Saturation of counterterms by resonances in $K \to \pi e^+ e^-$ decays

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ABSTRACT

The decays $K^+ \to \pi^+ e^+ e^-$, $K_S \to \pi^0 e^+ e^-$ and $K_L \to \pi^0 e^+ e^-$ are reinvestigated within the framework of chiral perturbation theory. The counterterms induced by strong, electromagnetic and weak interactions are determined assuming the resonance exchange. The weak deformation model, the factorization model and the large $N_c$ limit are used to create a weak Lagrangian. It is found that the results of the first two approaches depend on the $H_1$ coupling, defined in the effective chiral Lagrangian of the $O(p^4)$ order. The set of parameters used in the extended Nambu and Jona-Lasinio model can accommodate $K^+ \to \pi^+ e^+ e^-$ decay rate within the factorization approach. The CP violating $K_L \to \pi^0 e^+ e^-$ decay rate is discussed.
1 Introduction

The decays $K \to \pi e^+e^-$ inspire many theoretical and experimental studies due to possibility to observe CP violation \cite{1, 2, 3, 4}. There are following possible decays: $K^+ \to \pi^+e^+e^-$, $K_S \to \pi^0e^+e^-$ and $K_L \to \pi^0e^+e^-$. The CP conserving $K^+ \to \pi^+e^+e^-$ and $K_S \to \pi^0e^+e^-$ are dominated by virtual photon exchange. The amplitudes of order $O(p^2)$ vanish at the tree level \cite{2} and therefore the leading amplitudes for these transitions are of $O(p^4)$ order in the chiral perturbation theory (CHPT). At this order there are both one-loop contributions and tree level contributions. The one-loop contribution was determined by G. Ecker, A. Pich and E. de Rafael \cite{1, 2, 3, 4}, but the tree level contribution (or better the counterterm contribution) motivates many studies \cite{5, 6}. The decay $K_L \to \pi^0e^+e^-$ proceeding via virtual $\gamma^*$ is forbidden in the limit of CP conservation. The CP conserving process proceeds through two photon exchanges. A CP violating term is proportional to the known $\epsilon$ parameter \cite{3}. In addition, there is a direct $\Delta S = 1$ CP violating effect \cite{3, 7}. The amplitudes for processes $K^+ \to \pi^+\gamma\gamma$ and $K_S \to \pi^0\gamma\gamma$ involve the same counterterm couplings as processes with one virtual photon exchange. The effective theory contains a large number of unknown parameters. The phenomenological parameters appearing in the strong sector can all be determined, while many of the weak couplings cannot be fixed by experiment.

The authors of ref. \cite{8} have considered the resonance contribution to the coupling constants of the $O(p^4)$ effective chiral Lagrangian. They find clear evidence for the importance of vector meson contributions, which account
for the bulk of the low-energy coupling constants. The same idea has been
questioned in the weak interactions sector \([9, 10, 11, 12, 13]\). The authors in
\([9, 12, 13]\) have studied the most general case of the \(SU(3) \times SU(3)\)
invariant chiral Lagrangian using symmetry principles only, and they found that
there appear 37 independent terms. These couplings cannot be determined
by present experiments. Therefore, additional assumptions are needed to
describe the weak interactions. There are two procedures available:
a) the “weak deformation model”
b) the “factorization model”.
Both models can be formulated without any reference to resonances. Be-
cause the strong couplings of the \(O(p^4)\) order in the chiral Lagrangian seem
to be saturated by resonance exchange \([8]\), it seems natural that the weak
interactions at \(O(p^4)\) order in the chiral Lagrangian might be explained by
resonance contributions.
The purpose of this paper is to clarify the role of resonances in the counter-
terms for \(K \rightarrow \pi e^+ e^-\) decays.

The paper is organized as follows: In Sect. 2 we shall repeat the general re-
results for \(K \rightarrow \pi \gamma^*\) and \(K \rightarrow \pi \gamma \gamma\) amplitudes and we explain the electroweak
counterterm-couplings by resonance exchange. In Sect. 3 we shall analyse
these terms using ”factorization model”, ”weak deformation models” and
we shall apply the extended Nambu and Jona-Lasinio model for the couplings
in the strong chiral Lagrangian of \(O(p^4)\) order. In Sect. 4 the large \(N_c\) model
is combined with the resonance saturation. The \(K_L \rightarrow \pi^0 e^+ e^-\) decay rate is
briefly analysed in Sect. 5. In Sect. 6 we give summary of our results.
2 $K \to \pi \gamma^*$ and $K \to \pi \gamma \gamma$ decays in CHPT

It was shown that in the chiral perturbation theory at $O(p^2)$ $K \to \pi \gamma^*$ transitions are forbidden for a virtual photon $\gamma^*(q)$ for any value of $q^2$ \[1\]. Combining the contributions coming from one-loop and the counterterms, induced by strong, weak and electromagnetic interactions, \[2, 3\], the amplitudes for $K^+ \to \pi^+ \gamma^*$ and $K_S \to \pi^0 \gamma^*$ can be written as

\[ A(K^+ \to \pi^+ \gamma^*) = \frac{G_8 e}{(4\pi)^2} q^2 \Phi_+(q^2) \epsilon^\mu (p' + p)_\mu \] (1)

\[ A(K_S^0 \to \pi^0 \gamma^*) = \frac{G_8 e}{(4\pi)^2} q^2 \Phi_S(q^2) \epsilon^\mu (p' + p)_\mu \] (2)

where $p$ and $p'$ are pion's and kaon's momenta, $G_8 = \sqrt{\frac{1}{2}} G_F s_1 c_1 c_3 g_8$ is defined in \[1\] and from $K \to \pi \pi$ it was found that $|g_8| = 5.1$. The calculated one-loop correction reduces this value to $|g_8^{\text{loop}}| = 4.3$ \[5, 13\]. The factors $\Phi_{+,S}$ are split into one-loop and counterterm contributions

\[ \Phi_+ = W_+ + \Phi_K + \Phi_\pi \] (3)

\[ \Phi_S = W_S + 2\Phi_K \] (4)

The loop contributions $\Phi_K$ and $\Phi_\pi$ are determined in \[1, 2, 3\]. The $W_{+,S}$ have been defined as \[2, 3, 4\]

\[ W_+ = -\frac{16}{3} \pi^2 [W_1^r + 2W_2^r - 12L_9^r(\mu)] + \frac{1}{3} \log \frac{\mu^2}{M_K M_\pi} \] (5)
\[
W_S = \frac{-16}{3} \pi^2 [W_1 - W_2] + \frac{1}{3} \log \frac{\mu^2}{M_K^2}
\] (6)

In these equations \( L_9(\mu) \) is a coupling constant at \( O(p^4) \) strong Lagrangian defined in [8, 14, 15]

\[
\mathcal{L}_4 = \ldots - iTr L_9(f^{\alpha\beta}_+ u_{\alpha} u_{\beta}) + \frac{1}{4}(L_{10} + 2H_1)Tr(f_{+\alpha\beta} f^{\alpha\beta}_+) \\
- \frac{1}{4}(L_{10} - 2H_1)Tr(f_{-\alpha\beta} f^{\alpha\beta}_-)
\] (7)

with \( u^2 = U = exp(\frac{i}{4} \sum \lambda_i \phi_i) \) and \( f \simeq f_\pi = 0.0933 GeV \). The notation here is defined below:

\[
f^{\alpha\beta}_+ = u F^{\alpha\beta}_L u^\dagger \pm u^\dagger F^{\alpha\beta}_R u
\] (8)

\[
F^{\alpha\beta}_L = \partial^\alpha l^\beta - \partial^\beta l^\alpha - i[l^\alpha, l^\beta]
\] (9)

\[
F^{\alpha\beta}_R = \partial^\alpha r^\beta - \partial^\beta r^\alpha - i[r^\alpha, r^\beta]
\] (10)

\[
u = iu^\dagger D_\alpha U u^\dagger
\] (11)

\[
D_\alpha = \partial_\alpha U - ir_\alpha U + iU l_\alpha
\] (12)

and \( l_\alpha \) and \( r_\alpha \) denote left and right matrix external fields, with spin 1. In the presence of the external electromagnetic fields only \( l_\alpha = r_\alpha = |e|QA_\alpha \). The coupling constant \( L_9 \) is connected with the renormalized \( L^r_9 \) as

\[
L_9 = L^r_9 + \Gamma_9 \frac{\rho^\nu}{(4\pi)^2 \epsilon}, \quad \Gamma_9 = \frac{1}{4}
\] (13)
In this paper we choose $\nu = m_\rho$ for the renormalization scale $\nu$. In a similar way $W_1$ and $W_2$ counterterms of weak and electromagnetic origin can be written in the form \[ W_1 = W_1^r + \Gamma_9 \frac{\nu - \epsilon}{(4\pi)^2 \epsilon}, \quad \Gamma_9 = \frac{1}{4} \] (14)

In ref. \cite{2, 3} it was found that in the case of $K \to \pi \gamma^*$ and $K \to \pi \gamma \gamma$ decays there are three relevant local counterterms:

\[
L^{\Delta S = 1, \text{em}}_4 = \frac{ieG_f^2}{2} F^{\mu \nu} [W_1 Tr(Q \lambda_{6-i7} U^\dagger D_\mu U U^\dagger D_\nu U) \\
+ W_4 Tr(Q U^\dagger D_\mu U \lambda_{6-i7} U^\dagger D_\nu U)] \\
+ \frac{e^2 f^2}{2} G_8 F^{\mu \nu} F_{\mu \nu} W_4 Tr(\lambda_{6-i7} Q U^\dagger Q U) + h.c. \] (15)

The analysis of $K^+ \to \pi^+ \gamma \gamma$ decay shows that there is the following combination of coupling constants

\[
\hat{c} = 32\pi^2 [4(L_9 + L_{10}) - \frac{1}{3}(W_1 + 2W_2 + 2W_4)]. \] (16)

The combination $L_9 + L_{10}$ and the weak contribution $W_1 + 2W_2 + 2W_4$ are a renormalization scale invariant \cite{3, 14}. The loop contribution is finite in $K \to \pi \gamma \gamma$ decays and for $K^0 \to \pi^0 \gamma \gamma$ decay $O(p^4)$ counterterms contribution vanishes.

In order to determine the resonance saturation of the counterterm couplings in the chiral Lagrangian at $O(p^4)$, the lowest order couplings in the chiral expansion are needed. All these couplings are of $O(p^2)$, and resonance exchange will automatically produce contributions of $O(p^4)$. Using the equations of motion for resonances, the authors of ref. \cite{8} have found that $L_9, L_{10}$
and $H_1$ get contributions from vector and axial-vector resonances. They are

$$L_9^V = \frac{F_V G_V}{2M_V^2}, \quad L_{10}^V = -\frac{F_V^2}{4M_V^2}, \quad \quad H_1^V = -\frac{F_V^2}{8M_V^2}$$

(17)

$$L_9^A = 0, \quad L_{10}^A = \frac{F_A^2}{4M_A^2}, \quad H_1^V = -\frac{F_A^2}{8M_A^2}$$

(18)

where $M_V$ and $M_A$ are the octet masses of vector and axial-vector mesons. The octet couplings $F_V$ and $G_V$ can in principle be determined from the decay rates $\rho \rightarrow l^+l^-$ and $\rho \rightarrow 2\pi$, respectively, and they are

$$|F_V| = 0.154 \text{ GeV}, \quad |G_V| = 0.069 \text{ GeV}.$$  

(19)

For the determination of $F_A$ and the octet mass $M_A$ in the chiral limit the authors of [8] have used Weinberg's sum rules [18]. These two sum rules connect axial vector meson with vector meson parameters,

$$F_V^2 = F_A^2 + f^2$$

(20)

$$M_A^2 F_V^2 = M_A^2 F_A^2,$$  

(21)

$$F_A^2 = 0.128 \text{ GeV}$$

(22)

$$M_A^2 = 0.968 \text{ GeV}$$

(23)

The relevant $O(p^4)$ couplings $L_9$, $L_{10}$ and $H_1$ are in general divergent, like the rest of $O(p^4)$ couplings. They depend on the renormalization scale $\nu$ which is not seen in the observable quantities. The results we use, like [8, 14, 15] are obtained at $\nu = M_\rho$.

$$L_9^\nu = 6.9 \times 10^{-3}$$

(24)
\[ L_{10}^r = -6.0 \times 10^{-3} \]  
(25)
\[ H_1^r = 7.0 \times 10^{-3} \]  
(26)

The \( H_1^r \) is not accessible experimentally, but vector and axial vector mesons determine this coupling at scale \( \nu = M_\rho \) and, once the form of strong chiral Lagrangian at \( O(p^4) \) is chosen in the form \([8, 14, 15]\), this term is fixed by resonance exchange.

3 Counterterms and models for the weak Lagrangian at \( O(p^4) \) order

The weak deformation model and the factorization model are used in the literature \([3, 9]\) towards constructing the weak Lagrangian of \( O(p^4) \) order. Both models rest on the assumption that the strong chiral Lagrangian already determines the dominant features of \( \Delta S = 1 \) effective Lagrangian. It is obvious that such a procedure cannot be expected to yield the complete weak Lagrangian, since the short-distance contribution has no equivalence in the strong sector. One might expect that the dominant long-distance contribution results from resonances exchange like in the strong sector. We remark that there are two possible ways to obtain the resonance contribution to \( L_w \): One might first calculate the resonance contributions to the strong Lagrangian and then use a weak deformation or factorization procedure. Or, one might apply the weak deformation or factorization to the strong resonance Lagrangian of lowest order first and then integrate out the resonances, using their equations of motion. If the second approach is used, the resonance contribution recognized in \( H_1 \) counterterm of the Lagrangian \([7]\), is undoubtfully present.
a) Weak deformation model

This model is inspired by the geometry of the coset space $G/SU(3)_V$ [4]. It can easily be obtained starting with the strong Lagrangian of $O(p^2)$

$$\mathcal{L}_2 = \frac{f^2}{4} Tr(u_\mu u^\mu), \quad (27)$$

and making the replacement

$$u_\mu \rightarrow u_\mu + G_8 f^2 \{u_\mu, \Delta\} - \frac{2}{3} G_8 f^2 Tr(u_\mu \Delta), \quad (28)$$

where

$$\Delta = u_\lambda u^\dagger. \quad (29)$$

It is useful to introduce two forms

$$l_\mu = u[\partial - i(v_\mu - a_\mu)]u^\dagger = \Gamma_\mu + \frac{1}{2} i u_\mu \quad (30)$$

$$r_\mu = u[\partial - i(v_\mu + a_\mu)]u^\dagger = \Gamma_\mu - \frac{1}{2} i u_\mu. \quad (31)$$

In this model $\Gamma_\mu$ is deformed to

$$\Gamma_\mu \rightarrow \Gamma_\mu + \frac{1}{2} i G_8 f^2 \{u_\mu, \Delta\} - \frac{1}{3} i G_8 f^2 Tr(u_\mu \Delta) \quad (32)$$

After performing the weak deformation on $\mathcal{L}_4$ in (7) the counterterm coupling defined in (15) are found to be

$$W^r_1 = 4(L^r_9 + L^r_{10} + 2 H^r_1) \quad (33)$$
With the numerical results for $L_9^r$, $L_{10}^r$ and $H_1^r$ from (24), (25) and (26) we find

\begin{align*}
W_{1WDM}^{WDM} &= -0.0524 \quad (36) \\
W_{2WDM}^{WDM} &= 0.0276 \quad (37) \\
W_{4WDM}^{WDM} &= 0.004 \quad (38)
\end{align*}

what leads, using (5) and (6), to $W_+^{WDM} = -5.01$ $W_-^{WDM} = 4.50$. Among three possible decays only the decay rate of $K^+ \rightarrow \pi^+e^+e^-$ has been observed. The experimental bound for the branching ratio $K^+ \rightarrow \pi^+e^+e^-$ is obtained from Brookhaven experiment [19]

\[ BR(K^+ \rightarrow \pi^+e^+e^-) = (2.99 \pm 0.22) \times 10^{-7}, \quad (39) \]

resulting in bounds $W_+^{WDM} = 0.89^{+0.24}_{-0.14}$. It is obvious that the numerical value obtained by using the weak deformation model does not explain this experimental limit.

In the case of $K^+ \rightarrow \pi^+\gamma\gamma$ the counterterm coupling defined in (16) is $\delta^{WDM} = 0$. In ref. [3] the relation (34) was criticized as a result of assumption which is not part of the CHPT. We point out that this result originally was derived using the octet dominance hypothesis. Namely, the authors of ref.
have noticed that the one-loop amplitudes for $K^+ \to \pi^+ \gamma^*$, $K^0 \to \pi^0 \gamma^*$ and $\eta \to \bar{K}^0 \gamma^*$, if $m_K = m_\pi$, satisfy the relation

$$A(K^+ \to \pi^+ \gamma^*)|_l = -\sqrt{2}A(K^0 \to \pi^0 \gamma^*)|_l = \sqrt{2} \frac{2}{3}A(\eta \to \bar{K}^0 \gamma^*)|_l.$$ (40)

This relation restricts two pseudoscalars to be in the pure octet in the $SU(3)$ limit. We find that without $SU(3)$ symmetry these amplitudes satisfy

$$A(K^+ \to \pi^+ \gamma^*)|_l = -\frac{1}{\sqrt{2}}(A(K^0 \to \pi^0 \gamma^*)|_l + \sqrt{1} \frac{1}{3}A(\eta \to \bar{K}^0 \gamma^*)|_l).$$ (41)

Generally, as it was noticed in ref. [1], it means that two pseudoscalars can be in state of representation 8, 10, and $\bar{10}$ of $SU(3)$, and with the help of Clebsch-Gordon coefficients for $8 \times 10$ of $SU(3)$ group it can be seen that decuplet components cancel out in (41). Imposing (40), if $SU(3)$ limit holds, or (41) when $SU(3)$ is broken, on the amplitudes containing the weak counterterms, we find that relation (34) exists in both cases, with and without $SU(3)$ limit. Although our result for $W_+$ disagrees with the experimental bound, we cannot rule out the weak deformation model. The model itself contains very important feature: the relations (33), (34) and (35) are scale independent [4] and further study of this model would be useful.

b) Factorization model

The effective $\Delta S = 1$ weak Hamiltonian is given by

$$H_{eff} = \frac{G}{\sqrt{2}}C_i(\mu^2)Q_i + h.c.$$ (42)
where $Q_i$ are four-quark operators and $C_i(\mu^2)$ are Wilson coefficients depending on the QCD renormalization scale $\mu$. To lowest order in CHPT the left-current is realized as

$$J_{\mu}^{(1)} = \frac{\delta S_2}{\delta l_{\mu}},$$  \hspace{1cm} (43)$$

where $S_2$ is the action determined by $L_2$, what gives

$$J_{\mu}^{(1)} = \frac{f^2}{2} u^\dagger u_{\mu} u$$

$$= -i \frac{f^2}{2} \{U^\dagger \partial_{\mu} U + ieA_{\mu} U^\dagger [U, Q]\}. \hspace{1cm} (44)$$

To $O(p^4)$, the factorization model is defined for the dominant octet part of the weak Lagrangian by

$$L_{W^4}^{FM} = 4g_8 Tr(\lambda_6 - i7 \{J_{\mu}^{(1)}, J_{\mu}^{(3)}\}), \hspace{1cm} (45)$$

where $J_{\mu}^{(3)}$ is determined from

$$J_{\mu}^{(3)} = \frac{\delta S_4}{\delta l_{\mu}}. \hspace{1cm} (46)$$

In our case it leads to

$$J_{\mu}^{(3)} = -L_9 \{eF_{\mu\nu}(L_{\nu}, U^\dagger QU) + [L_{\nu}, Q]\}$$

$$+ iL_{9\nu} \partial_{\nu} \{[L_{\mu}, L_{\nu}] + ieA_{\mu} (-[L_{\nu}, Q] + [L_{\nu}, U^\dagger QU])\}$$

$$+ 2L_{10}\partial_{\nu}(eF_{\mu\nu} U^\dagger QU) + 4H_1 \partial_{\nu}(eF_{\mu\nu} Q) \hspace{1cm} (47)$$

where $L_{\mu} = U^\dagger \partial_{\mu} U$. Using these two currents, we easily find

$$W_1^r = 8(L_9^r + L_{10}^r + 2H_1^r) \hspace{1cm} (48)$$

$$W_2^r = 8L_9^r \hspace{1cm} (49)$$
\[ W_4^r = 8(L_{10}^r - H_1^r) \] (50)

With the help of the numerical results for \( L_9^r \), \( L_{10}^r \) and \( H_1^r \) from (24), (25) and (26) we derive

\[ W_{FM}^1 = -0.1048 \] (51)

\[ W_{FM}^2 = 0.0552 \] (52)

\[ W_{FM}^4 = 0.008, \] (53)

what results in \( W_{FM}^+ = -4.87 \) and \( W_{FS}^F = 8.67 \). The result for \( W_+ \), like the one calculated in the case of the weak deformation model, disagrees with the experimental one. The combination of the counterterm couplings derived in (16) gives \( \hat{c}_{FM} = -1.14 \). At present there is only an upper bound for the branching ratio of this process \( BR(K^+ \rightarrow \pi^+ \gamma \gamma) < 1 \cdot 10^{-6} \) \([21]\). This upper bound and the analysis of ref. \([3]\) lead to the limits \(-7.02 < \hat{c} < 1.89\) and the obtained value for \( \hat{c}_{FM} = -1.14 \) is allowed by these bounds.

Now, we search for the model which might accommodate this experimental result and we apply the extended Nambu and Jona-Lasinio model described in ref. \([20]\). In this reference, the low-energy effective action of an extended Nambu and Jona-Lasinio model to \( O(p^4) \) in the chiral counting is derived. The couplings of our interest are \( L_9 \), \( L_{10} \) and \( H_1 \). It was found \([21]\) that these couplings can be expressed as

\[ L_9 = \frac{N_c}{16\pi^2} \frac{1}{6} \left[ (1 - g_A^2) \Gamma_0 (1 + \gamma_{03}) + 2g_A^2 \Gamma_1 (1 + \frac{3}{2} \gamma_{12} - \frac{1}{2} \gamma_{13}) \right] \] (54)
\begin{align*}
L_{10} &= -\frac{N_c}{16\pi^2} \frac{1}{6} [(1 - g_A^2)\Gamma_0(1 + \gamma_{03}) + g_A^2\Gamma_1(1 + \gamma_{13})] \\
H_1 &= -\frac{N_c}{16\pi^2} \frac{1}{12} [(1 - g_A^2)\Gamma_0(1 + \gamma_{03}) - g_A^2\Gamma_1(1 + \gamma_{13})].
\end{align*}

In these equations we use

\begin{align*}
\gamma_{03} &= -\frac{\Gamma(2, x)}{\Gamma(0, x)} \frac{3}{5} g \\
\gamma_{12} &= \frac{\Gamma(3, x)}{\Gamma(1, x)} \frac{1}{5} g \\
\gamma_{13} &= -\frac{\Gamma(3, x)}{\Gamma(1, x)} \frac{3}{5} g,
\end{align*}

where \( \Gamma(n, x) \) denotes the incomplete gamma function

\[ \Gamma(n - 2, x = \frac{M_Q^2}{\lambda^2}, x) = \int_{\frac{M_Q^2}{\lambda^2}}^{\infty} \frac{dz}{z} e^{-z} z^{n-2}, \quad n = 1, 2, 3, \ldots, \]

\( g_A \) can be identified as a constant of the constituent chiral quark model and \( g \) might be connected to the gluon vacuum condensate \cite{20}, \( \lambda \) is a cut-off scale, \( M_Q \) is the constituent chiral quark-mass. For the most favorable set of parameters \( g_A = 0.61, M_Q = 0.265 GeV \) and \( \lambda = 1.165 GeV \) \cite{20}, it is easy to calculate \( L_0^r = 7.0 \cdot 10^{-3}, L_{10}^r = -5.9 \cdot 10^{-3} \) and \( H_1^r = -4.7 \cdot 10^{-3} \). These values lead to

\[ W_{1}^{WDM} = -0.0327 \]

\[ W_{2}^{WDM} = 0.028 \]
\[ W_{4}^{W_{DM}} = -0.0049 \]  \hspace{1cm} (63)

and consequently \( W_{+}^{W_{DM}} = 3.91 \), \( W_{S}^{W_{DM}} = 6.69 \) and \( \hat{c}^{W_{DM}} = 0 \). For the factorization model the values of \( W_{i} \) are doubled and physically relevant values are \( W_{+}^{FM} = 2.69 \), \( W_{S}^{FM} = 6.69 \) and \( \hat{c}^{FM} = -1.41 \). The numerical values of \( W_{i} \) are calculated at the scale \( \nu = m_{\rho} \). In Table 1 we present \( W_{+}, W_{S} \) and \( \hat{c} \) counterterms calculated for some of the values of the scale \( \nu \). Analysing fits from ref. [20] we find that their fit 4 gives most favorable value of the \( W_{+} \). In this fit there is no explicit vector (axial-vector) degree of freedom. Using this fit, it was found \( L^{r}_{9} = 5.8 \cdot 10^{-3} \), \( L^{r}_{10} = -5.1 \cdot 10^{-3} \) and \( H^{r}_{1} = -2.4 \cdot 10^{-3} \). These values leads to

\[ W_{1}^{FM} = -0.0328 \]  \hspace{1cm} (64)

\[ W_{2}^{FM} = 0.0464 \]  \hspace{1cm} (65)

\[ W_{4}^{FM} = -0.0216, \]  \hspace{1cm} (66)

resulting in \( W_{+}^{FM} = 1.22 \), \( W_{S}^{FM} = 4.46 \) and \( \hat{c}^{FM} = -0.88 \) for the scale \( \nu = m_{\rho} \). For the scale \( \nu = 0.265 \) GeV these values are \( W_{+}^{FM} = 0.51 \), \( W_{S}^{FM} = 3.75 \) and \( \hat{c}^{FM} = -0.88 \). The strenght of the counterterm coupling \( \hat{c}^{FM} = -0.88 \) is within the experimental limits. We can conclude that the factorization model combined with the parameters used in the extended Nambu and Jona-Lasinio model can accommodate the experimentally measured decay rate \( BR(K^{+} \rightarrow \pi^{+}e^{+}e^{-}) \) and the bounds on counterterm coupling \( \hat{c} \) obtained from \( BR(K^{+} \rightarrow \pi^{+}\gamma\gamma) \) decay rate.

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4 Large $N_c$ limit and counterterms

Using the large $N_c$ limit, $\Delta S = 1$ effective Hamiltonian is given by [16, 17]

$$H_{\Delta S=1}^{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* Q_2 + \text{h.c.}$$  \hspace{1cm} (67)

where $Q_2 = 4(\bar{s}_L \gamma_\mu u_L)(\bar{u}_L \gamma_\mu d_L)$. The full effective Hamiltonian at the leading order of large $N_c$ is

$$H_{\Delta S=1}^{\text{eff}} = 4 \left(G_8 \left(\frac{1}{2}\right) H_{8}^{\left(\frac{1}{2}\right)} + G_{27} \left(\frac{1}{2}\right) H_{27}^{\left(\frac{1}{2}\right)} + G_{27} \left(\frac{3}{2}\right) H_{27}^{\left(\frac{3}{2}\right)}\right),$$  \hspace{1cm} (68)

where in the strict large-$N_c$ $g_8 \left(\frac{1}{2}\right) |_{N_c \to \infty} = \frac{3}{5}$, $g_{27} \left(\frac{1}{2}\right) |_{N_c \to \infty} = \frac{1}{15}$, $g_{27} \left(\frac{3}{2}\right) |_{N_c \to \infty} = \frac{1}{3}$.

In this expression

$$H_{27} = -\frac{2}{3} (L_\mu)_{21} (L^\mu)_{13} - (L_\mu)_{23} (L^\mu)_{11}. \hspace{1cm} (69)$$

$H_{27}$ induces both $|\Delta I| = \frac{1}{2}$ and $|\Delta I| = \frac{3}{2}$ transitions via its components

$$H_{27} = \frac{1}{9} H_{27}^{\left(\frac{1}{2}\right)} + \frac{5}{9} H_{27}^{\left(\frac{3}{2}\right)}. \hspace{1cm} (70)$$

In our calculation we kept both components of $Q_2$ with $|\Delta I| = \frac{1}{2}$ and $|\Delta I| = \frac{3}{2}$, the octet and twenty-sevenplet. With the use of the expressions (43) and (47) we derive

$$A(K^+ \to \pi^+ \gamma^*) = \sqrt{\frac{1}{2}} G_F s_1 c_1 c_3 4 L_9 q^2 e^\mu (p' + p)_\mu$$  \hspace{1cm} (71)

$$A(K^0 \to \pi^0 \gamma^*) = 0. \hspace{1cm} (72)$$

For the $K^+ \to \pi^+ \gamma \gamma$ decay we obtain

$$G_8 \hat{c} = -32 \pi^2 \sqrt{\frac{1}{2}} G_F s_1 c_1 c_3 4 (L_9 + L_{10}). \hspace{1cm} (73)$$

15
The authors of ref. [16] have calculated the octet component of the isospin $\frac{1}{2}$ effective Hamiltonian. In the large $N_c$ limit the $G_8$ coupling is $g_8 = \frac{3}{5}$ (to be compared with the experimental value 5.1). They found that the factorization of the $G_8 W_i$, where $i = 1, 2, 4$, calculated at order $O(p^4)$ of the chiral Lagrangian is not valid at order $N_c$ in the $\frac{1}{N_c}$ expansion. We confirm their result

\begin{equation}
W_1 = W_2
\end{equation}

\begin{equation}
G_8 W_2 = 8 G_8 |_{\frac{1}{N_c} L_9}
\end{equation}

\begin{equation}
G_8 W_4 = 12 G_8 |_{\frac{1}{N_c} L_{10}}
\end{equation}

where the subscript $\frac{1}{N_c}$ explains that the given coupling constant is determined at the leading order of the $\frac{1}{N_c}$ expansion. We derive numerical results, taking into account the values of $L_9$ and $L_{10}$ when they are dominated by resonance exchange (24) and (25). They correspond to the expressions of $G_8 W_i$

\begin{align*}
G_8 W_1 & \rightarrow \sqrt{\frac{1}{2}} G_{FS_1 c_1 c_3} 0.0552 \\
G_8 W_2 & \rightarrow \sqrt{\frac{1}{2}} G_{FS_1 c_1 c_3} 0.0552 \\
G_8 W_4 & \rightarrow -\sqrt{\frac{1}{2}} G_{FS_1 c_1 c_3} 0.072
\end{align*}

or $G_8 W_S \rightarrow -\sqrt{\frac{1}{2}} G_{FS_1 c_1 c_3} 0.237$, and $G_8 W_+ \rightarrow -\sqrt{\frac{1}{2}} G_{FS_1 c_1 c_3} 0.737$. The result corresponding to $G_8 W_+$ is quite close to the lower experimental bound.
The result which corresponds to \( \sqrt{2} G_F s_1 c_1 c_3 \hat{c} \) is also within the experimental bound \( G_8 \hat{c} = -\sqrt{\frac{2}{2}} G_F s_1 c_1 c_3 1.01 \). The analysis of ref. [10] contains the effective the Hamiltonian calculated at the order \( p^4 \) in the chiral expansion and at the order \( N_c \) in the \( \frac{1}{N_c} \)-expansion. Their result leads to the conclusion that the factorization in \( G_8 W_i \) is not valid when next-to-leading corrections in \( \frac{1}{N_c} \) are calculated. Here, we have modified the leading term in \( \frac{1}{N_c} \)-expansion taking into account the contribution of 27 component of the \( Q_2 \) operator. Results are in slightly better agreement (factor of \( \frac{5}{3} \) increase) than the ones derived in [10] for their leading term in \( \frac{1}{N_c} \)-expansion. It is interesting in this approach the counterterm contributions do not depend on \( H_1 \) when calculated at the leading order in \( \frac{1}{N_c} \).

5 \( K_L \to \pi^0 e^+ e^- \) and CP violation

The decay \( K_L \to \pi^0 e^+ e^- \) is being investigated as a signal of direct \( \Delta S = 1 \) CP violation. In addition to a CP conserving process, which proceeds through two photon exchanges, there are two kinds of the CP violating decay: one proportional to the well known parameter \( \epsilon \) and the other direct CP violating effect. The direct CP violating component is the simplest. Uncertainties are coming from poor knowledge of the Standard model parameters, top quark mass and CKM matrix elements. The QCD short distance corrections to this mode have been analyzed to next-to-leading order by Buras et al. [7]. The weak operator responsible for this decay is

\[
\mathcal{H}_w = \frac{G_F}{\sqrt{2}} [C_{7V} + C_{7A}],
\]  

(80)
where

\[ Q_{7V} = (s\bar{d})_{V-A}(\bar{e}e)_V \]

(81)

\[ Q_{7A} = (s\bar{d})_{V-A}(\bar{e}e)_A. \]

(82)

The amplitude for \( K_L \rightarrow \pi^0 e^+ e^- \) can be written as

\[ A(K_L \rightarrow \pi^0 e^+ e^-) = -\frac{G_8 \alpha}{4\pi}(p + p')^\mu \bar{u}[\phi_V \gamma^\mu + \phi_A \gamma^\mu \gamma_5]v \]

(83)

with

\[ \phi_V = \epsilon \phi_S - \frac{16\pi^2}{3} i \text{Im} W_1 \]

(84)

and

\[ \phi_A = \frac{16\pi^2}{3} i \text{Im} W_5. \]

(85)

\( W_5 \) is defined by the effective Lagrangian counterterm

\[ \mathcal{L}'_w = -\frac{i2\pi\alpha}{3} G_8 W_5 e^\gamma^\mu \gamma_5 e Tr(\lambda_6 - i\tau)(\partial U - ie[A_\mu, U])U^\dagger. \]

(86)

Taking the \( \phi_V \) and \( \phi_A \) as in

\[ \phi_V = e^{i\pi/4}(0.57 \times 10^{-3}) - i 1.0 \times 10^{-3} \]

(87)

and

\[ \phi_A = i 1.0 \times 10^{-3} \]

(88)

we calculate the branching ratio \( BR(K_L \rightarrow \pi^0 e^+ e^-) \) for the counterterms couplings. For \( W_+ = 1.22 \) and \( W_S = 4.46 \), obtained in the case of the extended Nambu and Jona-Lasinio model, we find \( BR(K_L \rightarrow \pi^0 e^+ e^-) = \)
1.15 \cdot 10^{-10} \text{ and for } W_{s} = 0.51 \text{ and } W_{s} = 3.75 \text{ the decay rate is } BR(K_{L} \rightarrow \pi^{0}e^{+}e^{-}) = 7.77 \cdot 10^{-11}. \text{ From the large } N_{c} \text{ approach to the CP violating process we get } BR(K_{L} \rightarrow \pi^{0}e^{+}e^{-}) = 4.79 \cdot 10^{-9}. \text{ Experimentally was determined only the upper limit } BR(K_{L} \rightarrow \pi^{0}e^{+}e^{-}) \leq 4.3 \cdot 10^{-9}, \text{ a result from } [22], \text{ and } BR(K_{L} \rightarrow \pi^{0}e^{+}e^{-}) \leq 4.3 \cdot 10^{-11} \text{ obtained by } [23]. \text{ From our analysis it follows that indirect CP violation is rather dependent on the } W_{s} \text{ counterterm coupling. It was noticed } [4] \text{ that it is very difficult to distinguish between direct and indirect CP violation. These authors point out that the existence of direct CP violating term in the branching ratio } BR(K_{L} \rightarrow \pi^{0}e^{+}e^{-}) \text{ can not be seen without observing branching ratio } BR(K_{S} \rightarrow \pi^{0}e^{+}e^{-}). \text{ The CP conserving process proceeds via two photon exchange. The contribution of vector and scalar mesons to } K_{L} \rightarrow \pi^{0}\gamma\gamma \text{ is completely determined to } O(p^{6}) [4, 24]. \text{ The authors of ref. } [5] \text{ made a fit to the } \gamma\gamma \text{ spectrum in } K_{L} \rightarrow \pi^{0}\gamma\gamma \text{ in order to fix the parameter describing the role of vector mesons at this order. They obtain rather large value of the relevant parameter } a_{V}, \text{ comparing the results when the weak deformation model was used like in } [4, 24].

6 Summary

We have investigated counterterm couplings required by CHPT in the decays \( K^{+} \rightarrow \pi^{+}e^{+}e^{-}, \quad K_{s} \rightarrow \pi^{0}e^{+}e^{-} \) and \( K_{L} \rightarrow \pi^{0}e^{+}e^{-} \). The counterterms induced by the strong, weak and electromagnetic interactions have been fixed by resonance exchange.

- In our approach the weak deformation model, used for the weak interac-
tions, cannot reproduce the experimental result for $K^+ \to \pi^+e^+e^-$ decay rate.

- The branching ratio $BR(K^+ \to \pi^+e^+e^-)$ and $\hat{c}$ defined in $(K^+ \to \pi^+\gamma\gamma)$ decay amplitude, are in agreement with the experimental results, when the factorization model combined with the set of the parameters determined in the extended Nambu and Jona-Lasinio model is applied.

- The counterterm couplings depend on a physically unobservable coupling $H_1$. At the leading order in large $N_c$ there is no dependence on the $H_1$ coupling.

- The $BR(K_L^0 \to \pi^0e^+e^-)$ can be predicted within these approaches. The decay rate is very parameter dependent.

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Table 1: The counterterms $W_+, W_S$ calculated using the extended Nambu and Jona-Lasinio model for these values of the renormalization scale. The first three columns contain the values in the case of weak deformation model and in the last three columns there are values obtained in the case of factorization model.

| $\nu$ | $m_\rho$ | 0.265GeV | 1.165GeV | $m_\rho$ | 0.265GeV | 1.165GeV |
|-------|----------|----------|----------|----------|----------|----------|
| $W_+$ | 3.91     | 3.20     | 4.19     | 2.69     | 1.96     | 4.19     |
| $W_S$ | 3.49     | 2.78     | 3.77     | 6.69     | 5.98     | 6.96     |
| $c$   | 0        | 0        | 0        | $-1.41$  | $-1.41$  | $-1.41$  |
References

[1] G. Ecker, A. Pich and E. de Rafael, Phys. Lett. B 189 (1987) 363.

[2] G. Ecker, A. Pich and E. de Rafael, Nucl. Phys. B 291 (1987) 692.

[3] G. Ecker, A. Pich and E. de Rafael, Nucl. Phys. B 303 (1988) 665.

[4] G. Ecker, A. Pich and E. de Rafael, Phys. Lett. B 237 (1990) 481.

[5] John F. Donoghue and F. Gabbiani, Phys. Rev D 51 (1995) 2187.

[6] G. D’Ambrossio, G. Ecker, G. Isidori and H. Neufeld, preprint IFNNA-IV-94/24, DSF-94/24 UWThPh-1994-20, Oct. 1994.

[7] A. J. Buras, M. E. Lautenbacher, M. Misiak and M. Münz, Nucl. Phys. B 423 (1994) 349.

[8] G. Ecker, J. Gasser, A. Pich and E. de Rafael, Nucl. Phys. B 321 (1989) 311.

[9] G. Ecker, J. Kambor and D. Wyler, Nucl. Phys. B 394 (1993) 101.

[10] G. Isidori and A. Pugliese, Nucl. Phys. B 385 (1993) 437.

[11] S. Fajfer, Z. Phys C 61 (1994) 645.

[12] J. Kambor, J. Missimer and D. Wyler, Nucl. Phys. B 346 (1990) 17.

[13] J. Kambor, J. Missimer and D. Wyler, Phys. Lett. B 261 (1991) 496.

[14] J. Gasser and H. Leutwyler, Nucl. Phys. B 321 (1985) 465, 517.
[15] J. Bijnens and F. Cornet, Nucl. Phys. B296 (1986) 557.

[16] C. Bruno and J. Prades, Z. Phys. C 57 (1993) 585.

[17] A. Pich, E. de Rafael, Nucl. Phys. B358 (1991) 341.

[18] S. Weinberg, Phys. Rev. Lett. 18 (1967) 507.

[19] C. Alliegro et al., Phys. Rev. Lett. 68 (1992) 278.

[20] J. Bijnens, C. Bruno and E. de Rafael, Nucl. Phys. B390 (1993) 501.

[21] M. S. Atiya et al., (BNL-E787), Phys. Lett. 65 (1990) 1188.

[22] D. A. Harrris et al., Phys. Rev. Lett. 71 (1993) 3918.

[23] K. E. Ohl et al., Phys. Rev. Lett. 64 (1990) 2755.

[24] S. Fajfer, Phys. Rev. D 51 (1995) 1101.