Measuring the internal clock of the electron

Martín Rivas
Theoretical Physics Department, The University of the Basque Country,
Apdo. 644, 48080 Bilbao, Spain
E-mail: martin.rivas@ehu.es

Abstract. The existence of an internal frequency associated to any elementary particle conjectured by de Broglie is compared with a classical description of the electron, where this internal structure corresponds to the motion of the centre of charge around the centre of mass of the particle. This internal motion has a frequency twice de Broglie’s frequency, which corresponds to the frequency found by Dirac when analysing the electron structure. To get evidence of this internal electron clock a kind of experiment as the one performed by Gouanère et al. [2] will show a discrete set of momenta at which a resonant scattering effect, appears. The resonant momenta of the electron beam are given by \( p_k = \frac{161.748}{k} \, \text{MeV}/c \), \( k = 1, 2, 3, \ldots \), where only, the corresponding to \( k = 2 \), was within the range of Gouanère et al. experiment. The extension of the experiment to other values of \( p_k \), would show the existence of this phenomenon.

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1. Introduction

The main conjecture in de Broglie’s thesis is ... the existence of a periodic phenomenon of an unknown nature, associated to every portion of isolated energy, and related to its proper mass by Planck-Einstein’s equation \[ E = mc^2, \] so that a particle of mass \( m \) at rest should have an internal frequency \( \nu_0 = mc^2/h \) and when the particle moves at a velocity \( v \) this frequency will be \( \nu = mc^2/h\gamma \), where \( \gamma = (1 - v^2/c^2)^{-1/2} \) is the relativistic time dilation factor. Properly speaking, a moving electron has a greater energy \( \gamma mc^2 \) and therefore a greater frequency \( \gamma mc^2/h \) according to the above statement, but if this internal periodic motion has a physical reality will have the frequency proposed by de Broglie \( \nu = mc^2/h\gamma \) when measured by an inertial observer who sees the electron moving at the speed \( v \), and according to the time dilation of special relativity.

Using this conjecture, Gouanèrè et al. \[2\] tried to get evidence of this electron internal clock frequency by analysing the interaction of a beam of electrons with the atoms of a 1\( \mu \)m thick silicon crystal aligned along the \(< 110 >\) direction. If the momentum of the electron beam is the appropriate, they will interact in a kind of transversal resonant scattering, whenever the distance travelled by the electron during a period \( L = v/\nu \) will be related to the separation \( d = 3.84 \text{Å} \) between the atoms of the crystal. When

\[
L = \frac{v}{\nu} = \frac{h\gamma v}{mc^2} = \frac{hp}{m^2c^2} = d
\]

this will produce a lateral force such that the outgoing beam in the forward direction will show a decrease in the number of outgoing electrons for that resonant momentum \( p \). This resonant momentum is \( p = 80.874 \text{ MeV}/c \) while their experiment shows a value of \( p_{\text{exp}} = 81.1 \text{ MeV}/c \), a 0.28% higher than the predicted theoretical value (see Figure 1).

Their Monte Carlo calculation, using parabolic potentials to describe the atomic structures, shows a bad fitting with the experimental result for this internal frequency (curve (a) of Figure 1), while a clear matching (curve (b) of Figure 1), except for the above 0.28% shift, between the expected and the experimental outcome for a momentum range between 80.0 MeV/c and 82.0 MeV/c and when the electron frequency is taken twice de Broglie’s frequency \( 2\nu \) or \( \nu/2\nu = d \). This resonant momentum is 161.748 MeV/c, it is outside of the experiment range, and the observed peak corresponds to the second harmonic. This frequency corresponds to the zitterbewegung frequency or frequency found by Dirac of the internal motion of the electron.

We are going to produce this analysis based upon a classical electron model found from a very general formalism for describing spinning particles \[3\]. Recently \[4\], we have given different arguments justifying that the centre of charge and centre of mass of a spinning particle are different points, and, for the electron at rest, the centre of charge is moving in circles, at the speed of light, around the centre of mass with a frequency \( \nu_0 = 2mc^2/h \), twice as much as de Broglie’s frequency and coherently matched with Dirac’s internal frequency. This internal frequency changes as \( \nu = \nu_0/\gamma \) when the electron moves, because of the special relativity time dilation. It is this relative
motion of the centre of charge which has a clear interpretation as the Schroedinger’s zitterbewegung, and therefore a clear internal periodic phenomenon associated to a spinning particle.

2. The classical electron model

In the kinematical formalism [3], an elementary particle is, by definition, a mechanical system which in addition to being indivisible, as a consequence of the atomic hypothesis [5], it can never be deformed so that all allowed states are only kinematical modifications of any one of them. This means that when the state of an elementary particle changes there exists another inertial observer who measures the particle in the same state as before. This means that in a variational approach of classical physics, the initial \( x_1 \) and final \( x_2 \) states of the evolution of an elementary particle must be related by a transformation of the kinematical group \( x_2 = gx_1 \). Therefore, the boundary variables of the variational approach, necessarily span a homogeneous space of the kinematical group of space-time symmetries. When quantizing all classical systems characterized by such homogeneous spaces, their Hilbert space of pure states carries a projective unitary irreducible representation of the kinematical group [6]. It thus satisfies Wigner’s definition of a quantum elementary particle.

In this way, the parameters of the kinematical group become the classical variables we use, as the boundary values of the variational formalism for describing an elementary particle. In the relativistic and non-relativistic approach, these variables are reduced to the ten variables \( t, r, u, \alpha \), interpreted respectively as the time, position of the charge, velocity of the charge and orientation of the particle. In the relativistic case we have three disjoint, maximal homogeneous spaces of the Poincaré group spanned by
these variables with the constraint either \( u < c, u = c \) or \( u > c \). It is the manifold with \( u = c \), as suggested by the kinematical analysis of reference [4], which leads to Dirac’s equation when quantizing the system. This kinematical analysis states the possibility that the centre of mass and centre of charge of a spinning particle could be different points. In that case, the centre of charge has necessarily a velocity of constant absolute value, unreachable by any inertial observer, so that only the relativistic approach is selected and the charge moves at the speed \( c \).

The Lagrangian depends on these ten kinematical variables and also on the acceleration of the point \( r \) and on the angular velocity. For a Dirac particle, the charge located at point \( r \) is moving at the speed of light \( u = c \). The classical expression linear in the energy \( H \) and in the linear momentum \( P \), which gives rise to Dirac’s equation is

\[
H = P \cdot u + \frac{1}{c^2} S \cdot \left( \frac{du}{dt} \times u \right),
\]

where the energy \( H \) is expressed as the sum of two terms, \( P \cdot u \), or translational energy and the other, which depends on the spin of the system, or rotational energy. The spin comes from the dependence of the Lagrangian \( L \) of both, the acceleration \( \dot{u} \), and the angular velocity \( \omega \), and if we define

\[
U = \frac{\partial L}{\partial \dot{u}}, \quad W = \frac{\partial L}{\partial \omega},
\]

it takes the form

\[
S = u \times U + W = Z + W.
\]

The first part \( Z = u \times U \), or zitterbewegung part, is related to the separation between the centre of charge from the centre of mass and takes into account this relative orbital motion. It quantizes with integer values. The second part \( W \) is the rotational part of the body frame and quantizes with both integer and half-integer values. The total angular momentum with respect to the origin of observer’s frame is

\[
J = r \times P + S,
\]

so that the spin \( S \) is the angular momentum of the system with respect to the centre of charge \( r \), and not with respect to the centre of mass \( q \). By this reason, it is not a conserved quantity for a free particle, but satisfies the dynamical equation

\[
\frac{dS}{dt} = P \times u. \quad (1)
\]

This is the same dynamical equation satisfied by Dirac’s spin operator in the quantum case.

When expressed Dirac’s spin \( S \) and the centre of mass position \( q \) in terms of the velocity and acceleration of the charge they take, respectively, the form

\[
S = \left( \frac{H - u \cdot P}{(du/dt)^2} \right) \frac{du}{dt} \times u, \quad q = r + \frac{c^2}{H} \left( \frac{H - u \cdot P}{(du/dt)^2} \right) \frac{du}{dt}
\]

Dirac’s spin is always orthogonal to the osculator plane of the trajectory of the charge \( r \), in the direction opposite to the binormal for a positive energy particle, and in the
opposite direction for the antiparticle. This implies a difference in chirality between matter and antimatter. Point \( q \) is always different from point \( r \), because the observable \( H - u \cdot P \neq 0 \) thus justifying the conjecture that for a spinning particle the centre of mass and centre of charge are different points. The acceleration of the charge is pointing from \( r \) to the centre of mass \( q \), as it corresponds to a helix. It is shown that the dynamical equation of point \( r \) for the free particle and in the centre of mass frame is given by

\[
    r = \frac{1}{mc^2} S \times u,
\]

where the spin vector \( S \) is constant in this frame, as depicted in Fig. 2. The radius of the zitterbewegung motion is \( R = S/mc \), and the angular velocity \( \omega = mc^2/S \). When quantizing the system the classical parameter \( S = \hbar/2 \). The frequency of this internal motion and in the centre of mass frame is \( \nu_0 = 2mc^2/h \), i.e., twice de Broglie’s frequency, and the radius is \( R = \hbar/2mc \), is half Compton’s wavelength.

When seen from an arbitrary observer (see Figure 3), the motion of the charge for a longitudinally polarized electron is a helix, so that according to (1) Dirac’s spin precess around the direction of the conserved linear momentum \( P \). The spin with respect to the centre of mass is defined as

\[
    S_{CM} = S + (r - q) \times P.
\]

It is a conserved quantity for a free particle. The centre of mass velocity is \( v = dq/dt \), and the linear momentum is written as usual as \( P = \gamma(v)mv \). This means that the
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Figure 3. Precession of Dirac’s spin $S$ along the linear momentum $P$ for a moving electron. The transversal motion of the charge takes a time $\gamma(v)$ longer than when the centre of mass is at rest, to complete a turn and, therefore, the frequency of the internal motion of the charge is $\nu = \nu_0/\gamma$. The spin with respect to the centre of mass $S_{CM}$, is a constant of the motion for a free particle.

transversal motion of the charge is at the velocity $\sqrt{c^2 - v^2}$. The centre of charge of a moving electron takes a time $\gamma(v)$ times longer than for an electron at rest to complete a turn, which is a clear consequence of the time dilation. The faster the centre of mass of the electron moves the slower is the rotation frequency of the centre of charge around the centre of mass. The internal clock of a fast electron is running slower and we are going to take advantage of this fact to get the resonant interaction with the Si atoms.

3. Polarized beams

In figure 4 we show the unit cell of a silicon crystal and the separation $d$ between neighbouring atoms on the $XOY$ plane along which the electron beam is sent in the Gouanère et al. experiment [2]. In the figure 5 we show two possible free motions of a polarized electron. In part (a) we describe the motion of the centre of mass and centre of charge of an electron polarized along the direction of motion $OY$. In part (b) the electron is polarized perpendicularly to the page along the $OZ$ axis. In both cases the centre of mass motion is in the direction $<110>$ of the crystallographic plane, where the dots represent the location of the Si nuclei on that plane. For electrons polarized along the $OX$ direction, where the centre of charge motion is contained in the plane $YOZ$, the description is equivalent to the last one, where the atoms will be arranged on the $YOZ$ plane. The frequency of the internal motion of a moving electron is $\nu = 2mc^2/\hbar\gamma$, so that the distance travelled by the centre of mass during a period is $L = \hbar\gamma v/2mc^2 = \hbar p/2m^2c^2$. There will be a resonant scattering whenever $L = nd$ or when $d = kL$ for any natural number $n, k = 1, 2, 3, \ldots$. In the first case, the particle
moves faster and there will be a resonant interaction every $n$–th atom, while in the second case the momentum will be smaller, the particle will interact with each atom but every $k$ periods of the electron. The previsible values of these momenta are

$$p_n = n\frac{2m^2c^2d}{h}, \quad p_k = \frac{1}{k} 2m^2c^2d \frac{1}{h}, \quad n, k = 1, 2, 3, \ldots$$

$$p_n = 161.748, \quad 323.496, \quad 485.244, \quad 646.992, \ldots \text{MeV}/c$$

$$p_k = 161.748, \quad 80.874, \quad 53.916, \quad 40.437, \quad 32.3496, \quad 26.958, \ldots \text{MeV}/c$$

In the Gouanére et al. experiment [2] the range of the electron beam was prepared between 54 to 110 MeV/c, so that only the resonant frequency for $k = 2$, 80.874 MeV/c was available within that range, although they mention that an observation could also be possible at harmonic frequencies, for example at corresponding momenta of 161.748 MeV/c and 40.437 MeV/c, etc., probably with reduced intensity.

Our conjecture is that the momenta of small value $p_k$ are more favorable to produce a larger transversal scattering because the velocity of the beam is smaller, the particle remains in the crystal a longer time and the interaction is produced with every atom, and, therefore, the ratio of the transversal momentum transfer to the longitudinal momentum of the beam is larger, so that a greater number of electrons will be withdrawn from the forward direction, while for the larger momenta $p_n$ the ratio of the transversal perturbation to the longitudinal velocity is smaller and only a smaller number of atoms participate in the resonant scattering. This means that smaller the momenta the larger the depth of the counts of the resonant peak.

For very small $p_k$, and depending on the experimental resolution, it will be difficult to discriminate among the corresponding resonance peaks. For example, for $k = 99, 100,$ and 101, the corresponding momenta are

$$p_{99} = 1.63382 \text{ MeV}/c, \quad p_{100} = 1.61748 \text{ MeV}/c, \quad p_{101} = 1.60147 \text{ MeV}/c$$

which differ from each other of around 0.01 MeV/c which seems to be smaller than the resolution of the experiment.

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Figure 4. Silicon crystal unit cell, showing the direction $<110>$ on the $XY$ plane, and the separation $d = 3.84\text{\AA}$ between neighbouring atoms on that plane.

Figure 5. Part (a) describes a free motion of an electron polarized longitudinally along the $OY$ axis, while part (b) describes the free motion of an electron polarized transversally in the direction $OZ$, perpendicular to the picture and the motion of the centre of charge is contained on the $XY$ plane. The straight line represents the centre of mass motion and the curly line is the trajectory described by the centre of charge. The range $\lambda_C = 3.86 \times 10^{-13}\text{m}$ of the transversal motion of the centre of charge is Compton’s wave length, and $d = 3.84 \times 10^{-10}\text{m}$ represents the separation between atoms of the Si crystal on the plane corresponding to the $<110>$ direction of the electron beam. Both lengths are clearly not represented at the same scale. $L$ is the distance travelled by the electron during a period. There will be resonant scattering whenever $L = nd$ or $d = kL$, for $n, k = 1, 2, 3, \ldots$
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