Reducing Ground-Based Astrometric Errors with \textit{Gaia} and Gaussian Processes

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See W. F. Fortino \textit{et al} 2021 \textit{AJ} 162 106
Also see arXiv:2103.09881 (Léget \textit{et al}) for similar work on HSC
The Dark Energy Survey and the Dark Energy Camera

- Victor M. Blanco Telescope (Blanco) at CTIO
- Dark Energy Camera (DECam)
  - Given to CTIO in exchange for observation time
  - 62 2048×4096 CCD tiles (520MP total)
  - iPhone 12 Pro has a 12MP camera.
- >5000 deg²
- 758 Observing Nights over 6 years
- Goal: Looking for evidence of Dark energy and Dark matter, and more

NGC 1365, 60 Mly away
Focal plane

Utility Trunk—houses support electronics and utilities

Cryostat—contains focal plane & its electronics

L3 Lens
Filter
L2 Lens
L1 Lens

1.65 m (5'-5'')

1.65 m (5'-5'')
The Vera C. Rubin Observatory
and the Legacy Survey for Space and Time

- Near CTIO
- The successor to DES
- >18000 deg$^2$
- 3200MP Camera
- 10 year survey
- Goal: Dark Energy, Dark Matter, +
Atmospheric Turbulence

Why do stars twinkle?
The Method:

1. Registered Y6 DES to *Gaia* DR2
   • Constructed the residual field of positions

2. Trained a Gaussian Process regression model
   • Minimized correlated variance in the residual field
   • Used a custom kernel based on known physics

3. Calculated the reduction in correlated variance
Gaussian Process Regression (GPR)

Gaussian Process

• “A collection of random variables, any finite number of which has a joint gaussian distribution.”

• Can be thought of as a “distribution over functions.”

\[
\begin{align*}
    f(x) &\sim \mathcal{GP}(m(x), k(x, x'; \Theta)) \\
    m(x) &= \mathbb{E}[f(x)] \\
    k(x, x'; \Theta) &= \mathbb{E}[(f(x) - m(x))(f(x') - m(x'))]
\end{align*}
\]
The GPR Model

Model Inputs, $X$

- DES Astrometric Positions
  \[ x_i^{\text{DES}} = (\alpha_i, \delta_i) \]

Model Targets, $y$

- DES – Gaia Residuals
  \[ y_i = x_i^{\text{DES}} - x_i^{\text{Gaia}} \]

Kernel

- $k(x, x'; \Theta)$

Joint distribution of the observed target values, $y$, and the function values, $f_*$, at the test locations, $X_*$.

\[
\begin{bmatrix} y \\ f_* \end{bmatrix} \sim \mathcal{N} \left( 0, \begin{bmatrix} K(X, X) & K(X_*, X) \\ K(X_*, X) & K(X_*, X_*) \end{bmatrix} \right)
\]

$K(X_*, X)$ is an $n_* \times n$ matrix of covariances evaluated at all pairs of training and test points.

Posterior Predictive Mean

\[
\bar{f}_* = K(X, X_*)^T K(X, X)^{-1} y
\]

Noisy Posterior Predictive Mean

\[
\tilde{f}_* = K(X, X_*)^T (K(X, X) + W_{\text{DES}} + W_{\text{Gaia}})^{-1} y
\]
Atmospheric Turbulence

• Coherence length of ~10 arcmin
• Amplitude and patterns change unpredictably
• Clearly anisotropic in both amplitude and coherence length

Gaia DR2 Astrometry

• < 1 mas RMS error for G < 20 mag
• ~ 1 Star per arcmin$^2$
Curl-Free = gradient of some field

Exposure 228645, z-band
30 seconds
$\phi$

Wind

$y = \nabla \phi$
A model for astrometric distortions

Caused by fluctuations in the index of refraction integrated along line of sight, $\phi(x)$

- $y(x) = \nabla \phi(x)$
- $\phi$ is well approximated as a Gaussian random field, with power spectrum $P_\phi(k)$

$\rightarrow$ Gaussian-process interpolation will be optimal if we choose the right power spectrum or correlation function for the field.

**IF** the turbulence is confined to a single layer in the atmosphere, then we expect a power spectrum of index fluctuations following this model:

$$P_\phi(k) \propto (k^2 + k_0^2)^{-\frac{11}{6}} \left( \frac{J_1(kD/2)}{kD/2} \right)^2 \text{sinc}^2 \left( \frac{k \cdot w}{2} \right)$$

**Parameters**
- Outer Scale, $\theta = 2\pi/k_0$
- Diameter, $D$
- Wind Vector, $w$
- Total variance, $\sigma^2$

von Karman turbulence  
Integration over telescope aperture  
Time integration of wind motion
\[ \xi(r) \equiv \langle y(x)y(x + r) \rangle \]
Non-standard GPR aspects

• Anisotropic kernel - not hard to include in the linear algebra

• Dimensions of $y$ are not independent - expect a curl-free field.
  • $x$ and $y$ data can inform each other.
  • We derived a variant of the GPR formulae which enforces/exploits this, by forming a single $(2N \times 2N)$ covariance matrix for $N$ points.
  • Also allows for anisotropic measurement errors (as in Gaia)
Procedure

• Acquire DES and Gaia information for bright stars of every exposure, calculate $y$ for these & clip outliers.

• Calculate $\xi(x)$ of $y$

• Fit von Kármán model to $\xi_{\text{raw}}$
  • Execute GP, clip outliers in residuals

• Run an optimizer to minimize output $\xi_{\text{resid}}(x \lesssim 0.5^\prime)$ of validation set over kernel parameters
  • This is slow since it requires 50-100 evaluations of GP
Results - avg of ~300 exposures spread over years

\[
\xi_+(r) \text{ [mas}^2]\]

Pair Separation [deg]

$R_0 = 1.15'$

$R_0 = 1.34'$

$R_0 = 5.70'$
Mean $\xi_0$ Reduction
- r band: $11.689 \pm 1.675$
- i band: $12.446 \pm 1.772$
- z band: $11.956 \pm 1.604$
- Y band: $14.329 \pm 2.264$
A stringent test: calculate residuals to a Trans-Neptunian Object orbit fit as it moves a few degrees across the field over 5 years (Note it’s not so bright that shot noise is negligible)
Key Takeaways

- **Turbulence induced variance:**
  - 7 mas RMS → 2 mas RMS
- **Correlation length:**
  - 5.7 arcmin → 1.2 arcmin
- **Orbit Fitting — Eris:**
  - 10 mas RMS → 5 mas RMS
- **Room for Improvement:**
  - Y6 DES → LSST
  - Simultaneous solution for turbulence/proper-motion
  - von Kármán Kernel is not necessarily optimal
Thank you for listening!