Investigation on Effective Sampling Strategy for Multi-objective Design Optimization of RBCC Propulsion Systems via Surrogate-assisted Evolutionary Algorithms

Tuan Quang Hoa,*, Hideaki Ogawa, Cees Bilb

School of Aerospace, Mechanical and Manufacturing Engineering, RMIT University, Melbourne, VIC 3001, Australia

Abstract

Rocket-based combined cycle (RBCC) engines are an airbreathing propulsion technology that offers considerable potential for efficient access-to-space. Successful design of RBCC-powered space transport systems requires reliable databases for both vehicle and engine performance, calling for an effective sampling method to accurately resolve non-linear characteristics in vast design space. This paper presents an optimal sampling strategy based on the function gradients to realize efficient database construction based on evolutionary algorithms and assesses its effectiveness by applying the methodology to various test functions with multiple objectives as well as surrogate models representing scramjet intake characteristics for validation.

Keywords: optimal sampling ; evolutionary algorithms ; rocket-based combined cycle ; hypersonic airbreathing propulsion

1. Introduction

Economical access to space is of increasing importance for the promotion of space development. A wide variety of propulsion systems have been contrived and examined to achieve efficient space transport. Rocket-based combined cycle (RBCC) engines are a class of propulsion technology that incorporate air-breathing elements into a rocket engine, operating in several different modes including the ejector, ramjet, scramjet and rocket mode to enable...
thrust production at various speeds and altitudes. By optimally transition through various operating modes, they can produce high trajectory-averaged specific impulse during ascension resulting in lower propellant mass fractions and lighter gross weight vehicles for a given mission, as compared to traditional rocket engines [1]. JAXA’s E3 (Fig. 1), in particular, was designed as a prototype for an RBCC engine to achieve four different combustion cycles, and tested in the Ramjet Engine Test Facility (RJTF) in Mach 0, 4 and 6 flight conditions [2].

While offering numerous advantages in efficiency and flexibility, the RBCC system inherently represents complexity in various aspects including the operation, structure and thermal protection due to the inclusion of multiple components. Integrating the engine into an airframe, an RBCC-powered vehicle is inevitably associated with highly nonlinear characteristics for both aerodynamic and propulsion performance. The development of RBCC-powered space transport consequently poses a challenge to a conventional design approach where separately optimized components are combined together, leading to the need for concurrent engineering where multi-objective optimization (MOO) or vector optimization plays a crucial role [3, 4].

Genetic algorithms offer practical advantages especially for complex optimization problems that would otherwise represent a formidable challenge to conventional local search strategies [5]. In particular, evolutionary algorithms (EAs) are relatively easy to implement and often provide adequate and desired solutions by generating a set of multiple Pareto optimal fronts in a single run due to its population-based nature, self-adaption and robustness [6, 7]. Despite these advantages, however, EAs are associated with several drawbacks such as high computational demand and difficulty of parameter adjustment [8]. Surrogate modeling has been introduced to efficiently mitigate these drawbacks by replacing expensive function evaluations with approximation from meta-analysis models [9].

An MOO study was recently conducted for the trajectory optimization of TSTO (two-stage-to-orbit) space transport with RBCC propulsion via evolutionary algorithms incorporating pseudospectral methods [10]. The aerodynamic characteristics were estimated by interpolating the aerodynamic coefficients from a database generated by first-order CFD (computational fluid dynamics) simulations in conjunction with wind tunnel experiments [11], whereas the engine performance was predicted based on a database from separate analytical evaluations [12]. While considerable potential of RBCC-powered TSTO for access-to-space was demonstrated, databases with enhanced reliability are desired in order to assess the capability of the system more accurately by resampling the aerodynamic and propulsion characteristics in the design space via high-fidelity CFD simulations. The Monte-Carlo method or Latin Hypercube Sampling (LHS) are typically used for DOE (design of experiments) sampling, which tends to be prohibitive due to vast design space and three-dimensional viscous CFD simulations including chemical reactions.

The current study is undertaken to develop an optimal sampling strategy to effectively reduce the computational cost while keeping fidelity of the evaluations by focusing on the domains in the design space where the objective functions vary rapidly or change in tendency. This can be achieved by exploring the design space, directed by the
gradients or sensitivity of the output (performance parameters) to the design parameters, so that sampling can be
focused on the regions of interest by producing more offspring in the EA-based optimization process. The sampling
strategy is applied to test functions for various MOO problems as well as the sampling of scramjet intake
characteristics using surrogate data from a past study in order to verify its effectiveness.

### Nomenclature

- $x$: decision variable
- $f$: output of design problem
- $n_x$: number of decision variables
- $n_f$: number of objective functions
- $N$: population size
- $\alpha$: angle of $\partial f / \partial x$
- $\Delta \Gamma$: gradient function
- $F$: objective function
- $\eta_B$: compression efficiency
- $dp/ds$: pressure gradient

2. **Approaches**

2.1. **Optimization process**

The optimization study is conducted by employing the state-of-the-art surrogate-assisted evolutionary algorithms
(SAEAs), where individuals in the population pool evolve over generations via various genetic operations including
selection, recombination, reproduction and mutation. The optimization process is described in Fig. 2. The
optimization algorithms assess and rank the individuals in order to determine the population members for the next
generation. Multiple surrogate models are employed to assist the process by estimating the values of the objective
and constraint functions to replace actual function evaluations with various meta models such as response surface
models, radial basis function networks, kriging and multilayer perceptions [13].

In this study the design space is explored via the optimal sampling strategy, depending on the sensitivity of the
output of the design problem to the input (design parameters). This allows the search to adapt to and focus on the
region of interest by producing more individuals in the direction to maximize the sensitivity of the objective
functions, which is defined as their gradients to the design parameters.

![Fig. 2. Optimization chain for optimal sampling.](image)

2.2. **Objective functions**

This section describes the formulation of the objective functions for optimal sampling defined as the sensitivity
of the output of the considered design problem to the design parameters as the input. At the beginning of each
generation in the optimization, the output values ($f$) for the original problem are calculated for given input
parameters ($x$). To allow fair evaluation, both input and output values are normalized with respect to their minimum
and maximum values so that they all fall within the range $[0, 1]$. 
The gradients of the output to input are calculated to obtain the objective functions of the present study. An inverse trigonometric function \( \tan^{-1} \) is introduced here for the slope angle \( \alpha \) so as to confine the range within \([ -\pi/2, \pi/2 ]\) rather than \([ -\infty, \infty ]\), where infinity occurs in the case of identical input or output values. The gradient for an individual with respect to another individual is thus calculated as:

\[
\alpha_{i,j}(k,n) = \tan^{-1} \left( \frac{\partial f_n(i,j)}{\partial x_k(i,j)} \right)
\]

(1)

Where \( k \) and \( n \) are the indices for the decision variables and objective functions, respectively.

The angle function is defined as the amplitude of the summation of the angle function squared for all individuals.

The gradient function \( \Delta \Gamma \) of the present individual \( i \) for the \( n \)th objective function is then calculated as the sum of the angle function for all decision variables as follows:

\[
\Delta \Gamma_{i,n} = \sum_{k=1}^{n_x} \sum_{j=1}^{N} (\alpha_{i,j})^2
\]

(2)

Where \( n_x \) is the number of the decision variables.

The arithmetic mean of the gradient function is used as the quantity to be maximized for optimal sampling:

\[
F_n = \frac{1}{N} \sum_{i=1}^{N} \Delta \Gamma_{i,n}
\]

(3)

By converting from the maximization problem, the objective functions to be minimized simultaneously are defined as:

\[
F_{\text{objective}} = \begin{bmatrix} F_1 & F_2 & \ldots & F_{n_f} \end{bmatrix}
\]

(4)

Where \( n_f \) is the number of the objective functions.

3. Results

3.1. Test functions

A set of test functions for MOO are employed in order to verify the effectiveness of the optimal sampling strategy developed here. Table 1 shows the three selected test functions, namely, the Schaffer function N.1, Poloni’s bi-objective function, and Viennet function, along with the number of the objective functions chosen among those commonly used for optimization studies [14]. The definitions of the functions are formulated in Table 2. Both EA-based optimization and optimal sampling are performed by employing a population size of 400 over 20 generations for all test functions.

| Test Functions            | Decision variables | Objective functions | Constraint functions |
|--------------------------|--------------------|---------------------|---------------------|
| Schaffer function N.1    | 1                  | 2                   | 0                   |
| Poloni’s bi-objective function | 2              | 2                   | 0                   |
| Viennet function         | 2                  | 3                   | 0                   |

Table 1. Summary of test functions.
Table 2. Definition of test functions.

| Test Functions          | Decision variables | Objective functions                                                                 |
|-------------------------|--------------------|-------------------------------------------------------------------------------------|
| Schaffer function N.1   | $-10 \leq x_i \leq 10$ | Minimize: $f_1(x_i) = x_i^2$ 
$ f_2(x_i) = (x_i - 2)^2$ |
|                         |                    | Minimize: $f_1(x_i, x_j) = [1 + (A_i - B_i (x_i, x_j))^2 + (A_j - B_j (x_i, x_j))^2]$ 
$ f_2(x_i, x_j) = (x_i + 3)^2 + (x_j + 1)^2$ |
| Poloni’s bi-objective function | $-\pi \leq x_i, x_j \leq \pi$ | $A_i = 0.5\sin(1) - 2\cos(1) - \sin(2) - 1.5\cos(2)$ 
$A_j = 1.5\sin(1) - \cos(1) + 2\sin(2) - 0.5\cos(2)$ 
$B_i (x_i, x_j) = 0.5\sin(x_i) - 2\cos(x_i) + \sin(x_j) - 1.5\cos(x_j)$ 
$B_j (x_i, x_j) = 1.5\sin(x_i) - \cos(x_i) + 2\sin(x_j) - 0.5\cos(x_j)$ |
|                         |                    | Minimize: $f_1(x_i, x_j) = 0.5(x_i^2 + x_j^2) + \sin(x_i^2 + x_j^2)$ 
$ f_2(x_i, x_j) = \frac{(3x_i - 2x_j + 4)^2}{8} + \frac{(x_i - x_j + 1)^2}{27} + 15$ 
$ f_3(x_i, x_j) = \frac{1}{x_i^2 + x_j^2 + 1} - 1.1\exp\left(-\left(x_i^2 + x_j^2\right)\right)$ |
| Viennet function        | $-3 \leq x_i, x_j \leq 3$ | Minimize: $f_1(x_i, x_j) = 0.5(x_i^2 + x_j^2) + \sin(x_i^2 + x_j^2)$ 
$ f_2(x_i, x_j) = \frac{(3x_i - 2x_j + 4)^2}{8} + \frac{(x_i - x_j + 1)^2}{27} + 15$ 
$ f_3(x_i, x_j) = \frac{1}{x_i^2 + x_j^2 + 1} - 1.1\exp\left(-\left(x_i^2 + x_j^2\right)\right)$ |

3.2. Optimal sampling with Schaffer function N.1

Plotted in Fig. 3 are the optimization results comprising a distinct Pareto optimal front for the Schaffer function N.1 defined by 2 objectives and 1 decision variable on the left (a), along with the results from the present optimal sampling strategy on the right hand side (b). It can be seen that the sampling efforts are concentrated on the upper side of the curve as the population evolves (indicated by symbols in darker color), eventually converging at the top-right corner of the curve located at $(x_1, x_2) = (100, 144)$. In order to verify this solution, the first-order differentiation is performed for the two quantities of the Schaffer function N.1 with respect to the sole decision variable $x_1$. The graphs in Fig. 4(a) indicate that the amplitude of the differentiated functions become maximum at $x_1=0$ for both $|\partial f_1/\partial x_1|$ and $|\partial f_2/\partial x_1|$ simultaneously. The variations of the two functions plotted in Fig. 4 (b) shows that the values at $x_1=0$ are $(f_1, f_2) = (100, 144)$, indeed corresponding to values at the top-right corner in Fig. 3 (a).

Fig. 3. (a) Optimization results of Schaffer function N.1; (b) Optimal sampling results for Schaffer function N.1.
3.3. Optimal sampling with Poloni’s bi-objective function

Poloni’s function with 2 objectives and 2 decision variables is selected as the second test function. The results from the optimization via evolutionary algorithms are shown in Fig. 5 (a), while the sampled points via the present methodology are displayed in Fig. 5 (b). The distributions of the differentiated functions are plotted in Fig. 6 (a) and Fig. 6 (b) for the first-order differentiations of the first equation \( f_1 \) with respect to \( x_1 \) and \( x_2 \), respectively. It can be seen that \( \partial f_1 / \partial x_1 \) is characterized by peaks at \( (x_1, x_2) = (-1.84, -0.94) \) and \( (0.56, -1.04) \), while \( \partial f_1 / \partial x_2 \) becomes maximum at \( (x_1, x_2) = (-0.54, -2.24) \) and \( (-0.64, 0.16) \). The concentric contours around \( (x_1, x_2) = (-0.64, -1.04) \) in Fig. 6 (c) indicate that the function \( f_1 \) varies between about 10 and 61.6 in the circumference of the regions where \( |\partial f_1 / \partial x_1| \) and \( |\partial f_1 / \partial x_2| \) mark peaks. The first-order differentiations of the second function \( f_2 \), on the other hand, is a linear function of \( x_1 \) and \( x_2 \), as seen in Fig. 7 (a) and Fig. 7 (b), where it is found that \( |\partial f_2 / \partial x_1| \) and \( |\partial f_2 / \partial x_2| \) become maximum at \( x_1 = \pi \) and \( x_2 = \pi \), respectively. The function \( f_2 \) marks its peak value of 53.2 at \( (x_1, x_2) = (\pi, \pi) \). The net effects are observed in Fig. 5 (b), where the optimal sampling in the effort of maximizing both \( |\partial f_1 / \partial x| \) and \( |\partial f_2 / \partial x| \) has resulted in a front along the upper curve, confined by the upper bound of the function \( f_2 \), while the function \( f_1 \) has more freedom to vary to maximize \( |\partial f_1 / \partial x| \).
indicates that the first function has peaks on the longitudinal circumference of the circles with radii of approximately 2.5 and 3.5, while becomes maximum on the latitudinal circumference of the same circles. The distribution in Fig. 9 (c) also has a maximum value of 60 at (x_1, x_2) = (3, -3), as seen in Fig. 10 (b) and Fig. 10 (c). The function has a maximum value of 60 at (x_1, x_2) = (3, -3), as seen in Fig. 10 (c). The first-order differentiations of the third function displayed in Fig. 11 feature similar characteristics to those of the first function in Fig. 9, in that the maximum occurs on the longitudinal and latitudinal circumference of a circle for the differentiation with respect to x_1 and x_2, respectively. The distribution plotted Fig. 11 (c) indicates that f_3 varies between 0 and 0.15 around the zones where \( |\partial f_3/\partial x_1| \) and \( |\partial f_3/\partial x_2| \) become maximum. The compound effects of the trends observed are the progression of the sampled points in Fig. 8 (b), where optimal sampling has advanced in the direction to maximize \( |\partial f_1/\partial x|, |\partial f_2/\partial x| \) and \( |\partial f_3/\partial x| \) all, yet restricted by the definition of the functions.

Fig. 6. (a) Distribution of 1st-order differentiation of Poloni’s function \( \partial f_1/\partial x_1 \); (b) Distribution of 1st-order differentiation of Poloni’s \( \partial f_1/\partial x_2 \) (c) Distribution of Poloni’s bi-objective function \( f_1 \) with respect to \( x_1 \) and \( x_2 \).

Fig. 7. (a) Distribution of 1st-order differentiation of Poloni’s function \( \partial f_2/\partial x_1 \); (b) Distribution of 1st-order differentiation of Poloni’s \( \partial f_2/\partial x_2 \); (c) Distribution of Poloni’s bi-objective function \( f_2 \) with respect to \( x_1 \) and \( x_2 \).

3.4. Optimal sampling with Viennet function

The Viennet function comprising 3 objective functions for 2 decision variables is employed as the last test function to apply the optimal sampling method. Fig. 8 shows the results obtained from the tri-objective optimization (a) and the points that have resulted from optimal sampling (b). The distributions of the differentiated first function \( f_1 \) are plotted in Fig. 9 (a) and Fig. 9 (b) for the differentiations with respect to \( x_1 \) and \( x_2 \), respectively. \( |\partial f_1/\partial x_1| \) has peaks on the longitudinal circumference of the circles with radii of approximately 2.5 and 3.5, while \( |\partial f_1/\partial x_2| \) becomes maximum on the latitudinal circumference of the same circles. The distribution in Fig. 9 (c) indicates that the first function \( f_1 \) varies between 4 and 8 on the concentric circles in the regions of maximum \( |\partial f_1/\partial x_1| \) and \( |\partial f_1/\partial x_2| \) bounded by the upper and lower limits of \( x_1 \) and \( x_2 \). The differentiations of the second function \( f_2 \), on the other hand, are characterized by linear distributions for both \( x_1 \) and \( x_2 \), peaking both at \((x_1, x_2) = (3, -3)\), as seen in Fig. 10 (b) and Fig. 10 (c). The function \( f_2 \) also has a maximum value of 60 at \((x_1, x_2) = (3, -3)\), as seen in Fig. 10 (c). The first-order differentiations of the third function \( f_3 \) displayed in Fig. 11 feature similar characteristics to those of the first function in Fig. 9, in that the maximum occurs on the longitudinal and latitudinal circumference of a circle for the differentiation with respect to \( x_1 \) and \( x_2 \), respectively. The distribution plotted Fig. 11 (c) indicates that \( f_3 \) varies between 0 and 0.15 around the zones where \( |\partial f_3/\partial x_1| \) and \( |\partial f_3/\partial x_2| \) become maximum. The compound effects of the trends observed are the progression of the sampled points in Fig. 8 (b), where optimal sampling has advanced in the direction to maximize \( |\partial f_1/\partial x|, |\partial f_2/\partial x| \) and \( |\partial f_3/\partial x| \) all, yet restricted by the definition of the functions.
Fig. 8. (a) Optimization results of Viennet function; (b) Optimal sampling results for Viennet function.

Fig. 9. (a) Distribution of 1st-order differentiation of Viennet function $\frac{\partial f_1}{\partial x_1}$; (b) Distribution of 1st-order differentiation of Viennet $\frac{\partial f_1}{\partial x_2}$; (c) Distribution of Viennet function $f_1$ with respect to $x_1$ and $x_2$. 
Optimal sampling for scramjet intake performance

The methodology developed here is applied to the sampling of the axisymmetric scramjet intake in order to assess the capability for a practical example toward the application to the RBCC E3 propulsion system. The surrogate models trained in a preceding study [15] are used to represent the characteristics of the scramjet intake, where the compression efficiency, drag, and maximum adverse pressure gradient are predicted as functions of 6 design parameters representing the intake geometry in lieu of CFD evaluations (the optimization problem is simplified from the original one by removing the constraint on the exit temperature). Displayed in Fig.5 are the results from a surrogate-assisted tri-objective design optimization for the scramjet intake [15], along with all individuals considered in the process of optimal sampling due to the present approach (optimization has been performed up to 50 generations with a population size of 96 in both cases). The results from the optimal sampling are also presented in Fig. 13 in planar views from three perspectives. It is notable that the sampling has been advanced with a distinct orientation, steered by the present sampling strategy. The sampled points have converged and collapsed on to a pronounced string at the final generation, forming a ridge-shaped curve, which is assumed to be a cluster of the design points where the intake performance parameters (i.e., compression efficiency, intake drag, maximum adverse pressure gradient) are the most sensitive to the change in the intake geometry represented by the decision variables. Retraining of the surrogate models by solely using the sampled points have resulted in root-mean-square errors of 0.79%, 1.00%, and 0.84% via the kriging model for the compression efficiency, intake drag, and maximum adverse pressure gradient, respectively. This demonstrates the constructability of the database by applying surrogate modeling to the information gathered by means of the optimal sampling technique.
Fig. 12. (a) Original results from scramjet intake optimization [15]; (b) Results from optimal sampling for scramjet intake optimization.

Fig. 13. Results from optimal sampling for scramjet intake optimization from (a) perspective 1; (b) perspective 2; (c) perspective 3.

4. Conclusions and future work

A sampling strategy has been developed, aiming to enable effective database construction to be used for the design of space transport systems. Employed in conjunction with population-based optimization via evolutionary algorithms, it steers the direction of data point sampling toward the domain where the design output parameters vary sensitively to the input parameters in the exploration of the design space. The gradients of the design output are used as the objective functions to be maximized in the optimization process.

The optimal sampling approach proposed here has been applied to three test functions consisting of a set of functions, which are used as to yield design output for given input parameters represented by the decision variables of the multi-objective design optimization problems. It resulted in the concentrations of sampled data points in the regions where the gradients of the output parameters tend to be maximum according to analytical evaluations for all test functions, validating the effectiveness of the current sampling methodology. It has then been utilized for the sampling of the characteristics of an axisymmetric scramjet intake predicted by surrogate models trained in a preceding study. A reasonable concentration of sampled points has been produced as a result, demonstrating its capability in a practical design problem.

Future work includes the extension of the present sampling method to constrained design problems so that it can be used to build a database for the design and trajectory optimization of spaceplanes powered with rocket-based combined cycles by making use of high-fidelity computational fluid dynamics simulations.
References

[1] J. R. Olds, J. E. Bradford, SCCREAM: A Conceptual Rocket-Based Combined-Cycle Engine Performance Analysis Tool, Journal of Propulsion and Power. 2001, 17(2) 333-339.

[2] S. Hasegawa, K. Tani, Numerical Analysis of Fuel Injection Effects of the RBCC Engine in Ramjet Mode under Flight Mach 4 Condition, 46th AIAA/ASME/SAE/ASEE Joint Propulsion Conference & Exhibit. Nashville, TN, 2010.

[3] R. T. Marler, J. S. Arora Survey of multi-objective optimization methods for engineering, Structural and multidisciplinary optimization, 2004, 26(6) 369-395.

[4] D.A. Savic, Single-objective vs. multiobjective optimisation for integrated decision support, Proceedings of the First Biennel Meeting of the International Environmental Modeling and Software Society, 2002, 1, 7-12.

[5] Grosan, A. Abraham, H. Ishibuchi, Hybrid Evolutionary Algorithms, Studies in Computational Intelligence, Vol. 75, Springer, Berlin, Germany, 2007.

[6] C. C. Coello, G. B. Lamont, D. A. Van Veldhuizen, Evolutionary Algorithms for Solving Multi-Objective Problems. 2nd ed, Genetic and Evolutionary Computation Series, Springer, US, 2007

[7] C. M. Fonseca, P. J. Fleming, An Overview of Evolutionary Algorithms in Multiobjective Optimization, Evolutionary computation, 1995, 3(1) 1-16.

[8] T. Blickle, Theory of Evolutionary Algorithms and Application to System Synthesis, Computer Engineering and Networks Laboratory, vdf Hochschulverlag AG, 1997

[9] Y. S. Ong, P. B. Nair, A. J. Keane, Evolutionary Optimization of Computationally Expensive Problems via Surrogate Modeling, AIAA Journal, 2003, 41(4) 687-696.

[10] H. Ogawa, M. Kodera, S. Tomioka, S. Ueda, Multi-Phase Trajectory Optimization for Access-to-Space with RBCC-Powered TSTO via Surrogated-Assisted Hybrid Evolutionary Algorithms Incorporating Pseudo-Spectral Methods, AIAA Aviation. Atlanta, GA, 2014.

[11] S. Nomura, K. Hozumi, I. Kawamoto, Y. Miyamoto, Experimental studies on aerodynamic characteristics of SSTO vehicle at subsonic to hypersonic speeds, 16th International Symposium on Space Technology and Science, Sapporo, JP, 1988.

[12] S. Tomioka, T. Hiraia, T. Saito, K. Kato, M. Kodera, K. Tani, System Analysis of a Hydrocarbon-fueled RBCC engine applied to a TSTO Launch Vehicle, 29th International Symposium on Space Technology and Science, Nagoya, JP, 2013.

[13] N. V. Queipo, R. T. Haftka, W. Shyy, T. Goel, R. Vaidyanathan, and P. Kevin Tucker, Surrogate-based analysis and optimization, Progress in Aerospace Sciences, 2005, 41(1) 1-28.

[14] Test functions for optimizations, 2014 [cited 2014 Aug 10], Available from: http://en.wikipedia.org/wiki/Test_functions_for_optimization.

[15] H. Ogawa, R. R. Boyce, Physical Insight into Scramjet Inlet Behavior via Multi-Objective Design Optimization, AIAA Journal, 2012, 50(8) 1773-1783.