Partial differential equations

Stability by rescaled weak convergence for the Navier–Stokes equations

Stabilité faible pour le système de Navier–Stokes incompressible

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\textbf{A B S T R A C T}

We study a weak stability problem for the three-dimensional Navier–Stokes system: if a sequence \((u_0, n)\) of initial data, bounded in some scaling invariant space, converges weakly to an initial data \(u_0\) which generates a global regular solution, does \(u_0, n\) generate a global regular solution? Because of the invariances of the Navier–Stokes equations, a positive answer in general to this question would imply global regularity for any data, so we introduce a new concept of weak convergence (rescaled weak convergence) under which we are able to give a positive answer. The proof relies on profile decompositions in anisotropic spaces and their propagation by the Navier–Stokes equations.

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\textbf{R É S U M É}

On étudie la stabilité faible pour le système de Navier–Stokes: si une suite de données de Cauchy \((u_0, n)\) bornée dans un espace invarient par échelle, converge faiblement vers une donnée \(u_0\) engendrant une solution globale régulière, est-ce que \(u_0, n\) engendre une solution globale régulière? À cause des invariances de l’équation de Navier–Stokes, une réponse positive en toute généralité à cette question impliquerait la régularité globale pour toutes les données. Dans ce travail, nous fournissons une réponse positive dans le cadre d'un nouveau concept de convergence faible. La preuve est basée sur des décompositions en profils dans des espaces anisotropes et leur propagation par les équations de Navier–Stokes.

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\textbf{Version française abrégée}

On s’intéresse dans cette note à la question de la stabilité faible pour le système de Navier–Stokes homogène incompressible

\textbf{(NS)} \begin{align*}
\partial_t u + u \cdot \nabla u - \Delta u & = -\nabla p \quad \text{dans} \ \mathbb{R}^+ \times \mathbb{R}^3 \\
\text{div} u & = 0 \\
u|_{t=0} & = u_0,
\end{align*}

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 où \( p = p(t, x) \) et \( u = (u^1, u^2, u^3)(t, x) \) sont respectivement la pression et la vitesse d'un fluide visqueux incompressible. Rappelons que dans [1] (voir [10] dans le cadre des espaces de Besov) les auteurs ont montré un résultat de stabilité forte de dans \( \text{BMO}^{-1} \). Ici, nous alimerions aborder la question de la stabilité faible suivante : si \((u_{0,n})_{n \in \mathbb{N}}\), une suite bornée dans un \( \text{espace invariant par échelle, converge vers} \ u_0 \) au sens des distributions, avec \( u_0 \) engendrant une solution globale régulière, est-il de même pour \( u_{0,n} \) lorsque \( n \) est assez grand ?

Un premier pas dans cette direction a été réalisé dans [5] moyennant deux hypothèses supplémentaires. Comme il a été remarqué dans [5], une réponse positive à cette question impliquerait, à cause des invariances de (NS), la régularité globale pour toutes les données. Rappelons ces invariances : définissant pour tout nombre réel positif \( \lambda \) tout point \( x_0 \) de \( \mathbb{R}^3 \) l'opérateur

\[
A_{\lambda, x_0} \phi(t, x) := \frac{1}{\lambda} \phi \left( \frac{t}{\lambda^2} - \frac{x - x_0}{\lambda} \right).
\]

si \( u \) est une solution de (NS) associée à la donnée \( u_0 \), alors \( A_{\lambda, x_0} u \) résout (NS) avec la donnée \( A_{\lambda, x_0} u_0 \). Comme la résolution du système de Navier–Stokes pour toute donnée initiale régulière semble hors de portée, on est conduit naturellement à l'introduction d'une nouvelle notion de convergence faible, que nous appellerons du système de Navier–Stokes pour toute donnée initiale régulière semble hors de portée, on est conduit naturellement à l'introduction d'une nouvelle notion de convergence faible, que nous appellerons (pour « Rescaled convergence »).

Définition 1. On dit qu'une suite \((\phi_n)_{n \in \mathbb{N}}\) R-converge vers \( \phi_0 \) si, pour toute suite \((\lambda_n)_{n \in \mathbb{N}}\) de nombres positifs réels et toute suite \((x_n)_{n \in \mathbb{N}}\) de points dans \( \mathbb{R}^3 \), la suite \((A_{\lambda_n, x_n}(\phi_n - \phi_0))_{n \in \mathbb{N}}\) converge vers zéro dans le sens des distributions lorsque \( n \) tend vers l'infini.

Dans cet article, nous résolvons la question de la stabilité faible sous l'hypothèse de R-convergence. Notre résultat principal s'énonce comme suit (voir la définition 2 ci-dessous, où sont introduits les espaces de Besov anisotropes) :

Théorème 1. Soit \( q \) dans \( [0, 1] \) et soit \( u_0 \) dans \( B^{1,q}_{1,1} \) engendrant une unique solution globale pour (NS). Soit \((u_{0,n})_{n \in \mathbb{N}}\) une suite de champs de vecteurs à divergence nulle, bornée dans \( B^{1,1}_{1,1}(\mathbb{R}^3) \), telle que \( u_{0,n} \) R-converge vers \( u_0 \). Alors pour \( n \) assez grand, \( u_{0,n} \) engendre une unique solution globale pour (NS) dans l'espace \( L^2(\mathbb{R}^3; B^{1,1}_{1,1}) \).

Un exemple d'une telle suite \((u_{0,n})_{n \in \mathbb{N}}\) est fourni dans (6). La preuve de ce résultat repose en premier lieu sur une décomposition en profils anisotropes de la suite des données de Cauchy \((u_{0,n})_{n \in \mathbb{N}}\) dans l'esprit des travaux [4,6,11,13]. Cette décomposition se formule comme suit : \( u_{0,n} \) est égale, moyennant un terme de reste petit, à une somme finie de suites orthogonales de champs de vecteurs à divergence nulle (voir le théorème 2 ci-dessous). Ces suites sont obtenues à partir de la décomposition en profils établie dans [5] en groupant ensemble tous les profils de même échelle horizontale \( \lambda_n \). Ces suites sont de la forme

\[
A_{\lambda_n, x_0} \left( u_{\lambda_n, x_0}^j, w_{\lambda_n, x_0}^j, w_{\lambda_n, x_0}^{j,3} \right)_{j \in \mathbb{N}}.
\]

où on a noté \( f(x) = f(\lambda_n x, \beta x_3) \), avec \( h_{\lambda_n} \) une famille de suites de réels positifs tendant vers zéro et \( \lambda_{\lambda_n} \) une famille de suites de réels positifs, orthogonale au sens de (7).

Notons que la R-convergence de \( u_{0,n} \) vers \( u_0 \) intervient de manière cruciale dans cette étape : elle exclut de la décomposition en profils de \( u_{0,n} \) les suites de type (1) avec \( h_n \equiv 1 \), pour lesquelles, comme indiqué ci-dessus, le résultat semble hors de portée. Notons aussi que la condition de divergence nulle sur \( u_{0,n} \) permet d'inclure les termes de type (1) avec \( h_n \) tendant vers l'infini dans le terme de reste. Le fait que les seuls profils qui restent dans la décomposition sont du type (1) avec \( h_n \) tendant vers zéro est essentiel pour établir le théorème 1.

Une fois la décomposition en profils anisotropes établie, le point clé dans la preuve de la stabilité faible consiste à démontrer que chaque profil intervenant dans la décomposition de la donnée initiale engendre une solution globale pour (NS) dès que \( n \) est assez grand. Nous sommes ainsi amenés à établir deux résultats d'existence globale pour le système de Navier–Stokes associé à de nouvelles larges classes de données de Cauchy généralisant les exemples traités dans [7–9]. Enfin, on conclut la preuve du théorème en montrant que les solutions associées à chaque profil n'interagissent pas entre elles, grâce à l'orthogonalité des échelles \( \lambda_{\lambda_n} \), et ainsi que la somme de ces solutions est une solution approchée globale de (NS) pour la donnée \( u_{0,n} \).

1. Introduction

We are interested in the Cauchy problem for the three-dimensional, homogeneous, incompressible Navier–Stokes system

\[
\begin{cases}
\partial_t u + u \cdot \nabla u - \Delta u = -\nabla p & \text{in} \ \mathbb{R}^+ \times \mathbb{R}^3 \\
\text{div} u = 0 \\
u(t=0) = u_0,
\end{cases}
\]

(1.1)
where \( p = p(t, x) \) and \( u = (u^1, u^2, u^3)(t, x) \) are respectively the pressure and velocity of an incompressible, viscous fluid. The question of the existence of global, smooth solutions is a long-standing open problem since the pioneering works of J. Leray [15,16]. We refer for instance to [3] or [14] and the references therein, for recent surveys on the subject. An important point in the study of (NS) is its scale invariance, which reads as follows: defining the scaling operator, for any positive real number \( \lambda \) and any point \( x_0 \) of \( \mathbb{R}^3 \),

\[
A_{\lambda, x_0} \phi(t, x) \equiv \frac{1}{\lambda} \phi \left( \frac{t}{\lambda^2}, \frac{x - x_0}{\lambda} \right),
\]

(2)

if \( u \) solves (NS) with data \( u_0 \), then \( A_{\lambda, x_0} u \) solves (NS) with data \( A_{\lambda, x_0} u_0 \). In the following, we shall denote \( A_{1, 0} \) defined \( A_{1, 0} \). In this paper, we focus on the stability of global solutions. Let us recall that a strong stability result in BMO\(^{-1}\) was proved in [1] (see [10] for the Besov setting). Here we would like to address the question of weak stability:

If \( (u_{0, n})_{n \in \mathbb{N}} \), bounded in some scale invariant space, converges to \( u_0 \) in the sense of distributions, with \( u_0 \) giving rise to a global smooth solution, is it also the case for \( u_{0, n} \) for \( n \) large enough?

A first step in that direction was achieved in [5], under two additional assumptions to the weak convergence, one of which was an assumption on the asymptotic separation of the horizontal and vertical spectral supports. As remarked in [5], an example of a sequence \( (u_{0, n})_{n \in \mathbb{N}} \) bounded in a scale invariant space \( X_0 \) and converging weakly to 0 is

\[
u_{0, n} = n \varphi_0(n \cdot) = A_n \varphi_0
\]

with \( \varphi_0 \) an arbitrary divergence-free vector field. If the weak stability result were true, then since the weak limit of \( (u_{0, n})_{n \in \mathbb{N}} \) is zero, then for \( n \) large enough \( u_{0, n} \) would give rise to a unique, global solution. By scale invariance then so would \( \varphi_0 \), and this for any \( \varphi_0 \), so that would solve the global regularity problem for (NS). Another natural example is the sequence

\[
u_{0, n} = \varphi_0(-x_3) = A_{1, x_0} \varphi_0
\]

with \( (x_n)_{n \in \mathbb{N}} \) a sequence of \( \mathbb{R}^3 \) going to infinity. Thus sequences built by rescaling fixed divergence free vector fields according to the invariances of the equations have to be excluded from our analysis, since solving (NS) for any smooth initial data seems out of reach. This leads naturally to the following definition of rescaled weak convergence, which we shall call R-convergence.

**Definition 1.** We say that a sequence \( (\varphi_n)_{n \in \mathbb{N}} \) R-converges to \( \varphi_0 \) if for all sequences \( (\lambda_n)_{n \in \mathbb{N}} \) of positive real numbers and for all sequences \( (x_n)_{n \in \mathbb{N}} \) in \( \mathbb{R}^3 \), the sequence \( (A_{\lambda_n, x_n} (\varphi_n - \varphi_0))_{n \in \mathbb{N}} \) converges to zero in the sense of distributions, as \( n \) goes to infinity.

Remark that if the sequences defined by (3) and (4) R-converge to zero, then clearly \( \varphi_0 \equiv 0 \). On the other hand, the sequence

\[
u_{0, n}(x) = \varphi_0 \left( x_1, x_2, \frac{x_3}{n} \right)
\]

(5)

is easily seen to R-converge to zero for any \( \varphi_0 \) satisfying \( \varphi_0(x_1, x_2, 0) \equiv 0 \).

In this paper we solve the weak stability question under the R-convergence assumption instead of classical weak convergence. Actually the choice of the function space in which to pick the sequence of initial data is crucial, as for instance contrary to the examples (3) and (4), the sequence of initial data defined in (5) is not bounded in \( B^{-1+\frac{1}{p}}_p,\infty \) for finite \( p \) (one can consult [3] for an introduction to Besov spaces \( B^{s, q}_p \)). The function spaces we shall be working with are the anisotropic homogeneous Besov spaces \( B^{s, q}_{p, q'} \) which generalize the more usual isotropic Besov spaces \( B^s_p \) (for further details, see for instance [3,12,17]):

**Definition 2.** Let \( \hat{\chi} \) (the Fourier transform of \( \chi \)) be a radial function in \( \mathcal{D} (\mathbb{R}) \) such that \( \hat{\chi}(t) = 1 \) for \( |t| \leq 1 \) and \( \hat{\chi}(t) = 0 \) for \( |t| > 2 \). For \( (j, k) \in \mathbb{Z}^2 \), the horizontal resp. vertical truncations are defined by

\[
\begin{align*}
S^h_k f (\xi) &\equiv \hat{\chi} (2^{-k} \langle \xi_1, \xi_2 \rangle) \hat{f} (\xi) \quad \text{and} \quad \Delta^h_k \equiv S^{+h}_{k+1} - S^h_k, \\
S^v_j f (\xi) &\equiv \hat{\chi} (2^{-j} |\xi_3|) \hat{f} (\xi) \quad \text{and} \quad \Delta^v_j \equiv S^{+v}_{j+1} - S^v_j.
\end{align*}
\]

For all \( p \in [1, \infty] \) and \( q \in [0, \infty] \), and all \( (s, s') \in \mathbb{R}^2 \), \( s < 2/p, s' < 1/p \), the anisotropic homogeneous Besov space \( B^{s, q}_{p, q'} \) is defined as the space of tempered distributions \( f \) such that

\[
\|f\|_{B^{s, q}_{p, q'}} = \left\| 2^{ks+js'} \Delta^h_k \Delta^v_j f \right\|_{L^p} < \infty.
\]

In all other cases of indexes \( s \) and \( s' \), the Besov space is defined similarly, up to taking the quotient with polynomials.
Our main result states as follows.

**Theorem 1.** Let \( q \) be given in \([0, 1]\) and let \( u_0 \) in \( B^{1,1}_{q, q} \) generate a unique global solution to (NS). Let \((u_{0,n})_{n \in \mathbb{N}}\) be a sequence of divergence-free vector fields bounded in \( B^{1,1}_{1,1} \), such that \( u_{0,n} \) \( R\)-converges to \( u_0 \). Then for \( n \) large enough, \( u_{0,n} \) generates a unique, global solution to (NS) in the space \( L^2(\mathbb{R}^+; B^{1,1}_{2,1}) \).

An example of such a sequence of an initial data \((R\text{-converging to zero})\) is

\[
 u_{0,n}(x) := \left( u_0^1 \left( x_1, x_2, \frac{x_3}{n} \right), u_0^2 \left( x_1, x_2, \frac{x_3}{n} \right), 0 \right)
\]

(6)

where \((u_0^1, u_0^2)\) is a two-component divergence free vector field.

### 2. Ideas of the proof

We just give some basic ideas of our strategy, and we refer the reader to [2] for the complete proof. To prove Theorem 1, the first step consists of the proof of an anisotropic profile decomposition of the sequence of initial data \((u_{0,n})_{n \in \mathbb{N}}\), in the spirit of [4,6,11,13]. This result states as follows, where we have denoted \([f]_p(x) \defeq f(x_0, \beta x_3)\).

**Theorem 2.** Under the assumptions of Theorem 1 and up to the extraction of a subsequence, the following holds. There is a family of sequences \((\lambda_j^k)_{k \in \mathbb{N}}\) of positive real numbers, orthogonal in the sense that

\[
 j \not= k \implies |\log(\lambda_j^k/\lambda_{j'}^{k'})| \to \infty,
\]

(7)

and for all \( L \geq 1 \) there is a family of sequences \((h_{ij}^k)_{i \in \mathbb{N}}\) going to zero such that for any real number \( \alpha \) in \([0, 1]\) and for all \( L \geq 1 \), there are families of sequences of smooth divergence-free vector fields (for \( j \) ranging from 0 to \( L \)), \((v_{n,\alpha,L}^j, w_{n,\alpha,L}^j)_{n \in \mathbb{N}}\), \((v_{n,\alpha,L}^i, w_{n,\alpha,L}^i)_{n \in \mathbb{N}}\), such that the sequence \((u_{0,n})_{n \in \mathbb{N}}\) can be written under the form

\[
 u_{0,n} = u_0 + \sum_{j=1}^L A^i_{\alpha,L} \left( v_{n,\alpha,L}^j + h_{ij}^k \rho_{n,\alpha,L} \right)
\]

where the remainder term satisfies

\[
 \lim_{L \to \infty} \lim_{\alpha \to 0} \limsup_{n \to \infty} \sup_{L \geq 1} \sup_{\alpha \in [0, 1]} \sup_{n \in \mathbb{N}} \left\| e^{t\Delta} \rho_{n,\alpha,L} \right\|_{L^2(\mathbb{R}^+; B^{1,1}_{2,1})} = 0,
\]

(8)

while the following bound holds:

\[
 \sup_{L \geq 1} \sup_{\alpha \in [0, 1]} \sup_{n \in \mathbb{N}} \sum_{j=0}^L \left\| v_{n,\alpha,L}^{j,3} \right\|_{B^{1,1}_{2,1}} < \infty.
\]

(9)

These sequences are obtained from the profile decomposition derived in [5] by grouping together all the profiles having the same horizontal scale \( \lambda_n \). The \( R\)-convergence of \( u_{0,n} - u_0 \) to zero arises in a crucial way in that step: it excludes, in the profile decomposition of \( u_{0,n} \), sequences of types \((3)\) and \((4)\). Note also that the divergence free assumption on \( u_{0,n} \) allows us to guarantee that \( h_n \) goes to zero, which is essential to establish Theorem 1.

Once the anisotropic profile decomposition is established, the main step of the proof of Theorem 1 consists in proving that each individual profile involved in the decomposition of the initial data does generate a global solution to (NS) as soon as \( n \) is large enough. This leads to establishing global existence results for (NS) associated with new classes of arbitrarily large initial data generalizing the examples dealt in [7–9], and where the regularity is sharply estimated, in particular in scaling invariant (anisotropic) norms. More precisely, thanks to the scale invariance of (NS), we need to prove, on the one hand, that an initial data of the type

\[
 \Phi_{0,n} := \left( v_{0,n}^h + h_n w_{0,n}^h, w_{0,n}^3 \right)
\]

gives rise to a global solution to (NS) as soon as \( n \) is large enough (hence \( h_n \) is small enough). Following [7], we prove that an approximate, global solution to (NS) with data \( \Phi_{0,n} \) can be written under the form

\[
 \Phi_{n, app} := \left( v_{n}^h + h_n w_{n}^h, w_{n}^3 \right)
\]

where \( v_{n}^h \) solves the two-dimensional Navier–Stokes equations.
\[(N\text{S}2D)_{k_3}\begin{align*}
\partial_t v_n^h + v_n^h \cdot \nabla v_n^h - \Delta_h v_n^h &= -\nabla h \cdot p_n \quad \text{in } \mathbb{R}^+ \times \mathbb{R}^2, \\
\text{div}_h v_n^h &= 0, \quad v_{n|t=0} = v_{0,n}(\cdot, x_3),
\end{align*}\]

while \(w_n^3\) solves the transport–diffusion equation

\[(T_{h_3}) \begin{align*}
\partial_t w_n^3 + v_n^h \cdot \nabla w_n^3 - \Delta_h w_n^3 &= -h_n^3 \Delta_h^{-1} \partial_3 w_n^3 \quad \text{in } \mathbb{R}^+ \times \mathbb{R}^3, \\
w_{n|t=0}^3 &= w_{0,n}^3.
\end{align*}\]

and \(w_n^h\) is determined by the divergence-free condition on \(w_n\) which gives \(w_n^h \overset{\text{def}}{=} -\nabla h \Delta_h^{-1} \partial_3 w_n^3\). On the other hand, we also have to deal with initial data of the type

\[\Phi_{0,n} \overset{\text{def}}{=} u_0 + \left(\left(v_{0,n}^h + h_n w_{0,n}^h, w_{0,n}^3\right)\right)_{h_3}\]

and that is possible using the methods of [8] and [9], thanks to additional localization properties on the profiles in Theorem 2; we refer to [2] for details.

Finally, we prove that the sum of each global solution associated with each profile gives rise to a global, approximate solution to (NS). This is proved using an orthogonality argument showing that the profiles do not interact one with the other. Actually denoting by \(A_{n,j} \Phi_n^k\) the global solution associated with

\[A_{n,j}\left(\left(u_{n,\alpha,L}^{j,h} + h_n w_{n,\alpha,L}^{j,h}, w_{n,\alpha,L}^{j,3}\right)\right)_{h_n}\]

it is enough to prove that for each \(j \neq k\),

\[\limsup_{n \to \infty} \left\| A_{n,j} \Phi_n^{j,e} \otimes A_{n,k} \Phi_n^{k,e} \right\|_{L^1(\mathbb{R}^+; B_{2,1}^{\frac{1}{2}})} = 0.\]

Using the fact that \(B_{2,1}^{\frac{1}{2}}\) is an algebra along with the Hölder inequality, we have that for \(\gamma > 0\),

\[\left\| A_{n,j} \Phi_n^{j,e} \otimes A_{n,k} \Phi_n^{k,e} \right\|_{L^1(\mathbb{R}^+; B_{2,1}^{\frac{1}{2}})} \leq \left\| A_{n,j} \Phi_n^{j,e} \right\|_{L^{\frac{2}{1+\gamma}}(\mathbb{R}^+; B_{2,1}^{\frac{1}{2}})} \cdot \left\| A_{n,k} \Phi_n^{k,e} \right\|_{L^{\frac{2}{1+\gamma}}(\mathbb{R}^+; B_{2,1}^{\frac{1}{2}})}.
\]

Scaling arguments give easily that

\[\left\| A_{n,j} \Phi_n^{j,e} \right\|_{L^{\frac{2}{1+\gamma}}(\mathbb{R}^+; B_{2,1}^{\frac{1}{2}})} \sim \left(\frac{\lambda_n}{\lambda_0}\right)^{\gamma} \left\| \Phi_n^{j,e} \right\|_{L^{\frac{2}{1+\gamma}}(\mathbb{R}^+; B_{2,1}^{\frac{1}{2}})}\]

and

\[\left\| A_{n,k} \Phi_n^{k,e} \right\|_{L^{\frac{2}{1+\gamma}}(\mathbb{R}^+; B_{2,1}^{\frac{1}{2}})} \sim \frac{1}{\left(\frac{\lambda_n}{\lambda_0}\right)^{\gamma}} \left\| \Phi_n^{k,e} \right\|_{L^{\frac{2}{1+\gamma}}(\mathbb{R}^+; B_{2,1}^{\frac{1}{2}})}.
\]

so

\[\left\| A_{n,j} \Phi_n^{j,e} \otimes A_{n,k} \Phi_n^{k,e} \right\|_{L^1(\mathbb{R}^+; B_{2,1}^{\frac{1}{2}})} \leq \min\left\{ \left(\frac{\lambda_n}{\lambda_0}\right)^{\gamma}, \left(\frac{\lambda_n}{\lambda_0}\right)^{\gamma}\right\}.
\]

The orthogonality property (7) gives the result.

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