Composite control for trajectory tracking of wheeled mobile robots with NLESO and NTSMC

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Abstract
This paper proposes a control strategy integrating the non-linear extended state observer (NLESO) and the non-singular terminal sliding mode control (NTSMC) for the trajectory tracking of wheeled mobile robots subject to bounded disturbances. A new transformation method of chained model in terms of Lie derivative is presented to simplify the controller design. A specific NLESO combining linear term and non-linear term is designed to estimate the disturbances with a faster convergence performance. A scheme for determining the gain range of NLESO is explicitly given to facilitate the tuning of experimental parameters. Meanwhile, the NTSMC achieves finite time convergence of the tracking error system and the chattering phenomenon in NTSMC is dramatically alleviated with the compensation from NLESO. The experimental results validate the strong robustness and good performance of the proposed control strategy.

1 | INTRODUCTION

Wheeled mobile robots (WMRs) have been widely investigated over the decades due to its broad applications in military, civilian and other fields. Various advanced algorithms were proposed for the research on WMRs [1–3]. Adaptive control [4], sliding mode control [5], finite time control [6] and other reliable methods were commonly used to solve the intractable difficulties of WMRs. Among them, tracking control captures considerable attention. [7] handled the issue of trajectory tracking with lateral and longitudinal slippage. In [8], the tracking control with non-holonomic constraints in the presence of external disturbances and unknown parameters has been discussed. It is well known that underactuated characteristics, model uncertainty and external disturbances bring great challenges in both theoretical analysis and practical application for WMRs. In order to deal with these issues, researchers have come up with different approaches. [9] addressed the compensation method for underactuated problem by active disturbance rejection control (ADRC), while [10] presented an integral sliding mode controller which has an extra degree of freedom to deal with this problem. In [11], an adaptive fuzzy variable structure control was proposed to solve uncertainties and disturbances. To improve real-time robustness, the model predictive control was applied for trajectory tracking with external disturbances [12]. On the other hand, WMRs are subject to non-holonomic constraints, which means that WMRs cannot move in every direction freely. To this end, simplifying the model with appropriate transformations becomes a viable solution. Actually, WMRs are non-linear affine systems. In [13], a method of transforming the non-linear affine system into a chained form was developed. Due to the simple structure of the chained model, controller design and theoretical analysis become more convenient. [14] gave a specific chained transformation and attitude control of a space robot, while it cannot meet the requirement of the global transformation. To solve this problem, we have put forward a simple and practical method of global transformation for WMRs. In general, it is interesting to develop an appropriate control strategy to guarantee accurate tracking control for WMRs with uncertainties, disturbances and non-holonomic constraints. Furthermore, most of the existing results focus on the theoretical analysis and it is hard to apply them in practice. Therefore, this paper aims to design a practical trajectory tracking control strategy.

Sliding mode control has been widely studied due to its excellent disturbances rejection performance [15–17]. [18] proposed a novel sliding surface for the robust control with matched uncertainties. An event-triggered sliding mode control...
algorithm was investigated in [19]. These results provide some theoretical support for the research of sliding mode control. The key of sliding mode control is to design a suitable sliding mode manifold. Ordinary linear slide mode manifold can only guarantee asymptotic stability or exponential stability. In terms of non-singular terminal sliding mode control (NTSMC), [20] proposed a finite-time convergence strategy. As we all know, the chattering problem is inevitable for the aforementioned methods. This greatly hinders the application of sliding mode control in practice, although it has good robustness in theory. Fortunately, some tools have been developed to handle it. [21] utilized filtering techniques to schedule the switching control gain automatically when system entered the sliding motion. [22] switched on the derivative of control instead of the control input itself to reduce chatter phenomenon. A dynamic sliding mode control strategy was given to solve unmatched perturbations and chattering problem [23]. In [24], a robust two-dimensional observer for estimation of the state-dependent uncertainty in the sliding variable was considered for chattering reduction. It is undeniable that observer compensation is an effective technique, but a well-designed observer remains difficult. There are some advanced disturbance observers such as adaptive disturbance observer [25], finite-time disturbance observer [26] and sliding mode observer [27]. The above-mentioned disturbance observers have good performance, but they bring certain difficulties for practical application due to the complex parameters and difficulties in implementation. On the other hand, ADRC has received more and more attention for its superior capability of disturbance rejection [28]. The key component named extended state observer (ESO) in ADRC [29] provides an alternative for us. ESO estimates the total disturbance including modelling deviation, external disturbances and other uncertainties. Then the total disturbance is compensated by suitable feedback control. ESO has good accuracy and real-time performance when the gains are designed properly. Many types of ESO have been reported in the literature. In [30], a high gain ESO was constructed to guarantee that the output tracks the reference signal practically. [31] proposed an adaptive ESO to reduce the estimation errors. In [32], a non-linear ESO (NLESO) constructed from piece-wise smooth functions has been given, which consists of linear and fractional power functions. The convergence of NLESO has also been discussed in [33] which provides us a method of stability analysis. Furthermore, the practical stability of ADRC for the closed-loop system was considered in [34]. A higher-order ESO was designed to reconstruct the unmeasured states and disturbances online for brushless DC motor control [35],[36] presented a radial basis function neural network ESO to estimate the unknown wind gusts. The linear ESO has been well studied and successfully applied in engineering practice in [37] for parameter tuning.

Motivated by the aforementioned discussions, our work aims to design a controller with disturbance rejection ability and accurate tracking performance without chattering for WMRs under bounded perturbations. More specific, we develop a tracking control strategy which integrates the advantages of NTSMC and NLESO. NLESO is used to estimate the total disturbance.

\[
\begin{bmatrix}
\hat{\theta} \\
\dot{X} \\
\dot{Y}
\end{bmatrix} = \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix} \omega + \begin{bmatrix}
0 \\
\cos \theta \\
\sin \theta
\end{bmatrix} p
\]

(1)

NTSMC guarantees high-precision tracking with NLESO. Its superiority can be effectively verified in theory and practice. The main contributions of this paper are summarised as below.

i. A targeted NLESO is developed. Meanwhile, a scheme to determine the parameter range is explicitly given, which facilitates the practical applications.

ii. A finite time control strategy in terms of NTSMC is proposed with designed NLESO to reduce the chattering phenomenon which generally appears in traditional sliding mode control.

The remainder of this paper is organized as follows. In Section 2, the kinematic model of WMRs and the chain transformation method are formulated. In addition, the explicit control strategy and some related lemmas for stability analysis are given. Section 3 concentrates on the proof of convergence about NLESO and stability of the closed-loop system. In Section 4, experimental results are given to verify the validity of the proposed strategy. Finally, Section 5 makes a summary of this paper.

**Notations.** \( \text{sign} \) denotes the sign function, \( \| \cdot \| \) represents the Euclidean norm, \( I_{\omega, b} = \nabla b \cdot g \) is the Lie derivative of function \( b \) and vector field \( g \), \( ad_{\omega} g = \nabla g \cdot f - \nabla f \cdot g \) denotes Lie bracket of vector fields \( f \) and \( g \).

## 2 | PROBLEM FORMULATION

The kinematic model with non-holonomic constraints of WMRs is plotted in Figure 1.

![Kinematic model of WMRs](image)

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where \((X, Y)\) is the centre position of WMRs in Cartesian coordinates, \(\theta\) is the heading angle measuring the wheel orientation with respect to the positive direction of axis \(X\). The angular velocity \(\omega\) around the vertical axis and the linear velocity \(v\) are two control variables for trajectory tracking.

We can see that (1) is a two-input three-output non-linear affine system as

\[
\dot{x} = g_1(x)u_1 + g_2(x)u_2
\]

(2)

where \(x = [x_1, x_2, x_3]^T = [\theta, X, Y]^T\), \(g_1(x) = [1, 0, 0]^T\) and \(g_2(x) = [0, \cos x_1, \sin x_1]^T\) are smooth and linearly independent vector fields defined on an open set \(U\). We aim to find appropriate control variables \(u_1 = \omega\), \(u_2 = v\) to steer (2) from the initial state \(x_0 \in U\) to the final state \(x_f \in U\). In order to effectively use NLESO, the system needs to be transformed into a chained form. To this end, we define the following three distributions associated with system (2) as

\[
\Delta_0 := \text{span}\{g_1, g_2, ad_{g_1}g_2\}
\]

\[
\Delta_1 := \text{span}\{g_2, ad_{g_1}g_2\}
\]

\[
\Delta_2 := \text{span}\{g_2\}
\]

In [13], sufficient conditions were proposed to determine whether a non-linear affine system can be converted to the chained form or not. The detailed information is as follows:

- \(\text{dim} \Delta_0 = 3\)
- \(\Delta_0, \Delta_2\) are involutive on \(U\)
- \(h_1(x), h_2(x) : \mathbb{R}^3 \to \mathbb{R}\)
- \(h_1(x), h_2(x)\) are independent
- \(db_1 \cdot \Delta_1 = 0\) and \(db_1 \cdot g_1 = 1\)
- \(db_2 \cdot \Delta_2 = 0\) and \(db_2 \cdot ad_{g_1}g_2 \neq 0\)

Obviously, (1) satisfies conditions (3) and (4). Choosing \(h_1 = x_1\), \(h_2 = x_2 \sin x_1 - x_3 \cos x_1\), it can be easily seen that \(b_1\) and \(b_2\) meet conditions (5)–(8). Using the transformation

\[
\begin{align*}
\varphi_1 &= h_1 = \theta \\
\varphi_2 &= L_{g_1}h_2 = X \cos \theta + Y \sin \theta \\
\varphi_3 &= b_2 = X \sin \theta - Y \cos \theta
\end{align*}
\]

(9–11)

we can convert (2) into the following chained form

\[
\begin{align*}
\dot{\varphi}_1 &= u_1 = \omega \\
\dot{\varphi}_2 &= (L_{g_1}^T b_2)u_1 + (L_{g_1}^T L_{g_1} b_2)u_2 = -\zeta_3 \omega + \nu \\
\end{align*}
\]

(12–13)

where \(\varphi_1, \varphi_2, \varphi_3\) are new state variables and \(u_1, u_2\) are new control variables.

Note that transformation (9)–(13) is globally differential homeomorphism. Then we give a feasible and smooth desired reference trajectory of the WMRs centre position in Cartesian coordinates, that is \(X_r, Y_r\). From the kinematic model (1), the reference trajectory \(X_r, Y_r\) is straightforward to obtain the reference chained system

\[
\begin{align*}
\dot{\varphi}_1 &= v_1 \\
\dot{\varphi}_2 &= v_2 \\
\dot{\varphi}_3 &= \zeta_3 v_1
\end{align*}
\]

(14–16)

Simple calculation shows that \(v_1 = (\dot{Y}_r \dot{X}_r - \dot{X}_r \dot{Y}_r) (\dot{X}_r^2 + \dot{Y}_r^2)^{-1}\), \(v_2 = -\zeta_3 v_1 + (\dot{X}_r^2 + \dot{Y}_r^2)^{1/2}\). Then we define tracking errors as

\[
\begin{align*}
e_1 &= \varphi_1 - \varphi_1^* \\
e_2 &= \varphi_2 - \varphi_2^* \\
e_3 &= \varphi_3 - \varphi_3^*
\end{align*}
\]

(17)

According to (17), the kinematics of tracking errors become

\[
\begin{align*}
\dot{e}_1 &= e_1 - \nu_1 \\
\dot{e}_2 &= e_2 - \nu_2 \\
\dot{e}_3 &= \zeta_3 e_1 - \zeta_2 \nu_1
\end{align*}
\]

(18–20)

Analyzing the tracking error system (18)–(20) we can find that \(e_1\) is controlled by \(e_1\) independently. \(e_2\) converges to zero leading \(e_3\) to approach an invariant set. Furthermore, considering the external disturbances, we change (18)–(20) into a new form

\[
\begin{align*}
\dot{e}_1 &= e_1^* + f_1 \\
\dot{e}_2 &= e_2 - \nu_2 \\
\dot{e}_3 &= e_4 \\
\dot{e}_4 &= e_2^* + f_2
\end{align*}
\]

(21–24)

where \( e_1 \) is a new error state which is equal to the derivative of \( e_3 \) and \( e_1^* = e_1 + e_2 \), \( e_2 = \epsilon_1 e_3 + \zeta_2 \). \( f_1 \) is the unknown total disturbance contains system uncertainties and external disturbances and \( f_2 = (\zeta_2 e_4)^{(1)} + f_2^* \) includes reference trajectory, system uncertainties and external disturbances, which is state-dependent too. Both of these two disturbances are continuously differentiable.

Figure 2 is the architecture of the closed-loop system from reference trajectory to WMRs.

Controller 1, ESO 1 and Controller 2, ESO 2 for subsystem (21) and subsystem (23) and (24) will be designed respectively. NLESO reconstructs the total disturbance in terms of error and control signal. Feedback control signal is calculated through the sliding mode controller. Then combining the control signal with disturbance estimation, we get \( v_1^* \) and \( v_2^* \). The chattering phenomenon caused by sign function in sliding mode controller can be reduced via decreasing the gain of sign function. Accurate estimation of disturbances ensures its rationality. Finally, through (9)–(13) we can obtain the actual control inputs \( \omega = T_1 (r_1) = v_1 \) and \( e = T_2 (r_2, r_3, \zeta) = \zeta_3 r_1 + e_2^* - \zeta_2 r_1 \) to achieve trajectory tracking of WMRs.

For subsystem (21), NLESO 1 and Controller 1 are designed to regulate the angular velocity \( \omega \) of WMRs. Their specific forms are as follows:

**NLESO 1**:

\[
\begin{align*}
\dot{\hat{e}}_1 &= \dot{\hat{e}}_1^* + \frac{\alpha_{01}}{\xi_{01}} (e_1 - \hat{e}_1) + \frac{\varphi (e_1 - \hat{e}_1)}{\xi_{01}} + \epsilon_1^* \\
\dot{\hat{e}}_2 &= \frac{\alpha_{02}}{\xi_{02}} (e_1 - \hat{e}_1) + \frac{\varphi (e_1 - \hat{e}_1)}{\xi_{02}}
\end{align*}
\]

where \( \hat{e}_1, \dot{\hat{e}}_1 \) are estimated states and \( \alpha_{01}, \alpha_{02}, \xi_{01} \) are constant gains and pertinent constants of NLESO 1, respectively. \( \varphi (\epsilon) \) is a segmented continuous function described as

\[
\varphi (\epsilon) = \begin{cases} 
-1/R^* & \epsilon \in (-\infty, -\pi/2) \\
1/R \sin \epsilon & \epsilon \in (-\pi/2, \pi/2) \\
1/R^* & \epsilon \in (\pi/2, +\infty)
\end{cases}
\]

where \( R \) is a positive constant.

**Controller 1**:

\[
v_1^* = -k_1 \text{sign}(e_1) |e_1|^\alpha - \hat{f}_1
\]  

where \( \alpha \in (0, 1) \) and \( k_1 > 0 \).

NLESO 2 and Controller 2 for subsystem (23) and (24) with the aim of manipulating the linear velocity \( e \) are of the form:

**NLESO 2**:

\[
\begin{align*}
\dot{\hat{e}}_3 &= \hat{e}_4 + \frac{\beta_{01}}{\xi_{02}} (e_3 - \hat{e}_3) + \xi_{02} \varphi \left( \frac{e_3 - \hat{e}_3}{\xi_{02}} \right) \\
\dot{\hat{e}}_4 &= \hat{f}_2 + \frac{\beta_{02}}{\xi_{02}} (e_3 - \hat{e}_3) + \varphi \left( \frac{e_3 - \hat{e}_3}{\xi_{02}} \right) + v_2^* \\
\hat{f}_2 &= \frac{\beta_{03}}{\xi_{02}} (e_3 - \hat{e}_3) + \frac{1}{\xi_{02}} \varphi \left( \frac{e_3 - \hat{e}_3}{\xi_{02}} \right)
\end{align*}
\]

where \( \hat{e}_3, \dot{\hat{e}}_3, \hat{f}_2 \) are estimated states and \( \beta_{01}, \beta_{02}, \beta_{03}, \xi_{02} \) are constant gains and pertinent constants of NLESO 2.

**Controller 2**:

\[
\begin{align*}
\epsilon &= e_3 + \frac{1}{\beta} v_2^q \\
v_2^q &= -\left[ \beta_2 \frac{2q}{q} + k_2 \hat{e} + \left( \overline{D} + \eta \right) \text{sign}(\epsilon) \right] - \hat{f}_2
\end{align*}
\]

where \( k_2, \beta, \eta, \overline{D} \) are positive scalars, \( 1 < q \leq 2, p \) and \( q \) are positive odd numbers.

For stability analysis, the following Lemmas are needed.

**Lemma 1.** ([33]): If \(|f| + |\epsilon|\) is bounded (i = 1, 2, ..., n).

(1) \(|f| + |\epsilon|\) is bounded (i = 1, 2, ..., n).

(2) \(|f| + |\epsilon|\) is bounded (i = 1, 2, ..., n).

(3) \(|f| + |\epsilon|\) is bounded (i = 1, 2, ..., n).

**Lemma 2.** ([34]): If there exists a continuously differentiable, positive infinite function \( V(\epsilon) \), positive scalars \( \alpha \in (0, 1) \) and \( k > 0 \) such that \( V'(\epsilon) \leq -k V(\epsilon)^\alpha \). Then the system states converge to zero in finite time \( T \leq \frac{1}{k(1-\alpha)} V(e(0))^{1-\alpha} \).
Lemma 3. ([20]): If the controller is designed as the form of (31), it ensures that \( e_3 \) and \( e_4 \) converge to the sliding mode manifold within a finite time.

3 Stability Analysis

In this section, our main results are formulated. Firstly, we prove the convergence of NLESO. Then the closed-loop system stability theorem is given.

Let

\[
\begin{bmatrix}
A_1 \\
A_2 \\
A_3
\end{bmatrix} = \begin{bmatrix}
\begin{array}{ccc}
2a+b-rac{3}{2} & b & 1 \\
\frac{3}{2} & 1 & 0 \\
b & 0 & 1
\end{array}
\end{bmatrix}
\]

\[
\begin{bmatrix}
C_1 \\
C_2 \\
C_3
\end{bmatrix} = \begin{bmatrix}
\begin{array}{ccc}
a+2b-rac{3}{2} + 2 \beta_0 & b & 1 \\
2a+b-rac{3}{2} & a & 1 \\
-2b-rac{3}{2} & b & 1
\end{array}
\end{bmatrix}
\]

where \( \xi_2, \xi_3 \) are positive constants.

Theorem 1. Consider the tracking error systems (23) and (24) and the NLESO (28)–(30). If

\[
a > 0, \quad ab > 1/4
\]

\[
4abc - a - 2b - c - 4b^3 > 0
\]

then NLESO (28)–(30) is effective. The observation error satisfies

\[
| f_1 - \hat{f}_1 | \leq \bar{M}_1
\]

Proof. In actual experiments, when the coordinate origin is determined, the system states \( X, Y \) and \( \theta \) are bounded, which means \( z_1, z_2 \) and \( z_3 \) are bounded. At the same time, since the reference signal \( \bar{z}_1, \bar{z}_2 \) and \( \bar{z}_3 \) are bounded, the errors \( e_i \) are bounded eventually. For the external disturbances in the experiment, its amplitude and derivative are bounded such that \( f_1 \) and \( f_2 \) are bounded, which means \( H^2 \) holds. On the other hand, the control signals \( r \) and \( \omega \) are calculated based on the error \( e_i \). So the boundedness of \( e_i \) ensures the boundedness of the control signals. The bounded derivative of external disturbances and the bounded system state errors guarantee that the derivative of \( f_1 \) and \( f_2 \) are bounded. As a result, there exists suitable parameters \( C_0, c, \) and \( k \) such that \( H2 \) holds.

Let

\[
\begin{align*}
J_1(t) &= \frac{\epsilon_1 (\hat{z}_1 - \bar{z}_1) - \hat{\epsilon}_1 (\bar{z}_1 - \hat{z}_1)}{\hat{z}_1}, \\
J_2(t) &= \frac{\epsilon_2 (\hat{z}_2 - \bar{z}_2) - \hat{\epsilon}_2 (\bar{z}_2 - \hat{z}_2)}{\hat{z}_2}, \\
J_3(t) &= f_2 (\xi_2 t) - \hat{f}_2 (\xi_2 t).
\end{align*}
\]

From (28)–(30) and (23) and (24), the estimation error of NLESO 2 can be expressed as

\[
\begin{align*}
J_1 &= J_2 - \xi_2 (y_1) \\
J_2 &= J_3 - \xi_2 (y_1) \\
J_3 &= J_4 - \xi_2 (y_1)
\end{align*}
\]

where \( \xi_1, \xi_2, \xi_3, \xi_4 \) are constants. Let \( y = [y_1, y_2, y_3]^T \) and there exist positive definite function \( V(y) = \sum \frac{\partial V}{\partial y_i} (y_{i+1} - \xi_2 (y_i)) - \frac{\partial V}{\partial y_1} \xi_2 (y_1) \)

\[
\begin{align*}
J_1 &= J_2 - \xi_2 (y_1) \\
J_2 &= J_3 - \xi_2 (y_1) \\
J_3 &= J_4 - \xi_2 (y_1)
\end{align*}
\]

It is easy to see that we can always find a set of positive constants \( a, b, c \) satisfying (32)–(34) which makes sure that \( V(y) \) is positive definite. Therefore, condition (1) in \( H2 \) is satisfied and (1), (1) of \( H2 \) are clearly established. Finally, all the conditions of Lemma 1 hold so that the convergence of NLESO (28)–(30) is guaranteed. And the range of NLESO (28)–(30) gain is only determined by \( a, b \) and \( c \).
Let

\[
\bar{P} = \begin{bmatrix}
\frac{a}{2} & -\frac{1}{2} \\
-\frac{1}{2} & b
\end{bmatrix},
\begin{bmatrix}
\bar{A}_1 \\
\bar{A}_2
\end{bmatrix} = \begin{bmatrix}
-\frac{a}{2} & 1 - b \\
\frac{1}{2} & -b
\end{bmatrix}
\]

\[
\begin{bmatrix}
\alpha_0_1 \\
\alpha_0_2
\end{bmatrix} = \begin{bmatrix}
\frac{1}{2} + \frac{1}{R_1^2} (1 - b) - 1 \\
\frac{\bar{a}}{b} - 1 + (1 - b)\bar{\xi}_2
\end{bmatrix}
\]

where \(\bar{\xi}_2\) is a positive constant.

**Corollary 1.** For system (21) and NLESO (25) and (26), if

\[
\tilde{a} > 0, \quad \tilde{b} > 1/4
\]

\[
\bar{A}_1, \bar{B} + \bar{C}_i < 0, \quad i = 1, 2
\]

then the NLESO (25) and (26) is effective. The observation error admits

\[
|\hat{f}_2 - \hat{\xi}_2| \leq M_2.
\]

**Theorem 2.** For the tracking error system (21)–(24), states \(e_1, e_3\) converge to some invariant sets near zero and \(e_3\) converges to zero by the control strategy (25)–(31).

**Proof.** Introducing (27) into the tracking error equation (21) yields the closed-loop system

\[
e_1 = -k_1e_1^p + f_1 - \hat{f}_1
\]

Let the Lyapunov function be

\[
V_1 = \frac{1}{2}e_1^2
\]

The derivative of \(V_1\) along (21) is

\[
\dot{V}_1 = -k_1e_1^p + (f_1 - \hat{f}_1)e_1
\]

\[
\leq -(k_1 + k_2 - M_1(2V_1)^{\frac{a}{2}})(2V_1)^{\frac{a+1}{2}}
\]

where \(k_1\) and \(k_2\) are two parts of \(k_1\). It should be ensured that

\[
k_2 - M_1(2V_1)^{\frac{a}{2}} \geq 0.
\]

This constraint requires \(|e_1| \geq \left(\frac{M_1}{k_2}\right)^{\frac{a}{2}}\) which means that \(e_1\) converges to a certain invariant set. Then we have

\[
\dot{V}_1 \leq -k_1e_1^p
\]

where \(\alpha^* = \frac{a+1}{2} \in (0, 1), \quad k_1^* = k_12^{a^*} > 0\).

Following Lemma 2, \(e_1\) converges to an invariant set within time

\[
T_1 \leq \frac{1}{k_1^*(1-a^*)}V_1(e_1(t_0))^1-a^*.
\]

Theorem 1 guarantees that \(M_1\) is sufficiently small so that the error \(e_1\) can converge to an invariant set near the zero.

For the closed-loop system (23) and (24) and (31), we choose the Lyapunov function as

\[
V_2 = \frac{1}{2}e_2^2.
\]

Take the derivative of \(V_2\) along (23) and (24) to get

\[
\dot{V}_2 = i\left(e_4 + \frac{1}{p}q^{-1}e_4^p\right)
\]

\[
= \frac{1}{p}q^{-1}e_4^p\left(f_2 - \hat{\xi}_2 - (\bar{D} + \eta)|s| - k_2e_2^p\right)
\]

\[
\leq \frac{1}{p}q^{-1}e_4^p\left(s\bar{M}_2 - (\bar{D} + \eta)|s|\right)
\]

Choosing \(\bar{D} \geq \bar{M}_2\), then \(V_2 \leq -\eta^*|s|\), where \(\eta^* = \frac{1}{p}q^{-1}q\eta\). By Lemma 3, states \(e_3, e_4\) reach the sliding mode manifold within finite time \(t_r\). It indicates the time when the states converge from the sliding mode manifold to zero, which means \(e_1(t_r + t_s) = 0\). During the period from \(t_r\) to \(t_r + t_s\), \(s \neq 0\), that is

\[
e_3 = -\beta e_3^q/p
\]

Integrating (35), we have

\[
\int_{e_3(t_r)}^{e_3(t_r + t_s)} -\frac{q}{p} \frac{e_3^{-1}}{q} dt = \int_{t_r}^{t_r + t_s} -\beta dt = -\beta t_s
\]

\[
e_3 = \frac{p}{\beta(p-q)} \left|e_3(t_r)\right|^{1-\frac{q}{p}}
\]

The finite time is thus \(T_2 = t_r + t_s\). From (18)–(20) and (27)–(29) we have

\[
e_1 = e_1(t) + \bar{z}_2e_1 + \hat{\xi}_2.\]

If \(e_1 \neq 0\), then \(e_2 \to |\hat{\xi}_2|\) as \(e_3 \to 0\). We know that \(e_3, e_2\) are bounded, so \(e_2\) is bounded. This completes the proof of Theorem 2.

**Remark 1.** ([39]) It should be noted that the finite time convergence here is only for the tracking errors \(e_1, e_2, e_3\) rather than the whole closed-loop system due to the existence of NLESO. However, the asymptotic convergence characteristic of NLESO does not hinder the finite time convergence result of the tracking error. Even without NLESO compensation, the tracking error can still converge in a finite time, which is guaranteed by NTSMC. It is undeniable that tracking errors are precisely the most important performance indicators.

## 4 EXPERIMENTAL RESULTS AND DISCUSSION

Figure 3 is the experimental platform of WMRs. The whole WMRs experimental platform is made by Quanser company for
convenient controller verification. So WMRs have good wire-controlled chassis such that the upper controller only needs to design the linear velocity and angular velocity without considering the two-wheel coupling problem. The lower controller of experimental platform uses the inverse kinematics model for WMRs to transform the commanded linear velocity and angular velocity to a desired Left/Right wheel velocity command. For different linear velocities, the inverse model will calculate what the forward velocity would be for each wheel and generate a velocity command. In terms of sensors, the wheels of WMRs are equipped with two encoders with 2578 counts/rotation to measure linear velocity and transmit it to the host computer in real time via WiFi. The measurement noise can be attenuated by a digital filter to reduce the impact on the controller. In order to obtain accurate pose information, there are multiple reflective balls attached to the WMRs. Then optical cameras called optitrack camera measure the pose information of WMRs by tracking the reflective balls. The information measured by the optitrack camera is transmitted to the host computer via USB interface. In the host computer, the reference trajectory can be parameterized. Combining with the feedback information from the sensor module, the controller is designed according to the proposed strategy and the control signals are sent to WMRs via WiFi to execute the trajectory tracking task.

According to Theorem 1, a positive definite matrix $P$ that satisfying (32)–(34) is calculated, the corresponding NLESO gain range can be determined accordingly. Figure 4 depicts the gain range of NLESO with $a = 30$, $b = c = 1$. In fact, there are many choices for matrix $P$, and the gain range will be large or small. The larger the range, the more the combination, as long as it does not exceed the boundary, the convergence of the NLESO can be guaranteed. The parameters of controllers and NLESOs (25)–(31) are listed in Table 1. And the trial and error procedure of parameter tuning is given as follows:

Step 1) Initialize the parameters properly in terms of Theorem 1 and Theorem 2.
Step 2) Keep $R_1$ and $R_2$ unchanged, then fine-tune observers’ gains $\alpha_{01}, \alpha_{02}, \xi_{01}$, and $\beta_{01}, \beta_{02}, \beta_{03}, \xi_{02}$.
Step 3) Keep $\alpha, \beta, \eta, \bar{D}, p$ and $q$ unchanged and increase $k_1$ and $k_2$.
Step 4) If the control performance is not significantly improved, keep the controller and observer gains unchanged, then fine-tune $R_1, R_2$ and $\alpha, \beta, \eta, \bar{D}, p$ and $q$.
Step 5) Repeat Steps (2)–(4) until a satisfactory control performance is available.

Experimental results for tracking a circular trajectory $X_r = 0.8 \cos(0.2t + 3\pi/2)$, $Y_r = 0.8 \sin(0.2t + 3\pi/2)$ with bounded disturbances at $20$ s are depicted in Figure 5. WMR starts from the point $(0, -0.8)$ and tracks the circular trajectory counterclockwise with a constant linear velocity and angular velocity. Here, we compare the proposed method with classical NTSMC in [20] and the ESO-based method in [40]. It can be seen from Figure 5 that all these methods can track the reference trajectory stably before the bounded disturbance appears. However, our proposed controller can tend to the reference trajectory more quickly when there is external disturbance,
indicating its stronger robustness. Although the ESO-based method in [40] can modify the control signal, there is still a large tracking error because the convergence speed of classical ESO estimation is not as fast as NLESO. While the NTSMC method has the largest error since there is no disturbance compensation. The tracking error results are plotted in Figure 6. The real-time peak error of the existing methods exceeds 37% of the tracking radius, while the tracking error for our proposed method is significantly smaller than this value, which just only about 13%.

Furthermore, our method can re-track the reference trajectory more quickly, while the other methods require longer transient time. Figure 7 shows the estimated disturbances. We emphasize that [40] is the traditional NLESO based on \( f\alpha(\cdot) \) function. While this paper incorporates both linear and non-linear terms for NLESO design, which can provide faster convergence speed. Both disturbance observers can realize disturbances estimation. However, due to the lack of linear term compensation, the convergence speed of [40] is slower and the disturbances estimation effect is not good enough. From the control signals in Figure 8, it is found that the proposed controller can adjust the signal amplitude in real time to deal with bounded disturbances, so as to achieve good tracking effect. The performance indexes are listed in Tables 2 and 3, where \( \text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} e_i^2}, \) and \( \text{ISE} = \int_0^t e^2 \, dt. \)

The experimental results suggest that the proposed control strategy has satisfactory control precision and fast compensation capability for bounded disturbances.

5 | CONCLUSION

A NLESO combining linear term and non-linear term has been constructed to estimate external disturbances. The convergence of NLESO has been proved and a scheme of determining the NLESO gains range is suggested. Compared with traditional ESO, it has a faster convergence rate. A controller involving

| Index  | This paper | [20] | [40] |
|--------|------------|------|------|
| RMSE   | 0.0426     | 0.2109 | 0.0726 |
| SD     | 0.0423     | 0.2005 | 0.0675 |
| ISE    | 0.0545     | 1.3338 | 0.1581 |

| Index  | This paper | [20] | [40] |
|--------|------------|------|------|
| RMSE   | 0.0338     | 0.1646 | 0.0407 |
| SD     | 0.0334     | 0.1075 | 0.0404 |
| ISE    | 0.0343     | 0.8132 | 0.0498 |
NTSMC and NLESO has been designed for tracking control of WMRs. The control strategy not only guarantees strong robustness, but also relieves the chattering phenomenon of NTSMC. A finite time convergence result of tracking error system is obtained based on NTSMC technology. Through comparative experiments with the existing results, the effectiveness and superiority of the presented method in practical trajectory tracking control with bounded disturbances are verified.

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