A review of landmark articles in the field of co-evolutionary computing

Noe Casas
Email: research@noecasas.com

Abstract—Coevolution is a powerful tool in evolutionary computing that mitigates some of its endemic problems, namely stagnation in local optima and lack of convergence in high dimensionality problems. Since its inception in 1990, there are multiple articles that have contributed greatly to the development and improvement of the coevolutionary techniques. In this report we review some of those landmark articles dwelling in the techniques they propose and how they fit to conform robust evolutionary algorithms.

I. INTRODUCTION

Evolutionary algorithms mimic some of the dynamics of natural evolution. This technical report focuses on certain special forms of such dynamics: co-evolution. It is defined as an evolutionary scenario where the survival and the very evolutionary changes experiences by an individuals across it genetic path are affected by other individuals.

There is evidence of co-evolution in natural species. For instance, bees and flowers mutually provide benefits: bees feed from the flower nectar while helping their cross pollination. This is an example of cooperative coevolution. On the other hand, species can also coevolve competitively, much like predators and preys, respectively improving their attack and defense abilities. For instance, fossil records have proven that snails shells have become thicker at the same rate the claws of their predators have become stronger (and hence able to crush more easily the snail shell).

In order to study the topic, we have chosen seven articles that we consider to be relevant to the development of the field’s body of knowledge. The criteria used to do the selection combine historical relevance with number of citations (taken from google scholar[1] and citeseerx[2]).

This report is structured in three parts: first, section II provides a thorough review of each of the selected articles, in chronological order; then, section III discusses their strong and weak points; finally, section IV provide a brief overview of the state of the field, based on the reviewed articles.

II. REVIEW OF THE SELECTED ARTICLES

A. Co-evolving Parasites Improve Simulated Evolution As an Optimization Procedure (Hillis, 1990)

Hillis was the first to propose the use of co-evolution applied to evolutionary computation in [1] in 1990. His approach consisted of a competitive evolution scenario where there are two species, referred to as hosts-parasites or prey-predators, and where the goal of the system was to improve the performance of sorting networks.

A sorting network is an algorithm that sorts the input data using fixed comparisons. They differ from generic sorting algorithms in that they are not able to handle variable number of input data and that their comparisons are pre-defined. They consist of an assemblyment of wires and comparators, as shown in figure.[4]

![Fig. 1: Structure of a 4-input sorting network.](https://i.imgur.com/3.png)

The horizontal wires have the inputs of the netowrk on the left side and its outputs (i.e. the sorted inputs) on the right. The vertical connectors are comparators that switch the wire data if the upper connection is greater than the other one. This behaviour is illustrated in figure.[2]

![Fig. 2: Behaviour of a sorting network.](https://i.imgur.com/4.png)

The goodness of a sorting network is judged based on its correction (i.e. the results are always properly sorted) but also

[1] Images from mediawiki under license Creative Commons Attribution 3.0 Unported.
on the number of comparators (i.e. the less comparators, the better), in order to minimize execution time and cost (in case of deploying a hardware version of the network).

It is worth mentioning that sorting networks have recently regained popularity because they are used in GPU computing [2].

On his first attempts to approach the problem without co-evolution, Hillis found some of the typical problems attributed to soft computing techniques: stagnation in local optima and overfitting. In order to address these problems, Hillis introduced some changes to improve population diversity, but only after introducing competitive co-evolution he achieved notable results.

In the co-evolutionary algorithm proposed by Hillis, hosts represented configurations of the sorting network, while parasites represented test data to be supplied to a host as input. The fitness of each group is opposed to the other group: the fitness of the hosts depends on how many test cases (i.e. parasites) an individual has succeeded in sorting, while the fitness of the parasites depends on how many times it made a host fail sorting.

Despite the validity and novelty of the approach proposed by Hillis, we believe his article deserve some fair criticism:

- From the literary composition point of view, Hillis’ paper lacks proper structure. It has three sections named “Introduction”, “Sorting networks” and “The co-evolution of parasites”. In order to better deliver the information it contains, it should contain a section devoted to explain the experiments, another section exposing their results and another one drawing its conclusions and describing future research lines.

- From the scientific point of view, Hillis paper lacks proper presentation of the results and proper measurement of their performance. One would expect tables and graphics with comparisons of different algorithm tunings and argumentations for misbehaviours of the algorithms. All the appreciations in the article do not seem to be the result of rigorous study but appear rather intuitive and loose.

B. Co-evolutionary constraint satisfaction (Paredis, 1994)

Jan Paredis applied in [3] the approach pioneered by Hillis to a Constraint Satisfaction Problem (CSP).

The motivation for proposing a coevolving approach is that constraint satisfaction GA techniquet used in previous studies (i.e. Genetic Repair [4], Decoders that always lead to valid representations [5], Penalty Functions [6]) where all problem specific.

This way, he devised a new generic approach for constraint handling termed Co-evolutionary Constraint Satisfaction (CCS). In this approach, there is no domain knowledge to actively enforce the satisfaction of constraints, but only checks to know whether they are met.

The problem we tried to solve is the N queens arrangement on a chess board. Paredis’ algorithm maintains two populations: one with potential solutions and another one with constraints. They act much like Hillis’ predator-prey model, leading to an "arms race" between both populations.

Although the presentation and structure of the article are correct, the presentation of its results lacks proper measurements of the algorithm performance.

C. Evolving Complex Structures via Cooperative Coevolution (De Jong and Potter, 1995)

De Jong and Potter used coevolution in [7] to address the problem of designing evolutionary solutions that exhibit modularity (as opposed to the classic evolutionary algorithm where the solution emerges as-is).

Their generic approach consists in maintaining different species evolving separately in their own populations (following a schema inspired in the island model [8]). Each of these species represent a submodule that can be combined with the others to form a solution to the problem. Apart from them, there is an extra species that merges representative individuals from the former populations into a single individual, which is then subject to evaluation. Its credit flowed back to the original component individuals’ fitness.

The intent of this approach is to provide the individual species selective pressure to cooperate instead of compete, while keeping the competition among individuals of the same population.

They applied this generic schema to two different problems: function optimization and robot task learning.

Regarding function optimization, De Jong and Potter have each species to determine a specific parameter of the function to be optimized. When combined, they conform the complete parameter set that fully qualifies the function solution. They tested their approach using two functions: the highly multimodal Rastrigin function and the Rosenblock function [9] defined by two highly interdependent variables with a very deep valley containing the global optimum. Their results were mixed, as some of the variations of their approach outperformed a standard GA while others did not.

Regarding the robot learning problem, they used a system that maintained a model of the world and a set of production rules that were fed into a rules engine in order to determine the proper actions to be performed. In this case the genetic algorithm is in charge of evolving the rule set, taking into

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2The Rosenblock function was originally defined in the De Jong test function suite [9].
account the feedback loop to evaluate the fitness of the newly evolved rules. The different populations created different rule sets that were later combined to conform the robot’s behaviour.

The structure of the article is the standard one, with different sections to describe the problems to be solved, the results obtained, and a discussion about them. De Jong and Potter use best fitness value plots to compare the performance of their architecture with other reference approaches. However, they fail to provide evidence of the benefits of their proposal, but only sketch its apparent potential, leaving more concrete results for further studies.

Potter and De Jong continued this research line, both exploring individually the experiments performed in this article, like in [10] and [11], and dwelling deeper into the generic sub-component evolutionary framework and extending its applicability to other domains, like in [12].

D. New Methods for Competitive Coevolution (Rosin and Belew, 1996)

In their article [13], Rosin and Belew used competitive coevolution in the frame of game theory, following the previous experiences of John Maynard Smith [14] and Robert Axelrod [15].

The games Nim and 3D tic-tac-toe are used as problems to test the technique used in this article. The authors use the term host to refer to individuals whose fitness it being tested, and the term parasite for individuals used to the the hosts’ fitness.

Three novel co-evolutionary techniques were introduced by the authors in this article:

- **Competitive Fitness Sharing**: defeating a parasite awards an amount of points, that are shared among all hosts that defeated it. This rewards hosts that can defeat parasites that no other host could beat, hence decreasing the probability of parasites that no host can defeat.

- **Shared Sampling**: in order to keep the needed computational power as low as possible, not all hosts would fight against every parasite, but only against a sample of them. In order to select a strong parasite sample, the individual that defeated most opponents during the previous generation is selected, let us call it A; then, those hosts beating parasites that defeated A are selected until the sample is large enough.

- **Hall of Fame**: given the fact that we use finite populations but we do not want to lose strong parasites, a record is kept with the best parasite in each generation. Hosts are tested against the current parasite generation and the aforementioned best parasites of all times, which are referred to as hall of fame. They play a role analogous to that of elitism, but with the purpose of improving the testing instead of improving the population fitness.

The article provides a thorough study of the dynamics of the populations under the aforementioned techniques, identifying equilibrium conditions as well as extinction probabilities.

We consider this paper by Rosin and Belew to be of utmost quality, both regarding its correct structuration and exposition of their ideas, and also on the scientific evaluation and argumentation of the different techniques and the comparison of their performances.

E. Coevolving Predator and Prey Robots: Do “Arms Races” Arise in Artificial Evolution? (Nolfi and Floreano, 1998)

Stefano Nolfi and Dario Floreano researched in [16] the so-called “arms race” in competitive coevolutionary populations. This term refers to the property of those populations for trying to get fitter over generations to beat the individuals from the other populations; given that populations’ fitnesses are coupled by the competition, an increase in the fitness of one population usually leads to a decrease in the fitness of the other populations, normally alternating such a trend cyclically.

For their experiments, the authors used the classical Khepera robots to set up a predator-prey scenario. The intelligence of the robot is implemented in a perceptron with recurrent connections in the output layer. In order to determine the network weights, Nolfi and Floreano use an evolutionary algorithm where such weights are encoded as alleles in the genome (i.e. direct encoding of the neural network). The role of the predator is to detect the prey with its sensors and chase it until touching it. Two populations are maintained: one to optimize the weights of the predator and another one for the prey, both co-evolving in competition, using simple fitness functions: for the predator, 1 if it catches the prey, 0 otherwise; vice versa for the prey.

The authors first define the metric under which they shall evaluate their techniques. For this, they study the effects of the Red Queen Effect, by which the evolutionary benefits obtained by some individuals are reduced or eliminated by other population; this is possible because the fitness of an individual is also coupled to its competitors’ performance. To avoid this problem, they propose a variation to the Current Individual vs. Ancestral Opponents [17], called Master Tournament, which consists of testing the performance of the best individual of each generation against each best competitor of all generations.

They then elaborate on the proposal of Rosin and Belew [13] (covered in section II-D), hypothesizing that their Hall of fame approach may progressively lead to having less and less selective pressure to devise strategies effective against the current enemies and more and more biased towards past generations.

Their conclusions are that continuous increase in objective goodness is not guaranteed by competitive co-evolution, as populations can cycle between strategies that only provide
temporary advantage over the other populations and not long-term improvements. That this effect can be reduced by keeping ancestors to test individuals from the current generation, but this may hinder the effects of co-evolution themselves, because it leads to give more and more importance to devising a strategy that is successful with the ancestors instead of the current generation.

Although the article contains plenty of information and data, the organization makes it less than obvious to fit together the conclusions from each section. Furthermore, despite they state that the goal of the article is to determine the credibility of the "arms race" hypotheses, other topics are mixed in the exposition of the experiments (e.g. Red Queen Effect, Hall of Fame problems, the Bootstrap problem) without proper introduction or justification. We believe that the contents of the paper would have benefited remarkably from a classical section arrangement that would have enabled clearer conveyance of the authors’ conclusions.

F. Pareto coevolution: Using performance against coevolved opponents in a game as dimensions for Pareto selection (Noble and Watson, 2001)

Jason Noble and Richard A. Watson studied in [18] the applicability of Pareto optimality to coevolutionary algorithms. They use this idea to play Texas Hold’em Poker. Each individual encodes in its genome the strategies to play under different conditions, and plays against its fellow players, getting fitness reward in case of winning.

Their approach to introduce Pareto Optimality in a coevolutionary scenario is to consider each opponent to be a dimension that has to be optimized. Given that a Pareto-optimal solution is one where none of the dimensions can be improved without damaging the performance of the other dimensions, trying to find the pareto front actually means devising individuals that play well against all other opponents.

In order to avoid the red queen effect (see section II-E), the add to the set of opponents some reference poker strategies, that is, hardcoded (by themselves) poker strategies that remained constant over time. They were not optimal in any case, but they acted as a fixed reference point to objectively measure the fitness of the population.

Although their approach overperformed a normal GA, their results were not remarkably good, but they introducing Pareto optimality in the coevolutionary frame proved to be a successful line of research (e.g. [19]).

III. DISCUSSION

As seem in the summary of the selected articles presented in section II, there are two different branches for coevolutionary algorithms: competitive and cooperative.

In competitive coevolution approaches there are several subpopulations having the fitness of the individuals of each population defined so that advantage of one population implies disadvantage of the others. Under certain conditions, this leads to an "arms race" among the subpopulations, which is presumed to mitigate the stagnation in local minima typical from classical evolutionary algorithms thanks to the variation in time of the overall fitness landscape. The different subpopulations normally play different roles, like predator-prey (e.g. [16], in section II-E) or host-parasite (e.g. [1] in section I-A). Among such roles, we tend to see one that represents the individuals being tested (i.e. predator, host) and one that represents the tests themselves (i.e. prey, parasite). This way, improving in the fitness of a test means that the tested individual performs worse, and vice versa.

Although the arms race usually leads to improvement of the fitness, there are some pathological behaviours in this approach. The first one is the situation where the populations cycle over alternative fitness increasing periods, but do not achieve objective improvement; this problem is usually referred to as mediocre objective stasis. This, as pointed out by Floreano and Nolfi in [16], can be mitigated by keeping an archive of ancestor opponents (e.g. Hall of fame) and by introducing some objective measurements that evaluate the objective improvement of the populations, like the reference poker strategies by Noble and Watson in [18]. These two techniques also help with another typical problem of competitive coevolution: the lose of gradient, commonly referred to as the Red Queen Effect, which happens when one population achieves a level so superior compared to the other, that nothing can be learned by either population by competing.

Another problem in competitive coevolution is the lack of promotion of strong tests (i.e. tests that make tested individuals perform poorly). This happens either because tested individuals are not paired against them or because their effect in the fitness of tested individuals is very small due to the larger amount of favorable tests. For these two problems, Rosin and Belew in [13] used respectively Competitive Fitness Sharing and Shared Sampling. The former makes that the easier a test is (e.g. a lot of testing individuals performed well against it), the less fitness it awards. The latter makes that tests are paired with the individuals that have shown weaker against them in previous generations. Multiobjective evolution has also been used in this regard (e.g. [18]) by having each test be considered one of the criteria to be optimized. Hence, finding the Pareto front is analogous to find opponents that perform well against all tests.

In cooperative coevolution approaches there are several populations that combine their efforts to achieve fitter solutions. Their most typical use case is where each population has individuals of certain species that only represent part of the final solution. Representatives of each species are then combined to form a complete solution and the credit derived from its fitness flows down to the original subparts. This effectively consists in decomposing the problem in modular parts, hence achieving reduction in the dimensionality of the original solution space.
IV. CONCLUSIONS

We have traversed some of the remarkable articles regarding coevolutionary algorithms. Competitive and cooperative approaches offer complementary techniques to address complex problems where conventional evolutionary computation falls short regarding stagnation in local optima.

Despite this progress, there is still a significant need for crafting the algorithm and tuning it with problem-dependent considerations.

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