A Possible Explanation of Vanishing Halo Velocity Bias

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Abstract

Recently, Chen et al. accurately determined the volume weighted halo velocity bias in simulations and found that the deviation of velocity bias from unity is much weaker than the peak model prediction. Here we present a possible explanation of this vanishing velocity bias. The starting point is that halos are peaks in the low redshift non-Gaussian density field with smoothing scale $R_\Delta$ (virial radius), instead of peaks in the high-redshift initial Gaussian density field with a factor of $O(\Delta^{1/3})$ larger smoothing scale. Based on the approximation that the density field can be Gaussianized by a local and monotonic transformation, we extend the peak model to the non-Gaussian density field and derive the analytical expression of velocity dispersion and velocity power spectrum of these halos. The predicted deviation of velocity bias from unity is indeed much weaker than the previous prediction, and the agreement with the simulation results is significantly improved.

Key words: cosmology; observations – dark energy – dark matter – large-scale structure of universe

1. Introduction

The volume weighted halo/galaxy velocity bias at a large scale ($\gtrsim 10$ Mpc) is a long standing problem in modern cosmology. It is not only of theoretical importance in understanding the structure formation of the universe, but also of practical importance in constraining dark energy with peculiar velocity (e.g., redshift space distortion). Recently, Chen et al. (2018) managed to measure the halo velocity bias in simulations, with $0.1\%$–$1\%$ accuracy. This was achieved by a novel method that circumvents the sampling artifact problem (Bernardeau & van de Weygaert 1996; Bernardeau et al. 1997; Schaan & van de Weygaert 2000), prohibiting accurate velocity measurement (Pueblas & Scoccimarro 2009; Zheng et al. 2013, 2015; Zhang et al. 2015). A major finding is that the deviation of velocity bias $v_v$ from unity ($v_v - 1$) is very weak at $k \sim 0.1h$/Mpc. For example, all $z = 0$ halos in the mass angle $5 \times 10^{11} < M/\text{M}_\odot < 10^{14}$ have $|v_v - 1| < 0.1\%$ at $k \leq 0.1h$/Mpc (Table 2, Chen et al. 2018). This finding validates the usual assumption of $v_v = 1$ in the data analysis of peculiar velocity cosmology and eliminates a potential systematic error associated with the velocity bias.

However, these numerical findings disagree with the theoretical prediction of the peak model (Bardeen et al. 1986; Desjacques & Sheth 2010). The peak model is based on the correspondence of halos with peaks in the initial density field and the existence of correlation between density gradient and velocity. Based on these two facts, the seminal BBKS paper (Bardeen et al. 1986) predicted $\sigma_{v,\text{halo}}^2/\sigma_{\text{matter}}^2 < 1$ and the deviation reaches $10\%$ for $10^{13} \text{M}_\odot$ halos. Desjacques & Sheth (2010; hereafter DS10) extended BBKS to two-point statistics, and derived an elegant expression $v_v(k) = 1 - R_\Delta^2(M)k^2$. For $10^{13} \text{M}_\odot$ halos, the deviation from unity is $\sim 5\%$ at $k = 0.1h$/Mpc and larger at smaller scales. The peak model predictions (BBKS and DS10 to leading order) are theoretically exact, in the context of proto-halos defined in the linear and Gaussian initial conditions. Furthermore, the DS10 prediction has been verified in N-body simulations (Elia et al. 2012; Baldauf et al. 2015) and supported by further theoretical discussions (Chan et al. 2012; Biagetti et al. 2014; Chan 2015).

The discrepancies between the peak model prediction and the Chen et al. (2018) numerical finding then require explanations. (1) The difference in the halo/proto-halo definitions is likely the dominant cause. Chen et al. (2018) adopted the usual definition of halos, identified by the Friends-of-Friends (FoF) algorithm with linking length $b = 0.2$ at the investigated redshifts. These halos correspond to peaks in the late epoch non-Gaussian density field, smoothed with scale of the halo virial radius $R_\Delta$. Here $\Delta \sim O(100)$ is the mean halo density within radius $R_\Delta$, in the term of the mean cosmological matter density. On the other hand, theoretical works of the peak model focus on “proto-halos,” which are peaks in the initial Gaussian density field, smoothed with scale $R_\Delta = \Delta^{1/3}$. Numerical works of the peak model (Elia et al. 2012; Baldauf et al. 2015) adopt a different definition of “proto-halos,” as groups of particles that are members of $z = 0$ halos. Halos defined in the first way are hosts of galaxies in astronomical surveys, and therefore are directly related to the interpretation of galaxy velocity measurement. (2) Another difference is that Chen et al. (2018) defined the velocity bias as the ratio of halo velocity and matter velocity at the same epoch (e.g., $z = 0$, 1, 2). It is directly related to the galaxy velocity measurement at these redshifts, and its applications in measuring the structure growth rate and constraining gravity (e.g., Li et al. 2013). On the other hand, Elia et al. (2012) only measured the velocity bias at the initial redshifts ($z = 50$, 70), and Baldauf et al. (2015) defined the velocity bias with respect to the linearly evolved matter velocity. Furthermore, the velocity bias measured by Chen et al. (2018) is volume weighted, while that by Baldauf et al. (2015) is (halo number density) weighted.

Motivated by these possibilities, we present a quantitative derivation of the volume weighted halo velocity bias at late epoch. What we need is the non-Gaussian joint PDF of the density, density gradient, and velocity fields with smoothing scale $R_\Delta$. Due to complexities in the nonlinear evolution, no exact analytical expression exists. However, due to the fact that...
the non-Gaussian density field can be Gaussianized to a good
approximation (Coles & Jones 1991; Kofman et al. 1994;
Taylor & Watts 2000; Kayo et al. 2001; Neyrinck et al. 2009,
2011; Sato et al. 2010, 2011; Scherrer et al. 2010; Yu et al.
2011), we are able to write down the joint PDF analytically.
This allows us to capture major impact of the nonlinear
evolution on the halo velocity bias, and provides a possible
explanation on the observed $b_v \approx 1$.

2. The Velocity Bias in the Gaussianized Field

The N-point joint PDF of the density field is fully captured
by the one-point PDF and the N-point Copula (Scherrer et al.
2010). The Copula function is invariant under local and
monotonic transformation of density field. Scherrer et al.
(2010) found in N-body simulations that, despite significant
non-Gaussianity in the one-point PDF, the two-point Copula
is nearly Gaussian. This means that, once we perform a local
transformation to render the one-point PDF Gaussian, the
two-point PDF will be Gaussian as well. The well-known log-
normal transformation (Coles & Jones 1991; Kofman et al.
1994; Taylor & Watts 2000; Kayo et al. 2001; Neyrinck et al.
2009) is an approximation to the local Gaussianization
transformation.

We will work under this Gaussian Copula hypothesis. We
denote the Gaussianization transformation as $G = G(\delta)$, where
$\delta$ is the matter overdensity and $G$ is the Gaussianized field.
By the construction, both the one-point PDF $P(G)$ and the two-
point PDF $P(G_1, G_2)$ are Gaussian. The velocity field may have
a non-negligible, non-Gaussian component at a small scale
(e.g., internal motions within halos). Fortunately, the halo
velocity smooths and suppresses the non-Gaussian velocity
components below the scale of the halo virial radius. Furthermore,
we are interested in the large-scale velocity statistics. Therefore,
we neglect non-Gaussianities in the velocity field. We are then able to
derive the non-Gaussian joint PDF of $v$, $\delta$, and the density gradient $\nabla \delta$. It is essential to include the density gradient in the joint PDF,
since it is correlated with the velocity field. Halos only reside at
regions of zero density gradient (density peaks), resulting in the
velocity bias.

Since the joint PDF of $G$, $\nabla G$, and $v$ is Gaussian, the
expression of velocity bias is identical to that of BBKS and
DS10, once we replace the initial linear density $\delta_l$ by $G$,
$\nabla \delta_l$ by $\nabla G$, and the smoothing scale $R_S = (3M/(4\pi \rho_m))^{1/3}$
by $R_\Delta = (3M/(4\pi \Delta \rho_m))^{1/3}$. Here $\rho_m$ is the (comoving)
mean cosmological matter density. At high redshift, $\Delta \to 178$. At
redshift zero, $\Delta \approx 100 h/\bar{\rho}_m \approx 350$ due to the non-zero
cosmological constant (Eke et al. 1996). Extending BBKS to
the Gaussianized $G$ field, we obtain

$$ \frac{\sigma_{vG}^2}{\sigma_v^2} = 1 - r^2, \quad r^2 \equiv \frac{\langle v \cdot \nabla G \rangle^2}{\sigma_v^2 \sigma_{vG}^2}. $$

(1)

Extending DS10 to the $G$ field, we obtain

$$ b_v(k) \approx 1 - R_c^2 k^2, \quad R_c^2 \equiv \frac{\sigma_{vG}^2}{\sigma_{vG}^2}. $$

(2)

Here $\sigma_{vG}^2 \equiv \langle \alpha^2 \rangle$ ($\alpha = \delta, \nabla \delta, G, \nabla G, v, v_0$). We emphasize again that all properties are the smoothed properties with
smoothing scale $R_\Delta$.

For heuristic purposes, we show the derivation of Equation (1) here. The full derivation should be done in 3D,

which is lengthy. However, as found in BBKS and DS10, the
expression of velocity bias in 1D can be converted into the
realistic 3D case in a straightforward manner. Therefore, we
will only briefly present the derivation of the 1D case, where
the 3D gradient $\nabla$ is replaced by the 1D $\nabla' \equiv d/dx$. The joint
one-point PDF is

$$ P(v, \delta, \delta') = P(v, G, G') \left( \frac{dG}{d\delta} \right)^2. $$

(3)

Since $\langle G G' \rangle = 0$ and $\langle G v \rangle = 0$, the Gaussian PDF $P(v, G, G')$
is separable ($P(v, G, G') = P(G)P(v, G')$). The relevant PDF
for the velocity bias is

$$ P(v, G') = \frac{1}{2\pi \sqrt{1/C_G}} \exp \left\{ - \frac{1}{2} \left( \frac{v - \langle v G' \rangle}{\sigma_v^2} \right)^2 \right\}. $$

(4)

The covariance matrix between $v$ and $G'$ is

$$ C = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} = \begin{pmatrix} \sigma_v^2 & \langle v G' \rangle \\ \langle v G' \rangle & \sigma_{G'}^2 \end{pmatrix}. $$

(5)

Replacing the 1D gradient $G'$ with the 3D gradient $\nabla G$, we
obtain Equation (1). The derivation of $b_v(k)$ requires the
two-point joint PDF and is more lengthy. We refer the readers to
DS10 for details.

The velocity bias arises from a correlation between $v$ and $\nabla \delta$. Both the nonlinear evolution and smaller smoothing scale
$R_\Delta$ weaken such a correlation. We then expect a weaker
deviation of velocity bias from unity, and therefore better
agreement with simulations. Now we proceed to a numerical
evaluation using Equations (1) and (2).

2.1. Numerical Results under the Log-normal Approximation

The density field is known to be close to log-normal (Coles
& Jones 1991; Kofman et al. 1994; Taylor & Watts 2000; Kayo
et al. 2001; Neyrinck et al. 2009). Therefore, to a good
approximation,

$$ G(\delta) = \ln(1 + \delta) - \langle \ln(1 + \delta) \rangle. $$

(6)

Using the cumulant expansion theorem, we obtain

$$ 1 + \delta = e^{G - \sigma_G^2/2}, \quad 1 + \sigma_G^2 = \exp(\sigma_G^2), $$

$$ \sigma_{vG}^2 = \exp(\sigma_G^2) \sigma_{vG}^2, \quad \langle v \cdot \nabla G \rangle = \langle v \cdot \nabla \delta \rangle. $$

(7)
Now \( r \) and \( R_v \) in Equations (1) and (2) can be expressed with statistics of the density field,

\[
\begin{align*}
r^2 &= \frac{\langle v \cdot \nabla \delta \rangle^2}{\sigma_v^2} (1 + \sigma_v^2), \\
R_v^2 &= \frac{(1 + \sigma_v^2) \ln(1 + \sigma_v^2)}{\sigma_v^2}. \tag{8}
\end{align*}
\]

The corresponding properties above are determined by the nonlinear matter power spectrum \( P_\delta(k) \) through

\[
\begin{align*}
\langle v \cdot \nabla \delta \rangle &= \int \frac{k^3}{2 \pi^2} P_\delta(k) W^2_{\ell \Delta}(kR_{\Delta}) \frac{dk}{k}, \\
\sigma_v^2 &= \int \frac{k^3}{2 \pi^2} P_\delta(k) W^2_{\ell \Delta}(kR_{\Delta}) \frac{dk}{k}, \\
\sigma_v^2(v, v) &= \int \frac{k^3}{2 \pi^2} P_\delta(k) W^2_{\ell \Delta}(kR_{\Delta}) k^{0.2} \frac{dk}{k}. \tag{9}
\end{align*}
\]

\( P_\delta \) is evaluated using the CAMB web interface,\(^5\) which uses halolit (Smith et al. 2003) for the nonlinear power spectrum. We adopt a flat \( \Lambda \)CDM cosmology with \( \Omega_m = 0.268, \Omega_\Lambda = 1 - \Omega_m, \Omega_b = 0.044, \sigma_8 = 0.83, n_s = 0.96, \) and \( h = 0.71. \) \( W_{\ell \Delta}(x) = 3(\sin(x) - x \cos(x))/x^3 \) is the top-hat window function. The function \( \hat{W}(k) \leq 1, \) introduced in Zhang et al. (2013), describes the impact of nonlinear evolution in the density–velocity relation. We adopt the fitting formula in Zheng et al. (2013) to evaluate it. The nonlinear evolution weakens the density–velocity correlation, and leads to weaker deviation of \( b_v \) from unity. Including this effect is also essential for correct prediction of halo velocity bias.

Numerical evaluations of Equations (1) and (2) at \( z = 0 \) using Equation (8) are shown in Figures 1 and 2. The predicted \( 1 - \sigma^2_{\delta_v}/\sigma^2_{\delta_v} = r^2 \) increases with the halo mass. It is

\(^5\) https://lambda.gsfc.nasa.gov/toolbox/tb_camb_form.cfm

**Figure 1.** Predicted difference between the \( z = 0 \) halo velocity dispersion and matter velocity dispersion \( 1 - \sigma^2_{\delta_v}/\sigma^2_{\delta_v} = r^2 \), as a function of halo mass \( M \). Our prediction is a factor of \(~4\) smaller than the BBKS prediction.

**Figure 2.** Predicted \( R_v^2 (b_v = 1 - R_v^2k^2) \) at \( z = 0 \), as a function of halo mass \( M \). At \( k = 0.1 h/\text{Mpc}, \) we predict \( 1 - b_v \sim 0.1\% \), consistent with the recent simulation finding (Chen et al. 2018).

0.3\%, 0.9\%, 1.9\%, 3.6\%, and 5.5\%, for halos of mass \( 10^{11.1,12,13,14,15}M_\odot/h \) respectively (Figure 1). As a comparison, the BBKS prediction is a factor of 4 higher. The difference in \( R_v^2 \) is even larger (Figure 2). Our model predicts \( R_v^2 = 0.098(M_/h)^2 \) for \( M = 10^{12}M_\odot/h \). This means that \( 1 - b_v(k = 0.1h/\text{Mpc}) \sim 0.1\% \). For \( M = 10^{13}M_\odot/h \) halos, \( R_v^2 = 0.31(M_/h)^2 \) and \( 1 - b_v(k = 0.1h/\text{Mpc}) \sim 0.3\% \). These predictions agree well with the finding of \( \mathcal{O}(0.1\%) \) deviation of \( b_v \) from unity at \( k \leq 0.1 h/\text{Mpc} \) (Chen et al. 2018). For comparison, \( R_v^2 \) in DS10 is a factor of 10–20 larger.

3. Discussion and Conclusion

Our model extends the peak model methodology to a nonlinear and non-Gaussian density field and is capable of dealing with halos instead of proto-halos. The non-Gaussianity of the density field, the smaller smoothing scale (halo virial radius \( R_\Delta \) versus \( R_\chi \)), and the weaker density–velocity correlation all impact the velocity bias. The non-Gaussianity tends to amplify \( r^2 \) by a factor of \( 1 + \sigma_v^2 \), and amplify \( R_v^2 \) by a factor of \((1 + \sigma_v^2)/(1 + \sigma_v^2)/\sigma_v^2 \) (Equation (8)), comparing to the BBKS and DS10 expressions. In contrast, the nonlinearity and the smaller smoothing scale suppress \( r^2 \) and \( R_v^2 \), by increasing \( \sigma_v^2 \) in the denominator (Equation (8)). It further suppresses \( r^2 \) through the \( \hat{W} < 1 \) factor in the numerator term \( \langle v \cdot \nabla \delta \rangle \). Competitions of these opposite effects result in a weak deviation of \( b_v \) from unity.\(^6\)

Therefore, we provide a feasible explanation of the vanishing volume weighted halo velocity bias observed by Chen et al. (2018). Nevertheless, our model may miss other necessary ingredients because it does not explain all behaviors.

\(^6\) Interestingly, if we neglect the non-Gaussianity and nonlinearity and simply replace \( R_\Delta \) in the BBKS and DS10 expressions by \( R_\chi \), we also obtain a weak deviation of \( b_v \) from unity. But from the above discussions, this is a coincidence caused by the neglected non-Gaussianity and nonlinearity.
of velocity bias observed in simulations. First, it predicts incorrect redshift dependence of velocity bias. Chen et al. (2018) found that the halo velocity bias monotonically increases with decreasing redshift, regardless of halo mass. However, in our model, the redshift dependence of halo velocity bias is not only weaker, but may also be nonmonotonic (for less massive halos). Second, it cannot explain the observed $b_v > 1$ of $\lesssim 10^{12} M_\odot$ halos at $z = 0$. Our model, along with those of BBKS and DS10, always predicts $b_v < 1$. Both failures are likely related to the imperfection of Gaussianization. Furthermore, approximating the Gaussianization function with a log-normal transformation can result in further error (Figure 2, Neyrinck et al. 2009). Another possible cause is the subtlety in halo definitions. The halo catalog (Jing 2018) used by Chen et al. (2018) identifies halos with the FOF algorithm of linking length $b = 0.2$. The corresponding $\Delta$ varies with the halo mass (e.g., More et al. 2011), while the virial overdensity $\Delta$ adopted in our model is mass independent. The two $\Delta$ also have different redshift dependences. Furthermore, the Jing (2018) halo catalog excludes unbound particles in halos after FOF. This further complicates the correspondence between halos in simulations and in theory. We are not able to address these possibilities quantitatively and therefore leave them for future investigation.

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