Fixed Point SU(3) Gauge Actions: Scaling Properties and Glueballs

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We present a new parametrization of a SU(3) fixed point (FP) gauge action using smeared (“fat”) gauge links. We report on the scaling behaviour of the FP action on coarse lattices by means of the static quark-antiquark potential, the hadronic scale $r_0$, the string tension $\sigma$ and the critical temperature $T_c$ of the deconfining phase transition. In addition, we investigate the low lying glueball masses where we observe no scaling violations within the statistical errors.

1. Introduction

It is well known that on rather coarse lattices with lattice spacings $a \geq 0.1$ fm the Wilson gauge action produces quite large cut-off effects. On the other hand, decreasing the lattice spacing below 0.1 fm becomes increasingly difficult for simulations including fermions. Therefore using a better, i.e. improved gauge action might well be important.

One possible way of constructing non-perturbatively improved lattice gauge theories is provided by the fixed point (FP) action approach\textsuperscript{1}. FP actions are classically perfect, i.e. they have no lattice artefacts on classical configurations. In particular, they possess exactly scale invariant instanton solutions. Together with fermion FP actions they form appealing formulations of non-perturbatively improved lattice QCD\textsuperscript{2}.

Here we present a new parametrization for FP gauge actions which we apply to SU(3). The new parametrization is much more flexible and has a richer structure than the loop parametrizations studied so far\textsuperscript{3,4} and therefore allows to better reproduce the classical properties of the theory. In order to check the scaling behaviour of the parametrized FP action we perform a series of simulations in which we determine the critical temperature of the deconfining phase transition, the static $\bar{q}q$ potential and glueball masses. This paper summarizes some of the results\textsuperscript{5}.

2. FP gauge actions

We consider SU(3) pure gauge theory in four dimensional Euclidean space-time on a periodic lattice. For asymptotically free theories the determination of the FP action reduces to a saddle point problem encoded in the FP equation\textsuperscript{1}

$$A^{FP}(V) = \min_{\{U\}} \{ A^{FP}(U) + T(U, V) \} ,$$

where $A^{FP}$ is the FP action, $T(U, V)$ is the blocking kernel of a renormalization group (RG) transformation and $U$ and $V$ are the gauge fields on the fine and coarse lattice, respectively. We use the RG transformation of ref.\textsuperscript{6} where the parameters of the transformation have been optimized for a short interaction range of the FP action and improved rotational invariance. Eq. (1) represents an implicit equation for the FP action which, in principle, contains infinitely many couplings. In order to use it in MC simulations one has to find an appropriate parametrization.

3. The new parametrization

The main novelty in the new parametrization of the FP gauge action is the use of smeared (“fat”) links which are used to build simple Wilson loop plaquettes\textsuperscript{7}. To build a plaquette in the $\mu\nu$-plane from smeared links we introduce asymmetrically smeared links: instead of summing up all

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staples contributing to a smeared link we are suppressing the staples lying in the $\mu\nu$-plane relative to those in the orthogonal planes $\mu\lambda$, $\lambda \neq \nu$. In this way we are able to effectively generate long and complicated loops which, however, are still very compact.

From these “smeared” loops $W_{\mu\nu}^\text{pl}$ and from the unsmeared standard Wilson loop $U_{\mu\nu}^\text{pl}$ we construct smeared and unsmeared plaquette variables $w_{\mu\nu}$ and $u_{\mu\nu}$, respectively,

$$
\begin{align*}
u_{\mu\nu} &= \text{Re Tr}(1 - U_{\mu\nu}^\text{pl}), \\
w_{\mu\nu} &= \text{Re Tr}(1 - W_{\mu\nu}^\text{pl}),
\end{align*}
$$

which in turn are used in a mixed polynomial ansatz for the FP action

$$
A[U] = \frac{1}{N_c} \sum_{x,\mu<\nu} \sum_{k,l} p_{kl} u_{\mu\nu}(x)^k w_{\mu\nu}(x)^l. 
$$

(2)

The linear parameters $p_{kl}$ and the non-linear ones entering the smearing are determined through eq. (1) which is used recursively connecting coarse configurations to smoother and smoother ones. On the smoothest configuration we use the analytically calculable quadratic approximation to the FP action as the starting point [5].

An additional novelty is the use of local information stored in the fine configurations minimizing the r.h.s. of eq. (1) for the fitting procedure. Obviously the minimizing configurations contain much more information than just the total value of the action. To explore this information we calculate the derivatives of the FP action with respect to the gauge links in a given colour direction at each site and, together with the action values, include them in the fitting procedure for determining the parameters.

The final parametrization obtained in this way contains five non-linear parameters entering the smearing and 14 linear ones $\{p_{kl}, 0 < k + l \leq 4\}$ describing the mixed polynomial in eq. (2).

4. Physical results

Using the parametrized FP action we perform a large number of simulations on coarse lattices with lattice spacings $a$ between 0.1 and 0.33 fm in order to explore the scaling behaviour of different physical quantities. In all simulations we generate independent gauge configurations using a mixture of Metropolis and overrelaxation updates. Wherever it is applicable we make use of smearing techniques and employ variational techniques to increase the overlap of wave functions with the ground state [5].

4.1. The critical temperature

We first measure the critical temperature of the deconfining phase transition using the Polyakov loop $L$ as an order parameter. To do so we determine the critical couplings on lattices with temporal extent $N_\tau = 2, 3$ and 4 and various spatial volumes from the location of the peak in the Polyakov loop susceptibility,

$$
\chi_L \equiv V_\sigma \left(\langle |L|^2 \rangle - \langle |L| \rangle^2 \right), \quad V_\sigma = N_c^3.
$$

The location of the peak can be determined very precisely by a spectral density reweighting which combines the data of many simulations at values of the gauge coupling near the critical value. In order to perform the thermodynamic limit we resort to the finite scaling behaviour linear in $V_\sigma$ as expected for a first order phase transition. The results from these extrapolations are $\beta_c^{N_\tau = 2} = 2.361(1)$, $\beta_c^{N_\tau = 3} = 2.680(2)$ and $\beta_c^{N_\tau = 4} = 2.927(4)$. Finally, the critical temperature is determined through $a(\beta_c) T_c = 1/N_\tau$.

4.2. The static $\bar{q}q$ potential

One way of examining the scaling of the parametrized FP action is by measuring the static $\bar{q}q$ potential $V(r)$ in terms of the hadronic scale $r_0$ and comparing the quantity $r_0 (V(r) - V(r_0))$ versus $r/r_0$ at several values of $\beta$ (see figure [5]). We would like to stress that on coarse lattices this a non-trivial scaling test. The quality of our measurements can be seen for example by comparing our fit (dashed line) of the form Coulomb plus linear term with the result (dotted line) from measurements with an anisotropic tadpole/tree level Symanzik improved action [5].

The hadronic scale used above is determined from the force $F(r)$ between two static quarks. We use

$$
r_0^2 V'(r_0) = r_0^2 F(r_0) = 1.65, 
$$

(3)

where $r_0 \approx 0.49 \text{ fm} = (395 \text{ MeV})^{-1}$. However, since eq. (3) needs an interpolation of the force in
between lattice sites, on coarse lattices this definition of the scale is plagued by ambiguities stemming from the discreteness of the lattice points.

Another physical quantity of interest is the string tension $\sigma$ which describes the asymptotic long range behaviour of the potential.

Note, that here we do not aim at testing the rotational invariance of the potential. For the RGT used here this has been done at finite temperature, however, using a different parametrization of the FP action [4].

4.3. Scaling behaviour

We are now in a position to investigate the scaling behaviour of the parametrized FP action by means of the RG invariant quantities $r_0 T_c, T_c/\sqrt{\sigma}$ and $r_0 \sqrt{\sigma}$. We find that these quantities scale even on lattices as coarse as $a \simeq 0.33$ fm [3]. As an example we display the results for $T_c/\sqrt{\sigma}$ in figure 2 together with results obtained with different other actions [9–14].

4.4. Glueball masses

In the pure gauge glueball spectrum the lowest-lying $0^{++}$ state shows particularly large cutoff effects when measured with the Wilson action. It therefore provides an excellent candidate for sizing the improvements achieved with the parametrized FP action. To do so we perform simulations at three different lattice spacings in the range $0.1 \text{ fm} \leq a \leq 0.18 \text{ fm}$ and spatial volumes between $(1.4 \text{ fm})^3$ and $(1.8 \text{ fm})^3$. The scale is set by $r_0$ as determined in the previous section. We measure all length eight loops on five smearing levels in order to construct a wave function with a large overlap to the ground state.

In figure 3 we compare the results for the lowest lying $0^{++}$ glueball mass measured with the Wilson action [15–17], the anisotropic tadpole/tree level Symanzik improved action [18,19] and the FP action. Performing the continuum limit and using $r_0 \simeq 0.49$ fm we obtain a mass of $m_{0^{++}} = 1627(83)$ MeV for the $0^{++}$ glueball. Finally, we also determine the $2^{++}$ glueball mass at a lattice spacing $a = 0.1$ fm and obtain $m_{2^{++}} = 2354(95)$ MeV [5].

Measuring glueball states on rather coarse lattices immediately calls for anisotropic gauge actions. The construction of such an action using the FP approach is under way [20].
5. Conclusions

The new parametrization presented here provides a method for approximating FP gauge actions which is very general and flexible but still simple. It is possible to describe the FP actions accurately enough so that their classical properties are preserved. However, it is necessary to check the range of validity of the parametrized FP action. In view of future applications of the action in connection with a parametrized FP Dirac operator [21] this is of crucial importance. In addition, one has to assure that no pathologies are introduced through the parametrization. For the quantities and lattice spacings investigated so far we find that this is indeed the case. In particular, we find that these physical quantities scale even on lattices as coarse as $a = 0.33$ fm.

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