Quantum phase transitions with parity-symmetry breaking and hysteresis

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Symmetry-breaking quantum phase transitions play a key role in several condensed matter, cosmology and nuclear physics theoretical models1–3. Its observation in real systems is often hampered by finite temperatures and limited control of the system parameters. In this work we report, for the first time, the experimental observation of the full quantum phase diagram across a transition where the spatial parity symmetry is broken. Our system consists of an ultracold gas with tunable attractive interactions trapped in a spatially symmetric double-well potential. At a critical value of the interaction strength, we observe a continuous quantum phase transition where the gas spontaneously localizes in one well or the other, thus breaking the underlying symmetry of the system. Furthermore, we show the robustness of the asymmetric state against controlled energy mismatch between the two wells. This is the result of hysteresis associated with an additional discontinuous quantum phase transition that we fully characterize. Our results pave the way to the study of quantum critical phenomena at finite temperature4, the investigation of macroscopic quantum tunnelling of the order parameter in the hysteretic regime and the production of strongly quantum entangled states at critical points5.

Parity is a fundamental discrete symmetry of nature6 conserved by gravitational, electromagnetic and strong interactions7. It states the invariance of a physical phenomenon under mirror reflection. Our world is pervaded by robust discrete symmetries, spanning from the imbalance of matter and antimatter to the homo-chirality of DNA of all living organisms8. Their origin and stability is a subject of active debate. Quantum mechanics predicts that asymmetric states can be the result of phase transitions occurring at zero temperature, named in the literature as quantum phase transitions (QPTs)1,4. The breaking of a discrete symmetry via a QPT provides also asymmetric states that are particularly robust against external perturbations. Indeed, the order parameter of a continuous-symmetry-breaking QPT can freely (with no energy cost) wander along the valley of a `mexican hat' Ginzburg–Landau potential (GLP) by coupling with gapless Goldstone modes9. In contrast, the order parameter of discrete-symmetry-breaking QPTs is governed by a one-dimensional double-well GLP10. The reduced dimensionality suppresses Goldstone excitations, and the order parameter can remain trapped at the bottom of one of the two wells. This provides a robust hysteresis associated with a first-order QPT.

Evidence of parity-symmetry breaking has been reported in relativistic heavy-ions collisions11 and in engineered photonic crystal fibres12. Observation of parity-symmetry breaking in a QPT has been reported for neutral atoms coupled to a high-finesse optical cavity13. However, this is a strongly dissipative system, with no direct access to the symmetry-breaking mechanism necessary to study the robustness of asymmetric states. In addition, previous theoretical studies14,15 have interpreted the puzzling spectral properties of a gas of pyramidal molecules that date back to the 1950s (ref. 16), in terms of the occurrence of a QPT with parity-symmetry breaking.

In our system, the atomic ground state depends on two competing energy terms in the Hamiltonian \( H = H_a + g H_b \), where \( H_a = \int \mathrm{d} \mathbf{r} \, \Psi^\dagger(\mathbf{r}) \left(-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r})\right) \Psi(\mathbf{r}) \) includes kinetic and potential energy, and \( H_b = (2\pi\hbar^2 a_0/m) \int \mathrm{d} \mathbf{r} \, \Psi^\dagger(\mathbf{r}) \Psi(\mathbf{r}) \Psi(\mathbf{r}) \Psi^\dagger(\mathbf{r}) \) accounts for contact interaction between the atoms. Here, \( \Psi(\mathbf{r}) \) is the many-body wavefunction, \( \Psi^\dagger(\mathbf{r}) \) its hermitian conjugate (in the following we consider normalization \( \langle \Psi^\dagger(\mathbf{r}) \Psi(\mathbf{r}) \rangle = 1 \)), \( m \) the atomic mass, \( a_0 \) is the Bohr radius, \( \hbar \) the reduced Planck constant and \( V(\mathbf{r}) \) is a double-well trapping potential in the \( x \) direction (see Fig. 1a) and a harmonic trap in the orthogonal plane. The adimensional control parameter \( g = N a_0/\alpha \) is the product of the total number of atoms \( N \) and the scattering length \( a_0 < 0 \) characterizing the interatomic attractive interaction. The full many-body Hamiltonian is invariant under \( x \leftrightarrow -x \) mirror reflection. This parity symmetry imposes a spatially symmetric ground state for any value of the control parameter \( g \). Because \( H_a \) and \( H_b \) do not commute, the corresponding ground states are quite different. \( H_a \) is minimized by each atom equally spreading on both wells. A finite energy gap, specified as the tunnelling energy \( J \), separates the ground and the first (antisymmetric) excited state of \( H_a \). \( J \) can be tuned by controlling the height of the potential barrier between the two spatial wells. In contrast, \( g H_b \) is minimized by a linear combination of two degenerate states, one having all atoms localized in one well, the second with all atoms localized in the other well. Thanks to the competition in the Hamiltonian between the effective repulsion due to the kinetic energy and the attractive interatomic interaction, the energy gap between the two low-lying
phase transition can be theoretically described by an effective GLP \( W(z) \) within error bars. Below the critical value, that is, for \( g < g_c \), an ordered phase emerges with a non-zero order parameter that, in our case, is the normalized atomic population imbalance \( z = (N_l - N_h)/N \), where \( N_l \) and \( N_h \) are the number of atoms occupying the left and the right well, respectively.

To measure \( z \) across the phase transition (see Fig. 1b) we adopt the following experimental procedure. We start by cooling a gas of \( N = 4,500 \) atoms well below the Bose–Einstein condensation point until no thermal fraction can be detected (see Supplementary Information). The atoms are initially trapped in a harmonic potential, with a positive scattering length \( a_0 = 3a_0 \). We reach different target values of \( g \), above and below the critical point, by continuously transforming the harmonic trap into a double well, up to a certain barrier height and tunnelling \( J \approx 40 \) Hz, and by tuning the interatomic scattering length to negative values (see Supplementary Information). When \( g > g_c \), we find the system in a parity-symmetric disordered phase, with order parameter \( z \approx 0 \) within error bars. Below the critical value, that is, for \( g < g_c \), an ordered phase emerges with \( z \) driven away from zero. The phase transition can be theoretically described by an effective GLP \( W(z) = (\tilde{g}/2)z^2 - \sqrt{1-z^2} \), where \( \tilde{g} = gU/J \) is the normalized control parameter and \( U \) is the bulk energy (see Supplementary Information). When \( \tilde{g} \) crosses the critical value \( \tilde{g}_c = -1 \), the shape of \( W(z) \) continuously changes from a parabola to a double well (see Fig. 1c) with minima located at \( z = \pm \sqrt{1-\tilde{g}^2} \) (ref. 19). The continuous variation of \( z \) indicates the occurrence of a continuous phase transition. From the experimental measurements of the order parameter we obtain a critical value \( \tilde{g}_c = -1.3 \pm 0.2 \) (corresponding to a critical value of the interaction strength \( a_0 = -1.8 \pm 0.3a_0 \), in fair agreement with the theoretical prediction. The error bar is the quadratic sum of the errors coming from the fit in Fig. 1b, the atom number measurements and the estimation of the lattice depths (see Supplementary Information). We believe that the slight disagreement with respect to the expected value is mainly due to uncontrolled jumps in the value of the magnetic field whose origin has not been identified. This noise could result in a systematic shift of the interaction strength of approximately \( \pm 10\% \) (see Supplementary Information).

A one-dimensional GLP, which depends on a single real parameter, describes a phase transition with the breaking of a discrete symmetry as, for instance, the left–right symmetry in our case, or the spin-up/spin-down symmetry in the paramagnetic-to-ferromagnetic transition4. In these systems, a controlled symmetry-breaking term (in our case provided by an energy gap \( \delta \) between the two wells, see Fig. 2a) drives a first-order QPT in the ordered region of the phase diagram. This can be understood from the sudden variation of the absolute minimum of the GLP \( W(z) + \delta z \)
Figure 3 | Susceptibility and measurement of the critical exponent.
Susceptibility \( \chi = |dz/d\delta| \) of the system to potential energy gap \( \delta \) between the two wells. The measurement is performed close to the critical point for values \( g > g_c \). The curve is a fit to the data (see text) providing a critical exponent \(-1.0 \pm 0.1\) in excellent agreement with the theoretical prediction, equal to \(-1\). The error bar in the critical exponent comes from the indetermination of \( g_c \) (grey region).

Discrete-symmetry models are characterized by metastability and hysteresis when driving the system across the first-order transition. Both follow directly from the 1D nature of the effective GLP. We notice that bifurcation and hysteresis are typical phenomena in the dynamics of systems governed by a nonlinear equation of motion\(^{19,24,25}\). In our system, the origin of hysteresis can be understood from the shape of the GLP \( W(z) + \delta z \), which for \( g < -1 \) and \(|\delta| < \delta_c = [(g - 1)^{1/3} - 1]^{1/3} \) shows an absolute minimum and a local minimum (see Fig. 4b and Supplementary Information). The latter corresponds to a metastable point with a lifetime depending on the macroscopic quantum tunnelling rate of the order parameter through the effective GLP. This rate is exponentially smaller than the interwell tunnelling rate of the single atoms in the double-well trap. To demonstrate hysteresis in our system, we set \( J \approx 30 \text{ Hz}, \) add an energy gap \( \delta_0 \), and prepare a condensate in the well with lower energy (for example the right one, with \( \delta_0 = -4J < 0 \) and \( z \approx 1 \)). We then shift the relative energy of the two wells to a final value \( \delta \) in 500 ms, keeping \( J \) constant, and measure the order parameter after a short waiting time of 10 ms (green squares in Fig. 4c). The experiment is performed for different values of the control parameter \( g \). When \( g < g_c \), the strong attractive interaction between atoms forces the condensate to remain localized in the right well even when its energy minimum is lifted above the left well. When \( \delta \) overcomes a critical value \( \delta_c \), a spinoidal instability drives the gas down to the left towards the absolute minimum of the trapping potential in a timescale of approximately 10 ms, a fraction of \( 1/J \). An analogous behaviour is observed with an initial imbalance \( z \approx -1 \) (orange circles in Fig. 4c), forming a hysteresis loop. The area of the hysteresis loop decreases with increasing \( g \), and disappears for \( g > g_c \) (see Fig. 4c).

This work paves the way to the study of macroscopic quantum tunnelling in the hysteretic regime in the context of the quantum-to-classical transition problem\(^{26}\). Furthermore, it will be interesting to explore spontaneous symmetry breaking in gas mixtures as a function of the interspecies interactions\(^{27,28}\). Finally, our system will allow investigation of the creation of quantum
fluctuations$^{29}$ and entanglement$^2$ at the critical points as a resource for precision measurements$^{30}$ and other quantum technologies$^{31}$.

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Author contributions

A.T.F., G.S tingoli and M.F. designed the experiment and performed the measurements. I.P. and A.S. worked on the theoretical model. All authors participated in the data analysis, discussion of the results and writing of the manuscript.

Additional information

Supplementary information is available in the online version of the paper. Reprints and permissions information is available online at www.nature.com/reprints. Correspondence and requests for materials should be addressed to M.F.

Competing financial interests

The authors declare no competing financial interests.