Development of coordination system model on single-supplier multi-buyer for multi-item supply chain with probabilistic demand

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Abstract. Nowadays, the level of competition between supply chains is getting tighter and a good coordination system between supply chains members is very crucial in solving the issue. This paper focused on a model development of coordination system between single supplier and buyers in a supply chain as a solution. Proposed optimization model was designed to determine the optimal number of deliveries from a supplier to buyers in order to minimize the total cost over a planning horizon. Components of the total supply chain cost consist of transportation costs, handling costs of supplier and buyers and also stock out costs. In the proposed optimization model, the supplier can supply various types of items to retailers whose item demand patterns are probabilistic. Sensitivity analysis of the proposed model was conducted to test the effect of changes in transport costs, handling costs and production capacities of the supplier. The results of the sensitivity analysis showed a significant influence on the changes in the transportation cost, handling costs and production capacity to the decisions of the optimal numbers of product delivery for each item to the buyers.

Keywords: single supplier-multi buyer; coordination system; supply chain; multi-item; probabilistic demand

1. Introduction
Due to the increasing challenge of cost efficiency in order to enhance organization competitive edge in the global market, optimization in all aspects of organization functions has been the main concern of every organization. One way to overcome this challenge is by creating a coordination system between suppliers and buyers in a supply chain. Good coordination system between suppliers and buyers can support optimal decision making in an integrated supply chain. Chang and Chou [1] has developed an optimization model for the coordination system between single supplier and multi buyer to determine the optimal number of deliveries with the objective function to minimize the total supply chain cost. Buyer demand patterns considered in their model are deterministic, and supplier might produce only a single item.
In Chang and Chou [1] model, the coordination model has considered several important factors. However, there are some aspects that need to be taken into account in the coordination system model. According to Tersine [6], a deterministic demand pattern is rarely found in real conditions. Therefore, this study develops an optimization model of the coordinate system with probabilistic demand patterns. In addition, the development of the proposed model is intended for a supplier that can produce and supply various types of product to buyers.

Furthermore, this paper will be divided into five sections. Section 2 will discuss the literature review while Section 3 will describe the research method. The development as well as the discussion of the proposed coordination system model for a supplier and buyers with multi-item and probabilistic demand will be described in Section 4. Section 5 will describe the conclusions and further research.

2. Literature review
Several papers have discussed coordination systems of supplier and buyer. Table 1 shows a comparison of the characteristics of the coordination system model of previous studies and this research.

| Characteristic             | (Hejazi et al.) [3] | (Zavanella and Zanoni) [7] | (Chang and Chou) [1] | (This research, 2014) |
|----------------------------|---------------------|-----------------------------|----------------------|----------------------|
| Demand pattern             | Deterministic       | Deterministic               | Deterministic        | Probabilistic        |
| Product variation          | Single-item         | Single-item                 | Single-item          | Multi-item           |
| Number of entity           | Single supplier     | Single supplier             | Single supplier      | Single supplier      |
|                            | single buyer        | multi buyer                 | multi buyer          | multi buyer          |
| Total cost component       | Purchasing cost,    | Set-up cost, order cost,    | Set-up cost,         | Transportation cost, |
|                            | handling cost in    | handling cost in            | transportation,       | handling cost in     |
|                            | supplier and buyer  | supplier and buyer          | order, handling in   | supplier and buyer,  |
|                            |                     |                             | supplier and buyer,  | receiving            |
|                            |                     |                             |                      |                      |

Hejazi et al. [3], Zavanella and Zanoni [7] as well as Chang and Chou [1] developed a model with deterministic demand data for single item supply chain. Hejazi et al. [3] and Zavanella and Zanoni [8] using annual demand data, while Chang and Chou [1] using monthly demand data.

The proposed optimization model in this paper is coordination between single supplier that can produce and supply various types of item with buyers with probabilistic demand pattern. The components of the total cost that is considered in this paper are transportation cost, handling cost in supplier and buyer as well as stock out cost.

3. Research methodology
The model development in this paper will be conducted in two stages. The first stage is to create some adjustment from mathematical modeling from Chang and Chou [1]. The adjustments made in this stage are to remove ordering cost, receiving cost and set up cost from a component of total cost since these costs do not affect the decision in optimization model. Meanwhile, the second stage is to develop a proposed model by considering probabilistic demand pattern and multiple items. The completion of a mathematical model in this paper is using Lingo 11.0. Model validation and sensitivity analysis are also conducted for the proposed model in this paper.

4. Results and discussion
Some assumptions used in the initial model adjustment are: the supplier only produces one type of product, and demand data pattern is deterministic. Adjustments made for the initial model consist of the elimination of ordering cost, order receiving cost and set-up cost from total cost components. In a coordinated replenishment system, the order process is done only once in the beginning of planning
horizon. Ordering cost formulation according to Chang and Chou [1] is \( \sum_{i=1}^{n} \alpha_i \), and setup cost is \( C_x S \) in which there are no decision variables involved in that formulations. For order receiving cost, Chang and Chou [1] formulate it as \( \sum_i V_i(\sum_i Q_{ij}x_{ij}) \). As long as demand data is the same, total order receiving cost would be the same in a year, and it will not influence the decision of optimization model. Briefly, this initial model has objective function to minimize total cost. The Initial model also ensures that there is no shortage allowed, and the delivery quantity will not exceed supplier’s stock level.

This proposed model is developing a mathematical model for the coordination system between a supplier and buyers. The supplier can produce and supply various types of item to multiple buyers who have probabilistic demand pattern. There are some assumptions used in this research, such as supplier’s lead time is 0, meaning that the goods will be delivered immediately after the supplier finish their production and the delivery can be done simultaneously for all kind of goods. Another assumption is that a buyer is willing to accept all the delivery quantity from the supplier. The demand data in this research have normal distribution pattern, and there is no limit on warehouse and shipping capacity. In overall, the proposed model can be described as follows:

\begin{align*}
& i = \text{index of period}; \quad T = \text{number of period}; \\
& j = \text{index of buyer}; \quad B = \text{number of buyer}; \\
& k = \text{index of number of product type (item)} \quad P = \text{number of product}; \\
& D_{jk} = \text{Total annual demand of item-} k \text{ for buyer-} j \\
& d_{ijk} = \text{Demand of item-} k \text{ in period-} i \text{ for buyer-} j \\
& F_j = \text{Fixed transportation cost for buyer-} j \text{ per trip} \\
& h_{bk} = \text{Handling cost per unit of item-} k \text{ for buyer-} j \\
& h_{sk} = \text{Handling cost per unit of item-} k \text{ for supplier} \\
& Y_{ik} = \text{Stock position item-} k \text{ in supplier after production process finished in period-} i \\
& (Y_{ik} - \sum_{j} Q_{ijk}) = \text{Final stock supplier for item-} k \text{ in period-} i \\
& Y_{ijk} = \text{Stock position item-} k \text{ in buyer-} j \text{ after receiving delivery in period-} i \\
& (Y_{ijk} - d_{ijk}) = \text{Final stock buyer-} j \text{ for item-} k \text{ in period-} i \\
& R_{ik} = \text{Production quantity of item-} k \text{ in period-} i \\
& K = \text{Production capacity} \\
& Q_{ijk} = \text{Optimal shipping quantity item-} k \text{ in period-} i \text{ for buyer-} j \\
& x_{ijk} = \text{Binary variable} \\
& x_{ijk}= 1, \text{ if there is delivery of item-} k \text{ in period-} i \text{ for buyer-} j \\
& x_{ijk}= 0, \text{ otherwise} \\
& Z_{ijk} = \text{Binary variable} \\
& Z_{ijk}= 1, \text{ if there is stockout of item-} k \text{ for buyer-} j \text{ in period-} i \\
& Z_{ijk}= 0, \text{ otherwise} \\
& N_{ij} = \text{Binary decision} \\
& N_{ij}= 1, \text{ if there is delivery activity from buyer-} j \text{ in period-} i \\
& N_{ij}= 0, \text{ otherwise} \\
\end{align*}

Objective function

Minimize:

\begin{align*}
& \sum_{k=1}^{P} h_{bk} \sum_{i=1}^{T} (Y_{ijk} - d_{ijk})(1 - Z_{ijk}) + \sum_{k=1}^{P} h_{sk} \sum_{i=1}^{T} (Y_{ik} - \sum_{i=1}^{T} Q_{ijk}) + \sum_{j=1}^{B} F_j \sum_{i=1}^{T} N_{ij} + \\
& \sum_{j=1}^{B} \sum_{k=1}^{P} S_{ijk} \sum_{i=1}^{T} [(d_{ijk} - Y_{ijk})]Z_{ijk} \\
& \text{constraints} \\
& \sum_{j=1}^{B} Q_{ijk} = D_{jk}, \quad i = 1, \ldots, T; j = 1, \ldots, B; k = 1, \ldots, P \\
& Y_{ik} = (Y_{(i-1)k} - \sum_{j=1}^{B} Q_{(i-1)jk}) + R_{ik}, \quad i = 1, \ldots, T; j = 1, \ldots, B; k = 1, \ldots, P \\
\end{align*}
\( x_{ijk} = 1, j = 1, \ldots, B; k = 1, \ldots, P \)  
\( Y_{ik} - \sum_{j=1}^{B} Q_{ijk} \geq 0, \quad i = 1, \ldots, T; j = 1, \ldots, B; k = 1, \ldots, P \)  
\( Q_{ijk} \leq x_{ijk} \cdot M, \quad i = 1, \ldots, T; j = 1, \ldots, B; k = 1, \ldots, P \)  
\( Y_{ijk} = (Y_{(i-1)jk} - d_{ijk}) + Q_{ijk}, \quad i = 1, \ldots, T; j = 1, \ldots, B; k = 1, \ldots, P \)  
\( Y_{ijk} - d_{ijk} \leq (1 - Z_{ijk}) \cdot M, \quad i = 1, \ldots, T; j = 1, \ldots, B; k = 1, \ldots, P \)  
\( d_{ijk} - Y_{ijk} \leq Z_{ijk} \cdot M, \quad i = 1, \ldots, T; j = 1, \ldots, B; k = 1, \ldots, P \)  
\( \sum_{k=1}^{P} x_{ijk} \leq N_{ij} \cdot M, \quad i = 1, \ldots, T; j = 1, \ldots, B; k = 1, \ldots, P \)  
\( \sum_{i=1}^{T} R_{ik} \leq K, \quad i = 1, \ldots, T; k = 1, \ldots, P \)  
\( R_{ik} + (Y_{(i-1)jk} - \sum_{j=1}^{B} Q_{(i-1)jk}) \geq \sum_{j=1}^{B} Q_{ijk}, \quad i = 1, \ldots, T; j = 1, \ldots, B; k = 1, \ldots, P \)  

The objective function in this model is a total cost minimization which consists of transportation cost, handling cost in supplier and buyer, and also shortage cost, and it can be seen from equation (1). Equation (2) states that total delivery item-\( k \) to buyer-\( j \) in all of the period should be same with total demand item-\( k \) for buyer-\( j \). The supplier’s inventory position before delivering the goods is shown in equation (3), which is equal to the initial supplier’s stock of item-\( k \) in period-\( d \) and the production quantity of item-\( k \) in period-\( i \). Equation (4) ensures that there is a delivery on period 1 for all buyers and all items. Equation (5) ensures that there is enough quantity of item-\( k \) in period-\( i \) for supplying item-\( k \) to all buyers. Binary variable \( x_{ijk} \) on equation (6) states whether there is delivery of item-\( k \) to buyer-\( j \) in period-\( i \) or not. \( x_{ijk}=1 \) means that there is a delivery of item-\( k \) for buyer-\( j \) in period-\( i \), and otherwise. The amount of inventory position of item-\( k \) in buyer-\( j \) after receiving delivery in period-\( i \) is the sum of initial stock buyer-\( j \) for item-\( k \) and delivery quantity item-\( k \) for buyer-\( j \) in period-\( i \) which stated in equation (7). Equation (8) and (9) ensure that when \( Z_{ijk}=1 \), it means that there is stock out of item-\( k \) for buyer-\( j \) in period-\( i \), and otherwise. \( N_{ij} \) in equation (10) is a binary variable that states whether there is delivery for buyer-\( j \) in period-\( i \) or not.\( N_{ij}=1 \) when there is a delivery for buyer-\( j \) in period-\( i \), and otherwise when \( N_{ij}=0 \). Equation (11) ensures that the production quantity does not exceed production capacity of the supplier. Equation (12) ensures that sum of production quantity and initial stock of item-\( k \) in period-\( i \) should be able to meet delivery quantity of item-\( k \) to buyer-\( j \) in period-\( i \).

Case study for the proposed mathematical model consists of one supplier and five buyers that can be applied for consumer goods manufacturer. The supplier produces three items: item A, item B and item C with a production capacity of 4200 items/month. For a case study in this proposed model, demand data is probabilistic, and safety stock should be calculated. After calculating safety stock, total demand from each buyer can be determined by summing up the demand and safety stock for each buyer.

| \( d_{ijk} \) | Buyer 1 | Buyer 2 | Buyer 3 | Buyer 4 | Buyer 5 |
|-------------|---------|---------|---------|---------|---------|
| A | B | C | A | B | C | A | B | C | A | B | C |
| 1 | 234 | 209 | 204 | 301 | 227 | 268 | 281 | 152 | 308 | 243 | 244 | 280 | 179 | 291 | 257 |
| 2 | 207 | 143 | 196 | 257 | 227 | 222 | 223 | 229 | 269 | 231 | 191 | 254 | 203 | 286 | 252 |
| 3 | 238 | 194 | 243 | 270 | 186 | 256 | 287 | 249 | 255 | 191 | 279 | 353 | 186 | 263 | 201 |
| 4 | 198 | 216 | 293 | 248 | 161 | 284 | 152 | 253 | 211 | 173 | 241 | 311 | 255 | 263 | 198 |
| 5 | 204 | 207 | 226 | 279 | 179 | 252 | 193 | 173 | 281 | 176 | 266 | 236 | 209 | 206 | 257 |
| 6 | 164 | 215 | 235 | 293 | 206 | 270 | 159 | 210 | 213 | 165 | 193 | 276 | 199 | 244 | 203 |
| 7 | 178 | 165 | 288 | 249 | 190 | 242 | 241 | 239 | 268 | 265 | 279 | 250 | 145 | 202 | 204 |
| 8 | 267 | 195 | 266 | 208 | 234 | 294 | 298 | 256 | 231 | 160 | 253 | 241 | 217 | 233 | 259 |

| Table 2. Demand buyer-\( j \) for item-\( k \) in period-\( i \)(unit). |
Table 3. The handling cost in supplier and buyer for each item.

| Item | Buyer’s handling cost | Supplier’s handling Cost |
|------|-----------------------|--------------------------|
| Item A | IDR 117 | IDR 90 |
| Item B | IDR 157 | IDR 121 |
| Item C | IDR 196 | IDR 151 |

Table 4. Fix transportation cost.

| Buyer | Fix transportation cost (IDR/trip) |
|-------|-----------------------------------|
| 1.    | 600.000                           |
| 2.    | 500.000                           |
| 3.    | 450.000                           |
| 4.    | 600.000                           |
| 5.    | 450.000                           |

Table 5. Optimal delivery quantity for item-\(k\) for buyer-\(j\) in period-\(i\).

| Q\(_{ijk}\) | Buyer 1 | Buyer 2 | Buyer 3 | Buyer 4 | Buyer 5 |
|------------|--------|--------|--------|--------|--------|
| A          | 362    | 317    | 287    | 433    | 322    | 374    | 54     | 217    | 194    | 391    | 125    | 66     | 300    | 409    | 350    |
| B          | 0      | 0      | 0      | 0      | 0      | 0      | 445    | 151    | 276    | 1036   | 1186   | 1106   | 0      | 0      | 0      |
| C          | 279    | 280    | 320    | 400    | 332    | 370    | 219    | 222    | 263    | 0      | 0      | 0      | 156    | 978    | 382    |
| A          | 270    | 221    | 418    | 550    | 326    | 569    | 353    | 212    | 278    | 0      | 0      | 0      | 546    | 0      | 457    |
| B          | 0      | 0      | 0      | 0      | 0      | 0      | 93     | 81     | 60     | 1154   | 1646   | 1052   | 113    | 0      | 0      |
| C          | 619    | 866    | 920    | 0      | 0      | 0      | 487    | 544    | 764    | 0      | 0      | 0      | 0      | 0      | 0      |
| A          | 0      | 0      | 0      | 0      | 0      | 0      | 1116   | 853    | 486    | 0      | 0      | 0      | 0      | 254    | 700    | 791    |
| B          | 0      | 0      | 0      | 0      | 0      | 0      | 617    | 372    | 1509   | 0      | 0      | 0      | 424    | 516    | 763    |
| C          | 981    | 568    | 1067   | 0      | 0      | 1085   | 264    | 235    | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      |
| A          | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 890    | 574    | 368    | 0      |
| B          | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      |
| C          | 11     | 206    | 394    | 280    | 157    | 594    | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      |
| A          | 12     | 58     | 580    | 99     | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      |
| B          | 31     | 3285   | 3236   | 3164   | 2689   | 2628   | 3344   | 2581   | 2957   | 3114   | 2366   | 2971   | 2742   | 0      | 0      | 0      |
| C          | 2543   | 2310   | 3012   | 3285   | 2326   | 3164   | 2689   | 2628   | 3344   | 2581   | 2957   | 3114   | 2366   | 2971   | 2742   | 0      |

\(d_{ijk}\) | Buyer 1 | Buyer 2 | Buyer 3 | Buyer 4 | Buyer 5 |
|------|--------|--------|--------|--------|--------|
| A    | 9      | 223    | 184    | 270    | 305    | 185    | 266    | 179    | 273    | 313    | 253    | 263    | 268    | 199    | 242    | 243    |
| B    | 10     | 220    | 200    | 260    | 251    | 192    | 271    | 210    | 214    | 353    | 230    | 274    | 261    | 207    | 273    | 267    |
| C    | 11     | 208    | 198    | 369    | 285    | 183    | 193    | 266    | 172    | 371    | 216    | 240    | 203    | 138    | 163    | 193    |
| A    | 12     | 202    | 184    | 162    | 339    | 156    | 341    | 200    | 208    | 271    | 278    | 234    | 181    | 229    | 305    | 208    |
| Total| 2543   | 2310   | 3012   | 3285   | 2326   | 3164   | 2689   | 2628   | 3344   | 2581   | 2957   | 3114   | 2366   | 2971   | 2742   | 0      |
Table 6. Production quantity of item-\(k\) in period-\(i\).

| \(i\) | Item A (unit) | Item B (unit) | Item C (unit) | TOTAL (unit) |
|------|--------------|---------------|---------------|-------------|
| 1    | 1.539        | 1.390         | 1.270         | 4.200       |
| 2    | 1.481        | 1.337         | 1.382         | 4.200       |
| 3    | 1.053        | 1.811         | 1.335         | 4.200       |
| 4    | 1.719        | 759           | 1.722         | 4.200       |
| 5    | 1.360        | 1.727         | 1.112         | 4.200       |
| 6    | 1.106        | 1.411         | 1.683         | 4.200       |
| 7    | 1.370        | 1.553         | 1.277         | 4.200       |
| 8    | 1.040        | 888           | 2.272         | 4.200       |
| 9    | 1.246        | 803           | 2.151         | 4.200       |
| 10   | 574          | 368           | 890           | 1.832       |
| 11   | 363          | 988           | 280           | 1.631       |
| 12   | 612          | 157           | 0             | 769         |

The result of this proposed model is a decision for optimal shipping quantity using the scenario that has been defined. The optimal shipping quantity can be seen in Table 5, while the production quantity is in Table 6.

5. Conclusion
This study developed a mathematical model for coordination system between single supplier and multi buyer for multi item supply chain with probabilistic demand pattern. The objective function in this proposed model was to minimize total cost which consists of transportation cost, handling cost in supplier and buyers as well as stockout cost. Sensitivity analysis showed that the changing parameters have a significant influence on the value of decision variables and objective function. The transportation cost was inversely proportional to the total delivery frequency for one year, and the handling cost was directly proportional to the frequency of delivery within one year, and the production capacity was directly proportional to the handling cost.

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