Harmonic detection algorithm based on Nuttall double-window all-phase Fast Fourier Transform trispectral interpolation

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Abstract. In harmonic detection of power grid, spectrum leakage of traditional FFT algorithm affects the accuracy of detection results. In this paper, a Nuttall double-window all-phase FFT trispectral interpolation algorithm is proposed. The phase of the spectral line with the maximum amplitude at the sampling point is taken as the initial phase of the signal, and the position of the main harmonic line is determined by using the amplitudes of the three spectral lines near the harmonic peak frequency point. Simulation results show that this algorithm can reduce the error of harmonic parameter detection more effectively and reduce the influence of white noise.

1. Introduction

At present, Fast Fourier Transform (FFT) is widely used in harmonic detection of power grid. However, spectrum leakage reduces the accuracy of harmonic detection results and cannot meet the requirements of harmonic detection. Dongmei LIU[1] obtained a window function with better sidelobe characteristics through convolution, but convolution also increased the width of the main lobe of the window and increased the loss of frequency resolution. Xuming WANG[2] used more spectral line correction to improve the accuracy and greatly improve the algorithm complexity, which affected the real-time performance of harmonic analysis of power grid.

On the basis of traditional window-enamel FFT algorithms, Xiaohong HUANG[3] first proposed the concept of all-phase Fast Fourier Transform (apFFT) algorithm, whose phase detection accuracy is higher than FFT. Qian CHEN[4] proposed a combinational optimization algorithm combining quad-spectral interpolation and apFFT, which used the improved FFT algorithm to calculate the amplitude and frequency, and the all-phase FFT algorithm to calculate the phase. Hongbo ZHANG[5] proposed a double-window apFFT trispectral line correction algorithm, and extended the ratio correction method of FFT to the apFFT algorithm, achieving better harmonic detection accuracy. Wenyu ZHANG[6] applied the trispectral interpolation algorithm to all-phase FFT, which has higher accuracy than the traditional FFT trispectral interpolation.

In this paper, we use the interpolation correction of the three spectral lines closest to the peak frequency point of apFFT, combined with the phase invariance of apFFT, and verify the accuracy of the algorithm under the influence of complex harmonics, interharmonics and white noise by comparison of simulation examples.
2. Nuttall window all-phase Fast Fourier Transform trispectral interpolation algorithm

2.1. All phase FFT algorithm principle

According to the nature of the constant system, the algorithm flow of the \( \text{apFFT} \) is shown in Figure 1.

![Figure 1. apFFT algorithm process.](image)

In Figure 1, the convolution window \( w_c \) is obtained by the front window \( f(n) \) and the flipped rear window \( b(n) \) is obtained, i.e.

\[
\text{if } n \in [-N+1, N-1] 
\]

(1)

If the front window \( f \) and the rear window \( b \) are the same and are symmetric Windows, the discrete Fourier transform corresponding to the convolution window can be obtained by the time-domain convolution theorem.

\[
W_c(\omega) = F(\omega) \cdot B(\omega) = |W(\omega)|^2 
\]

(2)

Where, \( |W(\omega)|^2 \) is the amplitude frequency domain expression of the window function.

According to Figure 1, \( \text{apFFT} \) discrete spectrum is obtained by weighted summation of shifted sequence by rear window \( b(n) \) on the basis of FFT processing by front window \( f(n) \). Then, the Fourier transform of \( \text{apFFT} \) algorithm is:

\[
Y(k\Delta f) = A \left( W \left( 2\pi \frac{k-k_0}{N} \right) \right)^2 e^{jk\phi} 
\]

(3)

By comparing Equations (2) and (3), it can be seen that the phase of FFT algorithm is affected by its frequency offset, while the phase value of all-phase FFT algorithm is not. Therefore, the phase value calculated by all-phase FFT does not need to be modified to obtain high accuracy.

2.2. Properties of the Nuttall window function

Nuttall window with good sidelobe performance is a cosine combination window whose time domain expression is:

\[
w(n) = \sum_{m=0}^{M-1} (-1)^m b_m \cos(2\pi m \cdot n / N) 
\]

(4)

In Formula (4), the amplitude-frequency function obtained by Fourier transform is:

\[
W(k) = e^{-jk\phi} \sin(\pi k) \left[ \sum_{m=0}^{M-1} (-1)^m b_m \cdot \frac{\sin(2\pi k / N)}{2 \sin(\pi(k-m)/N) \sin(\pi(k+m)/N)} \right] 
\]

(5)
When $N$ is much greater than 1, the amplitude-frequency function of the window function can be simplified to:

$$W(k) = \frac{Nk}{\pi} \sin(k\pi)e^{-i\pi\left[\sum_{m=0}^{M-1} (-1)^m \frac{b_m}{k^2 - m^2}\right]}$$

(6)

2.3. Three-spectral line interpolation algorithm

Suppose that a single sinusoidal frequency signal with frequency $f_0$, amplitude $A$ and initial phase $\varphi_0$ is sampled uniformly at sampling frequency $f_s$, and the discrete time signal obtained is:

$$x(n) = A \sin(2\pi \frac{f_0}{f_s} n + \varphi_0)$$

(7)

Where, $n = 0, 1, 2 \cdots N - 1, N$ is the sampling point.

The maximum value of the amplitude obtained near the peak frequency $k_0$ is $k$, the left line is $k - 1$, the right line is $k + 1$, remember $\alpha = k_0 - k$, there is $0.5 < \alpha < 0.5$, accurately seek $\alpha$ is solved key.

Remember: $y_1 = |X(k-1)|, y_2 = |X(k)|, y_3 = |X(k+1)|$.

In this case, substitute $\beta = \frac{y_3 - y_1}{y_2}$ into equation (3) to obtain:

$$\beta = \frac{W(1-\alpha)^2 - W(-1-\alpha)^2}{W(-\alpha)^2}$$

(8)

When $N$ is large, Equation (8) can be simply written as $\beta = t(\alpha)$, and its inverse function is $\alpha = t^{-1}(\beta)$. In Matlab, ployfit function $\alpha = t^{-1}(\beta)$ is used for polynomial approximation, and the fitting polynomial of $\alpha$ can be obtained, and then the modified formula of harmonic and interharmonic frequencies can be obtained:

$$f_i = k\Delta f = (k + \alpha)\Delta f$$

(9)

Brought into the formula (3), a correction formula of the amplitude is obtained:

$$A = \frac{2(y_1 + 2y_2 + y_3)}{W(-1-\alpha)^2 + 2W(-\alpha)^2 + W(1-\alpha)^2}$$

(10)

When $N$ is large, the above polynomial approximation method of fitting function is also adopted, and Equation (10) can be further simplified as:

$$A = \left(N^2\right)^{-1}(y_1 + 2y_2 + y_3)v(\alpha)$$

(11)

Due to the phase invariance of apFFT, the phase value of the main line of apFFT can be directly taken as the initial phase of the signal, and more accurate measurement results can be obtained without correction.

The overall flow chart of the algorithm in this paper is shown in Figure 2:
3. Typical signal model and simulation analysis under noise interference

3.1. Simulation analysis of complex harmonic and interharmonic signals

The harmonic and interharmonic signal models given in literature[7] are adopted:

$$x(n) = \sum_{m=1}^{9} A_m \sin \left( 2\pi \frac{f_m}{f_s} n + \phi_m \right)$$  \hspace{1cm} (12)

Where: Sampling frequency $f_s$=3000Hz, actual fundamental frequency 50.1Hz, sampling number $N$ =1024, which belongs to asynchronous sampling. The amplitude $A_m$ and phase $\phi_m$ of each harmonics and interharmonics are shown in Table 1:

| Harmonic and interharmonic times | Amplitude $A_m$/V | Phase $\phi_m$/° | Harmonic and interharmonic times | Amplitude $A_m$/V | Phase $\phi_m$/° |
|---------------------------------|------------------|-----------------|---------------------------------|------------------|-----------------|
| 0.5                             | 3.2              | 20              |                                 | 5                | 16.4            | 100              |
| 1                               | 380              | 10              | 6.4                             | 1.8              | 120             |
| 3                               | 20               | 30              | 7                               | 12               | 150             |
| 3.6                             | 2.8              | 25              | 7.6                             | 1.4              | 180             |

Nuttall window trispectral FFT interpolation algorithm in literature[8] and Hanning double-window apFFT trispectral interpolation algorithm in literature[9] are used for simulation comparison with the algorithm in this paper. Figure 3-Figure 4 shows the simulation results.
As can be seen from the above simulation comparison results, the detection accuracy of amplitude and frequency of this method is improved by 1~3 orders of magnitude on average compared with the other two algorithms. Therefore, the algorithm proposed in this paper has good comprehensive performance and can obviously improve the precision of complex harmonics and interharmonics parameter detection.

### 3.2. Simulation analysis of signal containing white noise

The signal model shown in Formula (12) is selected, and the SNR changes within 20dB~100dB with a step size of 10dB, which is compared with the other two algorithms. The relative errors of fundamental wave amplitude and frequency calculated by different algorithms after adding white noise are shown in Figure 5 and Figure 6.

As can be seen from Figure 4 and 5, this algorithm can effectively reduce the effect of Gaussia white noise on amplitude and phase detection, and have a certain advantage of other algorithms in anti-white noise interference.

### 4. Conclusion

Aiming at the shortcomings of windowed interpolation FFT harmonic detection algorithm, this paper proposes a double-window apFFT trispectral interpolation algorithm. In order to improve the accuracy of harmonic detection, the frequency and amplitude of the sampled signal are interpolated by three spectral lines near the peak frequency point, and the phase invariance of apFFT is used to suppress spectrum leakage. The simulation results show that this method has high application value in complex harmonic and interharmonic detection and overcoming white noise pollution, and can provide more accurate and reliable harmonic parameters and information for power harmonic control and compensation. With the large increase of nonlinear load in power system, especially the introduction of high proportion of renewable energy and high proportion of power electronic equipment ("double high" system), a large number of harmonics are generated in power system, which leads to the deterioration
of power quality. Therefore, how to correctly and effectively evaluate the harmonic problem generated after the new energy access system is worth the focus of scholars.

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References
[1] Dongmei L, Chongliang Y, Yan Z, Hengrong M. (2018) Electricity harmonic analysis method based on Nuttall self-convolution window four-spectrum-line interpolation FFT. J. Journal of Hefei University of Technology (Natural Science), 41(01): 12-17.
[2] Xuming W, Kun T, Kejun L, Xiangming W, Xi Y. (2020) FFT harmonic analysis method based on Blackman window six-spectrum-line interpolation. J. Laboratory research and exploration, 39(06): 22-26.
[3] Xiaohong H, Zhaohua W. (2007) A novel spectral estimation method for reducing leakage. J. The signal processing, (01): 144-147.
[4] Qian C, Weiqing W, Haiyun W, Jiajun Z. (2020) Harmonic detection combined optimization algorithm based on self-convolution window four-spectrum-line interpolation and improved all phase. J. Electricity system and automation of automation, 32(08): 1-6.
[5] Hongbo Z, Xiaofeng C, Gaifeng L. (2015) Power harmonic analysis based on double-window apFFT binline correction. J. Department of Instrumentation, 36(12): 2835-2841.
[6] Wenyu Z, Baohua T, Chenxu H, Jiaqi H, Beitian G, Gui M, Xin R, Shuo Z. (2021) Power grid harmonic and interharmonic detection algorithm based on all-phase FFT trispectral line correction. J/OL. Journal of Central China Normal University (Natural Science edition): 1-8[2021-10-16]. http://kns.cnki.net/kcms/detail/42.1178.N.20210617.1132.002.html.
[7] Dabin L, Yaqun J, Chun H, Juan H. (2012) A new parameter estimation algorithm for harmonics and interharmonics. J. The grid technology, 36(06): 170-174.
[8] Wei L, Chunhong L, Xin Y. (2017) FFT harmonic measurement based on Nuttall window trispectral interpolation. J. Value engineering, 36(04): 79-81.
[9] Yongln P, Rui L, Xihao M, Yabin L. (2021) Hanning double-window all-phase FFT trispectral line interpolation algorithm for harmonic detection. J. Electric Power Science and Engineering, 37(04): 25-29.