The muon g-2 discrepancy: errors or new physics?

M. Passera*, W. J. Marciano† and A. Sirlin**

*Istituto Nazionale Fisica Nucleare, Sezione di Padova, I-35131, Padova, Italy
†Brookhaven National Laboratory, Upton, New York 11973, USA
**Department of Physics, New York University, 10003 New York NY, USA

Abstract. After a brief review of the muon g−2 status, we discuss hypothetical errors in the Standard Model prediction that could explain the present discrepancy with the experimental value. None of them looks likely. In particular, an hypothetical increase of the hadroproduction cross section in low-energy e+e− collisions could bridge the muon g−2 discrepancy, but is shown to be unlikely in view of current experimental error estimates. If, nonetheless, this turns out to be the explanation of the discrepancy, then the 95% CL upper bound on the Higgs boson mass is reduced to about 130 GeV which, in conjunction with the experimental 114.4 GeV 95% CL lower bound, leaves a narrow window for the mass of this fundamental particle.

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INTRODUCTION

The anomalous magnetic moment of the muon, aμ, is one of the most interesting observables in particle physics. Indeed, as each sector of the Standard Model (SM) contributes in a significant way to its theoretical prediction, the precise aμ measurement by the E821 experiment at Brookhaven [1] allows us to test the entire SM and scrutinize viable “new physics” appendages to this theory [2].

The SM prediction of the muon g−2 is conveniently split into QED, electroweak (EW) and hadronic (leading- and higher-order) contributions: aSMμ = aQED μ + aEW μ + aHLO μ + aHμO. The QED prediction, computed up to four (and estimated at five) loops, currently stands at aQED μ = 116584718.10(16)×10−11[3], while the EW effects provide aEW μ = 154(2)×10−11[4]. The latest calculations of the hadronic leading-order contribution, via the hadronic e+e− annihilation data, are in good agreement: aHLO μ = 6909(44)×10−11[5], 6894(46)×10−11[6, 7], 6921(56)×10−11[8], and 6944(49)×10−11[9]. The higher-order hadronic term is further divided into two parts: aHμO = aHμO(vp) + aHμO(lbl). The first one, −98(1)×10−11[6], is the O(α3) contribution of diagrams containing hadronic vacuum polarization insertions [10]. The second term, also of O(α3), is the hadronic light-by-light contribution; as it cannot be determined from data, its evaluation relies on specific models. Recent determinations of this term vary between 80(40)×10−11[11] and 136(25)×10−11[12]. The most recent one, 110(40)×10−11[13], lies between them. If we add this result to aHμO, for example the value of Ref. [6] (which also provides the hadronic contribution to the effective fine-structure constant, later required for our discussion), and the rest of the SM contributions, we obtain aSMμ = 116591778(61)×10−11. The difference with the experimental value aμEXP = 116592080(63)×10−11 [1] is Δaμ = aμEXP − aSMμ = ±302(88)×10−11, i.e., 3.4σ (all errors were added in quadrature). Similar discrepancies are found employing the aHLO μ values reported in Refs. [5, 8, 9]. For recent reviews of aμ see [7, 14, 15].

The term aHLO μ can alternatively be computed incorporating hadronic τ-decay data, related to those of hadroproduction in e+e− collisions via isospin symmetry [16, 17]. Unfortunately there is a large difference between the e+e− and τ-based determinations of aHLO μ, even if isospin violation corrections are taken into account [18]. The τ-based value is significantly higher, leading to a small (∼1σ) Δaμ difference. As the e+e− data are more directly related to the aHLO μ calculation than the τ ones, all recent analyses do not include the latter. Also, we note that recently studied additional isospin-breaking corrections somewhat reduce the difference between these two sets of data (lowering the τ-based determination) [19, 20], and a new analysis of the pion form factor claims that the τ and e+e− data are consistent after isospin violation effects and vector meson mixings are considered [21].

The 3.4σ discrepancy between the theoretical prediction and the experimental value of the muon g−2 can be explained in several ways. It could be due, at least in part, to an error in the determination of the hadronic light-by-light contribution. However, if this were the only cause of the discrepancy, aHμO(lbl) would have to move up by many standard deviations to fix it – roughly eight, if we use the aHμO(lbl) result of Ref. [13] (which includes all known uncertainties), and more than ten if the estimate of Ref. [12] is employed instead. Although the errors assigned to aHμO(lbl) are only educated guesses, this solu-
tion seems unlikely, at least as the dominant one.

Another possibility is to explain the discrepancy $\Delta a_\mu$ via the QED, EW and hadronic higher-order vacuum polarization contributions; this looks very improbable, as one can immediately conclude inspecting their values and uncertainties reported above. If we assume that the $g-2$ experiment E821 is correct, we are left with two options: possible contributions of physics beyond the SM, or an erroneous determination of the leading-order hadronic contribution $a^{HLO}_\mu$ (or both). The first of these two explanations has been extensively discussed in the literature; following Ref. [22] we will study whether the second one is realistic or not, and analyze its implications for the EW bounds on the mass of the Higgs boson.

**ERRORS IN THE HADRONIC CROSS SECTION?**

The hadronic leading-order contribution $a^{HLO}_\mu$ can be computed via the dispersion integral [23]

$$a^{HLO}_\mu = \frac{1}{4\pi^2} \int_{4m^2_h}^{s_\mu} ds K(s) \sigma(s),$$  

(1)

where $\sigma(s)$ is the total cross section for $e^+e^-$ annihilation into any hadronic state, with extraneous QED corrections subtracted off, and $s$ is the squared momentum transfer. The well-known kernel function $K(s)$ (see [24]) is positive definite, decreases monotonically for increasing $s$ and, for large $s$, behaves as $m^2_h/(3s)$ to a good approximation. About 90% of the total contribution to $a^{HLO}_\mu$ is accumulated at center-of-mass energies $\sqrt{s}$ below 1.8 GeV and roughly three-fourths of $a^{HLO}_\mu$ is covered by the two-pion final state which is dominated by the $\rho(770)$ resonance [17]. Exclusive low-energy $e^+e^-$ cross sections were measured at colliders at Frascati, Novosibirsk, Orsay, and Stanford, while at higher energies the total cross section was determined inclusively.

Let’s now assume that the discrepancy $\Delta a_\mu = a^{\text{EXP}}_\mu - a^{\text{SM}}_\mu = +302(88) \times 10^{-11}$, is due to $\sigma(s)$, and let us increase this cross section in order to raise $a^{HLO}_\mu$, thus reducing $\Delta a_\mu$. This simple assumption leads to interesting consequences. An upward shift of the hadronic cross section also induces an increase of the value of the hadronic contribution to the effective fine-structure constant at $M_Z$ [25],

$$\Delta \alpha^{(5)}_{\text{had}}(M_Z) = \frac{M_Z^2}{4\alpha \pi^2} P \int_{4m^2_h}^{s_\mu} ds \frac{\sigma(s)}{M_Z^2 - s},$$  

(2)

($P$ stands for Cauchy’s principal value). This integral is similar to the one we encountered in Eq. (1) for $a^{HLO}_\mu$. There, however, the weight function in the integrand gives a stronger weight to low-energy data. Let us define $a_i = \int_{4m^2_h}^{s_i} ds f_i(s) \sigma(s)$ ($i = 1, 2$), where the upper limit of integration is $s_i = M_i^2$, and the kernels are $f_1(s) = K(s)/(4\pi^2)$ and $f_2(s) = [M_i^2/(M_i^2 - s)]/(4\alpha \pi^2)$. The integrals $a_i$ with $i = 1, 2$ provide the contributions to $a^{HLO}_\mu$ and $\Delta \alpha^{(5)}_{\text{had}}(M_Z)$, respectively, from $4m^2_h$ up to $s_i$ (see Eqs. (1,2)). An increase of the cross section $\sigma(s)$ of the form $\Delta \sigma(s) = \epsilon \sigma(s)$ in the energy range $\sqrt{s} \in [\sqrt{s}_0 - \delta/2, \sqrt{s}_0 + \delta/2]$, where $\epsilon$ is a positive constant and $2m_\mu + \delta/2 < \sqrt{s}_0 < \sqrt{s}_0 - \delta/2$, increases $a_1$ by $\Delta a_1(\sqrt{s}_0, \delta, \epsilon) = \epsilon \int_{\sqrt{s}_0 - \delta/2}^{\sqrt{s}_0 + \delta/2} 2t \sigma(t^2) f(t^2) dt$. If we assume that the muon $g-2$ discrepancy is entirely due to this increase in $\sigma(s)$, so that $\Delta a_1(\sqrt{s}_0, \delta, \epsilon) = \Delta a_\mu$, the corresponding increase in $\Delta \alpha^{(5)}_{\text{had}}(M_Z)$ is

$$\Delta a_2(\sqrt{s}_0, \delta) = \Delta a_\mu \int_{\sqrt{s}_0 - \delta/2}^{\sqrt{s}_0 + \delta/2} 2t \sigma(t^2) f(t^2) dt = \int_{\sqrt{s}_0 - \delta/2}^{\sqrt{s}_0 + \delta/2} 2t \sigma(t^2) f(t^2) dt.$$

The shifts $\Delta a_2(\sqrt{s}_0, \delta)$ were studied in Ref. [22] for several bin widths $\delta$ and central values $\sqrt{s}_0$.

The present global fit of the LEP Electroweak Working Group (EWWG) leads to the Higgs boson mass $M_H = 84^{+34}_{-26}$ GeV and the 95% confidence level (CL) upper bound $M_H^{\text{UB}} \approx 154$ GeV [26]. This result is based on the recent preliminary top quark mass $M_t = 172.4(1.2)$ GeV [27] and the value $\Delta \alpha^{(5)}_{\text{had}}(M_Z) = 0.02758(35)$ [28]. The LEP direct-search 95% CL lower bound is $M_H^{\text{UB}} = 114.4$ GeV [29]. Although the global EW fit employs a large set of observables, $M_H^{\text{UB}}$ is strongly driven by the comparison of the theoretical predictions of the W boson mass and the effective EW mixing angle $\sin^2 \theta^\text{eff}_\text{lep}$ with their precisely measured values. Convenient formulæe providing the $M_V$ and $\sin^2 \theta^\text{eff}_\text{lep}$ SM predictions in terms of $M_H$, $M_t$, $\Delta \alpha^{(5)}_{\text{had}}(M_Z)$, and $\alpha_s(M_Z)$, the strong coupling constant at the scale $M_Z$, are given in [30]. Combining these two predictions via a numerical $\chi^2$-analysis and using the present world-average values $M_H = 80.399(25)$ GeV [31], $\sin^2 \theta^\text{eff}_\text{lep} = 0.23153(16)$ [32], $M_t = 172.4(1.2)$ GeV [27], $\alpha_s(M_Z) = 0.118(2)$ [33], and the determination $\Delta \alpha^{(5)}_{\text{had}}(M_Z) = 0.02758(35)$ [28], we get $M_H = 89^{+27}_{-21}$ GeV and $M_H^{\text{UB}} = 156$ GeV. We see that indeed the $M_H$ values obtained from the $M_V$ and $\sin^2 \theta^\text{eff}_\text{lep}$ predictions are quite close to the results of the global analysis.

The $M_H$ dependence of $a^{HLO}_\mu$ is too weak to provide $M_H$ bounds from the comparison with the measured value. On the other hand, $\Delta \alpha^{(5)}_{\text{had}}(M_Z)$ is one of the key inputs of the EW fits. For example, employing the more recent (and slightly higher) value $\Delta \alpha^{(5)}_{\text{had}}(M_Z) = 0.02768(22)$ [6] instead of 0.02758(35) [28], the $M_H$ prediction shifts down to $M_H = 88^{+32}_{-24}$ GeV and $M_H^{\text{UB}} = 145$ GeV. In [22] we considered the new values of $\Delta \alpha^{(5)}_{\text{had}}(M_Z)$ obtained shifting 0.02768(22) [6] by $\Delta a_2(\sqrt{s}_0, \delta)$ (including their un-
certainties), and computed the corresponding new values of $M_{H}^{UB}$ via the combined $\chi^{2}$-analysis based on the $M_{W}$ and $\sin^{2}\theta_{\text{eff}}^{\mu}$ inputs (for both $\Delta\alpha_{\text{em}}^{\mu}(M_{Z})$ and $a_{\mu}^{\text{HLO}}$ we used the values reported in [6]). Our results show that an increase $\varepsilon\sigma(s)$ of the hadronic cross section (in $\sqrt{s} \in [\sqrt{s}_{0} - \delta/2, \sqrt{s}_{0} + \delta/2]$), adjusted to bridge the muon $g-2$ discrepancy $\Delta\alpha_{\mu}$, decreases $M_{H}^{UB}$ further restricting the already narrow allowed region for $M_{H}$. We concluded that these hypothetical shifts conflict with the lower limit $M_{H}^{UB}$ when $\sqrt{s}_{0} \gtrsim 1.2$ GeV, for values of $\delta$ up to several hundreds of MeV. In [22] we pointed out that there are more complex scenarios where it is possible to bridge the $\Delta\alpha_{\mu}$ discrepancy without significantly affecting $M_{H}^{UB}$, but they are considerably more unlikely than those discussed above.

If $\tau$ data are incorporated in the calculation of the dispersive integrals in Eqs. (1.2), $a_{\mu}^{\text{HLO}}$ significantly increases to $7110(58) \times 10^{-11}$[17], $a_{\mu}^{\text{HLO}}$ (vp) slightly decreases to $-101(1) \times 10^{-11}$[6, 15], and the discrepancy drops to $\Delta\alpha_{\mu} = +89(95) \times 10^{-11}$, i.e. $\sim 1\sigma$. While using $\tau$ data almost solves the $\Delta\alpha_{\mu}$ discrepancy, it increases $\Delta\alpha_{\text{had}}^{\mu}(M_{Z})$ to $0.02782(16)$ [34, 17]. In [34] it was shown that this increase leads to a low $M_{H}$ prediction which is suggestive of a near conflict with $M_{H}^{UB}$, leaving a narrow window for $M_{H}$. Indeed, with this value of $\Delta\alpha_{\text{had}}^{\mu}(M_{Z})$ and the same above-discussed other inputs of the $\chi^{2}$-analysis, we find an $M_{H}^{UB}$ value of only 133 GeV.

Recently computed isospin-breaking violations, improvements of the long-distance radiative corrections to the decay $\tau^{-} \rightarrow \pi^{-}\pi^{+}\nu_{\tau}$ [19], and differentiation of the neutral and charged $\rho$ properties [20] reduce to some extent the difference between $\tau$ and $e^{+}e^{-}$ data, lowering the $\tau$-based determination of $a_{\mu}^{\text{HLO}}$. Moreover, a recent analysis of the pion form factor below 1 GeV claims that $\tau$ data are consistent with the $e^{+}e^{-}$ ones after isospin violation effects and vector meson mixings are considered [21]. In this case one could use the $e^{+}e^{-}$ data below $\sim 1$ GeV, confirmed by the $\tau$ ones, and assume that $\Delta\alpha_{\mu}$ is accommodated by hypothetical errors occurring above $\sim 1$ GeV, where disagreement persists between these two data sets. Reference [22] shows that this assumption would lead to $M_{H}^{UB}$ values inconsistent with $M_{H}^{BR}$.

In the above analysis, the hadronic cross section $\sigma(s)$ was shifted up by amounts $\Delta\sigma(s) = \varepsilon\sigma(s)$ adjusted to bridge $\Delta\alpha_{\mu}$. Apart from the implications for $M_{H}$, these shifts may actually be inadmissibly large when compared with the quoted experimental uncertainties. Consider the parameter $\varepsilon = \Delta\sigma(s)/\sigma(s)$. Clearly, its value depends on the choice of the energy range $[\sqrt{s}_{0} - \delta/2, \sqrt{s}_{0} + \delta/2]$ where $\sigma(s)$ is increased and, for fixed $\sqrt{s}_{0}$, it decreases when $\delta$ increases. Its minimum value, $\sim 4\%$, occurs if $\sigma(s)$ is multiplied by $(1 + \varepsilon)$ in the whole integration region, from $2m_{\pi}$ to infinity. Such a shift would lead to $M_{H}^{UB} \sim 70$ GeV, well below $M_{H}^{UB}$. Higher values of $\varepsilon$ are obtained for narrower energy bins, particularly if they do not include the $\rho-\omega$ resonance region. For example, a large $\varepsilon \sim 52\%$ increase is needed to accommodate $\Delta\alpha_{\mu}$ with a shift of $\sigma(s)$ in the region from $2m_{\pi}$ up to 500 MeV (reducing $M_{H}^{UB}$ to 139 GeV), while an increase in a bin of the same size but centered at the $\rho$ peak requires $\varepsilon \sim 8\%$ (lowering $M_{H}^{UB}$ to 127 GeV). As the quoted experimental uncertainty of $\sigma(s)$ below 1 GeV is of the order of a few per cent (or less, in some specific energy regions), the possibility to explain $\Delta\alpha_{\mu}$ with these shifts $\Delta\sigma(s)$ appears to be unlikely. Lower values of $\varepsilon$ are obtained if the shifts occur in energy ranges centered around the $\rho-\omega$ resonances, but also this possibility looks unlikely, since it requires variations of $\sigma(s)$ of at least $\sim 6\%$. If, however, such shifts $\Delta\sigma(s)$ indeed turn out to be the solution of the $\Delta\alpha_{\mu}$ discrepancy, then $M_{H}^{UB}$ is reduced to about 130 GeV [22].

It is interesting to note that in the scenario where $\Delta\alpha_{\mu}$ is due to hypothetical errors in $\sigma(s)$, rather than "new physics", the reduced $M_{H}^{UB} \lesssim 130$ GeV induces some tension with the approximate 95% CL lower bound $M_{H} \gtrsim 120$ GeV required to ensure vacuum stability under the assumption that the SM is valid up to the Planck scale [35] (note, however, that this lower bound somewhat decreases when the vacuum is allowed to be metastable, provided its lifetime is longer than the age of the universe [36]). Thus, one could argue that this tension is, on its own, suggestive of physics beyond the SM.

We remind the reader that the present values of $\sin^{2}\theta_{\text{eff}}^{\mu}$ derived from the leptonic and hadronic observables are respectively $\langle\sin^{2}\theta_{\text{eff}}^{\mu}\rangle = 0.23113(21)$ and $\langle\sin^{2}\theta_{\text{eff}}^{\mu}\rangle_{h} = 0.23222(27)$ [32]. In Ref. [22] we pointed out that the use of either of these values as an input parameter leads to inconsistencies in the SM framework that already require the presence of "new physics". For this reason, we followed the standard practice of employing as input the world-average value for $\sin^{2}\theta_{\text{eff}}^{\mu}$ determined in the SM global analysis. Since $M_{H}^{UB}$ also depends sensitively on $M_{t}$, in [22] we provide simple formulae to obtain the new values derived from different $M_{t}$ inputs.

**CONCLUSIONS**

We examined a number of hypothetical errors in the SM prediction of the muon $g-2$ that could be responsible for the present discrepancy $\Delta\alpha_{\mu}$ with the experimental value. None of them looks likely. In particular, following Ref. [22] we showed how an increase $\Delta\sigma(s) = \varepsilon\sigma(s)$ of the hadroproduction cross section in low-energy $e^{+}e^{-}$ collisions could bridge $\Delta\alpha_{\mu}$. However, such increases lead to reduced $M_{H}$ upper bounds (lower than 114.4 GeV – the LEP lower bound – if they occur in energy regions centered above $\sim 1.2$ GeV). Moreover, their amounts
are generally very large when compared with the quoted experimental uncertainties, even if the latter were significantly underestimated. The possibility to bridge the muon $g-2$ discrepancy with shifts of the hadronic cross section therefore appears to be unlikely. If, nonetheless, this turns out to be the solution, then the 95\% CL upper bound $M_{\mu^B}$ drops to about 130 GeV.

If $\tau$-decay data are incorporated in the calculation of $a_\mu^{SM}$, the muon $g-2$ discrepancy decreases to $\sim 1\sigma$. While this almost solves $\Delta a_\mu$, it raises the value of $\Delta a_\mu^{had}(M_z)$ leading to $M_{\mu^B} = 133$ GeV, increasing the tension with the LEP lower bound. One could also consider a scenario, suggested by recent studies, where the $\tau$ data confirm the $e^+e^-$ ones below $\sim 1$ GeV, while a discrepancy between them persists at higher energies. If, in this case, $\Delta a_\mu$ is fixed by hypothetical errors above $\sim 1$ GeV, where the data sets disagree, one also finds values of $M_{\mu^B}$ inconsistent with the LEP lower bound.

It has been suggested [37] that a $P$-wave electromagnetic bound state of $\pi^+\pi^-$(pionium) could enter the dispersion relations through 1\% mixing with the $\rho$ in a way that significantly increases the hadronic contribution to $a_\mu$. If so, such a state would give little change to the Higgs mass determination and would seem to refute our claims. However, Ref. [37] is in error. The required mixing is actually 0.1, not the erroneous 0.01 claim in [37], and such large mixing is not possible. The actual effect of pionium on $a_\mu$ is negligible [38].

If the $\Delta a_\mu$ discrepancy is real, it points to “new physics”, like low-energy supersymmetry where $\Delta a_\mu$ is reconciled by the additional contributions of supersymmetric partners and one expects $M_{\mu} \lesssim 135$ GeV for the mass of the lightest scalar [39]. If, instead, the deviation is caused by an incorrect leading-order hadronic contribution, it leads to reduced $M_{\mu^B}$ values. This reduction, together with the LEP lower bound, leaves a narrow window for the mass of this fundamental particle. Interestingly, it also raises the tension with the $M_{\mu}$ lower bound derived in the SM from the vacuum stability requirement.

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