Analysis of student’s mathematical reasoning ability materials quadratic equation on selected topics subject of secondary school

R Widiyasari1* and E Nurlaelah2

1Departemen Pendidikan Matematika, Universitas Pendidikan Indonesia, Jl. Dr. Setiabudhi No. 229, Bandung 40154, Indonesia
2Departemen Pendidikan Matematika, Universitas Pendidikan Indonesia, Jl. Dr. Setiabudhi No. 229, Bandung 40154, Indonesia

*ririn.widiyasari@upi.edu

Abstract. The purpose of this research is to know the students mathematical reasoning ability in solving on the concept quadratic equation on Selected Topics Subject of Secondary School in terms of initial student mathematical ability. This research uses qualitative approach with descriptive research type. Sources of data used in this study is a sixth semester students of Mathematics Education FIP UMI. Technique of collecting data through written test and interview. Based on the result of the research, it can be concluded that First, students with high initial ability categories have a tendency to use inductive and deductive reasoning elements well. Students can solve the problem of quadratic equations in the form of story problems in accordance with the steps of problem solving. Second, students with the medium initial ability categories have a tendency to use inductive and deductive reasoning elements well, but are less able to determine other ways to find answers. Third, students with low initial ability categories have a tendency to use inductive and deductive reasoning elements less well. Students are less able to solve the problem of quadratic equations according to problem solving steps and unable to determine other ways to find answers.

1. Introduction

Reasoning is a standard mathematical ability that has a close connection to mathematics. Mathematical reasoning is a thinking activity that has certain characteristics in finding truth. Mathematical reasoning should be given to all learners as early as from elementary schools to equip them with logical, analytical, systematic, creative thinking and cooperative ability. It is a process of thinking from various developmental insights in a phenomenon. People tend to think with various patterns, structures, or regularities in the real world and the symbolic situations of objects. Therefore, mathematical proof is the formal way of revealing certain types of reasoning [1].

Mathematics is a very important science for the development and improvement of a person’s intellectual competence in logical reasoning, spatial visualization, analysis, and abstract thought because mathematics is one of the basic science that plays an important role in the mastery and development of science and technology. The development of skilled and technology-based skilled human resources requires a strong foundation in mathematics. Students develop numeracy, reasoning,
thinking skills, and problem solving skills through learning and application of mathematics. It is valued not only in science and technology, but also in commerce, industry and everyday living [2].

In addition, mathematics is an effective tool to be used in identifying the environment and solving the problem that one faces. Throughout history, people have applied mathematics when they encounter problems. Mathematics can be considered a language that expresses abstract thinking, symbolic use, and reflection of conceptual ideas. It is also a means of understanding and exploring the real world with imagination. All these aspects make the teaching of mathematics a must. Due its importance, mathematics-related behavior has been involved in all areas of all levels of pre-programs for higher education [3].

Based on the observation conducted by the researcher at the Mathematics Education Study Program of the Faculty of Educational Sciences (FIP), UMJ, in the course of capita selecta of secondary school mathematics on the selected topics of quadratic equation, it was found that the students’ difficulty in learning lies in the reasoning to solve word problems on the quadratic equation topic. Some students were still confused about the meaning contained in the word problems and had difficulty to make a mathematical model from the problems. This is in line with Wahyuni’s opinion [4] that one of the difficulties experienced by learners in mathematics learning is solving word problems.

Reasoning is a thought process that attempts to link known facts to a conclusion [5]. Meanwhile, according to King reasoning is a mental activity that changes information to achieve a conclusion. Thus, reasoning is a very important process and is used for students in solving math problems. This corresponds to Nathaniel’s statement [6] that the kind of mathematical reasoning expected from students is the one in which students have the ability of making decisions about how to approach a problem; using strategies, skills, and concepts in finding solutions; determining the solution comprehensively; and solving the problem in an orderly manner [5].

Of course, student reasoning in solving mathematical problems is influenced by several factors. One of the factors that influence the level of student reasoning is students’ initial ability. Initial ability is the knowledge, skills, and capabilities students have mastered to learn new tasks. Initial ability is a necessary prerequisite for following the next learning material. The speed at which students master mathematics subject matter is influenced by the level of students’ initial ability. Students who have high or moderate initial ability do not have difficulty in understanding mathematics subjects so they have better learning achievements, whereas students with low initial ability will have many difficulties in understanding mathematics subject matter which will result in low student achievements in mathematics [7].

Mathematics materials and mathematical reasoning are two inseparable things, that is mathematics materials are understood through reasoning, and reasoning is trained to learners through the learning of mathematics materials, so mathematical reasoning is very important and necessary in learning mathematics materials. Thus, mathematical learning should emphasize the development of students’ potentials, including the ability to reason, create something, and solve problems. Based on the above description, it is necessary to conduct research that aims to determine the mathematical reasoning ability of Mathematics Education Study Program students of the Faculty of Educational Sciences (FIP), UMJ, in solving the problem of quadratic equation with regard to their initial ability.

2. Method
The research employed a qualitative approach with descriptive research type to find about the mathematical reasoning ability of Mathematics Education Study Program students, of FIP, UMJ, in solving quadratic equation problems in the course of capita selecta of secondary school mathematics in terms of their initial ability. Sugiyono said that defined qualitative research as one to examine the patterns of natural conditions of an object in which the researchers are key instruments. Qualitative research is analytical and descriptive [8].

The data obtained in this study were in the forms of answer sheets of student work in solving the problems of quadratic equation based on their ability and transcripts of interviews with the research subjects. Before conducting the research, the researcher first observed the students. Arikunto argued
that observation is a data-gathering technique carried out by conducting careful research and systematic recording [9].

The researcher then prepared a prerequisite test, research instruments, and interview guidelines validated by expert validators and some lecturers of mathematics education before being tested to students. Prerequisite test was conducted to determine the research subjects. The subjects were 6 (six) students of Mathematics Education Study Program of FIP, UMJ, who were selected based on their initial ability. They were further grouped into two students of high initial ability category, two of medium initial ability, and two of low initial ability. After determining the research subjects, the researcher tested the instruments to find the level of reasoning of the subjects in solving the problems of quadratic equation followed by the interview session. The data obtained were then validated using triangulation techniques and analyzed by data reduction, data presentation, and data verification [8]. Observations were conducted as support to collect data. Observations were made before the study and also during the learning activities took place.

3. Result and discussion

Based on the scores of the prerequisite test on quadratic equation, six subjects were selected, consisting of two students with high initial ability, two students with medium initial ability, and two students with low initial ability. Hereinafter, they will be called subject I, subject II, subject III, subject IV, subject V, and subject VI. The list of research subject values of observation can be seen in table 1 and the scores of their written test can be seen in Table 2:

| Subject Number | Score | Criteria       |
|----------------|-------|----------------|
| Subject 1      | 18,5  | Very Good      |
| Subject 2      | 17    | Very Good      |
| Subject 3      | 14    | Good           |
| Subject 4      | 13    | Good           |
| Subject 5      | 8,5   | Bad Enough     |
| Subject 6      | 8     | Bad Enough     |

| Subject Number | Student’s Initial Ability | Score | Criteria    |
|----------------|---------------------------|-------|-------------|
| Subject 1      | 100                       | High  |
| Subject 2      | 95                        | High  |
| Subject 3      | 85                        | Medium|
| Subject 4      | 80                        | Medium|
| Subject 5      | 55                        | Low   |
| Subject 6      | 45                        | Low   |

Following is the finding showing the students’ reasoning ability in solving the problems of quadratic equation:

Based on the answers provided by some of these subjects, the researcher can reveal 5 types of reasoning used by the subjects in solving the problem of quadratic equation, Inductive reasoning and deductive reasoning which will be explained in the next sections:

3.1. Creating an analogy

Creating an analogy is included into inductive reasoning. Based on the sample of subjects’ answers, it can be concluded that the subjects can conclude what is applicable to one event will also apply to
others. This ability can be seen from how the subjects are able to give an example of reduced distance, mention what is known and asked in a problem, and determine the relationship between what is known and asked. Students use source problem to solve target problem. Students are able to identify the relationship between the target problem and the previous knowledge.

The subjects with high, medium, and low initial abilities can use analogical reasoning well in solving quadratic equation problems. This is because the subject can identify whether there is a relationship between the problem (the target problem) and the knowledge it already has (the source problem), the student can identify a structure of the source problem in accordance with the target problem, and the student can know how to use the source problem in solve the target problem.

3.2. Creating generalizations
Creating generalization is included into inductive reasoning. The subjects’ answers demonstrate that they are able to draw conclusions from similar examples, objects, or events. This is apparent in how the subjects are able to use other means in determining the answer to a question. The subjects with high initial ability can use generalizations well in solving quadratic equation problems, while the subjects with medium and low initial abilities are not really able to use generalizations in solving quadratic equation problems.

In addition, subjects with high and medium initial ability are able to recognize a rule or pattern and are able to identify it, while subjects with low initial ability are less able to recognize a rule or pattern and have not been able to identify it. Subjects with high initial ability are able to use pattern identification results to determine the structure or subsequent data, while subject with medium and low initial ability are less able to use pattern identification results to determine the structure or subsequent data.

3.3. Categorical syllogism
Categorical syllogism is included into deductive reasoning. Based on the subjects’ answers, it is found that the subjects are able to draw conclusions from the specific to the general by mentioning three stages of reasoning: the major premise as indicated by specifying the formula, the minor premise by providing a solution of the formula used to then substitute the result, and the conclusion drawn from the result of the formula. This ability is seen in the subjects’ ability to determine the quadratic equation from the known, determine the roots of quadratic equations, substitute them, and draw conclusions from the answers.

In addition, the subjects with high and medium initial abilities are quite able to use categorical syllogism reasoning in solving the problem of quadratic equation, while the subjects with low initial ability are less capable of using the categorical syllogism reasoning in solving the quadratic equation problems. Subjects with high and medium initial abilities are able to gather facts, construct and define assumptions, assess or test assumptions, establish generalizations, construct arguments in favor, examine or test the truth of arguments and establish conclusions.

3.4. Linear syllogism
Linear syllogism is included into deductive reasoning. Based on the subjects’ answers, it can be concluded that the subjects are able to sequentially or linearly arrange objects stated in the premises; in other words, the subjects can write down the solution in sequence or linearly, starting from declaring what is known and asked in the question, creating a mathematical model, to working and determining the answer. The subjects with high, medium, and low initial abilities can all use linear syllogistic reasoning well in solving quadratic equation problems.

3.5. Proportional reasoning
Proportional reasoning is included into deductive reasoning. Based on the subjects’ answers, it can be inferred that the subjects are able to convert objects into mathematical representations through symbols using a formula. The subjects with high, medium, and low initial abilities in the early stages
attempts at quantifying do less well, and quite well on stage recognition of multiplicative relationships, and very good at accommodating covariance and invariance. The subjects with high, medium, and low initial abilities can use proportional reasoning well in solving quadratic equation problems. The results of analysis of the interviews can be seen in Table 3.

Table 3. List of research subject values of interviews.

| Subject Number | Score 1 | Score 2 | Score 3 | Score 4 | Score 5 | Total Score | Criteria |
|----------------|--------|--------|--------|--------|--------|-------------|----------|
| Subject 1      | 4      | 3      | 4      | 4      | 3      | 18          | High     |
| Subject 2      | 3      | 3      | 3      | 4      | 4      | 17          | High     |
| Subject 3      | 3      | 3      | 2      | 3      | 3      | 14          | Medium   |
| Subject 4      | 3      | 2      | 2      | 2      | 3      | 12          | Medium   |
| Subject 5      | 1      | 1      | 2      | 2      | 2      | 8           | Low      |
| Subject 6      | 2      | 2      | 1      | 1      | 1      | 7           | Low      |

Based on the data displayed in Table 3, it is found that there is disparity in the reasoning abilities of students when solving word problems of quadratic equation. Students with high initial ability can solve the problems well, students with medium initial ability can solve the problem quite well, and students with low initial ability are still not really able to solve the problems well because in evaluating problem-solving students with low initial ability tend to use one solution; hence, they cannot find similar answers in different ways and are less capable in planning the steps in problem solving.

The reasoning ability of students with low initial ability category needs special attention. Therefore, teachers are advised to use learning with problem-solving approach to improve students’ mathematical reasoning ability [10]. Based on the analysis of the written tests and interviews in Table 2, the use of reasoning in solving the problems of quadratic equation can be observed: Polya explained that in solving the problem there are four stages: understanding the problem, planning the solution, solving the problem according to the plan made in the second stage, rechecking the result. [11].

4. Conclusion
Based on the results of research and discussion obtained can be drawn a conclusion of the mathematical reasoning ability of students of Mathematics Education Study Program FIP UMJ in solving the problem of quadratic equations in terms of initial ability of students as follows:

First, students with high initial abilities have a tendency to use inductive and deductive reasoning elements well. Students can solve the problem of quadratic equations in the form of story problems properly and appropriately in accordance with the steps of problem solving. Second, students with moderate proficiency have a tendency to use inductive and deductive reasoning elements quite well. In general, early students are able to solve the problem of quadratic equations in the form of a story with a good but less able in determining other ways to find answers from the question. Third, students with low initial abilities have a tendency to use inductive and deductive reasoning elements less favorably. In general, low-skilled students are less able to solve the problem of quadratic equations in the form of story problems according to problem-solving steps and unable to determine other ways to find answers from the questioned.

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