$K\bar{K}N$ molecule state with $I = 1/2$ and $J^P = 1/2^+$ studied with three-body calculation

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A $K\bar{K}N$ system with $I = 1/2$ and $J^P = 1/2^+$ is investigated with non-relativistic three-body calculations by using effective $K\bar{N}$, $K\bar{K}$ and $KN$ interactions. The $KN$ interaction describes the $\Lambda(1405)$ as a $K\bar{N}$ molecule, and the $K\bar{K}$ interaction is adjusted to give $f_0(980)$ and $a_0(980)$ states as $K\bar{K}$ molecules. The present investigation suggests that a bound $K\bar{K}N$ state can be formed below the $K\bar{K}N$ threshold (1930 MeV) with a 90 $~$ 100 MeV width of three-hadron decays, which are dominated by $K\bar{K}N \to K\pi\Sigma$ and $\pi\eta N$. It is found that the $K\bar{K}N$ state is a weakly bound hadron molecular state with a size larger than an $\alpha$ particle because of the repulsive $KN$ interactions.

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I. INTRODUCTION

Exploring composite systems of mesons and baryons is a challenging issue both in theoretical and experimental hadron-nuclear physics. One of the historical examples in two-hadron systems is $\Lambda(1405)$ as a quasi-bound state of $K\bar{N}$ [1]. For mesonic resonances, the scalar mesons, $f_0(980)$ and $a_0(980)$, are also the candidates of the hadronic molecular states [2]. Baryon resonances as three-hadron systems have been also investigated theoretically for systems of $\pi K\bar{N}$ [3, 4, 5], $\pi K\bar{K}$ and $K\bar{K}N$ [6, 7]. Based on the idea to regard $\Lambda(1405)$ as a $K\bar{N}$ quasi-bound state [9, 10], bound systems of a few nucleons with anti-kaon were investigated in Refs. [7, 8, 9, 11, 12] and [4, 13, 14, 15, 16].

Recently a baryonic resonance with $J^P = 1/2^+$ and $S = -2$ composed by $K\bar{K}N$ has been studied in details by the authors in Ref. [8] based on three-body calculation with attractive $KN$ interactions given by Refs. [7, 12, 17]. In this system, the anti-kaons play unique roles, because they have enough attraction with the nucleon to form a quasi-bound state as $\Lambda(1405)$ and possess so heavy mass to provide small kinetic energy in the system. The quasi-bound state of $K\bar{K}N$ has a characteristic structure that one of the anti-kaons forms $\Lambda(1405)$ with the nucleon ($\Lambda$,cluster) as seen also in $K^-p p$ system [12], and the other anti-kaon spreads for long distance. This structure is caused by strong $KN$ attraction with $I = 0$.

In this paper, we explore quasi-bound states of the $K\bar{K}N$ system with $I = 1/2$ and $J^P = 1/2^+$, assuming that the $KN$ and $K\bar{K}$ systems have enough attractions to form quasi-bound states of $\Lambda(1405)$ in $I = 0$ and $f_0(980)$ ($a_0(980)$) in $I = 0$ ($I = 1$), respectively. We use the effective interactions of $K\bar{N}$ extracted by Akaishi-Yamazaki (AY) [9, 13] and Hyodo-Weise (HW) [17] in phenomenological way and chiral dynamics, respectively. These interaction provide $\Lambda(1405)$ as a quasi-bound state with $I = 0$ and also weak attraction in the $I = 1$ channel. The effective $K\bar{K}$ interactions are adjusted to reproduce the masses and the widths of $f_0(980)$ and $a_0(980)$ as the $K\bar{K}$ molecular states. The $KN$ interactions are known to have strong repulsion in $I = 1$ channel. We use the $KN$ potential fitted by observed scattering lengths.

The “fate” of the $K\bar{K}N$ molecular state strongly depends on its binding energy. If the energy of the $K\bar{K}N$ system is above the lowest threshold of the subcomponents, the $K\bar{K}N$ states can decay to the subcomponents and the width gets very large. If the $K\bar{K}N$ state is bound with moderate binding energy below all the thresholds of $\Lambda(1405)$+$K$, $f_0(980)+N$ and $a_0(980)+N$, the state is quasi-stable against these decay modes and has comparable decay width with those of the two-particle subsystems. For deeply bound $K\bar{K}N$ system, since the constituents largely overlap each other, the molecular picture may be broken down and two-body decays are enhanced.

Having strong attractions in $KN$ and $K\bar{K}$ subsystems, it is naturally expected that $K\bar{K}N$ forms a hadron molecule below the thresholds of $\Lambda(1405)$-$K$ and $f_0(a_0)$-$N$. The question arising here is whether or not the attractions are so strong that the hadronic molecular picture breaks down in deeply bound state and the quasi-bound state has large width, or in opposite direction, whether or not the repulsion of $KN$ is too strong for spoiling the bound state.

In Sec. III we describe the framework of the present calculations. We apply a variational approach with a Gaussian expansion method [18] to solve the Schrödinger equation of the three-body system. By treating the imaginary potentials perturbatively, we find the $K\bar{K}N$ quasi-bound state. In Sec. IV we present our results of the three-body calculation. In analysis of the wave functions, we discuss the structure of the $K\bar{K}N$ state. Section V is devoted to summary of this work.

II. FORMULATION

We apply a non-relativistic three-body potential model for the $K\bar{K}N$ system. The effective two-body interactions are given in local potential forms. The $K\bar{K}N$ wave function is calculated by solving Schrödinger equation with a Gaussian expansion method for the three-body system. In this section, we briefly explain the formulation and interactions used in the present work. The details of the formulation and the $KN$ interaction are
discussed in Ref. [8].

A. Hamiltonian

In the present work, the Hamiltonian for the \( K\bar{K}N \) system is given by

\[
H = T + V_{\bar{K}N}(r_1) + V_{KN}(r_2) + V_{K\bar{K}}(r_3), \tag{1}
\]

with the kinetic energy \( T \), the effective \( \bar{K}N \) interaction \( V_{\bar{K}N} \), the \( KN \) interaction \( V_{KN} \) and the \( K\bar{K} \) interaction \( V_{K\bar{K}} \). These interactions are given by \( \ell \)-independent local potentials as functions of \( K-N, \bar{K}-N \) and \( K-K \) distances, \( r_1, r_2 \) and \( r_3 \) defined by \( r_1 = |\mathbf{x}_\bar{K} - \mathbf{x}_N|, r_2 = |\mathbf{x}_N - \mathbf{x}_K| \) and \( r_3 = |\mathbf{x}_K - \mathbf{x}_\bar{K}| \), respectively, with spatial coordinates \( \mathbf{x}_K, \mathbf{x}_\bar{K}, \mathbf{x}_N \) for the kaon, the anti-kaon and the nucleon. For convenience, we introduce Jacobian coordinates, \( r_c \) and \( R_c \), in three rearrangement channels \( c = 1, 2, 3 \) as shown in Fig. 1. We assume isospin symmetry in the effective interactions, and we also neglect the mass differences among \( K^\pm, \bar{K}^0 \) and \( K^0 \), and that between proton and neutron by using the averaged masses, \( M_K = 495.7 \) MeV and \( M_N = 938.9 \) MeV. We do not consider three-body forces nor transitions to two-hadron decays, which will be important if the constituent hadrons are localized in a small region.

The kinetic energy \( T \) is simply given by the Jacobian coordinates with one of the rearrangement channels as

\[
T = \frac{-1}{2\mu_{r_c}} \nabla_r^2 r_c + \frac{-1}{2\mu_{R_c}} \nabla_{R_c}^2 R_c, \tag{2}
\]

with the reduced masses \( \mu_{r_c} \) and \( \mu_{R_c} \) for the corresponding configuration, for instance, \( \mu_{r_1} = M_K M_N/(M_K + M_N) \) and \( \mu_{R_1} = M_K(M_K + M_N)/(2M_K + M_N) \) for the rearrangement channel \( c = 1 \).

The effective interactions, \( V_{\bar{K}N}, V_{KN}, \) and \( V_{K\bar{K}} \), are obtained by s-wave two-body scattering with isospin symmetry. The explicit expression of the effective interactions will be given in Sec. II B. Open channels of \( \bar{K}N \) and \( K\bar{K} \) (\( \pi\Lambda \) and \( \pi\Sigma \) for \( \bar{K}N \), and \( \pi\pi \) and \( \pi\eta \) for \( K\bar{K} \)) are implemented effectively to the imaginary parts of the interactions \( V_{\bar{K}N} \) and \( V_{K\bar{K}} \). Consequently, the Hamiltonian (1) is not hermitian. In solving Schrödinger equation for \( K\bar{K}N \), we first take only the real part of the potentials and obtain wavefunctions in a variational approach. Then we calculate bound state energies \( E \) as expectation values of the total Hamiltonian (1) with respect to the obtained wave functions. The widths of the bound states are evaluated by the imaginary part of the complex energies as \( \Gamma = -2 \text{Im}E \).

B. Effective interactions

In this subsection, we explain the details of the effective interactions of the \( \bar{K}N, K\bar{K} \) and \( KN \) two-body subsystems in our formulation. The interaction parameters and the properties of the two-body subsystems are listed in Table I.

1. \( \bar{K}N \) interaction

In this work, we use the same \( \bar{K}N \) potential as used in Ref. [8] for the \( \bar{K}\bar{K}N \) calculations. We consider two different effective \( \bar{K}N \) interactions to estimate theoretical uncertainties. These two interactions were derived in different ways. One of the \( \bar{K}N \) interaction that we use is by Hyodo and Weise in Ref. [17] and was derived based on the chiral unitary approach for s-wave scattering amplitude with strangeness \( S = -1 \). The other interaction is Akaishi-Yamazaki (AY) potential derived phenomenologically by using \( \bar{K}N \) scattering and kaonic hydrogen data and reproducing the \( \Lambda(1405) \) resonance as a \( \bar{K}^{-}p \) bound state at 1405 MeV [9, 15]. Both \( \bar{K}N \) interactions have very strong attraction in \( I = 0 \) as to provide the \( \Lambda(1405) \) as a quasi-bound state of the \( \bar{K}N \) system, and have weak attraction in \( I = 1 \). Hereafter we refer the quasi-bound \( \bar{K}N \) state as \( \{\bar{K}N\}_{I=0} \).

The potential is written in a one-range Gaussian form as

\[
V_{\bar{K}N} = U_{\bar{K}N}^{r_0} \exp \left[-(r/b)^2\right] P_{\bar{K}N}(I = 0) + U_{\bar{K}N}^{I=1} \exp \left[-(r/b)^2\right] P_{\bar{K}N}(I = 1), \tag{3}
\]

with the isospin projection operator \( P_{\bar{K}N}(I = 0,1) \) and the range parameter \( b \). The potential depth \( U_{\bar{K}N}^{I}\bar{K}N \) are given in Table I. For the Hyodo-Weise potential, we use the parameter set referred as HNJH in Ref. [18], which was obtained by the chiral unitary model with the parameters of Ref. [19]. We refer this potential as “HW-HNJH potential”. The energy of the HW-HNJH potential is fixed at \( \omega = M_K + M_N - 11 \) MeV which is the resonance position of \( \Lambda(1405) \), since the energy dependence in the potential is small in the region of interest and it was found in Ref. [8] that the results were not sensitive to the choice of the energy for the \( K\bar{K}N \) system.

The important difference between the two interactions is the binding energy of the \( \bar{K}N \) system. In chiral unitary approaches for the meson-baryon interactions, the
Λ(1405) resonance is described as a \( \bar{K}N \) quasi-bound state \[20\] located at \( \omega \sim 1420 \) MeV in \( KN \) scattering amplitude \[21\]. This is a consequence of the double pole nature that \( \Lambda(1405) \) is described by superposition of two poles as found in Refs. \[21, 22, 23\]. For the \( AY \) potential, the \( \Lambda(1405) \) resonance was reproduced at \( \sim 1405 \) MeV as PDG reported. Thus, the \( AY \) potential has stronger a

b

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\( \{ \bar{K}K \} \}_{I=0,1} \).

We take the one-range Gaussian form,

\[ V^{I=0,1}_{\bar{K}K}(r) = U_{\bar{K}K}^{I=0,1} \exp \left[-(r/b)^2 \right] P_{\bar{K}K}(I = 0,1), \tag{4} \]

where the range parameter \( b \) is chosen to be the same value as that of the \( \bar{K}N \) interaction. We adjust the strength \( U_{\bar{K}K}^{I=0,1} \) to fit the \( f_0(980) \) and \( a_0(980) \) masses and the widths with the energies of two-body calculations of the \( \bar{K}K \) system. The particle data group (PDG) reports \[24\] the \( f_0(980) \) and \( a_0(980) \) have \( 980 \pm 10 \) MeV and \( 984.6 \pm 1.2 \) MeV masses with the \( 40 - 100 \) MeV and \( 50 - 100 \) MeV widths, respectively, in average of the compilation of the experimental data. The dominant decay modes are \( \pi\pi \) for \( f_0(980) \) and \( \pi\eta \) for \( a_0(980) \). We take the mass \( 980 \) MeV and the width \( 60 \) MeV as the inputs to determine the \( \bar{K}K \) interactions in both the \( I = 0 \) and \( I = 1 \) channels. Then we get \( U_{\bar{K}K}^{I=0,1} = -1155 - 283i \) MeV for \( b = 0.47 \) fm and \( U_{\bar{K}K}^{I=0,1} = -630 - 210i \) MeV for \( b = 0.66 \) fm, by fitting the energy of the \( \bar{K}K \) bound state to the meson mass and the width. We refer the former potential as “\( \bar{K}K(A) \)” and the latter as “\( \bar{K}K(B) \)”.

In this phenomenological single-channel interaction, the effect of the two-meson decays such as \( \pi\pi \) and \( \pi\eta \) decays is incorporated in the imaginary part of the effective \( \bar{K}K \) interaction.

In the present parametrization of the \( \bar{K}K \) potential, we have fitted the potential strengths to reproduce the PDG values of the \( f_0(980) \) and \( a_0(980) \) masses as bound state energies of \( \bar{K}K \) calculated with the perturbative treatment of the imaginary potential. When we directly calculate the pole position of the \( \bar{K}K \) scattering amplitude in Lippmann-Schwinger equation with the present potential, we get the value \( 998 - 32i \) MeV for the parameter set (A). This is obtained above the \( K \) threshold in the first Riemann sheet as a virtual state. This pole is consistent with the pole threshold of scattering amplitude obtained by the chiral unitary approach \[23, 26\]. In the chiral unitary approach, s-wave scattering amplitudes with \( I = 0 \) and \( I = 1 \) were reproduced well by coupled channels of \( \pi\pi, \pi\eta \) and \( K\bar{K} \). The \( f_0(980) \) and \( a_0(980) \) meson are obtained as the resonance poles at \( 993.5 \) MeV for \( f_0(980) \) and \( 1009.2 \) MeV for \( a_0(980) \) \[20\]. The \( f_0 \) is described dominantly by \( K\bar{K} \) scattering, while for \( a_0 \) the \( \pi\eta \) scattering is also important as well as \( K\bar{K} \) scattering. These values are slightly higher than the masses reported by PDG, which are given by the peak position of the spectra. We will discuss ambiguity of the \( K\bar{K} \) interactions in later section.

3. \( K\bar{N} \) interaction

We construct the \( K\bar{N} \) interaction based on observed \( KN \) scattering lengths \[27\]. The \( KN \) interactions are known to be strong repulsion in the \( I = 1 \) channel and very weak in the \( I = 0 \) channel. The experimental values of the scattering lengths for the \( I = 0 \) and \( I = 1 \) channels are \( a^{\perp}_{I=0} = -0.325 \) fm and \( a^{\perp}_{I=1} = -0.310 \pm 0.003 \) fm \[27\]. In the present calculation, we assume no interaction in the \( I = 0 \) channel. For the \( I = 1 \) channel, we use phenomenological interaction with the one-range Gaussian form again,

\[ V^{I=1}_{KN}(r) = U_{KN}^{I=1} \exp \left[-(r/b)^2 \right] P_{KN}(I = 1), \tag{5} \]

where the range parameter \( b \) is chosen to be the same value as that of the \( K\bar{N} \) interaction. We adjust the strength \( U_{KN}^{I=1} \) to reproduce the experimental scattering strength and obtain \( U_{KN}^{I=1} = 820 \) MeV and \( U_{KN}^{I=1} = 231 \) MeV for \( b = 0.47 \) fm and \( b = 0.66 \) fm, respectively. We refer the former parametrization as “\( KN(A) \)” and the latter one as “\( KN(B) \)”.

C. Three-body wave function

The three-body \( K\bar{K}N \) wave function \( \Psi \) is described as a linear combination of amplitudes \( \Phi^{(c)}_{I_{\bar{K}K}}(r_c, R_c) \) of three rearrangement channels \( c = 1, 2, 3 \) (Fig. 1). In the present calculation, we take the model space limited to \( l_c = 0 \) and \( l_c = 0 \) of the orbital-angular momenta for the Jacobian coordinates \( r_c \) and \( R_c \) in the channel \( c \) owing to the fact that the effective local potentials used in the present calculations are derived in consideration of the s-wave two-body dynamics. Then the wave function of the \( \bar{K}K\bar{N} \) system with \( I = 1/2 \) and \( J^P = 1/2^+ \) is written as

\[ \Psi = \sum_{c: I_{\bar{K}K}} \Phi^{(c)}_{I_{\bar{K}K}}(r_c, R_c) \left[[\bar{K}K]\right]_{I_{\bar{K}K}N} I_{1/2} \tag{6} \]

where the \( \left[[\bar{K}K]\right]_{I_{\bar{K}K}N} I_{1/2} \) specifies the isospin configuration of the wave function \( \Phi^{(c)}_{I_{\bar{K}K}}(r_c, R_c) \), meaning that the total isospin \( 1/2 \) for the \( \bar{K}K\bar{N} \) system is given by combination of total isospin \( I_{\bar{K}K} \) for the \( \bar{K}K \) subsystem and isospin \( 1/2 \) for the nucleon.
TABLE I: The interaction parameters and the properties of two-body systems. The energies (E) are evaluated from the corresponding two-body threshold. They are calculated by treating the imaginary part of the two-body potentials perturbatively. We also list the root-mean-square two-body distances of the $KN(I=0)$, $K\bar{K}(I=0)$ and $K\bar{K}(I=1)$ states, which correspond to $\Lambda(1405)$ and $f_0(980), a_0(980)$, respectively. For the $K\bar{K}$ interactions, we show the scattering lengths obtained in the present parameters.

| parameter set of interactions | (A)  | (B)  |
|-------------------------------|------|------|
| $KN$                         | HW-HNJH | AY    |
| $b$ (fm)                     | 0.47 | 0.66 |
| $U_{KN}^{I=0}$ (MeV)         | $-908 - 181 i$ | $-595 - 83 i$ |
| $U_{KN}^{I=1}$ (MeV)         | $-415 - 170 i$ | $-175 - 105 i$ |
| $K\bar{K}$                  |       |      |
| $b$ (fm)                     | 0.47 | 0.66 |
| $U_{K\bar{K}}^{I=0,1}$ (MeV) | $-1155 - 283 i$ | $-630 - 210 i$ |

The wave function of the $K\bar{K}N$ system is obtained by solving the Schrödinger equation,

$$[T + V_{KN}(r_1) + V_{KN}(r_2) + V_{K\bar{K}}(r_3) - E] \Psi = 0. \quad (7)$$

In solving the Schrödinger equation for the $K\bar{K}N$ system, we adopt the Gaussian expansion method for three-body systems given in Ref. [18] as same way as done in Ref. [8] with the parameters $r_{\text{min}}, R_{\text{min}} = 0.2 \text{ fm}$ and $r_{\text{max}}, R_{\text{max}} = 20 \text{ fm}$, and $r_{\text{max}}, N_{\text{max}} = 15$ for all the channels, $c = 1, 2, 3$. We treat the imaginary part of the potentials perturbatively. We first calculate the wave function for the real part of the Hamiltonian ($H_{\text{Re}}$) with variational principle in the model space of the Gaussian expansion. After this variational calculation, we take the lowest-energy solution. The binding energy $B(K\bar{K}N)$ of the three-body system is given as $B(K\bar{K}N) = -E_{\text{Re}}$.

Next we estimate the imaginary part of the energy for the total Hamiltonian by calculating the expectation value of the imaginary part of the Hamiltonian with the obtained wave function $\Psi$:

$$E_{\text{Im}} = \langle \Psi | \text{Im}V_{\bar{K}N} + \text{Im}V_{K\bar{K}} | \Psi \rangle. \quad (8)$$

The total energy is given as $E = E_{\text{Re}} + E_{\text{Im}}$, and the decay width is estimated as $\Gamma = -2E_{\text{Im}}$. In the present calculation, we have only three-body decays such as $\pi \Sigma K$, $\pi \Lambda \bar{K}$, $\pi \pi N$ and $\eta \pi N$ decays for the $K\bar{K}N$ state by the model setting.

The perturbative treatment performed above is justified qualitatively in the case of $|\langle \Psi | \text{Im}V | \Psi \rangle| \ll |\langle \Psi | \text{Re}V | \Psi \rangle|$. In the two-body systems, $KN$ and $K\bar{K}$, we find that this condition is satisfied reasonably, observing that $|\langle \text{Im}V_{KN} \rangle| \sim 20 \text{ MeV}$ is much smaller than $|\langle \text{Re}V_{KN} \rangle| \sim 100 \text{ MeV}$, and $|\langle \text{Im}V_{K\bar{K}} \rangle| \sim 30 \text{ MeV}$ is also much smaller than $|\langle \text{Re}V_{K\bar{K}} \rangle| \sim 100 \text{ MeV}$. Also in the case of the $K\bar{K}N$ system, it is found that the absolute values of the perturbative energy $|\langle \Psi | \text{Im}V | \Psi \rangle|$ $\sim 50 \text{ MeV}$ is much smaller than the real potential energy $|\langle \Psi | \text{Re}V | \Psi \rangle| \sim 200 \text{ MeV}$ in the present calculations.

We also calculate quantities characterizing the structure of the three-body system, such as spatial configurations of the constituent particles and probabilities to have specific isospin configurations. These values are calculated as expectation values of the wave functions.

The root-mean-square (r.m.s.) radius of the $K\bar{K}N$ state is defined as the average of the distribution of $K$, $\bar{K}$ and $N$ by

$$r_{K\bar{K}N} = \sqrt{\langle \Psi | \frac{1}{3}(x_K^2 + x_{\bar{K}}^2 + x_N^2) | \Psi \rangle}, \quad (9)$$

which is measured from the center of mass of the three-body system. We also calculate the r.m.s. values of the relative distances between two particles,

$$d_{KN} = \sqrt{\langle \Psi | r_{12}^2 | \Psi \rangle}, \quad (10)$$

$$d_{K\bar{K}} = \sqrt{\langle \Psi | r_{13}^2 | \Psi \rangle}, \quad (11)$$

$$d_{KN} = \sqrt{\langle \Psi | r_{23}^2 | \Psi \rangle}. \quad (12)$$

Here $r_1$, $r_2$ and $r_3$ are the $KN$ distance, $KN$ distance and the $K\bar{K}$ distance, respectively.

We also introduce the probabilities for the three-body system to have the isospin $I_{K\bar{K}}$ states as

$$\Pi \left( [K\bar{K}] I_{K\bar{K}} \right) \equiv \langle \Psi | P_{K}(I_{K\bar{K}}) | \Psi \rangle, \quad (13)$$

where $P_{K}(I_{K\bar{K}})$ is the projection operator for the isospin configuration $[[K\bar{K}]I_{K\bar{K}}N]_{I=1/2}$, as introduced
before. We calculate the probabilities that the three-body system has the isospin configurations of \([\{K N\}_{K N}]_{I=1/2}\), where the total isospin 1/2 is given by combination of total isospin \(I_{K N}\) for the \(K N\) subsystem and the kaon isospin 1/2:

\[
\Pi\left(\{K N\}_{I_{K N}}\right) = \langle \Psi \left| P_{K N}(I_{K N}) \right| \Psi \rangle ,
\]

where \(P_{K N}(I_{K N})\) is again the isospin projection operator.

III. RESULTS

In this section, we show the results of investigation of the \(KKN\) system with \(I = 1/2\) and \(J^P = 1/2^+\). We consider two parameter sets (A) and (B) for the two-body interactions listed in Table I. For the \(K N\) interactions, we use (A) the HW-HNJH potential and (B) the AY potential. For the \(K\) and \(K N\) interactions, we use the phenomenological interactions derived in Sec. II B 2 and II B 3: KK(A) and KN(A) for set (A), and KK(B) and KN(B) for set (B). In addition, we study the effect of the \(K N\) repulsion by switching off the \(K N\) interaction in the parameter sets (A) and (B).

1. Properties of \(KKN\) state

First of all, we find that, in both calculations (A) with the HW-HNJH and (B) with the AY potentials, the \(K K N\) bound state is obtained below all threshold energies of the \{\(K N\)\}_{\{K \}}_{I=0}+K, \{\(K \bar{K}\)\}_{I=0}+N and \{\(K \bar{K}\)\}_{\{K \}}_{I=0}+N channels, which correspond to the \(\Lambda(1405)+K, f_0(980)+N\) and \(a_0(980)+N\) states, respectively.\(^1\) This means that the obtained bound state is stable against breaking up to the subsystems. We show the level structure of the \(K K N\) system measured from the \(K + \bar{K} + N\) threshold in Fig. 2. The values of the real and imaginary parts of the obtained energies are given in Table II. The imaginary part of the energy is equivalent to the half width of the quasi-bound state. The contribution of each decay mode to the imaginary energy is shown as an expectation value of the imaginary potential \(\langle \text{Im}\psi \rangle\) and the results obtained without the \(K N\) interaction are also given. For the HW-HNJH potential, since the original HW-HNJH potential is moderately dependent on the energy of the \(\bar{K} N\) subsystem, we have calculated the bound state energy with the \(K N\) potential for the \(K N\) energy at \(\omega = M_K = M_N\) and found that the energy \(\langle \omega \rangle\) dependence of the HW-HNJH potential is small in the result.

Let us discuss first the results with the \(K N\) interaction in detail. The binding energy of the \(K K N\) state measured from the three-body \(K + \bar{K} + N\) threshold is larger in the result with (B) than that with (A), as found to be \(-19\) MeV and \(-41\) MeV in the cases of (A) and (B), respectively. This is because the AY potential gives a deeper binding of the \{\(K N\)\}_{\{K \}}_{I=0} state than the HW-HNJH potential due to the stronger \(K N\) attraction. These values have meaning just for the position of the quasi-bound state in spectrum. It is more physically important that the \(K K N\) bound state appears about \(10\) MeV below the lowest two-body threshold, \{\(K N\)\}_{\{K \}}_{I=0}+K, in both cases (A) and (B). This energy is compatible to nuclear many-body system, and it is considered to be weak binding energy in the energy scale of hadron system. This weak binding system has the following significant feature. The width of the \(K K N\) state is estimated to be \(\Gamma \sim 90\) MeV from the imaginary part of the energy. Comparing the results of the \(K K N\)

\(^1\) In the present calculation, because the \(K K\) interaction is adjusted to reproduce the \(f_0\) and \(a_0\) scalar mesons having the same mass and width, it is independent of the total isospin of the \(K K\) subsystem and the thresholds of \(f_0(980)+N\) and \(a_0(980)+N\) are obtained as the same value.
with the properties of the two-body subsystems shown in Table I. It is found that the real and imaginary energy of the $K\bar{K}N$ state is almost given by the sum of those of $\Lambda(1405)$ and $a_0(980)$ (or $f_0(980)$), respectively. This indicates that two subsystems, $KN$ and $K\bar{K}$, are as loosely bound in the three-body system as they are in two-body system.

The decay properties of the $K\bar{K}N$ state can be discussed by the components of the imaginary energy. As shown in Table II among the total width $\Gamma = -2E^{\text{Im}} \sim 90$ MeV, the imaginary potentials of the $\bar{K}N$ with $I = 0$ and the $K\bar{K}$ with $I = 1$ give large contributions as about 40 MeV and 50 MeV, respectively. The former corresponds to the $\Lambda(1405)$ decay channel and gives the $\bar{K}N \to \pi\Sigma$ decay mode with $I = 0$. The latter is given by the $a_0(980)$ decay, which is dominated by $K\bar{K} \to \pi\eta$. Contrary, the $\bar{K}N$ $(I = 1)$ and the $K\bar{K}$ $(I = 0)$ interactions provide only small contributions to the imaginary energy. This is because, as we will see later, the $\bar{K}N$ subsystem is dominated by the $I = 0$ component due to the strong $\bar{K}N$ attraction and the $K\bar{K}$ subsystem largely consists of the $I = 1$ component as a result of the three-body dynamics. The small contributions of the $\bar{K}N$ $(I = 1)$ and the $K\bar{K}$ $(I = 0)$ interactions to the imaginary energy implies that the decays to $\pi\Lambda\bar{K}$ and $\pi\eta\bar{N}$ are suppressed. Therefore, we conclude that the dominant decay modes of the $K\bar{K}N$ state are $\pi\Sigma\bar{K}$ and $\pi\eta\bar{N}$. This is one of the important characters of the $K\bar{K}N$ bound system.

Although the obtained $K\bar{K}N$ state is located below the thresholds of $\Lambda(1405) + K$, $f_0(980) + N$ and $a_0(980) + N$, there could be a chance to access the $K\bar{K}N$ state energetically by observing the $\Lambda(1405) + K$, $f_0(980) + N$ and $a_0(980) + N$ channels in the final states, because these resonances have as large widths as the $K\bar{K}N$ state. Since, as we will show later, the $K\bar{K}N$ state has the large $\Lambda(1405) + K$ component, the $K\bar{K}N$ state could be confirmed in its decay to $\Lambda(1405) + K$ by taking coincidence of the $\Lambda(1405)$ out of the invariant mass of $\pi\Sigma$ and the three-body invariant mass of the $\pi\Sigma\bar{K}$ decay.

Here we comment on theoretical uncertainty of the energy of the $K\bar{K}N$ state. In the present calculations, the $K\bar{K}$ interactions are obtained under the assumption that the $K\bar{K}$ attractive potentials provide $f_0(980)$ and $a_0(980)$ as quasi-bound states and are phenomenologically adjusted to reproduce the masses and the widths of $f_0(980)$ and $a_0(980)$. As discussed above, in the present result, the $K\bar{K}$ interaction with $I = 1$ gives the dominant contribution to the total width of the $K\bar{K}N$ state. We estimate theoretical uncertainty of the width of the $K\bar{K}N$ state by changing the inputs of the $a_0(980)$ width in the range from $\Gamma = 50$ to 100 as reported in PDG. We obtain the $\Gamma = 80 - 130$ MeV for the $K\bar{K}N$ state. We also find that the $K\bar{K}N$ state becomes unbound if the $K\bar{K}$ interaction with $I = 1$ is less attraction than 70% of the present values, in which $K\bar{K}$ with $I = 1$ is not bound in the two-body system.

Finally we discuss the role of the $KN$ repulsion in the $K\bar{K}N$ system. In Fig. 2 and Table II, we show the results calculated without the $KN$ interaction. We find that the binding energy of the $K\bar{K}N$ state is 20 MeV larger than the case of the calculation with the $KN$ repulsion in both (A) and (B) cases, and that the absolute value of the imaginary energy also becomes larger as $E^{\text{Im}} = -72$ MeV and $-63$ MeV for (A) and (B), respectively. These values correspond to the $\Gamma = 130 - 140$ MeV width for the $K\bar{K}N$ state. The reason that the three-body system without the $KN$ interaction has the more binding and the larger width is as follows. In general, the three-body system has less kinetic energy than the two-body system because of larger reduced mass in the three-body system. With less kinetic energy the system can localize more. As a result of the localization of the system, the system can gain more potential energy and larger imaginary energy in the case of no $KN$ interaction than the case with the $KN$ repulsion. In other words, thanks to the $KN$ repulsion, the $K\bar{K}N$ state is weakly bound and its width is suppressed to be as small as the sum of the widths of the subsystems.

### 2. Structure of $K\bar{K}N$ state

We discuss the structure of the $K\bar{K}N$ state with $I = 1/2$. For this purpose, we analyze the wave functions obtained in the present few-body calculation in terms of the spatial structure and the isospin configuration of the $K\bar{K}N$ system.

We first investigate the isospin configuration of the $K\bar{K}N$ state. We show the isospin components of sub-

| $V_{KN}$ | $\text{Off}$ | $\text{On}$ | $\text{Off}$ | $\text{On}$ | $\text{Off}$ |
|----------|-------------|-------------|-------------|-------------|-------------|
| $\Pi \left( [\bar{K}N]_0 \right)$ | 0.93 | 1.00 | 0.99 | 1.00 | 0.99 | 1.00 |
| $\Pi \left( [\bar{K}N]_1 \right)$ | 0.07 | 0.00 | 0.01 | 0.00 | 0.01 | 0.00 |
| $\Pi \left( [K\bar{K}]_0 \right)$ | 0.09 | 0.25 | 0.17 | 0.25 | 0.17 | 0.25 |
| $\Pi \left( [K\bar{K}]_1 \right)$ | 0.91 | 0.75 | 0.83 | 0.75 | 0.83 | 0.75 |

**TABLE III: Isospin and spatial structure of the $K\bar{K}N$ state**

The results without the $KN$ repulsive interaction are also shown. The r.m.s. radius of the $K$, $\bar{K}$, and $N$ distribution, and the r.m.s. values for the $K-N$, $K-\bar{K}$ and $K-N$ distances are listed. The isospin components of the subsystems, $KN$ and $K\bar{K}$ are also shown. The detailed definitions are described in Sec. HTC. The three-body wavefunction is obtained in the same way as that in Table I.
systems $\bar{K}N$ and $KK$ in Table III. It is found that the $\bar{K}N$ subsystem has a dominant $I = 0$ component because of the strong $\bar{K}N$ interaction in the $I = 0$ channel. In the $KK$ subsystem, the $I = 1$ configuration is dominant while the $I = 0$ component gives minor contribution. This isospin configuration is caused by the following reason. In both $I = 0$ and $I = 1$ channels, the $KK$ attraction is strong enough to provide quasi-bound $\bar{K}N$ states of $f_0(980)$ and $a_0(980)$. In addition, since these scalar mesons have similar masses and widths, the $KK$ interactions in $I = 0$ and $I = 1$ adjusted to these masses and widths are similar to each other. In fact, we use the same parametrization for the $KK$ interactions in the present calculation, which gives isospin-blind potential. Therefore, the $\bar{K}N$ interaction plays a major role to determine the isospin configuration of the $\bar{K}N$ state. Since the $\bar{K}N$ interaction has stronger attraction in the $I = 0$ channel than in the $I = 1$ channel, the system prefers to have $I = 0$ in the $\bar{K}N$ subsystem. If the $\bar{K}N$ subsystem has pure $I = 0$ configuration, which is the case without the $\bar{K}N$ repulsion, the $KK$ subsystem should be composed by $I = 0$ and $I = 1$ with the ratio of 1:3 to have $I = 1/2$ of $\bar{K}N$. Thus, the $\bar{K}N$ with $I = 0$ dominates the $\bar{K}N$ system, and simultaneously the $KK$ with $I = 1$ is dominant component. The small deviation from the pure $\bar{K}N(I = 0)$ configuration in the $\bar{K}N$ state originates in the $\bar{K}N$ repulsion.

Next we discuss the spatial structure of the $\bar{K}N$ bound system. In Table III, we show the root-mean-square (r.m.s.) radius of $\bar{K}N$, $r_{\bar{K}N}$ defined in Eq. (9), and r.m.s. values for the $K-N$, $K-K$ and $K-N$ distances, $d_{\bar{K}N}$, $d_{KN}$, $d_{KK}$ defined in Eqs. (10), (11) and (12), respectively, in the $\bar{K}N$ state. The r.m.s. distances of the two-body systems, $\{\bar{K}N\}_{I=0}$ and $\{KK\}_{I=1}$, are shown in Table III. It is interesting that the present result shows that the r.m.s. $\bar{K}-N$ and $K-K$ distances in the three-body $\bar{K}N$ state have values close to those in the quasi-bound two-body states, $\{\bar{K}N\}_{I=0}$ and $\{KK\}_{I=1}$, respectively. This implies again that the two subsystems of the three-body state have very similar characters with those in the isolated two-particle systems.

The r.m.s. $K-N$ distance is relatively larger than the r.m.s. $K-N$ and $K-K$ distances due to the $\bar{K}N$ repulsion. The effect of the repulsive $\bar{K}N$ interaction is important in the present system. Without the repulsion, we obtain smaller three-body systems as shown in Table III. Especially the distances of the two-body subsystems are as small as about 1.5 fm, which is comparable with the sum of the charge radii of proton (0.8 fm) and $K^+$ (0.6 fm). For such a small system, three-body interactions and transitions to two particles could be important. In addition, the present picture that the system is described in non-relativistic three particles might be broken down, and one would need relativistic treatments and two-body potentials with consideration of internal structures of the constituent hadrons.

Combining the discussions of the isospin and spatial structure of the $\bar{K}N$ system, we conclude that the structure of the $\bar{K}N$ state can be understood simultaneous coexistence of $\Lambda(1405)$ and $a_0(980)$ clusters as shown in Fig. III.2 This does not mean that the $\bar{K}N$ system is described as superposition of the $\Lambda(1405) + K$ and $a_0(980) + N$ states, because these states are not orthogonal to each other. The probabilities for the $\bar{K}N$ state to have these states are 90% as seen in Table III. It means that $\bar{K}$ is sheared by both $\Lambda(1405)$ and $a_0$ at the same time.

It is interesting to compare the obtained $\bar{K}N$ state with nuclear systems. As shown in Table III, the hadron-hadron distances in the $\bar{K}N$ state are about 2 fm, which is as large as nucleon-nucleon distances in nuclei. In particular, in the case (A) of the HW-HNJH potential, the hadron-hadron distances are larger than 2 fm and the r.m.s. radius of the three-body system is also as large as 1.7 fm. This is larger than the r.m.s. radius 1.4 fm for $^4$He. If we assume a uniform sphere density of the three-hadron system with the r.m.s. radius 1.7 fm, the mean hadron density is evaluated as 0.07 hadrons/(fm$^3$). Thus the $\bar{K}N$ state has large spatial extent and dilute hadron density.

Order of two-body decay widths can be estimated by geometrical argument. Let us suppose that transitions of three-body bound state to two particles are induced by contact interactions. In such cases, the transition probability is proportional to square of density, $1/r^6$, where $r$ is the radius of the three-body system. In the present calculation, the system of the radius is obtained as 1.7 fm in the parameter set (A). Assuming a typical decay width and radius of baryon resonances as 300 MeV and 0.8 fm, we estimate the two-body decay width as $300 \times (0.8/1.7)^6 \sim 3$ MeV. This is much smaller than the expected three-body decays.

IV. SUMMARY

We have investigated the $\bar{K}N$ system with $J^P = 1/2^+$ and $I = 1/2$ in non-relativistic three-body calculation. We have used the effective $KN$ potentials proposed by Hyodo-Weise and Akaishi-Yamazaki, which reproduce the $\Lambda(1405)$ as a quasi-bound state of $\bar{K}N$. The $\bar{K}N$ interactions are determined so as to reproduce $f_0(980)$ and $a_0(980)$ as quasi-bound states in $\bar{K}N$ with $I = 0$ and $I = 1$ channels, respectively. The potentials of $KN$ are adjusted to provide the observed $\bar{K}N$ scattering lengths,
having strong repulsion in $I = 1$ and no interaction in $I = 0$. The present three-body calculation suggests that a weakly quasi-bound state can be formed below all threshold energies of the $\Lambda(1405)^+ K$, $f_0(980)^+ N$ and $a_0(980)^+ N$. The calculated energies of the quasi-bound state are $-19$ MeV and $-41$ MeV from the $K\overline{K}N$ threshold in the results with HW and AY potentials, respectively. The width for three-hadron decays is estimated to be $90 \sim 100$ MeV. It has been found that the binding energy and the width of the $K\overline{K}N$ state is almost the sum of those in $\Lambda(1405)$ and $a_0(980)$.

Investigating the structure of the $K\overline{K}N$ system, we have found that, in the $K\overline{K}N$ state, the subsystems of $KN$ and $K\overline{K}$ dominate the $I = 0$ and $I = 1$, respectively, and that these subsystems have very similar properties with those in the two-particle systems. This leads to the $K\overline{K}N$ quasi-bound system can be interpreted as coexistence state of $\Lambda(1405)$ and $a_0(980)$ clusters and $\overline{K}$ is a constituent of both $\Lambda(1405)$ and $a_0(980)$ at the same time. As a result of this feature, the dominant decay modes are $\pi\Sigma K$ from the $\Lambda(1405)$ decay and $\pi\eta N$ from the $a_0(980)$ decay, and the decays to $\pi\Lambda K$ and $\pi\pi N$ channels are suppressed.

We also have found that the root-mean-square radius of the $K\overline{K}N$ state is as large as 1.7 fm and the inter-hadron distances are larger than 2 fm. These values are comparable to, or even larger than, the radius of $^4\text{He}$ and typical nucleon-nucleon distances in nuclei, respectively. Therefore, the $K\overline{K}N$ system more spatially extends compared with typical hadronic systems. These features are caused by weakly binding of the three hadrons, for which the $K\overline{N}$ repulsive interaction plays an important role.

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