Next-to-next-to-leading order QCD analysis of combined data for $xF_3$ structure function and higher-twist contribution.

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Abstract

The simultaneous QCD analysis of the $xF_3$ structure functions measured in deep-inelastic scattering by several collaborations is done up to 3-loop order of QCD. The $x$ dependence of the higher-twist contribution is evaluated and turns out to be in a qualitative agreement with the results of "old" CCFR data analysis and with renormalon approach predictions. The Gross–Llewellyn Smith sum rule and its higher-twist corrections are evaluated.

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1. The experimental data of the CCFR collaboration (we’ll call them “old”) obtained at Fermilab Tevatron \cite{1} for the $xF_3$ structure functions of deep-inelastic scattering of neutrinos and antineutrinos on an iron target provide an important means of accurate comparison of QCD with experiment. However, in view of revision of “old” data announced by CCFR collaboration \cite{2} the question arises: what can we say about the comparison of the QCD predictions on $Q^2$ dependence of the $xF_3(x, Q^2)$ structure function (SF) based on the data of neutrino DIS experiments different from those of CCFR?

In the present note, a combined fit of the experimental data of the CDHS \cite{3}, SCAT \cite{4}, BEBC-WA59 \cite{5}, BEBC–Gargamile \cite{6} and JINR-IHEP \cite{7} collaborations for the $xF_3$ structure functions is done in order to determine the $x$ dependence of the SF, higher twist (HT) contribution and the value of the scale parameter $\Lambda_{\overline{MS}}$.

2. We’ll use, for the QCD analysis, the Jacobi polynomial expansion method proposed in \cite{8}. It was developed in \cite{8}-\cite{14} and applied for the 3–loop order of pQCD to fit $F_2$ \cite{13} and $xF_3$ data \cite{14, 15}.

The $Q^2$ - evolution of the moments $M_3^{pQCD}(N, Q^2)$ is given by the well known perturbative QCD \cite{16, 17} formula:

$$M_3^{pQCD}(N, Q^2) = \left[\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)}\right]^{d_N} H_N(Q_0^2, Q^2) M_3^{pQCD}(N, Q_0^2), \quad N = 2, 3, ...$$  \hspace{1cm} (1)

$$d_N = \gamma^{(0), N/2} \beta_0, \cdots$$

The factor $H_N(Q_0^2, Q^2)$ contains all next– and the next–to–next–to–leading order QCD corrections \cite{14} and is constructed in accordance with \cite{14} based on theoretical results of \cite{19}.

The expression (1) provides an input for reconstruction of the SF by the Jacobi polynomial method. Following the method \cite{10, 11}, we can write the structure function $xF_3$ in the form:

$$xF_3^{pQCD}(x, Q^2) = x^\alpha(1 - x)^\beta \sum_{n=0}^{N_{max}} \Theta_n^\alpha(x) \sum_{j=0}^{n} c_j^{(n)}(\beta) M_3^{QCD}(j + 2, Q^2),$$  \hspace{1cm} (2)

where $\Theta_n^\alpha(x)$ is a set of Jacobi polynomials and $c_j^{(n)}(\alpha, \beta)$ are coefficients of the series of $\Theta_n^\alpha(x)$ in powers of $x$:

$$\Theta_n^\alpha(x) = \sum_{j=0}^{n} c_j^{(n)}(\beta)x^j.$$  \hspace{1cm} (3)

The unknown coefficients $M_3(N, Q_0^2)$ in (1) could be parametrised as Mellin moments of some function:

$$M_3^{QCD}(N, Q_0^2) = \int_0^1 dx x^{N-2} A x^b(1 - x)^c(1 + \gamma x), \quad N = 2, 3, ...$$  \hspace{1cm} (4)

1For reviews and references on higher order QCD results see \cite{18}.
To extract the HT contribution, the nonsinglet SF is parameterized as follows:

\[ xF_3(x, Q^2) = xF_3^{QCD}(x, Q^2) + h(x)/Q^2, \]  

where the \( Q^2 \) dependence of the first term in the r.h.s is determined by perturbative QCD. Constants \( h(x_i) \) (one per \( x \)-bin) parameterize the HT \( x \) dependence. We put \( x_i = 0.03, 0.05, 0.08, 0.15, 0.25, 0.35, 0.45, 0.50, 0.55, 0.65, 0.80 \) for \( i = 1, 2...11 \). The HT contribution for \( F_2 \) was determined in [20]. The values of constants \( h(x_i) \) as well as the parameters \( A, b, c, \gamma \) and scale parameter \( \Lambda \) are determined by fitting the combined set of data of 192 experimental points of \( xF_3 \) in a wide kinematic region: \( 0.5 \, GeV^2 \leq Q^2 \leq 196 \, GeV^2 \) and \( 0.03 \leq x \leq 0.80 \) and \( Q_0^2 = 10 \, GeV^2 \). We have put the number of flavors to equal 4. In accordance with the result of [3] concerning the disagreement of their data with perturbative QCD at small \( x \), a cut \( x \geq 0.35 \) was used for CDHS data. The TMC are taken into account to the order \( o(M_{nucl}^4/Q^4) \).

The nuclear effect of the relativistic Fermi motion is estimated from below by the ratio \( R_{D/N} = F_D^{3}/F_N^{3} \) obtained in the covariant approach in light-cone variables [21].

|   | LO         | NLO        | NNLO       |
|---|------------|------------|------------|
| \( \chi^2_{d.f.} \) | 312/176   | 316/176   | 312/176   |
| A  | 6.68 ± 0.38 | 6.92 ± 1.43 | 7.11 ± 0.38 |
| b  | 0.760 ± 0.027 | 0.768 ± 0.072 | 0.778 ± 0.027 |
| c  | 4.03 ± 0.07  | 3.97 ± 0.17  | 3.82 ± 0.07  |
| \( \gamma \) | 0.675 ± 0.156 | 0.452 ± 0.624 | 0.189 ± 0.128 |
| \( \Lambda_{MS} \) [MeV] | 191 ± 46   | 159 ± 39   | 163 ± 31   |

Table I. Results of the 1-, 2- (\( N_{Max} = 10 \)) and 3- order (\( N_{Max} = 8 \)) QCD fit (with TMC) of the combined \( xF_3 \) SF data for \( f = 4, Q^2 > 0.5 GeV^2 \) with the corresponding statistical errors, normalization coefficients and values of the HT contribution \( h(x_i) \).

3. Results of the fit are presented in Table 1 and Figures 1-3. The theoretical prediction for \( h(x) \) from [22] is presented Figure 3.

The experimental values of \( xF_3 \) for each collaboration were multiplied by the normalization factors \( C^{coll} \) which were considered as free parameters. Their values are not
sensitive to the order of pQCD in use and was found to be equal to: \( C^{BEBC−W A59} = 0.92 \pm 0.03, \ C^{SCAT} = 1.06 \pm 0.03, \ C^{JINR−IHEP} = 1.02 \pm 0.05 \) and \( C^{BEBC−Garg.} = 0.97 \pm 0.04 \). The value of \( C^{CDHS} = 1 \) was fixed.

The obtained value of \( \Lambda_{\overline{MS}} \) is larger than that given by a similar analysis of CCFR data \[15\] \( \Lambda_{\overline{MS}} = 134 \pm 57 \ MeV \) but exhibits relatively small statistical errors. Results of the NLO and NNLO fit give the constant of strong interaction \( \alpha_S^{NLO}(M_Z^2) = 0.105 \pm 0.004 \) and \( \alpha_S^{NNLO}(M_Z^2) = 0.107 \pm 0.003 \) in agreement, within the errors, with usual DIS results \[24\]. Additional uncertainties to the value of \( \alpha_S(M_Z^2) \) due to extrapolation of the \( Q^2 \) dependence of the SF with four flavors (\( f=4 \)) in a wide kinematic interval \( 0.5 \ GeV^2 \leq Q^2 \leq 196 \ GeV^2 \) were found to be 0.001 in \[26\].

The value of the perturbative part of the GLS sum rule \[27\] at \( Q^2 = 10 \ GeV^2 \) estimated by using results of Table 1 is equal to \( \int_0^{1} \frac{x F_{PQCD}(x)}{x} \ dx = 2.60 \pm 0.23 \) in agreement with results of the ”old” CCFR data analysis \[23, 12\].

The shape of \( h(x) \) is in qualitative agreement with theoretical predictions of the dispersion method of the renormalon approach \[22\] (for reviews and references see \[24\]) and with results of the QCD analysis of ”old” CCFR data presented in \[15\]. They obviously differ from the precise values of \( h(x) \) for singlet \( F_2 \) presented in \[24\].

Based on the results of Table 1, one can estimate the value of the first moment of \( h(x) \) which contributes to the GLS sum rule \[27\]: \( h_1 = \int_0^{1} \frac{h(x)}{x} \ dx \). The obtained values: \( h_1^{LO} = -0.42 \pm 0.27 \), \( h_1^{NLO} = -0.29 \pm 0.28 \) and \( h_1^{NNLO} = -0.26 \pm 0.27 \) are in agreement with theoretical predictions of \[29\] \( h_1 = -0.29 \pm 0.14 \) and \[30\] \( h_1 = -0.47 \pm 0.04 \) as well as with the recent result of \[31\].

4. In conclusion it should be stressed that combined fit provides still a more precise determination of \( \Lambda_{\overline{MS}} \) and \( h(x_i) \) in comparison to the analysis of ”old” CCFR data \[15\], while the shape of the SF ruled by parameters A, b, c and \( \gamma \) is determined less accurate. The most discrepancy with the ”old” CCFR data analysis takes place for the HT contribution to the GLS sum rule and for the HT x dependence at large x.

For a more precise determination of the HT contribution to SF, the role of the nuclear effect should be clarified and a more realistic approximation for \( R_{Fe/N} = F_{3Fe} / F_{3N} \) is needed. We also did not take into account the threshold effects on \( Q^2 \) evolution of SF due to heavy quarks \[32\] which is necessary owing to a wide kinematic region of data under consideration and could be realized based on the mass-dependent MOM-scheme \[20\].

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\(^2\)Hereafter present the value of \( h(x) \) in \([GeV^2]\).
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Figure captions.

Fig.1. Higher-twist contributions from LO fit.

Fig.2. Higher-twist contributions from NLO fit.

Fig.3. Higher-twist contributions from NNLO fit and the theoretical prediction for $h(x)$ from [22].
Fig. 3.
