Chiral Extrapolation of Lattice Data for Heavy Baryons

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Abstract

The masses of heavy baryons containing a $b$ quark have been calculated numerically in lattice QCD with pion masses which are much larger than its physical value. In the present work we extrapolate these lattice data to the physical mass of the pion by applying the effective chiral Lagrangian for heavy baryons, which is invariant under chiral symmetry when the light quark masses go to zero and heavy quark symmetry when the heavy quark masses go to infinity. A phenomenological functional form with three parameters, which has the correct behavior in the chiral limit and appropriate behavior when the pion mass is large, is proposed to extrapolate the lattice data. It is found that the extrapolation deviates noticeably from the naive linear extrapolation when the pion mass is smaller than about 500MeV. The mass differences between $\Sigma_b$ and $\Sigma^*_b$ and between $\Sigma_b^{(+)}$ and $\Lambda_b$ are also presented. Uncertainties arising from both lattice data and our model parameters are discussed in detail. We also give a comparison of the results in our model with those obtained in the naive linear extrapolations.

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I. Introduction

The spectrum of some hadrons has been calculated numerically in lattice QCD over the past few years. These hadrons include light mesons and baryons [1], heavy mesons [2, 3], and heavy baryons [2, 4]. Using non-relativistic QCD (NRQCD) on the lattice [5] for heavy quarks and the tadpole-improved clover action for light quarks, the authors of Refs.[2, 3] studied extensively the spectra of heavy mesons and heavy baryons (including doubly heavy baryons). These lattice data were obtained in the region where the mass of the pion is much larger than the physical mass of the pion. Hence one needs to extrapolate these data to the physical pion mass in order to obtain the heavy hadron masses in the real world. Naively, this is done by linear extrapolations which are inconsistent with the model independent, non-analytic behavior of hadron properties in the chiral limit. In order to overcome this problem, pion-hadron loops are included in the study of light hadron properties [6, 7, 8, 9]. This yields the correct leading and next-to-leading non-analytic terms in the light quark masses and leads to rapid variation at small pion masses. In general, lattice data extrapolated to the physical pion mass this way yield quite different results from linear extrapolations. Based on this, we considered previously the chiral extrapolation of the lattice data for heavy $D$ and $B$ mesons and discussed the important hyperfine splittings [10]. Here we generalize our approach to the case of heavy baryons and extrapolate the lattice data for heavy $b$-baryons obtained in Ref.[2].

We work in two opposite limits of quark masses. One is the zero quark mass limit while the other is the infinite quark mass limit. When the masses of the light quarks, $u$, $d$, and $s$, go to zero the QCD Lagrangian has a chiral $SU(3)_L \times SU(3)_R$ symmetry, which is spontaneously broken into $SU(3)_V$ plus eight Goldstone bosons. When the masses of the heavy quarks $c$ and $b$ go to infinity, we have an effective theory, heavy quark effective theory (HQET), which is invariant under heavy quark flavor and
heavy quark spin transformations, $SU(2)_f \times SU(2)_s$. Thus the interactions of heavy baryons with the light pseudoscalar mesons should be described by an effective chiral Lagrangian for heavy baryons which is invariant under both $SU(3)_L \times SU(3)_R$ and $SU(2)_f \times SU(2)_s$ transformations. This chiral Lagrangian will be applied in the small pion mass region while we extrapolate the lattice data to the physical pion mass.

The remainder of this paper is organized as follows. In Section II we give a brief review of the chiral Lagrangian for heavy baryons and including the propagators of heavy baryons. In Section III we apply this Lagrangian to calculate pion loop contributions to the self-energy of heavy baryons. Then we propose a phenomenological functional form with three parameters for extrapolating the lattice data to the physical region. In Section IV we use this form to fit the lattice data and give numerical results. Finally, Section V contains a summary and discussion.

II. Chiral perturbation theory for heavy baryons

When the light quark mass, $m_q$, approaches zero, the QCD Lagrangian possesses an $SU(3)_L \times SU(3)_R$ chiral symmetry. The light pseudo-Goldstone bosons associated with spontaneous breaking of chiral symmetry can be incorporated into a $3 \times 3$ matrix

$$\Sigma = \exp \left( \frac{2iM}{f_\pi} \right),$$

where $f_\pi$ is the pion decay constant, $f_\pi = 132$MeV, and $M$ is a matrix which includes the eight Goldstones

$$M = \begin{pmatrix}
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & \pi^+ & K^+ \\
\pi^- & -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & K^0 \\
K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}} \eta
\end{pmatrix}. \quad (2)$$

Under $SU(3)_L \times SU(3)_R$ transformations, $\Sigma$ is required to transform linearly,

$$\Sigma \to L\Sigma R^+, \quad (3)$$

where $L \in SU(3)_L$ and $R \in SU(3)_R$. 
While discussing the interactions of Goldstone bosons with other matter fields it is convenient to introduce

$$\xi = \sqrt{\Sigma}. \quad (4)$$

Under $SU(3)_L \times SU(3)_R$ transformations

$$\xi \rightarrow L\xi U^+ = U\xi R^+,$$  

where the unitary matrix $U$ is a complicated nonlinear function of $L$, $R$, and the Goldstone fields, and is invariant under the parity transformation.

A heavy baryon is composed of a heavy quark $Q$ ($Q = b, \text{ or } c$) and two light quarks $q_a q_b$ ($a$ ($b$) equals 1, 2, 3 for $u$, $d$, $s$ quarks, respectively). When the heavy quark mass, $m_Q$, is much larger than the QCD scale, $\Lambda_{\text{QCD}}$, the light degrees of freedom in a heavy baryon become blind to the flavor and spin quantum numbers of the heavy quark because of the $SU(2)_f \times SU(2)_s$ symmetries. Therefore, the light degrees of freedom have good quantum numbers which can be used to classify heavy baryons. The angular momentum and parity $J^P$ of the two light quarks may be $0^+$ or $1^+$, which correspond to $SU(3)_{L+R}$ antitriplet and sextet, respectively. The lowest-lying heavy baryons in the $\bar{3}$ representation have spin 1/2, and are denoted by fields which destroy these baryons, $T_a$ ($T_3 = \Lambda_Q$, $T_{1,2} = \Xi'_Q$). The lowest-lying heavy baryons in the 6 representation have spin 1/2 or 3/2, and are denoted by field operators $S^{ab}$ and $S^{ab}_\mu$, respectively, where $S^{(*)}_{(a)}^{11,12,22} = \Sigma^{(*)}_Q$, $S^{(*)}_{(a)}^{13,23} = \Xi^{(*)}_Q$, and $S^{(*)}_{(a)}^{33} = \Omega^{(*)}_Q$. $T_a$ transforms under $SU(3)_L \times SU(3)_R$ as

$$T_a \rightarrow T_b U^+_{ba}, \quad (6)$$

and under heavy quark spin symmetry

$$T_a \rightarrow ST_a, \quad (7)$$

where $S \in SU(2)_s$. $T_a$ also satisfies

$$T_a = \bar{\gamma}T_a. \quad (8)$$
where $v$ is the velocity of the heavy baryon.

It is convenient to combine $S^{ab}$ and $S^{*ab}_\mu$ into the field $S^{ab}_\mu$ [11]

$$S^{ab}_\mu = \frac{1}{\sqrt{3}}(\gamma_\mu + v_\mu)\gamma_5 S^{ab} + S^{*ab}_\mu. \quad (9)$$

Then under $SU(3)_L \times SU(3)_R$

$$S^{ab}_\mu \rightarrow U^a_c U^b_d S^{cd}_\mu, \quad (10)$$

while under heavy quark spin symmetry

$$S^{ab}_\mu \rightarrow 3 S^{ab}_\mu. \quad (11)$$

$S^{ab}_\mu$ satisfies the constraints

$$S^{ab}_\mu = \gamma^5 S^{ab}, \quad v^{\mu} S^{ab}_\mu = 0. \quad (12)$$

It is convenient to introduce a vector field $V^{\mu}_{ab}$,

$$V^{\mu}_{ab} = \frac{1}{2}(\xi^+ \partial^\mu \xi + \xi \partial^\mu \xi^+)_{ab}. \quad (13)$$

and an axial-vector field $A^{\mu}_{ab}$,

$$A^{\mu}_{ab} = \frac{i}{2}(\xi^+ \partial^\mu \xi - \xi \partial^\mu \xi^+)_{ab}. \quad (14)$$

Under $SU(3)_L \times SU(3)_R$, $V^{\mu} \rightarrow UV^{\mu}U^+ + U\partial^\mu U^+$, and $A^{\mu} \rightarrow UA^{\mu}U^+$. Defining the covariant derivative

$$(D^{\mu} T)_a = \partial^{\mu} T_a - T_b (V^{\mu})_a^b, \quad (15)$$

and

$$(D^{\mu} S^b_\nu)^{ab} = \partial^{\mu} S^{ab}_\nu + (V^{\mu})_a^c S^{cb}_\nu + (V^{\mu})^b_c S^{ac}_\nu, \quad (16)$$

we can show that under $SU(3)_L \times SU(3)_R$, $D^{\mu} T_a \rightarrow D^{\mu} T_b (U^+)_a^b$ and $D^{\mu} S^{ab}_\nu \rightarrow U^a_c U^b_d D^{\mu} S^{cd}_\nu$.

In the limit where light quarks have zero mass and heavy quarks have infinite mass, the Lagrangian for the strong interactions of heavy baryons with Goldstone
pseudoscalar bosons should be invariant under both chiral symmetry and heavy quark symmetry. It should also be invariant under Lorentz and parity transformations as required in general. The most general form for the Lagrangian satisfying these requirements is [12]

$$\mathcal{L} = i\bar{T}^a v_\nu (D^\nu T)_a - i\bar{S}_a^{\mu} v_\nu (D^\nu S_{\mu})^a_{\nu} + \Delta M \bar{S}_a^{\mu} S_{\mu}^a + ig_1 \epsilon_{\mu\sigma\lambda} \bar{S}_a^{\mu} v^\nu (A^\sigma)_b \epsilon^{abc} S_{\lambda}^{bc} - \frac{1}{3} \epsilon_{\mu\nu\rho\lambda} \bar{S}_a^{\mu} v^{\nu} - \frac{2}{3} v_\mu v_\nu,\] (17)

where $g_1$ and $g_2$ are coupling constants describing the interactions between heavy baryons and Goldstone bosons and $\Delta M$ is the mass difference between sextet and antitriplet heavy baryons in the heavy quark limit. As a consequence of heavy quark symmetry, $g_1$ and $g_2$ are universal for different heavy baryons. Since they contain information about the interactions at the quark and gluon level, they cannot be fixed from chiral perturbation theory, but should be determined by experiments.

In the limit $m_Q \to \infty$, the propagator for $\Lambda_Q$ is

$$\frac{i}{v \cdot p - \frac{1}{2} \phi},$$

where $p$ is the residual momentum of the heavy baryon. It can also be shown that for $\Sigma_Q$, the propagator is

$$\frac{i}{v \cdot p - \frac{1}{2} \phi},$$

and the propagator for $\Sigma^*_Q$ is

$$\frac{1 + \frac{1}{2} \phi}{v \cdot p - \frac{1}{2} \phi} \left( \frac{1}{2} \epsilon_{\mu\nu\rho\lambda} \bar{S}_a^{\mu} v^{\nu} - \frac{2}{3} v_\mu v_\nu,\right) + \frac{1 + \frac{1}{2} \phi}{v \cdot p - \frac{1}{2} \phi}.$$

In the limit $m_Q \to \infty$, there is no mass difference between $\Sigma_Q$ and $\Sigma^*_Q$.

In HQET, the leading term which is responsible for a mass difference between $\Sigma_Q$ and $\Sigma^*_Q$ is the color-magnetic-moment operator, $\frac{1}{m_Q} \bar{h}_v \sigma_{\mu\nu} G_{\mu\nu} h_v$ (where $h_v$ is the heavy quark field operator in HQET and $G_{\mu\nu}$ is the gluon field strength tensor). This term is singlet under $SU(3)_L \times SU(3)_R$ and leads to the following correction term to $\mathcal{L}$ in Eq.(17):

$$i \frac{\alpha}{m_Q} \bar{S}_a^{\mu} \sigma_{\mu\nu} S_{\nu}^{ab},$$ (18)
where $\alpha$ is a constant which also contains interaction information at the quark and gluon level, and which is the same for $\Sigma_Q$ and $\Sigma_Q^*$ at the tree level because of heavy quark symmetry. When QCD loop corrections are included, $\alpha$ depends on $m_Q$ logarithmically.

The term (18) enhances the mass of $\Sigma_Q^*$ by $\alpha/m_Q$ and lowers that of $\Sigma_Q$ by $2\alpha/m_Q$. Therefore, the propagators for $\Sigma_Q$ and $\Sigma_Q^*$ become

$$
\frac{i}{v \cdot p - \Delta M + \frac{2\alpha}{m_Q} \frac{1 + \not{v}}{2}},
$$

and

$$
\frac{1 + \not{v} - i(g_{\mu\nu} - \frac{1}{3}\gamma_\mu \gamma_\nu - \frac{2}{3}v_\mu v_\nu) 1 + \not{v}}{2} \frac{1 + \not{v}}{v \cdot p - \Delta M - \frac{\alpha}{m_Q} \frac{1 + \not{v}}{2}},
$$

respectively.

Substituting $\xi = \exp(iM/f_\pi)$ into Eqs.(13, 14) and making a Taylor expansion we obtain the following expressions for $V_\mu$ and $A_\mu$:

$$
V_\mu = \frac{1}{2f_\pi^2} [M, \partial_\mu M] + O(M^4), \quad (19)
$$

$$
A_\mu = -\frac{1}{f_\pi} \partial_\mu M + O(M^3). \quad (20)
$$

Substituting Eq.(9) and Eqs.(19, 20) into Eq.(17) we have the following explicit form for the interactions of heavy baryons with Goldstone bosons:

$$
-\frac{i}{2f_\pi^2} \bar{T}^a[M, v \cdot \partial M]_a^b T_b + \frac{i}{2f_\pi^2} \left\{ \bar{S}_{ab}[M, v \cdot \partial M]_c^a S_{cb} + \bar{S}_{ab}[M, v \cdot \partial M]_c^b S^{ac} \right\} - \frac{i}{f_\pi} g_1 \epsilon_{\mu\nu\lambda\sigma} v^\nu (\partial^\sigma M)^a_b \times \left[ -\frac{1}{3} \bar{S}_{ac} \gamma_\mu \gamma_\lambda \gamma_\nu S^{bc} + \bar{S}_{ac} \gamma_\mu \gamma_\lambda \gamma_5 S^{bc} + \frac{1}{\sqrt{3}} \bar{S}_{ac} \gamma_\mu \gamma_\lambda S^{abc} - \frac{1}{\sqrt{3}} \bar{S}_{ac} \gamma_\mu \gamma_\lambda S^{abc} \right] - \frac{g_2}{f_\pi} \left[ \frac{1}{\sqrt{3}} \epsilon_{abc} \bar{T}^a (\partial^\mu M)_d^b (\gamma_\mu + v_\mu) \gamma_5 S^{cd} + \epsilon_{abc} \bar{T}^a (\partial^\mu M)_d^b S^{*cd} \right. \\
- \left. \epsilon_{abc} \bar{S}^{*cd} \gamma_5 (\gamma_\mu + v_\mu) (\partial_\mu M)_d^b T^a + \epsilon_{abc} \bar{S}^{*cd} (\partial_\mu M)_d^b T^a \right], \quad (21)
$$

where $O(M^3)$ terms are ignored.
Chiral symmetry can be broken explicitly by nonzero light quark masses. This leads to the following leading order terms in the explicit chiral symmetry breaking masses:

\[ \lambda_1 \bar{S}_{ab} \left( \xi m_q \xi + \xi^+ m_q \xi^+ \right) \xi_{ba} \bar{S}_{ac} \mu + \lambda_2 \bar{S}_{ab} \xi_{ba} \text{Tr}(mq \Sigma^+ + \Sigma m_q) \]

\[ + \lambda_3 \text{Tr} \left( \xi m_q \xi + \xi^+ m_q \xi^+ \right)_{ab} a \bar{T} b + \lambda_4 \text{Tr}(\bar{T} a T a) \text{Tr}(mq \Sigma^+ + \Sigma m_q), \]

(22)

where \( \lambda_i (i = 1, 2, 3, 4) \) are parameters which are also independent of the heavy quark mass in the limit \( m_Q \to \infty \).

III. Formalism for the extrapolation of lattice data for heavy baryon masses

From the chiral Lagrangian for the interactions of heavy baryons with light Goldstone bosons, Eq.(21), we can calculate pion loop contributions to the heavy baryon propagators near the chiral limit - i.e., when the pion mass is not far from the chiral limit. This leads to a dependence of the heavy baryon masses on the pion mass. We will concentrate on \( \Sigma_Q \) and \( \Sigma^*_Q \), but other heavy baryons can be treated in the same way.

From Eq.(21) we can see that there are four diagrams for pion loop corrections to the propagator of either \( \Sigma_Q \) or \( \Sigma^*_Q \), and three diagrams for \( \Lambda_Q \). These diagrams are shown in Fig. 1 for \( \Sigma_Q \), in Fig. 2 for \( \Sigma^*_Q \), and in Fig. 3 for \( \Lambda_Q \). It can be easily seen that Fig. 1(a), Fig. 2(a) and Fig. 3(a) do not contribute (this is because the integrand is of the form \( k^\mu f(k^2) \) where \( k \) is the momentum of the pion in the loop, and \( f(k^2) \) is a function of \( k^2 \) and we will not consider them further.

Fig. 1(b) arises from the \( \Sigma_Q \pi \Sigma_Q \) vertex. In momentum space it can be expressed as

\[ \frac{i}{v \cdot p - \Delta M + \frac{2\alpha}{m_Q}} \left( -i \Sigma_1 \right) \frac{i}{v \cdot p - \Delta M + \frac{2\alpha}{m_Q}} \left( 1 + \frac{i}{2} \right), \]

(23)

where \( p \) is the residual momentum of the heavy baryon \( \Sigma_Q \). From Eq.(21), Fig. 1(b)
takes the following form:
\[
- \frac{g_\pi^2}{18 f_\pi^2} \varepsilon_{\mu\nu\sigma\lambda} \varepsilon^{\mu\nu'\sigma'\lambda'} v^\nu v'^\nu' \left( \frac{1 + \gamma^\mu \gamma^\lambda}{2} + \frac{1 + \gamma^\mu' \gamma^\lambda'}{2} \right) 
\left( v \cdot p - \Delta M + \frac{2a}{m_Q} \right)^2 \int \frac{d^4k}{(2\pi)^4} \frac{k^\sigma k'^\sigma'}{[v \cdot (p - k) - \Delta M + \frac{2a}{m_Q}](k^2 - m^2_\pi)},
\]
where again \( k \) is the momentum of the pion in the loop, and \( m_\pi \) is the pion mass.

As discussed in Ref.[10], the integral
\[
X^{\mu\nu} \equiv \int \frac{d^4k}{(2\pi)^4} \frac{k^\mu k'^\nu}{[v \cdot (k - \delta)](k^2 - m^2_\pi)},
\]
where \( \delta \) is some constant, can be written as
\[
X^{\mu\nu} = X_1(\delta) g^{\mu\nu} + X_2(\delta) v^\mu v'^\nu,
\]
where \( X_1 \) and \( X_2 \) are Lorentz scalars, which are functions of \( \delta \). Obviously, only the \( X_1 \) term contributes in Eq.(24). In the evaluation of \( X_1 \), the integration over \( k_0 \) was made first by choosing the appropriate contour. Then a cutoff \( \Lambda \), which characterizes the finite size of the source of the pion, was introduced in the three dimensional integration since pion loop contributions are suppressed when the Compton wavelength of the pion is smaller than the source of the pion. Since the leading non-analytic contribution of these loops is associated with the infrared behavior of the integral, it does not depend on the details of the cutoff. In this way, \( X_1(\delta) \) has the following expression [10]:
\[
X_1(\delta) = \frac{i}{72\pi^2} \left\{ 12(m^2_\pi - \delta^2)^{3/2} \left[ \frac{\Lambda + \sqrt{\Lambda^2 + m^2_\pi} - \delta}{\sqrt{m^2_\pi - \delta^2}} - \frac{\Lambda - \sqrt{\Lambda^2 + m^2_\pi} - \delta}{\sqrt{m^2_\pi - \delta^2}} \right] \right. 
+ 3\delta(2\delta^2 - 3m^2_\pi) \ln \frac{\Lambda + \sqrt{\Lambda^2 + m^2_\pi} - \delta}{\sqrt{m^2_\pi - \delta^2}} + 3\delta \Lambda \sqrt{\Lambda^2 + m^2_\pi} + 6(\delta^2 - m^2_\pi) \Lambda + 2\Lambda^3 \right\},
\]
when \( m^2_\pi \geq \delta^2 \):
\[
X_1(\delta) = \frac{i}{72\pi^2} \left\{ 6(\delta^2 - m^2_\pi)^{3/2} \ln \frac{(\Lambda + \sqrt{\Lambda^2 + m^2_\pi} - \delta - \sqrt{\delta^2 - m^2_\pi})m^2_\pi - \delta + \sqrt{\delta^2 - m^2_\pi}}{(\Lambda + \sqrt{\Lambda^2 + m^2_\pi} - \delta + \sqrt{\delta^2 - m^2_\pi})m^2_\pi - \delta - \sqrt{\delta^2 - m^2_\pi}} \right. 
+ 3\delta(2\delta^2 - 3m^2_\pi) \ln \frac{\Lambda + \sqrt{\Lambda^2 + m^2_\pi}}{m^2_\pi} + 3\delta \Lambda \sqrt{\Lambda^2 + m^2_\pi} + 6(\delta^2 - m^2_\pi) \Lambda + 2\Lambda^3 \right\},
\]
when \( m^2_\pi \geq \delta^2 \).
when $m_\pi^2 \leq \delta^2$. In the case where $\delta = 0$,

$$X_1 = \frac{i}{36\pi^2} \left(3m_\pi^3 \arctan \frac{\Lambda}{m_\pi} - 3m_\pi^2 \Lambda + \Lambda^3 \right).$$  \hspace{1cm} (29)

From Eqs.(23) and (24) we have

$$\Sigma_1 = i \frac{2g_1^2}{3f_\pi^2} X_1(\Delta_1),$$  \hspace{1cm} (30)

where $\Delta_1 = v \cdot p - \Delta M + \frac{2\alpha}{m_Q}$.

Figs. 1(c), (d) have the same expression as in Eq.(23), except for $\Sigma_1$ being replaced by $\Sigma_2$ and $\Sigma_3$, respectively. In the same way, we have

$$\Sigma_2 = i \frac{g_1^2}{3f_\pi^2} X_1(\Delta_2),$$  \hspace{1cm} (31)

where $\Delta_2 = v \cdot p - \Delta M - \frac{\alpha}{m_Q}$.

Fig. 1(d) arises from the $\Sigma_Q \pi \Lambda_Q$ vertex. In this paper we only consider $\Sigma_b^{(*)}$ since lattice data are available for them. For $\Sigma_b^\pm$, $\pi^\pm$ appears in the pion loop, then we have

$$\Sigma_3^{\Sigma_b^\pm} = i \frac{g_1^2}{f_\pi^2} X_1(\Delta_3),$$  \hspace{1cm} (32)

where $\Delta_3 = v \cdot p$. For $\Sigma_b^0$, $\pi^0$ appears in the pion loop, and we have

$$\Sigma_3^{\Sigma_b^0} = i \frac{g_2^2}{2f_\pi^2} X_1(\Delta_3).$$  \hspace{1cm} (33)

Defining $\Sigma$ as the sum of $\Sigma_1$, $\Sigma_2$, and $\Sigma_3$, the propagator of $\Sigma_b$ becomes

$$\frac{i}{v \cdot p - \Delta M + \frac{2\alpha}{m_Q} - \Sigma \frac{1 + \phi}{2}},$$  \hspace{1cm} (34)

where

$$\Sigma^{\Sigma_b^\pm} = i \frac{2g_1^2}{3f_\pi^2} X_1(\Delta_1) + i \frac{g_1^2}{3f_\pi^2} X_1(\Delta_2) + i \frac{g_2^2}{f_\pi^2} X_1(\Delta_3),$$  \hspace{1cm} (35)

and

$$\Sigma^{\Sigma_b^0} = i \frac{2g_1^2}{3f_\pi^2} X_1(\Delta_1) + i \frac{g_1^2}{3f_\pi^2} X_1(\Delta_2) + i \frac{g_2^2}{2f_\pi^2} X_1(\Delta_3).$$  \hspace{1cm} (36)
Pion loop contributions to the propagator of $\Sigma_b^*$ can be calculated in the same way. Fig. 2(b), (c), (d) can be expressed as

$$\frac{-i}{v \cdot p - \Delta M - \frac{\alpha}{m_Q}} i \Pi_i \frac{-i}{v \cdot p - \Delta M - \frac{\alpha}{m_Q}} \frac{1 + \frac{\not{\!q}_i}{2} \left( g_{\mu\nu} - \frac{1}{3} \gamma_\mu \gamma_\nu - \frac{2}{3} v_\mu v_\nu \right)}{1 + \frac{\not{\!q}_i}{2}},$$

where $i = 1, 2, 3$ for Fig. 2(b), (c), and (d), respectively. After some tedious derivations, we obtain

$$\Pi_1 = \frac{i g_1^2}{6 f_\pi^2} X_1(\Delta_2),$$

$$\Pi_2 = -i \frac{g_1^2}{6 f_\pi^2} X_1(\Delta_1),$$

and

$$\Pi_{3b}^{\Sigma^+} = \frac{i g_1^2}{f_\pi^2} X_1(\Delta_3),$$

$$\Pi_{3b}^{\Sigma^0} = \frac{i g_2^2}{2 f_\pi^2} X_1(\Delta_3).$$

Define $\Pi$ as the sum of $\Pi_1$, $\Pi_2$, and $\Pi_3$, the propagator of $\Sigma_b^*$ becomes

$$\frac{1 + \frac{\not{\!q}_i}{2} - i (g_{\mu\nu} - \frac{1}{3} \gamma_\mu \gamma_\nu - \frac{2}{3} v_\mu v_\nu) \frac{1 + \frac{\not{\!q}_i}{2}}{v \cdot p - \Delta M - \frac{\alpha}{m_Q} - \Pi} \frac{1 + \not{\!q}_i}{2}},$$

where

$$\Pi_{3b}^{\Sigma^+} = \frac{i g_1^2}{6 f_\pi^2} X_1(\Delta_2) - i \frac{g_1^2}{6 f_\pi^2} X_1(\Delta_1) + \frac{i g_2^2}{f_\pi^2} X_1(\Delta_3),$$

and

$$\Pi_{3b}^{\Sigma^0} = \frac{i 5 g_1^2}{6 f_\pi^2} X_1(\Delta_2) - i \frac{g_1^2}{6 f_\pi^2} X_1(\Delta_1) + i \frac{g_2^2}{2 f_\pi^2} X_1(\Delta_3).$$

In the same way, we can calculate pion loop contributions to the propagator of $\Lambda_b$. Fig. 3(b) and (c) can be expressed as

$$\frac{i}{v \cdot p} \left( -i K_i \right) \frac{i}{v \cdot p},$$

where $i = 1, 2$ for Fig. 3(b) and (c), respectively. We obtain

$$K_1 = i \frac{3 g_1^2}{f_\pi^2} X_1(\Delta_1),$$

and

$$K_2 = i \frac{6 g_2^2}{f_\pi^2} X_1(\Delta_2).$$
If we define $K = K_1 + K_2$, then the propagator of $\Lambda_b$ becomes

$$i \frac{v \cdot p - K}{v \cdot p - K} \frac{1 + \phi}{2},$$

(48)

where

$$K = i \frac{3g_2^2}{f_\pi^2} X_1(\Delta_1) + i \frac{6g_2^2}{f_\pi^2} X_1(\Delta_2).$$

(49)

After the correction from $\Sigma$ is added, the propagator of $\Sigma_b$ is proportional to

$$\frac{1}{v \cdot p - m_0 - \Sigma(v \cdot p)},$$

(50)

where $m_0$ is the mass term without $\Sigma$ correction.

The physical mass of $\Sigma_b$, $m$, is defined by

$$[v \cdot p - m_0 - \Sigma(v \cdot p)]|_{v \cdot p = m} = 0.$$

(51)

Therefore, to order $O(g_1^2, g_2^2)$ we have

$$m = m_0 + \Sigma(v \cdot p = m_0).$$

(52)

For $\Sigma^*_b$ and $\Lambda_b$, $\Sigma$ is replaced by $\Pi$ and $K$, respectively, in Eqs.(50-52). For $\Sigma_b$, $\Sigma^*_b$, and $\Lambda_b$, $m_0$ is $\Delta M - \frac{2\alpha}{m_b}$, $\Delta M + \frac{\alpha}{m_b}$, and 0, respectively. Consequently, the pion loop contribution to the mass of $\Sigma_b$, $\sigma_{\Sigma_b}$, has the following expression:

$$\sigma_{\Sigma^\pm_b} = i \frac{2g_1^2}{3f_\pi^2} X_1(0) + i \frac{g_1^2}{3f_\pi^2} X_1(-\frac{3\alpha}{m_b}) + i \frac{g_2^2}{2f_\pi^2} X_1(\Delta M - \frac{2\alpha}{m_b}),$$

(53)

and

$$\sigma_{\Sigma^0_b} = i \frac{2g_1^2}{3f_\pi^2} X_1(0) + i \frac{g_1^2}{3f_\pi^2} X_1(-\frac{3\alpha}{m_b}) + i \frac{g_2^2}{2f_\pi^2} X_1(\Delta M - \frac{2\alpha}{m_b}).$$

(54)

In the same way,

$$\sigma_{\Sigma^\pm_b} = i \frac{5g_1^2}{6f_\pi^2} X_1(0) - i \frac{g_1^2}{6f_\pi^2} X_1(\frac{3\alpha}{m_b}) + i \frac{g_2^2}{2f_\pi^2} X_1(\Delta M + \frac{\alpha}{m_b}),$$

(55)

and

$$\sigma_{\Sigma^0_b} = i \frac{5g_1^2}{6f_\pi^2} X_1(0) - i \frac{g_1^2}{6f_\pi^2} X_1(\frac{3\alpha}{m_b}) + i \frac{g_2^2}{2f_\pi^2} X_1(\Delta M + \frac{\alpha}{m_b}).$$

(56)
For $\Lambda_b$, we have

$$\sigma_{\Lambda_b} = i \frac{3g_2^2}{f_\pi^2} X_1(-\Delta M + \frac{2\alpha}{m_b}) + i \frac{6g_2^2}{f_\pi^2} X_1(-\Delta M - \frac{\alpha}{m_b}). \quad (57)$$

In Eqs.(53-57), $X_1$ is given by Eqs.(27-29).

In order to extrapolate the lattice data from large $m_\pi$ to the physical value of the pion mass, we follow the arguments proposed in Ref.[10] where we dealt with heavy mesons. These arguments can be generalized to the case of heavy baryons straightforwardly. Eqs.(53-57) are valid when $m_\pi$ is not far away from the chiral limit - i.e., when $m_\pi \leq \Lambda$. As pointed out in Refs.[6, 7, 8, 9, 10], pion loop contributions vanish in the limit $m_\pi \to \infty$, and the heavy baryon mass becomes proportional to $m_\pi^2$ when $m_\pi$ becomes large. This behaviour is consistent with lattice simulations. Following Refs.[6, 7, 8, 9, 10], we propose the following phenomenological, functional form for the extrapolation of lattice data for heavy baryons:

$$m_B = a_B + b_B m_\pi^2 + \sigma_B, \quad (58)$$

for $B = \Sigma_b, \Sigma_b^* or \Lambda_b$.

The advantage of fitting the lattice data in this way is that we can guarantee that our formalism has both the correct chiral limit behavior and the appropriate behavior when $m_\pi$ is large, with only three parameters ($a$, $b$, and $\Lambda$) to be determined in the fit.

Chiral symmetry is explicitly broken by the terms in Eq.(22). Substituting Eqs.(1, 4, 9) into Eq.(22) we have the following explicit expression:

$$2\lambda_1 \sum_{a,b=1}^{3} [m_q^a (-\bar{S}_{ab} S^{ab} + \bar{S}_{ab} S^{*ab})] + 2\lambda_2 \sum_{a=1}^{3} m_q^a \sum_{a,b=1}^{3} (-\bar{S}_{ab} S^{ab} + \bar{S}_{ab} S^{*ab})$$

$$+ 2\lambda_3 \sum_{a=1}^{3} m_q^a T_a T^a + 2\lambda_4 \sum_{a=1}^{3} m_q^a \sum_{a=1}^{3} T_a T^a, \quad (59)$$

where we have made a Taylor expansion for $\xi$ and omitted $O(1/f_\pi^2)$ terms. It can be seen that Eq.(59) does not contribute to the mass difference between $\Sigma_Q$ and $\Sigma_Q^*$ to order $m_q$. Corrections to this statement are of order $m_q O(1/f_\pi^2)$, with extra
suppression from $m_q$ with respect to the pion loop effects. They will therefore be ignored. Eq.(59) may contribute to the mass different between $\Sigma^{(*)}_Q$ and $\Lambda_Q$. Such effects will be considered to be effectively included in the parameter $\Delta M$ in Eq.(17).

IV. Extrapolation of lattice data for heavy baryon masses

The masses of $\Sigma_b$, $\Sigma_b^*$, and $\Lambda_b$ were calculated with the aid of NRQCD in quenched approximation in Ref.[2]. Since the mass of the heavy quark is much larger than $\Lambda_{QCD}$, it becomes an irrelevant scale for the dynamics inside a heavy hadron and is removed from NRQCD. This makes it possible to simulate heavy baryons when the lattice spacing is larger than the Compton wavelength of the heavy quark. The lattice space used is $1/a = 1.92$ GeV. For light quarks the tadpole-improved clover action was used which has discretization errors of order $\alpha_s a$. The value of $\beta$ which is related to the bare gauge coupling is 6.0 and the lattice size is $16^3 \times 48$. In the simulations, three different values for the hopping parameter $\kappa$, 0.1369, 0.1375, and 0.13808, were used. The light quark mass is related to $\kappa$ through the definition $m_q = \frac{1}{2a}(1/\kappa - 1/\kappa_c)$, with $\kappa_c = 0.13917$. These three hopping parameters correspond to three values of $m_\pi^2$: 0.6598GeV$^2$, 0.4833GeV$^2$, and 0.3141GeV$^2$, respectively.

The heavy baryon masses were calculated for five different values of $aM^0$ ($M^0$ is the bare heavy quark mass): 1.6, 2.0, 2.7, 4.0, 7.0, and 10.0, where the data for the last two values are less reliable because of large discretization errors [2]. The best estimate for $aM^0_b$, 2.31, was obtained by matching the lattice data to the mass of the $B$ meson. Consequently, in our fit we first extrapolate the lattice data for $aM^0_b=1.6$, 2.0, 2.7, 4.0 to $aM^0_b = 2.31$. This can be done by linear extrapolation with respect to $1/M^0$ with the form $c + \frac{d}{M^0}$, where $c$ and $d$ are constants. This is because $aE_{\text{sim}}$, which is the simulation mass in NRQCD and which is related to the heavy baryon mass, depends on $1/M^0$ linearly (note that in the case of $b$-baryons, $O((1/M^0)^2)$ can be safely ignored). Then from the data in Table XV of Ref.[2], we obtain the values
Table 1: Extrapolated values of $aE_{\sim}$ at $aM_b^0 = 2.31$.

| $\kappa$ | $aE_{\sim}(\Lambda_b)$ | $aE_{\sim}(\Sigma_b)$ | $aE_{\sim}(\Sigma_b^*)$ |
|---------|------------------------|-----------------------|------------------------|
| 0.13690 | 0.816(33)              | 0.877(28)             | 0.889(27)              |
| 0.13750 | 0.779(43)              | 0.845(32)             | 0.856(33)              |
| 0.13808 | 0.733(63)              | 0.818(40)             | 0.827(37)              |

of $aE_{\sim}$ for the three hopping parameters at $aM_b^0 = 2.31$, which are shown in Table 1. In the following, we will extrapolate these values to the physical pion mass with the formulas in Eq.(58).

In our fit we have to determine three parameters in our formalism, $(a_{\Sigma_b}, b_{\Sigma_b}, \Lambda)$ in Eq.(58), for example). These parameters are related to $\Delta M$, $\alpha$, $g_1$, and $g_2$, which represent interactions at the quark and gluon level and cannot be determined from the chiral Lagrangian for heavy baryons. In our fit, we treat them as effective parameters and assume that their possible slight $m_Q$ dependence, which results from QCD corrections and $1/m_Q$ corrections, has been taken into account effectively in this way.

$\Delta M$ is the mass difference between sextet and antitriplet heavy baryons. Since we do not have experimental data for the masses of $\Sigma_b^{(*)}$, we use the data for $\Sigma_c^{(*)}$ to determine $\Delta M$ [13]. The spin-averaged mass of $\Sigma_c^{(*)}$ is $\frac{1}{6}(2m_{\Sigma_c} + 4m_{\Sigma_c^*})$, which is bigger than $m_{\Lambda_c}$ by 0.213GeV. In our fit, we let $\Delta M$ vary between 0.17GeV and 0.23GeV, which are given by $m_{\Sigma_c} - m_{\Lambda_c}$ and $m_{\Sigma_c^*} - m_{\Lambda_c}$, respectively. The mass difference $m_{\Sigma_c^*} - m_{\Sigma_c}$, which is equal to $\frac{2m_c}{m_c}$ to order $1/m_c$, leads to $\alpha = 0.032$GeV$^2$ if we choose $m_c = 0.15$GeV. To see the dependence of our fit on $\alpha$, we let it vary from 0.025GeV$^2$ to 0.035GeV$^2$.

The coupling constant $g_2$ can be determined from the decay width for $\Sigma_c^* \rightarrow \Lambda_c\pi$, which has the following explicit form:

$$
\Gamma_{\Sigma_c^* \rightarrow \Lambda_c\pi} = \frac{g_2^2}{12\pi f_\pi^2} \left[ \frac{(m_{\Sigma_c^*}^2 - m_{\Lambda_c}^2)^2 - 2m_{\Sigma_c^*}^2(m_{\Sigma_c^*}^2 + m_{\Lambda_c}^2)}{4m_{\Sigma_c}^2} + m_\pi^4 \right] \frac{\alpha}{2} \left( m_{\Sigma_c^*} + m_{\Lambda_c} \right)^2 - m_{\pi}^2.
$$

(60)
From $\Gamma_{\Sigma_c^{++} \rightarrow \Lambda_c\pi^+} = 18 \pm 5\text{GeV}$, we have $g_2^2 = 0.559 \pm 0.155$, while from $\Gamma_{\Sigma_c^{0} \rightarrow \Lambda_c\pi^-} = 13 \pm 5\text{GeV}$, we have $g_2^2 = 0.404 \pm 0.155$. Hence, in our fit we choose the range $0.249 \leq g_2^2 \leq 0.714$.

Since $\Sigma_c^*$ cannot decay to $\Sigma_c\pi$, we cannot fix $g_1$ from decays. However, $g_1$ can be related to the matrix of the axial-vector current between sextet heavy baryon states where a $u \rightarrow d$ transition is involved. By assuming $g_{u,d}^A = 0.75$, which corresponds to $g_A^{\text{nucleon}} = 1.25$ in neutron $\beta$-decays and using spin-flavor wave functions for heavy baryons, the authors in Ref.[14] found that $g_1 = 0.38$. Based on this, we let $g_1^2$ vary from 0.1 to 0.2 in our fit.

As discussed in Section III, the parameter $\Lambda$ characterizes the size of the source of the pion. In principle, the value of $\Lambda$ can be determined by fitting the lattice data. However, since $\Lambda$ is mainly related to the data at small pion masses and the current lattice data are only available at large pion masses, the error in the determination of $\Lambda$ is very large. The size difference between $\Sigma_b$ and $\Sigma_b^*$ is caused by effects of order $1/m_b$ which are small. The size difference between $\Sigma_b$ and $\Lambda_b$ is caused by the difference between $0^+$ and $1^+$ light degrees of freedom, which is also the main reason for a size difference between $N$ and $\Delta$. It has been pointed out that the values of $\Lambda$ for $N$ and $\Delta$ are very close to each other [6]. Hence we expect that the difference between the values of $\Lambda$ for $\Sigma_b$ and $\Lambda_b$ should also be small. Since the integrand in $X_1$ becomes small near the cutoff $\Lambda$, a small variation in $\Lambda$ will only lead to an even smaller change in $X_1$. Based on these arguments, we will ignore the differences among the values of $\Lambda$ for $\Sigma_b$, $\Sigma_b^*$, and $\Lambda_b$. To see the dependence of our analysis on $\Lambda$, we let $\Lambda$ vary between 0.4GeV and 0.6GeV.

Using the three masses for $\Sigma_b$, $\Sigma_b^*$, and $\Lambda_b$ in Table 1, we fix the other two parameters ( $a_{\Sigma_b}$ and $b_{\Sigma_b}$ for $\Sigma_b$, for example) in Eq.(58) with the least squares fitting method. The values for these two parameters in the cases of $\Sigma_b^\pm$ and $\Sigma_b^{*\pm}$ are shown in Table 2, where we choose $\Lambda = 0.5\text{GeV}$, $\alpha = 0.032\text{GeV}^2$, $\Delta M = 0.213\text{GeV}$, $g_1^2 = 0.15$, and $g_2^2 = 0.48$. The extrapolated masses for $\Sigma_b$, $\Sigma_b^*$, and $\Lambda_b$ at the physical
pion mass are also shown in this table. The spin-averaged mass $m_{\Sigma_b}^{\text{ave}}$ is defined as $\frac{1}{6}(2m_{\Sigma_b} + 4m_{\Sigma_b^*})$.

With the parameters in Table 2 we obtain the masses of $\Sigma_b$, $\Sigma_b^*$, and $\Lambda_b$ as a function of the pion mass. These results are shown in Figs. 4, 5 and 6, respectively, for $\Lambda = 0.4$ and 0.6. The difference between $m_{\Sigma_b}^{\text{ave}}$ and $m_{\Lambda_b}$ is plotted in Fig. 7. The results of linear extrapolation are also shown in these figures.

It can be seen from Table 2 that the extrapolated mass difference between $\Sigma_b$ and $\Sigma_b^*$ has a very large error. This is caused by the uncertainty in the lattice data. A better way to obtain the mass difference between $\Sigma_b$ and $\Sigma_b^*$ is to extrapolate the lattice data for this mass difference which were obtained from ratio fits, since these data have much smaller errors. The mass difference between $\Sigma_b$ and $\Sigma_b^*$, $\Delta E$, was also given in Ref.[2] for five different values of $aM^0$. We use the data at $aM^0 = 1.6$, 2.0, 2.7, and 4.0 to obtain the value of $\Delta E$ at $aM^0 = 2.31$ with the formula

$$a\Delta E = \frac{e}{aM^0};$$

where $e$ is a constant. Eq.(61) arises from the fact that the mass splitting between $\Sigma_b$ and $\Sigma_b^*$ is caused by $1/m_Q$ effects. With the least squares fitting method we obtain results for $\Delta E$ at $aM^0 = 2.31$, for different values of $\kappa$. These are shown in
Table 3: Extrapolated values of $\Delta E$, the mass difference between $\Sigma_b^*$ and $\Sigma_b$, at $aM_b^0 = 2.31$ using Eq.(61).

| $\kappa$ | $a\Delta E$ | $\Delta E$ |
|----------|-------------|-------------|
| 0.13690  | 0.0093(5)   | 0.0179(10)  |
| 0.13750  | 0.0095(7)   | 0.0182(13)  |
| 0.13808  | 0.0096(7)   | 0.0184(14)  |

In order to extrapolate the values in Table 3 to the physical mass of the pion, we use the following formula:

$$m_{\Sigma_b^*} - m_{\Sigma_b} = \bar{a} + \bar{b}m_\pi^2 + \sigma_{\Sigma_b^*} - \sigma_{\Sigma_b}. \quad (62)$$

With $\Lambda = 0.5\text{GeV}$, $\alpha = 0.032\text{GeV}^2$, $\Delta M = 0.213\text{GeV}$, $g_1^2 = 0.15$, and $g_2^2 = 0.48$, we obtain $\bar{a} = 0.0188(26)$, $\bar{b} = -0.00166(470)$, and the extrapolated mass difference between $\Sigma_b^\pm$ and $\Sigma_b^{*\pm}$: $m_{\Sigma_b^{*\pm}} - m_{\Sigma_b^\pm} = 0.0180(25)$, which is listed in Table 2 as $(m_{\Sigma_b^{*\pm}} - m_{\Sigma_b^\pm})^*$. In Fig. 8, we show $m_{\Sigma_b^{*\pm}} - m_{\Sigma_b^\pm}$ obtained in this way as a function of the pion mass. For comparison, in Fig. 9 we plot the result for $m_{\Sigma_b^{*\pm}} - m_{\Sigma_b^\pm}$ which is obtained from Eq.(58), although it has very large errors. Because these errors are so large, the extrapolation from Eq.(58) is consistent with the extrapolation directly from the lattice data for the mass difference between $\Sigma_b$ and $\Sigma_b^*$.

In addition to the uncertainties which are caused by the errors in the lattice data, the fitted results can also vary a little in the range of the parameters $\alpha$, $\Delta M$, $g_1^2$, $g_2^2$, and $\Lambda$. In Table 4 we list these uncertainties.

In the naive linear extrapolations pion loop corrections are ignored. Hence the results do not depend on the parameters $\alpha$, $\Delta M$, $g_1^2$, $g_2^2$, and $\Lambda$. In Table 5 we list the results of linear extrapolations for comparison. We note that there is no difference between the results for $\Sigma_b^{(*)\pm}$ and $\Sigma_b^{(*)0}$ in the linear extrapolations.

Comparing the uncertainties listed in Table 2 and Table 4 we can see clearly that the main uncertainties in our fit are caused by the errors in the lattice data. In fact,
Table 4: Uncertainties for the extrapolated quantities for $\Sigma^+_b$ and $\Sigma^{*+}_b$ which are caused by the uncertainties associated with parameters in the fitting function.

| Quantities | $\alpha$ | $\Delta M$ | $g_1^2$ | $g_2^2$ | $\Lambda$ |
|------------|----------|-----------|---------|---------|---------|
| $m_{\Sigma^+_b}$ | 0.02% | 0.3% | 0.06% | 0.8% | 0.9% |
| $m_{\Sigma^{*+}_b}$ | 0.007% | 0.3% | 0.04% | 0.9% | 1.0% |
| $m_{\Lambda_b}$ | 0.0% | 0.2% | 0.0% | 1.7% | 1.8% |
| $m_{\Sigma^{*0}_b} - m_{\Sigma^0_b}$ | 3.1% | 0.8% | 1.6% | 12.6% | 10.2% |
| $m_{\Sigma^{*0}_b} - m_{\Lambda_b}$ | 0.0% | 3.4% | 0.3% | 3.7% | 9.3% |
| $(m_{\Sigma^{*0}_b} - m_{\Sigma^0_b})^*$ | 2.2% | 0.0% | 1.1% | 7.7% | 1.7% |

Table 5: Fitted parameters, extrapolated masses of $\Sigma_b$, $\Sigma^{*}_b$, and $\Lambda_b$ and mass differences at $m_{\pi}^{\text{phys}}$ for linear extrapolations. Numbers in brackets are the errors caused by the errors of the lattice data.

| | $\Sigma_b$ | $\Sigma^{*}_b$ | $\Lambda_b$ |
|------------|------------|------------|------------|
| $a$(GeV) | 1.465(0.143) | 1.479(0.187) | 1.63(0.208) |
| $b$(GeV$^{-1}$) | 0.330(0.265) | 0.346(0.326) | 0.460(0.366) |
| $\bar{a}$(GeV) | 0.0190(0.0025) | | |
| $\bar{b}$(GeV$^{-1}$) | -0.00172(0.00470) | | |
| $m$(GeV) | 1.4714(0.1384) | 1.4854(0.1803) | 1.2724(0.2008) |
| $m_{\Sigma^{*0}_b} - m_{\Sigma^0_b}$(GeV) | 0.0140(0.2272) | | |
| $m_{\Sigma^{*0}_b} - m_{\Lambda_b}$(GeV) | 0.2084(0.2385) | | |
| $(m_{\Sigma^{*0}_b} - m_{\Sigma^0_b})^*$(GeV) | 0.0190(0.0025) | | |
the errors of lattice data for heavy baryons are much larger than those for heavy mesons [3]. Indeed, the uncertainties in the extrapolated heavy baryon masses are about one order larger than those in the case of heavy mesons. However, because of the small errors in the lattice data for the mass splitting between Σ_±^b and Σ^*±_b the extrapolated mass difference at the physical pion mass also has a smaller error, about 28%.

From Figures 4 to 9 we see that when the pion mass is smaller than about 500MeV the extrapolations begin to deviate from linear behavior. This is because the pion loop corrections begin to affect the extrapolations around this point. As the pion mass becomes smaller and smaller, pion loop corrections become more and more important. For the masses of Σ_±^b, Σ^*_±^b, Λ_b, and the mass difference between Σ_±^b and Σ^*_±^b the extrapolated values are smaller than those obtained by linear extrapolation. For the difference between the spin-averaged mass of Σ^*(±)_b and the mass of Λ_b, the extrapolated value is larger than that obtained by linear extrapolation. We have checked that such behavior is independent of the uncertainties in the parameters in our model.

Comparing the results in the naive linear extrapolations with those with pion loop corrections being included we find that the difference between them is much smaller than that found in the case of mesons. For example, the splitting between Σ_±^b and Σ^*±_b is about 5% smaller if pion loop effects are taken into account, while the hyperfine splitting in the case of B mesons is about 20% smaller when pion loop effects are taken into account [10]. For the masses of Σ_±^b, Σ^*_±^b, and Λ_b, the extrapolated values with pion loop effects being included are about only 1% smaller than those in linear extrapolations, while for B and B^* the corresponding number is about 3% [10]. Hence in the case of heavy baryons, the linear extrapolation is a better approximation than in the case of heavy mesons.

For Σ_0^b and Σ^*0_b we should use Eqs.(54, 56) in the extrapolation of lattice data. Repeating the same procedure as that for Σ_±^b and Σ^*±_b we find that, in addition
Table 6: Fitted parameters, extrapolated masses of $\Sigma_b^0$ and $\Sigma_b^{*0}$ and mass difference at $m_{\pi}^{\text{phys}}$. Numbers in brackets are errors caused by the errors in the lattice data.

|                  | $\Sigma_b^0$        | $\Sigma_b^{*0}$       |
|------------------|----------------------|------------------------|
| $a$(GeV)         | 1.469(0.143)         | 1.482(0.187)           |
| $b$(GeV$^{-1}$)  | 0.327(0.265)         | 0.342(0.326)           |
| $a$(GeV)         | 0.0187(0.0026)       |                        |
| $b$(GeV$^{-1}$)  | -0.00151(0.00470)    |                        |
| $m$(GeV)         | 1.4638(0.1384)       | 1.4774(0.1803)         |
| $m_{\Sigma_b^{*+}} - m_{\Sigma_b^{+}}$(GeV) | 0.0135(0.2272)       |
| $m_{\Sigma_b^{*0}} - m_{\Lambda_b}$(GeV)    | 0.2226(0.2385)       |
| $(m_{\Sigma_b^{*+}} - m_{\Sigma_b^{+}})^*$ (GeV) | 0.0187(0.0025)       |

Table 7: Uncertainties for the extrapolated quantities for $\Sigma_b^0$ and $\Sigma_b^{*0}$ which are caused by the uncertainties in the parameters of the fitting function.

| Quantities       | $\alpha$ | $\Delta M$ | $g_1^2$ | $g_2^2$ | $\Lambda$ |
|------------------|----------|------------|---------|---------|-----------|
| $m_{\Sigma_b^{*+}}$ | 0.01%    | 0.2%       | 0.06%   | 0.4%    | 0.4%      |
| $m_{\Sigma_b^{*0}}$ | 0.007%   | 0.1%       | 0.04%   | 0.5%    | 0.4%      |
| $m_{\Lambda_b}$   | 0.0%     | 0.2%       | 0.0%    | 1.7%    | 1.8%      |
| $m_{\Sigma_b^{*+}} - m_{\Sigma_b^{+}}$ | 1.5%     | 0.0%       | 2.2%    | 5.9%    | 1.5%      |
| $m_{\Sigma_b^{*0}} - m_{\Lambda_b}$   | 0.0%     | 2.3%       | 0.3%    | 6.6%    | 7.9%      |
| $(m_{\Sigma_b^{*+}} - m_{\Sigma_b^{+}})^*$ | 1.6%     | 0.0%       | 1.1%    | 0.7%    | 0.5%      |

to some minor changes in numerical results, all the quantitative results remain the same. In Tables 6 and 7 we list our numerical results for $\Sigma_b^0$ and $\Sigma_b^{*0}$. Comparing the results in Table 6 with those in Tables 4 and 5 we can see that the naive linear extrapolations work even better in the case of $\Sigma_b^0$ and $\Sigma_b^{*0}$ than in the case of $\Sigma_b^\pm$ and $\Sigma_b^{*\pm}$.

V. Summary and discussion

The masses of heavy baryons $\Sigma_b$, $\Sigma_b^*$, $\Lambda_b$, and the mass difference between $\Sigma_b$ and $\Sigma_b^*$ have been calculated numerically in lattice QCD with unphysical pion masses
which are larger than about 560MeV. In order to extrapolate these data to the physical mass of the pion in a consistent way, we included pion loop effects on the heavy baryon masses by applying the effective chiral Lagrangian for heavy baryons when the pion mass is smaller than the inverse radii of heavy baryons. This chiral Lagrangian is invariant under both chiral symmetry (when the light quark masses go to zero) and heavy quark symmetry (when the heavy quark masses go to infinity). In order to study mass difference between $\Sigma_b$ and $\Sigma_b^*$, we took the color-magnetic-moment operator at order $1/m_Q$ in HQET into account since this operator is the leading one to cause splitting between $\Sigma_b$ and $\Sigma_b^*$. When $m_\pi$ becomes large, lattice data show that heavy baryon masses depend on $m_\pi^2$ linearly in the range of interest.

Based on these considerations, we proposed a phenomenological functional form to extrapolate the lattice data.

The advantage of our formalism is that it has the correct chiral limit behavior as well as the appropriate behavior when $m_\pi$ is large and that there are only three parameters to be determined in the fit to lattice data. It is found that when the pion mass is smaller than about 500MeV the extrapolations begin to deviate from the naive linear extrapolations. However, the differences between the extrapolations with and without pion loop effects being included is smaller than those in the case of heavy mesons. Hence the linear extrapolation is a better approximation in the case of heavy baryons. We carefully analysed uncertainties in our extrapolations which are caused by both lattice data errors and uncertainties in several parameters in our model and found that the main uncertainties are caused by the errors of the lattice data. By directly extrapolating the lattice data for $m_{\Sigma_b^*} - m_{\Sigma_b}$, which has much smaller errors, we found that the extrapolated mass difference between $\Sigma_b^\mp$ and $\Sigma_b^{*\mp}$ at the physical mass of the pion is 18.0 MeV, with an uncertainty of 28% caused by lattice data errors. For $\Sigma_b^0$ and $\Sigma_b^{0*}$ this difference is 18.7MeV with 26% uncertainty from lattice data errors. The uncertainties associated with the parameters in our model are at most a few percent. For the difference between the spin-averaged mass
of $\Sigma_b^{(*)\pm}$ and the mass of $\Lambda_b$, the extrapolated value has a very large error. This needs to be improved when the lattice data become more accurate. Furthermore, we should bear in mind that our extrapolations are based on the lattice data in the quenched approximation. From our experience in the cases of light and heavy mesons [15, 10], the quenched approximation may affect the mass splitting between $\Sigma_b$ and $\Sigma_b^*$. In addition, the lattice results for $m_{\Sigma_b^*} - m_{\Sigma_b}$ may be sensitive to both the coefficient of the $\sigma \cdot \mathbf{B}$ term in NRQCD [3] and the clover coefficient in the clover action for light quarks. This may also influence the lattice data and consequently affect our extrapolations.

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Figure Captions

Fig. 1 Pion loop corrections to the propagator of $SU(3)$ sextet heavy baryons with spin-1/2, where $S_Q^{(*)}$ represent spin-1/2(3/2) $SU(3)$ sextet heavy baryons with heavy quark $Q$ and $T_Q$ represents $SU(3)$ antitriplet heavy baryons.

Fig. 2 Pion loop corrections to the propagator of $SU(3)$ sextet heavy baryons with spin-3/2. Same notation as in Fig. 1.

Fig. 3 Pion loop corrections to the propagator of $SU(3)$ antitriplet heavy baryons. Same notation as in Fig. 1.

Fig. 4 Phenomenological fits to the lattice data for the masses of $\Sigma_b^{\pm}$ as a function of the pion mass. The solid (dashed) line corresponds to $\Lambda = 0.4(0.6)$GeV and the dotted line represents the fit using a linear extrapolation.

Fig. 5 Phenomenological fits to the lattice data for the masses of $\Sigma_b^{*\pm}$ as a function of the pion mass. Same notation as in Fig. 4.

Fig. 6 Phenomenological fits to the lattice data for the masses of $\Lambda_b$ as a function of the pion mass. Same notation as in Fig. 4.

Fig. 7 Difference between the spin-averaged mass of $\Sigma_b^{(\ast)\pm}$ and the mass of $\Lambda_b$ as a function of the pion mass, which is obtained from Figs. 4, 5 and 6 (same notation as in Fig. 4).

Fig. 8 Phenomenological fits to the lattice data for the mass difference between $\Sigma_b^{\pm}$ and $\Sigma_b^{*\pm}$ as a function of the pion mass (same notation as in Fig. 4).

Fig. 9 $m_{\Sigma_b^{\ast\pm}} - m_{\Sigma_b^{\pm}}$ as a function of the pion mass, which is obtained from Figs. 4 and 5. The large errors of lattice data are not shown (same notation as in Fig. 4).
Fig. 1

Fig. 2

Fig. 3
Fig. 4

Fig. 5
Fig. 6

Fig. 7
Fig. 8

Fig. 9