Investigation of effect of different geometric parameters on fundamental natural frequency of composite laminate plate

Ashish N Gadhve1,3 and Nishant S Kulkarni2

1 M. Tech student, Mechanical Design Engineering, Vishwakarma Institute of Technology, Pune, Maharashtra-411037, India
2 Assistant Professor, Mechanical Engineering Department, Vishwakarma Institute of Technology, Pune, Maharashtra-411037, India

3 ashishg.gadhve@gmail.com

Abstract: Composite materials have remarkable properties such as high strength to weight ratio, wear resistance, good electrical and thermal properties compared to metals. A laminated composite material consists of several layers of a composite laminas made of matrix and fibers. Each layer with different fiber orientations may have similar or dissimilar material properties. In this study, the effect of the fiber orientations on the fundamental natural frequency of laminated composite plate is investigated. The problem is analyzed and solved using the finite element method and Matlab. A variety of boundary conditions are used in the analysis. Numerical results were obtained for orthotropic symmetric laminated plate. Fundamental natural frequency of composite plates depends upon various characteristics like aspect ratio, boundary condition and fiber orientation. The effect of cut outs on fundamental natural frequency is analysed. It is observed that the ply angles are playing major role while deciding the fundamental natural frequency of composite laminate plate.

Keyword: Composite laminates, Finite element analysis.

1. Introduction

Composite materials are manufactured by mixture of two or more materials. The reinforcing and matrix phase are two phases of the composite. The reinforcing materials are in the different forms such as particles, and fibers. The fiber and matrix material can be a ceramic, polymer, and metal respectively. The composite material properties are taken from geometry, constituent of material and phase distribution. Umut Topal [1] studied the optimization of 4-layered composite plate having different ply angle orientation with a intermediate line support. In this study, different fibre orientations in the layers are considered as design variables. Mohammad Zanon [2] studied the fundamental natural frequency of composite material having laminated thick spherical shell. This paper gives analysis of mode shapes and natural fundamental frequency of thick spherical shell having simply support as a boundary condition. Muzaffer Topcu [3] deals with effect of different stacking sequence on fundamental frequency of composite laminated beams. Rubem Matimoto [4] analysed the effect of different stacking sequence on cylindrical laminated composite shells with ant colony algorithm. Hai Huang, Shenyang Chen, Haichao An [5] studied hybrid composite laminates for multiple objects such as optimizing frequency gap and maximal natural frequency with least cost. This article deals with investigating the effect of different ply angle direction on the natural fundamental frequency of laminated composite plates. This literature study shows that stacking sequence plays
major role in deciding natural frequency of laminate. Based on the current literature review, following scope is finalized for the present work.

1.1. Scope
- To determine the natural fundamental frequency of laminate composite plate using FEA along with numerical method.
- To investigate different boundary condition effects on fundamental frequency.
- To investigate effect of different cut outs sizes on fundamental natural frequency.
- To investigate aspect ratio effect on fundamental natural frequency of laminate composite plate.

2. Mathematical formulation
From differential of an element, the different equation of motion are derived. An element with moment resultants \( (M_x, M_y, \text{and} M_{xy}) \), shearing force \( q \), and internal forces like \( N_x, N_y \) \( \text{and} N_{xy} \). The equilibrium plate governing differential equations can be expressed as [18]

\[
\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = \rho \frac{\partial^2 v}{\partial t^2}\\
\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = \rho \frac{\partial^2 u}{\partial t^2}\\
\frac{\partial^2 M_x}{\partial x^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + q = \rho \frac{\partial^2 w}{\partial t^2}
\]

\( N_{xo} \) and \( N_{yo} \) are the exterior loadings in the X and Y directions and \( \rho \) is density.

Constitutive Relations: The fibers of every lamina are embedded in a matrix material which are supposed to be continuous and parallel. Each layer of the composite is to be taken as homogeneous and orthotropic. There are a number of thin laminates in the fiber-reinforced composite plate. Elements of stiffness matrix for individual lamina can be calculated using equations given below [17]

\[
Q_{11} = \frac{E_{11}}{(1-\mu_{12}\mu_{21})} \quad Q_{22} = \frac{E_{22}}{(1-\mu_{12}\mu_{21})} \quad Q_{12} = \mu_{21}Q_{11} \quad Q_{66} = Q_{12}
\]

Corresponding to the fiber direction, The elastic stiffness matrix is as follows

\[
\begin{bmatrix}
Q_{11} & Q_{12} & Q_{13} \\
Q_{12} & Q_{22} & Q_{23} \\
Q_{13} & Q_{23} & Q_{33}
\end{bmatrix}
\]

To acquire the elastic stiffness matrix, coordinate standard transformation is essential. The off-axis elastic constant matrix is derived with help of the on-axis elastic stiffness matrix as

\[
Q_{xx} = Q_{22}n^4 + Q_{11}m^4 + 2(2Q_{66} + Q_{12})m^2n^2 \\
Q_{xy} = Q_{12}m^4 + n^4 + (Q_{22} + Q_{11} - 4Q_{66})m^2n^2 \\
Q_{yy} = Q_{22}m^4 + Q_{11}n^4 + 2(Q_{66} + Q_{12})m^2n^2 \\
Q_{xs} = (2Q_{66} + Q_{12} - Q_{22})mn^3 + (Q_{11} - Q_{12} - 2Q_{66})m^3n
\]
\[ Q_{ys} = (2Q_{66} + Q_{12} - Q_{22})m^3n + (Q_{11} - Q_{12} - 2Q_{66})mn^3 \]
\[ Q_{ss} = Q_{66}(n^4 + m^4) + (Q_{22} + Q_{11} - 2Q_{12} - 2Q_{66})m^2n^2 \]

Where \( n = \sin \theta \) and \( m = \cos \theta \)

The constituent relationships of a laminated plate can be written as

\[
\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \varepsilon \\ k \end{bmatrix}
\]

Where \( A_{ij} \), \( B_{ij} \), \( D_{ij} \) and \( S_{ij} \) are the stiffness membrane matrix, stiffness coupling matrix, stiffness flexural matrix and shear stiffness transverse matrix.

2.1. Free vibration governing equation

The finite element formulation is based on the 1st order shear deformation theory. In the current analysis an 8 nodded element is assumed with 6 degrees of freedom i.e. \( u, v, w, \theta_x, \theta_y \) and \( \theta_z \) at every node. The eigen value expression of composite laminate plate for free vibration analysis is given as [16]

\[
([K] - \omega^2[M])\{\phi\} = \{0\}
\]

Where \([M] = \) global mass matrices, \([K] = \) global stiffness matrix, \( \phi = \) mode shape i.e. corresponding eigenvectors, \( \omega^2 = \) fundamental frequency.

The natural frequencies for a special orthotropic composite laminated plate is given as [18]

\[
\omega_{mn} = \frac{\pi^2}{\sqrt{\rho R^2 b^2}} \sqrt{D_{11} m^4 + 2(D_{12} + 2D_{66})m^2n^2R^2 + D_{22} n^4R^4}
\]

Where \( R = \frac{a}{b} \) (aspect ratio)

3. Simulation work

In the present work, square plate of composite laminate are modelled and simulation is done in ANSYS software

3.1. Modeling in ANSYS

The symmetric laminate plate with assumed stacking sequence 0°/90°/90°/0° made of E-glass epoxy is considered for analysis. The layers of composite are arranged to create 4-ply laminates with dimensions of plates 300 mm × 300 mm × 1mm. The plies are orthotropic in nature. The available default properties of E glass-epoxy laminate in Ansys are used in the analysis and are shown in following table 1.
Table 1. Test specimen material properties

| Property                                      | Value     |
|----------------------------------------------|-----------|
| Youngs modulus in axial direction $E_1$       | 56 GPa    |
| Youngs modulus in transverse direction $E_2$ | 26 GPa    |
| Shear stiffness $G$                          | 8 GPa     |
| Poisson's ratio $\mu_{12}$                   | 0.31      |
| Minor poisson's ratio                        | 0.048     |
| Density $\rho$                               | 1.6 e3 Kg/m$^3$ |

3.2. Meshing
Meshing in Ansys is a quick, general-purpose, and computerized high-performance calculations. For present work, the geometry used is of square cross section. As the geometry is simple in construction, the quadratic mesh is used with element size as 5. The mesh model of the plate is shown in figure 1.

![Figure 1. Meshing of plate](image1)

| Statistics                  |
|-----------------------------|
| Nodes                       | 3500      |
| Elements                    | 3481      |

3.3. Boundary condition
The finite element model of the plate after applying boundary conditions is as shown in Figure 2. For initial modal analysis of composite plate, simply supported boundary condition is considered.

![Figure 2. Boundary condition](image2)
3.4. Modal analysis
The modal analysis results obtained using Ansys are shown in Figure 3 and provided in Table 2 for validation purpose. The obtained fundamental natural frequency of composite plate is 468.06 Hz.

![Figure 3. Modal analysis](image)

3.5. Numerical analysis for natural frequency
A Matlab program is written to calculate natural frequency of simply supported composite laminate plate. Algorithm of the same is as given below

- Step 1 - Start
- Start 2 - Initialize variable (Ply angle, Thickness of ply, Number of ply, material properties, Dimension of plate)
- Step 3 - Calculation of minor poisons ratio and Stiffness matrix
- Step 4 - Check for condition.
- Step 5 - If the condition is true then go for step no. 6 otherwise go for step no. 9
- Step 6 - Calculate A, B, D matrix
- Step 7 - Go for step no. 3
- Step 8 - Loop end and calculate Fundamental natural frequency
- Step 9 - Print value of Fundamental natural frequency
- Step 10 - Stop

4. Data analysis
Natural frequencies obtained for simply supported plate using Ansys and Matlab code are compared in Table 2

| Fundamental Natural Frequency | FEA Analysis (Hz) | Numerical Analysis (Hz) | Error (%) |
|------------------------------|-------------------|-------------------------|-----------|
| Mode 1                       | 468.26            | 462.08                  | 1.31      |
| Mode 2                       | 786.64            | 774.12                  | 1.59      |
| Mode 3                       | 1036.3            | 1017.4                  | 1.83      |
| Mode 4                       | 1408.8            | 1374.7                  | 2.4       |
| Mode 5                       | 1527.3            | 1492.12                 | 2.29      |

The maximum difference observed between FEA and numerical results is 2.29%. This validates FEA model developed for simulation purpose. This validated model is now used to investigate different parameters effect such as boundary conditions, aspect ratio, as well as cut-outs on fundamental natural frequency of same composite plate.
5. Results and discussion

After getting initial results, the different geometrical parameter effects such as boundary conditions, aspect ratio as well as cut-outs on fundamental frequency of composite plate is studied in this section. The obtained results are provided in the subsequent sub-sections.

5.1. Boundary conditions effect

In this analysis different boundary conditions such as simply supported, cantilever and clamped edges is assumed for square shaped cross ply laminated plate having orientation as (0°/90°/90°/0°). In this simulation aspect ratio is kept constant as 1. The results obtained are shown in table 3 and figure 4.

| Mode | SSSS  | CCCC  | Cantilever |
|------|-------|-------|------------|
| 1    | 468.26| 629.05| 82.487     |
| 2    | 786.64| 1011.4| 143.79     |
| 3    | 1036.3| 1481.6| 358.11     |
| 4    | 1408.8| 1664.3| 512.07     |
| 5    | 1527.3| 1793.3| 599.48     |

Figure 4. Effect of boundary condition

5.2. Aspect ratio effect

The variation of aspect ratio is in between 0.5 to 2. The composite plate width is varied and the length of the plate is kept constant as 300 mm. Obtained first five natural frequencies for different aspect ratio are shown in table 4 and compared in figure 5.

| Aspect Ratio | Mode 1  | Mode 2  | Mode 3  | Mode 4  | Mode 5  |
|-------------|---------|---------|---------|---------|---------|
| 0.5         | 1092.4  | 1561.6  | 2242.9  | 2542    | 2850    |
| 1           | 468.26  | 786.64  | 1036.3  | 1408.8  | 1527.3  |
| 1.5         | 381     | 513.71  | 808.8   | 952.43  | 1125.4  |
| 2           | 357     | 424.88  | 570.64  | 774.57  | 924.24  |
5.3. Effect of different cut out shapes
The composite laminate plate with square cross ply (0°/90°/90°/0°) having different type of hole shapes i.e. hexagonal, circular, and square is analysed for calculation of fundamental natural frequency. For these cut outs the surface area is kept same. In all of the cases, the aspect ratio is varied from 0.5 to 2. The subsequent table 5 shows the fundamental natural frequencies.

Table 5. Effect of different cut out shapes on fundamental natural frequency

| Aspect ratio | Circular     | Square       | Hexagonal   |
|--------------|--------------|--------------|-------------|
| 0.5          | 1098.4       | 1103.45      | 1085.12     |
| 1            | 472          | 473.61       | 463.61      |
| 1.5          | 320          | 321          | 318         |
| 2            | 173.69       | 227          | 275.77      |

6. Conclusion
Composite materials have outstanding generous properties. In the initial phase of analysis, the natural frequency of composite plate for known stacking sequence is obtained using FEA. The natural frequencies obtained using FEA and numerical method are closely matching with each other, thus validates the developed FEA model. It is seen that ply angles are playing a major role while deciding the fundamental natural frequency. Effect of aspect ratio, boundary conditions and cut-out shapes on fundamental natural frequency is then investigated.
Simulation results show that, fundamental natural frequency is highest for all edges clamped, followed by all edges simply supported boundary condition. The natural frequency is lowest for the cantilever boundary condition. As the aspect ratio goes on decreasing, the fundamental natural frequency of composite plate increases. In case of plate with cut-outs, the plate having a square cut-out showed higher natural frequencies as compared to others and the plate having a hexagonal cut-out showed the lowest natural frequencies.

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