Quantitative wake analysis of a freely swimming fish using 3D synthetic aperture PIV

Leah Mendelson · Alexandra H. Techet

1 Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, MA, USA

Received: 21 October 2014 / Revised: 12 May 2015 / Accepted: 21 May 2015 / Published online: 17 June 2015
© Springer-Verlag Berlin Heidelberg 2015

Abstract Synthetic aperture PIV (SAPIV) is used to quantitatively analyze the wake behind a giant danio (Danio aequipinnatus) swimming freely in a seeded quiescent tank. The experiment is designed with minimal constraints on animal behavior to ensure that natural swimming occurs. The fish exhibits forward swimming and turning behaviors at speeds between 0.9 and 1.5 body lengths/second. Results show clearly isolated and linked vortex rings in the wake structure, as well as the thrust jet coming off of a visual hull reconstruction of the fish body. As a benchmark for quantitative analysis of volumetric PIV data, the vortex circulation and impulse are computed using methods consistent with those applied to planar PIV data. Volumetric momentum analysis frameworks are discussed for linked and asymmetric vortex structures, laying a foundation for further volumetric studies of swimming hydrodynamics with SAPIV. Additionally, a novel weighted refocusing method is presented as an improvement to SAPIV reconstruction.

1 Introduction

Fish are nature’s quintessential swimmers. With a clear and classic relationship between form and function, fish have evolved to survive in challenging environments and persist unrivaled by biomimetic swimming robots. The design of vehicles, such as those developed by Barrett et al. (1999), Fish et al. (2003), and Epps et al. (2009), can be significantly improved with detailed quantitative analysis of the momentum transfer between the fish and the fluid during swimming behaviors. Fish swimming behavior is inherently three-dimensional (3D), with multifarious fin and body motions and fin–wake interactions. The complexity of these swimming behaviors suggests that 3D rendering, with sufficient spatial and temporal resolution, is necessary to simultaneously capture the relevant kinematics and hydrodynamics to facilitate the quantification of propulsive performance.

Early work on fish swimming hydrodynamics employed qualitative shadowgraphy techniques, suggesting a series of linked 3D vortex rings in the wake of a steadily swimming fish, as well as during a “push and coast” swimming mode (McCutchen 1977). Looking to quantify wake hydrodynamics behind swimming fish, researchers turned to noninvasive techniques such as particle image velocimetry (PIV). PIV allows for non-intrusive measurements of the velocity field in a fluid by imaging, and then tracking, the motion of reflective flow tracers over time. Fundamental details on the implementation of algorithms used in PIV can be found in Raffel et al. (1998).

PIV measurements taken behind steadily swimming fish along the horizontal mid-plane of the body reveal clear reverse Kármán street vortex configurations in the fish wake (e.g., Stamhuis and Videler 1995; Wolfgang et al. 1999; Müller et al. 2000). Using multiple planes of 2D PIV measurements behind the pectoral fin of a steadily swimming bluegill sunfish, Drucker and Lauder (1999) reconstruct the wake of this fish revealing a series of staggered and interconnected vortex rings. Along the horizontal center plane of these linked vortex rings, the telltale reverse Kármán street of a propulsive thrust wake is present.
Striving to quantify swimming performance with only wake velocimetry has led researchers to analyze predominately 2D PIV data for a better understanding of the forces generated by swimming fish. The fluid impulse is one measure of the momentum in a vortex. Using an ideal vortex ring model for the fish wake, the impulse of the wake can be compared to the change in momentum of the fish body (Müller et al. 1997; Epps and Techet 2007). Impulse can be determined for an ideal vortex ring from the vortex ring circulation and diameter as

\[ I = \rho \Gamma \frac{\pi D^2}{4}, \]  

(1)

where \( \rho \) is the fluid density and \( D \) is the vortex ring diameter measured core to core in a 2D slice. This model assumes an axisymmetric vortex ring for all behaviors and requires accurate circulation \( \Gamma \) measurements. \( \Gamma \) for a vortex ring is calculated by taking the line integral of tangential velocity on a closed contour around the vortex

\[ \Gamma = \oint_C \mathbf{u} \cdot d\mathbf{l}, \]  

(2)

which minimizes error in calculation. Alternate methods for calculating circulation involve applying Stokes’ theorem and integrating vorticity over area, which requires first calculating vorticity as the curl of the velocity field, thus propagating unnecessary uncertainty (error) into the calculation of circulation.

Epps and Techet (2007) introduce an additional contribution to the fluid impulse model that results due to the thickness of a finite vortex core:

\[ I_1 = \rho \Gamma \frac{\pi D^2}{4} \left( 3D_0^2 - 4D^2 \right), \]  

(3)

where \( D_0 \) is the diameter of the finite vortex core. This additional impulse must be considered for a complete formulation of the wake vortex dynamics. Dabiri (2005) and Peng et al. (2007) further suggest frameworks for analyzing PIV data within the constraints of available measurement data taking into account added mass effects, which ultimately cannot be neglected.

The ideal vortex ring framework is the simplest wake model available that can be applied to the calculation of hydrodynamic impulse from PIV data. However, it does not take into account the effect of the vortex ring linking that is seen in steady forward swimming cases. Two-dimensional PIV vorticity measurements taken along a horizontal mid-plane of the fish wake contain the signature reverse Kármán vortex street described above. The vortices seen in this reverse vortex street contain combined circulation from the two linked rings. Using the linked cores could lead to over estimation of the thrust produced by the swimming fish, further making a case for investigations using 3D PIV methods.

The case for 3D data has also been made through many studies that acknowledge the limitations of using planar measurements to study swimming hydrodynamics (e.g., Tytell 2006, 2008). Lauder (2009) further cites 3D interactions as some of the most critical aspects of swimming analysis for engineers and biologists to consider as instrumentation develops. Three-dimensional considerations must also be extended to the development of biomimetic robots and vehicles (Lauder and Madden 2006), particularly in determining the importance of secondary fin and propulsor configurations.

The rise of advanced 3D imaging techniques and the fall of hardware costs make it feasible to study fish in their natural environments and measure most aspects of their behavior with noninvasive 3D imaging methods: from gross body kinematics to detailed wake hydrodynamics. Accurate analysis of swimming hydrodynamics requires synchronized knowledge of the fish behavior along with the fluid kinematics. Ideally, the fish swims in a natural manner unperturbed by external disturbances, such as bright flashing lasers, rapid movements outside the tank and large ambient flow disturbances (e.g., Stamhuis et al. 2002). In particular, the green wavelengths of laser light (\( \lambda = 527–532 \) nm), typically used during PIV in water, can disturb fish and provoke unnatural swimming behavior. Near-infrared illumination (\( \lambda = 808–810 \) nm) is invisible to fish and appears not to adversely affect their behavior. Near-IR has been successfully used to visualize unsteady fish maneuvering (Epps and Techet 2007), feeding (Adhikari and Longmire 2013), and plankton flow (Murphy et al. 2012).

The evolution of volumetric near-IR lasers now facilitates noninvasive 3D imaging as well.

Three-dimensional PIV measurements can be obtained using a number of imaging and reconstruction techniques, including tomographic PIV, stereo PIV, and defocusing digital PIV (DDPIV). Kittohofer et al. (2011) and Scarano (2013) provide detailed reviews of these methods and their advantages. Using stereoscopic PIV, Sakakibara et al. (2004) resolves substantial out-of-plane velocity during rapid fish turning and proves the need to consider three-dimensionality in maneuvering applications. The first volumetric PIV on fish, taken using defocusing DDPIV by Flammang et al. (2011a), provides complete visualization of 3D vortex rings shed by the caudal fins of bluegill sunfish and cichlid fish swimming in a flume. Results also show smaller vortices generated by the dorsal and anal fins interacting with the caudal fin wake, further emphasizing the need for volumetric measurements over planar ones. DDPIV is employed for the study of the caudal fin wake of a dogfish shark by Flammang et al. (2011b), revealing a novel, highly 3D, dual-linked vortex ring structure that is...
far more complex than any 2D slice through the flow would suggest. Such findings further accentuate that biological flow structures are more complex than how they are often modeled in 2D.

Volumetric techniques are well suited to study behaviors where the organism cannot be constrained to a single motion plane. Adhikari and Longmire (2013) use tomo-graphic PIV to study predator–prey dynamics in zebrafish feeding. The authors introduce a visual hull method for object reconstruction, which facilitates accurate fish body reconstruction, and masking that enables 3D PIV to resolve near-body flows as well as wake features.

Belden et al. (2010) introduce synthetic aperture PIV (SAPIV) as a new method for 3D flow imaging using camera arrays, based on light field imaging and synthetic aperture refocusing methods. The work presented herein applies SAPIV to the problem of swimming fish, providing fast and accurate reconstruction of 3D particle and velocity fields. Giant danio are allowed to freely swim in a quiescent, seeded tank and are tracked using 3D SAPIV methods. The fish body and fin motions are reconstructed using visual hull methods combined with body tracking algorithms. Three-dimensional velocimetry data is used to quantitatively analyze the fish wake in steady swimming and maneuvering, highlighting the advantages of volumetric impulse calculation methods and limitations of 2D analyses of 3D wake structures.

2 Materials and methods

2.1 Experiment setup

To further understand the fully three-dimensional wake of a swimming fish, giant danio (*Danio aequipinnatus*) are studied using 3D synthetic aperture PIV. Anderson (1996), Wolfgang et al. (1999), and Epps and Techet (2007) present detailed 2D PIV on the giant danio revealing classic wake patterns that warrant further study in 3D. Zhu et al. (2002) presents 3D numerical models of swimming danio, paving the way for possible comparisons with 3D PIV data.

Experiments are conducted in a five-gallon tank with dimensions 400 mm long × 200 mm wide × 250 mm in height. The experiment tank is filled to a water level of 160 mm, with water taken from the fish’s home tank. Slotted acrylic dividers are used to restrict the fish to swim in the center 300 mm of the tank without preventing flow from passing through. An array of nine cameras is positioned to view through the side of the tank. The experimental setup is shown in Fig. 1. SAPIV is performed on five individual fish. Herein, data are presented for a single specimen with body length BL = 58 mm and mass 4.8 g. The absolute distance between the two tips of the caudal fin is 14 mm.

The camera array used for the SAPIV setup consists of nine Manta CCD cameras by Allied Vision Technologies with 1292 × 964 pixel grayscale resolution. Each camera is equipped with a 35 mm C-mount Tamron lens set to f/ 5.6 to ensure a sufficient depth of field such that the entire measurement volume is in focus. The Manta cameras record at a fixed frame rate of 30 Hz. The array is positioned 635 mm from the front of the tank in a 3 × 3 camera arrangement with 230 mm horizontal spacing and 190 mm vertical spacing between cameras. The measurement volume where all camera fields of view overlap is 70 mm × 60 mm × 40 mm. As shown in Fig. 1, the coordinate system of the measurement volume is defined with X and Y parallel to the front tank wall and Z perpendicular to the front tank wall. The tank is seeded with 50 µm polylamide particles to a seeding density of C = 230 particles/cm³, such that the image seeding density is N = 0.03 particles/pixel.

Laser illumination is provided by a 1000 W Oxford Lasers Firefly Volumetric Laser with 808 nm wavelength. In general, near-IR light attenuates faster in water than green light, as discussed for 3D PIV by Adhikari and Longmire (2013); thus, a first surface mirror is placed at the end of the tank to reflect the beam back into the volume. Given the high level of light attenuation, further mirrored passes of the beam, as done by Ghaemi and Scarano (2010), do not provide significant additional illumination. The laser can be pulsed in sync with the cameras at a constant interframe time Δt = 0.033 s or used in a frame-straddling mode with an interpulse time Δt = 0.020 s. Both laser timings ensure particle displacements of at least a focal plane in the Z
direction. At shorter interframe times, the SAPIV system will successfully resolve $X$–$Y$ velocities, but not register particle displacement or velocity in the $Z$ direction. The laser pulse duration is set to $50 \mu s$ to eliminate any motion blur in the images and maximize particle illumination.

The SAPIV methods used in this study are validated with a benchmark experiment of a vortex ring generated by a mechanical piston. The piston has diameter $D_{\text{outer}} = 26$ mm, orifice diameter $D_{\text{orifice}} = 17$ mm, stroke length $L_{\text{stroke}} = 10$ mm and stroke velocity $U_{\text{piston}} = 20$ mm/s. The camera and laser setup are the same as described above, but the laser interframe time in the validation experiment is reduced to $\Delta t = 0.004$ s due to the speed of the vortex ring. The validation experiment is used to assess spatial resolution, percentage of valid velocity vectors, and the uncertainty in integral quantities determined from the velocity measurements, such as circulation.

### 2.2 Synthetic aperture PIV

Synthetic aperture PIV is employed to reconstruct the 3D velocity fields and body kinematics of the swimming fish. SAPIV methods are presented in detail by Belden et al. (2010), and applications of SAPIV to flapping wings are presented by Langley et al. (2014). SAPIV uses arrays of cameras, each with a different line of sight, to image a scene from multiple viewpoints. Images from each camera are refocused into a volumetric stack of images for processing with 3D PIV algorithms. Synthetic aperture refocusing simulates the effects of a camera with a narrow depth of field scanning through the light field. By spatially relating all the cameras to a global coordinate system (divided into finely spaced focal planes throughout the measurement volume), combining images and determining where features are in focus, the 3D location of a particle can be determined.

Image refocusing is performed using a modified version of the additive map-shift-average algorithm introduced by Belden et al. (2010). Since imaging is performed through the tank wall and water (and thus through media with two different refractive indices), the camera array is mapped using a calibration procedure that compensates for refractive effects to determine an accurate homography at any depth in the tank (Belden 2011). A homography transformation maps each of the nine raw camera images, at each time step, to the projected coordinates of each focal plane in $Z$. These transformed images can then be multiplied or added to reconstruct the entire image volume.

The depth resolution of a SAPIV setup is a function of the camera optics and the baseline spacing parameter, $D = \frac{\Delta X}{s_0}$, where $\Delta X$ is the distance between camera centers and $s_0$ is the distance from the cameras to the scene (in this case the center of the measurement volume). The ratio of $\delta Z$ to $\delta X$, the size of a single voxel in physical units (mm for this study) in the $Z$ and $X$–$Y$ directions, is given by

$$\frac{\delta Z}{\delta X} \approx \frac{1}{D} + \frac{Z}{D s_0}. \quad (4)$$

To prevent the formation of excessively elongated particles in the $Z$ direction, which in turn reduces $Z$ velocity resolution, the baseline spacing must be sufficiently large. In this study, $D = 0.33$, $\delta X = 0.0619$ mm, and the focal plane spacing is resultantly set to $\delta Z \approx 0.2$ mm. In order to achieve the high baseline spacing required for optimum resolution of particle displacements in the $Z$ direction, the camera array is positioned close to the tank with relatively wide baseline spacing between cameras. Due to the camera locations, the fish takes up a large portion of the field of view.

Before refocusing, raw images are preprocessed to remove reflections off the fish body, enhance contrast, reduce ghosting, and improve reconstruction quality. Large, high-intensity reflections and bright regions of the fish body are eliminated by subtracting the results of a $5 \times 5$ pixel median filter from the image. This filter is also used by Jeon and Sung (2012) and Adhikari and Longmire (2012) for object removal in tomographic PIV images. Images are then convolved with a $3 \times 3$ pixel Gaussian kernel; this Gaussian filter is used to enlarge particle image diameters for better reconstruction. Particle image sizes are small ($\leq 3$ pixel mean diameter) in the raw images due to the high numerical aperture ($f$#) required for sufficient depth of field to capture the measurement volume. To equalize intensity uniformly, both within a single image and across all images, normalization is performed with a local min/max filter ($10 \times 10$ pixel windows). This filter accounts for variations in the laser beam intensity over the volume, especially in regions where the fish body reflects more light into its immediate surroundings. To reduce amplified background intensity from the normalization, a sliding minimum (taken over $10 \times 10$ pixel windows) is subtracted from the normalized images.

### 2.3 Weighted refocusing

Previous studies using SAPIV have implemented a multiplicative algorithm to enhance the signal-to-noise ratio of the refocused images (e.g., Belden et al. 2012). However, when an object such as the fish body is present in the particle field, occlusions can occur in the imaged volume where particles are blocked from view in some subset of the cameras. A multiplicative algorithm requires perfect convergence of a particle in all cameras and thus fails to reconstruct any particles in crucial near-body regions where there are any occlusions. As a result, the multiplicative algorithm has been difficult to implement in SAPIV.
applications where moving bodies are present. Belden et al. (2010) and Langley et al. (2014) therefore use an additive method, in lieu of a multiplicative algorithm, such that the refocused image on a given focal plane $k$ is

$$I_{SAk} = \frac{1}{N} \sum_{i=1}^{N} I_{FP_{ik}},$$

where $N$ is the number of cameras, and $I_{FP_{ik}}$ is the transformed image from each camera on the $k^{th}$ focal plane.

After refocusing, SAPIV methods typically filter each reconstructed focal plane separately to retain only particles with intensities above a certain threshold, determined by a Gaussian fit to the focal plane intensity distribution. This process is relatively inefficient because every reconstructed focal plane must be thresholded separately. Additionally, the thresholding is inconsistent across focal planes when no single global threshold level is used, and thus, the original SAPIV thresholding method in Belden et al. (2010) requires substantial computational time. In this paper, additive refocusing of weighted images is introduced as a lower-cost alternative.

Weighted refocusing is achieved by applying a weighting function to each sequence of raw camera images separately for each camera, after the preprocessing steps but before the volumetric reconstruction is performed. The average value of each of the preprocessed images is used as a threshold value, below which all intensity levels are set to the maximum intensity value times $-1$. For example, given an image with an intensity scale between 0 and 1, all values below the calculated average would be set to $-1$. For 8-bit images, with intensity values ranging from 0 to 255, all values below the image average are set to $-255$. Applying the weight function significantly penalizes the dark regions of an image where no particles exist. When summed in the additive algorithm, the weight function from one image can cancel the ghosting from another image at a location where a particle does not exist. A particle is still successfully reconstructed when it is present in more camera views than it is absent from, and the intensity of the final reconstructed particle is proportional to the number of cameras that contribute to the reconstruction. The refocusing of weighted images acts effectively as a refocusing cost function. The cost function method reduces computational time by thresholding only as many images as there are cameras (nine for the present study) for each time step, instead of 100–500 refocused planes at each time step.

One significant benefit of the cost function method is that the reconstructed particles retain a Gaussian intensity profile. When applied to a single particle, the refocusing cost function results in a 3D Gaussian particle (Fig. 2), which allows for better subvoxel displacement estimation in 3D space. The original thresholding methods used by Belden et al. (2010) assumed a Gaussian model for the intensity histogram of each refocused image and simply cut off all light below $3\sigma$ above the mean, eliminating all details at the edges of refocused particles, in addition to any image noise, and encouraging peak locking during velocimetry calculations.

To evaluate reconstruction quality using the cost function method, a nine camera array is simulated to image 14,000 particles seeded over a 50 mm × 40 mm × 40 mm volume such that the image density $N$ is 0.015 particles/pixel. Reconstruction quality, as defined in Elsinga et al. (2006), is classically used to determine how well-synthetic images are reconstructed into 3D volumes. Using the refocusing cost function described above, the reconstruction quality is 0.98 for the simulated particle field. This level of reconstruction quality indicates very successful reconstruction of almost all of the synthetic particles and is comparable to reconstruction qualities obtained using the original SAPIV thresholding methods. The validation experiment on mechanically generated vortex rings is also performed using weighted refocusing to provide experimental validation of this reconstruction method.

### 2.4 Fish body reconstruction

Numerous studies have shown the importance of masking PIV data to obtain an accurate velocity field near a solid body. Since the fish is arbitrarily moving, it is also necessary to reconstruct the fish body to understand the relationship between its kinematics and the wake it creates. When experiments are run in a controlled flume, such as those performed by Flammang et al. (2011a) using DDPIV, the measurement volume can be positioned behind the fish by a 3D Gaussian kernel in voxel space and localized using this intensity distribution, analogous to a 2D PIV particle image in pixel space.

![Fig. 2 Intensity profile of a single synthetic particle refocused using the cost function over focal planes with spacing $\delta Z = 0.15$ mm. The particle center lies at $Z = 20.00$ mm. The particle can be represented as a 3D Gaussian kernel in voxel space and localized using this intensity distribution, analogous to a 2D PIV particle image in pixel space.](image-url)
body such that masking needs are minimal. In contrast, the fish in this study is moving freely, requiring a unique mask of the entire body for each image pair. Given the large number of cameras, defining the mask manually is also less than ideal, prompting the implementation of an automated masking procedure.

Masking of the fish is performed using the visual hull method (Adhikari and Longmire 2012). First, the fish body is identified in raw images from each camera at each timestep, e.g., Fig. 3a. In this study, reflections along the body are frequent and create high-intensity image regions along the body. Typical body identification algorithms struggle to detect both bright and dark regions of the fish body, requiring image segmentation for successful body identification. For example, Jeon and Sung (2012) and Im et al. (2015) separate particles from an arbitrary background based on the brightness and size in pixels of features in the image, but this segmentation method is strongly dependent on high-intensity contrast between the particles and a much fainter background, and the bright body reflections present in the SAPIV images of the danio could not be similarly separated. Edge detection procedures and morphological operations also can be used to remove objects (e.g., Adhikari and Longmire 2012; Langley et al. 2014). These methods do not directly address the issues with bright and dark spots in images as PIV image segmentation methods do. The numerous markings on the giant danio body create spurious edges within the raw images that often coalesce with particle edges during image stitching operations.

To address the body patterning of fish scales, Siddiqui (2007) introduces an adaptive thresholding algorithm specifically for fish. The algorithm segments the image by separately extracting both bright and dark portions of the body while morphologically minimizing coalesced particles. The two partial masks are combined and smoothed morphologically. A similar algorithm is implemented here, with changes in the initialization threshold and morphological filter sizes made to account for volumetric illumination and the body size of the fish in this study. Siddiqui reports a success rate of >90% using this algorithm; using this method on SAPIV data results in 93% success for all processed data where the fish is entirely within the measurement volume. For the 7% of processed images where the algorithm fails, masks are manually defined. Raw masks for a run imaging the caudal fin (from Fig. 3a) are shown in Fig. 3b.

Raw mask binary images are refocused using a multiplicative algorithm (Belden et al. 2012) to determine where all masks overlap on a given focal plane, which effectively

---

![Fig. 3](image-url)

**Fig. 3**  
(a) Raw image of the caudal fin from the center camera of the array. (b) Binary mask images from all nine cameras (arranged the same as the physical camera array) show several different silhouettes of the caudal fin as the fish swims across the tank. (c) Three slices of the refocused mask. One tip of the caudal fin lies at $Z = -2.0$ mm, the second tip at $Z = 6.0$ mm, and the caudal peduncle at $Z = 14.0$ mm. (d) Reconstructed caudal fin visual hull from the refocused mask planes. The mask is displayed at full resolution in $Z$ with equivalent spatial resolution in $X$ and $Y$. The elongation and tapering to a point in $Z$ created by the discrete blur pattern of partial mask convergence are prominent.
represents the portion of the body that physically occupies each plane, e.g., Fig. 3c. These focal planes are stacked to determine a visual hull of the fish body in the 3D volume (Fig. 3d). The mask is displayed at full resolution in Z and equivalent resolution in X and Y. The mask reveals the general shape of the caudal fin, with some elongation in Z that tapers to a point as the overlap between transformed camera views decreases. The elongation is an artifact created by the partial convergence of masks outside the actual plane of focus and is referred to in Murphy et al. (2013) as the “shadow” of the visual hull. Scharfman et al. (2013) eliminate the effects of this shadowing for spherical droplets using knowledge of the in-plane geometry. The addition of an overhead camera would allow for truncation of the visual hull based on the silhouette of the fish, but this is not possible in current experiment due to hardware constraints. Despite some elongation in the Z direction, the approximate size and orientation of the fish body is clear, and the mask is adequately large to block any regions within the body from biasing PIV results. While similar body reconstructions can be performed using fewer cameras (e.g., Adhikari and Longmire 2012; Murphy et al. 2012, 2013), the additional cameras help smooth the mask edges and reduce the overestimation of body volume. The reduction in visual hull volume as a result of using all nine cameras instead of only the four outermost cameras is between 5% (caudal fin mask only) and 20% (full body mask).

During the 3D cross-correlation step of SAPIV processing, if the midpoint of an interrogation window falls within the coordinates of the visual hull, it is included as part of the mask. Masked voxels are excluded from velocity vectors calculation and treated as a region in which no velocity vectors can pass through.

### 2.5 Swimming kinematics

Swimming kinematics are estimated by tracking five key points on the fish body: the left and right eyes, the frontmost junction of the dorsal fin and the body, and the two tips of the caudal fin (Fig. 4). The junction of the dorsal fin and the body is located at approximately half of the body length and is the closest point to the fish’s center of mass that can be consistently tracked in all viewpoints. Each of the five body points is manually located in the raw images (large displacements and orientation changes between successive timesteps made automated tracking inefficient to implement), from which a local binary mask of the body part is generated. Multiplicative refocusing is then used to create a local visual hull of each body feature at each timestep. The centroid of each local hull is tracked over time in the volume. The mean absolute position error of each voxel in a hull relative to the centroid is used as a metric of the body tracking uncertainty. The uncertainty is 0.24 mm in X (8% of the mean displacement between images), 0.20 mm in Y (7%), and 0.66 mm in Z (23%). Since the fish body occupies a large portion of the field of view, tracking the body over large time intervals (more than 10–15 images) is not possible without restricting the fish to swim in an unnaturally small area.

The Strouhal number, defined as $St = f A/U$, where $f$ is the tailbeat frequency in Hz, $A$ is the double amplitude of the caudal fin, and $U$ is the body speed, is determined from the body tracking results. The paths of the eyes and dorsal fin/body junction are used to determine the mean body velocity, while the motion plane and amplitude of the tailbeat are determined from the 3D position of the two caudal fin tips. Even within a single run, inconsistency in amplitude and velocity is observed as a result of stroke-to-stroke variation in the fish’s kinematics. Strouhal numbers presented represent averaged values over 2–3 tailbeat cycles due to the field of view.

Allowing the fish to swim freely enables SAPIV measurements to be obtained for a variety of fish behaviors and swimming speeds. Results are presented for cases of forward swimming at two speeds, 0.9 and 1.5 body lengths/second (BL/s), and a 75° turning maneuver entered into at 1.0 BL/s and exited at 1.3 BL/s. For each of these behaviors, the 3D structure of the wake is visualized and circulation and impulse are calculated. In the cases of 1.5 BL/s swimming and turning, the circulation and impulse are also measured over several consecutive timesteps to determine how quickly the wake features of these faster maneuvers develop and break down in an otherwise-static tank. The energy balance between the fish and its wake must be considered when the wake is at full strength, and energy has not dissipated in order to develop an accurate picture of performance.

---

**Fig. 4** Marker points positioned on the fish’s right eye, caudal fin tips, and the base of the dorsal fin. The mean body velocity and caudal fin amplitude are determined by refocusing and tracking these locations on the body.
2.6 Velocity field reconstruction and analysis

Velocity fields are determined from reconstructed 3D particle image volumes using a multipass normalized cross correlation. Processing is performed using a customized 3D version of MatPIV. MatPIV is an open source, MATLAB-based, 2D PIV code developed by Sveen (2004). The interrogation size used in this study is $128 \times 128 \times 32$ voxels for the first pass and $64 \times 64 \times 16$ for the two successive passes, all with 50 % overlap between interrogation windows. The window is sized such that the $X$, $Y$, and $Z$ dimensions are of similar physical size. The final resolution is a velocity vector every 1.85 mm in $X$ and $Y$ and every 1.60 mm in $Z$.

The velocity field is post-processed using a filter based on the height ratio between the correlation peak and the second highest peak and a local median filter with $3 \times 3 \times 3$ vector neighborhoods. Filtered vectors are replaced using iterative linear interpolation. Interpolation begins at points surrounded entirely by valid vectors. Points with one spurious neighbor are considered next, and the interpolation progresses in decreasing order of number of valid neighbors until the vector field is filled to completion in all unmasked regions. Approximately 5 % of the total vectors in the volume are replaced by post-processing. Once the velocity field is determined, additional quantities such as vorticity and circulation can be readily determined. Before computing vorticity, the velocity field is smoothed once with a $3 \times 3 \times 3$ Gaussian kernel. In 2D PIV, smoothing of velocity fields has been shown to increase the accuracy of velocity gradient-based calculations such as vorticity (Luff et al. 1999).

Circulation around coherent vortical structures in the flow is calculated, using Eq. (2), as the line integral of tangential velocity about rectangular contours of increasing enclosed area until a local maximum is reached. The line integral method for circulation is chosen since the method of integration of all vorticity in a set area above a threshold level (e.g., Epps and Techet 2007) is extremely sensitive to the chosen threshold level. Langley et al. (2014) show that the circulation magnitude is substantially underestimated at higher threshold levels even though the vorticity threshold level does not influence general temporal trends in vortex circulation as measured in a SAPIV experiment.

Circulation is calculated for multiple cuts through a vortex ring. The overall ring diameter $D$ and vortex core diameter $D_0$ are approximated on each slice, following canonical definitions for these parameters [e.g., see Fig. 2 in Müller et al. (1997)]. For slices through a vortex that are not inherently parallel to the $X$–$Z$, $Y$–$Z$ or $X$–$Y$ measurement planes, velocity components are transformed by rotation matrices to be tangent and normal to the slice plane required for circulation calculation. The transformed velocity fields are linearly interpolated to the smallest vector spacing of the original measurement (1.6 mm between vectors) in order to ensure that similar integration intervals are used in all slices, regardless of orientation. The ring diameter $D$ is measured as the difference between the two zero crossings in the axial velocity profile taken through the two vortex cores in a given slice, similar to methods described in Spedding et al. (1984). When both cores are not within the measurement volume, the ring radius is measured as the distance between one zero crossing in the axial velocity profile and the peak velocity vector in the center jet. The vortex core diameter $D_0$ is approximated as the diameter of a circle with equivalent area to the region on each slice where the Q-criterion (Hunt et al. 1988) is greater than zero. The size of the vortex core is used to characterize the symmetry of the vortex ring and assess the significance of additional terms in estimating impulse.

3 Results and discussion

3.1 Vortex ring

The mechanically generated vortex ring is used to validate the momentum analysis procedures described above in Sect. 2.6 and evaluate the uncertainty of measuring circulation, vortex geometry, and impulse in slices taken from the volumetric velocity field. As seen in Fig. 5a, slices of the ring velocity field taken at several angles about the center axis show an azimuthally uniform velocity field around a circular vortex core. The circulation slices are taken about an axis that is parallel to the peak velocity in the center jet, which is aligned with the $Y$-axis of the measurement volume. The ring’s overall shape is illustrated by the isovorticity surface. Since the ring extended beyond the depth of the measurement volume in $Z$, circulation analysis is performed on eleven slices taken about half its circumference. The azimuthal spacing of the slice planes is not uniform to better align with the original PIV measurement grid, in which the $X$ and $Y$ spacing is larger than the $Z$ spacing. The circulation integral is taken on each slice through the ring until it remains consistent over increasing area (Fig. 5b).

The measured ring radius $R = 1.13 \pm 0.06$ cm (5 % variation), core diameter $D_0 = 1.19 \pm 0.08$ cm (7 % variation), and peak circulation $\Gamma = 61.7 \pm 5.0$ cm$^2$/s (8 % variation) each individually show good agreement across all slices. From the mean values, the resultant impulse calculated is 262 g cm/s. Using Eq. (1) for the impulse of an axisymmetric vortex ring, and appropriate error propagation formulas [e.g., Taylor 1997], the final impulse measurement has approximately 13 % uncertainty.

The ratio of the vortex core diameter to the overall diameter of the vortex ring is the key parameter to determine the
relative importance of Eq. (3) to the overall impulse of the ring. In the case of the axisymmetric vortex ring, the ratio of $D$ and $D_0$ is 0.5. The additional impulse is 19% of the impulse determined from Eq. (2).

In addition to the measurement uncertainty benchmark, the vortex ring study shows that slices taken through planes not parallel to the PIV reference frame axes are consistent with those taken directly on the Cartesian grid. In SAPIV, it is possible to see velocity field distortion in the $Z$ direction if baseline spacing between cameras is insufficient. When planes in the transformed coordinate system match the untransformed data closely, this agreement indicates that the chosen baseline spacing is sufficient. If significant velocity field distortion existed, it would be possible to determine the extent of distortion using this benchmark experiment. For studies on freely swimming live fish, flexibility in analysis reference frame is crucial when alignment of the fish and the tank coordinate systems cannot be guaranteed and projections of the wake structures onto a laboratory reference frame fail to completely describe the wake topology (Fig. 6).

### 3.2 Forward swimming

#### 3.2.1 Single ring

Impulse analysis of the wake behind the fish is first performed for a case where SAPIV shows a single vortex ring behind the caudal fin within the measurement volume (Fig. 7). The swimming speed is $U = 0.9$ BL/s, and the unit tangent vector of the body velocity in the measurement reference frame is $(-0.25, 0.56, -0.79)$. The tailbeat frequency is $f = 3.4$ Hz, and the caudal fin tip
Double amplitude is 9.1 mm such that the Strouhal number is $St = 0.6$. The peak velocity in the ring center axial thrust jet is 0.6 BL/s (36 mm/s). The unit tangent of the jet is $(0.92, -0.32, -0.20)$. Thus, the absolute angle between the fish body trajectory and the jet is 105°.

Since the fish body velocity is not parallel to the tank, it is necessary to perform momentum analysis in a different reference frame than the one in which the PIV measurements are made. Any single slice through the measurement volume can provide as much information as a planar PIV experiment, and the analysis geometry can be defined relative to either the fish body or the vortex ring itself. Multiple contours through the ring are used to characterize the uncertainty of performing circulation analysis in a single plane, in particular the effects of asymmetries and the method by which a slice plane are identified.

For the single vortex ring reconstructed during 0.9 BL/s swimming, the ring geometry and circulation are analyzed on four slices. All slices are taken using the point within the ring’s center jet with maximal speed as the origin. To connect observed swimming kinematics and wake momentum, slices are taken normal to the mean fish body velocity (Body Motion Normal plane in Fig. 8) and parallel to the motion plane of the caudal fin (Caudal Motion plane in Fig. 8). This plane is determined using least squares to fit a plane to the points detected by tracking the tips of the fish body.

**Fig. 7** A single vortex ring behind the fish during forward swimming at $U = 0.9$ BL/s. The single red arrow indicates the mean fish body velocity. The solid orange trace is the path of the upper tip of the caudal fin. The solid brown trace is the path of the lower caudal fin tip. The nodes on the traces indicate the position of the tail tips in the current frame and show agreement with the fish body mask. The isovorticity contour is drawn at 5 s⁻¹. The peak velocity magnitude is 36 mm/s (0.6 BL/s). Every other vector along the Z-axis is plotted for clarity.

**Fig. 8** Slices of the ring used to determine circulation and impulse. Slices through the ring are taken parallel to the motion plane of the caudal fin, normal to the fish body motion (with mean body velocity indicated by the red arrow), and in two planes about an axis defined by the center thrust jet. The center of all slices is the point of maximum velocity in the jet. For each slice location, velocity vectors tangent to the plane (full grid resolution) and contours of the three-dimensional vorticity vector magnitude are displayed.
The two vortex cores in each slice plane are identified by the sign (+ or − by right hand rule) of the vorticity component normal to the slice plane. The ring diameters range from 1.47 to 1.80 cm on all the slices taken. Core diameters range from 0.68 to 0.97 cm. The circulation about each core ranges from 4.08 to 5.36 cm²/s. Measured circulation and core diameter are uncorrelated. The impulse as estimated from each plane is calculated using the mean of the two circulations in the slice. The dot product of the vorticity vector and the slice normal is used as a metric for alignment between the vortex core and the slice, with a value of 1 indicating perfect alignment.

caudal fin. The normal of the caudal fin motion plane is (−0.01, 0.84, 0.54). The angle between the normal of the caudal fin motion plane and the mean body velocity vector is 87°, indicating that the body velocity and the plane in which tail oscillations occur are effectively perpendicular.

The normal of the caudal fin motion plane is (0.18, 0.74, 0.68). The angle between the normal of the caudal fin motion plane and the mean body velocity vector is 87°, indicating that the body velocity and the plane in which tail oscillations occur are effectively perpendicular.

Table 1 summarizes the values for diameters, circulation, and impulse measured in the fish wake during forward swimming at $U = 0.9 \text{BL/s}$.

| Slice location     | $D$ (cm) | $D_{x}+ (cm)$ | $D_{y}- (cm)$ | $\Gamma+ (cm^2/s)$ | $\Gamma- (cm^2/s)$ | $I$ (g cm/s) | $\cos \theta \cdot \hat{n}$ |
|--------------------|----------|---------------|---------------|-------------------|-------------------|-------------|----------------------|
| Caudal Motion      | 1.80     | 0.74          | 0.88          | 4.58              | 4.68              | 11.8        | 0.93                 |
| Body Motion Normal | 1.66     | 0.79          | 0.97          | 4.84              | 4.38              | 9.6         | 0.94                 |
| Jet Axis slice 1   | 1.47     | 0.83          | 0.68          | 5.36              | 4.28              | 8.2         | 0.91                 |
| Jet Axis slice 2   | 1.72     | 0.74          | 0.83          | 4.08              | 4.60              | 10.0        | 0.80                 |

The mean circulation from the eight sampling locations (two per slice plane) is $4.60 \text{cm}^2/\text{s}$. The maximum variation between any calculated circulation and the mean is 17 % of the circulation magnitude. Using the diameter and mean circulation magnitude for the two cores in each slice, the impulse ranges from 8.2 to 11.8 g cm/s with an average of 9.9 g cm/s and a maximum variation between any single slice and the average of 19 %. The impulse values show variation beyond the predicted measurement uncertainty, suggesting that some of the variation is due to either asymmetries in the vortex ring itself or the choice of slice on which circulation and diameter are computed. $I \sim D^2$, and thus, errors estimating the vortex geometry are particularly detrimental to an accurate impulse estimate.

As a measure of alignment, we consider the dot product of the normalized 3D vorticity vector in each slice through the vortex core and the normal of the plane used to define the slice. In the case of the second slice, defined using the ring’s axial jet, there is substantial misalignment between the vortex core and the slice plane, a consequence of asymmetries in the fish wake that the axisymmetric vortex ring model does not account for. Using only the axial jet to define the path for circulation analysis does not guarantee that the contour is well aligned with the vortex core in that particular plane. However, it is not possible to completely decouple asymmetry from measurement uncertainty on the basis of alignment alone.

3.2.2 Multiple vortex rings

Steady forward fish swimming hydrodynamics within a range of optimal Strouhal numbers are characterized by a series of linked vortex rings (e.g., Triantafyllou et al. 1993; Borazjani and Sotiropoulos 2008). Here classical vortex ring linking is seen in the wake of a danio swimming at a speed $U = 1.5 \text{BL/s}$ (Fig. 9a). In this case, the danio tail-beat frequency $f = 3.0 \text{Hz}$, double amplitude of the caudal fin $A = 6.8 \text{mm}$, and speed $U$ yield $St = 0.24$, which is within the optimal range presented by Triantafyllou et al.
The unit tangent of the mean body velocity is $\mathbf{\hat{v}} = (-0.65, 0.60, -0.47)$.

To assess the use of the axisymmetric vortex ring model when linked rings are present, the circulation and ring geometry must be considered in both the linked and unlinked cores of each vortex ring. In a slice parallel to the mean body velocity, taken through the motion plane of the caudal fin (Fig. 9b), the velocity field is comprised of the flow about the linked cores and resembles the reverse Kármán street. When only the velocity tangent to the plane is visualized, the axial jets in rings 1 and 3 are parallel and have a component in the $-X'$ direction (opposite the body velocity of the fish and thus contributing forward thrust). The jet in ring 2, however, is strongest near the shared core.

Fig. 9  a Linked wake vortex rings observed at a swimming speed of $U = 1.5$ BL/s. The swimming direction of the fish is indicated by $U_{\text{fish}}$, the solid orange trace is the trajectory of the top tip of the caudal fin, and the solid brown trace is the path of the bottom tip of the caudal fin. The isovorticity contour is drawn at 4 s$^{-1}$. The velocity and vorticity fields of the shaded slice are shown in (b). b A single slice taken parallel to the motion plane of the caudal fin illustrates the reverse Kármán street formed by the conjoined cores. The vorticity contours show the magnitude of the full three-dimensional vorticity vector. The blue axes $X'$ (parallel to the body velocity) and $Y'$ (parallel to the caudal fin motion) denote the coordinate system for the vorticity profiles seen in (c). c Velocity profiles between the first two cores in the vortex street plotted in the reference frame are shown in (b). The components parallel and perpendicular to the body velocity are $v$ (along $Y'$) and $u$ (along $X'$), while $w$ is the out-of-plane component (positive out of the page).
of rings 2 and 3 (instead of in the center of the ring) and is oriented toward the \(+X'\) direction with minimal contribution to the forward thrust of the fish. The profiles in Fig. 9c show the velocity components on the slice plane with respect to the two linked cores \((u, v)\), along with the out-of-plane velocity component \((w)\). The \(w\) component is nearly equal in magnitude to the in-plane component \(u\). These profiles illustrate how the mean motion of the fish can be approximated by a planar measurement, and yet, the stroke-to-stroke variation in the out-of-plane velocity component is still significant.

Based on the results from analyzing a single vortex ring, defining analysis planes based on the body kinematics ensures more consistent alignment with the vortex core than using planes defined using just the vortex ring axis. For comparison to the linked cores, the linked vortex rings are also analyzed independently using slices through each individual center jet taken normal to the mean fish body velocity (Fig. 10). In each of the slices, only one of the vortex cores is used for analysis, as the other cores are too close to the edges of the measurement volume to be reliably analyzed.

Circulation is plotted versus time for the linked cores seen in the motion plane of the caudal fin and for the cores from slices of each individual ring taken normal to the fish body velocity (Fig. 11). The first frame in which ring 1 is observed to be completely detached from the fish body is used to define \(t = 0\) s. The circulation in the linked core between rings 1 and 2 \((\Gamma_{1,2})\) is substantially higher than the individual circulation in either ring \((\Gamma_1\) and \(\Gamma_2)\). The circulation in the linked core between rings 2 and 3 \((\Gamma_{2,3})\) increases as the third ring is shed. Both the circulation and core diameter over time are more consistent when taken for an individual ring.

The circulations in the linked cores \((\Gamma_{1,2}\) and \(\Gamma_{2,3}\)) are analogous to wake measurements made using planar PIV for two successive fin strokes. Taking only information from the caudal motion slice (Fig. 9b), it appears that circulation is not conserved \((\Gamma_{1,2,max} - \Gamma_{2,3,max} = 1.9\) cm\(^2\)/s). Since ring 1 \((\Gamma_1)\) is stronger than ring 3 \((\Gamma_3)\), the combined circulation of ring 1 and 2 is greater than that of rings 2 and 3. Considering a circulation balance of the first two rings and their linked core instead, the sum \(\Gamma_1 + \Gamma_2 - \Gamma_{1,2,linked}\) ranges from 0.5 to 1.4 cm\(^2\)/s, indicating that almost all of the circulation contained in each individual ring is contained in the linked core.

The vortex core diameter tracked over time (Fig. 11b) in the linked core between rings 1 and 2 is also larger than in the individual cores of either ring 1 or ring 2. The linked core between rings 2 and 3 grows over time after ring 3 has been shed (between \(t = 0.1\) and 0.17 s). The overall ring diameters are similar from ring to ring, \(D_{\text{Ring 1}} = 1.78 \pm 0.13\) cm and \(D_{\text{Ring 2}} = 1.76 \pm 0.15\) cm, despite the variance in calculated circulation. The vortex ring diameter measured between the two linked cores \((1, 2\) and \(2, 3\)) is \(1.78 \pm 0.04\) cm when measured over all times. There is no temporal trend in the ring diameters, indicating that the timescale over which this analysis is performed is shorter than the timescale over which the change in diameter due to vortex breakdown would be observed. The impulse derived using \(\Gamma_{1,2}\) ranges from 13.8 to 16.0 g cm/s for all times measured. The impulses derived using \(\Gamma_1\) and \(\Gamma_2\) range from 7.7 to 11.9 and 5.8 to 8.1 g cm/s, respectively. Even with the consistent ring diameters measured in the linked rings, on the basis of circulation alone, calculating the impulse using the linked cores overestimates the value compared to calculating the impulse using the isolated cores of the same vortex ring. The individual ring
impulses show more variation with each other than with the impulses measured at $U = 0.9$ BL/s, once again emphasizing how much local variation within the wake can be observed during steady forward swimming.

### 3.3 Turning

While efficient forward locomotion is one hallmark of fish propulsion, the maneuverability and agility of fish is even more impressive. During a turning behavior, there is no longer a single direction that can characterize the swimming velocity and both angular and linear accelerations must be considered within a propulsive momentum balance. Results are presented here for a $75^\circ$ turn executed over 0.3 s. The fish enters the turn swimming steadily at 1.0 BL/s with initial direction $(-0.13, -0.08, -0.99)$ and exits the turn at a steady velocity of magnitude 1.3 BL/s and direction $(0.91, 0.16, -0.39)$.

Velocity fields from two perspectives at three timesteps during and after the turn can be seen in Fig. 12. The time $t = 0$ s corresponds to the frame in which the fish initiates the turn. By $t = 0.25$ s, the body is fully curved and the beginnings of a jet can be seen behind the caudal fin. At $t = 0.45$ s, the trajectory of the dorsal fin base, located slightly behind the fish center of mass, shows that the fish has completed turning and resumed translating linearly at a steady velocity. The slice through the $X$–$Y$ plane shows a canonical vortex ring on which 2D momentum analysis could be performed. The simultaneous slice in the $X$–$Z$ plane, however, shows no independent vortex core, suggesting that the vortex ring is still attached to the caudal fin. The thrust jet behind the tail can be seen below the body as the fish turns through the volume. At $t = 0.65$ s, the vortex ring in the $X$–$Y$ plane appears relatively unchanged, and a vortex core independent of the fish body can be seen for the first time in the $X$–$Z$ plane. The center jet observed in this plane is significantly wider than it appears in the $X$–$Y$ slices. The unit tangent of the jet in the tank reference frame is $(-0.97, -0.20, -0.10)$. The absolute angle of the jet with the entry velocity of the turn (as measured at the dorsal fin/body junction) is $76^\circ$, and the angle with the exit velocity is $151^\circ$.

Identifying a plane for circulation and geometric analysis is complicated by several factors in this case. First, the axial jet is misaligned with both the initial and final body velocities. Secondly, the resultant vortex loop (Fig. 13) is more elliptical than observed during forward swimming, which produced more axisymmetric rings. These two factors make it difficult to define the center and orientation of analysis slices that are both coupled to the fish’s kinematics and aligned with the vorticity vector.

To perform impulse analysis using the full vortex geometry, it is first necessary to characterize that geometry without any assumption of symmetry. The vortex core line can be defined as a line representing the center of a vortex filament and is readily detected using algorithms to identify the local maxima of vorticity in a plane normal to the vorticity vector (e.g., Strawn et al. 1999). At each point along the core line, a slice is taken through the point with the normal parallel to the vorticity vector at that location.
The circulation and vortex core diameter are measured on each of these slice planes. Fig. 13 shows the core line at $t = 0.65$ s and a sample slice through the vortex at one point along the core line.

Statistics from all circulation cuts can be used to better define a representative circulation for the entire vortex filament. To approximate impulse based on the results from the core line, the median circulation, which is less sensitive to outliers than the mean, is used in conjunction with the projected area of the vortex filament onto a plane normal to the axial jet. Upper and lower bounds on the impulse can be set by using the minimum and maximum circulation seen anywhere along the core instead of the median.

The circulation along the core line is analyzed at three timesteps spaced 0.20 s apart (Fig. 14). All three analysis times fall after $t = 0.3$ s, the time at which the dorsal base trajectory is no longer turning. At $t = 0.45$ s, the vortex loop is still attached to the fish body, and the vortex core line passes through the caudal fin as a result. The time $t = 0.65$ s is the first frame in which the isovorticity surface is observed to form a closed contour instead of connecting with the caudal fin, indicating that the vortex has been shed completely from the fish body. At $t = 0.85$ s, the caudal fin has performed an additional tailbeat, the effects of which may contribute to the circulation calculated in the slices nearest to the tail. The vortex cores at the edge of the filament have also begun to break down.

Circulation is plotted along the length of the core line at each time (Fig. 14b). The circulations at the two endpoints of the vortex core line are highest at $t = 0.45$ s and drop by approximately 2 cm$^2$/s in each time interval. The circulation calculated where the vortex is still attached to the caudal fin is much lower. At $t = 0.65$ s, the circulation near the caudal fin (approximately 3 cm along the core line) is much higher and comparable with the circulation at the end points, but never reaches the maximum strength seen at $t = 0.45$ s. The caudal fin kinematics show a small oscillation of the tail as the fish emerges from the turn. Vorticity shed during this tailbeat and interactions with the primary turning vortex may be responsible for the local maxima in

Fig. 12 Wake during turn execution and after completion as visualized using SAPIV during a 75° turn executed over 0.3 s. The turn is initialized at $t = 0$ s. Simultaneous slices through the caudal fin and below the fish body show the vorticity forming along the body and the wide thrust jet left behind in the wake. The red arrows indicate the velocity and position of the dorsal fin base at the beginning ($t = 0$ s) and end ($t = 0.3$ s) of the observed turn; these velocity vectors are taken to be the entrance and exit velocities, respectively. The solid orange traces show the path of the two caudal fin tips.
circulation seen at halfway along the core line at \( t = 0.65 \) and 0.85 s. Sets of 2D PIV measurements obtained in either plane would present very different pictures of the vortex dynamics after the turn. SAPIV is able to capture that at different points along its length the vortex filament is simultaneously growing and decaying.

The projected area of the vortex core line in the direction of the axial jet has a maximum value of 3.4 cm\(^2\), over 25% greater than the enclosed area of a ring with the diameter estimated in the \( X-Y \) plane (2.5 cm\(^2\)). Given the truncation of the vortex filament, the actual enclosed area will be even larger. Table 2 summarizes the impulse at each timestep as estimated from the median, maximum, and minimum circulation. The lower bound is not calculated for \( t = 0.45 \) s when the ring is still attached to the body, and the lower circulation where the vortex is attached to the caudal fin does not actually represent the vortex filament itself. The bounds on the circulation at each time show significant range and highlight the very different understandings of the wake that can occur if circulation is only taken at a single slice location.

4 Conclusions

Synthetic aperture PIV provides an excellent tool for analysis of unsteady 3D biological flows. Use of automatic masking algorithms to track the fish in arbitrary positions throughout the measurement volume and rapid reconstruction algorithms make SAPIV a very attractive technique. The weighted cost function method presented herein, combined with additive refocusing, yields superior results when lighting is uneven and particles are occluded by a swimming body. SAPIV is readily scalable to even more complex maneuvers, such as jumping and rapid escape responses, where higher time resolution is necessary. Replacing the 30 Hz camera array with high-speed cameras, capable of 2000 Hz or greater frame rates, is a natural step-up in terms of 3D imaging with full time resolution. Better temporal resolution can be leveraged to further explore the process by which vortex rings form and link the energy balance between the fish and the wake in these scenarios. With an appropriate mapping function, synthetic aperture refocusing can ultimately be performed in any reference frame. Iterative refocusing procedures to first determine the high-level swimming kinematics of the fish followed by detailed refocusing in a fish-centric reference frame would remove many of the coordinate transformations and interpolations implemented herein. Iterative refocusing of multiple visual hulls, each with a different blur pattern, and detection of the union between body reconstructions may also be useful to reduce the shadowing in the body masks observed using a single visual hull.

Once volumetric PIV fields are generated, it is possible to expand conventional planar PIV analysis methods for fish wakes, such as the calculation of circulation and impulse, to utilize the additional information that 3D PIV provides. These techniques have been applied over multiple slices through the ring to characterize the geometry and symmetry of these volumetric features and assess the assumptions made during planar analysis. Through detailed 3D PIV measurements behind swimming fish, it is clear that analysis of simple 2D planes behind a swimming fish may be inadequate to fully characterize the propulsive performance of the animal. While on the basis of geometry, an axisymmetric vortex ring model is appropriate for analyzing the dynamics of fish wakes during forward swimming,

![Figure 13](image-url)
and the circulation used in this model must be determined such that it is representative of the entire ring.

Linked rings present a unique challenge in analysis where the merging of vortex cores yields higher circulation levels when viewed only in a 2D slice, even in the presence of a classic reverse Kármán street, than if analyzed holistically. Impulse calculated using the circulation of the linked core does not describe thrust in the direction of either jet specifically and is not characteristic of the circulation in the rest of the vortex ring, such that the acting impulse of an individual ring may be overestimated. The ability to segment individual contributions to wake momentum is significant in characterizing the stroke-to-stroke variation in observed steady swimming or in assessing the contributions of multiple propulsors to overall thrust. In cases with linked vortex rings, analysis of each individual vortex ring is insufficient to describe the evolution of the linked vortex cores. An advantage of volumetric PIV techniques such as SAPIV is that the dynamics of individual wake structures and their interactions can be resolved simultaneously.

Using 2D techniques to analyze 3D flow fields requires thoughtful consideration of the plane in which calculations are made and the geometry of the structures being analyzed. Three-dimensional PIV measurements also provide

![Fig. 14](image)

**Table 2** Impulse estimated from the median and maximum circulations of the vortex filament at three instances in time

| t (s) | $\Gamma_{\text{med}}$ (cm$^2$/s) | $\Gamma_{\text{max}}$ (cm$^2$/s) | $\Gamma_{\text{min}}$ (cm$^2$/s) | $A$ (cm$^2$) | $I_{\text{med}}$ (g cm/s) | $I_{\text{max}}$ (g cm/s) | $I_{\text{min}}$ (g cm/s) |
|------|----------------|----------------|----------------|-------------|----------------|----------------|----------------|
| 0.45 | 10.0           | 14.2           | n/a            | 1.8         | 18.3           | 25.6           | n/a            |
| 0.65 | 10.9           | 12.9           | 7.0            | 3.4         | 36.7           | 43.9           | 23.8           |
| 0.85 | 8.9            | 12.0           | 6.4            | 3.6         | 32.1           | 43.2           | 23.1           |

Upper bound and lower bounds on the impulse are also set by the maximum circulation value determined along the core line. A lower bound is not provided at $t = 0.45$ s because the vortex filament is still attached to the body, and the lower bound is clearly not representative of the vortex filament.
a sense of the uncertainty created by 2D PIV analysis of wake dynamics on arbitrary 2D slice locations through the vortex ring. In highly asymmetric behaviors, such as turning, a planar slice cannot fully characterize the wake geometry and the temporal and spatial trends in circulation. Further potential for volumetric PIV exists in generating analysis models that exploit this 3D data, such as direct integration of impulse along contours of arbitrary shape.

Acknowledgments The authors would like to thank Juliana Wu for her assistance with fish handling and experiments. The authors also acknowledge Barry Scharfman and Abhishek Bajpayee for discussions on object and particle reconstruction using synthetic aperture imaging.

References

Adhikari D, Longmire EK (2012) Visual hull method for tomographic PIV measurement of flow around moving objects. Exp Fluids 53(4):943–964
Adhikari D, Longmire E (2013) Infrared tomographic PIV and 3D motion tracking system applied to aquatic predator-prey interaction. Meas Sci Technol 24(2):024011
Anderson J (1996) Vorticity control for efficient propulsion. Ph.D. thesis, Massachusetts Institute of Technology
Barrett DS, Triantafyllou MS, Yue DKP, Ma Grosenbaugh, Wolfgang MU (1999) Drag reduction in fish-like locomotion. J Fluid Mech 392(1):183–212
Belden JL (2011) Synthetic aperture imaging for three dimensional resolution of fluid flows. Ph.D. thesis, Massachusetts Institute of Technology
Belden J, Truscott TT, Axiak MC, Techet AH (2010) Three-dimensional synthetic aperture particle image velocimetry. Meas Sci Technol 21(12):125403
Belden J, Ravela S, Truscott TT, Techet AH (2012) Three-dimensional bubble field resolution using synthetic aperture imaging: application to a plunging jet. Exp Fluids 53(3):839–861
Borazjani I, Sotiropoulos F (2008) Numerical investigation of the hydrodynamics of carangiform swimming in the transitional and inertial flow regimes. J Exp Biol 211(10):1541–1558
Dabiri JO (2005) On the estimation of swimming and flying forces from wake measurements. J Exp Biol 208(18):3519–32
Druce R, Lauder G (1999) Locomotor forces on a swimming fish: three-dimensional vortex wake dynamics quantified using digital particle image velocimetry. J Exp Biol 202(18):2393–2412
Elsinga G, Scaroa F, Wienke B, van Oudheusden B (2006) Tomographic particle image velocimetry. Exp Fluids 41(6):933–947
Epps BP, Techet AH (2007) Impulse generated during unsteady maneuvering of swimming fish. Exp Fluids 43(5):691–700
Epps BP, Valdivia y Alvarado P, Youcef-Toumi K, Techet AH (2009) Swimming performance of a biomimetic compliant fish-like robot. Exp Fluids 47(6):927–939
Fish F, Lauder G, Mittal R, Techet A, Triantafyllou M, Walker J, Webb P (2003) Conceptual design for the construction of a biorobotic AUV based on biological hydrodynamics. In: International symposium on unmanned untethered submersible technology. New Hampshire, USA
Flammang B, Lauder G, Troolin D, Strand T (2011a) Volumetric imaging of fish locomotion. Biol Lett 7(5):695–698
Flammang BE, Lauder GV, Troolin DR, Strand T (2011b) Volumetric imaging of shark tail hydrodynamics reveals a three-dimensional dual-ring vortex wake structure. Proc R Soc B 278(1725):3670–3678
Ghaemi S, Scarano F (2010) Multi-pass light amplification for tomographic particle image velocimetry applications. Meas Sci Technol 21(12):127002
Hunt JC, Wray A, Moin P (1988) Eddies, streams, and convergence zones in turbulent flows. Cent Turb Rep CTR 88:193–208
Im S, Jeon YJ, Sung HJ (2015) Tomo-PIV measurement of flow around an arbitrarily moving body with surface reconstruction. Exp Fluids 56(2):1–16
Jeon YJ, Sung HJ (2012) Three-dimensional PIV measurement of flow around an arbitrarily moving body. Exp Fluids 53(4):1057–1071
Kitzhofer J, Nunn T, Brücker C (2011) Generation and visualization of volumetric PIV data fields. Exp Fluids 51:1471–1492
Langley KR, Hardsester E, Thomson SL, Truscott TT (2014) Three-dimensional flow measurements on flapping wings using synthetic aperture PIV. Exp Fluids 55(10):1–16
Lauder GV (2009) Swimming hydrodynamics: ten questions and the technical approaches needed to resolve them. Exp Fluids 51(1):23–35
Lauder GV, Madden PGA (2006) Learning from fish: kinematics and experimental hydrodynamics for roboticists. Int J Autom Control 3(4):325–335
Luff J, Drouillard T, Rompage A, Linne M, Hertzberg J (1999) Experimental uncertainties associated with particle image velocimetry (PIV) based vorticity algorithms. Exp Fluids 26(1–2):36–54
McCutchon D (1977) Froude propulsive efficiency of a small fish, measured by wake visualization. In: Pedley T (ed) Scale effects in animal locomotion. Academic Press, London, pp 339–363
Müller U, Heuvel B, Stammhuis E, Videler J (1997) Fish foot prints: morphology and energetics of the wake behind a continuously swimming mullet (Chelon labrosus Risso). J Exp Biol 200(22):2893–2906
Müller U, Stammhuis E, Videler J (2000) Hydrodynamics of unsteady fish swimming and the effects of body size: comparing the flow fields of fish larvae and adults. J Exp Biol 203(2):193–206
Murphy D, Webster D, Yen J (2012) A high-speed tomographic PIV system for measuring zooplanktonic flow. Limnol Oceanogr Methods 10:1096–1112
Murphy DW, Webster DR, Yen J (2013) The hydrodynamics of hovering in antarctic krill. Limnol Oceanogr Fluids Environ 3(1):240–255
Peng J, Dabiri JO, Madden PG, Lauder GV (2007) Non-invasive measurement of instantaneous forces during aquatic locomotion: a case study of the bluegill sunfish pectoral fin. J Exp Biol 210(4):685–698
Raffel M, Willert C, Willert CE, Kompenhans S (1998) Particle image velocimetry. Springer, Berlin
Sakakibara J, Nakagawa M, Yoshida M (2004) Stereo-PIV study of flow around a maneuvering fish. Exp Fluids 36(2):282–293
Scarnano H, Longmire EK (2013) Tomographic PIV: principles and practice. Meas Sci Technol 24(1):012001
Scharfman B, Hart D, Techet A (2013) Light field imaging of turbulent liquid sheet breakup in air. In: 10th international symposium on particle image velocimetry, Delft, The Netherlands
Siddiqui MHK (2007) Velocity measurements around a freely swimming fish using PIV. Meas Sci Technol 18(1):96–105
Spedding G, Rayner J, Pennycuick C (1984) Momentum and energy in animal locomotion. Academic Press, London, pp 339–363
Steinberg J, Wray A, Moin P (1988) Eddies, streams, and convergence zones in turbulent flows. Cent Turb Rep CTR 88:193–208
Stamhuis E, Videler J, van Duren L, Müller U (2002) Applying digital particle image velocimetry to animal-generated flows: traps, hurdles and cures in mapping steady and unsteady flows in rotating, aplanar slice cannot fully characterize the wake geometry and the temporal and spatial trends in circulation. Further potential for volumetric PIV exists in generating analysis models that exploit this 3D data, such as direct integration of impulse along contours of arbitrary shape.
Strawn RC, Kenwright DN, Ahmad J (1999) Computer visualization of vortex wake systems. AIAA J 37(4):511–512
Sveen J (2004) An introduction to matpiv v. 1.6.1. eprint no. 2, ISSN 0809–4403, Department of Mathematics, University of Oslo. http://www.math.uio.no/~jks/matpiv
Taylor J (1997) Introduction to error analysis, the study of uncertainties in physical measurements. University Science Books, Sausalito
Triantafyllou G, Triantafyllou M, Grosenbaugh M (1993) Optimal thrust development in oscillating foils with application to fish propulsion. J Fluid Struct 7(2):205–224
Tytell ED (2006) Median fin function in bluegill sunfish Lepomis macrochirus: streamwise vortex structure during steady swimming. J Exp Biol 209(8):1516–1534
Tytell ED, Standen EM, Lauder GV (2008) Escaping Flatland: three-dimensional kinematics and hydrodynamics of median fins in fishes. J Exp Biol 211(2):187–195
Wolfgang M, Anderson J, Grosenbaugh M, Yue D, Triantafyllou M (1999) Near-body flow dynamics in swimming fish. J Exp Biol 202(17):2303–2327
Zhu Q, Wolfgang MJ, Yue DKP, Triantafyllou MS (2002) Three-dimensional flow structures and vorticity control in fish-like swimming. J Fluid Mech 468:1–28