Moving Media as Photonic Heat Engine and Pump
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ABSTRACT

A system consisting of two slabs with different temperatures can exhibit a non-equilibrium lateral Casimir force on either one of the slabs, when Lorentz reciprocity is broken in at least one of them. This system constitutes a photonic heat engine that converts radiative heat energy into work done by the non-equilibrium Casimir force. Inversely, by sliding two slabs at a sufficiently high relative velocity, heat is pumped from the slab at a lower temperature to the other one at a higher temperature. Hence the system operates as a photonic heat pump. In this work, we study the thermodynamic performance of such photonic heat engine and pump via the exact fluctuational electrodynamics formalism. The propulsion force due to the non-reciprocity and the drag force due to the Doppler effect were revealed as the physical mechanism behind the heat engine. We also show that the heat pump can be achieved only by the Doppler effect and non-reciprocal materials can help further reduce the required velocity to achieve heat pumping. Furthermore, we derive a relativistic version of the thermodynamic efficiency for our heat engine and show that the Carnot limit is independent of the frame of reference. We explore an ideal material dispersion to reach that efficiency. Our work serves as a conceptual guide for the realization of photonic heat engines based on fluctuating electromagnetic fields and relativistic thermodynamics and shows the important role of electromagnetic non-reciprocity in operating them.

Keywords: Near-field radiative heat transfer, non-equilibrium Casimir force, photonic heat engine, photonic heat pump, non-reciprocity, relativistic thermodynamics.

1. INTRODUCTION

Near-field radiative heat transfer and non-equilibrium Casimir forces between objects having different temperatures occur due to the exchange of energy and momentum of thermally emitted photons from the objects, respectively [1-20]. Because the same thermally emitted photons cause both energy and momentum transfer, it should be of interest to investigate a heat engine that operates by the conversion between radiative heat transfer and mechanical work driven by a non-equilibrium Casimir force. It was shown that non-equilibrium lateral Casimir forces act persistently between two geometrically symmetric objects such as spheres and plates when at least one object in the system breaks Lorentz reciprocity [21-28]. Using this lateral force, a heat engine that converts radiative heat transfer into non-equilibrium lateral Casimir force was proposed and investigated for a single gyrotropic sphere in the environment [27] and two semi-infinite parallel plates [28]. However, the detailed analysis of two semi-infinite parallel slabs at relative motion as heat engine and heat pump has not been performed. In this work [29], we investigate thermodynamic performance of a system consisting of two semi-infinite parallel plates in thermal non-equilibrium as photonic heat engine and heat pump.

2. FLUCTUATIONAL ELECTRODYNAMICS FORMALISM

We consider two semi-infinite, linear, and non-magnetic parallel slabs as shown in Fig. 1. Slab 1 is at rest whereas slab 2 can move laterally in the x-direction at the velocity \( V \) with respect to slab 1. We refer to the frames in which slabs 1 and 2 are at rest as the rest frame and the co-moving frame, respectively, and we use primes on the physical quantities in the co-moving frame. We consider the two slabs at the proper temperatures \( T_1 \) and \( T_2 \), respectively. For the observer in the rest frame, the net radiative heat flux \( \varphi_{1-2} \) from slabs 1 to 2 and the net shear stress \( f_{x,2} \) on slab 2, i.e., non-equilibrium Casimir force per unit surface area, are given as

\[
\left[ \frac{\varphi_{1-2}}{f_{x,2}} \right] = \int_0^{\infty} \frac{d\omega}{2\pi} \int \frac{d\mathbf{q}}{(2\pi)^2} \frac{\hbar \omega}{\hbar q x} \left[ n_B(\omega, T_1) - n_B(\omega', T_2) \right] \tau_{1-2}(\omega, \mathbf{q}; V),
\] (1)
where $\omega$ and $q = (q_\times, q_\gamma)$ are the angular frequency and the in-plane wavevector components of the electromagnetic waves, $n_B(\omega, T) = \frac{1}{\exp\left(\frac{\hbar \omega}{k_B T}\right) - 1}$ is the Bose-Einstein distribution, and $\hbar$ and $k_B$ are the reduced Planck constant and the Boltzmann constant, respectively. $\tau_{1-2}$ is the transmission coefficient for the electromagnetic waves emitted from slab 1 and absorbed by slab 2. For the propagative waves ($q = |q| < k_0 = \frac{\omega}{c}$) and evanescent waves ($q > k_0$), the transmission coefficients are given as

$$
\tau_{1-2}(\omega, q; V) = \begin{cases} 
\text{Tr} \left[ \left( I - R_2^\dagger R_2 \right) \frac{1}{D_{12}} \left( I - R_1 R_1^\dagger \right) \frac{1}{D_{12}^\dagger} \right], & q < k_0 = \frac{\omega}{c}, \\
\text{Tr} \left[ \left( \tilde{R}_2^\dagger - R_2 \right) \frac{1}{D_{12}} \left( R_1^\dagger - R_1 \right) \frac{1}{D_{12}^\dagger} e^{-2ik_{\gamma}d} \right], & q > k_0 
\end{cases},$$

(2)

where $I$ is the 2 by 2 identity matrix, $D_{12} = I - R_1 R_2 e^{2ik_{\gamma}d}$, $k_\gamma$ is the z-component of the wavevector in the vacuum, $k_\gamma = \sqrt{q^2 - k_0^2}$, and $\tilde{R}_2 = LR_2^\dagger(I^\dagger)^{-1}$. $R_1(\omega, q)$ and $R_2(\omega', q')$ are the reflection matrices of slabs 1 and 2 in the linear polarization basis, respectively. The angular frequency and wavevector in the two frames are related by the Lorentz transformation as

$$
\omega' = \gamma(\omega - q_x V), \quad q_x' = \gamma(q_x - \beta k_0), \quad q_y' = q_y, \quad k_z' = k_z,
$$

(3)

where $\beta = \frac{V}{c}$ and $\gamma^{-1} = \sqrt{1 - \beta^2}$. The matrix $L$ transforms the electric fields of forward propagating waves in the vacuum, i.e., the waves propagating towards the positive z-direction, from the rest frame to the co-moving frame via the Lorentz transformation. It is expressed as

$$
L = \frac{k_0'}{k_0 q_\gamma q_x} \begin{bmatrix} q_x^2 - \beta k_0 q_x & \beta k_0 q_y \\ -\beta k_0 q_x & q_y^2 - \beta k_0 q_x \end{bmatrix},
$$

(4)

where $k_0' = \frac{\omega'}{c}$ and $q' = |q'|$.

Figure 1. Two semi-infinite parallel slabs separated by a vacuum gap $d$ that are moving relative to each other at constant velocity $V$ along the $x$-direction. (a): the system operating as a heat engine where the net radiative heat flux $\varphi_{1-2}$ from slabs 1 to 2 is converted into work driven by non-equilibrium lateral Casimir force $f_{x,2}$ on slab 2 under proper directions of external magnetic fields. (b): the system operating as a heat pump where the external work done by the force $f_{\text{ext}}$ pumps heat from slab 2 at a low proper temperature to slab 1 at a higher proper temperature.

3. OPERATION AS HEAT ENGINE

We study the two slabs as shown in Fig. 1 (a) as a heat engine that converts radiative heat transfer into mechanical work driven by non-equilibrium lateral Casimir force. We consider that the temperatures of slabs 1 and 2 are $T_1 = 305\text{K}$ and $T_2 = 300\text{K}$, respectively, and the vacuum gap between the slabs is 10nm. In order to operate as a heat engine, non-equilibrium lateral Casimir force must act on slab 2 when the two slabs are at rest, which requires that one of the slabs must break Lorentz reciprocity. Thus, we consider a $n$-doped indium antimonide ($n$-InSb) as slab materials under external static magnetic fields in the $y$-direction as shown in Fig. 1 (a).
Figure 2 (a) and (b) show the radiative heat flux from slabs 1 to 2 and shear stress on slab 2 as a function of the velocity of relative motion under different magnitudes of external static magnetic fields. For a fixed magnitude of the magnetic fields, the radiative heat flux decreases at increasing velocities due to Doppler shift in the angular frequency. Compared to the radiative heat flux under no magnetic fields, the heat flux under the anti-parallel magnetic fields is smaller due to the interplay of the evolution of surface plasmon and phonon-polariton waves, as well as the hyperbolic modes that appear as the magnitude of magnetic fields increases. We also consider applying an external static magnetic field to only one of the slabs, i.e., \( B_1 = 0 \)T and \( B_2 = -3 \)T, as a more practical system. The radiative heat flux shows qualitatively similar result.

Figure 2 (b) shows the shear stress on slab 2. In the absence of the external magnetic field, the shear stress is zero at rest. When slab 2 moves as a result of external work, the shear stress acts on slab 2 in the direction opposite to the motion due to Doppler shift in the angular frequency. In contrast, in the presence of the external magnetic fields on at least one of the slabs, the non-zero shear stress acts on slab 2 even at rest, allowing the heat engine to self-start. At non-zero velocities, the shear stress acts in the same direction as the relative motion, accelerating slab 2. Thus, the two-slab system operates as a heat engine. As the velocity of slab 2 increases, the shear stress acting on slab 2 decreases and becomes zero at a steady-state velocity. For the case of \( B_1 = -B_2 = 3 \)T, the steady-state velocity is around \( V = 1.4 \times 10^3 \) m/s. To go beyond the steady-state velocity, external work must be applied, and the two-slab structure no longer operates as a heat engine.

The thermodynamic efficiency of the heat engine operating at non-relativistic velocities is defined for \( T_1 > T'_2 \) as

\[
\eta = \frac{f_{x,2}V}{\varphi_{1\rightarrow 2}},
\]

where this definition is meaningful only when \( f_{x,2}V > 0 \). We analytically show that the thermodynamic efficiency approaches the Carnot efficiency \( \eta \rightarrow 1 - T'_2/T_1 \) when the slabs support the modes only for \( q_x > 0 \), but not for \( q_x < 0 \), and the dispersion satisfies \( \omega = V_c / \left( 1 - \frac{T'_2}{T_1} \right) q_x \) where \( V_c \) is the operating velocity of the heat engine at which the efficiency approaches Carnot efficiency.

\[
\eta \rightarrow 1 - \frac{T'_2}{T_1} \quad \text{for} \quad q_x > 0
\]

\[
\eta \rightarrow 0 \quad \text{for} \quad q_x < 0
\]

\[
\omega = V_c / \left( 1 - \frac{T'_2}{T_1} \right) q_x
\]

\[
\eta \rightarrow \frac{V_c}{T_1} \quad \text{for} \quad \omega \text{ is the operating velocity}
\]

Figure 2. (a) Radiative heat flux from slabs 1 to 2 and (b) non-equilibrium lateral Casimir force per unit surface area on slab 2. Two slabs made of \( n \)-InSb are at \( T_1 = 305 \)K and \( T'_2 = 300 \)K and separated by \( d=10 \)nm. The dash-dot line in panel (b) describes \( f_{x,2} = 0 \).

### 4. Operation as Heat Pump

In the same two-slab system, we consider the situation where external work is applied on slab 2 to further increase the velocity beyond the steady state velocity of the heat engine. At sufficiently high velocity, we show that the radiative heat flows from slab 2 at a lower temperature to slab 1 at a higher temperature. Hence, the system operates as a photonic heat pump that converts external mechanical work to radiative heat from a lower to a higher temperature slab.

Figure 3 (a) and (b) show the radiative heat flux from slabs 1 to 2 and the shear stress on slab 2, respectively, for the same materials and temperatures as the heat engine shown in Fig. 2, but in a range of higher velocity. The shear stress on slab 2 is negative and external work needs to be applied in order to move slab 2 at such velocities. In the absence of the external magnetic field, the radiative heat flux becomes negative for the velocity greater than \( 10^5 \) m/s, where the heat flows from slab 2 at a lower proper temperature to slab 1 at a higher proper temperature. This shows that structures made of reciprocal material can operate as a heat pump due to the non-reciprocal wave propagations as induced by the Doppler shift in the angular frequency.

\[
\eta \rightarrow 1 - \frac{T'_2}{T_1} \quad \text{for} \quad q_x > 0
\]

\[
\eta \rightarrow 0 \quad \text{for} \quad q_x < 0
\]

\[
\omega = V_c / \left( 1 - \frac{T'_2}{T_1} \right) q_x
\]

\[
\eta \rightarrow \frac{V_c}{T_1} \quad \text{for} \quad \omega \text{ is the operating velocity}
\]
effect. The application of the external static magnetic fields can lower the velocity at which the cooling occurs; with the presence of anti-parallel magnetic fields of 3 T, the onset of the cooling occurs at a velocity of \( V \approx 6 \times 10^4 \) m/s. Comparing the cases of \( B_1 = B_2 = 0 \) T and \( B_1 = -B_2 = 3 \) T, the greater amount of radiative heat can be pumped by applying less amount of external mechanical work when the velocity is below \( V = 4.5 \times 10^5 \) m/s. This shows that the non-reciprocal materials can enhance the performance of the heat pump. The radiative heat flux in the case of \( B_1 = 0 \) T, \( B_2 = -3 \) T shows the qualitatively similar result to the case of \( B_1 = -B_2 = 3 \) T and the onset of cooling occurs at a lower velocity compared to the case of \( B_1 = B_2 = 0 \) T. The magnitude of the shear stress, however, is larger and the performance as a heat pump is lower.

![Figure 3. (a) Radiative heat flux from slabs 1 to 2 and (b) non-equilibrium lateral Casimir force per unit surface area on slab 2. The system parameters are the same as those in Fig. 2. The dash-dot line in panel (a) describes \( \varphi_{1-2} = 0 \).](image)

5. RELATIVISTIC THERMODYNAMIC EFFICIENCY OF HEAT ENGINE

The thermodynamic efficiency of the heat engine when the velocity of relative motion is non-relativistic is defined as Eq. (5). When a heat engine made of two slabs operates at relativistic velocities, the efficiency as defined in Eq. (5) results in unphysical consequences. We show that for the observer in the rest frame, Eq. (5) is bounded by \( \eta \leq 1 - T_2 / \gamma T_1 \). This indicates that the thermodynamic efficiency limit depends on the choice of reference frame. Moreover, in the limit of \( V \to c \), the efficiency reaches unity in the rest frame since \( \gamma \to \infty \), which is unphysical. Landsberg pointed out [30] that the Carnot limit should not depend on the choice of a particular reference frame and two contributions that appear in a heat engine operating at relativistic velocities. In this work, we derived the thermodynamic efficiency incorporating the two relativistic effects by using the relativistic thermodynamics [31]. By considering these contributions to the definition of useful work, we show that the relativistic thermodynamic efficiency is bounded by the Carnot efficiency which is independent of the frame of reference.

The first contribution originates from the change of momentum of slab 2 as a result of heat transfer. In the operation of a heat engine, the system must return to the same thermodynamic state as the initial state after one cycle. In relativistic thermodynamics, it means that the system must retain not only the same energy but also the same momentum after one cycle. Thus, in order to keep the momentum of the moving medium unchanged after one cycle, a certain amount of energy must be rejected to the heat sink and cannot be used as useful work. The second contribution is from the additional work that can be extracted as a result of the energy increase of slab 2 after one cycle. Consider the acceleration and deceleration of slab 2 before and after we operate the heat engine at a given velocity. In non-relativistic physics, the work required for the two processes are the same because the mass of slab 2 is invariant before and after the operation of the heat engine. However, in the relativistic cases, the work done by slab 2 in the deceleration is greater than the work done to slab 2 to accelerate due to the energy increase of slab 2 after the acceleration process. In principle, this difference between the two can be extracted as useful work.

By considering these two contributions, we derive the relativistic thermodynamic efficiency for \( T_1 > T_2 / \gamma \) as:

\[
\eta_{rel} = \frac{\gamma f_{x,2} V - (\gamma - 1) \varphi_{1-2}}{\varphi_{1-2}},
\]

and we showed that the relativistic thermodynamic efficiency is bounded by the Carnot efficiency \( \eta_{rel} \leq 1 - T_2 / T_1 \), which is independent of the frame of reference.
6. CONCLUSIONS

In summary, we showed that a system consisting of two semi-infinite parallel slabs with different temperatures can work as a photonic heat engine driven by non-equilibrium lateral Casimir forces when one of the materials breaks Lorentz reciprocity. Also, in this system a sufficiently high velocity of relative motion can enable radiative heat pump where radiative heat from the low to high temperature objects. Non-reciprocal wave propagations induced by the relative motion can realize the heat pump and the use of non-reciprocal materials can further reduce the amplitude of the required velocity of relative motion to achieve the heat pump. We showed that the relativistic thermodynamic efficiency of the photonic heat engine and pump is bounded by the Carnot efficiency and revealed the ideal dispersion of materials that approaches the limit. Our results point to a way of thermal energy harvesting and cooling by non-equilibrium Casimir forces enabled by breaking Lorentz reciprocity.

Reference

[1] E. G. Cravalho, C. L. Tien, and R. P. Caren, Journal of Heat Transfer 89, 351 (1967).
[2] C. M. Hargreaves, Physics Letters A 30, 491 (1969).
[3] D. Polder and M. Van Hove, Physical Review B 4, 3303 (1971).
[4] J.-P. Mulet, K. Joulain, R. Carminati, and J.-J. Greffet, Applied Physics Letters 78, 2931 (2001).
[5] J.-P. Mulet, K. Joulain, R. Carminati, and J.-J. Greffet, Microscale Thermophysical Engineering 6, 209 (2002).
[6] S. Shen, A. Narayanawamy, and G. Chen, Nano Letters 9, 2909 (2009).
[7] E. Rousseau, A. Siria, G. Jourdan, S. Volz, F. Comin, J. Chevrier, and J.-J. Greffet, Nature Photonics 3, 514 (2009).
[8] R. S. Ottens, V. Quetschke, S. Wise, A. A. Alemi, R. Lundock, G. Mueller, D. H. Reitze, D. B. Tanner, and B. F. Whiting, Physical Review Letters 107, 014301 (2011).
[9] T. Kralik, P. Hanzelka, V. Musilova, A. Srnka, and M. Zobac, Review of Scientific Instruments 82, 055106 (2011).
[10] K. Kim et al., Nature 528, 387 (2015).
[11] H. B. Casimir, in Proc. Kon. Ned. Akad. Wet. 1948), p. 793.
[12] H. B. G. Casimir and D. Polder, Physical Review 73, 360 (1948).
[13] E. M. Lifshitz, in Soviet Physics 1956), pp. 73.
[14] S. K. Lamoreaux, Physical Review Letters 78, 5 (1997).
[15] U. Mohideen and A. Roy, Physical Review Letters 81, 4549 (1998).
[16] C. Henkel, K. Joulain, J. P. Mulet, and J. J. Greffet, Journal of Optics A: Pure and Applied Optics 4, S109 (2002).
[17] M. Antezza, L. P. Pitaevskii, and S. Stringari, Physical Review Letters 95, 113202 (2005).
[18] M. Antezza, L. P. Pitaevskii, S. Stringari, and V. B. Svetovoy, Physical Review Letters 97, 223203 (2006).
[19] M. Antezza, L. P. Pitaevskii, S. Stringari, and V. B. Svetovoy, Physical Review A 77, 022901 (2008).
[20] A. O. Sushkov, W. J. Kim, D. A. R. Dalvit, and S. K. Lamoreaux, Nature Physics 7, 230 (2011).
[21] M. G. Silveirinha, S. A. H. Gangaraj, G. W. Hanson, and M. Antezza, Physical Review A 97, 022509 (2018).
[22] S. A. Hassani Gangaraj, G. W. Hanson, M. Antezza, and M. G. Silveirinha, Physical Review B 97, 201108 (2018).
[23] D. Pan, H. Xu, and F. J. Garcia de Abajo, Physical Review A 99, 062509 (2019).
[24] M. F. Maghrebi, A. V. Gorshkov, and J. D. Sau, Physical Review Letters 123, 055901 (2019).
[25] C. Khandekar and Z. Jacob, New Journal of Physics 21, 103030 (2019).
[26] C. Khandekar, S. Buddhiraju, P. R. Wilkinson, J. K. Gimzewski, A. W. Rodriguez, C. Chase, and S. Fan, Physical Review B 104, 245433 (2021).
[27] Y. Guo and S. Fan, ACS Photonics 8, 1623 (2021).
[28] D. Gelbwaser-Klimovsky, N. Graham, M. Kardar, and M. Krüger, Physical Review Letters 126, 170401 (2021).
[29] Y. Tsurimaki, R. Yu, and S. Fan, arXiv:2211.05193, (2022) (manuscript under review).
[30] P. T. Landsberg and K. A. Johns, Journal of Physics A: General Physics 5, 1433 (1972).
[31] R. C. Tolman, Relativity, thermodynamics, and cosmology (Courier Corporation, 1987).