Anisotropic motion of a dipole in a photon gas

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The behaviour of a single neutral particle with nonzero electric dipole moment placed in the bulk of a blackbody cavity at fixed temperature is here examined in the realm of quantum field theory. The interaction of the dipole with the thermal bath of photons leads to quantum dispersions of its linear and angular momenta, whose magnitudes depend on the cavity temperature and also on the dipole moment. It is shown that the amount of energy held by the dipole rotation is expressively larger than the one related to the center of mass translation. It is also unveiled a subtle behaviour of the kinetic energy of the dipole that initially increases in magnitude above the level of its late-time residual value. In spite of the smallness of the effects here discussed it is suggested that they could have observable implications.

I. INTRODUCTION

The question of how does the classical notion of observables "emerge" from a particular quantum system is certainly one of the most elusive ones in modern physics, which permeates equally all systems, from quantum gravity to condensed matter. Nevertheless, a natural mechanism for this process relies on the interaction of quantum systems with a thermal environment, through decoherence (see Ref. [1] for a review). The fluctuations induced by the environment suppress quantum interference effects, turning the quantum system into a statistical mixture, which occurs on a time scale smaller than the one the system takes to reach equilibrium (see Ref. [2], and also [3] for an example in semiclassical gravity). Alongside its importance, this interaction may be prevalent in our universe, as probably all naturally occurring quantum systems are open and interacting with a thermal environment. For instance, every system is immersed in a thermal bath of long wavelength gravitons [4].

It is then clear that if one is interested in quantum-based technology, one must be aware of the behavior of thermal quantum fields. Not only that, but in the detection of fundamental weak phenomena, such as the Hawking and Unruh effects [5, 6], for which one must have a deep understanding of the present thermal contributions in order to disentangle the former in a measured signal. Notwithstanding, theoretical models exist for probing the Unruh effect [7, 8], and the recent measurement of Hawking-like radiation in the context of analogue gravity [9] is an unquestionable milestone. Moreover, of particular importance for the present work is the notion of subvacuum phenomena [10], for which classically positive quantities assume negative values after renormalization. It was recently shown [11] that temperature can enhance subvacuum effects in some systems of boundary physics. Yet, the possibility of detection requires the thermal fluctuations to be well-known and distinguished from the boundary contributions.

Examining the behavior of systems under influence of thermal fluctuations can also lead to conclusions about the coupling between gravity and physical fields. In fact, first principles analysis shows that the interplay between vacuum and thermal local averages for scalar radiation near a reflecting wall implies in a natural restriction on the possible values the curvature coupling parameter ξ can take [12]. Particularly, it was found that in more than three spacetime dimensions such a range contains the conformal coupling, but it does not contain the minimal coupling.

In this paper, we propose to study the local thermal behaviour of the radiation field through a test neutral particle with nonzero dipolar moment, which models one of the most usual couplings between radiation and ordinary matter. We work in the stochastic regime, in which the decoherence has already played its part and we neglect the quantum internal degrees of freedom of matter, by treating the dipole as a classical particle. Such models are well suited to probe the construction of a bottom up thermodynamics, as the interacting point-system initially in non-equilibrium with the reservoir is a very different setting from the usual statistical mechanics [13, 14]. For the sake of simplicity, we assume that the background field is isotropic and homogeneous.

Due to the field-matter interaction, aspects of the background field can be tested through the motion of the test particle, and this induced motion resembles the modified vacuum induced motion studied in Refs. [11, 15–20]. In these works, a point-like charged particle was shown to perform a sort of random walk due to a transition between states of the electromagnetic field. Furthermore, for the particular case of Ref. [18], the charged particle entered a region in which the electromagnetic vacuum state was modified by the presence of a plane perfectly
conductor. This models a transition between a homogeneous isotropic background to one in which both symmetries are broken by the presence of the boundary, which is thus reflected by an anisotropy in the particle’s random walk. For the present case of the dipole in the thermal bath, velocity and angular momentum fluctuations are expected to occur with the transition between the empty space to the homogeneous isotropic thermal bath. We note also that because the particle’s dipolar orientation in perfect analogy with the charged counterpart.

\[ F = q \mathbf{E}(x,t), \]

In the presence of an external electric field \( \mathbf{E}(x,t) \), the particle is subjected to a force of interaction given by \( \mathbf{F}(x,t) = -\nabla U(x,t) \), where \( U(x,t) = -p \cdot \mathbf{E}(x,t) \) denotes its potential energy. Additionally, the dipole is affected by an origin-independent torque \( \mathbf{T} = \mathbf{p} \times \mathbf{E}(x,t) \). Force induces a translation of the dipole, while torque is the source of its rotation. Furthermore, in this work we are assuming that no other degrees of freedom are excited, e.g., internal vibrations are negligible. This hypothesis imposes some constraints on the system temperature, which will be addressed later on.

As for the electric field, we take it to be composed of a background field, which is the thermal bath, and a contribution from the field originated by the dipole. The backreaction would be the usual radiation reaction force (classically) or, in our case, a quantum dissipative force. The fluctuations then would be comprised of an intrinsic part, a damped oscillatory motion induced by the backreaction, and a driven contribution coming from the background field [14]. Nonetheless, we assume here to work in the weak coupling limit and with a time scale greater than the relaxation time of the intrinsic fluctuations. With that the reservoir is taken to be unperturbed, and the motion occurs solely due to fluctuations of the background thermal bath.

In what follows, we also assume the dipole has negligible spatial dimensions when compared to any other distance scales involved in the system, and also that the change in its centre of mass position caused by the interaction can be neglected. This last assumption is always justified in view of the smallness of the quantum effects [18]. Its dynamics is obtained by integrating the expression for the force due to the interaction with the external field (Newton’s second law for translation) which, in component notation reads,

\[ v_i(t) = \frac{3}{m} \sum_{j=1}^{3} \frac{p_j}{m} \int_0^t dt \partial_i E_j(x,t). \] (1)

This expression assumes that at an initial time \( t = 0 \) the background field is suddenly changed from vacuum to a thermal bath, and the effect of the interaction between the dipole and the thermal environment over the particle velocity is then calculated after a time \( \tau \). Analogously, from Newton’s second law for rotation, the dipole angular momentum is obtained as,

\[ L_i(\tau) = \sum_{j,k=1}^{3} \epsilon_{ijk} p_j \int_0^\tau dt E_k(x,t), \] (2)

where \( \epsilon_{ijk} \) is the Levi-Civita symbol in the 3-dimensional space, which is a completely antisymmetric tensor defined by \( \epsilon_{123} = 1 \). As the dipole is here considered to be a linear system, there will be only two rotational degrees of freedom: no rotation along the dipole symmetry axis.

**II. PRELIMINARY ASPECTS**

We shall assume a non-relativistic neutral particle with dipolar moment \( p = qa \), where \( a = |a| \) is the distance separating the two opposite charges of equal magnitude \( q \). In the presence of an external electric field \( \mathbf{E}(x,t) \), the particle is subjected to a force of interaction given by \( \mathbf{F}(x,t) = -\nabla U(x,t) \), where \( U(x,t) = -p \cdot \mathbf{E}(x,t) \) denotes its potential energy. Additionally, the dipole is affected by an origin-independent torque \( \mathbf{T} = p \times \mathbf{E}(x,t) \). Force induces a translation of the dipole, while torque is the source of its rotation. Furthermore, in this work we are assuming that no other degrees of freedom are excited, e.g., internal vibrations are negligible. This hypothesis imposes some constraints on the system temperature, which will be addressed later on.

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\[ v_i(t) = \frac{3}{m} \sum_{j=1}^{3} \frac{p_j}{m} \int_0^t dt \partial_i E_j(x,t). \] (1)

III. QUANTUM DISPERSIONS

As thermal electromagnetic radiation can only be treated as a quantum system, the electric field appearing in the above expression is now regarded as a quantum operator. Accordingly, the dipole is treated as a semi-classical particle, i.e., the width of its wave function is negligible when compared to the distance scales of the system, whereas its velocity and angular momentum are now operators acting on the Fock space of the field. As in any observable in a quantum system, measurement predictions are given through expectation values of these operators. Moreover, as we are dealing with a field in thermal equilibrium and not pure states, the expectation value of an observable \( O \) is given by \( \langle O \rangle_\beta = tr[\rho O] \).

The density operator \( \rho = (1/Z)\exp(-\beta H) \) describes the grand-canonical ensemble (zero chemical potential) with temperature \( T = 1/\beta \), where \( Z = \sum \exp[-\beta E_i] \) denotes the partition function, and \( \{E_i\} \) the set of eigenvalues of the Hamiltonian \( H \) [21].

Notice that the dipole is not in thermal equilibrium with the photons of the radiation field. Yet, the average force and torque on the particle is zero because of the system thermal state: \( \langle E_i \rangle_\beta = 0 \). It also follows that \( \langle v_i \rangle_\beta = 0 \) and \( \langle L_i \rangle_\beta = 0 \). Therefore, the squared mean deviations are simply \( \langle (\Delta v_i)^2 \rangle_\beta = \langle v_i^2 \rangle_\beta \) and \( \langle (\Delta L_i)^2 \rangle_\beta = \langle L_i^2 \rangle_\beta \), and these quantities describe how measurements of the corresponding observables are distributed around their zero average. It is shown in Appendix A that the quantum dispersion of the dipole velocity \( v \) is anisotropic. Specifically, the dispersion of the velocity component perpendicular to the dipole axis \( \langle v_{\perp}^2 \rangle_\beta \) is twice the dispersion along its axis (here called parallel direction) \( \langle v_{\parallel}^2 \rangle_\beta \). Hence, the mean value of the square of the velocity results to be \( \langle v^2 \rangle_\beta = 5 \langle v_{\parallel}^2 \rangle_\beta \). On the other hand, assuming the moment of inertia of the dipole is \( I \), and also that it is unidimensional, there will be only two degrees of freedom of rotation, and the dispersions of its angular velocity \( \omega \) in the directions perpen-
diucal to the dipole are equal, so that $\langle \omega^2 \rangle_\beta = 2 \langle \omega_\perp^2 \rangle_\beta$. These quantities are given by

$$\langle \omega^2 \rangle_\beta = \frac{2\pi^2 p^2}{45 m^2 \beta^3} f_\beta(\tau), \quad (3)$$

$$\langle \omega^2 \rangle_\beta = \frac{2p^2}{3I^2 \beta^2} g_\beta(\tau), \quad (4)$$

where we have defined the dimensionless functions of $\tau$,

$$f_\beta(\tau) = 1 + \frac{45\beta^4}{\pi^2 \tau^4} - 15 \left[ 2 + \cosh \left( \frac{2\pi\tau}{\beta} \right) \right] \text{csch}^4 \left( \frac{\pi\tau}{\beta} \right),$$

$$g_\beta(\tau) = 1 - \frac{3\beta^2}{\pi^2 \tau^2} + 3\text{csch}^2 \left( \frac{\pi\tau}{\beta} \right).$$

The behaviour of $f_\beta(\tau)$ and $g_\beta(\tau)$ is depicted in Fig. 1. Notice that both functions $f_\beta(\tau)$ and $g_\beta(\tau)$ vanish when $\tau \to 0$ and go to 1 when $\tau \to \infty$. Thus, the coefficients of these functions in Eqs. (3) and (4) already give the values of the corresponding quantum fluctuations in the late-time regime, i.e., when equilibrium is reached.

The dipole is assumed to be placed initially (at $t = 0s$) at rest in the bulk of a blackbody cavity at temperature $T$ and begins to gain energy from the radiation field until it reaches an stationary regime at late-times $\tau \gg \beta$, when the mean value of its energy becomes constant. We stress that the absorbed energy produces translational, rotational, and possibly vibrational motion of the dipole. Recall that we will be concerned here only with the first two types of effects. Vibration is only significant when the resonance frequency of a dipole is about the frequency of the driven electric field. As the main frequency of the radiation field is related to the temperature of the cavity, it can always be arranged to keep the vibrational contribution unimportant. Hence, neglecting any contribution to vibrational modes, the expectation value of the kinetic energy of the dipole as function of the interaction time $\tau$

![FIG. 1. Behavior of the auxiliary functions $f_\beta(\tau)$ and $g_\beta(\tau)$ as function of time.](image)

is given by

$$\langle K \rangle_\beta = \frac{p^2}{m\beta^4} \left[ \frac{\pi^2 f_\beta(\tau)}{45} + \frac{4\gamma^2}{3} g_\beta(\tau) \right],$$

where $\gamma^2 = m\beta^2/4I$. In the above result for $\langle K \rangle_\beta$ the first term is identified as $\langle mv^2/2 \rangle$ while the second one is $\langle I\omega^2/2 \rangle$. In the case of a dipole with a bond length $a$, its moment of inertia can be presented as $I = ma^2/4$, which leads to $\gamma = \beta/a$. As $f_\beta(\tau)$ and $g_\beta(\tau)$ have the same value in the late-time regime, $\gamma$ is a parameter that measures how much energy is stored by means of rotation of the dipole as compared to the energy held by its translational motion. Notice that the late-time value of the rotational energy is proportional to $\gamma^2$. So, when $\gamma \gg 1$ the translational energy is completely negligible when compared to the rotational one. Figure 2 depicts the total kinetic energy (solid curve) as function of $\tau/\beta$. The value $\gamma = 1$ was chosen just to be possible to visualise the behaviour of the different contributions (translational and rotational) to the kinetic energy in a same figure. It is interesting to observe that the translational contribution (dashed curve in Fig. 2) achieves a transient value that is larger than its late-time regime value. It means that after the dipole is placed in contact with the thermal bath of photons it initially gains more energy than it keeps when it achieves the stationary regime. It should be noticed that the mean values of the components of the particle velocity are related to the spatial derivatives of the electric field. Hence, the corresponding dispersions depend on the time evolution of the spatial variation of the field fluctuations, which are larger at the transition between vacuum and thermal states. Finally, part of the energy initially transferred to the linear motion of the dipole eventually returns to the radiation field or is converted in rotational energy.

![FIG. 2. Expectation value of the kinetic energy of the dipole in a radiation field. The behaviours of the rotational and translational contributions are separately depicted. The late-time regime is shortly achieved. The late-time value of the rotational contribution is proportional to the square of $\gamma$ parameter.](image)
IV. ESTIMATES

In order to have some estimates of the relative magnitude of energy contributions, let us express \( \gamma \) as,

\[
\gamma = \frac{1}{aT} = 2.29 \times 10^6 \left( \frac{1\text{nm}}{a} \right) \left( \frac{1\text{K}}{T} \right).
\]

As we see \( \gamma \gg 1 \) for most realistic configurations. At room temperatures of about 300K it follows that \( \gamma \approx 10^3 \) for typical molecular dipoles, which confirms that the energy absorbed by means of rotation is much greater than the energy absorbed by means of translational movement.

Suppose for instance that a molecule of potassium chloride (KCl) is used as a possible probe for the effects above discussed. It has a mass \( m_{\text{KCl}} = 1.2 \times 10^{-22} \text{g} \), a bond length of \( a = 2.5\text{Å} \), and a dipole moment \( p_{\text{KCl}} = 10.27\text{D} \), where D denotes debye (1D \( \approx 3.36 \times 10^{-30}\text{Cm} \)). After the system achieves the late-time regime, that occurs in a temperature dependent time interval of about \( 10^{-10}(1\text{K}/T) \) seconds after it is placed in contact with the thermal environment, the uncertainty in its linear \( \Delta v \) and angular \( \Delta \omega \) velocities can be obtained directly from the square-roots of Eqs. (3) and (4), respectively. For the linear velocity it results,

\[
\Delta v = 8.6 \times 10^{-15} \left( \frac{p}{p_{\text{KCl}}} \right) \left( \frac{m_{\text{KCl}}}{m} \right) \left( \frac{T}{1\text{K}} \right)^2 \text{ms}^{-1},
\]

which, at room temperature \( T = 300\text{K} \), gets about \( 10^{-9}\text{ms}^{-1} \). On the other hand the uncertainty in the angular velocity is such that,

\[
\Delta \omega = 7.9 \times 10^3 \left( \frac{p}{p_{\text{KCl}}} \right) \left( \frac{m_{\text{KCl}}}{m} \right) \left( \frac{T}{1\text{K}} \right) \left( \frac{1\text{Å}}{a} \right)^2 \text{s}^{-1},
\]

which achieves a value of the order of \( 10^6\text{s}^{-1} \) at room temperature.

A possible observable to this system would be the radiation emission by the dipole rotation. If the dipole is initially placed at rest in the bulk of a cavity with a fixed temperature \( T \), it is expected that after the system reaches its stationary regime, the uncertainty in its rotation frequency will be \( \Delta \omega \), whose estimate can be obtained from the formula above. Therefore, it is expected that radiation may be emitted within a frequency interval \( 0 \leq \omega \leq \Delta \omega \). The total power \( P \) radiated by the system can be estimated by assuming an idealised model of an electric dipole rotating in a plane with angular velocity \( \omega \). As shown in standard textbooks [22], the dominant contribution to this quantity is given by \( P = p^2 \omega^4 / 6 \epsilon_0 c^3 \).

Hence, the mean value of the corresponding quantum observable can be estimated as \( \langle P \rangle \sim 10^{-46}\text{Js}^{-1} \) for the potassium chloride in STP conditions, which is a tiny effect, as expected. It should be stressed, however, that this is related to the radiation emitted by a single dipole.

As another possibility to measure the effect here discussed, an experiment could be devised such that a molecule with a dipole moment initially prepared in a given direction is sent with constant velocity through the interior of a thermal cavity at temperature \( T \). As quantum dispersions of the velocity are not equal in all directions, after the dipole comes out the cavity it will present an anisotropic dispersion in its direction of motion. The deviation with respect to the original direction of motion could be eventually detected and related to the velocity uncertainty acquired during its interaction with the thermal fluctuations. This kind of reasoning was recently used as a possible way of detecting modified vacuum fluctuations in a similar system [18].

V. FINAL REMARKS

Concluding, some remarks are in order. At the stochastic level, the fluctuations of an isotropic gas of photons induce an anisotropic motion of a dipole, the fluctuations being greater in the direction perpendicular to the orientation of the dipole moment. This is an interesting result, because even though the dipole introduces a preferential direction, the driven stochastic force comes from an isotropic system. This effect is similar to the case of a charged particle initially at rest near a perfectly reflecting wall. Vacuum fluctuations of the modified vacuum state will produce an uncertainty in the parallel component of the particle velocity [16, 18].

When dispersive effects are neglected, fluctuations of thermodynamic quantities usually have a random walk behaviour being proportional to the interaction time [23]. In such cases, the thermal reservoir continuously gives away energy to the system, increasing its motion, and a dissipative force is needed so that this energy is given back to the environment and the dispersions settle to their usual thermal equilibrium value where the state of the particle is also a Gibbs state, with the same temperature as the environment. Nonetheless, in this model the thermal environment only gives away a finite amount of energy to the dipole. In Ref. [11] it was shown that this is so because the fluctuations of the gas of photons have anti-correlations, which, when integrated over an infinite interaction time, amount to a finite positive contribution. These anti-correlations arise because, contrarily to the usual thermal case, the field does not only push, but also pulls the dipole. Moreover, it dismisses the discomfort due to the lost energy when the fluctuations in the dipole velocity decay from the peak to a constant late-time value.

Furthermore, due the coupling with the field, which is not negligible when compared to the free dipole Hamiltonian, the late-time equilibrium state of the dipole is not a Gibbs thermal state [13]. The effective temperature, given by the velocity fluctuations, is different from the temperature of the KMS state of the electric field. This highlights the difference with a usual statistical mechanical system.
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Appendix A: Calculation of the dispersions

Renormalized thermal quantum fluctuations of the dipole velocity can be obtained by calculating the expectation value of the square of \( v_i \) given by Eq. (1), i.e.,

\[
\langle v_i^2 \rangle_\beta = \sum_{k,l=1}^3 \frac{p_k p_l}{2m} \int_0^\tau dt \int_0^\tau dt' G^{(1)}_{\beta kl \text{ Ren}}(x, t; x', t'),
\]

where the Hadamard function \( G^{(1)}_{\beta kl}(x, t; x', t') \) is the anticommutator of the field components at different spacetime points,

\[
G^{(1)}_{\beta kl}(x, t; x', t') = \langle \{ E_k(x, t), E_l(x', t') \} \rangle_\beta = \frac{1}{2\pi^2} P_{kl} \sum_{n=1}^\infty \frac{1}{(\Delta t - in\beta)^2 - \Delta x^2},
\]

with the definitions \( \Delta x = a' - a \), and \( P_{ij} = \delta_{ij} \nabla^2 + \partial_i \partial_j \).

It is understood that the free vacuum does not contribute to expectation values of observable quantities. Then, the Hadamard function must be renormalized, i.e., the divergent vacuum contribution must be subtracted, which results,

\[
G^{(1)}_{\beta kl \text{ Ren}}(x, t; x', t') = \frac{1}{\pi^2} \text{Re} P_{kl} \sum_{n=1}^\infty \frac{1}{(\Delta t - in\beta)^2 - \Delta x^2}.
\]

With that, a local description of the fluctuations of the thermal state of the field is given, together with its influence on a test dipole. The usual global thermodynamic properties of the photon gas can be obtained through the analysis of the energy momentum tensor of the quantum field.

In order to evaluate the integrals in Eq. (A1) it is worth noticing that, if x-axis is chosen to coincide with the dipole direction, only the field correlations parallel to it will be needed, which reads,

\[
G^{(1)}_{\beta x_i x_j \text{ Ren}}(x, t; x', t') = \frac{4}{\pi^2} \sum_{n=1}^\infty \frac{[(\Delta x_1)^2 - (\Delta x_2)^2 + (\Delta t - il\beta)^2]}{[(\Delta t - in\beta)^2 - (\Delta x)^2]^3}.
\]

Moreover,

\[
\lim_{\tau \to \infty} \frac{\partial}{\partial x_\parallel} G^{(1)}_{\beta x_i x_i \text{ Ren}}(x, t; x', t') = -\frac{16}{\pi^2} \sum_{n=1}^\infty \frac{1}{(\Delta t - in\beta)^6},
\]

\[
\lim_{\tau \to \infty} \frac{\partial}{\partial x_\perp} G^{(1)}_{\beta x_i x_i \text{ Ren}}(x, t; x', t') = -\frac{32}{\pi^2} \sum_{n=1}^\infty \frac{1}{(\Delta t - in\beta)^6}.
\]

Finally, integrating over \( t \) and \( t' \) it is found that,

\[
\langle v_\parallel^2 \rangle_\beta = \frac{\beta^2}{15\pi^2 m^2 \beta^4} \left[ 2\psi^{(3)}(1) - \psi^{(3)}(1 + \frac{i\tau}{\beta}) - \psi^{(3)}(1 - \frac{i\tau}{\beta}) \right] = \frac{2\pi^2 \beta^2}{225 m^2 \beta \Gamma} f_\beta(\tau),
\]

and \( \langle v_\perp^2 \rangle_\beta = 2 \langle v_\parallel^2 \rangle_\beta \). In this result, \( f_\beta(\tau) \) is the function appearing in Eq. (3), and is obtained by simplifying the polygamma functions \( \psi^{(3)}(x) \) in the above equation.

The fluctuations of the angular velocity can be obtained by using closely the same procedure. Setting \( L_i = I\omega_i \) in Eq. (2) and calculating the thermal expectation value of the square of \( \omega_i \), one can obtain,

\[
\langle \omega_i^2 \rangle_\beta = \frac{1}{T_2} \sum_{j,k,m,n=1}^3 \epsilon_{ijk} \epsilon_{imn} P_{j} P_{m} \int_0^\tau G^{(1)}_{\beta kn \text{ Ren}}(x, t; x', t') dt dt'.
\]

As before, the x-axis is chosen to coincide with the dipole direction, i.e., \( p_i = p_i \). Thus, direct inspection of the above equation reveals that the parallel component \( \langle \omega_i^2 \rangle_\beta = \langle \omega_i^2 \rangle \) is identically null. As for the perpendicular component, \( \langle \omega_\perp^2 \rangle_\beta = \langle \omega_\perp^2 \rangle = \langle \omega_\parallel^2 \rangle \), observing the identity \( \epsilon_{2jk} \epsilon_{2mn} = \delta_{jm} \delta_{kn} - \delta_{jn} \delta_{km} \), and after performing the integrals, it follows that,

\[
\langle \omega_\perp^2 \rangle_\beta = \frac{\beta^2}{3\pi^2 T_2 \beta^2} \left[ 2\psi^{(1)}(1) - \psi^{(1)}(1 + \frac{i\tau}{\beta}) - \psi^{(1)}(1 - \frac{i\tau}{\beta}) \right] = \frac{\beta^2}{3\pi^2 \beta} g_\beta(\tau),
\]

where \( g_\beta(\tau) \) is the function appearing in Eq. (4), which is obtained after simplifying the above polygamma functions \( \psi^{(1)}(x) \).
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