Remarks on the instability of black Dp-branes

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Abstract

We show that for black Dp-branes having charge $Q$ and Hawking temperature $T$, the product $QT^{7-p}$ is bounded from above for $p \leq 5$ and is unbounded for $p = 6$. While the maximum occurs at some finite value of a parameter for $p \leq 4$, it occurs at infinity of the parameter for $p = 5$. As a consequence, for fixed charge, there are two black Dp-branes (for $p \leq 4$) at any given temperature less than its maximum value, and when the temperature is maximum there is one black Dp-brane. For $p = 5$, there is only one black D5-brane at a given temperature less than its maximum value, whereas, for $p = 6$, since there is no bound for the temperature, there is always a black D6-brane solution at a given temperature. Of the two black Dp-branes (for $p \leq 4$), one is large which is shown to be thermodynamically unstable and the other is small which is stable. But for $p = 5, 6$, the black Dp-branes are always thermodynamically unstable. The stable, small black Dp-brane, however, under certain conditions, can become unstable quantum mechanically and decay either to a BPS Dp-brane or to a Kaluza-Klein “bubble of nothing” through closed string tachyon condensation. The small D5, D6 branes, although classically unstable, have the same fate under similar conditions.

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Black Dp-branes are the low energy solutions of string theory [1, 2] and can be considered as the higher dimensional analog (without the matter fields) of black holes in general theory of relativity. It is true that the black holes in four dimensions are classically stable, but, it is not so for their higher dimensional cousins. This was first shown by Gregory and Laflamme [3] for the neutral black string solution in space-time dimensions $d = 5$. They showed that this space-time under linearized perturbation suffers from a long wavelength instability. This feature is quite generic and remains true even when one introduces charge to the black string solution [4] except when the extremal point is reached [5]. However, the stability can be restored when the string direction is compactified and the size of the compact direction is made smaller than the horizon radius. The end point of the Gregory-Laflamme instability is believed to be the non-uniform or inhomogeneous string [6, 7] as the full dynamics of the transition is difficult to study in the linearized perturbation and remains somewhat speculative. Later it has been shown by Gubser and Mitra [8, 9] that the (uncompactified) charged black string will suffer from Gregory-Laflamme instability exactly when their specific heat is negative known as the correlated stability conjecture. Similar analysis also holds true for the black and neutral as well as charged p-brane solutions in higher dimensions with $p > 1$ [10] (also see [11] for a recent review).

In this letter we will specifically consider the black Dp-brane solutions of type IIA/B string theories 3. We will show how some simple observations can make the classical stability analysis of these solutions much easier. In particular, we point out that the charge $Q$ and the Hawking temperature $T$ of the black Dp-branes [1, 12, 13] are correlated in such a way that the product $QT^{7-p}$ has an upper bound for $p \leq 5$. However, no such bound exists for D6-brane. This implies that for fixed charge, the temperature of the Dp-branes (for $p \leq 5$) has an upper bound, but not for D6-brane 4. The temperature becomes maximum at a finite value of a parameter ($\theta$) related to the ADM mass and the charge of the solution only for $p \leq 4$, but for $p = 5$, the temperature becomes maximum when $\theta$ becomes infinite. For $p = 6$, there is no upper limit for the temperature. As a consequence we will show that when $p \leq 4$, there are two black Dp-brane (one small and the other large) solutions at any given temperature less than the maximum temperature and one black Dp-brane solution when the temperature is the maximum. For $p = 5$ there is one black D5-brane solution for the temperature less than its maximum value and for D6-brane, as there is no maximum temperature, there is always a solution for any given

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3 The black string solution in $d = 5$ considered by Gregory and Laflamme [3] is nothing but the double dimensionally reduced black D6-brane solution of type IIA string theory. Whereas the strings in $d > 5$ dimensions are the double dimensionally reduced Dp-branes (where $p = 11 - d$) in string theory.

4 This remains also true if we exchange the role between the charge and the temperature.
temperature. Using thermodynamics of the black Dp-brane we calculate their specific heat \[14, 9\] and found that the specific heat is always negative for the large black Dp-brane with \( p \leq 4 \) and it diverges for the black brane whose temperature is the maximum, signalling a possible critical point of second order phase transition. But for the small Dp-brane with \( p \leq 4 \), the specific heat is positive. So, based on the correlated stability conjecture mentioned earlier, the small Dp-brane is classically stable for \( 0 < p \leq 4 \) whereas the large one is classically unstable\[^5\]. For D5 and D6, the specific heat is always negative and so, they are always classically unstable. The unstable branes would of course decay to more stable and entropically more favorable solutions, the dynamics of which may be difficult to understand in the linearized approximation. But one can argue from the earlier studies that the large black Dp-branes, would either decay into small black Dp-branes or to non-uniform (inhomogeneous) Dp-branes and D5, D6 branes would decay only into non-uniform (inhomogeneous) branes \[^7\].

Thus far we have only discussed about the classical stability of the black Dp-branes and mentioned that the small black Dp-branes are classically stable for \( 0 < p \leq 4 \). The large black Dp-branes (for \( 0 < p \leq 4 \)) as well as the black D5 and D6 branes are classically unstable. However, under certain conditions the classically stable small black Dp-branes could become quantum mechanically unstable. This can happen when we reduce the temperature of the black Dp-brane keeping the charge fixed (or when we reduce the charge while keeping the temperature fixed). By this, the parameter \( \theta \) will become very large and the size of the black Dp-brane will get reduced. At this point the issue of quantum instability for the black Dp-brane arises. As a result, the black Dp-brane will start to Hawking radiate and will smoothly make a transition to the stable BPS Dp-brane. This can happen for all \( p \leq 4 \). However, when \( p > 4 \), there can be another kind of quantum mechanical instability if one of the brane directions is compact. When the fermions satisfy periodic boundary condition along the compact direction, the black brane will again make a transition to BPS Dp-brane. On the other hand, when the fermions satisfy antiperiodic boundary condition, then a fundamental string wound around the compact direction can become tachyonic \[^{16}\] and in that case the black Dp-brane can make a transition to the static Kaluza-Klein (KK) “bubble of nothing” (BON) \[^{17}\] by closed string tachyon condensation \[^{18, 19}\]. As we have mentioned, D5 and D6 branes are always classically unstable. However, for very large \( \theta \) (i.e., for small black branes), both D5 and D6 brane can either decay to non-uniform (or inhomogeneous) branes or

\[^5\]In order to relate the classical instability to local thermodynamic instability, the gravitational system must be of infinite extent and this excludes the case \( p = 0 \).

\[^6\]The \( p = 0 \) case, though not quite relevant in the present discussion, can also be addressed as in \[^{15}\].
they can smoothly make a transition to BPS branes. However, when one of the directions of the brane is compact, then the black brane can make a transition to BPS brane or dynamical KK BON, depending on whether the fermions in the theory satisfy periodic or antiperiodic boundary condition along the compact direction, respectively.

The electrically charged black Dp-brane solution in low energy type IIA/B string theory is given as [1],

\[
\begin{align*}
    ds^2 &= H^{-\frac{1}{2}} \left( -f dt^2 + \sum_{i=1}^{p} (dx^i)^2 \right) + H^{\frac{1}{2}} \left( \frac{dr^2}{f} + r^2 d\Omega^2_{8-p} \right) \\
    e^{2\phi} &= g_s^2 H^{\frac{3-p}{2}}, \quad A_{012...p} = \frac{1}{g_s} \left( H^{-1} - 1 \right) \coth \theta \tag{1}
\end{align*}
\]

where

\[
H = 1 + \frac{r_0^{7-p} \sinh^2 \theta}{r^{7-p}}, \quad f = 1 - \frac{r_0^{7-p}}{r^{7-p}}
\]  

are two harmonic functions and \(r_0\) and \(\theta\) are two parameters characterizing the solution related to the ADM mass and the charge of the black Dp-brane. Note that the metric is written in the string frame and \(g_s\) is the string coupling constant. The black brane has a horizon at \(r = r_0\). The electric charge associated with the black Dp-brane can be obtained as,

\[
Q = \frac{1}{2\kappa^2} \int_{\Omega_{8-p}} *F_{[p+2]} = \frac{(7-p)\Omega_{8-p} r_0^{7-p} \sinh \theta \cosh \theta}{2\kappa^2 g_s} \tag{3}
\]

where \(2\kappa^2 = (2\pi)^7 \alpha'^4\), with \(2\pi \alpha' = 1/T\), i.e., the inverse of string tension, and \(\Omega_n = 2\pi^{(n+1)/2}/\Gamma((n + 1)/2)\) is the volume of the \(n\)-dimensional unit sphere. \(*F_{[p+2]}\) is the Hodge dual of the field-strength associated with the gauge field \(A_{012...p}\). The Hawking temperature of the black Dp-brane can be obtained from the metric in (1) as usual by first Euclideanizing the time coordinate and then calculating the inverse of the periodicity of the Euclidean time direction to avoid the conical singularity. The temperature has the form,

\[
T = \frac{7 - p}{4\pi r_0 \cosh \theta} \tag{4}
\]

Eliminating \(r_0\) from (3) and (4) we have

\[
QT^{7-p} = \frac{\Omega_{8-p} (7-p)^{8-p} \sinh \theta \cosh^{8-p} \theta}{2\kappa^2 g_s (4\pi)^{7-p}} = \frac{\Omega_{8-p} (7-p)^{8-p}}{2\kappa^2 g_s (4\pi)^{7-p}} C(\theta) \tag{5}
\]

From (5) we notice that \(C(\theta) \propto QT^{7-p}\) is positive in general and vanishes for both \(\theta \to 0\) and \(\theta \to \infty\) for \(p \leq 4\). But for \(p = 5, 6\), \(C(\theta)\) vanishes only for \(\theta \to 0\). \(C(\theta)\) becomes

\(^{7}\)Without any loss of generality, we will assume that the branes are positively charged and so, \(\theta \geq 0\).
constant as $\theta \to \infty$ for $p = 5$, while $C(\theta)$ increases monotonically with $\theta$ for $p = 6$. It can be checked from the form of $C(\theta)$ in (3) that while $C(\theta)$ does not have a maximum for $p = 6$, there is a unique maximum of $C(\theta)$ for $p \leq 5$ at $\sinh^2 \theta = 1/(5-p)$. So, for $p \leq 4$, the maximum occurs at a finite $\theta$, but for $p = 5$ the maximum occurs when $\theta \to \infty$. The maximum value or the upper bound of $QT^{7-p}$ can be obtained as,

$$
(QT^{7-p})_{\text{max}} = \frac{\Omega_{8-p} (7-p)^{8-p}}{2\kappa^2 g_s} \frac{C(\theta)_{\text{max}}}{(4\pi)^{7-p}} = \frac{\Omega_{8-p} (7-p)^{8-p}(5-p)^{5-p}}{2\kappa^2 g_s} (4\pi)^{7-p}(6-p)^{5-p/2}
$$

(6)

So for any given product $QT^{7-p} \propto C(\theta) < C_{\text{max}}$, we will have two solutions for $\theta$ (the smaller $\theta$ is called $\theta_s$ and the larger $\theta$ is called $\theta_\ell$) for $p \leq 4$ and one solution for $p = 5, 6$. These features of the black Dp branes for various values of $p$ are depicted in Figure 1.

Figure 1: The plot shows how $C(\theta)$ varies with $\theta$ in some suitable units for various black Dp-branes. The lowest curve is for $p = 0$ and then $p = 1, p = 2$, and so on up to $p = 6$. The horizontal line represents a particular value of $C(\theta)$ and shows that it cuts the curves at two points for $0 \leq p \leq 4$ corresponding to large (the left one) and small (the right one) black Dp-branes. For $p = 5, 6$, the line cuts the curve once for either case.

Thus for a fixed charge $Q$, as we lower the temperature of the black D$p$ brane from its maximum value $T_{\text{max}}$, at a given temperature (denoted by the straight horizontal line in Figure 1) there are two black Dp-branes (for $p \leq 4$) corresponding to $\theta_s$ and $\theta_\ell$ but there is only one black D5 or D6 brane. Note that when $Q$ is fixed the size of the black brane corresponding to $\theta_s$ and $\theta_\ell$ can be obtained from (3) and this shows that smaller $\theta$ (or $\theta_s$) corresponds to larger black-brane and larger $\theta$ (or $\theta_\ell$) corresponds to smaller black brane.

Now we try to understand the classical stability of the black Dp-branes from thermodynamics. For this purpose we need the expression for the ADM mass [20] and the
entropy \([13]\) of the black Dp-brane which can be calculated from the metric in \([11]\) and they have the forms,

\[
M = \frac{(8-p)\Omega_{8-p}V_p r_0^{7-p}}{2\kappa^2 g_s^2} \left(1 + \frac{7-p}{8-p} \sinh^2 \theta\right), \quad S = \frac{4\pi\Omega_{8-p}V_p r_0^{8-p} \cosh \theta}{2\kappa^2 g_s^2} \tag{7}
\]

From \([3]\), \([11]\) and \([7]\), it is clear that the ADM mass satisfies the Smarr relation (see, for example, \([14]\)),

\[
(7-p)M = (8-p)TS + (7-p)\mu Q \tag{8}
\]

where \(\mu = \tanh \theta/g_s\) is the chemical potential. Now using the thermodynamical relation,

\[
dM = TdS + \mu dQ \tag{9}
\]

we obtain

\[
C_Q \equiv \left(\frac{\partial M}{\partial T}\right)_Q = T \left(\frac{\partial S}{\partial T}\right)_Q = \frac{4\pi\Omega_{8-p}V_p \cosh \theta r_0^{8-p} [(9-p) \sinh^2 \theta + (8-p)]}{2\kappa^2 g_s^2} [((5-p) \sinh^2 \theta - 1] \tag{10}
\]

where \(C_Q\) is the specific heat of the black Dp-brane and in writing the second expression in \([10]\) we have made use of \(Q\) in \([3]\), \(T\) in \([11]\) and \(S\) in \([7]\). Now it is clear that the sign of the specific heat will depend on the factor \((5-p) \sinh^2 \theta - 1\) in the denominator of \([10]\), since everything else is positive. It should be noted that the factor \((5-p) \sinh^2 \theta - 1\) is always negative for \(p = 5, 6\) and so the black D5 and D6 branes are classically unstable.

For \(p \leq 4\) the sign of this factor depends on the value of the parameter \(\theta\). However we find that in general \(((5-p) \sinh^2 \theta - 1)/ \cosh^{7-p} \theta = -(dC(\theta)/d\theta)\), where \(C(\theta)\) is defined in \([5]\). So, when the slope of the curve \(C(\theta)\) vs \(\theta\) is positive the specific heat is negative and when it is negative the specific heat is positive. In particular, when \(\theta = 0\) (i.e., for chargeless black Dp-brane), the specific heat is always negative for \(p = 5, 6\) and so the black D5 and D6 branes are classically unstable.

By looking at Figure 1, it is clear that for \(p \leq 4\), the slope \(dC(\theta)/d\theta\) remains positive from \(\theta = 0\) up to the point where \(C(\theta)\) becomes maximum \((C_{\max})\) and so, the black brane will remain unstable throughout this region. Then at the maximum point the slope vanishes and the specific heat diverges. After that the slope becomes negative from \(\theta\) corresponding to \(C_{\max}\) up to \(\theta \to \infty\). In this entire region the specific heat remains positive and the black brane is classically stable. In particular, when \(\theta \to \infty\) and also \(r_0 \to 0\) in such a way, that \(Q\) remains unchanged, we get back BPS Dp-brane solution from \([11]\) which is obviously stable. This can happen also for D5 and D6 branes. Since the smaller \(\theta\) is for the larger black Dp-brane (for \(0 < p \leq 4\)) which is unstable while the larger \(\theta\) is for smaller one which is stable due to \([3]\) for fixed charge, we therefore find that large black branes are unstable and small ones are stable classically for \(0 < p \leq 4\).
The unstable black branes will of course decay to a more stable or entropically more favorable solution. As we have mentioned in the introduction the unstable large black $D^p$-branes (for $p \leq 4$) can decay either into the stable small black $D^p$-branes or they will decay to non-uniform or inhomogeneous black brane solution as has been argued in [6, 7]. To substantiate our claim, we can compute the free energy of the two configurations and see whether the large black $D^p$-branes has more free energy than the small black $D^p$-branes. The Helmholtz free energy of the black $D^p$-branes per unit $p$-brane volume is given as,

$$F = \frac{M - TS}{V_p} = \frac{2Q}{(7-p)gs \sinh 2\theta} \left[ 1 + (7-p) \sinh^2 \theta \right]$$ \hspace{1cm} (11)

where we have used the expressions for the ADM mass, the entropy given in eq.(7), the temperature given in eq.(4) and the expression for the charge in eq.(3). In the following we will look at the reduced free energy, which is only relevant for the discussion when the charge and the temperature are fixed, and is defined as,

$$\frac{(7-p)gs}{2Q} F \equiv \tilde{F} = \frac{1 + (7-p) \sinh^2 \theta}{\sinh 2\theta}$$ \hspace{1cm} (12)

We will first consider the case $p = 4$ since in this case we can compute the difference of reduced free energy explicitly for all values of $C < C_{\text{max}}$. For $p < 4$, as we will mention, we can compare the free energies of the two black branes only for $C \ll C_{\text{max}}$. From eq.(6), we find that for $p = 4$, $C_{\text{max}}$ has the value $1/2$. So, for any $C(\theta) < 1/2$, we have two values of $\theta$, namely, $\theta_\ell$ (large $\theta$) corresponding to small black D4-brane and $\theta_s$ (small $\theta$) corresponding to large black D4-brane. The values of $\theta_\ell, s$ can be computed from eq.(5) as

$$\sinh \theta_\ell = \frac{1}{2C} \left( 1 + \sqrt{1 - 4C^2} \right)$$

$$\sinh \theta_s = \frac{1}{2C} \left( 1 - \sqrt{1 - 4C^2} \right)$$ \hspace{1cm} (13)

Substituting (13) in (12) we find,

$$\Delta \tilde{F} = \tilde{F}_{\text{large D4}} - \tilde{F}_{\text{small D4}} = \frac{\sqrt{2} (1 - 2C)}{4C} \left[ \left( 1 + \sqrt{1 - 4C^2} \right)^{\frac{1}{2}} - \left( 1 - \sqrt{1 - 4C^2} \right)^{\frac{1}{2}} \right]$$ \hspace{1cm} (14)

So, $\Delta \tilde{F}$ is always positive as $C < 1/2$. Thus for given charge $Q$ and temperature $T$, we find that the large black D4-brane has higher free energy than the small black D4-brane, consistent with the specific heat analysis.
For $p < 4$, it is difficult to solve the general $C(\theta)$ equation (see eq.(5))

$$\sinh \theta = C \cosh^{6-p} \theta$$

(15)

explicitly to find out $\theta_\ell$ and $\theta_s$. However, the equation (15) can be solved when $C(\theta) \ll C_{\text{max}}$. In this case the small $\theta$ satisfies $\theta_s \ll 1$ while the large $\theta$ satisfies $\theta_\ell \gg 1$. So, for small $\theta$, we find, to the leading order (from eq.(15))

$$\theta_s = C$$

(16)

while for large $\theta$ we find to the leading order,

$$e^{\theta_\ell} = 2C^{-\frac{1}{6-p}}$$

(17)

So, to the leading order

$$\bar{F}_{\text{large brane}} = \frac{1}{2C}$$

$$\bar{F}_{\text{small brane}} = \frac{T-p}{2}$$

(18)

From here it is obvious that the large black brane has larger free energy than the small black brane since $1/C \gg 1/C_{\text{max}}$. For example, if we take $1/C = 10/C_{\text{max}}$, then certainly large black brane has larger free energy. Once again, this is consistent with the specific heat analysis.

Next we will look at the small black Dp-brane solutions which are classically stable for $0 < p \leq 4$ and unstable for $p = 5, 6$. Small black branes even for $p \leq 4$ can become quantum mechanically unstable. If we do not impose any further conditions, all these black branes (including the classically stable ones (for $p \leq 4$) as well as the classically unstable ones (for $p = 5, 6$)) will Hawking radiate and if by this process $\theta \to \infty$ such that the charge remains fixed, the black brane will make a smooth transition to the stable BPS Dp-brane configuration (as is clear from (1), (2), (3)). On the other hand, if one of the brane directions is made compact, then the fermions in the theory can be either periodic or antiperiodic along the compact direction. For the periodic boundary conditions, the black branes will Hawking radiate and eventually go over to the stable BPS Dp-brane solution as before. But, for the antiperiodic boundary condition, the fundamental string wound along the compact direction can become tachyonic. As argued in ref.[21], if the tachyon is localized it can trigger a topology changing transition and the black Dp-brane

\[8\] Since black D5, D6 branes are classically unstable even when they are small, there is a possibility for them to decay to non-uniform (or inhomogeneous) branes.
can decay into either a static or a dynamical KK BON (for $0 < p \leq 4$) or only to a dynamical KK BON (for D5 and D6 branes) by a closed string tachyon condensation \cite{18,19}. However, in order for the black brane to decay through the perturbative stringy process of closed string tachyon condensation several conditions have to be satisfied and we will outline them briefly here for completeness, the details of which is given in ref.\cite{19}. First of all, note from (1) that, the size of the compact circle (let us assume that $x^1$ is the compact direction) varies from a finite value ($L \gg l_s$, where $l_s$ is the fundamental string length) at infinity to zero at the singularity ($r = 0$). So, in between the size of the circle becomes of the order of $l_s$, and so the winding fundamental string along this direction can become tachyonic. We will further assume that this happens on the horizon$^9$ such that,

$$L = l_s \cosh^{1/2} \theta$$

Since $L \gg l_s$, eq.\textcolor{red}{(19)} implies that the parameter $\theta$ must be very large or the size of the black brane is very small (this is the classically stable region for $0 < p \leq 4$ as can be seen from Figure 1). In that case the closed string tachyon will cause the circle to pinch off and the black D$p$-brane will make a transition to KK BON. The final KK BON configuration can be either static or dynamical as we have argued in detail in \cite{19}. In order to have a static bubble the curvature on the horizon must remain much smaller than the string scale, otherwise the supergravity description will break down and the black D$p$-brane will make a transition to open string modes (correspondence point) \cite{22}. Further, in order for the transition to occur, the size of the horizon must match with the size of the bubble, the charge of the black D$p$-brane must be equal to the flux associated with the bubble and also the size of the $x^1$-circle at infinity must match for the two configurations. We have seen in \cite{19} that all these conditions can be satisfied for the black D$p$-branes with $0 < p \leq 4$ and the black D5 and D6 branes can only make transitions to dynamical bubbles.

We have thus seen how some simple observations on the black D$p$-brane parameters make their stability analysis much easier. In particular, we have shown that the charge, $Q$, and the Hawking temperature, $T$ of the black D$p$-branes are such that the product $QT^{7-p}$ has an upper bound for all $p \leq 5$, but black D6-brane does not have such a bound. For $p \leq 4$, the maximum occurs at a finite value of the parameter $\theta$, but for $p = 5$, the maximum occurs when $\theta \to \infty$. This implies that, for fixed charge, there

\textcolor{red}{9}If the size of the circle is greater than $l_s$ on the horizon, there will be no closed string tachyon. However, as the black D$p$-brane Hawking radiate, the horizon size gets reduced and the size of the $x^1$-circle will also get reduced and eventually, will attain the size $l_s$ and then the closed string tachyon will appear \textcolor{red}{18}. }
are two black Dp-brane solution (for $p \leq 4$), one small and the other large, at a given temperature below the maximum value, whereas, for $p = 5, 6$, there is only one Dp-brane solution. By looking at the slope of the curve $C(\theta) \text{ vs. } \theta$ (depicted in Figure 1), which is proportional to the negative of the specific heat of the black Dp-branes, we found that, the large black Dp-branes are classically unstable, whereas, the small black Dp-branes are classically stable (for $0 < p \leq 4$) and D5 and D6-branes are always classically unstable. The classically unstable black branes have been argued to make a transition either to a stable small black brane (for $0 < p \leq 4$) or to a non-uniform or inhomogeneous stable brane configurations. Indeed, we have seen from the computation of the free energies of the two black brane configurations that the large black brane has larger free energy indicating that the transition from the large black D-brane to the small black D-brane is possible. However, when the parameter $\theta$ becomes very large or the horizon size of the black Dp-brane becomes very small (this implies that the temperature becomes very small for $p \leq 4$, but not for $p = 5, 6$), both the classically stable Dp-branes (for $0 < p \leq 4$) and classically unstable D5, D6-branes can become quantum mechanically unstable. So, the black branes (for all $p \leq 6$) will Hawking radiate and will make a smooth transition to BPS Dp-branes if we do not impose any further conditions. But if one of the brane directions is made compact (this can occur for $p > 0$), then depending on the periodicity or the antiperiodicity of the boundary conditions on the fermions along the compact direction, the black brane will make a transition either to a BPS Dp-brane or to a KK BON. The KK BON could be static or dynamical for $0 < p \leq 4$, but for $p = 5, 6$, the KK BON will be dynamical.

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