Beamsplitting attack to the revised KKKP protocol and a possible solution

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We show that the revised KKKP protocol proposed by Kye and Kim [Phys. Rev. Lett. 95, 040501(2005)] is still insecure with coherent states by a type of beamsplitting attack. We then further revise the KKKP protocol so that it is secure under such type of beamsplitting attack. The revised scheme can be used for not-so-weak coherent state quantum key distribution.

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Quantum key distribution (QKD) can help two legal parties (Alice and Bob) to accomplish unconditionally secure communications which is an impossible task by any classical method [1]. The security of QKD is guaranteed by known principles of quantum mechanics [2, 3, 4] rather than the assumed computational complexity in classical secure communication. It is one of the most promising applications of quantum information science and the gap between theory and practice has become narrower.

Single-photon QKD was the first being theoretically investigated [5], experimentally realized [6] and proved to be secure [7]. However, the imperfect single-photon source leads the QKD protocol vulnerable to the photon number splitting (PNS) attack [8] which limits its application in practice. Therefore, several revised protocols have been proposed using weak-coherent states [3, 8, 10, 11]. In particular, the classical process of basis reconciliation may be an important source of information leakage to an eavesdropper, known as Eve [9]. Very recently Kye et al. [12] proposed a blind polarization, where the sender and the receiver share key information by exchanging qubits with arbitrary polarization angles without basis reconciliation. (There is another protocol where basis reconciliation is not necessary [13, 14].) The so-called KKKP protocol, named after the inventors’ initials [12], was thought to be secure even when a key is embedded in a not-so-weak coherent-state pulse because only randomly polarized photons are exchanged as another important advantage. The KKKP protocol has generated considerable interests [14, 15].

Despite its advantages, the KKKP has been found vulnerable to impersonation attacks, due to the fact that a key has to travel three times between the legitimate users. As shown in [14], the original KKKP protocol [12] is insecure even if because of a mathematical loophole even for single-photon keys. The loophole has then been immediately filled up [12] and the protocol has been made secure for single-photon keys. However, the strength of the KKKP protocol lies in the possibility to use coherent-state keys of reasonable intensity. If Alice is limited to a single photon source, the protocol seems to be rather inefficient compared with the prior art standard protocols. In this Letter, we show the revised KKKP protocol is still insecure for coherent-state keys, and furthermore, we give a solution, which is robust against impersonation attacks.

Consider the revised KKKP protocol [16]. (Here we slightly simplify the protocol to perform (π/2) rotations, rather than (π/4) rotations.) : K1. Alice sends Bob two coherent pulses with polarization angles θ0, θ1. (For the ease of presentation, we shall use subscripts “0, 1” rather than “1,2”.) K2. After reception, Bob applies random shuffling \( U_\phi[\phi + s_0(\pi/2)] \otimes U_\phi[\phi + s_1(\pi/2)] \) where \( s_i \) is randomly chosen from \{0, 1\} and \( (i = 0, 1) \). Bob sends the two pulses back to Alice. K3. Upon reception, Alice applies \( U_\phi[-\theta_0 + k(\pi/2)] \otimes U_\phi[-\theta_1 + (k \oplus 1)(\pi/2)] \) and \( k \in \{0, 1\} \) is the key bit. Alice blocks one pulse and sends the other one back to Bob. The polarization angle of the surviving pulse is \( \phi + (s_b \oplus b \oplus \hat{s}_b) \pi/2 \), and \( b \) is the blocking factor for Alice to send out pulse 0 (b=0) or pulse 1 (b=1). K4. Bob applies \( U_\phi(-\phi) \) to the only pulse he receives and measures the polarization angle. The measurement outcome reveals the value \( l = s_b \oplus k \oplus b \). Alice announces \( b \) and Bob uses \( k = l \oplus b \oplus s_b \) as the secret bit shared with Alice.

We show our attack by two arguments. A: If Eve knows the parity value, \( s_0 \oplus s_1 \), she can attack the protocol successfully. B: There is indeed a way for Eve to know the value \( s_0 \oplus s_1 \) without causing any disturbance to Alice or Bob’s detection. The B can be expected from the following fact: When \( s_0 \oplus s_1 = 0 \), the protocol reduces to the single pulse KKKP protocol [12] actually, because Bob shuffles both qubits by the same degree. When \( s_0 \oplus s_1 = 1 \), the protocol also reduces to the (double pulse) KKKP protocol [12], because Bob shuffles the qubits always differently. In both cases, the protocol can be successfully attacked by Eve. The former and the latter cases are dealt with in Refs. [12] and [14].
respectively.

Now we show A with an impersonation attack where to Alice, Eve pretends herself to be Bob, while to Bob, Eve pretends herself to be Alice in their quantum communication channel. We assume that Eve does not attack the classical communication between Alice and Bob.

**Protocol A: A1.** After K1, Eve intercepts both pulses from Alice and stores them in set E1. Meanwhile, Eve prepares two coherent pulses by herself with polarization angles $\theta_0, \theta_1$. A2. After K2, Eve intercepts both pulses from Bob. After the treatment in the subprotocol As, Eve stores the remaining pulses intercepted from Bob in set E2. Suppose with subprotocol As, Eve now knows the value $s_0 \oplus s_1$. A3. If $s_0 \oplus s_1 = 0$, Eve rotates the polarization of pulses in set E1 by $U_y[\phi' + s_0(\pi/2)] \otimes U_y[\phi' + s_0'(\pi/2)]$ and sends them to Alice. Note that $s_0'$ is set by Eve herself. After K3, Eve intercepts the only pulse from Alice and measures its polarization after a rotation of $U_y(-\phi')$. The outcome reveals $l' = k \oplus s_0'$. Since $s_0'$ is set by Eve herself, Eve knows the value $k$ already. Eve rotates pulse 0 in set E2 by $U_y(-\theta_0 + k(\pi/2))$ and sends it to Bob. As Alice has expected in the protocol, Bob will obtain $l = k \oplus s_0$ for sure after he measures the polarization. If $s_0 \oplus s_1 = 1$, Eve rotates the polarization of pulses in set E1 by $U_y[\phi' + s_0(\pi/2)] \otimes U_y[\phi' + (s_0' + 1)(\pi/2)]$ and sends them to Alice. After K3, Eve intercepts the only pulse from Alice and measures its polarization after a rotation of $U_y(-\phi')$. As one may easily see, the outcome is simply $l' = s_0' \oplus k \oplus b$. Since $s_0'$ is set by herself, Eve already knows the value $k \oplus b$. Eve rotates pulse 0 in set E2 by $U_y(-\theta_0 + (k \oplus b)(\pi/2))$ and sends it to Bob. Bob will for sure obtain $l = k \oplus b \oplus s_0$, as Alice has expected in the protocol. Since $b$ is announced later, Eve can obtain $k$ by $k = (k \oplus b) \oplus b$.

Protocol A shows that Eve may have full information about Bob’s result without causing any noise, if she knows the value $s_0 \oplus s_1$. We now show that she can indeed know this by subprotocol As: Consider Fig. 1. After K2, Bob sends two pulses $B_0$ and $B_1$ back to Alice. Eve intercepts them and splits each of them by a beamsplitter. To know the value of $s_0 \oplus s_1$ is simply to know whether the two pulses (defined as E0 and E1) have the same polarization angle. Eve first observes whether each pulse contains at least one photon by a quantum non-demolition measurement. If yes, she takes one photon from each pulse and then guides them to a 50:50 beamsplitter after rotating the polarization of pulse E0 and pulse E1 by $-\theta_0, -\theta_1$, respectively. As it has been well known 17, if the polarization of two input photons are the same, one output beam must be vacuum. Therefore, if she observes one photon on each output ports, she concludes that $s_0 \oplus s_1 = 1$, otherwise, the result is inconclusive. Before guiding pulses E0 and E1 into a 50:50 beamsplitter, Eve may choose to rotate the polarization of beam E0 by $\pi/2$. In such a case, if she observes one photon on each output ports, she concludes that $s_0 \oplus s_1 = 0$, otherwise, the result is inconclusive. Also, if initially E0 or E1 is vacuum, the result is inconclusive. If the result is inconclusive, Eve intercepts and discards everything from Alice and Bob in protocol A. If the result is conclusive, Eve continues her protocol A with the exact information of $s_0 \oplus s_1$. (After subprotocol B, the pulses E0 and E1 are consumed already and Eve shall store pulses 0 and 1 in set $E_2$ for protocol A.) This attack protocol may sound plausible. However, there is a limitation for it due to a high probability of inconclusive events. The total of 75\%, Eve gets inconclusive events, and only 25\% of the case can be used. The best strategy for Eve is to prepare each pulses of $B0$ and $B1$ in photon-number eigenstates containing at least 2 photons. In this case, only with the probability of 25\%, Bob will receive any photons. Alice and Bob know in advance the bit rate, depending on their channel efficiency and coherent pulse amplitude Alice initially prepares. In 12, when the channel efficiency is as low as $\eta^2 = 0.5$ and the initial amplitude of Alice’s pulse is 2.83, the successful detection of a photon by Bob is calculated as 63.5\%. Thus 25\% is too low to be unnoticed by Alice and Eve. If Eve prepares $B_0$ and $B_1$ in coherent states of amplitude $\gamma$, the problem will become more serious as the coherent states already have non-zero probability of there being no photons.

Here, we confirm that what we discussed above is the optimum discrimination of the two qubits in parallel or anti-parallel polarizations (See 18). Eve can have more setups to measure $s_0 \oplus s_1$. For example, Eve increases the photon numbers of $B_0$ and $B_1$ and take two set of single photons from the pulses returning from Bob. She then have two beam splitter setup: one setup to give a conclusive event from the two qubits being in the same polarization and the other from them being in the anti-parallel polarization. This can however take enormous resources from Eve. Of course, Eve can use spy pulses in a singlet state to find if Bob’s operations are same or orthogonal for the two pulses but we stick to only impersonation attack as this will suffice our needs as seen later.

Moreover, in the present form of the revised KKKP protocol, Bob does not randomly change the intensity or phase of the two pulses before he sends them back to Alice therefore the two pulses sent out from Bob have the same intensity and they are phase-locked. In such a case, we can improve it using coherent state attack and photon number discriminator detector is not to be necessary.

If two coherent fields of the same amplitude are inputs to a beam splitter, 19, we know that the coherent state will be driven into only one output port. Let us assume that Eve prepares with a relatively large intensity fields (amplitude is $\gamma$). Eve sends them to Bob as she impersonates Alice to Bob. Upon reception of the pulses, Bob adds rotations and shuffling factors. As is
shown in Fig. 2, receiving the pulses from Bob, Eve will first split the pulses by a beamsplitter 1 (BS1). The coherent fields reflected by BS1 (amplitude \(r\gamma\), where \(r\) is the reflectivity of BS1) will be used to measure \(s_0 \oplus s_1\) and the transmitted field (amplitude \(t\gamma\), where \(t\) is the transmissivity) will be stored and eventually sent back to Bob. The reflected pulses will be further split into two by a 50:50 beam splitter (BS2). The reflected pulses of BS2 will be used to measure if the two pulses are parallel to each other, while the transmitted pulses will be used to check if they are orthogonal. Here we only consider the orthogonal measure. The first and second are split by the dynamic reflector (DR) to give a time delay to the orthogonal measure. The first and second are split by BS3). If they are of the same polarization, both pulses will be detected in photon detector 1 (PD1). The amplitude of the coherent field to PD1 is \(\sqrt{2}r\gamma\) and if they are orthogonal, they do not interfere and the amplitude of coherent fields to PD1 and PD2 are both \(r\gamma\). If Eve detects any photon, then it is due to the fact that the two were orthogonal.

Here, we remind that for a non-zero amplitude coherent field, there is non-zero probability of no photon detected so there is also a chance of inconclusive events. The success probability is calculated using the Poissonian nature of coherent state. \(P_{success} = (1 - e^{-\gamma})^2\). Similarly, Eve measures if two pulses are parallel to each other by rotating the polarization angle of one of the pulses by \(\pi/2\). The success probability is again \(P_{success}\). Therefore, the total probability of success is also \(P_{success}\).

Eve then let the pulses go to Bob in the conclusive cases for which Bob still has a probability of receiving zero photons as a coherent state of \(\gamma\) will travel to Bob. The total probability of Bob receiving any photons then is \(P_B = P_{success}(1 - e^{-\gamma})^2\). Eve knows the bit rate so she can make \(P_B\) to make it equal to the bit rate by changing \(\gamma\). Eve then has implemented the subprotocol successfully without having her action noticed by Alice and Bob. The revised KKKP protocol, with the idea being interesting, in its present form does not offer the security as it has been supposed therefore a more careful investigation is needed for real applications. Now we offer a possible solution.

Consider the following protocol \(W\):

**W1:** same with step K1 of KKKP protocol. **W2:** Upon reception, Bob applies random shuffling \(U_y[\phi + s_0(\pi/2)] \otimes U_y[\phi + \delta(\pi/4) + s_1(\pi/2)]\) where \(s_i \in \{0, 1\}\) \((i \in 0, 1)\) are random numbers. Bob sends the two pulses to Alice. **W3:** Upon reception, Alice decides randomly to either use the received pulses for test or continue the protocol for sharing a secret bit with Bob. If she decides to continue, she applies \(U_y[-\theta_0 + k(\pi/2)] \otimes U_y[-\theta_1 + k(\pi/2)]\) and \(k \in \{0, 1\}\) is the key bit. Alice blocks one pulse and sends the other one back to Bob. (Alice also detects the blocked pulse to make sure that this is not in vacuum.) The polarization angle of the surviving pulse is \(\phi + (s_0 \oplus k \oplus b)(\pi/2) + b\delta(\pi/4)\), and \(b\) is the blocking factor for Alice to send out pulse 0 (\(b=0\)) or pulse 1 (\(b=1\)). Alternatively, Alice may decide to consume the pulses from Bob to inspect the presence of Eve who tries to detect one or nothing events and if it is less than desired she will have to abolish the keys assuming there having been Eve in the middle. **W4:** Bob applies \(U_y[-\phi - \Delta(\pi/4)]\) to the only pulse he receives and measures the polarization angle. Pulse 1 is randomly chosen from 0 or \(\delta\). If \(\Delta\) happens to be equal to \(b\delta\), the measurement outcome reveals the value \(l = k \oplus b \oplus s_b\). Alice announces \(b\) and Bob uses \(k = l \oplus b \oplus s_b\) as the secret bit shared with Alice if
\[ \Delta = b \delta. \] If \( \Delta \neq b \delta \), they discard the data. \textbf{W5.} They run the above program for \( N \) times. \textbf{W6.} Alice announces which times she has used the pulses for test in step \textsc{W3}. Bob announces the values of \( \delta \) and \( s_0, s_1 \) he has chosen for those times. Alice only needs to consider those testing results where her rotation angle \( \omega(\pi/4) \) happens to be equal to \( [2(s_0 + s_1) - \delta](\pi/4) \). If they are all single-clickings, she judges that there is no Eve, otherwise, she aborts the protocol. \textbf{W7.} Alice and Bob also compare some of their bits through classical communications to see whether they have indeed shared the same key.

In the protocol above, there are two different types of error tests. Step \textbf{W6} is to test whether there is Eve who tries to detect the blocking factor \( b \) by sending Alice two different pulses. \textbf{W7} is to test whether there is Eve who tries to know \( k \) or equivalently, \( s_0 \oplus s_1 \). Given protocol \( W \), Eve cannot use the subprotocol B to obtain \( s_0 \oplus s_1 \) without causing any noise. Say, no matter how she rotates beam \( E_0 \) or \( E_1 \), any value of \( s_0 \oplus s_1 \) can cause the event of two-fold clicking. Without the exact information about \( s_0 \oplus s_1 \) at that time, she cannot continue protocol \( A \) because she would cause noise in Bob’s key and this may be detected in step \textbf{W7}.

In summary, we have shown that the revised KKKP protocol in its present form fails under a type of beamsplitter attack. We also propose a protocol which is robust against any impersonation attack.

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