Analysis of Magnetic Plucking Configurations for Frequency Up-Converting Harvesters

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Abstract. Magnetic plucking applies the strategy of frequency up-conversion in inertial energy harvesting when the energy source, such as human motion, only provides excitations with very low and irregular frequencies. This paper presents an analysis of three different magnet configurations to achieve magnetic plucking based on a three-dimensional analytical cube permanent magnet model: direct repulsive configuration, orthogonal configuration and indirect repulsive configuration. Simulation and experimental results indicate that the indirect repulsive configuration generates the largest tip displacement given the practical constraints in designing a wearable energy harvester. We have implemented this configuration in a wrist-worn rotational energy harvester to pluck multiple piezoelectric beams. Other configurations, however, can potentially be advantageous in applications with alternative constraints.

1. Introduction
Energy harvesting for wearable wellness sensors could provide the potential for continuous health monitoring by eliminating the need to replace or recharge batteries manually. The inherent limitation of utilizing human motion as the source for inertia energy harvesting is that it only provides excitations with very low and irregular frequencies. Frequency up-conversion is a commonly used strategy to tackle this issue by transforming the low-frequency input motion into high-frequency resonance of the transducer. Plucking is one technique which applies such a strategy. Eccentric rotor-based wearable energy harvesters have been demonstrated in previous endeavors either using magnets [1][2] or pins [3][4] to pluck piezoelectric beams. Magnetic coupling provides better reliability since it can be designed contact-free. Usually magnets are arranged to pluck the beam in the direction of magnet motion with either a direct repelling or an attractive configuration [5], i.e. in-plane plucking. While the in-plane plucking introduces a stronger jump phenomenon in the cantilever beam, the out-of-plane deflection configuration provides the capability of fabricating multiple beams in the harvester on a single substrate. Furthermore, mechanical design complexity is reduced and space utilization is increased which ultimately improves power output. However, if the magnets are simply aligned, as in the direct repulsive configuration in Figure 1, the overall thickness of the harvester will grow. Additionally, in this configuration the opposing magnets can get stuck in a side by side orientation, i.e. there is a pull-in effect. Thus exploring alternative magnet configurations with the ability to trigger out-of-plane plucking as well is worthwhile.

Among all the possible combination of orientations, we present two alternative magnet configurations: the orthogonal and the indirect repulsive configuration, which not only have demonstrated the capability of triggering plucking but also are free of the pull-in effect. As shown in...
Figure 1, the direct repulsive configuration is listed alongside for later comparison as we have applied this orientation in our previous harvester prototyping with off-the-shelf components [2]. This paper will introduce the modelling, experimental validation and finally an implementation of the alternative magnetic configuration in a frequency up-converting energy harvester.

Figure 1. Magnet configurations: (a) Direct Repulsive Configuration, (b) Orthogonal Configuration, (c) Indirect Repulsive Configuration.

2. Modelling of the Magnetic Interaction

The magnetic interaction model is a vital component in the coupled system model to accurately predict the actuation to the beam. In previous studies on magnetic plucking, the Gilbert model or the Ampère model is often applied which assumes the permanent magnet as either a magnetic charge or magnetic dipole. As a result, the force between magnets follows an inverse squared or cubic relationship with respect to the distance, which is a valid assumption when the size of magnets are negligible compared to the distance between them. However when devices are miniaturized as in our case, the size of magnets are often found to be on the same order of magnitude as the distance. In addition, torques are usually neglected, which could potentially provide a nontrivial excitation. The finite element method is a common approach to a more accurate approximation, which serves well as a validation, whereas the analytical expression provides fast calculation and integration with the dynamic simulation. Our study applies the distributed Gilbert model [6] to obtain the forces and torques between cube magnets analytically. The permanent cube magnet is modelled as two surfaces of evenly distributed magnetic charges. The total potential energy between two cube magnets shown in Figure 2 is given by:

\[
E = -\frac{JJ'}{2\pi\mu} \int \int \int \left( (\alpha + X - x)^2 + (\beta + Y - y)^2 + \gamma^2 \right)^{\frac{1}{2}} dxdydXdY + \frac{JJ'}{4\pi\mu} \int \int \int \left( \sum \left( (\alpha + X - x)^2 + (\beta + Y - y)^2 + (\gamma + 2L)^2 \right)^{\frac{1}{2}} dxdydXdY \right)
\]

(1)

Where the \( J \) and \( J' \) are the magnetic polarizations for each cube magnet of length \( 2L \), \( \mu \) is the permeability of the media and the origin of the O'XYZ frame is located at (\( \alpha, \beta, \gamma \)) with respect to the Oxyz frame (see Figure 2). The distance between two magnets \( d \) is defined as the length OO'. The forces and torques between two magnets are the derivative of the potential energy with respect to each direction. A similar formula can be derived for the case where magnetic polarizations are perpendicular as shown in [7].

Figure 2. Schematic of the generic magnetic configuration.
We use a static force profile to analyze the actuation mechanism of different magnetic plucking configurations. The force profile is given as a function the displacement normalized by the length of magnet. As illustrated in Figure 1, the beam is assumed to be rigid such that the magnet attached is considered static and does not move. The moving magnet on the rotor is assumed to follow a linear path as the small angles between the opposing surfaces of the magnets are neglected. Note that the model is created within the size of a wrist-worn device. The length of the cube permanent magnet (N42) is 1.59 mm (1/16 inch). A 1 mm gap between magnets is given to all the configurations for comparison as in the direct repulsive configuration such space is required to avoid the pull-in. In the alternative configurations the gap can be further reduced to increase the coupling. Particularly in the indirect repulsive configuration, a 0.5 mm offset is applied to induce the transverse force. We used the Radia software developed by ESRF to provide a numerical validation for the analytical model, which is plotted in the corresponding figures as well. The torque components are neglected as their contribution to the dynamics of the beam are approximately one order of magnitude smaller than the forces.

In general, the transverse force $F_z$ and the axial force $F_y$ (directionality shown in Figure 1) are of interest to the plucking. The transverse force directly deflects the beam while the axial force softens the beam and provides a bending torque when the beam is already deflected. The lateral force $F_x$ is irrelevant in terms of plucking. As shown in Figures 3 to 5, the transverse force exists among all the configurations and its magnitude is the greatest in the direct repulsive configuration given the same gap between the magnets. The indirect repulsive configuration benefits from a large axial force while the orthogonal configuration operates at a pure deflecting mode, i.e. only the transverse force is generated. Although the direct repulsive configuration seems to be the optimal mode to trigger the plucking based on the static force profile alone, as mentioned earlier, it requires a larger gap to avoid the pull-in effect and a larger space for beam deflection since the magnets are aligned in the direction of device thickness, which makes this configuration less appealing for a wearable device as it is size-sensitive. There is no possibility for the magnets to touch each other in the alternative magnetic configurations, thus we are able to implement a smaller gap between magnets which results in a larger transverse force. The offset in the indirect repulsive configuration can be optimized to increase the transverse force as well. As shown in Figure 6, the transverse force is in the optimal range and becomes less sensitive to the change in offset when the offset is in the range of 50% to 80% of the size of magnet.

![Figure 3. Static force profile of the direct repulsive configuration (gap = 1 mm).](image1.png)

![Figure 4. Static force profile of the orthogonal configuration (gap = 1 mm).](image2.png)
3. Experimental Validation

We created a simple cantilever beam structure with magnet attached on the tip to corroborate our model since the beam tip displacement is the direct result of the magnetic interaction. The scale is slightly increased compared to the previous model parameters for a more convenient measurement with a laser displacement sensor. A permanent cube magnet (N50) of 2 mm in length is glued to the tip of beam with corresponding orientation. The cantilever beam a brass reinforced piezoelectric bending actuator made by Piezo Systems with the dimension of 0.38 mm (t) × 3.2 mm (w) × 30 mm (l). The gap between magnets for the direct repulsive configuration is 2 mm as the pull-in occurs with a smaller gap while we can safely lower the gap in the other two configurations to 0.8 mm for a larger transverse force. The offset in the indirect repulsive configuration falls in the optimal range found in Figure 6.

Figure 7 illustrates our preliminary results compared with simulated profile. The indirect repulsive configuration outperforms other two configurations with the largest displacement when practical constraints are considered, i.e. the gap and the offset allowed in each configuration. While the shape of the direct repulsive and orthogonal configurations matches calculations well, the model seems to calculate a smaller tip displacement for the indirect repulsive configuration. The discrepancy is due to the softening effect caused by the axial force neglected in the beam model. In terms of designing a frequency up-converting harvester, all of these configurations are capable of plucking the beam as long as the magnet is moving fast enough such that the frequency of the magnetic force profile is well above the natural frequency of the beam.

4. Implementation of the Alternative Magnetic Configuration

As mentioned the main purpose of choosing out-of-plane plucking with alternative magnetic configuration is to allow the use of multiple beams fabricated on a single substrate and assembled with a simple fixture. We have designed and fabricated a wrist-worn eccentric rotor-based piezoelectric energy harvester shown in Figure 8 implementing the indirect repulsive magnetic configuration to pluck 6 bimorph piezoelectric beams. The thin-film piezoelectric beams are fabricated on a nickel substrate. A peak to peak output voltage of 5 volts has been achieved from a single unimorph beam across a 47 kΩ load under the excitation of intense walking when the device is worn on wrist. This will ultimately result in a total power output of over 40 µW given 6 working bimorph beams, which is enough to power a wearable wellness sensor continuously.
5. Conclusions
This paper has explored alternative configurations to achieve the magnetic plucking in frequency up-converting harvesters. Simulation and experimental results indicate that the indirect repulsive configuration is preferred for a size (and especially thickness) sensitive device such as a wearable energy harvester, which has been demonstrated by a wrist-worn piezoelectric harvester prototype. In a broader sense, however, these configuration can be implemented in other applications. For instance, the direct repulsive configuration should be considered when the device thickness is not a primary concern; the orthogonal configuration can potentially be utilized in precision sensing or actuation due to its pure actuation in the transverse direction.

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References
[1] Pillatsch P, Yeatman E M and Holmes A S 2013 J. Phys. Conf. Ser. 476 012010
[2] Xue T, Ma X, Rahn C and Roundy S 2014 J. Phys. Conf. Ser. 557 012090
[3] Pozzi M and Zhu M 2011 Smart Mater. Struct. 20 055007
[4] Lockhart R, Janphuang P, Briand D and de Rooij N F 2014 2014 IEEE 27th Int. Conf. Micro Electro Mech. Syst. 370–3
[5] Pillatsch P, Yeatman E M and Holmes A S 2014 Smart Mater. Struct. 23 025009
[6] Akoun G and Yonnet J 1984 IEEE Trans. Magn. MAG-20 1962–4
[7] Allag H and Yonnet J 2011 LDIA 2011 8th Int. Conf. Linear Drives Ind. Appl. 2011 1–5