Natural and model-independent conditions for evading the limits on the scale of new physics

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One of the most interesting phenomenological quantities connected the physics expected to underlie the Standard Model is its scale \( \Lambda \). In this paper I argue that the limits on this quantity obtained using model-independent parameterizations contain an tacit assumption that could be invalidated under a variety of situations. As a specific example, existing limits on \( \Lambda \) would be decreased by at least an order of magnitude if the underlying physics has a symmetry under which all Standard Model particles are singlets but none of the heavy excitations are. In this case current experiments would see no clear indications of new physics using precision measurements while future colliders capable of directly producing the heavy particles would only occur in pairs.

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I. INTRODUCTION

One of the central issues in the many attempts to describe the physics underlying the Standard Model is the determination of the scale at which these prophesied interactions will be directly observed. To date no (unambiguous) non-Standard Model effects have been found either through direct production, or indirectly through virtual effects[1]; accordingly the data has been used to provide limits on the scales and couplings of these interactions[2, 3]. As this data becomes more precise and experiments probe higher energies, the limits on the scale of new interactions is constantly being pushed to higher values. In some cases these limits have become so stringent that the scales of new physics are stated to lie beyond the reach (through direct observation) of near-future accelerators such as the LHC.

In many instances limits on the scale(s) associated with the new interactions are obtained within the context of specific models [2], but an alternative non-model specific approach has also been repeatedly followed [3], where these limits are obtained using generic couplings limited only by naturality constraints [5].

Within this “generic” approach it is possible to understand the absence of non-Standard Model effects by the assumption that the scale of the new interactions is so large it lies outside the sensitivity limit of all current experiments (involving real or virtual heavy excitations) 2. But an alternative possibility, which will be examined below, is that (i) the masses of the heavy excitations are too large to have been directly produced at the energies available, and (ii) that the couplings in the underlying theory are naturally suppressed so as to insure that current experiments are blind to all leading virtual processes involving the heavy physics. Under these conditions current precision data would exhibit only very small deviations from the Standard Model predictions, even when the scale of new physics is quite close to the energies currently being probed directly, and near-future colliders would produce new particles without any premonition of their existence form LEP or Tevatron data.

In this paper I will investigate the conditions under which this last possibility can be realized. For definiteness I will consider the case where the physics underlying the Standard Model is decoupling and weakly coupled (for non-decoupling theories, the scale of new physics is constrained by the consistency of the theory to be of order \( 4\pi v \sim 3\text{TeV} \),

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2 For example, the limits on the scale of the interactions responsible for 4-fermion operators involving two quarks and two leptons (all left-handed) is \( \gtrsim 25\text{TeV} \) [4].

2 This assumption is consistent whenever the heavy physics decouples [6]; if it does not the scale of the new physics will be directly accessible at the LHC [7].
where $v$ denotes the Standard Model vacuum expectation value [7]). To this end I will first parameterize the new physics effects by an effective Lagrangian [8], then I will characterize the leading contributions (Sect. II), and use this to provide constraints that would naturally insure their absence (Sect. III). Parting comments and some numerical estimates are presented in section IV. A calculational detail is relegated to the appendix.

II. EFFECTIVE THEORY AND TREE-LEVEL GENERATED OPERATORS

I will assume the existence of non-Standard Model interactions that can be described by a gauge theory whose scale $\Lambda$ lies significantly above the energies currently probed directly. Furthermore I will restrict the discussion to the case where the corresponding new interactions are weakly coupled, and such that they decouple in the limit $\Lambda \to \infty$ [6]. In this context the Standard Model is the low-energy limit of this more fundamental theory; I will refer to all Standard Model particles as “light”, and assume that all the other excitations have a mass $\gtrsim \Lambda$.

At low-energies (i.e. below $\Lambda$) the heavy excitations will manifest themselves through virtual processes. These effects can be parameterized by a set of effective operators involving only light fields and respecting the local symmetry of the Standard Model. [8] The condition that the heavy physics decouples implies that all observable effects (that is, all the virtual effects that cannot be absorbed in a renormalization of the Standard Model parameters) will disappear in the limit where the scale of the heavy physics $\Lambda$ is sent to infinity. [6]

If the heavy physics is also weakly coupled, then the larger the (canonical) dimension of an operator the smaller its impact. This is so because the anomalous dimensions of all operators will be small, so that to a good approximation an operator of canonical dimension $n$ will appear with a coefficient proportional to $\Lambda^{4-n}$, and its contribution to a process with characteristic energy $E$ will be suppressed by a factor $\sim (E/\Lambda)^{4-n}$ [8]. Note that $E \ll \Lambda$ is a tacit but central (and unavoidable) assumption when using an effective Lagrangian parameterization.

This observation generates a simple hierarchy among effective operators. The leading effects are generated by operators of dimension $\leq 4$ that respect the $U(1)_Y \times SU(2)_L \times SU(3)_c$ gauge symmetry; by definition these correspond to the terms in Standard Model Lagrangian. The most important subleading terms are generated by operators of dimension 5 and 6, which have been listed,[9, 10] Naturality [11] constraints can now be used to further refine the above hierarchy, which is important when considering the phenomenological applications of this approach. An operator of dimension $n$ appears with a coefficient of the form $f/\Lambda^{n-4}$ where $f$ is a numerical factor. When the underlying theory is weakly coupled, $f$ will equal a product of coupling constants for those operators that can be generated at tree level. $^3$ In contrast, the coefficients for operators that are only generated via loops contain additional coupling constants and a numerical suppression factor $\sim 1/(4\pi)^2$.

It follows that in the absence of tree-level generated operators the leading virtual effects produced by the heavy physics (to processes allowed within the Standard Model) will be of order $1/[G_F(4\pi\Lambda)^2]$, which is of the same order as the Standard Model radiative effects suppressed by an additional factor of $(m_W/\Lambda)^2$. In this case the limits on $\Lambda$ obtained form precision data will not improve on the direct ones.

The diagrams responsible for tree-level generated operators can be characterized as follows. $^4$ Consider an arbitrary graph with $I$ internal lines, $E$ external lines and vertices labeled $V_n$, and let $h_n$ be the number of heavy legs in $V_n$. For the situations considered, all internal lines are heavy and all external lines are light. If the graph contains $L$ loops then we have the relations

$$L = I + 1 - \sum V_n; \quad \sum h_n V_n = 2I \quad \Rightarrow \quad \sum (h_n - 2)V_n = 2(L - 1) \quad (1)$$

Tree level graphs ($L = 0$) must then contain at least one vertex with $h_n = 1$.

A theory will produce no tree-level generated operators when the heavy excitations are integrated out if and only if the Lagrangian (after spontaneous symmetry breaking) contains no terms with a single heavy field, that is, if the following vertices are absent:

$$\psi \psi \Phi \quad \bar{\psi} \bar{\psi} \bar{\Phi} \quad \psi \phi \Phi \quad \psi \bar{\psi} \Phi \quad \psi \bar{\psi} \bar{\Phi} \quad \psi \bar{\psi} \bar{X} \quad \psi A \Psi \quad \phi \phi X \quad (2)$$

$^3$ Coefficients generated at tree level will also receive radiative corrections, but these are small by assumption.

$^4$ In extracting the conditions for the absence of tree-level generated operators I will ignore any operator of dimension $\geq 5$ in the underlying theory. This is reasonable not because these terms are necessarily absent, but because, if present, they would be the result of the virtual effects of some yet heavier physics with scale $\Lambda' \gg \Lambda$. The coefficients of these operators will then be then suppressed by powers of $\Lambda/\Lambda' \ll 1$. 

(Ψ, Φ, X denote heavy fermions, scalars and vector bosons respectively, and ψi, φ, A their light counterparts); the vertex XAφφ, is not included since it follows form the absence of vertex Xφφ (see the appendix).

The list (2) contains only vertices of dimension ≤ 4. This is not because higher-dimensional terms are necessarily absent in the theory underlying the Standard Model, but because, if present, they would result form virtual interactions whose typical scale Λ′ is much larger than Λ. The coefficients of these higher-dimensional terms would be the suppressed by powers of Λ/Λ′ ≪ 1 and can be ignored.

The elimination of tree-level generated operators by excluding the list in (2) can of course be restricted to vertices satisfying certain symmetry properties, such as, for example, being baryon or lepton number conserving. Note also that requiring the absence of tree-level generated operators of dimension 5 and 6 implies the absence of tree-level generated operators to all orders in 1/Λ.

A. Operators of dimension 5 and 6

It is straightforward to verify these general arguments for the case of dimension 5 and 6 operators. Assuming the particle content of the Standard Model with a single scalar doublet, and with the addition of right-handed neutrinos, all gauge-invariant operators of dimension 5 are necessarily lepton-number violating [9]:

\[ O^{(5)}_1 = \left( \bar{\nu} \nu \right) (\bar{\phi} \ell \ell) \quad O^{(5)}_2 = \left( \bar{\nu} \nu \right) (\bar{\phi} \phi) \]  

(3)

where \( \ell \) denotes a left-handed fermion iso-doublet, \( \phi \) the Standard Model scalar doublet (and \( \bar{\phi} = i \sigma^2 \phi^* \)), \( \nu \) a right-handed neutrino singlet, and \( \sigma^I \) the usual Pauli matrices; family indices have been suppressed and the superscript \( c \) denotes the charge-conjugate fields.

\( O^{(5)}_1 \) can be generated at tree level by the exchange of a heavy scalar SU(2)L triplet of unit hypercharge 5 or by a zero-hypercharge heavy fermion singlet or triplet. \( O^{(5)}_2 \) can be generated by the exchange of a heavy scalar singlet of zero hypercharge, or of a fermion triplet of hypercharge 1/2. The corresponding graphs must then include two vertices in the list (2).

Operators of dimension 6 are much more plentiful: for a single family, assuming lepton and baryon number conservation and a single scalar doublet, they number 82 [10]. The list of tree-level generated operators is shorter, though still numerous (45 operators); the generic forms of these operators are

\[ \phi^6, \quad D^2 \phi^4, \quad \psi^2 \phi^3, \quad D \psi \phi^2, \quad (\bar{\psi} \psi)^2, \quad (\bar{\psi} \gamma^\mu \psi)^2 \]  

(4)

where φ, ψ and D denote scalars, fermions and covariant derivatives respectively. The detailed list of operators is presented in Ref. [5], where the graphs responsible for their generation is also displayed. It is then a simple matter to verify that the absence of the vertices (2) excludes the operators (4).

In this case, however, there is a simplification: none of the dimension 6 effective operators containing fermions and vector-bosons (but no scalars) can be generated at tree level [5] 7 and because of this the vertex of type \( \Psi \psi A \) in (2) is redundant when considering operators of dimension ≤ 6.

III. ELIMINATION OF TREE-LEVEL GENERATED OPERATORS

The minimal statement leading to the absence of tree-level generated operators is the one made above, namely, that all vertices in (2) be absent. This can be achieved in a natural way by assuming that the underlying theory has certain symmetry properties.

The simplest case corresponds to that where the heavy fields are all non-singlets under a certain symmetry while all light fields are invariant. This is realized in the MSSM [12] provided we label both scalar doublets as light, (the

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5 The conventions used are such that φ, ℓ and e have hypercharges 1/2, −1/2 and −1 respectively

6 It must be emphasized that (4) represents operators that might be generated by an underlying gauge theory taking into account only the most general constraints imposed by gauge invariance. Specific models might contain additional restrictions that prevent the appearance of one or more tree-level generated operators. In contrast, operators not in this list cannot be generated at tree-level by any gauge theory (triple-vector-boson operators fall in this category).

7 The reason is that due to gauge invariance an operator involving only vectors and fermions such as \( i \sigma^I \gamma^\mu D^\mu W_{\mu \nu}^I \), (where \( W_{\mu \nu}^I \) denotes the SU(2) field strength), must contain a term with only three fields, and no tree-level graph with 3 external legs and \( \geq 1 \) internal lines can be contracted.
“new” symmetry is then essentially R-parity [13]). In this case Λ is set by the scale of the soft-breaking terms and by the dimensional coefficient in the superpotential.

Another example is provided by the so called “universal” higher-dimensional theories [14]. In these models space time is assumed to have $4 + \delta$ dimensions such that its ground state consists of the topological product of our non-compact Euclidean 4-dimensional space-time and a compact Riemannian manifold $\mathcal{R}$ of dimension $\delta$. All heavy modes are associated with a non-zero momentum in the directions corresponding to $\mathcal{R}$, while all light (Standard Model) particles have zero momenta along these directions. As a result there are no vertices containing a single heavy mode.

### A. Constraints on the underlying local symmetry

It is also possible to arrange for the local symmetry of the full theory to eliminate the vertices (2), but the models are more convoluted. I will therefore present only a simplified discussion not intended to generate realistic theory. Specifically, I will consider a generic renormalizable gauge with scalar fields $s$, left and right-handed fermions $\chi_{L,R}$ and gauge bosons $V$, whose local symmetry group $\mathcal{G}$ is broken to $\mathcal{H}$ at scale $\Lambda$. I will make no attempt to insure that the light theory corresponds to the Standard Model, not will I include the interactions that generate masses for the light excitations.

I will assume that all scalar $\mathcal{G}$-multiplets receive a vacuum expectation value $\langle s \rangle = O(\Lambda)$, that all the heavy fermions and vector-boson masses are generated in this manner, and that all the resulting physical scalars are heavy. This last condition would not be realized in cases where the scalar potential for the $s$ has a symmetry group larger than $\mathcal{G}$; this type of models are explicitly excluded.

These constraints, together with the fact that the unbroken generators annihilate $\langle s \rangle$ and that the generators of $\mathcal{H}$ close into an algebra, eliminate all the vertices in (2) except those of the form $\Psi \psi A$ and $X \psi \psi$. In particular, vertices of the form $\Phi \psi \psi$ are excluded by the condition that all fermions receive a mass through spontaneous symmetry breaking and that all scalar multiplets receive a vacuum expectation value of order $\Lambda$.

The remaining $\Psi \psi A$ and $X \psi \psi$ vertices can be eliminated only by appropriate choice of the representations for the scalar and fermion fields. Note that if one is interested only in suppressing the leading heavy physics effects as generated by operators of dimension $\leq 6$ only vertices of type $\psi \psi X$ need be eliminated, see section II A. I will consider several possibilities when $\mathcal{G} = SU(N)$.

- If $\chi_L$ transform according to the fundamental representation and $\chi_R$ is a singlet, or if $\chi_L$ carries the adjoint and $\chi_R$ the fundamental, and if in either case $s$ carries the fundamental representation, then $\mathcal{H} = SU(N - 1)$, and vertices of the form $\Psi \psi A$ and $X \psi \psi$ are absent given this pattern of symmetry breaking (most easily seen by choosing a gauge where $\langle s \rangle = (0, \ldots, 0, v)$).

- This is also trivially the case in a vector-like theory where $\chi_{L,R}$ transform according to the fundamental representation and $s$ in the adjoint, for in this case there are no light scalars.

- If both $\chi_L$ and $s$ carry the adjoint representation and $\chi_R$ is a singlet, then $\mathcal{H} = SU([N/2]) \otimes SU(N - [N/2]) \otimes U(1)[16]$, and though vertices of the type $\Psi \psi A$ are excluded, those of the form $X \psi \psi$ will be present.

Theories where vertices of type $\psi \psi X$ are not eliminated will exhibit relatively large heavy physics effects at low energies in the form of chirality-preserving four-fermion interactions.

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8 Light masses are presumably generated by introducing other scalar field(s). As usual there will be a hierarchy problem [15] when attempting to maintain the gap between $\Lambda$ and the low energy scale.
B. Constraints form $SU(2)_L \times U(1)_Y$

As a final example I will consider the conditions under which the Standard Model gauge symmetry eliminates all the vertices in (2). This can be implemented by requiring that the none of the heavy fields in each term in (2) carries the same $SU(3) \times SU(2) \times U(1)$ representation as the factor containing light fields. For example, when considering the vertex $\psi \bar{\psi}X$, the product $\psi \bar{\psi}$ is either a fermion triplet of hypercharge $0$ or $\pm 1/3$, or an $SU(2)$ singlet of hypercharge $\pm n/3$, $0 \leq n \leq 5$ (assuming the vertex does not violate B-L and the presence of right-handed neutrinos). Then $\psi \bar{\psi}X$ is excluded if no $X$ carries any of these representations. Table I all the representations excluded in this way.

Note in particular that if the underlying theory is a spontaneously broken gauge theory and if absence of the vertices $\psi \bar{\psi}X$ and $\phi \bar{\phi}X$ is a consequence of the Standard Model gauge symmetry, then the underlying theory cannot contain a heavy vector-boson which is a Standard Model singlet. But the absence of such heavy vector-boson singlets implies that none of the group generators broken at scale $\Lambda$ commute with those of $SU(3) \times SU(2)_L \times U(1)$, and this implies that the Standard Model $SU(3)_c \times SU(2)_L \times U(1)_Y$ is the gauge group of the full theory.

IV. NUMERICAL CONSIDERATIONS AND CONCLUSIONS

The above arguments indicate that for weakly-coupled and decoupling theories there are (at least) two ways of hiding the heavy physics effects from current data. The first simply assumes that the scale of new physics is very large, thereby suppressing all heavy particle effects. The second assumes the absence of the vertices in (2) (protected by a symmetry) so as to strongly suppress all virtual effects, and a moderately high value for $\Lambda$ in order to explain the absence of direct heavy particle production.

These two approaches are not necessarily mutually exclusive. For example, we can assume that all effective operators that involve lepton-number violation are generated at scale $\Lambda_E$, while all operators that respect all discrete symmetries of the Standard Model are generated by physics with typical scale $\Lambda \ll \Lambda_E$. Then lepton-number violating effects can be suppressed by having $\Lambda_E^2 G_F \gg 1$, while the effects of operators generated at scale $\Lambda$ can be suppressed by insuring the absence of the vertices in (2), where the quantum numbers involved in each such vertex respects all the discrete symmetries of the Standard Model (this assumes that appropriate transformation properties can be assigned to the heavy particles).

Assuming the absence of tree-level effects one might reasonably ask how would existing limits be modified for models of the type considered in this paper. To estimate this note that the surviving effects occur via loops and appear with an additional factor of $\sim 1/(4\pi)^2$, so that a limit $\Lambda > M_{\exp}$, obtained when assuming the presence of tree-level generated operators, is replaced by $\Lambda > M_{\exp}/(4\pi)$: the existing limit is decreased by an order of magnitude. For processes that are not suppressed within the Standard Model the contributions from the heavy physics are suppressed by a factor $\sim (m_W/\Lambda)^2$ with respect to the Standard Model radiative corrections. Sensitivity to such effects requires a precision below 0.01%.

As a specific example consider the limits on “compositeness” obtained using operators of the form $(f/\Lambda^2) (\bar{\psi}_1 \gamma_\mu \psi_2) (\bar{\psi}_3 \gamma_\mu \psi_4)$, involving light left-handed fermions [4]. For example, for left-handed electrons, the existing limit is $\Lambda > 10^{10}\text{GeV}$ obtained using $f = 2\pi^2$; in the absence of tree-level generated operators, for which $f \sim 1/(4\pi^2)$, this is relaxed to $\Lambda > 360\text{GeV}$. Similarly, the limit on $\Lambda$ presented in [4] for all 4-fermion (current-current) operators are decreased by a factor $\sim 30$.

These results imply that there is a class of heavy-physics models for which all leading virtual effects can be naturally suppressed, and this leads to a considerable relaxation of the existing limits on the scale of new physics (by an order or magnitude or more). The simplest way of realizing this situation is to assume that all heavy excitations transform non-trivially under a certain (unknown) symmetry under which all Standard Model particles are singlets; which also implies that that near future accelerators will only pair-produce these heavy particles. Should this class of models describe the physics underlying the Standard Model, experiments will have at most weak indications of the presence and properties of any heavy excitations before they are directly produced.

APPENDIX A: ABSENCE OF VERTICES OF TYPE $XA\bar{\phi}\phi$

It is worth noting that the vertex $XA\bar{\phi}\phi$, is not included since it follows form the absence of vertex $X\phi\bar{\phi}$. To see this collect all the scalars in a large vector $\chi$ such that $\chi^T = (\phi^T, \Phi^T)$ (where $\Phi$ denotes the heavy scalar fields). Denote now the group generators for the (in general reducible) representation carried by $\chi$ by $\{T_a\}$, of which $\{T_I\}$ generate
the Standard Model gauge group while \( \{ T_r \} \) are the remaining ones. I can then choose

\[
T_I = \begin{pmatrix} \tau_I & 0 \\ 0 & t_I \end{pmatrix}, \quad T_r = \begin{pmatrix} U_r & N_r \\ N_r^\dagger & M_r \end{pmatrix}
\]

(A1)
since the Standard Model scalars transform irreducibly under the Standard Model gauge group.

Now, the scalar kinetic energy is

\[
|\partial \chi + iT_a V^a \chi|^2,
\]

(A2)

where \( V \) denotes all the gauge fields, \( X \) or \( A \), and where the gauge couplings have been absorbed into the gauge fields. Then the \( X \phi \phi \) and \( X A \phi \phi \) vertices are explicitly given by

\[
X^r \cdot \phi \partial^\mu U_r \phi \quad (X^r \cdot A^I) \phi \{U_r, \tau_I\} \phi
\]

(A3)
The first will be absent provided the matrix \( U_r \) vanishes, but this then implies that the vertex \( X A \phi \phi \) is also absent.

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