Rank-Enhanced Low-Dimensional Convolution Set for Hyperspectral Image Denoising

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Abstract—This paper tackles the challenging problem of hyperspectral (HS) image denoising. Unlike existing deep learning-based methods usually adopting complicated network architectures or empirically stacking off-the-shelf modules to pursue performance improvement, we focus on the efficient and effective feature extraction manner for capturing the high-dimensional characteristics of HS images. To be specific, based on the theoretical analysis that increasing the rank of the matrix formed by the unfolded convolutional kernels can promote feature diversity, we propose rank-enhanced low-dimensional convolution set (Re-ConvSet), which separately performs 1-D convolution along the three dimensions of an HS image side-by-side, and then aggregates the resulting spatial-spectral embeddings via a learnable compression layer. Re-ConvSet not only learns the diverse spatial-spectral features of HS images, but also reduces the parameters and complexity of the network. We then incorporate Re-ConvSet into the widely-used U-Net architecture to construct an HS image denoising method. Surprisingly, we observe such a concise framework outperforms the most recent method to a large extent in terms of quantitative metrics, visual results, and efficiency. We believe our work may shed light on deep learning-based HS image processing and analysis.

Index Terms—Hyperspectral imagery, denoising, deep learning, feature diversity, feature extraction

I. INTRODUCTION

Owing to the rich spectral and spatial information towards real-world scenes/objects, hyperspectral (HS) images have been adopted in numerous fields, such as military [1], [2], agriculture [3], [4], and marine monitoring [5], [6]. Unfortunately, the captured HS images are inevitably corrupted by various noises due to the limitations of hardware and environment effects (e.g., water absorption and terrible atmosphere), which may severely degrade downstream applications. Thus, there is a gradually growing pursuit for HS image denoising algorithms.

Over the past decades, many HS image denoising methods have been proposed [7]–[16]. The early works explicitly formulate HS image denoising as constrained optimization problems by employing prior knowledge, such as sparsity [8], non-local similarity [7], [17], total variation [18], [19], and low-rank tensors [9]–[11], [20]. However, due to the limited representation ability, these methods are insufficient to model such an ill-posed inverse problem, making the quality of recovered HS images still unsatisfied. Recently, owing to the powerful representational ability, deep learning-based HS image denoising methods have significantly improved the restoration quality [12]–[16], [21]–[23]. Particularly, most of existing deep learning-based methods extract spatial-spectral features of HS images by by applying 2-D conventional filters with multiple channels on each cube slice separately [12], 3-D conventional filters to simultaneously convolve in spatial and spectral domains [23], or a 2-D and 3-D conventional filters combined network [21]. However, these empirical designed feature extraction modules summarized in Fig. 1 are hard to provide quantitative instructions for network designing and may be not optimal, thus limiting performance. Besides, most of them tend to stack off-the-shelf modules to build complicated and large-capacity networks for pursuing high performance, such as multi-scale strategy [21], [23], [24], attention mechanism [24], and atrous convolution [22], [24].

In contrast to existing works, we aim to embed the high-dimensional spatial-spectral information of HS images both efficiently and effectively via theoretical analysis. Specifically, we first figure out the bottleneck that limits feature diversity by means of matrix rank analysis. Under the guidance of the theoretical analysis, motivating us to propose Re-ConvSet, which separately performs 1-D convolution along the three dimensions of an HS image side-by-side, and then aggregates the resulting spatial-spectral embeddings via a compression layer. Such a manner improves the upper bound of the rank of the matrix formed by the convolution kernels involved in Re-ConvSet to promote diverse features which are expected to be beneficial to image reconstruction. Meanwhile, Re-ConvSet reduces the number of network parameters. Finally, we realize an efficient, concise, and compact denoising method by incorporating Re-ConvSet into the widely-used U-Net architecture. Extensive experiments on both synthetic and real noisy HS images demonstrate the significant superiority of the proposed denoising method over state-of-the-art ones.

In summary, the main contributions of this paper are three-fold:

• an efficient and effective spatial-spectral feature extraction module based on theoretical analysis;
• comprehensive and quantitative investigations on various combinations of low-dimensional convolution that may shed light on deep learning-based HS image processing;
• a concise and compact HS image denoising method that reveals the importance of feature extraction and achieves

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the current state-of-the-art.

The remainder of the paper is organized as follows. Section II reviews the existing related works. Section III describes our proposed ReConv-Set in detail, followed by extensive experiments and analysis in Section IV. Finally, Section V concludes this paper.

II. RELATED WORK

A. Optimization-based Methods

This kind of methods generally formulates the HS image denoising as complex optimization problems, relying on the well-designed handcrafted priors, such as non-local similarity [7], [17], [25], total variation [18], [19] and low-rank priors [9]–[11], [20], [26]–[28]. To be specific, Qian et al. [7] proposed a sparse representation-based method by introducing the nonlocal similarity and spectral-spatial structure of HS imagery. Peng et al. [17] presented a novel tensor dictionary learning (TDL) model by taking the non-local similarity in space and global correlation in spectrum into account. He et al. [18] proposed a total variation regularized low-rank matrix factorization (LRTV) method, in which the nuclear norm, TV regularization, and \(L_1\) norm were integrated. Owing to the consideration of modeling the intrinsic property of HS images, several nonlocal low-rank tensor-based models achieved remarkable denoising performance. Xie et al. [9] proposed an intrinsic tensor sparsity regularization (ITSReg) model, in which the nuclear norm, TV regularization, and \(L_1\) norm were integrated. Chaud et al. [20] designed a hyper-Laplacian regularized unidirectional low-rank tensor recovery (LLRT) method, in which the nuclear norm, TV regularization, and \(L_1\) norm were integrated. On the other hand, he et al. [11] provided a unified paradigm to fuse spatial non-local similarity and global spectral low-rank properties by utilizing the low-dimensional orthogonal basis.

B. Deep Learning-based Methods

A considerable number of deep learning-based methods for HS image denoising have been presented, which improve the restoration quality of traditional optimization-based methods dramatically. For example, Chang et al. [12] first tackled HS image denoising by a deep neural network, in which the learned 2-D filters with multiple channels were utilized. Yuan et al. [21] designed a novel spatial-spectral network with both 2D and 3D convolutional kernels to fully exploit spatial and spectral features. Liu et al. [22] presented a 3-D atrous denoising convolution neural network (3DADCNN), in which 3-D kernels are integrated with the atrous convolution for enlarging the receptive fields in both the spatial and spectral dimensions simultaneously. Inspired by the separable 3-D spatial-temporal convolution [29], Dong et al. [13] proposed a separable 3-D convolution network to explore spatial-spectral correlations by decomposing 3-D convolution into the concatenation of 2-D spatial convolution and 1-D spectral convolution. Zhang et al. [23] designed a multi-scale spatial-spectral convolutional network, in which the spatial gradient and spectral gradient are jointly incorporated. Maffei et al. [30] proposed a single network, taking a full HS image cube as input, to explore the spatial-spectral correlation. Wei et al. [14] proposed a 3-D quasi-recurrent neural network (QRNN3D) to simultaneously explore the structural spatiotemporal correlation and global correlation along spectral. Besides, an alternating directional structure was integrated to alleviate the spatiotemporal dependence modeling. Shi et al. [24] designed a 3-D attention denoising network (3-D-ADNet) with two parallel branches, including spatial branch with the position attention module and spectral branch with the channel attention module. Cao et al. [15] designed a deep spatial-spectral global reasoning network (GRN) to explore the contextual information by combining the local and global spatial-spectral information of HS images. Rui et al. [31] presented a data-driven method to capture the general weighting principle of HS image denoising model. Bodrito et al. [16] proposed a trainable spectral-spatial sparse coding (T3SC) model by employing sparse coding and deep
Fig. 2. Illustration of our HS image denoising framework, which is constructed by incorporating the proposed Re-ConvSet into a residual U-Net architecture.

### III. PROPOSED METHOD

#### A. Problem Statement

Let \( X \in \mathbb{R}^{B \times H \times W} \) be a noisy HS image, and \( Y \in \mathbb{R}^{B \times H \times W} \) the corresponding noise-free one, where \( H \) and \( W \) are the spatial dimensions, and \( B \) is the number of spectral bands. The degradation process of \( Y \) to \( X \) could be generally formulated as

\[
X = Y + N_z, \tag{1}
\]

where \( N_z \in \mathbb{R}^{B \times H \times W} \) denotes the additive noise. Recovering \( Y \) from \( X \) is an ill-posed inverse problem in high-dimensional space, making it very challenging. Owing to the powerful representation ability and large capacity, recent deep CNN techniques have shown the great potential in addressing this problem [13]–[15], [21], [23]. Particularly, designing an efficient and effective learning backbone, i.e., the feature extraction layers, is one of the most critical issues.

Intuitively, 3-D convolution is the most direct choice for capturing the high-dimensional spatial-spectral information of HS images. However, it leads to a significant increase in the parameter size, which may potentially cause over-fitting and consume huge computing resources. Actually, the increase in the number of parameters does not bring about obvious performance improvement (see the results in Section IV-D). Although some low-dimensional convolution-based feature extraction manners have been proposed, as reviewed in Section II, they were empirically designed on the basis of the HS data structure, making it hard to provide quantitative instructions. Besides, they may not be optimal, thus limiting performance. Different from existing manner, we propose an efficient and effective feature extraction manner named Re-ConvSet from the theoretical perspective of promoting feature diversity.

We then incorporate the proposed Re-ConvSet into the widely-used U-Net [32] architecture for constructing an efficient and compact HS image denoising method, as illustrated in Fig. 2. We train the network by minimizing the \( \ell_1 \) distance between the recovered HS image \( \hat{Y} \) and the corresponding noise-free HS image \( Y \):

\[
\mathcal{L}_1(\hat{Y}, Y) = \frac{1}{B \times H \times W} \| \hat{Y} - Y \|_1. \tag{2}
\]

#### B. Feature Diversity Analysis

Feature diversity can effectively reflect the information richness of feature maps that is positively correlated with network performance [33]. Besides, some recent studies [33]–[36] indicate that learning diverse feature maps in the context of deep CNNs show great capability in reducing overfitting and improving the generalization ability of networks. Here, taking the 3-D convolutional layer as an example, we theoretically analyze the learned feature map with the rank of matrices and figure out the bottleneck limiting feature diversity, which further motivates us to design both efficient and effective feature representation module.

Let \( A \in \mathbb{R}^{M \times C \times k \times k \times k} \) denote a typical 3-D convolutional layer equipped with \( M \) 3-D kernels of size \( k \times k \times k \), where \( C \) is the number of channels. When feeding an HS feature map \( I \in \mathbb{R}^{C \times B \times H \times W} \) into \( A \), we can obtain an output feature volume \( F \in \mathbb{R}^{M \times B' \times H' \times W'} \), whose \( m \)-th feature map \( F_m \in \mathbb{R}^{B' \times H' \times W'} \) is obtained as

\[
F_m = \sum_{c=1}^{C} A'_m * I^c, \quad m = 1, 2, 3, \ldots, M, \tag{3}
\]

where \(*\) is the convolution operator, and \( A_m \) stands for the \( m \)-th convolutional kernel consisting of a stack of 3-D kernels \( A'_{m} \in \mathbb{R}^{k \times k \times k} \) \( (1 \leq c \leq C) \). By unfolding the high-dimensional tensors, we can equivalently re-write Eq. (3) in...
the form of 2-D matrix multiplication, i.e.,
\[ F = A \cdot I, \]  
where \( F \in \mathbb{R}^{M \times B' H' W'} \) is the matrix form of \( F \), \( A \in \mathbb{R}^{M \times k^3 C} \) is the kernel matrix generated by flattening each of the 3-D kernels as a row vector and then stacking all vertically, and \( I \in \mathbb{R}^{k^3 C \times B' H' W'} \) denotes the matrix form of \( I \) generated by sliding the kernel across the features along the horizontal, vertical-spatial, and spectral directions.

To quantitatively estimate the feature diversity, we analyze the rank of the feature matrix \( F \), which indicates the independence/freedom of elements of \( F \), and has demonstrated its effectiveness [37]. Specifically, based on the property of matrix multiplication, from Eq. (4) we have
\[ \text{Rank}(F) \leq \min\{\text{Rank}(A), \text{Rank}(I)\}, \]  
where \( \text{Rank}(\cdot) \) returns the rank of a matrix. Moreover, as the values of \( M \) and \( C \) are usually comparable, we have \( M \ll k^3 C \ll B' H' W' \), resulting in \( \text{Rank}(F) \leq M \). In other words, the feature diversity is severely limited by the massive gap between the two dimensions of matrix \( A \). Therefore, we can relive such an unbalanced matrix form of \( A \) to boost its rank upper bound, and likewise \( \text{Rank}(F) \), i.e., more elements of \( F \) are independent.

An intuitive way to boost the upper bound of \( \text{Rank}(A) \) is to increase \( M \). However, it can be seen from Eq. (3) that the computational cost for a 3-D convolutional layer is \( O(CMk^3 B' H' W') \), indicating that increasing \( M \) directly will further lead to a significant increase of the parameter size and computational complexity. Alternatively, we aim to improve the upper limit of \( \text{Rank}(A) \) without introducing extra computational burden to the network.

### C. Rank-Enhanced Low-Dimensional Convolution Set

Based on the above analysis, the problem boils down to how to utilize the same or fewer elements shown in Fig. 3 (a) to form a kernel matrix with a higher rank upper bound. To realize the goal, we can make the row-column sparse via filling these elements into different rows and columns of a larger zero matrix. Moreover, considering that the principal components of the 3-D kernel play a crucial role during the feature embedding process, we only employ the elements of \( A \) located at the principal components to fill the larger zero matrix, forming a new kernel matrix \( A_{rc} \in \mathbb{R}^{3M \times k^3 C} \). Formally, we can write the matrix form of Re-ConvSet as
\[ F_{rc} = A_{rc} \cdot I \quad \text{with} \quad A_{rc} = [A_{rc1}; A_{rc2}; A_{rc3}], \]  
where \( F_{rc} \in \mathbb{R}^{3M \times B' H' W'} \) is the matrix form of the output feature volume extracted by our Re-ConvSet, and \( A_{rc1}, A_{rc2}, \) and \( A_{rc3} \in \mathbb{R}^{M \times k^3 C} \) are the augmented matrix representations of low-dimensional kernels with zeros in various spatial-spectral domains, corresponding to the kernels of size \( 1 \times 1 \times k \), \( 1 \times k \times 1 \), and \( k \times 1 \times 1 \), respectively.

As illustrated in Fig. 3 (c), \( A_{rc} \) is filled with many elements equal to zero, resulting in \( 7C \) valid columns. Due to \( 3M < 7C \) in practical implementations, we generally have \( \text{Rank}(A_{rc}) \leq 3M \), and thus \( \text{Rank}(F_{rc}) \leq 3M \). Compared with the original kernel matrix \( A \), our Re-ConvSet boosts the rank upper bound from \( M \) to \( 3M \), thus potentially promoting more diverse features. Besides, the zero elements in \( A_{rc} \) do not contribute to the network parameters. Therefore, our Re-ConvSet not only learns the diverse spatial-spectral features of HS image, but also reduces the parameter size of the network.

### D. More Analysis

We also analyze the existing low-dimensional convolution-based feature extraction manners by using our formulation from the perspective of feature diversity to deeply understand them. Particularly, we take "Sequential 1-D and 2-D convolution" and "1-D + 2-D convolution", respectively; \( A_{c1} \in \mathbb{R}^{M \times k^3 C} \), \( A_{c2} \in \mathbb{R}^{M \times k^3 M} \), \( A_{c3} \in \mathbb{R}^{M \times k^3 C} \) and \( A_{c4} \in \mathbb{R}^{M \times k^3 C} \) are the corresponding matrices of 2-D and 1-D kernels, respectively; \( I_{1D} \in \mathbb{R}^{M \times B' H' W'} \) is the matrix form of intermediate feature maps; \( I_{2D} \in \mathbb{R}^{M \times B' H' W'} \) is generated by sliding 1-D kernels across the intermediate features; \( A_{c} \in \mathbb{R}^{2M \times k^3 C} \) denotes the matrix of combined 1-D and 2-D kernels, which is illustrated in Fig. 3 (b).

According to Eqs. (7) and (8), we have \( \text{Rank}(F_{c1}) \leq M \) and \( \text{Rank}(F_{c2}) \leq 2M \). That is, such feature extraction manners expand the rank upper bound from \( M \) to \( 2M \) at most, which is still limited.

**Remarks.** For the feature extraction manner involving parallel branches, e.g., Figs. 1 (d) and (e), we utilize a \( 1 \times 1 \times 1 \)
TABLE I

| σ  | Metrics | Methods |
|----|---------|---------|
|    | Noisy   | BM4D    | TDL   | ITSReg | LLRT   | GRN    | QRNN3D | T3SC   | Ours   |
| 30 | MPSNR↑  | 18.59   | 38.29 | 40.87  | 41.53  | 41.83  | 41.52  | 42.28  | 43.19  | 43.68  |
|    | MSSIM↑  | 0.1034  | 0.9342| 0.9557 | 0.9571 | 0.9653 | 0.9698 | 0.9701 | 0.9718 | 0.9747 |
|    | SAM↓    | 0.7269  | 0.1177| 0.0634 | 0.0929 | 0.0541 | 0.0690 | 0.0617 | 0.0616 | 0.0448 |
| 50 | MPSNR↑  | 14.15   | 35.54 | 38.50  | 39.19  | 38.84  | 39.86  | 40.22  | 40.81  | 41.39  |
|    | MSSIM↑  | 0.0429  | 0.8929| 0.9323 | 0.9350 | 0.9422 | 0.9535 | 0.9544 | 0.9567 | 0.9607 |
|    | SAM↓    | 0.9096  | 0.1535| 0.0841 | 0.1010 | 0.0734 | 0.0810 | 0.0733 | 0.0720 | 0.0534 |
| 70 | MPSNR↑  | 11.23   | 33.71 | 36.91  | 37.48  | 37.22  | 38.27  | 38.29  | 39.27  | 39.85  |
|    | MSSIM↑  | 0.0228  | 0.8545| 0.9104 | 0.9192 | 0.9264 | 0.9333 | 0.9326 | 0.9431 | 0.9478 |
|    | SAM↓    | 1.0273  | 0.1815| 0.1002 | 0.1144 | 0.0853 | 0.0933 | 0.0943 | 0.0810 | 0.0606 |
|    | Noisy   | 14.83   | 35.94 | 38.86  | 39.52  | 39.21  | 40.06  | 40.48  | 40.98  | 41.72  |
|    | MSSIM↑  | 0.0534  | 0.8979| 0.9353 | 0.9389 | 0.9452 | 0.9551 | 0.9559 | 0.9579 | 0.9626 |
|    | SAM↓    | 0.8800  | 0.1484| 0.0828 | 0.1037 | 0.0712 | 0.0801 | 0.0737 | 0.0723 | 0.0527 |

A convolutional layer to compress the multiple output feature volumes before feeding it into the subsequent layer in order to avoid channel explosion. Thus, the feature volumes extracted by different convolutional manners are finally with the equal size, i.e., the rank upper bounds of finally-output feature matrices are equal. However, our Re-ConvSet can boost the rank upper bound of feature matrix \( F \) from \( M \) to \( 3M \) during the feature extraction process, thus potentially promoting more diverse features. Here, we compared the singular value distributions of the feature maps extracted by various convolution manners in Fig. 4, where it can be clearly seen that the singular values of the feature matrix by our Re-ConvSet decrease more slowly than those of other schemes, indicating that our Re-ConvSet can balance the singular values to avoid only a few larges ones dominating the feature space (i.e., the degree of freedom of entries of the feature matrix that is approximately low-rank is limited), thus promoting feature diversity.

IV. EXPERIMENTS

A. EXPERIMENT SETTINGS

1) Datasets: We employed four commonly-used HS image benchmark datasets for evaluation, including two natural HS image datasets, i.e., ICVL\(^1\) [38] and CAVE\(^2\) [39], and two remote sensing HS images, i.e., Pavia University\(^3\) and Urban\(^4\), whose details are listed as follows:

- The ICVL dataset consists of 201 HS images of spatial dimensions \( 1392 \times 1300 \) and spectral dimension \( 31 \) covering the wavelength in the range of 400 to 700 nm, acquired by a Specim PS Kappa DX4 HS camera. We utilized 100 HS images as the training set, and the rest as the testing set.
- The CAVE dataset contains 32 HS images of spatial dimensions \( 512 \times 512 \) and spectral dimension \( 31 \) covering the wavelength in the range of 400 to 700 nm, collected by a generalized assorted pixel camera. Note that we randomly selected 10 HS images from this dataset only for testing.
- Pavia University contains \( 610 \times 610 \) pixels and 103 spectral bands gathered by the ROSIS sensor. This image is only used for testing.
- Urban contains \( 307 \times 307 \) pixels and 210 spectral bands collected by the HYDICE hyperspectral system. This image is corrupted by real unknown noise and widely used for real HS image denoising testing.

Following previous works [14], [15], we considered two kinds of noise settings, i.e., the Gaussian noise and the complex noise, which were applied to ICVL and CAVE datasets and Pavia University to simulate noisy HS images. Specifically, for the Gaussian noise, we set various noise levels, i.e., \( \sigma = 30, 50, 70 \), and “Blind (the value of \( \sigma \) is in the range of 30 to 70 but unknown)”. We generated five types of complex noises to imitate the real-world noise cases, including Non-i.i.d. Gaussian Noise, Gaussian and Stripe Noise, Gaussian and Deadline Noise, Gaussian and Impulse Noise, and Mixture Noise, referred as “Case 1” to “Case 5”. We refer the readers to [14], [15] for more details about the noise settings.

2) IMPLEMENTATION DETAILS: We implemented all the experiments with PyTorch on a machine with NVIDIA GeForce RTX 3080 GPU, Intel(R) Core(TM) i7-10700 CPU of 2.90GHz and 64-GB RAM. We employed the ADAM optimizer [40] with the exponential decay rates \( \beta_1 = 0.9 \) and \( \beta_2 = 0.999 \). The total training process was 25 epochs for both two kinds of noise experiments. We initialized the learning rate as \( 5 \times 10^{-4} \), which was halved every 5 epochs. We set the batch size to 4 i.i.d.

\(^1\)http://icvl.cs.bgu.ac.il/hyperspectral/
\(^2\)http://www.cs.columbia.edu/Cave/Databases/
\(^3\)http://www.ehu.eus/ccwintco/index.php/Hyperspectral_Remote_Sensing_Scenes/
\(^4\)https://rslab.ut.ac.ir/data
TABLE II
Quantitative comparisons of different methods under five complex noise cases over the ICVL dataset. The best and second best results are highlighted in bold and underline, respectively. “↑” (resp. “↓”) means the larger (resp. smaller), the better.

| Case | Metrics | Noisy | LRMR [26] | LRTV [18] | NMoG [28] | TDTV [19] | GRN [15] | QRNN3D [14] | T3SC | Ours |
|------|---------|-------|-----------|-----------|------------|-----------|-----------|-------------|------|------|
| 1    | MPSNR↑  | 17.80 | 28.64     | 33.96     | 34.96      | 37.95     | 39.97     | 42.79       | 43.51| 44.25|
|      | MSSIM↑  | 0.1516| 0.5153    | 0.8987    | 0.8279     | 0.9377    | 0.9587    | 0.9752       | 0.9776| 0.9799|
|      | SAM↓    | 0.7911| 0.3235    | 0.0647    | 0.1260     | 0.0671    | 0.0685    | 0.0430       | 0.0441| 0.0330|
| 2    | MPSNR↑  | 17.77 | 28.52     | 34.05     | 34.60      | 37.65     | 39.90     | 42.64       | 43.20| 44.28|
|      | MSSIM↑  | 0.1545| 0.5155    | 0.8996    | 0.8184     | 0.9348    | 0.9598    | 0.9750       | 0.9770| 0.9804|
|      | SAM↓    | 0.7895| 0.3250    | 0.0662    | 0.1793     | 0.0731    | 0.0672    | 0.0437       | 0.0488| 0.0323|
| 3    | MPSNR↑  | 17.36 | 27.78     | 32.72     | 33.60      | 35.67     | 38.74     | 42.31       | 41.42| 44.27|
|      | MSSIM↑  | 0.1473| 0.5075    | 0.8908    | 0.8212     | 0.9181    | 0.9548    | 0.9735       | 0.9724| 0.9801|
|      | SAM↓    | 0.8109| 0.3398    | 0.1021    | 0.1885     | 0.0937    | 0.0702    | 0.0455       | 0.0639| 0.0332|
| 4    | MPSNR↑  | 14.86 | 24.19     | 32.41     | 29.09      | 36.60     | 37.63     | 40.49       | 37.93| 42.31|
|      | MSSIM↑  | 0.1118| 0.3805    | 0.8722    | 0.6751     | 0.9265    | 0.9410    | 0.9533       | 0.9353| 0.9641|
|      | SAM↓    | 0.8480| 0.4681    | 0.1983    | 0.4510     | 0.0874    | 0.0952    | 0.0762       | 0.1669| 0.0650|
| 5    | MPSNR↑  | 14.07 | 23.79     | 31.39     | 28.45      | 34.51     | 38.01     | 39.42       | 35.84| 41.77|
|      | MSSIM↑  | 0.0936| 0.3817    | 0.8649    | 0.6746     | 0.9076    | 0.9473    | 0.9448       | 0.9248| 0.9615|
|      | SAM↓    | 0.8587| 0.4668    | 0.2135    | 0.4568     | 0.1063    | 0.0904    | 0.0809       | 0.1804| 0.0673|

TABLE III
Comparisons of #Param and #FLOPs of deep learning-based methods over the ICVL dataset. Since T3SC [16] was built based on sparse coding and deep learning, we could not calculate the #FLOPs like other pure deep learning-based methods.

| Metrics | Methods |
|---------|---------|
|         | GRN [15]| QRNN3D [14]| T3SC [16]| Ours |
| #Param (M) | 1.07 | 0.86 | 0.83 | 0.66 |
| #FLOPs (×) | 0.22 | 1.26 | N/A  | 0.95 |

B. Evaluation on Natural HS Images

Tables I and II show the quantitative results of different methods applied to handle HS images with the Gaussian noise and complex noise, respectively, where it can be observed that
- our method consistently achieves the best performance in terms of all the three metrics under all noise scenarios. Particularly, our method improves the MPSNR of the second best methods by 0.49 dB, 0.58 dB, 0.55 dB, and 0.74 dB under four Gaussian noise levels, and 0.74 dB, 1.08 dB, 1.96 dB, 1.82 dB, and 2.35 dB under five types of complex noises, respectively;
- the impressive performance of our method under both the blind Gaussian noise and all the complex noise scenarios demonstrates that it has better resilience on severely corrupted HS images; and
- existing deep learning-based methods, i.e., GRN [15], QRNN3D [14], and T3SC [16], surpass the traditional denoising methods to a large extent under the complex noise scenario. Moreover, our method further boosts their performance.

Fig. 5 shows the visual comparisons of denoising results by different methods, demonstrating the advantage of our method again, where we can observe that the denoised images by our method is cleaner and retain the original high-frequency details better. In addition, Fig. 6 illustrates the PSNR value of each band of the denoised HS images shown in Fig. 5, where it can be seen that our method achieves the highest PSNR values on almost all spectral bands.

Besides, we also compared the number of network parameters (#Param) and floating point of operations (#FLOPs) of deep learning-based methods in Table III, where it can be
observed that our method consumes fewer network parameters and has comparable #FLOPs, compared with state-of-the-art methods, demonstrating that the excellent performance of our method does not come at the cost of a larger capacity and...
higher computational complexity but is credited to elegant feature extraction technique.

Finally, Table IV lists the quantitative results of different methods on the CAVE dataset under the five complex noise scenarios, where for the deep learning-based methods, we directly applied the models trained on the ICVL dataset. From Table IV, it can be seen that our method achieves superior denoising performance to other state-of-the-art methods, demonstrating its strong generalization ability.

C. Evaluation on Remote Sensing HS Images

To demonstrate the generalization ability of the proposed method, we also conducted experiments on remote sensing HS images, whose spectral characteristics are significantly different from those of the previous natural HS images. Note that for all the deep learning-based methods, we directly applied the models trained on the ICVL dataset under this scenario.

1) Synthetic noisy data: Table V shows the results of different methods on Pavia University with the synthetic mixture noise; where as GRN [15] and T3SC [16] cannot handle the HS images with a different number of spectral bands from the training data, we did not report their results under this scenario. From Table V, it can be seen that our method still achieves the best quantitative performance among all methods, which strongly demonstrates the advantage and generalization.
Fig. 8. Visual comparison of different methods on Urban with real unknown noise. Here we selected the 38th, 112th, and 128th bands to form a pseudo RGB image to enable the visualization.

### TABLE VI

| Methods | #Params (M) | #FLOPs (T) | Rank upper bound | MPSNR↑ | MSSIM↑ | SAM↓ |
|---------|-------------|------------|------------------|--------|--------|------|
| 3-D conv. (Fig. 1 (a)) | 1.201 | 1.562 | M | 39.34 | 0.9430 | 0.0693 |
| Seq. 1-D conv. (Fig. 1 (c)) | 0.614 | 0.765 | M | 39.42 | 0.9430 | 0.0658 |
| Seq. 1-D and 2-D conv. (Fig. 1 (b)) | 0.717 | 0.889 | M | 39.58 | 0.9445 | 0.0652 |
| 1-D + 2-D conv. (Fig. 1 (d)) | 0.740 | 1.082 | 2M | 39.71 | 0.9459 | 0.0610 |
| Re-ConvSet | 0.658 | 0.953 | 3M | **39.85** | **0.9478** | **0.0606** |

ability of our method. Besides, the advantage of our method is further demonstrated by the visual comparison in Fig. 7, where it can be seen that it is hard for the compared methods to completely remove the mixture noise. By contrast, our method produces the denoised image with clearer and sharper textures.

2) Real noisy data: Fig. 8 provides the visual comparisons of denoised HS images by different methods on Urban, a real-world noisy HS image, where we can see that most of the compared methods fail to remove the unknown noise completely. By contrast, our method successfully tackles this unknown noise and produces a clearer and visually pleasing image.

### D. Ablation Study

We also directly and comprehensively compared the various combinations of low-dimensional convolutional kernels illustrated in Fig. 1. For fair comparisons, we built various denoising methods by only replacing the 1-D convolutional kernels of our method with the variants and retaining all the other settings (e.g., connections, aggregation, etc.). Besides, we also provided the results of 3-D convolution for reference. As listed in Table VI, it can be seen that compared with the 3-D convolution, all the combinations of low-dimensional convolution show their advantages on either quantitative performance or network compactness (#Params) and complexity (#FLOPs). Generally, a higher upper bound of the rank produces better reconstruction quality, which is consistent with our theoretical analysis. Particularly, our Re-ConvSet equipped with the second fewest number of network parameters has the highest rank upper bound during the feature embedding process and thus achieves the best quantitative performance, convincingly demonstrating its superiority and the importance of filter diversity in designing feature extraction module.

### V. Conclusion and Future Work

In this paper, we first proposed Re-ConvSet, an efficient and effective module for extracting high-dimensional spatial-spectral information of HS images. Specifically, based on the theoretical analysis that improving the rank of the matrix formed by unfolded convolutional filters can promote feature diversity, we designed Re-ConvSet by separately performing 1-D convolution along the three dimensions of HS images side-by-side and then aggregating the output, thus making it not only well capture the high-dimensional spatial-spectral information of HS images but also reduce the computational complexity. Furthermore, we built our HS image denoising method by incorporating Re-ConvSet into the widely-used U-Net architecture. We conducted extensive experiments on both synthetic and real noisy HS images and demonstrated the significant superiority of such a concise method over state-of-the-art methods both quantitatively and visually.

In the future, we will explore the potential of the proposed Re-ConvSet in other kinds of HS image processing tasks, e.g., HS image spatial resolution, classification, segmentation, etc.
