Constraints on Chaplygin quartessence from the CLASS gravitational lens statistics and supernova data

Abha Dev\textsuperscript{1}, Deepak Jain\textsuperscript{2}, and J. S. Alcaniz\textsuperscript{3}

\textsuperscript{1} Department of Physics and Astrophysics, University of Delhi, Delhi 110007, India
e-mail: abha@ducos.ernet.in
\textsuperscript{2} Deen Dayal Upadhyaya College, University of Delhi, Delhi 110015, India
e-mail: deepak@physics.du.ac.in
\textsuperscript{3} Departamento de Física, Universidade Federal do Rio Grande do Norte, C.P. 1641, Natal - RN, 59072-970, Brasil
e-mail: alcaniz@dfte.ufrn.br

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Abstract. The nature of the dark components (dark matter and dark energy) that dominate the current cosmic evolution is a completely open question at present. In reality, we do not even know if they really constitute two separated substances. In this paper we use the recent Cosmic All Sky Survey (CLASS) lensing sample to test the predictions of one of the candidates for a unified dark matter/energy scenario, the so-called generalized Chaplygin gas (Cg) which is parametrized by an equation of state \( p = -A/\rho^\alpha_{Cg} \) where \( A \) and \( \alpha \) are arbitrary constants. We show that, although the model is in good agreement with this radio source gravitational lensing sample, the limits obtained from CLASS statistics are only marginally compatible with the ones obtained from other cosmological tests. We also investigate the constraints on the free parameters of the model from a joint analysis between CLASS and supernova data.

Key words. Cosmology: theory – dark matter – cosmological parameters

1. Introduction

As is well known, there is mounting observational evidence that our Universe is presently dominated by two exotic forms of matter or energy. Cold, nonbaryonic dark matter, which accounts for \( \approx 30\% \) of the critical mass density and whose leading particle candidates are the axions and the neutralinos, was originally proposed to explain the general behavior of galactic rotation curves that differ significantly from the one predicted by Newtonian mechanics. Later on, it was also realized that the same concept is necessary for explaining the evolution of the observed structure in the Universe from density inhomogeneities of the size detected by a number of Cosmic Microwave Background (CMB) experiments. Dark energy or quintessence, which accounts for \( \approx 70\% \) of the critical mass density and whose leading candidates are a cosmological constant \( \Lambda \) and a relic scalar field \( \phi \), has been inferred from a combination of astronomical observations which includes distance measurements of type Ia supernovae (SNe Ia) indicating that the expansion of the Universe is speeding up not slowing down (Perlmutter et al. 1999; Riess et al. 1998), CMB anisotropy data suggesting \( \Omega_T \approx 1 \) (de Bernardis et al. 2000; Spergel et al. 2003), and clustering estimates providing \( \Omega_m \approx 0.3 \) (Calberg et al. 1996; Dekel, Burstein & White 1997). While the combination of the last two results implies the existence of a smooth component of energy that contributes with \( \approx 2/3 \) of the critical density, the SNe Ia results require this component to have a negative pressure, which leads to a repulsive gravity.

Despite the good observational evidence for the existence of these two forms of energy, it has never been shown that in fact they constitute two separate substances. In this concern, some authors have proposed the so-called Unified Dark Matter/Energy scenarios (UDME) or quartessence, i.e., models in which these two dark components are seen as different manifestations of a single fluid (see, for instance, Matos & Ureña-Lopez 2000; Davidson, Karasik & Lederer 2001; Watterich 2002; Kasuya 2001; Padmanabhan & Choudhury 2002). Among these theoretical proposals, an interesting attempt of unification was originally suggested by Kamenshchik et al. (2001) and developed by Bilić et al. (2002) and Bento et al. (2002), namely, the Chaplygin gas (Cg), an exotic fluid whose equation of state is given by

\[
p_{Cg} = -A/\rho^\alpha_{Cg},
\]
with $\alpha = 1$ and $A$ a positive constant. In actual fact, the above equation for $\alpha \neq 1$ constitutes a generalization of the original Chaplygin gas equation of state proposed by Bento et al. (2002). The idea of a dark-matter-energy unification from an equation of state like Eq. (1) comes from the fact that the Cg can naturally interpolate between nonrelativistic matter ($p = 0$) and negative-pressure ($p = -\text{const.}$) dark energy regimes (see Bento et al. 2002; Alcaniz, Jain & Dev 2002 for details).

Very recently, there has been a wave of interest in exploring theoretical (Bordemann & Hoppe, 1993; Hoppe 1993; Jackiw 2000; Gonzalez-Diaz 2003a; 2003b; Kremer 2003; Khalatnikov 2003; Balakin et al. 2003; Bilić et al. 2003) and observational consequences of the Chaplygin gas, not only as a possibility of unification for dark matter/energy but also as a new candidate for dark energy only. These models have been tested for a number of cosmological data sets, including SNe Ia data (Fabris, Goncalves & de Souza, 2002; Colistete Jr. et al. 2003; Avelino et al. 2003; Makler, de Oliveira & Waga 2003), statistical properties of gravitationally lensed quasars (Dev, Alcaniz & Jain 2003; Silva & Bertolami 2003), CMB measurements (Bento, Bertolami & Ścibor 2003a; 2003b; 2003c; Carturan & Finelli 2002; Amendola et al. 2003), age and angular size - redshift tests (Alcaniz, Jain & Dev 2002; Alcaniz & Lima 2003), measurements of X-ray luminosity of galaxy clusters (Cunha, Lima & Alcaniz 2003), future lensing and SNe Ia experiments (Avelino et al. 2003; Silva & Bertolami 2003; Sahni et al. 2003), as well as by observations of large scale structure (Mültamäki, Manera & Gaztanaga 2003; Bilić et al. 2003). The present situation is somewhat controversial, with some tests indicating good agreement between observational data and the theoretical predictions of the model and others ruling out the model as an actual possibility of description for our Universe (Sandvik et al. 2002; Bean & Dore 2003) (see, however, Beća et al. 2003).

The aim of the current paper is to check the validity of such models with radio-selected gravitational lens statistics and also with a combination of gravitational lensing and SNe Ia data. To do so, we adopt the most recent radio source gravitational lensing sample, namely, the Cosmic All Sky Survey (CLASS) statistical data which consists of 8958 radio sources out of which 13 sources are multiply imaged (Browne et al. 2002; Chae et al. 2002). Here, however, we work only with those multiply imaged sources whose image-splittings are known (or likely) to be caused by single galaxies, which reduces the total number of lenses to 9. For the cosmological background we consider a flat scenario in which the generalized Cg together with the observed baryonic content are responsible for the dynamics of the present-day Universe (UDME or quintessence models). In our computations we adopt $\Omega_r = 0.04$, in accordance with the latest measurements of the Hubble parameter (Freedman et al. 2002) and of the baryon density at nucleosynthesis (Burles, Nollett & Turner 2001).

This paper is organized as follows. In Sect. 2 we present the distance formulae necessary to our analysis. In Sect. 3 we discuss the CLASS statistical sample, especially the observational criteria used as well as the restrictions adopted in our analysis. In Sect. 4, we derive the corresponding limits on Cg scenarios from CLASS lensing statistics. We also examine the constraints obtained from the statistical combination of lensing data with recent SNe Ia observations and compare our constraints with others derived from independent analyses. Finally, in Sect. 5, we finish the paper by summarizing its main results.

### 2. Basic equations

By inserting Eq. (1) into the energy conservation law ($u_r T_{rr}^{\nu \nu} = 0$) one finds,

$$\rho_{\text{Cg}} = \left[ A + B \left( \frac{R}{R_o} \right)^{3(1+\alpha)} \right]^{\frac{1}{3\alpha}}, \quad (2)$$

or, equivalently,

$$\rho_{\text{Cg}} = \rho_{\text{Cg}_0} \left[ A_s + (1 - A_s) \left( \frac{R}{R_o} \right)^{3(1+\alpha)} \right]^{\frac{1}{3\alpha}}, \quad (3)$$

where $\rho_{\text{Cg}}$ stands for the Cg energy density, the subscript $o$ denotes present day quantities, $R(t)$ is the cosmological scale factor, $B = \rho_{\text{Cg}}^0 - A$ is a constant and $A_s = A/\rho_{\text{Cg}_0}^0$ is a quantity related to the sound speed of the Chaplygin gas today ($v_s^2 = A_s$).

A fundamental quantity related to the observables here considered is the distance - redshift relation, given by

$$X = \frac{c}{H_o R_o} \int_{(1+z)}^{1} \frac{dx}{x^2 E(\Omega_b, A_s, \alpha, x)}, \quad (4)$$

where $x = \frac{R(t)}{R_o} = (1 + z)^{-1}$ is a convenient integration variable, $\Omega_b$ stands for the baryonic matter density parameter, and the dimensionless function $E(\Omega_b, A_s, \alpha, x)$ is written as

$$E = \left\{ \frac{\Omega_b}{x^3} + (1 - \Omega_b) \left[ A_s + \frac{(1 - A_s)}{x^{3(\alpha+1)}} \right] \right\}^{1/2}. \quad (5)$$

For the lensing statistics developed in the next section, two concepts are of fundamental importance, namely, the angular diameter distance, $D_{\text{LS}}(z_L, z_S) = R_o(1+z_L)^2/(1+z_S)$, between two objects, for example a lens at $z_L$ and a source (galaxy) at $z_S$,

$$D_{\text{LS}}(z_L, z_S) = \frac{cH^{-1}_o}{(1+z_S)} \int_{z_L}^{z_S} \frac{dx}{x^2 E(\Omega_b, A_s, \alpha, x)}. \quad (6)$$
Subsequently, it was realized that a comparison of theoretical lensing probabilities with gravitational lensing observations provided an efficient constraint on the cosmological constant (Fukugita, Futamase & Kasai 1990; Turner 1990; Fukugita et al. 1992; Kochanek 1996) or more generally on the density and the equation of state of the dark energy component (Zhu 1998; 2000a; 2000b; Sarbu, Rusin & Ma 2001; Chae et al. 2002; Huterer & Ma, 2003). However, the absence of an unbiased statistical sample of sources that is complete to within well-defined observational selection criteria and the uncertainties in the luminosity function (LF) of galaxies have seriously complicated the application of such methods.

3. Gravitational lensing statistics of the CLASS Sample

Gravitational lensing directly probes the mass distribution in the Universe so that an investigation of lensing events of sources at high redshifts can provide important information about the global cosmological parameters and the structure of the Universe. The use of gravitational lensing statistics as a cosmological tool was first considered in detail by Turner, Ostriker & Gott (1984). Subsequently, it was realized that a comparison of theoretical lensing probabilities with gravitational lens observations provided an efficient constraint on the cosmological constant (Fukugita, Futamase & Kasai 1990; Turner 1990; Fukugita et al. 1992; Kochanek 1996) or more generally on the density and the equation of state of the dark energy component (Zhu 1998; 2000a; 2000b; Sarbu, Rusin & Ma 2001; Chae et al. 2002; Huterer & Ma, 2003). However, the absence of an unbiased statistical sample of sources that is complete to within well-defined observational selection criteria and the uncertainties in the luminosity function (LF) of galaxies have seriously complicated the application of such methods.

3.1. The CLASS Statistical Sample

Recently, the CLASS collaboration \(^1\) reported the so far largest lensing sample suitable for statistical analysis, in which 13 out of the 8958 radio sources are multiply imaged (Myers et al. 2002; Browne et al. 2002). This sample is well defined through the following observational selection criteria (Myers et al. 2002; Chae 2002; Browne et al. 2002): (i) the spectral index between 1.4 GHz and 5 GHz is flatter than \(-0.5\), i.e., \(\alpha \geq -0.5\) with \(S_\nu \propto \nu^\alpha\), where \(S_\nu\) is the flux density measured in milli-jansky; (ii) the total flux density of each source is \(\geq 20\) mJy at 8.4 GHz; (iii) the total flux density of each source is \(\geq S_\nu^0 \equiv 30\) mJy at 5 GHz; (iv) the image components in lens systems must have separations \(\geq 0.3\) arcsec. The sources probed by CLASS at \(\nu = 5\) GHz are well represented by a power-law differential number-flux density relation: \(dN/dS \propto S^\eta\) with \(\eta = 2.07 \pm 0.02\) (1.97 \pm 0.14) for \(S \geq S_\nu^0\) \((\leq S_\nu^0)\). The redshift distribution of unlensed sources in the sample is adequately described by a Gaussian model with a mean redshift \(z = 1.27\) and dispersion of 0.95 (Chae 2002). Guided by the above information about (i) the number-flux density relation and (ii) the redshift distribution of unlensed sources, we simulate the unlensed radio sources (8945 in number) of the CLASS statistical sample using the Monte-Carlo technique (rejection method).

In this paper, following Dev, Jain & Mahajan (2003), we work only with those multiply imaged sources whose images are known (or likely) to be caused by single galaxies. This means that our database is constituted by 9 lenses out of a sample of 8954 radio sources.

3.2. Lensing Statistics

We start our analysis by assuming the singular isothermal sphere (SIS) model for the lens mass distribution. As has been discussed elsewhere this assumption is a good approximation to the real mass distribution in galaxies (see, e.g., Turner, Ostriker & Gott 1984). In this case, the cross-section for lensing events is given by

\[
\sigma_{\text{SIS}} = 16\pi^4 \left(\frac{v_c}{c}\right)^4 \left(\frac{D_{\text{OL}} D_{\text{LS}}}{D_{\text{OS}}}\right)^2, \tag{8}
\]

where \(v\) represents the velocity dispersion and \(D_{\text{OL}}, D_{\text{OS}}\) and \(D_{\text{LS}}\) are, respectively, the angular diameter distances from the observer to the lens, from the observer to the source and between the lens and the source. By ignoring evolution of the number density of galaxies and assuming that the comoving number density is conserved, the differential probability of a lensing event can be expressed as

\[
d\tau = n_o (1 + z_L)^3 \sigma_{\text{SIS}} \frac{c dt}{dz_L}, \tag{9}
\]

where the quantity \(dt/dz_L\) can be easily obtained from Eq. (7) and the present-day comoving number density of galaxies is

\[
n_o = \int_0^\infty \phi(L) dL. \tag{10}
\]

\(^1\) The Cosmic Lens All Sky Survey (CLASS): [http://www.aoc.nrao.edu/~smyers/class.html](http://www.aoc.nrao.edu/~smyers/class.html)
The differential optical depth of lensing in traversing $dz_L$ with angular separation between $\phi$ and $\phi + d\phi$ is (Fukugita, Futamase & Kasai 1990; Turner 1990; Fukugita et al. 1992):

$$\frac{d\tau}{dz_L d\phi} d\phi dz_L = F^*(1 + z_L)^3 \left( \frac{D_{OL} D_{LS}}{R_o D_{OS}} \right)^2 \frac{1}{R_0} \frac{cdt}{dz_L} \times \frac{\gamma/2}{\Gamma(\alpha + 1 + \frac{2}{\gamma})} \left( \frac{D_{OS}}{D_{LS}} \phi \right)^{\frac{\gamma}{\alpha + 1 + \frac{2}{\gamma}}} \times \exp \left[ -\left( \frac{D_{OS}}{D_{LS}} \phi \right)^{\frac{\gamma}{\alpha} + 1 + \frac{2}{\gamma}} \right] \frac{d\phi}{\phi} dz_L. \quad (11)$$

where the function $F^*$ is defined as

$$F^* = \frac{16 \pi^3}{c H_0^2} \phi_v \nu_v \Gamma \left( \alpha + \frac{4}{\gamma} + 1 \right). \quad (12)$$

In Eq. (10), $\phi(L)$ is the Schechter LF (Schechter 1976) given by

$$\phi(L) dL = \phi_c \left( \frac{L}{L_*} \right)^{\alpha} \exp(-L/L_*) \frac{dL}{L_*.} \quad (13)$$

In order to relate the characteristic luminosity $L_*$ to the characteristic velocity dispersion $\nu_*$, we use the Faber-Jackson relation (Faber & Jackson 1976) for E/S0 galaxies ($L_* \propto \nu_*^3$), with $\gamma = 4$. For the analysis presented here we neglect the contribution of spirals as lenses because their velocity dispersion is small when compared to ellipticals.

The two large-scale galaxy surveys, namely, the 2dFGRS $^2$ and the SDSS $^3$ have produced converging results on the total LF. The surveys determined the Schechter parameters for galaxies (all types) at $z \leq 0.2$. Chae (Chae 2002) has worked extensively on the information provided by these recent galaxy surveys to extract the local type-specific LFs. For our analysis here, we adopt the normalization-corrected Schechter parameters of the 2dFGRS survey (Folkes et al. 1999): $\alpha = -0.74$, $\phi^* = 0.82 \times 10^{-22} h^2 \text{Mpc}^{-3}$, $\nu^* = 185 \text{ km/s}$ and $F^* = 0.014$.

The normalized image angular separation distribution for a source at $z_S$ is obtained by integrating $\frac{d\tau}{dz_L d\phi}$ over $z_L$:

$$\frac{dp}{d\phi} = \frac{1}{\tau(z_S)} \int_0^{\infty} \frac{d\tau}{dz_L d\phi} d\phi dz_L. \quad (14)$$

The corrected (for magnification and selection effects) image separation distribution function for a single source at redshift $z_S$ is given by (Kochanek 1996; Chiba & Yoshii 1999)

$$P'(\Delta \theta) = \mathcal{B} \frac{\gamma}{2 \Delta \theta} \int_0^{\infty} \left[ \frac{D_{OS}}{D_{LS}} \phi \right]^{\frac{\gamma}{\alpha + 1 + \frac{2}{\gamma}}} F^* \frac{cdt}{dz_L} \times \exp \left[ -\left( \frac{D_{OS}}{D_{LS}} \phi \right)^{\frac{\gamma}{\alpha} + 1 + \frac{2}{\gamma}} \right] \frac{(1 + z_L)^3}{\Gamma(\alpha + \frac{2}{\gamma} + 1)} \times \left( \frac{D_{OL} D_{LS}}{R_o D_{OS}} \right)^2 \frac{1}{R_0} \frac{d}{dz_L}. \quad (15)$$

Similarly, the corrected lensing probability for a given source at redshift $z$ is given by

$$P' = \tau(z_S) \int \frac{dp}{d\phi} \mathcal{B} \frac{d\phi}{\phi}. \quad (16)$$

Here $\phi$ and $\Delta \theta$ are related as $\phi = \frac{\Delta \theta}{\text{arcsec} \times \frac{h}{\text{cMpc}}}$, and $\mathcal{B}$ is the magnification bias. This is taken into account because, as widely known, gravitational lensing causes a magnification of images and this transfers the lensed sources to higher flux density bins. In other words, the lensed sources are over-represented in a flux-limited sample. The magnification bias $\mathcal{B}(z_S, S, \nu)$ increases the lensing probability significantly in a bin of total flux density $S_*$ by a factor

$$\mathcal{B}(z_S, S_*) = \frac{dN_{z_S}(> S_*)}{dS_*} \times \int_{\mu_{\text{min}}}^{\mu_{\text{max}}} \frac{dN_{z_S}(> S_*/\mu)}{dS_*} p(\mu) \frac{1}{\mu} \frac{d\mu}{\mu}. \quad (17)$$

Here $N_{z_S}(> S_*)$ is the intrinsic flux density relation for the source population at redshift $z_S$. $N_{z_S}(> S_*)$ gives the number of sources at redshift $z_S$ having flux greater than $S_*$. For the SIS model, the magnification probability distribution is $\rho(\mu) = 8/\mu^3$. The minimum and maximum total magnifications $\mu_{\text{min}}$ and $\mu_{\text{max}}$ in Eq. (17) depend on the observational characteristics as well as on the lens model. For the SIS model, the minimum total magnification is $\mu_{\text{min}} = 2$ and the maximum total magnification is $\mu_{\text{max}} = \infty$. The magnification bias $\mathcal{B}$ depends on the differential number-flux density relation $\frac{dN_{z_S}(> S_*)}{dS_*}$. The differential number-flux relation needs to be known as a function of the source redshift. At present redshifts of only a few CLASS sources are known. We, therefore, ignore redshift dependence of the differential number-flux density relation. Following Chae (2002), we further ignore the dependence of the differential number-flux density relation on the spectral index of the source.

An important selection criterion for the CLASS statistical sample is that the ratio of the flux densities of the fainter to the brighter images $R_{\text{min}}$ is $\geq 0.1$. Given such an observational limit, the minimum total magnification for double imaging for the adopted model of the lens is (Chae 2002):

$$\mu_{\text{min}} = \frac{1 + R_{\text{min}}}{1 - R_{\text{min}}}. \quad (18)$$

Another selection criterion is that the image components in lens systems must have separations $\geq 0.3$ arcsec. We incorporate this selection criterion by setting the lower limit of $\Delta \theta$ in Eq. (16) as 0.3 arcsec.

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$^2$ The 2dF Galaxy Redshift Survey (2dFGRS): [http://msowww.anu.edu.au/2dFGRS/](http://msowww.anu.edu.au/2dFGRS/)

$^3$ Sloan Digital Sky Survey: [http://www.sdss.org/](http://www.sdss.org/)
4. Testing Cg Scenarios against Observations

The expected number of lensed radio sources is \( N_{\text{lens}} = \sum P'_i \), where \( P'_i \) is the lensing probability of the \( i^{th} \) source and the sum is over the entire adopted sample. The expected number of lensed sources is thus a function of the parameters \( A_s \) and \( \alpha \). We have done a grid search for those combinations \((A_s, \alpha)\) by fixing \( N_{\text{lens}} = 9 \). In Fig. 1 we show the contour for 9 lensed radio sources in the parametric space \( A_s - \alpha \). As can be seen, while the entire range of \( \alpha \) is allowed, the parameter \( A_s \) is tightly restricted to the interval 0.48 \( \leq A_s \leq 0.66 \). This particular range for \( A_s \) is not compatible with the one obtained from a SNe analysis involving 92 events of the Supernova Cosmology Project and High-z supernova Search Team, i.e., \( A_s > 0.69 \) at 95% confidence level (Avelino et al. 2003) and is only marginally compatible with the limits from age + SNe performed by Makler, de Oliveira & Waga (2003). A comparison between the above interval with the one restricted by age estimates of high-z objects shows that the \( N_{\text{lens}} \) test for the CLASS sample is compatible with the existence of the radio galaxy LBDS 53W091 (3.5 Gyr at \( z = 1.55 \)) which implies \( A_s \geq 0.52 \) but that it is not in accordance with the existence of the 4.0-Gyr-old radio galaxy 53W069 (at \( z = 1.43 \)) and the 2.0-Gyr-old quasar APM 08279+5255 (at \( z = 3.91 \)) which requires, respectively, \( A_s \geq 0.72 \) and \( A_s \geq 0.82 \) (Alcaniz, Dev & Jain 2003). The above interval from the \( N_{\text{lens}} \) test is also not in agreement with the tight limits obtained by Silva & Bertolami (2003) from future SNe and lensing data, i.e., \( 0.75 \leq A_s \leq 0.79 \) at 2\( \sigma \) (\( \alpha \leq 0.2 \)).

The likelihood function for lensing can be written as

\[
\mathcal{L} = \prod_{i=1}^{N_l} (1 - P'_i) \prod_{k=1}^{N'_l} P'_{i}(\Delta \theta),
\]

(19)

Here \( N_l \) is the observed number of multiple-imaged lensed radio sources and \( N'_l \) is the number of unlensed sources in the adopted sample. The results of our analysis for the Cg model are displayed in Fig. 2a. The contours correspond to the 68.3% and 95.4% confidence level (cl) in the \((A_s, \alpha)\) plane. Although the entire range of \( \alpha \) is permissible within the 68.3% confidence level, the parameter \( A_s \) is constrained to be \( \leq 0.8 \) at 68.3% cl and \( \leq 0.9 \) at 95.4% cl. For this analysis the best fit model occurs for \( A_s = 0.36 \) and \( \alpha = 0.35 \), which corresponds to an accelerating scenario with a deceleration parameter \( q_o = -0.02 \) and a total expanding age of \( 7.33 h^{-1} \text{Gyr} \). Although not very restrictive, the constraints on the parameter \( A_s \) from the CLASS lensing sample of radio sources are more stringent than those obtained from the optical gravitational lensing surveys for quasars (Dev et al. 2003). However, it still limits the parameter \( \alpha \) very weakly.

4.1. Joint analysis with supernova data

In order to perform a joint analysis with CLASS and SNe data sets, we first follow the conventional magnitude-redshift test (see, for example, Goliath et al. 2001; Dicus & Repko 2003; Padmanabhan & Choudhury 2003) and use the SNe Ia data set that corresponds to the primary fit \( C \) of Perlmutter et al. (1999) together with the highest redshift supernova observed so far, i.e., the 1997ff at \( z = 1.755 \) and effective magnitude \( m^{\text{eff}} = 26.02 \pm 0.34 \) (Benitez et al. 2002) and two newly discovered SNe Ia, namely,
SN 2002dc at \( z = 0.475 \) and \( m_{\text{eff}} = 22.73 \pm 0.23 \) and SN 2002dd at \( z = 0.95 \) and \( m_{\text{eff}} = 24.68 \pm 0.2 \) (Blakeslee et al. 2003). We thus work with a total of 57 supernovae. The apparent magnitude of a given SNe is related to the luminosity distance \( d_L \) by the known relation \( m = M + 5 \log d_L \), where \( d_L = H_0 d_L \) and \( M \) is given by the intercept obtained by fitting the low-redshift data set to \( m(z) = M + 5 \log(cz) \) (Hamuy et al. 1996). The value obtained is \( M = -3.325 \) and confirms the results of Perlmutter et al. (1999). For the sake of completeness, we perform such SNe analysis. The best fit model occurs for \( A_s = 0.52 \) and \( \alpha = -0.2 \) with a minimum value of \( \chi^2_{\text{min}} = 64.31 \) (which corresponds to \( \chi^2 = 1.17 \)). When this magnitude-redshift test is combined with CLASS lensing statistics, tighter constraints on the \( A_s \) parameter can be obtained. As was shown, the index \( \alpha \) is highly insensitive to SNe Ia data (Makler, de Oliveira & Waga 2003). Figure 2b shows the result of this joint analysis. For the combined \( \chi^2 \) analysis we used \( \chi^2_{\text{total}} = \chi^2_{\text{SN}} - 2\ln l \), where \( l = L_{\text{lens}}/L_{\text{max}} \) is the normalized likelihood for lenses. As can be seen, the limits on \( A_s \) are more restrictive now than those imposed by the gravitational lensing statistics of the CLASS sample (Fig. 2a). Within 68.3% cl, the constraints on the parameter \( A_s \) are as follows: 0.39 \( \leq \) \( A_s \) \( \leq \) 0.71 at 68.3% cl and 0.35 \( \leq \) \( A_s \) \( \leq \) 0.74 at 95.4% cl. In particular, the best fit model occurs for \( A_s = 0.58 \) and \( \alpha = 0.5 \), corresponding to a 7.93h\(^{-1}\)-Gyr-old, accelerating Universe with a deceleration parameter \( q_o = -0.33 \).

### 5. conclusion

A considerable amount of observational evidence suggests that the current evolution of our Universe is fully dominated by two dark components, the so-called dark matter and dark energy. The nature of these components, however, is a tantalizing mystery at present, and it is not even known if they constitute two separate substances. In this paper we have investigate some observational predictions of cosmologies driven by an exotic component named the generalized Chaplygin gas. These models constitute an interesting possibility of unification for dark matter/energy, where these two dark components are seen as different manifestations of a single fluid (UDME). We have investigated observational constraints from lensing statistics on spatially flat UDME scenarios. Since gravitational lensing statistics constitutes an independent way of constraining cosmological parameters we have used the most recent lensing data, namely, the Cosmic All Sky Survey (CLASS) sample to obtain the 68.3% and 95.4% confidence intervals on the parameters of the Cg equation of state. Our statistical analysis shows that the best fit scenario for these data occurs at \( A_s = 0.36 \) and \( \alpha = 0.35 \). At 68.3% cl, parameter \( A_s \) is restricted to \( \leq 0.8 \) while the entire range of \( \alpha \) is allowed. By considering the observed number of lensed radio galaxies we tightly constrain \( A_s \) to the interval 0.48 \( \leq \) \( A_s \) \( \leq \) 0.66. From a joint \( \chi^2 \) analysis with SNe Ia data we obtain 0.35 \( \leq \) \( A_s \) \( \leq \) 0.74 at 95.4% cl with the best fit model occurring for \( A_s = 0.58 \) and \( \alpha = 0.5 \), which corresponds to an accelerating scenario with a deceleration parameter \( q_o = -0.33 \) and a total expanding age of 7.93h\(^{-1}\) Gyr. As has been commented earlier, such results are only marginally consistent with those obtained from independent cosmological tests. It means that only with a more general analysis, possibly a joint investigation involving different classes of cosmological data, it will be possible to delimit the \( A_s - \alpha \) plane more precisely, as well as to test more properly the consistency of these scenarios as a viable possibility of unification for the dark matter and dark energy scenarios.

![Confidence regions in the plane \( A_s - \alpha \)](image-url)
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References

Alcaniz, J. S., Jain D. & Dev A. 2003, Phys. Rev. D 67, 043514
Alcaniz J. S. & Lima J. A. S. 2003, astro-ph/0308465
Avelino P. P., Beça L. M. G., de Carvalho J. P. M., Martins C. J. A. P. & Pinto P. 2003, Phys. Rev. D 67, 023511
Beça L. M. G., Avelino P. P., de Carvalho J. P. M. & Martins C. J. A. P. 2003, Phys. Rev. D67, 101301(R)
Bento M. C., Bertolami O & Sen A. A. 2002, Phys. Rev. D 66, 043507
Bento M. C., Bertolami O & Sen A. A. 2003a, astro-ph/0303538
Bento M. C., Bertolami O & Sen A. A. 2003b, gr-qc/0305086
Benitez N., Riess A. G., Nugent P. E., Dickinson M., Chornock R. & Filippenko A. V. 2002, ApJ577, L1
Bilić N., Tupper G. B. & Viollier R. D. 2003, astro-ph/0304314
Blakeslee J. P. et al. 2003, ApJ, 589, 693
Bordemann M. and Hoppe J. 1993, Phys. Lett. B 317, 315 ; Hoppe J. 1993, hep-th/9311059
Browne I. W. A. et al. 2002, MNRAS, in press astro-ph/0211069
Burles S., Nollett K. M. & Turner M. S. 2004, ApJ, 609, 31
Calberg R. G. et al. 1996, Astrophys. J. 462, 32 ; Dekel A., Burstyn D. & White S., In Critical Dialogues in Cosmology, edited by N. Turok (World Scientific, Singapore (1997))
Carturan D. & Finelli F. 2002, astro-ph/0112626
Chae K. -H. et al. 2002, Phys. Rev. Lett., 89, 151301
Chae K. -H. 2002, astro-ph/0211244
Chiba M. & Yoshii Y. 2001, ApJ, 510, 42
Cunha J. V., Lima J. A. S. & Alcaniz J. S. 2003, astro-ph/0306319
de Bernardis P. et al. 2000, Nature, 404, 955; D. N. Spergel et al. 2003, astro-ph/0302099
Dev A., Alcaniz J. S. & Jain D. 2003, Phys. Rev D 67, 023515
Dev A., Jain D. & Mahajan S. 2003, astro-ph/0307441
Faber S. M. & Jackson R. E. 1976, ApJ, 204, 668
Fabris J. C., Gonçalves S. V. B. & de Souza P. E. 2002, astro-ph/0211422
Fabris J. C., Gonçalves S. V. B., de Sá Ribeiro R. 2003, astro-ph/0307028
Avelino P. P., Beça L. M. G., de Carvalho J. P. M., Martins C. J. A. P. 2003, astro-ph/0307427
Folkes S. et al. 1999, MNRAS, 308, 459
Freedman W. L. et al. 2001, ApJ, 553, 47
Fukugita M., Futamase T & Kasai M. 1990, MNRAS, 246, 24
Fukugita M., Futamase T, & Kasai M. 1999, MNRAS, 298, 42
Galli G., Ananian R., Astier P., Goobar A. & Pain R. 2001, A&A., 380, 6; Dicus D. A. & Repko W. W., Phys. Rev. D 67, 083520; Padmanabhan T & Choudhury T. R., MNRAS, 344, 823
Hamyu M., Davidson A., Karasik D. & Lederer Y., Huterer D. and Ma C. -P. 2003, astro-ph/0211244
Kamenshchik A., Moschella U. & Pasquier V. 2001, Phys. Lett. B 511, 265
Kocharov C. S. 1996, ApJ, 466, 638
Matos T. & Urena-Lopez L. A. 2000, Class. Quantum Grav. 17, L75; 2001 Phys. Rev. D 63, 063506; Davidson A., Karasik D. & Lederer Y., 2001, gr-qc/0111107
Makler M., de Oliveira S. Q. & Waga I. 2003, Phys. Lett B 555, 1
Myers S. T. et al. 2002, MNRAS, in press astro-ph/0111073
Multamaki T., Manera M. & Gaztanaga E. 2003, astro-ph/0307533
Bilic N., Lindebaum R. J., Tupper G. B. & Viollier R. D. 2003, astro-ph/0307214
Perlmuter S. et al. 1999, Astrophys. J. 517, 565
Riess A. et al. 1998, AJ 116, 1009
Sahni V., Saini T. D., Starobinsky A. A. & Alam U. 2003, JETP Lett. 77, 201
Sandvik H., Tegmark M., Zaldarriaga M. & Waga I. 2002, astro-ph/0212114
Sarbu N., Rusin D. & Ma C. -P. 2001, ApJ, 561, L147
Schechter P. L. 1976, ApJ, 203, 297
Silva P. T. & Bertolami O. 2003, astro-ph/0303535
Turner E. L., Ostriker J. P. & Gott J. R. 1984, ApJ, 284, 1
Zhu Z. -H. 1998, A&A, 338, 777; 2000a, Mod. Phys. Lett. A15, 1023; 2000b, Int. J. Mod. Phys. D9, 591