Nuclear symmetry energy and proton-rich reactions at intermediate energies

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Based on an isospin dependent transport model IBUU, effects of high density behavior of nuclear symmetry energy in the proton-rich reaction $^{22}$Si+$^{22}$Si at a beam energy of 400 MeV/nucleon are studied. It is found that the symmetry energy affects $\pi^+$ production more than $\pi^-$. More interestingly, comparing with neutron-rich reactions, for $\pi^-/\pi^+$ and dense matter's N/Z ratios, effects of symmetry energy in the proton-rich reaction both show contrary behaviors. The practical experiment by using the proton-rich reaction $^{22}$Si+$^{40}$Ca to study nuclear symmetry energy are also provided.

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Recently, pion production in heavy-ion collisions has attracted much attention in nuclear physics community [1][2]. One important reason is that pion production is connected with the high density behavior of nuclear symmetry energy [3][4]. The latter is crucial for understanding many interesting issues in both nuclear physics and astrophysics [7][8]. The high density behavior of nuclear symmetry energy, however, has been regarded as the most uncertain property of dense neutron-rich nuclear matter [14][15]. Many microscopic and/or phenomenological many-body theories using various interactions [16][17] predict that the symmetry energy increases continuously at all densities. On the other hand, other models [3][18][30] predict that the symmetry energy first increases to a maximum and then may start decreasing at certain supra-saturation densities. Thus, currently the theoretical predictions on the symmetry energy at supra-saturation densities are extremely diverse. To make further progress in determining the symmetry energy at supra-saturation densities, what is most critically needed is some guidance from dialogues between experiments and transport models, which have been done extensively in the studies of nuclear symmetry energy at low densities [31][32].

Using $\pi^-/\pi^+$ to probe the high density behavior of nuclear symmetry energy has evident advantage within both the $\Delta$ resonance model and the statistical model [30][35]. And several hadronic transport models have quantitatively shown that $\pi^-/\pi^+$ ratio is indeed sensitive to the symmetry energy [5][6][38][50], especially around pion production threshold. These transport models, however, usually simulate neutron-rich reactions to study the effect of symmetry energy, few proton-rich reactions on this subject were reported. Here we study the effects of symmetry energy on pion production, as well as the value of $\pi^-/\pi^+$ in the proton-rich collision because the National Superconducting Cyclotron Laboratory at Michigan State University, Rikagaku Kenkyusho (RIKEN, The Institute of Physical and Chemical Research) of Japan, and the Cooler Storage Ring in Lanzhou, China, are planning to do experiments of pion production to study the high density behavior of nuclear symmetry energy. In the framework of an Isospin-dependent Boltzmann-Uehling-Uhlenbeck (IBUU) transport model, as an example, we studied the effects of symmetry energy on $\pi^-/\pi^+$ in the proton-rich reaction $^{22}$Si+$^{22}$Si at a beam energy of 400 MeV/nucleon. It is found that the symmetry energy affects the value of $\pi^-/\pi^+$ and dense matter's N/Z ratios. As one expected, comparing with neutron-rich reactions, effect of symmetry energy in the proton-rich reaction shows contrary behaviors.

The isospin and momentum-dependent mean field potential used in the present work is [40][41]

$$U(\rho, \delta, \mathbf{p}, \tau) = A_{0}(x) \frac{\rho_{\tau}}{\rho_{0}} + A_{1}(x) \frac{\rho_{\tau}}{\rho_{0}} + B \left( \frac{\rho}{\rho_{0}} \right)^{\alpha} \left( 1 - x \delta^{2} \right) - 8x\tau \frac{B}{\sigma + 1} \rho^{\sigma-1} \delta \rho_{\tau} \left( \frac{2}{\rho_{0}} \right)^{4} \frac{\int d^{3} \mathbf{p} \prime \frac{f_{\tau}(\mathbf{r}, \mathbf{p} \prime)}{1 + (\mathbf{p} - \mathbf{p} \prime)^{2}/\Lambda^{2}}}{\int d^{3} \mathbf{p} \prime} \right), \quad (1)$$

where $\rho_{n}$ and $\rho_{p}$ denote neutron ($\tau = 1/2$) and proton ($\tau = -1/2$) densities, respectively. $\delta = (\rho_{n} - \rho_{p})/(\rho_{n} + \rho_{p})$ is the isospin asymmetry of nuclear medium. All parameters in the above equation can be found in refs. [41]. The variable $x$ is introduced to mimic different forms of the symmetry energy predicted by various many-body theories without changing any property of symmetric nuclear matter and the value of symmetry energy at normal density $\rho_{0}$. In this article we let the variable $x$ be 1. With these choices the symmetry energy obtained from the above single particle potential is consistent with the Hartree-Fock prediction using the original Gogny force [40] and is also favored by recent studies based on FOPI experimental data [1]. To study the effect of symmetry energy, we also select the stiff symmetry energy parameter $x = 0$ [32] as reference. The main reaction channels related to pion production and absorption are

$$\begin{align*}
NN & \rightarrow NN \\
NR & \rightarrow NR \\
NN & \leftrightarrow NR \\
R & \leftrightarrow N\pi,
\end{align*} \quad (2)$$

where $R$ denotes $\Delta$ or $N^{\ast}$ resonances. In the present work, we use the isospin-dependent in-medium reduced
NN elastic scattering cross section from the scaling model according to nucleon effective mass [42–45] to study the effect of symmetry energy on pion production. Assuming in-medium NN scattering transition matrix is the same as that in vacuum [42], the elastic NN scattering cross section in medium $\sigma_{NN}^{\text{medium}}$ is reduced compared with their free-space value $\sigma_{NN}^{\text{free}}$ by a factor of

$$R_{\text{medium}}(\rho, \delta, p) = \frac{\sigma_{NN}^{\text{medium}}}{\sigma_{NN}^{\text{free}}} = \frac{\mu_{NN}^*}{\mu_{NN}}^2.$$ (3)

where $\mu_{NN}$ and $\mu_{NN}^*$ are the reduced masses of the colliding nucleon pair in free space and medium, respectively. For in-medium NN inelastic scattering cross section, even assuming in-medium NN $\to$ NR scattering transition matrix is the same as that in vacuum, the density of final states $D_{NR}$ of NR is very hard to calculate due to the fact that the resonance’s potential in matter is presently unknown. The in-medium NN inelastic scattering cross section is thus quite controversial [16–50]. Because the purpose of present work is just study the effect of symmetry energy on pion production and charged pion ratio, to simplify the question, for the NN inelastic scattering cross section we use the free NN inelastic scattering cross section. The effective mass of nucleon in isospin asymmetric nuclear matter is

$$\frac{m^*}{m_{\pi}} = \left(1 + \frac{m_{\pi}}{p} \frac{dU_{\pi}}{dp} \right)^{-1}. $$ (4)

From the definition and Eq. (1), we can see that the effective mass depends not only on density and asymmetry of medium but also the momentum of nucleon.

Fig. 1 shows density distributions of protons and neutrons of the nucleus $^{22}\text{Si}$.

![Fig. 1: (Color online) Density distributions of protons and neutrons of the nucleus $^{22}\text{Si}$](image)

Fig. 2 shows central baryon density (upper window) and isospin asymmetry $\frac{N}{Z}$ of high density region (lower window) for the reaction $^{22}\text{Si} + ^{22}\text{Si}$ at a beam energy of 400 MeV/nucleon and an impact parameter of 0 fm.

![Fig. 2: (Color online) Central baryon density (upper window) and isospin asymmetry $\frac{N}{Z}$ of high density region (lower window) for the reaction $^{22}\text{Si} + ^{22}\text{Si}$ at a beam energy of 400 MeV/nucleon and an impact parameter of 0 fm.](image)

Fig. 2 shows the central baryon density (upper window) and the average $(N/Z)_{\rho > \rho_0}$ ratio (lower window) of all regions with baryon densities higher than $\rho_0$. It is seen that the maximum baryon density is about 2 times normal nuclear matter density. Moreover, the compression is rather insensitive to the symmetry energy because the latter is relatively small compared to the EOS of symmetric matter around this density. The high density phase lasts for about 11 fm/c from 2 to 13 fm/c for this reaction. It is interesting to see that the isospin asymmetry of the high density region is quite sensitive to the symmetry energy. The soft symmetry energy ($x = 1$) leads to a significantly lower value of $(N/Z)_{\rho > \rho_0}$ than the stiff one ($x = 0$). This is consistent with the well-known isospin fractionation phenomenon. Because of the $E_{\text{sym}}(\rho)\delta^2$ term in the EOS of asymmetric nuclear matter, it is energetically more favorable to have a larger isospin asymmetry $\delta$ in the high density region with a softer symmetry energy functional $E_{\text{sym}}(\rho)$. In the supernormal density region, as shown in Fig. 1 of reference [54], the symmetry energy changes from being soft to stiff when the param-
The contribution due to are produced through the decay of $\Delta(1232)$ resonances. threshold in nucleon-nucleon scatterings, almost all pions which is just about 100 MeV above the pion production states of the $\Delta(1232)$. Because the value of \(\frac{N}{Z}\) comes stable after 30 fm/c (can be seen from Fig. 4), the multiplicity of $\Delta(1232)$ resonances shown in the figure includes all four charge states while in the model we do treat and follow separately different charge states of the $\Delta(1232)$. Because the value of $\pi^-/\pi^+$ becomes stable after 30 fm/c (can be seen from Fig. 3), here we just give evolutions of $\Delta(1232)$ and charged pions till 30 fm/c. At a beam energy of 400 MeV/nucleon which is just about 100 MeV above the pion production threshold in nucleon-nucleon scatterings, almost all pions are produced through the decay of $\Delta(1232)$ resonances. The contribution due to $N^*$ resonances is negligible. It is interesting to see that the $\pi^+$ multiplicity depends more sensitively on the symmetry energy. This is because the $\pi^+$ mesons are mostly produced from proton-proton collisions, where asymmetry is always larger in the reaction induced by the proton-rich nuclei $^{22}\text{Si}+^{22}\text{Si}$. We can also see more $\pi^+$ than $\pi^-$ mesons are produced. Our finding that in proton-rich reactions $\pi^+$ mesons are more sensitive to the symmetry energy than $\pi^-$ contradicts the results of Ref. [54].

To reduce the systematic errors in simulations, especially in experimental analysis, one usually studies the $\pi^-/\pi^+$ instead of $\pi^-$ or $\pi^+$ only. Shown in Fig. 4 is effect of symmetry energy on the $(\pi^-/\pi^+)$ as a function of time in the central reaction $^{22}\text{Si}+^{22}\text{Si}$ at a beam energy of 400 MeV/nucleon. In the dynamics of pion resonance productions and decays the $(\pi^-/\pi^+)$ reads [54]

$$
(\pi^-/\pi^+)_\text{like} = \frac{\pi^- + \Delta^- + \frac{1}{3}\Delta^0}{\pi^+ + \Delta^+ + \frac{1}{3}\Delta^+}.
$$

This ratio naturally becomes $\pi^-/\pi^+$ ratio at the freeze-out stage [54]. From Fig. 4 we can see that sensitivity of $(\pi^-/\pi^+)_\text{like}$ to the effect of symmetry energy is clearly shown after $t = 10\,\text{fm/c}$. With the stiff symmetry ($x = 0$) the value of $\pi^-/\pi^+$ is higher than that with the soft symmetry ($x = 1$), this is understandable within the statistical model for pion production [54].
Shown in Fig. 6 is the differential $\pi^-/\pi^+$ ratios versus the kinetic energy. In the low energy ($E_{\text{kin}} \sim 45$ MeV) region, around the Coulomb peak the $\pi^-/\pi^+$ ratio is clearly separable with the $x$ parameter varying from 1 to 0. Sensitivity of $\pi^-/\pi^+$ ratio to the symmetry energy around the Coulomb peak is about 20%.

In the practical experiments, the proton-rich reaction $^{22}\text{Si}+^{40}\text{Ca}$ may be difficult to carry out. The only purpose of choosing this reaction is that we just want to show our studies more clearly. In the practical experimental plan, a reaction with proton-rich nucleus ($Z > N$) and stable nucleus ($Z \sim N$) (such as $^{22}\text{Si}+^{40}\text{Ca}$) is feasible. Shown in Fig. 6 is the effect of symmetry energy on the production and the value of $\pi^-/\pi^+$. As one expected, comparing with neutron-rich reactions, effect of symmetry energy in the proton-rich reaction shows contrary behavior. Studying proton-rich reactions can not only help us to probe the symmetry energy, but also check the theories about nuclear matter.

In conclusion, based on an isospin dependent transport model IBUU, effects of high density behavior of nuclear symmetry energy on $\pi^-/\pi^+$ in the proton-rich reactions $^{22}\text{Si}+^{22}\text{Si}$, $^{22}\text{Si}+^{40}\text{Ca}$ at a beam energy of 400 MeV/nucleon are studied. It is found that the symmetry energy evidently affects $\pi^+$ production and the value of $\pi^-/\pi^+$.

The $\pi^-/\pi^+$ ratio is approximately 1, roughly equal to $(5N^2 + N/Z)/(5Z^2 + NZ) \sim (N/Z)^2 = (44/44)^2 = 1$ in central heavy-ion reactions, with $N$ and $Z$ being the total neutron and proton numbers in the participant region.

FIG. 6: (Color online) Evolution of the $\pi^-/\pi^+$ ratio in the reaction $^{22}\text{Si}+^{40}\text{Ca}$ at a beam energy of 400 MeV/nucleon and an impact parameter of 0 fm.

FIG. 7: (Color online) Evolution of the $\pi^-/\pi^+$ ratio in the reaction $^{22}\text{Na}+^{22}\text{Na}$ at a beam energy of 400 MeV/nucleon and an impact parameter of 0 fm.

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