Learning activities using worksheet characterized by the recognition of mathematical symbols

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Abstract. One of the purposes of learning mathematics in college is to improve the ability of critical thinking and problem-solving. Before solving the problems, the college students need to recognize the problems not excepted the mathematics symbols written in the problem. Therefore it is important for the problem solver to aware and recognizes the meaning of mathematics symbols, called recognition of mathematical symbols. This educational design research aims to describe the development of learning activities using a worksheet to recognize the mathematical symbol. The design research phase consists of four successive phases: knowledge, designing, experimenting and analyzing. Findings reveal that: College students can distinguish between the form of functions and the value of functions; College students transform $\frac{\partial f(x,y)}{\partial y}$ to $f_{y}(x,y)$ when solving problems; College students can read $\frac{\partial^2 z}{\partial x^2}$ symbol but need more time to interpret that symbol rather than $f_{xx}(x,y)$ symbol. At the end of the learning practice, college students that doing error to recognize mathematics symbols are less than at the first teaching practice. The college students can recognize mathematical symbols well and then solve the problem correctly using correct mathematics symbols.

Keywords: recognition of mathematics symbols, using worksheets, learning mathematics.

1. Introduction

Mathematics is one of the learning subjects that connected with another subject such as physics, science, civil and others. Therefore, mathematics is learned and taught even in college and civil college students. To solve mathematics problems, students must understand the problem and the mathematical symbol. The ability to understand the meaning of the mathematical symbol and be aware of the mathematical symbol is called recognition of mathematical symbols [1].

The mathematical symbol is important to share or communicate the idea to others while solving problems. The symbols itself, form the foundation to communicate the idea [2]. Understanding the mathematical symbols also describe the student’s understanding of the mathematics problem. It is seen that the failure of understanding the mathematical symbol may considerably hinder the formation, understanding, and communication of concept [3]. Symbol as a cognitive function helps a person to resolve the subjectively difficult situation by modeling the problem for practical application [4].

Knowing that there is a mathematical symbol is not enough. Students have to aware of the meaning of that mathematical symbols and correctly use that mathematical symbol in right place. It is called the recognition of mathematical symbols. But, most students fail to understand or interpret the meaning of mathematical symbol due to read, pronounce and use them [3] not except for college students [2] [5].
Compared with the mathematical symbol at school, the mathematical symbol at university more symbolic, it is more subtle and requires flexibility from the reader [2]. It causes college students to fail to recognize the mathematical symbol. For example, when learning derivative symbols at school students recognize the derivative symbol of function \( y \) is written as \( \frac{dy}{dx} \), but in the university, they find the symbol \( \frac{\partial y}{\partial x} \). It was found that freshman of civil engineering college students, failed to determine the first derivative of a multivariable function. They did not know the symbol and not used the right symbols, therefore, they derivate in the wrong variable, and used the wrong symbol in solving problems.

The lecturer and researcher prepare the learning activity and worksheet that characterized by mathematical symbols to improve student’s ability to recognize mathematical symbols. This worksheet contains some questions about multivariable function before they solve the derivative of the multivariable function. Worksheet as a media was chosen because it can help the lecturer to manage the material, making students actively find the information or material they need, and provide an opportunity of conducting practices on the theoretical subject [6]. This worksheet working together with a learning activity that designs students to write what they had been prepared before class, discussion about the material, and solving related problems as a reflection activity.

The learning activities collaborate with worksheets consist of apperception, main activities, and closing. The apperception step contains activities that ask students to answer some questions about material that will be learned or about previous material. The apperception is important to gather, focused college student’s knowledge, and prepare college students for the new material [7]. The main activities were designed by group discussion activity after the lecturer gives a short explanation about the material that will be learned. The group discussion activities help college students to improve academic achievement, tolerance, and acceptance of diversity, and development of social skills [8] [9] [10]. After group discussion, college students solve some problems as a reflection test individually.

This paper aims to describe the learning activities while using a worksheet characterized by the recognition of mathematical symbols. The worksheet was used to guide college students to know the symbol, the meaning of symbols such that college students have better recognition of the mathematics symbols. Our chosen topic here is derivative for multivariable functions.

2. Method

This design research includes development studies that aim to develop a research-based solution for complex problems in educational practice [11]. The design research phase consists of four successive phases: knowledge, designing, experimenting and analyzing [12]. These four phases iterate to the next iteration consist of those four phases.

This research starts with teacher intention to correct students writing and reading of the mathematical symbols. The teacher found that a lot of college students did not recognize the mathematical symbol they had found. Most college students used the symbol that was used in school, then they did not solve the problem correctly because confused. Next, lecturer collaborates with the researcher to design the learning activities that using the worksheet to improve college student’s ability of recognition the mathematical symbols. Each part of the worksheet synchronous to the learning activities. We design three activities that students do: (a) apperception, which college students must solve some questions about previous material individually to prepare college students to the main material, (b) main activities, lecturer and college students discussed material that will learn globally, then college students solved group problems that were written in their worksheet, and (c) closing, college students solved some problems that had been learned. The questions in the group discussion’s task start from simple questions to the complicated questions.

Learning activities and the worksheet that was designed, implemented in three meetings in learning activities. All data will be collected from college student’s activities while using the worksheet, audio recording from the discussion of college students and lecturer while main activities and college student’s worksheets. We evaluate the data based on the student’s answers in their worksheets and the discussion in main activities. The data analyze based on the material and mathematical symbols that college
students had to know and college students had written in their worksheet. It was also used to design the content of the worksheet to the next meeting. Another data were college student’s responses about learning activities and the worksheet was collected by e-questioner. The subject of this research is 40 college students of the civil department that take the 2nd Calculus course.

3. Results and Discussion

The learning activities apply based on the lesson plan. The lesson plan and the worksheet had been validated and alleged valid with the validator, mastery of mathematical education that also tech calculus in the civil department. But this paper would not discuss the validity.

3.1. Learning Activities Using Worksheet

Generally, learning activities in each meeting contain three steps: apperception, main activities, and closing. Lecturer asked college students to have a seat with their group members, the first time they entered the class in each meeting. The apperception step asked college students to write what they had been read about the material or previous material. They wrote in the first pages of the worksheet (we called it “Rangkuman”) by answering the questions. When main activities, teachers together with college students discussed material that would be learned and discussed the problems in the Rangkuman section in the worksheet. After discussed material globally, college students asked to work in a group and solved some problems in the worksheet. While they having a discussion, the lecturer actively watched and guided college students to solve problems. At the end of the activities, they solved some problems in the last pages (we called it “Refleksi”) individually. The worksheet was used in each step of learning. The worksheet contains individual task and group task, therefore each college students have their worksheet. But, while group discussion, in main activities, students discuss to solve problems with their group that had been chosen and writes the group’s answer on their worksheet.

3.1.1. Functions of Two or More Variables (First Meeting).

The first material of the topic was identifying the multivariable function and determine d its domain. After gave an introduction about learning material and learning activities that would be done, the teacher gave the worksheet and asked college students to solve the Rangkuman section. They had around 30 minutes to solve the Resume task that contains 4 questions. Based on the observation, some college students that not read the material at home, could not answer the problem briefly and secretly saw their friends answer.

The main activities did well. While the lecturer explained the material, most students gave attention and asked to make sure that their answers in the Rangkuman section are right. The explanation was taken 30 minutes. The observer notes that the college students feel familiar with the domain of a function, so it caused the lecturer to explaine that sub-topic more than multivariable function. While group discussion activities, they enthusiastic solved problems and asked her/his friend to compare their answers. Some groups discussed the answer first, then write the same answer in each worksheet. While watched group discussion, it found that college students wrote the wrong symbol of domain function such that the lecturer explained once in that group.

The Refleksi section takes 30 minutes before class over. They solved six problems about multivariable function, the domain of the functions, and the value of the functions. Some college students solved it before the time was over. Some others tried to see her friend’s answers secretly but stopped when the lecturer remind them.

Overall, the learning activities did 90% as the lesson plan. Lecturer and observer found that college students were interested to write mathematical symbols correctly. They know the differences between \( f(x, y, z) \) and \( f(0, 1, -2) \). But some of them wrote not right about the set of domain functions. They know the answer (the domain function) but feel difficult to write it in mathematical symbols. The data interpret that 28 % of college students have bad recognition of mathematical symbols.

We agreed to design the worksheet for the next meeting with the first derivate of multivariable functions. We asked them to write the symbols of the first derivate, determine which symbol that
represents the first derivate of multivariable function, determine the value function, and determine the first derivate of a trigonometric function. We assumed they can solve the problems with the allocated time since they had been learned about chain rule in derivate one variable function and derivate trigonometric function.

3.1.2. First Derivative for Multivariable Functions (Second Meeting).

Based on the first meeting evaluation, we design the worksheet that contained 4 problems in the Rangkuman section, 6 problems to discuss in a group, and 3 problems in the Refleksi section. They were given 30 minutes to solve the problems in the Rangkuman section. Some students answer correctly and others did not. After explained the first derivative of multivariable function around 30 minutes, the lecturer asked them to answer the problem in group discussion. The lecturer stopped group discussion when 30 minutes before class is over.

While group discussing, the lecturer found that a college student can different the symbol of derivative multivariable and the symbol of one variable. Some college students confused when they should determine \( f_x(1, -1) \), let \( f_x(x, y) = 2 \). It was shown that students did not have a good understanding of value function. Based on the observation, most of them difficult to find the derivate of function that forms as \( f(x, y) = (2x^2 - 3x)^2 \). They forgot and used the chain rule, and some of them doubtful to use it. This phenomenon makes the lecturer explain the material (gave example case) about chain rule in front of the class. The observer found that students more enjoy when they found the \( f_y(x, y) \) or \( f_y(x, y) \) symbols then \( \frac{\partial f(x,y)}{\partial x} \) or \( \frac{\partial y(x,y)}{\partial y} \), therefore they transform \( \frac{\partial f(x,y)}{\partial x} \) to \( f_y(x, y) \) when solving problems with \( \partial \) symbol.

The Refleksi section shown that 22% of college students have good recognition in mathematical symbols. Based on the data by the lecturer and observer, the next meeting is designed with easy functions to derivate. It is caused by most college students need more time to derivate multivariable function correctly. The problems must be designed from the easiest functions, so students accustomed to derivate multivariable function. We decided to avoid trigonometric function in the next meeting to reduce their stress caused most of them not friendly with trigonometric function.

3.1.3. Second Derivative for Multivariable Functions (Third Meeting).

Based on the data analysis and interpreting data collected in the second meeting, we designed 4 questions in the Rangkuman section, 5 questions to discuss in the group, and 2 questions as Refleksi section. The third meeting’s material is determining the second derivative of multivariable functions. The first problems asked college students to recognize the order of derivate the multivariable function based on the given symbol. We design 10 problems of it. Then the second question asked them to write the differences between first derivate and second derivate. The third until fifth questions asked them to find the second derivative of functions. The class activities conducted with 30 minutes Rangkuman section, 30 minutes Refleksi section, and the rest time used to explanation and group discussion.

The learning activities were done well. College students and the lecturer can do all the learning steps. But, when college students were doing group discussions, they need more than one hour. They need more time to find the second derivative of the multivariable function, such that the course was finished more than the time should be.

The lecturer found that some college students were confused with the symbol written by \( \partial \). They need more time to recognize the \( \frac{\partial^2 z}{\partial x \partial y} \) or \( \frac{\partial^2 z}{\partial x^2} \). They can read the symbol by said “\( \frac{\partial^2 z}{\partial x^2} \) means \( z \) function is derivate 2 times concerning \( x \) variable”, but they still confused what should they do. The lecturer helps the students by giving the scaffolding question “if you derivate the function two times, you must find the first derivative of the function first. What is the first derivative function of the function?” This phenomenon showed that college students have no confidence in their answer such that they asked to make it sure. The observation notes that most students can follow the group discussion activities and has no problems to solve the questions.
3.2. Student’s Recognition of Mathematical Symbols

Each activity of the worksheet contains about recognizing mathematical symbols. Based on the lecturer and the observer, the college student’s ability to recognize mathematical symbols are better than before. Figure 1 below shows the percentage of students that have good recognition of mathematical symbols and has not. This data was obtained from the Refleksi section.

![Figure 1. Student’s Ability to Recognize Mathematical Symbols](image)

Most college students have a good ability to recognize mathematical symbols in the first meeting. They can determine the variables, notation of multivariable functions, and the domain of a function (73%). About 28% of the college students were wrong while making the multivariable functions, they wrote $f(x) = 2x + 3xy - 4$ for the example of two-variable functions. They also make a mistake to write the domain set of the function such as $D_f = \{(x+y) | x, y \in \mathbb{R}\}$. Some of them can determine the domain but cannot write the right symbols.

The material in the second meeting was continued from the first meeting. If students understand the variable and multivariable function’s notation, they can determine the first derivative of that function. About 22% of college students make mistakes in writing the first derivative symbol, they write $f'(x) = \frac{d}{dx}(x^3y) + \frac{d}{dx}(-3y^3x)$. This student written describes that this student doesn’t understand well about multivariable function. Some of them write $\frac{\partial z}{\partial x}(1,-2)$ as $-12 (2x^3 - 3xy)^3$ when finding the value of $\frac{\partial z}{\partial x}(x,y)$ at $x = 1$ and $y = -2$. At the third meeting, 20% of college students were still wrong to write $\partial$ symbols. They write $\partial$ as $a$. Some of them write $\frac{dz}{dxdy}$ as the second derivative of function $z$. Based on the data above, we can conclude that the student’s ability to recognize a mathematical symbol is better than the first meeting even though the difference is not significant.

3.3. Student’s Responses

Learning activities responses were gathered from e-questioner that college students fill after the third meeting. Figure 2 below shows the student’s responses to learning activities. Based on the data, we know that most students did not study at home or not prepared to learn the material. This phenomenon describes the student’s motivation and attractiveness to mathematics. The explanation after the Rangkuman section helps students to a better understanding of the material. College students agreed that group discussion helps them understand the material and exercises to solve some problems. But this group discussion not fully give good achievement to college students as [8] said. Among 24% of students, that think group discussion did not help them, 38% said that they were just copying another student’s worksheet. On the other hand, the Refleksi section effectually makes them responsible to understand the material.
Figure 2. Student’s Responses about Learning Activities

Figure 3 below shows the student’s responses about the worksheet that recognizes the mathematical symbol. They said that the worksheet can help them to be aware and understand the mathematical symbol, especially the derivative of a multivariable function. The students also recommend using the worksheet to the next material. Some of them that agreed to used worksheet said that it was more efficient than solving problems from a slide. But some of them that not agreed to used worksheet said that the task in the worksheet was too much and they need more time than the allocated time to finished by their self.

Figure 3. Student’s Responses about Worksheet

4. Conclusion
These design research findings based on the observer and lecturer reflection after evaluating all three meetings learning activities using the worksheet. The college student’s responses to the worksheet indicate that the teacher could implement the learning activities using the worksheet to the next material. The lecturer detected that: (a) College students can distinguish between the form of functions and the value of functions; (b) College students transform \( \frac{\partial f(x,y)}{\partial y} \) to \( f_y(x,y) \) when solving problems; and (c) College students can read \( \frac{\partial^2 z}{\partial x^2} \) symbol but need more time to interpret that symbol rather than \( f_{xx}(x,y) \) symbol. This finding shows the lecturer that recognition of mathematical symbols helps college students to understand the problems.

This research shows many phenomena of college student’s learning in this era. We found that they were still familiar and friendly with a worksheet based on paper. We would like to try to develop the electronic worksheet to make them more visual and animated effect. This research shows that the teacher
also has to aware of mathematical symbol, therefore the teacher can encourage their students in school to recognize the mathematical symbols. It would be interesting to design the research to encourage school students to recognize the mathematical symbols.

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