Three lepton decay mode of the proton

Patrick J. O’Donnell
Physics Department
University of Toronto
Toronto, Ontario M5S 1A7, Canada

and

Utpal Sarkar
Theory Group
Physical Research Laboratory
Ahmedabad - 380 009, India

Abstract

We consider the three lepton decay modes of the proton within the proton decay interpretation of the atmospheric neutrino anomaly. We construct higher dimensional operators in the framework of the standard model. The operators which allow the interesting decay modes are of dimension 10 involving $SU(2)_L$ non-singlet higgs. We show how these operators can be comparable to the dimension 9 operators. We then present a simple model which can give rise to the desired proton decay modes of the right order of magnitude.
Muon neutrinos produced by cosmic rays in the atmosphere are expected to be almost twice as many as electron neutrinos (where neutrinos are not distinguished from antineutrinos). But the results [1, 2] from the two large water-Cerenkov detectors are \( R_{\text{obs}}/R_{\text{MC}} = 0.60 \pm 0.07 \pm 0.05 \) from the Kamiokande experiment [1] and \( R_{\text{obs}}/R_{\text{MC}} = 0.54 \pm 0.05 \pm 0.12 \) from IMB [2], i.e., the observed ratio \( R = N(\nu_\mu)/N(\nu_e) \) is almost half the expected ratio. The experiments look for “contained” events which are caused by neutrinos of energy below 2 GeV. Although the more popular explanation of this anomaly is neutrino oscillation [3], there is another explanation in terms of proton decay [4]. It has been proposed that if proton decays into a positron and two neutrinos with a lifetime of \( 4 \times 10^{31} \) years (this value is consistent with the present proton decay limit for this decay mode [5]), then the excess “contained” electron events can actually be proton decay events. Since the energy of these electrons peak around 350 MeV with a distribution ranging up to 1 GeV, the decay mode has to be \( P \rightarrow e^+ \nu \nu \). We are not interested in neutron decay since neutron decay events cannot explain the atmospheric neutrino anomaly; neutron decay events have to have at least two charged leptons and hence give two leg “contained” events, which have not been observed. The possibility of the three lepton decay mode for the proton has been discussed earlier [6, 7], but it is difficult to incorporate this particular decay mode in those theories [8].

For this mechanism to work in any theoretical model, the main problem is to have this decay mode with only light neutrinos. In most theories left
handed neutrinos are light and right handed neutrinos are heavy. The decay modes are thus restricted to

$$P \rightarrow e^+ \nu_L \nu_L \text{ or } P \rightarrow e^+ \nu_L^c \nu_L \text{ or } P \rightarrow e^+ \nu_L \nu_L^c.$$  \hspace{1cm} (1)$$

These processes require six fermion operators [6, 7, 8] and hence are usually more suppressed [9] than decay modes of the type $P \rightarrow e^+ \pi^0$ (for which $\tau_P > 5 \times 10^{32}$ years [10]), which are allowed by four fermion operators. Thus the next problem is to make (1) more dominate the decay modes.

In this article we study this decay mode in detail. We first make an operator analysis [8, 11] for (1). We write down the effective operators of higher dimension $n$ allowed by the standard model, which are suppressed by $M^{(n-4)}$, where $M$ is the mass scale in the theory, and which depends on the details of the model. We consider the higgs scalars which break the standard model, namely, a higgs doublet and a higgs triplet. Although a triplet is not present in the minimal standard model it naturally exists in left-right symmetric theories [12].

We construct operators involving only the known fermions and which are invariant under the standard model gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$. In ref [8] these operators (not including higgs scalars) in the form $QQQLLL$ were discussed. We review these operators in some detail since there are subtleties that are important. Consider the operator $[\psi_{iL}^c \psi_{jL}][\psi_{kL} \psi_{lL}]$, antisymmetric in $[ij]$, which vanishes if the $i$th and the $j$th particles belong to the same generation. However the operator $[\psi_{iL}^c \psi_{jL}][\psi_{kL} \psi_{lL}]$ may be antisymmetric...
in [ij] and non-vanishing for one generation, even though both the operators look like $\psi_L^c\psi_L^c\psi_L^c\psi_L^c$ with two of them antisymmetric. In fact, if $A$ and $B$ are the same two fields which enter antisymmetrically in an operator $O = [\bar{A}^c B][\bar{C}^c D]$, then the operator vanishes only when the other fields $C$ and $D$ are of different helicity from $A$ and $B$. Otherwise one can always write another operator $O' = [\bar{A}^c D][\bar{C}^c B]$ which can be Fierz transformed to a combination of $O$ and an operator of the form $[\bar{A}^c \sigma_{\mu\nu} B][C^c \sigma_{\mu\nu} D]$, which is non-vanishing for one generation. This is because

$$\epsilon_{ij}(\bar{\psi}_{iaL}^c\psi_{jbL}) = -\epsilon_{ij}(\bar{\psi}_{iaL}^c\psi_{jaL})$$ and
$$\epsilon_{ij}(\bar{\psi}_{iaL}^c\sigma_{\mu\nu}\psi_{jbL}) = \epsilon_{ij}(\bar{\psi}_{jbL}^c\sigma_{\mu\nu}\psi_{jaL}).$$

We now write the detailed form of the operators and discuss the possibility of the decay mode (1). We work in the context of conventional models. This means that the neutrinos are left-handed. In the standard model there is no right handed neutrino and in left-right symmetric models the right handed neutrinos are too heavy. The other fields can be either left-handed or right handed.

There are only four dimension nine operators involving six fermions, which could allow (1). These are

\begin{align*}
O^1 &= (q_{iaL}^c q_{j\beta R})(d_{\gamma cR}^e l_{keL}^c e_{dR}^c)\epsilon_{\alpha\beta\gamma}(\tau^I\epsilon)_{ij}(\tau^I\epsilon)_{kl} \\
O^2 &= (q_{iaL}^c q_{j\beta R})(d_{\gamma cR}^e l_{kdL}^c)\epsilon_{\alpha\beta\gamma}(\tau^I\epsilon)_{ij}(\tau^I\epsilon)_{kl} \\
O^3 &= (u_{\alpha aR}^c u_{\beta bR})(q_{\gamma cL}^e l_{j\delta L}^c)\epsilon_{\alpha\beta\gamma}(\tau^I\epsilon)_{ij}(\tau^I\epsilon)_{kl} \\
O^4 &= (u_{\alpha aR}^c u_{\beta bR})(q_{\gamma cL}^e l_{j\delta L}^c)\epsilon_{\alpha\beta\gamma}(\tau^I\epsilon)_{ij}(\tau^I\epsilon)_{kl}
\end{align*}

(2)
where, $\alpha, \beta, \gamma$ are $SU(3)_c$ indices, $i, j, k, ... [I, J, ...]$ are $SU(2)_L$ doublet [triplet] indices and $a, b, c, ...$ are generation indices. Since by Fierz transformation the gauge boson mediated operators of type $$(\overline{\psi}_L \gamma^\mu \psi_R)(\overline{\psi}_L \gamma^\mu \psi_R)$$ are contained in $$(\overline{\psi}_L \psi_L)(\overline{\psi}_R \psi_R)$$ we have not written them separately.

In $O^1$ and $O^2$ the left handed quarks $q_L$ have to be up quarks. In all the four operators the up-quarks are antisymmetric in the $SU(3)_c$ index while symmetric in all other indices so they have to be antisymmetric in the generation indices. As a result all of these operators can only give rise to proton decay into charmed mesons, which is kinematically forbidden.

For an antisymmetric combination of left-handed [right-handed] fields to be non-vanishing, two more left-handed [right-handed] fields are necessary as discussed earlier. In the present case the neutrino fields have opposite helicity to the up quarks since $$(\psi^c)_L = (\psi^c)_R, \overline{(\psi)_R} = (\overline{\psi})_L$$ and $$(\overline{(\psi)_L})^c = (\overline{(\psi^c)_R}) = (\overline{(\psi^c)_L})$$ and hence $$(\overline{q_L}^c l^c_L) = 0.$$ Thus there cannot be any term of the form $$(\overline{q_L}^c l^c_L)(\overline{l_R} q_R)$$ except the gauge boson mediated ones, which can be Fierz transformed to the operators we have already listed. Unless one can make a model which has a light right handed neutrino [8] it is not possible to have the proton decay mode (I) without higgs scalars.

We now study the higher dimensional operators which are possible with those higgs scalars whose vacuum expectation values (vevs) give rise to the decay modes (I). We consider two types of higgs scalars which are non-singlets under the standard model. One type is the usual $SU(2)_L$ doublet...
field $\phi$, which gives masses to the fermions. The other possibility is a $SU(2)_L$ triplet $\Delta_L$ in addition to $\phi$. This field $\Delta_L$ is always present in left-right symmetric theories and can give Majorana mass to the left-handed neutrinos.

The operators without the higgs scalars are of dimension 9 and so this decay mode is suppressed by a factor $M^5$ in the amplitude where $M$ is the mass scale in the theory. Thus,

$$\tau_P \sim M^{10}$$

(3)

and for the experimental value $\tau_P \sim 4 \times 10^{31}$ yrs, we find $M \sim 10^6$ GeV. A dimension 10 operator can give rise to these decay modes when the higgs bosons $\phi$ or $\Delta_L$ acquires a $vev$, $\eta$, say. The amplitude is suppressed by a factor of $M^6/\eta$, i.e., the lifetime for the process becomes,

$$\tau_P \sim \frac{M^{12}}{\eta^2}.$$  

(4)

For a doublet field $\eta = \langle \phi \rangle \sim 250$ GeV, while for a triplet field $\eta = \langle \Delta_L \rangle = v_L \sim O(1)$ GeV, which implies $M \sim 10^5$, not much different from the dimension 9 case. Hence these dimension 10 operators can be as important as the dimension 9 operators; we present below an explicit model where, with reasonable choices of parameters, such dimension 10 operators give rise to the desired proton decay mode \( \Box \).

We first consider the doublet field $\phi$ transforming as $(2, -1)$ under $SU(2)_L \times U(1)_Y$. Starting with a dimension 9 operator (such as $\psi_L \psi_L \psi_R \psi_R \psi_L \psi_L$) one

\footnote{The upper bound on $v_L \sim O(1)$ GeV comes from LEP data \cite{13}.}
can contract the $SU(2)_L$ index of a left-handed field with that of the $\phi$. Then one of the remaining $\psi_L$ has to be replaced by a right-handed field for $SU(2)_L$ invariance. The bilinear forms in any operator can be of the form $\overline{\psi^L}_L \psi_L$, $\overline{\psi^R}_R \psi_R$ or $\overline{\psi^R}_R \psi^L_L$. Now if only one of the left-handed fields changes to a right-handed field then this one, or at least one other left-handed field, has to be charge conjugated to keep the bilinear form from vanishing. For example, $\psi^L_L \psi^L_L$ can be replaced by $\psi^R_R \overline{\psi^L}_L \phi$ or $\psi^R_R \phi \psi^L_L$. This is because the $\psi^L_L$ and $\psi^L_R \phi \psi^L_L$ are $SU(2)_L$ doublets, while $\psi^R_R$ and $\psi^R_R \phi \psi^L_L$ are singlets. Since only a neutral component of $\phi$ can acquire a vev, by charge conservation only a neutrino field can undergo such helicity flip. With this criteria we can now write down a set of dimension 10 operators involving the higgs doublet $\phi$ as

\begin{align}
O^1 &= (\overline{q_{iaa}}^{L} q_{jbd}^{L})(\overline{d_{cde}}^{R} l_{kd}^{L} c)(\overline{l_{me}}^{L} l_{jf}^{L}) \phi_{n} \epsilon_{\alpha \beta \gamma} (\tau^{I} \epsilon)_{ij} (\tau^{I} \epsilon)_{kl} \epsilon_{mn} \\
O^2 &= (\overline{q_{iaa}}^{L} l_{jbd}^{L})(\overline{d_{cde}}^{R} l_{md}^{L} c)(\overline{l_{ke}}^{L} q_{f}^{L} l_{j}^{L}) \phi_{n} \epsilon_{\alpha \beta \gamma} (\tau^{I} \epsilon)_{ij} (\tau^{I} \epsilon)_{kl} (\tau^{K} \epsilon)_{mn} (\tau^{K} \epsilon)_{IJ} \\
O^3 &= (\overline{q_{iaa}}^{L} q_{jbd}^{L})(\overline{l_{ke}}^{L} l_{md}^{L})(\overline{l_{le}}^{L} l_{f}^{R} l_{j}^{L}) \phi_{n} \epsilon_{\alpha \beta \gamma} (\tau^{I} \epsilon)_{ij} (\tau^{I} \epsilon)_{kl} \epsilon_{mn} \\
O^4 &= (\overline{q_{iaa}}^{L} l_{jbd}^{L})(\overline{q_{kec}}^{L} q_{ld}^{L})(\overline{l_{le}}^{L} l_{f}^{R} l_{j}^{L}) \phi_{n} \epsilon_{\alpha \beta \gamma} (\tau^{I} \epsilon)_{ij} (\tau^{I} \epsilon)_{kl} \epsilon_{mn} \\
O^5 &= (\overline{u_{ab}}^{R} u_{\beta d}^{R})(\overline{d_{cde}}^{R} l_{id}^{L}) (\overline{l_{ke}}^{L} l_{j}^{L}) \phi_{l} \epsilon_{\alpha \beta \gamma} (\tau^{I} \epsilon)_{ij} (\tau^{I} \epsilon)_{kl} \\
O^6 &= (\overline{u_{ab}}^{R} l_{ibd}^{L})(\overline{d_{cde}}^{R} u_{\beta d}^{R}) (\overline{l_{ke}}^{L} l_{j}^{L}) \phi_{l} \epsilon_{\alpha \beta \gamma} (\tau^{I} \epsilon)_{ij} (\tau^{I} \epsilon)_{kl} \\
O^7 &= (\overline{u_{ab}}^{R} u_{\beta d}^{R})(\overline{q_{kec}}^{L} q_{ld}^{L}) (\overline{l_{ke}}^{L} l_{f}^{R}) \phi_{l} \epsilon_{\alpha \beta \gamma} (\tau^{I} \epsilon)_{ij} (\tau^{I} \epsilon)_{kl} \\
O^8 &= (\overline{u_{ab}}^{R} e_{b}^{R})(\overline{q_{jbd}}^{L} l_{j}^{L}) (\overline{l_{ke}}^{L} u_{\gamma}^{R} l_{f}^{R}) \phi_{l} \epsilon_{\alpha \beta \gamma} (\tau^{I} \epsilon)_{ij} (\tau^{I} \epsilon)_{kl}
\end{align}

From these dimension 10 operators it is clear that the higgs $\phi$ allows only
$(B - L)$ conserving proton decays. The $SU(2)$ indices have been contracted in only one way for each of these operators. There are other operators which differ in the way the $SU(2)$ indices are contracted. However the main features of these operators are present in all other operators. In particular operators $O^1$, $O^3$, $O^5$ and $O^7$ vanish for one generation, irrespective of their $SU(2)_L$ contraction, $i.e.$, these operators do not give the decay modes (1). The other operators still allow for the decays. Both the operators $O^1$ and $O^2$ are of the form $QQd_RLL\bar{\phi}$ and similarly $O^3$ and $O^4$ are of the form $QQQe_RLL\bar{\phi}$; $O^5$ and $O^6$ are of the form $u_Ru_Rd_RLL\bar{\phi}$; and $O^7$ and $O^8$ are of the form $u_Ru_RQe_RLL\bar{\phi}$. But while $O^n$ (with odd $n$) can not give the decay modes (1), the $O^{(n+1)}$ operators can allow decays of (1).

In the minimal left-right symmetric model [6, 7, 12] there exists a natural choice of the higgs scalar masses which lets us develop a simple model which gives one of the decay modes of (1) with the right order of magnitude. The $vev$ of the right handed triplet higgs field $\Delta_R (1,1,3,-2)$, breaks $G_{LR} \equiv SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ to the standard model. From left-right parity ($D-$parity) there is a $\Delta_L$ which transforms as $(1,3,1,-2)$ under $G_{LR}$. The standard model higgs doublet field $\phi$ transforms as $(1,2,2,0)$ under $G_{LR}$.

The proton decay modes allowed by the dimension 10 operators can be mediated only if there exists $SU(3)_c$ color non-singlet higgs scalars. These scalars are present when $G_{LR}$ is embedded in a larger group $G_{PS} \equiv SU(4)_c \otimes SU(2)_L \otimes SU(2)_R$ or other larger groups. The field $\phi$ belongs to $(1,2,2)$ of $G_{PS}$, but this does not give correct mass relations [3] and one requires another.
field $\xi$ transforming as $(15,2,2)$ under $G_{PS}$. The fields $\Delta_L$ and $\Delta_R$ are contained in larger representations $(10,3,1)$ and $(10,1,3)$ of $G_{PS}$. For the decay mode of the operator $O^2$ we need the $SU(3)_c$ color triplet components of the fields $\Delta_L$ and $\xi$, which we represent by $\Delta^3_L$ and $\xi^3$ respectively. Then the Yukawa couplings

$$L_{Yuk} = f_{ql} (q_{i\alpha L} l_{jL}) \Delta^3_{\alpha IL} \tau^I \epsilon_{ij} + f_{dl} (d_{\alpha R} \tilde{e}_{i} l_{dL}) \xi^3_{\alpha^* i\hat{i}}$$

(where $\hat{i}, \hat{j}, ...$ are the $SU(2)_R$ indices) and the quartic scalar coupling

$$L_s = \lambda \Delta^3_{\alpha iLL} \Delta^3_{\beta jL} \xi^3_{\gamma \hat{i} i} \epsilon_{\alpha \beta \gamma} (\tau^I \epsilon_{ij})_{lL} (\tau^K \epsilon_{i\hat{j}})_{dL}$$

(give the $(B-L)$ conserving proton decay $P \to e_L^+ \nu_L \nu_L^c$ through the operator $O^2$ (at the level of standard model there is no distinction between the $\phi$ and the $\xi^1$) through the diagram of figure 1. The amplitude for the process is given by,

$$A = \frac{\lambda f_{ql}^2 f_{dl} \langle \xi^1 \rangle}{m^2_{\xi^1} m^4_{\Delta^3}}$$

where, $\langle \xi^1 \rangle = \langle \phi \rangle = 250$ GeV. The mass of $\xi^3$ can be as low as $\sim 100$ GeV. Then for a typical value of the quartic and the Yukawa couplings parameters, $\lambda \sim 10^{-2}$ and $f \sim 10^{-3}$, and for $m_{\Delta^3} \sim 6 \times 10^4$ GeV we obtain the lifetime for the proton decay in this particular mode to be $4 \times 10^{31}$ years which can explain the atmospheric neutrino anomaly.

The mass scale $m_{\Delta^3}$ need some explanation. In theories where left-right symmetry is broken at a very low energy $m_R \sim 1 - 10$ TeV, one can

\footnote{The lower bound on the left-right symmetry breaking scale comes from the LEP data to be $\sim 10^3$ GeV.}
have $m_\Delta \sim 10^4$ GeV. Otherwise in theories where the left-right $D-$parity is broken spontaneously [15], this scale can have another explanation. In this scenario the $D$-parity is broken by the $vev$ of the singlet field $\eta$ (1,1,1,0), which transforms under $D$ as $\eta \to -\eta$. The scalar and the fermionic fields transform under $D-$parity as $\Delta_{L,R} \to \Delta_{R,L}$ and $\psi_{L,R} \to \psi_{R,L}$, while $\phi$ and $\xi$ stay the same. With the field $\eta$ the lagrangian now contains terms,

$$L_{\eta\Delta} = M_\eta \eta (\Delta_R^\dagger \Delta_R - \Delta_L^\dagger \Delta_L) + \lambda_\eta \eta^2 (\Delta_L^\dagger \Delta_L + \Delta_R^\dagger \Delta_R)$$

which can then allow a different scenario for the masses. The masses of the fields $\Delta_L$ and $\Delta_R$ are then given by,

$$m_{\Delta_L}^2 = m_\Delta^2 - M_\eta \langle \eta \rangle + \lambda_\eta \langle \eta \rangle^2 \quad \text{and} \quad m_{\Delta_R}^2 = m_\Delta^2 + M_\eta \langle \eta \rangle + \lambda_\eta \langle \eta \rangle^2$$

where $m_\Delta$ is the mass parameter for $\Delta_{L,R}$ and generates the left-right symmetry breaking. One can now fine tune parameters to get a solution

$$\langle \eta \rangle \sim \langle \Delta_R \rangle \gg \langle \Delta_L \rangle \quad \text{and} \quad M_\eta \approx m_{\Delta_L} \approx \langle \Delta_R \rangle \approx m_\Delta \quad \text{and} \quad m_{\Delta_L} \ll \langle \Delta_R \rangle$$

We can thus have $m_\Delta \sim m_{\Delta_L} \sim 10^4$ GeV, even when $\langle \Delta_R \rangle$ is as large as $10^{10}$ GeV.

For completeness we shall now write down the operators in the presence of the triplet higgs scalar $\Delta_L$ which transforms as $(3, -2)$ under $SU(2)_L \times U(1)_Y$. These are given by,
These operators can give many possible diagrams which will allow three lepton decay mode of the proton which can explain the atmospheric neutrino problem. The first two operators $O^{1,2}$ has an antisymmetric combination of two $u'$s, which gives charmed meson decay modes for the proton. All the remaining operators can, in principle, give rise to the desired decay modes of the proton (1).

To summarize, we have given an operator analysis for the three lepton decay mode of the proton. The proton decay process which can explain the atmospheric neutrino problem is not allowed by dimension 9 operators.
This is allowed when a $SU(2)_L$ doublet or a triplet field acquire a \textit{vev}, which requires dimension 10 operators and sometimes can be comparable to dimension 9 operators. We write down a set of possible operators which can give this proton decay mode. In particular a left-right symmetric model which can allow this proton decay mode with an amplitude of right magnitude for a natural choice of the parameters.

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**Figure Caption**

**Figure 1** Diagram giving $P \rightarrow e_L^+\nu_L\nu_L^c$. 
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