Abstract

In this paper, an underlay cognitive radio network that consists of two secondary users (SU) and one primary user (PU) is considered. The SUs employ best effort transmission, whereas the PU uses Type-I Hybrid ARQ. Exploiting the redundancy in PU retransmissions, each SU receiver applies forward interference cancelation to remove a successfully decoded PU message in the subsequent PU retransmissions. The knowledge of the PU message state (Known or Unknown) at the SU receivers in addition to the ACK/NACK message from PU receiver are sent back to the transmitters on the error free feedback channels. With this approach and using a Constrained Markov Decision Process (CMDP) model and Constrained Multi-agent MDP (CMMDP), centralized and decentralized optimum access policies for two SUs are proposed to maximize their average long term sum throughput under an average long term PU throughput constraint. In the decentralized case, we assume that the channel access decision of each SU is unknown to the other SU. Numerical results demonstrate the benefits of the proposed optimal policies in terms of sum throughput of SUs. The results also reveal that the centralized access policy design outperforms the decentralized design especially when the PU can tolerate a low average long term throughput. Finally, extensions to an arbitrary number of SUs as well as the difficulties in decentralized access policy design with partial state information are described.

I. INTRODUCTION

The advent of new technologies and services in wireless communication has increased the demand for spectrum resources so that the traditional fixed frequency allocation will not be
able to meet these bandwidth requirements. However, most of the spectrum frequencies assigned to licensed users are under-utilized. Thus, cognitive radio is proposed to improve the spectral efficiency of wireless networks [2]. Cognitive radio enables the licensed primary users (PUs) and unlicensed secondary users (SUs) to coexist and transmit in the same frequency band [3], [4]. For a literature review on spectrum sharing and cognitive radio, the reader is referred to [5]-[7].

In the underlay cognitive radio approach, the smart SUs are allowed to simultaneously transmit in the licensed frequency band allotted to the PU. The PU is oblivious to the presence of the SUs while the SU needs to control the interference it causes at the PU receiver. Exploiting the Type-I HARQ retransmissions implemented by the PU is employed in [8], [9] and [10]. [8] considers a cognitive radio network composed of one PU and one SU, and does not utilize interference cancelation (IC) at the SU receiver. [9] employs Type-I HARQ with an arbitrary number of retransmissions and applies backward and forward IC after decoding the PU message at the SU receiver. The network considered in [10] is similar to [8], where the SU is also allowed to selectively retransmit its own previous corrupted message and apply a chain decoding protocol to derive the SU access policy. [11] applies Type-II Hybrid ARQ with at most one retransmission, where the SU receiver tries to decode the PU message in the first time slot and, if successful, it removes this PU message in the second time slot to improve the SU throughput. Extending the work in [11] to IR-HARQ with multiple rounds is addressed in [12], where several schemes are proposed. [13] proposes SU transmission schemes when the SU is able to infrequently probe the channel using the PU Type-II HARQ feedback with Chase combining (CC-HARQ). Exploiting primary Type-II HARQ in CRN has also been studied in [14] and [15].

In this paper, an optimum access policy for two SUs is designed, which exploits the redundancy introduced by the Type-I Hybrid-ARQ protocol in transmitting copies of the same PU message and interference cancelation at the SU receivers. The aim is to maximize the average long term sum throughput of SUs under a constraint on the average long term PU throughput degradation. We assume that the number of retransmissions is limited to at most $T$ times and both SUs have a new packet to transmit in each time slot. Two design scenarios are considered where in the first one, each SU is aware of the access decision made by the other SU, whereas in the second scenario, each SU does not know whether or not the other secondary user accesses the channel. We call them respectively as centralized and decentralized scenarios. Noting the
PU message knowledge state at each of the SU receivers and also the ARQ retransmission time, the $PU-SU_1-SU_2$ network is modeled using Markov Decision Process (MDP) and Multi-agent Markov Decision Process (MMDP) models [16], respectively in centralized and decentralized scenarios. Due to the constraint on the average long term PU throughput, we then have a constrained MDP (CMDP) and Constrained MMDP (CMMDP).

In the centralized case, the access policy in one state shows the joint probability of accessing or/and not accessing the channel by the two SUs. Using [17] and [18], it follows that the optimal policy may be obtained by the solution of the corresponding LP problem. In the decentralized scenario, there is an access policy for each SU describing the probability of accessing the channel by that SU. It is noteworthy that we are interested in random access policies instead of only deterministic access policies. Hence, the optimum policies in the centralized case can not be applied to a decentralized scenario. To propose local optimum access policies for the CMMDP model, we employ Nash Equilibrium.

The simulation results demonstrate that due to the use of forward IC (FIC), a cognitive radio network composed of two symmetric SUs converges to the upper bound faster than a cognitive radio network with one SU for large enough SNR of the channels from the PU transmitter to SU receivers. The results also reveal that our proposed centralized access policy design significantly outperforms the decentralized one when the average PU throughput constraint is low.

The paper is organized as follows. Following the system model in Section II, the rates and the corresponding outage probabilities are computed in Section III. Optimal access policies for two SUs in centralized and decentralized scenarios are proposed respectively in Sections IV and V. The numerical results are presented in Section VI and some extensions to the paper are discussed in Section VII. Finally the paper is concluded in Section VIII.

II. SYSTEM MODEL

In the system we consider, there exist one primary and two secondary transmitters denoted by $PU_{tx}, SU_{tx1}$ and $SU_{tx2}$, respectively. These transmitters transmit their messages with constant power over block fading channels. In each time slot (one block of the channel), the channels are considered to be constant. The signal to noise ratio of the channels $PU_{tx} \rightarrow PU_{rx}, PU_{tx} \rightarrow SU_{rx1}, PU_{tx} \rightarrow SU_{rx2}, SU_{tx1} \rightarrow SU_{rx1}, SU_{tx1} \rightarrow SU_{rx2}, SU_{tx2} \rightarrow SU_{rx1}, SU_{tx2} \rightarrow SU_{rx2}, SU_{tx1} \rightarrow PU_{rx}$ and $SU_{tx2} \rightarrow PU_{rx}$ are denoted by $\gamma_{pp}, \gamma_{ps1}, \gamma_{ps2}, \gamma_{s1s1}, \gamma_{s1s2}, \gamma_{s2s1}, \gamma_{s2s2}, \gamma_{s1p}$
and $\gamma_{s2p}$, respectively.

We assume that no Channel State Information (CSI) is available at the transmitters except the ACK/NACK message and the PU message knowledge state. Thus, transmissions are under outage, when the selected rates are greater than the current channel capacity.

PU is unaware of the presence of the SUs and employs Type-I HARQ with at most $T$ transmissions of the same PU message. We assume that the ARQ feedback is received by the PU transmitter at the end of a time-slot and a retransmission can be performed in the next time-slot. Retransmission of the PU message is performed if it is not successfully decoded at the PU receiver until the PU message is correctly decoded or the maximum number of transmissions allowed, $T$, is reached. In each time-slot, each SU, if it accesses the channel, transmits its own message, otherwise it stays idle and does not transmit. This decision is based on the access policy described later. The activity of the SUs affects the outage performance of the PU, by creating interference to the PU receiver. The objective is to design access policies for two SUs to maximize the average sum throughput of the SUs under a constraint on the PU average throughput degradation.

As denoted, we consider the centralized and decentralized scenarios. In the centralized scenario, there exists a central unit which receives the PU message knowledge states of two SUs as well as ACK/NACK message from PU receiver. This unit then computes the secondary access probabilities and provides them to the two SUs. In the decentralized scenario, there exists no central unit. However, the PU message knowledge state at each SU receiver is transmitted to both SU transmitters, but the two SU transmitters make channel access decisions independently, based on the PU message knowledge states and the ACK/NACK message from PU receiver. Thus, in the decentralized design each SU is not aware of the accessibility of the other SU to the channel.

If $SU_{rx1}$ or $SU_{rx2}$ succeeds in decoding the PU message, it can cancel the PU message from the received signal in future retransmissions. We refer to this as Forward Interference Cancelation (FIC) [9]. We call the PU message knowledge state as $\phi \in \{(K,K),(K,U),(U,K),(U,U)\}$, which denotes the knowledge of the PU message at the two SU receivers. For example, if $\phi = (K,K)$ then $SU_{rx1}$ and $SU_{rx2}$ both know the PU message and thus can perform FIC.

In the centralized scenario, we have four different combinations of the accessibility of the SUs to the channel, listed in the accessibility set $\varphi = \{(0,0),(1,0),(0,1),(1,1)\}$. The $a^{th}$ element
of the accessibility set denoted by $\phi(a) = (\phi(a, 1), \phi(a, 2))$ is referred to as accessibility action $a \in A$, where $A = \{0, 1, 2, 3\}$. For example, $\phi(1) = (1, 0)$ shows that only $SU_{tx1}$ accesses the channel. On the contrary in the decentralized case, $SU_i$ has its action $a_i$ which belongs to $A_i = \{0, 1\}$, where $a_i = 0$ means that $SU_i$ is not allowed to access the channel.

III. RATES AND OUTAGE PROBABILITIES

First we consider the centralized scenario, where we have accessibility action $a \in A = \{0, 1, 2, 3\}$ and then we address the decentralized scenario with two accessibility actions $a_1 \in A_1 = \{0, 1\}$ and $a_2 \in A_2 = \{0, 1\}$.

A. Centralized Scenario

The PU transmission rate, indicated by $R_p$, is considered fixed. However, based on PU message knowledge state $\phi$ and accessibility action $a$, the rate of the secondary user $i$ can be adapted and is denoted by $R_{s_i,a,\phi}$, $a \in \{1, 2, 3\}$. (The rates in accessibility action $a = 0$ are zero.) Since, for $a \in \{1, 2\}$, only one of the SUs transmits, we have $R_{s_1,2,\phi} = 0$ and $R_{s_2,1,\phi} = 0$; and furthermore,

$$R_{s_1,1,(\theta,K)} = R_{s_1,1,(\theta,U)} \triangleq R_{s_1,1,\theta} \quad \theta \in \{U, K\},$$

$$R_{s_2,2,(K,\theta)} = R_{s_2,2,(U,\theta)} \triangleq R_{s_2,2,\theta} \quad \theta \in \{U, K\}. \quad (1)$$

We also define $R_{s_1,3,(K,K)} \triangleq R_{s_1,3,K}$ and $R_{s_2,3,(K,K)} \triangleq R_{s_2,3,K}$.

The outage probability of the channel $PU_{tx} \rightarrow PU_{rx}$ in SU accessibility action $a$ is denoted by $\rho_{p,a}$. Noting that the $SU_1$ and $SU_2$ transmissions are considered as background noise at the $PU_{rx}$, we have

$$\rho_{p,a} = 1 - Pr \left( R_p \leq C \left( \frac{\gamma_{pp}}{1 + \varphi(a, 1)\gamma_{s_1p} + \varphi(a, 2)\gamma_{s_2p}} \right) \right) \quad a \in A = \{0, 1, 2, 3\}; \quad (3)$$

where

$$\varphi(a, 1) = \begin{cases} 0 & \text{if } a \in \{0, 2\} \\
1 & \text{otherwise} \end{cases} \quad (4)$$

$$\varphi(a, 2) = \begin{cases} 0 & \text{if } a \in \{0, 1\} \\
1 & \text{otherwise} \end{cases} \quad (5)$$
To compute the outage probability of the channel $SU_{txi} \rightarrow SU_{rxi}$, $i \in \{1, 2\}$, we first define the following SNR regions associated with decidability of $SU_i$ and $PU$ messages at $SU_{rxi}$.

$$\Gamma_{s_1,1,K}(R_{s_1,1,K}) \triangleq \left\{ (\gamma_{s_1s_1}) : R_{s_1,1,K} \leq C(\gamma_{s_1s_1}) \right\}$$

$$\hat{\Gamma}_{s_1,1,U}(R_p, R_{s_1,1,U}) \triangleq \left\{ (\gamma_{s_1s_1}, \gamma_{ps_1}) : R_{s_1,1,U} \leq C(\gamma_{s_1s_1}), R_p \leq C(\gamma_{ps_1}), R_{s_1,1,U} + R_p \leq C(\gamma_{s_1s_1} + \gamma_{ps_1}) \right\}$$

$$\bigcup \left\{ (\gamma_{s_1s_1}, \gamma_{ps_1}) : R_p > C(\gamma_{ps_1}), R_{s_1,1,U} \leq C\left(\frac{\gamma_{s_1s_1}}{1 + \gamma_{ps_1}}\right) \right\}$$

$$\Gamma_{s_1,3,(K,\theta)}(R_{s_1,3,(K,\theta)}) \triangleq \left\{ (\gamma_{s_1s_1}, \gamma_{s_2s_1}) : R_{s_1,3,(K,\theta)} \leq C(\gamma_{s_1s_1}) \right\}$$

$$\bigcup \left\{ (\gamma_{s_1s_1}, \gamma_{s_2s_1}) : R_{s_1,3,(K,\theta)} \leq C(\gamma_{s_2s_1}), R_{s_1,3,(K,\theta)} + R_{s_2,3,(K,\theta)} \leq C(\gamma_{s_1s_1} + \gamma_{s_2s_1}) \right\}$$

$$\bigcup \left\{ (\gamma_{s_1s_1}, \gamma_{s_2s_1}) : R_{s_1,3,(K,\theta)} + R_{s_2,3,(K,\theta)} \leq C(\gamma_{s_1s_1} + \gamma_{s_2s_1}), R_{s_1,3,(U,\theta)} + R_{s_2,3,(U,\theta)} + R_p \leq C(\gamma_{s_1s_1} + \gamma_{s_2s_1} + \gamma_{ps_1}) \right\}$$

$$\Gamma_{s_1,3,(U,\theta)}(R_p, R_{s_1,3,(U,\theta)}, R_{s_2,3,(U,\theta)}) \triangleq \left\{ (\gamma_{ps_1}, \gamma_{s_1s_1}, \gamma_{s_2s_1}) : R_p \leq C(\gamma_{ps_1}), R_{s_1,3,(U,\theta)} \leq C(\gamma_{s_1s_1}), R_{s_2,3,(U,\theta)} \leq C(\gamma_{s_2s_1}) \right\}$$

$$\bigcup \left\{ (\gamma_{ps_1}, \gamma_{s_1s_1}, \gamma_{s_2s_1}) : R_p \leq C(\gamma_{ps_1}), R_{s_1,3,(U,\theta)} \leq C\left(\frac{\gamma_{s_1s_1}}{1 + \gamma_{ps_1}}\right), R_{s_2,3,(U,\theta)} \leq C\left(\frac{\gamma_{s_2s_1}}{1 + \gamma_{ps_1}}\right) \right\}$$

$$\bigcup \left\{ (\gamma_{ps_1}, \gamma_{s_1s_1}, \gamma_{s_2s_1}) : R_{s_1,3,(U,\theta)} + R_{s_2,3,(U,\theta)} \leq C\left(\frac{\gamma_{s_1s_1} + \gamma_{s_2s_1}}{1 + \gamma_{ps_1}}\right) \right\}$$

$$\bigcup \left\{ (\gamma_{ps_1}, \gamma_{s_1s_1}, \gamma_{s_2s_1}) : R_{s_1,3,(U,\theta)} + R_p \leq C\left(\frac{\gamma_{s_1s_1} + \gamma_{ps_1}}{1 + \gamma_{s_2s_1}}\right) \right\}$$

$$\bigcup \left\{ (\gamma_{ps_1}, \gamma_{s_1s_1}, \gamma_{s_2s_1}) : R_{s_1,3,(U,\theta)} + R_p \leq C\left(\frac{\gamma_{s_1s_1} + \gamma_{ps_1} + \gamma_{s_2s_1}}{1 + \gamma_{s_2s_1} + \gamma_{ps_1}}\right) \right\}$$
\[
\Gamma_{s_2,2,K}(R_{s_2,2,K}) \triangleq \left\{ (\gamma_{s_2s_2}) : R_{s_2,2,K} \leq C(\gamma_{s_2s_2}) \right\} \tag{14}
\]

\[
\hat{\Gamma}_{s_2,2,U}(R_p, R_{s_2,2,U}) \triangleq \left\{ (\gamma_{s_2s_2}, \gamma_{ps_2}) : R_{s_2,2,U} \leq C(\gamma_{s_2s_2}) \right\},
\]

\[
R_p \leq C(\gamma_{ps_2}), R_{s_2,2,U} + R_p \leq C(\gamma_{s_2s_2} + \gamma_{ps_2})
\]

\[
\bigcup \left\{ (\gamma_{s_2s_2}, \gamma_{ps_2}) : R_p > C(\gamma_{ps_2}), R_{s_2,2,U} \leq C\left(\frac{\gamma_{s_2s_2}}{1 + \gamma_{ps_2}}\right) \right\} \tag{15}
\]

\[
\Gamma_{s_2,3,(\theta,K)}(R_{s_1,3,(\theta,K)}, R_{s_2,3,(\theta,K)}) \triangleq \left\{ (\gamma_{s_1s_2}, \gamma_{s_2s_2}) : R_{s_1,3,(\theta,K)} \leq C(\gamma_{s_1s_2}), \right. \\
R_{s_2,3,(\theta,K)} \leq C(\gamma_{s_2s_2}), R_{s_2,3,(\theta,K)} + R_{s_2,3,(\theta,K)} \leq C(\gamma_{s_1s_2} + \gamma_{s_2s_2})
\]

\[
\bigcup \left\{ (\gamma_{s_1s_2}, \gamma_{s_2s_2}) : R_{s_1,3,(\theta,K)} > C(\gamma_{s_1s_2}), R_{s_2,3,(\theta,K)} \leq C\left(\frac{\gamma_{s_2s_2}}{1 + \gamma_{s_1s_2}}\right) \right\} \tag{16}
\]

\[
\hat{\Gamma}_{s_2,3,(\theta,U)}(R_p, R_{s_1,3,(\theta,U)}, R_{s_2,3,(\theta,U)}) \triangleq \left\{ (\gamma_{ps_2}, \gamma_{s_1s_2}, \gamma_{s_2s_2}) : R_p \leq C(\gamma_{ps_2}), R_{s_1,3,(\theta,U)} \leq C(\gamma_{s_1s_2}), \right. \\
R_{s_2,3,(\theta,U)} \leq C(\gamma_{s_2s_2}), R_{s_2,3,(\theta,U)} + R_p \leq C(\gamma_{s_1s_2} + \gamma_{s_2s_2})
\]

\[
R_{s_1,3,(\theta,U)} + R_{s_2,3,(\theta,U)} \leq C(\gamma_{s_1s_2} + \gamma_{s_2s_2}), R_{s_1,3,(\theta,U)} + R_{s_2,3,(\theta,U)} + R_p \leq C(\gamma_{s_1s_2} + \gamma_{s_2s_2} + \gamma_{ps_2})
\]

\[
\bigcup \left\{ (\gamma_{s_1s_2}, \gamma_{s_1s_2}, \gamma_{s_2s_2}) : R_p > C(\gamma_{ps_2}), R_{s_1,3,(\theta,U)} \leq C\left(\frac{\gamma_{s_1s_2}}{1 + \gamma_{ps_2}}\right), R_{s_2,3,(\theta,U)} \leq C\left(\frac{\gamma_{s_2s_2}}{1 + \gamma_{s_1s_2}}\right), \right. \\
R_{s_1,3,(\theta,U)} + R_{s_2,3,(\theta,U)} \leq C\left(\frac{\gamma_{s_1s_2} + \gamma_{s_2s_2}}{1 + \gamma_{ps_2}}\right)
\]

\[
\bigcup \left\{ (\gamma_{ps_2}, \gamma_{s_1s_2}, \gamma_{s_2s_2}) : R_{s_1,3,(\theta,U)} > C(\gamma_{s_1s_2}), R_{s_2,3,(\theta,U)} \leq C\left(\frac{\gamma_{s_2s_2}}{1 + \gamma_{s_1s_2}}\right), \right. \\
R_{s_2,3,(\theta,U)} + R_p \leq C\left(\frac{\gamma_{s_2s_2} + \gamma_{ps_2}}{1 + \gamma_{s_1s_2}}\right)
\]

\[
\bigcup \left\{ (\gamma_{ps_2}, \gamma_{s_1s_2}, \gamma_{s_2s_2}) : R_{s_1,3,(\theta,U)} > C(\gamma_{s_1s_2}), R_p > C(\gamma_{ps_2}), R_{s_2,3,(\theta,U)} \leq C\left(\frac{\gamma_{s_2s_2}}{1 + \gamma_{s_1s_2} + \gamma_{ps_2}}\right) \right\} \tag{21}
\]
The SNR region \( \tilde{\Gamma}_{s_i,1,U}(R_p, R_{s_i,1,U}) \), \( i \in \{1, 2\} \) is the union of two regions. The first region guarantees that the \( SU_i \) and PU messages respectively transmitted at rates \( R_{s_i,1,U} \) and \( R_p \) are both correctly decoded at \( SU_{rx_i} \) via joint decoding. On the other hand, in the second region, only the \( SU_i \) message can be successfully decoded by assuming the interference from PU as background noise. Note that the other source is idle.

The SNR region \( \Gamma_{s_i,3,\phi}(R_{s_1,3,\phi}, R_{s_2,3,\phi}) \), \( i \in \{1, 2\} \), where \( \phi = (K, \theta) \) for \( i = 1 \) and \( \phi = (\theta, K) \) for \( i = 2 \), guarantees that the \( SU_i \) message transmitted at rate \( R_{s_i,3,\phi} \) is successfully decoded at \( SU_{rx_i} \) regardless of the decoding of another SU message transmitted at rate \( R_{s_j,3,\phi} \). Note that here the PU message received at \( SU_{rx_i} \) is canceled using FIC. On the other hand, if the PU message is not decoded at \( SU_{rx_i} \), the SNR region \( \tilde{\Gamma}_{s_i,3,\phi}(R_p, R_{s_1,3,\phi}, R_{s_2,3,\phi}) \), where \( \phi = (U, \theta) \) for \( i = 1 \) and \( \phi = (\theta, U) \) for \( i = 2 \), guarantees that the \( SU_i \) message transmitted at rate \( R_{s_i,3,\phi} \) is successfully decoded at \( SU_{rx_i} \) irrespective of the decoding of another SU and PU messages transmitted at rates \( R_{s_j,3,\phi} \) and \( R_p \), respectively.

We now return to the calculation of the outage probability of the channel \( SU_{tx_i} \rightarrow SU_{rx_i} \), \( i \in \{1, 2\} \) at the PU message knowledge state \( \phi \) and accessibility action \( a \) which is denoted by \( \rho_{s_i,a,\phi} \). In PU knowledge state \((K, K)\) or \((K, U)\), the PU message is known at \( SU_{rx_1} \) and therefore the PU message may be canceled at this receiver. Thus, for accessibility action \( a = 1 \) we have \( \rho_{s_1,1,(K,K)} = \rho_{s_1,1,(K,U)} \triangleq \rho_{s_1,1,K} \), where

\[
\rho_{s_1,1,K} = Pr \left( \gamma_{s_1s_1} \notin \Gamma_{s_1,1,K}(R_{s_1,1,K}) \right). \tag{22}
\]

In contrast, in PU knowledge state \((U, K)\) or \((U, U)\), where the PU message is not decoded at \( SU_{rx_1} \), the outage probability of the channel from \( SU_{tx_1} \) to \( SU_{rx_1} \) is under the influence of the received PU message. Thus, for accessibility action \( a = 1 \), we have \( \rho_{s_1,1,(U,K)} = \rho_{s_1,1,(U,U)} \triangleq \rho_{s_1,1,U} \), where

\[
\rho_{s_1,1,U} = Pr \left( \gamma_{s_1s_1} \notin \tilde{\Gamma}_{s_1,1,U}(R_p, R_{s_1,1,U}) \right). \tag{23}
\]

In a similar way we obtain \( \rho_{s_2,2,(K,K)} = \rho_{s_2,2,(U,K)} \triangleq \rho_{s_2,2,K} \) and \( \rho_{s_2,2,(K,U)} = \rho_{s_2,2,(U,U)} \triangleq \rho_{s_2,2,U} \) where,

\[
\rho_{s_2,2,K} = Pr \left( \gamma_{s_2s_2} \notin \Gamma_{s_2,2,K}(R_{s_2,2,K}) \right) \tag{24}
\]

\[
\rho_{s_2,2,U} = Pr \left( \gamma_{s_2s_2} \notin \tilde{\Gamma}_{s_2,2,U}(R_p, R_{s_2,2,U}) \right). \tag{25}
\]
For accessibility action $a = 3$ and $\theta \in \{U, K\}$, we have

$$\rho_{s_1,3,(K,\theta)} = Pr\left((\gamma_{s_1s_1}^{s_1}, \gamma_{s_2s_1}^{s_1}) \notin \Gamma_{s_1,3,(K,\theta)}(R_{s_1,3,(K,\theta)}, R_{s_2,3,(K,\theta)})\right);$$  \hfill (26)

$$\rho_{s_2,3,(\theta,K)} = Pr\left((\gamma_{s_1s_2}^{s_2}, \gamma_{s_2s_2}^{s_2}) \notin \Gamma_{s_2,3,(\theta,K)}(R_{s_2,3,(\theta,K)}, R_{s_2,3,(\theta,K)})\right);$$  \hfill (27)

$$\rho_{s_1,3,(\theta,U)} = Pr\left((\gamma_{ps_1}^{s_1}, \gamma_{s_1s_1}^{s_1}, \gamma_{s_2s_1}^{s_1}) \notin \tilde{\Gamma}_{s_1,3,(\theta,U)}(R_p, R_{s_1,3,(\theta,U)}, R_{s_2,3,(\theta,U)})\right);$$ \hfill (28)

$$\rho_{s_2,3,(\theta,U)} = Pr\left((\gamma_{ps_2}^{s_2}, \gamma_{s_1s_2}^{s_2}, \gamma_{s_2s_2}^{s_2}) \notin \tilde{\Gamma}_{s_2,3,(\theta,U)}(R_p, R_{s_1,3,(\theta,U)}, R_{s_2,3,(\theta,U)})\right)$$ \hfill (29)

and moreover $\rho_{s_i,3,(K,K)} \triangleq \rho_{s_i,3,K}$, $i \in \{1, 2\}$. Note that (26) to (29) include the effect of the mutual interference between the SUs.

The following SNR regions demonstrate the decoding of the PU message at SU receivers.

$$\tilde{\Gamma}_{ps_1,1,U}(R_p, R_{s_1,1,U}) \triangleq \left\{(\gamma_{s_1s_1}^{s_1}, \gamma_{ps_1}) : R_{s_1,1,U} \leq C(\gamma_{s_1s_1}^{s_1}), R_p \leq C(\gamma_{ps_1})\right\}$$  \hfill (30)

$$R_{s_1,1,U} + R_p \leq C(\gamma_{s_1s_1}^{s_1} + \gamma_{ps_1})$$

$$\bigcup\left\{(\gamma_{s_1s_1}^{s_1}, \gamma_{ps_1}) : R_{s_1,1,U} > C(\gamma_{s_1s_1}^{s_1}), R_p \leq C\left(\frac{\gamma_{ps_1}}{1 + \gamma_{s_1s_1}^{s_1}}\right)\right\}$$

$$\tilde{\Gamma}_{ps_1,2,U}(R_p, R_{s_2,2,U}) \triangleq \left\{(\gamma_{s_2s_1}^{s_2}, \gamma_{ps_1}) : R_{s_2,2,U} \leq C(\gamma_{s_2s_1}^{s_2}), R_p \leq C(\gamma_{ps_1})\right\}$$  \hfill (31)

$$R_{s_2,2,U} + R_p \leq C(\gamma_{s_2s_1}^{s_2} + \gamma_{ps_1})$$

$$\bigcup\left\{(\gamma_{s_2s_1}^{s_2}, \gamma_{ps_1}) : R_{s_2,2,U} > C(\gamma_{s_2s_1}^{s_2}), R_p \leq C\left(\frac{\gamma_{ps_1}}{1 + \gamma_{s_2s_1}^{s_2}}\right)\right\}$$

$$\tilde{\Gamma}_{ps_2,1,U}(R_p, R_{s_1,1,U}) \triangleq \left\{(\gamma_{s_1s_2}^{s_1}, \gamma_{ps_2}) : R_{s_1,1,U} \leq C(\gamma_{s_1s_2}^{s_1}), R_p \leq C(\gamma_{ps_2})\right\}$$  \hfill (32)

$$R_{s_1,1,U} + R_p \leq C(\gamma_{s_1s_2}^{s_1} + \gamma_{ps_2})$$

$$\bigcup\left\{(\gamma_{s_1s_2}^{s_1}, \gamma_{ps_2}) : R_{s_1,1,U} > C(\gamma_{s_1s_2}^{s_1}), R_p \leq C\left(\frac{\gamma_{ps_2}}{1 + \gamma_{s_1s_2}^{s_1}}\right)\right\}$$

$$\tilde{\Gamma}_{ps_2,2,U}(R_p, R_{s_2,2,U}) \triangleq \left\{(\gamma_{s_2s_2}^{s_2}, \gamma_{ps_2}) : R_{s_2,2,U} \leq C(\gamma_{s_2s_2}^{s_2}), R_p \leq C(\gamma_{ps_2})\right\}$$  \hfill (33)

$$R_{s_2,2,U} + R_p \leq C(\gamma_{s_2s_2}^{s_2} + \gamma_{ps_2})$$

$$\bigcup\left\{(\gamma_{s_2s_2}^{s_2}, \gamma_{ps_2}) : R_{s_2,2,U} > C(\gamma_{s_2s_2}^{s_2}), R_p \leq C\left(\frac{\gamma_{ps_2}}{1 + \gamma_{s_2s_2}^{s_2}}\right)\right\}$$
\[\tilde{\Gamma}_{p_{s_1},(\theta,\gamma)}(R_p, R_{s_1,3},(\theta,\gamma), R_{s_2,3},(\theta,\gamma)) = \left\{ (\gamma_{p_{s_1}}, \gamma_{s_{1}}, \gamma_{s_{2}}) : R_p \leq C(\gamma_{p_{s_1}}) \leq C(\gamma_{s_{1}}), R_{s_{2},3},(\theta,\gamma) \right\} \]

\[R_{s_{2},3},(\theta,\gamma) \leq C(\gamma_{s_{1}}), R_{s_{1},3},(\theta,\gamma) + R_p \leq C(\gamma_{s_{1}} + \gamma_{p_{s_1}}) \leq C(\gamma_{s_{2}}) + R_{s_{2},3},(\theta,\gamma) + R_p \leq C(\gamma_{s_{1}} + \gamma_{s_{2}} + \gamma_{p_{s_1}}) \]

\[R_{s_{1},3},(\theta,\gamma) + R_{s_{2},3},(\theta,\gamma) \leq C(\gamma_{s_{1}} + \gamma_{s_{2}}), R_{s_{1},3},(\theta,\gamma) + R_{s_{2},3},(\theta,\gamma) + R_p \leq C(\gamma_{s_{1}} + \gamma_{s_{2}} + \gamma_{p_{s_1}}) \]

(34)

\[\bigcup \left\{ (\gamma_{p_{s_1}}, \gamma_{s_{1}}, \gamma_{s_{2}}) : R_{s_{1},3},(\theta,\gamma) > C(\gamma_{s_{1}}), R_p \leq C(\frac{\gamma_{p_{s_1}}}{1 + \gamma_{s_{1}}}), R_{s_{2},3},(\theta,\gamma) \leq C(\frac{\gamma_{s_{2}}}{1 + \gamma_{s_{2}}}) \right\} \]

(35)

\[R_p + R_{s_{2},3},(\theta,\gamma) \leq C(\frac{\gamma_{p_{s_1}} + \gamma_{s_{2}}}{1 + \gamma_{s_{1}}}) \]

\[\bigcup \left\{ (\gamma_{p_{s_2}}, \gamma_{s_{1}}, \gamma_{s_{2}}) : R_{s_{2},3},(\theta,\gamma) > C(\gamma_{s_{2}}), R_{s_{1},3},(\theta,\gamma) \leq C(\frac{\gamma_{s_{1}}}{1 + \gamma_{s_{2}}}), R_p \leq C(\frac{\gamma_{p_{s_2}}}{1 + \gamma_{s_{2}}}) \right\} \]

(36)

\[R_{s_{1},3},(\theta,\gamma) + R_{s_{2},3},(\theta,\gamma) \leq C(\gamma_{s_{1}} + \gamma_{s_{2}}), R_{s_{1},3},(\theta,\gamma) + R_{s_{2},3},(\theta,\gamma) + R_p \leq C(\gamma_{s_{1}} + \gamma_{s_{2}} + \gamma_{p_{s_2}}) \]

(37)

\[\tilde{\Gamma}_{p_{s_2},(\theta,\gamma)}(R_p, R_{s_1,3},(\theta,\gamma), R_{s_2,3},(\theta,\gamma)) = \left\{ (\gamma_{p_{s_2}}, \gamma_{s_{1}}, \gamma_{s_{2}}) : R_p \leq C(\gamma_{p_{s_2}}) \leq C(\gamma_{s_{1}}), R_{s_{2},3},(\theta,\gamma) \right\} \]

\[R_{s_{2},3},(\theta,\gamma) \leq C(\gamma_{s_{2}}), R_{s_{1},3},(\theta,\gamma) \leq C(\gamma_{s_{1}} + \gamma_{p_{s_2}}) \leq C(\gamma_{s_{2}}) + R_{s_{2},3},(\theta,\gamma) \leq C(\gamma_{s_{1}} + \gamma_{s_{2}} + \gamma_{p_{s_2}}) \]

\[R_{s_{1},3},(\theta,\gamma) + R_{s_{2},3},(\theta,\gamma) \leq C(\gamma_{s_{1}} + \gamma_{s_{2}}), R_{s_{1},3},(\theta,\gamma) + R_{s_{2},3},(\theta,\gamma) + R_p \leq C(\gamma_{s_{1}} + \gamma_{s_{2}} + \gamma_{p_{s_2}}) \]

(38)

\[\bigcup \left\{ (\gamma_{p_{s_2}}, \gamma_{s_{1}}, \gamma_{s_{2}}) : R_{s_{1},3},(\theta,\gamma) > C(\gamma_{s_{1}}), R_p \leq C(\frac{\gamma_{p_{s_2}}}{1 + \gamma_{s_{1}}}), R_{s_{2},3},(\theta,\gamma) \leq C(\frac{\gamma_{s_{2}}}{1 + \gamma_{s_{1}}}) \right\} \]

(39)

\[R_p + R_{s_{2},3},(\theta,\gamma) \leq C(\frac{\gamma_{p_{s_2}} + \gamma_{s_{2}}}{1 + \gamma_{s_{1}}}) \]

\[\bigcup \left\{ (\gamma_{p_{s_2}}, \gamma_{s_{1}}, \gamma_{s_{2}}) : R_{s_{2},3},(\theta,\gamma) > C(\gamma_{s_{2}}), R_{s_{1},3},(\theta,\gamma) \leq C(\frac{\gamma_{s_{1}}}{1 + \gamma_{s_{2}}}), R_p \leq C(\frac{\gamma_{p_{s_2}}}{1 + \gamma_{s_{2}}}) \right\} \]

(40)

\[R_{s_{1},3},(\theta,\gamma) + R_p \leq C(\frac{\gamma_{s_{1}} + \gamma_{p_{s_2}}}{1 + \gamma_{s_{2}}}) \]

(41)
The SNR region $\tilde{\Gamma}_{p;i,j,U}(R_p, R_{s;j,j,U})$, $i, j \in \{1, 2\}$, guarantees that the PU message transmitted at rate $R_p$ is successfully decoded at $SU_{rxi}$ regardless of the decoding of $SU_j$ message transmitted at rate $R_{s;j,j,U}$ when only $SU_j$ accesses the channel. The SNR region $\tilde{\Gamma}_{ps;i,3,\phi}(R_p, R_{s;i,3,\phi}, R_{s;j,3,\phi})$, $i \in \{1, 2\}$ where $\phi = (U, \theta)$ for $i = 1$ and $\phi = (\theta, U)$ for $i = 2$, guarantees that the PU message transmitted at rate $R_p$ is successfully decoded at $SU_{rxi}$ without considering the decoding of the SU messages at rates $R_{s;i,3,\phi}$ and $R_{s;j,3,\phi}$.

Since the value of $R_{s;i,i,K}$ does not affect the outage performance at $PU_{rx}$ and $SU_{tx}$, $\{i, j\} \in \{\{1, 2\}, \{2, 1\}\}$ and the PU message can be canceled at $SU_{rxi}$, this rate is chosen so as to maximize the $SU_i$ throughput. Rate $R_{s;i,3,(K,K)}$ does not affect the outage performance at $PU_{rx}$ and again there is no PU message interfering at the $SU_i$ receiver. Thus, the values of $R_{s;i,3,(K,K)}$ and $R_{s;j,3,(K,K)}$ are jointly selected such that the SU sum throughput is maximized, whereas the same argument can not be applied for the states with unknown PU message, because in this case there is a tradeoff between the SU sum throughput and helping the SU receivers to decode the PU message.

**B. Decentralized Scenario**

As denoted in the decentralized case, each SU can not know the accessibility action selected by the other user. In this case each user has two accessibility actions $a_1 \in A_1 = \{0, 1\}$ and $a_2 \in A_2 = \{0, 1\}$. Since the action selected by each user is unknown to the other user in the decentralized scenario, the rate for $SU_i$ at state $a_i$ and PU message knowledge state $\phi \in \{(U,U), (U,K), (K,K), \{K,U\}\}$ is defined by $R_{s;i,a_i,\phi}$. There is the following correspondence between the actions $a_1$ and $a_2$ in the decentralized scenario and action $a$ in the centralized one:

$$a = \begin{cases} 0 & \text{if } a_1 = 0, a_2 = 0 \\ 1 & \text{if } a_1 = 1, a_2 = 0 \\ 2 & \text{if } a_1 = 0, a_2 = 1 \\ 3 & \text{if } a_1 = 1, a_2 = 1. \end{cases}$$

Hence, the outage probabilities as defined in Section III-A by substituting $a_1, a_2$ into $a$ can be used. Thus, the outage probability of the channel $PU_{tx} \to PU_{rx}$ and the outage probability of the channel $SU_{tx} \to SU_{rx}$, $i \in \{1, 2\}$ in SU accessibility action $a_1$ and $a_2$ are respectively denoted by $\rho_{p,a_1,a_2}$ and $\rho_{s;i,a_1,a_2,\phi}$.
In the next section, we propose optimal centralized access policies for SU transmitters to maximize the average SU sum throughput under a constraint on the PU throughput degradation.

IV. CENTRALIZED OPTIMAL ACCESS POLICIES FOR TWO SUs

The state of the PU – SU1 – SU2 system may be modeled by a Markov Decision Process \( s = (t, \phi) \), where \( t \in \{1, 2, \ldots, T\} \) is the primary ARQ state and \( \phi \in \{(U, U), (U, K), (K, K), \{K, U\}\} \) denotes the PU message knowledge state. The set of all states is indicated by \( S \).

The policy \( \mu \) maps the state of the network \( s \) to the probability that the secondary users take accessibility action \( a \in \{0, 1, 2, 3\} \). The probability that action \( a \) is selected in state \( s \) is denoted by \( \mu(a, s) \). For example, with probability \( \mu(1, s) \), \( SU_{tx1} \) transmits while \( SU_{tx2} \) does not access the channel; and with probability \( \mu(0, s) = 1 - \mu(1, s) - \mu(2, s) - \mu(3, s) \), they are both idle.

If accessibility action \( a \) is selected, the expected throughputs of \( SU_1 \) and \( SU_2 \) in state \( s = (t, \phi) \) are respectively computed as

\[
T_{s_1, a, \phi} = \begin{cases} 
R_{s_1, a, \phi}(1 - \rho_{s_1, a, \phi}) & \text{for } a \in \{1, 3\} \\
0 & \text{for } a \in \{0, 2\}
\end{cases}
\]

(43)

\[
T_{s_2, a, \phi} = \begin{cases} 
R_{s_2, a, \phi}(1 - \rho_{s_2, a, \phi}) & \text{for } a \in \{2, 3\} \\
0 & \text{for } a \in \{0, 1\}
\end{cases}
\]

(44)

Since the model considered here is a stationary Markov chain, the average long term SU sum throughput can be obtained as

\[
\bar{T}_{su,c} = E_{a,s=(t,\phi)} \left[ T_{s_1, a, \phi} + T_{s_2, a, \phi} \right]
= E_{a=(t,\phi)} \left[ \mu(1, s)R_{s_1,1,\phi}(1 - \rho_{s_1,1,\phi}) + \mu(2, s)R_{s_2,2,\phi}(1 - \rho_{s_2,2,\phi}) \\
\mu(3, s)(R_{s_1,3,\phi}(1 - \rho_{s_1,3,\phi}) + R_{s_2,3,\phi}(1 - \rho_{s_2,3,\phi})) \right],
\]

(45)

where \( E_{a,s} \) denotes the expectation with respect to \( a \) and \( s \). The outage probabilities \( \rho_{s_1,1,\phi}, \rho_{s_2,2,\phi}, \rho_{s_1,3,\phi} \) and \( \rho_{s_2,3,\phi} \) are given in (22) to (29).

The aim is to maximize the average long term sum throughput of the SUs under the long term average PU throughput constraint, where the average long term PU throughput is given by

\[
\bar{T}_{pu} = R_p \left( 1 - \sum_{a=0}^{3} E_{s=(t,\phi)} [\mu(a, s)] \rho_{p,a} \right).
\]

Using \( \mu(0, s) = 1 - \mu(1, s) - \mu(2, s) - \mu(3, s) \), the
average long term PU throughput $\bar{T}_{pu}$ is rewritten as follows:

$$
\bar{T}_{pu} = R_p \left( 1 - \sum_{a=1}^{3} E_{s=(t,\phi)} [\mu(a, s)] \rho_{p,a} \right) \\
- R_p \left( \rho_{p,0} - \sum_{a=1}^{3} E_{s=(t,\phi)} [\mu(a, s)] \rho_{p,0} \right) \\
= T^I_{pu} - R_p \left( \sum_{a=1}^{3} E_{s=(t,\phi)} [\mu(a, s)] (\rho_{p,a} - \rho_{p,0}) \right) \\
= T^I_{pu} - R_p \left( E_{a,s=(t,\phi)} [\rho_{p,a} - \rho_{p,0}] \right),
$$

where

$$
T^I_{pu} = R_p (1 - \rho_{p,0});
$$

and $\rho_{p,0}$, $\rho_{p,1}$, $\rho_{p,2}$ and $\rho_{p,3}$ are given in (3).

Thus, if we request that $\bar{T}_{pu} \geq T^I_{pu} (1 - \epsilon_{PU})$, the PU throughput degradation constraint is computed as follows

$$
T^I_{pu} - \bar{T}_{pu} = R_p E_{a,s=(t,\phi)} [\rho_{p,a} - \rho_{p,0}] \leq R_p (1 - \rho_{p,0}) \epsilon_{PU}.
$$

Now we can formalize the optimization problem as follows:

**Problem 1:**

$$
\text{maximize } \bar{T}_{su,c}(\mu) = E_{a,s=(t,\phi)} [T_{s_1,a,\phi} + T_{s_2,a,\phi}] \text{ s.t.}
$$

$$
E_{a,s=(t,\phi)} [\rho_{p,a} - \rho_{p,0}] \leq (1 - \rho_{p,0}) \epsilon_{PU} \triangleq \epsilon_{\omega},
$$

where $\mu(a, s)$ is the probability that accessibility action $a$ is selected in state $s$.

The constraint (49) is referred to as the normalized PU throughput degradation constraint.

To give a solution to Problem 1 we provide the following definition, which identifies the boundary between low and high access rate regimes.

**Definition 1:** Let $\mu_{\text{init}} = \{\mu_{0,\text{init}}, \mu_{1,\text{init}}, \mu_{2,\text{init}}, \mu_{3,\text{init}}\}$ be the policy such that the secondary user 1 or/and secondary user 2 in all states $s \in S_K = \{(t, (K, K)) : t \in \{1, 2, ..., T\}\}$ access the channel as follows

$$
\mu_{\text{init}} = \begin{cases} 
\{0, 1, 0, 0\} & \text{if } \max(u, v, w) = u \\
\{0, 0, 1, 0\} & \text{if } \max(u, v, w) = v \\
\{0, 0, 0, 1\} & \text{if } \max(u, v, w) = w
\end{cases}
$$
and for all other states \((s \notin \mathcal{S}_K)\), \(\mu_{\text{init}} = \{1, 0, 0, 0\}\), where

\[
\begin{align*}
u &= R_{s_1,1,K}(1 - \rho_{s_1,1,K}) \min\left(\frac{\epsilon_{\omega}}{\rho_{p,1} - \rho_{p,0}}, 1\right) \\
v &= R_{s_2,1,K}(1 - \rho_{s_2,1,K}) \min\left(\frac{\epsilon_{\omega}}{\rho_{p,2} - \rho_{p,0}}, 1\right) \\
w &= (R_{s_3,1,K}(1 - \rho_{s_3,1,K}) + R_{s_2,3,K}(1 - \rho_{s_2,3,K})) \min\left(\frac{\epsilon_{\omega}}{\rho_{p,3} - \rho_{p,0}}, 1\right).
\end{align*}
\]

(51)

For access policy \(\mu_{\text{init}}\), we compute the normalized PU throughput degradation constraint in (49) and refer to it as \(\omega_{\text{init}}\). Hence, replacing (50) in (49) and then computing the expectation with respect to \(a\) and \(s\), \(\omega_{\text{init}}\) can be obtained as follows:

\[
\omega_{\text{init}} = \begin{cases} 
(\rho_{p,1} - \rho_{p,0}) \sum_{t=1}^{T} \pi(t, (K, K)) & \text{if } \max(u, v, w) = u \\
(\rho_{p,2} - \rho_{p,0}) \sum_{t=1}^{T} \pi(t, (K, K)) & \text{if } \max(u, v, w) = v \\
(\rho_{p,3} - \rho_{p,0}) \sum_{t=1}^{T} \pi(t, (K, K)) & \text{if } \max(u, v, w) = w,
\end{cases}
\]

(54)

where \(\pi(t, (K, K))\) is the steady-state probability of being in state \(s = (t, (K, K))\); and \(u\), \(v\) and \(w\) are given in (51) to (53).

In the sequel, we address the upper bound to the average long term sum throughput of SUs, the low SU access rate regime \(\epsilon_{\omega} \leq \omega_{\text{init}}\) and high SU access rate regime \(\epsilon_{\omega} > \omega_{\text{init}}\).

A. Upper Bound to the Average Long Term SU Sum Throughput in Centralized Access Policy Design

An upper bound to the average long term SU sum throughput is achieved when the receivers are assumed to be aware of the PU message, so that they can always cancel the PU interference. Since each SU always knows the PU message, as in [9] there exists an optimal access policy which is independent of the ARQ state, and therefore is the same in each slot. We refer to this policy as \(\mu = \{\mu_0, \mu_1, \mu_2, \mu_3\}\). Thus, noting that \(\mu_1 + \mu_2 + \mu_3 \leq 1\), Problem [1] may be rewritten as follows:

**Problem 2:**

\[
\begin{align*}
\max_{\mu_1, \mu_2, \mu_3} T_{su,c}(\mu) &= \mu_1 R_{s_1,1,K}(1 - \rho_{s_1,1,K}) + \mu_2 R_{s_2,1,K}(1 - \rho_{s_2,1,K}) \\
&+ \mu_3 (R_{s_3,1,K}(1 - \rho_{s_3,1,K}) + R_{s_2,3,K}(1 - \rho_{s_2,3,K}))
\end{align*}
\]

(55)
subject to
\[ \mu_1(\rho_{p,1} - \rho_{p,0}) + \mu_2(\rho_{p,2} - \rho_{p,0}) + \mu_3(\rho_{p,3} - \rho_{p,0}) \leq \epsilon_w \]
\[ \mu_1 + \mu_2 + \mu_3 \leq 1, \] (56)
where, \(0 \leq \mu_1, 0 \leq \mu_2\) and \(0 \leq \mu_3\).

Proposition 1 below provides a solution to Problem 2.

**Proposition 1:** An access policy to achieve the upper bound is given by

\[ \mu^u = \begin{cases} 
1 - \min\left(\frac{\epsilon_w}{\rho_{p,1} - \rho_{p,0}}, 1\right), \min\left(\frac{\epsilon_w}{\rho_{p,1} - \rho_{p,0}}, 1\right), 0, 0 \right) & \text{if } \max(u, v, w) = u \\
1 - \min\left(\frac{\epsilon_w}{\rho_{p,2} - \rho_{p,0}}, 1\right), 0, \min\left(\frac{\epsilon_w}{\rho_{p,2} - \rho_{p,0}}, 1\right), 0 \right) & \text{if } \max(u, v, w) = v \\
1 - \min\left(\frac{\epsilon_w}{\rho_{p,3} - \rho_{p,0}}, 1\right), 0, 0, \min\left(\frac{\epsilon_w}{\rho_{p,3} - \rho_{p,0}}, 1\right) \right) & \text{if } \max(u, v, w) = w.
\] (57)

Furthermore, the upper bound to the average long term SU sum throughput is obtained as

\[ T^u_{su,c} = \begin{cases} 
\min\left(\frac{\epsilon_w}{\rho_{p,1} - \rho_{p,0}}, 1\right) R_{s_1,1,K} (1 - \rho_{s_1,1,K}) & \text{if } \max(u, v, w) = u \\
\min\left(\frac{\epsilon_w}{\rho_{p,2} - \rho_{p,0}}, 1\right) R_{s_2,2,K} (1 - \rho_{s_2,2,K}) & \text{if } \max(u, v, w) = v \\
\min\left(\frac{\epsilon_w}{\rho_{p,3} - \rho_{p,0}}, 1\right) \sum_{i=1}^{2} R_{s_i,3,K} (1 - \rho_{s_i,3,K}) & \text{if } \max(u, v, w) = w,
\end{cases} \] (58)

where \(u, v\) and \(w\) are defined in (51) to (53); and the other parameters are given in Sections II and III.

**Proof:**

Using Lagrangian multipliers \(\lambda_1\) and \(\lambda_2\), the Lagrangian for Problem 2 is

\[ L = \mu_1 R_{s_1,1,K} (1 - \rho_{s_1,1,K}) + \mu_2 R_{s_2,2,K} (1 - \rho_{s_2,2,K}) + \mu_3 R_{s_3,3,K} (1 - \rho_{s_3,3,K}) \]
\[ + R_{s_2,3,K} (1 - \rho_{s_2,3,K}) - \lambda_1 \left( \mu_1 (\rho_{p,1} - \rho_{p,0}) \right) \]
\[ + \mu_2 (\rho_{p,2} - \rho_{p,0}) + \mu_3 (\rho_{p,3} - \rho_{p,0}) - \epsilon_w \right) - \lambda_2 (\mu_1 + \mu_2 + \mu_3 - 1) \] (59)

Please note that the “\(\min\)” operation in (57) and (58) (which was erroneously not included in (11)) is needed to ensure that \(\mu^u\) is a valid probability distribution when \(\frac{\epsilon_w}{\rho_{p,i} - \rho_{p,0}} > 1\).
and then the Kuhn-Tucker conditions are as follows:

\[
\frac{\partial L}{\partial \mu_i} \leq 0, \quad \mu_i \geq 0, \quad \mu_i \frac{\partial L}{\partial \mu_i} = 0 \quad i \in \{1, 2, 3\} \tag{60}
\]

\[
\mu_1(\rho_{p,1} - \rho_{p,0}) + \mu_2(\rho_{p,2} - \rho_{p,0}) + \mu_3(\rho_{p,3} - \rho_{p,0}) - \epsilon_\omega \leq 0, \quad \lambda_1 \geq 0,
\]

\[
\lambda_1 \left( \mu_1(\rho_{p,1} - \rho_{p,0}) + \mu_2(\rho_{p,2} - \rho_{p,0}) + \mu_3(\rho_{p,3} - \rho_{p,0}) - \epsilon_\omega \right) = 0 \tag{61}
\]

\[
\mu_1 + \mu_2 + \mu_3 - 1 \leq 0, \quad \lambda_2 \geq 0, \quad \lambda_2(\mu_1 + \mu_2 + \mu_3 - 1) = 0. \tag{62}
\]

To solve the problem, we need to consider different situations for inequalities. The proof is completed in Appendix A.

**B. Low SU Access Rates Regime in Centralized Access Policy Design**

Now we consider the low SU access rate regime \( \epsilon_\omega \leq \omega_{\text{init}} \), where \( \epsilon_\omega \) is defined in (49).

**Proposition 2:** In the low SU access rate regime \( \epsilon_\omega \leq \omega_{\text{init}} \), the optimal access policy \( \forall s \in S_K \) is given by

\[
\mu^* = \begin{cases} 
1 - \frac{\epsilon_\omega}{\omega_{\text{init}}}, \frac{\epsilon_\omega}{\omega_{\text{init}}}, 0, 0 & \text{if } \max(u, v, w) = u \\
1 - \frac{\epsilon_\omega}{\omega_{\text{init}}}, 0, \frac{\epsilon_\omega}{\omega_{\text{init}}}, 0 & \text{if } \max(u, v, w) = v \\
1 - \frac{\epsilon_\omega}{\omega_{\text{init}}}, 0, 0, \frac{\epsilon_\omega}{\omega_{\text{init}}} & \text{if } \max(u, v, w) = w,
\end{cases} \tag{63}
\]

and

\[
\forall s \notin S_K, \mu^* = \{1, 0, 0, 0\}. \tag{64}
\]

Furthermore, the average long term SU sum throughput is obtained as

\[
\bar{T}_{su,c}^* = \begin{cases} 
\frac{\epsilon_\omega}{\rho_{p,1} - \rho_{p,0}} R_{s_1,1,K}(1 - \rho_{s_1,1,K}) & \text{if } \max(u, v, w) = u \\
\frac{\epsilon_\omega}{\rho_{p,2} - \rho_{p,0}} R_{s_2,2,K}(1 - \rho_{s_2,2,K}) & \text{if } \max(u, v, w) = v \\
\frac{\epsilon_\omega}{\rho_{p,3} - \rho_{p,0}} \sum_{i=1}^2 R_{s_i,3,K}(1 - \rho_{s_i,3,K}) & \text{if } \max(u, v, w) = w
\end{cases}, \tag{65}
\]

where the parameters are given in Section II.

**Proof:** With \( \mu_{\text{init}} \) in (50), the constraint (49) is equal to \( \omega_{\text{init}} \) as given in (54). However, for the low SU access rate regime, \( \epsilon_\omega \) is equal or lower than \( \omega_{\text{init}} \). To meet this stricter constraint,
we can scale the access policy $\mu_{\text{init}}$ in (50) by $\frac{\epsilon}{\omega_{\text{init}}}$ such that (49) is satisfied with equality. Therefore, $\mu^*$ in (63) satisfies the constraint. Replacing $\mu^*$ in (48) we obtain

$$
\bar{T}_{su,c}(\mu) = E_{a,s=(t,\phi)} [T_{s_1,a,\phi} + T_{s_2,a,\phi}]
$$

$$
= \begin{cases} 
\frac{\epsilon}{\omega_{\text{init}}} R_{s_1,1,K} (1 - \rho_{s_1,1,K}) \sum_{t=1}^{T} \pi(t, (K, K)) & \text{if } \max(u, v, w) = u \\
\frac{\epsilon}{\omega_{\text{init}}} R_{s_2,2,K} (1 - \rho_{s_2,2,K}) \sum_{t=1}^{T} \pi(t, (K, K)) & \text{if } \max(u, v, w) = v \\
\frac{\epsilon}{\omega_{\text{init}}} \sum_{i=1}^{3} R_{s_i,3,K} (1 - \rho_{s_i,3,K}) \sum_{t=1}^{T} \pi(t, (K, K)) & \text{if } \max(u, v, w) = w.
\end{cases}
$$

(66)

Thus, substituting $\omega_{\text{init}}$ in (66) results in SU sum throughput as given in (65). Since the SU sum throughput (65) is equal to the upper bound (58) in the low SU access rate regime $\epsilon \leq \omega_{\text{init}}$, the proposed access policy (63) and (64) is optimal. Note that in the low SU access rate regime since $\epsilon \leq \omega_{\text{init}}$, we have

$$
\frac{\epsilon_{\omega}}{\rho_{p,1} - \rho_{p,0}} \leq 1 \quad \text{if } \max(u, v, w) = u
$$

(67)

$$
\frac{\epsilon_{\omega}}{\rho_{p,2} - \rho_{p,0}} \leq 1 \quad \text{if } \max(u, v, w) = v
$$

(68)

$$
\frac{\epsilon_{\omega}}{\rho_{p,3} - \rho_{p,0}} \leq 1 \quad \text{if } \max(u, v, w) = w.
$$

(69)

C. High SU Access Rates Regime in Centralized Access Policy Design

In Problem [1], we are looking for an optimum policy for the CMDP problem. Therefore, for high SU access rate regime, we employ the equivalent LP formulation corresponding to CMDP, e.g., see [17], [18]. To provide the equivalent LP, we need the transition probability matrix of the Markov process denoted by $P$, where $P_{s\hat{s},a}$ is the probability of moving from state $s$ to $\hat{s}$ if accessibility action $a$ is chosen. To obtain the transition probability matrix $P_{s\hat{s},a}$, we need to compute the transition probability matrix of the PU Markov model $Q_{t\hat{t},a}$ as given in (70) which is the probability that the primary ARQ state $t$ is transferred to $\hat{t}$ if accessibility action $a$ is
TABLE I
PROBABILITY THAT PU MESSAGE KNOWLEDGE STATE $\phi = \{\theta, \eta\}$ IS CHANGED TO STATE $\dot{\phi} = \{\dot{\theta}, \dot{\eta}\}$ GIVEN ACTION $a$, WHERE $\theta, \eta, \dot{\theta}, \dot{\eta} \in \{U, K\}$.

| $T_0 \rightarrow$ | $(U, U)$ | $(U, K)$ | $(K, U)$ | $(K, K)$ |
|------------------|-----------|-----------|-----------|-----------|
| $\phi = (U, U)$  | $p_{ps1,a,\phi}p_{ps2,a,\phi}$ | $p_{ps1,a}(1 - p_{ps2,a,\phi})$ | $(1 - p_{ps1,a,\phi})p_{ps2,a,\phi}$ | $(1 - p_{ps1,a,\phi})(1 - p_{ps2,a,\phi})$ |
| $\phi = (U, K)$  | 0         | $p_{ps1,a,\phi}$ | 0         | $(1 - p_{ps1,a,\phi})$ |
| $\phi = (K, U)$  | 0         | 0         | $p_{ps2,a,\phi}$ | $(1 - p_{ps2,a,\phi})$ |
| $\phi = (K, K)$  | 0         | 0         | 0         | 1         |

selected.

\[
Q_{t\dot{t},a} = \begin{cases} 
1 & \text{if } \dot{t} = 1, t = T \\
1 - \rho_{p,a} & \text{if } \dot{t} = 1, t \neq T \\
\rho_{p,a} & \text{if } \dot{t} = t + 1 \\
0 & \text{otherwise.}
\end{cases}
\]

Thus, $P_{s\dot{s},a} = P_{(t,\phi)(\dot{t},\dot{\phi}),a}$ is given by

\[
P_{(t,\phi)(\dot{t},\dot{\phi}),a} = Q_{t\dot{t},a} Pr(\dot{\phi}|\phi, a),
\]  
(70)

where, $Pr(\dot{\phi}|\phi, a)$, the probability that the PU message knowledge state $\phi$ is changed to state $\dot{\phi}$ given action $a$, is expressed in Table I. For example if $s = (t, (U, U))$, $\dot{s} = (t + 1, (U, K))$ and $a = 1$, then $P_{s\dot{s},a} = \rho_{ps1,1}(1 - \rho_{ps2,1})\rho_{p,1}$. Note that $\rho_{ps_i,a,\phi}$ is the probability that $SU_{rx_i}$ is not able to decode the PU message in PU message knowledge state $\phi$ if accessibility action $a$ is selected.

For any unichain Constrained Markov Decision Process, there exists an equivalent LP formulation, where a MDP is unichain if it contains a single recurrent class plus a (perhaps empty) set of transient states. Thus, the following problem formalizes the equivalent LP for Problem [17]

\[
\text{Problem 3:} \\
\maximize \sum_s \sum_{a \in A} x(s, a) \left( T_{s1,a,\phi} + T_{s2,a,\phi} \right) s.t. \quad (71)
\]
\[ \sum_{s \in S} \sum_{a \in A} (\rho_{p,a} - \rho_{p,0}) x(s, a) \leq \epsilon \omega \quad (72) \]
\[ \sum_{a \in A} x(s', a) - \sum_{s \in S} \sum_{a \in A} P_{s',a} x(s, a) = 0 \quad \forall s' \in S \quad (73) \]
\[ \sum_{s \in S} \sum_{a \in A} x(s, a) = 1 \quad (74) \]
\[ x(s, a) \geq 0 \quad \forall s \in S, \ a \in A. \quad (75) \]

The relationship between the optimal solution of LP Problem 3 and the solution to the considered Problem 1 is obtained as follows [17]:

\[
\mu(a, s) = \begin{cases} 
\frac{x(s,a)}{\sum_{\hat{a} \in A} x(s, \hat{a})} & \text{if } \sum_{\hat{a} \in A} x(s, \hat{a}) > 0 \\
\text{arbitrary} & \text{otherwise}.
\end{cases} \quad (76)
\]

All cases of practical interest considered in this paper correspond to a unichain CMDP. For the equivalent linear problem corresponding to the general case of a multichain CMDP, the reader is referred to [18].

V. DECENTRALIZED ACCESS POLICIES FOR TWO SUs IN MMDP MODEL

In this section, we assume that there is no central unit to control the access policy of the SU transmitters. Therefore, each SU has to control its own access policy. We also assume that the PU message knowledge state of each SU receiver is known for the other SU transmitter. In fact, the SU1 sends back its PU message knowledge state on an error free feedback channel. The transmitted PU message knowledge state is heard by both SU transmitters. Hence, the state \( s \) defined in Section IV is known to both transmitters. However, since there is no central unit, \( SU_i \) does not know the action selected by \( SU_j, j \neq i \). Thus, each user knows the state of the MDP but not the action selected by the other user and our problem hence may be considered as a Multi-agent MDP (MMDP) [16].

The state of the \( PU - SU_1 - SU_2 \) system may be modeled by an Multi-agent Markov Decision Process \( s = (t, \phi) \), where \( t \in \{1,2,...,T\} \) is the primary ARQ state and \( \phi \in \{(U,U),(U,K),(K,K),(K,U)\} \) denotes the PU message knowledge state. The set of all states is indicated by \( S \). In contrast to the centralized scenario, we have two policies \( \mu_1 \) and \( \mu_2 \) which map the state of the network \( s \) to the probabilities that the secondary users 1 and 2 take accessibility actions \( a_1 \in A_1 = \{0,1\} \) and \( a_2 \in A_2 = \{0,1\} \), respectively. The probability that
action \( a_i \) is selected by \( SU_i \) in state \( s \) is denoted by \( \mu_i(a_i, s) \). We use the notation \( \mu = (\mu_1, \mu_2) \) for the access policy of the system in the decentralized case. As denoted, the objective is to maximize the average long term sum throughput of the SUs under the long term average PU throughput constraint, where the throughputs are influenced by actions selected by the two users. Note that with the PU throughput degradation constraint, the access policy designed in Section IV is a randomized policy [17]. Therefore, in general we can not find an access policy for each SU from the proposed centralized access policy. For example, assume the optimum policy designed in the centralized case is \( \mu = [0.3, 0, 0, 0.7] \). This means that the probability that both users are not allowed to access the channel is 0.3 and the probability that both access the channel is 0.7. As observed, there is no situation in which one of the secondary users accesses the channel and the other does not. Now assume there is no a central unit and \( SU_1 \) and \( SU_2 \) select actions \( a = 0 \) and \( a = 3 \), respectively. This is possible because based on the policy mentioned above the probability of selecting actions \( a = 0 \) and \( a = 3 \) are respectively 0.3 and 0.7. Thus, \( SU_1 \) does not transmit and it assumes that the other user does not transmit, as well. In contrast, \( SU_2 \) transmits and supposes the other user also accesses the channel. Thus, the policy selected at SU transmitters will not be optimum as obtained by the centralized optimization design.

If accessibility actions \( a_1 \) and \( a_2 \) are selected, the expected throughputs of \( SU_1 \) and \( SU_2 \) in state \( s = (t, \phi) \) are respectively computed as

\[
T_{s_1,a_1,a_2,\phi} = \begin{cases} 
R_{s_1,a_1,a_2,\phi}(1 - \rho_{s_1,a_1,a_2,\phi}) & \text{for } a_1 = 1 \\
0 & \text{for } a_1 = 0
\end{cases} \quad (77)
\]

\[
T_{s_2,a_1,a_2,\phi} = \begin{cases} 
R_{s_2,a_1,a_2,\phi}(1 - \rho_{s_2,a_1,a_2,\phi}) & \text{for } a_2 = 1 \\
0 & \text{for } a_2 = 0
\end{cases} \quad (78)
\]

and the average long term SU sum throughput can be obtained as

\[
\bar{T}_{SU,d}(\mu_1, \mu_2) = E_{a_1,a_2,s=(t,\phi)} [T_{s_1,a_1,a_2,\phi} + T_{s_2,a_1,a_2,\phi}]
= E_{s=(t,\phi)} \left[ \sum_{a_1,a_2} \mu_1(a_1, s) \mu_2(a_2, s) \{ R_{s_1,a_1,a_2,\phi}(1 - \rho_{s_1,a_1,a_2,\phi}) + R_{s_2,a_1,a_2,\phi}(1 - \rho_{s_2,a_1,a_2,\phi}) \} \right],
\]

where \( E_{a_1,a_2,s} \) denotes the expectation with respect to \( a_1, a_2 \) and \( s \). The outage probabilities \( \rho_{s_i,a_1,a_2,\phi}, i \in \{1, 2\} \) can be obtained from (22) to (29) by applying (42).
Using $\mu_1(0, s) = 1 - \mu_1(1, s)$ and $\mu_2(0, s) = 1 - \mu_2(1, s)$, the average long term PU throughput is rewritten as follows:

$$
\hat{T}_{pu}(\mu_1, \mu_2) = R_p \left( 1 - \sum_{a_1, a_2} E_{s=(t,\phi)} [\mu_1(a_1, s) \mu_2(a_2, s)] \rho_{p,a_1,a_2} \right)
$$

$$
= T_{pu}^I - R_p \left( E_{a_1,a_2,s=(t,\phi)} [\rho_{p,a_1,a_2} - \rho_{p,0,0}] \right), \tag{80}
$$

where $T_{pu}^I$ is the PU throughput when there is no interference from SU transmitters as already defined and here it is given by the following notation:

$$
T_{pu}^I = R_p(1 - \rho_{p,0,0}). \tag{81}
$$

By properly translating from $(a_1, a_2)$ to $a$ defined in (42), the outage probabilities $\rho_{p,a_1,a_2}$, $a_1 \in A_1$ and $a_2 \in A_2$ can be obtained from (3). As denoted, we have the constraint $\hat{T}_{pu}(\mu_1, \mu_2) \geq T_{pu}^I(1 - \epsilon_{PU})$. Thus, we may have the optimization problem as follows:

\textbf{Problem 4:}

$$
\text{maximize} \quad \hat{T}_{su,d}(\mu_1, \mu_2) = E_{a_1,a_2,s=(t,\phi)} [T_{s_1,a_1,a_2,\phi} + T_{s_2,a_1,a_2,\phi}] \quad \text{s.t.} \tag{82}
$$

$$
D(\mu_1, \mu_2) = E_{a_1,a_2,s=(t,\phi)} [\rho_{p,a_1,a_2} - \rho_{p,0,0}] \leq \epsilon_\omega, \tag{83}
$$

where $\epsilon_\omega$ is defined in Section [IV] and $\mu_i(a_i, s)$ is the probability that accessibility action $a_i$ is selected in state $s$ at transmitter $SU_i$.

In the sequel, a scheme based on Nash Equilibrium is proposed, which finds the local optimum policies by converting the CMMDP to a CMDP [19], [20].

\subsection{A. Decentralized Access Policy Design Using Nash Equilibrium}

We employ Nash Equilibrium in which no user has an interest in changing its policy. In fact, $SU_1$ transmitter designs its optimal policy by assuming a fixed policy for $SU_2$ and vice versa. This procedure continues until there is no benefit in employing more iterations. Assuming a fixed policy $\mu_j$ for $SU_j$, the problem for $SU_i$, $i \neq j$ can be considered as a CMDP, referred to as $CMDP_i$. The state space of the new model is the same as the system state $S$. In fact, since the system state $s$ is known for two users, the state of $CMDP_i$ is $s = (t, \phi_1, \phi_2)$. The $SU_{txi}$ chooses action $a_i$ from the set $A_i = \{0, 1\}$, where $a_i = 0$ shows the $SU_i$ is not allowed
to transmit and \( a_i = 1 \) represents the accessibility to the channel for \( SU_{txi} \). Proposition 3 and Problem 5 below are utilized to be able to represent the new model \( MDP_i \) in Proposition 4.

**Proposition 3:** Consider the fixed stationary policy \( \mu_j \) for \( SU_j \). For every stationary policy \( \mu_i \), the random process is unichain Markov chain with stationary transition probability

\[
Pr(s'|s, a_i) = \sum_{a_j} Pr(s'|s, a_i, a_j) \mu_j(a_j, s) = \sum_{a_j} P_{s', a_i, a_j} \mu_j(a_j, s),
\]

where by converting \( a_1 \) and \( a_2 \) to corresponding \( a \), \( P_{s', a_i, a_j} \) can be computed as described in Section IV-C.

**Proof:** Noting that the underlying system is Markov and also the fact that the action selected by \( SU_i \) in a given state only depends on that state, the transition probability from the history of states and actions to a new state is given as follows:

\[
Pr(s^{t+1}|s^t, a^t_i) = \sum_{a^t_j} Pr(s^{t+1}|s^t, a^t_i, a^t_j) Pr(a^t_j|s^t) = \sum_{a^t_j} P_{s^t, a^t_i, a^t_j} \mu_j(a^t_j, s),
\]

where \( s^t = \{s^1, s^2, ..., s^t\} \) and \( a^t_i = \{a^1_i, a^2_i, ..., a^t_i\} \). \( s^t \) and \( a^t_i \) are respectively the state and the action selected by \( SU_{txi} \) at time \( t \). Hence, if action \( a^t_i \) is chosen at \( SU_i \) at time \( t \), the next state depends only on the given state, or equivalently the state transition is Markov. Since the transition probability of moving from every state to state \( s = (1, \{0, 0\}) \) is not zero, we see that the Markov chain is unichain. Noting the stationary transition probability of the underlying system model, stationary policy and \( (86) \), we obtain

\[
Pr(s^{t+1}|s^t, a^t_i) = \sum_{a^t_j} Pr(s^{t+1}|s^t, a^t_i, a^t_j) \mu_j(a^t_j, s) = \sum_{a^t_j} P_{s^t, s^{t+1}, a^t_i, a^t_j} \mu_j(a^t_j, s),
\]

Therefore, the proof is complete.
By considering the fixed policy \( \mu_j \), we have

\[
E_{a_1,a_2,s=(t,\phi)} [T_{s_1,a_1,a_2,\phi} + T_{s_2,a_1,a_2,\phi}] = E_{a_1,s=(t,\phi)} \left[ \sum_{a_j} (T_{s_1,a_1,a_2,\phi} + T_{s_2,a_1,a_2,\phi}) Pr(a_j|s) \right]
\]

and

\[
E_{a_1,a_2,s=(t,\phi)} [\rho_{p,a_1,a_2} - \rho_{p,0,0}] = E_{a_1,s=(t,\phi)} \left[ \sum_{a_j} (\rho_{p,a_1,a_2} - \rho_{p,0,0}) Pr(a_j|s) \right]
\]

and therefore optimization Problem [4] can be rewritten as follows:

**Problem 5:**

\[
\max_{\mu_i(a_i,s)} \mathcal{T}_{s_u,d}(\mu_i) \triangleq E_{a_1,s=(t,\phi)} \left[ \sum_{a_j} (T_{s_1,a_1,a_2,\phi} + T_{s_2,a_1,a_2,\phi}) \mu_j(a_j, s) \right] \quad \text{s.t.} \quad (90)
\]

\[
E_{a_1,s=(t,\phi)} \left[ \sum_{a_j} (\rho_{p,a_1,a_2} - \rho_{p,0,0}) \mu_j(a_j, s) \right] \leq \epsilon_\omega, \quad (91)
\]

where \( \epsilon_\omega \) is defined in Section [IV] and \( \mu_i(a_i, s) \) is the probability that accessibility action \( a_i \) is selected in state \( s \) by transmitter \( SU_j \).

**Proposition 4:** Assume a fixed policy \( \mu_j \) for \( SU_j \), \( j \in \{1, 2\} \). The problem for \( SU_i, i \in \{1, 2\}, i \neq j \) is a CMDP characterized by tuple \( (s, \hat{P}^i, \hat{r}^i, \hat{d}^i) \), where

\[
\hat{P}_{s^i,a_i}^i = \sum_{a_j} P_{s^i,a_1,a_2} \mu_j(a_j, s), \quad (92)
\]

\[
\hat{r}^i(s, a_i) = \sum_{a_j} (T_{s_1,a_1,a_2,\phi} + T_{s_2,a_1,a_2,\phi}) \mu_j(a_j, s), \quad (93)
\]

\[
\hat{d}^i_{s,a_i} = \sum_{a_j} (\rho_{p,a_1,a_2} - \rho_{p,0,0}) \mu_j(a_j, s). \quad (94)
\]

\( \hat{P}^i \), \( \hat{r}^i \) and \( \hat{d}^i \), respectively are the transition matrix probability, the instantaneous reward function and the instantaneous cost function in the new model and \( P_{s^i,a_1,a_2} \) is the transition probability of the system.
Proof: Using Proposition 3 and Problem 5, the proof is straightforward. 

As explained in Section IV-C, there is an equivalent LP formulation for any unichain CMDP, and the LP formulation corresponding to CMDP \textsubscript{i} described in Problem 5 is given by 

**Problem 6:**

\[
\begin{align*}
\text{maximize} & \quad \sum_{s \in S} \sum_{a_i \in A_i} \left( \sum_{a_j \in A_j} (T_{s_1, a_1, a_2, \phi} + T_{s_2, a_1, a_2, \phi})\mu_j(a_j, s) \right) x^i(s, a_i) \\
\text{s.t.} & \quad \sum_{s \in S} \sum_{a_i \in A_i} \left( \sum_{a_j \in A_j} (\rho_{p, a_1, a_2} - \rho_{p, 0, 0})\mu_j(a_j, s) \right) x^i(s, a_i) \leq \epsilon \omega \\
& \quad \sum_{s \in S} \sum_{a_i \in A_i} x^i(\bar{s}, a_i) = 0 \quad \forall \bar{s} \in S \\
& \quad \sum_{s \in S} \sum_{a_i \in A_i} x^i(s, a_i) = 1 \\
& \quad x^i(s, a_i) \geq 0 \quad \forall s \in S, \ a_i \in A_i.
\end{align*}
\]

The relationship between the optimal solution of LP Problem 6 and the solution to the considered Problem 5 is also obtained as follows

\[
\mu_i(a_i, s) = \begin{cases} 
\frac{x^i(s, a_i)}{\sum_{\hat{a}_i \in A_i} x^i(s, \hat{a}_i)} & \text{if } \sum_{\hat{a}_i \in A_i} x^i(s, \hat{a}_i) > 0 \\
\text{arbitrary} & \text{otherwise.}
\end{cases}
\]

As denoted, SU \textsubscript{i} computes the optimum policy as given in (100) by considering a fixed policy for SU \textsubscript{j}. In fact, in this case \((i, j) = (1, 2)\). By changing \((i, j)\) from \((1, 2)\) to \((2, 1)\) and vice versa, this procedure iteratively continues such that an equilibrium is achieved. We prove later in Proposition 6 that an equilibrium point is always achieved. Algorithm 1 below describes the local optimal solution to Problem 4 based on Nash Equilibrium. The obtained access policies are local optimum solutions. We have to restart Algorithm 1 for several random initiations and see whether the resulting SU sum throughput is higher. We have the two following propositions related to Nash Equilibrium.

**Proposition 5:** Optimum access policies \(\mu^*_1\) and \(\mu^*_2\) solution to Problem 4 are a fixed point or an equilibrium point.

**Proof:** If \(\mu^*_1\) and \(\mu^*_2\) are the optimum solutions to problem 4, then

\[
\bar{T}_{su,d}(\mu^*_1, \mu^*_2) \geq \bar{T}_{su,d}(\mu_1, \mu_2),
\]

October 16, 2014 DRAFT
Algorithm 1 Local Optimum Policy using Nash Equilibrium

1) Choose initial stochastic policies $\mu_1$ and $\mu_2$ and select $i = 1$, $j = 2$ and $n = 1$.

2) Provide the solution to Problem 5 as given in (100) for a given fixed $\mu_j$ and compute optimum policy $\mu_i$ for $SU_i$.

3) Select $n = n + 1$ and $\mu^n = (\mu_1, \mu_2)$;

4) Change the role of $i$ and $j$, i.e., $k = i$, $i = j$ and $j = k$.

5) If $\mu^n = \mu^{n-1}$, then go step 6. Else go step 2.

6) $\mu_1$ and $\mu_2$ are the local optimum solution to original DEC-MMDP Problem 4.

where $(\mu_1, \mu_2)$ belongs to every feasible solution and $T_{su,d}(\mu_1^*, \mu_2^*)$ is given in Problem 4. Feasible solution means every set of polices that satisfies the constraint in Problem 4. Now suppose that the policy for $SU_2$ is fixed to $\mu_2^*$. Note that $\bar{T}_{su,d}(\mu_1)$ in Problem 5 is equal to $\bar{T}_{su,d}(\mu_1, \mu_2^*)$ in Problem 4. Thus, noting (101), we have

$$\bar{T}_{su,d}(\mu_1) \leq \bar{T}_{su,d}(\mu_1^*, \mu_2^*) \quad (102)$$

and if $\mu_1$ is equal to $\mu_1^*$, equality occurs. Thus, point $(\mu_1^*, \mu_2^*)$ is a fixed point. In other words, this fixed point is an equilibrium where either user can not get more benefit in SU sum throughput by more iterations. This concludes the proof.

Proposition 6: The SU sum throughput obtained by solving Algorithm 1 improves as the iteration index $n$ increases and furthermore, the iterative procedure based on Algorithm 1 converges to a fixed point.

Proof: Suppose $\mu_1^n$ and $\mu_2^n$ are the resulting policies in iteration $n$ of the algorithm and the resulting SU sum throughput is given by $\bar{T}_{su,d}(\mu_1^n, \mu_2^n)$. Now we consider $\mu_2^n$ to be fixed and improve $\mu_1^n$ to $\mu_1^{n+1}$ according to the algorithm. Therefore, $\mu_1^{n+1}$ is the optimum solution to Problem 5 and we have

$$T_{su,d}(\mu_1^{n+1}) \geq T_{su,d}(\mu_1^n) \quad (103)$$

or equivalently

$$\bar{T}_{su,d}(\mu_1^{n+1}, \mu_2^n) \geq \bar{T}_{su,d}(\mu_1^n, \mu_2^n). \quad (104)$$

Since $\bar{T}_{su,d}(\mu_1^n, \mu_2^n)$ and $\bar{T}_{su,d}(\mu_1^{n+1}, \mu_2^n)$ are the SU sum throughput respectively in iterations $n$ and $n + 1$, it is observed that the SU sum throughput can not decrease as the algorithm proceeds.
The same approach could be seen when the policy for $SU_1$ is constant and that of $SU_2$ improves. This shows that the SU sum throughput is an increasing function with respect to $n$. Since the performance is bounded by that of the centralized access policy design, it is proved that the proposed algorithm converges.

The performance of decentralized access policy design using Nash equilibrium is studied in Section VI.

VI. NUMERICAL RESULTS

We consider Rayleigh fading channels. Thus, the SNR $\gamma_x$, is an exponentially distributed random variable with mean $\bar{\gamma}_x$, where $x \in \{pp, ps_1, ps_2, s_1s_1, s_1s_2, s_2s_1, s_2s_2, s_1p, s_2p\}$. We consider the following parameters throughout the paper, unless otherwise mentioned. Following [9], we consider the average SNRs $\bar{\gamma}_{pp} = 10$, $\bar{\gamma}_{s_is_i} = 5$, $\bar{\gamma}_{s_is_2} = 3$, $\bar{\gamma}_{s_2s_1} = 3$, $\bar{\gamma}_{ps_i} = 5$, $\bar{\gamma}_{s_i,p} = 2$, $i \in \{1, 2\}$. The ARQ deadline is $T = 5$. The PU rate $R_p$ is selected such that the PU throughput is maximized when both SUs are idle, i.e., $R_p = \arg\max R T_{pu}^I(R)$. Thus, we set $R_p = 2.52$ and $T_{pu}^I = 1.57$. The PU throughput constraint is set to $(1 - \epsilon_{PU})T_{pu}^I$, where $\epsilon_{PU} = 0.2$. In the centralized case the rates $R_{s_1,a,\phi}^*$ and $R_{s_2,a,\phi}^*$ are computed as $(R_{s_1,a,\phi}^*, R_{s_2,a,\phi}^*) = \arg\max_{R_{s_1}, R_{s_2}} T_{s_1,a,\phi} + T_{s_2,a,\phi}$ so as to maximize the SU sum throughput, where $T_{s_i,a,\phi}$ is a function of $R_P$ (only if the PU message knowledge state is unknown for receiver $SU_i$), $R_{s_1}$ and $R_{s_2}$. In the decentralized case, the rate $R_{s_i,a,\phi}$ is selected so as to maximize $T_{s_i,a,\phi}$, irrespective of the other SU transmission.

The scheme “Forward Interference Cancelation” discussed here is called “FIC”. The centralized and decentralized access policy designs are respectively referred to as “FIC Decentralized” and “FIC Centralized”. For the centralized policy design, the performance bound described in Section IV-A is referred to as “PM already Known”. In addition, we also consider the scenario without using FIC, referred to as “No FIC” in the centralized access policy design. Note that “One Secondary User” denotes the case that only one SU exists in the CRN.

The SU sum throughput with respect to the PU throughput by varying the value of $\epsilon_{PU}$ is depicted in Fig. [1] Obviously, as the PU throughput $T_{pu}^I(1 - \epsilon_{PU}) = 1.57(1 - \epsilon_{PU})$ increases, the average sum throughput of SUs decreases. PU throughputs greater than 1.286 and 1.224 ($\epsilon_{PU} < 0.22$ and $\epsilon_{PU} < 0.18$) correspond to the low SU access rate regime respectively for centralized and decentralized scenarios. The FIC performance is the same as that of the upper
bound ("PM already Known" scheme) for the low SU access rate regime. Thus, for PU throughput equal to 1.255 ($\epsilon_{PU} < 0.2$) the FIC performance is the same as that of the upper bound in the centralized scenario. As observed, the CRN with two SUs in both centralized and decentralized scenarios provides the same SU sum throughput as the CRN with one SU for a large enough value of the constraint on the PU throughput. There is also a performance loss in applying the decentralized case with respect to the centralized one especially in the low PU throughput constraint. Our simulation results show that this loss in the decentralized scenario is because the assigned rate to each SU does not account for the decision made by the other SU, whereas in the centralized case the rates are jointly assigned. In fact, when the rates assigned to the SUs in the decentralized case are the same as those in the centralized case, our proposed decentralized design has the same performance as the centralized design.

The average sum throughput of SUs as a function of $\bar{\gamma}_{s1p}$ is depicted in Fig. 2 where $\bar{\gamma}_{s2p} = 2$. As observed, the SU sum throughput decreases as $\bar{\gamma}_{s1p}$ increases. This is because $\bar{\gamma}_{s2p} = 2$ and hence, the PU throughput degradation constraint is always active for the two SUs. A similar plot for the case $\bar{\gamma}_{s2p} = \bar{\gamma}_{s1p}$ is depicted in Fig. 3. As observed, for $\bar{\gamma}_{s1p} < 0.5$, $\bar{\gamma}_{s1p} < 0.25$ and $\bar{\gamma}_{s1p} < 0.45$ respectively in the CRN with one SU, centralized and decentralized cases, we have a different result. In fact, because the interference power of SUs has little effect on
the PU receiver, initially the PU throughput degradation constraint is not active and therefore $SU_{tx1}$ and $SU_{tx2}$ may utilize their powers to maximize their own throughput. The constraint becomes active for $\bar{\gamma}_{s1p} > 0.5$, $\bar{\gamma}_{s1p} = \bar{\gamma}_{s2p} > 0.25$ and $\bar{\gamma}_{s1p} = \bar{\gamma}_{s2p} > 0.45$, respectively in the CRN with one SU, centralized and decentralized cases; therefore, above those values, the SU sum throughput diminishes. As expected, in the cognitive radio with two symmetric SUs either centralized or decentralized scenarios, the PU throughput degradation constraint becomes active sooner than in the cognitive radio with one SU by increasing the SNR of the channels from the SU transmitters to the PU receiver. A similar observation is seen when $\bar{\gamma}_{ps1} = \bar{\gamma}_{ps2} = 2$ as depicted in Fig. 4. It is noteworthy that because the $\bar{\gamma}_{ps1} = \bar{\gamma}_{ps2} = 2$ are neither strong enough to be successfully decoded, nor weak to be considered as noise at the SU receivers, the SU sum throughput provided by the centralized case has more performance loss with respect to the upper bound compared with that in Fig. 3. This observation is clearly seen in the next two figures as discussed later.

Figs. 5 and 6 show the average SU sum throughput with respect to $\bar{\gamma}_{ps1}$ for $\bar{\gamma}_{ps2} = 5$ and $\bar{\gamma}_{ps2} = \bar{\gamma}_{ps1}$, respectively. Note that $R^*_{s1,a,\phi=(U,\theta)}$ and $R^*_{s2,a,\phi=(\theta,U)}$ respectively depend on $\bar{\gamma}_{ps1}$ and $\bar{\gamma}_{ps2}$. As expected, $\bar{\gamma}_{ps1}$ does not have any influence on “PM already Known” scheme. This is because in this scheme the PU message is previously known and can always be canceled by the
SU receiver in future retransmissions. It is observed that for large enough values of $\tilde{\gamma}_{ps_1}$, the upper bound is achievable by the FIC scheme in the centralized scenario. In fact, the SU receiver can successfully decode the PU message, remove the interference and decode its corresponding message. Note that the upper bound is computed in the centralized scenario. The sum throughput is minimized at $\tilde{\gamma}_{ps_1} = 2$ in the CRN with one SU, centralized and decentralized cases, where
the PU message is neither strong enough to be successfully decoded, nor weak to be considered as noise. It is also evident that the FIC scheme in Fig. 5 converges to the upper bound faster than in Fig. 6. The reason is that $\bar{\gamma}_{sp1}$ and $\bar{\gamma}_{sp2}$ increase simultaneously in Fig. 6 whereas the value of $\bar{\gamma}_{sp1}$ is considered to be equal to zero in Fig. 5 resulting in no interference to the PU receiver. It is also observed from Fig. 6 that a cognitive radio with two symmetric SUs converges to the upper bound faster than the network with one SU for large enough SNR of the channels from the PU transmitter to SU receivers. This is because of the use of the FIC scheme at the SU receivers.

VII. EXTENSIONS

In this section, we first provide the outlines to extend the access policy design for CRN with the arbitrary number of SUs, indicated by $N$. Then, we give the model for the decentralized scenario when the PU message knowledge state is known partially for the SUs in addition to the action selected by the SU is unknown for the other one.

A. CRN with $N$ Secondary Users

In order to extend the design to CRNs with $N$ SUs, we need to define the accessibility action and PU message knowledge state. PU message knowledge state $\phi = (\phi(1), ..., \phi(N))$ belongs to
the set of $2^N$ possible combinations of PU message knowledge states of all users, where $\phi(i)$ is PU message knowledge state of $SU_i$. In the centralized case we have $2^N$ possible combinations of accessibility of users to the channel. Each is an N-dimensional vector $(\varphi(a, 1), ..., \varphi(a, N))$ equal to the binary expansion of $a$, $1 \leq a \leq 2^N$. For the accessibility index $a$, $\varphi(a, i) = 1$ means that $SU_i$ is allowed to access the channel. In the decentralized case, the accessibility action is $a_i \in \{0, 1\}$ for secondary user $i$, where $a_i = 1$ means that this user is allowed to transmit.

The rate is defined as $R_{s_i,a,\phi}$. The SNR region $\Gamma_{s_i,a,\phi}(R_{s_1,a,\phi}, ..., R_{s_N,a,\phi})$, $i \in \{1, ..., N\}$, where $\phi(i) = K$, guarantees that the $SU_i$ message transmitted at rate $R_{s_i,a,\phi}$ is successfully decoded at $SU_{txi}$ regardless of the decoding of other SUs messages transmitted at rate $R_{s_j,a,\phi}$, $\forall j \neq i$. Moreover, the SNR region $\Gamma_{s_i,a,\phi}(R_p, R_{s_1,a,\phi}, ..., R_{s_N,a,\phi})$, where $\phi(i) = U$, guarantees that the $SU_i$ message transmitted at rate $R_{s_i,a,\phi}$ is successfully decoded at $SU_{rx i}$ irrespective of the decoding of other SUs and PU messages transmitted at rates $R_{s_j,a,\phi}$ and $R_p$ respectively.

Thus, the outage probability of the channel $SU_{txi} \rightarrow SU_{rx i}$, $i \in \{1, 2\}$ denoted by $\rho_{s_i,a,\phi}$ is computed as

$$
\rho_{s_i,a,\phi}=\{\phi(1),...,\phi(i)=K,...,\phi(N)\} = P(\gamma_{s_1s_i, \gamma_{s_2s_i}}) \notin \Gamma_{s_i,a,\phi}(R_p,R_{s_1,a,\phi},...,R_{s_N,a,\phi})
$$

(105)
and
\[
\rho_{s_i,a,\phi} = \{\phi(1), \ldots, \phi(i)=U, \ldots, \phi(N)\} = Pr\left((\gamma_{ps_i}, \gamma_{s_1s_i}, \ldots, \gamma_{s_Ns_i}) \notin \tilde{\Gamma}_{s_i,a,\phi}(R_p, R_{s_1,a,\phi}, \ldots, R_{s_N,a,\phi})\right).
\] (106)

The outage probability of the channel $PU_{tx} \rightarrow PU_{rx}$ in SU accessibility action $a$ is given as
\[
\rho_{p,a} = 1 - Pr\left(R_p \leq C\left(\frac{\gamma_{pp}}{1 + \sum_{i=1}^{N} \varphi(a, i)\gamma_{s_i}}\right)\right),
\] (107)
where $\varphi(a, i) = 0$ if $SU_i$ is idle, otherwise $\varphi(a, i) = 1$.

Thus, the state of the $PU - SU_1 - \ldots - SU_N$ system may be modeled by a Markov Decision Process $s = (t, \phi)$, where $t \in \{1, 2, \ldots, T\}$ is the primary ARQ state and $\phi$ is the PU message knowledge state defined above and therefore, the average long term SU sum throughput is computed as follows
\[
\bar{T}_{su}(\mu) = E_{a,s=(t,\phi)} \left[ \sum_{i=1}^{N} T_{s_i,a,\phi} \right] = E_{a,s=(t,\phi)} \left[ \sum_{i=1}^{N} R_{s_i,a,\phi} (1 - \rho_{s_i,a,\phi}) \right].
\] (108)

Thus, the problem is to maximize (108) under the constraint
\[
E_{a,s=(t,\phi)} [\rho_{p,a} - \rho_{p,0}] \leq (1 - \rho_{p,0})\epsilon_{PU} \triangleq \epsilon_{\omega},
\] (109)
and therefore, we can use LP formulation corresponding to CMDP or CMMDP as described in Sections IV and V. However, due to the exponential scaling of the problem size, this formulation can only be solved for $N$ not too large.

B. Decentralized Access Policy Design for Partially State Information

In Section V, the PU message knowledge state of each SU is also known to the other one, which makes the whole state of the system known to all SUs. Now we assume that each user can only observe its own PU message knowledge state. When there is an uncertainty about the state of the system, the problem is called “Distributed Partial State Information MDP” (DEC-PSI-MDP) which is a type of “Partially Observable MDP” (DEC-POMDP). For a literature review on the decentralized control of DEC-POMDP, the reader is referred to [21]. In this model, the shared objective function is used (here the SU Sum throughput) and the action is selected based
on the partial state observation at each SU. Because each secondary user is unaware of the belief states of the other user, it is impossible for each user to properly estimate the state of the system. Thus, a DEC-POMDP can not be formulated as a continuous state MDP. It is shown that DEC-POMDP is nondeterministic exponential (NEXP) complete even for two users [22] and, hence, only approximate solutions can be applied [20]. Consideration of this type of system is left as future work.

VIII. CONCLUSION

In this paper, an optimal access policy for two cognitive secondary users was proposed, under a constraint on the interference from the secondary users to the primary receiver. Leveraging the redundancy in ARQ retransmissions implemented by the PU, each SU receiver can cancel a successfully decoded PU message in the following ARQ retransmissions, thereby improving its own throughput. The centralized and decentralized scenarios were considered. In the first scenario, there is a centralized unit which controls both SU access capability to the channel to maximize the average sum throughput of SUs under the average PU throughput degradation constraint. In the decentralized scenario, there exists no central unit and therefore each SU is not aware of the action selected by the other one, while the state of the system is known to both secondary users. In the centralized case, the upper bound was formulated and a close form solution was provided. Our studies confirm that the centralized and decentralized scenarios may be modeled as CMDP and MMDP and therefore solved by linear programming. At the end, extensions of the problem to CRN with an arbitrary number of SUs as well as to CRN with partial state information were discussed.

APPENDIX A

PROOF OF PROPOSITION [1]

Let define:

\[ d_i = R_{s_i,i,K}(1 - \rho_{s_i,i,K}) \quad i \in \{1, 2\} \]  \hspace{1cm} (110)

\[ d_3 = R_{s_1,3,K}(1 - \rho_{s_1,3,K}) + R_{s_2,3,K}(1 - \rho_{s_2,3,K}) \]  \hspace{1cm} (111)

All situations are given as follows:
1) $\lambda_1 = 0$ and $\lambda_2 = 0$. From (60), it is necessary to have

$$d_i = 0 \quad i \in \{1, 2, 3\}$$

(112)

Hence, this case is not acceptable.

2) $\mu_i = 0$, $i \in \{1, 2, 3\}$. This case gives the SU sum throughput equal to zero and it does not provide the optimum solution.

3) $\lambda_1 > 0$, $\lambda_2 = 0$, $\mu_i > 0$, $\mu_j = 0$, $\mu_k = 0$, $(i, j, k) \in \{(1, 2, 3), (2, 1, 3), (3, 1, 2)\}$. It is observed from condition (60) that $\frac{\partial L}{\partial \mu_i} = 0$, $\frac{\partial L}{\partial \mu_i} \leq 0$ and $\frac{\partial L}{\partial \mu_k} \leq 0$. This occurs if

$$\frac{d_j}{\rho_{p,j} - \rho_{p,0}} \leq \frac{d_i}{\rho_{p,i} - \rho_{p,0}}$$

(113)

and

$$\frac{d_k}{\rho_{p,k} - \rho_{p,0}} \leq \frac{d_i}{\rho_{p,i} - \rho_{p,0}}$$

(114)

Noting (61) and (62), we have $\mu_i(\rho_{p,i} - \rho_{p,0}) = \epsilon \omega$ and $\mu_i \leq 1$; or equivalently

$$\mu_i = \frac{\epsilon \omega}{\rho_{p,i} - \rho_{p,0}} \leq 1.$$  

(115)

Thus, the resulting maximum SU sum throughput is equal to $d_i$.

4) $\lambda_1 = 0$, $\lambda_2 > 0$, $\mu_i > 0$, $\mu_j = 0$, $\mu_k = 0$, $(i, j, k) \in \{(1, 2, 3), (2, 1, 3), (3, 1, 2)\}$. It is observed from condition (60) that $\frac{\partial L}{\partial \mu_i} = 0$, $\frac{\partial L}{\partial \mu_j} \leq 0$ and $\frac{\partial L}{\partial \mu_k} \leq 0$. This occurs if

$$d_k \leq d_i$$

(116)

and

$$d_j \leq d_i.$$  

(117)

Noting (61) and (62), we have $\mu_i(\rho_{p,i} - \rho_{p,0}) \leq \epsilon \omega$ and $\mu_i = 1$; or equivalently

$$\mu_i = 1 \leq \frac{\epsilon \omega}{\rho_{p,i} - \rho_{p,0}}.$$  

(118)

Thus, the resulting maximum SU sum throughput is equal to $d_i$.

5) $\lambda_1 > 0$, $\lambda_2 > 0$, $\mu_i > 0$, $\mu_j = 0$, $\mu_k = 0$, $(i, j, k) \in \{(1, 2, 3), (2, 1, 3), (3, 1, 2)\}$. It is observed from condition (60) that $\frac{\partial L}{\partial \mu_i} = 0$, $\frac{\partial L}{\partial \mu_j} \leq 0$ and $\frac{\partial L}{\partial \mu_k} \leq 0$. This occurs if

$$d_k \geq d_i \text{ if } \rho_{p,k} - \rho_{p,0} \geq \rho_{p,i} - \rho_{p,0}$$

(119)

$$d_k < d_i \text{ if } \rho_{p,k} - \rho_{p,0} < \rho_{p,i} - \rho_{p,0}$$

(120)

$$d_j \geq d_i \text{ if } \rho_{p,j} - \rho_{p,0} \geq \rho_{p,i} - \rho_{p,0}$$

(121)

$$d_j < d_i \text{ if } \rho_{p,j} - \rho_{p,0} < \rho_{p,i} - \rho_{p,0}.$$  

(122)
Noting (61) and (62), we have
\[ \mu_i = 1 = \frac{\epsilon_\omega}{\rho_{p,i} - \rho_{p,0}}. \] (123)

Thus, the resulting maximum SU sum throughput is equal to \( d_i \).

6) \( \lambda_1 > 0, \lambda_2 = 0, \mu_1 > 0, \mu_2 > 0, \mu_3 > 0 \). It is observed from condition (60) that \( \frac{\partial L}{\partial \mu_1} = \frac{\partial L}{\partial \mu_2} = \frac{\partial L}{\partial \mu_3} = 0 \). This occurs if
\[ \frac{d_1}{\rho_{p,1} - \rho_{p,0}} = \frac{d_2}{\rho_{p,2} - \rho_{p,0}} = \frac{d_3}{\rho_{p,3} - \rho_{p,0}}. \] (124)

Noting (61) and (62), \( \mu_1(\rho_{p,1} - \rho_{p,0}) + \mu_2(\rho_{p,2} - \rho_{p,0}) + \mu_3(\rho_{p,3} - \rho_{p,0}) = \epsilon_\omega \) and \( \mu_1 + \mu_2 + \mu_3 \leq 1 \). These two conditions impose that
\[ \epsilon_\omega \leq \rho_{p,3} - \rho_{p,0}. \] (125)

Thus, the resulting maximum SU sum throughput is equal to \( d_1 \).

7) \( \lambda_1 = 0, \lambda_2 > 0, \mu_1 > 0, \mu_2 > 0, \mu_3 > 0 \). It is observed from condition (60) that \( \frac{\partial L}{\partial \mu_1} = \frac{\partial L}{\partial \mu_2} = \frac{\partial L}{\partial \mu_3} = 0 \). This occurs if
\[ d_1 = d_2 = d_3. \] (126)

Noting (61) and (62), \( \mu_1(\rho_{p,1} - \rho_{p,0}) + \mu_2(\rho_{p,2} - \rho_{p,0}) + \mu_3(\rho_{p,3} - \rho_{p,0}) \leq \epsilon_\omega \) and \( \mu_1 + \mu_2 + \mu_3 = 1 \). The conditions impose that
\[ \min(\rho_{p,1} - \rho_{p,0}, \rho_{p,2} - \rho_{p,0}) \leq \epsilon_\omega. \] (127)

Thus, the resulting maximum SU sum throughput is equal to \( d_1 \).

8) \( \lambda_1 > 0, \lambda_2 > 0, \mu_1 > 0, \mu_2 > 0, \mu_3 > 0 \). It is observed from condition (60) that \( \frac{\partial L}{\partial \mu_1} = \frac{\partial L}{\partial \mu_2} = \frac{\partial L}{\partial \mu_3} = 0 \). This occurs if
\[ d_i \leq d_3 \quad i \in \{1, 2\} \] (128)
\[ d_1 \geq d_2 \quad \text{if} \quad \rho_{p,1} - \rho_{p,0} \geq \rho_{p,2} - \rho_{p,0} \] (129)
\[ d_1 < d_2 \quad \text{if} \quad \rho_{p,1} - \rho_{p,0} < \rho_{p,2} - \rho_{p,0} \] (130)
\[ \frac{d_3}{\rho_{p,3} - \rho_{p,0}} < \min\left\{ \frac{d_1}{\rho_{p,1} - \rho_{p,0}}, \frac{d_2}{\rho_{p,2} - \rho_{p,0}} \right\} \] (131)
\[ \frac{d_1}{\rho_{p,1} - \rho_{p,0}} \geq \frac{d_2}{\rho_{p,2} - \rho_{p,0}} \quad \text{if} \quad \rho_{p,1} - \rho_{p,0} \leq \rho_{p,2} - \rho_{p,0} \] (132)
\[ \frac{d_1}{\rho_{p,1} - \rho_{p,0}} < \frac{d_2}{\rho_{p,2} - \rho_{p,0}} \quad \text{if} \quad \rho_{p,1} - \rho_{p,0} > \rho_{p,2} - \rho_{p,0} \] (133)
Noting (61) and (62), $\mu_1(\rho_{p,1} - \rho_{p,0}) + \mu_2(\rho_{p,2} - \rho_{p,0}) + \mu_3(\rho_{p,3} - \rho_{p,0}) = \epsilon_\omega$ and $\mu_1 + \mu_2 + \mu_3 = 1$. The conditions impose that

$$\min (\rho_{p,1} - \rho_{p,0}, \rho_{p,2} - \rho_{p,0}) \leq \epsilon_\omega \leq \max (\rho_{p,1} - \rho_{p,0}, \rho_{p,2} - \rho_{p,0})$$  \hspace{1cm} (134)
$$\epsilon_\omega \leq \rho_{p,3} - \rho_{p,0}.$$  \hspace{1cm} (135)

Thus, the resulting maximum SU sum throughput is equal or lower than $\epsilon_\omega \max \left( \frac{d_1}{\rho_{p,1} - \rho_{p,0}}, \frac{d_2}{\rho_{p,2} - \rho_{p,0}} \right)$ and the equality is achieved when $\frac{d_1}{\rho_{p,1} - \rho_{p,0}} = \frac{d_2}{\rho_{p,2} - \rho_{p,0}}$.  

**9) $\lambda_1 > 0$, $\lambda_2 = 0$, $\mu_i = 0$, $\mu_j > 0$, $\mu_k > 0$, $(i, j, k) \in \{(1, 2, 3), (2, 1, 3), (3, 1, 2)\}$.** It is observed from condition (60) that $\frac{\partial L}{\partial \mu_i} \leq 0$, $\frac{\partial L}{\partial \mu_j} = 0$ and $\frac{\partial L}{\partial \mu_k} = 0$. This occurs if

$$\frac{d_i}{\rho_{p,i} - \rho_{p,0}} \leq \frac{d_j}{\rho_{p,j} - \rho_{p,0}} = \frac{d_k}{\rho_{p,k} - \rho_{p,0}}.$$  \hspace{1cm} (136)

Noting (61) and (62), $\mu_j(\rho_{p,j} - \rho_{p,0}) + \mu_k(\rho_{p,k} - \rho_{p,0}) = \epsilon_\omega$ and $\mu_j + \mu_k \leq 1$. The conditions impose that

$$\epsilon_\omega \leq \max (\rho_{p,j} - \rho_{p,0}, \rho_{p,k} - \rho_{p,0}),$$  \hspace{1cm} (137)

and the resulting maximum SU sum throughput is equal to $\frac{\epsilon_\omega d_j}{\rho_{p,j} - \rho_{p,0}}$.

**10) $\lambda_1 = 0$, $\lambda_2 > 0$, $\mu_i = 0$, $\mu_j > 0$, $\mu_k > 0$, $(i, j, k) \in \{(1, 2, 3), (2, 1, 3), (3, 1, 2)\}$.** It is observed from condition (60) that $\frac{\partial L}{\partial \mu_i} \leq 0$, $\frac{\partial L}{\partial \mu_j} = 0$ and $\frac{\partial L}{\partial \mu_k} = 0$. This occurs if

$$d_i \leq d_j = d_k.$$  \hspace{1cm} (138)

Noting (61) and (62), $\mu_j(\rho_{p,j} - \rho_{p,0}) + \mu_k(\rho_{p,k} - \rho_{p,0}) \leq \epsilon_\omega$ and $\mu_j + \mu_k = 1$. The conditions impose that

$$\min (\rho_{p,j} - \rho_{p,0}, \rho_{p,k} - \rho_{p,0}) \leq \epsilon_\omega.$$  \hspace{1cm} (139)

The resulting maximum SU sum throughput is equal to $d_j$.

**11) $\lambda_1 > 0$, $\lambda_2 > 0$, $\mu_i = 0$, $\mu_j > 0$, $\mu_k > 0$, $(i, j, k) \in \{(1, 2, 3), (2, 1, 3), (3, 1, 2)\}$.** It is
observed from condition (60) that \( \frac{\partial L}{\partial \mu_i} \leq 0, \frac{\partial L}{\partial \mu_j} = 0 \) and \( \frac{\partial L}{\partial \mu_k} = 0 \). This occurs if
\[
d_j \geq d_k \text{ if } \rho_{p,j} - \rho_{p,0} \geq \rho_{p,k} - \rho_{p,0} \tag{140}
\]
\[
d_j < d_k \text{ if } \rho_{p,j} - \rho_{p,0} < \rho_{p,k} - \rho_{p,0} \tag{141}
\]
\[
\frac{d_j}{\rho_{p,j} - \rho_{p,0}} \geq \frac{d_k}{\rho_{p,k} - \rho_{p,0}} \text{ if } \rho_{p,j} - \rho_{p,0} \leq \rho_{p,k} - \rho_{p,0} \tag{142}
\]
\[
\frac{d_j}{\rho_{p,j} - \rho_{p,0}} < \frac{d_k}{\rho_{p,k} - \rho_{p,0}} \text{ if } \rho_{p,j} - \rho_{p,0} > \rho_{p,k} - \rho_{p,0} \tag{143}
\]
\[
d_i \geq d_j \text{ if } \rho_{p,i} - \rho_{p,0} \geq \rho_{p,j} - \rho_{p,0} \tag{144}
\]
\[
d_i < d_j \text{ if } \rho_{p,i} - \rho_{p,0} < \rho_{p,j} - \rho_{p,0} \tag{145}
\]
\[
d_i \geq d_k \text{ if } \rho_{p,i} - \rho_{p,0} \geq \rho_{p,k} - \rho_{p,0} \tag{146}
\]
\[
d_i < d_k \text{ if } \rho_{p,i} - \rho_{p,0} < \rho_{p,k} - \rho_{p,0} \tag{147}
\]

Noting (61) and (62), \( \mu_j (\rho_{p,j} - \rho_{p,0}) + \mu_k (\rho_{p,k} - \rho_{p,0}) = \epsilon_\omega \) and \( \mu_j + \mu_k = 1 \). The conditions impose that
\[
\min \left( \rho_{p,j} - \rho_{p,0}, \rho_{p,k} - \rho_{p,0} \right) \leq \epsilon_\omega \leq \max \left( \rho_{p,j} - \rho_{p,0}, \rho_{p,k} - \rho_{p,0} \right). \tag{148}
\]

The resulting maximum SU sum throughput is equal or lower than \( \epsilon_\omega \max \left( \frac{d_j}{\rho_{p,j} - \rho_{p,0}}, \frac{d_k}{\rho_{p,k} - \rho_{p,0}} \right) \) and the equality is achieved when \( \frac{d_j}{\rho_{p,j} - \rho_{p,0}} = \frac{d_k}{\rho_{p,k} - \rho_{p,0}} \).

Noting items 1 to 11, it is observed that items 3, 4, 6, 7, 9, 10 provide optimum solutions and hence, the optimum access policy and SU Sum throughput can be summarized in (57) and (58) respectively. Thus, the proof is complete.

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