VSS-LMS: LMS Algorithm Experimental Approach

M. O. Momoh1, O. C. Ubadike1, I. A. Kachalla2, M. A. Isa-Bello3, P. U. Chinedu4, M. B. Abdullahi5

1Faculty of Air Engineering, Air Force Institute of Technology Kaduna, Nigeria.
2Faculty of Ground and Communication Engineering, Air Force Institute of Technology Kaduna, Nigeria.
3Ahmadu Bello University, Zaria, Nigeria.
4Department of Computer Engineering, Edo State University Uzairue, Nigeria
5Department of Electrical and Electronics Engineering, Abdu-Gusau Polytechnic Talata Mafara, Nigeria

INTRODUCTION

Digital filters are significant tools used in extracting targeted signal in electronic data processing system, where desired signals are separated from undesired ones (signal) such as interferences or noises. Filters can be used to modify signal parameters by certain ratio relative to its computation. This modification can be done automatically by examining current and previous state of the filter, this is known as adaptive filtering or time-varying signal filtering [1-2]. Least mean square (LMS) algorithm is one of the classical adaptive filtering algorithms that has good computational simplicity, minimum mean square error and stability. LMS has been widely adopted for several applications because of these features and its simple structure, however, a typical LMS algorithm with a fixed step-size is characterized with slow convergences when tracked with high accuracy [3]. To curtail the slow convergence issue, variable step size was incorporated into LMS algorithms for adaptive filtering. Many variable step-size LMS algorithm have been developed over the years [4-8] which have yielded improved performance when compared to the conventional fixed step size LMS. Nonetheless, majority of the work, varies the step-size with a larger value of μ at the early stage of the adaptation process while using a smaller value towards convergence, hence, the converging speed was increased while sustaining good stability [3],[9]. However, in this experimental approach, a new formulation of adjusting the step size is introduced with less computational complexity compared to existing variable step size techniques.

LEAST MEAN SQUARE (LMS) ALGORITHM

The LMS algorithm is type of adaptive filter known as stochastic gradient-based algorithms as it utilise the gradient vector of the filter tap weights to converge to an optimal solution [10]. It is an iterative principle that involves adjusting and updating the filter tap weight to minimize the system output error.

The output y(n) of the filter structure can be obtain using [11]:

\[ y(k) = \sum_{n=0}^{N-1} w(k) x(n - k) = w^T x(n) \]  

(1)

Where n is number of iteration

The error signal is calculated by;

\[ e(k) = d(k) - y(k) \]  

(2)

The filter weights are updated from the error signal e(k) and input signal x(k) as expressed by;
\[ w(k+1) = w(k) + \mu e(k)x(k) \]  

Where \( w(k) \) is the current weight value vector, \( w(k+1) \) is the next weight value vector, \( x(k) \) is the input signal vector, \( e(k) \) is the filter error vector and \( \mu \) is the convergence factor which determine the filter convergence speed and overall behaviour [11].

**FORMULATED VARIABLE STEP SIZE LMS ALGORITHM**

The concept behind the variable step size (VSS-LMS) algorithm is to estimate the output error so as to update the step size in order to significantly reduce the output error. Instead of using a fixed step size, VSS-LMS varies the step size over certain range. Figure 1 is the block diagram of the proposed VSS-LMS algorithm.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{block_diagram.png}
\caption{Bock diagram of the formulated Variable Step Size LMS algorithm}
\end{figure}

The formulation of the proposed VSS-LMS algorithm is based on experimentation. The experiments compared the present error signal value \( e(n) \) to previous second to last error signal values \( e(n-2) \). Also, the variation in the value of the step size selection is reduced by half of its value for every ten (10) iteration where \( e(n) \) is greater than \( e(n-2) \).

The new step size selection is formulated as;

\[
\begin{align*}
\text{if } e(n) < e(n-2) & \quad u(n) \\
\text{if } e(n) > e(n-2) & \quad \text{then count } +1 \\
\text{if count } > 10 & \quad \frac{u(n)}{2}
\end{align*}
\]  

\[ (4) \]

**EXPERIMENTAL FORMULATION**

In this work, experimental simulation was used in characterizing and analysing the behaviour of the formulated VSS-LMS algorithm.

Table 1: Formulation analysis of VSS-LMS algorithm

| count | MSE | \( \mu = 0.005 \) | \( \mu = 0.003 \) | \( \mu = 0.07 \) |
|-------|-----|-----------------|-----------------|-----------------|
| >5    | 0.2146 | 0.2479 | 0.2330 |
| >8    | 0.1928 | 0.2349 | 0.2424 |
| >10   | 0.1823 | 0.2076 | 0.2300 |
| >12   | 0.1940 | 0.2127 | 0.2512 |
| >15   | 0.1971 | 0.2122 | 0.2593 |
Table 1 shows the variation of the filter design parameters for best performance. At count greater 10, the algorithm performs best.

**TESTING AND EVALUATION**

To evaluate the formulated VSS-LMS algorithm, sine wave signal was generated to represent desired signal $s(t)$ using general sinusoidal equation given by;

$$s(t) = A\sin(\omega t + \varphi)$$

(5)

Where $A$ is the amplitude, $t$ is time, $\varphi$ is phase angle and $\omega = 2\pi F$ with $F$ being the frequency.

Random noise was also generated using randn function to corrupt the desired signal. A correlated noise signal is also generated. Both the resultant noisy signal and the corrected signal are used as the input signals. These signals are then simulated by the MATLAB tool box that implements the LMS adaptive filtering under evaluation. By experimentation, the step size $\mu(n)$ and filter length $L$, are selected and adjusted based on the observed output signal of the system. The simulation result of formulated variable step size LMS algorithm for sinusoidal wave signal is given in the Figure 2.

![Figure 2](image.png)

**APPLICATION OF FORMULATED VSS-LMS IN ADAPTIVE NOISE CANCELLATION (ANC)**

Adaptive Noise Cancellation (ANC) method serves to filter out the interference component by identifying a model between a measurable noise source and the corresponding signal corrupted by the interferences [12]. ANC is a noise reduction technique which involve passing of corrupted signal through a filter that tends to suppress the noise while leaving the signal unchanged. ANC employ the principle of adaptive filtering that incorporate an adaptive algorithm to adjust the filter parameter with little or no priori information about the inputs signal characteristics [13].

In a typical ANC shown as in Figure 1 earlier, two input signals, $d(k)$ and $x(k)$, are applied simultaneously to the adaptive filter. The signal $d(k)$ is the corrupted signal containing both the desired signal, $s(k)$ and the noise $n(k)$, assumed to be uncorrelated with each other. The signal, $x(k)$ is a measure of the contaminating signal which is correlated in a way with $n(k)$, $x(k)$ is processed by the digital filter to produce an estimate $y(k)$ of $n(k)$. An estimate of the desired signal, $e(k)$ is then obtained by subtracting the digital filter output $y(k)$, from the contaminated signal [13-14].

To evaluate the performance of the new formulated VSS-LMS in ANC, speech signal is considered. Based on the selected parameters for the best system performance, speech signal is collected directly from the physical environment (room) through a microphone with presences of working fan as a background noise, and feed into to new formulated variable step LMS to implement adaptive noise cancelling.

The performance of the new formulated VSS-LMS algorithm was compared to the conventional fixed step size with different selected step size and filter order as shown in Table 2.
The performance parameter for this analysis is the mean square error (MSE) of the output signal. The mean square error (MSE) is a performance measure that indicates how well a system can adapt to a given solution. A small minimum MSE is an indication that the adaptive system is accurately modelled and can predict, adapt and converge to a given solution. However, a very large MSE usually indicates that the adaptive filter is not accurately model or the initial state of the adaptive filter is an inadequate starting point to cause the adaptive filter to converge. From the experimental result, the new formulated variable step size give a minimal MSE when compare with the fixed step size. The simulation result of the formulated variable step size LMS compared to the fixed step size for speech signal is given in Figure 3.

| Filter order | MSE for fixed step size | MSE for formulated VSS-LMS | MSE for fixed step size | MSE for formulated VSS-LMS |
|--------------|-------------------------|-----------------------------|-------------------------|---------------------------|
| 2            | 0.3169                  | 0.2502                      | 0.4542                  | 0.1810                    |
| 3            | 0.3927                  | 0.2122                      | 0.8439                  | 0.1971                    |
| 5            | 0.8439                  | 0.1971                      | 2.7715                  | 0.3606                    |
| 7            | 1.8231                  | 0.2964                      | 0.8439                  | 0.1971                    |

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**Figure 4.** Simulation result of the formulated variable step size LMS compared to the fixed step size for speech signal.

**CONVERGENCE SPEED ANALYSIS OF VSS-LSM ALGORITHM**

The convergence speed of the VSS-LSM algorithm is evaluated in time domain. Convergence speed measures how quickly an algorithm adapts and cancels the noise. In this analysis, the convergence speed is determined in terms of how the filter coefficients vector converges to a particular value(s). During the adaptation operation of the filter weight, the filter coefficients are updated continuously to obtained steady coefficient vector. At the point when the coefficient vector assume a particular value(s), the filter is said to have converged. Figure 5 shows the filter coefficient vector convergence of the VSS-LMS algorithm.
The VSS-LMS is observed to have started converging at about 120 iterations. At about 140 iterations, the filter coefficient vector converges completed and the noise is completely cancelled. As compared to the conventional LMS, the filter coefficient vector convergence is presented in Figure 5.

CONCLUSION

In conclusion a variable step size for LMS (VSS-LSM) algorithm was formulated, presented and implemented in this paper. The formulated model is based on experimental observation of the results obtained in varying different step sizes for filtering operation of the LMS. The model was simulated using sinusoidal signal and applied in adaptive noise cancellation using speech signal. Performance evaluation was based on the obtained mean square error (MSE) and convergence speed. Significant improvement was observed in the system output error and faster convergence time was also achieved using the formulated model.

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