Adiabatic Inversion in the SQUID, Macroscopic Coherence and Decoherence

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A procedure for demonstrating quantum coherence and measuring decoherence times between different fluxoid states of a SQUID by using “adiabatic inversion” is discussed. One fluxoid state is smoothly transferred into the other, like a spin reversing direction by following a slowly moving magnetic field. This is accomplished by sweeping an external applied flux, and depends on a well-defined quantum phase between the two macroscopic states. Varying the speed of the sweep relative to the decoherence time permits one to move from the quantum regime, where such a well-defined phase exists, to the classical regime where it is lost and the inversion is inhibited. Thus observing whether inversion has taken place or not as a function of sweep speed offers the possibility of measuring the decoherence time. Estimates with some typical SQUID parameters are presented and it appears that such a procedure should be experimentally possible. The main requirement for the feasibility of the scheme appears to be that the low temperature relaxation time among the quantum levels of the SQUID be long compared to other time scales of the problem, including the readout time. Applications to the “quantum computer”, with the level system of the SQUID playing the role of the qbit, are briefly examined.

I. INTRODUCTION

One of the first and certainly probably the most discussed systems in which one tries to demonstrate coherence between apparently macroscopically different states is the SQUID [1].

In the last decade a number of beautiful experiments at low temperature [2] have seen effects connected with the quantized energy levels [3] expected in the SQUID, showing that it does in fact in many ways resemble a “macroscopic atom”. A fast sweeping method [4] has also seen the effects of these quantized levels even at relatively high temperature.

If we could further show in some way that the quantum phase between two fluxoid states of the SQUID, where a great number of electrons go around a ring in one direction or the other, is physically meaningful and observable it would certainly provide even more impressive evidence for the applicability of quantum mechanics on large scales and help put to sleep any ideas about the existence of some mysterious scale where quantum mechanics stops working.

At this meeting, results using microwave spectroscopy have indeed been reported showing the repulsion of energy levels expected from the quantum mechanical mixing of different fluxoid states [5]. These results indicate by implication that two opposite fluxoid states can exist in meaningful linear combinations and so that their relative phase is significant.

Here we would like to propose another method for demonstrating the coherence between opposite fluxoid states and the meaningfulness of the phase between them. Furthermore the method to be described allows a measurement of the “decoherence time”, the time in which the definite phase or the coherence between the two states is lost. This is interesting of itself since it never has been done and is also relevant to the “quantum computer” where decoherence is the main obstacle to be overcome.

Our basic idea is to use the process of adiabatic inversion or level crossing where a slowly varying field is used to reverse the states of a quantum system. In its most straightforward realization, our proposal consists of starting with the SQUID in its lowest state, making a fast but adiabatic sweep, and reading out to see if the final state is the same or the opposite fluxoid state. If the state has switched to the other fluxoid, the system has behaved quantum mechanically, with phase coherence between the two states. If it stays in the original fluxoid state, the phase coherence between the states was lost and the system behaved classically (see Figs. 3 and 4).

In order for these effects to be unambiguously observ-
able, relaxation towards thermal equilibrium must be small on the time scales involved in the experiment. As is discussed in section IV, this seems to be obtainable at low temperature with suitably chosen sweep times.

A very interesting aspect of the present proposal, as we shall discuss below, is the possibility of passing between these two regimes, quantum and classical, by simply varying the sweep speed. This allows us to obtain, for adiabatic conditions, the decoherence time as the longest sweep time for which the inversion is successful.

II. ADIABATIC INVERSION IN THE RF SQUID

Most workers in this field are probably accustomed to discussing level crossing problems such as ours in a Landau-Zener picture where one exhibits the spacing of the two crossing energy levels as some external parameter or field is varied. This is of course a perfectly good and useful picture for many applications. However I would like to urge consideration of another perhaps more intuitive visualization, one which is particularly well suited to time dependent problems as we have here, and which also has the advantage that it gives a complete representation of the state of the system at a given time and not just the level splitting.

This picture utilizes the fact that any two-state system may viewed as constituting the two components of a “spin” . Looking at this “spin” and its motions then provides an easy visualization, one which has been used in many contexts [6]. That is, if we have two states \( |L> \) and \( |R> \), which for us will be the two lowest opposing fluxoid states, then the general state

\[
\alpha|L>+\beta|R>
\]  

(1)

has the interpretation of a spin pointing in some direction. Even the relative phase is represented: the numbers \( \alpha \) and \( \beta \) are complex numbers and the spin points in different directions when we change their relative phase. Thus the spin visualization gives a complete picture of the state (up to an irrelevant overall phase).

To identify \( |L> \) and \( |R> \) here, consider (Fig. 1) the familiar double well potential \( V \) for the rf SQUID biased with an external flux \( \Phi_x \)

\[
U = U_0[1/2(\Phi-\Phi_x)^2 + \beta_L \cos \Phi].
\]  

(2)

The horizontal axis, the “position” coordinate of the system, is \( \Phi \) the flux in the SQUID ring. Depending on whether the system is in the left or right well, the flux through the ring has a different sign and the current goes around in opposite directions. The potential can be varied by altering the external \( \Phi_x \), becoming symmetric for \( \Phi_x = 0 \). We identify the lowest level in each well with \( |L> \) and \( |R> \). Quantum mechanical linear combinations of them result from tunneling through the barrier. In the adiabatic inversion procedure to be discussed, the external \( \Phi_x \) is swept from a maximum to an minimum value, passing through zero, such that the initially asymmetric left and right wells exchange roles, the originally higher well becoming the lower one and vice versa. The asymmetry of the configurations, however, is kept small so that we effectively have only a two-state system, composed of the lowest state in each well.

![Figure 1](https://via.placeholder.com/150)

**FIG. 1.** Double potential well with harmonic level spacing \( \omega_0 \) and initial spacing \( \epsilon \) between lowest levels.

Various influences such as the tunneling energy or the external flux may affect the orientation of the spin. Pursuing the analogy, these influences may be thought of as creating a kind of pseudo-magnetic field \( V \) which causes the spin to precess. Representing the spin by a “polarization” \( P \) we get the picture of Fig. 2. The equation governing the motion of \( P \) is

\[
\dot{P} = V \times P - DP_{tr},
\]  

(3)

where \( V \) can be time dependent. The quantity \( D \) is the decoherence parameter, which we neglect for the moment and will deal with below. Note that in the absence of \( D \), according to Eq [3] \( P \) cannot change its length, so the density matrix, which \( P \) parameterizes, retains its degree of purity. In using Eq [3] one assumes the temperature low enough so that relaxation processes like barrier hopping or jumps between principal levels may be neglected, thus there is damping of only the transverse components, \( P_{tr} \) (see below).

Now it is a familiar fact under adiabatic conditions, where \( V \) varies slowly, that the “spin” \( P \) will tend to “follow” a moving magnetic field \( V(t) \). This is a completely familiar procedure when rotating the spin of say an atom or a neutron by a magnetic field. If we wish, we can completely invert the state by having \( V \) swing from “up” to “down”.

In the present problem, we can create a moving \( V \) by sweeping \( \Phi_x \). This is because the vertical component of \( V \), \( V_{vert} \) corresponds to the difference in the two lowest energy levels. Hence we can induce a level crossing and reverse the potential wells (see Figs. 3 and 4) by reversing \( \Phi_x \) and so the direction of \( V_{vert} \). For small asymmetry of the wells the splitting is linear in \( \Phi_x \). Thus if \( \epsilon \) is the initial level splitting, obtained when \( \Phi_x = \Phi_x^{max} \), we can
FIG. 2. Precession of the “spin” vector $\mathbf{P}$ around the pseudo-magnetic field vector $\mathbf{V}(t)$. In adiabatic inversion $\mathbf{V}$ swings from “up” to “down” and carries $\mathbf{P}$ with it.

write $V_{\text{vert}}(t) = \epsilon(\Phi_x(t)/\Phi_{x,\text{max}})$. As $\Phi_x$ sweeps from its positive maximum value to its negative minimum value, $V_{\text{vert}}$ reverses direction.

In doing so, $V_{\text{vert}}$ passes through zero where $\mathbf{V}$ is horizontal with only a transverse component $V_{\text{tr}}$. $V_{\text{tr}}$ corresponds to the tunneling energy between the two quasi-degenerate states, $V_{\text{tr}} = \omega_{\text{tunnel}}$. As $V_{\text{vert}}$ passes through zero at $\Phi_x = 0$, the $|L\rangle$ and $|R\rangle$ states are strongly mixed and the splitting of the resulting energy eigenstates is determined by $V_{\text{tr}}$, alone, as is also familiar in the Landau-Zener picture. The magnitude of $V(t)$ at a given time, $|V| = \sqrt{V_{\text{vert}}^2 + V_{\text{tr}}^2}$ gives the instantaneous splitting of the two levels. This varies from approximately $\epsilon$ in the vertical position of $\mathbf{V}$ to $\omega_{\text{tunnel}}$ in the horizontal position.

Having identified the components of $\mathbf{V}$, our next task is to ascertain the meaning of “adiabatic” or “slow” for the motion of $\mathbf{V}$. Adiabatic conditions obtain when the time variation in question does not contain significant frequencies or Fourier components corresponding to the energy splitting between levels. Expressed in terms of time, the rate of variation of $\mathbf{V}$ should be slow on the time scale corresponding to the tunneling time between the two states. Thus we have the requirement on $\mathbf{V}$ that its relative rate of variation $\dot{V}/V$ always be small compared to $V$ itself. Since the varying component of $\mathbf{V}$ is $V_{\text{vertical}}$ (neglecting the indirect effect of $\Phi_x$ on $\omega_{\text{tunnel}}$), we require $V_{\text{vertical}}/V \ll V$. Thus taking the near degenerate configuration where $V \approx \omega_{\text{tunnel}}$ we find

$$\frac{\dot{\Phi}_x(t)}{\Phi_{x,\text{max}}} \approx \epsilon \omega_{\text{sweep}} \ll \omega_{\text{tunnel}}^2 \quad (4)$$

as the condition for adiabaticity.

If the adiabatic condition is violated then $\mathbf{P}$ cannot follow $\mathbf{V}$ (See Fig. 6 below). We stress that for adiabatic inversion there must be a well-defined quantum phase between the two states, and that when this phase is lost the inversion is suppressed.

FIG. 3. A successful inversion, starting from the upper figure and ending with the lower figure. The black dot indicates which state is occupied. The system starts in the lowest energy level and stays there, reversing fluxoid states. It behaves as a quantum system with definite phase relations between the two states.

In Figs. 3 and 4 we give a schematic representation of the whole procedure. In Fig. 3 a successful inversion takes place. Starting (upper sketch) with the system in the lowest state since we are at low temperature, a sweep is performed. After the level crossing has been performed (lower sketch), the system has reversed flux, remaining in the lowest energy state. In the visualization of Fig. 2, $P_{\text{vert}}$ has reversed direction. This is the behavior to be expected of a quantum mechanical system with well-defined phase relations.

In Fig. 4 we have the same starting situation but the inversion is unsuccessful. The system ends up in the same fluxoid state, $P_{\text{vert}}$ does not reverse. This is the behavior to be expected classically, when decoherence is significant and there is no well-defined quantum phase between the two states.

More detailed information, as well as intermediate cases will be discussed in terms of the behavior of $\mathbf{P}$, as shown in Fig. 5 and Fig. 6 below. For this a discussion of the role of damping or decoherence is necessary. This is the topic of the next section.

III. DAMPING

We now turn to a quantitative discussion of dissipative or damping effects, those effects tending to destroy the quantum coherence of the system; that is we consider the role of the $D$ term in Eq. [3]. While the vertical
component of $\mathbf{P}$ characterizes the relative amounts of the two states which are present in a probabilistic sense, the quantity $P_{\text{tr}}$ measures the degree of phase coherence between the two states. $D$ gives the rate of loss of this phase coherence.

One could also have included a damping parameter for the vertical component of $\mathbf{P}$. Instead of the loss of phase coherence, this would represent direct transitions from one well to the other. Such relaxation processes, like jumping the potential barrier due to thermal effects, or “radiative transitions” from one well to another with emission or absorption of some energy can be minimized by low temperatures and fast sweep times, as discussed in the next section.

In the present problem we are in particular interested in the effect of $D$ on the inversion process. With an increasing loss of phase coherence, we expect the situation to become more and more “classical” and finally when the $D$ is large, for the inversion to be inhibited. Indeed, in solving Eq. 8 in the limit of large $D$ one finds the inversion is strongly blocked and that one arrives in the “Turing-Watched Pot-Zeno” regime where $P_{\text{vert}}$ is essentially “frozen”. (See the lower right panels of Figs. 5 and 6).

These general expectations are confirmed by a numerical study of Eq. 8 with a moving $V(t)$. Figs. 5 and 6 show results of the numerical study. The horizontal axis represents the time, running from the beginning to the end of the sweep. The two curves in each picture represent $P_{\text{vert}}$ (labeled $P_3$) and $P_{\text{trans}}$. $P_{\text{trans}}$ is represented in absolute value, while $P_{\text{vert}}$ can change sign. In Fig. 5 an adiabatic sweep is performed. In the first panel, where $D = 0$, one sees the succesful inversion of $P_{\text{vert}}$ as it moves from +1 to -1. $P_{\text{trans}}$ passes through 1 as $\mathbf{P}$ passes through the horizontal position. In the next panel, where $D$ is increased somewhat, to 0.01, the picture is essentially unchanged. In the third panel where $D = 0.2$, decoherence takes effect since the decoherence time, given by $1/D$, is on the order of the sweep time. $P$ shrinks to zero during the sweep, showing the loss of phase coherence. Finally, with very large $D$, $P_{\text{vert}}$ evolves hardly at all during the sweep and $P_{\text{trans}}$ stays at zero. This final case is the “Turing-Watched Pot-Zeno” behavior.

We see three regimes for the result of the sweep: A) the system stays in the original state (classical case), B) goes to the opposite state (quantum case), or C) ends sometimes in one state and sometimes in the other (intermediate case). These correspond to $P_{\text{vert}}$ starting as +1 and ending as +1, -1, or 0, respectively. Or in terms of Fig. 5. A) corresponds to the last panel (lower right), B) to the first two panels, and C) to the middle panel (lower left).

We stress that all this arises simply from solving Eq. 8 and that no further notions or assumptions are necessary.

In Fig. 6 we show the effects of non-adiabaticity, using a sweep a factor of ten faster than in Fig. 5. Even with $D = 0$ the inversion is incomplete, with $\mathbf{P}$ keeping unit length, but ending up rotating roughly in the horizontal plane. As $D$ is turned on, $P$ shrinks and finally for very large $D$ the “Turing-Watched Pot-Zeno” behavior sets in.

Given adiabatic conditions, the important relation is that between the sweep speed and the decoherence time
1/D. As one sees from Fig. 5, there is no effective decoherence until 1/D is on the order of the sweep time. This relation determines if the system has time to "decohere" during the sweep. Roughly speaking, one enters the classical regime when $P_{tr}$, characterizing the phase coherence, has time to shrink during the inversion. This is very interesting for us, since it means, if the experimental situation is favorable, that we can pass from the quantum regime to the classical regime simply by varying the sweep speed.

The next two parameters concern the extent and speed of the sweep. The initial asymmetry $\epsilon$ (Fig. 1) should be small compared to $\omega_0$ in order to retain the approximate two-state character of the system, but large compared to $\omega_{tunnel}$ to avoid initial mixing of the two states, say $\epsilon/\omega_0 \sim 10^{-1}$ to $10^{-2}$. The speed of the sweep, $\omega_{sweep}$ will be the easiest experimental parameter to control, and the behavior of the results as $\omega_{sweep}$ is varied will be an important check on the theory. It must not be so fast as to lose adiabaticity, but not so slow as to allow relaxation processes to mask the results. We may suppose it to be in the range $10^{-3} \omega_0 - 10^{-4} \omega_0 = 10 - 100$ MHZ. With these numbers it seems possible to satisfy the adiabatic condition Eq [3] on the one hand and to sweep fast relative to the relaxation time (see below) on the other.

We now come to the dissipative parameters: $D$, and $\omega_{relax}$, the relaxation rate for transitions among the SQUID levels. The calculation of dissipative effects in the SQUID is usually approached in terms of the Caldeira-Leggett model [1], where a coupling to a pseudoboson field, related to the resistance of the device, represents the dissipative effects. The distinction between relaxation and decoherence in the SQUID is principally a question of energy scale. For the former, influences (e.g. the pseudobosons), involving jumps between levels, energies on the order of the level splitting or more are involved. For the latter, where there is only a "dephasing", low energies, those below the level splitting, are important. In general, of course, both processes are present, but at low enough temperature relaxation should become small. For this reason the damping term in Eq [3] is taken to only affect the transverse components of $\mathbf{P}$.

Weiss and Grabert [10], have given a calculation of the effects of dissipation on coherence, and we may identify their "decay rate" at weak dissipation with $D$. This gives the estimate $D = D/Re^2$ (their $\Gamma$). For $R = 5 M\Omega$, we find $D = 0.08 mk = 9.6$ MHZ at $T = 100$ mk, and $D = 0.008 mk = 960$ kHZ for $T = 10$ mK. (Units: $1 K = 8 meV = 120$ GHZ, $1/e^2 = 4 k\Omega$). With these estimates, say 1-10 MHZ, we are in an interesting range since as mentioned, this offers the interesting possibility of being able to choose sweep speeds either slow or fast with respect to the decoherence time, while still retaining the adiabatic condition. The resulting switches between classical and quantum behavior would provide persuasive evidence for the correctness of our general picture, and allow the measurement of $D$ and its temperature dependence.

Our last parameter is the relaxation rate $\omega_{relax}$, characterizing, as said, the rate of conventional kinetic relaxation processes. We wish this to be small for two reasons. One is that relaxation should not take place during the sweep, which would obviously obscure our effects. Secondly, relaxation should also be small during the readout time, otherwise for example, the final configuration of Fig. 4 might turn into that of Fig. 3 before it could be detected.

Here we may use calculations which have been made in

IV. PARAMETER VALUES

We now discuss some some typical parameters of our system. In particular since we envision working at time scales shorter than have been common in this field, it may be useful to give a qualitative discussion of the various time scales involved. The highest frequency or shortest time present is the ordinary harmonic frequency $\omega_0$ (Fig. 1) giving the approximate level spacing in the well. For typical conditions this spacing may be on the order of several Kelvin ($K$), or some hundred GHZ. Since we suppose working in the Kelvin to milliKelvin range, this implies that for the equilibrium system at the start of the sweep only the lowest level is populated. The next parameter is the tunneling frequency through the barrier. This is a sensitive function of the SQUID parameters, with sample values $|\beta_L| = 1.19$, $C = 0.1 pF, L = 400 \mu H, R = 5 M\Omega$ we have $\omega_{tunnel} \approx 600$ MHZ. This corresponds to $\omega_{tunnel}/\omega_0 \sim 10^{-2}$, the tunneling introduces small energy shifts on the scale of the principal level splitting.
connection with the tunneling to the voltage state \cite{11}. The rate one finds depends on the separation of the states in question. If we call \( \omega_{\text{relax}} \) the value with the level separation at the beginning or end of the sweep we obtain in our example \( \sim 20 \text{ kHz} \). This characterizes the time in which the readout must occur.

During the sweep itself the level spacing is changing. Taking this into account we obtain the curve of Fig. 7, which shows the probability of an inter-level jump as a function of \( \omega_{\text{sweep}} \). It appears that for sweeps shorter than a microsecond relaxation is indeed negligible.

To indicate the general relationship of the various time scales, the frequencies \( \omega_{\text{relax}} \), \( \omega_{\text{dec}} = 1/D \) and \( \omega_{\text{tunnel}} \) are marked for the horizontal axis. As may be seen, we may indeed be able to obtain the favorable situation of being able to sweep slow or fast relative to the decoherence time, while retaining small relaxation.

![Fig. 7. The curve shows the probability of relaxation during a sweep, as a function of the sweep frequency (inverse sweep time). Important time scales or frequencies are marked for the parameters indicated. It appears possible to have sweeps either slow or fast relative with respect to the decoherence time, while retaining small relaxation.](image)

Concerning the readout, we have examined a scheme involving a switchable flux linkage to a DC SQUID, which would be sufficiently fast for the above estimates \cite{12}. Here one profits from the fact that observation of the system is only necessary after the procedure is completed.

V. NONADIABATIC PROCEDURES

In general, we need not necessarily limit ourselves to slow, adiabatic processes. There is, say, the opposite case of the “sudden approximation” where \( \mathbf{V} \) is changed very quickly and \( \mathbf{P} \) tends to stays put, as was beginning to happen in Fig. 6. This could be of interest for example, if we find experimentally that we always get an inversion (Fig. 3), and never a failed inversion (Fig. 4) for the range of parameters available to us operationally. There could be two reasons for this behavior. It could be a true result in the sense that the decoherence is very small; we are always in the quantum situation and the adiabatic inversion always works.

But we might worry that instead we have a rapid relaxation from the upper state to the lower one after or during the sweep. That is, perhaps we really had no inversion and Fig. 4 applies. The relaxation rate is greater than we think and the system just falls back to the lowest energy state with the emission of some energy before we can read out.

A way to clarify the situation would be to start our adiabatic inversion procedure from the upper state. Then a successful inversion means we end in the upper state. If relaxation was not the problem, we should then detect the upper state. On the other hand if relaxation is significant, we should always end in the lower state, regardless of where we start.

Since at low temperature everything is in the lowest state, in order perform this check we need a method of getting the system into the upper state at the start of the sweep. We can accomplish this by a sudden, nonadiabatic, reversal of \( V_{\text{vertical}} \), that is, of the applied flux. Since the wavefunction or \( \mathbf{P} \) will (approximately) not change, we will have the system in the upper state to start with. We now proceed with the adiabatic sweep as before and we can check if the results were due to relaxation or true quantum coherence.

Other interesting configurations and procedures can undoubtedly be arrived at by combining various operations in this way.

VI. QBITS AND THE QUANTUM COMPUTER

The two-state system under discussion here suggests itself as a physical embodiment of the “quantum computer”. The “qbit” itself is naturally represented by \( L \) and \( R \) playing the role of 0 and 1. A linear combination of \( L \) and \( R \) may be created by adiabatically rotating \( \mathbf{P} \) from some starting position. Adiabatic inversion is evidently an embodiment of NOT since it will turn one linear combination into another one with the weights of \( L \) and \( R \) interchanged. As for CNOT, the other basic operation, a NOT is performed or not performed on a “target bit” according to the state of a second, “control bit”, which itself does not change its state. One straightforward realization of this would be to perform the NOT operation as just described in the presence of an additional linking flux supplied by a second SQUID nearby. The magnitude and direction of this linking flux supplied by a second SQUID nearby. The magnitude and direction of this linking flux supplied by a second SQUID nearby.
SQUID systems like these would seem to be particularly well suited for the embodiment of the quantum computer, where we wish to generate a series of unitary transformations for the various steps of computation. This may be done by creating a “moving landscape” of potential maxima and minima, as in our simplest one-dimensional example of the adiabatic inversion. This imaginary landscape can be produced and manipulated by controlling various external parameters (as with our sweeping flux), performing the various operations in one physical device. Naturally, practicality will depend very much on the relation between the speed of these operations and the decoherence/relaxation times which we hope to determine by the present methods.

Although our interest here has been the SQUID, it will be evident that the principle of determining decoherence times through the inhibition of adiabatic inversion could be applied to many other types of systems as well.

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