A note on the RG flow in $(N = 1, \ D = 4)$ Supergravity and applications to $Z_3$ orbifold/orientifold compactification

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Abstract

We apply the standard approach of RG flow for the gauge couplings in $N=1 \ D=4$ Supergravity to show how to match its results with the heterotic $Z_3$ orbifold and Type IIB $Z_3$ orientifold-based models. Using only supergravity, anomaly cancellation and the requirement of unification we determine the part of the Kähler potential of the model invariant under the symmetries of the model. For heterotic orbifolds/type IIB orientifolds, this shows that the lowest order Kähler term of the dilaton has the structure $-\ln(S+S)$ in agreement with string calculations. The structure of the holomorphic coupling is also found from arguments of unification and anomaly cancellation under the conjectured $SL(2,\mathbb{Z})_{T_i}$ symmetries of the models. A consequence of the latter is that in the case of the $Z_3$ orientifold the holomorphic coupling necessarily contains a part with coefficient proportional to the one loop beta function, in agreement with string calculations which do not however assume this symmetry. This gives circumstantial evidence for the existence of this symmetry at string level in $Z_3$ orientifold. Finally, we comment on the values of the unification scale and examine the possibility of mirage unification in which the effective unification scale may be situated far above the string scale.

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1 Introduction

Considerable interest has recently been attracted by the initial investigation \[1\] of sigma-model symmetries in Type II B D=4, N=1 orientifolds at the level of the effective Lagrangian. The motivation for the study of such symmetries is suggested by the case of D=4, N=1 heterotic orbifold models. There an \(SL(2, R)_{T}\) transformation leaving invariant the classical Lagrangian also corresponds to an exact \(SL(2, Z)_{T}\) symmetry of the underlying string (T duality). What do we know on the orientifold side?

The proposed “strong-weak” duality \[2\] between the heterotic and type I vacua in ten dimensions suggested the existence of links between heterotic and type I models in lower dimensions \[3\]. This has lead in particular to a generalised heterotic-type II B orientifold duality in four dimensions in which both models are weakly coupled \[3\] for some regions of the moduli space. An example of this latter duality is provided by \(Z_3\) orbifold-orientifold models \[3, 4\] without Wilson lines and these are the models we will consider in the following. Their very similar spectrum strengthens the suggestion that they are indeed dual. The presence of the sigma model symmetry and its anomaly cancellation in the \(Z_3\) heterotic case thus suggests its existence and associated anomaly cancellation in the \(Z_3\) orientifold model as well. Thus studying the anomaly cancellation on the orientifold side may help us find out more about the proposed “weak-weak” duality in four dimensions. Here we examine the consequences of this \(SL(2, Z)_{T}\) symmetry in the \(Z_3\) orientifold model and discuss its implications for the RG flow of the gauge couplings, intimately connected to its anomaly cancellation. Previous analyses and associated difficulties of \(SL(2, Z)_{T}\) symmetry on the orientifold side were discussed in \[1, 4, 5, 6, 7, 8, 9\].

In the case of the heterotic string, the models based on the \(Z_3\) orbifold have the gauge group \(SU(12) \times SO(8) \times U(1)_A\) where \(U(1)_A\) is anomalous. This anomaly, which is universal, is cancelled by the shift of the dilaton. The Fayet Iliopoulos term, dilaton dependent, is cancelled by shifts of some additional fields. These are the so-called “blowing up” modes of the orbifold which are charged only under the \(U(1)_A\) and compensate the Fayet Iliopoulos term when they acquire some expectation value, without breaking further the gauge group. The mass of the anomalous gauge boson becomes equal to the string scale and decouples from the RG flow of the model.

For the case of the orientifold (we mainly refer to the \(Z_3\) orientifold, but some aspects in the following are more general) the situation is slightly different. In this case the gauge group is also \(SU(12) \times SO(8) \times U(1)_A\) where \(U(1)_A\) is again anomalous. However, in this case the anomalies are gauge group dependent and may be cancelled by a generalised Green Schwarz mechanism. This can be realised by the shifting \[1, 4\] of the twisted axions which are singlets under the gauge group. The anomalous vector superfield mixes with these states and acquires a mass which is again of string scale order, regardless of the particular choice of the Kähler potential for the twisted moduli \[4\].

After determining that the two models have a similar spectrum and the same gauge group below the string scale, it is interesting to analyse whether the weak coupling - weak coupling duality really holds beyond tree level. This was the purpose of the investigation in \[6\] at the one loop level. Starting from the RG flow in the string (linear) basis and using the linear-chiral duality relation \[9\] it was shown \[6\] that the results for the low energy couplings raise doubts about the existence of the \(Z_3\) orbifold-orientifold duality \[6\]. There is however a difficulty in analysing the relation between the couplings of the \(Z_3\) orbifold and \(Z_3\) orientifold models. This is due to the fact that the string scale in the latter case was not invariant under the proposed \(SL(2, Z)_{T}\) symmetry. Moreover, in the orientifold limit, this means that the low energy physics, as it emerges from the linear basis formula \[10\] is not invariant under the symmetry. In the linear basis \[9\] for the \(Z_N\) orientifold

\[
g_a^{-2}(\mu) = l^{-1} + \sum_{k=1}^{(N-1)/2} s_{ak} m_k + \frac{b_a}{8\pi^2} \ln \frac{M_I}{\mu}
\]

1
where \( l \) is the string coupling, \( s_{ak} \) is a coefficient proportional to one loop beta function \( b_k \) and \( m_k \) is the scalar component of the twisted linear multiplet describing the blowing up modes of the associated orbifold. In the following we will drop the index \( k \) whenever we refer to the particular case of the \( Z_3 \) orientifold. The orientifold limit \( m_k \rightarrow 0 \) together with the dependence of the tree level string scale \( M_I \) on the \( T_i \) moduli\(^1\) leads to the conclusion that \( g_a(\mu \sim M_z) \) changes under \( SL(2, Z)_{T_i} \). This is obviously inconsistent with the fact that \( g_a(\mu \sim M_z) \) is a physical observable and it must remain invariant under this transformation if it is a symmetry of the theory.

There are three possible resolutions to this problem. The first is that there may be additional terms in eq.(1) which render \( g_a(\mu \sim M_z) \) invariant. The second possibility is that the string scale definition to be used in (1) is changed from the tree level value \( M_I \) and its one-loop improved value is invariant under the transformation of \( T_i \). This is similar to the heterotic case where the tree level string scale \( M_H \) depends on \( S, M_H^2 \sim M_T^2/(S + \bar{S}) \); the dilaton changes under \( SL(2, Z)_{T_i} \) (for universal anomaly cancellation) so \( M_H \) is not invariant, but the one-loop improved heterotic string scale is invariant under this symmetry, as we will discuss later. The last possibility is that the proposed symmetry \( SL(2, Z)_{T_i} \) is not there at all in the case of \( Z_3 \) orientifold models, putting into doubt the existence of the proposed \( Z_3 \) orbifold-orientifold duality.

In this work we investigate these possibilities adopting a different perspective. We work only in the chiral basis of \( N = 1, D = 4 \) Supergravity and do not use the (questionable) linear-chiral multiplet duality relation\(^2\). We assume there is an effective supergravity theory below some cut-off scale \( \Lambda \). As we will show, this approach provides relevant information for anomaly cancellation and the unification of the gauge couplings. This is interesting because it will give us some understanding as to whether the proposed mirage unification scenario initially suggested by\(^3\) can be made to work for \( Z_3 \) orientifolds. This requires a careful investigation of the RG flow for both models based on the \( Z_3 \) orbifold/orientifold.

The RG flow in \( N = 1, D = 4 \) Supergravity models was introduced in\(^4\) (see also\(^5, 6\) for applications to the heterotic case) and we will briefly review some aspects of it in the next section, with emphasis on the importance of the various terms contributing to the (perturbatively exact) running. A more illustrative form of the RG flow is presented in Section\(^8\) which makes manifest how string theories renormalise, through the Kähler terms for the moduli fields, the bare coupling and the (high) scale to which the effective (canonical) gauge couplings run. This discussion will show that the assumption there is gauge coupling unification in supergravity models suggests the existence of a link at a deeper level in string theory between the structure of the Kähler potential for moduli fields (other than the dilaton) and that of the dilaton itself. In Section\(^9, 10\) we discuss the case of the \( Z_3 \) orbifold. We stress the importance of the one loop improved string scale which emerges as the natural cut-off in the RG flow of the (effective) gauge couplings and which should be used in models which want to test the weak coupling - weak coupling duality, at one-loop level. In Section\(^11\) we perform a similar analysis to investigate the gauge couplings running in the case of \( Z_3 \) orientifold model. Using only anomaly cancellation arguments in \( D = 4 \), we recover on purely field theoretic grounds, the result from string theory\(^4, 5\) that the coefficient of the twisted moduli entering the RG flow is indeed proportional to the one loop beta function for the case of \( Z_3 \) orientifold. The calculation also shows that the effective unification scale could be far above the string scale (mirror unification), perhaps close to the Planck scale. Unfortunately our analysis cannot say more about this value since the v.e.v. of the twisted moduli is not determined by the Fayet Iliopoulos mechanism. This is due to our present lack of understanding of the linear-chiral duality relation, which relates the string result\(^4\) to that of Supergravity RG flow for \( g_a(\mu \sim M_z) \), the question of invariance of the string scale under the

\(^1\)At string tree level this is given by \( M_I^2 = (ReS/\prod_{i=1}^3 ReT_i)^{1/2} M_T^2/(2ReS) \).
assumed $SL(2, Z)_{T_i}$ symmetry and the cancellation of anomalies in type IIB orientifolds [4]. These issues may affect the results obtained from the Fayet Iliopoulos mechanism in the orientifold limit. This mechanism has been investigated in ref. [4] but its conclusions seem to be in conflict with the proposed linear-chiral multiplet duality relation [4] suggesting further study is necessary.

To avoid some of these problems for the $Z_3$ orientifold models we present the results in terms of an unknown function $G$ invariant under the $SL(2, Z)_{T_i}$ symmetry and speculate about its effects on the value of the unification scale. Our conclusions are presented in the last section.

2 RG flow, symmetries and the link with string theory

The standard procedure for analysing the RG flow equations in local supersymmetric theories and their matching with string theory was introduced by Kaplunovsky and Louis in ref. [11, 12] (see also [13]). Its application to string models in general requires a knowledge of the Kähler potentials of the moduli fields as well as the structure of the holomorphic couplings.

Let us consider the link between the supergravity RG flow and string theory - in our case heterotic orbifolds and Type II B orientifolds. There are two important differences between these cases. Firstly, the ultraviolet cut-off $\Lambda$ of the RG flow of the Wilsonian coupling in the effective theory of supergravity may not be the same in both cases. However, whatever its value may be, by definition [11] it must be independent of all fields/moduli [11, 12]. From the point of view of the effective supergravity theory $\Lambda$ must be constant in Planck units, and it can therefore be set equal to the Planck scale $M_P$. This applies to the effective supergravity models of the heterotic strings [11, 12]. It is not clear to us whether this should also be the case for the type I string models we consider here, although certainly $\Lambda$ must be a moduli/field independent quantity. Secondly, for the two classes of string models we consider, the expressions for the holomorphic function $f_a$ and the dependence on the moduli fields of the Kähler potentials may be different. Their input from string theory in both cases realises the connection between effective supergravity and string theory, allowing a test of string unification at (N=1, D=4) supergravity level. However, throughout this paper, we will keep the string input at minimum and use only supergravity arguments for most of the calculations. A great deal of the structure of the Wilsonian coupling and even their relationship to the Kähler moduli can be understood on pure supergravity grounds together with gauge unification which we assume applies. This will be detailed in the following sections.

The integral of the RG flow for the effective gauge couplings $g_a$ in rigid supersymmetry follows from the exact beta function for $g_a$ introduced by Novikov, Shifman, Vainshtein, Zakharov (NSVZ), [14, 15]. The result after integration is

$$F_a = g_a^{-2} \left( \mu \right) + \frac{b_a}{8\pi^2} \ln \mu - \frac{T(G_a)}{8\pi^2} \ln g_a^{-2} \left( \mu \right) + \sum_r \frac{T_a^{(r)}}{8\pi^2} \ln Z_r (\rho, \mu) \tag{2}$$

where condition $dF_a/d\mu = 0$ recovers the “NSVZ” beta function. Here $b_a = -3T_a(G) + \sum_r T_a^{(r)}$. After applying this equation at two different scales, one recovers the running of the couplings between these scales to all orders in perturbation theory. The coefficients $Z$ are normalised to unity at some arbitrary scale $\rho$. At two loop level one can reproduce the structure of the familiar RG flow of the Minimal Supersymmetric Standard Model (MSSM).

For the Wilsonian couplings of the local supersymmetric theories we have the following integral of

\footnote{for the heterotic case}
the renormalisation group

\[ F_a = \text{Re} f_a + \frac{b_a}{8\pi^2} \ln \Lambda + \frac{c_a}{16\pi^2} \mathcal{K}, \quad \mathcal{K} = \kappa^2 K \]  

(3)

where \( \Lambda \) is the ultraviolet cut-off scale. Equation (3) simply reflects that Wilsonian couplings do not renormalise beyond one-loop. The last term in (3) accounts for the super-Weyl anomaly induced by the rescaling of the metric necessary to separate gravity from quantum field theory effects in the supergravity action [11, 16]. We use the notation \( c_a = -T_a(G) + \sum_r T_a^{(r)} \), and \( \kappa^2 = 8\pi/M_P^2 \). \( K \) is the full Kähler potential which includes a term for moduli fields and one for the charged matter fields [11]. Its expression is given by

\[ K = K_{\text{mod}} + K_{\text{matter}} = k^{-2} \mathcal{K} + K_{\text{matter}} \]  

(4)

where the moduli part of the Kähler potential \( K_{\text{mod}} \sim \mathcal{O}(M_P^2) \) is the only relevant part in (3), the charged matter term \( K_{\text{matter}} \sim ZGQ \) being strongly suppressed relative to the moduli contribution. Therefore \( \mathcal{K} \approx \mathcal{K} \). Equations (2), (3) above set the integral of the RG flow in models with local supersymmetry in the following form [11, 12, 13]

\[ g_a^{-2}(\mu) = \text{Re} f_a + \frac{b_a}{16\pi^2} \ln \frac{\Lambda^2}{\mu^2} + \frac{1}{16\pi^2} \left\{ c_a \mathcal{K} + 2T(G_a) \ln g_a^{-2}(\mu) - 2 \sum_r T^{(r)}(\mu) \ln Z_r(\rho, \mu) \right\} \]  

(5)

A comment on the structure of equation (3) and how it is linked to string theory is in place here. Although this discussion is made in the context of the weakly coupled heterotic models, it is in fact more general. Usually the link with string theory is established by taking the Kähler potential for the dilaton and other moduli as given to some order of perturbation in string theory. Consider, for example, the Kähler potential for the dilaton \( \mathcal{K} \sim -\ln(S + \overline{S}) \). Its contribution in the curly braces of (3) is usually split into two terms, one given by \(-b_a/(16\pi^2) \ln(S + \overline{S})\) which combines with the cut-off \( \Lambda \) to give the heterotic string scale, \( M_P^2 \sim \Lambda^2/(S + \overline{S}) \), while the remaining part \(-c_a - b_a/(16\pi^2) \ln(S + \overline{S})\) combines with \( \ln g_a^{-2}(\mu) \) to give \( 2T_a(G) \ln g_a^{-2}(\mu) \) in the curly braces. Further, this last term (pure gauge) is usually ignored as being considered a higher order (two loop and beyond) term in the effective field theory. This is not correct because it is inconsistent to split the logarithmic dependence of the dilaton into two pieces and to retain only \( b_a \ln(S + \overline{S}) \) while ignoring the part proportional to \( 2T_a(G) \ln(S + \overline{S}) \). Since the string expansion parameter is \( \sim 1/\text{Re}S \) truncating such an expansion to keep only part of \( \ln(S + \overline{S}) \) term makes the whole calculation only valid in \( \mathcal{O}(\text{Re}S) \) order i.e. string tree level.

The presence of the terms \( Z_a(\rho, \mu) \) is due to wavefunction renormalisation at/above and below some cut-off \( \overline{\Lambda} \) of the charged matter fields. Usually only their value at/above the cut-off is retained to give contribution to the so called one-loop string threshold effects (dependent on the \( T \) moduli). This contribution is due to the Kähler terms for charged matter fields which are not canonically normalised and comes with what can be considered as a wavefunction coefficient of string origin, \( Z_r(\rho, \overline{\Lambda}) \sim \prod_{i=1}^3 (T_{\rho} + T_{\overline{\Lambda}})^{n_r} \) where \( n_r \) are modular weights of the matter fields, \( \rho \) is some arbitrary scale\(^3\) and \( \overline{\Lambda} \) is the string scale. The contribution (computable in field theory) to \( Z \) below the cut-off (later we will identify it with the string scale, \( \overline{\Lambda} \)) is needed to account for matter and mixed gauge-matter contributions to the running of the \( \text{effective} \) couplings. At one loop order there is a gauge contribution of the form \( \ln Z(\overline{\Lambda}, \mu) \sim \ln g_a(\overline{\Lambda})/g_a(\mu) \). This together with the fact \( g_a^2(\overline{\Lambda}) \sim (S + \overline{S})^{-1} \) shows it is of comparable magnitude to the dilaton contribution \( \ln(S + \overline{S}) \) and must be included.

\(^3\)which turns out to be the string scale, see later.

\(^4\)The scale \( \rho \) is not physical, it simply corresponds to a particular point in the moduli space where \( Z \) is normalised to unity.
more elegant motivation for including \( \ln Z \) terms below the cut-off scale is due to the fact that they originate from anomaly cancellation (from the rescaling of chiral superfields) just as the terms \( \ln g_a^{-2} \) do \( [15] \) (from the rescaling of vector superfields), so they are on equal footing.

The effective couplings \( g_a \) are physical quantities, and therefore they must be invariant under the symmetries of the theory. In particular we require the invariance of the low energy physics under the Kähler symmetry transformations of the Kähler potential \( K \) and of the superpotential \( W \), given by \( (\phi \) stands for the moduli fields and \( Q \) for the charged matter fields) \( [11] \)

\[
\begin{align*}
K(\phi', \bar{\phi}', Q', \overline{Q}') &\rightarrow K(\phi, \bar{\phi}, Q, \overline{Q}) + J(\phi, Q) + \overline{J}(\phi, \overline{Q}) \\
W(\phi', Q') &\rightarrow W(\phi, Q) \exp(-\kappa^2 J(\phi, Q))
\end{align*}
\]

(6) (7)

At the classical level, these transformations leave the action invariant \( [11, 12, 13] \). At the quantum level these symmetries imply a transformation of the holomorphic coupling \( f_a \) (Kähler anomaly) which is shifted by a quantity proportional to \( J(\phi) \) as follows from \( (8) \). In a similar way, a symmetry transformation of the matter fields of the form

\[
Q' \rightarrow Q'' = Y_{r}^{\phi}(\phi)Q''
\]

induces another anomaly with \( Q^\dagger ZQ = Q^\dagger Z'Q' \), so \( Z \rightarrow Z' = Y^{-1}ZY^{-1} \). This transformation together with that of the wavefunction coefficients leaves the Kähler potential for charged matter fields invariant, but at the quantum level this rescaling of the fields induces an anomalous term. For low-energy physics (i.e. \( g_a(\mu) \)) to stay invariant under the combined effect of transformations \( (3) \) and \( (8) \) one finds the following transformation for the holomorphic coupling \( f_a(\phi) \)

\[
f_a(\phi) \rightarrow f_a(\phi') = f_a(\phi) - \frac{c_a}{8\pi^2} \kappa^2 J(\phi) - \sum_r \frac{T^{(r)}_a}{4\pi^2} \ln Y^{(r)}(\phi)
\]

(9)

This is a very important relation as it relates two values of the Wilsonian coupling \( f_a \) at different points in the moduli space \( [11] \).

3 \( N = 1 D = 4 \) Supergravity and RG flow

From eq.\( (2) \) evaluated at two different scales \( \mu \) and \( \mu' \) \( (\mu \leq \mu') \) one finds to all orders in perturbation theory that the effective couplings run as follows

\[
g_a^{-2}(\mu) = g_a^{-2}(\mu') + \frac{3T_a(G)}{8\pi^2} \ln \frac{\mu'}{\mu} \left( g_a^2(\mu') / g_a^2(\mu) \right)^{1/3} + \sum_r \frac{T^{(r)}_a}{8\pi^2} \ln \frac{\mu'}{\mu Z_r(\mu', \mu)}
\]

(10)

Here we have used the fact that \( Z_r(\rho, \mu) = Z_r(\rho, \mu')Z_r(\mu', \mu) \). In the above RG flow the second term on the r.h.s. is the pure gauge term (exact to all orders) while the last term is the mixed matter-gauge and matter only (Yukawa) term again exact to all orders in perturbation theory. Computing \( Z \) at one loop, eq.\( (10) \) reproduces the structure of the well-known two-loop running in the MSSM in the presence of Yukawa couplings \( [14, 15] \).

If the gauge couplings unify at some scale (in this case \( \mu' \) which also provides the cut-off) \( g_a(\mu') = g_0 \). Eq.\( (10) \) gives the RG flow in rigid supersymmetry, while we know that the effective theory close to the string scale should actually be that of supergravity. However, the phenomenological success of eq.\( (10) \) in the context of the MSSM in relating low energy values of the gauge couplings suggests any corrections to it should be small. It would therefore be good to have a deeper understanding, at the
supergravity level, of why eq. (10) is such a good approximation and of how the cut-off $\mu'$ and the bare coupling $g_0$ emerge.

In string theory the ratio of the gauge couplings is predicted. The effective (N=1, D=4) supergravity theory leads to a test of such string predictions using the RG flow. In this the gauge couplings unification condition must be imposed as a boundary condition.

The RG flow in supergravity is similar to that in eq. (10) as long as the space-time is nearly flat, but at scales of order $O(\mu')$ gravity effects become more important. At such scales the running of the couplings is given by (11) which we may present in a a form similar to (10)

$$g_a^{-2}(\mu) = Re f_a + \frac{-3T_a(G)}{8\pi^2} \ln \frac{\Lambda e^{K/2}}{\mu (e^K/g_a^2(\mu))^{1/3}} + \sum_r \frac{T_a^{(r)}}{8\pi^2} \ln \frac{\Lambda e^{K/2}}{\mu Z_r(\rho, \mu)}. \quad (11)$$

This equation is very similar to (11) in its structure, and leads to the interpretation of the “cut-off” as $\Lambda e^{K/2}$, the scale present in the second and third terms in the r.h.s. of equation (11). Comparing (11) and (10) we see that the second term on the r.h.s. of (10) is the (all orders) pure gauge contribution while the last term is the matter contribution (again to all orders in perturbation theory). The presence of the “cut-off” $\Lambda e^{K/2}$ instead of $\Lambda$ may be interpreted as a gravitational effect corresponding to the rescaling of the metric (11). The only effect of such a rescaling is to change the scale up to which the effective couplings run as compared to the holomorphic coupling running, eq. (12); the structure of the running is similar in both cases, eqs. (11), (10). Moreover, evaluating (11) at two different scales $\mu$ and $\mu'$ and subtracting, we recover eq. (10) which corresponds to flat space-time RG flow, strengthening our assertion that only the cut-off of the running and the bare coupling (i.e. the boundary conditions) change when going to the effective supergravity case. Further, since the leading contribution in $K$ is that of moduli fields which are intrinsically of string origin, eq. (11) makes manifest how string theory normalises the cut-off and the bare coupling of the effective supergravity theory through the Kähler potential of moduli fields.

The unification condition for the gauge couplings is of the following form $Re f_a = e^{-K} = g_a^{-2}(\Lambda e^{K/2})$ up to scheme dependence terms. If this condition is met, then eq. (11) is of the same form as eq. (10). Thus eq. (11) together with the gauge unification requirement provides an explanation why corrections close to the string scale preserve the successful (rigid supersymmetry) form of eq. (10).

The cut-off of the RG flow for the effective couplings plays an important role in our discussion. It must correspond to a physical threshold and thus must be invariant under the various symmetries of the theory. To clarify this we make a separation in the Kähler potential contributions

$$K = K_S + K_T \quad (12)$$

where $K_S$ is the $T$ independent part while $K_T$ changes under the symmetry transformation (10). As an example, in the $Z_3$ heterotic orbifold, $K_S$ stands for the dilaton Kähler term $\sim -\ln (S+S)$ and $K_T$ for the Kähler term of the untwisted moduli $T, U$ and their higher order (i.e. one loop and beyond in string coupling) mixing with the dilaton and other moduli. A similar structure exists for the $Z_3$ orientifold models due to an $SL(2, \mathbb{Z})$ symmetry (11). Using this separation we may write eq. (11) as

$$g_a^{-2}(\mu) = Re f_a + \sigma_a + \frac{-3T_a(G)}{8\pi^2} \ln \frac{\Lambda e^{K_S/2}}{\mu (e^{K_S/g_a^2(\mu)})^{1/3}} + \sum_r \frac{T_a^{(r)}}{8\pi^2} \ln \frac{\Lambda e^{K_S/2}}{\mu Z_r(\Lambda, \mu)}. \quad (13)$$

If we considered instead $\Lambda e^{K/2}$ as a cut-off and with $e^{K_S}$ in the denominator of the second term in the r.h.s. of (11) acting as a bare coupling ($w, v$ are arbitrary constants), the condition $b_0 w + 2T_a(G)v = c_0$ gives $b_0 (w-1) = 2T_a(G)(1-v)$ with (unique) gauge group independent solution $w = v = 1$. This motivates our interpretation of $\Lambda e^{K/2}$ as a potential “cut-off” in eq. (11).

For this see eq. (11).
where we used the notation

\[
\sigma_a = \frac{1}{16\pi^2} \left[ c_a \mathcal{K}_T - \sum_r 2T_a^{(r)} \ln Z_r(\rho, \tilde{\Lambda}) \right]
\]  

(14)

In eq. (13) \( \tilde{\Lambda} \) corresponds to the scale at which string theory gives the Kähler term for the charged matter fields in a non-canonically normalised form \( K_{\text{matter}} = Z_{\nu} \overline{Q}_r Q_r \approx \prod_{i=1}^{3} (T_i + T_i) \overline{T}_i Q_r \) where the approximation ignores higher order string corrections. We therefore identify the scale \( \tilde{\Lambda} \) with the string scale of the theory.

Any variation of \( \sigma_a \) induced by changes of \( \mathcal{K}_T \) or \( Z_r(\rho, \tilde{\Lambda}) \) under the symmetries of the string theory, must be cancelled by that of \( \text{Re} f_a \) (eq. (8)) otherwise the predictions for the low energy physics (i.e. \( g_a(\mu) \)) would not be invariant. We conclude that the quantity \( F_a \) defined by

\[
F_a \equiv \text{Re} f_a + \sigma_a
\]  

(15)

is invariant under symmetry transformations (3), (8). Implementing such symmetries in (13) and (14) to all orders in perturbation theory is a very difficult task. This is so because \( \mathcal{K}_T \) and \( Z_r(\rho, \tilde{\Lambda}) \) (present in \( \sigma_a \)) are real functions of the moduli which in general have contributions from all orders in perturbation theory in the (string) coupling and we only know their first few perturbative terms. Their expressions at the string scale are

\[
\mathcal{K}_T = \mathcal{K}_T^{(0)} + \mathcal{K}_T^{(1)} + \cdots = -\sum_{i=1}^{3} \ln(T_i + T_i) + \mathcal{K}_T^{(1)} + \cdots = -\sum_{i=1}^{3} \ln(T_i + T_i) \left( 1 + \mathcal{O}(g_s^2) + \cdots \right)
\]  

(16)

where only the tree level term is explicitly given. Higher order terms usually mix the dilaton with the rest of moduli fields. Here \( g_s^2 \) is the string coupling. Similarly

\[
Z_r = Z_r^{(0)} + Z_r^{(1)} + \cdots = \prod_{i=1}^{3} (T_i + T_i)^{n_1} + Z_r^{(1)} + \cdots = \prod_{i=1}^{3} (T_i + T_i)^{n_1} \left( 1 + \mathcal{O}(g_s^2) + \cdots \right)
\]  

(17)

where \( n_1 \) are the modular weights and again only the tree level term is given. Our lack of knowledge of the higher order string corrections in the above equations is reflected in the accuracy to which \( \sigma_a \) and the gauge couplings are calculated and this also affects the unification analysis to follow. Note that using only string tree level values for \( Z \) and \( \mathcal{K} \) only determines the gauge couplings at the string scale at one loop level.

The requirement that \( F_a \) be invariant under the symmetries of the theory strongly constrains the form of \( f_a \) and \( \sigma_a \) in specific string models, with additional implications for the unification of the couplings as we now discuss. Firstly, this requirement relates the Kähler potential for the untwisted moduli \( T_i \) (present in \( \sigma_a \)) to the real part of the holomorphic coupling. Secondly, the condition of gauge coupling unification further relates the latter to the Kähler term for the dilaton as we explicitly show in the next two sections for specific examples. This leads to the suggestion that there must be a deeper relationship in string theory between the Kähler term \( \mathcal{K}_T \) for untwisted moduli and that of the dilaton \( \mathcal{K}_S \). Invariance of \( F_a \) can also provide a consistency check for string models with the type of symmetries outlined in the previous section. Other symmetries of the Kähler potential (for example under \( S \rightarrow 1/S \)) may be implemented in a similar manner.
3.1 Gauge couplings in $Z_3$ heterotic orbifolds

The heterotic string with gauge group $SO(32)$ compactified on the orbifold $T_6/Z_3$, has gauge group $SU(12) \times SO(8) \times U(1)_A$ where $U(1)_A$ is anomalous. This anomaly is universal and is cancelled by shifts of the dilaton. A dilaton dependent Fayet Iliopoulos term is then generated and this is cancelled by v.e.v.’s of the fields $M_{\alpha \beta \gamma}$, the blowing-up modes of the orbifold which are present in the twisted sector and transform as $(1,1)_{-4}$ under the gauge group. Additional twisted states $V_{\alpha \beta \gamma}$ are present which transform as $V_{\alpha \beta \gamma} = (1,8_s)_{+2}$. These fields become massive through superpotential couplings \[19\]. For the untwisted string sector there are three families of states $Q_\alpha = (12,8_r)_{-1}$ and $\phi_\alpha = (\mathbf{66},1)_{+2}$, the dilaton $S$ and the $T_i$ moduli.

In heterotic orbifolds the “sigma-model” symmetry\[ transformation of the $T_i$ moduli is given by

$$T_i \rightarrow a_i T_i - i b_i, \quad a_i d_i - b_i c_i = 1$$

The Kähler potential of charged matter fields at the string scale has the form

$$K_{\text{matter}} = Q_i^\dagger Z_i Q_r$$

Its invariance under $SL(2,R)_{T_i}$ determines $Y(\phi)$ given the form of $Z_r$, eq.(17). At the tree level for $Z$ this implies

$$Z_r = \prod_{i=1}^{3}(T_i + \overline{T_i})^n_i + \cdots, \quad Y_r = \prod_{k=1}^{3}(d_i + ic_i T_i)^n_i$$

The string theory form of the Kähler term of the $T$ moduli is given at the tree level by

$$K_T = -\sum_{i=1}^{3}\ln(T_i + \overline{T_i}) + \cdots, \quad J = \kappa^{-2} \sum_{i=1}^{3}\ln(ic_i T_i + d_i)$$

Thus we find

$$\sigma_a = -\frac{1}{16\pi^2}\sum_{i=1}^{3} b'^i_a \ln(T_i + \overline{T_i}) ; \quad b'^i_a = c_a + 2 \sum_r n_a T^r_a$$

For the class of $Z_3$ heterotic orbifolds of interest here we have \[12\] $b'^i_a \equiv b'^i$ i.e. gauge group independent\[ implying the same for $\sigma_a$. The change of $\sigma_a$ under moduli transformation is compensated by that of $f_a$ (eq.(1)) which must therefore also contain a gauge group independent part. This is indeed the case in heterotic string theory where $f_a$ is given by the universal dilaton $f_a = S$. Therefore

$$F_a = \frac{1}{2} \left[ S + \overline{S} - \frac{1}{8\pi^2} \sum_{i=1}^{3} b'^i \ln(T_i + \overline{T_i}) \right] = f_a$$

and the dilaton is shifted and $F_a$ is $SL(2,Z)_{T_i}$ invariant. Eqs.(23), (13) together with the requirement of the existence of a unified (i.e. gauge group independent) bare coupling give $\exp(-K_S) = F_a$ in agreement with the gauge group independence of $F_a$. This gives

$$K_S = -\ln \left\{ \frac{1}{2} \left[ S + \overline{S} - \frac{1}{8\pi^2} \sum_{i=1}^{3} b'^i \ln(T_i + \overline{T_i}) \right] \right\}$$

\[Only the discrete subgroup $SL(2,Z)_{T_i}$ survives at the string level.

\[We consider only Kac-Moody level one string theory.
While this result is expected in string theory under the presence of $SL(2, R)_{T_i}$ symmetry, we find it interesting that we recovered it using field theory arguments (anomaly cancellation) and the condition of unification imposed on the RG flow (13) for the gauge couplings. The fact that string theory gives a similar ($SL(2, Z)_{T_i}$ invariant) expression for the dilaton potential provides a consistency check of our approach. Moreover, the definition of $F_a$ eq.(23) includes some dependence of the Kähler potential for $T$ moduli (21). From the invariance of the former under $SL(2, R)_{T_i}$ symmetry together with the unification condition relating $F_a$ to $K_S$ we conclude that the Kähler potentials for $T$ and $S$ are related at a deeper level in string theory where the unification of the gauge couplings is respected.

From eq.(13) we find the final form for the RG flow ($\mu \sim M_z$)

$$g_a^{-2}(\mu) = f_0 + \frac{-3T_a(G)}{8\pi^2} \ln \frac{\Lambda/\sqrt{f_0}}{\mu \left(f_0^{-1}/g_a^2(\mu)\right)^{1/3}} + \sum_r \frac{T_a^{(r)}}{8\pi^2} \ln \frac{\Lambda/\sqrt{f_0}}{\mu Z_r(\Lambda, \mu)}$$

We thus identify the unification scale $\Lambda_U$ and the RG flow cut-off in (string inspired) supergravity models as in eq.(26) below. $\Lambda$ is taken equal to $M_P$ as the natural, moduli independent cut-off of RG flow of the holomorphic coupling in (3) [12] so

$$\Lambda_U = \frac{\Lambda}{\sqrt{f_0}} \approx \frac{M_P}{\left(S + \sum_{i=1}^{3} \epsilon_i T_i \right)^{1/2}}$$

where the approximation sign stands for additional numerical factors depending on the regularisation scheme [20]. This value of the unification scale, the heterotic string scale, is slightly different from that usually quoted $M_H = M_P/(S+S)^{1/2}$ [12]. This difference is due to the mixing at one loop level between the dilaton and $T$ moduli which we considered, so we see eq.(26) is the one-loop improved heterotic string scale. The numerical effect of the presence of the $T$ dependence in this definition is small, since $ReT = O(1)$ in weakly coupled heterotic strings. Our definition of $\Lambda$ as the string scale makes the whole unification picture at the string scale self-consistent since if $\Lambda = \Lambda_U$, then $Z_r(\Lambda, \mu = \Lambda_U) = 1$ with the unified coupling $g_a^{-2}(\mu = \Lambda_U) = f_0$ and eq.(25) is respected. Note that both the string scale and the bare coupling are manifestly $SL(2, Z)_{T_i}$ invariant, as one would expect in a theory with such a symmetry. Since $ReT \sim O(1)$ we can expand the unified coupling $f_0$

$$f_0 \approx Re(S + \sum_{i=1}^{3} \epsilon_i T_i)$$

which recovers a result of heterotic string theory for the structure of the one loop holomorphic coupling. If we ignore the approximations made in (20) and in (21) for $K_T$, equations (25) and (26) are perturbatively exact. More explicitly, eq.(25) uses an input from string theory, the boundary value of $f_0$ for the RG flow below the unification scale, and $f_0$ is known to one loop order only, however below this scale eq.(25) is perturbatively exact. This concludes our examination of the RG flow for the $Z_3$ heterotic orbifold model. The method developed here applies generally to the class of heterotic string models without a $N=2$ sector.

Note that eq.(24) justifies the use of the flat space-time RG flow, being equivalent to all orders in perturbation theory below the unification scale to eq.(10). Moreover it determines the cut-off scale and the bare coupling in terms of the fundamental string quantities.
3.2 Gauge couplings in $Z_3$ orientifolds

The models based on $Z_3$ orientifolds are similar to those based on the heterotic counterpart, the $Z_3$ orbifold. The gauge group is again $SU(12) \times SO(8) \times U(1)_A$ with the following structure of the spectrum. The closed string sector contains 27 twisted moduli $M_{\alpha \beta \gamma}$ corresponding to the blowing up modes of the associated orbifold and with their linear symmetric combination labelled by $M$. The closed string sector also includes the untwisted moduli $T_i$ and the dilaton $S$. The open string sector has three families of states $Q_a = (12, S_a)_{-1}$, $A_a = (66,1)_{+2}$ due to strings stretching between the 9-branes. The $U(1)_A$ is again anomalous and the anomaly is this time non-universal and cancelled by shifting the twisted pseudoscalar axions which are in the same chiral multiplets $M$ as the scalars corresponding to the blowing up modes of the orientifold. A combination of the twisted states with the anomalous vector superfield forms a heavy vector multiplet, which after decoupling at a high scale\(^9\) “leaves” a $SU(12) \times SO(8)$ gauge group, just as in the $Z_3$ orbifold model. We see that below this scale there is a match of the spectrum and of the gauge groups of $Z_3$ and $Z_3$ orientifolds, although some of our results could apply to more general cases. Since in the heterotic case such anomalies are cancelled, the proposed orientifold-orientifold duality implies that these anomalies are also cancelled \(^1\) for $Z_3$ and $Z_7$ orientifolds. The duality symmetry which prompted the study of the cancellation of these anomalies in type IIB orientifolds was investigated beyond tree level in \(^3\). However, this analysis was based on the linear - chiral multiplet transformation \(^3\) which was assumed to hold at one loop level and this was proved using only the tree-level string scale in the linear basis for the gauge couplings eq.(1), rather than its one-loop improved value. This tree level definition of the string scale is not invariant under the symmetry transformation of $T_i$’s and from eq.(1) this means that low energy physics is not invariant either. In the heterotic case the string scale changes at one loop\(^9\), giving an invariant form \(^26\). We expect something similar should apply to the orientifold case as well. We therefore conclude that the linear-chiral multiplet duality relation may prove to be more complicated than assumed and that the two models $Z_3$ orbifold/orientifold could still be dual to each other. We do not make explicit use of this duality, but we will later discuss its compatibility with our results in the effective field theory approach where all states considered are in the chiral basis. From this (“string based”) effective field theory point of view it is useful to investigate the phenomenological consequences for the running couplings and discuss the unification of the gauge couplings in the presence of the $SL(2, Z)_{T_i}$ symmetry transformation \(^17\) of the $T_i$ moduli, eq.(18).

As in the heterotic case we take as input from the $Z_3$ orientifold string theory the Kähler terms for the untwisted moduli $T$

$$K_T = -\kappa^2 \hat{\kappa}^{-2} \sum_{i=1}^{3} \ln(T_i + \overline{T}_i) + \cdots$$

(28)

where $\hat{\kappa}^{-2}$ is the coefficient in front of the Kähler potential as given after compactification in string theory for $Z_3$ orientifold while $\kappa^2$ is due to definition \(^3\). In this class of orientifolds we also have a Kähler term for the charged matter fields of the form \(^17\) with \(^6\), \(^8\) respected. Under the combined

\(^9\)This is of the order of the string scale, just as in the case of the heterotic orbifold \(^3\).

\(^1\)For this see previous section.

\(^11\)Note that transformation \(^18\) is not a $T$ duality transformation in type I vacua. The latter exchanges different type of D branes, the three $T_i$’s and the dilaton \(^20\). Clearly, transformation \(^18\) is not of this type. Further, in the orientifold model we examine with the proposed symmetry only D9 branes are present.
Finally, we note that ReM can be shifted by a universal (gauge group independent) term as well compensated by a gauge group dependent part and, if unification exists, a gauge group independent part as well. Therefore, we have no counterpart of the states SU function of the associated gauge group.

Compatibility with the string calculation for \( b^i_a \) for the Z_3 orientifold requires that \( \epsilon \equiv \kappa^2 \tilde{\kappa}^{-2} \) be equal to 1. Unlike the case of the Z_3 heterotic orbifold, \( b^i_a \) in the Z_3 orientifold is gauge group dependent due to the different spectrum content. We have \( b^i_a = b_a / 3 \) where \( b_a \) is the one loop beta function of the associated gauge group SU(12) × SO(8). This is because in the Z_3 orientifold there is no counterpart of the states \( V_{\alpha\beta\gamma} \) of the Z_3 heterotic orbifold.

The requirement that \( F^a \), eq. (13) be invariant requires that \( f_a \) have indeed a non-zero gauge group dependent part and, if unification exists, a gauge group independent part as well. Therefore \( f_a = S + \sum_k s_{ak} M_k \) which establishes the structure of the holomorphic coupling for this model. For the gauge group dependent part the presence in \( f_a \) of the fields \( M_k \) is required on pure field theory grounds to cancel the gauge group dependent variation of \( \sigma_a \) under \( SL(2, Z) \) (anomaly cancellation). Further, we know from the string theory that the fields \( M_k \) are indeed singlets under the gauge group of the orientifold. The gauge independent part of \( f_a \) cannot be proportional to moduli other than \( S \) like for example \( T_i \)'s because the latter would transform non-linearly under (18) and invariance of \( F^a \) (15) would not be respected. Further, we may consider that \( S \) (identified as the dilaton) is not shifted under transformation (18). These results are indeed in agreement with type II B Z_N models, \( N = odd \) where the holomorphic coupling \( f_a \) in the presence of 9-branes only is given by

\[
f_a = S + \sum_{k=1}^{(N-1)/2} s_{ak} M_k
\]

with \( M_k \) twisted moduli (chiral basis). For Z_3 orientifold there is only one field \( M_k \equiv M \). From now on, whenever possible we will consider the more general case of Z_N orientifold and in the results for the Z_3 case we drop the index \( k \). We then find from (15), (13) that the twisted moduli transform according to

\[
M_k \rightarrow M'_k = M_k - \frac{1}{8\pi^2} \sum_{i=1}^{3} \delta^i_k \ln(d_i + ic_i T_i)
\]

where coefficients \( s_{ak} \) satisfy the relationship

\[
\sum_k s_{ak} \delta^i_k = b^i_a
\]

The result (31) recovers the initial string-based suggestion of (11) that anomalies can be cancelled by the transformation of the twisted moduli (31) for Z_3 and Z_7 orientifolds. However, this is only true if \( \epsilon = 1 \) when condition (21) is identical to that proposed in string theory (1). This therefore fixes the coefficient in front of the Kähler term for the moduli, eq. (28) for the agreement of field-string theory to hold, and the situation is then very similar to that of the heterotic string. However eq.(31) assumes that the dilaton is inert under \( SL(2, Z) T_i \) symmetry and plays no role in anomaly cancellation. In principle the dilaton could be shifted by a universal (gauge group independent) term as well compensated by an opposite shifting of the twisted moduli part of \( f_a \), but this is not allowed in string theory (6).  

Finally, we note that \( ReM_k \) cannot represent a vacuum state invariant under \( SL(2, Z) T_i \) and cannot be

\footnote{This is an input from string theory (1). See later for discussion on this point.}
set to zero as in such a case anomaly cancellation and invariance of $F_a$ eq.(13) would not be respected, and low energy physics would not be invariant under the transformation of $T_i$ eq.(18).

For the $Z_3$ orientifold with a single field $M$ present, eq.(13) shows that $s_{ak} \equiv s_a \times b_k^i = b_a/3$ on pure anomaly cancellation grounds (also $\delta_i = 6$). The proportionality of $s_a$ coefficient to the one loop beta function is again in agreement with explicit string calculations of these coefficients \[9, 1\]. We would like to stress that unlike the string calculation \[9\], this proportionality is here a consequence of imposing the anomaly cancellation under the conjectured $SL(2, Z)_{T_i}$ symmetry. This provides a check for our approach, circumstantial evidence for the presence of this symmetry at string level and further motivation for studying its implications.

We can now proceed to investigate the RG flow for the $Z_3$ orientifold and the unification of the gauge couplings. Their values at the string scale $\Lambda$ are given by (using (13), (16), (17), (30), (32))

$$g_a^{-2}(\Lambda) = Re f_a + \frac{3T_a(G)}{8\pi^2} \ln \frac{\Lambda e^{K/2}}{\Lambda \left(e^K/g_a^2(\Lambda)\right)^{1/3}} + \sum_r \frac{T_a^{(r)}}{8\pi^2} \ln \frac{\Lambda e^{K/2}}{\Lambda Z_r(\rho, \Lambda)}$$

$$g_a^{-2}(\Lambda) = Re S + \frac{3T_a(G)}{8\pi^2} \ln \frac{\Lambda e^{K/2}}{\Lambda \left(e^K/g_a^2(\Lambda)\right)^{1/3}} + \sum_r \frac{T_a^{(r)}}{8\pi^2} \ln \frac{\Lambda e^{K/2}}{\Lambda Z_{string}} + \frac{c_a}{16\pi^2} \left[\mathcal{K}^{(1)} + \cdots\right]$$

(33)

where we used the notation

$$Z_{string} = \{1 + \mathcal{O}(g_a^2)\}, \quad G = \frac{2\pi^2}{9} \left\{ M + M - \frac{1}{8\pi^2} \sum_{i=1}^3 \delta_i \ln(T_i + T_i) \right\}$$

(34)

while the effective couplings at low energy scales ($\mu \sim M_z$) are given by

$$g_a^{-2}(\mu) = g_a^{-2}(\bar{\Lambda}) + \frac{3T_a(G)}{8\pi^2} \ln \frac{\bar{\Lambda}}{\mu \left(g_a^2(\bar{\Lambda})/g_a^2(\mu)\right)^{1/3}} + \sum_r \frac{T_a^{(r)}}{8\pi^2} \ln \frac{\bar{\Lambda}}{\mu Z_r(\bar{\Lambda}, \mu)}$$

(35)

The low energy physics represented by $g_a^2(\mu \sim M_z)$ is indeed invariant under $SL(2, Z)_{T_i}$ transformation since $G$ itself is invariant. This fact can be seen explicitly by adding eqs.(13) and (36) and ignoring the extra terms due to higher order (one-loop) string corrections to $Z$ and $K_T$ at/above the string scale (denoted in (13) by $Z_{string}$ and $K_T^{(1)} \sim \mathcal{O}(g_a^2)$ respectively). Such extra terms would affect some of the two loop corrections (of string origin!) for the gauge couplings. Even though the string scale $\bar{\Lambda}$ and consequently $g_a(\bar{\Lambda})$ are not invariant under an $SL(2, Z)_{T_i}$ symmetry, the low energy physics ($g_a(\mu), \mu \sim M_z$) is still invariant (in this approximation), as should be the case. To see explicitly if this true beyond this approximation we would need an explicit string calculation of $Z_{string}$ and $K_T^{(1)}$.

If the couplings unify beyond one-loop level one sees from eq.(14) that $\mathcal{K}_S = -\ln(S + S')$. This identification recovers the structure for the Kähler potential given by a string calculation. In this case the couplings unify at the scale $\Lambda'$ where

$$\Lambda' = \Lambda e^{K/2} e^G$$

(37)

Up to the terms $\mathcal{O}(g_a^2)$ the unified coupling is “fixed” by the dilaton alone, $g^{-2}(\Lambda') = Re S$ rather than in combination with the twisted moduli. This is supported by the fact that (unlike $M_k$) $S$ is not involved in anomaly cancellation and is invariant under the $SL(2, Z)_{T_i}$.

\[13\]One-loop order (and beyond) string-induced corrections to $Z$ and $K_T$, normalised to their tree level values, induce two-loop-like (and beyond) terms in the RG flow for the effective gauge couplings, due to physics at/above the string scale. These corrections have the structure similar to but distinct from that of field theory two loop terms.
3.2.1 The unification scale and the linear-chiral multiplet relation

The result (37) for the unification scale certainly requires some discussion. From the supergravity point of view it has been argued that the scale Λ entering the definition of Λ' eq.(37), should be identified with the Planck mass, $M_P$. In the heterotic string this leads to the conclusion that the couplings unify at the (one-loop improved) heterotic string scale given by eq.(26). For the effective supergravity RG flow eqs.(33),(34) applied to the $Z_3$ orientifold model the important point is that Λ does not bring any moduli dependence in definition (37).

If the unification scale is larger than the string scale, $\Lambda' > \Lambda$, the structure of eq.(34) mimics the field theory running of the gauge couplings above $\Lambda$. This is not RG flow in the field theory sense because the radiative corrections are of string origin (moduli contributions) which do not have a field theory correspondence. The effect of these moduli contributions is to renormalise the wavefunctions of the gauge sector (through the presence of $\exp(K_S/2)$ on the r.h.s. of (34)) and those of the matter sector (through the presence of $Z_{string} \equiv \{1 + O(g_s^2)}\}$ on the r.h.s. of (34)). This means that there may be a stage of “mirage unification”\(^{14}\) at $\Lambda'$ induced by string effects only. It is possible however that $\Lambda' = \Lambda$ and unification actually takes place at the string scale $\Lambda$. To distinguish between these two cases of unification one needs to know the exact value of $G$.

The value of $G$ can be fixed by the Fayet Iliopoulos mechanism\(^{15}\) in the following way. In general the D term in the Lagrangian contains in addition to the Fayet Iliopoulos term, proportional to $G$, a contribution from fields charged under the anomalous $U(1)_A$. However at least for $Z_3$ orientifold model we considered, fields charged under $U(1)_A$ are also charged under the non-Abelian group. A non-vanishing v.e.v. of these fields would therefore trigger a breaking of the non-Abelian symmetry group factors. In the following we will discuss two possibilities corresponding to whether we insist on the presence or absence of the full non-Abelian gauge symmetry of the model.

If we insist that the full non-Abelian symmetry be unbroken, then the D term in the Lagrangian depends only on the contribution given by $G$. This leads to the conclusion that the vanishing of the D term contribution in the Lagrangian (to preserve supersymmetry) implies the vanishing of $G$ as well\(^{16}\) in the orientifold limit. To illustrate this case\(^{16}\) consider the twisted moduli Kähler potential $K(M_k, \overline{M}_k)$ which must be invariant under the $SL(2,Z)_{T_i}$ symmetry and the anomalous $U(1)_A$, conditions which lead to the following change of its original form

$$K_M(M_k, \overline{M}_k) \equiv K_M(M_k + \overline{M}_k) \rightarrow K_M(M_k + \overline{M}_k - \delta_k V_A - \frac{1}{8\pi} \sum_{i=1}^3 \delta_k^i \ln(T_i + \overline{T}_i)) \quad (38)$$

This contribution is not present in the running couplings (eq.(33)) because it is considered a higher order contribution in the moduli. However it does control the value of $G_k$ ($G$ for $Z_3$) through the Fayet Iliopoulos mechanism. This requires, in the case of unbroken non-Abelian symmetry of $Z_N$ orientifold

$$\sum_{k=1}^{(N-1)/2} \delta_k \frac{\partial K_M}{\partial M_k} \bigg|_{V=0, \theta=\overline{\theta}=0} = 0 \quad (39)$$

and therefore a sufficient (and necessary for $Z_3$ orientifold) condition is that

$$\frac{\partial K_M}{\partial M_k} \bigg|_{V=0, \theta=\overline{\theta}=0} = 0 \quad (40)$$

14 The origin of these corrections is in eq.(17).
15 For more on Fayet Iliopoulos mechanism in type IIB orientifolds see\(^{23}\).\(^{1}\)
16 In the following we will consider the more general case of $Z_N$ orientifolds, N odd. For $Z_3$ orientifold one must suppress the indices $k$.\(^{1}\)
As was shown in [3] this corresponds (for nonsingular potential) to the condition

$$G_k = \frac{2\pi^2}{9} \left\{ M_k + \overline{M}_k - \frac{1}{8\pi^2} \sum_{i=1}^{3} \delta^k_i \ln(T_i + \overline{T}_i) \right\} = 0$$

(41)

For the case of the $Z_3$ orientifold ($G_k \equiv \mathcal{G}$) we thus find that the unification scale $\Lambda' = \Lambda e^{K_{S}/2}$. However, it is possible that $K_M$ has additional $SL(2,Z)_{T_i}$ invariant contributions to its argument, eq.(38), leading to non-zero value for $G_k$ ($\mathcal{G}$ for $Z_3$) (and which may give a large value for $\Lambda'$). Moreover, the result of eq.(11) for $G_k$ ($\mathcal{G}$) is questionable for two additional reasons.

Apparently, an unification scale of value $\Lambda e^{K_{S}/2} = M_P/(S + \overline{S})^{1/2}$ would be very encouraging from a phenomenological point of view and would preserve some similarities with the heterotic case. In fact an explicit string calculation [3] has shown that in orientifold models the contribution of the $N=2$ sector indeed extends beyond the string scale up to the winding mode scale, which could be close to the Planck scale. However in the present case there does not appear to be a physical threshold associated with this, so this possibility appears implausible.

Secondly, the vanishing of $G_k$ is not compatible with the linear-chiral multiplet duality relation [4] which relates eqs.(1), (33)

$$2m_k = M_k + \overline{M}_k - \frac{1}{8\pi^2} \sum_{i=1}^{3} \delta^k_i \ln(T_i + \overline{T}_i) + \frac{b_k}{8\pi^2} \ln \left[ \prod_{i=1}^{3} \frac{ReT_i}{ReS} \right]^{1/2}$$

(42)

with $b_a = \sum_k b_k s_{ak}$, $(b_k \equiv b = 18$ for $Z_3$). The origin of this disagreement was highlighted in the Introduction, and is due to the $T_i$ dependence of the tree level string scale in $Z_N$ orientifold models, $M_T^2 = 2M_P^2(S + \overline{S})^{-1/2} \prod_{i=1}^{3} (T_i + \overline{T}_i)^{-1/2}$. This extra $T_i$ dependence brought in solely by the string scale definition is manifest in the linear-chiral multiplet relation, the last term on the r.h.s. of (42).

The $T_i$ dependence of this term is not present in the linear basis of the Lagrangian leading to $G_k = 0$ in ref.[3] in the orientifold limit, $m_k \to 0$. One could attempt to preserve the multiplet duality eq.(12), and for this the Lagrangian of linear multiplets of [3] should contain an additional term [6], given by

$$\Delta L_2 = \frac{1}{16\pi^2} \sum_{k=1}^{(N-1)/2} b_k \hat{L}_k \ln \left[ \hat{L}_k \prod_{i=1}^{3} (T_i + \overline{T}_i) \right]$$

(43)

in the notation of [1] with $\hat{L}_k$ standing for the linear multiplet basis. This brings in the third term in the rhs of (42) and leads to $G_k \neq 0$ in the orientifold limit, $m_k \to 0$. However, the value of $G_k$ in this limit as given by (12) is not $SL(2,Z)_{T_i}$ invariant, contrary to what we already established, eq.(33). This is because the last term in (12) is not invariant and ruins the anomaly cancellation of the model as observed in [3]. All these difficulties are caused by the fact that the tree level definition of the string scale, $M_T$ responsible for the last term on the rhs of (12) is not $SL(2,Z)_{T_i}$ invariant. As we observed after eq.(1), for the same reason $g_\mu(\mu \sim M_z)$ in (1) is not invariant under $SL(2,Z)_{T_i}$.

It is possible that the tree level string scale $M_T$ is made $SL(2,Z)_{T_i}$ invariant by one loop corrections due to the moduli, similarly to the heterotic case, eq.(23); this should actually be the case since this scale is a physical one and must be invariant if the symmetry $SL(2,Z)_{T_i}$ is present. In this case the invariance of $g_\mu(\mu \sim M_z)$ in (1) is assured, if we replace $M_T$ by $\hat{\Lambda}$, where the latter is assumed invariant. As $S$ does not play any role in anomaly cancellation in $Z_N$ orientifold, and is therefore fixed (to all orders) this means that the $T_i$ dependence of the one loop improved value of the string scale must be $SL(2,Z)_{T_i}$ invariant. One possibility is that this value is given by

$$\hat{\Lambda} = M_T \left[ \gamma(T_1, T_2, T_3) \right]^{-1/2}$$

(44)
where $\gamma(T_1, T_2, T_3)$ is such as to keep $\tilde{\Lambda}$ invariant under the transformation of $T_i$. An example is $\gamma(T_1, T_2, T_3) = \prod_{i=1}^{3} |\eta(iT)|^2$. However, one needs a confirmation of this, based on a string calculation of the radiative corrections to eq.(17). The new linear-chiral multiplet relation relating (3) to (1) (with $M_f$ replaced by $\tilde{\Lambda}$ to keep $\eta_{a}(\mu\sim M_z)$ $SL(2, Z)_{T_I}$ invariant) is in this case similar to (42), but with an additional factor $\gamma(T_1, T_2, T_3)$ under the last log in (42). As the linear-chiral relation in the orientifold limit (eq. (I) with $M_f \to \tilde{\Lambda}$, $m_k \to 0$) essentially imposes unification at the “new” string scale $\tilde{\Lambda}$, $\mathcal{G}$ will be such that $\Lambda' = \tilde{\Lambda}$, and there is no “mirage” unification. We conclude with the remark that in the absence of additional string corrections to the tree level string scale $M_f$ in eq. (I), of the nature discussed above, the $SL(2, Z)_{T_1}$ symmetry seems implausible, casting doubts on the $Z_3$ orbifold/orientifold duality.

So far we have considered that the full non-Abelian gauge symmetry of the $Z_3$ orientifold model was preserved. However it is possible that this symmetry will be broken if matter fields charged under the $U(1)_A$ symmetry, also charged under the non-Abelian gauge group are present and develop a v.e.v. This is indeed possible for the $Z_3$ orientifold model considered. In this situation the D term in the Lagrangian contains additional contributions from the charged matter fields. The condition of preserving supersymmetry (the vanishing of the D term) will then lead to a non-vanishing value for $\mathcal{G}$ different from the case discussed above, due to the additional v.e.v. contributions. These contributions lead to the “mirage unification” scenario in the sense that $\Lambda'$ may be situated above the string scale $\tilde{\Lambda}$. The “mirage” unification would not mean an effective “running” of the couplings above the string scale, but simply a change from $\tilde{\Lambda}$ to a possibly higher scale due to the presence of the v.e.v. of fields charged under $U(1)_A$ and the initial non-Abelian group.

To conclude, the most likely possibility we envisage is that, if $SL(2, Z)_{T_1}$ symmetry is indeed present, the couplings unify at the string scale ($\tilde{\Lambda}$), if the initial non-Abelian group is unbroken, but the linear-chiral multiplet relation must be changed. It is also possible that the gauge group is broken to a non-Abelian subgroup so $\mathcal{G}$ receives additional corrections due to the v.e.v. of the fields involved in the gauge symmetry breaking. In this case it is possible that $\Lambda'$ be situated above the string scale due to the v.e.v. which modify the value of $\mathcal{G}$ from the previous case. (Of course, an alternative possibility is that the symmetry $SL(2, Z)_{T_1}$ is not present at all for the $Z_3$ orientifold.)

4 Conclusions

In this paper we investigated the RG flow in two string inspired models based on the $Z_3$ orbifold and orientifold respectively. The RG flow in Supergravity proves to be a strong tool for exploring various aspects of these models. In $Z_3$ orbifold based models we have shown that RG flow together with the requirement of gauge coupling unification provides information on the structure of the Kähler potential for the dilaton and the values of the unified coupling/unification scale. In the case of $Z_3$ orientifold model we have shown that anomaly cancellation requires the presence of additional fields $M$ to cancel the anomalies induced by the $SL(2, Z)_{T_1}$ symmetry. Moreover, their contribution to the gauge couplings comes with a coefficient proportional to the one loop beta function, just as is found in string calculations. Finally, the invariance of the dilaton under the proposed symmetry and the presence of unification suggests that the dilaton can act as the bare coupling on its own (rather than in linear combination with the twisted moduli), at a scale of order $\Lambda/(S + \mathcal{G})^{1/2} e^{\mathcal{G}}$. The exact value of this scale depends on the value of $\mathcal{G}$ leading to two possibilities. The first of these preserves unification

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17This may be difficult to check. Unlike the heterotic case where the sigma model symmetry has an underlying string equivalent (T duality), it is not clear if this symmetry holds exactly in type I string [4]. The heterotic-type I duality in 10D would suggest this symmetry does not survive in type I perturbation theory and therefore the appearance of the term $\ln \eta(iT)$ in [4] would be of non-perturbative origin.
at the string scale, but the linear-chiral multiplet duality must be changed. The second possibility may realise a “mirage” unification due to v.e.v. of charged matter fields breaking the initial group to a non-Abelian subgroup. These v.e.v.’s bring additional corrections to $\mathcal{G}$ from the previous case, and thus $\Lambda'$ may be larger than $\tilde{\Lambda}$.

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