A Full-Newton AC-DC Power Flow Methodology for HVDC Multi-Terminal Systems and Generic DC Network Representation

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Abstract: The use of Direct Current (DC) transmission links in power systems is increasing continuously. Thus, it is important to develop new techniques to model the inclusion of these devices in network analysis, in order to allow studies of the operation and expansion planning of large-scale electric power systems. In this context, the main objective of this paper is to present a new methodology for a simultaneous AC-DC power flow for a multi-terminal High Voltage Direct Current (HVDC) system with a generic representation of the DC network. The proposed methodology is based on a full Newton formulation for solving the AC-DC power flow problem. Equations representing the converters and steady-state control strategies are included in a power flow problem formulation, resulting in an expanded Jacobian matrix of the Newton method. Some results are presented based on HVDC test systems to confirm the effectiveness of the proposed approach.

Keywords: high-voltage direct current; multi-terminal HVDC systems; control modes on multi-terminal HVDC system; power flow; Newton method

1. Introduction

With the prospect of the increasing use of Direct Current (DC) transmission links in power systems, it has become increasingly important to have techniques to model the devices responsible for the AC-DC interconnection in power system analysis software, particularly in the power flow (PF), to allow correct network modeling as a whole to improve the quality of simulation results that could be used in studies of operation and transmission expansion planning of electric power systems.

DC transmission has become an alternative that is technically and economically competitive in the transport of large amounts of active power over long distances, in underwater crossings with the use of cables, and in asynchronous connections within a large variety of lengths, including zero, between two areas [1–5].

The DC transmission links generally are characterized by the interconnection of two systems of Alternating Current (AC) by two converter stations: a rectifier and an inverter. The connection between these stations is established by one or more DC transmission lines: single or double polarity. The rectification and inversion are performed in the converter bridges composed by semiconductor elements.

High Voltage Direct Current (HVDC) transmission technology was used for the first time in Brazil in the 1980s to integrate the energy generated by the Itaipu Hydroelectric Power Plant, connecting the Foz do Iguaçu and Ibiúna converter stations, utilizing two
dipoles at ±600 kV, over about 800 km. In 2009, Brazil resumed the use of HVDC trans-
mission technology with the construction of the ±600 kV transmission system associated
with the Rio Madeira hydroelectric complex (Santo Antônio and Jirau hydroelectric power
plants), composed of two bipoles designed to transmit 6300 MW over approximately
2400 km, connecting Coletora Porto Velho and Araraquara stations. By the year 2020,
an important initiative to consider is the establishment of the Belo Monte HVDC links
(2539 km-long transmission), with the construction of the ±800 kV transmission system
associated with the Belo Monte hydroelectric power plant.

From this context, it appears that the study of severe operating conditions is important to
support studies on the reliability of hydroelectric plants. These studies can assist in assessing
the Availability Factor (AFA) [6]. Thus, modeling the electrical system as close as possible to
reality is of fundamental importance. This is the case of the Santo Antônio hydroelectric plant,
which may be impacted by the behavior of the HVDC transmission system.

The power flow software is the most frequently used tool in both operation and
expansion planning studies of electrical power systems. It is used to determine the equip-
ment rating, electrical equipment loading and system losses, bus voltage magnitudes and
angles, and reactive power support requirements to maintain voltages within limits for
a given scenario and a contingency list. For each case study, the network configuration
and parameters, the bus active and reactive load, and MW generation must be specified.
The increasing complexity of power systems, introduced by new large-scale AC and HVDC
interconnections and by the application of Flexible AC Transmission System (FACTS)
devices in such systems, have imposed new challenges to power system engineers and
software developers. The reference [7] describes a generic methodology for representing
control devices and the associated challenges.

The commonly used techniques in studies of power flow alternate the convergence
process between AC and DC system models. This method is carried out until the global
convergence of the two systems is obtained. It is important to point out that the inclusion
of DC links can cause, in general, a significantly slower convergence of power flow. One of
the main reasons for this slow convergence is the different rates of convergence of the DC
and AC systems, which interfere with each other, slowing system global convergence.

In reference [8], a simultaneous (full Newton) methodology for the representation of
DC transmission links based on the DC transmission link of the hydro electrical unit Itaipu
(Brazil) was proposed by including equations that model the converters, the DC network
(considering only two DC bus), and control modes to the set of equations that model the
power system. The proposed model aims to (i) improve the convergence characteristics
of systems with coupling between the AC-DC set of equations in the Newton-Raphson
method and (ii) allow the adjustment of the control modes automatically during the
iterative process.

In [9], the authors present a versatile approach to AC/DC system representation,
which is able to model several types of converter structures, including the conventional
two-terminal AC-DC power flow, and to connect to more DC terminals. The sequential
(alternate) method is used to solve the AC-DC power flow, in which the AC network
solution is carried out through the conventional Newton-Raphson method, and in the DC
network model, the system presents five variables per terminal. It is also proposed that the
variation of taps be limited, which is an improvement compared to other methodologies.
Another advantage is the possibility of solving DC systems connected in series by adjusting
some equations of the DC network.

Another study of DC transmission links [10] uses the eliminated variable method,
which is a simple and reliable tool for the study of DC transmission links in multi-terminal
systems, in which the DC and AC variables are treated separately. In this work, the active
and reactive powers demanded by inverters are treated as voltage-dependent loads. Thus,
the DC steady-state equations are solved, and the DC variables are eliminated from the
PF equations. Voltage-dependent PQ buses were adopted in [11] to represent the HVDC
system in the power flow problem. The work [12] proposed a modified power flow
formulation considering the HVDC system as a voltage-dependent load on the AC side at both DC system sides in two-terminal systems.

In [13], a nodal voltage-based universal steady-state PF formulation is proposed. The main goal is to consider the bipolar VSC-MTDC (Voltage Source Converter Multi-Terminal HVDC). A power flow alternating iterative method is proposed to obtain the positive/negative-pole DC network power flow. A series of nodal equivalent methods involving various control strategies are also proposed for the power flow algorithm. A comparative study of full Newton and sequential AC-DC power flow formulations for a VSC-MTDC is presented in the paper [14].

In [15], the authors use a sequential methodology to present a study where the solution of the AC-DC power flow is intrinsically related to the characteristics of a linear coefficient matrix $G$, which integrates the information of the network and HVDC control modes. The main contribution of this paper is to demonstrate that a necessary condition to solve the HVDC system is that the coefficient matrix $G$ must be nonsingular. In the study, the conditions for the characteristic of a $G$ matrix under feasible combinations of control modes and the parameters of the HVDC systems are introduced. The reference [16] proposed an alternative approach also based on a sequential method for the AC-DC power flow solution to handle multi-infeed DC systems.

The authors of [17] present some methods based on the correlation analysis between the steady-state security region and operational constraints. The reference [18] proposes an operation strategy of the hybrid multi-terminal high voltage DC to increase the utilization of the AC transmission corridors in parallel with HVDC systems considering the available transmission capacity (ATC) between two electrical areas. An analysis of distribution networks is carried out in the reference [19]. This work proposed an integrated load flow approach for AC-DC distribution systems.

In this context, the main objective of this paper is to propose a new methodology for a full Newton AC-DC power flow formulation for an MTDC system with a generic methodology for representing the DC network in stationary studies. The main motivation is the increasing interest in the operational feasibility and potential application of multi-terminal HVDC systems. It is sought to demonstrate that the proposed methodology provides an efficient and generic way to represent any DC network in HVDC multi-terminal systems through a system of equations. Finally, some results are presented based on HVDC test systems to validate the effectiveness and robustness of the proposed methodology.

From the above, the main contributions of this paper are

• to present an alternative power flow formulation based on a full Newton methodology for MTDC transmission systems, where all DC variables are considered in the formulation, allowing for a generic representation of HVDC control modes;
• to describe the AC/DC base transformation in the Jacobian matrix calculation;
• to propose a formulation that represents the generic DC network.

2. Modeling Background

The steady-state model for power flow studies of the HVDC utilized in this work is constituted by three subdivisions: the converter model itself, more specifically the rectifier and inverter model, the DC transmission line/network model, and the control system models for the Multi-Terminal HVDC (MTDC).

• converter model;
• DC transmission line/network model;
• control model in Multi-Terminal HVDC (MTDC) systems.

2.1. Mathematical Model of the Converter

The model for the converter used in Multi-Terminal HVDC systems can be conveniently represented by Figure 1. The variables are adequately represented in a p.u. system.
The values of rectifier and inverter commutation resistors, \( R_r \) and \( R_i \), are given by Equations (1) and (2). Note that \( R_i \) is conveniently adopted as a negative value.

\[
R_r = \frac{3X_r}{\pi} \tag{1}
\]
\[
R_i = -\frac{3X_i}{\pi} \tag{2}
\]

The representation of the rectifiers and inverters is based on the following premises [20]:
- the current \( I_c \) is ripple-free;
- the AC systems consist of constant-frequency, perfectly sinusoidal, balanced voltage sources behind balanced impedances, which assumes that all harmonic currents and voltages introduced by the commutation system do not propagate into the AC system because of filtering;
- the converter transformers do not saturate.

The equations that model the rectifier and inverter for MTDC systems are the same as those used in the DC link models for systems with only two DC terminals [8].

2.1.1. Mathematical Model of the Rectifier

The equations that model the behavior of the rectifier in the DC link are given by [1,20,21]:

\[
V_d - k_r a_r V_r \cos(\alpha) + \frac{3}{\pi} X_r I_r = 0 \tag{3}
\]
\[
\frac{2 \mu_r + \sin(2\alpha) - \sin(2(\alpha + \mu_r))}{\cos(2\alpha) - \cos(2(\alpha + \mu_r))} - \tan(\phi_r) = 0 \tag{4}
\]
\[
\cos(\alpha) - \cos(\alpha + \mu_r) - \frac{2R_r I_r}{k_r a_r V_r} = 0 \tag{5}
\]

2.1.2. Mathematical Model of the Inverter

Similarly, the equations that model the behavior of the inverter in the DC link are given by [1,20,21] are

\[
V_d - k_i a_i V_i \cos(\gamma) - \frac{3}{\pi} X_i I_i = 0 \tag{6}
\]
\[
\frac{2 \mu_i + \sin(2\gamma) - \sin(2(\gamma + \mu_i))}{\cos(2\gamma) - \cos(2(\gamma + \mu_i))} - \tan(\phi_i) = 0 \tag{7}
\]
\[
\cos(\gamma) - \cos(\gamma + \mu_i) - \frac{2R_i I_i}{k_i a_i V_i} = 0 \tag{8}
\]
2.2. DC Transmission Network Model

Considering a circuit that contains \( N \) nodes with only independent current sources, the simplified nodal equations have the following form:

\[
\begin{bmatrix}
    I_1 \\
    I_2 \\
    \vdots \\
    I_{N-1}
\end{bmatrix}
= \begin{bmatrix}
    G_{11} & -G_{12} & \cdots & -G_{1,N-1} \\
    -G_{21} & G_{22} & \cdots & -G_{2,N-1} \\
    \vdots & \vdots & \ddots & \vdots \\
    -G_{N-1,1} & -G_{N-1,2} & \cdots & G_{N-1,N-1}
\end{bmatrix}
\begin{bmatrix}
    V_1 \\
    V_2 \\
    \vdots \\
    V_{N-1}
\end{bmatrix}
\] (9)

where \( V_1 \) and \( V_{N-1} \) denote the voltages of \( N-1 \) nodes with respect to the reference node.

The proposed methodology provides an efficient and generic way to represent any DC network through the system of equations presented in (9). This approach has become useful in some studies, for example, in research related to “Tap HVDC”, which seeks to disfigure the DC link as a transmission point-to-point system and make it flexible regarding the extraction or injection along the transmission line.

2.3. Control Model in MTDC Systems

The basic control principle for MTDC systems is a generalization of the one used in two-terminal systems [8]. As is shown in [20], the control characteristic for each converter is composed of piece-wise functions representing constant-current control (CC) and constant-firing angle control (Constant Extinction Angle (CEA) for inverters and Constant Ignition Angle (CIA) for rectifiers). In addition, an optimal constant-voltage segment may be included. The following is a general discussion of significant aspects related to the control of parallel-connected and series-connected systems.

The equations that model the controls are as follows:

- **CIA/CEA Control with Constant-Voltage (CV):**
  \[
  \cos(\delta_{sch}^c) = \cos(\delta_c) \\
  V_{d_{sch}^c} = V_{d_c}
  \] (10)

  The use of the variable \( \cos(\delta_c) \) instead of \( \delta_c \) improves the linearity of the problem [9].

- **Constant-Current Control (CC):**
  \[
  I_{sch}^c = I_c \\
  V_{d_c} + 0.97.(-k_c.a_c.V_c.\cos(\delta_{min}^c) + R_c.I_c) = 0
  \] (13)

- **Constant-Power Control (CP):**
  \[
  P_{c_{sch}^c} = P_{c_c} \\
  V_{d_c} + 0.97.(-k_c.a_c.V_c.\cos(\delta_{min}^c) + R_c.I_c) = 0
  \] (15)

where:

\[
  k_c = \left(\frac{3\sqrt{2}}{\pi}\right).n_b
  \]

For the converter operating with current or power control mode, it is common to adjust the transformer tap control with the phase control. Consequently, the converter will operate at some DC voltage below its minimum extinction or ignition angle characteristic to prevent mode shifts from occurring with normal AC voltage changes. Typically, a 3% voltage margin is provided, with the normal \( \alpha_{min} \) or \( \gamma_{min} \) given in (13) and (15), and typical values of the control angles \( \alpha \) and \( \gamma \) are 15° and 20°, respectively, for those DC terminals with a scheduled current or power control. From a practical point of view, this common voltage margin of 3% can be considered in the PF computation by adjusting the DC voltage equations for such converters with a coefficient of 0.97 [9,22].
3. Proposed Methodology

The proposed method of solution consists of including the equations that model the HVDC systems in the conventional power flow formulation. In order to achieve this goal, six new state variables will be included for each converter: $\phi_c$, $I_c$, $\mu_c$, $a_c$, $\delta_c$, and $V_{dc}$. The assumptions considered for Newton’s method to solve simultaneously the AC-DC PF, based on the system of equations $\Delta y = J \Delta x$, are presented below.

Considering the equations described in Section 2, the system of equations related to the HVDC system ($\Delta y$) is structured as shown in Table 1.

**Table 1.** Mismatch vector of equations in AC-DC Power Flow ($\Delta y$).

| Converter | Mismatch | Equation |
|-----------|----------|----------|
| Converter 1 | $\Delta y_{\text{conv}.1}^1$ | Equation (3) or (6) |
| | $\Delta y_{\text{conv}.1}^2$ | Equation (4) or (7) |
| | $\Delta y_{\text{conv}.1}^3$ | Equation (5) or (8) |
| | ... | ... |
| Converter $n$ | $\Delta y_{\text{conv}.n}^1$ | Equation (3) or (6) |
| | $\Delta y_{\text{conv}.n}^2$ | Equation (4) or (7) |
| Control equation | $\Delta y_{\text{cont}.1}^1$ | Equation (5) or (8) |
| | $\Delta y_{\text{cont}.1}^2$ | Equation (10) or (12) or (14) |
| | $\Delta y_{\text{cont}.1}^3$ | Equation (11) or (13) or (15) |
| | ... | ... |
| Converter $n$ | $\Delta y_{\text{cont}.n}^1$ | Equation (10) or (12) or (14) |
| | $\Delta y_{\text{cont}.n}^2$ | Equation (11) or (13) or (15) |
| DC network equations | $\Delta y_G$ | Equation (9) |

Based on Table 1, the vector of state variables related to the HVDC system ($\Delta x$) will be updated according to the order presented in Table 2.

The general linear system to be solved at each iteration in the solution process is shown in Equation (16).

$$
\begin{bmatrix}
\vdots \\
\Delta P_c' \\
\Delta Q_c' \\
\vdots \\
\text{ine} \Delta y_{\text{conv}.1}^1 \\
\text{ine} \Delta y_{\text{conv}.1}^2 \\
\vdots \\
\text{ine} \Delta y_{\text{cont}.1}^1 \\
\text{ine} \Delta y_{\text{cont}.1}^2 \\
\vdots \\
\text{ine} \Delta y_G
\end{bmatrix}
= 
\begin{bmatrix}
\begin{bmatrix}
I_{AC-AC} & I_{AC-DC} \\
I_{DC-AC} & I_{DC-DC}
\end{bmatrix}
\end{bmatrix}
\begin{bmatrix}
\vdots \\
\Delta \theta_c \\
\Delta V_c \\
\vdots \\
\text{ine} \Delta x_{\text{conv}.1}^1 \\
\text{ine} \Delta x_{\text{conv}.1}^2 \\
\vdots \\
\text{ine} \Delta x_{\text{cont}.1}^1 \\
\text{ine} \Delta x_{\text{cont}.1}^2 \\
\vdots \\
\text{ine} \Delta x_G
\end{bmatrix}
\right)
= (16)
where:

\[
\begin{array}{c}
\begin{bmatrix}
\mathbf{J}_{\text{ac-ac}}
\end{bmatrix} = \\
\begin{bmatrix}
\cdots & \cdots & \cdots & \cdots \\
\cdots & \frac{\partial P^1}{\partial Q^1} & \frac{\partial P^1}{\partial V_{\text{dc1}}^{\text{conv}}} & \cdots \\
\cdots & \cdots & \frac{\partial P^1}{\partial V_{\text{dc2}}^{\text{cont}}} & \cdots \\
\cdots & \cdots & \cdots & \cdots
\end{bmatrix}
\end{array}
\]

\[
\begin{array}{c}
\begin{bmatrix}
\mathbf{J}_{\text{ac-dc}}
\end{bmatrix} = \\
\begin{bmatrix}
\frac{\partial P^1}{\partial x_1} & \frac{\partial P^1}{\partial x_2} & \frac{\partial P^1}{\partial x_3} & \cdots & \frac{\partial P^1}{\partial x_N} \\
\frac{\partial P^1}{\partial x_1} & \frac{\partial P^1}{\partial x_2} & \frac{\partial P^1}{\partial x_3} & \cdots & \frac{\partial P^1}{\partial x_N} \\
\frac{\partial P^1}{\partial x_1} & \frac{\partial P^1}{\partial x_2} & \frac{\partial P^1}{\partial x_3} & \cdots & \frac{\partial P^1}{\partial x_N} \\
\cdots & \cdots & \cdots & \cdots & \cdots
\end{bmatrix}
\end{array}
\]

\[
\begin{array}{c}
\begin{bmatrix}
\mathbf{J}_{\text{dc-ac}}
\end{bmatrix} = \\
\begin{bmatrix}
\cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots \\
\end{bmatrix}
\end{array}
\]

\[
\begin{array}{c}
\begin{bmatrix}
\mathbf{J}_{\text{dc-dc}}
\end{bmatrix} = \\
\begin{bmatrix}
\frac{\partial Q^1}{\partial x_1} & \frac{\partial Q^1}{\partial x_2} & \frac{\partial Q^1}{\partial x_3} & \cdots & \frac{\partial Q^1}{\partial x_N} \\
\frac{\partial Q^1}{\partial x_1} & \frac{\partial Q^1}{\partial x_2} & \frac{\partial Q^1}{\partial x_3} & \cdots & \frac{\partial Q^1}{\partial x_N} \\
\frac{\partial Q^1}{\partial x_1} & \frac{\partial Q^1}{\partial x_2} & \frac{\partial Q^1}{\partial x_3} & \cdots & \frac{\partial Q^1}{\partial x_N} \\
\cdots & \cdots & \cdots & \cdots & \cdots
\end{bmatrix}
\end{array}
\]
Table 2. Vector of state variables related to the High Voltage Direct Current (HVDC) system (Δx).

| State variables related to the converter equations | Converter 1 | Δx_conv.1 | Δφ_c_conv.1 |
|-----------------------------------------------|-------------|-----------|-------------|
| Converter n                                  | Δx_conv.1   | ΔI_c_conv.1 |
| Converter n                                  | Δx_conv.2   | Δµ_c_conv.1 |

| State variables related to the Control equations | Converter 1 | Δx_cont.1 | Δα_c_cont.1 |
|-----------------------------------------------|-------------|-----------|-------------|
| Converter n                                  | Δx_cont.1   | Δδ_c_cont.1 |
| Converter n                                  | Δx_cont.2   | Δφ_c_cont.1 |

| State variables related to the DC network equations | Converter 1 | Δx_G | ΔVd1 |
|-----------------------------------------------|-------------|------|------|
| Converter n                                  | Δx_G        | ΔVd_n |

By observing Equation (16), the original Jacobian matrix of the problem is preserved. The new derivatives are located in additional rows and columns. The AC-AC block is the original Jacobian matrix and contains the derivatives of the equations of MW and MVAr power from the AC system in relation to the variables of the original system state. In the AC interface buses, injections of active and reactive power regarding the DC system should be considered. These injections are given by [1]

\[ p_{dc} = V_d.r \cdot I_r \]  \hspace{1cm} (17)

\[ Q_{dc} = V_d.r \cdot I_r \cdot \tan(\phi_r) \]  \hspace{1cm} (18)

\[ p_{dc} = V_d.i \cdot I_i \]  \hspace{1cm} (19)

\[ Q_{dc} = -V_d.i \cdot I_i \cdot \tan(\phi_i) \]  \hspace{1cm} (20)

From there, the residuals of the interface buses are given by

\[ \Delta P' = P_{sch} - P' \]  \hspace{1cm} (21)

\[ \Delta Q' = Q_{sch} - Q' \]  \hspace{1cm} (22)

where:

\[ P' = P_{calc} + P_{dc}.MVA_{dc} \]  \hspace{1cm} (23)

\[ Q' = Q_{calc} + Q_{dc}.MVA_{dc} \]  \hspace{1cm} (24)

\[ MVA_{dc} = \frac{MVA_{dc}}{MVA_{base}} \]  \hspace{1cm} (25)

**Tutorial Example**

This tutorial example shows with more details the basic principles of the proposed AC-DC power flow with a HVDC multiterminal system with three converter terminals.
(connected to infinity buses), one intermediate DC bus (called X in this example), and three HVDC lines, shown in Figure 2, which is composed of a DC network (represented by a generic methodology), with control modes that are shown below.

![Figure 2. Tutorial Example: System test multiterminal HVDC.](image)

For the case in study, the converters are working under the control modes shown in Table 3.

Table 3. Control mode configuration for the HVDC multi-terminal tutorial example test system.

| Rectifier 1 | Rectifier 2 | Inverter 1 |
|-------------|-------------|------------|
| Control modes | Constant-Power (CP) | Constant-Current (CC) | CEA with Constant-Voltage (CV) |

The system of equations that models, in a generic form, the DC network in this case is

\[
\begin{bmatrix}
I_{r1} \\
I_{r2} \\
I_{i1} \\
I_x
\end{bmatrix} =
\begin{bmatrix}
\frac{3}{R_{dc}} & \frac{1}{R_{dc}} & 0 & -\frac{2}{R_{dc}} \\
-\frac{1}{R_{dc}} & \frac{2}{R_{dc}} & \frac{1}{R_{dc}} & 0 \\
0 & -\frac{1}{R_{dc}} & \frac{3}{R_{dc}} & -\frac{2}{R_{dc}} \\
-\frac{2}{R_{dc}} & 0 & -\frac{2}{R_{dc}} & \frac{4}{R_{dc}}
\end{bmatrix}
\begin{bmatrix}
V_{d1} \\
V_{d2} \\
V_{d1} \\
V_{d2}
\end{bmatrix}
\]

The residuals equations related to the DC system (\(\Delta y^{dc}\)) are

\[
\Delta y_1^{dc} = -V_{d1} + k_r a_{r1} V_{r1} \cos(\alpha_1) - \frac{3}{\pi} X_{r1} I_{r1}
\]

\[
\Delta y_2^{dc} = -\frac{2 \mu_{r1} + \sin(2 \alpha_1) - \sin(2 (\alpha_1 + \mu_{r1}))}{\cos(2 \alpha_1) - \cos(2 (\alpha_1 + \mu_{r1}))} + \tan(\phi_{r1})
\]

\[
\Delta y_3^{dc} = -\cos(\alpha_1) + \cos(\alpha_1 + \mu_{r1}) + \frac{2 R_{r1} I_{r1}}{k_r a_{r1} V_{r1}}
\]

\[
\Delta y_4^{dc} = -V_{d2} + k_r a_{r2} V_{r2} \cos(\alpha_2) - \frac{3}{\pi} X_{r2} I_{r2}
\]

\[
\Delta y_5^{dc} = -\frac{2 \mu_{r2} + \sin(2 \alpha_2) - \sin(2 (\alpha_2 + \mu_{r2}))}{\cos(2 \alpha_2) - \cos(2 (\alpha_2 + \mu_{r2}))} + \tan(\phi_{r2})
\]
\[ \Delta y^{dc}_6 = -\cos(a_2) + \cos(a_2 + \mu_{r2}) + \frac{2.R_{r2}.I_{r2}}{K_{r,a12}.V_{r1}} \]  
(32)

\[ \Delta y^{dc}_7 = -Vd_{11} + k_1.a_{i1}.V_{i1}.\cos(\gamma_{11}) + \frac{3}{\pi}.X_{i1}.I_{i1} \]  
(33)

\[ \Delta y^{dc}_8 = -\frac{2.\mu_{i1} + \sin(2.\gamma_{11}) - \sin(2.(\gamma_{11} + \mu_{i1}))}{\cos(2.\gamma_{11}) - \cos(2.(\gamma_{11} + \mu_{i1}))} + \tan(\phi_{i1}) \]  
(34)

\[ \Delta y^{dc}_9 = -\cos(\gamma_{11}) + \cos(\gamma_{11} + \mu_{i1}) + \frac{2.R_{i1}.I_{i1}}{K_{i,a11}.V_{i1}} \]  
(35)

\[ \Delta y^{dc}_{10} = p^{dc}_{r1} - p^{dc}_{r1} \]  
(36)

\[ \Delta y^{dc}_{11} = -Vd_{r1} + 0.97.(k_{r1}.a_{r1}.V_{r1}.\cos(\alpha_{r1}^{min}) - R_{r1}.I_{r1}) \]  
(37)

\[ \Delta y^{dc}_{12} = 1d^{dc}_{r2} - 1d^{dc}_{r2} \]  
(38)

\[ \Delta y^{dc}_{13} = -Vd_{r2} + 0.97.(k_{r2}.a_{r2}.V_{r2}.\cos(\alpha_{r2}^{min}) - R_{r2}.I_{r2}) \]  
(39)

\[ \Delta y^{dc}_{14} = \cos(\gamma_{r1}^{dc}) - \cos(\gamma_{i1}) \]  
(40)

\[ \Delta y^{dc}_{15} = Vd^{dc}_{r1} - 1d^{dc}_{r1} \]  
(41)

\[ \Delta y^{dc}_{16} = I_{r1} - \frac{(Vd_{r1} - Vd_{r2})}{R_{dc}} - \frac{2.(Vd_{r1} - Vd_{s})}{R_{dc}} \]  
(42)

\[ \Delta y^{dc}_{17} = I_{r2} - \frac{(Vd_{r2} - Vd_{r1})}{R_{dc}} - \frac{2.(Vd_{r2} - Vd_{s})}{R_{dc}} \]  
(43)

\[ \Delta y^{dc}_{18} = I_{i1} - \frac{(Vd_{i1} - Vd_{i2})}{R_{dc}} - \frac{2.(Vd_{i1} - Vd_{s})}{R_{dc}} \]  
(44)

\[ \Delta y^{dc}_{19} = I_{i2} - \frac{(Vd_{i2} - Vd_{i1})}{R_{dc}} - \frac{2.(Vd_{i2} - Vd_{s})}{R_{dc}} \]  
(45)

The DC variables of the problem are included in the following order: \( \phi_{i1}, I_{r1}, \mu_{r1}, \phi_{r2}, I_{r2}, \mu_{r2}, \phi_{i1}, I_{i1}, \mu_{i1}, a_{r1}, a_{r2}, a_{i1}, \gamma_{i1}, V_{d11}, V_{d21}, V_{d12}, V_{d22}, \) and \( V_{ds} \).

The solution of the general linear system of Equation (16) provides the values for the variables in the steady state of the AC/DC system.

### 4. Results

The proposed methodology is verified through the study of two test systems. The first one is a modified version of the IEEE 14-bus system, and the second one is a 4-bus test system considering three HVDC systems with different DC networks.

#### 4.1. Modified IEEE 14-Bus

The first study is an AC-DC system with three terminals and two HVDC lines, shown in [15], based on the well-known IEEE 14-bus. The system topology is shown in Figure 3. The base power is 100 MVA.
Figure 3. IEEE 14-bus test system with a radial three-converter HVDC multiterminal [15].

The parameters of the AC system are shown in Tables 4–8.

Table 4. Capacitor shunt compensation (MVAr).

| AC Bus No. | 9 | 2 | 4 | 5 |
|------------|---|---|---|---|
| Rated reactive power | 15.0 | 20.0 | 20.0 | 20.0 |

Table 5. Reactance and tap of transformers (%).

| From Bus No. | To Bus No. | Ratio | $X_t$ |
|--------------|------------|-------|-------|
| 4            | 7          | 0.978 | 20.912 |
| 4            | 9          | 0.969 | 55.618 |
| 5            | 6          | 0.932 | 25.202 |

Table 6. Generator Parameters.

| Bus No. | $V_g$ (p.u.) | $Q_{max}$ (MVAr) | $Q_{min}$ (MVAr) | $P_g$ (MW) |
|---------|--------------|------------------|------------------|------------|
| 1       | 1.060        | 150.00           | −150.00          | 232.38     |
| 2       | 1.045        | 50.00            | −40.00           | 40.00      |
| 3       | 1.010        | 40.00            | 0.00             | 0.00       |
| 6       | 1.070        | 40.00            | −6.00            | 0.00       |
| 8       | 1.090        | 40.00            | −6.00            | 0.00       |

$V_g$ = generator terminal voltage; $Q_{max}$ = maximum reactive power generation; $Q_{min}$ = minimum reactive power generation; $P_g$ = output active power generation.
Table 7. System load configuration.

| Load Bus No. | \( P_l \) (p.u.) | \( Q_l \) (p.u.) |
|-------------|------------------|------------------|
| 2           | 0.217            | 0.127            |
| 3           | 0.942            | 0.190            |
| 4           | 0.478            | −0.039           |
| 5           | 0.076            | 0.016            |
| 6           | 0.112            | 0.075            |
| 9           | 0.295            | 0.166            |
| 10          | 0.090            | 0.058            |
| 11          | 0.035            | 0.018            |
| 12          | 0.061            | 0.016            |
| 13          | 0.135            | 0.058            |
| 14          | 0.149            | 0.050            |

\( P_l \) = active load power; \( Q_l \) = reactive load power.

Table 8. Parameters of transmission lines and transformers.

| From Bus No. | To Bus No. | \( R \) (%) | \( X \) (%) | \( y \) (MVAR) |
|--------------|------------|-------------|-------------|---------------|
| 1            | 2          | 1.938       | 5.917       | 5.28          |
| 1            | 5          | 5.403       | 22.304      | 4.92          |
| 2            | 3          | 4.699       | 19.797      | 4.38          |
| 3            | 4          | 6.701       | 17.103      | 1.28          |
| 4            | 5          | 1.335       | 4.211       | 0.00          |
| 6            | 11         | 9.498       | 19.890      | 0.00          |
| 6            | 12         | 12.291      | 25.581      | 0.00          |
| 6            | 13         | 6.615       | 13.027      | 0.00          |
| 7            | 8          | 0.000       | 17.615      | 0.00          |
| 7            | 9          | 0.000       | 11.001      | 0.00          |
| 9            | 14         | 12.711      | 27.038      | 0.00          |
| 9            | 10         | 0.03181     | 08.450      | 0.00          |
| 10           | 11         | 8.205       | 19.207      | 0.00          |
| 12           | 13         | 22.092      | 19.988      | 0.00          |
| 13           | 14         | 17.093      | 34.802      | 0.00          |

\( R \) = series resistance; \( X \) = series reactance; \( y \) = total shunt admittance.

The parameters of the HVDC system and the control modes in this study are shown in Tables 9 and 10, respectively.

Table 9. The HVDC system configuration.

| Converter No. | AC Bus No. | \( a_c \) (p.u.) | Type   | \( X_c \) (p.u.) | HVDC Line | \( R_d \) (p.u.) |
|---------------|------------|------------------|--------|------------------|-----------|-----------------|
| 1             | 5          | 0.950            | Rect.  | 0.083            | 1–2       | 0.02            |
| 2             | 2          | 0.975            | Rect.  | 0.125            | 2–3       | 0.02            |
| 3             | 4          | 1.000            | Inv.   | 0.125            |           |                 |

Table 10. Control modes configuration for Cases 1 and 2.

| Converter No. | Case 1   | Case 2   |
|---------------|----------|----------|
|               | 1 2 3 1 2 3 |          |
| Control mode  | CP CC CEA CIA CP CEA |
| Scheduled value| 0.47 0.47 17 | 14 0.56 17 |

Using the proposed methodology, it is possible to solve the problem in both operating conditions. The results for Cases 1 and 2 are shown in Tables 11–14. It is important to point
out that the proposed methodology converged in eight iterations in both cases. The adopted solution tolerances were \(10^{-6}\) p.u. for the AC system and \(10^{-9}\) p.u. for the DC system. All results were validated with the reference [15].

Table 11. AC bus voltage result.

| Bus No. | Case 1 | Case 2 |
|---------|--------|--------|
|         | Magnitude (p.u.) | Ang (°) | Magnitude (p.u.) | Ang (°) |
| 1       | 1.0600 | 0.000  | 1.0600 | 0.000  |
| 2       | 1.0450 | −4.084 | 1.0450 | −4.077 |
| 3       | 1.0100 | −13.550| 1.0100 | −13.561|
| 4       | 0.9940 | −12.189| 0.9938 | −12.213|
| 5       | 0.9940 | −12.132| 0.9937 | −12.162|
| 6       | 1.0700 | −17.380| 1.0700 | −17.409|
| 7       | 1.0493 | −15.546| 1.0492 | −15.571|
| 8       | 1.0900 | −15.546| 1.0900 | −15.571|
| 9       | 1.0425 | −17.267| 1.0425 | −17.293|
| 10      | 1.0400 | −17.570| 1.0399 | −17.597|
| 11      | 1.0514 | −17.595| 1.0514 | −17.623|
| 12      | 1.0541 | −18.188| 1.0541 | −18.217|
| 13      | 1.0485 | −18.197| 1.0485 | −18.226|
| 14      | 1.0270 | −18.687| 1.0270 | −18.715|

Table 12. Active and reactive generation result.

| Bus No. | Case 1 | Case 2 |
|---------|--------|--------|
|         | P (MW) | Q (MV Ar) | P (MW) | Q (MV Ar) |
| 1       | 233.1400 | 0.0028  | 233.1800 | 0.0044  |
| 2       | 39.3671  | 41.7683 | 39.3652  | 41.3397 |
| 3       | 0.0000   | 38.8600 | 0.0000   | 38.9700 |
| 4       | 0.0000   | 0.0787  | 0.0000   | 0.0005  |
| 5       | 0.0000   | 0.1516  | 0.0000   | 0.0388  |
| 6       | 0.0000   | 30.3600 | 0.0000   | 30.5100 |
| 8       | 0.0000   | 25.2000 | 0.0000   | 25.2600 |

Table 13. HVDC system result for Case 1.

| Parameter | Converter No.—Control Mode |
|-----------|----------------------------|
|           | 1—CP | 2—CC | 3—CEA |
| \(V_{dc}\) (p.u.) | 1.2069 | 1.1991 | 1.1819 |
| \(\phi_c\) (°) | 18.7800 | 29.2480 | 27.8680 |
| \(I_c\) (p.u.) | 0.3894 | 0.4700 | −0.8594 |
| \(\delta_c\) (°) | 14.0600 | 24.1800 | 17.0000 |
| \(\mu_c\) (°) | 8.7690 | 9.6480 | 19.5290 |
| \(a_c\) (p.u.) | 0.9500 | 0.9750 | 1.0000 |
| \(P_{dc}^c\) (MW) | 46.9967 | 56.3571 | −101.5734 |
| \(Q_{dc}^c\) (MV Ar) | 15.9810 | 31.5594 | 53.7087 |

The comparison of Tables 10 and 13 shows that the proposed methodology was effective in controlling the scheduled values in Case 1 considering the CP, CC, and CEA modes, respectively.

Regarding Case 2, the comparison of Tables 10 and 14 also shows that the proposed methodology was effective in controlling the scheduled values considering the CIA, CP, and CEA modes, respectively.
Table 14. HVDC system result for Case 2.

| Parameter                | Converter No.—Control Mode |
|--------------------------|----------------------------|
|                          | 1—CIA                      | 2—CP                      | 3—CEA                      |
| \(V_{dc}(p.u.)\)        | 1.2069                     | 1.1990                    | 1.1818                     |
| \(\phi_{dc}(\degree)\)  | 18.7740                    | 29.2620                   | 27.8770                    |
| \(I_{dc}(p.u.)\)        | 0.3932                     | 0.4671                    | -0.8603                    |
| \(\delta_{dc}(\degree)\)| 14.0000                    | 24.2300                   | 17.0000                    |
| \(\mu_{dc}(\degree)\)   | 8.8620                     | 9.5820                    | 19.5440                    |
| \(a_{dc}(p.u.)\)        | 0.9500                     | 0.9750                    | 1.0000                     |
| \(P_{dc}^c(MW)\)        | 47.4562                    | 56.0053                   | -101.6720                  |
| \(Q_{dc}^c(MVAR)\)      | 16.1312                    | 31.3798                   | 53.7806                    |

4.2. Four-Bus Test System

Figure 4 shows the topology of the four-bus test system. This system consists of three parallel HVDC systems, each of them with a different DC network configuration. However, it is important to point out that the three DC networks are equivalent. Therefore, the main objective here is to demonstrate that the proposed methodology is generic, allowing the calculation of voltage and current in any internal DC bus or circuit.

Figure 4. Four-bus test system including three HVDC with different DC networks.

The proposed power flow converged in four iterations. The converged DC voltages in the intermediate bus are \(V_x'' = V_x''' = 0.9772\ p.u.\). The DC voltages in the inverter are \(V_d'' = V_d''' = V_d'''' = 0.954\ p.u.\), and the currents are proportional to the circuit resistance. Moreover, it is important to highlight that the rectifier and inverter solutions in each HVDC are the same, as expected. This result shows that the proposed methodology is capable of solving a generic DC network.

A similar test was carried out using a large-scale system corresponding to a heavy load condition of the Brazilian Interconnected System topology for August 2018. This system comprises 6520 buses and 9487 circuits with a total active power demand of 81,865.4 MW and a reactive power demand of 20,383.7 MVAR. The results were consistent with those presented for the four-bus test system.

4.3. Discussion

The results obtained by the proposed methodology demonstrate the efficiency and the robustness of the tool for the simultaneous solution of the AC-DC power flow for analyzing...
MTDC systems. The proposed methodology uses the simultaneous strategy for the AC-DC power flow solution. This choice is related mainly to the following factors. First, because of the work proposed in [8], in which an analysis for the simultaneous solution of AC-DC power flow for steady-state studies is developed, for a DC system of only two terminals, based on the DC transmission link of Itaipu, which has served as the premise for the studies proposed in this paper. Second, the full Newton methodology is a well-established tool in the general literature on the subject, and, depending on the system feature, the choice of a sequential strategy could result in a delay in the AC convergence process, so the simultaneous strategy might be more interesting in these scenarios. Third, for the purposes of the study, the chosen method is best suited to the needs of this work. Finally, another decisive factor in this choice is related to the fact that there are few studies nowadays about the solution of Multi-Terminals HVDC systems using the simultaneous strategy.

5. Conclusions

This paper addresses the main assumptions involved in the modeling of Multi-Terminal HVDC systems in the AC-DC power flow problem for steady-state studies. The proposed methodology is based on a full Newton formulation of the AC-DC power flow problem for a multi-terminal HVDC system with a generic methodology for representing the DC network in steady-state studies.

The results show that the proposed methodology is valid and effective in power transmission studies via Multi-Terminal HVDC systems. It can be stated that the methodology and the proposed models constitute valuable tools in solving current problems of AC-DC power flow, as a result of the increase in electricity demand, the interconnection of systems operating in different frequencies, and a more intensive use of Multi-Terminal HVDC transmission structures in electric power systems.

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Abbreviations

The following abbreviations are used in this manuscript:

HVDC: High Voltage Direct Current
MTDC: Multi-Terminal HVDC
VSC: Voltage Source Converter
CSC: Current Source Converter
DC: Direct Current
AC: Alternating Current

The following notation are used in this manuscript:

\( \hat{V}_c \): Primary phasor voltage of the converter transformer
\( \hat{E}_c \): Secondary phasor voltage of the converter transformer
\( a_c \): Converter transformer turns ratio
\( X_c \): Converter commutating reactance per bridge/phase
\( R_c \): Converter equivalent commutating resistance
\( \phi_c \): Converter power factor angle
\( \mu_c \): Commutating converter angle
\( \delta_c \): Converter firing angle
\( \alpha \): Rectifier ignition angle
\( \gamma \): Inverter extinction angle
\( V_{dc} \): Converter direct voltage per pole
\( I_c \): Converter direct current per pole
\( R_{dc} \): DC resistance
\( P_{dc} \): Injection of active power on the AC interface bus associated with the converter
\( Q_{dc} \): Injection of reactive power on the AC interface bus associated with the converter
\( n_b \): Number of series-connected bridges in a terminal

superscripts:

+ for positive pole quantities
− for negative pole quantities
c for converter
r for rectifier
i for inverter
dc for direct current
max for maximum values
min for minimum values
sch for scheduled values

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