The analysis of binary matrix symmetry properties in the tasks concerned with test control of digital devices

G Petrukhnova
Department of Automated and Computing Systems, Voronezh State Technical University, Russian Federation
E-mail: gvpetrukhnova@mail.ru

Abstract. The object of research is the properties of binary matrix symmetry. The constructed model of a binary matrix enabled us to determine the measure of matrix symmetry. Entropy criteria are obtained on the basis of the symmetry measure, which allows us to rank objects, represented by binary structures, in the order of their preference, and to solve various optimization tasks. The use of the obtained results in the tasks concerned with test control of digital devices has been shown. The "black box" is type model of a digital device was described. The tests were solved, relating to optimizing the probability distribution of input signals during random testing of discrete devices. With the help of the generator of pseudo-random numbers, the tests were devised covering single faults, modeled as «stuck-at faults» and «bridging faults» at control points. The resulting entropy criteria allowed us to reduce the duration of control tests of digital devices. The experimental data analysis enables us to make a conclusion about the expediency of using the entropy criteria in the theory and practice of digital device testing.

1. Introduction
Translated from Greek, symmetry means proportionality. Since ancient times, this concept has been intuitively perceived by people as a certain aesthetic criterion. In some world national cultures, strict order, the proportionality of constituent elements of an object and the balance between the parts of the whole were considered as beauty signs of the object. In other cultures, preference was given to asymmetry, symbolizing movement and energy. Symmetry was originally understood as one of its forms, which is now called geometric symmetry.

In science, the concept of symmetry came to be used in the 19th century, in connection with the development of crystallography and the discovery of crystallographic classes, as well as the emergence of the group theory in mathematics. In the general case, symmetry is invariance of the mathematical or physical structure of an object with respect to selected transformations. Symmetry allows us to evaluate and compare the objects of different shape and study their properties. For example, a cube-shaped dice and a roulette wheel with six sectors have different geometric shapes, but the same probabilistic symmetry.

The language of symmetry is very flexible and convenient for solving practical problems, as it allows for studying the structures of both the object and the problem being solved. It is impossible to find the knowledge area and the sphere of human existence, in which the ideas of symmetry would not be reflected.
In this article, the object of research is the properties of binary matrix symmetry. In linear algebra, a symmetric matrix is a square matrix that does not change when transposed. The elements of the symmetric matrix are symmetric elements with respect to its main diagonal.

In the article, the binary matrix symmetry is considered as the property of its structural invariance with respect to selected transformations. The examples of transformations include the reciprocal permutations of identical matrix rows, the reciprocal permutations of zero column elements, the reciprocal permutations of one column elements, and the combination of these permutations. Such operations are called bijective functions from a symbol set by to itself [1].

For the purpose of research, you can choose a matrix of arbitrary dimensionality, not necessarily square, as it is in the traditional approach. To identify a binary matrix of the specific structure, the concept of symmetry measure is used. The less the measure of matrix symmetry is, the more diverse the structural matrix elements are.

The main purpose of this article is to obtain a quality criterion, convenient for practical use, on the basis of binary matrix symmetry. The criterion should permit ranking binary matrices in the order of their preference in terms of structure. The above-mentioned permutations are selected as transformations preserving the structure.

The article considers structural entropy [2] and introduces a generalized criterion of entropy quality. A structure is understood as a relationship between the elements within the system, that is necessary and sufficient for the system to achieve its goal. Three specific criteria are obtained on the basis of the generalized entropy criterion. At the end of the article, we show the practical use of entropy criteria in the tasks, concerned with test control of digital devices, and present the data, related to experimental criteria study.

2. Materials and method
Let us introduce the binary matrix symmetry measure and, then, the generalized criterion of entropy quality. It will be shown how the theoretical results obtained can be used for test control of digital devices.

2.1. A binary matrix symmetry measure
Let there be a binary matrix consisting of $K$ columns and $N$ rows:

$$
\begin{array}{cccc}
Y_{11} & Y_{12} & \cdots & Y_{1K} \\
Y_{21} & Y_{22} & \cdots & Y_{2K} \\
\vdots & \vdots & \ddots & \vdots \\
Y_{N1} & Y_{N2} & \cdots & Y_{NK}
\end{array}
$$

(2.1)

We assume that a matrix row is a minimal unit of binary matrix’s partitioning into structural elements. A row is a binary set. The number of possible types of binary sets, consisting of $K$ elements is $2^K$. Let $R = 2^K$.

Let’s divide the matrix rows into classes. Binary sets are combined into the same class if the Hamming distance (module 2) between them is equal to zero:

$$
\sum_{i=1}^{K} y_{ji} \oplus y_{ki} = 0,
$$

(2.2)

where $y_{ji}$ is the $i$ element of the $j$ row, $y_{ki}$ is the $i$ element of the $k$ row, $K$ is the number of matrix columns.

For example, if a binary matrix contains the rows consisting only of zeros and those exclusively consisting of ones, then binary sets containing all zeros are combined into one class, and binary sets containing all ones are combined into another class. Note that the classes may contain a different number of rows, and some classes are empty if the corresponding types of rows are not in the matrix (2.1).
The measure of the object’s symmetry is the number of its automorphisms (i.e. transformations preserving each of the partitioning classes) [3]. The reciprocal permutations of identical matrix rows are selected as transformations. Under such transformations, each class of matrix partitioning (2.1) is preserved.

The number of permutations in a partitioning class containing the number \( l \) of elements is the factorial \( l! \). Then the symmetry measure \( f \) of the matrix (2.1), the partitioning elements of which are distributed among \( R \) classes, is calculated by the formula:

\[
f = \prod_{i=1}^{R} l_i!,
\]

where \( l_i \) is the number of rows in the \( i \) partitioning class; \( R \) is the total number of possible partitioning classes.

The condition must be met:

\[
\sum_{i=1}^{R} l_i = N,
\]

where \( N \) is the number of binary matrix rows.

Note that the number of partitioning classes increases with the growing number of matrix columns (2.1):

\[
R = 2^K,
\]

where \( K \) is number of binary matrix columns.

It can be shown that if the matrix (2.1) consists of elements belonging to \( s \) partitioning classes, \( s \leq R \), and each of the classes is not an empty set, then the measure of matrix symmetry reaches the minimal value, provided that \( l_1 = l_2 = \ldots = l_s \), i.e., in case all the classes that make up the matrix contain the same number of elements. This fact is in good agreement with the known results [3].

The above-mentioned approach to calculating the measure of matrix symmetry does not take into account the internal structure of its columns. For a more detailed analysis of the binary matrix structure, its co-partitioning must be fulfilled [3].

With reference to partitioning \( \{T_1, T_2, \ldots, T_m\} \), co-partitioning \( \{\theta_1, \theta_2, \ldots, \theta_r\} \) is a partitioning \( \{\theta_1, \theta_2, \ldots, \theta_r\} \) having the following properties:

- the intersection of elements \( t_i \cap t_j \), in which \( t_i \in T_i, t_j \in \theta_j \), contains no more than one matrix element \( y_{ij} \);
- any enlarged partitioning no longer possesses this property;
- if \( t_i \cap t_j \neq \emptyset \), then \( t_j \) intersects with any element \( t_p, t_p \in T_p \), if \( T_p \neq \emptyset \).

A binary matrix column is selected as the smallest element of its co-partitioning. A column is a binary set. Let the \( i \) class of matrix co-partitioning include the columns containing the \( i \) number of ones. Since the number of rows in the matrix is \( N \), the number of binary sets' classes will be \( N+1 \). Co-partitioning contains the classes, beginning with a class with columns composed of zeros only and ending with a class with columns exclusively containing ones.

Note that co-partitioning classes can contain a different number of columns. Some classes are empty if the corresponding column types are not present in the matrix (2.1).

Let's divide the matrix columns into classes. There are reciprocal permutations of ones and the reciprocal permutations of zeros which are chosen as transformations that preserve the structure of each binary matrix column. Then, the binary matrix symmetry measure with respect to partitioning performed is defined as follows:
where $n_j$ is the number of ones in the matrix column $j$; $K$ is the number of matrix columns, $N$ is number of matrix rows.

It is possible to show that the symmetry measure $g$ of a matrix reaches a minimum when the number of ones (zeros) in each of the matrix columns, is equal to the integer value of the expression $0.5 \cdot K$:

$$
\arg \min_{n_{k_1}, \ldots, n_{k_K}} (g) = ([0.5 \cdot K], \ldots, [0.5 \cdot K])
$$

This fact is in good agreement with the known results [3].

It is possible to appraise the matrix symmetry using the measure proposed in [3], taking into account its partitioning and co-partitioning:

$$
S = f^\gamma \cdot g^\beta,
$$

where $\gamma$ and $\beta$ are fixed parameters, $f$ and $g$ are the measures of binary matrix symmetry, according to formulas (2.3) and (2.6). Then, we will get:

$$
S = \left( \prod_{i=1}^{R} l_i! \right)^\gamma \cdot \left( \prod_{j=1}^{K} n_j! \cdot (N - n_j)! \right)^\beta,
$$

where $l_i$ is the number of rows in the $i$ partitioning class; $R$ is the total number of possible partitioning classes, $n_j$ is the number of ones in the $j$ matrix row; $K$ is the number of matrix columns, $N$ is matrix row number, $\gamma$ and $\beta$ are fixed parameters.

Let us assume that $\gamma=1$, if the binary matrix structure analysis requires its partitioning into rows, otherwise $\gamma=0$. Let us assume that $\beta = 1$, if the binary matrix structure analysis requires its co-partitioning (i.e. the partitioning into columns), otherwise, $\beta = 0$.

The resulting expression (2.9) for calculating the symmetry measure of the binary matrix is the basis for the synthesis of entropy quality criteria. These criteria permit ranking the objects, represented by the binary matrix, in the order of their preference, and allow us to solve various problems of optimization.

2.2. Synthesis of quality criteria

In various technical tasks, concerned with control and modeling of complex systems, it becomes necessary to analyze the structure and evaluate the quality of objects represented by a binary matrix. For example, binary matrices may reflect the qualitative nature of the relationship between objects, and the availability of their structural elements.

It is inconvenient to use the formula (2.9) for practical research of a binary matrix. Let us make use of the fact that the function logarithm has extremal points, located in the same place where the function itself is. Let us transform the expression (2.9) by the Stirling formula:

$$
\ln n! = n \cdot \ln n - n + O(\ln n)
$$

After transformation of the formula (2.9), we will obtain the expression:

$$
\ln S \approx \gamma \cdot \left( \sum_{j=1}^{R} (l_j \cdot \ln l_j) - N \right)
$$
Let \( q_i = n_i/N \), \( i=1, \ldots, K \). Let \( p_j = l_j/N \), \( j=1, \ldots, R \). Then, after inserting \( q_i \) and \( p_j \) in the formula (2.11) and a series of subsequent transformations, we will obtain the following expression:

\[
\ln S \approx \gamma \cdot N \left( \sum_{j=1}^{R} (p_j \cdot \ln p_j) + \ln N - 1 \right) + \beta \cdot N \left( \sum_{i=1}^{K} (q_i \cdot \ln q_i) + \sum_{i=1}^{K} ((1 - q_i) \cdot \ln(1 - q_i)) + K \cdot \ln N \right)
\]  \hspace{1cm} (2.12)

The variable-free terms are discarded, and the quality criterion, convenient for practical use, is obtained:

\[
H = \frac{\gamma}{K} \sum_{j=1}^{R} (p_j \cdot \ln p_j) + \frac{\beta}{K} \left( \sum_{i=1}^{K} (q_i \cdot \ln q_i) + \sum_{i=1}^{K} ((1 - q_i) \cdot \ln(1 - q_i)) \right)
\]  \hspace{1cm} (2.13)

where \( R \) is the total number of possible partitioning classes for a binary matrix (2.1), calculated by the formula (2.5); \( p_j \) is the probability with which an element of class \( j \) occurs in the matrix (2.1); \( K \) is the number of binary matrix columns; \( q_i \) is probability of one, occurring in the \( i \) column of the binary matrix; \( N \) is the number of binary matrix rows; \( \gamma \) and \( \beta \) are fixed parameters.

Herewith, the following condition must be met for probabilities \( p_j \):

\[
\sum_{j=1}^{R} p_j = 1
\]  \hspace{1cm} (2.14)

In this case, \( \gamma = 1 \), if the binary matrix structure analysis requires its partitioning into rows, otherwise \( \gamma = 0 \); \( \beta = 1 \), if the binary matrix structure analysis requires its co-partitioning (i.e. the partitioning into columns), otherwise \( \beta = 0 \).

When trying all possible combinations of \( \beta \) and \( \gamma \) on the basis of the generalized criterion, it is possible to obtain three specific criteria.

\[
H_1 = \sum_{j=1}^{R} (p_j \cdot \ln p_j)
\]  \hspace{1cm} (2.15)

\[
H_2 = \sum_{i=1}^{K} (q_i \cdot \ln q_i) + \sum_{i=1}^{K} ((1 - q_i) \cdot \ln(1 - q_i))
\]  \hspace{1cm} (2.16)

\[
H_3 = \sum_{j=1}^{R} (p_j \cdot \ln p_j) + \sum_{i=1}^{K} (q_i \cdot \ln q_i) + \sum_{i=1}^{K} ((1 - q_i) \cdot \ln(1 - q_i))
\]  \hspace{1cm} (2.17)

Note the parameter \( R \) is determined by formula (2.5). The software implementation of the criteria (2.15) and (2.17) is a multidimensional problem for a matrix of large dimensionality. To solve this problem, it is necessary to consider the probability \( 2^K \) of binary sets, where \( K \) is the number of rows in a binary matrix. The expression (2.16) has lower computational complexity. To implement the expression (2.16), it is necessary to determine the probability of one, occurring in matrix rows, i.e., the probability of one in \( K \) binary sets.
Thus, the comparative quality analysis of objects, represented by the binary matrix, is conducted on the basis of the symmetry theory system, which implies a certain rule for matrix’s division into structural units, the identification of their classes, and the availability of some general criteria allowing for ranking objects in the order of their preference. The obtained criteria allow us to state that the greatest structural variety is possessed by binary matrices, matching the measure of symmetry, close to the minimal value, which is calculated by the formula (2.13). Let us describe the practical use of the proposed mathematical apparatus, using the synthesis of digital device control tests as an example.

2.3. A in-circuit and functional control of digital device

There are two types of digital device test control, namely, functional and in-circuit. These two types of control can be regarded as information operations [4]. It can be stated that both in-circuit and functional control operations in the quality assurance system of various products are regarded as control tests [5].

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The in-circuit control is a type of control, which allows us to apply testing impacts to inner contacts of printed node connections, and get the feedback from inner printed connections [5]. The purpose of this control is to identify the faults which are modeled as «stuck-at faults» and «bridging faults» [5]. The «stuck-at fault» type models are manifested in setting one of the logical levels on the circuit signal distribution line.

The in-circuit control is based on the assumption that the object functions in a regular way, if it meets the technical requirements. In this case, we do not need the knowledge of features relating to interaction within the entire node being tested; it is only sufficient to know the list and parameters of the components being checked, as well as the circuits of their connections. In the particular case, the in-circuit control can be considered as the initial stage of functional control.

The in-circuit control has the advantage over the functional one, which consists in the fact that it helps to localize the fault at a component-wise level with lower costs. Herewith, the in-circuit control tests should, whenever possible, detect the greatest possible number of faults. This requirement helps to reduce the overall cost of digital circuit testing.

Suppose there is a digital device with independent primary inputs. We will assign numbers to all circuit lines of a digital device, being investigated for the presence of «stuck-at faults» and «bridging faults», in a sequence convenient for the test developer. The number of control points can include the inputs of a digital device, its outputs and accessible internal points. In the same sequence, we will record the output logical signals, corresponding to these points, which are obtained after applying some input action to a digital device and getting its response. The model of the testing process, constructed in this way, makes it possible to represent the digital device response in the form of a matrix (2.1) having a great number of rows.

The digital device circuit can be checked by feeding all possible test patterns into its inputs. However, such a test will be redundant and very long. We will feed the test patterns, created by a random number generator, into the inputs of a device, and then, record and analyze the response at the outputs. Logic zeros and ones will be fed into each input of a digital device with specified probability. This approach is called weighted random pattern testing. The proposed model allows you to optimize the length of a random test.

Numerous approaches to implementing weighted random testing of digital devices and optimizing the test length have been proposed [6] – [13]. Particularly, in the work by V.D. Agraval [4], it is suggested that the information approach, based on output entropy maximization $H(p_1,...,p_n)$ should be used to shorten the test length. The expression $H(p_1,...,p_n)$ matches the criterion (2.15), obtained above, except for the sign. In their work, V.D. Speransky and N.V. Cherevkko [7] consider a task similar to the one, described in the paper [6], but a different target function is maximized. This target function looks
the same as the above-mentioned criterion (2.16), but with a different sign. The absence of the minus sign in expressions (2.13, 2.15 - 2.17) is explained by the fact that symmetry reflects some orderliness of the parts of an object examined. The minimal symmetry correlates with the maximal variety of the object’s structural elements.

In this case, such an object is the response of the tested device to random impacts applied to its inputs. The output reaction is represented in the form of a binary matrix (2.1), structurally divided into rows and columns. Thus, during the synthesis of tests, we will proceed from the fact that the quality of control tests is determined by their structure. Hereewith, the top-quality test among those of the same length will correspond to the smallest measure of symmetry.

It is also worth noting that the criteria (2.15) and (2.16) are rather well-studied, and the results of these studies are presented in [6], [7]. The criterion (2.17) was tested by the author of the article for solving the problem of synthesizing the control test of «stuck-at fault» and «bridging fault» types, and the results of these studies will be presented below.

2.4. The fault control test length determination of the «stuck-at fault» and «bridging fault» types

The assessment of test quality should consider the completeness in coverage of possible malfunctions of «stuck-at fault» and «bridging fault» types. Among the control tests of the same length, the more effective is the one, covering all possible malfunctions in less time (i.e. through smaller number of test actions applied to the object). This test will correlate with minimal symmetry.

When testing a digital device, there is always a problem of marking the end of this process. To solve it, it is proposed to use the entropy criterion considered in the paper [13]:

\[ H(k_1, k_2, ..., k_K) = \sum_{i=1}^{S} k_i \cdot \ln k_i, \]  

(2.18)

where \( k_i \) is the number of \( i \)-class columns of the output reaction, presented in the form of a binary matrix (2.1); \( S \) is the number of classes, different from an empty set.

In this case, the same class combines the columns, the Hamming distance between which is equal to zero:

\[ \sum_{i=1}^{N} y_{ij} \oplus y_{ik} = 0, \]  

(2.19)

where \( N \) is the test length (the number of binary matrix rows (2.1)); \( y_{ij} \) is the \( i \)-element of \( j \)-column, \( y_{ik} \) is \( i \)-element of \( k \)-column.

In this case, the following condition must be met:

\[ \sum_{i=1}^{S} k_i = K, \]  

(2.20)

where \( K \) is the number of columns of the binary matrix.

This criterion is convenient for use in assessing the quality of control tests, covering «stuck-at faults» and «bridging faults» types. Among the two in-circuit control tests, more faults are covered by the one with the lower value of the criterion (2.18).

When all possible the «bridging faults» are covered by the test, the criterion value (2.18) becomes equal to 0, and all matrix columns will be different, if permitted by the digital device architecture. To cover possible «stuck-at faults», it is necessary for the matrix to contain no columns consisting of logical ones only, and the columns exclusively composed of logical zeros.

The criterion (2.18) can also be used to reduce the length of the control test for detecting the «bridging faults». If the next, \((n+1)\) test action does not carry any useful information about such faults, the value of the criterion (2.18) calculated for the test of \((n+1)\) in length will not change compared
with the value obtained for the test of \( n \) length. This means that the last \((n+1)\) input and the response to it must be analyzed for exclusion from the control test.

2.5. The optimization of the digital device test length

The task of optimizing the weight set for a pseudo-random control test can be formulated as follows. Let the primary inputs of this circuit be independent and have different weights (that is, with pseudo-random testing, single logical signals are fed into these inputs with different probabilities). Thus, there is a vector of weights. Let \( q_i(u), (i = 1, ..., K) \), be the probability of a logical «1», occurring at the \( i \) output. Let \( p_j(u), (j = 1, ..., 2^K) \), is the probability of the occurring \( j \) output binary set.

Thus, we will formulate the following problem, related to optimizing the probability distribution of input signals for pseudo-random testing of digital circuits; it is required to find the vector \( u^* = (u^*_1, ..., u^*_i) \), located in the region of permitted values \( u^* \in U^k = \{ u = (u_1 ... u_i), \ 0 < u_i < 1, \ i = 1, ..., L \} \), at which the selected objective function \( H \) has the minimal value. One of the functions (2.15) – (2.17), obtained on the basis of the generalized criterion (2.13), must be selected as an objective function.

According to the formula (2.13), the most effective test correlates with the symmetry value, close to the minimal one. Theoretically, the smallest value of symmetry (the objective function) is achieved if \( q_i(u) = 0.5, \ (i = 1, ..., K) \), and \( p_j(u) = 2^{-K}, \ (j = 1, ..., 2^K) \). But the architecture of digital devices is quite complex and can prevent such distribution of output signals’ probabilities. It is also worth noting that the pseudo-random sequence is always redundant.

The probabilities \( q_i(u), (i = 1, ..., K) \), and \( p_j(u), (j = 1, ..., 2^K) \), are unknown quantities and their explicit obtaining, even for simple digital circuits, is a difficult task. Therefore, we will replace them with corresponding frequencies calculated using sufficient length, selected in a random way.

Each of the functions (2.15) – (2.17) is random, and its explicit form is unknown, since output probabilities were substituted with frequencies. The direct calculation of values of these functions and their derivatives is impossible. To solve the problem in question, it is advisable to apply numerical methods that use observations of how the optimized random function is implemented. Such methods include the method of coordinate descent. The description of the method and the issues of its convergence are described in detail in many papers, in particular, in the literary sources [14], [15].

The implementation of the method of coordinate descent in the problem under study requires assessing the probability \( q_i(u), \ (i = 1, ..., K) \), and \( p_j(u), (j = 1, ..., 2^K) \). For this purpose, as indicated by the literary source [7], it is sufficient to assess the probability of a logical one at each output of the device by its relative frequency with an accuracy of 0.95 and an error of no more than 0.03. Such indicators can be achieved using a sample containing about 1000 test suites.

3. Results and discussion

The optimization algorithm study of the weight sets was carried out for digital circuits, the characteristics of which are given in table 1.

| Table 1. Digital circuits characteristics |
|------------------------------------------|
| Circuit number | Input number | Control point number |
|----------------|--------------|----------------------|
| 1              | 4            | 9                    |
| 2              | 3            | 8                    |
| 3              | 6            | 11                   |
| 4              | 3            | 11                   |

The tested devices were presented as "black box" models. The number of checked points could include digital device inputs, its outputs and internal points. The control points were accessible.
Models of «stuck-at faults» and «bridging faults» were viewed as a class of possible faults. In this case, it was assumed that a «bridging fault» could occur between any control points of the tested digital circuit. The tests were devised, checking all possible single faults of the above-mentioned types at control points.

The synthesis of the control test consisted of three stages:

- a set of the most informative test actions were distinguished by solving the problem of weight set optimization using the coordinate descent method;
- a test was identified, covering all possible faults of the above-mentioned types at control points;
- impacts that do not carry useful information were removed from test.

The sequence of actions, described above, can be represented by the following scheme:

$$X \rightarrow X^* \rightarrow T^* \rightarrow T,$$

where $X$ is the initial set of test patterns, $X^*$ is the set of the most informative test patterns, selected from $X$, $T^*$ is random test sequence, selected from $X^*$, covering all possible faults, however, containing redundant test sets; $T$ is the test, close to optimal one.

At the beginning of the experimental studies, the problems of weight set optimization were solved using the coordinate descent method. In order to conduct a comparative analysis of results, each of the objective functions (2.15) – (2.17) was used to optimize the weight sets. In other words, three optimization problems were solved for the same digital device on the basis of specified objective functions.

At the first stage of solving this problem, test impacts were generated with frequency of a logical «1», equal to 0.5 for each input of the tested device. Then the frequencies were optimized. The results of solving optimization problems are presented in table 2.

### Table 2. Results of solving optimization problems

| Circuit number | Entropy criterion | The resulting weight set |
|----------------|-------------------|--------------------------|
| 1              | $H_1$             | 0.7 0.6 0.6 0.6          |
|                | $H_2$             | 0.5 0.5 0.5 0.5          |
|                | $H_3$             | 0.6 0.6 0.6 0.6          |
| 2              | $H_1$             | 0.7 0.7 0.7              |
|                | $H_2$             | 0.6 0.4 0.4              |
|                | $H_3$             | 0.6 0.6 0.4              |
| 3              | $H_1$             | 0.6 0.6 0.5 0.6 0.6      |
|                | $H_2$             | 0.5 0.5 0.5 0.5 0.5      |
|                | $H_3$             | 0.6 0.6 0.5 0.6 0.5      |
| 4              | $H_1$             | 0.4 0.5 0.6              |
|                | $H_2$             | 0.5 0.5 0.5              |
|                | $H_3$             | 0.4 0.5 0.6              |

The experimental data showed that, for each of the schemes, except for the last one, we have three different weight sets. This result is explained by the fact that various objective functions have been optimized. On the basis of each objective function, various informative test actions were selected from the region of permissible values, and thus we managed to reduce the length of the pseudo-random test.

On the basis of the results obtained, at the next stage of research, pseudo-random test actions with frequencies close to optimal ones were applied to digital devices. Test impacts were selected from the range of acceptable values. Three experiments were performed for each digital device, if the results of optimization problem solution with various objective functions were different. The moment of test
completion was determined on the basis of the criterion (2.18). The coverage of «stuck-at faults» was subsequently analyzed. The results of the experiments are presented in the third column of table 3.

Such development of tests produces good results, but the resulting control test always contains excessive input actions and corresponding device responses. Therefore, at the next stage, the output reactions of the devices were analyzed using the criterion (2.18). This made it possible to exclude binary sets from the test that do not carry useful information about the faults. The experimental data are presented in the fourth column of table 3.

Table 3. Test length

| Circuit number | Objective function | Test length after solution optimization tasks | Test length after remove redundant test patterns |
|----------------|--------------------|-----------------------------------------------|-----------------------------------------------|
| 1              | $H_1$              | 20                                            | 6                                            |
|                | $H_2$              | 37                                            | 6                                            |
|                | $H_3$              | 20                                            | 6                                            |
| 2              | $H_1$              | 10                                            | 7                                            |
|                | $H_2$              | 7                                             | 6                                            |
|                | $H_3$              | 10                                            | 6                                            |
| 3              | $H_1$              | 12                                            | 6                                            |
|                | $H_2$              | 14                                            | 7                                            |
|                | $H_3$              | 12                                            | 8                                            |
| 4              | $H_1$              | 5                                             | 5                                            |
|                | $H_2$              | 33                                            | 5                                            |
|                | $H_3$              | 5                                             | 5                                            |

Let us analyze the third column of the table, which shows the test length after solving the optimization problem. In other words, a test is analyzed that contains both informative test patterns and the test patterns that do not carry useful information. You may notice that the criterion $H_2$ shows the same results as the criterion $H_1$. Since the solution of optimization problems for each device produces different weight sets, then we get three different tests for the same device (for the 4th device, there are two tests, respectively). After excluding the test patterns not carrying useful information, the tests of various lengths were obtained for schemes 2 and 3.

The analysis of the third and the fourth columns of the table shows:
- for scheme 1, the results obtained using the objective function $H_2$ can be considered the worst, although in the end the resulting test has the same length as all the others;
- for scheme 2, the best results were obtained using the objective function $H_2$, and the worst ones were obtained using the objective function $H_1$;
- for scheme 3, the best results were obtained using the objective function $H_1$ and the worst ones were obtained using the objective function $H_3$;
- for scheme 4, the results obtained using the target function $H_2$ can be considered the worst, although in the end the resulting test has the same length as all the others.

Each objective function indirectly reflects certain structural features of a digital device. This explains the difference in the results obtained.

The criterion $H_3$ obtained by the author of the article showed good results and can be used to analyze the structural features of objects represented by a binary matrix. In comparison with the criteria $H_1$ and $H_2$, its implementation is specified by greater computational complexity.
4. Conclusion
Symmetry is certain ordering of parts of the object under study. In turn, such ordering allows us to study the structure of the object, select the individual parts of its blocks, and determine the rules for their construction. Symmetry allows you to compress information about the structure of the object, which is essential in the synthesis of tests for digital devices.

The article defines the symmetry measure of a binary matrix. The resulting expression has a fairly great computational complexity and is inconvenient in practical use. The generalized entropy quality criterion was obtained on the basis of the symmetry measure of the binary matrix.

Based on the generalized entropy criterion, three particular criteria were obtained. Two of them are widely used in the theory and practice of digital device testing, and the third criterion was under investigation.

The entropy quality criteria allow us to rank the objects represented by the binary matrix in the order of their preference. The use of the results obtained is demonstrated, using the example of control test planning for a digital device.

The models of "stuck-at fault" and "bridging faults" types were considered as a class of possible faults. In this case, it was assumed that a "bridging faults" could occur between any control points of the digital circuit under test. The tests were devised, checking all possible single faults of specified types at control points. The experimental studies have shown good results.

The models proposed in the article can be used to analyze the objects structural features of different nature represented by a binary matrix. The using of symmetry principles opens up possibilities for improving the object structure from the standpoint of the object achieving its goals.

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