ON UNUSUAL INTERACTIONS OF THE $\left(\frac{1}{2}, 0\right) \oplus \left(0, \frac{1}{2}\right)$ PARTICLES

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We continue to study the ‘fermion – 4-vector potential’ interactions in the framework of the McLennan-Case construct which is a reformulation of the Majorana theory of the neutrino. This theory is shown after applying Majorana-like anzatzen to give rise to appearance of unusual terms as $\sigma \cdot [A \times A^*]$, which were recently discussed in non-linear optics.

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As a result of extracting solid data certifying the existence of the mass of neutrino in the LANL experiment [1] the interest in the Majorana-like models has grown considerably. The McLennan-Case reformulation [2] of the Majorana theory [3] got further development in the papers of Ahluwalia [4] and myself [5,6]. In 1996 I received private communications from D. V. Ahluwalia [7] about unusual interactions of neutral particles in his model which is closely related with the Case consideration. Even before I learnt about the possible importance of phase factors of corresponding field functions in defining the structure of the mass term [8]. They gave initial impulse in writing this work. Further investigations from different standpoints [10] (compare also with results of non-linear optics [9]) produced simultaneously with this work were also very incentive in my attempts to solve the problem rigorously.\(^1\)

The main result of the present paper is the theoretical proof of possible physical signifi-

\(^1\)Obviously, the Evans et al. derivation of similar terms [“The Enigmatic Photon. Vol. 3” (Kluwer Academic, Dordrecht, 1996), pp. 9-16, 187-189] has no any sense in the presented form. It should be regarded as completely erroneous until that time when needed clarifications and corrections would be given. But, the Esposito derivation of the term $\sim \sigma \cdot [A \times A^*]$ is correct. I am grateful to him for sending me the alternative proof before the publication.
cance of the term $\mathbf{c} \cdot [\mathbf{A} \mathbf{A}^*]$ in the interaction of $(1/2, 0) \oplus (0, 1/2)$ fermions. In the process of calculations we use the notation and the metric of ref. [2b]. The Dirac equation is written

$$\left(\gamma^\mu \partial_\mu + \kappa\right)\psi = 0,$$

where $g^{\mu \nu} = \text{diag}(-1, 1, 1, 1)$ and $\gamma^\mu$ are the Dirac matrices. Their explicit form can be chosen as follows

$$\gamma^0 = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}, \gamma^i = \begin{pmatrix} 0 & i\sigma^i \\ -i\sigma^i & 0 \end{pmatrix}, \gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (2)$$

The Pauli charge-conjugation $4 \times 4$ matrix is then

$$C = \begin{pmatrix} 0 & \Theta \\ -\Theta & 0 \end{pmatrix}, \text{ where } \Theta = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \quad (3)$$

It has the properties

$$C = C^T, \quad C^* = C^{-1}, \quad \quad (4a)\quad C\gamma^\mu C^{-1} = \gamma^\mu^*, \quad C\gamma^5 C^{-1} = -\gamma^5^*. \quad (4b)$$

As opposed to K. M. Case we introduce the interaction with the 4-vector potential in the beginning and substitute $\partial_\mu \rightarrow \nabla_\mu = \partial_\mu - ieA_\mu$ in the equation (1). For the sake of generality we assume that the 4-vector potential is a complex field what is the extension of this concept comparing with the usual quantum-field consideration. (In the classical (quantum) field theory the 4-vector potential in the coordinate representation is usually considered to be pure real function (functional)). After introducing projections onto subspaces of the chirality quantum number

$$\psi_\pm = \frac{1}{2}(1 \pm \gamma^5)\psi, \quad \gamma^\mu_\pm = \frac{1}{2}(1 \pm \gamma^5)\gamma^\mu \quad (5)$$

we re-write the equation (1) and Eq. (3) of ref. [2b]

$$\left(\gamma^\mu \nabla^*_\mu + \kappa\right)C^{-1}\psi^* = 0,$$

which already describe the interactions of (anti) fermion with the complex 4-vector potential, to the following set
\[ \gamma_+^\mu \nabla_\mu \psi_- + \kappa \psi_+ = 0, \]  
\[ \gamma_-^\mu \nabla_\mu \psi_+ + \kappa \psi_- = 0, \]  
\[ \gamma_+^\mu \nabla_\mu \psi_- C^{-1} + \kappa C^{-1} \psi_+ = 0, \]  
\[ \gamma_-^\mu \nabla_\mu \psi_+ C^{-1} + \kappa C^{-1} \psi_- = 0. \]  

(7a) \( \text{On using the matrices } \eta^\mu = C \gamma_+^\mu \text{ and } \eta^{\mu*} = \gamma_-^\mu C^{-1} \text{ and } \varphi = \psi_+ = \frac{1}{2}(1 + \gamma^5) \psi, \ \chi = C^{-1} \psi_- \text{ we obtain} \)

\[ \eta^{\mu*} \nabla_\mu \chi^* + \kappa \varphi = 0, \]  
\[ \eta^\mu \nabla_\mu \varphi + \kappa \chi^* = 0, \]  
\[ \eta^{\mu*} \nabla_\mu^* \varphi^* + \kappa \chi = 0, \]  
\[ \eta^\mu \nabla_\mu^* \chi + \kappa \varphi^* = 0. \]  

(8a) \( \text{in the sub-space of the positive chirality quantum number. And with the matrices } \zeta^\mu = \gamma_+ C^{-1}, \zeta^{\mu*} = \gamma_-^\mu \text{ and the notation } \eta = \psi_-, \xi = C^{-1} \psi_+ \text{ we obtain the set} \)

\[ \zeta^{\mu*} \nabla_\mu \eta + \kappa \xi^* = 0, \]  
\[ \zeta^\mu \nabla_\mu \xi^* + \kappa \eta = 0, \]  
\[ \zeta^{\mu*} \nabla_\mu^* \xi + \kappa \eta^* = 0, \]  
\[ \zeta^\mu \nabla_\mu^* \eta^* + \kappa \xi = 0. \]  

(9a) \( \text{for the negative chirality quantum number. One can use four equations of these sets to describe the physical system. If now apply the Majorana condition given by Case } \psi_- = C^{-1} \psi_+ \text{ (see Eq. (8) in [2b]) one can arrive at } \chi = \varphi \text{ and} \)

\[ \nabla_\mu^* \varphi \equiv \nabla_\mu \varphi, \text{ hence, } A_\mu = -A_\mu^* \]  

as a consequence of the compatibility condition of the set of equations (8a-8d). The 4-potential becomes to be pure imaginary. This model seems to be perfectly possible after redefining the phase factor between positive- and negative- energy solutions in the field operator of the 4-vector potential.
Furthermore, it is difficult to extract the new physical content from the modification of the Majorana anzatz such that \( \psi_\pm = e^{i\alpha(x)}C^{-1}\psi^*_{\pm} \) and, therefore, \( \chi = e^{-i\alpha(x)}\varphi \). We come to

\[
\eta^\mu \nabla_\mu \varphi + \kappa e^{i\alpha(x)}\varphi^* = 0, \tag{11a}
\]

\[
\eta^\mu \nabla_\mu \varphi^* + \kappa e^{-i\alpha(x)}\varphi = 0, \tag{11b}
\]

and

\[
\partial_\mu \alpha = e(A_\mu + A^*_\mu), \tag{11c}
\]

thus recovering (with \( \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu] \))

\[
\left[ \nabla^\mu \nabla_\mu - i\sigma^{\mu\nu} \nabla_\mu \nabla_\nu - \kappa^2 \right] \varphi = 0, \tag{12}
\]

and its complex conjugate.

From the above consideration it seems that we failed to derive the needed term. But, we wish to insist on the general case. In order to proceed let us observe that in the set (8a-8d) and (9a-9d) the second and the third equations of each set are complex conjugates each other; the first equation and the fourth equation are also complex conjugates each other. If we do not want to introduce such strong restrictions on the 4-vector potential as above it is logical to introduce different Majorana-like anzatzen for these subsets. This is perfectly possible after one reminds that the subspaces of different \( CP \) quantum number are independent ones for certain states. On this basis, firstly, we re-write the Dirac equation and its charge conjugate to another set (\( \varphi \) and \( \chi \) are two-component spinors):

\[
\eta^\mu \nabla_\mu \varphi + \kappa C\varphi = 0, \tag{13a}
\]

\[
\eta^\mu \nabla^*_{\mu} \varphi^* + \kappa C^{-1}\varphi^* = 0, \tag{13b}
\]

\[
\eta^\mu \nabla^*_{\mu} \chi + \kappa C\chi = 0, \tag{13c}
\]

\[
\eta^\mu \nabla_\mu \chi^* + \kappa C^{-1}\chi^* = 0. \tag{13d}
\]

Next, we set up the following anzatz

\[
C^{-1}\psi^*_\pm = \mp P\psi^*_\pm,
\]
where $P$ is the space inversion operator. Finally, marking the resulting subsets of equation by some discrete quantum number (we denote them as “$s$” and “$a$”) one obtains

\begin{align}
\eta^\mu \nabla^*_\mu \chi_s + \kappa \chi_s^* &= 0, \\
\eta^* \nabla_\mu \chi_s + \kappa \chi_s &= 0.
\end{align}

and

\begin{align}
\eta^\mu \nabla_\mu \varphi_a - \kappa \varphi_a^* &= 0, \\
\eta^* \nabla^*_\mu \varphi_a^* - \kappa \varphi_a &= 0.
\end{align}

As a result we obtain second-order equations for $\chi_s$ and $\varphi_a$:

\begin{align}
\left[ \nabla^\mu \nabla^*_\mu - i\sigma^{\mu\nu} \nabla_\mu \nabla^*_\nu - \kappa^2 \right] \chi_s(x^\mu) &= 0, \\
\left[ \nabla^*_\mu \nabla_\mu - i\sigma^{\mu\nu} \nabla^*_\mu \nabla_\nu - \kappa^2 \right] \varphi_a(x^\mu) &= 0
\end{align}

and their complex conjugates.

One can proceed further with transformations of these equations to the accustomed forms. This is only algebraic exercises. One can see the existence of “new terms” in the equations: we proved that some physical states of the spin-1/2 fermion have interactions of the form

\[ i\epsilon^{ijk} \sigma^k \nabla_i \nabla_j^* \rightarrow +i\epsilon^2 \sigma \cdot [A \times A^*], \]

for “$s$” states, and with the inverse sign, for the “$a$” states. (The unit system $c = \hbar = 1$ is used.)

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\footnote{One could also obtain similar subsets of equations after the application of the modified Majorana \textit{anzatz}}

\[ \psi_- = \varphi_{s,A} C^{-1} \psi_+^*. \]

Here, $\varphi_{s,A} = \pm 1$; the upper sign being used for the first subset, Eqs. (8b,8c) and the down sign being used for the second subset, Eqs. (9a,9d). This is possible because the sub-spaces of different chirality quantum numbers can also be considered as the independent subspaces and we can choose any two equations from the both subsets. But, this explanation can be still considered as obscure by someone due to the discussion in the first part of the Letter. In my opinion, the proper consideration of the theory of 4-vector potential is necessary to clarify this point.
At last we note that the $(1/2, 0) \oplus (0, 1/2)$ field operator is naturally decomposed into the parts $\Psi = \psi_s + \psi_A$

$$\psi_s(x^\mu) = \int \frac{d^3p}{(2\pi)^3 2E_p} \left\{ [u_i(p^\mu)c_i(p^\mu) + u_i(p^\mu)d_i(p^\mu)] e^{-i\phi} + 
+ \left[ Cu_i^{\dagger}(p^\mu)c_i^\dagger(p^\mu) + Cu_i^{\dagger}(p^\mu)d_i^\dagger(p^\mu) \right] e^{+i\phi} \right\}, \quad (19)$$

$$\psi_A(x^\mu) = \int \frac{d^3p}{(2\pi)^3 2E_p} \left\{ [u_i(p^\mu)d_i(p^\mu) + u_i(p^\mu)c_i(p^\mu)] e^{-i\phi} - 
- \left[ Cu_i^{\dagger}(p^\mu)d_i^\dagger(p^\mu) + Cu_i^{\dagger}(p^\mu)c_i^\dagger(p^\mu) \right] e^{+i\phi} \right\}, \quad (20)$$

where $\phi = (Et - \mathbf{p} \cdot \mathbf{x})/\hbar$. As easily demonstrated both parts satisfy (separately each other) the Dirac equation. Certain relations between creation/annihilation operators are assumed. They are dictated by the modified Majorana-like anzatzen.

Finally, I would like to present references to some works which, in my opinion, would be relevant to further discussions of the questions put forth here. Several works already revealed importance of the term $\sigma \cdot [\mathbf{A} \times \mathbf{A}^\ast]$ in the non-linear optics. Other works are [11], where the concept of two coordinate-space Dirac equations have been re-discovered independently (cf. [6b,d,e] and [12,13]); and ref. [14], where the matters of interface between gravity and quantum mechanics have been firstly discussed rigorously.

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