Influence of a constant field on a circular photovoltaic effect in two-dimensional superlattices

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Abstract. Using the relaxation time approximation, an analytical expression was obtained for the direct current density arising in a two-dimensional superlattice when, in addition to two elliptically polarized waves, a constant field acts on the sample along one of the directions.

1. Problem statement
One way to determine the characteristics of high-frequency electromagnetic oscillations (amplitude, frequency, phase) is to study the response in the form of direct current, which under certain conditions can occur in a solid-state structure with nonlinear properties (for example, [1-4]). Among these effects, the circular photovoltaic effect (CVHE) (see review [5]) studied in the surface layers of bulk materials, in thin films [6], sets of quantum wells [7-9], and silicon MOS structures should be distinguished first of all. [10], in graphene and graphene superlattices [1-4]. In general terms, this effect consists in the generation of direct current in the material under the action of an obliquely incident elliptically polarized electromagnetic wave in the direction perpendicular to the projection of the wave vector onto the plane of the sample. By the magnitude of the direct current, one can judge both the properties of radiation and the properties of the structure [11]. The effect is directly related to the presence of spin in charge carriers, and is best studied in materials with a Hamiltonian containing a term linear in modulus of the quasimomentum and explicitly taking into account the spin-orbit interaction (the so-called Rashba and Dresselhaus Hamiltonians). Note that CPGE is also possible in materials for which the spin-orbit interaction is uncharacteristic, in this case the appearance of direct current is associated with the transfer of the angular momentum of the photon to free charge carriers, and the effect is not due to spin orientation, but is purely orbital [8, 10]. Consider a superlattice, the energy spectrum of current carriers in which we write in the form:

\[ \varepsilon(p) = \Delta \left[ 1 - \cos \left( \frac{p_x d}{\hbar} \right) \cos \left( \frac{p_y d}{\hbar} \right) \right], \quad (1) \]

where $\Delta$ is the width of the miniband, $d$ is the period of the SL.

We assume that periodic potential is two-dimensional, and its period as well as miniband width in direction along X and Y axis are equal. Expression (1) is a model spectrum, which the most closely
correspond to 2D quantum dot superlattice and at certain condition can be used for description of GSL. Problem geometry is presented on figure 1. The intensity of the applied electric fields \( E_1 = E_{10} \cos(\omega t) \), \( E_2 = E_{20} \cos(\omega t + \varphi) \), the field \( E_1 \) is applied along the X axis, the field \( E_2 \) along the Y axis and along one of the directions a constant electric field is created \( E_c \). The vector of the intensity of the resulting electric field has the form

\[
E = \{E_c + E_{10} \cos(\omega t); E_{20} \cos(\omega t + \varphi)\}.
\]  (2)

We are interested in determine constant component of the current density \( j_y \) along Y axis.

\[ j_y = \lambda \langle [E_c, [E_1, E_2]] \rangle, \]

Angle brackets mean averaging over the period of the electromagnetic wave. The proportionality coefficient \( \lambda \) includes dependence on material parameters and temperature. Substituting here the expressions for the fields and averaging, we get

\[ j_y = \lambda E_c E_{10} E_{20} \cos \varphi / 2. \]

2. Investigation of the direct current density

The current density is determined as follows:

\[ j_y = e \sum_p \nu_p (p) f(p, t), \]  (3)

where \( \nu = \frac{\partial c(p)}{\partial p} = \frac{\Delta d}{h} \left\{ \sin \frac{p_x d}{h} \cos p_y \frac{d}{h}; \cos \frac{p_x d}{h} \sin p_y \frac{d}{h} \right\}. \]

For further calculations, we use the relaxation time approximation. Consider the Boltzmann kinetic equation in the approximation of a constant collision frequency:
\[
\frac{\partial f(p,t)}{\partial t} + E \frac{\partial f(p,t)}{\partial p} = - \nu \left( f(p,t) - f_0(p) \right),
\]

where \( f_0(p) \) - equilibrium distribution function; \( \nu \) - average frequency of collisions. The solution to equation (4) is a nonequilibrium distribution function. This solution can be found by the method of characteristics and has the form:

\[
f(p,t) = \nu \int_{-\infty}^{t} dt' \exp \left( -\nu(t-t') \right) f_0(p'(t';t)).
\]

Here \( p'(t';p,t) \) is the solution of the classical equation of motion \( dp'/dt = eE(t') \) with an initial condition \( t' = t, \ p' = p \) that has the following form:

\[
p'_x = p_x + eE_0 (t' - t) + \frac{eE_0}{\omega} \left( \sin \left( \omega t' \right) - \sin \left( \omega t \right) \right),
\]

\[
p'_y = p_y + \frac{eE_{20}}{\omega} \left( \sin \left( \omega t' + \varphi \right) - \sin \left( \omega t + \varphi \right) \right).
\]

The equilibrium distribution function \( f_0 = A \exp(-\varepsilon(p)/T) \) is normalized by the condition:

\[
A \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp \left\{ -\frac{\Delta}{T} \left[ 1 - \cos \left( \frac{p_x d}{h} \right) \cos \left( \frac{p_y d}{h} \right) \right] \right\} dp_x dp_y = n,
\]

where \( T \) is the temperature expressed in energy units, \( n \) is a surface concentration of charge carriers.

The final expression for normalization constant \( A \) is:

\[
A = \frac{\Delta}{2\pi \hbar^2} \left( \frac{\Delta}{2T} \right) \left( \frac{1}{2} \right) .
\]

Thus, the nonequilibrium function is represented as:

\[
f(p,t) = \frac{\nu}{\exp \left( \frac{\Delta}{T} \right)} \int_{-\infty}^{0} \exp \left( -\nu a \right) \exp \left( \frac{\Delta}{T} \cos \left( \frac{p_x d}{h} - eE_0 \frac{d}{\hbar \omega} \sin(t-a) \right) - \sin(t) \right) \cdot \cos \left( \frac{p_y d}{h} + \frac{eE_{20}}{\hbar \omega} \left( \sin(t-a + \varphi) - \sin(t + \varphi) \right) \right) da.
\]

where \( a = t - t' \).

By introducing a number of the denotation, presented below, and making the replacement, we get:
\[ f(p,t) = C \nu \int_{-\infty}^{0} d\epsilon \exp(-\nu \epsilon) \exp \left( \gamma \cos \left( q_{s} - \frac{eE_{d}}{\hbar \omega} + F_{1}g_{1} \right) \cos \left( q_{s} + F_{2}g_{2} \right) \right), \]  

(10)

where \( C = A / \exp(\gamma) = \hbar^{4} / d^{2} J_{0}^{2} (\gamma / 2) \), \( \gamma = \Delta / T \), \( q_{s,v} = p_{s,v} \, d / \hbar \), \( F_{1,2} = eE_{0,20} d / h \omega \), \( g_{1} = \sin(t-a) - \sin(t) \), \( g_{2} = \sin(t - a + \varphi) - \sin(t + \varphi) \), \( a \rightarrow \omega t' \), \( t \rightarrow \omega t \).

We substitute the expression for the nonequilibrium distribution function in the current density formula (3), where we go from summative to integration in the standard way. Taking into account the introduced notation, we obtain:

\[ j_{s} = j'_{y_{0}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp(-\nu \epsilon) \exp \left( \gamma \cos \left( q_{s} + F_{1}g_{1} + \frac{eE_{d}}{\hbar \omega} \right) \cos(q_{s} + F_{2}g_{2}) \right) \cdot \sin \left( \frac{p_{d}}{\hbar} \right) \cos \left( \frac{p_{d}}{\hbar} \right) dp_{d} dp_{d} da, \]

(11)

where \( j'_{y_{0}} = \frac{C \epsilon d \Delta \nu}{h} \).

Thus, taking the integrals entering into the expression and averaging over the period of the electromagnetic wave, we bring the expression to the following form:

\[ j_{s} = j_{0} \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} \sum_{l_{0}=0}^{k} \frac{1}{J_{k}(F_{1}) J_{l}(F_{2}) J_{m}(F_{3})} \exp(i\nu(l+m)) \delta(Q,0) \left[ \frac{(-1)^{k+l}-(-1)^{k+l}}{in+il+iF_{0}+v} + \frac{(-1)^{n+m}-(-1)^{n+m}}{in+il-iF_{0}+v} \right] \]

(12)

where \( j_{0} = C \epsilon \hbar \Delta \nu / 4d \), \( Q = i\nu(n+k+l+m) \), \( \nu \rightarrow \omega / \omega \), \( F_{0} = eE_{d} d / \hbar \omega \).

We plot the current density \( j_{s} \) versus the dimensionless constant field strength \( F_{0} \) for different values of the phase difference between the components of an elliptically polarized wave. For convenience, when constructing the graphs \( E_{1} \) and \( E_{2} \), we represent in the form \( E \sin(\alpha) \) and \( E \cos(\alpha) \), where \( E \) is some voltage. Correspondingly \( F_{1} \) and \( F_{2} \) are equal to \( F \sin(\alpha) \) and \( F \cos(\alpha) \). \( F \) taken equal to 0.1.

The graph shows that the dependence is essentially nonlinear and has an oscillating character. The maximum current density is observed at \( \alpha = \frac{\pi}{4} \).
Figure 2. Current density dependence in units of $j_y$ from the field $F_0$.

The value $F_0$ is the ratio of the energy $eE, d$, accumulated by the electron when it moves in a constant electric field at the superlattice period $d$, to the photon energy. The transverse current plots on figure 2 have features near integer values of the ratio $\frac{eE, d}{\hbar \omega} = \frac{\Omega_0}{\omega}$ (here the notation $eE, d = \Omega_0$ is introduced - the Stark frequency). Near these values of $E_0$, the current graph should show discontinuities of the second kind as the collision frequency $\nu$ tends to zero. Due to the nonadditivity of the energy spectrum of the graphene superlattice, electrons perform some complex motion under the action of applied electric fields; therefore, under conditions corresponding to the Stark resonance, the transverse current also acquires a non-monotonic character. Note that the current density in the transverse direction is approximately an order of magnitude lower than the current density in the direction of the axis of the superlattice.

3. Special cases analysis
We shall analyze some limiting cases for the expression of the density of the longitudinal current.

The case of weak fields. Since the expansion of the Bessel functions $J_n(F_2), J_0(F_2), J_1(F_1), J_m(F_1)$ begins with the terms of the order $F_1, F_1^m, F_2, F_2^k$, it suffices to leave only the terms with $n, k, l, m = -1, 0, 1$ in the sum. Considering cases that satisfy the condition, the expression for current density takes the following form:

$$j_y = j_0 \frac{F_0 F_1 (1 + 3\nu^2)}{\nu^2 (-i + \nu)^2 (i + \nu)^2}.$$

This expression is obtained taking into account the fact that the phase difference is 0. But phase dependent expression for the current in the limit of weak electric fields is...
The case of a high collision frequency ($\nu \gg 1$). In this case, the greatest contribution to the value of the current density will be given by elements with small quantities $n, k, l, m$. To show this, consider the following factor:

$$
\left( \frac{(-1)^{k+m} - (-1)^{k+l}}{in + il + iF_0 + \nu} + \frac{(-1)^{n+m} - (-1)^{n+l}}{in + il - iF_0 + \nu} \right).
$$

Due to the fact that the expression $(\nu)^{-1} << 1$, the terms $in + il - iF_0$ and $in + il + iF_0$ must be taken the smallest. It follows from the condition $n + k + l + m = 0$ that

$$
j_y = j_0 \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} \frac{1}{i} J_n(F_1)J_k(F_1)J_l(F_2)J_m(F_2) \left[ \frac{(-1)^{k+m} - (-1)^{k+l}}{in + il + iF_0 + \nu} + \frac{(-1)^{n+m} - (-1)^{n+l}}{in + il - iF_0 + \nu} \right]
$$

We obtain the expression for the density of the longitudinal current:

$$
j_y = j_0 \frac{16F_0(1 + 3\nu^2)J_n(F_1)J_o(F_1)J_l(F_2)J_m(F_2)}{\nu^2(-i + \nu^2)(i + \nu^2)}.
$$

Expanding in this expression the Bessel functions into series on the fields $F_1, F_2$ limiting by to the first non-vanishing approximation, we obtain the expression for the current density for the case of weak fields:

$$
j_y = j_0 \frac{F_0F_1F_2(l + 3\nu^2)}{\nu^2(-i + \nu^2)(i + \nu^2)}.
$$

### 4. Conclusion

Thus, using the relaxation time approximation, an analytical expression was obtained for the direct current density arising in a two-dimensional superlattice when two waves with mutually perpendicular planes of polarization and a constant field along one of the directions act on the sample, the ratio of the frequencies of the variable fields is 1. On The general analytical solution considers special cases of small amplitudes of the incident waves and high frequencies.

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