Identifying discreditable firms in a large-scale ownership network

Tao Zhou\textsuperscript{1}, Yan-Li Lee\textsuperscript{2}, Qian Li\textsuperscript{3}, Duanbing Chen\textsuperscript{1,4}, Wenbo Xie\textsuperscript{4,5}, Tong Wu\textsuperscript{3}, Tu Zeng\textsuperscript{3}

\textsuperscript{1}CompleX Lab, University of Electronic Science and Technology of China, Chengdu 711731, P. R. China
\textsuperscript{2}School of Computer and Software Engineering, Xihua University, Chengdu 610039, China
\textsuperscript{3}Business Big Data Inc., Chengdu 610095, P. R. China
\textsuperscript{4}Union Big Data Inc., Chengdu 610299, P. R. China
\textsuperscript{5}School of Computer Science, Southwest Petroleum University, Chengdu 610500, People's Republic of China

Abstract: Violations of laws and regulations about food safety, production safety, quality standard and environmental protection, or negative consequences from loan, guarantee and pledge contracts, may result in operating and credit risks of firms. The above illegal or trust-breaking activities are collectively called discreditable activities, and firms with discreditable activities are named as discreditable firms. Identification of discreditable firms is of great significance for investment attraction, bank lending, equity investment, supplier selection, job seeking, and so on. In this paper, we collect registration records of about $1.13 \times 10^8$ Chinese firms and construct an ownership network with about 6 million nodes, where each node is a firm who has invested at least one firm or has been invested by at least one firm. Analysis of publicly available records of discreditable activities show strong network effect, namely the probability of a firm to be discreditable is remarkably higher than the average probability given the fact that one of its investors or investees is discreditable. In comparison, for the risk of being a discreditable firm, an investee has higher impact than an investor in average. The impact of a firm on surrounding firms decays along with the increasing topological distance, analogous to the well-known “three degrees of separation” phenomenon. The uncovered correlation of discreditable activities can be considered as a representative example of network effect, in addition to the propagation of diseases, opinions and human behaviors. Lastly, we show that the utilization of the network effect largely improves the accuracy of the algorithm to identify discreditable firms.
Perceiving the risk of a firm is a basis for banks to approve loans, for institutional investors to make investments, for job seekers to evaluate career prospects, for another firms to judge whether or not to cooperate, and so on. If risks of a huge number of firms can be estimated by a low-cost method, one can then evaluate the overall risk of a region or an industry, which helps financial institutions avoid the risk in advance and win more time for government to resolve the risk [1]. Financial institutions have already developed successful methods to comprehensively evaluate risks of firms, based on firms’ elementary attributes (industry, region, size, category, etc.) and balance sheets [2]. Recently, thanks to the explosive development of information technologies, we could obtain some microscopic data that records growing processes of firms and interactions between firms. The former includes real-time cash flows, online recruitment, bidding results, media reports, announcements of court sessions, consumption of power and water, and so on, and the latter includes supply chains, cash transactions, equity relationships, joint directors, and so on. Combining these data and advanced algorithms of data mining and machine learning, one could propose more timely and effective methods to quantify risks of individual firms, to characterize the propagation of risks in firm networks, and to predict developing tendencies of individual firms and the whole industry [3][4].

This paper will analyze a kind of very important yet rarely reported data associated with risks of firms, namely the records of firms’ discreditable activities. Those records include administrative penalties resulted from violations of laws and regulations about food safety, production safety, quality standard and environmental protection, executed records because of loan, guarantee and pledge contracts, negative records due to the delay or deficiency of tax, social insurance or house fund, negative judicatory adjudications related to the labour law, and so on. For convenience, firms with those discreditable records are named as discreditable firms. Notice that, a discreditable firm is not necessarily risky or recessionary, but undoubtedly we should be more cautious in potential cooperation. Meanwhile, it is a serious warning if discreditable activities frequently occur in a specific industry or many investees of a firm become discreditable.

Although not for discreditable activities, some scientists have already studied network effects of firm risks, including the theoretical understanding of how risks cascade in firm networks [5][6], the detection of early-warning signals of and pathways towards instability in financial networks [7][8], the identification of the most risky nodes in loan networks [9], the rating of credit risks of firms in payment networks [10], and so on. This paper focuses on the so-called ownership network [11][12], where each node denotes a firm and a directed link from $i$ to $j$ represents the ownership that $i$ is $j$’s investor.

We obtained the basic information of 113,450,085 firms from the public API of the National Enterprise Credit Information Publicity System (www.gsxt.gov.cn). The discreditable records of these firms are directly obtained from executive information disclosed by the Supreme People’s Court of the People’s Republic of China (http://zxgk.court.gov.cn/). A firm’s basic information includes the unified social credit code (i.e., ID of the firm), name, category, investors (together with their respective shares), registration date, registered capital, registered address, and so on. Detailed data description is presented in Supplementary Text S1. The unified social credit code can guarantee the uniqueness of a firm (names may be duplicated), based on which we can construct the ownership
network. Most firms have not invested other firms and only have natural person investors, behaving as isolated nodes in the ownership network, and thus are omitted in the later analyses. There are in total 5,932,130 firms having been invested by or having invested at least one firm, and 462,880 firms have at least one discreditable record. Those 5,932,130 firms constitute the ownership network $G$, which has as many as 1,306,309 components. Here, a component is a maximum weakly connected subgraph of $G$. The largest component, denoted by $G^m$, contains 2,097,683 firms, and other components are much smaller. For example, the second largest component has 23,346 firms, the third has 1,119 firms, and all others are of less than 1000 firms. As shown in figure 1A, except for the largest and second largest components, the size distribution of all remaining components follows the Zipf’s law [13], with Zipf’s exponent being about 0.51. According to the quantitative relationship between Zipf’s law and power law [14], the probability density function of component sizes obeys a power law with exponent about $1+1/0.51=2.96$. The Supplementary Text S2 presents the probability density function of the component sizes, the simplified derivation of the mathematical relationship between Zipf’s law and power law, and the comparison between theoretical predicted and real power-law exponents. The largest component $G^m$ is very sparse, with only 2,519,230 directed links and an average degree about 1.2. $G^m$ contains 208,107 discreditable firms, accounting for about 10% of total firms in $G^m$. As shown in figure 1B and figure 1C, $G^m$ exhibits the scale-free property [15], where its in-degree distribution well follows a power law with exponent about 3.2 according to the maximum likelihood estimation [16], and its out-degree distribution has a heavy tail, looks like a power law, but cannot be fitted by a single power-law function. If we treat $G^m$ as an undirected network, its clustering coefficient [17] and assortativity coefficient [18] are 0.038 and -0.0064, respectively.

Figure 1. Topological properties of the ownership network. (A) Zipf’s plot of the component sizes, where the x-axis denotes ranks of components according to their sizes and y-axis shows the corresponding sizes. As visual examples, four small-size components are illustrated, with discreditable firms being emphasized by orange color. (B) The in-degree distribution of $G^m$ in a log-log plot, which can be well fitted by a power law with exponent 3.2. (C) The out-degree distribution of $G^m$ in a log-log plot, which exhibits a heavy-tailed feature but cannot be well fitted by a single power law.
We further analyze how the discreditable firms distribute in the ownership network: whether they exhibit a certain pattern or follow a random manner. Firstly, we test if the discreditable firms tend to form some clusters. For each component \( c \) in \( G \), we calculate the size of the largest discreditable component \( S_c \) (i.e., the size of the largest component of the subgraph induced by the discreditable firms in \( c \)). If \( c \) contains \( N_c \) discreditable firms, we say the aggregation degree of discreditable firms in \( c \) is \( \tau_c = S_c / N_c \). If all discreditable firms constitute a single component, then \( \tau_c = 1 \); if every discreditable firm is isolated in the induced subgraph, then \( \tau_c = 1 / N_c \). To guarantee all components under consideration are non-trivial, we remove the components whose nodes are all discreditable firms and the components with < 2 discreditable firms. The number of remaining components is 27,673. Figure 1A marks the aggregation degrees of the four example components.

Figure 2. Discreditable firms exhibit an aggregation pattern in the ownership network. (A) Distributions of aggregation degrees in the ownership network \( G \) (orange, \( p(\tau_c) \), left y-axis) and the null networks \( \tilde{G} \) (cyan, \( p(\tilde{\tau}_c) \), right y-axis). Aggregation degrees in the real ownership network are statistically larger than those in the null networks, indicating that the discreditable firms tend to aggregate together. The 15th largest component is visualized in the top-left corner, which shows a quasi-tree structure with 439 nodes and 439 links. Normal and discreditable firms are colored in cyan and orange, links connecting two discreditable firms are colored in orange and other links are colored in cyan. This component has 67 discreditable firms and the aggregation degree is 0.7015 (i.e., the largest component in the subgraph induced by discreditable firms has 47 firms), while the aggregation degree of the null networks, averaged over 1000 independent implementations, is only 0.1892. (B) Zipf’s plot of the component sizes in the discreditable subgraphs of \( G \) (orange) and a corresponding null network (cyan), where the x-axis denotes ranks of components according to their sizes and y-axis shows the corresponding sizes. (C) Zipf’s plot of the component sizes in the discreditable subgraphs of \( G^{in} \) (orange) and a corresponding null network (cyan).

We build a null network for each of the 27,673 components. In the null network, the topology and the number of discreditable firms are fixed, while discreditable firms are randomly selected from all nodes in the corresponding component. For each component, we independently generate 1000
null networks and calculate the average aggregation degree of discreditable firms, denoted by $\tilde{r}_c$. Figure 2A reports the two distributions of aggregation degrees, $p(r_c)$ and $p(\tilde{r}_c)$, for the 27,673 components and their null networks. For the convenience of comparison, we put the two distributions in the same plot but with different scales in y-axis. As shown in figure 2A, aggregation degrees in real components are statistically larger than those in null networks. In particular, in 58.32% real components, $r_c = 1$, while this ratio is only 0.35% in null networks. For the largest component, $r_c = 0.1991$, while the corresponding $\tilde{r}_c$ is only 0.0034. Figure 2A visualizes the 15th largest components, where $r_c = 0.7015$ and $\tilde{r}_c = 0.1892$. If we simply calculate the average aggregation degree over all real components or null networks, then $\langle r_c \rangle = 0.7922$ and $\langle \tilde{r}_c \rangle = 0.6823$, the former is obviously larger. The Supplementary Text S3 directly compares $p(r_c)$ and $p(\tilde{r}_c)$ in a unified y-axis, and contrastively visualizes the 15th largest component and one realization of the corresponding null model. Figure 2B compares component size distributions in the discreditable subgraphs of $G$ and a corresponding null network, where the discreditable subgraph is the subgraph induced by all discreditable firms, and the null network of $G$ is generated by redistributing all discreditable firms while keeping the topology unchanged. As shown in figure 2B, discreditable firms of the real network tend to form larger components than the null network. Figure 2C compares $G^m$ and its null network, showing the similar result. In a word, in the macro level, comparing with null networks, discreditable firms in the real ownership network are more likely to form large components.

Figure 3. Local clustering of discreditable firms. (A) How $L(m)$ changes as the increase of $m$ when treating $G^m$ as a directed network. The orange and cyan curves represent the cases with neighbors defined as investees and investors, respectively. (B) How $L(m)$ changes as the increase of $m$ when treating $G^m$ as an undirected network. (C) The increment rate of the probability that an arbitrary firm $i$ is a discreditable firm given the information that at least one $d$-order neighbor of $i$ is discreditable, comparing with the probability that $i$ is a discreditable firm without any reference information. The orange, cyan and red colors respectively denote the cases with $d$-order neighbors being defined as firms reached by directed paths with length $d$ starting from $i$, directed paths with length $d$ ending at $i$, and paths with length $d$ when treating links as undirected.

Next, we analyze how a discreditable firm will affect its neighborhood in the ownership network.
Our hypothesis is that the occurrences of discreditable activities are partially governed by the network effect, namely a firm neighboring to discreditable investors or investees will have a high probability to be discreditable itself. As most components are of very small size, we concentrate on the largest component $G^m$. To characterize the network effect, we denote $L(m)$ the probability that a firm is discreditable given the fact that there are no less than $m$ discreditable firms of its neighbors. Given a firm $i$, its neighbors can be defined in three different ways: (i) the firms being pointed by an out-link starting from $i$ (i.e., all $i$'s investees); (ii) the firms having an in-link pointing to $i$ (i.e., all $i$'s investors); (iii) the neighbors of $i$ by treating $G^m$ as an undirected network (i.e., both $i$'s investees and investors). Obviously, when $m = 0$, $L(m)$ equals to the fraction of discreditable firms in $G^m$, say $L(0) = 208,107/2,097,683 = 0.0992$. If our hypothesis is correct, we should observe an increasing $L(m)$ as the increase of $m$ from 0. Figure 3A shows $L(m)$ according to the definitions (i) and (ii) where $G^m$ is treated as a directed network, and figure 3B shows $L(m)$ according to the definition (iii) where $G^m$ is treated as an undirected network. Those plots uncover two important phenomena. Firstly, $L(m)$ increases as the increasing of $m$ for all the three definitions, supporting the above-mentioned hypothesis. Secondly, the impacts from neighbors are asymmetric, specifically, the impacts from a discreditable investee is higher than that from a discreditable investor. The Supplementary Text S4 provides the results from the whole network $G$, which are similar to the results from $G^m$.

As shown in Figure 3A and Figure 3B, the influence from direct neighbors is tremendous. Specifically, the probability to be discreditable is increased by about 80% in average if one of the target firm’s direct neighbors is known to be discreditable. Accordingly, a natural inference is that the neighbors’ neighbors are also influential, because if a neighbor’s neighbor is discreditable, the expected probability to be discreditable of the neighbor will increase, and thus the target firm will be more likely to be discreditable. In general, we consider the influence from $d$-order neighbors, namely how much the probability to be discreditable increases if we known one of the target firm’s $d$-order neighbors (i.e., firms with distance $d$) is discreditable. Figure 3C presents three typical cases that respectively define $d$-order neighbors of the target firm $i$ as firms reached by directed paths with length $d$ starting from $i$, directed paths with length $d$ ending at $i$, and paths with length $d$ when treating links as undirected (the last case is called undirected $d$-order neighbors). One can observe the well-known “three degree of separation” phenomenon [19][20], namely the influence decays as the increase of distance, and when $d>3$, the influence becomes insignificant. Readers should be aware of that the three cases shown in figure 3C are only a part of possible neighboring relationships. For example, when treating $G^m$ as a directed network, in addition to the investors’ investors and investees’ investees, we can also analyze the investors’ investees and investees’ investors. The Supplementary Text S5 gives the results of all possible definitions of $d$-order neighbors ($d \leq 3$). In despite of complication, the results are qualitatively the same to those reported in figure 3C.

The above network effect of firms’ discreditable activities can help us identify risky firms. To demonstrate such advantage, we study a typical classification problem: to predict which firms are discreditable. For each firm, we consider two kinds of features, the individual features and the network features. The individual features include five aspects: (i) registered capital; (ii) type (according to the official document No. 105 [2008] issued by the State Council, there are in total 158 firm types); (iii) size (according to the standard from the Ministry of Industry and Information
Technology, the National Bureau of Statics, the National Development and Reform Commission, and the Ministry of Finance of the People’s Republic of China, there are five size groups, say large, medium, small, micro and unidentified; (iv) registered address (including the State Administration for Industry and Commerce of the People’s Republic of China, and 31 provinces, autonomous regions, municipalities directly under the central government of China); (v) industrial category (according to the standard from the National Bureau of Statistics, there are 21 industrial categories and our data covers 19 of them). The registered capital is a numerical feature, and others are all categorical features. For each categorical feature, if it has $h$ values, we use the one-hot encoding to generate the corresponding $h$ binary features. For example, the registered address is expanded to 32 binary features. If a firm is registered in Beijing, then the feature associated to Beijing takes value 1 and other 31 features take value 0. Eventually, we obtain 215 individual features, including 1 numerical feature and 214 binary features. The industrial category cannot be obtained from the industrial and commercial registration information, and thus we infer firms’ categories by semantic analysis, based on firms’ names and business scopes. The Supplementary Text S6 provides detailed introduction of every individual feature, as well as the algorithm to infer industrial categories. To ensure the understandability, we only consider 12 network features that can be calculated based on the information of 2-order neighbors of any target firm $i$, including the number of discreditable firms and the fraction of discreditable firms in (i) $i$’s investors, (ii) $i$’s investees, (iii) $i$’s undirected 1-order neighbors, (iv) investors of $i$’s undirected 1-order neighbors, (v) investees of $i$’s undirected 1-order neighbors, and (vi) $i$’s undirected 2-order neighbors. The Supplementary Text S7 provides precise definitions and examples of how to calculate those network features.

We apply the logistic regression, a highly interpretable predictor good at heterogeneous features. If the feature vector of a firm is $x$, the likelihood that this firm is discreditable is

$$E(x, w) = \left(1 + e^{-w^T x}\right)^{-1},$$

where $w$ is a 227-dimensional (215 individual features and 12 network features) vector for feature weights. If the training data contains $n$ firms with known status (discreditable or not), say $(x_1, y_1), \ldots, (x_n, y_n)$, where the label $y_i$ denotes whether the $i$th firm is discreditable (1-discreditable; 0-not discreditable), then the weighting vector $w$ can be obtained by maximizing the likelihood $\prod_{i=1}^n [E(x, w)]^{y_i}[1 - E(x, w)]^{1-y_i}$. In $G^m$, there are in total 1,851,993 firms (about 88.29%) with complete individual features. We use the 10-fold cross validation, that is, we randomly divide the 1,851,993 firms into 10 equal-size groups, treat each group as the testing set once (the other 9 groups constitute the training set), and take the average prediction precision over the 10 runs to evaluate the performance. In each run, we obtain the weighting vector $w$ by the training set and rank firms in the testing set according to their likelihoods $E$. If we predict the top-$S$ firms with the highest likelihoods as discreditable firms, the precision is $P = R/S$, where $R$ is the number of truly discreditable firms among the $S$ predicted firms. Figure 4A shows how the precision changes with increasing $S$. Obviously, the prediction based on network features is better than that based on individual features, and by combining individual and network features, the performance can be remarkably improved.

In the logistic regression, weights indicate importances of features. Figure 4B lists weights of considered features, from which one can observe that the top-4 features with the highest weights are all network features, and there are 7 network features out of the top-10 highest weighted features.
The feature with the highest weight is the number of discreditable firms in the target firm’s undirected 1-order neighbors, whose weight is 49.4 times of the highest weight of individual features. Although differences in weights cannot fully represent differences in feature importances, the observation still demonstrate the significance of network effect in identifying discreditable firms. In a word, the network features play a significant role in the prediction, and can capture risky signals that cannot be reflected by individual features.

Figure 4. Significant roles of network features, reflected by the improvement of prediction
**precision and high weights in the regression model.** (A) How precision changes with increasing $S$, where blue squares, green triangles and red circles denote results based on individual features, network features, and combined features. (B) Weights of features when using combined features. Features are ranked anticlockwise by their absolute weights, and the height of a feature is proportional to the logarithm of its absolute weight. The features with negative weights are marked by *, and the features with zero weights are not shown.

Although the fact that risks of firms can propagate through ownerships is commonsensible, large-scale empirical studies are rare. Based on the ownership network with millions of firms, we uncover the notable network effect in firms’ discreditable activities, namely the probability of a firm to be discreditable is remarkably higher than the average probability if one or more of its neighbors are discreditable. We further show the asymmetry in the network effect that the influence from investees is higher than that from investors, as well as the “three degrees of separation” phenomenon that the influence of a firm decays along with the increasing topological distance, and will become insignifican for neighbors with distances larger than three. Network effects of risky behaviors, such as smoking [20], substance use [21] and drinking [22], are widely observed, yet an unsolved challenge is that the peer influence (one learns risky behaviors from friends) and peer selection (one makes friends because they share same risky behaviors) are not easy to be distinguished [23][24].

This work extends the scope of network effects and will not encounter the “peer influence vs. peer selection” problem, because an investor may invest a discreditable firm for other advantages of this firm, but will not invest a firm just because it is discreditable, namely the effect from peer selection should be absent or insignificant.

The estimation of the likelihood that a target firm will be discreditable in the near future can help banks decide whether to approve the loan of this firm, help institutional investors decide whether to invest this firm, help local governments decide whether to recruit this firm, help regulators (like the State Administration for Market Regulation and the Ministry of Ecology and Environment) detect early premonition of possible discreditable activities of this firm, help job seekers to decide whether to apply or accept a position of this firm, help other firms decide whether to cooperate with this firm, and so on. By a logistic regression model, we demonstrate that the introduction of a few very simple network features can largely increase the prediction precision. We expect the addition of more elaborately designed network features can further improve the prediction, such as features from 3-order neighbors of, certain motifs associated with, and higher-order structures involving a target firm [25][26][27]. For example, The Supplementary Text S8 shows that a firm’s likelihood to be discreditable will sharply increase if we know it belongs to some directed cycles. The discreditable activities can be divided into different classes (e.g., finance, safety, environment, etc.), and the in-depth analyses on class-specified discreditable activities may provide additional insights. This work offers an elementary yet solid starting point towards comprehensive understanding of risky behaviors of firms.

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