The Throat as a Randall-Sundrum Model
with Goldberger-Wise Stabilization

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Abstract

An interesting feature of type IIB flux compactifications is the natural presence of strongly warped regions or ‘throats’. These regions allow for a 5d Randall-Sundrum model interpretation with a large hierarchy between the UV and IR brane. We show that, in the 5d description, the flux stabilization of this hierarchy (or, equivalently, of the brane-to-brane distance) can be understood as an implementation of the Goldberger-Wise mechanism. This mechanism relies on the non-trivial bulk profile of the so-called Goldberger-Wise scalar, which in addition has fixed expectation values at the boundaries and thereby stabilizes the size of the 5d interval. The Goldberger-Wise scalar is realized microscopically by the continuously varying flux of the Neveu-Schwarz 2-form potential $B_2$ on the $S^2$ cycle in the throat. Its back-reaction on the 5d geometry leads to a significant departure from a pure AdS$_5$ background. We also find that, for a wide range of parameters, the universal Kähler modulus of the 10d compactification plays the role of a UV-brane field in the equivalent 5d model. It governs the size of a large 4d curvature term localized at the UV brane. We hope that our simple 5d description of the stabilized throat will be useful in various phenomenological and cosmological applications and that refined versions of this construction will be able to account for all relevant details of the 10d model.
1 Introduction

Flux compactifications of type IIB string theory, or rather its low-energy limit type IIB supergravity, provide an attractive path towards potentially realistic 4d models of particle physics. Remarkably, by turning on fluxes in the supergravity background it is possible to stabilize all complex structure moduli and the dilaton in a controlled manner (see [1,2] for some of the original papers and [3] for recent reviews). In addition, in flat 4d compactifications of this type, spacetime is in general the warped product (rather than the direct product) of Minkowski space and the internal manifold. This can naturally induce a large hierarchy of scales.

In the work of Giddings, Kachru and Polchinski [2] the construction of such warped supergravity compactifications with fluxes and large hierarchies has been analyzed in detail (see also [4,5]). Spacetime is taken to be the warped product of Minkowski space with a compact Calabi-Yau 3-fold. To understand the part of the construction relevant to the hierarchy, consider first the unwarped case and a 3-fold which has a conifold singularity. This means that the neighborhood of a certain point has the geometry of a real cone over the compact 5d Einstein manifold $T^{1,1}$ (see e.g. [6]). By smoothing this singularity and turning on fluxes on certain cycles of the internal manifold, this region is deformed into a ‘throat’, i.e. a strongly warped region of spacetime whose geometry is approximately $\text{AdS}_5 \times T^{1,1}$. The radial coordinate of the cone becomes the radial coordinate of $\text{AdS}$; the throat connects the remainder of the compact manifold at one end with the blown-up tip of the cone at the other end [7–9]. The dilaton and the single complex structure modulus of the model are stabilized, and for a suitable choice of fluxes, a large hierarchy between the two ends is generated [2]. At this level of the construction, the universal Kähler modulus governing the size of the compact space remains massless. It can then be stabilized by nonperturbative effects [10,11] (see, however, [12]), by the interplay of perturbative and nonperturbative physics [13], or even in an entirely perturbative fashion [14,15].

The close relation between type IIB throat geometries and the warped 5d models of Randall and Sundrum [16,17] was emphasized early on [18,19]. Compactifications with an infinite $\text{AdS}_5 \times S^5$ region can be identified precisely with the Randall-Sundrum II model. By contrast, the 5d geometry of the stabilized $T^{1,1}$ throat relevant here deviates from $\text{AdS}_5$ and is only similar to the Randall Sundrum I model. The role of the Randall-Sundrum ‘UV brane’ is played by the compact manifold, and the ‘IR brane’ is the bottom of the throat.\footnote{We alert the reader that we will be using the term ‘brane’ with two different meanings, both for string-theoretic D-branes and for the 4d boundaries of a slice of AdS$_5$ space as is common in field theory model building with extra dimensions. It should always be clear from the context what kind of ‘brane’ we are referring to.} The stabilization of the brane-to-brane distance requires that the homogeneity of pure AdS space be broken. In the present case, this is realized by the deviation from AdS$_5$ geometry observed in the $T^{1,1}$ throat.

It is now natural to inquire about the details of the effective field theory describing the 5d physics and, in particular, the stabilization mechanism. We consider the
understanding of the 5d dynamics to be important in view of the wide range of phenomenological applications that such constructions might have. We recall the immense amount of cosmological and particle physics model building triggered by the Randall-Sundrum proposals (see [20] for a selective list of papers and reviews). More recently, many interesting applications of the type IIB flux-induced hierarchies have emerged (see e.g. [4, 5, 10, 21–24]). In particular, it is conceivable that the natural generation of hierarchies by fluxes affects the relation of particle phenomenology to the string landscape and its statistics [25]. One of the meeting points of these efforts is the 5d effective theory, which, we believe, should therefore be understood beyond the leading (pure AdS) approximation. Such an effective field theory will be useful independently of whether the Standard Model is eventually found to reside on the IR or the UV brane.

In the present paper, we derive some of the essential properties of the effective Randall-Sundrum-type model within the following approach: Starting from the well-known 10d solution, we identify the coordinate measuring physical distances along the throat in 5d units. The fundamental dynamical quantity varying along this coordinate is the flux of the Neveu-Schwarz 2-form potential $B_2$ on the $S^2$ cycle of the $T^{1,1}$ (or, equivalently, the 5-form flux on the $T^{1,1}$, or the radius of the $T^{1,1}$). This information is encoded in a 5d scalar field $H$ with a well-defined 5d profile, which is accompanied by a corresponding profile of the 5d curvature. It turns out that, within the 5d model, such a background is easily realized by postulating an appropriate 5d potential for $H$. Thus, we find ourselves precisely in the setting of the Goldberger-Wise stabilization mechanism [26] (see also [27]), with a slowly varying Goldberger-Wise scalar $H$. (This can be seen as a simplified description of one of the multi-scalar solutions of [8].) The back reaction of the varying scalar potential on the metric deforms the AdS space, and by fixing the values of $H$ on the boundaries (introducing very steep brane potentials) the length of the throat is also fixed. Furthermore, it turns out that, within the validity range of the 5d model, the unfixed universal Kähler modulus corresponds to a UV brane field – the radius of the 6d compact manifold at the UV end of the throat. Thus, we are able to write down an effective 5d action (with appropriate brane contributions) incorporating $H$, the universal Kähler modulus $\rho$ and, of course, the 5d radion governing the length of the 5d compactification interval.

The paper is organized as follows: In Sect. 2, we briefly review, following Refs. [6–9] and [2], the 10d solution for the conifold throat to the extent that it will be needed later on. We include a simplified derivation of the radial variation of the 5-form flux in the throat since the proper 5d description of this effect is one of the main objectives pursued in the rest of the paper. We also emphasize that, generically, a purely conical region is present between compact space and throat.

In Sect. 3, we turn to the effective 5d model. We relate the 5d coordinate measuring the invariant distance in the 5d Einstein frame to the radial coordinate of the conifold throat. Having identified the geometry and the profile of the 5-form flux in terms of this coordinate, we are able to construct the corresponding Goldberger-Wise-type stabilization model (the potential being $V(H) \sim H^{-8/3}$) and to relate the Goldberger-Wise scalar to the 10d quantities varying along the throat.
Section 4 deals with the 5d equivalent of the universal Kähler modulus. According to [5], where the explicit dependence of warp factor and metric on this modulus have been identified, the strongly warped region disappears in the limit of an extremely large volume. However, we emphasize and demonstrate quantitatively that, in a large intermediate range of the volume modulus, the length of the throat is effectively independent of this field. In this parameter range, the universal Kähler modulus is a brane field which has the peculiar feature of governing the size of a very large 4d curvature term localized at the UV brane.

Our results can be collected in the form of an explicit 5d effective action of Randall-Sundrum type, which includes both bulk and brane contributions. This action is displayed in Sect. 5 where we also give the relation between the main parameters of our 5d formulation and the usual string moduli and comment on a possible more complete identification in terms of the superfield formulation of the Randall-Sundrum model.

Section 6 contains our conclusions and some further comments concerning a possible manifestly supersymmetric description in 5 dimensions.

2 The supergravity solution in the warped region

To set our notation, we recall that the bosonic sector of type IIB supergravity contains the RR and NS 3-forms (we follow the conventions of [2, 28])

\[ F_3 = dC_2 \quad \text{and} \quad H_3 = dB_2 \]  

as well as the RR 5-form

\[ F_5 = dC_4. \]  

These forms enter the leading-order Lagrangian only in the combinations

\[ \tilde{F}_5 = F_5 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3, \]  

\[ G_3 = F_3 - \tau H_3 \quad \text{where} \quad \tau = C_0 + i \exp(-\phi). \]  

We will focus on solutions (or regions of solutions) where the RR 0-form potential \( C_0 \) vanishes and the dilaton \( \phi \) is constant.

The simplest strongly warped region or ‘throat’ available in type IIB supergravity is the AdS\(_5 \times S^5\) geometry arising in the vicinity of a stack of \( N \) D3 branes (see e.g. [18,19,29] and refs. therein). It can be understood as the deformation of flat 10d Minkowski space caused by the D3 branes. More specifically, the equations of motion demand [30,31] that the \( \tilde{F}_5 \) flux on the \( S^5 \) submanifolds enclosing the branes be accompanied by a warp factor \( h(r)^{-1/2} \),

\[ h(r) = 1 + \frac{R^4}{r^4}, \quad R^4 = 4\pi g_s N \alpha'^2 \frac{\pi^3}{\text{Vol } S^5} = 4\pi g_s N \alpha'^2, \quad g_s = e^\phi, \]  

where

\[ V = \text{Vol } S^5 = N \pi^3 / 4. \]
which enters the metric in the form

\[ ds^2 = h(r)^{-1/2}dx^2 + h(r)^{1/2}(dr^2 + r^2d\Omega_5^2) . \]  

(6)

Here and in the following, \( dx^2 \) stands for \( \eta_{\mu \nu} dx^\mu dx^\nu \), where \( \eta_{\mu \nu} \) is the 4d Minkowski metric. The metric of Eq. (6) is asymptotically flat, but near the branes, i.e. for \( r \ll R \), becomes the \( \text{AdS}_5 \times S^5 \) metric

\[ ds^2 = \frac{r^2}{R^2} dx^2 + \frac{R^2}{r^2} dr^2 + R^2 d\Omega_5^2 . \]  

(7)

It is crucial that the original flat metric (Eq. (6) with \( h(r) \) set to unity) is deformed only in a very mild way, which is fully specified by the single function \( h(r)^{1/2} \). This mildness of the deformation and the direct connection between 5-form flux and warp factor persist in the much more general geometries considered in [2, 5].

A very similar situation arises in the case of the conifold with branes at the singularity. The conifold is a real cone over the base \( T^{1,1} = (SU(2) \times SU(2)) / U(1) \); it has a Calabi-Yau metric described in [32]. Topologically, \( T^{1,1} \cong S^3 \times S^2 \), with both spheres shrinking to zero size at the singularity at the tip of the cone. Now consider the product of 4d Minkowski space with the conifold, such that the spacetime metric reads

\[ ds^2 = dx^2 + dr^2 + r^2 ds_{T^{1,1}}^2 . \]  

(8)

Placing \( N \) D3-branes at the conifold singularity corresponds to turning on \( N \) units of \( \tilde{F}_5 \) flux on \( T^{1,1} \) in the supergravity background. This leads to a deformation analogous to the \( \text{AdS}_5 \times S^5 \) case above,

\[ ds^2 = h(r)^{-1/2}dx^2 + h(r)^{1/2}(dr^2 + r^2 ds_{T^{1,1}}^2) , \]  

(9)

with the same function \( h(r) \), but with the radius \( R \) now given by [33]

\[ R^4 = 4\pi g_s N \alpha'^2 \frac{\pi^3}{\text{Vol} T^{1,1}} = 4\pi g_s N \alpha'^2 \frac{27}{16} . \]  

(10)

For large \( r \), far away from the branes, the geometry reduces to that of Eq. (8), while near the branes it is given by \( \text{AdS}_5 \times T^{1,1} \).

The AdS/CFT correspondence states that supergravity on this background is dual to a 4d conformal field theory (for a review of the gauge theory side of the models we are discussing, see [34],.) The radial coordinate \( r \) of AdS space translates into the renormalization group scale on the gauge theory side, with small \( r \) corresponding to infrared physics and large \( r \) to the ultraviolet. To let the throat end in the infrared, i.e. at some small but non-zero \( r \) and to stabilize the distance to that point, the conformality of the above solution in the infrared regime has to be broken. This can be realized by turning on \( F_3 \) flux on the \( S^3 \) cycle of the \( T^{1,1} \). As will become clear in a moment, the result will be a radial variation of the \( \tilde{F}_5 \) flux \( N \), which is promoted to a function \( N_{\text{eff}}(r) \).

One possibility to introduce \( F_3 \) flux is by placing, in addition to the \( N \) D3 branes, \( M \) fractional D3 branes at the conifold singularity. These are actually D5 branes wrapped
over the collapsing 2-cycle and are constrained to reside at the singular point. They source $M$ units of flux of the RR 3-form field strength $F_3$ on the $S^3$.

Another possibility to introduce $F_3$ flux is to blow up the conifold singularity to give the deformed conifold geometry, in which the $S^3$ remains of finite size. Now, $M$ units of $F_3$ flux can be put on the $S^3$ cycle. This creates a geometry as in the Klebanov-Strassler region [9] at the end of the throats of [7, 8]. Furthermore, $N_{D3}$ D3 branes may be placed in this end-region of the throat.\(^2\)

Our main interest is not in the precise way in which the throat is cut off in the infrared, but in the dependence of $N_{\text{eff}}$ on $r$ caused by the $F_3$ flux. This dependence, analyzed in detail in [7, 8], is in fact very easy to understand. The key observation is that, with $F_3$ flux being present only on the $S^3$ cycle of the $T^{1,1}$,

$$
(4\pi^2\alpha')^2 N_{\text{eff}}(r) = \int_{T^{1,1}} \tilde{F}_5 \sim \left( \int_{S^3} F_3 \right) \left( \int_{S^2} B_2 \right).
$$

(11)

Now, while the $F_3$ flux can not change continuously, the $B_2$ flux will generically have a non-trivial radial dependence, which is at the origin of the the radial variation of $N_{\text{eff}}$.

To be more precise, observe that

$$
(4\pi^2\alpha')^2 \left( N_{\text{eff}}(r_2) - N_{\text{eff}}(r_1) \right) = \int_{T^{1,1}} \tilde{F}_5 - \int_{T^{1,1}} \tilde{F}_5 = \int_{T^{1,1} \times (r_1, r_2)} d\tilde{F}_5 = \int_{T^{1,1} \times (r_1, r_2)} H_3 \wedge F_3,
$$

(12)

where the final integrals are over a segment of the throat, which corresponds to an interval $(r_1, r_2)$ in terms of the variable $r$. In compactifications leading to 4d Minkowski space, $G_3$ is imaginary self-dual [2]. Since the throat will ultimately have to be part of such a compactification, we can restrict ourselves to imaginary self-dual $G_3$, which implies that $H_3 = g_s \ast_6 F_3$. As one can easily see by counting the powers of the metric and its inverse, the warp factor drops out of this expression for $H_3$. Thus, the last integral in Eq. (12) can be evaluated in terms of $F_3$ and the simple conifold metric: We obtain

$$
\frac{dN_{\text{eff}}(r)}{dr} \sim \frac{g_s}{\alpha'^2} \int_{T^{1,1}} d^5y \sqrt{g_{\text{con}}(r, y)} F_{mnp} F^{mnp}.
$$

(13)

The quantization condition

$$
\frac{1}{2\pi\alpha'} \int_{S^3} F_3 = 2\pi M
$$

(14)

implies the scaling $F_3 \sim M\alpha'$ for the non-vanishing components of $F_3$. Taking into account the $r$ dependence introduced by the conifold metric and the fact that $F_3$ has no

\(^2\)The characteristic radius of the Klebanov-Strassler region is given by $R^4 \sim M^2 \alpha'^2 g_s^2$. Thus, for $g_s M^2 \gg N_{D3}$, the Klebanov-Strassler region is relatively flat compared to the curvature caused by the $N_{D3}$ D3 branes. One can then interpret this situation as a ‘throat within a throat’ [23], the second throat being a pure $\text{AdS}_5 \times S^5$ region with $N_{D3}$ units of $F_5$ flux. However, we will not pursue this more extended geometric picture here.
components in the $r$ direction, Eq. (13) evaluates to
\[
\frac{dN_{\text{eff}}(r)}{dr} = \frac{ag_s M^2}{r} \quad \text{or} \quad N_{\text{eff}}(r) = ag_s M^2 \ln(r/r_s),
\]
where $r_s$ is an integration constant. The $O(1)$ numerical prefactor $a = 3/(2\pi)$ can be determined using the explicit conifold metric and flux forms [33]. Apart from this prefactor, our simplified discussion provides the exact result.

Thus, we have now finally arrived at the geometry of the ‘conifold throat’ with varying warp factor. It is characterized by
\[
ds^2 = \tilde{h}(r)^{-1/2} dx^2 + \tilde{h}(r)^{1/2} (dr^2 + r^2 ds_{T_{1,1}}^2),
\]
with
\[
\tilde{h}(r) = 1 + \frac{a' \alpha'^2 g_s^2 M^2 \ln(r/r_s)}{r^4},
\]
where $a' = 4\pi a \cdot 27/16 = 81/8$. This follows by simply inserting $N_{\text{eff}}(r)$ in Eq. (10). The simplicity of this solution is not surprising since it is known that the $\tilde{F}_5$ flux, determined by $N_{\text{eff}}$, fully specifies the warp factor [2]. Even though this solution is exact, it is not physical. It has a singularity at $r = r_s$, which we assume to be resolved by a Klebanov-Strassler region. In other words, we can trust the above solution for $N_{\text{eff}} \gg g_s M^2$ or, equivalently, $r \gg r_s$.

If the Klebanov-Strassler region contains $N_{D3}$ explicit (i.e. not simulated by flux) D3 branes, then the throat ends at larger $r$. This is clear since the effective D3 charge visible near the end-region is $N_{D3} + O(g_s M^2)$. Another way to think about this is that, when moving along the throat to smaller $r$, one encounters $N_{D3}$ D3 branes. The effective charge then suddenly drops by $N_{D3}$ and, subsequently, the throat is terminated by a Klebanov-Strassler region ‘earlier than expected’. In summary, if $N_{D3}$ D3 branes are present, we can trust Eq. (17) only for $N_{\text{eff}} \gg N_{D3} + g_s M^2$.

In the UV-region, at large $r$, the throat will end when $\tilde{h}(r)$ approaches unity, i.e. when $\alpha'^2 g_s N_{\text{eff}}(r) \simeq r^4$. Just from the knowledge of $M$ and $N_{\text{eff}}$ at a certain $r$, it is impossible to tell where the throat will end in the UV. From the dual CFT perspective, this knowledge corresponds to information about higher-dimension operators, which is usually hard to access for the low-energy observer. In the conical region that follows at larger $r$, the integrated $\tilde{F}_5$ flux $N_{\text{eff}}$ continues to grow with $r$ as before, but the back-reaction is not strong enough to affect the geometry. We assume that at some still larger $r = R_c$, the approximate conifold geometry goes smoothly over to a compact Calabi-Yau orientifold geometry. The compactification radius can thus be approximately identified with $R_c$. Clearly, the total D3 charge of fluxes and localized sources in the bulk of this space has to compensate the $\tilde{F}_5$ flux present at the end of the conifold region at $r = R_c$.

\[3\]Note that, just from knowledge of $N_{\text{eff}}$ and $M$ at large $r$, it is impossible to tell whether the throat will end with or without explicit D3 branes. This is so because contributions to the overall $\tilde{F}_5$ flux from $F_5$ (first term on the r.h. side of Eq. (10)) and from 3-form fluxes (the second and third terms) mix under gauge transformations.
We thus arrive at the overall picture illustrated in Fig. 1. The compact space (e.g. a Calabi-Yau orientifold) has a conical region (cf. Eq. (8)) with non-vanishing $\tilde{F}_5$ flux. Going to smaller $r$, one reaches the throat region, where the back reaction of the flux deforms the geometry significantly and which is finally smoothly terminated by a Klebanov-Strassler solution.

![Diagram](image)

**Figure 1** Illustration of the way in which the throat is part of the overall compact space. The Klebanov-Strassler (KS) region may contain explicit D3 branes. In anticipation of the 5d description we have indicated which regions will later become the UV and IR branes and the 5d bulk.

Apart from describing our setting and introducing some notation, the purpose of this section was to emphasize the following points for later use: The running of $N_{\text{eff}}$ with $r$ is driven by the $M$ units of $F_3$ and can be derived in a very simple manner, independently of the details of the geometry and, in particular, the warp factor. In general, the throat is connected to the Calabi-Yau via a conical region, where the running continues just as within the throat. The IR end of the throat may contain explicit D3 branes and may therefore occur at any value of $N_{\text{eff}}$ above $N_{\text{eff, min}} \sim g_s M^2$.

## 3 The equivalent 5-dimensional model

Consider now the ‘throat’ region of Fig. 1 where, over sufficiently small distances in the radial direction, the geometry is approximately $\text{AdS}_5 \times T^{1,1}$. This is the region where Eqs. (16) and (17) apply and where, in the expression for the warp factor $\tilde{h}(r)$, the term $\sim 1/r^4$ dominates over the additive constant 1. The geometry deviates from $\text{AdS}_5 \times T^{1,1}$ in that the AdS curvature, characterized by the length scale $R_{\text{eff}}$,

$$R_{\text{eff}}^4(r) = g_s N_{\text{eff}}(r) \alpha'^2 \frac{27}{16} = d' \alpha'^2 g_s^2 M^2 \ln(r/r_s),$$

(18)
varies slowly along the radial direction. The region in question is terminated by a Klebanov-Strassler resolution of the conifold in the IR (at small $r$) and by the purely conical region together with the compact manifold in the UV (at large $r$).

At a position characterized by $r$, we can give an effective 5-dimensional description on length scales $L \gg R_{\text{eff}}(r)$. Furthermore, as long as $L$ is not too large, the change of $R_{\text{eff}}$ will be insignificant on length scales $L$, so that the 5d geometry will be approximately AdS$_5$. Note, however, that there is no length scale at which flat 5d space would provide a good approximation. If we want this 5d effective description to be valid in the vicinity of the UV brane, we also have to require that the size $R_c$ of the compact space (the ‘brane thickness’) is smaller than $L$. Of course, this size is governed by the universal Kähler modulus, which will be the subject of the next section. For the purpose of this section, we simply assume that it has been fixed at an appropriate value by some dynamics not relevant in the throat region.

To characterize the throat as a whole, the variation of $R_{\text{eff}}$ has to be taken into account. In other words, the negative 5d cosmological constant $\Lambda_5$, characteristic of AdS$_5$, has to be replaced by a potential energy $V(H)$, which must be a function of at least one 5d scalar field $H$ to allow for a spatial variation. It is intuitively clear that this field $H$, which must have a non-trivial profile $H(r)$ in the bulk, carries the information of the quantity $N_{\text{eff}}(r)$ or, equivalently, $R_{\text{eff}}(r)$ of the full 10d picture. As discussed at length in the previous section, the microscopic origin of this field is the flux of the NS 2-form potential $B_2$ on the $S^2$ cycle of the $T^{1,1}$.

Working in a 5d Einstein frame with canonically normalized $H$,

$$\mathcal{L}_5 = \frac{1}{2} M_5^3 R_5 - \frac{1}{2} (\partial H)^2 - V(H) + \cdots, \quad (19)$$

we can now inquire about the appropriate function $V(H)$. The profile $H(r)$ induced by this potential will give rise to a certain scalar-field energy density. The back reaction of this energy density has to modify the AdS$_5$ geometry in a way reproducing the metric of Eq. (16).

To find the potential $V(H)$, it is convenient first to identify an alternative radial coordinate which directly measures physical distances along the throat. Calling this coordinate $y$, such an infinitesimal distance, measured in units of the 5d reduced Planck mass, is given by $M_5 \, dy$. To compare this with the coordinate $r$, observe that a straightforward dimensional reduction of a model with the metric of Eq. (16) to 5d would give rise to an $r$-dependent coefficient of the 5d Ricci scalar, which we call $M_5^{3,\text{eff}}(r)$. A model with the Lagrangian of Eq. (19) could only result after a Weyl rescaling by an appropriate function of a radially varying scalar field. However, we can avoid this procedure by working with the $r$-dependent infinitesimal distance in units of $M_5,\text{eff}(r)$ and demanding

$$M_5 \, dy = M_5,\text{eff}(r) \sqrt{g_{rr}} \, dr = [M_5^{10} R_{\text{eff}}(r) \, \text{Vol} \, T^{1,1}]^{1/3} [R_{\text{eff}}(r)/r] \, dr. \quad (20)$$

Using $M_8^{10} = 2/[(2\pi)^7 \alpha'^4]$ and $\text{Vol} \, T^{1,1} = 16\pi^3/27$, this is further evaluated to give

$$M_5 \, dy = \left(\frac{(g_s M)^4}{3\pi^4}\right)^{1/3} (\ln(r/r_s))^{2/3} d(\ln(r/r_s)). \quad (21)$$
By the above calculation, $y$ is only determined up to an arbitrary additive constant. One possibility of fixing this constant of integration is to define

$$M_5 y = b \left( g_s^2 M^2 \right)^{2/3} \left( \ln(r/r_s) \right)^{5/3} = b \left( g_s^2 M^2 \right)^{-1} (g_s N_{\text{eff}}(r)/a)^{5/3},$$

(22)

where we have introduced the numerical coefficient $b = \frac{3^{2/3} 5^{1/3} - \pi^{-4/3}}{2}$. This is an important intermediate result: We have expressed the basic 5d quantity, the physical distance $y$ in units of $M_5$, through $g_s$, $N_{\text{eff}}(r)$ and $M$, the fundamental dimensionless parameters of the 10d geometry. To make the above equations more readable, we define the length scale

$$R_s = M_5^{-1} b \left( g_s^2 M^2 \right)^{2/3},$$

(23)

which, up to $O(1)$ factors, corresponds to the size of the $T^{1,1}$ in the Klebanov-Strassler region. (Of course, our definition remains useful if the throat ends at some $r_{IR} \gtrsim r_s$ and no actual Klebanov-Strassler region is present.) We can then write

$$(y/R_s) = \left( \ln(r/r_s) \right)^{5/3} = \left( \frac{N_{\text{eff}}(r)}{a g_s M^2} \right)^{5/3}.$$

(24)

In the following, we will treat $y/R_s$ as parametrically large.

The 5d metric can now be written as

$$ds_5^2 = e^{2A(y)} dx^2 + dy^2,$$

(25)

where the warp factor, following from Eqs. (16), (17) and (21) together with the Weyl rescaling used to go to the 5d Einstein frame, reads

$$A(y) \simeq (y/R_s)^{3/5} + O(\ln(y/R_s)) + \text{const}.\ (26)$$

Here the constant term is irrelevant since it can be absorbed in a rescaling of Minkowski space. We may also neglect the subleading logarithmic term, writing the warp factor as

$$A(y) = k(y) y, \quad k(y) \simeq R_s^{-1} (y/R_s)^{-2/5}.$$

(27)

Note that, up to the sign of $y$ (which is chosen to avoid a large number of ‘$-$’ signs in the following equations) and the slow variation of $k$, this corresponds to the now widely used metric conventions of [17].

We are now looking for a potential $V(H)$ such that the back-reaction of the varying scalar $H$ induces a varying curvature as in Eq. (27). In general, such an analysis requires the solution of the coupled equations of motion for the metric and $H$. However, in the present case, a simplified computation will be sufficient: Because the warp factor varies slowly, we can use the equation of motion of a scalar field with potential $V(H)$ in an AdS background, allowing for a $y$ dependence of the curvature scale $k$:

$$\left( \partial_y^2 + 4A'(y) \partial_y \right) H - \frac{\partial V}{\partial H} = 0.$$

(28)

4This corresponds to the ‘special case solution’ of Sect. 5 of [8], which we therefore find to be the appropriate description of the actual throat between the Klebanov-Strassler and the conical regions.
Furthermore, if the typical length scale for the variation of $H$ is larger than the curvature radius, we can neglect the second-derivative term:

$$k(y)\partial_y H \sim \frac{\partial V}{\partial H}.$$  (29)

The value of $k(y)$ is determined by an effective 5D bulk cosmological constant coming mainly from the potential term with $H$ set to its local VEV. In AdS space, the relation between cosmological constant and curvature is [17]

$$V(H) = -24 M_5^3 k^2,$$  (30)

which can be solved for $k$ and inserted in Eq. (29). Using in addition the relation $\partial_y H = (dV/dy)(\partial V/\partial H)^{-1}$, we arrive at

$$\frac{\partial V}{\partial H} \sim \left( \frac{-V}{M_5^3} \right)^{1/4} \left( \frac{dV}{dy} \right)^{1/2}.$$  (31)

Since Eq. (27) implies that

$$V \sim -M_5^3 R_s^{-2} (y/R_s)^{-4/5},$$  (32)

we can rewrite Eq. (31) as

$$\frac{\partial V}{\partial H} \sim M^{-21/8} R_s^{3/4} (-V)^{11/8}.$$  (33)

This now finally determines the desired functional dependence of $V$ on $H$:

$$V(H) \sim -M_5^7 R_s^{-2} H^{-8/3}.$$  (34)

Thus, we conclude that 5d gravity coupled to a scalar field $H$ with the potential of Eq. (34) reproduces the effective 5d geometry of the throat of the 10D compactification. The $y$-dependence of $H$ follows straightforwardly from the above:

$$H \sim M_5^{3/2} (y/R_s)^{3/10}.$$  (35)

It can also be easily verified that the conditions

$$|\partial_y^2 H| \ll |A'(y)\partial_y H| \quad \text{and} \quad (\partial_y H)^2 \ll |V|,$$  (36)

which justify our simplified treatment, are satisfied.

To describe the throat as a whole (cf. Fig. 2), we need to add an IR and UV brane with specific tensions and boundary conditions for $H$ to our 5d model. We assume the tensions to be positive and negative for the UV and IR brane respectively and the values to be such that both branes are static in an AdS space with curvature determined by the boundary values of $H$ and $V(H)$. To discuss the boundary conditions on $H$ explicitly, recall that
Figure 2: The throat with the values of \( N_{\text{eff}}(r) \) and \( R_{\text{eff}}(r) \) at several positions \( r \). The dotted line indicates that, in the presence of explicit D3 branes, the throat may end at \( r_{\text{IR}} > r_s \).

\( H \) substitutes the parameters \( R_{\text{eff}} \) (or, equivalently, \( N_{\text{eff}} \)) of the 10d construction. The explicit relations read

\[
H \sim M_5^{3/2} (R_{\text{eff}}/R_s)^2 \sim M_5^{3/2} (N_{\text{eff}}/N_s)^{1/2} \quad \text{with} \quad N_s = a g_s M^2.
\] (37)

Thus, the boundary condition \( H(y_s) \sim M_5^{3/2} \) will reproduce the IR end corresponding to a Klebanov-Strassler region with \( M \) units of \( F_3 \) flux. Field-theoretically, such a boundary condition can be realized by an appropriate brane potential for \( H \) with an extremely steep minimum.

Recall that, more generally, the throat may end at \( r_{\text{IR}} > r_s \) if explicit D3 branes are present at the IR end. The corresponding boundary condition reads \( H(y_{\text{IR}}) \sim M_5^{3/2} (N_{\text{IR}}/N_s)^{1/2} \), where \( N_{\text{IR}} = N_{\text{D3}} + a g_s M^2 \) can be chosen freely.

At the UV end, we can define \( N \) as the number of \( \tilde{F}_5 \) flux units on the \( T^{1,1} \) cycle at the ‘large’ or UV end of the conical region. This number is determined by localized sources, e.g. O3 planes and D3 branes, and regions with 3-form flux within the remainder of the compact space.\(^5\) In the conical region, this flux number changes according to Eq. (15). Assuming that the conical region is not too large, this change is very small compared to the change that occurs within the throat, so that we can identify the \( \tilde{F}_5 \) flux \( N_{\text{UV}} \) at the UV end of the throat (the IR end of the conical region) with the flux number \( N \) defined above. (A more detailed discussion of the subtle effect of the running inside the conical region will be given in the next section.) Thus, the UV boundary condition of the 5d model reads \( H(y_{\text{UV}}) \sim M_5^{3/2} (N_{\text{UV}}/N_s)^{1/2} \sim M_5^{3/2} (N/N_s)^{1/2} \).

\(^5\)Note that in the literature \( N = M K \) is frequently used to designate the effective D3 charge from \( M \) units \( F_3 \) flux on the \( S^3 \) cycle and \( K \) units of \( H_3 \) flux on its dual. This definition coincides with ours if the 3-form flux in question is mainly concentrated outside the ‘compact manifold’ of Fig. 1.
In summary, we have presented a 5d model (gravity plus minimally coupled scalar field, with the potential given above) which, upon compactification on an interval with boundary conditions $H(y_{UV}) \sim M_5^{3/2} (N_{IR/UV}/N_s)^{1/2}$, provides the 5d description of the conifold throat. In particular, the 5d bulk profile of $H$ fixes, together with the boundary conditions, the throat length $y_{UV} - y_{IR}$. Thus, in the case of the conifold throat, flux stabilization corresponds to the familiar Goldberger-Wise stabilization mechanism [26,27] of 5d Randall-Sundrum models. There is however an important difference: in the conifold throat the back-reaction of the scalar field on the geometry is crucial. It describes the effect of the $M$ units of $F_3$ flux – the duality cascade in the dual gauge theory.

The most important shortcoming of the effective 5d model presented so far is the absence of the universal Kähler modulus common to such type IIB supergravity compactifications [2]. We have avoided this issue by simply assuming that the typical radius of the compact space at the UV end is somehow stabilized. In the next section, we will relax this assumption and discuss the interplay of this degree of freedom with our 5d model of the throat.

4 The universal Kähler modulus as a brane-field

One of the more relevant effects of fluxes in compactifications of type IIB supergravity is to generate a potential for complex structure moduli and for the dilaton. However, at least one Kähler modulus remains massless, even in these constructions. In the limit where the warping goes to zero, this modulus is simply an overall scaling of the internal metric and hence corresponds to a change of the volume of the compact manifold.

In the presence of warping the scaling behavior is more subtle [5]. In terms of the metric of Eq. (16), the flat direction corresponds to a shift $\tilde{h}(r) \rightarrow \tilde{h}(r) + c - 1$ for an arbitrary value of the constant $c$. As in the unwarped case, this affects the volume of the manifold.

This realization of the volume modulus can, in fact, be understood very easily: The metric of Eq. (16) contains only two dimensionful parameters, $\alpha'$ and $r_s$. A volume modulus, if present, can only change the ratio of these two scales. Indeed, a rescaling $r_s \rightarrow r_s c^{1/4}$ corresponds, together with an appropriate rescaling of $r$ and $x$, to the shift in $\tilde{h}$ specified above.

If $c$ becomes extremely large, larger than $\tilde{h}(r_{IR})$, the throat disappears and the variation of $c$ corresponds to an overall scaling of the entire compact space. In this regime, the radius $R_c$ is bigger than the length scale $L$ at which our 5d effective description is defined. In other words, the “brane thickness” of the UV brane is so large that the 5d picture is lost.

Let us consider instead values of $c$ such that $R_c < L$. In the ‘compact manifold’, $\tilde{h}$ is approximately constant and the variation of $c$ corresponds again to a simple scaling. As far as the throat is concerned, it is crucial that the $\tilde{F}_3$ flux $N$ at the UV end of the conical region is fixed; it does not depend on the volume of the compact space. To a
very good approximation, this is also true for the $\tilde{F}_5$ flux $N_{UV}$ at the UV end of the throat (i.e. at the IR end of the conical region). Furthermore, neither the $F_3$ flux nor the presence of explicit D3 branes at the IR end of the throat are affected by the volume scaling. Thus, neither the boundary condition $N_{UV}$ nor $N_{IR}$ changes when $c$ varies and then, as we discussed in Sect. 3, the length of the throat is fixed. This means that, in the 5d description, the Kähler modulus plays the role of a massless UV-brane field while the 5d radion is already stabilized.

However, this picture is correct only at first approximation. The key to the $c$-independence of $N$ was its definition as the flux at the transition point between the conical and the more general compact geometries. This definition does not depend on the overall scaling. By contrast, $N_{UV}$ is defined at the transition point between conical and throat geometries. As we will now demonstrate, this transition point has non-trivial $c$-dependence, which is reflected in a weak $c$-dependence of $N_{UV}$. The resulting effect on the length of the throat is small compared to the effect on the compact region, as we will show explicitly. Nevertheless, for extremely large $c$ this effect will cut into the length of the throat such that, eventually, the throat disappears. This is consistent with the limit of weak warping discussed above.

We refer to the combination of compact space and conical region as the UV brane. From the 10d point of view, this is the area where the warp factor is, to a good approximation, constant. Thus, the universal Kähler modulus simply corresponds to an overall rescaling of this space. In particular, the flux number

$$N = N_{\text{eff}}(R_c) \simeq \frac{1}{(4\pi^2\alpha')^2} \int_{T^{1,1}} \tilde{F}_5$$

is invariant under this rescaling. From the point of view of the throat, it is determined, once and for all, by the localized sources and flux in the compact space, i.e. ‘on the other side’ of the $T^{1,1}$ submanifold at $r = R_c$.

We now focus on the conical region and the throat. If we insist on always using coordinates such that the warp factor in the conical region is unity, as in Eqs. (16) and (17), then $R_c$ can be identified with the universal Kähler modulus. Given the general $r$-dependence of $N_{eff}$ in throat and conical region, Eq. (15), one finds the constraint

$$N = a g_s M^2 \ln (R_c/r_s) ,$$

which fixes $r_s$ in terms of $R_c$. This gives the warp factor (cf. Eq. (17))

$$\tilde{h}(r) = 1 + \left( a'/a \right) \alpha^2 g_s \left[ N - a g_s M^2 \ln (R_c/r) \right] \frac{1}{r^4} .$$

The boundary between conical region and throat, $r = r_{UV}$, is then determined by the solution of the equation $\tilde{h}(r_{UV}) = 1$. Assuming that, at this boundary, the logarithmic term in Eq. (40) is small relative to $N$ and working to leading order in this small term, we find

$$r_{UV}^4 \simeq \left( a'/a \right) \alpha^2 g_s \left[ N - a g_s M^2 \frac{4}{4} \ln \left( \frac{R_c^4}{\left( a'/a \right) \alpha^2 g_s N} \right) \right] .$$
Thus, the conical region shrinks to zero size if $R_c^4$ takes the value

$$R_{c,\min}^4 = (\alpha'/a)\alpha^2 g_s N,$$

(42)

and our approximation remains valid as long as $(R_c/R_{c,\min}) \ll \exp(N/a g_s M^2)$. In the large-hierarchy case, which is our main interest in this paper, the last expression is roughly equal to the inverse warp factor and is thus very large. In other words, there is a large range in which the variation of $R_c$ (i.e. of the universal Kähler modulus) has very little effect on the throat length, as expressed by Eq. (41). In this domain, it is mainly just a scaling of the compact manifold at the UV end of the throat. Thus, we are led to the conclusion that, from the 5d point of view, the universal Kähler modulus is a field localized at the UV brane.

Let us now translate the above discussion to the 5d picture in a more quantitative way. From the 5d perspective, the fundamental scale is the reduced 5d Planck mass defined by $M_5$. Near the UV brane, $M_5$ is related to $M_{10}$ by $M_5^3 \approx M_{10}^8 R_{UV}^5$. For not too large $R_c$, we can identify $R_{UV}$ with $R_{c,\min}$, with the result that

$$R_c M_5 \sim (g_s N)^{2/3} (R_c/R_{c,\min}).$$

(43)

We can think of this as of the UV brane thickness in units of $M_5$. In the same units, the physical length $L_{\text{throat}}$ of the throat is given by

$$L_{\text{throat}} M_5 \sim (y_{UV} - y_{IR}) M_5 \sim (g_s M)^{4/3} \left[ \left( \frac{N_{UV}}{ag_s M^2} \right)^{5/3} - \left( \frac{N_{IR}}{ag_s M^2} \right)^{5/3} \right],$$

(44)

where $N_{UV}$ is the flux at the small end of the conical region or, equivalently, at the large end of the throat. Our interest is in the dependence of $L_{\text{throat}}$ on $R_c$. Thus, we cannot simply identify $N_{UV}$ with $N$, but rather we have to take care of this subtle distinction which is due to the running in the conical region:

$$N = N_{UV} + ag_s M^2 \ln(R_c/R_{c,\min}).$$

(45)

We now assume that $R_c$ grows by a factor $1 + \epsilon$ (where $\epsilon \ll 1$). Then, on the one hand, the thickness of the UV brane in units of $M_5$ increases by $\sim \epsilon (g_s N)^{2/3} R_c/R_{c,\min}$. On the other hand, the length of the throat, also measured in units of $M_5$, shrinks by $\sim \epsilon (g_s M)^{4/3} (N_{UV}/ag_s M^2)^{2/3}$. The ratio of these two quantities is $\sim R_c/R_{c,\min} > 1$, i.e. the throat shrinks less than the brane thickness grows.

This can be turned into an even more explicit argument for the Kähler modulus being a brane field: From the 5d perspective, it is perfectly acceptable to define the throat length either by Eq. (44) or, including the UV brane thickness into the size of the 5d interval, by the sum of Eqs. (43) and (44). When $R_c$ grows, the throat length shrinks according to the first and grows according to the second definition. Thus $R_c$ cannot be consistently identified with the length of the 5d interval. Instead, it has to be modelled by a field localized at the UV brane. Of course, because our 5d effective theory is valid only at length scales above $L$, we should be careful not to increase $R_c$ above $L$. Otherwise, the 5d description of the UV end becomes meaningless.
5 The explicit Randall-Sundrum-type 5d action

We are now finally in a position to construct the 5d effective action including bulk and brane fields. For $R_c \gg R_{c,\text{min}} \simeq R_{\text{UV}}$, the integral over the compact space at the UV end of the throat contributes

$$\mathcal{L}_{\text{UV}} = \frac{1}{2} M^8_{10} R^{6}_c (\mathcal{R}_4 + 30 (\partial \ln R_c)^2 + \cdots)$$

(46)

to the 4d effective action. We can view this as a precise definition of $R_c$, which is chosen such that $R^6_c$ is the volume of the compact space. Note that, at this point, the 4d metric implicit in $\mathcal{R}_4$ is simply the 4d part of the 10d metric in the Einstein frame, i.e. no Weyl rescaling has been performed.

The bulk part was already given in Eq. (19). Writing the 5d metric as

$$ds^2_5 = e^{2A(y)-2A(y_{\text{UV}})} g_{\mu \nu} dx^\mu dx^\nu + dy^2,$$

(47)

and integrating from $y_{\text{IR}}$ to $y_{\text{UV}}$, this contributes the following piece to the Einstein-Hilbert-term of the 4d action:

$$\frac{1}{2} \left( M^8_5 \int_{y_{\text{IR}}}^{y_{\text{UV}}} dy \exp \left[ 2 \left( \frac{y}{R_s} \right)^{3/5} - 2 \left( \frac{y_{\text{UV}}}{R_s} \right)^{3/5} \right] \right) \mathcal{R}_4 \approx \frac{5}{12} M^3_5 R_s \left( \frac{y_{\text{UV}}}{R_s} \right)^{2/5} \mathcal{R}_4.$$ (48)

Here $\mathcal{R}_4$ is to be evaluated with the 4d metric $g_{\mu \nu}$. The warp factor in Eq. (47) has been normalized to ensure consistency with the 4d metric in Eq. (46).

Note that the relative normalization of the coefficients of the $\mathcal{R}_4$ and the $(\partial \ln R_c)^2$ terms in Eq. (46) is intimately linked to the fact that $R^4_c$ is the imaginary part of a superfield $\rho$ [2], which is part of a no-scale supergravity model [35]. This relation is apparently destroyed by the addition of the 4d Einstein-Hilbert contribution of Eq. (48). However, this term is subdominant in the large-$R_c$ limit in which Eq. (46) was derived. Corrections to this equation are indeed expected since, near the IR end of the conical region, $R_c$ loses its interpretation as an overall scaling modulus of the compact space. For consistency with Eq. (48), the coefficient of the $\mathcal{R}_4$ term in Eq. (46) has to be modified according to

$$M^8_{10} R^{6}_c \rightarrow M^8_{10} R^{6}_c - \frac{5}{6} M^3_5 R_s \left( \frac{y_{\text{UV}}}{R_s} \right)^{2/5}.$$

(49)

After these remarks we now give the full 5d action to the extent that it can be inferred from the present analysis. In doing so, it is convenient to absorb a factor $g_s M$ into the definition of the scalar field. Thus, we define

$$\tilde{H} = cg_s M H,$$ (50)

$^6$The prefactor 30 arises as $k(k - 1)$, with $k = 6$ the number of compact dimensions.
where the $\mathcal{O}(1)$ numerical constant $c$ is chosen to ensure that $\bar{H}(y) = M_5^{3/2} (g_s N_{\text{eff}}(y))^{1/2}$ for the solution of Sect. 3. The action now reads

$$S_{5d} = \int d^5 x \sqrt{-g_5} \left( \frac{1}{2} M_5^2 \mathcal{R}_5 - \frac{1}{2(c g_s M)^2} (\partial \bar{H})^2 + c_V M_5^2 H^{-8/3} + \cdots \right)$$

$$+ \int d^4 x \sqrt{-g_{4,\text{UV}}} (K_{\text{UV}} + \mathcal{L}_{\text{UV}}) + \int d^4 x \sqrt{-g_{4,\text{IR}}} (K_{\text{IR}} + \mathcal{L}_{\text{IR}}),$$

where $K_{\text{UV/IR}}$ is the trace of the extrinsic curvature (the Gibbons-Hawking surface term [36]) and $(g_{4,\text{UV/IR}})_{\mu\nu}$ is the induced metric at each of the 4d boundaries. The value of the positive $\mathcal{O}(1)$ numerical constant $c_V$ is not important for our purposes. The brane Lagrangians are

$$\mathcal{L}_{\text{UV}} = \frac{c_1}{2} M_5^2 (g_s N_{\text{UV}})^{-10/3} \left[ ((R_c M_5)^6 - c_2 (g_s N_{\text{UV}})^4) \mathcal{R}_4 + 30 (R_c M_5)^6 (\partial \ln R_c)^2 \right] - V_{\text{UV}}(\bar{H}) - \Lambda_{4,\text{UV}} + \cdots$$

(52)

and

$$\mathcal{L}_{\text{IR}} = -V_{\text{IR}}(\bar{H}) - \Lambda_{4,\text{IR}} + \cdots,$$

(53)

with numerical coefficients $c_1 = 32 \pi^{1/3}/9$ and $c_2 = 3 \cdot 2^{2/3}/(32 \pi)$. Here $V_{\text{UV}}$ and $V_{\text{IR}}$ are steep potentials setting $\bar{H}$ to its values at the UV and IR brane respectively, for example,

$$V_{\text{UV/IR}} = \mu^2 \left[ \bar{H} - M_5^{3/2} (g_s N_{\text{UV/IR}})^{1/2} \right]^2,$$

(54)

with a very large coefficient $\mu$. The brane tensions or 4d brane cosmological constants $\Lambda_{\text{UV}}$ and $\Lambda_{\text{IR}}$ have values

$$\Lambda_{\text{UV}} = +M_4^4 \sqrt{6/c_V} (g_s N_{\text{UV}})^{-2/3} \quad \text{and} \quad \Lambda_{\text{IR}} = -M_4^4 \sqrt{6/c_V} (g_s N_{\text{IR}})^{-2/3}.$$

(55)

The fundamental dynamics of the throat can now be easily understood from the 5d action of Eq. (51): The scalar field $\bar{H}$ governs, via the potential term, the (approximately AdS) curvature and hence the warping. The rapidity with which the curvature changes as one moves along the 5th dimension is determined by the coefficient of the kinetic term for $\bar{H}$. In the limit of vanishing $M$, no change is possible – this is the pure AdS$_5$ case. The boundary or brane values of $\bar{H}$ are determined by steep brane potentials. The IR-brane potential models the way in which the Klebanov-Strassler region (or a more complicated corresponding geometry) determines the value of $N_{\text{eff}}$ in the IR regime. The UV-brane potential models the way in which the various stringy and field-theoretic sources of D3-brane flux in the compact space determine $N_{\text{eff}}$ in the conical region. The combined dynamics of UV/IR-brane and 5d bulk actions then stabilizes the length of the interval and fixes the hierarchy.

In the above 5d effective action, $R_c$ appears as a brane field localized at the UV-boundary. However, it is a brane field of very peculiar type. In the 5d Einstein frame, $R_c$ is part of the coefficient of the brane-localized Ricci-scalar and has a wrong-sign kinetic term. Of course, this can be remedied by performing an appropriate $R_c$-dependent Weyl
rescaling of the 5d metric. However, in such a Weyl frame \( R_c \) would cease to be a UV-brane field. Note furthermore that \( R_c \) can easily be parametrically larger than its lower bound (in the present analysis) \( R_{c, \text{min}} \approx R_{UV} \). In this case, our 5d model develops a parametrically large gravitational brane-kinetic term. This interesting possibility [37] has been considered for phenomenological reasons in field-theoretic model building (see e.g. [38]).

Before closing, we would like to explicitly relate the most important parameters of our 5d description, the boundary scalar \( R_c \) and the 5d radion \( \Delta y = y_{UV} - y_{IR} \), to the corresponding standard string moduli. Focussing on the universal Kähler modulus \( \rho \) and a single complex structure modulus \( z \) (and neglecting the warping for the moment), the 4d \( \mathcal{N} = 1 \) superfield action is determined by the Kähler potential

\[
\mathcal{K}(\rho, z) = -3 \ln[-i(\rho - \bar{\rho})] - \ln \left(-i \int \Omega \wedge \bar{\Omega}\right),
\]

and the superpotential

\[
W(z) = \int G_3 \wedge \Omega.
\]

The holomorphic (3,0) form \( \Omega \) is normalized using some 3-cycle of the compact space at the UV end of the throat, and \( z \) is defined via the \( S^3 \) cycle in the throat discussed in Sect. 2,

\[
z = \int_{S^3} \Omega.
\]

It is well-known that the imaginary part of the universal Kähler modulus governs the compactification volume. More precisely, the 4d no-scale field \( \rho \) of [2] is related to \( R_c \) by

\[
\text{Im} \rho \sim R_c^4.
\]

We can leave the constant of proportionality arbitrary since we do not intend to fix a possible additive constant in \( \mathcal{K} \).

In [2] the relation of the complex structure modulus \( z \) to the relative warping between the UV and IR region is found to be

\[
e^{A(r_{IR}) - A(r_{UV})} \approx |z|^{1/3}.
\]

Here \( \exp[2A(r)] = \tilde{h}(r)^{-1/2} \) (cf. Eq. [10]) is the 10d warp factor, which differs from the 5d warp factor \( \exp[2A(y)] \) of Eq. [23] by an insignificant (non-exponential) correction related to the 5d Weyl rescaling. The relative 5d warping is

\[
e^{A(y_{IR}) - A(y_{UV})} \approx \exp \left[-(\Delta y/R_s)^{3/5}\right],
\]

which allows us to express \( z \) through the 5d radion:

\[
|z|^{1/3} \approx \exp \left[-(M_5 \Delta y/b)^{3/5}(g_s M)^{-4/5}\right].
\]

This concludes our comparative discussion of \( R_c \) and \( \Delta y \) and the string moduli \( \rho \) and \( z \).
Of course, it would be desirable not to stop here but rather to go on and identify the superfield description of the stabilized Randall-Sundrum model [39] with the moduli description of the 10d flux compactification. At present, we can only offer some comments which may lead in this direction:

The essential quantity on the 5d side is the radion superfield $T$ with $\text{Re} T \sim \Delta y$.

The Kähler potential in terms of $T$ is expected to be [40] (see also [39, 41])

$$K_{5d} \simeq -3 \ln \left[ \int_{y_{UV} - \text{Re} T}^{y_{UV}} dy e^{2A(y)-2A(y_{UV})} \right],$$

i.e. it is proportional to the logarithm of the coefficient of the Ricci scalar in the 4d effective action before Weyl rescaling. We now consider $y_{UV}$ to be constant and focus exclusively on the $T$ dependence entering through the lower integration limit $y_{IR} = y_{UV} - \text{Re} T$. This $T$ dependence corresponds to the $z$ dependence in the language of 10d moduli (cf. Eqs. (60) and (61)) so that we can write

$$\int_{y_{UV} - \text{Re} T}^{y_{UV}} dy e^{2A(y)-2A(y_{UV})} = \text{const.} \cdot |z|^{2/3} \int_{-\infty}^{y_{IR}} dy e^{2A(y)-2A(y_{IR})} \simeq \text{const.} \cdot \frac{|z|^{2/3}}{2A'(y_{IR})}. \quad (64)$$

Since $A'(y_{IR}) \sim (-\ln |z|)^{-2/3}$, this implies for the $z$-dependent part of the Kähler potential

$$K_{5d} \simeq -3 \ln \left[ \text{const.} - |z|^{2/3}(-\ln |z|)^{2/3} \right] \sim |z|^{2/3}(-\ln |z|)^{2/3}, \quad (65)$$

where the prefactor and subdominant terms have been suppressed.

This is to be compared with the $z$-dependent part of Eq. (60) which, following [42], can be computed as follows: To account for warping, $\Omega \wedge \bar{\Omega}$ is replaced with $e^{-4A}\Omega \wedge \bar{\Omega}$ [4]. The dominant $z$-dependent contribution comes from the tip of the throat and depends only on two period integrals. The relevant cycles of the compactification manifold are the conifold 3-cycle with period $z$, cf. Eq. (58), and its dual $\tilde{S}^3$ with period

$$\int_{\tilde{S}^3} \Omega = \frac{z}{2\pi i} \ln z + \text{holomorphic} \quad (66)$$

($\tilde{S}^3$ will extend outside the throat into the compact manifold, whose precise form determines the holomorphic part). There will in general be other pairs of 3-cycles with period integrals that depend purely holomorphically on $z$. With the warp factor contribution at the tip given by $e^{-4A} \sim |z|^{-4/3}$, we obtain for the $z$-dependent part

$$- \ln \left(-i \int e^{-4A} \Omega \wedge \bar{\Omega} \right) \simeq \ln \left[ \text{const.} - |z|^{2/3} \ln(z\bar{z}) + \cdots \right] \sim |z|^{2/3}(-\ln |z|). \quad (67)$$

Here the ellipses stand for higher-order terms of the form $f(z)\bar{g}(\bar{z})$ with $f, g$ holomorphic. As before, the prefactor and subdominant terms have been suppressed.

We see that, in the small-$z$-limit, the structure of Eqs. (65) and (67) agrees (in the sense that the logarithms of the derivatives of $K$ coincide). Going beyond this approximation (which on the 5d side corresponds to constant warping), the two results are still
intriguingly close but not quite the same. The failure to fully match the string-theoretic with the 5d field-theoretic result is not unexpected in many ways. On the one hand, it may be necessary to account for subleading warping corrections on the 10d side. On the other hand, calculating the Kähler potential on the basis of Eq. (63) and using the naive identification of $\Delta y$ in terms of $|z|$ may be too simplistic. It may be necessary and it would certainly be highly desirable to start with a manifestly supersymmetric 5d Lagrangian which reproduces the correct 5d scalar potential governing the profile of the Goldberger-Wise scalar $H$ and hence the warp factor. We leave the detailed analysis of these issues to future work.

6 Conclusions and Outlook

In this paper, we have derived what we believe to be the main characteristics of the 5d effective theory describing the throat region of a type IIB flux compactification. It is well known that, at first approximation, the throat can be viewed as a Randall-Sundrum I model, where the UV brane is the compact space and the IR brane is the Klebanov-Strassler region of the throat. We take this analogy beyond leading order by identifying the 5d dynamics that leads to the deviation from the AdS$_5$ geometry observed in flux compactifications and to the stabilization of the size of the 5th dimension, i.e. the radion.

To be more specific, we find that in the effective 5d theory the radion is stabilized by a variant of the Goldberger-Wise mechanism: The 5d bulk scalar $H$ has a potential $V(-H) \sim H^{-8/3}$ which induces a non-trivial 5d bulk profile of $H$. Via gravitational back-reaction, this gives rise to a 5d curvature consistent with the known 10d throat solution. The non-trivial profile of $H$ reflects the variation of the size of the $T^{1,1}$ transverse space as one moves along the throat (or, equivalently, of the 5-form flux on the $T^{1,1}$, or of the NS 2-form flux on the $S^2$ cycle of the $T^{1,1}$). Together with the UV and IR boundary values of $H$, which are fixed by flux numbers and (anti-) D3 brane charges, this profile determines the length of the throat.

From the 5d perspective, the universal Kähler modulus (which can be left unfixed for our purposes) is a UV brane field. It governs the coefficient of a UV-brane-localized 4d Ricci-scalar [37]. As the Kähler modulus grows, a very large brane-localized gravitational kinetic term develops, which might have interesting phenomenological and cosmological implications [38]. At the same time, the effective brane thickness of the UV brane grows. It is then clear that, for extremely large volumes, the UV brane ‘eats up the throat’ and both the hierarchy and the 5d picture are lost. However, in a large intermediate range of volumes, the length of the throat is practically independent of the universal Kähler modulus.

Clearly, our analysis leaves many questions unanswered. First of all, it would certainly be desirable to derive the 5d action by an explicit dimensional reduction rather than by consistency arguments, as we have done. This would, in particular, enable us to include the full set of light 5d fields in the 5d Lagrangian and to characterize the dynamics of the UV and IR brane in more detail. More importantly, such an explicit calculation may
open the way to a better understanding of the supersymmetry that should be a feature of our 5d bulk action.

More specifically, we are faced with the following problems as far as 5d supersymmetry is concerned. The conifold throat with constant warping is known to have 4d $\mathcal{N}=1$ superconformal symmetry [6,31], which is referred to as $\mathcal{N}=2$ (or 5d $\mathcal{N}=1$) SUSY in the literature concerned with the supersymmetric Randall-Sundrum model [43]. Since the potentials in such a theory are highly constrained, it should be non-trivial and interesting to understand how our effective $H^{-8/3}$ potential can arise. Unfortunately, the recently discussed supersymmetric Goldberger-Wise models [44] based on massive bulk hypermultiplets do not appear to generate such a potential in any obvious way. In fact, one might consider the alternative possibility that, because of the $M$ 3-form flux units on the $S^3 \subset T^{1,1}$, SUSY is always broken from the 5d point of view. In this case, there would be no 5d supersymmetric Lagrangian. However, it is then unclear how the effective 4d (non-conformal) $\mathcal{N}=1$ SUSY, which is known to be present in the 4d effective theory, arises. We consider these to be interesting and important problems for the future.

A better understanding of our stepwise (10d to 5d to 4d) dimensional reduction in a manifestly supersymmetric approach may be relevant for the analysis of SUSY breaking mediation in this framework. Given the large expectations that have been placed on geometric or conformal sequestering and the corresponding interest in its possible violation (see e.g. [39,45]), we consider it important to confront those models with the explicit string-theory realization of the Randall-Sundrum scenario discussed here. We hope that new insights in 5d and 10d SUSY breaking phenomenology will be possible along the lines of the 5d effective field theory approach discussed in this paper.

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