Racetrack Fixed Field Alternating Gradient accelerator for variable energy extraction

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In this paper, a novel concept of fixed field alternating gradient (FFAG) accelerator is presented which solves the major problems of conventional FFAG and cyclotron accelerators. This concept combines stable racetrack fixed field configuration with dispersion free straight sections to enable variable energy extraction. A time-varying synchrotron-like alternating gradient focusing structure makes it possible for the first time to correct the beam optics of an FFAG accelerator in order to heuristically maximize the beam transmission by minimizing the tune excursion and avoiding the crossing of harmful betatron resonances. This concept makes it possible to design a compact variable energy extraction machine which is particularly useful for research and medical applications such as heavy ion therapy.

I. INTRODUCTION

A Fixed Field Alternating Gradient (FFAG) accelerator is a concept that was invented in the 1950s almost independently in the US, Japan and Russia [1][2]. Several electron machines were built in the US. However, it was not until the 1990s that the interest for FFAGs was revived in Japan. One main feature of FFAG and cyclotrons is that they both use fixed field combined function magnets in order to achieve a stable particle motion in the accelerator. Therefore, for heavy ion therapy accelerators requiring energies up to several 100 MeV/nucleon, they are the most viable option for a compact design that fits best into a hospital environment. In addition, with the recent advances in the superconducting magnet technology, the latter has become canonical for many applications.

An FFAG accelerator has more design flexibility compared to other types of fixed field accelerators. Unlike cyclotrons where the main target is to achieve isochronism for all energies, the magnetic field of an FFAG can obey any radial or azimuthal dependence. Thus, with the worldwide interest many FFAG designs emerged tailored to areas ranging from industrial processes to medical [3][4][5][6] and fundamental research [7][8][9]. In the early days, almost all designs were based on highly symmetric rings to reduce the operating restrictions that may arise with systematic field errors. However, recently, several racetrack concepts were proposed [10][11]. This is generally considered as a solution to practical issues such as the construction cost, the injection and extraction problem, as well as to accommodate and simplify the design of the RF cavity.

Although recently built FFAGs demonstrated the concept, several problems remain. In [12] it is shown that small gradient field errors in a ring FFAG accelerator lead to a non-scaling of the orbits. The latter contributes to the change of the average horizontal restoring force as well as the magnetic flutter and induces a variation of the betatron wave number leading to the crossing of transverse resonances during the acceleration cycle. For instance, a gradient field error of the order of 5% yields a variation of the betatron wave number of the order of 7% leading to the losses of the majority of the beam [13]. This is mainly due to the large fringing field caused by the yoke free magnets [14] and to the slow acceleration, i.e. to the low energy gain per turn. The main challenge with optics correction in fixed field accelerators is that the beam moves outward radially during the acceleration. Therefore, a correction scheme should be implemented along the radius of the magnet to produce the desired field profile and is practically expensive and difficult to implement since it modifies simultaneously the bending as well as the focusing of the beam in both the horizontal and vertical plane. In addition, other errors may contribute as well to the beam losses such as misalignment errors.

This letter shows that a racetrack FFAG configuration can achieve a fixed tune even in presence of field and alignment errors and thus overcome resonance crossings problem. The method is based on creating dispersion free straight sections where a time varying alternating gradient focusing structure is placed. The merit of this approach is that the time varying field structure can be programmed in a way to maximize the overall beam transmission of the built machine.

The idea of creating a dispersion-free long insertion in an FFAG was first discussed by Meads in 1983 [15]. However, the design was not optimized and focused on using scaling FFAG type magnets to achieve a fixed tune machine. In addition, the sensitivity to misalignments as well as the correction of the optics were not discussed. The design discussed in this paper is greatly simplified with the use of both scaling and non-scaling type of magnets to create a racetrack and compact microtron-like configuration.

As illustrated in Fig. 1, a configuration with 4 identical radial sector magnets (m1) is privileged since it provides enough space between the magnets to place injection/extraction devices as well as other elements discussed below. In order to have the most compact design, no reverse bend is considered. Even though cyclotrons are the natural choice for high intensity applications due to their cw operation, the turn separation for outer orbits becomes small when the beam becomes more relativistic, thus posing a serious beam extraction issue. In general,
it is considered that 1 GeV is the energy limit for cyclotrons which justifies the development of synchrotron accelerators for heavy ion therapy requiring energies up to 450 MeV/nucleon. In this study, one privileges a different class of accelerator, an FFAG since the fixed field operation allows high repetition rate and eases the problem of extracting the beam. However, one main objective of medical heavy ion accelerators is to target tumors of various depths therefore requiring to tune the beam energy. This is particularly challenging with fixed field accelerators since the beam orbit moves radially outwards during the acceleration. For this reason, it is common to use a beam degrader instead. This reduces the energy of the output beam but also reduces its quality due to multiple scattering. Thus, activation due to beam losses is an issue that requires more shielding of the accelerator. Besides, the beam intensity becomes strongly dependent on the beam energy which complicates the heavy ion treatment and justifies once again the wide use of synchrotrons for this application. In order to remediate this problem and extract the beam at various energies, the idea is to transform the circular conventional FFAG into a racetrack configuration by replacing the 4-fold symmetry with a 2-fold symmetry machine: the main idea is to add a second magnet (m2) for each sector and optimize its field profile in order for the pair (m1+m2) to create a 90 degree bending angle for all energies. This is achieved using the beam dynamics tracking code ZGOUBI: a fitting method is employed which consists in tracking particles with different momenta and the same initial coordinates (point A in figure 1) to the screen where their final coordinates are recorded. The field along the radius of magnet (m2) is adjusted until all particles exit perpendicular to the screen. Once this is achieved, the ring is completed by creating mirror symmetries with respect to the screen and to the line passing by point A and perpendicular to the screen. A microtron-like racetrack FFAG configuration with dispersion-free straight sections is thus achieved.

The advantages of the dispersion-free sections are numerous: first one can place a stripping foil at that location. Since the lattice is designed for a specific charge to mass ratio (in this case $q/A = 1/2$), in order to produce protons, one can inject $H2^+$ ions instead, accelerate them to an intermediate energy and strip them into protons at the location of the straights. Since all heavy ions leaving the straight sections are on stable orbits, they can be accelerated again to the top energy corresponding to the top magnetic rigidity. In addition, the straight section can be used to dump the beam after tripping during the acceleration which helps reduce the activation of the accelerator components. This is a common problem for cyclotrons for instance. Furthermore, tight space for the RF cavity causes coupling with the leakage field of the main lattice magnets leading to closed orbit distortion issues.

The field profile for each magnet writes as a separable function in radial and azimuthal coordinates in the following way: $B(R, \theta) = R(R)F(\theta)$ where $B$ is the vertical component of the magnetic field in the median plane, $R$ is the radial coordinate with respect to the center of the ring and $\theta$ is the azimuthal coordinate. The fringe field falloff of the magnets is represented by the Enge model: $R(R) = B_0(R/R_0)^k$ where $k$ is the average field index of the magnet and $B_0$ is the field at $R = R_0$. This guarantees a large transverse dynamic acceptance. The first optimization step is to adjust just the edge angle as well as the average field index $k$ of (m1) in order to ensure enough strong focusing and avoid the crossing of integer and half integer resonances. The main idea is that the bare lattice, i.e. without any added time-varying element, be stable in order to characterize its beam optics and to mitigate the associated risks as part of the construction process. The parameters of the test lattice are listed in Table I. Note that only the exit edge angle of (m1) is optimized since it has the largest impact on reducing the tune excursion mainly in the vertical plane. In summary, the transverse tunes of the optimized lattice are shown in Fig. 2 in red where one can see that the integer and half integer resonances are avoided.

![FIG. 1. Vertical component of the magnetic field in the median plane of the accelerator with some of the closed orbits between injection and extraction energies (shown in dashed black curves). QF and QD refer to the focusing and defocusing trim quadrupoles respectively.](image-url)

| Parameter                          | Value | Unit |
|-----------------------------------|-------|------|
| Charge to mass ratio              | 1/2   |      |
| Circumference                     | 15    | m    |
| Length of field-free straight section | 1    | m    |
| Field index k of m1               | 0.75  |      |
| Injection/extraction kinetic energy | 12/75 | MeV/nucleon |
| Maximum field                     | 8     | T    |

TABLE I. Main parameters of the test lattice.
nances are avoided. Next, one aims at reducing the tune excursion in order to avoid the crossing of any type of resonance, up to the 3rd order, known to be detrimental to the beam. For this, one constructs the one turn transfer map of the bare lattice. This is achieved by tracking particles with small displacements from each closed orbit, i.e. each different particle energy. Then, one adds the trim quadrupoles $Q_F$ and $Q_D$ in a way that maintains the 2-fold symmetry of the accelerator as shown in Fig. 1. This is of particular importance in order not to increase the number of betatron resonances that may be driven. The transfer map of the time-varying lattice is thus re-calculated using a MATHEMATICA code and the gradients of the trim quadrupoles are determined for each particle energy in order to match the tunes of a chosen working point. The solution obtained is finally implemented in the tracking code ZGOUBI by adding the time-varying elements to the bare lattice. The results of the tune calculation from the accelerated orbit is shown in green in Fig. 2 where one proved that the resonance crossing problem is overcome. In summary, the time varying field of the trim quadrupoles compensates for the monotonic behavior of the tunes in the bare lattice. For instance, increasing simultaneously the gradients of $Q_F$ and $Q_D$ allows to increase both horizontal and vertical tunes. This applies to any bare lattice in the presence of field or misalignment errors of its main magnets. In addition, the working point of the lattice can be optimized in a way to heuristically maximize the overall beam transmission. This is of utmost importance for the study of the resonance crossing problem in fixed field accelerators. The main challenge for the implementation of such a method is to achieve the ramping rate of the trim quadrupoles. However, the technological challenge can be reduced by increasing the total length of the elements, therefore increasing the length of the straight sections. Witte and Berg already proposed adequate design for rapid cycling quadrupole magnets [20]. Next, one evaluates the Dynamic Aperture (DA) of the accelerator in presence of misalignment errors. The DA is defined here as the maximum initial horizontal normalized amplitude that the particle can have without any losses due to single particle dynamics effects over 30000 turns of the entire acceleration cycle. This is a fundamental criterion in order to determine the tolerance of the lattice to imperfections. Each of the 12 magnets is transversely (horizontally and vertically) offset randomly using a Gaussian distribution with a standard deviation $\sigma$ and a cutoff at $3\sigma$. For each error, 100 different patterns are tested and an initial normalized vertical displacement at 20 mm.mrad is chosen. The effect of the misalignment on the DA is shown in Fig. 3 where one can see that with reasonable alignment errors ($\sigma \approx 100\mu m$) the DA is essentially unchanged. However, the DA of the lattice with trim quadrupoles is almost 3 times larger than the DA of the bare lattice. This is mainly due to the third integer resonance crossing problem with the bare lattice, i.e. $Q_z = 1/3$. As expected the lattice with the optimized working point has the largest DA.

Since one of the main objectives of this design is to achieve a variable energy extraction, one decided to calculate the maximum deviation of the accelerated orbit from the ideal one at the location of the extraction device during the entire acceleration cycle and in presence of misalignment errors as described above. In general, the maximum is reached near the injection cycle due to the adiabatic damping of the amplitude of particle oscillations with the momentum. The accelerated maximum rms orbit distortion for various misalignment errors is shown in Fig. 4. It can be seen that the vertical plane is the limiting problem for the variable energy extraction. However, this is still acceptable with a reasonable alignment error of the order of $\sigma \approx 100\mu m$ corresponding to a maximum average orbit distortion of 1.8 mm.

In this paper, a novel concept of compact racetrack
FFAG accelerator with dispersion free straight sections was proposed. It was shown that this is of particular interest for a heavy ion accelerator with variable energy extraction and enables dual acceleration of carbon ions as well as protons through stripping. In addition, one demonstrated for the first time that it is possible to overcome the problem of resonance crossings by timel-varying the field of the trim quadrupoles during the acceleration cycle. This is of particular interest for high intensity applications and is a step further towards designing a tunable FFAG accelerator with minimum beam losses. The study of the dynamics of this concept were investigated. In addition, the sensitivity to random misalignment errors of the magnets was explored and it was found that the mean DA is almost insensitive to the alignment error.

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