Using Loaded N-Port Structures to Achieve the Continuous-Space Electromagnetic Channel Capacity Bound

Zixiang Han\textsuperscript{a}, Member, IEEE, Shanpu Shen\textsuperscript{a}, Member, IEEE, Yujie Zhang\textsuperscript{a}, Member, IEEE, Shiwen Tang\textsuperscript{a}, Graduate Student Member, IEEE, Chi-Yuk Chiu\textsuperscript{a}, Senior Member, IEEE, and Ross Murch\textsuperscript{a}, Fellow, IEEE

Abstract—A method for achieving the continuous-space electromagnetic channel capacity bound using loaded \textit{N}-port structures is described. It is relevant for the design of compact multiple-input multiple-output (MIMO) antennas that can achieve channel capacity bounds when constrained by size. The method is not restricted to a specific antenna configuration and a closed-form expression for the channel capacity limits are provided with various constraints. Furthermore, using loaded \textit{N}-port structures to represent arbitrary antenna geometries, an efficient optimization approach is proposed for finding the optimal MIMO antenna design that achieves the channel capacity bounds. Simulation results of the channel capacity bounds achieved using our MIMO antenna design with one square wavelength size are provided. These show that at least 18 ports can be supported in one square wavelength and achieve the continuous-space electromagnetic channel capacity bound. The results demonstrate that our method can link continuous-space electromagnetic channel capacity bounds to MIMO antenna design.

Index Terms—Beamspace, channel capacity, continuous space, electromagnetic field, information theory.

I. INTRODUCTION

Information theory has been widely applied to the analysis and design of wireless communication systems to approach theoretical capacity limits [1], [2], [3]. However, the physical realization of wireless communication systems is based on the implementation of antennas and radio-frequency (RF) circuits [4], [5]. Therefore, the joint study of information theory and electromagnetic field theory can be utilized to further extend the theoretical channel capacity limits of the entire wireless communication system. This has led to the development of electromagnetic information theory (EIT) [6], [7]. Utilizing EIT, the concept of aerial degrees-of-freedom (ADOF) has been introduced to estimate the number of orthogonal basis functions required for describing electromagnetic fields around antenna systems [8]. The number of ADOF can be used to determine the channel capacity because it refers to the maximum number of parallel sub-channels that can be used for transmission [9]. To estimate the channel capacity bound for excitations restricted to a given volume or area, continuous current sources limited to a region, exciting the electromagnetic channel, have been analyzed using continuous-space approaches [10], [11], [12]. In addition, using Nyquist sampling and ADOF [13], a bound on the number of independent samples needed for reconstructing an electromagnetic field can be found. However, the continuous-space and Nyquist results do not provide the corresponding antenna designs to achieve their bounds and there is no link between the bounds and antenna geometry. When full electromagnetic modeling of the antenna geometry is performed, that accounts for issues such as mutual coupling, polarization and distributed antenna currents, establishing this link is a challenging task.

Multiple-input multiple-output (MIMO) antenna systems, which exploit ADOF by using multiple antennas at transceivers, play a critical role in approaching the predicted channel capacity bounds in wireless communication systems [14], [15]. Significant effort has therefore been focused on designing suitable MIMO antenna systems [16], [17], [18], [19], [20], [21], [22], [24]. In practice, when antennas are separated by half a wavelength, mutual coupling between antennas is non-negligible and cannot be ignored [5]. This phenomenon is more significant in a two-dimensional antenna array where the central antenna will experience mutual coupling from each of the 8 neighboring antennas (16 if two polarizations are used). This is because the antennas have a finite size (not ideal point sources) and are surrounded in all directions by neighboring antennas. Therefore, in practical array based implementations, the antenna spacings are normally designed to be larger than half a wavelength so that mutual coupling is reduced and subsequently the density of the antenna array is lower than that expected from Nyquist sampling [16], [17], [18], [19], [20], [21]. As a result, the potential ADOF cannot
be fully utilized and the system capacity reduces with a significant performance gap compared to that of the MIMO capacity bounds. To overcome these issues, there has been an emphasis on novel design approaches that achieve as many antennas as possible within a certain volume or area [22], [23], [24]. One example has provided up to 22 antennas per square wavelength in which element coupling is constrained to be less than -10 dB [25]. These designs have to tradeoff strong mutual coupling effects with performance and this has led to design limits on the antennas possible per unit volume or area [26]. As a result, the analysis of the effects of mutual coupling on channel capacity performance have also been well studied [27], [28]. However there has been no direct link that can connect the design of MIMO antennas with the continuous-space approaches utilized in EIT [7], [29] or the Nyquist bound [13]. Such a link would also provide a systematic design methodology for compact MIMO antennas that reach the capacity bounds.

In this paper we introduce a method, by using loaded $N$-port structures [30], [31], [32], to link the antenna geometry and continuous-space electromagnetic channel capacity together. These are typically two distinct research areas that have not been reconciled in previous literature [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25]. Our work is an attempt to bridge the gap between the practical design of MIMO antennas and their design limits predicted by EIT. Our method is not restricted to a specific antenna configuration and it provides a systematic methodology for the analysis and design of compact MIMO antennas that can approach the predicted capacity bounds. In our method, by using a beamspace representation of MIMO systems, orthogonal radiation patterns form independent sub-channels in a similar manner to the spatial separation of antennas that form spatial sub-channels [33]. Therefore, field distributions in the far-field can be linked to the capacity analysis in beamspace MIMO and then to antenna geometry by using the loaded $N$-port structure [34], [35]. The specific contributions of this work are summarized as follows:

1. We propose to use loaded $N$-port structures to characterize arbitrary antenna geometries given constrained size. These can form the required current distribution on a surface for channel capacity maximization. They are also beneficial for finding the appropriate discrete feeds required in MIMO systems since the feeds for the final potential MIMO antenna are already incorporated at the beginning of the analysis through the loaded $N$-port structure.

2. We use an approach based on the Theory of Characteristic Modes (TCM) and a beamspace representation of MIMO systems to formulate the channel capacity problem. This formulation bridges the gap between electromagnetic propagation and information theory. The method is general and can be applied to arbitrary antenna types that are lossy or lossless while being immersed in an arbitrary power angular spectrum (PAS) [36].

3. We provide closed-form expressions for capacity bounds considering practical constraints including the current norm, radiated power as well as dual constraints. In particular, the number of effective ADoF can be directly obtained given these practical constraints, and these can be used in the antenna design for finding the maximum number of antenna ports that can be set in a given structure with size constraint.

4. We propose an efficient optimization method, based on alternating optimization, for finding the optimal MIMO antenna with negligible mutual coupling among ports. By using the optimization method, we can obtain the optimal MIMO antenna configuration with the desired number of feeding ports while achieving performance close to the channel capacity bounds. In the design, the feeding ports are decoupled with each other so that further signal processing to handle coupling is not needed at the receiver side.

5. We simulate the channel capacity of a 2D planar surface using our proposed method, which matches well with previous work. We also simulate a realistic MIMO antenna design having one square wavelength size taking polarization and PAS into account [36]. Simulation results show that with practical constraints, at least 18 ports can be supported in a one wavelength square structure to achieve the continuous-space electromagnetic channel capacity bound. An example design for a 20-port antenna in a one square wavelength area that achieves the capacity bounds is also provided, demonstrating the effectiveness of the proposed method.

It is important to note that previous studies only focus on obtaining the channel capacity bound for a given aperture [6], [7], [8], [9], [10], [11], [12], [13] or designing MIMO antennas without linking to theoretical performance bounds [22], [23], [24], [25]. The approach in our work provides a systematic method for designing compact MIMO antennas that can achieve channel capacity bounds, and bridges the gap between the practical design of MIMO antennas and their performance limits predicted by EIT and this has not done before.

**Organization:** Section II formulates the MIMO system model for the loaded $N$-port structures. Section III provides the derivation for the resultant channel capacity with different constraints. Section IV introduces the link with antenna design and describes efficient optimization approaches to obtain the optimal antenna configuration. In Section V, we provide numerical results of channel capacity for a proposed MIMO antenna design to demonstrate the potential of the technique. Section VI concludes the work.

**Notation:** Bold lower and upper case letters denote vectors and matrices respectively. Upper case letters in calligraphy refer to the transpose, conjugate transpose, $A^T$, $A^H$, $[A]_{i,j}$, $\text{Tr}(A)$ refer to the transpose, conjugate transpose, (i,j)th entry, determinant, and trace of a matrix $A$, respectively. $\mathbb{C}$ denotes complex number sets and $\sqrt{-1}$ denotes an imaginary number. $CN(\mu, \sigma^2)$ denotes complex Gaussian distribution with mean $\mu$ and variance $\sigma^2$. $U_N$ denotes an $N \times N$ identity matrix. $\text{diag}(a_1, \ldots, a_N)$ is a diagonal matrix with diagonal entries being $a_1, \ldots, a_N$. $\langle a, b \rangle = a^H b$ refers to the inner product of two vectors $a$ and $b$. It should also be noted that the dependence of all variables on frequency is assumed and it is not explicitly shown for brevity.
feeding ports while achieving performance close to the channel current distribution that achieves the channel capacity bound. The challenge is to find structures that can create the necessary current distribution \( j(\mathbf{r}) \) with discrete feeding ports required in MIMO antenna design.

Fig. 1. Illustration of a current distribution \( j(\mathbf{r}) \) in a confined volume \( V \) represented by (a) continuous space and (b) approximated by a loaded \( N \)-port discretized structure.

**II. SYSTEM MODEL**

In Fig. 1(a), an arbitrary transmit element is shown, confined to a volume \( V \), with current distribution \( j(\mathbf{r}) \), where \( \mathbf{r} \) is the spatial coordinate. When matched with the proper receive element and channel, and with the current distribution \( j(\mathbf{r}) \) appropriately set, the resulting system can achieve the continuous-space electromagnetic channel capacity bound [11]. Our objective in this work is to design the optimal MIMO antenna that can best approximate the required current distribution over all frequencies of interest.

By setting the separations between ports in the structure to \( \mathbf{r} \in [r_1, r_2, \ldots, r_N] \) and interpreted as being related to the ADoF of the structure. The problem of finding the optimal \( N \)-port structure to approach the continuous-space electromagnetic channel capacity bound \([11]\). Our objective in this work is to design the optimal MIMO antenna that can best approximate the required current distribution over all frequencies of interest.

In Fig. 1(b), an arbitrary transmit element is shown, confined to a volume \( V \), with current distribution \( j(\mathbf{r}) \), where \( \mathbf{r} \) is the spatial coordinate. When matched with the proper receive element and channel, and with the current distribution \( j(\mathbf{r}) \) appropriately set, the resulting system can achieve the continuous-space electromagnetic channel capacity bound [11].

For the purpose of analysis, we write \( \mathbf{i} = [i(r_1), i(r_2), \ldots, i(r_N)]^T \in \mathbb{C}^N \times 1 \) as a vector consisting of current (with amplitude and phase) at each of the ports in Fig. 1(b) [39]. Correspondingly, the far-field radiation patterns arising from the unit port currents are written as \( \mathbf{e}_T = [e_{T,1}, e_{T,2}, \ldots, e_{T,N}] \in \mathbb{C}^{2K \times N} \), where \( e_{T,n} = [e_{T,\theta,n}, e_{T,\phi,n}]^T \in \mathbb{C}^{2K \times 1} \) is the radiation pattern of the \( n \text{th} \) port excited by a unit current when all the other ports are open-circuited. \( e_{T,\theta,n} \in \mathbb{C}^{K \times 1} \) and \( e_{T,\phi,n} \in \mathbb{C}^{K \times 1} \), refer to the \( \theta \) and \( \phi \) polarization components of \( e_{T,n} \) over \( K \) uniformly sampled three-dimensional (3D) spatial angles. In our work, \( \mathbf{E}_T \) can be found using CST studio suite electromagnetic field simulation software [40].

We use the results in [36] to decompose the radiation patterns, \( \mathbf{E}_T \), into a natural set of orthonormal modal functions similarly to TCM [31], [42]. These orthonormal modal functions can then be used for analysis of the channel capacity and interpreted as being related to the ADoF of the structure. Specifically, for an arbitrary excitation current \( \mathbf{i} \) (which can be regarded as the coefficient imposed on \( \mathbf{E}_T \)), the resulting overall radiation pattern can be expressed as \( \mathbf{e} = \mathbf{E}_T \mathbf{i} = [e_{\theta,n}, e_{\phi,n}]^T \in \mathbb{C}^{2K \times 1} \) [39] so that the radiated power of the loaded \( N \)-port structure is given by

\[
\frac{1}{\eta} \langle \mathbf{e}, \mathbf{e} \rangle = \frac{1}{\eta} \langle \mathbf{E}_T \mathbf{i}, \mathbf{E}_T \mathbf{i} \rangle = \frac{1}{\eta} \mathbf{i}^H \mathbf{E}_T^H \mathbf{E}_T \mathbf{i} = \mathbf{i}^H \mathbf{K}_T \mathbf{i},
\]
where $\eta$ is the free space impedance, $K_T$ is the correlation of the steering matrix at the transmitter side [36], which is defined as

$$K_T = \frac{1}{\eta} E_T^H E_T.$$  

(2)

$K_T$ can be intuitively interpreted as the radiation resistance matrix. The diagonal entries of $K_T$ refer to the radiation resistance of each port and the off-diagonal entries of $K_T$ indicate the mutual coupling between ports. As a result, $K_T$ becomes diagonal when there is no mutual coupling among all ports while $K_T$ has both diagonal and non-diagonal entries when ports are coupled with each other. It is different from the resistance part of the input $N$-port impedance matrix which includes antenna losses consisting of ohmic and dielectric losses [36], [43].

We can perform eigenvalue decomposition (EVD) on $K_T$ to decompose the spatial correlation of radiation patterns $E_T$ as

$$K_T = \Lambda Q A \Lambda^H,$$  

(3)

where $\Lambda = \text{diag} (\lambda_1, \lambda_2, \ldots, \lambda_N)$ is a real diagonal eigenvalue matrix. It is assumed that there is no inner resonance in the antenna losses consisting of ohmic and dielectric losses [36], [43].

The orthonormal basis set with unity radiated power can be formed since the inner product of two radiation patterns excited by $q_i$ and $q_j$ is given by

$$\frac{1}{\eta} (f_{T,i}, f_{T,j}) = \frac{1}{\eta} (E_T q_i, E_T q_j) = \frac{1}{\eta} q_i^H E_T^H E_T q_j = \frac{1}{\eta} q_i^H \Lambda q_j = \delta_{ij} \lambda_i,$$  

(4)

where $\delta_{ij}$ is the Kronecker delta function (0 if $i \neq j$ and 1 if $i = j$). That is, the radiation patterns excited by different column vectors in $Q$ are orthogonal. The $\lambda_i, i = 1, 2, \ldots, N$, can be regarded as the radiation resistance of the $i$th radiation pattern since when excited by $q_i$ (whose $l_2$-norm is unity), the radiated power of $f_{T,i}$ is $\lambda_i$. It should be highlighted that this decomposition is general for arbitrary propagation environments characterized by PAS [36], where the only difference in (3) is that the current basis in $Q$ and the corresponding eigenvalues in $\Lambda$ would be changed.

The orthonormal basis set with unity radiated power can be written as

$$B_T = E_T Q \Lambda^{-\frac{1}{2}},$$  

(5)

so that we have $\frac{1}{\eta} B_T^H B_T = U_N$ with $B_T = [b_{T,1}, b_{T,2}, \ldots, b_{T,N}] \in \mathbb{C}^{N \times N}$. The diagonal entries in $\Lambda^{-\frac{1}{2}}$ are scaling factors to form the orthonormal basis. Therefore, for those basis with small radiation resistances, large currents are required to radiate patterns with unity power.

By replicating the approach for the receive volume, we respectively denote the steering matrices of the $M$ receive antennas as $E_R = [e_{R,1}, e_{R,2}, \ldots, e_{R,M}] \in \mathbb{C}^{K \times 1}$ where $e_{R,m} \in \mathbb{C}^{K \times 1}, m = 1, 2, \ldots, M$.

The final step is to connect the transmitter and receiver using an appropriate channel model. The beamspace domain or virtual channel representation [33], [44] is a well established approach that decomposes the channel into orthogonal beams. This model fits naturally with the modal decompositions we have formed at the transmitter and receiver. Using the beamspace approach, the equivalent channel matrix in the angular domain can be expressed as

$$H = \frac{1}{\eta} E_R^H H_v E_T,$$  

(6)

where $H_v \in \mathbb{C}^{K \times K}$ is the virtual channel whose entries refer to the channel gain from each angle of departure (AoD) to each angle of arrival (AoA) [44]. By taking (6) we can write the overall system model as

$$y = \frac{1}{\eta} E_R^H H_v E_T i + n,$$  

(7)

where $n \in \mathbb{C}^{M \times 1}$ is the additive Gaussian noise and satisfies the complex Gaussian distribution $\mathcal{CN} (0, \sigma^2 I_N)$ with noise power $\sigma^2$ and $y \in \mathbb{C}^{M \times 1}$ is the received signal.

III. CHANNEL CAPACITY FORMULATION

Using (6) and (7), channel capacity of the system in the beamspace domain can be written as

$$C = \log_2 \left| U_M + \frac{\text{HR}_v H^H}{\sigma^2} \right|,$$  

(8)

where $R_v = E \{ i | H \}$ is the covariance matrix of current.

We assume the $M$ receive antennas ($N \leq M$ for tractability) are ideally isolated so that the steering matrix $E_R$ is an exactly orthonormal basis set of receive antennas satisfying $\frac{1}{\eta} E_R^H E_R = U_M$. Using (5), we can rewrite the channel matrix (6) as

$$H = \frac{1}{\eta} E_R^H H_v B_T \Lambda^{-\frac{1}{2}} Q^H = H_{\text{id}} \Lambda^{-\frac{1}{2}} Q^H,$$  

(9)

where $H_{\text{id}} = \frac{1}{\eta} E_R^H H_v B_T \in \mathbb{C}^{M \times N}$ with each entry following i.i.d. distribution due to the orthonormality among radiation patterns in $E_R$ and $B_T$. The capacity (8) is then rewritten using (9) as

$$C = \log_2 \left| U_M + \frac{H_{\text{id}} \Lambda^{-\frac{1}{2}} Q^H R_v Q \Lambda^{-\frac{1}{2}} H_{\text{id}}^H}{\sigma^2} \right|.$$  

(10)

Channel capacity is constrained by physical limits consisting of 1) the radiated power constraint $\text{Tr} \left( \frac{1}{\eta} E_T R_T E_T^H \right) = \text{Tr} (R_T K_T) \leq P_{\text{rad}}$ with $P_{\text{rad}}$ being the upper bound of radiated power and 2) the maximum currents that can exist at the input of the ports of the reactively loaded structure, $\text{Tr} (R_T) \leq I_{\text{in}}$ with $I_{\text{in}}$ being the upper bound of the current norm. In practice, both constraints need to be applied to obtain an implementable antenna. A constraint on the input power (it would be the same as $P_{\text{rad}}$ if the antenna was lossless) cannot
be formulated since an expression for the input impedances are not known as we do not yet know the final feed arrangement.

In the following subsections, the formulation of capacity with two constraints are derived, which is general and can be applied to arbitrary $N$-port antenna systems.

A. Capacity With Radiated Power Constraint

We firstly constrain the radiated power for the derivation of channel capacity. This optimization problem can be formulated as

$$\max_{R_i} \log_2 \left( U_M + \frac{H_{iid} \Lambda \gamma^H R_i Q \Lambda \gamma H_{iid}}{\sigma^2} \right)$$

subject to

$$\text{Tr} \left( R_i K_T \right) \leq P_{\text{rad}}.$$  \hspace{1cm} (11)

By decomposing the arbitrary radiation pattern $e$ onto the set of orthonormal radiation pattern basis in $B_T$, we have

$$e = E_T i = B_T \Lambda \gamma^H i = B_T \beta$$

with

$$\beta = \Lambda \gamma^H i$$

so that each entry in $\beta$ refers to the magnitude allocated to each orthonormal pattern basis. The radiated power constraint (12) can be transformed to

$$\text{Tr} \left( R_i K_T \right) = \text{Tr} \left( \Lambda \gamma^H R_i Q \Lambda \gamma^{H/2} \right) = \text{Tr} \left( \{ \beta \beta^{H/2} \} \right) = \text{Tr} \left( R_{\beta} \right) \leq P_{\text{rad}}.$$  \hspace{1cm} (15)

where $R_{\beta} = \{ \beta \beta^{H/2} \}$ is the covariance matrix of orthonormal pattern basis magnitude. Accordingly, the optimization problem (11) and (12) is re-formulated as

$$\max_{R_{\beta}} \log_2 \left( U_M + \frac{H_{iid} \Lambda \gamma^H R_{\beta} Q \Lambda \gamma H_{iid}}{\sigma^2} \right)$$

subject to

$$\text{Tr} \left( R_{\beta} \right) \leq P_{\text{rad}}.$$  \hspace{1cm} (16)

It can be noticed that the problem (16) and (17) is the same as the capacity maximization problem of ideal $M \times N$ MIMO. However, due to some extremely small eigenvalues in $\Lambda$, the norm of current $i$ could be extremely large and not implementable [45]. In other words, the maximum current is unconstrained.

B. Capacity With Current Constraint

Next, we consider imposing a constraint on the current norm $\text{Tr} \left( R_i \right)$. The optimization problem of channel capacity can then be formulated as

$$\max_{R_i} \log_2 \left( U_M + \frac{H_{iid} \Lambda \gamma^H R_i Q \Lambda \gamma H_{iid}}{\sigma^2} \right)$$

subject to

$$\text{Tr} \left( R_i \right) \leq I_{in}^2.$$  \hspace{1cm} (18)

By decomposing the current $i$ onto the set of current basis $q_i$, $i = 1, 2, \ldots, N$, we can obtain the coefficient for the decomposition as $\gamma_i = \{ q_i, i \} = q_i^H i$ which refers to the magnitude allocated to the $i$th current basis. We collect the coefficient $\gamma_i$ into a vector $\gamma = \{ \gamma_1, \gamma_2, \ldots, \gamma_N \}^T$ which can be related to $i$ by

$$\gamma = Q^H i.$$

$\gamma$ has the same norm as $i$ so that the current constraint (19) can be transformed to

$$\text{Tr} \left( R_i \right) = \text{Tr} \left( Q^H R_i Q \right) = \text{Tr} \left( E \left\{ \gamma \gamma^H \right\} \right) = \text{Tr} \left( R_{\gamma} \right) \leq I_{in}^2,$$  \hspace{1cm} (21)

where $R_{\gamma} = E \left\{ \gamma \gamma^H \right\}$ is the covariance matrix of the current basis magnitude. Then the problem (18) and (19) can be transformed to

$$\max_{R_{\gamma}} \log_2 \left( U_M + \frac{H_{iid} \Lambda \gamma^H R_{\gamma} Q \Lambda \gamma H_{iid}}{\sigma^2} \right)$$

subject to

$$\text{Tr} \left( R_{\gamma} \right) \leq I_{in}^2.$$  \hspace{1cm} (23)

The optimal solution for the currents can be found by performing equal power (EP) or water-filling (WF) allocation directly on $R_{\gamma}$.

It can be observed that due to the existence of $\Lambda^{1/2}$ in (22), the radiated power of each orthogonal basis in $B_T$ is not equal and proportional to the corresponding eigenvalues in $\Lambda$. Therefore, in this formulation, the WF method tends to allocate more power to those basis with larger eigenvalues since it increases radiated power and resulting system capacity. In other words, the radiated power is unconstrained.

C. Capacity With Dual Constraints

Finally, we consider dual constraints imposed by the current norm and radiated power simultaneously. That is, in practical setups the radiated power is limited and the norm of current $i$ must also be restricted. This optimization problem is formulated as

$$\max_{R_{\gamma}} \log_2 \left( U_M + \frac{H_{iid} \Lambda \gamma^H R_{\gamma} Q \Lambda \gamma H_{iid}}{\sigma^2} \right)$$

subject to

$$\text{Tr} \left( R_{\gamma} \right) \leq I_{in}^2,$$  \hspace{1cm} (24)

$$\text{Tr} \left( R_i \right) \leq P_{\text{rad}}.$$

We follow the transformation in (14) as well as the formulation in (16) and (17), so the problem (24) to (26) can be transformed to

$$\max_{R_{\gamma}} \log_2 \left( U_M + \frac{H_{iid} \Lambda \gamma^H R_{\gamma} Q \Lambda \gamma H_{iid}}{\sigma^2} \right)$$

subject to

$$\text{Tr} \left( R_{\gamma} \right) \leq I_{in}^2,$$  \hspace{1cm} (27)

$$\text{Tr} \left( R_i \right) \leq P_{\text{rad}}.$$  \hspace{1cm} (29)

It can be observed that the capacity formulation is only related to $\Lambda$, which is a diagonal matrix with diagonal entries being the eigenvalues of $K_T$.

To solve the problem (27) to (29), we firstly consider EP allocation method. Although the system provides $N$ orthogonal radiation pattern basis, some eigenvalues in $\Lambda$ are too small so that large currents are required to radiate their
corresponding basis (5) which cannot be used for practical MIMO antenna design. To avoid generating large currents, i.e. satisfying the current constraint (29), \( P_{\text{rad}} \) should be allocated to those bases with largest radiation resistance (i.e. larger eigenvalues in \( \Lambda \)). Therefore, we equally allocate power \( P_{\text{rad}} \) to the first \( N_{\text{eff}} \) (\( N_{\text{eff}} \leq N \)) basis and drop the remaining \( N - N_{\text{eff}} \) basis. These \( N_{\text{eff}} \) basis can be effectively used with maximum current norm \( I_{\text{in}}^2 \) and radiated power \( P_{\text{rad}} \), and thus can be regarded as the effective ADoF of the transmitter [35].

The diagonal entries in \( \mathbf{R}_\beta \) are then given by

\[
[\mathbf{R}_\beta]_{i,i} = \begin{cases} 
\frac{P_{\text{rad}}}{N_{\text{eff}}} & i = 1, 2, \ldots, N_{\text{eff}}, \\
0 & \text{otherwise}.
\end{cases}
\] (30)

The constraint (29) can be written as

\[
\text{Tr} \left( \mathbf{R}_\beta \Lambda^{-1} \right) = \sum_{i=1}^{N_{\text{eff}}} \frac{P_{\text{rad}}}{N_{\text{eff}}} \lambda_i^{-1} \leq I_{\text{in}}^2.
\] (31)

We define \( \epsilon = \frac{I_{\text{in}}^2}{P_{\text{rad}}} \) in (31) and it can be interpreted loosely as a conductance which has the unit of \( \Omega^{-1} \). In essence for a given \( P_{\text{rad}} \), a larger \( I_{\text{in}} \) (\( \epsilon \) high) indicates that the antenna will have a smaller input resistance overall and if it is too small the antenna will not be implementable. Alternatively if \( I_{\text{in}} \) is too low ( \( \epsilon \) low), the input impedance will be required to be too high and also not implementable. Therefore we should set it to be in a range centered around 1/50 \( \Omega^{-1} \) so that it is in line with the antennas desired input impedance.

The effective ADoF \( N_{\text{eff}} \) is related to \( \epsilon \) for a fixed SNR by

\[
\sum_{i=1}^{N_{\text{eff}}} \frac{\lambda_i^{-1}}{N_{\text{eff}}} \leq \epsilon
\] (32)

where the left term refers to the average of the reciprocal of eigenvalues for the first \( N_{\text{eff}} \) basis. It can be observed from (32) that a larger \( \epsilon \) indicates that the number of available orthogonal basis, i.e. \( N_{\text{eff}} \), and the consequent capacity bound can be larger. The maximum \( N_{\text{eff}} \) can be obtained by solving (32) for a fixed \( \epsilon \). That is effective ADoF \( N_{\text{eff}} \) is dependent on \( \epsilon \) (the maximum allowable current norm given \( P_{\text{rad}} \)), indicating the number of basis that can be used with the constraint of current norm and radiation power.

Next we consider using the WF method to maximize capacity in problem (27) to (29). We start with the channel gain of \( N \) sub-channels in channel matrix \( \mathbf{H}_{\text{fid}} \). By performing EVD on \( \mathbf{H}_{\text{fid}}^H \mathbf{H}_{\text{fid}} \), we obtain the channel gain matrix \( \mathbf{S} = \text{diag} \left( s_1, s_2, \ldots, s_N \right) \) which is a real diagonal matrix with \( s_i \), \( i = 1, 2, \ldots, N \), being the channel gain of the \( i \)th sub-channel. Then we use the Lagrangian method whose function is given by

\[
L = -\log_2 \left| \mathbf{U}_N + \frac{\mathbf{S}}{\sigma^2} \mathbf{R}_\beta \right| + \mu_{\text{rad}} \text{Tr} \left( \mathbf{R}_\beta - P_{\text{rad}} N \mathbf{U}_N \right) + \mu_{\text{in}} \text{Tr} \left( \mathbf{R}_\beta \Lambda^{-1} - \frac{I_{\text{in}}^2}{N} \mathbf{U}_N \right),
\] (33)

where \( \mu_{\text{rad}} \) and \( \mu_{\text{in}} \) are Lagrangian multipliers. Taking the partial derivative of \( L \) with respect to \( \mathbf{R}_\beta \), we have

\[
\frac{\partial L}{\partial \mathbf{R}_\beta} = -\log_2 e \cdot \mathbf{S} \left( \sigma^2 \mathbf{U}_N + \mathbf{S} \mathbf{R}_\beta \right)^{-1} + \mu_{\text{rad}} \mathbf{U}_N + \mu_{\text{in}} \mathbf{I}_N = 0.
\] (34)

Combining the solution of \( \mathbf{R}_\beta \) in (34), two constraints (28), (29) can be written as [28]

\[
\text{Tr} \left( \mathbf{R}_\beta \right) = \text{Tr} \left( \log_2 e \left( \mu_{\text{rad}} \mathbf{U}_N + \mu_{\text{in}} \mathbf{I}_N \right)^{-1} - \sigma^2 \mathbf{S} \right) = P_{\text{rad}},
\] (35)

\[
\text{Tr} \left( \mathbf{R}_\beta \Lambda^{-1} \right) = \text{Tr} \left( \log_2 e \left( \mu_{\text{rad}} \mathbf{A} + \mu_{\text{in}} \mathbf{U}_N \right)^{-1} - \sigma^2 \left( \mathbf{A} \mathbf{S} \right)^{-1} \right) = I_{\text{in}}^2.
\] (36)

The optimal multipliers \( \mu_{\text{rad}}^* \) and \( \mu_{\text{in}}^* \) in (36) and (35) can be found via binary search [28] and the optimal \( \mathbf{R}_\beta \) can be solved in (34).

It should be noted that the above closed-form expressions for system capacity are formulated without considering the physical realization and practical excitation for MIMO antennas. Therefore, the MIMO antenna structure should be carefully designed and optimized to achieve the capacity bounds with dual constraints, which will be described in the next section.

IV. LOADED N-PORT STRUCTURE AND ANALYSIS

In the previous section, we have provided an optimization formulation to obtain the required port currents on the loaded \( N \)-port structure for capacity maximization with various constraints. In this section, we link those currents to MIMO antenna design with only \( Q \leq N \) feeding ports in the loaded \( N \)-port structures.

A. Network Analysis

To begin to construct the MIMO antenna, we divide the \( N \) ports in Fig. 1(b) into \( Q \) active feeding ports and \( N - Q \) loaded ports. The advantage of this approach is that the feeds for the final potential \( Q \)-port MIMO antenna are incorporated at the beginning of the analysis through the \( N \)-port structure. The equivalent circuit model for the \( N \)-port network with \( Q \) active feeding ports (numbered 1 to \( Q \)) and \( N - Q \) parasitic loaded ports (numbered \( Q + 1 \) to \( N \)) is shown in Fig. 2. The loaded ports are either lossless with inductive and capacitive reactance or open. The active feeding ports are connected to a matching network to achieve the necessary 50 \( \Omega \) input impedance. By optimizing the \( N - Q \) load reactances, we wish to generate \( Q \) orthogonal radiation patterns from the designated \( Q \) feeding ports so that the capacity (8) can be maximized. In essence we will attempt to find loads that create the required current distributions for approaching the capacity bounds. The design procedure for selecting which ports are fed and which are loaded to achieve the optimum channel capacity using (10) is described later.

In Fig. 2, the \( q \)th feeding port is excited by a voltage source \( v_q \), \( q = 1, 2, \ldots, Q \), and the \( (Q + p) \)th parasitic port is loaded with reactance \( x^*_{Q+p} \), \( p = 1, 2, \ldots, N - Q \).
We group the voltage source excitation \( v_q, q = 1, 2, \ldots, Q \), into a vector as \( \mathbf{v}_A = [v_1, v_2, \ldots, v_Q]^T \in \mathbb{C}^{Q \times 1} \) and define \( \mathbf{v}_P = [0, 0, \ldots, 0]^T \in \mathbb{C}^{(N-Q) \times 1} \) as a \((N-Q)\)-dimension zero vector due to there being no excitation at the parasitic ports. We also group current at the feeding and parasitic ports into vectors as \( \mathbf{i}_A = [i_1, i_2, \ldots, i_Q]^T \in \mathbb{C}^Q \) and \( \mathbf{i}_P = [i_{Q+1}, i_{Q+2}, \ldots, i_N]^T \in \mathbb{C}^{(N-Q) \times 1} \), respectively. The voltage and current in the \( N \)-port network are then related by [46]

\[
\begin{bmatrix}
\mathbf{v}_A \\
\mathbf{v}_P 
\end{bmatrix} =
\begin{bmatrix}
\mathbf{Z}_A & \mathbf{Z}_{AP} \\
\mathbf{Z}_{PA} & \mathbf{Z}_P + j\mathbf{X}_L 
\end{bmatrix}
\begin{bmatrix}
\mathbf{i}_A \\
\mathbf{i}_P 
\end{bmatrix},
\tag{37}
\]

where \( \mathbf{X}_L = \text{diag}(x_{Q+1}^L, x_{Q+2}^L, \ldots, x_N^L) \) represents the load reactance connected to each parasitic port. \( \mathbf{Z}_A \in \mathbb{C}^{Q \times Q} \) and \( \mathbf{Z}_P \in \mathbb{C}^{(N-Q) \times (N-Q)} \) are the impedance sub-matrix of the \( Q \) feeding ports and the \( N - Q \) parasitic ports, respectively. \( \mathbf{Z}_{AP} \in \mathbb{C}^{Q \times (N-Q)} \) and \( \mathbf{Z}_{PA} \in \mathbb{C}^{(N-Q) \times Q} \) are the sub-matrix referring to the mutual impedance between feeding and \( N - Q \) parasitic ports with \( \mathbf{Z}_{AP} = \mathbf{Z}_{PA}^T \). From (37) we can obtain the relationship between the current on the feeding and parasitic ports as

\[
\mathbf{i}_P = - (\mathbf{Z}_P + j\mathbf{X}_L)^{-1} \mathbf{Z}_{PA}\mathbf{i}_A.
\tag{38}
\]

Our interest is obtaining \( Q \) orthogonal far-field radiation patterns to form the beamspace MIMO system. To that end, we collect the radiation pattern of each feeding port \( \mathbf{e}_{T,q}(\Omega) \), \( q = 1, 2, \ldots, Q \), in the antenna system into matrix form \( \mathbf{E}_T = [\mathbf{e}_{T,1}(\Omega), \mathbf{e}_{T,2}(\Omega), \ldots, \mathbf{e}_{T,Q}(\Omega)] \), which can be given as

\[
\mathbf{E}_T(\Omega, \mathbf{X}_L) = \mathbf{E}_{A}(\Omega) - \mathbf{E}_P(\Omega)(\mathbf{Z}_P + j\mathbf{X}_L)^{-1}\mathbf{Z}_{PA},
\tag{39}
\]

where \( \Omega = (\theta, \phi) \) denotes the spatial angle with \( \theta \) and \( \phi \) representing the elevation and azimuth angles in spherical coordinates, respectively. \( \mathbf{E}_{A}(\Omega) = [\mathbf{e}_{A,1}(\Omega), \ldots, \mathbf{e}_{A,Q}(\Omega)] \) collects \( Q \) open-circuit radiation patterns of the feeding ports with \( \mathbf{e}_{A,q}(\Omega) \), \( q = 1, 2, \ldots, Q \) being the open-circuit radiation pattern of the \( q \)th feeding port excited by a unit current when all the other feeding and parasitic ports are open-circuit. Similarly, \( \mathbf{E}_P(\Omega) = [\mathbf{e}_{P,Q+1}(\Omega), \ldots, \mathbf{e}_{P,N}(\Omega)] \) collects \( N - Q \) open-circuit radiation patterns of the parasitic ports with \( \mathbf{e}_{P,q}(\Omega) \), \( p = 1, 2, \ldots, N - Q \), being the open-circuit radiation pattern of the \((Q + p)\)th parasitic port excited by a unit current when all the other feeding and parasitic ports are open-circuited. It can be observed that \( \mathbf{E}_T \) consists of the original open-circuit radiation pattern of the \( Q \) feeding ports \( \mathbf{E}_A \) and a perturbation term affected by the load reactance \( \mathbf{X}_L \) across the parasitic ports.

We use a full electromagnetic solver, CST studio suite [40], to simulate the open-circuit radiation patterns of all \( N \) ports of the loaded \( N \)-port structure. It should be noted that the simulation only needs to be performed once because any radiation pattern excited by any current distribution can then be found by using (39) [47]. This reduces the computation complexity enormously as full electromagnetic simulation is not needed during the load optimization and capacity simulation stages.

### B. Optimization

To obtain the optimal design of the MIMO antenna, we wish to select \( Q \) active feeding ports and optimize the load reactances at the \( N - Q \) parasitic ports. The objective is to generate a set of orthogonal radiation patterns in \( \mathbf{E}_T \) at \( Q \) feeding ports so that the resulting channel capacity can be maximized.

To achieve an optimization result that also meets practical design constraints, we need to note that not all ports would be suitable for use as feeding. For example, ports positioned on the edges of the structure are more likely to be accessible for feeds than those completely surrounded by ports. For this reason, we partition the \( N \) ports into two sets for optimization. Assume \( S (S > Q) \) out of \( N \) ports are feasible feeding ports and whose indices in the set are given by \( S = \{1, 2, \ldots, S\} \). The objective is therefore to find the optimum \( Q \) ports from the \( S \) feasible ports where the set of indices for the \( Q \) feeding ports is given by \( \mathcal{P} = \{p_1, p_2, \ldots, p_Q\} \subset S \). In addition, we must find the optimum loads for the remaining \( N - S \) ports so that together with the \( S - Q \) unselected feeding ports, the load reactance matrix \( \mathbf{X}_L \) can meet the requirements of orthogonality for the \( Q \) selected feeding ports.

We use the correlation coefficient between the radiation patterns of two feeding ports, i.e. \( \rho_{r,j}(\Omega, \mathbf{X}_L) \) of the \( j \)th feeding port and \( \mathbf{e}_{r,k}(\Omega, \mathbf{X}_L) \) of the \( k \)th feeding port, to evaluate their similarity, and this is defined as

\[
\rho_{jk}(\mathbf{X}_L) = \frac{\langle \mathbf{e}_{r,j}(\Omega, \mathbf{X}_L), \mathbf{e}_{r,k}(\Omega, \mathbf{X}_L) \rangle}{\| \mathbf{e}_{r,j}(\Omega, \mathbf{X}_L) \| \| \mathbf{e}_{r,k}(\Omega, \mathbf{X}_L) \|}
\tag{40}
\]

where \( \rho_{jk} \) satisfies \( 0 \leq |\rho_{jk}| \leq 1 \). When \( |\rho_{jk}| = 0 \), \( \mathbf{e}_{r,j} \) and \( \mathbf{e}_{r,k} \) are orthogonal to each other. Leveraging the correlation coefficient (40), we can formulate the optimization problem as

\[
\min_{\mathbf{P}, \mathbf{X}_L} \sum_{j \neq k} \sum_{p \in \mathcal{P}, p \neq j} |\rho_{jk}(\mathbf{X}_L)|.
\tag{41}
\]

It should be noted that the optimization variables \( \mathcal{P} \) and \( \mathbf{X}_L \) are highly coupled with each other in this problem. This is because altering feeding port indices greatly changes the
impedance matrix \( Z_A, Z_{AP}, \text{ and } Z_P \) in (37) and the resulting optimal \( X^L \). In addition, the diagonal entries in \( X^L \) are continuous variables while entries in \( P \) are discrete variables, making the problem (41) complicated to solve. To overcome this difficulty and meet these constraints, we propose the use of the alternating optimization method to iteratively optimize the load reactances and selection of the feeding ports [48], [49]. For simplicity, the \( S-Q \) unused feeding ports are left open in the entire optimization process to ease the computational burden. However they could also be included in the load optimization process if deemed necessary.

To start with, we randomly select \( Q \) feeding ports, out of \( S \), as the initial guess at iteration 0. The set consisting of the indices for \( Q \) initial random feeding ports is given by \( P^{(0)} = \{ p_1^{(0)}, p_2^{(0)}, \ldots, p_Q^{(0)} \} \subset S \) with \( p_q^{(0)} \) being the index of the \( q \)th initial feeding port. Presetting the \( S-Q \) unused feeding ports to be open, we then need to optimize \( N-S \) load reactances at the parasitic ports where the initial guess for the load reactance matrix is given as \( X^{L(0)} = \text{diag}(\infty, \ldots, \infty, 0, \ldots, 0) \) with the first \( S-Q \) diagonal entries being fixed as infinity (open).

At the \( i \)th iteration, we wish to find the load reactances \( X^{L(i)} \) to produce near orthogonal radiation patterns with feeding ports indices set \( P^{(i-1)} \). This can be performed by solving the optimization problem

\[
\min_{X^{L(i)}} \sum_{j \neq k} \sum_{k \in P^{(i-1)}} \sum_{j \in P^{(i-1)}} |\rho_{jk}(X^{L(i)})|,
\]

This makes the correlation coefficient norm between any two patterns in \( E_F \) as close to zero as possible. To solve the unconstrained optimization problem (42), we can use the quasi-Newton method [50] which guarantees convergence to a stationary point of the problem (42). The optimal load reactances at the \( i \)th iteration are denoted by \( X^{L(i)} \).

We then use \( X^{L(i)} \) from (42) to optimize the indices of the feeding ports \( P^{(i)} \), that is,

\[
\min_{P^{(i)}} \sum_{k \in P^{(i)}} \sum_{j \in P^{(i)}} |\rho_{jk}(X^{L(i)})|,
\]

which is an NP-hard optimization problem. To solve this problem, we propose a low-complexity approach which is based on sequentially optimizing each entry in \( P^{(i)} \), i.e. \( p_1^{(i)}, p_2^{(i)}, \ldots, p_Q^{(i)} \), one-by-one. Specifically, we define \( P_q^{(i)} = \{ p_1^{(i)}, \ldots, p_q^{(i)} \} \) as the intermediate state of feeding port indices at the \( i \)th iteration where the indices of the first \( q \) feeds have been optimized and the remaining \( Q-q \) feeds not optimized. When optimizing index \( p_q^{(i)} \) for the \( q \)th feeding port, we fix the other \( Q-1 \) indices and select the optimal \( p_q^{(i)} \) from the remaining \( S-Q+1 \) indices in \( S \) to minimize the correlation coefficient (40) among all \( Q \) ports in \( P_q^{(i)} \), which can be formulated as

\[
\min_{p_q^{(i)}} \sum_{k \in P_q^{(i)} \setminus P_{q-1}} \sum_{j \in P_{q-1}} |\rho_{jk}(X^{L(i)})|,
\]

\[ \text{s.t. } p_q^{(i)} \in S \setminus \left( P_{q-1} - P_{q-1} \right), \]

with \( q \) sequentially taken as \( q = 1, 2, \ldots, Q \) where \( P_{q-1} = P^{(i-1)} \) when \( q = 1 \). It should be noted that by sequentially optimizing each entry in \( P^{(i)} \), the value of the objective function (44) is descending since the optimal feeding port index can be selected to minimize (44) by replacing the corresponding entry in \( P^{(i)} \).

By iteratively optimizing the load reactances and selecting feeding ports, the objective function (42) and (44) can converge to a local optimal solution [49]. Therefore, we obtain the optimal load reactances \( X^{L*} \) of the loaded \( N \)-port structures with \( Q \) feeds with indices of \( P^{*} \). Algorithm 1 summarizes the overall algorithm for optimizing the load reactances and feeding ports. The performance of the algorithm in finding optimal MIMO antenna configuration will be shown in the next section.

As a final note, it is important to highlight that signal processing approaches for conventional MIMO cannot be directly applied to deal with the loaded \( N \)-port structure. This is because a very large number of RF front ends, with signal processing, would be required for all \( N \) ports. In contrast, our approach for the \( N \)-port structure aims to optimize the reactances at the loaded ports and the feeding port positions of the loaded \( N \)-port structure. The resulting MIMO antenna has external feeds limited in number by capacity bounds and from that design conventional signal processing methods can then be applied.

V. NUMERICAL RESULTS

In this section, we firstly compare our proposed method with previous work for a single-polarized 2D square planar surface [11] to verify its accuracy and correctness. Afterwards, we propose a practical dual-polarized MIMO antenna that can approach the continuous channel capacity bound.

For channel capacity simulations with the current constraint, we use the same SNR definition as in [11] which is the ratio of the current norm \( I_{in}^2 \) to the noise power \( \sigma^2 \). While for capacity simulation with the radiated power constraint and also dual constraints, we use the conventional SNR definition, i.e. the ratio of maximum radiated power \( P_{rad}^* \) to the noise power \( \sigma^2 \).

A. Comparison of Channel Capacity

In the first set of simulation results we wish to verify our approach by comparing with previous results [11]. The

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Algorithm 1: The Alternating Optimization method

**Input**: \( P^{(0)}, X^{L(0)} \) (initial values described in text);

1: **Initialization**: \( i = 0; \)
2: **repeat**
   \( i = i + 1; \)
3: Find \( X^{L(i)} \) with \( P^{(i-1)} \) by (42);
4: for \( q = 1; Q \)
5: Find \( p_q^{(i)} \) subject to (45) with \( X^{L(i)} \) by (44);
6: Update \( P_q^{(i)} \);
7: **end**
8: \( P^{(i)} = P_Q^{(i)} \)
9: **until** \( P^{(i)} = P^{(i-1)}; \)

**Output**: \( X^{L*} = X^{L(i)}, P^* = P^{(i)}; \)
previous results consider a continuous source current on a 2D square planar surface with one square wavelength size at both the transmitter and receiver. The 2D surface is discretized into an $N$-port structure as shown in Fig. 3 where discretizations with 4 and 180 ports are shown. This corresponds to dividing the one wavelength square surface into 4 and 100 sub-elements respectively similarly to the discretizations of previous work [11].

To compare our method to previous work [11], we only need to find the required currents at all $N$ ports and do not need to consider finding $N - Q$ loads or $Q$ feeding ports. We can therefore use the results in (22) and (23) directly without considering the loads and feeds to find the port currents.

The same channel setup utilized previously [11] is also invoked and includes single polarization in a rich scattering environment with Rayleigh fading and 2D uniform PAS on the azimuth plane [36]. In addition, we use the current norm constraint so that the capacity optimization problem is formulated as (22) and (23), with WF to find the currents. This is similar to equation (21) in [11].

Simulation results of the system capacity are shown in Fig. 4 for various levels of discretizations. In the figure, the number of discretizations per dimension is used to align with previous work [11]. That is 10 sub-elements per wavelength correspond to 100 sub-elements in total. It can be observed that by using our method, the system capacity approaches that using the method in [11] when the number of sub-elements per wavelength is more than four. It should be noted that there are small fluctuations in the simulated capacity compared with the method in [11]. This is because current on the discrete ports is an approximation to the continuous current on the surface, which is related to the position of the ports in the $N$-port structure. However, the numerical gap becomes smaller when increasing the number of sub-elements per wavelength which provides more accurate modeling of the current distributions on the surface. Therefore, we can conclude that our method can obtain the continuous-space electromagnetic channel capacity bound if the dimension of the sub-element and the separation distance between ports is less than a wavelength (from Fig. 3 greater than 5 sub-elements per wavelength).

### B. MIMO Antenna Capacity Bound

Next, we use the technique in Section IV to design a practical MIMO antenna that approaches the capacity bounds. The frequency considered in the following is 2.4 GHz so that the wavelength is 125 mm.

The first step is to propose a structure that is implementable with $Q$ feeds but general enough to allow the formation of arbitrary current distributions for capacity maximization. As such our proposed $N$-port structure is shown in Fig. 5 where a 2D square copper planar surface with one square wavelength size is again utilized. The copper surface is mounted on a substrate where the copper has electric conductivity of $5.8 \times 10^7$ S/m while the substrate is made with Rogers 5880C which has permittivity of 2.2 and loss tangent of 0.0009. The ohmic and dielectric losses of the material leads to a decrease of eigenvalues in $K_T$ since these losses result in power loss as heat which is not included in the open-circuit radiation patterns in $E_T$. Therefore, those basis with small
eigenvalues (related to superdirectivity phenomena \cite{45}) are severely affected by ohmic losses. However, the ohmic losses have little impact on the basis with large eigenvalues so that the radiation efficiencies of the basis with large eigenvalues are high. In our work, we aim to design a MIMO antenna with limits on the current norm and the excitation of basis patterns with large eigenvalues.

We discretize the 2D copper surface into a 11 × 11 sub-element array where the size of the sub-element is $10 \times 10$ mm$^2$. To implement the feeds, an additional copper plane is placed underneath as a ground plane so that a conventional feed can be connected between the ground and the center of each sub-element as shown in the elevation view in Fig. 5. The $N$ ports of the structure are defined as the ports across each pair of adjacent sub-elements on the surface and also between the ground and the center of each sub-element. That is there are 220 ports on the surface and 121 ports between the ground and sub-elements making up a total of $N = 341$ ports. The $Q$ feeding ports need to be selected from the 121 ports between the ground and sub-elements.

While the ultimate aim is to select $Q$ feeding ports from the ground to sub-element ports, we firstly need to find the required currents on the $N$ ports to optimize capacity and approach the capacity bounds similarly as in Section V.A. This allows us to first determine the capacity performance of the structure with ideal port currents. The final load reactances and feeds arrangements of the antenna that can approximate these currents are found in the next subsection.

In the capacity simulations, we obtain them as a function of $P$ ($P = 1, 2, 3$, etc.) basis with the highest eigenvalues (out of $N = 341$) for transmission. This allows us to determine the number of effective ADoF provided by the structure. In the simulations, we use $P$ ideally isolated antennas at the receiver side. We also assume a rich scattering environment so that entries in $H_v$ satisfy $|H_{ij}| \sim \mathcal{CN}(0, 1)$ for $i, j = 1, 2, \ldots, K$. In addition, dual polarization in a rich scattering environment with Rayleigh fading and 3D uniform PAS over full sphere is utilized. For these reasons the capacity results obtained for this channel will be much greater than that obtained in Section V.A which used only one polarization, a 2D PAS and a receiver antenna that was the same as the transmitter.

1) Capacity Bound With Individual Constraints: We simulate the capacity of the proposed structure when we use the $P$ basis functions with the highest eigenvalues to create a $P \times P$ system with the two individual constraints of current norm and radiated power. The simulated capacity with the two constraints using EP allocation and WF method are shown in Fig. 6. These are benchmarked with the results of an ideal $P \times P$ MIMO system where the transmitter is equipped with $P$ spatially isolated antennas. It can be observed that if we only consider the constraint of radiated power, the simulated capacity of the structure is the same as that of ideal MIMO. However, the current norm in this case can be extremely large and not implementable.

On the other hand, if we only consider the constraint of current norm, the capacity of the loaded structure is higher than the ideal MIMO systems for the first few basis. This is because those basis with large eigenvalues have larger radiated power than the ideal MIMO system. However, when we consider more basis, e.g. more than 50, the increase in capacity is negligible since those basis have small eigenvalues and only radiate limited power. Therefore, in this case the WF method can allocate more power to these 50 basis with large eigenvalues to improve the capacity.

2) Capacity Bound With Dual Constraints: Next we consider imposing dual constraints for the simulation of channel capacity and this leads us to consider $\epsilon$ and $N_{\text{eff}}$. In Fig. 7, we plot the minimum $\epsilon$ for the first $N_{\text{eff}}$ basis as calculated.
The WF method allocates most power to the first decrease since equality in dual constraints cannot be achieved. It can be noticed that $\epsilon$ is an increasing function of $N_{\text{eff}}$ as expected.

For an ideal lossless antenna, the input impedance is $Z_{\text{in}} = 50\ \Omega$, i.e., $P_{\text{rad}} = Z_{\text{in}} I_{\text{in}}^2$. Therefore, a region of interest on Fig. 7, is when $\epsilon = I_{\text{in}}^2/P_{\text{rad}} = 1/Z_{\text{in}} = 0.02$. In this region $N_{\text{eff}}$ is approximately 18 and therefore provides an estimate of the minimum effective ADoF to expect. In essence we can expect to achieve at least 18 ports in the one square wavelength area. We consider it as a lower bound in practice, depending on the exact relation between $I_{\text{in}}^2$ and $P_{\text{rad}}$. We may be able to achieve more ports because the internal currents in the antenna can be higher than those at the feeds.

In Fig. 8 we plot the resulting capacity when we use the first $P$ basis functions to create a $P \times P$ system for various $\epsilon$ using EP and WF. It can be observed again that the capacity of the MIMO system using the $N$-port structure increases as $\epsilon$ becomes larger. When considering basis $P$ smaller than $N_{\text{eff}}$, the system capacity of MIMO using the loaded structure is the same as that of ideal $P \times P$ MIMO. However when $P$ approaches $N_{\text{eff}}$, the capacity using EP allocation starts to decrease since equality in dual constraints cannot be achieved. The WF method allocates most power to the first $N_{\text{eff}}$ basis so that the capacity slightly increases when adding more basis to the $P \times P$ MIMO system. In addition, the $\epsilon$ value points where the WF and EP lines separate indicate the number of available basis $N_{\text{eff}}$ (i.e., effective ADoF) provided by the $\epsilon$ value structure as calculated in (32) and approximately match with the results in Fig. 7.

The results in Fig. 7 and Fig. 8 are useful references in designing MIMO antenna for approaching the capacity bound. The use of $\epsilon$ provides us with an estimate of the number of feeding ports or $N_{\text{eff}}$. In particular, we can expect to be able to provide at least 18 ports in the one square wavelength size antenna structure shown in Fig. 5.

C. MIMO Antenna Design

To provide a useful antenna, we need to select feeding ports and the loads for all the ports in the $N$-port loaded structure. The feeding ports and loads must be selected to achieve a close match to the optimum currents found in the previous section for a feasible $P$. This then would provide the optimal antenna configuration that can achieve the channel capacity bound with dual constraints.

As discussed previously for an ideal lossless antenna, the input impedance is $Z_{\text{in}} = 50\ \Omega$ and therefore $\epsilon = I_{\text{in}}^2/P_{\text{rad}} = 1/Z_{\text{in}} = 0.02$ so that $N_{\text{eff}}$ is at least 18. Expecting to have higher currents internally on the antenna, we aim for 20 ports in our design. That is we will need to re-create the currents obtained when $P = 20$ from the previous section.

To reduce the computational effort in finding the loads, we preset the sub-elements in the center row and center column of the 2D copper surface as being connected together (shorted) so there is a single large cross element centered on the surface. That is ports $\{56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 6, 17, 28, 39, 50, 72, 83, 94, 105, 116\}$ in Fig. 5 are shorted together. The cross element separates the other sub-elements into four sub-square arrays. The cross is also isolated from the other sub-elements so that the ports between the cross and the other adjacent sub-elements are open without loads. In effect, we are presetting some of the ports to be shorted or open on the surface. This reduces the computational load of the optimization by reducing the number of load reactances to be found from 220 to 160. In addition, to reduce the search space for the $Q = 20$ feeds we also restrict it to the $S = 64$ ports between the ground plane and the sub-elements on the edge of each sub-square array (instead of all 121 possible locations). For those ports underneath the surface that were not selected as feeds, we set them to be open.

To find the optimum reactances across the 160 ports on the surface and the indices of the 20 feeding ports underneath, we use the method as described in Section IV so
that the excited radiation patterns of the 20 feeding ports are closest to being orthogonal (42). The final indices of the 20 feeding ports after the alternating optimization are given by \{1,3,5,7,9,11,29,33,49,55,67,69,75,89,95,111,113,117,119, 121\} and their physical location can be found from Fig. 5. The 20 feeding ports are then each fed by a feeding probe across the common ground and the center of the selected sub-elements as shown in the elevation view in Fig. 5. It should be noted that the feeding ports of the final optimized MIMO antenna have already been decoupled with each other by the load reactances between sub-elements. Therefore, instead of using complicated multi-port conjugate matching networks [26], the impedance matching for the 20 feeding ports can be performed separately with a conventional II-type matching network at each port to achieve matching to 50 Ω. The return loss (S11) results of the 20 ports are provided in Fig. 9 where the reflected power at 2.4 GHz is around -15 dB and the bandwidth is around 40 MHz.

By using the 20 excited radiation patterns from the resultant loaded structure with 20 feeds, and a beamspace channel model (7), the capacity of the 20 × 20 MIMO system is found and the simulated results are shown in Fig. 10. These are also benchmarked with the capacity bound of the loaded structure without feeds obtained by WF method in Section III-C and the capacity of ideal MIMO. It can be observed that the MIMO antenna using the loaded structure with 20 feeds achieves performance close to ideal MIMO and the capacity bound predicted by Fig. 8 for loaded structure without feeds. The capacity gap is due to the small correlations among the radiation patterns of the feeding ports and power loss from mutual coupling. However, the average advantage of the proposed method is that the optimal antenna structure for the MIMO antennas that can approach channel capacity bounds when constrained by size.

In the proposed method, we derive the closed-form expressions for the channel capacity limits using a beamspace channel model with a current constraint, radiated power constraint as well as with dual constraints. We also introduce a method for antenna design using the loaded ports structure and provide an efficient alternating optimization approach for finding the optimal MIMO antenna configuration to generate orthogonal radiation patterns. This can be used to construct a beamspace MIMO system that approaches the channel capacity bounds.

Simulation results of the channel capacity using our proposed method matches well with previous work. Furthermore, by optimizing the load reactance and feeding port positions in our proposed antenna design with one square wavelength size, the achieved capacity performance is close to the continuous-space channel capacity bounds, demonstrating the effectiveness of the proposed method. In particular we show that at least 18 ports can be supported in a one wavelength square structure to achieve the continuous-space electromagnetic channel capacity bound. It is also shown that the limit on the number of ports is constrained by the maximum current that the antenna can handle. An example design for a 20-port antenna in one square wavelength area that achieves the capacity bounds is also provided. One challenge of the final antenna is the limited bandwidth and this can be addressed by increasing the height in the antenna design.

The approach we have provided allows a wide variety of compact MIMO antenna designs to be investigated and prototyped. Designing a testbed and implementing the loaded N-port structures in a MIMO system together with the application of circuit theory [46] needs to be performed in field experiments to demonstrate the capacity bounds in the future. The connection between the proposed technique and holographic MIMO approaches [51] is also an interesting avenue to explore. Moreover, linking capacity bounds with antenna geometry designs with the effect of 4D electromagnetics [37], [38] in a wideband scenario can also be considered in the future where the maximum bandwidth for the radiation pattern basis can be derived.

VI. CONCLUSION

In this paper, we have proposed a novel method for designing antennas that approach the continuous-space electromagnetic channel capacity bounds. The method can link continuous-space electromagnetic channel capacity bounds to MIMO antenna design. The method is not restricted to a specific antenna configuration and uses loaded N-port structures to discretize the continuous space and represent arbitrary antenna geometries. It is useful for designing compact MIMO antennas that can approach channel capacity bounds when constrained by size.

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Shanpu Shen (Member, IEEE) received the bachelor’s degree in communication engineering from the Nanjing University of Science and Technology, Nanjing, China, in 2013, and the Ph.D. degree in electronic and computer engineering from The Hong Kong University of Science and Technology (HKUST), Hong Kong, in 2017. He was a Visiting Ph.D. Student with the Microsystems Technology Laboratories, Massachusetts Institute of Technology, Cambridge, MA, USA, in 2016. He was also a Post-Doctoral Fellow with HKUST from 2017 to 2018. He was also a Post-Doctoral Research Associate with the Communications and Signal Processing Group, Imperial College London, London, U.K., from 2018 to 2020. He is currently a Research Assistant Professor with HKUST. His current research interests include RF energy harvesting, wireless power transfer, reconfigurable intelligent surface, the Internet of Things, MIMO systems, and antenna design and optimization.

Yujie Zhang (Member, IEEE) received the bachelor’s degree in optoelectronic information science and engineering from the Huazhong University of Science and Technology, Wuhan, China, in 2017, and the Ph.D. degree in electronic and computer engineering from The Hong Kong University of Science and Technology (HKUST), Hong Kong, in 2021. He was a Post-Doctoral Fellow with HKUST from 2021 to 2023. He is currently a Research Fellow with the Department of Electrical and Computer Engineering, National University of Singapore, Singapore. His research interests include the antenna design on the Internet of Things applications, reconfigurable intelligent antenna and surface, MIMO systems, millimeter wave, RF energy harvesting, wireless power transmission, and 6G.

Shiwen Tang (Graduate Student Member, IEEE) received the bachelor’s degree in radio wave propagation and antenna from the University of Electronic Science and Technology of China, Chengdu, China, in 2016, the M.Sc. degree in electronic and electrical engineering from the University of Strathclyde, Glasgow, U.K., in 2018, and the master’s degree in circuits and systems from the University of Electronic Science and Technology of China in 2019. She is currently pursuing the Ph.D. degree with the Department of Electronic and Computer Engineering, The Hong Kong University of Science and Technology (HKUST), Hong Kong. Her current research interests include the millimeter-wave antenna design, MIMO antennas, reconfigurable antenna design, and 6G.

Ross Murch (Fellow, IEEE) received the bachelor’s and Ph.D. degrees in electrical and electronic engineering from the University of Canterbury, New Zealand. He is currently a Chair Professor with the Department of Electronic and Computer Engineering and a Senior Fellow with the Institute of Advanced Study, The Hong Kong University of Science and Technology (HKUST). He is known for his research on multiple antenna technology, including multianisotropic MIMO, compact multiport antennas, and multiport energy harvesting. His current research interests include creating new RF wave technology for making a better world and this includes RF imaging, ambient RF systems, energy harvesting, electromagnetic information theory, 6G, the IoT, multiport antenna systems, and reconfigurable intelligent surfaces. His unique expertise lies in his combination of knowledge from both wireless communication systems and electromagnetics and he publishes in both areas. In total his research contributions include nearly 200 journal publications and more than 20 patents while successfully supervising more than 50 research students. He also has a strong interest in education, enjoys teaching, and has won five teaching awards. He was the Department Head of HKUST from 2009 to 2015. He joined HKUST in 1992 as an Assistant Professor and has remained with HKUST, Hong Kong, since then, where he is also a Chair Professor. From 1990 to 1992, he was a Post-Doctoral Fellow with the Department of Mathematics and Computer Science, University of Dundee, U.K. Prof. Murch is also an IET, HKIE, and FHKEng Fellow and has won several awards, including the Computer Simulation Technology (CST) University Publication Award. He has been a David Bensted Fellow, Simon Fraser University, Canada, an HKTITI Fellow with Southampton University, U.K., and has spent sabbaticals with MIT, USA; AT&T, USA; Allgon Mobile Communications, Sweden; and Imperial College London. He has served IEEE in various positions, including an IEEE area editor, a technical program chair, a distinguished lecturer, and a fellow evaluation committee.

Chi-Yuk Chiu (Senior Member, IEEE) received the B.Eng. and M.Eng. degrees in electronic engineering from the City University of Hong Kong in 2001, 2001, and 2005, respectively. He joined the Department of Electronic and Computer Engineering (ECE), The Hong Kong University of Science and Technology (HKUST), as a Research Associate in 2005. Then, he worked with Sony Mobile Communications, Beijing, as a Senior Antenna Engineer in 2011. He joined ECE, HKUST, as a Research Assistant Professor in 2015. He has published over 100 technical papers, two book chapters, and holds several patents related to antenna technology. His main research interests include the design and analysis of small antennas, MIMO antennas, applications of characteristic modes, and energy harvesting. He is currently the Vice Chair of IEEE Antennas and Propagation Society (AP-S)/Microwave Theory and Technology Society (MTT-S) Hong Kong Joint Chapter, a member of IEEE AP-S Education Committee, IEEE AP-S C. J. Reddy Travel Grant Assistant Coordinator, and a Lead Guest Editor of a Special Section in IEEE OPEN JOURNAL OF ANTENNAS AND PROPAGATION (OJAP).