New Windows based Color Morphological Operators for Biomedical Image Processing

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Abstract. Morphological image processing is well known as an efficient methodology for image processing and computer vision. With the wide use of color in many areas, the interest on the color perception and processing has been growing rapidly. Many models have been proposed to extend morphological operators to the field of color images, dealing with some new problems not present previously in the binary and gray level contexts. These solutions usually deal with the lattice structure of the color space, or provide it with total orders, to be able to define basic operators with required properties. In this work we propose a new locally defined ordering, in the context of window based morphological operators, for the definition of erosions-like and dilations-like operators, which provides the same desired properties expected from color morphology, avoiding some of the drawbacks of the prior approaches. Experimental results show that the proposed color operators can be efficiently used for color image processing.

1. Introduction
The Mathematical Morphology (MM) field of image processing was founded by Matheron [1] and Serra [2] and became very popular in non-linear image processing. Initially, the MM had been introduced as a processing technique for binary images, which were regarded as sets; therefore, its elementary operations are based on the set theory [3]. However, the extension to gray scale images, using the umbra concept [4-5], introduced a generalization of the basic morphological operations which were subsequently used in many image processing and analysis tasks e.g. morphological filtering [6], watershed segmentation [7], etc. Gray scale morphology is based on the lattice theory, which implies a partial ordering of the data within the gray scale images. In this case, the lattice structure is not difficult to obtain, since the gray scale images are real functions, and the set of real numbers implicitly possesses a lattice structure. However, while the extension from binary to gray scale images is a natural one, the extension to color, and multi-spectral, images is not straightforward, because of the vectorial nature of the data, and the difficulty in finding a suitable ordering for it. Barnett introduced four types of vector orderings: marginal (M-order), reduced (R-order), conditional (C-order) and partial (P-order) [8]. When applied to color data, all these orderings have certain disadvantages, depending on the application. For instance, the marginal ordering introduces false colors; and the reduced and partial orderings are either relying on pre-orderings, thus lacking the anti-symmetry property, or behave like conditional orderings, generating perceptual nonlinearities [9]. Compared with gray-scale images, color images can provide richer information. With the wide use of color in medical image equipment, the interest on the color perception and processing has been growing rapidly. In this work we proposed morphological color operators, based on a new locally defined ordering, avoiding some of the drawbacks of the prior approaches. First, we apply these new operators to synthetic image, comparing the results against marginal and lexicographical ordering. Then we present two examples of application, of the new operators, to previous biomedical image processing problems: detection of exudates in ocular images and bone marrow biopsies segmentation. Experimental results show that the proposed color morphological operators can be efficiently used in color image processing.
2. Theoretical Concepts

Initially, the MM had been introduced as a processing technique for binary images, which were regarded as sets; therefore, its elementary operations are based on the set theory.

For binary images, modeled as subsets of a domain \( E \), dilation and erosion of an image \( A \) by the structuring element \( B \) are defined, respectively, as:

\[
\delta_B(A) = \bigcup_{y \in B} A_y, \quad (1)
\]

\[
\varepsilon_B(A) = \bigcap_{y \in B} A_y. \quad (2)
\]

When modeling binary images by their characteristic functions, the equivalent to a set \( A \) is a function \( \chi_A : E \rightarrow \{0,1\} \), defined by \( \chi_A(x) = 1 \Leftrightarrow x \in A \), the dilatation and erosion by a flat structuring element, defined by a set \( B \), can also be defined by:

\[
\delta_B(\chi_A)(x) = \sup_{y \in B} \{\chi_A(x - y)\} \quad (3)
\]

\[
\varepsilon_B(\chi_A)(x) = \inf_{y \in B} \{\chi_A(x + y)\} \quad (4)
\]

Immediately after, in a natural way, were extended for gray level images. The dilation and erosion of an image \( f : E \rightarrow L \), by a flat structuring element \( B \) is defined, similarly, by:

\[
\delta_B(f)(x) = \sup_{y \in B} \{f(x - y)\} \quad (5)
\]

\[
\varepsilon_B(f)(x) = \inf_{y \in B} \{f(x + y)\} \quad (6)
\]

These definitions are based on the existence of an order relationship in the space of gray levels \( L \), where the minimum and maximum are obtained. More generally, if the range is a more complex complete lattice, the same definition can still be applied, where the minimum is replaced by the infimum and the maximum by the supremum.

Existing approaches for color mathematical morphology need to deal with the fact that usual color spaces are vector spaces, usually with three color components, like red, green and blue intensities for the classic RGB model.

A color image can be modeled as a function \( f : D \subset \mathbb{R}^2 \rightarrow \mathbb{R}^3 \) where \( \mathbb{R}^3 \) that represents a color space. Usually the order on the space of images is derived from an order in the color space \( \mathbb{R}^3 \). Let be \( \mathbb{R}^3 \subset \mathbb{R}^3 \) the unit cube (RGB color space) and let \( \leq \) be an order in \( \mathbb{R}^3 \) that provides a structure of complete lattice. The normal extension of this order to the space of images, or functions \( f : D \subset \mathbb{R}^2 \rightarrow \mathbb{R}^3 \), with an order \( \leq \), is by defining it, for \( f, g : D \subset \mathbb{R}^2 \rightarrow \mathbb{R}^3 \), as \( f \leq g \) if and only if \( f(x) \leq g(x) \forall x \in D \) [10]. This order provides the images with a lattice structure too. Once a lattice structure is defined on the space of color images, the basic operations erosion (\( \varepsilon_B^n \)) and dilation (\( \delta_B^n \)) can be defined: for a color image \( f : D \subset \mathbb{R}^2 \rightarrow \mathbb{R}^3 \) its dilation and erosion by a structuring element \( B \) is defined as:

\[
\varepsilon_B^n(f) = \inf_{s \in B} \{f \circ \tau_s\} \quad (7)
\]

\[
\delta_B^n(f) = \sup_{s \in B} \{f \circ \tau_s\} \quad (8)
\]

being \( \tau_s : \mathbb{R}^2 \rightarrow \mathbb{R}^2 \) traslation function by the element \( s \in \mathbb{R}^2 \), \( \tau_s(x) = x + s \).

The results of erosion and dilation of a color image depends strongly on the order established in the color space. After defining the basic operators in the domain of color images, we can define the morphological opening (\( \gamma_B^n \)) and closing (\( \phi_B^n \)) filters in the same way as defined for binary images and gray levels.

\[
\gamma_B^n(f) = \delta_B^n(\varepsilon_B^n(f)) \quad (9)
\]

\[
\phi_B^n(f) = \varepsilon_B^n(\delta_B^n(f)) \quad (10)
\]

An important issue with this lattice structure for the RGB color space, inherited by the color images space, it that it results on the creation of false colors when applying the basic operator, since the infimum for a set colors may be a new color different from any of them [11]. For example, the
infimum between \((0.5,1,1)\), a kind of light blue and \((1,1,0.5)\), a light yellow, is \((0.5,1,0.5)\), a light green (figure 1).

To solve this situation, many approaches were proposed in literature. Between them we can list, P-ordering and R-ordering. In C-ordering, there is a total order, obtained by sequential application of the order of the components. An example of such order is the lexicographic order. In R-ordering, the RGB vectors are mapped to a scalar (for example by its distance to a reference point), and a total order is applied. The drawback with this approach lies in the need of a reference point, and the fact that the map is not one to one [11-12].

In [11] the proposed method of lexicographic cascade depends on the distance to a reference color. Because that distance defines a partial ordering, since different colors can have the same distance to the reference color, a lexicographic cascade is applied to decide on these cases. This solution, as other related ones, suffers from the need to choose a reference color and a lexicographic order. Other solution, proposed in [12], define a total order over all the RGB colors observed in the image, solving the problem without defining a total order over the whole RGB space. This last solution deviates from the theoretical foundations of MM as operators on lattices, but erosion and dilation operators result in no false colors. Other solutions, like the one proposed in [13] provides another total ordering in the HSI color space, weighting the components using a cost function, and solving the ties by a lexicographic ordering. This approach solves the need for a reference color, but still depends on a lexicographic order chosen ad-hoc.

On another field of study of Mathematical Morphology, windows based operators have been studied for many years, as a way to represent locally defined morphological operators, and for their automatic design from examples [14]. In this context, translation invariant locally defined Morphological Operators can be represented as w-operators, like the case of erosions and dilations. This family of morphological operators includes and extends that subfamily of classical morphological operators. One of the advantages of designing w-operators is the freedom from the need to define complete lattices on the image space, based on the lattices defined in the color space. Any mapping from a window observation to the color space defines valid w-operators on images [15]. These advantages are used, for example, for the statistical design of morphological operators based on examples [16].

3. Proposed Approach

In this work we propose an approach, for color morphology, that deviates from the global lattice or total order used in previous works, by implementing color w-operators and defining erosion-like and dilation-like operators using a local total order on the observed windows.

Given the existence of a lattice structure in the color space, by applying the minimum (or maximum) operation on the colors observed in a symmetric window \(W\), around a pixel \(x\), and replacing, on the resulting image, that pixel by the computed value, it is equivalent to erode (or dilate) the color image by the structuring element \(B\) defined by \(W\) (\(B\) and \(W\) as the same subsets).

In our approach, to avoid the issues related to the need to define an ad-hoc total ordering in the color space, we define a local minimum and local maximum based on a ordering of the observed values on the window. With this local definition, equations (1) and (2) can be applied. In this case the resulting morphological w-operators are not strictly erosions and dilations, since they are not defined as infimum and supremum on complete lattices over the images, but just on a total order on the window, so they will be called erosion-like and dilation-like operators.

![Example of the creation of false color: a) light blue \((0.5,1,1)\), b) light yellow \((1,1,0.5)\) and c) result of the infimum, light green \((0.5,1,0.5)\).](attachment:figure1.png)
The key properties of the local order is that, because it does not depend on a global order, it can be made independent of ad-hoc decisions, like the election of a reference color, or the selection of a lexicographic order and, in the other hand, it can be made dependent on spatial properties of the window. The local order defined will have three main properties: a) it compares first the average intensity of the three channels; b) in case of ties, it compares the closeness, as color, to the central pixel; c) in case of ties, it compares the spatial closeness to the central pixel. The first option is based on the need to specify an order that is related to the luminosity of the pixels, as in [13]. The second option is based on that, if two candidates to the minimum (or maximum) have the same luminosity, then we want to choose the one closest, as color, to the original pixel being replaced. This would avoid color invasions in extreme situations. Finally, in the unlikely situation where two colors candidate to be the minimum have the same distance, as color, to the central pixel, then we want to avoid spatial color invasion.

Formally, the process is described in the following way. The first step consists on providing an order on the elements of $B$. Let say that $B = \{ y_1, ..., y_n \}$. This order of the pixel should reflect the distance between the pixels and the central pixel. The first elements should be the closest, spatially, to the central pixel. Let $x$ a pixel of the image where we want to compute the erosion or dilation by $B$. We list the observed colors in the neighbor of pixel $x$, as $v_1, ..., v_n$, where $v_j = f(x + y_j)$, and the index of the central one pixel is $k_0 = 1$ (since the spatial order should always position the central pixel first).

The minimum between the values $v_i$ is computed based on the following local total order:

$$v_i \leq v_j \iff \begin{cases} v_i \leq v_j \quad \text{or} \\ v_i = v_j \quad \text{and} \quad |v_i - v_{k_0}| \leq |v_j - v_{k_0}| \\ v_i = v_j \quad \text{and} \quad |v_i - v_{k_0}| = |v_j - v_{k_0}| \quad \text{and} \quad i \leq j \end{cases}$$

Finally, the erosion-like of an image by a structuring element $B$ is obtained by computing, for each pixel $x$, the minimum based on the newly defined order, on the window defined by $B$. For dilations, since a maximum will be selected, the equation for the local order is the following:

$$v_i \geq v_j \iff \begin{cases} v_i \geq v_j \quad \text{or} \\ v_i = v_j \quad \text{and} \quad |v_i - v_{k_0}| \leq |v_j - v_{k_0}| \\ v_i = v_j \quad \text{and} \quad |v_i - v_{k_0}| = |v_j - v_{k_0}| \quad \text{and} \quad i \leq j \end{cases}$$

The dilation-like of an image by a structuring element $B$ is obtained by computing, for each pixel $x$, the maximum based on the newly defined order, on the window defined by $B$.

This change prioritizes the closest colors and pixels by positioning them higher on the local order. It is important that the ordering of the elements of the window reflects the distance to the central pixel. Usually an indexing in spiral, from inside to outside, would provide an appropriate indexing.

As an example of application, let’s assume that we are working with a 3x3 structuring element $B$. It defines a window $W$, and the index of the pixels in $W$ is given by the following schema, where the central pixel has index 1:

| 7 | 3 | 6 |
|---|---|---|
| 4 | 1 | 2 |
| 8 | 5 | 9 |

Let’s assume then that we observe an image with the windows $W$, and we observe the following RGB colors around the central pixel in the original image:

| (10,10,10) | (10,10,10) | (10,10,10) |
| (10,10,10) | (5,10,11) | (9,5,11) |
| (5,8,12) | (10,10,10) | (10,10,10) |

The list of values, based on the order on $W$, would be:
The final order on these color values would be: $v_1 \geq v_4 \geq v_5 \geq v_6 \geq v_7 \geq v_i \geq v_8 \geq v_9$. The first six values are obtained because $v_1$ is the largest intensity value, they all have the same distance (as color) to $v_2$, so that they keep the pixel order defined in $B$. The following value, $v_5$, is selected because its intensity is larger than the rest. Finally, $v_6$ and $v_9$ have the same intensity, but $v_6$ is closer to $v_1$, as color, than for $v_9$, so that $v_6$ is the next in list ($v_6 \leq v_9$). Finally, the minimum value for this list of colors is $v_9 = (5,8,12)$, selected as the new value for the central pixel.

| Color     | Intensity (square) | Distance to Central (square) |
|-----------|--------------------|-----------------------------|
| $v_1$ = (5,10,11) | 8.66               | 0                           |
| $v_2$ = (9,5,11)  | 8.33               | 9                           |
| $v_3$ = (10,10,10) | 10.0              | 6                           |
| $v_4$ = (10,10,10) | 10.0              | 6                           |
| $v_5$ = (10,10,10) | 10.0              | 6                           |
| $v_6$ = (10,10,10) | 10.0              | 6                           |
| $v_7$ = (10,10,10) | 10.0              | 6                           |
| $v_8$ = (10,10,10) | 10.0              | 6                           |
| $v_9$ = (5,8,12)  | 8.33               | 3                           |

Figure 2. Comparing results to M-order using a synthetic image containing colored balls.
This new approach departs from the previous work in this area by moving from a total order on the RGB space to a total order on the RGB colors in the observed window. Because of this, the result of the erosion-like and dilation-like are expected to deviate less from the original pixel color, when more than one candidate (to minimum or maximum) is present, avoiding the use of ad-hoc lexicographic orders or selection of a global reference color.

To compare this approach and the previously ones, we applied basic operators: erosion and dilation based on lattice order (or M-order), erosion and dilatation based on lexicographic order, erosion-like and dilation-like operators, on a synthetic image containing colored balls. Figure 2 shows the comparison between M-order and our approach. In the figures a region of the same image was scaled in order to appreciate better the effects of the different operators. It can be shown that when applying M-order that false colors appear as it was expected unlike using the new operators.

Figure 3 shows the comparison between lexicographical order and our approach. The figure shows a region of the image in a larger scale in order to appreciate better the differences between the different operators. It can be shown that when applying lexicographic order, no false colors appear but the quality of the original image is not preserved. For erosion the border looks strongly pixelated, while for the dilation, the interior of the object looses its smooth coloring. Both of these issues don’t appear for our proposed approach.

Like with standard erosions and dilation, the new erosion-like and dilation-like operators, here defined, can be combined to generate more complex operators, like opening, closing, to-hats, etc. This property allows us to port previously designed algorithms, based on gray level images, to the context of color biomedical images, as will be described in the next section.
A. Detection of Exudates

As an example of application of the new morphological operators to previous biomedical image processing problems, we applied them to the detection of exudates in ocular images, which was done in a previous work using the M-order [17]. Hard and soft exudates are the main signs of diabetic macular edema (DME). The segmentation of both kinds of exudates generates valuable information not only for the diagnosis of DME, but also for treatment, which helps to avoid vision loss and blindness. Here we present the results obtained by applying the same method in Pastore et al. [17], but using erosion-like and dilation-like operators for the definition of opening-like and closings-like operators. The images were obtained from the Diabetic Retinopathy Database and Evaluation Protocol [18]. The algorithm presented can be summarized in the following three stages:

Step 1: We apply successive morphological opening-like with growing structuring elements (SE). This removes the objects that are smaller than the initial size of the SE. When this operator is applied again, increasing the size of the SE, the objects remained in the previous stage are removed. This process continues until only an average color background stays in the image.

Step 2: In order to highlight the bright objects, we subtract the image obtained in the previous stage to the original image.

Step 3: We binarize the resulting image using the Otsu's algorithm to segment the most relevant components [19].

Figure 4-a shows that, in retinopathy images, the exudates have a light color with respect to the background. Because of this feature, when applying the morphological opening-like, defined for color images, it removes the exudates that are smaller than the initial size SE (figure 4-b). If this operator is applied again, increasing the size of SE, the exudates remaining in the previous stages are removed. When, continuing this process $n$ times, only the background of the image is retained (figure 4-c). To highlight the exudates, we subtract the image obtained in the previous stage from the original image (figure 4-d). Since the RGB image has the relevant information in the green component, this component is binarized using Otsu's method (figure 4-e). Finally, to visualize the results, a Canny Filter [20] is applied to the resulting image (figure 4-f), and the result is superimposed to the original image (figure 4-g).

Figure 4. Steps of the proposed method. (a) Original image, (b) initial opening, (c) successive openings of the original image, (d) subtraction between the original image and the image (c), (e) binarization of the green component, (f) Canny Filter, (g) resulting image overlay to the original image.
Figure 5 shows the results obtained for three processed images of the Diabetic Retinopathy Database and Evaluation Protocol [18]. The first column shows the original images, the second column shows the segmentations performed by the proposed method, the third column shows the segmentation overlaid to the original image in order to assess the results.

In this example we can see how more complex operators, like openings-like operators, can be generated from the erosion-like and dilation-like operators proposed here, producing similar results than with previously defined morphological operators.

B. Segmentation of Bone Marrow Biopsies

Another example of porting a previously algorithm is the identification of trabecular structures, as done previously in [21]. The anatomic and pathologic reports of the histological cuts given results expressed in percentages, indicating the presence of the trabeculae, hematopoietic and fatty cells. This allows evaluating the existence or grade of some metabolic disorder, comparing normal values with pathologic ones [22]. To segment dark objects (trabecular tissue), we applied the same method than in the previous section, but using successive closing-like operators.

Figure 6 shows the results obtained in three bone marrow biopsies images. The first column shows the original image, the second column shows the segmentation obtained with the new color operators, and the third column shows the two of them together for a better visualization. The results of this application of like-morphological operators are similar to the ones obtained previously in [21].
Figure 6. Results obtained by the proposed method. First column, original images. Second column, segmented images. Last column, segmented images overlaid to the original image.

4. Conclusions
In this work we presented new color morphological-like operators, named erosion-like and dilation-like operators, which can also be combined to generate more complex operators. To visualize the differences between this new approach and the previously proposed ones, we applied the basic operators: erosion and dilation based on lattice order (or M-order), erosion and dilatation based on lexicographic order, erosion-like and dilation-like operators on synthetic images. The new like-operators do not introduce false colors preserving the quality of the original image. Both of these issues are not simultaneously obtained with other approaches. Finally, as an example of its use, we show how more complex operators can be used on biomedical image processing, via operators similar to morphological opening, closing and top-hat. The new operators, part of the family of the morphological w-operators, outperform previously proposed color morphology approaches, regarding quality of the resulting image, and can be used to define more complex operators.

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