Nonphotonic electrons at RHIC within $k_t$-factorization approach and with experimental semileptonic decay functions.

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Abstract

We discuss production of nonphotonic electrons in proton-proton scattering at RHIC. The distributions in rapidity and transverse momentum of charm and bottom quarks/antiquarks are calculated in the $k_t$-factorization approach. We use different unintegrated gluon distributions from the literature. The hadronization of heavy quarks is done by means of Peterson and Braaten et al. fragmentation functions. The semileptonic decay functions are found by fitting recent semileptonic data obtained by the CLEO and BABAR collaborations. We get good description of the data at large transverse momenta of electrons and find a missing strength concentrated at small transverse momenta of electrons. Plausible missing mechanisms are discussed.

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I. INTRODUCTION

Recently the PHENIX and STAR collaborations has measured transverse momentum distribution of so-called nonphotonic electrons [1, 2]. It is believed that the dominant contribution to the nonphotonic electrons/positrons comes from the semileptonic decays of charm and beauty mesons. These processes have three subsequent stages. First $c\bar{c}$ or $b\bar{b}$ quarks are produced. The dominant mechanisms being gluon-gluon fusion and quark-antiquark annihilation. Next the heavy quarks/antiquarks are turned to heavy charmed mesons $D, D^*$ or $B, B^*$. The vector $D^*$ and $B^*$ mesons decay strongly producing $D$ and $B$ mesons. Finally the heavy pseudoscalar mesons decay semileptonically producing electrons/positrons.

The inclusive heavy quark/antiquark production can presently be calculated at Fixed-Order plus Next-to-Leading-Log (FONLL) level [3]. The predictions for electron spectra in proton-proton collisions at RHIC can be found in Ref. [4]. An alternative approach for inclusive heavy quark production is $k_t$-factorization [5, 6, 7, 8, 9, 10, 11]. In this approach emission of gluons (see Fig. 1) is encoded in so-called unintegrated gluon distributions (UGDFs). The latter approach is very efficient for description of $Q\bar{Q}$ correlations [12].

The hadronization of heavy quarks is usually done with the help of fragmentation functions. The Peterson fragmentation functions are often used in this context [13]. The parameters of the Peterson fragmentation functions are adjusted to $e^+e^-$ or $p\bar{p}$ production of heavy mesons. Another perturbative fragmentation model has been proposed in Ref. [14] (BCFY).

The last ingredient are semileptonic decays of heavy mesons. Until recently this component was treated by modeling the decay [15, 16, 17]. Only recently the CLEO [18] and BABAR [19] collaborations has measured very precisely the spectrum of electrons/positrons coming from the decays of $D$ and $B$ mesons, respectively. This is done by producing resonances: $\Psi(3770)$ which decays into $D$ and $\bar{D}$ mesons (CLEO) and $\Upsilon(4S)$ which decays into $B$ and $\bar{B}$ mesons (BABAR). In both cases the heavy mesons are almost at rest, so in practice one measures the meson rest frame distributions of electrons/positrons.

In the present analysis we shall apply the $k_t$-factorization approach. At relatively low RHIC energies rather intermediate $x$-values become relevant. The Kwiecinski unintegrated gluon (parton) distributions seem relevant in this case [20]. We shall use also Ivanov-Nikolaev distributions which were fitted to deep-inelastic HERA data including intermediate-$x$ region [21]. We shall use both Peterson and BCFY fragmentation functions. The electron/positron decay functions will be fitted to the recent CLEO and BABAR data.

II. FORMALISM

Let us consider the reaction $h_1 + h_2 \rightarrow Q + \bar{Q} + X$, where $Q$ and $\bar{Q}$ are heavy quark and heavy antiquark, respectively. In the leading-order (LO) approximation within the collinear approach the quadruply differential cross section in the rapidity of $Q$ ($y_1$), in the rapidity of $\bar{Q}$ ($y_2$) and the transverse momentum of one of them ($p_t$) can be written as

$$
\frac{d\sigma}{dy_1dy_2dp_t} = \frac{1}{16\pi^2s^2} \sum_{i,j} x_1p_i(x_1, \mu^2) x_2p_j(x_2, \mu^2) |M_{ij}|^2.
$$

(2.1)

Above, $p_i(x, \mu^2)$ and $p_j(x, \mu^2)$ are the familiar (integrated) parton distributions in hadron $h_1$ and $h_2$, respectively. There are two types of the LO $2 \rightarrow 2$ subprocesses which en-
ter Eq. (2.1): $gg \rightarrow QQ$ and $q\bar{q} \rightarrow QQ$. The first mechanism dominates at large energies and the second one near the threshold. The parton distributions are evaluated at:

$$x_1 = \frac{m_u}{\sqrt{s}} (\exp(y_1) + \exp(y_2)), \quad x_2 = \frac{m_d}{\sqrt{s}} (\exp(-y_1) + \exp(-y_2)),$$

where $m_t = \sqrt{p_t^2 + m_Q^2}$. The formulae for matrix element squared averaged over the initial and summed over the final spin polarizations can be found e.g. in Ref. [22].

If one allows for transverse momenta of the initial partons, the sum of transverse momenta of the final $Q$ and $\bar{Q}$ no longer cancels. Formula (2.1) can be easily generalized if one allows for the initial parton transverse momenta. Then

$$\frac{d\sigma}{dy_1 dy_2 dp_{1,t} dp_{2,t}} = \sum_{i,j} \int \frac{d^2 \kappa_{1,t}}{\pi} \frac{d^2 \kappa_{2,t}}{\pi} \frac{1}{16\pi^2 (x_1 x_2 s)^2} |\mathcal{M}_{ij}|^2 \delta^2 (\vec{\kappa}_{1,t} + \vec{\kappa}_{2,t} - \vec{p}_{1,t} - \vec{p}_{2,t}) \mathcal{F}_i (x_1, \kappa_{1,t}^2) \mathcal{F}_j (x_2, \kappa_{2,t}^2), \quad (2.2)$$

where now $\mathcal{F}_i (x_1, \kappa_{1,t}^2)$ and $\mathcal{F}_j (x_2, \kappa_{2,t}^2)$ are the so-called unintegrated gluon (parton) distributions. The extra integration is over transverse momenta of the initial partons. The two extra factors $1/\pi$ attached to the integration over $d^2 \kappa_{1,t}$ and $d^2 \kappa_{2,t}$ instead over $dk_{1,t}^2$ and $dk_{2,t}^2$ as in the conventional relation between the unintegrated ($\mathcal{F}$) and the integrated ($\mathcal{g}$) parton distributions. The two-dimensional Dirac delta function assures momentum conservation. Now the unintegrated parton distributions must be evaluated at: $x_1 = \frac{m_u}{\sqrt{s}} \exp(y_1) + \frac{m_d}{\sqrt{s}} \exp(y_2)$, $x_2 = \frac{m_u}{\sqrt{s}} \exp(-y_1) + \frac{m_d}{\sqrt{s}} \exp(-y_2)$, where $m_{i,t} = \sqrt{p_{i,t}^2 + m_Q^2}$. In general, the matrix element must be calculated for initial off-shell partons. The corresponding formulae for initial gluons were calculated in [5, 6] (see also [7]). It is easy to check [12] that in the limit $\kappa_{1,t} \rightarrow 0$, $\kappa_{2,t} \rightarrow 0$ the off-shell matrix elements converge to the on-shell ones.

Introducing new variables:

$$\bar{Q}_t = \vec{\kappa}_{1,t} + \vec{\kappa}_{2,t}, \quad \bar{q}_t = \vec{\kappa}_{1,t} - \vec{\kappa}_{2,t} \quad (2.3)$$

we can write:

$$\frac{d\sigma_{ij}}{dy_1 dy_2 dp_{1,t} dp_{2,t}} = \int d^2 q_t \frac{1}{4\pi^2} \frac{1}{16\pi^2 (x_1 x_2 s)^2} |\mathcal{M}_{ij}|^2 \mathcal{F}_i (x_1, \kappa_{1,t}^2) \mathcal{F}_j (x_2, \kappa_{2,t}^2). \quad (2.4)$$

This formula is very useful to study correlations between the produced heavy quark $Q$ and heavy antiquark $\bar{Q}$ [12].

For example

$$\frac{d\sigma_{ij}}{dp_{1,t} dp_{2,t}} = \int d\phi_1 d\phi_2 p_{1,t} p_{2,t} \int dy_1 dy_2 \int d^2 q_t \frac{1}{4\pi^2} \frac{1}{16\pi^2 (x_1 x_2 s)^2} |\mathcal{M}_{ij}|^2 \mathcal{F}_i (x_1, \kappa_{1,t}^2) \mathcal{F}_j (x_2, \kappa_{2,t}^2)$$

$$= 4\pi \frac{1}{2} \frac{1}{2} \int d\phi_- p_{1,t} p_{2,t} \int dy_1 dy_2 \int d^2 q_t \frac{1}{4\pi^2} \frac{1}{16\pi^2 (x_1 x_2 s)^2} |\mathcal{M}_{ij}|^2 \mathcal{F}_i (x_1, \kappa_{1,t}^2) \mathcal{F}_j (x_2, \kappa_{2,t}^2). \quad (2.5)$$

1 In this paper we shall use the following convention of unintegrated gluon distributions: $\int_0^{\mu^2} \mathcal{F}(x, \kappa^2) dk^2 \sim x q(x, \mu^2)$.
In the last equation we have introduced $\phi_- \equiv \phi_1 - \phi_2$, where $\phi_- \in (-2\pi, 2\pi)$. The factor $4\pi$ comes from the integration over $\phi_+ \equiv \phi_1 + \phi_2$. The first factor $1/2$ comes from the jacobian transformation while the second factor $1/2$ takes into account an extra extension of the domain when using $\phi_+$ and $\phi_-$ instead of $\phi_1$ and $\phi_2$.

At the Tevatron and LHC energies the contribution of the $gg \to Q\bar{Q}$ subprocess is more than an order of magnitude larger than its counterpart for the $q\bar{q} \to Q\bar{Q}$ subprocess. At RHIC energy the relative contribution of the quark-antiquark annihilation is somewhat bigger. Therefore in the following we shall take into account not only gluon-gluon fusion process i.e. $i=0$ and $j=0$ but also the quark-antiquark annihilation mechanism.

The purely perturbative\footnote{when both UGDFs are generated perturbatively} $k_t$-factorization formalism to $h_1h_2 \to Q\bar{Q}$ applies if $\kappa_{1,t}^2, \kappa_{2,t}^2 > \kappa_0^2$. The choice of $\kappa_0^2$ is to a large extent arbitrary. In Refs.\cite{8} a rather large $\kappa_0^2$ was chosen and the space $\kappa_{1,t}^2 \times \kappa_{2,t}^2$ was subdivided into four disjoint regions. For example the contribution when both $\kappa_{1,t}^2$ and $\kappa_{2,t}^2$ are small was replaced by the leading-order collinear cross section. Such an approach assures that $\sigma_{Q\bar{Q}}^{\text{tot}} > \sigma_{Q\bar{Q}}^{\text{tot}}(\text{collinear LO})$ by construction.

It is rather obvious that the resulting cross section strongly depends on the choice of $\kappa_0^2$ which makes the procedure a bit arbitrary. Our philosophy here is different. Many models of UGDF in the literature treat the soft region explicitly. Therefore we use the $k_t$-factorization formula everywhere on the $\kappa_{1,t}^2 \times \kappa_{2,t}^2$ plane.

The production of electrons/positrons is a multi-step process. The whole procedure of electron/positron production can be written in the following schematic way:

$$\frac{d\sigma^e}{dy d^2p} = \frac{d\sigma^Q}{dy d^2p} \otimes D_{Q\to D} \otimes f_{D\to e},$$

where the symbol $\otimes$ denotes a generic convolution. The first term responsible for production of heavy quarks/antiquarks is calculated in the $k_t$-factorization approach. Some details were already discussed above. Next step is the process of formation of heavy mesons. We follow a phenomenological approach and take Peterson and Braaten et al. fragmentation functions with parameters from the literature (see e.g. \cite{26}). The electron decay function should account for the proper branching fractions. The latter are known experimentally (see e.g. \cite{18, 19, 27}). These functions can in principle be calculated \cite{15, 16}. This introduces, however, some model uncertainties and requires inclusion of all final state channels explicitly. An alternative is to use experimental input. The decay functions have been measured only recently \cite{18, 19}. How to use the recent experimental information will be discussed in the next section.

\textbf{III. RESULTS}

In principle, the semileptonic decays can be modeled (see e.g. \cite{15, 16, 17}). Since there are many decay channels with different number of particles this is not an easy task. In our approach we take less ambitious but more pragmatic approach. In Fig.\ref{fig} we show our purely mathematical fit to not absolutely normalized data of the CLEO \cite{18} and BABAR \cite{19} collaborations. We find a good fit with:

$$f_{\text{CLEO}}(p) = 12.55(p+0.02)^{2.55}(0.98-p)^{2.75}$$

(3.1)
for the CLEO data [18] and

\[
f_{BABAR}(p) = \left( 126.16 + 14293.09 \exp(-2.24 \ln(2.51 - 0.97p)^2) \right) \\
\left( -41.79 + 42.78 \exp(-0.5(|p - 1.27|)/1.8)^{8.78} \right)
\] (3.2)

for the BABAR data [19]. In these purely numerical parametrizations \( p \) must be taken in GeV.

After renormalizing to experimental branching fractions for \( D \to e \) (about 10 \% [3] and \( B \to e \) (10.36 ± 0.06(stat.) ± 0.23(syst.%) [19]) we shall use them to generate electrons/positrons in the rest frame of the decaying \( D \) and \( B \) mesons in a Monte Carlo approach. We shall neglect a small effect of the non-zero motion of the \( D \) mesons in the case of the CLEO experiment and of the \( B \) mesons in the case of the BABAR experiment. This effect is completely negligible.

For illustration of the whole procedure in Fig.3 we show as an example two-dimensional distributions in rapidity and transverse momentum for charm quarks, \( D \) mesons and electrons from the decay of \( D \) mesons. Both fragmentation and semileptonic decays cause degradation of transverse momentum. On average \( p_{t,e} < p_{t,D} < p_{t,c} \). The spectra of electrons are much softer than initial spectra of charm quarks. On the other hand the distributions in rapidity of electrons are much broader than the corresponding distributions of quarks/antiquarks.

Now we shall concentrate on invariant cross section as a function of electron/positron transverse momentum. Such distributions have been measured recently by the STAR and PHENIX collaboration at RHIC [1, 2]. In Fig.4 and Fig.5 we show results obtained with Kwieciński UGDF [20] and different combinations of factorization and renormalization scales as well as for different fragmentation functions (Peterson and BCFY). The differences between results obtained with different combinations quantify theoretical uncertainties. Similarly as for the standard collinear approach [4] one gets uncertainties of the order of a factor 2. We show individual contributions of electrons/positrons initiated by \( c/\bar{c} \) or \( b/\bar{b} \). The contribution of the \( c/\bar{c} \) (dashed) dominates at low transverse momenta of electrons/positrons. At transverse momenta of the order of 4 - 5 GeV the both contributions become comparable. We obtain rough agreement for large transverse momenta. Similarly as for the higher-order collinear approach [4] there is a missing strenght at lower transverse momenta. A better agreement is obtained with renormalization scale taken as transverse momentum of the initial gluon(s). There are two strong coupling constant in the considered order. In practice we take \( \alpha_s(k^2_1)\alpha_s(k^2_2) \), i.e. different argument for each running coupling constant. This is rather a standard prescription used in \( k_t \)-factorization approach (see e.g. [9, 10]) although does not have a deep theoretical foundation. In the latter case to avoid Landau pole we use analytic prescription of Shirkov and Solovtsov [23].

The situation for the Kwieciński UGDF is summarized in Fig.6 where we have shown uncertainty band of our theoretical calculation. The upper curves are for \( \mu^2_R = k^2_1 \) and \( \mu^2_F = 4m^2_Q \) and the lower curves are for \( \mu^2_R = 4m^2_Q \) and \( \mu^2_F = 4m^2_Q \). Up to now we have presented only the PHENIX collaboration data which span a broader range of lepton transverse

\[ \text{BR}(D^+ \to e^+ \nu_e X) = 16.13 \pm 0.20 \text{(stat.)} \pm 0.33 \text{(syst.%)}, \text{BR}(D^0 \to e^+ \nu_e X) = 6.46 \pm 0.17 \text{(stat.)} \pm 0.13 \text{(syst.%)}. \]

Because the shapes of positron spectra for both decays are identical within error bars we can take the average value and simplify the calculation.

\[ \text{BR}(D^+ \to e^+ \nu_e X) = 16.13 \pm 0.20 \text{(stat.)} \pm 0.33 \text{(syst.%)}, \text{BR}(D^0 \to e^+ \nu_e X) = 6.46 \pm 0.17 \text{(stat.)} \pm 0.13 \text{(syst.%)}. \]

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momenta. In Fig.6 we show also the STAR collaboration data. The experimental results of both groups are not completely consistent. In the interval $3 \text{ GeV} < p_t(\text{lepton}) < 6 \text{ GeV}$, the STAR data points are somewhat higher than the PHENIX data points. This disagreement needs further explanation. Our results are roughly consistent with both experimental sets at large $p_t(\text{lepton})$. There is a missing strength at small transverse momenta where only the PHENIX collaboration data exist. This will be discussed further in the following.

In Fig.7 we show results obtained with Ivanov-Nikolaev UGDF. Although there is some improvement at low transverse momenta, the cross section for larger transverse momenta exceed the experimental data.

It is not clear for the moment what is the missing strength. Up to now we have included only gluon-gluon fusion which is known to be dominant contribution at large center-of-mass energies (Tevatron, LHC). At RHIC energies the typical longitudinal momentum fractions of gluons are still not too small $x_1, x_2 \sim 0.01$ and the contribution of the quark-antiquark annihilation may be not negligible. Therefore in the following we shall include also quark-antiquark annihilation process. Those processes can be included in a similar way in the formalism of unintegrated parton distributions. The corresponding diagram is shown in Fig.8. The Kwieciński formalism \cite{20} allows to calculate unintegrated quark/antiquark distribution in the same framework as unintegrated gluon distributions. In Fig.9 we present the contribution of quark-antiquark annihilation $q\bar{q} \rightarrow c\bar{c}$ (dash-dotted line). This contribution is similar in size to the $gg \rightarrow b\bar{b}$ contribution. The contribution of $q\bar{q} \rightarrow b\bar{b}$ is negligible and is not shown here.

Study of nonphotonic $e^\pm$ and hadron correlations allows to ”extract” a fractional contribution of the bottom mesons $B/(D + B)$ as a function of electron/positron transverse momentum \cite{24}. Recently the STAR collaboration has extended the measurement of the relative $B$ contribution to electron/positron transverse momenta $\sim 10 \text{ GeV} \cite{25}$. In Fig.10 and 11 we present our results for different unintegrated gluon distributions and different fragmentation functions. There is a strong dependence on the factorization and renormalization scale in the case of the Kwieciński unintegrated gluon distributions. A better agreement is obtained with the Peterson fragmentation functions. The separation into charm and bottom contributions is very important in the context of identifying the missing strenght. A new correlation method was proposed recently to identify and separate charm and bottom production on a statistical basis \cite{25}. The method was tested using known event generators. An alternative method of extracting the relative $B$ contribution from azimuthal angular correlations of nonphotonic electrons and $D_0$ mesons was proposed \cite{26}. One can hope that application of the new methods will help in disantagling the contributions better.

IV. DISCUSSION OF THE RESULTS

We have calculated inclusive spectra of nonphotonic electrons/positrons for RHIC energy in the framework of the $k_t$-factorization. We have concentrated on the dominant gluon-gluon fusion mechanism and used two recent unintegrated gluon distribution functions from the literature. Special emphasis was devoted to the Kwieciński unintegrated gluon (parton) distributions. In this formalism, using unintegrated quark and antiquark distributions, one can calculate in addition the quark-antiquark annihilation process including transverse momenta of initial partons (quarks/antiquarks). In addition, we have used unintegrated gluon distributions constructed by Ivanov and Nikolaev to describe deep-inelastic data measured at HERA.
When calculating spectra of charmed ($D, D^*$) and bottom ($B, B^*$) mesons we have used Peterson and Braaten et al. fragmentation functions with model parameters from the literature. There are no big differences between results obtained with both fragmentation functions.

A very important ingredient, which influences the final spectra, is the distribution of electrons/positrons from the decay of $D$ and $B$ mesons. Here we have used recent results of the CLEO and BABAR collaborations. The momentum spectra of electrons/positrons from the decays of $D$ and $B$ mesons produced in the $e^+e^-$ collisions were used in the present calculation to generate distribution of electrons/positrons coming from the decays of $D$ and $B$ mesons produced in the hadronic reactions. This way we have avoided all uncertainties associated with modeling semileptonic decays of mesons.

We have compared results obtained in our approach with experimental data measured recently by the PHENIX collaboration at RHIC. We get a reasonable description of the data at large transverse momenta of electrons/positrons. Similarly as for the higher-order collinear approach there is a missing strength at lower transverse momenta.

Up to now there is no clear explanation of the enhanced production of electrons/positrons at low transverse momenta. The uncertainties related to the choice of factorization and renormalization scale seems to be insufficient. There can be several reasons of the unexplained strength at low transverse momenta.

The $k_t$-factorization approach includes many higher-order contributions which are embodied in unintegrated gluon (parton) distributions. Some higher-order contributions are definitely not included. A simple and transparent example are emissions of gluons of the heavy quarks/antiquarks. This contribution can be estimated in the standard collinear approach. This effect is, however, not limited to low transverse momenta.

It is commonly assumed that $D/\bar{D}$ mesons are produced via fragmentation of $c/\bar{c}$ quarks. However, at lower energies (fixed target experiments) an asymmetry between different species of $D$ mesons have been observed [28]. This asymmetry can be due to fragmentation of light ($u,d,s$) quarks/antiquarks [29] ($q \to D(q\bar{c})c$ or $\bar{q} \to D(\bar{q}\bar{c})\bar{c}$) or meson cloud effects [30]. The asymmetry increases with rapidity (or Feynman $x_F$). This makes questionable the common assumption that $D$ mesons are produced exclusively via fragmentation of $c$ quarks. In this context, it would be very useful to analyze electronic spectra at larger rapidities. If these mechanisms are responsible for the missing strength then the discrepancy there would be even larger. In the moment only muons were measured at forward rapidities [31] and there seems to be some enhancement, although systematic error bars are rather large.

The results of the PHENIX collaboration were obtained by subtraction of several components, including decays of vector mesons, so-called Dalitz decays, $K_{e3}$ decays and other mechanisms. All of them are concentrated at low transverse momenta [2]. Only a sketch of the subtruction procedure was presented [32]. The details of the subtraction are not presented in extenso. It is therefore not clear to us how reliable such subtraction is. In addition, there are several mechanisms which were not included. These are Drell-Yan processes, processes initiated by two photons (they are expected to be concentrated at low transverse momenta) and several other exclusive processes never calculated in the literature. It seems therefore difficult to draw definite conclusions before cross section for all these processes is evaluated. We leave such calculations for separate detailed studies. In principle, also

\footnote{There is a substantial fragmentation of light quarks ($q \neq s$) in the case of kaon production. Such a contribution for $D$ mesons is therefore also not excluded.}
analysis of coincidence spectra, e.g. in invariant mass of the dilepton pair $M_{ee}$, could help to pin down the missing mechanisms.

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FIGURES

FIG. 1: A basic diagram relevant for gluon-gluon fusion in $k_t$-factorization.

FIG. 2: Our fit to the CLEO [18] and BABAR [19] data.

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FIG. 4: Transverse momentum distribution of electrons/positrons with the Kwieciński UGDF. Different combinations of factorization and renormalization scales are used. On the left side we show results with Peterson fragmentation functions and on the right side with BCFY fragmentation functions.

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FIG. 5: The same as in the previous section but with different choices of factorization/renormalization scale.

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FIG. 6: Uncertainty band of our $k_t$-factorization calculation (both D and B decays) with unintegrated Kwieciński gluon distribution for Peterson fragmentation function (left panel) and BCFY fragmentation function (right panel). The open triangles represent the PHENIX collaboration data and the solid circles the STAR collaboration data.

FIG. 7: Transverse momentum distribution of electrons/positrons obtained with Ivanov-Nikolaev UGDF and Peterson (left panel) and BCFY (right panel) fragmentation functions.

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FIG. 8: A diagram for quark-antiquark annihilation.

FIG. 9: Transverse momentum distribution of electrons/positrons with Kwiecinski UPDFs. The dash-dotted line corresponds to the $q\bar{q} \rightarrow b\bar{b}$ contribution.
FIG. 10: The fraction of the B decays for the Kwieciński UGDF and different combinations of the factorization and renormalization scales.
FIG. 11: The fraction of the B decays for the Ivanov-Nikolaev UGDF and two different models of the fragmentation. In this calculation $\mu_R^2 = 4m_Q^2$. 

$W = 200$ GeV

Ivanov–Nikolaev

$\mu^2 = 4m_Q^2$

$\mu^2 = 4m_Q^2$

Peterson FF (solid)

BCFY FF (dashed)