Low-requrement fast gates enable quantum computation in long ion chains

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We present a model for implementing fast entangling gates (~1μs) with ultra-fast pulses in arbitrarily long ion chains, that requires low numbers of pulses and can be implemented with laser repetition rates well within experimental capability. We demonstrate that we are able to optimise pulse sequences that have theoretical fidelities above 99.99% in arbitrarily long ion-chains, for laser repetition rates on the order of 100 – 300 MHz. Notably, we find higher repetition rates are not required for gates in longer ion chains, which is in contrast to scaling analyses with other gate schemes. When pulse imperfections are considered in our calculations, we find that achievable gate fidelity is independent of the number of ions in the chain. We also show that pulse control requirements do not scale up with the number of ions. We find that population transfer efficiencies of above 99.9% from individual ultra-fast pulses is the threshold for realising high-fidelity gates, which may be achievable in near-future experiments.

INTRODUCTION

Trapped ion platforms are a promising platform for realising noisy intermediate-scale quantum (NISQ) computers [2, 3]. While there has been significant progress in demonstrating high-fidelity control for small numbers of qubits [4], scaling up trapped ion processors without slowing down gate speeds remains an open challenge.

High-fidelity entangling gates have been achieved using bichromatic light fields tuned near the motional sidebands to drive state-dependent trajectories through phase space [5], as first proposed by Mølmer and Sørensen [6–8]. The Mølmer Sørensen (MS) mechanism requires a phase space [5], as first proposed by Mølmer and Sørensen [5]. Fast gates with ultra-fast pulses in arbitrarily long ion chains require approximately the same number of ions in the trap is increased, the ion-mode coupling decreases and the trap frequency must be reduced to avoid buckling of the ion chain; both of these factors lead to a slower timescale for sideband resolution and thus longer gate times for gates in longer ion chains. This strongly limits the number of gates that can be performed before decoherence.

Outside of this ‘sideband-resolving’ regime, multiple motional modes are excited by the ion-light interaction. Fast gate mechanisms use sequences of ultra-fast broadband laser pulses or amplitude-modulated continuous pulses to control the trajectories of each of these modes, and realise geometric phase gates [9–12]. Continuous-pulse fast gates have recently been demonstrated by Schäfer et al. [13], who demonstrated a high-fidelity (99.8%) 1.6 μs gate in a two-ion system, as well as a lower-fidelity (~60%) 480 ns gate. Fast gates with ultra-fast pulses, which are the focus of this manuscript, have not yet been experimentally demonstrated, although several groups are currently working on their realisation [14, 15]. Ultra-fast pulses have been used to demonstrate single-qubit control [16] and preparation of two-qubit Bell states [17].

The scaling behaviour of pulsed fast gates in large ion crystals has been studied in recent years [1, 18, 19]. For chains of 10 or more ions confined in a Paul trap, existing gate schemes require repetition rates of order 10 – 20 GHz for implementing high-fidelity fast gates [1]. In microtrap arrays, fast gates scale more favourably, with pulse sequences optimised for gates in two-ion systems remaining robust when applied to arbitrarily large ion crystals [19]. While this means that much lower repetition rates are required (~1 GHz for gates on the same timescale as the motional frequencies), large number of pulses are typically required to entangle ions that are separated by ~100 μm. We have identified that pulse errors are likely to be the dominant source of errors in fast-gates [20], and thus while fast gates in microtrap arrays are promising for their scalability, their experimental realisation requires improvement of pulse control from current experiments by several orders of magnitude.

In this manuscript we show that recently developed gate schemes can allow scalable processing in existing linear traps and with lasers repetition rates well within current experimental capabilities. We find gate solutions that are faster than the trap period that can be implemented in long chains of ions with far fewer laser pulses than previous schemes. Notably, we find that pulse control requirements do not become restrictive as the number of ions increases. In fact we find that gates in arbitrarily long ion chains require approximately the same level of laser control as gates in two-ion systems. This result opens up a new pathway to scaling up trapped ion quantum computers without slowing down computation.

GATE FORMALISM AND OPTIMISATION

Pulsed fast gate schemes are composed of state-dependent momentum kicks (SDKs) from pairs of
counter-propagating π-pulses, interspersed with periods of free evolution of the ions. These pulses are resonant with the electronic transition of the ions and can be described by the following interaction picture Hamiltonian:

\[ H_I = \sum_{j=1}^{2} \frac{\hbar \Omega(t)}{2} \left( \sigma_+^j e^{i(k x^j + \phi)} + \sigma_-^j e^{-i(k x^j + \phi)} \right), \]  

where \( x^i \) is the deviation of the \( i \)-th ion from its equilibrium, \( k \) and \( \phi \) are the laser wavevector and phase, and \( \Omega(t) \) is the Rabi rate (satisfying \( \int_0^T \Omega(t) \, d\tau = \pi \)).

By expanding the ion’s position into the mode basis, \( k x^i = \sum_m b_m^i \eta_m (a_m + a_m^\dagger) \), the resulting unitary can be written in terms of the mode displacement operator \( D_m(\alpha) = e^{\alpha a_m^\dagger - \alpha^* a_m} \):

\[ U_\pi = \prod_{j=1}^{2} \left( \sigma_+^j e^{-i \phi} \prod_m D_m(i b_m^j \eta_m) + \text{h.c.} \right), \]

where \( b_m^i \) is the ion-mode coupling between the \( j \)-th ion and the \( m \)-th motional mode. The \( m \)-th motional mode has an associated frequency \( \omega_m \) and Lamb-Dicke parameter \( \eta_m = \sqrt{\hbar / 2 M \omega_m} \). A SDK can be built by applying a second π-pulse that is counter-propagating \( (k \rightarrow -k \) or equivalently \( b_m^j \rightarrow -b_m^j \)). Assuming the two pulses are split from a single large pulse so that the laser phase \( \phi \) perfectly cancels, the SDK unitary can be expressed:

\[ U_{\text{SDK}} = \prod_{j=1}^{2} \prod_m D_m \left( -2i \eta_m b_m^j \sigma_z^j \right). \]  

The action of a single SDK on a coherent motional state can be understood as creating a cat state with \( |\alpha| = 2 \eta_m b_m^j \) in each of the \( m \) motional modes. We have deliberately omitted the bounds on \( \eta_m \), as the number of relevant motional modes depends on the dimensionality of the system: in general there are \( 3N \) modes for a system of \( N \) ions, however by controlling direction of incident light, only modes along a particular trap axis are targeted.

The unitary for the gate can be expressed as \( N_p \) SDKs with free evolution between kicks:

\[ U_{\text{gate}} = \prod_{k=1}^{N_p} U_{\text{SDK}} \exp(i \sum_m \omega_m \delta t_k a_m^\dagger a_m), \]

where \( \delta t_k \) is the time between the \((k-1)\)-th and \( k \)-th SDK. In the ideal gate, a geometric phase gate is implemented on two ion-qubits indexed \( \mu \) and \( \nu \) with unitary \( U_{\text{gate}} = \exp(\frac{i \pi}{4} \sigma_z^\mu \sigma_z^\nu) \); with all motional modes decoupled from the ions electronic states by the end of the gate. Notably, this gate scheme applies outside of the Lamb-Dicke regime \( \eta^2 \langle 2(\alpha_a + 1) \rangle \ll 1 \), unlike continuous-pulse fast gate schemes \([11, 12]\).

We perform optimisation of the number of SDKs and their timings in order to minimise the state-averaged fidelity of the unitary \( U_{\text{gate}} \) with respect to the ideal unitary \( U_{\text{id}} \), following the two-step procedure outlined in Ref. \([20]\). In the first step, we optimise the gate infidelity assuming an infinite laser repetition rate and consider a family of gate schemes where groups of SDKs that occur simultaneously are separated by regular intervals in time. In the second step, we expand the SDKs composing each group onto a grid of timings specified by the laser repetition rate, and optimise the timings between the SDKs at the edge of adjacent groups. This second stage of optimisation is done using an ordinary differential equation (ODE) description of the gate dynamics to calculate the accumulated phase and residual coupling to the motional modes. For efficient integration of the ODEs in large ion chains, we truncate the Coulomb potential to second order and convert to a normal-mode basis where the ODEs decouple. For details of our fidelity calculations, and further discussion of our optimisation approach, we direct readers to Ref. \([20]\).

**MODEL AND RESULTS**

Previous analyses have identified that when the gate time \( T_G \) is much faster than the motional dynamics \( T_G \ll 2\pi / \omega_m \), only the motions of local ions are affected by the gate, leaving the rest of the ion chain untouched \([1, 11]\). We exploit this in our model; as the axial frequency is reduced to accommodate longer ion chains, the motional dynamics slow down and thus distant ions are increasingly unlikely to be affected by a gate with a constant speed. This approach is only valid for gates between neighbouring ions; gates between distant ions are generally unable to successfully restore the motion of all collective modes \([1]\).

In our model we consider a chain of ions in a Paul trap with fixed radial frequency \( \omega_r/2\pi = 5 \text{ MHz} \), and variable axial trap frequency \( \omega_z = \omega_r/0.65N^{0.865} \) which is sufficient to prevent buckling of a chain of \( N \) ions \([21]\). We take the light field to be oriented down the length of the ion chain, such that only the axial modes are excited by the SDKs.

We use the Anti-symmetric Pulse Group scheme \([20]\) with \( N_k \) groups of SDKs,

\[ z = \{ -z_{N_k/2}, \ldots, -z_2, -z_1, z_1, z_2, \ldots, z_{N_k/2} \}, \]

\[ t = \{ -t_{N_k/2}, \ldots, -t_2, -t_1, t_1, t_2, \ldots, t_{N_k/2} \}, \]

where \( z_j \) and \( t_j \) are the number of SDKs in the \( j \)-th SDK group and the time that group arrives at the ions. As a shorthand we will refer to this as the APG(\( N_k \)) scheme. The first stage of the optimisation is over the elements \( \{ z_1, z_2, \ldots, z_{N_k/2} \} \), with the timings set at constant intervals \( t_j = j T_G/N_k \). The second stage takes optimal
of pulse groups required for our optimisation to converge and is truncated to second order in pulse. This expression is based on a worst-case analysis for \( N \) \( \pi \) pulse errors reduce the fidelity as \( F \simeq |1 - N_p \epsilon|^2 F_0 \) \[20\]. Here \( F_0 \) is the ideal fidelity (\( \epsilon = 0 \)) and \( \epsilon \) is a characteristic error in the population transfer of a single ultra-fast \( \pi \) pulse. This expression is based on a worst-case analysis and is truncated to second order in \( \epsilon \), which is accurate for \( N_p \epsilon \ll 1 \).

The pulse errors place strong constraints on the maximum number of SDKs in the gate, so our global optimisation is first performed with tight bounds on the number of SDKs \( |z_j| \leq 1 \), and repeated for gradually loosened bounds up to \( |z_j| \leq 10 \). Of the resulting optimised pulse sequences, those with fewer pulses (smaller \( N_p \)) are found to be optimal when the effects of pulse errors are accounted for. We find that the minimal number of pulse groups required for our optimisation to converge to high-fidelity solutions is \( N_k = 16 \) for ions toward the edges of the chain, and \( N_k = 18 \) for ions closer to the middle. Somewhat surprisingly, we find this to be independent of the length of the chain for \( N > 6 \).

Restricting the number of pulses allowed in any given gate has the consequence of limiting the achievable gate speed for reasonable experimental repetition rates; we find that gate times between 0.7–1.2 \( \mu \)s are feasible with minimal numbers of pulses and repetition rates between 100 MHz–1 GHz. In terms of trap units, these gate times are roughly \( 4 \times (2\pi/\omega_t) \) for two-ion systems, and up to \( 0.2 \times (2\pi/\omega_t) \) in \( N = 100 \) ion chains in our model. For optimisations of gates in very large ion chains \( N \gtrsim 60 \), we find it is often optimal for the last SDK groups to contain no SDKs, effectively shortening the gate time by a factor of about 12.5% (11%) for \( N_k = 16(18) \) SDK groups. We stress that we can optimise for faster gate times than those we report in this manuscript, however only at the cost of allowing larger number of SDKs which in turn will require pulse control far beyond current experiments.

In Figure 2 we show the results of gate optimisations in this model for ion chains of different lengths. We find that even in chains of up to 100 ions the optimisation is able to find high-fidelity (above 99.9%) gate solutions with \( T_G \simeq 1 \mu \)s. We observe a difference in fidelities (in the absence of SDK imperfections) for gates in different locations in the chain: optimised gates for ions at the edge of the chain are best placed, and ions in the

![Simulated phase-space trajectories of the axial modes of a \( N = 5 \) ion chain during gate with 16 SDKs, for the two-qubit states \(|\uparrow\uparrow\rangle \) and \(|\uparrow\downarrow\rangle \). In the gate scheme, individual SDKs are grouped together so that some kicks can be made larger than others. The accumulated entangling phase is proportional to differences in areas enclosed by the \(|\uparrow\uparrow\rangle \) and \(|\uparrow\downarrow\rangle \) trajectories, summed for each motional mode. Here we represent each mode in position-velocity phase-space, rotating at its normal frequency (annotated). It is apparent that some of the trajectories do not close perfectly, which contributes to a non-zero infidelity of \( \sim 5 \times 10^{-5} \).](image-url)
middle of the chain are worst placed, with fidelities of about $1 - 10^{-7}$ and $1 - 10^{-3}$ in the large $N$ limit, respectively. Fig. 2 also shows that realistic gate fidelity (with pulse errors included) is roughly independent of the number of ions in the chain. In Fig. 3 we show that the achievable infidelity is roughly $1 - F \sim 10^2 \epsilon$; thus to realise high-fidelities of above $99.99\%$ pulse errors of about $\epsilon = 10^{-6}$ are required. This is beyond the capability of current experiments, however may be achievable in near-future experiments through the implementation of composite pulse sequences to improve the population transfer of the ultra-fast pulses. We discuss this notion further in the following section.

Notably these results are all for gates optimised for a repetition rate of $300\text{MHz}$, which is well within experimental feasibility [14,15]. In Figure 3 we show that repetition rates as low as $100 - 200 \text{MHz}$ are compatible with high-fidelity gate solutions for a long ion chain ($N = 60$). This is in stark contrast with the study of Ref. [1], where the authors reported the requirement of repetition rates $\sim 5 \text{GHz}$ to achieve high-fidelities in chains of $N > 5$ ions. This significant improvement is a key result of this manuscript and is in part due to the differences in their model from ours; the authors took the axial trap frequency $\omega_x$ to be fixed as the ion chain increases in length, as opposed to our more experimentally realistic model where we reduce $\omega_x$ and instead fix $\omega_y$. Moreover, this improvement can largely be attributed to the superiority of the two-stage protocol we use for gate optimisation, that has more degrees of freedom and fewer constraints on pulse sequences than previous schemes [20].

**EXPERIMENTAL ROBUSTNESS**

**Pulse errors.** We have shown in this manuscript that errors of at least $\epsilon = 10^{-5}(10^{-4})$ are needed to achieve fidelities above $90\%(99\%)$. To date, there have been only few experiments demonstrating single qubit control with ultra-fast pulses, with the state-of-the-art in pulse errors around $\epsilon = 5 \times 10^{-3}$ [16,17]. These pulse errors are typically due to intensity fluctuations of the laser; for square pulses $\epsilon \approx \Delta f/I$. We have previously commented [20] that replacing each $\pi$ pulse with a robust composite-pulse sequence [22] is a promising pathway to improving these errors. In particular, replacing each pulse with a four-pulse BB1 sequence that is insensitive to first and second order intensity fluctuations [23] can dramatically improve the precision of the SDKs in the gate. However, this approach is only effective and robust to phase-fluctuations if the constituent pulses are split from a larger parent pulse. Creating larger pulses will prove experimentally challenging for state-of-the-art lasers, and it will be necessary to reduce laser repetition rate to dedicate increased laser power to creating larger pulses given the significant improvements the BB1 sequence promises. In recent years, there has been a focus on improving laser repetition rates to enable pulsed fast gate schemes, however given that the gate solutions we present in this manuscript require low numbers of pulses and repetition rates $\sim 100 - 300\text{MHz}$, it is appropriate the focus is shifted to improving the transfer probability of the ultrafast $\pi$ pulses. Other prospects for improving these errors are the use of rapid-adiabatic passage [24], and pulse-shaping methods [25].

We assume the counter-propagating $\pi$ pulses composing each SDK are split from the same parent pulse, in which case laser phase cancels exactly. Therefore, this fast gate model is completely insensitive to phase fluctuations within the ultra-fast pulses, which can be a source of error for some continuous-pulse gate mechanisms [26].

**Coulomb non-linearity.** For the gates we propose, the maximum displacement from equilibrium due to SDKs is on the order of $10\text{nm}$. As this is two orders of magnitude
Cross-talk. A key experimental limitation to any computation with ion chains is excitation of neighbouring ions due to imperfect addressing of target ions, commonly referred to as ‘cross-talk’. In our model, we assume cross-talk can be made negligible by shelving all of the ions’ qubit states into a metastable/long-lived electronic state with a global envelope, and unshelving the target ions, prior to implementing the gate [27]. This can be done with microwave pulses with very high fidelity.

Hot motional states. This fast gate mechanism is insensitive to the initial motional state [6]. For the calculations in the manuscript we assume a thermal product state for each motional mode with an average phonon occupation of $\bar{n} = 0.1$, however these gates are robust to much higher occupations that are achievable with only Doppler cooling $\bar{n}=10$ [28]. This enables the use of the axial modes which are typically more challenging to cool near ground state. Notably, pulsed fast gate schemes are valid outside of the Lamb-Dicke regime $\eta^2(2(a^\dagger a) + 1) \ll 1$, and thus errors are not introduced by out-of-Lamb-Dicke effects which have dominated errors in continuous-pulse fast gates [13].

With regards to trap heating, previous studies have shown that gate fidelity is unlikely to be affected as long as a heating event does not occur during the gate operation [18], and thus the trap heating ultimately places a limit on the number of gates that can be performed in succession. However, as the gate speeds we have considered in this manuscript ($\sim 1\mu s$) are orders of magnitude faster than typical trap heating rates, there is a high ceiling for the total number of entangling gates that can be performed before improved laser control is required. If a heating event occurs before or after the gate operation, the fidelity will not be significantly affected as the motional state disentangles from the qubit state.

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