Scale modeling vibro-acoustics

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Abstract: This review provides a summary of the basic theory of scale modeling and some example applications. The article is written to introduce principles and the thinking process unique to scale modeling but not to provide a thorough literature review of relevant articles. Five different examples of scale modeling application to vibro-acoustic problems are given, including architectural acoustics and modeling a concert hall, acoustic streaming jets, acoustic reciprocity, vibration casting, and reduction of NOx by acoustic wave. The scaling laws for the first three examples reported there are introduced in this article based on the law approach, while for the last two examples of scaling laws were developed in this article.

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1. INTRODUCTION

Scale modeling involves the use of physical models whose size is either smaller or larger than that of the full scale system (which is called the prototype in the technical literature), to conduct experiments for the purpose of testing scaling law, attempting to validate them (confirmation tests), or use of the validated scale in various applications (production tests) [1,2]. Similarity to be achieved by scale modeling here can be defined as:

\[
\pi_i = \pi_i'
\]  (1)

where \( \pi \) is dimensionless number called as pi-number, subscript, \( i = 1, 2, 3, \ldots, n \), and prime (‘) stands for the model and non-prime for the full scale. Equation (1) provides the following meaning: suppose that the full scale phenomenon is governed by a particular set of physical laws, the model must be governed by the same set of physical laws. For example, many of our engineering problems can be described by governing equations of mass, momentum, energy, and chemical species. Some problems are governed by a balance between different forces, while others can be more appropriately described by a balance between different types of energy.

There are two different types of similarity: geometrical (or static) and dynamical [1,2]. The geometrical similarity describes similarity when things are not in action, while the dynamical similarity describes similarity when thing are in action. Since the geometrical similarity itself is not our current interest, we will provide two examples of dynamical similarity. The first one is a turbulent flow pattern created behind Mt. Fuji under over 100 m/s wind and its 1/25,000 scale model based on the Froude number scaling (a life time work of Professor Seiji Soma [3]), whose explanation will be detailed for the second example. Figure 1 shows a remarkable similarity in their flow pattern between the prototype Mt. Fuji and its scale model [1,3].

The second example of dynamical similarity: Water flow through horizontally placed straight pipes, adopted from [1,2]. This dynamical similarity is described among three different forces: viscous and inertia forces of fluid and pressure force acting on the fluid, while gravity and surface tension forces are assumed to be secondary, yielding two independent pi-numbers, \( \pi_1 = (\text{inertia force})/(\text{viscous force}) \), and \( \pi_2 = (\text{pressure force})/(\text{inertial force}) \), where \( \pi_1 \) is the Reynolds number, \( \text{Re} \), and \( \pi_2 \) is the Euler number, \( \text{Eu} \). By applying Eq. (1) for similarity, \( \text{Re} = \text{Re}' \) and \( \text{Eu} = \text{Eu}' \) will be obtained, which will be further converted to a functional relationship \( \text{Eu} = \Phi(\text{Re}) \), where \( \Phi \) is an arbitrary function. Nikuradse [4] conducted a series of scale model pipe flow experiments to obtain this functional relationship \( \Phi \) by changing the diameter of the pipe, flow velocity, and surface roughness. Figure 2 shows the
concept of his finding in the $Eu$ number as a function of the $Re$ number, where laminar, transition, and turbulent flow regimes were identified in an order from low to high $Re$ number. When the $Re$ numbers are relatively low, the flow is laminar dominated by viscous force over the inertia force, establishing $Eu = 64/Re$. With an increase in the $Re$ number, flow transitions from laminar to turbulent, where both viscous and inertial forces are important. With a further increase in the $Re$ number, flow will become fully developed turbulent and the $Eu$ number becomes approximately independent of the $Re$ number, i.e., $Eu = \text{const}$. This functional relationship suggests that the inertia force of fluid is dominant while the viscous force is negligible.

Scale modeling problems, however, are not always as straightforward as the above example. Williams [5], for example, theoretically derived the total of 27 pi-numbers for mass fires using the governing equations of the mass, momentum, energy and chemical reactions. He realized that keeping all of the 27 pi-numbers the same for the full scale and the corresponding model is practically impossible, therefore reducing them to some practically achievable numbers is necessary. Relaxation or partial scaling is a technique to approximately achieve similarity without significantly compromising the original similarity. Emori et al. [1,2] offer several specific methods, among which the following two techniques are worthy to mention here, i.e., keeping only the strong physical laws and ignoring weak ones, and circumventing the strong laws by breaking the total phenomena into a linear combination of smaller parts and developing achievable scaling laws for each of these parts. Interesting enough, Williams concluded the $Fr$ number ($= \text{inertial force/buoyancy force}$) is the single most important pi-number out of 27, and the $Fr$ number has been proven to be one of the most frequently applied pi-numbers in fire and combustion problems.

It may be beneficial to think about how to teach the relaxation technique to students, since it is the most difficult technique in scale modeling yet plays a key role in achieving similarity. The late Professor Emori, pioneer of the law approach, recommended a lot of practice in many different case studies until students can develop their engineering mind to capture the principle of relaxation. Following his recommendation, the University of Kentucky’s scale modeling course (ME565) offers a combination of fundamental theory, several case studies, and hands-on projects [6].

2. SCALE MODELING AND THE LAW APPROACH

There are at least three different ways to obtain scaling laws [1,2], namely the equation approach, the parameter approach, and the law approach. The equation approach requires the full governing equation to describe the phenomenon that we are facing. This approach is mathematically rigorous, but often faces a challenge since there are many phenomena where detailed mechanisms are still unknown or very complex in describing with equations. The parameter approach is probably most well-known among all three approaches in relation to Buckingham’s pi-theorem [1,2]. This approach requires identification of all the correct and only the correct parameters that are involved in the phenomenon. A successful selection of these correct parameters is the key in the parameter approach, but there is no method to achieve that goal. If the correct parameters are selected, the correct pi-numbers can be obtained by Buckingham’s pi-theorem, while if the selection is wrong by dropping one of important parameters or including an unimportant one, then the final pi-numbers are wrong.

This shortcoming of the parameter approach can be compensated by the law approach, which can help researchers to conduct their physical interpretation of the phenomenon that they are facing. Based on that interpretation, selection of the most influential forces or energies...
can be made, which will form pi-numbers which assist in the design of scale models and testing against the full scale or among differently scaled models. Agreement means the physical interpretation was correct, while disagreement means the physical interpretation was wrong. Therefore, the law approach is to directly test the validity of assumptions based on physical interpretations, scaling laws, and conformation tests.

Figure 3 shows a flow diagram that describes the concepts of scale modeling in relationship to those of numerical modeling. Both scale and numerical modeling begin with assumptions, which are the most important element in both methods.

There is, however, no standardized formula or methods to be applied to obtain “good assumptions,” and the author of this article found a strong connection between the inductive kufu approach and this good assumption making process, probably deeply rooted in the traditional Japanese culture where Apophatic tradition was emphasized (for readers who are interested in connection between philosophy and scale modeling, see Preface of [1] and D. T. Suzuki [7]). Furthermore, A. Einstein [8], E. B. Wilson [9] and Forman Williams [10] all discuss the character of good scientific research with the emphasis on researchers’ thinking process and philosophy rather than specific scientific techniques. Their baseline philosophy also resonates with the traditional Japanese “Monozukuri culture,” where the spiritual aspect of technology is emphasized in relation to Zen [1,7]. Scale modeling is a good way of finding this connection between philosophy and science/engineering by promoting researchers’ interpretation of nature’s mechanism [1] via developing scaling laws.

After the assumptions are made, then the deductive processes indicated in Fig. 3 offer two different paths, depending on whether the approach to be applied is scale modeling or numerical simulation. In other words, if we come up with “good assumptions,” then the final result can be right, independent of the technique, while if we start with “no good assumptions,” then the final result must be wrong. Importantly, the two approaches can be complementary (see Introduction in Ref. [11]). The vibro-acoustic reciprocity problem, one of the scale modeling applications to appear in the later section, uses both scale modeling and the boundary element method. In addition, the latest book: Progress in Scale Modeling Volume-2 [11] provides a good collection of the combined use of scale and numerical modeling.

Finally, scale modeling can help researchers when they are faced with the following four aspects: (1) when the problem is too complex to be explored either by numerically or theoretically; (2) when full scale experiments are not a viable option to validate numerical model predictions, (3) when technology improvement and new product development is sought, and (4) when the inductive approach is needed to capture the influential physical laws that govern the phenomenon.

Before moving to the next section in applications of scale modeling, it may be worthy of repeating Professor Emori’s emphasis on the heart of scale modeling, “it is fundamentally simple, yet requires researchers’ overall knowledge, know-how, wisdom, experience, and willpower to solve engineering problems. Thus, it is a type of art rather than strict science, and therefore whose principles can’t be easily passed from a teacher to his students by standardized class room teaching alone.”

The following sections provide five different examples of scale modeling in vibration and acoustics: Architectural acoustics and modeling a concert hall, acoustic streaming jets, acoustic reciprocity, vibration casting, and reduction of NO\textsubscript{x} (a major pollutant from combustion and power generation systems) by acoustic wave. The first three problems offer scaling laws, while the latter two do not. The author of this article re-interpreted those available scaling laws based on the law approach, while for the latter two problems new scaling laws are developed and discussed.

### 3. ARCHITECTURAL ACOUSTICS

Emori et al. [1,2] introduced scale modeling of architectural acoustics and provided scaling laws on three important elements, porous material, thin panels, and cavity resonators that construct the overall performance of the architectural acoustics. Here a summary of these scaling laws is offered. They state that the desired reverberation time, $t$, for concert halls is about 1.8 s for frequencies between 500 and 1,000 Hz, and 1.3 s for higher frequencies; but predicting the reverberation time theoretically is difficult, for it depends on the acoustical properties and the geometrical shapes of the walls, the ceiling, the floor, the stage, and the seats. Variations of these design parameters greatly change the reverberation time, $t$, and its
distribution over frequency, to affect such acoustical criteria as warmth, liveness, and brilliance. Sound is governed by inertia and adiabatic compression forces, while viscous force is negligible, therefore, forming \( \pi_3 = (\text{inertia force/adiabatic compression force}) \), which is Mach number, \( Ma \). Using the representative expression of the inertia and adiabatic compression force, \( Ma \) number can be written as \( Ma = u \gamma^{1/2} / \alpha \), where \( u \) is the characteristic velocity, \( \alpha \) is the speed of sound, and \( \gamma \) is the adiabatic compression ratio. Assuming \( \gamma \) and \( \alpha \) are kept constant between the full scale and model, then \( u \) becomes constant resulting in the scaling relationship: \( \ell = f' / f \), where \( \ell \) is the characteristic length and \( f \) is the characteristic frequency.

There are three common types of absorbing materials: porous structures, thin panels or membranes, and cavity resonators. The porous structures absorb sound most effectively at higher frequencies, whereas the panels and resonators produce high absorption around their natural frequencies, usually below 1,000 Hz. It has been shown that a combination of the porous structures with panels or resonators can absorb sound at almost any desired frequency. These three different types of sound absorbing materials absorb the acoustic energy and convert them to friction energy, heat, and internal damping of the vibrating structure, however, whose detailed mechanism is difficult to model because their governing laws are either not well known or, if known, lead to conflicting design requirements [1,2].

3.1. Porous-material

When a sound wave strikes porous material, the sound pressure will excite the air contained in the fine canals of the material. To overcome the frictional resistance between vibrating air and solid material, energy is taken away from the sound wave. It is impossible, however, to describe all the detail of the viscous losses due to the individual capillaries and pores, because the variations of the pores in shape, size, direction, their interconnections, and their distribution are too complex to scale down in detail. Here pores can be regarded as parallel capillaries with a representative length, \( \ell \), which can be determined indirectly by a permeability test where the permeability, \( k \), is defined as the measure of the ease with which a gas flows through a porous medium. Using an experimentally determined relationship, \( k \sim \ell^2 \), the pressure force can be written as \( F_p = p_k \ell^2 = pk \). The other acting force is the viscous force \( F_{\mu} = \mu \ell u \). These two forces form \( \pi_4 = F_p / F_{\mu} = pk / \mu \ell u \), which is the Lagrange number, \( La \).

3.2. Thin Panels

Thin panels mounted at some distance from a solid wall convert sound energy into vibrational energy, with the air between panel and wall acting as spring. The larger the movement of the panel, the larger absorption is obtained with the resonance frequency of the panel-air system. If the panel is thin enough to act as a membrane, its stiffness can be neglected; if not, the stiffness must be taken into account together with its mass. Ultimately, the absorbed energy can be dissipated as heat through internal damping, which can be achieved by soft porous material filled in the space between panel and wall, whose internal resistance is large compared to that of the panel so that the panel’s damping can be ignored.

Here the absorption mechanism by thin panels was interpreted as a combination of the sound wave, the elasticity and inertia of the panel, the elasticity and inertia of the air between panel and wall, and the d-e acoustical resistance of the porous material, and leading to two major forces: inertia force of the panel, \( F_i \) and elastic force of the panel, \( F_{\sigma} \). As a result, these two major forces can form a pi-number: \( \pi_5 = F_i / F_{\sigma} = \rho c^2 f^2 / \sigma \), which is the Cauchy number, \( Ca \).

3.3. Cavity Resonators

A cavity resonator acts as a spring-mass system, where the pressure of the sound waves striking the wall vibrates the mass of the air around and in the cavity’s throat, with the air in the cavity acting as a spring. The cavity resonator dissipates energy with viscous friction created when air is going through porous material placed inside the cavity. When this throat friction is ignored, then the inertia force of the air, the elasticity of the air and the absorption of the porous material become important, yielding \( Ca \) number (or \( \pi_5 \)), the same pi-number obtained for the thin panels.

3.4. Modeling a Concert Hall

Emori et al. [1,2], showed through the above examples that there are three basic pi-numbers, \( Ma \) number (\( \pi_3 \)), \( La \) number (\( \pi_4 \)), and \( Ca \) number (\( \pi_5 \)), that control architectural acoustics. They applied these scaling laws to an existing full scale Meister Singer Halle in Nuremberg, Germany and the tenth scale model (schematic is shown in Fig. 4). The \( Ma \) number scaling imposes, \( u = \text{constant} \). The \( La \) number scaling yields \( pk / \mu \ell u = \rho' k' / \mu' \ell' u' \). Applying the \( Ma \) number scaling result, \( u = \text{const.} \), to the \( La \) number scaling and assuming \( p / \mu = \text{const.} \), then \( k / \ell = k' / \ell' \). The \( Ca \) number scaling, \( \pi_4 = \rho u^2 / \sigma \) will be automatically satisfied under \( u = \text{const.} \), \( \rho = \rho' \) and \( \sigma = \sigma' \).

Using the above scaling laws as a guide, the tenth scale model of Meister Singer Halle was carefully designed to satisfy all the scaling conditions: the sound absorption of walls, seats, audience, and other factors. The audience was simulated by egg cartons, and the model air was dried to 4% relative humidity to adjust attenuation coefficient.
between the full scale and the model. Since the property of non-directivity of the sound source is important for determining the reverberation time, \( t \), a spark source was used for both the full scale and the model, and its decay was recorded by a microphone. The reverberation time for both the full scale and model was recorded as a function of frequency between 0.1 and 100 kHz. Subjective tests were also performed including short excerpts of a string quartet and a speech were recorded in an echo-free room and played back. The task of the listeners was to match the model recording and the original Meister Singer Halle recording. Figure 4 (bottom) shows excellent agreement between prototype and its \( \frac{1}{10} \) th scale model results.

4. VIIBRO-ACOUSTIC RECIPROCITY

Liu, Zhou and Herrin [12] studied vibro-acoustic reciprocity by designing two geometrically similar structures with a shaker source to model sound radiation. They determined transfer functions between sound pressure at a point in the far field and the velocity of a patch both for the large scale Plexiglas made cabin structure whose approximate dimension is \((1.2 \text{ m long} \times 0.45 \text{ m high} \times 0.3 \text{ m wide})\) and the corresponding half-scale structure. A point monopole source was used for the reciprocal measurements for both structures. After they confirmed reciprocity between these two scale models, they applied the boundary element method (BEM) to determine the reciprocal transfer functions and validated BEM prediction with the scale modeling measurement functions. Their scale modeling is based on the \( Ma \) number scaling which determines the frequency ratio between the full scale and the corresponding scale model: \( \ell' / \ell = f'/f \), which is explained in the section of architectural acoustics. The validated BEM model was then used to predict the sound pressure in the far field from the vibrating structure.

5. ACOUSTIC STREAMING JETS

Progressive acoustic waves have been used as an acoustic streaming jet method to generate flow from a far remote place to enhance heat and mass transfer or possibly chemical reactions (as shown later in an example of acoustic wave reduction of \( \text{NO}_x \)). Both numerical and scale modeling have been applied to find important parameters that can enhance acoustic streaming jet effects. For the numerical modeling approach, the Navier-Stokes equation was modified to add an acoustic term. The N-S equation was then non-dimensionalized to obtain pi-numbers. Basically the N-S equation has four independent forces: inertia, viscous, pressure and body (or external) forces, which can form three independent pi-numbers. After adopting several assumptions, the \( Re \) number was identified as one of the key pi-numbers [13] in the form of force \( Re = f \ell' / \nu \), and in the form of power \( Re = P / \nu \ell' \), where \( P \) (power) = \( \rho \ell' \nu^3 \) and \( \nu \) = kinematic viscosity. This scaling law suggests that both the inertia and viscous force of liquid metal dominates the phenomenon.

6. VIBRATION CASTING

The University of Kentucky formed an interdisciplinary research team consisting of researchers with expertise in materials, manufacturing, thermal-fluids and scale modeling to study vibration casting of aluminum alloy [14]. They decided to design a laboratory scale model apparatus and apply mechanical vibration to an Al–Si eutectic (Al–12.5% Si) alloy at a frequency of 100 Hz and variable amplitudes during solidification. They found a profound effect on the microstructure and mechanical properties of castings: the silicon morphology was strongly influenced by the level of vibration amplitude reducing the lamellar spacing and changing the silicon morphology to become more fibrous. However, there was a certain critical amplitude value beyond which the silicon morphology coarsened. The maximum elongation is more influenced by vibration than the tensile strength for the range of conditions tested. The inertia energy of vibration was transferred to the alloy which influenced the morphology of the Al–Si eutectic (Al–12.5% Si) alloy. Four different cases may be possible when the major governing forces are: (1) \( F_i \) and \( F_o \), (2) \( F_i \), \( F_p \), and \( F_o \), (3) \( F_i \), \( F_\mu \), and \( F_o \), and (4) \( F_i \),
Here $F_i$ = the inertial force of liquid mold, $F_{\mu}$ = the viscous force of the liquid mold, $F_g$ = gravity force acting on the liquid mold, $F_s$ = the material strength of the casted alloy. A careful interpretation of each case and justification of the use of the above physical laws are obviously required, but that is not possible for this short article. Therefore, only possible scaling laws to be developed for each case are suggested for the benefit of future studies. The simplest case (1) will lead to $Ca = \frac{F_i}{\rho u^2} = \frac{F_s}{\sigma}$, where $\rho$ = material density, $u$ = representative velocity, $\sigma$ = material stress constant. By using the relationship $u = \ell / t = \ell f$, $\sigma = (\ell f)^2$. Using the characteristic length, $\ell$, as the amplitude of vibration, and $f$ as frequency of vibration, $\sigma \sim (\text{the amplitude})^2$ will be obtained. For case (2), an additional pi-number, $Fr$, will be formed. The $Fr$ number imposes $u \sim \ell^{1/2}$, and when it is combined with the $Ca$ number, $\sigma \sim (\text{the amplitude})$ will be obtained. For case (3), a combination of the $Ca$ and the $Re$ number provide $\sigma \sim (\text{the amplitude})^{-2}$. For case (4), the achievable scaling laws require relaxation. The scaling law prediction that has the closest agreement with their experimental results is case (2) with a few factors off.

7. REDUCTION OF NO\textsubscript{x} BY ACOUSTIC WAVE

Application of acoustic wave to reduce pollutants (NO\textsubscript{x} and CO) from hydrocarbon fuel burning power generation systems was experimentally investigated by M. I. Ali et al. [15]. They designed a laboratory scale premixed burner system, as shown in Fig. 5, equipped with a vertically oriented combustion tube and a horizontally oriented acoustic excitation tube to study the effects of the following three parameters on NO\textsubscript{x} and CO concentration: (1) frequency of the acoustic wave, (2) methane reburn with post air injection, and (3) the amount of injected post-flame air. Their results showed that 95% NO\textsubscript{x} concentration was achieved by a combination of acoustic wave excitation with reburn and post air-injection compared with the baseline methane reburn technology capable of reducing NO\textsubscript{x} by 65%. The CO concentration was also shown to have a significant reduction with an application of 50 Hz acoustic excitation.

This technique may be scaled up to large power generation systems by developing scaling laws, however, the detail mechanism of NO\textsubscript{x} and CO reduction by acoustic wave is still unknown. Here the first attempt was made based on the following interpretation: the propane-air premixed flame generates heat by chemical reaction and high temperature combustion by-products are released from the flame tip and move upwardly flowing through a vertically oriented combustion tube by buoyancy force. This flow generated by a relatively small premixed flame will meet with acoustic wave at a certain height, where the acoustic wave moves horizontal to the upward combustion by-product flow. At this interaction point, the acoustic wave driven by the pressure force will be governed by the inertia and viscous force, while the upward combustion by-product flow will be governed by the inertia and buoyancy force. Based on the above assumptions, the $Re$ number for the horizontal direction and the $Fr$ number for the vertical direction may be separately applied. The suggested scaling laws wait for future scale model experiments for testing the validity.

8. CONCLUSION

In the age of high speed computation, scale modeling plays an important role by providing a validation tool to numerical modeling and helping researchers to promote their understanding of the control mechanism of nature and enhancing their imagination to capture the essence of nature’s mechanism. Five different examples of scale modeling applied to vibro-acoustic problems are briefly introduced and their scaling laws are discussed based on the law approach. There are newly suggested scaling laws in this article, and the author of this article wishes some researchers to conduct scale modeling research to validate these scaling laws.
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