Goldstino and sgoldstino in microscopic models
and the constrained superfields formalism.

I. Antoniadis\textsuperscript{a,}\textsuperscript{†}, E. Dudas\textsuperscript{b, c}, D. M. Ghilencea\textsuperscript{a, b, d,}\textsuperscript{‡}

\textsuperscript{a} CERN - Theory Division, CH-1211 Geneva 23, Switzerland.
\textsuperscript{b} CPHT (UMR CNRS 7644) Ecole Polytechnique, F-91128 Palaiseau, France.
\textsuperscript{c} LPT (UMR CNRS 8627), Bat 210, Université de Paris-Sud, F-91405 Orsay Cedex, France.
\textsuperscript{d} DFT, National Institute of Physics and Nuclear Engineering (IFIN-HH) Bucharest MG-6, Romania.

Abstract

We examine the exact relation between the superconformal symmetry breaking chiral superfield ($X$) and the goldstino superfield in microscopic models of an arbitrary Kahler potential ($K$) and in the presence of matter fields. We investigate the decoupling of the massive sgoldstino and scalar matter fields and the offshell/onshell-SUSY expressions of their superfields in terms of the fermions composites. For general $K$ of two superfields, we study the properties of the superfield $X$ after integrating out these scalar fields, to show that in the infrared it satisfies (offshell) the condition $X^3 = 0$ and $X^2 \neq 0$. We then compare our results to those of the well-known method of constrained superfields discussed in the literature, based on the conjecture $X^2 = 0$. Our results can be used in applications, to couple offshell the (s)goldstino fields to realistic models such as the MSSM.

\textsuperscript{†}on leave from CPHT (UMR CNRS 7644) Ecole Polytechnique, F-91128 Palaiseau, France.
\textsuperscript{‡}E-mail: ignatios.antoniadis@cern.ch, dumitru.ghilencea@cern.ch, Emilian.Dudas@cpht.polytechnique.fr
1 Introduction

A consequence of supersymmetry breaking is the presence of a Goldstone fermion - the goldstino - and its scalar superpartner, the sgoldstino. In supergravity the goldstino is the longitudinal component of the gravitino which becomes massive (of mass $\sim f/M_{\text{Planck}}$), while the sgoldstino field acquires mass and decouples at low energy, just like Standard Model (SM) superpartners. If the scale of supersymmetry breaking ($\sqrt{f}$) is low (an example is gauge mediation) the goldstino couplings ($\sim 1/\sqrt{f}$) are much stronger than the Planck-suppressed couplings of transverse gravitino and one can work in the gravity-decoupled limit, with a massless goldstino.

The study of the interactions of the goldstino and sgoldstino with matter is an interesting research area that started with the pioneering work of Akulov and Volkov [1] and were investigated extensively in the early days of supersymmetry [2]-[6], with renewed interest in [7]-[21]. One way to describe Goldstino interactions is using the formalism of constrained superfields which has a long history, that started with [3], [4], [5], [6], and more recently in [7]. For a review of the constrained superfields method with new insight and detailed examples, we refer the reader to [8]. The role of the constraints applied to the matter and goldstino superfields is to project out (i.e. integrate out) the heavy SM superpartners and sgoldstino that decouple at low energy. The method is useful to describe in a superfield formalism, nonlinear effective Lagrangians obtained after integrating out, in a Susy model, all heavy superpartners.

To learn more about the couplings of goldstino/sgoldstino to matter fields one starts from the conservation equation of the Ferrara-Zumino current $J$ [22] that has the form $\bar{D} \bar{\alpha} J_{\alpha \dot{\alpha}} = D_\alpha X$. $X$ is a chiral superfield that breaks supersymmetry and conformal symmetry and was only recently proposed to be identified [8] in the infrared (IR) limit, to the goldstino superfield. By considering an IR constraint of the form $X^2 = 0$, one shows that in the IR the sgoldstino becomes a composite of goldstino fields, $\phi \propto \psi \psi / F$. This constraint, when added to a Lagrangian $L = \int d^4 \theta X^1 X + \left[ \int d^2 \theta f X + h.c. \right] + O(1/\Lambda)$ provides, onshell, a superfield description of the Akulov-Volkov Lagrangian for the goldstino [7, 8]. The constraint $X^2 = 0$ brings interactions to the otherwise free theory of $L$ and projects out the sgoldstino. Such constrained $L$ can arise in the low energy limit from an effective Lagrangian with additional non-renormalizable $O(1/\Lambda)$ corrections to $L$, that provide a large sgoldstino mass; this is then integrated out via the eqs of motion and generates the constraint ($\Lambda$ is a UV scale). This is the situation in the absence of matter fields.

In the presence of matter (super)fields the situation is more subtle. In general, if one would like to couple the goldstino (super)field to the MSSM, this can be done via the divergence of the supersymmetric current and the equivalence theorem [2]. Alternatively, one can use effective polynomial interactions between the MSSM and goldstino superfields [8], as detailed in [20]. In the presence of matter fields, the superfield constraint that projects out the sgoldstino was conjectured [8] to remain (in IR) if the form $X^2 = 0$, while the constraint that projects out
the massive scalar component $\phi^j$ of a matter superfield $\Phi_j$ can be taken $X \Phi_j = 0$. These can be used to identify the sgoldstino and scalar matter fields after they decouple, as functions (composites) of the (light) fermions present; for particular examples see [8] but also some counterexamples in [21]. The onshell/offshell validity limits of these conjectured constraints in the presence of matter fields are discussed in this paper in a general setup of microscopic models with an arbitrary Kahler potential.

For a model independent approach to the goldstino physics, we use the solution for $X$ of the Ferrara-Zumino conservation equation, that can be found in [23] for an arbitrary Kahler $K$ and superpotential $W$; this shows that $X$ is a combination of all the fields present. Surprisingly, the implications of this result for goldstino microscopic models were little studied and in this work we explore this direction. Taking advantage of this result, we compute in a model independent way for an arbitrary $K$, the expression of $X$ after integrating out, via the eqs of motion, the massive sgoldstino and scalar matter fields that decouple in IR. We prove that for two superfields case and a linear superpotential the field $X$ then satisfies (offshell) the condition $X^3 = 0$ while $X^2 \neq 0$ (for more superfields and additional interactions this can be generalized to higher order conditions). This result is in disagreement with the conjecture $X^2 = 0$ mentioned earlier, as the general condition to identify the sgoldstino field. Further, for an arbitrary $K$, we evaluate the offshell/onshell sgoldstino and the scalar matter fields as composites of the light fermions (goldstino and matter fermions). We show that these scalar fields have expressions such that their superfields satisfy (offshell) higher order polynomial conditions, such as $\Phi_1^3 = \Phi_2^3 = \Phi_1 \Phi_2^2 = \Phi_2^3 = 0$. This was also noticed previously in particular examples in [21]. We consider these conditions can be more fundamental than the property $X^3 = 0$ mentioned, which depends on assumptions about the so-called improvement term, due to which $X$ is not unique. Let us stress that while we also investigate various onshell-Susy conditions for superfields, the constraints are really interesting and most relevant when valid offshell. Finally, the non-linear goldstino superfield can be used to couple it offshell to a non-linear Susy realization of the SM (see Section 5 of [21]).

The plan of the paper is as follows. Section 2 computes the component fields of the superfield $X$ for arbitrary $K, W$. Section 3 reviews the case of one superfield (goldstino) and its exact relation to $X$, using both the method of eqs of motion and the constrained superfield method. Applications to two specific models are also provided. Section 4 evaluates, in the presence of matter fields, the properties of $X$ after integrating out the sgoldstino and the scalar matter field. The expressions of these scalar fields in terms of the light fermions present are also computed and the properties of the corresponding non-linear superfields investigated. The results are illustrated for the case of two specific models. Comparison with the method of constrained superfields is performed in Section 5 to examine the validity limits of the latter. Our conclusions are presented in Section 6.
2 The Lagrangian and the superfield $X$ for general $K$, $W$.

In this section we consider a general action of arbitrary Kahler $K(\Phi^i, \Phi_j^\dagger)$ and superpotential $W(\Phi^i)$ and compute the components of the $X$ superfield \cite{23}; $\Phi^i$, $i = 1, 2, ...$ denotes a goldstino chiral superfield ($i = 1$) and additional matter fields ($i > 1$), with components ($\phi^i, \psi^i, F^i$).

The general action is, after a Taylor expansion in Grassmann variables:

\[
L = \int d^4\theta K(\Phi^i, \Phi_j^\dagger) + \left[ \int d^2\theta W(\Phi^i) + \int d^2\bar{\theta} W(\Phi_j^\dagger) \right]
\]

\[
= K_i^j \left[ \partial_\mu \phi^i \partial^\mu \phi_j^\dagger \right] + \frac{i}{2} \left( \psi^i \sigma^\mu D_\mu \bar{\psi}_j^\dagger - D_\mu \psi^i \sigma^\mu \bar{\psi}_j^\dagger \right) + F^i F_j^\dagger
\]

\[
+ \frac{1}{4} K_{ij}^k \psi^i \psi^j \bar{\psi}_k \bar{\psi}_l + \left[ (W_k - \frac{1}{2} K_{ij}^k \bar{\psi}_i \bar{\psi}_j) F^k - \frac{1}{2} W_{ij} \psi^i \psi^j + h.c. \right]
\]

(1)

where we ignored a $(-1/4) \Box K$ in the rhs and a sum over repeated indices is understood. We denoted $K_i \equiv \partial K/\partial \phi^i$, $K^n_i \equiv \partial^2 K/\partial \phi^i \partial \phi^n_j$, $W_j = \partial W/\partial \phi^i$, $W^j = (W_j)^\dagger$, etc, with $W = W(\phi^i)$, $K = K(\phi^i, \phi_j^\dagger)$. The derivatives acting on the fermionic fields are

\[
D_\mu \psi^i \equiv \partial_\mu \psi^i - \Gamma^i_{jk} \left( \partial_\mu \phi^j \right) \psi^k,
\]

\[
D_\mu \bar{\psi}_l \equiv \partial_\mu \bar{\psi}_l - \Gamma^l_{jk} \left( \partial_\mu \phi^j \right) \bar{\psi}_k,
\]

\[
\Gamma^i_{jk} = (K^{-1})^i_m K^m_{jk}
\]

\[
\Gamma^l_{jk} = (K^{-1})^l_m K^m_{jk}
\]

(2)

Eq. (1) is the offshell form of the Lagrangian. The eqs of motion for auxiliary fields

\[
F^i_m = -(K^{-1})^i_m W_i + \frac{1}{2} \Gamma^i_{mj} \bar{\psi}_m \bar{\psi}_j
\]

\[
F^m_i = -(K^{-1})^m_i W^i + \frac{1}{2} \Gamma^m_{lj} \psi^l \psi^j
\]

(3)

can be used to obtain the onshell form of $L$. That leads to a scalar potential

\[
V = (K^{-1})^i_j W_i W^j
\]

(4)

that shall be used later on. After a Taylor expansion about the ground state, one finds the mass of the sgoldstino and additional scalar matter fields present.

Let us introduce a chiral superfield ($X$) that is a measure of superconformal symmetry breaking

\[
D^a \mathcal{J}_{a\dot{a}} = D_a X,
\]

\[
X \equiv (\phi_X, \psi_X, F_X)
\]

(5)

Here $\mathcal{J}$ is the Ferrara-Zumino current \cite{22}; the component $\psi_X$ is related to the supersymmetry current and $F_X$ to the energy-momentum tensor. For the general, non-normalizable action in (1), $X$ has been calculated in \cite{23} (see also \cite{8} for a discussion) and has the form

\footnote{The derivation of this formula is using the eqs of motion \cite{23}. Here we take this formula as general, and consider that it applies/can be “continued” offshell too.}
\[ X = 4W - \frac{1}{3} \mathcal{D}^2 K - \frac{1}{2} \mathcal{D}^2 Y \tag{6} \]

that is valid for arbitrary \( K \) and \( W \). The last term is the so-called improvement term where \( Y(\Phi) \) is a holomorphic function related to a Kahler transformation \( K \to K + 3/2(Y + Y^\dagger) \), that does not change the eqs of motion. The component fields of \( \mathcal{D}^2 Y \) are \( \mathcal{D}^2 Y^\dagger = (-4F_j^\dagger; -4i\theta \overline{\psi}_j Y; 4\Box \phi^\dagger_j) \). In principle with a carefully chosen \( Y \) one could in principle try to simplify the form\(^2\) of \( X \). In fact the presence of the \( Y \)-dependent terms renders \( X \) rather arbitrary. In the following we ignore the improvement term effect on \( X \) and set \( \mathcal{D}^2 Y^\dagger = 0 \), and return to this later in the text. Note that \( X \) has mass dimension 3. To obtain a component form of \( X \) that is needed later on, we use

\[
\mathcal{D}^2 K = (-4) \left[ K^j F_j^\dagger - \frac{1}{2} K^{ij} \overline{\psi}_i \overline{\psi}_j \right] + (4i) \sqrt{2} \theta \left[ \sigma^\mu \left( K^j \overline{\partial}_\mu \overline{\psi}_j + K^{ij} \overline{\psi}_j \overline{\partial}_\mu \phi^i_j \right) + i \psi^k \left( K^j_k F^\dagger_j - \frac{1}{2} K^{ij}_k \overline{\psi}_i \overline{\psi}_j \right) \right] + (-4) \theta \left[ L_{W=0} - \partial_\mu \left( K^j \overline{\partial}_\mu \phi^j_i - \frac{i}{2} K^{ij}_k \overline{\psi}_i \overline{\psi}_j \right) \right] \tag{7} \]

Here \( L_{W=0} \) is \( L \) of eq.\(^1\) with \( W \) and all its derivatives set to 0. We then find from \( \text{eq.}\(^6\) \)

\[
X = 4W(\phi^j) + \frac{4}{3} \left[ K^j F_j^\dagger - \frac{1}{2} K^{ij} \overline{\psi}_i \overline{\psi}_j \right] + \sqrt{2} \theta \left\{ 4W_i \psi^i - \frac{4i}{3} \left[ \sigma^\mu \left( K^j \overline{\partial}_\mu \overline{\psi}_j + K^{ij} \overline{\psi}_j \overline{\partial}_\mu \phi^i_j \right) + i \psi^k \left( K^j_k F^\dagger_j - \frac{1}{2} K^{ij}_k \overline{\psi}_i \overline{\psi}_j \right) \right] \right\} + \theta \left\{ 4\left[ W_i F^i - \frac{1}{2} W_{ij} \psi^i \psi^j \right] + \frac{4}{3} \left[ L_{W=0} - \partial_\mu \left( K^j \overline{\partial}_\mu \phi^j_i - \frac{i}{2} K^{ij}_k \overline{\psi}_i \overline{\psi}_j \right) \right] \right\} \tag{8} \]

From this one immediately identifies the field components (\( \phi_X, \psi_X, F_X \)) of the superfield \( X \) for a general Lagrangian, and we shall use this information in the following sections.

In the infrared, it was recently noted that \( X \) “flows” to a chiral superfield that for a single field case satisfies \( X^2 = 0 \), leading to an Akulov-Volkov action in superfields \( \text{eq.}\(^3\) \). In calculations, to take the IR limit one should effectively impose \( X^2 = \mathcal{O}(1/\Lambda) \). In the presence of additional fields, a constraint for \( X \) would actually mean a constraint for a combination of these fields, see eq.\( \text{eq.}\(^5\) \).

In the remaining sections we study the relation between \( X \) and the goldstino superfield (\( \Phi_1 \)), for the case of one or more superfields present and for arbitrary \( K \). The properties of \( X \) are also discussed. We then analyze the relation with the constrained superfields formalism.

\(^2\)For example one could “cancel” the \( W \) dependence, by choosing \( Y^\dagger = (-1/2)(\mathcal{D}^2/\Box)W \). However, in this case the solution for \( Y \) would be non-local and is unacceptable. See also discussion in Section \( \text{eq.}\(^4\) \).
3 The case of one superfield: Goldstino (Φ) and the X field.

3.1 General results.

To begin with, consider that we have only one superfield, the goldstino itself Φ = (φ, ψ, F), and no matter superfields. We would like to clarify for an arbitrary K, the offshell and onshell link between X and Φ, in the IR limit of setting ∂μ derivatives to 0.

The action considered is that of (1) with one superfield only (Φ) and below we simplify the notation into O where

\[ \frac{∂K}{∂φ} \]

and no matter superfields. We would like to clarify for an arbitrary K, the offshell and onshell link between X and Φ, in the IR limit of setting ∂μ derivatives to 0.

\[ V = W_φ W^φ (K^{-1})^φ \]

The minimum conditions give \( k^φ_φ \), \( w_φ \) \( k^φ_φ \), \( w^φ_φ \) \( k^φ_φ \), \( w^φ_φ \) where \( k^φ_φ \), \( w_φ \), etc denote the values of \( K^φ_φ \), \( W_φ \), etc evaluated on the ground state, i.e. \( k_φ, w_φ, k^φ_φ \), ..., are numbers.

As mentioned, we take \( W = f \), then \( w_φ = 0 \) so it follows that on the ground state \( k^φ_φ = k^φ_φ = 0 \) and the goldstino is indeed massless. The conditions for local minimum are\(^3\)

\[ (k^φ_φ)^2 - |k^φ_φ|^2 \geq 0, \quad k^φ_φ < 0 \]

and the scalar masses of real component fields of goldstino \( φ = 1/√(φ_1 + iφ_2) \) are

\[ m^2 = (k^φ_φ)^2 - k^φ_φ w^2 - k^φ_φ k^φ_φ - k^φ_φ w^2 \]

\[ + |w_φ|^2 (k^φ_φ k^φ_φ - k^φ_φ w^2 ± |w_φ| k^φ_φ w^2 - k^φ_φ w^2) \]
Assuming that $K$ is such as $m_{1,2}^2$ are both positive and since $\psi$ is massless, we can integrate out the sgoldstino via the equations of motion at zero momentum. From our general Lagrangian for one field only we find the eqs of motion for $\phi, \phi^\dagger$ which we combine to obtain

$$K^\phi_{\phi\phi} K^\phi_{\phi\phi} - K^\phi_{\phi\phi} K^\phi_{\phi\phi} = \frac{\psi \psi^\dagger}{2F} \left[ K^\phi_{\phi\phi} \right]^2 - \frac{\psi \psi^\dagger \psi \psi^\dagger}{4 F F^\dagger} \left[ K^\phi_{\phi\phi} K^\phi_{\phi\phi} - K^\phi_{\phi\phi} K^\phi_{\phi\phi} \right]$$

One expands this about the ground state $\langle \phi \rangle = 0$, up to linear fluctuations in $\phi$, to find

$$\phi = \frac{\psi \psi}{2 F} - \frac{\psi \psi^\dagger \psi \psi^\dagger}{4 F F^\dagger} \frac{k^\phi_{\phi\phi} k^\phi_{\phi\phi} - k^\phi_{\phi\phi} k^\phi_{\phi\phi}}{|k^\phi_{\phi\phi}|^2 - (k^\phi_{\phi\phi})^2} + O(\phi^2; \phi^2; \phi^\dagger)$$

According to (12), the denominator in the middle term is proportional to the product of the two masses $-m_{1,2}^2/f^4$ and this presence will be encountered again in the case of more fields. Relative to the denominator, the numerator of the same term has an extra $1/\Lambda$ due to extra derivative. However, the error $O(\phi^3)$ simply makes the coefficient of the middle term undetermined. So the final result is, after eliminating $F$ (by $F \rightarrow -f/k^\phi$):

$$\phi = -(k^\phi_{\phi\phi} \frac{\psi \psi}{2 f^2} + O(1/\Lambda))$$

We can use eq. (13) in eq. (9) to compute $X$, after integrating out the sgoldstino $\phi$. The result is, for $W = f \Phi$ and $K^\phi = \phi + O(1/\Lambda)$

$$X = (4 f + 4/3 F^\dagger) \left[ \frac{\psi \psi}{2 F} + \sqrt{2} \theta \psi + F \theta \theta \right] + O(1/\Lambda)$$

which satisfies offshell $X^2 = 0$ up to terms $O(1/\Lambda)$ that vanish in the IR limit. Onshell

$$X = -(8/3 f) \left[ \frac{\psi \psi}{2 F} - \sqrt{2} \theta \psi + f \theta \theta \right] + O(1/\Lambda)$$

This concludes our review of the case of one superfield present only, the goldstino itself.

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4For $\phi$ and any $W$ this is $K^\phi_{\phi\phi} F F^\dagger - 1/2 (K^\phi_{\phi\phi} \psi \psi F^\dagger + K^\phi_{\phi\phi} \psi \psi F) + 1/4 K^\phi_{\phi\phi} \psi \psi \psi \psi + W_{\phi\phi} F - 1/2 W_{\phi\phi} \psi \psi = 0$
3.2 A simple example.

To illustrate the previous results, let us briefly apply them to some particular model. First we review the model in [8], with

\[ K = \Phi^\dagger \Phi - \frac{c}{\Lambda^2} \Phi^2 \Phi^\dagger + \frac{\tilde{c}}{\Lambda^2} (\Phi^3 \Phi^\dagger + \Phi \Phi^\dagger) + O(1/\Lambda^3), \quad W = f \Phi \]  

(18)

The higher dimensional D-terms ensure that the sgoldstino acquires a mass, to decouple in the IR, while the goldstino remains massless. Indeed, from (10) one finds a scalar potential

\[ V = f^2 \left[ 1 + 4 \frac{c}{\Lambda^2} \phi \phi^\dagger + 3 \frac{\tilde{c}}{\Lambda^2} (\tilde{c} \phi^2 + h.c.) + O(1/\Lambda^3) \right] \]  

(19)

From this or from (12) the masses are

\[ m_{1,2}^2 = f^2 (4 \frac{c}{\Lambda^2} \pm 6 \frac{\tilde{c}}{\Lambda^2}) \]  

with the choice \( |\tilde{c}| < (2/3) c \) one ensures stability and positive scalar (masses). Using the eqs of motion at zero momentum for \( \phi \), one finds [8]

\[ \phi = \frac{\psi \psi}{2F} + O(1/\Lambda) \]  

(20)

in agreement with the previous general results.

3.3 O’Raifeartaigh model with small supersymmetry breaking.

Let us make a side remark. One may ask how to generate the higher dimensional terms in \( K \) that ensure that the sgoldstino becomes massive, together with a linear superpotential which brings the Susy breaking, while the goldstino fermion remains massless. This can be done in a standard O’Raifeartaigh model. In such model, at tree level the sgoldstino is massless, but it acquires a mass via a one loop renormalization of \( K \), induced by the other (massive) superfields of the model. To see this, consider an O’Raifeartaigh model with

\[ K = \Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2 + \Phi_3^\dagger \Phi_3, \quad W = \frac{1}{2} h \Phi_1 \Phi_2^2 + m_s \Phi_2 \Phi_3 + f \Phi_1 \]  

(21)

so \( \Phi_{2,3} \) have a large Susy mass \( (m_s) \) while \( \Phi_1 \) is massless. One computes the one-loop correction to the Kahler potential of \( \Phi_1 \) (this becomes the goldstino superfield) and then integrate out the two massive superfields \( \Phi_{2,3} \). The result is shown below, under the simplifying assumption of small Susy breaking i.e. \( f h < |m_s|^2 \). For details see Appendix A of [25]. One finds

\[ K = \Phi_1^\dagger \Phi_1 - \epsilon (\Phi_1^\dagger \Phi_1)^2 + O(\epsilon^2), \quad W_{eff} = f \Phi_1, \quad \text{with} \quad \epsilon = \frac{1}{12} \left( \frac{h^2}{4\pi} \right)^2 \frac{1}{|m_s|^2} \]  

(22)

For a reliable effective theory approach, the mass of the sgoldstino which is \( m_1^2 = 4 \epsilon f^2 \) should be of order \( \sim f \), which ultimately means \( h^2 \sim O(4\pi) \) i.e. a nearly strongly coupled regime. As seen from the previous example (18) with \( \tilde{c} = 0 \) and \( c/\Lambda^2 \rightarrow \epsilon \), one finds that \( \phi^1 = \psi^1 \psi^1/(2F^1) \), in agreement with the general discussion. Other methods to generate a mass term for sgoldstino may be possible and in general strong dynamics is preferred.
3.4 The results using the method of constrained superfields.

The method of constrained superfields takes the property $X^2 = 0$ that we saw is satisfied in IR after integrating out the sgoldstino $\phi$ and actually imposes it, from the beginning, as an input constraint for the model. In fact it is enough to impose a weaker condition

$$F X^2 = 0, \quad \Rightarrow \quad \phi X = \frac{\psi \psi^X}{2 F X} \Rightarrow \quad \phi X = \frac{\psi \psi}{2 F} \cdot \partial \phi$$

(23)

So $F X^2 = 0$ implies $X^2 = 0$. The last eq in (23) is obtained after a Taylor expansion of the denominator in the previous step, in which we used $F$ of (9). One then finds

$$W(\phi) + \frac{1}{3} \left[K^\phi F^\dagger - \frac{1}{2} K^\phi K^\phi \frac{\bar{\psi} \psi}{\psi} \right] = \frac{\psi \psi}{2 F} \left[W_\phi + \frac{1}{3} \left[K^\phi F^\dagger - \frac{1}{2} K^\phi K^\phi \frac{\bar{\psi} \psi}{\psi} \right] \right]$$

(24)

which for $W = f \Phi$ gives

$$\phi = -\frac{K^\phi F^\dagger}{3 f} + K^\phi \frac{\bar{\psi} \psi}{6 f} + \frac{1}{2 F} \left[1 + \frac{1}{3 f} K^\phi F^\dagger \right] \psi \psi - \frac{K^\phi K^\phi}{12 f F} \frac{\bar{\psi} \psi}{\psi}$$

(25)

Since all $K^\phi, K^\phi, K^\phi$ depend on $\phi, \phi^\dagger$, this is an implicit expression of $\phi$ in terms of the $\psi$, for any $K$ and can be used in applications. We take canonical kinetic terms $K = \Phi^\dagger \Phi + O(1/\Lambda)$ and do not need to detail $O(1/\Lambda)$ terms, but only ensure that they render $\phi$ massive as seen earlier. From (25) one finds

$$\phi = \frac{\psi \psi}{2 F} + O(1/\Lambda)$$

(26)

valid offshell. Eq.(26) implies that $\Phi^2 = O(1/\Lambda)$, which together with $K = \Phi^\dagger \Phi + O(1/\Lambda)$ can reproduce in IR the Akulov-Volkov action for $\Phi$.

Going to onshell supersymmetry, eq (25) gives

$$\phi = \frac{K^\phi}{3 K^\phi} - \frac{\bar{\psi} \psi}{6 f K^\phi} \left[K^\phi K^\phi - K^\phi K^\phi \right] - \frac{\psi \psi}{3 f} K^\phi$$

(27)

This is the implicit expression for the sgoldstino $\phi$ as a goldstino composite. It can be used for specific applications, such as for example [14]. There is dependence on both $\bar{\psi} \psi$ and $\psi \bar{\psi}$, but also possible dependence on $\phi^\dagger$, so for specific $K$ one solves this for $\phi, \phi^\dagger$ in terms of fermion composites. Taking $K = \Phi^\dagger \Phi + O(1/\Lambda)$ where $1/\Lambda$ suppresses the dimensionful derivatives of $K$, from (27) one recovers (20) with $F$ replaced by $-f$.

As an application to Section 3.2, using (25) in which we replace the derivatives of $K$ by their expressions derived with (18), we find an eq for $\phi$ (not shown), that is solved easily by trying a solution of the form $\phi = a_1 \psi \bar{\psi} + a_2 \bar{\psi} \psi + a_3 \psi \bar{\psi} \psi$ ($a_{1,2,3}$ are constants to be identified). The final offshell solution turns out to be exactly eq (20) found instead via the eqs of motion. Similar considerations apply for Section 3.3 whose result $\phi^1 = \psi^1 \bar{\psi}^1/(2 F^1)$ is also recovered from (25). This concludes our review of the one-field case.
4 Goldstino and sgoldstino in the presence of matter fields.

4.1 General results.

We proceed to study more realistic cases with a goldstino superfield in the presence of matter superfields. The starting point is $L$ of (1) in which sgoldstino and scalar matter fields acquire mass through higher dimensional Kahler terms and decouple at low energy. It is our purpose to examine the expressions of these scalar fields at the low energies, as functions (composites) of the light fermions present (goldstino and matter fermions). Using these expressions, we then examine at low energy, the expression of X and its properties.

To this end, from eq.(8) we identify, in the limit of vanishing space-time derivatives $\partial_\mu \to 0$, the components of $X = (\phi_X, \psi_X, F_X)$:

$$
\phi_X = 4 W(\phi^i) + \frac{4}{3} \left[ K^j F^i_j - \frac{1}{2} K^{ij} \bar{\psi}_i \psi_j \right]
$$

$$
\psi_X = \psi^k \frac{\partial \phi_X}{\partial \phi^k} - \frac{4 i}{3} \sigma^\mu \left( K^j \partial_\mu \bar{\psi}_j + K^{ij} \bar{\psi}_j \partial_\mu \phi^i \right) = \psi^k \frac{\partial \phi_X}{\partial \phi^k} + O(\partial_\mu),
$$

$$
F_X = F^i \frac{\partial \phi_X}{\partial \phi^i} - \frac{1}{2} \psi^i \psi^j \frac{\partial^2 \phi_X}{\partial \phi^i \partial \phi^j} + \frac{4 i}{3} \left( K^j \partial_\mu \phi^i \frac{\partial \phi^j}{\partial \phi^i} + \frac{i}{2} \left( \psi^i \sigma^\mu D_\mu \bar{\psi}^j - D_\mu \psi^i \sigma^\mu \bar{\psi}^j \right) \right)
$$

To evaluate $X$ after the decoupling of the sgoldstino/scalar matter fields, we consider, for simplicity, the Lagrangian (1) for only two (super)fields, one goldstino and one matter scalar field, in order $O(1/\Lambda^3)$; we thus neglect corrections with more than four derivatives of $K$. We also assume, for simplicity, that

$$
W = f \Phi_1,
$$

so $\Phi_1$ breaks supersymmetry. The scalars $\phi^{1,2}$ acquire mass via higher dimensional D-terms and decouple at low energy, while with above $W$, $\psi^1$ remains massless (is the goldstino$^5$).

Let us integrate the scalars, by using the eqs of motion for $\phi^i_m$, $m = 1, 2$, which (at zero momentum) have the form

$$
K^{i1} F^i F^i_j - \frac{1}{2} K^{ij} \psi^i \psi^j F^i_k - \frac{1}{2} K^{ij} \bar{\psi}_i \bar{\psi}_j F^i_k + O(1/\Lambda^3) = 0,
$$

$$
K^{i2} F^i F^i_j - \frac{1}{2} K^{ij} \psi^i \psi^j F^i_k - \frac{1}{2} K^{ij} \bar{\psi}_i \bar{\psi}_j F^i_k + O(1/\Lambda^3) = 0, \quad i, j, k = 1, 2. \quad (30)
$$

To solve this for $\phi^{1,2}$ one expands (30) about the ground state (assumed $\langle \phi_i \rangle = 0$), so the field dependent Kahler derivatives become

$^5$See next section for an example.
\[
K_{ij}^{jm} = k_{ij}^{jm} + \phi^1 k_{i1}^{jm} + \phi^2 k_{i2}^{jm} + \cdots \\
K_{il}^{jm} = k_{il}^{jm} + \cdots, \quad K_k^{ij} = k_k^{ij} + \cdots
\]

(31)

where the dots represent contributions which are of higher order $O(1/\Lambda^3)$. As in the one-field case, $k_{i\cdots}^{\cdot\cdot}$ denote numerical values of the field-dependent Kahler derivatives $K_{ij}^{jm}$ on the ground state. To simplify the calculation we work in normal coordinates basis, where $k_{ij}^{jm} = k_{k}^{ij} = 0$, $k_i = \delta_i^j$, $R_{ij}^{k} = k_{ij}^{k}$ for the curvature tensor, etc. Then the solution of system (30) for $\phi^{1,2}$ is, after using the permutation symmetries of some of the indices:

\[
\begin{align*}
\phi^1 &= \frac{\psi^1 \psi^1}{2 F^1} - \frac{c_1}{2 F^1} (F^2 \psi^1 - F^1 \psi^2)^2 + O(1/\Lambda) \\
\phi^2 &= \frac{\psi^2 \psi^2}{2 F^2} - \frac{c_2}{2 F^2} (F^2 \psi^1 - F^1 \psi^2)^2 + O(1/\Lambda)
\end{align*}
\]

(32)

where $c_{1,2}$ are

\[
\begin{align*}
c_1 &= \frac{\det [k_{2m}^{kn} F_k^1]}{\det [k_{ij}^{il} F_i^l F_j^1]} \\
c_2 &= \frac{\det [k_{1m}^{kn} F_k^1]}{\det [k_{ij}^{il} F_i^l F_j^1]}
\end{align*}
\]

(33)

In $c_{1,2}$ the free indices \{m, n\}, \{l, p\} are of the 2x2 matrices whose determinant is evaluated. The $O(1/\Lambda)$ correction originates from eq. (30) with (31). In $c_{1,2}$ one can replace $k_{ij}^{im}$ by corresponding curvature tensor $R_{ij}^{k}$, to obtain the result for general coordinates. These results for $\phi^{1,2}$ together with corresponding $\psi^{1,2}$ and $F^{1,2}$ define non-linear superfields $\Phi_{1,2}$ that can couple offshell the goldstino to matter, see for example Section 5 of [21].

Intriguingly, one notices by direct calculation that the scalar fields expressions in (32) are such that the corresponding superfields of components $\Phi_i = (\phi^i, \psi^i, F^i)$ with $i = 1, 2$ and with $\phi^i$ as in (32) satisfy, for arbitrary $c_{1,2}$, the following generalized, higher order polynomial superfield constraints

\[
\Phi_1^2 = \Phi_2^2 \quad \Phi_1 \Phi_2^2 = \Phi_2^3 = 0
\]

(34)

which are valid offshell in IR (when we ignore $O(1/\Lambda)$ in the rhs of (32)). To check (34) one shows by direct calculation that their scalar, fermion and auxiliary components vanish, provided that $\phi^{1,2}$ have the form shown. This property was noticed recently in [21] for particular cases, and as shown above, is actually valid for general $K$. Note also that offshell

\[
\Phi_1^2 \neq 0, \quad \Phi_1 \Phi_2 \neq 0
\]

(35)

The relation is $R_{ij}^{k} = k_{ij}^{k} - k_i^j (k^{-1})^j_k k_{ij}^{k}$, in normal coordinates $k_{ij}^{k} = 0 = k_i^j$, so $R_{ij}^{k} = k_{ij}^{k}$. In the complex geometry convention, $R_{ij}^{k}$ is actually replaced by $(R_{\bar{\nu}})^{\gamma}_{\eta} = K_{\gamma\nu} - K_{\nu\gamma} K^\eta_{\bar{\eta}} K_{\bar{\eta} \eta}$. 

10
Let us also present some onshell results. Using that the scalar potential is $V = (K^{-1})^i_j W_i W^j$ one can show that the denominators of $c_{1,2}$ are, with $F^j$ replaced by their vev’s, the determinant of the squared mass matrix in the bosonic sector of sgoldstino ($\phi^1$) and scalar matter field ($\phi^2$). As shown by the scalar potential, these acquire mass via the Kahler metric. Going to onshell supersymmetry, one finds

$$c_1 = \frac{1}{f^2} \det \left[ R^{ij}_{2m} \right], \quad c_2 = \frac{1}{f^2}$$

so the onshell result for $\phi^{1,2}$ is

$$\phi^1 = -\frac{\psi^1 \psi^1}{f} + \frac{\det R^{ij}_{2m}}{f^2} \psi^2 \psi^2 + O(1/\Lambda)$$

$$\phi^2 = -\frac{\psi^1 \psi^2}{f} + O(1/\Lambda)$$

Note that the ratio of the two determinants is dimensionless and independent of the UV scale $\Lambda$. The absence in (37) of any similarity in the structure of $\phi^{1,2}$ apparent in previous (32) is due to $c_{1,2}$ and to the fact that only $F^1$ is non-vanishing onshell in the approximation considered. In the presence of more matter fields, the solutions $\phi^j, j \geq 2$ have a similar form. Note that even onshell, $\Phi^1 \neq 0$. In the formal limit of infinite scalar masses, onshell $c_1 = 0$. With $c_{1,2}$ one can investigate the properties of $X$ after decoupling $\phi^{1,2}$ (Section 4.3).

### 4.2 A simple example.

Let us first illustrate the results for $\phi^{1,2}$ of (32) for a particular model \[21\] with

$$K = \Phi_1^{\dagger} \Phi_1 + \Phi_2^{\dagger} \Phi_2 - \epsilon_1 (\Phi_1^{\dagger} \Phi_1)^2 - \epsilon_2 (\Phi_2^{\dagger} \Phi_2)^2$$

$$- \epsilon_3 (\Phi_1^{\dagger} \Phi_1)(\Phi_2^{\dagger} \Phi_2) - \epsilon_4 [(\Phi_1^{\dagger})^2 \Phi_2^2 + h.c.] + O(1/\Lambda^3)$$

and superpotential

$$W = f \Phi_1, \quad \epsilon_i = O(1/\Lambda^2)$$

The scalar potential is

$$V = (K^{-1})^k_p W_k W^p = (K^{-1})^i_j f^2 = f^2 (1 + \epsilon_3 |\phi^2|^2 + 4 \epsilon_1 |\phi^1|^2)$$

\footnote{The trace of the mass matrix is $-2 R_{il}^{im} F_i F^l$ \[21\].}
\[ m_{\phi_1}^2 = \epsilon_3 f^2, \quad m_{\phi_2}^2 = 4 \epsilon_1 f^2 \] for the goldstino (\( \phi^1 \)) and scalar matter field (\( \phi^2 \)), respectively (we choose \( \epsilon_{1,3} > 0 \)) and the ground state is indeed at \( \phi_{1,2} = 0 \).

The solution for \( \phi^{1,2} \) is that of (42); using eq. (33) one finds [21]

\[
\begin{align*}
c_1 &= \frac{1}{\Delta} \epsilon_3 (\epsilon_2 F_1^{12} - \epsilon_4 F_1^{2}) = -\frac{\epsilon_4}{\epsilon_1 F_1^2} + \mathcal{O}(1/|F_1|^4) \\
c_2 &= \frac{1}{\Delta} \epsilon_3 (\epsilon_1 F_1^{12} - \epsilon_4 F_2^{12}) = \frac{1}{F_1^2} + \mathcal{O}(1/|F_1|^4), \quad \text{where} \\
\Delta &= \epsilon_3 F_1^2 (\epsilon_1 F_1^{12} - \epsilon_4 F_2^{12}) + \epsilon_3 F_2^2 (\epsilon_2 F_2^{12} - \epsilon_4 F_1^{12}) + 4(\epsilon_1 \epsilon_2 - \epsilon_4^2)|F_1|^2|F_2|^2 \tag{41}
\end{align*}
\]

The offshell result for \( \phi^{1,2} \) is given in (42) with these \( c_{1,2} \) and has \( \mathcal{O}(1/\Lambda) \) correction that originates from the eqs of motion for \( \phi^{1,2} \) that are valid in \( (1/\epsilon_1) \times \mathcal{O}(1/\Lambda^3) = \mathcal{O}(1/\Lambda) \) (since \( \epsilon_1 \) is multiplying \( \phi^{1,2} \) in these eqs). The rhs expansion of \( c_{1,2} \) in \( 1/|F_1| \) is allowed onshell, after taking account that onshell \( F^1 = -f + \mathcal{O}(\epsilon_i) \) and \( F^2 = \mathcal{O}(\epsilon_i) \). Note that \( c_{1,2} \) are ratios of \( \epsilon_i \), so \( c_{1,2} = \mathcal{O}(1/\Lambda^0) \). The onshell result is then

\[
\begin{align*}
\phi^1 &= -\frac{\psi^1 \psi^1}{2 f} - \frac{\epsilon_4}{\epsilon_1 f} \frac{\psi^2 \psi^2}{2} + \mathcal{O}(1/\Lambda) \\
\phi^2 &= -\frac{\psi^1 \psi^2}{f} + \mathcal{O}(1/\Lambda) \tag{42}
\end{align*}
\]

in agreement with (37). Obviously, \( \Phi^2_1 \neq 0 \) since \( (\phi^1)^2 \neq 0 \).

### 4.3 The properties of the field \( X \) after integrating out the scalar fields.

Returning now to the field \( X = (\phi_X, \psi_X, F_X) \) of (28), we use the solution of the eqs of motion for \( \phi^{1,2} \) of (42), to find the expression of \( X \) after integrating out these scalars. For canonical kinetic terms \( K = \Phi_1 \Phi_1 + \Phi_2 \Phi_2 + \mathcal{O}(1/\Lambda) \) and with \( W = f \Phi_1 \) we find after some algebra

\[
X^2 = \rho \left[ (\psi^1 \psi^1)(\psi^2 \psi^2) - 4 \sqrt{2}(\psi^1 \psi^2)(F^1 \theta \psi^2 + F^2 \theta \psi^1) + 2 (\theta \theta)(\psi^1 F^2 - \psi^2 F^1) \right] + \mathcal{O}(1/\Lambda) \tag{43}
\]

where \( \mathcal{O}(1/\Lambda) \) involves terms that contain \( \bar{\psi} \); from (43) one can read the components of \( X^2 = (\phi_X^2, \psi_X^2, F_X^2) \). Also

\[
\begin{align*}
\rho &= \frac{1}{2 F_1 F_2} \left[ \sigma_1 \sigma_2 - (F^1 \sigma_1 + F^2 \sigma_2)(c_1 \sigma_1 F^2 + c_2 \sigma_2 F^1) \right] + \mathcal{O}(1/\Lambda) \\
\sigma_1 &= 4 f + 4/3 F_1^4, \quad \sigma_2 = 4/3 F_2^4 \tag{44}
\end{align*}
\]
where again $O(1/\Lambda)$ suppresses $\psi$-dependent terms and $c_{1,2}$ are those of [33]. Obviously $X^2 \neq 0$, unless $\rho = 0$. However, with $c_{1,2}$ of [33] this is not possible in general. In specific cases with particular values for the individual Kahler curvature terms, one may have a vanishing $X^2$ but this is not true in general. Thus the conjecture $X^2 = 0$ is not verified in general.

Let us evaluate $X^2$ onshell. One finds, using (36) that

$$\rho \bigg|_{\text{onshell}} = -\frac{1}{2} c_1 \left(\frac{8}{3} f\right)^2 = -\frac{32}{9} \frac{\det R^{ll}_{2m}}{\det R^{ll}_{1m}} + O(1/\Lambda) \quad (45)$$

As a result

$$X^2 \bigg|_{\text{onshell}} = -\frac{64}{9} \left[\frac{\det R^{ll}_{2m}}{\det R^{ll}_{1m}}\right] f \left(\psi^2 \psi^2\right) \left[\frac{\psi^1 \psi^1}{2 f} - \sqrt{2} (\theta \psi^1) + (\theta \theta) f\right] + O(1/\Lambda) \quad (46)$$

which clearly does not vanish, except in specific cases when $c_1 = 0$, i.e. if the determinant in the numerator vanishes or that in the denominator is infinite. The last factor in the rhs is the (onshell) goldstino superfield in the absence of matter superfields and after integrating out the sgoldstino, see [17].

So far we found that $X^2$ is not vanishing offshell or onshell. Next, let us investigate the offshell value of $X^3$. One finds using (28) together with canonical kinetic terms for $\Phi_{1,2}$ and $W = f \Phi_1$, that

$$\phi_{X^3} = \phi_X \phi_{X^2} \propto (\phi^1 \sigma_1 + \phi^2 \sigma_2) (\psi^1 \psi^1) (\psi^2 \psi^2) + O(1/\Lambda) = O(1/\Lambda)$$

$$\psi_{X^3} = 3 \phi_X^2 \psi_X = (3/2) \phi_X \psi_X \propto (3/2) (\phi^1 \sigma_1 + \phi^2 \sigma_2) (\psi^1 \psi^2) (F^1 \theta \psi^2 + F^2 \theta \psi^1) = O(1/\Lambda)$$

$$F_{X^3} = 3 \phi_X (F_X \phi_X - \psi_X \psi_X) = 3 (\phi^1 \sigma_1 + \phi^2 \sigma_2) [(\phi^1 \sigma_1 + \phi^2 \sigma_2) (F^1 \sigma_1 + F^2 \sigma_2)$$

$$- (\psi^1 \sigma_1 + \psi^2 \sigma_2)^2] = O(1/\Lambda) \quad (47)$$

The last step in the rhs of each of these eqs uses that $\phi^{1,2}$ contain bilinears in $\psi^{1,2}$ as seen from (32) and also the expressions of the components of $X^2 = (\phi_{X^2}, \psi_{X^2}, F_{X^2})$ as shown in [13]. Finally, for $F_{X^3}$ we used (32) (without replacing $c_{1,2}$ by their values [33]). In conclusion, $X^3 = 0$ in the infrared (offshell-Susy).

One could reverse the arguments and take the property $X^3 = 0$ and consider it as an input condition, in a constrained superfields formalism, to identify the goldstino superfield (and to replace the conjectured $X^2 = 0$ constraint, see last section). We chose not to do so, for the following reasons. Let us remind that these results are for a vanishing “improvement” term in (6). In the presence of an arbitrary non-vanishing such term, even the property $X^3 = 0$ shown above for 2 fields can be violated, since [28] is changed. This stresses that the properties of $X$ are not uniquely defined, not even in onshell-Susy for $X$ or its powers. As a result, conclusions

13
derived from assuming some constraints on X have to be regarded with due care. Note that while the properties of X such as \(X^3 = 0\) are affected by the improvement term, those of \(\Phi_{1,2}\) superfields are independent of this, and can be considered as more fundamental and of more use in practice. Finally, in the presence of more matter fields (\(n\)) or (non-linear) superpotential terms, this condition is likely to change into \(X^{n+1} = 0\), see also [21].

For future reference, we provide below onshell-supersymmetry results, without an approximation in \(1/\Lambda\) and also for an arbitrary \(W\), without integrating out the scalar fields \(\phi^j\). These are obtained from the onshell structure of \(\phi_X, F_X, \psi_X\). Using (28) one finds

\[
\phi_X = \sigma + \sigma^{mn} \psi_m \overline{\psi}_n, \quad \psi_X = \frac{8}{3} \psi^k W_k
\]

\[
F_X = \beta + \beta_{mn} \psi_m \psi^n + \beta_{kl}^{mn} (\psi_m \psi^n) (\overline{\psi}_k \overline{\psi}_l)
\]

where

\[
\sigma = 4W - (4/3) K^i (K^{-1})^i_k W_k, \quad \sigma^{mn} = (2/3) [K^i (K^{-1})^i_k K^m_{kn} - K^{mn}]
\]

\[
\beta = -(8/3) W_m (K^{-1})^m_k W^k, \quad \beta_{mn} = 2 \left[ W_k \Gamma^k_{mn} - W_{mn} \right], \quad \beta_{kl}^{mn} = (1/3) R_{mn}^{kl}
\]

\(R_{mn}^{kl}\) denotes the curvature tensor, \(R_{mn}^{kl} = K_{mn}^{kl} - K_{kn}^{lj} (K^{-1})^i_j K_{lm}^{ik}\) and we considered an arbitrary superpotential \(W\). Here all coefficients of the fermions are scalar field-dependent.

4.4 Another example.

For more insight into the properties of the field \(X\) and the relation to \(\Phi_{1,2}\) consider a particular model [21] with

\[
K = \Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2 - \epsilon (\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2)^2, \quad W = f \Phi_1,
\]

One can easily extend this to \(N\) fields by extending in \(K\) the two sums from 2 to \(N\) and also \(f \Phi_1 \rightarrow \sum_{i=1}^{N} f_i \Phi_i\) (similar for the formulae below). Using a zero-momentum integration of the heavy scalars \(\phi^{1,2}\), one finds

\[
\Phi_i = \left\{ \frac{1}{|F|^2} \left( \psi^i - \frac{F^i}{2|F|^2} \psi^k F_k^\dagger \psi^n \right) F_n^\dagger \psi^n \right\} + \sqrt{2} \theta \psi^i + \theta^2 F_i, \quad |F|^2 = |F_1|^2 + |F_2|^2.
\]

where \(\phi^i\) defined by the curly bracket is obtained from eq. (32), (33).

As also seen in (35), note that \(\Phi_i^2 \neq 0, i = 1, 2\). However, define the superfield

\[
\tilde{X} = \frac{1}{|F|} F_i^\dagger \psi^i = \frac{(F_i^\dagger \psi)^2}{2|F|^2} + \sqrt{2} \theta \frac{F_i^\dagger \psi^i}{|F|} + \theta^2 |F|
\]

Interestingly, this superfield satisfies the constraint \(\tilde{X}^2 = 0\), with goldstino defined by the \(\theta\) component. Further, any linear combination \(\Phi_i' = c_{ij} \Phi_j\) satisfies the constraint \(\tilde{X} \Phi_i' = 0\). It
should be recognized that this is a very symmetric example and the constraint $\tilde{X}^2 = 0$ seen here is not generally valid, as already known from previous example, Section 4.2.

The natural question is: what is the link of $\tilde{X}$ with $X$ of (6), (28), or why $X^2 \neq 0$ in this case. Indeed, according to [13], [14] for this model $X^2 \neq 0$, unless one formally takes $f = 0$ (unacceptable) or, alternatively, chooses a non-zero improvement term that cancels $W(\phi_i)$ in $\phi_X$ of (28) but leaves $\psi_X, F_X$ unchanged, up to space-time derivatives. Such an improvement term would require a non-local $Y$, and is therefore not acceptable, see discussion and footnote immediately after eq.(6). In conclusion $X$ of (6), (28) cannot satisfy $X^2 = 0$, even though there does exist another, unrelated superfield $\tilde{X}$ with this property. Nevertheless, the existence of $\tilde{X}$ with such property is specific to this very special model.

5 The constrained superfields method and its validity.

For a better understanding of the results so far, one would like to have a closer look at the infrared conjecture $X^2 = 0$ in the presence of matter fields [8], as a definition for the sgoldstino. For the sake of the argument, in this section we therefore assume that this is true and then analyze its implications, to see more closely its validity limits, by comparing with the results of the previous sections. In fact it is sufficient to impose $F_{X^2} = 0$. One has

$$F_{X^2} = 0 \quad \Rightarrow \quad \phi_X = \frac{\psi_X \psi_X}{2 F_X} \quad \Rightarrow \quad X^2 = 0. \quad (53)$$

After using (28), one obtains from the middle relation

$$\phi_X = \left(\frac{\partial \phi_X}{\partial \phi^j} \psi^j\right)^2 \left(2 F_k \frac{\partial \phi_X}{\partial \phi^k}\right)^{-1} \left\{1 + \sum_{j \geq 1} \left[\psi^k \psi^l \frac{\partial^2 \phi_X}{\partial \phi^k \partial \phi^l} \left(2 F^k \frac{\partial \phi_X}{\partial \phi^k}\right)^{-1}\right]^j\right\} \quad (54)$$

where $O(\partial_\mu)$ terms, that vanish in the IR, are not displayed from now on. The sum over $j$ has actually a finite number of terms, for a large enough power of the Grassmann variables $\psi^k$.

Together with the definition of $\phi_X$ in (28) used to evaluate the derivatives, (54) is an implicit definition of $\phi^1$ to be identified later with the sgoldstino ($\psi^1$ is the goldstino) if $W = f \Phi_1$.

To integrate out the scalars fields $\phi^j$ ($j \neq 1$) other than sgoldstino, one imposes $\Phi$

$$F_X \phi_j = 0, \quad \Rightarrow \quad \phi^j = \frac{\psi^j \psi_X}{F_X} - \phi_X \frac{F^j}{F_X} \quad \Rightarrow \quad X \Phi_j = 0 \quad (55)$$

where $\Phi_j$ is an arbitrary matter superfield and eq.(53) was used. The middle relation can be re-written as

$$\phi^j = \frac{\psi^j \psi_X}{2 F^j} - \frac{1}{2 F^j} \frac{1}{F_X} (F^j \psi_X - F_X \psi^j)^2, \quad j : \text{fixed, } j \neq 1. \quad (56)$$
This form anticipates the structure of the offshell solution for scalar matter fields and goldstino (see later). From this one finds

$$\phi^j = \frac{\psi^j \psi^j}{2 F^j} - \frac{1}{2 F^j} \frac{1}{F_X} \left[(F^k \psi^j - F^j \psi^k) \frac{\partial \phi_X}{\partial \phi^k} - \frac{1}{2} (\psi^m \psi^k) \psi^j \frac{\partial^2 \phi_X}{\partial \phi^m \partial \phi^k} \right]^2$$  (57)

where for the derivatives of $\phi_X$ one uses the first line in (28). Such derivatives can only bring 4-fermion or more corrections that are $\mathcal{O}(1/\Lambda)$. Similar considerations for $\phi_X$ of (54).

Eq. (54), (57) are general results. They simplify if we ignore the $\mathcal{O}(1/\Lambda)$ terms to give

$$\phi^j = \frac{\psi^j \psi^j}{2 F^j} - \frac{1}{2 F^j} \left[F^k \alpha_k - 2 W_{mn} \psi^m \psi^n\right]^{-2} \left[\alpha_k (F^k \psi^j - F^j \psi^k) - 2 W_{ls} (\psi^l \psi^s) \psi^j \right]^2 + \mathcal{O}(1/\Lambda)$$  (58)

where

$$\alpha_k = 4 W_k + (4/3) K^m F^j_m$$  (59)

where $\mathcal{O}(1/\Lambda)$ accounts for $\geq 4$ fermions terms. In the first bracket of $\phi^j$ one Taylor expands to a finite series in spinors. If $W_{mn} \neq 0$ four-fermion terms can be present even in order $\mathcal{O}(1/\Lambda^0)$. For $W_{mn} = 0$, eq. (56) becomes

$$\phi^j = \frac{\psi^j \psi^j}{2 F^j} - \frac{1}{2 F^j} \left[\alpha_k (F^k \psi^j - F^j \psi^k) \right]^2 + \mathcal{O}(1/\Lambda)$$  (60)

which is a form that we shall use shortly.

### 5.1 The case of two scalar fields $\phi^{1,2}$ - offshell results.

Let us now consider the implications of the above results for the case of two fields only (goldstino plus a matter field) but for an arbitrary Kahler potential with canonical kinetic terms and a “standard” goldstino superpotential:

$$K = \Phi^1_i \Phi^1_i + \Phi^1_i \Phi^2_i + \mathcal{O}(1/\Lambda), \quad W = f \Phi^1$$  (61)

$\mathcal{O}(1/\Lambda)$ terms stand for Kahler terms that are higher dimensional and that we do not need to specify explicitly here; they involve derivatives $K^m_i, K^k_{ij}$, etc in (1), and are thus suppressed by powers of $1/\Lambda$. Such terms also give mass to sgoldstino $\phi^1$ and scalar matter field $\phi^2$ as we saw earlier. After using eqs. (58), (60) with $W$ of (61), one finds
\begin{align}
\phi^1 &= \frac{\psi^1 \psi^1}{2 F^1} - \frac{c_1}{2 F^1} (F^2 \psi^2 - F^1 \psi^1)^2 + \mathcal{O}(1/\Lambda) \\
\phi^2 &= \frac{\psi^2 \psi^2}{2 F^2} - \frac{c_2}{2 F^2} (F^2 \psi^1 - F^1 \psi^2)^2 + \mathcal{O}(1/\Lambda) \\
\end{align}

where the “new” \(c_{1,2}\) denote some functions of \(F_{1,2}\):

\begin{align}
\quad c_1 = \frac{\alpha^2_2}{(F^k \alpha_k)^2} = \frac{[(1/3)F^1_{12}]^2}{[F^k (\delta_{k1} f + 1/3 F^1_1)]^2}, \quad c_2 = \frac{\alpha^2_1}{(F^k \alpha_k)^2} = \frac{(f + 1/3 F^1_1)^2}{[F^k (\delta_{k1} f + 1/3 F^1_1)]^2}
\end{align}

This is the offshell form of sgoldstino \(\phi^1\) and scalar matter field \(\phi^2\). Note the similarity with \([50]\), \([51]\) and the symmetry of \(c_{1,2}\) in indices 1, 2, for the formal limit of vanishing supersymmetry scale \(f\). The result in \([62]\) takes into account that offshell and in the IR limit \(\phi_X\) is not \(\phi_1\) but rather a combination of \(\phi^1\) and \(\phi^2\), with no relative suppression in \(\Lambda\). This is seen from the definition of \(\phi_X\) which contains terms such as \(\phi_X \supset K^k F^1_j = \phi^1 F^1_1 + \phi^2 F^1_2 + \mathcal{O}(1/\Lambda)\) which involves both \(\phi_{1,2}\).

Eqs. \((62), (63)\) represent the main result of this section and should be compared to those in \([32]\) with \([33]\). One immediately sees that while the structure of the solution is similar, the offshell values of the coefficients \(c_{1,2}\) are different in the two cases. The ultimate reason of this difference is due to the fact that the conjecture \(X^2 = 0\) in IR has a limited validity and is at the origin of this discrepancy.

As seen in the previous section, one notices that the scalar fields expressions in \([62]\) are such that the corresponding superfields of components \(\Phi_i = (\phi^i, \psi^i, F^i)\) with \(i = 1, 2\) and with \(\phi^i\) as in \([62]\) satisfy, for arbitrary \(c_{1,2}\), the higher order polynomial superfield constraints, valid offshell, shown in \([64]\), which are independent of the exact values of \(c_{1,2}\) (and ignoring \(\mathcal{O}(1/\Lambda)\) in the rhs of \([62]\)). Also notice that offshell \([35]\) is also respected.

To understand the relation with the result in \([8]\) consider the formal limit of large scale of supersymmetry breaking (\(\sqrt{f}\)). One should be aware of the restricted validity of this limit, since we are already within a \(\mathcal{O}(1/\Lambda)\) expansion, which can contain in particular \(\mathcal{O}(\sqrt{f}/\Lambda)\) terms. Nevertheless, in this case \([62]\) with \([64]\) gives

\begin{align}
\phi^1 &= \left\{ \frac{\psi^1 \psi^1}{2 F^1} - \frac{F^2_{12}}{18 f^2 (F^1)^3} (F^2 \psi^2 - F^1 \psi^1)^2 + \mathcal{O}(1/f^3) \right\} + \mathcal{O}(1/\Lambda) \\
\phi^2 &= \left\{ \frac{\psi^2 \psi^2}{F^1} - \frac{F^2 \psi^1 \psi^2}{2 (F^1)^2} + \frac{1}{3 f (F^1)^3} (F^2 \psi^1 - F^1 \psi^2)^2 \right. \\
&\quad - \left. \frac{1}{18 f^2 (F^1)^4} (2 |F^1|^2 + 3 |F^2|^2) (F^2 \psi^1 - F^1 \psi^2)^2 + \mathcal{O}(1/f^3) \right\} + \mathcal{O}(1/\Lambda) \\
\end{align}

*\textsuperscript{8}\textsuperscript{A}A consistency check: with these values of \(c_{1,2}\), \(X^2 \propto \rho\) of \([35]\) is indeed vanishing.*
For infinite \( f \) (i.e. order \( 1/f^0 \)) only the first (first two) term in \( \phi^1 (\phi^2) \) contribute respectively, and one recovers the result \[8\]

\[
\phi^1 = \psi^1 \psi^1 / (2F^1) \quad \phi^2 = \psi^1 \psi^2 / F^1 - F^2 \psi^1 / (2(F^1)^2)
\] (65)

Let us now discuss some onshell-Susy results. Eq. (62) gives onshell

\[
\phi^1 = -\psi^1 \psi^1 / (2 f) + O(1/\Lambda), \quad \phi^2 = -\psi^1 \psi^2 / f + O(1/\Lambda)
\] (66)

Note that in the onshell case \( \phi_X = 4f \phi_1 - 4/3K^j(K^{-1})^j f = 8/3\phi_1 f + O(1/\Lambda) \) so \( \phi_X \propto \phi^1 \) while offshell \( \phi_X \) is a mixture of both \( \phi^{1,2} \) as already mentioned. Note also that onshell \( \Phi_{1,2} \) (i.e. auxiliary \( F^{1,2} \) replaced by their solution of eqs of motion and the scalar components as in \[66\]) satisfy

\[
\Phi_1^2 = \Phi_1 \Phi_2 = 0 \text{ in IR}
\] (67)

in contradiction with \( \Phi_1^2 \neq 0 \) of \[37\].

The onshell results of the last two eqs can also be obtained using directly the onshell form of \( X \), after imposing \( \phi_X = \psi_X \psi_X / (2F_X) \) that comes from the IR conjecture \( X^2 = 0 \). From this together with \[48\], \[49\] one finds

\[
\phi_X = \frac{32}{9} \frac{\beta^{kl}}{\beta^{mn}} \psi^k \psi^l W_k W_l + \cdots
\] (68)

which can be Taylor expanded into a finite series. For our simple case with only 2 fields (goldstino and a matter field) with canonical kinetic terms in \( K \) and \( W = f \Phi_1 \) one finds

\[
\phi_X = \frac{32}{9} \frac{f^2}{\beta} \psi^1 \psi^1 + (\geq 4 \text{ fermions})
\] (69)

which has no \( \psi^2 \psi^2 \) contribution in the onshell-Susy case. Using \[48\], \[49\] one finds

\[
\phi^2 = \frac{8f}{3\beta} (\psi^1 \psi^2) + \frac{32}{9} \frac{f^3}{\beta^2} (K^{-1})_1^2 (\psi^1 \psi^1) + (\geq 4 \text{ fermions}) = -\frac{\psi^1 \psi^2}{f} + O(1/\Lambda)
\] (70)

and that

\[
\phi_1 = -\frac{\psi^1 \psi^1}{f} + O(1/\Lambda)
\] (71)

in agreement with \[66\]. From this we find again that \( \phi^1 \) cannot contain, onshell, a \( \psi^2 \psi^2 \) fermionic pair, as also seen in \[66\]. This is in contradiction with the result found in \[56\],
As a result the IR constraint \( X^2 = 0 \) leads to results that onshell/offshell are not correct, except when \( c_1 = 0 \) when agreement with the results of Section 4 exists (onshell).

One possibility for \( c_1 = 0 \) is in the limit the denominator in \( c_1 \) that is proportional (onshell) to the scalar masses product (see (36)), is infinite, i.e. the scalar masses that are projected out by the constraints are infinite. This may not be too surprising, given that the constraints \( X^2 = 0 \) and \( X\Phi_2 = 0 \), not involving the curvature tensor, are not sensitive to the spectrum of integrated scalars, which must thus be decoupled.

A second possibility for \( c_1 = 0 \) is when \( \det(R_{12}^{1k}) = 0 \), in which case agreement with Section 4 is again obtained onshell, even when the masses of the integrated scalars are finite. Such agreement exists if the Kahler potential has a symmetry that enforces \( c_1 \propto \det(R_{12}^{1k}) = 0 \), with \( \det(R_{11}^{1k}) \neq 0 \) and finite. For example one can consider a discrete R-symmetry, with \( \Phi_{1,2} \) of different R-charges \( q_{1,2} \); if such symmetry of \( L \) exists, then these conditions can indeed be realized, with \( q_1 \neq q_2 \). In particular, different discrete R-charges for \( \Phi_{1,2} \) forbid the term in \( K \supset \epsilon_4 (\Phi_1^1)^2 (\Phi_2)^2 + h.c. \) that is present in (42) and then agreement with Section 4 exists. Thus in the presence of such symmetry the constrained superfields method based on the infrared constraint \( X^2 = 0 \) can give correct results onshell. One should keep however in mind that it is the offshell constraint that is relevant when coupling the goldstino field to matter, in a superfield language.

6 Conclusions.

For a general nonlinear sigma model, we studied the properties of the superfield \( X \) that violates the conservation of the Ferrara-Zumino supercurrent and breaks supersymmetry and the conformal symmetry. We investigated the properties that this field satisfies for microscopic models of an arbitrary Kahler potential, in the presence of matter fields. We also investigated the decoupling of the massive scalars (sgoldstino and matter scalars) and the effect of this on the properties of \( X \). The study can also be relevant for identifying the offshell couplings of the goldstino/sgoldstino to matter fields by using an effective approach to couple their corresponding non-linear superfields.

As it is known, in the absence of matter superfields, the action with \( K = X^\dagger X \) and an effective superpotential \( W = fX \) and a constraint \( X^2 = 0 \), provide a superfield description of the Akulov-Volkov action for the goldstino fermion and a non-linear realization of supersymmetry. This constraint essentially integrates (projects) out the scalar partner of goldstino, the sgoldstino. Such scenario can be realized in an effective model in which \( K \) has higher dimensional Kahler terms which provide a mass term for the sgoldstino; this can then be integrated out via the eqs of motion, to become a goldstino composite and enforce the condition \( X^2 = 0 \). An example in this direction is provided at one-loop level by the familiar O’Raifeartaigh model of spontaneous supersymmetry breaking, with small supersymmetry breaking, upon integrating out the other two massive superfields.
In the presence of additional matter fields beyond the goldstino superfield, the situation is more subtle and was analyzed in detail in this work. Offshell the scalar component of $X$, $\phi_X$, is now a mixture of the scalars present in the theory, sgoldstino and scalar matter field (squark, slepton). Using the general form of $X$ for an arbitrary $K$ and a linear superpotential in the goldstino superfield, we computed the form of $X$ after integrating out the scalar degrees of freedom (sgoldstino and scalar matter fields), which had acquired mass via the higher dimensional Kahler terms. As a result of this, for the two-fields case $X$ has the property that offshell $X^3 = 0$ while $X^2 \neq 0$. Thus the previous conjecture in the literature $X^2 = 0$ used as a condition to identify the sgoldstino has a restricted validity. It was also shown that the sgoldstino and scalar matter field have expressions that are such that their corresponding non-linear superfields satisfy offshell higher order polynomial constraints, such as cubic conditions $\Phi_1^3 = \Phi_1^2 \Phi_2 = \Phi_1 \Phi_2^2 = \Phi_2^3 = 0$ where $\Phi_1$ ($\Phi_2$) denote the goldstino (matter) superfields, respectively. The offshell expressions of $\Phi_{1,2}$ can be used to couple these non-linear multiplets to matter. The cubic conditions shown above can change in the case of more complicated superpotentials such as the R-parity violating ones or in the presence of more superfields. The cubic conditions of $\Phi_{1,2}$ mentioned are also more general because, unlike the field $X$ and its properties, are independent of the choice for the improvement term.

For a better understanding of our results, we examined the consequences of imposing the superfield constraints $X^2 = X \Phi_j = 0$ with $\Phi_j$ a matter superfield; these constraints were in the past conjectured to project out the sgoldstino and the scalar component of $\Phi_j$, which are however implicitly assumed to be infinitely massive. As a result, while $\Phi_{1,2}$ do satisfy offshell cubic conditions as those mentioned above, their exact form is different from the general case discussed above, due to different values of some coefficients $c_{1,2}$. Therefore the results derived from the superfield constraints have a restricted validity. For two fields case correct onshell results are possible when the curvature tensor $R_{ij}^{kl}$ of the Kahler manifold satisfies the condition $c_1 \sim \det R_{2k}^{ij} = 0$, with $\det R_{1k}^{ij} \neq 0$. This may be possible if the action has a discrete R-symmetry under which the superfields have different $R$-charges. One should keep however in mind that it is actually the offshell constraint that is relevant when coupling the goldstino field to matter, in a “non-linear” superfield language.

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