Power Dependent Resonant Frequency of a Microwave Cavity Due to Magnetic Levitation

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Abstract—The fact that a magnet levitates above a superconductor is a fascinating consequence of the interaction between electromagnetic fields and the macroscopic quantum state of the superconductor. Such field-matter interactions, which include the Meissner and Josephson effects, combined with superconducting devices, such as low-loss superconducting microwave cavities, are emerging as key elements used in quantum computers. One technique that can be used to monitor the levitation of a magnet is to place the magnet within a superconducting microwave cavity and monitor the microwave resonance of that cavity. Here, we report measurements of the change in resonance frequency of the microwave cavity with the Meissner-levitated permanent magnet. The cavity resonance changes because the magnet perturbs the radio-frequency field inside the microwave cavity. The changes in resonant frequency and quality factor were measured as functions of input power and temperature. The observed power-dependent variations are explained in terms of the power dependence of the London penetration depth.

Index Terms—RF power, magnetic levitation, superconducting transition, quality factor, microwave cavity.

I. INTRODUCTION

LEVITATION of a magnet above a type-II superconductor is the most commonly observed example of superconducting magnetic levitation [1]. The magnetic flux is partially expelled and partially trapped within the superconductor below the superconducting critical temperature [2]. The trapped flux creates a frozen image inside the superconductor, holding the magnet at a fixed position [3], [4]. Superconducting levitation removes the need to mechanically hold the object, thereby reducing mechanical and thermal coupling between the levitated object and its environment [5]. This type of levitation has been used as a new mechanical transduction platform for the spin qubit in a diamond NV center by coupling the magnetic field to the qubit [6]. In the experiment [6], the levitation height of the micro-magnet is controlled, enabling adjustment of the coupling between the two systems. The main challenge in this experiment is to adjust the levitation height, which requires heating and cooling of the superconductor [7].

Magnetic levitation above a type-I superconductor has an advantage compared with the type-II case in that there is no loss due to flux creep [8], [9]. However, levitation height and magnet position control become difficult because of the absence of trapped flux [10]. In this paper, we report on the manipulation of the levitation height of a magnet levitated within a type I superconducting radio-frequency cavity. The levitation height is adjusted by adjusting the radio-frequency power injected into the cavity. Our measurements show changes in the cavity resonance frequency and quality factor as functions of the input power and temperature. Our setup could be useful to understand the quantum mechanical phenomenon of cavity-magnet coupled systems.

The resonance frequency of a coaxial quarter-wave cavity can be approximated in terms of capacitance and inductance as [11]:

\[
f = \frac{1}{2\pi \sqrt{LC}}. \tag{1}
\]

The capacitance term includes all the energy storing component of the cavity, whereas the inductance term include all the loss mechanism [11]. The total inductance (L) can be divided into the geometry (Lg) and kinematic (Lk) inductance [12]. The Lg value depends upon the shape of the cavity and is constant, whereas the Lk term is dependent upon the external magnetic field as \( L_k(B) \propto \lambda(B) \), where \( \lambda \) is the penetration depth [13]. According to the two-fluid model, the density of the cooper pair (n_s) changes as a function of temperature as: \( \frac{n_s}{n} = 1 - (\frac{T}{T_C})^4 \), where n is the total number of electrons, and \( T_C \) is the critical temperature of the superconductor [5]. The London penetration depth estimates the length up to which an external magnetic field penetrates [14]. It also depends on the density of the cooper pairs as: \( \lambda_L = \sqrt{\frac{m_e c^2}{4\pi n_e e^2}} \). Here, \( m_e \) is the bare electron mass in the vacuum, e is the electronic charge, and c is the velocity of the light. The higher power that is put inside the superconducting cavity could reduce the density of the cooper pairs [15]. This in turn increases the penetration depth [16]. Accordingly, we propose that the power dependence in our experimentally-measured resonance frequency and quality factor arises because increasing the radio-frequency power within the cavity changes \( \lambda_L \) thereby allowing the levitated magnet to change its height.
The cavity was fabricated from a single piece of bulk 6061 aluminum, a type-I superconductor of transition temperature, $T_c \sim 1.2$ K. The magnet is restricted on the stub by a snugly fitted plastic sleeve. The cavity-magnet system is fixed on the base plate of the dilution refrigerator. The fridge is cool-down in the automated way to its base temperature. In our experiment, the fridge is heated up and cooled down in a controlled way to the desired temperature and cooling rate. We have used a heater that is set in the mixing chamber to control the temperature. The temperature is adjusted by circulation of the He$_3$:He$_4$ mixture. For the data acquisition, a vector network analyzer (VNA) HP8720 is used. Full 2-port calibration is performed before each set of measurements using a special calibration kit.

The electric field of the cavity is localized around the edge of the stub ($1.5 \text{ mm} < x < 2 \text{ mm}$). Moreover, the electromagnetic field of the cavity decays exponentially ($e^{-\beta z}$) from the stub to the open end of the cavity, where $\beta$ is the propagation constant and $z$ is the vertical distance from the stub [18]. The high field concentration at a small area is extremely sensitive to the external perturbation. Such interaction results into a large change in the resonance frequency of the cavity [19]. In this work, a magnet is levitated from the edge of the stub. Here, the amount of interaction between the magnet and localized field determines the amount of frequency shift.

To better understand the frequency shift, finite-element calculations in COMSOL 5.5 were done using Electromagnetic, frequency domain (emw) physics. In the simulations, the eigenfrequency and the associated eigenmode in the resonant cavity are obtained by solving wave equation, $\mu_0 \Delta \times (\Delta \times \vec{E}) = \mu_r k_0^2 (\epsilon_r - \frac{\partial^2}{\partial x^2}) \vec{E}$. Here, $\mu_r$ is the relative permeability, $\epsilon_r$ is the relative permittivity, $k_0 = \frac{2\pi}{\lambda}$ is the wavenumber, and $\sigma$ is the conductivity of the material. The resonant frequency of a coaxial quarter-wave cavity is determined by the length of the stub of the cavity [18]. They are inversely related as $f \propto \frac{1}{l}$, where $f$ is the frequency, $c$ is the velocity of light, and $l$ is the length of the stub. When a magnet changes its position on the stub, it changes the effective length of the stub. Hence, it changes the frequency of the cavity. The change in the frequency with respect to the position of the magnet is shown in Fig. 1(c). The difference in frequency is calculated from the bare cavity frequency. In the simulations, the magnet is manually shifted from the center ($x, z = 0 \text{ mm}, 0 \text{ mm}$) to the edge ($x, z = 0 \text{ mm}, 2 \text{ mm}$) of the stub. The same procedure is repeated to the distance of 2 mm above the stub. The region enclosed by the black lines is the region of our interest. We expect magnetic levitation in this region [20]. There are two unique frequency shift patterns due to the radial and vertical displacements of the magnet. When the magnet moves radially, its interaction with the concentrated electric field increases. This type of magnetic displacement cause downshift in the frequency. However, the trend of the frequency shift is opposite for magnetic levitation. The lift of the magnet induces upward shift in the frequency. In both the cases, the amount of frequency change is highest at the edge of the stub [21].

### III. Cryogenic Measurements

#### A. Effect of the Input Power

The cavity with a magnet resting on the stub is first let automated cooldown to its base temperature ($\sim 135$ mK). Then, it is slowly warmed up to 1 K using the heater attached to the mixing chamber. Same procedure is repeated for different input powers in test. The change in frequency as a function of temperature at different input power level is shown in Fig. 2(a). The frequency shows the sudden jumps at higher temperature indicating the large change in penetration depth due to the change in effective
inductance of the cavity-magnet system. The largest downshift is due to the phase transition of the superconductor into the normal state [22]. Fig. 2(a) shows the change in resonance frequency as a function of temperature at different power level. The three low powers (−15 dBm, −5 dBm, and 0 dBm) caused the phase transition around 800 mK. However, the significant change in the transition temperature is seen at the input power of 5 dBm. The 5 dBm power induced the phase transition at lower temperature. A detail study of the effect of the input power on the resonance frequency of the cavity shown in Fig. 2(b). The temperature of the cavity was held fixed and the power was ramped up from −15 dBm to 5 dBm.

**B. Quality Factor as a Function of Power**
We have seen significant effect of the input power on the resonance frequency of the cavity at 757 mK and 785 mK. To further investigate this effect, quality factor as a function of power was measured. The total quality factor of the cavity quantifies the total amount of energy that the cavity can store compared to the total losses in one radio frequency (RF) cycle [23]. Fig. 3 shows the total Q of the cavity-magnet system as a function of the power at 757 mK and 785 mK. The power is ramped up from −15 dBm to 5 dBm and is ramped back down to −15 dBm. The data acquisition is done every minute. At 757 mK, improvement in the Q is seen during the increase of power [24]. The trend of the Q was sustained during the power ramp-down as well. However, at 785 mK a sharp drop in Q is observed around −2.5 dBm. There was no recovery of Q after that. We conclude the superconductor is robust enough against the external magnetic field at 757 mK. However, the input power has a destructive effect on Q at 785 mK. The hysteresis in Q likely due to the trapped magnetic field during the power ramp down.

**C. Power Variation at 757 Mk**
As shown above, the cavity-magnet system shows its robustness at 757 mK. At this temperature, we have measured the change in frequency at different power level. The data were recorded for 5 minutes of duration. This allows us to thermally stabilized the cavity before the measurement were performed. As shown in Fig. 4(a), the large change in frequency was observed for input power ≥ 5 dBm. The second set of measurement were done with switching the power from high to low and repeating the measurements at different power level as shown in Fig. 4(b). The switching feature in frequency is likely due to the robustness of the cavity-magnet system. It is to be noted that this switching in frequency was not observed at 785 mK, where the change in quality factor was observed due to the power switching as shown in Fig. 3.
We have probed the resonance frequency of a superconducting quarter-wave cavity containing a levitated magnet as functions of the temperature and the input microwave power. We find in our experiments that the levitation height of the magnet shifts downward suddenly when the microwave power is increased beyond a certain value that depends on cavity temperature (e.g., 5 dBm at 757 mK). This sudden shift is reversible on the 1-minute timescale by reducing the microwave cavity power to −5 dBm while keeping the refrigerator temperature constant.

These added methods of levitated object control could be useful for quantum information applications of cavity-magnet coupled systems.

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