Equations of Motion with Multiple Proper Time: A New Interpretation of Basic Quantum Physics

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Equations of motion for single particle under two proper time model and three proper time model have been proposed and analyzed. The motions of particle are derived from pure classical method but they exhibit the same properties of quantum physics: the quantum wave equation, de Broglie equations, uncertainty relation, statistical result of quantum wave-function. This shows us a possible new way to interpret quantum physics. We will also prove that physics with multiple proper time does not cause causality problem.

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I. INTRODUCTION

From Thirring [1] and Kaluza [2]'s 5-dimensional space-time to today’s superstring theory, Physics with extra space-time dimensions have been studied for about 90 years. One of questions the theories have to face is: “do we have any observation of extra dimension in nature? ” For more than 90 years in another area, from Einstein to Bohm [3]'s quantum hidden variable theory, people have been looking for a way to find classical interpretation of quantum physics. The purpose of this paper is to show that it is possible to interpret quantum physics by adding extra time dimension (or dimensions) in classical physics.

The extra space-time dimensions in most theories are space dimensions. People believed that extra time dimension could cause causality problem [4]. I.Bars and etc [5] [6] [7] proposed two timelike dimensions in string theory which called “two time physics”. I.Bars’ papers could raise questions like: “Does two time-like dimensions necessarily mean two dimensional time? ”; “Does that create causality problem? ” One of the most important questions is: if there are two dimensional time, a particle should be able to move under two independent “proper time”, how does the motion look like in basic physics?

In this paper, multiple proper times are introduced in classical physics. In addition, a particle is always treated as a point particle in this paper. Section I will introduce the model of two independent proper time. The world lines equations and motion of equation of single particle under two proper time are derived. Section II will introduce the model of three dimensional time. In section III, the interpretation of quantum physics is discussed. In section IV Causality under multiple dimensional time is discussed. Section V will show that multiple dimensional time will not cause causality problem.

II. TWO PROPER TIME MODEL FOR CLASSICAL SINGLE PARTICLE

Relativity introduced symmetry between time and space into physics. In Riemann Space, the only difference between time and space are their signatures (+ and -): but there is one asymmetrical properties which time is different from space – proper time. τ. τ plays as a special affine parameter in Relativity. In fact, τ is the one we called “time” in our common life(in low speed world). If there is n-dimensional time in the world, there should be n-dimensional proper time.

In this section, we will study how the motion of single particle looks like if there are two proper time. Throughout this paper, τ indicates the first proper time which is the proper time in relativity. σ indicates the second proper time. We call world line τ if particle moves along world line with proper time τ, and world line σ if particle moves along world line with proper time σ. We made some reasonable assumptions to second proper time: 1)World lines σ are orthogonal to world lines τ. 2)σ and τ are independent; i.e., when particle’s position on world line τ is unchanged, the particle can still move on world line σ and vice versa. 3)Similar to Kaluza [2] and Klein [3]'s idea, we apply cylinder condition on σ: σ is a loop with value from 0 to 2π.

For free particle with constant velocity, let \( x_{\alpha \tau} (\alpha = 0, 3) \) be 4-dimension coordinates on world line τ; \( u = \sqrt{u_{i}u^{i}}, (i = 1, 2, 3) \) which is speed of particle on world line τ; \( x_{\alpha \sigma} \) be 4-dimension coordinates on world line σ; \( v = \sqrt{v_{i}v^{i}}, (i = 1, 2, 3) \) which is speed of particle on world line σ. \( t_{\tau} \) as time on world line τ, \( t_{\sigma} \) as time on world line σ, then

\[
x_{0\tau} = ct_{\tau} \quad x_{i\tau} = u_{i}t_{\tau} \quad t_{\tau} = \frac{\tau}{\sqrt{1 - \frac{u^{2}}{c^{2}}}}
\]

(1)

where \( i=1,2,3; \) c is the speed of light. Momentum and energy can be defined as

\[
p_{i} = mu_{i} \quad E = mc^{2}
\]

(2)

where \( i=1,2,3; \) m is mass of particle. Then from special
FIG. 1: World line $\tau$ and world line $\sigma$ on $t - x_i$ plane in Minkowski space. Particle can move along both world lines, slope of $\tau$ is $u/c$. At $t = 0$, the single particle will be shown at many positions: $-x_1, -x_2, ..., -x_n$ with different values of $\tau$ and $\sigma$ ($-\tau_1, \sigma_1), (...-\tau_n, \sigma_n$). Also the single particle will be shown at $x = 0$ at different time: $t_1, t_2, ..., t_n$ with different $\tau$ and $\sigma$ values; where $x_n = h/mu$ and $t_n = h/mc^2$ which are de Broglie wavelength and period.

relativity, world line $\tau$ satisfies condition

$$\tau = \frac{i}{m_0} (Et - p_1 x_1 - p_2 x_2 - p_3 x_3)$$  \hspace{1cm} (3)$$

where $m_0$ is static mass. Fig1 draws world line $\tau$ and world line $\sigma$ on $x_0 - x_i$ plane in Minkowski Space.

Slope of world line $\tau$ is $x_1/0\tau$, Slope of world line $\sigma$ is $x_1/0\sigma$, and $x_1/0\tau = x_1/0\sigma$ since they are perpendicular to each other, so

$$v_i = \frac{c^2}{u_i}$$  \hspace{1cm} (4)$$

$v_i > c$ and $\sigma$ is space like, $\tau$ and $\sigma$ are independent; On each point of world line $\tau$, particle can move on world line $\sigma$.

Fig2. draws world line $\tau$ and world line $\sigma$ on $x_0 - x_i$ plane in Riemann Space.

The equations of motion of particle are:

$$x_0(\tau, \sigma) = \frac{\tau}{\sqrt{1 - \frac{c^2}{v^2}}} + i \frac{\sigma}{\sqrt{1 - \frac{c^2}{v^2}}} + x_0(\tau_0, \sigma_0)$$  \hspace{1cm} (5)$$

$$x_i(\tau, \sigma) = \frac{u_i \tau}{\sqrt{1 - \frac{c^2}{v^2}}} - i \frac{\sigma}{\sqrt{1 - \frac{c^2}{v^2}}} + x_i(\tau_0, \sigma_0)$$  \hspace{1cm} (6)$$

where $i=1,2,3$; $x_0(\tau_0, \sigma_0)$ is initial value of $x_0$, $x_i(\tau_0, \sigma_0)$ is initial value of $x_i$; imaginary number $i$ is to keep $\sigma$ and $x_n$ to be real value since $v >> c$. Under equation $\textbf{6}$, particle’s position is determined by two proper time $(\tau, \sigma)$, but the particle’s position can not be localized by each of them individually. As the result, particle’s spatial position $x_i$ is not localized at fixed time $t$. Therefore, different from Relativity, the physics of single particle is not localized by 4-dimensional space-time.

From Fig1 we see that at $t = x_0 = 0$, particle’s spatial position can be at $-x_n$ through the path: from $-\tau_n$ to $-x_n$; and can be at $x_2$ through the path: first from $-\tau_n > -\tau_2$, then from $-\tau_2 > x_2$; and can be at $x_1$ through the path: first from $-\tau_n > -\tau_1$, then from $-\tau_1 > x_1$. At $t=0$ and $\frac{mv^2}{h^2} = -2\pi$, we have

$$x_i = \frac{h}{p_i}$$  \hspace{1cm} (7)$$

similarly at $x_i = 0$ and $\frac{mv^2}{h^2} = 2\pi$,

$$t = \frac{h}{E}$$  \hspace{1cm} (8)$$

FIG. 2: World line $\tau$ and world line $\sigma$ on $X_0 - X_i$ plane in Riemann space. At each points of world line $\tau$, the particle can move along world lines $\sigma$, slope of $\tau$ is $u/c$. Each world lines $\sigma$ parallel to each other with slope: $v/c = c/u$.

In Riemann space, $\tau$ and $\sigma$ are orthogonal to each other. At $t = 0$, the single particle will be shown at many positions: $x_1, x_2, ..., x_n$ with different values of $\tau$ and $\sigma$ ($\tau_1, \sigma_1), (...\tau_n, \sigma_n$). Also the single particle will be shown at $x = 0$ at different time: $t_1, t_2, ..., t_n$ with different values of $\tau, \sigma$; where $x_n = h/mu$ and $t_n = h/mc^2$ which are de Broglie wavelength and period.
FIG. 3: World lines $\sigma$ is a infinitesimal loop to fixed point $(x_0, t_0)$. $\tau$ perpendicular to loop $\sigma$, so $\tau$ can point to any direction, the slope of $\tau$ is from $-\infty$ to $\infty$ which means the momentum is from $-\infty$ to $\infty$.

If $m_0 \tau$ satisfies periodic condition, then equation (1) and (8) become de Broglie equations, but $\tau$ is the proper time associate with time dimension $t$, and we never observed periodic properties in classical physics for $t$, so we need to add one more time dimension and proper time in the next section.

The Above equations illustrate the motion of single particle under two proper time with constant energy and momentum. The single particle spreads out everywhere in space-time, i.e., with fixed energy and momentum, the particle’s position and time are uncertain. That is, because at each fixed $\tau$, particle can move by $\sigma$ to another position. To localize a particle, we need to make all $\sigma$ “stay” only at one position $(x_{i0}, t_0)$. From equation (1), $v_i$ can not be zero, so world line $\sigma$ must be a infinitesimal loop around $(x_{i0}, t_0)$ as shown in Fig3. But world line $\tau$ perpendicular to world line $\sigma$, and the slope of $\tau$ is $\frac{\lambda}{\pi}$; momentum $p_i = m_0 v_i$. From Fig3, we see that because at each points on circle $\sigma$, particle can move perpendicular to $\sigma$ which create a world line $\tau$, and the slope of $\tau$ is from $-\infty$ to $\infty$, so the momentum becomes $-\infty$ to $\infty$. I.e. if we localized particle’s position and time, the momentum and energy will become uncertain.

FIG. 4: The loop of extra time dimension on complex plane

III. THREE PROPER TIME MODEL – APPROACH TO QUANTUM PHYSICS

Let’s introduces new time coordinate $x_4$ and new proper time $\phi$. Fig4. draws time loop on a complex-plane. and $\phi$ is the angle from 0 to $2\pi$. The equation of loop is $e^{i\phi}$. Let

$$\phi = \frac{m_0 \tau}{\hbar} \quad (9)$$

i.e., particle moves around the loop of $\phi$ with angular velocity $\frac{m_0}{\hbar}$, uses equation (8), the equation of loop becomes:

$$e^{-\frac{im_0}{\hbar} \phi} = e^{\frac{i}{\hbar} (Et - p_1 x_1 - p_2 x_2 - p_3 x_3)} \quad (10)$$

It is wave function of quantum particle with fixed 4-momentum. In fact, in Fig1, at $t=0$, particle’s positions are any points from $-\lambda = \frac{\hbar}{p_i}$ to 0 with $x_4$ from $-2\pi$ to 0; At $x=0$, particle stays position $(x=0)$ at time from 0 to $\frac{2\pi}{\hbar}$ with $x_4$ from 0 to $2\pi$. Therefore, Fig1. and Fig 2. illustrate a plane wave. It is created by single particle’s motion under proper time $\tau$, $\sigma$, $\phi$.

Assume the geometry of new time dimension $x_4$ is a loop with fixed radius (we can also assume the radius of loop is very small, for example, the length of Planck constant $h$, to be convenience, here we choose radius =1 ), then

$$x_4 = e^{i\phi} \quad (11)$$

On $x_i$ coordinate, when $x_4$ moves around a circle, $x_i$ moves from 0 to $\lambda$, this can be interpreted as $x$ oscillating along $x_4$ direction which is perpendicular to x-t plane.
Let’s go back to proper time $\sigma$, and let the time dimension which associates with $\sigma$ be $x_5$. Particle moves along world line $\sigma$ with speed $\frac{\sigma}{u}$ which is phase velocity of de Broglie wave. Similar to $x_4$, assume $x_5$ is also a loop as illustrated in Fig4., we can build 1 to 1 relationship between loop $x_5$ and world line $\sigma$, let

$$x_5 = e^{i\tau}(Et-p_1x_1-p_2x_2-p_3x_3)$$  \hspace{1cm} (12)

In real world, we only have knowledge of one dimensional time $t$. In experiment, we measure one time dimension by using “clock”. We do not know how to synchronize each particle’s 2nd and 3rd time dimensions $x_4$ and $x_5$. When a particle of apparatus arrives at $(x, t)$ with 2nd time dimension $x_4a$, the $x_4$ value of the particle to be measured can be any value on loop $e^{i\tau}$; But to “meet” the particle at location $X(x_i)$, the apparatus’s particle and the particle $p$ must arrive at location $X(x_i)$ at the same three dimensional time, i.e. $x_{4a} = x_{4p}$, and $x_{5a} = x_{5p}$. The possibility of $x_{4a} = x_{4p}$ is

$$\psi = P_\phi = \frac{e^{i\tau}}{\int e^{i\tau}}$$  \hspace{1cm} (13)

Similarly, the possibility of $x_{5a} = x_{5p}$ is

$$P_\sigma = \frac{e^{-i\tau}}{\int e^{-i\tau}} = \psi^*$$  \hspace{1cm} (14)

So the total possibility to find particle at $(x, t)$ is

$$P = P_\phi P_\sigma = \psi \psi^*$$  \hspace{1cm} (15)

For the plane wave of single photon, proper time $\tau = 0$, equation (15) is no longer valid. Then proper times $\sigma$ and $\phi$ are not related to $\tau$. Instead, we separate the motions of photon by three proper time. 1) Photon moves along world line $\tau$. 2) Photon has oscillation $E = E_0 e^{-i\omega t + \lambda x}$ by $\sigma$ which perpendicular to $\tau$, where $E$ is electric field. 3) Photon has oscillation $B = B_0 e^{-i\omega t + \lambda x}$ by $\phi$ which perpendicular to $\tau$, where $B$ is magnetic field. The possibility to find photon is proportional to:

$$S = E \times B$$  \hspace{1cm} (16)

In another paper [11], I proved that by choosing 6-dimensional space-time metric as

$$(g_{AB}) = \begin{pmatrix}
g_{\alpha\beta} & \psi \\
\psi^* & -1
\end{pmatrix}$$  \hspace{1cm} (17)

where metric elements $g_{\alpha\beta}$ is 4-dimensional metric. We can derive Klein-Gordon equation directly from Einstein field equation:

$$\hat{G}_{\alpha\beta} = \kappa \hat{T}_{\alpha\beta}$$  \hspace{1cm} (18)

Under this metric, for spinless free particle, we have equation of world line $\tau$:

$$ds^2 = dx_\alpha dx^\alpha + e^{i\sigma}(-m_0 x_\alpha x^\alpha)dx_4 dx_4 - dx_5 dx_5$$  \hspace{1cm} (19)

Equation for world line $\sigma$

$$ds^2 = dx_\alpha dx^\alpha - dx_4 dx_4 + e^{i\sigma}(\sigma^\alpha x_\alpha - m_0 x_\alpha)dx_5 dx_5$$  \hspace{1cm} (20)

In general, for spinless particle, equation (19) becomes

$$ds^2 = dx_\alpha dx^\alpha + \psi^2 dx_4 dx_4 - dx_5 dx_5$$  \hspace{1cm} (21)

Put it into Einstein field quation, it satisfied wave equation [11]:

$$\partial_\alpha \partial^{\alpha} \psi = 0$$  \hspace{1cm} (22)

where $\alpha = 0..5$, $\psi$ is wave function. The wave equation (22) is derived directly from Einstein field equation where Planck constant plays the same role as gravitational constant [11]. It means that quantum phenomena can be understood as pure geometry effect of 6-dimensional space-time.

IV. INTERPRETATION OF QUANTUM PHYSICS

Non-local property of single quantum particle is one of the most important reasons why quantum physics does not fit in classical physics theory. In quantum physics, a single particle can stay at different places at the same time. In double slits interference experiment, if we try to use classical paths to describe particle’s motion, the particle must have to pass both slits at the same time. In condensed matter physics, an electron’s spatial positions will be everywhere in lattice at any time; i.e. the electron’s must be able to stay in may spatial positions inside lattice at the same time; in the experiment about Bell’s inequality, a single particle must stay in two different spatial places even though the distance between those two places are “far”. Those all conflict with our knowledge in classical physics (including Relativity). In classical physics with one dimensional time, a particle can not stay in more than one place at the same time.

Section [11] demonstrates that by introducing multiple proper time, single particle’s motion shows non-local properties in classical physics. The particle can move to different places by extra proper time $\sigma$, in Fig1, at time $t=0$, particle’s spatial positions are $x_1$, $x_2$, ... $x_n$. In fact, section [11] and [11] draw two different pictures:

1) Along world line $\tau$, free particle moves like classical particle with constant velocity with classical energy and momentum.

2) World lines of $\sigma$ and $\phi$ are also straight lines. Relations between $\phi$ and 2nd time dimension $x_4$, $\sigma$ and 3rd time dimension $x_5$ are $x_4 = e^{i\phi} = e^{i\sigma}$, $x_5 = e^{i\sigma} = e^{-i\phi}$. The equations come from the geometry of $x_4$ and $x_5$, which are loops in complex plane.

Put 1) and 2) together, then with equation it created plane wave function for single particle. Single particle’s position is not unique under 1 dimension time $t$, but the position is unique under three dimensional time ($t$, $x_4$, $x_5$).
x_5). Because of the uniqueness, one particle can not contribute two energies, one electron can not contribute two electronic charges. In addition, at each point (t, x_4, x_5) of three dimensional time, the particle’s energy, momentum and charge are the same definition as in classical physics.

To measure particle p at (x,t), the values x_4 and x_5 of apparatus’s particle must by the same as the values of p. That is because two things can only meet at the same time (here are three dimensional times). In section [11] we have discussed that: because we do not have apparatus to measure 2nd and 3rd dimensional time, we can not determine particle’s location by our 4-dimensional apparatus. Instead, the particle’s position is statistical in 4-dimensional space-time description. The possibility of finding the particle is proportional to \( \psi \psi^* \) in equation [15]. Equation [15] is derived by single free particle with constant velocity. In general cases, the map between 4-dimensional coordinates are not necessary to be plane wave function. Instead, it could be the combination of plane wave function with different frequencies and wave-lengths. For example, the case of Fig3. Then the world lines of particle become multiple lines of world lines \( \tau \) with different slopes. But on each individual world line \( \tau \), x_4 and x_5 are plane waves with the same frequency and wave-length. That means in general:

\[
x_4 = \Psi = \psi(\tau_0) + \psi(\tau_1) + ... \psi(\tau_n)
\]

Then

\[
x_5 = \psi^*(\tau_0) + \psi^*(\tau_1) + ... \psi^*(\tau_n) = \Psi^*
\]

So the possibility of finding particle is always \( |\Psi|^2 \). When we found the particle, the particle’s momentum, energy and other observables are defined along \( \tau \), so the average value of classical observable F is:

\[
< F > = \int F|\psi|^2
\]

Fig1. and Fig2. show that when particle’s momentum is fixed, particle’s position is uncertain because particle can move by \( \sigma \). Fig3. shows that when particle’s position is localized, particle’s momentum becomes uncertain because of the changing slopes of \( \tau \) by world line \( \sigma \), i.e. we can not find a world lines distribution of \( \tau \) and \( \sigma \) such that both position and momentum are constant. That is the reason we have uncertainty relationship for x and p in quantum physics.

Now let’s consider the double slits interference experiment for particle. In x-y plane, let particle move along x coordinate with \( y = 0 \), the double slits at x=d, two slit’s coordinates are \( S_1(d, y/2) \) and \( S_2(d, -y/2) \), and screen at x = S. When particle reaches x=d, we know that even though the particle’s y component of velocity \( u_y \) is zero: \( u_y = 0 \), particle still moves in y direction by proper time \( \sigma \), so at x=d, particle can move from S1 to S2 by \( \sigma \), i.e. the particle passes both slits at the same time t. At S1, the particle’s \( x_4 \) and \( x_5 \) value are \( (e^{i\phi}, e^{-i\phi}) \), and at \( S_2 \) with values \( (e^{i(\phi+\delta)}, e^{-i(\phi+\delta)}) \), \( \delta \) is a small number since the distance between \( S_1 \) and \( S_2 \) is small. After particle passes \( S_1 \) and \( S_2 \), the world line \( \tau \) splits into two paths with world lines \( \tau_1, \tau_2 \). Let \( x_{14}, x_{51} \) be value of \( x_4 \) and \( x_5 \) on path 1; \( x_{42}, x_{52} \) be value of \( x_4 \) and \( x_5 \) on path 2. Suppose \( \tau_1 \) and \( \tau_2 \) meet at p where p is a point on screen. At p, we have

\[
x_{41} = x_{42} \quad x_{51} = x_{52} \quad (26)
\]

Let \( \Delta L = \) path(from \( S_2 \) to p) - path(from \( S_1 \) to p), then to get equation [26], \( \Delta L \) must satisfies:

\[
\Delta L = (n + \frac{\delta}{2\pi})\lambda \quad (27)
\]

where \( n \) is any integer and \( \lambda \) is wave length, particle can not reach those points, which does not satisfy equation [27], so we get interference pattern on the screen.

V. CAUSALITY IN THREE DIMENSIONAL TIME

In real world, time has direction. Here proper time \( \tau \) and proper time \( \sigma \) have directions too.

From Fig1., Fig2. and Fig5., one can see that \( \tau \) and \( \sigma \) both move to positive direction of t. Along world line \( \tau \) when \( \sigma \) is unchanged, if event 1 happens at \( \tau_1 \) and event 2 happens at \( \tau_2 \) and \( \tau_1 > \tau_2 \), then corresponding universal time \( t_1 > t_2 \), so, on world line \( \tau \), the causality is preserved. Along world line \( \sigma \) when \( \tau \) is unchanged, if event 1 happens at \( \sigma_1 \) and event 2 happens at \( \sigma_2 \) and \( \sigma_1 > \sigma_2 \), then corresponding universal time \( t_1 > t_2 \), so on world line \( \sigma \), the causality is also preserved. In general, event 1 happens at \( (\tau_1, \sigma_1) \), and event 2 happens at \( (\tau_2, \sigma_2) \), if \( \tau_1 > \tau_2 \) and \( \sigma_1 > \sigma_2 \), then we have \( t_1 > t_2 \), causality is preserved. But what will happen when \( \sigma_1 > \sigma_2 \) and \( \tau_1 < \tau_2 \)? On Fig5, if event 1 happens on \( x_1 \), event 2 happened on \( x_2 \), then on “local static” reference frame, event 1 happened after event 2 because \( \tau_1 > \tau_2 \), on universal time, event 1 happened at the same time as event 2 because both happened at \( t = 0 \), on world line \( \sigma \), event 1 happened before event 2 because \( \sigma_1 < \sigma_2 \), does it conflict with causality?

Look at Fig5. Suppose universal time t at \( t = -t_2 \), particle reached \( \tau_2 \) along world line \( \tau \); then particle moves to \( x_2 \) at \( t=0 \) along world line \( \sigma \), \( \sigma = \sigma_2 \), on “local rest” reference frame (world line \( \tau \) which is still at \( \tau = \tau_2 \), the particle goes to future because \( t = 0 > t_2 \). If at \( x_2 \), particle does not have any interaction with other particles, then the particle can’t “see” anything in future, when particle moves back to \( \tau = \tau_1, \sigma = 0 \), there is nothing related to causality, no event occurred. But if event 2 happened at \( x_2 \); i.e. particle interacts with other particle, then the particle’s physical state is changed by interaction (Remember: We can not measure a particle without affect its physical status). Suppose the particle’s momentum
FIG. 5: Current universal time is $t = -t_2$, particle reaches $x_2$ at $t=0$, it is in future since $t = 0 > -t_2$. When event 2 happens at $x_2$, the particle interacts with other particles, so particle’s world lines is changed, its next movement will be based on new world line $\tau'$ has a small changes: $\delta p$, then the particle’s next move will start on a new world line $\tau'$, the particle can not go back to original world line $\tau$, this phenomenon is corresponding to wave-packet collapse in quantum physics, so any event happened on this particle after event 2 will be on time $t > 0$, the causality is still preserved. Oriented $\tau$ and $\sigma$ and wave-packet collapse are key factors to keep causality preserved.

All physical reference frames still move along world line $\tau$ with speed $u < c$, so causality is preserved in any reference frame. Although particle’s speed $v > c$ along world line $\sigma$, we can not observe or measure this speed because we can not determine particle’s position without affecting particle’s velocity (momentum). From all above, we see that three dimensional time contain the basic properties of quantum physics, one can understand that three dimensional time will not conflict with causality law unless quantum physics itself conflicting causality law. If two identical particles are correlated each other, and we separate the wave to two parts with certain distance, if we affect one part of wave, then the other part on the other place will be affected on the same time (i.e. the information passed without time change). This is the well known Bell’s inequality. There are already many papers discussing about causality law in this phenomenon [9].

VI. SOME DISCUSSIONS

First, it is interesting to see the relation between three dimensional time and String theory. Actually Fig1. looks like a world sheet in String theory. If we let $\sigma$ be space dimension instead of proper time, it turns to bosonic String theory. There are two major differences between three dimensional time and String theory.

1) In String theory, the motion in $\sigma$ direction is compacted. It can not be very large since we never see extra space dimensions in real world. In three dimensional time, the distance traveled by world line $\sigma$ can be very large, this is a very important property which provides non-local properties of three proper time physics.

2) Three dimensional time has different statistical results from String theory. Because of the special character of time (which different from space), it demonstrates the same statistical results as quantum physics.

But we still can use some results of String theory. Considering two proper time case, put Lagrangian

$$L = -\frac{1}{2}m[(\dot{x} \cdot x')^2 - (\dot{x} \cdot \dot{x})(x' \cdot x')]^{1/2}$$

(28)

Where $\dot{x} = \frac{dx}{d\tau}$; $x' = \frac{dx}{d\phi}$. The Lagrangian above is the same as the Lagrangian in string theory [4]. The classical equation of motion is

$$\frac{\partial}{\partial \tau}(\delta L / \delta \dot{x}_a) + \frac{\partial}{\partial \phi}(\delta L / \delta \dot{x}_a)) = 0$$

(29)

Add constraints [4]:

$$\dot{x} \cdot x' = 0, \quad \dot{x} \cdot x + x' \cdot x' = 0$$

(30)

The equation of motion (29) becomes wave equation:

$$\ddot{x}_a = x''_a$$

(31)

The above result is the same as bosonic string theory [4]. For free particle with constant momentum, we choose solution:

$$x_a(\tau, \phi) = e^{-\frac{im_0}{\hbar}(\tau - \phi)}$$

(32)

Since $x_a(\tau, \phi)$ must be real number, the above equations have solutions only when

$$\phi = \tau = \frac{1}{m_0}(Et - p_1x_1 - p_2x_2 - p_3x_3)$$

(33)

We see that equation corresponding to proper time $\phi$.

Second, there are possible some interesting relations between three dimensional time and quantum field theory. Feynman [10] interpreted negative energy state of particle as: negative energy state represents the particle
moving to negative time direction. It is hard to understand or illustrate this in 1 dimensional time theory, but it can be often seen in three dimensional time: looking at Fig5. particle goes to $x_2$ (future) at $t=0$, then go back to $t_1(\tau_1,\sigma = 0)$. In addition, in section II Fig3, when $\sigma$ accrossing $X_i$ coordinate, momentum becomes infinite, that is because the particle moves from one location to another location without changing universal time $t$ – the particle moves by $\sigma$ only. That is to say We get infinity momentum because we only use space and first time dimension $t$ to calculate momentum. If we uses $\sigma$, the infinity will be gone. It is possible to use this to deal with the infinities in quantum field theory in future.

Third, this paper is only dealing with “basic” quantum physics. I.e., it is only discussing spinless particle. I believe that spin is coming from the motion of extra time dimension. To discuss the particle with integer spin and half-integer spin, we have to find a way to explain the results of Bose-Einstein statistics and Fermi-Dirac statistics. We will discuss that in another paper.

I used three dimensional time to interpret quantum physics in two other papers before [12] [11]. This paper provides more details and clear pictures for three dimensional time theory.

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Equations of motion for single particle under two proper time model and three proper time model have been proposed and analyzed. The motions of particle are derived from pure classical method but they exhibit the same properties of quantum physics: the quantum wave equation, de Broglie equations, uncertainty relation, statistical result of quantum wave-function. This shows us a possible new way to interpret quantum physics. We will also prove that physics with multiple proper time does not cause causality problem.

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I. INTRODUCTION

From Thirring [1] and Kaluza [2] 's 5-dimensional space-time to today’s superstring theory, Physics with extra space-time dimensions have been studied for about 90 years. One of questions the theories have to face is: “do we have any observation of extra dimension in nature? ” For more than 90 years in another area, from Einstein to Bohm [3]’s quantum hidden variable theory, people have been looking for a way to find classical interpretation of quantum physics. The purpose of this paper is to show that it is possible to interpret quantum physics by adding extra time dimension (or dimensions) in classical physics.

The extra space-time dimensions in most theories are space dimensions. People believed that extra time dimension could cause causality problem [4]. I.Bars and etc [5] [6][7] proposed two timelike dimensions in string theory which called “two time physics”. I.Bars’ papers could raise questions like: “Does two time-like dimensions necessarily mean two dimensional time? ” “Does that create causality problem? ” One of the most important questions is: if there are two dimensional time, a particle should be able to move under two independent “proper time”, how does the motion look like in basic physics?

In this paper, multiple proper times are introduced in classical physics. In addition, a particle is always treated as a point particle in this paper. Section II will introduce the model of two independent proper time. The world lines equations and motion of equation of single particle under two proper time are derived. Section III will introduce the model of three dimensional time. In section IV, the interpretation of quantum physics is discussed. In section V, Causality under multiple dimensional time is discussed. Section IV will show that multiple dimensional time will not cause causality problem.

II. TWO PROPER TIME MODEL FOR CLASSICAL SINGLE PARTICLE

Relativity introduced symmetry between time and space into physics. In Riemann Space, the only difference between time and space are their signatures (+ and -); but there is one asymmetrical properties which time is different from space – proper time. \( \tau \) plays as a special affine parameter in Relativity. In fact, \( \tau \) is the one we called “time” in our common life (in low speed world). If there is \( n \)-dimensional time in the world, there should be \( n \)-dimensional proper time.

In this section, we will study how the motion of single particle looks like if there are two proper time. Throughout this paper, \( \tau \) indicates the first proper time which is the proper time in relativity. \( \sigma \) indicates the second proper time. We call world line \( \tau \) if particle moves along world line with proper time \( \tau \), and world line \( \sigma \) if particle moves along world line with proper time \( \sigma \). We made some reasonable assumptions to second proper time:

1)World lines \( \sigma \) are orthogonal to world lines \( \tau \).

2)\( \sigma \) and \( \tau \) are independent; i.e., when particle’s position on world line \( \tau \) is unchanged, the particle can still move on world line \( \sigma \) and vice versa.

3)Similar to Kaluza [2] and Klein [8]’s idea, we apply cylinder condition on \( \sigma \): \( \sigma \) is a loop with value from 0 to \( 2\pi \).

For free particle with constant velocity, let \( x_{\alpha \tau} (\alpha = 0,3) \) be 4-dimension coordinates on world line \( \tau \); \( u = \sqrt{u_i u_i}, (i = 1, 2, 3) \) which is speed of particle on world line \( \tau \); \( x_{\alpha \sigma} \) be 4-dimension coordinates on world line \( \sigma \); \( v = \sqrt{v_i v_i}, (i = 1, 2, 3) \) which is speed of particle on world line \( \sigma \). \( t_{\tau} \) as time on world line \( \tau \), \( t_{\sigma} \) as time on world line \( \sigma \), then

\[
x_{0\tau} = ct_{\tau}, \quad x_{i\tau} = u_i t_{\tau}, \quad t_{\tau} = \frac{\tau}{\sqrt{1 - \frac{u^2}{c^2}}} \tag{1}
\]

where \( i = 1, 2, 3 \); \( c \) is the speed of light. Momentum and energy can be defined as

\[
p_i = m u_i, \quad E = mc^2 \tag{2}
\]

where \( i = 1, 2, 3 \); \( m \) is mass of particle. Then from special
relativity, world line τ satisfies condition

\[ \tau = \frac{i}{m_0} (Et - p_1 x_1 - p_2 x_2 - p_3 x_3) \]  (3)

where \( m_0 \) is static mass. Fig1 draws world line \( \tau \) and world line \( \sigma \) on \( x_0 - x_1 \) plane in Minkowski Space.

Slope of world line \( \tau \) is \( x_{i\tau}/x_{0\tau} \), Slope of world line \( \sigma \) is \( x_{i\sigma}/x_{0\sigma} \), and \( x_{i\tau}/x_{0\tau} = x_{0\sigma}/x_{i\sigma} \) since they are perpendicular to each other, so

\[ v_i = \frac{c^2}{u_i} \]  (4)

\( v_i > c \) and \( \sigma \) is space like, \( \tau \) and \( \sigma \) are independent; On each point of world line \( \tau \), particle can move on world line \( \sigma \).

Fig2. draws world line \( \tau \) and world line \( \sigma \) on \( x_0 - x_i \) plane in Riemann Space.

The equations of motion of particle are:

\[ x_0(\tau, \sigma) = \frac{\tau}{\sqrt{1 - \frac{\sigma^2}{c^2}}} + i \frac{\sigma}{\sqrt{1 - \frac{\sigma^2}{c^2}}} + x_0(\tau_0, \sigma_0) \]  (5)

\[ x_i(\tau, \sigma) = \frac{u_i \tau}{\sqrt{1 - \frac{\sigma^2}{c^2}}} - i \frac{\sigma}{\sqrt{1 - \frac{\sigma^2}{c^2}}} + x_i(\tau_0, \sigma_0) \]  (6)

where \( i=1,2,3 \); \( x_0(\tau_0, \sigma_0) \) is initial value of \( x_0, x_i(\tau_0, \sigma_0) \) is initial value of \( x_i \); imaginary number \( i \) is to keep \( \sigma \) and \( x_0 \) to be real value since \( v >> c \). Under equation (6), particle’s position is determined by two proper time \( (\tau, \sigma) \), but the particle’s position can not be localized by each of them individually. As the result, particle’s spatial position \( x_i \) is not localized at fixed time \( t \). Therefore, different from Relativity, the physics of single particle is not localized by 4-dimensional space-time.

From Fig1 we see that at \( t = x_0 = 0 \), particle’s spatial position can be at \( -x_n \) through the path: from \( -\tau_n \) to \( -x_n \); and can be at \( x_2 \) through the path: first from \( -\tau_n \) to \( -\tau_2 \), then from \( -\tau_2 \) to \( x_2 \); and can be at \( x_1 \) through the path: first from \( -\tau_n \) to \( -\tau_1 \), then from \( -\tau_1 \) to \( x_1 \). At \( t=0 \) and \( \frac{m_0 \pi}{\hbar} = -2\pi \), we have

\[ x_i = \frac{\hbar}{p_i} \]  (7)

similarly at \( x_i = 0 \) and \( \frac{m_0 \pi}{\hbar} = 2\pi \),

\[ t = \frac{\hbar}{E} \]  (8)
FIG. 3: World lines $\sigma$ is a infinitesimal loop to fixed point $(x_0, t_0)$. $\tau$ perpendicular to loop $\sigma$, so $\tau$ can point to any direction, the slope of $\tau$ is from $-\infty$ to $\infty$ which means the momentum is from $-\infty$ to $\infty$.

If $\frac{\Delta x}{\Delta t}$ satisfies periodic condition, then equation (7) and (8) become de Broglie equations, but $\tau$ is the proper time associate with time dimension $t$, and we never observed periodic properties in classical physics for $t$, so we need to add one more time dimension and proper time in the next section.

The Above equations illustrate the motion of single particle under two proper time with constant energy and momentum. The single particle spreads out everywhere in space-time, i.e., with fixed energy and momentum, the particle’s position and time are uncertain. That is, because at each fixed $\tau$, particle can move by $\sigma$ to another position. To localize a particle, we need to make all $\sigma$ “stay” only at one position $(x_{i0}, t)$. From equation (4), $v_i$ can not be zero, so world line $\sigma$ must be a infinitesimal loop around $(x_{i0}, t_0)$ as shown in Fig 3. But world line $\tau$ perpendicular to world line $\sigma$, and the slope of $\tau$ is $\frac{\Delta \phi}{\Delta \tau}$; momentum $p_i = \hbar v_i$. From Fig 3, we see that because at each points on circle $\sigma$, particle can move perpendicular to $\sigma$ which create a world line $\tau$, and the slope of $\tau$ is from $-\infty$ to $\infty$, so the momentum becomes $-\infty$ to $\infty$. I.e. if we localized particle’s position and time, the momentum and energy will become uncertain.

FIG. 4: The loop of extra time dimension on complex plane

III. THREE PROPER TIME MODEL – APPROACH TO QUANTUM PHYSICS

Let’s introduces new time coordinate $x_4$ and new proper time $\phi$. Fig 4. draws time loop on a complex-plane, and $\phi$ is the angle from 0 to $2\pi$. The equation of loop is $e^{i\phi}$. Let

$$\phi = \frac{m_0 \tau}{\hbar}$$

i.e., particle moves around the loop of $\phi$ with angular velocity $\frac{m_0}{\hbar}$, uses equation (3), the equation of loop becomes:

$$e^{i\frac{m_0 \tau}{\hbar}} = e^{\frac{i}{\hbar}(E t - p_1 x_1 - p_2 x_2 - p_3 x_3)}$$

It is wave function of quantum particle with fixed 4-momentum. In fact, in Fig 1, at $t=0$, particle’s positions are any points from $-\lambda = \frac{\hbar}{p_i}$ to 0 with $x_4$ from $-2\pi$ to 0; At $x=0$, particle stays position $(x=0)$ at time from 0 to $\frac{\lambda}{2}$ with $x_4$ from 0 to $2\pi$. Therefore, Fig 1. and Fig 2. illustrate a plane wave. It is created by single particle’s motion under proper time $\tau$, $\sigma$, $\phi$.

Assume the geometry of new time dimension $x_4$ is a loop with fixed radius (we can also assume the radius of loop is very small, for example, the length of Planck constant $\hbar$, to be convenience, here we choose radius = 1), then

$$x_4 = e^{i\phi}$$

On $x_i$ coordinate, when $x_4$ moves around a circle, $x_i$ moves from 0 to $\lambda$, this can be interpreted as $x$ oscillating along $x_4$ direction which is perpendicular to x-t plane.
Let’s go back to proper time $\tau$, and let the time dimension which associates with $\sigma$ be $x_5$. Particle moves along world line $\sigma$ with speed $c/\hat{u}$ which is phase velocity of de Broglie wave. Similar to $x_4$, assume $x_5$ is also a loop as illustrated in Fig.4., we can build 1 to 1 relationship between loop $x_5$ and world line $\sigma$, let

$$x_5 = \frac{1}{\hbar}(Et-p_1x_1-p_2x_2-p_3x_3)$$  \hspace{1cm} (12)

In real world, we only have knowledge of one dimensional time $t$. In experiment, we measure one time dimension by using “clock”. We do not know how to synchronize each particle’s 2nd and 3rd time dimensions with $x_4$ and $x_5$. When a particle of apparatus arrives at $(x,t)$ with 2nd time dimension $x_4a$, the $x_4$ value of the particle to be measured can be any value on loop $e^{i\tau}$; But to “meet” the particle at location $X(x_i)$, the apparatus’s particle and the particle $p$ must arrive at location $X(x_i)$ at the same three dimensional time, i.e. $x_{4a} = x_{4p}$, and $x_{5a} = x_{5p}$. The possibility of $x_{4a} = x_{4p}$ is

$$\psi = P_\phi = \frac{e^{i\tau}}{\int e^{i\tau}}$$  \hspace{1cm} (13)

Similarly, the possibility of $x_{5a} = x_{5p}$ is

$$P_\sigma = \frac{e^{-i\tau}}{\int e^{-i\tau}} = \psi^*$$  \hspace{1cm} (14)

So the total possibility to find particle at $(x,t)$ is

$$P = P_\phi P_\sigma = \psi\psi^*$$  \hspace{1cm} (15)

For the plane wave of single photon, proper time $\tau = 0$, equation (9) is no longer valid. Then proper times $\sigma$ and $\phi$ are not related to $\tau$. Instead, we separate the motions of photon by three proper time. 1) Photon moves along world line $\tau$. 2) Photon has oscillation $E = E_0e^{-i\omega t + \lambda x}$ by $\sigma$ which perpendicular to $\tau$, where $E$ is electric field. 3) Photon has oscillation $B = B_0e^{-i\omega t + \lambda x}$ by $\phi$ which perpendicular to $\tau$, where $B$ is magnetic field. The possibility to find photon is proportional to:

$$S = E \times B$$  \hspace{1cm} (16)

In another paper [11], I proved that by choosing 6-dimensional space-time metric as

$$(g_{AB}) = \begin{pmatrix} g_{\alpha\beta} & \psi \\ \psi & -1 \end{pmatrix}$$  \hspace{1cm} (17)

where metric elements $g_{\alpha\beta}$ is 4-dimensional metric. We can derive Klein-Gordon equation directly from Einstein field equation:

$$\hat{G}_{AB} = \kappa T_{AB}$$  \hspace{1cm} (18)

Under this metric, for spinless free particle, we have equation of world line $\tau$:

$$ds^2 = dx_\alpha dx^\alpha + e^{\mp i(x^0x_0 - m_0x_5)} dx_4 dx^4 - dx_5 dx^5$$  \hspace{1cm} (19)

Equation for world line $\sigma$

$$ds^2 = dx_\alpha dx^\alpha - dx_4 dx^4 + e^{\mp i(x^0x_0 - m_0x_4)} dx_5 dx^5$$  \hspace{1cm} (20)

In general, for spinless particle, equation (19) becomes

$$ds^2 = dx_\alpha dx^\alpha + \psi^2 dx_4 dx^4 - dx_5 dx^5$$  \hspace{1cm} (21)

Put it into Einstein field equation, it satisfied wave equation [11]:

$$\partial_A \partial^A \psi = 0$$  \hspace{1cm} (22)

where $A = 0..5$, $\psi$ is wave function. The wave equation (22) is derived directly from Einstein field equation where Planck constant plays the same role as gravitational constant[11]. It means that quantum phenomena can be understood as pure geometry effect of 6-dimensional space-time.

IV. INTERPRETATION OF QUANTUM PHYSICS

Non-local property of single quantum particle is one of the most important reasons why quantum physics does not fit in classical physics theory. In quantum physics, a single particle can stay at different places at the same time. In double slits interference experiment, if we try to use classical paths to describe particle’s motion, the particle has to pass both slits at the same time. In condensed matter physics, an electron’s spatial positions will be everywhere in lattice at any time; i.e. the electron’s must be able to stay in may spatial positions inside lattice at the same time; in the experiment about Bell’s inequality, a single particle must stay in two different spatial places even though the distance between those two places are “far”. Those all conflict with our knowledge in classical physics(including Relativity). In classical physics with one dimensional time, a particle can not stay in more than one place at the same time.

Section II demonstrates that by introducing multiple proper time, single particle’s motion shows non-local properties in classical physics. The particle can move to different places by extra proper time $\sigma$, in Fig1, at time $t=0$, particle’s spatial positions are $x_1, x_2, ..., x_n$. In fact, section II and III draw two different pictures:

1) Along world line $\tau$, free particle moves like classical particle with constant velocity with classical energy and momentum.

2) World lines of $\sigma$ and $\phi$ are also straight lines. Relations between $\phi$ and 2nd time dimension $x_4$, $\sigma$, and third time dimension $x_5$ are $x_4 = e^{i\phi} = e^{\frac{\tau}{\hbar}}$, $x_5 = e^{i\sigma} = e^{\frac{\sigma}{\hbar}}$. The equations come from the geometry of $x_4$ and $x_5$, which are loops in complex plane.

Put 1) and 2) together, then with equation it created plane wave function for single particle. Single particle’s position is not unique under 1 dimension time $t$, but the position is unique under three dimensional time $(t, x_4, x_5)$.
though the particle’s y component of velocity at $x = S$. When particle reaches $x = d$, we know that even coordinates are important for particle. In x-y plane, let particle move along x in quantum physics. The reason we have uncertainty relationship for $x$ and $p$ is localized, particle’s momentum becomes uncertain because of the changing slopes of world lines. But on each individual world line $\tau$, $x_4$ and $x_5$ are plane waves with the same frequency and wave-length. That means in general:

$$x_4 = \Psi = \psi(\tau_0) + \psi(\tau_1) + \ldots \psi(\tau_n) \quad (23)$$

Then

$$x_5 = \psi^*(\tau_0) + \psi^*(\tau_1) + \ldots \psi^*(\tau_n) = \Psi^* \quad (24)$$

So the possibility of finding particle is always $|\Psi|^2$. When we found the particle, the particle’s momentum, energy and other observables are defined along $\tau$, so the average value of classical observable $F$ is:

$$< F > = \int F|\psi|^2 \quad (25)$$

Fig1. and Fig2. show that when particle’s momentum is fixed, particle’s position is uncertain because particle can move by $\sigma$. Fig3. shows that when particle’s position is localized, particle’s momentum becomes uncertain because of the changing slopes of $\tau$ by world line $\sigma$, i.e. we can not find a world lines distribution of $\tau$ and $\sigma$ such that both position and momentum are constant. That is the reason we have uncertainty relationship for $x$ and $p$ in quantum physics.

Now let’s consider the double slits interference experiment for particle. In x-y plane, let particle move along x coordinate with $y = 0$, the double slits at $x = d$, two slit’s coordinates are $S_1(d, y/2)$ and $S_2(d, -y/2)$, and screen at $x = S$. When particle reaches $x = d$, we know that even though the particle’s y component of velocity $u_y$ is zero: $u_y = 0$, particle still moves in y direction by proper time $\sigma$, so at $x = d$, particle can move from S1 to S2 by $\sigma$, i.e. the particle passes both slits at the same time $t$. At $S_1$, the particle’s $x_4$ and $x_5$ value are $(e^{i\theta}, e^{-i\theta})$, and at $S_2$ with values $(e^{i(\theta+\delta)}, e^{-i(\theta+\delta)})$, $\delta$ is a small number since the distance between $S_1$ and $S_2$ is small. After particle passes $S_1$ and $S_2$, the world line $\tau$ splits into two paths with world lines $\tau_1$, $\tau_2$. Let $x_{41}, x_{51}$ be value of $x_4$ and $x_5$ on path 1: $x_{42}, x_{52}$ be value of $x_4$ and $x_5$ on path 2. Suppose $\tau_1$ and $\tau_2$ meet at $p$ where $p$ is a point on screen. At p, we have

$$x_{41} = x_{42}, \quad x_{51} = x_{52} \quad (26)$$

Let $\Delta L =$ path(from S2 to p) - path(from S1 to p), then to get equation (26), $\Delta L$ must satisfies:

$$\Delta L = (n + \frac{\delta}{2\pi})\lambda \quad (27)$$

where $n$ is any integer and $\lambda$ is wave length, particle can not reach those points, which does not satisfy equation (27); so we get interference pattern on the screen.

V. CAUSALITY IN THREE DIMENSIONAL TIME

In real world, time has direction. Here proper time $\tau$ and proper time $\sigma$ have directions too.

From Fig1., Fig2. and Fig5., one can see that $\tau$ and $\sigma$ both move to positive direction of $t$. Along world line $\tau$ when $\sigma$ is unchanged, if event 1 happens at $\tau_1$ and event 2 happens at $\tau_2$ and $\tau_1 > \tau_2$, then corresponding universal time $t_1 > t_2$, so, on world line $\tau$, the causality is preserved. Along world line $\sigma$ when $\tau$ is unchanged, if event 1 happens at $\sigma_1$ and event 2 happens at $\sigma_2$ and $\sigma_1 > \sigma_2$, then corresponding universal time $t_1 > t_2$, so on world line $\sigma$, the causality is also preserved. In general, event 1 happens at $(\tau_1, \sigma_1)$, and event 2 happens at $(\tau_2, \sigma_2)$, if $\tau_1 > \tau_2$ and $\sigma_1 > \sigma_2$, then we have $t_1 > t_2$, causality is preserved. But what will happen when $\sigma_1 > \sigma_2$ and $\tau_1 < \tau_2$? On Fig5, if event 1 happens on $x_1$, event 2 happened on $x_2$, then on “local static” reference frame, event 1 happened after event 2 because $\tau_1 > \tau_2$, on universal time, event 1 happened at the same time as event 2 because both happened at $t = 0$, on world line $\sigma$, event 1 happened before event 2 because $\sigma_1 < \sigma_2$, does it conflict with causality?

Look at Fig5. Suppose universal time t at $t = -t_2$, particle reached $\tau_2$ along world line $\tau$; then particle moves to $x_2$ at $t = 0$ along world line $\sigma$, $\sigma = \sigma_2$, on “local rest” reference frame (world line $\tau$) which is still at $\tau = \tau_2$, the particle goes to future because $t = 0 > t_2$. If at $x_2$, particle does not have any interaction with other particles, then the particle can’t “see” anything in future, when particle moves back to $\tau = \tau_1, \sigma = 0$, there is nothing related to causality, no event occurred. But if event 2 happened at $x_2$; i.e. particle interacts with other particle, then the particle’s physical state is changed by interaction (Remember: We can not measure a particle without affect its physical status). Suppose the particle’s momentum
FIG. 5: Current universal time is $t = -t_2$, particle reaches $x_2$ at $t=0$, it is in future since $t > 0$. When event 2 happens at $x_2$, the particle interacts with other particles, so particle's world lines is changed, its next movement will be based on new world line $\tau'$ has a small changes: $\delta p$, then the particle's next move will start on a new world line $\tau'$, the particle can not go back to original world line $\tau$, this phenomenon is corresponding to wave-packet collapse in quantum physics, so any event happened on this particle after event 2 will be on time $t > 0$, the causality is still preserved. Oriented $\tau$ and $\sigma$ and wave-packet collapse are key factors to keep causality preserved.

All physical reference frames still move along world line $\tau$ with speed $u < c$, so causality is preserved in any reference frame. Although particle's speed $v > c$ along world line $\sigma$, we can not observe or measure this speed because we can not determine particle's position without affecting particle’s velocity (momentum). From all above, we see that three dimensional time contain the basic properties of quantum physics, one can understand that three dimensional time will not conflict with causality law unless quantum physics itself conflicting causality law. If two identical particles are correlated each other, and we separate the wave to two parts with certain distance, if we affect one part of wave, then the other part on the other place will be affected on the same time (i.e. the information passed without time change). This is the well known Bell’s inequality. There are already many papers discussing about causality law in this phenomenon [9].

VI. SOME DISCUSSINONS

First, it is interesting to see the relation between three dimensional time and String theory. Actually Fig.1. looks like a world sheet in String theory. If we let $\sigma$ be space dimension instead of proper time, it turns to bosonic String theory. There are two major differences between three dimensional time and String theory.

1) In String theory, the motion in $\sigma$ direction is compacted. It can not be very large since we never see extra space dimensions in real world. In three dimensional time, the distance traveled by world line $\sigma$ can be very large, this is a very important property which provides non-local properties of three proper time physics.

2) Three dimensional time has different statistical results from String theory. Because of the special character of time (which different from space), it demonstrates the same statistical results as quantum physics.

But we still can use some results of String theory. Considering two proper time case, put Lagrangian

$$ L = -\frac{1}{2}m[(\dot{x} \cdot x')^2 - (\dot{x} \cdot x)(x' \cdot x')]^{1/2} $$

(28)

Where $\dot{x} = \frac{dx}{d\tau}$; $x' = \frac{dx}{d\phi}$. The Lagrangian above is the same as the Lagrangian in string theory [4]. The classical equation of motion is

$$ \frac{\partial}{\partial \tau} \left( \frac{\delta L}{\delta \dot{x}_\alpha} + \frac{\partial}{\partial \phi} \frac{\delta L}{\delta x'_\alpha} \right) = 0 $$

(29)

Add constraints [4]:

$$ \dot{x} \cdot x' = 0, \quad \dot{x} \cdot x + x' \cdot x' = 0 $$

(30)

The equation of motion (29) becomes wave equation:

$$ \ddot{x}_\alpha = \dddot{x}_\alpha $$

(31)

The above result is the same as bosonic string theory [4]. For free particle with constant momentum, we choose solution:

$$ x_\alpha(\tau, \phi) = e^{-im_0(\tau-\phi)} $$

(32)

Since $x_\alpha(\tau, \phi)$ must be real number, the above equations have solutions only when

$$ \phi = \tau = \frac{1}{m_0}(Et - p_1x_1 - p_2x_2 - p_3x_3) $$

(33)

We see that equation(33) corresponding to proper time $\phi$.

Second, there are possible some interesting relations between three dimensional time and quantum field theory. Feynman [10] interpreted negative energy state of particle as: negative energy state represents the particle...
moving to negative time direction. It is hard to understand or illustrate this in 1 dimensional time theory, but it can be often seen in three dimensional time: looking at Fig5. particle goes to $x_2$ (future) at $t=0$, then go back to $t_1(t_1, \sigma = 0)$. In addition, in section II, Fig3, when $\sigma$ accrossing $X_i$ coordinate, momentum becomes infinite, that is because the particle moves from one location to another location without changing universal time $t$ – the particle moves by $\sigma$ only. That is to say We get infinity momentum because we only use space and first time dimension $t$ to calculate momentum. If we uses $\sigma$, the infinity will be gone. It is possible to use this to deal with the infinities in quantum field theory in future.

Third, this paper is only dealing with “basic” quantum physics. I.e., it is only discussing spinless particle. I believe that spin is coming from the motion of extra time dimension. To discuss the particle with integer spin and half-integer spin, we have to find a way to explain the results of Bose-Einstein statistics and Fermi-Dirac statistics. We will discuss that in another paper.

I used three dimensional time to interpret quantum physics in two other papers before [12] [11]. This paper provides more details and clear pictures for three dimensional time theory.

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