Non Linear Breaking of the Electroweak Symmetry and Large Extra Dimensions

D. Dominici$^{a,b}$

$^a$Dipartimento di Fisica, Università di Firenze, I-50125 Firenze, Italia
$^b$I.N.F.N., Sezione di Firenze, I-50125 Firenze, Italia

Abstract

We consider the coupling to gravity in $4 + \delta$ dimensions of a non linear electroweak symmetry breaking sector, with $\delta$ compactified dimensions, and derive an effective lagrangian by integrating over the KK excitations of the graviton and of the dilaton. The effective chiral lagrangian describes the interactions of the electroweak gauge bosons and the goldstone bosons from the electroweak breaking.


1 Introduction

There is been recently a growing interest in theories in $4 + \delta$ dimensions with $\delta$ compactified dimensions and an effective Planck scale $M_{\text{Pl}(4+\delta)}$ which can be lower than the Planck scale \([1, 2]\). In these models gravity propagates in $4 + \delta$ dimensions while strong and electroweak interactions are bounded on a three dimensional brane. Experimental tests of gravity exclude an effective Planck scale $M_{\text{Pl}(5)}$ down to one TeV for $\delta = 1$ and a recent experiment has also put a limit $M_{\text{Pl}(6)} \geq 3.5 \text{ TeV}$ for the case $\delta = 2$ \([3]\). The phenomenology of these scenarios has been largely studied \([4, 5, 6]\), concentrating mainly on the KK excitations of the gravitons \([4, 5, 6]\) but also on KK excitations of the dilatons \([6, 7, 8]\). If the gravity is brought down to one TeV scale then there could be some intertwining between gravity and electroweak symmetry breaking. A new mechanism of spontaneous symmetry breaking induced by the Kaluza Klein (KK) excitations have been recently proposed \([7]\). The phenomenological consequences of the graviscalar Higgs boson mixing have been analysed \([6, 8]\). In this note we consider a non linear breaking of the electroweak symmetry and derive an effective lagrangian by integrating over the KK excitations of the graviton and of the dilaton. The effective chiral lagrangian describes the interactions of the electroweak gauge bosons and the goldstone bosons from the electroweak breaking.

The starting point of the model is the Einstein lagrangian in $4 + \delta$ dimensions \([3, 4]\)

$$\frac{1}{\kappa^2} \sqrt{-\hat{g}} \hat{R}$$

One first expands around the flat metric

$$\hat{g}_{\hat{\mu}\hat{\nu}} = \eta_{\hat{\mu}\hat{\nu}} + \kappa \hat{h}_{\hat{\mu}\hat{\nu}} + O(\kappa^2)$$

($\hat{\mu}, \hat{\nu} = 0, 1, 2, 3, \ldots, 3 + \delta$) and then one performs a dimensional reduction using the following ansatz for the metric

$$\hat{h}_{\hat{\mu}\hat{\nu}} = V_{\delta}^{-1/2} \left( \frac{h_{\mu\nu} + \eta_{\mu\nu} \phi}{A_{\mu i}} A^{\nu j} 2 \phi_{ij} \right)$$

with $\kappa V_{\delta}^{-1/2} = \kappa \equiv \sqrt{16\pi} / M_{\text{Pl}}$, where $M_{\text{Pl}}$ is the usual Planck mass for three spatial dimensions and $V_{\delta} = L^\delta$ ($\kappa \sim M_{\text{Pl}(4+\delta)}^{-\delta/2+1}$) and $\phi = \phi_{ii}$ denotes...
the dilaton mode. After compactification on a δ dimensional torus $T^δ$ one expands the fields in Fourier modes $h_{\mu\nu}^\vec{n}$, $A_{\mu i}^\vec{n}$ and $\phi_{ij}^\vec{n}$.

The lagrangian for the massive $\vec{n}$-KK states at the leading order in $\kappa$ is given by [6, 4]

$$
\mathcal{L}^\vec{n} = \frac{1}{2} \left( \partial^\mu \tilde{h}_{\nu\rho,\vec{n}} \partial_\mu \tilde{h}_{\nu\rho} - \partial^\mu \tilde{h}_{\nu,\vec{n}} \partial_\mu \tilde{h}_{\nu} - 2 \tilde{h}_{\mu,\vec{n}} \tilde{h}_{\mu} + 2 \tilde{h}_{\mu,\vec{n}} \partial_\mu \tilde{h}_{\mu} - m_n^2 \tilde{h}_{\mu,\vec{n}} \tilde{h}_{\mu} \right) + \sum_{i=1}^\delta (-\frac{1}{2} \tilde{F}_{\mu,\vec{n}} \tilde{F}_{\mu} + m_n^2 \tilde{A}_{\mu,\vec{n}} \tilde{A}_{\mu} - \vec{n}) + \delta \sum_{(ij)=1} (\partial^\mu \tilde{\phi}_{ij} \partial_\mu \tilde{\phi}_{ij} - m_n^2 \tilde{\phi}_{ij} \tilde{\phi}_{ij}) \tag{4}
$$

where $\tilde{h}_{\rho} = \partial^\nu \tilde{h}_{\nu,\vec{n}}$ and

$$
m_n^2 = \frac{4\pi^2 n^2}{L^2} \tag{5}
$$

with $n^2 = \vec{n}$, $\vec{n} = (n_1, n_2, \ldots, n_\delta)$. For the definition of the fields $\tilde{h}_{\mu,\vec{n}}, \tilde{A}_{\mu,\vec{n}}$ and $\tilde{\phi}_{ij}$ in terms of $h_{\mu,\vec{n}}, a_{\mu,\vec{n}}$ and $\phi_{ij}$ see [6, 4].

The fields are subjected to the constraints

$$
\partial^\mu \tilde{A}_{\mu,\vec{n}} = 0, \quad n_i \tilde{A}_{\mu,\vec{n}} = 0 \quad, \quad n_i \tilde{\phi}_{ij} = 0 \tag{6}
$$

The total lagrangian $\mathcal{L}$ is obtained by summing $\mathcal{L}^\vec{n}$ only over $\vec{n}$ values such that the first non-zero $n_k$ is positive. We will denote this sum by $\sum_{\vec{n}>0} \mathcal{L}^\vec{n}.$

Matter fields leave in four dimensions and their action is given by

$$
\int d^4x \sqrt{-\hat{g}_{\text{ind}}} \mathcal{L}(\hat{g}_{\text{ind}}, S, V, F) \tag{7}
$$

for scalar (S), vector (V) and fermions (F). Expanding the metric $(\hat{g}_{\text{ind}})^{\mu\nu} = \eta_{\mu\nu} + \kappa (h_{\mu\nu} + \eta_{\mu\nu} \phi)$, the order $\kappa$ of eq.(7) is given by

$$
- \frac{\kappa}{2} \int d^4x (h^{\mu\nu} T_{\mu\nu} + \phi T_{\mu}^\mu) \tag{8}
$$

where

$$
T_{\mu\nu} = -\eta_{\mu\nu} \mathcal{L} + 2 \left( \frac{\partial \mathcal{L}}{\partial \hat{g}^{\mu\nu}} \right)_{\hat{g}^{\mu\nu} = \eta_{\mu\nu}} \tag{9}
$$
is the energy-momentum tensor.

The interactions of the massive $\vec{n}$-KK states to the matter fields are given by

$$L_{\text{mix}}^{\vec{n}} = -\frac{\kappa}{2} \left[ (\vec{h}^{\mu\nu,\vec{n}} + \vec{h}^{\mu\nu,-\vec{n}}) T_{\mu\nu} + \omega_\delta \left( \vec{\phi}^{\vec{n}} + \vec{\phi}^{-\vec{n}} \right) T_{\mu}^{\mu} \right]$$

(10)

where

$$\omega_\delta = \left[ \frac{2}{3(\delta + 2)} \right]^{1/2}$$

(11)

In conclusion we will consider the lagrangian

$$L = \sum_{\vec{n} > 0} \left[ L^{\vec{n}} + L_{\text{mix}}^{\vec{n}} \right]$$

(12)

Integrating out $\vec{h}^{\vec{n}}$, $\vec{A}^{\vec{n}}$ and $\vec{\phi}^{\vec{n}}$ fields, one gets an effective lagrangian given by

$$L_{\text{eff}} = \frac{\kappa^2}{4} \sum_{\text{all } \vec{n}} D \left[ T^{\mu\nu} P_{\mu\nu;\rho\sigma} T_{\rho\sigma} + \frac{\omega_\delta^2(\delta - 1)}{2} T_{\mu}^{\mu} T_{\nu}^{\nu} \right]$$

(13)

where

$$D = \frac{1}{\Box + m_n^2}$$

(14)

and $P_{\mu\nu;\rho\sigma}$ in the momentum space is

$$P_{\mu\nu;\rho\sigma} = \frac{1}{2} (g_{\mu\rho} g_{\nu\sigma} + g_{\mu\sigma} g_{\nu\rho} - g_{\mu\nu} g_{\rho\sigma})$$

$$- \frac{1}{2m_n^2} (g_{\mu\rho} k_{\nu} k_{\sigma} + g_{\mu\sigma} k_{\nu} k_{\rho}) + (\mu \rightarrow \nu)$$

$$+ \frac{1}{6} (g_{\mu\nu} + \frac{2}{m_n^2} k_{\mu} k_{\nu})(g_{\rho\sigma} + \frac{2}{m_n^2} k_{\rho} k_{\sigma})$$

(15)

The integration over the fields $\vec{A}^{\vec{n}}$ is trivial, because these fields decouple. By using in the effective action the conservation of $T_{\mu\nu}$ (or the classical equations of motion), one can rewrite $L_{\text{eff}}$ as

$$L_{\text{eff}}' = \frac{\kappa^2}{4} \sum_{\text{all } \vec{n}} D \left( T^{\mu\nu} T_{\mu\nu} - \frac{1}{3} T_{\mu}^{\mu} T_{\nu}^{\nu} + \frac{\omega_\delta^2(\delta - 1)}{2} T_{\mu}^{\mu} T_{\nu}^{\nu} \right)$$

(16)
The sum $\frac{\kappa^2}{4} \sum_{\vec{n}} D$ is ultraviolet divergent for $\delta \geq 2$. Different procedures of regularization have been suggested \[4, 6\]. For instance using an ultraviolet cutoff $M_S$ one gets, for $M_S \gg \sqrt{s}$, \[6\]

$$
\frac{\kappa^2}{4} \sum_{\vec{n}} D = \frac{\kappa^2}{2} \sum_{\vec{n}>0} D \sim \frac{1}{2M_S^4(\delta - 2)} + O(s/M_S^2) \quad \text{for } \delta > 2
$$

(17)

$$
\frac{\kappa^2}{4} \sum_{\vec{n}} D = \frac{\kappa^2}{2} \sum_{\vec{n}>0} D \sim \frac{1}{4M_S^4} \log \frac{M_S^2}{s} + O(s/M_S^2) \quad \text{for } \delta = 2
$$

(18)

2 Coupling to scalars

For a general complex scalar field $\Phi$, we have the following energy-momentum tensor

$$
T^S_{\mu\nu} = -\eta_{\mu\nu} D^\rho \Phi^\dagger D_\rho \Phi + \eta_{\mu\nu} V(\Phi^\dagger \Phi) + D_\mu \Phi^\dagger D_\nu \Phi + D_\nu \Phi^\dagger D_\mu \Phi
$$

(19)

where the gauge covariant derivative is defined as

$$
D_\mu = \partial_\mu + ig A^a_\mu T^a
$$

(20)

with $g$ the gauge coupling, $A^a_\mu$ the gauge fields, $T^a$ the Lie algebra generators and $V(\Phi^\dagger \Phi)$ is the potential.

If we are interested in the non linear breaking of the $SU(2) \times U(1)$ symmetry it is usual to work with the matrix

$$
M = \sqrt{2} \begin{pmatrix} \varphi_0 & -\varphi_*^* \\ \varphi_- & \varphi_0^* \end{pmatrix} = \sqrt{2} (\Phi, -i\sigma_2 \Phi^*)
$$

(21)

if $\Phi$ denotes the SM Higgs doublet. The description of the non linear breaking is obtained with the condition $M^\dagger M = v^2$, $v^2 = 1/(\sqrt{2}G_F)$ and therefore it is more convenient to introduce a unitary matrix $U$, such that $M = vU$. By requiring a custodial $SU(2)$ the effective scalar field lagrangian at the lowest order is then written as \[9\]

$$
\mathcal{L} = \frac{v^2}{4} g_{\sigma\rho} \text{tr}(D^\sigma U^\dagger D^\rho U) - \frac{1}{2} g_{\mu\nu} g_{\rho\sigma} \text{tr}(W^{\mu\rho} W^{\nu\sigma}) - \frac{1}{2} g_{\mu\nu} g_{\rho\sigma} \text{tr}(Y^{\mu\rho} Y^{\nu\sigma}) + \mathcal{O}(p^4)
$$

(22)
where $D_\mu$ denotes the $SU(2) \times U(1)$ covariant derivative

$$D_\mu U = (\partial_\mu + \frac{i}{2}gW_\mu^a \tau^a)U - \frac{i}{2}g'UY_\mu \tau^3$$

and

$$W_{\mu\nu} = \frac{\tau_i}{2} (\partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\epsilon^{ijk}W^j_\mu W^k_\nu)$$

$$Y_{\mu\nu} = \frac{\tau_3}{2} (\partial_\mu Y_\nu - \partial_\nu Y_\mu)$$

Notice that the nonlinear realization of the scalar field excludes terms of the type $R\Phi^\dagger \Phi$, $R$ being the scalar curvature.

The energy momentum tensor, using eqs.(9) and (22), can be expressed as

$$T_{\mu\nu} = T^S_{\mu\nu} + T^G_{\mu\nu}$$

where

$$T^S_{\mu\nu} = -\eta_{\mu\nu} \frac{v^2}{4} \text{tr}[(D_\mu U)\dagger D_\mu U] + \frac{v^2}{4} \left\{\text{tr}[(D_\mu U)\dagger D_\nu U] + \text{tr}[(D_\nu U)\dagger D_\mu U]\right\}$$

is the contribution from the scalar field lagrangian and

$$T^G_{\mu\nu} = \frac{1}{2} \eta_{\mu\nu} [\text{tr}(W^\rho_\sigma W^\sigma_\rho) + \text{tr}(Y^\rho_\sigma Y^\sigma_\rho)] - 2[\text{tr}(W^\rho_\mu W^\rho_\nu) + \text{tr}(Y^\rho_\mu Y^\rho_\nu)]$$

is the contribution from the gauge field kinetic term lagrangian. Notice that $(T^G)^\mu_\mu = 0$.

### 3 Effective lagrangian

By using the eqs.(16) and (26) one finds the following expression for the effective lagrangian

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{KK}$$

where $\mathcal{L}_0$ is given in eq.(22),

$$\mathcal{L}_{KK} = \mathcal{L}^S + \mathcal{L}^G$$

6
with
\[ L^S = \frac{\kappa^2 v^4}{8} \sum_{\vec{n}>0} D [(\text{tr}[D_\mu U^\dagger D_\nu U])^2 \]
\[ + \left( -\frac{1}{3} + \omega_5 \frac{\delta - 1}{2} \right) \text{tr}(D_\rho U^\dagger D^\rho U)^2 \]
\[ + \frac{\kappa^2}{2} \sum_{\vec{n}>0} D \left[ \frac{v^2}{2} \text{tr}(D_\rho U^\dagger D^\rho U)[\text{tr}(W^{\mu\nu} W_{\mu\nu}) + \text{tr}(Y^{\mu\nu} Y_{\mu\nu})] \right] \]
\[ - 2v^2 \text{tr}[D^\mu U^\dagger D^\nu U][\text{tr}(W_{\mu\rho} W_{\nu\rho}) + \text{tr}(Y_{\mu\rho} Y_{\nu\rho})] \]
\[ + \frac{\kappa^2}{2} \sum_{\vec{n}>0} D \left[ \frac{v^2}{2} \text{tr}(W_{\mu\rho} W_{\nu\rho})[\text{tr}(W^{\mu\rho} W^{\nu\rho}) + \text{tr}(W^{\mu\rho} W^{\nu\rho})] \right] \] (31)

and
\[ L^G = -\frac{\kappa^2}{2} \sum_{\vec{n}>0} D [(\text{tr}(W^{\mu\nu} W_{\mu\nu}))^2 + (\text{tr}(Y^{\mu\nu} Y_{\mu\nu}))^2 \]
\[ + 2\text{tr}(W^{\mu\nu} W_{\mu\nu})\text{tr}(Y^{\mu\nu} Y_{\mu\nu}) \]
\[ + 2\kappa^2 \sum_{\vec{n}>0} D [\text{tr}(W_{\mu\rho} W_{\nu\rho})\text{tr}(W_{\mu\rho} W_{\nu\rho}) \]
\[ + \text{tr}(Y_{\mu\rho} Y_{\nu\rho})\text{tr}(Y_{\mu\rho} Y_{\nu\rho}) \]
\[ + 2\text{tr}(W_{\mu\rho} W_{\nu\rho})\text{tr}(Y_{\mu\rho} Y_{\nu\rho}) \] (32)

where now \( \sum_{\vec{n}>0} D \) is given by the expansions in eqs. (17, 18). \( L_{KK} \) is the term coming from integrating the KK excitations of the graviton and of the dilaton. \( L^S \) contains terms of dimension four and six, while \( L^G \) contains dimension eight operators. These contributions from KK excitations will add to other terms in eq. (22) of order \( O(p^4) \) and higher order in principle already existing. For simplicity we have neglected these terms.

Let us now consider the standard parametrization for \( SU(2)_L \times SU(2)_R \) invariant Lagrangian up to the order \( p^4 \),
\[ L' = \frac{v^2}{4} \text{tr}(D_\rho U^\dagger D^\rho U) \]
\[ + \alpha_4 [\text{tr}(D_\mu U^\dagger D_\nu U)]^2 \]
\[ + \alpha_5 [\text{tr}(D_\rho U^\dagger D^\rho U)]^2 \]
\[ + \alpha_4^{KK} L_{KK}^2 \frac{L_{KK}^2}{16\pi^2} = \frac{\kappa^2 v^4}{8} \sum_{\vec{n}>0} D = \frac{v^4}{2M_5^4(\delta - 2)} \] (33)

Assuming \( \delta > 2 \) one has has
\[ \alpha_4^{KK} = \frac{L_{KK}^2}{16\pi^2} = \frac{\kappa^2 v^4}{8} \sum_{\vec{n}>0} D = \frac{v^4}{2M_5^4(\delta - 2)} \] (34)
\[\alpha_{KK} = \frac{L_{KK}}{16\pi^2} = \frac{\kappa^2 v^4}{8} \sum_{\vec{n} > 0} \mathcal{D} \left(-\frac{1}{3} + \omega_8^2 \delta - \frac{1}{2}\right) = -\frac{1}{\delta + 2} \frac{\kappa^2 v^4}{8} \sum_{\vec{n} > 0} \mathcal{D}\]

Here we have also introduced the alternative parameterization \[11\].

The dimension six remaining terms have been also considered \[12\]. These contain anomalous fourlinear and higher order gauge couplings and WW goldstone boson vertices. In the list of possible operators of \[12\] these correspond to \( k_w^w, k_c^w, k_b^b, k_c^b \) terms and contribute to \( WW\gamma\gamma, WWZ\gamma \) and \( ZZ\gamma \) vertices

\[
\begin{align*}
&\frac{k_w^w}{\Lambda^2 g^2} \text{tr}(W_{\mu \nu} W^{\mu \nu}) \text{tr}(V^\alpha V_\alpha) + \frac{k_c^w}{\Lambda^2 g^2} \text{tr}(W_{\mu \nu} W^{\mu \alpha}) \text{tr}(V^\nu V_\alpha) \\
&+ \frac{k_b^b}{\Lambda^2 g^2} \text{tr}(Y_{\mu \nu} Y^{\mu \nu}) \text{tr}(V^\alpha V_\alpha) + \frac{k_c^b}{\Lambda^2 g^2} \text{tr}(Y_{\mu \nu} Y^{\mu \alpha}) \text{tr}(V^\nu V_\alpha)
\end{align*}
\]

with

\[V_\alpha = (D_\alpha U) U^\dagger\]  

Since

\[\text{tr}(V^\nu V_\alpha) = -\text{tr}[(D^\nu U)^\dagger (D_\alpha U)]\]

one has

\[\frac{k_w^w}{\Lambda^2 g^2} = -\frac{\kappa^2 v^2}{4} \sum_{\vec{n} > 0} \mathcal{D} = -\frac{v^2}{4M_\delta^4(\delta - 2)} \quad k_c^w = -4k_0^w\]

and

\[\frac{k_b^b}{\Lambda^2 g^2} = -\frac{\kappa^2 v^2}{4} \sum_{\vec{n} > 0} \mathcal{D} = -\frac{v^2}{4M_\delta^4(\delta - 2)} \quad k_c^b = -4k_0^b\]

In principle these can be studied at LEP2 and future linear colliders with the processes

- \( e^+ e^- \to W^+ W^- \gamma \)
- \( e^+ e^- \to Z \gamma \gamma \)
- \( e^+ e^- \to ZZ \gamma \)
Present limits from LEP2 are $k^w,b_0,c/\Lambda^2 \sim 10^{-2} - 10^{-3} \text{GeV}^{-2}$ and increase to $k^w,b_0,c/\Lambda^2 \sim 10^{-5} \text{GeV}^{-2}$ at a linear collider with $\sqrt{s} = 500 \text{ GeV}$ and an integrated luminosity of $L = 500 \text{ fb}^{-1}$ [12]. Similar bounds are obtained at LHC by studying the processes $pp \rightarrow \gamma\gamma W^*(\rightarrow l\nu)$ and $pp \rightarrow \gamma\gamma Z^*(\rightarrow ll)$ [14]. However assuming

$$\kappa^2 \sum_{n>0} D \sim \frac{1}{M_S^4}$$  \label{41}

and $M_S \sim 1 \text{ TeV}$ the expected value of these anomalous couplings is of order $k^w,b_0,c/\Lambda^2 \sim 10^{-7} - 10^{-8} \text{GeV}^{-2}$. This turns out much smaller than the possible reaches of LHC and LC with $\sqrt{s} = 500 \text{ GeV}$.

The $\alpha_4$ and $\alpha_5$ terms contribute to $WW$ scattering with strength which assuming $M_S \sim 1 \text{ TeV}$ are $O(10^{-3})$; therefore they are at the border of the regions which can be be tested at LHC with $L = 100 \text{ fb}^{-1}$ and at a LC with $\sqrt{s} = 1.8 \text{ TeV}$ and $L = 200 \text{ fb}^{-1}$ [13] (see also [16] for reviews on future measurements of electroweak symmetry breaking effective lagrangian parameters).

The dimension eight operators give additional contribution to $WW$ scattering and furthermore contain a $\gamma^4$ coupling which can be studied at $\gamma\gamma$ collider. For instance at a $\gamma\gamma$ option of a $\sqrt{s} = 500 \text{ GeV}$ LC one can test $M_S \sim 4 \text{ TeV}$ [17]; at a $\sqrt{s} = 1 \text{ TeV}$ LC with $L = 200 \text{ fb}^{-1}$ one can test masses $M_S \sim 3 \text{ TeV}$ studying $e^+e^- \rightarrow \gamma\gamma e^+e^-$ [18].

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