Dynamical population synthesis: constructing the stellar single and binary contents of galactic field populations

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Accepted 2011 July 27. Received 2011 July 27; in original form 2011 February 22

ABSTRACT
The galactic field’s late-type stellar single and binary populations are calculated on the observationally well-constrained supposition that all stars form as binaries with invariant properties in discrete star formation events. A recently developed tool (Marks, Kroupa & Oh) is used to evolve the binary star distributions in star clusters for a few million years until an equilibrium situation is achieved which has a particular mixture of single and binary stars. On cluster dissolution the population enters the galactic field with these characteristics. The different contributions of single stars and binaries from individual star clusters, which are selected from a power-law-embedded star cluster mass function, are then added up. This gives rise to integrated galactic field binary distribution functions (IGBDFs), resembling a galactic field’s stellar content (dynamical population synthesis). It is found that the binary proportion in the galactic field of a galaxy is larger the lower the minimum cluster mass, $M_{\text{ecl, min}}$, the lower the star formation rate, SFR, the steeper the embedded star cluster mass function (described by index $\beta$) and the larger the typical size of forming star clusters in the considered galaxy. In particular, period, mass ratio and eccentricity IGBDFs for the Milky Way (MW) are modelled using $M_{\text{ecl, min}} = 5 \, M_\odot$, SFR = 3 $M_\odot$ yr$^{-1}$ and $\beta = 2$ which are justified by observations. For $r_h \approx 0.1–0.3$ pc, the half-mass radius of an embedded cluster, the aforementioned theoretical IGBDFs agree with independently observed distributions, suggesting that the individual discrete star formation events in the MW generally formed compact star clusters. Of all late-type binaries, 50 per cent stem from $M_{\text{ecl}} \lesssim 300 \, M_\odot$ clusters, while 50 per cent of all single stars were born in $M_{\text{ecl}} \gtrsim 10^4 \, M_\odot$ clusters. Comparison of the G-dwarf and M-dwarf binary populations indicates that the stars are formed in mass-segregated clusters. In particular, it is pointed out that although in the present model all M-dwarfs are born in binary systems, in the MW’s Galactic field the majority ends up being single stars. This work predicts that today’s binary frequency in elliptical galaxies is lower than that in spiral and dwarf galaxies. The period and mass-ratio distributions in these galaxies are explicitly predicted.

Key words: methods:numerical – binaries: general – open clusters and associations: general – solar neighbourhood – Galaxy: stellar content – star clusters: general.

1 INTRODUCTION
The Milky Way’s (MW) Galactic field stellar late-type population (spectral types G to M) consists of roughly 50 per cent single stars and 50 per cent binaries (Duquennoy & Mayor 1991; Fischer & Marcy 1992; Mayor et al. 1992; Raghavan et al. 2010). Throughout this work we will use the Duquennoy & Mayor (1991, hereafter DM91) results for G-dwarfs as the canonical Galactic field population to which data are being compared, since it is the only existing long-term (13 yr) survey performed by a single team, i.e. the DM91 data provide a homogeneous data set.

In a spectroscopic survey of halo, thick and thin disc populations, Carney et al. (2005) find for high proper motion stars that 28 ± 3 per cent of all metal-poor ([Fe/H] $\leq -1$) centre-of-mass systems (cms)\textsuperscript{1} are binaries. A similar number (26 ± 3 per cent) is found for the more metal-rich stars in their sample ([Fe/H] $> -1$). These

\textsuperscript{1} A system refers either to a single star or to a binary.
values compare well with a binary fraction of 22 percent identified by DM91 over the same period range (1.9 < P < 7500 d). So there is no evidence for the variation of binary properties in the Galactic field over cosmological time.

The origin of the Galactic field composition is a result of star formation but has not been predicted with success. Indeed, Fisher (2004) finds in his theory of isolated star formation a bell-shaped binary period distribution function (period BDF) broadly similar to that of the Galactic field, but is not able to make specific predictions concerning the form of the period and other BDFs. As discussed in Kroupa (2011), the Moeckel & Bate (2010) seminal smoothed particle hydrodynamics simulations result in a semimajor axis BDF with too many binaries with orbits around a few astronomical units (au). Furthermore, their mass-ratio distribution shows too few binaries with \( q = m_2/m_1 < 0.8 \), where \( m_1 \) and \( m_2 \) are the primary- and secondary-component masses, respectively. The reason for this might be a collapse which is too deep resulting in a too dense cluster and thus too efficient binary disruption. Additionally, after 10 Myr of dynamical evolution including instantaneous residual-gas expulsion, orbits with semimajor axes \( a > 10 \text{ au} \) are underrepresented. Stellar feedback (e.g. Bate 2009), i.e. self-regulated star formation, might help to remedy this by reducing the depth of the collapse through an increased pressure.

Indeed, direct cloud collapse calculations are very limited in predicting binary star properties owing to the severe computational difficulties of treating the magnetohydrodynamics together with correct radiation transfer and evolving atomic and molecular opacities during collapse. Such computations are also prohibitively expensive. There is, therefore, currently no sufficient numerical framework to derive the multiplicity properties of stars. While such computations are important for understanding the physical processes in observed star formation regions, they do not have predictive power yet. In particular, larger systems, e.g. galaxies, cannot be synthesized.

Furthermore, the outcome of star formation computations cannot be expected to result in the binary properties of the Galactic field since clustered star formation is the dominant discrete star formation event (e.g. Lada & Lada 2003; Bressert et al. 2010; Lada 2010), and binaries are dynamically processed in star clusters before they become part of a galactic field. Therefore, a galactic field stellar population is the sum over all discrete star formation events, which on dissolution contribute a number of single stars and binaries dependent on the star formation conditions and dynamical history (Kroupa 1995b; Goodwin 2010). This is the topic of the present investigation.

Observations of low-density pre-main-sequence populations, i.e. ‘distributed’ star formation, show that most, if not all stars, form as members of binaries and that they exhibit a period BDF that is rising towards long periods (Simon et al. 1995; Ghez et al. 1997; Kohler & Leinert 1998; Duchêne 1999; Connelley, Reipurth & Tokunaga 2008; Kroupa & Petr-Gotzens 2011). Indeed, there is a simple but powerful argument that the vast majority of stars must form in binaries: the lack of a significant single-star population in dynamically not evolved star-forming regions means that stars cannot form in higher order multiple systems. These would decay on a system crossing time (10^5 yr, Goodwin & Kroupa 2005).

\( N \)-body computations of initially binary-dominated star clusters have shown that a rising period BDF can be turned into a bell-shaped one by gravitational interactions among the cluster members within a few million years (terminated simulated evolution; Kroupa 1995a,b, Marks, Kroupa & Oh 2011; Oh et al. 2011). Here, an analytical treatment for the change of orbital-parameter BDFs in star clusters, which initially obeyed an invariant rising period BDF, is used (Marks et al. 2011, hereafter Paper I) to efficiently calculate the stellar single and binary contents in individual star clusters. Then the populations coming from all star clusters of a galaxy’s freshly formed star cluster system are summed which yields galactic field stellar populations once the star clusters have dissolved (dynamical population synthesis).

We note that the method developed here to calculate the integrated galactic field binary distribution function (IGBDF, equation 15) underlies similar concepts as the theory of the integrated galactic (stellar) initial mass function (IGIMF), which sums up the initial mass functions (IMFs) in all discrete star formation events showing that galaxy-wide IMFs are steeper at the high-mass end than the invariant IMF in star clusters (Kroupa & Weidner 2003; Weidner & Kroupa 2005). This theory has been proven extraordinarily successful in describing and predicting observational properties of galaxies (Köppen, Weidner & Kroupa 2007; Pflamm-Altenburg & Kroupa 2008, 2009; Pflamm-Altenburg, Weidner & Kroupa 2009; Recchi, Calura & Kroupa 2009; Calura et al. 2010).

In Section 2 the model to calculate IGBDFs is devised and in Section 3 the results are presented and compared to observations. Finally, Section 4 discusses and shows model predictions and Section 5 summarizes the main points of this investigation.

2 MODEL

In order to integrate over all stellar populations in discrete star formation events (Section 2.2), i.e. in embedded star clusters, it is necessary to first understand the evolution of binary populations in them (Section 2.1). Note that the terms discrete star formation event, star cluster and embedded star cluster are here used synonymously.

In the following reference will be made to birth or pre-main-sequence and initial BDFs. The birth distributions (Kroupa 1995a), including random pairing of component masses for late-type stars, describe the properties of binary populations after they were born. But binaries are still embedded in their circumstellar material, i.e. the components have not yet reached the main-sequence stage. The initial distributions (Kroupa 1995b) describe the corresponding statistical properties of a young binary population after birth binaries have undergone a phase of re-distribution of energy and angular momentum within their circumstellar material, called pre-main-sequence eigenvolution, acting on a time-scale of \( <10^4 \text{ yr} \). This mechanism introduces correlations between the orbital parameters of short-period binaries (such as between period and eccentricity; Kroupa 1995b) as seen in observations. For a summary of these processes the reader is referred to Paper I.

2.1 Binary distributions in star clusters

In order to construct the field population by adding up the single stars and binaries in individual clusters, the evolution of binary populations has to be understood in terms of the initial properties of their host cluster (\( M_{\text{cl}}, r_h \)).

In Paper I an efficient method is provided to analytically describe the first 5 Myr of the evolution of orbital-parameter BDFs in \( N \)-body computations of star clusters. The computations start with 100 percent binaries distributed according to an initially rising period BDF derived in Kroupa (1995b), which is consistent with constraints for pre-main-sequence and Class I protostellar binary populations (see Paper I). Their method quantifies a stellar-dynamical operator (Kroupa 2002, 2008), \( \Omega_{0,\text{dyn}}^{\text{M}_{\text{cl}}}(t) \), based on the initial cluster mass density, \( \rho_{\text{cl}} = 3M_{\text{cl}}/8\pi r_h^3 \), where \( M_{\text{cl}} \) is the...
total mass in stars that formed in the embedded cluster and $r_b$ is its initial half-mass radius. This operator transforms an initial ($t = 0$), perhaps invariant (see Paper I), orbital-parameter BDF, $D_{n_{\text{orb}}}^b$, into an evolved one, $D_{n_{\text{orb}}}^{M_{\text{ecl}}} (t)$, after some time $t$ of stimulated evolution,

$$D_{n_{\text{orb}}}^{M_{\text{ecl}}} (t) = \Omega_{\text{dyn}}^{M_{\text{ecl}}} (t) \times D_{n_{\text{orb}}}^b.$$  (1)

In the formulation of Paper I, $\Omega_{\text{dyn}}^{M_{\text{ecl}}} (t)$ acts in particular on the initial BDF for binding energies, $\Phi_{\text{dyn}}^{M_{\text{ecl}}} (m_1, m_2)$, i.e.,

$$\Phi_{\text{dyn}}^{M_{\text{ecl}}} (m_1, m_2) = \Omega_{\text{dyn}}^{M_{\text{ecl}}} (t) \times \Phi_{\text{dyn}} (m_1, m_2),$$  (2)

but extraction of other BDFs (period, semimajor axis, mass ratio, eccentricity) is also possible with their model, given the interrelation between the orbital parameters via Kepler’s laws (Section 2.4).

Let $N_{\text{sys}} = N_b + N_h$ be the number of systems, i.e. the sum of all single stars and binaries with primary-star mass near $m_1$ in a stellar population. Then a BDF, $\Phi_{\text{ecl}}^{M_{\text{ecl}}} (m_1) (x = \log_{10} E, \, a, \, e, \, q, \ldots)$, is defined as the distribution of binary fractions, $f_b = N_b / N_{\text{sys}}$, as a function of $x$,

$$\Phi_{\text{ecl}}^{M_{\text{ecl}}} (m_1) = \frac{d f_b (x)}{dx} = \frac{1}{N_{\text{sys}}} \frac{d N_b (x)}{dx},$$  (3)

where $d N_b (x)$ is the number of binaries in the interval $[x, \, x + dx]$. The total binary fraction equals the area below the BDF:

$$f_b (m_1) = \int_{-\infty}^{\infty} \Phi_{\text{ecl}}^{M_{\text{ecl}}} (m_1) \, dx.$$  (4)

### 2.2 Integrated orbital-parameter distributions

The galactic field’s binary population is the sum over the populations in all star clusters that ever formed in a galaxy, having evolved for at least a time-span $t_{\text{freeze}}$ after which the binary orbital-parameter properties become frozen-in,

$$D_{\text{ecl}}^b = \int_{m_{\text{cl}, \text{min}}}^{m_{\text{cl}, \text{max}}} \Omega_{\text{dyn}}^{M_{\text{ecl}}} (t_{\text{freeze}}) \, \xi_{\text{cl}} (M_{\text{cl}}) \, d M_{\text{cl}},$$  (5)

where $\xi_{\text{cl}} (M_{\text{cl}})$ is the embedded cluster mass function (ECMF). For simplicity, it is assumed that all star clusters have formed with comparable half-mass radii, $r_b$. The integration ranges from a minimum cluster mass, $M_{\text{cl}, \text{min}}$, to some maximum cluster mass, $M_{\text{cl}, \text{max}} = M_{\text{cl}, \text{max}} (\text{SFR})$. The maximum cluster mass that can form in a galaxy which has a given star formation rate (SFR) is determined by (Weidner, Kroupa & Larsen 2004)

$$M_{\text{cl}, \text{max}} (\text{SFR}) = \frac{84793 \times \left( \frac{\text{SFR}}{\dot{M}_\odot \text{yr}^{-1}} \right)^{0.75}}{M_\odot}.$$  (6)

Star clusters are distributed according to a power-law ECMF with index $\beta$,

$$\xi_{\text{cl}} (M_{\text{cl}}) = k_{\text{cl}} \times M_{\text{cl}}^{-\beta},$$  (7)

which is normalized such that the sum of the masses of all clusters equals the total mass of the freshly formed star cluster system,

$$M_{\text{SCS}} = \int_{m_{\text{cl}, \text{min}}}^{m_{\text{cl}, \text{max}} (\text{SFR})} M_{\text{cl}} \, \xi_{\text{cl}} (M_{\text{cl}}) \, d M_{\text{cl}}.$$  (8)

The total mass, $M_{\text{SCS}}$, needed to find the normalization constant $k_{\text{cl}}$ is determined from the SFR and the formation time-scale, $\delta t$, of the star cluster system:

$$M_{\text{SCS}} = \text{SFR} \times \delta t.$$  (9)

Weidner et al. (2004) found that about every $\delta t = 10 \text{Myr}$ of ongoing star formation an ECMF is fully populated.

The binary population will enter a galactic field with characteristics set at time $t_{\text{freeze}}$. This is reached after sufficient time for cluster internal stimulated evolution when only hard binaries are left (Paper I), or if a cluster suddenly expands rapidly, e.g. as a result of residual-gas expulsion, which inhibits further stimulated evolution. This state is already reached after a few million years or even in less than 1 Myr for dense configurations (Kroupa 1995a; Duchêne, Bouvier & Simon 1999; Fregeau, Ivanova & Rasio 2009; Parker et al. 2009, Paper I). For our purpose, we choose the freeze-in time to coincide with the time of the occurrence of the first supernovae, $t_{\text{freeze}} \approx 3 \text{Myr}$. If the cluster is still embedded, supernovae are expected to drive out the residual gas of the embedded cluster rapidly, leading to cluster expansion and destruction of the majority of clusters in a star cluster system ($\approx 90$ per cent; e.g. Lada & Lada 2003). The exact time star clusters are allowed to evolve their population is not that important since the time-scale on which binary dissolution occurs is short. In particular, the difference in the binary fraction between 3 and 5 Myr in the $N$-body models used in Paper I is of the order of a few per cent only.

### 2.3 Normalization

Since the stellar-dynamical operator (equation 2) transforms between BDFs independently of the number of systems in a star cluster, one has to take care to preserve the definition and normalization for individual clusters (equations 3 and 4) also for the integrated population. Consider therefore as an example two clusters consisting of $N_{\text{sys}} = 10$ and 100 systems, with $f_b = 40$ and 60 per cent, respectively. Evaluating equation (5) for the two clusters would result in $f_b = 50$ per cent for the combined (or integrated) population, but the true resulting population has $f_b = 64/110 = 58$ per cent. Thus, the number of systems making up the population has to be taken into account.

Define a BDF for a galaxy analogous to equation (3),

$$\Phi_{\text{ecl}}^{\text{GF}} (m_1) = \frac{d f_b^{\text{GF}} (x)}{dx} = \frac{1}{N_{\text{sys}}^{\text{GF}}} \frac{d N_b^{\text{GF}} (x)}{dx},$$  (10)

where $d N_b^{\text{GF}} (x)$ and $N_{\text{sys}}^{\text{GF}}$ are the numbers of binaries in the interval of size $dx$ and the number of cms in a whole galaxy, respectively. The goal is thus to calculate the number of binaries per $x$-interval and number of systems (single+binary) in a galaxy from the respective numbers in star clusters separately.

For an initial cluster mass, $M_{\text{cl}}$, the total number of freshly hatched stars in that particular cluster is calculated from

$$N_c (M_{\text{cl}}) = \frac{M_{\text{cl}}}{\overline{m}},$$  (11)

where $\overline{m} \approx 0.4 M_\odot$ is the average mass of the canonical stellar IMF (Kroupa 2001). The total number of binaries is related to $N_c (M_{\text{cl}})$ via

$$N_b (M_{\text{cl}}) = N_c (M_{\text{cl}}) - N_{\text{sys}} (M_{\text{cl}}).$$  (12)

Inserting this in $f_b = N_b / N_{\text{sys}}$ and rearranging gives the number of cms in that particular cluster,

$$N_{\text{sys}} (M_{\text{cl}}) = \frac{N_c (M_{\text{cl}})}{1 + f_b}.$$  (13)

Since $\Phi_{\text{ecl}}^{M_{\text{ecl}}}$ is known (equation 2) and therefore $f_b$ (equation 4), $N_{\text{sys}}$ and $N_b$ can be calculated. The number of binaries per interval $dx$ becomes

$$\frac{d N_b (x)}{dx} = \frac{N_{\text{sys}} (M_{\text{cl}}) \times \Phi_{\text{ecl}}^{M_{\text{ecl}}}}{dx}.$$  (14)

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Table 1. Adopted mass ranges for single stars or primary components of binaries with different spectral types (SpTs).

| SpT | F  | G  | K  | M  |
|-----|----|----|----|----|
| m_1/M_⊙ | 1.04–1.04 | 0.8–1.04 | 0.45–0.8 | 0.08–0.45 |

Therefore, equation (10) becomes

\[ \Phi^\text{GF}_{\text{cum}}(m_1) = \frac{\int_{M_\text{cl,min}}^{M_\text{cl,max}(\text{SFR})} \frac{dn(M_\text{cl})(dN_{\text{cum}})(\xi_{\text{cl}}) dM_{\text{cl}}}{N_{\text{cum}}(M_{\text{cl}})(\xi_{\text{cl}})(M_{\text{cl}}) dM_{\text{cl}}} }{\int_{M_\text{cl,min}}^{M_\text{cl,max}(\text{SFR})} \frac{dn(M_{\text{cl}})}{N_{\text{cum}}(M_{\text{cl}}) dM_{\text{cl}}}}. \]  

Equation (15) is referred to as the IGBDF. It can be calculated for single stars and binaries with primary mass being in an interval \( \Delta m \) around \( m_1 \), e.g., 0.8–1.04 \( M_\odot \) or for all late-type stellar systems \( m_1/M_\odot \in [0.08, 2] \) (Table 1).

2.4 \( P, e, q \) and \( a \) IGBDFs

In order to extract IGBDFs for different orbital parameters (period, \( P \), eccentricity, \( e \), mass ratio, \( q \), and semimajor axis, \( a \)) from the known energy IGBDF, the procedure is basically as described in Section 4.2 of Paper I for orbital-parameter distributions in single clusters. The idea is to compile a large library of binaries (\( N_{\text{lib}} = 10^7 \) binaries for the present purpose), whose properties are selected according to the recipe in Kroupa (1995b, see Paper I). Here, the library consists only of binaries with primary masses up to 2 \( M_\odot \) in order to resemble a galactic field and it contains the values for \( m_1, m_2, E_\text{s}, P, e, q \) and \( a \). The total number of initial binaries in the integrated population, calculated according to Section 2.3, is scaled so as to match the library size of \( N_{\text{lib}} \) binaries. Following this, the final distributions are constructed by removing an appropriate amount of binaries with a given binding energy from the library according to the number ratio of binaries in the resulting and initial energy IGBDFs. The remaining binaries are used to construct the \( P, e, q \) and \( a \) IGBDFs. In order to extract subdistributions, such as for a special spectral type or period range, only those remaining binaries are used which fulfill the additional criteria. In the forthcoming sections, mass ranges for single stars or primary-component masses for binary stars, respectively, are adopted, as shown in Table 1.

3 RESULTS

According to equation (15), the properties of an IGBDF depends on the minimum embedded cluster mass and the SFR (i.e. the maximum cluster mass, equation 6) of the considered galaxy, the index, \( \beta \), of the ECMF (equation 7) and the average half-mass radius, \( r_\text{h} \), of the star clusters. The influence of these parameters on the integrated binary properties by means of the global binary fraction will be investigated. A discussion of the Galactic field binary population of the MW and what can be learned about the initial star cluster system of the MW from the observed binary population ends this section.

3.1 The importance of low-mass clusters for the Galactic field binary population

From which type of cluster do most Galactic field binaries originate? Dissolving low-mass embedded clusters will each contribute only a small number of systems to a Galactic field population but a large fraction of binaries will be among them due to inefficient stimulated evolution (Paper I). For high-mass clusters the situation is the other way round. However, the ECMF (equation 7) is steep, i.e. there are many more low-mass than high-mass clusters. Thus, this question is not simple to answer qualitatively.

Therefore the number of systems, binaries and single stars which are added to the Galactic field by all clusters of a given mass \( M_{\text{cl}} \) is calculated by evaluating,

\[ \frac{dN_{\text{cum}}(b/s)}{dM_{\text{cl}}} = N_{\text{cum}}(b/s)(M_{\text{cl}}) \xi_{\text{cl}}(M_{\text{cl}}). \]  

The result is depicted in the left panel of Fig. 1 for the MW star cluster system with \( \beta = 2.0, r_\text{h} = 0.2 \) pc, \( M_{\text{cl,min}} = 5 \) \( M_\odot \) and SFR=3 \( M_\odot \) yr\(^{-1} \) (see Section 3.3 below). It shows that the steepness of the ECMF dominates such that most systems in the Galactic field stem from low-mass clusters. For the same star cluster system parameters, the right panel of Fig. 1 plots the normalized cumulative number

\[ N_{\text{cum}}(b/s)(M_{\text{cl}}) = \int_{M_{\text{cl,min}}}^{M_{\text{cl,max}}} \frac{dN_{\text{cum}}(b/s)}{dM_{\text{cl}}} dM_{\text{cl}}, \]  

where \( N_{\text{cum}}(b/s) \) is the total number of systems, binaries or singles in the Galactic field, respectively, i.e. equation (16) integrated from \( M_{\text{cl,min}} \) to \( M_{\text{cl,max}} \). It demonstrates that about 50 per cent of all single stars in the Galactic field formed in clusters with initial masses \( M_{\text{cl}} < 2 \times 10^3 \) \( M_\odot \), while 50 per cent of all Galactic field binaries originate from clusters with \( M_{\text{cl}} < 300 \) \( M_\odot \) only. For single stars the situation is inverted. Roughly 50 per cent of all single stars come from star clusters with masses \( M_{\text{cl}} > 10^3 \) \( M_\odot \).

3.2 Parameter study and degeneracy

Fig. 2 depicts the global binary fraction (colour-coded), i.e. including all late-type binaries (\( m_1 \leq 2 \) \( M_\odot \)), of a galaxies’ stellar population based on the four parameters \( (M_{\text{cl,min}}, \text{SFR}, \beta \) and \( r_\text{h} \) in equation (15). We learn the following from these diagrams, if all other parameters are fixed.

(i) The higher the SFR, the smaller the field-binary population. A higher SFR yields a larger \( M_{\text{cl}} \) (equation 6). A higher mass cluster is generally denser than a lower mass cluster and will therefore contribute a population with a lower binary fraction (Paper I). Therefore, \( f_\text{b} \) drops with increasing SFR.

(ii) The larger the \( r_\text{h} \), the larger the galactic field binary fraction. Larger typical half-mass radii imply lower densities, resulting in dynamically less evolved binary populations before dissolution (higher \( f_\text{b} \), Paper I). Therefore, \( f_\text{b} \) increases with increasing \( r_\text{h} \).

(iii) The steeper the ECMF (larger index \( \beta \)), the larger the field-binary population. A steeper ECMF amounts to a lesser proportion of high mass, i.e. higher density clusters. Therefore, on average populations with larger binary fractions enter the field and \( f_\text{b} \) increases.

(iv) The larger the minimum cluster mass, the fewer binaries exist in the field. Increasing \( M_{\text{cl,min}} \) means cutting out of low-density clusters which would contribute populations with large binary proportions. The remaining higher mass clusters shed dynamically more evolved populations into a galactic field.

Fig. 2 distinctly shows that the parameter space is degenerate. For example, the increase in \( f_\text{b} \) due to a steeper ECMF can be compensated for by introducing smaller cluster sizes (left panel) or assuming a larger \( M_{\text{cl,min}} \) (right panel). Similarly, a larger typical \( r_\text{h} \) can be counteracted with a higher SFR (middle panel), and so on. Therefore, constraints on at least some of the four parameters
Figure 1. Left panel: number of all singles (dotted), binaries (dash-dotted) and all systems (single+binary, solid) per cluster mass in the Galactic field (equation 16), respectively, that form within one star cluster system formation time-scale δt (equation 9). Lines are drawn for the MW star cluster system (β = 2.0, rh = 0.2 pc, Mecl,min = 5M⊙ and SFR = 3M⊙ yr⁻¹). Low-mass clusters contribute most systems to the Galactic field population owing to the steepness of the ECMF. The intersection between the single-star and binary-star lines is the cluster mass at which fb = 50 per cent for the used parameters. Right panel: cumulative number of singles, binaries and all systems as a function of Mecl normalized to the respective total number (equation 17) for the same star cluster system parameters as in the left panel. Low-mass clusters (Mecl ≲ 300 M⊙) are the dominant binary contributors while high-mass clusters (Mecl ≳ 10⁴ M⊙) donate most single stars.

Figure 2. Study of the influence of the parameters in equation (15) determining the integrated stellar population. In each panel the colour-coding shows the global binary fraction, fb, i.e. including all late-type binaries (m₁ ≤ 2 M⊙), according to the bar on the right of each panel (in per cent). The solid overlaid lines indicate curves of constant binary fraction. The big solid white dot in each panel marks the position of the MW (Section 3.3). The binary population increases with decreasing SFR, with increasing cluster half-mass radii, rh, with steepening of the ECMF (increasing index β) and with decreasing minimum cluster mass, Mecl,min.

from independent sources are needed to reduce the allowed solution space, when comparing model predictions with observed distributions. Those are available for the MW.

3.3 The Milky Way

In order to calculate a synthetic stellar population for the MW’s Galactic field, we need to estimate the parameters entering the IGBDF for our Galaxy.

The current global SFR of the MW using different methods is determined to lie between ≈1 and 5 M⊙ yr⁻¹ (Smith, Biermann & Mezger 1978; Diehl et al. 2006; Misiriotis et al. 2006; Calzetti et al. 2009; Murray & Rahman 2010; Robitaille & Whitney 2010). These values are compatible with the assumption that the total stellar mass in the disc and bulge of the MW (5 × 10¹⁰ M⊙, Binney & Tremaine 2008) has assembled continuously during the last 13.7 Gyr with an SFR of 3.6 M⊙ yr⁻¹. Assuming SFR = 3 M⊙ yr⁻¹ yields Mecl,max = 1.9 × 10⁵ M⊙ (equation 6) for the most massive cluster having formed in the MW disc, comparable to the most massive open cluster in the Piskunov et al. (2007) sample of 236 open clusters within 1 kpc from the Sun (1.1 × 10⁵ M⊙ for Sco OBS). Taking the minimum cluster mass similar to that of a Taurus-Auriga-like group (Kroupa & Bouvier 2003, Mecl,min = 5 M⊙) and an ECMF index β = 2 similar to the observed slopes for Galactic embedded clusters (Lada & Lada 2003; de la Fuente Marcos & de la Fuente Marcos 2004; Gieles et al. 2006), we can evaluate the properties
of the Galactic field binary population by numerically integrating equation (15) for different embedded cluster half-mass radii.

3.3.1 Energy distribution

Fig. 3 shows the energy IGBDF for the above parameters, for different cluster half-mass radii and for primaries of all masses. It is found that the binary fraction in the Galactic field (the area under the distributions) is smaller if the typical cluster radii are smaller. Smaller average radii lead to higher initial densities in star clusters which results in more efficient binary dissolution and thus a lower binary fraction (Section 3.2). For $r_h = 0.1$ pc the overall binary fraction is $f_b = 0.34$ and for $r_h = 0.8$ pc, $f_b = 0.71$.

3.3.2 Period distribution

For the adopted initial conditions in star clusters (Paper I) and the above values for the SFR, $\beta$ and $M_{\text{crit}}$, in the MW, the period IGBDFs in Fig. 4 for different values of $r_h$ demonstrate that the initially rising period distribution in star clusters translates into a bell-shaped form in the Galactic field, at least if the star clusters from which the Galactic field population originates, were rather compact. Comparison with the corrected observed Galactic field period BDF for G-dwarfs (DM91) suggests that clusters formed quite compact (left panel). The G-dwarf field binary fraction of $\approx 57$ per cent is best reproduced with a typical initial cluster size of $r_h = 0.1$ pc, where $f_b(G) = 0.58$. A similar study by Raghavan et al. (2010) investigating the multiplicity properties of 454 solar-type stars selected from the Hipparcos catalogue is in agreement with the DM91 data and thus with the compact formation of star clusters.

Fischer & Marcy (1992) studied the M-dwarf binary population within 20 pc from the Sun, showing that the period distribution of M-dwarfs can also be described by a lognormal distribution. Comparison with the IGBDF models (Fig. 4, right panel) favours their formation in slightly more extended clusters. For $r_h = 0.3$ pc the binary frequency of 41 per cent compares well with the observational result ($f_b = 0.42$). Recently, Bergfors et al. (2010) surveyed 124 M-type stars for binary separations $a \lesssim 200$ au for their multiplicity properties. The observed semimajor axis distribution has been translated into periods using Kepler’s laws and are incorporated in Fig. 4 (right panel). Their data are compatible with the Fischer & Marcy (1992) data. For a subsample of the model binary population solutions is discussed in Section 4. The line types are as in Fig. 3.
with \( a \lesssim 200 \) au, as in the observations, the \( r_h = 0.2 \) pc model binary frequency is \( f_b = 0.31 \) and agrees with the observed binary fraction of \( \approx 32 \) per cent. Bergfors et al. (2010) note, however, that there might be some overabundance of systems with \( P < 20 \) d, corresponding to \( a \lesssim 80 \) au for an average system mass of \( \approx 0.5 \) M\( \odot \), due to the sample selection and that corrections might be necessary for non-physical (optical) pairs.

### 3.3.3 Mass-ratio distribution

The result that star formation in rather compact structures is favoured is independently confirmed by considering the distribution of mass ratios. Fig. 5 depicts the mass-ratio IGBDFs for the complete field population (left), IGBDF for all binaries with primary masses \( m_1 \leq 2 \) M\( \odot \), for G-dwarf binaries only (middle) and for M-dwarfs (right). It is apparent that the complete distribution is flat between roughly \( q = 0.2 \) and 0.9, while the G-type binary sub-distribution is decreasing and the M-dwarf IGBDF is increasing with increasing \( q \). We emphasize that all three mass-ratio IGBDFs result from initially sampling the two birth components of a binary randomly from the same underlying stellar IMF (e.g. Kroupa 1995a). The decreasing trend is also evident for F- and K-type primaries (Fig. 8) and is seen in the observations of G-dwarfs by DM91 (the middle panel of Fig. 5). The observational data compare very well with the compact formation models, as above for the period IGBDFs. Addition of the mass-ratio IGBDFs for the different spectral types (Table 1 and Fig. 8) results in the flat distribution for the complete mass-ratio IGBDF (left panel in Fig. 5). The binary fraction of 35 per cent in the observations of 106 G- to M-type systems in the analysis by Reid & Gizis (1997) are best compatible with the \( r_h = 0.1 \) pc model where \( f_b = 0.34 \).

In contrast to the declining \( q \)-distribution for G-dwarfs found by DM91, note that the results of Raghavan et al. (2010) suggest a mass-ratio distribution for binaries with a solar-type primary which is flat between \( q = 0.2 \) and 0.9, while finding a similar period distribution (see the discussion in Section 4). Fischer & Marcy (1992) and Bergfors et al. (2010) also extracted a mass-ratio distribution for their respective samples of M-dwarfs whose rising shapes are well reproduced by the models (right panel). As expected from the period IGBDFs (Fig. 4), the same \( r_h = 0.3 \) and 0.2 pc models, respectively, are consistent with the observations, being again larger than for G-dwarfs.

#### 3.3.4 Eccentricity distribution

The eccentricity IGBDF for G-dwarf binaries is in agreement with the observational data (Fig. 6). The distribution of eccentricities is bell shaped for \( \log_{10} P_{\text{circ}} \sim 1.06 < \log_{10} P < 3 \), where \( P_{\text{circ}} \) is the circularization period identified by DM91 below which the orbits are circular (\( e \approx 0 \)). Circularization occurs through pre-main-sequence eigenevolution (Kroupa 1995b, Paper I). The eccentricity IGBDF follows the thermal distribution for \( \log_{10} P > 3 \) because it is invariant to stimulated evolution.

#### 3.3.5 Single and binary populations based on spectral type

The fraction of binaries in the field is a function of the spectral type (i.e. mass) of the primary. The later the primary spectral type, the lower is the resulting binary fraction in the field (Fig. 7). Especially, M-dwarfs have a lower binary fraction than binaries with a F-, G- or K-type primary. The reason for this is twofold.

First, due to the shape of the stellar IMF M-type stars are most numerous so that upon random pairing of the binary components at birth and after eigenevolution (Section 2) about \( \approx 90 \) per cent of all initial binaries carry a companion of spectral type M. Thus, every time a binary dissolves, in nine out of 10 cases at least one M-dwarf will end as a single star (two if an M-dwarf binary dissolves). Each M-dwarf contributes to \( N_{\text{ HMS}}(M) \), and therefore \( f_b(M) = N_b(M)/N_{\text{ HMS}}(M) \) shrinks.

Secondly, binaries with a lower binding energy, \( E_b \), are more prone to dissolution (Paper I) and the binding energy is proportional to the mass of the primary, \( E_b \propto m_1 \). Since, by construction, all binaries initially follow the same initial period distribution (Kroupa 1995b, Fig. 4), the M-dwarf (low \( m_1 \)) energy IGBDF is shifted to slightly lower energies compared to F-, G- and K-type binaries. Thus, it is generally easier to dissolve M-dwarf binaries than binaries with a primary of an earlier spectral type. Furthermore, low-mass binaries are more frequent, again owing to the shape of the stellar IMF. In the model, \( \approx 57 \) per cent of all binaries have an M-dwarf primary.

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**Figure 5.** Mass-ratio IGBDFs for all late-type binaries (\( m_1 \leq 2 \) M\( \odot \), left panel), G-dwarf (middle panel) and M-dwarf binaries (right panel), for the MW model (Section 3.3) with different half-mass radii (histograms). The line types are as in Fig. 3. While the distribution between \( q = 0.2 \) and 0.9 is flat for the complete distribution it declines for the G-dwarfs and increases for M-type binaries with increasing \( q \). The peak at mass ratios in the model close to unity (an effect of pre-main-sequence eigenevolution) is also evident in the observational data for all primary-masses combined (Reid & Gizis 1997, left) and M-dwarf binaries (Fischer & Marcy 1992; Bergfors et al. 2010, right). It is less pronounced, if at all, for the G-dwarf observations by DM91 (middle). Note that agreement is obtained for similar \( r_h \) as for the period IGBDFs (Fig. 4).
Figure 6. Eccentricity IGBDFs for G-dwarf binaries in comparison with the observations by DM91 in the indicated period ranges. \( P_{\text{circ}} = 11.6 \text{ d} \) is the circularization period derived from the observational data. Note that in both panels the data is (exceptionally) normalized to the total number of binaries instead of systems, in order to be able to compare to the observations. This is also why IGBDF models with different \( r_h \) (histograms, different line types) can hardly be distinguished. Left panel: the eccentricity BDF is bell shaped for orbital periods below \( 10^3 \text{ d} \) due to pre-main-sequence eigen-evolution as in the observational data. Right panel: for \( P \geq 10^3 \text{ d} \) the \( e \)-distribution follows the thermal distribution (\( f_b(e) = 2e \), solid line) for the IGBDF model as well as in the observations. The thermal eccentricity distribution is invariant of stimulated evolution.

Figure 7. Comparison of the binary fractions among systems of one spectral type (SpT) for IGBDF models with different \( r_h \) (filled and open circles for an ECMF slope \( \beta = 2 \) and 2.4, respectively, in each column from top to bottom: 0.8, 0.6, 0.4, 0.2 and 0.1 pc). The initial binary fraction for all spectral types is \( f_b = 1 \) (dotted horizontal line). M-dwarf binaries have the lowest binary fraction when they enter the Galactic field. The flatter the ECMF (smaller \( \beta \)), the lower is the binary fraction (Section 3.2). Observed binary fractions agree with the \( \beta = 2, r_h = 0.1 \)–0.2 pc IGBDF model best (data taken from DM91; Fischer & Marcy 1992; Mayor et al. 1992; Kroupa, Tout & Gilmore 1993; Delfosse et al. 2004; Bergfors et al. 2010; Raghavan et al. 2010). All observational data in Fig. 7 again compare best with star formation in compact star clusters (\( r_h \approx 0.1\)–0.2 pc and \( \beta = 2 \)). Although all stars are locked up in binaries initially, more than half of all the systems in the Galactic field end up as single stars due to stimulated evolution in star clusters before they dissolve (Fig. 2). M-dwarfs in the IGBDF model constitute about \( \approx 80\% \) of the single star population in the Galactic field. Only \( \approx 13, 3 \) and 2 per cent of all single stars in the IGBDF model have spectral type K, G and F, respectively (the remaining 2 per cent are of spectral type A, for a Galactic field population which consists of systems with \( m_1 \leq 2 \text{M}_\odot \), Section 2.4).

4 DISCUSSION AND MODEL PREDICTIONS

4.1 Formation in compact star clusters

Comparison of the observational data with the model has suggested that MW star clusters typically formed quite compact (\( r_h \approx 0.1\)–0.3 pc, Section 3.3). Such small radii compare with the observational lower end of the sizes of dense cores in giant molecular clouds. However, the sizes of these dense cores range up to 2 pc and the spatial extent of embedded clusters are typically comparable (e.g. Lada & Lada 2003). But there is evidence that the forming stars within the embedded cluster start dynamically cold (Walsh, Myers & Burton 2004; Peretto, André & Belloche 2006; Lada et al. 2008) and that protostellar objects are more confined than more evolved young stellar objects (Teixeira et al. 2006; Muench et al. 2007). Thus, an embedded cluster will collapse to a smaller configuration. In that sense the derived half-mass radii may be interpreted as the primary initially. Dynamical encounters including M-dwarfs will therefore occur often in the clusters. In turn, each disruption of an M-dwarf binary will reduce \( N_b(M) \) by one, increase \( N_{\text{cm}}(M) \) by one and, in turn, reduce \( f_b(M) \).
sizes of clusters when they reach their peak stellar density and stimulated evolution is most efficient.

Additionally the IGBDF model implicitly assumed that a typical half-mass radius for all clusters exists (Section 2.2). If a mass–radius relation for embedded clusters better describes reality (e.g. Harris & Pudritz 1994, for virialized gas cores in giant molecular clouds), the typical \( r_h \) for MW clusters can be seen as an average value for all clusters of any mass and size. Note that if such a mass–radius relation exists, upon averaging the inferred best \( r_h \) will be closer to the true value for low-mass clusters, which are most important for the Galactic field binary population (Section 3.1). Higher mass clusters will then occupy a somewhat larger radius range. However, a trend of radius with luminosity (or mass) in young star clusters has been shown to be shallow for the galaxy merger NGC 3256 (Zepf et al. 1999), for stellar clusters in 18 spiral galaxies (Larsen 2004) and for young clusters in M 51 (Scheepmaker et al. 2007).

The typical \( r_h \) is arrived at assuming \( \beta = 2 \) down to \( 5 M_\odot \). The ECMF might however flatten (\( \beta \to 0 \)) below \( \approx 50-100 M_\odot \) (Lada & Lada 2003), implying that fewer low-mass clusters are present compared to the numbers used in the model. According to a computation with a broken power-law ECMF, where \( \beta = 0 \) for \( M_{\text{ext}} \leq 100 M_\odot \) and \( \beta = 2 \) otherwise was chosen, typical half-mass radii need to be larger by \( \approx 0.1 \) pc in order to agree with the observations.

A small difference in the solutions for \( r_h \) (0.1 versus 0.3 pc) in the solutions for G- and M-dwarf binaries might be evident (Fig. 4). Since the estimated cluster size is also a measure of how dense the region is in which the respective subpopulation has formed, the possible difference might simply indicate their formation in different locations of the same cluster. A primordial mass-segregated cluster, where the G-dwarfs would be more centrally confined to a region of higher density, while M-dwarfs form out to larger radii, would naturally account for the apparently different ranges of allowed cluster radii.

The uncertainty in the inferred typical cluster size of 0.2 pc does not appear to be very large, given the estimates for \( r_h \) by comparison of the model with independent observations (Section 3.3). Even if a possible error in the ECMF index, \( \beta \), of up to 0.5 is considered (Larsen 2009) the uncertainty in \( r_h \) is of the order of 0.1–0.3 pc only (Fig. 2, left panel).

To calculate the composition of the Galactic field stellar population a (constant, average) global SFR was adopted. A declining SFR history might instead be better suitable to describe the evolution of the MW disc (Boissier & Prantzos 1999; Naab & Ostriker 2006; Schönrich & Binney 2009). If the SFR has been higher than average, clusters more massive than allowed for the adopted SFR (equation 6) would have been able to form, which would have contributed a larger number of single stars. Later, when the SFR sank below the average SFR the highest mass clusters would not be able to form any more and more binaries would enter the field originating in the low-mass clusters. These effects might eventually compensate each other, but will depend on the actual history, which cannot be tested without a modification of the used code. Also, the global history might be non-representative for the solar neighbourhood (Boissier & Prantzos 1999). However, even if the SFR has been much higher in the past only and settled to the present value, the inferred typical \( r_h \) would not change strongly. Reducing or enhancing the SFR by up to two orders of magnitude and at the same time retaining the observed binary fraction requires, respectively, a \( r_h \) smaller by \( \approx 0.1 \) pc or larger by \( \approx 0.2–0.3 \) pc only (as evident from Fig. 2, middle panel).

In this sense, the typical cluster size seems to be well constrained by the IGBDF model.

### 4.2 MW orbital-parameter BDFs

Since the IGBDF models for the MW (Section 3.3) with typical star cluster sizes of \( r_h \approx 0.2 \) pc agree well with independent observational data, these models are used to predict the period, semimajor axis, mass ratio and eccentricity IGBDFs for the solar neighbourhood.

Fig. 8 depicts the resulting distributions for binaries of different spectral types and the combined distributions (\( m_1 \leq 2 M_\odot \); no additional cuts, e.g. in period, are applied). For the period, mass ratio and eccentricity IGBDFs the distributions for F-, G-, and K-type binaries are very similar and therefore probably hard to distinguish by observations. However, the mass-ratio IGBDF might hold the ability to test the model prediction for different spectral types at low \( q \approx 0.1–0.2 \) and high \( q \approx 0.9–1 \) mass ratios since differences are more pronounced there. In particular, there is a larger gap between the G- and K-binaries around \( q \approx 0.15 \) and between the F- and later-type binaries at \( q \approx 0.05 \) which might be visible in observations, too.

The M-dwarf binary IGBDFs for all quantities are distinct from the corresponding distributions for earlier spectral types since breaking-up of binaries having at least one M-type component happens frequently before their birth clusters dissolve (Section 3.3.5). Thus, by the time the M-dwarfs emerge from the clusters their majority are single stars consistent with observations (Lada 2006, Section 3.3.5), despite being born as binaries. Differences in observed distributions for M-dwarf binary populations and those of later types should be apparent and are thus suited to test the IGBDF predictions. The combined IGBDFs lie typically between the M-dwarf distribution and the earlier types.

Note that it is not possible to show a common initial mass-ratio distribution since it depends, in contrast to the other IGBDFs, on the considered spectral type (see also Kouwenhoven et al. 2009). This is a result of the random selection of birth binary component masses and the upper limit for the mass of the primary star when a spectral-type-limited sample is investigated.\(^2\) While at least the shape of the initial mass-ratio distributions is similar for F-, G- and K-binaries, it is completely different for M-dwarfs (compare, e.g., the middle with the right panel in Fig. 5 for the initial distributions of G- and M-dwarfs, respectively). The IGBDFs \( \text{decline with increasing mass ratio for F-, G- and K-type binaries, while the mass-ratio IGBDF for M-dwarfs is increasing with increasing } q \). We explicitly note that the same observational data for M-dwarfs, for which typically a flat \( q \)-distribution is inferred, are consistent with the increasing trend in the models. The mass-ratio distribution is \( \text{flat only if all primary masses are combined to construct a mass-ratio IGBDF.} \)

As opposed to DM91, Raghavan et al. (2010) find a mass-ratio distribution for binaries with a solar-type primary that is flat between \( q \approx 0.2 \) and 0.9, while finding a similar period distribution as DM91. This cannot be expected within the framework of the IGBDF model. Since the range of considered primary masses in the Raghavan et al. (2010) and DM91 study are comparable, their mass-ratio distributions should be in agreement if the period

\(^2\) By definition the secondary mass can only be lower.
Figure 8. Predictions for the period ($\log_{10}P$), semimajor-axis ($a$), mass ratio ($q$) and eccentricity ($e$) IGBDFs for different spectral types and the combined distributions in the solar neighbourhood for the MW models (Section 3.3) which best fit the observational data ($r_h \approx 0.2$ pc). The $\log_{10}P$, $a$ and $e$ IGBDFs for F-, G- and K-binaries appear very similar, but can probably be distinguished at low and high $q$-values in the mass-ratio IGBDF. The peaks seen for $q = 0.9 - 1$ in the mass-ratio distributions are due to pre-main-sequence eigen-evolution (Section 2). All IGBDFs for M-dwarfs and the all-primary mass IGBDFs are distinct from those for F- to K-binaries. Note that a common initial distribution for the $q$-IGBDFs cannot be given since they are different for different spectral types (see the discussion in Section 4).

4.3 Dependence on galaxy morphology

The binary properties of galaxies within the IGBDF model are strongly dependent on the SFR (Section 3.2). Since SFRs in galaxies are observed to cover a large range from $\approx 0 \, M_\odot \, yr^{-1}$ for giant ellipticals (Es) to $\gtrsim 1000 \, M_\odot \, yr^{-1}$ for ultraluminous infrared galaxies (ULIRGS; Grebel 2011), binary frequencies and binary properties are expected to vary between galaxies of different morphology. This might have cosmological implications, such as for the SN Type Ia rates in galaxies.

4.3.1 Elliptical galaxies

Ellipticals are nowadays more or less free of cold gas and are pressure or random stellar motion supported with low or no star
formation activity. Even the suspected precursors of giant Es, quasars at high redshift ($z \approx 6$, $t_{\text{Universe}} \lesssim 1$ Gyr), reveal supersolar metallicities indicating a starburst that quickly enriched the material with metals (Fan et al. 2001). This suggests that Es had a large SFR initially until their gas reservoir was depleted. One of the highest redshift quasars known has an SFR of $\approx 1000 M_\odot$ yr$^{-1}$ as derived for ULIRGS (Fan 2006). For SFR = $10^3 M_\odot$ yr$^{-1}$, from the middle panel of Fig. 2, a binary fraction of the order of $\approx 30$–$40$ per cent for a typical cluster size of $r_\text{cl} = 0.2$ pc can be inferred. It is noted that a non-shallow mass–radius relation (Section 4.1), which is not considered in the present models, might affect the results for such a high SFR. If, additionally, during star bursts low-mass clusters are not able to form, i.e. $M_{\text{cl,min}}$ is larger, this would further lower the binary fraction in Es (Fig. 2, right panel). Thus, if E galaxies formed rapidly they ought to have low binary fractions.

4.3.2 Spiral galaxies

In terms of the SFR, spiral galaxies are intermediate objects between Es and dwarf galaxies. SFRs in spirals like the MW lie between 0.1 and $10 M_\odot$ yr$^{-1}$ (e.g. Lee et al. 2009, 2011). For the same cluster size and an SFR of $1 M_\odot$ yr$^{-1}$, from Fig. 2 a global binary frequency of $\approx 40$–$50$ per cent is expected, similar to what is seen in the solar neighbourhood (Section 3.3).

4.3.3 Dwarf irregular galaxies

Dwarf irregular (dIrr) galaxies have very low SFRs ranging down to $10^{-5} M_\odot$ yr$^{-1}$ (Lee et al. 2009, 2011). For $r_\text{cl} \approx 0.2$ pc, dIrrs would be expected to exhibit a significantly larger binary fraction, which is of the order of 70–80 per cent.

4.3.4 IGBDFs for galaxy types

The above results hold if cluster formation in the MW is representative for other galaxy types (Section 3.3, $r_\text{cl} = 0.2$ pc, but with different SFRs). Then, the expected period and mass-ratio IGBDF for Es, spirals and dIrrs for a population of binaries with $m_1 \lesssim 2 M_\odot$ (no cuts in period or primary mass) are expected to be as in Fig. 9. The period IGBDF of Es peaks at shorter periods than the corresponding distributions for S and dIrr galaxies. It is due to the initial starburst in Es (high SFR), namely that more dynamically evolved binary populations from high-mass clusters, which are not present in the lower SFR spirals and dIrrs (equation 6), contribute to the galactic field of Es (Section 3.2). The mass-ratio IGBDFs for all morphological types look alike, a difference only appearing through the varying binary fraction between the galaxies (the area below the distributions).

4.4 Model limitations

The validity of the results and predictions outlined in this analysis depend on the accuracy of the assumptions entering the IGBDF model. While the model formulated in Section 2.2 appears robust, the results obtained by performing the integration in equation (15) depend on the properties of the birth and initial binary population (see Section 2, Kroupa 1995a,b, Paper I) and the resulting analytical description of the evolution of binary properties in the $N$-body models of Paper I, which use this initial binary population. The two major assumptions are that (i) the birth binary population is formed via random pairing of the two binary components from the canonical stellar IMF and that (ii) the birth population evolves into the initial population via pre-main-sequence eigenevolution, which was specifically parametrized for G-dwarf binaries (Kroupa 1995b). If, e.g., instead of the declining mass-ratio distribution found in DM91 the flat mass-ratio distribution in Raghavan et al. (2010) for solar-type stars were correct, this would possibly imply a birth pairing method for binaries which is different from random-pairing and, in turn, would require alteration of the binary birth population (see also Kouwenhoven et al. 2009). Alternatively, the eigenevolution model might need adjustments, e.g., through primary star-dependent eigenevolution parameters $\lambda = \lambda(m_1)$ and $\chi = \chi(m_1)$, so that the initial binary population may be mildly different.

Figure 9. Predictions for the period ($\log_{10} P$) and mass-ratio ($q$) IGBDFs based on galaxy morphology. Model parameters are as those for the MW (Section 3.3, $r_\text{cl} = 0.2$ pc) except that SFRs are used according to the galaxy type (Section 4.3). Elliptical (E) galaxies have the lowest binary fraction (the area below the distribution) and the period IGBDF peaks at shorter periods than for spirals (S) and dwarf irregulars (dIrr). The shapes of the mass-ratio IGBDFs for the different galaxies resemble each other. The pre-main-sequence eigenevolution peak in the mass-ratio IGBDF is visible, as in Fig. 8.
for other spectral types. These limitations should always be kept in mind, but the overall results of the IGBDF model would not be affected.

5 SUMMARY AND OUTLOOK

Following observational evidence which implies all stars less massive than about 2 M☉ to form as binaries with component masses picked randomly from the stellar IMF in discrete star formation events (i.e. embedded star clusters) and allowing for pre-main-sequence eigenvolution (Section 2) and stimulated evolution, the concept of IGBDFs is introduced. Adding up the stellar populations that ever formed in star clusters which are selected from an ECMF yields the statistical binary properties of a galactic field population (Dynamical Population Synthesis). This approach is similar to the IGIMF theory which adds up the stellar IMFs in individual clusters to calculate the galaxy-wide IMF (Section 1).

The IGBDFs depend on the minimum cluster mass, M_min, the galaxy-wide SFR, the steepness of the ECMF and the typical, or average value for cluster half-mass radii, r_h, in a star cluster system. The galactic field binary fraction increases with decreasing M_min and SFR, with increasing index β of the ECMF (equation 7) and with increasing r_h (Fig. 2). It is found that low-mass (i.e. low-density) clusters contribute most binaries to the field since stimulated evolution is least effective in them and low-mass clusters are most numerous due to the steep ECMF. High-mass (high-density) clusters donate most single stars since binary destruction is effective and the number of stars forming in a high-mass cluster is larger than in a low-mass cluster.

Applying the IGBDF model to the MW, i.e. estimating M_min and the SFR from observations, the period, mass ratio and eccentricity IGBDFs are constructed for different typical cluster sizes, r_h. The models independently agree with observed distributions for late-type binaries in the solar neighbourhood, solely adjusting the typical r_h to 0.1–0.3 pc, which is the single remaining free model parameter. This suggests that MW clusters typically form quite compact. M-dwarf binaries appear to have formed in slightly more extended clusters than G-dwarfs, a possible sign of mass segregation. The integrated populations show that the majority of all Galactic field primaries end up being single stars despite being born in binary systems. In particular, the Galactic field binary fraction is a function of the spectral type, i.e. f_0(M) < f_0(K) < f_0(G) < f_0(F), and the binary-frequencies derived from binary-star formation and stimulated evolution in compact clusters agree with observational data.

It has been pointed out that the shape of the mass-ratio distribution depends on the considered primary mass (Fig. 5). While F- to K-type binaries show a decreasing trend with increasing q, according to the model the M-dwarf mass-ratio distribution increases as q increases. A flat distribution is only obtained if primaries from the full mass range are combined when constructing the mass-ratio distribution.

Using the best-fitting model, the named integrated distributions for late-type binary populations (F to M) in the solar neighbourhood are predicted (Fig. 4). While the IGBDFs for F-, G- and K-type binaries are probably hard to distinguish by observation, M-dwarf binary and the all-binary cumulative distributions appear rather distinct from them. Therefore, the M-dwarf and cumulative distributions are probably the best populations in comparison with the ones for F-, G- and K-binaries, to test the IGBDF predictions.

Assuming star formation in the MW (in terms of the ECMF and cluster radii) is typical also for other galaxy morphologies, the binary population in elliptical galaxies (high SFR) is predicted to be significantly smaller than in spiral (intermediate SFR) and dwarf galaxies (low SFR) and their period- and mass-ratio IGBDFs are calculated.

The IGBDF model will be extended to allow us the calculation of binary populations in galaxies which had a strongly varying star formation history, i.e. a time dependence will be incorporated, via SFR(t). It will be possible to synthesize binary properties in individual nearby (dwarf) galaxies, and the case of the MW may be revisited. A full synthetic galaxy might be constructed and ‘observed’ in the computer to mimic real data in order to provide more sophisticated means of comparison with observations. It will also allow us to test their influence on observationally derived properties, especially in populations where binaries cannot be resolved, such as velocity dispersions (used to calculate dynamical masses).

ACKNOWLEDGMENTS

MM was supported for this research through a stipend from the International Max Planck Research School (IMPRS) for Astronomy and Astrophysics at the Universities of Bonn and Cologne. The authors thank K. M. Menten and D. Raghavan for useful discussions and help with their data.

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