Analysis of blade tip timing data from fan blades with synchronous and non-synchronous vibration

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Abstract. Blade tip timing (BTT) as an alternative method is capable of directly measuring the blade tip displacement and inferring the vibration characteristics of rotating blades. Its use is growing in popularity due to its ease of application and the ability to deliver information concerning all blades in a rotor. Although the interpretation of the collected data remains an issue, this paper puts forward a process of using such data to analyse the vibrational characteristics of fan blades under different operating conditions and find the blade endurance margin combined with finite element analysis (FEA). A test system with eight casing-mounted optical probes and one OPR (once per revolution) sensor is built to collect the blades’ arrival time of an axial fan. Results indicate that flutter occurs at engine order (EO) 1.5 with a combination of three nodal diameters (2, 3, 4), however, nodal diameter 2 is dominant. A noticeable response (EO≈0.5) is considered as stall. Because every blade vibrates with a frequency lower than its natural frequency (eigenfrequency), and all of them undergo a similar vibration form, continuously rising and falling, which caused by the rotating stall cells. The 1st flex mode – EO2 crossing is found at 48% rotor speed and it’s consistent with the results of FEA. Finally, the blade endurance in Goodman diagram demonstrates that flutter poses a higher risk in comparison with the synchronous vibration.

1. Introduction

As regards the turbomachinery, there are two kinds of vibrations, which are referred to as the synchronous and non-synchronous vibration. The former one is defined by the blade vibrational frequency being an integer multiple of the rotor speed, and it’s normally due to the excitation of upstream stator blade-rows. Similarly, the non-synchronous vibration means the blade vibrational frequency is non-integral with respect to rotor speed, and it’s a result of aerodynamic instabilities, i.e., rotation stall and flutter. Both synchronous and non-synchronous vibrations can be the causes of blade failure. Current engine development involves a variety of compliance tests to ensure the safety of products. Over the years, strain gauges have been the first option for the measurement of blade stress for turbomachinery. Recently, BTT, a non-contact measurement method, has become a more promising alternative technique for inferring blade tip displacement using the arrival time of blades measured by a number of casing-mounted probes. Compared with the traditional measurement technique, one of its greatest strengths is the ability to measure the blade tip displacements of all rotor blades in one stage.

A typical BTT system has several probes mounted in the casing to obtain blades’ arrival time, and the OPR sensor located above the shaft, which provides the reference time and blade indexing.
information. Compared with capacitance and eddy current probes, the optical probe has the best spatial resolution [1]. Therefore, it is selected in this case and it’s readily available. The optical probe has two paths, one for laser light emission, and the other one to receive reflections from the blade tip. A timing system will record the time of arrival (TOA) of the blade. If there is no vibration, the blade arrival time can be calculated just by the rotor frequency (\( \Omega \)) and probe installation angle (\( \theta_i \)), and it is defined as the theoretical time (equation (1)). However, the blade vibration always exists, so the vibrating blades will reach the probe slightly earlier or later as shown in figure 1. With the knowledge of the rotor frequency and blade tip radius (\( r \)), the blade tip displacement (\( d_i \)) at each probe can be calculated by the time difference between measurement and theory as in equation (2).

There are many published methods with different configurations of probes and algorithms in order to identify the parameters of blade vibration. They can be divided into two groups according to the blade vibrational characteristics. For synchronous vibration, Zablotskiy and Korostelev [2] developed a “One parameter” method in the 1970s, which can obtain the vibrational amplitude based on its phase-frequency characteristics. Zielinski, Ziller [3] and Heath [4] proposed a two-parameter plot method used two probes to evaluate the blade amplitude and frequency, but it's sensitive to noise and has low anti-interference [5]. By using several probes circumferentially equispaced in the casing, Gallego-Garrido put forward a new class of methods for two simultaneous resonances based on an autoregressive framework, and it's validated by the experiment [6][7]. For non-synchronous vibration, the known techniques are single-blade and all-blade spectrum analysis presented by Heath. He suggested the resonance frequencies cannot be identified reliably using the single-blade spectra technique, on account of a restricted number of samples; and all-blade spectrum analysis is under the assumption that the bladed-disk assembly vibrates in nodal diameter modes [8].

In order to deal with the synchronous and non-synchronous vibration together, Russhard [9] developed a process to manipulate the raw data of blades’ arrival time and he successfully extracted the vibrational amplitude, frequency, and phase angle of every blade, which can shorten the development cycles of turbomachinery [10]. The complete equation to describe blade vibration at any probe is expressed as:

\[
d_i = P_i + (a_0 + a_1 \sin(EO \times \theta) + a_2 \cos(EO \times \theta)) + (b_1 \sin(feo \times \theta) + b_2 \cos(feo \times \theta)) + \bar{n}
\]  

(3)
Where $P_i$ represents individual probe steady positional offset, and $n_i$ is the noise term. The integer ($EO$) and non-integer ($feo$) are defined by the ratio of blade vibrational frequency to the rotor frequency. $a_0, a_1, a_2, b_1, b_2$ are unknown parameters that need to be calculated.

However, knowing blade vibrational parameters is not enough. The main goal of blade test is to find the stress distribution of every blade and ensure that each of them is qualified for application. Therefore, based on Russhard’s work, this paper presents a complete process to explain how to interpret BTT data from fan blades under different vibrational types as shown in figure 2. The first part includes all the preparatory work: FEA and raw data processing, etc. In phenomena identification section, both synchronous and non-synchronous response can be observed directly by the traveling wave plot. Then, the traditional sine fitting (Least Square Model) method is used to calculate the amplitude and phase through the estimated blade frequency, which can be obtained in Campbell diagram. The second part mainly focuses on how to use the measured displacement of one point to deduce the distribution of blade stress for a specific mode.

![Flow chart of BTT data processing.](image)

**Figure 2.** Flow chart of BTT data processing.

### 2. Measurement setup

Figure 3 shows the BTT testing system in the laboratory. Eight probes are mounted in the casing and one OPR sensor is employed for measuring the rotor speed. The first blade passing from probe 1 is labeled as blade 1, and the remaining blades are stamped successively against the rotating direction. All the probes are located in one plane perpendicular to the shaft. The laser wavelength is 625nm, and a minimum number of fibers are used allowing higher measurement resolutions to be obtained; in addition, the input bandwidth and timing resolution of timing card are 3MHz and 15ns respectively. When the laser signal triggers the timing unit, a time value will be created and stored for further processing. The operating temperature keeps below 250°C to ensure that all is well.
The change of rotor speed against revolution is presented in figure 4. With the slow increase in the speed, there are three marked regions where the blade vibrational amplitude is very large, in particular the third one. The reason for speed dropping after 5300 revolutions approximately is to protect blades. Data analysis only throws the emphasis on those three regions.

3. Methodology and results

3.1. Finite element analysis
The finite element analysis is a convenient way to obtain the blade natural frequencies and the corresponding mode shapes, and the probe axial position is chosen in accordance with those mode shapes. It should avoid the node at which there is no displacement. Given the blade axial movement, the probe axial position is selected near trailing edge. Furthermore, in order to precisely pinpoint the location of laser spot for the realization of the conversion from displacement to stress, 120 nodes are arranged in the chord direction so that the averaged distance between two nodes is about 2mm.

Figure 5 shows the first four modes of a blade obtained by ANSYS® modal analysis under the presstressing force as a result of rotor speed. Campbell diagram comprises the eigenfrequencies and EO lines (black lines) as in figure 6, and the intersections (e.g. red point) show the possible resonant
position. Therefore, it is clear to observe where the synchronous vibration might happen with the help of Campbell diagram before conducting any experiments.

3.2. Raw data processing

The circumferential spacing of the probes results in misaligned data arrangement by default. Therefore, the first collected arrival time by each probe corresponds to a different blade number as shown in figure 7. The first subscript of TOA represents the revolution, and the second one indicates which blade is passing from the probe.

![Figure 7. Data storage.](image)

Taking probe 1 as a reference, the number of blades ($N$) between two probes can be calculated by:

$$\alpha = \frac{360^\circ}{N_b}$$  \hspace{1cm} (4)

$$N = \left\lfloor \frac{\theta_1 - \theta_i}{\alpha} \right\rfloor$$ \hspace{1cm} (5)

Where $N_b$ is the number of blades, and $\alpha$ implies the pitch of cascade.

Therefore, the correct timing order for each probe is to move $N$ elements to the front respectively in each revolution as shown in figure 8.

![Figure 8. Data alignment (P2).](image)

3.2.1. Stack plot. In order to validate the procedure of data alignment, the stack plot representing the blade steady state displacement is introduced. By applying equation (2), TOA data can be converted into the displacement. In figure 9, because each blade in a rotor has a different position to others due to mechanical tolerances. This leads to the steady-state displacement measured in the non-vibrating blade to be a value other than zero. Although the displacements of each probe have some
discrepancies, the displacement trend line of each probe is similar. This result is in accord with the expectation, and the data has been aligned.

![Figure 9. Stack plot.](image)

### 3.2.2. Data zeroing

With the help of OPR sensor, it's convenient to identify the start of each revolution. Hence, the blade displacements are calculated by this reference time instead of its theoretical value. However, this manipulation can create a large DC offset (figure 10) by contrast with the blade vibrational displacement itself. Before the next step, the data must be zeroed.

![Figure 10. Data before and after zeroing.](image)

There are many different methods to conduct the procedure of data zeroing. In this case, the piecewise linear fitting is used for each probe's data as their respective DC offset. The major advantage of this method is that it only removes the DC offset and has no effects on dynamic content. Accordingly, only blade vibration information and noise term are left after data zeroing. The error caused by this zeroing procedure will be discussed in “Sine fitting” section.

### 3.3. Phenomena identification
A method to detect a response in BTT data is termed as the traveling wave plot, which produces similar data to that of inter-stage pressure transducers, only with better fidelity. Because those transducers are located in the flow field, they carry various unwanted signals with different frequencies. The traveling wave plot in figure 11 is developed by creating a series of FFTs (fast Fourier transforms) using all blades’ data from a BTT probe which has been processed to remove the differences in the inter-blade spacing observed in the stack plot.

![Figure 11. Traveling wave plot.](image)

The abscissa axis divisions of traveling wave plot represent the number of FFT revolutions. Each FFT revolution contains several revolutions, which are aligned to the probe and number of blades to preserve the phase relationship for nodal diameter calculation. The ordinate axis is made up of engine order and nodal diameter, which can be deduced from:

\[ f_p = f_b \pm ND \times \Omega \]  

(6)

Where \( f_p \) (observed frequency) and \( f_b \) (blade frequency) represent the vibration frequency in stationary and rotating coordinate systems respectively. The sign associated with the nodal diameter indicates the direction of propagation of traveling wave. (+: forward-traveling wave; −: backward-traveling wave). Multiplying \( 1/\Omega \) on both sides of equation (6):

\[ f_p = EO \pm ND \]  

(7)

The nodal diameter \( (ND) \) can be expressed by:

\[ ND = \frac{\phi_2 - \phi_1}{\theta_2 - \theta_1} = \frac{\Delta \phi}{\Delta \theta} \]  

(8)

Where \( \Delta \phi \) is the difference between the two-phase angles at the peak frequency, which can be obtained by a cross-power analysis using the data from two probes, and \( \Delta \theta \) is the circumferentially included angle of those probes [11]. \( ND \) is an integer value. If the observed frequency by a BTT probe is also an integer, which means this should be a synchronous response \( (EO \in \mathbb{Z}^+) \), otherwise it is a non-synchronous one. Therefore, both synchronous and non-synchronous vibration can be observed in traveling wave plot directly. In order to find the true blade frequency, it still requires data from an additional probe to determine the nodal diameter and remove it from the observed frequency. With the knowledge of \( EO \) and rotor speed, the resonance point and blade modal shape can be identified in Campbell diagram.

Using these methods, it is possible to identify a synchronous vibration associated with \( M1-EO2 \) and two non-synchronous events marked in the red circle, A and B. According to the blade frequency, it's known that the first one is a response with \( EO < 1 \), possibly a stall; whereas, the second one has \( EO \approx 1.5 \) with a number of associated nodal diameters, possibly a stall flutter.

3.4. Sine fitting (Least square model)

In the raw data processing section, the DC offset has been removed, therefore, only the vibrational and noise terms are left. It’s worth mentioning that the \( P_i \) contains lots of information, not only the blade
lean, untwist and axial movement, but also the axial position of the blade. To extract those parameters requires other techniques and functions that are topics of research at other universities. It’s not the target of this paper. With the known engine order associated with a vibration event and the installation angles of probes, the blade amplitude and phase can be calculated by sine fitting method. The Pearson correlation coefficient ($R$) is used to quantify the degree of correlation between the calculated and zeroed blade displacements [12].

$$R = \left( \frac{1}{N_p - 1} \sum \left( \frac{D_i - \overline{D}}{S_d} \right) \left( \frac{d_i - \overline{d}}{S_d} \right) \right)^2$$

(9)

In which $N_p$ is the number of probes and $D_i$ represents the calculated displacement; $S_d$ ($S_d$) and $\overline{D}$ ($\overline{d}$) are the corresponding standard deviation and average value of displacements. The difference between the calculated and zeroed displacements will be classified as an error, and evaluated by the uncertainty ($u$):

$$u = \left( \frac{\max \{[d_i - D_i]\} - \min \{[d_i - D_i]\}}{2} \right) \times \max(D_i) \times 100\%$$

(10)

3.4.1. Non-synchronous vibration. As suggested by the traveling wave plot, a cross-power analysis using two probes data and the FFT are performed in area A (figure 12). The actual nodal diameters corresponding to the three relatively high peaks are calculated by equation (8).

Table 1. The observed and true blade frequency.

| Order | Frequency (Hz) | True |
|-------|----------------|------|
| 3.5   | 153.1          | 1.5  | 2   |
| 4.5   | 197.4          | 1.5  | 3   |
| 5.5   | 241.8          | 1.5  | 4   |

Figure 12. Fourier spectrum (FFT Revs=5300).

Table 1 shows the observed and true blade frequency including the corresponding nodal diameters. All nodal diameters are positive, which indicates they should belong to the forward traveling wave. Since there are more than one nodal diameter, the observed frequencies have several different values while blade frequency is a constant. Although the blade vibrational frequency at the current situation is not an integer multiple of the rotor speed, it's equal to the blade natural frequency of M1 according to the results of FEA. Therefore, it's concluded that flutter is encountered, and some synchronous responses in Fourier spectrum imply this is not pure flutter.

The FFT result of area B is presented in figure 13. There are two main spikes having nodal diameter 0, therefore the corresponding blade vibrational frequencies are far less than its natural frequency. It's a very distinctive phenomenon, and the Fourier spectrum containing several frequency components also indicates the internal flow fields are quite complex at this time.
Figure 13. Fourier spectrum (FFT Revs=2610).

Figure 14. The change of amplitudes during rotating stall.

Figure 15. Synchronous vibration from 1000 to 2400 revolutions.

Figure 14 shows the blade maximum amplitudes (peak-to-peak) are quite large, which exceeds our design objective. All blades experience a similar vibration form, continuously rising and falling, but there is a delay, not all blades reaching their own peak value at the same time. This vibrational behaviour can be explained by the movement of rotating stall cells. The propagation direction of stall cells is in the reverse direction to the rotor, hence every blade suffers from loading and unloading alternately. In addition, there are several blades that have relatively larger amplitudes and need to be checked further.

3.4.2. Synchronous vibration. Regarding the synchronous vibration, figure 15 shows the amplitude of each blade, together with their corresponding fitting coherence and EO. Compared with the non-synchronous vibration, the maximum vibration amplitude is smaller. The coherence values are in excess of 0.9 in the region from 1600 to 1950 revolutions. It can be found that EO2 crosses M1 at 48% rotor speed from the Campbell diagram, which demonstrates the results of experiment and FEA are consistent.
As can be seen from the figure 16, the uncertainty values are below 5% for the majority of revolutions. At high uncertainty regions caused by the zeroing process as well as measurement error, the coherence is quite low, which reduces the reliability of fitting results to some extent. But the amplitudes at those regions are comparatively small, hence it’s not significant for the evaluation of the blade stress.

![Figure 16. Uncertainty.](image)

3.5. Blade endurance

The FEA is capable of providing the information of the non-dimensional displacement and stress of each node. It’s assumed that the relationship between nodal displacement and its averaged stress is linear. Accordingly, a scaling factor \( k \) of nodal displacement can also be used for the nodal stress.

\[
k = \frac{D_{\text{FEM}}}{D_{\text{BTT}}}
\]

In Eqn. (11), \( D_{\text{FEM}} \) and \( D_{\text{BTT}} \) refer to the displacement calculated by FEA and BTT system respectively at the same node. The blade real stress of each node can be obtained by its corresponding modal stress multiplied by the inverse of \( k \).

![Figure 17. Goodman diagram.](image)

Then the stress clouds of flutter and synchronous vibration for blade mode 1 can be plotted in figure 17. The blue line is known as the Goodman line that represents the cycle number of the blade failure. The intercept of X and Y axis indicates the material’s ultimate yield strength and alternating stress (fully reversed) respectively. All nodal stresses have been normalized by the ultimate yield strength, and there is an error bar for each of them. Because the amplitude obtained from the curve
fitting for each individual response is described with an uncertainty value. It’s obvious that the flutter margin (15%) is much less than that of synchronous vibration.

4. Conclusions
With the help of FEA, blade tip timing technique is able to obtain the blade stress without using strain gauges, which enhances the application of BTT. This paper presents a process that explains how to analyze the vibration characteristics of the fan blades by the raw TOA data in detail. Key conclusions are:

- The use of a standard process is important to reduce the processing errors introduced by data zeroing and improve the accuracy of calculated blade stresses.
- In this case, flutter occurs with a dominant nodal diameter 2 at 92% rotor speed approximately, and 1st flex mode – 2 engine order crossing is found at 48% rotor speed. In the high response region, the coherence between calculated and measured displacement is beyond 0.9, which ensures the reliability of the analysis method.
- The blade endurance in Goodman diagram shows that the flutter has less margin compared with the synchronous vibration. That’s the reason why we have to avoid a flutter scenario in reality.

References
[1] García I, Beloki J, Zubia J, et al. An optical fiber bundle sensor for tip clearance and tip timing measurements in a turbine rig[J]. Sensors, 2013, 13(6): 7385-7398.
[2] Zablotskiy I Y, Korostovlev Y A. Measurement of Resonance Vibrations of Turbine Blades with the Elura Device[R]. FOREIGN TECHNOLOGY DIV WRIGHT-PATTERSON AFB OH, 1978.
[3] Zielinski M, Ziller G. Noncontact vibration measurements on compressor rotor blades[J]. Measurement Science and Technology, 2000, 11(7): 847.
[4] Heath S. A new technique for identifying synchronous resonances using tip-timing[J]. ROLLS ROYCE PLC-REPORT-PNR, 1999.
[5] Dimitriadis G, Carrington I B, Wright J R, et al. Blade-tip timing measurement of synchronous vibrations of rotating bladed assemblies[J]. Mechanical Systems and Signal Processing, 2002, 16(4): 599-622.
[6] Gallego-Garrido J, Dimitriadis G, Carrington I B, et al. A Class of Methods for the Analysis of Blade Tip Timing Data from Bladed Assemblies Undergoing Simultaneous Resonances—Part II: Experimental Validation[J]. International Journal of Rotating Machinery, 2007, 2007.
[7] Gallego-Garrido J, Dimitriadis G, Wright J R. A class of methods for the analysis of blade tip timing data from bladed assemblies undergoing simultaneous resonances—Part I: theoretical development[J]. International Journal of Rotating Machinery, 2007, 2007.
[8] Heath S, Imregun M. A survey of blade tip-timing measurement techniques for turbomachinery vibration[J]. Journal of engineering for gas turbines and power, 1998, 120(4): 784-791.
[9] Russhard P. Development of a blade tip timing based engine health monitoring system[D]. Thesis, Faculty of Engineering and Physical Sciences, School of Mechanical Aerospace and Civil Engineering, Manchester University, 2010.
[10] Russhard P. The Rise and Fall of the Rotor Blade Strain Gauge[M]/Vibration Engineering and Technology of Machinery. Springer, Cham, 2015: 27-37.
[11] WATKINS W, CHI R. Noninterference blade-vibration measurement system for gas turbine engines[J]. Journal of Propulsion and Power, 1989, 5(6): 727-730
[12] Russhard P. Timing analysis: U.S. Patent 8457909 B2[P]. 2009-10-22.