Evaluation of Position-related Information in Multipath Components for Indoor Positioning

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Abstract—Location awareness is a key factor for a wealth of wireless indoor applications. Its provision requires the careful fusion of diverse information sources. For agents that use radio signals for localization, this information may either come from signal transmissions with respect to fixed anchors, or from multipath propagation. Using a-priori knowledge of a floor plan of the environment, specular multipath components can be exploited, based on a geometric-stochastic channel model. In this paper, a unified framework is presented for the quantification of this type of position-related information, using the concept of equivalent Fisher information. We derive analytical results for the Cramér-Rao lower bound of multipath-assisted positioning, considering bistatic transmissions between agents and fixed anchors, monostatic transmissions from agents, cooperative measurements in-between agents, and combinations thereof, including the effect of clock offsets and missing synchronization. Awareness of this information enables highly accurate and robust indoor positioning. Computational results show the applicability of the framework for the characterization of the localization capabilities of some environment, quantifying the influence of different system setups, signal parameters, and the impact of path overlap.

I. INTRODUCTION

Location awareness is a key component of many future wireless applications. Achieving the needed level of accuracy robustly is still elusive, especially in indoor environments which are characterized by harsh multipath conditions. Promising candidate systems thus either use sensing technologies that provide remedies against multipath or they fuse information from multiple information sources [1], [2]. Multipath-assisted indoor positioning employs both of these strategies. Ultra-wideband (UWB) signals are used to facilitate the separation of multipath components (MPCs). Knowing the floor plan, these MPCs can be associated to the local geometry and used as additional (virtual) anchors (VAs). In this way, additional position-related information is exploited that is contained in the radio signals.

This is in contrast to competing approaches, which either detect and avoid non-line-of-sight (NLOS) measurements [3], mitigate errors induced by strong multipath conditions [4], or employ more realistic statistical models for the distribution of the range estimates [5]. Cooperation between agents is another method to increase the amount of available information [6] and thus to reduce the localization outage. Actual exploitation of multipath propagation requires prior knowledge [7].

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Fig. 1. Illustration of multipath geometry using VAs for (i) bistatic transmissions (blue) between an anchor at $p_1^{(j)}$ and an agent at $p_1^{(m)}$ and for (ii) a monostatic measurement (gray) by an agent at $p_1^{(j)}$.

Insight on the position-related information that is conveyed in the signals [11] can be gained by an analysis of performance bounds. Using the concept of equivalent Fisher Information Matrices (EFIMs) [12], [13], allows for analytic evaluation of the Cramér-Rao lower bound (CRLB) for vector parameters from multiple information sources [9]. In an inverse problem, the room geometry can be inferred from the multipath and known measurement locations [10].

Fig. 1 illustrates the geometric model for multipath-assisted positioning. A signal transmitted between an anchor at position $p_1^{(j)}$ and an agent at $p_1^{(m)}$ includes specular reflections at the
room walls, indicated by the blue lines. These reflections can be modeled geometrically using VAs $p_{j}^{(i)}$, mirror images of the anchor w.r.t. walls that can be computed from the floor plan [22], [23], [26]. We call this the bistatic setup, where the fixed anchors and the floor plan constitute the available infrastructure. In a cooperative setup, agents localize themselves using bistatic measurements in between them. Here, the node at $p_{1}^{(j)}$ is an agent that plays the role of an anchor (and thus provides a set of VAs) for the agent at $p_{1}^{(m)}$. If the agents are equipped accordingly, they can use monostatic measurements, indicated by the gray lines. Here, the node at $p_{1}^{(j)}$ acts as anchor for itself with its own set of VAs.

For these measurement setups, we identify the following scenarios of interest: (i) Multipath-ToA (time of arrival) with known synchronization between anchors and agents, (ii) Multipath-TDOA (time difference of arrival) with unknown synchronization between either anchors or agents or also between the individual anchors, and (iii) Multipath-Coop with cooperation between the anchors, monostatic measurements, and possibly additional fixed anchors. For a tracking application, we have coined the terms multipath-assisted indoor navigation and tracking (MINT) for the bistatic setup [15], and Co-MINT [27] for the cooperative setup. The robustness and accuracy of MINT have been reported in [22], [24] and references therein. Also, a real-time demonstration system has been realized [23].

The key contributions of this paper are:

- We present a mathematical framework for the quantification of position-related information contained in geometrically modeled specular reflections in (ultra) wideband wireless signals under DM.
- This information is quantified for conventional bistatic, monostatic, and cooperative measurement scenarios, optionally including unknown clock offsets, allowing for important insights that can be used in the design of a localization system.
- The results show the relevance of a site-specific, position-related channel model for indoor positioning and the components it comprises of.

The paper is organized as follows: Section II introduces the CRLB on the position estimation error. Section III describes the relationship between signal parameters and node positions in a generic form. These results are used in Section IV to derive the CRLB for the different scenarios. Finally, Sections V and VI wrap up the paper with results, discussions, and conclusions.

**Mathematical notations:** For a random variable $z$, $\text{tr}\{\cdot\}$ and $\text{diag}\{\cdot\}$ are the trace and the diagonal of a square matrix, respectively. $|A|_{m,n}$ is the $(n,m)$-th element of matrix $A$; $A_{N \times M}$ indicates the size of a matrix. $\|\cdot\|$ is the Euclidean norm, $\cdot$ is the absolute value, and $(\ast)$ denotes convolution. $A \succeq B$ means that $A - B$ is positive semidefinite. $I_N$ is the identity matrix of size $N$. $(\cdot)^{H}$ is the Hermitian conjugate.

II. SIGNAL MODEL

In Sections II and III we simplify the setup—for the ease of readability—to a single (fixed) anchor located at position $p_1 \in \mathbb{R}^2$ and one agent at position $p \in \mathbb{R}^2$. Note that two-dimensional position coordinates are used throughout the paper, for the sake of simplicity. A baseband UWB signal $s(t)$ with effective pulse duration $T_p$ is exchanged between the anchor and the agent. The corresponding received signal is modeled as

$$r(t) = r_{\text{det}}(t) + r_{\text{diff}}(t) + w(t)$$

$$= \sum_{k=1}^{K} \alpha_k s(t - \tau_k) + (s \ast \nu)(t - \epsilon) + w(t). \quad (1)$$

The first term $r_{\text{det}}(t)$ describes a sum of $K$ deterministic MPCs with complex amplitudes $\{\alpha_k\}$ and delays $\{\tau_k\}$. We model these delays by VAs at positions $p_k \in \mathbb{R}^2$, yielding $\tau_k = \frac{1}{c}\|p - p_k\| + \epsilon$, with $k = 1 \ldots K$, where $c$ is the speed of light and $\epsilon$ represents the clock-offset due to clock asynchronism. $K$ is equivalent to the number of visible VAs at the agent position $p$. We assume the energy of $s(t)$ is normalized to one.

The second term $r_{\text{diff}}(t)$ denotes the convolution of the transmitted signal $s(t)$ with the DM $\nu(t)$, which is modeled as a zero-mean Gaussian random process. We assume uncorrelated scattering along the delay axis $\tau$, hence the auto-correlation function (ACF) of $\nu(t)$ is given by

$$K_{\nu}(\tau, u) = E_{\nu}\{\nu(\tau)\nu^*(u)\} = S_{\nu}(\tau)\delta(\tau - u), \quad (2)$$

where $S_{\nu}(\tau)$ is the power delay profile (PDP) of DM at the agent position $p$. The DM process is assumed to be quasi-stationary in the spatial domain, which means that $S_{\nu}(\tau)$ does not change in the vicinity of position $p$ [28].

Finally, the last term $w(t)$ denotes an additive white Gaussian noise (AWGN) process with double-sided power spectral density (PSD) of $N_0/2$.

In the following, we will drop the clock-offset $\epsilon$. We will re-introduce it in Section V-B where the TDOA measurement scenario is studied.

III. CRAMÉR-RAO LOWER BOUND

The goal of multipath-assisted indoor positioning is to estimate the agent’s position $p$ from the signal waveform, exploiting the knowledge of the VA positions $\{p_k\}$, in presence of diffuse multipath and AWGN with known statistics. Let $\theta$ denote the estimate of the position-related parameter vector $\theta = [p^T \Re \alpha^T \Im \alpha^T]^T \in \mathbb{R}^{2+d}$, where $\Re \alpha = [\Re \alpha_1, \ldots, \Re \alpha_K]^T$ and $\Im \alpha = [\Im \alpha_1, \ldots, \Im \alpha_K]^T$ are the real and imaginary parts of the complex amplitudes $\alpha$, respectively, which are nuisance parameters. According to the information inequality, the error covariance matrix of $\theta$ is bounded by

$$\mathbb{E}_{\theta}\left\{ (\hat{\theta} - \theta)(\hat{\theta} - \theta)^{H} \right\} \geq \mathbb{I}_{\theta}^{-1}, \quad (3)$$
where $\mathcal{I}_\theta \in \mathbb{R}^{D_\theta \times D_\theta}$ is the Fisher information matrix (FIM) and its inverse represents the CRLB of $\theta$. We apply the chain rule to derive this CRLB (cf. [12], [14]), i.e., the FIM $\mathcal{I}_\theta$ is computed from the FIM of the signal parameter vector $\psi = [\tau^T, \mathbf{R}_n^T, 3\alpha^T]^T \in \mathbb{R}^{D_\theta}$, where $\tau = [\tau_1, \ldots, \tau_K]^T$ represents the vector of position-related delays. We get

$$\mathcal{I}_\theta = J^T J$$

with the Jacobian

$$J = \frac{\partial \psi}{\partial \theta} \in \mathbb{R}^{D_\psi \times D_\theta}, \quad \text{(5)}$$

The FIM $\mathcal{I}_\psi \in \mathbb{R}^{D_\psi \times D_\psi}$ of the signal model parameters can be computed from the likelihood function $f(r|\psi)$ of the received signal $r$ conditioned on parameter vector $\psi$,

$$\mathcal{I}_\psi = \mathbb{E}_{r|\psi} \left\{ \left[ \frac{\partial}{\partial \psi} \ln f(r|\psi) \right]^T \right\} \quad \text{A. Likelihood Function}$$

$$f(r|\psi) \propto \exp \left\{ -\frac{r - \mathbf{S}\alpha}{2}\mathbf{C}_n^{-1}(r - \mathbf{S}\alpha) \right\}$$

$$\mathcal{I}_\psi = \mathbb{E}_{r|\psi} \left\{ \left[ \frac{\partial}{\partial \psi} \ln f(r|\psi) \right]^T \right\} \quad \text{(6)}$$

where $\mathbf{S} = [s_1, \ldots, s_T] \in \mathbb{R}^{N \times K}$ is the signal matrix containing delayed versions $s_k = [s(T_n - \tau_k), s(2T_n - \tau_k), \ldots, s(NT_n - \tau_k)]^T$ of the sampled transmit pulse and $\mathbf{C}_n = \alpha_n^2 \mathbf{I}_N + \mathbf{C}_c \in \mathbb{R}^{N \times N}$ denotes the co-variance matrix of the noise processes. The vector of AWGN samples has variance $\sigma_n^2 = N_0/T_s$; the elements of the DM co-variance matrix are given by $[\mathbf{C}_c]_{n,m} = T_s \sum_{t=1}^N s_t(t) s_t(t)$ (see Appendix A). This equation can be used to numerically evaluate the likelihood function without further assumptions. It can be evaluated numerically, but the inverse of the covariance matrix $\mathbf{C}_n$, which is needed as a whitening operator to account for the non-stationary DM process, limits the insight it can possibly provide. With the assumption that the received deterministic MPCs $\{\alpha_k s(t - \tau_k)\}$ are orthogonal to one another, i.e. the columns of the signal matrix $\mathbf{S}$ are orthogonal, it can be approximated by

$$f(r|\psi) \propto \exp \left\{ -\frac{2}{\sigma_n^2} \mathbf{r}^H \left( \sum_{k=1}^K w_k^2 \alpha_k s_{\tau_k} \right) \right\}$$

$$\mathcal{I}_\psi = \mathbb{E}_{r|\psi} \left\{ \left[ \frac{\partial}{\partial \psi} \ln f(r|\psi) \right]^T \right\} \quad \text{(6)}$$

$$f(r|\psi) \propto \exp \left\{ -\frac{2}{\sigma_n^2} \mathbf{r}^H \left( \sum_{k=1}^K w_k^2 \alpha_k s_{\tau_k} \right) \right\}$$

The FIM $\mathcal{I}_\psi$ of the signal model parameters quantifies the information gained from the measured signal $r$ on the signal parameters. The position-related part of this information lies in the MPC delays $\tau$, which are a function of the position $p$. With appropriate prior knowledge, such as the room geometry, this information can be used beneficially for positioning.

To compute the position error bound (PEB), which represents the CRLB on the position error, we need the upper left $2 \times 2$ submatrix of the inverse of FIM $\mathcal{I}_\theta$,

$$\mathcal{I}_\theta^{-1} \in \mathbb{R}^{2 \times 2}$$

$$\mathcal{P}(p) = \sqrt{\text{tr} \left\{ \mathcal{I}_\theta^{-1} \right\}} = \sqrt{\text{tr} \left\{ \mathcal{I}_p^{-1} \right\}} \quad \text{(10)}$$

where $\mathbf{H} = \partial \mathbf{r}/\partial \mathbf{p}$ is the submatrix of Jacobian $\mathbf{S}$ that relates to the position-related information, the derivatives of the delay vector $\tau$ w.r.t. position $\mathbf{p}$. It describes the variation of the signal parameters w.r.t. the position and can assume different, scenario-dependent forms, depending on the roles of anchors and agents. General expressions for these spatial delay gradients are derived in the next section.

IV. SPATIAL DELAY GRADIENTS

The following notations are used to find the elements of matrix $\mathbf{H}$: $\mathbf{p}^{(m)} \in \mathbb{R}^2$ is the position of the $m$-th agent, where $m \in N_m = \{1, 2, \ldots, M\}$. $\mathbf{p}_k^{(j)} \in \mathbb{R}^2$ is the position of the $j$-th fixed anchor, $j \in N_f = \{M+1, \ldots, M+J\}$, with VAs at positions $\mathbf{p}_k^{(j)} \in \mathbb{R}^2$. In the cooperative scenario, we
defines the rotation matrix, $\mathbf{R}$, to cover fixed anchors as well as agents which act as anchors. The corresponding VAs are at $p_k^{(\xi)} \in \mathbb{R}^2$. To describe gradients w.r.t. anchor or agent position, we use an index $\eta$, introducing $p^{(\eta)} \in \mathbb{R}^2$.

The delay of the $k$-th MPC is defined by the distance between the $k$-th VA and the $m$-th agent,

$$\tau_k^{(\xi,m)} = \frac{1}{c} \| p^{(m)} - p_k^{(\xi)} \| = \frac{1}{c} \sqrt{(x^{(m)} - x_k^{(\xi)})^2 + (y^{(m)} - y_k^{(\xi)})^2}. \quad (11)$$

The angle of vector $(p^{(m)} - p_k^{(\xi)})$ is written as $\phi_k^{(\xi,m)}$. To describe the relation between the signal parameter $\tau_k^{(\xi,m)}$ and the geometry, we need to analyze the spatial delay gradient, the derivative of the delay $\tau_k^{(\xi,m)}$ w.r.t. position $p^{(\eta)}$,

$$h_k^{(\xi,n,m)} = \frac{\partial \tau_k^{(\xi,m)}}{\partial p^{(\eta)}} = \frac{1}{c} \frac{\partial \| p^{(m)} - p_k^{(\xi)} \|}{\partial p^{(\eta)}} = \frac{1}{c} \frac{\partial (x^{(m)} - x_k^{(\xi)})}{\partial p^{(\eta)}} \| p^{(m)} - p_k^{(\xi)} \| + \frac{1}{c} \frac{\partial (y^{(m)} - y_k^{(\xi)})}{\partial p^{(\eta)}} \| p^{(m)} - p_k^{(\xi)} \|$$

$$= \frac{1}{c} \left( \delta_{m,n} I_2 - \delta_{n,\xi} \frac{\partial p_k^{(\xi)}}{\partial p^{(\eta)}} \right)^T e(\phi_k^{(\xi,m)}) \quad (13)$$

where $e(\phi) := [\cos(\phi), \sin(\phi)]^T$ is a unit vector in direction of the argument angle and $\delta_{m,\eta}$ is the Kronecker delta.

In Appendix [3] we show that the Jacobian of a VA position w.r.t. its respective anchor’s position is given by

$$\frac{\partial p^{(\xi)}}{\partial p^{(\eta)}} = \text{Rot}(2\gamma_k^{(\xi)}) \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} q^{(\xi)} \quad (14)$$

where

$$\text{Rot}(\gamma) := \begin{bmatrix} \cos(\gamma) & -\sin(\gamma) \\ \sin(\gamma) & \cos(\gamma) \end{bmatrix} \quad (15)$$

defines the rotation matrix, $Q_k^{(\xi)}$ is the VA order, i.e. the number of signal reflections of MPC $k$, and $\gamma_k^{(\xi)}$ is the effective wall angle, an alternating sum of wall angles

$$\gamma_k^{(\xi)} := \sum_{q=1}^{Q_k^{(\xi)}} (-1)^q \alpha_{k,q}^{(\xi)} \quad (16)$$

where index $q$ iterates the order of occurrence of walls during MPC reflection or VA construction. Using $(14)$ in $(13)$, we get

$$h_k^{(\xi,n,m)} = \frac{1}{c} \left( \delta_{m,n} e(\phi_k^{(\xi,m)}) - \delta_{\eta,\xi} e((-1)^q \alpha_{k,q}^{(\xi)} \phi_k^{(\xi,m)} + 2\gamma_k^{(\xi)}) \right), \quad (17)$$

where the first summand represents the influence of the agent position while the second summand is linked to the anchor position. We stack the transposed gradient vectors [17] for the entire set of multipath components in the gradient matrix $H^{(\xi,n,m)} \in \mathbb{R}^{K^{(\xi,m)} \times 2M}$ and the matrices for all the agents’ derivatives into matrix $H^{(\xi,m)} \in \mathbb{R}^{K^{(\xi,m)} \times 2M}$.

The following specializations will be used:

1) Bistatic scenario: $k = 1, \ldots, K^{(\xi,m)}$

a) The gradient with respect to the agent: This case describes the derivatives of delay $\tau^{(\xi,m)}$ w.r.t. the agent position, i.e. $\eta = m$, yielding the gradient

$$h_k^{(\xi,m,m)} = \frac{\partial \tau_k^{(\xi,m)}}{\partial p^{(m)}} = \frac{1}{c} e(\phi_k^{(\xi,m)}) \quad (18)$$

in direction of the line connecting the two nodes. We define the gradient matrix $H^{(\xi,m,m)} = H^{(\xi,m,m)} \in \mathbb{R}^{K^{(\xi,m)} \times 2}$.

b) The gradient with respect to the anchor: In this case, the derivatives w.r.t. the anchor position $p^{(\xi)} = p_1^{(\xi)}$ are described, i.e. $\eta = \xi$, yielding for the $k$-th MPC

$$h_k^{(\xi,\xi,m)} = \frac{\partial \tau_k^{(\xi,m)}}{\partial p^{(m)}} = -\frac{1}{c} e((-1)^q \phi_k^{(\xi,m)} + 2\gamma_k^{(\xi)}) \quad (19)$$

again in direction of the line connecting the two nodes. The proof for the final equality can be obtained graphically. The gradient matrix is $H^{(\xi,m)} = H^{(\xi,m)} = H^{(\xi,\xi,m)} \in \mathbb{R}^{K^{(\xi,m)} \times 2}$.

2) Monostatic scenario: Here we restrict the VA set to $k = 2, \ldots, K^{(m,m)}$, the agent is as well the anchor, $\xi = m$, and both move synchronously, $\eta = m$, i.e., the two terms in [17] interact with each other. The gradient

$$h_k^{(m,m,m)} = \frac{\partial \tau_k^{(m,m)}}{\partial p^{(m)}} = \frac{1}{c} \left( e(\phi_k^{(m,m)}) - e((-1)^q \phi_k^{(m,m)} + 2\gamma_k^{(m)}) \right)$$

has been decomposed—as shown in Appendix [3]—into a magnitude term $0 \leq \| h_k^{(m,m,m)} \| \leq \frac{\pi}{2}$ and a resulting direction vector. Both depend on the angle $\phi_k^{(m,m)}$ itself, the VA order, and the angles of all contributing walls comprised in $\gamma_k^{(m)}$. The gradient matrix is $H^{(m,m)} = H^{(m,m)} \in \mathbb{R}^{(K^{(m,m)} - 1) \times 2}$.

Single reflections ($Q_k^{(m)} = 1$, $\gamma_k^{(m)} = \phi_k^{(m,m)}$ + $\frac{\pi}{2}$) and reflections on rectangular corners ($Q_k^{(m)} = 2$, $\gamma_k^{(m)} = \pm \frac{\pi}{2}$) constitute important types of monostatic VAs. Both have $\frac{\partial \gamma_k^{(m,m)}}{\partial p^{(m)}} = \pm \frac{\pi}{2} e(\phi_k^{(m,m)})$, which is twice as much spatial sensitivity of delays as in the bistatic cases [13] and [19], thus providing higher ranging information. The simplest case of a vanishing gradient (magnitude zero) is a second-order reflection between parallel walls ($Q_k^{(m)} = 2$, $\gamma_k^{(m)} = 0$).

V. CRLB ON THE POSITION ERROR

In this Section, the CRLB on the position error is derived for ToA, TDoA, and a cooperative scenario. In the latter, the agents execute radar-type (monostatic) measurements, cooperative (bistatic) measurements to other agents, and also ToA measurements to fixed (physical) anchors.

Using a stack vector $\Psi = [T^T, R^\mathcal{A}^T, S^A T]^T$ of the signal parameters for all relevant nodes, with $T$ combining the delays
and \( A \) combining the amplitudes, the Jacobian \( \mathbf{J} \) has the following general structure for any measurement scenario.

\[
\mathbf{J} = \frac{\partial \Psi}{\partial \Theta} = \begin{bmatrix}
\mathbf{H} & \mathbf{L} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{I}
\end{bmatrix}
\]  

(21)

where zero-matrices in the off-diagonal blocks are skipped for clarity and \( D_{\epsilon} = 2 \sum_{j=1}^{K} K(j) \). The subblocks \( \mathbf{H}(j) = \mathbf{H}_{\epsilon j} \) account for the geometry as described in Section IV. Due to the independence of the received signals, the EFIMs \( \mathbf{I}_p(j) \) from the \( J \) different anchors are additive. Using Equation (11) and the block structure of \( \mathbf{I}_p(j) \), we can write the EFIM as a canonical sum

\[
\mathbf{I}_p = \sum_{j \in N_j} (\mathbf{H}(j))^T \left( \mathbf{A}_A(j) - \mathbf{A}_B(j) (\mathbf{A}_C(j)^{-1}) \mathbf{A}_B(j)^T \right) \mathbf{H}(j)
\]  

(24)

where \( \mathbf{A}_A(j), \mathbf{A}_B(j), \) and \( \mathbf{A}_C(j) \) are subblocks of \( \mathbf{I}_p(j) \) defined in (6). Expression (24) simplifies when we assume no path overlap (i.e., orthogonality) between signals from different VAs. In this case, \( \mathbf{A}_B = \mathbf{0} \) and \( \mathbf{A}_C \) will be diagonal. Using the result of Appendix D we can then write

\[
\mathbf{I}_p = \sum_{j \in N_j} (\mathbf{H}(j))^T \mathbf{A}_A(j) \mathbf{H}(j)
\]  

\[
\approx \frac{8\pi^2 \beta^2}{e^2} \sum_{j \in N_j} \sum_{k=1}^{K(j)} \text{SINR}_k D_k(\phi_k(j))
\]  

(25)

where \( \beta \) is the effective (mean square) bandwidth of \( s(t) \) (see Appendix D).

\[
\text{SINR}_k^{(j)} := \left( \frac{w_k^{(j)}}{N_0} \right)^2 \left( \frac{1}{N_0 + T_N S_N^{(j)}(\tau_k^{(j)})} \right)
\]  

(26)

is the signal-to-interference-plus-noise ratio (SINR) of the \( k \)-th MPC of the \( j \)-th anchor, and

\[
\mathbf{D}_k(\phi_k^{(j)}) = \mathbf{e}(\phi_k^{(j)}) \mathbf{e}(\phi_k^{(j)})^T
\]  

(27)

is called ranging direction matrix (cf. [12]), a rank-one matrix with an eigenvector in direction of \( \phi_k^{(j)} \).

Valuable insight is gained from (25) and (26). In particular,

- Each VA (i.e. each deterministic MPC) adds some positive term to the EFIM in direction of \( \phi_k^{(j)} \) and hence reduces the PEB in direction of \( \phi_k^{(j)} \).
- The SINR is the magnitude of this contribution (cf. ranging intensity information (RII) in [12]). It is limited by diffuse multipath as quantified by coefficient \( w_k^{(j)} \), an effect that scales with effective pulse duration \( T_N \) (cf. [20]) and thus inversely with the bandwidth.
- The effective bandwidth scales the EFIM. Any increase corresponds to a decreased PEB.

Discussion of path overlap (cf. [12]):

- \( \tau_k - \tau_{k'} \ll T_N \): In this case the MPCs can not be distinguished and the position-related information is entirely lost.
- \( \tau_k - \tau_{k'} \approx T_N \): In this case the MPCs are correlated, but the position-related information can still partly be used. The discrete-time formulation of the CRLB based on (7) can quantify this information gain appropriately, in contrast to our previous, continuous formulation in [13].
- \( \tau_k - \tau_{k'} \gg T_N \): In this case, MPCs are considered to be orthogonal and (25) can be used if it holds for all \( k \neq k' \).

B. Derivation of the CRLB for Multipath-TDoA

Next we consider the same setup as before, but assume the clock offsets \( \epsilon \) to be unknown parameters. The differences between arrival times still provide position information in this case, hence it has been named TDoA.

When using multiple anchors, we distinguish two different scenarios where either the clocks of all anchors are synchronized among each other, or alternatively no synchronization is present at all. While this does not affect the signal parameter FIM, we need to take it into account when performing the parameter transformation. Apart from the partial derivatives \( \mathbf{L} = \partial \mathbf{T} / \partial \epsilon \), the terms of the Jacobian are identical for ToA and TDoA, resulting in

\[
\mathbf{J} = \begin{bmatrix}
\mathbf{H}^{(1)}_{K(1) \times 2} & \mathbf{L}^{(1)}_{K(1) \times D_k} \\
\mathbf{0} & \mathbf{0} \\
\mathbf{H}^{(2)}_{K(2) \times 2} & \mathbf{L}^{(2)}_{K(2) \times D_k} \\
\mathbf{0} & \mathbf{0} \\
\vdots & \vdots \\
\mathbf{H}^{(J)}_{K(J) \times 2} & \mathbf{L}^{(J)}_{K(J) \times D_k} \\
\mathbf{0} & \mathbf{0} \\
\mathbf{I}_{D_k \times D_k}
\end{bmatrix}
\]  

(28)

where \( \mathbf{L}(j) = \partial \mathbf{T}(j) / \partial \epsilon \) and \( D_k \) is the length of \( \epsilon \).

Synchronized anchors: When assuming \( \epsilon^{(1)} = \ldots = \epsilon^{(J)} = \epsilon \), the vector \( \epsilon \) reduces to \( \epsilon = \epsilon \). The derivatives
of the arrival times with respect to the clock offset are then given by \( L^{(j)} = I^{(j)}_{\text{syn}} = [1, \ldots, 1]^T \). Applying the parameter transformation and computing the block inverse similar as in (23) leads to additivity of the 3 × 3 EFIMs \( \mathbf{I}_{p,e}^{(j)} \) for the extended parameter vector \( [p^T, \epsilon]^T \) (see Appendix E). When neglecting path overlap this expression simplifies to

\[
\mathbf{I}_{p,e} = \sum_{j \in \mathcal{N}_j} \mathbf{I}_{p,e}^{(j)} = 8\pi^2 \beta^2 \sum_{j \in \mathcal{N}_j} \sum_{k=1}^{K^{(j)}} \text{SINR}_k^{(j)} \mathbf{D}_{r,e}(\phi_k^{(j)}),
\]

with

\[
\mathbf{D}_{r,e}(\phi_k^{(j)}) = v v^T, \quad v = \left[ \frac{1}{c} \cos(\phi_k^{(j)}), \frac{1}{c} \sin(\phi_k^{(j)}), 1 \right]^T.
\]

The inner sum in (29) reveals that the 3 × 3 EFIMs \( \mathbf{I}_{p,e}^{(j)} \) are in canonical form. Since \( \mathbf{D}_{r,e} \) is a positive semidefinite matrix, it highlights that each VA adds information for the estimation of \( p \) and \( \epsilon \), scaled by its SINR and \( \beta \).

The EFIM \( \mathbf{I}_p \) can be computed from \( \mathbf{I}_{p,e} \) by again applying the blockwise inversion lemma. When neglecting path overlap, the expression for \( \mathbf{I}_p \) simplifies to

\[
\mathbf{I}_p = \frac{8\pi^2 \beta^2}{c^2} \left[ \sum_{j \in \mathcal{N}_j} \sum_{k=1}^{K^{(j)}} \text{SINR}_k^{(j)} \mathbf{D}_{r,e}(\phi_k^{(j)}) - \mathbf{C} \right],
\]

where \( \mathbf{C} \) accounts for the (negative) influence of the clock offset estimation with

\[
\mathbf{C} = \frac{1}{\sum_{j \in \mathcal{N}_j} \sum_{k=1}^{K^{(j)}} \text{SINR}_k^{(j)}} \mathbf{c} \mathbf{c}^T,
\]

\[
\mathbf{c} = \sum_{j \in \mathcal{N}_j} \sum_{k=1}^{K^{(j)}} \text{SINR}_k^{(j)} \mathbf{e}(\phi_k^{(j)}).
\]

Note that TDoA can theoretically achieve equal performance as ToA under the rather unlikely condition \( \mathbf{c} = 0 \). Otherwise \( \mathbf{C} \) reduces the information, and thereby increases the PEB with respect to ToA.

**Asynchronous anchors:** When having \( \epsilon^{(i)} \neq \epsilon^{(j)}, \forall i \neq j \), \( i, j \in \mathcal{N}_j \), we stack all clock offsets in the vector \( \epsilon = [\epsilon^{(1)}, \ldots, \epsilon^{(J)}]^T \). The derivatives of the arrival times with respect to the clock offsets are then given by a gradient matrix \( \mathbf{L} = \partial \mathbf{T} / \partial \epsilon \) of size \( \sum_{j \in \mathcal{N}_j} K^{(j)} \times J \) which stacks submatrices \( \mathbf{L}_{\text{syn}}^{(j)} \) with one nonzero column \( \mathbf{L}_{\text{syn}}^{(j)}_{i,j} = 1, i = 1, \ldots, K^{(j)} \). This leads to the additivity of the 2 × 2 EFIM for TDoA as shown in Appendix E, i.e., \( \mathbf{I}_p = \sum_{j \in \mathcal{N}_j} \mathbf{I}_{p,c}^{(j)} \). When neglecting path overlap, \( \mathbf{I}_p \) takes the form of (30), but with

\[
\mathbf{C} = \sum_{j \in \mathcal{N}_j} \frac{1}{\sum_{k=1}^{K^{(j)}} \text{SINR}_k^{(j)}} \mathbf{e}^{(j)} \left( \epsilon^{(j)} \right)^T,
\]

\[
\mathbf{e}^{(j)} = \sum_{k=1}^{K^{(j)}} \text{SINR}_k^{(j)} \mathbf{e}(\phi_k^{(j)}).
\]

Again, equality with ToA is obtained if each \( \mathbf{e}^{(j)} = 0 \), otherwise the PEB is increased.

**C. Derivation of the CRLB for Multipath-Coop**

We assume \( M \) agents \( m \in \mathcal{N}_m = \{1, 2, \ldots, M\} \) and \( J \) fixed anchors \( j \in \mathcal{N}_j = \{M+1, \ldots, M+J\} \), which cooperate with one another. As outlined in the Introduction, every agent conducts a monostatic measurement, meaning it emits a pulse and receives the multipath signal reflected by the environment, and conventional bistatic measurements with all other agents and the fixed anchors. All measurements are distributed such that every agent is able to exploit information from any of its received and/or transmitted signals. The clock-offsets \( \epsilon \) are considered to be zero.

The signal parameter vectors for the \((j,m)\)-th received signal \( \mathbf{r}^{(j,m)} \) are defined as \( \mathbf{r}^{(j,m)} = [r_1^{(j,m)}, \ldots, r_{K^{(j,m)}}^{(j,m)}]^T \) and \( \mathbf{a}^{(j,m)} = [a_1^{(j,m)}, \ldots, a_{K^{(j,m)}}^{(j,m)}]^T \). For deriving the cooperative EFIM, we stack positions \( \mathbf{p}^m \) of the \( M \) agents into the vector

\[
\mathbf{P} = [(\mathbf{p}^{(1)}_1)^T, \ldots, (\mathbf{p}^{(M)}_1)^T] \in \mathbb{R}^{2M \times 1}
\]

and all measurements \( \mathbf{r}^{(j,m)} \) in the vector

\[
\mathbf{R} = [(\mathbf{r}^{(1,1)}_1)^T, \ldots, (\mathbf{r}^{(1,M)}_1)^T, \ldots, (\mathbf{r}^{(M+1,1)}_1)^T, \ldots, (\mathbf{r}^{(M+J,M)}_1)^T]^T \in \mathbb{C}^{D_R \times 1},
\]

where \( D_R = NM(M+J) \). Further, we stack the signal parameters correspondingly in the vectors

\[
\mathbf{T} = [(\mathbf{r}^{(1,1)}_1)^T, \ldots, (\mathbf{r}^{(1,M)}_1)^T, \ldots, (\mathbf{r}^{(M+J,M)}_1)^T]^T
\]

and

\[
\mathbf{A} = [(\mathbf{a}^{(1,1)}_1)^T, \ldots, (\mathbf{a}^{(1,M)}_1)^T, \ldots, (\mathbf{a}^{(M+J,M)}_1)^T]^T
\]

of length \( D_T = D_A = \sum_{j \in \{N_m \cup \mathcal{N}_j\}} \sum_{m \in \mathcal{N}_m} K^{(j,m)} \) to construct vector \( \mathbf{\Psi} = [\mathbf{T}^T, \mathbf{R} \mathbf{A}^T, \mathbf{A} \mathbf{A}^T]^T \). The corresponding joint log-likelihood function, assuming independent measurements \( \mathbf{r}^{(j,m)} \) between the cooperating nodes, is defined as

\[
\ln f(\mathbf{R} | \mathbf{\Psi}) = \sum_{j \in \{N_m \cup \mathcal{N}_j\}} \sum_{m \in \mathcal{N}_m} \ln f(\mathbf{r}^{(j,m)}) | \mathbf{r}^{(j,m)}, \mathbf{a}^{(j,m)}).
\]

The EFIM \( \mathbf{I}_p \) is described by (see Appendix F)

\[
\mathbf{I}_p = \sum_{j \in \{N_m \cup \mathcal{N}_j\}} \sum_{m \in \mathcal{N}_m} \mathbf{F}^{(j,m)} \mathbf{A}^{(j,m)} \mathbf{H}^{(j,m)}
\]

where

\[
\mathbf{A}^{(j,m)} = \mathbf{A}^{(j,m)} - \mathbf{A}^{(j,m)} \left( \mathbf{A}^{(j,m)} \right)^{-1} \left( \mathbf{A}^{(j,m)} \right)^T
\]

yields the sub-blocks \( \mathbf{I}_{p}^{(j,m)} \) of the FIM for the likelihood function (36), for independent measurements, and \( \mathbf{H}^{(j,m)} \) are the spatial delay gradients of the Jacobian

\[
\mathbf{J} = \begin{bmatrix}
\mathbf{H}^{(1,1)}_{K^{(1,1)} \times 2M} & \cdots & \\
\vdots & \ddots & \\
\mathbf{H}^{(1,M)}_{K^{(1,M)} \times 2M} & \cdots & \\
\mathbf{H}^{(M+J,M)}_{K^{(M+J,M)} \times 2M} & \cdots & \\
\end{bmatrix}
\]
TABLE I  
CHANNEL PARAMETERS FOR NUMERICAL EVALUATIONS.

| Parameter               | Value | Description                         |
|-------------------------|-------|-------------------------------------|
| Deterministic MPCs      |       | max. VA order attenuation per reflection |
| carrier frequency f_c  | 7 GHz |                                     |
| pulse duration T_p     | 1 ns  |                                     |
| RRC pulse shape        | 0.6   |                                     |
| PDP of diffuse multipath |     | normalized power                    |
| Noi                   | 1.16 · 10^{-6} |                              |
| τ_1                   | 20 ns |                                     |
| γ_1                   | 5 ns  |                                     |
| τ_0                   | 0.98  |                                     |
| AWGN                   |       | PSD                                 |

where D_f = 2DA^2 As shown in Appendix E, one gets the following final result for the EFIM \( \mathcal{I}_p \) for all agents:

\[
\mathcal{I}_p = \begin{bmatrix}
\mathbf{I}_s^{(1)} + 2\mathbf{I}_s^{(1)} + \mathbf{I}_s^{(1)}_{\text{m}} + 2\mathbf{I}_s^{(2)} + \cdots + 2\mathbf{I}_s^{(M)} \\
2\mathbf{I}_s^{(1,2)} \\
\vdots \\
2\mathbf{I}_s^{(M,1)} \\
\mathbf{I}_s^{(M)} + 2\mathbf{I}_s^{(M)}_{\text{m}} + 2\mathbf{I}_s^{(M)}_{\text{a}} + 2\mathbf{I}_s^{(M)}_{\text{a}}
\end{bmatrix}
\]

(40)

The diagonal blocks \( \mathbf{I}_s^{(m,n)} \) account for the bistatic measurements between agent \( \eta \) and all other agents, \( \mathbf{I}_s^{(m,n)} = \sum_{j \in \mathcal{N}_s} (\mathbf{H}_{\text{ag}}^{(m,n)})^T \Lambda^{(m,n)} \mathbf{H}_{\text{ag}}^{(m,n)} \) account for the bistatic measurements between agent \( \eta \) and all fixed anchors, and \( \mathbf{I}_s^{(m,n)}_{\text{m}} = (\mathbf{H}_{\text{mo}}^{(m,n)})^T \Lambda^{(m,n)} \mathbf{H}_{\text{mo}}^{(m,n)} \) account for the monostatic measurement of agent \( \eta \). The off-diagonal blocks \( \mathbf{I}_s^{(\eta,n')} \) account for the uncertainty about the cooperating agents in their role as anchors (cf. (53) and (54)). This has a negative effect on the localization performance of the agents. The factors of two in (40), related to the EFIM of measurements in-between agents, results from the fact that those measurements are performed twice. This simplifies the notations in this section. If such repeated measurements are avoided, the same result would apply but with these factors removed.

Finally, the CRLB on position \( \mathbf{p}^{(\eta)} \) of agent \( \eta \) is

\[
\mathcal{P}(\mathbf{p}^{(\eta)}) = \text{tr} \left\{ \mathcal{I}_p^{-1}(\eta,n) \right\}.
\]

VI. RESULTS

The computational results are shown for a room illustrated in Fig. 2. For the ToA and TDoA scenarios, we assume one or two fixed anchors at \( \mathbf{p}_1^{(1)} = [10, 7]^T \) and \( \mathbf{p}_1^{(2)} = [2, 1]^T \), and one moving agent at an arbitrary position. For the cooperative scenario, the two anchors act as cooperating agents with fixed (but unknown) positions, such that in total 3 agents cooperate. For all three setups, VAS up to order two are considered. For all evaluations of the PEB, we use a point grid with a resolution of 2 cm, resulting in 180,000 points covering the entire room.

As transmit signal \( s(t) \), we use a root-raised-cosine (RRC) pulse with unit energy and a roll-off factor \( \beta_R = 0.6 \), modulated on a carrier frequency of \( f_c = 7 \text{ GHz} \). The computations are done for pulse durations of \( T_p = 0.5 \text{ ns} \), \( T_p = 1 \text{ ns} \) and \( T_p = 2 \text{ ns} \). For all antennas, we assume isotropic antenna patterns in the azimuth domain and matching polarizations. The free-space pathloss is modeled by the Friis equation. To account for the material impact, we assume 3 dB attenuation per reflection. The PDP of the DM is considered to be a fixed double-exponential function, cf. [21, eq. (9)]. This choice has also been made in [13] as it reflects the common assumption of an exponential decay of the DM power and also accounts for the fact that the LOS component is not impaired by DM as severely as MPCs arriving later. All channel-specific parameters for the deterministic MPCs, DM and AWGN are summarized in Table I.

A. Multipath-ToA

Fig. 2 shows the PEB over the floor-plan of the room for ToA and \( T_p = 1 \text{ ns} \). Fig. 2(a) indicates that the PEB neglecting path-overlap (cf. [25]), where \( T_N = T_p = 1 \text{ ns} \) for an RRC pulse) is below 10 cm for most of the area for just one physical anchor at position \( \mathbf{p}_1^{(1)} \). The entire room is covered even though the anchor is partly not visible at the agent positions. One can clearly see the visibility regions of different VA-modeled MPCs encoded by the level of the PEB. If path-overlap is considered in the computation of the CRLB (using (24)), the adverse effect of room symmetries is observable, corresponding to regions where deterministic MPCs overlap, see Fig. 2(b). In case of unresolvable path overlap, i.e. the delay difference of two MPCs is less than the pulse duration \( \tau_k - \tau_{k'} \ll T_p \), the information of the corresponding components is entirely lost (see Section V-A). Fig. 2 also shows error ellipses for several positions computed from (10). These ellipses represent the geometrically decomposed PEB with twenty-fold error standard-deviation. Fig. 2(c) shows the PEB for the same parameters but for two anchors at \( \mathbf{p}_1^{(1)} \) and \( \mathbf{p}_1^{(2)} \), considering path-overlap. The error ellipses clearly indicate that the PEB is much smaller and the impact of path overlap has been reduced.

A quantitative assessment for the scenarios in Fig. 2 is given in Figs. 3 and 4 which show the CDFs of the PEB for different pulse durations (\( T_p = 0.5 \text{ ns}, T_p = 1 \text{ ns} \) and \( T_p = 2 \text{ ns} \)). One can observe that the position error increases vastly w.r.t. this parameter. The “no PO” results account for the proportional scalings of Fisher information with bandwidth and additionally for the increased interference power due to DM, both of which are clearly seen in approximation (25). The influence of path overlap, which is neglected in (25), magnifies this effect even further because its occurrence becomes more probable. Over all, the error magnitude scales by a factor of almost ten, while the pulse duration is scaled by a factor of four. The detrimental effect of path overlap diminishes at \( T_p = 0.5 \text{ ns} \).

Our work in [22-24] shows algorithms based on the presented signal model that can closely approach these bounds. I.e. cm-level accuracy is obtained for 90% of the estimates.
Fig. 2. Logarithmic PEB (10) for Multipath-ToA with $T_p = 1$ ns over the example room for VAs up to order two. (a) One anchor at $p_1^{(1)}$; path overlap neglected. (b) same as (a) but considering the influence of path overlap. (c) a second anchor has been introduced at $p_1^{(2)}$; path overlap included. At some sample points, 20-fold standard deviation ellipses are shown.

B. Multipath-TDoA

Fig. 3 shows the CDFs of PEB (10) for Multipath-TDoA with $T_p = 1$ ns, $T_p = 1$ ns and $T_p = 2$ ns, and one anchor at $p_1^{(1)}$. Path overlap is neglected in results marked by dashed lines.

A qualitative representation of the PEB over the example room is shown in Fig. 6, with two anchors at $p_1^{(1)}$ and $p_1^{(2)}$, $T_p = 1$ ns, and VAs up to order two. By comparing this result with the ToA-PEB shown in Fig. 2(c), one can observe that the bound for TDoA is in general larger than for ToA. Also, the impact of path overlap is increased.

Fig. 7 compares ToA and TDoA for the two-anchors case and $T_p = 1$ ns, considering VAs of order one or two, and also an NLOS scenario where the LOS component has been set to zero across the entire room. One can observe the importance of the LOS component which usually has a significantly larger SINR and provides thus more position-related information than MPCs arriving later. Increasing the VA order also leads to an information gain, except for very few positions, where it has a detrimental effect. The latter is explained by the fact that
Fig. 5. CDFs of the PEB (10) for Multipath-TDoA and different synchronization settings at pulse duration $T_p = 1 \text{ ns}$, for VAs up to order two. Either two anchors are used at $p_1^{(1)}$ and $p_1^{(2)}$, which can be synchronized or not, or just the first anchor.

Fig. 6. Logarithmic PEB (10) for Multipath-TDoA over the example room with $T_p = 1 \text{ ns}$, using two asynchronous anchors at $p_1^{(1)}$ and $p_1^{(2)}$ and VAs up to order two. 20-fold standard deviation ellipses are shown at some sample points.

Fig. 7. CDFs of the PEB in (10) for ToA and TDoA and $T_p = 1 \text{ ns}$ with two anchors at $p_1^{(1)}$ and $p_1^{(2)}$. VAs of order one or two are considered; for the latter case also for an artificial NLOS situation over the whole room.

a larger VA order leads to more positions with unresolvable path overlap. This corresponds in Figs. 2(c) and 6 especially to locations close to walls and in corners.

Fig. 8. Logarithmic PEB (41) with $T_p = 1 \text{ ns}$ over the example room for three cooperating agents, two of which are resting at positions $p_1^{(1)}$ and $p_2^{(2)}$. The PEB is decomposed into its (a) monostatic and (b) cooperative components. Plot (c) shows the total PEB for Multipath-Coop. In (c), also the 40-fold standard deviation ellipses are shown at some sample points for these three cases and—in addition—for the (bistatic) case with fixed anchors.

Fig. 9. CDFs of the PEB (41) for Multipath-Coop with $T_p = 1 \text{ ns}$, for VAs of order one and two, analyzing contributions of different measurements.

C. Multipath-Coop

Fig. 8 contains 2D-plots of the different contributions to the PEB in (41) for the cooperative case. The PEB has been evaluated for Agent 3 across the entire room with two resting, cooperating agents at $p_1^{(1)}$ and $p_2^{(2)}$, considering VAs up to order two. In Fig. 8(a), only the monostatic measurements of Agent 3 are considered, illustrating the adverse effect of room symmetries and resulting unresolvable path overlap. In particular, areas close to the walls are affected as well as the diagonals of the room. Fig. 8(b) shows the information
Fig. 10. CDFs of the PEB in (11) for Multipath-Coop with $T_p = 0.5$ ns and $T_p = 2$ ns for VAs of order two, showing contributions of different measurements types.

provided by the agents at $p^{(1)}$ and $p^{(2)}$ in their role as anchors. Their contribution is similar to the fixed-anchor case analyzed in Fig. 2(c), but due to uncertainties in their own positions, this information is not fully accessible. A robust, infrastructure-free positioning system is obtained if these two components can complement one another. Indeed Fig. 8(c) indicates excellent performance across the entire area. The distinction between the parts of the position-related information is further highlighted by the CRLB ellipses in Fig. 8(c), which also include the fixed-anchor (bistatic) case of Fig. 2(c). It shows the decreased information of the cooperative part in comparison to the bistatic case with fixed anchors. The monostatic ellipses are mostly oriented towards the nearest wall, where the most significant information comes from. In many cases, this seems to be nicely complemented by the cooperative contribution.

Fig. 9 shows the CDFs of the PEB in (11) for $T_p = 1$ ns and VAs of order one and two. It is interesting to note that Multipath-Coop does not benefit from taking into account second-order MPCs. This is explained by the large influence of the monostatic measurements, for which second-order reflections cause many regions with unresolvable path overlap (c.f. Fig. 8(a)). For cooperative measurements, increasing the VA order is still beneficial.

Fig. 10 illustrates the influence of bandwidth on Multipath-Coop. Using $T_p = 0.5$ ns and $T_p = 2$ ns for VAs of order two. Especially for the monostatic measurements, the occurrence of unresolvable path overlap is significantly reduced, leading to a clear advantage of a larger bandwidth.

VII. CONCLUSIONS AND OUTLOOK

In this article, we have introduced a unified framework for evaluating the accuracy (and indirectly also the robustness) of radio-based indoor-localization methods that exploit geometric information contained in deterministic multipath components. The analysis shows and quantifies fundamental relationships between environment properties and the position-related information that can potentially be acquired. This is due to two mechanisms: (i) diffuse multipath, which is related to physical properties of the propagation environment, acts as interference to useful specular multipath components. (ii) path overlap, which relates to system design choices as the placement of

agents but also to the given geometry of an environment, may render deterministic components useless. An increased signal bandwidth allows to counteract those effects since it improves the time-resolution of the measurements: The power of DM thus decreases and path overlap becomes less likely.

The framework allows for the analysis of different measurement setups: For instance, (i) in absence of synchronization, position information can be extracted from the time-difference between MPCs. The need for clock-offset estimation reduces thereby the positioning accuracy in comparison to a synchronized setup. (ii) Cooperation between agents increases the available position-related information, but the uncertainty of the unknown positions of agents acting as anchors partly levels this effect. (iii) With monostatic measurements, the VAs move synchronously with the agents, which leads to a scaling of the information provided by MPCs. These MPC-geometry-dependent scaling factors lie between zero and two w.r.t. a conventional bistatic measurement.

We have already shown that the quantification of position-related information, as provided by the presented framework, can be used efficiently for designing positioning and tracking algorithms (e.g. [22]–[24]). The proper parametrization of the underlying geometric-stochastic channel model optimizes such algorithms and provides valuable insight for system design choices such as antenna placements and signal parameters. Algorithms that can learn and extract these environmental parameters online from measurements may achieve such optimization without the need for manual system optimization and are thus an important topic for further research on robust indoor localization.

APPENDIX A

LIKELIHOOD FUNCTION FOR ORTHOGONAL PULSES

For a sampled received signal, the covariance of AWGN and the DM is written as

$$C_n = \sigma_n^2 I_N + C_c = \sigma_n^2 I_N + \bar{S}^H S \bar{S}$$

(A.1)

where the full signal matrix $\bar{S} \in \mathbb{R}^{N \times N}$ is given by

$$\bar{S} = \begin{bmatrix} S_1^H \\ \vdots \\ S_N^H \end{bmatrix} = \begin{bmatrix} s(T_s - T_\nu) & \ldots & s(NT_s - T_\nu) \\ s(T_s - 2T_\nu) & \ldots & s(NT_s - 2T_\nu) \\ \vdots & \vdots & \vdots \\ s(T_s - NT_\nu) & \ldots & s(NT_s - NT_\nu) \end{bmatrix}$$

(A.2)

with $s_i = [s(T_s - iT_\nu) \ldots s(NT_s - iT_\nu)]^H$. So we can obtain for the covariance matrix of DM (c.f. Section II-A)

$$[C_c]_{n,m} = \bar{S}^H S \bar{S}$$

$$= \sum_{i=1}^{N} T_s S_\nu (iT_\nu) s(nT_s - iT_\nu) s(mT_s - iT_\nu).$$

(A.3)

Using the Woodbury matrix identity, the inverse of the covariance matrix $C_n$ can be written as

$$Q_n = \frac{1}{\sigma_n^2 I_N} - \frac{1}{\sigma_n^2} \bar{S}^H (S_\nu^{-1} + \frac{1}{\sigma_n^2} \bar{S} S^H)^{-1} \bar{S} \frac{1}{\sigma_n^2} I_N$$

$$= \frac{1}{\sigma_n^2 I_N} - \frac{1}{\sigma_n^2} \bar{S}^H S_\nu \left( \sigma_n^2 I_N + S_\nu S^H \right)^{-1} S.$$
We now introduce the same assumptions as in [15], i.e. \( T_r \) is at Nyquist rate (denoted here as \( T_N \)), the transmit signal has a block spectrum, and the deterministic MPCs are orthogonal. This means that the full signal matrix \( SS^H \) is an orthogonal matrix with \( SS^H = \frac{1}{T_0} I_N \) and the signal matrix \( S \) is also orthogonal. So, (A.4) is given by

\[
Q_n = \frac{1}{\sigma_n^2} I_N - \frac{1}{\sigma_n^2} S^H S_p T_N (N_0 I_N + S_p)^{-1} S \tag{A.5}
\]

which has a simple diagonal matrix to be inverted. We re-write the summands of likelihood function (7) using (A.5), yielding

\[
\Re \{ r^H Q_n s \alpha \} = \Re \left\{ \sum_{k=1}^{K} \alpha_k r^H Q_n s_{r_k} \right\}
= \Re \left\{ \frac{1}{\sigma_n^2} \sum_{k=1}^{K} \alpha_k r^H \left( I_N - S^H S_p T_N \left( N_0 I_N + S_p \right)^{-1} S \right) s_{r_k} \right\}
\]

and

\[
\alpha^H S^H Q_n s \alpha = \sum_{k=1}^{K} \sum_{k'=1}^{K} \alpha_k^* \alpha_{k'}^* \alpha_k r^H S_{r_k}^H Q_n s_{r_k}
= \frac{1}{\sigma_n^2} \sum_{k=1}^{K} \sum_{k'=1}^{K} \alpha_k^* \alpha_{k'}^* \alpha_k r^H \left( I_N - S^H \right) S_p T_N \left( N_0 I_N + S_p \right)^{-1} S_{r_k} s_{r_k}. \tag{A.6}
\]

The factor \( S S_{r_k} \) on the very right of these expressions is an autocorrelation vector of the transmitted signal shifted to delay time \( \tau_k \). The desired properties of \( s(t) \)—a large bandwidth and favorable autocorrelation properties—imply that this autocorrelation function has most of its energy concentrated at delay \( \tau_k \). It hence samples the nonstationary PDP at time \( \tau_k \) and we can approximate the summands of each summand

\[
S^H S_p T_N \left( N_0 I_N + T_N S_p \right)^{-1} S \approx \frac{T_N S_p (\tau_k)}{N_0 + T_N S_p (\tau_k)} I_N.
\]

With this approximation, (A.6) and (A.7) reduce to

\[
\Re \{ r^H Q_n s \alpha \} = \Re \left\{ \frac{1}{\sigma_n^2} \sum_{k=1}^{K} w_k^2 \alpha_k s_{r_k} \right\}
\]

and

\[
\alpha^H S^H Q_n s \alpha = \frac{1}{\sigma_n^2} \sum_{k=1}^{K} w_k^2 |\alpha_k|^2 s_{r_k}^H s_{r_k}
\]

respectively, where \( w_k^2 = N_0 / (N_0 + T_N S_p (\tau_k)) \). The double sum in (A.7) disappears due to the orthogonality of \( S \). So the likelihood function (7) can be re-written as (8), which is equivalent to (14) in [15].

**APPENDIX B**

**JACOBIAN OF VA POSITION w.r.t. Anchor Position**

We want to find a simple expression for \( \frac{\partial p_{\text{VA}}^{(\xi)}}{\partial p_{\text{VA}}} \). We restrict our derivation on a single VA of a specific node \( \xi, k \)-indexing and use a simpler notation \( \partial p_{\text{VA}} / \partial p \). As explained in Section II, \( p_{\text{VA}} \) is obtained by mirroring \( p \) on walls \( Q \) times where \( Q \) is the VA order. We use index \( q \) for this iteration and refer to the intermediate positions as \( p_q \) where \( p_0 = p \) and \( p_Q = p_{\text{VA}} \). We need to express \( p_{\text{VA}} \) as a function of \( p \) and room geometry. We account for the latter by considering walls with line equations

\[
y - y_q = \tan(\gamma_q) \cdot (x - x_q) \tag{B.1}
\]

where \( \gamma_q \) is the wall angle and \( d_q = (x_q, y_q)^T \) is an offset vector. We obtain the \( q \)-th position by mirroring position \( q - 1 \) on the \( q \)-th wall, or more formally

\[
p_q = \text{Mir}(p_{q-1}, \gamma_q, d_q). \tag{B.2}
\]

The mirroring operation is given by

\[
\text{Mir}(p_{q-1}, \gamma_q, d_q) = M(\gamma_q) \cdot (p_{q-1} - d_q) + d_q
= M(\gamma_q) \cdot p_{q-1} + (I - M(\gamma_q)) \cdot d_q \tag{B.3}
\]

where we use a mirror matrix that acts w.r.t. a line through the origin at angle \( \gamma_q \).

\[
M(\gamma_q) = \begin{bmatrix} \cos(2\gamma_q) & \sin(2\gamma_q) \\ \sin(2\gamma_q) & -\cos(2\gamma_q) \end{bmatrix}
= \text{Rot}(2\gamma_q) \cdot [1 \ 0; 0 \ -1]. \tag{B.5}
\]

\( M(\gamma_q) \) has eigenvalues \( \{-1, +1\} \) and bears analogies to rotation. \( F \) represents a sign-flip in the second dimension. For breaking down (B.3), we prefer the latter form of (B.3) because of the separated \( p_{q-1} \)-summand. By carefully repeated application, we obtain a formula

\[
p_{\text{VA}} = M(\gamma_q) \cdot p_{Q-1} + (I - M(\gamma_q)) d_Q
= M(\gamma_q)M(\gamma_{Q-1}) \cdot p_{Q-2} +
M(\gamma_q)(I - M(\gamma_{Q-1})) d_{Q-1} + (I - M(\gamma_q)) d_Q
= \ldots = \left( \prod_{q=0}^{Q-1} M(\gamma_{Q-q}) \right) p +
\sum_{q=1}^{Q} \left( \prod_{q=1}^{Q-q} M(\gamma_{Q-q+1}) \right) (I - M(\gamma_q)) d_q \tag{B.6}
\]

where the derivative w.r.t. \( p \) is just the leading product of mirror matrices. Transposition reverses multiplication order

\[
\left( \frac{\partial p_{\text{VA}}}{\partial p} \right)^T = \prod_{q=1}^{Q} M(\gamma_q). \tag{B.7}
\]

To resolve this product, we derive a pseudo-homomorphism property of the mirror matrix. We note that both \( F \) and \( M(\gamma) \) are symmetric, orthogonal, and self-inverse. Thus, \( M(\gamma) = \text{Rot}(2\gamma) \cdot F \) implies \( M(\gamma) \cdot F = \text{Rot}(2\gamma) \). We rearrange the product of two mirror matrices

\[
M(\gamma)M(\beta) = M(\gamma)M(\beta)^T = \text{Rot}(2\gamma)^T F F^T \text{Rot}(2\beta)^T
= \text{Rot}(2\gamma) \cdot I \cdot \text{Rot}(-2\beta) = \text{Rot}(2(\gamma - \beta))
\]
and obtain the property
\[
\mathbf{M}(\gamma)\mathbf{M}(\beta) = \mathbf{M}(\gamma - \beta) \cdot \mathbf{F}. \tag{B.8}
\]
Applying (B.8) to (B.7) \((Q - 1)\)-times yields
\[
\left(\frac{\partial \mathbf{p}_A}{\partial \mathbf{p}}\right)^T = \mathbf{M}(\gamma) \cdot \mathbf{F}^{Q-1} = \mathbf{Rot}(2\gamma) \cdot \mathbf{F}^Q \tag{B.9}
\]
where we refer to \(\tilde{\gamma} := \gamma_1 - \gamma_2 + \ldots + (-1)^Q - 1 \cdot \gamma_Q\) as the effective wall angle.

**APPENDIX C**

**DELAY GRADIENT FOR THE MONOSTATIC SETUP**

We transform the initial gradient from Appendix \(B\) into a magnitude-times-unit-vector form by component-wise application of basic trigonometric identities. This yields an insightful expression for the monostatic case, cf. (20). We consider
\[
e(\phi) - e((-1)^Q \phi + 2\tilde{\gamma}) = \begin{bmatrix} \cos(\phi) - \cos((-1)^Q \phi + 2\tilde{\gamma}) \\ \sin(\phi) - \sin((-1)^Q \phi + 2\tilde{\gamma}) \end{bmatrix}.
\]
Using the even/odd parity of \(Q\)
\[
O := \frac{(-1)^Q - 1}{2} \phi + \tilde{\gamma} = \begin{cases} \tilde{\gamma} & \text{If } Q \text{ is even} \\ \tilde{\gamma} - \phi & \text{If } Q \text{ is odd} \end{cases}
\]
\[
E := \frac{(-1)^Q + 1}{2} \phi + \tilde{\gamma} = \begin{cases} \tilde{\gamma} + \phi & \text{If } Q \text{ is even} \\ \tilde{\gamma} & \text{If } Q \text{ is odd} \end{cases}
\]
we further get
\[
e(\phi) - e((-1)^Q \phi + 2\tilde{\gamma}) = 2\sin(O) \begin{bmatrix} \sin(E) \\ -\cos(E) \end{bmatrix}
\]
\[
= \begin{cases} 2\sin(O) \cdot E \left(-\pi \frac{\pi}{2} \right) & \text{If } Q \text{ is even} \\ 2\sin(\tilde{\gamma}) \cdot E \left(-\pi \frac{\pi}{2} \right) & \text{If } Q \text{ is odd} \end{cases} \tag{C.1}
\]

**APPENDIX D**

**FIM FOR ORTHOGONAL PULSES**

If we assume no path overlap (i.e. orthogonality) between signals from different VAs, \(\mathbf{A}_B = \mathbf{0}\) and \(\mathbf{A}_A\) is a diagonal matrix. Using the resulting likelihood function \(\mathbf{M}(\gamma)\) derived in Appendix \(A\), \(\mathbf{A}_A\) can be rewritten as
\[
[A_A]_{k,k} = \mathbb{E}_{\mathbf{r}\mid \psi} \left\{ \frac{\partial^2 \ln f(\mathbf{r}\mid \psi)}{\partial \tau_k \partial \tau_k} \right\}
\]
\[
= 2\Re \left\{ \frac{\alpha^2 k^2 w_k^2}{\sigma^2_n} \left( \frac{\partial s_{\tau_k}}{\partial \tau_k} \right)^H \frac{\partial s_{\tau_k}}{\partial \tau_k} \right\}
\]
\[
= 2\Re \left\{ \frac{\alpha^2 k^2 w_k^2}{\sigma^2_n} \frac{\partial^2}{\partial \tau_k^2} \sum_{i=1}^N s(iT_s - \tau_k)s(iT_s - \tau_k) \right\}
\]
Approximating the sum by an integral, we get
\[
[A_A]_{k,k} \approx 2\Re \left\{ \frac{\alpha^2 k^2 w_k^2}{\sigma^2_n} \frac{\partial^2}{\partial \tau_k^2} \int_0^{NT_s} s(t - \tau_k)s(t - \tau_k)dt \right\}
\]
\[
= 2\Re \left\{ \frac{\alpha^2 k^2 w_k^2}{\sigma^2_n} \int_{-f}^{+f} |2\pi f|^2 |S(f)|^2 df \right\}
\]
\[
= 8\Re \beta^2 \alpha^2 k^2 w_k^2 / N_0
\]
where \(\beta = \int_{-f}^{+f} f^2 |S(f)|^2 df\) is the effective (mean square) bandwidth for \(s(t)\) normalized to unit energy.

**APPENDIX E**

**DERIVATION OF THE TDOA CRLB**

**Synchronized anchors:** In order to derive the \(3 \times 3\) EFIM \(\mathbf{I}_{\text{e}}\) we need to repartition the transformation matrix \(\mathbf{J}\) by combining the submatrices \(\mathbf{H}(j)\) and \(\mathbf{L}(j) = \mathbf{l}_{\text{syn}}(j)\) to \(\mathbf{G}(j) = [\mathbf{H}(j), \mathbf{l}_{\text{syn}}(j)]\). Applying the transformation leads to
\[
\mathbf{I}_{\theta} = \mathbf{J}^T \mathbf{I}_{\psi} \mathbf{J} = \begin{bmatrix} \sum_{j \in \mathcal{N}_j} (\mathbf{G}(j))^T \mathbf{A}_A^{(j)} \mathbf{G}(j) & (\mathbf{G}(j))^T \mathbf{A}_B^{(j)} \mathbf{G}(j) & \ldots & (\mathbf{G}(j))^T \mathbf{A}_C^{(j)} \mathbf{G}(j) \\ (\mathbf{A}_B^{(j)})^T \mathbf{G}(j) & (\mathbf{A}_B^{(j)})^T \mathbf{G}(j) & \ldots & (\mathbf{A}_B^{(j)})^T \mathbf{G}(j) \\ \vdots & \vdots & \ddots & \vdots \\ (\mathbf{A}_C^{(j)})^T \mathbf{G}(j) & (\mathbf{A}_C^{(j)})^T \mathbf{G}(j) & \ldots & (\mathbf{A}_C^{(j)})^T \mathbf{G}(j) \end{bmatrix}. \tag{E.1}
\]
The \(3 \times 3\) EFIM is then given as the sum over the EFIMs of the corresponding anchors
\[
\mathbf{I}_{\text{e}} = \sum_{j \in \mathcal{N}_j} (\mathbf{G}(j))^T \mathbf{A}_A^{(j)} \mathbf{G}(j), \tag{E.2}
\]
When neglecting path overlap, this reduces to
\[
\mathbf{I}_{\text{e}} = \sum_{j \in \mathcal{N}_j} (\mathbf{G}(j))^T \mathbf{A}_A^{(j)} \mathbf{G}(j), \tag{E.3}
\]
which leads finally to (29).

**Asynchronous anchors:** The result for \(\mathbf{I}_{\theta} \tag{E.1}\) is also valid when considering asynchronous anchors, provided that we respect \(\mathbf{L}(j) = \mathbf{l}_{\text{syn}}(j)\) and \(\mathbf{G}(j) = [\mathbf{H}(j), \mathbf{l}_{\text{syn}}(j)]\). We apply the blockwise inversion lemma twice, first to derive the EFIM \(\mathbf{I}_{\text{e}}\) (note that now \(\epsilon\) is a vector), and then again to prove the additivity of the EFIMs \(\mathbf{I}_{\text{e}}^{(j)}\).

The EFIM \(\mathbf{I}_{\text{e}}^{(j)}\) is now a square matrix of order \(2 + J\). It can be expressed as in \(\tag{E.2}\), but taking account of the changed definition of \(\mathbf{G}(j)\). We can write its structure as
\[
\mathbf{I}_{\text{e}}^{(j)} = \sum_{j \in \mathcal{N}_j} \begin{bmatrix} \mathbf{I}_{\text{A}}^{(j)} & \mathbf{I}_{\text{B}}^{(j)} \\ \mathbf{I}_{\text{B}}^{(j)^T} & \mathbf{I}_{\text{D}}^{(j)} \end{bmatrix}, \tag{E.4}
\]
with \(\mathbf{I}_{\text{A}}^{(j)} \in \mathbb{R}^{2 \times 2}, \mathbf{I}_{\text{B}}^{(j)} \in \mathbb{R}^{2 \times J}, \text{ and } \mathbf{I}_{\text{D}}^{(j)} \in \mathbb{R}^{J \times J}\). Further evaluation yields, that only the \(j\)-th column of \(\mathbf{I}_{\text{B}}^{(j)}\) is nonzero, and the sum over \(\mathbf{I}_{\text{B}}^{(j)}\) can be written as
\[
\sum_{j \in \mathcal{N}_j} \mathbf{I}_{\text{B}}^{(j)} = \begin{bmatrix} \mathbf{b}^{(1)} \cdot \cdot \cdot \mathbf{b}^{(J)} \end{bmatrix}, \quad \mathbf{b}^{(j)} \in \mathbb{R}^2, \tag{E.5}
\]
meaning that each column is determined by the contribution of a different anchors. Similarly, $\mathbf{I}_D^{(j)}$ has only one nonzero entry $[\mathbf{I}_D^{(j)}]_{j,j}$, leading to

$$\sum_{j \in N_j} \mathbf{I}_D^{(j)} = \text{diag}\left(\left[\mathbf{I}_D^{(j)}\right]_{1,1}, \ldots, \left[\mathbf{I}_D^{(j)}\right]_{j,j}\right). \quad (E.6)$$

Rewriting $\mathbf{I}_P$ (E.4) and again applying the blockwise inversion lemma yields the additivity of the EFIMs $\mathbf{I}_P^{(j)}$:

$$\mathbf{I}_P = \sum_{j \in N_j} \mathbf{I}_A^{(j)} - \frac{1}{\left[\mathbf{I}_D^{(j)}\right]_{j,j}} \mathbf{b}^{(j)} \mathbf{b}^{(j)^T} = \sum_{j \in N_j} \mathbf{I}_P^{(j)}. \quad (E.7)$$

The involved terms are defined by

$$\mathbf{I}_A^{(j)} = \mathbf{H}^{(j)^T} \left(\mathbf{A}_A^{(j)} - \mathbf{A}_B^{(j)} \left(\mathbf{A}_C^{(j)}\right)^{-1} \left(\mathbf{A}_B^{(j)}\right)^T\right) \mathbf{H}^{(j)},$$

$$\left[\mathbf{I}_D^{(j)}\right]_{j,j} = \sum_{u=1}^{K^{(j)}} \sum_{v=1}^{K^{(j)}} \left[\mathbf{A}_A^{(j)} - \mathbf{A}_B^{(j)} \left(\mathbf{A}_C^{(j)}\right)^{-1} \left(\mathbf{A}_B^{(j)}\right)^T\right]_{u,v},$$

and

$$\mathbf{b}^{(j)} = \left(\mathbf{H}^{(j)^T}\left[\mathbf{A}_A^{(j)} - \mathbf{A}_B^{(j)} \left(\mathbf{A}_C^{(j)}\right)^{-1} \left(\mathbf{A}_B^{(j)}\right)^T\right][1 \ldots 1]^T_{\times K^{(j)}}\right)[\mathbf{1}_M].$$

**APPENDIX F: DERIVATION OF THE MULTIPATH-COOP CRLB**

The EFIM for the cooperative setup is defined as

$$\mathbf{I}_P = \mathbf{H}^T \text{diag} \left(\Lambda^{(1,1)}, \ldots, \Lambda^{(1,M)}, \ldots, \Lambda^{(M+J,M)}\right) \mathbf{H}, \quad (F.1)$$

being of size $2M \times 2M$. It can be written with subblock $\mathbf{H}$ from (39) in the canonical form (37). Matrix $\Lambda^{(j,m)}$ is defined in (38). The canonical form decomposes the EFIM $\mathbf{I}_P$ into contributions from independent transmissions inbetween the agents or between agents and fixed anchors. Matrix $\mathbf{I}_P$ consists of the following subblocks for $\eta, \eta' \in N_m = \{1, \ldots, M\}$,

$$\mathbf{I}_P^{\eta,\eta'}_{2 \times 2} = \sum_{j \in (N_\text{Mo} \cup N_\text{Ag})} \sum_{m \in N_m} \left(\mathbf{H}^{(j,m)}\right)^T \Lambda^{(j,m)} \mathbf{H}^{(j,m)^T} \quad (F.2)$$

where $\mathbf{H}^{(j,m)}$ stacks the spatial delay gradients (17) as defined in Section IV. Considering that only summand $(j, m)$ of $\mathbf{I}_P$ contributes to a block, for which either index $j$ or index $m$ equals $\eta$ or $\eta'$, we get the following subblocks:

1. **Off-diagonal blocks** $\eta \neq \eta'$:

$$\mathbf{I}_P^{\eta,\eta'}_{2 \times 2} = \left(\mathbf{H}^{(j,m)}\right)^T \Lambda^{(j,m)} \mathbf{H}^{(j,m)^T} = \sum_{j \in N_m} \left(\mathbf{H}^{(j,m)}\right)^T \Lambda^{(j,m)} \mathbf{H}^{(j,m)^T} \quad (F.2)$$

using the definitions for $\mathbf{H}_\text{An}^{(\eta,\eta')}$ and $\mathbf{H}_\text{Ag}^{(\eta,\eta')}$ from Section IV.1.

With $\mathbf{H}_\text{An}^{(\eta,\eta')} = \mathbf{H}_\text{Ag}^{(\eta',\eta)}$ (Section IV.1), and $\Lambda^{\eta,\eta'} = \Lambda^{(\eta',\eta)}$, we get

$$\mathbf{I}_P^{\eta,\eta'}_{2 \times 2} = 2 \mathbf{I}_C^{\eta,\eta'} = 2 \left(\Lambda^{(\eta',\eta)}\right)^T \mathbf{H}_\text{Ag}^{(\eta',\eta')} \mathbf{H}_\text{Ag}^{(\eta',\eta')} \quad (F.3)$$

2. **Diagonal blocks** $\eta = \eta'$:

$$\mathbf{I}_P^{\eta,\eta'}_{2 \times 2} = \left(\mathbf{H}^{(j,m)}\right)^T \Lambda^{(\eta,\eta)} \mathbf{H}^{(\eta,\eta)} + \sum_{j \in N}\sum_{m \neq \eta} \left(\mathbf{H}^{(j,m)}\right)^T \Lambda^{(j,m)} \mathbf{H}^{(j,m)}$$

and

$$\sum_{j \in N_m} \left(\mathbf{H}_\text{An}^{(\eta,m)}\right)^T \Lambda^{(\eta,m)} \mathbf{H}_\text{An}^{(\eta,m)} + \sum_{j \in N_\text{Ag}} \left(\mathbf{H}_\text{Ag}^{(\eta)}\right)^T \Lambda^{(\eta)} \mathbf{H}_\text{Ag}^{(\eta)}$$

(using again $\mathbf{H}_\text{An}^{(\eta,\eta')}$ and $\mathbf{H}_\text{Ag}^{(\eta,\eta')}$ from Section IV.1) and $\Lambda^{\eta,\eta'} = \Lambda^{(\eta,\eta)}$ due to reciprocity, we get

$$\mathbf{I}_P^{\eta,\eta'}_{2 \times 2} = \mathbf{I}_C^{\eta,\eta'} + 2 \sum_{m \in N_m} \mathbf{I}_A^{\eta,m} + \sum_{j \in N_j} \mathbf{I}_A^{j,j}$$

which implicitly defines the contributions from monostatic measurements, bistatic measurements in between agents, and bistatic measurements between agents and fixed anchors.

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Position error bound [log(m)]
cont. $s(t)$, $\beta_R=0.6$
cont. $s(t)$, $\beta_R=0$
sampled $s(nT_s)$, $M=1$, $\beta_R=0.6$
sampled $s(nT_s)$, $M=1$, $\beta_R=0$
sampled $s(nT_s)$, $M=2$, $\beta_R=0.6$
sampled $s(nT_s)$, $M=2$, $\beta_R=0$
VA order 1
VA order 1, no PO
VA order 2
VA order 2, no PO
VA order 2, NLOS
VA order 2, NLOS, no PO
