Supersymmetry and DLCQ Limit
of
Lie 3-algebra Model of M-theory

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Abstract

In arXiv:1003.4694 we proposed two models of M-theory, Hermitian 3-algebra model and Lie 3-algebra model. In this paper, we study the Lie 3-algebra model with a Lorentzian Lie 3-algebra. This model is ghost-free despite the Lorentzian 3-algebra. We show that our model satisfies two criteria as a model of M-theory. First, we show that the model possesses $\mathcal{N} = 1$ supersymmetry in eleven dimensions. Second, we show the model reduces to BFSS matrix theory with finite size matrices in a DLCQ limit.

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1 Introduction

BFSS matrix theory \cite{1} is one of the strong candidates of non-perturbative definition of superstring theory. It is conjectured to describe infinite momentum frame (IMF) limit of M-theory and many evidences were found. Because only D0-branes in type IIA superstring theory survive in this limit, BFSS matrix theory is defined by the one-dimensional maximally supersymmetric Yang-Mills theory. Since the theory is a gauge theory, a matrix representation is allowed and dynamics of a many-body system can be described by using diagonal blocks of matrices. However, it seems impossible to derive full dynamics of M-theory from BFSS matrix theory because it treats D0-branes as fundamental degrees of freedom. For example, we do not know the manner to describe longitudinal momentum transfer of D0-branes. Therefore, we need a matrix model that treats membranes as fundamental degrees of freedom in order to derive full dynamics of M-theory.

IIB matrix model \cite{2} is also one of the strong candidates of non-perturbative definition of superstring theory. It starts with the Green-Schwartz type IIB superstring action in order to treat strings themselves as fundamental degrees of freedom. If we fix the $\kappa$ symmetry to Schild gauge $\theta_1 = \theta_2$, the action reduces to that of the zero-dimensional maximal supersymmetric Yang-Mills theory with area preserving diffeomorphism (APD) symmetry. Since the resultant action is a gauge theory, it describes dynamics of many-body systems. IIB matrix model is defined by replacing the APD algebra with $u(N)$ Lie algebra in the action.

In paper \cite{3}, we obtained matrix models of M-theory in an analogous way to obtain IIB matrix model. We started with the Green-Schwartz supermembrane action in order to obtain matrix models of M-theory that treat membranes themselves as fundamental degrees of freedom. We showed, by using an approximation, that the action reduces to that of a zero-dimensional gauge theory with volume preserving diffeomorphism (VPD) symmetry \cite{4,5} if we fix the $\kappa$ symmetry of the action to a semi-light-cone gauge, $\Gamma_{012}\Psi = \Psi$. We proposed two 3-algebra models of M-theory which are defined by replacing VPD algebra with finite-dimensional 3-algebras in the action. Because the 3-algebra models are gauge theories, they are expected to describe dynamics of many-body systems as in the other matrix models.

One of the two models is based on Hermitian 3-algebra \cite{6,7} (Hermitian 3-algebra model), whereas the another is based on Lie 3-algebra \cite{10,22} (Lie 3-algebra model). The Hermitian 3-algebra model with $u(N) \oplus u(N)$ symmetry was shown to reduce to BFSS matrix theory.
with finite size matrices when a DLCQ limit is taken in [3]. A supersymmetric deformation of the Lie 3-algebra model with the $A_4$ algebra was investigated by adding mass and flux terms in [23].

In this paper, we study the Lie 3-algebra model with a Lorentzian Lie 3-algebra. We show that this model satisfies two criterion as a model of M-theory. In section two, we show that the model possesses $\mathcal{N} = 1$ supersymmetry in eleven dimensions. In section three, we show the model reduces to BFSS matrix theory with finite size matrices in a DLCQ limit as it should do: it is generally shown that M-theory reduces to such BFSS matrix theory in a DLCQ limit [24–27].

2 $\mathcal{N} = 1$ Supersymmetry Algebra in Eleven Dimensions

In [3], we proposed the Lie 3-algebra model of M-theory, whose action is given by

\[
S_0 = \left\langle -\frac{1}{12}[X^I, X^J, X^K]^2 - \frac{1}{2}(A_{\mu ab}[T^a, T^b, X^I])^2 - \frac{1}{3}E^{\mu\nu\lambda}A_{\mu ab}A_{\nu cd}A_{\lambda ef}[T^a, T^c, T^d][T^b, T^e, T^f] - \frac{i}{2}\bar{\Psi}\Gamma^\mu A_{\mu ab}[T^a, T^b, \Psi] + \frac{i}{4}\bar{\Psi}\Gamma_{IJ}[X^I, X^J, \Psi] \right\rangle. \tag{2.1}
\]

The fields are spanned by Lie 3-algebra $T^a$ as $X^I = X^I_aT^a$, $\Psi = \Psi_a T^a$ and $A^\mu = A^\mu_{ab}T^a \otimes T^b$, where $I = 3, \cdots, 10$ and $\mu = 0, 1, 2$. $<>$ represents a metric for the 3-algebra. $\Psi$ is a Majorana spinor of SO(1,10) that satisfies $\Gamma_{012}\Psi = \Psi$. $E^{\mu\nu\lambda}$ is a Levi-Civita symbol in three-dimensions. In this section, we will show that this action possesses $\mathcal{N} = 1$ supersymmetry in eleven-dimensions.

The action is invariant under 16 dynamical supersymmetry transformations,

\[
\delta X^I = i\epsilon\Gamma^I\Psi
\]
\[
\delta A_{\mu ab}[T^a, T^b, \ ] = i\epsilon\Gamma_\mu\Gamma_I[X^I, \Psi, \ ]
\]
\[
\delta \Psi = -A_{\mu ab}[T^a, T^b, X^I]\Gamma^\mu\Gamma_I\epsilon - \frac{1}{6}[X^I, X^J, X^K]\Gamma_{IJK}\epsilon,
\] \tag{2.2}
where $\Gamma_{012} = -\epsilon$. These supersymmetries close into gauge transformations on-shell,

$$[\delta_1, \delta_2] X^I = \Lambda_{cd}[T^c, T^d, X^I]$$

$$[\delta_1, \delta_2] A_{\mu ab}[T^a, T^b, ] = \Lambda_{ab}[T^a, T^b, A_{\mu cd}[T^c, T^d, ]] - A_{\mu ab}[T^a, T^b, \Lambda_{cd}[T^c, T^d, ]] + 2i\tilde{\epsilon}_2 \Gamma^\nu \epsilon_1 O_{\mu \nu}^A$$

$$[\delta_1, \delta_2] \Psi = \Lambda_{cd}[T^c, T^d, \Psi] + (i\tilde{\epsilon}_2 \Gamma^\mu \epsilon_1 \Gamma_{\mu} - i\frac{1}{4} \tilde{\epsilon}_2 \Gamma_{KL} \epsilon_1 \Gamma_{KL}) O_{\mu \nu}^\Psi,$$

(2.3) implies that a commutation relation between the dynamical supersymmetry transformations is

$$\delta_2 \delta_1 - \delta_1 \delta_2 = 0,$$

(2.5) up to the equations of motions and the gauge transformations.

Lie 3-algebra with an invariant metric is classified into four-dimensional Euclidean $A_4$ algebra and Lie 3-algebras with indefinite metrics in [16–18, 28, 29]. We do not choose $A_4$ algebra because its degrees of freedom are just four. We need an algebra with arbitrary dimensions $N$, which is taken to infinity to define M-theory. Here we choose the most simple indefinite metric Lie 3-algebra, so called Lorentzian Lie 3-algebra associated with $u(N)$ Lie algebra,

$$[T^{-1}, T^a, T^b] = 0$$

$$[T^0, T^i, T^j] = [T^i, T^j] = f^{ij}{}_k T^k$$

$$[T^i, T^j, T^k] = f^{ijk} T^{-1},$$

(2.6) where $a = -1, 0, i$ ($i = 1, \cdots, N^2$). $T^i$ are generators of $u(N)$. A metric is defined by a symmetric bilinear form,

$$<T^{-1}, T^0> = -1$$

$$<T^i, T^j> = h^{ij},$$

(2.7) (2.8)
and the other components are 0. The action is decomposed as

\[ S = \text{Tr}( -\frac{1}{4}(x_0^I)^2(x^I, x^J)^2 + \frac{1}{2}(x_0^I[x_0^I, x^J])^2 - \frac{1}{2}(x_0^Ib_\mu + [a_\mu, x^I])^2 - \frac{1}{2}E^{\mu\nu\lambda}b_\mu[a_\nu, a_\lambda] \\
+ i\bar{\psi}_0\Gamma^\mu b_\mu\psi - \frac{i}{2}\bar{\psi}\Gamma^\mu[a_\mu, \psi] + \frac{i}{2}x_0^I\bar{\psi}\Gamma_{IJ}[x^J, \psi] - \frac{i}{2}\bar{\psi}_0\Gamma_{IJ}[x^I, x^J]\psi), \]

(2.9)

where we have renamed \(X^I_0 \rightarrow x_0^I, X^I_I \rightarrow x^I, \Psi_0 \rightarrow \psi_0, \Psi_I T^i \rightarrow \psi, 2A_{\mu 0} T^i \rightarrow a_\mu, \) and \(A_{\mu ij}[T^i, T^j] \rightarrow b_\mu.\) In this action, \(T^{-1}\) mode, \(X^I_{-1}, \Psi_{-1} \) or \(A^\mu_{-1a}\) does not appear, that is they are unphysical modes. Therefore, the indefinite part of the metric (2.7) does not exist in the action and our model is ghost-free like a model in \([30]\). This action can be obtained by a dimensional reduction of the three-dimensional \(\mathcal{N} = 8\) BLG model \([13–15]\) with the same 3-algebra. The BLG model possesses a ghost mode because of its kinetic terms with indefinite signature. On the other hand, our model does not possess a kinetic term because it is defined as a zero-dimensional field theory like IIB matrix model \([2]\).

This action is invariant under the translation

\[ \delta x^I = \eta^I, \quad \delta a^\mu = \eta^\mu, \]

(2.10)

where \(\eta^I\) and \(\eta^\mu\) belong to \(u(1).\) This implies that eigen values of \(x^I\) and \(a^\mu\) represent an eleven-dimensional space-time.

The action is also invariant under 16 kinematical supersymmetry transformations

\[ \tilde{\delta}\psi = \tilde{\epsilon}, \]

(2.11)

and the other fields are not transformed. \(\tilde{\epsilon}\) belong to \(u(1)\) and satisfy \(\Gamma_{012}\tilde{\epsilon} = \tilde{\epsilon}.\) \(\tilde{\epsilon}\) and \(\epsilon\) should come from 16 components of 32 \(\mathcal{N} = 1\) supersymmetry parameters in eleven dimensions, corresponding to eigen values \(\pm 1\) of \(\Gamma_{012},\) respectively, as in the case of the semi-light-cone supermembrane. Its target-space \(\mathcal{N} = 1\) supersymmetry consists of remaining 16 target-space supersymmetries and transmuted 16 \(\kappa\)-symmetries in the semi-light-cone gauge, \(\Gamma_{012}\Psi = \Psi [3, 31, 32].\)

A commutation relation between the kinematical supersymmetry transformations is given by

\[ \tilde{\delta}_2\tilde{\delta}_1 - \tilde{\delta}_1\tilde{\delta}_2 = 0. \]

(2.12)
The 16 dynamical supersymmetry transformations (2.2) are decomposed as

\[ \delta x^I = i \bar{\epsilon} \Gamma^I \psi \]
\[ \delta x_0^I = i \bar{\epsilon} \Gamma^I \psi_0 \]
\[ \delta x_{-1}^I = i \bar{\epsilon} \Gamma^I \psi_{-1} \]

\[ \delta \psi = -(b_\mu x_0^I + [a_\mu, x^I]) \Gamma^\mu \Gamma_I \epsilon - \frac{1}{2} x_0^I [x^J, x^K] \Gamma_{IJK} \epsilon \]
\[ \delta \psi_0 = 0 \]
\[ \delta \psi_{-1} = - \text{Tr}(b_\mu x^I) \Gamma^\mu \Gamma_I \epsilon - \frac{1}{6} \text{Tr}([x^I, x^J] x^K) \Gamma_{IJK} \epsilon \]

\[ \delta a_\mu = i \bar{\epsilon} \Gamma_\mu \Gamma_I (x_0^I \psi - \psi_0 x^I) \]
\[ \delta b_\mu = i \bar{\epsilon} \Gamma_\mu \Gamma_I [x^I, \psi] \]
\[ \delta A_{\mu-1i} = i \bar{\epsilon} \Gamma_\mu \Gamma_I \frac{1}{2} (x_{i-1}^I \psi_i - \psi_{-1} x_i^I) \]
\[ \delta A_{\mu-10} = i \bar{\epsilon} \Gamma_\mu \Gamma_I \frac{1}{2} (x_{-1}^I \psi_0 - \psi_{-1} x_0^I), \quad (2.13) \]

and thus a commutator of dynamical supersymmetry transformations and kinematical ones acts as

\[ (\tilde{\delta}_2 \delta_1 - \delta_1 \tilde{\delta}_2) x^I = i \bar{\epsilon}_1 \Gamma^I \tilde{\epsilon}_2 \equiv \eta^I \]
\[ (\tilde{\delta}_2 \delta_1 - \delta_1 \tilde{\delta}_2) a^\mu = i \bar{\epsilon}_1 \Gamma^\mu \Gamma_I x_0^I \tilde{\epsilon}_2 \equiv \eta^\mu \]
\[ (\tilde{\delta}_2 \delta_1 - \delta_1 \tilde{\delta}_2) A_{\mu-1i} = \frac{1}{2} i \bar{\epsilon}_1 \Gamma^\mu \Gamma_I x_{i-1}^I \tilde{\epsilon}_2, \quad (2.14) \]

where the commutator that acts on the other fields vanishes. Thus, the commutation relation for physical modes is given by

\[ \tilde{\delta}_2 \delta_1 - \delta_1 \tilde{\delta}_2 = \delta_\eta, \quad (2.15) \]

where \( \delta_\eta \) is a translation.

If we change a basis of the supersymmetry transformations as

\[ \delta' = \delta + \tilde{\delta} \]
\[ \tilde{\delta}' = i (\delta - \tilde{\delta}), \quad (2.16) \]
we obtain
\[\begin{align*}
\delta_2'\delta_1' - \delta_1'\delta_2' &= \delta_\eta, \\
\bar{\delta}_2\bar{\delta}_1' - \bar{\delta}_1'\bar{\delta}_2 &= \delta_\eta, \\
\bar{\delta}_2\delta_1' - \delta_1'\bar{\delta}_2 &= 0.
\end{align*}\] (2.17)

These 32 supersymmetry transformations are summarised as \(\Delta = (\delta', \bar{\delta}')\) and (2.17) implies the \(\mathcal{N} = 1\) supersymmetry algebra in eleven dimensions,
\[\Delta_2\Delta_1 - \Delta_1\Delta_2 = \delta_\eta.\] (2.18)

### 3 DLCQ limit

In this section, we will take a DLCQ limit of our model and obtain BFSS matrix theory with finite size matrices as desired.

First, we separate the auxiliary fields \(b^\mu\) from \(A^\mu\) and define \(X^\mu\) by
\[A^\mu = X^\mu + b^\mu.\] (3.1)

We identify space-time coordinate matrices by redefining matrices as follows. By rescaling the eight matrices as
\[X^I = \frac{1}{T}X'^I\]
\[X^\mu = X'^\mu,\] (3.2)
we adjust the scale of \(X^I\) to that of \(X^\mu\). \(T\) is a real parameter. Next, we redefine fields so as to keep the scale of nine matrices:
\[X'^\mu = X'^\mu\]
\[X'^i = X'^{i_5}\]
\[X'^{0} = \frac{1}{T}X'^{0}\]
\[X'^{10} = \frac{1}{T}X'^{10}\] (3.3)
where \(p = 1, 2\) and \(i = 3, \ldots, 9\). We also redefine the auxiliary fields as
\[b^\mu = \frac{1}{T^2}b'^\mu.\] (3.4)
DLCQ limit of M-theory consists of a light-cone compactification, \( x^- \approx x^- + 2\pi R \), where \( x^\pm = \frac{1}{\sqrt{2}} (x^{10} \pm x^0) \), and Lorentz boost in \( x^{10} \) direction with an infinite momentum. We define light-cone coordinates as

\[
X'^0 = \frac{1}{\sqrt{2}} (X^+ - X^-) \\
X'^{10} = \frac{1}{\sqrt{2}} (X^+ + X^-)
\]  \hspace{1cm} (3.5)

We treat \( b''^\mu \) as scalars. A matrix compactification \[33\] on a circle with a radius \( R \) imposes following conditions on \( X^- \) and the other matrices \( Y \), which represent \( X^+ \), \( X''^p \), \( X''^i \), \( b''^\mu \), and \( \Psi \):

\[
X^- - (2\pi R)1 = U^\dagger X^- U \\
Y = U^\dagger Y U,
\]  \hspace{1cm} (3.6)

where \( U \) is a unitary matrix. After the compactification, we cannot redefine fields freely. A solution to (3.6) is given by \( X^- = \bar{X}^- + \tilde{X}^- \), \( Y = \tilde{Y} \) and

\[
U = \bar{U} \otimes \mathbf{1}_{\text{Lorentzian}},
\]  \hspace{1cm} (3.7)

where \( U(\mathbb{N}) \) part is given by,

\[
\bar{U} = \begin{pmatrix}
0 & 1 & 0 \\
\vdots & \ddots & \ddots \\
0 & 0 & 1
\end{pmatrix} \otimes \mathbf{1}_{n \times n}.
\]  \hspace{1cm} (3.8)

A background \( \bar{X}^- \) is

\[
\bar{X}^- = -T^3 \bar{x}_0 T^0 - (2\pi R) \text{diag}(\cdots, s - 1, s, s + 1, \cdots) \otimes \mathbf{1}_{n \times n},
\]  \hspace{1cm} (3.9)

and a fluctuation \( \tilde{x} \) that represents \( u(\mathbb{N}) \) parts of \( \bar{X}^- \) and \( \tilde{Y} \) is

\[
\begin{pmatrix}
\tilde{x}(0) & \tilde{x}(1) & \cdots \\
\tilde{x}(-1) & \ddots & \ddots \\
\vdots & \ddots & \tilde{x}(1) \\
\tilde{x}(-1) & \tilde{x}(0)
\end{pmatrix}.
\]  \hspace{1cm} (3.10)

Each \( \tilde{x}(s) \) is a \( n \times n \) matrix, where \( s \) is an integer. That is, the \( (s, t) \)-th block is given by \( \tilde{x}_{s,t} = \tilde{x}(s - t) \).
We make a Fourier transformation,

$$\tilde{x}(s) = \frac{1}{2\pi R} \int_0^{2\pi R} d\tau x(\tau) e^{is\tilde{\tau}/R}, \quad (3.11)$$

where $x(\tau)$ is an $n \times n$ matrix in one-dimension and $RR' = 2\pi$. From (3.9), (3.10) and (3.11), the following identities hold:

$$\sum_t \tilde{x}_{s,t} x'_{t,u} = \frac{1}{2\pi R} \int_0^{2\pi R} d\tau x(\tau)x'(\tau)e^{i(s-u)\tilde{\tau}/R}$$

$$\text{tr} \left( \sum_{s,t} \tilde{x}_{s,t} x'_{t,s} \right) = \frac{1}{2\pi R} \int_0^{2\pi R} d\tau \text{tr} (x(\tau)x'(\tau))$$

$$[\tilde{x}^-, \tilde{x}]_{s,t} = \frac{1}{2\pi R} \int_0^{2\pi R} d\tau \partial_\tau x(\tau)e^{i(s-t)\tilde{\tau}/R}, \quad (3.12)$$

where $\text{tr}$ is a trace over $n \times n$ matrices and $V = \sum_s 1$. We will use these identities later.

Next, let us boost the system in $x^{10}$ direction:

$$\tilde{X}^+ = \frac{1}{T} \tilde{X}^{m+}$$
$$\tilde{X}^- = T \tilde{X}^{m-}$$
$$\tilde{X}^{mp} = \tilde{X}^{mp}$$
$$\tilde{X}^m = \tilde{X}^m. \quad (3.13)$$

IMF limit is achieved when $T \to \infty$. The second equation implies that $X^- = -T^3 x_0 T^0 + TX^{m-}$, where $X^{m-} = \tilde{X}^{m-} + \tilde{X}'^{m-}$ and $\tilde{X}'^{m-} = -(2\pi R')\text{diag}(\cdots, s-1, s, s+1, \cdots) \otimes 1_{n \times n}$. $R' = \frac{1}{T} R$ goes to zero when $T \to \infty$. To keep supersymmetry, the fermionic fields need to behave as

$$\Psi = \frac{1}{T} \Psi^m. \quad (3.14)$$
To summarize, relations between the original fields and the fixed fields when $T \to \infty$ are

\[
\begin{align*}
    a^0 &= \frac{1}{\sqrt{2}} \left( \frac{1}{T^2} x^{m+} - x^{m-} \right) \\
    a^p &= x^{mp} \\
    x^i &= \frac{1}{T} x^{ni} \\
    x^{10} &= \frac{1}{\sqrt{2}} \left( \frac{1}{T^3} x^{m+} + \frac{1}{T} x^{m-} \right) \\
    x^{0} &= \frac{1}{T} x^{m0} \\
    x^{00} &= \frac{1}{\sqrt{2}} \left( \frac{1}{T^3} x^{m+} + \frac{1}{T} x^{m-} \right) - \frac{1}{\sqrt{2}} T \bar{x}^{-} \\
    b^\mu &= \frac{1}{T^2} b^{m\mu} \\
    \psi &= \frac{1}{T} \psi^m \\
    \psi_0 &= \frac{1}{T} \psi_0^m. 
\end{align*}
\]

By using these relations, equations of motion of the auxiliary fields $b^\mu$,

\[
b^\mu = \frac{1}{(x^0)^2} (-x_0^I [a^\mu, x_I] - \frac{1}{2} E_{\mu\nu\lambda} [a_\nu, a_\lambda] + i \bar{\psi}_0 \Gamma^\mu \psi) \tag{3.16}
\]

are rewritten as

\[
\begin{align*}
    b^{m0} &= -\frac{2}{(x_0)^2} [x^{m1}, x^{m2}] + O\left(\frac{1}{T}\right) \\
    b^{m1} &= -\frac{\sqrt{2}}{(x_0)^2} [x^{m2}, x^{m-}] + \frac{1}{x_0} [x^{m1}, x^{m-}] + O\left(\frac{1}{T}\right) \\
    b^{m2} &= \frac{\sqrt{2}}{(x_0)^2} [x^{m1}, x^{m-}] + \frac{1}{x_0} [x^{m2}, x^{m-}] + O\left(\frac{1}{T}\right). \tag{3.17}
\end{align*}
\]

If we substitute them and (3.15) to the action (2.9), we obtain

\[
\begin{align*}
    S &= \frac{1}{T^2} \text{Tr} \left( \frac{1}{2(x_0)^2} [x^{m-}, x^{mp}]^2 + \frac{1}{4} [x^{m-}, x^{m\mu}]^2 - \frac{1}{2(x_0)^2} [x^{mp}, x^{mq}]^2 - \frac{1}{2} [x^{mp}, x^{m\mu}]^2 - \frac{(x_0^{-})^2}{8} [x^{mi}, x^{mj}]^2 \\
        &\quad - \frac{i}{2\sqrt{2}} \bar{\psi}^m \Gamma^0 [x^{m-}, \psi^m] - \frac{i}{2 \sqrt{2}} \bar{\psi}^m \Gamma^p [x^{m}, \psi^m] - \frac{i}{2 \sqrt{2}} \bar{x}_0^{-} \bar{\psi}_0^m \Gamma_{10i} [x^{m}, \psi^m] \right) + O\left(\frac{1}{T^3}\right). \tag{3.18}
\end{align*}
\]

Therefore, the action reduces to

\[
\begin{align*}
    \hat{S} &= \frac{1}{T^2} \text{Tr} \left( \frac{1}{2(x_0)^2} [x^{m-}, x^{mp}]^2 + \frac{1}{4} [x^{m-}, x^{m\mu}]^2 - \frac{1}{2(x_0)^2} [x^{mp}, x^{mq}]^2 - \frac{1}{2} [x^{mp}, x^{m\mu}]^2 - \frac{(x_0^{-})^2}{8} [x^{mi}, x^{mj}]^2 \\
        &\quad - \frac{i}{2\sqrt{2}} \bar{\psi}^m \Gamma^0 [x^{m-}, \psi^m] - \frac{i}{2 \sqrt{2}} \bar{\psi}^m \Gamma^p [x^{m}, \psi^m] - \frac{i}{2 \sqrt{2}} \bar{x}_0^{-} \bar{\psi}_0^m \Gamma_{10i} [x^{m}, \psi^m] \right) \tag{3.19}
\end{align*}
\]
in $T \to \infty$ limit. By redefining
\[
x^{m_i} \rightarrow \frac{2^{\frac{3}{4}} \sqrt{T} x^{m_i}}{\sqrt{x_0}} \\
x^{m_p} \rightarrow \frac{\sqrt{x_0 T}}{2^{\frac{3}{4}}} x^{m_p} \\
x^{m_r} \rightarrow 2^{\frac{1}{4}} \sqrt{x_0 T} x^{m_r} \\
\psi^{m} \rightarrow \frac{2^{\frac{3}{4}} T^{\frac{3}{4}}}{(x_0^{-})^{\frac{1}{4}}} \psi^{m},
\]
we obtain
\[
S = \text{Tr}(\frac{1}{2}[x^{m-}, x^{m}]^2 - \frac{1}{4}[x^{m}, x^{m}]^2 - i \frac{\bar{\psi} \Gamma^0 [x^{m-}, \psi^{m}]}{2} - i \frac{\bar{\psi} \Gamma^p [x^{m}, \psi^{m}]}{2} - i \bar{\psi} \Gamma^{10i} [x^{m}, \psi^{m}]). 
\]
(3.20)
The background in $x^{m-}$ is modified, where $\frac{1}{\sqrt{T}} R' \to R'$. By using the identities (3.12), we can rewrite (3.21) and obtain the action of BFSS matrix theory with finite $n$,
\[
S = \int_{-\infty}^{\infty} d\tau \text{tr}(\frac{1}{2}(D_0 x^I)^2 - \frac{1}{4}[x^I, x^J]^2 + \frac{1}{2} \bar{\psi} \Gamma^0 D_0 \psi - i \frac{\bar{\psi} \Gamma^p [x^I, \psi]}{2} - i \bar{\psi} \Gamma^{10i} [x^I, \psi]). 
\]
(3.22)
We have used $\tilde{R}' = \infty$ because $R' \to 0$ when $T \to \infty$. In DLCQ limit of our model, we see that $X^-$ disappears and $X^+$ changes to $\tau$ as in the case of the light-cone gauge fixing of the membrane theory.

The way to take DLCQ limit (3.2) - (3.15) is essentially the same as in the case of the Hermitian model [3] because the limit realizes the ”novel Higgs mechanism” [34].

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