The formation of disk-bulge-halo systems
and the origin of the Hubble sequence

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ABSTRACT

We investigate the formation of disk-bulge-halo systems by including bulges in the Fall & Efstathiou theory of disk formation. This allows an investigation of bulge dominated disk galaxies, such as S0s and disky ellipticals. These latter systems, which consist of an elliptical spheroid with an embedded disk with a scale-length of typically a few hundred parsecs, seem to form a smooth sequence with spirals and S0s towards lower disk-to-bulge ratio. The aim of this paper is to examine whether spirals, S0s, and disky ellipticals all can be incorporated in one simple galaxy formation scenario. We investigate an inside-out formation scenario in which subsequent layers of gas cool and form stars inside a virialized dark halo. The inner, low angular momentum material is assumed to form the bulge. Stability arguments are used to suggest that this bulge formation is a self-regulating process in which the bulge grows until it is massive enough to allow the remaining gas to form a stable disk component. We assume that the baryons that build the disk do not loose their specific angular momentum, and we search for the parameters and physical processes that determine the disk-to-bulge ratio, and therewith explain to a large extent the origin of the Hubble sequence. The spread in halo angular momenta coupled with a spread in the formation redshifts can explain the observed spread in disk properties and disk-to-bulge ratios from spirals to S0s. If galaxy formation is efficient, and all available baryons are transformed into the disk-bulge system, cosmologies with $\Omega_0 \lesssim 0.3$ can be excluded, since stable spiral disks would not be allowed to form. However, if we assume that the efficiency with which galaxies form depends on the formation redshift, as suggested by the small amount of scatter in the observed Tully-Fisher relation, and we assume that the probability for a certain baryon to ultimately end up in the disk or bulge is independent of its specific angular momentum, spirals are allowed to form, but only at small formation redshifts ($z \lesssim 1$). At higher formation redshifts, stability arguments suggest the formation of systems with smaller disk-to-bulge ratios, such as S0s. Since density perturbations in clusters will generally collapse earlier than those in the field, this scenario naturally predicts a density-morphology relation, the amplitude of which depends on the baryon fraction of the Universe. Disky ellipticals are too compact to be incorporated in this scenario, and they thus do not form a continuous sequence with spirals and S0s, at least not in the sense of the galaxy formation scenario envisioned in this paper. Alternative formation scenarios for the disky ellipticals, such as gas-rich mergers or an internal mass loss origin for the embedded disks, are much more viable.

Subject headings: galaxies: formation — galaxies: general — galaxies: halos — instabilities
1. Introduction

One of the most compelling puzzles in present day astronomy is the question how galaxies formed. In particular, we need to understand the wide variety of sizes, masses and morphologies of galaxies observed, as well as their dynamics, ages and metallicities. In addition, we need to understand the origin of the scaling relations, such as the fundamental plane relations for ellipticals and the Tully-Fisher relation for spirals, as well as the so-called density-morphology relation (Dressler 1980). The latter shows that the more compact galaxies, such as ellipticals and S0s, are preferentially found in overdense regions such as galaxy clusters.

The classical way of depicting the different morphologies of galaxies is by means of the Hubble diagram, which reveals the gross distinction of three sorts of galaxies: spirals, S0s, and ellipticals. Several studies over the past years have shown that elliptical galaxies can be roughly divided in two subclasses: the rotation supported, low luminosity disky ellipticals and the pressure supported, bright, boxy ellipticals (e.g., Bender 1988; Bender et al. 1989; Capaccioli, Caon & Rampazzo 1990). This dichotomy has recently been strengthened by properties observed in their central regions (e.g., Nieto, Bender & Surma 1991; Jaffe et al. 1994; Ferrarese et al. 1994; Gebhardt et al. 1996; Faber et al. 1997). Based on this dichotomy amongst elliptical galaxies, it has been suggested that the classical Hubble diagram should be revised (Kormendy & Bender 1996), to use a more physical subdivision of the class of the ellipticals, rather than the apparent flattening. Kormendy & Bender proposed to use the velocity anisotropy which is well measured by the isophote shapes (Bender 1988; Bender et al. 1989).

The main morphological parameter that sets the classification of galaxies in the (revised) Hubble diagram is the disk-to-bulge ratio $D/B$. Understanding the origin of the Hubble sequence is thus intimately related to understanding the parameters and processes that determine the ratio between the masses of disk and bulge. The diskiness of the low luminosity ellipticals is generally interpreted as due to an embedded disk. These disks, which we term ‘embedded disks’ in the following, are smaller and brighter than disks in S0s and spirals (Scorza & Bender 1995), and it has been emphasized many times that the disky ellipticals build a continuous sequence with S0 galaxies (Capaccioli 1987; Carter 1987; Bender 1988, 1990; van den Bergh 1990; Capaccioli et al. 1990; Rix & White 1990; Capaccioli & Caon 1992; Bender et al. 1993; Scorza & Bender 1995, 1996; Scorza & van den Bosch 1998). This continuity suggests similar formation histories, whereby one or several parameters of the proto-galaxy vary smoothly.

Disk dominated systems such as spirals are believed to have formed by cooling of the baryonic matter inside a virialized dark halo. As the gas cools, its specific angular momentum is conserved, and the amount of angular momentum of the dark halo thus determines the size of the disk (Fall & Efstathiou 1980; see Section 2 for more details on this disk-formation scenario). The formation of bulge dominated disk systems is far less clear. The aim of this paper is to investigate to what extent the disky ellipticals and S0s can be incorporated in the Fall & Efstathiou theory for the formation of galactic disks, by incorporating a simple picture for the formation of the bulge.

We envision an inside-out formation scenario for the bulge. It is assumed that the bulge forms out of the low-angular momentum material in the halo, which cools and tries to settle into
a small, compact disk. Such disks are however unstable, and we assume here that this instability, coupled with the continuous supply of new layers of baryonic matter that cool and collapse, forms the bulge. This inside-out bulge formation is self-regulated in that the bulge grows until it is massive enough to allow the remaining gas to form a stable disk component. We do not describe the bulge formation in any detail but merely use empirical relations of the characteristic structural parameters of bulges and ellipticals to describe the end result of the formation process as a realistic galaxy. We use this simple formation scenario to investigate the predicted disk-to-bulge ratios and disk scale-lengths as a function of the halo angular momentum, and as a function of formation redshift and cosmology. The main focus of this paper is to investigate whether this inside-out formation scenario can account for two orders of magnitude variation in disk-to-bulge ratio, required in order to incorporate a wide variety of disk-bulge systems: from late-type spirals with $D/B \gtrsim 10$ to S0s ($D/B \sim 0.1$) to disky ellipticals ($D/B \sim 0.1$).

This paper is organized as follows. In Section 2 we describe the formation scenario, the galaxy formation efficiency, and the stability criterion for our disk-bulge-halo models. In Section 3 we use these models to investigate the position of different sorts of disks in the parameter space of disk central surface brightness versus disk scale-length. In Section 4 we discuss to what extent S0s and disky ellipticals can be incorporated in this formation scenario, and what parameters may be responsible for the origin of (the major part of) the Hubble sequence. In Section 5 we briefly discuss alternative formation scenarios for disky ellipticals. Our results are summarized and discussed in Section 6.

2. The formation scenario

A remarkably successful model for the formation of galaxies is the White & Rees (1978) theory, wherein galaxies form through the hierarchical clustering of dark matter and subsequent dissipational settling of gaseous matter within the dark halo cores. Coupled with the notion of angular momentum gain by tidal torques induced by nearby proto-galaxies (Hoyle 1953; Peebles 1969), the White & Rees theory provides the background for a model for the formation of galactic disks (Fall & Efstathiou 1980; Faber 1982; Gunn 1982; Fall 1983; van der Kruit 1987). In this model, disks form through the collapse of a uniformly rotating proto-galaxy. Owing to the non-dissipative character of the dark matter, its collapse halts when the system virializes. At regions where the baryonic density is high enough, the baryons can cool and decouple from the dark matter. The angular momentum of the baryons, acquired from tidal torques from nearby proto-galaxies, prevents the collapse from proceeding all the way to the center, and causes the baryons to settle in a rapidly rotating disk. This simple picture of disk formation yields realistic disk sizes and surface brightness profiles provided that there is little angular momentum transfer from the baryonic matter to the dark halo, and that the dark halo has a density profile such that the resulting galaxy has a flat rotation curve. Fall (1983) extended the original Fall & Efstathiou disk formation scenario by placing it in a cosmological context, and by deriving Freeman’s law (Freeman 1970) and the Tully-Fisher relation. Recently, two papers reexamined this disk formation model and included the effects of halo-contraction due to the accumulation of baryonic matter in
the center of the galaxy: Dalcanton, Spergel & Summers (1997) showed that the disk formation model outlined above can not only explain the properties of normal, high surface brightness (HSB) disks, but also those of the class of low surface brightness (LSB) disks (see also Jimenez et al. 1998); Mo, Mao & White (1998) compared the outcome of disks in different cosmologies, and used this to investigate the effects of disk evolution and the predicted population of Lyα absorbers in QSO spectra. The assumptions we make are similar to those made by Dalcanton et al. (1997) and Mo et al. (1998), but we extend upon their work by including bulges. We investigate how the disky ellipticals and S0s may be fitted into this particular formation scenario, by comparing the differences in properties of the proto-galaxies that result in spirals, S0s, and disky ellipticals.

2.1. Inside-out bulge formation

The turn-around, virialization, and subsequent dissipational settling of the baryonic matter of a proto-galaxy is an inside-out process. First the innermost shells virialize and heat its baryonic material to the virial temperature. Because of the relatively high density and low angular momentum of this gas, it will rapidly cool and (try to) settle in a very small, compact disk. However, self-gravitating disks are violently unstable and form a bar (e.g., Hohl 1971; Ostriker & Peebles 1973). Meanwhile the next shell of the proto-galaxy cools and tries to settle into a disk structure at a radius determined by its angular momentum. The resulting structure in the center of the dark halo consequently becomes more and more unstable.

Bars are efficient in transporting gas inwards (e.g., Friedli & Martinet 1993; Wada & Habe 1995), and can cause vertical heating by means of a collective bending instability (e.g., Combes et al. 1990; Pfenniger 1984; Pfenniger & Friedli 1991; Raha et al. 1991). Both these process lead ultimately to the dissolution of the bar; first the bar takes a hotter and triaxial shape, but is later on transformed in a spheroidal bulge component (e.g., Combes et al. 1990; Pfenniger & Norman 1990; Pfenniger & Friedli 1991; Friedli 1994; Martinet 1995; Norman, Sellwood & Hasan 1996). Here we assume that this bar instability and subsequent dissolution, coupled with the continuous supply of new layers of baryonic matter that cool and collapse, forms the bulge. The numerical simulations performed by the numerous studies listed above, have been able to show that bars can build small and close to exponential bulges (see also Courteau, de Jong & Broeils 1996 for observational evidence for bulge formation out of disks). However, no simulations have been able to use bar instabilities to produce very large bulges or even ellipticals. On the other hand, all these simulations started from a normal, marginally unstable disk, whereas the process we are describing starts from a highly unstable, compact disk, includes the continuous supply of baryonic matter from prolonged cooling flows, and is likely to have additional heating sources due to the star formation and feedback processes which are expected to take place in this phase of the bulge formation. Detailed numerical simulations that take all these processes into account are required in order to investigate whether this can indeed lead to massive bulges as assumed here. Finally, we emphasize that the high densities of the inner shells yield cooling time-scales that may be significantly shorter than the dynamical time-scale. This can add to the inside-out formation of a bulge component, as suggested by Kepner (1997).
We assume that the bulge formation process discussed above continuous until the bulge has become massive enough such that the subsequent layers of baryonic material can cool and form a disk which is stable against bar-formation. The process of disk-bulge formation is thus a self-regulating one in that the bulge grows until it is massive enough to sustain the remaining gas in the form of a stable disk. Since the stability is directly related to the amount of angular momentum of the gas (see Section 2.4), we expect a clear correlation between the disk-to-bulge ratio and the angular momentum of the dark halo out of which the galaxy is assembled. We will show that this correlation is present, and that it nicely follows the marginal stability curve as expected from the self-regulating mechanism proposed here.

2.2. A disk-bulge-halo model

The galaxies under investigation in this paper, varying from spirals to disky ellipticals, are assumed to consist of a disk plus bulge embedded in a dark halo. Virialized dark halos can be quantified by a mass $M$, a radius $r_{200}$, and an angular momentum $J$ (originating from cosmological torques). Spherical collapse models (e.g., Gunn & Gott 1972) show that virialized halos have an over-density of approximately 200. In a recent series of papers Navarro, Frenk & White (1995; 1996; 1997, hereafter NFW) showed that virialized dark halos have universal equilibrium density profiles, independent of mass or cosmology (but see Moore et al. 1997), which can be well fit by

$$\rho(r) = \rho_{\text{crit}} \frac{\delta_0}{(r/r_s)(1 + r/r_s)^2},$$

where

$$\delta_0 = \frac{200}{3} \ln(1 + c) - c/(1 + c),$$

with $c = r_{200}/r_s$ the concentration parameter, $r_s$ a scale radius, and $\rho_{\text{crit}}$ the critical density for closure. The radius $r_{200}$ is defined as the limiting radius of a virialized halo, and corresponds to the radius within which the mean density is $200 \rho_{\text{crit}}$. Throughout we only consider spherical halos. Rather than specifying the halo by its mass $M$, it is customary to specify it by its circular velocity $V_{200} = \sqrt{GM/r_{200}}$. Given the mass $M$, the formation redshift $z_{\text{form}}$, and the specific cosmology, the concentration parameter $c$ can be calculated using the procedure outlined in Appendix A of Navarro, Frenk & White (1997). In this paper we compare two different cosmologies: a standard cold dark matter cosmology (SCDM) with a total matter density of $\Omega_0 = 1.0$, a Hubble constant of $H_0 = 100h \text{ km s}^{-1} \text{Mpc}^{-1}$ with $h = 0.5$, and with a rms linear over-density at $z = 0$ in spheres of radius $8h^{-1} \text{ Mpc}$ of $\sigma_8 = 0.6$. This latter parameter normalizes the CDM power spectrum. The second cosmology discussed here is an open cold dark matter model (OCDM) with $\Omega_0 = 0.3$, $h = 0.6$, and $\sigma_8 = 0.68$. Both models have a zero cosmological constant and the parameters are typical of those favored by large-scale structure constraints. The baryon density is set to be $\Omega_{\text{bar}} = 0.0125h^{-2}$, in agreement with the nucleosynthesis constraints (e.g., Walker et al. 1991). The baryonic mass fraction is then given by $\Omega_{\text{bar}}/\Omega_0$.

We assume the final system to have a disk-to-bulge ratio $t \equiv M_d/M_b$, such that the masses of
disk, $M_d$, and bulge, $M_b$, can be written as

$$M_d = \epsilon_{gf} \frac{t}{1 + t} \frac{\Omega_{\text{bar}}}{\Omega_0} M \equiv m_d M,$$

(3)

and

$$M_b = \epsilon_{gf} \frac{1}{1 + t} \frac{\Omega_{\text{bar}}}{\Omega_0} M \equiv m_b M.$$

(4)

Here $\epsilon_{gf}$ is the galaxy formation efficiency, expressed as the fraction of the available baryons that ultimately end up in the galaxy (see Section 2.3). As mentioned in Section 2.1, we assume that the bulge is formed out of the low-angular momentum material. Furthermore, we assume that the material that builds the disk does not lose its specific angular momentum. Under these conditions we have that $J_d \equiv j_d J$, where

$$j_d = f(t, \epsilon_{gf})[m_d + m_b].$$

(5)

The function $f(t, \epsilon_{gf})$ depends on the disk-to-bulge ratio and on the detailed physics that describe how the fraction $\epsilon_{gf}$ of the baryonic material ends up in the disk-bulge system (see Section 2.3). This function is derived in the Appendix. In the following we parameterize $J$ by the dimensionless spin parameter $\lambda$, defined by

$$\lambda = \frac{|J|E|^{1/2}}{GM^{5/2}}.$$  

(6)

Here $E$ is the halo’s total energy. Several studies, both analytical and numerical, have shown that the distribution of spin angular momentum of collapsed dark matter halos is well approximated by a log normal distribution:

$$p(\lambda)d\lambda = \frac{1}{\sigma\lambda\sqrt{2\pi}} \exp\left(-\frac{\ln^2(\lambda/\bar{\lambda})}{2\sigma^2}\right) \frac{d\lambda}{\lambda},$$

(7)

(e.g., Barnes & Efstathiou 1987; Ryden 1988; Cole & Lacey 1996; Warren et al. 1992). We consider a distribution that peaks around $\bar{\lambda} = 0.05$ with $\sigma = 0.7$. These values are in good agreement with the $N$-body results of Warren et al. (1992).

We assume the disk to be an infinitesimally thin exponential disk, with surface brightness

$$\Sigma_d(R) = \Sigma_{d,0} \exp(-R/R_d).$$

(8)

and total luminosity

$$L_d = 2\pi\Sigma_{d,0}R_d^2.$$  

(9)

In order to convert the disk parameters to an observer’s frame we need to assume a mass-to-light ratio $\Upsilon_d$. Bottema (1997) found $\Upsilon_B = (1.79 \pm 0.48)$ for $h = 0.75$ for HSB disks. Using $B - V \approx 0.8$ (e.g., de Jong 1996b) and $(B - V)_\odot = 0.65$ this yields $\Upsilon_d = 2.1h$ in the $V$-band.

Although bulges and elliptical galaxies are reasonably well fit by a $r^{1/4}$ ‘de Vaucouleurs’ law, it has recently been shown that a better fit is obtained by using a generalized $r^{1/n}$ law. This fitting function, introduced by Sersic (1968), has the form

$$\Sigma_b(r) = \Sigma_{b,0} \exp\left[-a\left(\frac{r}{r_e}\right)^{1/n}\right].$$

(10)
where $\Sigma_{b,0}$ is the central surface brightness, $r_e$ is the effective radius, and $n$ is the exponential variable. For $n = 1$ one has a pure exponential, while for $n = 4$ one recovers the de Vaucouleurs profile. The parameter $a$ is determined by the requirement that $r_e$ is the radius encircling half of the total luminosity and is very well approximated by

$$a = 2.0n - 0.324,$$

(Ciotti 1991). The total luminosity of a system with a $r^{1/n}$ luminosity profile (assuming sphericity) is given by

$$L_b = \frac{n\Gamma(2n)}{a^{2n}}2\pi\Sigma_{b,0}r_e^2,$$

where $\Gamma$ is the Gamma function. It was found that the best fitting exponent $n$ is correlated with the total luminosity of the spheroid: Andredakis, Peletier & Balcells (1995) used the $r^{1/n}$ law to fit the luminosity profiles of bulges of spiral galaxies, and found the effective radius $r_e$ to be related to the total luminosity of the bulge by the empirical relation

$$M_B = -19.75 - 2.8\log(r_e),$$

(13)

In addition, Caon, Capaccioli & D’Onofrio (1993) found that the exponent $n$ is correlated with the effective radius through the empirical relation

$$\log(n) = 0.28 + 0.52\log(r_e).$$

(14)

Although this empirical relation has only been shown to fit early-type galaxies, we assume here that it is also valid for bulges. We take $n$ to be limited between $n = 1$ (exponential) and $n = 4$ (de Vaucouleurs). None of our results are significantly influenced by this assumption however. The main parameter for the bulge is its total mass; changes in its actual density distribution are only a second order effect. The relations (10) – (14) allow us to describe the bulges by a single parameter; once the total luminosity of the bulge is known, we can use the empirical relations to determine all other bulge parameters! Van der Marel (1991) found that ellipticals have $\Upsilon_R = 6.64h$ (Johnson $R$). Using $V - R_f \approx 0.68$ (e.g., van der Marel 1991) and $(V - R_f)_\odot = 0.52$ this yields $\Upsilon_b = 7.7h$ in the $V$-band. For our combined disk-bulge model we assume that the mass-to-light ratio is constant over the entire galaxy with a value equal to the luminosity weighted average of the mass-to-light ratios of the disk and bulge. Alternatively, we could have assigned the bulge and disk separate mass-to-light ratios of $\Upsilon_d$ and $\Upsilon_b$ respectively, but the mass-to-light ratio of the disks in S0s and disky ellipticals is clearly larger than for spiral discs. Assuming a constant, luminosity weighted ratio is thus more realistic.

Once the baryons inside the virialized dark halo start to cool, this induces a contraction of the inner region of the dark halo. We follow Blumenthal et al. (1986) and Flores et al. (1993) and assume that the halo responds adiabatically to the slow assembly of the baryonic matter in disk and bulge, and that it remains spherical (see also Dalcanton et al. 1997 and Mo et al. 1998). Under these assumptions the angular action of the dark halo material is an adiabatic invariant, such that a particle initially at a radius $r_i$ ends up at mean radius $r$ according to

$$r_i M_i(r_i) = r M_f(r).$$

(15)
Here $M_i(r)$ is the initial mass distribution (the NFW profile) given by

$$M_i(r) = M \left[ \ln(1 + cx) - cx/(1 + c) \right] / \ln(1 + c) - c/(1 + c),$$

with $x = r/r_{200}$, and $M_f(r)$ is the final mass distribution given by

$$M_f(r) = M_d(r) + M_b(r) + (1 - m_d - m_b)M_i(r_i).$$

Here

$$M_d(r) = M_d \left[ 1 - \left(1 + \frac{r}{R_d}\right) \exp(-r/R_d) \right],$$

and

$$M_b(r) = \frac{2a^{2n+1}}{\pi n^2 \Gamma(2n)} M_b \int_0^{r/e} dy \int_0^\infty \frac{y^{\beta-1} \exp(-ax^\beta)}{\sqrt{x^2 - y^2}} dx,$$

with $\beta = 1/n$.

Mo et al. (1998) have shown that the scale-length of the disk can be written as

$$R_d = \frac{1}{\sqrt{2}} \left( \frac{j_d}{m_d} \lambda r_{200} f_c^{-1/2} f_R^{-1} \right),$$

where

$$f_R = \frac{1}{2} \int_0^\infty u^2 e^{-u} \frac{V_c(R_d u)}{V_{200}} du,$$

a parameter that indicates the amount of self-gravity of the disk, and

$$f_c = \frac{c}{2} \frac{1 - 1/(1+c)^2 - 2\ln(1+c)/(1+c)}{[c/(1+c) - \ln(1+c)]^2}.$$

The circular velocity $V_c(r)$ is given by

$$V_c^2(r) = V_{c,d}^2(r) + V_{c,b}^2(r) + V_{c,DM}^2(r),$$

with

$$V_{c,d}^2(r) = \frac{GM_b(r)}{r},$$

$V_{c,d}^2(r)$ as given by equation [2.196] in Binney & Tremaine (1987), and

$$V_{c,DM}^2(r) = \frac{G[M_f(r) - M_d(r) - M_b(r)]}{r}.$$

We can solve this set of equations in an iterative way, following the procedure used by Mo et al. (1998). For a given $M$, $\lambda$, $z_{form}$ and $t$ we first calculate the concentration parameter $c$ of the dark halo before collapse of the baryons. We then calculate the parameters of the bulge using equations (11) – (14). Next we start with a guess for $f_R$, by setting $f_R = 1$, and we calculate the disk parameters $R_d$ and $\Sigma_{0,d}$ using equations (24) and (11) respectively. Subsequently we solve for $r_i$ as a function of $r$ using equations (15) – (19), and thus obtain $M_f(r)$. We then calculate $V_c^2(r)$, and substitute this in equation (22) to obtain a new value for $f_R$. Convergence is rapidly achieved: 3 to 5 iterations yield an accuracy of $R_d$ already better than one percent.
2.3. Estimating the galaxy formation efficiency

Since we have defined the mass of the dark halo such that the average over-density of the halo is 200 times the critical density for closure at the virialization redshift $z$, we have that

$$M = \frac{V_{200}^3}{10GH(z)}.$$  \hspace{1cm} (26)

This halo mass can be related to the disk luminosity (assuming no bulge, i.e., $t = \infty$), by

$$M = \frac{\Upsilon_d L_d}{\epsilon_{gf} (\Omega_{\text{bar}}/\Omega_0)},$$  \hspace{1cm} (27)

(cf. equation 3). Combining the above equations yields

$$L_d = \frac{\epsilon_{gf} (\Omega_{\text{bar}}/\Omega_0) V_{200}^3}{10G\Upsilon_d H(z)}.$$  \hspace{1cm} (28)

This is similar to the Tully-Fisher relation: $L = A (V_c/220)^\gamma$, whereby the zero-point, $A$, is inversely proportional to $H(z)$. However, current data suggest no zero-point evolution (Vogt et al. 1996, 1997; Mao, Mo & White 1997). In other words, if we assume a significant scatter in the formation redshifts of spiral galaxies (some massive spirals are already in place at $z \sim 1$, see Vogt et al. 1996), the dependence of equation (28) would yield a scatter in the Tully-Fisher relation much larger than observed: if spirals form between $z = 0$ and $z = 1$ (and they are not significantly modified since their formation) a scatter of $\sim 1.1$ mag is predicted for an Einstein-de Sitter Universe, much larger than the observed scatter of $\lesssim 0.3$ mag. This suggests that the redshift dependence of the Tully-Fisher zero-point has to be corrected for by either imposing $\Upsilon_d \propto H(z)^{-1}$ or $\epsilon_{gf} \propto H(z)$. We consider the former option unlikely, since this predicts spirals at $z \sim 1.0$ to be much bluer than observed (see Vogt et al. 1996). We therefore consider a scenario in which the galaxy formation efficiency is redshift dependent. Combining the observed Tully-Fisher relation with equation (28) yields

$$\epsilon_{gf} = 0.678 h^2 \Omega_0 \left( \frac{A}{L^*} \right)^{3/\gamma} \left( \frac{L}{L^*} \right)^{(1-3/\gamma)} \frac{H(z)}{H_0},$$ \hspace{1cm} (29)

where $L^* = 1.0 \times 10^{10} h^{-2} L_\odot$. Using the Tully-Fisher relation of Giovanelli et al. (1997), converted to the $V$-band using $V - I = 1.0$ as an average for spirals (de Jong 1996b), yields

$$\epsilon_{gf} = 1.562 h^2 \Omega_0 \frac{H(z)}{H_0},$$ \hspace{1cm} (30)

(i.e., Giovanelli et al. find $\gamma \approx 3.0$). Here we assumed that the HI-linewidth equals twice the circular velocity $V_{200}$.

The important factor here is

$$\frac{H(z)}{H_0} = \sqrt{\Omega_\Lambda + (1 - \Omega_\Lambda - \Omega_0)(1 + z)^2 + \Omega_0 (1 + z)^3},$$  \hspace{1cm} (31)
which shows that the galaxy formation efficiency depends on the specific cosmology and the formation redshift. Here $\Omega_\Lambda = \Lambda / (3H_0^2)$, with $\Lambda$ the cosmological constant. In the SCDM model with $h = 0.5$ and $\Omega_0 = 1.0$, equation (30) yields $\epsilon_{gf} = 0.39(1 + z)^{3/2}$. Galaxies that were assembled at higher redshift thus formed more efficiently. More efficiently here means that a larger fraction of the baryonic matter available in the dark halo actually made it into the disk-bulge system. At $z = 0$ this fraction is approximately forty percent, and this increases to hundred percent at $z \geq 0.87$. Halos of a given mass are denser at higher redshift, and higher density implies higher escape velocity and higher cooling efficiency, both of which could give a plausible explanation for an increase of the galaxy formation efficiency with increasing redshift.

In what follows we consider two extreme cases of this galaxy formation (in)efficiency. In the first scenario, which we call the ‘cooling’-scenario, we assume that only the inner fraction $\epsilon_{gf}$ of the baryonic mass is able to cool and form the disk-bulge system: the outer parts of the halo, where the density is lowest, but which contain the largest fraction of the total angular momentum, never gets to cool. In the second scenario, referred to hereafter as the ‘feedback’-scenario, we assume that the feedback from star formation and evolution is such that each baryon in the dark halo, independent of its specific angular momentum, has a probability of $\epsilon_{gf}$ to ultimately end up as a constituent of the disk or bulge. The choice of either of these scenarios enters our equations in the function $f(t, \epsilon_{gf})$ (see the Appendix).

2.4. Stability

As is well known, disks that are self-gravitating tend to be unstable against bar formation. Halos with smaller $\lambda$ yield disks with smaller scale-lengths (see equation [20]). For a given disk-to-bulge ratio, there is thus a critical $\lambda$ such that for $\lambda < \lambda_{crit}$ the disk will be bar unstable. Christodoulou, Shlosman & Tohline (1995) derived a stability criterion for bar formation based on the angular momentum content of the disk, rather than the energy content (such as the well known Ostriker & Peebles (1973) criterion). For a disk-bulge-halo system this stability criterion can be written such that disks are stable against bar formation if

$$\alpha = \frac{V_d}{2V_c} < \alpha_{crit},$$

(32)

with $V_d$ and $V_c$ the characteristic circular velocities of the disk and the composite disk-bulge-halo system (equation [23]), respectively. This criterion is similar to the one proposed by Efstathiou, Lake & Negroponte (1982). As characteristic velocities we consider the circular velocities at $R = 3R_d$, since Mo et al. (1998) have shown that this is the radius where typically the rotation curve of the composite system reaches a maximum. For a thin exponential disk, approximately 80 percent of the disk mass is located inside $3R_d$, and

$$V_d^2(3R_d) \approx 0.359 \frac{GM_d}{R_d},$$

(33)
Substituting equation (33) in (32), and using equation (20) we find that disks are stable if the spin parameter of their halos obeys

$$\lambda > \sqrt{2} \varepsilon_{\text{crit}} \left( \frac{m_d^2}{j_d} \left( \frac{V_{200}}{V_c(3R_d)} \right) \right)^2 f_1^{1/2} \equiv \lambda_{\text{crit}},$$

with

$$\varepsilon_{\text{crit}} = \frac{\sqrt{0.359}}{2\alpha_{\text{crit}}}.$$  (35)

This reduces to $\varepsilon_{\text{crit}} = 1.15$ for stellar disks and $\varepsilon_{\text{crit}} = 0.86$ for gaseous disks, using $\alpha_{\text{crit}} = 0.26$ and $\alpha_{\text{crit}} = 0.35$, respectively (see Christodoulou et al. 1995). This is in excellent agreement with the empirical value of $\varepsilon_{\text{crit}} = 1.1$ found by Efstathiou et al. (1982) based on N-body experiments. Throughout we take $\varepsilon_{\text{crit}} = 1.0$ to take into account that disks are often made up of a mix of gas and stars.

The critical value of $\lambda$ given by equation (34) depends on the galaxy formation efficiency $\epsilon_{\text{gf}}$, and thus on the specific cosmology and formation redshift according to equation (31). Furthermore, since $\lambda_{\text{crit}} \propto j_d^{-1}$, and $j_d$ is different for the cooling and feedback scenarios, $\lambda_{\text{crit}}$ depends on the particular model we choose for the galaxy formation (in)efficiency. In Figure 1 this dependence is shown for three different cosmologies: SCDM, OCDM, and $\Lambda$CDM. The latter is a model with a non-zero cosmological constant with $\Omega_0 = 0.3$, $\Omega_{\Lambda} = 0.7$, $h = 0.6$ and $\sigma_8 = 0.97$.

Results are shown for both the feedback scenario (left panel) and the cooling scenario (right panel). Figure 1 plots the logarithm of the critical spin parameter versus redshift. The dotted lines outline the probability distribution of the spin parameter (equation [7]); the indices (indicated in the right panel only) denote the probability of finding a halo below that line. At low formation redshifts $\lambda_{\text{crit}}$ increases with redshift, due to the increasing galaxy formation efficiency with redshift. The turnovers to a flat line at higher redshift are caused by the requirement that $\epsilon_{\text{gf}} \leq 1.0$. Note that in a SCDM cosmology with $z_{\text{form}} \geq 0.87$ (for which $\epsilon_{\text{gf}} = 1.0$) about 50 percent of all halos yield unstable disks, and thus require the formation of a bulge component in order to stabilize the disk. This increases to $\sim 90$ percent in models with $\Omega_0 = 0.3$. Furthermore, it is apparent that $\lambda_{\text{crit}}$ is much less redshift dependent in the case of the cooling scenario, than in the case of the feedback scenario. The implications of this are discussed in Section 4. As can be seen, the results are very similar for the two low $\Omega_0$ models. Since $\lambda_{\text{crit}}$ is the important parameter for what follows, we only focus on a comparison of SCDM with OCDM cosmologies, and note that results for the $\Lambda$CDM model are similar as for the OCDM model.

### 2.5. Isothermal halo profiles

It has recently been argued that the results of Navarro, Frenk & White concerning the universal density profiles of dark halos may be incorrect. Very high resolution simulations by Moore et al. (1997) indicate that the central density cusp of dark halos may be even steeper than $r^{-1}$. We therefore investigate the sensitivity of our results to the assumption of a NFW halo profile by comparing our results with those obtained considering isothermal halo profiles, i.e., halos with...
a density distribution given by
\[ \rho(r) = \frac{V_c^2}{4\pi G r^2}. \]  
(36)

For such halo profiles the circular velocity, \( V_c \), is independent of radius and equal to the circular velocity at the virial radius, \( V_{200} \). The results for the isothermal density profiles can be obtained from the equations in this paper by taking \( f_c = 1 \). We have rerun all models with an isothermal halo, and compared the results with those of the NFW halo profile. None of the results presented in this paper are significantly influenced by the assumption of a NFW halo profile.

3. Towards understanding the \( \mu_0–R_d \) diagram

One of the strongest suggestions for a smooth transition from spirals and S0s towards disky ellipticals along a sequence of decreasing disk-to-bulge ratio is provided by the \( \mu_0–R_d \) diagram, in which the central surface brightness of disks, \( \mu_0 \), is plotted against the disk’s scale-length \( R_d \) (see Scorza & Bender 1995). Scorza & van den Bosch (1998) investigated the continuity of disk properties of a variety of disk galaxies and extended the diagram towards much higher \( \mu_0 \) and smaller \( R_d \) by including the nuclear disks discovered from Hubble Space Telescope photometry in a number of early-type galaxies (van den Bosch et al. 1994; Kormendy et al. 1996; van den Bosch, Jaffe & van der Marel 1997). This extended \( \mu_0–R_d \) diagram, already published in Scorza & van den Bosch (1998), is shown in Figure 2. It plots the central surface brightness (in V-band) as a function of the logarithm of the scale-length of a vast variety of disks, ranging from the extraordinary large disk of Malin I (data taken from Bothun et al. 1987), to the very small nuclear disks (data taken from Scorza & van den Bosch 1998). In addition, Figure 2 plots the parameters of embedded disks (solid circles, data taken from Scorza & Bender 1995 and Scorza et al. 1998), and of a combined sample of S0s (stars), HSB disks (open circles), and LSB disks (triangles). Data are taken from Kent (1985), de Jong (1996a), Sprayberry et al. (1995), de Blok, van der Hulst & Bothun (1995), and McGaugh & Bothun (1994). Figure 2 combines the parameters of disks that span 4 orders of magnitude in both scale-length and central surface brightness!

Although there is considerable ‘scatter’, the \( \mu_0(V)–R_d \) diagram of Figure 2 seems to reveal an almost linear relation. It is tempting to believe that this may teach us something about the formation of galactic disks. However, it should be noted that selection effects play an important role at the lower left part of the diagram: Most of the spirals and S0s in Figure 2 are taken from large diameter limited surveys, and are thus strongly biased towards the largest scale-lengths. Furthermore, one is always strongly biased against low surface brightness disks, especially if they are embedded in a much brighter spheroid. Even more, the fact that selection effects are different for the various data sets combined in Figure 2 prohibits a statistical conclusion on this lower left part of the diagram (e.g., see detailed discussions in de Jong 1996b and Dalcanton et al. 1997). However, the absence of disks in the upper right part of the diagram is real, and should in principle be related to the formation and evolution of disk systems.

We now use our disk-bulge-halo models to investigate the locations of the different sorts of disks in the \( \mu_0(V)–R_d \) diagram. First we note that spirals and S0s with the highest central surface
brightness for given scale-length lie approximately along a line of constant disk luminosity. Part of the upper boundary in the $\mu_0(V) - R_d$ diagram thus seems related to a cut-off in disk luminosity. This is to be expected from the cut-off in the galaxy luminosity function, which is well described by the Schechter (1976) function. This tells us that there are only very few galaxies with luminosities exceeding $3L^*$. We thus assume that the upper boundary is a line along which the sum of bulge and disk luminosities is equal to $3L^*$. The dotted line in Figure 2 (which is partly drawn as a solid thick line) corresponds to $L_d = 3L^*$. The position of a disk along a line of constant $L_d$ is determined by the spin parameter $\lambda$: disks originating from halos with smaller $\lambda$ have smaller scale-lengths, and higher central surface brightness. As shown in Section 2.4, there is a critical spin parameter such that for $\lambda < \lambda_{\text{crit}}$ disks are unstable. The disk for which $L_d = 3L^*$ and $\lambda = \lambda_{\text{crit}}$ is indicated by a thick solid dot in Figure 2. If we want to build disks with scale-lengths smaller than this, we need a bulge to help stabilize the disk. The curved thick solid line, that originates at the thick solid dot and extends towards smaller scale-lengths, corresponds to a line with $L_d + L_b = 3L^*$, $\lambda = \lambda_{\text{crit}}$, and with a disk-to-bulge ratio $t$ running from $t = \infty$ (big solid dot) towards $t = 0$. The dashed line is the line with $\lambda = \lambda_{\text{crit}}$, $t = \infty$, and $L_d$ running from $3L^*$ (big solid dot) to $L_d = 0$. Using these limits on the expected scale-lengths and central surface brightnesses of disks in our disk-bulge-halo models, we can distinguish 4 different regions in the $\mu_0(V) - R_d$ diagram, labeled I to IV in Figure 2. Region I is the region where the disk luminosity exceeds $3L^*$. Region II is the region where $L_d < 3L^*$ and $\lambda > \lambda_{\text{crit}}$: disks in this region are stable against bar formation even without a bulge. Disks in region III, however, require a bulge to be stable. Region IV, finally, is the region where systems with $L_d + L_b < 3L^*$ have an unstable disk despite the presence of the bulge. The position of the big solid dot, and therewith of the boundary between regions II and III, depends strongly on the formation redshift and cosmology (cf. Figure 1). Figure 2 is based on a SCDM cosmology with zero formation redshift. Although quite a number of assumptions have gone into our disk-bulge-halo models, the upper boundary (indicated by the thick solid line) nicely separates the region without disks, from the region most densely occupied by disks. Of course, the fact that the number density is highest just below the boundary is an observational bias effect as discussed above. One has to keep in mind that the $\mu_0 - R_d$ diagram can be rather confusing since there are three parameters that determine the position of a disk in this diagram, namely the disk luminosity, the spin parameter, and the disk-to-bulge ratio. In the following we therefore focus on another diagram in which the luminosity dependence has been removed.

4. Clues to the formation of bulge-disk systems

We now ask ourselves whether we can learn something about the formation of disks and bulges from a comparison of our disk-bulge-halo models with real galaxies. We do this by calculating, for each galaxy in our combined sample, the spin parameter $\lambda$ of the dark halo which, under a given set of assumptions, yields the observed disk properties (scale-length and central surface brightness). We thus use our formation scenario to link the disk parameters to those of the dark halo, and use the statistical properties of dark halos to discriminate between different cosmogonies. We start by choosing a cosmology and a formation redshift and we read in the disk-to-bulge ratio, $t$, and the scale-length and central surface brightness of the disk. From this we determine the
luminosities of the disk and bulge, which, upon using equations (3), (4), and (30), yield the total mass $M$ and the particular density distribution of the disk-bulge-halo system. We then calculate $\lambda$ from equation (20) and plot this against the disk-to-bulge ratio $t$. This $\lambda$ is the value of the spin parameter that the halo would have had if the observed disk-bulge system formed out of that halo in the way envisioned here. One of the advantages of this diagram is that the dependence on luminosity is eliminated: the position of a disk in this diagram is virtually independent of the disk’s total luminosity.

In Figures 3 (SCDM) and 4 (OCDM) we plot the location of disks of LSB spirals (triangles), HSB spirals (open circles), S0s (stars), and disky ellipticals (solid circles) in a log[\lambda] vs. log[$t$] diagram. The five small-dashed lines in Figures 3 and 4 correspond to the value of $\lambda$ below which we expect 1, 10, 50, 90, and 99 percent of the halos for a distribution function of spin parameters given by equation (7) with $\bar{\lambda} = 0.05$ and $\sigma_{\lambda} = 0.7$ (see also Figure 1). These lines are drawn to guide the eye. If our assumptions for the formation of disk-bulge-halo systems are correct, the distribution of inferred halo spin parameters should be similar to this log normal distribution.

The thick solid lines in Figures 3 and 4 correspond to $\lambda = \lambda_{\text{crit}}$ (equation [34]). Disks that lie below this line are unstable against bar formation. This critical value of the spin parameter has been calculated for a halo with a total mass of $10^{13} \, M_\odot$, but is virtually identical for other halo masses. The crowding of disks at $t = 10$ is due to the fact that we have assigned a disk-to-bulge ratio of ten to systems for which no value of $t$ was determined (these are mainly a number of the LSB spirals). The panels on the left correspond to $z_{\text{form}} = 0.0$, the panels in the middle to $z_{\text{form}} = 1.0$, and the panels on the right to $z_{\text{form}} = 3.0$. The upper three panels correspond to models in which $\epsilon_{g, f} = 1.0$, i.e., in which all the baryons in the dark halo make it into the galaxy. The panels in the middle correspond to the cooling scenario, and the lower panels to the feedback scenario. The main difference between the upper and the middle and lower panels is that the former have $\lambda_{\text{crit}}$ independent of formation redshift (there is a non-zero but negligible dependence on $z_{\text{form}}$ left due to the fact that the concentration parameter $c$ of the NFW halo profiles depends on formation redshift), whereas $\lambda_{\text{crit}}$ increases with $z_{\text{form}}$ in the middle and lower panels (according to the curves in Figure 1).

Since $R_d \propto r_{200}$ (equation [20]) and $r_{200} = 0.1V_{200}/H(z)$, disks forming in a halo with mass $M$ and spin parameter $\lambda$ are smaller for higher $z_{\text{form}}$. A higher formation redshift thus implies a larger spin parameter in order to yield the observed value of $R_d$, something which is evident from Figures 3 and 4. Furthermore, disks seem to follow the $\lambda = \lambda_{\text{crit}}$ curve, i.e., disk-bulge systems do not have bulges that are significantly more massive than required by disk-stability. This is at least consistent with the inside-out bulge formation scenario proposed in Section 2.1, in which the formation of disk and bulge is self-regulating and governed by disk stability.

The OCDM model with constant and near to unity galaxy formation efficiency (upper panels in Figure 4) can be ruled out. In this scenario systems with a large disk-to-bulge ratio, such as spirals, are rare; only $\sim 10$ percent of the dark halos is expected to yield such systems. The stability line $\lambda = \lambda_{\text{crit}}$ crosses the average $\bar{\lambda}$ of the spin parameter distribution at $D/B \approx 1.0$, and thus half of the halos is expected to form systems with a disk-to-bulge ratio less than unity. This is in conflict with the fact that the major fraction of field galaxies are spirals. Furthermore,
stable spirals can only form at high redshift \( z_{\text{form}} \gtrsim 3 \), since for lower formation redshifts the spirals fall below the stability line. In the OCDM cooling scenario, spirals that formed recently tend to be slightly more stable, but still, the probability that a certain halo yields a system with a large disk-to-bulge ratio is rather small, rendering this scenario improbable. However, for the feedback OCDM cosmogony, a promising scenario unfolds: proto-galaxies that collapse at high redshifts are preferentially found in overdense regions such as clusters. The densities of these virialized halos are high, and this (somehow) causes the galaxy formation to be very efficient; nearly all the baryons in the dark halos make it into the galaxy. As a consequence, the systems that form require relatively small disk-to-bulge ratios (e.g., similar to S0s) in order to be stable. As the Universe evolves, and proto-galaxies with smaller over-densities (which are predominantly located in the field) start to turn around and collapse, the galaxy formation efficiency decreases, and systems with larger disk-to-bulge ratios are allowed to form. This scenario thus automatically yields a morphology-density relation, in which systems with smaller disk-to-bulge ratios (e.g., S0s) are preferentially to be found in overdense regions. Unfortunately, the disky ellipticals seem to not fit in this scenario. Even at formation redshifts of three, the average disky elliptical lies near the line with \( p(< \lambda) = 0.01 \).

The scenario outlined above for our OCDM cosmology also applies to the SCDM model with \( \Omega_0 = 1 \), albeit somewhat less restricting. After all, \( \lambda_{\text{crit}} \) only gets as large as \( \bar{\lambda} \). Consequently, in a SCDM Universe our model predicts less of a morphology-density relation than in a low density Universe; i.e., the magnitude of the morphology-density relation depends on the baryon fraction \( \Omega_{\text{bar}}/\Omega_0 \).

4.1. Formation redshifts

We define the typical formation redshift of a subset of galaxies as that redshift for which the average disk yields \( \lambda = \bar{\lambda} \). In Figure 5 we plot histograms of inferred formation redshifts for spirals (HSB and LSB disks combined), S0s, and disky ellipticals. The three dotted lines are drawn to guide the eye and correspond to redshifts of 1, 3, and 10. The inferred formation redshift is determined by calculating the redshift at which a halo with \( \lambda = \bar{\lambda} = 0.05 \) yields the observed disk parameters. If a disk lies already above the \( \lambda = 0.05 \) line at \( z = 0 \), a formation redshift of zero is assigned. The arrows in Figure 5 indicate the mean of the inferred formation redshifts for each sample. As can be seen, spiral disks are consistent with having formed fairly recently: \( \langle z_{\text{form}} \rangle \approx 0.3 \) with a mean formation redshift of zero. This conclusion was already reached by Mo et al. (1998) based on the same disk formation scenario as used here, but on somewhat different arguments. The distribution of \( z_{\text{form}} \) is much more spread out for S0s, with a mean formation redshift of \( \sim 7 \). Note that misclassifications may be partially responsible for this large spread. Finally, the inferred formation redshifts for the disky ellipticals are unrealistically high, with a mean of \( \sim 38 \). We thus conclude that unless there is a significant loss in specific angular momentum of the material that forms these systems, disky ellipticals seem to not form a continuous sequence with S0s and spirals, at least not in the sense of the formation scenario investigated here.

The histograms in Figure 5 correspond to the feedback SCDM cosmogony. However, the
results are very similar for the other cosmogonies discussed in this paper: although the details of the histograms are somewhat different, the averages are the same within the errorbars.

5. Alternative formation scenarios for disky ellipticals

As discussed above it seems unlikely that disky ellipticals can be fit in the simple bulge-disk formation scenario envisioned here. The small disk-to-bulge ratios in these systems renders an inside-out formation scenario for the bulges in disky ellipticals unlikely. Two alternative formation scenarios for disky ellipticals have been proposed: (i) disky ellipticals are the outcome of gas-rich mergers (e.g., Bender, Burstein & Faber 1992; Scorza & Bender 1996), and (ii) embedded disks are formed out of internal mass loss from the old stars in the spheroid (Scorza 1993; Brighenti & Mathews 1997).

Hernquist & Barnes (1991) have shown how in gas-rich mergers the gas becomes segregated in two components. The first component looses most of its specific angular momentum and collapses to form a dense central cloud. When converted into stars, these compact blobs of gas may well account for the high central densities in disky ellipticals (Barnes 1998). The second component gets propelled to larger radii in the tidal tails originating from the merger. This gas may either escape or reaccrete (through cooling) onto the newly formed ‘elliptical’ and form a new disk component, resembling the embedded disks. The numerous physical processes at play and the large parameter space available make it difficult to come up with detailed predictions for the resulting merger remnant. However, at present there seem to be no clear observations that rule out gas-rich mergers as the origin for disky ellipticals (but see Kissler-Patig, Forbes & Minniti 1998).

In the internal mass loss scenario one expects a clear correlation between the mass of the disk and the mass of the spheroid. This correlation is indeed present and Scorza & Bender (1998) showed that typical mass loss rates for the old spheroid population are in excellent agreement with the typical ratio between the masses of the embedded disk and the spheroid. Another prediction from this scenario is that the spheroid and disk rotate in the same direction and that their specific angular momenta are similar (at least if we assume that the gas blown into the interstellar medium by the spheroid does not loose its specific angular momentum when cooling to form the embedded disk). Scorza & Bender (1995) found this to be the case from kinematic decompositions of line-of-sight velocity profiles of a number of disky ellipticals from which they determined estimates of the specific angular momenta of the embedded disk and the spheroid. Their data, which we plot in Figure 6, yields an average ratio of

$$\langle \frac{J}{M_d} \rangle = 1.1 \pm 0.6.$$  \(37\)

A third prediction associated with this scenario is that the disks are younger than the spheroids. Recently, de Jong & Davies (1997) reported that disky ellipticals have higher Hβ line indices than the more luminous boxy ellipticals, and suggested this to be due to a young population, most likely the embedded disk (see also Brighenti & Mathews 1997). A final prediction for the
internal mass loss scenario is that the disks have surface brightness profiles that are steeper than exponential, since the gas does not cool from a uniform sphere in solid body rotation, but from a differentially rotating, centrally concentrated gas distribution (Scorza 1993; Brighenti & Mathews 1997). Indeed, Scorza & Bender (1995) and Scorza et al. (1998) have found that the embedded disks often have surface brightness profiles that are significantly steeper than exponential.

The masses, specific angular momenta, surface brightness profiles, and ages of embedded disks are thus all consistent with them having formed out of mass loss from the old stellar population of the spheroid. More accurate determinations of the specific angular momenta of embedded disks and their spheroids in a larger sample of disky ellipticals will prove helpful in constraining this further.

6. Conclusions & Discussion

Understanding galaxy formation is intimately linked with understanding the origin of the Hubble sequence. An important clue is provided by comprehending the formation of disky ellipticals and S0s, simply because these systems form the transitional class from the classical ellipticals to the spirals. Disky ellipticals seem to form a continuous sequence with spirals and S0s towards smaller disk-to-bulge ratio, and it is thus tempting to believe that disky ellipticals, S0s and spirals all formed in a similar fashion. The aim of this paper has been to investigate a simple formation scenario for disk-bulge-halo systems, and to search for the main parameters and/or processes that determine the disk-to-bulge ratio and thus explain to a large extent the origin of (the major part of) the Hubble sequence.

We considered the disk formation scenario originally proposed by Fall & Efstathiou (1980), in which the size of the disk, which is formed by cooling of the gas in a dark halo, is determined by the amount of angular momentum of the halo. We have extended upon this formation scenario by including a simple picture for the inside-out formation of an additional bulge component out of the inner, low angular momentum material. Stability arguments are used to suggest that the formation of the bulge is a self-regulating process in which the bulge grows until it is massive enough to allow the remaining gas to form a stable disk component. We do not describe the bulge formation in any detail but merely use empirical relations which allow us to describe the bulge by a single parameter, namely its mass.

Each dark halo contains a fraction $\Omega_{\text{bar}}/\Omega_0$ of baryons. We introduced a galaxy formation efficiency $\epsilon_{gf}$ which describes the fraction of those baryons that actually build up the disk-bulge system. The theory of spherical collapse coupled with the definition of a virialized halo predicts a Tully-Fisher relation of the form $L \propto V_c^3$ as observed, with a zero-point that depends on the Hubble constant $H(z)$. Recent observations, however, suggest that the Tully-Fisher zero-point is independent of redshift, implying that the galaxy formation efficiency is proportional to $H(z)$. A physical explanation for this redshift dependence may be the higher escape velocities and cooling efficiencies at higher redshifts.

For a combined sample of $\sim 200$ galaxies, varying from spirals to disky ellipticals, we
calculated the value of the halo’s spin parameter which yields the observed disk properties under the assumption that disk-bulge systems form in the way envisioned here. We compared two cosmologies (SCDM vs. OCDM) and investigated the differences between assuming a galaxy formation efficiency of unity and two (extreme) scenarios in which $\epsilon_{gf} \propto H(z)$: a cooling scenario, in which we assume that only the inner fraction $\epsilon_{gf}$ of the available baryons cools to form the disk and bulge, and a feedback scenario, in which each baryon, independent of its specific angular momentum, has a probability $\epsilon_{gf}$ of ultimately ending up in the disk or bulge. Our main conclusions are the following:

- Disk-bulge systems do not have bulges that are significantly more massive than required by stability of the disk component. This suggests a coupling between the formation of disk and bulge, and is consistent with the self-regulating, inside-out bulge formation scenario proposed here.

- If we live in a low-density Universe ($\Omega_0 \lesssim 0.3$), the only efficient way to make spiral galaxies is by assuring that only a relatively small fraction of the available baryons makes it into the galaxy, and furthermore that the angular momentum distribution of those baryons is similar to that of the entire system; i.e., the probability that a certain baryon becomes a constituent of the final galaxy has to be independent of its specific angular momentum. In the cooling scenario, most of the angular momentum of the system remains in the outer layers, and most halos form disk-systems with massive bulges, such as S0s. If, however, the galaxy formation efficiency is regulated as described by our ‘feedback’ model, a promising scenario unfolds: At formation redshifts $z > 3.0$ the galaxy formation efficiency is unity, and systems that form build up a large bulge to support the disk that assembles around them. These galaxies resemble S0s. Galaxies that form later, at $z \approx 0$, no longer require a massive bulge, and spirals are preferentially formed. Coupled with the notion that density perturbations that collapse early are preferentially found in high density environments such as clusters, this scenario automatically predicts a morphology-density relation in which S0s are most likely to be found in clusters. In a SCDM Universe a similar, albeit less restrictive, mechanism is at work, which predicts a morphology-density relation of smaller amplitude.

- A reasonable variation in formation redshift and halo angular momentum can yield approximately one order of magnitude variation in disk-to-bulge ratio, and our simple formation scenario can account for both spirals and S0s. However, disky ellipticals have too large bulges and too small disks to be incorporated in this scenario. Apparently, their formation and/or evolution has seen some processes that caused the baryons to lose a significant amount of their angular momentum. Merging and galaxy harassment (Moore et al. 1996) are likely to play a major role for these systems.

Finally we wish to emphasize of few of the major shortcomings of the oversimplified formation scenario discussed here. First of all, we have neglected the fact that the merging of halos is an ongoing process, and that this is very effective in destroying disks (e.g., Toth & Ostriker 1992). Gas-dynamical simulations that do not involve the energy and momentum feedback to the gas from supernova explosions, stellar winds, UV radiation etc. produce galactic disks that are some
two orders of magnitude smaller than the observed spiral disks (e.g., Navarro & Steinmetz 1997): merging (and also harassment) are very effective in transporting angular momentum out into the halo, thus yielding more and more compact galaxies. This problem with forming galactic disks of proper dimensions is often referred to as the angular momentum problem. Despite our ignorance regarding the effects of merging, harassment, feedback, and (re)-ionization of the Universe, the observed sizes of (spiral) disks clearly suggest however, that the combine effect of all these processes is apparently such that the material that forms galactic disks has not lost much of its original angular momentum acquired from cosmological torques. The use of the Fall & Efstathiou disk formation scenario thus seems justified, despite the aforementioned shortcomings.

In the past semi-analytical simulations of galaxy formation have been mainly based on the assumption that all bulges result from the merging of disk galaxies (e.g., Kauffmann, White & Guiderdoni 1993; Cole et al. 1994; Baugh, Cole & Frenk 1996; Somerville & Primack 1998). In this paper we have examined an inside-out bulge formation scenario, which should be regarded as complimentary to this merging scenario. Our formation scenario can account for both spirals and S0s, but fails to build systems that are even more bulge dominated. Although it has been shown that bar-instabilities can lead to the formation of small (and close to exponential) bulges, a more detailed study is required to investigate whether the inside-out bulge formation scenario discussed here can yield more massive bulges as well. It is at least intriguing that the critical spin parameter is of the same order of magnitude as the typical spin parameter for halos, suggesting that a significant fraction of halos will yield unstable disks, unless part of the baryonic material is transformed into a bulge component as suggested here. So despite the clearly oversimplified nature of the formation scenario envisioned here, it may provide a useful framework for future investigations of galaxy formation.

7. Acknowledgments

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A. Angular momentum distribution of the disk

Mestel (1963) noticed that disks of spirals have specific angular momentum distributions similar to that of a uniform sphere in solid body rotation. Exponential disks are thus a natural consequence of the Fall & Efstathiou disk formation scenario, if we assume that proto-galaxies have a specific angular momentum distribution similar to that of a uniformly rotating, uniform sphere, and that the material that forms the disk does not lose its specific angular momentum (e.g., Gunn 1982; Dalcanton et al. 1997).

The total angular momentum of a uniformly rotating, uniform sphere with radius $a$ and constant angular velocity $\omega$ is

$$ J = \frac{2}{5} M \omega a^2, $$

and the mass with specific angular momentum less than $j$ is given by

$$ M(< j) = M \left[ 1 - \left( 1 - \frac{j}{\omega a^2} \right)^{2/3} \right]. $$

Since the baryonic and dark matter are well mixed, the angular momentum distribution for the baryonic matter component is identical to that of the total mass of the system, and $J_{\text{bar}} = (\Omega_{\text{bar}}/\Omega_0)J$. The bulge forms out of the low angular momentum material with a cylindrical radius $R < R_{\text{bulge}}$. Solving $M_{\text{bar}}(< j) = M_b$ yields

$$ R_{\text{bulge}} = a \sqrt{1 - \left( \frac{t}{1 + t} \right)^{3/2}}, $$

and the total angular momentum of the bulge is

$$ J_b = \frac{2}{5} M_{\text{bar}} a^2 \omega \left[ 1 - \left( 3 \left( \frac{R_{\text{bulge}}}{a} \right)^2 + 1 \right) \left( 1 - \left( \frac{R_{\text{bulge}}}{a} \right)^2 \right)^{3/2} \right], $$

which is the total angular momentum of the intersection of a solid sphere with radius $a$ and a solid cylinder with radius $R_{\text{bulge}} \leq a$ (where the sphere and cylinder have a common center). The disk forms out of the material with $R_{\text{bulge}} < R < a$, and has a total angular momentum $J_d = J_{\text{bar}} - J_b \equiv j_d J$. We assume that the specific angular momentum of this material is conserved during disk formation.

In the feedback scenario, in which each baryon has a probability $\epsilon_{gf}$ of ending up in the final disk-bulge system, we find upon substituting $R_{\text{bulge}}$ (equation [A3]) in equation (A4), and with the use of equations (3) and (4)

$$ j_d = [m_d + m_b] \frac{t}{1 + t} \left[ \frac{5}{2} - \frac{3}{2} \left( \frac{t}{1 + t} \right)^{2/3} \right] \equiv f(t, \epsilon_{gf}) [m_d + M_b] $$

In the cooling scenario, only the baryonic material with $r < r_{\text{cool}}$ cools to form the disk and bulge, where

$$ r_{\text{cool}} = \epsilon_{gf}^{1/3} a, $$
and one finds

\[ j_d = \epsilon_{gf}^{2/3}(m_d + m_b) \frac{t}{1+t} \left[ \frac{5}{2} - \frac{3}{2} \left( \frac{t}{1+t} \right)^{2/3} \right] \equiv f(t, \epsilon_{gf})(m_d + M_b). \]  

(A7)
REFERENCES

Andredakis, Y. C., Peletier, R. F., & Balcells, M. 1995, MNRAS, 275, 874

Barnes, J. E. 1998, in Galaxies: Interactions and Induced Star Formation; 26th SAAS-FEE Advanced Course, eds. J. Barnes, R. Kennicutt, & F. Schweizer (Springer-Verlag).

Barnes, J. E., & Efstathiou, G. 1987, ApJ, 319, 575

Baugh, C. M., Cole, S., & Frenk, C. S. 1996, MNRAS, 283, 1361

Bender, R. 1988, A&A, 193, L7

Bender, R. 1990, in Dynamics and Interactions of Galaxies, ed. R. Wielen (Berlin: Springer-Verlag), p. 232

Bender, R., Surma, P., Döbereiner, S., Möllenhoff, C., & Madejsky, R. 1989, A&A, 217, 35

Bender, R., Burstein, D., & Faber, S.M. 1992, ApJ, 399, 462

Bender, R., et al. 1993, in Structure, Dynamics, and Chemical Evolution of Elliptical Galaxies, eds., I.J. Danziger, W.W. Zeilinger & K. Kjär, (Garching: ESO), p. 3

Binney, J. J., & Tremaine S. 1987, Galactic Dynamics (Princeton: Princeton University Press)

Blumenthal, G. R., Faber, S. M., Flores, R., & Primack, J. R. 1986, ApJ, 301, 27

Bothun, G. D., Impey, C. D., Malin, D. F., & Mould, J. R. 1987, AJ, 94, 23

Bottema, R. 1997, A&A, 328, 517

Brighenti, F., & Mathews, W. G. 1997, ApJ, 490, 592

Caon, N., Capaccioli, M., & D’Onofrio, M. 1993, MNRAS, 265, 1013

Capaccioli, M. 1987, in IAU Symposium 127, Structure and Dynamics of Elliptical Galaxies, ed. T. de Zeeuw (Dordrecht: Reidel), p. 47

Capaccioli, M., Caon, N., & Rampazzo, R. 1990, A&A, 242, 24P

Capaccioli, M., & Caon, N. 1992, in Morphological and Physical Classification of Galaxies, eds. G. Longo, M. Capaccioli & G. Busarello (Dordrecht: Kluwer), p. 99

Carter, D. 1987, ApJ, 312, 514

Ciotti, L. 1991, A&A, 249, 99

Cole, S., Aragón-Salamanca, A., Frenk, C. S., Navarro, J. F., & Zepf, S. E. 1994, MNRAS, 271, 781

Cole, S., & Lacey, S. 1996, A&A, 281, 716

Combes, F., Debbasch, F., Friedli, D., & Pfenniger, D. 1990, A&A, 233, 82

Courteau, S., de Jong, R. S., & Broeils, A. H., 1996, ApJ, 457, L73

Christodoulou, D. M., Shlosman, I., & Tohline, J. E. 1995, ApJ, 443, 551

Dalcanton, J. J., Spergel, D. N., & Summers, F. J. 1997, ApJ, 482, 659

de Blok, W. J. G., van der Hulst, J. M., & Bothun, G. D. 1995, MNRAS, 274, 235
de Jong, R. S. 1996a, A&AS, 118, 557
de Jong, R. S. 1996b, A&A, 313, 45
de Jong, R. S., & Davies, R. L. 1997, MNRAS, 285, L1
Dressler, A. 1980, ApJ, 236, 351
Efstathiou, G., Lake, G., & Negroponte, J. 1982, MNRAS, 19, 1069
Faber, S. M. 1982, in Astrophysical Cosmology: Proc Study Week on Cosmology and Fundamental Physics, ed. H.A. Brück, G.V. Coyne & M.S. Longair, (Vatican: Pontifical Sci Acad), p. 191
Faber, S. M., et al., 1997, AJ, 114, 1771
Fall, S. M., & Efstathiou, G. 1980, MNRAS, 193, 189
Fall, S. M. 1983, in Internal Kinematics and Dynamics of Galaxies, IAU Symposium 100, ed. E. Athanassoula (Dordrecht: Reidel), p. 391
Ferrarese, L., van den Bosch, F. C., Ford, H. C., Jaffe, W., & O’Connell, R. W. 1994, AJ, 108, 1598
Flores, R., Primack, J. R., Blumenthal, G. R., & Faber, S. M. 1993, ApJ, 412, 443
Freeman, K. C. 1970, ApJ, 160, 811
Friedli, D. 1994, in Mass-Transfer Induced Activity in Galaxies, ed. I. Shlosman (Cambridge University Press), p. 268
Friedli, D., & Martinet, L. 1993, A&A, 277, 27
Gebhardt, K., et al. 1996, AJ, 112, 105
Giovanelli, R., Haynes, M. P., da Costa, L. N., Freudling, W., Salzer, J. J., & Wegner, G. 1997, ApJ, 477, L1
Gunn, J. E. 1982, in Astrophysical Cosmology, ed. H.A. Bruck, G.V. Coyne, M.S. Longair, Vatican: Potifical Academia Scientiarum, p. 191
Gunn, J. E., & Gott, J. R. 1972, ApJ, 176, 1
Hernquist, L., & Barnes, J. E. 1991, Nature, 354, 210
Hohl, F. 1971, ApJ, 168, 343
Hoyle, F. 1953, ApJ, 118, 513
Jaffe, W., Ford, H. C., Ferrarese, L., van den Bosch, F. C., & O’Connell, R. W. 1994, AJ, 108, 1567
Jimenez, R., Padoan, P., Matteucci, F., & Heavens, A. F. 1998, preprint (astro-ph/9804049)
Kauffmann, G., White, S. D. M., & Guiderdoni, B. 1993, MNRAS, 264, 201
Kent, S. 1985, AJ, 59, 115
Kepner, J. V. 1997, preprint (astro-ph/9710329)
Kissler-Patig, M., Forbes, D. A., & Minniti, D. 1998, preprint (astro-ph/9804261)
Kormendy, J., & Bender, R. 1996, ApJ, 464, L119
Kormendy, J., et al. 1996, ApJ, 459, L57
Mao, S., Mo, H. J., & White, S. D. M. 1997, preprint [astro-ph/9712167]
Martinet, L. 1995, Fund. Cosmic Physics, 15, 341
McGaugh, S. S., & Bothun, G. D. 1994, AJ, 107, 530
Mestel, L. 1963, MNRAS, 126, 553
Mo, H. J., Mao, S., & White, S. D. M. 1998, MNRAS, 295, 319
Moore, B., Katz, N., Lake, G., Dressler, A., & Oemler, A. Jr. 1996, Nature, 379, 613
Moore, B., Governato, F., Quinn, T., Stadel, J., & Lake, G. 1997, preprint [astro-ph/9709051]
Navarro, J. F., Frenk, C. S., & White, S. D. M. 1995, MNRAS, 275, 720
Navarro, J. F., Frenk, C. S., & White, S. D. M. 1996, ApJ, 462, 563
Navarro, J. F., Frenk, C. S., & White, S. D. M. 1997, ApJ, 490, 493
Nieto, J.-L., Bender, R., & Surma, P. 1991, A&A, 244, L37
Norman, C. A., Sellwood, J. A., & Hasan, H. 1996, ApJ, 462, 114
Ostriker, J. P., & Peebles, P. J. E. 1973, ApJ, 186, 467
Peebles, P. J. E. 1969, ApJ, 155, 393
Pfenniger, D. 1984, A&A, 134, 373
Pfenniger, D. 1993, in Galactic Bulges, IAU Symposium 153, eds. H. Dejonghe & J. J. Habing (Dordrecht: Kluwer), p. 387
Pfenniger, D., & Norman, C. 1990, ApJ, 363, 391
Pfenniger, D., & Friedli, D. 1991, A&A, 252, 75
Raha, N., Sellwood, J. A., James, R. A., & Kahn, F. D. 1991, Nature, 352, 411
Rix, H.-W., & White, S. D. M. 1990, ApJ, 254,389
Ryden, B. S. 1988, ApJ, 329, 589
Schechter, P. 1976, ApJ, 203, 297
Scorza, C. 1993, Ph.D. Thesis, Landessternwarte Heidelberg
Scorza, C., & Bender, R. 1995, A&A, 293, 20
Scorza, C., & Bender, R. 1996, in New Light on Galaxy Formation, IAU Symposium 171, eds. R. Bender & R. L. Davies (Dordrecht: Kluwer), p. 55
Scorza, C., & Bender, R. 1998, A&A, submitted
Scorza, C., & van den Bosch, F. C. 1998, MNRAS, in press
Scorza, C., Bender, R., Winkelmann, C., Capaccioli, M., Macchetto, F. D., & Nieto, J.-L. 1998, A&A, in press
Sersic, J. L. 1968, Atlas de galaxias australes. Observatorio Astronomico, Cordoba.

Somerville, R. S., & Primack, J. R. 1998, preprint (astro-ph/9802268)

Sprayberry, D., Impey, C. D., Bothun, G. D., & Irwin, M. J. 1995, AJ, 109, 558

van den Bergh, S. 1990, ApJ, 348, 57

van den Bosch, F. C., Ferrarese, L., Jaffe, W., Ford, H. C., & O’Connell, R. W. 1994, AJ, 108, 1579

van den Bosch, F. C., Jaffe, W., & van der Marel, R. P. 1998, MNRAS, 293, 343

van der Kruit, P. C. 1987, A&A, 173, 59

van der Marel, R. P. 1991, MNRAS, 253, 710

Vogt, N. P., Forbes, D. A., Phillips, A. C., Cronwall, C., Faber, S. M., Illingworth, G. D., & Koo, D. 1996, ApJ, 465, L15

Vogt, N. P., et al. 1997, ApJ, 479, L121

Wada, K., & Habe, A. 1995, MNRAS, 277, 433

Walker, T. P., Steigman, G., Schramm, D. N., Olive, K. A., & Kang, H.-S. 1991, ApJ, 376, 51

Warren, M. S., Quinn, P. J., Salmon, J. K., & Zurek, W. H. 1992, ApJ, 399, 405

White, S. D. M., & Rees, M. J. 1978, MNRAS, 183, 341

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Fig. 1.— The critical value $\lambda_{\text{crit}}$ of the halo’s spin parameter (plotted logarithmically) as a function of redshift $z$ for both the feedback scenario (left panel) and the cooling scenario (right panel). Results are plotted for three different cosmologies: SCDM (solid line), OCDM (dot-dashed line), and ΛCDM (dashed line). The five dotted lines are labeled by the probability of finding a halo below that line for the spin parameter distribution discussed in Section 2.2. The increase of $\lambda_{\text{crit}}$ with increasing redshift is due to the increasing galaxy formation efficiency. When this efficiency becomes unity, $\lambda_{\text{crit}}$ becomes constant with redshift as apparent in the figure. Note the similarity between the OCDM and ΛCDM cosmologies compared to the SCDM scenario.

Fig. 2.— The $\mu_0 - R_d$ diagram for a large variety of disks: LSB spirals (triangles), HSB spirals (open circles), S0s (stars), disky ellipticals (solid circles), and nuclear disks (squares). The dotted line (partly plotted as a thick solid line) corresponds to the line with $L_d = 3L^*$. The dashed line corresponds to $\lambda = \lambda_{\text{crit}}$ for the case without bulge (i.e., $t = \infty$). The curved thick solid line corresponds to $\lambda = \lambda_{\text{crit}}$ and $L_d + L_b = 3L^*$, with $t$ running from zero to infinity. These lines border 4 regions, the meaning of which are discussed in the text.

Fig. 3.— Results for a SCDM cosmology. Plotted are the logarithm of the spin parameter versus the logarithm of the disk-to-bulge ratio ($D/B = t$). Solid circles correspond to disky ellipticals, stars to S0s, open circles to HSB spirals, and triangles to LSB spirals. The thick solid line is the stability margin $\lambda_{\text{crit}}$; halos below this line result in unstable disks. As can be seen, real disks avoid this region, but stay relatively close to the stability margin, in agreement with the self-regulating bulge formation scenario discussed in Section 2.1. The dashed curves correspond to the 1, 10, 50, 90, and 99 percent levels of the cumulative distribution of the spin parameter according to equation (7) with $\bar{\lambda} = 0.05$ and $\sigma_{\lambda} = 0.7$. Upper panels have a galaxy formation efficiency equal to unity, middle panels correspond to the cooling scenario, and lower panels to the feedback scenario. Panels on the left correspond to $z_{\text{form}} = 0$, middle panels to $z_{\text{form}} = 1$, and panels on the right to $z_{\text{form}} = 3$.

Fig. 4.— Same as Figure 3, except now for the OCDM universe with $\Omega_0 = 0.3$. The main difference is that $\lambda_{\text{crit}}$ reaches higher values here. Both in the case with $\epsilon_{gf} = 1.0$ (upper panels), and in the cooling scenario (middle panels), $\sim 90$ percent of the halos is expected to result in unstable disks unless a significant bulge is formed (in the cooling scenario this reduces to $\sim 75$ percent for $z_{\text{form}} = 0$). The majority of the halos yields systems with a significant bulge such as S0s, and both these models can thus be ruled out, given the relatively large numbers of spirals in the present Universe. In the feedback scenario (lower panels) stable spiral systems can form, albeit only at low redshifts. At higher redshifts systems with smaller disk-to-bulge ratios form, such as S0s.

Fig. 5.— Histograms of the (normalized) distribution of formation redshifts (plotted as log[1 + $z_{\text{form}}$]) determined as described in the text for spirals (left panel, HSB and LSB spirals combined), S0s (middle panel), and disky ellipticals (right panel). The arrows indicate the means for each sample (the mean for the spirals is zero). The number $n$ of galaxies in each of the separate samples is indicated in the corresponding boxes. The dotted lines indicate formation redshifts of $z = 1$,
$z = 3$, and $z = 10$. These histograms correspond to the SCDM model with feedback, but are similar for other cosmogonies.

Fig. 6.— The specific angular momentum $J/M$ (in kpc km s$^{-1}$) of the disk versus that of the bulge. As can be seen, within the errorbars the specific angular momenta of disk and bulge are roughly similar. (Data taken from Scorza & Bender 1995).
