Functional versus canonical quantization of a nonlocal massive vector-gauge theory

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Abstract

It has been shown in literature that a possible mechanism of mass generation for gauge fields is through a topological coupling of vector and tensor fields. After integrating over the tensor degrees of freedom, one arrives at an effective massive theory that, although gauge invariant, is nonlocal. Here we quantize this nonlocal resulting theory both by path integral and canonical procedures. This system can be considered as equivalent to one with an infinite number of time derivatives and consequently an infinite number of momenta. This means that the use of the canonical formalism deserves some care. We show the consistency of the formalism we use in the canonical procedure by showing that the obtained propagators are the same as those of the (Lagrangian) path integral approach. The problem of nonlocality appears in the obtainment of the spectrum of the theory. This fact becomes very transparent when we list the infinite number of commutators involving the fields and their velocities.

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1 Introduction

It is widely accepted that the forces of nature are described by gauge theories. These theories are characterized by the gauge symmetries which are related to massless fields. However, sometimes it is necessary that these fields become massful, as it occurs, for instance, in the case of the Salam-Weinberg theory. Nowadays, it has also been widely accepted that spontaneous symmetry breaking together with the Higgs mechanism are the most probable explanation for the origin of the acquisition of mass by gauge fields. However, if this is actually true, the Higgs bosons must exist in nature. The point is that there is no precise theoretical prediction on the mass scale where these fields could be found and experiments till now have shown no evidence about them.

In this way, alternative mechanisms of mass generation for gauge fields that do not spoil what is well established and that do not contain Higgs bosons are welcome. This might be the case of vector-tensor gauge theories \[1\], where vector and tensor fields are coupled in a topological way by means of a kind of Chern-Simons term. The general idea of this mechanism resides in the following: tensor gauge fields \[3\] are antisymmetric quantities and consequently in \(D = 4\) they exhibit six degrees of freedom. By virtue of the massless condition, the number of degrees of freedom goes down to four. Since the gauge parameter is a vector quantity, this number would be zero if all of its components were independent. This is nonetheless the case because the system is reducible (which means that the gauge transformations are not all independent) and we mention that the final number of physical degrees of freedom is one. It is precisely this degree of freedom that can be absorbed by the vector gauge field in the vector-tensor gauge theory in order to acquire mass \[1, 3\]. This peculiar structure of constraints involving tensor gauge theories implies that quantization as well as its non-Abelian formulation deserve some care and a reasonable amount of work has been done on these subjects \[4, 5, 6, 7\].

Usually, the treatment of the vector-tensor gauge theory is carried out with both vector and tensor fields placed together and just at the end the integration over the tensor field is done in order to obtain the effective result for the vector theory. This procedure usually hides an important aspect of this effective theory, that is its nonlocality. We mention that it is equivalent to a theory with an infinite number of higher derivative terms and, consequently, an infinite number of momenta also.

It is not our purpose here to advocate if a vector-tensor gauge theory with topological coupling is more suitable to explain the mass generation than the usual spontaneous symmetry breaking together with the Higgs mechanism. Our intention in the present paper is to study the quantization of massive vector gauge field directly by means of the nonlocal effective Lagrangian \[8\]. We do it both by path integral, where we use a Lagrangian formulation, as well as by the canonical approach. This is the subject of Sections 2 and 3 respectively. We would like also to add that the non-Abelian formulation for the vector tensor gauge theory is not a simple task. This is so because the non-Abelian version loses the reducibility condition unless we consider that the Maxwell stress tensor as zero \[8\]. Another possibility is to
introduce a kind of Stuckelberg field, that disappears in the Abelian limit, in order to keep the same number of degrees of freedom in both sectors (Abelian and non-Abelian) of the theory. We shall consider only the Abelian case in this paper. We left Sec. 4 for some concluding remarks and include four appendices to present details of some calculations.

2 Brief review of the vector-tensor gauge theory and the path integral quantization of the effective vector theory

The Abelian theory for vector and tensor fields coupled in a topological way is described by the Lagrangian density:

\[ L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{m}{2} \epsilon^{\mu\nu\rho\lambda} B_{\mu\nu} F_{\rho\lambda}, \] (2.1)

where \( F_{\mu\nu} \) and \( H_{\mu\nu\rho} \) are totally antisymmetric tensors written in terms of the potentials \( A_\mu \) and \( B_{\mu\nu} \) (also antisymmetric) through the stress tensors

\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \] (2.2)
\[ H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\rho B_{\mu\nu} + \partial_\nu B_{\rho\mu}. \] (2.3)

In expression (2.1), \( \epsilon^{\mu\nu\rho\lambda} \) is the totally antisymmetric symbol and \( m \) is a mass parameter. It is easy to see, by using the (coupled) Euler-Lagrange equations for \( A_\mu \) and \( B_{\mu\nu} \) (also antisymmetric) through the stress tensors, that \( F_{\mu\nu} \) satisfy a massive Klein-Gordon equation, with a mass parameter \( m \).

We observe that the Lagrangian (2.1) is invariant under the gauge transformations

\[ \delta A_\mu = \partial_\mu \Lambda, \] (2.4)
\[ \delta B^{\mu\nu} = \partial_\mu \Lambda^\nu - \partial_\nu \Lambda^\mu, \] (2.5)

where \( \Lambda \) and \( \Lambda^\mu \) are (before fixing the gauge) generic functions of spacetime. This is a reducible theory, what means that not all the gauge transformations above are independent. In fact, if we choose the gauge parameter \( \Lambda^\mu \) as the gradient of some scalar \( \Omega \) we have that \( B^{\mu\nu} \) does not change under the gauge transformation (2.5).

Functionally integrating over the antisymmetric tensor field \( B_{\mu\nu} \) we get, after a convenient gauge fixing procedure, the effective action.
\[
S_0 [A_\mu] = - \frac{1}{4} \int d^4 x \, F_{\mu\nu} \left( 1 + \frac{m^2}{\Box} \right) F^{\mu\nu}.
\]  

(2.6)

The action (2.6), although nonlocal, is gauge invariant. It is important to emphasize that this theory is renormalizable, a characteristic that is lost when a mass term is directly put by hand as in the Proca theory.

Let us calculate the (covariant) propagator for the field \( A_\mu \). We opt to use a Lagrangian formulation in order to avoid the problem of the infinite number of momenta. Let us use the Batalin-Vilkovisky (BV) formalism [5, 9]. The nonminimum BV action can be written as

\[
S = S_0 + \int dx \left( A^*_\mu \partial^\mu c + b \bar{c}^* \right),
\]

(2.7)

where the \( A^*_\mu \) and the pair \((c^*, \bar{c}^*)\) are respectively the antifields of the gauge field \( A^\mu \) and of the ghosts \((c, \bar{c})\). The auxiliary field \( b \) was introduced in order to fix the gauge in a covariant way. This can be done, for instance, with the aid of the gauge-fixing fermion functional

\[
\Psi = \int dx \, \bar{c} \left( - \alpha b + \partial^\mu A_\mu \right),
\]

(2.8)

with \( \alpha \) being a parameter. The vacuum functional is defined by [5, 9]

\[
Z_\Psi = \int [dA_\mu][d\bar{c}][dc][db][dA^*_\mu][dc^*][\delta \phi^* - \delta \Psi \delta \phi] \exp \{ iS \}.
\]

(2.9)

The action \( S \) is given by (2.7) and \( \phi \) is generically referring to gauge and ghost fields. After functionally integrating over the antifields as well as over the auxiliary field \( b \), we arrive at

\[
\bar{S} [J] = \int d^4 x \left[ - \frac{1}{4} F_{\mu\nu} \left( 1 + \frac{m^2}{\Box} \right) F^{\mu\nu} - \partial_\mu \bar{c} \partial^\mu c + \frac{1}{2\alpha} \left( \partial^\mu A_\mu \right)^2 + J_\mu A^\mu \right],
\]

(2.10)

where we have introduced an external source \( J^\mu \) in order to calculate the propagator. This can be directly obtained by a straightforward calculation. The result, written in momentum space, is

\[
K^{\mu\nu} = - \frac{1}{k^2 + m^2} \left[ \eta^{\mu\nu} + \left( \frac{\alpha - 1}{k^2} + \frac{m^2}{k^4} \right) k^\mu k^\nu \right].
\]

(2.11)

We notice that there is actually a mass pole at \( k_0^2 = \vec{k}^2 + m^2 \).

In the next Section we are going to see how this and other features appear in terms of a canonical quantization procedure.
3 Canonical quantization

Let us consider the Lagrangian for vector fields of Eq. (2.10)

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} \left(1 + \frac{m^2}{\Box} \right) F^{\mu\nu} - \frac{1}{2\alpha} \left(\partial^\mu A_\mu\right)^2. \quad (3.1)$$

To implement the process of canonical quantization for such system, it is necessary to obtain the canonical momenta. So, we have to isolate the time derivatives from the nonlocal operator $\Box^{-1}$. We then conveniently write

$$\frac{1}{\Box} = -(\nabla^2 - \partial_t^2)^{-1},$$

$$= -\frac{1}{\nabla^2} - \frac{\partial_t^2}{\nabla^4} - \cdots \quad (3.2)$$

As one observes, the system described by (3.1) is effectively a system with an infinite number of time derivatives and consequently it contains an infinite number of momenta [10, 11]. A practical way of obtaining the momentum expressions is to consider the variation of the action by fixing the fields and their velocities at just one of the extreme times, say, $\delta A_\mu(\vec{x}, t_0) = 0 = \delta \dot{A}_\mu(\vec{x}, t_0) = \delta \ddot{A}_\mu(\vec{x}, t_0) = \cdots$. After some algebraic calculation, we get (please, see Appendix A)

$$\delta \int_{t_0}^t dt \int d^3 \vec{x} \mathcal{L} = \int_{t_0}^t dt \int d^3 \vec{x} \left[ \left(1 + \frac{m^2}{\Box} \right) \partial^\mu F_{\mu\nu} + \frac{1}{\alpha} \partial_\mu \partial_\nu A^\mu \right] \delta A^\nu$$

$$- \int d^3 \vec{x} \left[ \left(1 + \frac{m^2}{\Box} \right) F_{0\nu} + \frac{m^2}{2\Box} \nabla^2 \dot{F}_{i\nu} + \frac{1}{\alpha} \partial_\mu A^\mu \eta_{0\nu} \right] \delta A^\nu$$

$$+ \frac{m^2}{2} \int d^3 \vec{x} \left[ \frac{1}{\nabla^2} F_{i\nu} \delta \dot{A}^\nu \right]$$

$$- \frac{m^2}{2} \int d^3 \vec{x} \left[ \frac{1}{\nabla^4} \left( \frac{1}{\nabla^2} F_{0\nu} + \frac{\partial^i}{\nabla^4} \dot{F}_{i\nu} \right) \delta \ddot{A}^\nu \right]$$

$$+ \frac{m^2}{2} \int d^3 \vec{x} \left[ \frac{1}{\nabla^4} F_{i\mu} \delta \dddot{A}^\nu \right] + \cdots \quad (3.3)$$

The coefficient of $\delta A^\nu$ in the first term is the equation of motion, namely,

$$\left(1 + \frac{m^2}{\Box} \right) \partial^\mu F_{\mu\nu} + \frac{1}{\alpha} \partial_\mu \partial_\nu A^\mu = 0. \quad (3.4)$$

In the remaining terms, the coefficients of $\delta A^\nu$, $\delta \dot{A}^\nu$, $\delta \ddot{A}^\nu$ etc. are the canonical momentum conjugate to $A^\nu$, $\dot{A}^\nu$, $\ddot{A}^\nu$ etc. Denoting these momenta by $\pi_\nu$, $\pi_\nu^{(1)}$, $\pi_\nu^{(2)}$ etc., we have
\[ \pi_\nu = -\left(1 + \frac{m^2}{\Box}\right) F_{0\nu} - \frac{m^2}{2\Box} \partial^i \dot{F}_{i\nu} - \frac{1}{\alpha} \partial_\mu A^\mu \eta_{0\nu}, \]

\[ \pi^{(1)}_\nu = \frac{m^2}{2\Box} \partial_\mu F_{\mu\nu}, \]

\[ \pi^{(2)}_\nu = -\frac{m^2}{2\Box} \left( F_{0\nu} + \frac{\partial^i}{\Delta^2} \dot{F}_{i\nu} \right), \]

\[ \pi^{(3)}_\nu = \frac{m^2}{2\Box} \partial_\mu F_{\mu\nu}, \]

\[ \pi^{(4)}_\nu = -\frac{m^2}{2\Box} \frac{1}{\Delta^4} \left( F_{0\nu} + \frac{\partial^i}{\Delta^2} \dot{F}_{i\nu} \right), \]

\[ \vdots \]

(3.5)

Systems with higher derivatives have fields and their velocities as independent coordinates. For example, a system with two derivatives has its fields (denoting them generically by \( \phi \)) and their velocities \( \dot{\phi} \) as independent coordinates. If there are no constraints in the theory, the Poisson brackets (P.B.) are the bridge to the quantum commutator. Thus, we must have \([\phi, \dot{\phi}] = 0\). The commutators that might not be zero are those (in this example with two derivatives) involving \( \phi \) and \( \dot{\phi} \) with the higher derivatives \( \ddot{\phi} \) and \( \dddot{\phi} \).

The problem that comes out in the system we are studying is that there is an infinite number of time derivatives and it is not clear a priori which commutators are not zero. In order to try to figure them out we take the Lagrangian (3.4), expanded in \( t \) derivatives, till a certain limit order \( n \) and at the end we let \( n \) go to infinity. Let us then consider the expansion (3.2) till \( \partial^2_t \), which is the first nontrivial order,

\[ \mathcal{L}_3 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^2}{4} F_{\mu\nu} \left(1 + \frac{\partial^2}{\Delta^2}\right) \frac{1}{\Delta^2} F^{\mu\nu} - \frac{1}{2\alpha} (\partial_\mu A^\mu)^2. \] (3.6)

The equation of motion and the momenta are given by

\[ \partial^\mu F_{\mu\nu} - m^2 \left(1 + \frac{\partial^2}{\Delta^2}\right) \frac{\partial^\mu}{\Delta^2} F_{\mu\nu} + \frac{1}{\alpha} \partial_\mu \partial_\nu A^\mu = 0, \] (3.7)

\[ \pi_\nu = -F_{0\nu} + m^2 \left(1 + \frac{\partial^2}{\Delta^2}\right) \frac{1}{\Delta^2} F_{0\nu} + \frac{m^2}{2} \partial^i \partial_\nu F_{i\nu} - \frac{1}{\alpha} \eta_{0\nu} \partial_\mu A^\mu, \] (3.8)

\[ \pi^{(1)}_\nu = -\frac{m^2}{2} \frac{\partial_\mu}{\Delta^4} F_{\mu\nu}, \] (3.9)

\[ \pi^{(2)}_\nu = \frac{m^2}{2} \frac{1}{\Delta^4} F_{0\nu}. \] (3.10)

In this order, the phase-space coordinate is given by \((A_\mu, \pi_\nu) \oplus (\dot{A}_\mu, \pi^{(1)}_\nu) \oplus (\ddot{A}_\mu, \pi^{(2)}_\nu)\). We thus observe that relations (3.9) and (3.10) are constraints, as well as the zero component of \( \pi_\nu \). The fundamental nonvanishing P.B. are
\[
\{A_\mu(x, t), \pi_\nu(y, t)\} = \delta_\nu^\mu \delta(x - y), \\
\{\dot{A}_\mu(x, t), \pi_\nu^{(1)}(y, t)\} = \delta_\nu^\mu \delta(x - y), \\
\{\ddot{A}_\mu(x, t), \pi_\nu^{(2)}(y, t)\} = \delta_\nu^\mu \delta(x - y).
\] (3.11)

In order to calculate the P.B. matrix of the constraints, it is convenient to develop them separating all the velocities. The result is

\[
T_0 = \pi_0 + \left(\frac{1}{\alpha} + \frac{m^2}{2\sqrt{t}}\right) \dot{A}_0 + \frac{m^2}{2\sqrt{t}} \partial^i \ddot{A}_i + \frac{1}{\alpha} \partial_i A^i, \\
T_\nu^{(1)} = \pi_\nu^{(1)} - \frac{m^2}{2\sqrt{t}} \left[\nabla^2 A_\nu + \delta_\nu^0 \partial_i \ddot{A}_i - \delta_\nu^i (\ddot{A}_i - \partial_i \dot{A}_0 - \partial_i \partial_j A_j)\right], \\
T_\nu^{(2)} = \pi_\nu^{(2)} + \delta_\nu^i \frac{m^2}{2\sqrt{t}} \left(\partial_i A_0 - \dot{A}_i\right).
\] (3.12, 3.13, 3.14)

We observe that the last constraint for \(\nu = 0\) becomes \(T_0^{(2)} = \pi_0^{(2)}\). We also observe that the other constraints do not contain \(\ddot{A}_0\) and consequently the P.B. matrix for the constraints above will be singular (in fact, \(\ddot{A}_0\) does not play any role in the theory). Thus, instead of the constraint (3.14), we take

\[
T_i^{(2)} = \pi_i^{(2)} + \frac{m^2}{2\sqrt{t}} \left(\partial_i A_0 - \dot{A}_i\right).
\] (3.15)

The P.B. matrix of the constraints reads

\[
S = \begin{pmatrix}
0 & m^2 \delta_\nu^0 \left(\frac{1}{\alpha} + \frac{1}{\sqrt{t}}\right) & m^2 \partial_j \\
- m^2 \delta_\mu^0 \left(\frac{1}{\alpha} + \frac{1}{\sqrt{t}}\right) & - m^2 \left(\delta_\mu^0 \eta^{\nu \rho} + \delta_\nu^0 \eta^{0 \rho}\right) \frac{\partial_\rho}{\sqrt{t}} & m^2 \partial^i \frac{1}{\sqrt{t}} \\
m^2 \partial^i & - m^2 \delta_\nu^i \frac{1}{\sqrt{t}} & 0
\end{pmatrix} \delta(x - y).
\] (3.16)

Since this matrix involves space and time indices separately, the calculation of its inverse requires some care. The details of the calculation is presented in Appendix B. The result reads

\[
S^{-1} = \begin{pmatrix}
0 & - \alpha \delta_\nu^0 & \alpha \partial^j \\
\alpha \delta_\mu^0 - \alpha (\delta_\mu^0 \partial_k \partial_j + \delta_k^0 \delta_\nu^j \partial_j) & - \delta_\mu^0 \left(\frac{\delta_j^\mu \Sigma^j_{\mu \nu}}{m^2} - \alpha \partial_k \partial^j\right) & \alpha \partial^j \\
\alpha \partial_i & \delta_\nu^j \left(\frac{\delta_j^i \Sigma^j_{\mu \nu}}{m^2} - \alpha \partial_k \partial^j\right) & 0
\end{pmatrix} \delta(x - y).
\] (3.17)
With this inverse, we directly obtain the following Dirac brackets (D.B.) \[12\]

\[
\{ \dot{A}_\mu(\vec{x}, t), A^\nu(\vec{y}, t) \}_D = \alpha \delta_\mu^0 \delta_\nu^0 \delta(\vec{x} - \vec{y}),
\]

\[
\{ \ddot{A}_\mu(\vec{x}, t), A^\nu(\vec{y}, t) \}_D = \alpha \delta_\mu^i \delta_\nu^0 \partial_i \delta(\vec{x} - \vec{y}).
\] (3.18)

The bracket \(\{\ddot{A}_\mu, A^\nu\}\) is obtained from \(\{\pi_\mu, A^\nu\}\). The result is

\[
\{ \ddot{A}_\mu(x, t), A^\nu(\vec{y}, t) \} = - \delta_\mu^\nu \frac{\nabla^4}{m^2} \delta(\vec{x} - \vec{y}).
\] (3.19)

The commutators follow directly from expressions (3.18) and (3.19), i.e.

\[
[\dot{A}_\mu(x, t), A^\nu(\vec{y}, t)] = i \alpha \delta_\mu^0 \delta_\nu^0 \delta(\vec{x} - \vec{y}),
\]

\[
[\dot{A}_\mu(x, t), A^\nu(\vec{y}, t)] = i \alpha \delta_\mu^i \delta_\nu^0 \partial_i \delta(\vec{x} - \vec{y}),
\]

\[
[\ddot{A}_\mu(x, t), A^\nu(\vec{y}, t)] = - i \delta_\mu^\nu \frac{\nabla^4}{m^2} \delta(\vec{x} - \vec{y}).
\] (3.20)

It might be opportune and instructive to call our attention for the following fact. We notice that from the first commutator above we get \([\dot{A}_i(x, t), A^j(\vec{y}, t)] = 0\). Since there is no dependence of this result with the mass parameter \(m\), it may appear that there is a conflict with the limit case when \(m \rightarrow 0\), where the Maxwell theory is obtained. We know that this commutator is not zero in the Maxwell theory. What happens it that in the limit of \(m \rightarrow 0\), the structure of constraints is not the same as in the massful case, and consequently, the results we obtain in one sector cannot be kept in the other. We find important to explain this point with details in order to reinforce the formalism we are using. We use the Appendix C to do this.

Let us now consider the propagator calculation. One can directly show by using the path integral formalism that the propagator corresponds to the inverse of the operator that appears in the equation of motion. Considering (3.7), we have

\[
\left\{ \eta_{\mu\nu} \left[ 1 - \left( 1 + \frac{\partial^2}{\nabla^2} \right) \frac{m^2}{\nabla^2} \right] \Box + \left[ \frac{1}{\alpha} - 1 + \left( 1 + \frac{\partial^2}{\nabla^2} \right) \frac{m^2}{\nabla^2} \right] \partial_\mu \partial_\nu \right\} A^\nu = 0.
\] (3.21)

This means that the propagator must satisfies the equation

\[
\left\{ \eta_{\mu\nu} \left[ 1 - \left( 1 + \frac{\partial^2}{\nabla^2} \right) \frac{m^2}{\nabla^2} \right] \Box + \left[ \frac{1}{\alpha} - 1 + \left( 1 + \frac{\partial^2}{\nabla^2} \right) \frac{m^2}{\nabla^2} \right] \partial_\mu \partial_\nu \right\} \times T(A^\nu(x)A^\rho(x')) = i \delta_\mu^\rho \delta(\vec{x} - \vec{x}').
\] (3.22)

If what we have done till now is consistent, that is to say, if the quantization embodied in the commutators (3.20), the expression (3.22) ought to be verified.
In fact, after a hard algebraic calculation, we show that this actually occurs (see Appendix D for some details).

Let us now consider the Lagrangian with the next term of the expansion of $\Box^{-1}$,

$$\mathcal{L}_4 = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{m^2}{4} F_{\mu \nu} \left(1 + \frac{\partial_i^2}{\Box^2} + \frac{\partial_i^4}{\Box^4}\right) \frac{1}{\Box^2} F^{\mu \nu} - \frac{1}{2\alpha} \left(\partial_\mu A^\mu\right)^2. \quad (3.23)$$

Proceeding as before we obtain the equation of motion

$$\partial^\mu F_{\mu \nu} - m^2 \left(1 + \frac{\partial_i^2}{\Box^2} + \frac{\partial_i^4}{\Box^4}\right) \frac{\partial^\mu}{\Box^2} F_{\mu \nu} + \frac{1}{\alpha} \partial_\mu \partial_\nu A^\mu = 0 \quad (3.24)$$

and the momentum expressions

$$\pi_\nu = -F_{0 \nu} + m^2 \left(1 + \frac{\partial_i^2}{\Box^2} + \frac{\partial_i^4}{\Box^4}\right) \frac{1}{\Box^2} F_{0 \nu}
+ \frac{m^2}{2} \left(1 + \frac{\partial_i^2}{\Box^2}\right) \frac{\partial^i}{\Box^4} F_{\nu \mu} - \frac{1}{\alpha} m_0 \partial_\mu A^\mu,
\pi^{(1)}_\nu = -\frac{m^2}{2} \left(1 + \frac{\partial_i^2}{\Box^2}\right) \frac{1}{\Box^4} F_{\nu \mu} F_{\mu \nu},
\pi^{(2)}_\nu = \frac{m^2}{2} \left(1 + \frac{\partial_i^2}{\Box^2}\right) \frac{1}{\Box^2} F_{0 \nu} + \frac{m^2}{2} \partial_i \partial^i F_{\nu \mu},
\pi^{(3)}_\nu = -\frac{m^2}{2 \Box^6} \partial^\mu F_{\mu \nu},
\pi^{(4)}_\nu = \frac{m^2}{2 \Box^6} F_{0 \nu}. \quad (3.25)$$

The set of independent constraints is now given by

$$T_0 = \pi_0 + \frac{1}{\alpha} \partial_i A^i + \left(\frac{m^2}{2 \Box^2} + \frac{1}{\alpha}\right) \dot{A}_0 + \frac{m^2}{2} \partial^i \ddot{A}_i
+ \frac{m^2}{2 \Box^4} \ddot{A}_0 + \frac{m^2}{2 \Box^6} \dddot{A}_i,
T^{(1)}_\mu = \pi^{(1)}_\mu - \frac{m^2}{2 \Box^2} \left(A_\mu + \delta_\mu^i \partial_i A^j \right) - \frac{m^2}{2} \delta_\mu^i \partial^j \ddot{A}_i
- \frac{m^2}{2} \delta_\mu^i \partial_i \dddot{A}_0 - \frac{m^2}{2} \delta_\mu^i \partial_\nu \partial_\mu A^j
- \frac{m^2}{2} \delta_\mu^i \partial^j \ddot{A}_i - \frac{m^2}{2} \delta_\mu^i \partial_i \dddot{A}_0 + \frac{m^2}{2} \delta_\mu^i \partial^j \dddot{A}_i,
T^{(2)}_i = \pi^{(2)}_i + \frac{m^2}{2 \Box^4} A_0 + \frac{m^2}{2 \Box^6} \partial_\nu \partial_\mu A^j
+ \frac{m^2}{2 \Box^4} \ddot{A}_0 - \frac{m^2}{2 \Box^6} \dddot{A}_i,
\[
T_i^{(3)} = \pi_i^{(3)} - \frac{m^2}{2\nabla^4} A_i - \frac{m^2}{2} \frac{\partial_i \partial_j}{\nabla^6} A^j
- \frac{m^2}{2} \frac{\partial_i}{\nabla^6} \dot{A}_0 + \frac{m^2}{2\nabla^6} \ddot{A}_i,
\]
\[
T_i^{(4)} = \pi_i^{(4)} + \frac{m^2}{2} \frac{\partial_i}{\nabla^6} A_0 - \frac{m^2}{2\nabla^6} \dot{A}_i,\]

where, as before, velocities were conveniently separated. In the last constraint, we have not considered the index \( \mu = 0 \) because there is no other term involving \( \ddot{A}_0 \). We have not also considered the zero components of \( T_\mu^{(2)} \) and \( T_\mu^{(3)} \) because these components do not constitute independent constraints.

With these constraints, we calculate the D.B. involving fields and their velocities, in the same way we have done in the previous approximation. The quantization of the present approximation is expressed by the following commutators:

\[
[\dot{A}_\mu (\vec{x}, t), A' (\vec{y}, t)] = i \alpha \delta^0_\mu \delta^0_\nu \delta (\vec{x} - \vec{y}),
\]
\[
[\ddot{A}_\mu (\vec{x}, t), A' (\vec{y}, t)] = i \alpha \delta^k_\mu \delta^0_\nu \partial_k \delta (\vec{x} - \vec{y}),
\]
\[
[A_\mu (\vec{x}, t), A' (\vec{y}, t)] = 0,
\]
\[
[A_\mu (\vec{x}, t), A'' (\vec{y}, t)] = 0,
\]
\[
[\dot{A}_\mu (\vec{x}, t), A'' (\vec{y}, t)] = -i \delta^\nu_\mu \nabla^6 \delta (\vec{x} - \vec{y}),
\]

where \( A_\mu \) stands for five time derivatives over \( A_\mu \). Using the commutators above, one can also show that the propagator satisfies a similar relation like (3.23) with the operator that appears in the equation of motion (3.25). This shows that the commutators above are also consistent relations.

Now it is not difficult to infer the commutator relations when all the terms of the operator \( \Box^{-1} \) are taken into account. These are given by

\[
[\dot{A}_\mu (\vec{x}, t), A'' (\vec{y}, t)] = i \alpha \delta^0_\mu \delta^0_\nu \delta (\vec{x} - \vec{y}),
\]
\[
[\ddot{A}_\mu (\vec{x}, t), A'' (\vec{y}, t)] = i \alpha \delta^k_\mu \delta^0_\nu \partial_k \delta (\vec{x} - \vec{y}),
\]
\[
[A_\mu (\vec{x}, t), A'' (\vec{y}, t)] = 0,
\]
\[
[A_\mu (\vec{x}, t), A''' (\vec{y}, t)] = 0,
\]
\[
[\dot{A}_\mu (\vec{x}, t), A''' (\vec{y}, t)] = 0,
\]
\[
\vdots
\]
\[
\lim_{n \to \infty} \left( A_\mu^{(2n-1)} (\vec{x}, t), A'' (\vec{y}, t) \right) = -i \delta^\nu_\mu \frac{\nabla^2n}{m^2} \delta (\vec{x} - \vec{y}).
\]
We see in this way that the canonical structure, despite its nonlocality, is perfectly consistent with the functional procedure, generating propagators for the vectorial theory which display the presence of a massive field. To develop the theory furthermore, trying to construct the Fock space by introducing creation and annihilation operators for the vectorial fields, this seems to be nontrivial. This is so because there is no way, if \( m \) does not vanish, to avoid a canonical dependence between the vectorial field and its derivative of order \( 2n - 1 \), in the limit when \( n \) goes to infinity. We may say that the set of equations (3.28) shows us where the nonlocality problem appears in the process of quantization of these theories.

4 Conclusion

In this work we have considered the quantization of a nonlocal massive vector gauge invariant field theory, which can be effectively obtained from a vector-tensor theory with topological coupling. We have quantized this nonlocal system first by using the BV Lagrangian functional formalism, where the propagator could be obtained without major problems. After that, we have considered its canonical quantization, where the nonlocality becomes a more difficult problem to be circumvented. The non-localibility manifests itself through the canonical independence, at commutator level, between the gauge field and its derivative of order \( n \), in the limit when \( n \) goes to infinity. We have shown, however, that a systematic use of the canonical quantization procedure order by order permitted us to generate the same Greens functions as those obtained from the functional formalism. On the other hand, the Fock space structure seems difficult to be displayed, due to the odd canonical structure generated by the system. As it would be expected, when the mass parameter goes to zero, the tensor and vector sector of the theory decouple, and in the effective vector theory new constraints arise as a consequence of this limit. We have also shown in an appendix that a careful analysis of these constraints leads to a canonical structure that is identical to the usual massless gauge theory.

We could argue about the functional Hamiltonian quantization, due to Batalin, Fradkin and Vilkovisky (BFV) \[13\], of this nonlocal system. The use of this formalism here appears to be a nontrivial task. Even with the procedure of how to circumvent the infinity number of momenta, we still have an additional problem because velocities have to be considered as independent canonical coordinates in higher derivative systems. Consequently, it is necessary to distinguish in the Hamiltonian path integral formalism what is a time derivative of a coordinate and what is an independent coordinate itself \[14\]. This problem is presently under study and possible results shall be reported elsewhere \[15\].

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Appendix A

In this Appendix, we present some details of the calculation of Eq. (3.3). Considering the Lagrangian (3.1), we have for a general variation of the corresponding action,

\[
\delta \int_{t_0}^{t} dt \int d^3 \vec{x} \mathcal{L} = -\int_{t_0}^{t} dt \int d^3 \vec{x} \left[ \frac{1}{2} \partial_{\mu} \delta A_{\nu} \left( 1 + \frac{m^2}{\Box} \right) F^{\mu\nu} + \frac{1}{2} F_{\mu\nu} \left( 1 + \frac{m^2}{\Box} \right) \partial^{\mu} \delta A^{\nu} + \frac{1}{\alpha} \left( \partial_{\mu} A^{\mu} \right) \partial_{\nu} \delta A^{\nu} \right].
\]

(A.1)

Let us consider each term of the above expression in a separate way. The development of the first term leads to

\[
-\frac{1}{2} \int_{t_0}^{t} dt \int d^3 \vec{x} \partial_{\mu} \delta A_{\nu} \left( 1 + \frac{m^2}{\Box} \right) F^{\mu\nu}
\]

\[= -\frac{1}{2} \int_{t_0}^{t} dt \int d^3 \vec{x} \left\{ \partial_{\mu} \left[ \delta A_{\nu} \left( 1 + \frac{m^2}{\Box} \right) F^{\mu\nu} \right] - \delta A_{\nu} \left( 1 + \frac{m^2}{\Box} \right) \partial_{\mu} F^{\mu\nu} \right\},
\]

\[= -\frac{1}{2} \int d^3 \vec{x} \delta A_{\nu} \left( 1 + \frac{m^2}{\Box} \right) F^{0\nu}
\]

\[+ \frac{1}{2} \int_{t_0}^{t} dt \int d^3 \vec{x} \delta A_{\nu} \left( 1 + \frac{m^2}{\Box} \right) \partial_{\mu} F^{\mu\nu}. \quad (A.2)
\]

For the second term, we have

\[
-\frac{1}{2} \int_{t_0}^{t} dt \int d^3 \vec{x} F_{\mu\nu} \left( 1 + \frac{m^2}{\Box} \right) \partial^{\mu} \delta A^{\nu}
\]

\[= -\frac{1}{2} \int_{t_0}^{t} dt \int d^3 \vec{x} \left\{ \partial^{\mu} \left[ F_{\mu\nu} \left( 1 + \frac{m^2}{\Box} \right) \delta A^{\nu} \right] - \partial^{\mu} F_{\mu\nu} \left( 1 + \frac{m^2}{\Box} \right) \delta A^{\nu} \right\},
\]

\[= -\frac{1}{2} \int d^3 \vec{x} F_{0\nu} \left( 1 + \frac{m^2}{\Box} \right) \delta A^{\nu} + \frac{1}{2} \int_{t_0}^{t} dt \int d^3 \vec{x} \partial^{\mu} F_{\mu\nu} \left( 1 + \frac{m^2}{\Box} \right) \delta A^{\nu},
\]

\[= -\frac{1}{2} \int d^3 \vec{x} F_{0\nu} \delta A^{\nu} + \frac{1}{2} \int_{t_0}^{t} dt \int d^3 \vec{x} \partial^{\mu} F_{\mu\nu} \delta A^{\nu}
\]
\[
+ \frac{m^2}{2} \int d^3 \vec{x} \, F_{0 \nu} \left( \frac{1}{\nabla^2} + \frac{\partial_t^2}{\nabla^4} + \cdots \right) \delta A^\nu \\
- \frac{m^2}{2} \int_{t_0}^t d\tau \int d^3 \vec{x} \, \partial^\mu F_{\mu \nu} \left( \frac{1}{\nabla^2} + \frac{\partial_t^2}{\nabla^4} + \cdots \right) \delta A^\nu,
\] (A.3)

where we have used the expansion (3.2). We observe that in the last term of the expression above, there is an integration over time and an infinite number of time derivatives acting over \(\delta A^\nu\). It is necessary some care to deal with these terms. Let us consider some of them isolately

\[
- \frac{m^2}{2} \int_{t_0}^t d\tau \int d^3 \vec{x} \, \partial^\mu F_{\mu \nu} \frac{\partial^2}{\nabla^4} \delta A^\nu \\
= - \frac{m^2}{2} \int_{t_0}^t d\tau \int d^3 \vec{x} \left\{ \partial_t \left[ \frac{\partial^\mu}{\nabla^4} F_{\mu \nu} \partial_t \delta A^\nu \right] - \frac{\partial_t \partial^\mu}{\nabla^4} F_{\mu \nu} \partial_t \delta A^\nu \right\},
\]

\[
= - \frac{m^2}{2} \int d^3 \vec{x} \frac{\partial^\mu}{\nabla^4} F_{\mu \nu} \delta A^\nu + \frac{m^2}{2} \int d^3 \vec{x} \frac{\partial^\mu}{\nabla^4} \dot{F}_{\mu \nu} \delta A^\nu \\
- \frac{m^2}{2} \int_{t_0}^t d\tau \int d^3 \vec{x} \frac{\partial^\mu}{\nabla^4} \ddot{F}_{\mu \nu} \delta A^\nu.
\] (A.4)

In a similar way, we would have for the next term

\[
- \frac{m^2}{2} \int_{t_0}^t d\tau \int d^3 \vec{x} \, \partial^\mu F_{\mu \nu} \frac{\partial^2}{\nabla^6} \delta A^\nu \\
= - \frac{m^2}{2} \int d^3 \vec{x} \frac{\partial^\mu}{\nabla^6} F_{\mu \nu} \delta \dddot{A}^\nu + \frac{m^2}{2} \int d^3 \vec{x} \frac{\partial^\mu}{\nabla^6} \dddot{F}_{\mu \nu} \delta A^\nu \\
- \frac{m^2}{2} \int d^3 \vec{x} \frac{\partial^\mu}{\nabla^6} \dot{F}_{\mu \nu} \delta A^\nu + \frac{m^2}{2} \int d^3 \vec{x} \frac{\partial^\mu}{\nabla^6} \ddot{F}_{\mu \nu} \delta A^\nu \\
- \frac{m^2}{2} \int_{t_0}^t d\tau \int d^3 \vec{x} \frac{\partial^\mu}{\nabla^6} \partial^\nu F_{\mu \nu} \delta A^\nu,
\] (A.5)

and so on. Introducing these results into the initial expression (A.3), we obtain

\[
- \frac{1}{2} \int_{t_0}^t d\tau \int d^3 \vec{x} \, F_{\mu \nu} \left( 1 + \frac{m^2}{\Box} \right) \partial^\mu \delta A^\nu \\
= \frac{1}{2} \int_{t_0}^t d\tau \int d^3 \vec{x} \left( 1 + \frac{m^2}{\Box} \right) \partial^\mu F_{\mu \nu} \delta A^\nu
\]
\[ + \frac{1}{2} \int d^3 \vec{x} \left(-F_{0\nu} + m^2 \frac{1}{\nabla^2} F_{0\nu} + m^2 \frac{\partial^\mu}{\nabla^4} \bar{F}_{\mu\nu} + \ldots\right) \delta A^\nu \]
\[ - \frac{m^2}{2} \int d^3 \vec{x} \left( \frac{\partial^\mu}{\nabla^4} F_{\mu\nu} + \frac{\partial^\mu}{\nabla^6} \bar{F}_{\mu\nu} + \frac{\partial^\mu}{\nabla^8} \bar{F}_{\mu\nu} + \ldots\right) \delta A^\nu \]
\[ + \frac{m^2}{2} \int d^3 \vec{x} \left( \frac{1}{\nabla^4} F_{0\nu} + \frac{\partial^\mu}{\nabla^6} \bar{F}_{\mu\nu} + \frac{\partial^\mu}{\nabla^8} \bar{F}_{\mu\nu} + \ldots\right) \delta A^\nu \]
\[ - \frac{m^2}{2} \int d^3 \vec{x} \left( \frac{\partial^\mu}{\nabla^6} F_{\mu\nu} + \frac{\partial^\mu}{\nabla^8} \bar{F}_{\mu\nu} + \frac{\partial^\mu}{\nabla^{10}} \bar{F}_{\mu\nu} + \ldots\right) \delta \bar{A}^\nu \]
\[ + \ldots \quad (A.6) \]

We notice in the expression above that some terms can be put together to reobtain the nonlocal operator \( \Box^{-1} \). For example,
\[ \frac{1}{\nabla^2} F_{0\nu} + \frac{\partial^\mu}{\nabla^4} \bar{F}_{\mu\nu} + \frac{\partial^\mu}{\nabla^6} \bar{F}_{\mu\nu} + \ldots \]
\[ = \left( \frac{1}{\nabla^2} + \frac{\partial^2}{\nabla^4} + \ldots \right) F_{0\nu} + \left( \frac{1}{\nabla^4} + \frac{\partial^2}{\nabla^6} + \ldots \right) \partial^i \bar{F}_{i\nu} \]
\[ = - \frac{1}{\Box} \Box F_{0\nu} - \frac{1}{\nabla^2} \Box \partial^i \bar{F}_{i\nu}, \quad (A.7) \]
\[ \frac{\partial^\mu}{\nabla^4} F_{\mu\nu} + \frac{\partial^\mu}{\nabla^6} \bar{F}_{\mu\nu} + \frac{\partial^\mu}{\nabla^8} \bar{F}_{\mu\nu} + \ldots \]
\[ = - \frac{1}{\nabla^2} \Box F_{\mu\nu}. \quad (A.8) \]

and so on. Using these results into (A.6), we obtain the final form of the second term of (A.1).
\[ - \frac{1}{2} \int_{t_0}^t d\tau \int d^3 \vec{x} F_{\mu\nu} \left( 1 + \frac{m^2}{\Box} \right) \partial^\mu \delta A^\nu \]
\[ = \frac{1}{2} \int_{t_0}^t d\tau \int d^3 \vec{x} \left( 1 + \frac{m^2}{\Box} \right) \partial^\mu F_{\mu\nu} \delta A^\nu \]
\[ - \frac{1}{2} \int d^3 \vec{x} \left[ \left( 1 + \frac{m^2}{\Box} \right) F_{0\nu} + \frac{m^2}{\nabla^2} \Box F_{\nu\nu} \right] \delta A^\nu \]
\[ + \frac{m^2}{2} \int d^3 \vec{x} \frac{1}{\nabla^2} \Box F_{\mu\nu} \delta A^\nu \]
\[ - \frac{m^2}{2} \int d^3 \vec{x} \left( \frac{1}{\nabla^2} \Box F_{0\nu} + \frac{1}{\nabla^4} \Box \partial^i \bar{F}_{i\nu} \right) \delta A^\nu \]
\[ + \frac{m^2}{2} \int d^3 \vec{x} \frac{1}{\nabla^4} \Box F_{\nu\nu} \delta \bar{A}^\nu \]
\[ + \ldots \quad (A.9) \]
We finally consider the last term of expression (A.1).

\[- \frac{1}{\alpha} \int_{t_0}^{t} d\tau \int d^3 \bar{x} \partial_\mu A^\mu \partial_\nu \delta A^\nu \]
\[= - \frac{1}{\alpha} \int_{t_0}^{t} d\tau \int d^3 \bar{x} \left[ \partial_\nu (\partial_\mu A^\mu \delta A^\nu) - \partial_\nu \partial_\mu A^\mu \delta A^\nu \right], \]
\[= - \frac{1}{\alpha} \int_{t_0}^{t} d\tau \int d^3 \bar{x} \partial_\mu A^\mu \delta A^\nu + \frac{1}{\alpha} \int_{t_0}^{t} d\tau \int d^3 \bar{x} \partial_\nu \partial_\mu A^\mu \delta A^\nu. \quad (A.10)\]

Introducing the results given by expressions (A.2), (A.9) and (A.10) into the initial expression (A.1), the equation (3.3) is obtained.

**Appendix B**

In this appendix, we calculate the inverse of the matrix (3.16). First we notice it has the following block structure

\[
S = \begin{pmatrix}
\begin{pmatrix} 1 \times 1 \end{pmatrix} & \begin{pmatrix} 1 \times 4 \end{pmatrix} & \begin{pmatrix} 1 \times 3 \end{pmatrix} \\
\begin{pmatrix} 4 \times 1 \end{pmatrix} & \begin{pmatrix} 4 \times 4 \end{pmatrix} & \begin{pmatrix} 4 \times 3 \end{pmatrix} \\
\begin{pmatrix} 3 \times 1 \end{pmatrix} & \begin{pmatrix} 3 \times 4 \end{pmatrix} & \begin{pmatrix} 3 \times 3 \end{pmatrix}
\end{pmatrix}
\]

B.1

Of course, since the inverse $S^{-1}$ has the same block structure, we consider it is generically given by

\[
S^{-1} = \begin{pmatrix}
A & B^\rho & C^k \\
D_\nu & E_\nu^\rho & F_\nu^k \\
G_j & H_j^\rho & I_j^k
\end{pmatrix}
\]

B.2

We then must have

\[
\int d^3 \bar{y} \ S(\bar{x}, \bar{y}) S^{-1}(\bar{y} - \bar{z}) = \begin{pmatrix}
1 & 0 & 0 \\
0 & \delta_\mu^\rho & 0 \\
0 & 0 & \delta_j^k
\end{pmatrix} \delta(\bar{x} - \bar{z}). \quad (B.3)
\]
The combination of Eqs. (3.16), (B.2) and (B.3) gives us the following set of equations (after integrating over the intermediary variable \( \vec{y} \) and summing on mudding indices):

\[
\left( \frac{m^2}{\nabla^2} + \frac{1}{\alpha} \right) D_0 + \frac{m^2}{\nabla^4} \delta^j G_j = \delta(\vec{x} - \vec{z}),
\]

\[
\left( \frac{m^2}{\nabla^2} + \frac{1}{\alpha} \right) E_0^\rho + \frac{m^2}{\nabla^4} \delta^j H_j^\rho = 0,
\]

\[
\left( \frac{m^2}{\nabla^2} + \frac{1}{\alpha} \right) F_k^0 + \frac{m^2}{\nabla^4} \delta^j I_j^k = 0
\]

\[
\delta_0 \mu \left( \frac{m^2}{\nabla^2} + \frac{1}{\alpha} \right) A + \delta_0 \mu \frac{m^2}{\nabla^4} \partial_j D^j + \delta_0 \mu \frac{m^2}{\nabla^4} \partial_j D^0 - \delta_0 \mu \frac{m^2}{\nabla^4} G_j = 0
\]

\[
\delta_0 \mu \left( \frac{m^2}{\nabla^2} + \frac{1}{\alpha} \right) B^\rho + \delta_0 \mu \frac{m^2}{\nabla^4} \partial_j E_j^\rho + \delta_0 \mu \frac{m^2}{\nabla^4} \partial_j E_j^0 - \delta_0 \mu \frac{m^2}{\nabla^4} H_j^0 = \delta_0 \mu \delta(\vec{x} - \vec{y}),
\]

\[
\delta_0 \mu \left( \frac{m^2}{\nabla^2} + \frac{1}{\alpha} \right) C_k + \delta_0 \mu \frac{m^2}{\nabla^4} \partial_j F_j^k + \delta_0 \mu \frac{m^2}{\nabla^4} \partial_j F_j^0 - \delta_0 \mu \frac{m^2}{\nabla^4} I_j^k = 0
\]

\[
\partial_i A - D_i = 0,
\]

\[
\partial_i B^\rho - E_i^\rho = 0
\]

\[
\partial_i C^k - F_i^k = \nabla^4 \frac{\delta^k}{m^2} \delta(\vec{x} - \vec{z}),
\]

\[
\partial_i E_0^0 - H_i^0 = 0,
\]

\[
\partial_i E_0^k - H_i^k = -\nabla^4 \frac{\delta^k}{m^2} \delta(\vec{x} - \vec{z}),
\]

\[
\partial_i D_0 - G_i = 0,
\]

\[
\partial_i F_0^k - I_i^k = 0.
\]

The inverse \( S^{-1} \) is obtained by solving these equations. This is just a matter of algebraic work and the solution is

\[
A = 0,
\]
\[
B^0 = -\alpha \delta(x - z),
\]
\[
B^k = 0,
\]
\[
C^k = \alpha \partial^k \delta(x - z),
\]
\[
D_0 = \alpha \delta(x - z),
\]
\[
D_i = 0,
\]
\[
E^0_0 = 0,
\]
\[
E^0_i = -\alpha \partial_i \delta(x - z),
\]
\[
E^k_0 = -\alpha \partial^k \delta(x - z),
\]
\[
E^k_i = 0,
\]
\[
F_{k0} = 0,
\]
\[
F_{ki} = \left(\alpha \partial_i \partial^k - \delta^k_i \frac{\nabla^4}{m^2}\right) \delta(x - z),
\]
\[
G_j = \alpha \partial_j \delta(x - z),
\]
\[
H^0_j = 0,
\]
\[
H^k_j = -\left(\alpha \partial_j \partial^k - \delta^k_j \frac{\nabla^4}{m^2}\right) \delta(x - z),
\]
\[
I^k_j = 0.
\]
(B.8)

These are the elements of the matrix \(S^{-1}\) given by \((3.17)\).

**Appendix C**

When we take the limit \(m \to 0\) in expressions \((3.5)\), we get the following expressions for the momenta

\[
\pi_\nu = -F_{0\nu} - \frac{1}{\alpha \eta_{0\nu}} \partial_\mu A^\mu,
\]
\[
\pi_\nu^{(1)} = 0.
\]
(C.1)

The remaining momenta (which are all zero) do not make sense to be considered because in the limit \(m \to 0\) the system does not have infinite derivatives anymore. We observe that relations \((C.1)\) are constrains. So, the commutators cannot come from the P.B. of \(A_\mu\) and \(\dot{A}_\nu\), that is actually zero, but from the Dirac one. Let us calculate the D.B. of \(A_\mu\) and \(\dot{A}_\nu\). First, we need the P.B. matrix of the constraints. We denote these constraints by

\[
T_{1\mu} = \pi_\mu + F_{0\mu} + \frac{1}{\alpha \eta_{0\mu}} \partial_\nu A^\nu,
\]
\[
T_{2\mu} = \pi_\mu^{(1)}.
\]
(C.2)
Thus

$$
(S^\mu_\nu) = \begin{pmatrix}
\{T^\mu_1, T^\nu_1\} & \{T^\mu_1, T^\nu_2\} \\
\{T^\mu_2, T^\nu_1\} & \{T^\mu_2, T^\nu_2\}
\end{pmatrix}
$$

$$
= \begin{pmatrix}
0 & 1 \\
-1 & 0
\end{pmatrix}
\left(\eta^\mu_\nu + \frac{1-\alpha}{\alpha} \eta^0_\mu \eta^0_\nu\right) \delta(\vec{x} - \vec{y}).
$$

(C.3)

To calculate the D.B. we need the inverse of the matrix above. This is give by

$$
(S^-\nu_\mu)^{-1} = \begin{pmatrix}
0 & -1 \\
1 & 0
\end{pmatrix}
\left(\eta^\mu_\nu + (\alpha - 1) \eta^0_\mu \eta^0_\nu\right) \delta(\vec{x} - \vec{y}).
$$

(C.4)

Now, the D.B. can be directly calculated. The result is

$$
\{A_\mu(\vec{x}, t), \dot{A}^\nu(\vec{y}, t)\} = -\left(\delta^\nu_\mu + (\alpha - 1) \eta^0_\mu \delta^0_\nu\right) \delta(\vec{x} - \vec{y}).
$$

(C.5)

The commutator between $A_i$ and $\dot{A}^j$ can be directly obtained from the D.B. above and it is actually non zero when $m \to 0$, what makes consistent the procedure we are developing.

**Appendix D**

Considering that

$$
T\left(A^\nu(x)A^\rho(x')\right) = \theta(t - t') A^\nu(x)A^\rho(x') + \theta(t' - t) A^\rho(x')A^\nu(x)
$$

(D.1)

and using the commutators given by expression (3.20) we can obtain the following relations

$$
\frac{\partial^2}{\partial t^2} T\left(A^\nu(x)A^\rho(x')\right) = i \alpha \delta^\nu_\rho \eta^0_\mu \delta(x - x') + T\left(\dot{A}^\nu(x)A^\rho(x')\right),
$$

$$
\nabla^2 T\left(A^\nu(x)A^\rho(x')\right) = T\left(\nabla^2 A^\nu(x)A^\rho(x')\right),
$$

$$
\Box T\left(A^\nu(x)A^\rho(x')\right) = i \alpha \delta^\nu_\rho \eta^0_\mu \delta(x - x') + T\left(\Box A^\nu(x)A^\rho(x')\right),
$$

$$
\frac{1}{\nabla^2} \Box T\left(A^\nu(x)A^\rho(x')\right) = i \alpha \delta^\nu_\rho \eta^0_\mu \frac{1}{\nabla^2} \delta(x - x') + T\left(\frac{1}{\nabla^2} \Box A^\nu(x)A^\rho(x')\right),
$$

$$
\frac{\partial^2}{\nabla^2} \Box T\left(A^\nu(x)A^\rho(x')\right) = i \left[\alpha \delta^\nu_\rho \eta^0_\mu \frac{1}{\nabla^2} \Box + \alpha \eta^\mu_\nu \delta^\rho_0 \frac{\partial_i \partial_k}{\nabla^2}\right]
$$

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\[
- \eta^{\nu\rho} \nabla^2 \frac{m^2}{\delta(x - x')} + T \left( \frac{\partial^2}{\nabla^2} A^\nu(x) A^\rho(x') \right),
\]
\[
\partial_\mu \partial_\nu T \left( A^\nu(x) A^\rho(x') \right) = i \alpha \delta^0_\mu \delta^0_\rho \delta(x - x') + T \left( \partial_\mu \partial_\nu A^\nu(x) A^\rho(x') \right),
\]
\[
\frac{\partial_\mu \partial_\nu \partial^2}{\nabla^4} T \left( A^\nu(x) A^\rho(x') \right) = i \left[ \alpha \delta^0_\mu \delta^0_\rho \frac{\partial^2}{\nabla^4} + \alpha \delta^i_\mu \delta^0_\rho \frac{\partial_i \partial_t}{\nabla^4} - \delta^0_\mu \delta^0_\rho \frac{1}{m^2} \right. 
\]
\[
- \alpha \delta^0_\mu \delta^0_\rho \frac{1}{\nabla^2} \delta(x - x') + T \left( \frac{\partial_\mu \partial_\nu \partial^2}{\nabla^4} A^\nu(x) A^\rho(x') \right). \quad (D.2)
\]

Using the relations above in the left side of Eq. (3.22) we can show that the identity is actually satisfied.
References

[1] E. Cremmer and J. Scherk, Nucl Phys. B72 (1974) 117. See also, T.J. Allen, M.J. Bowick and A. Lahiri, Mod. Phys. Lett. A6 (1991) 559.

[2] M. Kalb and P. Ramond, Phys. Rev. D9 (1974) 2273.

[3] R. Amorim and J. Barcelos-Neto, Mod. Phys. Lett. A10 (1995) 917.

[4] R.K. Kaul, Phys. Rev. D18 (1978) 1127. C.R. Hagen, Phys. Rev. D19 (1979) 2367; V.O. Rivelles and L. Sandoval, Rev. Bras. Física 21 (1991) 274; A. Lahiri, Mod. Phys. Lett. A8 (1993) 2403; J. Barcelos-Neto and M.B.D. Silva, Int. J. Mod. Phys. A10 (1995) 3759; Mod. Phys. Lett. A11 (1996) 515.

[5] See also M. Henneaux and C. Teitelboim, Quantization of gauge systems (Princeton University Press, New Jersey, 1992) and references therein.

[6] D.Z. Freedman and P.K. Townsend, Nucl. Phys. B177 (1981) 282.

[7] J. Barcelos-Neto, A. Cabo and M.B.D. Silva, Z. Phys. C72 (1996) 34; A. Lahiri, Phys. Rev. D55 (1997) 5045.

[8] We mention that quantization of nonlocal theories has been carried out by other authors, but not in the same context that will be presented here. See, for example, E.C. Marino and R.L.P.G. Amaral, J. Phys. A25 (1992) 5183; D.G. Barci, L.E. Oxman, and M. Rocca, Int. J. Mod. Phys. A11 (1996) 2111.

[9] I.A. Batalin and G.A. Vilkovisky, Phys. Lett. B102 (1981) 27; Phys. Rev. D28 (1983) 2567.

[10] For a detailed discussion on systems with higher derivatives, see D. Musik, Degenerate systems in generalized mechanics (Beograd, Publ. Inst. Math., 1992) and references therein.

[11] J. Barcelos-Neto and N.R.F. Braga, Mod. Phys. Lett. A4 (1989) 2195.

[12] P.A.M. Dirac, Can. J. Math. 2 (1950) 129; Lectures on quantum mechanics (Yeshiva University, New York, 1964).

[13] G.A. Vilkovisky, Phys. Lett. B55 (1975) 224; I.A. Batalin, G.A. Vilkovisky, Phys. Lett. B69 (1977) 309; E.S. Fradkin and T.E. Fradkina, Phys. Lett. B72 (1978) 343.

[14] J. Barcelos-Neto and C.P. Natividade, Z. Phys. C51 (1991) 313.

[15] R. Amorim and J. Barcelos-Neto, work in progress.