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Isospin Mixing and the Cubic Isobaric Multiplet Mass Equation in the Lowest
\( T = 2, \ A = 32 \) Quintet

M. Kamil,\(^{1}\) S. Triambak,\(^{1,*}\) A. Magilligan,\(^{2}\) A. García,\(^{3}\) B. A. Brown,\(^{2}\) P. Adsley,\(^{4,5}\) V. Bildstein,\(^{6}\) C. Burbadge,\(^{6}\) A. Díaz Varela,\(^{6}\) T. Faestermann,\(^{7}\) P. E. Garrett,\(^{6}\) R. Hertenberger,\(^{6}\) N. Y. Kheswa,\(^{5}\) K. G. Leach,\(^{5}\) R. Lindsay,\(^{1}\) D. J. Marín-Lambarri,\(^{1}\) F. Ghazi Moradi,\(^{6}\) N. J. Mukwevho,\(^{1}\) R. Neveling,\(^{5}\) J. C. Nzobadila Ondze,\(^{1}\) P. Papka,\(^{10,5}\) L. Pellegrin,\(^{5,5}\) V. Pesudo,\(^{1}\) B. M. Rebeiro,\(^{1}\) M. Scheck,\(^{11}\) F. D. Smit,\(^{5}\) and H.-F. Wirth\(^{8}\)

\(^{1}\)Department of Physics and Astronomy, University of the Western Cape, P/B X17, Bellville 7535, South Africa
\(^{2}\)Department of Physics and Astronomy and National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824-1321, USA
\(^{3}\)Department of Physics and Center for Experimental Nuclear Physics and Astrophysics, University of Washington, Seattle, Washington 98195, USA
\(^{4}\)School of Physics, University of the Witwatersrand, Johannesburg 2050, South Africa
\(^{5}\)iThemba LABS, P.O. Box 722, Somerset West 7129, South Africa
\(^{6}\)Department of Physics, University of Guelph, Guelph, Ontario N1G 2W1, Canada
\(^{7}\)Physik Department, Technische Universität München, D-85748 Garching, Germany
\(^{8}\)Fakultät für Physik, Ludwig-Maximilians-Universität München, D-85748 Garching, Germany
\(^{9}\)Department of Physics, Colorado School of Mines, Golden, Colorado 80401, USA
\(^{10}\)Department of Physics, Stellenbosch University, Private Bag X1, Matieland, 7602, South Africa
\(^{11}\)School of Computing, Engineering, and Physical Sciences, University of the West of Scotland, Paisley PA1 2BE, United Kingdom
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The Isobaric Multiplet Mass Equation (IMME) is known to break down in the first \( T = 2, A = 32 \) isospin quintet. In this work we combine high-resolution experimental data with state-of-the-art shell-model calculations to investigate isospin mixing as a possible cause for this violation. The experimental data are used to validate isospin-mixing matrix elements calculated with newly developed shell-model Hamiltonians. Our analysis shows that isospin mixing with non-analog \( T = 1 \) states contributes to the IMME breakdown, making the requirement of an anomalous cubic term inevitable for the multiplet.

The isobaric multiplet mass equation (IMME) [1, 2]

\[ M(T_z) = a + bT_z + cT_z^2, \]  
relates the masses of an isobaric multiplet, with \( T_z = (N - Z)/2 \) being the isospin projection of each multiplet member. The above quadratic relation results from isospin symmetry breaking (ISB) due to two-body charge-dependent interactions. It only holds if the ISB interactions are described at tree-level as the sum of an isoscalar, isovector and isotensor operator of rank 2 [3].

Over the years, the general success of the IMME over a large mass range made it a reliable tool to address a variety of research problems. For example, it was used to test recent advances in nuclear theory [4–6], map the proton dripline [7], identify candidates for two-proton radioactivity [8, 9], search for physics beyond the standard model [10], infer rapid proton capture (\( rp \)) nuclear reaction rates relevant for studies of novae and x-ray bursts [11–13], assess global nuclear mass model predictions [14] and constrain calculations relevant for CKM unitarity tests [15].

In this context, the lowest isospin \( T = 2 \) quintet for \( A = 32 \) (with spin and parity \( J^z = 0^+ \)) is an interesting case. The \( \beta \) decay of \( ^{32}\)Ar, the most proton-rich member of the quintet was previously used for searches of exotic scalar [10] and tensor [20] weak interactions as well as for benchmarking ISB corrections [17] important for obtaining a precise value of \( V_{ud} \), the up-down element of the CKM quark-mixing matrix [15]. In fact, the \( A = 32 \) quintet is one of the most extensively studied and precisely measured multiplets to date [18, 21–25]. It remains an anomalous case, for which the IMME breaks down significantly [26]. A satisfactory fit to the measured masses is only obtained with an additional cubic \( dT_z^3 \) term, with \( d = 0.89(11) \) keV (c.f. Table I). This is the

* striambak@uwc.ac.za

\begin{table} [!h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
Isobar & \( T_z \) & \( M_{\exp} \) (keV)\(^{a}\) & \( M_{\text{IMME}} \) (keV) \\
\hline
\(^{32}\)Ar & \(-2\) & \(-2200.4(1.8)\) & \(-2200.35(158)\) \\
\(^{32}\)Cl & \(-1\) & \(-8288.4(7)^{1}\) & \(-8288.43(47)\) \\
\(^{32}\)Si & \(0\) & \(-13967.58(28)^{c}\) & \(-13967.57(25)\) \\
\(^{32}\)P & \(+1\) & \(-19232.44(7)^{d}\) & \(-19232.43(7)\) \\
\(^{32}\)Si & \(+2\) & \(-24077.69(30)\) & \(-24077.69(30)\) \\
\hline
\end{tabular}
\caption{Cubic IMME fit to measured mass excesses of the lowest \( T = 2 \) quintet in \( A = 32 \). The fit yields \( d = 0.89(11) \) keV, with \( P(\chi^2, \nu) = 0.95 \).
\(^{a}\) Ground state masses are taken from Ref. [16].
\(^{b}\) \( E_x = 5046.3(4) \) keV from Ref. [17].
\(^{c}\) \( E_x = 12047.96(28) \) keV from Ref. [18].
\(^{d}\) \( E_x = 5072.44(6) \) keV from Ref. [19].}
\end{table}
**smallest and most precisely** determined violation of the IMME observed so far. Unlike other multiplets, where apparent violations of the IMME were resolved through subsequent measurements [27–32], the $A = 32$ anomaly has persisted over several years, despite high-precision re-measurements of ground state masses [21, 22, 33] as well as excitation energies [18, 28]. A recent compilation [26] showed the $A = 32$ quintet to be a unique case, in which the $\chi^2$ value for a cubic fit yields 95% probability that it is the correct model to describe the data. Since there are no known fundamental reasons that preclude a cubic IMME term, it is interesting that the magnitude of the extracted $d$ coefficient for this case agrees well with theoretical estimates that used a simple nonperturbative model [34] or a three-body second-order Coulomb interaction [35], both of which allow a non-vanishing cubic term, with $|d| \approx 1$ keV. Alternatively, the role of isospin-mixing with non-analog $0^+$ states was also theoretically investigated in the recent past [24, 25].

We delve into the above aspect here, via an analysis of high-resolution experimental data and a comparison with calculations that use recently developed shell model Hamiltonians [36]. For the former, we mainly rely on data from a previous $^{32}$Ar $\beta$ decay experiment at CERN-ISOLDE [10], that acquired $\beta$-delayed protons from unbound states in the daughter $^{32}$Cl ($S_p \approx 1581$ keV) with high-resolution (full widths at half maximum of $\sim 6$ keV). The primary goal of the ISOLDE experiment was to search for scalar currents in the weak interaction, by determining the $\beta\nu$ angular correlation ($a_{3\beta\nu}$) for the decay, via a precise analysis of the shape of the superallowed $\beta$-delayed proton peak [10]. Part of the proton spectrum is shown in Fig. 1.

The high resolution nature of the ISOLDE data allow an identification of potential isospin admixtures to the $T = 2$ isobaric analog state (IAS) in $^{32}$Cl. The nature of each $\beta$ transition is encoded in the shapes of the proton groups, which would be different if the transitions were Fermi ($0^+ \rightarrow 0^+$), with $a_{3\beta\nu} = 1$ or Gamow-Teller ($0^+ \rightarrow 1^+$), with $a_{3\beta\nu} = -1/3$. We analyzed these data using the R-matrix formalism described in Refs. [37, 38]. In the analysis, the proton peaks were grouped as $p_0$, $p_1$, $p_2$ or $p_3$ depending on whether the proton emission left the residual $^{31}$S nucleus in its ground state or any of its first three excited states at 1249, 2234 and 3077 keV (see Fig. 9 in Ref. [17]). Interference was allowed between all levels that had the same quantum numbers, transition type (Fermi or Gamow-Teller), and final states in $^{31}$S. The R-matrix fits folded in the detector response function and lepton recoil effects (described in Ref. [10]), and were parameterized using various $J^\pi$ values for the daughter $^{32}$Cl states and associated $a_{3\beta\nu}$ coefficients. The fits yielded relative intensities, $^{32}$Cl excitation energies and intrinsic widths. They were repeated for different values of $a_{3\beta\nu}$, spin-parity combinations and $p_0$, $p_1$, $p_2$, $p_3$ assignments for the daughter levels to obtain optimal results. A few important features of the analysis are described below.

Peaks C, E and H were assumed to be from the $p_1$ group. These assignments were based on data reported by independent $^{32}$Ar $\beta$-delayed proton-$\gamma$ coincidence measurements [17, 39]. We observe that a reasonably good R-matrix fit is attained (Fig. 1) with the parameters listed in Table II. The fit assumes that peak B arises from a Fermi transition, while the others (apart from peak I) are exclusively from Gamow-Teller decays. Based purely on $\chi^2$ values from independent fits, peak I could be either from a Fermi or Gamow-Teller decay.

We compared these results with $^{32}$S($^{4}$He,$t$) data that were independently obtained at the MLL tandem accelerator facility in Garching, Germany. The experiment

![FIG. 1. $^{32}$Ar $\beta$-delayed proton spectrum from the ISOLDE experiment [10] and its corresponding R-matrix fit. The inset shows a magnified portion of the spectrum.](image-url)
used $\sim$300 enA of 33 MeV $^3$He$^{++}$ ions, incident on a 120 $\mu$g/cm$^2$-thick natural ZnS target. The tritons exiting the target were momentum analyzed using the high-resolution Q3D magnetic spectrograph [40, 41]. A sample triton spectrum in the energy range of interest is shown in Fig. 2. These data provided an important confirmation of the $p_0$ assignments for peaks A, D, F and G in our R-matrix analysis. Additionally, since the $^{32}$S($^3$He,$t$) reaction predominantly populates $J^* = 1^+$, $T = 1$ levels at forward angles [42], the states observed at these energies in both $^{32}$Ar $\beta$ decay and the $^{32}$S($^3$He,$t$) reaction can be ruled out as sources of $J^* = 0^+$ isospin impurity. This comparative analysis leaves only the 4443 and 5302 keV levels (c.f. Table II) as potential admixed states. We find from the $\beta$ decay data that the $p_1$ intensity for the latter is around 1.2 times larger than its $p_0$ group. In comparison, the $p_1$ intensity for the IAS is roughly 80 times smaller than the $p_0$. This is due to the low penetrability of $l = 2$ protons from the $J^* = 0^+$ IAS. The above discrepancy makes it highly unlikely for the 5302 keV state to have spin-parity $0^+$, which rules it out as a source of isospin mixing.

We next used our measured $\beta$-delayed proton intensities in Table II, together with shell model calculations of isospin mixing to investigate the matter further. For the latter we used newly developed isospin non-conserving (INC) USDC and USDI interactions, described extensively in Ref. [36]. The INC parameters in the new USD Hamiltonians were obtained from a fit to several mirror displacement energies and stringently tested via a comparison with experimental data [36]. The isospin-mixing matrix elements calculated with the new Hamiltonians were robustly validated [36] with results from independent high-precision $^{31,32}$Cl $\beta$ decay experiments [43–45], where large isospin-mixing in the daughter $^{31,32}$S states were observed. More recently, such calculations were used together with a $^{32}$Ar $\beta$ decay measurement [39], that acquired valuable proton-gamma coincidence data, albeit with lower proton energy resolution. Ref. [39] identified two possible sources of $T = 1$ isospin mixing at 4799 and 4561 keV. However, their measured proton branches were significantly lower than calculated values. We show below that the higher-resolution ISOLDE data justifies ruling out these proposed levels, while providing a viable alternative for the admixed $T = 1,0^+$ state, which is consistent with both theory predictions as well as experimental observations.

Our present shell model calculations show that the isospin mixing within the $T_z = 1$, 0, and $-1$ members of the quintet occurs primarily with a single $T = 1$ state, located few hundred keV below the $T = 2$ IAS in each isobar. The results are summarized in Table III, which lists the energy differences ($\Delta E = E_i - E_{IAS}$) between the admixed $T = 1$ and $T = 2$ states for each nucleus, and the calculated isospin-mixing matrix element ($v$) for $^{32}$Cl. The evaluated mixing matrix elements for each of the three nuclei are plotted in a separate supplemental file. We note that the mixing matrix elements obtained (for all three isobars) with the older USDB-CD interactions [46] are nearly a factor of two smaller than the ones obtained with the newer interactions. This is consistent with previous observations for $^{31,32}$Cl $\beta$ decay [36].

The predicted $J^*; T = 0^+$; 1 level in $^{32}$Cl can be identified by obtaining an experimental value of $v$ from the data in Fig. 1 and Table II. For two-state mixing, $v_{\text{expt}}$ is simply

$$v_{\text{expt}} = \Delta E_{\text{expt}} \left[ \frac{B(F)_{\text{admix}}}{B(F)_{\text{SA}}} \right]^{1/2},$$

where the ratio in the square bracket is the (Fermi) strength to the admixed $T = 1$ state, relative to the superallowed decay. This is easily determined from the measured $I^o_{\beta}$ values in Table II, the ratio of calculated phase-space factors, a small ISB correction [17] and the $p_0$ contribution to the total superallowed intensity. On applying this prescription to the remaining candidate $0^+$ level at 4443 keV, we obtain a $v_{\text{expt}} = 39.0(24)$ keV, in excellent agreement with the calculations. The results in Table III, together with our aforementioned observations

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**TABLE III.** Calculated energy differences between the $T = 2$ IAS and the nearest $0^+$, $T = 1$ state in $^{32}$Cl, $^{32}$S, and $^{32}$P. The isospin mixing matrix element in $^{32}$Cl is listed for comparison.

| Interaction | $\Delta E$ (keV) | $v$ (keV) |
|-------------|------------------|-----------|
| $^{32}$Cl   |                  |           |
| USDC        | -226             | -186      | -237     | 40     |
| USDI        | -308             | -266      | -326     | 41     |
| USDCm       | -324             | -239      | -293     | 46     |
| USDIm       | -405             | -321      | -383     | 47     |
| USDB-CD     | -440             | -378      | -427     | 22     |
| Expt (this work) | -603           | 39.0(24)  |          |        |
and the experimental values listed in Table II allow a credible identification of the 4443 keV level as the predicted admixed $T = 1$ state. The discrepancy between theory and experiment for $\Delta E$ should not be surprising, given the $\sim$150 keV root-mean-square (rms) deviation for energies in USD interactions [36]. As further tests of our calculations, we also evaluated amplitudes for isospin-forbidden proton emission from the two admixed $J^\pi = 0^+$ levels in $^{32}$Cl and the effect of the $T = 1$ isospin mixing on the superallowed Fermi decay of $^{32}$Ar. Our results again show reasonable agreement with experiment. These aspects are discussed in the supplemental file.

We next investigated additional cubic ($dT_3^3$) and quartic ($eT_4^4$) terms to the IMME due to such isospin mixing. One can determine the exact solutions for the $d$ and $e$ coefficients by modifying Eq. (1) to incorporate such terms, such that

$$d = \frac{1}{12}(M_2 - 2M_1 + 2M_{-1} - M_{-2})$$

$$e = \frac{1}{24}(M_2 - 4M_1 + 6M_0 - 4M_{-1} + M_{-2}),$$

(3)

where the $M_{T_i}$ are isobar masses in the quintet. The results for $d$ and $e$ using the calculated values of $v$ and $\Delta E$ are shown in Fig. 3, and labeled as “unshifted”. We repeated these evaluations by shifting the $T = 2$ states in $^{32}$Cl, $^{32}$S and $^{32}$P by the amount needed to reproduce our experimentally determined 603 keV energy difference in $^{32}$Cl. The same $\Delta E$ was used for the three isobars due to the lack of similar experimental information for $^{32}$S and $^{32}$P. The shifts were accomplished by adding a $T^2$ term to the Hamiltonian that shifts the $T = 2$ states relative to the others, without changing the isospin-mixing. As evident in Fig. 3, the shifts mildly affect the $e$ coefficient (due to changes in the $T = 0$ mixing with the IAS in $^{32}$S), but significantly decrease the calculated $d$ coefficient to $\approx 0.3-0.4$ keV for the new interactions. The single-state contributions from $T = 0$ and $T = 1$ levels are

$$d_i = -\frac{1}{6}s_P + \frac{1}{6}s_{Cl}$$

$$e_i = -\frac{1}{6}s_P + \frac{1}{4}s_S - \frac{1}{6}s_{Cl},$$

(4)

where $s = -v^2/\Delta E$ is the shift in each IAS due to two-state mixing. Thus, one can remove the $T = 1$ mixing contribution for further investigation (labeled as “removed” in Fig. 3). We observe that on doing so, the extracted coefficients are mostly consistent with zero. The negative $e$ coefficient from the USDI calculation is due to mixing with a $T = 0$ state in $^{32}$S. However such $T = 0$ mixing would not explain the non-zero $d$ coefficient required for the quintet, as evident from Eq. (4).

The above analysis validates the contention that isospin mixing with predicted $T = 1$ levels necessitates a small cubic term for the multiplet. Our extracted $d$ coefficients for the “shifted” calculations from different USDc and USDI Hamiltonians agree reasonably well with one another, but are smaller than the experimental value $d = 0.89(11)$ keV, from Table I.

In summary, we used high-resolution experimental data to validate newly-developed shell model calculations of isospin mixing in $^{32}$Cl. This analysis is used to investigate the observed IMME violation in the first $T = 2$; $A = 32$ quintet. We show that isospin mixing with shell-model-predicted $T = 1$ states below the IAS necessarily result in a break down of the IMME, leading to the requirement of a small cubic term. However, this alone cannot explain the magnitude of the experimental $d$ coefficient in Table I. Additionally, our observations pertaining to $^{32}$Ar $\rightarrow$ $^{32}$Cl superallowed Fermi decay may be useful to benchmark theory calculations [17] of model-dependent ISB corrections that are important for top-row CKM unitarity tests [15]. This is particularly relevant in light of recent evaluations of radiative corrections [47] that show an apparent violation of CKM unitarity at the $>3\sigma$ level [48].

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