High-temperature differential emission measure and altitude variations in the temperature and density of solar flare coronal X-ray sources

Natasha L. S. Jeffrey¹, Eduard P. Kontar¹, and Brian R. Dennis²

¹ School of Physics & Astronomy, University of Glasgow, Glasgow G12 8QQ, UK
e-mail: natasha.jeffrey@glasgow.ac.uk
² Solar Physics Laboratory, Code 671, Heliophysics Science Division, NASA Goddard Space Flight Center, Greenbelt, MD 20771, USA

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ABSTRACT

The detailed knowledge of plasma heating and acceleration region properties presents a major observational challenge in solar flare physics. Using the Ramaty High Energy Solar Spectroscopic Imager (RHESSI), the high temperature differential emission measure, DEM(T), and the energy-dependent spatial structure of solar flare coronal sources were studied quantitatively. The altitude of the coronal X-ray source was observed to increase with energy by ~+0.2 arcsec/keV between 10 and 25 keV. Although an isothermal model can fit the thermal X-ray spectrum observed by RHESSI, such a model cannot account for the changes in altitude, and multi-thermal coronal sources are required where the temperature increases with altitude. For the first time, we show how RHESSI imaging information can be used to constrain the DEM(T) of a flaring plasma. We developed a thermal bremsstrahlung X-ray emission model with inhomogeneous temperature and density distributions to simultaneously reproduce i) DEM(T); ii) altitude as a function of energy; and iii) vertical extent of the flaring coronal source versus energy. We find that the temperature-altitude gradient in the region is ~+0.08 keV/arcsec (~1.3 MK/Mm). Similar altitude-energy trends in other flares suggest that the majority of coronal X-ray sources are multi-thermal and have strong vertical temperature and density gradients with a broad DEM(T).

Key words. Sun: flares – Sun: corona – Sun: X-rays, gamma rays – Sun: particle emission

1. Introduction

Over the past decade, the X-ray imaging spectroscopy of the Ramaty High Energy Solar Spectroscopic Imager (RHESSI; Lin et al. 2002) has allowed changes in the spatial properties of solar flare X-ray sources to be studied in detail. Along with high energy resolution X-ray spectroscopy (≤1 keV at 3 keV increasing to 5 keV at 5 MeV), RHESSI is capable of indirect X-ray imaging using nine rotating modulation collimators (RMCs) giving angular resolutions between 2°3 and 183″. In practice, the angular resolution is usually ≥5″ (full width half maximum – FWHM) due to finite counting statistics and image reconstruction uncertainties. However, RHESSI is capable of inferring changes in the positions of X-ray sources down to the sub-arcsecond level. Sub-arcsecond measurements of X-ray footpoint locations have been achieved using X-ray visibilities (see e.g. Kontar et al. 2008, 2010; Jeffrey & Kontar 2013), improving upon forward fitting a Gaussian source model to the RHESSI modulated lightcurves (e.g. Aschwanden et al. 2002).

Recently, many studies have investigated flares where the majority of X-ray emission comes from a coronal thick-target source (e.g. Veronig & Brown 2004), in contrast to the usual flare that is dominated by footpoint hard X-ray (HXR) emission. The first energy dependent spatial study of such coronal X-ray sources (Xu et al. 2008) and further works (e.g. Guo et al. 2012, 2013) examined how X-ray source lengths (the direction that appears to lie parallel to a guiding magnetic field) changed with X-ray energy. Such observations, with the help of numerical simulation (Jeffrey et al. 2014), have allowed the estimation of coronal plasma number density and the acceleration rate of electrons within the region. Kontar et al. (2011b) found increasing X-ray source widths (defined in the direction perpendicular to the guiding field) with energy, a trend consistent with magnetic turbulence in the flaring coronal source. Jeffrey & Kontar (2013) studied the temporal evolution of radial positions (altitudes), lengths and widths of such coronal X-ray sources.

Changes in coronal X-ray source radial position (or altitude) with X-ray energy have not been studied extensively in the RHESSI era. Unlike the changes in X-ray source size, an increasing altitude with time is often observed for all types of flaring coronal X-ray sources, and the trend is often seen in other wavelengths such as UV, extreme-UV (EUV) and soft X-rays (e.g. Forbes & Acton 1996). In a standard flare model, the upward motion of coronal X-ray sources with time is often explained by the upward motion of a magnetic reconnection site, with loops reconnecting continuously at increasing altitude and then cooling (e.g. Svetska et al. 1987; Tsuneta et al. 1992; Svetska 1996; Gallagher et al. 2002). Sometimes an initial decrease in altitude is followed by an increase in altitude after the impulsive phase of the flare (Sui & Holman 2003; Sui et al. 2004; Veronig et al. 2006; Liu et al. 2009; Joshi et al. 2009; Reznikova et al. 2010; Gosain 2012; Jeffrey & Kontar 2013), and sometimes even more complicated motions are observed (e.g. Liu et al. 2013). These observations are viewed as an argument to support the standard flare scenario involving magnetic reconnection above the coronal source. Despite RHESSI’s unprecedented spectral resolution, the temperature structure (differential
emission measure DEM($T$) above $T \sim 1$ keV remains poorly determined (e.g. Prato et al. 2006).

In this paper, we present a spatial-spectral study of flaring coronal X-ray sources. A limb flare SOL2013-05-13T02:12 is studied in detail (the time of 02:12 UT is the RHESSI peak flux time). We deduce the changes in coronal X-ray source location with energy at a given time and find a relationship between X-ray source energy and altitude. During this study, we are not concerned with the physical cause of the trend, only how RHESSI imaging information can be used to further constrain the DEM($T$), so that it is consistent with both RHESSI spectral and imaging observations. Further, an isothermal model, often used to fit the X-ray spectrum at low energies, is shown to be inconsistent with RHESSI imaging observations for the flares studied.

2. The observation of SOL2013-05-13T02:12

GOES X1.7 flare SOL2013-05-13T02:12 was chosen for detailed analysis because it had a strong coronal X-ray source located at the eastern limb. A limb flare was chosen so that the radial distance changes correspond to height changes with minimal projection effects. This flare has one visible northern footpoint that can be seen up to energies of \(~100\) keV. Figure 1 shows a loop filled with hot plasma imaged with the Solar Dynamics Observatory Atmospheric Imaging Assembly (SDO AIA; Lemen et al. 2012) in the 94 Å passband. RHESSI 10–20 keV and 60–80 keV contours are displayed. The 10–20 keV emission comes from the flare coronal source located above the 94 Å loop, while the bulk of the 60–80 keV emission comes from a northern footpoint source located close to the footpoint of the 94 Å loop. The RHESSI and GOES lightcurves for the flare are shown in Fig. 2. There is a gradual rise in the X-ray flux up to 50 keV from 01:56 UT onwards, with sharper increases in the 50–100 keV and 100–300 keV bands between 01:56 and 02:30 UT.

3. RHESSI spectroscopy and imaging

A spectral analysis of the flare was performed for the time interval from 02:08 to 02:10 UT. The count rate for this X-class flare is high so spectroscopy using a single detector could be performed. Comparison of individual spectra from detectors 1 to 9 showed that detector 6 had the best energy resolution, and shows the spectral features between 6 to 10 keV. As expected, the spectrum during this time interval has a strong thermal component below 30 keV and a power-law spectrum at higher energies up to \(~150\) keV (see Fig. 3). Since RHESSI is in attenuator state A3 (the thin and thick attenuators cover the detectors), the lower energy limit was set to 6 keV. The majority of the counts recorded below 6 keV are not incident photons at that energy; they are from high energy photons producing K-shell escape photons from the germanium detector itself (see e.g. Phillips et al. 2006).

Using the Object Spectral Executive (OSPEX software, Schwartz et al. 2002), the following five functions describing the thermal component were fitted to the background-subtracted count fluxes in the energy range between 6 and 100 keV:

1. The isothermal function (f$_{vth}$), “Fit 1”, provides the temperature $T$ [keV] and emission measure $EM$ [cm$^{-3}$] of the
Table 1. A parameter comparison of the thermal fitting functions shown in Fig. 3.

| Fit function          | $T_1$ [keV] | $T_2$ [keV] | $T_{\text{peak}}$ [keV] | $EM_1 \times 10^{49}$ [cm$^{-3}$] | $EM_2 \times 10^{49}$ [cm$^{-3}$] | DEM (2 keV) $\times 10^{49}$ [keV$^{-1}$ cm$^{-3}$] | DEM ($T_{\text{peak}}$) $\times 10^{49}$ [keV$^{-1}$ cm$^{-3}$] | $\chi^2$ |
|-----------------------|-------------|-------------|--------------------------|----------------------------------|----------------------------------|-------------------------------------------------|------------------------------------------------|-------|
| vth                   | 2.5         | X           | X                        | 0.9                              | X                                | X                                               | X                                               | 1.3   |
| 2vth                  | 1.7         | 2.7         | X                        | 3.4                              | 0.4                              | X                                               | X                                               | 1.0   |
| multi_therm_pow       | X           | X           | 1.4                      | X                                | X                                | 2.0                                             | X                                               | 1.0   |
| multi_therm_gauss     | X           | X           | 1.4                      | X                                | X                                | X                                               | X                                               | 1.0   |
| multi_therm_pow_exp   | X           | X           | 1.4                      | 5.2                              | X                                | X                                               | X                                               | 1.0   |

thermal source. The emission measure and temperature are free parameters while the relative iron abundance is fixed at 1.0 times the CHIANTI atomic database (Dere et al. 1997; Landi et al. 2013) coronal value, which is the default OSPEX value (investigating different relative iron abundances is beyond the scope of the paper). The relative iron abundance is fixed at this value for all fits (1) to (5) (see Table 1 and Fig. 3 for all parameters and fits).

2. The double isothermal function (f_2vth), “Fit 2”, is the sum of two isothermal functions each with their own emission measures ($EM_1, EM_2$) and temperatures ($T_1, T_2$), but with the same, fixed relative iron abundance. $EM_1, EM_2, T_1, T_2$ are all free parameters.

3. A multi-thermal power-law function (f_multi_therm_pow), “Fit 3”, relating the differential emission measure DEM($T$) [cm$^{-3}$ keV$^{-1}$] to the temperature $T$ by,

$$DEM(T) = DEM(T = 2 \text{ keV}) \left(\frac{2}{T}\right)^\beta$$

where $\beta$ is the power-law index and DEM(2 keV) is the differential emission measure at a temperature of 2 keV. The function also provides a minimum and maximum value of temperature $T$. The relative iron abundance is fixed, while all other parameters are free.

4. A multi-thermal Gaussian in log10 $T$ (f_multi_therm_gauss), “Fit 4”, relates the DEM to the temperature $T$ using,

$$DEM(T) = DEM(T_{\text{peak}}) \exp\left(-\frac{(\log_{10} T - \log_{10} T_{\text{peak}})^2}{2\sigma^2}\right)$$

where the DEM is a Gaussian in logarithmic temperature space, $T_{\text{peak}}$ [keV] is the peak temperature of the DEM, $T_{\text{peak}}$ is the DEM at $T_{\text{peak}}$ and $\sigma$ is the standard deviation of the Gaussian in units of log$_{10}$ keV. The relative iron abundance, and the minimum and maximum temperatures, are fixed. DEM($T_{\text{peak}}$), $T_{\text{peak}}$ and $\sigma$ are free parameters.

5. Finally a multi-thermal power-law and exponential function (f_multi_therm_pow_exp), “Fit 5” relates the DEM to the temperature using,

$$DEM(T) = \frac{EM}{\xi T_{\text{peak}}^2 (\xi - 1)} \left(\frac{T_{\text{peak}}}{T}\right)^\xi \exp\left(-\frac{\xi T_{\text{peak}}}{T}\right)$$

where $\Gamma(x)$ is the gamma function, $EM$ [cm$^{-3}$] is the total emission measure integrated from the minimum to the maximum temperature, $T_{\text{peak}}$ is the peak temperature of the DEM and $\xi$ is the power-law index. $EM$, $T_{\text{peak}}$ and $\xi$ are free parameters while the relative iron abundance, and minimum and maximum temperatures, are fixed.

The following functions were also included in each case (see Fig. 3): thick-target bremsstrahlung (f_thick2_vnorm) to account for non-thermal emission from a power-law distribution of electrons above ~30 keV, a pileup correction$^1$, a function to fine tune the RHESSI detector response matrix (f_drm_mod), and line components (f_line) to account for instrumental features near ~10 keV.

Our spectral fit results show that it is impossible to constrain the overall shape of the DEM($T$) below $\sim T = 1.5$ keV with RHESSI data. The isothermal function (which is commonly used in RHESSI spectral analysis, see e.g. Holman et al. 2011; Kontar et al. 2011a, as reviews) is a marginally worse fit in terms of the reduced $\chi^2$ and the residuals below 15 keV (Table 1 and Fig. 3). Otherwise, all thermal models can adequately fit the thermal part of the spectrum. This result clearly demonstrates that a wide range of temperature distributions (summarised in Fig. 4) with various DEM($T$) are consistent with the measured RHESSI count flux spectrum. In Fig. 4, the DEM for Fit 3 (grey), Fit 4 (orange) and Fit 5 (blue) are shown. Over the temperature range of ~1.6 keV to 3.0 keV, the OSPEX model DEMs agree within the model errors (shaded areas), suggesting that RHESSI can constrain the form of the DEM($T$) between this temperature range, regardless of the chosen DEM model. The shaded areas are derived from the formal uncertainties on each free parameter in the models determined assuming Poisson statistics are the only source of error and that all are orthogonal. A more detailed Monte Carlo analysis would be needed to determine these uncertainties more accurately using the methods described in Ireland et al. (2013), but the range of applicability of each model shown in Fig. 4 is adequate for our current purposes.

3.1. RHESSI imaging at various energies in the thermal range

The flare was imaged using RHESSI detectors 3–7 over five energy bands (10–11, 11–13, 13–16, 16–20 and 20–25 keV), where the coronal X-ray emission dominated (Fig. 1). Detectors 1, 2, 8 and 9 were not used. Detector 1 showed no modulation (the source was over-resolved with significant noise), detector 2 is only sensitive above ~20 keV, and the resolutions of detectors 8 and 9 (106″ and 183″ respectively) are larger than the image area of 64″ × 64″. The images were created using two imaging algorithms: CLEAN (Högbom 1974; Hurford et al. 2002) and Visibility Forward Fitting (Vis-FwdFit; Schmahl et al. 2007) using a 1 arcsecond pixel size. CLEAN images show a loop-like structure visible up to the 20–25 keV bin. Such a loop-like structure is well suited for fitting a simple curved

$^1$f_pileup_mod The pile-up component accounts for those photons arriving at the detector at nearly the same time, that are detected as a single count with an energy equal to the sum of the individual photon energies.
Fig. 3. Photon flux spectrum fitted with different functions describing the thermal component. Top left: single isothermal, Top right: double isothermal, Bottom left: multi-thermal Gaussian in log $T$ and bottom right: multi-thermal power-law and exponential. The normalised residuals are plotted below each spectrum, and each is created using only detector 6 and the functions fit the count spectrum between 6 and 100 keV for a chosen time interval of 02:08 to 02:10 UT. The values of all thermal parameters are shown in the figure and in Table 1. The gold lines in each panel represent Gaussian line fits (line) compensating for instrumental anomalies in the RHESSI spectrum. A thick2_vnorm function (light blue) is used to account for the non-thermal X-ray emission at higher energies.

elliptical Gaussian fit, using Vis_FwdFit$^2$, to RHESSI’s X-ray visibilities, so that the location and the characteristic sizes of the X-ray source can be found. The Vis_FwdFit $\chi^2$ values determine whether the parameters provided by the fit are acceptable. We note that Vis_FwdFit actually provides the fitted curve centre of mass and this is not equivalent to the loop top position which is required for the study. Instead, the image peak is found using the SolarSoftWare (SSW) routine parakeak.pro. This program estimates the peak position by performing a parabolic fit to the image data. An error is found using the standard deviation of the parabolic fit. The CLEAN image peak is also found using the same method so that the positions provided by each algorithm can be compared. Figure 5 shows a CLEAN image for an energy range of 13–16 keV. Vis_FwdFit 50% contours at 10–11 keV, 13–16 keV and 20–25 keV are displayed. Figure 5 also shows a footpoint source at 60–80 keV.

3.2. The relationship between X-ray energy and radial distance

Using the CLEAN and Vis_FwdFit image peak positions $(x, y)$ provided by parakeak.pro, we calculated the radial distances $R = \sqrt{x^2 + y^2}$ (measured from the solar disk centre) for each algorithm at each energy range. An error for $R$ is found from error propagation. Figure 5 (right) shows that the calculated $R$ errors are small for this flare (approximately half an arcsecond or less) at the chosen energy ranges and time, and that the data from two imaging algorithms give a similar trend – the higher the energy, the higher the location of the peak. Both results can be well-fitted by a straight line, so that the observed relationship between radial distance $R$ and energy $\epsilon$ is approximated by

$$\frac{dR(\epsilon)}{d\epsilon} = \alpha, \quad \text{[arcsec keV}^{-1}\text{]}$$

where $\alpha$ is a constant gradient. Both algorithms give a similar gradient $dR/d\epsilon$ so close to $0.24 \pm 0.02$ arcsec/keV for Vis_FwdFit and $0.20 \pm 0.01$ arcsec/keV for CLEAN, which is important for the analysis. Comparing both algorithms, the absolute values of $R(\epsilon)$ are shifted by $\pm 0.73 - 0.74$, with the CLEAN values at a lower height than the Vis_FwdFit values. This is due to differences in the way each algorithm reconstructs an image. The CLEAN algorithm creates an image by finding point sources and convolving them with the instrumental point spread function (PSF). The image peak simply corresponds to the brightest point smoothed out by the PSF. Since Vis_FwdFit fits a chosen distribution such as a Gaussian to the X-ray visibilities, the peak of a given source is determined by the fitted distribution to

2 http://hesperia.gsfc.nasa.gov/rhessi3/software/imaging-software/vis-fwdfit/index.html
The peak of X-ray emission and DEM(T)

From RHESSI observations (see Fig. 5), we found that the flare coronal X-ray source altitude increases with energy, according to the approximate linear relation (4). Hence

$$\frac{dz}{d\epsilon} = \alpha$$

(5)

where \(z\) is the height above the solar limb.

Our task is to find an analytical model that can produce the observed \(z\) versus \(\epsilon\) trend and relate such changes to a temperature \(T(z)\) structure and a number density \(n(z)\) structure in the flaring coronal region. To do this, we derive a relationship between the altitude \(z\) of the X-ray source and energy \(\epsilon\).

Since the spectroscopy results for the flare show that the spectrum in the range \(\sim 6\) to 25 keV can be well-fitted by multi-thermal models (Fig. 3), we assume that all the coronal X-rays in this range are emitted as multi-thermal bremsstrahlung. This assumption is supported by the fact that the thermal emission dominates the non-thermal emission in this range. The spectroscopy results (Fig. 3) show that the thermal and non-thermal components are equal at \(\sim 28\) keV with the thermal component being increasingly dominant at lower energies. Consequently, we assume that all the emission in the 6 to 25 keV energy range is multi-thermal bremsstrahlung from the flaring coronal source.

Consider an emitting volume \(dV = Adz\), where \(A\) is the X-ray source area perpendicular to the radial direction (altitude) and image plane, as shown in Fig. 7. The photon flux emitted as thermal bremsstrahlung per unit energy \(\epsilon\) per unit height \(z\) from a plasma characterised by a temperature \(T\) [keV] and number density \(n\) [cm\(^{-3}\)] is given by e.g. Tandberg-Hanssen & Emslie (1988)

$$J(\epsilon, z) = n^2(z)A(z)\frac{\epsilon}{e^{\frac{\epsilon}{T(z)}}} \exp\left(-\frac{\epsilon}{T(z)}\right)$$

(6)

where \(J(\epsilon, z)\) is the photon flux per unit of height [photons \(s^{-1} \text{ cm}^{-2}\text{ keV}^{-1}\text{ cm}^{-1}\)].

The total photon flux [photons \(s^{-1} \text{ cm}^{-2}\text{ keV}^{-1}\)] integrated over all \(z\) is then given by,

$$I(\epsilon) = \int_0^\infty J(\epsilon, z)dz.$$  

(7)

At any observed energy \(\epsilon\), X-ray flux \(J(\epsilon, z)\) should have maximum at \(z(\epsilon)\) determined by the derivative

$$\frac{\partial J}{\partial z} = \frac{n^2 A}{e^{\frac{\epsilon}{T(z)}}} \exp\left(-\frac{\epsilon}{T(z)}\right)$$

(8)

Re-arranging Eq. (8) for \(\epsilon(z)\) one finds,

$$\epsilon(z) = \frac{T(z)}{2} = \frac{d(n^2 A)}{dz} T^2 \frac{T}{n^2 A}$$

(9)

and we can write:

$$\epsilon(z) = \frac{T(z)}{2} \frac{d(n^2 A)}{dT} T^2 n^2 A$$

(10)

The plasma density \(n(z)\) and temperature \(T(z)\) also determine the differential emission measure

$$\text{DEM}(T) = n^2 \frac{dV}{dT}$$

Using the definition of \(\text{DEM}(T)\), one can write

$$\text{DEM}(T) = n^2 \frac{dV}{dT} |_{z=T(T)} = n^2 A \frac{dz}{dT} |_{z=T(T)}$$

(11)

where \(z = z(T)\) is the height at a temperature \(T\).

Both \(\text{DEM}(T)\) and the peak position of the X-ray flux at a given energy \(\epsilon(z)\) are uniquely determined by \(T(z)\) and \(n^2(z)A(z)\).
4.1. Isothermal plasma

Let us first assume a simple case. For an isothermal plasma \( dT/dz = 0 \) and hence \( d(n^2A)/dT = 0 \), Eq. (10) becomes

\[
\epsilon_0 = \frac{T}{2} = \frac{T_0}{2}
\]

over all \( z \), where \( T = T_0 \) is a constant. Therefore, an isothermal plasma cannot account for the observed changes in peak X-ray energy with height \( z \). Importantly, this shows that an isothermal plasma in the flaring coronal source is ruled out by imaging data, even though an isothermal bremsstrahlung function can reasonably fit the solar flare spectrum found using RHESSI (see Fig. 3).

4.2. Height-dependent temperature with constant \( n^2A \)

If \( n^2A \) is constant with height \( z \), i.e. \( d(n^2A)/dz = 0 \), then Eq. (9) yields

\[
\epsilon_0 = \frac{T(z)}{2}
\]

Since the observed X-ray energies are 10 to 25 keV, the temperatures \( T(z) \) in a constant \( n^2A \) plasma would have to be of the order 20 to 50 keV, i.e. 230 to 580 MK. From the RHESSI spectrum, we see that the majority of the emitted thermal X-rays are emitted at lower energies corresponding to lower temperature sources of 1 to 3 keV, and hence a constant \( n^2A \) is unlikely.

4.3. Height-dependent \( T(z) \) and \( n^2(z)A(z) \)

In Eq. (10), if the gradient \( d(n^2A)/dT \neq 0 \), then \( d(n^2A)/dT = (d(n^2A)/dz)(dz/dT) \) must be negative for energy \( \epsilon \) to be positive, or the second term must be smaller than the first term on the RHS of Eq. (10). Since we observe higher energy X-ray sources above lower energy X-ray sources, the temperature must increase with altitude, so that \( dT/dz > 0 \). Hence, in this scenario \( d(n^2A)/dz < 0 \) for \( d(n^2A)/dT < 0 \). However, \( d(n^2A)/dz \) does not have to be negative over all \( z \), only over a portion of \( z \) where we see the X-ray peak positions at a given energy \( \epsilon \). Actually, if \( d(n^2A)/dz \) is negative over the entire region, then the peak positions may appear at a lower altitude than suggested by a given observation, but this is further investigated via modelling. It is more difficult to determine the form of \( \epsilon \) versus \( z \) for the case where \( d(n^2A)/dz \) is positive, and the second term is smaller than the first term. Hence modelling is required to further determine the forms of \( T(z) \) and \( n^2(z)A(z) \) that can account for both RHESSI imaging and spectroscopic observations.

5. Modelling the vertical temperature and number density distributions

Using X-ray imaging and spectroscopy data as a guide, we develop a model of \( T(z) \) and \( n^2(z)A(z) \), which can simultaneously explain flare X-ray imaging and spectroscopic observations.

Firstly, we note that the density \( n(z) \) and the area perpendicular to the image plane \( A(z) \) (see Fig. 7 for the geometry) appear in Eqs. (10) and (11) as a combination \( n^2(z)A(z) \). Therefore, the observations cannot say anything further about the individual distributions of \( n(z) \) and \( A(z) \), only about the combination \( n^2(z)A(z) \). However, we can deduce the form of \( n^2(z)A(z) \) and the temperature structure \( T(z) \) from the observations.

The modelled \( n^2(z)A(z) \) and \( T(z) \) should reproduce the following main characteristics of X-ray emission determined from RHESSI observations:

1. The observed X-ray source widths (using a Gaussian fit) should have a FWHM ~ 11" with visible X-ray emission over ~20" in the radial direction between the energies of 10 to 25 keV. The minimum visible X-ray emission should be at a height ~10" above the limb.
2. Altitude \( z \) versus peak energy \( \epsilon \) should be well-fitted by Eq. (10), relating energy, height, \( T(z) \) and \( n^2(z)A(z) \). The
The average width or vertical size of an X-ray source in the radial direction \( R \) is \( (W) = 11.2 \pm 0.5 \). The width was also found using three larger energy bins of 10–12.5 keV, 12.5–18 keV and 18–25 keV, increasing the number of counts and hence reducing the uncertainty. This gives an average width of \( (W) = 11.0 \pm 0.3 \). The Vis_FwdFit image profiles along a line perpendicular to the solar surface and through the centroid position of the coronal source, for the energy bin of 10–11 keV. Both curves are divided by the maximum value for comparison. Vis_FwdFit (blue) fits a Gaussian distribution to the X-ray visibilities, and although the real X-ray profile may deviate from a Gaussian form assumed by Vis_FwdFit (and the CLEAN PSF), the low Vis_FwdFit \( \chi^2 \) values tell us a Gaussian is an adequate approximation for the study.

\[
\sigma_{\text{FWHM}} = \frac{1}{\sqrt{2\ln 2}} \cdot \text{FWHM} = 2 \sqrt{\ln 2} \cdot \sigma
\]

The model emission measure \( EM \)

\[
EM = \int_T \text{DEM}dT
\]
Fig. 7. Left: sketch of the coronal X-ray source geometry, as seen by RHESSI. The lower temperature plasma is located below the higher temperature plasma. A small volume of emitting plasma dV = A dz = LdA/z varies with altitude z. Right: cartoon of the X-ray distribution versus z at energies ε1 < ε2 < ε3 created by varying temperature T and n^2A distributions with altitude z. The peak X-ray positions at a given energy, as viewed by RHESSI, are related to a T and n^2A value at that position.

\[ \langle T \rangle = \frac{\int T \, DEM \, dT}{\int DEM \, dT} \]  
(17)

are found to be \( EM = 6.7 \times 10^{49} \) cm\(^{-3}\) and \( \langle T \rangle = 1.4 \) keV (~16 MK).

Over the temperature range between 1.6 and 2.5 keV, the model z varies from around ~15'' to ~25'' (see Fig. 8 Panel 2). This leads to a model temperature gradient of \( dT/dz \sim 0.08 \) keV/arcsec. From the model, it is suggested that the form of the DEM at lower temperatures below 1.6 keV is closer to both the Gaussian and power-law exponential OSPEX models, which could not be confidently determined by spectroscopy alone.

In Fig. 9 (right most panel), we also show that such a trend is common at different times during the studied flare, with the gradient \( dR/de \) varying at different times. However, changes in time are beyond the scope of this paper. The temporal changes are the subject of ongoing studies. In Fig. 9 (middle and left most panels), we also show that the coronal X-ray source altitude increases with energy for two other flares studied at a single time range close to the peak flux of each flare.

### 6. Summary

A detailed analysis of solar flare SOL2013-05-13T02:12, during a single time interval, was performed using RHESSI imaging and spectroscopy observations. We investigated quantitatively the increase in altitude of the flare coronal X-ray source with energy at a single time interval during the flare impulsive phase, a trend that has been noted previously (e.g. Jeffrey & Kontar 2013).

The RHESSI imaging analysis shows that the peaks of X-ray emission in the coronal X-ray source are located close together over a small distance of 3'' between the X-ray energies of 10 and 25 keV. The increase in source height with photon energy can be well-fitted by a linear function with a gradient of \( dz/de = 0.2 \) arcsec/keV. RHESSI spectroscopy showed that a number of different thermal models (both isothermal and multi-thermal) can fit the spatially integrated X-ray spectrum adequately well. At the same time, the analysis of the flare data shows that the emitting flare plasma in the corona cannot be isothermal, as an isothermal plasma is unable to account for the observed changes in altitude with X-ray energy, as shown by RHESSI imaging. This is an important result for deriving the properties of hot X-ray emitting plasma in the flaring solar corona, which is often performed with an isothermal model.

Further, our study shows how RHESSI imaging can be used to constrain the properties of flaring plasma beyond what is possible using X-ray spectroscopy alone. Coronal X-ray emission in the range of 10 to 25 keV can be well explained using model distributions in temperature, number density, and area that vary with altitude. Modelling can adequately explain all the main observations: radial position (gradient \( dz/de \) and approximate height above the X-ray limb), vertical source size, and differential emission measure. For our model, the temperature and \( n^2A \) gradients (\( dT/dz \) and \( dn^2A/dz \)) must have opposite signs and the \( n^2A \) distribution must decrease over a portion of the region with increasing altitude z, in order to produce the RHESSI imaging results. The modelling suggested that \( dT/dz \) between 1.6 and 2.5 keV should be ~0.08 keV/arcsec. Since this gradient occurs over most of the observable region (~15''), it may be used in future analysis to distinguish between different cooling and heating processes occurring within the flaring coronal X-ray source. A constant area A suggests that the number density should decrease over an order of magnitude with altitude, although A might also vary with z, as suggested by observations of X-ray source length and width variations. Again, such a prediction should help constrain the processes occurring in the coronal region of solar flares.

We have analysed a number of other limb flares (see Fig. 9 for some examples) with bright coronal X-ray sources, and they all show the source altitude increasing with increasing X-ray energy. This suggests that the observations presented here are common for a majority of coronal sources, and hence vertical temperature and density gradients with rather broad multi-thermal DEM(T)s are common.
Fig. 8. Temperature and density-altitude distribution model (see Sect. 5 for details) with 6 different panels. In all panels, the best model result is represented by the red line and points. 1: $n^2 A$ vs. height $z$ given by Eq. (14). 2: Temperature $T$ vs. $z$ given by Eq. (15). 3: X-ray distribution at various photon energies $\epsilon$ vs. altitude. The vertical lines indicate the peak locations. 4: The model X-ray distribution peak position vs. energy (red dots) and Eq. (9) using the model $T(z)$ and $n^2(z) A(z)$ (pink dashed-dotted line). 5: X-ray distribution widths (FWHM) vs. energy. 6: Differential emission measure, DEM($T$). Panels 4–6 show both the results of the RHESSI data analysis (in blue) and the model predictions (in red). The grey areas show the range of each parameter that can adequately match the measured values using the range of modelled $T(z)$ shown in Panel 2.

One notable limitation of RHESSI is the insensitivity to plasma at temperatures below $\sim 1$ keV. Different imaging instruments should be used to extend the range of temperatures (e.g. Battaglia & Kontar 2013). We plan to better constrain the electron number density and temperature distributions of a flaring coronal region, supplementing existing RHESSI data with EUV imaging and spectroscopy data. This will allow us to extend our imaging approach to temperatures down to a few MK.

In conclusion, we have demonstrated how imaging should be used to improve spectral analysis, allowing the shape of the
Fig. 9. Coronal X-ray sources in different energy bands, for two additional flares: SOL2005-08-23T14:32 from 14:30 to 14:33 UT (left) and SOL2013-05-13T16:05 from 16:02 to 16:04 UT (middle). Both flares are imaged during the time of peak X-ray flux, and the source altitude increases with X-ray photon energy in these two flares. This trend also occurs for the analysed flare SOL2013-05-13T02:12 at different times (right). Here the imaging results are shown for the time of 02:04 to 02:06 UT during the rise phase of the flare, showing that the gradient $dR/d\epsilon$ varies with time.

DEM($T$) to be better deduced, with X-ray imaging providing a relatively simple guidance to the $n^2(z)A(z)$ and temperature $T(z)$ profiles within a flaring coronal X-ray source.

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