CFD Based Model to Identify the Minimum Length of a Gradual Rectangular Expansion Operated under Supercritical Flow

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Abstract. Computational Fluid Dynamics (CFD) has been used to derive a mathematical model to relate the length of a gradual expansion (Le) with the others of hydraulic and geometric parameters of the flow in rectangular channels under supercritical flow conditions. Five models with different length of expansion 30 cm, 50 cm, 75 cm, 100 cm, and 125 cm were simulated in the CFD v18.2 software and verified by experimental data which have been measured into a laboratory flume. All models of CFD software were processed depending on k – epsilon viscous models while the mesh was built by adapting the multi-zone method. Five values of inlet velocity of flow were applied in the runs of the program to each model of 1.5 m/s, 1.6 m/s, 1.7 m/s, 1.8 m/s, and 2 m/s. Depths of water along the channel were measured in multi-section, before expansion, within expansion and after expansion. The results of CFD analysis showed that the minimum length of expansion to maintain the flow within supercritical regime was 0.35 m and the ratio of (Le/W) equal to 1.167. The results of the non-dimensional relationship were compared with the experimental results and the comparison showed a significant correlation where the highest percentage difference was 13.8%. The coefficient of determination for this equation was 0.973.

1. Introduction
To design an expansion structure (transition) under conditions of supercritical flow is challenging due to the possibility of separation of flow from the boundaries in addition to local standing waves [1]. The transition of a channel may be defined as the structure that connect the cross section when it changed from size to another. Transitions are categorized into three types:

- Changing in form but the magnitude of the cross-sectional area is the same.
- The decrease in cross-sectional area and the change in form will occur due to contraction.
- Increasing the cross-sectional area and the change in form will occur due to expansion [2].

The flow through transitions is very complex due to the complexity of velocity profile from section to another and the boundaries is very slightly affected on the velocity. The velocity profile and turbulence distributions of an open channel due to decelerating flow in a gradual expansion were studied by measuring it using LDV equipment. The results indicated that the law of logarithmic can describe well the average velocity in the bottom of the channel while the law of Coles can be applied to the outer region. The shear velocity can be calculated by the logarithmic fit of the average velocity and by the stress distributions of Reynolds fit. The intensities of turbulence
reach the maximum values at the ratio of \( z / h \) ranged between 0.1 to 0.15. The intensities of turbulence increase along the expansion reach especially in the outer region and this is clearly pronounced in the larger values of Reynolds number [3].

Both the intensity of turbulence and Reynolds stress distributions are measured to deviate from the empirical and theoretical curves for uniform flow.

Experiments of flow in expansions section with or without a hump were carried out using a recirculating laboratory flume to measure the depth of flow, cross-sectional average velocities, and pressure. The results of measurements were analyzed by the concept of energy for direct estimates of the coefficient of energy loss. Without the hump, measured pressures of water showed an adverse gradient in the expansion portion, opposing the approaching flow. Estimating values of coefficient of the energy loss ranged between 0.46 to 0.62. These results represent an indication for the design of the expansions of channels, in addition, to calibrate and validate the numerical models of hydrodynamics. The models with the presence of the hump have been used to make the flow more accelerated, converting an adverse pressure gradient and reducing the coefficients of energy loss values more than 50% when compared with the other cases of corresponding values without using the hump [4].

A modifications of hydraulics of constrictions of an open channel to analyze the loss of energy in open channel expansions. The modified results have been compared with the other methods of analysis using data resulted from the laboratory work which conducted on open channel expansions with vertical walls and baffles with triangular-shaped. In addition, the procedure of design for baffled outlet structures has been modified [5].

A singular integral equations method was used to estimate the profile of the free surface of potential flow in the contraction or expansion of open channels. The application of such free surface hydraulics of the fluid motion is too complex. For the numerical analysis, the singular integral equations are used depending on the Singular Integral Operators Method (SIOM) with constant and linear elements. The results of free-surface profile in a special transition were compared with the others numerical results from the SIOM [6].

An experimental work was carried out to investigate the effect of hydraulic and geometric parameters on the dissipation of energy, hydraulic jump location with a change in the height of elements of roughness and the divergence of walls in different values of discharges. All experiments were carried out in a horizontal rectangular flume with the gradual expansion of 0.5 m wide and 10 m long with four physical models. The measured characteristics of the hydraulic jump with different non-dimensional divergences ratio \( B = b_1/b_2 \) with values of 0.4, 0.6, 0.8 and 1 and the inflow Froude numbers \( (Fr_1) \) ranged between 6 to 12 were compared with each other and with the corresponding values of the classic hydraulic jump. The results proved that the tailwater depth which is required to form a hydraulic jump generated in a gradual expansion basin with the rough bed was smaller when compared with the other when generated in a rectangular channel with a smooth bed. Depending on the experimental results, the empirical formula was derived to relate the characteristics of the hydraulic jump with the height of roughness elements and divergence ratio of the wall [7].

In this research, an empirical formula was derived to calculate the minimum length of a gradual expansion in rectangular channels when the flow under supercritical conditions. The modelling was performed by computational fluid dynamic (Ansys, R18.2) for a flume with dimensions of (30 cm width and 60 cm depth). The results of CFD modelling were compared with an experimental data resulted from experiments were carried out in a flume with same dimensions.

2. The Models of CFD

The software of computational fluid dynamics (CFD) provides a qualitative and quantitative prediction with the high reality of the flow of fluids by introducing mathematical models (partial differential equations), numerical solutions using different methods of solution techniques and high reliability when the flow was represented.

The volume of fluid (VOF) represents the suitable simulation method where the phase interaction was based on the model of the force of continuum surface and localization of compressive scheme phase[8].

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For both experimental work and CFD models, all the calculations and work was built depending on a laboratory channel with a cross-section of 0.3 m width * 0.6 m height. Five models were applied in the ANSYS software of CFD to simulate the flow in a gradual expansion with different dimensions under supercritical flow conditions. According to the length of the gradual expansion, the models were classified into three types according to the length of expansion. The transition structure is installed in the flume to connect between rectangular reach at 0.1 m in wide and 0.5 m in length and another wider rectangular reach its width and length respectively 0.3 m and 0.75 m as shown in figure 1. The length of expansion of models undertaken are 30 cm, 50 cm, 75 cm, 100 cm and 150 cm. All models were examined under five different of discharges ranged between 30 l/s to 55 l/s. The general description of the typical model is shown in figure 1.

Figure 1. (a) Top view of the expansion (b) Water surface profile before, within and after the expansion.

2.1. Geometry and mesh of models
Figure 2 shows the geometry of a CFD three - dimensional model (ANSYS, R18.2) using space claim then editing the scheme by design modular. For the second step in preparing the CFD model, i.e. mesh, the method used is a multi-zone method (Figure 3) with dimension for each cell of 0.01 m for maximum Tet size, 0.01 m for maximum face size and 0.01 m for minimum size.

2.2. Setup of models
The first step in setting the model is selecting the volume of fluid (VOF) method. This method used widely in CFD models and viscous model of k – epsilon (2 eqn) to compute free surface profile with the complex flow. Considering two phases of fluids, the primary phase is air and the secondary phase is water. The fraction for each cell of mesh is computed and the location of the water surface is also computed with partial fraction size of water [9]. The solution initialization depends on initial region of water body by adapting a region from x = 0 m to x = 0.5 m, from y = 0 m to y > 0.3 m (depending on the discharge value) and from z = 0 m to z = 0.1 m.
3. Functional dimensionless relationship
By the help of Buckingham method, a dimensionless formula may be derived to describe the relationship between the maximum length of expansion ($L_e$) and other parameters ($y_1$, $y_e$, $y_2$, $Fr_2$, $\Theta$). After elimination and delimitation, the following dimensionless relationship between dependent and the governing independent parameters is restated:

$$L_e \div y_2 = f \left( \frac{y_1}{y_2}, y_e, Fr_2, \Theta \right)$$

Where $L_e$ is the length of expansion, $y_1$ is the depth upstream the expansion, $y_2$ is the depth after the jump downstream the expansion, $y_e$ is the critical depth in the expansion zone, $Fr_2$ is Froude number downstream the expansion and $\Theta$ is the inclination angle for both sides of expansion.

4. Results and discussion
The results showed that for all inlet velocity values (i.e. velocity at upstream), the water depth, $y_1$ decreases when the flow enters the zone of contraction and the critical flow occurs.
The hydraulic jump is achieved at the outlet of expansion. So, depending on how far this jump will dissipate the energy of flow, Froude number value for sequent depth of jump \( (F_{r2}) \) was calculated to indicate the type of flow after expansion zone. Table 1 illustrates the output data of CFD software for five models and for five values of inlet velocities.

Figure 4 shows the relationship between the length of expansion \( (L_e) \) and Froude number value after expansion zone \( (F_{r2}) \). Depending on the figure 4, the analysis of CFD models shows that the maximum length of expansion to produce a subcritical flow after expansion zone ranged between 0.3 m to 0.34 m depending on the value of inlet velocity (i.e. \( L_e/W \) ratio ranged between 1 to 1.13). While the minimum length of expansion part to maintain the flow regime within a supercritical flow is 0.35 m with \( (L_e/W) \) equal to 1.167.

### Table 1. Results of CFD models.

| Mean Velocity Value (m/s) | \( L_e \) (m) | \( y_1 \) (m) | \( y_e \) (m) | \( y_2 \) (m) | \( V_2 \) (m/s) | \( F_{r2} \) | \( \Theta^o \) |
|--------------------------|---------------|---------------|---------------|---------------|----------------|-------------|-------------|
| 1.5                      | 0.3           | 0.205         | 0.074         | 0.109         | 0.939          | 0.908       | 18.43       |
|                          | 0.5           | 0.201         | 0.092         | 0.085         | 1.187          | 1.304       | 11.31       |
|                          | 0.75          | 0.2           | 0.082         | 0.076         | 1.325          | 1.54        | 7.59        |
|                          | 1             | 0.2           | 0.085         | 0.075         | 1.333          | 1.554       | 5.71        |
|                          | 1.5           | 0.198         | 0.072         | 0.068         | 1.447          | 1.767       | 3.81        |
| 1.6                      | 0.3           | 0.205         | 0.075         | 0.11          | 0.994          | 0.957       | 18.43       |
|                          | 0.5           | 0.218         | 0.095         | 0.09          | 1.292          | 1.375       | 11.31       |
|                          | 0.75          | 0.207         | 0.093         | 0.079         | 1.398          | 1.587       | 7.59        |
|                          | 1             | 0.207         | 0.092         | 0.078         | 1.413          | 1.615       | 5.71        |
|                          | 1.5           | 0.206         | 0.077         | 0.069         | 1.589          | 1.927       | 3.81        |
| 1.7                      | 0.3           | 0.208         | 0.076         | 0.114         | 1.034          | 0.977       | 18.43       |
|                          | 0.5           | 0.22          | 0.101         | 0.092         | 1.361          | 1.436       | 11.31       |
|                          | 0.75          | 0.21          | 0.096         | 0.08          | 1.493          | 1.689       | 7.59        |
|                          | 1             | 0.209         | 0.1           | 0.077         | 1.534          | 1.763       | 5.71        |
|                          | 1.5           | 0.209         | 0.082         | 0.071         | 1.675          | 2.013       | 3.81        |
| 1.8                      | 0.3           | 0.211         | 0.076         | 0.119         | 1.067          | 0.988       | 18.43       |
|                          | 0.5           | 0.2203        | 0.103         | 0.095         | 1.396          | 1.448       | 11.31       |
|                          | 0.75          | 0.2123        | 0.102         | 0.08          | 1.59           | 1.794       | 7.59        |
|                          | 1             | 0.2117        | 0.101         | 0.08          | 1.588          | 1.792       | 5.71        |
|                          | 1.5           | 0.2113        | 0.085         | 0.071         | 1.776          | 2.122       | 3.81        |
| 2                        | 0.3           | 0.213         | 0.077         | 0.12          | 1.185          | 1.091       | 18.43       |
|                          | 0.5           | 0.221         | 0.104         | 0.1           | 1.473          | 1.486       | 11.31       |
|                          | 0.75          | 0.214         | 0.102         | 0.08          | 1.778          | 2.002       | 7.59        |
|                          | 1             | 0.213         | 0.102         | 0.08          | 1.766          | 1.992       | 5.71        |
|                          | 1.5           | 0.212         | 0.086         | 0.072         | 1.967          | 2.343       | 3.81        |
5. **Derivation of a new empirical equation**

Depending on non-dimensional parameters which were formed in article 2 using the statistical software (SPSS v21), for the group of data listed in table 1. The following empirical formula was derived depending on 75% of results to derive the relationship of $\frac{L_e}{y_2}$:

\[
\frac{L_e}{y_2} = 19.209 + 7.353 \times \frac{y_1}{y_2} - 22.63 \times \frac{y_e}{y_2} + 5.477 \times F_{r2} - 1.379 \times \Theta
\]

The coefficient of determination ($R^2$) is 0.973

6. **Accuracy of the equation**

6.1. **Comparison with CFD results**

To check the accuracy of equation (2), CFD data which are not used in the non–linear regression will be used in this equation, and the results are compared with the others resulted from equation 2 as shown in table 2 and figure 6:
Table 2. A Comparison between CFD results and the results of equation 2.

| \(\frac{L_e}{y_2}\) from Equation (2) | \(\frac{L_e}{y_2}\) from CFD | Percentage of Difference % |
|-------------------------------------|-----------------------------|---------------------------|
| 14.865                             | 13.333                      | 10.31                     |
| 10.063                             | 9.494                       | 5.66                      |
| 4.104                              | 5.465                       | 3.31                      |
| 11.583                             | 12.953                      | 11.83                     |

Figure 6. Calculated \(\frac{L_e}{y_2}\) vs. Resulted \(\frac{L_e}{y_2}\) from CFD.

6.2. Comparison with experimental results

Table 3 illustrates a group of data for the experimental work was conducted in a laboratory flume with dimensions of 0.3 m * 0.6 m in cross section and 15 m long to verify the results of CFD simulation. The expansion zone located at a distance of 0.5 m after channel intake. The boundary conditions for the experimental work are the same in the simulation but only three models the laboratory tests with three specified values of velocity have been conducted as follows:

Table 3. Experimental Results.

| Velocity value (m/s) | \(L_e\) (m) | \(y_1\) (m) | \(y_e\) (m) | \(y_2\) (m) | \(V_2\) (m/s) | \(F_{r2}\) | \(O^*\) |
|----------------------|-------------|-------------|-------------|-------------|---------------|-----------|--------|
| 1.5                  | 1           | 0.198       | 0.078       | 0.0627      | 1.579         | 2.013     | 5.71   |
| 1.6                  | 0.75        | 0.2         | 0.09        | 0.0668      | 1.597         | 1.973     | 7.59   |
| 1.7                  | 0.5         | 0.203       | 0.11        | 0.105       | 1.096         | 1.079     | 11.31  |
| 1.7                  | 0.75        | 0.21        | 0.097       | 0.0584      | 2.038         | 2.692     | 5.71   |

Another checking the accuracy of equation (2), \(\frac{L_e}{y_2}\) resulted from equation 2 (table 2) now are compared with other data were obtained from an experimental work for the same dimensions, which were conducted by the same researcher on the laboratory models which have the value of \(L_e\) equal to 50 cm and 75 cm., as shown in table 4 and figure 7:

Table 4. A Comparison between CFD results and experimental work.

| \(\frac{L_e}{y_2}\) from Equation (2) | \(\frac{L_e}{y_2}\) from Experimental Work | Percentage of Difference % |
|-------------------------------------|---------------------------------------------|---------------------------|
| 14.865                             | 15.958                                      | 6.85                      |
| 10.063                             | 11.223                                      | 10.34                     |
| 4.104                              | 4.761                                       | 13.8                      |
| 11.583                             | 12.836                                      | 9.76                      |
7. Conclusions
- The difference percentage values of \((L_e/y_2)\) obtained from CFD models and the others which were calculated from equation (2) ranged between 3.31% to 11.83% and this results represent a good agreement between the results of CFD models and the results of equation 2.
- When the length of the expansion part decreases, the effect of the hydraulic jump generated after the expansion is clearer in dissipating the flow energy.
- Froude number decreases when the value of the ratio of \((L_e/W)\) approach to 1 and the flow regime dislocated toward the subcritical zone.
- The intensities of turbulence increase along with the expansion, especially when the length of expansion decreases. This is obvious for larger Froude numbers. The turbulence intensity may be increased as a result of a slight increase of shear velocity at the beginning of the expansion especially when the value of inclination is larger.

8. References
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Figure 7. Calculated \(L_e/y_2\) vs. Resulted \(L_e/y_2\) from experimental work