Force and deformation response analysis of dual structure slope excavation and support

Xuhe Gao, Wei-ping Tian, Jiachun Li, Hongliang Qi, Zhipei Zhang and Shiyang Li

School of Mechanics, Civil Engineering & Architecture, Northwestern Polytechnical University, Xi’an, PR China; Key Laboratory of Highway Engineering in Special Region, Ministry of Education, Chang’an University, Xi’an, PR China; College of Geology and Environment, Xi’an University of Science and Technology, Xi’an, PR China;

ABSTRACT
Aiming at the problem of fuzzy selection basis of dual-structure slope simulation calculation parameters and the complicated force and deformation of the slope body in the supporting process. The study relies on the treatment project of the right side slope of the K5+220-K5+770 section of the TJ1A section of the Jiangwen Expressway. The slope is a gravel soil-weathered bedrock dual structure slope, which is referred to as Hetangba slope in the study. Use Midas/GTS NX to build a trial calculation model. A new method for checking simulation parameters of deep displacement monitoring data is proposed. The superposition calculation method of pore water pressure is proposed to realize the slope stress-seepage coupling. The analysis method of slope excavation and support process is put forward. The analysis shows that the most dangerous working condition in the slope excavation and support process is the second-level excavation. When there is a local displacement and plastic strain concentration area on the slope, the reference significance of the extreme value and the safety factor is reduced. Compared with the application of pore water pressure and no pore water pressure, there is no obvious change in the position of the extreme value of horizontal displacement and plastic strain in working conditions 2–8, but the extreme value changes are different. The slope safety factor of working conditions 2–8 is reduced. The continuous distribution zone of plastic strain inside the slope disappeared in condition 7. There is a very small unstable area on the top of the third-level slope of working conditions 9–11. The research results have great theoretical guiding significance for the measurement and control simulation and early warning analysis of the dual structure slope construction process.

Abbreviations: Kn+xyz: n Kilometers plus xyz meters; TJ1A: Civil Construction Section 1A; Midas: Multi-tier Distributed Applications Services; GTS NX: New eXperience of Geotechnical and Tunnel analysis System; et al.: and others; FLAC3D: Fast Lagrangian
1. Introduction

The slope composed of the upper gravel soil and the lower weathered bedrock is called the dual structure slope (Zheng et al. 2010; Editorial Board of Engineering Geology Manual 2018). In the existing research on dual slopes, there is little involved in the analysis of the stress-seepage coupling conditions of the simulation parameter check and the analysis of the response characteristics of the slope body’s force and deformation in the process of excavation and support.

In previous studies on slope analysis, Tian et al. (2013) used FLAC3D to study a soft rock composite slope and revealed the deformation and failure mechanisms of the slope. Tian et al. (2013) carried out a three-dimensional numerical simulation and stability analysis of a soil-rock dual-structure slope in a water conservancy project to predict the occurrence and evolution of slope failure. Chen et al. (2013) performed a case-study of the dual-structure slope of an open-pit mine. While they studied and discussed the evolution of slope instability and the associated mechanism, and identified the optimal excavation support structure for the slope, there is no discussion of the excavation and support process or of the effects of seepage. Tang et al. (2013) found that the upper part of a binary slope with a soft deposit layer and a hard bottom layer will rotate and topple about the upper layer, whereas the lower layer will slip along the soil-rock interface. Liang et al. (2014) conducted research on the seismic resistance of anti-slide piles in dual-structure slopes, determined the failure characteristics of the slopes, and demonstrated the earthquake resistance before and after the construction of anti-slide pile supports. Mei et al. (2015) used model experiments and numerical simulations to study the failure characteristics of dual-structure slopes and to evaluate the effects of different material interfaces on slope behaviour under applied load. Xu et al. (2021) used discrete element methods to carry out simulation calculations for rock deformation, damage and rupture and achieved fruitful results (Yu et al. 2021; Yuan et al. 2021; Zhou et al. 2022, 2020).

With regards to modelling of slopes, Zhang and Gao (2016) established finite element models of loess landslides based on the strength reduction method and the limit equilibrium method. This work also determined potential sliding surface positions, calculated the overall stability coefficient of the slope and identified relatively weak zones on the slope through comparative analysis. Furthermore, Xu et al. (2016) carried out studies on the impact of rainfall and reservoir water grades on slope
stability using numerical simulation methods. While informative, this work lacked a comprehensive process analysis (Guo et al. 2017; Li et al. 2017; Lu et al. 2017; Tang et al. 2017; Yu et al. 2017; Lei et al. 2018; Gholampour and Johari 2019; Johari and Kalantari 2021).

In summary, a variety of previous studies have been conducted on slope stability and failure mechanisms. However, this work has largely investigated simple slope body compositions and relied on calculation processes that are highly dependent on the shear strength parameters of the sliding body. This approach falls short for dual-structure slopes where the complex geological composition of the slope makes it more difficult to select the appropriate parameters for calculating slope stability. Therefore, a study on the calculation of parameters for dual-structure slope analysis is needed. In addition, the influence of seepage on slope stability has become increasingly of interest, but there are currently few studies on the force and deformation characteristics of dual-structure slopes during excavation and reinforcement that account for seepage conditions. Based on this, this article carries out the stress and deformation analysis of the dual structure slope excavation and supports process under the condition of steady seepage, proposes and uses the deep displacement monitoring data to check the simulation parameters. The variation characteristics of horizontal displacement, plastic strain and safety factor in the process of dual structure slope excavation and support are analyzed.

2. Materials and methods

2.1. Study area

This study examined the right slope of the TJ1A K5+220–K5+770 section of the Jiangwen Expressway Governance project. The first support structure was designed for the left and right sides of the K5+327 to K5+520 section. The grade of the first slope was 1:0.75 and it had a height of 8 m and a window-type retaining wall for protection. The second slope had a grade of 1:0.75, a slope height of 8 m, and grass planted in a prestressed anchor cable frame for protection. The third slope had a grade of 1:1, a slope height of 8 m, and grass planted in a prestressed anchor cable frame for protection. Finally, the fourth slope had a grade of 1:1 and reached the top of the slope. The fourth slope was protected by a grass block and a stone grid. Anti-sliding piles were added at the third-level platform for additional reinforcement. The cross-sectional dimensions of the anti-sliding piles were 3×2.4 m, the distance between the piles was 5.0 m, and the length of the piles was 30 m. Three levels of anchor cables were added for slopes 2 and 3. The slope was reinforced every 3 m and the anchor cable was a 4 Φs 15.24 scattered-type cable with a downward tilt of 20° and a drilling hole diameter of 130 mm. The total length of the anchor cable was 20 m, anchor section was 10 m long, and prestress was 500 kN.

On 28 September 2014, cracking occurred on the right slope of the K5+385 to K5+500 section. The cracks were intermittently continuous arcs ranging from a 2–12 m long, 1–3 cm wide, and 0.4–2.0 m deep. The second stage of reinforcement was thus designed and constructed between October 2015 and December 2016 in the K5+336 to K5+500 section. The construction of the three-level platform anti-slide
piles in this section has been completed. The cross-section of the anti-sliding piles was 3 m × 2.4 m and the pile spacing was 5.0 m. An additional anchor beam was added to the top of the anti-sliding pile using a 6 Φs 15.24 anchor cable tilted 28° downwards with a hole diameter of 130 mm. The anti-sliding piles were numbered Z11–Z31, the length of the anchor cable was 33 m, the anchoring section was 11 m long, and 750 kN of prestress was applied. A row of anchor cable anti-sliding piles (Z32–Z48) was also added to the secondary platform of the slope, and the tops of the piles were connected by beams. These anti-sliding piles were 33–34 m long. The pile top anchor cable was a 6 Φs 15.24 dispersion type cable angled 20° downhill with a drilling hole diameter of 130 mm, an anchoring section length of 12 m and a pre-stress of 750 kN. The tops of piles Z47 and Z48 were not anchored by cable. The first grade used a window hole-type retaining wall, and the fourth grade used rhombic grid planting grass to protect the slope.

A row of drainage holes was arranged on the first grade of the K5 +370 to K5 +420 section. The vertical distance between the drain hole and the bottom platform is 4 m, and the horizontal distance between the hole is 5 m. A second row of drainage holes was arranged on the second grade. The water outlet was 1.5 m higher than the platform and the holes were spread across a horizontal distance of 5 m. Construction on this slope section avoided the drainage holes leading from the anti-slide pile body. The drainage hole water inlet was 1 m deep in the sliding zone and tilted by 6°. The diameter of the drainage hole was 110 mm and the diameter of the drainage pipe, which was fabricated with PVC plastic, was 100 mm. The spacing and depth of the drainage holes was determined to allow slight position adjustments according to the actual groundwater distribution and the positions of the anti-slide piles after slope excavation.

2.2. Construction monitoring

2.2.1. Monitoring point layout

The layout of the monitoring points for deep slope displacement is shown in Figure 1 and a cross-section of the landslide area is shown in Figure 2. The slope slip surface is an interface formed between the landslide body and the immovable (parent) body when the landslide body moves. The potential slip surface of the slope was identified by a preliminary analysis of the monitoring data.

2.2.2. Inclination monitoring

Inclination monitoring can be used to determine the vertical orientation of shallow soil at a fixed point on the slope, which can in turn be used to calculate the lateral displacement of the slope. The lateral displacement data obtained by this method are consistent with the lateral horizontal displacement data determined by numerical simulation and it offers good contrast and mutual proof characteristics.

Inclination monitoring technology and equipment setups are relatively mature. In this study, a movable inclinometer was used for on-site monitoring, and the inclinometer tube matched with the inclinometer was installed at the deep displacement monitoring points listed in Figure 1. There are two sets of guide chutes perpendicular to each
other on the inside of the inclinometer used here. Put the guide wheel on the inclinometer into the bottom of the tube along one of the guide grooves in the inclinometer tube. When measuring, lift the signal line marked with the scale of the inclinometer from the bottom of the tube and record the reading of the inclinometer. The inclinometer

Figure 1. Schematic of deep hole displacement monitoring locations.

Figure 2. Cross-sectional view of the slope at K5 + 420.
indicates the inclination of each scaled position of the signal line according to the angle between the measuring tube and the gravity line $\theta_i$. The measuring and reading equipment converts the inclination into the position difference $\Delta d = L \sin \theta_i$ between the upper and lower guide wheels of the measuring position, where $L$ is the measuring point segment length. The horizontal displacement of each point can then be obtained by summing $d = \sum l \sin \theta_i$ from bottom to top, as shown in Figure 3.

### 2.3. Basic law of seepage – Darcy’s law

Darcy concluded through experiments that the law describing the linear relationship between the seepage velocity of water in saturated soil and the hydraulic slope is as follows (Liu 2008):

$$Q = kAi = -kA \frac{dh}{ds}.$$  \hspace{1cm} (1)

where $Q$ is the seepage flow, $A$ is the cross-sectional area of seepage flow, $i$ is the loss rate along the path $s$ (i.e. the drop in permeability ratio), and $h$ denotes the height difference. It can be determined from hydraulics that the seepage flow $Q$ through a certain section is equal to the product of the velocity $v$ and the cross-section $A$, such that $Q = Av$. Therefore, Darcy’s law can also be expressed as $v = ki$, where $v$ represents the seepage velocity. Equation (1) shows that seepage velocity is linearly related to the permeability ratio drop; thus, this equation is also referred to as the linear seepage law.

### 2.4. Model design

#### 2.4.1. Simulation assumptions and regions

Midas/GTS software was used for modelling. Assume as follows:
1. Rock and soil obeys the Mohr–Coulomb strength criterion.
2. The groundwater and seepage were simplified to the steady-state seepage.
3. The supporting pile and anchor cables obey an elastic constitutive model.
4. The pile-soil structure interface is obtained by calculation of the pile contact properties, and the anchor cable-rock structure interface is obtained by the simulation of the anchor section element.

The strata were divided into sliding layers, gravelly soil (accumulation), strongly weathered mudstone, and medium weathered mudstone. The specific layout of the geometric design model is shown in Figure 4. As the fully weathered mudstone layer was too thin, a ‘pink out’ phenomenon occurred in the study area. As thin layers do not play a significant role in determining the overall stability of the slope under seepage conditions, this layer was simplified in the model.

In the first pore water pressure simulation, the boundary node water head of 80 m is applied on the right side, and the slope water head of 54 m is applied on the left side. Pore water pressure was calculated from based on the steady-state seepage conditions and applied to the four-grade excavation model, and the model of the three-grade anti-slide pile of the primary support structure. In the second pore water pressure simulation, the boundary node water head of 80 m is applied on the right side, and the slope water head of 46 m is applied on the left side. The pore water pressure was calculated based on steady-state seepage and applied to the three-grade excavation model and the model of the three-grade anchor cable component of the primary support structure. In the third pore water pressure simulation, the boundary node water head of 80 m is applied on the right side, and the slope water head of 38 m is applied on the left side. The pore water pressure was calculated from the steady-state seepage conditions and applied to the secondary excavation model and the model of the secondary anchor cable for the primary support. In the fourth pore water pressure simulation, the boundary node water head of 80 m is applied on the right side, and the slope water head of 30 m is applied on the left side. The pore water pressure was calculated from the steady-state seepage conditions and applied to the secondary excavation model and the model of the secondary anchor cable for the primary support.
pressure simulation, the boundary node water head of 80 m is applied on the right side, and the slope water head of 30 m is applied on the left side. The water head of the slope drainage node in the above stages was 0, and is modelled as a pressure head. The pore water pressure calculated from the steady-state seepage condition was applied to the models of the first-grade excavation, the third-grade anchor cable for the second support, the second-grade anti-slide pile for the second support, and the second-grade anchor cable for the second support.

2.4.2. Model boundary conditions
The boundary conditions of the model were as follows:

1. The x-direction of the left and right boundaries of the model is a constraint boundary.
2. The z-direction of the front and back boundaries of the model is a constraint boundary.
3. The upper boundary of the model is a free boundary.

The model grid division followed the applicability principle; thus, the rock and soil body was modelled with a quadrilateral and triangular plane element grid, the anti-slide piles were modelled with a linear beam element grid, and the prestressed anchor cable anchor section was modelled with a linear truss element grid. Grid densification was performed on the excavation slope, around the anti-slide pile and at the interface of the rock–soil layers. Based on the above assumptions and considerations, a model with a length, width and height of $150 \times 1 \times 80$ m, respectively, was established. The model had a total of 11,932 units and 23,923 nodes, as shown in Figure 5. The

![Figure 5. Mesh model diagram of the K5+420 slope section.](image)
subgrade centre line was used as the boundary line during modelling; thus, the x-direction \((u = 0)\) constraint was imposed on the subgrade boundary line.

### 2.4.3. Process analysis

Slope treatment engineering is not just an issue of the final status of a project; at the same time, it also involves the construction process. Slope treatment engineering problems rarely occur at completion. On the contrary, most slope engineering problems arise during construction. Aiming at this problem, an analysis method of slope excavation and support under different seepage conditions is proposed. The simulation design conditions were divided into two parts: no pore water pressure and impose pore water pressure.

In the simulations where no pore water pressure was applied, 11 working conditions were considered as follows:

- **Condition 1:** Initial state
- **Condition 2:** Excavation of the fourth grade of the slope
- **Condition 3:** Addition of the third grade anti-slide piles for the primary support
- **Condition 4:** Excavation of the third grade of the slope
- **Condition 5:** Addition of anchor cable for the primary support of the third-grade slope
- **Condition 6:** Excavation of the second grade of the slope
- **Condition 7:** Addition of anchor cable for the primary support of the secondary grade slope
- **Condition 8:** Excavation of the first grade of the slope
- **Condition 9:** Construction of the secondary support and addition of the third-grade anchor cable
- **Condition 10:** Construction of the secondary support and addition of the secondary grade anti-slide piles
- **Condition 11:** Construction of the secondary support and addition of the secondary grade anchor cables

Steady-state seepage was considered for four simulation conditions, where each condition represents the completion of excavation at one grade of the slope (beginning at the fourth grade). The application of pore water pressure was considered for several simulation conditions as follows:

- Excavation of the fourth grade of the slope: considering the pore water pressure generated by the excavation of the fourth grade.
- Addition of the third grade anti-slide piles for the primary support: considering the pore water pressure generated by the excavation of the fourth grade.
- Excavation of the third grade of the slope: considering the pore water pressure generated by the excavation of the third grade.
- Addition of anchor cable for the primary support of the third grade slope: considering the pore water pressure generated by the excavation of the third grade.
Excavation of the second grade slope: considering the pore water pressure generated by the excavation of the second grade.

Addition of anchor cable for the primary support of the secondary grade slope: considering the pore water pressure generated by the excavation of the second grade.

Excavation of the first grade of the slope: considering the pore water pressure generated by the excavation of the first grade.

Construction of the secondary support and addition of the third grade anchor cable: considering the pore water pressure generated by the excavation of the first grade.

Construction of the secondary support and addition of the secondary anti-slide piles: considering the pore water pressure generated by the excavation of the first grade.

Construction of the secondary support and addition of the secondary anchor cables: considering the pore water pressure generated by the excavation of the first grade.

Based on statistical data for anchor cable prestress loss and the real-world conditions, the most unfavourable situation for anchor cable prestress loss was considered. Accordingly, the long-term prestress loss rates of the 750 and 500 kN prestressed anchor cables were taken as 30% and 20%, respectively. Thus, the pre-stress values of the anchor cables in the model were reduced to 525 and 400 kN, respectively. The general modelling and analysis process is summarized in Figure 6.

2.4.4. p Value testing and parameter verification

Hypothesis testing is an important concept in inferential statistics where the p value is used as a basis for selecting or rejecting a hypothesis. Specifically, the p value reflects the probability of an event occurring by chance. Using the significance test method, $p < 0.01$ is generally considered to be a significant statistical difference, which indicates that the likelihood the difference between samples is caused by sampling error or chance is less than 1%. That is, when the deviation between the result obtained by the simulation trial calculation and the monitoring value is within a certain range (in this case, $p > 0.01$), the parameter selection is reasonable.

3. Results

3.1. Deep displacement monitoring

The positions of the deep displacement monitoring holes 9, 12 are shown in Figures 1 and 2. The monitoring curves are shown in Figures 7 and 8.

3.2. Model validation

It can be seen from Figure 9 that the maximum horizontal displacement of the slope body after the initial support is 0.102191 m. Use the accumulated horizontal displacement data of No. 9 and No. 12 deep displacement monitoring holes on the K5 + 420 section before the second support (29 August 2015) to carry out a check and analysis.

Use $p$ value to test the results (Figure 10), in which the hole 9 $p = 0.039 > 0.01$; the hole 12 $p = 0.793 > 0.01$. Initially verify that the parameters selected for the simulation are valid.
3.3. Parameter determination

The parameters of concrete and anchor cables are determined according to the specifications. Poisson’s ratio, gravity, permeability coefficient, and volumetric water
content are determined through geotechnical tests. The elastic modulus, friction angle, and cohesive force are selected based on on-site geotechnical tests, using $p$ value inspection. Tables 1–3 list the parameters required for the calculation model.

### 3.4. Model results

Overall slope stability is dependent not only on whether the stress on each point of the sliding surface exceeds the limit state but also on whether the strain on each point of the sliding surface has reached the limit state. When a dual-structure slope is disturbed by external force, such as excavation, the sliding body creeps or slips out. This results in increased displacement and plastic strain in some areas of the slope. Concurrently, as the main supporting structure of the slope, the bending moment, and shear force on the
anti-slide pile increase dramatically. This deformation and failure process under stress is consistent with the general slope failure process. Therefore, this article uses the nodal plastic strain and the change in x-direction displacement of the sliding surface are used as quantitative indicators of the overall instability of the slope.

3.4.1. Ignoring steady-state seepage

Figure 11(a) shows that the maximum horizontal displacement of the slope is $-4.6874e-2$ (Figure 11(a1)) and that there is a continuous distribution of plastic strain along the slope (Figure 11(a2)).
The comparative analysis of Figure 11(a,b) shows that when the slope is affected by the disturbance of the excavation of the fourth grade, the maximum horizontal displacement of the upper slope increases to $1.1522 \times 10^0$ m (Figure 11(b1)). The maximum effective plastic strain also increases to $4.75157 \times 10^{-1}$, and a weakly penetrating plastic strain distribution zone forms inside the slope (Figure 11(b2)). In addition, the first stage of excavation reduces the slope safety factor to 1.0688.

The comparative analysis of Figure 11(b,c) shows that the construction of anti-slide piles further disturbs the slope. In this condition, the maximum horizontal displacement reaches $1.82109 \times 10^0$ m (Figure 11(c1)). The penetrating characteristic of the plastic strain zone becomes more pronounced (Figure 11(c2)), and the maximum effective plastic strain further increases. However, due to the action of anti-slide piles, the overall stability of the slope improves slightly in this condition compared to the excavated slope without anti-slide piles.

The comparative analysis of Figure 11(c,d) shows that the primary supporting anti-slide piles are effective at reducing further slope movement. The maximum horizontal displacement of the upper slope body is reduced from $1.82109 \times 10^0$ m to $1.07793 \times 10^{-1}$ m (Figure 11(c1,d1)), and the maximum plastic strain is reduced from $6.59860 \times 10^{-1}$ to $6.49579 \times 10^{-2}$ (Figure 11(c2,d2)). Furthermore, the penetrating plastic

![Figure 9. Horizontal displacement of the formation after initial support.](image)

**Figure 10.** Horizontal displacement monitoring data and simulation data from (a) hole 9 ($p=0.039$) and (b) hole 12 ($p=0.793$).
strain feature seen in Figure 11(c) disappears, and the plastic strain band is redistributed. The overall stability of the slope is also improved.

The comparative analysis of Figure 11(d,e) shows that the maximum horizontal displacement of the upper slope increased from $1.07793 \times 10^{-1}$ m to $2.72755 \times 10^{-1}$ m in this condition (Figure 11(d1,e1)), despite the addition of a supporting anchor cable. However, the maximum horizontal displacement and effective plastic strain were transferred to the newly excavated third grade slope angle (The maximum value of horizontal displacement and effective plastic strain in working condition 4 is near the potential slip surface). The maximum plastic strain also increased from $6.49579 \times 10^{-2}$ to $1.61592 \times 10^{-1}$ (Figure 11(d2,e2)).

The comparative analysis of Figure 11(e,f) shows that the horizontal displacement and plastic strain areas of the slope were transferred to the area below the newly excavated secondary and tertiary slopes during the excavation of the second grade. The maximum displacement increased to $9.98443 \times 10^{-1}$ m (Figure 11(f1)), and the maximum plastic strain increased to $3.97754 \times 10^{-1}$ (Figure 11(f2)). The slope safety factor decreased to 0.7 in this condition.

### Table 1. Rock and soil parameters.

| Soil layer                  | Elastic modulus (kN/m²) | Poisson’s ratio | Angle of internal friction (°) | Cohesion forces (kN/m²) | Natural unit weight (kN/m³) | Saturated unit weight (kN/m³) | Permeability coefficient (m/d) |
|-----------------------------|-------------------------|-----------------|---------------------------------|-------------------------|-----------------------------|-------------------------------|--------------------------------|
| Sliding layer               | 160,000                 | 0.28            | 10.5                            | 12                      | 18.0                        | 18.5                          | 0.10                           |
| Gravel Soil                 | 80,000                  | 0.30            | 15.0                            | 30                      | 17.5                        | 18.0                          | 0.10                           |
| Strongly Weathered Mudstone | 200,000                 | 0.25            | 19.0                            | 50                      | 19.5                        | 20.0                          | 0.20                           |
| Medium Weathered Mudstone  | 1,200,000               | 0.20            | 28.0                            | 310                     | 20.5                        | 21.0                          | 0.02                           |

Note: The sliding layer includes a gravel soil layer, strongly weathered mudstone and some medium weathered mudstone; thus, the elastic modulus will be higher than that of the gravel soil layer. In the process of parameter checking, the simulated displacement and plastic strain of gravel soil and weathered bedrock need to be adjusted continuously to meet the actual deformation in the slope excavation and support process. Therefore, the elastic modulus and internal friction angle after checking are the values that satisfy the potential slip surface formation and the overall creep (slight slip) of the slope, which is smaller than the data obtained in the normal laboratory.

The longitudinal spacing of the anti-slide piles is 5 m and the longitudinal width of the anti-slide piles is 2.4 m. In theory, each meter of vertical pile needs to bear the remaining sliding force of the 2.08 m-thick sliding body. Therefore, it is necessary to multiply the sliding weight by 2.08 in the calculation.

### Table 2. Mechanical parameters of the support structure.

| Structure      | Elastic modulus (kN/m²) | Unit weight (kN/m³) | Poisson’s ratio |
|----------------|-------------------------|--------------------|-----------------|
| Anti-slide pile| 2.4E7                   | 25.0               | 0.3             |
| Anchor         | 2.0E7                   | 78.5               | 0.2             |

### Table 3. Parameters of the anchor cables.

| Anchor cable classification | Slope spacing (m) | Total length (m) | Anchoring section (m) | Prestress (kN) | Tilt angle (°) | Aperture (mm) |
|-----------------------------|-------------------|------------------|-----------------------|----------------|---------------|---------------|
| Initial support             |                   |                  |                       |                |               |               |
| Third grade                 | 1                 | 3                | 20                    | 10             | 500           | 20            | 130           |
| 2                           | 3                 | 20               | 10                    | 500            | 20            | 130           |
| 3                           | 3                 | 20               | 10                    | 500            | 20            | 130           |
| Second support              |                   |                  |                       |                |               |               |
| Grade 1                     | 1                 | 3                | 20                    | 10             | 500           | 20            | 130           |
| Grade 2                     | 3                 | 20               | 10                    | 500            | 20            | 130           |
| Secondary support           | Grade 3           | –                 | 11                    | 750            | 28            | 130           |
| Grade 2                     | –                 | 12                | 750                   | 20             | 130           |

The longitudinal spacing of the anti-slide piles is 5 m and the longitudinal width of the anti-slide piles is 2.4 m. In theory, each meter of vertical pile needs to bear the remaining sliding force of the 2.08 m-thick sliding body. Therefore, it is necessary to multiply the sliding weight by 2.08 in the calculation.
The comparative analysis of Figure 11(f,g) shows that the maximum horizontal displacement returned to the upper slope with the addition of a secondary anchor cable for the primary support (Figure 11(g1)). The interior of the upper slope redeveloped to form a weakly continuous plastic strain distribution zone (Figure 11(f2)). In addition, a new concentrated area of horizontal displacement distribution and plastic strain formed near the secondary slope platform.

The comparative analysis of Figure 11(g,h) shows that the maximum horizontal displacement and plastic strain of the slope were reduced during the excavation of the first grade. The concentrated distribution of horizontal displacement and effective plastic strain in the lower portion of the first and second platforms is still evident, and the regional maximum values increased near the newly excavated first and second platforms. A local penetrating plastic strain distribution zone formed under the primary slope in this condition, resulting in a reduction in the safety factor of the slope, which reflects the stability of the most unfavourable local sliding surface. However, the plastic strain distribution in the upper slope disappeared.

**Figure 11.** The horizontal displacement and effective plastic strain extreme value of the slope without considering the steady-state seepage.
(1) Horizontal displacement

(2) Effective plastic strain.

b. Working condition 2: excavation of the fourth grade.

Figure 11. (Continued)
(2) effective plastic strain

c. Working condition 3: addition of the third grade anti-slide piles for the primary support

(1) Horizontal displacement

(2) effective plastic strain

d. Working condition 4: excavation of the third grade

Figure 11. (Continued)
In the simulation of the second support stage, the loss of anchor cable prestress over time was considered. To this end, the prestress of the second and third-grade anchor cables for both the primary support and secondary support structures in the subsequent simulations were reduced according to the statistical calculation of long-term prestress loss determined by field monitoring.

The comparative analysis of Figure 11(h,i) shows that the horizontal displacement and the maximum plastic strain of the slope were slightly reduced by the addition of the third grade anchor cable for the secondary support. The addition of the anchor cable also transferred the maximum horizontal displacement from the first grade of the slope to the toe of the third grade (Figure 11(h1,i1)). Similarly, the maximum plastic strain was transferred from the first grade of the slope to the third grade of

Figure 11. (Continued)
(2) effective plastic strain

f. Working condition 6: excavation of the second grade

(1) Horizontal displacement

(2) effective plastic strain

g. Working condition 7: addition of anchor cable for the primary support of the secondary grade slope

Figure 11. (Continued)
the slope (Figure 11(h2,i2)). A localized zone of penetrating plastic strain formed under the tertiary slope, and the plastic strain distribution expands in this condition. However, the overall strain penetrability is reduced, which results in local increases in
(2) effective plastic strain

i. Working condition 9: construction of the secondary support and addition of the third grade anchor cable

(1) Horizontal displacement

(2) effective plastic strain

j. Working condition 10: construction of the secondary support and addition of the secondary grade anti-slide piles
the safety factor of the slope. Finally, the plastic strain distribution on the upper slope reappears in this condition.

The comparative analysis of Figure 11(I,j) shows that the addition of secondary grade anti-slide piles effectively eliminated the continuous distribution of plastic strain under the third grade of the slope. However, this addition transferred the maximum displacement of the slope back to the first grade, along with the maximum plastic strain. These maximum values of horizontal displacement and plastic strain are significantly reduced compared to the previous working condition. However, because this condition was based on limited local data, the reference value of the maximum displacement and strain is reduced. Importantly, the continuous distribution of plastic strain on the upper slope reappeared in this condition (Figure 11(j2)), but the safety factor increased to 1.1004.

The comparative analysis of Figure 11(j,k) shows that the locations of the maximum horizontal displacement and plastic strain are unchanged from the previous working condition. The maximum horizontal displacement decreased from $-1.06173e0$ m to $-1.68235e-1$ m (Figure 11(ji,ki)), and the maximum plastic strain
decreased from 9.05025e-1 to 1.39265e-1 (Figure 11(jii,kii)). However, this decrease is of little significance due to the limited available data. The safety factor for the slope in this condition reflects local stability, and it unexpectedly decreases compared to the previous working condition. The continuous distribution of plastic strain is still present on the upper slope; however, no perforating sliding zone is formed due to the effects of the supporting structure. Therefore, the slope is stable overall.

The maximum values of horizontal displacement and effective plastic strain of the slope for the different working conditions during the excavation and reinforcement processes are summarized in Table 4.

### 3.4.2. Considering steady-state seepage

Figure 12(a) shows that, affected by the disturbance of the excavation of the fourth grade, the extreme horizontal displacement of the upper slope is $-1.14295e0$ m (Figure 12(a1)). The effective plastic strain extreme value is 5.02907e-1 (Figure 12(a2)). A weakly penetrating plastic strain distribution zone is formed inside the slope. The slope safety factor is 1.0500. Compared with the absence of pore water pressure (Figure 11(b)), the horizontal displacement and effective plastic strain extreme values of the slope both increase, and the safety factor decreases.

The comparative analysis of Figure 12(a,b) shows that, after the anti-slide pile is applied, the horizontal displacement extreme value is reduced to $-1.08653e0$m (Figure 12(b1)). The plastic strain zone still shows weak penetration. The effective plastic strain limit is reduced to 4.67608e-1 (Figure 12(b2)). The overall stability of the slope is increased to 1.0754. Compared with the absence of pore water pressure (Figure 11(c)), the extreme value of horizontal displacement, the extreme value of effective plastic strain and the safety factor of the slope are reduced.
The comparative analysis of Figure 12(b,c) shows that the extreme value of the horizontal displacement of the upper slope increases to $1.21743 \times 10^0$ m (Figure 12(c1)). The extreme value of plastic strain increases to $5.28746 \times 10^{-1}$ (Figure 12(c2)).

**Figure 12.** The horizontal displacement and effective plastic strain extreme value of the slope considering the steady-state seepage.

The comparative analysis of Figure 12(b,c) shows that, the extreme value of the horizontal displacement of the upper slope increases to $-1.21743e0$ m (Figure 12(c1)). The extreme value of plastic strain increases to $5.28746e-1$ (Figure 12(c2)). The slope plastic
(2) effective plastic strain

b. Working condition 3: addition of the third grade anti-slide piles for the primary support

(1) horizontal displacement

c. Working condition 4: excavation of the third grade

Figure 12. (Continued)
(1) horizontal displacement

(2) effective plastic strain

d. Working condition 5: addition of anchor cable for the primary support of the third grade slope

(1) horizontal displacement

Figure 12. (Continued)
(2) effective plastic strain

e. Working condition 6: excavation of the second grade

(1) horizontal displacement

(2) effective plastic strain

f. Working condition 7: addition of anchor cable for the primary support of the secondary grade slope

Figure 12. (Continued)
strain zone is still weakly connected. Compared with no pore water pressure (Figure 11(d)), the extreme value of horizontal displacement of the slope increases, the extreme value of effective plastic strain decreases, and the safety factor does not change.

Figure 12. (Continued)
Figure 12. (Continued)

(2) effective plastic strain

h. Working condition 9: construction of the secondary support and addition of the third grade anchor cable

(1) horizontal displacement

i. Working condition 10: construction of the secondary support and addition of the secondary grade anti-slide piles
The comparative analysis of Figure 12(c,d) shows that, the horizontal displacement extreme value of the upper slope body is reduced to $-7.93354\times10^{-1}$ m (Figure 12(d1)). The extreme value of plastic strain is reduced to $5.04945\times10^{-1}$ (Figure 12(d2)). At this time, the maximum value of the horizontal displacement and the effective plastic strain are transferred to the position of the slope angle formed by the new excavation. The plastic strain distribution band is greatly reduced. The safety factor is reduced to 1.0344. Compared with the absence of pore water pressure (Figure 11(e)), the horizontal displacement and effective plastic strain extreme values of the slope increased, and the safety factor decreased.

The comparative analysis of Figure 12(d,e) shows that, the horizontal displacement extreme value of the upper slope body increases to $-1.08979\times10$ m (Figure 12(e1)). The extreme value of plastic strain increased to $5.50583\times10^{-1}$ (Figure 12(e2)). The plastic strain distribution zone of the slope disappears. The safety factor is reduced to 0.6250. Compared with the absence of pore water pressure (Figure 11(f)), the...
horizontal displacement and effective plastic strain extreme values of the slope increased, and the safety factor decreased.

The comparative analysis of Figure 12(e,f) shows that, the extreme value of the horizontal displacement of the upper slope body is reduced to $-4.55753e-1$ m (Figure 12(f1)). The extreme value of plastic strain is reduced to $2.79647e-1$ (Figure 12(f2)). The extreme value of the horizontal displacement of the slope and the extreme value of the effective plastic strain both occur at the toe of the third grade slope. The safety factor is increased to 0.9777. Compared with no pore water pressure applied (Figure 11(g)), the horizontal displacement and effective plastic strain extreme value of the slope body are reduced, and the safety factor is reduced.

The comparative analysis of Figure 12(f,g) shows that, the horizontal displacement extreme value of the upper slope body is reduced to $-3.2321e-1$ m (Figure 12(g1)). The extreme value of plastic strain is reduced to $2.62366e-1$ (Figure 12(g2)). The extreme value of the horizontal displacement of the slope and the extreme value of the effective plastic strain both occur at the shear exit position of the first grade slope. The safety factor is reduced to 0.9250. Compared with no pore water pressure applied (Figure 11(h)), the horizontal displacement and effective plastic strain extreme value of the slope body are reduced, and the safety factor is reduced.

The comparative analysis of Figure 12(g,h) shows that, the extreme value of the horizontal displacement of the upper slope body is reduced to $-5.6339e-1$ m (Figure 12(h1)). The extreme value of plastic strain increased to $7.77206e-1$ (Figure 12(h2)). The extreme value of the horizontal displacement of the slope and the extreme value of the effective plastic strain both occur at the position where the third-level anchor cable is applied for the secondary support. The safety factor is increased to 0.9523. Compared with the absence of pore water pressure (Figure 11(i)), the horizontal displacement and effective plastic strain extreme values of the slope increased, and the safety factor decreased.

The comparative analysis of Figure 12(h,i) shows that, the horizontal displacement extreme value of the upper slope body is reduced to $-3.8860e-1$ m (Figure 12(i1)). The extreme value of plastic strain is reduced to $5.74272e-1$ (Figure 12(i2)). The extreme value of the horizontal displacement of the slope and the extreme value of the effective plastic strain both occur at the position where the third-level anchor cable is applied for the secondary support. The safety factor is increased to 1.0039. Compared with no pore water pressure applied (Figure 11(j)), the extreme values of horizontal displacement and effective plastic strain of the slope are reduced, and the safety factor is reduced.

The comparative analysis of Figure 12(i,j) shows that, the horizontal displacement extreme value of the upper slope body increases to $-5.73812e-1$ m (Figure 12(j1)). The extreme value of plastic strain increased to $8.81764e-1$ (Figure 12(j2)). The extreme value of the horizontal displacement of the slope and the extreme value of the effective plastic strain both occur at the position where the third-level anchor cable is applied for the secondary support. The safety factor is increased to 0.9531. Compared with the absence of pore water pressure (Figure 11(k)), the extreme values of horizontal displacement and effective plastic strain of the slope increased, and the safety factor decreased.
According to the comparative analysis of the horizontal displacement and plastic strain of the slope before and after the pore water pressure is applied, the extreme values of the horizontal displacement and effective plastic strain in the slope excavation and support process are shown in Table 5.

Note: Limited to the length of the article, the force characteristics of the slope support structure are discussed separately in another manuscript.

4. Discussion

Stability analysis of dual-structure slopes often focuses on monitoring statistics or status analysis, without performing a verification of the simulation parameters used or a corresponding process analysis. Existing slope stability analysis methods and calculations are mainly dictated by geotechnical parameters, which have caused the determination of simulation parameters to be a contentious issue in this field. The current methods are to determine parameters through geotechnical tests, statistical data obtained from a large number of similar strata, or empirical data. However, these methods each have significant limitations. The parameters obtained by geotechnical testing often need to be revised as the testing conditions differ from the actual project conditions. The use of statistical data is only applicable to ordinary strata and requires the accumulation of significant amounts of engineering data. Finally, empirical data is convenient to use, but lacks rigor. Importantly, all three methods lack generalizability to slopes in special geological environments. In order to address these limitations, this paper uses deep displacement monitoring data and $p$ value testing to verify the simulation parameters, and puts forward an analysis method for dual-structure slope excavation and reinforcement processes.

The simulation in Section 3.3.1 is carried out without considering the steady-state seepage. In working conditions 1–8, it was found that the calculated slope safety factor may be limited by local failure or near failure. Thus, determining the overall stability of the slope required further analysis of the horizontal displacement and
effective plastic strain of the slope. Due to greater excavation at the lower slope relative to the upper slope, the early stages of excavation led to further increases in the maximum horizontal displacement inside the broken body of the slope and expansion of the continuous plastic strain distribution. This was the root cause of the many cracks that appeared above the slope during the initial stages of excavation. In the later stages of slope excavation, the horizontal displacement and plastic strain of the soil above the slope were controlled by the anchor cables and the anti-slide piles in the upper slope. However, the local stability of the lower part of the excavated slope worsened. Thus, timely reinforcement is needed to prevent a new instance of sliding.

The analysis of working conditions 9–11 showed that when there is locally increased displacement and plastic strain, the maximum values of these parameters decreases with the size of the strained area. Furthermore, the reference value of the safety factor decreases under these conditions. Analysing the overall safety factor of the slope during the excavation and support construction processes, it was found that the most dangerous conditions occurred during the secondary excavation stage. At this stage, the areas of maximum horizontal displacement and plastic strain are superimposed, which is extremely unfavourable for the overall stability of the slope, and the safety factor of the slope drops accordingly. By analysing the maximum values and distribution characteristics of the horizontal displacement and plastic strain of the slope during the excavation and reinforcement process, it becomes clear that, in some cases, a comparison of these parameters between different slope states is necessary to make an accurate judgement regarding potential slope treatments.

The simulation in Section 3.3.2 is carried out with considering the steady-state seepage. Analysis of the slope before and after the application of pore water pressure at different stages of slope excavation and primary support construction (conditions 2–8) revealed that pore water pressure did not cause a significant change in the location of the maximum horizontal displacement or plastic strain. However, the magnitude of the maximum values changed after considering pore water pressure, indicating that the influence of seepage flow on the deformation of the slope during excavation and reinforcement is complex. Based on this finding, the phased fluctuation in horizontal displacement and the resulting strain cannot be used as the criterion of overall slope stability when performing design calculations and monitoring construction. Significantly, the safety factor of the slope was reduced when considering pore water pressure, which provides a clear indication of the adverse effects of seepage on overall slope stability. The inclusion of pore water pressure also caused the continuous distribution of plastic strain in the upper slope to disappear for the model of the slope after the implementation of the secondary anchor cable for the primary support structure.

Under steady-state seepage conditions, the application of pore water pressure makes the top of the third grade of the slope appear unstable when the secondary support structure is added to the model (conditions 9–11). This effect interferes with the horizontal displacement and plastic strain analysis. The calculation results show that the true distribution of the maximum horizontal displacement and plastic strain is at the shear exit of the potential slip surface, which is the first grade of the slope. Therefore, the lowest slope safety factors were calculated for the top of the third grade of the slope for the conditions considering the secondary support structure; the
overall safety factor of the slope was ‘covered’. In these cases, the horizontal displacement and plastic strain at the shear exit position of the potential sliding surface were used as the quantitative analysis standards. No statistical analysis was performed for the maximum horizontal displacement, the maximum plastic strain or the safety factor. In addition, the continuous distribution of plastic strain along the upper slope observed without consideration of steady-state seepage for conditions 10 and 11 disappears when seepage is considered.

5. Conclusions

In this study, the horizontal displacement, plastic strain, and safety factor of the Hetang Dam dual structure slope before and after considering the pore water pressure were compared and analysed. The conclusions of this work are as follows:

1. Carrying out on-site deep displacement monitoring, and proposing a numerical simulation method for the construction process of a dual structure slope. The theory of $p$ value verification based on the results of on-site deep displacement monitoring is established. Propose and use the superposition calculation of pore water pressure to realize the seepage-stress coupling of the dual structure slope.

2. When pore water pressure was not applied, early-stage slope excavation caused extreme horizontal displacement and continuous plastic strain inside the broken body. The analysis shows that this is because the lower slope of the excavation is relatively similar to the upper slope of the excavated slope foot. Importantly, the expansion of the displacement distribution zone is extremely detrimental to the overall stability of the slope. In the later stages of slope excavation, due to the action of the anchor cables and anti-slide piles during the third and fourth grades of excavation and support construction, the horizontal displacement and plastic strain of the soil above the slope are controlled, but the local stability of the lower part of the excavated slope worsens. Local displacement and plastic strain on a confined area of the slope reduced the slope safety factor. The most dangerous working condition in the slope engineering project was the secondary excavation stage. At this stage, the magnitude of the maximum value of the horizontal displacement and the plastic strain, the location, and the plastic penetration zone are superimposed. This is extremely unfavourable to the overall stability of the slope and caused the safety factor of the slope to decrease abruptly.

3. When pore water pressure was considered, there is no obvious change in the extreme position of the horizontal displacement and plastic strain in the working condition 2–8, but the extreme value changes differently. Seepage has a complex influence on the stress and deformation of a slope during excavation and reinforcement. The safety factor of the slope was reduced when pore water pressure was considered compared with the safety factor of the slope without considering pore water pressure. Thus, seepage has an adverse effect on the overall stability of the slope. When the secondary anchor cable of the primary support was considered, the continuous distribution zone of plastic strain in the upper slope disappeared. When considering the secondary supporting structure, a small
area at the top of the third-grade slope was unstable, which interfered with the horizontal displacement and plastic strain analysis. Thus, the horizontal displacement and plastic strain data at the shear exit position of the potential slip surface should be used as the quantitative analysis standard.

The method of validating simulation parameters based on deep displacement monitoring data and \( p \) value inspection that was used in this work is a simple parameter verification method. Thus, this approach can be easily extended to other commercial or self-programming software tools and may help bridge the gap between field monitoring and numerical simulation. However, due to uncontrollable factors such as the simulation boundary determination, grid division and construction definition, the supporting structure process analysis method proposed in this research needs to be further investigated prior to broader adoption of this approach.

**Ethical statement**

This manuscript has not been published or presented elsewhere in part or in entirety and is not under consideration by another journal. We have read and understood your journal’s policies, and we believe that neither the manuscript nor the study violates any of these. There are no conflicts of interest to declare.

**Disclosure statement**

No potential conflict of interest was reported by the authors.

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**Data availability statement**

The data that support the findings of this study are available from the corresponding author, Xuhe Gao, upon reasonable request.

**References**

Chen P, Zhou Z, Cao L. 2013. Study on shape optimization and instability mechanism of soil-rock composite slope in open-pit mine. World Sci Technol Res Dev. 6:690–693.

Editorial Board of Engineering Geology Manual. 2018. Handbook of engineering geology. 5th ed. Beijing, China: China Construction Industry Press.

Gholampour A, Johari A. 2019. Reliability-based analysis of braced excavation in unsaturated soils considering conditional spatial variability. Comput Geotech. 115:103112–103163.
Guo Z, Yin K, Tang Y, Huang F, Fu X. 2017. Geological engineering risks and safety investment decision. Geol Sci Technol Inf. 4:260–265.

Johari A, Kalantari AR. 2021. System reliability analysis of soldier-piled excavation in unsaturated soil by combining random finite element and sequential compounding methods. Bull Eng Geol Environ. 80(3):2423–2485.

Lei D, Yi W, Liu Q, Xia J. 2018. Reliability and sensitivity analysis of Woshaxi Landslide stability in Three Gorges Reservoir Area. Saf Environ Eng. 25:23–28.

Liang Q, Wang L, Sun W, Liu G. 2014. Seismic performance of anti-slide piles in layered slopes with dual structure. Earthq Eng Eng Vib. 34:248–255.

Liu Y. 2008. Hydraulics. Beijing, China: China Water Resources Press.

Li Z, He Y, Sheng J, Li H, Li Z, Yang Y. 2017. Landslide model for slope of reservoir bank under combined effects of rainfall and reservoir water grade. Chin J Geotech Eng. 39:452–459.

Lu B, Guo Y, Zhao E, Gong Z, Wang C. 2017. Analysis of seepage characteristics and stability of a slope under conditions of reservoir water grade fluctuation and rainfall. J China Three Gorges Univ (Nat Sci). 39:54–59.

Mei L, Wang S, Tong K, Cai Z. 2015. Experimental study on the influence of different interfaces on the failure of dual structure slopes. J Water Resour Arch Eng. 13:6–11. issn. 1672-1144.2015.03.002

Tang X, Zheng Y, Tang H. 2013. Numerical analysis of slope deformation and failure evolution characteristics. J Chongqing Univ. 10:101–113. 2013.10.016

Tang Y, Yin K, Tang Z. 2017. Research on the regulation of rain infiltration in the Sanzhouxi landslide based on HYDRUS. Hydrogeol Eng Geol. 44:152–156. 163.

Tian S, Guo Q, Wu D. 2013. Deformation mechanism analysis of soft rock composite slope based on FLAC3D. Open-Pit Mining Technol. 8:31–35.

Tian W, Wu B, Du M, Wang C. 2013. Stability analysis of soil-rock composite slope based on three-dimensional numerical simulation. J Water Resour Arch Eng. 2:52–57.

Xu T, Fu M, Yang SQ, Heap MJ, Zhou GL. 2021. A numerical meso-scale elasto-plastic damage model for modeling the deformation and fracturing of sandstone under cyclic loading. Rock Mech Rock Eng. 54(9):4569–4591.

Xu Y, Qi X, Zhang N. 2016. Numerical simulation and stability analysis for the seepage flow in the Sanzhouxi landslide under the associative action of reservoir water grade fluctuations and rainfall infiltration. Hydrogeol Eng Geol. 43:111–118.

Yuan Y, Xu T, Heap MJ, Meredith PG, Yang T, Zhou G. 2021. A three-dimensional mesoscale model for progressive time-dependent deformation and fracturing of brittle rock with application to slope stability. Comput Geotech. 135:104160.

Yu S, Zhang J, Wang J, Wang T, Zhu W, Hu N. 2017. Seepage and slope stability analysis under different rainfall patterns based on Fredlund & Xing parameters. J China Three Gorges Univ (Nat Sci). 39:46–51.

Yu XY, Xu T, Heap MJ, Baud P, Reuschle T, Heng Z, Zhu WC, Wang XW. 2021. Time-dependent deformation and failure of granite based on the virtual crack incorporated numerical manifold method. Comput Geotech. 133:104070.

Zhang Z, Gao X. 2016. Midas/GTS-based loess slope stability analysis method and its application. J Water Resour Archit Eng. 4:182–185. 3969/j. issn. 1672-1144. 2016. 02. 036

Zheng Y, Chen Z, Wang G, Ling T. 2010. Engineering treatment of slope & landslide. 2nd ed. Beijing, China: People’s Communication Press.

Zhou GL, Xu T, Konietzky H, Zhu W, Heng Z, Yu XY, Zhao Y. 2022. An improved grain-based numerical manifold method to simulate deformation, damage and fracturing of rocks at the grain size level. Eng Anal Boundary Elem. 134(1):107–116.

Zhou GL, Xu T, Heap MJ, Meredith PG, Mitchell TM, Sesnic ASY, Yuan Y. 2020. A three-dimensional numerical meso-approach to modeling time-independent deformation and fracturing of brittle rocks. Comput Geotech. 117:103274.