Hamiltonian Reduction of General Relativity and Conformal Unified Theory

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Abstract

We discuss the application of the method of the gaugeless Hamiltonian reduction to general relativity. This method is based on explicit resolving the global part of the energy constraint and on identification of one of the metric components with the evolution parameter of the equivalent unconstrained (reduced) system.

The Hamiltonian reduction reveals a possibility to unify General Relativity and Standard Model of strong and electro-weak interactions with the modulus of the Higgs field identified with the product of the determinant of 3D metric and the Planck constant.

We give the geometrical foundation of the scalar field, derive and discuss experimental consequences of this unified model: the cosmic Higgs vacuum, the Hoyle-Narlikar cosmology, a σ-model version of Standard Model without Higgs particle excitations and inflation.

1. Introduction

An identification of physical quantities in General Relativity (GR) is a long-time problem which stimulated Dirac to formulate, for this aim, the general Hamiltonian theory for constrained systems [1] developed later by many authors (see e.g. monographs [2, 3, 4]).

Main difficulties of this identification are the mixing of the parameters of general coordinate transformations with the true dynamic variables and the problem of gauge ambiguities. In the last years there appeared some new ideas to overcome these difficulties.

The first idea is the idea of the gaugeless Hamiltonian reduction. It belongs to Shanmugadhasan [5] (see also [6, 7]) who paid attention to the fact that, in the theory of differential equations with partial integrals of the type of the first class constraints, Levi-Civita [8] managed to remove ”gauge ambiguities” by explicit resolving ”constraints” without any additional second class constraints of the type of gauge fixing in the Dirac terminology [1]. Levi-Civita used canonical transformations to convert the first class constraints into new superfluous momenta [9]. In this case, the Hamiltonian of the system on constraints automatically does not depend on the corresponding superfluous variables and gauge fixing is not needed.

The second idea is the idea of classification of times of the Hamiltonian reduction [11]. It follows from the application of the Levi-Civita prescription to the system with invariance under the...
time reparametrization transformations \[12, 13, 14, 15\]. The explicit resolution of the energy-type constraints shows that one of the former (superfluous) variables of a constrained (extended) system leaves the extended phase space to become the evolution parameter of the reduced system. This is just the variable with negative contributions to the energy-type constraints. In addition, every action of relativistic theory has to be supplemented by a geometrical convention which relates a measurable invariant interval with parameters and variables of the extended system. One of the variables plays the role of the Lagrange multiplier in the Dirac general Hamiltonian description. Thus, we face with three "times" of the Hamiltonian reduction of the constrained extended system. The first is the coordinate time in the initial extended action. The second is the evolution parameter of the reduced system which is one of former variables of the extended system. The third time is a product of the first coordinate time and the Lagrange factor (or the lapse function in the Dirac-ADM foliation of the metric in GR). In Special Relativity (SR) the third time it is the proper time of an observer. The gaugeless Hamiltonian reduction represents explicit resolving of the energy-like constraint. As a result, the extended system is reduced to the subsystem where the role of evolution parameter is played by the superfluous variable. In addition, we have "proper time dynamics" described by two equations of ES for the superfluous variable and its momentum. In SR, the proper time dynamics is nothing but the relationship between the proper time of an observer and the proper time of a particle (i.e. the Lorentz transformation). In cosmological models, the proper time dynamics is just the Friedmann-Hubble law of evolution of the Universe. Here the role of superfluous variable with a negative contribution to the energy constraint is played by the cosmic scale factor. The Hubble law is a consequence of the dependence of the proper time of a comoving observer on the evolution parameter, the cosmic scale factor.

In this classification of times, the main problem of the Hamiltonian reduction in GR is to pick out the global variable which can play the role of the evolution parameter of the corresponding reduced system. The third idea is the idea of identification of this evolution parameter with the global component of the determinant of the 3D metric of the Dirac-ADM foliation of the space time \[14, 15\]. We can extract this parameter by solving the global part of the energy constraint where the role of the superfluous momentum is played by the second fundamental form. Note that the idea to consider the trace of the second fundamental form as the time-like variable was discussed in \[16, 17\].

To represent 3D space metric determinant as an independent variable of the extended system, one should use the so-called conformal invariant Lichnerowicz variables \[18\] that are suited for studying the problem of initial data in GR \[16\]. The Hamiltonian reduction in terms of Lichnerowicz variables reveals a possibility to unify General Relativity and the Standard Model with the modulus of Higgs scalar field identified with the product of the determinant of 3D metric and the Planck constant. The obtained unified theory is described by the Lagrangian without any dimensional parameter and it is conformally invariant. In this Conformal Unified Theory (CUT) \[19, 14, 15\], the observer can measure only conformally invariant observables including the corresponding interval \[17\] \[14\]. Although gravitational parts of GR and CUT are mathematically equivalent, they differ by the chosen physical convention about measurable quantities. The geometrical foundation of this theory can be obtained in the scalar version of the Weyl geometry of similarity \[21\] where a scalar field is the measure of change of the length of a vector in its parallel transfer (like, in the Riemann geometry, the metric is the measure of change of the direction of a vector).

The present paper is devoted to the discussion of the gaugeless Hamiltonian reduction in General Relativity and to the derivation of physical consequences of Conformal Unified Theory. The content of the paper is the following: In Section 2, we recall the method of the Hamiltonian reduction using as examples classical mechanics, special relativity, and cosmological models with invariance under the time reparametrizations. We give the classification of times of the Hamiltonian reduction. Then, in Section 3, we discuss the extraction of the global variable from the metric in GR. Section 4 is devoted
2. Hamiltonian reduction

2.1. Classical mechanics

The main problem of General Relativity theory is a complete separation of true physical variables from parameters of general coordinate transformations.

The simplest example of a general coordinate transformation is reparametrization of time. To formulate the problem, we will first consider the version of classical mechanics with time reparametrization invariance.

To get this example, we start with a generic classical system given by the Hamilton action:

$$W^{RS}[p_i, q_i|q_0] = \int_{q_0}^{q_0(2)} dq_0 \left[ \sum_i p_i \frac{dq_i}{dq_0} - H^{RS}(p_i, q_i) \right].$$  \hfill (1)

The superscript "RS" means a "reduced system"; it was introduced in order to distinguish it from the "extended system" (ES) defined below. Here we have time $q_0$ which is the evolution parameter. The system is invariant with respect to the displacement in time $q_0 \to q_0 + \delta$ (as $H^{RS}$ is $q_0$ - independent) but it is not invariant with respect to the general time reparametrization $q_0 \to q'_0(q_0)$.

An extended reparametrization - invariant system can be constructed if we introduce a "superfluous" pair of canonical variables $(p_0, q_0)$ and the Lagrange factor $N$:

$$W^{ES}[p_i, q_i; p_0, q_0|t, N] = \int_{t_1}^{t_2} dt \left[ \sum_i p_i \dot{q}_i - p_0 \dot{q}_0 - NH^{ES} \right]$$  \hfill (2)

where

$$H^{ES}(q_0, p_0, q_i, p_i) = [-p_0 + H^{RS}(p_i, q_i)].$$

is the extended Hamiltonian.

The coordinate time $t$ has been introduced. The extended system is invariant with respect to its general reparametrization $t \to t' = t'(t)$.

The Hamiltonian reduction means explicit solution of the equations for the "superfluous" variable:

$$\frac{\delta W^{ES}}{\delta N} = 0 \Rightarrow -p_0 + H^{RS}(p_i, q_i) = 0,$$

$$\frac{\delta W^{ES}}{\delta q_0} = 0 \Rightarrow \frac{dp_0}{dt} = 0,$$

$$\frac{\delta W^{ES}}{\delta p_0} = 0 \Rightarrow dq_0 = Ndt.$$  \hfill (3)

The first of them is a constraint, the second is a conservation law. If we substitute the solution of this equation into the extended action (2), we get the conventional action for classical mechanics (1) with the parameter of evolution as a former variable of ES.

The third equation for the superfluous momentum $p_0$ (3) is the relation between the evolution parameter $q_0$ and a combination of the Lagrange factor $N$ and the coordinate time $t$. If we introduce the notion of proper time $T$
\[ dT \equiv N dt, \]

then equation (3) converts into the *proper time dynamics* (PTD)

\[ dT = dq_0. \]

The proper time dynamics relation (4) is very simple in the present case of a classical - mechanics system. It is not the case, in general, as we will see below in SR and cosmology. There, instead of "coincidence", we get the Lorentz transformation for SR and the Hubble law for cosmological models. Our aim is General Relativity.

The equation for the "superfluous" canonical momentum establishes the relation between the evolution parameter \( q_0 \) of RS and invariant time \( T \) constructed with the use of the Lagrange factor.

Thus we have three times of the Hamiltonian reduction:

i) the coordinate time \( t \),

and the two times that are parametrized by the former one and are reparametrization - invariant:

ii) the evolution parameter \( q_0 \) of RS as the former variable of ES and

iii) the proper time \( T \) defined by (4).

In the general case, in the process of Hamiltonian reduction, any extended system

\[
W^{ES}[p_i, q_i; p_0, q_0 | t, N] = \int_{t_1}^{t_2} dt \left( -p_0 \dot{q}_0 + \sum_i p_i \dot{q}_i - N H^{ES}(q_0, p_0, q_i, p_i) \right)
\]

is split into two parts. The first part is a set of reduced subsystems

\[
W^{RS}_{(1, 2, \ldots, \ldots)}[p_i, q_i | q_0] = \int \frac{dq_0}{q_0} \left[ \sum_i p_i \frac{dq_i}{dq_0} - H^{RS}_{(1, 2, \ldots, \ldots)} \right]
\]

corresponding to the set of different solutions of the energy constraint

\[ H^{ES} = 0 \Rightarrow P_{0(1, 2, \ldots, \ldots)} = H^{RS}_{(1, 2, \ldots, \ldots)}. \]

The second part is given by the "proper time dynamics" determined by the equation for superfluous momentum

\[
\frac{\delta W^{ES}}{\delta p_0} = 0 \Rightarrow \frac{dq_0}{dT} = -\frac{\partial H^{ES}}{\partial p_0} \quad \text{def} \quad \sqrt{\rho(q_0)} \quad \Rightarrow \quad T(q_0) = \int_0^{q_0} \frac{dq_0}{\sqrt{\rho(q_0)}}.
\]

This is the evolution of the proper time [given by the relation (4) which is the chosen convention] with respect to the evolution parameter of the reduced system.

There are two facts of the considered gaugeless Hamiltonian reduction worth to emphasize:

I. The evolution parameter \( q_0 \) of RS is one of the initial variables of ES.

II. Variation principle is added by the convention about measurable time.

As will be shown later, we can change the action independently keeping the convention unchanged or we can change the convention keeping the action fixed.
2.2. Proper time dynamics in special relativity

Here we will present two examples of the proper time dynamics. The first of them is given by SR. The Hamiltonian action reads

\[
W_{ES} = \int_{t_1}^{t_2} dt \left( -p_0 \dot{q}_0 + \sum_i p_i \dot{q}_i - \frac{N}{2m} \left[ -p_0^2 + H_c \right] \right) \tag{5}
\]

and the proper time of an observer is given by

\[
dt = N dt
\]

according to our convention (4) and to the ordinary description.

Variation of (5) with respect to \( p_0 \) leads to the proper time dynamics:

\[
\frac{\delta W_{ES}}{\delta p_0} = -dq_0 dt + N \frac{p_0}{m} = 0, \Rightarrow T(q_0)_\pm = \pm \int_0^{q_0} dq_0 \sqrt{\frac{m^2}{p_f^2 + m^2}} = \pm q_0 \sqrt{1 - v^2}. \tag{6}
\]

If we recall the relation between \( p \) and \( v \) in SR, we get the relation between the proper time and the evolution parameter. This relation represents the Lorentz transformation of the proper time of a particle into the proper time of an observer.

2.3. Proper time dynamics in cosmology

The time reparametrization - invariant cosmological models follow from the Einstein-Hilbert action for the FRW metric with a constant 3-D curvature

\[
W = \int d^4 x \sqrt{-g} \left[ -\frac{\mu^2}{6} R(g) + L_{\text{mat}} \right]; \quad (3) R(\gamma^c) = \frac{6k}{a_0^2}, \quad \mu = M_{\text{Pl}} \sqrt{3/8\pi}
\]

Here, the role of evolution parameter is played by the cosmic scale factor \( a_0 \):

\[
ds^2 = a_0^2(t) [N^2 dt^2 - \gamma_{ij} dx^i dx^j], \quad ds|_{dx=0} = a_0 dT_c = dT_F. \tag{6}
\]

where \( T_F \) is the FRW time and \( T_c \) is the conformal proper time introduced according to our convention.

With the above notation the action of ES is given by

\[
W_{ES} = \int_{t_1}^{t_2} dt \left( -p_0 \dot{a}_0 + \sum f p_f \dot{f} - N_c \left[ -\frac{p_0^2}{4V_0} + H_c \right] \right) \tag{7}
\]

where

\[
H_c(a_0, p_f, f) = -V_0 K a_0^2 + H_{\text{mat}}(p_f, f), \quad K = \mu^2 k a_0^{-2}
\]

The proper time dynamics (or the equation for superfluous momentum) describes the evolution of proper time of an observer (8) with respect to the evolution parameter, which is the scale factor

\[
\frac{\delta W}{\delta p_0} = 0 \Rightarrow \frac{da_0}{dT_c} = \pm \rho^{1/2} \Rightarrow T_c(a_0)_\pm = \pm \int_0^{a_0} da_0 \rho^{-1/2}. \tag{8}
\]
where $\rho$ is the density of matter in cosmological models

$$\rho = \frac{H_c}{V_0} = \frac{\rho_{\text{anisotropic}}}{a_0^2} + \rho_{\text{rad}} + \rho_{\text{dust}}a_0 - Ka_0^2 + \Lambda a_0^4.$$  

Inverting the PTD relation (8) we get the Hubble law which reflects the evolution of scale factor:

$$a_0 = a_0(T_F) \Rightarrow Z = \frac{a_0(T_F - D/c)}{a_0(T_F)} - 1 \simeq (D/c)H_{\text{Hub}}(T_F) + \ldots.$$

While a photon emitted by a star atom some time ago is flying, the Universe is expanding with all lengths, including the wave-length of this photon, which becomes more red than a photon emitted by the same standard atom on the Earth.

Afterwards we’ll show that the convention about measurable time can be changed and another cosmological picture with a similar Hubble law can be obtained.

2.4. Gaugeless Hamiltonian reduction and quantum cosmology

To demonstrate the facilities of gaugeless Hamiltonian reduction, let us consider quantum cosmology using as an example the Universe filled with photons.

We have the extended action (7) with the matter Hamiltonian $H_{\text{mat}}$

$$H_{\text{mat}} = H_{\text{photon}} = \sum K \left( \frac{p_{K}^2}{V_0} + \omega_{K}^2 q_{K}^2 \right)$$

The PTD based on convention (6) and the constraint $H_{\text{ES}} = 0$ describes the Hubble law for the radiation stage.

The same constraint $H_{\text{ES}} = 0$ can be used to get the WDW equation in the conventional approach to quantization.

$$\left( -\left( \frac{\hat{P}_0^2}{4V_0} + V_0 k \frac{\hat{a}_0^2}{r_0^2} \right) + \hat{H}_{\text{photon}} \right) \Psi_{\text{WDW}} = 0 \quad \left( \hat{P}_0 = \frac{1}{i \frac{d}{da}} \right)$$

There arise the questions: what is the physical interpretation of the wave function $\Psi_{\text{WDW}}(a)$, what is the reason of its nonnormalizability, what is its connection with the Hubble law $T(a)$?

To reply all these questions, it is enough to recall the convention for the proper time (6) and to make the Levi-Civita canonical transformation [8, 5, 6] which converts the constraint into a new momentum

$$(P_0, a_0) \Rightarrow (\Pi, \eta); \quad \frac{P_0^2}{4V_0} + \frac{k a_0^2}{r_0^2} V_0 = \Pi.$$ 

so that the new scale variable (as the new parameter of evolution) coincides with the proper time (like in classical mechanics).

In this case we have two maps of the canonical transformations and two Universes.

The action in terms of the new variables reads

$$W_{\pm}^{\text{ES}} = \int_{t_1}^{t_2} dt \left[ \sum K p_{K} q_{K} \pm \Pi \dot{\eta} - N_c (-\Pi + H_{\text{photon}}) \right].$$

We get the constraint

$$\frac{\delta W_{\pm}^{\text{ES}}}{\delta N_c} = 0 \Rightarrow \Pi = H_{\text{photon}}$$
and the simplest proper time dynamics
\[ \frac{\delta W_{ES}^\pm}{\delta \Pi} = 0 \Rightarrow d\eta = \pm N_c dt = \pm dT \]

In this version the WDW equation coincides with the Schrödinger equation
\[ \frac{d}{i d\eta} \Psi_{HR}(\eta|q) = H_{\text{photon}} \Psi_{HR}(\eta|q); \quad \hat{\Pi} = \frac{d}{i d\eta} \]

and we get \([12, 13]\) the spectral decomposition over normalizable eigenfunctions
\[ \Psi_{HR}(\eta|q) = \sum_E \left[ e^{iET} < E|q > \alpha^{(+)} + e^{-iET} < E|q >^* \alpha^{(-)} \right] = \sum_E \Psi_E(T); \quad (10) \]

The obtained wave function (10) can be simply interpreted as the wave function of photons in a box and it bears a direct relation to the Hubble law. The derivative of each term of the spectral decomposition with respect to the measurable time \(T_F\) (9) gives the measurable energy of the red shift
\[ \frac{d}{i dT_F} \Psi_E = \frac{E}{a(T_F)} \Psi_E. \]

The wave function becomes normalizable as one of variables (\(\eta\)) leaves the phase space.

2.5. Statement of problems

We will formulate the statement of problems using SR as an example.

In SR, the reduced Hamiltonian is a square root of the sum of squares of the momentum and mass. The fundamental question of GR is: what is the reduced Hamiltonian of this theory?

In SR we have the reduced action with the evolution parameter as a former dynamic variable of ES. What is the evolution parameter in GR? Is it the scale factor, or the second fundamental form or something else? What is the proper time dynamics of GR?

If we fulfill the Hamiltonian reduction program in SR, we can quantize and perform the spectral decomposition of the normalizable wave function. What are similar quantization, spectral decomposition and normalizable wave function in GR?

3. General Relativity and Conformal Unified Theory

We will to present our version of solving these problems in GR.

GR is based on the Einstein-Hilbert action
\[ W^{ES}[g_{\mu\nu}, F] = \int d^4x \sqrt{-g} \left[ \frac{\mu^2}{6} (R(g) + \mathcal{L}_{\text{mat}}(g, F)) \right], \quad \mu = M_{Pl} \sqrt{3}/8\pi. \quad (11) \]

and the convention about measurable interval
\[ (ds)^2 = g_{\mu\nu} dx^\mu dx^\nu. \quad (12) \]

Both the action and the convention are invariant under general coordinate transformations.
For the Hamiltonian description one conventionally uses the Dirac-ADM foliation of four-dimensional metric with pointing out the time is of an observer

\[ ds^2 = N^2 dt^2 - \langle 3 \rangle g_{ij} \dot{d}x^i \dot{d}x^j; \quad \dot{d}x^i = dx^i + N^i dt. \] (13)

We have also used the conformal invariant variables of Lichnerowicz that are convenient for studying the problem of initial data and the Hamiltonian dynamics [10, 17]:

\[ F_c^{(n)} = F^{(n)} \langle 3 \rangle g \langle n/6 \rangle; \quad \varphi_c = \mu \langle 3 \rangle g \langle 1/6 \rangle \] (14)

\[ N_c = N \langle 3 \rangle g^{-1/6}; \quad g_{ij}^{c} = \langle 3 \rangle g \langle -1/3 \rangle; \quad (\langle g \rangle = 1) \] (15)

where \( F^{(n)} \) is a matter field of the conformal weight \( n \).

Then, the Einstein-Hilbert action assumes the form

\[ W_E^H = \int d^4 x \sqrt{-g} \left[ -\frac{\mu^2}{6} \langle (4) R \rangle \right] = \int d^4 x \left[ -N_c \frac{\varphi_c^2}{6} \langle (4) R \rangle + \varphi_c \partial_\mu (N_c \partial_\mu \varphi_c) \right] \] (16)

which coincides with the action of the conformal invariant Penrose-Chernikov-Tagirov (PCT) theory of a scalar field \( \Phi \) in terms of the Lichnerowicz variables (14), (15) \( (\varphi_c = \Phi \langle 3 \rangle g \langle 1/6 \rangle) \).

However, in contrast with GR the observables in PCT theory are conformally invariant quantities, in particular, an observer measures the conformally invariant interval

\[ (ds_c)^2 = N_c^2 dt^2 - g_{ij}^{c} \dot{d}x^i \dot{d}x^j. \] (17)

The PCT version is preferable from the point of view of unification of gravity with other interactions. There is a possibility to identify the PCT scalar field with the modulus of the Higgs doublet and to add the matter fields as the conformal invariant part of the Standard Model for strong and electro-weak interactions [19, 20]. The obtained model was called the Conformal Unified Theory (CUT) [14, 19, 15].

In the first-order formalism the actions of both theories can be represented in the form which is a continual local generalization of the extended mechanics:

\[ W^{ES}[P_f, f; P_\varphi, \varphi_c] = \int_{t_1}^{t_2} dt \int d^3 x \left[ \sum_{f=\varphi_c, F} P_f D_t f - P_\varphi D_t \varphi_c - N_c (-\frac{P^2}{4} + H_f) \right], \] (18)

where \( D_t f, D_t \varphi_c \) are the time covariant derivatives

\[ D_t \varphi_c = \partial_t \varphi_c - \partial_k (N^k \varphi_c) + \frac{2}{3} \varphi_c \partial_k N^k. \]

Action (18) is invariant under reparametrization of the coordinate time \( t \). According to the considered gaugeless Hamiltonian reduction, we should pick out the superfluous variable of the extended system, which plays the role of the evolution parameter for the corresponding reduced system.

Our idea is to carry out the global Hamiltonian reduction. This means extracting a global component from the local scalar field as the evolution parameter of RS

\[ \varphi_c(t, x) = \varphi_0(t)a(t, x); \quad D_t \varphi_c = \frac{d \varphi_0}{dt} a + \varphi_0 D_t a. \] (19)

We shall extract also a global component from the conformal invariant lapse function

\[ N_c(t, x) = N_0(t) \mathcal{N}(t, x). \] (20)
As we have introduced one more variable, we should impose one more constraint and take the constraint
\[ \int d^3x a \frac{D_\alpha a}{N_c} = 0, \]  
which diagonalizes the kinetic part of the Lagrangian.

New variables require the corresponding momenta \( P_0 \) and \( P_a \). We define the decomposition of the old momentum \( P_\phi \) over the new momenta \( P_0 \) and \( P_a \)
\[ P_\phi \overset{\text{def}}{=} P_0 \frac{a}{NV_0} + \frac{1}{\varphi_0} P_a; \quad V_0 = \int d^3x \frac{a^2}{N}. \]  
to get the conventional canonical structure for the new variables
\[ \int d^3x P_\phi D_t \varphi = P_0 \frac{d\varphi_0}{dt} + \int d^3x P_a D_t a. \]

The substitution of these definitions into the old action produces a superfluous momentum term:
\[ W^{ES} = \int_{t_1}^{t_2} dt \left( \int d^3x \sum_{f=a,g,c,F} P_f D_t f - P_0 \varphi_0 - N_0 \left( -P_0^2 + H_f \right) \right) \]

Finally, the obtained action \( W^{ES} \) acquires the structure of the extended cosmological model with two functionals: the functional of the volume \( V_0 \) \( (22) \), and the functional of the Hamiltonian:
\[ H_f[\varphi_0] = \int d^3x N[-\frac{P_0^2}{4\varphi_0} + H_f]. \]

Resolving the constraint
\[ \frac{\delta W^{ES}}{\delta N_0} = 0 \Rightarrow (P_0)_\pm = \pm 2\sqrt{V_0 H_f} \]
we get the reduced action
\[ W^{RS}(\pm) = \int_{\varphi_1=\varphi_0(t_1)}^{\varphi_2=\varphi_0(t_2)} d\varphi_0 \left\{ \left( \int d^3x \sum_f P_f D_\varphi f \right) \mp 2\sqrt{V_0 H_f} \right\}. \]

The variation of the RS action determines the dependence of all variables on the evolution parameter \( P_f = P_f(\varphi_0), \quad f = f(\varphi_0) \) and completely reproduces the Einstein equations.

Proper time dynamics gives the integrals of motion as functionals of field variables \( \frac{\delta W^{ES}}{\delta P_0} = 0 \Rightarrow (P_0)_\pm = \pm \sqrt{V_0 H_f(\varphi_0)}; \]
\[ \frac{d\varphi_0}{dT} \pm = \frac{(P_0)_\pm}{2\sqrt{V_0}} = \pm \sqrt{\rho(\varphi_0)}, \]
where
\[ dT = N_0 dt; \quad \rho = \frac{H_f}{V_0}; \]
\[ T(\varphi_0)_{\pm} = \pm \frac{\varphi_0}{\int_0^1 d\varphi \rho^{-1/2}}. \]  

(26)

It is easy to check that the local part of the scalar field in perturbation theory is nothing but the potential of the Newton interaction. To reproduce the homogeneous FRW cosmology, it is sufficient to neglect this interactions.

Finally, we get the Hubble law in two versions, GR and CUT.

We emphasise that the definition of observables in General Relativity (12) has some defects as compared with CUT (17).

i) In contrast with CUT, in GR there is mixing of the evolution parameter with metric.

ii) For a space with positive curvature the Friedmann time violates causality [12, 22] but the conformal one does not!

iii) In CUT, an observable 3D-volume is an integral of motion; in GR we have a singularity at the beginning.

Therefore, it is worth to thoroughly consider physical consequences of the Conformal Unified Theory.

4. Conformal World

4.1. Hoyle-Narlikar cosmology

The convention of measurable interval in CUT (17) leads to the Hoyle-Narlikar cosmology [23] where the red shift is explained by the evolution of all masses rather than by the expansion of the Universe. A photon emitted by an atom on a star remembers the size of this atom, and after billions of years, the wavelength of this star photon can be compared with that of a photon emitted by a standard atom on the Earth, the size of which is much less than the size of the star atom. As result, we arrive at the red shift

\[ Z = \frac{\varphi_0(T - D/c)}{\varphi_0(T)} - 1 \simeq \frac{(D/c)H_{\text{Hub}}(T)}{\varphi_0(T)} + \ldots \]

with the Hubble parameter determined by the proper time dynamics (23)-(26)

\[ H_{\text{Hub}} = \frac{1}{\varphi_0(T)} \frac{d\varphi_0(T)}{dT} = \frac{\sqrt{\rho}}{\varphi_0(T)}. \]

(27)

From the last equation (27) it follows that

\[ \varphi_0 = \frac{\sqrt{\rho}}{H_{\text{Hub}}}. \]

The substitution of the observational data

\[ \rho = \rho_{\text{critical}} \Omega_{\text{exp}}; \quad \rho_{\text{critical}} = 3M_{\text{Pl}}^2 H_{\text{Hub}}^2 / 8\pi \]

leads to the present - day value of the scalar field which coincides with the Newton constant in the action (14) and (16)

\[ \varphi_0^2(T = T_0) = \frac{M_{\text{Pl}}^2}{16\pi} \Omega_{\text{exp}} = \frac{\mu^2}{6} \Omega_{\text{exp}} \]

in agreement with astrophysical data [24].

\[ 0.02 < \Omega_{\text{exp}} < 2. \]
4.2. Cosmic Higgs vacuum

It is clear that the present-day state of the Universe provides the laboratory vacuum. One can only suppose that at the present day stage the total Hamiltonian can be divided into the part forming evolution of the Universe (the Universe expectation value) and the Laboratory part (for which the expectation value equals zero)

\[ H_f[\varphi_0] \overset{\text{def}}{=} \rho_{\text{Un}}V_0 + (H_f - \rho_{\text{Un}}V_0) = \rho_{\text{Un}}(\varphi_0)V_0 + H_L; \]

\[ \langle \text{Universe}|H_f|\text{Universe} \rangle = \rho_{\text{Un}}V_0; \]

\[ \langle \text{Universe}|H_L|\text{Universe} \rangle = 0. \]

It was known that \( \rho_{\text{Un}} \sim 10^{79} \text{m}^{\text{proton}} \) while \( H_L \sim 1 \text{m}^{\text{proton}} \), therefore, we can apply the nonrelativistic type approximation for decomposing the square root over the inverse volume

\[ \sqrt{V_0H_f} = \sqrt{V_0^2}\rho_{\text{un}} + V_0H_L = 2V_0\sqrt{\rho_{\text{un}}} + \frac{H_L}{\sqrt{\rho_{\text{un}}}}. \]

Finally, we get a splitting of the action on the cosmological and laboratory parts

\[ W^R_{(+)}(p_f,f|\bar{\varphi}_0) = W^G_{(+)}(\bar{\varphi}_0) + W^L_{(+)}(p_f,f|\bar{\varphi}_0). \]

The proper time dynamics

\[ \frac{d\varphi_0}{\sqrt{\rho_{\text{un}}}} = dT \]

allows us to rewrite the laboratory action in terms of the observable time

\[ W^L_{(+)}(p_f,f|\bar{\varphi}_0) = \int_{T_1}^{T_2}dT \left( \int d^3x \sum_f p_fD_Tf - H_L(p_f,f|\varphi_0(T)) \right). \]

(28)

In this case, the laboratory part of the total action in terms of the conformal time coincides with the \( \sigma \) model version of SM (within the gravitational interactions).

As the time of the laboratory experiment \( \Delta T \) is much smaller than the age of the Universe \( T_0 \):

\[ T_1 = T_0 - \frac{\Delta T}{2}; \quad T_2 = T_0 + \frac{\Delta T}{2}; \quad \Delta T \ll T_0, \]

we can neglect the change of the scalar field in laboratory experiments:

\[ \varphi_0(T) \approx \varphi_0(T_0) = \sqrt{\frac{3}{8\pi}}; \quad T_0 - \frac{\Delta T}{2} < T < T_0 + \frac{\Delta T}{2}. \]

In perturbation theory, corrections to the scale factor \( a \) represent the Newton potential

\[ \varphi_c(T,x) = \varphi_0(T) a(T,x); \quad a(T,x) = 1 + (\text{Newton potential}). \]

like in QED the time component of the electromagnetic field gives the Coulomb potential. We see that the particle-like excitations of the scalar field are absent, as predicted in [10].
4.3. **σ-model version of SM**

Theoretically, the status of the Higgs sector in conventional SM is still mysterious. The physical motivation for its existence as a consequence of the first symmetry principles is unclear. The imaginary mass in the Higgs potential is rather unexpected. There is also a number of difficulties caused by the scalar mode of a nonvanishing vacuum expectation value in cosmology (a great vacuum density [25], a monopole creation [26], domain walls [27]).

Almost all we know about the Higgs particle comes from collider experiments. In direct search the Higgs particle is looked in the process

\[ e^+ e^- \rightarrow Z^{(*)} \rightarrow Z^* H \rightarrow X f \bar{f}. \]

The search for the Higgs particle is a main motivation for building new high energy accelerators. The LHC will be able to give a definite answer to the question concerning the existence of the SM Higgs particle. Some information can be derived also from the upgraded Tevatron. The main Higgs production mechanisms at hadron colliders are the following:

1) \( gg \rightarrow H \) **gluon – gluon fusion**
2) \( WW(ZZ) \rightarrow H \) **weak boson fusion**
3) \( q\bar{q} \rightarrow W(Z) + H \) **association with W/Z**
4) \( gg(q\bar{q}) \rightarrow t \bar{t} + H \) **association with t\bar{t}**

There arises a fundamental experimental problem how to discriminate between the SM and an effective sigma model obtained from the Conformal Unified Theory considered here [28]. Some perspectives that open with the new HE accelerators, mainly LHC, are in principle well known and are widely presented in the literature [29] but never in the present context.

A new class of tests is the analysis of properties of the virtual Higgs (or rather a UV-regulator) that can be compared on different experimental energy scales.

Our proposition is based on the observation that the role of the Higgs particle as a regularizing parameter of the model can be played by an effective parameter that is in principle dependent on the energy of the considered process as we are working with the \( S \)-matrix with a finite interval of time [28]. This assumed energy dependence can be tested in high precision experiments planned in the nearest future [30].

4.4. **Beginning**

In the beginning of the Universe (at \( \varphi_0 = 0 \)) we could not separate the evolution of the scalar field (or the scale factor in GR) from the evolution of matter fields.

To study the beginning stage, we need the classification of Hamiltonians, which follows from the classification of times, considered in the beginning of the present paper. This classification is given in Table 1.
Three Times

COORDINATE TIME
\[ t \rightarrow t' = t'(t) \]

EVOLUTION PARAMETER
\[ \varphi_0 \]

"MEASURABLE" TIME
\[ dT = N_0 dt \]

Three Hamiltonians

CONSTRAINT
\[ H^{ES} = 0 \]

EVOLUTION HAMILTONIAN
\[ P_0 = 2\sqrt{V_0}\dot{H}_f \]

"MEASURABLE HAMILTONIAN"
\[ H^M_{(hj)} = -\frac{\partial W^{RS}}{\partial T} \]

Table 1. Classification of Hamiltonians

The energy constraint corresponds to the noninvariant nonobservable coordinate time. Superfluous momentum on the constraint can be called the evolution Hamiltonian. Using the Hamiltonian-Jacobi prescription we can also introduce the "measurable Hamiltonian" as the derivative of the action with respect to the "measurable" time.

We have considered the beginning of the Universe in the QFT approximation. This means:

i) neglecting all interactions

ii) the decomposition over the inverse volume, and

iii) the use of oscillator-like variables (as in QFT).

Details of our approximation are the following.

We considered only photons and gravitons and, finally, got the set of oscillators (\( K \) stands for photons; \( L \) for gravitons):

\[
\int_{V_0} d^3x \frac{1}{2} \left( P^2_{(A)} + (\partial_i A^\perp)^2 \right) = \sum_{K=(k,\alpha)} \frac{1}{2} \left( p_K^2 + \omega_K^2 q_K^2 \right); \\
\int d^3x \frac{1}{2} \left( \frac{6P^2_{(h)}}{\varphi_0^2} + \frac{\varphi_0^2}{24} (\partial_i h^\perp)^2 \right) = \sum_{L=(l,\alpha)} \frac{1}{2} \left( p_L^2 + \omega_L^2 q_L^2 \right). 
\]

Gravitons differ from photons by an additional (Hubble-like) term in the action with a singularity at the beginning.
We get the extended system action:

\[
W^{ES} = \int dt \left[ \sum I \dot{q}_I P_I - \dot{\varphi}_0 (P_0 + \sum L \frac{q^L P^L}{\varphi_0}) + \frac{d}{dt} \left( \frac{\varphi_0 P_0}{2} \right) + N_0 H^{ES} \right]
\]

where

\[
H^{ES} = \left( -\frac{P^2}{4V_0} + \sum_{I=K,L} \frac{1}{2} (p^2_I + \omega^2 q^2_I) \right)
\]

We can in detail consider only one mode of gravitons with a wavelength of an order of the background cosmic radiation one.

Apart from, the conventional stage of radiation of the Universe we got also the stage of "inflation" with respect to the conformal time (see Appendix B.).

The measurable energy for photons completely coincides with the conventional energy of photons in QFT in the flat space-time.

The measurable energy for gravitons in the present - day asymptotics coincides with the Tolman definition of the energy momentum tensor \[31\]. Due to the Hubble - like term, the measurable energy can become zero at the point where \(\varphi(b) = \varphi_0(T_0 = 0) \neq 0\) before singularity, and then accepts negative values.

This means that an observer at this moment sees the creation of the Universe from nothing with the set of non-zero quantum numbers, integrals of motion.

Negative values can be treated correctly only in quantum theory where they can be removed by replacing the creation to annihilation ones and vice versa.

5. Conclusions

We considered three new ideas related to

i) gaugeless Hamiltonian reduction of General Relativity with the internal evolution parameter,

ii) classification of "times", and

iii) resolving the global energy constraint.

These three ideas give us a chance to convert GR into the Conformal Unified Theory with the set of predictions:

a) the Hoyle-Narlikar cosmology,

b) the cosmic Higgs effect for formation of masses of elementary particles and the Newton constant of gravity, which are defined by the cosmological data on the density of matter and the Hubble parameter (the Mach principle),

c) the \(\sigma\)-model version of SM without the Higgs particle-like excitations,

d) the inflation stage as consequence of the graviton dynamics at the beginning of the Universe.

The Lagrangian of CUT does not contain any dimensional parameters, as masses of all fields are changed by a scalar field (multiplied by the corresponding dimensionless coupling constants). The scalar field in CUT restores the conformal symmetry of the action, like the vector gauge fields restore the gauge symmetry. In accordance with the “gauge ideology” of Weyl, Yang–Mills, Utiyama and Kibble, the minimal conformal - invariant dynamics of the scalar field corresponds to the PCT action with negative sign. This action, the scalar field, the conformal invariant variables and measurable time can be based in the scalar version of the Weyl geometry of similarity where we can measure only the ratio of lengths of two vectors at the same point.

In the conventional approach, one tries to describe large-scale phenomena by the theory with the Higgs spontaneous symmetry breaking mechanism invented to describe physics of elementary particles.
Here we were trying to describe the generation of elementary particle masses by the conformal version of the Einstein theory proposed to describe large-scale phenomena including generation of the Universe.

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Appendix A: Fermions in CUT

We consider the Fock action for \( (n) \) fermions \[ W = \int d^4x \sqrt{-g} \left[ -\bar{\Psi}^I \gamma^\sigma (D_\sigma) \Psi^I - \Phi \bar{\Psi}^I \hat{X}^{IJ} \Psi^J \right]. \] (A.1)

where \( D_\sigma \) is the Fock covariant derivative, \( \hat{X}^{IJ} \) is the matrix of dimensionless coupling constants in the unitary gauge of SM and \( \Phi \) is the modulus of the Higgs doublet field.

In terms of the Lichnerowicz variables \( \bar{\Psi}_c, \varphi_c \) and \( g^c \) [14], [13] and for the triad form of the Dirac-ADM parametrization

\[ g^c_{ij} = \omega_i(t)\omega_j(t); \quad \omega_i(t)\tilde{\omega}_i^j = \delta^j_i; \quad (||g^c|| = 1) \] (A.2)

the Fock action is

\[ W^F = \int d^3x dt \left[ \frac{1}{i} \bar{\Psi}_c \gamma^\sigma (D(t) \psi)^I - J^{[kl]} (D(t) \omega)_{(k)}(l) - N_c \mathcal{H}_\psi \right] \] (A.3)

where

\[ \mathcal{H}_\psi = \varphi_c \bar{\Psi}_c \hat{X}^{IJ} \psi_c - [\bar{\Psi}_c \gamma^\sigma (D(t) \psi)^I + J^0_k b - \partial_k J^k] \] (A.4)

\[ D(t) \psi^I_c = (\partial_0 - N^k \partial_k + \frac{1}{2} \partial_k N_t) \psi^I_c; \quad D(t) \psi^I_c = (\partial_0 - \frac{1}{2} \partial_k \tilde{\omega}^k) \psi^I_c \] (A.5)

\[ J^{[kl]} = \frac{i}{2} (\bar{\psi}_c \gamma_5 \gamma^\sigma (\psi)_c \gamma^\sigma (\psi)_c) \] (A.6)

\[ (D(t) \omega)_{(k)}(l) = \tilde{\omega}^{n}_{(k)} \partial_0 \omega_n(l) - N_i D_\omega(l)n + D(l)N_n; \quad b = D(t) \omega(n) \tilde{\omega}^{n}_{(j)} \tilde{\omega}^{l}_{(k)} \] (A.7)

In the first-order formalism we have the sum of the PCT action and the Fock one

\[ W = W^{PCT} + W^F = \int dt d^3x \left[ \sum_{f = \psi, \omega, \tilde{\omega}} P_f D_t f - \frac{1}{2} \partial_t (P_\varphi \varphi) - N_c \mathcal{H} + S \right] \] (A.8)
where

\[
\mathcal{H} = -\frac{P^2}{4} + 3 \frac{P^2}{2 \phi_c^2} + \frac{\phi_c^2}{6} \tilde{R} + H \psi; \quad S = -\frac{1}{3} \partial_j (\phi_c \partial^j (\phi_c N_c)) + \partial_j (J^j N_c);
\]  

\[
\bar{R} = \left[ (3) R(g)^c \right] + 8 \phi_c^{-1/2} \partial^2 \phi_c^{1/2}; \quad D_t \phi_c = \partial_0 \phi_c + \frac{2}{3} \partial_k (N^k \phi_c); \quad \partial_0 \phi_c = \partial_0 \phi_c - \partial_k (N^k \phi_c) \tag{A.10}
\]

The Lagrange equations for momentum \( P_f \) are

\[
P^{(k)(l)}_\omega = \frac{\phi_c^2}{6N_c} \left[ (D_l \omega)(k)(l) + (D_l \omega)(l)(k) \right] + J^k_5; \tag{A.11}
\]

\[
P^{(l)}_{\phi_c} = \frac{2D_t \phi_c}{N_c}; \quad P^I_{\Psi} = \bar{\Psi}^I \gamma_0 \tag{A.12}
\]

We keep here all surface terms of the PCT actions and use the equality \( D_i N_i = \partial_i N_i \) for the metric with \( ||g^c|| = 1 \).

The extraction of the global components of a scalar field and the lapse function (19) - (21) and the Hamiltonian reduction lead to the reduced action of the type of (24) with the time surface term:

\[
W^{RS}_{\pm} = \int d^3x d\phi_0 \left\{ \left( \int d^3x \sum_f P_f D_f \phi_f \right) \mp 2\sqrt{V_0 H_f} \pm \frac{d}{d\phi_0} \phi_0 \sqrt{V_0 H_f} \right\}, \tag{A.13}
\]

where \( f \) runs over \( a, \omega, \Psi \),

\[
H_f = \frac{H(-2)}{\phi_0} + H(0) + \phi_0 H(1) + \phi_0^2 H(2)
\]

\[
H(-2) = \int d^3x N \left[ -\frac{P^2}{4} + 3 \frac{P^2}{2 \omega^2} \right]
\]

\[
H(0) = \int d^3x [N H_\Psi + J^k \partial_k N]
\]

\[
H(1) = \int d^3x N a \bar{\Psi}^I X^I \Psi^J_c
\]

\[
H(2) = \int d^3x \left[ N a^2 \frac{\tilde{R}}{6} + \frac{1}{3} \partial_j (a \partial^j (a N)) \right]
\]

**Appendix B: Inflation**

Consider the equation for a single graviton mode. The reduced system is described by two equations:

\[
\frac{d q_L}{d \phi_0} = \frac{q_L}{P_t \phi_0} + \frac{q_L}{\phi_0}; \quad (\rho_t = \rho_0 + \rho_g); \tag{B.1}
\]

\[
- \frac{d p_L}{d \phi_0} = \frac{p_L}{P_t \phi_0} + \frac{p_L}{\phi_0}; \quad (\rho_g = \frac{1}{2V_0} (p_L^2 + \omega_L^2 q_L^2)), \tag{B.2}
\]

where \( \rho_0 \) is a conserved density of the photon radiation.
The reduced system is supplemented with proper time dynamics

\[ T(\varphi_0) = \int_{[0]}^{\varphi_0} \frac{d\varphi}{\sqrt{\rho_t(\varphi)}}; \quad H_{\text{Hub}} = \frac{1}{\varphi_0} \frac{d\varphi_0}{dT} = \frac{\sqrt{\rho_t(\varphi_0)}}{\varphi_0}. \]  

(B.3)

From comparison of two terms on the right hand side of eqs. (B.1) and (B.2), we can see that there are two regimes:

i) \( \varphi_0^2 \gg \rho_t/\omega_L^2 \); ii) \( \varphi_0^2 < \rho_t/\omega_L^2 \).

In the first regime, we can neglect the last terms of eqs. (B.1) and (B.2) and then we get the usual gravitational wave with conserved density \( \rho_g \) (\( \frac{d}{d\varphi} \rho_g = 0 \)):

\[ q_L = \sqrt{2V_0\rho_g} \sin(\omega_L T + \delta_0); \quad p_L = \sqrt{2V_0\rho_g} \cos(\omega_L T + \delta_0); \quad T(\varphi_0) = \frac{\varphi_0}{\sqrt{\rho_0 + \rho_g}}. \]  

(B.4)

In the second regime [neglect of the first terms in eqs. (B.1) and (B.2)] we get

\[ \omega q_L = \sqrt{2V_0} \frac{\varphi_0}{T_0}; \quad p_L = \sqrt{2V_0} \frac{A}{\varphi_0} \]  

(B.5)

so that the density and the PTD are of the forms:

\[ \rho_t = \rho_0 + \frac{A^2}{\varphi_0^2} + \frac{\varphi_0^2}{T_0^2}; \]  

(B.6)

\[ \varphi_0(T) = \frac{T_0}{\sqrt{2}} \left[ \rho_0 [\cosh(2T/T_0) - 1] + 2 \frac{A}{T_0} \sinh(2T/T_0) \right]^{1/2}. \]  

(B.7)

The density (B.6) shows that solutions (B.5) can be treated as a spontaneous generation of space geometry due to the 3D space curvature. From (B.7) we can see that there is the period of inflation in terms of the conformal time (here measurable): see Fig. 1. where one represents the dependence of measurable energy

\[ E_c = 2\rho_t + \sqrt{\rho_t} \frac{p_L q_L}{\phi_0} - \sqrt{\rho_t} \frac{d}{d\varphi_0} (\phi_0 \sqrt{\rho_t}) \]  

on the evolution parameter \( \varphi_0 \) as the result of the numerical computations for solutions of (B.1) and (B.2).
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