Emergent Universe from A Composition of Matter, Exotic Matter and Dark Energy

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ABSTRACT

A specific class of flat Emergent Universe (EU) is considered and its viability is tested in view of the recent observations. Model parameters are constrained from Stern data for Hubble Parameter and Redshift (H(z) vs. z) and from a model independent measurement of BAO peak parameter. It is noted that a composition of Exotic matter, dust and dark energy, capable of producing an EU, can not be ruled out with present data. Evolution of other relevant cosmological parameters, viz. density parameter (Ω), effective equation of state (EOS) parameter (ωeff) are also shown.

Key words: Emergent Universe, Cosmological Parameters, Observations.

1 INTRODUCTION

It is known from observational cosmology that our universe is passing through a phase of acceleration. Unfortunately, the present phase of acceleration of the universe is not clearly understood. Standard Big Bang cosmology with perfect fluid assumption fails to accommodate the observational fact. However, an accelerating universe is permitted if a small cosmological constant (Λ) be included in the Einstein’s gravity. There is, however, no satisfactory theory that explains the origin of Λ which is required to be unusually small. Moreover, Standard Big Bang model without a cosmological constant is inevitably plagued with a time like singularity in the past. The Big Bang model is also found to be entangled with some of the observational features which do not have explanation in the framework of perfect fluid model. Consequently an inflationary epoch in the early universe is required (Guth 1981) to resolve the outstanding issues in cosmology. It is not yet understood when and how the universe entered the phase. However, the concept of inflation is taken up to build a consistent scenario of the early universe. Inflation may be realized in a semiclassical theory of gravity where one requires an additional inputs like existence of a scalar field which describes the matter in the universe. An alternative approach is also followed where gravitational sector of the Einstein field equation is modified by including higher order terms in the Einstein-Hilbert action (Sotiriou 2007). To address the present accelerating phase of the universe once again attempts are made where theories with a modification of the gravitational sector taking into account higher order terms that are relevant at the present energy scale are considered. There are other approaches generally adopted considering modification of the matter sector by including very different kind of matter known as exotic matter namely, Chaplygin gas and its variations (Bento, Bertolami & Sen 2002; Bilic, Tupper & Viollier 2001), models consisting one or more scalar field and tachyon fields (Lyth 2003). While most of these models address dark energy part of the universe, other models based on non-equilibrium thermodynamics and Boltzmann formulation, which do not require any dark energy (Zimdahl et al. 2001; Balakin et al. 2003; Lima, Silva & Santos 2008), are also considered suitable for describing late universe. A viable cosmological model should accommodate an inflationary phase in the early universe with a suitable accelerating phase at late time. An interesting area of cosmology is to consider models which are free from the initial singularity also. Emergent Universe (EU) scenario is one of the well known choices in this field. EU models are proposed in different framework like Brans-Dicke theory (del Campo, Herrera & Labranda 2007), brane world cosmology (Banerjee, Bandyopadhyav & Chakraborty...
Note that an EU model is permitted with BAO peak parameter measurement and CMB shift parameters from observational data namely, SNIa data, configuration may be phenomenologically realized with an EOS found that the possibility of $A_\alpha > 0$ and a string specific configuration is permissible even with $A < 0$ and a string specific configuration may be phenomenologically realized with an EOS considered by Mukherjee et al. (2006). Recenty, using eq. (1) for an EU model proposed by Mukherjee et al. (2006), we determined various constraints that are imposed on the EOS parameters from observational data namely, SNIa data, BAO peak parameter measurement and CMB shift parameter measurement (Paul, Thakur & Ghose 2010). It was noted that an EU model is permitted with $A < 0$. It is found that the possibility of $A = 0$ case is also permitted when we probe the contour diagram of $A - B$ plane with 95% confidence. The case $A = 0$ corresponds to a composition of dust, exotic matter and dark energy in the universe which is certainly worth exploring. In this paper a specific EU model is taken up where the matter energy content of the universe comprises of dust, exotic matter and dark energy. Using Stern data (Table. 1), the admissibility of model parameters are determined from $H(z)$ vs. $z$ (Stern et al. 2006) and using measurement of model independent BAO peak parameter $A$. We also plotted evolution of cosmologically relevant parameters in our model. The paper is organized as follows: in section 2 field equations for the model are discussed, in section 3 and 4 we the model is constrained with Stern data and Stern+BAO data respectively. Finally in section 5 the findings are summarized with a discussion.

### 2 FIELD EQUATIONS

We consider Robertson-Walker (RW) metric which is given by:

$$ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right)$$

(2)

where $k = 0, +1(-1)$ is the curvature parameter in the spatial section representing flat or closed (open) universe and $a(t)$ is the scale factor of the universe, $r, \theta, \phi$ are the dimensionless comoving co-ordinates. The Einstein field equation is

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}R = 8\pi GT_{\mu\nu}$$

(3)

where $R_{\mu\nu}$, $R$ and $T_{\mu\nu}$ represent Ricci tensor, Ricci scalar and energy momentum tensor respectively. Using RW metric in Einstein field equation we obtain time-time component which is given by

$$3 \left( \frac{\dot{a}}{a} + \frac{k}{a^2} \right) = \rho$$

(4)

where we consider natural units i.e., $c = 1$, $8\pi G = 1$. Another field equation is the energy conservation equation which is given by

$$\frac{dp}{dt} + 3H(p + p) = 0,$$

(5)

where $p$, $\rho$ and $H$ are respectively pressure, energy density, Hubble parameter ($H = \frac{\dot{a}}{a}$). The Hubble parameter ($H$) can be expressed in terms of redshift parameter ($z$) which is given by

$$H(z) = \frac{1}{1 + z} \frac{dz}{dt}.$$

(6)

Since the components of matter and dark energy (exotic matter) are conserved separately, we may use energy conservation equation together with EOS given by eq. (1) to determine the energy density which is obtained on integrating eq. (5):

### Table 1. Stern Data ($H(z)$ vs. $z$)

| $z$ Data | $H(z)$ | $\sigma$ |
|---------|--------|----------|
| 0.00    | 73     | 8.0      |
| 0.10    | 69     | 12.0     |
| 0.17    | 83     | 8.0      |
| 0.27    | 77     | 14.0     |
| 0.40    | 95     | 17.4     |
| 0.48    | 90     | 60.0     |
| 0.88    | 97     | 40.4     |
| 0.90    | 117    | 23.0     |
| 1.30    | 168    | 17.4     |
| 1.43    | 177    | 18.2     |
| 1.53    | 140    | 14.0     |
| 1.75    | 202    | 40.4     |
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3 CONSTRAINTS ON MODEL PARAMETERS FROM OBSERVATIONAL DATA

In this section we consider an EU model implemented in a flat universe using EOS given by eq. (1). A special case \(A = 0\) is taken up here to explore EU scenario with a definite composition of matter namely, dust, exotic matter and dark energy. In this case eq. (10) can be represented in functional form given by

\[
H^2(H_0, B, K, z) = H_0^2 E^2(B, K, z),
\]

(11)

where

\[
E(B, K, z)^2 = \left( \Omega_{const} + \Omega_1 (1 + z)^{-\frac{2}{A_1}} + \Omega_2 (1 + z)^{\frac{2A_2}{A_1}} \right)^2.
\]

(12)

In the present case \(\Omega_{const}\) denotes a constant density parameter which corresponds to energy density described by a cosmological constant \(\Lambda\). We denote the above density parameter by \(\Omega_{\Lambda}\). \(\Omega_1\) corresponds to some exotic matter which we denote by \(\Omega_e\). \(\Omega_2\) corresponds to a dust like fluid which we denote by \(\Omega_d\). Using the above in eq. (12) we obtain :

\[
E(B, K, z)^2 = \left( \Omega_\Lambda + \Omega_e (1 + z)^{-\frac{2}{A_1}} + \Omega_d (1 + z)^{\frac{2}{A_1}} \right)^2.
\]

(13)

The above functions will be used in the next section for analysis with observational results and to determine the model parameters.

3.1 Analysis with Stern \((H(z) vs.z)\) data

In this section we define \(\chi^2\) function as :

\[
\chi_{stern}^2(H_0, A, B, K, z) = \sum \left( \frac{H(H_0, B, K, z) - H_{obs}(z))^2}{\sigma_z^2} \right)
\]

(14)

where \(H_{obs}(z)\) is the observed Hubble parameter at redshift \(z\) and \(\sigma_z\) is the error associated with that particular observation. The present day Hubble parameter \((H_0)\) is a nuisance parameter here. The objective of the analysis is to determine the constraints imposed on the model parameters namely, \(B\) and \(K\) from the observational input. So we can safely marginalize over \(H_0\), defining a function

\[
L(A, B, K, z) = \int \mathcal{E} \left[ \frac{H(H_0, B, K, z) - H_{obs}(z))^2}{\sigma_z^2} \right] P(H_0) dH_0
\]

where \(P(H_0)\) represents a prior distribution function. Here we consider a Gaussian Prior with \(H_0 = 72 \pm 8\). In the
theoretical model it is demanded that the model parameters should satisfy the inequalities (i) $B > 0$, (ii) $K > 0$. Therefore, the model parameters obtained from the best fit analysis with observational data are determined in the theoretical parameter space. The best fit values obtained for the parameters here are: $B = 0.2615$ and $K = 0.4742$ together with $\chi^2_{\text{min}} = 1.02593$ (per degree of freedom). The plots of 68.3%, 95% and 99.7% confidence level contours are shown in fig. 2. The following range of values are permitted: $0.003 < B < 0.5996$ and $0.303 < K < 0.63$ within 68.3% confidence level.

3.2 Analysis with Stern+BAO data

For a flat universe BAO peak parameter may be defined as in a low redshift region such as $0 < z < 0.35$ (Eisenstein et al. 2005):

$$A = \sqrt{\frac{\Omega_m}{E(z)}} \left( \frac{\int_{z_0}^{z_1} \frac{dz}{E(z)}}{z_1 - z_0} \right)^{2/3}$$

(15)

where $\Omega_m$ is the total density parameter for matter content of universe. One has to consider a constant $\omega$ (EOS parameter). Even if $\omega$ is not strictly constant, it is quite reasonable to take a constant $\omega$ value over a small redshift interval. It would not be strictly the value of $\omega$ at $z = 0$ but rather some average value in the region $0 < z < 0.35$. Here we use a technique adopted by Eisenstein et al. (2005) to explore the parameter $A$ which is independent of dark energy model. The value of $A$ for a flat universe is $A = 0.469 \pm 0.017$ as measured in Eisenstein et al. (2005) using SDSS data. We define $\chi^2_{\text{BAO}} = \frac{A - 0.469^2}{(0.017)^2}$. For a joint analysis scheme we consider $\chi^2_{\text{tot}} = \chi^2_{\text{stern}} + \chi^2_{\text{BAO}}$. The best fit values found in the joint analysis are: $B = 0.2599$ and $K = 0.4751$ along with $\chi^2_{\text{min}} = 1.1681$ (per degree of freedom). Contours of 68.3%, 95% and 99.7% confidence level are shown in fig. 2. Here we found that the range of values permitted within 68.3% confidence is a bit elevated: $0.009 < B < 0.606$ and $0.3126 < K < 0.6268$.

4 RELEVANT COSMOLOGICAL PARAMETERS

The range of permitted values of model parameters $B$ and $K$ are determined above. In this section we determine the variation of both the density parameter and the equation of state. Note that the model is an asymptotically de Sitter model and a late time phase of acceleration is assured. 68.3%, 95% and 99.7% confidence level contours in $\Omega_d - \Omega_e$ plane are shown in fig. 3. It is found that the best fit value ($\Omega_d + \Omega_e = 0.3374$) permits $\Omega_k = 0.6626$. It is also noted that the generally predicted values $\Omega_k \approx 0.72$ and $\Omega_d \approx 0.04$ are permitted here within 68.3% confidence level. We also plot the evolution of the effective EOS in fig. 4. As expected it remains negative throughout. EU, as we have noted, is an asymptotically de Sitter universe and it is evident from fig. 5 that $\rho$ decreases to a very small value at late universe.

5 DISCUSSIONS

In this paper considering a very specific model of flat EU, we determine the observational constraints on the model parameters. For this recent observational data namely, Stern data, measurement of BAO peak parameter are used. The specific form of EOS given by eq. (1) to obtain EU scenario in a flat universe is employed here for the purpose. We set $A = 0$ in the eq. (1) to begin with. A equal to zero represents a universe with a composition of exotic matter only. This kind of EOS has been considered in Nojiri, Odinstov & Tsujikawa (2005). In our previous work Paul, Thakur & Ghose (2010) on EU model it is noted that a small non zero value of A (although zero is not ruled out) is permitted. As a result the analysis was done with non zero $A$. In this paper since we are interested in a specific composition of matter energy content of the universe corresponding to $A = 0$ analysis is carried out for EOS given by (1)
with $A = 0$ only. As suggested by [Mukherjee et al., 2006], it corresponds to the content of the universe which is a composition of dark energy, dust and exotic fluid. The above composition is reasonable to obtain a viable scenario of the universe considering the observational facts. It seems that the exotic part of the EOS may also contribute in the budget of dark energy content of the universe. We found that the observationally favoured amount of dark energy present in universe today $\Omega_\Lambda \approx 0.72$ is permitted in our model within 68.3% confidence level. However, it may be mentioned here that the model may be extended even if $|A| \ll 1$ and so that we can write $A + 1 \approx 1$. We found that a composition of dust, exotic matter and dark energy may produce an EU model within the framework of Einstein's gravity with a non-linear equation of state. It is also noted that this kind of model can accommodate many other composition of matter energy depending on the value of $A$. The viability for those will be taken up elsewhere.

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