The drag of photons by electric current in quantum wells

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The flow of electric current in quantum well breaks the space inversion symmetry, which leads to the dependence of the radiation transmission on the relative orientation of current and photon wave vector, this phenomenon can be named current drag of photons. We have developed a microscopic theory of such an effect for intersubband transitions in quantum wells taking into account both depolarization and exchange-correlation effects. It is shown that the effect of the current drag of photons originates from the asymmetry of intersubband optical transitions due to the redistribution of electrons in momentum space. We show that the presence of dc electric current leads to the shift of intersubband resonance position and affects both transmission coefficient and absorbance in quantum wells.

I. INTRODUCTION

A wide variety of optical and transport phenomena can be observed in semiconductor structures. Often the determining factor in whether a particular phenomenon can be observed in a specific structure is the symmetry of this structure. The presence of heterointerfaces in nanostructures by itself leads to a reduction of spatial symmetry in comparison with bulk materials\textsuperscript{[1]} and allows the emergence of new effects absent in bulk semiconductors. The symmetry of the structure can be further controlled and lowered in various ways, e.g. deformation, a gradient of temperature, magnetic field, or dc electric current. The latter can lead to a number of electro-optical effects, such as current-induced optical activity\textsuperscript{[2–3]} or second harmonic generation\textsuperscript{[4–7]}, which are observed both in bulk and low-dimensional structures.

In the present manuscript, we theoretically study the variation of the refractive index induced by dc electric current linear in current magnitude for the resonant optical transition between subbands in quantum well. The in-plane electric current in quantum well breaks inversion-symmetry and leads to nonequivalence of direction along and against the current. As a result of this, the dielectric function of the quantum well can change depending on the orientation of the radiation wave vector with respect to the current direction. This phenomenon is the opposite to the well-known photon drag effect of electrons: the generation of a direct electric current caused by the absorption of radiation due to the transfer of photon momentum to the free charge carriers. Photon drag effect was widely studied in both bulk semiconductors\textsuperscript{[8–11]} and quantum well\textsuperscript{[12–16]} and used for characterization of semiconductor structures kinetic properties as well as ultrafast infrared detectors\textsuperscript{[17–19]}. By analogy, the effect under study can be named the current drag of photons (CDOP), since optical path length of radiation transmitted through quantum well changes in presence of dc current as if photons are “dragged” by electrons. A similar phenomenon caused by hole current at the optical transition between light-hole and heavy-hole subbands was previously discovered in bulk Ga\textsubscript{1–x}In\textsubscript{x}As\textsuperscript{[20]}. We start with the microscopic model to demonstrate physics behind the CDOP. The geometry of the problem is shown in Fig. 1. We consider direct optical transitions between the first and second subbands of the quantum well. For such transitions, the radiation frequency typically is in the infrared region. Direct intersubband optical transitions are induced by the oblique and p-polarized incident radiation, so its electric field has both in-plane and perpendicular to the quantum well components. Dc electric current affects the radiation transition through the quantum well, which results in the dependence of the $E_\text{r}(t)$ and $E_\text{t}(t)$ on the current magnitude and direction for the fixed angle of incidence. To explain the reason for this dependence, we first turn to a formal description of the electronic states in the quantum well. The Hamiltonian of electron in quantum well is given by

$$
\hat{H}_0 = \frac{\hat{p}^2}{2m^*} + U(z),
$$

where $m^*$ is the electron effective mass, $U(z)$ is barrier potential. The solution of this well-known Shrödinger equation gives a series of electron subbands energies

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Figure schematically demonstrates the geometry of the problem. Incident radiation described by electric field $E(t)$ is partially reflected and transmitted from quantum well with dc electric current, which are described by $E_r(t)$ and $E_t(t)$, respectively.}
\end{figure}
\[ E_i(k) = E_i^{(0)} + \hbar^2 k^2/(2m^*) \] with corresponding wave functions \(|i, k\rangle\), in coordinate representation

\[ \langle r | i, k \rangle = \Phi_{i,k}(r) = \varphi_i(z) e^{ikr} \frac{1}{\sqrt{S}} , \]

where \( k \) is in-plane electron wave vector and \( S \) is the surface area of the quantum well.

Radiation induces dipole moment along \( z \)-direction in quantum well oscillating with the frequency of the incident electromagnetic wave. Dipole moment density along \( z \)-direction is given by

\[ \Pi_z = 2 \sum_k f_k \Pi_{k'k} , \]

where \( f_k \) is electron distribution function and \( \Pi_{k'k} \) contribution from individual transitions of electrons with initial state in the first subband with wave vector \( k \) and final state in the second subband with wave vector \( k' \). For the direct optical transition the initial electron wave vector \( k \) differs from the final \( k' \) by the in-plane component of radiation wave vector \( q \parallel \) due to in-plane momentum conservation law. The contributions to the dipole moment induced by optical transitions is proportional to

\[ \Pi_{k'k} \propto \frac{e z_{21}}{E_2(k + q \parallel) - E_1(k) - \hbar \omega - i\hbar \Gamma} , \]

where \( z_{21} \) is matrix element of coordinate between first and second subband, \( e \) is electron charge, \( \hbar \omega \) is photon energy and \( \Gamma \) is optical transition dephasing rate. The difference between initial energy of electron and final is given by

\[ E_2(k') - E_1(k) = E_{21} + \frac{\hbar^2}{2m^*} q_{\parallel}^2 + \frac{\hbar^2}{m^*} q_{\parallel} k \],

where \( E_{21} = E_2^{(0)} - E_1^{(0)} \) is energy difference between the ground and second subbands, we assume that linewidth \( \hbar \Gamma \) is much smaller than \( E_{21} \). Last term of the right-hand side of Eq. (4) shows that the contribution of the transition to the \( \Pi_z \) depends on the relative direction of wave vectors \( k \) and \( q \parallel \). If we set the direction of \( q \parallel \) along \( x \)-axis the values of \( \Pi_{k'k} \) will depend on the sign of \( k_x \) which is schematically shown in Fig. 2(a) by thick and dashed blue arrows, thus there is a definite asymmetry of optical transitions.

The physics behind CDOP can be most clearly demonstrated for the case of low temperatures for degenerated electron gas described by the Fermi step function. For spin degeneracy, the initial and second subband, we assume that linewidth \( \Gamma \) is much smaller than \( E_{21} \). Last term of the right-hand side of Eq. (3) shows that the contribution of the transition to the \( \Pi_z \) depends on the relative direction of wave vectors \( k \) and \( q \parallel \). If we set the direction of \( q \parallel \) along \( x \)-axis the values of \( \Pi_{k'k} \) will depend on the sign of \( k_x \) which is schematically shown in Fig. 2(a) by thick and dashed blue arrows, thus there is a definite asymmetry of optical transitions.

The Hamiltonian of the electrons excited by electromagnetic wave is given by

\[ \hat{H} = \hat{H}_0 + \hat{V} e^{-i\omega t} + \hat{V}^\dagger e^{i\omega t} , \]

where \( \hat{V} \) is perturbation induced by incident radiation. By solving equation for density matrix one obtains for non-diagonal component \( \rho_{21}(k) = \gamma_{21}(k) \exp(-i\omega t) \), where

\[ \gamma_{21}(k) = \frac{-i(h \Delta \omega + \hbar \Gamma)}{\hbar \Delta \omega - i\hbar \Gamma} f^{(0)}(k) , \]

\[ h \Delta \omega = E_2(k + q \parallel) - E_1(k) - \hbar \omega \] is the offset frequency, we assume that without perturbation caused by radiation the second subband is completely empty and all electrons are in the first subband. The interaction of electrons confined in quantum well with radiation causes oscillations of both electronic charge given by

\[ \delta \rho_e(r) = e \varphi_1(z) \varphi_2(z) \tau |q\rangle \langle r| e^{-i\omega t} + c.c. , \]

and current along \( z \)-direction the main contribution to which has the form

\[ j_z = \frac{-i\hbar e}{2m^*} \Phi(z) \gamma e^{i\eta} r e^{-i\omega t} + c.c. , \]

and current along \( x \)-axis, while dc current may have arbitrary direction. For p-polarized electromagnetic wave nonzero components of electric and magnetic field are \( E_x \), \( E_z \) and \( H_y \).
and the fields themselves have the form
\[ E(\mathbf{r}, t) = (\mathbf{a}_z E_z(z) + \mathbf{a}_x E_x(z))e^{i\mathbf{q} \cdot \mathbf{r} - i\omega t} + \text{c.c.}, \]
\[ H(\mathbf{r}, t) = \mathbf{a}_y H_y(z)e^{i\mathbf{q} \cdot \mathbf{r} - i\omega t} + \text{c.c.}. \]
Maxwell equations for the components of electric induction, electric field and magnetic field are given by
\[ iq_1 D_x(z) + \partial_z D_y(z) = 4\pi e \varphi_1(z) \varphi_2(z) \mathrm{e}^\gamma, \]
\[ \partial_z E_x(z) - iqE_z(z) = \frac{i\omega}{c} E_y(z), \]
\[ iq_1 H_y(z) = \frac{4\pi}{c} \mathbf{e} \Phi(z) \mathrm{e}^\gamma - \frac{i\omega}{c} D_z(z), \tag{9} \]
electric induction and electric field are related by \( D = \varepsilon_i \varepsilon E \), where \( \varepsilon_i \) is permittivity of medium, \( \varepsilon_1 \) corresponds to the barrier and \( \varepsilon_2 \) is dielectric function of the material of the quantum well. For simplicity we assume that the quantum well has infinite barriers, such that the wave functions do not penetrate into the barriers. In this case straightforward calculation of (9) yields equation for \( E_z(z) \)
\[ \frac{2\pi e^2}{m^* \varepsilon_2} \left( \frac{\partial^2}{\partial z^2} + i\varepsilon_2 \frac{\omega^2}{c^2} \right) \Phi(z) + \left( \frac{\partial^2}{\partial z^2} + k_{2z}^2 \right) E_z(z) = 0, \tag{10} \]
where \( k_{2z}^2 = \frac{\varepsilon_2}{c^2} - q_0^2 \) is radiation wave vector along \( z \)-axis. Electric field outside of the quantum well is described by
\[ E_z(z) = E_0 e^{i k_{2z} z} + R E_0 e^{-i k_{2z} z}, \quad z < 0 \]
\[ E_z(z) = T E_0 e^{i k_{1z} z}, \quad z > a \tag{11} \]
where \( a \) is quantum well width, \( R \) and \( T \) are reflection and transmission coefficients, respectively. This coefficients allow to obtain the quantum well absorbance, which is given by \( \eta = 1 - |T|^2 - |R|^2 \). Electric field inside quantum well for \( 0 < z < a \) can be found using Green’s function method
\[ E_z(z) = A E_0 e^{i k_{2z} z} + B E_0 e^{-i k_{2z} z} - \int dz' G(z - z') \frac{2\pi e^2}{m^* \varepsilon_2} \left( \frac{\partial^2}{\partial z^2} + i\varepsilon_2 \frac{\omega^2}{c^2} \right) \Phi(z'), \tag{12} \]
where \( G(z - z') = e^{i k_{2z} |z - z'|} / (2i k_{2z}) \) is Green’s function, coefficients \( A \) and \( B \) describe general solution and found from boundary conditions at \( z = 0 \) and \( z = a \), which are \( \varepsilon_1 E_z(z) = \partial_z E_z(z) \) being continuous. By solving (10) using (11) and (12) one obtains, that reflection and transmission are given by
\[ T = 1 + R = 1 + \frac{i\pi e^2}{k_1 m^* \varepsilon_2} E_0 q_0^2 \int_0^a dz e^{i k_{2z} \Phi(z)}, \tag{13} \]
here the presence of electric current in the quantum well is taken into account in \( \Phi \). Latter is found by summing over all \( k \) of (11), where the part of the Hamiltonian describing perturbation is given by
\[ \hat{\mathcal{V}} = \frac{\epsilon}{c} \frac{\partial^2 A_z(z) + A_z(z) \partial^2}{2m^*} + \frac{\partial U_{xc}[n]}{\partial n}, \tag{14} \]
where first term of the right-hand side stands for interaction of electrons with alternating electric field and second term takes into account exchange-correlation effects. Here we chose a gauge \( A_z(z) = c E_z(z) / (i\omega) \), where electric potential is static and interaction of electrons is described by the means of vector potential. Equation (14) takes into account depolarization effect (see Refs.19,20), since \( E_z(z) \) consists not only of the field of incident radiation, but also of the field induced by oscillations of electron density along \( z \)-direction. \( U_{xc}[n(r)] \) in the Eq. (14) is the exchange-correlation energy in the local density approximation (for details see Refs.19,21,22), \( n(r) \) is electron gas density and \( \delta n(r) = \rho_\delta(r) / e \) is oscillating part of electron density. Self consistent equation for \( \gamma \) has the form
\[ \gamma = \sum_k 2 \frac{f^{(0)}(k)}{\hbar \Delta \omega - i\hbar \Gamma} \times \left[ \frac{\hbar c}{2m^* \omega} \left( \frac{e_1}{e_2} E_0 \int_0^a dz \Phi(z) + \frac{2\pi e^2}{m^* \varepsilon_2} \int_0^a dz \Phi^2(z) \right) - \frac{\pi}{a} \int_0^a dz \varphi_1^2(z) \varphi_2^2(z) \frac{\partial U_{xc}[n(z)]}{\partial n} \right]. \tag{15} \]
To describe electron distribution function in the presence of dc electric current we use drift velocity ansatz
\[ f(k) = f^{(0)}(k - k_{dr}) = \frac{1}{\hbar^2 (k - k_{dr})^2 - E_F} - \frac{2m^*}{\hbar^2 T} + 1, \tag{16} \]
where \( E_F \) is the Fermi energy, \( T \) is temperature in energy units, \( N_e \) total electron gas density, \( \hbar k_{dr} = m^* v_{dr} \) and \( v_{dr} \) is electron gas drift velocity. The relation between drift velocity and electric current is given by \( j = e N_e v_{dr} \). Leaving leading correction terms in denominator, which are linear both in \( q_0 \) and \( v_{dr} \) we obtain
\[ \gamma = \frac{\hbar c}{2m^* \omega} E_0 \frac{e_1}{e_2} N_e \int_0^a dz \Phi(z) \left( \frac{\partial^2}{\partial z^2} + i\varepsilon_2 \frac{\omega^2}{c^2} \right) \Phi(z'), \tag{17} \]
where \( E_{21}^2 = E_{21}^2(1 + \alpha - \beta) \) is shifted resonance energy,
\[ \alpha = -\frac{8\pi N_e e^2}{e^2 E_{21}} \int_0^a dz \left( \int_0^z dz' \varphi_1(z') \varphi_2(z') \right)^2 \tag{18} \]
describes depolarization effect20 and
\[ \beta = \frac{2N_e}{E_{21}} \int_0^a dz \varphi_1^2(z) \varphi_2^2(z) \frac{\partial U_{xc}(n)}{\partial n} \tag{19} \]
exchange-correlation effect. Finally substitution of (17) into Eq. (13) leads to the transmission coefficient given by

\[ T = 1 + \frac{e^2\hbar}{2m^*c}\frac{\varepsilon_1^2}{\varepsilon_2\sqrt{\varepsilon_1}}\frac{\sin^2\theta}{\cos\theta} \frac{1}{4N_e} \frac{1}{E_{21} + h\mathbf{q}_\parallel \cdot \mathbf{v}_{dr} - \hbar\omega - i\hbar\Gamma}, \]

and absorbance given by

\[ \eta = \frac{e^2\hbar}{m^*c}N_e\frac{\varepsilon_1^2}{\varepsilon_2\sqrt{\varepsilon_1}}\frac{\sin^2\theta}{\cos\theta} \frac{\hbar\Gamma}{(E_{21} + h\mathbf{q}_\parallel \cdot \mathbf{v}_{dr} - \hbar\omega)^2 + (\hbar\Gamma)^2}, \]

where

\[ \zeta = \left[ \frac{\pi^2\hbar^2}{m^*\hbar\omega} \left( \int_0^a dz \Phi(z) \right)^2 \right] \]

is numerical coefficient. For the constant barrier potential inside the quantum well (particle in a box) close to the intersubband resonance frequency \( \zeta = 512/27 \). It is worth noting that finite height of the barriers, static Hartree potential and exchange-correlation effects alter subband energy difference \( E_{21} \), electron wave function and coefficient \( \zeta \). This effects are widely discussed in the literature but not related to the CDOP and not in the focus of the present manuscript.

### IV. DISCUSSION

Considering optical properties quantum well can be treated as a uniaxial crystal where in–plane permittivity tensor components differ from perpendicular to the quantum well \( \varepsilon_{zz} \). The CDOP manifests itself as a shift in resonance frequency as can be seen from Eqs. (20) and (21). As a result the dielectric constant \( \varepsilon_{zz} \) and refractive index of quantum well changes if an electric current is present in the quantum well. For CDOP the permittivity tensor component along z-direction in the slab model is given by

\[ \varepsilon_{zz} = \varepsilon_2 + \frac{e^2\hbar}{m^*a_{eff}} \frac{N_e}{\omega} \frac{1}{\pi} \frac{1}{\Delta \omega + \mathbf{q}_\parallel \cdot \mathbf{v}_{dr} - i\hbar\Gamma}, \]

where \( \Delta \omega = E_{21}/\hbar - \omega \) and \( a_{eff} \approx a \) is the effective thickness of the transition layer.

Since permittivity tensor depends on \( \mathbf{v}_{dr} \), by altering dc current magnitude and direction one can change the optical path of transmitted radiation through quantum well. The difference between the phase shift in the presence of dc current and when current is absent is a measurable quantity that can be observed experimentally. This addition phase shift caused by CDOP is designated \( \delta \phi \).

![Graph showing the dependence of phase shift induced by CDOP on the radiation frequency for GaAs/AlAs quantum well.](image)

**FIG. 3:** Figure shows dependence of additional phase shift induced by CDOP on the radiation frequency for GaAs/AlAs quantum well. Curve is calculated after Eq. (23) for quantum well width \( a = 10 \text{ nm} \), dephasing rate \( \hbar\Gamma = 10 \text{ meV} \), electron density \( N_e = 5 \times 10^{11} \text{ cm}^{-2} \) and drift electron velocity \( v_{dr} = 10^6 \text{ cm/s} \). The inset shows the setup of multiple-reflection wave-guide geometry, which can be used for detecting of the CDOP.

and has the form

\[ \delta\phi = -\zeta \frac{e^2\hbar}{2m^*c}N_e\frac{\varepsilon_1^2}{\varepsilon_2\sqrt{\varepsilon_1}}\frac{\sin^2\theta}{\cos\theta} \frac{h\mathbf{q}_\parallel \cdot \mathbf{v}_{dr}}{\pi} \times \frac{1}{\pi} \frac{[(E_{21} - \hbar\omega)^2 - (\hbar\Gamma)^2]}{[(E_{21} - \hbar\omega)^2 + (\hbar\Gamma)^2]^2}, \]

where we assumed total phase shift \( \varphi \approx \text{Im}(T - 1) \), which is typically relevant for quantum wells since absorption of single quantum well is weak. The dependence of \( \delta\phi \) on radiation frequency is shown in Fig. 3. Phase shift induced by CDOP exhibits maximum exactly at intersubband resonance position \( E_{21} = \hbar\omega \), it changes sign at \( \hbar\Delta \omega = \pm\hbar\Gamma \) and decays to zero away from resonance.

To increase the phase shift \( \delta\phi \) the sample with multiple quantum wells in wave-guide geometry can be used (see the inset in Fig. 3). For \( N \) quantum wells and \( M \) reflections phase shift caused by CDOP is \( N \cdot M \cdot \delta\phi \). Modern technology allows to grow structure with both \( N \approx 100 \) and \( M \approx 100 \), which yields the induced by CDOP phase shift \( \sim 1^\circ \), which can be reasonably well observed in the experiment.

Another quantity which can be experimentally observed is absorbance. Relative correction to the absorbance induced by CDOP is given by

\[ \frac{\delta\eta}{\eta} = -\frac{2h\mathbf{q}_\parallel \cdot \mathbf{v}_{dr}}{(E_{21} - \hbar\omega)^2 + (\hbar\Gamma)^2}. \]

The dependence of \( \delta\eta/\eta \) is shown in Fig. 4 the main features of relative correction to the absorbance are as well observed at \( \hbar\Delta \omega = 0 \) and \( \hbar\Delta \omega = \pm\hbar\Gamma \). However, unlike the phase shift caused by CDOP \( \delta\eta/\eta \) is zero at
FIG. 4: The dependence of relative absorbance change caused by CDOP on the radiation frequency for GaAs/AlAs quantum well. Curve is calculated after Eq. (24) for quantum well width $a = 10$ nm, dephasing rate $\Gamma = 10$ meV, electron density $N_e = 5 \cdot 10^{11}$ cm$^{-2}$ and drift electron velocity $v_{dr} = 10^6$ cm/s.

intersubband resonance and has maximum at detuning $\hbar \Delta \omega = \pm \Gamma$. The characteristic value of $\delta \eta/\eta$ of 0.1% is within reach of modern experimental capabilities.

Finally we point out that shift of resonance position $\hbar q_{\parallel} \cdot v_{dr}$ is actually exactly matches the Doppler shift in the frame of reference moving with $v_{dr}$, which further supports the title of "current drag of photons".

V. SUMMARY

Microscopic theory of the drag of photons by electric current in quantum wells have been developed. It has been shown that absorbance, transmission, and reflection coefficients are affected by dc electric current in quantum well. We demonstrated that the CDOP is due to the asymmetry of intersubband transitions, this asymmetry appears by taking into account the shift of electron distribution in momentum space with respect to the wave vector of light. The main feature of the CDOP is the shift of intersubband resonance position by the value of $\hbar q_{\parallel} \cdot v_{dr}$. The corrections caused by CDOP to the phase shift and radiation absorption are estimated for GaAs based quantum wells.

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