Hadron Amplitudes in Composite Superconformal String Model

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Abstract—We discuss dynamical and covariant introduction of isospin quantum number into a composite string model.

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1. INTRODUCTION

We discuss a new type of string model which is able to describe spectrum of mesons and baryons and calculate hadron interaction amplitudes at low and intermediate energy region 0.1–10 GeV. It is known that string models give linear Regge trajectories in a tree approximation. The next loop corrections will lead to deviation from the linearity. However if we construct the model in a consistent way, we can expect that this deviation will be a small one.

Initially string models grew up from dual resonance models and were constructed for description of hadrons. However, they did not succeed on this way. To be selfconsistent classical strings require unphysical value of intercept of the leading meson trajectory \( \alpha_0 = 1 \). While the experimental value of intercept of p-meson trajectory equals \( \frac{1}{2} \). In the case of \( \alpha_0 = 1 \) the open string sector contains in its spectrum of states a massless vector particle. It is natural to interpret it as a gluon which is not a hadron state. The closed string sector contains in its spectrum of states a massless tensor particle i.e. graviton. Moreover, scale parameters for open and closed strings are dependent \( \alpha'_{\text{open}} = 2\alpha'_{\text{closed}} \). The coupling constant of the graviton is proportional to \( \alpha'_{\text{closed}} \) and it leads to the scale parameter for closed sector of order of Planck scale. Hence classical strings have problems to describe hadron interaction and gravity simultaneously.

Another problem was to introduce the quantum numbers into a string model. The well-known Chan-Paton factor introduces an isospin into Veneziano model in nondynamical way and leads to isotopic degeneracy in the spectrum.

2. COMPOSITE SUPERCONFORMAL STRING MODEL

Composite superconformal string model was suggested [1] as a string model which is able to describe hadrons. It overcomes troubles of classical strings due to new topology. While classical string has one two-dimensional surface, our model has several two-dimensional surfaces. It has basic two-dimensional surface and two edging two-dimensional surfaces. In the case of baryons there is one more two-dimensional ridge surface (Fig. 1) [2]. The edging and ridge surfaces carry quark quantum numbers and in some sense correspond to quarks. The model is self consistent at the value of intercept of leading meson trajectory \( \alpha_0 = \frac{1}{2} \).

Here we have realistic p-meson trajectory [3]. Due to complicated topology composite string allows to introduce two independent scales. Hadron scale \( \alpha'_H \) is defined on the edging surfaces, Planck scale \( \alpha'_{\text{Pl}} \) is

Fig. 1. Classical string, meson composite string, baryon composite string.
defined on the ridge surfaces [4]. The supersymmetry conditions are satisfied on two-dimensional surface only and the model does not predict any supersymmetric particles in a target space.

We use $W$-formalism to formulate the model. In this case the symmetry of the model is defined by the super Virasoro algebra generator. For Neveu–Schwarz case the operator $\hat{W} \sim e^{iK}:$ is a normal ordered exponent of two-dimensional fields. The vertex 1 commutates with $G$ generators. The superconformal symmetry is the main symmetry of the model. However, to make physical state spectrum free from negative norm states we have to introduce additional symmetry condition:

$$[\hat{V}, \Xi] = 0.$$  \hspace{1cm} (2)

This supercurrent condition leads to massless $\pi$-meson in tree approximation.

In terms of ground state emission vertices (1) the interaction amplitude for tree approximation can be expressed as follows:

$$A_N = \prod d\zeta \left< 0 \right| V(z_1) \ldots V(z_N) \left| 0 \right>.$$ \hspace{1cm} (3)

Let us consider $G$ generators of the model:

$$G = \sum_{\mu=0}^{3} \partial X_\mu H_\mu + \sum_{a=1}^{6} I^a \theta^a$$

(on the basic surface)

$$+ \sum_{\mu, i} (Y^{(i)}_\mu f^{(i)}_\mu + \phi^{(i)}_1 \psi^{(i)}_2 \phi^{(i)}_3)$$

(on the edge surfaces)

$$+ \sum_{\mu, i} (Y^{(i)}_\mu f^{(i)}_\mu)$$ (on the ridge surfaces).

Each surface has the set of two-dimensional fields.

On the basic surface there are fields $\partial X_\mu$, $I^a$, and anticommutating superpartners $H_\mu$, $\theta^a$. Lorentz index takes values $\mu = 0 \ldots 3$, index $a$ takes values $1 \ldots 6$. Field $I^6$ plays a definite role for strong interaction, $I^1 \ldots I^5$ fields gives negligible contribution into a tree approximation of hadron interaction. On the $i$th edging surface there are fields $Y^{(i)}_\mu$ and anticommutating superpartners $f^{(i)}_\mu$, anticommutating fields $\phi^{(i)}_1, \phi^{(i)}_2, \phi^{(i)}_3$ realize supersymmetry on the edging surface in a nonlinear way. On the ridge surface there are fields $Y^{(f)}_\mu$ and anticommutating superpartners $f^{(f)}_\mu$.

To calculate interaction amplitudes we have to define vertex operators of ground states. We start from the description of nucleons. From the experiment we know that $N\bar{N}$ can transit to odd number of $\pi$-mesons. To make this transition possible it is necessary to introduce the nucleon vertex operator as a sum of two terms $V_N = V^{NS} + V^{BH}$:

$$\hat{V}^{NS}_{i,j} = z_j I^0_{i,j} \left( G_r \hat{W}_{i,j+1} \right)_z I^0_{i,j},$$

$$\hat{V}^{BH}_{i,j} = z_j I^0_{i,j} \left( G_r \hat{F}_{i,j+1} \right)_z I^0_{i,j},$$ \hspace{1cm} (5)

where Neveu–Schwarz type vertex $\hat{V}^{NS}$ is the product of odd number of anticommutating field components, Bardakci–Halpern type vertex $\hat{V}^{BH}$ is the product of even number of anticommutating field components, operator $\hat{F}$ is the sum of two-dimensional anticommutating fields with conformal spin $J = \frac{1}{2}$. The operator $\hat{F}$ ought to satisfy the symmetry condition:

$$[\left[ G, \hat{F} \right], \hat{W}] = 0.$$ \hspace{1cm} (6)

The vertex operator should have conformal spin $J = 1$.

The expressions (5), (6) define the conformal properties of the vertices. If we want to calculate the interaction amplitudes of a certain states, we need to define the wavefunction which carry quantum numbers of a certain internal states.

Each additional surface carries spinor and isospinor $\lambda_i$. It is not a field of point-like particle in Minkowski-space but it is a usual constant operator on the edging or ridge surface. In terms of $\lambda_i$ we can express spin-parity and isospin structure of nucleon $J^P = \frac{1}{2}^+$ in four ways:

$$(\check{\lambda}_i, \gamma_\mu \lambda_j) \lambda_k, \quad (\check{\lambda}_i, \gamma_5 \gamma_\mu \lambda_j) \lambda_k \gamma_5, \quad \sum_a (\check{\lambda}_i, \gamma_5 \tau^a \lambda_j) \lambda_k \tau^a,$$

$$\sum_a (\check{\lambda}_i, \gamma_5 \tau^a \lambda_j) \lambda_k \gamma_5,$$ \hspace{1cm} (7)

where indices $i$ and $j$ refer to edging surfaces, index $r$ refers to ridge surface, $\tau^a$ is Pauli matrix.

For two types of nucleon vertex (5) we choose the following realization of nucleon quantum numbers:

$$(\check{\lambda}_i, \lambda_j) \lambda_f \quad \text{for } \hat{V}^{BH},$$

$$\sum_a (\check{\lambda}_i, \gamma_5 \tau^a \lambda_j) \lambda_k \gamma_5 \tau^a \quad \text{for } \hat{V}^{NS}.$$ \hspace{1cm} (8)

3. QUANTUM NUMBERS

In this model a special role play the eigenvalues of the zeroth components of two-dimensional fields $Q(z) = \sum_a Q_a z^a$. The eigenvalues of $Y^{(f)}_\mu$ from the edging surfaces are the momenta $\sqrt{Q^{(f)}_\mu}$ with hadron
The parity of intermediate states is defined by parity of external states associated with the spinor and isospinor \( \lambda^{(f)} \) on the ridge surface. In the rest system for the external nucleon state it gives:

\[
\lambda^{(f)} \hat{p}_N = m_N \lambda^{(f)}. \tag{10}
\]

Since \( \lambda^{(f)} \) does not depend on two-dimensional coordinate \( z \), we can rewrite (10) in noncovariant form:

\[
\lambda^{(f)} \gamma_0 = \lambda^{(f)}. \tag{11}
\]

\( \lambda^{(f)} \) can appear at another intermediate state. In that case (10) can be rewritten in terms of momenta \( \hat{p}_M \) of state under consideration:

\[
\lambda^{(f)} \hat{p}_M = M \lambda^{(f)}, \tag{12}
\]

where \( M \) is the mass of the state. For the ridge surface in the Bardrcki–Halpern component of the nucleon we have positive parity in (11). In contrary for the ridge surface in the Neveu–Schwarz component of the nucleon the parity is negative:

\[
\lambda^{(f)} \gamma_5 \hat{p}_M = -M \lambda^{(f)} \gamma_5. \tag{13}
\]

The spin-parity of \( \lambda^{(f)} \) on the ridge surface is fixed by the external states. This fact leads to defined spin-parity of the intermediate state. Thus we have nondegenerated in parity description for intermediate baryon states.

4. CONCLUSIONS

The composite superconformal string model has dynamical introduction of the isospin dependence. Due to different parity of the ridge surface in the Neveu–Schwarz and Bardrcki–Halpern component of the nucleon vertex the model gives nondegenerated in paritybaryon states. Because of these facts and the experiment-consistent description of \( \pi, \eta \) and nucleon masses we can expect a satisfactory description of hadron amplitudes and hadron spectrum.
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