\{Q\bar{s}\}\{Q^{'i}\bar{s}\} Molecular States in QCD Sum Rules\* 

ZHANG Jian-Rong (张建荣)\dagger and HUANG Ming-Qiu (黄明球) 

Department of Physics, National University of Defense Technology, Hunan 410073, China 

(Received March 8, 2010; revised manuscript received April 26, 2010) 

Abstract We systematically investigate the mass spectra of \{Q\bar{s}\}\{Q^{'i}\bar{s}\} molecular states in the framework of QCD sum rules. The interpolating currents representing the molecular states are proposed. Technically, contributions of the operators up to dimension six are included in operator product expansion (OPE). The masses for molecular states with various \{Q\bar{s}\}\{Q^{'i}\bar{s}\} configurations are presented. The result 4.36±0.08 GeV for the $D_s^*\bar{D}_{s0}^*$ molecular state is consistent with the mass 4350±4.0 ± 0.7 MeV of the newly observed $X(4350)$, which could support $X(4350)$ interpreted as a $D_s^*\bar{D}_{s0}^*$ molecular state. 

PACS numbers: 11.55.Hx, 12.38.Lg, 12.39.Mk 

Key words: mass spectra, molecular states, QCD sum rules 

1 Introduction 

The field of heavy hadron spectroscopy is experiencing a rapid advancement mainly propelled by the continuous observations of hadronic resonances, for example, $X(3872)$,[1] $Y(3930)$,[2] $Y(4260)$,[3] $Z(3930)$,[4] $X(3940)$,[5] $Z^+(4430)$,[6] $Z^0(4050)$,[7] $Z^+(2520)$,[7] $Y(4140)$[8] etc. (for experimental reviews, e.g., see Refs. [9–10]), some of which are not easy to accommodate within the quark model picture and may not be conventional charmonium states. Masses for some of these hadrons are very close to the meson-meson thresholds, for which are interpreted as possible \{Q\bar{q}\}\{Q^{'i}\bar{q}\} molecular candidates in [11–15]. For instance, the charmonium-like state $Y(3930)$ has been interpreted as a $D_s^*\bar{D}_s^*$ molecular state.[14–16] Considering the SU(3) symmetry of the light flavor quarks, there may also exist and have a rich spectroscopy for \{Q\bar{s}\}\{Q^{'i}\bar{s}\} molecular states acting as the corresponding partners of \{Q\bar{q}\}\{Q^{'i}\bar{q}\} molecular states. In fact, some authors have deciphered the newly observed $Y(4140)$ as a $D_s^*\bar{D}_s^*$ molecular state,[15–16] just as the molecular partner of $D_s^*\bar{D}_s^*$. Furthermore, QCD itself does not exclude the existence of \{Q\bar{s}\}\{Q^{'i}\bar{s}\} molecular states besides conventional mesons and baryons, so studies of them may deepen one’s understanding of the strong interaction. On all accounts, it is interesting to study mass spectra for the \{Q\bar{s}\}\{Q^{'i}\bar{s}\} molecular states. However, it is far from clear how to generate hadron masses from first principles in QCD since it is highly nonperturbative in the low energy region where futile to attempt perturbative calculations, and then one has to treat a genuinely strong field in nonperturbative methods. Under such a circumstance, one could resort to QCD sum rule[17] (for reviews see [18–21] and references therein), which is a comprehensive and reliable way for evaluating the nonperturbative effects. Up to now, there have been some works testing the $D_s^*\bar{D}_s^*$ state from QCD sum rules.[22–24] Presently, we extend the work on $(Q\bar{s})^{(*)}(\bar{Q}s)^{(s)}$ molecular states[24] to various \{Q\bar{s}\}\{Q^{'i}\bar{s}\} molecular states. 

The paper is organized as follows. In Sec. 2, QCD sum rules for the molecular states are introduced, and both the phenomenological representation and QCD side are derived, followed by the numerical analysis to extract the hadronic masses in Sec. 3, and a brief summary in Sec. 4. 

2 Molecular State QCD Sum Rules 

2.1 Interpolating Currents 

Following the standard scheme,[10] the $Q\bar{s}$ mesons with $J^P = 0^-, 1^-, 0^+$, and $1^+$ are named $D_s, D_s^*, D_{s0}^*$, and $D_{s1}$ for charmed mesons, with $B_s, B_s^*, B_{s0}^*$, and $B_{s1}$ for bottom mesons, respectively. In this work, the corresponding configurations for these mesons are represented as $(Q\bar{s}), (Q\bar{s})^*, (Q\bar{s})_{0}^*,$ and $(Q\bar{s})_{1}$. In full theory, the interpolating currents for these mesons can be found in Refs. [25–26]. Presently, one constructs the molecular state current from meson-meson type of fields, while constructs the tetraquark state current from diquark-antidiquark configuration of fields. The currents constructed from meson-meson type of fields can be related to those composed of diquark-antidiquark type of fields by Fiertz rearrangements. However, the relations are suppressed by a typical color and Dirac factors so that one could obtain a reliable sum rule only if one has chosen the appropriate current to have a maximum overlap with the physical state, which is expected to be particularly true for multiquark configuration with special molecular or diquark structures. Concretely, it will have a maximum
For the former case, the starting point is the two-point correlator
\[
\Pi(q^2) = i \int d^4x e^{iq\cdot x} \langle 0|T[j^\mu(x)j^{\nu}\bar{\psi}(0)]|0\rangle.
\]  
(1)

The correlator can be phenomenologically expressed as
\[
\Pi(q^2) = \frac{\lambda_H^2}{M_H^2 - q^2} + \frac{1}{\pi} \int_0^\infty ds \frac{\text{Im} \Pi^{\text{phen}}(s)}{s - q^2} + \text{subtractions},
\]  
(2)

where \(M_H\) denotes the mass of the hadronic resonance, and \(\lambda_H\) gives the coupling of the current to the hadron \(\langle 0|j|H\rangle = \lambda_H\). In the OPE side, the correlator can be written as
\[
\Pi(q^2) = \int_{(m_Q+m_{Q'}+2m_j)^2}^{\infty} ds \rho_{\text{OPE}}(s) \frac{d}{s-q^2}
\]
\[
(m_Q = m_{Q'} \text{ or } m_Q \neq m_{Q'}) ,
\]  
(3)

where the spectral density is given by
\[
\rho_{\text{OPE}}(s) = \frac{1}{\pi} \text{Im} \Pi^{\text{OPE}}(s) .
\]

After equating the two sides, assuming quark-hadron duality, and making a Borel transform, the sum rule can be written as
\[
\lambda_H^2 e^{-M_H^2/M^2} = \int_{(m_Q+m_{Q'}+2m_j)^2}^{\infty} ds \rho_{\text{OPE}}(s) e^{-s/M^2} ,
\]  
(4)

where \(M^2\) indicates Borel parameter. To eliminate the hadronic coupling constant \(\lambda_H\), one reckons the ratio of derivative of the sum rule and itself, and then yields
\[
M_H^2 = \frac{\int_{(m_Q+m_{Q'}+2m_j)^2}^{\infty} ds \rho_{\text{OPE}}(s) e^{-s/M^2}}{\int_{(m_Q+m_{Q'}+2m_j)^2}^{\infty} ds \rho_{\text{OPE}}(s) e^{-s/M^2} } .
\]

For the latter case, one starts from
\[
\Pi^{\mu\nu}(q^2) = i \int d^4x e^{iq\cdot x} \langle 0|T[j^\mu(x)j^{\nu}\bar{\psi}(0)]|0\rangle .
\]
(6)

Lorentz covariance implies that the correlator (6) can be generally parameterized as
\[
\Pi^{\mu\nu}(q^2) = (q^\mu q^\nu - g^{\mu\nu}) \Pi^{(1)}(q^2) + \frac{4q^\mu q^\nu}{q^2} \Pi^{(0)}(q^2) .
\]
(7)

The \(\Pi^{(1)}(q^2)\) of the correlator proportional to \(g^{\mu\nu}\) will be chosen to extract the mass sum rule here. Similarly, one can finally yield
\[
M_H^2 = \frac{\int_{(m_Q+m_{Q'}+2m_j)^2}^{\infty} ds \rho_{\text{OPE}}(s) e^{-s/M^2}}{\int_{(m_Q+m_{Q'}+2m_j)^2}^{\infty} ds \rho_{\text{OPE}}(s) e^{-s/M^2} } ,
\]
(8)

with
\[
\rho_{\text{OPE}}(s) = \frac{1}{\pi} \text{Im} \Pi^{(1)}(s) .
\]

2.3 Spectral Densities

Calculating the OPE side, one works at leading order in \(\alpha_s\) and considers condensates up to dimension six, utilizing the similar techniques in Refs. [28–29]. The \(s\) quark is dealt as a light one. To keep the heavy-quark mass finite, one uses the momentum-space expression for the heavy-quark propagator. One calculates the light-quark part of the correlation function in the coordinate space, which is then Fourier-transformed to the momentum space in \(D\) dimension. The resulting light-quark part is combined with the heavy-quark part before it is dimensionally regularized at \(D = 4\). For the heavy-quark propagator with two and three gluons attached, the momentum-space expressions given in Ref. [25] are used. After some tedious calculations, the concrete forms of spectral densities can be derived, which are collected in the Appendix. In detail,
some different currents lead to the similar OPE, for example, the terms for \( D_s \bar{D}_s \) and \( D_s^* \bar{D}_s^* \) are similar, the ones for \( D_s^* \bar{D}_s^* \) and \( D_{s1} \bar{D}_{s1} \) are similar and so on. Although the terms for them are similar respectively, the corresponding signs may be different, such as a term for \( D_s \bar{D}_s \) may be “plus” sign while the related one for \( D_{s0}^* \bar{D}_{s0}^* \) may be “minus”, which caused by the differences of \( \gamma \)-matrices in the interpolating currents and the differences of the trace results. Numerically, the two quark condensate \( \langle \bar{s}s \rangle \) is the most important condensate correction, the absolute value of which is bigger than the absolute values of the four quark condensate \( \langle \bar{q}q \rangle \) as well as the mixed condensate \( \langle g\bar{s}\sigma \cdot Gs \rangle \). Meanwhile, the two gluon condensate \( \langle g^2G^2 \rangle \) and the three gluon condensate \( \langle g^3G^3 \rangle \) are very small and almost negligible.

3 Numerical Analysis

In this part, the sum rules (5) and (8) will be numerically analyzed. The input values are taken as \( m_c = 1.23 \) GeV, \( m_b = 4.20 \) GeV, and \( m_s = 0.13 \) GeV,[10] with \( \langle \bar{q}q \rangle = -(0.23)^3 \) GeV, \( \langle \bar{s}s \rangle = 0.8 \langle \bar{q}q \rangle \), \( \langle g\bar{s}\sigma \cdot Gs \rangle = m_0^2 \langle \bar{s}s \rangle \), \( m_0^2 = 0.8 \) GeV, \( g^2G^2 = 0.88 \) GeV, and \( g^3G^3 = 0.045 \) GeV.[20,28] Complying with the standard procedure of sum rule analysis, the threshold \( s_0 \) and Borel parameter \( M^2 \) are varied to find the optimal stability window. Namely, we try to consider the Borel curve stability’s dependence on the Borel plateaus (the threshold \( s_0 \) and Borel parameter \( M^2 \)), and find the Borel windows where the perturbative contribution should be larger than the condensate contributions in the OPE side while the pole contribution should be larger than continuum contribution in the phenomenological side. Thus, the regions of thresholds are taken as values presented in the related figure captions, with \( M^2 = 3.5 \sim 4.5 \) GeV for \( D_s^* \bar{D}_s^* \), \( D_{s1} \bar{D}_{s1} \), \( D_s \bar{D}_s \), \( D_{s0}^* \bar{D}_{s0}^* \), and \( D_{s1} \bar{D}_{s1} \), \( M^2 = 7.5 \sim 9.0 \) GeV for \( B_s \bar{D}_s \), \( B_s^* \bar{D}_s^* \), \( B_{s1} \bar{D}_{s1} \), \( B_s \bar{D}_s \), \( B_s^* \bar{D}_s^* \), \( B_{s1} \bar{D}_{s1} \), \( D_s^* \bar{D}_s^* \), \( D_{s1} \bar{D}_{s1} \), \( D_s \bar{D}_s \), \( D_{s0}^* \bar{D}_{s0}^* \), \( D_{s1} \bar{D}_{s1} \), and \( D_s \bar{D}_s \), \( M^2 = 9.5 \sim 11.0 \) GeV for \( B_{s0}^* \bar{B}_{s0}^* \), \( B_{s1}^* \bar{B}_{s1}^* \), \( B_s \bar{B}_s \), \( B_s^* \bar{B}_s \), \( B_{s1}^* \bar{B}_{s1}^* \), respectively. Tables 1–2 collect all the numerical results. Note that uncertainties are owing to the sum rule windows (variation of the threshold \( s_0 \) and Borel parameter \( M^2 \)), not involving the ones from the variation of quark masses and QCD parameters for which are appreciably smaller in comparison with the ones from the sum rule windows here. The numerical result \( 4.13 \pm 0.10 \) GeV for \( D_s^* \bar{D}_s^* \) agrees well with the mass \( 4134.0 \pm 2.9 \pm 1.2 \) MeV for \( Y(4140) \),[24] which supports the \( D_s^* \bar{D}_s^* \) molecular configuration for \( Y(4140) \). After the completion of the calculations here, evidence for a new resonance (named as \( X(4350) \)) has been observed by the Belle Collaboration.[30] Note that the predicted value 4.36 \pm 0.08 GeV for the \( D_s^* \bar{D}_s^* \) molecular state here is consistent with the mass \( 4350^{+5}_{-4.6} \pm 0.7 \) MeV of the newly observed structure.

| Table 1 | The mass spectra of molecular states with same heavy quarks. |
|---------|----------------------------------------------------------|
| Hadron  | Configuration | Mass/GeV       | Hadron  | Configuration | Mass/GeV |
| \( D_s \bar{D}_s \) | \((cs)(\bar{c}s)\) | 3.91 \pm 0.10[24] | \( B_s \bar{B}_s \) | \((bs)(\bar{b}s)\) | 10.70 \pm 0.10[24] |
| \( D_s^* \bar{D}_s^* \) | \((cs)^* (\bar{c}s)^*\) | 4.01 \pm 0.10[24] | \( B_s^* \bar{B}_s^* \) | \((bs)^* (\bar{b}s)^*\) | 10.71 \pm 0.11[24] |
| \( D_s^* \bar{D}_s \) | \((cs)^* (\bar{c}s)\) | 1.13 \pm 0.10[24] | \( B_s^* \bar{B}_s \) | \((bs)^* (\bar{b}s)\) | 10.80 \pm 0.10[24] |

| Table 2 | The mass spectra of molecular states with differently heavy quarks. |
|---------|----------------------------------------------------------|
| Hadron  | Configuration | Mass/GeV       | Hadron  | Configuration | Mass/GeV |
| \( B_s \bar{D}_s \) | \((bs)(\bar{c}s)\) | 7.31 \pm 0.09 | \( B_s^* \bar{D}_s^* \) | \((bs)^* (\bar{c}s)^*\) | 7.71 \pm 0.07 |
| \( B_s \bar{D}_s \) | \((bs)^* (\bar{c}s)\) | 7.37 \pm 0.09 | \( B_s^* \bar{D}_s^* \) | \((bs)^* (\bar{c}s)\) | 7.78 \pm 0.08 |
| \( B_s^* \bar{D}_s^* \) | \((bs)^* (\bar{c}s)^*\) | 7.46 \pm 0.09 | \( B_s^* \bar{D}_s^* \) | \((bs)(\bar{c}s)\) | 7.30 \pm 0.09 |
| \( B_s^* \bar{D}_s^* \) | \((bs)^* (\bar{c}s)^*\) | 8.07 \pm 0.09 | \( B_s^* \bar{D}_s^* \) | \((bs)^* (\bar{c}s)^*\) | 8.07 \pm 0.09 |
| \( B_s \bar{D}_s \) | \((bs)(\bar{c}s)\) | 8.14 \pm 0.09 | \( B_s^* \bar{D}_s^* \) | \((bs)(\bar{c}s)\) | 7.73 \pm 0.07 |
| \( B_s \bar{D}_s \) | \((bs)(\bar{c}s)\) | 8.17 \pm 0.11 | \( B_s^* \bar{D}_s^* \) | \((bs)(\bar{c}s)\) | 7.78 \pm 0.08 |
| \( B_s \bar{D}_s \) | \((bs)(\bar{c}s)\) | 7.65 \pm 0.07 | \( B_s^* \bar{D}_s^* \) | \((bs)(\bar{c}s)\) | 7.79 \pm 0.08 |
| \( B_s \bar{D}_s \) | \((bs)(\bar{c}s)\) | 7.80 \pm 0.08 | \( B_s^* \bar{D}_s^* \) | \((bs)(\bar{c}s)\) | 7.86 \pm 0.08 |
Fig. 1  The dependence on $M^2$ for the masses of $D_s^0 \bar{D}_{s0}$ and $B_s^0 \bar{B}_{s0}$ from sum rule (5). The continuum thresholds are taken as $\sqrt{s_0} = 5.0 \sim 5.2$ GeV and $\sqrt{s_0} = 11.8 \sim 12.0$ GeV, respectively.

Fig. 2  The dependence on $M^2$ for the masses of $D_{s1} \bar{D}_{s1}$ and $B_{s1} \bar{B}_{s1}$ from sum rule (5). The continuum thresholds are taken as $\sqrt{s_0} = 5.3 \sim 5.5$ GeV and $\sqrt{s_0} = 12.2 \sim 12.4$ GeV, respectively.

Fig. 3  The dependence on $M^2$ for the masses of $D_{s1} \bar{D}_{s0}$ and $B_{s1} \bar{B}_{s0}$ from sum rule (8). The continuum thresholds are taken as $\sqrt{s_0} = 5.1 \sim 5.3$ GeV and $\sqrt{s_0} = 11.9 \sim 12.1$ GeV, respectively.
Fig. 4  The dependence on $M^2$ for the masses of $D_sD_{s0}^*$ and $B_sB_{s0}^*$ from sum rule (5). The continuum thresholds are taken as $\sqrt{s_0} = 4.6 \sim 4.8$ GeV and $\sqrt{s_0} = 11.5 \sim 11.7$ GeV, respectively.

Fig. 5  The dependence on $M^2$ for the masses of $D_1sD_s$ and $B_1sB_s$ from sum rule (8). The continuum thresholds are taken as $\sqrt{s_0} = 4.8 \sim 5.0$ GeV and $\sqrt{s_0} = 11.6 \sim 11.8$ GeV, respectively.

Fig. 6  The dependence on $M^2$ for the masses of $D_{s1}^*D_{s0}^*$ and $B_{s1}^*B_{s0}^*$ from sum rule (8). The continuum thresholds are taken as $\sqrt{s_0} = 4.8 \sim 5.0$ GeV and $\sqrt{s_0} = 11.6 \sim 11.8$ GeV, respectively.
Fig. 7  The dependence on $M^2$ for the masses of $D_s^* \bar{D}_{s1}$ and $B_s^* \bar{B}_{s1}$ from sum rule (5). The continuum thresholds are taken as $\sqrt{s_0} = 4.9 \sim 5.1$ GeV and $\sqrt{s_0} = 11.6 \sim 11.8$ GeV, respectively.

Fig. 8  The dependence on $M^2$ for the masses of $B_s \bar{D}_s$ and $B_s^* \bar{D}_s^*$ from sum rule (5). The continuum thresholds are taken as $\sqrt{s_0} = 7.7 \sim 7.9$ GeV and $\sqrt{s_0} = 7.9 \sim 8.1$ GeV, respectively.

Fig. 9  The dependence on $M^2$ for the masses of $B_{s0}^* \bar{D}_{s0}$ and $B_{s1} \bar{D}_{s1}$ from sum rule (5). The continuum thresholds are taken as $\sqrt{s_0} = 8.5 \sim 8.7$ GeV and $\sqrt{s_0} = 8.8 \sim 9.0$ GeV, respectively.
Fig. 10 The dependence on $M^2$ for the masses of $D_s^* \bar{B}_s$ and $B_s^* \bar{D}_s$ from sum rule (8). The continuum thresholds are taken as $\sqrt{s_0} = 7.7 \sim 7.9$ GeV and $\sqrt{s_0} = 7.8 \sim 8.0$ GeV, respectively.

Fig. 11 The dependence on $M^2$ for the masses of $D_{s1}^* \bar{B}_{s0}$ and $B_{s1}^* \bar{D}_{s0}$ from sum rule (8). The continuum thresholds are taken as $\sqrt{s_0} = 8.5 \sim 8.7$ GeV and $\sqrt{s_0} = 8.6 \sim 8.8$ GeV, respectively.

Fig. 12 The dependence on $M^2$ for the masses of $D_{s1}^* \bar{B}_{s0}$ and $B_{s1}^* \bar{D}_{s0}$ from sum rule (8). The continuum thresholds are taken as $\sqrt{s_0} = 8.2 \sim 8.4$ GeV and $\sqrt{s_0} = 8.1 \sim 8.3$ GeV, respectively.
Fig. 13  The dependence on $M^2$ for the masses of $D_s^* B_s$ and $B_s^* D_s$ from sum rule (5). The continuum thresholds are taken as $\sqrt{s_0} = 8.3 \sim 8.5$ GeV and $\sqrt{s_0} = 8.2 \sim 8.4$ GeV, respectively.

Fig. 14  The dependence on $M^2$ for the masses of $D_s B_s^*$ and $B_s D_s^*$ from sum rule (5). The continuum thresholds are taken as $\sqrt{s_0} = 8.1 \sim 8.3$ GeV and $\sqrt{s_0} = 8.0 \sim 8.2$ GeV, respectively.

Fig. 15  The dependence on $M^2$ for the masses of $D_{s1} B_s$ and $B_{s1} D_s$ from sum rule (8). The continuum thresholds are taken as $\sqrt{s_0} = 8.2 \sim 8.4$ GeV and $\sqrt{s_0} = 8.2 \sim 8.4$ GeV, respectively.
4 Summary

In summary, QCD sum rules have been employed to compute the masses of molecular states, including the contributions of operators up to dimension six in OPE. Ultimately, we have arrived at mass spectra for molecular states with various \(\{Q\bar{s}\}\{Q'\bar{s}\}\) configurations. The numerical result 4.13 ± 0.10 GeV for \(D_s^* D_{s0}^*\) agrees well with the mass 4143.0 ± 2.9 ± 1.2 MeV for \(Y(4140)\),[24] which supports the interpretation of \(Y(4140)\) as a \(D_s^* D_{s0}^*\) molecular state. The predicted value 4.36 ± 0.08 GeV for the \(D_s^* D_{s0}^*\) molecular state is consistent with the mass 4350\(^{+4.6}_{-5.1}\) ± 0.7 MeV of the newly observed \(X(4350)\), which could support \(X(4350)\) interpreted as a \(D_s^* D_{s0}^*\) molecular state. More experimental evidence on \(\{Q\bar{s}\}\{Q'\bar{s}\}\) molecular states besides \(Y(4140)\) and \(X(4350)\) may appear if they do exist, and the data on molecular states are expecting further experimental identification, which may be searched for experimentally at facilities such as Super-B factories in the \(J/\psi\bar{\phi}\) mass spectrum in the future.

Appendix

It is defined that \(r(m_Q, m_{Q'}) = \alpha m_Q^2 + \beta m_{Q'}^2 - \alpha \beta s\) (\(m_Q = m_{Q'}\) or \(m_Q \neq m_{Q'}\)). With

\[
\rho_{\text{pert}}(s) = \frac{3}{211 \pi^6} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \int_{\beta_{\text{min}}}^{1} d\alpha d\beta (1 - \alpha - \beta) \left[ \frac{1}{\alpha^3 \beta^3} r(m_Q, m_{Q'}) \right]^2
- \frac{2^2 m_Q^2 m_{Q'}^2}{\alpha^3 \beta^3} r(m_Q, m_{Q'}) - \frac{2^2 m_{Q'}^2 m_Q^2}{\alpha^3 \beta^3} r(m_{Q'}, m_Q) + \frac{3 \cdot 2^2 m_Q m_{Q'} \beta}{\alpha^2 \beta^2} \right] r(m_Q, m_{Q'})^2,
\]

\[
\rho^{(g\bar{s})}(s) = \frac{3}{2^7 \pi^4} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \int_{\beta_{\text{min}}}^{1} d\alpha d\beta \left[ - \frac{m_Q^2}{\alpha^2 \beta^2} r(m_Q, m_{Q'}) - \frac{m_{Q'}^2}{\alpha \beta^2} r(m_{Q'}, m_Q) + \frac{2 m_Q m_{Q'} \beta}{\alpha \beta^2} \right] r(m_Q, m_{Q'})^2,
\]

\[
\rho^{(g\bar{s})}(s) = \frac{3}{2^7 \pi^4} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \int_{\beta_{\text{min}}}^{1} d\alpha d\beta \left[ - \frac{m_Q^2}{\alpha^2 \beta^2} r(m_Q, m_{Q'}) + \frac{m_{Q'}^2}{\alpha \beta^2} r(m_{Q'}, m_Q) + \frac{2 m_Q m_{Q'} \beta}{\alpha \beta^2} \right] r(m_Q, m_{Q'})^2,
\]

\[
\rho^{(g\bar{s}G)}(s) = \frac{3}{2^7 \pi^4} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \int_{\beta_{\text{min}}}^{1} d\alpha d\beta \left[ - \frac{m_Q^2}{\alpha^2 \beta^2} r(m_Q, m_{Q'}) - \frac{m_{Q'}^2}{\alpha \beta^2} r(m_{Q'}, m_Q) - \frac{2 m_Q m_{Q'} \beta}{\alpha \beta^2} \right] r(m_Q, m_{Q'})^2,
\]

\[
\rho^{(g\bar{s}G)}(s) = \frac{3}{2^7 \pi^4} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \int_{\beta_{\text{min}}}^{1} d\alpha d\beta \left[ - \frac{m_Q^2}{\alpha^2 \beta^2} r(m_Q, m_{Q'}) - \frac{m_{Q'}^2}{\alpha \beta^2} r(m_{Q'}, m_Q) - \frac{2 m_Q m_{Q'} \beta}{\alpha \beta^2} \right] r(m_Q, m_{Q'})^2,
\]

for \(\{Q\bar{s}\}(Q'\bar{s})\).
\[-\frac{m_s}{\alpha \beta} r(m_Q, m_{Q'}) + \frac{2 m_Q m_{Q'} m_s}{\alpha \beta} + \frac{m_{Q'}^2 m_s^2}{\alpha} r(m_Q, m_{Q'}) \]

\[\rho(\bar{s}s) = \frac{3}{2 \pi^2} \left[ \left(2 m_Q m_{Q'} - 2 m_Q m_s\right) \sqrt{\left(s - m_Q^2 + m_{Q'}^2\right)^2 - 4 m_Q^2 s} + 3 m_Q^2 m_s^2 \right] \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha(-2 m_Q^2 m_s + 3 m_Q^2 \alpha(1 - \alpha)), \]

\[\rho(\bar{q}s \cdot Gs)(s) = \frac{3}{2 \pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\alpha_{\min}}^{1 - \alpha} d\beta(1 - \alpha - \beta) \left[ \frac{m_Q^2 m_s}{\alpha^3} (1 + \alpha + \beta) r(m_Q, m_{Q'}) \right. \]

\[\rho(\bar{q}G^2(s)) = \frac{3}{2 \pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\alpha_{\min}}^{1 - \alpha} d\beta(1 - \alpha - \beta) \left[ \frac{m_Q^2 m_s}{\alpha^3} (1 + \alpha + \beta) r(m_Q, m_{Q'}) \right. \]

\[\rho(\bar{q}s \cdot Gs)(s) = \frac{3}{2 \pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\alpha_{\min}}^{1 - \alpha} d\beta(1 - \alpha - \beta) \left[ \frac{m_Q^2 m_s}{\alpha^3} (1 + \alpha + \beta) r(m_Q, m_{Q'}) \right. \]
\[
\rho^{(q^3G^3)}(s) = \frac{(q^3G^3)}{2^{11\pi^6}} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta (1 - \alpha - \beta) \left[ \frac{1}{\alpha^3} r(m_Q, m_{Q'}) + \frac{2m_Q^2}{\alpha^3} \right] r(m_Q, m_{Q'})^2,
\]

\[
\rho^{(\bar{s}s)}(s) = \frac{3(\bar{s}s)}{2\pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta (1 - \alpha - \beta) \left[ \frac{m_Q^2}{\alpha^3} r(m_Q, m_{Q'}) + \frac{m_Q}{\alpha^3} r(m_Q, m_{Q'}) \right] r(m_Q, m_{Q'})^2;
\]

for \((Q\bar{s})^* (Q'\bar{s})^*\),

\[
\rho^{(q\bar{s} - G\bar{s})}(s) = \frac{3(q\bar{s} - G\bar{s})}{2\pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta (1 - \alpha - \beta) \left[ \frac{1}{\alpha^3} r(m_Q, m_{Q'}) + \frac{2m_Q^2}{\alpha^3} \right] r(m_Q, m_{Q'})^2,
\]

\[
\rho^{(q^2G^2)}(s) = \frac{(q^2G^2)}{2^{11\pi^6}} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta (1 - \alpha - \beta) \left[ \frac{m_Q^2}{\alpha^3} r(m_Q, m_{Q'}) + \frac{m_Q^2}{\alpha^3} r(m_Q, m_{Q'}) \right] r(m_Q, m_{Q'})^2;
\]

\[
\rho^{(g^2G^2)}(s) = \frac{(g^2G^2)}{2^{13\pi^6}} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{\beta_{\max}} d\beta (1 - \alpha - \beta) \left[ \frac{1}{\alpha^3} r(m_Q, m_{Q'}) + \frac{2m_Q^2}{\alpha^3} \right] r(m_Q, m_{Q'})^2;
\]

for \((Qg\bar{s})^* (Q'g\bar{s})^*\).
\[
\rho^{(g\sigma \cdot G \sigma)}(s) = \frac{\langle g\sigma \cdot G \sigma \rangle}{2^7 \pi^4} \left\{ \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{2} \left\{ \frac{3m_{Qr}}{1 - \alpha} \right\} \left[ \alpha m_Q^2 + (1 - \alpha) m_{Qr}^2 - \alpha(1 - \alpha)s \right] \\
+ 2^2 m_s [2 \alpha m_Q^2 + 2(1 - \alpha) m_{Qr}^2 - 3\alpha(1 - \alpha)s] \\
+ \left[ 3 \cdot 2^2 m_Q m_Q' m_s + (m_Q + m_Q') m_s^2 \right] \sqrt{(s - m_Q^2 + m_{Qr}^2)^2 - 4m_{Qr}^2 s^2/s} \right\},
\]

\[
\rho^{(g \cdot G^2)}(s) = \frac{\langle g^2 G^2 \rangle}{2^9 \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{2} \int_{\beta_{\min}}^{1 - \alpha} \frac{d\beta}{2} \left[ \frac{m_Q^2}{\alpha^3} r(m_Q, m_{Qr}) \\
+ \frac{3m_Q m_s r(m_Q, m_{Qr})}{2\alpha^3} + \frac{m_Q^2 m_s}{\alpha^2} + \frac{3m_Q m_s}{\beta^3} r(m_Q, m_{Qr}) + \frac{m_Q m_s}{\beta^2} \right],
\]

\[
\rho^{(g^3 G^3)}(s) = \frac{\langle g^3 G^3 \rangle}{2^{11} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{2} \int_{\beta_{\min}}^{1 - \alpha} \frac{d\beta}{2} \left[ \frac{1}{\alpha^3} r(m_Q, m_{Qr}) + \frac{2m_Q m_s}{\alpha^3} r(m_Q, m_{Qr}) \\
+ \frac{1}{\beta^3} r(m_Q, m_{Qr}) + \frac{2m_Q m_s}{\alpha^3} + \frac{3m_Q m_s}{\beta^3} + \frac{m_Q m_s}{\beta^2} \right],
\]

for \((Q\bar{s})_1(Q's)_1\).

\[
\rho^{\text{pert}}(s) = \frac{3}{2^7 2^4 \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{2} \int_{\beta_{\min}}^{1 - \alpha} \frac{d\beta}{2} \left[ \frac{1}{\alpha^3} (1 + \alpha + \beta) r(m_Q, m_{Qr})^2 \\
+ \frac{2m_Q m_s}{\beta^3} (1 + \alpha + \beta) r(m_Q, m_{Qr}) + \frac{2m_Q m_s}{\alpha^2} r(m_Q, m_{Qr}) + \frac{3 \cdot 2^3 m_Q m_{Qr} m_s^2}{\alpha^2 \beta^3} r(m_Q, m_{Qr})^2 \right],
\]

\[
\rho^{(s\bar{s})}(s) = \frac{3\langle s\bar{s} \rangle}{2^7 \pi^4} \left\{ \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{2} \int_{\beta_{\min}}^{1 - \alpha} \frac{d\beta}{2} \left[ \frac{m_Q^2}{\alpha^3} (1 + \alpha + \beta) r(m_Q, m_{Qr}) + \frac{m_Q^2}{\alpha^2} r(m_Q, m_{Qr}) \\
- \frac{m_Q}{\alpha} r(m_Q, m_{Qr}) + \frac{2m_Q m_s}{\beta^3} m_s m_{Qr} - \frac{m_Q m_s}{\alpha^2} \right] r(m_Q, m_{Qr}) \\
+ \left\{ \frac{m_s}{\alpha(1 - \alpha)} \left[ \alpha m_Q^2 + (1 - \alpha) m_{Qr}^2 - \alpha(1 - \alpha)s \right] + \frac{m_s^2}{\alpha} \right\} \left[ \alpha m_Q^2 + (1 - \alpha) m_{Qr}^2 - \alpha(1 - \alpha)s \right] \right\},
\]

\[
\rho^{(s\bar{s})}(s) = \frac{\langle s\bar{s} \rangle^2}{2^5 \pi^2} \left( 2m_Q m_{Qr} + m_Q m_s \right) \sqrt{(s - m_Q^2 + m_{Qr}^2)^2 - 4m_{Qr}^2 s^2/s} + \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{2} \left( m_Q m_s + \frac{3m_{Qr}^2}{2} (1 - \alpha) \right),
\]

\[
\rho^{(g\sigma \cdot G \sigma)}(s) = \frac{3 \langle g\sigma \cdot G \sigma \rangle}{2^7 \pi^4} \left\{ \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{2} \int_{\beta_{\min}}^{1 - \alpha} \frac{d\beta}{2} \left[ \frac{m_Q^2}{\alpha^3} (1 + \alpha + \beta) r(m_Q, m_{Qr}) \\
+ \left( 1 - \alpha \right) m_{Qr}^2 - \alpha(1 - \alpha)s \right] + \frac{m_Q^2}{\alpha^2} \right\} \left( m_Q^2 + (1 - \alpha)m_{Qr}^2 - 2\alpha(1 - \alpha)s \right) \\
+ \frac{m_Q^2 m_s^2}{3} (1 - \alpha) \right\} + \left( 2m_Q m_s m_s + \frac{3m_{Qr}^2 m_s}{3} \right) \sqrt{(s - m_Q^2 + m_{Qr}^2)^2 - 4m_{Qr}^2 s^2/s} \right\},
\]

\[
\rho^{(g \cdot G^2)}(s) = \frac{\langle g^2 G^2 \rangle}{2^{12} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{2} \int_{\beta_{\min}}^{1 - \alpha} \frac{d\beta}{2} \left[ \frac{m_Q^2}{\alpha^3} (1 + \alpha + \beta) r(m_Q, m_{Qr}) \\
+ \frac{m_Q^2 m_s}{\alpha^3} \right] \left( 1 + \alpha + \beta \right) r(m_Q, m_{Qr}) + \frac{3m_Q m_s m_s}{\alpha^2} \right\} \left( 1 + \alpha + \beta \right) r(m_Q, m_{Qr}) \\
+ \frac{m_Q^2 m_s}{\beta^3} \left( 1 + \alpha + \beta \right) + \frac{6m_Q m_s m_s}{\alpha^3} \right\} \left( 1 + \alpha + \beta \right) r(m_Q, m_{Qr}) + \frac{2m_Q^2 m_s}{\beta^3} m_s \\
+ \frac{m_Q^2 m_s}{\alpha^2} \left( 1 + \alpha + \beta \right) + \frac{6m_Q m_s m_s}{\alpha^2} m_s + \frac{6m_Q m_{Qr} m_s}{\beta^2} \right\},
\]

\[
\rho^{(g^3 G^3)}(s) = \frac{\langle g^3 G^3 \rangle}{2^{14} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{2} \int_{\beta_{\min}}^{1 - \alpha} \frac{d\beta}{2} \left[ \frac{1}{\alpha^3} (1 + \alpha + \beta) r(m_Q, m_{Qr}) \\
+ \frac{2m_Q^2 m_s}{\alpha^3} (1 + \alpha + \beta) + \frac{1}{\beta^3} (1 + \alpha + \beta) r(m_Q, m_{Qr}) + \frac{2m_Q^2 m_s}{\beta^3} (1 + \alpha + \beta) \\
+ \frac{6m_Q m_{Qr} m_s}{\alpha^3} (1 + \alpha + \beta) + \frac{3 \cdot 2^2 m_Q m_s}{\beta^3} m_s + \frac{2m_Q m_s}{\alpha^2} + \frac{m_Q^2 m_s}{\beta^2} (1 + \alpha + \beta) \right].
\]
\[
\rho(g^2G^2)(s) = \frac{g^2G^2}{24\pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta (1-\alpha-\beta) \left\{ \frac{m_{\bar{G}}^2}{\alpha^3} (1+\alpha+\beta) r(m_Q,m_{\bar{Q}}) \\
+ \frac{m_Q^2}{\beta^3} (1+\alpha+\beta) r(m_Q,m_{\bar{Q}}) - \frac{3m_Qm_s}{\alpha^3} (1+\alpha+\beta) r(m_Q,m_{\bar{Q}}) \\
- \frac{m_Q^2m_s\beta}{\alpha^3} (1+\alpha+\beta) + \frac{6m_Qm_s}{\beta^3} r(m_Q,m_{\bar{Q}}) + \frac{2m_Q^2m_s\alpha}{\beta^3} + \frac{2m_Q^2m_s^2m_s}{\alpha^2} \\
- \frac{m_Q^2m_s}{\beta^2} (1+\alpha+\beta) - \frac{6m_Qm_s^2}{\alpha^2} - \frac{6m_Qm_s^2}{\beta^2} \right\},
\]

\[
\rho(g^3G^3)(s) = \frac{g^3G^3}{24\pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta (1-\alpha-\beta) \left\{ \frac{1}{\alpha^3} (1+\alpha+\beta) r(m_Q,m_{\bar{Q}}) \\
+ \frac{2m_Q^2\beta}{\alpha^3} (1+\alpha+\beta) + \frac{1}{\beta^3} (1+\alpha+\beta) r(m_Q,m_{\bar{Q}}) + \frac{2m_Q^2\alpha}{\beta^3} (1+\alpha+\beta) \\
- \frac{6m_Q^2m_s\beta}{\alpha^3} (1+\alpha+\beta) + \frac{3 \cdot 2^2m_Qm_s\alpha}{\beta^3} + \frac{2m_Qm_s^2m_s}{\alpha^2} - \frac{m_Qm_s}{\beta^2} (1+\alpha+\beta) \right\},
\]

for \((Q\bar{s})_1(\bar{Q}s)\),

\[
\rho^{pert}(s) = \frac{3}{24\pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta (1-\alpha-\beta) \left\{ \frac{1}{\alpha^3} (1+\alpha+\beta) r(m_Q,m_{\bar{Q}}) \right\}^2,
\]

\[
\rho^{(Q\bar{s}s)} = \frac{3}{24\pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} \left\{ \frac{m_Qm_s\beta}{\alpha^2\beta^3} (1+\alpha+\beta) r(m_Q,m_{\bar{Q}}) - \frac{m_Q}{\alpha^2\beta} \right\} r(m_Q,m_{\bar{Q}}),
\]

\[
\rho^{(Q\bar{s}s)} = \frac{3}{24\pi^6} \left\{ \frac{m_s}{\alpha(1-\alpha)} [am_Q^2 + (1-\alpha)m_Q^2] - [am_Q^2 + (1-\alpha)m_Q^2 - \alpha(1-\alpha)] ) \right\},
\]

\[
\rho^{(Q\bar{s}G\bar{s})} = \frac{3}{24\pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \left\{ \frac{m_Qm_s\beta}{\alpha^2\beta^3} (1+\alpha+\beta) r(m_Q,m_{\bar{Q}}) + \frac{m_Q}{\alpha^2\beta} \right\},
\]

\[
\rho^{(g^2G^2)}(s) = \frac{g^2G^2}{24\pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta (1-\alpha-\beta) \left\{ \frac{m_Q^2}{\alpha^3} (1+\alpha+\beta) r(m_Q,m_{\bar{Q}}) \\
+ \frac{m_Q^2}{\beta^3} (1+\alpha+\beta) r(m_Q,m_{\bar{Q}}) - \frac{3m_Qm_s}{\alpha^3} (1+\alpha+\beta) r(m_Q,m_{\bar{Q}}) \\
+ \frac{m_Q^2m_s\beta}{\alpha^3} (1+\alpha+\beta) - \frac{6m_Qm_s}{\beta^3} r(m_Q,m_{\bar{Q}}) - \frac{2m_Q^2m_s\alpha}{\beta^3} - \frac{2m_Q^2m_s^2m_s}{\alpha^2} \\
+ \frac{m_Q^2m_s}{\beta^2} (1+\alpha+\beta) - \frac{6m_Qm_s^2}{\alpha^2} - \frac{6m_Qm_s^2}{\beta^2} \right\},
\]

\[
\rho^{(g^3G^3)}(s) = \frac{g^3G^3}{24\pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta (1-\alpha-\beta) \left\{ \frac{1}{\alpha^3} (1+\alpha+\beta) r(m_Q,m_{\bar{Q}}) \\
+ \frac{2m_Q^2\beta}{\alpha^3} (1+\alpha+\beta) + \frac{1}{\beta^3} (1+\alpha+\beta) r(m_Q,m_{\bar{Q}}) + \frac{2m_Q^2\alpha}{\beta^3} (1+\alpha+\beta) \\
+ \frac{6m_Qm_s\beta}{\alpha^3} (1+\alpha+\beta) - \frac{3 \cdot 2^2m_Qm_s\alpha}{\beta^3} + \frac{2m_Qm_s^2m_s}{\alpha^2} + \frac{m_Qm_s}{\beta^2} (1+\alpha+\beta) \right\},
\]
for \((Q\bar{s})(\bar{Q}'s)_1\), and

\[
\rho_{\text{pert}}(s) = \frac{3}{2\pi^6} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \mathrm{d}\alpha \int_{\beta_{\text{min}}}^{1-\alpha} \mathrm{d}\beta \left[ \frac{1}{\alpha \beta^3} r(m_Q, m_{Q'})^2 + \frac{2m_{Q'} m_s}{\alpha^3 \beta^2} r(m_Q, m_{Q'}) \right.
\]

\[
\left. - \frac{2m_Q m_s}{\alpha^3 \beta^2} r(m_Q, m_{Q'}) - \frac{3 \cdot 2^2 \alpha^4 m_Q m_{Q'} m_s^2}{\alpha^2 \beta^2} r(m_Q, m_{Q'})^2, \right.
\]

\[
\rho(\bar{s}s)(s) = \frac{3}{2\pi^4} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \mathrm{d}\alpha \int_{\beta_{\text{min}}}^{1-\alpha} \mathrm{d}\beta \left[ \frac{m_Q}{\alpha^2 \beta} r(m_Q, m_{Q'}) - \frac{m_Q}{\alpha^2 \beta^2} r(m_Q, m_{Q'}) \right.
\]

\[
\left. - \frac{2^3 \alpha^4 m_Q m_s^2}{\alpha \beta} r(m_Q, m_{Q'}) + \left\{ \frac{2m_Q}{\alpha (1-\alpha)} \right\} \left[ m_Q^2 + (1-\alpha) m_{Q'}^2 - \alpha (1-\alpha) s \right] \right\},
\]

\[
\rho(\bar{s}s)^2(s) = \frac{3}{2\pi^4} \left\{ -2m_Q m_{Q'} - \frac{(m_Q - m_{Q'}) m_s}{2} \sqrt{(s - m_Q^2 + m_{Q'}^2)^2 - 4m_Q^2 s/s} + 3m_s \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \mathrm{d}\alpha (1-\alpha) \right\},
\]

\[
\rho(g\bar{s}s-Gs)(s) = \frac{3}{2\pi^4} \left\{ \frac{2}{3} [2m_Q^2 + 2(1-\alpha) m_{Q'}^2 - 3\alpha (1-\alpha) s] + [-2m_Q m_{Q'} m_s \right\}
\]

\[
\left. - \left[ m_Q - m_{Q'} \right] m_s^2 \right\} \sqrt{(s - m_Q^2 + m_{Q'}^2)^2 - 4m_Q^2 s/s},
\]

\[
\rho(g^2G^2)(s) = \frac{3}{2\pi^6} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \mathrm{d}\alpha \int_{\beta_{\text{min}}}^{1-\alpha} \mathrm{d}\beta \left[ \frac{m_Q}{\alpha^2 \beta^3} r(m_Q, m_{Q'}) + \frac{m_Q^2}{\beta^3} r(m_Q, m_{Q'}) \right.
\]

\[
\left. + \frac{3m_{Q'} m_s^2}{2\alpha^3} r(m_Q, m_{Q'}) - \frac{3m_Q m_s^2}{2\alpha^3} r(m_Q, m_{Q'}) - \frac{m_Q m_s^2}{\beta^3} \right\},
\]

\[
\rho(g^2G^3)(s) = \frac{3}{2\pi^6} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \mathrm{d}\alpha \int_{\beta_{\text{min}}}^{1-\alpha} \mathrm{d}\beta \left[ \frac{m_Q}{\alpha^2 \beta^3} \right.
\]

\[
\left. + \frac{1}{\beta^3} r(m_Q, m_{Q'}) + \frac{2m_Q m_s^2}{\beta^3} + \frac{3m_Q m_s^2 \alpha}{\alpha^3} - \frac{3m_Q m_s^2 m_s}{\beta^3} - \frac{m_Q m_s^2}{2\alpha^2} + \frac{m_Q m_s^2}{2\beta^2} \right\},
\]

for \((Q\bar{s})(\bar{Q}'s)_1\). The integration limits are given by

\[
\alpha_{\text{min}} = \frac{s - m_Q^2 + m_{Q'}^2 - \sqrt{(s - m_Q^2 + m_{Q'}^2)^2 - 4m_Q^2 s}}{2s},
\]

\[
\alpha_{\text{max}} = \frac{s - m_Q^2 + m_{Q'}^2 + \sqrt{(s - m_Q^2 + m_{Q'}^2)^2 - 4m_Q^2 s}}{2s}, \quad \beta_{\text{min}} = \frac{\alpha m_Q^2}{s \alpha - m_Q^2}.
\]

Acknowledgments

J.R. Zhang is very indebted to Ming Zhong for helpful discussions.

References

[1] S.K. Choi, et al., (Belle Collaboration), Phys. Rev. Lett. 91 (2003) 262001; V.M. Abazov, et al., (D0 Collaboration), Phys. Rev. Lett. 93 (2004) 162002; D. Acosta, et al., (CDF Collaboration), Phys. Rev. Lett. 93 (2004) 072001; B. Aubert, et al., (BaBar Collaboration), Phys. Rev. D 71 (2005) 071103.

[2] S.K. Choi, et al., (Belle Collaboration), Phys. Rev. Lett. 94 (2005) 182002; B. Aubert et al. (BaBar Collaboration), Phys. Rev. Lett. 101 (2008) 082001.

[3] B. Aubert, et al., (BaBar Collaboration), Phys. Rev. Lett. 95 (2005) 142001; Q. He, et al. (CLEO Collaboration), Phys. Rev. D 74 (2006) 091104(R); C.Z. Yuan, et al. (Belle Collaboration), Phys. Rev. Lett. 99 (2007) 182004.

[4] S. Uehara, et al., (Belle Collaboration), Phys. Rev. Lett. 96 (2006) 082003.

[5] K. Abe, et al., (Belle Collaboration), Phys. Rev. Lett. 98 (2007) 082001.
[6] K. Abe, et al., (Belle Collaboration), Phys. Rev. Lett. 100 (2008) 142001.
[7] R. Mizuk, et al., (Belle Collaboration), Phys. Rev. D 78 (2008) 072004.
[8] T. Aaltonen, et al., (CDF Collaboration), Phys. Rev. Lett. 102 (2009) 242002.
[9] E.S. Swanson, Phys. Rep. 429 (2006) 243.
[10] C. Amsler, et al., (Particle Data Group), Phys. Lett. B 667 (2008) 1.
[11] F.E. Close and P.R. Page, Phys. Lett. B 578 (2004) 119; M.B. Voloshin, Phys. Lett. B 579 (2004) 316; C.Y. Wong, Phys. Rev. C 69 (2004) 055202; E.S. Swanson, Phys. Lett. B 590 (2004) 189; N.A. Törnqvist, Phys. Lett. B 598 (2004) 197.
[12] X. Liu, X.Q. Zeng, and X.Q. Li, Phys. Rev. D 72 (2005) 054023; X. Liu, Y.R. Liu, W.Z. Deng, and S.L. Zhu, Phys. Rev. D 77 (2008) 034003; X. Liu, Y.R. Liu, W.Z. Deng, and S.L. Zhu, Phys. Rev. D 77 (2008) 094015.
[13] J.L. Rosner, Phys. Rev. D 76 (2007) 114002; C. Meng and K.T. Chao, arXiv:0708.4222; S.H. Lee, A. Mihara, F.S. Navarra, and M. Nielsen, Phys. Lett. B 661 (2008) 28; C.E. Thomas and F.E. Close, Phys. Rev. D 78 (2008) 034007.
[14] X. Liu, Z.G. Luo, Y.R. Liu, and S.L. Zhu, Eur. Phys. J. C 61 (2009) 411.
[15] X. Liu and S.L. Zhu, Phys. Rev. D 80 (2009) 017502.
[16] N. Mahajan, arXiv:0903.3107; T. Branz, T. Gutsche, and V.E. Lyubovitskij, arXiv:0903.5424; G.J. Ding, arXiv:0904.1782.
[17] M.A. Shifman, A.I. Vainshtein, and V.I. Zakharov, Nucl. Phys. B 147 (1979) 385; B 147 (1979) 448; V.A. Novikov, M.A. Shifman, A.I. Vainshtein, and V.I. Zakharov, Fortschr. Phys. 32 (1984) 585.
[18] M.A. Shifman, Vacuum Structure and QCD Sum Rules, North-Holland, Amsterdam (1992).

[19] B.L. Ioffe, in The Spin Structure of the Nucleon, ed. by B. Frois, V.W. Hughes, and No. de Groot, World Scientific, Singapore (1997), arXiv:9511401.
[20] S. Narison, QCD Spectral Sum Rules, World Scientific, Singapore (1989).
[21] P. Colangelo and A. Khodjamirian, in At the Frontier of Particle Physics: Handbook of QCD, Vol. 3, M. Shifman Ed., Boris Ioffe Festschrift, World Scientific, Singapore (2001) pp. 1495–1576, arXiv:0010175; A. Khodjamirian, Talk Given at Continuous Advances in QCD 2002/ARKADYFEST (2002), arXiv:0209166.
[22] Z.G. Wang, Eur. Phys. J. C 63 (2009) 115.
[23] R.M. Albuquerque, M.E. Bracco, and M. Nielsen, Phys. Lett. B 678 (2009) 186.
[24] J.R. Zhang and M.Q. Huang, J. Phys. G: Nucl. Part. Phys. 37 (2010) 025005.
[25] L.J. Reinders, H.R. Rubinstein, and S. Yazaki, Phys. Rep. 127 (1985) 1.
[26] L.J. Reinders, H.R. Rubinstein, and S. Yazaki, Nucl. Phys. B 186 (1981) 109.
[27] M. Nielsen, F.S. Navarra, and S.H. Lee, arXiv:0911.1958.
[28] R.D. Matheus, S. Narison, M. Nielsen, and J.M. Richard, Phys. Rev. D 75 (2007) 014005; S.H. Lee, K. Morita, and M. Nielsen, Phys. Rev. D 78 (2008) 076001; M.E. Bracco, S.H. Lee, M. Nielsen, and R.R. daSilva, Phys. Lett. B 671 (2009) 240; S.H. Lee, K. Morita, and M. Nielsen, Nucl. Phys. A 815 (2009) 29; R.M. Albuquerque and M. Nielsen, Nucl. Phys. A 815 (2009) 53.
[29] J.R. Zhang and M.Q. Huang, Phys. Rev. D 77 (2008) 094002; Phys. Rev. D 78 (2008) 094007; Phys. Rev. D 78 (2008) 094015; Phys. Lett. B 674 (2009) 28.
[30] C.P. Shen, et al., (Belle Collaboration), Phys. Rev. Lett. 104 (2010) 112004; C.Z. Yuan, (on behalf of Belle Collaboration), arXiv:0910.3138 [hep-ex]; C.P. Shen, (for the Belle Collaboration), arXiv:0912.2386 [hep-ex].