Baryon effects on the dark matter halos constrained from strong gravitational lensing

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\textbf{ABSTRACT}

Simulations are expected to be the powerful tool to investigate the baryon effects on dark matter (DM) halos. Recent high resolution, cosmological hydrodynamic simulations (Di Cintio et al. 2014, DC14) predict that the inner density profiles of DM halos depend systematically on the ratio of stellar to DM mass ($M_*/M_{\text{halo}}$) which is thought to be able to provide good fits to the observed rotation curves of galaxies. The DC14 profile is fitted from the simulations which are confined to $M_{\text{halo}} \leq 10^{12} M_\odot$, in order to investigate the physical processes that may affect all halos, we extrapolate it to much larger halo mass, including that of galaxy clusters. The inner slope of DC14 profile is flat for low halo mass, it approaches 1 when the halo mass increases towards $10^{12} M_\odot$ and decreases rapidly after that mass. We use DC14 profile for lenses and find that it predicts too few lenses compared with the most recent strong lensing observations SQLS (Inada et al. 2012). We also calculate the strong lensing probabilities for a simulated density profile which continues the halo mass from the mass end of DC14 ($\sim 10^{12} M_\odot$) to the mass that covers the galaxy clusters (Schaller et al. 2015, Schaller15), and find that this Schaller15 model predict too many lenses compared with other models and SQLS observations. Interestingly, Schaller15 profile has no core, however, like DC14, the rotation curves of the simulated halos are in excellent agreement with observational data. Furthermore, we show that the standard two-population model SIS+NFW cannot match the most recent SQLS observations for large image separations.

\textbf{Key words:} Cosmology: theory—dark matter—galaxies: halos—gravitational lensing: strong

\section{1 INTRODUCTION}

It is well-known that General Relativity (GR) is very successful on small scales like our solar system. When applied to scales of galaxies and larger, however, some exotic ingredients are needed to explain our observations. For example, when GR (its weak gravitational field form) is applied to galaxies, we need cold dark matter (CDM) to explain the observational data of rotation curves; when applied to cosmology, we need dark energy (DE) to explain the ongoing accelerating expansion of our Universe. To account for the cosmic structure formation and gravitational lensing in the context of ΛCDM cosmology, both CDM and DE are needed. There are no direct observational evidences supporting the existence of CDM and DE. Their properties are assumed so that the usual astronomical observations can be interpreted reasonably based on GR (and thus Newtonian theory of gravity).

In this paper, we focus on the properties of CDM, and consider only the observational constraints arising from rotation curves and strong lensing. Early optical and 21 cm line of neutral hydrogen observations for late-type disk galaxies all indicate the property of having an almost constant rotation velocity in their outer parts. If Newtonian theory of gravity is correct, the flat rotation curves suggest the existence of some non-baryonic matter, called dark matter (DM), surrounding each observable galaxy as a dark halo. Other observations and structure formation theory require that DM is cold, that is, the DM particles are massive and their random velocity is small. Furthermore, it turns out that the amount of CDM is at least several times larger in mass than observable baryonic matter. Therefore, the total density profiles of galaxies and clusters of galaxies are CDM dominated, the usual baryonic matter (usually resides in the central region) can play the role of changing the inner slope, the importance of which depends on the amount it contributes to the total mass (Schaller et al. 2015a, Xu et al. 2016, Yannick et al. 2017). For later considerations, we use...
the density profiles of DM halos to stand for the total mass distributions. The observational data for flat rotation curves can be well-described if the mass density profile of CDM particles is modeled as the singular isothermal sphere (SIS): \( \rho \propto r^{-2} \), when we observe the outer parts of disk galaxies. Interestingly, this steep, singular power-law model is preferred by strong gravitational lensing for giant elliptical galaxies. On the other hand, however, recent high-resolution rotation velocity associated with dark matter in the inner parts of disk galaxies indicates the presence of constant density DM cores. In fact, it is now established that the cored isothermal sphere (CIS) fit well the observed rotation curves, both in the inner and outer parts of disk galaxies (de Blok 2010).

\[ \rho_{\text{CIS}}(r) = \frac{\rho_0}{1 + (r/r_c)^2}, \]

where \( \rho_0 \) is the central density, and \( r_c \) is the core radius of a halo. Unfortunately, CIS model cannot match strong lensing observations.

Gravitational lensing provides a powerful tool to detect DM, although it is not sensitive to whether the mass doing the lensing is baryonic or dark, but rather simply depends on the total. For a certain given mass of a lensing galaxy or galaxy cluster, strong lensing efficiency is very sensitive to the slope \( \gamma \) of the central total mass density profile \( (\rho \propto r^{-\gamma}) \). For example, a cored density profile like CIS for a reasonable value of the core radius \( r_c \) (usually determined by rotation curves) would lead to an extremely low lensing rate compared with lensing observations, while singular isothermal sphere (SIS, \( \gamma = 2 \), for elliptical galaxies) matches observations well. As for galaxy clusters as lenses, NFW (Navarro, Frenk, & White 1996, 1997; \( \gamma = 1 \)) has been used as a good model, but as will be demonstrated in this paper, the most recent strong lensing observations require a steeper slope. It should be pointed out that dark matter halos are triaxial rather than spherical (Jing et al. 2002; Despali et al. 2014; Bonamigo et al. 2015; Despali et al. 2017). The ellipticity would significantly increase the lensing efficiency (Bartelmann et al. 1998; Meneghetti et al. 2001; 2003; Hennawi et al. 2007; Broadhurst & Barkana 2008), but not so important compared with the inner slope. For example, Giocoli et al. (2013) present MOKA, a new algorithm for simulating the gravitational lensing signal from cluster-sized halos, and find that the strong lensing cross sections depend most strongly on the concentration and on the inner slope of the density profile of a halo, followed in order of importance by halo triaxiality and the presence of a bright central galaxy.

The observations of rotation curves and strong lensing can only be used to constrain the density profile of halos, not directly the properties of CDM particles. It is the structure formation theory, mainly through computer simulations, that determines what the properties of CDM should be assumed so that the density profile can be correctly predicted. For example, if CDM is self-interacting, or DM particles are warm (e.g., Shao et al. 2012), a cored density profile can be created even without baryons. In the standard, hierarchical, CDM paradigm of cosmological structure formation, galaxy formation begins with the gravitational collapse of over dense regions into bound, virialized halos of CDM. In this ACDM paradigm, halos form from purely collisionless DM particles with primordial power spectrum of fluctuations predicted by inflationary model. Small halos are the first to form, and larger halos form subsequently by mergers of pre-existing halos and by accretion of diffuse dark matter that has never been part of a halo. In a simplified picture (White & Rees 1978), baryonic gas is initially well mixed with the DM particles, then participates in the gravitational collapse of DM and is heated by shocks to the virial temperature of the DM halos. Bound in the potential wells of DM halos, baryonic gas proceed to cool radiatively due to bremsstrahlung, recombination and collisionally exited line emission (Frenk & White 2012).

A full analytic description of the development of such dissipationless hierarchical clustering came in the early 1990’s with extensions of the original Press-Schechter model based on excursion set theory (Bower 1991; Bond et al. 1991; Lacey & Cole 1993; Kauffmann & White 1993). The halo mass function derived in this analytic theory is in well agreement with that from DM only simulations (Sheth & Tormen 1999; Jenkins et al. 2001; Warren et al. 2006; Reed et al. 2007; Tinker et al. 2008; Crocce et al. 2010; Courin et al. 2014; Angulo et al. 2014; Watson et al. 2013; Despali et al. 2016). We need such mass function in our lensing probability calculations.

The current computer simulations are the most robust tools to explore the formation and evolution of the large scale structure of the universe (Frenk & White 2012). In the ACDM paradigm, purely CDM N-body simulations can reproduce the observed cosmic web as demonstrated by Sloan Digital Sky Survey (SDSS). In this scenario, observable galaxies made-up of baryons form at the centers of DM halos. Unfortunately, the cosmic gas (the initial form of baryons), and the subsequent star formation processes, are poorly understood. Based on N-body technique, there are mainly two complementary methods for simulating the galaxy formation. The direct inclusion of the baryonic component and all the astrophysical processes affecting it, known as numerical hydrodynamic method, is too computationally taxing to perform for large samples of galaxies (Teyssier 2002; Springel 2005; Commeron, Debou & Teyssier 2014; Tescari et al. 2014; Pakmor et al. 2016; Katsianis et al. 2017). This method can treat reliably only a subset of the relevant gas physics, such as the shock heating of gas and its subsequent radiative cooling. Another method is known as semi-analytic modeling. The main difference with the direct simulation is that, instead of solving the equations of hydrodynamics directly, one employs a simple, spherically symmetric model in which the gas is assumed to have been fully shock-heated to the virial temperature of each halo, so that its cooling and accretion can be accurately calculated. This phenomenological treatments of baryonic processes are based on physical insights gained from simulations of individual systems and from observations. Uncertainty parameters such as the efficiency of star formation and stellar feedback can be adjusted to reproduce the observed properties of all types of galaxies and clusters of galaxies.

Over the past decades, a range of studies have associated galaxies with DM halos at a given epoch, using a variety of techniques, including halo occupation distribution modeling (e.g., Berlind & Weinberg 2002; Bullock et al. 2002), the conditional luminosity function modeling (e.g.,
Yang et al. (2003), and variants of the abundance matching technique (e.g., Kravtsov & Klypin 1999; Neyrinck et al. 2004, Tasitsiomi et al. 2004, Conroy et al. 2006), which relate the observed properties of galaxies to their formation histories in a hierarchical manner.

Recent high resolution cosmological hydrodynamic simulations (Di Cintio et al. 2014, DC14) introduce a mass-dependent density profile to describe the distribution of dark matter within galaxies, which takes into account the stellar-to-halo mass ratio (\(M_*/M_{\text{halo}}\)) dependence of baryon effects on DM. Using a Markov Chain Monte Carlo approach, Katz et al. (2016) found that the DC14 model provides better fits to the most recent observed rotation curves of galaxies over a large range of luminosity and surface brightness than do halo models which neglect baryonic physics (i.e., NFW).

In this paper we apply the DC14 model to the strong lensing probability calculations and compare the results with the most recent strong lensing observations. Although the most recent rotation curves for disk galaxies support the D14 model, however, there are no direct evidences arising from the simulations show us that all the central galaxies of the final halos are disk galaxies. In fact, similar to the previous work in the literature (e.g., Shapiro, Iliev & Raga 1999, Mashchenko, Couchman & Wadsley 2008), the morphological types of galaxies are unconcerned in these simulations. The combined baryon effects such as gas cooling, stellar feedback and dynamical friction are modeled to reshape the inner slope of the density profile of DM halos which initially have the functional form of NFW, rather than to distinguish disk galaxies from ellipticals. The main difference with previous similar work is that DC14 profile depends on the halo mass. At low mass end, each halo display a central core, and for halos with increasing mass, some astrophysical processes erase the central cores and steepen the inner slopes of the DM density profiles. So it would be interesting to check whether or not the DC14 model, which is centrally steepened for \(M_{\text{halo}} \sim 10^{12} M_\odot\), is able to describe the massive galaxies.

Clearly, it would be very helpful for us to understand the baryon effects on DM distributions if we have a simulated density profile which continues the halo mass from the mass end of DC14 (\(\sim 10^{12} M_\odot\)) to the mass that covers the galaxy clusters. One such example is the investigation for the internal structure and density profiles of halos of mass \(10^{10} - 10^{14} M_\odot\) in the Evolution and Assembly of Galaxies and their Environment (EAGLE) simulations (Schaller et al. 2013, Schaller15). These follow the formation of galaxies in a LCDM cosmology and include a treatment of the baryon physics thought to be relevant. As desired, in this mass range the total density profile is similar to NFW in the inner and outer parts, but has a slope of \(-2\) at some radius \(r_\sim 2.27\) kpc (approximately independent of the total mass) relatively near the centers of halos. We calculate the lensing probabilities corresponding to the Schaller15 model and compare the results with other models and observations.

For comparisons, we also demonstrate the lensing probabilities for SIS + NFW and DC14(\(\beta = \gamma = 2\)) + NFW models. We compare these results with observation of JVAS/CLASS survey and the Sloan Digital Sky Survey Quasar Lens Search (SLS, Inada et al. 2012). We adopt the most generally accepted values of the parameters for flat LCDM cosmology, for which, with usual symbols, \(\Omega_m = 0.27, \Omega_\Lambda = 0.73, h = 0.75\) and \(\sigma_8 = 0.8\).

This paper is organized as follows: in Section 2 we present the lensing equations for DC14 and Schaller15 models. We calculate the lensing probabilities for different profiles and compare them with observations in Section 3. The discussions and conclusions are presented in Section 4.

## 2 Lensing Equations

### 2.1 DC14 Model

The DC14 model is derived by fitting the so-called \((\alpha, \beta, \gamma)\) double-power-law model to the simulations, the resultant density profile is

\[
\rho(r) = \frac{\rho_s}{\left(\frac{r}{r_s}\right)^\beta \left[1 + \left(\frac{r}{r_s}\right)^\alpha\right]^{(\beta-\gamma)/\alpha}},
\]

where \(\rho_s\) is the scale density and \(r_s\) the scale radius, and

\[
\alpha = 2.94 - \log_{10}[(10^{X+2.33})^{-1.08} + (10^{X+2.33})^{2.29}]
\]

\[
\beta = 4.23 + 1.34X + 0.26X^2
\]

\[
\gamma = -0.06 + \log_{10}[(10^{X+2.56})^{-0.68} + (10^{X+2.56})^{3}]
\]

where \(X = \log_{10}(M_*/M_{\text{halo}})\) and the mass range of validity of \(\alpha, \beta, \gamma\) is \(-4.1 < \log_{10}(M_*/M_{\text{halo}}) < -1.3\). For our purpose, we need to know \(\alpha, \beta, \gamma\) as functions of the halo mass \(M_{\text{halo}}\). Fortunately, we have a good fitting formula at hand (Guo et al. 2011)

\[
M_*/M_{\text{halo}} = 0.129 \left[\left(\frac{M_{\text{halo}}}{M_0}\right)^{-0.926} + \left(\frac{M_{\text{halo}}}{M_0}\right)^{0.261}\right]^{-2.440}
\]

where \(M_0 = 10^{11.4} M_\odot\). This formula is valid when the halo mass ranges from \(10^{10.8} M_\odot\) to \(10^{14.9} M_\odot\), a range that dominates the strong lensing probabilities.

As usual, we define the mass of a halo to be the mass within \(r_{200}\) (which is the radius of a sphere around a DM halo within which the average mass density is 200 times the critical mean mass density of the universe),

\[
M_{\text{halo}} = 4\pi \int_0^{r_{200}} \rho r^2 dr = 4\pi \rho_s r_s^2 f(c_1),
\]

with \(c_1 = r_{200}/r_s\) the concentration parameter, and

\[
f(c_1) = \int_0^{c_1} x^2 dx / [x^\gamma (1 + x^\alpha)^{\beta/\gamma}]^{1/\alpha}.
\]

In flat LCDM cosmology, the parameters \(\rho_s\) and \(r_s\) can be expressed as (Chen 2003a),

\[
\rho_s = \rho_c \left[\Omega_m (1 + z)^3 + \Omega_\Lambda\right]^{200} c_1^3 f(c_1),
\]

\[
r_s = 1.626 \left(\frac{M_1}{\Omega_m (1 + z)^3 + \Omega_\Lambda}\right)^{1/3} h^{-1}\text{Mpc},
\]

where \(\rho_c\) is the present value of the critical mass density of the universe, and \(M_1\) is the reduced mass of a halo defined as \(M_1 = M_{\text{halo}}/(10^{13} h^{-1} M_\odot)\).

The surface mass density for the DC14 profile is

\[
\Sigma(x) = 2\rho_s r_s V(x)
\]

where

\[
\rho_s = \rho_c \left[\Omega_m (1 + z)^3 + \Omega_\Lambda\right]^{200} c_1^3 f(c_1),
\]

\[
r_s = 1.626 \left(\frac{M_1}{\Omega_m (1 + z)^3 + \Omega_\Lambda}\right)^{1/3} h^{-1}\text{Mpc},
\]

\[
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\]

\[
r_s = 1.626 \left(\frac{M_1}{\Omega_m (1 + z)^3 + \Omega_\Lambda}\right)^{1/3} h^{-1}\text{Mpc},
\]

\[
\Sigma(x) = 2\rho_s r_s V(x)
\]
\[ V(x) = \int_0^{\infty} \left( x^2 + z^2 \right)^{-\gamma/2} \left[ \left( x^2 + z^2 \right)^{\alpha/2} + 1 \right]^{(\gamma-\beta)/\alpha} \, dz, \]

and \( x = |\vec{x}|, \vec{x} = \vec{\xi}/r_s, \vec{\xi} \) is the position vector in the lens plane. We thus obtain the lensing equation for a DC14 halo

\[ y = x - \mu_s \frac{g(x)}{x}, \quad (10) \]

where \( y = |\vec{y}|, \vec{y} = \vec{y}_s D_S / D_L \) is the position vector in the source plane, and

\[ g(x) \equiv \int_0^x u V(u) \, du, \quad (11) \]

and

\[ \mu_s \equiv 4 \rho_s r_s \Sigma_{cr}, \quad (12) \]

where \( \Sigma_{cr} = (c^2/4\pi G)(D_S/D_L D_{LS}) \) is the critical surface mass density; \( D_L, D_S \) and \( D_{LS} \) are the angular diameter distances from the observer to the lens, to the source and from the lens to the source, respectively.

We can get some simple but important results about the lensing efficiency for DC14 model even before calculating the lensing probabilities. We notice that, from equation (2), NFW profile is a specific form that has \((\alpha, \beta, \gamma) = (1, 3, 1)\), and SIS profile is similar to \((\alpha, \beta, \gamma) = (2, 2, 2)\), where in the latter case, \( \alpha \) can be any non-zero number and we let \( \alpha = 2 \) for definiteness. Therefore, it would be helpful to plot the parameters \( \alpha, \beta, \gamma \) versus \( M_{\text{halo}} \) from \( 10^{11} M_\odot \) to \( 10^{14.9} M_\odot \), as shown in Fig. 1. We find that the inner slope has \( \gamma \sim 1 \), an NFW like value, only in a narrow range of halo mass around \( 10^{12} M_\odot \), and it flattens for lower and higher mass ranges. This would result in a much lower lensing efficiency compared with that of NFW model and even further lower compared with SIS model.

Another parameter that helps us to understand the lensing efficiency is \( \mu_s \) in lensing equation (10). According to strong lensing theory, multiple images can occur only for sufficiently large values of \( \mu_s \). Fig. 2 shows how \( \mu_s \) changes with \( M_{\text{halo}} \) for both DC14 and NFW models, with a fixed typical value 0.45 of the lens redshift and the average value 1.56 of the source redshift (Inada et al. 2012). We find that the values of \( \mu_s \) for NFW model uniformly surpass that of DC14 in the whole range of halo mass of \( (10^{11} M_\odot, 10^{14.9} M_\odot) \), and the difference increases markedly for \( M_{\text{halo}} > 10^{12} M_\odot \). This would also means an obviously lower lensing efficiency for DC14 model compared with the NFW model.

### 2.2 Schaller15 model

The Schaller15 model is derived from EAGLE simulations, the total (baryons plus DM) density profile consists of two terms (Schaller et al. 2015)

\[ \rho(r) = \frac{\rho_{cr}}{\left( \frac{r}{r_s} \right)^2 + \left( \frac{r}{r_s} \right)^4} \left[ \frac{\rho_{cr}}{\left( \frac{r}{r_s} \right)^2 + \left( \frac{r}{r_s} \right)^4} + \frac{\rho_{si}}{1 + \left( \frac{r}{r_i} \right)^2} \right], \quad (13) \]

where the first term is the NFW profile, and the second term is NFW-like in that it shares the same asymptotic behavior at small and large radii and has a slope of -2 at its scale radius, \( r = r_s \). We write the surface mass densities corresponding to the two terms as

\[ \Sigma_i(x) = 2 \rho_{cr} \delta_i r_s V_i(x), \quad (14) \]
where
\[
V_1(x) = \int_0^\infty \left( x^2 + z^2 \right)^{-1/2} \left[ \left( x^2 + z^2 \right)^{1/2} + 1 \right]^{-2} dz,
\]
and
\[
\Sigma_2(x) = 2 \rho_{\text{c}} \delta_i V_2(x),
\]  
with
\[
V_2(x) = \int_0^\infty \left( x^2 + z^2 \right)^{-1/2} \left[ \left( x^2 + z^2 \right)^{1/2} + 1 \right]^{-1} dz.
\]

For later calculations, we need to know the characteristic densities \( \delta_c, \delta_i \) and characteristic radii \( r_s, r_i \) as functions of the total mass \( M_{200} \), which reads \( \text{Schaller et al. 2015} \):
\[
M_{200} = 2 \pi \rho_{\text{c}} \left\{ 2 \delta_c r_0^3 \left[ \ln \left( 1 + \frac{r_{200}}{r_s} \right) - \frac{r_{200}}{r_{200} + r_s} \right] + \delta_i r_i^3 \ln \left( 1 + \frac{r_{200}}{r_i^2} \right) \right\}.
\]

In practice, however, it is impossible to derive so many parameters from the only equation (16), we thus fit \( \delta_c, \delta_i \) etc. to \( M_{200} \) with the data given by \( \text{Schaller et al. 2015} \). The fitted results are displayed in Fig. 3, Fig. 4 and Fig. 5. The fitting formulas are
\[
\delta_c = 10^{0.65 X + 2.75 X^2 + 1.75 X^3 + 1.75 X^4 + \delta_{\text{cc}}},
\]
where \( X = \log 10(M_{200}) \) and \( \delta_{\text{cc}} = 0.06, \delta_{\text{cb}} = -1.65, \delta_{\text{cc}} = 15.22; \delta_i = 10^{0.59 X + 1.18}; r_s = 0.75 \times 10^5 (r_{\text{cm}} X^3 + r_{\text{cb}} X^2 + r_{\text{cc}} X + r_{\text{cd}}) h^{-1}\text{Mpc} \)
(17)
(18)
(19)
where \( r_{\text{cm}} = 2.08, r_{\text{cb}} = -61.0, r_{\text{cc}} = 598.57, r_{\text{cd}} = -1960.43; \) and for \( r_i \) we adopt the average value \( r_i = 2.27 \text{kpc} \) \( \text{Schaller et al. 2015} \), or
\[
r_i = 1.70 \times 10^{-3} h^{-1} \text{Mpc}.
\]

We thus obtain the lensing equation for a Schaller15 halo
\[
y = x - \mu_x \frac{g_1(x)}{x} - \mu_y \frac{g_2(x)}{x},
\]
where \( y \) and \( x \) are defined in the same way as for DC14 model, \( \mu_x = 4\rho_{\text{c}} \delta_i r_i/\Sigma_{\text{cr}}, \mu_y = 4\rho_{\text{c}} \delta_i r_i/\Sigma_{\text{cr}} \) and
\[
g_1(x) \equiv \int_0^x u V_1(u) du,
\]
(22)
\[
g_2(x) \equiv \int_0^x u V_2(u) du.
\]
(23)

As indicated by \( \text{Schaller et al. 2015} \), Schaller15 profile has two lengthscales, \( r_s \) and \( r_i \), where the former describes the NFW-like outer parts of the halo, and the latter the deviations from NFW in the inner regions. The second term in Eq. (15) is the inner component, which is characterized by two quantities, a scale radius \( r_i \) and a density contrast \( \delta_i \). This inner profile is an empirical model that describes the deviation from NFW due to the presence of stars and some contraction of the DM. So we expect that the lensing efficiency for Schaller15 model should be higher than NFW.
3 LENSING PROBABILITIES

The quasars of redshift \(z_s\) are lensed by foreground CDM halos of galaxy clusters and galaxies, the lensing probability with image separations larger than \(\Delta \theta\) is (Schneider et al. 1992)

\[
P(> \Delta \theta) = \int_0^{\infty} \frac{dI_L(z)}{dz} dz \times \int_0^{\infty} \tilde{n}(M, z) \sigma(M, z) B(M, z) dM,
\]

where \(D_L(z)\) is the proper distance from the observer to the lens located at redshift \(z\)

\[
D_L(z) = \frac{c}{H_0} \int_0^z \frac{dz}{(1 + z)\sqrt{\Omega_m(1 + z)^3 + \Omega_L}},
\]

here \(c\) is the speed of light in vacuum and \(H_0\) is the current Hubble constant. We make \(z_s = 1.56\) for statistical sample SQLS (Mada et al. 2012), and \(z_s = 1.27\) which is the mean value of the redshift distribution for quasars approximated by a Gaussian model (Helbig et al. 1999; Marlow et al. 2000; Myers et al. 2003). The physical number density \(\tilde{n}(M, z)\) of virialized DM halos of masses between \(M\) and \(M + dM\) is related to the comoving number density \(n(M, z)\) by \(\tilde{n}(M, z) = n(M, z)(1 + z)^3\), the latter was originally given by Press & Schechter (1974), and the improved version is (Sheth & Tormen 1999)

\[
n(M, z) dM = \frac{\rho_{\text{crit}}}{M} f(M, z) dM,
\]

where

\[
f(M, z) = -\frac{1}{2} \left( \frac{\Delta}{M} \frac{d\ln \Delta}{d\ln M} \right) \exp \left[ -\frac{\delta^2(z)}{2\Delta^2} \right]
\]

is PS mass function. In Eq. (24) above, \(\Delta^2(M)\) is the present variance of the fluctuations in a sphere containing a mean mass \(M\),

\[
\Delta^2(M) = \frac{1}{2\pi^2} \int_0^{\infty} P(k) W^2(kr_M) k^2 dk,
\]

where \(P(k)\) is the power spectrum of density fluctuations, \(W(kr_M)\) is the Fourier transformation of a top-hat window function

\[
W(kr_M) = 3 \left[ \frac{\sin(kr_M)}{(kr_M)^3} \right. - \left. \frac{\cos(kr_M)}{(kr_M)^2} \right],
\]

and

\[
r_M = \left( \frac{3M}{4\pi\rho_0} \right)^{1/3}.
\]

In Eq. (27), \(\delta_c(z)\) is the over density threshold for spherical collapse by redshift \(z\) (Navarro, Frank, & White 1997);

\[
\delta_c(z) = \frac{1.68}{D(z)}
\]

where \(D(z)\) is the linear growth function of density perturbation (Carroll & Press 1992)

\[
D(z) = \frac{g(\Omega(z))}{g(\Omega_m)(1 + z)},
\]

in which

\[
g(z) = \frac{5}{2} x \left( \frac{1}{70} + \frac{209x}{140} - \frac{x^2}{140} + \frac{x^4/7}{140} \right)^{-1},
\]

and

\[
\Omega(z) = \frac{\Omega_m(1 + z)^3}{1 - \Omega_m + \Omega_m(1 + z)^3}.
\]

We use the fitting formulae for CDM power spectrum \(P(k)\) given by Eisenstein & Hu (1999)

\[
P(k) = A k T^2(k),
\]

where \(A\) is the amplitude normalized to \(\sigma_8 = \Delta(r_M = 8h^{-1}\text{Mpc}) = 0.8\), and

\[
T = \frac{L}{L + C q_{\text{eff}}},
\]

with

\[
L \equiv \ln(e + 1.8q_{\text{eff}}),
\]

\[
q_{\text{eff}} \equiv \frac{k}{\Omega_m h^2 \text{Mpc}^{-1}},
\]

\[
C \equiv 14.4 + \frac{325}{1 + 60.5q_{\text{eff}}^2}.
\]

The cross-section is

\[
\sigma(M, z) = \pi y_{c r}^2 r_s^2 \theta(M - M_{\text{min}}),
\]

where \(y_{c r}\) is the maximum value of \(y\), the reduced position of a source, such that when \(y < y_{c r}\) multiple images can occur; \(\theta(x)\) is a step function, and \(M_{\text{min}}\) is determined by the lower limit of image separation

\[
\Delta \theta = \frac{r_s \Delta x}{D_M} \approx \frac{2x_0 r_s}{D_L},
\]

and Eq. (15) for DC14 model as

\[
M_{\text{DC14}}^{\text{min}} = 8.927 \times 10^{-8} M_{15}
\]

\[
\times \left( \Omega_m(1 + z)^3 + \Omega_\Lambda \right) \left( \frac{c_1 D_L \Delta \theta}{x_0} \right)^3,
\]

and Eq. (14) for Schaller15 model as

\[
M_{\text{Schaller15}} = 10^{m_y + m_s} + m_c Y^2 + m_c Y^2 + m_s Y + M_{15} - 15,
\]

where \(Y = \frac{10^3}{0.97} \times r_s\), \(m_y = 3.62 \times 10^{-6}\), \(m_s = -0.001\), \(m_c = 0.09\), \(r_s = 9.92\). Note that, for Schaller15 model, we define \(M = M_{200/10^{15} M_{15}}\). In Eq. (11), we have approximated the image separation \(\Delta x\) to be \(2x_0\), where \(x_0\) is the positive zero position of function \(y(x)\).

The magnification bias \(B(M, z)\) should be calculated by considering the actual flux ratio and differential luminosity of quasar sources (e.g., Oguri et al. 2008; Yang & Chen 2008), however, since we investigate only the order of magnitudes of lensing probabilities, we adopt a simple model (Li & Ostriker 2002): \(B \approx 2.2 A_m^{1.3}\), with \(A_m = D_L \Delta \theta / (r_s y_{c r})\).

We first present, in Fig. 6 the lensing probabilities predicted by Eq. (23) with the survey results of JVAS/CLASS, which is a subset of 8958 sources from the combined JVAS/CLASS survey that forms a well-defined statistical sample containing 13 multiply imaged sources suitable for analysis of the lens statistics (Myers et al. 2003; Browne et al. 2003; King et al. 1999). The lensing probability for DC14 model (the dotted line) is much lower than the
observations, which verifies our previous predictions simply based on the inner slope and parameter $\mu_s$. Also shown in Fig. 4 is the well known two-population SIS+NFW model, which has long been used as a standard model in strong lensing statistics (Sarbu, Rusin, & Ma 2001; Li & Ostriker 2002; Chen 2003a,b, 2004a,b; Zhang 2004). In this model, SIS is used for lensing galaxies (mostly giant ellipticals) and NFW for lensing clusters of galaxies (Bolton et al. 2013), and the transition occurs at $M_{\text{halo}} \sim 10^{13} M_\odot$. We can conclude that the SIS+NFW model fits the observations reasonably well, in the sense that the SIS predictions fit the small image separations well, whilst the NFW predictions are below the upper limit put by JVAS/CLASS survey for $6^\prime \leq \Delta \theta \leq 15^\prime$ (Li & Ostriker 2002). We know that the density profile for SIS model is $\rho_{\text{SIS}}(r) = \sigma_v^2/(2\pi Gr^2)$, where $\sigma_v$ is the velocity dispersion; alternatively, if we set $\beta = \gamma = 2$ in Eq. (2) we have $\rho(r) = \rho_s r^2/r^2$. They are both proportional to $1/r^2$, and differ only in a constant. The latter case is denoted as “DC14($\beta = \gamma = 2$)+NFW” (the dot-dash line) in Fig. 6 which is approximately equivalent to SIS+NFW model. This reflects a very important fact about strong lensing statistics that we have repeatedly emphasized: the inner slope of density profile for lensing halos is the most important factor compared with others (e.g., shapes and substructures). The lensing probabilities for CIS model have been investigated in detail (Chen & McGaugh 2010), we paste the line (dot-dash) in Fig. 6. We find that the lensing probabilities for DC14 are lower than NFW but higher than the CIS model, which can be explained by the steepening tendency of the inner slope of the DC14 profile when the halo mass increases towards $\sim 10^{12} M_\odot$, as displayed in Fig. 4.

We also compare the lensing probabilities predicted with Eq. (21) for various density profiles with the most recent observations of SQLS in Fig. 7. The statistical sample for SLS (Inada et al. 2012) consists of 26 quasar lenses selected from 50836 source quasars in the redshift range $0.6 < z < 2.2$ with Galactic extinction corrected (Schlegel et al. 1998), magnitudes brighter than $i = 19.1$. Note that the predicted lensing probabilities for each model in Fig. 7 are obviously higher than their counterparts displayed in Fig. 6 due to the different redshifts $z_s$ of quasars we have chosen, $z_s = 1.56$ for SLS and 1.27 for JVAS/CLASS. We find from Fig. 7 that SIS profile can still match the SLS observations well, whilst NFW predicts the lensing probabilities that are about an order of magnitude lower than the observations (Gioioli et al. 2016). The usually employed standard model SIS+NFW breaks down for large image separations. As pointed out previously, something like the ellipticity and substructure, which deviate from the spherical and smooth NFW model, cannot compensate for the large discrepancy, we thus tend to believe that a steeper inner slope than NFW may achieve the large image separation observations (Chen & McGaugh 2010).

The predicted lensing probabilities for Schaller15 model (dot-dashed line) are shown in Fig. 5 together with the observations for SLS sample (thick histogram), the predictions for the models of SIS+NFW (dashed line) and DC14 (dotted line). Surprisingly, we find that Schaller15 model predicts too many lenses compared with SLS observations and all other models.

![Figure 6. Lensing probabilities with image separations larger than $\Delta \theta$ for DC14 model and Schaller15 model, and compared the results with observations and other models. As expected, the lensing efficiency for DC14 is much less than SIS (which fit the observations for galactic lenses quite well), and even less than NFW. The reason is that the lensing efficiency is very sensitive to the inner slope of the density profile of the lens halos, which actually dominates the predictions. Despite that the inner slope $\gamma$ of DC14 profile approaches 1 (which is NFW like) when the halo mass increases towards $10^{12} M_\odot$, it decreases dramatically after that mass as shown in Fig. 7. We know that the DC14 model is fitted from the simulations which are confined to $M_{\text{halo}} \leq 10^{12} M_\odot$, and thus should be valid only in this range; however, there are no evidences arising from the treatments of astrophysical processes for the simulations show us that we cannot extrapolate to larger halos. What is more important about DC14 profile is that it is very far from the SIS like density profile in the galactic mass range, namely around $M_{\text{halo}} \sim 10^{12} M_\odot$, which is required to explain strong lensing observations. This phenomena is, in fact, genuine in the literature: up to now, all the simulations claimed to have explained reasonably the observations of rotation curves fail to explain the observations of strong lensing, whatever the valid halo mass ranges declared. For example, one possible solution to the cusp-core problem is the turbulence driven by stellar feedback during galaxy formation (Mashchenko, Couchman & Wadsley).](image-url)
Figure 7. Lensing probabilities with separation larger than $\Delta \theta$: observations for SQLS sample (thick histogram), and the predictions for the models of SIS+NFW (dashed line), DC14($\beta = \gamma = 2$) + NFW (dot-dashed line) and DC14 (dotted line). Predicted lensing probabilities are calculated with $z_s = 1.56$.

Figure 8. Lensing probabilities with separations larger than $\Delta \theta$: observations for SQLS sample (thick histogram), and the predictions for the models of SIS+NFW (dashed line), Schaller15 (dot-dashed line) and DC14 (dotted line). Predicted lensing probabilities are calculated with $z_s = 1.56$.

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2008; Mashchenko, Wadsley & Couchman 2008), which leads to a final halo file with a finite core radius for all galaxies, including giant ellipticals. Such a situation is consistent with essentially all observations of rotation curves (McGaugh et al. 2007), but contradicts with strong lensing observations (Chen & McGaugh 2010).

One may argue that, disk galaxies from which we observe the rotation curves and the giant ellipticals which dominate strong lensing phenomena, are of very different galaxy types and form in very different environments and histories. In the spirit of hierarchical CDM structure formation paradigm, however, they are formed from the same initial conditions (gas mixes with dark matter) and undergo the same subsequent hierarchical sequences, and thus cannot form separately and independently. Clearly, any valuable and theoretically significant predictions of the properties of galaxies should be of those for any simulations that cover the mass range from dwarf galaxies to giant ellipticals, and should have the volume size large enough to include the statistically well-defined samples of galaxies. This is necessary to ensure the hierarchical galaxy formation theory be faithfully, coherently and self-consistently implemented in the ΛCDM paradigm. Therefore, for any simulations, whatever the manners are assumed in which baryon effects are modeled to modify the initially pure DM halo profiles, it is difficult, if not impossible, to identify the final galaxies hosting in the center of DM halos only with disk galaxies, in particular when the halo mass is as large as $10^{12} M_\odot$. That is, when the redshift $z \sim 0$, there should exist other galaxy types apart from disk galaxies. We thus emphasize that, it is meaningful for DC14 and any other similar density profiles to be tested against rotation curves, only if we assume that the halo mass density profiles are unconcerned with morphological types of the hosted galaxies. Accordingly, the density profiles can also, and should be, tested by other available observations, in particular by the observations of strong lensing (Chen 2005; Li & Chen 2009; Chen & McGaugh 2010).

In practice, however, limited by the computer capabilities, more details about the inner structure of each halo need higher resolutions (i.e., smaller particle mass) which would strongly restrict the sample size under considerations. Consequently, simulations can only be designed to tackle a certain specific problem (usually determined by observations). For example, disk galaxies and giant ellipticals are often simulated independently, usually among very different communities. The baryons have two opposite effects on the central mass density of DM halos. While stellar feedback and dynamical friction can induce expansion of the DM halo and produce a core, the adiabatic contractions can steepen central density to the SIS type (Blumenthal et al. 1986; Gnedin et al. 2004; Gustafsson, Fairbairn & Sommer-Larsen 2006). The uncertainties of the parameters appear in different models for the baryon effects allow us to calibrate the parameters with observations. This inevitably leads to the simulation results which are strongly observation-dependent. It is thus no surprise that the baryon processes modeled for simulations that can produce the CIS profile cannot naturally proceed to produce SIS (Chen & McGaugh 2014).

The most recent seemingly comprehensive simula-
Baryon effects on DM halos

For halos with mass $\sim 10^{12} M_\odot$, their core structure is often treated specifically to fit some specific aspects of the $\Lambda$CDM cosmology, such as the rotation curves and lensing effects from strong lensing. In the context of current simulation tools, the Schaller15 model has difficulties in reproducing both the rotation curves and the lensing efficiencies. For halos with mass $> 10^{12} M_\odot$, the central regions of halos with mass $> 10^{12} M_\odot$ are best described by the DC14 model. However, the over-predicted lensing efficiencies mean that the baryon effects on DM suggested by the Schaller15 model cannot be true.

We conclude that, it is difficult for current simulations to reconcile the DM distributions derived from the observations of rotation curves and that from strong lensing. In the context of ACMD cosmology, if baryon effects, in the computer simulations, are treated specifically to fit some specific aspects of the $\Lambda$CDM cosmology, the over-predicted lensing efficiencies mean that the baryon effects on DM suggested by the Schaller15 model cannot be true.

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