Calculations of Quadrupole Deformation Parameters for Nuclei in fp shell
Ahmed H. Ali,¹ Samah O. Hassoon,² Haitham T. Tafash ³

¹Dept. Of Medical Physics, College of Applied Sciences, University of Fallujah

²Faculty of agriculture, University of Kufa, Iraq

³College of medicine, University of Fallujah, Iraq

Corresponding Author E-mail: dr.ahmedphysics@uofallujah.edu.iq

Abstract. The quadrupole transition rates and intrinsic quadrupole moment are calculated for some fp shell nucleus. For fp-shell nuclei with neutron number \( N > 20 \) and with proton number \( p > 20 \), fp shell model space and fpd6 effective interaction are used. The calculations are performed using effective charges obtained from Bohr – Mottelson model and also the standard proton and neutron effective charges, and the results are compared with the available experimental values. The results of the quadrupole deformation parameter calculated from transition probabilities are better from the intrinsic quadrupole moment with ST effective charges.

1. Introduction

The shell model built at the end of 1940-th, having explained great amount of data associated with the ground and weakly excited states of atomic nuclei, faced substantial problems. Particularly, the measured values of quadrupole momenta for several nuclei appeared to be much higher the same values calculated due to the shell model [1].

The nuclear deformation can be linked to the shell structure of single particle levels in a spherical potential [2].

The spherical nucleus shape becomes less and less stable when the number of particles or holes in the unfilled shells is increased [3]. The concept ideas concerning non spherical nuclei have been most completely described by A. Bohr [4]. The elongation of the nucleus is related to the interaction between the nucleons outside closed shells and the surface. The deformation of nucleur characterized by the moment of inertia around the axis vertical to the symmetry axis of the nucleus, its electric quadrupole moment its and magnetic dipole moment.

Richter et al. [5] studied similar approaches to determine the four single particle energies of the fp shell and the195 two-body matrix elements. In one approach a semiempirical interaction was obtained by using one-boson exchange potentials plus core polarization correction as in Ref. [6]. The final interaction obtained in this study was denoted FPD6 in Ref. [5]. The symbol \( \beta_2 \) to indicate reduced quadrupole deformation parameter calculated from transition probabilities in contrast to \( \delta \), which indicates reduced quadrupole deformation parameter calculated from the intrinsic quadrupole moment \( Q_0 \). The quadrupole deformed nuclei are labelled as prolate \( (Q_0 > 0) \) or oblate \( (Q_0 < 0) \) and are spherical nuclei when the quadrupole momemt equal zero \( (Q_0=0) \). For the light nuclei, the deformation parameters are somewhat larger. The odd \(-\)A and odd-odd nuclei are found to have shapes similar to neighboring even-even nuclei [5].
2. Theory

The electric matrix element related between the initial state nuclear and final state nuclear is [7]:

\[ M(EJ) = \left| \langle J_f \| \sum_k e(k) \hat{Q}_j(\vec{r}_k) \| J_i \rangle \right|. \]  

(1)

where \( e(k) \) is the electric charge for the k-th nucleon and \( e(k) = 0 \) for neutron. The addition of a valence neutron will induce polarization of the core into configurations outside the adopted model space. The reduced electric matrix element can be written in terms of the proton and neutron

\[ M(EJ) = \sum_{t_z} e_{eff}(t_z) \langle J_f \| \hat{O}_j(\vec{r}_z, t_z) \| J_i \rangle \]  

(2)

where \( \langle J_f \| \hat{O}_j(\vec{r}_z, t_z) \| J_i \rangle \) is the electric matrix element which is expressed as the sum of the products of the one-body density matrix \((OBDM)\) times the single-particle matrix elements,

\[ \langle J_f \| \hat{O}_j(\vec{r}_z, t_z) \| J_i \rangle = \sum_{j,j'} OBDM(J_i, J_f, J_z, t_z, j, j') \langle j' \| \hat{O}_j(\vec{r}_z, t_z) \| j \rangle \]  

(3)

where \( j \) and \( j' \) denote single-particle states for the shell model space.

The matrix element can be written in terms by assigning effective charges \( e_{eff}(t_z) \) to the protons and neutrons which are out closed shell,

\[ M(EJ) = \sum_{t_z} e_{eff}(t_z) \langle J_f \| \hat{O}_z(\vec{r}_z, t_z) \| J_i \rangle_{MS} \]  

(4)

The effective charge for nucleon \( e_{eff}(t_z) \) can be written from

\[ e_{eff}(t_z) = e(t_z) + \frac{\langle J_f \| \Delta \hat{O}_j(\vec{r}_z, t_z) \| J_i \rangle_{CP}}{\langle J_f \| \hat{O}_j(\vec{r}_z, t_z) \| J_i \rangle_{MS}} \]  

(5)

\[ e_{eff}(t_z) = e(t_z) + \delta e(t_z) \]  

(6)

where \( \delta e(t_z) \) is the polarization charge for nucleon.

The different between the effective charge and the charge of the single nucleons is referred to as the polarization charge. The effective charge values deduced from the observed static and transition moments. It is to be noted that the value of may depend somewhat on the orbit of the nucleon; in particular, the polarization effect decrease when the binding energy of the nucleon becomes small, since the nucleon, when outside the nuclear surface, is less effective in polarizing the core [8].

The reduced electric transition probability from \( J_i \) to \( J_f \) be defined as [7]:

\[ B(EJ) = \left( \frac{|M(EJ)|}{2J_i+1} \right)^2 \]  

(7)

and define the electric quadrupole moment in a state \( |J = 2, M = 0 \rangle \) for \( J_i = J_f \) is [7]:

\[ Q(J = 2) = \left( \frac{I_i}{-J_i} 0 \right) \frac{16\pi}{5} M(EJ) \]  

(8)

The intrinsic quadrupole moment \( Q_0 \) can be calculated from reduced transition probability \( B(E2) \) defined as [9,10]:

\[ Q_0 = \frac{16\pi}{5} [B(E2)]^{1/2} \]  

(9)

Also intrinsic moment \( Q_0 \) values were calculated from static electric quadrupole moment \( Q \) values measured in the laboratory system is defined as [4]:

\[ Q_0 = \frac{(+1)(2I+3)}{(3I^2 - 1)(I+1)} Q \]  

(10)

where \( I \) is the spin of the state the member of rotational band based on nucleus ground state, \( K \) is \( I \)'s projection on symmetry axis [4].

The quadrupole deformation parameters can be calculated by two methods: from reduced transition probability \( B(E2) \) is defined as [9]:

\[ \beta_2 = \left( \frac{4\pi}{3ZE^2} \right) [B(E2; 0^+ \rightarrow 2^+_1)]^{1/2} \]  

(11)

where \( Z \) is the atomic number, \( R_0 = 1.2A^{1/3} \text{ fm} \) and from intrinsic quadrupole moments \( Q_0^* \) is given by [10, 11]:

\[ \beta_2 = \left( \frac{4\pi}{3ZE^2} \right) [Q_0^*]^{1/2} \]
$\delta = \frac{0.750e}{Z \langle r^2 \rangle}$

The value of $\langle r^2 \rangle$ was evaluated using the following expression [11]

$\langle r^2 \rangle = 0.6R_i^2 (1 + 10/(3\pi \alpha_e/R_e)^2)/(1 + (\pi \alpha_e/R_e)^2)$ \hspace{1cm} (For $A \leq 100$)

$\langle r^2 \rangle = 0.6(1.2A^{1/3})^2$ \hspace{1cm} (For $A > 100$)

Parameters of radial Woods-Saxon potential form-factor [4], $R = 1.07A^{1/3} \text{fm}$ and $\alpha_e = 0.55 \text{ fm}$

The relationship between the two parameters $\delta$ and $\beta_2$ is given by equation (16); to leading order, we have $\delta = 0.95\beta_2$ in the limit of a sharp surface. The advantage of the deformation parameter $\delta$ is its rather direct relationship to the experimentally determined quadrupole moment [4].

$$\delta \approx 0.94\beta_2 \left(1 - \frac{4}{3}\pi^2 \left(\frac{\alpha_e}{R_e}\right)^2\right) + 0.34\beta_2^2$$ \hspace{1cm} (15)

$$\delta \approx 0.95\beta_2$$ \hspace{1cm} (16)

3. Results and Discussion

However, some important excitations are still not included. Thus the interactions as well as the observable operators should be renormalized accordingly to the chosen model space, which leads to effective interactions and operators. Usually the electric quadrupole operators used in normal shell model calculations are empirically fitted [12]. There are also efforts to derive the effective operators microscopically [13, 18]. Some experimental efforts have been made in this way to pin down the effective charges in the $fp$ shell nuclei [19, 22].

In the shell model calculations the NushellX@MSU code [23], with $fp$ model space and $fpd6$ interaction [5], is used. Also adopted two set effective charges, the Bohr-Mottelson effective charges (B-M) [4] which are calculated for each nuclei according equation below:

$$e_{\text{eff}}(t_x) = e(t_x) + e\delta e(t_{x})\delta e(t_x) = Z/A - 0.32(N-Z)/A - 2t_2\delta[0.32 - 0.3(N-Z)/A]$$

and the standard effective charges (ST) [24] which are 0.45 $e$ for the neutron and 1.36 $e$ for the proton. The radial wave functions of the single-particle matrix elements $M$ were calculated with the harmonic oscillator potential with size parameters $b$ calculated for each nucleus with mass number $A$ as shown in Table 1 [25]

$$b = \sqrt{\frac{\hbar}{M\omega}} = \sqrt{\frac{\hbar}{M \cdot 45A^{1/3} - 25A^{2/3}}}$$

The reduced quadrupole transition B(E2) for nuclei $A = 44, 46, 48, 50,$ and 52 in $fp$ shell were calculated with two effective charges sets, set one B-M effective charges and set two ST effective charges ($e_p = 1.36, e_n = 0.45$). The experimental and theoretical B(E2) values are shown in Table 1.

Calculation of the deformation parameter are performed by two methods, from B(E2) one and two from the intrinsic quadrupole moment. The deformation parameters for nuclei in the present work are calculated from B(E2) and by using Eq. (12) where adopted two effective charges are used; results were compared with values experimental [26] as shown in Table 1 and Fig.1. Calculations of B(E2) with B-M effective charges are underestimate with experimental values and $^{50}$Cr is overestimate while $^{48}$Ti nucleus and $^{52}$Cr nucleus agree with experimental values as shown Fig.1.a. Calculations of the excitation energy were overestimate with experimental values, except $^{52}$Cr nucleus is close to experimental values as shown Fig.1.b. Excitation energies are increasing for $^{44}$Ca nucleus and $^{52}$Cr nucleus because magic number for proton and number, respectively. Calculations of the deformation parameters are underestimate with experimental values [26] except $^{48}$Ti nucleus and $^{52}$Cr nucleus agree with experimental values as shown Fig.1.c.

Calculations of B(E2) with ST effective charges [24] are underestimate with experimental values, except $^{50}$Cr nucleus is overestimate while $^{52}$Cr nucleus agrees with experimental values as shown Fig.2.a. Calculations of the excitation energies were overestimate with experimental values, except $^{52}$Cr nucleus is close to experimental values as shown Fig.2.b. Calculations of the deformation parameters are underestimate with experimental values [26], except $^{48}$Ti nucleus is overestimate while $^{52}$Cr nucleus agrees with experimental values as shown Fig.2.c.
The quadrupole deformation parameters were calculated by two methods and using B-M effective charges. The first method from B(E2) which adopts to calculate the quadrupole deformation parameters $\beta_2$ as according equation (9) and the theoretical results with experimental values [9] were compared. The second method from the intrinsic quadrupole moment ($Q_0$) which adopts to calculate quadrupole deformation parameters $\delta$ as according equation (12) and value of $\langle r^2 \rangle$ was calculated from equation (13). The theoretical results and experimental values [9] are tabulated in Table 3 and as shown Fig.3 were compared. Calculations of the theoretical quadrupole deformation parameters $\beta_2$ are underestimate with experimental values, except $^{50}$Cr nucleus is overestimate while $^{52}$Cr nucleus agrees with experimental values. Calculations of the theoretical quadrupole deformation parameters $\delta$ are close with experimental values, except $^{44}$Ca nucleus is underestimate with experimental values.

The quadrupole deformation parameters were calculated by two methods and using ST effective charges. The first method from B(E2) which adopts to calculate quadrupole deformation parameters $\beta_2$ as according equation (11) and the theoretical results with experimental values [9] were compared. The second method from the intrinsic quadrupole moment ($Q_0$) which adopts to calculate quadrupole deformation parameters $\delta$ as according equation (12) and value of $\langle r^2 \rangle$ was calculated from equation (13). The theoretical results and experimental values [9] are tabulated in Table 4 and as shown Fig.4 were compared. Calculations of the theoretical quadrupole deformation parameters $\beta_2$ are underestimate with experimental values, except $^{50}$Cr nucleus is overestimate while $^{52}$Cr nucleus agrees with experimental values. Calculations of the theoretical quadrupole deformation parameters $\delta$ are underestimate with experimental values, except $^{50}$Cr nucleus and $^{52}$Cr nucleus agree with experimental values.

4. Conclusions

Shell model calculations are performed with the harmonic oscillator potential single particle wave functions to calculate the B(E2) of fp shell nuclei. The shell model is less successful in describing transition rates and unless taking the core polarization effects into account through effective charges taken for proton and neutron. Calculations of B(E2) with B-M effective charge are better than of standard effective charges. The deformation parameter values of $^{44}$Ca and $^{52}$Cr indicates that these nuclei are nearly spherical closure in accordance of previous conclusions of the magic number 20 and 28, respectively.

The results of the deformation parameter from the intrinsic quadrupole moment with ST effective charges better than results of the deformation parameter from the B(E2) with B-M effective charges or ST effective charges.

5. References

[1] A.N. Vodin, E.G. Kopanets, L.P. Korda, and V.Yu. Korda. 2003. The influence of nuclear deformations on the probabilities of electromagnetic transitions in 1d2s-shell nuclei. Nuclear Physics Investigations.

[2] Hadyńska-Klk, K., Napiorkowski, P. J., Zielińska, M. et. 2018. Quadrupole collectivity in $^{42}$Ca from low-energy Coulomb excitation with AGATA. Physis Review C 97, 024326.

[3] S. B. Doma, K. A. Kharroube, A. D. Tefiya, and H. S. El-Gendy. 2011. The Deformation Structure of the Even-Even p- and s-d Shell Nuclei.

[4] A. Bohr and B. R. Mottelson. 1975. Nuclear Structure, Benjamin, New York, Vol.2.

[5] W. A. Richter, M. J. Van Der Merwe, R. E. Julies, and B. A. Brown. 1991. Nuclear Physics A523, 325.

[6] B. A. Brown, W. A. Richter, R. E. Julies, and H. B. Wildenthal. 1988. Annals Physics (N.Y.) 182, 191.

[7] P.J. Brussaard and P. W. M. Glademans. 1977. Shell- model Application in Nuclear Spectroscopy” North- Holland Publishing Company, Amsterdam.

[8] A. Bohr and B. R. Mottelson. 1969. Nuclear Structure, Benjamin, New York. Vol.1.

[9] S. Raman, C. W. Nestor, Jr., and P. Tikkanen. 2001. Transition Probability from the Ground to the First–Excited 2+ State of Even–Even Nuclides. Atomic Data and Nuclear Data Tables, 78, 28
[10] H. Friedrichs, S. Lindenstruth, B. Schlitt, C. Wesselborg, P. von Brentano, R.-D. Herzberg, A. Zilges. 1993. Deformation dependence of low lying MI strengths in even Nd isotopes. Phys. Rev. C 47, 1474.

[11] I. Boboshin, B. Ishkhanov, S. Komarov, V. Orlin, N. Peskov, and V. Varlamov. 2007. Investigation of quadrupole deformation of nucleus and its surface dynamic vibrations. International Conference on Nuclear Data for Science and Technology.

[12] Caurier E, Martinez-Pinedo G, Nowacki F, et al. 2005. The shell model as a unified view of nuclear structure. Review of Modern Physics 77: 427.

[13] Petrovich F, McManus H, Borysowicz J, and Hammerstein G R. 1977. Core polarization in inelastic scattering. Phys. Rev. C, 16: 839

[14] Sagawa H and Brown B A. 1984. E2 core polarization for sd-shell single-particle states calculated with a skyrme-type interaction. Nuclear Physics A, 430: 84

[15] Hamamoto I and Sagawa H. 1996. Core polarization charges of quadrupole transitions in neutron drip line nuclei. Phys. Rev. C, 54: 2369

[16] Zhang C L, Zhang H Q, Zhang X Z, H Sagawa, F R Xu. 2006. Polarization charge of O isotopes. Journal of Physics G: Nuclear Particle Physics, 32: 2261

[17] Hai-liang Ma, Bao-guo Dong, Yu-liang Yan, Xi-zhen Zhang. 2009. Polarization charges and electric quadrupole transitions of yrast terminating bands in $^{44,46,48}$Ti. Physics Rev. C, 80: 014316.

[18] Ma H L, Dong B G, Yan Y L. 2010. Polarization effects and application to E2 transitions in even carbon isotopes. Physics Letter B, 688: 150

[19] Dafni E, Mahnke H E, Noel W, M. H. Rafaelovich, and G. D. Sprouse. 1981. E2 moments of the $^{43}$Sc($19/2^+$) and the $^{45}$Ti($19/2^+$) states and the $^{40}$Ca core deformation. Phys. Rev. C, 23: 1612

[20] Rudolph D, Hoischen R, HellstrAam M, S. PietriZs. PodolyákP. H. ReganA. B. GarnsworthyS. J. SteerF. BeckerP. BednarczykL. CáceresP. DoornenbalJ. GerlM. GórskaJ. GreboszI. Hamamoto I and Sagawa H. 1993. Evidence for an isomeric 3/2 state in $^{52}$Co. European Physics Journal A, 36: 131

[21] R. du Rietz, J. Ekman, D. Rudolph, C. Fahlander, A. Dewald, O. Möller, B. Saha, M. Axiotis, M. A. Bentley, C. Chandler, G. de Angelis, F. Della Vedova, A. Gadea, G. Hammond, S. M. Lenzi, N. Márjinean, D. R. Napoli, M. Nespolo, C. Rusu, D. Tonev. 2004. Effective Charges in the fp Shell. Physics Review Letter, 93: 222501.

[22] Valiente-Dobon J J, Mengoni D, Gadea A, E. Farnea, S. M. Lenzi, S. Lunardi, A. Dewald, Th Pissulla, S. Szilner, R. Broda, F. Recchia, A. Algara, L. Angus, D. Bazzacco, G. Benzoni, P. G. Bizzeti, A. M. Bizzeti-Sona, P. Boutachkov, L. Corradi, F. Crespi, G. De Angelis, E. Fioretto, A. Görgen, M. Gorsea, A. Gottardo, E. Grodner, B. Guiot, A. Howard, W. Králas, S. Leoni, P. Mason, R. Menegazzo, D. Montanari, G. Montagnoli, D. R. Napoli, A. Obertino, T. Pawlat, B. Rubio, E. Sahin, F. Scarlassara, R. Silvestri, A. M. Stefanini, J. F. Smith, D. Steppenbeck, C. A. Ur, P. T. Wady, J. Wrzesiński, E. Maglione, I. Hamamoto. 2009. Lifetime Measurements of the Neutron-Rich N=30 Isotones $^{50}$Ca and $^{51}$Sc: Orbital Dependence of Effective Charges in the fp Shell. Physics Review Letter, 102: 242502

[23] B. A. Brown, W. D. M. Rae. 2014. The Shell-Model Code NuShellX@MSU. Nuclear Data Sheet.

[24] W. A. Richter, S. Mkhize, B. Alex Brown. 2008. sd-shell observables for the USDA and USDB Hamiltonians. Physical Review C.

[25] B. A. Brown, R. Radhi, B.H. Wildenthal. 1983. Electric quadrupole and hexadecupole nuclear excitations from the perspectives of electron scattering and modern shell-model theory, Physics Reports. 101, 313.

[26] B. Pritychenko, M. Birch, B. Singh, M. Horoi. 2016. Tables of E2 transition probabilities from the first +2 states in even–even nuclei. Data Nuclear Data Tables 107 1.
Table 1. The calculated transition rates and deformation parameters in some nuclei in fp shell using Bhor–Mottelson effective charges (B-M) [4] and \(fpd_6\) interaction [10]. Experimental B(E2) and \(\beta_2\)exp. are taken from Ref. [26].

| Nucleus | \((E_x)_{\text{theo}}\) (MeV) | \((E_x)_{\text{exp}}\) (MeV) | \(b\) | \(e_p\) | \(e_n\) | \(B(E2)_{\text{B-M}}\) | \(B(E2)_{\text{exp.}}\) [26] | \(\beta_2\)theo. [26] | \(\beta_2\)exp. [26] |
|---------|-------------------------------|-------------------------------|-------|-------|-------|------------------------|-----------------------------|----------------------|----------------------|
| \({}_{20}^{44}\text{Ca}\) | 1.6                           | 1.15                          | 1.96  | 1.13  | 0.72  | 144                    | 467±20                     | 0.14                 | 0.252                |
| \({}_{22}^{46}\text{Ti}\)  | 0.97                          | 0.88                          | 1.97  | 1.16  | 0.77  | 830                    | 951±25                     | 0.3                  | 0.317                |
| \({}_{22}^{48}\text{Ti}\)  | 1.1                           | 0.98                          | 1.98  | 1.14  | 0.73  | 655                    | 662±29                     | 0.25                 | 0.257                |
| \({}_{24}^{50}\text{Cr}\)  | 0.88                          | 0.78                          | 2.0   | 1.16  | 0.78  | 1310                   | 1052±32                    | 0.32                 | 0.289                |
| \({}_{24}^{52}\text{Cr}\)  | 1.37                          | 1.43                          | 2.01  | 1.14  | 0.73  | 607.3                  | 622±24                     | 0.21                 | 0.217                |
Figure 1. Calculated and measured $B(E2; 0^+ \rightarrow 2^+)$, excitation energy and deformation parameters of some nuclei in $fp$ shell. The experimental values are taken from Ref. [26].
Table 2. The calculated transition rates and deformation parameters in some nuclei in fp shell using standard effective charges (ST) $e_p = 1.36$ and $e_n = 0.45$ [24] and $fpd6$ interaction [10]. Experimental $B(E2)$ and $\beta^2_{\exp}$ are taken from Ref. [26].

| Nucleus | $(E_x)_{\text{theo.}}$ (MeV) | $(E_x)_{\text{exp}}$ (MeV) | $b$  | $e_p$ | $e_n$ | $B(E2)_{\text{ST}}$ | $B(E2)_{\exp}$ | $\beta^2_{\text{theo.}}$ | $\beta^2_{\exp}$ |
|---------|-----------------|-----------------|------|------|------|----------------|----------------|----------------|----------------|
| $^{44}_{20}Ca$ | 1.6  | 1.15  | 1.96 | 1.36 | 0.45 | 57  | 467±20 | 0.08  | 0.252 |
| $^{46}_{22}Ti$ | 0.97 | 0.88  | 1.978 | 1.36 | 0.45 | 638 | 951±25 | 0.26  | 0.317 |
| $^{48}_{22}Ti$ | 1.1  | 0.98  | 1.984 | 1.36 | 0.45 | 521 | 662±29 | 0.228 | 0.257 |
| $^{50}_{24}Cr$ | 0.88 | 0.78  | 2.0  | 1.36 | 0.45 | 1331 | 1052±32 | 0.32  | 0.289 |
| $^{52}_{24}Cr$ | 1.37 | 1.43  | 2.01 | 1.36 | 0.45 | 633.9 | 622±24 | 0.219 | 0.217 |
**Figure 2.** Calculated and measured $B(E2; 0^+ \rightarrow 2^+)$, excitation energy and deformation parameters by using standard effective charges [24] of some nuclei in $fp$ shell. The experimental values are taken from Ref. [26].
Table 3. The calculated deformation parameters in some nuclei in fp shell using B-M effective charges [4] and fpd6 interaction [10]. Experimental B(E2), $\beta_{\text{exp}}$ and $\delta_{\text{exp}}$ are taken from Ref. [9].

| Nucleus  | $b$   | $B(E2)_{\text{exp.}}$ | $B(E2)$ B-M | $\beta_{\text{theo.}}$ | $\beta_{\text{exp.}}$ | $Q^+_\text{exp}$ | $Q^+_{\text{theo}}$ | $\delta_{\text{theo}}$ | $\delta_{\text{exp}}$ |
|----------|-------|--------------------------|-------------|-------------------------|------------------------|-----------------|-----------------|------------------------|------------------------|
| $^{44}_{20}Ca$ | 1.96  | 470±20                   | 144         | 0.14                    | 0.25                   | 68.7±15         | 38.09           | 0.11                   | 0.22                   |
| $^{46}_{22}Ti$ | 1.978 | 950±5                    | 830         | 0.29                    | 0.317                  | 97.7±26         | 91.3            | 0.25                   | 0.27                   |
| $^{48}_{22}Ti$ | 1.984 | 720±40                   | 655         | 0.256                   | 0.269                  | 85±24           | 81.1            | 0.22                   | 0.23                   |
| $^{50}_{24}Cr$ | 2.0   | 1080±6                   | 1310        | 0.32                    | 0.29                   | 104.2±20        | 114.8           | 0.27                   | 0.25                   |
| $^{52}_{24}Cr$ | 2.01  | 660±30                   | 607.3       | 0.214                   | 0.22                   | 81.4±19         | 78.1            | 0.18                   | 0.19                   |
Figure 3. The calculated of the deformation parameters by using B-M effective charges and by using two methods. The results with experimental values [9] were compared.
Table 4. The calculated deformation parameters in some nuclei in fp shell using ST effective charges [4] and \textit{fspd6} interaction [10]. Experimental B(E2), $\beta_{\text{exp}}$, and $\delta_{\text{exp}}$ are taken from Ref. [9].

| Nucleus | \( b \)  | \( B(E2)_{\text{exp}} \) | \( B(E2)_{\text{ST}} \) | \( \beta_{\text{theo}} \) | \( \beta_{\text{exp}} \) | \( Q_{\text{exp}}^\circ \) | \( Q_{\text{theo}}^\circ \) | \( \delta_{\text{theo}} \) | \( \delta_{\text{exp}} \) |
|---------|---------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \({}_{{}^{44}}{}^{20}\text{Ca} \) | 1.96 | 470±20 | 57 | 0.088 | 0.253±5 | 68.7±15 | 23.85 | 0.07 | 0.22 |
| \({}_{{}^{46}}{}^{22}\text{Ti} \) | 1.978 | 950±5 | 638 | 0.26 | 0.317 ±8 | 97.7±26 | 80.1 | 0.22 | 0.27 |
| \({}_{{}^{48}}{}^{22}\text{Ti} \) | 1.984 | 720±40 | 521 | 0.228 | 0.269±7 | 85±24 | 72.4 | 0.19 | 0.23 |
| \({}_{{}^{50}}{}^{24}\text{Cr} \) | 2.0 | 1080±6 | 1331 | 0.32 | 0.293±8 | 104.2±20 | 106.6 | 0.25 | 0.25 |
| \({}_{{}^{52}}{}^{24}\text{Cr} \) | 2.01 | 660±30 | 633.9 | 0.219 | 0.223±42 | 81.4±19 | 79.7 | 0.19 | 0.19 |
Figure 4. The calculated of the deformation parameters by using ST effective charges and by using two methods. The results with experimental values [9] were compared.