Synchronization Effects in Intrinsic Josephson junctions by Non-equilibrium Heating

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Abstract.
In order to study a role of the non-equilibrium heating on intrinsic Josephson junctions, we derive a model equation including the thermal heat diffusion process along c-axis and discuss how the heating affects the dynamics of stacked Josephson junctions. We suggest that the heating gives rise to a new type of coupling which are different from the established ones, i.e., the capacitive and inductive couplings. The coupling becomes rather large when the heating becomes dominant. We suggest that the coupling makes it possible to synchronize all the stacked junctions and to radiate a strong electromagnetic wave emission.

1. Introduction
The observation of an electromagnetic wave emission from High-Tc intrinsic Josephson junctions (IJJ’s) has been one of the most active target in IJJ research community since the confirmation of intrinsic Josephson effects [1]. In IJJ’s, its stacking structure composed of thousands of junctions offers a great opportunity to radiate a high power and monochromatic electromagnetic wave if all the stacked junctions or a part of them synchronize. Thus, several theoretical and experimental works have been intensively made for more than a decade. However, there has been no clear and direct observation works until a recent report by Ozyuzer et al [2].

The coupling mechanism between stacked junctions in intrinsic Josephson junctions has been one of the mostly debated theoretical issues for the latest decade because the collective dynamics of IJJ’s arises from the coupling [3]. The inductive coupling has established soon after the suggestion because the idea is very clear, namely, the magnetic field screening is incomplete within a superconducting layer because the magnetic penetration depth is much larger than the layer thickness [4, 5]. On the other hand, the capacitive coupling has took a long period to spread the idea to the community of IJJ’s because of its novelty and uniqueness to IJJ’s [6, 7]. In addition to the above two essential couplings, a coupling induced by the charge imbalance has been suggested as a finite temperature effect [8]. The coupling can be basically incorporated into the capacitive one. Recently, one of authors (Machida) and Sakai suggested a unified framework for these couplings [3].
Such fundamental theoretical works on IJJ’s have stimulated several numerical simulations in order to explore whether the synchronization among all the stacked junctions is possible or not by the suggested couplings. At first, a pioneering work [9] and more systematic ones [10],[11] including solely the inductive coupling clarified that a large emission is possible in the rectangular vortex flow [11]. The phase-locked flow state appears as a resonant mode when the vortex flow speed matches with the phase velocity of the highest plasma wave. This result has inspired several experimentalists, but its clear and direct confirmation has been not reported yet except for a few works [12]. This is because it requires to reach a relatively high voltage state without the heating and related breakdown effects.

The idea using the vortex flow state has been not successful yet, while a clear and direct confirmation of a large power emission has been achieved in a different situation by Ozyuzer et al [2]. They employed a large mesa sample whose ab-plane’s sides are comparable to the magnetic penetration depth along c-axis and measured a sign of the large emission by simply applying the current along c-axis with no magnetic field. Since the observation, the emission conditions have been intensively studied [13]. Consequently, the cavity resonance of AC Josephson emission has been believed as a basic mechanism. However, there have still remained some unsolved issues, e.g., a classification of the excited main mode, a stability of the emission, a small conversion rate of the input energy, and so on.

In addition to the AC Josephson emission assisted by the cavity resonance, a different type of strong emission has been very recently reported [14]. The emission is observable in a bended I-V characteristics seen in the high-voltage return path when decreasing the current [15]. Normally, such a bending in I-V characteristics appears due to the superconducting gap suppression by the overheating. Thus, the strong emission in the region is unexpected from the standard theories on IJJ’s. Very recently, an intuitive idea due to its negative resistance has been presented [16].

In this paper, we suggest a theoretical framework in order to approach the heated range in the I-V characteristics based on the established theories on IJJ’s [3]. The framework is just an extension from the treatment using a global temperature over all the stacked junctions to that using the local temperature depending on its location. We show two frameworks, one of which assumes the superconducting phase to be homogeneous along the ab-plane for simplicity, and another of which includes its variation along ab-plane. Theoretically, the former one is based on the capacitive coupling solely, and the latter one includes both the capacitive and the inductive couplings [3].

The contents of this paper are as follows. In section 2, a theoretical treatment including the heating on the single junction is presented. The all relations associated with the local temperature are applied to stacked systems, i.e., IJJ’s. Section 3 derives its extension version to stacked IJJ’s without the coupling between stacked junctions. Section 4 devotes the inclusion of the couplings intrinsic to IJJ’s.

2. A Theoretical Framework of the Heating in Single Junction
The superconducting phase dynamics of the single Josephson junction is described by the RCSJ model,

\[ \frac{d^2 \phi}{dt^2} + \beta(T, V) \frac{d\phi}{dt} + \sin \phi = \frac{I}{j_c(T, V)}, \]

where \( \phi \) is the superconducting phase, \( \beta \) is the damping constant related to the McCumber parameter \( \beta_c \) as \( \beta = \frac{1}{\sqrt{3} \beta_c} \), and \( j_c \) is the critical current. In Eq.(1), the time \( t \) is normalized by the plasma frequency \( \omega_p \). The normalization is also used for all the following equations. Here, we note that \( \beta \) and \( j_c \) are dependent on the temperature given by \( T \) and the voltage \( V(= \frac{d\phi}{dt}) \). Thus, the superconducting phase dynamics in the single Josephson junction is found to be influenced by the temperature variation.
Let us write down the temperature and voltage dependences of $\beta$ and $J_c$. At first, that of $\beta$ is given by

$$\beta(T^*, V) = \beta_N - A_0 \frac{1}{1 + e^{\frac{V - V_G(T^*)}{\gamma}}},$$  \hspace{1cm} (2)

where $\beta_N$ means $\beta$ in the normal state, $\gamma$ and $A_0$ are constants which originate from quasiparticle excitations in the superconducting state, and $V_G(T^*)$ is the temperature dependent gap voltage, whose dependence is given by

$$V_G(T^*) = V_G(0)(1 - \frac{T^*}{T_c}) \quad (T < T_c), \quad V_G = 0 \quad (T > T_c).$$  \hspace{1cm} (3)

In the above relation Eq.(2), when $T^*$ is above $T_c$, then $\beta$ is $\beta_N$. On the other hand, when $T^*$ is below $T_c$, then $\beta$ becomes smaller than $\beta_N$. The temperature dependence of $J_c$ is given by the Ambegaokar-Baratoff relation. Although these relations should be derived from the microscopic theory, the above phenomenological relations is enough to qualitatively capture the heating effects on the superconducting phase dynamics.

Next, we derive an equation describing the temperature variation. The heat balance equation (see Figure 1) is given as

$$W(I/j_c(0)\frac{\partial \phi}{\partial t}) = C_v \frac{\partial T^*}{\partial t} + H(T^* - T_{env}),$$  \hspace{1cm} (4)

where $W$ is the Joule heat, $C_v$ is the heat capacity of the junction system, the parameter $H$ is the heat energy extraction rate from the heat bath whose temperature is is given by $T_{env}$. The parameter $H$ is flexibly fixed by executing calculations for I-V characteristics, i.e., a suitable value to observe the bending behaviors is selected. Generally, the coupling with the heat bath is composed of not only the first order of the temperature difference but also its higher orders. Here, we note that we take into account the first order solely for simplicity. Eq.(4) describes the dynamics of $T^*$ associated with the Joule-heat energy production, and the solution of $T^*$ becomes an input parameter of Eq.(1) through the temperature dependence of $\beta(T^*)$ (Eq.(2)) and $j_c(T^*)$ (Eq.(3)).

![Figure 1. A schematic figure for the heat balance in a single Josephson junction.](image)

By combining Eq.(1) with Eq.(4), we can obtain the I-V characteristics influenced by the heating. The typical example showing back-bending I-V characteristics is given by Figure 2. The calculation details will be published elsewhere [17]. In Fig.2, the back-bending is observable in the return path of I-V characteristics. This feature is very similar to that observed in IJJ’s.
3. A Theoretical Framework of the Heating in Intrinsic Josephson Junction

The superconducting phase dynamics in IJJ’s is described by an equation incorporating both the capacitive and inductive couplings [3]. In this paper, we firstly exhibit the framework without their couplings, and clarify how the heating induces a new coupling. Afterwards, we touch the heating effects on the system including the capacitive and inductive couplings. More details will be published elsewhere [17].

3.1. The Heating Effects without Capacitive and Inductive Couplings

Except for the last subsection, we do not take into account any variation of the superconducting phase along the in-plane direction. In this case, the superconducting dynamics without the capacitive and inductive couplings is described by the independent RCSJ model equation written as

$$\frac{\partial^2 \phi_{l+1,l}}{\partial t^2} + \beta(T^*_{l+1,l}, V_{l+1,l}) \frac{\partial \phi_{l+1,l}}{\partial t} + \sin \phi_{l+1,l} = \frac{I}{j_c(T^*_{l+1,l})},$$

(5)

where $l$ means $l$-th superconducting layer, and $\phi_{l+1,l}$ is the superconducting phase difference between $l$-th and $l + 1$-th superconducting layers. We note that the temperature is also defined on each IJJ as $T^*_{l+1,l}$ together with $\phi_{l+1,l}$. The dynamics of $\phi_{l+1,l}$ is completely independent if the temperature is a fixed constant over all junctions as usual treatments.

Let us derive the heat balance equation on IJJ’s. The equation is basically given by adding thermal diffusion process along c-axis into Eq.(4),

$$W(I/j_c(T^*_{l+1,l}) \frac{\partial \phi_{l+1,l}}{\partial t}) = C_v \frac{\partial T^*_{l+1,l}}{\partial t} + H(T^*_{l+1,l} - T_{env}) - \frac{\kappa}{D^2}(T^*_{l+2,l+1} + T^*_{l,l-1} - 2T^*_{l+1,l}),$$

(6)

where $\kappa$ is c-axis thermal conductivity and $D$ is the layer periodicity. In Eq.(6), the third term in the right hand side is a new term, which is essential to IJJ’s. If one drops the term, then the set of equation (Eq.(5) and (6)) has no coupling and the system never shows any synchronized behaviors. In contrast, if one turns on the thermal diffusion, then the temperature variation associated with the Joule heat generation can propagate along c-axis. Namely, the variation of $\beta(T^*_{l+1,l}, V_{l+1,l})$ and $j_c(T^*_{l+1,l}, V_{l+1,l})$ propagates to the neighbor’s ones. This is a kind of synchronization of the parameters in the coupled oscillator equation (Eq.(5)). Especially, since $\beta$ has a role of brake on the rotating, its synchronization works as a synchronized brake for
several junctions. Thus, one can expect the synchronization of the superconducting dynamics along c-axis by the heating. In addition, it is found that the coupling strength is relatively large compared to the capacitive coupling \[6, 7\]. The non-dimensionalized coupling constant is given by \( \frac{v_F \ell}{\omega_p D^2} \), where \( v_F \) and \( \ell \) are the Fermi velocity and the mean free path, respectively, and its value is comparable to \( 10^2 \), which is much larger than the capacitive coupling \( \sim \mu^2/D^2 \sim 0.2 \) for Bi-2212, where \( \mu \) is the charge screening length \[6\] and an order of smaller than the inductive coupling \( \sim \lambda^2/D^2 \sim 10000 \) for Bi-2212 \[4, 5\]. Thus, when the heating becomes dominant, the new coupling is not a negligible one.

\[ \frac{\partial^2 \phi_{l+1,l}}{\partial t^2} + \beta(T^*_{l+1,l}) \frac{\partial \phi_{l+1,l}}{\partial t} + \sin \phi_{l+1,l} = I/jc(T^*_{l+1,l}) + \alpha(T^*_{l+1,l}, V_{l+1,l}) \Delta^2 \sin \phi_{l+1,l}, \quad (7) \]

where \( \alpha \) is the capacitive coupling constant \( \equiv \frac{\epsilon \mu^2}{D^2} \), where \( \epsilon \) and \( \mu \) are the dielectric constant and the charge screening length, respectively), and \( \Delta^2 \) is the discrete second derivative operator along c-axis direction. It is known that the constant goes down to zero value when the voltage goes to the gap energy value \[18\]. Thus, the capacitive coupling becomes negligible in the heated region like the bended I-V characteristics and Eq.(7) then goes to Eq.(5).

3.3. Heating Effects and Inductive Coupling

In this subsection, we skip the direct derivation of the equation including the inductive coupling and just discuss typical dynamics of IJJ’s in the heated range due to the space limitation. In the theoretical framework for the heating effects on IJJ’s, the inclusion of the inductive coupling naturally requires a variation of the local temperature along ab-plane as well as c-axis. This extension is straightforward and the heat balance equation has another new term describing the

![Figure 3. A schematic figure for the heat balance in an IJJ.](image-url)
thermal conduction along the ab-plane. Thus, the coupled sine-Gordon equation has a damping parameter depending on the temperature variation along the ab-plane. The capacitive coupling diminishes close to the gap energy, while the inductive one still remains as a large contribution because the original one at zero temperature is much larger than other couplings. We can expect that in addition to the large coupling, a synchronization of $\beta$ via the heat diffusion along both c-axis and ab-plane directions emphasizes the synchronization of the superconducting phase dynamics of all the junctions. This numerical confirmation will an important future issue [17].

4. Summary and Conclusion
We studied the heating effects on the single Josephson junction and IJJ’s. For the single junction, the heat balance is closed inside the junction except for an energy flow into the heat bath, while the heat balance in IJJ’s includes the heat diffusion along c-axis and ab-plane and a new type of coupling appears. This new coupling brings about a synchronization of the damping parameters on rotations of several junctions. Thus, we can expect a large emission of electromagnetic wave from IJJ due to the synchronization of the superconducting phases in the heated region. We concludes that the heating has an important role on the synchronization of all stacked junctions and AC Josephson emission.

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References
[1] Kleiner R., Steinmeyer F., Kunkel G., and Muller P., 1992 Phys. Rev. Lett. 68 2394; Oya G., Aoyama N., Irie A., Kishida S., and Tokutaka H., 1992 Jpn. J. Appl. Phys. 31 L829.
[2] Ozyuzer L., Koshelev A. E., Kurter C., Gopalsami N., Li O. A., Tachiki M., Kadowaki K., Yamamoto T., Tachiki T., Gray K. E., and Welp U., 2007 Science 318 1291.
[3] Machida M., and Sakai S., 2004 Phys. Rev. B70 144520.
[4] Sakai S., Bodin P., and Pedersen N. F., 1993 J. Appl. Phys. 73 2411.
[5] Bulaevskii L. N., Zamora M., Baeriswyl D., Beck H., Clem J. R., 1994 Phys. Rev. B 50 12831.
[6] Koyama T., and Tachiki M.,1996 Phys. Rev. B54 16183.
[7] Machida M., Koyama T., and Tachiki M., 1999 Phys. Rev. Lett. 83 4618.
[8] See e.g., Ryndyk D. A., Pozdnjakova V. I., Shereshevskii I. A., and Vdovicheva N. K., 2001 Phys. Rev. B 64 052508.
[9] Kleiner R., 1994 Phys. Rev. B 50 6919.
[10] Ustinov A. V., and Sakai S., 1998 Appl. Phys. Lett. 73 686.
[11] Machida M., Koyama T., Tanaka A., and Tachiki M., 2000 Physica C 330 85.
[12] Bae M. H., Lee H. J., and Choi J. H., 2007 Phys. Rev. Lett. 98 027002.
[13] Matsumoto H., Koyama T., and Machida M., (in press); Bulaevskii L. N., and Koshelev A. E., 2007, Phys. Rev. Lett. 99 057002.
[14] Kadowaki K., (presentation file in Dubna Nano 2008).
[15] For a pioneering work in the emission in the bended I-V characteristics, see e.g., Lee K., Wang W., Iguchi I., Tachiki M., Hirata K., and Mochiku T., 2000 Phys. Rev. B61 3616.
[16] Pedersen N. F., (presentation file in Dubna Nano 2008).
[17] Machida M., Koyama T., and Matsumoto H., in preparation.
[18] Machida M., Koyama T., Tanaka A., and Tachiki M., 2000Physica C331 85.