Branching Ratios and $CP$ Asymmetries of $B \to a_1(1260) \pi$ and $a_1(1260) K$ Decays

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Abstract

We present the studies of the decays $B \to a_1(1260) \pi$ and $a_1(1260) K$ within the framework of QCD factorization. Due to the G-parity, unlike the vector meson, the chiral-odd two-parton light-cone distribution amplitudes of the $a_1$ are antisymmetric under the exchange of quark and anti-quark momentum fractions in the SU(2) limit. The branching ratios for $a_1 \pi$ modes are sensitive to tree–penguin interference. The resultant $B(B^0 \to a_1^\pm \pi^\mp)$ are in good agreement with the data. However, using the current Cabibbo–Kobayashi–Maskawa angles, $\beta = 22.0^\circ$ and $\gamma = 59.0^\circ$, our results for the mixing-induced parameter $S$ and $\alpha_{\text{eff}}$ differ from the measurements of the time-dependent CP asymmetries in the decay $B^0 \to a_1^\pm \pi^\mp$ at about the 3.7$\sigma$ level. This puzzle may be resolved by using a larger $\gamma \gtrsim 80^\circ$. For $a_1 K$ modes, the annihilation topologies give sizable contributions and are sensitive to the first Gegenbauer moment of the leading-twist tensor (chiral-odd) distribution amplitude of the $a_1$ meson. The $B \to a_1 K$ amplitudes resemble the corresponding $B \to \pi K$ ones very much. Taking the ratios of corresponding CP-averaged $a_1 K$ and $\pi K$ branching ratios, we can extract information relevant to the electroweak penguins and annihilations. The existence of new-physics in the electroweak penguin sector and final state interactions during decays can thus be explored.
I. INTRODUCTION

The first charmless hadronic $B$ decay involving a $1^{3}P_{1}$ axial-vector meson that has been observed is $B^0 \to a_1^+(1260)\pi^\mp$ [1, 2, 3, 4, 5], which goes through $b \to u\bar{u}d$. The measurements of time-dependent CP asymmetries in hadronic $B$ decays originating from $b \to u\bar{u}d$ can provide the information directly related to the Cabibbo-Kobayashi-Maskawa (CKM) weak phase $\alpha \equiv \arg(-V_{td}V_{tb}^*/V_{ud}V_{ub}^*)$ (or called $\phi_2$), for which some results have been given from the data of $B \to \pi^+\pi^-, \rho^\pm\pi^\mp$ and $\rho^\pm\rho^\mp$ [6]. The BaBar collaboration recently reported the observation of $B^0 \to a_1^\pm(1260)\pi^\mp$, including CP-violating parameters, branching fractions, and $\alpha_{\text{eff}}$, where the bound on the difference $\Delta\alpha = \alpha - \alpha_{\text{eff}}$ can be constrained by using the broken SU(3) flavor symmetry [7, 8] or isospin analysis [9, 10, 11].

In this paper, we present the phenomenological studies of $B \to a_1\pi$ and $a_1K$ within the framework of QCD factorization, where the former processes are tree-dominated, while the latter are penguin-dominated. The $a_1(1260)$, which will be denoted by $a_1$ for simplicity, is the $1^{3}P_{1}$ state. Due to the G-parity, the chiral-even two-parton light-cone distribution amplitudes (LCDAs) of the $a_1$ are symmetric under the exchange of quark and anti-quark momentum fractions in the SU(2) limit, whereas, unlike the vector meson, the chiral-odd two-parton LCDAs are antisymmetric. Ref. [12] is the only literature so far for the calculation of LCDAs of $1^{3}P_{1}$ axial-vector mesons. The large first Gegenbauer moment of the leading-twist tensor distribution amplitude of the $a_1$ meson [12] could have a sizable impact on the annihilation amplitudes. On the other hand, it is interesting to note that, for the axial-vector mesons with quantum number $1^{1}P_{1}$, their chiral-even LCDAs are anti-symmetric under the exchange of quark and anti-quark momentum fractions in the SU(3) limit, while the chiral-odd two-parton LCDAs are symmetric [12, 13]. The hadronic $B$ decays involving such a meson are sensitive to the new-physics search [14, 15].

Because the axial-vector and pseudoscalar penguin contributions interfere constructively in the dominant decay amplitudes of $B \to a_1K$, for which the $K$ is emitted and $a_1$ shares the same spectator quark within the $B$ meson, the $B \to a_1K$ amplitudes resemble very much the corresponding $B \to \pi K$ ones. Moreover, larger CP asymmetries could be expected in the $a_0^0K^-$ and $a_0^0K^0$ modes due to the much larger decay constant of the $a_1(1260)$, as compared with $\pi K$ channels.

To resolve the puzzle about the observations of the decays $B \to \pi K$ and $\pi\pi$ within the Standard Model (SM) [6], some approaches were proposed, including considerations of final state interactions (FSIs) [16, 17, 18], and use of SU(3) flavor symmetry to extract hadronic parameters from the $\pi\pi$ data and then to predict $K\pi$ channels [19, 20, 21]. On the other hand, it was argued that new-physics with a large CP-violating phase may exist in the electroweak penguin sector [19, 20, 22]. The present studies for $B \to a_1\pi$ and $a_1K$ modes can offer further tests for the above theories.

The layout of the present paper is as follows. In Sec. II, we discuss light-cone distribution amplitudes for an axial-vector meson. A brief description for applying QCD factorization to the decays $B \to a_1\pi$ and $a_1K$ is given in Sec. III, where some relevant formulas are collected in Appendices A and B. In terms of the notations $\alpha_i^p$ and $\beta_i^p$, which were given in Ref. [23],
one can find that the amplitudes for $AP$ modes have the same expressions with those for $PP$ and $VP$ modes (where $A \equiv$ the axial-vector meson, $P \equiv$ the pseudoscalar meson, and $A \equiv$ the vector meson). Sec. IV contains the numerical analysis for the branching ratios and CP asymmetries. Our conclusions are summarized in Sec. V.

II. TWO-PARTON LCDAS OF THE $a_1$ AND PROJECTION OPERATORS ON THE LIGHT-CONE

For decays involving an axial-vector meson (denoted as $A$) in the final state, the QCD corrections can turn the local quark-antiquark operators into a series of nonlocal operators as

$$\langle A(P, \lambda) | \bar{q}_1(\lambda) q_2(y) |0\rangle = -\frac{i}{4} \int_0^1 du e^{i(u_y + \bar{y}x)} \left\{ f_{AM} \left( \frac{e^{(\lambda)}_y}{p_z} \Phi_{\parallel}(u) + \frac{e^{(\lambda)}_y}{p_z} \gamma_5 \Phi_{\perp}(u) \right) + \frac{1}{4} \epsilon_{\mu \rho \nu} \epsilon^{\ast}_{(\lambda)} p^\mu z^\sigma \frac{g^{(\ast)}_{\perp}(u)}{4} - \frac{1}{4} \epsilon_{\perp} \epsilon^{(\ast)}_{(\lambda)} z^\mu \frac{h^{(\ast)}_{\parallel}(u)}{2} \right\},$$

where the chiral-even LCDAs are given by

$$\langle A(P, \lambda) | \bar{q}_1(y) \gamma_5 q_2(x) |0\rangle = i f_{AM} \int_0^1 du e^{i(u_y + \bar{y}x)} \left\{ \epsilon_{\mu \rho \sigma} \epsilon^{(\ast)}_{(\lambda)} p^\mu z^\rho \frac{g^{(\ast)}_{\perp}(u)}{4} \right\},$$

$$\langle A(P, \lambda) | \bar{q}_1(y) \gamma_5 q_2(x) |0\rangle = -if_{AM} \epsilon_{\mu \rho \sigma} \epsilon^{(\ast)}_{(\lambda)} p^\mu z^\rho \int_0^1 du e^{i(u_y + \bar{y}x)} \frac{g^{(\ast)}_{\perp}(u)}{4},$$

with $u (\bar{u} = 1 - u)$ being the momentum fraction carried by $q_1(\bar{q}_2)$, and the chiral-odd LCDAs are given by

$$\langle A(P, \lambda) | \bar{q}_1(y) \gamma_5 q_2(x) |0\rangle = f_{AM} \int_0^1 du e^{i(u_y + \bar{y}x)} \left\{ \epsilon^{(\ast)}_{\perp \mu} p_\nu - \epsilon^{(\ast)}_{\perp \nu} p_\mu \right\} \Phi_{\parallel}(u),$$

$$\langle A(P, \lambda) | \bar{q}_1(y) \gamma_5 q_2(x) |0\rangle = f_{AM} \epsilon^{(\ast)}_{\perp \mu} \frac{m_A^2}{2} \frac{g^{(\ast)}_{\parallel}(u)}{2}.$$
respectively. The LCDAs $\Phi_\parallel, \Phi_\perp$ are of twist-2, and $g_\perp^{\perp}, g_\perp^{(a)}, h_\perp^{(t)}, h_\perp^{(p)}$ of twist-3. For the $a_1$ meson, due to G-parity, $\Phi_\parallel, g_\perp^{\perp}$ and $g_\perp^{(a)}$ are symmetric with the replacement of $u \leftrightarrow 1 - u$, whereas $\Phi_\perp, h_\perp^{(t)}$ and $h_\perp^{(p)}$ are antisymmetric in the SU(2) limit [12]. Here, we restrict ourselves to two-parton LCDAs with twist-3 accuracy.

Assuming that the axial-vector meson moves along the negative z-axis, the derivation for the light-cone projection operator of an axial-vector meson in the momentum space is in complete analogy to the case of the vector meson. We separate the longitudinal and transverse parts for the projection operator:

$$M_{\alpha A}^\perp = M_{\alpha \|}^A + M_{\alpha \perp}^A,$$

where only the longitudinal part is relevant in the present study and given by

$$M_{\parallel}^A = -\frac{i f_A}{4} \frac{m_A (e^* n_\perp)}{2} \not\! k \cdot \gamma_5 \Phi_\parallel(u) - \frac{i f_A}{4} \frac{m_A (e^* n_\perp)}{2 E} \left\{ \frac{i}{2} \sigma_{\mu \nu} \gamma_5 n_\perp \frac{\partial}{\partial k_{\perp \nu}} - \gamma_5 \frac{h_\perp^{(\mu)}(u)}{2} \right\} |_{k=\uparrow},$$

with the momentum of the quark $q_1$ in the $A$ meson being

$$k_\mu^\uparrow = u E n_\perp^\mu + k_\perp^\mu + \frac{k_\perp^2}{4uE} n_+^\mu,$$

for which $E$ is the energy of the axial-vector meson and the term proportional to $k_\perp^2$ is negligible. Here, for simplicity, we introduce two light-like vectors $n_\perp^\mu \equiv (1, 0, 0, -1)$, and $n_+^\mu \equiv (1, 0, 0, 1)$. In general, the QCD factorization amplitudes can be written in terms of the form $\int_0^1 du \text{Tr}(M_\parallel \cdots)$.

In the following, we will give a brief summary for LCDAs of the $a_1$ mesons, for which the detailed properties can be found in Ref. [12]. $\Phi_{\parallel \perp}^{a_1}(u)$ can be expanded in Gegenbauer polynomials:

$$\Phi_{\parallel \perp}^{a_1}(u) = 6 u \bar{u} \sum_{i=0}^{\infty} a_i^{(\perp),a_1} C_i^{3/2} (2u - 1).$$

For the $\Phi_{\parallel \perp}^{a_1}(u)$, due to the G-parity, only terms with even (odd) Gegenbauer moments survive in the SU(2) limit. In the present work, we consider the approximations:

$$\Phi_{\parallel}^{a_1}(u) = 6 u \bar{u} \left\{ 1 + a_2^{a_1} \frac{3}{2} \left[ 5(2u - 1)^2 - 1 \right] \right\},$$

$$\Phi_{\perp}^{a_1}(u) = 18 a_1^{a_1} u \bar{u}(2u - 1).$$

Note that we have defined $f_{a_1}^\perp = f_{a_1}$ since the product $f_{a_1}^{a_1} a_1^{a_1} a_1$ always appears together. Neglecting the three-parton distributions and terms proportional to the light quark masses, we can relate the twist-3 distribution amplitudes to the twist-2 ones by Wandzura-Wilczek
relations \[12, 24\] and then obtain:

\[
\begin{align*}
  h_{\|}^{(t)}(v) &= (2v - 1) \left[ \int_0^v \frac{\Phi_{\perp}(u)}{u} du - \int_v^1 \frac{\Phi_{\perp}(u)}{u} du \right] \equiv (2v - 1) \Phi_{\sigma}(v), \\
  h_{\|}^{(p)}(v) &= -2 \left[ \int_0^v \frac{\Phi_{\perp}(u)}{u} du - \int_v^1 \frac{\Phi_{\perp}(u)}{u} du \right] \equiv -2 \Phi_{\sigma}(v), \\
  \int_0^v du (\Phi_{\perp}(u) - h_{\|}^{(t)}(u)) &= v \bar{v} \left[ \int_0^v \frac{\Phi_{\perp}(u)}{u} du - \int_v^1 \frac{\Phi_{\perp}(u)}{u} du \right] \equiv v \bar{v} \Phi_{\sigma}(v). \quad (2.13)
\end{align*}
\]

The normalization conditions for LCDAs are

\[
\begin{align*}
  \int_0^1 du \Phi_{\|}(u) &= 1, \quad \int_0^1 du \Phi_{\perp}(u) = 0, \quad (2.14) \\
  \int_0^1 du h_{\|}^{(t)}(u) &= 0, \quad \int_0^1 du h_{\|}^{(p)}(u) = 0. \quad (2.15)
\end{align*}
\]

For the pseudoscalar meson \((P)\) with the four-momentum \(P_\mu\), the light-cone projection operator in the momentum space reads

\[
M_P = \frac{i f_P}{p_\perp} \gamma_5 \Phi_P(u) + \frac{i f_P \mu_P}{4} \left\{ \frac{i}{2} \sigma_{\mu\nu} \gamma_5 n^\mu n^\nu \frac{\phi'_\sigma(u)}{6} - \frac{\phi_\sigma(u)}{6} \sigma_{\mu\nu} \gamma_5 n^\mu \frac{\partial}{\partial k_{\perp\nu}} - \gamma_5 \frac{\phi_\sigma(u)}{2} \right\}_{k=up}, \quad (2.16)
\]

where \(\mu_P = m_2^2 / (m_1 + m_2)\) is proportional to the chiral condensate (with \(m_{1,2}\) the masses of quarks) and the approximate forms of LCDAs that we use are

\[
\begin{align*}
  \Phi_P(u) &= 6u\bar{u} \left\{ 1 + 3a_P(2u - 1) + a_P^2 \frac{3}{2} \left[ 5(2u - 1)^2 - 1 \right] \right\}, \\
  \Phi_p(u) &= 1, \quad \frac{\Phi_\sigma(u)}{6} = u(1 - u). \quad (2.17)
\end{align*}
\]

### III. DECAY AMPUTERES

Within the framework of QCD factorization, in general the effective weak Hamiltonian matrix elements for \(\overline{B} \to M_1 M_2\) decays can be expressed in the form \[23\]

\[
\langle M_1 M_2 | H_{\text{eff}} | \overline{B} \rangle = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \langle M_1 M_2 | T_A^p + T_B^p | \overline{B} \rangle, \quad (3.1)
\]

where \(\lambda_p \equiv V_{pb} V_{pq}^*\) with \(q \equiv d\) or \(s\), \(M_2\) is the emitted meson, and \(M_1\) shares the same spectator quark within the \(\overline{B}\) meson. Considering a generic \(b\)-quark decay, \(T_A^p\) describe contributions from naive factorization, vertex corrections, penguin contractions and spectator scattering, whereas \(T_B^p\) contain the weak annihilation topologies.
For $\overline{B}$ decay processes, the QCD factorization approach advocated in \cite{25, 26} allows us to compute the nonfactorizable corrections in the heavy quark limit since only hard interactions between the $(\overline{B}M_1)$ system and $M_2$ survive in the $m_b \to \infty$ limit. Naive factorization is recovered in the heavy quark limit and to the zeroth order of QCD corrections. In this approach, the LCDAs play an essential role. In the present study using the notations $\alpha_i^p$ and $\beta_i^p$ given in Ref. \cite{23}, the amplitudes for $AP$ modes have the same expressions with those for $PP$ and $VP$ modes; $\overline{B} \to a_1\pi$, $a_1\overline{K}$ decay amplitudes in terms of $\alpha_i^p$ and $\beta_i^p$ can be obtained from $\overline{B} \to \rho\pi$, $\rho\overline{K}$ \cite{23} by setting $\rho \to a_1$. However, one should note that the determination of the relative signs of the detailed amplitudes behind the coefficients $\alpha_i^p$ and $\beta_i^p$ is non-trivial.

**A. Decay amplitudes due to $T_A^p$**

In general, $T_A^p$ can be expressed in terms of $c \alpha_i^p(M_1M_2) X^{(\overline{B}M_1,M_2)}$, where $c$ contains factors of $\pm 1$ and $\pm 1/\sqrt{2}$ arising from flavor structures of final-state mesons, $\alpha_i$ are functions of the Wilson coefficients (see Eq. (3.7)), and

$$X^{(B.A.P)} = \langle P(p)|(V - A)_{\mu}|0\rangle\langle A(p)|(V - A)^{\mu}|\overline{B}(p_B)\rangle = -2if_{P}m_{A}V^{BA}_{0}(q^{2})(e^{*}_{\lambda})_{pB},$$

$$X^{(\overline{B}.A)} = \langle A(q)|(V - A)_{\mu}|0\rangle\langle P(p)|(V - A)^{\mu}|\overline{B}(p_B)\rangle = -2if_{A}m_{A}F^{BP}_{1}(q^{2})(e^{*}_{\lambda})_{pB}.$$  

Here the decay constants of the pseudoscalar meson $P$ and the axial-vector meson $A$ are defined by \cite{27}

$$\langle P(p)|\overline{q}\gamma_{\mu}\gamma_{5}q|0\rangle = -if_{P}p_{\mu}, \quad \langle A(p,\lambda)|\overline{q}\gamma_{\mu}\gamma_{5}q|0\rangle = if_{A}e^{(\lambda)*}. \quad (3.4)$$

The form factors for the $B \to A$ and $P$ transitions are defined as \cite{27}

$$\langle A(p,\lambda)|A_{\mu}|\overline{B}(p_B)\rangle = i\frac{2}{m_{B} + m_{A}}e_{\nu\alpha\beta\epsilon^{*}_{\lambda}}p^{\beta}_{pB}(V^{BA}_{A}(q^{2})),$$

$$\langle A(p,\lambda)|V_{\mu}|\overline{B}(p_B)\rangle = -\left[(m_{B} + m_{A})e^{*(\lambda)*}V_{1}^{BA}(q^{2}) - (\epsilon^{(\lambda)*}_{\lambda})_{pB}(p_B + p)_{\mu}\frac{V_{2}^{BA}(q^{2})}{m_{B} + m_{A}}\right]$$

$$+2m_{A}\frac{e^{(\lambda)*}_{\lambda}}{q^{2}}p_{B}\epsilon_{\mu} V_{3}^{BA}(q^{2}) - V_{0}^{BA}(q^{2})\right],$$

$$\langle P(p)|V_{\mu}|\overline{B}(p_B)\rangle = \left[(p_B + p)_{\mu} - \frac{m_{B}^{2} - m_{P}^{2}}{q^{2}}q_{\mu}\right]F_{1}^{BP}(q^{2}) + \frac{m_{B}^{2} - m_{P}^{2}}{q^{2}}q_{\mu}F_{0}^{BP}(q^{2}), \quad (3.5)$$

where $q = p_{B} - p$, $V_{3}^{BA}(0) = V_{0}^{BA}(0)$, $F_{1}^{BP}(0) = F_{0}^{BP}(0)$ and

$$V_{3}^{BA}(q^{2}) = \frac{m_{B} + m_{A}}{2m_{A}}V_{1}^{BA}(q^{2}) - \frac{m_{B} - m_{A}}{2m_{A}}V_{2}^{BA}(q^{2}). \quad (3.6)$$
The coefficients of the flavor operators \( \alpha_i^p \) can be expressed in terms of \( \alpha_i^p \) as follows:

\[
\begin{align*}
\alpha_1(M_1M_2) &= a_1(M_1M_2), \\
\alpha_2(M_1M_2) &= a_2(M_1M_2), \\
\alpha_3^p(M_1M_2) &= a_3^p(M_1M_2) - a_5^p(M_1M_2), \\
\alpha_4^p(M_1M_2) &= \begin{cases} a_{10}^p(M_1M_2) + r_{\chi}^p a_6^p(M_1M_2) & \text{for } M_1M_2 = AP, \\ a_{10}^p(M_1M_2) - r_{\chi}^p a_6^p(M_1M_2) & \text{for } M_1M_2 = PA, \end{cases}
\end{align*}
\] (3.7)

\[
\begin{align*}
\alpha_{3,EW}^p(M_1M_2) &= a_6^p(M_1M_2) - a_7^p(M_1M_2), \\
\alpha_{4,EW}^p(M_1M_2) &= \begin{cases} a_{10}^{p,EW}(M_1M_2) + r_{\chi}^{p,EW} a_6^{p,EW}(M_1M_2) & \text{for } M_1M_2 = AP, \\ a_{10}^{p,EW}(M_1M_2) - r_{\chi}^{p,EW} a_6^{p,EW}(M_1M_2) & \text{for } M_1M_2 = PA, \end{cases}
\end{align*}
\]

where

\[
\begin{align*}
&\ r_{\chi}^p(\mu) = \frac{2m_p^2}{m_b(\mu)(m_2 + m_1)(\mu)}, \\
&\ r_{\chi}^A(\mu) = \frac{2m_A}{m_b(\mu)}.
\end{align*}
\] (3.8)

The effective parameters \( \alpha_i^p \) in Eq. (3.7) to next-to-leading order in \( \alpha_s \) can be expressed in forms of \[23\].

\[
a_i^p(M_1M_2) = \left( c_i + \frac{c_i \pm 1}{N_c} \right) N_i(M_2) \\
+ \frac{c_i \pm 1}{N_c} \frac{C_F \alpha_s}{4\pi} \left[ V_i(M_2) + \frac{4\pi^2}{N_c} H_i(M_1M_2) \right] + P_i^p(M_2),
\] (3.9)

where \( c_i \) are the Wilson coefficients, \( p = u, c \), \( C_F = (N_c^2 - 1)/(2N_c) \) with \( N_c = 3 \), the upper (lower) signs refer to odd (even) \( i \), \( M_2 \) is the emitted meson, \( M_1 \) shares the same spectator quark within the \( B \) meson, and

\[
N_i = \begin{cases} 0 & \text{for } i = 6, 8, \text{ and } M_2 = a_1, \\ 1 & \text{for the rest.} \end{cases}
\] (3.10)

\( V_i(M_2) \) account for vertex corrections, \( H_i(M_1M_2) \) for hard spectator interactions with a hard gluon exchange between the emitted meson and the spectator quark of the \( B \) meson and \( P_i(M_2) \) for penguin contractions. The detailed results for the above quantities are collected in Appendix A. Note that in the present case, some relative signs change in \( H_i \) as compared with the \( PP \) and \( VP \) modes.

### B. Decay amplitudes due to \( T_B^p \) — annihilation topologies

The \( \bar{B} \to AP \) amplitudes governed by the annihilation topologies read

\[
\frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \langle AP|T_B^p|\bar{B} \rangle = -i\frac{G_F}{\sqrt{2}} f_B f_A f_P \sum_{p=u,c} \lambda_p \left[ \sum_{i=1}^4 e_i b_i + e_3 b_{3,EW} + e_6 b_{4,EW} \right],
\] (3.11)
where the coefficients $e_i$ are process-dependent and weak annihilation contributions are parameterized as

\[ b_1 = \frac{C_F}{N_c^2}c_1 A_i^1, \quad b_3 = \frac{C_F}{N_c^2} \left[ c_3 A_i^1 + c_5 (A_3^1 + A_4^1) + N_c c_6 A_3^f \right], \]

\[ b_2 = \frac{C_F}{N_c^2}c_2 A_1^1, \quad b_4 = \frac{C_F}{N_c^2} \left[ c_4 A_i^1 + c_6 A_2^f \right], \]

\[ b_{3,EW} = \frac{C_F}{N_c^2} \left[ c_9 A_i^1 + c_7 (A_3^1 + A_4^1) + N_c c_8 A_3^i \right], \]

\[ b_{4,EW} = \frac{C_F}{N_c^2} \left[ c_7 A_i^1 + c_8 A_2^1 \right]. \]  

(3.12)

The subscripts 1, 2 and 3 of $A_{i,f}^n$ denote the annihilation amplitudes induced from $(V-A)(V-A)$, $(V-A)(V+A)$ and $(S-P)(S+P)$ operators, respectively, and the superscripts $i$ and $f$ refer to gluon emission from the initial and final-state quarks, respectively. For decays $B \to AP$, the detailed expressions for $A_{i,f}^n$ are given in Appendix B.

**IV. NUMERICAL RESULTS**

**A. Input parameters**

In the numerical analysis, we use the next-to-leading Wilson coefficients in the naive dimensional regularization (NDR) scheme [28]. The relevant parameters are summarized in Table I [29, 30, 31, 32, 33]. The value of $f_B$ that we use is consistent with the lattice average [34]. The current value of $F_{B\pi}(0)$ becomes a little smaller, and is more suitable to explain the $\pi\pi$ data [6]. We use the light-cone sum rule results for the $B \to \pi, K$ [31] and $B \to a_1$ [32] transition form factors, for which the momentum dependence is parametrized as [35]

\[ f(q^2) = f(0) \left( \frac{1}{1 - q^2/m_{B^*}^2} + \frac{r_{BZ(Y)q^2/m_{B^*}^2}}{1 - \alpha_{BZ(Y)q^2/m_{B^*}^2}} \right), \]  

(4.1)

where $m_{B^*}$ is the lowest-resonance in the corresponding channel. Note that since the mass of the $a_1$ meson is not small, we have, for instance, $[F_{B\pi}(m_{a_1}^2)/F_{B\pi}(0)]^2 \simeq 1.2$. It means that the $q^2$ dependence of $B \to \pi, K$ form factors cannot be ignored in the prediction. As for the $B \to a_1$ form factor, its $q^2$ dependence can be negligible due to the small mass of pseudoscalar mesons. However, to be consistency, I also consider its $q^2$ dependence in the analysis. Our light-cone sum rule result for $V_0^{Ba_1}(0)$ is a little larger than the previous QCD sum rule calculation, 0.23 $\pm$ 0.05 [36]. It is interesting to compare with other quark model calculations in the literature. The magnitude of $V_0^{Ba_1}(0)$ is about 0.13 and 1.02 $\sim$ 1.22 in the quark model calculations in Ref. [37] and Refs. [38, 39], respectively. The magnitude of the former is too small and the latter is too large if using them to compute the branching ratios.
of $B^0 \rightarrow a_1^\pm \pi^\mp$ and then comparing with the data. The values of the Gegenbauer moments of leading-twist LCDAs for the $a_1$ meson are quoted from Ref. [12]. The integral of the $B$ meson wave function is parameterized as [25]

$$\int_0^1 \frac{d\rho}{1-\rho} \Phi^B_1(\rho) \equiv \frac{m_B}{\lambda_B},$$  \hfill (4.2)

where $1-\rho$ is the momentum fraction carried by the light spectator quark in the $B$ meson. Here we use $\lambda_B(1 \text{ GeV}) = (350 \pm 100)$ MeV.

There are three independent renormalization scales for describing the decay amplitudes. The corresponding scale will be specified as follows: (i) the scale $\mu_v = m_b/2$ for loop diagrams contributing to the vertex and penguin contributions to the hard-scattering kernels, (ii) $\mu_H = \sqrt{\mu_v \Lambda_h}$ for hard spectator scattering, and (iii) $\mu_A = \sqrt{\mu_v \Lambda_h}$ for the annihilation with the hadronic scale $\Lambda_h \approx 500$ MeV. We follow [25] to parameterize the endpoint divergences $X_A \equiv \int_0^1 dx/\bar{x}$ and $X_H \equiv \int_0^1 dx/\bar{x}$ in the annihilation and hard-spectator diagrams, respectively, as

$$X_{A(H)} = \ln \left( \frac{m_B}{\Lambda_h} \right) (1 + \rho_{A(H)} e^{i\phi_{A(H)}}),$$  \hfill (4.3)

with the unknown real parameters $\rho_A, \rho_H$ and $\phi_A, \phi_H$. We adopt the moderate value $\rho_{A,H} \leq 0.5$ and arbitrary strong phases $\phi_{A,H}$ with $\rho_{A,H} = 0$ by default, i.e., we assign a 50\% uncertainty to the default value of $X_{A(H)}$ (with $\rho_{A,H} = 0$) [40, 41]; with the allowed ranges of $\rho_{A,H}$, the theoretical predictions for $\pi K$ modes are consistent with the data. Note that the $a_1 K$ rates could be sensitive to the magnitude of $\rho_A$.

### B. Results

We follow the standard convention for the direct CP asymmetry

$$A_{CP}(\bar{f}) \equiv \frac{\mathcal{B}(\bar{B} \rightarrow f) - \mathcal{B}(B^0 \rightarrow f)}{\mathcal{B}(\bar{B} \rightarrow f) + \mathcal{B}(B^0 \rightarrow f)}. \hfill (4.4)$$

The branching ratios given in the present paper are CP-averaged and simply denoted by $\mathcal{B}(\bar{B} \rightarrow f)$. The numerical results for CP-averaged branching ratios and direct CP asymmetries are summarized in Tables II and III respectively. The results for time-dependent CP parameters of the decay $B(t) \rightarrow a_1^\pm \pi^\mp$ are shown in Table IV.

1. $\bar{B} \rightarrow a_1 \pi$

The decay of the $B^0$ meson to $a_1^\pm \pi^\mp$ was recently measured by the BaBar and Belle groups [1, 2, 3, 4, 5]. A recent updated result by BaBar yields [4]

$$\mathcal{B}(B^0 \rightarrow a_1^\pm \pi^\mp \rightarrow \pi^\mp \pi^\pm \pi^\pm) = (16.6 \pm 1.9 \pm 1.5) \times 10^{-6}. \hfill (4.5)$$
Assuming that $B(a_1^+ \to \pi^+ \pi^0 \pi^\pm)$ equals to $B(a_1^+ \to \pi^+ \pi^0 \pi^0)$ and $B(a_1^- \to \pi^- \pi^0 \pi^\pm)$ equals to 100%, they have obtained

$$B(B^0 \to a_1^+ \pi^\pm) = (33.2 \pm 3.8 \pm 3.0) \times 10^{-6}. \quad (4.6)$$

Very recently, the measurements of time-dependent CP asymmetries in the decay $B^0 \to a_1^+ \pi^\pm$ have been reported by the BaBar collaboration [5]. From the measurements, the individual branching ratios of $\overline{B}^0 \to a_1^+ \pi^-$ and $a_1^- \pi^+$ can be obtained. As given in Table III our theoretical results are in good agreement with experiment. It was shown in Ref. [42] that tree(T)-penguin(P) interference depends on the sign of $\sin \gamma$ for $\Delta(a_1^- \pi^-)$ and $\Delta(a_1^+ \pi^0)$, while it is constructive in $B^- \to a_1^- \pi^0$. Because the amplitudes of $a_1 \pi$ and $\rho \pi$ modes are dominated by the terms with $\alpha_1$ and $\alpha_2^p$, and $\text{Re}[\alpha_1^p(\pi a_1)] \approx \text{Re}[\alpha_2^p(\pi a_1)] \approx -0.34$, one can easily obtain the following relations,

$$\frac{B(\overline{B}^0 \to a_1^- \pi^+)}{B(\overline{B}^0 \to a_1^+ \pi^-)} \approx \left( \frac{F_1^{B \pi}(m_{a_1}^2) f_{a_1}}{F_1^{B \pi}(m_{\rho}^2) f_{\rho}} \right)^2,$$

which can offer constraints on the magnitudes of $f_{a_1}$ and $V_{B1a1}(m_{a_1}^2)$. Moreover, the ratio $B(\overline{B}^0 \to a_1^- \pi^+)/B(\overline{B}^0 \to a_1^+ \pi^-)$ is

$$\frac{B(\overline{B}^0 \to a_1^- \pi^+)}{B(\overline{B}^0 \to a_1^+ \pi^-)} = \left( \frac{F_1^{B \pi}(m_{a_1}^2) f_{a_1}}{V_{B1a1}(m_{a_1}^2) f_{\pi}} \right)^2 \left\{ 1 + \text{Re} \left[ \frac{\lambda_t}{\lambda_u} \left( \frac{\alpha_4(\pi a_1) - \alpha_4(a_1 \pi) + \beta_3(\pi a_1) - \beta_3(a_1 \pi)}{\alpha_1(\pi a_1)} \right) \right] \right\} + O(\rho_{4,EW}, \beta^p_{4,3,EW}, \beta^p_{4,EW}), \quad (4.8)$$

which is not only sensitive to the form factor and decay constant of the $a_1$ meson but also to the weak phase $\gamma$. The measurement of the above ratio allow us to obtain the further constraint on the value of $\gamma$.

The large direct CP asymmetries may result from the non-zero value of the weak annihilation parameter ($\rho_4$) and its corresponding phase. See Table III. With default parameters, the direct CP asymmetries for $a_1^+ \pi^-, a_1^- \pi^+, a_1^- \pi^0, a_1^+ \pi^-$ are only at a few percent level, whereas
it can be very remarkable for the $a_1^0\pi^0$ mode. At the present time, the large errors in the measurements for CP asymmetries do not allow us to draw any particular conclusion in comparison with theoretical predictions. (See Tables III and IV.)

2. Time-dependent CP for $B(t) \rightarrow a_1^\pm \pi^\mp$

Following Ref. [8], we define

$$
A_+ \equiv A(B^0 \rightarrow a_1^+ \pi^-), \quad A_- \equiv A(B^0 \rightarrow a_1^- \pi^+),
$$

$$\overline{A}_+ \equiv A(B^0 \rightarrow a_1^- \pi^+), \quad \overline{A}_- \equiv A(B^0 \rightarrow a_1^+ \pi^-).$$

Neglecting CP violation in the $B^0 - \overline{B}^0$ mixing and the width difference in the two $B^0$ mass eigenstates, time-dependent decay rates for initially $B^0$ decaying into $a_1^\pm \pi^\mp$ can be parameterized by

$$
\Gamma(B^0(t) \rightarrow a_1^\pm \pi^\mp) = e^{-\Gamma t} \frac{1}{2} \left( |A_\pm|^2 + |\overline{A}_\mp|^2 \right) \times \left[ 1 + (C \pm \Delta C) \cos \Delta mt - (S \pm \Delta S) \sin \Delta mt \right],
$$

where

$$C \pm \Delta C \equiv \frac{|A_\pm|^2 - |\overline{A}_\mp|^2}{|A_\pm|^2 + |\overline{A}_\mp|^2}. $$
TABLE II: CP-averaged branching fractions for the decays $B \to a_1(1260)\pi$ and $a_1(1260)K$ (in units of $10^{-6}$). The theoretical errors correspond to the uncertainties due to variation of (i) Gegenbauer moments, decay constants, (ii) quark masses, form factors, and (iii) $\lambda_B, \rho_{A,H}, \phi_{A,H}$, respectively, added in quadrature.

| Mode         | Theory                                      | Expt. (BaBar) [4, 5] | Expt. (Belle) [3] |
|--------------|---------------------------------------------|----------------------|-------------------|
| $B^0 \to a_1^+\pi^-$ | $8.7^{+0.2+2.4+2.1}_{-0.2-2.0-1.3}$          |                      |                   |
| $B^+ \to a_1^-\pi^+$ | $25.1^{+2.5+6.5+2.6}_{-2.4-5.8-1.6}$          |                      |                   |
| $B^0 \to a_1^0\pi^+$ | $33.8^{+2.6+8.9+4.7}_{-2.6-7.8-2.9}$          | $21.0 \pm 5.4$      |                   |
| $B^0 \to a_1^0\pi^0$ | $0.7^{+0.1+0.2+0.7}_{-0.1-0.1-0.3}$          |                      |                   |
| $B^- \to a_1^-\pi^0$ | $14.9^{+1.9+3.7+2.4}_{-1.7-3.3-2.1}$          |                      |                   |
| $B^- \to a_1^0\pi^-$ | $7.3^{+0.3+1.7+1.3}_{-0.3-1.5-0.9}$          |                      |                   |

| Mode         | Theory                                      |                      |                   |
|--------------|---------------------------------------------|----------------------|-------------------|
| $B^0 \to a_1^+K^-$ | $15.1^{+1.2+12.7+21.2}_{-1.2-6.3-7.2}$          | $12.2 \pm 4.5$      |                   |
| $B^0 \to a_1^-\bar{K}^0$ | $6.0^{+0.4+5.6+9.7}_{-0.4-2.6-3.1}$          |                      |                   |
| $B^- \to a_1^-\bar{K}^0$ | $19.1^{+1.3+15.5+24.5}_{-1.3-7.8-11.0}$       |                      |                   |
| $B^- \to a_1^0\bar{K}^-$ | $11.8^{+1.0+8.7+13.1}_{-1.0-4.6-4.8}$         |                      |                   |

TABLE III: Direct CP asymmetries for the decays $B \to a_1(1260)\pi$ and $a_1(1260)K$ (in %). See Table II for errors.

| Mode         | Theory                                      |                      |                   |
|--------------|---------------------------------------------|----------------------|-------------------|
| $B^0 \to a_1^+\pi^-$ | $-3.2^{+0.1-0.3+20.1}_{-0.0-0.5-19.5}$          | $7 \pm 21 \pm 15$ |                   |
| $B^0 \to a_1^-\pi^+$ | $-1.7^{+0.1+0.1+13.6}_{-0.1-0.1-13.4}$          | $15 \pm 15 \pm 7$ |                   |
| $B^0 \to a_1^0\pi^0$ | $+9.3^{+5.4+6.9+25.0}_{-6.1-8.9-74.7}$          |                      |                   |
| $B^- \to a_1^-\pi^0$ | $-0.4^{+0.4-0.1-11.1}$                      | $-0.4 \pm 1.6 \pm 1.1$ |                   |
| $B^- \to a_1^0\pi^-$ | $-0.5^{+0.3+1.5+13.0}_{-0.3-2.4-14.6}$          |                      |                   |

and

$$S \pm \Delta S \equiv \frac{2\text{Im}(e^{-2i\beta}\overline{A}_{\pm}A_{\pm}^*)}{|A_{\pm}|^2 + |\overline{A}_{\pm}|^2}.$$  \hspace{1cm} (4.12)$$

Here $\Delta m$ denotes the neutral $B$ mass difference and $\Gamma$ is the average $B^0$ width. For an initial $B^0$ the signs of the $\cos \Delta mt$ and $\sin \Delta mt$ terms are reversed. The four decay modes define five asymmetries: $C, S, \Delta C, \Delta S$, and the overall CP violating $A_{CP}^{a_1\pi}$

$$A_{CP}^{a_1\pi} \equiv \frac{|A_+|^2 + |\overline{A}_-|^2 - |A_-|^2 - |\overline{A}_+|^2}{|A_+|^2 + |\overline{A}_-|^2 + |A_-|^2 + |\overline{A}_+|^2}.$$  \hspace{1cm} (4.13)$$

Two $\alpha$-related phases can be defined by

$$\alpha_{\text{eff}}^\pm \equiv \frac{1}{2}\arg(e^{-2i\beta}\overline{A}_{\pm}A_{\pm}^*),$$  \hspace{1cm} (4.14)$$
TABLE IV: Parameters of the time-dependent $B \to a_1^\mp \pi^\mp$ decay rate asymmetries. $S$ and $\Delta S$ are computed for $\beta = 22.0^\circ$, corresponding to $\sin(2\beta) = 0.695$, and $\gamma = 59.0^\circ$. See Table III for errors.

| $\alpha^\mp_{CP}$ | Theory | Experiment (BaBar) \cite{5} |
|-------------------|--------|-----------------------------|
| $A^\mp_{CP}$ | $0.01 +0.00+0.00+0.05$ | $-0.07 \pm 0.07 \pm 0.02$ |
| $C$ | $0.02 +0.00+0.00+0.13$ | $-0.10 \pm 0.15 \pm 0.09$ |
| $S$ | $-0.55-0.22-0.06-0.13$ | $0.37 \pm 0.21 \pm 0.07$ |
| $\Delta C$ | $0.48+0.04+0.04+0.05$ | $0.26 \pm 0.15 \pm 0.07$ |
| $\Delta S$ | $-0.01-0.00-0.00-0.03$ | $-0.14 \pm 0.21 \pm 0.06$ |

which coincide with $\alpha$ in the limit of vanishing penguin amplitudes. The average of $\alpha^+_{\text{eff}}$ and $\alpha^-_{\text{eff}}$ is called $\alpha_{\text{eff}}$:

$$
\alpha_{\text{eff}} \equiv \frac{\alpha^+_{\text{eff}} + \alpha^-_{\text{eff}}}{2} = \frac{1}{4} \left[ \arcsin \left( \frac{S + \Delta S}{\sqrt{1 - (C + \Delta C)^2}} \right) + \arcsin \left( \frac{S - \Delta S}{\sqrt{1 - (C - \Delta C)^2}} \right) \right].
$$

The numerical results for the time-dependent CP parameters are collected in Table IV. The magnitudes of $A^\mp_{CP}$, $C$ and $\Delta S$ are small in the QCD factorization calculation, where $C$ is sensitive to the annihilations and can be $\sim 10\%$ in magnitude. $\Delta C$ describes the asymmetry between $B(B^0 \to a_1^\mp \pi^\mp) + B(B^0 \to a_1^- \pi^+) + B(B^0 \to a_1^- \pi^-)$ and $B(B^0 \to a_1^\mp \pi^\mp) + B(B^0 \to a_1^\mp \pi^\mp)$, and thus can be read directly from Tables II and III. Neglecting penguin contributions, $S$ and $\alpha_{\text{eff}}$, which depend on $\alpha(= \pi - \beta - \gamma)$, coincides with $\sin 2\alpha$ and $\alpha$, respectively, in the SM. Using $\alpha = 99.0^\circ$, i.e., $\gamma = 59.0^\circ$, the numerical results for $S$ and $\alpha_{\text{eff}}$ differ from the experimental values at about the 3.7$\sigma$ level. This puzzle may be resolved by using a smaller $\alpha = \pi - \beta - \gamma \lesssim 78^\circ$. In Fig. II, we plot $S$ versus $\gamma$ (and $\alpha$), where we parameterize $V_{ub} = 0.00368 e^{-i\gamma}$. The best fitted value is $\gamma = (87^{+33}_{-13})^\circ$, corresponding to $\alpha = (71^{+17}_{-83})^\circ$, for $\beta = 22^\circ$.

3. $\overline{B} \to a_1(1260)K$ decays

The decays $\overline{B} \to a_1 K$ are penguin-dominated. Because the dominant axial-vector and pseudoscalar penguin coefficients, $a^p_4(a_1 K)$ and $a^p_6(a_1 K)$, are constructive in the $a_1 K$ modes, $\overline{B} \to a_1 K$ and the corresponding $\overline{B} \to \pi K$ decays should have similar rates. It is instructive
FIG. 1: $S$ and $\alpha_{\text{eff}}$ versus $\gamma$ (and $\alpha$) for adopting $\beta = 22^\circ$. The solid curves are obtained by using the central values (default values) of input parameters. The region between two dashed lines is the theoretical variation within the allowed range of input parameters.

to consider the four ratios:

$$R_1 = \frac{B(B^0 \to a_1^+ K^-)}{B(B^0 \to \pi^+ K^-)} = \left( \frac{V_{B^0 a_1}}{F_{B^0 \pi}} \right)^2 \left( \frac{\alpha_4(a_1 K)}{\alpha_4(\pi K)} \right)^2 \times \left[ 1 + 2 \text{Re} \left( \frac{\beta_3(a_1 K)}{\alpha_4(a_1 K)} - \frac{\beta_3(\pi K)}{\alpha_4(\pi K)} + \cdots \right) \right],$$

$$R_2 = \frac{B(B^- \to a_1^- K^0)}{B(B^- \to \pi^- K^0)} = \left( \frac{V_{B^0 a_1}}{F_{B^0 \pi}} \right)^2 \left( \frac{\alpha_4(a_1 K)}{\alpha_4(\pi K)} \right)^2 \times \left[ 1 + 2 \text{Re} \left( \frac{\beta_3(a_1 K)}{\alpha_4(a_1 K)} - \frac{\beta_3(\pi K)}{\alpha_4(\pi K)} + \cdots \right) \right],$$

$$R_3 = \frac{B(B^0 \to a_1^- K^0)}{B(B^0 \to \pi^- K^0)} = \left( \frac{V_{B^0 a_1}}{F_{B^0 \pi}} \right)^2 \left( \frac{\alpha_4(a_1 K)}{\alpha_4(\pi K)} \right)^2 \left[ 1 - 3 \text{Re} \left( \frac{\alpha_4(\pi K)}{\alpha_4(a_1 K)} \right) r_1 - \frac{\alpha_4(\pi K)}{\alpha_4(a_1 K)} r_2 \right] + 2 \text{Re} \left( \frac{\beta_3(a_1 K)}{\alpha_4(a_1 K)} - \frac{\beta_3(\pi K)}{\alpha_4(\pi K)} + \cdots \right),$$

$$R_4 = \frac{B(B^- \to a_1^0 K^-)}{B(B^- \to \pi^0 K^-)}$$
\[
\left( \frac{V_{0Ba1}(m_{K}^2)}{F_{0}^{Ba1}(m_{K}^2)} \right)^2 \left( \frac{\alpha_i(a_1K)}{\alpha_i^c(a_1K)} \right)^2 \left[ 1 + 3 \text{Re} \left[ \frac{\alpha_3^c(\overline{K}a_1)}{\alpha_i(a_1K)} r_1 - \frac{\alpha_3^c(\overline{K}\pi)}{\alpha_i^c(\pi K)} r_2 \right] \right] + 2 \text{Re} \left( \frac{\beta_5(a_1K) + \beta_5^c(\overline{K}a_1)}{\alpha_i^c(a_1K)} - \frac{\beta_5(\pi K) + \beta_5^c(\overline{K}\pi)}{\alpha_i^c(\pi K)} \right) + \cdots \right], \\
(4.16)
\]

where

\[
r_1 = \frac{F_{0}^{BK}(m_{a_1}^2) f_{a_1}}{V_{0}^{Ba1}(m_{K}^2) f_{K}} \approx 1.9, \\
r_2 = \frac{F_{0}^{BK}(m_{a_1}^2) f_{K}}{F_{0}^{Ba1}(m_{K}^2) f_{K}} \approx 1.1,
\]

and the dots stand for the neglected terms which are numerically estimated to be less than 1% in magnitude. The ratios \(R_{1,2,3,4}\), which are very insensitive to \(\gamma\), are approximately proportional to \(\left| \frac{V_{0}^{Ba1}(m_{K}^2)}{F_{0}^{Ba1}(m_{K}^2)} \right|^2\) and receive corrections mainly from the electroweak penguin and annihilation topologies. The value of the annihilation \(\beta_3\) is sensitive to \(a_1^{\perp,a_1}\). The contributions originating from electroweak penguin and annihilation amplitudes can be further explored by taking into account the following measurements for ratios,

\[
\frac{R_1}{R_2} \approx 1 - 3 \text{Re} \left( \frac{\beta_3^c(\overline{K}a_1)}{\alpha_i(a_1K)} - \frac{\beta_3^c(\overline{K}\pi)}{\alpha_i^c(\pi K)} \right), \\
(4.19)
\]

\[
\frac{R_1 - R_3}{R_2 - R_4} \approx 6 \text{Re} \left[ \frac{\alpha_3^c(\overline{K}a_1)}{\alpha_i(a_1K)} r_1 - \frac{\alpha_3^c(\overline{K}\pi)}{\alpha_i^c(\pi K)} r_2 \right], \\
(4.20)
\]

\[
\frac{R_1}{R_3} \approx \frac{R_4}{R_2} \approx 1 + 3 \text{Re} \left[ \frac{\alpha_3^c(\overline{K}a_1)}{\alpha_i(a_1K)} r_1 - \frac{\alpha_3^c(\overline{K}\pi)}{\alpha_i^c(\pi K)} r_2 \right] \approx 1 + \frac{1}{2} \left( \frac{R_1}{R_2} - \frac{R_3}{R_4} \right). \\
(4.21)
\]

Although the above ratios are parameterized according to the QCD factorization, they can be treated in a model-independent way. It is worth stressing that because \(\Phi_{a_1}^\perp(u)\) is anti-symmetric under interchange of the quark and antiquark momentum fractions in the SU(2) limit, the weak annihilations (and hard spectator interactions), which could contribute sizable corrections to the decay amplitudes, enter the \(\overline{B} \rightarrow a_1\overline{K}\) amplitude in a very different pattern compared with \(\overline{B} \rightarrow \pi\overline{K}\) decays. More relevant information about \(X_A\) and \(a_1^{\perp,a_1}\) can thus be provided by the measurement of \(R_1/R_2\).

With default parameters, the direct CP asymmetries are analogous to the corresponding \(\overline{B} \rightarrow \pi\overline{K}\) modes; because \(A_{CPs}\) are dominated by \(\text{Re}(V_{ud}V_{ub}^*) \text{Im}(\alpha_i^c + \beta_3^c) \text{Im}(V_{us}^*V_{ub})\) times \(\text{Re}[\alpha_1 + \alpha_2 F_{1BK}f_{a_1}/(V_{0}^{Ba1}f_{K})]\) and \(-\text{Re}[\alpha_2 F_{1BK}f_{a_1}/(V_{0}^{Ba1}f_{K})]\) terms for \(a_1^0K^-\) and \(a_1^0\overline{K}\) modes, respectively, their direct CP asymmetries are thus a little larger than the corresponding \(\pi\overline{K}\) modes in magnitude due to the decay constant enhancement. Note that the value of \(\beta_3\) is sensitive to the first Gegenbauer moment of \(\Phi_{a_1}^\perp(u)\) and the annihilation parameters \(\rho_A\) and \(\phi_A\). On the other hand, an outstanding problem is the determination of the signs for direct CP observations in the \(\pi\overline{K}\) modes. The experimental results are
\[ A_{CP}(B^0 \rightarrow \pi^+K^-) = -0.095 \pm 0.013 \] and \[ A_{CP}(B^- \rightarrow \pi^0K^-) = 0.046 \pm 0.026 \text{[6]} \]. Some proposals, for instance the contribution due to new-physics in the SM electroweak penguin sector [19, 20, 22] or due to FSIs [17, 18], were advocated for the resolution. The ratio measurements for \( R_1/R_2 - R_3/R_4, R_1/R_3, \) and \( R_1/R_2 \) directly probe the electroweak penguins. Moreover, the approximate relation given in Eq. (4.21) will be violated if the FSI patterns are different between \( a_1K \) and \( \pi K \) modes.

V. CONCLUSIONS

We have studied \( \mathcal{B} \rightarrow a_1(1260) \pi, a_1(1260)\bar{K} \) decays. This paper is the first one in the literature using the QCD factorization approach to study \( B \rightarrow AP \) decays. Interestingly, due to the G-parity, the leading-twist LCDA \( \Phi_+^{a_1} \) of the \( a_1(1260) \) defined by the nonlocal tensor current is antisymmetric under the exchange of quark and anti-quark momentum fractions in the SU(2) limit, whereas the \( \Phi_1^{a_1} \) defined by the nonlocal axial-vector current is symmetric. The large magnitude of the first Gegenbauer moment \( (a_1^{+,a_1}) \) of \( \Phi_+^{a_1} \) could have a sizable impact on the annihilation amplitudes. If one ignores \( \Phi_1^{a_1} \), i.e., letting \( a_1^{+,a_1} = 0 \), with default parameters (where \( \rho_A = 0 \)), the branching ratio for \( a_1^{0}\bar{K} \) mode becomes 1.8 times smaller, while the changes of branching ratios for \( a_1 \pi \) and the remaining \( a_1 \bar{K} \) modes are at the level of 5% and 10%, respectively.

Our main results are summarized as follows.

- Our results for \( \mathcal{B}(\mathcal{B}^0 \rightarrow a_1^+ \pi^-, a_1^- \pi^+) \) are in good agreement with the data. Theoretically, the rates for \( \mathcal{B} \rightarrow a_1(1260) \pi \) are close to the corresponding ones for \( \mathcal{B} \rightarrow \rho \pi \). The differences between the above two modes are mainly caused by different magnitudes of form factors \( (V_0^{Ba_1} \text{ and } A_0^{B\rho}) \) and decay constants \( (f_{a_1} \text{ and } f_\rho) \), and by different patterns of tree–penguin interference. For \( \sin \gamma > 0 \), the T-P interference is destructive in \( \mathcal{B}^0 \rightarrow a_1^+ \pi^+, B^- \rightarrow a_1^0 \pi^- \), but constructive in \( B^- \rightarrow a_1^- \pi^0 \). Because the amplitudes of \( a_1 \pi \) and \( \rho \pi \) modes are dominated by terms with \( \alpha_1 \) and \( \alpha_2^\rho \), and \( \text{Re}[\alpha_4^\rho(\pi a_1)] \approx \text{Re}[\alpha_4^\rho(\pi a_1)/3] \approx \text{Re}[\alpha_4^\rho(\rho \pi)] \approx -\text{Re}[\alpha_4^\rho(\rho \pi)] \approx -0.034 \), we obtain the relations as given in Eqs. (4.7) and (4.8). Thus estimates for form factors and decay constants as well as the weak phase \( \gamma \) can thus be made from these ratio measurements.

- For \( CP \) asymmetries, the large experimental errors do not allow us to draw any particular conclusion in comparison with theoretical predictions. The time-dependent CP asymmetry measurement in \( B^0 \rightarrow a_1^\pm \pi^\mp \) can lead to the accurate determination of the CKM angle \( \gamma \). Using the current fitted value \( \gamma = 59.0^\circ \), i.e., \( \alpha = 99.0^\circ \) corresponding to \( \beta = 22.0^\circ \) in the SM, our results show that \( S \) and \( \alpha_{\text{eff}} \) differ from the present data at about the 3.7\( \sigma \) level. This puzzle may be resolved by using a larger \( \gamma \gtrsim 80^\circ \). Further measurements can clarify this discrepancy.

- The branching ratios for the decays \( B \rightarrow a_1 \pi \) and \( a_1 K \) are highly sensitive to the magnitude of \( V_0^{Ba_1}(0) \). Using the LC sum rule result, \( V_0^{Ba_1}(0) = 0.28 \pm 0.03 \text{[32]} \), the
resultant branching ratios for $a_1^\pm \pi^\mp$ modes consist with the data very well. Nevertheless, the value of $V_{Ba}^{B_a}(0)$ is about 0.13 and 1.02 $\sim$ 1.22 in the quark model calculations in Ref. [37] and Refs. [38, 39], respectively. If the quark model result is used in the calculation, $B(\overline{B}^0 \rightarrow a_1^\mp \pi^-)$ will be too small or large as compared with the data.

- The $\overline{B} \rightarrow a_1 K$ amplitudes resemble the corresponding $\overline{B} \rightarrow \pi K$ amplitudes very much. Taking the ratios of corresponding CP-averaged $a_1 K$ and $\pi K$ branching ratios, we can extract information about the transition form factors, decay constants, electroweak penguin ($\alpha_{3,EW}(K a_1)$), and annihilation topology ($\beta_{3,EW}(a_1 \overline{K})$). See Eqs. (3.19)-(3.21). Thus, the possibilities for existing new-physics in the electroweak penguin sector and for final state interactions during decays can be explored.

**Note added.** Recently Belle has updated the following measurement [44]: $B(\overline{B}^0 \rightarrow a_1^+ \pi^- + a_1^- \pi^+) = (29.8 \pm 3.2 \pm 4.6) \times 10^{-6}$ which is in good agreement with our result. On the other hand, BaBar has reported new measurements on $a_0^0 \pi^0$, $a_1^0 K^-$, $a_1^- K^0$ modes [45, 46], where $B(\overline{B}^0 \rightarrow a_1^+ K^-) = (16.3 \pm 2.9 \pm 2.3) \times 10^{-6}$ is also in good agreement with our prediction, whereas the central values of branching ratios for the remaining modes are about $2 \sim 3$ times larger than our predictions. The latter discrepancies should be clarified by the improved measurements in the future.

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**APPENDIX A: THE COEFFICIENTS $a_i^p$**

In the below discussion, we set $\Phi_\parallel^P \equiv \Phi^P$. In Eq. (3.9), the expressions for effective parameters $a_i^p$ are

$$a_i^p(M_1 M_2) = \left( c_i + \frac{c_i \pm 1}{N_c} \right) N_i(M_2) + \frac{c_i \pm 1}{N_c} \frac{C_F \alpha_s}{4\pi} \left[ V_i(M_2) + \frac{4\pi^2}{N_c} H_i(M_1 M_2) \right] + P_i^p(M_2). \quad (A1)$$

$N_i$ is given in Eq. (3.10). The vertex corrections have the same expressions as those for $VP$ modes [23] with LCDA$s$ of the vector meson being replaced by the corresponding ones of the $a_1$ meson. For the penguin contractions $P_i^p(M_2)$, one can perform the same replacements but needs to add an overall minus sign to $P_i^0(a_1)$ and $P_i^0(a_1)$. $H_i(M_1 M_2)$ have the expressions:

$$H_i(M_1 M_2) = \frac{-i f_{B} f_{M_1} f_{M_2}}{X(B M_1, M_2)} \int_0^1 d\rho \frac{\Phi^P(\rho)}{1 - \rho},$$
for $i = 1 - 4, 9, 10,$

$$H_i(M_1M_2) = \frac{i f_{Bf_{M_1f_{M_2}}}}{X(BM_1M_2)} \int_0^1 d\rho \frac{\Phi_B(\rho)}{1 - \rho}$$

$$\times \int_0^1 dv \int_0^1 du \left( \frac{\Phi^{M_1}_{\parallel}(v)\Phi^{M_2}_{\parallel}(u)}{uv} \pm r^{M_1}_{X} \Phi_{m_1}(v)\Phi^{M_2}_{\parallel}(u) \right),$$

(A2)

for $i = 5, 7,$ and $H_i(M_1M_2) = 0$ for $i = 6, 8,$ where the upper (lower) signs apply when $M_1 = P \ (M_1 = A).$ Here $\Phi^B_i(\rho)$ is one of the two LCDAs of the $B$ meson [25].

**APPENDIX B: THE ANNIHILATION AMPLITUDES $A_{i}^{f}$**

For $A_{i}^{f}$ (see Eq. (3.12)), some signs change in comparison with the results of $B \to PP$ and $PV.$ We obtain

$$A_1^i = \pi \alpha_s \int_0^1 dx dy \left\{ \Phi^{M_2}_{\parallel}(x) \Phi^{M_1}_{\parallel}(y) \left[ \frac{1}{y(1 - xy)} + \frac{1}{x^2y} \right] - r^{M_1}_{X} r^{M_2}_{X} \Phi_{m_1}(x) \Phi_{m_1}(y) \frac{2}{xy} \right\},$$

$$A_1^f = A_2^f = 0,$$

$$A_2^i = \pi \alpha_s \int_0^1 dx dy \left\{ \Phi^{M_2}_{\parallel}(x) \Phi^{M_1}_{\parallel}(y) \left[ \frac{1}{x(1 - xy)} + \frac{1}{xy^2} \right] - r^{M_1}_{X} r^{M_2}_{X} \Phi_{m_1}(x) \Phi_{m_1}(y) \frac{2}{xy} \right\},$$

$$A_3^i = \pm \pi \alpha_s \int_0^1 dx dy \left\{ r^{M_1}_{X} \Phi^{M_2}_{\parallel}(x) \Phi_{m_1}(y) \frac{2y}{xy(1 - xy)} + r^{M_1}_{X} \Phi^{M_1}_{\parallel}(y) \Phi_{m_2}(x) \frac{2x}{xy(1 - xy)} \right\},$$

$$A_3^f = \pm \pi \alpha_s \int_0^1 dx dy \left\{ r^{M_1}_{X} \Phi^{M_2}_{\parallel}(x) \Phi_{m_1}(y) \frac{2(1 + x)}{x^2y} - r^{M_2}_{X} \Phi^{M_1}_{\parallel}(y) \Phi_{m_2}(x) \frac{2(1 + y)}{xy^2} \right\},$$

(B1)

where the upper (lower) signs apply when $M_1 = P \ (M_1 = A)$ and the detailed definitions of the distribution amplitudes of the axial-mesons have been collected in Sec. [II]. Again, here we have set $\Phi^{\parallel}_{\parallel} \equiv \Phi^P.$

Using the asymptotic distribution amplitudes of $\Phi^{a_1}_{\perp}(u)$ and $\Phi_P(u),$ and the approximation $\Phi^{a_1}_{\perp}(u) = 18u\bar{u}(2u - 1)a^{\perp}_{4X},$ we obtain the annihilation amplitudes

$$A_1^i \approx 6 \pi \alpha_s \left[ 3 \left( X_A - 4 + \frac{\pi^2}{3} \right) - a^{\perp}_{4X} r^{a_1}_{X} r^{P}_{X} X_A(X_A - 3) \right],$$

(B2)

$$A_1^f \approx 6 \pi \alpha_s \left[ 3 \left( X_A - 4 + \frac{\pi^2}{3} \right) - a^{\perp}_{4X} r^{a_1}_{X} r^{P}_{X} X_A(X_A - 3) \right],$$

(B3)

$$A_3^i \approx 6 \pi \alpha_s \left[ r^{P}_{X} \left( X_A^2 - 2X_A + \frac{\pi^2}{3} \right) + 3a^{\perp}_{4X} r^{a_1}_{X} \left( X_A^2 - 2X_A - 6 + \frac{\pi^2}{3} \right) \right],$$

(B4)

$$A_3^f \approx 6 \pi \alpha_s \left( 2X_A - 1 \right) \left[ r^{P}_{X} X_A - 3a^{\perp}_{4X} r^{a_1}_{X} \left( X_A - 3 \right) \right].$$

(B5)
1. B. Aubert et al. [BABAR Collaboration], arXiv:hep-ex/0408021.
2. B. Aubert et al. [BaBar Collaboration], arXiv:hep-ex/0507029.
3. K. Abe et al. [Belle Collaboration], arXiv:hep-ex/0507096.
4. B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 97, 051802 (2006) arXiv:hep-ex/0603050.
5. B. Aubert et al. [BABAR Collaboration], arXiv:hep-ex/0612050.
6. See Heavy Flavor Averaging Group, http://www.slac.stanford.edu/xorg/hfag/ and references therein.
7. M. Gronau and J. Zupan, Phys. Rev. D 70, 074031 (2004) arXiv:hep-ph/0407002.
8. M. Gronau and J. Zupan, Phys. Rev. D 73, 057502 (2006) arXiv:hep-ph/0512148.
9. Y. Grossman and H. R. Quinn, Phys. Rev. D 58, 017504 (1998) arXiv:hep-ph/9712306.
10. J. Charles, Phys. Rev. D 59, 054007 (1999) arXiv:hep-ph/9806468.
11. H. J. Lipkin, Y. Nir, H. R. Quinn and A. Snyder, Phys. Rev. D 44, 1454 (1991).
12. K. C. Yang, Nucl. Phys. B 776, 187 (2007) arXiv:0705.0692 [hep-ph].
13. K. C. Yang, JHEP 0510, 108 (2005) arXiv:hep-ph/0509337.
14. K. C. Yang, Phys. Rev. D 72, 034009 (2005) [Erratum-ibid. D 72, 059901 (2005)] arXiv:hep-ph/0506040.
15. P. K. Das and K. C. Yang, Phys. Rev. D 71, 094002 (2005) arXiv:hep-ph/0412313.
16. W. S. Hou and K. C. Yang, Phys. Rev. Lett. 84, 4806 (2000) [Erratum-ibid. 90, 039901 (2003)] arXiv:hep-ph/9911528.
17. C. K. Chua, W. S. Hou and K. C. Yang, Mod. Phys. Lett. A 18, 1763 (2003) arXiv:hep-ph/0210002.
18. H. Y. Cheng, C. K. Chua and A. Soni, Phys. Rev. D 71, 014030 (2005) arXiv:hep-ph/0409317.
19. A. J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, Phys. Rev. Lett. 92, 101804 (2004) arXiv:hep-ph/0312259.
20. A. J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, Nucl. Phys. B 697, 133 (2004) arXiv:hep-ph/0402112.
21. R. Fleischer, S. Recksiegel and F. Schwab, arXiv:hep-ph/0702275.
22. S. Baek and D. London, arXiv:hep-ph/0701181.
23. M. Beneke and M. Neubert, Nucl. Phys. B 675, 333 (2003).
24. S. Wandzura and F. Wilczek, Phys. Lett. B 72, 195 (1977).
25. M. Beneke, G. Buchalla, M. Neubert, and C.T. Sachrajda, Phys. Rev. Lett. 83, 1914 (1999); Nucl. Phys. B 591, 313 (2000).
26. M. Beneke, G. Buchalla, M. Neubert, and C.T. Sachrajda, Nucl. Phys. B 606, 245 (2001).
27. M. Bauer, B. Stech and M. Wirbel, Z. Phys. C 34, 103 (1987).
28. G. Buchalla, A.J. Buras, and M.E. Lautenbacher, Rev. Mod. Phys. 68, 1125 (1996).
29. Particle Data Group, W. M. Yao et al., J. Phys. G 33, 1 (2006).
30. CKMfitter Group, J. Charles et al., Eur. Phys. J. C 41, 1 (2005); http://ckmfitter.in2p3.fr.
[31] P. Ball and R. Zwicky, Phys. Rev. D 71, 014015 (2005) arXiv:hep-ph/0406232.
[32] K.C. Yang, in preparation.
[33] P. Ball, V. M. Braun and A. Lenz, JHEP 0605, 004 (2006) arXiv:hep-ph/0603063.
[34] M. Bona et al. [UTfit Collaboration], JHEP 0610, 081 (2006) arXiv:hep-ph/0606167.
[35] P. Ball, Phys. Lett. B 644, 38 (2007) arXiv:hep-ph/0611108.
[36] T. M. Aliev and M. Savci, Phys. Lett. B 456, 256 (1999) arXiv:hep-ph/9901395.
[37] H. Y. Cheng, C. K. Chua and C. W. Hwang, Phys. Rev. D 69, 074025 (2004) arXiv:hep-ph/0610359.
[38] D. Scora and N. Isgur, Phys. Rev. D 52, 2783 (1995) arXiv:hep-ph/9503486.
[39] A. Deandrea, R. Gatto, G. Nardulli and A. D. Polosa, Phys. Rev. D 59, 074012 (1999) arXiv:hep-ph/9811259.
[40] H. Y. Cheng, C. K. Chua and K. C. Yang, Phys. Rev. D 73, 014017 (2006) arXiv:hep-ph/0508104.
[41] H. Y. Cheng, C. K. Chua and K. C. Yang, arXiv:0705.3079 [hep-ph].
[42] K. C. Yang, Phys. Rev. D 69, 054025 (2004) arXiv:hep-ph/0308005.
[43] W. S. Hou and K. C. Yang, Phys. Rev. D 61, 073014 (2000) arXiv:hep-ph/9908202.
[44] K. Abe et al. [Belle Collaboration], arXiv:0706.3279 [hep-ex].
[45] D. Walker [BABAR Collaboration], arXiv:0708.0050 [hep-ex].
[46] D. Brown, invited talk presented at the XXIII International Symposium on Lepton and Photon Interactions at High Energy, August 13-18, 2007, Daegu, Korea.