Betatron resonance electron acceleration and generation of relativistic electron beams using 200fs Ti:sapphire laser pulses

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Abstract
The generation of relativistic electron beams with quasi-thermal energy distribution (maximum energy: 30 MeV) by the interaction of a Ti:sapphire laser pulse of 200 fs duration, focussed to an intensity of \( \sim 2.1 \times 10^{18} \) W cm\(^{-2} \), with an underdense (electron density \( \sim 3.6 \times 10^{19} \)) to \( \sim 1.1 \times 10^{20} \) cm\(^{-3} \) He gas-jet plasma was observed. We observed two stages of self-focusing of the laser pulse in the plasma. Two groups of accelerated electrons associated with these two stages of laser channeling were also observed, and are attributed to the betatron resonance acceleration mechanism. This is supported by 2D PIC simulations performed using the EPOCH code and a detailed theoretical analysis. Further, the generation of quasi-monoenergetic electron beams with a peak energy of \( \sim 17–22 \) MeV was also observed, even with such long laser pulses, although with a low probability of occurrence.

Keywords: laser plasma electron acceleration, betatron electron acceleration, laser self focusing, laser channeling

(Some figures may appear in colour only in the online journal)

Introduction
Laser driven plasma-based electron acceleration is seen as a potential candidate for the development of future compact accelerators. In this regard, the laser wakefield acceleration (LWFA) technique [1–5] has been successfully used for the generation of high quality quasi-monoenergetic (QM), near GeV, and more than GeV energy, electron beams using the ‘bubble regime’ [6–15] of wakefield acceleration. A laser pulse of short duration, such that \( L = c \tau \lesssim \lambda_p \) is used, where \( L \) is laser pulse length, \( c \) is speed of light, \( \tau \) is FWHM (full width at half maximum) pulse duration, and \( \lambda_p \) is plasma wavelength. However, the generation of QM electron beams of a few tens of MeV energies have also been demonstrated by the self-modulated laser wakefield acceleration (SM-LWFA) [16–23] regime at comparatively higher plasma densities (\( L \gg \lambda_p \)) by using laser pulses as long as 90 fs, and it was suggested that small laser pulselets that formed due to the self-modulation of the laser pulse could create bubble regime conditions [16]. Earlier reported experiments with comparatively longer laser pulses (a few hundreds of fs to ps) also generated relativistic electron beams through SM-LWFA but with a broad continuous spectrum [24–27]. In similar conditions of \( L \gg \lambda_p \), another possible mechanism could be betatron resonance acceleration where resonant transfer of energy from the laser field to the oscillating electrons in a laser channel occurs and is known as direct laser acceleration (DLA) and was also observed experimentally [28–30]. In some of the reports on SM-LWFA, the applicability of the DLA mechanism has been discussed [31–33].

In recent studies on the bubble/blowout regime of wakefield acceleration with ionization induced injection (using a mixed gas target of He and N\(_2\)) where the laser pulse length used was of the order of the bubble length, Shaw et al [34] showed that electrons gain energy (particularly higher
energy electrons) both from the wakefield and the DLA mechanism. The applicability of the DLA mechanism has also been reported in the case of a clustering Ar gas-jet target [35, 36]. In most of the experiments on DLA using underdense gas-jet targets, the electron beam spectrum was continuous [30, 34–36]; however, in some of the early experiments a quasi-thermal [28, 29] energy spectrum was also observed. The generation of non-Maxwellian electron beams accelerated by DLA in near critical plasmas, produced using few mm thick targets, have also been reported [37]. It may be emphasized here that the DLA mechanism of acceleration is particularly attractive from the point of view of betatron radiation generation [35, 38–41].

In this paper, we report an experimental study on electron acceleration along with the channeling of a ~200 fs duration Ti:sapphire laser pulse, focussed to an intensity of ~2.1 × 10^{18} W cm^{-2}, in an underdense He plasma produced from a 1.2 mm long gas-jet target (plasma density ~3.6 × 10^{19} to ~1.1 × 10^{20} cm^{-3}). The generation of two prominent groups of electron beams was observed, with their peak energies in the range of 8–10 MeV and 15–25 MeV, (divergence ~10–20 mrad), with an overall quasi-thermal electron spectrum. Interestingly, the generation of single QM electron beams (peak energy ~17–22 MeV, energy spread ~20%) was also observed, but with comparatively poor reproducibility. The two groups of accelerated electrons could be associated with the observed two stages of laser self-focusing and channeling. 2D PIC simulations using EPOCH code [42], along with detailed theoretical analysis, suggest the applicability of betatron resonance acceleration i.e. the DLA mechanism. This is corroborated by earlier reported PIC simulations [28, 43, 44] by other groups, suggesting the applicability of DLA in similar experimental conditions, and also recent simulations [40, 45–47] where it has been clearly shown that in the case of L ~ l_{\gamma}, the dominant acceleration mechanism is DLA, not wakefield.

As stated above, studies on betatron resonance electron acceleration in laser plasma interaction is desirable and of interest for betatron radiation generation associated with this particular regime of electron acceleration [35, 38, 40]. Further, the characterization of electron beams generated is also important, as it may be pointed out here that the generation of QM electron beams using such long (200 fs) laser pulses in underdense gas-jet plasma, where the dominant acceleration mechanism could be betatron resonance acceleration, have not been reported earlier.

Experimental results

Long laser pulses of ~200 fs duration were focussed on a helium gas-jet target of 1.2 mm length and the generation of relativistic electron beams was studied by varying the plasma density in the range of ~3.6 × 10^{19} cm^{-3}–1.1 × 10^{20} cm^{-3} with a peak intensity of ~2.1 × 10^{18} W cm^{-2}. The spatial profile of electron beams recorded for various plasma densities are shown in figure 1. The generation of relativistic electron beams in the forward direction were observed above a threshold plasma density of ~3.6−4 × 10^{19} cm^{-3} (figure 1(a)). For densities of up to ~7 × 10^{19} cm^{-3}, similar electron beams with full angle divergence of ~40 mrad (FWHM) were observed (figures 1(b)–(e)). At higher density of 9 × 10^{19} cm^{-3}, the overall beam divergence increased (~120 mrad), however, an intense central spot was still seen (figure 1(f)).

The energy of the electron beam was measured by dispersing the electrons using a magnetic spectrograph. The typical spectra recorded in the density range used are shown in figure 2. The generation of two groups of electrons, one with peak energy in the range of 8–10 MeV, and a second group of electrons with a comparatively higher energy of ~15–25 MeV, was observed. The maximum electron energy extended up to ~30 MeV (at 10% of the peak energy amplitude value). Figures 2(i) and (ii) show the first group of lower energy electrons. As can be seen, with the increase in plasma density, the energy of this group of electrons increased. Along with lower energy electrons a second group of electrons with higher energy was also observed. A series of ten raw images showing two groups of electron beams recorded are shown in figure 2(iii) and the corresponding spectra are shown in figure 2(iv). The generation of single and both groups of electrons together were observed with an almost equal probability, ~45% of the shots during the experiment. As shown in figures 2(iii) and (iv), in many shots quasi-thermal electron energy distribution was also observed. Using the DRZ high calibration data available in the literature [48], the total charge within the 1/e^2 area of the undispersed electron beams (figure 1) was in the range of 10–50 pC for density in the
range of $4 \times 10^{19} \text{cm}^{-3}$ to $7 \times 10^{19} \text{cm}^{-3}$, which increased to >450 pC at a higher density of $9 \times 10^{19} \text{cm}^{-3}$. The total charge contained within the dispersed electron beams showing single electron spectra (figures 2(i) and (ii)) above 7 MeV, increases from $\sim 0.7 \text{pC}$ at a density of $4 \times 10^{19} \text{cm}^{-3}$ to $\sim 12 \text{pC}$ at a density of $1 \times 10^{20} \text{cm}^{-3}$. In the case of dual electron beams (figures 2(iii) and (iv)), the total charge above 7 MeV varied in the range of $\sim 2$–8.5 pC, and shot-to-shot fluctuation in the fraction of the higher energy (15–25 MeV) component was in the range of $\sim 15$%–65% of the total charge.

Interestingly, in almost $\sim 10$% of the shots, single, comparatively with lower divergence $\sim 5$–10 mrad, QM electron beam (peak energy $\sim 17$–22 MeV, energy spread $\sim 20$%) was observed as shown in figures 3(i) and (ii).

Laser propagation inside plasma was studied using 90° Thomson side scattering imaging. Figure 4 shows the typical laser channels recorded for the range of plasma density used at a fixed laser power of 7.5 TW. For the density range used, $P_{\text{GW}}/P_{c}$ was in the range of 9–28, where $P_{c}$ (GW) =17.4$(n_{e}/n_{c})$ is the critical power for self-focusing [49], $n_{e}$ is the critical density and $n_{c}$ is the electron density. Relativistic self-focusing and guiding of the laser pulse was observed. For plasma density of $\sim 3.6$–4$ \times 10^{19} \text{cm}^{-3}$, the channel length was $\sim 255 \mu \text{m}$ ($\sim 1.4 Z_{0}$) (figure 4(a)). With an increase in the plasma density, the channel length increased and for the range of plasma density used, the maximum propagation length was in the range of $\sim 450$–550 $\mu \text{m}$ ($\sim 2.5$–3 $Z_{0}$) (figures 4(b) to (e)). Two stages of laser focusing and channeling were observed: after initial propagation the laser pulse slightly defocuses in the middle (bulging of the channel) for

Figure 1. Typical electron beam profiles for various densities (a) $3.6 \times 10^{19} \text{cm}^{-3}$, divergence: $\sim 40$ mrad, (b) $4 \times 10^{19} \text{cm}^{-3}$, divergence: $\sim 39$ mrad, (c) $5 \times 10^{19} \text{cm}^{-3}$, divergence: $\sim 30$ mrad, (d) $6 \times 10^{19} \text{cm}^{-3}$, divergence: $\sim 32$ mrad, (e) $7 \times 10^{19} \text{cm}^{-3}$, divergence: $\sim 33$ mrad and (f) $9 \times 10^{19} \text{cm}^{-3}$, divergence: $\sim 137$ mrad.

Figure 2. (i) Raw images of quasi-thermal electron beams with single peak electron energy at different densities of (a) $4 \times 10^{19} \text{cm}^{-3}$, (b) $6 \times 10^{19} \text{cm}^{-3}$, (c) $7 \times 10^{19} \text{cm}^{-3}$, (d) $1 \times 10^{20} \text{cm}^{-3}$, (ii) Quasi-thermal spectra of the corresponding electron beams, (iii) Raw images of two groups of electrons at different densities, (a) $5 \times 10^{19} \text{cm}^{-3}$, (b)–(f) $6 \times 10^{19} \text{cm}^{-3}$, (g)–(i) $8 \times 10^{19} \text{cm}^{-3}$, (j) $1.1 \times 10^{20} \text{cm}^{-3}$, (iv) Spectra of the corresponding electron beams.
a small distance, followed by a further self-focusing in the later stage of propagation. The extents of the second stage of channeling varied from shot to shot, and have been marked in the images of figure 4. Further, the average channel radius (FWHM) increased from the initial $\sim 4$–$5 \mu m$, to $\sim 6$–$10 \mu m$ in the middle, and then converged to $\sim 4 \mu m$ at the end.

During the experiment, in a few percent of the shots a slight bending in the later part of the laser channel was observed and, associated with it, two electron beams were observed on the phosphor screen (without a magnet in the path) as shown in figure 5. Otherwise, for the straight channels (figure 4), single intense electron beams with a slight background halo, as shown in figure 1, were observed.

Discussions

One of the possible mechanisms of electron acceleration in laser channels for $L > \lambda_p$ is SM-LWFA. Considering which, for the highest density of $1.1 \times 10^{20} \text{cm}^{-3}$, the dephasing...
limited maximum energy of the electrons [7] is expected to be only \( \sim 5\) MeV. In contrast, in the present experiment the maximum electron energy is up to \( \sim 30\) MeV, along with the observed increase in electron energy with density. Further, earlier simulations carried out for similar experimental conditions have shown that the wakefields are effective only at the foot of the laser pulse, signifying a dominant electron acceleration mechanism of betatron resonance acceleration [28, 43, 44]. For a laser pulse duration of 150 fs, intensity \( \sim 10^{19}\) W cm\(^{-2}\), and plasma density of \( 1.7 \times 10^{20}\) cm\(^{-3}\), in a 2D PIC simulation using VLPL code, Pukhov et al [43] did not find any regular plasma wave and thus the only possible mechanism for electron energy gain was identified as betatron resonance acceleration. In another 3D simulation [44] for a comparatively longer laser pulse of 460 fs, and a lower density of \( 4 \times 10^{19}\) cm\(^{-3}\), electron acceleration was studied for different values of \( P/P_c \). The generation of energetic electrons was observed for \( P > P_c \). For large values of \( P/P_c \) (= 6), it was observed that only the head of the laser pulse is modulated at the plasma period and only a few regular plasma periods exist where the SM-LWFA mechanism could be applicable. However, in the remaining unmodulated region of the laser pulse no regular wakefield was observed, and electron acceleration in this region was attributed to the DLA. Further, Gahn et al [28] also performed 3D VLPL PIC simulation to explain their experimental observations on betatron resonance acceleration. For a laser pulse of 200 fs duration, plasma density of \( 2 \times 10^{20}\) cm\(^{-3}\) and \( P/P_c \sim 8 \), they also showed that regular wakefield exists only at the foot of the laser pulse, and the dominant mechanism of electron acceleration was identified as direct laser acceleration. Further, recently, through extensive 2D PIC simulations carried out using OSIRIS code [40, 45], and also through VLPL code [46, 47], it has been clearly shown that in the case of \( L \gg \lambda_p \) DLA contributes a significant portion of the total energy gained by the electrons. In particular, Lemos et al [40] found that for long laser pulses of 700 fs duration and a plasma density of \( \sim 1 \times 10^{19}\) cm\(^{-3}\), nonlinear plasma waves are driven only at the front of the laser pulse. The contribution of DLA versus the wakefield mechanism to electron acceleration was found to also be dependent on the value of \( P/P_c \), and it was observed that for high values of \( P/P_c \) (>6) the dominant acceleration mechanism is DLA. In earlier simulations [43] also it was suggested that DLA is applicable for large values of \( P/P_c \) (>6). A high value of \( P/P_c \) is favorable for DLA as it leads to strong relativistic self-focusing of the laser pulse. The front of the laser pulse itself leads to almost complete blowout of the electrons from the laser axis, and the formation of a near hollow waveguide through which the trailing part of the laser pulse propagates and therefore does not drive a strong plasma wave i.e. wakefield [40]. In the present experiment too, we have observed electron acceleration at high values of \( P/P_c \) (>28), consistent with the above-reported simulations, and therefore support the applicability of DLA.

In the present experiment both laser pulse duration and plasma density were varied to obtain the optimum regime for relativistic electron beam generation. One regime was obtained for the laser pulse duration of \( \sim 200\) fs, and plasma density in the range of \( 3.6-4 \times 10^{19} - 1.1 \times 10^{20}\) cm\(^{-3}\), as described above. Another regime of electron acceleration was observed at the comparatively lower laser pulse duration of \( \sim 55\) fs, where the generation of QM electron beams of peak energy \( \sim 30\) MeV with almost 100% reproducibility (similar to our earlier observation [23] and various other reports [16–23]) was observed, and this could be attributed to the SM-LWFA mechanism. Figure 6 shows a typical QM electron beam spectrum recorded at the laser pulse duration of \( \sim 55\) fs during the same experimental run. The observed distinct change in the electron beam energy and spectrum at the lower pulse duration of \( \sim 55\) fs compared to the longer laser pulse duration of 200 fs during the same experimental campaign also suggests the applicability of different electron acceleration mechanisms in the two cases.

To understand the applicable acceleration mechanism and to support experimental observations we also performed a 2D PIC simulation using EPOCH code [42]. The 2D PIC simulation was performed, with a simulation box of 450 \( \mu \)m (x-direction) \( \times 60 \mu \)m (y-direction). A laser pulse of 200 fs duration, at a wavelength of 800 nm, and intensity of \( 2 \times 10^{18}\) W cm\(^{-2}\), propagating along the x-direction with polarization along the y-direction, enters the simulation box from the left and interacts with the plasma. The plasma length within the simulation box was modeled with a linear density ramp in the initial 100 \( \mu \)m of the simulation box with density varying from 0 to \( n_e = 7 \times 10^{19}\) cm\(^{-3}\), followed by a uniform plasma density of \( 7 \times 10^{19}\) cm\(^{-3}\) for the remaining 350 \( \mu \)m. A resolution of \( \lambda/20 \) was used both in the longitudinal (x-direction, laser propagation direction) and transverse direction (y-direction), where \( \lambda \) stands for laser wavelength, which corresponds to the total number of cells of \( 11,250 \times 1500 \). Number of macro particles per cell was 30. Each macro particle in the simulation box corresponds to \( 1.33 \times 10^6 \) electrons. The time step is chosen to satisfy the Courant condition and is further reduced by a factor of 0.8 to improve the accuracy of simulation. The simulation data was saved after every 20 fs. Further, field plots have not been averaged.
over grid cells, hence the inherent grid-cell variation is also visible and the results show the actual data values of each grid. After post-processing the generated data, the relevant parts were zoomed for better visualization by a factor of almost two in both dimensions.

Figures 7(a(i)–(iii)) shows the electron density profile at different time steps in the laser propagation direction which clearly shows channel formation associated with relativistic self-focusing. Immediately after entering the plasma, self-focusing of the laser pulse occurs and for the initial \( \sim 150 \, \mu m \) of plasma length, a channel radius of \( \sim 4–5 \, \mu m \) is observed (figure 7(a(i))). In the middle of the plasma channel, defocusing of the laser pulse occurs (channel radius \( \sim 8–10 \, \mu m \)) as shown in figure 7(a(ii)). Finally, a second stage of laser self-focusing occurs (channel radius \( \sim 4 \, \mu m \)) during the remaining part of the \( \sim 150 \, \mu m \) of laser propagation inside the plasma (figure 7(a(iii))). This is consistent with the experimentally recorded laser channels, as shown in figure 4. For the entire self-focused region of the laser propagation, the generation of laser-driven wakefield is not observed, except in the middle of the propagation, where slight defocusing and a modulation of the laser pulse is observed (figure 7(b)), that too within a small portion at the front part of the laser pulse (figure 7(c)), which also could not sustain in the later stage of propagation. Similar laser channel formation, using a long laser pulse of 700 fs and plasma density of \( 1 \times 10^{19} \, \text{cm}^{-3} \), has been reported in a recent 2D PIC simulation using OSIRIS code [40] (cf figure 2(e) of [40]), and also the generation of wakefield, was found only at the front part of the laser pulse (cf figure 2(c) of [40]), and hence DLA was concluded to be the dominant acceleration mechanism. Also, as is evident from figure 7(c), the maximum value of the normalized wakefield is \(<1\), i.e. wave breaking limit, and hence minimizes the possibility of self-injection of electrons and acceleration through wakefield. Hence, in the present case, the applicable mechanism could only be betatron resonance acceleration.

Figures 7(d(ii)–(iii)) shows \( P_x \) (momentum) versus \( x \) (propagation distance) at different time steps corresponding to channels shown in figures 7(a(i)–(iii)). In the first stage of channeling corresponding to \( \sim 150 \, \mu m \) of laser propagation, acceleration of electrons up to \( \sim 11–15 \, \text{MeV} \) is observed.
(figure 7(d)(ii)). With further propagation, the formation of multiple bunches was observed along with a slight increase in electron energy (figure 7(d)(ii)). Finally, associated with the second stage of laser focusing and channeling of \(\sim 150 \mu\text{m}\) (figure 7(d)(iii)), enhancement of electron energy up to \(\sim 30–37\text{ MeV}\) occurred, along with retaining the feature of formation of multiple electron bunches. This is consistent with the two groups of electrons observed experimentally. Two groups of electrons are associated with the above described two stages of laser channeling. The first group of electrons with a peak energy of \(\sim 8–10\text{ MeV}\) (figure 2) is accelerated in the first stage of the laser channel. Higher energy electrons with a peak energy of \(\sim 15–25\text{ MeV}\), and a maximum energy of \(\sim 30\text{ MeV}\), are generated due to acceleration in the later part. The observation of two spectrum patterns could be because of a not very stable second stage of laser self-focusing and channeling which fluctuates shot-to-shot, as is also evident in the laser channels shown in figure 4. As a result, higher energy components (peak energy \(\sim 15–25\text{ MeV}\)) are visible as clear peaks in some of the shots or as bulging in other shots (figures 2(iii) and (iv)).

Mangles et al [30] have also reported through simulation two stages of electron acceleration in the case of DLA, where enhancement in the electron energy was attributed to the acceleration in the magnetically constricted part of the laser channel in the later stage of the propagation. In the present experiment, observation of two electron beams associated with the bending of the laser channel in the later part of laser propagation, as shown in figure 5, also supports the two-stage acceleration scenario.

Further, we plot the variation of the normalized maximum transverse laser field \((eE_x/\text{mc}\omega)\) and the longitudinal wakefield \((eE_z/\text{mc}\omega_v)\) along the propagation distance (figure 8(a)). The plot has been drawn by spline fitting the data points corresponding to the maximum value of fields at 20 fs intervals. This shows that during the entire propagation length the laser field is higher compared to the wakefield. The separate contributions of the maximum transverse \((\gamma_x)\) and longitudinal \((\gamma_z)\) energy gain
\[
\left\{ \begin{array}{l}
\gamma_x = -\int_0^r \frac{2epE_z}{(mc)^2} \, dr, \\
\gamma_z = -\int_0^r \frac{2epE_x}{(mc)^2} \, dr
\end{array} \right.
\]
to the total energy gain \(\gamma^2 = 1 + \gamma_x + \gamma_y\) were studied and plotted along the propagation distance (figure 8(b)). This shows that \(\gamma_y\) from the laser field (DLA) is much greater than \(\gamma_x\) and also that the gain is significantly higher in the later stage of propagation. The variation of \(\gamma_y\) with total \(\gamma\) follows a straight line with slope close to 1, thereby emphasizing the dominant contribution of DLA over wakefield to the total energy gain of electrons inside the channels (figure 8(c)).

The increase in the longitudinal momentum with distance also shows electrons oscillating in the laser field gain transverse momentum by resonant transfer of energy from the field, which is converted to the longitudinal direction via the \(v \times B\) force.

Next, we performed a detailed theoretical analysis of betatron resonance acceleration in our experimental conditions to support our observations. In betatron resonance, the acceleration energy of the accelerated electrons \(\gamma\) with phase \(\phi\) is given by [50]:
\[
\frac{d\gamma}{d\phi} = -\frac{eA_0v_{\nu A} \cos \phi}{2mc^2\left(\omega - \frac{\omega_{t0}}{\sqrt{\gamma}} - kv_x\right)}.
\]

Integrating equation (1) we get \(F(\gamma) = -P \sin \phi + C_i\), where \(F(\gamma)\) is given by
\[
F(\gamma) = \gamma - \eta\sqrt{(1 - \alpha_0)\gamma^2 - 1} - \eta \cos^{-1} \frac{1}{\gamma\sqrt{1 - \alpha_0}}
-2\gamma^{1/2}\frac{\omega_{t0}}{\omega}.
\]

Here \(A_0\) is the electric field amplitude, \(v_{\nu A}\) and \(m\) are the on-axis velocity and the rest mass of the electrons, \(\omega\) is the laser frequency, \(\omega_{t0} = \omega_0\sqrt{\eta}\) represents the bounce frequency of the oscillation, \(k = (\omega/c)\eta\) is the wave number, \(\eta = (1 - \omega_p^2/\omega^2(1 + a_0^2/2)^{-1/2})^{1/2}\) is the ratio of the group velocity of the laser in plasma to that in vacuum, which also gives a measure of the electron density, \(v_i = c(1 - 1/\gamma^2 - v_{\nu A}^2/2c^2)^{1/2}\) is the axial velocity of the electron, \(P = a_0v_{\nu A}/2c\), \(C_i = F(\gamma_0) + P\sin\phi_0\) is the integration constant, \(\gamma_0\) is the initial energy of electrons, and \(\phi_0\) is the initial phase of the wave seen by the electron, and \(\alpha_0 = v_{\nu A}^2/2c^2\). Figure 9(a) shows the plot of \(F(\gamma)\) versus \(\gamma\) for \(\eta = 0.983\) (average value for the density range used in the experiment).

Next, we discuss trapping of the electron beams in the laser field by plotting separatrix \((\gamma\ \text{versus} \ \phi)\) (figure 9(b))
using equation [50]:

$$F(\gamma) - F_{\text{min}} = P(1 - \sin \phi)$$  \hspace{1cm} (3)$$

where $F_{\text{min}}$ is the minimum value of $F$ obtained from figure 9(a). The trapping of electrons in the laser field and the subsequent energy gain depends entirely on the phase space evolution of the electrons with respect to the laser field. The largest value of the right hand side of equation (3) is equal to $2P$. Therefore, a horizontal line was drawn in figure 9(a) at a height of $2P$ from $F_{\text{min}}$, which cuts the $F(\gamma)$ curve at two points, $R$ and $S$, which corresponds to the bottom and top of the separatrix (figure 9(b)), respectively. The cross points of the separatrix occur at $\gamma = \gamma_{\text{opt}}$. Figures 9(a) and (b) show that the maximum energy acquired by a trapped electron is $\sim 34$ MeV ($\gamma(S) \sim 67.5$), close to the energy observed in the present experiment. Figure 9(b) shows that the electrons with $\gamma = \gamma_{\text{opt}}$ are in phase and trapped with the laser field and hence gain energy from the laser field with propagation up to the dephasing limit, whereas electrons with $\gamma > \gamma_{\text{opt}}$ and $\gamma < \gamma_{\text{opt}}$ are untrapped and hence only oscillate with the laser field.

Further, we discuss the divergence of the electron beams, which, in the case of betatron resonance acceleration, is expected to be large. Gahn et al [28] observed electron beams with divergence of $\sim 15^\circ$ FWHM. In the present experiment we have observed a comparatively lower divergence of $\sim 30-40$ mrad (1.7°-2.3°), which was also the case with Mangles et al [30], who reported divergence of $\sim 50$ mrad (2.8°). In this regard we estimate the transverse energy gain $\gamma_T$ of the electrons in the combined fields of the laser and the static fields of the laser channel, is given by [50]:

$$\gamma_T = 1/(1 - \eta^2)[-Q\eta + (1 + Q^2 - \eta^2)^{1/2}]$$  \hspace{1cm} (4)$$

where,

$$Q = \eta\left[1 + \frac{a_0^2}{2}e^{-s\gamma_0/\gamma_c^2\eta^2} \right] - \left(1 + \frac{a_0^2}{2}\right) + \frac{\omega_c \pi r_0}{\omega \eta \lambda} \left(e^{-s\gamma_0/\gamma_c^2\eta^2} - 1\right) - \frac{1}{\eta^2} \sqrt{\frac{\gamma_0}{\gamma_c^2\eta^2} - 1 + \gamma_0}$$.  \hspace{1cm} (5)$$

Here, $x_T$ is the turning point radius (the maximum transverse amplitude of oscillation), $r_0$ is laser focal spot, $\omega_c$ is the cyclotron frequency, and $\lambda$ is laser wavelength. In figure 9(c) we have plotted $\gamma_T$ versus $x_T/r_0$ showing significant energy gain even for smaller values of $x_T/r_0$ i.e. from lower electron oscillation amplitude, in our case, compared to the other
previous reports [28, 30, 50]. This suggests that in the present experimental conditions, significant energy gain occurs from regions very close to the laser channel axis, where the laser intensity also remains constant to its peak value. This is because of the comparatively larger laser focal spot used in the present experiment, and it is found to be favorable for generating comparatively lower divergence electron beams through the betatron resonance acceleration mechanism. Hence, for a given laser pulse duration and intensity, expression (4) could be used for the analytical estimation of suitable conditions i.e. plasma density (through the variable \( \eta \)) and laser focal spot size \((r_0)\), for estimating electron energy gain with oscillation amplitude and hence for generating lower divergence electron beams.

Finally, we discuss the quasi-thermal nature of the electron spectra (figure 2) recorded in the present experiment. Similar quasi-thermal/non-Maxwellian electron energy distribution was observed by Gahn et al [28] and Mangles et al [30] in the experiments with underdense plasma where the betatron resonance acceleration mechanism was applicable. Recently, Toncian et al [37] have also reported the generation of non-Maxwellian energy distribution of electrons through DLA. In this regard, we also estimate the dephasing length \( (L_d) \) for the betatron resonance acceleration mechanism. For this we solve a pair of coupled differential equations describing the energy gain and the phase change of the electrons with the propagation distance given by following equation:

\[
\frac{d\gamma}{d\xi} = - \frac{a_0 (\alpha_0 / 2)^\frac{1}{2}}{(1 - \alpha_0 - 1 / \gamma)^\frac{1}{2}} \cos \phi \quad (6)
\]

\[
\frac{d\phi}{d\xi} = \frac{1 - \omega \eta}{\omega / \gamma^\frac{1}{2}} - \eta. \quad (7)
\]

Here, \( \xi = z \omega / c \) and \( z \) is the propagation distance. \( L_d \) was estimated by plotting \( \gamma \) versus \( z \) with initial conditions at \( \xi = 0, \phi = \pi / 2 + \Delta \), where \( \Delta = 0.02, \) and \( \gamma_0 = \gamma_{\text{opt}}. \) For the range of plasma density used, \( L_d \) was in the range of 100–150 \( \mu \text{m} \) (as shown in figure 9(d) for \( \eta = 0.983, \) the average of the present density regime), and the corresponding maximum energy gain is \( \sim 40 \text{MeV}. \) The maximum channel length of \( \sim 500 \mu\text{m} \) was observed in the present experiment but, as discussed above, both experimentally and through PIC simulations, the acceleration that occurred in the two stages of laser channeling and observed channel lengths of \( \sim 100–150 \mu\text{m} \) in both the stages are comparable to \( L_d. \) This could probably lead to phase space rotation of the trapped electrons [7, 51] and hence bunching in the laser field and the appearance of quasi-thermal features in the electron beam spectra. It may be pointed out here that, in recent 2D PIC simulations [46, 47] it has been observed that even with DLA as the dominant acceleration mechanism along with wakefield acceleration, the energy spread of the electrons could be retained (even reduced), and in fact the energy distribution of the DLA electrons were found to be considerably lower than that of the non-DLA electrons.

Finally, in the experiment we have also observed the generation of QM electron beams (figure 3), although only in a few percent of the shots. Due to the poor reproducibility of such QM beams, at this stage, their generation could not be attributed convincingly to any specific mechanism (i.e. betatron resonance acceleration or wakefield acceleration) and further investigation is required.

**Conclusion**

In conclusion, we have observed relativistic electron beams of up to \( \sim 30 \text{MeV} \) energy and divergence \( \leq 40 \text{mrad} \), through the interaction of a 200 fs duration Ti:Sapphire laser pulse with underdense (plasma density \( \sim 3.6 \times 10^{19} \) to \( \sim 1.1 \times 10^{20} \text{cm}^{-3} \)) helium plasma. Two stages of acceleration, associated with the corresponding two stages of laser self-focusing and propagation, was observed. Various experimental observations and 2D PIC simulations suggest that the dominant acceleration mechanism of electrons is DLA (betatron resonance acceleration). Various observations are supported and explained using detailed theoretical analysis of the betatron resonance acceleration mechanism. Further, the generation of QM (\( \Delta E / E \sim 10\%-20\% \)) electron beams with a peak energy of \( \sim 17–22 \text{MeV} \) was also observed, even with such long laser pulses. The study could be useful in betatron radiation generation [38–40] in the laser plasma channel associated with direct laser accelerated electrons [41], and will be a subject of our future investigations.

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