LightestScalarResonancesandtheDynamicsofthe$\gamma\gamma \to \pi\pi$Reactions

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The high-statistics Belle data on the $\gamma\gamma \to \pi^+\pi^-$ and $\gamma\gamma \to \pi^0\pi^0$ reactions have been jointly analyzed. The main dynamical mechanisms of these reactions for energies below 1.5 GeV have been revealed. It has been shown that the direct coupling constants of the $\sigma(600)$ and $f_0(980)$ resonances with a $\gamma\gamma$ pair are small and that the $\sigma(600) \to \gamma\gamma$ and $f_0(980) \to \gamma\gamma$ decays are four-quark transitions due primarily to $\pi^+\pi^-$ and $K^+K^-$ loop mechanisms, respectively. The role of the chiral shielding of the $\sigma(600)$ resonance is emphasized. The widths of the $f_0(980) \to \gamma\gamma$ and $\sigma(600) \to \gamma\gamma$ decays averaged over the resonance mass distributions, as well as the width of the $f_2(1270) \to \gamma\gamma$ decay, are estimated as $(\Gamma_{f_0 \to \gamma\gamma})_{\pi\pi} \approx 0.19$ keV, $(\Gamma_{\sigma \to \gamma\gamma})_{\pi\pi} \approx 0.45$ keV, and $\Gamma_{f_2 \to \gamma\gamma}(m_{f_2}^2) \approx 3.8$ keV.

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The investigation of the lightest scalar resonances $\sigma(600)$, $\kappa(800)$, $a_0(980)$, and $f_0(980)$ is one of the main goals of nonperturbative QCD, because the elucidation of their nature is important for understanding both the physics of confinement and the means of the breaking of the chiral symmetry at low energies, which are the main consequences of QCD for hadron physics. The nontrivial nature of these states is commonly accepted. In particular, there is plenty of evidence of their four-quark ($q\bar{q}^2$) structure (see, e.g., [1] and references therein). One of these evidences is the suppression of the production of the $a_0(980)$ and $f_0(980)$ resonances in the $\gamma\gamma \to \pi^0\eta$ and $\gamma\gamma \to \pi\pi$ reactions, respectively, which was predicted more than 25 years ago [2] and observed in the experiment [3]. The problem of the mechanisms of the production of the $\sigma(600)$, $f_0(980)$, and $a_0(980)$ resonances in the $\gamma\gamma$ collisions is closely associated with the problem of their internal quark structure. This explains the long-term theoretical and experimental interest in the $\gamma\gamma \to \pi\pi$ reactions at low energies. Recently, the Belle Collaboration obtained new data on the cross sections for the $\gamma\gamma \to \pi^+\pi^-$ [4] and $\gamma\gamma \to \pi^0\pi^0$ [5] reactions with statistics two orders of magnitude larger than all previous experiments and revealed a pronounced signal from the $f_0(980)$ resonance [4,5]. The preceding indications of the production of the $f_0(980)$ resonance in the $\gamma\gamma$ collisions were much less definite [6–8]. The signal from the $f_0(980)$ resonance appears to be small, which is in good agreement with the prediction of the four-quark model [1,2].

In this paper, we report the results of the investigation of the main dynamical mechanisms of the $\gamma\gamma \to \pi^+\pi^-$ and $\gamma\gamma \to \pi^0\pi^0$ reactions on the basis of the analysis of the Belle data [4,5] and our previous investigations of the physics of the scalar mesons in the $\gamma\gamma$ collisions [2,9–13].

The Belle data on the cross sections for the $\gamma\gamma \to \pi^+\pi^-$ and $\gamma\gamma \to \pi^0\pi^0$ reactions obtained for invariant mass $\sqrt{s}$ of the $\pi\pi$ systems from 0.8 to 1.5 GeV are shown in Fig. 1, where the data of other groups [6–8] are also shown for $\sqrt{s}$ from 2$m_{\pi}$ to 0.85 GeV. All existing data correspond to the incomplete solid angle of the detection of the final pions such that $|\cos\theta| \leq 0.6$ and $|\cos\theta| \leq 0.8$ for the production of the $\pi^+\pi^-$ and $\pi^0\pi^0$ pairs, respectively, where $\theta$ is the polar angle of the pion emission in the cms of the initial photons. The pronounced peaks attributed to the production of the $f_0(980)$ and $f_2(1270)$ resonances are observed in the cross sections for both reactions. The background under these peaks is nearly absent in the $\gamma\gamma \to \pi^0\pi^0$ channel. On the contrary, the resonances in the $\gamma\gamma \to \pi^+\pi^-$ channel are seen against a large smooth background, which is primarily attributed to the mechanism of the charged one-pion exchange [11–16]. The pure Born cross section for the $\gamma\gamma \to \pi^+\pi^-$ process at $|\cos\theta| \leq 0.6$, the total cross section $\sigma^{\text{Born}} = \sigma_0^{\text{Born}} + \sigma_2^{\text{Born}}$, and the cross sections $\sigma_0^{\text{Born}}$ and $\sigma_2^{\text{Born}}$, where the subscript $(\lambda = 0$ or 2) is the absolute value of the difference between the helicities of the initial photons, are shown in Fig. 1a for comparison. Owing to the Low theorem and chiral symmetry, the one-pion Born contribution should dominate near the threshold of the $\gamma\gamma \to \pi^+\pi^-$ reaction. As seen in Fig. 1a, this expectation does not contradict the near-threshold data; however, these data were obtained with large errors. The cross section $\sigma_0^{\text{Born}}$ decreases rapidly with an increase in $\sqrt{s}$, so that the contribution $\sigma_2^{\text{Born}}$ dominates completely in $\sigma^{\text{Born}}$ at $\sqrt{s} > 0.5$ GeV (see Fig. 1a). Note that the contributions from the $S$ and $D_{\lambda=2}$ partial waves dominate in the region $\sqrt{s} < 1.5$ GeV in $\sigma_0^{\text{Born}}$ and $\sigma_2^{\text{Born}}$, respectively. These partial Born contributions are strongly modified due to the strong interaction between pions in the final state, because the $\pi\pi$ interaction at $\sqrt{s} < 1.5$ GeV is strong only in the $S$ and $D$ waves. The inclusion of the final-state interaction in the $S$-wave Born amplitudes of the $\gamma\gamma \to \pi^+\pi^-$ (and $\gamma\gamma \to K^+K^-$) reaction leads to certain predictions for the $S$-wave amplitude of the $\gamma\gamma \to \pi^0\pi^0$ reaction.

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FIG. 1: Cross sections for the $\gamma\gamma \to \pi^+\pi^-$ and $\gamma\gamma \to \pi^0\pi^0$ reactions. Only statistical errors are shown for the Belle data [4,5]. The curves in panel (a) are described in the main text and on the figure. The curves in panel (b) are the approximations of the data on the $\gamma\gamma \to \pi^0\pi^0$ reaction.

Figure 2 shows the Belle experimental data for the angular distributions in the $\gamma\gamma \to \pi^0\pi^0$ reaction [5]. They are excellently reproduced by the simple two-parametric expression $|a|^2 + |b d_{20}(\theta)|^2$, where $d_{20}(\theta)$ is the $d$ function [3] and $l$ is the orbital angular momentum of the final $\pi\pi$ system. Therefore, the cross section for the $\gamma\gamma \to \pi^0\pi^0$ at $\sqrt{s} < 1.5$ GeV is described by contributions only from the $S$ and $D_2$ partial waves [17].

Thus, let us consider a model for the helicity, $M_0$, and partial, $M_{1,2}$, amplitudes of the $\gamma\gamma \to \pi\pi$ reaction, where the electromagnetic Born contributions from point-like charged $\pi$ and $K$ exchanges modified in the $S$ and $D_2$ waves by strong final-state interactions, as well as the contributions due to the direct interaction of the resonances with photons (see also [11,13]), are taken into account:

$$M_0(\gamma\gamma \to \pi^+\pi^-; s, \theta) = M_0^{\text{Born}}(s, \theta) + \tilde{I}_{\pi^+\pi^-}(s) T_{\pi^+\pi^- \to \pi^+\pi^-}(s) + \tilde{I}_{K^+K^-}(s) T_{K^+K^- \to \pi^+\pi^-}(s) + M_{\text{res}}^{\text{direct}}(s),$$

$$M_2(\gamma\gamma \to \pi^+\pi^-; s, \theta) = M_2^{\text{Born}}(s, \theta) + 80\pi d_{20}(\theta) M_{\gamma\gamma \to f_2(1270)} \to \pi^+\pi^-(s),$$

$$M_0(\gamma\gamma \to \pi^0\pi^0; s, \theta) = M_{00}(\gamma\gamma \to \pi^0\pi^0; s) = \tilde{I}_{\pi^0\pi^0}(s) T_{\pi^0\pi^0 \to \pi^0\pi^0}(s) + \tilde{I}_{K^+K^-}(s) T_{K^+K^- \to \pi^0\pi^0}(s) + M_{\text{res}}^{\text{direct}}(s),$$
Here, \(M^\text{Born}_{0}(s, \theta) = (32\pi\alpha s)/(1 - \rho_{\pi}^2(s) \cos^2 \theta)\) and \(M^\text{Born}_{2}(s, \theta) = 8\pi\alpha \rho_{\pi}^2(s) \sin^2 \theta/(1 - \rho_{\pi}^2(s) \cos^2 \theta)\) are the Born helicity amplitudes of the \(\gamma\gamma \to \pi^+ \pi^-\) reaction, \(\rho_{\pi}(s) = (1 - 4m_{\pi}^2/s)^{1/2}\), and \(\alpha = 1/137\). The function \(\tilde{I}_{\pi+\pi-}(s)\) at \(s \geq 4m_{\pi}^2\) has the form \(\tilde{I}_{\pi+\pi-}(s) = 8\alpha \left\{ \frac{m_{\pi}^2}{s} \left[ \pi + i \ln \frac{1 + \rho_{\pi}(s)}{1 - \rho_{\pi}(s)} \right]^2 - 1 \right\}\). The functions \(\tilde{I}_{\pi+\pi-}(s)\) and \(\tilde{I}_{K+K-}(s)\) are the amplitudes of the production of the \(\pi\pi\) and \(K\bar{K}\) waves amplitudes of the corresponding reactions; \(T_{\pi+\pi-\to\pi+\pi-}(s) = 2T_{0}^\prime(s) + T_{0}^\prime(s)/3\) and \(T_{\pi+\pi-\to\pi+\pi-}(s) = 2T_{0}(s) - T_{0}^\prime(s)/3\), where \(T_{0}(s) = \eta^0(s) \exp[2i\delta^0(s)] - 1/2i\rho_\pi(s)\) are the phases, and \(\eta^0(s)\) are the inelasticity factors of the S wave \(\pi\pi\) scattering in the channels with isospin \(I = 0\) and 2.  

The parametrization of the amplitudes \(T_{0}^\prime(s)\) and \(T_{K+K-\to\pi+\pi-}(s)\), which was used in the joint analysis of the data on the \(\pi^0\pi^0\) mass spectrum in the \(\phi\to\pi^0\pi^0\) decay, \(\pi\pi\) scattering at \(2m_{\pi} < \sqrt{s} < 1.6\) GeV, and \(\pi\pi\to K\bar{K}\) reaction, was described in detail in [18]. This parametrization is based on the concept that the amplitude \(T_{0}^\prime(s)\) must include the contribution from mixed \(\sigma\) (600) and \(f_0(980)\) resonances and the contribution from the background, which has a large negative phase due to the chiral symmetry; the latter contribution shields (hides) the \(\sigma\) resonance [1,18,20]. Formulas (1) and (3) transfer the effect of the chiral shielding of the \(\sigma\) resonance from the \(\pi\pi\) scattering to the \(\gamma\gamma\to\pi\pi\) amplitudes. If this shielding were absent, then the \(\gamma\gamma\to\pi^0\pi^0\) cross section (see Fig. 1b) would be about 100 nb rather than 10 nb due to the \(\pi^+\pi^-\) loop mechanism of the \(\sigma\) decay [12]. According to [18],

\[
T_{0}^\prime(s) = T_{0}^\prime(s) + e^{2i\delta^0_B(s)}T_{\text{res}}(s),
\]

\[
T_{K+K-\to\pi+\pi-}(s) = e^{i\delta^0_B(s) + \delta^0_K(s)}T_{\text{res}}^{K+K-\to\pi+\pi-}(s),
\]

and

\[
T_{0}^\prime(s) = \{\exp[2i\delta^0_B(s)] - 1\}/[2i\rho_\pi(s)],
\]

where \(\delta^0_B(s)\) and \(\delta^0_K(s)\) are the phases of the elastic S-wave background in the \(\pi\pi\) and \(K\bar{K}\) channels with \(I = 0\), respectively. The amplitudes of the \(\sigma\) (600) – \(f_0(980)\) resonance complex have the form [13,18]

\[
T_{\text{res}}(s) = (\eta^0(s) \exp[2i\delta^0(s)] - 1)/[2i\rho_\pi(s)] = \frac{g_{\sigma\pi+\pi-} - \Delta_\sigma(s)}{32\pi[D_\sigma(s)D_{f_0(s)} - \Pi_{f_0\sigma}(s)]},
\]

\[
T_{\text{res}}^{K+K-\to\pi+\pi-}(s) = \frac{g_{\pi\pi\to\pi^+\pi^-} - \Delta_\pi(s)}{16\pi[D_\sigma(s)D_{f_0(s)} - \Pi_{f_0\sigma}(s)]},
\]

\[
M_{\text{res}}(s) = s e^{i\delta^0_B(s)} \frac{g_{\sigma\gamma\gamma}^0D_{f_0(s)} + g_{f_0\gamma\gamma}^0\Delta_\sigma(s)}{D_\sigma(s)D_{f_0(s)} - \Pi_{f_0\sigma}(s)},
\]

where \(\Delta_{f_0}(s) = D_{f_0}(s)g_{\sigma+\pi^-} + \Pi_{f_0}(s)g_{\sigma+\pi^-} - \Delta_\sigma(s) = D_{f_0}(s)g_{\sigma+\pi^-} + \Pi_{f_0}(s)g_{\sigma+\pi^-}\), and \(\eta^0(s) = \delta^0_B(s) + \delta^0\res(s)\). The expressions presented in [18] were used for \(\delta^0_B(s)\), propagators \(1/D_\sigma(s)\) and \(1/D_{f_0(s)}\) of the \(\sigma\) resonance and \(f_0(980)\) resonances, respectively, and the matrix element of the polarization operator \(\Pi_{f_0\sigma}(s)\). The values of the parameters in the strong amplitudes \((m_\pi, g_{\pi+\pi^-}, g_{f_0\gamma\gamma}, ... )\) correspond to variant 1 from Table 1 in [18].

Thus, according to Eqs. (1), (3), and (7), the \(\sigma\) (600) – \(\gamma\gamma\) and \(f_0(980)\) – \(\gamma\gamma\) decays are described by the triangle \(\pi^+\pi^-\) and \(K^+\bar{K}^-\) loop diagrams (the resonances – \(\pi^+\pi^-\), \(K^+\bar{K}^-\) – \(\gamma\gamma\)), which correspond to the four-quark transitions [12,13], and by the direct coupling constants of the resonances with the photons \(g_{\sigma\gamma\gamma}^0\) and \(g_{f_0\gamma\gamma}^0\) [9–14].

The amplitudes of the production of the \(f_2(1270)\) resonance in Eqs. (2) and (4), \(M_{\gamma\gamma\to f_2(1270)\to\pi^+\pi^-}(s) = M_{\gamma\gamma\to f_2(1270)\to\pi^0\pi^0}(s)\), have the form

\[
M_{\gamma\gamma\to f_2(1270)\to\pi^+\pi^-}(s) = 6\pi\theta^2(\theta)M_{\gamma\gamma\to f_2(1270)\to\pi^0\pi^0}(s).
\]
\( \sqrt{s} G_2(s) \sqrt{2\Gamma_{f_2-\pi\pi}(s)/3} / [m_{f_2}^2 - s - i\sqrt{3}\Gamma_{f_2}^{\text{tot}}(s)], \)

where

\[ G_2(s) = \sqrt{\Gamma_{f_2-\gamma\gamma}(s) + \frac{M_{\text{Born}}^2(s)}{16\pi}} \sqrt{\frac{2}{3} \rho_{\pi^+}(s) \Gamma_{f_2-\pi\pi}(s)}, \]

\[ \Gamma_{f_2}^{\text{tot}}(s) = \Gamma_{f_2-\pi\pi}(s) + \Gamma_{f_2-\gamma\gamma}(s) + \Gamma_{f_2-4\pi}(s). \]

By definition

\[ \Gamma_{f_2-\gamma\gamma}(s) = |G_2(s)|^2 \quad \text{and} \quad \Gamma_{f_2}^{(0)}(s) = \frac{m_{f_2}^2}{\sqrt{s}} \Gamma_{f_2-\gamma\gamma}(s) \frac{s^2}{m_{f_2}^2}. \]

Here, the factor \( s^2 \) and the factor \( s \) in Eq. (7) appear due to the gauge invariance. The second term in \( G_2(s) \) corresponds to the \( f_2(1270) \to \pi^+\pi^- \to \gamma\gamma \) transition with real pions in the intermediate state and ensures the satisfaction of the Watson theorem for the \( \gamma\gamma \to \pi\pi \) amplitude with \( \lambda = \ell = 2 \) and \( J = 0 \) below the first inelastic threshold. This makes a small contribution (less than 6%) to \( \Gamma_{f_2-\gamma\gamma}(m_{f_2}^2) \) [13]. It is commonly accepted that the quark-antiquark transition \( q\bar{q} \to \gamma\gamma \), i.e., the \( \Gamma_{f_2-\gamma\gamma}(m_{f_2}^2) \) contribution dominates in the \( f_2(1270) \to \gamma\gamma \) decay. As shown in [12,13] and noted below, the situation for the scalar mesons is opposite.

The leading contribution to \( \Gamma_{f_2}^{\text{tot}}(s) \) comes from the partial decay width \( f_2(1270) \to \pi\pi, \)

\[ \Gamma_{f_2-\pi\pi}(s) = \frac{\Gamma_{f_2}^{\text{tot}}(m_{f_2}^2)}{B(f_2 \to \pi\pi)} \frac{m_{f_2}^2}{s} \frac{\rho_{\pi^+}(s)}{D_2(q_{\pi^+}(s) R_{f_2})}. \]

where \( D_2(x) = 1/(9 + 3x^2 + x^4), q_{\pi^+}(s) = \sqrt{s} \rho_{\pi^+}(s)/2, R_{f_2} \) is the interaction radius, and \( B(f_2 \to \pi\pi) = 0.847 \). Small contributions from \( \Gamma_{f_2-\gamma\gamma}(s) \) and \( \Gamma_{f_2-4\pi}(s) \) are the same as in [13]. The parameter \( R_{f_2} \) controls the relative shape of the wings of the \( f_2(1270) \) resonance and is important particularly for the approximation of the data with small errors.

We use the following notation and normalizations for the cross sections

\( \sigma(\gamma\gamma \to \pi^+\pi^+; |\cos \theta| \leq 0.6) \equiv \sigma = \sigma_0 + \sigma_2 \quad \text{and} \quad \sigma(\gamma\gamma \to \pi^0\pi^0; |\cos \theta| \leq 0.8) \equiv \sigma = \sigma_0 + \sigma_2, \)

where

\[ \sigma_\lambda = \frac{\rho_{\pi^\pm}(s)}{64\pi s} \int_{-0.6}^{0.6} |M_\lambda(\gamma\gamma \to \pi^+\pi^+; s, \theta)|^2 d\cos \theta \quad \text{and} \quad \sigma_\lambda = \frac{\rho_{\pi^\pm}(s)}{128\pi s} \int_{-0.8}^{0.8} |M_\lambda(\gamma\gamma \to \pi^0\pi^0; s, \theta)|^2 d\cos \theta. \]

First, we consider the approximation of the data only on the cross section for the \( \gamma\gamma \to \pi^0\pi^0 \) reaction (see Fig. 1b); as mentioned above, the background situation in this channel is more pure than in the \( \gamma\gamma \to \pi^+\pi^- \) one. The solid line in Fig. 1b, which well describes these data, corresponds to the following model parameters:

- \( m_{f_2} = 1.269 \) GeV,
- \( \Gamma_{f_2}^{\text{tot}}(m_{f_2}^2) = 0.182 \) GeV, \( R_{f_2} = 8.2 \) GeV^{-1},
- \( \Gamma_{f_2-\gamma\gamma}(m_{f_2}^2) = 3.62 \) keV,
- \( \sigma_{\sigma\gamma} = 0.969 \) GeV,
- \( g_{\sigma\gamma}^{(0)} = 0.536 \) GeV^{-1}, and \( g_{\sigma\gamma}^{(0)} = 0.652 \) GeV^{-1}. The approximation indicates that the direct constants \( g_{\sigma\gamma}^{(0)} \) and \( g_{\sigma\gamma}^{(0)} \) are small in agreement with the prediction in [2]:

\( \Gamma_{f_2-\gamma\gamma}^{(0)}(m_{f_2}^2) = |m_{f_2}^2 g_{\sigma\gamma}^{(0)}(m_{f_2}^2)/(16\pi m_\sigma)| = 0.012 \) keV and

\( \Gamma_{f_2-\gamma\gamma}^{(0)}(m_{f_2}^2) = |m_{f_2}^2 g_{\sigma\gamma}^{(0)}(m_{f_2}^2)/(16\pi m_\sigma)| = 0.008 \) keV. In turn, this indicates the dominance of the \( \pi^+\pi^- \) and \( K^+K^- \) loop mechanisms of the coupling of \( \sigma(600) \) to \( f_0(980) \) with photons. Indeed, according to estimates [11,12], the width of the \( \sigma(600) \to \pi^+\pi^- \to \gamma\gamma \) decay through the \( \pi^+\pi^- \) loop mechanism is approximately 1–1.75 keV in the region \( 0.4 < \sqrt{s} < 0.5 \) GeV [12], and the width of the \( f_0(980) \to K^+K^- \to \gamma\gamma \) decay through the \( K^+K^- \) loop mechanism after averaging over the resonance mass distribution is approximately 0.15–0.2 keV [11].

However, such an approximation of the \( \gamma\gamma \to \pi^0\pi^0 \) cross section leads to a contradiction with the data for \( \gamma\gamma \to \pi^+\pi^- \) (see the solid line for \( \sigma = \sigma_0 + \sigma_2 \) in Fig. 1a). This is associated with a large Born contribution to \( \sigma_2 \) and a strong constructive (destructive) interference of this contribution with the contribution from the \( f_2(1270) \) resonance at \( \sqrt{s} < m_{f_2} \). Note that these contributions are absent in the \( \gamma\gamma \to \pi^0\pi^0 \) reaction. The problem of the joint description of the data for the \( \gamma\gamma \to \pi^+\pi^- \) and \( \gamma\gamma \to \pi^0\pi^0 \) reactions was pointed out in [13], where the solution of this problem was proposed. The situation can be significantly corrected by multiplying the \( \gamma\gamma \to \pi^+\pi^- \) Born amplitudes for photon pairs, \( M_\lambda^{\text{Born}}(s, \theta) \), by the common suppressing form factor \( G(t, u) \) [7,8,10,13,16,21], where \( t \) and \( u \) are
the normal Mandelstam variables for the $\gamma\gamma \to \pi^+\pi^-$ process. To demonstrate this, we use the following expression proposed in [21]:

$$G(t, u) = \frac{1}{s} \left[ \frac{m^2_\pi^+ - t}{1 - (u - m^2_\pi^+)/x^2_1} + \frac{m^2_\pi^- - u}{1 - (t - m^2_\pi^-)/x^2_2} \right],$$

where $x_1$ is the free parameter. This ansatz is acceptable in the physical region of the $\gamma\gamma \to \pi^+\pi^-$ reaction. Changing $m_\pi^+$ to $m_{K^+}$ and $x_1$ to $x_2$, we also obtain the form factor for the Born amplitudes of the $\gamma\gamma \to K^+K^-$ reaction.

The solid lines for the cross sections $\sigma = \sigma_0 + \sigma_2$ and $\tilde{\sigma} = \tilde{\sigma}_0 + \tilde{\sigma}_2$ in Figs. 3a and 3b, respectively, demonstrate the joint approximation of the data for the $\gamma\gamma \to \pi^+\pi^-$ reaction in the region $0.85 < \sqrt{s} < 1.5\text{GeV}$ and for the $\gamma\gamma \to \pi^0\pi^0$ reaction in the region $2m_\pi < \sqrt{s} < 1.5\text{GeV}$ including the form factors modifying the Born contributions for point particles. The resulting description is more than satisfactory, but only with the inclusion of the total (statistical and systematic) errors in the Belle data, which are shown in Figs. 3a and 3b in the form of shaded bands. We believe that this treatment is justified. The statistical errors of two Belle measurements are small so that it is impossible to obtain the formally acceptable $\chi^2$ values in the joint approximation of the $\pi^+\pi^-$ and $\pi^0\pi^0$ data without the inclusion of the systematic errors. The lines in Figs. 3a and 3b correspond to the parameters $m_{f_2} = 1.272$ GeV, $\Gamma^{\text{tot}}_{f_2}(m^2_{f_2}) = 0.196$ GeV, $R_{f_2} = 8.2$ GeV$^{-1}$, $\Gamma_{f_2 \to \gamma\gamma}(m_{f_2}) = 3.84$ keV, $m_{f_0} = 0.969$ GeV, $g_{\gamma\gamma} = 0.049$ GeV$^{-1}$ ($\Gamma^{(0)}_{\gamma\gamma}(m^2_{\pi^\pm})$ is negligible), $g_{f_0\gamma\gamma} = 0.718$ GeV$^{-1}$ ($\Gamma^{(0)}_{f_0 \to \gamma\gamma}(m^2_{f_0}) \approx 0.01$ keV), $x_1 = 0.9$ GeV and $x_2 = 1.75$ GeV. A comparison of Figs. 1b and 3b shows that the effect of the form factors on the cross section for the $\gamma\gamma \to \pi^0\pi^0$ reaction is weak in contrast to the cross section for the $\gamma\gamma \to \pi^+\pi^-$ (see Figs. 1a and 3a). We emphasize that our conclusions on the mechanisms of the two-photon decays (productions) of the $\sigma(600)$ and $f_0(980)$ resonances remain valid.

It is interesting to consider the $\gamma\gamma \to \pi^+\pi^-$ cross section attributed only to the resonance contributions, i.e.,

$$\sigma_{\text{res}}(\gamma\gamma \to \pi^+\pi^-; s) = \frac{\rho_{\pi^+\pi^-}(s)}{32\pi s} |\tilde{T}_{\pi^+\pi^-}(s)|^2 \left[ \tilde{I}_{\pi^+\pi^-}(s) e^{-2\delta_{\pi^+\pi^-}(s)} \right] + \tilde{I}_{K^+K^-}(s) T_{K^+K^- \to \pi^+\pi^-}(s) + M_{\text{res}}^{\text{direct}}(s),$$

[see Eqs. (1) and (5)-(7)]. Here, the superscript ff means that the functions $\tilde{T}(s)$ are obtained with the inclusion of the form factors [10]. The cross section $\sigma_{\text{res}}(\gamma\gamma \to \pi^+\pi^-; s)$ has a pronounced peak near 1 GeV from the $f_0(980)$ resonance, which is due primarily to the contribution from the $\gamma\gamma \to K^+K^- \to \pi^+\pi^-$ transition. Following [9,11], we
determine the width of the $f_0(980) \rightarrow \gamma\gamma$ decay averaged over the resonance mass distribution in the $\pi\pi$ channel:

$$\langle \Gamma_{f_0 \rightarrow \gamma\gamma} \rangle_{\pi\pi} = \int_{0.8 \text{ GeV}}^{1.1 \text{ GeV}} \frac{3s}{8\pi^2} \sigma_{\text{res}}(\gamma\gamma \rightarrow \pi^+\pi^-;s) \, d\sqrt{s}. \quad (8)$$

This quantity is an adequate characteristic of the coupling of the $f_0(980)$ resonance with a $\gamma\gamma$ pair [11]. For the present joint approximation, $\langle \Gamma_{f_0 \rightarrow \gamma\gamma} \rangle_{\pi\pi} \approx 0.19 \text{ keV}$. Accepting that $2m_\pi < \sqrt{s} < 0.8 \text{ GeV}$ is the region of the wide $\sigma(600)$ resonance, we obtain $\langle \Gamma_{\sigma \rightarrow \gamma\gamma} \rangle_{\pi\pi} \approx 0.45 \text{ keV}$ by analogy with Eq. (8).

Note that the contributions from the $\omega(782)$ and $h_1(1170)$ exchanges to the $S$-wave amplitude of the $\gamma\gamma \rightarrow \pi^0\pi^0$ reaction have opposite signs and cancel each other.

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