Estimation of electric conductivity of the quark gluon plasma via asymmetric heavy-ion collisions

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Abstract

We show that in asymmetric heavy-ion collisions, especially off-central Cu+Au collisions, a sizable strength of electric field directed from Au nucleus to Cu nucleus is generated in the overlapping region, because of the difference in the number of electric charges between the two nuclei. This electric field would induce an electric current in the matter created after the collision, which result in a dipole deformation of the charge distribution. The directed flow parameters $v_1^\pm$ of charged particles turn out to be sensitive to the charge dipole and provide us with information about electric conductivity of the quark gluon plasma.

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**Introduction.**—The quark gluon plasma (QGP), which consists of deconfined quarks and gluons, is expected to have filled the early universe \(^1\). We are now at the stage to study properties of the QGP experimentally through the relativistic heavy-ion collisions at Relativistic Heavy Ion Collider (RHIC) in BNL and Large Hadron Collider (LHC) in CERN. One of the most interesting observations is the very strong elliptic flow in off-central collisions, which indicates a very small ratio of shear viscosity to entropy density \(\eta/s\) \(^2\). We are now trying to learn more detailed properties of the QGP by constraining transport coefficients such as shear viscosity, bulk viscosity, and charge diffusion constants.

In this Letter, we propose a new way of estimating the electric conductivity \(\sigma\) of the QGP via asymmetric nucleus-nucleus collisions at ultrarelativistic energies. Theoretically, lattice QCD simulations \(^6\)–\(^8\) and perturbative QCD calculations \(^9\) have been utilized to estimate electric conductivity of the QGP. So far, the estimated values of \(\sigma\) have differed significantly from each other and experimental information is very much awaited. Very recently, asymmetric collisions between Copper (Cu) and Gold (Au) nuclei have been performed at RHIC and the PHENIX Collaboration reported their first results \(^10\). We show that Cu+Au collisions can be useful for extracting the electric conductivity of the QGP. In off-central Cu+Au collisions, a substantial magnitude of electric field directed from a colliding Au nucleus to Cu nucleus is generated in the overlapping region. This happens only when colliding two nuclei carry the different number of electric charge. This electric field would induce a current in the matter created after the collision, resulting in a dipole deformation of the charge distribution in the medium. Later, the time evolution of the system is dominated by the strong radial flow, which is an outward collective motion of the medium. Henceforth the charge asymmetry formed in the early stage is frozen. Thus, we argue that charge-dependent directed flow of the observed hadrons are sensitive to the charge dipole formed at the early stage, which reflects the electric conductivity of the QGP.

Conventionally the electric conductivity of the QGP is estimated from experiments via the Kubo formula \(^11\). The production rate of thermal dilepton is expressed by the electric current-current correlation function \(^12\)–\(^13\) and its small frequency region is governed by the transport peak \(^14\). Thus one can estimate the electric conductivity through comparison of theoretical results with dilepton invariant mass spectra \(^15\). Compared to this method, the present approach is a rather direct one, in which the response of the matter to an applied electric field is directly quantified.
The effects of transient strong electromagnetic fields are under intensive discussions recently, especially in the context of the chiral magnetic effect \[16–18\]. So far, there has been no experimental evidence that strong fields actually exist. Observation of a charge-dependent directed flow would also provide evidence that a strong electromagnetic field is actually created in heavy-ion collisions.

**Electric fields in Cu+Au collisions.**— Here, we show that, in off-central collisions between copper and gold nuclei, a sizable strength of electric field is generated in the overlapping region of two nuclei. Because of the difference in the number of protons between the two nuclei, the generated electric field tends to the copper nucleus. The situation is different from the electromagnetic fields in the collisions of the same species of nuclei \[19, 20\]. In symmetric collisions like Au+Au or Cu+Cu, the event-averaged electric field does not have a specific direction, although the magnitude of the electric fields generated in each event is considerably large \(|e\vec{E}| \sim |e\vec{B}| \sim O(m_\pi^2)|. We have performed event-by-event calculations of the electromagnetic fields in Cu+Au collisions to show that there should be a significantly large copper-tended electric field.

The electromagnetic fields are generated by the protons in nuclei. If we regard protons as point particles, the electric and magnetic fields at a position \(\vec{x}\) and time \(t\) is written by the Liénard-Wiechert potentials,

\[
|e|\vec{E}(t, \vec{x}) = \alpha_{EM} \sum_n \frac{1 - \vec{v}_n^2}{R_n^3 \left[1 - (\vec{R}_n \times \vec{v}_n)^2/R_n^2\right]^{3/2}} \vec{R}_n,
\]

\[
|e|\vec{B}(t, \vec{x}) = \alpha_{EM} \sum_n \frac{1 - \vec{v}_n^2}{R_n^3 \left[1 - (\vec{R}_n \times \vec{v}_n)^2/R_n^2\right]^{3/2}} \vec{v}_n \times \vec{R}_n,
\]

where \(\vec{R}_n \equiv \vec{x} - \vec{x}_n(t)\) with \(\vec{x}_n(t)\) the position vector of \(n\)-th proton, \(\vec{v}_n\) is the velocity vector of \(n\)-th proton, \(|e|\) is the electric charge of a proton, and \(\alpha_{EM}\) is the fine structure constant. We have defined the origin of the spatial coordinate as the middle of the centers of the nuclei and \(x\) and \(y\) axes as in Fig. 1. The summation is taken over all the protons in the colliding two nuclei. The positions of the protons inside a nucleus are sampled from the Woods-Saxon distribution with the standard parameters \[21\].

Figure 2 shows the event-averaged electric fields in Cu+Au collisions at impact parameter \(b = 4\) fm. Each vector represents direction and magnitude of the electric field at that point.
We find that the electric field in the central region of the overlapping area has a specific tendency to go from Au to Cu. Although the direction of electric fields fluctuates on an event-by-event basis because of the fluctuation in the proton positions inside colliding nuclei, the direction is correlated with the reaction plane for asymmetric collisions. The magnitude of the electric fields is as large ($|e\vec{E}| \sim O(m^2_\pi)$) as the electric and magnetic fields in Au+Au collisions at the same collision energy.

We have also calculated the time dependence of the averaged electric fields as shown in Fig. 3. The strength of the fields decays as the spectators fly away. Nevertheless, it is notable that even at $t = 1$ fm/$c$ the electric field is considerably larger than the so-called “critical field” for electrons, $|e|B_c = |e|E_c = m^2_e$.

Electric dipole of the plasma and charge-dependent directed flow.— The strong electric field toward the Cu nucleus at the early stage would induce an electric current in the medium that consists of the QGP after the thermalization time. As a result, the charge distribution would be modified and a charge dipole would be formed. One can expect that the dipole-like deformation of the charge distribution at the early stage would also be present in the observed charge distribution. This is because the electromagnetic charge is an exactly conserved quantity and an inhomogeneity relaxation of a conserved charge density takes a long time. Once a radial flow starts, the medium expands rapidly and the charge dipole created at the early stage would be frozen. Thus, we can reasonably assume that the dipole-deformation
FIG. 2: (Color online) Event-averaged electric field in the transverse plane in off-central Cu+Au collisions at $t = 0$ (the collision time) with impact parameter $b = 4$ fm at $\sqrt{s_{NN}} = 200$ GeV. Vectors are shown only in $|y| < 6$ fm. The average is taken over $10^4$ events.

FIG. 3: (Color online) Event average of the time evolution of electric fields in off-central Cu+Au collisions ($b = 4$ fm) at $\vec{x} = \vec{0}$. The average is taken over $10^4$ events.

in the plasma remains intact in the observed charge distribution.

The azimuthal angle distribution of the net charge, to the leading order in the multipole expansion, can be written as

$$\frac{d}{d\phi} \left( N_+ - N_- \right) = \left( \bar{N}_+ - \bar{N}_- \right) \left( 1 + 2e \cos \phi \right),$$

where the azimuthal angle $\phi$ is measured from the $x$-axis and $\bar{N}_{\pm}$ is defined as the angle
average of the number distribution,

$$N_{\pm} \equiv \int \frac{d\phi}{2\pi} \frac{dN_{\pm}}{d\phi}. \quad (4)$$

The dipole deformation of the medium is quantified by the value of $d_e$. We assume that the azimuthal distribution of the total number of particles is still written by the $v_1$ without the effect of the electromagnetic fields as

$$\frac{d (N_+ + N_-)}{d\phi} = (\bar{N}_+ + \bar{N}_-) (1 + 2v_1 \cos \phi), \quad (5)$$

since electromagnetic fields are not expected to change the bulk flow significantly. From Eqs. (3) and (5), the distribution of charged particles can be written as

$$\frac{dN_+}{d\phi} = \bar{N}_+ \left[ 1 + \frac{\bar{N}_+ + \bar{N}_-}{2\bar{N}_+} 2(v_1 + Ad_e) \cos \phi \right] \quad (6)$$

$$\frac{dN_-}{d\phi} = \bar{N}_- \left[ 1 + 2[v_1 - A(d_e - v_1)] \cos \phi + O[(Ad_e)^2] \right]. \quad (7)$$

Thus, the directed-flow coefficients $v_1$ for positively and negatively charged particles are written as

$$v_1^\pm = v_1 \pm Ad_e', \quad (8)$$

where we have defined $d'_e \equiv d_e - v_1$. The values $v_1^\pm$ are linear functions of $A$ and their slopes are given by the dipole-like deformation parameter $d'_e$, which is written as

$$d'_e = \frac{1}{\bar{N}_+ - \bar{N}_-} \int r dr d\phi \cos \phi \left[ j_e^0(r, \phi) - j_{e,E=B=0}^0(r, \phi) \right], \quad (9)$$

where $j_e^0(r, \phi)(j_{e,E=B=0}^0(r, \phi))$ is the transverse charge density in the presence (absence) of electromagnetic fields.

**Estimate of the charge-dependent directed flow.**— Let us make an order-of-magnitude estimate of the value of the charge-dependent directed flow parameter $Ad'_e$. For that purpose, we first roughly evaluate the total charge that is transferred from the gold-side to copper-side in the presence of an electric field. The total charge $Q$ transferred across a plane $S$ from $t = 0$ to $\tau$ is written as

$$Q = \int_0^\tau dt \int_S \mathbf{J} \cdot d\mathbf{S} = \int_0^\tau dt \int_S \sigma \mathbf{E} \cdot d\mathbf{S}, \quad (10)$$
where we have used the constitutive relation $\vec{J} = \sigma \vec{E}$ with $\sigma$ the electric conductivity. Let $S$ be the plane which includes the origin and is perpendicular to the line which connects the centers of the two colliding nuclei at $t = 0$, the moment two nuclei contact. Neglecting the space-time dependence of $\sigma$, $Q$ is rewritten as

$$Q \sim \sigma \tau \int_S \vec{E} \cdot d\vec{S}.$$  \hspace{1cm} (11)

The integral in Eq. (11) is just the total electric flux that goes through the plane $S$. Hence, the total transferred charge $Q$ is roughly given by

$$\int_S \vec{E} \cdot d\vec{S} \sim \frac{Z_{Au} - Z_{Cu}}{2} \frac{|e|}{\epsilon},$$ \hspace{1cm} (12)

where $Z_{Au}$ and $Z_{Cu}$ are the number of protons in two nuclei, and $\epsilon$ is the dielectric constant of the QGP.

According to lattice QCD simulations, the electric conductivity of the QGP is estimated as \cite{20}

$$\sigma \sim B C_{EM} T, \quad C_{EM} \equiv \sum_f e_f^2,$$ \hspace{1cm} (13)

where the sum in the electromagnetic vertex factor is taken over the flavors and $B$ is a coefficient that depend on the calculations. If we consider $u$, $d$, and $s$ quarks, $C_{EM} = \frac{8\pi\alpha_{EM}}{3}$. The value of the coefficient $B$ differs among calculations: $B \simeq 0.4$ in Refs. \cite{7,8} and $B \simeq 7$ in Ref. \cite{9}. On the other hand, perturbative QCD calculations predicts $\sigma \simeq 6T/e^2$ \cite{9}, which is much larger than the values from lattice QCD simulations.

As for $\tau$, we take the time scale that the radial flow starts, $\tau \sim 1 \text{ fm}/c$. If we take typical values for the other parameters, $T \sim 200 \text{ MeV}$, $\epsilon \sim 1$, and $B \sim 1$, the total transferred charge is estimated as

$$Q \sim BC_{EM} T \tau \frac{Z_{Au} - Z_{Cu}}{2} \frac{|e|}{\epsilon} \sim 1 \cdot \frac{8\pi}{3} \alpha_{EM} \cdot 200 \text{ MeV} \cdot 1 \text{ fm}/c \cdot 25|e| \sim 1.7 |e|.$$  \hspace{1cm} (14)

Now we can roughly evaluate $Ad_{\epsilon}$. Let us choose the events in which the numbers of positive and negative hadrons are equal, $N_+ = N_-$, and assume that $n$ charges have been
transferred by the electric field. Then, the number $n$ can be written as

$$n = -\frac{1}{2} \int_{-\pi/2}^{\pi/2} d\phi \frac{d(N_+ - N_-)}{d\phi} = -2Ad_e' (\bar{N}_+ + \bar{N}_-).$$

(15)

Therefore, the directed flow parameter $Ad_e'$ is written by $n$ as

$$Ad_e' = -\frac{\pi n}{N_{tot}},$$

(16)

where $N_{tot} \equiv 2\pi (\bar{N}_+ + \bar{N}_-)$ is the total number of charged particles. The number $n$ is related to the total transferred charge roughly as $n \sim Q/|e|$. Therefore, $Ad_e'$, the charge-dependent part of the directed flow parameter, and the electric conductivity of the plasma are parametrically related as

$$Ad_e' \sim -\frac{\pi \sigma \tau}{N_{tot}|e|} \int_S \vec{E} \cdot d\vec{S}.$$  

(17)

If one takes $N_{tot} \sim 10^3$ and $n \sim 1$ (Eq. (14)), the order of magnitude of the directed-flow parameter is estimated as

$$Ad_e' \sim -10^{-3}.$$

(18)

This value would be within experimental reach. Note that the value (18) is negative since the electric field tends the Cu nucleus.

Let us comment on potential uncertainties in the estimate above. It would be possible that the charge dipole formed at the early stage can be obscured in the later stages, namely the hydrodynamic evolution and hadronic collisions. In order to quantify these effects, we have to calculate the time evolution of the charge density under an electric field. Hence, it is necessary to use a hydrodynamic model which includes electromagnetic fields and a hadronic afterburner. One should also consider the effects of fluctuations of the generated electric fields on an event-by-event basis, although the fields have a tendency to direct from Au to Cu nucleus on average. The charge dipole could be smeared out by the fluctuations. Event-by-event simulations are necessary to consider the effect of such fluctuations. Finally, although we have assumed the dielectric constant is a constant, it can in general depend on frequency and wave length. Consideration of such effect is left as a future work.

**Summary.**— We have pointed out that, in Cu+Au collisions, a sizable strength of electric field directed from Au to Cu nucleus is generated in the overlapping region. We have shown
this by performing event-by-event numerical calculation of the produced electromagnetic fields. We have also pointed out that the electric field would induce an electric current in the matter created after the collision and it would result in a dipole deformation of the charge distribution in the medium. We have shown that the charge-dependent directed flow of hadrons is sensitive to the charge dipole in the medium and is useful in estimating the electric conductivity of the QGP.

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