Evidence for sub-Chandrasekhar-mass progenitors of Type Ia supernovae at the faint end of the width–luminosity relation

Stéphane Blondin,1∗ Luc Dessart,2 D. John Hillier3 and Alexei M. Khokhlov4

1Aix Marseille Univ, CNRS, LAM, Laboratoire d’Astrophysique de Marseille, Marseille, France
2Unidad Mixta Internacional Franco-Chileno de Astronomía (CNRS UMI 3386), Departamento de Astronomía, Universidad de Chile, Camino El Observatorio 1515, Las Condes, Santiago, Chile
3Department of Physics and Astronomy & Pittsburgh Particle Physics, Astrophysics, and Cosmology Center (PITT PACC), University of Pittsburgh, Pittsburgh, PA 15260, USA
4Department of Astronomy & Astrophysics, the Enrico Fermi Institute, and the Computation Institute, The University of Chicago, Chicago, IL 60637, USA

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ABSTRACT
The faster light-curve evolution of low-luminosity Type Ia supernovae (SNe Ia) suggests that they could result from the explosion of white dwarf (WD) progenitors below the Chandrasekhar mass ($M_{\text{Ch}}$). Here we present 1D non-local thermodynamic equilibrium time-dependent radiative transfer simulations of pure central detonations of carbon–oxygen WDs with a mass ($M_{\text{tot}}$) between 0.88 and 1.15 M⊙ and a $^{56}\text{Ni}$ yield between 0.08 and 0.84 M⊙. Their lower ejecta density compared to $M_{\text{Ch}}$ models results in a more rapid increase of the luminosity at early times and an enhanced γ-ray escape fraction past maximum light. Consequently, their bolometric light curves display shorter rise times and larger post-maximum decline rates. Moreover, the higher $M^{(56}\text{Ni})/M_{\text{tot}}$ ratio at a given $^{56}\text{Ni}$ mass enhances the temperature and ionization level in the spectrum-formation region for the less luminous models, giving rise to bluer colours at maximum light and a faster post-maximum evolution of the $B-V$ colour. For sub-$M_{\text{Ch}}$ models fainter than $M_B \approx -18.5$ mag at peak, the greater bolometric decline and faster colour evolution lead to a larger $B$-band post-maximum decline rate, $\Delta M_{15}(B)$. In particular, all of our previously published $M_{\text{Ch}}$ models (standard and pulsational delayed detonations) are confined to $\Delta M_{15}(B) < 1.4$ mag, while the sub-$M_{\text{Ch}}$ models with $M_{\text{tot}} \lesssim 1$ M⊙ extend beyond this limit to $\Delta M_{15}(B) \approx 1.65$ mag for a peak $M_B \approx -17$ mag, in better agreement with the observed width–luminosity relation (WLR). Regardless of the precise ignition mechanism, these simulations suggest that fast-declining SNe Ia at the faint end of the WLR could result from the explosion of WDs whose mass is significantly below the Chandrasekhar limit.

Key words: radiative transfer – supernovae: general.

1 INTRODUCTION
The finding that the peak magnitudes of Type Ia supernovae (SNe Ia) are tightly correlated with their post-maximum decline rate in several optical bands (Pskovskii 1977; Phillips 1993) revolutionized the use of these events as extragalactic distance indicators and formed the cornerstone for the discovery of dark energy (Riess et al. 1998; Perlmutter et al. 1999).

Due to its determining role in observational cosmology, this so-called width–luminosity relation (hereafter WLR when considering the B-band) has received extensive theoretical attention. The slower post-maximum decline of SNe Ia with larger peak $M_B$ has been thought to arise from an increase in opacity with $^{56}\text{Ni}$ mass, which affects the time-scale for the radiation to escape the ejecta (e.g. Hoeflich et al. 1996; Mazzali et al. 2001; Pinto & Eastman 2001). However, in contrast with observational studies that show a significantly weaker relation between peak magnitude and post-maximum decline rate for the integrated (U)BVRI flux (e.g. Contardo, Leibundgut & Vacca 2000), one would expect this relation to also hold for the bolometric luminosity.

A later study by Kasen & Woosley (2007) showed that the decay heating from $^{56}\text{Ni}$ affects the onset of $\gamma\rightarrow\mu$ recombination of iron-group elements (IGEs) in the ejecta, which causes a faster colour evolution for the least luminous events (in particular the $B-V$ colour), and hence a larger decline rate in $B$ relative to other bands. The WLR is then interpreted purely as a colour effect related to the level of Fe I/Co I line blanketing, independent of the evolution of the bolometric luminosity.

The mass of the exploding white dwarf (WD) (hereafter $M_{\text{tot}}$) is known to affect the bolometric evolution around maximum light (Pinto & Eastman 2000). However, most theoretical studies of

∗ E-mail: stephane.blondin@lam.fr

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the WLR have either focused exclusively on Chandrasekhar-mass (M\textsubscript{Ch}) progenitors (e.g. Mazzali et al. 2001; Pinto & Eastman 2001; Höflich et al. 2002; Kasen & Woosley 2007) or excluded sub-M\textsubscript{Ch} double-detonation models based on their apparent colour mismatch (see e.g. Hoeflich et al. 1996). A generic problem of these sub-M\textsubscript{Ch} models resides in the composition of the detonated He shell, which is needed to trigger a secondary detonation in the carbon–oxygen (C–O) core. The synthesis of \textsuperscript{56}Ni and other IGEs in this external shell causes discrepant colours and spectra at early times (Nugent et al. 1997; Kromer et al. 2010). Nonetheless, Sim et al. (2010) have shown that pure central detonations in single sub-M\textsubscript{Ch} WDs without an outer He shell are able to reproduce the overall trend of the observed WLR, although it is still unclear whether sub-M\textsubscript{Ch} models fare better than the standard M\textsubscript{Ch} scenario, in particular for low-luminosity SNe Ia. However, observational studies have shown that the narrow light curves of fast-declining SNe Ia likely result from sub-M\textsubscript{Ch} ejecta, and that the WLR is naturally explained as a relation between ejected mass and \textsuperscript{56}Ni mass (Scalzo et al. 2014a; Scalzo, Ruiter & Sim 2014b).

While the WLR imposes very strict constraints on SN Ia models, its reproduction by no means guarantees a high fidelity to observed events (see e.g. Blondin et al. 2011). Moreover, the emphasis on the importance of the ionization balance in SN Ia ejecta to explain the WLR highlights the need for an accurate determination of the atomic level populations via a solution to the statistical equilibrium equations, i.e. a fully non-local thermodynamic equilibrium (non-LTE) solution to the radiative transfer problem, often approximated in previous studies. Here we present 1D non-LTE time-dependent radiative transfer simulations of sub-M\textsubscript{Ch} models for SNe Ia (Section 2), performed using the CMFGEN radiative-transfer code (Hillier & Dessart 2012).

To better illustrate the impact of a low ejecta mass on the radiative display, we confront these sub-M\textsubscript{Ch} models to our previously published M\textsubscript{Ch} models, which include ‘standard’ M\textsubscript{Ch} delayed-detonations (DDC series; Blondin et al. 2013) and pulsational M\textsubscript{Ch} delayed-detonations (PDDEL series; Dessart et al. 2014b). Despite the 1D treatment of the explosion, these M\textsubscript{Ch} models were found to capture most of the salient features of more elaborate multidimensional simulations, in particular the asymptotic kinetic energy and chemical stratification (see e.g. Seitenzahl et al. 2013). More importantly, they provided an excellent match to the observed properties of both normal and low-luminosity SNe Ia at maximum light (Blondin et al. 2013), and to the spectral evolution of normal events out to \textlessthanoreq\textless 100 d past explosion (SN 2002bo, Blondin, Dessart & Hillier 2015; SN 2005cf, Dessart et al. 2014c; SN 2011fe, Dessart et al. 2014b).

We start by analysing the bolometric evolution of the sub-M\textsubscript{Ch} models, highlighting the differences in rise time and post-maximum decline compared to M\textsubscript{Ch} models at a given peak luminosity, i.e. similar \textsuperscript{56}Ni mass but different WD mass, \textit{M}_{\text{tot}} (Section 3). We then discuss the impact of a sub-M\textsubscript{Ch} ejecta on the maximum-light \textit{B} - \textit{V} colour (Section 4), and on its post-maximum evolution for low-luminosity SNe Ia (Section 5). Last, we assess the merits of sub-M\textsubscript{Ch} progenitors in reproducing the observed WLR all the way to the faint end (Section 6). We discuss possible progenitor scenarios leading to the detonation of a sub-M\textsubscript{Ch} WD in Section 7, followed by our conclusions.

2 SUB-CHANDRASEKHAR-MASS MODELS

The sub-M\textsubscript{Ch} models studied here correspond to pure central detonations of WDs with a mass ranging between 0.88 and 1.15 M\textsubscript{\odot} (Table 1). As such they are similar to the sub-M\textsubscript{Ch} models of Sim et al. (2010), albeit with some additional numerical treatment and radiative-transfer post-processing (see below). The initial WD is assumed to be in hydrostatic equilibrium, and is composed of equal amounts of \textsuperscript{12}C and \textsuperscript{16}O by mass, with traces of \textsuperscript{22}Ne and solar composition for all other isotopes. Importantly, we do not consider the presence of an external He shell, required in the double-detonation scenario to trigger a detonation in the C–O core. Despite the artificial nature of this setup, the models are nonetheless intrinsically coherent and enable us to qualitatively assess the impact of a lower ejecta mass on the radiative display. The 1D hydrodynamical treatment of the explosion phase is analogous to that used in our previous SN Ia studies based on M\textsubscript{Ch} delayed-detonation models (see Blondin et al. 2013, their section 2). The calculation is carried out until the ejecta mass shells reach a ballistic regime (homologous expansion), less than \textless 20 s past explosion. We smooth sharp variations in the composition and density profiles of the hydrodynamical input through convolution with a Gaussian kernel of width \textit{\sigma} = 800 km s\textsuperscript{-1} (see also Blondin et al. 2015).

One important difference is the absence of an initial deflagration phase, required to pre-expand the WD in M\textsubscript{Ch} models and hence synthesize intermediate-mass elements during the secondary detonation phase. In sub-M\textsubscript{Ch} models the burning can proceed directly in the form of a detonation wave, since the lower WD density ensures the burning will be incomplete in the outer ejecta layers with no prior pre-expansion. As a result, the initial WD structure sets the density at which the C–O fuel is burned, which is lower for lower WD masses. In particular, the amount of \textsuperscript{56}Ni synthesized in the explosion is directly proportional to the WD mass, as is the asymptotic kinetic energy (see Table 1). In our sub-M\textsubscript{Ch} model series (SCH), the initial range of WD masses (0.88–1.15 M\textsubscript{\odot}) translates into a \textsuperscript{56}Ni yield between 0.08 and 0.84 M\textsubscript{\odot}.

The lower WD density also limits the production of neutron-rich stable isotopes of IGEs (e.g. \textsuperscript{52}Fe, \textsuperscript{58}Ni) in the inner ejecta, while these are more abundantly produced during the deflagration phase of M\textsubscript{Ch} models. In the outer ejecta layers, however, the burning proceeds at similar densities in both the sub-M\textsubscript{Ch} and M\textsubscript{Ch} models considered here, resulting in similar abundance profiles. At a given \textsuperscript{56}Ni yield, the sub-M\textsubscript{Ch} models can thus be considered as low-mass analogs of the M\textsubscript{Ch} models when discussing their radiative properties at early times, including the maximum-light phase discussed in the present paper.

The long-term evolution is computed with the 1D, time-dependent, non-LTE radiative-transfer code CMFGEN (Hillier & Dessart 2012; Dessart et al. 2014c), which includes the treatment of non-local energy deposition and non-thermal processes (e.g. Dessart et al. 2014a). Apart from the two-step \textsuperscript{56}Ni\rightarrow\textsuperscript{56}Co\rightarrow\textsuperscript{56}Fe decay chain, we treat eight additional two-step decay chains associated with \textsuperscript{57}K, \textsuperscript{44}Ti, \textsuperscript{56}Cr, \textsuperscript{54}Cr, \textsuperscript{51}Mn, \textsuperscript{52}Fe, \textsuperscript{58}Co, \textsuperscript{57}Ni, and a further six one-step decay chains associated with \textsuperscript{41}Ar, \textsuperscript{42}K, \textsuperscript{43}Sc, \textsuperscript{47}Sc, \textsuperscript{60}Co (see Dessart et al. 2014c). All atom/ion-level populations are determined explicitly through a solution of the time-dependent statistical equilibrium equations, which are coupled to the energy and radiative-transfer equations. We consider the following ions: C I–III, O I–III, Ne I–III, Na I, Mg II–III, Al II–IV, Si II–IV, S II–IV, Ar I–III, Ca II–IV, Sc II–III, Ti II–III, Cr II–IV, Mn II–III, Fe I–VII, Co II–VII and Ni II–VII. The number of levels considered for each ion is given in Appendix A, Table A1. We assume a constant effective Doppler width (including both thermal and turbulent velocities) of 50 km s\textsuperscript{-1} for our line-absorption profiles. While this value largely overestimates the true effective Doppler width (likely a few km s\textsuperscript{-1} at most) and hence the ejecta opacity, we have run extensive tests imposing an effective Doppler width of 10 km s\textsuperscript{-1} and found a negligible impact (at the few per cent level) on the predicted rise times and post-maximum
### 3 BOLOMETRIC RISE AND POST-MAXIMUM DECLINE

Pinto & Eastman (2000) have emphasized the impact of the total mass\(^1\) on the bolometric light curves of SNe Ia, through its effect on the ejecta density and \(\gamma\)-ray escape. A decrease in \(M_{\text{tot}}\) results in a lower column depth that favours a more rapid release of radiation at early times and an enhanced \(\gamma\)-ray escape fraction past maximum light, leading to a shorter rise time and a faster post-maximum decline. This is indeed the case for the sub-\(M_{\text{Ch}}\) models presented here, which display systematically shorter bolometric rise times than the \(M_{\text{Ch}}\) models for a given \(56\text{Ni}\) mass (Fig. 1, top panel).

Due to the shorter rise time, the instantaneous rate of energy deposition by radioactive decays (\(\dot{L}_{\text{decay}}\)) at maximum light is larger (as is the peak luminosity, \(L_{\text{bol}}\)), resulting in a more efficient heating of the ejecta compared to a \(M_{\text{Ch}}\) model with the same \(56\text{Ni}\) mass. The lower \(M_{\text{tot}}\) exacerbates this effect, as the specific heating rate at maximum light (\(\dot{L}_{\text{decay}}/M_{\text{tot}}, \text{noted } \dot{\epsilon}_{\text{decay}}\) in Table 1) is even larger. This latter parameter is key in explaining the bluer maximum-light colours of the sub-\(M_{\text{Ch}}\) models (see Section 4).

Moreover, all of the sub-\(M_{\text{Ch}}\) models have a bolometric decline rate \(\Delta M_{15}(\text{bol}) \geq 0.80\) mag (and up to 0.98 mag), where the \(M_{\text{Ch}}\) models are confined to smaller values (0.54–0.76 mag; see Fig. 1, bottom panel). This larger bolometric decline for the sub-\(M_{\text{Ch}}\) models will naturally affect all photometric bands, including the \(B\) band on which the WLR is based (Section 6).

That said, both the \(M_{\text{Ch}}\) and sub-\(M_{\text{Ch}}\) models share a weak dependence of their post-maximum bolometric decline rate on the \(56\text{Ni}\) mass: the variation in \(\Delta M_{15}(\text{bol})\) is less than \(-0.2\) mag across a factor of \(\sim 7–10\) variation in \(56\text{Ni}\) mass. When excluding the faintest sub-\(M_{\text{Ch}}\) model (SCH1p5), the difference in \(\Delta M_{15}(\text{bol})\) drops to a mere 0.07 mag for the \(M_{\text{Ch}}\) series. This contrasts with the far stronger dependence of the post-maximum \(B\)-band decline rate \(\Delta M_{15}(B)\) on \(56\text{Ni}\) mass (and hence peak luminosity; see Table 1), and lends support to the interpretation of the WLR as a colour effect as opposed to an increase of the diffusion time with \(56\text{Ni}\) mass (see Kasen & Woosley 2007).

In fact, the bolometric evolution more naturally follows an opposite trend to the \(B\)-band WLR, the more luminous models declining more rapidly (this is especially true in the \(56\text{Ni}\) mass range

| Model    | \(M_{\text{tot}}\) (\(M_{\odot}\)) | \(E_{\text{kin}}\) (erg) | \(v_{\text{max}}(56\text{Ni})\) (km s\(^{-1}\)) | \(\dot{\epsilon}_{\text{decay}}\) (erg s\(^{-1}\) g\(^{-1}\)) | \(t_{\text{rise}}(\text{bol})\) (day) | \(L_{\text{bol}}(B)\) (mag) | \(M_{\text{Ni}}\) (mag) | \(\Delta M_{15}(\text{bol})\) (mag) | \(\Delta M_{15}(B)\) (mag) | \((B - V)_{15}\) (mag) |
|----------|-------------------------------|-------------------------|-----------------------------|-----------------------------|---------------------------------|-----------------------------|-----------------|---------------------------------|-------------------------------|-----------------------------|
| SCH1p5   | 0.88                          | 3.62 (4)                | 16.7                        | 17.7                        | 1.85 (43)                       | 1.02 (43)                   | 0.08            | 1.02 (43)                       | 0.74                           | 0.02                        | 0.20                        |
| SCH2p0   | 0.90                          | 4.63 (4)                | 17.1                        | 17.7                        | 1.38 (43)                       | 1.93 (43)                   | 0.12            | 1.05 (43)                       | 0.74                           | 0.04                        | 0.36                        |
| SCH2p5   | 0.93                          | 4.63 (4)                | 17.1                        | 17.8                        | 1.38 (43)                       | 1.95 (43)                   | 0.17            | 1.08 (43)                       | 0.74                           | 0.04                        | 0.50                        |
| SCH3p5   | 0.98                          | 5.47 (4)                | 17.6                        | 17.2                        | 1.44 (43)                       | 1.96 (43)                   | 0.30            | 1.14 (43)                       | 0.72                           | 1.03                        | 0.65                        |
| SCH4p5   | 1.03                          | 5.67 (4)                | 17.6                        | 17.2                        | 1.44 (43)                       | 1.96 (43)                   | 0.46            | 1.20 (43)                       | 0.72                           | 1.04                        | 0.90                        |
| SCH5p5   | 1.08                          | 5.67 (4)                | 17.6                        | 17.2                        | 1.44 (43)                       | 1.96 (43)                   | 0.63            | 1.24 (43)                       | 0.72                           | 1.28                        | 1.14                        |
| SCH6p5   | 1.13                          | 5.67 (4)                | 17.6                        | 17.2                        | 1.44 (43)                       | 1.96 (43)                   | 0.80            | 1.28 (43)                       | 0.72                           | 1.52                        | 1.34                        |
| SCH7p0   | 1.15                          | 5.89 (4)                | 17.6                        | 17.2                        | 1.44 (43)                       | 1.96 (43)                   | 1.02            | 1.32 (43)                       | 0.72                           | 1.85                        | 1.59                        |
| SCH1p5   | 0.88                          | 3.62 (4)                | 16.7                        | 17.7                        | 1.85 (43)                       | 1.02 (43)                   | 0.08            | 1.02 (43)                       | 0.74                           | 0.02                        | 0.20                        |
| SCH1p5   | 0.88                          | 3.62 (4)                | 16.7                        | 17.7                        | 1.85 (43)                       | 1.02 (43)                   | 0.08            | 1.02 (43)                       | 0.74                           | 0.02                        | 0.20                        |
| SCH1p5   | 0.88                          | 3.62 (4)                | 16.7                        | 17.7                        | 1.85 (43)                       | 1.02 (43)                   | 0.08            | 1.02 (43)                       | 0.74                           | 0.02                        | 0.20                        |
| SCH1p5   | 0.88                          | 3.62 (4)                | 16.7                        | 17.7                        | 1.85 (43)                       | 1.02 (43)                   | 0.08            | 1.02 (43)                       | 0.74                           | 0.02                        | 0.20                        |

Notes: Numbers in parenthesis correspond to powers of 10. The \(56\text{Ni}\) mass corresponds to \(t \approx 0\), and \(v_{\text{max}}(56\text{Ni})\) is the velocity of the ejecta shell that bounds 99 per cent of the total \(56\text{Ni}\) mass. \(\dot{\epsilon}_{\text{decay}}\) is the specific heating rate at bolometric maximum, corresponding to the instantaneous rate of decay energy actually deposited in the ejecta at this time (\(\dot{\epsilon}_{\text{decay}}\)) divided by the total mass (\(M_{\text{Ni}}\)). The last two columns give the \(B - V\) colour at \(B\)-band maximum and 15 d later.

\(^{1}\)In this paper we only consider explosions that completely unbind the WD, hence the ejecta mass is equal to the WD mass.
This is in part due to the larger outward MB to the faint end (peak events, and holds for both M Type Ia supernovae obey a strong relation between their peak MB 4 MAXIMUM-LIGHT COLOURS

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Figure 1. Bolometric rise time (top) and decline rate ΔM15(bol) (bottom) versus 56Ni mass for the MCh (DDC series, circles; PDDEL series, squares) and sub-MCh models (diamonds).

0.2–0.6 M⊙; see Fig. 1). The bolometric decline rate is related to the magnitude and rate of change of the γ-ray escape fraction past maximum light, which is larger for higher 56Ni mass (see Stritzinger et al. 2006). This is in part due to the larger outward extent of the 56Ni distribution (see the vspa56Ni column in Table 1), which favours the earlier and more rapid escape of γ-rays, thereby limiting the amount of decay energy actually deposited in the ejecta (see also Pinto & Eastman 2001). This effect is modulated by the release of stored radiation past maximum light, which leads to a secondary bolometric maximum whose timing can affect the value of ΔM15(bol).

4 MAXIMUM-LIGHT COLOURS

Type Ia supernovae obey a strong relation between their peak MB and their B − V colour at maximum light, brighter events displaying bluer colours (e.g. Tripp 1998). This relation simply reflects the larger magnitude of decay heating from 56Ni for more luminous events, and holds for both MCh and sub-MCh models (Fig. 2).

Nonetheless, one would expect the greater specific heating rate (εdecay; see the previous section) for the sub-MCh models at a given 56Ni mass to result in even bluer colours. As seen in Fig. 2, this effect is present and more pronounced at the faint end (up to ~0.6 mag bluer B − V colour at a given 56Ni mass), where the relative difference in εdecay is largest (see Table 1). In contrast, all models brighter than M0 = −19 mag at peak have very similar B − V colours at maximum (~0.03 mag standard deviation).

The sub-MCh models follow the observed relation all the way to the faint end (peak MB ≳ −17 mag), while the MCh models are systematically too red at a given peak MB. This is also the case for the least luminous sub-MCh model (SCH1p5), in which the 56Ni mass is too low (<0.08 M⊙), where all the other models have ≥0.12 M⊙ of 56Ni; see Table 1) to efficiently heat the spectrum-formation layers at maximum light. In fact, the only model to have an even redder B − V colour at maximum is the MCh model DDC25, which has the lowest specific heating rate of all (εdecay < 10^9 erg s^-1 g^-1).

5 THE POST-MAXIMUM COLOUR EVOLUTION OF LOW-LUMINOSITY SNe Ia

We focus on three models with similar 56Ni mass (within ±0.02 M⊙ of 0.23 M⊙) to better illustrate the impact of Mtot on the post-maximum colour evolution of low-luminosity SNe Ia: the standard MCh delayed-detonation model DDC22, the pulsational MCh delayed-detonation model PDDEL12, and the sub-MCh model SCH3p0, corresponding to the pure central detonation of a 0.95 M⊙ C–O WD. Their B-band light curves (normalized to the same peak magnitude) and B − V colour evolution is shown in Fig. 3. One clearly sees the faster evolution of the B-band light curve for the sub-MCh model (in part due to the faster bolometric evolution; see Fig. 3 inset), with a ~2 d shorter rise time and a >0.3 mag larger ΔM15(B) (see Table 1), accompanied by a faster evolution of the B − V colour around maximum light. This results in a gradual decrease in the difference in B − V colour compared to the MCh models, from >0.5 mag at B-band maximum to <0.3 mag 15 d later.

The greater specific heating rate of the sub-MCh model (εdecay; see Section 3) results in a relatively modest increase of the temperature in the spectrum-formation region at maximum light compared to the MCh models (on the order of 10–15 per cent). Combined with the lower ejecta density, this slightly larger temperature is nonetheless sufficient to induce a change in the mean ionization state of the gas, enhancing the n/α ionization ratio of IGEs (Sc, Ti, Cr, Fe and Co) that would otherwise effectively block the flux in the B band and emit in redder bands in their once-ionized state (especially Fe ii; see Appendix B).

The bluer B − V colour of the sub-MCh model thus results from both an overall shift of the SED to the blue (akin to a purely thermal effect) and a modulation of line-blanketing from changes in
ionization. Both effects are clearly visible in the maximum-light spectra shown in Fig. 4. By 15 d past B-band maximum, the temperature in the spectrum-formation region of the sub-M\textsubscript{Ch} model has decreased sufficiently to enhance the absorption by singly ionized IGEs to a level comparable to that in the M\textsubscript{Ch} models. The differences in B-band flux as well as in the overall SED shape are greatly reduced at this phase, although the sub-M\textsubscript{Ch} model remains slightly bluer, thereby limiting the B-band post-maximum decline rate.

6 THE WIDTH-LUMINOSITY RELATION

The WLR for the M\textsubscript{Ch} and sub-M\textsubscript{Ch} models is shown in Fig. 5. At the bright end (peak \( M_B \lesssim -18.5 \) mag), both M\textsubscript{Ch} and sub-M\textsubscript{Ch} models follow the observed WLR (grey data points in Fig. 5), albeit with slightly steeper slopes (similar for the DDC and SCH series, and somewhat shallower for the PDD series). At the faint end (peak \( M_B \gtrsim -18.5 \) mag), the M\textsubscript{Ch} models display a turnover to an anti-WLR, reaching a maximum \( \Delta M_{15}(B) \) value of \( \sim 1.4 \) mag (DDC series) and \( \sim 1.3 \) mag (PDDEL series). These models correspond to the same ones that have a too red \(-B-V\) colour at maximum light in Fig. 2 (highlighted with red circles in both figures), due to the less efficient heating of the spectrum-formation region from the lower \( \dot{\varepsilon}_{\text{decay}} \) (Section 4). Combined with a modest post-maximum evolution of their \(-B-V\) colour, the resulting B-band decline rate is then too low for the M\textsubscript{Ch} models to reproduce the faint end of the WLR despite their low luminosity.

In the sub-M\textsubscript{Ch} models, the more rapid colour evolution around maximum light combined with a larger bolometric decline rate result in a higher \( \Delta M_{15}(B) \) for the same \( \Delta M_{15}(B) \) value than the M\textsubscript{Ch} model with the same \( 56\text{Ni} \) mass, DDC25 (\( M(56\text{Ni}) = 0.12 \text{ M}_\odot \) for both).

Note, however, that the sub-M\textsubscript{Ch} models also transition to an anti-WLR, albeit at much fainter magnitudes (peak \( M_B \gtrsim -17 \) mag), and that they fail to reach the highest observed \( \Delta M_{15}(B) \) values. We speculate that small variations in the \( M(56\text{Ni})/M_{\text{tot}} \) ratio or in the \( 56\text{Ni} \) distribution within the ejecta (due to explosion asymmetries or large-scale mixing) could enhance the rate of \( \gamma\)-ray escape and lead to a better agreement with the data in this respect. We will discuss this in more detail in a companion paper in which we compare the SCH2p0 model to the low-luminosity SN 1999by.

The sub-M\textsubscript{Ch} models of Sim et al. (2010) yield systematically larger \( \Delta M_{15}(B) \) values compared to our SCH models at a given peak \( M_B \), by up to \( \sim 0.6 \) mag for their two most luminous models and \( \sim 0.3 \) mag for their two least luminous models (Fig. 5). Their most luminous model (\( M(56\text{Ni})/M_{\text{tot}} = 0.81/1.15 \text{ M}_\odot \), similar to our SCH7p0 model) declines too slowly for its luminosity, with \( \Delta M_{15}(B) \approx 1.3 \) mag for a peak \( M_B \approx -20 \), where SNe Ia of comparable luminosity (and our SCH7p0 model) have \( \Delta M_{15}(B) \approx 0.8 \) mag. Their least luminous model, however, is in better agreement with the observed WLR, with \( \Delta M_{15}(B) \approx 1.8 \) mag, where our SCH1p5 model only has \( \Delta M_{15}(B) \approx 1.5 \) mag for the same peak \( M_B \approx -16.6 \). Differences in the nucleosynthetic post-processing of

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**Figure 3.** Normalized B-band light curves (top) and \( B-V \) colour evolution (bottom) for the standard M\textsubscript{Ch} delayed-detonation model DDC22 (dashed line), the pulsational M\textsubscript{Ch} delayed-detonation model PDDEL12 (dotted line), and the sub-M\textsubscript{Ch} model SCH3p0 (\( M_{\text{tot}} = 0.95 \text{ M}_\odot \); solid line), all of which have a similar \( 56\text{Ni} \) mass of 0.23 \( \text{ M}_\odot \). The inset shows the normalized bolometric light curves, where the time axis now corresponds to days from bolometric maximum.

**Figure 4.** Optical spectra at B-band maximum and 15 d later, for the standard M\textsubscript{Ch} delayed-detonation model DDC22 (red line), the pulsational M\textsubscript{Ch} delayed-detonation model PDDEL12 (yellow line), and the sub-M\textsubscript{Ch} model SCH3p0 (\( M_{\text{tot}} = 0.95 \text{ M}_\odot \); blue line), all of which have a similar \( 56\text{Ni} \) mass of 0.23 ± 0.02 \( \text{ M}_\odot \). The spectra have been normalized to their mean flux in the range 3000–10 000 Å. The upper panel shows the normalized transmission curves for the B and V filters.
the explosion models and in the radiative-transfer treatment are the likely causes of these offsets (Sim, private communication).

As was the case for the bolometric light curves (Section 3), the faster $B$-band post-maximum decline of the sub-$M_{\text{Ch}}$ models is associated with a shorter $B$-band rise time, as observed for low-luminosity SNe Ia (Ganeshalingam, Li & Filippenko 2011). To match the current practice of observers, the time reference is set to maximum light. Studies of the WLR thus ignore the scatter in rise times for SNe Ia with different peak $M_B$. While determining the precise time of explosion is observationally complicated, the recent examples of SN 2011fe (Nugent et al. 2011), SN 2013dy (Zheng et al. 2013) and three SN Ia candidates discovered during the Kepler mission (Olling et al. 2015) illustrate the potential of high-cadence searches to constrain the rise time. Future studies of the WLR will then need to consider the pre-maximum rise as well as the post-maximum decline when distinguishing between fast and slow light-curve evolution.

7 DISCUSSION AND CONCLUSIONS

The predicted observational signatures of pure central detonations in sub-$M_{\text{Ch}}$ WDs corroborate a number of distinctive properties of low-luminosity SNe Ia. At a given $^{56}\text{Ni}$ mass, the lower ejecta mass compared to $M_{\text{Ch}}$ models results in a faster bolometric evolution around maximum light, which affects all photometric bands. Furthermore, the larger $M(^{56}\text{Ni})/M_{\text{tot}}$ ratio leads to a bluer $B - V$ colour at maximum light and a higher post-maximum $B$-band decline rate, $\Delta M_{15}(B)$, in better agreement with the observed WLR. Conversely, the lower specific heating rate in the $M_{\text{Ch}}$ models considered here results in a $B - V$ colour at maximum that is too red compared to observations (through excess absorption by singly ionized IGEs and weaker emission in the blue), and hence a more modest post-maximum colour evolution that imposes an upper limit on the $\Delta M_{15}(B)$ value.

Even then, the sub-$M_{\text{Ch}}$ models fail to match the fastest decline rates observed for low-luminosity, 91bg-like SNe Ia. Variations in the $M(^{56}\text{Ni})/M_{\text{tot}}$ ratio or in the $^{56}\text{Ni}$ distribution within the ejecta could enhance the rate of $\gamma$-ray escape and result in a more efficient cooling of the spectrum-formation region at these phases. Sim et al. (2010) also find that a modest change in the initial WD composition (replacing 7.5 per cent by mass of $^{12}\text{C}$ with $^{22}\text{Ne}$) has a non-negligible impact on the resulting $\Delta M_{15}(B)$ value. We note that a similar $M(^{56}\text{Ni})/M_{\text{tot}}$ ratio as our low-luminosity sub-$M_{\text{Ch}}$ models can, in principle, be achieved in the context of $M_{\text{Ch}}$ models if the explosion is too weak to completely unbind the WD, leaving behind a bound remnant (see Fink et al. 2014).

The sub-$M_{\text{Ch}}$ (and $M_{\text{Ch}}$) models studied here remain somewhat artificial for genuine quantitative predictions to be made, yet the level of sophistication enabled by the 1D treatment of the radiative transfer enables us to qualitatively assess the impact of a lower ejecta mass on the ionization balance for a given $^{56}\text{Ni}$ mass, which is crucial in explaining the observed WLR. Ultimately, our results will need to be confirmed by more realistic, multidimensional models, although radiative-transfer simulations with the same level of

Figure 5. WLR for the $M_{\text{Ch}}$ (DDC series, circles; PDDEL series, squares) and sub-$M_{\text{Ch}}$ models (filled diamonds; the numbers associated with each symbol corresponds to the $^{56}\text{Ni}$ and progenitor WD mass, respectively). Models with a similar $^{56}\text{Ni}$ mass (within $\pm 0.02 M_\odot$) are connected with a dotted line. Also shown are the pure C–O sub-$M_{\text{Ch}}$ models of Sim et al. (2010) (open diamonds), as well as measurements taken from Hicken et al. (2009) and Taubenberger et al. (2008) (grey points). Models that have transitioned to an anti WLR are highlighted with a red circle. These correspond to the same models that have a too red $B - V$ colour at maximum light in Fig. 2. Only the sub-$M_{\text{Ch}}$ models yield $B$-band light curves with a decline-rate $\Delta M_{15}(B) > 1.4$ mag.

7 DISCUSSION AND CONCLUSIONS

The predicted observational signatures of pure central detonations in sub-$M_{\text{Ch}}$ WDs corroborate a number of distinctive properties of low-luminosity SNe Ia. At a given $^{56}\text{Ni}$ mass, the lower ejecta mass compared to $M_{\text{Ch}}$ models results in a faster bolometric evolution around maximum light, which affects all photometric bands. Furthermore, the larger $M(^{56}\text{Ni})/M_{\text{tot}}$ ratio leads to a bluer $B - V$ colour at maximum light and a higher post-maximum $B$-band decline rate, $\Delta M_{15}(B)$, in better agreement with the observed WLR. Conversely, the lower specific heating rate in the $M_{\text{Ch}}$ models considered here results in a $B - V$ colour at maximum that is too red compared to observations (through excess absorption by singly ionized IGEs and weaker emission in the blue), and hence a more modest post-maximum colour evolution that imposes an upper limit on the $\Delta M_{15}(B)$ value.

Even then, the sub-$M_{\text{Ch}}$ models fail to match the fastest decline rates observed for low-luminosity, 91bg-like SNe Ia. Variations in the $M(^{56}\text{Ni})/M_{\text{tot}}$ ratio or in the $^{56}\text{Ni}$ distribution within the ejecta could enhance the rate of $\gamma$-ray escape and result in a more efficient cooling of the spectrum-formation region at these phases. Sim et al. (2010) also find that a modest change in the initial WD composition (replacing 7.5 per cent by mass of $^{12}\text{C}$ with $^{22}\text{Ne}$) has a non-negligible impact on the resulting $\Delta M_{15}(B)$ value. We note that a similar $M(^{56}\text{Ni})/M_{\text{tot}}$ ratio as our low-luminosity sub-$M_{\text{Ch}}$ models can, in principle, be achieved in the context of $M_{\text{Ch}}$ models if the explosion is too weak to completely unbind the WD, leaving behind a bound remnant (see Fink et al. 2014).

The sub-$M_{\text{Ch}}$ (and $M_{\text{Ch}}$) models studied here remain somewhat artificial for genuine quantitative predictions to be made, yet the level of sophistication enabled by the 1D treatment of the radiative transfer enables us to qualitatively assess the impact of a lower ejecta mass on the ionization balance for a given $^{56}\text{Ni}$ mass, which is crucial in explaining the observed WLR. Ultimately, our results will need to be confirmed by more realistic, multidimensional models, although radiative-transfer simulations with the same level of
sophistication as ours are currently not feasible in 3D (see e.g. Sim et al. 2013).

But do such low-mass explosions occur in Nature? Detonations of sub-$M_{\odot}$ WDs require an external trigger, usually via thermal instabilities in a layer of accreted He (double detonation mechanism; see e.g. Woosley & Weaver 1994). This layer can detonate at a lower mass than previously thought (Bildsten et al. 2007), but the resulting IGEs synthesized at high velocities cause discrepant colours and spectra at early times (Kromer et al. 2010). More recent calculations by Shen & Moore (2014), however, show that a detonation in a 0.005 $M_{\odot}$ He shell on a 1.0 $M_{\odot}$ WD only produces $^{26}$Si and $^1$He, and that significant $^{44}$Ti/$^{48}$Cr production only occurs for shells more massive than $\sim 0.01 M_{\odot}$. Double detonations thus remain an attractive scenario for fast-declining SNe Ia for which empirical determinations of the ejecta mass indicate sub-$M_{\odot}$ progenitors (Scalzo et al. 2014a,b).

Likewise, He-ignited violent mergers (Pakmor et al. 2013), where the dynamical detonation of 0.01 $M_{\odot}$ of He is sufficient to trigger a detonation in the C–O core, also synthesize very little IGEs in the outer layers ($\sim 10^{-8} M_{\odot}$). In particular, Pakmor et al. (2013) argue that low-luminosity SNe Ia result from the violent merger of a C–O+He WD system, which is less massive (and hence results in a more rapid light-curve evolution) than double C–O WD mergers. The predicted rate of such systems can, in principle, account for the observed SN Ia rate (Ruiter et al. 2011), and their long delay times would corroborate the association of low-luminosity SNe Ia with older stellar populations (e.g. Howell 2001).

Another possibility, suggested by van Kerkwijk, Chang & Justham (2010), involves the merger of two low-mass WDs whose combined mass is less than the Chandrasekhar limit (a 0.6 + 0.6 $M_{\odot}$ merger in their paper). The explosion does not occur during the merger event, but at a later time via convectional heating through accretion of the thick disc from the merger remnant (see Lorén-Aguilar, Isern & García-Berro 2009). However, this scenario is unlikely to hold for a combined merger mass $\lesssim 1.2 M_{\odot}$, given the high-temperature and high-density conditions required to ignite carbon (van Kerkwijk, private communication), and hence would fail to reproduce the fast-declining SNe Ia at the faint end of the WLR, which we argue result from ejecta masses $\lesssim 1 M_{\odot}$. In this respect, head-on collisions of two 0.5 $M_{\odot}$ WDs represent an interesting alternative: Kushnir et al. (2013) find that such a model yields 0.11 $M_{\odot}$ of $^{56}$Ni, comparable to our SCH2p0 model. However, most studies of this progenitor channel conclude that WD collisions can account for at most a few per cent of the observed SN Ia rate (see e.g. Papish & Perets 2016).

Regardless of the precise ignition mechanism, the results presented in this paper strongly suggest that explosions of sub-$M_{\odot}$ WDs are a viable scenario for fast-declining SNe Ia at the faint end of the WLR, whose colours cannot be reproduced by $M_{\odot}$ models. In several upcoming companion papers, we will present an in-depth study of the sub-$M_{\odot}$ model SCH2p0 compared to the low-luminosity SN 1999by, and explore more generally the feasibility of such sub-$M_{\odot}$ models to reproduce the observed properties of more luminous events, to address the question of multiple progenitor channels for Type Ia supernovae.

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APPENDIX A: MODEL ATOMS

Table A1 gives the number of levels (both super levels and full levels; see Hillier & Miller 1998 and Dessart & Hillier 2010 for details) for the model atoms used in the radiative-transfer calculations presented in this paper.

Oscillator strengths for CNO elements were originally taken from Kurucz (2009); other sources include Berrington (1983) and the Iron Project (Hummer et al. 1993). Unfortunately Ni and Co photoionization data are generally unavailable, and we have utilized crude approximations. Charge exchange cross-sections are from the tabulation by Kingdon & Ferland (1996).

2 Data are available online at http://kurucz.harvard.edu
Figure B1. Contribution of individual ions (bottom panels) to the full optical synthetic spectra of the standard $M_{\text{Ch}}$ delayed-detonation model DDC22 (red line), the pulsational $M_{\text{Ch}}$ delayed-detonation model PDDEL12 (yellow line), and the sub-$M_{\text{Ch}}$ model SCH3p0 ($M_{\text{tot}} = 0.95 M_{\odot}$; blue line), all of which have a similar $^{56}\text{Ni}$ mass of $0.23 \pm 0.02 M_{\odot}$, at $B$-band maximum light (left) and 15 d later (right). The spectra have been normalized to their mean flux in the range 3000–10 000 Å. Only ions that impact the flux at the >10 per cent level at either phase are shown. The Ca II ion spectra marked with a "*" have been scaled down for clarity.

**APPENDIX B: CONTRIBUTION OF INDIVIDUAL IONS TO THE TOTAL OPTICAL FLUX**

Fig. B1 reveals the contribution of individual ions (bottom panels) to the full optical synthetic spectra of the standard $M_{\text{Ch}}$ delayed-detonation model DDC22, the pulsational $M_{\text{Ch}}$ delayed-detonation model PDDEL12 and the sub-$M_{\text{Ch}}$ model SCH3p0 ($M_{\text{tot}} = 0.95 M_{\odot}$), all of which have a similar $^{56}\text{Ni}$ mass of $0.23 \pm 0.02 M_{\odot}$, at $B$-band maximum light and 15 d later. Only ions that impact the flux at the >10 per cent level at either phase are shown.

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