Influence of Liquid Sloshing on FLNG Motion

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Summary

This paper provides the rational explanation of influence of liquid sloshing on the FLNG motion using spring-mass type mechanical model to represent the liquid cargo behavior and the coupling effects on the FLNG motion. The mechanical model of sloshing was originally developed in the mid-1960s and used to represent the coupling effects on the spacecraft motion for predicting and controlling the spacecraft behavior and stability. Authors believe that the mechanical model is a useful tool for better understanding the interaction of liquid cargo and the floating body motion. Firstly, the concept and associated formula of the mechanical model are explained and the parameters of the mechanical model for a rectangular tank are obtained analytically. Secondly, the equation of floating body motion and the mechanical model are coupled. Next, the numerical demonstration is performed and compared to the experiment and the calculation by other means. Finally, a rational explanation of the influence of liquid sloshing on the FLNG motion is provided.

1. Introduction

Over the past decade, the demand for liquefied natural gas (LNG) has increased sharply. It is expected to increase further in the near future, as LNG is considered “cleaner” than other fossil fuels (e.g. oil and coal) and supply of which is more stable than renewable (e.g. wind, solar, etc.). The governments and major oil and gas companies’ great interest in LNG development has resulted in technical innovations in the LNG supply chain, such as increase in capacity of LNG tank systems, development of FLNG (floating LNG production, storage and offloading units), FSRU (floating storage and regasification units), etc. The advancement of FLNG system has created high expectations in development of offshore stranded gas fields and monetizing the gas from the existing offshore oil fields.

One of the major issues to be overcome in realization of FLNG is the motion prediction of floating bodies. This is essential to estimate the reliability and availability of FLNG. Research and development of motion prediction have been performed by a number of researchers and research groups over a long period of time. Thanks to the increased computing power, the study of motion prediction has achieved a sufficient level of sophistication for practical application—even though there are still technical issues in improving the accuracy, such as the need for methods to incorporate the viscous effect, gap resonance, wind and current load estimation, liquid cargo effects including sloshing, etc.

The present paper focuses on the effect of liquid cargo on the FLNG motion caused by the liquid movement including sloshing, free surface and free rotation. The effect due to contribution of liquid cargo to the mass distribution and mass inertia is excluded.

There are a number of preceding researches focusing on the coupling of liquid motion in tanks with floating body motion. The barge motion in beam waves influenced by the rectangular tanks partially filled with water was investigated by Yamashita and Molin et al. by means of numerical calculation and test conducted on an experimental model. Malenica et al. presented the calculation method for the dynamic coupling between sloshing and seakeeping and validated against the results from Molin et al. Their calculation method is based on the linear potential theory in the frequency domain. The floating body motion affected by the non-linear sloshing in tanks partially filled with liquid was investigated by Journée, Rognebakke and Faltinsen, Lee et al., Wang and Arai and Rocha et al. The liquid behavior in tanks is represented by the fluid dynamic methods such as the boundary element method, finite difference method, particle method, etc. and it is coupled with the floating body motion.

The authors intend to provide rational explanation of influence of liquid sloshing on the FLNG motion by the simple motion calculation method considering liquid sloshing. As a result, it is possible to simulate accurately the complex phenomena of floating body motion influenced by sloshing using numerical calculation, but at the same time such a highly elaborate numerical calculation becomes an opaque process and the correctness of the calculation results is difficult to verify. Comparison of results from experiment conducted on a model is the most reliable verification method; however, this is time consuming and expensive.

An alternative verification method is to confirm if the major characteristic indicator such as the natural period or the period corresponding to the peak response is close to the one predicted by other method. Although this method only provides partial reliability, it can be done easily. Another benefit of this method is that it is possible to identify the fundamental mechanism of the phenomenon of interest.

The spring-mass type mechanical model prescribes the mechanism of the phenomena of interest. This method can only
realize the phenomena of the prescribed mechanism, but its calculation process is simple and transparent; the causal relationship between input parameters and output is easy to trace.

For the purpose of explaining the complex phenomena of floating body motion influenced by the sloshing, this one is the preferred choice over CFD based numerical calculation method.

2. Mechanical Model of Liquid Cargo

2.1 General

The effect of lateral sloshing, which is dynamic in nature, is mainly due to a horizontal oscillation of center of liquid mass relative to the tank. This can be well represented by a mechanical model.

The mechanical model of sloshing was originally developed in the mid-1960s and used to represent the coupling effects with the spacecraft motion for predicting and controlling the spacecraft behavior and stability\(^{10,11}\).

Similar mechanical model can also be used to represent the coupling effects with the floating body motion as shown in Fig. 1.

![Fig. 1 Mechanical Model Application for FLNG.](image1)

**Fig. 1** Mechanical Model Application for FLNG.

\[
\begin{align*}
F_{x,\text{amp}} &= M \left( \sum_{n=1}^{\infty} m_n \omega_n^2 \omega_n \right) + h \alpha \left( \sum_{n=1}^{\infty} m_n g \frac{H_n}{h \omega_n^2} \omega_n^2 \right) + h \alpha \left( \sum_{n=1}^{\infty} m_n g \frac{H_n}{h \omega_n^2} \omega_n^2 \right) \\
M_{x,\text{amp}} &= M \left( \sum_{n=1}^{\infty} m_n \frac{g}{h \omega_n^2} \omega_n^2 \omega_n \right) + h \alpha \left( \sum_{n=1}^{\infty} m_n g \frac{H_n}{h \omega_n^2} \omega_n^2 \right) + h \alpha \left( \sum_{n=1}^{\infty} m_n g \frac{H_n}{h \omega_n^2} \omega_n^2 \right) \\
&+ h \alpha \left( \sum_{n=1}^{\infty} m_n g \frac{H_n}{h \omega_n^2} \omega_n^2 \right)
\end{align*}
\]

\[I_y = I_R - I_d\]  \hspace{1cm} (3)

where \(m_T, \omega_n, h, g\) and \(I_R\) are total liquid mass, \(n\)-th natural angular frequency, liquid height, gravity constant and mass inertia of rigid cargo, respectively.

2.2 Description of Mechanical Model

The dynamic behavior of liquid in a tank is represented by the mechanical model as shown in Fig. 2. The liquid inside the tank is replaced by masses, springs, dampers and a disk.

![Fig. 2 Spring-Mass Model of Liquid in Tank.](image2)

**Fig. 2** Spring-Mass Model of Liquid in Tank.

The fixed mass and mass inertia are represented by \(m_0\) and \(I_0\), respectively. Each of slosh modes corresponding to the sloshing natural frequencies is represented by the spring-mass and damper system. The mass for each slosh mode is located at \(H_n\) which is the height from the center of gravity of the liquid. Note that throughout this paper, the positive direction of \(H_n\) is opposite to the height of slosh masses. \(H_n\), i.e., the height of \(m_n\), is positive when the \(n\)-th mode mass is located above the COG, but \(H_n\) is positive when it is located below the COG. The damping is neglected. In addition, a disk \(I_d\) is introduced as the mass inertia of free rotating liquid which does not contribute to the mass inertia of the liquid.

When the tank is excited at angular frequency \(\omega\) by a small time-varying linear displacement \(X_0\) along the x axis and angular rotation \(\alpha_0\) about an axis through the COG, the spring masses deflect a distance \(x_n\) relative to the tank walls as a result of the tank motion. The amplitudes of horizontal force and moment exerted by \(X_0\) and \(\alpha_0\) are obtained as follows:

\[F_{x,\text{amp}} = L \left( \sum_{n=1}^{\infty} m_n \omega_n^2 \omega_n \right) + h \alpha \left( \sum_{n=1}^{\infty} m_n g \frac{H_n}{h \omega_n^2} \omega_n^2 \right) + h \alpha \left( \sum_{n=1}^{\infty} m_n g \frac{H_n}{h \omega_n^2} \omega_n^2 \right) \]

\[M_{x,\text{amp}} = M \left( \sum_{n=1}^{\infty} m_n \frac{g}{h \omega_n^2} \omega_n^2 \omega_n \right) + h \alpha \left( \sum_{n=1}^{\infty} m_n g \frac{H_n}{h \omega_n^2} \omega_n^2 \right) + h \alpha \left( \sum_{n=1}^{\infty} m_n g \frac{H_n}{h \omega_n^2} \omega_n^2 \right) \]

\[I_y = I_R - I_d\]  \hspace{1cm} (3)

where \(m_T, \omega_n, h, g\) and \(I_R\) are total liquid mass, \(n\)-th natural angular frequency, liquid height, gravity constant and mass inertia of rigid cargo, respectively.

2.3 Determination of Model Parameters

To determine the parameters of a mechanical model, horizontal force and moment shall be obtained by analytical, numerical or experimental means.

For simple tank geometry such as a rectangular tank as shown in Fig. 3, an analytical solution can be obtained.

![Fig. 3 Rectangular Tank.](image3)

**Fig. 3** Rectangular Tank.
Assuming the liquid in the tank is incompressible, inviscid and irrotational, liquid motion can be described by velocity potential, $\Phi$ which satisfies the following basic equations.

$$\nabla^2 \Phi = \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0$$  \hspace{1cm} (4)

$$\frac{\partial \Phi}{\partial t} + \frac{p}{\rho_c} + gz = 0$$  \hspace{1cm} (5)

where $p$ and $\rho_c$ are fluid pressure and fluid density, respectively.

When the tank is excited at an angular frequency $\omega$ by a small time-varying linear displacement $X_0$ along x axis the boundary conditions are specified as follows:

$$\left[ \frac{\partial \Phi}{\partial x} \right]_{x=\pm h} = 0 \quad \text{(at free surface)},$$  \hspace{1cm} (6)

$$\left[ \frac{\partial \Phi}{\partial z} \right]_{z=\pm \frac{h}{2}} = 0 \quad \text{(at tank bottom)},$$  \hspace{1cm} (7)

$$\left[ \frac{\partial \Phi}{\partial y} \right]_{y=\pm \frac{h}{2}} = X_0 \omega \exp(i\alpha) \quad \text{(at tank wall)},$$  \hspace{1cm} (8)

$$\left[ \frac{\partial \Phi}{\partial y} \right]_{y=\pm \frac{h}{2}} = 0 \quad \text{(at tank wall)}$$  \hspace{1cm} (9)

First, by solving the eigenvalue problem, natural frequencies are determined as follows:

$$\omega_n^2 = g \lambda_n \tanh(\lambda_n h), \quad \lambda_n = \frac{(2n-1)\pi}{a}$$  \hspace{1cm} (10)

Then, solving the basic differential equation to satisfy the boundary conditions, the velocity potential is obtained as:

$$\Phi = X_0 \omega \exp(i\alpha) \times \left\{ x + \sum_{\alpha=0}^\infty \frac{\omega^2}{\omega_\alpha^2 - \omega^2} \frac{4a}{(2n-1)^2} \cosh \left[ \frac{\lambda_n (z + h/2)}{2} \right] \sin (\lambda_n x) \right\}$$  \hspace{1cm} (11)

The unsteady liquid pressure is:

$$p = -\rho \frac{\partial \Phi}{\partial t} = -i\rho X_0 \omega \exp(i\alpha) \times \left\{ x + \sum_{\alpha=0}^\infty \frac{\omega^2}{\omega_\alpha^2 - \omega^2} \frac{4a}{(2n-1)^2} \cosh \left[ \frac{\lambda_n (z + h/2)}{2} \right] \sin (\lambda_n x) \right\}$$  \hspace{1cm} (12)

The horizontal force and moment in the rectangular tank excited horizontally are obtained as equations (12) through (14) in a similar manner.

$$F_{x,\text{exp}} = X_0 \left\{ 1 + 8a \sum_{\alpha=0}^\infty \frac{\tanh(\lambda_n h)}{\pi^2(2n-1)^2} \frac{\omega^2}{\omega_\alpha^2 - \omega^2} \right\}$$  \hspace{1cm} (13)

$$M_{x,\text{exp}} = hX_0 \left\{ \frac{1}{12} \frac{a}{h} + \sum_{\alpha=0}^\infty \frac{8(a/h)\tanh(\lambda_n h)}{\pi^2(2n-1)^2} \left[ \frac{1}{2} - \frac{2(a/h)}{\pi(2n-1)} \tan \left( \frac{\lambda_n h}{2} \right) + \frac{g}{\omega_\alpha^2} \frac{\omega^2}{\omega_\alpha^2 - \omega^2} \right] \right\}$$  \hspace{1cm} (14)

$$I_x = \frac{1}{12} m_r \left[ a^2 + h^2 \right] \left[ 1 + \frac{4}{1 + (a/h)^2} + \frac{768a/h}{\pi^2(2n-1)^2} \sum_{\alpha=0}^\infty \frac{\tanh [(2n-1)\pi h/2a]}{(2n-1)^2} \right]$$  \hspace{1cm} (15)

By equating equation (1) to the sum of the equations (10) and (12), and equation (2) to the sum of the equations (11) and (13), and equation (3) to equation (14), slosh mass, associated height and mass inertia of the disk are determined as follows:

$$m_s = \frac{8a \tan h(\lambda_n h)}{h \pi^2(2n-1)^2}$$  \hspace{1cm} (16)

$$H_s = \frac{4}{1 + (a/h)^2} + \frac{768a/h}{\pi^2(2n-1)^2} \sum_{\alpha=0}^\infty \frac{\tanh [(2n-1)\pi h/2a]}{(2n-1)^2}$$  \hspace{1cm} (17)

$$I_s = \frac{1}{12} m_r \left[ a^2 + h^2 \right] \left[ 1 + \frac{1}{1 + (a/h)^2} + \frac{768a/h}{\pi^2(2n-1)^2} \sum_{\alpha=0}^\infty \frac{\tanh [(2n-1)\pi h/2a]}{(2n-1)^2} \right]$$  \hspace{1cm} (18)

Remaining parameters, i.e., spring constant, fixed mass and its height are determined as below:

$$K_s = m_s \omega_n^2 = \frac{8g \tan h(\lambda_n h)}{h \pi^2(2n-1)^2}$$  \hspace{1cm} (19)
\[ m_i = m_j - \sum_{a \neq i} m_a \]  
(20)

\[ H_i = \sum_{a = 0}^{n} \frac{m_a}{m_0} H_a \]  
(21)

3. Coupled Equation of Floating Body Motion and Liquid Cargo Effects

3.1 Floating Body Motion without Liquid Cargo

The motion of floating body relative to its COG is described by the following equation in frequency domain.

\[ [-\omega^2 (M_G + M_{A_G}) + i\omega B_{GL} + C_G] X_G = F_{GL} \]  
(22)

where \( X_G \) is the floating body motion matrix, \( M_G \) is the mass matrix of the floating body, \( M_{A_G} \) is the hydrodynamic added mass matrix, \( B_{GL} \) is the hydrodynamic damping matrix, \( C_G \) is the hydrostatic restoring matrix, and \( F_{GL} \) is the hydrodynamic excitation force matrix due to the incoming and diffraction waves.

3.2 Coupled Floating Body Motion with Liquid Cargo

To obtain the equation of coupled motion of the floating body and the liquid cargo effects, the forces induced by the liquid cargo are added to the equation of the floating body motion. When there are multiple tanks, all tank forces are added together.

\[ [-\omega^2 (M_{BG} + M_{A_G}) + i\omega B_{GL} + C_G] X_G = \sum F_{GL} \]  
(23)

where \( M_{BG} \) is the mass matrix of the floating body (excluding liquid cargo) relative to its COG (including liquid cargo), \( M_{A_G} \) is the mass matrix of the floating body including liquid cargo, \( M_{A_G} \) is the hydrodynamic added mass matrix, \( B_{GL} \) is the hydrodynamic damping matrix, \( C_G \) is the hydrostatic restoring matrix, and \( F_{GL} \) is the hydrodynamic excitation force matrix relative to the local tank origin.

Here,

\[ m_4 = m_4 = 0, \quad m_5 = m_5 = M_G(x_{COG} - z_0), \quad m_{16} = m_{16} = M_G(y_{COG} - x_0), \quad m_{20} = m_{20} = M_G(y_{COG} - x_0), \quad m_{24} = m_{24} = M_G(z_{COG} - y_0), \quad m_{28} = m_{28} = M_G(z_{COG} - y_0) \]

\[ I_{11} = I_{11} + M_G \left[ (y_{COG} - x_0)^2 + (z_{COG} - y_0)^2 \right] \]
\[ I_{12} = I_{12} + M_G \left[ (y_{COG} - x_0)^2 + (z_{COG} - y_0)^2 \right] \]
\[ I_{13} = I_{13} + M_G \left[ (x_{COG} - z_0)^2 + (y_{COG} - x_0)^2 \right] \]
\[ I_{14} = I_{14} + M_G \left[ (x_{COG} - z_0)^2 + (y_{COG} - x_0)^2 \right] \]
\[ I_{15} = I_{15} + M_G \left[ (x_{COG} - z_0)^2 + (y_{COG} - x_0)^2 \right] \]
\[ I_{16} = I_{16} + M_G \left[ (x_{COG} - z_0)^2 + (y_{COG} - x_0)^2 \right] \]

\[ F_{FL} \] is tank liquid force matrix relative to local tank origin.

\[ F_{FL} = m_0 \omega^2 \times \]
\[ \begin{pmatrix}
1 + A_1 & 0 & 0 & 0 & B_1 & 0 \\
0 & 0 & 0 & -B_2 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & -B_3 & 0 & m_T & C_4 & \frac{1}{\omega^2} \\
B_4 & 0 & 0 & 0 & m_T & C_4 & \frac{1}{\omega^2} \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix} \]

where,

\[ A_1 = \frac{m_{14}}{m_2} \omega^2 \]
\[ A_2 = \frac{m_{24}}{m_2} \omega^2 \]
\[ B_1 = \frac{m_{16}}{m_2} \omega^2 \]
\[ B_2 = \frac{m_{17}}{m_2} \omega^2 \]
\[ C_4 = \frac{m_{28}}{m_2} \omega^2 \]
\[ D_4 = \frac{m_{29}}{m_2} \omega^2 \]

\[ I_{1} = I_{1} - I_{1d} \]
\[ I_{2} = I_{2} - I_{1d} \]

\( F_{TC} \) is coordinate transfer matrix of the tank liquid force from local tank origin to COG.

\[ F_{TC} = \begin{pmatrix}
0 & V_{F_{11}} & -F_{11} \mathbf{V} \\
V_{F_{11}} & F_{11} & -V_{F_{11}} \mathbf{V} \\
-F_{11} \mathbf{V} & -V_{F_{11}} \mathbf{V} & -F_{11} \mathbf{V} \\
\end{pmatrix} \]
where,
\[ \mathbf{F}_T^L = \begin{bmatrix} \mathbf{F}_{11}^{L1} & \mathbf{F}_{12}^{L2} \\ \mathbf{F}_{21}^{L1} & \mathbf{F}_{22}^{L2} \end{bmatrix} \]
\[ \mathbf{F}_{11}^{L1} = m_c \omega^2 \times \begin{bmatrix} 1 + A_1 & 0 & 0 \\ 0 & 1 + A_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \]
\[ \mathbf{F}_{12}^{L2} = m_c \omega^2 \times \begin{bmatrix} 0 & B_1 & 0 \\ -B_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \]
\[ \mathbf{F}_{21}^{L1} = m_c \omega^2 \times \begin{bmatrix} 0 & -B_2 & 0 \\ B_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \]
\[ \mathbf{F}_{22}^{L2} = m_c \omega^2 \times \begin{bmatrix} \frac{I_x}{m_y} + C_y + D_y \frac{1}{\omega^2} & 0 & 0 \\ 0 & \frac{I_y}{m_x} + C_x + D_x \frac{1}{\omega^2} & 0 \\ 0 & 0 & 0 \end{bmatrix} \]
\[ \mathbf{V} = \begin{bmatrix} 0 & -(z_{yG} - z_G) & y_{yG} - y_G \\ -(x_{yG} - y_G) & 0 & -x_{yG} - x_G \\ -(y_{yG} - y_G) & x_{yG} - x_G & 0 \end{bmatrix} \]

The forces induced by liquid cargo are finally divided into added mass part and hydrostatic restoring part.
\[ \mathbf{F}_T^L = \mathbf{F}_{11}^{L1} + \mathbf{F}_{12}^{L2} = \omega^2 \mathbf{M}_{1A,G} + \mathbf{C}_{L,G} \] (24)

And the equation of floating body motion coupled with the liquid cargo effects is obtained as follows;
\[ -\omega^2 \left( \mathbf{M}_{x,G} + \mathbf{M}_{A,G} + \sum \mathbf{M}_{1A,G} \right) + i \omega \mathbf{B}_G + \mathbf{C}_G - \sum \mathbf{C}_{L,G} \right] \cdot \mathbf{x}_G = \mathbf{F}_e^M \] (25)

4. Numerical Demonstration

4.1 Validation

In order to demonstrate the capabilities of the present method outlined in this paper, the moment generated by the rotational oscillation of rectangular tank and the RAO of sway and roll motion with and without the liquid cargo effects are calculated and compared with the results obtained by the experiment or calculation by other means.

4.1.1 Moment for rotational oscillation

One of the comparison targets of the moment for the rotational oscillation of the rectangular tank was obtained by Bosch et al.\(^{[12]}\) experimentally. Another was calculated by the 3D finite difference method (FDM) developed by Arai et al.\(^{[13]}\).

The parameters of rectangular tank with water for the validation are shown in Table 1 and Fig. 4. The results are presented using a non-dimensional form used by Bosch et al.\(^{[12]}\).

\[ M_{\text{sd-exp}} = \frac{M_{\text{exp}}}{\rho g B^2 L} \] (26)

Table 1 Parameters of rectangular tank for validation.

| Case   | h/B | h/H | s/B |
|--------|-----|-----|-----|
| Case 1 | 0.04| 0.06| 0.00|
| Case 2 | 0.32| 0.50| 0.00|

Fig. 4 Rectangular tank for validation.

(a) Case 1 (h/B=0.04) for rotational oscillation 1.9 deg.

(b) Case 1 (h/B=0.04) for rotational oscillation 0.1 deg.

(c) Case 2 (h/B=0.32) for rotational oscillation 1.9 deg.

Fig. 5 Moment for rotational oscillation of rectangular tank.
\[ \sigma = \omega \sqrt{\frac{B}{g}} \]  

(27)

where \( M_{\text{amp}}, \rho, g, \omega \) and \( L \) are moment amplitude, fluid density, gravity constant, angular frequency, and tank length, respectively.

Comparisons of the moment for the rotational oscillation of rectangular tank are shown in Fig. 5.

The moment in the vicinity of the sloshing natural frequency does not become as large as predicted by linear theory due to the non-linear liquid motion such as a train of small waves, hydraulic jump, etc. Fig. 5 (a) shows that the present method overestimates the moment calculated by the 3D FDM when the rotational oscillation is small. Fig. 5 (c) shows that for case of the intermediate water level, the present method reproduces the 3D FDM calculation better than that of low water level shown in Fig. 5 (a).

Actually, the simulation by the 3D FDM shows that even in the low and high frequency range apart from the sloshing natural frequency the non-linear liquid motion especially multi-modal behavior is observed. Therefore the moment calculated by the 3D FDM does not match exactly with the one calculated by the present method. However, it is also confirmed that the linear liquid motion is the major contributor.

### 4.1.2 RAO of sway and roll motion

Rocha et al.\(^8\) measured the 6-DOF FLNG motion coupled with and without the liquid cargo effects and calculated by the numerical method based on the linear potential theory (WAMIT). Their FLNG model is a barge type hull equipped with six rectangular tanks located inside the hull. The loading conditions corresponded to that all six tanks were filled with water up to 15%, 50% and 90% of the tank height.

For validation of this paper, the sway and roll motion for beam waves with 2 loading conditions of 15% and 50% are selected. The principal particulars and properties of the loading condition are presented in Tables 2 to 4. Fig. 6 shows the tank arrangement of the FLNG model.

As shown in Figs. 7 and 8, the numerical calculation by the present method is compared to the FLNG motion measured and calculated by Rocha et al. The RAO calculated by the present method is very close to that calculated by WAMIT.

While comparing the experimental measurement for the loading condition of 15%, the sway and roll motion around the sloshing natural period [Fig. 7 (c) and (d)] is not well reproduced by either the present method or WAMIT. As Rocha et al pointed out in their paper, this is due to the non-linear sloshing in the tanks and it is the limitation of the calculation method based on the linear potential theory. For the loading condition of 50%, the calculations reproduce the experimental measurement well.

The authors concluded that the sway and roll motion affected by the liquid cargo effects are reasonably evaluated by both the present method and WAMIT, except in case the effect of the non-linear sloshing is significant [Fig. 7 (c) and (d)].

One important observation is that the roll motion for the loading condition of 50% seems to be not affected much by the liquid cargo effects.

### Table 2 Principal particulars of FLNG

|                  |                |
|------------------|----------------|
| Length Overall   | 471 m          |
| Length BP        | 450 m          |
| Breadth          | 81 m           |
| Depth            | 38 m           |

### Table 3 Principal particulars of tank

|                   |                |
|-------------------|----------------|
| Length            | 58.41 m        |
| Width             | 45.10 m        |
| Depth             | 29.04 m        |
| Bottom elevation from keel | 5.00 m |

### Table 4 Loading condition

|                  |                |
|------------------|----------------|
| Draft            | 12.22 m, 16.60 m|
| Water level in tanks | 4.36 m (15%) 14.52 m (50%) |
| Displacement volume | 432,364 m³ 591,683 m³ |
| KG               | 22.99 m, 18.53 m |
| Roll Radius of Gyration (*) | 31.86 m 26.02 m |
| GMt              | 28.62 m, 23.19 m |
| Roll natural period without liquid cargo effects (**) | 15.2 s 14.0 s |
| Sloshing natural period of tank | 14.0 s 8.7 s |

(* Reference point of roll radius of gyration is COG of FLNG.
(**) Liquid cargo in tanks is treated as solid.

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Fig. 6 Tank arrangement of FLNG model (unit: m)

### 4.2 Discussion of Liquid Cargo Effects on FLNG motion

In order to identify effect of the liquid cargo on FLNG sway and roll motion, the equation of the sway and roll motion of the FLNG model is obtained from the equation (25).
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Fig. 7 Sway and Roll motion for loading condition of 15%.

(a) RAO of sway motion without liquid cargo effects.
(b) RAO of roll motion without liquid cargo effects.
(c) RAO of sway motion with liquid cargo effects.
(d) RAO of roll motion with liquid cargo effects.

Fig. 8 Sway and Roll motion for loading condition of 50%.

(a) RAO of sway motion without liquid cargo effects.
(b) RAO of roll motion without liquid cargo effects.
(c) RAO of sway motion with liquid cargo effects.
(d) RAO of roll motion with liquid cargo effects.
where, \( M \) is the mass of the floating body including the liquid cargo and is equal to \( pV \), \( p \) is the density of the surrounding water, \( V \) is the displacement volume, \( I_B^1 \) and \( G_M \) are the mass inertia and the transverse metacentric height of the floating body including the liquid cargo relative to its COG, \( \rho V B \) is the hydrodynamic added mass, damping and wave forces for the \( k \) and \( f \) components, respectively. The \( x_{24} \) and \( x_{44} \) are the sway and roll motion of the COG of the floating body, and \( j \) is the number of tanks. The term \( \sum_{j=1}^{6} m_j D_{ij} \) represents the free surface effect of the liquid cargo.

Further, the equation (28) is divided by \( \rho V w_2 a \omega^2 \) and the equation (29) is divided by \( \rho V B w_2 a \omega^2 \), and the non-dimensional equations of motion are obtained.

\[
-\left(1 + a^{22}\right) \dot{x}'_2 + i b^{22} \dot{x}'_2 - a^{244} + i b^{244} \dot{x}'_2 = F_{2}'\quad(30)
\]

\[
-\left(1 + a^{44}\right) \dot{x}'_4 + i b^{44} \dot{x}'_4 = \left( \frac{B \cdot G}{B} \right)^2 + a^{44} + i b^{44} \dot{x}'_4 = F_{4}'\quad(31)
\]

The non-dimensional hydrodynamic coefficients in equations (30) and (31) are as follows:

\[
a^{22} = \frac{m_{24}^2 + \sum_{j=1}^{6} m_j A_{ij}}{\rho V} - \frac{m_{24}^2 - \sum_{j=1}^{6} m_j A_{ij}}{\rho V} B_{ij} + B_{ij} + \frac{m_{24}^2 - \sum_{j=1}^{6} m_j A_{ij}}{\rho V} B_{ij} + B_{ij}
\]

\[
a^{44} = \frac{m_{44}^2 + \sum_{j=1}^{6} m_j A_{ij}}{\rho V} \left( \frac{B \cdot G}{B} \right)^2 + a^{44} + i b^{44}
\]

\[
= \frac{\sum_{j=1}^{6} m_j D_{ij}}{\rho VB_{ij} \omega^2}
\]

\[
x_2 = \frac{x_{24}}{w_2}, x_4 = \frac{x_{44}}{w_2}
\]

where, \( w_2 \) is the incident wave amplitude, \( B \) is the breadth of the FLNG model, \( R_{P1}^2 \) is the roll radius of gyration.

As shown in the equations (30) and (31), the sway and roll affect each other and the degree of sway-roll coupling is represented by the terms, \([-a^{244} + ib^{244}] - [-a^{424} + ib^{424}]\).

The absolute value of the non-dimensional terms expressed as below is used in the following section.

\[
A^{22} = \sqrt{\left(1 + a^{22}\right)^2 + \left(b^{22}\right)^2}
\]

\[
A^{44} = \sqrt{\left(1 + a^{44}\right)^2 + \left(b^{44}\right)^2}
\]

Figs 9 to 12 show the non-dimensional absolute values of the terms, \( A^{22}, A^{24}, A^{32}, A^{34}, A^{44} \), non-dimensional exciting forces, \( F_2' \) and \( F_4' \), RAOs of sway and roll excluding and including the liquid cargo effects in tanks, respectively. Two RAOs are presented: one is obtained with the sway-roll coupling and the other is obtained without the coupling.

For the sway and roll motion without the liquid cargo effects in tanks for the loading condition of 15%, the sway-roll coupling is not significant as shown in Fig. 9. The \( A^{22} \) and \( A^{32} \) terms [solid line in Fig. 9 (a) and (b)] are kept small relative to the other terms in all the period range and the RAOs of sway and roll are almost the same regardless of the sway-roll coupling [Fig. 9 (c) and (d)].

When the liquid cargo effects in tanks are added, the \( A^{24} \) and \( A^{42} \) terms [solid line in Fig. 10 (a) and (b)] become large in the whole period range, especially around the sloshing natural period. As the result, the sway-roll coupling is enhanced. This shall be paid attention in order to understand the coupled sway and roll motion with the liquid cargo effects.

The major contribution term for sway motion is the \( A^{22} \) term which is characterized by the mass. When the liquid cargo effects are considered, the \( A^{22} \) term varies significantly due to the sloshing in tanks [dotted line in Fig. 10 (a)]. The \( A^{22} \) term becomes extremely large at the sloshing natural period, 14.0 sec; the decoupled sway motion becomes very small [dotted line in Fig. 10 (b)]. Further, the equation (29) is divided by \( \rho V w_2 a \omega^2 \) and the equation (29) is divided by \( \rho V B w_2 a \omega^2 \), and the non-dimensional equations of motion are obtained.

\[
-\alpha \omega^2 \left( m_{24}^2 + \sum_{j=1}^{6} m_j A_{ij} \right) + i \omega b_{24}^2 \dot{x}_2 + \left( m_{24}^2 - \sum_{j=1}^{6} m_j A_{ij} \right) B_{ij} + i \omega b_{44}^2 \dot{x}_4 = F_{2}' \quad(30)
\]

\[
-\alpha \omega^2 \left( m_{44}^2 + \sum_{j=1}^{6} m_j A_{ij} \right) - i \omega b_{44}^2 \dot{x}_4 = \left( \frac{B \cdot G}{B} \right)^2 + \alpha \omega^2 \left( m_{44}^2 - \sum_{j=1}^{6} m_j A_{ij} \right) + i \omega b_{44}^2 \dot{x}_4 = F_{4}' \quad(31)
\]
Fig. 9 Sway and Roll motion without liquid cargo effects for loading condition of 15%.

(a) Non-dimensional term for sway.
(b) Non-dimensional term for roll.
(c) Sway motion.
(d) Roll motion.

Fig. 10 Sway and Roll motion with liquid cargo effects for loading condition of 15%.

(a) Non-dimensional term for sway.
(b) Non-dimensional term for roll.
(c) Sway motion.
(d) Roll motion.
Fig. 11 Sway and Roll motion without liquid cargo effects for loading condition of 50%.

Fig. 12 Sway and Roll motion with liquid cargo effects for loading condition of 50%.
The major contribution term for roll motion is the $A^{44}$ term which is characterized by the mass inertia and the restoring moment. When the liquid cargo effects are not considered, the $A^{44}$ term takes the minimum value at 15.2 sec and a roll resonant peak is observed [dotted line in Fig. 9 (b) and (d)].

When the liquid cargo effects are considered, the $A^{44}$ term has the local minimum value twice, at 13.0 sec and 18.4 sec. In addition, the $A^{44}$ term has a peak value at the sloshing natural period, 14.0 sec; the decoupled roll motion becomes very small [dotted line in Fig 10 (b) and (d)]. Accordingly, the two peaks at 13.0 sec and 18.4 sec and local minimum at 14.0 sec are observed in roll motion decoupled from sway motion [dotted line in Fig. 10 (d)]. When the coupling with sway motion is considered, the period corresponding to the peaks and the local minimum are shifted [solid line in Fig. 10 (d)].

The free surface effect shall be paid more attention. Due to the free surface effect, the restoring moment is smaller and as the result, roll motion decoupled from sway motion becomes larger. And then the sway coupling reduces the roll motion.

For the loading condition of 50% shown in Figs. 11 and 12, exactly the same reasoning holds; however appearance is different due to the difference of the natural period and degree of sloshing influence on roll motion. Because the degree of sloshing influence on roll motion is not so strong, the influence of roll motion on sway motion is not significant. But note that the roll motion is still affected significantly by the liquid cargo effects such as free surface effect and enhancement of the sway-roll coupling.

5. Conclusions

The calculation method based on the mechanical model is presented in this paper in order to estimate and understand the influence of the liquid cargo effects on the FLNG motion.

- The liquid cargo effects are represented by the spring-mass type mechanical model and they are coupled with motion of floating body.
- The parameters of a mechanical model for the liquid in a rectangular tank are determined analytically.
- The present method outlined in this paper is validated by comparing the moment for the rotational oscillation of the tank rectangular tank measured by Bosch et al and as calculated by the 3D FDM.
- The present method is also validated by comparing with the FLNG motion measured and calculated by Rocha et al. The sway and roll motion affected by the liquid cargo effects are reasonably evaluated, except in case that the effect of the non-linear sloshing is significant.
- The liquid cargo effects enhance the sway-roll coupling. This shall be paid attention in order to understand the coupled sway and roll motion with the liquid cargo effects.
- A rational explanation for observation of the peak and the local minimum in sway and roll RAO due to the effect of liquid cargo is provided by taking into account of the effect of the mass, mass inertia, restoring moment and sway-roll coupling.

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