

\[ \dot{M}_1 = \dot{M}_2 = f(P_1 - P_2) \]  \hspace{1cm} (22)

\[ q = c_v \frac{1}{11.7} \sqrt{\frac{P_1 - P_2}{5G}} \]

\[ \dot{M}_1 - \rho \dot{q} \]

Equations (10 to 20) are the complete set of equations that describe the behaviour of this spring. These are a set of non-linear differential equations and it is not possible to obtain an analytical solution for the purpose of determining the effect of the DPs on FRs.
This qualitative behaviour of the airspring with auxiliary volumes can be represented as lead compensator element, employing a spring in series with a parallel arrangement of spring and damper as shown in figure 5. The complex stiffness of the element is given as:

\[ K_{comp} = \frac{K_1 (K_2 + \beta_2 s)}{K_1 + K_2 + \beta_2 s} \]  

(23)

Numerical simulation based on the following vehicle parameters has been performed for a quarter car. The sprung mass stiffness is a variable stiffness of air spring; the parameters used for this analysis are taken from [9] and they are presented in table 1.

### Table 1. Airspring model parameter.

| Parameters                        | Amount          | Unit   |
|-----------------------------------|-----------------|--------|
| Sprung mass (Ms)                  | 8650            | kg     |
| Unsprung mass(Mun)                | 1350            | kg     |
| Operating pressure (Pg)           | 7 *105          | N/m²   |
| Absolute pressure (P)             | 8 *105          | N/m²   |
| Tire stiffness (Kt)               | 6.5*106         | N/m    |
| Suspension damping coefficient (Cs)| 43100           | Ns/m   |
| Maximum height (H)                | 0.3             | m      |
| Maximum diameter (D)              | 0.548           | m      |
| V1 (main airspring volume)        | 0.070758        | m³     |
| V2 airspring auxiliary volume     | 0.112287        | m³     |
| V3 airspring auxiliary volume     | 0.074858        | m³     |
| V4 airspring auxiliary volume     | 0.024953        | m³     |
| High stiffness                    | 8.8*105         | N/m    |
| Medium stiffness                  | 3.6483*105      | N/m    |
| Low stiffness                     | 2.2*105         | N/m    |

The volume ratio of V2, V3, and V4 is assumed as 1: 1.5: 4.5. Hence their value from the above analysis is given in table 1. In this part, for analysis purpose the damper element has been considered as a fixed value damper and the airspring stiffness varies with volume. The effect of the valve restriction needs a laboratory investigational justification and is not included in this paper. Based on
the numerical manipulation this valve restriction pressure drop is found to be small and it is disregarded as it has a negligible effect. Hence the compound spring stiffness is treated as an equivalent stiffness of springs connected in series.

4. Modelling of semi-active suspension system

The complexity to develop a mathematical model for a semi-active suspension system of a vehicle depends on the suspension system components model sophistication i.e. the model developed for MR damper and airspring. Hence, the mathematical model for semi-active suspension system involves analysis of a non-linear system. To simplify this model, it is assumed that the model is combination of linear and non-linear parts.

4.1. Mathematical modelling of Semi-Active suspension with Variable Spring Stiffness

The physical model of semi-active suspension system with variable spring stiffness is shown in figure 6. The variable stiffness of the spring is due to the arrangement of variable air bag volumes together with the main volume. The curve between stiffness and airbag volume is piece wise linear.

The damping coefficient (Cs) remains fixed. Thus, the equations of motion for the TDF quarter car model shown in figure 8 are:

\[
M\ddot{x} + K_{\text{con}}(x - x_u) + C_s(\dot{x} - \dot{x}_u) = 0
\]

\[
M_x\ddot{\dot{x}} + K_{\text{con}}(x - x_u) + C_s(\dot{x} - \dot{x}_u) + K_x = K_0 x_0
\]

Assume the following state variables: \(x_1 = x, x_2 = \dot{x}, x_3 = x_u, x_4 = \dot{x}_u\) its state space equations of motion are:

\[
[\begin{array}{c}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4
\end{array}] =
[\begin{array}{cccc}
0 & 1 & 0 & 0 \\
0 & 0 & K_0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & M_s
\end{array}]
[\begin{array}{c}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{array}]

+ [\begin{array}{c}0 \\
0 \\
0 \\
0
\end{array}]

\[
\dot{x} = (A + A^T)x + B \cdot x_0
\]

4.2. Mathematical modelling of Semi-Active suspension with Variable Spring Stiffness and Variable Stiffness Spring

The mathematical model of this system figure 7 is complex that involves two nonlinear sub systems. The damping coefficient is a function of current, and the spring stiffness is a function of volume. Their values can be represented as: \(C_s = C_s(t)\) and \(K_{\text{con}} = K_{\text{con}}(v)\).
The variable stiffness value $K_{\text{com}}(v)$ is determined using equation (21), Similarly the values of the variable damper $C_s(i)$ is determined using equations (1 to 3) and (4 to 6). The equations of motion are:

$$M_s \ddot{x}_s + K_{\text{com}}(v)(x_s - x_g) + C_s(i)(\dot{x}_s - \dot{x}_g) = 0$$

$$M_{\text{col}} \ddot{x}_c + K_{\text{com}}(v)(x_c - x_g) + C_s(i)(\dot{x}_c - \dot{x}_g) + K_r x_c = K_r x_s$$

The state space representation of the system is:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{K_{\text{com}}(v)}{M_s} & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ \frac{C_s(i)}{M_s} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix}$$

4.3. Mathematical modelling of Semi-Active suspension with MR Damper

The analysis of semi-active suspension system with both variable stiffness and variable damping becomes a bit complex due to the existence of the two nonlinear variables $C_s(i)$ and $K_{\text{com}}(v)$, especially this complexity made the development of the control law difficult. Hence in this section, the airspring stiffness is considered as a constant value represented as $K_s$ shown in figure 8.
The equations of motion are:

\[ M_0 \ddot{x}_2 + K_s (x_2 - \ddot{z}_u) + C_s \ddot{\omega}(x_2 - \dot{z}_u) = 0 \]  \hspace{1cm} (30)

\[ M_m \ddot{x}_1 + K_s (x_1 - \ddot{z}_u) + C_s \ddot{\omega}(x_1 - \dot{z}_u) + K_x x_1 = K_x x_0 \]  \hspace{1cm} (31)

Accordingly, the state space representation of the system is modified to equation (32).

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 \\
-\frac{1}{M_0} & 0 & \frac{K_s}{M_0} & 0 \\
0 & 0 & 1 & 0 \\
\frac{K_s}{M_m} & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
0 \\
\frac{K_x}{M_m}
\end{bmatrix} x_0 +
\begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix} u
\]

\[ x = (A + A') x + B \cdot u \]  \hspace{1cm} (32)

The model parameters used for this analysis are taken from Fox and Cebon [10], which are: \( M_s = 1000 \) kg \( n = 2 \), \( \gamma = 500 \) N/m, \( \nu = 613000 \) m\(^{-2} \), \( M_m = 100 \) kg, \( A = 30.56 \), \( k_1 = 540 \) N/m, \( K_t = 138000 \) N/m and \( K_s = 56850 \) N/m.

5. Control of semi-active suspension system

In this work three control laws have been applied to the semi-active suspension system equipped with MR damper. The simulation results of these laws (PID, Skyhook and Fuzzy logic controllers) have been analysed. From the simulation results it has been observed that the fuzzy logic law gives good result. This is due to: in classical control theory good model result depends on strength of differential equation. However, fuzzy logic control depends on heuristics and rules. Therefore, fuzzy logic controller has an advantage over classical controller when it is applied to complex systems. It can be developed with minimal knowledge about the system dynamics. Tsampardoukas [11] and Mehrdad N. [12] have considered a fuzzy logic controller design consisting of the following four steps:

1. Rule-base (a set of If-Then rules)
2. Inference, which emulates the expert’s decision in interpreting and applying knowledge about how best to control the system.
3. Fuzzification, in which the crisp inputs are transformed to fuzzy values.
4. Defuzzification in which the fuzzy output is converted back to the crisp values.

The fuzzy membership functions of input and output is:
The fuzzy controller rules presented in table 2 will give the amount of current required continuously to reasonably damp the unexpected disturbances imposed to the system. This current results a damping coefficient $C_s(i)$ according to the s-function developed as simulation subsystem of the main suspension system.

### Table 2. Fuzzy rule output.

| Suspension speed | Relative speed |
|------------------|----------------|
| PB               | L              |
| NB               | L              |
| NS               | L              |
| Z                | L              |
| PS               | M              |
| PS               | S              |
| PS               | M              |
| PS               | L              |
| PB               | L              |
| PB               | L              |
| PB               | L              |
| PB               | L              |

6. Simulation result

The two semi-active suspension systems: one with variable air spring stiffness and the other with both variable damper and variable stiffness airspring described have been analyzed for their time response to step input and transmissibility response. The semi-active suspension system with fixed airspring stiffness and MR damper has been analyzed with three control laws to a step input.

6.1. Variable Stiffness Airspring

The time response of the semi-active suspension system with variable airspring stiffness is given in figure 11. The response shows using a stiff spring in the system leads to an oscillatory response with higher initial overshoots, on the other hand low airspring stiffness results in small overshoot, reduced response time, with some steady state error. But the suspension system with medium airspring stiffness gives zero steady state error, small response time and larger overshoot.

![Figure 10. System responses for different stiffness values subjected to a step input excitation.](image-url)
The transmissibility of the semi-active suspension system with variable stiffness is given in figure 11. From the transmissibility curve: the airspring with high stiffness gives small vibration transmissibility in the low frequency range; the spring with small stiffness gives good vibration isolation in the medium frequency area. However, airspring with any stiffness value has nearly the same behaviour to respond against vibration in the high frequency range. Hence, the new design approach helps to have variable stiffness airspring responding to any disturbance frequency as shown by a black colour in figure 11.

6.2. Variable Stiffness and Variable Damper Results
The time response for semi-active suspension system with variable damper and variable stiffness is shown in figure 12. The result is similar in its nature to the results of figure 10 except here the overshoot and number of oscillation before steady state have increased.

![Figure 11](image1.png)

**Figure 11.** Effects of different stiffness in vibration isolation, and expected response improvements for designing variable stiffness air spring.

![Figure 12](image2.png)

**Figure 12.** System responses for variable stiffness and variable damping to a step input excitation.
One of the most important advantages of a semi-active suspension system is its capacity to shift the resonance frequency to the left. Secondly the resonance amplitude of vibration decreases substantially to a prominent value. In the low frequency area, when frequency is lower than 0.5 Hz, effective vibration isolation will be obtained by using high stiffness - high damping coefficient in the suspension system. In the medium frequency area where frequency is between 0.5 and 10 Hz the combined arrangement of low stiffness and high damping coefficient give better vibration isolation. Whereas the vibration isolation in the high frequency area, where frequency is greater than 10 Hz, almost all types of stiffness have equal effect of vibration isolation, however, a smaller damping coefficient results in good vibration isolation. Thus, introducing a variable stiffness airspring to the semi-active suspension system improves the ride comfort of vehicles as shown in figure13 drawn by a black colour.

6.3. Semi-Active Suspension System with MR Damper

Here the MR damper model parameters are simulated using three control rules (skyhook, fuzzy and PID). Both the fuzzy logic and skyhook control laws use an s-function for iterated analysis of MR damper. The time response of the three control laws are compared in figures 14 to 18. The steady state response of PID and fuzzy logic controllers are nearly the same except PID controller gives small response time with no oscillation after the first overshoot, but it has a bigger overshoot. The skyhook controller is accompanied by a maximum overshoot and oscillatory response with slow damping. Of the three control laws, the fuzzy logic controller has relatively good response both in terms of small overshoot, few unwanted oscillations after the first overshoot, and quick damping of oscillation.
The sprung mass velocity response of the semi-active suspension quarter car model under these control laws is presented in figure 15. Both fuzzy and PID response shows the system returns to its original state after being disturbed. However, fuzzy controller has reduced overshoot. From the value of variable damping coefficient shown in figure 17, the damping coefficient value for PID controller is the reverse to the requirements in reality. Both the sky-hook and fuzzy logic controller response looks reasonable.
From the relative velocity of sprung mass to unsprung mass response shown in figure17, the unsprung mass of the system is highly unstable for the fuzzy logic controller, which is due to high stiffness of the tire spring. The current required by the MR damper is shown in Figure 18. The skyhook controller is acting as on/off switch. But the real requirement of current by the damper is a continuous process. This continuous requirement and continuous supply of current is only achieved through fuzzy controller.
Looking the overall performance of semi-active suspension system with fuzzy controller shown in figures 15 to 18, it is the best control law suitable for semi-active suspension system equipped with variable damper (MR) that requires a continuous small current supply for its best performance.

7. Conclusion
Looking at the conflicting requirements of suspension for comfort and handling; a new conceptual design of variable stiffness airspring is developed to facilitate independent control of ride height and airspring stiffness. The brain of semi-active suspension system is laid on the robustness of its control laws; three control laws have been implemented to analyse the mathematical model of semi-active suspension system with different stiffness and damper arrangement. The fuzzy logic control law and skyhook control law are designed to work with the s-function code for MR damper iterative analysis. The numerical simulation results indicate the fuzzy logic control has got a good reputation for best ride behaviour. The performance of semi-active suspension system with MR damper and fuzzy logic controller is optimum.

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Figure 18. Current demand of skyhook and MR damper.
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