The determination of the operation parameters at the axial hydraulic turbine

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Abstract: In the operating point of the monitoring moment there are assumed from process the monitoring measured parameters: the active and reactive power, upstream and downstream water levels (after the intake trash rake and at the outlet of the turbine draft tube), wicket gate and runner opening blades, the differential pressure in the spiral chamber and the hydrounit speed. So, there was established the characteristic curves obtained on analytic basis and similitude and compared with the curves measured experimentally on the hydraulic machines from the power plant. The cavitation coefficient of the machine and the cavitation coefficient of the equipment are in function of the system parameters between them especially the suction head, the runner and wicket gates blades angles of opening. The solution proposed is a method of determining the operating turbine parameters and of the cavitation, by reducing the error caused by the similitude phenomenon, using an accurate estimation of the turbine operating parameters according to the universal diagram of the turbine. The numerical obtained values permit the necessary correlation through a complex function which is able to reduce or eliminate the unwished effects of the cavitation phenomena on the hydraulic turbines of the Iron Gates power plant.

1. Introduction

Cavitation is normally defined as the formation of bubbles filled with vapour, gas or their mixture and its collapse.

Cavitation is the main obstacle to develop the high performance hydraulic turbines. Cavitation erosion occurs, affecting the performance of hydraulic machines with noises, vibrations and oscillations of the whole system.

Iron Gates I Power Plant was functioning from 1971 and is located on the Danube at the 942,450 km upstream from Drobeta Turnu Severin town. Considering the long time of the hydrounit operation, and the volume of the reparations in the last years, it was actual the problem of the units refurbishment in order to prepare the next cycle of 30 years in exploitation.

To combat the cavitation, appropriate measures should be carefully considered and balanced at the turbine design, the machine selection, the materials and operation parameters, the determination of the turbine cavitation number and the machine reparations.

Hydro units operate at variable heads due to the limited retention of the lake thereof; the head is basically dependent on the flow. Turbine at the flows higher than inflow causes power reduction with the head reducing, also under certain conditions at the dam requires the protection against flooding by overflow. Hydrounits works actually under the variable heads, the combinatorial relationship is set to H = cst, at several values and then is approximated for other operating points.
If we evaluate properly the cavitation phenomenon and the operating parameters, the hydraulic turbine can avoid the unfavorable regimes in order to obtain optimized operation and to reduce the cavitation phenomenon.

| Parameters: | Measuring units | Meaning |
|-------------|-----------------|---------|
| $P$         | MW              | power   |
| $\varphi$   |                 | wicket gates opening |
| $\beta$     |                 | runner blades opening |
| $Q$         | m$^3$/s         | flow    |
| $n$         | rot/min         | rotative speed |
| $H$         | m               | head    |
| $H_S$       | m               | turbine suction head |
| $H_{St}$    | m               | static head |
| $D$         | m               | runner diameter |
| $Z_{av \, pr}$ | m               | downstream level from Adriatic sea |
| $p$         | pascals         | pressure |
| $P_V$       | pascals         | vapour pressure of water |
| $P_{at}$    | pascals         | atmosphere pressure |
| $\rho$      | kg/m$^3$        | density of water |
| $v$         | m/s             | water speed |
| $k_W$M, $k_V$M, $k_{u1}$ | -               | speed coefficients of the model |
| $\eta$      | %               | efficiency |
| $\sigma_{\text{inst}}$ | -               | plant cavitation coefficient |
| $\sigma_T$  | -               | turbine cavitation coefficient |
| $\eta_{Ta}$ | %               | draft tube efficiency |

Table 2 Subscripts.

| Parameters: | Meaning |
|-------------|---------|
| $P$         | prototype |
| $T$         | turbine |
| $m$         | measured |
| $c$         | calculated |

2. The axial hydraulic turbine operation with cavitation
At the axial turbine with great discharges if it is considered only the difference from blade axis to downstream level at the draft tube outlet. It is not real because results lower than the concordant value from the exploitation diagram.

To optimize the hydraulic axial turbine there are eliminated the differences between the operating and cavitation parameters resulted from the model data with the universal diagram and those obtained by the measured parameters in the turbine operation.

Coefficient of cavitation in hydraulics $\sigma$ is a dimensionless size that characterize the phenomenon of cavitation from such a point M of the fluid flow [1]:

$$\sigma = \sigma_{\text{inst}} - \sigma_T = \frac{\Delta p}{1/2 \rho \cdot v^2} \approx \frac{P_M - P_V}{\rho \cdot g \cdot H_T}$$ (1)
Cavitation occurs when the pressure of a point is equal to the vapor pressure: $p_M = p_V \Rightarrow \sigma_{\text{inst}} = \sigma_T = \sigma_{\text{crit}}$

To avoid the cavitation is necessary that in every points of the hydraulic layout the pressure must be greater than the vapor pressure: $p_M > p_V \Rightarrow \sigma_{\text{inst}} > \sigma_T$

where $\sigma_T$ is obtained from statistical formulae [2] (average formula) and [1]:

$$\sigma_{\text{inst}} = \frac{P_{at} - P_c - H_S}{H_T}$$  \hspace{1cm} (2)

$H_S = 31 - Z_{av\rho}$, where the blade axe level is 31 m from Adriatic Sea.

In figure 1 it is presented the universal cavitation diagram of the turbine model.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{The model cavitation diagram with the cavitation coefficient $\sigma_{0.5}$.}
\end{figure}
The cavitations phenomenon is examined with the cavitation coefficient \( \sigma_{\text{inst}} = \sigma_{0.5} \) on the universal cavitational diagram which is from the model tests of the producer.

So with the measured parameters and the similitude formulae, there are determined the double unitary parameters \( n_{11}, Q_{11} \), and then with the universal characteristic of the model (obtained from Sulzer Hydro documentation) the cavitation coefficient \( \sigma_{0.5} \) and for the prototype cavitations coefficient it is used the transposition formula [4]:

\[
\sigma_T = \sigma \times k_{P_{\text{max}3}} k_{W_3} \left( \frac{\eta_t}{\eta} - 1 \right) + \eta_{Ta} k_{v_3} \left( \frac{\eta_T^2}{\eta^2} - 1 \right)
\]  

(3)

Maximum speed coefficient of the model \( k_{\text{pmax}3} \equiv 0.3 \) and \( \eta_{Ta} \equiv 0.8 \) from [4]

\[
k_{\text{pmax}3} = \frac{w_{\text{max}}^2}{w_3^2 - 1}
\]  

(4)

\[
k_{w3M} = \frac{w_3}{(2gH)^{1/2}} \quad k_{v3} = \frac{v_3}{(2gH)^{1/2}}
\]  

(5)

\[
k_{u1} = \frac{u_1}{(2gH)^{1/2}} = \frac{2\pi r n}{60(2gH)^{1/2}}
\]  

(6)

The speeds distribution on the runner blade is represented in figure 2.

Figure 2. Speeds distribution on the runner blade.

\( u = u_1 = u_3 \) - tangential speed, \( v_m = v_{m1} = v_{m3} \) – characteristic speed in the point 1 and 3 of the axial turbine section. \( D = 0.308 \text{ m} \) – model diameter.

It is considered from [4]:

\[
k_{w3}^2 = k_{vm1}^2 + k_{u1}^2,
\]  

(7)

then it results:

\[
k_{vm} = k_{vm1} = k_{v3} = \frac{4Q}{4\pi r^2 (1 - \nu^2)(2gH)^{1/2}},
\]  

(8)

where the relative diameter of the runner:

\[
\]
\[ \nu = \frac{d}{D} = \frac{d_p}{D_p} = \frac{4.25}{9.5} = 0.45. \]  

(9)

It is considered in the formula (6) \( r = D/2 \) the zone of the runner blade with the anticavitational lip where is the most cavitation erosions at the Iron Gates I power plant.

In table 3 we have a selection of the registered data in the turbine operation at different angles of runner and wicket gates at the hydrounit 6 from January 2007 to June 2008 and the model parameters are determined with the similitude formulas.

**Table 3.** The registered data in the turbine operation

| \( \varphi \) [\( ^\circ \)] | \( \beta \) [\( ^\circ \)] | \( Q_{11} \) [m\(^3\)/s] | \( n_{11} \) [rot/min] | \( \eta_{11} \) [%] | \( \eta_T \) [%] | \( \sigma_{h_s} \) [-] | \( Z_{av_{pr}} \) [m] | \( H_{STM} \) [m] | \( H_T \) [m] | \( Q_{TM} \) [m\(^3\)/s] | \( P_{TM} \) [MW] | \( \sigma_{inst_T} \) [-] |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 27.53 | -3.84 | 0.74 | 124.51 | 0.188 | 91.14 | 94.13 | 0.252 | 40.36 | 28.73 | 28.73 | 368 | 97.12 | 0.678 |
| 30.00 | -1.60 | 0.86 | 126.91 | 0.238 | 91.34 | 94.30 | 0.311 | 41.16 | 27.66 | 27.65 | 415 | 104.39 | 0.733 |
| 32.03 | 0.26 | 0.95 | 128.71 | 0.271 | 91.47 | 94.43 | 0.352 | 41.45 | 26.90 | 26.89 | 454 | 112.54 | 0.765 |
| 34.08 | 3.94 | 1.09 | 125.16 | 0.313 | 91.8 | 94.90 | 0.397 | 40.80 | 28.44 | 28.43 | 536 | 139.19 | 0.700 |
| 36.00 | 6.60 | 1.21 | 126.04 | 0.342 | 91.97 | 94.98 | 0.436 | 41.10 | 28.05 | 28.04 | 593 | 151.57 | 0.721 |
| 38.00 | 8.07 | 1.32 | 129.54 | 0.388 | 91.84 | 94.84 | 0.496 | 41.76 | 26.56 | 26.55 | 627 | 151.41 | 0.786 |
| 40.00 | 12.45 | 1.50 | 124.67 | 0.475 | 91.4 | 94.49 | 0.591 | 40.57 | 28.66 | 28.64 | 740 | 190.90 | 0.687 |
| 42.00 | 12.67 | 1.58 | 129.32 | 0.514 | 91.1 | 94.17 | 0.646 | 41.96 | 26.64 | 26.62 | 753 | 179.65 | 0.791 |
| 44.00 | 14.19 | 1.71 | 131.57 | 0.588 | 90.74 | 93.67 | 0.738 | 41.88 | 25.74 | 25.72 | 802 | 181.78 | 0.816 |
| 46.18 | 16.67 | 1.87 | 129.81 | 0.679 | 89.78 | 92.86 | 0.845 | 41.88 | 26.43 | 26.41 | 887 | 192.70 | 0.795 |

To specify the model parameters from the exploitation parameters of the turbine it is considered the model head formula from the producer model tests.

\[ H = \frac{p_{st}}{\rho \cdot g} + \frac{Q^2}{2gS_{M1}^2} - \frac{Q^2}{2gS_{M2}^2} \]  

(10)

Where: \( p_{st} = 108.800 \) Pa – differential pressure between the intake and outlet section at the model tests, \( \rho \) - density of water, \( g = 9.80661 \) m/s\(^2\) – gravitation acceleration of the model.

From the formula (10) and similitude formula for the flow it results the flow formula of the transition from the model to the double unitary parameters:

\[ Q = \left( \frac{2 \cdot \frac{P_u}{\rho \cdot g} \cdot Q^2 \cdot (S_{M1}^2 - S_{M2}^2) D^3 \eta}{2 g (S_{M1}^2 - S_{M2}^2) H \cdot D^3 \eta_T - Q^2 \cdot D^3 \eta} \right)^{\frac{1}{2}} \]  

(11)

Considering \( \eta_{11} = \eta \) the similitude equation for transition from prototype to model [3]:

\[ n = n_p \cdot \frac{D_p}{D} \left( \frac{H}{H_p} \right)^\frac{\eta}{\eta_T} \]  

(12)

From the similitude formula of the flow and formula (10) it results the flow formula of the transition from the model to the double unitary parameters:
\[ Q_{11} = \left( \frac{2g \left( S_{M1}^2 - S_{M2}^2 \right) Q^2}{2 \cdot \frac{P_m}{\rho \cdot g} g(S_{M1}^2 - S_{M2}^2) D^4 + Q^2 D^4} \right)^{1/2} \]  

Likewise it results from the similitude formula of the rotative speed and the formula (10):

\[ n_{11} = nD \left( \frac{2g \left( S_{M1}^2 - S_{M2}^2 \right)}{2 \cdot \frac{P_m}{\rho \cdot g} g(S_{M1}^2 - S_{M2}^2) + Q^2} \right)^{1/2} \]  

The variations of the turbine and unit cavitations coefficient with the “measured” flow are shown in figure 3. It is noticed that the coefficient value \( \sigma_{0.5T} \) is greater for higher flows and lower cavitations conditions.

![Figure 3](image)

**Figure. 3** The variation of the turbine and unit cavitation coefficient with the “measured” flow.

3. **The determination of the “calculated” parameters of the axial hydraulic turbine**

In order to study and optimize the turbine operation and to reduce the cavitations effect, there are evaluated the calculated parameters using the similitude formulas for different measured parameters at constant static heads. At the beginning with the corresponding angle of the runner and wicket gates openings there are resulted from the universal diagram of the model the double unitary parameters and then with the similitude formulas, the model parameters.

In order to determine by calculation the prototype parameters, first there are determined the double unitary parameters of the model, knowing the opening angle of the wicket gates and runner.

On the universal characteristic of the turbine figure 4 resulted from stand tests, the corresponding point of the industrial turbine operating point is defined by values \( \beta_p = \beta \) and \( \varphi_p = \varphi \) of the model and there are determined with Lagrange interpolation, the universal model parameters.

From the similitude formula of the flow and formula (10) it results the flow formula for transition from the double unitary parameters to the model parameters:
\[
Q = \left( \frac{2 \frac{P_n}{\rho \cdot g} (S_{M1}^2 - S_{M2}^2) Q_{1,1}^2 D_1^4}{2g(S_{M1}^2 - S_{M2}^2)} - \frac{Q_{1,1}^2 D_1^4}{2} \right)^{1/2}
\]

\( S_{M1} = 1.22846 \, \text{m}^2 \) intake section of the model, \( S_{M2} = 0.25556 \, \text{m}^2 \) outlet section of the model draft tube, \( g = 9.80661 \, \text{N/kg} \) – gravitational acceleration.

For the rotative speed it is considered the equation (14) and results:

\[
n = \frac{n_1}{D} \left( \frac{P_n}{\rho \cdot g} + \frac{Q^2}{2g(S_{M1}^2 - S_{M2}^2)} \right)^{1/2}
\]

For the transition from model to prototype there are considered the next similitude formulas [3]:

\[
H_p = H \left( \frac{n_p}{n} \right)^2 \left( \frac{D_p}{D} \right)^2 \frac{\eta}{\eta_T} = H \left( \frac{71.43}{n} \right)^2 \left( \frac{9.5}{0.308} \right)^2 \frac{\eta}{\eta_T},
\]

\[
Q_p = Q \left( \frac{H_p}{H} \right)^{1/2} \left( \frac{D_p}{D} \right)^2 \left( \frac{\eta}{\eta_T} \right)^{1/2} = Q \left( \frac{H_p}{H} \right)^{1/2} \left( \frac{9.5}{0.308} \right)^2 \left( \frac{\eta}{\eta_T} \right)^{1/2},
\]

\[
.\, P_p = \gamma_T \cdot H_p \cdot Q_p \cdot \eta_T \cdot 10^{-6} = 998.3 \cdot 9.8066 \cdot H_p \cdot Q_p \cdot \eta_T \cdot 10^{-6}.
\]

**Figure 4.** Universal characteristic of the turbine model at the Iron Gates I power plant.
The model efficiency is determined by the universal diagram figure 4 with Lagrange polynomial as a function of the opening angles of the wicket gates and runner at the turbine exploitation, resulting the values $Q_{11}$ and $n_{11}$. The prototype efficiency is obtained with the formula $CEI_{995}$ recommended by the Sultzer Hydro documentation in the model tests:

$$
\eta_T = \eta_h + \frac{1 - \eta_{hop}}{\left(\frac{Re_{u_{opt}}}{Re_u}\right)^{0.16}} + \frac{1 - \nu'}{V} \left[\left(\frac{Re_{u_{ref}}}{Re_u}\right)^{0.16} - \left(\frac{Re_{u_{opt}}}{Re_u}\right)^{0.16}\right] \quad (20)
$$

$\nu = 1.0025 \times 10^{-6}$ - [m$^2$/s] - cinematic viscosity of the model with temperature 19.24 [$^\circ$C]

$\nu_p = 1.0036 \times 10^{-6}$ - [m$^2$/s] - cinematic viscosity of the prototype with the temperature 20 [$^\circ$C]

$Re_{u_{opt}} = 6.6011 \times 10^6$ - Reynolds model number at optimum operation conditions

$Re_{u_{ref}} = 7 \times 10^6$ - Reynolds number for the reference turbine

$n_p = 71.43$ [rot/min] - turbine rotative speed

$\eta_{hop} = 0.91738$ [-] - optimum hydraulic efficiency recommended by the producer

$V = 0.8$ - report between longitudinal and total losses of the turbines

From the equations (18) and (19) with $P_P = P_{TM}$ it was obtained the next formula:

$$
Q_T = \frac{D_P}{D} \times \left(\frac{10^3 \times P_{T_M} \times D_P \times Q_T^2}{\gamma_T \times D \times H \times \eta_T}\right)^{1/3} \quad (21)
$$

Considering the turbine head formula and the measured static head it results the formula:

$$
H_T = H_{STM} + \frac{\alpha_1 Q_T^2}{2gS_1^2} = \frac{\alpha_2 Q_T^2}{2gS_e^2} \quad (22)
$$

$\alpha_1 \approx \alpha_e = 1.024$ - Coriolis coefficient, $S_1$ - intake rectangular section of the turbine

$S_e$ - outlet rectangular section of the turbine

Finally it results the turbine power where there are replaced $Q_T$ and $H_T$ from previous formulas.

$$
P_T = \gamma_T \cdot H_T \cdot Q_T \cdot \eta_T \cdot 10^{-6} = 998.3 \cdot 9.8066 \cdot H_T \cdot Q_T \cdot \eta_T \cdot 10^{-6} \quad (23)
$$

Figure 5 represents the variation of the turbine and unit cavitation coefficient with the “calculated” flow of the prototype turbine at different static heads.

![Figure 5](image-url)

**Figure 5.** The cavitation coefficients variation with flow at $H_{STM} = 26.56$ m.
It is noticed that for the calculated parameters the cavitations coefficient \( \sigma_{0.5T} \) grows at greater flows and means the lower cavitations conditions.

4. The necessary corrections for the optimum operation of the axial hydraulic turbine

We have for the flow \( s = f(P_{Tm}, H_{STm}) \) which is an additional correction coefficient of flow resulting from the processing of a large number of measurements taken during hydrounits operation.

\[
Q_{Tc} = s^{1/2} Q_T ,
\]

\[
s = 1 \left[ 2g^3 H_T Q_T \eta_T \right]^2 + \left( \frac{\alpha_c Q_T^2}{2g S^2} - \frac{\alpha_c Q_T^2}{2g S^2} \right) \frac{P_{Tm}^2 Q_T^3}{2g S^2} \cdot \eta_T - gP_{Tm}^3 H_T Q_T \eta_T \left( \frac{\alpha_c Q_T^2}{2g S^2} - \frac{\alpha_c Q_T^2}{2g S^2} \right) \eta_T
\]

The correct evaluation of the cavitation phenomenon and of the operation parameters of the turbine is obtained with the correction coefficient of the flow which reduces the errors of flow calculation.

| Table 4. The measured parameters and calculated flow \( H_{STm} = 26.56 \text{ m} \). |
| --- |
| \( \varphi [^\circ] \) | \( \beta [^\circ] \) | \( \sigma_{0.5T} \) [-] | \( H_{Tm} \) [m] | \( ZAV \) [m] | \( Q_{Tm} \) [m³/s] | \( \eta_T \) [%] | \( P_{Tm} \) [MW] | \( \sigma_{inst T} \) [-] | \( s [-] \) | \( Q_{Tc} \) [m³/s] |
| 28.43 | -4.16 | 0.247 | 26.56 | 41.96 | 351 | 93.64 | 86.29 | 0.793 | 0.6745 | 342.17 |
| 33.34 | 1.30 | 0.370 | 26.55 | 41.57 | 479 | 94.66 | 115.30 | 0.779 | 0.7712 | 462.39 |
| 34.67 | 2.96 | 0.390 | 26.55 | 42.00 | 507 | 94.71 | 125.71 | 0.795 | 0.7870 | 499.68 |
| 38.00 | 8.07 | 0.493 | 26.55 | 41.76 | 627 | 94.90 | 151.41 | 0.786 | 0.8388 | 609.51 |
| 42.03 | 12.33 | 0.606 | 26.55 | 41.82 | 729 | 94.35 | 177.60 | 0.789 | 0.8720 | 723.59 |
| 44.27 | 15.02 | 0.714 | 26.54 | 42.10 | 805 | 93.83 | 193.41 | 0.799 | 0.8804 | 792.10 |

In the next diagrams (figure 6) there are presented, for the measured flows \( Q_{Tm} \) the variation of the turbine and unit cavitation coefficient \( \sigma_{inst T}, \sigma_{0.5T} \) at constant static measured head.

![Figure 6. The cavitation coefficient variation with flow at \( H_{STm} = 26.56 \text{ m} \).](image-url)

It is noticed that for the measured parameters the same as calculated parameters the cavitation coefficient \( \sigma_{0.5T} \) is greater at greater flows. In order to compare the measured with the calculated parameters it is observed that for the calculated parameters there are lower cavitation conditions from the measured parameters.
It is recommended to determine the real exploitation diagram using a correction coefficient of an accurate assessment of the flow at the turbine, which will reflect the reality by reducing the error caused by the phenomenon of similitude. If there are reduced differences between the measured parameters and the “calculated” prototype parameters, the cavitation phenomenon is minimized.

5. Conclusions
Studying the parameters of cavitation in turbine operation with internationally known formulae, there are observed some differences between data resulting from the calculation by universal diagram and those obtained from the measured parameters. Comparing the diagrams translated from model to prototype with the corresponding diagram of the measured data, it is shown that for the first situation, the turbine operates in lower conditions in terms of cavitation. The dynamic coefficient of supplementary correction of the flow ensures at the hydraulic turbines the permanent monitoring of the flow with a good precision. Hence we need to use a correction coefficient of the “calculated“ parameters in accordance with the cavitation erosion phenomenon observed and found to comply with the existing reality and with the parameters measured in turbine operation at the Iron Gates I hydropower plant.

6. References
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