A Principle to Determine the Number

\( (3 + 1) \) of Large Spacetime Dimensions

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ABSTRACT

We assume that our universe originated from highly excited and interacting strings with coupling constant \( g_s = \mathcal{O}(1) \). Fluctuations of spacetime geometry are large in such strings and the physics dictating the emergence of a final spacetime configuration is not known. We propose that, nevertheless, it is determined by an entropic principle that the final spacetime configuration must have maximum entropy for a given amount of energy. This principle implies, under some assumptions, that the spacetime configuration that emerges finally is a \((3 + 1)\)–dimensional FRW universe filled with \( w = 1 \) perfect fluid and with \( 6 \)–dimensional compact space of size \( l_s \); in particular, the number of large spacetime dimensions is \( d = 3 + 1 \). Such an universe may evolve subsequently into our universe, perhaps as in Banks – Fischler scenario.
In superstring theory the number of critical dimensions of the space-time is $9 + 1$ whereas the spacetime in the observed universe is $3 + 1$ dimensional. It is expected that 6 spatial dimensions will compactify to string size by some mechanism, resulting in a $3 + 1$ dimensional universe. One proposal for such a mechanism [1] involves winding modes of the strings and other recent ones [2] involve various $D$-branes.  

1. Consider an FRW universe. In string theory, it is described using low energy effective action for zero modes. In the past, as time decreases, the temperature of the universe eventually reaches the string scale at which the universe is to be described by stringy variables. In this context a stringy correspondence principle, analogous to that of Horowitz and Polchinski [4, 5], is formulated for the evolution of the state of the universe [6]. Briefly, according to this principle, at temperatures lower than string scale the universe state evolves as in FRW cosmology whereas at higher temperatures it evolves as highly excited strings. At the transition, the entropies and energies in these two descriptions differ by numerical factors of $O(1)$ as shown in [6] in the weak coupling limit where the string coupling constant $g_s \ll 1$.

This transition can be thought of as the universe turning into highly excited strings or, in reverse which is not well understood, as highly excited strings turning into an expanding FRW universe. Here, we assume that a similar picture holds for our universe also where the string coupling constant $g_s = O(1)$. Accordingly then, as one follows the evolution in the past, our universe turns into highly excited and, since $g_s = O(1)$, highly interacting strings; conversely, our universe originated from such highly excited and interacting strings.

However, in a system composed of highly excited strings with a large number of interacting degrees of freedom, fluctuations of spacetime geometry are large and associated spacetime concepts are not well defined. These fluctuations may be pictured as spacetime configurations emerging and dissolving back into strings from time to time. Finally a spacetime configuration, stable against dissolving back, must emerge leading to our universe. This is clearly a necessary requirement for the assumption here that our universe, where spacetime is well defined, originated from highly excited and interact-

1In the context of 11-dimensional supergravity with Freund–Rubin type compactifications, it is shown using quantum cosmology techniques that observed spacetime must be 4-dimensional [3].
The physics dictating the course of the emergence of such a final configuration is beyond our grasp. In this paper we propose that, nevertheless, it is determined by an entropic principle that the final spacetime configuration must have maximum entropy for a given amount of energy. This is a standard principle generically applicable for systems containing a large number of interacting degrees of freedom.

We then study a consequence of the entropic principle and show that it implies, under a few further assumptions, that the spacetime configuration that emerges finally is a \((3 + 1)\)–dimensional FRW universe filled with \(w = 1\) perfect fluid\(^2\) and with \(6\)–dimensional compact space of size \(l_s\); in particular, the number of large spacetime dimensions is \(d = 3 + 1\). Such an universe may be taken to evolve subsequently into our universe, perhaps as in Banks–Fischler scenario \([7]\).

This paper is organised as follows. In section 2 we present the relevant expressions for an FRW universe. In section 3 we present our proposal and study a consequence of the entropic principle. In section 4 we conclude with a few remarks.

\section{Consider the evolution of a \(d\)–dimensional spatially flat FRW universe containing a perfect fluid with density \(\rho\) and pressure \(p = w\rho\). We assume that such an universe originated from a ten dimensional superstring theory compactified on a \(p = (10 - d)\)–dimensional compact space. With \(\hbar = c = 1\), the \(d\)–dimensional Planck length \(l_{pl}\) is then given by

\[
\left(\frac{l_{pl}}{l_s}\right)^{d-2} \approx \frac{g_s^2 l_s^8}{V_p} \tag{1}
\]

where \(g_s\) is the string coupling constant and \(= \mathcal{O}(1)\) for our universe, \(l_s\) is the string length, \(V_p\) is the volume of the compact space and, here and in the following, \(\simeq\) denotes that numerical factors of \(\mathcal{O}(1)\) are omitted. If the compact space is \(e.g.\) toroidal with sizes \(L_1, \ldots, L_p\) then \(V_p \approx \prod_l^p L_i\).

The parameters of this FRW universe are \(w\), \(V_p\), and \(d\) and their ranges are restricted. The standard energy conditions imply that \(-1 \leq w \leq 1\). For toroidal compactification, T-duality symmetry of the string theory implies

\footnote{which can also be thought of as black hole fluid \([7]\)}
that $V_p > l_p^3$. We assume this to be the case in general also. In superstring theory $d \leq 10$. Also, gravity plays an important role in what follows and, hence, we further assume that $d \geq 4$ since gravity is not a propagating degree of freedom in lower dimensions.

The relevant line element $ds$ is given, in the standard notation, by

$$ds^2 = -dt^2 + a^2 \left( dr^2 + r^2 d\Omega_{d-2}^2 \right).$$  \hspace{1cm} (2)

Solving the equations of motion one gets, with $\alpha = \frac{2}{(d-1)(1+w)}$,

$$a(t) = \left( \frac{t}{l_{pl}} \right)^{\alpha}, \quad \rho(t) \simeq \frac{1}{l_{pl}^d} \left( \frac{a_{pl}}{a} \right)^{(d-1)(1+w)}. \hspace{1cm} (3)$$

The constant $w \leq 1$. For example, $w = \frac{1}{d-1}$ for radiation field whereas $w = 1$ for a massless scalar field. Such fields are present in string theory. When the universe contains many perfect fluids with different $w$‘s then that with the highest value of $w$ dominates in the past when the temperature $T$ is high.

It is natural to take the size of the universe to be given by the size of its horizon which encompasses the maximum region within which causal contact is possible. We do so in the following. The entropy $S$ and the energy $E$ of the universe can then be defined to be those contained within its horizon and are given by

$$S = \sigma V_{d-1} r_H^{d-1}, \quad E = \rho V_{d-1} L_H^{d-1}$$

where $\sigma = \frac{(\rho + p)}{T} a^{d-1}$ is the constant comoving entropy density, $V_n$ is the volume of an unit $n$-dimensional ball, and

$$r_H = \int_0^t \frac{dt}{a} = \frac{l_{pl}}{(1-\alpha)a_{pl}} \left( \frac{t}{l_{pl}} \right)^{1-\alpha}, \quad L_H = r_H a = \frac{t}{1-\alpha}$$

are the comoving coordinate and the physical size of the horizon respectively. Written in terms of $t$, the expressions for $S$ and $E$ become

$$S = C_S \left( \frac{t}{l_{pl}} \right)^{(d-1)(1-\alpha)}, \quad E = C_E \left( \frac{t}{l_{pl}} \right)^{d-3}$$

where $C_S$ and $C_E = \mathcal{O}(1)$ are numerical coefficients. Holographic principle \[8\] implies that $C_S = \mathcal{O}(1)$ also. See \[6\] more detailed expressions. The
entropy $S$ as a function of energy $E$ is then given, with $b = \frac{d-3+(d-1)\omega}{(d-3)(1+w)}$, by

$$S(E) \simeq (l_{pl}E)^b \simeq \left(\frac{g_s^2 l_s^8}{V_p}\right)^\frac{b}{d-2} E^b.$$  \hspace{1cm} (6)

3. The FRW description of the universe given above is obtained using low energy effective action for string zero modes. In the past, as time decreases, the temperature of the universe increases and reaches the string scale $\simeq \frac{1}{l_s}$. At such a scale a large number of higher modes of strings are excited and their effects must be included. The FRW description of the universe given above is then to be replaced by a stringy description.

In this context, a stringy correspondence principle is formulated for the evolution of the state of the universe [6], in analogy with that of Horowitz and Polchinski for black hole states [4, 5]. According to this principle, there is a correspondence between a FRW universe state and a highly excited string state. When the temperature is lower than string scale the universe state evolves as in FRW cosmology and is described by FRW variables. When the temperature is higher than string scale the universe state evolves as highly excited strings and is to be described by stringy variables. At the transition, the entropies and energies in these two descriptions differ by numerical factors of $\mathcal{O}(1)$.

This is shown for the transition from FRW description to the stringy one in the weak coupling limit where $g_s \ll 1$. The transition can be thought of as the universe turning into highly excited strings as one follows its evolution in the past. The reverse transition, namely highly excited strings turning into an expanding FRW universe, is not well understood. See [6] for a few remarks on this transition and [9] for a detailed study of the transition from strings to black hole.

Here, we assume that a similar picture holds for our universe also where the string coupling constant $g_s = \mathcal{O}(1)$. Accordingly then, as one follows the evolution in the past, our universe turns into highly excited and, since $g_s = \mathcal{O}(1)$, highly interacting strings; conversely, our universe originated from such a highly excited and highly interacting strings.

The dynamics of such highly excited and interacting strings are difficult to study at present. Some of the difficulties are that all excited modes and
their non trivial interactions must be included; the standard low energy effective actions are not applicable; fluctuations of spacetime geometry, described essentially by string zero modes, are large and associated spacetime concepts are not well defined.

These spacetime fluctuations may be pictured as some spacetime configuration emerging and after some time dissolving back into strings, then some other configuration emerging and dissolving back, and so on. A spacetime configuration here is to be parametrised by e.g. the number and sizes of spacetime dimensions, constituent fields in the spacetime, etcetera. These parameters are different for different configurations. Finally a spacetime configuration emerges from strings which is stable against dissolving back, and whose subsequent evolution, described by an effective action, will lead to our universe. ³ That such a final configuration, with well defined spacetime concepts, must emerge from strings is clearly a necessary requirement for the assumption here that our universe, where spacetime is well defined, originated from highly excited and interacting strings.

Although the physics dictating the course of the emergence of such a final configuration is beyond our grasp, it may be possible to determine the final configuration itself. We propose that it is determined by the following entropic principle:

\[ \text{The spacetime configuration that emerges finally from highly excited and interacting strings is the one which has maximum entropy for a given amount of energy.} \]

Here and in the following it is assumed that energy \( E \gg \frac{1}{l_s} \).

This is the standard entropic principle ⁴ and is applicable if the system is sufficiently ergodic and mixing so that various configurations can be sampled and the maximum entropic one picked out. This is generically the case if the system contains a large number of interacting degrees of freedom. The present case of highly excited and interacting strings with coupling constant \( g_s = \mathcal{O}(1) \) is indeed a system with a large number of interacting degrees of freedom. Hence, it is likely to be sufficiently ergodic and mixing so that

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³This picture is loosely analogous to a solid structure emerging from a liquid.

⁴In statistical mechanics where entropic principle is commonly applied, if microcanonical and canonical ensembles are equivalent then maximising entropy translates into minimising free energy; otherwise microcanonical ensemble, and therefore maximising entropy, is more fundamental.
entropic principle can be applied to it. Assuming this to be the case, we now apply the entropic principle and study a consequence.

Assume that a \( d \) -dimensional FRW universe described earlier is the spacetime configuration that emerges finally from the strings whose coupling constant \( g_s = \mathcal{O}(1) \) and string length is \( l_s \). For a given energy \( E \), the entropy \( S(E) \) of the universe is given in equation (6). The parameters of this spacetime are \( w \), \( V_p \), and \( d \) and their ranges are restricted. The standard energy conditions imply that \( w \leq 1 \). As per our assumptions, the T-duality symmetry of the string theory implies \( V_p > \sim l_p^4 \); and \( d \geq 4 \) since gravity is not a propagating degree of freedom in lower dimensions. According to the entropic principle, the values of these parameters will be such as to maximise \( S(E) \) for a given \( E \). It can be easily seen that the values which maximise \( S(E) \) are

\[
w = 1 , \quad V_p \simeq l_s^p , \quad d = 4
\]

and that the maximum entropy \( S_{\text{max}}(E) \) is

\[
S_{\text{max}}(E) \simeq l_p^2 E^2 \simeq g_s^2 l_s^2 E^2 . \quad (7)
\]

For open or closed FRW universe, or for an universe containing other perfect fluids also such as radiation \( (w = \frac{1}{d-1}) \) or a cosmological constant \( (w = -1) \), the \( w = 1 \) perfect fluid dominates the universe in the past. Therefore, to the leading order, its entropy \( S(E) \) is still given by that for \( w = 1 \) fluid, namely by equation (6) with \( w = 1 \). Hence, the entropic principle can not determine whether the universe that emerges finally is flat, open, or closed or the details of its subleading matter contents.

In reference [7], Banks and Fischler present a detailed scenario where an universe filled with \( w = 1 \) perfect fluid, also thought of as black hole fluid, evolves into radiation dominated FRW universe whose subsequent evolution proceeds as in the standard cosmology. Perhaps then the above \( (3 + 1) \) -dimensional FRW universe with \( w = 1 \) perfect fluid, which is similar to the one studied in [7] but is obtained here as a consequence of entropic principle, may also be taken to evolve as in Banks – Fischler scenario into radiation dominated universe and then subsequently into our universe.

Note that gravity plays an important role. In the presence of gravity, a space filled with \( e.g. \) radiation fluid \( (w = \frac{1}{d-1}) \) evolves as an FRW universe,
with the entropy $S$ and the energy $E$ contained within horizon related as in equation (6). In the absence of gravity, the entropy $S$ and the energy $E$ of radiation fluid are related as

$$S(E) \simeq (LE)^{d-1}$$

where $L$ is the spatial size. Entropic principle, *i.e.* maximising $S(E)$ with respect to the parameters $d \leq 10$ and $L$, would then give $d = 10$ and $L \to \infty$. This may perhaps be the case for free, or weakly interacting, strings where the coupling constant $g_s = 0$, or $g_s \ll 1$, but applying entropic principle in this context is questionable because the interactions are absent, or arbitrarily weak.

We are not aware of any other spacetime configuration $^6$ where gravity is present and whose entropy $S(E)$ is greater than $S_{\text{max}}(E)$ given in equation (7). If we assume that no such configuration exists then it follows that the entropic principle implies that the spacetime configuration that emerges finally from a highly excited and interacting strings is a $(3 + 1)$ - dimensional FRW universe filled with $w = 1$ perfect fluid and with 6 - dimensional compact space of size $l_s$. Such a spacetime would then evolve into our universe, perhaps as in Banks – Fischler scenario. In particular, the entropic principle would thus have determined the number, $d = 3 + 1$, of large spacetime dimensions.

4. We have proposed here that the spacetime configuration that emerges finally from highly excited and interacting strings is determined by the entropic principle. This is a standard principle and is generically applicable for systems containing a large number of interacting degrees of freedom.

Highly excited and interacting strings is a system with a large number of interacting degrees of freedom justifying thereby the application of entropic principle. The interaction effects are strong since $g_s = \mathcal{O}(1)$ and one may therefore expect that various spacetime configurations are sampled efficiently; that lower entropic configurations are short lived; and that the spacetime configuration which emerges finally is one of maximum entropy. The entropic principle implies, under the assumptions mentioned earlier, that the spacetime configuration that emerges finally is a $(3 + 1)$ - dimensional

$^6$The obvious case of black hole fluid can be thought of as equivalent to $w = 1$ fluid [7].
FRW universe filled with $w = 1$ perfect fluid and with 6-dimensional compact space of size $l_s$; in particular, the number of large spacetime dimensions is $d = 3 + 1$.

However, the entropic principle does not give the details of the dynamical features involved such as how the spacetime configurations emerge from and dissolve back into strings; how various configurations are sampled; the ‘life time’ of lower entropic configurations; or, the time scale over which the maximum entropic configuration emerges finally.

One may try to understand such details in the weak coupling limit where $g_s \ll 1$ and where perturbative techniques may be applied. But, such an understanding may not be possible because one usually assumes a background spacetime in a perturbative formulation and also because the interactions are arbitrarily weak when $g_s \ll 1$ and applying entropic principle may then be questionable. One will then need techniques applicable when $g_s = \mathcal{O}(1)$.

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