Let $G$ be a topological group and let $P \to M$ be a principal $G$-bundle over a manifold $M$. The isomorphism class of $P$ is uniquely determined by the homotopy class of its inducing map $\alpha : M \to BG$. The gauge group of $P$, denoted by $\mathcal{G}_\alpha(M)$, is the topological group consisting of $G$-equivariant automorphisms of $P$ that fix $M$. The study of the topology of gauge groups is closely related to hot topics in mathematical physics and the geometry of manifolds.

In this paper the author considers three types of high dimensional manifolds $M$, and studies the homotopy types of gauge groups of their principal $G$-bundles. The three types of high dimensional manifolds are: $(n-1)$-connected closed oriented combinatorial $2n$-manifolds, the total spaces of oriented sphere bundles of real vector bundles over spheres with cross sections, and highly connected closed oriented $2n$-manifolds. The author shows that under certain conditions a wedge decomposition of $\Sigma M$ induces a product decomposition of gauge groups of principal $G$-bundles over $M$ (Proposition 2.2). Since suspensions of the three types of manifolds are homotopy equivalent to wedges of smaller spaces, their gauge groups $\mathcal{G}_\alpha(M)$ are homotopy equivalent to products of recognizable spaces (Theorems 1.1-1.3). In Section 7 the author applies the gauge group decomposition to computing the homotopy exponents of $\mathcal{G}_\alpha(M)$. Given a prime $p$ the homotopy exponent $\exp_p(\mathcal{G}_\alpha(G))$ is the least power of $p$ annihilating all $p$-torsions in $\pi_*(\mathcal{G}_\alpha(G))$. Using Theorems 1.1-1.3 the author gives upper bounds to $\exp_p(\mathcal{G}_\alpha(M))$ (Lemma 7.1), and sharpens the estimates when the structural group $G$ is a certain low rank Lie group with respect to $p$ (Propositions 7.6 and 7.7).

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MSC:

- 55P15 Classification of homotopy type
- 55P40 Suspensions
- 54C35 Function spaces in general topology
- 55R25 Sphere bundles and vector bundles in algebraic topology
- 57R19 Algebraic topology on manifolds and differential topology
- 57S05 Topological properties of groups of homeomorphisms or diffeomorphisms

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gauge groups; homotopy suspensions; homotopy decompositions; homotopy exponents

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References:

[1] Adams, J. F.. On the groups $J(X)$-IV. Topology 5 (1966), 21-71. · Zbl 0145.19902
[2] Atiyah, M. and Bott, R.. The Yang-Mills equations over Riemann surfaces. Philos. Trans. R. Soc. Lond. A, Math. Phys. Eng. Sci. 308 (1983), 523-615. · Zbl 0509.14014
[3] Bott, R. and Samelson, H.. Application of the theory of Morse to symmetric spaces. Am. J. Math. 80 (1958), 964-1029. · Zbl 0101.39702
[4] Cohen, R. L. and Milgram, R. J.. The homotopy type of gauge theoretic moduli spaces, from: ‘Algebraic topology and its applications’. In Math. Sci. Res. Inst. Publ. (eds. Carlson, G. E., Cohen, R. L., Hsiang, W. C. and Jones, J. D. S.), vol. 27, pp. 15-55 (New York: Springer, 1994). · Zbl 0799.57019
[5] Cohen, F. R., Moore, J. C. and Neisendorfer, J. A.. The double suspension and exponents of the homotopy group of spheres. Ann. Math. 109 (1979), 549-565. · Zbl 0443.55009
[6] Crabb, M. C. and Sutherland, W. A.. Counting homotopy types of gauge groups. Proc. London Math. Soc. 81 (2000), 747-768. · Zbl 1024.55005
[7] Davis, D. M. and Theriault, S. D.. Odd-primary homotopy exponents of simple compact Lie groups. Geom. Topol. Mono-
[43] Whitehead, G.. Elements of homotopy theory GTM 62 (Berlin-Heidelberg, New York: Springer-Verlag, 1978).
[44] Wu, W. T.. On Pontrjagin classes, II. Sci. Sin. 4 (1955), 455-490. - Zbl 0068.37005
[45] Wu, J.. Homotopy theory and the suspensions of the projective plane. In Memoirs AMS, vol. 162, pp. 130, No. 769, 2003). - Zbl 1022.55009

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