Confronting quasi-exponential inflation with WMAP seven

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We confront quasi-exponential models of inflation with WMAP seven years dataset using Hamilton Jacobi formalism. With a phenomenological Hubble parameter, representing quasi-exponential inflation, we develop the formalism and subject the analysis to confrontation with WMAP seven using the publicly available code CAMB. The observable parameters are found to fair extremely well with WMAP seven. We also obtain a ratio of tensor to scalar amplitudes which may be detectable in PLANCK.

I. INTRODUCTION

Present-day cosmology is inflowing into an era where it is becoming more and more possible to constrain the models of the early universe by precise data coming from highly sophisticated observational probes like WMAP\textsuperscript{1}, PLANCK\textsuperscript{2}, SDSS\textsuperscript{3}, ACBAR\textsuperscript{4}, QUaD\textsuperscript{5}. Such observations are gradually leading theoretical cosmology towards the details of physics at very high energies, and the possibility of testing some of the speculative ideas of recent years. Inflation – the most fascinating among them – was first proposed back in 1981 by Guth\textsuperscript{6}, and has since been developed by a wide variety of probable models\textsuperscript{7–14}. However, the inflationary scenario is more like a paradigm than a theory, since a specific compelling model is yet to be separated out from the spectrum of possible models and alternatives.

So far the most appealing prediction of inflationary paradigm was that it may generate spectra of both density perturbations and gravitational waves. Before the detection of CMB anisotropies by COBE\textsuperscript{15}, cosmological observations had limited range of scales to access, and it was sufficient to predict from an inflationary scenario a scale-invariant spectrum of density perturbations and a negligible amplitude of gravitational waves. However, since COBE\textsuperscript{15} and, of late, WMAP\textsuperscript{1} came forward, the spectra are now well-constrained over a wide range of scales ranging from 1 Mpc upto 10,000 Mpc. So the inflationary predictions should now be very precise to incorporate latest observations, say, from WMAP seven year run. The ongoing satellite mission PLANCK\textsuperscript{2} will restrict the theoretical predictions further. It promises to survey the ratio of the tensor to scalar amplitudes $r$ up to the order of $10^{-2}$\textsuperscript{16} (orders below WMAP\textsuperscript{1} predictions $r < 0.36$ at 95\% Confidence Level [C.L.]\textsuperscript{17}) so as to comment more precisely on primordial gravitational waves and the energy scale of inflation. PLANCK may even discriminate single field inflationary models from multi-field models by detecting (provided $f_{NL} \geq 5$) large non gaussianity. So this is high time to confront different class of inflationary models with latest data and forthcoming predictions.

The usual technique used in the inflationary scenario to solve the dynamical equations is the slow roll approximations\textsuperscript{18}. But it is not the only possibility for successfully implementing models of inflation and solutions outside the slow roll approximations have been found\textsuperscript{20}. To incorporate all the models irrespective of slow roll approximations Hamilton Jacobi formalism\textsuperscript{21–23} turned out to be very useful. The formulation is imitative by considering the inflaton field itself to be the evolution parameter. The key advantage of this formalism is that here we only need the Hubble parameter $H(\phi)$ to be specified rather than the inflaton potential $V(\phi)$. Since $H$ is a geometric quantity, unlike $V$, inflation is more naturally described in this language\textsuperscript{22, 23}. Further, being first order in nature, these equations are easily tractable to explore the underlying physics. Once $H(\phi)$ has been specified, one can, in principle, derive a relation between $\phi$ and $t$ which will enable him to get hold of $H(t)$ and the scale factor $a(t)$ therefrom. Therefore Hamilton Jacobi formalism provides us with a straightforward way of exploring inflationary scenario and related observational aspects.

In this article, we intend to confront quasi-exponential inflationary models with WMAP seven years data\textsuperscript{1} with a phenomenological Hubble parameter following Hamilton Jacobi formalism. The absence of time dependence in the Hubble parameter produces de-Sitter inflation which was very appealing from theoretical point of view but its acceptability is more or less limited considering present day observations. So, certain deviation from an exact exponential inflation turns out to be a good move so as to go along with latest as well as forthcoming data. In general these models are called quasi-exponential inflation. Our primary intention here is to confront this quasi-exponential inflation with WMAP seven using the publicly available code CAMB\textsuperscript{24}. Nevertheless, as it will turn out, the analysis also predicts a detectable tensor to scalar ratio so as to reflect significant features of quasi-exponential inflation in forthcoming PLANCK\textsuperscript{2} data sets as well. Thus, PLANCK may directly verify (or put further constraint to) our analysis by detecting (or pushing further below the upper bound of) gravitational waves.

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II. MODELING QUASI-EXPONENTIAL INFLATION BY HAMILTON JACOBI

The Hamilton Jacobi formalism allows us to express the Friedmann equation in terms of a first order differential equation \[ \left[ H'(\phi) \right]^2 - \frac{3}{2 M_p^2} H(\phi)^2 = -\frac{1}{2 M_p^4} V(\phi) \] and the evolution of the scalar field \[ \dot{\phi} = -2 M_p^2 H'(\phi) \]
where a prime denotes a derivative with respect to the scalar field \( \phi \) and a dot a time derivative. The above two equations govern the inflationary dynamics in Hamilton Jacobi formalism. The shape of the associated potential can be obtained by rearranging terms of Eqn. (2.1) to give
\[ V(\phi) = 3 M_p^2 H^2(\phi) \left[ 1 - \frac{1}{3} \epsilon_H \right] \]
where \( \epsilon_H \) has been defined as
\[ \epsilon_H = 2 M_p^2 \left( \frac{H'}{H} \right)^2 \]
We further have
\[ \frac{\ddot{\phi}}{\dot{\phi}} = H(\phi)^2 \left[ 1 - \epsilon_H \right] \]
So \( \epsilon_H < 1 \) implies accelerated expansion. It is customary to define another parameter by
\[ \eta_H = 2 M_p^2 \left( \frac{H''}{H} \right) \]
It is worthwhile to mention here that the parameters \( \epsilon_H \) and \( \eta_H \) are not the usual slow roll parameters. But in the slow roll limit \( \epsilon_H \to \epsilon \) and \( \eta_H \to \eta - \epsilon \) \[18\], \( \epsilon \) and \( \eta \) being usual slow roll parameters.

Let us consider a phenomenological Hubble parameter representing quasi exponential inflation
\[ H(\phi) = H_{inf} \exp \left[ \frac{\rho}{M_p^4} \frac{\phi}{p(1 + \phi M_p)} \right] \]
where \( p \) is a dimensionless parameter and \( H_{inf} \) is a constant having dimension of Planck Mass. The value of the constants can be fixed from the conditions for successful inflation and the observational bounds. As it will turn out in subsequent analysis this Hubble parameter can indeed be cast into a form of quasi exponential inflation for some choice in the parameter space.

The two parameters \( \epsilon_H \) and \( \eta_H \) in the Hamilton Jacobi formalism take the form
\[ \epsilon_H = \frac{2}{p^2(1 + \phi M_p)^4}, \quad \eta_H = -\frac{2 (1 + 2 p + 2 p \phi M_p)}{p^2(1 + \phi M_p)^4} \]

Now, the condition for inflation may be put forward using equation of state parameter which has the form
\[ \omega(\phi) = \frac{P(\phi)}{\rho(\phi)} = -\frac{4}{3 p^2(1 + \phi M_p)^4} - 1 \]
\[ |p| > \frac{\sqrt{2}}{(1 + \phi_{end} M_p)^2} \]
From the constraint \( \omega < -\frac{1}{3} \) during inflation, we get
\[ |p| \leq \frac{\sqrt{2}}{(1 + \phi_{end} M_p)^2} \]
where \( \phi_{end} \) is the value of the inflaton field at the end of inflation. In order to implement a successful model of inflation both the restrictions on \( \omega \) should be satisfied.
First note that if $p$ is negative then from Eqn. (2.7) and Eqn. (2.14) we have $\phi > 0$, i.e. $\phi(t)$ increases with time which incorporates the so called graceful exit problem [26, 27]. We discard the values of $p$ greater or equal to $\sqrt{2}$ as well, since $p \geq \sqrt{2}$ would imply $\epsilon_H < 1$ for any value of $\phi$ giving rise to the same problem. As a result the feasible range for $p$ would be: $0 < p < \sqrt{2}$. Further, for sufficient inflation we also need $|\eta| < 1$ during inflation. And violation of that condition after inflation drags the inflaton towards its potential minima quickly. But if $p < 0.586$ then $|\eta|$ will always be less than one. Imposing this condition further restricts the range of the otherwise free parameter $p$ within $0.586 < p < \sqrt{2}$. We shall use a representative value for the parameter $p$ within the above range later on in this paper while confronting CAMB [24] outputs with WMAP seven [1].

Now to derive the expression for the scale factor we shall make use of the relation

$$\frac{\dot{\phi}^2}{a(\phi)} = H(\phi)$$

(2.12)

Plugging Eqn. (2.7) into Eqn. (2.2) and using Eqn. (2.12) we have

$$a(\phi) = a_e \exp \left[-p(1 + \frac{\phi}{M_P})^3 \right]$$

(2.13)

where $a_e \equiv a(\phi_{end}) \exp \left[-p(1 + \frac{\phi_{end}}{M_P})^3 \right]$, $a(\phi_{end})$ being the scale factor at the end of inflation.

The primary quantities related to inflation have been summarized in Table I. We see that $|\eta| \approx 1$ after the end of inflation, and so slow roll would be a very good approximation throughout the inflationary period, though we do not utterly require them in this formalism [24].

Table I: Different parameters for different values of $p$ within its allowed range

| $p$ | $\epsilon_H < 1$ | $|\eta|$ | $\phi_{end}$ | $\phi_{in}$ | N | $V(\phi_{in})^{1/4}$ | $10^{16}$ GeV |
|-----|----------------|---------|-------------|----------|---|-----------------|--------------|
| 0.60 | 0.535 | 0.385 | 0.535 | 7.260 | 50 | 1.005 |
| 0.70 | 0.421 | 0.414 | 0.421 | 6.845 | 56 | 0.901 |
|       |         |         |         | 7.448 | 70 | 0.907 |

That the above scale factor indeed represents quasi-exponential inflation will transpire from the following analysis. We first expand $H(\phi)$ into the power series of $\phi$

$$H(\phi) = H_1 \left[ 1 - \frac{1}{p(1 + \phi/M_P)} + \frac{1}{2!} p^2(1 + \phi/M_P)^2 - ... \right]$$

(2.14)

where we have defined $H_1 \equiv H_{infinite} \exp \left[ \frac{1}{p} \right]$. The above expansion, at next to leading order, gives rise to

$$\phi \approx M_P \left[ \frac{6H_1(t - t_e)}{p} \right]^{1/3} - 1$$

(2.15)

Here $t_e = \frac{6H_1 t_{end}}{p} + (1 + \frac{\phi_{end}}{M_P})^3$ and $t_{end}$ corresponds to the time at the end of inflation.

The combination of Eqn. (2.13) and Eqn. (2.15) gives the scale factor as a function of cosmic time $t$

$$a(t) \approx a_e \exp \left[ H_1(t - t_e) \right]$$

(2.16)

So the expression for the conformal time turns out to be

$$\eta \approx -\frac{1}{H_1 a(t)}$$

(2.17)

Thus, our analysis indeed deals with quasi-exponential inflation. The higher order terms in the expansion (2.14) will, in principle, measure further corrections to the scale factor but the analytical solutions are neither always obtainable nor utterly required. Rather, one can directly confront the observable parameters with WMAP seven in order to constrain quasi-exponential inflation, as done in the rest of the article.

III. PERTURBATIONS AND OBSERVABLE PARAMETERS

A. Scalar curvature perturbation

The $k^{th}$ Fourier mode of the comoving curvature perturbation, obtained by solving Mukhanov-Sasaki equation in the slow-roll limit and adopting standard Bunch-Davies [28] vacuum as initial condition, is approximately given by

$$\mathcal{R}_k \approx \frac{pH_1 \eta \sqrt{2 \kappa}}{2M_P} \left[ \frac{6 \ln (H_1 a_e \eta)}{p} \right]^{2/3} \exp \left[ -i k \eta \right] \sqrt{2k} \left( 1 - \frac{i}{k \eta} \right)$$

(3.1)

So the dimensionless spectrum for the comoving curvature perturbation is

$$P_k(k) = \frac{p^2 H_1^2}{16M_P^2 \pi^2} \left[ 1 + k^2 \eta^2 \right]^{2/3} \left[ \frac{6 \ln (H_1 a_e \eta)}{p} \right]^{4/3}$$

(3.2)

Now to evaluate the spectrum at horizon exit $k = aH$, we notice that

$$1 + k^2 \eta^2 = 2 - \frac{2}{6H_1 p^2(t_e - t)}$$

(3.3)

To arrive at Eqn. (3.3) we have used Eqn. (2.17) and Eqn. (2.19) respectively. This reflects the features of quasi-exponential behavior where the effect of the evolution of the scalar field has been directly taken into considerations, without using $k = -\eta^{-1}$ a priori. We shall,
of course, use the relation \( k = -\eta^{-1} \) finally while estimating parameters. Loosely speaking, the argument is analogous to putting a specific value for a variable after integration (which is more accurate) rather than putting it before (which may be a good approximation at the best). Somewhat similar treatment, though from a different physical perspective, has been employed in [29].

Therefore the power spectrum for \( \mathcal{R}_k \) evaluated at horizon crossing is now given by

\[
P_{\mathcal{R}}(k)_{|k = aH} = \frac{p^2 H^2}{8\pi^2 M_p^2} \left( \left( \frac{6A(k)}{p^4} \right)^{4/3} - \frac{6A(k)}{p^2} \right) \tag{3.4}\]

where we have defined \( A(k) \equiv \ln \left( H_1 a_e k^{-1} \right) \). Also at horizon crossing \( d \ln k = H_1 dt \), so the scalar spectral index looks

\[
n_S(k) = 1 - \frac{4}{3} \left[ \frac{6}{p^4} \right]^{4/3} A(k)^{1/3} - \frac{6}{p^2} A(k)^{1/3} - \frac{6A(k)}{p^2} \tag{3.5}\]

and its running

\[
n'_S(k) = -\frac{4}{3} \left[ \frac{6}{p^4} \right]^{8/3} A(k)^{2/3} - \frac{20}{p^2} \frac{6A(k)}{p^2} A(k)^{1/3} + \frac{36}{p^6} A(k) \left( \frac{6A(k)}{p^2} \right)^{4/3} - \frac{6A(k)}{p^2} \right) \tag{3.6}\]

Here note that we would get \( n_S = 1 - \frac{4}{3 A(k)} \) and \( n'_S = \frac{4}{3 A(k)} \) the expressions for the scalar spectral index and its running respectively if we had put \( k = -\eta^{-1} \) directly into Eqn. 3.3.

**B. Tensor perturbation**

The power spectrum for the tensor modes representing primordial gravitational waves, obtained in a similar way as before, is given by

\[
P_T(k) = 2P_h(k) = \frac{H_i^2}{\pi^2 M_p^2} \left( 1 + k^2 \eta^2 \right) \tag{3.7}\]

Now using Eqn. 3.3 we evaluate Eqn. 3.7 at horizon crossing

\[
P_T(k)_{|k = aH} = \frac{2H_i^2}{\pi^2 M_p^2} \left[ 1 - \frac{1}{6p^2 A(k)^{1/3}} \right] \tag{3.8}\]

Corresponding spectral index can be derived immediately from Eqn. 3.8 as

\[
n_T(k) = -\frac{1}{3} \left[ \frac{1}{(6p^2)^{1/3} A(k)^{4/3} - A(k)} \right] \tag{3.9}\]

and the running of the tensor spectral index is given by

\[
n'_T(k) = -\frac{1}{9} \left[ \frac{4(6p^2 A(k))^{1/3} - 3}{(6p^2)^{1/3} A(k)^{4/3} - A(k)} \right]^2 \tag{3.10}\]

TABLE II: Observable quantities as obtained from the theory of fluctuations

| \( p \)   | \( P_{\mathcal{R}}^{1/2} \) | \( n_S \) | \( n_T \) | \( n'_S \) | \( n'_T \) | \( r \) |
|----------|-----------------|----------|----------|----------|----------|-----|
| \( 10^{-5} \) | \( 0.60 \) | \( 0.49 \) | \( 0.97 \) | \( 1.51 \) | \( -4.25 \) | \( -3.85 \) |
| \( 10^{-4} \) | \( 0.64 \) | \( 0.47 \) | \( 0.95 \) | \( 1.50 \) | \( -4.20 \) | \( -3.80 \) |
| \( 10^{-5} \) | \( 0.70 \) | \( 0.41 \) | \( 0.94 \) | \( 1.43 \) | \( -4.33 \) | \( -3.40 \) |
| \( 10^{-2} \) | \( 0.74 \) | \( 0.35 \) | \( 0.92 \) | \( 1.37 \) | \( -4.46 \) | \( -3.00 \) |

Here tensor spectral index \( n_T = 0 \) and its running \( n'_T = 0 \), the results that we would obtain by the direct substitution of \( k = -\eta^{-1} \) into Eqn. 3.7.

We are now in a position of dealing with one of the most important observable parameters, namely, the ratio of tensor to scalar amplitudes. In the present context we have found

\[
r = \frac{16}{6A(k)^{4/3} p^{2/3}} \tag{3.11}\]

In the next section we shall comment on its numerical estimates.

The consistency relation can be obtained by combining Eqn. 3.9 and Eqn. 3.11, which in this case turns out to be

\[
r = -8n_T \left( 1 - \frac{r^{1/4}}{2p^{1/2}} \right) \tag{3.12}\]

The result is slightly different from the known relation \( r = -8n_T \). In the literature there are other techniques for obtaining a modified consistency relation. Such a modified consistency relation can be found in any analysis where higher order terms in the expansion of slow roll parameters are taken into account [30]. The consistency relation is also modified in the context of brane inflation [24] and non-standard models of inflation [32] where generalized propagation speed (less than one) of the scalar field fluctuations relative to the homogeneous background have been considered. Further deviation from the usual consistency relation can be found in [33] where tensor to scalar ratio has been shown to be a function of tensor spectral index, scalar spectral index and running of the tensor spectral index. Our approach is somewhat similar to these.

IV. CONFRONTATION WITH WMAP SEVEN

**A. Direct numerical estimation**

In Table II we have estimated the observable parameters from the first principle of the theory of fluctuation as derived in the previous section, for two sets of values of \( p \) within its allowed range, \( 0.58 < p < \sqrt{2} \). For estimation we have taken \( H_{inf} = 2.27 \times 10^{-6} \text{ Mpc}^{-1} \), \( a_e = 7.5 \times 10^{-31} \) and set the pivot scale at \( k_0 = 0.002 \text{ Mpc}^{-1} \). Table II reveals that the observable parameters as derived from our analysis are in excellent agreement with the current
TABLE III: Input parameters

\[
\begin{array}{cccccc}
\theta_0 & \Omega_{_B} & \Omega_{_C} & \eta_{_B} & \eta_{_C} & Mpc \\
Gyr & \Omega_{_m} & \Omega_{_\Lambda} & \eta_{_Rec} & \eta_{_0} & Mpc \\
13.708 & 0.2669 & 0.7331 & 0.0 & 285.15 & 14347.5
\end{array}
\]

TABLE IV: Different physical quantities as obtained from CAMB

\[
\begin{array}{cccccc}
H_0 & \tau_{_Reion} & \Omega_{_b}h^2 & \Omega_{_c}h^2 & T_{_CMB} & \Omega_0 \\
km/sec/Mpc & K \\
71.0 & 0.09 & 0.0226 & 0.1119 & 2.725 & \Omega_0
\end{array}
\]

observations as given by WMAP seven years data for \(\Lambda\)CDM background \([1]\).

Nevertheless, present day observations are eagerly waiting to detect primordial gravitational waves. The current observational bound on the ratio of tensor to scalar amplitudes as given by WMAP is \(r < 0.36\) at 95\% C.L. \([17]\). With the ongoing satellite experiment PLANCK promising to detect it down to the order of \(10^{-2}\) \([16]\) in near future, this parameter is now playing a pivotal role (along with non-gaussianity) to discriminate among different models \([34]\). The numerical estimation reveals that the parameter \(r\) is of the order \(10^{-2}\) for quasi-exponential inflation. Thus, PLANCK \([2]\) may directly verify (or put further constraint to) our analysis by detecting (or pushing further below the upper bound of) gravitational waves.

B. CAMB output and comparison

In what follows we shall make use of the publicly available code CAMB \([24]\) in order to confront our results directly with observational data. For CAMB, the Eqn. (3.4) has been set as initial power spectrum and the values of the initial parameters associated with inflation are taken from the Table II for \(p = 0.60\) within its allowed range \(0.586 < p < \sqrt{2}\). Also WMAP seven years dataset for \(\Lambda\)CDM background has been used in CAMB to obtain matter power spectrum and CMB angular power spectrum. We have set the pivot scale at \(k_0 = 0.002\) Mpc\(^{-1}\).

Table III shows input from the WMAP seven years dataset for \(\Lambda\)CDM background.

Table IV shows the output obtained from CAMB, which is in fine concord with WMAP seven years data. The results obtained here are for a representative value of the parameter \(p = 0.6\) within its allowed range.

The curvature perturbation is generated due to the fluctuations in the inflaton and remains almost constant on the super Hubble scales. Long after the end of inflation it makes horizon reentry creating matter density fluctuations through the gravitational attraction of the potential wells. These matter density fluctuations grew with time forming the structure in the universe. So the measurement of the matter power spectrum is very crucial as it is directly related to the formation of structure. In Fig 3 the CAMB output for the variation of the spectrum of the matter density fluctuations with the scale for quasi-exponential inflation and the best fit spectra of WMAP7 for \(\Lambda\)CDM + TENS \([35]\) have been shown and it represents true behavior indeed \([2]\).

In Fig 4 we confront CAMB output of CMB angular power spectrum \(C_{TT}^{\Lambda\)CDM + TENS\(}\) for quasi-exponential inflation, the best fit spectra of WMAP7 for \(\Lambda\)CDM + TENS and WMAP7 data with the multipoles \(l\)
The heights of the peaks are very susceptible to the baryon fraction. Also, the peak positions are sensitive to the curvature of the space and on the rate of cosmological expansion hence on the dark energy and other forms of the matter. In Fig. 4, the first and most prominent peak arises at $l = 220$ at a height of $5811 \mu K^2$ followed by two equal height peaks at $l = 537$ and $l = 815$. This is in excellent agreement with WMAP seven years data [1] for $\Lambda$CDM background. The direct comparison of our prediction for $p = 0.60$ in Fig. 5 shows fine match with WMAP data apart from the two outliers at $l = 22$ and $l = 40$. The above analysis is for a representative value of the parameter $p = 0.6$ within its allowed range.

The gravitational waves generated during inflation also remain constant on super Hubble scales having small amplitudes. But as its wavelength becomes smaller than horizon the amplitude begins to die off very rapidly. So the small scale modes have no impact in the CMB anisotropy spectrum only the large scale modes have little contribution and this is obvious from Fig. 5.

Further, in Fig. 6 and Fig. 7 we have plotted CMB TE and EE angular power spectrum respectively for quasi-exponential inflation and the best fit spectra of WMAP7 for $\Lambda$CDM + TENS and compared with WMAP seven years data. Both the plots resonate fairly well with the latest WMAP data [1].

Thus, from the entire analysis, it turns out that quasi-exponential inflation confronts extremely well with WMAP seven dataset. We expect that the forthcoming data from PLANCK would constrain quasi-exponential inflation further.

V. CONCLUSION

In this article we have confronted quasi-exponential models of inflation with WMAP seven year dataset using Hamilton-Jacobi formalism. We have first developed the formalism with a phenomenological Hubble parameter and have demonstrated how and to what extent the scenario measures deviation from standard exponential inflation (e.g., de-Sitter model). The deviation, incorporated through a new parameter $p$, has then been constrained by estimating the major observable parameters from the model and confronting them with WMAP seven year dataset. The observable parameters, as obtained from our analysis, turn out to be in good fit with the latest WMAP data.

We have further utilized the publicly available code CAMB [24] to compare with WMAP seven data [1] the matter power spectrum as well as CMB angular power spectra for the TT, TE and EE modes obtained from our analysis. This allows us to put stringent constraints on the model parameters. CAMB outputs for an emblematic value of the parameter $p$ within its allowed range $0.586 < p < \sqrt{2}$ are in excellent agreement with the latest WMAP data. Values of the most significant cosmological parameters have also been calculated using CAMB.
and have found to fair well with the observational bounds as given by WMAP seven years data. This leads us to conclude that quasi-exponential inflation confronts extremely well with WMAP seven within a certain parameter space.

Nevertheless, another appealing aspect of our analysis is the possibility of verifying quasi-exponential inflation by PLANCK. The current observational bound on the ratio of tensor to scalar amplitudes as given by WMAP is \( r < 0.36 \) at 95% C.L. [17]. With the ongoing satellite experiment PLANCK promising to detect it down to the order of \( 10^{-2} \) in near future, this parameter is now playing a pivotal role (along with non-gaussianity) to discriminate among different models. The numerical estimation reveals that the parameter \( r \) is of the order of \( 10^{-2} \) for quasi-exponential inflation. Thus, the forthcoming data from PLANCK can be confronted directly with quasi-exponential inflation. We are looking forward to confront this type of inflation with PLANCK and to have more exciting results in near future. We also plan to engage ourselves in estimating cosmological parameters using another highly useful code COSMOMC [36]. Results in this direction will be reported shortly.

Acknowledgments

BKP thanks Sourav Mitra and Dhiraj Hazra for useful discussions. BKP also thanks Council of Scientific and Industrial Research, India for financial support through Senior Research Fellowship (Grant No. 09/093 (0119)/2009). SP is supported by a research grant from Alexander von Humboldt Foundation, Germany, and is partially supported by the SFB-Tansregio TR33 “The Dark Universe” (Deutsche Forschungsgemeinschaft) and the European Union 7th network program “Unification in the LHC era” (PITN-GA-2009-237920).

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