Adiabatic multicritical quantum quenches: Continuously varying exponents depending on the direction of quenching

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Abstract – We study adiabatic quantum quenches across a quantum multicritical point (MCP) using a quenching scheme that enables the system to hit the MCP along different paths. We show that the power-law scaling of the defect density with the rate of driving depends non-trivially on the path, i.e., the exponent varies continuously with the parameter $\alpha$ that defines the path, up to a critical value $\alpha = \alpha_c$; on the other hand for $\alpha \geq \alpha_c$, the scaling exponent saturates to a constant value. We show that dynamically generated and path-dependent effective critical exponents associated with the quasicritical points lying close to the MCP (on the ferromagnetic side), where the energy gap is minimum, lead to this continuously varying exponent. The scaling relations are established using the integrable transverse XY spin chain and generalized to a MCP associated with a $d$-dimensional quantum many-body systems (not reducible to two-level systems) using adiabatic perturbation theory. We also calculate the effective path-dependent dimensional shift $d_0(\alpha)$ (or the shift in center of the impulse region) that appears in the scaling relation for special paths lying entirely in the paramagnetic phase. The numerically obtained results are in good agreement with analytical predictions.

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Following the Kibble-Zurek (KZ) [1,2] prediction of the scaling of the density of defect in the final state of a quantum many-body system following a slow quench [3,4] across a quantum critical point (QCP) [5,6], there has been an upsurge in theoretical studies on quantum quenching across critical points [7–14] (for a review see [15]). The KZ argument predicts that the scaling of the defect density ($n$) in the final state is universal and is given by $n \sim 1/\tau^{\nu d/(\nu z+1)}$ where $\tau$ is the inverse rate of driving across a QCP with the correlation length and dynamical exponents $\nu$ and $z$, respectively, and $d$ is the spatial dimension. The possibility of the experimental verification of Kibble-Zurek scaling (KZS) in a spin-1 Bose condensate [16], in ions trapped in optical lattices [17,18], and also in ultracold fermionic atoms in optical lattices [19,20] has paved the way for the above-mentioned theoretical studies.

Although the KZS for quenching through a quantum critical point is well understood, the scaling of the defect density following an adiabatic quantum quench across a quantum multicritical point (MCP) is relatively less studied. A non-KZS behavior ($n \sim 1/\tau^{1/6}$) of the density of defects (wrongly oriented spins) for quenching across the MCP of the spin-1/2 transverse XY chain was reported for the first time in ref. [8] which was later explained in ref. [21] introducing an effective dynamical exponent $z_2(=3)$ for Jordan-Wigner solvable spin chains [22] reducible to a collection of decoupled two-level systems in the Fourier space and applying Landau-Zener (LZ) transition formula [23]. This argument was extended to the non-linear quenching of a general Hamiltonian in ref. [24]. In a recent communication, Deng et al. [25], attributed this anomalous scaling behavior to the existence of quasicritical points close to a MCP where the energy gap is minimum and proposed a generic scaling for the multicritical quantum quenches in terms of the effective critical exponents associated with these quasicritical points. Studies on inhomogeneous quantum transitions across a MCP has also been reported recently [26]. But how does the scaling exponent depend on the direction of approaching the MCP? In this work, we address this particular question proposing a different quenching scheme that enables the system to cross the
multicritical point along different directions and derive the corresponding KZS which reduces to all the previous results in appropriate limits.

At the outset, we summarize our main result using the example of a one-dimensional spin-1/2 transverse XY Hamiltonian [27] given by

\[ H = \frac{1}{2} \sum_j (J_x \sigma_j^x \sigma_{j+1}^x + J_y \sigma_j^y \sigma_{j+1}^y + h \sigma_j^z), \]

where \( \sigma_j^x \), \( \sigma_j^y \), and \( \sigma_j^z \) are the Pauli spin matrices, \( J_x, J_y \) are the interactions along \( x \) and \( y \), respectively, and \( h \) denotes the strength of the transverse field and henceforth, we set \( J_x + J_y = 0 \). The parameter \( \gamma = J_x - J_y \) is the measure of the anisotropy of the interactions and \( J_x + J_y = \gamma = 1 \) refers to the transverse Ising limit [28]. The phase diagram of the Hamiltonian is well known and shown in fig. 1; we study the quenching across the MCP (A) at \( h = 1, \gamma = 0 \) with the correlation length exponents and the dynamical exponents \( \nu_1 = 1/2 \) and \( z_1 = 2, \) respectively.

We propose a generic quenching scheme where the parameters \( h \) and \( \gamma \) are both varied simultaneously following the relation

\[ h(t) = 1 + |\gamma(t)|^{\alpha} \text{sgn}(t); \quad \alpha > 0 \quad \text{and} \quad \gamma(t) = -t/\tau \]

with time \( t \) going from \(-\infty\) to \( \infty \); the system reaches the MCP A at \( t = 0 \). The quenching path hits the MCP vertically (i.e., \( |\partial \gamma / \partial h|_{\gamma \rightarrow 0} \rightarrow \infty \)) for \( \alpha > 1 \) and horizontally for \( \alpha < 1 \). The parameter \( \alpha \) introduces a non-linear temporal variation of \( h \), thereby determining the path of quenching across the MCP as shown in fig. 1; for \( \alpha = 1 \) (path I), the MCP is approached linearly and hence the path is always equidistant from the Ising and Anisotropic critical lines. Similarly, if the parameters \( h \) and \( \gamma \) are changed simultaneously by the same non-linear rate as \( \gamma(t) = h(t) - 1 = |t/\tau|^{\alpha} \) \( \text{sgn}(t) \) as studied in [25], the quenching path once again happens to be equidistant from these two critical lines. The same is not true for the quenching scheme (2) if \( \alpha \neq 1 \); paths with \( \alpha \to 0 \) and \( \alpha \to \infty \) correspond to the passage across the anisotropic and Ising critical lines, respectively. For \( \alpha = 1 \), the scheme (2) is the same as that of ref. [25].

Our studies reveal the existence of a critical value \( \alpha_c = 2 \) for model (1) for the quenching scheme (2). For \( 1 \leq \alpha < 2 \), the defect density \( n_2 \) scales as \( n_2 \sim \tau^{-\alpha/6} \), i.e., the exponent varies continuously with the parameter \( \alpha \) and saturates to the scaling 1/3 for \( \alpha \geq 2 \). We argue that the quenching path crosses the quasicritical points for \( \alpha < 2 \) but not for \( \alpha > 2 \) when we recover the scaling relation valid for quenching the system across the MCP by linearly varying the parameter \( \gamma \) along the Ising critical line [13] that amounts to setting \( \alpha \to \infty \) in eq. (2). We do also derive similar scaling relation for the special path III (fig. 1) and calculate the path-dependent dimensional shift appearing in the scaling. Finally, we extend our studies to a \( d \)-dimensional generic Hamiltonian using the adiabatic perturbation theory [4] and establish the scaling relation, \( n_2 \sim \tau^{-\alpha d_{2v}}/|\alpha(z_1 \nu_2(1)+z_1 \nu_2(1-\alpha))| \) where \( z_1, \nu_1 \) are the exponents associated with the MCP while \( z_2(\alpha) \) and \( \nu_2(\alpha) = \nu_2 z_1/z_2(\alpha) \) are path-dependent exponents associated with the quasicritical points. To the best of our knowledge this path-dependent \( z_2(\alpha) \) leading to a path-dependent continuously varying exponent of KZS were never reported before. It needs to be emphasized here that these path-dependent exponents are generally not useful in describing the equilibrium quantum critical behaviour though they do dictate the non-equilibrium dynamics of models as discussed above.

It is well known that using the Jordan-Wigner transformation [22], the Hamiltonian (1) can be reduced to direct product of decoupled \( 2 \times 2 \) Hamiltonians for each momentum \( k \) given by

\[ H_k = \begin{bmatrix} h + \cos k & i \gamma \sin k \\ -i \gamma \sin k & -h + \cos k \end{bmatrix} \]

and that the excitation gap is

\[ \Lambda_k = \sqrt{(h + \cos k)^2 + \gamma^2 \sin^2 k}. \]

In the vicinity of the quantum MCP A, with \( \pi - k = k \to 0 \), eq. (3) can be simplified to

\[ H_k = (h - 1 + k^2) \hat{\sigma}^2 + \gamma k \hat{\sigma}^z. \]
For the sake of comparison, let us briefly recall the results of quenching across the MCP A with \( \gamma(t) = h(t) - 1 = |t/\tau|^{\alpha} \text{sgn}(t) \) as studied in [25] using the linearization process. The Hamiltonian (3) gets modified to

\[
H_k = \left( \frac{|t|}{\tau} \text{sgn}(t) + k^2 \right) \hat{\sigma}^x + \left( \frac{|t|}{\tau} \text{sgn}(t) k \hat{\sigma}^z ,
\]

\[
= \left( \frac{|t|}{\tau} \text{sgn}(t) + k^2 \right) \hat{\sigma}^x + k^3 \hat{\sigma}^z .
\]  

In deriving (6) an appropriate unitary transformation is used so that the time dependence of the off-diagonal terms is removed and the LZ transition formula is directly applicable for \( \alpha = 1 \) [23]. We observe that the energy gap is minimum at a time \( t_0 \) so that \( |t_0/\tau|^\alpha \text{sgn}(t) + k^2 = 0 \); this defines a quasicritical point at \( t_0 \) on the ferromagnetic side of the MCP (i.e., \( t < 0 \)). The minimum gap scales as \( k^3 \) so that we have an effective dynamical exponent \( z_2 = 3 \) even though the dynamical exponent at the MCP (i.e., \( t = 0 \)) is still given by \( z_1 = 2 \); the equivalent correlation length exponent \( \nu_2 = \nu_1 z_1/z_2 = 1/3 \). We emphasize here that for this quenching scheme [25], the exponent \( z_2 \) is fixed (\( \approx 3 \)) irrespective of the value of \( \alpha \).

Since in the limit of \( \tau \to \infty \), the maximum contribution to the defect comes from the vicinity of \( t_0 \), we linearize the Hamiltonian (6) [24] around \( t = t_0 \)

\[
H_k = \left( \frac{|t-t_0|}{\tau \text{eff}} \right) \hat{\sigma}^x + k^3 \hat{\sigma}^z ,
\]  

with \( \tau \text{eff} = \tau k^{-2(\alpha-1)/\alpha} \) and \( \tau \text{eff} = \tau \) for \( \alpha = 1 \). The higher-order terms lead to a faster decay of the defect density and hence can be dropped in the limit of large \( \tau \). The excitation probability is now readily obtained using the Landau-Zener formula \( p_h = \exp(-\pi k^2 \tau \text{eff}) \) which is integrated over all modes \( k \) to derive the scaling of the defect density given as

\[
n = \int dk p_h \sim \left( \frac{\tau}{\alpha} \right)^{-\alpha/[6\alpha + 2(1-\alpha)]} \sim \left( \frac{\tau}{\alpha} \right)^{-\alpha/[2\alpha + 1]} .
\]  

It is straightforward to generalize to a d-dimensional quantum MCP with \( \nu_1 z_1 = 1 \) described by the Hamiltonian

\[
H_k = (\lambda_1 + k^{z_1}) \hat{\sigma}^x + \lambda_2 k^\beta \hat{\sigma}^x ,
\]

\[
(\frac{|t|}{\tau} \text{sgn}(t) + k^{z_2}) \hat{\sigma}^x + |t/\tau|^{\alpha} \text{sgn}(t) k^2 \hat{\sigma}^z ,
\]  

where the MCP is reached at time \( t = 0 \) due to the simultaneous variation of two parameters \( \lambda_1 \) and \( \lambda_2 \) under the quenching scheme \( \lambda_1 = \lambda_2 = |t/\tau|^{\alpha} \text{sgn}(t) \). Comparison with the Hamiltonian (5) gives \( \lambda_1 = h - 1 \), \( \lambda_2 = \gamma \), \( z_1 = 2 \) and \( \beta = 1 \). Linearizing around \( t_0 \) and using \( \tau \text{eff} = \tau k^{-2(\alpha-1)/\alpha} \), we similarly get

\[
n \sim \tau^{-\alpha d/[2z_2\alpha + z_1(1-\alpha)]} ; \quad z_2 = z_1 + \beta .
\]  

Although scaling given in eq. (11) matches identically with the scaling relation eq. (4) of ref. [25] for \( \alpha = 1 \) (with \( \nu_0 z_1 = \nu_2 z_2 = 1 \)), there is a small difference for \( \alpha \neq 1 \). This mismatch stems from the fact that we have used linearization of the Schrödinger equations in the vicinity of \( t_0 \) (or \( \tau \text{eff} \)) which necessarily yields a term \( z_1(1-\alpha) \) in the KZS for \( \alpha \neq 1 \). We do also note that for \( \alpha < 1 \), \( \tau \text{eff} \to 0 \) in the limit \( k \to 0 \) rendering the process of linearization inappropriate. In the following, we shall restrict our analytical calculations to \( \alpha \equiv 1 \).

Now for our quenching scheme (2), the Hamiltonian (3) takes the form

\[
H_k = \left( \frac{|t|}{\tau} \text{sgn}(t) + k^2 \right) \hat{\sigma}^x + \frac{t}{\tau} k \hat{\sigma}^z .
\]  

We note that the energy gap of Hamiltonian (12) is minimum at a quasicritical point for \( t = t_0 \) where \( |t_0/\tau|^\alpha \text{sgn}(t) + k^2 = 0 \) only if \( \alpha < 2 \) while for \( \alpha > 2 \), the minimum gap occurs right at the MCP (i.e., at \( t = 0 \)) (see figs. 1 and 2). Linearizing around \( t = t_0 \), we get

\[
H_k = \left( \frac{|t-t_0|}{\tau \text{eff}} \right) \hat{\sigma}^x + k^{2/\alpha} \hat{\sigma}^z ,
\]  

with \( \tau \text{eff} = \tau k^{-2(\alpha-1)/\alpha} \). Here, we have ignored the time dependence of the off-diagonal term in favor of the minimum value at \( t = t_0 \) scaling as \( k^{2/\alpha+1} \) which is appropriate for \( 1 \leq \alpha < 2 \) but not for \( \alpha \geq 2 \) since \( \tau \text{eff}/(\tau/k) \sim k^{(2-\alpha)/\alpha} \to 0 \) as \( k \to 0 \) if \( \alpha < 2 \). This implies that \( \tau \text{eff} \) dictates the scaling behavior of the defect for the dynamics of eq. (12) for \( 1 \leq \alpha < 2 \) while for \( \alpha > 2 \) a crossover to the scaling behavior dictated by \( kt/\tau \) is expected. Further, the exponent \( z_2 = z_2(\alpha) = 2/\alpha + 1 \) depends on \( \alpha \) though \( z_1 \) is completely independent of the path and fixed (\( \approx 2 \)). We emphasize that the existence of the path-dependent \( z_2 \) is the crux of the quenching scheme (2) which leads to striking consequences shown below. We already note that \( \alpha = 2 \) is a marginal case where \( z_2 \) becomes equal to \( z_1 \).

The application of the Landau-Zener formula to eq. (13) similarly yields a scaling relation for the defect density \( n_2 \) for the quenching scheme (2) given by

\[
n_2 = \int dk \exp(-\pi k^2 \tau \text{eff}) \sim \left( \frac{\tau}{\alpha} \right)^{-\alpha/6} .
\]
so that the exponent varies continuously with the parameter $\alpha$. In the limit $\alpha = 1$, we recover the well-known scaling $n \sim \tau^{-1/6}$ [21] while for $\alpha \geq 2$, we get $n \sim \tau^{-1/3}$ [13,14] the system hits the MCP vertically and does no longer cross any quasicritical point along the path and hence the quenching scheme boils down to driving the system across the MCP along the Ising critical line [13] when the term $|t/\tau|^{\alpha}$ in eq. (12) can essentially be ignored. The defect density as a function of $\tau$ for different $\alpha$ obtained by numerical diagonalization of the Schrödinger equations in the case of full path is shown in fig. 3, and the results are in good agreement with the prediction of eq. (14). We note that the linearization process retains only the slowest $\tau$-dependence of $n$, while contributions from the higher-order terms decay much faster. However, this can be numerically observed only in the limit of sufficiently large $\tau$. We reiterate that the scaling relation given in (14) is inappropriate for $\alpha < 1$ though our numerical results do indicate the existence of a continuously varying exponent that decreases with decreasing $\alpha$.

Generalizing to the $d$-dimensional Hamiltonian (9) with

$$\tau_{\text{eff}} = \tau k^{-z_1(\alpha-1)/\alpha}/\alpha,$$

we get the relation

$$n_2 \sim \tau^{-d/[2z_2\alpha+z_1(1-\alpha)]}.$$  (15)

Although relation (15) closely resembles (11), it is to be noted that in our quenching scheme the exponent $z_2(z_1/\alpha + \beta)$ and hence, the scaling exponent depends on $\alpha$ up to $\alpha = \alpha_c$ when $z_1(\alpha_c) = z_2(\alpha_c)$ i.e., $\alpha_c = z_1/(z_1 - \beta)$.

Finally, we consider the quenching scheme

$$h(t) = 1 + |\gamma|^\alpha$$

and $\gamma = t/\tau$,  (16)

in which the system lies in the paramagnetic phase throughout the quenching process and touches the MCP at $t = 0$ (see path III in fig. 1). The case $\alpha = 1$ was studied in ref. [25] and a scaling relation of the form $n \sim \tau^{-3/4}$ was obtained. This anomalous scaling behavior was justified using the argument of a dynamical shift of the center of the impulse region [25] which makes it asymmetric as opposed to the conventional KZS such that non-critical energy states contribute to the defect with $p_k = k^{2d(\alpha)}d_0(\alpha)$ where $d_0(\alpha)$ is the additional effective dimension which eventually appears in the KZS. In a similar spirit, we use the LZ formula [29] for the Hamiltonian (13) for the half path so that the singularity at $t = 0$ is avoided; this provides the correct scaling for the full path since the symmetry of the initial and final states are the same. Expanding the transition probability (eq. (7) of [29]) in Taylor series around $T_f = 0$, we get a dominant contribution given as $p_k = \omega/T_f^2$ where the dimensionless time $T_f = (k^{2d(\alpha)}/\tau)^{\sqrt{\tau_{\text{eff}}}}$ and the dimensionless coupling $\omega = k^{d(\alpha)}\sqrt{\tau_{\text{eff}}}$. Using $\tau_{\text{eff}} = k^{2d(\alpha)}/\alpha$, we get $p_k = (1/\alpha^2)k^{2d(\alpha)-1}/\alpha$. The approximation is valid up to $k = k_{\text{max}}$ for which $T_f = 1$ given by $k_{\text{max}} \sim \tau^{-\alpha/2(\alpha+1)}$. The scaling of the defect density ($n_{\text{sp}}$) for these special paths is hence given by

$$n_{\text{sp}} = \int_{k_{\text{max}}}^{k_{\text{max}}} \text{dk}p_k \sim \frac{k_{\text{max}}^{4\alpha}}{\Gamma(1+\alpha)} \sim \tau^{-\alpha/2(\alpha+1)},$$  (17)

where in the limit $\alpha = 1$, we recover the scaling $\tau^{-3/4}$ as in [25]. We conclude that for the quenching scheme (16) the dimensional shift $d_0(\alpha) = 2(2 - \alpha)/\alpha$, varies continuously with $\alpha$; $d_0(\alpha) = 2$ for $\alpha = 1$ and vanishes for $\alpha \geq \alpha_c(=2)$ when once again the impulse region becomes symmetrical. Interestingly, for these special paths also we obtain a continuously varying exponent which saturates to 1/3 in the limit $\alpha \rightarrow \alpha_c$. Figure 4 shows the defect density as a function of $\tau$ for different $\alpha$ for the special path III, as obtained from numerical integration.
We shall now derive the generalized scaling relation for a quantum system with \( \nu_1 z_1 \neq 1 \) using adiabatic perturbation theory [4]. We note from eq. (13) that the dynamics across a MCP can be described as a linear quench with a different \( \tau_{\text{eff}} \) for each \( k \) mode across a quasicritical point which provides the most dominant contribution to the defect.

For a general Hamiltonian parametrized by \( \lambda = t/\tau \), we can write the wave function as \( \psi = \sum_o a_p(t) \phi(\lambda) \). In the above basis the Schrödinger equation becomes

\[
\frac{i}{\tau} \frac{d a_p(t)}{d \lambda} + i \frac{1}{\tau} \sum_q a_q(t) \langle p | \frac{d}{d \lambda} | q \rangle = E_\nu(\lambda) a_p(t),
\]

where \( E_\nu(\lambda) \) is the eigenenergy of \( H(\lambda) \). A unitary transformation

\[
a_p(t) = \tilde{a}_p(\lambda) e^{-i\tau E_\nu(\lambda) d\lambda}
\]

transforms eq. (18) to

\[
\frac{d \tilde{a}_p(\lambda)}{d \lambda} = - \sum_q \tilde{a}_q(\langle p | \frac{d}{d \lambda} | q \rangle e^{i\tau E_\nu(\lambda) - E_\nu(\lambda')} d\lambda'.
\]

Now taking into account that initially the system was in the ground state, which makes a single term dominate the sum in eq. (20), and expressing the above equations in the momentum basis, we obtain,

\[
n \simeq \frac{1}{(2\pi)^d} \int_{-\infty}^{\infty} d\lambda \left| \langle \tilde{k} | \frac{\partial}{\partial \lambda} | 0 \rangle \right|^2 e^{i\tau (E_\nu(\lambda) - E_\nu(\lambda')) d\lambda'}
\]

where near a quasicritical point, \( \delta E_\nu(\lambda) \simeq |\lambda|^{-2z_2} \times F'(|k|^2/|\lambda|^{2z_2}) \). Here \( \lambda \) denotes the deviation from the quasicritical point and \( F'(x) \sim x \) for large \( x \). Again, we have \( \langle \tilde{k} | \frac{\partial}{\partial \lambda} | 0 \rangle = \frac{1}{2} G(k/\lambda^{1/z}) \) where \( G(0) \) is a constant. Using these relations, defining \( N' = k^{1/z_2} \) and eventually rescaling \( |\tilde{k}| \rightarrow |\tilde{k}|^{z_{21}/z_2} \), we get

\[
n_{21} \sim \tau^{-d_1 z_{21}/(\alpha z_{21} + z_1 z_2(1-\alpha))}
\]

which reduces to eq. (15) for \( z_2 = 1 \). Of course, the scaling relation (22) holds up to a value of \( \alpha = \alpha_c \) such that \( z_1(\alpha_c) = z_2(\alpha_c) \) and for \( \alpha > \alpha_c \) gets modified to \( n \sim \tau^{-d_1 z_{11}/(\alpha z_{11} + z_1 z_2(1-\alpha))} \). For special paths (e.g., path III), we note that the minimum gap is given by exponents associated with the MCP and derive the generalized scaling relation given by

\[
n_{\text{sp}} \sim \tau^{-d(\alpha_0)/2}/(\alpha_{11} z_1 z_2 + z_1 z_2 z_2(1-\alpha)),
\]

where we have included the dimensional shift \( d(\alpha_0) \) which varies continuously with \( \alpha_0 \) for \( \alpha < \alpha_c \), while for \( \alpha > \alpha_c \) the scaling saturates to \( n \sim \tau^{-\alpha_1 z_1 z_2(1-\alpha)} \).

The main results of the paper are summarized in eqs. (14), (15), (17) and (22), (23). In conclusion, we propose a different quenching scheme across the MCP and derive a KZS with an exponent continuously varying with the path as a consequence of path-dependent effective dynamical exponents associated with the quasicritical points. This continuously varying exponent up to a critical value of \( \alpha \) is the cardinal feature of this paper. Also, our studies establish that unlike the conventional KZS, the path-dependent effective exponents, which usually do not appear in studies of the equilibrium quantum critical behavior, do appear in the scaling relation of the defect density in special situations like quenching across a multicritical point.

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