Pseudogap in fermionic density of states in the BCS-BEC crossover of atomic Fermi gases

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Abstract We study pseudogap behaviors of ultracold Fermi gases in the BCS-BEC crossover region. We calculate the density of states (DOS), as well as the single-particle spectral weight, above the superfluid transition temperature $T_c$ including pairing fluctuations within a $T$-matrix approximation. We find that DOS exhibits a pseudogap structure in the BCS-BEC crossover region, which is most remarkable near the unitarity limit. We determine the pseudogap temperature $T^*$ at which the pseudogap structure in DOS disappears. We also introduce another temperature $T^{**}$ at which the BCS-like double-peak structure disappears in the spectral weight. While one finds $T^* > T^{**}$ in the BCS regime, $T^{**}$ becomes higher than $T^*$ in the crossover and BEC regime. We also determine the pseudogap region in the phase diagram in terms of temperature and pairing interaction.

Keywords atomic Fermi gas · BCS-BEC crossover · pseudogap

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1 Introduction

Recently, the BCS-BEC crossover has been realized in ultracold Fermi gases[1]. In this phenomenon, using a tunable pairing interaction associated with a Feshbach resonance, one can study Fermi superfluids from the weak-coupling BCS regime to the strong coupling BEC regime in a unified manner. Because of this advantage, superfluid Fermi gases would be also useful for the study of high-$T_c$ cuprates with a strong pairing interaction.

In the under-doped regime of high-$T_c$ cuprates, the so-called pseudogap structure has been observed in the density of states (DOS)[2]. As the origin of the pseudogap,
strong pairing fluctuations has been proposed\[3,4,5,6\]. However, because of the complexity of this system due to strongly correlated electrons, other possibilities, such as antiferromagnetic spin fluctuations and a hidden ordered state, have been also discussed. Thus, to confirm the pairing fluctuation scenario, another simple system only having superfluid fluctuations would be useful.

The cold Fermi gas system meets this demand. It is much simpler than high-$T_c$ cuprates, and pairing fluctuations dominate over the BCS-BEC crossover physics. Indeed, the pseudogap phenomenon in this system has been recently predicted\[3,6,7\]. Although the $s$-wave pairing symmetry of superfluid Fermi gas is different from the $d$-wave one in high-$T_c$ cuprates, we can still expect that the study of pseudogap phenomenon in cold Fermi gases would be helpful in understanding the under-doped regime of high-$T_c$ cuprates. Since a photoemission-type experiment has recently become possible in cold Fermi gases\[8\], observation of strong-coupling effects on single-particle excitations is now possible within the current technology.

In this paper, we investigate pseudogap behaviors of atomic Fermi gases above the superfluid transition temperature $T_c$. Including pairing fluctuations within a $T$-matrix approximation, we calculate DOS and single-particle spectral weight. We examine how pairing fluctuations affect them over the entire BCS-BEC crossover region. We also discuss. Thus, to confirm the pairing fluctuation scenario, another simple system only having superfluid fluctuations has been proposed\[3,4,5,6\]. However, because of the complexity of this system due to strongly correlated electrons, other possibilities, such as antiferromagnetic spin fluctuations and a hidden ordered state, have been also discussed. Thus, to confirm the pairing fluctuation scenario, another simple system only having superfluid fluctuations would be useful.

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## 2 Formalism

We consider a uniform two-component Fermi gas described by pseudospin $\sigma = \uparrow, \downarrow$. For a broad Feshbach resonance (which all the current experiments are using), it is known that one can safely study the interesting BCS-BEC crossover physics by using the ordinary BCS model\[1\], given by

$$H = \sum_{p,\sigma} \xi_p c_{p\sigma}^\dagger c_{p\sigma} - U \sum_{q, p, p'} c_{p+q/2\uparrow}^\dagger c_{p-q/2\uparrow}^\dagger c_{-p+q/2\downarrow} c_{p-q/2\downarrow}. \tag{1}$$

Here, $c_{p\sigma}$ is an annihilation operator of a Fermi atom with pseudospin $\sigma = \uparrow, \downarrow$. $\xi_p \equiv \varepsilon_p - \mu = p^2/2m - \mu$ is the kinetic energy, measured from the chemical potential $\mu$, where $m$ is an atomic mass. The pairing interaction ($U > 0$) is assumed to be tunable by a Feshbach resonance. In cold atom physics, the strength of pairing interaction is conveniently described in terms of the parameter $(k_F a_s)^{-1}$, where $a_s$ is the $s$-wave scattering length and $k_F$ the Fermi momentum. In this scale, the BCS limit and BEC limit are, respectively, given by $(k_F a_s)^{-1} \ll -1$ and $(k_F a_s)^{-1} \gg +1$. The region $-1 \lesssim (k_F a_s)^{-1} \lesssim +1$ is referred to as the crossover region. The relation between $U$ and $a_s$ is given by $4\pi a_s/m = -U/[1 - U \sum_p (m/p^2)]$.

The single-particle thermal Green’s function is given by $G_p(i\omega_n) = 1/(i\omega_n - \xi_p)$. Here, $\omega_n$ is the fermion Matsubara frequency, and $G_p(i\omega_n) = 1/(i\omega_n - \xi_p)$ is the non-interacting Fermi Green’s function. The self-energy $\Sigma(p, i\omega_n)$ involves effects of pairing fluctuations. In this paper, we include strong-coupling corrections within the $T$-matrix approximation\[3,4,6\]. The resulting self-energy has the form

$$\Sigma(p, i\omega_n) = T \sum_{q, \nu_n} \Gamma(q, i\nu_n) G_q^{\dagger} G_{q-p}(i\nu_n - i\omega_n + \delta). \tag{2}$$
where $\nu_n$ is the boson Matsubara frequency. The particle-particle scattering matrix $\Gamma$ (describing pairing fluctuations) is given by $\Gamma(q, i\nu_n) = -U/[1 - U\Pi(q, i\nu_n)]$, where $\Pi(q, i\nu_n) = T\sum_p,\omega_n G_p^0(p + q/2(i\nu_n + i\omega_n))G_{-p+q/2}^0(-i\omega_n)$ is a pair-propagator.

To discuss the pseudogap phenomenon above $T_c$, we need to determine $T_c$. Following Ref. [10], we employ the Thouless criterion [11], $\Gamma(q = 0, i\nu_n = 0, T = T_c)^{-1} = 0$, and solve this equation, together with the equation for the number of fermions,

$$N = 2T\sum_{p, \omega_n} e^{i\omega_n\delta} G_p(i\omega_n).$$  \hspace{1cm} (3)

The above treatment can describe the smooth crossover behavior of $T_c$ and $\mu$ in the BCS-BEC crossover [6, 10]. Namely, starting from the weak-coupling BCS regime, $T_c$ gradually deviates from the mean-field result to approach $T_c = 0.218\varepsilon_F$ of an $N/2$ ideal molecular Bose gas (where $\varepsilon_F$ is the Fermi energy). The chemical potential monotonically decreases from $\varepsilon_F$ in the crossover regime to be negative in the BEC regime ($(k_F a_s)^{-1} > 0.35$). The negative $\mu$ indicates the formation of two-body bound states, so that the BEC regime is well described by a molecular Bose gas, as expected.

Above $T_c$, we solve Eq. (3) to determine $\mu$. DOS $\rho(\omega)$ and the spectral weight $A(p, \omega)$ are, respectively, evaluated from the analytic continued Green’s function, as

$$\rho(\omega) = -\frac{1}{\pi}\sum_p \text{Im} G_p(i\omega_n \rightarrow \omega_+),$$  \hspace{1cm} (4)

$$A(p, \omega) = -\frac{1}{\pi}G_p(i\omega_n \rightarrow \omega_+).$$  \hspace{1cm} (5)
3 Results

Figure 1 shows DOS at $T_c$. In the BCS side ($(k_Fa_s)^{-1} < 0$) (panel (a)), we find a pseudogap (dip) structure around $\omega = 0$, which evolves as one approaches the unitarity limit ($(k_Fa_s)^{-1} = 0$). Since we only include pairing fluctuations, this pseudo gap purely originates from pre-formed pairs in the normal state.

In the BEC regime (where $\mu < 0$), when we only include the negative $\mu$ and ignore other strong coupling effects, DOS has a finite gap as $\rho(\omega) \propto \sqrt{\omega + |\mu|}$. Indeed, Fig 1(b) shows that DOS continuously changes into the fully gapped structure. Since $2|\mu|$ equals the binding energy of a molecule in the BEC limit [10], the almost fully gapped structure at $(k_Fa_s)^{-1} = 1$ in Fig 1(b) indicates that the system is rather close to a molecular Bose gas than a Fermi atom gas.

Figure 2 shows DOS above $T_c$. As expected, the pseudogap gradually disappears as one increases the temperature. When we define the pseudogap temperature $T^*$ at which the dip structure in DOS disappears, we obtain the phase diagram Fig 3. Although $T_c$ calculated in the mean-field theory is usually considered as a characteristic temperature at which preformed pairs appear, Fig 3 shows that the pseudogap temperature $T^*$ evaluated from DOS is actually much lower than the mean-field value of $T_c$ ($T_{BCS}$ in this figure).

In the BCS theory, the spectral weight below $T_c$ has the double-peak structure as

$$A(p, \omega) = \frac{1}{2} \left[ 1 + \frac{\xi p}{E_p} \right] \delta(\omega - E_p) + \frac{1}{2} \left[ 1 - \frac{\xi p}{E_p} \right] \delta(\omega + E_p).$$ (6)
Here, $E_p = \sqrt{\xi_p^2 + \Delta^2}$ is the Bogoliubov excitation spectrum, where $\Delta$ is the superfluid order parameter. The minimum value of peak-to-peak energy equals $2\Delta$. In the simple mean-field theory, this double-peak structure only exists below $T_c$. However, when one includes pairing fluctuations, the double-peak structure still remains above $T_c$, as shown in Fig. 4. This characteristic structure disappears at a certain temperature $T^{**}$ (See Fig. 4a and (b)), which we can define as another pseudogap temperature.

When we plot $T^{**}$ in Fig. 3 we find that it does not coincide with $T^*$. While the latter is higher in the BCS side, one finds $T^{**} > T^*$ in the BEC side. In the BCS side, although the double-peak structure is absent when $T \geq T^{**}$, pairing fluctuations still strongly affect $A(p, \omega)$ around the Fermi level $p \sim \sqrt{2m\mu}$, leading to a broad and low single peak structure. Then, since DOS is given by the momentum summation of $A(p, \omega)$, DOS around $\omega \sim 0$ (which is dominated by $A(p \sim \sqrt{2m\mu}, \omega)$) is suppressed, giving the dip structure. On the other hand, as shown in Fig. 4(c), the lower peak in $A(p, \omega)$ soon becomes broad above $T_c$ in the BEC regime, so that the effect of lower peak is easily smeared out in the momentum summation in Eq. (6). As a result, DOS does not reflect the double-peak structure in $A(p, \omega)$ when $T^* < T \leq T^{**}$.

The above result indicates that the pseudogap temperature depends on what we measure. When we measure a quantity where DOS is crucial, such as the specific heat, $T^*$ would be the crossover temperature between the pseudogap regime (PG) and normal Fermi gas regime (NF). On the other hand, when we consider a quantity dominated by the spectral weight, such as the photoemission-type experiment done by JILA group[8], $T^{**}$ would work as the crossover temperature between PG and NF.

As discussed previously, the pseudogapped DOS continuously changes into fully gapped one in the BEC regime, reflecting that the system reduces to an $N/2$ molecular Bose gas (MB). As a crossover temperature between PG and MB regime, the molecular binding energy $E_{\text{bind}}$ would be useful, because thermal dissociation of molecules is suppressed when $T \lesssim E_{\text{bind}}$. In the present case, noting that $E_{\text{bind}} \simeq |2\mu|$ in the BEC regime (where $\mu < 0$), one may conveniently determine the pseudogap regime as the region surrounded by $T_c$, $2|\mu|$, and $T^*$ or $T^{**}$ in Fig. 3.
Fig. 4 Spectral weight $A(p, \omega)$ as a function of $\omega$. In each panel, we take the momentum where the peak-to-peak energy is minimum: (a) $p/k_F = 0.91$, (b) 0.83, and (c) 0.01.

4 Summary

To summarize, we have discussed effects of pairing fluctuations on single-particle properties of a cold Fermi gas above $T_c$. Within the framework of $T$-matrix approximation, we showed how the pseudogap appears in the density of states and the single-particle spectral weight over the entire BCS-BEC crossover region. We have also determined the pseudogap regime in the BCS-BEC crossover phase diagram. This phase diagram would be useful for the observation of pseudogap phenomenon in this system.

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