Mesino Oscillation in MFV SUSY

Joshua Berger, Csaba Csáki, Yuval Grossman, and Ben Heidenreich

Department of Physics, LEPP, Cornell University, Ithaca, NY 14853

jb454,csaki,yg73,bjh77@cornell.edu

Abstract

$R$-parity violating supersymmetry in a Minimal Flavor Violation paradigm can produce same-sign dilepton signals via direct sbottom-LSP pair production. Such signals arise when the sbottom hadronizes and the resulting mesino oscillates into an anti-mesino. The first bounds on the sbottom mass are placed in this scenario using current LHC results.

I. INTRODUCTION

The 2011 and 2012 data from the Large Hadron Collider (LHC) place severe constraints on natural $R$-parity conserving models of supersymmetry (SUSY) [1–3]. While such models are not excluded by the data, if they are to solve the hierarchy problem of the Standard Model (SM), they are forced to have either non-generic spectra where only third-generation squarks are light [4–8] or nearly degenerate particles, either in the form of stealth SUSY [9] or a squashed [10] spectrum. On the other hand, the stubborn agreement between SM predictions and observations in channels with large missing transverse energy (MET) cuts may indicate that the assumption of exact $R$-parity conservation is incorrect.

Models with $R$-parity violation (RPV) have been proposed since the early days of SUSY [11]. More recently a possible connection between the problems of baryon and lepton number violation and large flavor changing operators was highlighted [12, 13]. The assumption of Minimal Flavor Violation (MFV) has been shown to be sufficient to prevent both rapid nuclear decay (and other baryon-number violating processes) and large corrections to flavor observables in the $B$, $D$, and $K$ systems. In models of MFV SUSY, sparticles are pair produced as in $R$-parity conserving models, while the lightest supersymmetric partner (LSP) is generally unstable on collider scales and will decay via the baryon-number violating $u^cd^c$ superpotential term. In this paper, we investigate one of the interesting scenarios that can arise in the model of [13].
In MFV SUSY models it is particularly compelling to consider the case when the LSP is a third generation squark: naturalness requires light third generation squarks in general, and we will see below that in the MFV scenario there is a high probability for this to be actually realized, due to the large top Yukawa coupling. The phenomenology of this scenario is also particularly rich, as the lifetime of an LSP stop or sbottom is long enough that the squark hadronizes to form a mesino by binding to a light quark pulled from the vacuum. It is, however, usually sufficiently short-lived to decay before reaching the detector. Observing squark production is challenging in this scenario, due to the lack of any obvious handles on the events, such as missing energy or displaced vertices. Instead we will make use of the idea of mesino-antimesino oscillations, following Sarid and Thomas [14]. We will demonstrate that sbottom-LSP pair production often allows for mesino-antimesino oscillations, which may lead to same-sign dilepton signals.

A sbottom LSP decays dominantly to a top quark and a strange quark [13]. If one of the sbottom mesinos oscillates before decaying, the tops will be of the same charge, and if both tops decay leptonically this leads to same-sign leptons. These events would also contain $b$ quarks from the top decays, providing further handles on the event. Recently, CMS searched for such events and placed bounds on their cross sections [15]. We will show that this CMS search already places some bounds on sbottoms, which should improve significantly with more data.

The rest of this paper is organized as follows. In Section 2, we study the typical squark spectra in MFV SUSY scenarios and demonstrate that the stop and sbottom are most often the lightest squarks. In Section 3, we present a calculation of the decay rate and oscillation time for a squark LSP in MFV SUSY and show that a significant oscillation probability is possible and occurs frequently. In Section 4, we comment on the sensitivity of existing LHC searches to this scenario. We conclude in Section 5. The details of the calculation of the mesino-antimesino oscillation rate is given in the appendix.

II. MFV SQUARK SPECTRA

In an MFV SUSY model the LSP decays and we are not restricted to models with a neutralino LSP, whereas the phenomenology of the model will depend on the identity of the LSP. In particular, the LSP can be colored and, as we consider below, can be a squark.

MFV requires that all flavor violation be proportional to the appropriate combination of Yukawa matrices, which are treated as spurions of the flavor symmetry. The squark mass
matrices are then required to have the following form [13]:

\[
M_{\tilde{u}}^2 = \begin{pmatrix}
m_{\tilde{q}1}^2 + (a_u + v_u^2)Y_uY_u^\dagger + b_uY_dY_d^\dagger + D_{uL} & A_uY_u \\
A_u^*Y_u^\dagger & m_{\tilde{u}1}^2 + (a_u + v_u^2)Y_u^\dagger Y_u + D_{uR}
\end{pmatrix},
\]

(1)

and

\[
M_{\tilde{d}}^2 = \begin{pmatrix}
m_{\tilde{q}1}^2 + a_uY_uY_u^\dagger + (b_d + v_d^2)Y_dY_d^\dagger + D_{dL} & A_dY_d \\
A_d^*Y_d^\dagger & m_{\tilde{d}1}^2 + (a_d + v_d^2)Y_d^\dagger Y_d + D_{dR}
\end{pmatrix},
\]

(2)

The \(D\) terms are automatically flavor diagonal and given by

\[
D_L = \left(T^3 - Qs_w^2\right)\cos(2\beta) m_Z^2, \quad D_R = Qs_w^2 \cos(2\beta) m_Z^2,
\]

(3)

where \(Q = +2/3\) \((-1/3)\) and \(T^3 = +1/2\) \((-1/2)\) for the up-type (down-type) squarks, \(s_w\) is the sine of the Weinberg angle, \(\tan \beta\) is the ratio of the Higgs VEVs, and \(m_Z\) is the mass of the \(Z\). The parameters \(m_i^2\), \(a_i\), \(b_i\), and \(A_i\) arise from supersymmetry breaking, and we therefore expect them to be of order \(m_{\text{soft}}^2\).

Given these constraints, we can perform a scan over the parameter space that determines the squark spectrum. We select random values for the undetermined dimension-two parameters uniformly in \([-m_{\text{soft}}^2, m_{\text{soft}}^2]\). For this scan, we choose \(m_{\text{soft}} = 1\) TeV and \(\tan \beta = 10\). The overall result is not very sensitive to this choice. We impose the constraint that the smallest eigenvalues of both squark mass matrices be greater than the top mass, \(m_t \approx 175\) GeV. In general, left-right mixing is not too large and we therefore use notation where \(\tilde{b}_L\) refers to the mass eigenstate of the sbottom that is mostly a left-handed sbottom. We also impose that

![FIG. 1. Distribution of lightest squark flavor over a random sampling of MFV SUSY parameter space.](image-url)
the lightest stop-like squark be lighter than 500 GeV as demanded by naturalness. Under these conditions, the distribution of lightest squark flavors is given by Fig. [1].

We observe that roughly 85% of parameter points have a third-generation lightest squark, out of which 15% have a sbottom squark at the bottom of the spectrum. The large likelihood of a third generation lightest squark can be explained by the relatively large left-right mixing for this generation. This mixing tends to drive the mass of the lighter third generation squark down, making it more likely to be lightest overall. (There is also a significant contribution from the naturalness cut, since requiring one light stop tends to reduce the incidence of both stops being made heavy by a large positive $a_q$.) Note that at large $\tan\beta$ this effect is enhanced for the sbottoms, making it even more likely to get a sbottom LSP. It is therefore natural to consider signatures of a sbottom LSP in MFV SUSY and we do so from this point on.

III. MESINO OSCILLATION IN MFV SUSY

The MFV SUSY scenario offers a rich phenomenology due to the naturally small decay width of the LSP, a consequence of approximate $R$-parity conservation. The couplings are sufficiently small to yield LSP lifetimes that are longer than the timescales of SM short-distance physics, such as hadronization, yet often shorter than the timescales set by macroscopic distances in the LHC detectors. In this intermediate range, it can be difficult to construct observables that are not overwhelmed by SM background. If the LSP carries color, however, then it lives sufficiently long to hadronize, an intriguing possibility. This process can yield additional phenomena that allow for efficient selection of SUSY events.

The case of a sbottom LSP is particularly fruitful. If the gluino is heavy, the dominant SUSY production mode will be sbottom pair production. The dominant decay of the sbottom in MFV SUSY is to top and strange. The top has a leptonic decay mode, which already suppresses many SM backgrounds. As we show, the fact that the sbottom hardronizes allows for the possibility of sbottom oscillations, which lead, some fraction of the time, to same-sign lepton events.

While other squark flavors can also oscillate, this turns out to be parametrically rarer. In addition, up-type squark LSPs do not decay leptonically, precluding the possibility of a same-sign dilepton signature. We do not consider these other possibilities further in this work.

We also do not consider the case where gluino pair production is significant. This would lead to additional same-sign lepton events due to the Majorana nature of the gluino, provid-
FIG. 2. The leading diagram for the $R$-parity violating sbottom decay.

ing a background to the case we are considering.

We begin by calculating the sbottom decay width. We denote the lightest sbottom mass eigenstate by $\tilde{b}$. Its decay width depends on an overall (generically order 1) coefficient that we denote by $\lambda''$. The Lagrangian terms that gives the decay is [13]:

$$\mathcal{L} = - (\lambda'')^* \epsilon_{ijk} \frac{m_u}{v c_\beta} U_{q_i}^D \frac{m_{u,j}}{v s_\beta} V^{*}_{i,j} \frac{m_{d,k}}{v c_\beta} \tilde{b} u^{c \dagger} d^{c \dagger} \sim - (\lambda'')^* V_{td} \frac{m_b m_s}{v^3 c_\beta s_\beta} \tilde{b} t^{c \dagger} s^{c \dagger}, \quad (4)$$

where $v = 174$ GeV, $V$ is the CKM matrix, and we use $\tilde{b}_1$ to denote the lightest down-type squark, which we assume is predominantly sbottom-like. The mixing matrix $U^D$ is defined such that

$$\tilde{q} = U^D q_i.$$  

(5)

In this notation, $\tilde{q}$ are the squark flavor-basis fields in the mass basis of the quarks and $\tilde{q}$ are the squark mass-basis fields. The approximation is valid if the lightest sbottom is mostly right-handed. Otherwise, there is an additional suppression from the left-right mixing. The partial decay width can then be calculated using the diagram in Fig. 2. The result (neglecting the mass of the down quark in the phase space integral) is:

$$\Gamma = \sum_{j',k} \frac{1}{32 \pi^2} |(\lambda'')^* \sum_{i,j,q} \epsilon_{ijk} \frac{m_u}{v c_\beta} U_{q_i}^D \frac{m_{u,j}}{v s_\beta} V^{*}_{i,j} \frac{m_{d,k}}{v c_\beta} \tilde{b} u^{c \dagger} d^{c \dagger}|^2 m_b \left(1 - \frac{m_{u,j}^2}{m_b^2}\right)^2.$$  

(6)

To gain some intuition about sbottom LSP decays, we now make a few approximations. The decay is dominated by $\tilde{b} \to t^c s^c$ provided there is sufficient phase space. In the interesting segment of parameter space, the LSP is made up almost entirely of some admixture of the left-handed and right-handed sbottom, so that the decay width is approximately:

$$\Gamma \approx \frac{1}{32 \pi^2} |\lambda''|^2 \sin^2 \theta \frac{m_b^2 m_s^2 m_t^2 |V_{td}|^2 m_b}{v^6 c_\beta^4 s_\beta^2} \left(1 - \frac{m_{u,j}^2}{m_b^2}\right)^2 \left(\frac{t_\beta}{10}\right)^4 \left(\frac{m_b}{300 \text{ GeV}}\right),$$  

(7)
where $\theta$ is the left-right mixing squark mixing angle.

The sbottom decay rate is much less than the hadronization scale $\Lambda_{QCD} \sim 0.2$ GeV. Thus, the sbottom squark will hadronize before decaying to form fermionic mesino bound states $\tilde{B}_q = \tilde{b}^* q$ and $\tilde{B}^c = \tilde{b}^c q$. If $q = d, s$, then the mesino is neutral, opening up the possibility for mesino oscillations, first discussed in [14]. Since few details of the calculation of the oscillation rate were given in [14], we elaborate on it in Appendix A explaining the necessary approximations. Our final result, eq. (A17), is in broad agreement with that of [14], and we restate it here:

$$\Delta m = \omega = g_\gamma^2 \left| (U^D_{dL,1})^2 + (U^D_{dR,1})^2 \right| f_B^2 \left( 1 - \frac{1}{N_c^2} \right) \frac{m_{\tilde{g}}}{m_{\tilde{g}}^2 - m_{\tilde{b}}^2}. \quad (8)$$

This result depends on the nature of the spectator quark. We can use MFV to approximate the ratio of the oscillation rates as we have $|U^q_M| \propto |V_{tq}V_{tb}|$ for $M = L, R$. In this approximation, we get:

$$\frac{\omega_s}{\omega_d} \approx \left| \frac{V_{ts}}{V_{td}} \right|^2 \approx 23 \quad (9)$$

The dependence of this ratio on the dimension-two parameters of the squark mass matrix is generically very weak.

With this factor in mind, we consider oscillation of the sbottom-down mesino. The oscillation rate can be estimated by

$$\omega \approx \frac{f_B^2}{2} \cos^2 \theta \left| V_{td}V_{tb}^* \right|^2 \frac{m_t^4}{m_{\tilde{g}}^4} \frac{m_{\tilde{g}}}{m_{\tilde{g}}^2 - m_{\tilde{b}_1}^2} \sim (4 \times 10^{-12} \text{ GeV}) \left( \frac{f_B}{28.7 \text{ MeV}} \right)^2 \cos^2 \theta \left( \frac{1000 \text{ GeV}}{m_{\tilde{g}}} \right). \quad (10)$$

These results are not too far from the decay rates, eqs. (7), but with different parametric dependence. Thus, we expect some parts of parameter space where the oscillation rate is comparable to or larger than the decay rate, leading to appreciable mesino oscillations.
To get a better sense of how common such a phenomenon is, we define the oscillation parameter

\[ x = \frac{\Delta m}{\Gamma}. \]  

(11)

The time-integrated probability for a sbottom mesino to oscillate into an anti-sbottom mesino before decaying is

\[ p(x) = P(\tilde{B} \rightarrow \tilde{B}^c) = \frac{x^2}{2(1 + x^2)}. \]  

(12)

The oscillation probability is small for \( x \ll 1 \) and becomes appreciable near \( x \sim 1 \), whereas for \( x \gg 1 \) the \( \tilde{B} \) oscillates very rapidly, and the mesino contains an equal mixture of sbottom and anti-sbottom components. We scan over parameter space using the same procedure as in Section II, selecting points with a sbottom LSP and calculating \( x_d \) (the \( \tilde{B}_d \) oscillation parameter) and \( \theta \) for each such point. The results of the scan are shown in Fig. 4. We observe that \( x_d > 1 \) in a significant portion of parameter space, particularly when the LSP is predominantly left-handed.

If the sbottom is the LSP and has a mesino oscillation time comparable to or larger than its lifetime, then there is a very distinct signature of direct sbottom pair-production. The sbottoms will hadronize and the resulting mesino may oscillate before decaying. The mesino must be neutral for oscillations to occur, which occurs when the spectator is a down or strange quark, or roughly half the time as estimated from the \( B \) system. If exactly one of the mesinos oscillates before decaying, then the resulting two halves of the final state will have the same charge. Furthermore, these final states each involve a top quark whose charge is easy to tag if it undergoes a leptonic decay. This final state has same-sign leptons, \( b \) jets, and a small, but non-negligible, amount of missing energy. The entire chain is illustrated in
FIG. 5. Diagram for $R$-parity violating sbottom decay that leads to same sign leptons.

The branching fraction for this mode is given by

$$\text{Br}(\tilde{b}^* \rightarrow b b \ell^+ \ell^+) = \text{Br}(W \rightarrow \ell \nu)^2 f(x_d, x_s) \approx f(x_d, x_s) \times 6.5\%,$$

where $f(x_d, x_s)$ denotes the probability that exactly one of the two mesinos oscillates. Here $h_i$ is the fraction of sbottoms that form mesinos with spectator $i$ and we use the effective leptonic rate for the $W$ which includes leptonic tau decays. Note that for $x_i \gg 1$ the rate is maximal, and since $h_d + h_s \approx 1/2$ we have $f(x_d, x_s) \approx 3/8$. Despite the modest branching fraction, this decay mode will likely be the most sensitive channel for discovering a sbottom LSP in MFV SUSY.
FIG. 6. Recasted CMS bounds on sbottom direct production in terms of the sbottom mass and $x_d$. Only the four most sensitive signal regions are shown: SR0 in dashed green, SR1 in dotted blue, SR2 in solid red, and SR4 in dash-dotted orange. The most conservative upper limit on the number of new physics events is used for each search region, though varying this number has little effect on the bounds.

IV. BOUNDS FROM A CMS SEARCH

CMS already has a search [15] that is quite sensitive to the above decay chain. The same-sign dilepton and $b$ jets search includes search regions with 0, 30 GeV and 50 GeV MET cuts, all of which can be sensitive to our scenario due to the neutrinos from leptonic top decays. The relevant bounds from this search are presented in Table 2 of [15]. In addition, they present efficiency fits for the various cuts in terms of parton-level objects, allowing for easy reinterpretation. In this section, we use this information to reinterpret their bounds in terms of MFV SUSY with a sbottom LSP, and comment on future prospects.

To obtain a bound, we generated $pp \rightarrow \tilde{b}\tilde{b}^*$ at 8 TeV using Pythia 8 with all showering and hadronization turned off. The events are decayed at the parton-level. The analysis cuts are applied using the efficiencies presented in Section 6 of [15]. No mixing is introduced in event generation, but an $x_d$-dependent factor is applied to the final efficiency to account for the branching fraction to same-sign leptons. The cross-section for pair production is calculated at NLO using Prospino 2.1 [16] [17]. The resulting bounds are shown in Fig. 6. SR2, which counts only positively charged same sign pairs, yields the strongest bound, since a presumed fluctuation in the data led to all observed same-sign events having negatively charged leptons. For a maximal same-sign branching fraction, the exclusion extends between
180 GeV and 305 GeV.

Note that obtaining same sign lepton events requires $x \gtrsim 1$. The reason that there is sensitivity in the small $x_d$ region is due to the possibility of producing strange mesinos. Even for $x_d \ll 1$, it is possible to get $x_s > 1$.

V. CONCLUSIONS

MFV SUSY is a compelling new paradigm for exploring supersymmetry without $R$-parity that offers many new and challenging channels to explore at the LHC. A systematic study of the phenomenology of all plausible scenarios in this framework is required to ensure full sensitivity to weak-scale supersymmetry. We have explored one interesting scenario with a sbottom-like squark LSP.

Direct squark production will be essential for probing all possible corners of natural SUSY parameter space. Our work has demonstrated that the LHC can be sensitive to directly produced sbottom LSPs in the MFV SUSY scenario using the important fact that in this framework strongly-interacting LSPs will hadronize. Using a CMS search for same sign dileptons and $b$-jets, we have put a bound on sbottoms with masses between 180 and 305 GeV that undergo large mesino oscillations, which is a plausible scenario for the case with sbottom LSPs.

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Appendix A: Determination of the Mesino Oscillation Frequency

In this appendix, we present the details of the calculation of the mesino oscillation frequency, carefully listing all approximations as they enter. Throughout the appendix, we will assume a down quark spectator, but the results extend trivially to the strange quark case. We further denote the lightest squark by $\tilde{b}$ and assume that it is sbottom-like.

In the quark and squark mass basis, there are two combinations of sbottom and down quark that correspond to light mesino Weyl fermions:

$$\tilde{B}_1 \equiv \tilde{B} = \tilde{b}^* d, \quad \tilde{B}_2 \equiv \tilde{B}^c = \tilde{b}^c d^c.$$  \hspace{1cm} (A1)
The most general quadratic Lagrangian for these mesino fields is given by:

\[ \mathcal{L} = \frac{1}{2} m_{ij} \tilde{B}^i \tilde{B}^j + \text{h.c.} \quad (A2) \]

Before including corrections due to the gluino, the diagonal entries of \( m_{ij} \) vanish, and the two Weyl fermions combine to form a Dirac fermion. The mass, corresponding to the off-diagonal terms in \( (A2) \), is given to leading order by

\[ m_{12} = m_\tilde{b} . \quad (A3) \]

The leading corrections are of order \( \Lambda_{\text{QCD}} \), which we neglect.

The diagonal elements \( m_{11} \) and \( m_{22} \), corresponding to Majorana masses for \( \tilde{B} \) and \( \tilde{B}^c \), are not in general equal, and are generated at leading order by tree-level gluino exchange, leading to an oscillation between mesinos and antimesinos. The oscillation frequency is equal to the mass splitting between the two mass eigenstates, whose squared masses are the eigenvalues of \( m^\dagger m \). We take \( m_{12} \) to be real by performing an appropriate field redefinition, in which case the eigenvalues of \( m^\dagger m \) are given by

\[ \frac{1}{2} \left( |m_{11}|^2 + |m_{22}|^2 + 2m_{12}^2 \pm \sqrt{|m_{11}|^2 + |m_{22}|^2 + 2m_{12}^2 - 4|m_{12}^2 - m_{11}m_{22}|^2} \right) . \quad (A4) \]

To leading order in \( m_{11} \) and \( m_{22} \), the resulting mass splitting is

\[ \omega = \Delta m = |m_{11} + m_{22}^*| . \quad (A5) \]

We work at leading order in the heavy squark approximation. Instead of determining \( m_{11} \) and \( m_{22} \) directly, we employ the simple and general formula:

\[ \omega = \frac{1}{m_{12}} |\langle \tilde{B}(\vec{0}, s) | H_{\text{eff}}(\vec{0}) | \tilde{B}(\vec{0}, s) \rangle| , \quad (A6) \]

for \( \omega \ll m_{12} \), where \( H_{\text{eff}}(\vec{x}) \) is the effective Hamiltonian density generated by integrating out the gluino and \( |\tilde{B}(\vec{p}, s)\rangle \) and \( |\tilde{B}(\vec{p}, s)\rangle \) denote one-particle mesino and antimesino states, respectively, with momentum \( \vec{p} \) and spin \( s \) with no sum over \( s \). (We use the standard covariant normalization for one-particle momentum eigenstates, \( \langle \vec{p}|\vec{q} \rangle = 2E_{\vec{p}}(2\pi)^3\delta^{(3)}(\vec{p}-\vec{q}). \)

The effective Hamiltonian density from integrating out the gluino is:

\[ H_{\text{eff}} = \frac{C_L}{2} (\tilde{b}^*d)(\tilde{b}^*d) + \frac{C_R}{2} (\tilde{b}d^c)(\tilde{b}d^c) + \text{h.c.} , \quad (A7) \]

for coefficients \( C_L \) and \( C_R \) to be determined, where the color indices are contracted as indicated by the parentheses. Thus,

\[ \omega = \frac{1}{m_\tilde{b}} \left| \frac{C_L}{2} \langle \tilde{B}|(\tilde{b}^*d)(\tilde{b}^*d)|\tilde{B} \rangle + \frac{C_R^*}{2} \langle \tilde{B}|(\tilde{b}^*d^c)(\tilde{b}^*d^c)|\tilde{B} \rangle \right| . \quad (A8) \]
The structure is very similar to (A5), and indeed the two terms within the absolute value in (A8) are precisely $m_\tilde{g}$ times the Majorana masses which appear in (A5).

To determine the $C_{L,R}$, we compare the short-distance amplitudes for oscillation obtained using the MSSM Lagrangian and using the effective Hamiltonian in (A7). The MSSM gluino exchange amplitudes $\mathcal{M}_L$ and $\mathcal{M}_R$ (Fig. 3) are given by

\[
\mathcal{M}_L = 2 [g_s^2 (U_{d_{L,1}}^*)]^2 \left[ \frac{m_{\tilde{g}} \delta_{ij}^a}{m_{\tilde{g}}^2 - m_{b_1}^2} \right] \left[ t^a_{ij} t^a_{ij'} + t^a_{ij} t^a_{ij'} \right],
\]
\[
\mathcal{M}_R = 2 [g_s^2 (U_{d_{R,1}}^*)]^2 \left[ \frac{m_{\tilde{g}} \delta_{ij}^a}{m_{\tilde{g}}^2 - m_{b_1}^2} \right] \left[ t^a_{ij} t^a_{ij'} + t^a_{ij} t^a_{ij'} \right], \tag{A9}
\]

where we work in a basis where the gluino mass is real, and an overall factor of two arises since the gluino-quark-squark vertex comes with a factor of $\sqrt{2}$. The color factors in these amplitudes simplify to [18]:

\[
t^a_{ij} t^a_{ij'} + t^a_{ij} t^a_{ij'} = \frac{1}{2} \left( \delta_{ij'} \delta_{ij'} + \delta_{ij} \delta_{ij'} - \frac{1}{N_c} \delta_{ij'} \delta_{ij'} - \frac{1}{N_c} \delta_{ij} \delta_{ij'} \right). \tag{A10}
\]

The effective operators in (A7) yield amplitudes:

\[
\mathcal{M}'_{L,R} = C_{L,R} (\delta_{ij} \delta_{ij'} + \delta_{ij} \delta_{ij'}). \tag{A11}
\]

By demanding that $\mathcal{M}_L (\mathcal{M}_R)$ from (A9) is equal to $\mathcal{M}'_L (\mathcal{M}'_R)$ from (A11), we extract the coefficients $C_L$ and $C_R$:

\[
C_L = g_s^2 (U_{d_{L,1}}^*)^2 \frac{m_{\tilde{g}}}{m_{\tilde{g}}^2 - m_{b_1}^2} \left( 1 - \frac{1}{N_c} \right), \quad C_R = g_s^2 (U_{d_{R,1}}^*)^2 \frac{m_{\tilde{g}}}{m_{\tilde{g}}^2 - m_{b_1}^2} \left( 1 - \frac{1}{N_c} \right). \tag{A12}
\]

The same result can be obtained in the large $m_{\tilde{g}}$ limit by integrating out the gluino in the Lagrangian, neglecting the kinetic term.

As QCD is parity invariant, the hadronic matrix elements in (A8) are equal. We estimate them using the vacuum insertion approximation. In this approximation, we insert the vacuum between the operators in all possible ways, giving [19, 20]:

\[
\langle \bar{B} | (\tilde{b}_i^* d^i) (\tilde{b}_j^* d^j) | \bar{B} \rangle \approx 2 \left[ \langle \bar{B} | (\tilde{b}_i^* d^i) | 0 \rangle \langle 0 | (\tilde{b}_j^* d^j) | \bar{B} \rangle + \langle \bar{B} | (\tilde{b}_i^* d^i) | 0 \rangle \langle 0 | (\tilde{b}_j^* d^j) | \bar{B} \rangle \right], \tag{A13}
\]

where we indicate color indices explicitly, and there are two ways to obtain each of the terms, yielding a prefactor of 2. The contraction with the color-neutral external state kills the terms with $i \neq j$ in the second term. Exactly one in every $N_c$ terms has $i = j$, so we get the relation:

\[
\langle \bar{B} | (\tilde{b}_i^* d^i) | 0 \rangle \langle 0 | (\tilde{b}_j^* d^j) | \bar{B} \rangle = \frac{1}{N_c} \langle \bar{B} | (\tilde{b}_i^* d^i) | 0 \rangle \langle 0 | (\tilde{b}_j^* d^j) | \bar{B} \rangle. \tag{A14}
\]
Our result is thus:

\[
\langle \bar{B}|(\bar{b}^* d^t)(\bar{b}^* d^t)|\bar{B} \rangle = \langle \bar{B}|(\bar{b}^* d^t)(\bar{b}^* d^t)|\bar{B} \rangle \\
\approx 2\frac{N_c + 1}{N_c} \langle \bar{B}|(\bar{b}_i^* d^i)|0\rangle \langle 0|(\bar{b}_j^* d^j)|\bar{B} \rangle \equiv 2\frac{N_c + 1}{N_c} f_B^2 m_{\bar{B}}. \tag{A15}
\]

The mesino decay constant \(f_{\bar{B}}\) can be estimated using the \(B\) meson decay constant and assuming heavy quark symmetry. Up to threshold corrections, the relationship is given by [21]:

\[
f_{\bar{B}} = f_B \sqrt{\frac{m_b}{m_{\bar{b}_1}}} \left( \frac{\alpha_s(m_b)}{\alpha_s(m_t)} \right)^{6/23} \left( \frac{\alpha_s(m_t)}{\alpha_s(m_{\bar{b}_1})} \right)^{6/21}.
\tag{A16}
\]

Using the latest values of \(f_B = 190.6\) MeV [22, 23] and \(\alpha_s(m_Z) = 0.1184\) [24], the \(\overline{MS}\) quark masses \(m_b = 4.19\) GeV and \(m_t = 160\) GeV [24], as well as with a numerical solution to the NNNLO beta function for \(\alpha_s\) [25], which we evaluate at \(m_{\bar{b}_1} = 300\) GeV, we find a value of \(f_{\bar{B}} = 28.7\) MeV.

Putting these pieces together, we arrive at our final expression:

\[
\Delta m = g^2_s \left| \left( U_{dL,1}^D \right)^2 + \left( U_{dR,1}^D \right)^2 \right| f_B^2(m_{\bar{b}_1}) \left( 1 - \frac{1}{N_c^2} \right) \frac{m_{\bar{g}}}{m_{\bar{g}}^2 - m_{\bar{b}_1}^2}, \tag{A17}
\]

with \(f_{\bar{B}}\) given by (A16). This agrees with [14] up to a factor of 8 and the dependence on the CP-violating phase in the squark mixing matrix. This result has some hadronic uncertainty, which we estimate to be of order 10\% based on estimates of the validity of the same approximations for the \(B\) meson systems.

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