Propagation of neutral mesons in asymmetric nuclear matter

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Abstract

We calculate dispersion relations and propagators for the $\sigma$, $\omega$, $\rho$ and $\delta$ mesons in relativistic nuclear matter with a proton-neutron asymmetry. In addition to the $\sigma$-$\omega$ and $\delta$-$\rho$ mixings already present in symmetric matter, mixing occur between all components of the $\sigma$-$\omega$-$\delta$-$\rho$ system when the proton fraction differs from 1/2.

1 Introduction

This work extends previous results on the dispersion relation of mesons in relativistic, dense, hot nuclear matter, to systems where there exists a proton-neutron asymmetry. We will be working in a model of the quantum hadrodynamics type (QHD) [1], where the nucleon-nucleon interaction is mediated by the exchange of $\omega$, $\sigma$, $\pi$... etc. mesons. In this work, the dispersion relations of the mesons are calculated by performing a linear response analysis around an asymmetric Hartree ground state. The propagators are then obtained by inverting the dispersion relations. The resulting expression corresponds to the random phase approximation.

A new feature of the asymmetric matter is that new meson mixing channels open. A standard case of meson mixing is that occurring for the $\sigma$-$\omega$ pair, where one meson is converted into the other by desintegration and recombination of a virtual NN loop. Moreover, the isoscalar mesons can couple with the isovector ones whenever the distribution function of proton and neutrons differ. This has been calculated earlier for the $\omega$-$\rho$ pair (see e.g. [2, 3]) in relation with the interpretation of dilepton production in heavy ion collisions [4, 5]. The $\delta$-$\rho$ mixing was recently investigated by [6] in symmetric matter as a further mechanism contributing to dilepton production, and also by [7] and [8]. We present here a derivation and systematical study of the dispersion relation and propagators of the coupled neutral meson system $\omega$-$\sigma$-$\rho^0$-$\delta^0$. The charged mesons will be studied elsewhere.

Our calculations may apply to the physics of neutron stars or of heavy ion collisions.

Nuclear physics in terrestrial conditions is for the most part well described by the $N = Z$ hypothesis. Nevertheless, new data begin to be available with sizeable deviations from symmetry, by performing experiments with nuclei at the border of the stability lines. Indeed, with the planned construction of radioactive beam facilities in several countries, exciting new possibilities will open of investigating the equation of state and dynamical properties of asymmetric nuclear matter. Such experiments will hopefully be able to shed some light on the behavior of the asymmetry energy at high density [9].
Among nuclear physics applications, one can think for example of observing the excitation of collective modes in neutron rich nuclei [10]. Another application concerns the description of the isospin dynamics in heavy ion collisions at intermediate energies. In particular, one of the accessible experimental observables, the balance energy, defined as the beam energy at which transverse collective flow disappears, is valued as providing a clean way to estimate the influence of medium effects on the nucleon-nucleon cross section [11]. Previous work [12] has shown that, besides short range effects described the Brueckner approximation, one cannot neglect the screening of the nuclear interaction arising from the exchange of dressed mesons. An interesting phenomenon would occur if a zero sound mode is excited, since it would appear as a pole in the meson propagator and a corresponding resonant behavior of the cross section [13]. This analysis can be extended in order to include the description of isospin effects [14, 15, 16].

Large neutron to proton ratio is the rule in astrophysical situations, where the matter is in $\beta$ equilibrium. In cold neutron stars, the proton fraction is about 10% or lower. In supernovae explosions and hot proto-neutron stars, the proton fraction is higher, about 30 to 40%, but deviations from the symmetric nuclear matter cannot be neglected.

In the context of the physics of supernova collapse and the early stages of proto-neutron star cooling, the screening of the nucleon-nucleon interaction has consequences on the neutrino opacities [17, 18, 19, 20]. In dense and hot matter, the neutrino-nucleon scattering rate is modified by the correlations which affect the nucleon current. In the random phase approximation, the neutrino-nucleon differential scattering cross section is given by

$$\frac{d\sigma}{dE_\nu d\Omega} = -\frac{G_F^2}{32\pi^3} E_\nu' \left\frac{1}{1-e^{-z}} \text{Im}(\bar{\Pi}_{WW}^{(R) \mu\nu} L_{\mu\nu}) \right\ (1)$$

where $\Pi_{WW}^{(R) \mu\nu}$ is the retarded polarization obeying the Dyson equation

$$\bar{\Pi}_{WW}^{\mu\nu} = \Pi_{WW}^{\mu\nu} + \Pi_{WS}^{\mu\alpha} \bar{D}_{SS \alpha\beta} \Pi_{SW}^{\beta\nu} \ (2)$$

$W$ and $S$ represent vertices for the weak and strong couplings respectively and $\bar{D}_{SS \alpha\beta}$ is the dressed meson propagator. This propagator involves the meson dispersion relation in the denominator. The magnitude of the scattering rate therefore depends crucially on whether or not zero-sound modes turns up in the meson propagation within the integration range. As an example, the Skyrme calculation of [19], which present a spin zero sound mode, predict an enhancement of neutrino opacities, whereas the relativistic approach of [17, 18, 20], where this mode is absent, predict a reduction of the opacities. Further study is necessary.

The calculation of [17, 18] did not take into account the full picture of meson propagation in asymmetric matter. We have applied [20] the propagators derived in the present work to the determination of the neutrino-nucleon scattering rate, and find that the reduction effected by RPA correlations is less efficient when all mixing channels are taken into consideration.

### 2 The Model

#### 2.1 Lagrangian density

We will use a version of the quantum hadrodynamics model developed by Walecka and coworkers in which the nucleon-nucleon interaction is mediated the six mesons taken into account by the Bonn potential model, i.e. $\sigma$, $\omega$, $\pi$, $\rho$, $\eta$ and $\delta$ mesons. The latter two mesons are often neglected in mean-field type calculations by arguing that they lead to comparatively
small corrections to the nucleon-nucleon interaction potential in the vacuum \[21, 22\]. Nevertheless, the $\delta$ meson can play a significant role in the context of dense asymmetric matter, since the value of the $\delta$ mean field controls the mass difference between proton and neutron. This field modifies the behaviour of the neutron star equation of state \[23\]. The value of the $\delta NN$ coupling has a strong influence on the proton fraction of matter in $\beta$ equilibrium. Another justification is the fact that an extrapolation of Dirac-Brueckner-Hartree-Fock calculations to asymmetric matter was shown to be equivalent to a density-dependent mean field theory with a delta meson \[24, 25, 26\].

The $\pi$ and $\eta$ will mix with each other, and separate from the $\omega$-$\sigma$-$\rho$-$\delta$ sector. We will not study the $\pi$-$\eta$ system here, since our main purpose was in fact to apply the results derived in this work to the calculation of the RPA reduction to the neutrino-nucleon scattering rate, where the pion does not contribute \[20\]. The calculation of the mixed $\pi$-$\eta$ dispersion relations would not anyway present any particular difficulty, using the same method as applied here to derive the equations which describe the $\sigma$-$\delta$ subsystem.

The hat on the Wigner operator and meson fields means that they are operators. The pion does not appear in this Lagrangian because it will not be investigated further in this work. Terms such as $\hat{\sigma}\hat{\pi}^2$ or $\hat{\rho}_\mu(\partial_\mu \hat{\pi} \times \hat{\pi})$ also do not appear, beacause we decided to focus in this work on meson mixing originating from NN loops, and will not treat meson loops. A more realistic treatment should include them as well.

This Lagrangian is non renormalizable stricto sensu due to the derivative couplings of the $\rho$ meson (and $\pi$ if included with a pseudovector coupling). Nevertheless, this coupling is necessary for a good description of experimental data. In fact, renormalizability is not a necessary requirement for effective theories \[27\]. Anyway, a renormalization procedure should be applied at the given level of approximation in order to avoid pathological behavior of the dispersion relation. To this end, we will add counterterms to the Lagrangian. A discussion of this issue is given in section §3.2 and Appendix B.

### 2.2 Relativistic Hartree equilibrium

The approach taken by \[28\] is applied here. The main steps leading to the dispersion relations are only briefly recorded; the reader is referred for details to \[28\]. It consists of writing a kinetic equation for the Wigner operator, and perform a linear response analysis of perturbations of the system around the equilibrium as obtained in the Hartree approximation. We first define the Wigner function

\[
\hat{F}(x,p) = \int \frac{d^4R}{(2\pi)^2} \exp(-ip.R) \hat{\psi} \left( x + \frac{R}{2} \right) \otimes \hat{\psi} \left( x - \frac{R}{2} \right)
\]  

(4)

In the Hartree approximation, we suppose that the meson field operators $\hat{\sigma}, \hat{\omega}, ...$ can be replaced by their classical expectation values $\sigma_H, \omega_H, ...$ and that the equilibrium is homogeneous. We will moreover assume that there does not exist charged condensates (only the
yielding a condition on the chemical potentials which determines served baryon current terms of the classical fields. They are self consistent relations which can be solved for a given
The middle two lines define the effective masses and momenta of the neutron and proton in
The first case would be relevant for heavy ion collisions, where the asymmetry is determined
average. It is given by
The equation of state is obtained by calculating the energy momentum tensor and con-
conserved baryon current
For further details on the equation of state, the reader may wish to consult the work of Kubis
The proton fraction is defined as
It can be given as an input parameter or be determined from the $\beta$ equilibrium condition. The
yielding a condition on the chemical potentials which determines $Y_p$

$$p + e^- \rightarrow n + \nu$$

$$\tilde{\mu} = \mu_n - \mu_p = \mu_e - \mu_\nu$$

$$Y_p = \frac{n_p}{n_n + n_p}$$

$$J^\mu = n_B u^\mu ; \quad n_B = n_n + n_p$$

$$T^{\mu\nu} = (\rho + P) u^\mu u^\nu - P g^{\mu\nu}$$

$$= \frac{1}{2} g^{\mu\nu} \left( m_\sigma^2 \sigma_H^2 + \frac{1}{3} b m_\sigma^2 \sigma_H^2 + \frac{1}{4} c \sigma_H^4 - m_\omega^2 \omega_H^2 + m_\delta^2 \delta_H^2 - m_\rho^2 \rho_H^2 \right)$$

$$F_H(p) = \begin{bmatrix}
\frac{1}{(2\pi)^3} \delta(P_p^2 - M_p^2)(\gamma.P_p + M_p)f_p(P_p) & 0 \\
0 & \frac{1}{(2\pi)^3} \delta(P_n^2 - M_n^2)(\gamma.P_n + M_n)f_n(P_n)
\end{bmatrix}$$

(5)

with $P_p^\mu = p^\mu - g_\omega \omega_H^\mu - g_\rho \rho_H^\mu$ ; $M_p = m - g_\sigma \sigma_H - g_\delta \delta_H$

(6)

$\omega_H^\mu = \frac{g_\omega}{m_\omega} \int d^4p \ Tr [\gamma^\mu F_H(p)]$ ; $\sigma_H = \frac{g_\sigma}{m_\sigma} \int d^4p \ Tr [F_H(p)]$

(8)

$\rho_H^\mu = \frac{g_\rho}{m_\rho} \int d^4p \ Tr [\gamma^\mu \gamma F_H(p)]$ ; $\delta_H = \frac{g_\delta}{m_\delta} \int d^4p \ Tr [\gamma F_H(p)]$

(9)

$$f_i(p) = \left[ \frac{\theta(p_0)}{e^{\beta(p_\mu - \mu_i)} + 1} + \frac{\theta(-p_0)}{e^{-\beta(p_\mu - \mu_i)} + 1} - \theta(-p_0) \right] , \ i \in \{p, n\}$$

(10)

The equation of state is obtained by calculating the energy momentum tensor and conserv-

The middle two lines define the effective masses and momenta of the neutron and proton in
terms of the classical fields. They are self consistent relations which can be solved for a given
total density, proton/neutron asymmetry and temperature.

The equation of state is obtained by calculating the energy momentum tensor and con-

$$J^\mu = n_B u^\mu ; \quad n_B = n_n + n_p$$

(11)

$$T^{\mu\nu} = (\rho + P) u^\mu u^\nu - P g^{\mu\nu}$$

(13)
In cold neutron stars, the neutrinos escape freely from the system, and we can moreover set the chemical potential of the neutrinos to zero. The proton fraction is of the order of 0.1 depending on the density which is considered. In the last stages of supernova collapse and early phase of protoneutron star cooling on the other hand, the matter is opaque to neutrinos and the density which is considered. In the last stages of supernova collapse and early phase of protoneutron star cooling on the other hand, the matter is opaque to neutrinos and the lepton fraction is determined by dynamical equations governing the lepton fraction $Y_L = Y_\nu + Y_e$. Charge conservation imposes $Y_e = Y_p$. For a typical value $Y_L = 0.4$, we obtain $\mu_\nu = (6\pi^2 n_B Y_\nu)^{1/3}$ of the order of 200 – 250 MeV and $Y_p \sim 0.3 – 0.36$.

An example of the thermodynamics which are obtained from this model is shown on Fig. 1, where the effective masses and proton fraction are represented as a function of density for neutrino-free matter in beta equilibrium. It was calculated with parameter set which fits the saturation point. Due to the coupling to the $\delta$ field, the proton effective mass is higher than that of the neutron in this model. For the same value of the asymmetry energy at saturation, larger proton fractions are obtained at high density with a finite value of the coupling to the delta field, e.g. $g_\delta = 5$, than $g_\delta = 0$.

3 Dispersion relations

3.1 Linear response

The Hartree solution was a particular approximation to the general evolution equations

$$\left[ i \frac{\gamma}{2} \partial + (\gamma \cdot p - m) \right] \hat{F}(x, p) = -\int \frac{d^4 R}{(2\pi)^4} d^4 \xi \, e^{-i(p - \xi - \frac{R}{2})} \hat{F}(x - \frac{R}{2}) \hat{F}(x, \xi) \quad (16)$$

with

$$\hat{F}(x) = g_\sigma \hat{\sigma}(x) + g_\delta \hat{\delta}(x), \hat{\sigma} - g_\omega \gamma^\mu \hat{\omega}_\mu(x)$$

$$- g_\rho \gamma^\mu \hat{\rho}_\mu(x), \hat{\sigma} + i \frac{f_\rho}{2m} \sigma^{\mu\nu} \partial_\mu \hat{\rho}_\nu(x), \hat{\sigma} \quad (17)$$

$$\partial_\mu \hat{F}^{\mu\nu} + m^2_\sigma \hat{\omega}^{\nu} = g_\omega \int d^4 p \, \text{Tr} \left[ \gamma^\nu \hat{F}(x, p) \right] \quad (18)$$

$$\left[ \partial_\mu \partial^\mu + m^2_\sigma \right] \hat{\sigma} + b m \sigma^2 + c \sigma^3 = g_\sigma \int d^4 p \, \text{Tr} \left[ \hat{F}(x, p) \right] \quad (19)$$

$$\partial_\mu \hat{R}^{\mu\nu} + m^2_\rho \hat{\rho}^{\nu} = g_\rho \int d^4 p \, \text{Tr} \left[ \gamma^\nu \hat{\tau} \hat{F}(x, p) \right] - i \frac{f_\rho}{2m} \partial_\mu \int d^4 p \, \text{Tr} \left[ \sigma^{\mu\nu} \hat{\tau} \hat{F}(x, p) \right] \quad (20)$$

$$\left[ \partial_\mu \partial^\mu + m^2_3 \right] \hat{\delta} = g_\delta \int d^4 p \, \text{Tr} \left[ \hat{\tau} \hat{F}(x, p) \right] \quad (21)$$

The hat on the Wigner operator and meson fields means that they are operators. The physical quantities are extracted by taking statistical averages. Up to now, these equations are exact. A first approximation will be made by neglecting correlations, i.e. replacing the statistical average of products of meson operators with the Wigner operator, by the product of statistical averages: $< \hat{F} \hat{F} > \rightarrow < \hat{F} > < \hat{F} >$.

Let us now consider a small perturbation around the Hartree equilibrium. We will develop $< \hat{F}(x, p) >= F_H(p) + F_1(x, p)$, $< \hat{\sigma}(x) >= \sigma_H + \sigma_1(x)$ ... etc. The Hartree equilibrium is obtained at zeroth order. At first order, we obtain equations for the perturbation which, after performing a Fourier transformation, are easily solved. The dispersion relations of the mesons are obtained by replacing the solution for the perturbation to the Wigner function

$$F_1 = G(p - \frac{q}{2}) \, S(q) \, F_H(p + \frac{q}{2}) + F_H(p - \frac{q}{2}) \, S(q) \, G(p + \frac{q}{2}) \quad (22)$$

with

$$S(q) = -g_\sigma \sigma_1(q) + g_\omega \gamma^\mu \omega_1(q) - g_\rho \hat{\delta}(q), \hat{\sigma} + g_\rho \gamma^\mu \hat{\rho}_1(q), \hat{\sigma} + \frac{f_\rho}{2m} \sigma^{\mu\nu} q_\rho \hat{\rho}_\nu(q), \hat{\sigma} \quad (23)$$

5
and
\[ G(p) = \begin{bmatrix} \frac{\gamma P_p + M_p}{P_p^2 - M_p^2} & 0 \\ 0 & \frac{\gamma P_n + M_n}{P_n^2 - M_n^2} \end{bmatrix} \] (24)
in the linearized, Fourier-transformed field equations for the mesons. For example, the dispersion relation for the sigma meson reads
\[ (-q^2 + m^2_\sigma + 2b m_\sigma \sigma_H + 3c \sigma_H^2) \sigma_1 = g_\sigma \int d^4 p \text{ Tr}[F_1(p,q)] \]
\[ = \Pi_{i\sigma} \sigma_1 + \Pi_{i\omega}^\mu \omega_{1\mu} + \Pi_{i\delta}^\mu \delta_{1\mu} + \Pi_{i\rho}^\mu \rho_{1\mu} \] (25)
This defines the polarizations. The full set of dispersion relations is given in matricial form in Eq. (23). The index \( a \) is an isospin index. When taking the traces, it can be seen that the dispersion relations of the neutral mesons \( \sigma, \omega, \rho, \delta^0, \pi^0 \) decouple from those of the charged mesons \( \rho^\pm, \delta^\pm, \pi^\pm \). In this paper we limit ourselves to the dispersion relations of the neutral mesons; the case of the charged mesons will be the subject of future work.

The nonlinear \( \sigma^3, \sigma^4 \) couplings contribute terms on the left hand side which we can absorb in a definition
\[ M_\sigma^2 = m_\sigma^2 + 2b m_\sigma \sigma_H + 3c \sigma_H^2. \] (26)

We point out again that there are no meson loops (such as would be contributed \( e.g. \) by a \( \pi-\pi \) loop insertion in the \( \sigma \) propagator) at this level of approximation in our formalism. They were discarded when we made the assumption \(< \hat{\Phi} \hat{F} > < \hat{\Phi} > < \hat{F} >\). One could obtain them by restoring meson-meson correlations, or insert such loops by hand basing ourselves on the diagrammatic approach. This will not be done in this work, since our first aim was to study meson mixing originating in NN loops.

### 3.2 Polarizations

Let us continue on the example of the \( \sigma \) meson. Explicitly, the \( \Pi_{i\sigma}^{(i)} \) polarization is given by
\[ \Pi_{i\sigma} = \Pi_{i\sigma}^{(p)} + \Pi_{i\sigma}^{(n)} \]
\[ \Pi_{i\sigma}^{(i)} = -\int d^4 p \text{ Tr} \left[ g_\sigma G \left( P_i + \frac{q}{2} \right) g_\sigma F_H \left( P_i - \frac{q}{2} \right) + (G \leftrightarrow F_H) \right] \]
At zero temperature, the result obtained through the application of the linear response analysis coincides with the one-loop approximation. The other polarizations are given by similar equations. Taking the traces and integrating over the angles, we finally arrive at the formulae given in the Appendix. Some remarks are in order about the imaginary part at finite temperature and about the contribution of vacuum fluctuations.

We will have to choose a prescription to go around the poles of the propagator Eq. (24). In Eq. (28), the prescription was chosen as follows
\[ \Pi_{i\sigma}^{(i)} = -g_\sigma^2 \int d^4 p \text{ Tr} \left[ \left\{ g_\sigma \right\} \left\{ \gamma_i \left( P_i - \frac{q}{2} \right) + M_i \right\} \left\{ g_\sigma \right\} \left\{ \gamma_i \left( P_i + \frac{q}{2} \right) + M_i \right\} \right] \times \]
\[ \times \left[ \frac{\delta \left( \left( P_i + \frac{q}{2} \right)^2 - M_i^2 \right) f_i \left( P_i + \frac{q}{2} \right) + \delta \left( \left( P_i - \frac{q}{2} \right)^2 - M_i^2 \right) f_i \left( P_i - \frac{q}{2} \right) }{(2\pi)^3 \left( P_i - \frac{q}{2} \right)^2 - M_i^2 + i\epsilon} \right] \]
\[ = 4g_\sigma^2 \int d^4 p \left[ p^2 - \frac{q^2}{4} + M_i^2 \right] \left\{ \frac{\varphi_i(p + q/2) - \varphi_i(p - q/2)}{2p.q - i\epsilon} \right\} \] (27)
with \( \varphi_i(p) = \frac{1}{(2\pi)^2} \delta(P_i^2 - M_i^2) f_i(p) \)

At finite temperature, the real time Green’s function formalism \( [29] \) defines several polarizations depending on the position of the time arguments on the Keldysh contour. Their real parts coincide, the imaginary parts are related to each other by relations such as

\[
\text{Im} \, \Pi^R(w,k) = \tanh \left( \frac{\omega}{2T} \right) \text{Im} \, \Pi^I(w,k).
\]

It can be verified that the usual retarded polarizations \( [29] \) can be obtained by taking \( \pm i \epsilon \text{sign}(p_0) \) instead of \( \pm i \epsilon \) in the above expression \( (27) \). The imaginary parts given in the Appendix as well as those used in the calculation of the propagators will be those of the retarded polarizations.

The last term \( \theta(-p_0) \) in the expression of the Wigner function Eq. \( (10) \) describes the contribution of the vacuum. When taken under the integration sign in the expression of the polarizations, it yields a divergent term. Some renormalization procedure has to be applied in order to remove the divergence. It is not sufficient to ignore this term (by performing a normal ordering), because the subtracted term depends on the effective masses and therefore on the thermodynamical conditions. In the following we will apply dimensional regularization and counterterm subtraction as described \( e.g \). in \( [30, 28, 31] \).

What is new with respect to these references is that, due to proton-neutron asymmetry, the contributions of the mesons cannot be renormalized independently.

The mixed polarizations \( \Pi^\mu_{\sigma \rho} \) and \( \Pi^\mu_{\delta \omega} \) are both proportional to \( n^\mu = u^\mu - (q.u/q^2)g^\mu \). They must vanish in vacuum, since the hydrodynamical velocity quadrivector is undefined in this case. Therefore, we can treat the renormalization of the \( \sigma-\delta \) subsystem independently from that of the \( \omega-\rho \) subsystem. We review in this section the main steps of the renormalization of the \( \sigma-\delta \) subsystem and give the expressions of all renormalized vacuum contributions in Appendix B.

After performing the dimensional regularization, the divergent contributions are extracted in the form \( (M_p^2 + M_n^2)/\epsilon \) and \( q^2/\epsilon \) in \( \Pi^{\mu \nu}_{\sigma \rho} \) or \( \Pi^{\mu \nu}_{\delta \omega} \), and in the form \( (M_p^2 - M_n^2)/\epsilon \) in \( \Pi^{\mu \nu}_{\sigma \delta} \) or \( \Pi^{\mu \nu}_{\delta \sigma} \). The fact that \( M_n^2 + M_p^2 = 2(m - g_\sigma \sigma)^2 + 2g_\delta^2 \delta^2 \) and \( M_p^2 - M_n^2 = 2(m - g_\rho \rho)g_\delta \delta \) dictates the form of the counterterm Lagrangian one has to introduce, in order to neutralize these divergences.

\[
L_{CT} = Z_\sigma \partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2} A_1 \sigma^2 + \frac{1}{3} A_2 \sigma^3 + \frac{1}{4} A_3 \sigma^4 \\
+ Z_\delta \partial_\mu \delta \partial^\mu \delta + B_1 \delta + B_2 \sigma \delta^2 + B_3 \sigma^2 \delta^2 + C \delta^4
\]  (28)

Each of the couplings of the counterterm Lagrangian \( (Z_i, A_i, B_i, C) \) is split \( (A_1 = \text{infinite} + a_1, \text{etc}) \) in an infinite part, which is chosen so as to cancel the divergence, and a residual finite part \( (Z_i, a_i, b_i, c) \), which has to be determined by imposing some physical conditions that the renormalized polarizations should verify.

There are several possible choices, each of which defines a renormalization procedure. One possibility is to impose that the polarization and its first derivatives with respect to \( \sigma \) and \( \delta \) vanish at some point, for example on the mass shell \( q^2 = m_\tau^2 \) (as \( e.g \). in the work of Diaz Alonso et al. \( [28] \)) or at \( q^2 = 0 \) (as in the work of Kurasawa and Suzuki \( [32] \)). Another possibility is to require that the initial structure of the expression in terms of the effective masses is preserved (The procedures of \( e.g \). Hatsuda et al. \( [33] \) or Sarkar et al \( [3] \) do have this property, as well as “scheme 3” of \( [31] \)). Here we will work with the scheme of Kurasawa and Suzuki (called “scheme A” in the following) and scheme 3 of \( [31] \) called here “scheme B”.

As was already pointed out in \( [31] \), the final result, and in particular the behavior of the effective meson masses as a function of density, is very sensitive to the choice of renormalization.
tion procedure. We must consider this as an unsolved problem for the moment. There is some hope that one could solve it by applying “naturalness” arguments \[27\] and considerations on the symmetries of the underlying more fundamental theory, of which the present effective one represents a low-energy approximation.

Actually, the asymmetric case offers us an example of how symmetry arguments do impose some further restrictions on the choice of the subtraction procedure. As a matter of fact, the mixing between the $\sigma$ and $\delta$ mesons requires that the same counterterms should simultaneously cancel divergences in various polarizations. For example, the $B_3$ is used to regulate the $\delta^2/\epsilon$ divergence in $\Pi_{\sigma\sigma}$, the $\sigma^2/\epsilon$ divergence in $\Pi_{\delta\delta}$ and the $\sigma\delta/\epsilon$ divergences in $\Pi_{\sigma\delta}$ and $\Pi_{\delta\sigma}$. The $B_2$ regulates divergences $\delta/\epsilon$ in $\Pi_{\sigma\delta}$ and $\Pi_{\delta\sigma}$, and $\sigma/\epsilon$ in $\Pi_{\delta\delta}$. The compatibility will impose additional relations between the finite $b_2$, $b_3$, etc. constants. It was found that the renormalization scheme used by \[28\] must be discarded on this basis, since the additional constraints necessary for the compatibility of the counterterms are not fulfilled. Both scheme A and B used in this paper fulfill the compatibility relations. Scheme B preserves the structure of the expression as a function of the effective masses $M_p$, $M_n$ but gives after renormalization $\Pi_{\sigma\delta} \neq \Pi_{\delta\sigma}$, whereas these two quantities were given by the same expression before renormalization. For scheme A the reverse occurs, we have $\Pi_{\sigma\delta} = \Pi_{\delta\sigma}$ after renormalization but the structure in terms of $M_p$, $M_n$ is not maintained.

Further details are given in Appendix B.

### 3.3 Dispersion relations

The equations for the dispersion relations can be cast in matricial form:

$$\left[D\right] [\Phi_1] = 0 \, , \quad D = \begin{bmatrix}
D_\sigma & D^\mu_{\sigma\omega} & D_\delta & D^\mu_{\sigma\rho} \\
D^\rho_{\omega\sigma} & D^\mu_{\omega\rho} & D_\omega & D^\mu_{\omega} \\
D_\delta & D^\rho_{\delta\omega} & D_{\delta\delta} & D^\rho_{\delta\rho} \\
D^\rho_{\rho\omega} & D^\mu_{\rho\rho} & D_\rho & D^\mu_{\rho} \\
\end{bmatrix}, \quad \Phi_1 = \begin{bmatrix}
\sigma_1 \\
\omega_\mu \\
\delta_1 \\
\rho_{1\mu} \\
\end{bmatrix} \tag{29}
$$

In a referential where the momentum transfer is equal to $q^\mu = (\omega, 0, 0, k)$, the $D$ matrix has the structure

\[
\begin{array}{|cccc|cccc|}
\hline
q & r & 0 & 0 & s & q_1 & r_1 & 0 & 0 & s_1 \\
\hline
r & a & 0 & 0 & b & t_1 & v & 0 & 0 & z \\
0 & 0 & c & 0 & 0 & 0 & x & 0 & 0 \\
0 & 0 & 0 & c & 0 & 0 & 0 & x & 0 \\
s & b & 0 & 0 & e & u_1 & z & 0 & 0 & y \\
q_1 & t_1 & 0 & 0 & u_1 & d & t & 0 & 0 & u \\
r_1 & v & 0 & 0 & z & t & f & 0 & 0 & j \\
0 & 0 & x & 0 & 0 & 0 & h & 0 & 0 \\
0 & 0 & 0 & x & 0 & 0 & 0 & h & 0 \\
s_1 & z & 0 & 0 & y & u & j & 0 & 0 & i \\
\hline
\end{array}
\]

with the following definitions of $a$, $b$, $c$, $d$ ... etc.
In writing the preceding equations, we used the decomposition of the polarizations on a basis of orthogonal tensors

\[
\begin{align*}
q &= q^2 - m^2_{\sigma} + \Pi_{\sigma} \\
r &= -k^2/q^2 \Pi_{\omega \sigma} \\
s &= -\omega k/q^2 \Pi_{\omega \sigma} \\
a &= k^2 + m^2_{\omega} + k^2/q^2 \Pi_{\omega L} \\
b &= \omega k + \omega k/q^2 \Pi_{\omega L} \\
c &= q^2 - m^2_{\omega} + \Pi_{H} \\
e &= \omega^2 - m^2_{\omega} + \omega^2/q^2 \Pi_{\omega L} \\
d &= q^2 - m^2_{\delta} + \Pi_{\delta} \\
t &= -k^2/q^2 \Pi_{\delta \rho} \\
u &= -\omega k/q^2 \Pi_{\delta \rho} \\
t_1 &= -k^2/q^2 \Pi_{\delta \rho} \\
u_1 &= -\omega k/q^2 \Pi_{\delta \rho}
\end{align*}
\]

with

\[
\begin{align*}
\Pi_{\mu \nu} &= -\Pi_{\omega T} T^{\mu \nu} - \Pi_{\omega L} L^{\mu \nu} \\
\Pi_{\rho \omega} &= -\Pi_{\rho \omega T} T^{\mu \nu} - \Pi_{\rho \omega L} L^{\mu \nu} \\
\Pi_{\mu \sigma} &= \Pi_{\mu \sigma} \eta^{\mu} \\
\Pi_{\delta \rho} &= \Pi_{\delta \rho} \eta^{\mu} \\
\Pi_{\omega \delta} &= \Pi_{\omega \delta} \eta^{\mu}
\end{align*}
\]

If the non linear couplings \(b, c\) are non zero, one should replace everywhere \(m^2_{\sigma}\) by \(M^2_{\sigma}\) as defined in Eq. \(26\). Taking the determinant of this matrix, we obtain the dispersion relation. It is of the form \(\text{det}(\mathcal{D}) = m^2_{\rho} m^2_{\omega} \delta^2_T \delta_L = 0\), the transverse propagation modes being given by

\[
\delta_T = (q^2 - m^2_{\rho} + \Pi_{\rho T})(q^2 - m^2_{\omega} + \Pi_{\omega T}) - \Pi^2_{\rho \omega T} = 0
\]  

(30)

and the longitudinal mode by

\[
\delta_L = \left[ (q^2 - m^2_{\sigma} + \Pi_{\sigma})(q^2 - m^2_{\rho} + \Pi_{\rho}) - \Pi^2_{\sigma \rho} \right] \left[ (q^2 - m^2_{\rho} + \Pi_{\rho L})(q^2 - m^2_{\omega} + \Pi_{\omega L}) - \Pi^2_{\rho \omega L} \right] \\
+ (q^2 - m^2_{\omega} + \Pi_{\omega L})(q^2 - m^2_{\omega} + \Pi_{\omega}) \Pi^2_{\delta \rho} \eta^2 + (q^2 - m^2_{\rho} + \Pi_{\rho L})(q^2 - m^2_{\omega} + \Pi_{\omega}) \Pi^2_{\delta \omega} \eta^2 \\
+ (q^2 - m^2_{\omega} + \Pi_{\omega L})(q^2 - m^2_{\delta} + \Pi_{\delta}) \Pi^2_{\sigma \rho} \eta^2 + (q^2 - m^2_{\rho} + \Pi_{\rho L})(q^2 - m^2_{\delta} + \Pi_{\delta}) \Pi^2_{\sigma \omega} \eta^2 \\
- 2 \Pi_{\rho \omega L} \left[ (q^2 - m^2_{\sigma} + \Pi_{\sigma}) \Pi_{\delta \rho} \Pi_{\delta \omega} \eta^2 + (q^2 - m^2_{\omega} + \Pi_{\omega}) \Pi_{\sigma \rho} \Pi_{\sigma \omega} \eta^2 \right] \\
- 2 \Pi_{\sigma \delta} \left[ (q^2 - m^2_{\omega} + \Pi_{\omega L}) \Pi_{\delta \rho} \Pi_{\sigma \rho} \eta^2 + (q^2 - m^2_{\rho} + \Pi_{\rho L}) \Pi_{\sigma \rho} \Pi_{\sigma \omega} \eta^2 \right] \\
- 2 \Pi_{\sigma \delta} \left[ (q^2 - m^2_{\omega} + \Pi_{\omega L}) \Pi_{\sigma \rho} \Pi_{\delta \rho} \Pi_{\sigma \omega} \eta^2 \right] + \eta^4 (\Pi_{\omega \omega} \Pi_{\sigma \rho} - \Pi_{\delta \rho} \Pi_{\sigma \omega})^2 = 0.
\]  

(31)
4 Propagators

The propagators are obtained by inverting the dispersion relation $G = D^{-1}$. The propagator matrix has the same structure as the dispersion matrix.

$$G = \begin{bmatrix}
G^\sigma & G^\sigma_\omega & G^\sigma_\delta & G^\sigma_\rho \\
G^\omega_\mu & G^\omega_\mu_\nu & G^\omega_\mu_\delta & G^\omega_\mu_\rho \\
G^\delta_\nu & G^\delta_\nu_\omega & G^\delta_\nu_\delta & G^\delta_\nu_\rho \\
G^\rho_\sigma & G^\rho_\sigma_\omega & G^\rho_\sigma_\delta & G^\rho_\sigma_\rho
\end{bmatrix}$$

The scalars isoscalar $G_\sigma$ and isovector $G_\delta$ propagators are given by

$$G_\sigma = \frac{1}{\delta L} \left\{ (q^2 - m_\sigma^2 + \Pi_\omega L) \Pi_{\delta \rho} \eta^2 + (q^2 - m_\rho^2 + \Pi_\rho L) \Pi_{\delta \omega} \eta^2 - 2 \Pi_{\delta \rho} \Pi_{\delta \omega} \Pi_{\rho \omega L} \eta^2 \\
+ (q^2 - m_\delta^2 + \Pi_\delta) \left\{ (q^2 - m_\omega^2 + \Pi_\omega L)(q^2 - m_\rho^2 + \Pi_\rho L) - \Pi_{\rho \omega L} \right\} \right\} \quad (32)$$

$$G_\delta = \frac{1}{\delta L} \left\{ (q^2 - m_\sigma^2 + \Pi_\omega L) \Pi_{\sigma \rho} \eta^2 + (q^2 - m_\rho^2 + \Pi_\rho L) \Pi_{\sigma \omega} \eta^2 - 2 \Pi_{\sigma \rho} \Pi_{\sigma \omega} \Pi_{\rho \omega L} \eta^2 \\
+ (q^2 - m_\sigma^2 + \Pi_\sigma) \left\{ (q^2 - m_\omega^2 + \Pi_\omega L)(q^2 - m_\rho^2 + \Pi_\rho L) - \Pi_{\rho \omega L} \right\} \right\} \quad (33)$$

Mixed $\sigma$-$\delta$ meson propagation in asymmetric matter is given by

$$G_{\sigma \delta} = \frac{1}{\delta L} \left\{ -(q^2 - m_\sigma^2 + \Pi_\omega L) \Pi_{\delta \rho} \Pi_{\sigma \omega} \eta^2 - (q^2 - m_\rho^2 + \Pi_\rho L) \Pi_{\delta \omega} \Pi_{\sigma \omega} \eta^2 \\
+ (\Pi_{\delta \omega} \Pi_{\sigma \rho} + \Pi_{\delta \rho} \Pi_{\sigma \omega} + \Pi_{\delta \rho} \Pi_{\sigma \omega} + \Pi_{\delta \rho} \Pi_{\sigma \omega}) \Pi_{\rho \omega L} \eta^2 \\
- \left\{ (q^2 - m_\omega^2 + \Pi_\omega L)(q^2 - m_\rho^2 + \Pi_\rho L) - \Pi_{\rho \omega L} \right\} \right\} \quad (34)$$

We have the usual $\sigma$-$\omega$ mixing in the isoscalar sector and $\delta$-$\rho$ mixing in the isovector sectors. Moreover, in asymmetric matter there are also mixing between the isoscalar and isovector mesons $\delta$-$\omega$ and $\sigma$-$\rho$:

$$G_{\sigma \omega}^{\mu} = -\frac{\eta^{\mu}}{\delta L} \left\{ (\Pi_{\delta \omega} \Pi_{\sigma \rho} - \Pi_{\delta \rho} \Pi_{\sigma \omega}) \Pi_{\delta \rho} \eta^2 - (q^2 - m_\rho^2 + \Pi_\rho L)(q^2 - m_\delta^2 + \Pi_\delta) \Pi_{\sigma \omega} \\
+ (q^2 - m_\delta^2 + \Pi_\delta) \Pi_{\delta \omega} \Pi_{\sigma \rho} + (q^2 - m_\rho^2 + \Pi_\rho L) \Pi_{\delta \sigma} \Pi_{\rho \omega L} - \Pi_{\delta \rho} \Pi_{\sigma \delta} \Pi_{\rho \omega L} \right\} \quad (35)$$

$$G_{\delta \rho}^{\mu} = -\frac{\eta^{\mu}}{\delta L} \left\{ (\Pi_{\delta \omega} \Pi_{\sigma \rho} - \Pi_{\delta \rho} \Pi_{\sigma \omega}) \Pi_{\delta \omega} \eta^2 - (q^2 - m_\omega^2 + \Pi_\omega L)(q^2 - m_\delta^2 + \Pi_\delta) \Pi_{\sigma \rho} \\
+ (q^2 - m_\delta^2 + \Pi_\delta) \Pi_{\delta \sigma} \Pi_{\omega \rho L} + (q^2 - m_\omega^2 + \Pi_\omega L) \Pi_{\delta \rho} \Pi_{\sigma \omega} \Pi_{\rho \omega L} \right\} \quad (36)$$

$$G_{\sigma \rho}^{\mu} = -\frac{\eta^{\mu}}{\delta L} \left\{ -(\Pi_{\delta \omega} \Pi_{\sigma \rho} - \Pi_{\delta \rho} \Pi_{\sigma \omega}) \Pi_{\delta \omega} \eta^2 - (q^2 - m_\omega^2 + \Pi_\omega L)(q^2 - m_\delta^2 + \Pi_\delta) \Pi_{\sigma \rho} \\
+ (q^2 - m_\delta^2 + \Pi_\delta) \Pi_{\delta \sigma} \Pi_{\rho \omega L} - \Pi_{\delta \rho} \Pi_{\sigma \delta} \Pi_{\rho \omega L} \right\} \quad (37)$$

$$G_{\delta \omega}^{\mu} = -\frac{\eta^{\mu}}{\delta L} \left\{ -(\Pi_{\delta \omega} \Pi_{\sigma \rho} - \Pi_{\delta \rho} \Pi_{\sigma \omega}) \Pi_{\delta \rho} \eta^2 - (q^2 - m_\rho^2 + \Pi_\rho L)(q^2 - m_\delta^2 + \Pi_\delta) \Pi_{\sigma \omega} \\
+ (q^2 - m_\delta^2 + \Pi_\delta) \Pi_{\delta \sigma} \Pi_{\omega \rho L} + (q^2 - m_\rho^2 + \Pi_\rho L) \Pi_{\delta \omega} \Pi_{\sigma \omega} \Pi_{\rho \omega L} \right\} \quad (38)$$

The propagators of the vector mesons $\omega$ and $\rho$ can be decomposed as

$$G_{\omega \mu}^{\nu} = G_{\omega L} L^{\mu \nu} + G_{\omega T} T^{\mu \nu} + G_{\omega Q} Q^{\mu \nu}$$

$$G_{\rho \mu}^{\nu} = G_{\rho L} L^{\mu \nu} - G_{\rho T} T^{\mu \nu} + G_{\rho Q} Q^{\mu \nu}$$

$$G_{\rho \omega}^{\mu} = G_{\rho L} L^{\mu \nu} + G_{\rho T} T^{\mu \nu}$$

(39)
with for the ω meson

\[ G_{\omega L} = \frac{-1}{\delta_L} \left\{ (q^2 - m_\omega^2 + \Pi_\delta) \Pi_\rho^2 \eta^2 + (q^2 - m_\omega^2 + \Pi_\rho) \Pi_\delta^2 \eta^2 - 2 \Pi_\delta \Pi_\rho \Pi_\sigma \eta^2 \right\} - \left( q^2 - m_\rho^2 + \Pi_\rho \right) \left[ (q^2 - m_\omega^2 + \Pi_\sigma)(q^2 - m_\delta^2 + \Pi_\delta) - \Pi_\delta^2 \right] \]

(40)

\[ G_{\omega T} = \frac{-1}{\delta_T} \left( q^2 - m_\rho^2 + \Pi_\rho \right) \]

(41)

\[ G_{\omega Q} = \frac{1}{m_\omega^2} \]

(42)

for the rho meson

\[ G_{\rho L} = \frac{-1}{\delta_L} \left\{ (q^2 - m_\rho^2 + \Pi_\delta) \Pi_\omega \eta^2 + (q^2 - m_\rho^2 + \Pi_\sigma) \Pi_\delta \eta^2 - 2 \Pi_\delta \Pi_\omega \Pi_\rho \Pi_\sigma \eta^2 \right\} - \left( q^2 - m_\rho^2 + \Pi_\omega \right) \left[ (q^2 - m_\sigma^2 + \Pi_\rho)(q^2 - m_\delta^2 + \Pi_\delta) - \Pi_\delta^2 \right] \]

(43)

\[ G_{\rho T} = \frac{-1}{\delta_T} \left( q^2 - m_\rho^2 + \Pi_\rho \right) \]

(44)

\[ G_{\rho Q} = \frac{1}{m_\rho^2} \]

(45)

and finally for the mixing between the ω and ρ mesons

\[ G_{\rho\omega L} = \frac{-1}{\delta_L} \left\{ (q^2 - m_\omega^2 + \Pi_\sigma) \Pi_\delta \Pi_\omega \eta^2 - (q^2 - m_\delta^2 + \Pi_\delta) \Pi_\rho \Pi_\omega \eta^2 \right\} + \left( \Pi_\delta + \Pi_\rho \Pi_\omega \right) \Pi_\omega \Pi_\delta \eta^2 \]

\[- \left[ (q^2 - m_\sigma^2 + \Pi_\sigma)(q^2 - m_\delta^2 + \Pi_\delta) - \Pi_\delta^2 \right] \Pi_{\rho\omega L} \]

(46)

\[ G_{\rho\omega T} = \frac{-\Pi_{\rho\omega T}}{\delta_T} \]

(47)

5 Numerical results

5.1 Choice of parameters

At the mean field level, the equation of state is determined by equations (11,12). The coupling constants were adjusted in order to reproduce the properties of the saturation point. It is enough to adjust five parameters in order to reproduce the five basic experimentally measured properties: saturation density \( n_{\text{sat}} \), binding energy \( B/A \), incompressibility modulus \( K \), effective nucleon mass \( m^* \) and asymmetry energy \( a_A \). Usually these parameters are taken to be the meson-nucleon coupling constants \( g_\sigma, g_\omega, g_\rho \) and the non-linear sigma couplings \( b, c \); and the coupling of the \( \delta \) meson is set equal to zero. The meson masses are kept equal to their value as quoted in the Particle Data Book \( m_\omega = 782.6 \text{ MeV}, m_\rho = 769 \text{ MeV}, m_\delta = 983 \text{ MeV} \), and the \( \sigma \) meson mass is taken to be equal to the standard result of the Bonn potential model \( m_\sigma = 550 \text{ MeV} \). The tensor coupling of the rho meson is derivative and does not enter in the equation of state at the mean field level. It will be taken equal to the Bonn potential value \( f_\rho/g_\rho = 6.1 \) or to the vector dominance model value \( f_\rho/g_\rho = 3.7 \).

If the \( \delta \) meson coupling is not equal to zero, we have one more parameter to play with. When keeping the parameters \( g_\sigma, g_\omega, b, c \) constant, varying \( g_\delta \) rescales the asymmetry energy, which has to be readjusted by changing the value of \( g_\rho \). The \( \delta \) is usually discarded, moreover some studies concluded that it does not noticeably affect the description of stable nuclei [34]. On the other hand, interest in the \( \delta \) meson has been revived since it appears that density
dependent field theory needs a $\delta$ with a large value of the coupling in order to reproduce Dirac-Brueckner calculations of asymmetric matter \[24, 26\]. Whether or not a $\delta$ meson is present can be important for the equation of state of neutron stars \[23\]. Among other things, a nonzero coupling to the $\delta$ manifests itself by a splitting of the proton and neutron effective masses. Hopefully, experiments planned with neutron rich nuclei will help clarifying this issue.

In principle, renormalization of vacuum fluctuations appearing in the expressions of the nucleon effective mass and energy density should be performed, yielding the relativistic Hartree approximation. The procedure is standard and does not present difficulties (see e.g. \[28\]). However it is known \[1, 35\] that, after effecting a readjustment of the coupling constants, the equation of state is very similar in the renormalized (Hartree) and unrenormalized (mean field) case. Since the thermodynamics enters the polarizations only through the effective masses $M_n, M_p$ and chemical potentials $\mu_n, \mu_p$, we take the pragmatic point of view that it will not change the dispersion relations whether the underlying thermodynamics were generated by the mean field or Hartree approximations, and limit ourselves to fit the mean field. Reasonable values for the experimental data are

$$n_{\text{sat}} = 0.17 \text{ fm}^{-3}, \quad B/A = -16 \text{ MeV}, \quad K = 250 \text{ MeV}, \quad m^*/m = 0.8, \quad a_A = 30 \text{ MeV} \quad (48)$$

and can be reproduced with the following parameter set

$$g_\sigma = 8.00, \quad g_\omega = 7.667, \quad b/g_\sigma^3 = 9.637 \times 10^{-3}, \quad c/g_\sigma^4 = 7.847 \times 10^{-3}$$

$$(g_\rho, g_\delta) = (3.685, 0.) \text{ or } (5.203, 5.) \quad (49)$$

Note that the parameter $c$ is positive, so that we are safe from any unpleasant instabilities which may appear \[36\] in commonly used parametrizations where this is not the case.

This parameter set will be referred to in the following as set (1). For the dispersion relations we will use moreover other parameter sets besides this one. As a matter of fact, at this level of approximation, there is no compelling reason why the couplings describing the thermodynamics should be the same as those entering the RPA. When the dressed meson propagator is used in calculations of the RPA corrections to neutrino opacities, the scattering processes are usually described by parametrizations such as given in Bonn potential model. For example, from \[22\]

$$g_\sigma = 10.20, \quad g_\omega = 15.85, \quad b = 0, \quad c = 0$$

$$g_\rho = 3.19, \quad f_\rho/g_\rho = 6.1, \quad g_\delta = 3.73$$

$$\Lambda_\sigma = \Lambda_\delta = \Lambda_\rho = 2000 \text{ MeV}, \quad n_\rho = 2, \quad \Lambda_\omega = 1500 \text{ MeV} \quad (50)$$

The cutoffs $\Lambda_\sigma, \Lambda_\omega, \Lambda_\rho$ and $\Lambda_\delta$ enter the definition of form factors which multiply the coupling constants in the calculation of the polarizations. They are introduced in order to take into account the effects of the finite size of the nucleon.

$$g_\alpha \to g_\alpha \left( \frac{\Lambda_\alpha^2 - m_\alpha^2}{\Lambda_\alpha^2 - q^2} \right)^{n_\alpha}, \quad \alpha \in \{\sigma, \omega, \rho, \delta\}, \quad n_\alpha = 1 \text{ except for } n_\rho = 1 \text{ or } 2.$$ 

This parameter set will be referred to in the following as set (2).

From \[51\] we can take the parameter set of Brockmann-Machleidt potential C which also fits the free nucleon-nucleon scattering data, but with a higher value of the coupling to the $\delta$ meson

$$g_\sigma = 10.044, \quad g_\omega = 15.853, \quad b = 0, \quad c = 0$$

$$g_\rho = 3.455, \quad f_\rho/g_\rho = 6.1, \quad g_\delta = 7.985$$

$$\Lambda_\sigma = 1800 \text{ MeV}, \quad \Lambda_\omega = \Lambda_\delta = 1500 \text{ MeV}, \quad \Lambda_\rho = 1300 \text{ MeV}, \quad n_\rho = 1 \quad (51)$$
This parameter set will be referred to in the following as set (3).

A way out of the dilemma between choosing parameters which fit the saturation point, or the scattering data in vacuum, could be working with one of the recent parametrizations of the results of Dirac-Brueckner calculations (which obtain the saturation point using the free potential as input) in terms of density-dependent coupling constants. A promising model is the density dependent relativistic hadron field theory (DDRH) which is lately being developed by Lenske and collaborators [25]. A serious study would involve performing linear response analysis on the field equations obtained from the DDRH lagrangian density. This is currently under consideration.

5.2 Transverse modes

Depending on the values of the coupling constants and cutoffs on the one hand, and of the renormalization scheme on the other hand, we may have a different structure of the branches. If the renormalization is not performed, we have spurious branches, as is known from previous studies [28, 31]. In particular, when we define the effective meson masses as the solution \( \omega \) of the dispersion relation on the axis \( k = 0 \), a kink structure is found, whatever strong form factor is applied. When the renormalization is performed, the kink disappears and normal branches emerge.

In asymmetric matter, the transverse mode of the omega meson mixes with that of the rho meson. With renormalization scheme A (using the method of Kurasawa-Suzuki), we obtain no more branches than the normal ones with parameter sets (2) or (3). On the other hand, with parameter set (1), there is also a zero sound branch at finite momentum, similar to the one conducing to the (spurious) pion condensation. This zero-sound mode is related to the high value of the rho meson coupling constants necessary in order to reproduce the correct asymmetry energy, together with the high value of \( f_\rho/g_\rho = 6.1 \) of the Bonn potential. As a matter of fact, the zero-sound mode is appreciably reduced when we take the vector dominance model value \( f_\rho/g_\rho = 3.7 \). It disappears completely for \( f_\rho = 0 \). On the other hand, recent calculations from QCD sum rules [38] would also favor a high value of \( f_\rho \) (\( f_\rho/g_\rho = 8.0 \pm 2.0 \)). The zero-sound mode is also present with parameter set (1) when using the other renormalization scheme (B), although much weaker. When present, the zero sound mode is somewhat reduced at larger proton-neutron asymmetry. In any case, it will be possible to eliminate this spacelike mode with a contact interaction [8] of the Landau-Migdal type (as in the \( \pi+\rho + g' \) model), which also helps to correct the singular \( \delta(r) \) behavior of the rho exchange potential at short distance.

The expression of the omega polarization is identical in schemes A and B. On the other hand, the result for the rho meson is quite different depending on the chosen renormalization scheme. As shown is [31], the effective \( \rho \) mass increases with increasing density in the renormalization scheme of Kurasawa-Suzuki (A), whereas it decreases in renormalization scheme (B) which keeps the formal structure of the vacuum terms as a function of the effective nucleon masses \( M_n, M_p \).

The position of the normal branches changes with asymmetry. It is best seen by plotting the variation of the effective rho and omega meson masses with asymmetry. The effective meson masses are defined as the solutions \( \omega \) of the dispersion relation \( \delta_T(\omega, \vec{k}) = 0 \) at vanishing three momentum \( \vec{k} \). They are shown on Figs. 2 and 3, (rightmost upper and lower panels), at four times saturation density and vanishing temperature, with parameter set (1). The continuous lines were obtained by keeping the \( \omega-\rho \) mixing while the dashed line is obtained by setting it to zero.

In renormalization scheme A, the omega meson effective mass is the lowest of the two
and decreases with increasing asymmetry (≡ smaller $Y_p$ values), whereas the rho meson mass increases with increasing asymmetry. In renormalization scheme B, the reverse occurs. The effective rho mass is now the lowest and decreases with increasing asymmetry. As in the case of the behavior of the $\rho$ mass with density \cite{[31]}, the uncertainty in the choice of a renormalization procedure once more precludes a reliable prediction.

In order to evaluate the strength of the mixing, we can define the mixing angle as in \cite{[31]}

$$\tan(2\theta_{\omega\rho T}) = \frac{2\sqrt{|\eta^2|}\Pi_{\rho\omega T}}{m_\rho^2 - m_\omega^2 - \Pi_{\rho T} + \Pi_{\omega T}}$$

(52)

The dependence of $\theta_{\omega\rho T}$ in the density and asymmetry parameters is represented on Fig. 4. The curves are labelled by the value of the proton fraction $Y_p$. The mixing angle is calculated at a fixed three-momentum $k = 500$ MeV, at the values of $\omega$ which are solutions of the dispersion relation. We have two angles, one for each meson branch. The mixing angle $\theta_{\omega\rho T}$ vanishes in symmetric matter and/or at zero momentum. Non negligible values are obtained in asymmetric matter, especially if renormalization scheme A is used.

5.3 Longitudinal modes

When studying the solutions of the longitudinal dispersion relation $\delta_L$, we obtain the normal branches corresponding to each of the four mesons involved in the longitudinal part of the dispersion relation. Again, the behavior of the corresponding effective masses as a function of density depends on the choice of the coupling constants, cutoffs and renormalization scheme. In both schemes, the sigma meson mass increases with density, more rapidly in scheme B. The rho meson mass increases in scheme A and decreases in scheme B. The delta meson mass decreases in scheme A and increases in scheme B. The omega meson mass coincides in both schemes, it first decreases with density and then starts increasing.

The behavior of the effective masses at a fixed density $n_B = 4$ $n_{\text{sat}}$ as a function of the proton fraction $Y_p$ is represented in Figs. 2 and 3. The effective masses obtained for the omega and rho meson from the longitudinal dispersion relation coincide with those obtained from the transverse modes, as it should be. The delta mass is larger at smaller $Y_p$ in both schemes. The sigma mass increases in scheme B but first decreases and then slightly increases with asymmetry in scheme A.

Besides the normal branches, we have also heavy rho meson branches (see also e.g. \cite{[31]}) which would disappear if a stronger cutoff were applied. These will not be further studied here. Finally, there are zero sound branches. The zero-sound branch due to $\sigma$-$\omega$ mixing is already well known from studies of symmetric matter. It is present in both renormalization schemes with parameter sets (2) and (3).

In Fig. 5 we show the dependence of the zero-sound mode due to $\sigma$-$\omega$ mixing with the asymmetry, at four times saturation density and vanishing temperature. It is seen that at high asymmetry the strength of the zero sound is only slightly reduced. The associated characteristic velocity increases, resulting in a small rotation of the zero sound branch in the $\omega$-$k$ plane. When studying the imaginary parts of the polarization, it is seen that at zero temperature the lower branch of the zero sound mode is suppressed by Landau damping, whereas the higher branch is not.

In the delta-rho sector, no zero sound branch similar to that existing in the $\sigma$-$\omega$ system was found, whatever parameter set was used. Some spurious branches may appear in the spacelike region at high momentum, if we work with parameter set (1) and in renormalization scheme A. They disappear when applying a moderate cutoff.
5.4 Effect of the mixing on the propagators

In the preceding paragraphs, we obtained results for the effective meson masses and mixing angles, which concern the timelike region of the $\omega$-$k$ plane. In order to characterize the influence of meson mixing in the spacelike part of the $\omega$-$k$ plane, we also studied the propagators at fixed $k$ for varying $\omega$ with the condition $\omega < k$. The knowledge of the behavior of the propagators for this parameter range can be of use for the study of RPA corrections to neutrino-nucleon scattering, since it enters the definition of the RPA correction to the polarization insertion of Eq. (2).

A sample of the results is displayed in Figs. 6 to 8. The thermodynamical conditions were chosen to be $n_B = 4 n_{\text{sat}}$, $T=20$ MeV, neutrino free matter in $\beta$ equilibrium $Y_\nu = 0$. The exchanged 3-momentum was fixed at $k = 50$ MeV. Similar features were observed for other values of $n_B$ and $k$.

The real and imaginary part of the $\sigma$ meson propagator are shown on Fig. 6. The $G_{\sigma \omega}$ and $G_{\omega \sigma}$ behave in the same way. The behavior of the real part of $G_\sigma$ is dominated by the pole at the location of the zero sound branch. The lower part of the zero sound branch falls into the region of Landau damping ($\omega/k = 0.67$ on the figure) so that it does not give rise to a pole. On the other hand, the upper branch falls on the fringe of the Landau damping zone at zero temperature, so that the corresponding pole ($\omega/k = 0.83$ on the figure) remains largely unscreened. The effect of mixing is not very strong.

As we saw in the preceding paragraph, there does not exist a $\delta$-$\rho$ zero sound branch. Due to the full $\sigma$-$\omega$-$\rho$-$\delta$ mixing however, the propagators of the $\delta$-$\rho$ subsystem get contaminated by the zero sound pole in the $\sigma$-$\omega$ subsystem, as can be seen on Fig. 7, where we plotted the $G_{\delta \rho}$ propagator. The $G_{\delta}$ and $G_{\rho \rho}$ propagators behave in a similar way.

Finally, Fig. 8 shows an example of the behavior of the transverse rho propagator $G_{\rho T}$. The mixing with the $\omega$ meson is responsible for a moderate modification of the real and imaginary parts.

6 Conclusion

The dispersion relations of the neutral mesons $\sigma$, $\omega$, $\rho$ and $\delta$ have been obtained in the random phase approximation in the framework of a relativistic hadronic field theory. It was shown that in nuclear matter with a proton-neutron asymmetry, new mixing channels open between mesons of different isospin number. A numerical evaluation indicates that the influence of this mixing on the effective meson masses and zero sound mode is moderate, of the order of 5% to 10%. An application of these to the calculation of RPA corrections to neutrino-nucleon scattering also leads to effects of this order of magnitude. A similar study concerning the charged mesons is presently underway.

The amount of corrections arising from meson mixing is somewhat larger in the $\rho$-$\omega$ (both transverse and longitudinal) and $\delta$-$\omega$ channels than in the $\rho$-$\sigma$ or $\sigma$-$\delta$ channels for the thermodynamical and kinematic conditions explored in this work. The role of these new mixing channels on the behavior of the dilepton production rate in heavy ion collisions remains to be investigated, and could bring further arguments to the discussion of the behavior of the $\rho$ meson in the medium.

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Appendix A - Real, Matter Part of Polarizations

The general expressions of the matter part of the polarizations can be written in terms of the integrals for $i \in \{p, n\}$

\[
I_{m(i)}(p, \omega, k) = \int_{-1}^{1} du \frac{(pu)^n}{(\omega\sqrt{p^2 + M_i^2} - pk - q^2/2)(\omega\sqrt{p^2 + M_i^2} - pk + q^2/2)}
\]

(53)

\[
\Pi_{m(i)}(\omega, k) = \int_{0}^{\infty} \frac{p^2 dp}{\sqrt{p^2 + M_i^2}} (p^2 + M_i^2)^n I_{m(i)}(p, \omega, k) \left(n_i(p) + (-1)^{n+m} \pi_i(p)\right)
\]

(54)

\[
X_{0}^{(i)} = \int_{0}^{\infty} \frac{p^2 dp}{\sqrt{p^2 + M_i^2}} \left(n_i(p) + \pi_i(p)\right)
\]

(55)

Expressions for $\Pi_{\sigma}, \Pi_{\mu\rho}^{\mu\nu}, \Pi_{\delta\omega}, \Pi_{\sigma}^{\mu\nu}$ and $\Pi_{\rho}^{\mu\nu}$ have already been given in Appendix A of [8] in the case of symmetric nuclear matter. In asymmetric matter, one should remove the degeneracy factor $d = 2$ of the formulae presented in [8] and sum instead on contributions of the proton and neutron loops. For example, the $\delta\rho$ mixing polarization is now given by

\[
\text{Re} \, \Pi_{\delta\rho}^{(i)} = \text{Re} \, \Pi_{\delta\rho}^{(p)} + \text{Re} \, \Pi_{\delta\rho}^{(n)} = -\frac{g_{\delta\rho}}{4\pi^2} \left(2M_i \, g_{\rho} + \frac{f_{\rho}}{2m} q^2 \eta^\mu \left[T_{0(i)}^{1} - T_{1(i)}^{0}\right]\right)
\]

(56)

In asymmetric matter, we will moreover have mixing polarizations between mesons of different isospin number. They are given by the difference between the contributions of the proton and neutron loops.

\[
\text{Re} \, \Pi_{\omega\rho L} = \text{Re} \, \Pi_{\omega\rho L}^{(p)} - \text{Re} \, \Pi_{\omega\rho L}^{(n)}
\]

\[
\text{Re} \, \Pi_{\omega\rho L}^{(i)} = -\frac{g_{\omega\rho}}{2\pi^2} \left[T_{0(i)}^{1} - T_{2(i)}^{0}\right] - \left(\frac{f_{\rho}}{2m} \right) \frac{g_{\rho}}{4\pi^2} M_i q^4 T_{0(i)}^{0}
\]

\[
\text{Re} \, \Pi_{\omega\rho T} = \text{Re} \, \Pi_{\omega\rho T}^{(p)} - \text{Re} \, \Pi_{\omega\rho T}^{(n)}
\]

\[
\text{Re} \, \Pi_{\omega\rho T}^{(i)} = -\frac{g_{\omega\rho}}{4\pi^2} \left[M_i^2 q^2 T_{0(i)}^{0} + (\omega^2 + k^2) T_{2(i)}^{0} + T_{0(i)}^{0} - 4\omega k T_{1(i)}^{1}\right]
\]

\[
\text{Re} \, \Pi_{\delta\omega}^{(i)} = \text{Re} \, \Pi_{\delta\omega}^{(p)} - \text{Re} \, \Pi_{\delta\omega}^{(n)}
\]

\[
\text{Re} \, \Pi_{\delta\omega}^{(i)} = -\frac{g_{\delta\omega}}{2\pi^2} \eta^\mu M_i q^2 \left[T_{0(i)}^{1} - \frac{\omega}{k} T_{1(i)}^{0}\right]
\]

\[
\text{Re} \, \Pi_{\sigma\rho}^{(i)} = \text{Re} \, \Pi_{\sigma\rho}^{(p)} - \text{Re} \, \Pi_{\sigma\rho}^{(n)}
\]

\[
\text{Re} \, \Pi_{\sigma\rho}^{(i)} = -g_{\sigma} \left(2M_i \, g_{\rho} + \frac{f_{\rho}}{2m} q^2 \right) \frac{q^2}{4\pi^2} \left[T_{0(i)}^{1} - \frac{\omega}{k} T_{1(i)}^{0}\right]
\]

\[
\text{Re} \, \Pi_{\sigma\delta} = \text{Re} \, \Pi_{\sigma\delta}^{(p)} - \text{Re} \, \Pi_{\sigma\delta}^{(n)}
\]

\[
\text{Re} \, \Pi_{\sigma\delta}^{(i)} = -\frac{g_{\sigma\delta}}{2\pi^2} \left[2X_{0}^{(i)} + \left(\frac{q^2}{4} - M_i^2 q^2 \right) T_{0(i)}^{0}\right]
\]
Appendix B - Renormalization

Let us first consider the scalar mesons:

The contribution of the vacuum to the $\sigma$ meson polarization is

$$\Re \Pi_{\sigma\sigma}^{\text{vac}} = \frac{g_\sigma^2}{2\pi^2} \left[ I_1^{(p)} + \left( \frac{q^4}{4} - M_p^2 q^2 \right) I_2^{(p)} + I_1^{(n)} + \left( \frac{q^4}{4} - M_n^2 q^2 \right) I_2^{(n)} \right]$$

with

$$I_1^{(i)} = \int d^4p \, \delta(p^2 - M_i^2) \theta(-p_0)$$

$$I_2^{(i)} = \int d^4p \, \frac{\delta(p^2 - M_i^2) \theta(-p_0)}{(p.k)^2 - k^2/4}$$

The vacuum part of the $\delta$ polarization $\Re \Pi_{\delta\delta}^{\text{vac}}$ has the same structure. The divergent part of the integrals $I_1^{(i)}$ and $I_2^{(n)}$ is extracted by dimensional regularization

$$I_1^{(i) \text{ reg}} = \pi M_i^2 \left[ \frac{-1}{\epsilon} + \ln \left( \frac{M_i}{m} \Lambda_1 \right) \right]$$

$$I_2^{(i) \text{ reg}} = -\frac{2\pi}{q^2} \left[ \frac{-1}{\epsilon} + \ln \left( \frac{M_i}{m} \Lambda_2 \right) - \frac{1}{2} + \theta(q^2, M_i^2) \right]$$

where $\theta$ is a finite valued function

$$\theta(q^2, M_i^2) = y \int_0^\infty \frac{dx}{(x^2 + y)^{1/2}} \quad \text{with} \quad y = 1 - \frac{q^2}{4M_i^2} \quad (57)$$

In order to simplify somewhat the expressions, we will use the following short notations

$$\theta_p = \theta(q^2, M_p^2) \quad , \quad \theta_n = \theta(q^2, M_n^2)$$

$$\theta_\sigma = \theta(m_\sigma^2, m_\sigma^2) \quad , \quad \theta_\delta = \theta(m_\delta^2, m^2)$$

$$\theta_{\sigma\delta} = \frac{\partial \theta(q^2, M_i^2)}{\partial q^2} |_{q^2=m_\sigma^2, M_i^2=m^2} \quad (58)$$

and work in units of the free nucleon mass (i.e., put $m = 1$). $\epsilon$ is a vanishingly small quantity and $\Lambda_1, \Lambda_2$ are finite arbitrary integration constants.

We obtain infinities of the type

$$\Pi_{\sigma\sigma}^{\text{vac}} = \frac{g_\sigma^2}{2\pi^2} \left[ \frac{M_p^2 + M_n^2}{\epsilon_1} + \frac{q^2}{\epsilon_2} + \Lambda_{\sigma\sigma} \right]$$

with $M_p^2 + M_n^2 = 2(m - g_\sigma \sigma)^2 + 2g_3^2 \delta^2$ and $\Lambda_{\sigma\sigma}$ is a finite contribution. Since the mixed polarization $\Pi_{\delta\delta}^{\text{vac}}$ depends on the difference of the contributions of proton and neutron loops, one finds that the $q^2/\epsilon$ singularities from both contributions cancel each other, and one is left with

$$\Pi_{\sigma\delta}^{\text{vac}} = \frac{g_\sigma g_\delta}{2\pi^2} \left[ \frac{M_p^2 - M_n^2}{\epsilon_3} + \Lambda_{\sigma\delta} \right]$$

with $M_p^2 - M_n^2 = 2(m - g_\sigma \sigma)g_\delta \delta$ and $\Lambda_{\sigma\delta}$ is a finite contribution. $\Pi_{\delta\delta}^{\text{vac}}$ and $\Pi_{\sigma\sigma}^{\text{vac}}$ have similar structures.

These infinite contributions can be cancelled by adding to the Lagrangian \((3)\) the counterterms

$$Z_\sigma \partial_\mu \sigma \partial^\mu \sigma + Z_\delta \partial_\mu \delta \partial^\mu \delta + \frac{1}{2} A_1 \sigma^2 + \frac{1}{3} A_2 \sigma^3 + \frac{1}{4} A_3 \sigma^4 + B_1 \delta^2 + B_2 \delta^2 + B_3 \sigma^2 \delta^2 + C \delta^4 \quad (59)$$
The infinities are neutralized by choosing the couplings of the counterterm Lagrangian so that contribute to the dispersion relation as e.g.

\[
(-q^2 + M^2)\sigma_1 = \left[ \Pi_{\sigma\sigma}^{\text{mat}} + \frac{g_3^2}{2\pi^2} \Lambda_{\sigma\sigma} \right] \sigma_1 + \left[ \Pi_{\sigma\delta}^{\text{mat}} + \frac{g_3 g_5}{2\pi^2} \Lambda_{\sigma\delta} \right] \delta_1 \\
+ \left[ Z_\sigma q^2 + A_1 + 2A_2 \sigma + 3A_3 \sigma^2 + 2B_3 \delta^2 + \frac{g_5^2}{2\pi^2} \left( \frac{2(m - g_3 \sigma)^2 + 2g_5^2 \delta^2}{\epsilon_1} + q^2 \right) \right] \sigma_1 \\
+ \left[ 2B_2 \delta + 4B_3 \sigma \delta + \frac{g_3 g_5}{2\pi^2} \left( \frac{2(m - g_3 \sigma)g_5}{\epsilon_3} \delta \right) \right] \delta_1
\]

The infinities are neutralized by choosing the couplings of the counterterm Lagrangian so that, for example

\[
2B_3 + \frac{g_2^2 g_3^3}{\pi^2} \frac{1}{\epsilon_1} = b_3^\sigma \\
4B_3 - \frac{g_2^2 g_3^3}{\pi^2} \frac{1}{\epsilon_3} = b_3^\delta \\
2B_3 + \frac{g_2^2 g_3^3}{\pi^2} \frac{1}{\epsilon_4} = b_3^{\delta\sigma} \\
4B_3 - \frac{g_2^2 g_3^3}{\pi^2} \frac{1}{\epsilon_6} = b_3^{\delta\delta}
\]

with \( b_3^\sigma, b_3^\delta, \ldots \) etc finite. The unknown finite remnants are defined so that the vacuum polarizations

\[
\Pi_{\sigma\sigma}^{\text{vac}} = \frac{g_2^2}{2\pi^2} \Lambda_{\sigma\sigma} + z_\sigma q^2 + a_1 + 2a_2 \sigma + 3a_3 \sigma^2 + 2b_3^{\sigma\sigma} \delta^2 \\
\Pi_{\sigma\delta}^{\text{vac}} = \frac{g_3 g_5}{2\pi^2} \Lambda_{\sigma\delta} + 2b_3^{\sigma\delta} \delta + 4b_3^{\delta\sigma}
\]

fulfill some physical conditions. One generally requires the polarization \( \Pi_{\sigma\sigma}^{\text{vac}} \) to cancel in the vacuum \( (M_\rho = M_n = m, \sigma = \delta = 0) \) on the mass shell \( q^2 = m^2 \). In \[28\] one further imposes that the first derivatives with respect to the \( \sigma \) and \( \delta \) fields vanish at the same point

\[
\frac{\partial \Pi_{\sigma\sigma}^{\text{vac}}}{\partial \sigma} \bigg|_{q^2=m^2,M_\rho=M_n=m,\sigma=0} = 0 , \quad \frac{\partial^2 \Pi_{\sigma\sigma}^{\text{vac}}}{\partial \sigma^2} \bigg|_{q^2=m^2,M_\rho=M_n=m,\sigma=0} = 0
\]

In the scheme of Kurasawa and Suzuki (scheme A in this work), one requires that these derivatives vanish at \( q^2 = 0 \)

\[
\frac{\partial \Pi_{\sigma\sigma}^{\text{vac}}}{\partial \sigma} \bigg|_{q^2=0,M_\rho=M_n=m,\sigma=0} = 0 , \quad \frac{\partial^2 \Pi_{\sigma\sigma}^{\text{vac}}}{\partial \sigma^2} \bigg|_{q^2=0,M_\rho=M_n=m,\sigma=0} = 0
\]

In the schemes of Shiozawa and Hatsuda \[33\] or scheme 3 of \[31\] (scheme B of this work), the structure of the expressions in terms of \( M_n, M_\rho \) is preserved, leading to conditions such as

\[
a_1 + 2a_2 \sigma + 3a_3 \sigma^2 + 2b_3^{\sigma\sigma} \delta^2 = a_1 \frac{1}{2m^2} \left[ 2(m - g_3 \sigma)^2 + 2g_5^2 \delta^2 \right] = a_1 \frac{1}{2m^2} \left[ M_n^2 + M_\rho^2 \right]
\]

These conditions yield expressions of the constants \( b_2^{\sigma\sigma}, b_2^{\sigma\delta}, \ldots \) etc in terms of the function \( \theta \) defined in Eqs. \[33\]. For example, in scheme B one obtains

\[
b_2^{\sigma\sigma} = \frac{g_2 g_3^2}{2\pi^2} m \left( -4\theta_\sigma + m^2 \sigma (4 - m^2) \theta_{\sigma\sigma} \right) = \frac{g_2 g_3^2}{2\pi^2} m \sigma , \quad b_2^{\sigma\delta} = -\frac{g_2 g_5^2}{2\pi^2} \sigma \\
b_2^{\sigma\delta} = -\frac{g_2 g_3^2}{2\pi^2} m \left( -4\delta_\sigma + m^2 (4 - m^2) \theta_{\sigma\delta} \right) = -\frac{g_2 g_3^2}{2\pi^2} m \delta , \quad b_3^{\sigma\delta} = \frac{g_2 g_5^2}{2\pi^2} \delta
\]
The constants determined by these conditions must moreover fulfill the compatibility conditions imposed by (64)

\[
b_3^{\alpha\delta} - b_3^{\delta\alpha} = \frac{g_\sigma}{m} \left( b_2^{\alpha\sigma} - b_2^{\delta\sigma} \right) , \quad 2b_3^{\delta\delta} - b_3^{\delta\delta} = \frac{g_\sigma}{m} \left( b_2^{\delta\delta} - b_2^{\delta\delta} \right)
\]

The constants determined by the sets of conditions (62) or (63) do indeed fulfill these compatibility conditions, whereas the set of conditions (61) do not.

We now give the full expressions of the renormalized polarizations in schemes A or B.

**Renormalization scheme A**

\[
\Pi_{\sigma\sigma}^{\text{vac}(A)} = \frac{g_\sigma^2}{2\pi^2} \left[ 3M_p^2 \ln M_p + 3M_n^2 \ln M_n - \frac{q^2}{2} (\ln M_p + \theta_p) - \frac{q^2}{2} (\ln M_n + \theta_n) \right. \\
+ 2M_p^2 \theta_p + 2M_n^2 \theta_n + (q^2 - m_\sigma^2) (\theta_\sigma - (4 - m_\sigma^2) \theta_{\sigma q}) - (4 - m_\sigma^2) \theta_\sigma \\
\left. + \left( 1 - 13 \frac{M_p^2 + M_n^2}{2} + 6(M_n + M_p) \right) \right] \tag{65}
\]

The expression of \( \Pi_{\delta\delta}^{\text{vac}(A)} \) can be obtained by replacing everywhere \( g_\sigma, m_\sigma, \theta_\sigma, \theta_{\sigma q} \) by \( g_\delta, m_\delta, \theta_\delta, \theta_{\delta q} \) in \( \Pi_{\sigma\sigma}^{\text{vac}(A)} \).

\[
\Pi_{\sigma\delta}^{\text{vac}(A)} = \frac{g_\sigma g_\delta}{2\pi^2} \left[ 3M_p^2 \ln M_p - 3M_n^2 \ln M_n - \frac{q^2}{2} (\ln M_p + \theta_p) + \frac{q^2}{2} (\ln M_n + \theta_n) \right. \\
+ 2M_p^2 \theta_p - 2M_n^2 \theta_n + \left( \frac{M_n - M_p}{2} \right) (-12 + 13(M_n + M_p)) \right] \tag{66}
\]

Again, the expression of \( \Pi_{\delta\sigma}^{\text{vac}(A)} \) is obtained by replacing everywhere the indices \( \sigma \) by \( \delta \) in \( \Pi_{\sigma\sigma}^{\text{vac}(A)} \).

For the vector mesons, we have

\[
\Pi_{\omega\omega}^{\text{vac}(A) \mu \nu} = \frac{g_\omega^2}{6\pi^2} \left[ 2M_p^2 (\theta_p - 1) + 2M_n^2 (\theta_n - 1) + q^2 (\ln M_p + \theta_p) + q^2 (\ln M_n + \theta_n) \right. \\
- \frac{4q^2}{m_\omega^2} (\theta_\omega - 1) - 2q^2 \theta_\omega \left] \times \left( g^{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \right.
\]

\[
\Pi_{\omega\rho}^{\text{vac}(A) \mu \nu} = \Pi_{\rho\omega}^{\text{vac}(A) \mu \nu}
\]

\[
= \left\{ \frac{g_\omega g_\rho}{6\pi^2} \left[ 2M_p^2 (\theta_p - 1) - 2M_n^2 (\theta_n - 1) + q^2 (\ln M_p + \theta_p) - q^2 (\ln M_n + \theta_n) \right] \right. \\
+ \left( \frac{f_\rho}{2m} \right) \frac{g_\omega}{2\pi^2} q^2 \left[ M_p (\ln M_p + \theta_p) - M_n (\ln M_n + \theta_n) + 2(M_n - M_p) \right] \left] \times \left( g^{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \right.
\]

\[
\Pi_{\rho\rho}^{\text{vac}(A) \mu \nu} = \left\{ \frac{g_\rho^2}{6\pi^2} \left[ 2M_p^2 (\theta_p - 1) + 2M_n^2 (\theta_n - 1) + q^2 (\ln M_p + \theta_p) + q^2 (\ln M_n + \theta_n) \right] \right.
\]

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\[ -\frac{4q^2}{m_p^2}(\theta_\rho - 1) - 2q^2\theta_\rho \]
\[ + \left( \frac{f_\rho}{2m} \right)^2 \frac{q^2}{6\pi^2} \left[ 3M_p^2 \ln M_p + 4M_\rho^2 \theta_\rho + \frac{q^2}{2} (\ln M_p + \theta_\rho) + 3M_n^2 \ln M_n + 4M_n^2 \theta_n \right. \]
\[ + \frac{q^2}{2} (\ln M_n + \theta_n) - (8 + m_\rho^2)\theta_\rho - (q^2 - m_\rho^2)(\theta_\rho + (8 + m_\rho^2)\theta_\rho) \]
\[ + 5 + 6(M_n + M_p) - 17\frac{M_p^2 + M_n^2}{2} \]
\[ + \left( \frac{f_\rho}{2m} \right)^2 \frac{g_\rho}{\pi^2} q^2 \left[ M_p (\ln M_p + \theta_\rho) + M_n (\ln M_n + \theta_n) - 2\theta_\rho + 4 \left( m - \frac{M_n + M_p}{2} \right) \right] \left\{ g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right\} \] (69)

**Renormalization scheme B**

\[ \Pi_{\sigma\sigma}^{\text{vac}} (\text{B}) = \frac{g_\sigma g_\delta}{2\pi^2} \left[ 3M_p^2 \ln M_p + 3M_n^2 \ln M_n - \frac{q^2}{2} (\ln M_p + \theta_\rho) - \frac{q^2}{2} (\ln M_n + \theta_n) \right. \]
\[ + 2M_\rho^2 \theta_\rho + 2M_n^2 \theta_n + \left( 1 - \frac{M_p^2 + M_n^2}{2} \right) \left( 4\theta_\sigma - m_\sigma^2 (4 - m_\sigma^2)\theta_{\sigma\sigma} \right) \]
\[ + (q^2 - m_\sigma^2) \left( \theta_\sigma - (4 - m_\sigma^2)\theta_{\sigma\sigma} \right) - (4 - m_\sigma^2)\theta_\sigma \] (70)

\[ \Pi_{\sigma\delta}^{\text{vac}} (\text{B}) = \frac{g_\sigma g_\delta}{2\pi^2} \left[ 3M_p^2 \ln M_p - 3M_n^2 \ln M_n - \frac{q^2}{2} (\ln M_p + \theta_\rho) + \frac{q^2}{2} (\ln M_n + \theta_n) \right. \]
\[ + 2M_\rho^2 \theta_\rho - 2M_n^2 \theta_n + \left( \frac{M_p^2 - M_n^2}{2} \right) \left( -4\theta_\sigma + m_\sigma^2 (4 - m_\sigma^2)\theta_{\sigma\sigma} \right) \] (71)

Once more, the expressions of \( \Pi_{\sigma\delta}^{\text{vac}} (\text{B}) \) and \( \Pi_{\sigma\sigma}^{\text{vac}} (\text{B}) \) are obtained by replacing everywhere the indices \( \sigma \) by \( \delta \) in \( \Pi_{\sigma\sigma}^{\text{vac}} (\text{B}) \) and \( \Pi_{\sigma\delta}^{\text{vac}} (\text{B}) \) respectively.

The expression of \( \Pi_{\omega\omega}^{\text{vac}\mu\nu} \) coincides in both schemes. For the polarizations involving a vertex with the rho meson, we have

\[ \Pi_{\omega\rho}^{\text{vac}(\text{B})\mu\nu} = \left\{ \frac{g_\omega g_\rho}{6\pi^2} \left[ 2M_\rho^2 (\theta_\rho - 1) - 2M_n^2 (\theta_n - 1) + q^2 (\ln M_p + \theta_\rho) - q^2 (\ln M_n + \theta_n) \right] \right. \]
\[ + \left( \frac{f_\rho}{2m} \right)^2 \frac{g_\omega}{2\pi^2} q^2 \left[ M_p (\ln M_p + \theta_\rho) - M_n (\ln M_n + \theta_n) + (M_n - M_p)\theta_\omega \right] \left\{ g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right\} \] (73)

and \( \Pi_{\rho\omega}^{\text{vac}(\text{B})\mu\nu} \) is obtained by replacing \( \theta_\omega \) by \( \theta_\rho \) in the last term.
\[\Pi_{\nu^0}^{\nu_0} = \left\{ \frac{g^2}{6\pi^2} \left[ 2M_p^2(\theta_p - 1) + 2M_n^2(\theta_n - 1) + q^2(\ln M_p + \theta_p) + q^2(\ln M_n + \theta_n) \right] - \frac{4q^2}{m_p^2}(\theta_p - 1) - 2q^2\theta_p \right\} + \left( \frac{f_0}{2m} \right)^2 \frac{g^2}{6\pi^2} \left[ 3M_p^2 \ln M_p + 4M_p^2\theta_p + \frac{q^2}{2} (\ln M_p + \theta_p) + 3M_n^2 \ln M_n + 4M_n^2\theta_n \right] + \frac{g^2}{2} (\ln M_n + \theta_n) - (8 + m_p^2)\theta_p - (q^2 - m_p^2)(\theta_p + (8 + m_p^2)\theta_{pq}) + \left( \frac{M_p^2 + M_n^2}{2} - 1 \right) \left[ -8\theta_p + m_p^2(8 + m_p^2)\theta_{pq} \right] + \left( \frac{f_0}{2m} \right) g \nu q^2 [M_p (\ln M_p + \theta_p) + M_n (\ln M_n + \theta_n) - \theta_p(M_n + M_p)] \right\} \times \left( g^{\mu\nu} - q^\mu q^\nu \right) \] (74)

As mentioned before, the renormalization constants fulfill the compatibility conditions (64). Note however that, in scheme B, \(\Pi_{\sigma\delta} \neq \Pi_{\delta\sigma}\) since they are renormalized on the mass shell of the \(\sigma\) and \(\delta\) respectively, i.e.

\[\Pi_{\sigma\delta}(q^2 = m_\sigma^2) = 0 ; \Pi_{\delta\sigma}(q^2 = m_\delta^2) = 0\]

This breaks the symmetry which was explicit on the initial divergent expression. The symmetry could be reestablished e.g. by imposing that the polarizations vanish at some intermediate common scale \(\mu_{\sigma\delta}\)

\[\Pi_{\sigma\delta}(q^2 = \mu_{\sigma\delta}^2) = 0 = \Pi_{\delta\sigma}(q^2 = \mu_{\delta\sigma}^2)\]

In renormalization scheme A on the other hand the symmetry was already preserved by the choice of the common renormalization point \(q^2 = 0\). The same remark applies to \(\Pi_{\omega\mu}^{\omega} \neq \Pi_{\mu\omega}^{\omega}\) in scheme B since they are renormalized on the mass shell of the \(\omega\) and \(\rho\) respectively. They coincide in scheme A.

**Appendix C - Imaginary Parts of Polarizations**

The imaginary part arise the factor \(\pm i\epsilon\) in the denominator of (24) after applying the formula \(1/(x + i\epsilon) = \mathcal{P}/x - i\pi\delta(x)\). Depending on the prescription applied for going around the pole in the propagator, we obtain the imaginary part of one of the definitions \(\Pi, \Pi^R, \Pi^L, \Pi^1\) ... all these being related to each other through factors such as \(\tanh(\beta\omega/2)\) (23). Here we chose to give the retarded polarizations. They are obtained by inserting \(\pm i\epsilon \ \text{sign}(p_0)\) in the denominator of Eq. (24).

All imaginary parts can be expressed in terms of three integrals \(E_1^{(i)}, E_2^{(i)}, E_3^{(i)}\) with \(i \in \{n, p\}\). At finite temperature, for the calculation of the retarded polarizations, we have

\[E_{1}^{(i)} = \int dy \left[ \left( y + \frac{\omega}{2} \right)^{n-1} \theta(y - y_{L}^{(i)}) \{ n \omega_i(y - n)(y + \omega) \} \right] \]

\[\quad + (-1)^n \left( y - \frac{\omega}{2} \right)^{n-1} \theta(y - y_{R}^{(i)}) \{ n \omega_i(y - \omega) \} \]

(75)
– for timelike momentum with \( q^2 < 4M^2 \), the imaginary parts vanish: \( E^{(i)}_n = 0 \),
– for timelike momentum with \( q^2 > 4M^2 \),

\[
E^{(i)}_n = (-1)^{n-1} \int_{M_i}^\infty dy \left(y - \frac{\omega}{2}\right)^{n-1} \left[ \theta(y - y_L^{(i)}) - \theta(y - y_U^{(i)}) \right] \left\{ n^{(i)}(\omega - y) + \pi^{(i)}(y) - (T_{\delta}) \right\}
\]

with

\[
n_i(y) = \left[ e^{\beta(y - \mu_i)} + 1 \right]^{-1}, \quad \pi_i(y) = \left[ e^{\beta(y + \mu_i)} + 1 \right]^{-1},
\]

\[
y_L^{(i)} = \left| \frac{k\sqrt{\Delta^{(i)}} - \omega}{2} \right|, \quad y_U^{(i)} = \left| \frac{k\sqrt{\Delta^{(i)}} + \omega}{2} \right|, \quad \Delta^{(i)} = 1 - 4\frac{M^2}{q^2}
\]

\[
\mathcal{I} \Pi^{(i)}_\sigma = -\frac{g_\sigma^2}{2\pi k} \left( M_i^2 - \frac{q^2}{4} \right) E^{(i)}_1
\]

\[
\mathcal{I} \Pi^{(i)}_{\omega L} = \frac{g_\omega}{2\pi k} \left[ \frac{q^2}{4} E^{(i)}_1 - \frac{q^2}{k^2} E^{(i)}_3 \right]
\]

\[
\mathcal{I} \Pi^{(i)}_{\omega T} = \frac{g_\omega}{4\pi k^2} \left[ \left( M_i^2 + \frac{q^2}{4} \right) E^{(i)}_1 + \frac{q^2}{k^2} E^{(i)}_3 \right]
\]

\[
\mathcal{I} \Pi^{(i)}_\rho = -\frac{g_\rho g_\omega}{2\pi k} q^2 M_i E^{(i)}_2
\]

\[
\mathcal{I} \Pi^{(i)}_\delta = -\frac{g_\delta^2}{2\pi k} \left( M_i^2 - \frac{q^2}{4} \right) E^{(i)}_1
\]

\[
\mathcal{I} \Pi^{(i)}_{\rho L} = \frac{1}{2\pi k} \left\{ g_\rho \left[ \frac{q^2}{4} E^{(i)}_1 - \frac{q^2}{k^2} E^{(i)}_3 \right] + g_\rho \frac{f_\rho}{2m} q^2 M_i E^{(i)}_1 + \left( \frac{f_\rho}{2m} \right)^2 \left[ M_i^2 q^2 E^{(i)}_1 + \frac{q^4}{k^2} E^{(i)}_3 \right] \right\}
\]

\[
\mathcal{I} \Pi^{(i)}_{\rho T} = \frac{1}{4\pi k^2} \left\{ g_\rho \left[ \left( M_i^2 + \frac{q^2}{4} \right) E^{(i)}_1 + \frac{q^2}{k^2} E^{(i)}_3 \right] + 2g_\rho \frac{f_\rho}{2m} q^2 M_i E^{(i)}_1 + \left( \frac{f_\rho}{2m} \right)^2 \left[ \left( M_i^2 q^2 + \frac{q^4}{4} \right) E^{(i)}_1 - \frac{q^4}{k^2} E^{(i)}_3 \right] \right\}
\]

\[
\mathcal{I} \Pi^{(i)}_{\delta \rho} = -\frac{g_\delta (2M_i g_\rho + (f_\rho/2m)q^2)}{4\pi k^3} q^2 E^{(i)}_2
\]

\[
\mathcal{I} \Pi^{(i)}_{\rho L} = \frac{g_\rho (2M_i g_\rho + (f_\rho/2m)q^2)}{4\pi k^3} q^2 E^{(i)}_2
\]

\[
\mathcal{I} \Pi^{(i)}_{\rho T} = -\frac{g_\rho g_\omega}{2\pi k} q^2 E^{(i)}_2
\]

\[
\mathcal{I} \Pi^{(i)}_\delta = \frac{g_\delta g_\omega}{2\pi k} \left( M_i^2 - \frac{q^2}{4} \right) E^{(i)}_1
\]

\[
\mathcal{I} \Pi^{(i)}_{\rho L} = \frac{1}{4\pi k} \left\{ g_\rho g_\omega \left[ \frac{q^2}{2} E^{(i)}_1 - 2\frac{q^2}{k^2} E^{(i)}_3 \right] + g_\rho \frac{f_\rho}{2m} q^2 M_i E^{(i)}_1 \right\}
\]

\[
\mathcal{I} \Pi^{(i)}_{\rho T} = \frac{1}{4\pi k} \left\{ g_\rho g_\omega \left[ \left( M_i^2 + \frac{q^2}{4} \right) E^{(i)}_1 + \frac{q^2}{k^2} E^{(i)}_3 \right] + g_\rho \frac{f_\rho}{2m} q^2 M_i E^{(i)}_1 \right\}
\]
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Fig. 1 Effective masses and proton fraction as calculated from parameter set [14] with $g_\delta = 5$ in neutrino-free matter in $\beta$ equilibrium at $T=0$. 
Fig. 2 Effective masses of mesons as calculated with renormalization procedure A as a function of asymmetry. The density was fixed at $n_B = 4 \, n_{sat}$ and the temperature at $T = 0$, and the coupling constants are those of (49).
Fig. 3 Effective masses of mesons as calculated with renormalization procedure B as a function of asymmetry. The density was fixed at $n_B = 4 n_{sat}$ and the temperature at $T = 0$, and the coupling constants are those of (49).
Fig. 4 $\rho$-$\omega$ mixing angle in the transverse mode as a function of density asymmetry, as calculated with renormalization procedures A (left) or B (right). The parameters are those of [22] and the temperature is set to $T=0$. The exchanged 3-momentum was fixed to $k = 500$ MeV. The curves are labelled by the value of $Y_\rho$.

Fig. 5 Effect of asymmetry on the zero sound mode due to $\sigma$-$\omega$ mixing. The figure was plotted for $n_B = 4n_{sat}$ and $T = 0$ and using renormalization scheme (A). The coupling constants are those of [22].
Fig. 6 Real and imaginary parts of the $\sigma$ meson propagator.

Fig. 7 Real and imaginary parts of the mixed $\delta$-$\rho$ meson propagator.

Fig. 8 Real and imaginary parts of the transverse $\rho$ meson propagator.