Fingerprints of a Bosonic Symmetry-Protected Topological State in a Quantum Point Contact
Rui-Xing Zhang and Chao-Xing Liu
Phys. Rev. Lett. 118, 216803 — Published 26 May 2017
DOI: 10.1103/PhysRevLett.118.216803
Fingerprints of bosonic symmetry protected topological state in a quantum point contact

Rui-Xing Zhang and Chao-Xing Liu

1 Department of Physics, The Pennsylvania State University, University Park, Pennsylvania 16802

In this work, we study the transport through a quantum point contact for bosonic helical liquid that exists at the edge of a bilayer graphene under a strong magnetic field. We identify “smoking gun” transport signatures to distinguish bosonic symmetry protected topological (BSPT) state from fermionic two-channel quantum spin Hall (QSH) state in this system. In particular, a novel charge insulator/spin conductor phase is found for BSPT state, while either charge insulator/spin insulator or charge conductor/spin conductor phase is expected for the two-channel QSH state. Consequently, a simple transport measurement will reveal the fingerprint of bosonic topological physics in bilayer graphene systems.

PACS numbers: 71.10.Pm, 72.15.Nj, 85.75.-d, 72.80.Vp

Introduction - Ever since the discovery of topological insulators (TIs) [1–4], intensive research has been focused on understanding the role of symmetry in protecting new topological states, which are known as “symmetry protected topological (SPT) states” [5, 6]. A grand challenge in this field is to understand the role of interaction in SPT states and to realize interacting SPT states in realistic materials. Recently, it was theoretically proposed that interaction has a dramatic effect on topological properties of bilayer graphene under a tilted magnetic field [7]. The strong magnetic field guarantees the spin conservation, and drives the system into a quantum spin Hall (QSH) state with edge states described by fermionic two-particle interaction has a dramatic effect on topological properties of bilayer graphene and identify key signatures to distinguish BSPT state from fermionic QSH state.

The aim of this work is to explore transport properties of bosonic helical liquid of BSPT state in bilayer graphene and identify key signatures to distinguish BSPT state from fermionic QSH state. First of all, the bosonic charge-2e edge excitation of BSPT state carries electric currents and a two-terminal measurement will also reveal $\frac{4e^2}{h}$ conductance, taking into account two edges in a realistic sample. Thus, the two-terminal transport measurements [8] cannot distinguish the BSPT state from QSH state in bilayer graphene. Several possible experimental probes, such as shot noise measurement of 2e-charge, have been considered in Ref. [7]. However, such noise measurement is experimentally challenging and sometimes controversial, and a simple transport detection of BSPT state is desirable.

In this work, we study a quantum point contact (QPC) between two edges of bilayer graphene under a tilted magnetic field, as shown in Fig. 1. With the help of this QPC setup, fingerprints of BSPT state are clearly revealed in the phase diagram of inter-edge tunneling physics. Based on realistic interaction in bilayer graphene, our main results show (1) a novel charge insulator/spin conductor phase [21, 22], labelled as IC phase [22], when BSPT state is formed, and (2) in contrast, either charge conductor/spin conductor or charge insulator/spin insulator phase, labelled as CC/II phase, for the fermionic two-channel QSH state, where BSPT state is not formed. Thanks to the unique transport properties in IC phase, we propose simple two-terminal conductance measurements in both vertical and horizontal directions in the bilayer graphene QPC. Perfect insulating behaviors in both directions will be the “smoking gun” signal for BSPT physics, unambiguously distinguishing BSPT state from fermionic QSH state.

Model Hamiltonian - We consider a bilayer graphene sample in a four-terminal configuration as shown in Fig. 1. Both in-plane magnetic field (B∥) and out-of-plane magnetic field (B⊥) are required to drive the system into the QSH regime with two-channel helical Luttinger liquid on the boundary [8, 23]. A strong asymmetric potential (VA) induced by a gate voltage can drive the system into a layer polarized insulating phase with a trivial gap [24, 26]. As a result, we can locally gate the sample and nontrivial edge modes exist at the interface between unbiased region (blue region) and biased region (orange region), as shown

The aim of this work is to explore transport properties of bosonic helical liquid of BSPT state in bilayer graphene and identify key signatures to distinguish BSPT state from fermionic QSH state. First of all, the bosonic charge-2e edge excitation of BSPT state carries electric currents and a two-terminal measurement will also reveal $\frac{4e^2}{h}$ conductance, taking into account two edges in a realistic sample. Thus, the two-terminal transport measurements [8] cannot distinguish the BSPT state from QSH state in bilayer graphene. Several possible experimental probes, such as shot noise measurement of 2e-charge, have been considered in Ref. [7]. However, such noise measurement is experimentally challenging and sometimes controversial, and a simple transport detection of BSPT state is desirable.

In this work, we study a quantum point contact (QPC) between two edges of bilayer graphene under a tilted magnetic field, as shown in Fig. 1. With the help of this QPC setup, fingerprints of BSPT state are clearly revealed in the phase diagram of inter-edge tunneling physics. Based on realistic interaction in bilayer graphene, our main results show (1) a novel charge insulator/spin conductor phase [21, 22], labelled as IC phase [22], when BSPT state is formed, and (2) in contrast, either charge conductor/spin conductor or charge insulator/spin insulator phase, labelled as CC/II phase, for the fermionic two-channel QSH state, where BSPT state is not formed. Thanks to the unique transport properties in IC phase, we propose simple two-terminal conductance measurements in both vertical and horizontal directions in the bilayer graphene QPC. Perfect insulating behaviors in both directions will be the “smoking gun” signal for BSPT physics, unambiguously distinguishing BSPT state from fermionic QSH state.

Model Hamiltonian - We consider a bilayer graphene sample in a four-terminal configuration as shown in Fig. 1. Both in-plane magnetic field (B∥) and out-of-plane magnetic field (B⊥) are required to drive the system into the QSH regime with two-channel helical Luttinger liquid on the boundary [8, 23]. A strong asymmetric potential (V A) induced by a gate voltage can drive the system into a layer polarized insulating phase with a trivial gap [24, 26]. As a result, we can locally gate the sample and nontrivial edge modes exist at the interface between unbiased region (blue region) and biased region (orange region), as shown

The aim of this work is to explore transport properties of bosonic helical liquid of BSPT state in bilayer graphene and identify key signatures to distinguish BSPT state from fermionic QSH state. First of all, the bosonic charge-2e edge excitation of BSPT state carries electric currents and a two-terminal measurement will also reveal $\frac{4e^2}{h}$ conductance, taking into account two edges in a realistic sample. Thus, the two-terminal transport measurements [8] cannot distinguish the BSPT state from QSH state in bilayer graphene. Several possible experimental probes, such as shot noise measurement of 2e-charge, have been considered in Ref. [7]. However, such noise measurement is experimentally challenging and sometimes controversial, and a simple transport detection of BSPT state is desirable.

In this work, we study a quantum point contact (QPC) between two edges of bilayer graphene under a tilted magnetic field, as shown in Fig. 1. With the help of this QPC setup, fingerprints of BSPT state are clearly revealed in the phase diagram of inter-edge tunneling physics. Based on realistic interaction in bilayer graphene, our main results show (1) a novel charge insulator/spin conductor phase [21, 22], labelled as IC phase [22], when BSPT state is formed, and (2) in contrast, either charge conductor/spin conductor or charge insulator/spin insulator phase, labelled as CC/II phase, for the fermionic two-channel QSH state, where BSPT state is not formed. Thanks to the unique transport properties in IC phase, we propose simple two-terminal conductance measurements in both vertical and horizontal directions in the bilayer graphene QPC. Perfect insulating behaviors in both directions will be the “smoking gun” signal for BSPT physics, unambiguously distinguishing BSPT state from fermionic QSH state.

Model Hamiltonian - We consider a bilayer graphene sample in a four-terminal configuration as shown in Fig. 1. Both in-plane magnetic field (B∥) and out-of-plane magnetic field (B⊥) are required to drive the system into the QSH regime with two-channel helical Luttinger liquid on the boundary [8, 23]. A strong asymmetric potential (V A) induced by a gate voltage can drive the system into a layer polarized insulating phase with a trivial gap [24, 26]. As a result, we can locally gate the sample and nontrivial edge modes exist at the interface between unbiased region (blue region) and biased region (orange region), as shown
in Fig. 1. The local gates can be designed to form a QPC configuration in this device and the tunneling between two edges only occurs at the QPC.

As justified in the supplementary materials [27], helical edge modes can exist in both edges and are labeled by the fermionic operators $\psi_{i,L}$ that are connected to the lead $i \in \{1, 2, 3, 4\}$ and characterized by a channel index $l \in \{I, II\}$ and a direction index $\lambda \in \{\text{in}, \text{out}\}$. Abelian bosonization technique is applied and the corresponding bosonic chiral fields $\chi_{i,l,\lambda}$ are defined as $\psi_{i,L} = \frac{F_{i,l,\lambda}}{\sqrt{2\pi a_0}} e^{i f(\lambda) \phi_{i,l,\lambda}}$, with the Klein factor $F_{i,l,\lambda}$, the coefficient $f(\lambda) = +1(-1)$ for a right (left) mover and the short-distance cut-off $a_0$. Let us define the edge that connects the leads 1 (3) and 2 (4) as the top (bottom) edge and the bosonic chiral fields on each edge are related to the $\chi_{i,l,\lambda}$ field by

\[
\chi(t,b,l,R) = \chi(1,4)_{l,\text{out}}(x)\Theta(-x) - \chi(2,3)_{l,\text{in}}(x)\Theta(x) \\
\chi(t,b,l,L) = \chi(1,4)_{l,\text{in}}(-x)\Theta(x) - \chi(2,3)_{l,\text{out}}(x)\Theta(x),
\]

(1)

with step function $\Theta(x)$. Here the $+x$ direction is directed along the edge from lead 1 (4) to lead 2 (3). The dual boson fields are introduced as $\phi_{t,b,l} = \chi(t,b,l,R) + \chi(t,b,l,L)$ and $\theta_{t,b,l} = -\chi(t,b,l,R) + \chi(t,b,l,L)$. Together with the unharmonic terms that respect both $U(1)_c$ and $U(1)_s$ symmetries, the full Hamiltonian is given by

\[
\mathcal{H} = \sum_{s \in \{t,b\}} \sum_{l=\pm} \frac{v_l}{2} [K_l (\partial_x \phi_{s,l})^2 + \frac{1}{K_l} (\partial_x \theta_{s,l})^2]
\]

\[+ g_1 \sum_s \cos 2\sqrt{2}\pi \phi_{s,+} + g_2 \sum_s \cos 2\sqrt{2}\pi \phi_{s,-} \tag{2}\]

where $\phi_{s,\pm} = \frac{1}{\sqrt{2}}(\phi_{s,l} \pm \phi_{s,\text{II}})$ and $\theta_{s,\pm} = \frac{1}{\sqrt{2}}(\theta_{s,l} \pm \theta_{s,\text{II}})$ are bonding and anti-bonding fields, respectively. When $g_1 = g_2 = 0$, this Hamiltonian describes the low-energy edge physics of QSH state with a spin Chern number $2$. Here $K_+ = \sqrt{\frac{2v_t + 2s + g_1}{2v_b + 2s + g_2}}$, and it is expected that $K_+ > 1$. An explicit definition of $g_3$ and $g_4$ can be found in the supplementary materials [27]. A non-zero $g_1$ term is relevant, which will freeze the $\phi_{s,-}$ field as $\phi_{s,-} = \frac{(2n_s + 1)\pi}{2\sqrt{2}}$ with $n_s \in \mathbb{Z}$, and gap out the anti-bonding boson modes. The pinning of $\phi_{s,-}$ field is dubbed BSPT condition, which mathematically distinguishes bosonic helical liquid from two-channel helical Luttinger liquid. We further introduce the notation of spin-charge basis as

\[
\phi_\sigma = \phi_{+,+}, \quad \phi_\sigma = \theta_{-,+}, \quad \phi_\sigma = \theta_{++,+}, \quad \phi_\sigma = \phi_{-,+}. \tag{3}\]

with $\phi_{s,\pm,\pm} = (\phi_{s,l} \pm \phi_{s,\text{II}})/\sqrt{2}$ and $\theta_{s,\pm,\pm} = (\theta_{s,l} \pm \theta_{s,\text{II}})/\sqrt{2}$. The corresponding Hamiltonian is

\[
\mathcal{H}_{\text{BSPT}} = \sum_{r,s=\pm} \frac{v_r}{2} [K_+ (\partial_x \phi_r)^2 + \frac{1}{K_+} (\partial_x \theta_r)^2], \tag{4}\]

Therefore, the remaining free bosonic bonding fields $\phi_{s,+}$ and $\theta_{s,+}$ form helical bosonic edge modes carrying spin-1 and charge-2e.

**Tunneling physics and Phase diagram -** For QPC structure, tunneling process is expected to take place at the contact point $x = 0$. Inter-edge tunnelling for a QSH state are only constrained by the symmetries of the system. In a BSPT QPC setup, however, tunneling terms are additionally constrained by the BSPT condition defined above. We will show that this requirement not only constrains the explicit form of tunneling process, but also modifies the scaling dimension of tunneling operators and greatly changes the phase diagram of tunneling process.

Let us start with the single-particle tunneling, and $U(1)_s$ symmetry requires that an electron must switch its velocity when hopping between different edges. Generally, the single-particle tunneling operator is

\[
T_{l,l'} = t_{l,l'} \psi_{l,l,L}^\dagger \psi_{b,l',R} + h.c. \tag{5}\]

In the bosonized language, $T_{l,l'} = t_{l,l'} \cos \sqrt{2}\pi [\phi_{+,+} + \theta_{-,+} - f_+(\phi_{-,+} + \theta_{-,+})] - f_-(\phi_{-,+} + \theta_{-,+}),$ where $f_\pm = \frac{1}{2}[(1)^l \pm (-1)^l]$. BSPT condition guarantees that the correlation function of its dual fields $(\theta_{-,+}(\tau)\theta_{-,+}(0))$ diverges as $g_1 \rightarrow \infty$. As a result, the correlation function of any vertex operator of $\theta_{-,+}$ vanishes under RG operation. Since $\theta_{-,+}$ always appears in $T_{l,l'}$, we conclude that single particle tunneling $T_{l,l'}$ is generally forbidden in the BSPT QPC. Physically, this implies that single-particle tunneling is in-compatible with the BSPT condition, and violates the bosonic nature of BSPT state.

Next, we examine the two-particle tunneling shown in Fig. 2(a), where a right mover on the top edge (spin-up) tunnels to a left mover on the bottom edge (spin-down), and a right mover on the bottom edge (spin-down) simultaneously tunnels to a left mover on the top edge (spin-up). As a result, the charge transfer between the top and bottom edges is zero, while the spin transfer is one. This type of spin-1 tunneling process is mathematically
described by
\[ V^\sigma = v^\sigma_{l_1,l_2,l_3,l_4} \psi^\dagger_{b,L,l_1} \psi_{l_1,R,l_2} \psi^\dagger_{l_3,L,l_4} \psi_{b,R,l_4} + h.c., \] (6)
where \( l_1, l_2, l_3, l_4 \in I, II \). Under BSPT condition, the absence of anti-bonding field \( \theta_{s,-} \) in \( V^\sigma \) yields a strong constraint on the channel index \( l_i \): \( l_1 = l_4 = l, \ l_2 = l_3 = l' \), which leads to
\[ V^\sigma = v^\sigma \cos 2\sqrt{\pi} \phi_{s,+} = v^\sigma \cos 2\sqrt{\pi} \phi_{s}. \] (7)
There exists another type of symmetry allowed two-particle tunneling term, which describes inter-edge transfer of 2e charge and zero spin, as shown in Fig. 2(b):
\[ V^\rho = v^\rho_{l_1,l_2,l_3,l_4} \psi^\dagger_{b,L,l_1} \psi_{l_1,R,l_2} \psi^\dagger_{b,R,l_3} \psi_{l_3,L,l_4} + h.c. \] (8)
The condition for a non-vanishing \( V^\rho \) can be similarly identified as \( l_1 \neq l_4, \ l_2 \neq l_3 \), leading to the bosonized expression of charge-2e tunneling as
\[ V^\rho = v^\rho \cos 2\sqrt{\pi} \phi_{s,-} = v^\rho \cos 2\sqrt{\pi} \phi_{s}. \] (9)
As shown in Ref. [2], the elementary bosonic excitations on the edge \( s \) are found to be either charge-2e spin-singlet Cooper pair \( \Phi_{s,q=2e} = \psi_{s,I,R} \psi_{s,I,L} - \psi_{s,I,L} \psi_{s,I,R} \sim e^{-i\sqrt{2\pi} \phi_{s,-}} \) or spin-1 chargeless spinon \( \Phi_{s,\sigma=1} = \psi_{s,I,L} + \psi_{s,I,R} \sim e^{-i(\pi \sigma) \sqrt{2\pi} \phi_{s,-}} \). For the definition of bosonic operator \( \Phi_{s,\sigma=1} \), we have used the convention \((-1)^l = -1 \) and \((-1)^b = 1 \), which originates from opposite spin-momentum locking at different edges. The above two-particle tunneling terms can be rewritten as,
\[ V^\sigma = v^\sigma \Phi^\dagger_{b,\sigma=2e} \Phi_{b,\sigma=2e} + h.c. \]
\[ V^\rho = v^\rho \Phi^\dagger_{b,q=2e} \Phi_{b,q=2e} + h.c. \] (10)
Therefore, two-particle tunneling \( V^\sigma \) and \( V^\rho \) are physically interpreted as the tunneling of bosonic quasiparticles across the QPC, as shown in Fig. 2(c) and (d). In other words, Eq. (10) demonstrates the minimal tunneling events allowed in a bosonic SPT system.

Now we are ready to analyze and compare the phase diagram of tunneling physics for bilayer graphene QPC structure with and without the formation of BSPT state. In a series of pioneering works, the QPC physics of fermionic 1-channel helical Luttinger liquid and fermionic 4-channel helical Luttinger liquid have been studied in a QSH system [20, 21, 28] and a bilayer graphene with domain walls [29]. The phase diagram of our bilayer graphene QSH state follows the paradigm in the above systems: (1) In the weak interaction limit, both single-particle and two-particle tunneling terms are small and irrelevant, which defines CC phase. However, a duality transformation of CC phase reveals another stable fixed point where the QPC is pinched off, giving rise to the so-called II phase [21]. Therefore, CC and II fixed points are separated by a QPC pinch-off transition in this parameter regime. (2) As the repulsive (attractive) interaction strengths exceed critical values, QPC is driven into the IC (I or charge conductor/spin insulator) phase where spin-1 (charge-2e) tunneling is relevant. We have mapped out the phase diagram of fermionic two-channel QSH state in QPC setup of bilayer graphene, as shown in Fig. 3(a). More details can be found in the supplementary materials [27].

When bulk BSPT state is formed, however, BSPT condition freezes the anti-bonding degree of freedom and removes the role of \( K^- \) in the phase diagram. Scaling dimensions of two-particle tunneling terms are further modified to \( \Delta(v^\sigma) = \frac{1}{K_+} \) and \( \Delta(v^\rho) = K_+ \), in comparison to the QSH case [27]. This change of scaling dimensions leads to different RG equations
\[ \frac{dv^\sigma}{da} = (1 - \frac{1}{K_+})v^\sigma, \ \ \ \frac{dv^\rho}{da} = (1 - K_+)v^\rho, \] (11)
with real space scaling factor \( a \) for \( v^\sigma,v^\rho \). For \( K_+ > 1 \), we find \( v^\sigma \) is relevant while \( v^\rho \) is irrelevant, leading to the IC phase. In contrast, the CI phase appears for \( K_+ < 1 \) and is separated from the IC phase by a critical point at \( K_+ = 1 \), as shown in Fig. 3(b). Comparing Fig. 3(a) and (b), we find two phase diagrams are completely different in the weak interaction limit \( K_+ \approx 1 \), thus providing a route to distinguish BSPT state and fermionic two-channel QSH state in bilayer graphene.

Experimental detection - Based on the phase diagram (Fig. 3(a) and (b)), we next turn to realistic bilayer graphene systems. First, we need to give an estimate of the Luttinger parameters \( K_{\pm} \), which can be extracted from the screened Coulomb interaction between two edge state electrons. As discussed in the supplementary materials [27], after mapping the screened Coulomb interaction into the four-fermion interactions in Luttinger liquids, we find that \( K_+ \) is determined by the ratio between interaction strength and kinetic energy of the edge modes, while \( K^- \) is related to the difference between intra- and inter-Landau level interactions. Assuming the out-of-plane magnetic field to be 2 Tesla and a substrate
configurations of the proposed two-terminal measurement are shown in (a) and (b). Voltage configurations of the proposed two-terminal measurement are shown in (c) and (d). Temperature dependence of conductance for QSH state in (e) and BSPT state in (f).
by gate voltages, as shown in Fig. 1, which is absent in other QSH systems. In the supplementary materials [27], a detailed calculation of extracting effective charge from shot noise spectrum is also presented. Bosonic 2e-charge is found, which originates from the instanton tunneling events of IC fixed point. Compared with this direct probe of bosonic electric charge, the transport measurements we proposed are much simpler and more feasible for experiment realization.

Acknowledgement We would like to thank Cenke Xu for useful discussions. C.-X.L. acknowledge the support from Office of Naval Research (Grant No. N00014-15-1-2675).

[1] L. Fu, C. L. Kane, and E. J. Mele, Physical Review Letters 98, 106803 (2007).
[2] H. Zhang, C.-X. Liu, X.-L. Qi, X. Dai, Z. Fang, and S.-C. Zhang, Nature physics 5, 438 (2009).
[3] M. Z. Hasan and C. L. Kane, Reviews of Modern Physics 82, 3045 (2010).
[4] X.-L. Qi and S.-C. Zhang, Reviews of Modern Physics 83, 1057 (2011).
[5] X. Chen, Z.-C. Gu, Z.-X. Liu, and X.-G. Wen, Science 338, 1604 (2012).
[6] X. Chen, Z.-C. Gu, Z.-X. Liu, and X.-G. Wen, Physical Review B 87, 155114 (2013).
[7] Z. Bi, R. Zhang, Y.-Z. You, A. Young, L. Balents, C.-X. Liu, and C. Xu, arXiv preprint arXiv:1602.03190 (2016).
[8] P. Maher, C. R. Dean, A. F. Young, T. Taniguchi, K. Watanabe, J. Hone, and P. Kim, Nature Physics 9, 154 (2013).
[9] C. L. Kane and E. J. Mele, Physical review letters 95, 226801 (2005).
[10] C. L. Kane and E. J. Mele, Physical review letters 95, 146802 (2005).
[11] B. A. Bernevig, T. L. Hughes, and S.-C. Zhang, Science 314, 1757 (2006).
[12] M. König, S. Wiedmann, C. Brüne, A. Roth, H. Buhmann, L. W. Molenkamp, X.-L. Qi, and S.-C. Zhang, Science 318, 766 (2007).
[13] C. Liu, T. L. Hughes, X.-L. Qi, K. Wang, and S.-C. Zhang, Physical review letters 100, 236601 (2008).
[14] I. Knez, R.-R. Du, and G. Sullivan, Physical review letters 107, 136603 (2011).
[15] Y.-Y. He, H.-Q. Wu, Y.-Z. You, C. Xu, Z. Y. Meng, and Z.-Y. Lu, Physical Review B 93, 115150 (2016).
[16] Y.-Z. You, Z. Bi, D. Mao, and C. Xu, Physical Review B 93, 125101 (2016).
[17] T. Yoshida and N. Kawakami, arXiv preprint arXiv:1604.00122 (2016).
[18] V. Mazo, C.-W. Huang, E. Shimshoni, S. T. Carr, and H. Fertig, Physical Review B 89, 121411 (2014).
[19] V. Mazo, E. Shimshoni, C.-W. Huang, S. T. Carr, and H. Fertig, Physica Scripta 2015, 014019 (2015).
[20] C.-Y. Hou, A.-A. Kim, and C. Chamon, Physical review letters 102, 076602 (2009).
[21] J. C. Teo and C. Kane, Physical Review B 79, 235321 (2009).
[22] Just to clarify, when we talk about BSPT state or fermionic QSH state, we refer to the intrinsic bulk topological state of the system, which is independent of the appearance or absence of QPC structure. When we talk about II/IC/CI/CC phases, we refer to the inter-edge tunneling phase which emerges only when QPC is present.
[23] A. F. Young, J. Sanchez-Yamagishi, B. Hunt, S. H. Choi, K. Watanabe, T. Taniguchi, R. Ashoori, and P. Jarillo-Herrero, Nature 505, 528 (2014).
[24] E. McCann, Physical Review B 74, 161403 (2006).
[25] E. V. Castro, K. Novoselov, S. Morozov, N. Peres, J. L. Dos Santos, J. Nilsson, F. Guinea, A. Geim, and A. C. Neto, Physical Review Letters 99, 216802 (2007).
[26] M. Kharitonov, Physical review letters 109, 046803 (2012).
[27] See Supplemental Material [url], which includes Refs. [31–34].
[30] In our work, Luttinger parameter $K_\pm$ is defined in analogous to the inverse of Luttinger parameter $g$ in Ref. [21]. To be specific, repulsive interaction implies $K_+ > 1$ in our notation and $g < 1$ in Ref. [21].
[31] E. McCann and M. Koshino, Reports on Progress in Physics 76, 056503 (2013).
[32] C. Kane and M. P. Fisher, Physical Review B 46, 15233 (1992).
[33] T. Martin, arXiv:cond-mat/0501208 (2005).
[34] J. Maciejko, C. Liu, Y. Oreg, X.-L. Qi, C. Wu, and S.-C. Zhang, Physical review letters 102, 256803 (2009).