Radiation effect on free convection flow between oscillating parallel plates with mass diffusion

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Abstract

This study investigates radiation effect on unsteady oscillatory free convection between two parallel plates with mass diffusion. Appropriate non-dimensional variables are used to reduce the dimensional governing equations to dimensionless equation along with imposed initial and boundary conditions. The solutions for velocity, temperature, and concentration profiles are obtained by using Laplace transform method. To illustrate the behavior of the fluid flow, graphical results are presented with influence of Schmidt number, Prandtl number, radiation parameter, oscillating parameter, Grashof number, and mass Grashof number. The corresponding expressions for skin friction, Nusselt number, Sherwood number are also calculated. It is observed that increasing radiation parameter, Prandtl, and Schmidt number will increase the Nusselt number but the skin friction will be reduced.

Keywords: Free convection, radiation, oscillating, parallel plates, mass diffusion

INTRODUCTION

Convective flows in parallel plates have received much attention due to the wide applications involving natural convection heat transfer [1]. The flow between parallel plates has various application such as in petroleum industry, purification of crude oil, pumps accelerators, and power generators [2]. Many researchers have investigated problem regarding free convection flow between two parallel plates [3-6]. Narahari [7] investigated natural convection flow in vertical channel with ramped wall temperature at one boundary while Jha et al. [8] investigated transient free convective flow in vertical channel with constant temperature and constant heat flux.

Free convection in vertical channel has also been studied with different physical effects such as radiation, heat generation/absorption, chemical reaction, magnetohydrodynamics (MHD), and porous medium [9-12]. Narahari [13] have investigated transient free convection between vertical parallel plates with ramped wall temperature at one boundary with presence of thermal radiation while Jha et al. [14] investigated unsteady natural convection Couette flow of heat generating/absorbing fluid between vertical parallel plates filled with porous material. Rajput et al. [15] investigated transient free convection MHD flow between two long vertical parallel plates with constant temperature and variable mass diffusion. The research found that the velocity and skin friction of the fluid increase with increasing the value of time t but decrease with increasing the value of the Prandtl number Pr, Schmidt number Sc and magnetic parameter M. Then, Rajput et al. [16] extended his investigation to investigate combined effects of chemical reactions and heat generation/absorption on unsteady transient free convection MHD flow between long vertical parallel plates through a porous medium with constant temperature and mass diffusion.

Oscillatory flow has received some attention in researchers as it was found can higher the rates of heat transfer between parallel plates [17]. Raju et al. [18] investigated unsteady MHD free convection oscillatory Couette flow through a porous medium with periodic wall temperature while Das et al. [19] investigated an oscillatory MHD convective flow in a vertical channel filled with porous medium with hall and thermal radiation effects. While MHD oscillatory flow through a porous channel saturated with porous medium was investigated by Falade et al. [20]. Bunyono et al. [17] investigate unsteady oscillatory Coette flow between vertical plates with constant radiative heat flux. The research show that the oscillatory parameter ω played a greater role causing the fluid to oscillate inside the Coette channel.

Motivated by above investigations, the present analysis is to investigate oscillatory free convection flow between two parallel plates with mass diffusion. Radiation effects is also considered in this research. The problem will be solved using exact method which are Laplace Transform method.

METHODOLOGY

Let us consider an unsteady free convection flow between two parallel plates with the presence of radiation. Oscillating plate at y’ = 0 will also be considered in this study. The x’-axis is considered along one of the vertical plates and the y’-axis is taken normal to the plates.
Initially, the time \( t' \leq 0 \), the temperature of the fluid and the plates are same as \( T_h \) and the concentration of the fluid is \( C_h \). At \( t' > 0 \) the temperature of the plate and concentration of the fluid at \( y' = 0 \) are raised to \( T_w' \) and \( C_w' \), respectively, causing the flow of free convection currents. The physical configuration of the problem is presented in Figure 1 and the governing equations under the normal Boussinesq’s approximation are as follows:

\[
\begin{aligned}
&\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g \beta (T' - T_h) + g \beta' (C' - C_h) \\
&\rho \frac{\partial C'}{\partial t'} = k \frac{\partial^2 C'}{\partial y'^2} - \frac{\partial q}{\partial y'} \\
&\frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2}
\end{aligned}
\]  

(1)

The volumetric coefficient of thermal expansion, \( \nu \) is the frequency of velocity of the wall.

\[
\begin{aligned}
t &\leq 0 : u' = 0 & T' = T_h & C' = C_h \quad \text{for} \quad 0 \leq y' \leq h \\
t > 0 : u' = U_0 \cos \omega t \quad \text{and} \quad U_0 \sin \omega t \quad T' = T_h' & C' = C_w' \quad \text{at} \quad y' = 0 \\
u' = 0 & T' = T_h' & C' = C_h \quad \text{at} \quad y' = h
\end{aligned}
\]

where \( u' \) is velocity of the fluid, \( g \) is acceleration due to gravity, \( \nu \) is the volumetric coefficient of thermal expansion, \( \nu \) is the species concentration in the fluid while \( C_h' \) is the specific heat at constant pressure, \( k \) is the thermal conductivity of the fluid, \( q' \) is the radiative heat flux, \( D \) is mass diffusion coefficient, \( T_w' \) and \( C_w' \) is the temperature and concentration of the plate at \( y' = 0 \) and \( \omega \) is the frequency of velocity of the wall.

The radiation heat flux under Rosseland approximation is given by

\[
q_r = -\frac{4\sigma^* \nu T^4}{3k^* \nu}
\]

(5)

where \( \sigma^* \) and \( k^* \) are the Stefan-Boltzmann constant and the mean spectral absorption coefficient, respectively. It is supposed that the temperature difference within the flow are sufficiently small, then equation (5) can be linearized by expanding \( T^4 \) into Taylor series about \( T_h \), and neglecting higher order terms, we find that

\[
T_h \approx 4T_h^3 - 3T_h^4
\]

(6)

Introducing the following non-dimensional quantities:

\[
\begin{aligned}
&y = \frac{y'}{h} & t = \frac{t'}{h} & u = \frac{u'}{U_0} & T = \frac{T' - T_h'}{T_{w'} - T_h'} \\
&C = \frac{C_h'}{C_w' - C_h'} & \mu = \rho v
\end{aligned}
\]

(7)

Then, from equation (5)-(7), equations (1)-(3) and boundary conditions (4) becomes:

\[
\begin{aligned}
&\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} + Gr \theta + Gm C \\
&\frac{\partial \theta}{\partial t} = \left( \frac{3R + 4\gamma}{3R Pr} \right) \frac{\partial^2 \theta}{\partial y^2}
\end{aligned}
\]

(8)

(9)

SOLUTIONS OF THE PROBLEM

The governing equations (8)-(10) with boundary conditions (11) were solved in the exact form by the Laplace transform technique and their solutions in the transform \((y', q)\) plane are given by

\[
\begin{aligned}
\bar{u}_c &= \sum_{n=0}^{\infty} \left( \frac{a_2 \nu^2 e^{-a_\gamma q} - a_2 \nu^2 e^{-b_\gamma q}}{\nu^2} \right) + \frac{1}{1} e^{-a_\gamma q} - \frac{1}{1} e^{-a_\gamma q} \\
&= \frac{1}{1} e^{-a_\gamma q} + \frac{1}{1} e^{-a_\gamma q} - \frac{1}{1} e^{-a_\gamma q}
\end{aligned}
\]

(10)

where \( Gr \) is the thermal Grashof number, \( Gm \) is the mass Grashof number, \( Pr \) is the Prandtl number, \( Sc \) the Schmidt number, \( R \) is the radiation parameter, and \( \omega \) is the oscillating parameter.

\[
\begin{aligned}
\bar{u}_s &= \sum_{n=0}^{\infty} \left( \frac{a_2 \nu^2 e^{-a_\gamma q} - a_2 \nu^2 e^{-b_\gamma q}}{\nu^2} \right) + \frac{1}{1} e^{-a_\gamma q} - \frac{1}{1} e^{-a_\gamma q} \\
&= \frac{1}{1} e^{-a_\gamma q} + \frac{1}{1} e^{-a_\gamma q} - \frac{1}{1} e^{-a_\gamma q}
\end{aligned}
\]

(11)

where the subscripts \( c \) and \( s \) in above equation refer to cosine and sine oscillations of the plate and

\[
\begin{aligned}
a &= 2n + y & b &= 2n + 2 - y & \bar{A} &= \frac{3R Pr}{3R + 4} \\
a_1 &= \frac{Gr}{\bar{A} - 1} & a_2 &= \frac{Gm}{Sc - 1} & a_3 &= a_1 + a_2
\end{aligned}
\]

Laplace inversion of above equation are as follows:
\[
\begin{align*}
\tau &= -\frac{\partial u}{\partial y} \bigg|_{y=0} \\
N\!u &= -\frac{1}{\theta(0,y)} \frac{\partial T}{\partial y} \bigg|_{y=0} \\
S\!h &= \frac{C}{\partial y} \bigg|_{y=0}
\end{align*}
\]

\section*{LIMITING CASES}

In order to underline the theoretical value of the general solutions for velocity, as well to gain physical insight of the flow regime, we consider some special cases whose technical relevance is well-known in the literature.

1. Solutions for stokes first problem

By taking \( \omega = 0 \), which corresponds to impulsive motion of the plate, then the velocity yield,

\[
\begin{align*}
u_c &= \sum_{n=0}^{\infty} \left( a_3 \left( \frac{a^2}{2} + t \right) \text{erfc} \left( \frac{a}{2\sqrt{x}} \right) - a \frac{e^{-a^2}}{a} - \left( b^2 + \right. \right) \\
t \text{erfc} \left( \frac{a}{2\sqrt{x}} \right) + b \left( \frac{e^{-b^2/a^2}}{b} \right) + \left( \frac{1}{2} \right) \left( \text{erfc} \left( \frac{a}{2\sqrt{x}} \right) + \sqrt{\text{erfc} \left( \frac{a}{2\sqrt{x}} \right)} \right) + e^{-a^2} \left( \frac{a}{2\sqrt{x}} + \right. \right. \right. \\
&\left. e^{-b^2/a^2} \text{erfc} \left( \frac{b}{2\sqrt{x}} + \sqrt{\text{erfc} \left( \frac{b}{2\sqrt{x}} \right)} \right) - \frac{1}{2} \left( \frac{1}{\text{erfc} \left( \frac{a}{2\sqrt{x}} \right) + \sqrt{\text{erfc} \left( \frac{a}{2\sqrt{x}} \right)} \right) + e^{-b^2/a^2} \left( \text{erfc} \left( \frac{b}{2\sqrt{x}} + \sqrt{\text{erfc} \left( \frac{b}{2\sqrt{x}} \right)} \right) - \frac{1}{2} \right) \left( \frac{1}{\text{erfc} \left( \frac{b}{2\sqrt{x}} \right) + \sqrt{\text{erfc} \left( \frac{b}{2\sqrt{x}} \right)} \right)
\end{align*}
\]

Respectively,

\[
\begin{align*}
u_s &= \sum_{n=0}^{\infty} \left( a_3 \left( \frac{a^2}{2} + t \right) \text{erfc} \left( \frac{a}{2\sqrt{x}} \right) - a \frac{e^{-a^2}}{a} - \left( b^2 + \right. \right) \\
t \text{erfc} \left( \frac{b}{2\sqrt{x}} \right) + b \left( \frac{e^{-b^2/a^2}}{b} \right) + \left( \frac{1}{2} \right) \left( \text{erfc} \left( \frac{a}{2\sqrt{x}} \right) + \sqrt{\text{erfc} \left( \frac{a}{2\sqrt{x}} \right)} \right) + e^{-a^2} \left( \frac{a}{2\sqrt{x}} + \right. \right. \right. \\
&\left. e^{-b^2/a^2} \text{erfc} \left( \frac{b}{2\sqrt{x}} + \sqrt{\text{erfc} \left( \frac{b}{2\sqrt{x}} \right)} \right) - \frac{1}{2} \left( \frac{1}{\text{erfc} \left( \frac{a}{2\sqrt{x}} \right) + \sqrt{\text{erfc} \left( \frac{a}{2\sqrt{x}} \right)} \right) + e^{-b^2/a^2} \left( \text{erfc} \left( \frac{b}{2\sqrt{x}} + \sqrt{\text{erfc} \left( \frac{b}{2\sqrt{x}} \right)} \right) - \frac{1}{2} \right) \left( \frac{1}{\text{erfc} \left( \frac{b}{2\sqrt{x}} \right) + \sqrt{\text{erfc} \left( \frac{b}{2\sqrt{x}} \right)} \right)
\end{align*}
\]

2. Solution in the absence of thermal radiation (\( R \to 0 \))

In the absence of thermal radiation, the corresponding solutions are directly obtained from the general solutions (8)-(11) by taking \( R \to 0 \).

\[
\begin{align*}
u_c &= \sum_{n=0}^{\infty} \left( a_3 \left( \frac{a^2}{2} + t \right) \text{erfc} \left( \frac{a}{2\sqrt{x}} \right) - a \frac{e^{-a^2}}{a} - \left( b^2 + \right. \right) \\
t \text{erfc} \left( \frac{b}{2\sqrt{x}} \right) + b \left( \frac{e^{-b^2/a^2}}{b} \right) + \left( \frac{1}{2} \right) \left( \text{erfc} \left( \frac{a}{2\sqrt{x}} \right) + \sqrt{\text{erfc} \left( \frac{a}{2\sqrt{x}} \right)} \right) + e^{-a^2} \left( \frac{a}{2\sqrt{x}} + \right. \right. \right. \\
&\left. e^{-b^2/a^2} \text{erfc} \left( \frac{b}{2\sqrt{x}} + \sqrt{\text{erfc} \left( \frac{b}{2\sqrt{x}} \right)} \right) - \frac{1}{2} \left( \frac{1}{\text{erfc} \left( \frac{a}{2\sqrt{x}} \right) + \sqrt{\text{erfc} \left( \frac{a}{2\sqrt{x}} \right)} \right) + e^{-b^2/a^2} \left( \text{erfc} \left( \frac{b}{2\sqrt{x}} + \sqrt{\text{erfc} \left( \frac{b}{2\sqrt{x}} \right)} \right) - \frac{1}{2} \right) \left( \frac{1}{\text{erfc} \left( \frac{b}{2\sqrt{x}} \right) + \sqrt{\text{erfc} \left( \frac{b}{2\sqrt{x}} \right)} \right)
\end{align*}
\]
oscillating parameter $G_m$, Schmidt number $Sc$, radiation parameter $R$

numerically in order to determine the effects of several involved

RESULTS AND DISCUSSION

In this section, the obtained exact solutions are studied
numerically in order to determine the effects of several involved
parameter such as Prandtl number $Pr$, Grashof number $Gr$, Mass
Grashof number $Gm$, Schmidt number $Sc$, radiation parameter $R$, and
oscillating parameter $\omega$. Skin friction, Nusselt number, and Sherwood
number are also plotted with different parameters.

Fig. 2 Concentration profile with different $Sc$ number with $t = 0.1$.

Fig. 3 Temperature profile with different $Pr$ number with $t = 0.1$.

Fig. 4 Temperature profile with different $R$ number with $t = 0.1$ and $Pr = 0.71$.

Fig. 5 Velocity profile with different $Sc$ number with $t = 0.1$, $Pr = 2$, $Gm = 2$, $\omega = \frac{\pi}{2}$, $R = 0.2$.

Fig. 6 Velocity profile with different $Pr$ number with $t = 0.1$, $Sc = 0.3$, $Gr = 2$, $Gm = 2$, $\omega = \frac{\pi}{2}$, $R = 0.2$.

Fig. 7 Velocity profile with different $R$ number with $t = 0.1$, $Pr = 0.71$, $Gr = 5$, $Gm = 5$, $\omega = \frac{\pi}{2}$, $Sc = 0.5$.
**Table 1** Skin friction with different parameter.

| t  | Pr   | Sc | Gr | Gm | $\pi$ | R | $\tau$ |
|----|------|----|----|----|------|---|--------|
| 0.1| 0.71 | 0.3| 2  | 2  | $\pi$ | 2 | -0.663458 |
| 0.1| 0.71 | 0.3| 2  | 2  | $2\pi$| 2 | 0.00182304 |
| 0.1| 0.71 | 0.3| 2  | 2  | $3\pi$| 2 | 1.01048   |
| 0.1| 3    | 0.3| 2  | 2  | $2\pi$| 2 | -0.124196 |
| 0.1| 5    | 0.3| 2  | 2  | $2\pi$| 2 | -0.167746 |
| 0.1| 3    | 0.3| 2  | 2  | $2\pi$| 1 | -0.0945283 |
| 0.1| 3    | 0.3| 2  | 2  | $2\pi$| 3 | -0.136619 |

**Table 2** Nusselt number with different parameter.

| t  | Pr   | R  | Nu  |
|----|------|----|-----|
| 0.1| 0.71 | 2  | 1.18092 |
| 0.1| 3    | 2  | 2.39365 |
| 0.1| 5    | 2  | 3.0919 |
| 0.1| 0.71 | 1  | 1.0311 |
| 0.1| 0.71 | 3  | 1.26002 |

Figure 2 shows concentration profiles graph with different Schmidt ($Sc$) number. From the graph, increasing the $Sc$ number will decrease the velocity. It is found that $Sc$ number relates to the relative thickness of the mass transfer boundary layer, if $Sc$ number is increasing, the thickness of boundary layer would increase and will cause the velocity decreases. While Figure 6 and 7 show the velocity profile for different $Pr$ and radiation parameter. Increasing both $Pr$ and radiation parameter will cause the velocity to decreases. Figure 8 show velocity profile for different oscillation parameter. The graph shows that increasing the oscillation parameter will cause the velocity to decreases. While Figure 9 and 10 show velocity field with different Grashof number ($Gr$) and mass Grashof number ($Gm$). It is observed that velocity increasing with increasing the value of $Gr$. Physically this is possible because as $Gr$ number increases, the contribution from the buoyancy force near the plate become significant and hence a rise in velocity is observed. Similar to mass Grashof number, when $Gm$ is increased, the velocity was found increased.

Figure 5-10 show the effects of different parameters on the velocity profile. From Figure 5, it is observed that increasing $Sc$ number will decrease the velocity. It is found that $Sc$ number relates to the relative thickness of the mass transfer boundary layer, if $Sc$ number is increasing, the thickness of boundary layer would increase and will cause the velocity decreases. While Figure 6 and 7 show the velocity profile for different $Pr$ and radiation parameter. Increasing both $Pr$ and radiation parameter will cause the velocity to decreases. While Figure 9 and 10 show velocity field with different Grashof number ($Gr$) and mass Grashof number ($Gm$). It is observed that velocity increasing with increasing the value of $Gr$. Physically this is possible because as $Gr$ number increases, the contribution from the buoyancy force near the plate become significant and hence a rise in velocity is observed. Similar to mass Grashof number, when $Gm$ is increased, the velocity was found increased.

Table 1 shows the skin friction for different $Pr$ and radiation parameter. It is depicted from the table that increasing $Pr$ and Radiation parameter causes the skin friction to decrease. Nusselt number with different $Pr$ and radiation parameter are presented in Table 2. It shows that increasing the $Pr$ and radiation will increase the Nusselt number. While Table 3 shows solution of Sherwood number against time with different $Sc$ number. The table shows that increasing the $Sc$ number will increase the Sherwood number.

**CONCLUSION**

An exact solution to the problem of oscillating free convection flow between two parallel plates with mass diffusion and radiation effects is investigated. Solutions of the problem are obtained by using Laplace transform technique. The effects of various parameters on velocity, temperature and concentration profiles as well as skin friction, Nusselt and Sherwood number are numerically studied. The following conclusions can be summarized from this study:

- Increasing Schmidt number will reduce the concentration of the fluid but increase the Sherwood number.
- Increasing Prandtl number will reduce the temperature, velocity and skin friction but will increase the Nusselt number.
- Increasing the oscillation parameter will lower the velocity.
- Increasing radiation parameter will reduce the temperature, velocity, and skin friction but will increase the Nusselt number.
- Increasing Grashof and mass Grashof number will increase the velocity of the fluid flow.
- The solutions satisfy the initial and boundary conditions.

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