The Study Contact Bending Stress Two-Layer Plates from Fiberglass with Interfacial Defects of Structure

Emad Toma Bane Karash*
Northern Technical University, Iraq

Submission: September 09, 2017; Published: October 16, 2017

Corresponding author: Emad Toma Bane Karash, Northern Technical University, Iraq, Email: Emadhoke207@yahoo.com

Abstract

The presented version of the nonlinear discrete-structural theory adequately reflects the work of real structures. Obtained are in satisfactory compliance with the theoretical and experimental data. The magnitude of the transverse shear stresses of contact increases 4-5 times in a zone bearing the circuit, which leads to interfacial failure of the adhesive layer. Thus, the proposed model allows the calculation to locate the contact area, touch, pressure, and the changing nature of the state stress at the interface.

Keywords: Structure; Bending stress; fiberglass; Anisotropic; Shell

Introduction

The significant difference between mechanical and physical properties of the individual components of the structure of layered thin-walled structural elements have driven to the creation of adequate discrete-structural theories to calculate this group of problems. Contact problem of mechanics of multilayered plates and shells is considered in [1-6], which, based on the discrete approach built functional and obtained a system of equations for solving such problems, provided non-ideal contact layers. A method for solving nonlinear problems of contact between the two shells of different shapes and equidistant layers can be found in [7-10], on the other hand detailed analysis of the latest results and trends in the development of the discrete-structural theory of laminated plates and shells found in [11].

In this paper, we propose a variant of the geometrically nonlinear dispersion specifically-layered structure theory of structural elements and study nature of the change of contact stresses in bending double-layer plates fiberglass with interfacial defects of the material structure. The derivation equilibrium equations and geometric ratio take into account the influence of transverse shear deformation and compression. The results of theoretical studies are compared with experimental data.

Formulation of Analytical Model

According to the theory of discrete structural, mathematical model considered, the multilayer shell consists of n thin anisotropic layers. The volume of (n) rigid layers is \( V = \sum V_i \). Each cladding layer deformed assigned to an orthogonal curvilinear coordinate system, \( \alpha_i \) (i=1,2), \( \alpha_3 \). Coordinate directed along the common normal \( \mathbf{m}(\mathbf{z}) \) to the middle surface \( S^{(k)} \) and equidistant surface \( S^{(k)} \) k-th layer. Index \( z \) with the introduction of other characters means that the corresponding values are to the point \( (\alpha_1, \alpha_2, z) \) equidistant surface \( S^{(z)} \).

Total displacement vector \( \mathbf{u}^{(k)} \) k-th point of the hard layer according to the refined Tomachinko shell theory can be represented as:

\[
\mathbf{u}^{(k)} = \mathbf{u}^{(k)} + z^{(k)} \mathbf{r}^{(k)} + \varphi^{(k)} (z) \psi^{(k)}, \tag{1}
\]

Where \( \mathbf{u}^{(k)} \) displacement vector points of the middle surface \( S^{(k)} \), \( \mathbf{r}^{(k)} \) - Vector function of the angles of rotation and compression of the fibers in the direction normal to the undeformed middle surface \( S^{(k)} \) during deformation. \( \varphi^{(k)} \) - Nonlinear continuous distribution function tangential displacements through the thickness of the k-th layer, analysis and approximation given in [11]

\[
\varphi^{(k)} (\alpha_1^{(k)}, \alpha_2^{(k)}) = \ldots
\]

Vector shift function. Covariant components of vectors \( \mathbf{u}^{(k)} \), \( \mathbf{r}^{(k)} \), \( \psi^{(k)} \) recorded using the following expressions:

\[
\mathbf{u}^{(k)} = \mathbf{u}^{(k)} + \mathbf{r}^{(k)} \mathbf{u}^{(k)} + \mathbf{h}^{(k)} \mathbf{w}^{(k)}, \quad \mathbf{r}^{(k)} = \mathbf{r}^{(k)} \mathbf{r}^{(k)} + \mathbf{h}^{(k)} \mathbf{r}^{(k)}
\]
Deformation tensor components \( \sigma_{\alpha\beta}(k) \), defined as half the corresponding metric tensor components before and after deformation:

\[
2\sigma_{\alpha\beta}(i) = \delta_{\alpha\beta}(k) - \delta_{\alpha\beta}(i), \quad 2\sigma_{\alpha\beta}(ii) = \delta_{\alpha\beta}(ii) - \delta_{\alpha\beta}(i), \quad 2\sigma_{\alpha\beta}(iii) = \delta_{\alpha\beta}(iii) - 1. \tag{3}
\]

The final form of the equilibrium equations and boundary conditions for the solution of contact problems of laminated shells are usually made on the basis of Reissner variational principle. Considering that in the direction normal to the middle surface of the individual layers of the shell an axial line of general and topological coordinate systems are combined and also bias with the topological coordinate surfaces of the middle surface layer, the variational equation for the Reissner principle multilayer shell write in the following form:

\[
\partial R = \sum \partial R^k - \sum \frac{\partial}{\partial \gamma} \left( \int_{\Omega_0} \sigma_{\alpha\beta}(k) \gamma_{\alpha\beta}(i) \right) dV = 0 \quad (\alpha, \beta = 1, 2) \tag{4}
\]

Punctuation starts from the layers of negative values of \( x \) from 1 to \( n \). Wherein \( P^k(i) \) - specific additional work strain \( k \)-th layer, \( \sigma_{\alpha\beta}(k) \), \( \varepsilon_{\alpha\beta}(k) \) - components of the stress and the strain tensors.

If we assume that the dual surfaces of the \( k \)-th layer of the conditions of full contact:

\[
U_{\bar{R}}^{(k,k-1)} = U_{\bar{R}}^{(k-1,k)}, \quad X_{\bar{R}}^{(k,k-1)} = X_{\bar{R}}^{(k-1,k)} \tag{5}
\]

in vector form:

\[
u^{(k-1)} = \nu^{(k-1,k)}, \quad X_{\bar{R}}^{(k-1,k)} = X_{\bar{R}}^{(k-1,k)} \tag{6}
\]

The difference of the elementary work of external forces \( \delta \bar{A}_R(k) \) represented by the following formula:

\[
\delta \bar{A}_R(k) = \sum \delta \overline{R}^k - \sum \frac{\partial}{\partial \gamma} \left( \int_{\Omega_0} \sigma_{\alpha\beta}(k) \gamma_{\alpha\beta}(i) \right) dV + \sum \left( \sigma_{\alpha\beta}(k) \gamma_{\alpha\beta}(i) \right) dI + \sum \left( \sigma_{\alpha\beta}(k) \gamma_{\alpha\beta}(i) \right) dI + \left( \varepsilon_{\alpha\beta}(k-1) \delta \bar{A}_R(k) + \left( \varepsilon_{\alpha\beta}(k) - \varepsilon_{\alpha\beta}(k) \right) dI \right) dV. \tag{7}
\]

Where \( \bar{S}_R(k) \) - middle surface of the \( k \)-th layer; \( \theta_{\alpha\beta}(k), \theta_{\alpha\beta}(k) \) - part of the loop \( \bar{R} \) (k). Vectors external forces \( \bar{F}_R(k) \) points \( M_{\bar{R}}(k) \) and additional points \( \bar{B}_R(k) \), are inclusive in the equation (7), are defined by:

\[
X_{\bar{R}}(k) - X_{\bar{R}}(k) + \int_{S_{\bar{R}}} \sigma^{(k)} dS_{\bar{R}} = \int_{S_{\bar{R}}} \sigma^{(k)} dS_{\bar{R}} + \int_{S_{\bar{R}}} \sigma^{(k)} dS_{\bar{R}}, \quad \bar{B}_R(k) = \bar{B}_R(k) + \int_{S_{\bar{R}}} \bar{B}_R(k) dS_{\bar{R}}. \tag{8}
\]

The vectors \( S_{\bar{R}}(k), \quad \bar{X}_{\bar{R}}(k) \), include the contravariant components of the tensor of contact stresses:

\[
\sigma^{(k)}_{\alpha\beta}(k), \quad \varepsilon^{(k)}_{\alpha\beta}(k), \quad (i = 1, 2, 3) \tag{9}
\]

Subscripts “+” and “-” indicate the top and bottom surfaces of the \( k \)-th layer. A similar entry are vectors of the external load \( \bar{Q}_R^{+}(k), \quad \bar{Q}_R^{-}(k) \), \( \bar{q}_R^{+}(k), \quad \bar{q}_R^{-}(k) \), \( \bar{q}_R^{+}(k), \quad \bar{q}_R^{-}(k) \), \( \bar{q}_R^{+}(k), \quad \bar{q}_R^{-}(k) \). \( i = l, 2 \).

Vector \( \bar{P}_R(k) \) takes into account the effect of its own weight. Contravariant components \( M_{\bar{R}}(k), \quad M_{\bar{R}}(k) \) moment vector \( M_{\bar{R}}(k) \) with respect to the basis vectors \( \bar{f}_{\alpha\beta}(k) \) and \( \bar{m}_{\alpha\beta}(k) \) as to the equation:

\[
\bar{M}_R(k) = M_{\bar{R}}^{(k)}(k) \bar{f}_{\alpha\beta}(k) + \bar{m}_{\alpha\beta}(k) \bar{M}_{\bar{R}}^{(k)}(k) \tag{11}
\]

Also, elementary work \( \bar{Q}_R(k) \) layer of the shell is differentiated generally by the vector \( \bar{F}_R(k) \), the main point \( \bar{G}_R(k) \) additional main point \( \bar{L}_R(k) \), associated with the tension at loop \( \bar{\theta}_{\alpha\beta}(k) \) by virtue of a predetermined displacement of the contour points \( \bar{a}_{\alpha\beta}(k) \).

The second term of equation (4) should be represented in the form:

\[
\left| \begin{array}{c}
\alpha_\gamma \\
\beta_\gamma \\
\end{array} \right| \sum_{i=1}^{n} \left( \bar{Q}_R(i) \right)_{\alpha\beta} + \sum_{i=1}^{n} \left( \bar{Q}_R(i) \right)_{\alpha\beta} \left( \int_{S_{\bar{R}}} \bar{Q}_R^{(i)} dS_{\bar{R}} \right) - \sum_{i=1}^{n} \left( \bar{Q}_R(i) \right)_{\alpha\beta} \left( \int_{S_{\bar{R}}} \bar{Q}_R^{(i)} dS_{\bar{R}} \right) \tag{12}
\]

Where

\[
\left| \begin{array}{c}
\alpha_\gamma \\
\beta_\gamma \\
\end{array} \right| \sum_{i=1}^{n} \left( \bar{Q}_R(i) \right)_{\alpha\beta} + \sum_{i=1}^{n} \left( \bar{Q}_R(i) \right)_{\alpha\beta} \left( \int_{S_{\bar{R}}} \bar{Q}_R^{(i)} dS_{\bar{R}} \right) - \sum_{i=1}^{n} \left( \bar{Q}_R(i) \right)_{\alpha\beta} \left( \int_{S_{\bar{R}}} \bar{Q}_R^{(i)} dS_{\bar{R}} \right) \times

\]
on the lateral contour of the \( k \)-th layer denoted shell. Due to the lack of space to show resolution systems, physical and geometrical ratio in developed forms not possible. Kinematic and static contact condition (5) facial surface \( k \)-th layer and associated surfsurfaces, \( k + 1 \) and \( k - 1 \)-th layer, according to the notation introduced earlier take this equation:

\[
2\eta_1^{(k)} + \eta_2^{(k)} + \eta_3^{(k)} + \eta_4^{(k)} - \frac{1}{2} \left( y^{(1)} - y^{(2)} \right) + \frac{1}{2} \left( y^{(3)} - y^{(4)} \right) - \frac{1}{2} \left( y^{(5)} - y^{(6)} \right) - \frac{1}{2} \left( y^{(7)} - y^{(8)} \right) = 0, \quad (i = 1, 2); \quad \eta_1^{(i)} = w^{(i)}(1) + w^{(i)}(2), \quad \eta_2^{(i)} = w^{(i)}(3) + w^{(i)}(4), \quad \eta_3^{(i)} = w^{(i)}(5) + w^{(i)}(6), \quad \eta_4^{(i)} = w^{(i)}(7) + w^{(i)}(8) \quad \text{for} \ i = 1, 2
\]

Presented the kinematic relations (14) in the construction of a system of equations (13) for the entire package element layers and proceed static conditions on personal contact mating surfaces (15) on the basis of the penalty function method [11], we can create an algorithm for solving the contact problem discrete structural theory of multilayer shells. If between \( k \)-th and \( k + 1 \)-th cladding layers. Assuming no kinematic connections at the surface interface of these layers may occur effort unknown vectors \( \tilde{q}_{(k)}, \tilde{q}_{(k+1)} \) contact interaction. According to the 3rd Newton’s second law is a dependence \( \tilde{q}_{(k)} = -\tilde{q}_{(k+1)} \). To account for the impact of the efforts of the contact interaction of the layers in the variational equation Reissner principle (4) it is necessary to introduce a term that takes into account the work force of contact interaction in the motion vector of each layer portion of the mating surface:

\[
\Lambda_q = \sum_{m=1}^{k+1} \int_{S_{m}} q_{(m)} \mu^{(m)} dS
\]

Efforts contact interaction \( \tilde{q}_{(k)} = q_{(k)} \mu^{(k)} + q_{(k)} m^{(k)} \) arise when the condition:

\[
(\mu_x^{(k)} - \mu_x^{(k+1)}) < 0 \quad (17)
\]

Coupling in areas harsh layers, in the case, where the inequality (17) is not performed while moving the field of points \( S^{(k)}, S^{(k+1)} \) during deformation, the contact pressure \( \tilde{q}_{(k)} \) in equation (13) takes the value \( \tilde{q}_{(k)} = 0 \). Solving the system of equations (13), it is easy to find the value of a given accuracy of the contact pressure on the basis of the iterative method suggested in [12].

**Results and Discussion**

Test cases investigated plate round shape in terms of diameter 0.16 m regular structure of fiberglass. Considered plates were made of 4 layers of glass TG 430 -C (100) (producer - Latvia). The binder used polyester orthophthalic resin with low styrene emission Cristic 2 - 446PA (producer - UK). Physical and mechanical properties of fiberglass plates determined as follows. Initially, according to GOST 25,601-80, determined mechanical properties of black glass-stick determined by the elastic modulus and Poisson’s ratio when stretching the sample is measured using dial gauges up to 0.01 mm. To measure the strain gage used KF4P 1-3-200. Sticker gages were placed according to the instructions on the label AZHV TO 2,782,001. To measure the output signals of strain gages and reporting digitally measuring system used SIIT-3. A mathematical model of calculation is a two-layer plan Steen composed of two layers of thick hard transtropich \( (1) = h (2) = 1.0 \times 10^{-3} m \).

![Figure 1](image1.png)  
**Figure 1**: radial stress on the front surface of the circular plate (1.2-rigid clamping circuit; 3.4-freely supported contour; ° - experimental data).

![Figure 2](image2.png)  
**Figure 2**: Transverse shear stresses along the mating surface contacting layers of a circular plate (1.2-free simple loop; 2-rigid clamping loop).

It is presumed that the interfacial contact area of the hard layer is very pliable, that is, they may be "slippage" relative to each other. The results of studies of plate bending under the action of uniform pressure \( q = 0.025 \text{ Mpa} \) are shown in Figure 1-2. The relative error of the theoretical value of the deflection in the center of the plate, when compared with the experimental data was less than 3%: \( wz = 0.2. 10^{-2} \text{ m} \) - A rigid clamping support; \( ws = 0.2. 10^{-2} \text{ m} \) - for a simply supported contour.

**Conclusion**

According to thenonlinear discrete - structural theory of multi-layer plates and shells the intense deformed case of
anisotropic elements with defects in the structure of the material is investigated. At the conclusion of the equations of balance and geometrical parity the influence of deformations of cross shift and cross-pressure is taken into account. The results of theoretical research are compared with experimental data.

Acknowledgment

This research was supported by Mechanical Engineering Research Program in the Northern Technical University/Technical Institute of Mosul funded by the Ministry of Higher Education and Scientific/Research/Republic of Iraq. (No. 2016-00432).

References

1. Brischetto S, Carrera E (2010) Coupled thermo-mechanical analysis of one-layered and multilayered plates. Composite Structures 92: 1793-1812.
2. Carrera E, Brischetto S (2010) Thermo-mechanical coupling in multilayered plates and shells. 27th international congress of the aeronautical sciences, pp. 1-10.
3. Guidault PA, Allix O, Champaney L, Navarro JP (2007) A two-scale approach with homogenization for the computation of cracked structures. Computers & Structures 85(17-18): 1360-1371.
4. Kim JS Cho M (2007) Enhanced first-order theory based on the mixed formulation and transverse normal effect. International Journal of Solids and Structures 44: 1256-1276.
5. Cen S, Long Y, Yao Z (2002) A new hybrid enhanced displacement based element for the analysis of laminated composite plates. Computers & Structures 80(9-10): 819-833.

6. Wang X, Zhong Z (2003) Three-Dimensional Solution of Smart Laminated Anisotropic Circular Cylindrical Shells with Imperfect Bonding. International Journal of Solids and Structures 40(22): 5901-5921.
7. Yu B, Yang L (2010) Elastic Modulus Reduction Method for Limit Analysis of Thin Plate and Shell Structures. Thin-Walled Structures 48(4-5): 291-298.
8. Rahman T, Jansen EL (2010) Finite Element Based Coupled Mode Initial Post-Buckling Analysis of a Composite Cylindrical Shell. Thin-Walled Structures 48(1): 25-32.
9. Sofiyev AH (2003) Torsional Buckling of Cross-Ply Laminated Orthotropic Composite Cylindrical Shells Subject to Dynamic Loading. European Journal of Mechanics 22(6): 943-951.
10. Li ZM, Lin ZQ (2010) Non-Linear Buckling and Post-buckling of Shear Deformable Anisotropic Laminated Cylindrical Shell Subjected to Varying External Pressure Loads. Composite Structures 92(2): 553-567.
11. Zurich E (2008) Model for Interlinear Normal Stresses in Doubly Curved Laminates. International Journal of Mechanical Sciences, DISS. ETH NO. 17725.
12. Fagiano C, Abdalla MM, Kassapoglou C, Gurdal Z (2011) Interlaminar Stress Recovery for Three-Dimensional Finite Elements.
13. Xiang X, Guoyong J, Wanyou Li (2014) A numerical solution for vibration analysis of composite laminated conical cylindrical shell and annular plate structures. Composite Structures 111: 20-30.

Your next submission with Juniper Publishers will reach you the below assets

- Quality Editorial service
- Swift Peer Review
- Reprints availability
- E-prints Service
- Manuscript Podcast for convenient understanding
- Global attainment for your research
- Manuscript accessibility in different formats
  (Pdf, E-pub, Full Text, Audio)
- Unceasing customer service

Track the below URL for one-step submission

https://juniperpublishers.com/online-submission.php