Deformation Mechanism of Thin and Medium-Thickness Cylinder and Its Collapse Strength under Bending

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Abstract. Considering the influence of flattening of casing cross-section on collapse, the stress equivalence principle was applied to conclude the triaxial stress and collapse strength. The method to determine casing collapse strength considering flattening of casing cross-section was established. The diameter-to-thickness ratio had a major influence on the flattening of the casing cross-section compared with curvature. For the 5 1/2”×7.72mm P110 casing, the flattening course by the 0.01/m curvature decreased the collapse strength concluded by API 5C3 by approximately 7.5%.

1. Introduction

The mechanical behavior of casing under bending is one of the basic problems in mechanical research. To solve the problem of section deformation of casing, Yu Tongxi conducted a series of research on the deformation, ultimate bending moment, and failure formation of circular casing. By using full quantity theory, the relationship among bending curvature, casing flatness, and bending moment in the elastic–plastic range was demonstrated [1-3]. Xue Jiaxing et al. designed a four-point bending experiment of casing and determined the relationship between the instability mode of casing and diameter-to-thickness ratio [4]. E. Daxin, et al. analyzed the stress and strain by the tangential direction of the bending line and the thickness direction of casing wall thickness through a large number of casing bending tests. They found that the strain neutral layer moves toward the bending center during the bending process, and the amount of internal displacement of the strain neutral layer is inversely proportional to the relative bending radius [5-6]. Huayong Zhong et al. established the calculation formula for the flattening of the cross-section of a straight circular casing [7].

In the design of the bending casing strength, the American Petroleum Institute (API) specification API 5C3 presents the calculation method of the casing cross-section to collapse under bending. The influence of casing section deformation caused by bending on the weakening of the casing cross-section to collapse was ignored [8]. Weian Gong considered the influence of flattening of the casing cross-section on collapse and got the critical load of the elliptical shell, but its axial stress was not considered [9]. Mengshi Jin, et al. assumed that the circumferential hoop strain of the circular casing is not zero, and demonstrated the relationship between collapse and the bending moment of the long cylindrical shell [10, 11]. E. Tsuru, et al. combined experiments and the finite element method to analyze the influence of different diameter-to-thickness ratios and flattening on the strength and failure mechanism of submarine pipelines and collapse [12]. Zhuang Baotang, et al. established a 3D model of horizontal well casing and analyzed the collapse of casing under different wellbore curvatures using the finite
element method [13]. Tongtao Wang, et al. calculated the bending stress caused by wellbore curvatures and deduced the formula of collapse on the casing of angle buildup interval that considered the wellbore curvatures [14]. Jun Wang, et al. calculated the axial stress of the casing and applied the energy method to infer the formula of the collapse of the casing of angle buildup interval considering axial stress [15]. Huang Genlu, et al. determined the influence of wellbore curvature and uneven extrusion force and got the formula of casing collapse strength strength in inclined section considering the influence of wellbore curvature and uneven extrusion force [16]. Shen Zhaoxi analyzed the uneven circumferential stress in the circumferential direction of non-circular casing and proposed a formula for casing collapse strength, but they did not consider the influence of bending effects [17].

Lin Yuanhua, et al. derived full-wall yielding collapse of the casing under uniform external pressure based on unified strength theory [18]. Andre C. Nogueira described the collapse formula for submarine pipelines under external pressure and bending in the fourth edition of API RP1111 [19]. A. Hilberink, et al. used ABAQUS to analyze the deformation of the composite casing during the bending process and analyzed the relationship between the bending curvature and bending moment of the section [20]. Chen Hongyuan, et al. studied the buckling failure of a 40-inch pipeline under bending load by an experimental method and determined its ultimate compressive strain [21].

To solve the problem of the deformation and failure of the casing under bending and external pressure, the influence of flattening of the casing cross-section on collapse, the stress equivalence principle was applied. This study provides a reference for the deformation mechanism and strength analysis of medium-thick wall casing under bending.

2. Strain-stress analysis of bending casing section under natural coordinates

In accordance with the generalized Hooke law, the axial stress and circumferential stress formulas of the casing cross-section under bending were built.

2.1. Analysis of circumferential strain of casing under bending

As shown in Fig. 1, the curvature of the wellbore causes the casing to bend and deform, and the casing section is deformed under bending stress [3].

\[
\varepsilon_s(s,t) = \frac{\varepsilon_{\theta\theta} + \frac{d\phi}{ds}t}{1 + \frac{d\theta}{ds}t}
\]  

(1)

To simplify the calculation, we assume that the length of the middle section of the casing is not changed before and after the deformation. The circumferential strain \(\varepsilon_s(s,t)\) at any point of the casing section can be simplified as follows:
\[
\varepsilon_z(s,t) = \frac{d\phi}{ds} = \frac{d\phi}{1 + \frac{d\theta}{ds} t}
\]

Where \( t \) is the distance from a point on the section of the casing to the midline of the section, m. \( \rho_s \) is the radius of the curvature of the line in the mid-section of the casing section, m.

\subsection*{2.2. Axial stress analysis of casing under bending}

The axial strain \( \varepsilon_{zo} \) of the midline of the bent casing section was obtained as follows:

\[
\varepsilon_{zo} = \frac{y(s)}{\rho_z}
\]

Where \( y(s) \) is the vertical distance from any point on the middle section of the casing section to \( x \)-axis.

Experiments revealed that the section of the casing was deformed from a circular shape to an elliptical shape [4, 10, 19]. The projection of the midlines of the casing section satisfies

\[
y(s) = R(1 - \zeta)\sin \varphi
\]

\subsection*{2.3. Analysis of axial stress and circumferential stress of casing under bending}

Assuming that the casing is in the line elastic phase, on the basis of the generalized Hooke law [22], the circumferential stress \( \sigma_s \) and axial stress \( \sigma_z \) expressions of the casing section were obtained.

\[
\sigma_s = \frac{E}{(1 - \mu^2)}(\varepsilon_s + \mu \varepsilon_z)
\]

\[
\sigma_z = \frac{E}{(1 - \mu^2)}(\varepsilon_z + \mu \varepsilon_s)
\]

\subsection*{3. Analysis of the relationship between flattening of the casing cross-section and curvature based on the minimum potential energy principle}

Through the deformation energy formula, the total strain energy formula \( U \) of the casing cross-section under bending was built:

\[
U = 2 \int_{s_1 - i/2}^{s_1 + i/2} \int_{0}^{\epsilon_z} \left( \int_{0}^{\epsilon_z} \sigma_s d\varepsilon_s + \int_{0}^{\epsilon_z} \sigma_z d\varepsilon_z \right) dtds
\]

In the process of bending deformation, the curvature \( \rho_z \) and the flattening of the casing cross-section \( \zeta \) shows one-to-one correspondence, and a multi-solution is absent. Therefore, in accordance with the minimum potential energy principle, \( \frac{\partial U}{\partial \zeta} = 0 \), the corresponding relationship between the flattening of the casing cross-section \( \zeta \) and the curvature \( \rho_z \) can be obtained as follows:
\[ \xi = \frac{4\left(\frac{R}{t}\right)^2 - \frac{R}{\rho_s}}{4\left(\frac{R}{t}\right)^2 + 3} \quad (8) \]

4. Analysis of the three-axis strength and critical load of the curvature and bending

A section with an unrounded degree was present under external pressure to produce additional bending moment \( M \), which can be expressed as

\[ M_f = \frac{p_0 D^2}{4} (1 + \xi) \xi \cos 2\phi \quad (9) \]

Where \( M_f \) is cross-sectional horizontal additional bending moment, N/m; \( p_0 \) is external pressure, Pa.

The critical collapse strength of casing considering bending is:

\[ p_{cr} = \frac{2D^2}{D^2 - d^2} \frac{\sigma_s}{t} \left[ 1 - 0.75 \frac{\sigma_s}{\sigma_s} - 0.5 \frac{\sigma_s}{\sigma_s} \right] \quad (10) \]

5. Analysis of the influence of bending curvature and diameter-to-thickness ratio on the flattening of casing cross-section and collapse strength

To investigate the influence of bending curvature and diameter-to-thickness ratio on the flattening of the casing cross-section and the critical load of the external pressure, P110 5 1/2"x7.72mm, P110 5 1/2"x9.17mm, and P110 5 1/2"x10.54mm were used as research objects.

5.1. Analysis of the influence of bending curvature and diameter-to-thickness ratio on the flattening of the casing cross-section

As shown in Fig. 2, the coefficient of flattening of the casing cross-section increased as the curvature of the wellbore increased. However, as the curvature of the wellbore continued to increase, the variation in the flattening coefficient of the section gradually decreased. As the curvature of the wellbore increased, the stress of the section approached the strength of the material. Plastic deformation occurred, and the bearing capacity of the casing decreased. However, in practice, if the curvature of the wellbore is less than 0.5/m, then the flattening of the casing cross-section of P110 5 1/2" does not exceed 10%.

![Figure 2. Relationship between the curvature \( \kappa \) and flattening \( \xi \).](image_url)

As shown in Fig. 3, under the same wellbore curvature constraint, the larger the diameter-to-thickness ratio, the greater the flattening of the casing cross-section. Moreover, the greater the curvature of the wellbore, the more significant the effect of the diameter-to-thickness ratio on the flattening of the casing cross-section.
Figure 3. Relationship between the diameter-to-thickness ratio (D/t) and flattening $\xi$.

5.2. Analysis of the influence of bending curvature and diameter-to-thickness ratio on the collapse of the casing

Fig. 4 shows that the curvature of 0.01/m reduced the safety design coefficient by approximately 0.075. Thus, the flattening of the casing cross-section was caused by the curvature of 0.01/m, and the collapse of the casing considering the bending stress was reduced by 0.075. Because the actual wellbore curvature was less than 0.1/m, the safety design factor of P110 5 1/2"×7.72mm casing in the range of 0–0.1/m curvature was given.

Figure 4. Relationship between the safety design factor $c$ and flattening $\xi$.

As the bending curvature increases, the flattening of the casing cross-section increases. Under the action of external pressure, the circumferential additional stress of the casing section increases, resulting in the deformation failure of the casing. Under the action of external pressure, the axial additional stress of the casing section increases, resulting in the deformation failure of the casing. For the P110 5 1/2"×7.72mm casing, the flattening of the casing cross-section due to the curvature of 0.01/m probably reduced the flattening of the casing cross-section by about 7.5% under the bending load of the API.

6. Conclusion

(1) Through the energy method and the minimum potential energy principle, the relationship between the bending curvature and flattening of the casing cross-section in the elastic range was established.

(2) The results of the comparison showed that the diameter-to-thickness ratio had a greater influence on the flattening of the casing cross-section than the bending curvature.

(3) With increasing bending curvature, the flattening of the casing cross-section increased. For the P110 5 1/2"×7.72mm casing, the flattening of the casing cross-section caused by the 0.01/m curvature may reduce the casing’s extrusion resistance strength under API bending load by 7.5%.
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