Accelerating multidimensional cosmologies with scalar fields

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Abstract

We study multidimensional cosmological models with a higher-dimensional product manifold $R \times M_0 \times M_1$ (where $M_0$ and $M_1$ are spherical and flat spaces, correspondingly) in the presence of a minimal free scalar field. The dimensions of spaces $M_i$ ($i = 0, 1$) are not fixed and the choice of an external (our) space in the product manifold is arbitrary. Dynamical behaviour of the model is analyzed both in Einstein and Brans-Dicke conformal frames. For a number of particular cases, it is shown that external space-time undergoes an accelerated expansion.

Keywords: Accelerated cosmologies, Einstein frame, Brans-Dicke frame, scalar field.

1 Introduction

Distance measurements of type Ia supernovae (SNe.Ia) [1] as well as cosmic microwave background (CMB) anisotropy measurements [2] performed during the last years give strong evidence for the existence of dark energy — a smooth energy density with negative pressure which causes an accelerated expansion of the Universe at present time. The challenge to theoretical cosmology consists in finding a natural explanation of such acceleration (dark energy).

In the present paper we consider a multidimensional cosmological model with a higher-dimensional product manifold which consists of spherical and flat spaces with dimensions $d_0$ and $d_1$ correspondingly in the presence of a minimally coupled free scalar field. We investigate this model to find under which circumstances the external spacetime undergoes an accelerated expansion with simultaneous compactification of the internal space.

To start with, let us consider a cosmological model with factorizable geometry

$$g = -\exp[2\gamma(\tau)]d\tau \otimes d\tau + a_0^2(\tau)g_0 + a_1^2(\tau)g_1,$$  

(1)
which is defined on the manifold

\[ M = R \times M_0 \times M_1 \]

and with minimal free scalar field \( \phi \). We assume that factors \( M_i \) are Einstein spaces: \( R[g_0] = R_0 > 0, \ R[g_1] = 0 \). The action for considered model reads

\[
S = \frac{1}{2\kappa^2} \int_M d^Dx \sqrt{|g|} R + \int_M d^Dx \sqrt{|g|} \left[ -\frac{1}{2} e^{-2\gamma(\tau)} \partial_M \phi \partial_N \phi \right] + S_{YGH},
\]

where \( S_{YGH} \) is the York-Gibbons-Hawking boundary term.

It can be easily seen \([3]\) that in the harmonic time gauge \( \gamma = \gamma_0 \) (see \([3]\)) equations of motion have the following solutions:

\[
a_0(\tau) = a_{(c)0} \exp \left( -\frac{1}{d_0-1} \sqrt{\frac{d_1(d_0-1)}{D-2}} p^1 \tau \right),
\]

\[
a_1(\tau) = a_{(c)1} \exp \left( \frac{1}{d_1} \sqrt{\frac{d_1(d_0-1)}{D-2}} p^1 \tau \right),
\]

\[
\tau = p^2 \tau + q^2,
\]

where \( a_{(c)0,1}, p^{1,2} \) and \( q^2 \) are constants of integrations and \( 2\epsilon = (p^1)^2 + (p^2)^2 > 0 \).

We shall analyze a dynamical behaviour of the model both in Brans-Dicke and Einstein conformal frames where metrics are connected as follows \([5]\)

\[
g = -e^{2\gamma} dt \otimes dt + a_i^2 g^{(c)} + a_i^2 g^{(i)}
\]

\[
= -dt \otimes dt + a_i^2 g^{(c)} + a_i^2 g^{(i)}
\]

\[
= \Omega^2 (-\tilde{d} \otimes \tilde{d} + \tilde{a}_i^2 g^{(c)} + \tilde{a}_i^2 g^{(i)}).
\]

Here, \( t (\tilde{t}) \) is a physical/synchronous time and \( a_\epsilon, a_i (\tilde{a}_\epsilon, \tilde{a}_i) \) are scale-factors of the external and internal spaces in the Brans-Dicke (Einstein) frame. Conformal factor \( \Omega = a_i^{-d_i} \) (in the case of 4-D external spacetime).

### 2 Dynamical behaviour in the Brans-Dicke frame

Here, the Hubble and the deceleration parameters can be expressed via harmonic time as follows:

\[
H_0 = \frac{1}{a_0} \frac{da_0}{dt} = -\frac{\xi_1 + \xi_2 \tanh(\xi_2 \tau)}{f(\tau)(d_0 - 1)}, \quad -q_0 = \frac{1}{a_0} \frac{d^2a_0}{dt^2} = -\frac{\xi_2 + \xi_1 \tanh(\xi_2 \tau)}{f^2(\tau)(d_0 - 1)},
\]

\[
H_1 = \frac{1}{a_1} \frac{da_1}{dt} = \frac{\xi_1}{f(\tau)d_1}, \quad -q_1 = \frac{1}{a_1} \frac{d^2a_1}{dt^2} = \frac{\xi_1((D-2)\xi_1 + d_0d_1 \xi_2 \tanh(\xi_2 \tau))}{f^2(\tau)d_1^2(d_0 - 1)},
\]

where \( f(\tau) = \frac{d\tau}{d\tau}, \ \xi_1 = \frac{d_1(d_0-1)}{D-2} p^1, \ \xi_2 = \sqrt{\frac{(d_0-1)2\epsilon}{d_0}}. \) We consider the expanding space as an external one. Thus, we arrived to two separate cases:
We have 2 particular cases where external space undergoes accelerated expansion:

1. $d_0\xi_2 > |\xi_1| > \xi_2$, Big Rip scenario:

In this case, the scale-factor $a_0$ is

$$a_0(t) \simeq a_{(c)0} \left[ \frac{(d_0\xi_2 - |\xi_1|)(c - t)}{(d_0 - 1)2^{\frac{d_0}{2(c - 1)}}a_{(c)0}^d a_{(c)1}^1} \right]^{\frac{|\xi_1| - \xi_2}{d_0\xi_2 - |\xi_1|}}. \quad (6)$$

It can be easily seen that the Universe behaves according to the Big Rip scenario, i.e. the scale-factor $a_0$ achieves infinite value in the finite period of time.

2. $|\xi_1| > d_0\xi_2$:

Here, for the scale factor of the external space we obtain the asymptote:

$$a_0(t) \simeq a_{(c)0} \left[ \frac{(|\xi_1| - d_0\xi_2)(t - c)}{(d_0 - 1)2^{\frac{d_0}{2(c - 1)}}a_{(c)0}^d a_{(c)1}^1} \right]^{\frac{|\xi_1| - \xi_2}{|\xi_1| - d_0\xi_2}}. \quad (7)$$

For these cases, the following relations between momentum $p^1$ and kinetic energy $\varepsilon$ hold:

- case $d_0\xi_2 > |\xi_1| > \xi_2$: $\frac{d_0 d_1 (p^1)^2}{(D - 2)} 2\varepsilon > \frac{d_4 (p^1)^2}{d_0 (D - 2)}$, i.e. real scalar field and Big Rip scenario

- case $|\xi_1| > d_0\xi_2$: $2\varepsilon < \frac{d_4 (p^1)^2}{d_0 (D - 2)}$, so $2\varepsilon < (p^1)^2$, i.e. it is possible only if $(p^1)^2 = (\dot{\phi}(\tau))^2 < 0$ and scalar field is imaginary

**ii. $M_1$ is the external space ($\xi_1 > 0$):**

In this case, $M_1$ undergoes an accelerating expansion (via Big Rip scenario) with simultaneous compactification of $M_0$. Such behaviour for $M_1$ can be easily traced from asymptotical behaviour of $a_1(t)$:

$$a_1(t) \simeq a_{(c)1} \left[ \frac{(\xi_1 + d_0\xi_2)(c - t)}{(d_0 - 1)2^{\frac{d_0}{2(c - 1)}}a_{(c)0}^d a_{(c)1}^1} \right]^{\frac{\xi_1(d_0 - 1)}{(\xi_1 + d_0\xi_2)d_1}}. \quad (8)$$

### 3 Dynamical behaviour in the Einstein frame

Now, we analyze the behaviour of the model in the Einstein frame. Here, we also consider two separate cases:

**i. $M_0$ is the external space:**

If $M_0$ is the external space, its deceleration parameter is

$$-\ddot{q}_0 = \frac{1}{a_0} \frac{d^2 a_0}{dt^2} = -\frac{\xi_0^2}{f_{E0}(\tau)(d_0 - 1)} < 0, \quad (9)$$
where \( f_{E0}(\tau) = \frac{d\tilde{t}}{dt} \). It can be easily seen that in this case an accelerated expansion of the external space is impossible.

**ii. \( M_1 \) is the external space:**

If \( M_1 \) is the external space, its deceleration parameter reads:

\[
-\tilde{q}_1 = \frac{1}{\tilde{a}_1} \left( \frac{d^2\tilde{a}_1}{dt^2} \right) = -\frac{1}{f_{E1}(\tau)} \frac{1}{d_1^2(d_0 - 1)^2(d_1 - 1)} \times \\
\times \left[ (D-2)\xi_1 + d_1d_0\xi_2 \tanh(\xi_2\tau) \right]^2 + \frac{d_0d_1^2(d_0 - 1)\xi_2^2}{\cosh^2(\xi_2\tau)} < 0, \tag{10} \]

where \( f_{E1}(\tau) = \frac{d\tilde{t}}{d\tau} \). Accelerated expansion of the external space is impossible in this case too.

**Summary**

Briefly, we can summarize our results as follows:

**In the Brans-Dicke frame:**

i. accelerated expansion of the positive curvature space can be archived either if it undergoes the Big Rip scenario or it contains scalar field with negative kinetic term.

ii. Ricci-flat space undergoes late time acceleration via Big Rip scenario and its qualitative behavior does not depends on a particular choice of parameters of the model.

**In the Einstein frame:**

it is impossible to achieve an accelerated expansion of the external space (either \( M_0 \) or \( M_1 \)).

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**References**

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