Response to rotating forcing of the von-Kármán disk boundary layer

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Abstract. In the present experimental investigation of the three-dimensional boundary layer due to a disk rotating in otherwise still air, the aim is to study the response to a radially localized perturbation applied with a prescribed relative frequency with respect to the disk.

The response to localized rotating forcing is measured with a hot-wire probe. The rotation rate of the forcing element is controlled independently of the disk rotation rate, and the dynamics of the spatial response is studied as a function of the ratio between the two rotation rates. The theoretically expected disturbance trajectories are derived from an instability analysis based on the exact local dispersion relations computed from the complete linearized Navier–Stokes equations. Theoretical predictions and experimental measurements are shown to be in good agreement.

1. Context

The flow produced by a disk rotating in otherwise still fluid has served as the canonical configuration for the study of three-dimensional boundary layers ever since von Kármán (1921) obtained the basic flow as an exact similarity solution of the Navier–Stokes equations. It is known to display a sharp transition from laminar to turbulent flow at a nondimensional radius $R \approx 500$ (Reed & Saric, 1989; Saric et al., 2003). Using linear stability analysis, this location was found by Lingwood (1995) to precisely coincide with the onset of local absolute instability. More recently (Pier, 2003b), a fully nonlinear analysis and a secondary stability analysis have further contributed to the understanding of the complex dynamics prevailing near the transition radius and have led to the design of an open-loop control strategy (Pier, 2003a) that has been theoretically shown (Pier, 2007) to be applicable in this context. The aim of this method is to delay onset of secondary perturbations, and thus transition, by a controlled modification of the primary nonlinear state.

In the above context, the aim of the experimental work presented here is to investigate the response to a radially localized perturbation applied with a prescribed relative frequency with respect to the disk.
2. Experimental setup

The experimental setup consists of a 50 cm diameter glass disk that is rotated at constant angular speeds, up to 1500 rpm. Disk rotation and viscosity produce a three-dimensional boundary layer of constant thickness $\delta = \sqrt{\nu/\Omega_d}$ (a typical value is $\delta \simeq 350 \mu m$), where $\nu$ is the kinematic viscosity and $\Omega_d$ the disk rotation rate. In the entire study, spatial and temporal coordinates are nondimensionalized based on $\delta$ and $\Omega_d$ respectively.

Forcing is applied via pins held by a rotating hollow cylinder placed above the disk which extend down into the boundary layer (see figure 1). The cylinder holding the pins is rotated by a second motor at an angular speed that can be varied independently of the disk rotation rate.

Flow measurements are carried out by hot-wire anemometry. The hot wire is positioned by a computer-controlled high-precision two-axes traversing mechanism and provides azimuthal velocity time series to characterize the entire flow. Different geometries (spherical, cylindrical, thin rod) for the pin heads are used to investigate their influence on the flow dynamics and in particular to vary the amplitude of the applied forcing.

3. Response to rotating forcing

In order to investigate the response of the boundary layer to localized rotating forcing, the rotation rate of the disk was chosen so that the non-dimensional forcing radius was just below the theoretical onset of convective instability, allowing us to study the development of the forced response over the whole convectively unstable region. The forcing elements were rotated at various frequencies $\omega_f$ in the range $0 \leq \omega_f \leq 1.0$. For $\omega_f = 1$ the forcing is stationary with respect to the disk surface, while the values $\omega_f < 1$ correspond to forcing in relative motion with respect to the disk.

Time-series of azimuthal velocity are recorded at different radial and axial locations for a large number of forcing revolutions. To extract those components that are periodic with respect to the forcing, phase-locked averages are computed, which can be interpreted as the mean perturbed flow in the frame of reference of the forcing device.

The data plotted in figure 2 show how the perturbation produced at $R_f = 250$ evolves while propagating radially outwards. The perturbation ‘humps’ are seen to follow a nearly linear trajectory in the $(R, \theta')$-plane. These slopes are found to increase with $\omega_f$ and to agree extremely well with predictions based on linear theory, as described in the next section.
radially outwards, away from the forcing. The spatial branches are governed by the dispersion relation for complex response of the flow to radially localized harmonic forcing. Mathematically, this involves solving the eigenvalue problem obtained after linearization by solving the complete Navier–Stokes equations around the corresponding three-dimensional boundary layer velocity profiles.

For simplicity we assume a local linear theory at a given radial location $R$ by solving the eigenvalue problem obtained after linearization of the complete Navier–Stokes equations around the corresponding three-dimensional boundary layer velocity profiles. In classical fashion, the total instantaneous flow field at a given radial location $R$ is separated into basic and perturbation quantities. Since the radial values of interest are large compared to the boundary layer thickness, the assumption of weak radial development is legitimate. Small-amplitude perturbations are then written in normal-mode form as $u(z) \exp i(\alpha r + \beta \theta - \omega t)$, where the radial wavenumber $\alpha$, the azimuthal mode number $\beta$ and the frequency $\omega$ are governed by the local dispersion relation

$$\omega = \Omega(\alpha, \beta; R),$$

computed at each radial location $R$ by solving the eigenvalue problem obtained after linearization of the complete Navier–Stokes equations around the corresponding three-dimensional boundary layer velocity profiles.

In the present context, it is the spatial problem that is of particular interest: the spatial response of the flow to radially localized harmonic forcing. Mathematically, this involves solving the dispersion relation for complex $\alpha$ with given real values of $\beta$ and $\omega$. This process yields two spatial branches $\alpha^\pm(\beta, \omega; R)$, and it is the $\alpha^+$ branch that pertains to a response developing radially outwards, away from the forcing.

For a localized forcing element at radial position $r_f$ rotating at a constant angular frequency $\omega_f$, the instantaneous location of the forcing element is at $r = r_f$ and $\theta = \omega_f t$. For simplicity we assume a local linear theory at $R = r_f$ and consider that the perturbation due to the forcing is steady in the frame of reference of the forcing element. For $r > r_f$, the full spatial response is then obtained as

$$u(r, \theta, z, t) \simeq \int_{\beta} d\beta \, u(z; \beta) \exp i[\alpha(\beta)(r - r_f) + \beta \theta'],$$

where $\alpha(\beta) \equiv \alpha^+(\beta, \omega = \omega_f; R = r_f)$ and $\theta' = \theta - \omega_f t$ is the azimuthal coordinate in the forcing frame; $u$ represents the associated spatial eigenmode.

Recognizing that this perturbation in $(\theta', r)$-space (figure 3a) is formally similar to a spatio-temporal wavepacket propagating in $(x, t)$-space (figure 3b) and applying a classic stationary-phase method (Huerre & Monkewitz, 1990), the maximum of the forced response is then expected to follow the ray $\Delta \theta'/\Delta r = \kappa_{max}$ where the slope $\kappa_{max}$ is given by

$$\kappa_{max} = -\frac{d\alpha}{d\beta}(\beta_{max}) \quad \text{with} \quad \frac{d\alpha}{d\beta}(\beta_{max}) = 0.$$
\[ \frac{\theta'}{r} = \kappa_t \quad \text{and} \quad \kappa_l \]

(a) Sketch of wavepacket in \((\theta', r)\)-space produced by a rotating forcing element. 
(b) Typical impulse response in \((x, t)\)-space.

5. Discussion

Figure 4 shows the slopes of the experimentally measured trajectories (open red symbols) plotted as a function of the forcing frequency, together with results derived from linear theory: the solid black line represents \(\kappa_{\text{max}}\) and green and blue lines are the similarly computed leading and trailing edges of a linear wavepacket. Despite significant nonlinear effects in the experiment, the maximum of the forced perturbation is found to be in good agreement with \(\kappa_{\text{max}}\), except for lower values of \(\omega_f\).

By using a forcing element fixed with respect to the disk surface, Jarre et al. (1996) have been able to measure the trajectory of the disturbance generated by such a roughness element.

Figure 4. Experimentally determined slopes \(\kappa = \Delta \theta' / \Delta R\) of the trajectories of perturbation maximum (open red symbols) plotted together with the theoretical predictions (black line) and leading and trailing edges (green and blue lines). Black full symbol indicates data point obtained at \(\omega_f = 1\) by Jarre et al. (1996).
Their data, corresponding to $\omega_f = 1$, is indicated by the black full symbol in figure 4. Our data are hence seen to be in excellent agreement and to extend this result to a wide range of relative forcing frequencies.

In order to study the influence of nonlinearities, investigations are in progress with smaller forcing elements.

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