Analysis of Differential Calculus in Economics

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Abstract. The differential is one of the mathematical material in calculus which is loaded with counts. Differential counts can be applied in economics for profit optimization. This study aims to analyze differential calculus in economics. This research is a descriptive qualitative study. This research analyzes the profit optimization in the entrepreneurial world with the second differential formula in calculus. Data analysis is to describe the results of the analysis of the second differential formula with economics in optimizing profits. Calculation of calculus uses the second differential of the mathematical model with the provision that the second differential result is negative which is smaller zero. The results of the analysis show that the second differential calculus smaller than zero, so it can be concluded that to obtain profit optimization in the economy it can be applied using the second differential calculus.

1. Introduction
The differential is one of the calculus material in mathematics that studies a derivative function. In determining the derivative of a function, the function must be differentiable. The derivative process is called differentiation so that the derivative is closely related to the differential. In the basic theorem of calculus states that differentiation is a reversal of integration. Differentials in calculus describe changes in a function with a small change in the function variable. In addition, differential calculus discusses solving problems related to a change that is the change in the form of independent variables and dependent variables. These variables are formed in a mathematical model. Mathematical models related to differential can be applied in various fields, for example in the fields of physics, biology, economics. Modern economics leads to the emergence of fundamental concepts and methods in economic theory, which allows the use of differential and integral calculus to describe economic phenomena, effects, and processes [1–3]. Calculus is becoming more accessible in the business because it uses calculus as a tool, a constructive calculus course in meeting economic needs [4,5]. Differential calculus in economics is used in simplifying economic problems including optimizing profits, balancing supply and demand prices. Problems in the economy can be simplified by limiting the number of variables by assuming that other variables remain constant. The results of previous studies [6] states that the buyer's economic order quantity and the vendor's optimal number of deliveries are derived by setting the first derivatives to zero and solving the simultaneous equations. In addition, the results of previous studies [7] states that such derivatives also are the key to the formulation of subproblems determining the response of a problem's solution when the data values on which the problem depends are perturbed. [8] states that the calculus theory and many economic models are dynamic models to relate those two subjects to motivate the use of the calculus of variations and optimal control problems on time scales in the study of economic models.
Regarding the descriptions that have been described, it is very interesting to discuss research related to calculus derivatives in economics and objectives of the research was to analyze derivative calculus in economics.

1.1. Calculus
Calculus material, for the concept and differential definition of calculus includes the understanding of functions, limits and derivatives [9], namely:

1.1.1. Function.
The concept of function is one of the most important concepts in mathematics, especially calculus. A function $f$ is a correspondence rule that connects each $x$ object in a set, called the origin (domain), with a single value $f(x)$ from a second set. The set of values obtained is thus called the function result range.

1.1.2. The limit.
The concept of limits can be understood through the question, what happens to the function $f(x)$ when $x$ approaches a constant $c$.

Intuitive definition of limits:
To state that $\lim_\limits{x \to c} f(x) = L$, means that when $x$ is close but differs from $c$ then $f(x)$ is close to $L$.

1.1.3. Derivative Function.
Derivatives are neutral words in mathematical terms. Derivatives as keywords in calculus and additional words to functions and limits.

Definition: The derivative function $f$ is another function $f'(read "f accent")$ whose value at any number $c$ is:

$$f'(c) = \lim_\limits{h \to 0} \frac{f(c + h) - f(c)}{h} \text{ or } f'(x) = \lim_\limits{h \to 0} \frac{f(x + h) - f(x)}{h}$$

(provided that this derivative exists and not $\infty atau - \infty$)

If this derivative does exist, it is said that $f$ is differentiated in $c$. The search for derivatives is called differentiation and the part associated with derivatives is called the differentiation calculus. The differentiation operation takes a function $f$ and produces a new function that is $f'$ (first derivative). If $f'$ is differentiated, it still produces other functions that are stated with $f''$ (second derivative). If $f''$ is differentiated again, it will still produce other functions called $f'''$ (third derivative), then the fourth derivative is declared $f''''$, the fifth derivative is expressed $f'''''$ and so on [10,11].

1.2. Derivative calculus
Calculus material, for the concept and differential definition of second derivative calculus relating to economics, namely:

1.2.1. Maximum and Minimum.
Definition:
Suppose that a function $f(x)$ is given with the area of origin $S$ and contains the point $c$, it is said that: $f(c)$ is the maximum value of $f$ on $S$ if $f(c) \geq f(x)$ for all $x$ in $S$, $f(c)$ is the minimum value $f$ on $S$ if $f(c) \leq f(x)$ for all $x$ in $S$. $f(c)$ is the extreme value of $f$ on $S$ if it is a maximum or minimum value and the function we want to maximize or minimize is an objective function

Theorem $A$ (the maximum-minimum theorem of existence): if $f$ is continuous at a closed interval $[a, b]$ then $f$ has a maximum value and a minimum value. Note the key words in theorem $A$: $f$ is required to be continuous and the set $S$ is required to be a closed interval.
Theorem B (critical point theorem): if \( f \) is defined in the interval \( I \) containing point \( c \). If \( f'(c) \) is an extreme value, \( c \) must be a critical point. In other words, \( c \) is one of: endpoint of \( I \), stationary point of \( f \) and that is the point where \( f''(c) = 0 \) or singular point of \( f \) and that is the point where \( f'(c) \) does not exist.

1.2.2. Monotonic and concave.

Theorem A (monotonic theorem): suppose that there are continuous \( f \) intervals of \( I \) and are defined at each inner point of \( I \) then: if \( f'(x) > 0 \) for all \( x \) in \( I \), then \( f \) rises at \( I \) and \( f'(x) < 0 \) for all points in \( I \) then \( f \) falls at \( I \).

Theorem B (concave theorem): suppose \( f \) is defined twice at the open interval \( I \) then: if \( f''(x) > 0 \) for all \( x \) in \( I \) then \( f \) concaves upward on \( I \) and if \( f''(x) < 0 \) for all \( x \) in \( I \) then \( f \) concaves downward on \( I \).

1.2.3. Maximum and Minimum with the First Derivative Test and the Second Derivative Test.

Theorem A (first derivative test): if \( f \) be continuous at an open interval \( (a, b) \) containing a critical point \( c \): if \( f'(x) > 0 \) for all \( x \) in \( (a, c) \) and \( f'(x) < 0 \) for all \( x \) in \( (c, b) \) then \( f(c) \) is the local maximum value of \( f \), if \( f'(x) < 0 \) for all \( x \) in \( (a, c) \) and \( f'(x) > 0 \) for all \( x \) in \( (c, b) \) then \( f(c) \) is the local minimum value \( f \), if \( f'(x) \) is marked the same on all parties of \( c \), then \( f(c) \) is not an extreme local value of \( f \).

Theorem B (second derivative test): suppose \( f' \) and \( f'' \) exist at each interval point \( (a, b) \) containing \( c \), and for example \( f'(c) = 0 \): if \( f''(c) < 0 \), then \( f(c) \) is the local maximum value of \( f \) and if \( f''(c) > 0 \), then \( f(c) \) is the local minimum value of \( f \).

1.3. Differential calculus in economics

In the economic field, the calculation uses the second derivative of calculus to obtain profit optimization. There are three functions in the economy, namely: the cost function of each production (\( C \)), the amount of production function (\( Q \)), the revenue function (\( R \)) and the profit function (\( \pi \)). If all product units are sold, the relationship of the functions is: \( \pi = R - C \). If \( \pi'' < 0 \) then in the economy the profit optimization is obtained. The profit function \( \pi \) is analyzed using the second derivative of \( \pi \) so that in the economy the profit optimization is obtained: if \( \pi'' < 0 \), then \( \pi \) is maximum, profit optimization is obtained, if \( \pi'' > 0 \), then \( \pi \) is minimum, profit (loss) minimization is obtained and if \( \pi'' = 0 \) then \( \pi \) is a turning point.

The result of the calculation on the second differential calculus was negative, which was smaller than zero \( \pi'' < 0 \), so in the economy the optimization of profits were obtained [12].

2. Method

This research was a qualitative research. The research was done by analyzing differential calculus in economics on food entrepreneurship. Differential calculus in economics is related to differential calculations applied in profit optimization. Differential calculus in economics is through analyzing the differential calculus formula with its application in obtaining the results of calculations on the second differential calculus is negative ie smaller than zero \( \pi'' < 0 \) by determining the cost function of each production (\( C \)), the demand function of each production (\( P \)), the number of production functions (\( Q \)), the revenue function (\( R \)) and the profit function (\( \pi \)) [13].

3. Result and Discussion

The results of research on food entrepreneurship obtained data on the cost function of each production (\( C \)), the demand function of each production (\( P \)), the number of production functions (\( Q \)), the revenue function (\( R \)) and the profit function (\( \pi \)) obtained the following data.

Fixed costs in production amounted to \( Rp \) 33,250,000 with details:
Monthly salary for employees = Rp 15,750,000, equipment maintenance = Rp 1,500,000, tax = Rp 13,000,000, promotional advertising = Rp 500,000, electricity, and water and telephone = Rp 2,500,000

Expenses in production with details:
Fried chicken = Rp 10,500,000, grilled chicken = Rp 13,250,000, crispy chicken = Rp 12,500,000, roasted chicken = Rp 12,750,000, spicy chicken = Rp 11,000,000, chicken roll = Rp 12,250,000, chicken steak = Rp 11,750,000

Production expenses in 2017 and 2018 on the Surabaya Indonesia crispy chicken entrepreneur with details in table 1:

| Table 1. Production Results |
|--------------------------------|
| Description | Production results in 2017 | Production results in 2018 |
|---------------|-----------------------------|-----------------------------|
| Volume | Unit price (Rp) | Volume | Unit price (Rp) |
| Fried chicken | 6700 | 9,500 | 5400 | 10,500 |
| Grilled chicken | 6730 | 11,000 | 4730 | 12,500 |
| Crispy chicken | 6500 | 9,500 | 4900 | 10,500 |
| Roasted chicken | 5250 | 10,500 | 3250 | 12,000 |
| Spicy chicken | 4300 | 8,500 | 3000 | 9,500 |
| Chicken roll | 5600 | 9,000 | 4300 | 10,200 |
| Chicken steak | 4550 | 10,500 | 3300 | 11,500 |

The average cost of each production variable can be obtained from the description of the table above, namely: fried chicken = Rp 1,944, grilled chicken = Rp 2,801, crispy chicken = Rp 2,551, roasted chicken = Rp 3,923, spicy chicken = Rp 3,666, chicken roll = Rp 2,849, chicken steak = Rp 3,561

Profit optimization analysis using differential calculus that is looking for the cost function of each production variable can be obtained from the description of the table above, namely: fried chicken = Rp 1,944, grilled chicken = Rp 2,801, crispy chicken = Rp 2,551, roasted chicken = Rp 3,923, spicy chicken = Rp 3,666, chicken roll = Rp 2,849, chicken steak = Rp 3,561

Production results in 2017 and 2018

| Table 2. Cost Functions, Demand Functions and Revenue Functions |
|---------------------------------------------------------------|
| Description | Cost function (C) | Demand function (P) | Revenue Function (R) |
|---------------|-------------------|-------------------|---------------------|
| Fried chicken | 33,2500,00 + 1,944Q | -0.77Q + 14,654 | -0.77Q^2 + 14,654Q |
| Grilled chicken | 33,2500,00 + 2,801Q | -0.75Q + 24,688 | -0.75Q^2 + 24,688Q |
| Crispy chicken | 33,2500,00 + 2,551Q | -0.625Q + 13,563 | -0.625Q^2 + 13,563Q |
| Roasted chicken | 33,2500,00 + 3,923Q | -0.75Q + 14,438 | -0.75Q^2 + 14,438Q |
| Spicy chicken | 33,2500,00 + 3,667Q | -0.77Q + 11,808 | -0.77Q^2 + 11,808Q |
| Chicken roll | 33,2500,00 + 2,849Q | -0.92Q + 14,169 | -0.92Q^2 + 14,169Q |
| Chicken steak | 33,500,00 + 3,561Q | -0.83Q + 14,250 | -0.83Q^2 + 14,250Q |

The description in table 2, the form of economic equations in the cost function (C), the demand function (P) and the revenue function (R) is obtained from the fixed costs of production, expenditure costs, production volume and unit price of the variable production. In the cost function, demand function, revenue function and profit function, we can get the function of production amount (Q) and profit function (π) in each production variable. The results in table 3:

| Table 3. Function of Total Production and Profit Function |
|--------------------------------------------------------|
| Description | Profit Function (π) | Profit Optimization (π’’) |
|---------------|-------------------|---------------------------|
| Fried chicken | π = -0.77Q^2 + 12,710Q - 33,2500,00 | π’’ = -1.54Q + 12,710 |
| Chicken roll | π = -0.75Q^2 + 14,438Q - 33,2500,00 | π’’ = -1.69Q + 14,438 |
| Chicken steak | π = -0.77Q^2 + 11,808Q - 33,2500,00 | π’’ = -1.55Q + 11,808 |

The results in table 3 show that the average cost of each production variable can be obtained from the description of the table above, namely: fried chicken = Rp 1,944, grilled chicken = Rp 2,801, crispy chicken = Rp 2,551, roasted chicken = Rp 3,923, spicy chicken = Rp 3,666, chicken roll = Rp 2,849, chicken steak = Rp 3,561

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In table 3, the results are fried chicken with \( \pi'' = -1.54 \), grilled chicken \( \pi'' = -1.5 \), crispy chicken \( \pi'' = -1.25 \), roasted chicken \( \pi'' = -1.5 \), spicy chicken \( \pi'' = -1.54 \), chicken roll \( \pi'' = -1.84 \), and chicken steak \( \pi'' = -1.66 \). This can be said to be a second differential calculus with \( \pi'' < 0 \). The total production function \( (Q) \) shows the optimization of production in each production variable and to obtain profit optimization by using the second differential calculus in \((\pi)\) and \((\pi'' < 0)\). This is in accordance with \([13]\) the opinion which states that when the optimization problem is represented by its objective function, the first and second-order conditions for optimality can be expressed in terms of the first and second epi-derivatives of that function. The statement is related to the opinion of \([7]\) who states that optimal production lot size are by using the differential calculus on the production inventory cost function with the need to prove optimality first.

Differential calculus in economics to obtain profit optimization in determining the cost function of each production \((C)\), the demand function of each production \((P)\), the total of production functions \((Q)\), the revenue function \((R)\) and the profit function \((\pi)\) with completion stages, namely: a) to determine the demand function, the equation of the line is used; b) the line equation in the economic equation with \(Q_1\) is the initial demand function, \(Q_2\) is the final demand function, \(P_1\) is the initial demand function, \(P_2\) is the initial demand function so that the equation is obtained: \(\frac{P-P_1}{P_2-P_1} = \frac{Q-Q_1}{Q_2-Q_1}; c)\) the revenue function with \(R = P \cdot Q\) and the profit function with \(\pi = R - C\). This is in accordance with \([14]\) the opinion which states the quantity of economic production using differential calculus and solving economic equations in two simultaneous equations (derived from assigning two first partial derivatives to zero) with the need to prove the optimality of conditions with second-order derivatives. In addition, \([5]\) the basic equation of equilibrium theory is supply equals demand. Since we are in a situation of several markets, there are several variables in this equation. \([15]\) Financial Derivatives remain the only ones that can attract people outside the mathematics and physics community because they explain how and why practical financial problems in economics.

### 4. Conclusion

Research conclusions from the results exposure and discussion in economics regarding optimal profit that the second calculus differential calculation can be used in economics to get profit optimization in entrepreneurship. The optimization of production in each production variable and to obtain profit optimization in economics using the second differential calculus in \((\pi)\) and \(\pi'' < 0\). Optimize the advantages of using differential calculus by making economic equations in mathematical models, solving mathematical models and distinguishing first and then differentiating the second with negative second differential results in production variables.
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