The Conductivity Tensor for the Hubbard Model

F. Mancini\textsuperscript{a} and D. Villani\textsuperscript{b}\textsuperscript{*}

\textsuperscript{a} Dipartimento di Scienze Fisiche “E.R. Caianiello” e Unità I.N.F.M. di Salerno
Università di Salerno, 84081 Baronissi (SA), Italy

\textsuperscript{b} Serin Physics Laboratory, Rutgers University, Piscataway, New Jersey 08855-0849

Abstract

A new theoretical analysis of the current-charge and charge-charge propagators is presented for the Hubbard model, using the static approximation for the Composite Operator Method. This approximation manifestly preserves a sum rule which governs the single-site dynamics. We compare our results with those obtained by numerical analysis.

\textbf{Keywords:} Drude weight, Ward-Takahashi identities, Hubbard model.

72.10.-d, 71.10.Fd, 74.72.-h, 71.10.-w

\textsuperscript{*}Corresponding author. E-mail address: dvillani@physics.rutgers.edu
The study of optical spectra is an important probe of the dynamics of strongly correlated electronic systems and gives information about the normal and superconducting states of these materials \[1,2\]. Also, the optical conductivity is of fundamental theoretical interest because the spectral weight at low frequencies might be a natural order parameter for the Mott transition \[3\].

The optical spectrum of undoped cuprate compounds, such as $La_{2}CuO_{4}$, $YBa_{2}Cu_{3}O_{6}$ and $Nd_{2}CuO_{4}$, is characterized by the $O2p - Cu3d_{x^2-y^2}$ charge-transfer excitation with a threshold energy in the $1.5 - 2.0eV$ range \[1,4\]. By doping charge carriers, a drastic change in the optical spectrum occurs. The charge-transfer excitation gap is rapidly suppressed and instead an edge shows up around $1.0eV$ in the reflectivity spectrum, indicating that the spectrum is dominated by low-energy excitations in the energy range lower than $1.0eV$ \[5\]. This kind of changes with carrier doping are universal for all known hole-doped and electron-doped cuprate superconductors. Besides, the appearance of optical anomalies is not unique to the doped copper oxides, but is almost universally observed in the spectra of the systems with strong electron correlation \[1,2,6\].

In the simplest form, the Hubbard model is believed to describe many properties of strongly correlated fermion systems. The applicability of the model to the superconducting copper-oxides is related to the fact that lower and upper Hubbard subbands can mimic the $O2p$ and $Cu3d_{x^2-y^2}$ bands of the cuprates and thus the charge-transfer gap can be described by an effective Hubbard on-site coupling constant.

Theoretical studies of the optical conductivity for the Hubbard model started long time ago \[7\]: it was investigated by the moment method, by the equation of motion method for the Green’s functions in the Hubbard I approximation and in the Hubbard III approximation. For the two-dimensional Hubbard model the strong coupling limit has been considered mostly by numerical methods based on exact diagonalization for small clusters \[8–14\]. A detailed discussion of optical and photoemission sum rules for one- and two-dimensional Hubbard model has been given recently by comparing a perturbation theory in the kinetic term with numerical calculations \[15\].
An analytical analysis of the frequency dependent conductivity has been given by perturbation method in weak-coupling limit [16,17] and by applying the memory function technique in terms of the Hubbard operators [1]. Recently the optical conductivity has been investigated within the dynamical mean-field approach [6]. However, for the realistic planar Hubbard model non-local corrections to the transport vertices and the self-energy are important [6]. Beyond this, vertex corrections are crucial to satisfy the relevant Ward-Takahashi identities [17].

In this Letter, we present a new theoretical analysis of the conductivity tensor by use of the Composite Operator Method [18] for the two-dimensional Hubbard model. By exploiting sum rules after the Ward-Takahashi identities and the Pauli principle, the calculation of the current-current propagator is reduced to the charge-charge one in the framework of a fully self-consistent scheme. Further, it will be shown that the charge-charge propagator preserves an important sum rule which governs the single-site dynamics avoiding double occupancy configurations unless to form local singlets.

The Letter is organized as follows. First, the model is set up and the formula for the conductivity tensor in the linear response theory is summarized. The Ward-Takahashi identities are given in the context of the two-dimensional Hubbard model and the hierarchy degrading the current-current propagator to the current-charge and charge-charge ones is built up. The calculation of the current-charge and charge-charge propagators is realized within the static approximation for the Composite Operator Method. Then, the results for the optical conductivity and the Drude weight are presented and compared with those obtained by numerical analysis of small clusters. Some concluding remarks are presented at the end.

2. In the presence of an electromagnetic field described by the vector potential $\vec{A}(\mathbf{r},t)$, the Hubbard model is as follows

$$H = \sum_{\langle ij \rangle} [t_{ij}(A) - \mu \delta_{ij}] c^\dagger(i)c(j) + U \sum_i n_{\uparrow}(i)n_{\downarrow}(i) \quad (1)$$

where we use a standard notation. The hopping matrix elements contain the Peierls factors.
which guarantee the gauge invariance.

In the framework of the linear response theory, the electric conductivity tensor is given as follows

$$\sigma_{ab}(k,\omega) = e^2 \frac{1}{i(\omega + i\eta)} \langle K_a \rangle \delta_{ab} - \frac{1}{i(\omega + i\eta)} g_{ab}(k,\omega)$$  \hspace{1cm} (2)

where $g_{ab}(k,\omega)$ is the retarded current-current propagator, with $K_a(i)$ being the kinetic energy density operator in the $a$ direction.

In particular, the frequency-dependent uniform, i.e. $k = 0$, electric conductivity $\sigma_{xx}(\omega)$ is given by

$$\sigma_1(\omega) = \text{Re} \sigma_{xx}(\omega) = D\delta(\omega) + \sigma_{inc}(\omega)$$  \hspace{1cm} (3)

where $D$ is the Drude weight

$$D = e^2 \pi \left[ -\langle K_x \rangle + \frac{1}{e^2} \text{Re} g_{xx}(0, \omega \to 0) \right]$$  \hspace{1cm} (4)

which measures the ratio of the density of the mobile charge carriers to their mass [3], and $\sigma_{inc}$ is the incoherent contribution

$$\sigma_{inc}(\omega) = -\frac{1}{\omega} \text{Im} g_{xx}(0,\omega)$$  \hspace{1cm} (5)

By defining the charge $\rho(i)$ and current $j(i)$ densities as

$$\rho(i) = ec^\dagger(i)c(i) \hspace{1cm} (6)$$

$$j(i) = \frac{tea^2}{i} c^\dagger(i) \left[ \nabla - \nabla \right] c(i) \hspace{1cm} (7)$$

it is immediate to obtain the conservation law

$$\nabla \cdot j(i) + \frac{\partial}{\partial t} \rho(i) = 0$$  \hspace{1cm} (8)

The symmetry content of the algebraic equation (8) manifests at level of observation as relations among matrix elements once a choice of the physical space of states has been made. Indeed, by defining the causal charge and current propagators as $\chi_{ab}(i,j) = \langle T[g_a(i)g_b(j)]\rangle$
where \( g_a(i) = (\rho(i), j_x(i), j_y(i)) \) for \( a = 0, x, y \), we can derive a series of Ward-Takahashi identities, which in momentum space read as

\[
ia \omega \chi_{00}(k, \omega) = \left[ 1 - e^{-ik_x a} \right] \chi_{x0}(k, \omega) + \left[ 1 - e^{-ik_y a} \right] \chi_{y0}(k, \omega) \tag{9}\]

\[
ia \omega \chi_{x0}(k, \omega) = -2te^2a^2\langle c^\dagger(i)c^a(i) \rangle \left[ 1 - e^{ik_x a} \right] \\
- \left[ 1 - e^{ik_x a} \right] \chi_{xx}(k, \omega) - \left[ 1 - e^{ik_y a} \right] \chi_{xy}(k, \omega) \tag{10}\]

\[
ia \omega \chi_{y0}(k, \omega) = -2te^2a^2\langle c^\dagger(i)c^a(i) \rangle \left[ 1 - e^{ik_y a} \right] \\
- \left[ 1 - e^{ik_x a} \right] \chi_{yx}(k, \omega) - \left[ 1 - e^{ik_y a} \right] \chi_{yy}(k, \omega) \tag{11}\]

\[
a^2 \omega^2 \chi_{00}(k, \omega) = 8ta^2e^2 \left[ 1 - \alpha(k) \right] \langle c^\dagger(i)c^a(i) \rangle \\
+ 2\chi_{xx}(k, \omega) \left[ 1 - \cos(k_x a) \right] + \chi_{xy}(k, \omega) \left[ 1 - e^{-ik_x a} \right] \left[ 1 - e^{ik_y a} \right] + 2\chi_{yy}(k, \omega) \left[ 1 - \cos(k_y a) \right] \tag{12}\]

\[
a^2 \omega^2 \chi_{00}(k, \omega) = 8ta^2e^2 \left[ 1 - \alpha(k) \right] \langle c^\dagger(i)c^a(i) \rangle \\
+ 2\chi_{xx}(k, \omega) \left[ 1 - \cos(k_x a) \right] + \chi_{xy}(k, \omega) \left[ 1 - e^{-ik_x a} \right] \left[ 1 - e^{ik_y a} \right] + 2\chi_{yy}(k, \omega) \left[ 1 - \cos(k_y a) \right] \tag{13}\]

By these identities we see that the current-current propagator \( \chi_{ab}(k, \omega) \) can be expressed in terms of the current-charge and the charge-charge ones.

In order to calculate the current-charge and the charge-charge propagators, we use the static approximation [19] for the composite field

\[
\psi(i) = \begin{pmatrix} \xi(i) \\ \eta(i) \end{pmatrix} \tag{14}\]

By defining

\[
N(k) = F.T.\langle T[n(i)n(j)] \rangle \quad S(k) = F.T.\langle T[\psi(i)\psi^\dagger(j)] \rangle \quad I = F.T.\langle \{\psi(i),\psi^\dagger(j)\} \rangle_{E.T.} = \begin{pmatrix} 1 - \frac{n}{2} & 0 \\ 0 & \frac{n}{2} \end{pmatrix} \tag{15}\]

we have

\[
N(k) = -in^2(2\pi)^3a^{-2}\delta^{(3)}(k) - \frac{n(2 - n)}{n - 2D} \left\{ I_{11}^{-1} [Q_{1111}(k) + Q_{1112}(k) + Q_{1211}(k) + Q_{1212}(k)] \\
+ I_{22}^{-1} [Q_{1212}(k) + Q_{1222}(k) + Q_{2212}(k) + Q_{2222}(k)] \right\} \tag{17}\]
with
\[ Q_{\alpha\beta\gamma\delta}(k, \omega) = \frac{i\Omega}{(2\pi)^3} \int d^2p d\Omega S_{\alpha\beta}(k+p, \omega + \Omega) S_{\gamma\delta}(p, \Omega) \]

(18)

where D is the double occupancy
\[ D = \frac{1}{2} \langle \eta^\dagger(i)\eta(i) \rangle \]

(19)

\( k \) is the four-dimensional vector \( k = (k, \omega) \); the symbol F.T. denotes the Fourier transform.

Then, it is direct to see that
\[ \chi_{00}(k) = e^2 N(k) \]

(20)

and
\[ \chi_{a0}(k) = (-ite^2a) \frac{1}{2} \left\{ I_{11}^{-1} \left[ \langle \xi^\alpha \xi^\dagger \rangle + \langle \xi^\alpha \eta^\dagger \rangle \right] - I_{22}^{-1} \left[ \langle \eta^\alpha \xi^\dagger \rangle + \langle \eta^\alpha \eta^\dagger \rangle \right] \right\} \]
\[ -2I_{22}^{-1}(-ite^2a) \left[ Q_{1212}^a(k) + Q_{2222}^a(k) \right] \]
\[ -2I_{22}^{-1}(-ite^2a) \left[ Q_{1222}^a(k) + Q_{2212}^a(k) \right] \]

(21)

where we have defined
\[ Q_{\alpha\beta\gamma\delta}^a(k, \omega) = \frac{i\Omega}{(2\pi)^3} \int d^2p d\Omega \left[ e^{i(p_a+k_a)a} - e^{-i(p_a)a} \right] S_{\alpha\beta}(k, \Omega) S_{\gamma\delta}(p + k, \Omega + \omega) \]

(22)

We have in the coordinate space
\[ N(i, j) = n^2 - \frac{n(2-n)}{n - 2D} \left\{ I_{11}^{-1} \left[ Q_{1111}^a(i, j) + Q_{1112}^a(i, j) + Q_{1211}^a(i, j) + Q_{1212}^a(i, j) \right] \right\} \]
\[ + I_{22}^{-1} \left[ Q_{1212}^a(i, j) + Q_{2222}^a(i, j) + Q_{2212}^a(i, j) + Q_{2222}^a(i, j) \right] \]

(23)

In particular, at equal sites (23) reduces to
\[ N(i, i) = \langle n(i)n(i) \rangle = n + 2D \]

(24)

We thus see that the charge propagator satisfies the sum rule
\[ \frac{ia^2}{(2\pi)^3} \int d^2k d\omega \chi_{00}(k) = e^2(n + 2D) \]

(25)
as it should be. The physical content of (25) is that of the Pauli principle because it governs the single-site dynamics of two fermions.

By means of the Ward-Takahashi identity (12) we obtain

\[ \chi_{xx}(k_x, k_y = 0, \omega) = \frac{a^2 \omega^2 \chi_{00}(k_x, k_y = 0, \omega)}{2 [1 - \cos(k_x a)]} + \frac{1}{2} a^2 e^2 K \]  

where \( K = -4t \langle c_i^\dagger c_i^\alpha (i) \rangle \) is the kinetic energy per site. From the analytical structure of equation (26), we note that \( \chi_{xx}(k_x, k_y = 0, 0) = a^2 e^2 K/2 \) does not depend on \( k_x \) and is proportional to the kinetic energy. It is remarkable that such a peculiar behaviour has been previously found by use of Quantum Monte Carlo techniques [8]. The correspondence with the notation of Ref. 8 is given by \( g_{xx} = -e^2 \Lambda_{xx} \).

By taking the limit \( k_x \to 0 \) in (26) and by considering the retarded function we have

\[ g_{xx}(0, \omega) = \frac{1}{2} a^2 e^2 K + \lim_{k_x \to 0} \frac{a^2 \omega^2 \chi_{00}^R(k_x, \omega)}{2 [1 - \cos(k_x a)]} \]  

Recalling (3) we have for the optical conductivity

\[ \sigma_1(\omega) = D\delta(\omega) + \sigma_{inc}(\omega) \]  

where the Drude weight \( D \) is given by

\[ D = \pi \text{Re} \lim_{\omega \to 0} \lim_{k_x \to 0} \frac{a^2 \omega^2 \chi_{00}^R(k_x, \omega)}{2 [1 - \cos(k_x a)]} \]  

and the incoherent part \( \sigma_{inc}(\omega) \) is as follows

\[ \sigma_{inc}(\omega) = -\frac{1}{\omega} \text{Im} \lim_{k_x \to 0} \frac{a^2 \omega^2 \chi_{00}^R(k_x, \omega)}{2 [1 - \cos(k_x a)]} \]  

As emphasized in the beginning, the whole problem of the response to an external electromagnetic field has been reduced to the evaluation of the charge-charge propagator. In the static approximation of the Composite Operator Method [18-20] the charge-charge propagator, as the spin-spin one, can be connected to convolutions of single-particle propagators [19]. It is important to note that the convolutions in (21) and (22) involve higher order single-particle Green’s functions. Then, such a scheme does not contradict the conclusion
reached by exact diagonalization analysis [11] that any attempt to compute the conductivity from a convolution of single-particle propagators is destined to fail in one dimension and is questionable in planar systems. In our case, the occurrence of convolutions is related to a linearized dynamics, but also involves choice of occupation dependent electronic excitations as basic fields [19]. In this way, the electron propagation is described as a repetition of composite excitations which automatically take into account scattering of electrons on spin and charge fluctuations due to strong correlations. Also, the use of a higher order basic field gives the advantage of implementing the Pauli principle and of a clear theoretical understanding of the terms originating from intraband or interband propagation. In fact, it turns out that only intraband excitations contribute to the Drude weight, whereas interband ones build up the incoherent part.

In the static approximation, by means of the previous results, the retarded charge-charge propagator is given by

$$\chi_R^{00}(k, \omega) = -i n^2 e^2 (2\pi)^3 a^{-2} \delta^{(3)}(k) - \frac{n(2-n)e^2}{n - 2D} \left\{ I_{11}^{-1} \left[ Q_{1111}^R(k) + Q_{1112}^R(k) \right. \\
+ Q_{1211}^R(k) + Q_{1212}^R(k) \bigg]\right. + I_{22}^{-1} \left[ Q_{1212}^R(k) + Q_{1222}^R(k) + Q_{2212}^R(k) + Q_{2222}^R(k) \right] \right\} \quad (31)$$

where

$$Q_{\alpha\beta\gamma\delta}^R(k, \omega) = \frac{\Omega}{(2\pi)^2} \int d^2p f[f(E_j(k+p))] - f[E_i(p)] \sigma^{(i)}_{\alpha\beta}(p) \sigma^{(j)}_{\gamma\delta}(k+p) \quad (32)$$

$$f(\omega)$$ is the Fermi distribution function. $$\sigma^{(i)}_{\alpha\beta}(p)$$ are the spectral functions of the single-particle propagator. These functions have been previously calculated [18, 19].

Let us define

$$A_{ij}(k, p) = I_{11}^{-1} \left[ \sigma^{(i)}_{11}(p) \sigma^{(j)}_{11}(k+p) + \sigma^{(i)}_{11}(p) \sigma^{(j)}_{12}(k+p) + \sigma^{(i)}_{12}(p) \sigma^{(j)}_{11}(k+p) + \sigma^{(i)}_{12}(p) \sigma^{(j)}_{12}(k+p) \right] + I_{22}^{-1} \left[ \sigma^{(i)}_{12}(p) \sigma^{(j)}_{12}(k+p) + \sigma^{(i)}_{12}(p) \sigma^{(j)}_{22}(k+p) + \sigma^{(i)}_{22}(p) \sigma^{(j)}_{12}(k+p) + \sigma^{(i)}_{22}(p) \sigma^{(j)}_{22}(k+p) \right] \quad (33)$$

and

$$X_{ij}(k, \omega) = \frac{\Omega}{(2\pi)^2} \int d^2p f[E_j(k+p)] - f[E_i(p)] \frac{A_{ij}(k, p)}{\omega + E_i(p) - E_j(k+p) + i\eta} \quad (34)$$
Then, $\chi^R_{00}(k, \omega)$ can be rewritten as

$$\chi^R_{00}(k, \omega) = -in^2 e^2 (2\pi)^3 a^{-2} \delta^{(3)}(k) - \frac{n(2-n)e^2}{n-2D} \sum_{i,j=1}^{2} X_{ij}(k, \omega)$$

and

$$D = -\frac{n(2-n)\pi e^2}{n-2D} \sum_{i,j=1}^{2} \lim_{\omega \to 0, k_x \to 0} \lim_{\omega \to 0, k_x \to 0} \frac{a^2 \omega^2 \text{Re} X_{ij}(k_x, \omega)}{2[1 - \cos(k_x a)]}$$

It is direct to see that the interband terms $X_{12}(k, \omega)$ and $X_{21}(k, \omega)$ do not contribute. Also, by means of straightforward calculations the contribution of the intraband terms gives

$$\lim_{\omega \to 0, k_x \to 0} \lim_{\omega \to 0, k_x \to 0} \frac{a^2 \omega^2 \text{Re} X_{ij}(k_x, \omega)}{2[1 - \cos(k_x a)]} = -\frac{a^2 t^2}{4k_BT} \Omega \frac{(2\pi)^2}{2} \int d^2 p \sin^2(ap_x) X_i(p)$$

and the Drude weight takes the expression

$$D = \frac{n(2-n)\pi e^2 a^2 t^2}{4k_BT(n-2D)} \sum_{i=1}^{2} \frac{\Omega}{(2\pi)^2} \int d^2 p \sin^2(ap_x) X_i(p)$$

where we defined

$$X_i(p) = \epsilon_i^2(p)[1 - T_i^2(p)]A_{ii}^{(0)}(p)$$

with

$$T_i(p) = \tanh \left( \frac{E_i(p) - \mu}{2k_BT} \right)$$

In the same way, it is straightforward to show that the incoherent part is given by

$$\sigma_{\text{inc}}(\omega) = \frac{1}{\omega} \frac{n(2-n)e^2}{n-2D} \sum_{i,j=1}^{2} \text{Im} \lim_{k_x \to 0} \lim_{\omega \to 0} \frac{a^2 \omega^2 X_{ij}(k_x, \omega)}{2[1 - \cos(k_x a)]}$$

where the intraband terms do not contribute. In particular, it is found that $\sigma_{\text{inc}}(\omega) = 0$ for $\omega < \omega_c$. For any finite value of $U$ there is a gap in the incoherent part of the optical conductivity, given by

$$\omega_c = 2\sqrt{I_{11}I_{22}}U \quad [\text{At half-filling } \omega_c = U = 2\mu]$$

In Fig. 1 we present results for the Drude weight and from numerical analysis of small clusters [13].
For $U = 15$ the Hubbard gap is clearly visible at half-filling because, as discussed by Kohn [3], one expects that in an insulating phase $D$ will vanish. Also, there is no spectral weight in the interval $0 \leq \omega \leq 15$ [cfr. Eq. (42)]. Upon doping, spectral weight is transferred from the region above the Hubbard gap to the Drude peak at zero frequency. This is up to a critical doping where there is a deviation downwards which may be taken as a signature of a drastic change in the nature of the carriers. Indeed, after this doping the system starts to resemble a gas of non-interacting electrons and the Drude weight follows the kinetic energy [12]. It is worth noting that the value of the Drude weight and of its slope in proximity of half-filling should be carefully interpreted within the scheme used. In fact, many numerical and theoretical indications [2] signal the onset of additional states in the gap due to doping, extending upwards from the lower Hubbard band, which absorb a large spectral weight. Then, the absence of a mid-gap band accompanied by a conserved total spectral weight, as in the approximation used [20], will very much affect the spectral weight transfer between the interband charge excitations and the intraband ones. Further, this occurrence can be responsible of a Drude weight larger then the non-interacting value near half-filling for some particular choice of the on-site Coulomb repulsion $U$.

The kinetic energy is shown in Fig. 2. The results have been obtained through the scheme given in Refs. 21 and 22 which makes extensive use of thermodynamic identities for the Helmholtz free energy per site.

These results to a large extent contradict data available both from numerical and experimental analysis [1, 2]. The observed ratio $\omega_c/U$, connected to the energy scale associated with interband charge excitations, is greater compared to what is expected from Lanczos’s techniques [12]. $\omega_c/U$ is depicted as function of the filling $n$ in Fig. 3.

However, the small system sizes and the lack of theoretical analysis of finite size corrections make interpretation of the results uncertain. A more troublesome discrepancy is related to the absence of a clear feature centered at low frequency beyond the Drude peak, quickly developing by varying the dopant concentration, as already discussed in relation with the Drude weight results. The presence of a mid-gap band is a genuine two-dimensional ef-
fect [11] that cannot be related to interband charge excitations which lead to an absorption band centered near $U$, as seen also in the strong coupling counterpart of the Hubbard model [2] (i.e., the $t - J$ model). This is the well known mid-infrared band, which has been observed in several cuprate superconductors and strongly correlated electronic systems by use of analytical and numerical techniques [1, 2]. We do not at present have a resolution of this discrepancy, but a possible one is as follows.

The energy scale on which the center of gravity of the mid-infrared band is centered is $4t^2/U$, that is the value of the Heisenberg exchange $J$ when derived from the one-band Hubbard model. Then, optical mid-gap effects can be observed only if the scheme presents in addition to the lower Hubbard band a quasi-particle peak separated by a gap or pseudogap of order $J$. On the contrary, we cannot reproduce the latter result since for our estimations of the single-particle propagator we have used a two-pole expansion for the one-electron spectral density. In this respect, an enlarged composite basis with a third asymptotic excitation can resolve this structure.

In conclusion, we have shown that the operatorial equation (8) expressing the charge conservation law has a direct corollary in terms of the existence of Ward-Takahashi identities, which hierarchically degrade the evaluation of the current-current propagator to the current-charge and charge-charge ones. Such an occurrence takes place also in the non interacting system (i.e., $U = 0$). However, it is worth pointing out the different role played by the Ward-Takahashi identities in non interacting systems compared to that in the framework of an approximate theory for interacting ones. In the first case the Ward-Takahashi identities are automatically satisfied and it is a matter of convenience to compute the conductivity from the current-charge or charge-charge propagators. In a theoretical scheme for an interacting system there is no way to avoid some approximation. Then, in addition to a clear convenience in computing a hierarchically degraded propagator, there is the possibility to build up a physical space of states where the general symmetry principle of charge conservation law can be enforced to model the dynamics. The scheme turns out to be even more coherent by preserving the fermionic character of the propagating charges or, in other words, by
implementing the Pauli principle. That is, by recovering such general symmetries the propagation of the charge and their intrinsic dynamics is constrained in a suitable Hilbert space where the charges propagate without degrading in number and avoiding double occupancy configurations unless to form local singlets.

The authors would like to express their thanks to Adolfo Avella for valuable discussions and Sarma Kancharla for a careful reading of the manuscript. One of us (D.V.) thanks Gabriel Kotliar for kind hospitality during his stay at the Rutgers University where a part of this work has been done.
REFERENCES

[1] S. Uchida, Jpn. J. Appl. Phys. 32, 3784 (1993).

[2] E. Dagotto, Rev. Mod. Phys. 66, 763 (1994), and references therein.

[3] W. Kohn, Phys. Rev. 133, A171 (1964).

[4] Y. Tokura, S. Koshihara, T. Arima, H. Takagi, T. Ido, S. Ishibashi, S. Uchida, Phys. Rev. B 41, 11657 (1990).

[5] S. Uchida, T. Ido, H. Takagi, T. Arima, Y. Tokura, S. Tajima, Phys. Rev. B 43, 7942 (1991).

[6] A. Georges, G. Kotliar, W. Krauth, M.J. Rozenberg, Rev. Mod. Phys. 68, 13 (1996).

[7] For a recent review, see, e.g., N. Plakida, J. Phys. Soc. Jpn. 65, 3964 (1996).

[8] D.J. Scalapino, S.R. White, S.C. Zhang, Phys. Rev. Lett. 68, 2830 (1992).

[9] A.J. Millis, S.N. Coppersmith, Phys. Rev. B 42, 10807 (1990).

[10] A. Moreo, E. Dagotto, Phys. Rev. B 42, 4786 (1990).

[11] W. Stephan, P. Horsch, Phys. Rev. B 42, 8736 (1990).

[12] E. Dagotto, A. Moreo, F. Ortolani, J. Riera, D.J. Scalapino, Phys. Rev. B 45, 10107 (1992).

[13] E. Dagotto, A. Moreo, F. Ortolani, D. Poilblanc, J. Riera, Phys. Rev. B 45, 10741 (1992).

[14] W. Stephan, P. Horsch, Int. J. Mod. Phys. B 6, 589 (1992).

[15] H. Eskes, A.M. Oles, M.B.J. Meinders, W. Stephan, Phys. Rev. B 50, 17980 (1994).

[16] S. Wernbter, L. Tewordt, Physica C 211, 132 (1993).

[17] W. Brenig, Z. Phys. B 89, 187 (1992).
[18] F. Mancini, S. Marra, H. Matsumoto, Physica C 244, 49 (1995); F. Mancini, D. Villani, H. Matsumoto, Phys. Rev. B 57, 6145 (1998); A. Avella, F. Mancini, D. Villani, L. Siurakshina, V.Yu. Yushankhai, Int. J. Mod. Phys. B 12, 81 (1998).

[19] F. Mancini, S. Marra, H. Matsumoto, Physica C 252, 361 (1995).

[20] F. Mancini, Phys. Lett. A 249, 231 (1998).

[21] F. Mancini, D. Villani, H. Matsumoto, Thermodynamics of the 2D Hubbard model, cond-mat/9709189.

[22] A. Avella, F. Mancini, D. Villani, The t-t’-U model as a minimal model for cuprate materials, cond-mat/9707088.
FIG. 1. The normalized Drude weight $D_n = D/(2\pi e^2)$ as a function of the filling $n$ for various couplings $U$. Full squares are zero temperature data using exact diagonalization techniques on $4 \times 4$ sites. Open squares and triangles indicate results for a $\sqrt{10} \times \sqrt{10}$ site cluster.

FIG. 2. Kinetic energy per site $-K$ as a function of the filling $n$ for various couplings $U$. Full rhombi are zero temperature data using exact diagonalization techniques on $4 \times 4$ sites. Open rhombi indicate results for a $\sqrt{10} \times \sqrt{10}$ site cluster.
FIG. 3. $\omega_c/U$ as function of the filling $n$. 