New model averaging approach in predicting mortality rate of intensive care unit patients

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Abstract. Model Averaging (MA) is one of the well-known statistical modelling approach to produce a fitted model in applied research. Even though it was proposed to overcome underestimation of parameter estimates issues in Model Selection (MS), the final best model of MA includes insignificant variables. The goal of this research is to propose an New Model Averaging (NMA) method which is based on MA approach with elimination of insignificant variables. Data of Intensive Care Unit (ICU) was studied to highlight the most influential factor of mortality rate. ICU is commonly associated with a high mortality rate due of its complexity of treatments. The guidelines of NMA method on ICU patient’s data were presented and the models obtain were compared using 10-Fold Cross-Validation. The results reveals that the performance of NMA is slightly better than MA. The most significant factors for mortality of ICU patients were concluded to be patient’s age, SAPS II score discharge and whether or not the patients use ventilation machine. In conclusion, the study showed that the elderly patients have a greater risk of mortality after discharge from the hospital and SAPS II score provide a good indication in predicting hospital mortality.

1. Introduction

Model-building approach is one of the key areas of interest in the application of statistical modelling. The goal of statistical modelling is to formed a model that will helps in making conclusion in order to solve problems as well as making prediction. Best model should have a good fit and summarize the data as close as possible in order to explain the relationship in the data. Model Averaging (MA) is a common statistical modelling tools which aims to overcome Model Selection (MS) issues regarding underestimation of parameter estimates. MS eliminates insignificant variables to produce a fitted model. Unlike MS, MA formed best model by decreasing the estimates of a weaker variables by averaging the weights of all possible models. Since MA will include all covariates being studied regardless of its significant in the final best model, this research will show a new approach of MA (NMA) which is based on MA approach but with elimination of insignificant variables.

For illustration purpose, data of Intensive Care Unit (ICU) patients were studied using both MA and NMA. In addition, two model selection criteria (MSC) were compared to determine which MSC works best in MA and NMA method.

ICU was recorded as unit with highest mortality according to [1] compared to other unit in a hospital. The average of mortality rate in United States (US) were ranges from 8% to 19%, or about 500,000 deaths per year. ICU is one of the unit where medical errors are most likely to occur due to its complexity.
of care [2]. This research summarizes the causes that influenced the patient’s survivability by using NMA approach and the whole procedures of obtaining the best model were explained step by step to provide a clear guideline of model-building.

2. Methodology

2.1. Multiple Binary Logit

Multiple Binary Logit (MBL) is another term for Logistic Regression. It is a commonly known method for modelling data with binominal outcome, which is presented as binary indicator taking on values 0 and 1 [3]. The general model of MBL [4] is

\[ Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \cdots + \beta_q X_{qi} + u_i \]  

where the binary dependent variable is denoted by \( Y \), \( X_j \) is the \( j^{th} \) independent variable where \( j = 1, 2, \ldots, q \), \( \beta_j \) is the \( j^{th} \) coefficient of \( j^{th} \) independent variable where \( j = 1, 2 \ldots q \), \( u \) is the random error of the model and \( P_i \) is the estimated probability of event occurs.

\[ \hat{P}_i = \frac{\exp(\beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \cdots + \beta_q X_{qi})}{1 + \exp(\beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \cdots + \beta_q X_{qi})} \]  

Unlike Multiple Regression, MBL will present the results of success/failure in forms of estimated probability where it takes values between 0 and 1. As an example, a probability of 0.80 means that there is 80% chance of outcome 1 (success) to occur and vice versa.

2.2. Intensive Care Unit Patient’s Dataset

Data of ICU patients collected from Hospital Sultanah Aminah (HSA) in Johor were studied to illustrate the model-building process. The data was collected from January 2001 until August 2002 with sample of 753 patients. The sample consist of seven independent variables with binary dependent variables. The focus is to formed a model where the covariates effect the survivability of ICU patient’s so that the best model will be able to pinpoint the most influential factor affecting response variable (\( Y \)).

| Variable | Descriptions |
|----------|--------------|
| \( Y \)  | Survival of ICU Patients:  |
|          | 1 = Alive     |
|          | 0 = Died      |
| \( X_1 \)| Age (Patient’s Age) |
| \( X_2 \)| S2sadm |
|          | SAPS II score during the first 24 hours in the wards (SAPS II score admit) |
| \( X_3 \)| S2disc |
|          | SAPS II score during the discharge from the ward/hospital (SAPS II score discharge) |
| \( X_4 \)| Race (Patient’s Race):  |
|          | 1 = Malay |
|          | 2 = Chinese |
|          | 3 = Indian |
|          | 4 = others races |
| \( X_5 \)| Sex (Patient’s Gender): |
|          | 1 = Male  |
|          | 2 = Female |
| \( X_6 \)| Organfail (Organ failure before and during the treatment in the ICU): |
|          | 1 = no organ failure |
|          | 2 = at least one organ fails |
| \( X_7 \)| Comorbid (Existence of Comorbid disease before being treated in the ICU): |
|          | 1 = not suffer from comorbid disease |
|          | 2 = suffer at least one comorbid disease |
| \( X_8 \)| Mecvent (Patients using mechanical ventilator machine): |
|          | 1 = No |
|          | 2 = Yes |

2.3. Model Selection Criteria
MSC is needed in MS to rank a set of possible models in order to pick the best model whereas in MA, it is needed to compute the weight for each possible model. In this paper, the performance of Corrected Akaike Information Criteria, \(AICc\) by [5] and Bayesian Information Criteria, \(BIC\) [6] were explored. \(AICc\) is an adjusted version for Akaike Information Criteria (\(AIC\)) [7] which is intended to overcome the problem of small sample size. \(AICc\) and \(BIC\) can be computed as follows [5] and [6].

\[
AICc = -2\log L(M) - 2p \frac{n}{n-p-1} \tag{3}
\]

\[
BIC = -2\log L(M) - p[\log(n)] \tag{4}
\]

where \(L(M)\) is the minimum value for likelihood function of model \(M\), \(n\) is the number of observation, \(p\) is the number of parameter and \(I_m\) is type of MSC. Different MSC were compared to identify which will produce model with minimum error. Model with minimum error indicate the best model.

2.4. Model Averaging

2.4.1. Traditional Model Averaging

MA is an alternative MS which intended to overcome the underestimation of standard errors that is a consequence of MS. Unlike MS, there are no elimination of insignificant variables in MA approach. Instead, MA will shrink the estimates of a weaker variables by averaging weights of all possible models. The weight is constructed so that the final model averaging estimator is optimal with respect to minimizing a Mallows criterion, Mean Square Error or other meaningful criteria [8,9,10].

The procedure of obtaining best model using MA begin by listing all possible models build by using combinations of variables. The total number of all possible models with no interaction variable is [11].

\[
M = \sum_{j=1}^{q} (\binom{q}{C_j}) = \frac{q!}{j!(q-j)!} \tag{5}
\]

where \(M\) is the total number of all possible models, \(q\) is the number of single independent variables and \(j = 1, 2, ..., q\). Next step is to compute weights for all possible models. MA aims to incorporate the estimates of potentially good model by averaging the weights of all possible models to produce a robust estimator for the best model. According to [9], by combining the estimates over set of candidate models, issue regarding uncertainty associated with MS is solved. The weight, \(W_m\) is

\[
W_m = \frac{\exp\left(I_m\right)}{\sum_{m=1}^{M} \exp\left(I_m\right)} \tag{6}
\]

where \(m\) is possible models, \(m = 1, 2, 3 ..., M\) and \(I_m\) is the type of model selection criterion (\(AICc\) or \(BIC\)). The coefficient estimates for each covariates based on \(W_m\) is

\[
\hat{\beta}_p = \frac{\sum_{m=1}^{M} W_m \hat{\beta}_{p,m}}{\sum_{m=1}^{M} W_m} \tag{7}
\]

where \(\hat{\beta}_{p,m}\) is the estimate of \(\beta_p\) under model for \(m = 1,2,..., M\). Coefficient estimates are computed for each covariate to form the best model. Best model is obtain after all coefficients for each covariates is computed.

To ensure the best model reliability, Pearson Chi-Square goodness-of-fit test and Deviance goodness-of-fit test as suggested by [4] will be carried out. Three scatter plots of residuals which are ordinary residual against estimated probability, Pearson residual against estimated probability and Deviance residual against estimated probability will be plotted to support the results of goodness-of-fit test. Residual scatter plot for best model should approximately result in horizontal line with zero intercept.
The procedure for goodness-of-fit test is based on guidelines explained in [12]. The process of model-building using MA as discussed by [13] is summarised as in figure 1.

2.4.2. New Model Averaging
Model-building procedure using MA is simple and convenient (as in figure 1). Despite from that, best model produced by this modelling method will include all covariates regardless of its significant. According to [14], minimization of covariates in the final best model will results in numerical stability and reliable results. MS is another well-known approach of statistical modelling which aims to eliminates insignificant variables. Even so, the model-building flow is lengthy and time consuming when compared with MA. The reliability of the result was also debated as MS is known for introducing additional uncertainty into the model-building process and to overcome this issue, MA was proposed.

Therefore, both modelling methods had disadvantage in producing best model. To solve this issue, this study will combine both MA and MS approach to produce practical model-building approach which incorporate best of both methods. The model build using the proposed New Model Averaging (NMA) method will consist of only significant variables where each estimates are formed based on weights of potentially good model.

Model-building using NMA is almost similar with traditional MA except for the removal of insignificant variables. After model was obtain from the combination of computed coefficients,
independent variable with p-value larger than 0.05 which indicate insignificant variable will be eliminated one by one. This process only allows one insignificant variable to be eliminated at a time and after the removal, the process of obtaining model will be rerun until best model with only significant variable is obtain. The flow of model-building using alternative method is simplified as in figure 2.

2.5. Accuracy Measure using 10-Folds Cross Validation

To compare the accuracy of predictive model produced by traditional MA and alternative method, 10-folds Cross Validation were computed. According to [15] internal validation is applied to a predictive model to determine how well the performance of the model in other data sets from a similar population and to estimate the expected prediction error of model. In this study, the expected prediction error used were Root Mean Square Error (RMSE), Mean Absolute Error (MAE) and Mean Square Error (MSE). Equation (8), (9) and (10) presents the formula for prediction error as suggested by [16].

\[
RMSE = \sqrt{\frac{\sum_{i=1}^{N}(Y_i - \hat{Y}_i)^2}{N}} \tag{8}
\]
\[
MAE = \frac{\sum_{i=1}^{N}|Y_i - \hat{Y}_i|}{N} \tag{9}
\]
\[
MSE = \frac{\sum_{i=1}^{N}(Y_i - \hat{Y}_i)^2}{N} \tag{10}
\]

where, \( N \) is the total number of sample, \( Y \) is the actual value of dependent variables and \( \hat{Y} \) is the estimated value of \( Y \).

3. Data Analysis

A complete explanation on the illustration of MA approach including goodness of-fit test was covered in [13]. The steps involved in obtaining best model using NMA is based on figure 2.

**Step 1:** All Possible Models

Since there are eight independent variables in ICU patient’s dataset, the total number of all possible models were computed using equation (1).

\[
M = 1^{(8)} + 1^{(8)} + 1^{(8)} + 1^{(8)} + 1^{(8)} + 1^{(8)} + 1^{(8)} = 255 \text{ possible models}
\]

**Step 2:** Compute Weights

The weights are computed for each possible model based on \( AIC_c \) and \( BIC \) values. Table 2 and table 3 shows some of the weights for possible models.

| Possible Models | \( AIC_c \) | Weight |
|-----------------|------------|--------|
| \( \hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_3 + \hat{\beta}_6 X_8 \) | -6.59 | 0.09 |
| \( \hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_3 + \hat{\beta}_4 X_4 + \hat{\beta}_6 X_8 \) | -6.3 | 0.08 |
| \( \hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_3 + \hat{\beta}_7 X_7 + \hat{\beta}_6 X_8 \) | -6.27 | 0.08 |
| \( \hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_3 + \hat{\beta}_4 X_4 + \hat{\beta}_7 X_7 + \hat{\beta}_6 X_8 \) | -6.26 | 0.08 |
| \( \hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_2 + \hat{\beta}_3 X_3 + \hat{\beta}_6 X_8 \) | -5.44 | 0.05 |
| \( \hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_3 + \hat{\beta}_4 X_4 \) | -0.33 | 0.00 |
Table 3. Weights based on BIC

| Possible Models | $BIC$ | Weight |
|-----------------|-------|--------|
| $\hat{Y} = \beta_0 + \beta_1 X_1 + \beta_3 X_3 + \beta_8 X_8$ | 16.45 | 0.52 |
| $\hat{Y} = \beta_0 + \beta_1 X_1 + \beta_3 X_3$ | 18.13 | 0.23 |
| $\hat{Y} = \beta_0 + \beta_1 X_1 + \beta_3 X_3 + \beta_4 X_4 + \beta_8 X_8$ | 21.33 | 0.05 |
| $\hat{Y} = \beta_0 + \beta_1 X_1 + \beta_3 X_3 + \beta_7 X_7 + \beta_8 X_8$ | 21.36 | 0.05 |
| $\hat{Y} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_8 X_8$ | 22.19 | 0.03 |
| $\hat{Y} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_7 X_7$ | 27.14 | 0.00 |

Step 3: Compute Coefficients
The estimated coefficients $\hat{\beta}$ can be obtained by using equation (3). For example, $\hat{\beta}_0$ can be computed as

$$\hat{\beta}_0 = \frac{\beta_{(0,1)W_1} + \beta_{(0,2)W_2} + \beta_{(0,3)W_3} + \cdots + \beta_{(0,255)W_{255}}}{W_1 + W_2 + W_3 + \cdots + W_{255}}$$

Since,
$$\sum_{m=1}^{M} W_{im} = 1$$

Hence,

$$\hat{\beta}_0 = \beta_{(0,1)W_1} + \beta_{(0,2)W_2} + \beta_{(0,3)W_3} + \cdots + \beta_{(0,255)W_{255}}$$

Each coefficient estimated were calculated based on coefficient value as well as weight obtain from each possible model. The final best model was formed using combination of each coefficient estimate computed in this step.

Step 4: Obtain Best Model
The best model was obtained after all the coefficients $\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2, \ldots, \hat{\beta}_8$ were calculated. The full model of ICU patients is shown in table 4.
Step 5: Elimination of Insignificant Variables

Table 5. Elimination of Insignificant Variables in Model NMA using $AIC_c$

| Variables in $MA(AIC_c)$ | Elimination 1 | Elimination 2 | Elimination 3 | Elimination 4 | Elimination 5 | Elimination 6 |
|-------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $X_1$                   | 4.15e-05      | 4.14e-05      | 3.86e-05      | 2.79e-05      | 1.8e-05       | 1.56e-05      |
| $X_2$                   | 0.6685        | 0.6687        | 0.6608        | -             | -             | -             |
| $X_3$                   | 2.00E-16      | 2e-16         | 2e-16         | 2e-16         | 2e-16         | 2e-16         |
| $X_4$                   | 0.4954        | 0.4951        | 0.4966        | 0.4994        | 0.517         | -             |
| $X_5$                   | 0.9736        | -             | -             | -             | -             | -             |
| $X_6$                   | 0.9117        | 0.9119        | -             | -             | -             | -             |
| $X_7$                   | 0.5105        | 0.5099        | 0.5099        | 0.5082        | -             | -             |
| $X_8$                   | 0.0111        | 0.0111        | 0.0113        | 0.0134        | 0.017         | 0.0151        |

Table 6. Elimination of Insignificant Variables in Model NMA using $BIC$

| Variables in $MA(BIC)$ | Elimination 1 | Elimination 2 | Elimination 3 | Elimination 4 | Elimination 5 | Elimination 6 | Elimination 7 |
|------------------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $X_1$                  | 2.22E-05      | 2.22e-05      | 2.2e05        | 2.05e-05      | 1.78e-05      | 3.31e-06      | 3.31e-06      |
| $X_2$                  | 0.9010        | 0.901         | 0.901         | -             | -             | -             | -             |
| $X_3$                  | 2.00E-16      | 2e-16         | 2e-16         | 2e-16         | 2e-16         | 2e-16         | 2e-16         |
| $X_4$                  | 0.8100        | 0.810         | 0.810         | 0.810         | 0.811         | -             | -             |
| $X_5$                  | 0.9870        | -             | -             | -             | -             | -             | -             |
| $X_6$                  | 0.9750        | 0.975         | -             | -             | -             | -             | -             |
| $X_7$                  | 0.8300        | 0.830         | 0.830         | 0.830         | -             | -             | -             |
| $X_8$                  | 0.1920        | 0.192         | 0.192         | 0.196         | 0.200         | 0.198         | -             |

Table 5 and 6 summarizes the elimination process for both best models in table 4. The elimination of insignificant variables shows removal of five insignificant variables in Model using $AIC_c$ whereas six insignificant variables were omitted in model obtained using $BIC$. The best models formed using NMA is as in table 7.
Table 7. Best Model using New Model Averaging

| Method        | Model                                                                 |
|---------------|----------------------------------------------------------------------|
| NMA using $AIC_c$ | $\hat{Y} = 1.0048 + 0.00207X_1 - 0.01168X_3 + 0.05812X_8$          |
| NMA using $BIC$  | $\hat{Y} = 1.02086 + 0.00224X_1 - 0.01193X_3$                     |

Table 8. Best Model for all Approach and Accuracy Measures

| Method        | Full Model                                                                 | RMSE | MSE  | MAE  |
|---------------|---------------------------------------------------------------------------|------|------|------|
| MA using $AIC_c$ | $\hat{Y} = 1.02152 + 0.00219X_1 + 0.00020X_2 - 0.01175X_3 -0.00647X_4 - 0.00030X_5 + 0.00149X_6 -0.01257X_7 + 0.06199X_8$ | 0.239| 0.145| 0.058|
| MA using $BIC$  | $\hat{Y} = 1.01200 + 0.00213X_1 + 0.00002X_2 - 0.01176X_3 -0.00111X_4 - 0.00005X_5 + 0.00015X_6 -0.00180X_7 + 0.04320X_8$ | 0.239| 0.144| 0.058|
| NMA using $AIC_c$ | $\hat{Y} = 1.0048 + 0.00207X_1 - 0.01168X_3 + 0.05812X_8$                      | 0.239| 0.141| 0.058|
| NMA using $BIC$  | $\hat{Y} = 1.02086 + 0.00224X_1 - 0.01193X_3$                               | 0.239| 0.143| 0.058|

Table 8 summarize the accuracy measures after 10-fold cross-validation for all methods. The results reveals that the performance of the proposed method is almost similar with traditional MA method. Even so, model produced using NMA show a slightly better performance in forms of MAE value, with difference range of 0.002 to 0.004 when compared with other models. From the results in Table 8, model formed using NMA($AIC_c$) is chosen as the best model of ICU patients. The final best model is

$$\hat{Y} = 1.0048 + 0.00207(Age) - 0.01168(S2disc) + 0.05812(Mecvent)$$

By using equation (2), best model showed that when the values for all the variables in the final best model above are 0, the probability of ICU patient’s survivability is

$$P_I = \frac{\exp^{1.0048}}{1 + \exp^{1.0048}} = 0.7320 \approx 0.73$$

One-unit increase in Age ($X_2$), will decrease the probability of patient’s survivability by 0.00207. Similarly, the probability of patient's survivability also will decrease if there is an increase in Mecvent ($X_3$). The probability of patient's survivability increases by 0.01168 when the SAPS II score discharge increases.

4. Conclusion and Discussion
This research focus on illustrating the guidelines of MA and NMA on medical data set where the dependent variable is the ICU patient’s survivability. The guidelines can be applied in any study involving binary dependent variable. The results conclude that model based on NMA approach shows a slightly better performance compared to traditional MA. Besides that, when researchers aim is to pinpoint the factors affecting the dependent variables, NMA approach has an advantage because the best model using MA will incorporate all covariate being studied even though it is not significant. NMA was
proposed as an alternative to model averaging, which will eliminate insignificant variables, making it easier for researchers to make conclusion.

The factors with high risk contributing to patient’s mortality are Age ($X_1$), S2sdisc ($X_3$) and Mecvent ($X_8$) are obtained through NMA model. The elder patients (age more than 60 year-old) have higher mortality rate when admitted in ICU and age is a significant factor in survivability of ICU patients. About two-thirds of non-elderly survived until they are discharged from the hospital. Research conducted by [19] also agreed that elder patients have greater risk of mortality after discharge from the hospital. S2sdisc or SAPS II score discharge is a scoring system used to predict risk of hospital mortality after patient’s is discharge and is proven to provide good calibration in predicting hospital mortality [17]. Mecvent is a system that was used for assessment of respiratory mechanics [18]. In this research, it describes the use of ventilation machine on patient. [19] clarified that mechanical ventilation as a good marker of hospital mortality.

In conclusion, NMA shows to have advantages over traditional MA as it produces best model with better performance and provide a good solution when the researcher’s aim is to highlight the most significant factor, as NMA approach eliminates the non-contributing covariates in the final best model. The performance of NMA and MA does not vary much as NMA is build based on MA approach. Although this research has achieved its original aim, the study was restricted with interaction between continuous variables. Interaction between categorical variables will introduces additional challenges. Therefore, the interaction between categorical variables can be explored. In addition, the analysis become more complicated with presence of missing data. Thus, the application of NMA should be explored for incomplete data.

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