A Novel Power-Based Orthogonal Signal Generator for Single-Phase Systems

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Abstract—Conventional orthogonal signal generators (OSGs) employed in single-phase systems exhibit significant delay in tracking the original signal. To tackle this problem, an OSG with fast synchronizing speed is proposed by applying the principle of power definition of sinusoids. The proposed OSG can accurately track the single-phase input signal in less than half cycle, which is one cycle faster than the widely used OSGs. Depending on the application, the proposed OSG can be modified to meet application-based requirements. The proposed OSG in a single phase frequency locked-loop (FLL) implementation is firstly introduced in this paper. In addition, the ability of a low-pass filter to act not just as a filter, but to eliminate steady-state error is revealed. Through experimental comparison with Park and SOGI OSG-based FLLs, it is shown that the proposed OSG-based FLL provides faster estimation of frequency and phase angle of a single-phase voltage.

Index Terms—Orthogonal signal generator, frequency-locked loops, dynamic response.

I. INTRODUCTION

Generation of orthogonal signals in three-phase systems is straightforward and can be achieved using the Clark transformation [1]. However, the availability of just one signal in single-phase systems makes the generation of orthogonal signals a tedious task. Even though some solutions, such as, Park and second-order generalized integrator (SOGI) based orthogonal signal generators (OSGs) are very popular in the literature to tackle this issue, they are characterized by slow speed and exhibit significant delay in tracking the original single-phase input voltage [2], [3].

In SOGI OSG, the orthogonal signals are generated using transfer functions which continuously change with the estimated frequency, while Park OSG utilizes feedback of the pseudo-generated signal. Interestingly, both OSGs are equivalent from the control point of view and since one has no comparative advantage over the other in terms of dynamic response, it can be said that only one class of OSG is widely employed in single-phase systems [4]. This necessitates the development of OSGs to track the original single-phase signal.

Therefore, this paper proposes a novel orthogonal signal generator block for single-phase systems. The proposed OSG relies on the principle of power definition of sinusoids and notch filtering for oscillation rejection. The proposed OSG is able to track a single-phase voltage with fast speed. Furthermore, an application of the proposed OSG for FLL implementation is carried out where it is shown that the proposed OSG may be modified as required. In the FLL implementation, it is revealed that incorporating a low-pass filter within the proposed OSG eliminates steady-state phase-angle error. Fast response of the proposed OSG for FLL implementation is validated through experimental studies in comparison to Park and SOGI-FLLs.

II. PROPOSED OSG

The first step in generating a set of orthogonal signals from the single-phase input voltage is by obtaining the magnitude and the phase angle error information ($V_d$ and $V_q$), respectively. This is done by passing the reference phase angle ($\theta$) through sine and cosine functions then multiplying the results with the original single-phase as shown below.

$$
\begin{align*}
V_d &= V \sin \theta \sin \hat{\theta} = \frac{V}{2} \left( \cos(\theta - \hat{\theta}) - \cos(\theta + \hat{\theta}) \right) \\
V_q &= V \sin \theta \cos \hat{\theta} = \frac{V}{2} \left( \sin(\theta - \hat{\theta}) + \sin(\theta + \hat{\theta}) \right)
\end{align*}
$$

(1)

where $V$ is the voltage amplitude and $\hat{\theta}$ is the phase angle of the single-phase input voltage.

Under phase locked condition ($\theta = \hat{\theta}$), the terms of $V_d$ and $V_q$ provide estimations of the voltage amplitude and phase error, respectively. However, both signals suffer from double frequency oscillation. These oscillations are removed by independently passing $V_d$ and $V_q$ through notch filters tuned at the double of the estimated frequency. The notch filter in [5] is utilized in this work since it effectively attenuate disturbances with $-300 \text{ dB}$ gain at frequencies of unwanted signals. Thus, the oscillation-free
magnitude and phase angle error information are:
\[
\begin{align*}
\bar{V}_d &= \frac{V}{2} \cos(\theta - \hat{\theta}) \\
\bar{V}_q &= \frac{V}{2} \sin(\theta - \hat{\theta})
\end{align*}
\]  
(2)

Multiplying \(\bar{V}_d\) and \(\bar{V}_q\) with the result of passing \(\hat{\theta}\) through sine and cosine functions, and after some manipulations, a set of orthogonal signals are generated as in (3).
\[
\begin{align*}
\bar{V}_\alpha &= \bar{V}_d \sin \hat{\theta} + \bar{V}_q \cos \hat{\theta} = \frac{V}{2} \sin \hat{\theta} \\
\bar{V}_\beta &= \bar{V}_q \sin \hat{\theta} - \bar{V}_d \cos \hat{\theta} = -\frac{V}{2} \cos \hat{\theta}
\end{align*}
\]  
(3)

Finally, the required set of orthogonal signals are obtained by introducing a factor of 2 to the amplitudes of \(\bar{V}_\alpha\) and \(\bar{V}_\beta\) as follow.
\[
\begin{align*}
V_\alpha &= V \sin \hat{\theta} \\
V_\beta &= -V \cos \hat{\theta}
\end{align*}
\]  
(4)

The complete realization of the procedure used in generating the proposed \(V_\alpha\) and \(V_\beta\) is shown in Fig. 1.

To demonstrate the fast-tracking capability of the proposed OSG, the single-phase input voltage is applied to the proposed OSG, PARK and SOGI OSGs at 0.2 s. as in Fig. 2. The result obtained for the in-phase signal shows that the proposed OSG tracks the single-phase input voltage in less than half cycle, and is about 1 cycle faster than Park and SOGI OSGs. A similar result can be obtained with the quadrature-phase signal. The above study demonstrated that the proposed OSG can track the original signal in less than half a cycle.

Under distorted voltage condition, any nth harmonic component in the supply voltage will appear as a sinusoidal signal with frequencies of the order \(n \pm 1\). These harmonic components can be rejected by replacing the notch filter within the OSG with a moving average filter. The applicability of the proposed OSG is demonstrated in the next section.

III. APPLICATION OF THE PROPOSED OSG IN FREQUENCY-LOCKED LOOP

A. Frequency and Phase Angle Estimation Algorithms

The frequency and phase estimation algorithms applied to the set of orthogonal signals in (4) is presented. Assuming unity amplitude voltage, the frequency can be estimated by taking the derivative of \(V_\alpha\) and \(V_\beta\), and performing some mathematical manipulations as follow.
\[
\begin{align*}
\begin{bmatrix}
V'_\alpha \\
V'_\beta
\end{bmatrix} &= \begin{bmatrix}
dV_\alpha \\
dV_\beta
\end{bmatrix} = \begin{bmatrix}
\omega \cos(\omega t) \\
\omega \sin(\omega t)
\end{bmatrix} \\
\hat{\omega} &= \sqrt{(V'_\alpha)^2 + (V'_\beta)^2}
\end{align*}
\]  
(5)

where \(\hat{\omega}\) is the estimated frequency.

Finally, a low-pass filter with cut-off frequency \((\omega_c)\) is applied to the estimated frequency to keep it free from high frequency disturbances. It should be noted that as long as the supply voltage is clean, noise introduced by the differentiation operation does not compromise the performance of the algorithm. In the presence of harmonics, cascaded notch filters tuned at harmonic frequencies can be utilized to keep the voltage clean.

The phase angle estimation can proceed by integrating the estimated frequency \((\hat{\omega})\) to give an estimation of the reference phase angle \((\hat{\theta})\).
\[
\hat{\theta} = \int_0^t \hat{\omega} dt = \hat{\theta} t
\]  
(6)

However, there exists a small phase angle difference between the actual \((\theta)\) and the reference \((\hat{\theta})\) phase angles. This phase difference is accounted for by taking the tangent inverse of the oscillation-free direct \((\bar{V}_d)\) and quadrature \((\bar{V}_q)\) voltages.
\[
\theta_{diff} = \tan^{-1} \left( \frac{\bar{V}_q}{\bar{V}_d} \right) = (\omega - \hat{\omega}) t
\]  
(7)

Thus, the estimated phase angle \((\hat{\theta}_1)\) can be expressed as:
\[
\hat{\theta}_1 = \hat{\theta} + \theta_{diff}
\]  
(8)
B. FLL Small-Signal Modeling

Using the proposed OSG of Fig. 1 with the frequency estimation algorithm presented, linearized model of the FLL’s frequency estimation loop is shown in Fig. 3(a). Dynamics associated with the oscillation rejection is totally avoided by choosing the corner frequency of the FLL sufficiently lower than two-fifth of the double frequency i.e. 40 Hz [5]. As such, neglecting the dynamics of the notch filter and applying block reduction techniques, a simplified model is obtained as in Fig. 3(b).

Since $\omega(s) = s\theta(s)$ and $\hat{\omega}(s) = s\hat{\theta}(s)$, the open-loop transfer function of the phase angle estimation loop $(G_{ol}(s))$ can be obtained as:

$$G_{ol}(s) = \frac{\hat{\theta}(s)}{\theta(s)} = \frac{\omega_0}{s}$$  \hspace{1cm} (9)

From (9), it can be observed that $G_{ol}(s)$ has only one integrator in its denominator polynomial, thereby resulting in a type-1 control system.

IV. OSG MODIFICATION FOR FLL IMPLEMENTATION

When the proposed OSG is applied in the FLL implementation, as per (9), it will suffer from steady-state phase angle error during phase angle jump. Also, when the assumption of unity voltage amplitude in the FLL algorithm of (5) is violated, the FLL’s frequency and phase angle estimations become erroneous. These issues are resolved by incorporating low-pass filters with cut-off frequency $(\omega_p)$ to introduce an extra pole in the FLL’s frequency and phase angle open loop transfer functions. Also, an amplitude normalization system (AN-block) is incorporated within the OSG to obtain the unity amplitude voltages employed in (5). This is achieved by normalizing $\bar{V}_d$ and $\bar{V}_q$ of (2). The procedure of amplitude normalization is implemented within the AN-block of Fig. 4 as:

$$\begin{align*}
\bar{V}_d^n &= \frac{V_d}{\sqrt{(V_d)^2 + (V_q)^2}} = \cos(\theta - \hat{\theta}) \\
\bar{V}_q^n &= \frac{V_q}{\sqrt{(V_d)^2 + (V_q)^2}} = \sin(\theta - \hat{\theta})
\end{align*}$$  \hspace{1cm} (10)

where $\bar{V}_d^n$ and $\bar{V}_q^n$ are the normalized estimates of the voltage amplitude and phase error, respectively.

The modifications involving low-pass filter and AN-block are highlighted in the modified OSG shown in Fig. 4 [5]. Thus, the...
block diagram of FLL implementation with the modified OSG is depicted in Fig. 5. Only the small signal model of phase estimation loop in the FLL implementation with the modified OSG is presented (Fig. 6) to show the effectiveness of the low-pass filter in eliminating the steady-state phase angle error. Specifically, it can be seen from Fig. 6(b) that with the introduction of a low-pass filter within the OSG, the open-loop transfer function of the phase angle estimation loop exhibits two (2) poles at its origin. This makes the phase angle estimation loop a type-2 control system, thereby eliminating the steady-state phase angle error in the FLL’s estimation during phase angle jump.

V. EXPERIMENTAL RESULTS

Ability of the proposed OSG (Fig. 4) in FLL implementation to provide fast response is evaluated by experimental comparison with Park and SOGI-FLLs. In this comparison, the control parameters of all the schemes are equivalent and tuned using $\zeta = 0.7071$ and $\omega_n = 200$ rad/s. Therefore, $\omega_p = 282.84$ rad/s, $\omega_o = 141.42$ rad/s and $k_{SOGI} = 0.9$. Digital implementation of all schemes was carried out in DS-1103 digital signal processor (DSP) from dSPACE and throughout the experiment, the sampling frequency ($f_s$), nominal frequency ($\omega_f$), and voltage amplitude ($V$) are kept at 15 kHz, 50 Hz, and 1 pu, respectively, except for the case of voltage sag.

Figs. 7(a) and (b) illustrate the frequency response experimental results when the single-phase input voltage undergoes a +5 Hz frequency step change and +20° phase angle jump, respectively. The frequency settling time (2% criteria) and peak frequency overshoot for the proposed power-based OSG FLL (PB-OSG FLL), PARK FLL, and SOGI FLL are: (30 ms, 55 ms, 55 ms); (1.2 Hz, 1.1 Hz, 1.1 Hz) during frequency step change and (39 ms, 67 ms, 67 ms); (4.6 Hz, 4.0 Hz, 4.0 Hz) during phase angle jump, respectively. From the results, it can be seen that the proposed OSG outperforms PARK and SOGI OSGs in terms of response time. Due to space restriction, the phase angle error response is not provided. However, the experimental results for the peak phase angle overshoot are obtained as: (8.4°, 9.7°, 9.7°) and (5.6°, 7.6°, 7.6°) for PB-OSG FLL, PARK FLL, and SOGI FLL during frequency step change and phase angle jump, respectively. Generally speaking, the proposed PB-OSG FLL provides the least phase angle overshoot.

VI. CONCLUSION

A new orthogonal signal generator (OSG) with fast tracking capability is proposed for single-phase systems by applying the power definition of sinusoids. It is shown that the proposed OSG tracks the single-phase voltage in less than half cycle and is about 1 cycle faster than Park and SOGI OSGs. Furthermore, an FLL implementation with the proposed OSG is carried out and the ability of a low-pass filter to act not just as filter, but to eliminate steady-state phase angle error is revealed. Through experimental validations, the proposed OSG when implemented with a FLL provides fast response in comparison to Park and SOGI FLLs.

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