Consensus ADMM-Based Distributed Simultaneous Imaging & Communication

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Abstract: This paper takes the first steps toward enabling wireless networks to perform both imaging and communication in a distributed manner. We propose Distributed Simultaneous Imaging and Symbol Detection (DSISD), a provably convergent distributed simultaneous imaging and communication scheme based on the alternating direction method of multipliers. We show that DSISD achieves similar imaging and communication performance as centralized schemes, with order-wise reduction in computational complexity. We evaluate the performance of DSISD via 2.4 GHz Wi-Fi simulations.

Keywords: ADMM, joint imaging and communication, MIMO, wireless imaging.

1. INTRODUCTION

In recent years, there has been growing interest in performing imaging with wireless networks usually tailored for communications (Vakalis et al., 2019; Guan et al., 2021). Wireless networks are ubiquitous and scalable by design, with powerful backend computing capabilities. This makes it possible to perform imaging alongside communication on a network-wide scale.

We study the problem of distributed simultaneous imaging and communication. We consider the system configuration shown in Fig. 1, with $N \geq 1$ multiple-input multiple-output (MIMO) base stations receiving transmissions from an uplink user. The base stations aim to: (i) perform uplink user symbol detection (communication) and (ii) estimate the reflective response of scatterers in the environment (imaging). We assume the base stations can cooperate over a static, undirected graph $G$, which models limited capacity backhaul links between the base stations. Due to the limited capacity of backhaul links, the base stations must perform the two operations (communication and imaging) in a distributed manner with local processing.

We propose Distributed Simultaneous Imaging and Symbol Detection (DSISD), a provably convergent distributed algorithm for simultaneous imaging and communication. DSISD is the distributed variant of decode-and-image, a centralized algorithm previously proposed in (Mehrotra and Sabharwal, 2022). We show that DSISD achieves similar imaging and communication performance as decode-and-image, with $O(N)$ reduction in computational complexity. We numerically evaluate the performance of DSISD via 2.4 GHz Wi-Fi simulations.

To the best of our knowledge, we are the first to consider the problem of distributed simultaneous imaging and communication. Prior work has largely focused on communication-only (no imaging) and imaging-only (a-priori known communication data) problems. For instance, distributed algorithms have been proposed for beamforming, symbol detection, and interference alignment (Kumar and Rajawat, 2016; Chen and Tao, 2017; Li et al., 2017), and for radar imaging (Zabolotsky and Mavrychev, 2018; Hu et al., 2021). In the absence of priors on the scatterers in the environment and uplink data symbols, imaging-only and communication-only problems are convex. Nevertheless, we show that simultaneous imaging and communication corresponds to a bi-convex problem. We show that DSISD converges to the stationary points of the bi-convex problem with sublinear rate and guarantees asymptotic consensus across all $N$ base stations.

This paper is organized as follows. In Section 2, we present the system model and problem formulation for Fig. 1. We present the proposed algorithm, DSISD, and associated theoretical results in Section 4. In Section 5, we show...
2. SYSTEM MODEL

Consider the system shown in Fig. 1 with \( N \geq 1 \) base stations receiving uplink transmissions. We adopt the system model proposed in (Mehrotra and Sahbarwal, 2022), where a similar configuration with a single base station was analyzed. In the sequel, we shall collectively refer to the set of scatterers that reflect uplink transmissions to the base stations as the **imaging scene**. We make the following assumptions about the system operation.

**Assumption 1:**
- Scatterers in the imaging scene remain static for \( T \) symbol durations, i.e., coherence interval.
- The uplink signaling is uni-polarized, with operating wavelength \( \lambda \).
- All \( N \) base stations are equipped with equal number of receive antennas, \( K_{\text{RX}} \), and operate synchronously over the same set of time-frequency resources.

Formally, let \( K_{\text{TX}} \) denote the number of transmit antennas at the uplink user, and let \( M \) denote the number of scatterers in the imaging scene. We denote the reflective coefficients of the \( k \)-th scatterer at the \( m \)-th antenna as \( P(k, m) \) and let \( X_T \) denote the matrix of all reflective coefficients across all antennas.

\[
Y_T(n) = P_{\text{RX}}^{(n)} \text{diag}(f) (P_{\text{TX}})_{\text{TX}}^\top X_T + N_T^{(n)}, \quad (1)
\]

where the two matrices \( P_{\text{TX}} \) and \( P_{\text{RX}}^{(n)} \) have sizes \( K_{\text{TX}} \times M \) and \( K_{\text{RX}} \times M \) respectively. The \((k, m)\)-th element of each path delay matrix is a scaled complex exponential that depends on the scattering wavelength \( \lambda \), and the locations of the \( k \)-th antenna and \( m \)-th scatterer in the scene,

\[
P(k, m) = \exp(-j \frac{2\pi}{\lambda} \| r_k - \bar{r}_m \|_2), \quad \forall P \in \{P_{\text{TX}}, P_{\text{RX}}^{(n)}\},
\]

where \( r_k \) and \( \bar{r}_m \) denote the position vectors of the \( k \)-th antenna and \( m \)-th scatterer in the imaging scene.

In the next section, we formulate the distributed simultaneous imaging and communication problem.

3. PROBLEM FORMULATION

Given the system model in (1), the base stations aim to collaboratively perform two functions:

1. **Imaging:** Estimate the reflectivity vector \( f \), and
2. **Communication:** Estimate the uplink symbols \( X_T \).

Our goal is to enable both functionalities with only local processing of the received symbols \( Y_T^{(n)} \) at every base station \( n \in \{1, \ldots, N\} \). Specifically, we make the following assumption on the prior knowledge at the base stations.

**Assumption 2:** The \( n \)-th base station has local knowledge of received symbols \( Y_T(n) \), path delay matrices \( \{P_{\text{TX}}, P_{\text{RX}}^{(n)}\} \) and discrete set \( X_{T,T} \) from which uplink data symbols \( X_{T,T} \) are drawn. Examples of \( X_T \) are \( X_T = \{\pm 1\}^{K_{\text{TX}} \times T} \) and \( X_T = \{(\pm 1)/\sqrt{2}\}^{K_{\text{TX}} \times T} \) for binary and quadrature phase-shift keying (BPSK and QPSK).

We assume the first \( T_1 \) symbols in \( X_T = [X_{T_1}, X_{T-T_1}] \) to be pilot symbols known to all \( N \) base stations. The base stations thus aim to collaboratively solve the problem:

\[
\min_{X_{T-T_1} \in \mathcal{X}_T, f, X_{T_1}} \frac{1}{2} \sum_{n=1}^{N} \|Y_T(n) - P_{\text{RX}}^{(n)} \text{diag}(f) (P_{\text{TX}})_{\text{TX}}^\top X_T\|^2_F. \quad (P1)
\]

Problem (P1) is an integer constrained bi-convex problem, since the objective function is non-convex and non-separable in \( f \) and \( X_{T-T_1} \), but convex in each variable separately. We simplify the problem by relaxing the integer constraints, i.e., solve the unconstrained problem.

To formulate the distributed version of (P1), we assume the base stations are connected over a backhaul network, modeled as a graph \( \mathcal{G} \).

**Assumption 3:** The graph \( \mathcal{G} = (\mathcal{V}, \mathcal{E}) \) is static and undirected, and every node knows who its neighbors are.

The distributed problem is formulated by defining local variables \( f^{(n)} \) and \( X^{(n)}_{T,T_1} \), with the constraint that local solutions of nodes connected by an edge in \( \mathcal{E} \) are identical,

\[
\min_{X^{(n)}_{T,T_1}, f^{(n)}} \frac{1}{2} \sum_{n=1}^{N} \|Y_T(n) - P_{\text{RX}}^{(n)} \text{diag}(f^{(n)}) (P_{\text{TX}})_{\text{TX}}^\top X^{(n)}_{T,T_1}\|^2_F, \quad \forall n \in \mathcal{V},
\]

\[
s.t. X^{(n)}_{T-T_1} = X^{(j)}_{T-T_1}, f^{(n)} = f^{(j)}, \forall (i, j) \in \mathcal{E}. \quad (P2)
\]

After solving (P2), nodes perform symbol detection via zero-forcing (Tse and Viswanath, 2005),

\[
\hat{X}^{(n)}_{T-T_1} = \Pi \hat{X}_{T-T_1} (X^{(n)}_{T-T_1}), \quad \forall n \in \mathcal{V},
\]

where \( \Pi \) denotes projection onto set \( \mathcal{S} \).

4. MAIN RESULTS

We begin by analyzing first-order optimality conditions in Section 4.1 to characterize the number of uplink pilots \( T_1 \) required to solve (P1). We subsequently present the proposed DSISD algorithm in Section 4.2, and derive associated convergence guarantees in Section 4.3.

4.1 Optimality Conditions for Problem (P1)

Let the objective function in (P1) be denoted by

\[
\mathcal{L}(f, X_{T-T_1}) = \frac{1}{2} \sum_{n=1}^{N} \|Y_T(n) - P_{\text{RX}}^{(n)} \text{diag}(f) (P_{\text{TX}})_{\text{TX}}^\top X_T\|^2_F.
\]

The KKT conditions for (P1) correspond to

\[
\nabla_f \mathcal{L}(f^*, X^*_T, X^*_T) = 0, \quad \nabla_{X_{T-T_1}} \mathcal{L}(f^*, X^*_T, X^*_T) = 0,
\]

where \( f^* \) and \( X^*_T, X^*_T \) are minimizers of (P1). Hence, the stationary points of (P1) are given by

\[
f^* = (H_{\text{img}}|_{X_{T-T_1} = X^*_T})^\dagger \text{vec}(Y_T),
\]

\[
X^*_T = (H_{\text{comm}}|_{f^*} \cdot Y_T - X^*_T),
\]

where \((\cdot)^\dagger\) denotes the pseudo-inverse and \(\text{vec}(\cdot)\) denotes the vectorization operator. The matrix \(H_{\text{img}}\) is given by

\[
H_{\text{img}} = (X_{T}^\top \otimes I_{NK_{\text{ox}}})(P_{\text{TX}} * P_{\text{RX}}).
\]
The bi-convexity of the objective function \( L \) naturally motivates using an alternating procedure to solve (P2). The proposed DSISD algorithm (Algorithm 1) is thus based on consensus ADMM (Boyd et al., 2011).

Let the concatenation of local solutions across all \( N \) nodes be denoted by variables \( \mathbf{x}^{(1)} = [(X_{1,k}^{(1)})^\top, \ldots, (X_{N,k}^{(1)})^\top] \) and \( \mathbf{f}^T = [(f^{(1)})^T, \ldots, (f^{(N)})^T] \). Moreover, let

\[
 g(\mathbf{X}, \mathbf{f}) = \frac{1}{N} \sum_{n=1}^{N} \|Y^{(n)} - P^{(n)}_R(\mathbf{f}) (P_T X^{(n)})^\top \|_F^2
\]

denote the objective function in (P2). Finally, \( L_G \in \mathbb{R}^{N \times N} \) denotes the Laplacian matrix for \( \mathcal{G} \).

With above defined notation, (P2) may be recast as

\[
 \min_{\mathbf{X}, \mathbf{f}} g(\mathbf{X}, \mathbf{f})
\]

s.t. \( (L_G \odot I_{K_T}) \mathbf{X} = 0 , \ (L_G \odot I_M) \mathbf{f} = 0 \). (P3)

At every iteration \( k \in \{ 0, 1, \cdots \} \), DSISD performs the following ADMM updates:

\[
 \mathbf{X}^{k+1} = \arg \min_{\mathbf{X}} A(\mathbf{X}, \mathbf{f}, \mathbf{D}_X, \mathbf{D}_f),
\]

\[
 \mathbf{X}^{k+1} = \arg \min_{\mathbf{X}} A(\mathbf{X}, \mathbf{f}, \mathbf{D}_X, \mathbf{D}_f),
\]

\[
 \mathbf{D}_f^{k+1} = \mathbf{D}_f^k + (L_G \odot I_M) \mathbf{f}^{k+1},
\]

\[
 \mathbf{D}_X^{k+1} = \mathbf{D}_X^k + (L_G \odot I_{K_T}) \mathbf{X}^{k+1},
\]

where \( A(\mathbf{X}, \mathbf{f}, \mathbf{D}_X, \mathbf{D}_f) \) denotes the augmented Lagrangian,

\[
 A(\mathbf{X}, \mathbf{f}, \mathbf{D}_X, \mathbf{D}_f) = g(\mathbf{X}, \mathbf{f}) + \rho_f \| (L_G \odot I_M) \mathbf{f} \|^2_F + \rho_X \| (L_G \odot I_{K_T}) \mathbf{X} \|^2_F.
\]

The variables \( \mathbf{D}_X \) and \( \mathbf{D}_f \) denote scaled dual variables, whereas \( \rho_X \) and \( \rho_f \) denote penalty parameters.

The updates in (A1) to (A4) can be performed locally at every base station since \( g(\mathbf{X}, \mathbf{f}) \) and \( A(\mathbf{X}, \mathbf{f}, \mathbf{D}_X, \mathbf{D}_f) \) are separable across pairs of local solutions \((f^{(n)}, X^{(n)})\). Below, we derive performance guarantees for DSISD.

\section{Performance Guarantees for DSISD}

In Theorem 1 and Corollary 1, we characterize convergence guarantees and algorithm complexity for DSISD.

\section{4.3 Performance Guarantees for DSISD}

In Theorem 1 and Corollary 1, we characterize convergence guarantees and algorithm complexity for DSISD.

\section{Theorem 1. Let Assumptions 1, 2, 3 and the conditions in Lemma 1 hold. Then, the updates of DSISD satisfy the following properties:

\textbf{Asymptotic consensus:} All \( N \) nodes in \( \mathcal{G} \) are in consensus asymptotically, i.e.,

\[
 \lim_{k \to \infty} \|(L_G \odot I_M) \mathbf{f}^{k+1} \|_2 = 0
\]

\[
 \lim_{k \to \infty} \|(L_G \odot I_{K_T}) \mathbf{X}^{k+1} \|_F = 0
\]

\textbf{Convergence to stationary points:} Limit points of iterates \((X^{k}, f^{k}, D^{k}_X, D^{k}_f)\) converge to a KKT point of (P3).

\textbf{Sublinear convergence rate:} Iterates \((X^{k}, f^{k}, D^{k}_X, D^{k}_f)\) converge to a KKT point of (P3) with rate \( O(1/K) \) in terms of the optimality gap.
Proof. Due to space constraints, we only provide a proof sketch with key steps and ideas for the complete proof.

**Result 1: Asymptotic Consensus**

We seek to prove

\[ \lim_{k \to \infty} \| D_{f}^{k+1} - D_{f}^{k} \|_2 = 0, \quad \lim_{k \to \infty} \| D_{X}^{k+1} - D_{X}^{k} \|_2 = 0. \]

We show the above result via two intermediate steps.

**Step 1:** We first derive upper bounds on successive dual update norms, \( \| D_{f}^{k+1} - D_{f}^{k} \|_2 \) and \( \| D_{X}^{k+1} - D_{X}^{k} \|_2 \). Consider the first-order optimality condition for (A2),

\[ \nabla \pi g(X_{k+1}, \mathcal{F}) + \rho X_{k+1} \mathcal{F}_{X} = 0. \]

Thus, an upper bound on \( \| D_{X}^{k+1} - D_{X}^{k} \|_2 \) is given by

\[ \| D_{X}^{k+1} - D_{X}^{k} \|_2 \leq \frac{\sqrt{2}}{\lambda_{2}(L_{f}^{+})} \| \nabla \pi g(X_{k}, \mathcal{F}) - \nabla \pi g(X_{k+1}, \mathcal{F}) \|_2, \]

where we have used the fact that \( \frac{1}{2} \| (L_{f} \otimes I_{KZ}) \mathcal{X} \|_2^2 = \lambda_{2}(L_{f}^{+}) \)-strongly convex.

On appropriate substitutions, the right hand side can be upper bounded in terms of successive primal update norms, \( \| X_{k+1} - X_{k} \|_2 \), \( \| \mathcal{F}_{k+1} - \mathcal{F}_{k} \|_2 \). Thus, we next show

\[ \lim_{k \to \infty} \| \mathcal{F}_{k+1} - \mathcal{F}_{k} \|_2 = 0 \] and \( \lim_{k \to \infty} \| X_{k+1} - X_{k} \|_2 = 0 \).

**Step 2:** We equivalently show \( \sum_{k=1}^{\infty} \| X_{k+1} - X_{k} \|_2 < \infty \) and

\[ \sum_{k=1}^{\infty} \| \mathcal{F}_{k+1} - \mathcal{F}_{k} \|_2 < \infty \] via two sub-results.

(i) First, we show that the augmented Lagrangian is upper bounded in terms of the primal update norms as

\[ A(X_{k+1}, \mathcal{F}_{k+1}, D_{f}^{k}, D_{X}^{k}, D_{f}^{k}) - A(X_{k}, \mathcal{F}_{k}, D_{X}^{k}, D_{f}^{k}) \leq -\lambda_{2}(L_{f}^{+}) \left( \rho X_{k+1} - X_{k} \right)^{2} + \rho_{f} \left( \mathcal{F}_{k+1} - \mathcal{F}_{k} \right)^{2}. \]

We show the above by upper bounding the left hand side and using the first-order optimality conditions corresponding to (A1) and (A2), the convexity of the objective function \( g(X, \mathcal{F}) \) in \( X \) (resp. \( \mathcal{F} \)) given fixed \( \mathcal{F} \) (resp. \( X \)), as well as the \( \lambda_{2}(L_{f}^{+}) \)-strong convexity of the terms \( \frac{1}{2} \| (L_{f} \otimes I_{KZ}) \mathcal{X} \|_2^2 \) and \( \frac{1}{2} \| (L_{f} \otimes I_{KZ}) \mathcal{F} \|_2^2 \).

(ii) Next, we show that the augmented Lagrange is lower bounded at every iteration, i.e.,

\[ \sum_{k=1}^{K} A(X_{k+1}, \mathcal{F}_{k+1}, D_{f}^{k}, D_{X}^{k}, D_{f}^{k}) > -\infty, \]
\[ \sum_{k=1}^{K} A(X_{k}, \mathcal{F}_{k}, D_{f}^{k}, D_{X}^{k}, D_{f}^{k}) > -\infty. \]

To show the above result, we utilize the dual updates in (A3) and (A4), as well as the equality

\[ \langle C, D \rangle = \frac{1}{2} \left( \| C + D \|_F^2 - \| C \|_F^2 - \| D \|_F^2 \right), \]

for any arbitrary \( C \) and \( D \).

On taking the limit \( K \to \infty \) in (i) and (ii) above, we obtain

\[ \sum_{k=1}^{\infty} \| X_{k+1} - X_{k} \|_2 < \infty \] and \( \sum_{k=1}^{\infty} \| \mathcal{F}_{k+1} - \mathcal{F}_{k} \|_2 < \infty. \)

**Result 2: Convergence to Stationary Points**

The KKT conditions corresponding to (P3) are

\[ \nabla \pi g(X_{k}, \mathcal{F}) + \rho X_{k} \mathcal{F}_{X} = 0, \]
\[ \nabla \pi g(X_{k}, \mathcal{F}) + \rho (L_{f} \otimes I_{KZ}) \mathcal{F}_{f} = 0, \]
\[ (L_{f} \otimes I_{KZ}) X_{k} = 0, \quad (L_{f} \otimes I_{KZ}) \mathcal{F}_{k} = 0. \]

We aim to show that the limit points corresponding to Algorithm 1 satisfy the above KKT conditions. To that end, observe that in the limit \( k \to \infty \), the update steps (A1) and (A2) satisfy

\[ \nabla \pi g(X_{k}, \mathcal{F}) + \rho X_{k} \mathcal{F}_{X} = 0, \quad \nabla \pi g(X_{k}, \mathcal{F}) + \rho (L_{f} \otimes I_{KZ}) \mathcal{F}_{f} = 0. \]

In addition, since \( \lim_{k \to \infty} \| (L_{f} \otimes I_{KZ}) \mathcal{F}_{k} \|_2 = 0 \) and \( \lim_{k \to \infty} \| (L_{f} \otimes I_{KZ}) X_{k} \|_2 = 0 \) (c.f., Result 1), convergence to stationary points follows.

**Result 3: Sublinear Convergence Rate**

To derive convergence rates, we bound the optimality gap defined in the theorem statement. Per Result 1, for large enough \( \epsilon > 0 \), the optimality gap is upper bounded as

\[ Q(X_{k+1}, \mathcal{F}_{k+1}, \mathcal{F}_{k}, \mathcal{F}_{f}) \leq \epsilon \| X_{k+1} - X_{k} \|_2 + \epsilon \| \mathcal{F}_{k+1} - \mathcal{F}_{k} \|_2. \]

Moreover, for some large enough \( \nu > 0 \), Result 1 implies

\[ A(X_{k+1}, \mathcal{F}_{k+1}, D_{f}^{k}, D_{X}^{k}, D_{f}^{k}) - A(X_{k}, \mathcal{F}_{k}, D_{f}^{k}, D_{X}^{k}, D_{f}^{k}) \leq -\nu \| X_{k+1} - X_{k} \|_2^2 + \nu \| \mathcal{F}_{k+1} - \mathcal{F}_{k} \|_2^2. \]

On averaging over indices \( k = 1, \ldots, K \), we obtain

\[ \frac{1}{K} \sum_{k=1}^{K} Q(X_{k+1}, \mathcal{F}_{k+1}, \mathcal{F}_{k}, \mathcal{F}_{f}) \leq \frac{\epsilon}{K \nu} \left( A(X_{1}, \mathcal{F}_{1}, D_{f}^{1}, D_{X}^{1}, D_{f}^{1}) - A(X_{K+1}, \mathcal{F}_{K+1}, D_{f}^{K+1}, D_{X}^{K+1}, D_{f}^{K+1}) \right). \]

In other words, the convergence in terms of the optimality gap is sublinear with rate \( O(1/K). \)

**Remark 1.** To the best of our knowledge, we are not aware of any existing convergence analysis for ADMM that is directly applicable to our problem, which is distributed, integer constrained, bi-concave, and non-separable. On the one hand, in (Xu and Yin, 2013), the authors consider the centralized bi-concave and non-separable function class and show sublinear convergence assuming the objective function satisfies the Kurdyka–Lojasiewicz (KL) inequality. On the other hand, in (Hong et al., 2016), the authors consider the distributed non-concave but separable function class and show convergence to stationary points. Neither analysis applies to our problem. However, in the absence of integer constraints, our problem is closely related to

\[ \langle C, D \rangle = \frac{1}{2} \left( \| C + D \|_F^2 - \| C \|_F^2 - \| D \|_F^2 \right). \]
matrix factorization (MF). Hence, we have adapted prior convergence analysis for MF (Hajinezhad et al., 2016; Hajinezhad and Shi, 2018; Hong et al., 2017) to our problem. Since our problem is not identical to MF, our results have certain minor differences. Specifically, convergence to stationary points for MF only holds under certain regimes on the ADMM penalty parameters (e.g., \( \rho > 1 \)). No such conditions are required in our results.

Remark 2. We remark that the applicability of ADMM to bi-convex problems is well-known from (Boyd et al., 2011). Since strong duality does not hold in bi-convex problems, we can only demonstrate approximate convergence to KKT points. To do so, we have used the optimality gap function from (Hajinezhad et al., 2016; Hajinezhad and Shi, 2018; Hong et al., 2017) since it quantifies both first-order optimality conditions and the consensus error.

We now characterize the algorithm complexity for DSISD.

Corollary 1. Let the same assumptions as in Theorem 1 hold, and let \( \epsilon > 0 \) be a desired accuracy. Then, the algorithm complexity for DSISD is

\[
O(2|E|(M+K_{TX}(T-T_1)) + (M+K_{RX}T)(NKN_{RX}T))/\epsilon.
\]

Proof. The communication complexity is given by

\[
O(2|E|(M+K_{TX}(T-T_1)))/\epsilon,
\]

which corresponds to the total number of real entries transferred across the network \( G \) over \( K = O(1/\epsilon) \) iterations as per the convergence rate in Theorem 1.

The computational complexity corresponds to the total cost of least-squares updates in every iteration and equals

\[
O(N(M(K_{RX}T)^2+M^2K_{RX}T+K_{RX}K_{TX}^2+K_{TX}K_{RX}^2))/\epsilon.
\]

Assuming the number of scatterers in the scene largely dominates the number of transmitting or receiving antennas, i.e., \( M \gg K_{TX}, K_{RX} \), the algorithm complexity is given by the statement in the corollary.

For comparison, consider decode-and-image. The communication complexity corresponds to every node transferring its \( K_{RX} \times T \) matrix of received symbols \( Y^{n}_{T} \) to the central server, and thus equals \( O(NK_{RX}T) \). The computational complexity corresponds to least-squares updates over \( N \times \) larger matrices, and equals \( O(K_{RX}NTM(M+K_{RX}NT)) \). Hence, DSISD has \( O(N) \) smaller computational complexity compared to decode-and-image.

In the next section, we numerically evaluate the performance of DSISD and compare it with decode-and-image.

5. NUMERICAL EVALUATION

We simulate the 2D planar configuration shown in Fig. 2(a) with \( N = 3 \) base stations receiving data transmissions from an uplink user in the 2.4 GHz Wi-Fi band (\( \lambda = 0.125 \text{ m} \)). We assume uncoded BPSK uplink transmissions, i.e., \( X_{T} = \{\pm 1\}^{K_{TX} \times N} \), with coherence interval \( T = 100 \) symbols. The uplink user is equipped with a uniform linear array (ULA) with \( K_{TX} = 4 \) antennas. The base stations are each equipped with a ULA with \( K_{RX} = 8 \) antennas, and are oriented at angles 0°, -30° and 45° with respect to the uplink user’s array.

For simplicity, we consider a star topology \( G \), where the base stations are all connected to a fusion node (not illustrated in Fig. 2(a)). The fusion node only processes the consensus information, i.e., local solutions \( \mathbf{f}_{k}^{(n)} \) and \( \mathbf{X}_{T-T_1:k},k \) exchanged by base stations, and does not process the received symbols \( Y_{T}^{(n)} \) directly. The updates at the fusion node correspond to

\[
\hat{f}_{k} = \frac{-\sum_{m} \lambda G(N+1,m)f_{m-1}^{(n)} + D_{f}^{(n+1)}}{\lambda G(N+1,N+1)},
\]

\[
\hat{X}_{T-T_1:k} = \Xi_{T-T_1} \left( \frac{-\sum_{m} \lambda G(N+1,m)X_{m-1}^{(n)} + D_{X}^{(n+1)}}{\lambda G(N+1,N+1)} \right),
\]

where the index \( (N+1) \) denotes the fusion node. All remaining steps are identical to Algorithm 1, with \( \bar{f} \) and \( \bar{X} \) corresponding to the concatenation of local solutions over all \( (N+1) \) nodes in the network.

First, we numerically evaluate the validity of our convergence analysis from Theorem 1. Fig. 2(b) shows the optimality gap for DSISD at various receive signal-to-noise ratio (SNR) values. We observe that the sublinear \( O(1/K) \) convergence predicted by Theorem 1 holds in the high SNR regime. In future work, we shall refine our convergence analysis to incorporate the effect of SNR.

In Figs. 3(a), (b) and (c), we show that DSISD achieves similar communication and imaging performance as decode-and-image, with \( O(N) \) smaller computational complexity. In Fig. 3(a), we show that DSISD achieves communication bit error rate (BER) within 2 dB of decode-and-image. In Fig. 3(b), we show that the imaging point spread functions (PSFs) for DSISD are similar to those for decode-and-image, with 3 dB main lobe widths of \( \lambda \) and 0.4\( \lambda \) in range (\( x \)) and cross-range (\( y \)). Moreover, compared to local imaging performed at every base station, DSISD achieves resolution gains equivalent to jointly processing measurements across all \( N = 3 \) base stations. Finally, Fig. 3(c) shows that the computational complexity (in total number of floating operations) for DSISD is \( O(N) \) smaller compared to that of decode-and-image.
Fig. 3. (a) DSISD achieves communication bit error rate (BER) within 2 dB of centralized performance across a wide range of receive signal-to-noise ratio (SNR) values. (b) DSISD has similar imaging point spread functions (PSFs) as decode-and-image, with 3 dB main lobe widths of $\lambda$ and 0.4A in range (x) and cross-range (y). Moreover, compared to local imaging performed at every base station, DSISD achieves resolution gains equivalent to jointly processing measurements across all $N = 3$ base stations. (c) DSISD has $O(N)$ smaller computational complexity (in total number of floating operations) compared to decode-and-image. All plots correspond to $K = 30$ iterations.

6. CONCLUDING REMARKS

We proposed DSISD, a provably convergent distributed algorithm based on consensus ADMM for simultaneous imaging and communication. We showed that DSISD achieves similar imaging and communication performance as centralized schemes with an order-wise reduction in computational complexity. We shall extend our convergence analysis to include the effects of integer constraints and SNR in future work. Moreover, we shall explore accelerated variants of DSISD with faster convergence rates.

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