Characteristics of Complex Networks in Neural Networks

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Abstract. The complex network characteristics of the neuronal functional network composed of individual neurons have been rarely studied, within even fewer recording the number of neurons. In this paper, we use a multi-electrode recording spike trains datasets recorded from slice cultures of rodent somatosensory cortex in monkeys. The number of neurons in a single recording process reached several hundred. The entire recording time was divided into non-overlapping time segments to construct a functional network over different times. We analyze the small world characteristics, the modular network and community structures of neuronal functional networks. The network characteristics are analyzed to determine the changing process with the evolution of different neuronal functional networks. We proposed a new similarity coefficient $S$ to compare the similarity indexes between the community structures. We found that neuronal networks have small world and modularity characteristics in common, when compared to a random network. The network characteristics are stronger when the networks retain a small number of edges. These findings contribute to the study of the neuronal network organization of individual neurons.

Keywords: Small World, Modularity, Community Structure

1. Introduction

An analysis of the structure of the brain can be mapped using a graph. Graph theory is widely used in the analysis of complex systems. The connection structures of the brain, such as the brain region connecting structure or the voxel connection structure can be converted into a network connection structure. The complexity of these network structures can be studied using the complex network methods. There have been a number of studies addressing the functional network structure of the brain, typically constructed by measuring the correlations between the BOLD signals of fMRI [1],[2]. Through the analysis of these brain functional networks, it was found that brain functional networks have some special characteristics of complex networks. By analyzing the different characteristics of the complex network, we can identify the network differences between the normal brain and the pathological brain [3],[4].

To date, there have been no reports of the small world characteristics in a neuronal functional network containing larger numbers of neurons. In this study, we use publicly available datasets for analysis. In these recordings, the functional networks could be constructed from the spike trains of...
hundreds of neurons. Compared to the previous studies, where only dozens of neurons were analyzed, these larger the networks are more likely to show the important characteristics of complex networks. We constructed a number of functional networks according to different time periods. We also analyzed the small-world and modular characteristics of the network and described the change process of the network characteristics among the different time periods.

2. Data sets and Methods

2.1. Experimental Data
In our study, we used a public multi-electrode recording data sets as the analysis data. The data contained the recordings of neuron spiking activity in mouse somatosensory cortex that was performed in organotypic slice cultures after 2 to 4 weeks in vitro. The recording process used 512 electrodes to record the neuronal spike trains, where hundreds of neurons could be recorded at once. The technical details of the recording process are described in detail from previous literature [5] (Ito et al., 2014). These data can be downloaded from the website (http://www.crcns.org/) [6].

2.2. Network Construction
The current research on brain network can be divided into three categories: anatomical connection, functional connection, and effective connection. Here, we calculate the correlation of neuronal spike trains to illustrate the functional connections between neurons. These calculations are generally done by either binning the firing time or binless [7]. Here, we consider the fire time of neuronal action potentials as input $X\{X_i, X_2, ..., X_J\}$, where $X_i$ represents the spike train of the $i$th neuron. We use the correlation coefficient to calculate the correlation between pairs of neurons:

$$R(i,j) = \frac{C(i,j)}{\sqrt{C(i,i)C(j,j)}}$$

where $C$ is the covariance matrix of $X$, $C = \text{Cov}(X)$. We took the absolute value of $R$ as the correlation strength between neurons. The values of $R$ are within $[0,1]$. A greater value of $R$ indicated more strength of the functional connectivity between neurons. The resulting network is a fully connected graph. We regard the edge density $p$ as a network construction parameter. We constructed the different functional networks using different values of $p$. The $p$ threshold values ranged from 0.1 to 0.9 in increments of 0.1. As shown in Figure 1, we used 20 neurons to illustrate the construction of a neuronal functional network according to the neuronal spike trains of neurons.

![Figure 1](image.png)

**Figure 1.** Construction of a neuronal functional network. (A) Spike trains of 20 neuronal, a line represent spike train of a neuron. (B) The correlation matrix of 20 neurons. (C) The functional connection between 20 neurons, the physical location of neurons is random.
2.3. Small-World Network
The small-world network was originally introduced by Watts and Strogatz [8]. It is used to analyze the characteristics of the real network which is different from the random network. In our study, it can be seen from Figure 1 that the neuronal functional network can be regarded as a connected graph G, where the nodes are single neurons, and the correlations of the spike trains are the edges of the graph. Here we study the binary undirected neuronal functional network.

The clustering coefficient of a node \( i \), \( C_i \), which was calculated as the number of actual connections between the number of neighbor’s nodes of the node \( i \) \((e_i)\) divided by the possible maximum number of connected edges \( \frac{k_i(k_i-1)}{2} \), is expressed as follows:
\[
C_i = \frac{2e_i}{k_i(k_i-1)} = \frac{\sum_{j,m} a_{im}a_{mj}}{k_i(k_i-1)}
\]

The clustering coefficient \( C_i \) ranges from 0 to 1. The clustering coefficient for the whole network is given by
\[
C = \langle C_i \rangle = \frac{1}{N} \sum_i C_i
\]

The path length between any pair of nodes (e.g., node \( i \) and node \( j \)) is defined as the sum of the edge lengths along this path. The shortest path length for whole network \( L \) describes the average length of the shortest path between two nodes in the network.
\[
L = \frac{1}{N(N-1)} \sum_{i,j} l_{ij}
\]

Shortest path length \( L \) is a measure of the transfer speed of parallel information in the brain; whereas clustering coefficient \( C \) is a measure of the information exchange of each subgraph.

To examine the small-world properties, the \( C \) and \( L \) of the brain networks were compared with those of random networks. The small-world network has a larger clustering coefficient and a smaller shortest path length. That is \( C_{\text{real}} / C_{\text{random}} \gg 1 \), \( L_{\text{real}} / L_{\text{random}} \approx 1 \). Humphries defines it as a single measurement value [9]. Small-World-Ness:
\[
s_{\text{sw}} = \frac{C_{\text{real}}/L_{\text{real}}}{C_{\text{random}}/L_{\text{random}}}
\]

A network is said to be “small-world” when \( s_{\text{sw}} > 1 \).

2.4. Modular Network
If a network is not a random network, the distribution of nodes in the network will be in a cluster structure, or modularity. The most common concept of modularity based on graph partitioning is modularity \( Q \) that was proposed by Newman [10]. In this concept, the \( Q \) value is used to identify the community partitioning in the network. Originally, \( Q \) was used for unweighted networks, where the \( Q \) value is defined as:
\[
Q = \frac{1}{2m} \sum_{ij} (A_{ij} - \frac{k_i k_j}{2m}) \delta(C_i, C_j)
\]

where \( k_i \) and \( k_j \) are the degree of nodes \( j \), \( i \), and \( m \) is the total number of edges in the network. The \( \delta(C_i, C_j) \) function is equal to 1 if nodes \( i \) and \( j \) are in the same community; if they are in a different community they are equal to 0. By this definition, \( Q \) can only be used for binary networks.
In the study of complex networks, a popular method is to divide the network into community structures through the modularity optimization method. Here, we used the external optimization method to get the maximum value of $Q$ for each network and the corresponding partitioning results [11]. In order to evaluate if the neural functional network is a modular network, we also compared the $Q_{real}$ values of the real network to the $Q_{rand}$ values of the same scale random network.

$$m = \frac{Q_{real}}{Q_{rand}}$$  \hspace{1cm} (7)

The larger the modular value $m$ is, the stronger the modularity of the neuronal functional network is, and the network becomes increasingly different from the random network.

### 2.5. Similarity of Community Structure

We have generated different neuronal functional networks at different times, where each functional network can be divided into several community structures. In order to compare the similarity indexes between the community structures, we proposed a new similarity coefficient $S$. In our study, the networks were constructed during different times, had a different number of nodes, and a different number of community structures. Therefore, it is not appropriate to use the mutual information to measure the similarity of community structures. In this paper, we propose a new similarity coefficient based on the set theory [12]. Let $A = \{A_i\}, i = 1, 2, ..., k_1$ and $B = \{B_j\}, j = 1, 2, ..., k_2$ be two community patterns of the two different networks $A$ and $B$, with a different number of nodes and they each contain a different number of communities. $k_j$ is the number of communities of network $A$ while $k_i$ is the number of communities of network $B$. The similarity of community $A_i$ and community $B_j$ can be calculated as:

$$S_{ij} = \frac{A_i \cap B_j}{A_i \cup B_j}$$  \hspace{1cm} (8)

Then the similarity coefficient of the network community structure pattern $A$ and $B$ are defined as:

$$S_{AB} = \frac{\sum_{i=1}^{k_1} \sum_{j=1}^{k_2} S_{ij} + \sum_{i=1}^{k_1} \sum_{j=1}^{k_2} S_{ji}}{k_1 + k_2}$$  \hspace{1cm} (9)

![Figure 2](image-url)  

**Figure 2.** Illustration of two community pattern networks with a different number of nodes. Network A is partitioned into three community structures. Network B is partitioned into two community structures.

As an example, Figure 2 depicts community patterns of two different networks: A and B. The number of nodes is different, where community A contains 8 nodes and community B contains 7 nodes. Network A is divided into 3 communities and network B is divided into 2 communities. From this, the similarity of patterns between A and B are calculated as: $S_{A1B1}=1$, $S_{A1B2}=0$, $S_{A2B1}=0$, $S_{A2B2}=2/3$, $S_{A3B1}=0$, $S_{A3B2}=1/4$ and $S_{B1A1}=1$, $S_{B1A2}=0$, $S_{B1A3}=0$, $S_{B2A1}=0$, $S_{B2A2}=2/3$, $S_{B2A3}=1/4$. Lastly, $S_{AB}=0.7667$. From this, the similarity coefficient can be used to quantify the similarity of community structure.
patterns. We can construct the neuronal functional networks at different times and calculate the similarity coefficient $S$ of its community patterns.

3. Results

We randomly selected four recordings to be used as the experimental data. The number of neurons obtained from each recording is shown in table 1. It can be seen that the number of neurons recorded at one time is much larger than previous studies, with a recording length of approximately 60 minutes. This was further divided into 6 minute windows, so that the total recording time can be divided into 10 non-overlapping trails. The neurons of each trial were joined into a functional network. In total, we constructed 10 functional networks at different moments ($T_1,T_2,...,T_{10}$) in a recording. The constructed network was a fully connected network. We retained the different number of edges according to the different edge densities of the $p$ ($p=0.1,0.2,...,0.9$) threshold, where we also constructed 9 networks that contained a different number of edges.

| Table 1. | The number of neurons in each recording |
|----------|-----------------------------------------|
|          | Data1 | Data2 | Data3 | Data4 |
| Neurons  |       |       |       |       |
| Trials   | 10    | 10    | 10    | 10    |

3.1. Small-world Characteristics

To evaluate the small-world network characteristics, we calculated the clustering coefficient $C$ and the shortest path length $L$ for each neuronal functional network, and then compared the values to those of a random network, averaged over 10 random data points.

| Table 2. | Small-world properties of four recordings |
|----------|-----------------------------------------|
|          | $p$ | T1 | T2 | T3 | T4 | T5 | T6 | T7 | T8 | T9 | T10 |
| Data1    | 0.1 | 0.00 | 0.00 | 0.00 | 2.31 | 1.99 | 2.11 | 1.98 | 0.00 | 2.06 | 0.00 |
|          | 0.2 | 1.82 | 0.00 | 0.00 | 1.92 | 0.00 | 0.00 | 1.80 | 1.93 | 1.78 | 1.58 |
|          | 0.3 | 1.58 | 0.00 | 0.00 | 1.65 | 0.00 | 0.00 | 1.58 | 1.60 | 1.47 | 1.37 |
|          | 0.4 | 1.36 | 1.46 | 1.40 | 1.39 | 0.00 | 0.00 | 1.37 | 1.38 | 1.27 | 1.17 |
|          | 0.5 | 1.20 | 1.28 | 1.24 | 1.29 | 0.00 | 0.00 | 1.21 | 1.20 | 1.13 | 1.10 |
|          | 0.6 | 1.18 | 1.16 | 1.15 | 1.18 | 0.00 | 0.00 | 1.10 | 1.15 | 1.12 | 1.09 |
|          | 0.7 | 1.18 | 1.11 | 1.15 | 1.18 | 0.00 | 0.00 | 1.09 | 1.15 | 1.12 | 1.09 |
|          | 0.8 | 1.13 | 1.07 | 1.14 | 1.14 | 0.00 | 0.00 | 1.10 | 1.15 | 1.12 | 1.09 |
|          | 0.9 | 1.08 | 1.08 | 1.06 | 1.06 | 0.00 | 0.00 | 1.08 | 1.06 | 1.07 | 1.06 |
| Data2    | 0.1 | 2.75 | 2.14 | 0.00 | 2.18 | 0.00 | 0.00 | 2.14 | 0.00 | 2.22 | 0.00 |
|          | 0.2 | 2.02 | 1.83 | 0.00 | 1.83 | 1.80 | 0.00 | 1.92 | 1.90 | 1.78 | 1.55 |
|          | 0.3 | 1.58 | 0.00 | 0.00 | 1.67 | 0.00 | 0.00 | 1.62 | 1.47 | 1.34 |
|          | 0.4 | 1.31 | 0.00 | 1.50 | 1.50 | 0.00 | 0.00 | 1.38 | 1.28 | 1.18 |
|          | 0.5 | 1.14 | 0.00 | 1.33 | 1.33 | 0.00 | 0.00 | 1.21 | 1.13 | 1.09 |
|          | 0.6 | 1.03 | 0.00 | 1.20 | 1.19 | 0.00 | 0.00 | 1.15 | 1.12 | 1.09 |
|          | 0.7 | 1.02 | 1.02 | 1.09 | 1.09 | 0.00 | 0.00 | 1.15 | 1.12 | 1.09 |
|          | 0.8 | 1.02 | 1.01 | 1.07 | 1.09 | 0.00 | 0.00 | 1.15 | 1.11 | 1.10 |
|          | 0.9 | 1.02 | 1.01 | 1.06 | 1.06 | 0.00 | 0.00 | 1.06 | 1.07 | 1.06 |
| Data3    | 0.1 | 2.31 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
|          | 0.2 | 1.91 | 0.00 | 2.03 | 1.95 | 2.00 | 1.86 | 1.87 | 1.89 | 0.00 | 0.00 |
|          | 0.3 | 1.59 | 0.00 | 1.73 | 1.64 | 1.66 | 1.58 | 1.63 | 1.61 | 1.58 | 1.48 |
|          | 0.4 | 1.38 | 1.42 | 1.46 | 1.43 | 1.42 | 1.37 | 1.37 | 1.40 | 1.31 | 1.29 |
|          | 0.5 | 1.21 | 1.26 | 1.29 | 1.27 | 1.23 | 1.21 | 1.21 | 1.22 | 1.19 | 1.19 |
|          | 0.6 | 1.15 | 1.14 | 1.15 | 1.16 | 1.17 | 1.15 | 1.15 | 1.15 | 1.17 | 1.13 |
|          | 0.7 | 1.15 | 1.11 | 1.13 | 1.16 | 1.17 | 1.15 | 1.15 | 1.15 | 1.17 | 1.13 |
It is impossible to compute the average path length $L$ and small-world characteristics because some constructed functional neuronal networks were no longer connected networks. When this was the case, we assumed these values to be 0. We were able to find all the small world properties ($sw=1$) in the different neuronal networks. It seems that the neural network is also a small world network, similar to other brain networks. The way the edge density parameters of $p$ were chosen directly affects the number of edges in the networks. When the value of $p$ is smaller, the resulting neuronal functional network is a sparse network. The small-world characteristic was stronger when $p$ is larger. Therefore, if we analyze the existence of a small world network, we should not only consider the number of network nodes, but also the number of connected edges in the network.

3.2. Modularity

We calculated the maximum value of modularity $Q$ on neuronal functional networks at different times and compared these values with the values obtained from a random network.

![Figure 3](image_url)

**Figure 3.** The modular characteristics $m$ of the different neural networks, compared with that of a random network.

The results show that in different recordings, $m$ is greater than 1. The modularity of the neuronal functional network is larger than the random network, suggesting that neuronal functional networks typically present a modular structure. In addition, when the edge density $p$ is small, the value of $m$ is larger. However, the network obtained with $p=0.1$ did not correspond to the maximum modularity value $m$.

4. Conclusion
A large number of studies have found that brain functional networks based on fMRI have both small-world and modular characteristics [13]. However, there are only a few studies based on neuronal functional networks substantiating this. In this study, we performed a complex network analysis on a large scale neuronal network, where we simultaneously analyzed the small-world characteristics and modularity. We found that neuronal functional networks have the complex network characteristics similar to other brain functional networks, and there was a positive correlation between small world networks and modularity. Specifically, we were interested in the characteristics of the complex network changing across the recording time. Our work demonstrated that networks of spiking neurons display specific non-random structures across time.

We used the spike trains data that was recorded from a spontaneous state monkey, which the neuronal firing process cannot correspond to a cognitive process. In the future, it would be interesting to observe neurons that were recorded from experimental objects corresponding to cognitive behavioral tasks. The network structure of cognitive behavior neuronal firing is an essential topic for future neuroscience research.

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