Calculation on Short-term Stiffness of Simply-supported Beams with Gradient Concrete and Glass Fiber Reinforced Plastic Bars

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Abstract: It is of the advantages of high bearing capacity and economy for the gradient concrete beam with glass fiber reinforced plastic(GFRP) bars, so it will be applied widely in practical engineering. In order to acquire the short-term stiffness of simply-supported beams with gradient concrete and GFRP bars, 14 simply-supported beams were designed. Based on material constitutive model, finite element models of the beams are established by ABAQUS software. The stiffness variation law of the specimens is acquired. The simulation results are in good agreement with the experimental results, which proves the correctness of finite element modeling. Based on numerical simulation the stiffness calculation formula of the beams is provided, which can provide reference for the application of the beams in the actual engineering.

1. Introduction

In order to make fully use of the advantages of high compressive strength of high strength concrete (HSC) and economy of normal strength concrete (NSC)[1-2], a kind of flexural members with gradient concrete is proposed, which the HSC is used in compression zone and the NSC is used in tension zone[3].

Masjedi et al. proposed the calculation method of composite beams with variable stiffness, and the differential equations of four degrees of freedom were established according to Euler Bernoulli theory. Later the beams with variable stiffness were tested, and the static response of composite beams with non-uniform stiffness was proposed according to Chebyshev collocation method[4-5]. Wang et al. studied the stiffness degradation law of steel-concrete beams under fatigue load, and it was found that with the increase of fatigue cycles, the stiffness degradation law of beams showed an obvious S-type monotonic decreasing trend[6-7]. Zhou et al. carried out seven bending tests of reinforced concrete short beams strengthened with CFRP and based on reasonable calculation assumption, a calculation method for the whole process curve of bending stiffness of reinforced concrete short beams strengthened with CFRP was proposed[8-9]. Four reinforced concrete (RC) beams with non-dismantling ultra-high performance concrete formwork were tested by Liang et al. Based on the assumption of plane section, the effective moment of inertia method was used to establish the calculation formula for bending stiffness of this kind of beams[10-12].

At present, although the stiffness of GFRP concrete beams has already been studied, the research on stiffness of simply-supported beams with gradient concrete and GFRP bars has not been reported and the relative theory is immature. Based on ABAQUS, the short-term stiffness of simply-supported
beams with gradient concrete and GFRP bars was analyzed. The calculation formula for short-term stiffness of this type of beams was put forward through statistical regression, which could provide theoretical support for its popularization and application in practical engineering.

2. Specimen design
In order to study the influence of different parameters on the beam stiffness, 14 simply-supported beams with gradient concrete and GFRP bars were designed as shown in table 1. The main parameters included the strength of HSC in compression zone \( f_{cu}^{k1} \), the reinforcement ratio of GFRP bars \( \rho_{GFRP} \) and the section heights of HSC in compression zone \( h_1 \). The calculated span of all specimens was 2100mm with a sectional size of 150mm×200 mm(\( b \times h \)). Two steel bars with a diameter of 12mm were adopted in the compression zone. Stirrups with a diameter of 8mm and a spacing of 80mm were adopted. The compressive strength of NSC \( f_{cu}^{k} \) was 30MPa, and the thickness of concrete cover was 20mm.

| Specimens | \( f_{cu}^{k} \) (MPa) | \( f_{cu}^{k1} \) (MPa) | GFRP bars | \( \rho_{GFRP} \) | \( h_1 \) (mm) |
|-----------|------------------------|------------------------|-----------|----------------|----------------|
| SBGC-1    | 30                     | 60                     | 2φ14      | 1.14%          | 90             |
| SBGC-2    | 30                     | 60                     | 2φ16      | 1.49%          | 90             |
| SBGC-3    | 30                     | 60                     | 2φ18      | 1.88%          | 90             |
| SBGC-4    | 30                     | 60                     | 2φ20      | 2.33%          | 90             |
| SBGC-5    | 30                     | 60                     | 2φ14      | 1.14%          | 120            |
| SBGC-6    | 30                     | 60                     | 2φ16      | 1.49%          | 120            |
| SBGC-7    | 30                     | 60                     | 2φ18      | 1.88%          | 120            |
| SBGC-8    | 30                     | 60                     | 2φ20      | 2.33%          | 120            |
| SBGC-9    | 30                     | 70                     | 2φ18      | 1.88%          | 90             |
| SBGC-10   | 30                     | 80                     | 2φ18      | 1.88%          | 90             |
| SBGC-11   | 30                     | 90                     | 2φ18      | 1.88%          | 90             |
| SBGC-12   | 30                     | 70                     | 2φ20      | 2.33%          | 120            |
| SBGC-13   | 30                     | 80                     | 2φ20      | 2.33%          | 120            |
| SBGC-14   | 30                     | 90                     | 2φ20      | 2.33%          | 120            |

3. ABAQUS finite element analysis and verification

3.1 Constitutive model
The linear elastic model was adopted as the constitutive model of GFRP bars, which the ultimate tensile strength was 606 Mpa and the elastic modulus was 40 Gpa. The ideal double-broken line model was adopted as the elastic-plastic constitutive model of steel bars and stirrups, and the constitutive model proposed in the Code for Design of Concrete Structures was adopted as that of NSC, while HSC adopted Ding's constitutive model.

3.2 Finite element model
The HSC, NSC and gaskets were established by C3D8R solid element, while steel bars and GFRP bars were established by T3D2 three-dimensional truss element. Steel bars were embedded into concrete by embedded command, and HSC and NSC were connected by binding. Besides, the displacement of gasket at the left end was constrained in X, Y and Z directions \( (U_1=U_2=U_3=0) \), and the displacement of gasket at the right end was constrained in Y and Z directions \( (U_2=U_3=0) \). Hexahedral element was used for the whole mesh generation. The finite element model was shown in figure 1.
3.3 Experiment verification of numerical analysis

Two simply-supported reinforced gradient concrete test beams by W. Zhou and one simply-supported concrete test beam with FRP bars by Y. F. Zheng[13] were selected to carry out finite element simulation. The comparative results of the ultimate load ($N_m$ and $N_u$) and the deflection ($f_m$ and $f_u$) obtained by simulation and experiment were shown in table 2, and the results were in good agreement.

| Specimens | $f_m$ (mm) | $f_u$ (mm) | $N_m$ (kN) | $N_u$ (kN) |
|-----------|------------|------------|------------|------------|
| B1        | 42.81      | 42.62      | 71.73      | 69.61      |
| B2        | 38.66      | 38.63      | 143.18     | 138.34     |
| BL2-1     | 20.71      | 20.81      | 88.91      | 93.65      |

4. Stiffness analysis of simply-supported beams

4.1 Formula of the short-term Stiffness

Bending rigidity of section:

$$ B_s = \frac{M_k}{\Phi} \quad (1) $$

Relationship between the strain of GFRP bars and concrete:

$$ \Phi = \frac{\varepsilon_f + \varepsilon_c}{h_0} \quad (2) $$

Substituting formula (2) into formula (1):

$$ B_s = \frac{M_k h_0}{\varepsilon_f + \varepsilon_c} \quad (3) $$

Where $\Phi$ is the curvature of the section of simply-supported beam, $\varepsilon_f$ and $\varepsilon_c$ are the average strain of GFRP bars and the average strain of high-strength concrete, respectively.

According to the cross-sectional balance, formula (4) can be obtained:

$$ M_k = \sigma_f A_f \eta h_0 \quad (4) $$

According to the physical relationship, formula (5) can be obtained:

$$ \bar{\varepsilon}_f = \psi \varepsilon_f = \psi \frac{\sigma_f}{E_f} \quad (5) $$

According to the equilibrium relationship of cross-sectional stress distribution, formula (6) and formula (7) can be obtained:

$$ M_k = C \cdot \eta h_0 = \omega \cdot \sigma_c \bar{\varepsilon} h_0 \quad (6) $$

$$ M_k = T \cdot \eta h_0 = \sigma_k A_f \eta h_0 \quad (7) $$

The compressive stress at the edges of high-strength concrete:
According to the balance of the physical relationship, formula (10) and formula (11) can be obtained:

\[ \varepsilon_c = \psi_c \sigma_c = \frac{M_k}{\zeta E_c b h_0^2} \]  
\[ \varepsilon_t = \frac{\sigma_{tk}}{E_t} = \frac{M_k}{E_t A_t \eta h_0} \]

Substituting formula (10), (11) into formula (3). Then short-term cross-sectional stiffness of the beam can be obtained by formula (12):

\[ B_s = \frac{E_t A_t h_0^2}{\eta + \frac{E_t \rho_{GFRP}}{E_c \zeta}} \]

The deflection formula of the beams:

\[ f = \alpha \frac{ML_0^2}{B_s} \]

Where \( \rho_{GFRP} \) is the GFRP reinforcement ratio, that is \( \rho_{GFRP} = A_t / bh_0 \). \( \eta \) is the internal force arm coefficient of the cracked section. \( \eta \) and \( \alpha \) are equal to 0.86 and 0.1375, respectively.

4.2 Comparisons between deflection values of calculated and finite element analysis

The deflection of 14 simply-supported beams \( f_{cal} \) is calculated according to formula (13), which is shown in table 3. \( f_s \) represents the mid-span deflection of the specimens obtained by the finite element method. The calculated value of the formula agrees well with the simulated result. The mean value of the ratio of the calculated value to the simulated value is 1.06.

| Specimens | \( f_{cu}^k \) (MPa) | \( f_{ku}^k \) (MPa) | \( \rho_{GFRP} \) | \( f_s \) (mm) | \( f_{cal} \) (mm) | \( \frac{|f_s - f_{cal}|}{f_s} \) |
|-----------|----------------|----------------|------------|-------------|--------------|------------------|
| SBGCG-1   | 30             | 60             | 1.14%      | 30.14       | 28.94        | 9.0%             |
| SBGCG-2   | 30             | 60             | 1.49%      | 25.22       | 25.23        | 0.1%             |
| SBGCG-3   | 30             | 60             | 1.88%      | 22.11       | 21.26        | 3.8%             |
| SBGCG-4   | 30             | 60             | 2.33%      | 18.11       | 17.14        | 5.3%             |
| SBGCG-5   | 30             | 60             | 1.14%      | 59.2        | 54.12        | 8.6%             |
| SBGCG-6   | 30             | 60             | 1.49%      | 43.13       | 38.98        | 9.5%             |
| SBGCG-7   | 30             | 60             | 1.88%      | 41.72       | 37.92        | 9.1%             |
| SBGCG-8   | 30             | 60             | 2.33%      | 33.14       | 30.08        | 9.2%             |
| SBGCG-9   | 30             | 70             | 1.88%      | 48.95       | 46.23        | 5.5%             |
| SBGCG-10  | 30             | 80             | 1.88%      | 61.73       | 58.03        | 5.9%             |
| SBGCG-11  | 30             | 90             | 1.88%      | 59.42       | 58.66        | 1.2%             |
| SBGCG-12  | 30             | 70             | 2.33%      | 33.7        | 36.25        | 7.4%             |
| SBGCG-13  | 30             | 80             | 2.33%      | 35.34       | 38.20        | 8.1%             |
| SBGCG-14  | 30             | 90             | 2.33%      | 38.61       | 41.35        | 7.1%             |

5. Conclusions

At present, although the stiffness of GFRP concrete beams has already been studied, the research on
stiffness of simply-supported beams with gradient concrete and GFRP bars had not been reported and the relative theory was immature. Based on ABAQUS, the short-term stiffness of simply-supported beams with gradient concrete and GFRP bars was analyzed. The conclusions are as follows:

(1) By numerical simulation of two simply-supported Reinforced concrete beams with gradient concrete and one simply-supported beam with NSC and GFRP bars, the simulated results and experimental data agree well.

(2) Based on the principle of minimum stiffness, the deflection calculation formula of simply-supported beams consisting of gradient concrete and GFRP bars is regressed statistically, which is in good agreement with the finite element calculation. It can provide reference for the application of the beams in the actual engineering.

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