Hadron multiplicity as the limit of jet multiplicity at high resolution

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Recently exact numerical results from the evolution equation for parton multiplicities in QCD jets have been obtained. A comparison with various approximate results is presented. A good description is obtained not only of the jet multiplicities measured at LEP-1 but also of the hadron multiplicities for cms energies above 1.6 GeV in $e^+e^-$ annihilation. The solution suggests that a final state hadron can be represented by a jet in the limit of small (nonperturbative) $k_\perp$ cut-off $Q_0$. In this description using as adjustable parameters only the QCD scale $\Lambda$ and the cut-off $Q_0$, the coupling $\alpha_s$ can be seen to rise towards large values above unity at low energies.

1. INTRODUCTION

The phenomena of multiparticle production in hard processes require for their description within QCD both perturbative and non-perturbative elements. Whereas the perturbative part – at least in principle – is rather well understood, there are different approaches to deal with the hadronization at large distances and also the characteristic scale for the transition between both regimes is not uniquely defined. Recently, we studied this transition region more quantitatively in case of a simple observable, the multiplicity of jets in an event.

At a large resolution scale $Q_c$ only a few jets are resolved and their rate can be calculated in fixed order perturbation theory; with increasing resolution more and more jets are resolved and in this case results can be obtained by a resummation of the leading and next-to-leading logarithmic terms of the perturbation theory. In these calculations the multiplicity is obtained in absolute normalization and the only free parameter is the QCD scale $\Lambda$. Increasing the resolution further the jet multiplicity coincides finally with the hadron multiplicity. The jet multiplicity is defined for a given resolution cut-off parameter $Q_c$ using the well known Durham/$k_\perp$ algorithm which counts some clusters of particles as separate if their relative transverse momenta are larger than $Q_c$. In $e^+e^-$ one also introduces the normalized scale $y_c = (Q_c/Q)^2$ where $Q$ denotes the total cms energy.

A description of hadron spectra and hadron multiplicity has been previously obtained from the modified leading log approximation (MLLA) of QCD assuming a close similarity of parton and hadron spectra with a small transverse momentum cut-off $Q_0$ for the parton cascade (Local Parton Hadron Duality – LPHD). In this description the parameter $Q_0$ represents the confinement scale which terminates the perturbative evolution. The multiplicity of hadrons is then found proportional (roughly twice in the fits) to the multiplicity of partons, also the energy dependences are slightly different (for a recent analysis, see [1]). One might therefore expect that with decreasing $k_\perp$ cut-off $Q_c$ the perturbative regime ends before the hadronic regime is reached at $Q_c \to Q_0$.

In our analysis we search for the existence of such an additional scale. Furthermore we investigate whether we can see from the experimental data some characteristic features of the perturbative analysis, in particular, the strong variation of the coupling $\alpha_s(k_\perp)$ when approaching the QCD scale $\Lambda$.

2. PERTURBATIVE ANALYSIS

The inclusive and exclusive characteristics of the multiparticle final states can be conveniently treated using the generating functional. It fulfills
a coupled system of evolution equations ("master equation") in the virtuality $\kappa = 2E \sin(\Theta/2)$ with $\Theta$ the opening angle and $E$ the energy of the quark or gluon jet \cite{3}. It takes into account the angular ordering inside the parton cascade as well as the running of the coupling with the transverse momentum of the emitted parton. For initialization of the jet evolution one takes the condition that at the threshold of the process there is only one parton in the jet which carries the full momentum. After appropriate functional differentiation one obtains the coupled integro-differential equation for quark and gluon jet multiplicities $N_q$ and $N_g$ \cite{1}.

The master equation yields the complete MLLA asymptotics, i.e. the correct leading and next-to-leading terms in an expansion of $\sqrt{\alpha_s}$. Furthermore it generates a perturbative expansion which allows for the fulfilment of the initial conditions at threshold. The master equation can be simplified by making high energy approximations already in the integral kernel ("MLLA evolution equation"\cite{4}). For this simplified equation analytic expressions can be obtained for the multiplicities in quark and gluon jets \cite{2}. Because of the high energy approximations the behaviour near threshold has some unphysical features (multiplicities smaller than one). To avoid this difficulty we have therefore solved the master equation numerically for given parameters $\Lambda$ and $Q_c$ (or $Q_0$).

In Fig. 1 we compare the results for the multiplicities in quark and gluon jets for different approximations to the master equation choosing a small value $\lambda = 0.02$ for the parameter $\lambda = \ln(Q_0/\Lambda)$ as obtained below from a fit to the hadron multiplicities. One can see in (a) that the quark jet multiplicities for the exact solution and the MLLA at the same $\lambda$ behave very similar already above the inelastic threshold; the result for the "limiting spectrum" at $\lambda = 0$ \cite{3} which has the limit $N_q \to 0$ at threshold $Y \to 0$ is smaller in absolute normalization by roughly a factor 2. It should be noted however that the close agree-
ment within 20% of the upper three curves is special for $\lambda = 0.02$ and disappears with increasing $\lambda$. In Fig. 1b we compare our exact solution of the master equation for the ratio of multiplicities in quark and gluon jets, again at $\lambda = 0.02$, with several approximations using and not using the initial conditions at threshold. One can see that at LEP energies ($Y \approx 5$) the exact solution is smaller than all quoted approximations.

A qualitative difference between the exact solution of the master equation and the analytic MLLA results is found in the behaviour for $\lambda \to 0$: whereas the multiplicity approaches a finite value in the latter case, it diverges for the exact solution as it does in case of the double log approximation (DLA) with $\ln$.

$$\mathcal{N} \sim K_0(A\sqrt{\kappa})I_1(A\sqrt{\ln(\kappa/\Lambda)}) + \ldots$$

which at high energies and small $\lambda_c$ behaves like $\ln \lambda_c \exp(A\sqrt{\ln(\kappa/\Lambda)})$ and exhibits a logarithmic singularity at $\lambda_c = 0$.

The behaviour near threshold can be improved by taking into account the full perturbative result at fixed order (see, for example, ref. [2]). Such results depend on the particular process. For $e^+e^-$ annihilation the $\mathcal{O}(\alpha_s)$ corrected solution has been calculated [3] as

$$\mathcal{N}_\text{corr}^{e^+e^-}(y_e) = 2N_q(y_e) - 2N_q^{(1)}(y_e) + N^{3-jet}(y_e)$$

where $N^{3-jet}(y_e)$ is the numerically integrated 3 jet cross section (for running $\alpha_s(k_\perp)$) and $2N_q^{(1)}(y_e)$ is the corresponding quantity in $\mathcal{O}(\alpha_s)$ from the master equation.

3. COMPARISON WITH $e^+e^-$ DATA

In Fig. 2 we show the data of the average jet multiplicity at $Q = 91$ GeV as a function of the resolution parameter $y_e$, obtained with the $k_\perp$ algorithm. The theoretical predictions from [3] for the jet data are given in absolute normalization in terms of the single parameter $\Lambda$. Also shown are the data on hadron multiplicities in the energy range $Q = 1.6 \ldots 91$ GeV taken as $\mathcal{N}_\text{all} = \frac{3}{2}N_{\text{ch}}$ as a function of the same scale parameter, now calculated as $y_e = Q_0^2/Q^2$, where the parameter $Q_0$ corresponding to the parton $k_\perp$ cutoff characterizing a hadronic scale according to the LPHD picture and is obtained from a fit to the data. Another adjustable parameter here is the overall normalization $K_{\text{all}}$ which relates the parton and hadron multiplicities in $\mathcal{N}_\text{all} = K_{\text{all}}\mathcal{N}_\text{corr}^{e^+e^-}$. A good description of the data is obtained with parameters

$$K_{\text{all}} = 1, \Lambda = 500 \pm 50 \text{ MeV}, \lambda = 0.015 \pm 0.005$$

($\lambda = \ln(Q_0/\Lambda)$) which correspond to the curves in Fig. 2. The fits describe the data within 5% or within the errors.

A remarkable result of our analysis with improved accuracy is the common normalization $K_{\text{all}} = 1$ which is possible without difficulty; the normalization parameter $K_{\text{all}}$ is correlated with $Q_0$ and can be varied within about 30%. This differs from the earlier results based on the limiting spectrum (approximate solution with $\lambda = 0$) which lead to the larger value $K_{\text{all}} \approx 2$ (see, for example, refs. [4,4]). This difference can be traced back to the different normalization at
threshold, see Fig. 1a. The solution with $K_{\text{eff}} = 1$ is natural as it provides the correct boundary conditions $N = 2$ at threshold for both hadrons and jets. Then a continuous connection between jet and hadron multiplicities results: both are described by the same equation (3) and one is obtained from the other at fixed $y_c$ by changing the absolute scale $Q_c$ down to $Q_0$. There is no evidence for another nonperturbative scale above $Q_0$. In this description a single hadron corresponds to a single parton of low virtuality $Q_c$.

An important role in this interpretation of the data is played by the running of $\alpha_s$. First we note that in a model with fixed $\alpha_s$ the multiplicity would depend only on the ratio of the available scales through the variable $y_c$ but not on the absolute scales $Q$ or $Q_c$ separately; such a model would predict at high energy a power like dependence on $Q/Q_c$, i.e. a straight line in Fig. 2 for both the hadron and jet multiplicities between the two curves shown.

On the other hand, in case of running $\alpha_s$ the additional scale $k_\perp/\Lambda$ appears which, for a given $y_c$, leads to an enhancement of multiplicities at small absolute scales $Q$ or $Q_c$ and hence to the different curves in Fig. 2. The effect of the running $\alpha_s$ is most pronounced at large $y_c$. Here the quantity $\mathcal{N} - 2$ is of $O(\alpha_s)$ and therefore reflects the typical coupling. From the separation of the jet and hadron data in Fig. 2 (see also data on $\mathcal{N} - 2$ in [3]) one finds that around $y_c \sim 0.1$ the typical couplings should differ by more than an order of magnitude. As the coupling to produce jets at LEP energies is around $\alpha_s \sim 0.1$ the coupling to produce particles at low energies is therefore

$$\alpha_s > 1 \quad \text{at} \quad Q \sim 1.6 \text{ GeV.} \quad (4)$$

The fact that both data sets are rather well described over the full kinematic region where data are available confirms that $k_\perp$ is a good choice for the scale of $\alpha_s$ and also supports the LPHD concept to describe hadrons by the $k_\perp$ cut-off $Q_0$. There is a certain ambiguity in the definition of $k_\perp$ which leads to different values for $\Lambda$. The value quoted above in (3) refers to the Durham-$k_\perp$ definition whereas the usual $k_\perp$ would yield a smaller value around 300 MeV [1].

Another characteristic sign of the running coupling is the strong rise of the jet rate towards small $y_c$. Only for very high resolution, if $Q_c$ is lowered from 900 MeV ($y_c \sim 10^{-4}$) to the final $Q_0 \sim 500$ MeV, about three quarters of the final multiplicity are produced, three times as many jets as in the large complementary kinematic region with $y_c > 10^{-4}$. The steep rise towards small $y_c$ in Fig. 2 reflects the Landau singularity in the coupling at $y_\Lambda = \Lambda^2/Q^2$, which is however screened by the hadronization scale $y_0 = Q_0^2/Q^2 \gtrsim y_\Lambda$. This behaviour is qualitatively described by the high energy DLA result [1] which shows the singularity at $\lambda = 0$. It will be interesting to study this behaviour experimentally, but some additional assumptions on mass effects are necessary in this region [1].

The prediction for the hadron multiplicity in gluon jets (Fig. 1b) is also well met by experimental data at LEP energies [1]; there are also predictions for the dependence of the subjet multiplicity in gluon jets on the scale parameter $y_c$.

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