Complex Diffusion Monte-Carlo method for the systems with complex wave function: test by the simulation of 2D electron in uniform magnetic field

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Abstract

On the base of Diffusion Monte-Carlo method it is developed a new Complex Diffusion Monte-Carlo (CDMC) method allowing to simulate the quantum systems with complex wave function. There are no approximations on the calculation of modulus and phase of wave function in contrast to other methods. We find that the averaged value of any quantity in CDMC will have no direct contribution from the phase of the distribution function but only from the phase of the Green function of the diffusion equation. This is most important and crucial point of CDMC.

We are testing CDMC by the calculation of the wave function and the ground state energy of two-dimensional electron placed into the external uniform magnetic field. There is an excellent agreement between simulations’ results and an analytical ones.

I. INTRODUCTION

Quantum-mechanical wave function is an essentially complex in many cases. There are well-known examples – electron in the external uniform magnetic field when vector potential has a central-symmetric form and system of anyons. The complexity of wave function does not give a possibility for the simulations of such kind of systems by using well-known Green Function Monte-Carlo method (see review [1]). This method essentially demand the reality of the system’s wave function which considered as probability weight in during stochastic process.

There had been undertaken the several attempts [2–5] to construct a Monte-Carlo method for the simulation of quantum systems with complex wave function. Authors of the works [2–5] suggested to take as a probability weight the module of the complex distribution function. All quantities are calculated in [2–5] by averaging over this complex distribution func-
tion. The phase of this function $\alpha$ is taken accordingly $\alpha = \sum \alpha_i$, where $\alpha_i$ is the phase at the $i$-time step of the stochastic process.

Another quantum Monte Carlo method using algorithm without branching for the simulating of complex problems was developed in [4]. The main trouble of this approach was an increasing of the statistical error as a function of the whole time of the simulation.

The basic difficulty of the numerical simulation of the fermions is essentially the same as for the systems with the complex wave function. Their wave function can change the sign and therefore can not be used as the probability weight in the simulation process.

For the simulation of the continuum (not on the lattice) fermionic systems it were developed widely used fixed node Monte-Carlo method [4] (see also reviews [1,7]) and recently proposed constrained path Monte-Carlo method [8]. In these both methods it was assumed the restriction on the random walks connected with the uncertainties in the space localization of the wave function node surfaces. The comparison of these methods was done in [8].

Our method is very close to fixed phase Diffusion Monte-Carlo method developed in [3], which was applied to two-dimensional electrons in magnetic field. Laughlin’s wave function [10] was used as a trial one. In the framework of Diffusion Monte-Carlo (DMC) method it was calculated only modulus of the system’s wave function. The phase of system’s wave function was not calculated but considered as fixed and equal to the phase of Laughlin wave function.

Here it is proposed new Complex Diffusion Monte-Carlo (CDMC) method including also the simulation of the phase factor of the wave function.

CDMC are tested first by the calculations of the ground state wave function and energy of two-dimensional electron placed into the external uniform magnetic field. There is an excellent agreement between simulations’ results and analytical ones. Namely, it is reproduced the ground state energy and his degeneracy on the orbital quantum number $m$ and also the simulated wave function is in exact correspondence with analytical prediction. The description of CDMC method is given at the section II of the paper. In the section III we represent the description of the simulation algorithm. Analytical expressions for the algorithm quantities are given at section IV. Section V represent the discussion of the simulation results.

II. COMPLEX DIFFUSION MONTE-CARLO METHOD FOR THE QUANTUM SYSTEMS WITH COMPLEX WAVE FUNCTION

Hamiltonian of the electron placed into external uniform magnetic field with vector potential $\vec{A} = \frac{1}{2} [ \vec{H}, \vec{r} ]$, where $\vec{H}$ - magnetic field, $\vec{r}$ - electron’s radius vector, and in external potential $V(r)$ has a form:

$$\hat{H} = \frac{1}{2M} (\vec{p} + \frac{|e|}{c} \vec{A})^2 + V(r).$$

(1)

Here $M$ - mass and $e = -|e|$ charge of electron, $\vec{p} = -i\hbar \vec{\nabla}$, where $\vec{\nabla} = \frac{\partial}{\partial x} \vec{i} + \frac{\partial}{\partial y} \vec{j}$, $c$ - light velocity. By taking in to account a relation $\vec{p}\vec{A} - \vec{A}\vec{p} = div \vec{A} = 0$ for the such kind of vector potential one can rewrite Hamiltonian (1) in the following form:
\[ \hat{H} = \frac{\vec{p}^2}{2M} + \frac{|e|}{Mc} \vec{A} \vec{p} + \frac{|e|^2}{M c^2} \vec{A}^2 + V(r). \] (2)

Let us introduce a complex distribution function

\[ f(\vec{r}, t) = \Psi_T^*(\vec{r}) \Psi(\vec{r}, t). \] (3)

Here \( \Psi_T^*(\vec{r}) \) is a complex conjugated trial wave function of electron in magnetic field. Wave function \( \Psi(\vec{r}, t) \) satisfies a Schrödinger equation with imaginary time (expressed in \( \hbar \) units)

\[ -\frac{\partial \Psi(\vec{r}, t)}{\partial t} = (\hat{H} - E_T)\Psi(\vec{r}, t). \] (4)

It is necessary to choose a trial energy \( E_T \) in such manner \([6]\) that at \( t \to \infty \) \( \Psi(\vec{r}, t) \to \Psi_0(\vec{r}) \) - an exact stationary wave function of ground state of Hamiltonian (1).

So, by accounting (4), one can write the equation for the distribution function \( f \equiv f(\vec{r}, t) \):

\[ -\frac{\partial f}{\partial t} = \frac{\vec{p}^2}{2M} f - \frac{\vec{p}}{M} (f \vec{F}_Q(\vec{r})) + \frac{|e|}{Mc} \vec{A} \vec{p} f + (E_L(\vec{r}) - E_T) f, \] (5)

where

\[ \vec{F}_Q(\vec{r}) = \Psi_T^{*-1}(\vec{r}) \vec{p} \Psi_T^*(\vec{r}), \] (6)

\[ E_L(\vec{r}) = \Psi_T^{*-1}(\vec{r}) \hat{H}' \Psi_T^*(\vec{r}), \] (7)

\[ \hat{H}' = \frac{1}{2M} (\vec{p} - \frac{|e|}{c} \vec{A})^2 + V(r). \] (8)

For the \( \vec{p} = -i\hbar \vec{\nabla} \) and \( D = \frac{\hbar^2}{2M} \), and by renaming

\[ \vec{F}_Q(\vec{r}) = 2\Psi_T^{*-1}(\vec{r}) \vec{\nabla} \Psi_T^*(\vec{r}), \] (9)

we have an equation:

\[ -\frac{\partial f}{\partial t} = -D \Delta f + D \vec{\nabla} (f \vec{F}_Q(\vec{r})) - \frac{i\hbar|e|}{Mc} \vec{\nabla} f + (E_L(\vec{r}) - E_T) f. \] (10)

Here \( \Delta = \vec{\nabla}^2 \).

In general case when \( \Psi_T^*(\vec{r}) \) is a complex wave function the quantity \( \vec{F}_Q(\vec{r}) \) has a form:

\[ \vec{F}_Q(\vec{r}) = \text{Re} \vec{F}_Q(\vec{r}) + i \text{Im} \vec{F}_Q(\vec{r}). \] (11)

So, a final view of an equation for the distribution function \( f \) is

\[ -\frac{\partial f}{\partial t} = -D \Delta f + D \vec{\nabla} (f \text{Re} \vec{F}_Q(\vec{r})) + i \vec{\nabla} (D f \text{Im} \vec{F}_Q(\vec{r})) - \frac{\hbar|e|}{Mc} \vec{\nabla} f + (E_L(\vec{r}) - E_T) f. \] (12)
Following [3], we assume: when the time step of integration of equation (12) \( \tau \to 0 \) the function \( \tilde{F}_Q(\vec{r}) \) remains constant, i.e., we will suppose that \( \tilde{F}_Q(\vec{r}) \equiv \tilde{F}_Q(\vec{r}') \) at \( \tau \to 0 \), where vector \( \vec{r} \) corresponds to time point \( t + \tau \) and \( \vec{r}' \) to point \( t \).

Let us introduce a new quantity

\[
\vec{A}_Q(\vec{r}, \vec{r}') = \frac{1}{2} \text{Im} \tilde{F}_Q(\vec{r}') - \frac{\hbar|e|}{2DMc} \vec{A}(\vec{r}).
\]  

(13)

Then at \( \tau \to 0 \) the Green function of equation (12) has a form:

\[
G(\vec{r}, \vec{r}'; \tau) = G_1(\vec{r}, \vec{r}'; \tau) \exp \left[ -\tau(\text{E}_L(\vec{r}) - \text{E}_T) \right] \exp \left[ i\vec{A}_Q(\vec{r}, \vec{r}')(\vec{r} - \vec{r}' - D\tau \text{Re} \tilde{F}_Q(\vec{r}')) \right],
\]  

(14)

where

\[
G_1(\vec{r}, \vec{r}'; \tau) = \frac{\exp[D\tau \vec{A}_Q(\vec{r}, \vec{r}')]}{4\pi D\tau} \exp \left[ \frac{-(\vec{r} - \vec{r}' - D\tau \text{Re} \tilde{F}_Q(\vec{r}'))^2}{4D\tau} \right].
\]  

(15)

One can see that the Green function \( G(\vec{r}, \vec{r}'; \tau) \) (14) and distribution function \( f \) given by (3) are the complex functions. These both quantities are related by usual integral equation:

\[
f(\vec{r}, t + \tau) = \int d\vec{r}' G(\vec{r}, \vec{r}'; \tau)f(\vec{r}', t).
\]  

(16)

From equation (16) it is followed that the modulus and the phase of the distribution function at consequent time point are determined by the modulus and the phase of the Green function and of ones of the distribution function at previous time point of integration of diffusion equation (12).

We have to note that for the general case of complex wave function \( \Psi_T(\vec{r}) \) the energy \( \text{E}_L(\vec{r}) \) is also a complex, so real and imaginary part of \( \text{E}_L(\vec{r}) \) contribute to ones of the Green function. Therefore the last two exponents in (14) has a form:

\[
\exp \left[ -\tau(\text{Re} \text{E}_L(\vec{r}) - \text{E}_T) \right] \times \exp \left[ i\vec{A}_Q(\vec{r}, \vec{r}')(\vec{r} - \vec{r}' - D\tau \text{Re} \tilde{F}_Q(\vec{r}')) - i\tau \text{Im} \text{E}_L(\vec{r}) \right].
\]  

(17)

III. THE DESCRIPTION OF THE SIMULATION ALGORITHM

1) Let us to have the \( N_c \) of the initial configurations \( \vec{r} \), i.e. a set of initial systems in which the electron position has random and uniform distribution. One can choose the configurations of \( \vec{r} \) and with the distribution function \( f(\vec{r}, 0) = |\Psi_T(\vec{r})|^2 \), because at \( t = 0 \) (3) is real.

The choice of the boundary condition depends on the problem. For the one electron in the external magnetic field a boundary is periodic, i.e., if electron cross a boundary from one side of simulated cell, it enters into cell from opposite side. For example, for the system of bosons in 2D parabolic well and in external magnetic field the boundaries must be free because parabolic well determines itself the boundary of space distribution of the particles.
2) It is performed the quantum drift and diffusion of the particle from \( k \)-th configuration, for example, in accordance with the formula

\[
\vec{r}_k = \vec{r}_{k}^\prime + D\tau \text{Re} \bar{F}_Q(\vec{r}_k^\prime) + \chi.
\] (18)

Here \( \chi \) - is a gaussian random number having mean value zero and dispersion \( 2\sqrt{D\tau} \).

3) The transition into new space point in this configuration is accepted with probability

\[
P(\vec{r}^\prime \to \vec{r}, \tau) \equiv \min(1, W(\vec{r}, \vec{r}^\prime)),
\] (19)

where

\[
W(\vec{r}, \vec{r}^\prime) = \frac{|\Psi^T(\vec{r})|^2 G_1(\vec{r}^\prime, \vec{r}; \tau)}{|\Psi^T(\vec{r}^\prime)|^2 G_1(\vec{r}, \vec{r}^\prime; \tau)}.
\]

Here \( G_1(\vec{r}, \vec{r}^\prime; \tau) \) is given by (15).

If transition of electron is accepted then in accordance with (17) it has new phase, if no then electron keep his old phase.

4) After changing of the electron position from \( k \)-th configuration into new space point, are calculated \( \text{Re} E_L(\vec{r}_k) \), \( \text{Im} E_L(\vec{r}_k) \) and other quantities of interest.

5) By using of the first exponential factor in (17) it is calculated the multiplicity \( M_k \) (the branching probability) for the configuration \( k \) accordingly

\[
M_k = \exp[-\tau(\text{Re} E_L(\vec{r}_k) - E_T)]
\] (20)

If \( M_k \) is not integer, we add an uniformly distributed random number between 0 and 1 to it and take \( M_k \) equal to nearest integer.

6) If \( M_k \neq 0 \) then \( M_k \) copies of new \( k \)-th configuration place in the list of the new \( N \) configurations that one is the initial at the next step \( \tau \) of integration of the diffusion equation. If \( M_k = 0 \) then there is no \( k \)-th configuration in the list of new \( N \) configuration for the next time step \( \tau \).

All quantities of interest as \( \text{Re} E_L(\vec{r}_k) \), \( \text{Im} E_L(\vec{r}_k) \) and so on are multiplied by the factor \( M_k \) for the calculating of mean values of these ones.

7) It is repeated steps 2)-6) of algorithm until all \( N_c \) configurations will not overlooked and electrons on these configurations will not simulated on the displacement and the having a new phase.

8) It are calculated a mean energy and others mean quantities on the \( N \) number of configurations, got in the point 6) of algorithm, at this time step \( \tau \) in accordance with formula (23) (see below, where it is necessary to change \( M \) by \( N \) and to take into account that the phase of electron is equal to the phase of the Green function \( \alpha_G \) and at the calculation of a mean energy that the energy \( E_L(\vec{r}_k) \) has a real \( \text{Re} E_L(\vec{r}_k) \) and an imaginary \( \text{Im} E_L(\vec{r}_k) \) parts.)

9) It are repeated steps 1)-8) an integer number of time steps \( \tau \) of integration of diffusion equation. After that it are determined the mean values of quantities \( \text{Re} \overline{E}, \text{Im} \overline{E} \) and others on this integer number too. It is redetermined the new value of \( E_T \) in accordance with \( (E_T)_{\text{new}} = [(E_T)_{\text{old}} + \text{Re} E]/2 \) accordingly with assumption that \( \text{Im} E \ll \text{Re} E \). An integer number of time steps \( \tau \) represents an one time block \( \Delta t \).

10) Every time block \( \Delta t \) has the \( N_c \) of initial configurations. The \( N_c \) of initial configurations are filled randomly by configurations of just ended time block. The random choice
of configurations consists of two steps:
a) a random choice of a number of time step $\tau$ in every time block $\Delta t$;
b) a random choice of configuration from $N$ configurations at this fixed time step $\tau$.
In this manner filled list of $N_c$ configurations will be initial one for the next time block $\Delta t$.

11) The repeating of big number of time blocks $\Delta t$ decreases essentially a correlation between configurations in neighbor time blocks and provides a right calculation of mean quantities.

Let us discuss in detail the calculation of the mean quantities. In general case (see [4,5]) the mean value of the some quantity $F(\vec{R}(t))$ with the complex distribution function $f(\vec{R}, t)$ at the time $t$ of the running process is

$$< F(t) > = \frac{\sum_{i=1}^{M} \exp[i\alpha(\vec{R}_i(t))] F(\vec{R}_i(t))}{\sum_{i=1}^{M} \exp[i\alpha(\vec{R}_i(t))]}. \quad (21)$$

Here $\vec{R}_i(t)$ is the coordinates $\vec{r}_1, \vec{r}_2, ..., \vec{r}_N$, $N$ - number of particles of system, $M$ - number of configurations at time point $t$. The particles coordinates $\vec{r}_1, \vec{r}_2, ..., \vec{r}_N$ in (21) are weighted with probability $|f(\vec{R}_i(t), t + \tau)|$, and the quantity $\alpha(\vec{R}_i(t))$ is a phase of the distribution function $f(\vec{R}_i, t)$.

As it is clear from the integral expression (14), the phase of the distribution function at consequent time moment determines through the phase of the Green function and the one of the distribution function at previous time moment. So, we have:

$$\alpha(\vec{R}_i(t + \tau)) = \alpha_G(\vec{R}_i(t + \tau), \vec{R}_j(t)) + \alpha(\vec{R}_j(t)). \quad (22)$$

Here $\alpha_G(\vec{R}_i(t + \tau), \vec{R}_j(t))$ is the phase of the Green function (14) (with account of (17)), and index $j$ shows that $\vec{R}_j$ taken from configurations at time moment $t$. By substituting (22) into (21) we have

$$< F(t + \tau) > = \frac{\sum_{i=1}^{M} e^{i\alpha_G(\vec{R}_i(t+\tau), \vec{R}_j(t))} F(\vec{R}_i(t + \tau))}{\sum_{i=1}^{M} e^{i\alpha_G(\vec{R}_i(t+\tau), \vec{R}_j(t))}} = \frac{\sum_{i=1}^{M} e^{i\alpha_G(\vec{R}_i(t+\tau), \vec{R}_j(t))} F(\vec{R}_i(t + \tau))}{\sum_{i=1}^{M} e^{i\alpha_G(\vec{R}_i(t+\tau), \vec{R}_j(t))}}. \quad (23)$$

The mean quantity $< F(t+\tau) >$ in (23) is determined only by the phase $\alpha_G$ of the Green function, because all quantities under sum in the numerator and the denominator are weighted with the probability $|f(\vec{R}_i(t + \tau), t + \tau)|$, i.e. at time moment $t + \tau$ with new configurations $\vec{R}_i(t + \tau)$.

Next, let us consider the expression for the phase of the Green function (17). We have
\[ \alpha_G = \bar{A}_Q(\vec{r}, \vec{r}')(\vec{r} - \vec{r}') - D\tau Re \bar{F}_Q(\vec{r}') - \tau Im E_L(\vec{r}). \]  

From (8) and (9) it is seen that the expressions \( Re \bar{F}_Q(\vec{r}') \) and \( E_L(\vec{r}) \) have no \( \tau \) dependence. From (18) one can show that at \( \tau \to 0 |\vec{r}| \to |\vec{r}'| \) as \( \tau^{1/2} \), because \( \chi \) has \( \tau^{1/2} \) dependence. At the same limit \( \bar{A}_Q(\vec{r}, \vec{r}') \to \bar{A}_Q(\vec{r}', \vec{r}') \) (see the expression (13) for the \( \bar{A}_Q(\vec{r}, \vec{r}') \)), i.e. there is no \( \tau \) dependence, also. So, we have \( \alpha_G \sim \tau^{1/2} \).

IV. ANALYTICAL EXPRESSIONS FOR THE ALGORITHM QUANTITIES

We take trial wave function of electron in the form:

\[ \Psi^*_T(\vec{r}) = \frac{C}{a_H} \left( \frac{\alpha x + i\beta y}{a_H} \right)^m \exp \left( -\gamma \frac{(x^2 + y^2)}{4a_H^2} \right). \]  

Here \( C \) is normalization constant; \( a_H = (\hbar / M w_H)^{1/2} \) is magnetic length, where \( w_H = |e| H / M c \) - cyclotron frequency; \( x \) and \( y \) the two spatial components of electron’s coordinate; \( m \) is orbital quantum number; \( \alpha, \beta \) and \( \gamma \) are arbitrary numerical constants.

At \( \alpha = \beta = \gamma = 1 \) (25) gives an exact analytical expression of electron ground state wave function. We deform exact electron wave function, i.e. take as the input \( \alpha, \beta \) and \( \gamma \) not equal to 1, and then study a relaxation of the trial function to the exact one via simulation process.

As length unit we take \( a_H \), as energy unit - \( \hbar w_H / 2 \), and as time unit - \( 2 / \hbar w_H \).

By substituting of the wave function (25) into (9), we have

\[ Re \bar{F}_Q(\vec{r}) = \frac{1}{\alpha^2 x^2 + \beta^2 y^2} \left[ i(2m\alpha^2 x - \gamma x^2 \alpha^2 - \gamma \beta^2 xy) + \right. \]
\[ + ji(2m\beta^2 y - \gamma \beta^2 x^2 - \gamma \beta^2 y^2) \left. \right], \]
\[ Im \bar{F}_Q(\vec{r}) = \frac{2m\alpha \beta}{\alpha^2 x^2 + \beta^2 y^2} [ - \gamma y + x \gamma ] . \]  

Here and below \( \vec{i} \) and \( \vec{j} \) the unit vectors in \( x \) and \( y \) directions correspondingly.

For the \( \bar{A}_Q(\vec{r}, \vec{r}') \) (13) gives

\[ \bar{A}_Q(\vec{r}, \vec{r}') = \frac{1}{\alpha^2 x^2 + \beta^2 y^2} \left[ i(-2m\alpha \beta y' + y(\alpha^2 x^2 + \beta^2 y^2)) + \right. \]
\[ + ji(2m\alpha \beta x' - x(\alpha^2 x^2 + \beta^2 y^2)) \left. \right]. \]  

The wave function (25) and eq. (7) (here \( V(r) = 0 \)) gives

\[ Re E_L(\vec{r}) = Re E^{(1)}_L(\vec{r}) + Re E^{(2)}_L(\vec{r}), \]  

where

\[ Re E^{(1)}_L(\vec{r}) = \gamma (m + 1) + (x^2 + y^2) \left[ 1 - \frac{x^2}{4} - \frac{m \alpha \beta}{\alpha^2 x^2 + \beta^2 y^2} \right], \]
\[ Re E^{(2)}_L(\vec{r}) = \frac{m(m - 1)(\beta^2 - \alpha^2)(\alpha^2 x^2 - \beta^2 y^2)}{(\alpha^2 x^2 + \beta^2 y^2)^2}; \]
and also

\[ ImE_L(\vec{r}) = \frac{mxy(\beta^2 - \alpha^2)}{\alpha^2x^2 + \beta^2y^2} \left( 1 - \frac{(m - 1)\alpha\beta}{\alpha^2x^2 + \beta^2y^2} \right). \] (29)

By using formulas (26)-(29), it is easy to write the analytical expressions for other quantities.

V. THE DISCUSSION OF THE SIMULATION RESULTS.

The simulation of the ground state of one electron in magnetic field is a simple problem and was performed on the usual personal computer.

Figs.1-4 are presented the result of the calculations of the real \( ReE \) and imaginary part \( ImE \) of the ground state energy for different orbital quantum numbers \( m \) and different starting trial functions defined by the parameters \( \alpha, \beta, \gamma \) as a functions of the number \( T \) of time blocks \( \Delta t \) (see point 9) of algorithm) (a full number of time blocks are chosen ten in each running).

In every time block \( \Delta t \) initial number of configurations \( N_c \) was chosen equal 1000 and number of time steps \( \tau \) equal 100. We everywhere suggested \( \tau \) equal 0.01. Typical mean number of the population number \( N \) (see point 6) of the algorithm) was around 2500 at initial blocks \( \Delta t \) and around 1000 at final blocks \( \Delta t \).

Fig.1 present \( ReE \) and Fig.2 present 1000 \( ImE \) for \( m = 0, 4, 8 \) with parameters \( \alpha = \beta = \gamma = 1 \) as a functions of \( T \).

It is seen from Figs.1,2 \( ReE \) is approaching fast to the exact value of ground state energy and reproduce very well an orbital quantum number \( m \) degeneracy, while the imaginary part of energy \( ImE \sim 10^{-3} ReE \).

The additional calculations shows, that the increasing of the statistics, i.e. the increasing of \( N_c \) and a number of steps \( \tau \) in each block \( \Delta t \), decreases \( ImE \) more.

The Figs.3,4 represent the \( ReE \) and \( ImE \) for the different choices of the \( m = 0 \) trial function. From these Figs. we can see that the deformation of wave function in some limits does not alter of the relaxation of the running process to exact final state.

Fig.5 present the initial uniform spatial distribution of electron before simulation, while Fig.6 - the final spatial distribution of electron after simulation for \( m = 13 \).

In this simulation the trial wave function \( \Psi_T^* \) is taken with parameters \( \alpha = \beta = \gamma = 1, m = 13 \) and \( N_c = 500 \). Since the trial function coincide with exact wave function we have to expect fast convergence of initial distribution to exact one if the algorithm is correct. In accordance with analytical solution (see [11]) the spatial distribution for one electron in this case must have a ring like form with mean radius \((2m+1)^{1/2}\) and wide 1 (in magnetic length \( a_H \) units). As it is seen from Fig.6, there is an excellent agreement between simulation and analytical results.

We suppose also further tests of the method by simulations of anyons [12].
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FIG. 1. The real part of the ground state energy $\text{Re}E$ as a functions of the number $T$ of time blocks $\Delta t$ (see point 9) of algorithm) for different orbital quantum numbers $m$. Solid line - $m = 0$, marked solid line - $m = 4$, dashed line - $m = 8$.

FIG. 2. The same as Fig.1 but for $1000\text{Im}E$. 
FIG. 3. The real part of the ground state energy $ReE$ for different starting parameters $\alpha, \beta, \gamma$ and the same $m = 0$ as a function of the number $T$ of time blocks $\Delta t$. Solid line - $\alpha = \beta = \gamma = 1$, marked solid line - $\alpha = 1.4, \beta = \gamma = 1$, dashed line - $\alpha = \beta = 1, \gamma = 0.9$.

FIG. 4. The same as Fig.3 but for $1000ImE$. 
FIG. 5. Initial spatial distribution of the electron.

FIG. 6. The result of the simulation: final spatial distribution of the electron in uniform magnetic field at $m = 13$. The length unity is $a_H$, initial number of points $N_c = 500$. 