Three-Dimensional SCFTs, Supersymmetric Domain Wall and Renormalization Group Flow

Changhyun Ahn and Jinsub Paeng

Department of Physics, Kyungpook National University, Taegu 702-701 Korea

ahn@knu.ac.kr

abstract

By analyzing $SU(3) \times U(1)$ invariant stationary point, studied earlier by Nicolai and Warner, of gauged $\mathcal{N} = 8$ supergravity, we find that the deformation of $S^7$ gives rise to nontrivial renormalization group flow in a three-dimensional boundary super conformal field theory from $\mathcal{N} = 8$, $SO(8)$ invariant UV fixed point to $\mathcal{N} = 2$, $SU(3) \times U(1)$ invariant IR fixed point. By explicitly constructing 28-beins $u, v$ fields, that are an element of fundamental 56-dimensional representation of $E_7$, in terms of scalar and pseudo-scalar fields of gauged $\mathcal{N} = 8$ supergravity, we get $A_1, A_2$ tensors. Then we identify one of the eigenvalues of $A_1$ tensor with “superpotential” of de Wit-Nicolai scalar potential and discuss four-dimensional supergravity description of renormalization group flow, i.e. the BPS domain wall solutions which are equivalent to vanishing of variation of spin $1/2, 3/2$ fields in the supersymmetry preserving bosonic background of gauged $\mathcal{N} = 8$ supergravity. A numerical analysis of the steepest descent equations interpolating two critical points is given.
1 Introduction

Few examples are known for three-dimensional interacting conformal field theories, mainly due to strong coupling dynamics in the infrared (IR) limit. In the previous papers \cite{1,2}, three-dimensional (super)conformal field theories were classified by utilizing the AdS/CFT correspondence \cite{3,4,5} and earlier, exhaustive study of the Kaluza-Klein supergravity \cite{6}.

The simplest spontaneous compactification of the eleven-dimensional supergravity \cite{7} is the Freund-Rubin \cite{8} compactification to a product of $AdS_4$ space-time and an arbitrary compact Einstein manifold $X_7$ of positive scalar curvature. The best known example is provided by round- and squashed-$S^7$. The standard Einstein metric of the round-$S^7$ yields a vacuum with $SO(8)$ gauge symmetry and $\mathcal{N} = 8$ supersymmetry. The second, squashed Einstein metric \cite{9}, yields a vacuum with $SO(5) \times SO(3)$ gauge symmetry and $\mathcal{N} = 1$ (or 0) supersymmetry, depending on the orientation of the $S^7$ \cite{10}. In \cite{1}, the well-known spontaneous (super)symmetry breaking deformation from round- to squashed-$S^7$ was mapped to a renormalization group (RG) flow from $\mathcal{N} = 0$ (or 1), $SO(5) \times SO(3)$ invariant fixed point in the ultraviolet (UV) to $\mathcal{N} = 8$, $SO(8)$ invariant fixed point in the IR. The squashing deformation corresponded to an irrelevant operator at the $\mathcal{N} = 8$ superconformal fixed point and a relevant operator at the $\mathcal{N} = 1$ (or 0) (super)conformal fixed point, respectively.

In contrast to the Freund-Rubin compactifications, the symmetry of the vacuum of Englert type compactification is no longer given by the isometry group of $X_7$ but rather by the group which leaves invariant both the metric and four-form magnetic field strength. By generalizing compactification vacuum ansatz to the nonlinear level, solutions of the eleven-dimensional supergravity were obtained directly from the scalar and pseudo-scalar expectation values at various critical points of the $\mathcal{N} = 8$ supergravity potential \cite{11}. They reproduced all known Kaluza-Klein solutions of the eleven-dimensional supergravity: round $S^7$ \cite{12}, $SO(7)^{-}$-invariant, \textit{parallelized} $S^7$ \cite{13}, $SO(7)^{+}$-invariant vacuum \cite{14}, $SU(4)^{-}$-invariant vacuum \cite{15}, and a new one with $G_2$ invariance. Among them, round $S^7$- and $G_2$-invariant vacua are stable, while $SO(7)^{+}$-invariant ones are known to be unstable \cite{16}. In \cite{2}, via AdS/CFT correspondence, deformation of $S^7$ was interpreted as renormalization group flow from $\mathcal{N} = 8$, $SO(8)$ invariant UV fixed point to $\mathcal{N} = 1$, $G_2$ invariant IR fixed point by analyzing de Wit-Nicolai potential.

In this paper, we will continue to analyze a vacuum of $\mathcal{N} = 8$ supergravity with $SU(3) \times U(1)$ symmetry, studied earlier by Nicolai and Warner \cite{17}, that was considered very briefly in \cite{2}. In section 2, by explicitly constructing 28-beins $u, v$ fields, that are an element of fundamental 56-dimensional representation of $E_7$, in terms of scalar and pseudo-scalar fields of $\mathcal{N} = 8$ supergravity, we get $A_1, A_2$ tensors (\cite{4}, \cite{4}). In section 3, we will be identifying a deformation which gives rise to a renormalization group flow associated with the symmetry breaking $SO(8) \rightarrow SU(3) \times U(1)$ (both of which are stable vacua) and find that the deformation
operator is relevant at the $SO(8)$ fixed point but becomes irrelevant at the $SU(3) \times U(1)$ fixed point. In section 4, we identify one of the eigenvalues of $A_1$ tensor with “superpotential” of de Wit-Nicolai scalar potential and discuss the BPS domain wall solutions. Finally in appendix, there exist some details. See also recent papers [18] on RG flows and AdS/CFT correspondence.

## 2 de Wit-Nicolai Potential and AdS$_4$ Supergravity Vacua

de Wit and Nicolai [19, 20] constructed a four-dimensional supergravity theory by gauging the $SO(8)$ subgroup of $E_7$ in the global $E_7 \times$ local $SU(8)$ supergravity of Cremmer and Julia [7]. In common with Cremmer-Julia theory, this theory contains self-interaction of a single massless $\mathcal{N}=8$ supermultiplet of spins $(2,3/2,1,1/2,0^+,0^-)$ but with local $SO(8) \times$ local $SU(8)$ invariance. It is well known [21] that the 70 real scalars of $\mathcal{N}=8$ supergravity live on the coset space $E_7/SU(8)$ since 63 fields may be gauged away by an $SU(8)$ rotation and are described by an element $\mathcal{V}(x)$ of the fundamental 56-dimensional representation of $E_7$:

$$\mathcal{V}(x) = \begin{pmatrix}
    u_{ij}^{IJ}(x) & v_{ijKL}(x) \\
    u^{ij}^{IJ}(x) & u_{ij}^{KL}(x)
\end{pmatrix}, \quad (1)$$

where $SU(8)$ index pairs $[ij], \cdots$ and $SO(8)$ index pairs $[IJ], \cdots$ are antisymmetrized and therefore $u$ and $v$ fields are $28 \times 28$ matrices and $x$ is the coordinate on 4-dimensional space-time. Complex conjugation can be done by raising or lowering those indices, for example, $(u_{ij}^{IJ})^* = u^{ij}_{IJ}$ and so on. Under local $SU(8)$ and local $SO(8)$, the matrix $\mathcal{V}(x)$ transforms as $\mathcal{V}(x) \rightarrow U(x)\mathcal{V}(x)O^{-1}(x)$ where $U(x) \in SU(8)$ and $O(x) \in SO(8)$ and matrices $U(x)$ and $O(x)$ are in the appropriate 56-dimensional representation. In the gauged supergravity theory, the 28-vectors transform in the adjoint of $SO(8)$ with resulting non-abelian field strength.

Although the full gauged $\mathcal{N}=8$ Lagrangian is rather complicated [20], the scalar and gravity part of the action is simple (we are considering a gravity coupled to scalar field theory since matter fields do not play a role in domain wall solutions of section 4) and maybe written as

$$\int d^4x \sqrt{-g} \left( \frac{1}{2} R - \frac{1}{96} |A_{ijkl}^\mu|^2 - V \right) \quad (2)$$

where the scalar kinetic terms are completely antisymmetric and self-dual in its indices:

$$A_{ijkl}^\mu = -2\sqrt{2} \left( u_{ij}^{iJ} \partial_\mu v^{kIJ} - v^{ijI} \partial_\mu u_{kI}^{jJ} \right) \quad (3)$$

and $|A_{ijkl}^\mu|^2$ is a product of $A_{ijkl}^\mu$ and its complex conjugation as above and $\mu$ is the 4-dimensional space-time index. Let us define $SU(8)$ so called T-tensor which is cubic in the 28-beins $u$ and $v$, manifestly antisymmetric in the indices $[ij]$ and $SU(8)$ covariant. This comes from naturally by introducing a local gauge coupling in the theory. Furthermore, other tensors...
coming from T-tensor play an important role in this paper and scalar structure is encoded in two $SU(8)$ tensors. That is, $A_{1}^{ij}$ tensor is symmetric in $(ij)$ and $A_{2i}^{ijk}$ tensor is antisymmetric in $[ijk]$:

$$T_{i}^{kij} = (u_{i}^{j}u_{j}^{I} + v_{i}^{j}u_{j}^{IJ}) (u_{m}^{JK}u_{m}^{km} - v_{m}^{JK}v_{m}^{kmKL}),$$

$$A_{1}^{ij} = -\frac{4}{21} T_{m}^{ijm}, \quad A_{2i}^{ijk} = -\frac{4}{3} T_{i}^{[ijk]}.$$  

(4)

Then de Wit-Nicolai effective nontrivial potential arising from $SO(8)$ gauging can be written as compact form:

$$V = -g^{2} \left( \frac{3}{4} |A_{1}^{ij}|^{2} - \frac{1}{24} |A_{2i}^{ijk}|^{2} \right)$$

(5)

where $g$ is a $SO(8)$ gauge coupling constant and it is understood that the squares of absolute values of $A_{1}, A_{2}$ are nothing but a product of those and its complex conjugation on 28-beins $u$ and $v$. The 56-bein $V(x)$ can be brought into the following form by the gauge freedom of $SU(8)$ rotation

$$V(x) = \exp \left( \begin{array}{c}
0 \\
\phi^{mpq}(x) \\
\phi_{ijkl}(x) \\
0 
\end{array} \right),$$

(6)

where $\phi^{ijkl}$ is a complex self-dual tensor describing the 35 scalars $35_{v}$ (the real part of $\phi^{ijkl}$) and 35 pseudo-scalar fields $35_{c}$ (the imaginary part of $\phi^{ijkl}$) of $\mathcal{N} = 8$ supergravity. After gauge fixing, one does not distinguish between $SO(8)$ and $SU(8)$ indices. The scalar potential of gauged $\mathcal{N} = 8$ supergravity has four stationary points with at least $G_{2}$ invariance [22]. The full supersymmetric solution where both $35_{v}$ scalars and $35_{c}$ pseudo-scalars vanish yields $SO(8)$ vacuum state with $\mathcal{N} = 8$ supersymmetry (Note that $SU(8)$ is not a symmetry of the vacuum).

It is known that, in $\mathcal{N} = 8$ supergravity, there also exists a $\mathcal{N} = 2$ supersymmetric, $SU(3) \times U(1)$ invariant vacuum [17]. To reach this critical point, one has to turn on expectation values of both scalar $\lambda(x)$ and pseudo-scalar $\lambda'(x)$ fields as

$$\langle \phi_{ijkl}(x) \rangle = \frac{1}{2\sqrt{2}} \left( \lambda(x) X_{ijkl}^{+} + i\lambda'(x) X_{ijkl}^{-} \right),$$

where $X_{ijkl}^{+}$ and $X_{ijkl}^{-}$ are the unique (up to scaling), completely antisymmetric self-dual and anti-self-dual tensors which are invariant under $SU(3) \times U(1)$,

$$X_{ijkl}^{+} = +[(\delta_{ijkl}^{1234} + \delta_{ijkl}^{5678}) + (\delta_{ijkl}^{1256} + \delta_{ijkl}^{3478}) + (\delta_{ijkl}^{1278} + \delta_{ijkl}^{3456})]$$

$$X_{ijkl}^{-} = -[(\delta_{ijkl}^{1357} - \delta_{ijkl}^{2468}) + (\delta_{ijkl}^{1368} - \delta_{ijkl}^{2457}) + (\delta_{ijkl}^{1458} - \delta_{ijkl}^{2367}) - (\delta_{ijkl}^{1467} - \delta_{ijkl}^{2358})].$$

Therefore 56-beins $V(x)$ can be written as $56 \times 56$ matrix whose elements are some functions of scalar and pseudo-scalars by exponentiating the vacuum expectation value $\phi_{ijkl}$. On the
other hand, 28-beins $u$ and $v$ are an element of this $\mathcal{V}(x)$ according to (1). One can construct 28-beins $u$ and $v$ in terms of $\lambda(x)$ and $\lambda'(x)$ fields explicitly and they are given in the appendix (20). Now it is ready to get the informations on the $A_1$ and $A_2$ tensors in terms of $\lambda(x)$ and $\lambda'(x)$ via (4).

It turned out that $A_1^{IJ}$ tensor has two distinct eigenvalues, $z_1$, $z_2$ with degeneracies 6, 2 respectively and has the following form

$$A_1^{IJ} = \text{diag} (z_1, z_1, z_1, z_1, z_2, z_2),$$  \hspace{1cm} (7)

where

$$z_1 = \frac{1}{4} (p + q) (c(3 + m') - 2s), \quad z_2 = \frac{1}{4} (p + q) (c(3 + m') - 2sm')$$

and

$$p \equiv \cosh(\lambda/2\sqrt{2}), \quad q \equiv \sinh(\lambda/2\sqrt{2}), \quad p' \equiv \cosh(\lambda'/2\sqrt{2}), \quad q' \equiv \sinh(\lambda'/2\sqrt{2})$$

$$m \equiv \cosh(\sqrt{2}\lambda), \quad n \equiv \sinh(\sqrt{2}\lambda), \quad m' \equiv \cosh(\sqrt{2}\lambda'), \quad n' \equiv \sinh(\sqrt{2}\lambda')$$

$$c \equiv \cosh(\lambda/\sqrt{2}), \quad s \equiv \sinh(\lambda/\sqrt{2}), \quad c' \equiv \cosh(\lambda'/\sqrt{2}), \quad s' \equiv \sinh(\lambda'/\sqrt{2}).$$ \hspace{1cm} (8)

The eigenvalue $z_2$ at the $SU(3) \times U(1)$ critical point is equal to $-\sqrt{-\frac{\Lambda_{SU(3)} \times U(1)}{6}} \frac{\ell_{\text{pl}}}{g}$ where $\Lambda_{SU(3) \times U(1)}$ is the cosmological constant at that point, as we will see later. So it has unbroken $\mathcal{N} = 2$ supersymmetry:the number of supersymmetries is equal to the number of eigenvalue of $A_1$ tensor for which $|\text{eigenvalue of } A_1| = \ell_{\text{pl}} \sqrt{-\frac{\Lambda}{6g^2}}$. On the other hand, at the $SO(8)$ critical point, $z_1 = z_2$ and since these 8 eigenvalues are equal to $-\ell_{\text{pl}} \sqrt{-\frac{\Lambda}{6g^2}}$, this gives $\mathcal{N} = 8$ supersymmetry as we expected.

Similarly $A_2,_{L}^{IJK}$ tensor can be obtained from the triple product of $u$ and $v$ fields by definition (4). It turns out that they are written in terms of five kinds of fields $y_1, y_2, y_3, y_4$ and $y_5$ and are given in the appendix:

$$y_1 = \frac{1}{4} (p + q) (c(1 - m') - 2s), \quad y_2 = \frac{1}{4} (p + q) (c(1 - m') + 2s) = \frac{2\sqrt{2}}{3} \frac{\partial z_2}{\partial \lambda},$$

$$y_3 = \frac{1}{4} (p + q) (c - m'(c - 2s)), \quad y_4 = -\frac{1}{4} i(p + q)cn',$$

$$y_5 = -\frac{1}{4} i(p + q)(c - 2s)n' = \frac{-i}{\sqrt{2}} \frac{\partial z_2}{\partial \lambda'}. \hspace{1cm} (9)$$

Note the expression of $z_2$ and its derivatives with respect to $\lambda$ and $\lambda'$ which are $y_2$ and $y_5$ up to some constant, respectively. This observation will be crucial as we discuss about supersymmetry variations in the context of BPS domain wall solutions in section 4. One of the eigenvalues of $A_1^{IJ}$ tensor, $z_2$ will provide a “superpotential” of $V$ in section 4.
Finally the scalar potential can be written by combining all the components of $A_1, A_2$ tensors using the form of (5) as

\[
V = -g^2 \left( \frac{3}{4} \left( 2|z_2|^2 + 6|z_1|^2 \right) - \frac{2}{24} \left( 36|y_1|^2 + 18|y_2|^2 + 18|y_3|^2 + 72|y_4|^2 + 24|y_5|^2 \right) \right)
= 2g^2 e^2 \left( (s^3 + c^3) s'^2 - 3c \right)
= \frac{1}{32} g^2 e^{-\frac{1}{\sqrt{2}} \lambda - 2\sqrt{2} \lambda'} \left( 1 + e^{\sqrt{2} \lambda'} \right)^2 \left[ 3 + e^{2\sqrt{2} \lambda} - 30e^{\sqrt{2} \lambda'} + 3e^{2\sqrt{2} \lambda'} - 24e^{\sqrt{2}(\lambda + \lambda')} + e^{2\sqrt{2}(\lambda + \lambda')} \right]
\]

which is exactly the same form obtained by Warner \[22\] sometime ago using $SU(8)$ coordinate system as an alternative approach.

\[
\text{Figure 1: Scalar potential } V(\lambda, \lambda'). \text{ The left axis corresponds to } \lambda \text{ and right one does } \lambda'. \text{ The extremum value } V = -9\sqrt{3}/2 = -7.79 \text{ for } SU(3) \times U(1) \text{ occurs around } \lambda = 0.78 \text{ and } \lambda' = 0.93 \text{ while the local maximum value } V = -6 \text{ for } SO(8) \text{ appears around } \lambda = 0 \text{ and } \lambda' = 0. \text{ We take } g^2 \text{ as 1 for simplicity.}
\]

The $AdS_4$-invariant ground-states correspond to $\lambda, \lambda'$ taking constant values and the space-time curvature maximally symmetric. The two vacua are as follows:

| Gauge symmetry     | $\lambda$ | $\lambda'$ | $V$       |
|--------------------|-----------|------------|-----------|
| $SO(8)$            | 0         | 0          | $-6g^2$   |
| $SU(3) \times U(1)$| $\sqrt{2} \sinh^{-1} \left( \frac{1}{\sqrt{3}} \right) = 0.78$ | $\sqrt{2} \sinh^{-1} \left( \frac{1}{\sqrt{2}} \right) = 0.93$ | $-\frac{9\sqrt{3}}{2}g^2 = -7.79g^2$ |
Table 1. *Summary of two critical points: symmetry group, vacuum expectation values of fields, and cosmological constants.*

The scalar potential \( V(\lambda, \lambda') \) depicted in Figure 1 exhibits the two critical points: \( SO(8) \) point is a maximum point while \( SU(3) \times U(1) \) is an other extremum point. The former is invariant under the full \( SO(8) \) group while the latter is invariant only under the \( SU(3) \times U(1) \) subgroup.

### 3 Three-Dimensional Super Conformal Field Theories

In this section, by exploiting the results of section 2 on the Kaluza-Klein spectrum under the deformation, we will find an operator that gives rise to a renormalization group flow associated with the symmetry breaking \( SO(8) \rightarrow SU(3) \times U(1) \) and get that the operator is relevant at the \( SO(8) \) fixed point but becomes irrelevant at the \( SU(3) \times U(1) \) fixed point.

#### 3.1 \( SO(8) \) Invariant Conformal Fixed Point

We will be identifying a renormalization group flow associated with symmetry breaking \( SO(8) \rightarrow SU(3) \times U(1) \) in a three-dimensional strongly coupled field theory. We will show that the perturbation operator is relevant at the \( SO(8) \) invariant UV fixed point corresponding to \( OSp(8|4) \) extended supersymmetry but becomes irrelevant at the \( SU(3) \times U(1) \) invariant IR fixed point corresponding to \( OSp(2|4) \) extended supersymmetry. To identify conformal field theory operator corresponding to the perturbation while preserving \( SU(3) \times U(1) \) symmetry, we will consider harmonic fluctuations of space-time metric and \( \lambda(x) \) scalar field and \( \lambda'(x) \) pseudo-scalar field around \( AdS_4 \times S^7 \). From the scalar potential Eq. (10), one finds that the cosmological constant \( \Lambda \) at \( SO(8) \) fixed point is given by

\[
\Lambda_{SO(8)} = -6g^2 \equiv -\frac{3}{r_{UV}^2 \ell_{pl}^2},
\]

where \( r_{UV} \) is the radius of \( AdS_4 \) and \( \ell_{pl} \) is the eleven-dimensional Planck scale. Conformal dimension of the perturbation operator representing this deformation is calculated by fluctuation spectrum of the scalar and pseudo-scalar fields. From the scalar kinetic terms of \(-|A_\mu^{IJKL}|^2/96 \) and the explicit forms of \( u_{KL}^{IJ} \) and \( v^{IJKL} \) in Eq. (20), the resulting kinetic term turns out to be

\[
-\frac{1}{2} \left[ \frac{3}{8} (\partial_\mu \lambda)^2 + (\partial_\mu \lambda')^2 \right].
\]

After rescaling the \( \lambda \) and \( \lambda' \) fields as \( \overline{\lambda} = \sqrt{\frac{3}{4}} \lambda, \overline{\lambda'} = \lambda' \), one finds that the mass spectrum of the \( \overline{\lambda} \) field around \( SO(8) \) fixed point in the unit of inverse radius of \( AdS_4 \) is given by:

\[
M_{\overline{\lambda}}^2(SO(8)) = \left[ \frac{\partial^2 V}{\partial \overline{\lambda}^2} \right]_{\overline{\lambda} = \overline{\lambda'} = 0} = -4g^2 \ell_{pl}^2 = -2 \frac{1}{r_{UV}^2}.
\]
Similarly, the mass spectrum of the $\mathcal{N}$ field around $SO(8)$ fixed point is given by:

$$M_{\mathcal{N}}^2(SO(8)) = \left[\frac{\partial^2 V}{\partial \mathcal{N}^2}\right]_{\mathcal{N}=\mathcal{N}=0} = -4g^2\ell^2_{pl} = -2\frac{1}{r^2_{UV}}. \quad (12)$$

Via AdS/CFT correspondence, one finds that in the corresponding $\mathcal{N} = 8$ superconformal field theory, the $SU(3) \times U(1)$ symmetric deformation ought to be a relevant perturbation of conformal dimension $\Delta = 1$ or $\Delta = 2$. Recall that, on $S^7$, mass spectrum of the representation corresponding to $SO(8)$ Dynkin label $(n, 0, 2, 0)$ is given by $\tilde{M}^2 = ((n + 1)^2 - 9)m^2$ where $m^2$ is mass-squared parameter of a given $AdS_4$ space-time and a scalar field $S$ satisfies $(\Delta_{AdS} + \tilde{M}^2)S = 0$. This follows from the known mass formula \cite{23} $M^2 = ((n + 1)^2 - 1)m^2$ for $0^+(1)$. For $35_c$ corresponding to $n = 0$, $\tilde{M}^2_{35_c} = -8m^2$ and this ought to equal to Eq. (12). Recalling that $r^2_{UV} = r^2_{S^7}/4 = 1/4m^2$,

$$\left[\frac{\partial^2 V}{\partial \mathcal{N}^2}\right]_{\mathcal{N}=\mathcal{N}=0} = -2\frac{1}{r^2_{UV}} = \tilde{M}^2_{35_c}. $$

The conformal dimensions of the corresponding chiral operators in the SCFT side are $\Delta = (n + 4)/2$. Some of these operators may be identified with a product of two fermions times $n$ scalars. Then $35_c$ pseudo-scalars correspond to the conformal primaries of $\Delta = 2$ which consist of quadratic of Majorana gauginos in the irreducible representations $8_c$ of $SO(8)$ of 3-dimensional $\mathcal{N} = 8$ $SU(N_c)$ gauge theory living on the worldvolume of $N_c$ coincident M2 branes.

On the other hand, the mass spectrum of the representation corresponding to $SO(8)$ Dynkin label $(n + 2, 0, 0, 0)$ is given by $\tilde{M}^2 = ((n - 1)^2 - 9)m^2$. This follows from the known mass formula \cite{23} $M^2 = ((n - 1)^2 - 1)m^2$ for $0^+(1)$. For $35_v$ corresponding to $n = 0$, $\tilde{M}^2_{35_v} = -8m^2$ and this ought to equal to Eq. (11):

$$\left[\frac{\partial^2 V}{\partial \mathcal{N}^2}\right]_{\mathcal{N}=\mathcal{N}=0} = -2\frac{1}{r^2_{UV}} = \tilde{M}^2_{35_v}. $$

The conformal dimensions of the corresponding chiral operators in the SCFT side are $\Delta = (n + 2)/2$. Some of these operators may be identified with a product of $(n + 2)$’s scalar fields in the vector multiplet. The $35_v$ scalars correspond to the conformal primaries of $\Delta = 1$ which consist of quadratic of real scalars in the irreducible representation $8_v$ of $SO(8)$ of 3-dimensional $\mathcal{N} = 8$ $SU(N_c)$ gauge theory.

### 3.2 $SU(3) \times U(1)$ Invariant Conformal Fixed Point

Let us next consider the conformal fixed point corresponding to the $SU(3) \times U(1)$ symmetry. Again, from the scalar potential Eq. (11), one finds that cosmological constant $\Lambda$ at $SU(3) \times U(1)$
fixed point is given by

\[ \Lambda_{SU(3) \times U(1)} = -\frac{9\sqrt{3}}{2} g^2 \equiv -\frac{3}{r_{IR}^2 \ell_{pl}^2}. \]

One calculates mass spectrum of the scalar and pseudo-scalar fields straightforwardly:

\[
\begin{align*}
M_{\lambda\lambda}^2(SU(3) \times U(1)) &= \left[ \frac{\partial^2 V}{\partial \lambda^2} \right]_{\lambda_{ext}, \bar{\lambda}_{ext}} = 3\sqrt{3}g^2\ell_{pl}^2, \\
M_{\lambda\lambda'}^2(SU(3) \times U(1)) &= \left[ \frac{\partial^2 V}{\partial \lambda \partial \lambda'} \right]_{\lambda_{ext}, \bar{\lambda}_{ext}} = 6\sqrt{3}g^2\ell_{pl}^2, \\
M_{\lambda\lambda'}^2(SU(3) \times U(1)) &= \left[ \frac{\partial^2 V}{\partial \lambda'^2} \right]_{\lambda_{ext}, \bar{\lambda}_{ext}} = 6\sqrt{3}g^2\ell_{pl}^2,
\end{align*}
\]

where \( \lambda_{ext} \) and \( \bar{\lambda}_{ext} \) takes the form of vacuum expectation values in Table 1.

\[
\sinh \left( \frac{\sqrt{2}}{3} \lambda_{ext} \right) = \frac{1}{\sqrt{3}}, \quad \sinh \left( \frac{1}{\sqrt{2}} \bar{\lambda}_{ext} \right) = \frac{1}{\sqrt{2}}.
\]

Diagonalizing the mass matrix, one obtains the mass eigenvalues as follows:

\[
M^2 = \frac{3}{2} \left( 3 - \sqrt{17} \right) \times \sqrt{3}g^2\ell_{pl}^2, \quad \frac{3}{2} \left( 3 + \sqrt{17} \right) \times \sqrt{3}g^2\ell_{pl}^2.
\]

One finds that the fluctuation spectrum for \( \lambda, \lambda' \) fields around \( SU(3) \times U(1) \) fixed point takes one negative value and one positive value:

\[
M_{\lambda\lambda}^2(SU(3) \times U(1)) = -\left( \sqrt{17} - 3 \right) \frac{1}{r_{IR}^2}, \quad M_{\lambda\lambda'}^2(SU(3) \times U(1)) = \left( 3 + \sqrt{17} \right) \frac{1}{r_{IR}^2},
\]

where \( \bar{\lambda} = \frac{1}{\sqrt{13}}(2\lambda + 3\bar{\lambda}) \) and \( \bar{\lambda}' = \frac{1}{\sqrt{13}}(3\lambda - 2\bar{\lambda}) \). Under \( SO(8) \rightarrow SU(3) \times U(1) \), the branching rule of a \( SO(8) \) Dynkin label \( (0, 0, 2, 0) \oplus (2, 0, 0, 0) \) corresponding to the representation \( 35_c \oplus 35_v \) in terms of \( SU(3) \) representation is given as follows:

\[
70 = 1(0) \oplus 1(0) \oplus 1(1) \oplus 1(0) \oplus 1(-1) \oplus 8(0) \oplus 8(0) \\
\oplus 3(-1/3) \oplus 3(1/3) \oplus 6(1/3) \oplus 6(-2/3) \oplus 6(-1/3) \oplus 6(2/3) \\
\oplus 3(2/3) \oplus 3(-1/3) \oplus 3(-1/3) \oplus 3(-2/3) \oplus 3(1/3) \oplus 3(1/3) \oplus 1(0)
\]

where the number in the brackets after \( SU(3) \) representation is the hypercharge, \( Y \), of it. Since the deformation preserves \( SU(3) \times U(1) \) group, the spectrum ought to correspond to that of the singlet.

From the above mass spectrum, one finds that, in \( N = 2 \) superconformal field theory, the \( SU(3) \times U(1) \) symmetric deformation ought to be an irrelevant perturbation of conformal dimension \( \Delta = (3 + \sqrt{21} + 4\sqrt{17})/2 = 4.5616 \). The corresponding eigenvector determines the
direction from which the flow approaches the fixed point. The irrelevant operator in the field
theory that controls this flow has this dimension. We thus conclude that the perturbation
operator dual to the $\tilde{\lambda}$ field induces nontrivial renormalization group flow from $\mathcal{N} = 8$ super-
conformal UV fixed point with $SO(8)$ symmetry to $\mathcal{N} = 2$ superconformal IR fixed point with
$SU(3) \times U(1)$ symmetry. Scaling dimension of other deformations is $\Delta = (3 \pm \sqrt{21} - 4\sqrt{17})/2$
for $\tilde{\lambda}$ field.

4 Supersymmetric Domain Wall and RG Flows

In this section, we investigate domain walls arising in supergravity theories with a nontrivial
superpotential defined on a restricted 2-dimensional slice of the scalar manifold. On the sub-
sector, one can write the supergravity potential in the canonical form. One of the eigenvalues
of $A_{IJ}^1$ tensor (7), $z_2$ provides a “superpotential” $W$ related to scalar potential $V$ by

$$V(\lambda, \lambda') = g^2 \left[ \frac{16}{3} \left( \frac{\partial W}{\partial \lambda} \right)^2 + 4 \left( \frac{\partial W}{\partial \lambda'} \right)^2 - 6W^2 \right]$$

(13)

where

$$W(\lambda, \lambda') = \frac{1}{16} e^{-\frac{1}{2\sqrt{2}} \sqrt{\lambda - \sqrt{2}\lambda'}} \left( 3 - e^{\sqrt{2}\lambda} + 6e^{\sqrt{2}\lambda'} + 3e^{2\sqrt{2}\lambda'} + 6e^{\sqrt{2}(\lambda + \lambda')} - e^{\sqrt{2}(\lambda + 2\lambda')} \right).$$

(14)

Note that superpotential $W$ is real rather than complex and this fact will make finding a BPS
solution easier. At the two critical points, the gradients of $W$ with respect to $\lambda, \lambda'$ vanish. That
is, supersymmetry preserving extrema of the potential satisfy $\frac{\partial W}{\partial \lambda} = 0$ and $\frac{\partial W}{\partial \lambda'} = 0$. This implies
that supersymmetry preserving vacua have negative cosmological constant: the scalar potential
$V$ at the two critical points becomes $V = -6g^2W^2$ in very simple form. The superpotential $W$
has the following values at the two critical points yielding stable $AdS_4$ vacua.

| Gauge symmetry       | $\lambda$ | $\lambda'$ | $W$ |
|----------------------|-----------|------------|-----|
| $SO(8)$              | 0         | 0          | 1   |
| $SU(3) \times U(1)$  | $\sqrt{2}\sinh^{-1}\left(\frac{1}{\sqrt{3}}\right) = 0.78$ | $\sqrt{2}\sinh^{-1}\left(\frac{1}{\sqrt{2}}\right) = 0.93$ | $\frac{3\sqrt{3}}{2}$ |

Table 2. Summary of two critical points in the context of superpotential: symmetry group,
vacuum expectation values of fields, and superpotential. The superpotential $W(\lambda, \lambda')$ also
exhibits the two critical points: $SO(8)$ point is a minimum while $SU(3) \times U(1)$ point is an
other extremum.

To construct the superkink corresponding to the supergravity description of the nonconfor-
mal RG flow from one scale to another connecting the above two critical points, the form of a
3d Poincare invariant metric but breaking the full conformal group $SO(3, 2)$ invariance takes the form:

$$ds^2 = e^{2A(r)} \eta_{\mu\nu} dx^\mu dx^\nu + dr^2, \quad \eta_{\mu\nu} = (-, +, +)$$

(15)
characteristic of space-time with a domain wall where \( r \) is the coordinate transverse to the wall. By change of variable \( U(r) = e^{A(r)} \) at the critical points, the geometry becomes \( AdS_4 \) space with a cosmological constant \( \Lambda \) equal to the value of \( V \) at the critical points: \( \Lambda = -3(\partial_r A)^2 \). In the dual theory this corresponds to a superconformal fixed point of the RG flow. We are looking for solutions that are asymptotic to \( AdS_4 \) space both for \( \lambda, \lambda' \rightarrow 0 \) for \( r \rightarrow \infty \) so that the background is asymptotic to the \( \mathcal{N} = 8 \) supersymmetric \( AdS_4 \) background at infinity with \( r_{UV} \) while \( \lambda \rightarrow \lambda_{IR} = \sqrt{2}\sinh^{-1}\left(\frac{1}{\sqrt{3}}\right), \lambda' \rightarrow \lambda'_{IR} = \sqrt{2}\sinh^{-1}\left(\frac{1}{\sqrt{2}}\right) \) for \( r \rightarrow -\infty \) with \( r_{IR} \) and so we approach a new conformal fixed point. The second order differential equations of motion for the scalars and the metric from (2) read

\[
\frac{\partial^2 \lambda}{\partial r^2} + 3 \left( \frac{dA}{dr} \right) \left( \frac{\partial \lambda}{\partial r} \right) = \frac{4}{3} \frac{\partial V}{\partial \lambda}, \\
\frac{\partial^2 \lambda'}{\partial r^2} + 3 \left( \frac{dA}{dr} \right) \left( \frac{\partial \lambda'}{\partial r} \right) = \frac{\partial V}{\partial \lambda'}, \\
6 \left( \frac{dA}{dr} \right)^2 - \frac{3}{4} \left( \frac{\partial \lambda}{\partial r} \right)^2 - \left( \frac{\partial \lambda'}{\partial r} \right)^2 + 2V = 0, \\
4 \frac{d^2 A}{dr^2} + 6 \left( \frac{dA}{dr} \right)^2 + \frac{3}{4} \left( \frac{\partial \lambda}{\partial r} \right)^2 + \left( \frac{\partial \lambda'}{\partial r} \right)^2 + 2V = 0. \tag{16}
\]

The last relation can be obtained by differentiating the third one and using other relations. Only three of them are independent. By substituting the domain wall ansatz (15) into the Lagrangian (2), the Euler-Lagrangian equations are the first, second and fourth equations of (16) for the functional \( E[A, \lambda, \lambda'] \) with the integration by parts on the term of \( \frac{d^2 A}{dr^2} \) where

\[
E[A, \lambda, \lambda'] = -\frac{1}{2} \int_{-\infty}^{\infty} dr e^{3A} \left[ -6 \left( \frac{dA}{dr} \right)^2 + \frac{3}{4} \left( \frac{\partial \lambda}{\partial r} \right)^2 + \left( \frac{\partial \lambda'}{\partial r} \right)^2 + 2V \right] \\
= -\frac{1}{2} \int_{-\infty}^{\infty} dr e^{3A} \left[ -6 \left( \frac{dA}{dr} \right)^2 + \sqrt{2}g W' \right] + \frac{3}{4} \left( \frac{\partial \lambda}{\partial r} - \frac{8}{3} \sqrt{2} g \frac{\partial W}{\partial \lambda} \right)^2 \\
+ \left( \frac{\partial \lambda'}{\partial r} - 2 \sqrt{2} g \frac{\partial W}{\partial \lambda'} \right)^2 \right] - 2\sqrt{2}ge^{3A}W|_{-\infty}^{\infty} \\
\geq -2\sqrt{2}g \left(e^{3A}W(\infty) - e^{3A}W(-\infty)\right) \tag{17}
\]

which is so-called topological charge or domain wall number. As \( r \) goes from large positive values(the UV) to large negative values(the IR) the change in \( e^{3A}W \) is a measure of the topological charge of the superkink.

Then \( E[A, \lambda, \lambda'] \) is extremized by the following so-called BPS domain wall solutions \footnote{In \cite{25}, static domain wall solutions in ungauged \( \mathcal{N} = 1 \) supergravity theories were found.}: \[
\frac{\partial \lambda}{\partial r} = \pm \frac{8}{3} \sqrt{2} g \frac{\partial W}{\partial \lambda}, \quad \frac{\partial \lambda'}{\partial r} = \pm 2\sqrt{2} g \frac{\partial W}{\partial \lambda'}, \quad \frac{dA}{dr} = \mp \sqrt{2} g W. \tag{18}
\]
It is evident that the left hand sides of the first and second relations vanish as one approaches the supersymmetric extrema, i.e. \( \frac{\partial W}{\partial \lambda} = \frac{\partial W}{\partial \lambda'} = 0 \) thus indicating a domain wall configuration. The asymptotic behaviors of \( A(r) \) are \( A(r) \to r/r_{UV} + \text{const} \) for \( r \to \infty \) and \( A(r) \to r/r_{IR} + \text{const} \) for \( r \to -\infty \). Then by differentiating \( A(r) \) with respect to \( r \) those of \( \partial_r A \) become \( \partial_r A \to 1/r_{UV} \) for \( r \to \infty \) and \( \partial_r A \to 1/r_{IR} \). At the two critical points, since \( V = -6g^2W^2 \), one can write the inverse radii of \( AdS_4 \) as cosmological constant or superpotential \( W \). Therefore we conclude that \( 1/r \) is equal to \( \pm \sqrt{2}gW \). This fact is encoded in the last equation of (18). It is straightforward to verify that (18) satisfy the gravitational and scalar equations of motion given by second order differential equations (16). Using (18), the monotonicity [26] of \( A \) which is related to the local potential energy of the superkink leads to

\[
\frac{d^2 A}{dr^2} = -4g^2 \left( \frac{4}{3} \left( \frac{\partial W}{\partial \lambda} \right)^2 + \left( \frac{\partial W}{\partial \lambda'} \right)^2 \right) = - \left( \frac{3}{8} \left( \frac{\partial \lambda}{\partial r} \right)^2 + \frac{1}{2} \left( \frac{\partial \lambda'}{\partial r} \right)^2 \right) \leq 0. \tag{19}
\]

One can understand the above bound (17) as a consequence of supersymmetry preserving bosonic background. In order to find supersymmetric bosonic backgrounds, the variations of spin \( 1/2, 3/2 \)-fields should vanish. From [20], the gravitational and scalar parts of these variations are:

\[
\delta \psi^i_\mu = 2D_\mu \epsilon^i - \sqrt{2}gA_{ij}^{\mu} \gamma_\mu \epsilon^j, \\
\delta \chi^{ijk} = -\gamma^\mu A_{\mu}^{ijkl} \epsilon^i - 2gA_{2l}^{ij} \epsilon^l,
\]

where

\[
D_\mu \epsilon^i = \partial_\mu \epsilon^i - \frac{1}{2} u^{i}_{\mu ab} \sigma^{ab} \epsilon^j + \frac{1}{2} B_\mu^i \epsilon^j, \quad B_\mu^i = \frac{2}{3} \left( u^{ik}_{I J} \partial_\mu u^{J}_{j k} - v^{ikIJ}_{\mu} \partial_\mu v^{J}_{j k I} \right)
\]

where \( \omega \) is a spin connection, \( \sigma \) a commutator of two gamma matrices and \( B \) is a \( SU(8) \) gauge field for local \( SU(8) \) invariance of the theory. In order to make the AdS/CFT correspondence completely, one should find the flow between \( SO(8) \) fixed point and the \( SU(3) \times U(1) \) fixed point. One should be able to preserve \( \mathcal{N} = 2 \) supersymmetry on the branes all along the flow. The vanishing of \( \delta \chi^{ijk} \) associates the derivatives of scalar \( \lambda \) and pseudo-scalar \( \lambda' \) with respect to \( r \) with the gradients of superpotential \( W \). The variation of 56 Majorana spinors \( \chi^{ijk} \) gives rise to the first order differential equations of \( \lambda \) and \( \lambda' \) by exploiting the explicit forms of \( A_{\mu}^{ijkl} \) and \( A_{2l}^{ij} \) in the appendix. Although there is a summation over the last index \( l \) appearing in \( A_{\mu}^{ijkl} \) and \( A_{2l}^{ij} \), nonzero contribution runs over only one index. When \( i = 1, j = 2, k = 7 \) and \( l = 8 \), the variation of \( \chi^{127} \) leads to

\[
\delta \chi^{127} = \frac{1}{2} \frac{\partial \lambda}{\partial r} \epsilon_8 - 2gy_2 \epsilon^8 = \frac{1}{2} \frac{\partial \lambda}{\partial r} \epsilon_8 - \frac{4\sqrt{2}}{3} g \frac{\partial W}{\partial \lambda} \epsilon^8.
\]
where we used the fact that $y_2$ in (I) can be written as gradient of superpotential. From this, we arrive at the first equation of BPS domain wall solutions (18). Similarly, when $i = 1, j = 6, k = 3$ and $l = 8$, the variation of $\chi^{163}$ leads to

$$
\delta \chi^{163} = -\frac{i}{2} \frac{\partial \lambda'}{\partial r} \epsilon_8 - 2gy_5 \epsilon_8 = -\frac{i}{2} \frac{\partial \lambda'}{\partial r} \epsilon_8 + \sqrt{2}gy \frac{\partial W}{\partial \lambda'} \epsilon_8.
$$

The vanishing of this is exactly same as the second equation of (18). One can check also there exist similar BPS domain wall solutions for nonzero $\epsilon_7$. From these first order differential equations, it is straightforward to check with the help of appendix that all other supersymmetric parameters $\epsilon_i$ where $i = 1, \ldots, 6$ vanish. As a result, the flow preserves $\mathcal{N} = 2$ supersymmetry, generated by $\epsilon_7$ and $\epsilon_8$, on the M2 brane. Moreover, the variation of gravitinos $\psi^i_r$ with vanishing $B_{\mu j}$ will lead to

$$
\delta \psi^8_r = -\frac{dA}{dr} \epsilon_8 - \sqrt{2}gW \epsilon_8.
$$

which will also produce the third equation of (18). Now we have shown that there exists a supersymmetric flow if and only if the equations (18) are satisfied, that is, the flow is determined by the steepest descent of the superpotential and the cosmology $A(r)$ is determined directly from this steepest descent.

Let us consider mass, $\widetilde{M}^2$ for the $\lambda, \lambda'$ at the critical points of superpotential $W$. By differentiating (13) and putting $\frac{\partial W}{\partial \lambda} = \frac{\partial W}{\partial \lambda'} = 0$, we get

$$
\widetilde{M}^2 = 2g^2W^2 \left( \frac{\mathcal{U}_{XX}}{\mathcal{U}_{XX}^2} (\mathcal{U}_{XX} - 3) + \mathcal{U}_{XX} \mathcal{U}_{XX} \mathcal{U}_{XX} (\mathcal{U}_{XX}^2 - 3) + \mathcal{U}_{XX} \mathcal{U}_{XX} \mathcal{U}_{XX} (\mathcal{U}_{XX}^2 - 3) + \mathcal{U}_{XX} \mathcal{U}_{XX} \mathcal{U}_{XX} \right),
$$

where

$$
\mathcal{U}_{XX} = \frac{2}{W} \left( \frac{\partial^2 W}{\partial \lambda^2} \right), \quad \mathcal{U}_{XX} = \frac{2}{W} \left( \frac{\partial^2 W}{\partial \lambda \partial \lambda'} \right), \quad \mathcal{U}_{XX} = \frac{2}{W} \left( \frac{\partial^2 W}{\partial \lambda'^2} \right).
$$

The mass scale is set by the inverse radius, $1/r$, of the AdS$_4$ space and this can be written as $1/r = \ell_p \sqrt{-V/3} = \sqrt{2}gW$ where we used $V = -6g^2W^2$. Via AdS/CFT correspondence, $\mathcal{U}$ is related to the conformal dimension $\Delta$ of the field theory operator dual to the fluctuation of the fields $\lambda, \lambda'$. Since the matrix $\mathcal{U}$ is real and symmetric, it has real eigenvalues $\delta_k$ and the eigenvalues of $\tilde{M}^2 r^2$ are given by $\delta_k (\delta_k - 3)$. Since a new radial coordinate $U(r) = e^{A(r)}$ is the renormalization group scale on the flow, we should find the leading contributions to the $\beta$ functions of the couplings $\lambda, \lambda'$ in the neighborhood of the end points of the flow. At a fixed point, the fields are constants and corresponding $\beta$ functions vanish. Since $\frac{d}{dr} = \frac{dA}{dr} U \frac{d}{dU}$ =
\(-\sqrt{2} g W U \frac{d}{d\tau}\), (18) becomes

\[
U \frac{d}{dU} \lambda = -\frac{2}{W} \frac{\partial W}{\partial \lambda} \approx -\left( U_{XX} \delta \lambda + U_{X\lambda} \delta \lambda' \right),
\]

\[
U \frac{d}{dU} \lambda' = -\frac{2}{W} \frac{\partial W}{\partial \lambda'} \approx -\left( U_{XX}' \delta \lambda + U_{X\lambda}' \delta \lambda' \right),
\]

where we expanded to first order in the neighborhood of a critical point. Thus \(U\) determines the behavior of the scalar \(\lambda, \lambda'\) near the critical points. The RG flow of the coupling constants of the field theory is encoded in the \(U\) dependence of the fields. To depart the UV fixed point \((U = +\infty)\) the flow must take place in directions in which the operators must be relevant and to approach the IR fixed point \((U \to 0)\) the corresponding operators must be irrelevant.

The contour maps of \(V\) and \(W\) on the \((\lambda, \lambda')\) parameter space are shown in Figure 2. The map of \(V\) shows two extrema. At \((\lambda, \lambda') = (0, 0)\), it is the maximally supersymmetric and locally maximum of \(V\) while minimum of \(W\). At \((\lambda, \lambda') = (0.78, 0.93)\), it is \(\mathcal{N} = 2\) supersymmetric and other extremum of both \(V\) and \(W\). A numerical solution of the steepest descent equation connecting these two critical points can be obtained numerically.

By realizing the fact that the scalar potential (13) has a symmetry of \(W \to -W\), the BPS domain wall solutions with \(-W\) also satisfy the minimization condition of energy (17) and satisfy equations of motion. By taking the opposite signs in the right hand sides of (18) and differentiating \(W\) with respect to \(\lambda\) and \(\lambda'\), one gets \(\lambda(r), \lambda'(r)\) and \(\partial_r A(r)\) which interpolate between the two supersymmetric vacua in Figure 3 numerically. By change of variable, \(\tanh r\) rather than \(r\), one draws them between \(-1(r = -\infty)\) and 1 \((r = \infty)\) in the horizontal axis. Starting IR fixed point, with initial data \(\lambda = 0.78, \lambda' = 0.93(\text{or } e^{\lambda/\sqrt{2}} = 1.32, e^{\lambda'/\sqrt{2}} = 1.39)\), they decrease monotonically and finally go to the expectation values, \(\lambda = 0 = \lambda'(\text{or } e^{\lambda/\sqrt{2}} = 1 = e^{\lambda'/\sqrt{2}})\) of UV fixed point. Similarly, starting IR fixed point, with initial condition \(\partial_r A = \sqrt{2} g W = 2.74\) for \(g = 1.7\), it decreases monotonically showing the property of (19) and finally goes to the expectation values, \(\partial_r A = 2.40\) of UV fixed point.

In summary, we have found an operator that gives rise to RG flow related to the symmetry breaking \(SO(8) \to SU(3) \times U(1)\) and got that the operator is relevant at the \(SO(8)\) fixed point but irrelevant at the \(SU(3) \times U(1)\) fixed point. The ability of writing de Wit-Nicolai scalar potential in terms of superpotential allowed us to determine BPS domain wall solutions easily. This superpotential originates from the structure of contracted T-tensor which was a cubic in 28-beins \(u\) and \(v\). From known supersymmetry variation of spin 1/2, 3/2 fields in gauged \(\mathcal{N} = 8\) supergravity, we were able to verify that supersymmetry preserving bosonic background in supergravity theory results in BPS domain wall solutions. The leading contributions to the \(\beta\) functions of the couplings of the field theory were encoded in a quantity, second derivatives of superpotential with respect to scalar and pseudo-scalars divided by superpotential itself.

So far, various critical points of gauged \(\mathcal{N} = 8\) supergravity potential are known for at
least $SU(3)$ invariance \cite{22}. It would be interesting to study the possibility of existence of other critical points of gauged $\mathcal{N} = 8$ supergravity by requiring smaller symmetry group invariance.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{contour_map.png}
\caption{The contour map of $V$ (on the left) and $W$ (on the right), with $\lambda'$ on the vertical axis and $\lambda$ on the horizontal axis. $V$ has vanishing first derivatives in all directions orthogonal to the plane. At $(\lambda, \lambda') = (0, 0)$, it is the maximally supersymmetric and locally maximum of $V$ while minimum of $W$. At $(\lambda, \lambda') = (0.78, 0.93)$, it is $\mathcal{N} = 2$ supersymmetric and other extremum of both $V$ and $W$.}
\end{figure}

5 Appendix

The 28-beins $u$ and $v$ fields can be obtained by exponentiating the vacuum expectation values $\phi_{ijkl}$:

\[ u_{KL}^{IJ} = \text{diag} \left( u_1, u_2, u_3, u_4, u_5, u_6, u_7 \right), \]
\[ v_{KL}^{IJ} = \text{diag} \left( v_1, v_2, v_3, v_4, v_5, v_6, v_7 \right), \]

where each submatrix is $4 \times 4$ matrix and we denote the antisymmetric index pairs $[IJ]$ and $[KL]$ explicitly for convenience.

\[ u_1 = \begin{pmatrix}
[12] \ A \ B \ B \ B \\
[34] \ B \ A \ B \ B \\
[56] \ B \ B \ A \ B \\
[78] \ B \ B \ B \ A \\
\end{pmatrix}, \quad u_2 = \begin{pmatrix}
[13] \ C \ -D \ -iE \ -iE \\
[24] \ -D \ C \ iE \ iE \\
[57] \ iE \ -iE \ C \ D \\
[68] \ iE \ -iE \ D \ C \\
\end{pmatrix}, \]
Figure 3: The plots of $e^{2\sqrt{2}}$ (on the left), $e^{\sqrt{2}}$ (on the middle) and $\partial_r A$ (on the right), with $\tanh r$ on the horizontal axis. They arrive at 1, 1, 2.40 respectively when $r \to \infty$ or $\tanh r\big|_{r \to \infty} = 1$. The value of $\lambda = 0 = \lambda'$ corresponds to the expectation values of fields of UV fixed point. The asymptotic value of last one is consistent with the value of $\partial_r A = \sqrt{2}gW$ at UV fixed point where $W = 1$. We took $g = 1.7$ for simplicity.

| $u_3$ | $u_4$ | $u_5$ | $u_6$ | $u_7$ | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $[14]$ $C$ $D$ $-iE$ $iE$ | $[15]$ $C$ $-D$ $iE$ $iE$ | $[16]$ $C$ $D$ $iE$ $-iE$ | $[17]$ $C$ $D$ $iE$ $-iE$ | $[18]$ $C$ $-D$ $iE$ $iE$ | $[12]$ $F$ $G$ $G$ $G$ | $[13]$ $H$ $-iI$ $iJ$ $iJ$ | $[14]$ $H$ $iJ$ $iJ$ $iJ$ | $[15]$ $H$ $-iI$ $-iJ$ $iJ$ | $[16]$ $H$ $I$ $-iJ$ $iJ$ |
| $[23]$ $D$ $C$ $-iE$ $iE$ | $[26]$ $-D$ $C$ $-iE$ $-iE$ | $[25]$ $D$ $C$ $iE$ $-iE$ | $[28]$ $D$ $C$ $iE$ $-iE$ | $[27]$ $-D$ $C$ $-iE$ $-iE$ | $[34]$ $G$ $F$ $G$ $G$ | $[24]$ $-I$ $H$ $-iJ$ $-iJ$ | $[23]$ $I$ $H$ $iJ$ $-iJ$ | $[26]$ $-I$ $H$ $iJ$ $iJ$ | $[35]$ $-iE$ $-iE$ $C$ $D$ |
| $[58]$ $iE$ $iE$ $C$ $-D$ | $[37]$ $-iE$ $iE$ $C$ $D$ | $[38]$ $-iE$ $-iE$ $C$ $-D$ | $[36]$ $-iE$ $iE$ $C$ $D$ | $[39]$ $iE$ $iE$ $D$ $C$ | $[56]$ $G$ $G$ $F$ $G$ | $[57]$ $iJ$ $-iJ$ $-I$ $-H$ | $[58]$ $iJ$ $iJ$ $-H$ $I$ | $[59]$ $iJ$ $-iJ$ $-I$ $-H$ | $[67]$ $-iJ$ $-iJ$ $I$ $-H$ |
| $[67]$ $-iE$ $-iE$ $-D$ $C$ | $[48]$ $-iE$ $iE$ $D$ $C$ | | | | $[78]$ $G$ $G$ $G$ $F$ | | | | | |
\[ v_6 = \begin{pmatrix} 17 & 28 & 35 & 46 \\ 17 & -H & -I & iJ -iJ \\ 28 & -I & -H & iJ -iJ \\ 35 & iJ & iJ & H -I \\ 46 & -iJ & -iJ & -I & H \end{pmatrix}, \quad v_7 = \begin{pmatrix} 18 & 27 & 36 & 45 \\ 18 & -H & I & iJ -iJ \\ 27 & I & -H & -iJ & -iJ \\ 36 & iJ & -iJ & H & I \\ 45 & iJ & -iJ & I & H \end{pmatrix}, \]

where
\[ A \equiv p^3, \quad B \equiv pq^2, \quad C \equiv pp'q^2, \quad D \equiv pq, \quad E \equiv qp'q' \]
and \( p, p', q \) and \( q' \) are given in (5). The nonzero components of \( A_2 \) tensor, \( A_{2,L}^{IJK} \) can be obtained from (4) and they are given by

\[
A_{2,1}^{256} = A_{2,1}^{234} = A_{2,2}^{165} = A_{2,2}^{143} = A_{2,3}^{456} = A_{2,3}^{412} = A_{2,4}^{365} = A_{2,4}^{321} =
\]
\[
A_{2,5}^{634} = A_{2,5}^{612} = A_{2,6}^{543} = A_{2,6}^{521} \equiv y_1,
\]
\[
A_{2,7}^{182} = A_{2,7}^{384} = A_{2,7}^{856} = A_{2,8}^{127} = A_{2,8}^{473} = A_{2,8}^{675} \equiv y_2,
\]
\[
A_{2,1}^{728} = A_{2,2}^{817} = A_{2,3}^{748} = A_{2,4}^{837} = A_{2,5}^{768} = A_{2,6}^{857} \equiv y_3,
\]
\[
A_{2,1}^{368} = A_{2,1}^{458} = A_{2,1}^{357} = A_{2,1}^{647} = A_{2,2}^{684} = A_{2,2}^{637} = A_{2,2}^{547} = A_{2,2}^{538} =
\]
\[
A_{2,3}^{861} = A_{2,3}^{726} = A_{2,3}^{175} = A_{2,3}^{285} = A_{2,4}^{682} = A_{2,4}^{185} = A_{2,4}^{725} = A_{2,4}^{716} =
\]
\[
A_{2,5}^{427} = A_{2,5}^{814} = A_{2,5}^{713} = A_{2,5}^{823} = A_{2,6}^{428} = A_{2,6}^{813} = A_{2,6}^{273} = A_{2,6}^{174} \equiv y_4,
\]
\[
A_{2,7}^{362} = A_{2,7}^{452} = A_{2,7}^{531} = A_{2,7}^{146} = A_{2,8}^{163} = A_{2,8}^{541} = A_{2,8}^{532} = A_{2,8}^{462} \equiv y_5,
\]

where \( y_i \)'s are given in (3). Notice that we did not write down other components of \( A_2 \) tensor which are interchanged between 2nd and 3rd indices because it is manifest that \( A_{2,L}^{IJK} = -A_{2,L}^{IKJ} \), by definition. Moreover there is a symmetry between the upper indices: \( A_{2,L}^{IJ,K} = A_{2,L}^{IK,J} \).

The kinetic term can be summarized as following block diagonal matrices:

\[
A_{\mu}^{IJ,KL} = \text{diag} (A_{\mu,1}, A_{\mu,2}, A_{\mu,3}, A_{\mu,4}, A_{\mu,5}, A_{\mu,6}, A_{\mu,7}),
\]

where

\[
A_{\mu,1} = \frac{1}{2} \partial_{\mu} \begin{pmatrix} 12 & 34 & 56 & 78 \\ 12 & -\lambda & -\lambda & -\lambda \\ 34 & -\lambda & 0 & -\lambda & -\lambda \\ 56 & -\lambda & -\lambda & 0 & -\lambda \\ 78 & -\lambda & -\lambda & -\lambda & 0 \end{pmatrix}, \quad A_{\mu,2} = \frac{1}{2} \partial_{\mu} \begin{pmatrix} 13 & 24 & 57 & 68 \\ 13 & 0 & \lambda & -i\lambda' & -i\lambda' \\ 24 & \lambda & 0 & i\lambda' & i\lambda' \\ 57 & -i\lambda' & i\lambda' & 0 & \lambda \\ 68 & -i\lambda' & i\lambda' & \lambda & 0 \end{pmatrix},
\]
\[
A_{\mu,3} = \frac{1}{2} \partial_{\mu} \begin{pmatrix} 14 & 23 & 58 & 67 \\ 14 & 0 & -\lambda & -i\lambda' & i\lambda' \\ 23 & -\lambda & 0 & -i\lambda' & i\lambda' \\ 58 & -i\lambda' & -i\lambda' & 0 & -\lambda \\ 67 & i\lambda' & i\lambda' & -\lambda & 0 \end{pmatrix}, \quad A_{\mu,4} = \frac{1}{2} \partial_{\mu} \begin{pmatrix} 15 & 26 & 37 & 48 \\ 15 & 0 & \lambda & i\lambda' & i\lambda' \\ 26 & \lambda & 0 & -i\lambda' & -i\lambda' \\ 37 & i\lambda' & -i\lambda' & 0 & \lambda \\ 48 & i\lambda' & -i\lambda' & \lambda & 0 \end{pmatrix},
\]

16
\[ A_{\mu,5} = \frac{1}{2} \partial_{\mu} \left( \begin{array}{cccc} [16] & [25] & [38] & [47] \\ 16 & 0 & -\lambda & i\lambda' - i\lambda' \\ 25 & -\lambda & 0 & i\lambda' - i\lambda' \\ 47 & -i\lambda' & -i\lambda' & -\lambda \end{array} \right), \quad A_{\mu,6} = \frac{1}{2} \partial_{\mu} \left( \begin{array}{cccc} [17] & [28] & [35] & [46] \\ 17 & 0 & -\lambda & i\lambda' - i\lambda' \\ 28 & -\lambda & 0 & i\lambda' - i\lambda' \\ 46 & -i\lambda' & -i\lambda' & -\lambda \end{array} \right), \]

\[ A_{\mu,7} = -\frac{1}{2} \partial_{\mu} \left( \begin{array}{cccc} [18] & [27] & [36] & [45] \\ 18 & 0 & \lambda & i\lambda' \lambda \end{array} \right), \quad \left( \begin{array}{cccc} [17] & [28] & [35] & [46] \\ 17 & 0 & -\lambda & i\lambda' & -i\lambda' \\ 28 & -\lambda & 0 & i\lambda' & -i\lambda' \\ 46 & -i\lambda' & -i\lambda' & -\lambda \end{array} \right) \right), \]

**Acknowledgments**

This work was supported by Korea Research Foundation Grant(KRF-1999-015-DI0019). CA thanks Yonsei Visiting Research Center(YVRC) where this work was initiated. We thank S.-J. Rey for helpful discussions on relating subjects and reading an earlier version of this manuscript. We are grateful to H. Nicolai for helpful correspondence on his work.

**References**

[1] C. Ahn and S.-J. Rey, Nucl.Phys. **B565** (2000) 210.

[2] C. Ahn and S.-J. Rey, Nucl.Phys. **B572** (2000) 188.

[3] J. Maldacena, Adv.Theor.Math.Phys. **2** (1998) 231.

[4] E. Witten, Adv.Theor.Math.Phys. **2** (1998) 253.

[5] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, Phys.Lett. **428B** (1998) 105.

[6] M.J. Duff, B.E.W. Nilsson and C.N. Pope, Phys. Rep. **130** (1986) 1.

[7] E. Cremmer, B. Julia and J. Scherk, Phys.Lett. **B76** (1978) 409; E. Cremmer and B. Julia, Nucl.Phys. **B159** (1979) 141.

[8] P.G.O. Freund and M.A. Rubin, Phys.Lett. **B97** (1980) 233.

[9] G. Jensen, Duke Math. J. **42** (1975) 397; J.-P. Bourguignon and H. Karcher, Ann.Sci.Normale Sup. **11** (1978) 71.

[10] M.A. Awada, M.J. Duff and C.N. Pope, Phys.Rev.Lett. **50** (1983) 294; M.J. Duff, B.E.W. Nilsson and C.N. Pope, Phys.Rev.Lett. **50** (1983) 2043; **51** (1983) 846(errata).
[11] B. de Wit, H. Nicolai and N.P. Warner, Nucl.Phys. B255 (1985) 29.

[12] M.J. Duff and C.N. Pope, *Kaluza-Klein supergravity and the seven sphere*, in: Supersymmetry and Supergravity ’82, eds. S. Ferrara, J.G. Taylor and P. van Nieuwenhuizen (World Scientific, Singapore, 1983).

[13] F. Englert, Phys.Lett. B119 (1982) 339.

[14] B. de Wit and H. Nicolai, Phys.Lett. B148 (1984) 60.

[15] C.N. Pope and N.P. Warner, Phys.Lett. B150 (1985) 352.

[16] B. de Wit and H. Nicolai, Nucl.Phys. B231 (1984) 506.

[17] H. Nicolai and N.P. Warner, Nucl.Phys. B259 (1985) 412.

[18] L. Girardello, M. Petrini, M. Porrati and A. Zaffaroni, Nucl.Phys. B569 (2000) 451; I. Bakas, A. Brandhuber, K. Sfetsos, hep-th/9912132; K. Pilch, N.P. Warner, hep-th/0002192; V.L. Campos, G. Ferretti, H. Larsson, D. Martelli and B.E.W. Nilsson, JHEP 0006 (2000) 023; A. Brandhuber, K. Sfetsos, hep-th/0004148; K. Pilch and N.P. Warner, hep-th/0004063; N. Evans and M. Petrini, hep-th/0006048; K. Pilch and N.P. Warner, hep-th/0006066; I. Bakas, A. Bilal, J.-P. Derendinger and K. Sfetsos, hep-th/0006222.

[19] B. de Wit and H. Nicolai, Phys.Lett. B108 (1982) 285.

[20] B. de Wit and H. Nicolai, Nucl.Phys. B208 (1982) 323.

[21] E. Cremmer and B. Julia, Phys.Lett. B80 (1978) 48.

[22] N.P. Warner, Phys.Lett. B128 (1983) 169; Nucl.Phys. B231 (1984) 250.

[23] B. Biran, A. Casher, F. Englert, M. Rooman and P. Spindel, Phys.Lett. B134 (1984) 179.

[24] K. Skenderis and P.K. Townsend, Phys.Lett. B468 (1999) 46.

[25] M. Cvetic, S. Griffies and S.-J. Rey, Nucl.Phys. B381 (1992) 301; M. Cvetic, S. Griffies and S.-J. Rey, Nucl.Phys. B389 (1993) 3.

[26] L. Girardello, M. Petrini, M. Porrati and A. Zaffaroni, JHEP 9812 (1998) 022; D.Z. Freedman, S.S. Gubser, K. Pilch and N.P. Warner, hep-th/9904017.