Abstract

In this note we present a recipe which transforms any pull-push along a span of categories into a push-pull along a cospan and vice versa, based on a theorem from Guitart.

A RECIPE FOR BASE CHANGE

Consider a lax commutative square

\[
\begin{array}{ccc}
A & \rightarrow^s & B \\
\downarrow_t & & \downarrow_f \\
C & \rightarrow^g & D
\end{array}
\]

of small categories and a bicomplete category $V$. The natural transformation $fs \Rightarrow gt$ induces a base change natural transformation $tsts^* \Rightarrow g^*f!$ between the pull-push $tsts^*$ and the push-pull $g^*f!$ which both send functors $B \rightarrow V$ to functors $C \rightarrow V$. Dually, using right extensions instead of left extensions, one obtains another base change transformation $f^*g_* \Rightarrow s,t^*$.

In this note, we address two questions:

- Given a span of categories, which category should one use so that the base change transformation be assured to be an isomorphism? In other words: is it possible to systematically rewrite a pull-push along a span of categories $C \leftarrow A \rightarrow B$ as a push-pull along a cospan $C \rightarrow \square \leftarrow B$ of categories?

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• Given a cospan of categories, which category should one use

\[ C \rightarrow B \leftarrow D \]

so that the base change transformation be assured to be an isomorphism? In other words: is it possible to systematically rewrite a pull-push along a cospan of categories \( C \rightarrow D \leftarrow B \) as a push-pull along a span \( C \leftarrow ? \rightarrow B \) of categories?

We give an affirmative answer to both questions, using a theorem of Guitart: one can always replace the box \( ? \) with the category universally fitting in the lax square diagram. In the first case, the category universally fitting in the lax square has seldom been called ‘co-comma’: we shall rename it flow sum for the occasion; it is obtained by adding arrows \( s(a) \rightarrow t(a) \) to the disjoint union \( B \sqcup C \). In the second case, the category universally fitting in the lax square is usually called ‘comma category’, we rename it flow product.

**Theorem** [1, 1.14]. The base change natural transformations

\[
t_s^* \Rightarrow g^* f_t^* \quad \text{and} \quad f^* g_s \Rightarrow s^* t^*
\]

are isomorphisms when the lax square is

• a flow sum square;

• or a flow product square.

Guitart studied base change transformations in the late 1970s; more recently his results have been extended to higher categories. Maltsiniotis gave an extension to derivators [2]. Then, Gepner, Haugseng and Kock showed that the base change transformation coming from a commutative square of \( \infty \)-groupoids is an equivalence if and only if the square is a fibre product [3, 2.1.6].

The note is organised in three sections. In the first section, we give the definitions of the base change natural transformation, the flow sum and the flow product. The second section is dedicated to proving the results, reducing to the case where \( C \) is punctual. In the last section, we give a direct application of the base change isomorphism, describing a context where the pull-push along a span of categories is functorial.

1 **BASE CHANGE**

1.1 **Pulling and pushing**

Let us fix a bicomplete category \( \mathcal{U} \). Given a functor \( f : A \rightarrow B \) between two small categories, composition by \( f \) induces a functor

\[
\text{Fun}(A, \mathcal{U}) \leftarrow \text{Fun}(B, \mathcal{U})
\]

called the pull-back functor. Since \( \mathcal{U} \) is bicomplete the pull-back functor admits two push-forward adjoints. We shall denote them \( f_! \dashv f^* \dashv f_* \).
1.2 Base change along lax squares

Consider a lax square of small categories, functors and natural transformations

\[
\begin{array}{ccc}
A & \xrightarrow{s} & B \\
\downarrow t & & \downarrow f \\
C & \xrightarrow{g} & D
\end{array}
\]

the natural transformation \(fs \Rightarrow gt\) induces another natural transformation \(s^*f^* \Rightarrow t^*g^*\). Using the unit of the adjunction \(f_! + f^*\) and the counit of the adjunction \(t_! + t^*\), one obtains

\[
t_!s^* \Rightarrow t_!s^*f_! \Rightarrow t_!t^*g_!f_! \Rightarrow g_!f_!
\]

a natural transformation \(t_!s^* \Rightarrow g_!f_!\) that we shall call the base change natural transformation. In a similar way, one also obtains a base change formula \(f_!g_! \Rightarrow s_!t^*\).

Remark 1.1. The natural transformation \(fs \Rightarrow gt\) also induces a natural transformation \(g_!t_! \Rightarrow f_!s_!\) which in turns gives us an alternative way

\[
t_!s^* \Rightarrow g_!t_!s^* \Rightarrow g_!f_!s_!s^* \Rightarrow g_!f_!
\]

d of writing the base change natural transformation.

1.3 Flow sum

Definition 1.2. The flow sum of a span of categories \(C \leftarrow A \rightarrow B\), is the category \(B \sqcup_A C\) obtained by enlarging the disjoint union of \(B\) and \(C\)

\[
B \sqcup A C \subset B \sqcup_A C
\]

with morphisms \(s(a) \xrightarrow{a} t(a)\) for each object \(a\) in \(A\) and relations generated by morphisms in \(A\).

Concretely,

- the set of objects of \(B \sqcup_A C\) is the disjoint union of the set of objects of \(B\) and the set of objects of \(C\);

\[
\text{Objects}
\left(B \sqcup_A C\right) := \text{Objects}(B) \sqcup \text{Objects}(C)
\]

- the sets of arrows are as follows. On the one hand, we have

\[
\begin{align*}
\text{Hom}_{B \sqcup_A C}(b, b') &:= \text{Hom}_B(b, b') \\
\text{Hom}_{B \sqcup_A C}(c, c') &:= \text{Hom}_C(c, c') \\
\text{Hom}_{B \sqcup_A C}(c, b) &:= \emptyset
\end{align*}
\]

for any objects \(b, b'\) in \(B\) and \(c, c'\) in \(C\).

On the other hand

\[
\text{Hom}_{B \sqcup_A C}(b, c) := \{\psi \theta\}
\]
is generated by words of the form $\psi a \phi$ where $a$ is an object of $A$, $\phi: b \rightarrow s(a)$ is an arrow in $B$ and $\psi: t(a) \rightarrow c$ is an arrow in $C$. All these morphisms are subject to an equivalence relation generated by

$$(\psi' \circ t(\theta))a \phi \sim \psi' a'(s(\theta) \circ \phi)$$

for every morphism $\theta: a \rightarrow a'$ in $A$, every morphism $\psi'$ in $C$ with source $t(a')$ and every morphism $\phi$ in $B$ with target $s(a)$.

Composition of morphisms is given by the composition in $B$ and the composition in $C$ in the obvious way.

$$b \xrightarrow{\phi} s(a) \xrightarrow{a} t(a) \xrightarrow{\psi} c$$

Figure 1: A morphism from $b$ to $c$ in the flow sum.

Remark 1.3. The relation $\sim$ is not transitive; two three letters words $\psi a \phi$ and $\psi' a' \phi'$ are thus equivalent if and only if there is a commutative hammock between them [Fig. 2].

Figure 2: A commutative hammock between $\psi_0 a_0 \phi$ and $\psi_n a_n \phi_n$; they represent the same arrow $b \rightarrow c$ in the flow sum.

Remark 1.4. The flow sum $B \coprod_A C$ fits in the lax commutative diagram

$$\begin{array}{ccc}
A & \xrightarrow{s} & B \\
\downarrow & \searrow & \downarrow \\
C & \xrightarrow{t} & B \coprod_A C
\end{array}$$

and is universal for that property. We shall call such a lax square a flow sum square. Notice that the flow sum is in general different from the lax push-out of the same span.
1.4 Flow product

**Definition 1.5.** The flow product of a cospan of categories \( B \xrightarrow{f} D \xleftarrow{g} C \) is the category \( B \times_D C \)

- whose objects are the triples \((b, c, \eta)\) with \( b \in B, c \in C \) and \( \eta : f(b) \to g(c) \) a map in \( D \);
- whose morphisms \((b, c, \eta) \to (b', c', \eta')\) are pairs \((\phi, \psi)\) where \( \phi : b \to b' \) is a morphism in \( B \) and \( \psi : c \to c' \) is a morphism in \( C \), such that the following square

\[
\begin{array}{ccc}
  f(b) & \xrightarrow{f(\phi)} & f(b') \\
  \eta & \downarrow & \downarrow \eta' \\
  g(c) & \xrightarrow{g(\psi)} & g(c')
\end{array}
\]

commutes.

Composition is given by the formula \((\phi, \psi) \circ (\phi', \psi') = (\phi \circ \phi', \psi \circ \psi')\) for two composable morphisms \((\phi, \psi)\) and \((\phi', \psi')\).

**Remark 1.6.** The flow product \( B \times_D C \) fits in the lax commutative diagram

\[
\begin{array}{ccc}
  B \times_D C & \longrightarrow & B \\
  \downarrow & \searrow & \downarrow f \\
  C & \xrightarrow{g} & D
\end{array}
\]

and is universal for that property. We shall call such a lax square a flow product square. Notice that the flow product is in general different from the lax product of the same cospan. The flow product is also called the comma category and is alternatively denoted \((f, g), f/g\) or \(B \downarrow D C\) in the literature.

**Remark 1.7.** For any two categories \( C \) and \( B \), one can define the category of cospans from \( B \) to \( C \) whose objects are cospans of functors \( f : B \to D \leftarrow C : g \) and whose morphisms from \((f, D, g)\) to \((f', D', g')\) are functors \( h : D \to D'\) so that \( f' = hf \) and \( g' = hg \). One can define in a similar way the category \( \text{Span}(B, C) \). The construction \( D \mapsto B \times_D C \) may be enhanced into a functor from \( \text{Cosp}(B, C) \) to \( \text{Span}(B, C) \). Similarly, the construction \( A \mapsto B \coprod_A C \) may be enhanced into a functor in the other way. The universal properties of the flow sum [1.4] and the flow product [1.6] yield

\[
\begin{array}{ccc}
  \text{Span}(B, C) & \xrightarrow{\text{Flow sum}} & \text{Cosp}(B, C) \\
  \text{Flow product} & \xleftarrow{} & \end{array}
\]

an adjunction between the flow sum functor and the flow product functor.

**Remark 1.8.** The fibre product is naturally a full subcategory

\[
B \times_D C \subset B \times_D C
\]

of the flow product: a triple \((b, c, \eta)\) belongs to the fibre product when \( \eta \) is an identity morphism.
1.5 Base change theorem

**Theorem 1.9.** Given a lax commutative square

\[
\begin{array}{ccc}
A & \xrightarrow{s} & B \\
\downarrow t & & \downarrow f \\
C & \xrightarrow{g} & D
\end{array}
\]

of small categories, the base change natural transformations

\[ t_!s^* \Rightarrow g^*f_! \quad \text{and} \quad f_!^*g^* \Rightarrow s_!t^* \]

are isomorphisms when the above square is either

- a flow sum square;
- or a flow product square.

**Proof.** The proof that \( t_!s^* \Rightarrow g^*f_! \) is an isomorphism can be deduced from the punctual case for both the flow product case [2.11] and the flow sum case [2.14]; it is the subject of the next section. For \( f_!^*g^* \Rightarrow s_!t^* \), one only needs to change \( U \) into \( U^{op} \).

**Remark 1.10.** In the literature, the base change isomorphism is also called the ‘projection formula’ or the ‘Beck-Chevalley condition’. Squares inducing a base change isomorphism are called exact by Guitart.

2 THE PROOF

2.1 Composition of base change transformations

Given two adjacent squares

\[
\begin{array}{ccc}
E & \xrightarrow{k} & A & \xrightarrow{s} & B \\
\downarrow l & & \downarrow t & & \downarrow f \\
F & \xrightarrow{r} & C & \xrightarrow{g} & D
\end{array}
\]

one can paste the two natural transformations \( fs \Rightarrow gt \) and \( g(tk \Rightarrow rl) \) into a natural transformation from \( fsk \) to \( grl \).

**Lemma 2.1** [4, 2.2]. Base change natural transformations can be composed along adjacent squares: given two adjacent squares the base change transformation \( l_!(sk)^* \Rightarrow (gr)^*f_! \) is the composite of \( (l_!k^* \Rightarrow r_!t_!)s^* \) and \( r_!(t_!s^* \Rightarrow g^*f_!) \).

2.2 Composition of flow products

Fibre products of categories can be composed along adjacent squares: given a diagram of small categories,

\[
\begin{array}{ccc}
E & \longrightarrow & A & \longrightarrow & B \\
\downarrow & & \downarrow & & \downarrow \\
F & \longrightarrow & C & \longrightarrow & D
\end{array}
\]
if the left and right squares are both fibre products, then the external rectangle is also a fibre product. This can be summarised

\[(B \times_D C) \times_C F = B \times_D F\]

in a short formula.

Similarly, flow products can be composed with fibre products, as explained in the following straightforward lemma.

**Lemma 2.2.** If the right square is a flow product square, then the external rectangle is a flow product square if and only if the left square is a fibre product square. In particular, one has

\[
(B \times_D C) \times_C F = B \times_D F
\]

in a short formula.

**Remark 2.3.** One has the same result

\[F \times_B (B \times_D C) = F \times_D C\]

if one would have stacked the fibre product square above the flow product square. Similarly, one has a dual result with flow sums and the disjoint union of small categories.

2.3 Pointwise base change

**Definition 2.4.** Let \(f : B \to D\) be a functor between two small categories and let \(d\) be an object of \(B\). The fibre \(f^{-1}(d)\) of \(f\) at \(d\) is the fibre product

\[
\begin{array}{ccc}
  f^{-1}(d) & \longrightarrow & B \\
  \downarrow^{d} & \swarrow & \downarrow^{f} \\
  * & \to & D
\end{array}
\]

while the flow \(\overrightarrow{f^{-1}}(d)\) of \(f\) to \(d\) is

\[
\begin{array}{ccc}
  \overrightarrow{f^{-1}}(d) & \longrightarrow & B \\
  \downarrow^{d} & \swarrow & \downarrow^{f} \\
  * & \to & D
\end{array}
\]

the flow product of the same cospan. Notice that by definition, the fibre \(f^{-1}(d)\) is a full subcategory of the flow \(\overrightarrow{f^{-1}}(d)\).

**Remark 2.5.** The flow of \(f\) to \(d\) is also denoted as a slice \(f_j/d\) in the literature.

**Definition 2.6.** A functor \(f : A \to B\) is said to be cofinal if any diagram of shape \(B\) valued in any category \(W\) can be restricted to a diagram of shape \(A\) along \(f\) without changing its colimit. This happens exactly when the flow \(d \times_D B\) of \(f\) from \(d\) is connected, for every object \(d \in D\).

When \(f\) is cofinal, the canonical map

\[
\pi: f_j/f^* \Rightarrow \pi!
\]

is an isomorphism, where \(\pi: B \to *\) is the canonical map to the punctual category.
**Definition 2.7.** Let \( f : A \to B \) be a functor. An arrow \( \phi : a \to a' \) in \( A \) is \( f \)-cocartesian if for every other arrow \( \phi' : a \to a'' \) such that \( f(\phi') = \psi \circ f(\phi) \), there exists a unique lift \( \overline{\psi} \) of \( \psi \) such that \( \phi' = \overline{\psi} \circ \phi \).

The functor \( f \) is called an opfibration if for every \( a \) in \( A \) and every \( \phi : f(a) \to b \), there exists a \( f \)-cocartesian lifting \( \overline{\psi} : a \to a' \) of \( \phi \).

**Lemma 2.8.** When \( f : B \to D \) is an opfibration, the inclusion \( f^{-1}(d) \subset f^{-1}(d) \) is cofinal.

**Proof.** Let \((b, \eta : f(b) \to d)\) be an object of the flow of \( f \) to \( d \). We shall show that the flow of the inclusion from \((b, \eta)\) (that is, it’s coslice by \((b, \eta)\)) is connected. Since \( f \) is an opfibration, \( \eta \) admits a cocartesian lift \( \theta : b \to e \) and we have thus a morphism \( \theta : (b, \eta) \to (e, \text{id}_d) \); it is initial and the flow from \((b, \eta)\) is thus connected. \( \square \)

**Lemma 2.9.** Consider a lax square

\[
\begin{array}{ccc}
A & \to & B \\
\downarrow & & \downarrow f \\
* & \to & D
\end{array}
\]

of small categories. The base change natural transformation is an isomorphism when either

- \( A \) is the flow \( f^{-1}(d) \) of \( f \) to \( d \in D \);
- or \( A \) is the fibre \( f^{-1}(d) \) of \( f \) at \( d \in D \) and \( f \) is an opfibration.

**Proof.** The first case comes from the punctual evaluation of \( f \):

\[
f_*F(d) = \lim_{(b, f(b) \to d)} F(b)
\]

where \( F \) is any functor from \( B \) to \( \mathcal{V} \); this is a classic result of category theory and can be found for example in *Categories for the Working Mathematician* [5, X.5.3]. The second case follows from the first case by cofinality [2.8] and composition of base change natural transformations [2.1]. \( \square \)

### 2.4 Flow product case

We are now ready to give the proof of the general case for the flow product. Consider the following

\[
\begin{array}{ccc}
B \times_D C & \to & B \\
\downarrow r & & \downarrow f \\
C & \to & D
\end{array}
\]

flow product square.

**Lemma 2.10.** The projection \( t : B \times_D C \to C \) is an opfibration.

**Proof.** For any object \((b, c, \eta : f(b) \to g(c))\) in \( B \times_D C \) and any map \( \phi : c \to c' \) in \( C \), a cocartesian lifting of \( \phi \) is given by the map \((\text{id}_b, \phi) : (b, c, \eta) \to (b, c', g(\phi) \circ \eta)\). \( \square \)
**Proposition 2.11.** In the case of a flow product square as above, the base change natural transformation is an isomorphism.

*Proof.* In order to show that the base change natural transformation is an isomorphism, it is enough to show that it gives an isomorphism at each $c$ in $C$. By pulling-back $t$ along $c : * \to C$

\[
\begin{array}{ccc}
\overline{f^{-1}(g(c))} & \xrightarrow{u} & B \times_D C \xrightarrow{s} B \\
\downarrow v & & \downarrow f \\
* & \xrightarrow{c} & C \xrightarrow{g} D
\end{array}
\]

one obtains an outer rectangle which is a flow product square [2.2]. As a consequence and since $t$ is an opfibration, the base change natural transformations

\[
v_1u^* \Rightarrow c^*t_1 \quad \text{and} \quad v_1u^*s^* \Rightarrow c^*g^*f_1
\]

are isomorphisms [2.9]. Then using the composition of base change natural transformations [2.1] and the 2-out-of-3 property of isomorphisms

\[
c^*t_1s^* \Rightarrow c^*g^*f_1
\]

is also an isomorphism as required. 

When $f$ is an opfibration, one can replace the flow product by the fibre product in order for the base change to be an isomorphism.

**Proposition 2.12** [6, 11.6]. Let

\[
\begin{array}{ccc}
C \times_D B & \longrightarrow & B \\
\downarrow & & \downarrow f \\
C & \xrightarrow{g} & D
\end{array}
\]

be a fibre product of small categories and assume $f$ is an opfibration, then the base change natural transformation is an isomorphism.

*Proof.* Following the same line of thoughts, we pull-back along $c : * \to C$ and end up with

\[
\begin{array}{ccc}
\overline{f^{-1}(g(c))} & \xrightarrow{u} & C \times_D B \xrightarrow{s} B \\
\downarrow v & & \downarrow f \\
* & \xrightarrow{c} & C \xrightarrow{g} D
\end{array}
\]

a pull-back rectangle and since the pull-back of an opfibration is again an opfibration, we can conclude using the punctual case. 

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**2.5 Flow sum case**

As in the flow product case, we shall show that the base change formula is a point-wise isomorphism by pulling back along each object $c : * \to C$.

Given a lax square

\[
\begin{array}{ccc}
A & \xrightarrow{s} & B \\
\downarrow t & & \downarrow f \\
C & \xrightarrow{g} & D
\end{array}
\]
and an object $c \in C$, the functor $s$ induces a functor

$$t^{-1}(c) \xrightarrow{s} f^{-1}(g(c))$$

sending a pair $(a, \phi : t(a) \to c)$ to the pair $(s(a), fs(a) \to gt(a) \xrightarrow{g(\phi)} g(c))$.

**Lemma 2.13.** If $D$ is the flow sum of the span $C \leftarrow A \rightarrow B$, then the functor $s : t^{-1}(c) \to f^{-1}(g(c))$ is cofinal for every $c \in C$.

**Proof.** Let $b$ be an object of $B$ and $\psi a \phi$ be an arrow from $b$ to $c$ in the flow sum. We shall show that the flow of $s$ from $(b, \psi a \phi)$ is connected.

It is not empty since there is a morphism $\phi : (b, \psi a \phi) \to (s(a), \psi a)$. Let $\phi' : (b, \psi a \phi) \to (s(a'), \psi'a')$ be another object of the flow. Since the two morphisms $\psi a \phi$ and $\psi'a'\phi'$ are equal in the flow sum, they are connected by a commutative hammock diagram [Fig. 2]. As a consequence, any two objects of the flow can be connected via a zigzag coming from a hammock diagram: the flow is connected.

**Proposition 2.14.** If the lax square above is a flow sum square, the base change natural transformation is an isomorphism.

**Proof.** We want to show that the natural transformation

$$c^*t_!s^* \Rightarrow c^*g^*f_!$$

is an isomorphism for every $c \in C$. For this, let us consider this cube

whose top and bottom faces are commutative and whose other faces are lax commutative.

Since base change for the left face is an isomorphism [2.9], by the 2-out-of-3 property of isomorphisms and the composition of base change natural transformations [2.1], we need to show that the base change natural transformation is an isomorphism for the front rectangle made of the left face and the front face.

Now, this rectangle is equal to the back rectangle made of the back face and the right face; by the previous lemma [2.13], base change is an isomorphism for the back face and we know that it is also the case for the right face [2.9], so it is an isomorphism for the back rectangle. \qed
3 Functorial Pull–Push

We give here a direct application of the base change isomorphism for flow product squares and flow sum squares.

Remark 3.1. Given two cospans \( A \rightarrow B \leftarrow C \) and \( C \rightarrow D \leftarrow E \), using previous computations [2.2, 2.3] we get canonical isomorphisms

\[
A \times_B (C \times_D E) = (A \times_B C) \times_C (C \times_D X) = (A \times_B C) \times_D E
\]

or in other words: the flow product is associative. Similarly, flow sums are also associative.

Definition 3.2. Let \( \text{Cat}_{\text{span}} \) be the pseudo-category whose objects are the small categories and whose morphisms between two categories \( A \) and \( C \) are the spans \( A \leftarrow B \rightarrow C \). Composition of \( A \leftarrow B \rightarrow C \) with \( C \leftarrow D \rightarrow E \) is given by the span \( A \leftarrow B \times_C D \rightarrow E \).

\[
\begin{array}{ccc}
B & \rightarrow & D \\
\downarrow & & \downarrow \\
A & \leftarrow & C \\
\end{array}
\]

Figure 3: Composition of spans of categories.

Remark 3.3. The associativity of the flow product insures that the composition of spans given in the definition is associative; it does not have a unit though. Using the associativity of flow sums, one could instead build a category whose morphisms are given by cospans.

The assignment

\[
\left( A \leftarrow B \rightarrow C \right) \mapsto \left( \text{Fun}(A, U) \xrightarrow{g \cdot f^*} \text{Fun}(C, U) \right)
\]

gives us a functor \( \text{Cat}_{\text{span}} \rightarrow \text{Fun}_{\text{2,1}} \) from the pseudo-category of small categories and spans to the (2,1)-category of categories, functors and natural isomorphisms.

The same thing can be done using cospans as morphisms, in which case one obtains a similar theorem. One could also change the formula \( g \cdot f^* \) to \( g \cdot f^* \).

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