Modeling of weak shock waves propagation in aqueous foam layer

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Abstract. Dynamics of low-intensity air shock waves in the shock tube containing an aqueous foam layer is theoretically investigated. Modeling of studied process is carried out using two-phase model of aqueous foam developed by the authors in single-pressure, single-speed and two-temperature approximations. The model takes into account the Ranz-Marshall interphase contact heat transfer, effective Herschel-Bulkley viscosity, which describes foam behavior as a non-Newtonian fluid, and elastic properties of aqueous foam under a weak shock impaction without destruction of foam structure. Properties of air and water as the foam components are described by realistic equations of state. Computer implementation of the aqueous foam model is carried out in the solver, developed by the authors in OpenFOAM software. The influence of aqueous foam viscoelastic properties on the intensity and structure of a shock wave has been investigated. When analyzing the obtained solutions, reliability of the proposed model and method of numerical modeling is estimated by comparative analysis of the found solutions and literature experimental data.

1. Introduction
Study of aqueous foam damping properties, which reduce the intensity of external influences, is of important scientific and practical interest, since aqueous foam barriers are one of the promising methods for protecting high-energy technological processes in industrial production.

Under conditions of quasi-static tests [1], the aqueous foam characteristics enable to consider it as an elastic-plastic-viscous system. Elasticity of foam structural bonds becomes apparent at small deformations and makes it possible to distort the initial shape of the bubble cells, but when the load is removed – it returns the cells to their original state. As the deformation increases, the foam structure has a delayed elastic deformation, characterized by destruction of initial bonds with their rearrangement and restoration of structure similar to original state. Further increase in deformation is accompanied by a complete rupture of bonds (destruction of the foam) with formation of a gas-liquid mixture.

For the purposes of this study it is of interest to apply the approaches, used to describe the elastic-viscous-plastic properties of aqueous foam in quasi-static regimes, for problems of modeling and studying the aqueous foam behavior under shock-wave impact of various intensities.

The features of shock waves (SW) dynamics in aqueous foams are considered in the works [2]–[9]. In [2], the mitigating processes of strong SW induced by the HE explosion during their propagation in dry aqueous foams are theoretically and experimentally investigated. It was
found, that when passing through a foam medium, there is a significant weakening of shock pulse and decreasing in its propagation speed.

Numerical investigation of spherical explosion in aqueous foam for the experimental conditions [2] was carried out in [3, 4]. When modeling the foam behavior, it was used the assumption of its destruction into microdroplets under the action of intense SW, and the model of a gas-liquid mixture is valid to describe the foam medium. Calculations, obtained by the proposed model numerical implementation in OpenFOAM software [5], coincide with the experimental data [2] in terms of SW velocity and amplitude.

In works [6], [7] the exploration of two-dimensional effects, arising from the interaction of strong SW with an aqueous foam layer, was carried out using computer modeling. The reasons, influencing for attenuation of shock pulse intensity by using foam barriers, are investigated. Dynamics of wave phenomena, leading to the formation of toroidal vortices in the gas region in front of the foam layer, is studied.

Computational study of SW propagation in the aqueous foam layer under the action of low-intensity air SW was carried out in [8] for experimental conditions [9]. The developed mathematical model describes the behavior of aqueous foam as a non-Newtonian liquid, taking into account the effective Herschel-Bulkley viscosity, interfacial heat transfer processes according to the Ranz-Marshall model and realistic equations of state. Reliability of the proposed aqueous foam model and method, used for its numerical implementation, has been confirmed in the form of satisfactory agreement between calculations and experimental data.

In this study, the experimental results on investigation the features of aqueous foam dynamic reaction to a weak impact, retaining its structure, become important. In [9], [10], experiments on the interaction of a small-amplitude SW with the aqueous foam layer are presented in the form of pressure histories, demonstrating the process of forming in the aqueous foam layer the elastic precursor overtaking the main SW.

In the present paper, mathematical modeling and numerical study of the plane air SW interaction with aqueous foam layer for experimental conditions [9] are performed. In comparison with [8], the model developed by the authors additionally takes into account the influence of aqueous foam elastic properties on weak SW impact, that do not destroy foam structure.

2. Governing equations

To study the process under consideration, the two-phase model of aqueous foam is proposed, which includes the conservation laws of mass and energy for each phase, momentum equation and equation of foam liquid volume fraction dynamics in accordance with single-pressure, single-speed, two-temperature approximations, based on the approaches [11], [12], [5].

Continuity equation for \( i \)-th phase

\[
\frac{\partial (\alpha_i \rho_i)}{\partial t} + \text{div}(\alpha_i \rho_i \vec{v}) = 0, \tag{1}
\]

Energy equation for \( i \)-th phase

\[
\frac{\partial (\alpha_i \rho_i (u_i + K_i))}{\partial t} + \text{div}(\alpha_i \rho_i (u_i + K_i) \vec{v}) = -p \frac{\partial \alpha_i}{\partial t} - \text{div}(\alpha_i \vec{v} p) + \text{div}(\alpha_i \vec{v} \cdot \tau) + \text{div}(\alpha_i \vec{v} \cdot \vec{s}) + \text{div}(\alpha_i \gamma_i \frac{c_{p,i}}{c_{V,i}} (\nabla h_i)) + K_{ht} (T_j - T_i), \tag{2}
\]

Momentum equation for mixture

\[
\frac{\partial (\rho \vec{v})}{\partial t} + \text{div}(\rho \vec{v} \vec{v}) = -\nabla p + \text{div} \tau + \text{div} \vec{s}, \tag{3}
\]
Equation of liquid volume fraction dynamics

$$\frac{\partial \alpha_1}{\partial t} + \text{div}(\alpha_1 \vec{v}) - \alpha_1 \text{div}(\vec{v}) = \alpha_1 \alpha_2 \left( \frac{1}{\rho_2} \frac{d\rho_2}{dt} - \frac{1}{\rho_1} \frac{d\rho_1}{dt} \right).$$ (4)

Here: $t$ — time; $\vec{v}$ — flow velocity vector; $p$ — pressure; $\alpha_i, \rho_i$ — volume fraction and density; $u_i, K_i$ — internal and kinetic energy; $c_{p,i}, c_{V,i}$ — specific heat capacities at constant pressure and volume; $\gamma_i$ — thermal conductivity, $h_i$ — enthalpy, $T_i$ — temperature. Subscript $i$ denotes $i$–th phase ($i = 1$ — water, $i = 2$ — air); $\rho = \alpha_1 \rho_1 + \alpha_2 \rho_2$ — aqueous foam density; $\alpha_1 + \alpha_2 = 1$; $\tau$ — viscous stress tensor, $s$ — stress tensor deviator.

The components of air and water that form the aqueous foam are described by realistic Peng-Robinson [13] and Mie-Gruneisen [14] equations of state, respectively.

Heat transfer coefficient $K_{ht}$ according to the Ranz-Marshall model [15] has the form:

$$K_{ht} = \kappa_1 \frac{\text{Nu}}{d_{20}} , \quad \text{Nu} = 2 + 0.6 \frac{Re^{1/2}}{Pr^{1/3}},$$

where $\kappa_1$ — thermal conductivity of water, $d_{20}$ — diameter of bubbles in the foam; $Re, Nu, Pr$ — Reynolds, Nusselt and Prandtl numbers, respectively.

Viscous stress tensor $\tau$ depends on the flow velocity vector $\vec{v}$:

$$\tau = \mu_{eff} (\nabla \vec{v} + \nabla \vec{v}^T) - \frac{2}{3} (\mu_{eff} \text{div} \vec{v}) I,$$

where $I$ — unit tensor.

Based on the experimental observations of [9], when describing the elastic properties of aqueous foam, in the proposed model it is assumed, that if the shearing stresses do not exceed the elastic limit $s_0$, the behavior of aqueous foam is described by Hooke’s law of elasticity [11]:

$$s = \mu_s (\nabla \vec{e} + \nabla \vec{e}^T) - \frac{2}{3} (\mu_s \text{div} \vec{e}) I,$$

where $\vec{e}$ — deformation vector, $\mu_s$ — shear modulus.

If the applied deformation is large enough, the shear stress limit is exceeded, and the foam changes from an elastic state to a viscoplastic one. This transition occurs if the von Mises yield criterion [11] is fulfilled:

$$|I_2(s)| - \frac{1}{3} s_0^2 > 0,$$

which requires the adjustment of the stress deviator $s$ components by the formula [11]:

$$\tilde{s}_{kl} = s_{kl} - \frac{s_0}{\sqrt{3}|I_2(s)|}.$$

Here $I_2(s)$ — second invariant of the stress deviator tensor $s$, $s_{kl}, \tilde{s}_{kl}$ — initial and normalized components of $s$ respectively.

The viscoplastic properties of aqueous foam, as a non-Newtonian liquid, are described in accordance with the Herschel-Bulkley model, which in terms of the effective viscosity $\mu_{eff}$ takes the form [16]:

$$\mu_{eff} = k |\dot{\gamma}|^{-n} + \tau_0 |\dot{\gamma}|^{-1},$$

where $\tau_0$ — yield stress, $\dot{\gamma}$ — shear rate, $k$ — consistency index, $n$ — flow index [16].
3. Problem definition

To study the proposed model capabilities, the process of weak SW propagation in aqueous foam is analyzed in accordance with the experiments, presented in [9].

The shock tube 4 m long, consisting of high (HP) and low (LP) pressure chambers, contains the transparent tube segment, in which the aqueous foam layer (AF) \( \approx 0.63 \) m long is located. Oscillograms of SW pressure are recorded by sensors 1–5, fixed at the distance of 0.1 m from each other. The sensor 1 is located in the gas region in front of the foam layer at the distance of \( l_1 = 2.5 \) m from the diaphragm (D); the sensors 2–5 are located in the aqueous foam area. The experimental setup is performed in Fig. 1, where the \( x \)–axis shows the pressure chambers sizes and sensors locations in relation to the diaphragm position.

In this work, in accordance with the experiments [9], the simulation of studied process was carried out under the following initial conditions for pressure: \( p_{0,HP} = 5.5 \) bar, \( p_{0,LP} = 1 \) bar (\( T_0 = 300 \) K). Liquid volume fraction of the foam: \( \alpha_{10} = 0.05 \). After the rupture of diaphragm, the SW formation and propagation into the depth of the shock tube, containing the gas region and the foam layer, was initiated.

The aqueous foam model (1)–(4) was numerically implemented in the free hydrodynamic software OpenFOAM by creating the new solver that simulates the SW propagation in aqueous foams with elastic-visco-plastic properties.

4. Results

The numerical simulation results and corresponding experimental data [9] of the plane SW dynamics in pure gas and in gas, containing the aqueous foam layer, are shown in Fig. 2 and Fig. 3 in the form of overpressure oscillograms \( \Delta p \), recorded by sensors 1–5. The zero point of the time axis coincides with the moment when the SW arrives at the first sensor.

In pure gas, the calculated and experimental time dependences of overpressure for sensors 1–5 show the plane SW propagation of amplitude \( \Delta p \approx 1.5 \) bar, moving with the velocity \( D_g = 500 \) m/s. The second pressure jump of amplitude \( \approx 4.0 \) bar, observed on sensors 3–5, is formed due to the reflection of main SW from the left boundary of shock tube.

The calculated and experimental oscillograms in the presence of a foam layer, obtained from the sensor 1, located in the gas region in front of the foam layer, have two-stage structure. The first increase in pressure occurs at the moment of the main air SW arrival, the second pressure jump is a consequence of the reflection of this wave from the boundary with aqueous foam, which is denser than the gaseous medium. Comparison of maximum SW amplitudes recorded by the sensor 2 in the foam (see Fig. 3) and gas (see Fig. 2) shows a slight increase in pressure by \( \Delta p \approx 0.5 \) bar in the foam layer, which is caused by foam compaction during interaction with SW.

Analysis of the obtained numerical solutions, taking into account the presence of foam layer in the shock tube, showed the decrease in the shock pulse front velocity to \( D_f = 200 \) m/s, which is \( \approx 2.5 \) times less than the SW velocity in the gas \( D_g \).

With the time, when moving along the foam layer, the SW takes the two-wave structure, consisting of the main SW and its leading elastic precursor \( S \). The formation of the precursor...
Figure 2. Time dependences of overpressure $\Delta p$ in sensor positions $l_1, \ldots, l_5$ in pure gas; 1 — numerical results; 2 — experimental data [9].

Figure 3. Time dependences of overpressure $\Delta p$ in sensor positions $l_1, \ldots, l_5$ in gas, containing the aqueous foam layer; 1 — numerical results; 2 — experimental data [9]; $S$ — elastic precursor on the experimental and calculated pressure histories.

becomes visible on the pressure profiles corresponding to the sensors 3–5 (see Fig. 3, $S$): both in calculations and experiments, the $S$ of amplitude $\Delta p \approx 0.5$ is fixed in front of the main SW of amplitude $\Delta p \approx 2$ bar.

The obtained numerical simulation results are in satisfactory agreement with the experimental data [9].

5. Conclusion
The two-phase model, describing the behavior of aqueous foam under the influence of weak SW without destroying its structure, has been developed. The model takes into account contact heat transfer at the interface, nonlinear Herschel-Bulkley viscosity, and elastic properties of
aqueous foam at small deformations. Numerical implementation of the model is carried out in the new solver created by the authors in free software OpenFOAM. The process of weak air SW propagation in the shock tube containing the aqueous foam layer is investigated under the conditions of experiments [9]. It was found, that the velocity of SW front in the foam layer is \( \approx 2.5 \) times lower than in gas. Accounting for the elastic properties of aqueous foam in the developed model made it possible to identify in the calculations two-wave structure of forming SW, consisting of the elastic precursor, that overtakes the main SW. Reliability of the numerical simulation results is confirmed by the agreement of obtained solutions and experimental data [9].

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