There is some observational evidence for earlier evolution of clusters of galaxies than predicted in the standard ΛCDM model with a Gaussian primordial density fluctuation field, and a low value for the mass variance parameter ($\sigma_8$). Particularly difficult in this model is the interpretation of possible excess CMB anisotropy on cluster scales as due to the Sunyaev-Zeldovich (S-Z) effect. We have calculated S-Z power spectra in the standard model, and in two alternative models which predict higher cluster abundance - a model with non-Gaussian PDF, and an early dark energy model. As anticipated, the levels of S-Z power in the latter two models are significantly higher than in the standard model, and in good agreement with current measurements of CMB anisotropy at high multipole values. Our results provide a sufficient basis for testing the viability of the three models by future high quality measurements of cluster abundance and the anisotropy induced by the S-Z effect.

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1. Introduction

The power spectrum of the primary CMB anisotropy falls sharply at multipoles $\ell > 1000$, where ‘excess’ power is primarily induced by the Sunyaev-Zeldovich (S-Z) effect. There are some indications that such excess power was measured by the CBI\cite{1}, ACBAR\cite{2}, and BIMA\cite{3} experiments. Attributing the measured power to the S-Z effect requires a higher cluster abundance than predicted in the standard ΛCDM cosmological model, particularly so for the low value of the mass variance parameter, $\sigma_8 = 0.74^{+0.05}_{-0.06}$, deduced from the WMAP 3-year data\cite{4}. An unrealistically high value $\sigma_8 \gtrsim 1$ would be required for consistency with the current CMB measurements. This possible discrepancy enhances interest in alternative models in which higher cluster abundances are predicted.
Other evidence for earlier and more abundant cluster population comes from radio observations of clusters at high redshift and larger values of the concentration parameter and Einstein radii than expected in the standard model, which imply earlier formation of high mass \( M \geq 10^{15} M_\odot \) clusters. The cluster angular two-point correlation function provides yet another measure of the abundance and evolved nature of the population. A recent analysis of Spitzer Space Telescope observations seems to indicate early formation of very massive galaxies, with number densities that are considerably higher than predicted in the standard ΛCDM model, and surprisingly high level of clustering.

The first of two alternative models, in which clusters are expected to form earlier than in the standard model, is an isocurvature CDM with scale-dependent non-Gaussian, \( \chi^2_m \) distributed primordial density fluctuation field, where \( m \) is the number of CDM fields added in quadratures to yield the \( \chi^2_m \) distribution. With increased number of random primordial density fluctuation (PDF) fields, their sum approaches a Gaussian distribution (in accord with the central limit theorem); thus, the degree of deviation from a normally distributed PDF is the largest for \( m = 1 \). The evolution of the large scale structure and primary CMB anisotropy in the \( \chi^2 \) family of models were explored in several studies, showing explicitly that primordial overdensities attain larger amplitudes with higher probabilities than in a Gaussian field. Correspondingly, cluster form earlier and are more abundant, thereby enhancing levels of S-Z observables. In the latter two papers the then current WMAP 1-year normalization was used, \( \sigma_8 = 0.9 \), and it was shown that already with this relatively high value it was difficult to reconcile the CMB power excess with the inferred cluster population, if the primordial fluctuation field was Gaussian.

We note that the degree of non-Gaussianity in the explored \( \chi^2 \) models is consistent with limits set by analyses of the WMAP data. This is largely due to the fact that the latter dataset yields information on scales that are much larger than those associated with clusters. On these large scales the low overdensities may be indistinguishable from a non-Gaussian distribution when the density field is not scale invariant.

Our previous work focused on the predicted levels of S-Z anisotropy and cluster number counts in the \( \chi^2 \) model, showing that S-Z power levels in this model are appreciably higher than indicated by current measurements. Here we explore predicted S-Z power spectra in the \( \chi^2 \) model.

Temporal variation of the dark energy density in early dark energy (EDE) models provides an alternative for generating an enhanced cluster abundance at higher redshifts. In these models the DE is appreciable already at early epochs and attains the observationally inferred value at present. The evolution of structure in the linear regime and CMB anisotropy in these models have been explored in several works. Two specific EDE models have recently been investigated in detail, resulting in explicit numerical determination of the linear growth factor of density perturbations, critical density for spherical collapse, \( \delta_c \), and overdensity at virializa-
tion, $\delta_v$, quantities needed to calculate the cluster mass function. The non-vanishing dark energy component at early times drives an early acceleration phase, implying a slower evolution of the linear growth factor and reduced values of $\delta_c$. Thus, for a given value of the mass variance normalization, $\sigma_M$, the corresponding quantity at early times should be larger than what is implied in the $\Lambda$CDM model, and since the critical overdensity for collapse at a given redshift is linearly extrapolated from an earlier time, the slower evolution of the growth factor in EDE models results in a lower $\delta_c$ as compared with its value in $\Lambda$CDM model. The reduced $\delta_c$ and higher $\sigma_M$ obviously yield a more abundant cluster population.

Here we update the predicted S-Z power spectra in the above two alternative cosmological models. In Section 2 we briefly describe the variant of the Press & Schechter mass function in the $\chi^2$-distributed PDF, and outline the properties of the EDE model adopted in our calculations. Results of the calculations of S-Z power spectra in the $\Lambda$CDM, EDE, and non-Gaussian models are presented and compared in Section 3, followed by a brief discussion in Section 4.

2. Calculations

The calculations of S-Z power spectra requires knowledge and modeling of global, large scale, and cluster quantities. To do so we adopt the methodology described in several papers. We refer to the $\Lambda$CDM, EDE, and non-Gaussian models as models I, II, and III, respectively. Models I and III were described by us, so our brief discussion here will include only the most essential aspects of these models. The EDE model we adopt here is characterized by the density parameter of early quintessence $\Omega_e = 0.03$, and the coefficient of the equation of state parameter $w_0 = -0.9$ at $z = 0$. The effective coefficient as function of redshift is

$$w(z) = \frac{w_0}{1 + u \log (1 + z)},$$

where

$$u \equiv \frac{-3w_0}{\log \left(\frac{1-\Omega_e}{\Omega_m}\right) + \log \left(\frac{1-\Omega_m}{\Omega_m}\right)},$$

and $\Omega_m$ is the matter density parameter.

The basic quantity in our calculations of S-Z power spectra is the Press & Schechter (1974) mass function,

$$n(M, z) = -\mu F(\mu) \frac{\rho_b}{M \sigma_M^2(z)} \frac{d\sigma}{dM} dM,$$

where $\mu \equiv \delta_c(z)/\sigma_M(z)$ is the ratio of the critical overdensity for collapse to the mass variance $\sigma_M$ at redshift $z$, and $\rho_b$ is the background density at $z = 0$. For a Gaussian PDF field assumed in models I and II, we have

$$F(\mu) = \sqrt{\frac{2}{\pi}} e^{-(\mu^2/2)}.$$
The corresponding expression for a PDF which obeys $\chi^2_2$ statistics of model III is particularly simple

$$F(\mu) = e^{-(1+\mu)}. \quad (5)$$

The mass function is normalized such that all the mass is included in halos, a normalization that in the original Press & Schechter mass function was affected by including a (‘fudge’) factor of 2. The functional form of the mass function is different in model III from that in models I and II, and the redshift dependence of the critical density for collapse and the linear growth factor both differ in model II than the corresponding quantities in the other two models. Explicit expressions for these quantities are given in our recent paper [15].

The mass variance $\sigma_M$ was calculated with a top-hat window function and CDM transfer functions taken from Bardeen et al. (1986) [22] - adiabatic transfer function for models I and II, and isocurvature transfer function for model III. The shape of the CDM transfer function in the EDE model is slightly different than in the standard model; this difference is ignored here. For the isocurvature model this function (of the wavenumber $k$) is

$$T(k)_{iso} = (5.6q)^2 \left[ 1 + \frac{(40q)^2}{1 + 215q + (16q)^2(1 + 0.5q)^{-1}} + (5.6q)^{8/5} \right]^{-5/4}, \quad (6)$$

where $q \equiv k/(\Omega_m h^2 \text{Mpc}^{-1})$, with $h = H_0/(100 \text{ km s}^{-1} \text{ Mpc}^{-1})$. Values of the global parameters were taken to be those deduced from the WMAP 3-year data, $\Omega_\Lambda = 0.76$ (which, for the case $w \neq -1$ is usually written as $\Omega_Q$), $\Omega_m = 0.24$, $h = 0.73$, and $\sigma_8 = 0.74^{+0.05}_{-0.06}$. The spectral index of the PDF spectrum is $n = 1$ in models I and II, and $n = -1.8$ in model III. In the EDE model the differential equations for the evolution of the linear growth factor, $\delta_c$, and $\delta_v$, are different than in the other two models. Full description of the of these equations and their numerical solutions can be found in our recent paper [15].

A meaningful comparison of the predicted S-Z spectra of the three models considered here requires proper and self-consistent normalization of the respective mass functions. This is accomplished by requiring that the cumulative cluster density at $z = 0$ is in agreement with that calculated in the standard $\Lambda$CDM model.

In addition to the global and large-scale parameters, the calculation of S-Z power spectra necessitates full description of the properties of IC gas. To do so clusters are assumed to be in hydrostatic equilibrium with $\beta$ profiles for the total mass and gas density profiles, and with a gas mass fraction of 0.1. The gas temperature is determined from the equation of hydrostatic equilibrium.

### 3. Sample Power Spectra

The behavior of the cumulative mass function in the three models at $z = 3$ and $z = 0.01$ is shown in Fig. 1 for the mass range $10^{13} M_\odot h^{-1} \leq M \leq 10^{16} M_\odot h^{-1}$. As is clear, the models are correctly normalized to yield the same cumulative cluster
density at low redshifts. Abundances of high-mass clusters are indeed higher in models II and III than in model I. As anticipated, the relative abundances with respect to those in ΛCDM increase with redshift. It is also apparent that at early times the excess of high overdensity fluctuations in the $\chi^2$ model has a stronger impact on the abundance of high mass clusters than the slower evolution of the linear growth factor and lower values of $\delta_c$ in model II, whereas their respective effects at present are about the same in these two models.

The enhanced cluster abundances in models II and III are directly manifested in higher S-Z power levels than in the standard model, as is immediately evident in Fig. 2 which shows the spectra at 31 GHz together with CBI, ACBAR, and BIMA measurements at this frequency. In addition, the (broad) peak power in the non-Gaussian model is reached at multipoles, $\ell \sim 7000 - 8000$ as compared with $\ell \sim 4000 - 5000$ in model I and II, a direct consequence of the higher abundances of distant clusters (with smaller apparent sizes) in model III.

The predictions of both the EDE and non-Gaussian models are more consistent with current measurements of the CMB power spectrum. The shape of their S-Z spectra are virtually identical for $\ell \geq 10^4$, with power levels somewhat lower at lower $\ell$ in the former model than in the non-Gaussian model.
4. DISCUSSION

The work reviewed here has been motivated by initial indications that there might be a significant discrepancy between current measurements of levels of CMB anisotropy on scales $\ell \geq 2000$, and predicted levels of S-Z power in the standard $\Lambda$CDM with gaussian PDF field. The discrepancy stems mainly from the fact that high-mass clusters, the largest contributors to S-Z power whose density decreases sharply with decreasing $\sigma_8$, are not sufficiently abundant if this important parameter is as low as deduced from the 3-year WMAP data. A higher cluster abundance is expected when the PDF field has the form predicted in the non-gaussian $\chi^2$ model. This is a result of higher probabilities for overdense regions at high $z$, leading to earlier collapse of proto-cluster halos. In the early quintessence model earlier cluster collapse is a manifestation of higher linear growth factor, and lower value of the critical density for collapse at high $z$. Accordingly, levels of S-Z power are higher in these two alternative models. Clearly, these two non-standard models are by no means the only viable alternatives to the standard model. As we noted, the non-Gaussian model considered here is just one of the $\chi^2_m$ family, but with decreasing non-Gaussianity with increasing $m$. Also, other early quintessence models with higher EDE densities result in slower evolution of the linear growth factor, and reduced value of $\delta_c$, thereby
leading to higher levels of S-Z power.

While levels of S-Z power spectra predicted in the standard model span an appreciable range, reflecting also uncertainties in the evolution of internal properties of clusters - such as IC gas density and temperature - their maximal values are still well below current observational results. This conclusion is based on extensive investigations of the cluster temperature-mass relations, IC gas models, including non-isothermal polytropic temperature profiles, and evolution of the gas mass fraction.

Our purpose here has been to show that the two alternative models explored can produce S-Z power levels that are substantially higher than in the standard model. It is too early to actually fit the predictions to the preliminary high ℓ results. This will have to be done in conjunction with other cluster observables, such as cluster (S-Z) number counts, and the two-point correlation function (which were considered in our previous work). Other cluster measures that are very much affected in these models are formation times and concentration parameters (whose observational manifestations include, e.g., mass profiles and Einstein ring sizes). Nonetheless, it is quite clear from our results that the χ² model, and EDE models with larger quintessence densities at early times than in the specific model adopted here, predict significantly higher levels of S-Z power than indicated by current observational results, and therefore seem to be non-viable.

We should know soon whether the apparent discrepancy is real, when results of more extensive and precise measurements of the high ℓ power will become known. But irrespective of these upcoming CMB and S-Z measurements, there seem to be other observational indications that cluster formed earlier and are more abundant than predicted in standard ΛCDM. The non-gaussian and EDE models explored here seem to be viable alternatives, should the discrepancy persist.

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