A Predicted Relation between the Temperature and Density Profile of Cluster Hot Gas

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Abstract Based on the assumptions that a fraction of cluster dark matter is composed of degenerate neutrinos and they are in hydrostatic equilibrium with other matter, we predict a relation between the density profile and temperature of the cluster hot gas. The predicted relation agrees with observational data of 103 clusters.

Keywords Intergalactic medium, Cluster, Dark Matter, Neutrinos

1 Introduction

Observational data on rotation curves of galaxies and mass profiles of clusters indicate that dark matter exists. For example, the rotation curves of dwarf galaxies indicate that their total masses are much greater than the visible mass (Swaters et al. 2000, Salucci et al. 2002). Also, the integrated total mass of a cluster is several times greater than the visible mass including hot gas and galaxies (Reiprich and Bohringer 2001). On the other hand, recent neutrino oscillation experiments indicate that neutrinos have finite but small rest mass. Therefore, at least some fractions of dark matter should compose of neutrinos, which is known as hot dark matter (HDM). It is commonly believed that neutrinos exist in clusters and affect their structures. Cowsik and McClelland (1973) provided a simple model to understand the virial mass discrepancy in the Coma cluster if neutrinos have rest mass. After the non-zero rest mass of neutrinos was confirmed by experiments (Fukuda et al. 1998, Bilenky et al. 1998), neutrinos being a candidate cluster dark matter has become a hot topic again. Treumann et al. (2000) presented a model to calculate the mass distribution in two clusters, Coma and A119, including cold dark matter, \( \sim 2 \text{ eV} \) neutrino dark matter and hot gas. Recently, Nakajima and Morikawa (2007) presented a model using 1-2 eV degenerate neutrinos in hydrostatic equilibrium to fit the observed flat core in A1689, which contradicts with the results obtained by numerical simulations of cold dark matter particles (Navarro et al. 1996, Moore et al. 1999). All the above results indicate that neutrino dark matter can be an important component in the mass distribution of clusters. Neutrinos alone cannot form structures as their free streaming scale is too large. However, with the help of cold dark matter, neutrinos can be gravitationally bound in the clusters and their effects may be observable (Chan and Chu 2006). In this article, we study a possible observable consequence of neutrinos in clusters. We assume that the degenerate neutrinos and hot gas particles are in hydrostatic equilibrium under the gravity of cold dark matter, galaxies and intergalactic hot gas. We derive an approximate relation among the parameters specifying the density profiles of the cluster hot gas and their temperatures in 103 clusters. We also make predictions about the density profiles of neutrinos in clusters.

2 Neutrinos in Clusters

Currently, there are not much data on cluster observables. All we have now are the average hot gas temperature \( T \), luminosity \( L \), core radius \( r_c \) and the parameter \( \beta \) in King’s \( \beta \)-model (Brownstein and Moffat 2005). In this section, we derive a relation among the cluster observables by assuming that neutrinos are bound in hydrostatic equilibrium by the overall mass profiles in clusters.
In King’s β-model (King 1972; Jones and Forman 1984), the hot gas number density is
\[
n_g = n_c \left(1 + \frac{r^2}{R_c^2}\right)^{-3/2},
\]
where \(n_c\) is the central number density. Suppose the hot gas is in hydrostatic equilibrium and the interactions between the baryons and the neutrinos are negligible; then the pressure gradients of the hot gas and the neutrinos are balanced by the total gravity inside a cluster independently. Therefore we have
\[
kT \frac{dn_g}{m_g n_g} \frac{dr}{r^2} = \frac{GM(r)}{r^2},
\]
where \(M(r)\) is the enclosed mass in a cluster including the masses of galaxies, hot gas, cold dark matter and neutrinos, \(m_g\) is the average mass of the hot gas particles, and we have assumed that the temperature \(T\) is constant throughout the hot gas. Although Vikhlinin et al. (2005) suggested the temperature may not be uniform especially near the center of some clusters, the variations may only amount to 20-30 %, which has little effect on our results.\(Tdn_g/ \frac{dr}{r^2} \approx 4 - 5\) times higher. Also, Chan and Chu (2007) show the temperature variation in cluster hot gas is not significant because energy transfer by conduction is highly efficient. Therefore, in the following analysis, we approximate the temperature as uniform. On the other hand, we suppose that the neutrinos with mass \(m_\nu\) are degenerate and in hydrostatic equilibrium inside the cluster. Therefore we have
\[
\frac{1}{\rho_\nu} \frac{dP}{dr} = -\frac{GM(r)}{r^2},
\]
where
\[
P = \frac{4\pi^2h^2}{5m_\nu^8/3} \left(\frac{3}{4\pi g_s}\right)^{2/3} \rho_\nu^{5/3} = K_v \rho_\nu^{5/3}
\]
being the degeneracy pressure of neutrinos, \(\rho_\nu\) their mass density and \(g_s\) the degree of freedom of each type of neutrinos. We assume \(g_s = 1\) and combine Eqs. (3) and (4) to get
\[
5K_v \rho_\nu^{-1/3} \frac{d\rho_\nu}{dr} = \frac{kT d}{m_g} \frac{d}{dr} \left(\ln n_g\right).
\]
Using the density profile of the hot gas in Eq. (1) and integrating Eq. (5), we finally obtain
\[
\rho_\nu^{2/3} = \rho_c^{2/3} - \frac{3kT\beta}{5K_v m_g} \ln \left(1 + \frac{r^2}{r_c^2}\right)
\]
for \(r < R (\rho_\nu = 0 \text{ for } r > R)\) and the total enclosed mass profile (Reiprich and Bohringer 2001)
\[
M(r) = \frac{3kT r^3 \beta}{m_g G(r_c^2 + r^2)},
\]
where
\[
\rho_c = \left[\frac{3kT \beta}{5K_v m_g} \ln \left(1 + \frac{R^2}{r_c^2}\right)\right]^{3/2}
\]
is the central neutrino density and \(R\) is the radius of the neutrino density profile. The total mass profile Eq. (7) has a soft core which is different from the NFW profile obtained by N-body simulation. Nevertheless, recent gravitational lensing data support the existence of soft cores in clusters, in contradiction to the NFW profile (Tyson et al. 1998; Sand et al. 2002; Broadhurst 2005). Since there is no robust definition of the radius and total mass of a cluster, we follow Brownstein and Moffat (2003) to define the radius and total mass of a cluster \(M_c\) by assuming a cut off radius where the total mass density = 250 times mean cosmological density of baryons (Brownstein and Moffat 2005). We can then obtain a relation \(M_{14} = (1.5 \pm 0.1) \log(\beta T_K) + (-10.7 \pm 0.4)\) (see Fig. 1), where \(M_{14} = M_c/10^{14}M_\odot\) and \(T_K\) is the temperature of the hot gas in K, or
\[
M_c \approx q(\beta T)^{3/2},
\]
where \(q \approx (870 - 5010)M_\odot K^{-3/2}\) is a constant which depends sensitively on the definition of the cut off radius.

In the following, we obtain a relation among the observables \(r_c, \beta\) and \(T\). We integrate the density profile in Eq. (6) to get the total mass of the neutrinos:
\[
M_\nu = \int_0^R 4\pi r^2 \rho_\nu \, dr = \tilde{\rho}_c^3 I(u_0),
\]
where
\[
I(u_0) = \int_0^{u_0} 4\pi u^2 \left[\ln \left(\frac{1 + u_0^2}{1 + u^2}\right)\right]^{3/2} \, du,
\]
and \(u = r/r_c, u_0 = R/r_c, \tilde{\rho} = (3kT\beta/5K_v m_g)^{3/2}\). In a cluster, we assume the ratio of \(M_\nu\) to \(M_c\) to be the same as the cosmological value
\[
\frac{M_\nu}{M_c} \approx \frac{\Omega_\nu}{\Omega_m} = \frac{m_\nu}{\alpha},
\]
where \(\alpha = 94\Omega_m h^2 \approx 12.6\ eV\) (Chan and Chu 2007). \(\Omega_\nu\) and \(\Omega_m\) are cosmological density parameters of neutrinos and total matter, and \(h \approx 0.7\) is the Hubble parameter. By combining Eqs. (8)-(12), we finally get:
\[ r_c = \left( \frac{q m_\nu}{\alpha I(u_0)} \right)^{1/3} \left( \frac{5 K_\nu m_g}{3 k} \right)^{1/2} \approx \frac{(2.0 - 3.6) \text{ Mpc}}{m_a I(u_0)^{1/3}}. \]

From Eq. (13), we notice that for fixed \( m_\nu \), \( r_c \) depends on \( u_0 \) only. Plotting \( R \) against \( r_c \), we see that the values of \( R \) are nearly constant for all clusters. \( R \) is approximately proportional to \( 1/m_\nu \) (see Fig. 2). If \( m_\nu \leq 2 \text{ eV} \), which is the current upper bound \( \text{[Elgaroy 2006; Sanders 2007]} \), then \( R \gg r_c \) for most clusters.

Suppose the total central density of a cluster \( \rho_0 \) is related to \( \rho_c \) by a power law \( \rho_c \propto \rho_0^\gamma \); by defining \( 4\pi r^2 \rho_0 = dM(r)/dr \) at \( r = 0 \) and rearranging Eq. (8), we obtain the key relationship between the core radius \( r_c \) and the product \( \beta T \):

\[ \ln r_c \approx \left( \frac{2\gamma - 3}{4\gamma} \right) \ln(\beta T) + \text{constant}, \]

where we have assumed that \( \ln(1 + R^2/r_c^2) \) is nearly a constant for all clusters. To verify the above prediction, we plot \( \ln r_c \) against \( \ln(\beta T) \) for 103 clusters in Fig. 3; an approximately linear relation is obtained which agrees with Eq. (14). The slope in Fig. 3 is \( 0.97 \pm 0.11 \) which corresponds to \( \gamma \approx -3/2 \) (correlation coefficient = 0.66). However, the uncertainties in \( M_c, \beta, T \) and \( r_c \) are quite large, and the total mass profile of a cluster (Eq. (7)) is only derived by using King’s \( \beta \)-model. Therefore our model can only give an approximate prediction of the relation between \( \ln r_c \) and \( \ln(\beta T) \) with \( \gamma \approx -3/2 \).

### 3 Discussion and summary

Neutrinos exist in clusters and they may form structures with help of cold dark matter \( \text{[Chan and Chu 2006]} \). By assuming the hydrostatic equilibrium of neutrinos and hot gas particles with total mass in clusters, we obtain the density profile of neutrinos in terms of \( \beta, T \) and \( r_c \), and we can thereby obtain an approximate relation among these parameters with \( m_\nu \leq 2 \text{ eV} \). If \( \rho_c \propto \rho_0^\gamma \), then a linear relationship between \( \ln r_c \) and \( \ln(\beta T) \) is obtained which agrees with the observed data with \( \gamma \approx -3/2 \). Our result is also compatible with Sanders (2007) that the core profiles in clusters can be explained by neutrinos as dark matter.

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Fig. 3  $\ln r_c$ vs. $\ln(\beta T)$ for 103 clusters, where $r_c$ and $T$ are in kpc and keV respectively. The crosses are the observed data and the solid line is the best fitted line. The slope obtained is $0.97 \pm 0.11$ with correlation coefficient $\sim 0.66$. 
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