**Observationally constrained accelerating model with higher power of non-metricity and squared trace**

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**Abstract** We have evaluated the cosmological parameters of $f(Q,T)$ gravity model obtained with the use of squared trace form of $f(Q,T)$ and at the backdrop of flat FLRW metric. Our derived model was coupled using recent $H(z)$ data, Pantheon data, and combined $H(z)$ and Pantheon data and BAO data sets. The current age of the Universe has been determined. The transitional behaviour of the Universe has been shown. The cosmographic parameters are parametrized and found that our generated models approaches to the concordance $ΛCDM$ model. The $Om(z)$ diagnostic has been examined in order to demonstrate the dark energy part of the Universe.

**Keywords:** $f(Q,T)$ gravity, Dark energy, $H(z)$ data, Cosmological data, $Om(z)$ diagnostic.

I. INTRODUCTION

The late time acceleration [1–6] is one of the most significant accomplishments in the recent research on cosmology and gravitational physics. This findings compels to go outside the frame to explain the repulsive nature of gravity on large cosmic scales. It is not known the source of the creation of the repulsive gravity, however the reason might be the presence of dark energy. The presence of dark energy, a non-standard component of the Universe with negative pressure, or a large-scale infra-red modification of General Relativity. Dark energy has major share in the mass-energy budget of the Universe and remains sub-dominant in prior epochs resulting difficulty in model building. The current dynamics of the Universe show the acceleration due to the dark energy component and the cosmological constant ($Λ$) is its most simple candidate. This has been originated from early vacuum quantum fluctuations. At late time of the cosmic dynamics, the concordance flat $ΛCDM$ paradigm has become most successful in explaining the late time acceleration. The equation of state parameter of dark energy enables to distinguish different phases of the cosmological models ($ω_{DE} ≈ −1$). The other two ideas are the quintessence and phantom phase, which can be respectively identified as, $−1 ≤ ω_{DE} < 0$ and $ω_{DE} ≤ −1$. However, the recent surveys ruled out the possibility of $ω_{DE} ≈ −1$, although $ω_{DE}$ could be a bit less than $−1$ [7–9]. Several corrections are suggested to $ω_{DE}$: CMB observations was limited to $ω_{DE} = −1.073^{+0.090}_{−0.089}$ by the 9-year WMAP survey [10], $ω_{DE} = −1.0840 ± 0.063$ is suggested by a combination of CMB and Supernova data [10]. Kumar and Xu constrained it to $ω_{DE} = −1.06^{+0.11}_{−0.13}$ based on a combined examination of the data sets of SNLS3, BAO, Planck, WMAP9, and WiggleZ [11]. Combining Planck data with additional astronomical data, including Type Ia supernovae, Ade et al. [12] suggested $ω_{DE} = −1.006 ± 0.045$.

We shall discuss here some of the recent findings of the cosmological models with the observational data sets. Wei and Zhang [13] have used the $H(z)$ data set to capture the key aspects of ten cosmological models; six of them failed to sustain with the observational data sets, however the remaining four were compatible with the data set. Further it has been extended to eleven interacting dark energy models with varying couplings to the observable $H(z)$ data [14]. However, none of these models outperforms the simplest $ΛCDM$ model. Seikel et al. [15] have studied novel consistency tests for the $ΛCDM$ model and the parameters are expressed explicitly in terms of $H(z)$. Magana et al. [16] have investigated five different dark energy models with variable equations of state as a function of redshift. Farooq et al. [17] constructed an updated Hubble parameter $H(z)$ at redshifts 0.07 < $z$ < 2.36 and used it to restrict model parameters in both spatially flat and curved dark energy cosmological models. Mukherjee and Banerjee [18] take a kinematic method to describe late time dynamics of the universe with constant value of the jerk parameter.

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Using a kinematic technique, Mamon et al. [19] have investigated accelerated expansion phase of the Universe. The deceleration parameter $q$ has been parametrized in a model-independent manner. Using the Hubble data set and latest joint light curves, Amirhashchi [20] has further tightened the model parameters range for the $\Lambda$CDM model. Cao et al. [21] have used the observational $H(z)$ data acquired from cosmic chronometers and BAO methods to evaluate the validity of $\Lambda$CDM using the two-point $Om_{h}^{2}(z_{2}; z_{1})$ diagnostic. In context of $f(Q,T)$ gravity, the best fit values of the model parameters are found using the $R_{1}^{2}$-test formula and the available observational data sets by Pradhan et al. [22]. Yadav et al. [23] studied a bulk viscous universe with dark energy dominance in Bianchi type I space-time and used observational $H(z)$ data (OHD) and a hybrid OHD and Pantheon compilation of SN Ia data to constrain the parameters. Also, a kinematic expression for the deceleration parameter was developed, and its current value $q_{0} = -0.58^{+0.02}_{-0.03}$ and transition redshift $z_{t} = 0.723^{+0.16}_{-0.18}$ were constrained. Using recent $H(z)$ and Pantheon compilation data, Goswami et al., [24], studied a bulk viscous anisotropic Universe and limited its model parameters. By bounding their derived model with recent $H(z)$ data, Pantheon data, and joint $H(z)$ and Pantheon data, estimate the current value of Hubble constant as $H_{0} = 69.39 \pm 1.54 \ km s^{-1} Mpc^{-1}$, $70.016 \pm 1.65 \ km s^{-1} Mpc^{-1}$ and $69.36 \pm 1.42 \ km s^{-1} Mpc^{-1}$ using cosmic chronometric approach.

Another geometric method to measure the rate of expansion of the Universe is the baryon acoustic oscillation (BAO) method. During recombination, sound waves in baryon-photon plasma are frozen as density fluctuations, with the sound horizon determining a different scale [25–27]. These sound waves appear as a peak in the matter correlation function, or equivalently as a series of oscillations in the power spectrum, at the scale of the sound horizon. CMB measurements, which yield the physical matter and baryon densities that control the sound speed, expansion rate, and recombination time in the early Universe. This can be used to predict the length scale, which corresponds to the sound horizon at the $r_{s}(z_{d})$ baryon drag epoch and the most recent determination is $r_{s}(z_{d}) = 153.3 \pm 2.0 \ Mpc$ [28]. From three separate galaxy surveys, significant BAO detections have been reported as i) The Sloan Digital Sky Survey (SDSS) [29–35], ii) The WiggleZ Dark Energy Survey (WiggleZ) [36], and iii) The 6-degree Field Galaxy Survey (6dFGS) [37]. The most exact BAO observations were analysed by comparing the SDSS, particularly the Luminous Red Galaxy (LRG) component [29]. Eisenstein et al. [29] reported a convincing discovery of the acoustic peak in the SDSS Third Data Release (DR3) LRG sample with effective redshift $z = 0.35$ using a two-point correlation function. Beutler et al. [37] have announced a BAO finding at $z = 0.1$ in the low-redshift Universe by the 6dFGS. At higher redshifts, the WiggleZ Survey quantified BAOs at $z = 0.6$, producing a $\sim 4$ percent measurement of the baryon acoustic scale.

Two equivalent representations of General Relativity are the curvature representation and teleparallel representation. In curvature representation, the torsion and nonmetricity vanishes whereas in teleparallel the curvature and non-metricity vanishes. Another equivalent representation can be realised where the geometrical variable that describes the properties of the gravitational interaction can be represented as nonmetricity; vanishing curvature and torsion. The nonmetricity would describe the variation of the length of a vector in teleparallel transport. This approach is known as symmetric teleparallel approach [38]. Further with the non-minimal coupling between the nonmetricity $Q$ and trace $T$ of energy momentum tensor, Xu et al. [39] have extended this symmetric teleparallel gravity, as $f(Q,T)$ gravity. The coupling between $Q$ and $T$ leads to non-conservation of the energy-momentum tensor in $f(Q,T)$ theory. Different aspects of the cosmological models are recently studied using the $f(Q,T)$ gravity. The scale factor based cosmological models [40, 41] provide the late time cosmic acceleration scenario with the parameters value as prescribed by the cosmological observations. The transient behaviour of the model can also be observed [42]. Also, with $H(z)$ and Pantheon data, $f(Q,T)$ gravity model can be fitted with $\Lambda$CDM model [43, 44]. In this paper, we have presented an accelerating cosmological model of the Universe in $f(Q,T)$gravity and parametrize the cosmological parameters with the available cosmological data sets. The paper is organised as follows: In Sec II, the field equations of $f(Q,T)$ gravity has been discussed and the field equations are derived in the functional form $f(Q,T) = -\lambda_{1}Q^{2} - \lambda_{2}T^{2}$ i.e. considering the squared trace. In sec III, the observational data sets are discussed and the parametrization of Hubble parameter is done along with the other geometrical parameters. In Sec IV, the baryon acoustic oscillation data set used to obtain the parameters. The results and discussions of the models are given in sec V.
II. FIELD EQUATIONS OF $f(Q, T)$ GRAVITY

The action of $f(Q, T)$ gravity [39],

$$S = \int \left( \frac{1}{16\pi} f(Q, T) + L_m \right) d^4x \sqrt{-g},$$  \hspace{1cm} (1)

where $Q$ and $T$ are respectively the nonmetricity and trace of the energy momentum tensor. $L_m$ be the matter Lagrangian and $g = det(g_{\mu\nu})$ be the determinant of the metric tensor. The nonmetricity $Q$ can be defined as,

$$Q \equiv -8^{\mu\nu}(L^k_{\mu
u} L^l_{\nu k} - L^k_{\nu k} L^l_{\mu \nu}),$$  \hspace{1cm} (2)

where the disformation, $L^k_{\mu
u} \equiv -\frac{1}{2} g^{k\lambda}(\nabla_{\gamma} g_{\lambda\mu} + \nabla_{\lambda} g_{\gamma\mu} - \nabla_{\mu} g_{\gamma\lambda})$. The field equations of $f(Q, T)$ gravity [39] by varying the gravitational action (1) is,

$$-\frac{2}{\sqrt{-g}} \nabla_k (f_Q \sqrt{-g} P^k_{\mu\nu}) - \frac{1}{2} f g_{\mu\nu} + f_T (T_{\mu\nu} + \Theta_{\mu\nu}) - f_Q (P_{\mu\nu} Q^{kl} - 2Q^{kl} P_{kl\nu}) = 8\pi T_{\mu\nu},$$  \hspace{1cm} (3)

where $f_Q = \frac{\partial f(Q, T)}{\partial Q}$ and $T_{\mu\nu} \equiv -2\frac{\delta (\sqrt{-g} L_m)}{\delta g_{\mu\nu}}$ be the energy momentum tensor. Also, $\Theta_{\mu\nu} \equiv g^{\alpha\beta} \frac{\partial T_{\alpha\beta}}{\partial g_{\mu\nu}}$ and $P_{\mu\nu} = -\frac{1}{2} T_{\mu\nu} + \frac{1}{4} (Q^k - Q^k) g_{\mu\nu} - \frac{1}{4} \delta^{k}_{(\mu} Q_{\nu)\lambda} \lambda$ is the super potential of the model. Now,

$$T = T_{\mu\nu} g^{\mu\nu}$$
$$Q_k = Q^{\mu}_{k \mu}, \quad \tilde{Q}_k = Q^{\mu}_{k \mu}$$  \hspace{1cm} (4)

respectively be the trace of the energy momentum tensor and non-metricity tensor. To frame the cosmological model, we consider here that the universe is spatially flat, homogeneous and isotropic in the form of FLRW space-time,

$$ds^2 = -N^2(t) dt^2 + a^2(t)(dx^2 + dy^2 + dz^2),$$  \hspace{1cm} (5)

where the lapse function is $N(t)$ and in standard case as, $N(t) = 1$. The scale factor $a(t)$ can assume any form of cosmic time to examine the accelerating behaviour of the Universe. The Hubble function describes the expansion rate and can be related to the scale factor as, $H(t) = \frac{\dot{a}(t)}{a(t)}$, an over dot denotes the time derivative. $\tilde{T} = \frac{N(t)}{N(t)}$ is the dilation rate and for the standard case it vanishes and the non-metricity becomes, $Q = 6H^2$. We consider the matter in the form of the perfect fluid distribution, whose energy momentum tensor can be written as, $T_{\mu\nu} = diag(-\rho, p, p, p)$. For the standard case, the field equations of $f(Q, T)$ gravity (3) for the flat, isotropic and homogeneous space-time can be expressed in an abstract form [39, 40] as,

$$p = -\frac{1}{16\pi} \left[ f - 12FH^2 - 4\zeta \right]$$  \hspace{1cm} (6)

$$\rho = \frac{1}{16\pi} \left[ f - 12FH^2 - 4\zeta k_1 \right],$$  \hspace{1cm} (7)

where $F \equiv f_Q = \frac{\partial f}{\partial Q}$ and $8\pi \kappa \equiv f_T = \frac{\partial f}{\partial T}, \kappa_1 = \frac{\kappa}{1 + \kappa}$ and $\zeta = FH$. The evolution equation of Hubble’s function can be obtained by adding eqns. (6) and (7),

$$\dot{\zeta} = 4\pi(p + \rho)(1 + \kappa)$$  \hspace{1cm} (8)
While comparing with the Friedmann equations, the effective pressure \( p_{\text{eff}} \) and effective energy density \( \rho_{\text{eff}} \) can be expressed as,

\[
2H + 3H^2 = \frac{1}{F} \left[ \frac{f}{4} - 2FH + 4\pi[(1 + \kappa)\rho + (2 + \kappa)p] \right] = -8\pi p_{\text{eff}} \tag{9}
\]

\[
3H^2 = \frac{1}{F} \left[ \frac{f}{4} - 4\pi[(1 + \kappa)\rho + \kappa p] \right] = 8\pi \rho_{\text{eff}} \tag{10}
\]

Keeping in mind, the cosmological applications of the \( f(Q, T) \) gravity, three forms of \( f(Q, T) \) has been suggested, such as (i) \( f(Q, T) = \lambda_1 Q + \lambda_2 T \), (ii) \( f(Q, T) = \lambda_1 Q^m + \lambda_2 T \), (iii) \( f(Q, T) = -\lambda_1 Q - \lambda_2 T^2 \), where \( \lambda_1 \) and \( \lambda_2 \) are two constants \([39]\).

We are considering the model \( f(Q, T) = -\lambda_1 Q^m - \lambda_2 T^2 \), from (8) one can write the equation for \( \dot{H} \)

\[
\dot{H} + \frac{\dot{F}H}{F} = \frac{4\pi}{(1 + \kappa)}(\rho + p) \tag{11}
\]

\( F = f_Q = \frac{\partial f}{\partial Q} = -\lambda_1 mQ^{m-1} \) and \( 8\pi \kappa \equiv f_T \Rightarrow \kappa = -\frac{\lambda_2 (3\omega - 1)\rho}{4\pi} \)

From eqns. (7) and (8) the energy density can be written as

\[
\rho = \frac{f - 12FH^2}{16\pi [1 + (1 + \omega)\kappa]} \tag{12}
\]

\[
\rho = \frac{(2m - 1)\lambda_1 Q^m - \lambda_2 (2m - 1)\rho^2}{16\pi [1 + \lambda_2(\omega + 1)(1 - 3\omega)\rho/4\pi]} \tag{13}
\]

\[
\rho = \frac{8\pi}{\lambda_2(1 - 3\omega)(\omega + 5)} [1 + \sqrt{1 + \lambda_1 \lambda_2(2m - 1)(1 - 3\omega)(\omega + 5)Q^m/64\pi^2}] \tag{14}
\]

Power expanding the square root term in the above equation provides \( \rho \propto Q^m \) if the condition \( \lambda_1 \lambda_2(2m - 1)(1 - 3\omega)(\omega + 5)Q^m/64\pi^2 \ll 1 \) is satisfied. As a result, we have the typical general relativistic result for \( m = 1 \) for matter dominated case in this limit.

\[
\dot{H} = -\frac{32\pi^2(1 + \omega)}{\lambda_1 \lambda_2(2m - 1)(1 - 3\omega)(\omega + 5)mQ^{m-1}} \times \left[ 1 + \frac{2}{\lambda_1 \lambda_2(2m - 1)(1 - 3\omega)(\omega + 5)Q^m/64\pi^2} \right] \times \left[ -1 + \sqrt{1 + \lambda_1 \lambda_2(2m - 1)(1 - 3\omega)(\omega + 5)Q^m/64\pi^2} \right] \tag{15}
\]

Consider \( \eta = \frac{64\pi^2}{\lambda_1 \lambda_2(2m - 1)(1 - 3\omega)(\omega + 5)} \)

\[
2\dot{H} = -\eta(1 + \omega) \frac{mQ^{m-1}}{\left[ 1 + \frac{2}{\sqrt{1 + Q^m/\eta}} \right] \left[ -1 + \sqrt{1 + Q^m/\eta} \right]} \tag{16}
\]

We have noticed that for the decreasing values of \( \lambda_1 \) and \( \lambda_2 \) in the positive domain, the value of \( \eta \) parameter increase. As a result, we have the limiting value \( Q^m/\eta \ll 1 \) for the small positive values of \( \lambda_1 \) and \( \lambda_2 \), hence, Eq. (16) can be approximated as

\[
\frac{dH}{dt} = -\frac{3(1 + \omega)H^2}{2m} \tag{17}
\]
Since we have observed that for \( m = 1 \) the condition get satisfied for matter dominated case, hence we will fix the value of \( m = 1 \).

On simplification, we can obtain, \( \dot{H} + \gamma H^2 = 0 \), where \( \gamma = \frac{3(1+\omega)}{2} \). On solving, we obtain

\[
H(t) = \frac{1}{\gamma t + c_1},
\]

(18)

where \( c_1 \) be the integrating constant. Subsequently we can find the scale factor as, \( a(t) = c_2 [\gamma t + c_1]^{\frac{1}{\gamma}} \). Using the relationship between the scale factor and the redshift parameter, \( a = \frac{1}{1+z} \), we can find the parametric form of \( H(z) \) as,

\[
H(z) = H_0 (1+z)^{\gamma},
\]

(19)

where \( H_0 \) be the present value of the Hubble parameter. For matter dominated phase \( (\omega = 0) \) and dark energy dominated phase \( (\omega \text{ is constant}) \) respectively we obtain the following relation for the Hubble parameter,

\[
H(z) = H_0 (1+z)^{\frac{3}{2}}
\]

\[
H(z) = H_0 (1+z)^{\frac{3(1+\omega)}{2}}
\]

(20)

The Friedman equation describes the rate of expansion \( H(z) \) at redshift \( z \) with constant equation of state parameter as,

\[
\frac{H^2(z)}{H_0^2} = \Omega_m (1+z)^3 + \Omega_{DE} (1+z)^{3(1+\omega)}
\]

(21)

Subsequently, we shall parametrize the parameter with the cosmological data sets.

### III. MODEL WITH OBSERVATIONAL DATA

We shall outline in this section, the cosmological data to be used in the problem to parametrize the model parameters. The data sets are related to the expansion history of the Universe i.e describing the distance-redshift relations. Mostly we shall use the expansion rate data from early type galaxies and Pantheon supernovae data.

- **Hubble Data**: Through estimations of their differential evolution, early type galaxies provide Hubble parameter measurements. The process of compilation of such observations is known as the cosmic chronometers and from the recent result, the redshift range, \( 0.07 < z < 2.36 \). For these measurements, the \( \chi^2_{OHD} \) estimator can be constructed with 55 data points from different data source defined in TABLE III,

\[
\chi^2_{OHD} = \sum_{i=1}^{55} \frac{[H_{th}(z_i, \theta) - H_{obs}(z_i)]^2}{\sigma^2_H(z_i)}
\]

(22)

where \( H_{th}(z_i, \theta) \) and \( H_{obs}(z_i) \) respectively represents the generated Hubble parameter and the observed Hubble parameter and \( \theta \) be the vector of the cosmological background parameters. \( \sigma^2_H(z_i) \) denotes the observational errors on the measured values \( H_{obs}(z_i) \).

- **Pantheon supernovae data**: Supernovae type Ia observation was the first to indicate the accelerated expansion of the Universe. Several new SNIa data sets have been developed and in this analysis, Pantheon Supernovae data is considered that contains 1048 data points and provide the estimated value of the distance moduli \( \mu_i \) in the redshift range \( 0.01012 < z < 2.26 \) [45]. The model parameters are to be fitted by comparing the observed and theoretical value of the distance moduli. The distance moduli can be defined as,

\[
\mu(z, \theta) = 5 \log_{10}[d_L(z, \theta)] + \mu_0,
\]

(23)
where $\mu_0$ is the nuisance parameter and $d_L$ is the dimensionless luminosity distance defined as,

$$d_L(z, \theta) = (1 + z) \int_0^z \frac{dz'}{E(z')}$$  \hspace{1cm} (24)

where $E(z) = H(z)/H_0$. Now the $\chi^2$ estimator reads,

$$\chi^2_{\text{Pantheon}(z, \theta)} = \sum_{i=1}^{1048} \left[ \frac{\mu(z_i, \theta)_{\text{th}} - \mu(z_i)_{\text{obs}}}{\sigma_\mu(z_i)} \right]^2,$$  \hspace{1cm} (25)

where $\sigma_\mu(z_i)$ is the standard error in the observed value. We shall use these data sets to constrain our models. For joint analysis, $\chi^2_{\text{joint}}$ can read as

$$\chi^2_{\text{joint}} = \chi^2_{\text{OHD}} + \chi^2_{\text{Pantheon}}$$  \hspace{1cm} (26)

FIG. 1. Plots of EoS parameter vs density parameter using $H(z)$ data (left panel) and Pantheon data (right panel).

FIG. 2. Plots of EoS parameter vs. density parameter with $H(z)$ + Pantheon data.
TABLE I. Summary of the observational results

| Source/Data | \( H(z = 0) \) | Pantheon | \( H(z = 0) + \text{Pantheon} \) |
|-------------|---------------|---------|-------------------------------|
| \( H_0(\text{km} \text{s}^{-1} \text{Mpc}^{-1}) \) | 70.7 | 69.95 | 70.7 |
| \( \omega_{DE} \) | \(-1.011^{+0.002}_{-0.002} \) | \(-1.034^{+0.003}_{-0.003} \) | \(-1.024^{+0.001}_{-0.001} \) |
| \( (\Omega_{DE})_0 \) | 0.7434^{+0.003}_{-0.003} | 0.69^{+0.0043}_{-0.0043} | 0.734^{+0.002}_{-0.002} |

The model created above is compared to observational data points from \( H(z) \) and pantheon data. We can see the best fitted values of the EoS parameter, \( \omega \) and the density parameter for dark energy, \( \Omega_{DE} \) on the contour map for 1\( \sigma \), 2\( \sigma \) and 3\( \sigma \). The density parameter \( \Omega_{DE} = 0.7434 \pm 0.003 \), \( \Omega_{DE} = 0.69 \pm 0.0043 \) and \( \Omega_{DE} = 0.734 \pm 0.002 \) form \( H(z) \) data, Pantheon data and \( H(z) + \text{Pantheon} \) data sets, respectively, whereas the EoS parameter becomes \( \omega = -1.011 \pm 0.002 \), \( \omega = -1.034 \pm 0.003 \) and \( \omega = -1.024 \pm 0.001 \).

where \( H_0 \) is the current rate of expansion, \( \Omega_m \) and \( \Omega_{DE} \) respectively represent that density parameter of matter and dark energy component and \( \omega(z) \) is the EoS parameter of dark energy component. The integral of \( 1/H(z) \) over redshift determines the luminosity distance. According to the analysis on the combination of low redshift data and CMB anisotropy, 30% of the matter energy density \( (\Omega_m) \) constitutes the cold dark matter and around 70% dark energy in the form of cosmological constant \( \Omega_{\Lambda} \). Also, \( \Omega_m = \Omega_{CDM} + \Omega_b \) with \( \Omega_{CDM} \approx 0.25 \) and the baryonic matter \( \Omega_b \approx 0.05 \). It has vanishing spatial curvature, \( \Omega_k \approx 0 \) and residual radiation, \( \Omega_r \approx 5 \times 10^{-5} \). The radiation density parameter has been set as, \( \Omega_r = 0 \). Here, we shall discuss the simple dark energy models such as (i) \( \Lambda \text{CDM} \) and (ii) \( \omega \text{CDM} \) with the \( H(z) \) as given in Eq. (18). The \( \omega \text{CDM} \) approach is an extension of \( \Lambda \text{CDM} \) model, where \( \omega \neq -1 \) exactly. Like the concordance model, a negative \( \omega \) is required to ensure that the universe accelerates. Now,

\[
H(z) = H_0 \sqrt{(1 - \Omega_{DE})(1 + z)^3 + \Omega_{DE}(1 + z)^{3(1+\omega)}}
\]

(27)

FIG. 3. Plots for \( H(z) \text{ km} \text{s}^{-1} \text{Mpc}^{-1} \) (left panel) and \( H(z)/(1+z) \text{ km} \text{s}^{-1} \text{Mpc}^{-1} \) (right panel) vs redshift from 55 data points of \( H(z) \) as in TABLE III, \( \Omega_{DE} = 0.74343 \) and \( \omega = -1.011 \). The points with error bars indicate the observed Hubble data. The solid blue line represents the derived model.

According to the present observations the value of Hubble parameter is \( H_0 = 71 \pm 3 \text{ km} \text{s}^{-1} \text{Mpc}^{-1} \), and in the present scenario we took the value of Hubble parameter as \( H_0 \approx 70 \text{ km} \text{s}^{-1} \text{Mpc}^{-1} \). FIG. 3 shows that the model presented here passes almost in the middle of the observational \( H(z) \) data points.

In the cosmographic series, Hubble parameter \( H \) is the first derivative form of the scale factor. The second, third and fourth derivative form of the scale factor respectively produces the deceleration, jerk and snap parameter. Determining the physical quantities \( H \) and \( q \) has become very crucial in characterising the evolution of the Universe. The Hubble parameter \( H \) gives us the rate of expansion of the Universe so that the age of the Universe can be determined and the sign of the deceleration parameter \( q \) indicates the accelerating or decelerating dynamics of the
Universe, however it does not account for the entire dynamics. The change of sign of the jerk parameter $j$ is important in the sense that the positive $j$ indicates the change of expansion at some point during the evolution. The deceleration and jerk parameter define the local dynamics but can not distinguish the cosmological models effectively. When $j_0 = 1$, the model favours the $\Lambda$CDM model and the universe continues to expand at an accelerated rate under the influence of a cosmological constant. On the other hand, the value of $s$ is required in order to establish if there is any evolution of dark energy. The functional dependence of dark energy on the redshift $z$ is affected by departures from the predicted value of $s$, which is evaluated in the concordance model, demonstrating that it evolves as the universe expands. The cosmographic parameters can be obtained as,

$$ q = -1 - \frac{H}{H^2} = -1 + \frac{3}{2} \left[ (1 - \Omega_{DE}) + \omega_{DE}(1 + \omega)(1 + z)^{3\omega} \right] $$

$$ j = \frac{\ddot{H}}{H^3} - 3q - 2 = \frac{\Omega_{DE} \left[ 9\omega(\omega + 1) + 2 \right] (z + 1)^{3\omega} - 2}{2\Omega_{DE} [(z + 1)^{3\omega} - 1] + 2} $$

$$ s = 6 + 4j + 3q(4 + q) + \frac{\ddot{H}}{H^4} $$

$$ s = \Omega_{DE}[(\Omega_{DE} - 1)(3\omega + 4) \left[ 3\omega(6\omega + 5) + 7 \right] (z + 1)^{3\omega} - 14(\Omega_{DE} - 1)^2 - \Omega_{DE}(3\omega + 1)(3\omega + 2)(9\omega + 7)(z + 1)^{6\omega}] $$

$$ 4 \left[ \Omega_{DE} [(z + 1)^{3\omega} - 1] + 1 \right]^2 $$

(30)

FIG. 4. Plots of deceleration parameter vs. redshift for $\omega = -1$ (blue line) and $\omega = -1.024$ (red line)
In FIG. 4, the deceleration parameter shows early deceleration to late time acceleration, the transition happens from deceleration to acceleration at $z_t = 0.76$ and the present value of deceleration parameter $\approx -0.62$ from the model. It is noteworthy to mention that the value of deceleration parameter is match with the present observable value of deceleration parameter which is $-0.56$ [46]. At late time, it approaches to $-1$ showing the accelerating behaviour of the model. Another important aspect of the model is its deviation from $\Lambda$CDM model, which can be identified through the state finder pair $(j, s)$ [47, 48]. When $(j, s) = (1, 0)$, the model approaches to $\Lambda$CDM and the same feature we can obtain in this model by assuming $\omega = -1$ at present value of $z$ (see FIG. 5). However when we consider $\omega = -1.024$, the pair approaches to $(1.11, -0.04)$ at the late time of evolution and at $z = 0$ i.e for the present value of $\omega$, it gives $(j, s) = (1.08, -0.2)$. We can say that the model discussed here extremely similar to the $\Lambda$CDM behaviour.

Another important cosmological diagnostic is the $O\!m(z)$ diagnostic, which is the combination of Hubble parameter and the cosmological redshift. This will provide a null test for dark energy being the cosmological constant. If the value of $O\!m(z)$ is the same at different redshifts, then dark energy is $\approx \Lambda$, the cosmological constant. We can find the expression for $O\!m(z)$ as,

$$O\!m(z) = \frac{E^2(z) - 1}{(1 + z)^3 - 1}$$

where $E(z) = \frac{H(z)}{H_0}$

$$O\!m(z) = \frac{(1 - \Omega_{DE})(1 + z)^3 + \Omega_{DE}(1 + z)^3(1 + \omega) - 1}{(1 + z)^3 - 1}$$

(31)

FIG. 5. Plots of jerk parameter(left panel) and snap parameter (right panel) vs. redshift for $\omega = -1$ (blue line) and $\omega = -1.024$ (red line)

FIG. 6. Graphical representation for $O\!m(z)$ vs. $z$ with $\Omega_{DE} = 0.734$ and $\omega = -1.024$.  

0.255
0.260
0.265
0.270
z
Om(z)
The slope of $\Omega_m(z)$ can distinguish between different dark energy models, even if the value of the matter density is not known. A positive slope of $\Omega_m(z)$ for the dark energy model indicates the phantom ($\omega < -1$) behaviour whereas the negative slope indicates the quintessence ($\omega > -1$) model. From FIG. 6 shows the phantom like behaviour of the model.

Now, we shall see the behaviour of distance modulus of the model with the Pantheon data set. The blue line in the panel of FIG. 7 shows the distance modulus of the model. In the plot, it is traversing well inside the error bar. The recent dataset has a higher number of data points and extends longer, to $z = 2.26$ instead of $z \approx 1.5$. However, the Pantheon data set is deceptive, since only six data points out of 1048 are over $z = 1.5$, and only one is above $z = 2$. The error bars have also been decreased. So, with higher number of data points, it has been anticipated more consistent results on the recreation of the expansion history of the Universe.

Now, we shall compute the age of the Universe as,

$$H_0(t_0 - t) = \int_0^z \frac{dx}{(1 + x)E(x)}, \quad E(z) = \frac{H(z)}{H_0},$$

where,

$$H_0t_0 = \lim_{z \to \infty} \int_0^z \frac{dx}{(1 + x)E(x)}$$

$H(z)$ is given in Eq. (27).

The behaviour of time with redshift is depicted in FIG. 8. It is found that for infinitely large $z$, $H_0(t_0 - t)$ converges to 0.9966. We can use this to calculate the current age of the Universe as $t_0 = 0.9966 \times 14.09 \text{ Gyrs}$, which is quite close to the age calculated from the Planck result $t_0 = 13.78 \pm 0.020 \text{ Gyrs}$. As a result, the Universe’s derived model is fairly consistent with current data.
IV. BARYON ACOUSTIC OSCILLATIONS AND COSMIC MICROWAVE BACKGROUND

Numerous galaxy surveys have demonstrated that BAO standard ruler measurements are self-consistent with the standard cosmology model obtained from CMB observations, and have resulted in new, more stringent cosmological parameter constraints. The measurement of baryon acoustic oscillations (BAOs) in the large-scale clustering pattern of galaxies, as well as their use as a cosmological standard ruler, is a very promising and complementary method for mapping the distance-redshift relation [49–53]. Giostri et al. [54] integrate type Ia supernovae (SN Ia) data with recent baryonic acoustic oscillations (BAO) and cosmic microwave background (CMB) measurements to limit a kink-like parametrization of the deceleration parameter. Eisenstein et al. [55] investigated the nonlinear degradation of the baryon acoustic signature using a number of methods. The Hubble parameter, \( H(z) \), and the angular diameter distance, \( D_A(z) \), can theoretically be derived concurrently from data in the radial and transverse directions using the baryon acoustic oscillation (BAO) scale, which gives galaxy clustering its power as a dark energy probe [51, 53, 56]. As described by Jarosik et al. [57] in their WMAP data, the value of the decoupling redshift, \( z_* \), is taken to be 1092.

In a cosmological model, \( D_V(z) \) is a composite of the physical angular-diameter distance \( D_A(z) \) and the Hubble parameter \( H(z) \), which determine tangential and radial separations, respectively:

\[
D_V(z) = \left( \frac{D_A^2(z)c_2}{H(z)} \right)^{\frac{1}{3}} \tag{34}
\]

\[
D_A(z_*) = c \int_0^{z_*} \frac{dz'}{H(z')} \tag{35}
\]

\[
d_z = \frac{r_s(z_d)}{D_V(z)} \tag{36}
\]
TABLE II. BAO measurements at six different redshifts are now included in the most recent BAO distance data set, which includes the 6dFGS, SDSS, and WiggleZ studies. Following table summarises the information.

| Sample   | Redshift (z) | $d_z$ | $D_A(z_*)/D_V(z_{BAO})$ |
|----------|--------------|-------|-------------------------|
| 6dFGS    | 0.106        | 0.3360 ± 0.0150 [36] | 30.95 ± 1.46 [54] |
| SDSS     | 0.200        | 0.1905 ± 0.0061 [36] | 17.55 ± 0.60 [54] |
| SDSS     | 0.350        | 0.1097 ± 0.0036 [36] | 10.11 ± 0.37 [54] |
| WiggleZ  | 0.440        | 0.0916 ± 0.0071 [36] | 8.44 ± 0.67 [54]  |
| WiggleZ  | 0.600        | 0.0726 ± 0.0034 [36] | 6.69 ± 0.33 [54]  |
| WiggleZ  | 0.730        | 0.0592 ± 0.0032 [36] | 5.45 ± 0.31 [54]  |

The plot for the distilled parameter and BAO/CMB constraints $D_A(z_*)/D_V(z_{BAO})$ has been made using the $H_0 = 70.7 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_{DE} = 0.734$ for the model and $\omega = -1.024$, that we obtained in our previous results. As can be seen from the graphs, the values we obtained in our results are best suited with the observational values of the BAO data.

V. CONCLUSIONS

In this paper, we have developed the scale factor using some algebraic manipulation of the field equations obtained in $f(Q,T)$ gravity. The governing equation can be solvable if the value of model parameter becomes very small. One case leads to the gravity with non-metricity only and the other to trace of energy momentum tensor only. We prefer to go with the trace part and assumed the trace as its square form. The cosmological parameters and EoS parameter have been constrained using the $H(z)$, $H(z)+$Pantheon and BAO data set. Using 55 data points and a $\chi^2$ minimization strategy, we rebuilt $H(z)$ and distance modulus for observable $H(z)$ values in the redshift range $0.07 < z < 2.36$. We also looked at the Pantheon compilation data, which included 1048 SNIa apparent magnitude measurements. The essential characteristics of the results are summarised as:

The cosmological model undergoes transition from deceleration to acceleration phase at a transition redshift $z_t = 0.76$. Some recent constraints are compatible with the extracted value of the transition redshift. At the present epoch, the transiting Universe derived in the model has deceleration parameters $q_0 = -0.62$. The Hubble parameter $H_0$ is $70.7 \text{ km s}^{-1} \text{ Mpc}^{-1}$, which is in the middle of the reported Hubble tension, according to a joint study of $H(z)$ and Pantheon compilation data. We calculated the age of the Universe using the restricted value of the Hubble parameter. We recreated the cosmographic parameters and the $\Omega m(z)$ parameter as a test of our derived models to
TABLE III. The observational data-set that was used in this paper

| No. | Redshift | H(z) | σ_H | Ref. | No. | Redshift | H(z) | σ_H | Ref. |
|-----|----------|------|-----|------|-----|----------|------|-----|------|
| 1.  | 0.07     | 69.0 | 19.6| [58] | 29. | 0.48     | 87.79| 2.03| [64] |
| 2.  | 0.09     | 69.0 | 12.0| [59] | 30. | 0.480    | 97   | 62  | [70] |
| 3.  | 0.12     | 68.6 | 26.2| [58] | 31. | 0.510    | 90.4 | 1.9 | [66] |
| 4.  | 0.17     | 83   | 8   | [60] | 32. | 0.52     | 94.35| 2.64| [64] |
| 5.  | 0.179    | 75.0 | 4.0 | [61] | 33. | 0.56     | 93.34| 2.3 | [64] |
| 6.  | 0.199    | 75.0 | 5.0 | [61] | 34. | 0.59     | 98.48| 3.18| [64] |
| 7.  | 0.200    | 72.9 | 29.6| [58] | 35. | 0.593    | 104.0| 13.0| [61] |
| 8.  | 0.24     | 79.69| 3.32| [62] | 36. | 0.6      | 87.9 | 6.1 | [68] |
| 9.  | 0.27     | 77   | 14  | [60] | 37. | 0.610    | 97.3 | 2.1 | [66] |
| 10. | 0.280    | 88.8 | 36.6| [58] | 38. | 0.64     | 98.02| 2.98| [64] |
| 11. | 0.30     | 81.7 | 5.0 | [63] | 39. | 0.680    | 92.0 | 8.0 | [61] |
| 12. | 0.31     | 78.18| 4.74| [64] | 40. | 0.73     | 97.3 | 7   | [68] |
| 13. | 0.34     | 83.8 | 2.96| [62] | 41. | 0.781    | 105.0| 12  | [61] |
| 14. | 0.35     | 82.7 | 9.1 | [65] | 42. | 0.875    | 125  | 17  | [61] |
| 15. | 0.352    | 83   | 14  | [61] | 43. | 0.880    | 90.0 | 40.0| [70] |
| 16. | 0.36     | 79.94| 3.38| [64] | 44. | 0.900    | 117  | 23.0| [60] |
| 17. | 0.38     | 81.5 | 1.9 | [66] | 45. | 1.037    | 154  | 20  | [61] |
| 18. | 0.3802   | 83.0 | 13.5| [67] | 46. | 1.300    | 168  | 17  | [60] |
| 19. | 0.4      | 95   | 17  | [60] | 47. | 1.563    | 160  | 33.6| [71] |
| 20. | 0.40     | 82.04| 2.03| [64] | 48. | 1.430    | 177  | 18  | [60] |
| 21. | 0.4004   | 77   | 10.2| [67] | 49. | 1.530    | 140  | 14  | [60] |
| 22. | 0.4247   | 87.1 | 11.2| [67] | 50. | 1.750    | 202  | 40  | [60] |
| 23. | 0.43     | 86.45| 3.27| [62] | 51. | 1.965    | 186.5| 50.4| [71] |
| 24. | 0.44     | 82.6 | 7.8 | [68] | 52. | 2.30     | 224  | 8   | [72] |
| 25. | 0.44     | 84.81| 1.83| [64] | 53. | 2.33     | 224  | 8   | [73] |
| 26. | 0.4497   | 92.8 | 12.9| [67] | 54. | 2.340    | 222  | 7   | [74] |
| 27. | 0.470    | 89   | 34  | [69] | 55. | 2.360    | 226  | 8   | [75] |
| 28. | 0.4783   | 80.9 | 9.0 | [67] |      |          |      |     |      |

see if they differ from the concordance $\Lambda$CDM model. It should be noted that a divergence from the $\Lambda$CDM model could indicate that the dark energy and dark matter components interact. At least at this epoch, the behaviour of the $\Omega_m(z)$ parameter in our models may favour a phantom phase.

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