On universality of the coupling of neutrinos to $Z$

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Abstract

We employ an effective Lagrangian approach and use LEP data to place severe bounds on universality violations of the couplings of $\nu_e$, $\nu_\mu$, and $\nu_\tau$ to the $Z$ boson. Our results justify the assumption of universality in these couplings that is usually made, as for example in the analysis of solar neutrinos detected at SNO.
In the analysis of the observations on solar neutrinos at the Sudbury Neutrino Observatory (SNO) \cite{1} one makes the assumption that neutrinos interact with nucleons and electrons according to the predictions of the electroweak Standard Model (SM). Of course, this assumption ought to be confirmed by experiment. Although many of the basic neutrino properties predicted by the SM and used in the SNO analysis have been tested in accelerator experiments, as we will argue this is not exactly true for all of them.

A crucial assumption in the SNO analysis involves the coupling of neutrinos to the neutral current. At SNO, neutral current interactions induce the process $\nu_i + d \rightarrow p + n + \nu_i$, where $i$ can be $e, \mu, \tau$, and $\nu_i + e^- \rightarrow \nu_i + e^-$ with $i = \mu, \tau$, and they also participate in the elastic scattering $\nu_e + e^- \rightarrow \nu_e + e^-$. To deduce the actual fluxes of neutrinos reaching the Earth it is assumed that $\nu_e, \nu_\mu, \text{and} \nu_\tau$ couple with the same strength to $Z$, i.e., that universality in the coupling of neutrinos to the $Z$ boson holds. We will concentrate in the observational evidence for this hypothesis.

Let us first make a very rough estimation of the precision level of SNO. Take for example the total flux measured with the neutral current reaction at SNO \cite{1},

$$\phi_{NC} = 5.09^{+0.63}_{-0.61} \times 10^6 \text{ cm}^{-2} \text{s}^{-1}$$

where we have added their statistical and systematic errors in quadrature. The result has a relative error of about 12%, and gives us an idea of the SNO precision. These are their very first results and, of course, with more statistics and refinements, the relative error is going to decrease in the near future.

To set the stage for our discussion, let us write the Lagrangian that in the SM describes the interaction of matter with the $Z$ boson,

$$\mathcal{L}_{NC} = -\frac{g}{\cos \theta_w} J^{NC}_\alpha Z^\alpha$$

with $g$ the $SU(2)$ coupling and $\theta_w$ the weak mixing angle. The neutral current $J^{NC}$ in the SM is given by

$$J^{NC}_\alpha (\text{SM}) = \frac{1}{2} \sum_i [1 + r_i] \bar{\nu}_i \gamma_\alpha \nu_i + \ldots$$

where the dots refer to other particles and the sum is over $i = e, \mu, \tau$. ($\nu_i$ is in fact $\nu_{Li}$; to simplify the notation we omit the left-handed $L$ subscript). In (3), $r_e, r_\mu, r_\tau$ are the usual radiative corrections arising in the SM. At this radiative level, there are universality
violations coming from vertex corrections, but they are small. For instance, we have

\[ r_\tau - r_e \approx \frac{\alpha}{4\pi} \log \frac{m_\tau^2}{m_e^2} \approx 0.009 \]  

(4)

and even smaller for \( r_\tau - r_\mu \) and \( r_\mu - r_e \) (see for example Ref.[2]). Certainly, this amount of universality violation is not a concern for the SNO analysis. In our analysis we can also ignore it, since we will end up with upper bounds on universality violations that are larger than (4).

In order to check the universality hypothesis in the neutrino coupling to \( Z \), we write the modified neutral current as

\[ J_{\text{NC}}^\alpha = \frac{1}{2} \sum_i [1 + \Delta_i] \bar{\nu}_i \gamma_\alpha \nu_i + \ldots \]  

(5)

The parameters \( \Delta_e, \Delta_\mu, \Delta_\tau \) are possible deviations coming from physics beyond the SM. We will constrain these parameters using experiment.

Data from LEP constitutes a very precise test for the SM. The couplings of neutrinos to \( Z \) are constrained by the invisible \( Z \) width, or equivalently, in the determination of the number of neutrinos \( N_\nu \). A combined fit to all LEP data gives

\[ N_\nu = 2.994 \pm 0.012 \]  

(6)

that we take from the 2001 update of the PDG [3]. Each \( \nu_i \) contributes

\[ (1 + \Delta_i)^2 \approx 1 + 2\Delta_i \]  

(7)

to \( N_\nu \). Thus, the result (6) leads to the relation

\[ |\Delta_e + \Delta_\mu + \Delta_\tau| \leq 0.009 \]  

(8)

To be conservative, we have taken the maximal deviation from \( N_\nu = 3 \) in (6) to put the bound (8). Also, here and in the following we work at first order in \( \Delta_i \).

If there is new physics that alter the \( Z\nu\nu \) coupling but respect universality, i.e., \( \Delta_e = \Delta_\mu = \Delta_\tau \), then (8) implies that each individual \( \Delta_i \) must be very small. However, if universality is violated then we may have cancellations in the sum (8) and, actually, there is no strict bound on individual \( \Delta_i \). Such possible cancellations cannot be banned from first principles. The purpose of the present note is to constrain the breaking of universality in \( Z\nu\nu \) interactions.
The direct way to test universality in the neutrino coupling to the $Z$ boson is through the analysis of the scattering $\nu_i + e^- \rightarrow \nu_i + e^-$. Available data on $\nu_\mu + e^- \rightarrow \nu_\mu + e^-$, together with LEP results, allow to place the limit [4]

$$|\Delta_\mu| \leq 0.037$$

(9)

However, the $\nu_e\nu_e Z$ coupling is known at a much worse level. We have the experimental limit on universality violation [5]

$$0.13 \leq \Delta_\mu - \Delta_e \leq 0.20$$

(10)

and of course no limits involving $\Delta_\tau$. At the view of the SNO precision, this limit (and absence of limit for the $\tau$ neutrino) is too loose to be useful.

We will now show that we can improve the limits on universality working with effective Lagrangians. The key point is the following. Deviations from the SM can be treated by using effective Lagrangians. The general idea of the effective Lagrangian approach is that theories beyond the SM, emerging at some characteristic energy scale $\Lambda$, have effects at low energies $E \leq G_F^{-1/2}$, and these effects can be taken into account by considering a Lagrangian that extends the SM Lagrangian, $\mathcal{L}_{\text{SM}}$:

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{eff}}.$$  

(11)

The effective Lagrangian $\mathcal{L}_{\text{eff}}$ contains operators of increasing dimension that are built with the SM fields including the scalar sector, and is organized as an expansion in powers of $(1/\Lambda)$.

The success of the electroweak SM at the level of quantum corrections can be considered as a check of the gauge symmetry properties of the model. To preserve the consistency of the low energy theory, with a Lagrangian given by (11), we will assume that $\mathcal{L}_{\text{eff}}$ is $SU(2) \otimes U(1)$ gauge invariant. Some of the problems that originate when dealing with non-gauge invariant interactions have been discussed in [6]. The gauge-invariant operators that dominate at low energies have dimension six and have been listed in [7].

There are two classes of dimension-six operators that may originate violations of universality in the neutrino sector of the neutral current:

$$A_i = i \left[ \Phi^\dagger D_\alpha \Phi \right] \left[ \bar{L}_i \gamma^\alpha L_i \right]$$

(12)

$$B_i = i \left[ \Phi^\dagger \left( D_\alpha \bar{\tau} + \bar{\tau} D_\alpha \right) \Phi \right] \cdot \left[ \bar{L}_i \gamma^\alpha \tau L_i \right]$$

(13)
Here $\Phi$ is the Higgs field and $L_i$ is the lepton isodoublet

$$L_i = \begin{pmatrix} l_i \\ \nu_i \end{pmatrix}_L$$

(14)

where now $l_i$ are the charged leptons, and the subscript $L$ is for left-handed. The operators contain the covariant derivative,

$$D_\alpha = \partial_\alpha + ig \frac{\vec{\tau}}{2} \cdot \vec{W}_\alpha + ig' \frac{Y}{2} B_\alpha$$

(15)

with the gauge bosons $\vec{W}, B$, the gauge couplings $g, g'$, and the Pauli matrices $\vec{\tau}$ and the hypercharge $Y$.

The effective Lagrangian relevant for our purposes can now be written as

$$\mathcal{L}_{\text{eff}} = \sum_i \left( \frac{\alpha_i}{\Lambda_i^2} A_i + \frac{\beta_i}{\Lambda_i'^2} B_i \right)$$

(16)

where $\Lambda_i, \Lambda_i'$ are high-energy scales and $\alpha_i, \beta_i$ are unknown strength coefficients accompanying the operators. Below the scale of spontaneous symmetry breaking the effective Lagrangian (16) induces contributions of the type shown in (5). Substituting

$$\Phi \rightarrow \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

(17)

with $v^2 = 1/(\sqrt{2} G_F) \simeq (246 \text{ GeV})^2$, in (16), we get

$$\Delta_i = -a_i + b_i$$

(18)

corresponding to the contributions of the two operators,

$$a_i = \alpha_i \frac{v^2}{2 \Lambda_i^2}$$

(19)

$$b_i = \beta_i \frac{v^2}{\Lambda_i'^2}$$

(20)

It is clear that unless the combinations $-a_i + b_i$, for $i = e, \mu, \tau$ are equal, we will have universality violations in the coupling of neutrinos to the Z boson.

The operators in the effective Lagrangian (16) have other effects at low energy. They contribute to the couplings of the $Z$ boson to the charged leptons $l_i$. Indeed we find,

$$J^{NC}_{\alpha} = \sum_i \left[ -\frac{1}{2} + \sin^2 \theta_w \left( \frac{a_i}{2} - \frac{b_i}{2} \right) \right] \bar{l}_i \gamma_\alpha l_i$$

(21)

$$+ \sin^2 \theta_w \sum_i \bar{l}_i R \gamma_\alpha l_i R + \ldots$$

(22)
where the dots indicate other particles than charged leptons. We see that the couplings to right-handed charged leptons $l_{iR}$ are not modified.

Also, the charged current coupling to the charged $W$ gets a contribution,

$$\mathcal{L}_{\text{CC}} = -\frac{g}{\sqrt{2}} J_{\alpha}^{\text{CC}} W^{+\alpha} + \text{h.c.} \quad (23)$$

$$J_{\alpha}^{\text{CC}} = \sum_i [1 + b_i] \bar{\nu}_i \gamma_\alpha l_{iL} + \ldots \quad (24)$$

where again the dots stand for the part involving other SM fields. In the charged current sector there are also violations of universality coming from radiative corrections in the SM framework, but numerically they have at most the value shown in (2) and we will neglect them.

Our purpose is to constrain universality violations in the couplings $\nu\nu Z$. We can reach our objective by considering the constraints on the non-standard contributions to the coupling of $Z$ to charged leptons (22) and on the similar contributions in the charged current sector (24). The experimental information we need is taken from the 2001 PDG update that uses LEP data [3].

For example, in our scheme we have

$$\frac{\Gamma(W \to \tau\nu)}{\Gamma(W \to e\nu)} \simeq 1 + 2(b_\tau - b_e) \quad (25)$$

where we work at first order in $b_i$ and neglect lepton masses in front of the $W$ mass. The experimental ratio

$$\frac{\Gamma(W \to \tau\nu)}{\Gamma(W \to e\nu)} = 1.002 \pm 0.029 \quad (26)$$

leads to the bound

$$2|b_\tau - b_e| \leq 0.031 \quad (27)$$

where again we conservatively use the maximal deviation from 1 in (26). We also consider

$$\frac{\Gamma(Z \to \tau^+\tau^-)}{\Gamma(Z \to e^+e^-)} \simeq 1 + \frac{1 - 2s_w^2}{8s_w^4 - 4s_w^2 + 1} 2(a_\tau + b_\tau - a_e - b_e) \quad (28)$$

($s_w = \sin \theta_w$) and use [3]

$$\frac{\Gamma(Z \to \tau^+\tau^-)}{\Gamma(Z \to e^+e^-)} = 1.0020 \pm 0.0030 \quad (29)$$

which leads to

$$|a_\tau + b_\tau - a_e - b_e| \leq 0.0019 \quad (30)$$
The inequality

$$|\Delta_{\tau} - \Delta_{e}| = | - a_{\tau} + b_{\tau} + a_e - b_e|$$

$$\leq |a_{\tau} + b_{\tau} - a_e - b_e| + 2|b_{\tau} - b_e|$$  \hspace{1cm} (31) (32)

allows, from (27) and (30), to get

$$|\Delta_{\tau} - \Delta_{e}| \leq 0.033$$  \hspace{1cm} (33)

This is our main result concerning the limit on universality violation for $\nu_e$ and $\nu_{\tau}$. We can do a totally parallel exercise for $\nu_e$ and $\nu_\mu$, with the result

$$|\Delta_{\mu} - \Delta_{e}| \leq 0.040$$  \hspace{1cm} (34)

We finally would like to mention that another assumption of SNO is the absence of neutrino flavor changing neutral currents (FCNC). Such exotic interactions would be a contribution to $J^{NC}$ of the form

$$\delta J^{NC}_\alpha = \Delta_{e\mu} \bar{\nu}_e \gamma_\alpha \nu_\mu + \text{h.c.}$$  \hspace{1cm} (35)

and similar for $\nu_e \nu_{\tau}$ and $\nu_\mu \nu_{\tau}$. The contributions of these FCNC parameters to $N_\nu$ have no interference, like the universality violations, see (2). It follows that now cancellations are no longer possible. We can assume non-zero $\Delta_{e\mu}$, $\Delta_{e\tau}$, and $\Delta_{\mu\tau}$ and employ the experimental limit on $N_\nu$ (6) to infer

$$\left( (\Delta_{e\mu})^2 + (\Delta_{e\tau})^2 + (\Delta_{\mu\tau})^2 \right)^{1/2} \leq 0.095$$  \hspace{1cm} (36)

In conclusion, we have placed severe constraints to universality violations of the couplings of neutrinos to $Z$. We have introduced the parameters $\Delta_i$ in the neutral current expression (3) and showed which operators in an effective Lagrangian approach may lead to universality breaking. We have constrained the effects of this Lagrangian in the sector involving charged leptons, and these constrains have been used to reach our numerical results (33) and (34). We have shown that universality holds at the level of 4% in the current that couples to $Z$. Our results can be interesting in all the analysis making the assumption of $\nu\nu Z$ universality, like in the solar neutrino experiment at SNO.
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