Sliding Mode Robust Control of a Wire-Driven Parallel Robot Based on HJI Theory and a Disturbance Observer

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ABSTRACT A sliding mode robust control law based on Hamilton-Jacobi Inequality (HJI) theory and a disturbance observer is proposed for a wire-driven parallel robot (WDPR) used in a wind-tunnel test. First, the wire-driven parallel robot is described, and its kinematics model established. Second, according to the uncertainty, external disturbance, and redundant drive of the system, the dynamic model of the end-effector and drive system, and the overall dynamic model of the system are established. Hamilton-Jacobi Inequality theory and the designed disturbance observer are applied to the designed sliding mode robust control law, and the anti-interference ability of the WDPR is verified. The stability of the closed-loop system is analyzed by Lyapunov’s second method, and the results show that the closed-loop system tends to be asymptotically stable. Finally, taking the dynamic trajectory simulation of compound motion and six-degree-of-freedom motion as examples, the designed sliding mode robust control law is verified by simulation, and the contrastive simulation analysis shows that the disturbance observer can effectively reduce the switching gain, thus effectively reducing chattering and improving the control accuracy of the system. The simulation results show that the designed sliding mode robust control law can effectively suppress the influence of external disturbance on the control error. The control input and the length of the wire change in a certain range. They also prove that the pose error is small, and the control accuracy is high. All of the foundings lay a theoretical foundation and technical support for the practical application of the prototype in a wind-tunnel test.

INDEX TERMS Wire-driven parallel robot, dynamic modeling, HJI theory, disturbance observer, sliding mode control, simulation analysis.

I. INTRODUCTION

As a new type of parallel robot, the wire-driven parallel robot (WDPR) uses a wire instead of the traditional connecting rod as the driving element to connect the moving platform (the end-effector) and the static platform [1]. The motion and force of the driver are thus converted to the motion and force of the end-effector in parallel with the wire as the medium. This combines the advantages of the parallel robot and the wire drive, and overcomes the defects of a small work space and complicated mechanical structure of the traditional parallel robot. Because of the advantages of simple structure, low cost, fast response speed, and good dynamic performance [2], as well as important research and application value, the wire-driven parallel robot has been applied to a wind-tunnel test [3], medical rehabilitation [4], [5], a lifting robot [6], a high-speed camera [7] and other fields [8].

Because the wire can only bear the unidirectional load and the system has the characteristics of redundant drive, the stability of the wire-driven parallel robot in high-speed and high-precision operation is a difficult problem to solve. At the same time, due to its characteristics of complexity, strong coupling, multi-input and multi-output, the choice of its control strategy is particularly important. Therefore, the design of the WDPR’s control method is highly challenging.
At present, experts and scholars have carried out studies on the control of the WDPRs, and made some achievements. However, due to the different application fields and work properties, the control methods are also different. Rogier de Rijk et al. proposed a sliding mode robust control strategy of multi-input and multi-output for the out-of-plane stability of the end-effector for a planar cable-driven parallel robot [9]. Ronghui Qi et al. proposed modeling and model predictive control for a hybrid cable-driven robot (HCDDR), and verified the correctness of the model and control strategy through control design, simulation, and experiment [10]. Xiaoqiang Tang et al. adopted a dual-thread PID force/position hybrid control method to control the cable tension of cable-driven parallel robots (CDPRs) in real-time, and verified the effectiveness of the proposed method through experiment and theoretical analysis [11]. In order to adapt to the inherent uncertainty of the system, H. D. Taghirad et al. proposed a control topology based on a cascade structure. The inner ring of the structure controls the cable force, and the outer ring tracks the precise position of the robot end-effector. In the design of the outer loop controller, a sliding mode robust controller based on the direct method of Lyapunov is proposed, and an experiment is carried out on a deployable suspension cable-driven robot. The experimental results show the controller’s effectiveness in the case of inherent uncertainty in the system [12]. Based on the established dynamic model of the cable drive unit, YUPENG ZOU et al. proposed a master dynamic control strategy and proved that the designed control strategy improved the loading precision and dynamic performance of the force servo system through experimental research [13]. Zane Zake et al. proposed a vision-based control method for a CDPR and discussed the stability analysis of the control scheme with uncertainty in the analysis model and test device [14]. For a CDPR, Weiwei Shang et al. combined cable-length space adaptive synchronization with work-space adaptive compensation to propose a new dual-space adaptive synchronization controller and verified the effectiveness of the designed controller through experiments [15]. Peter Racioppo et al. studied the control strategy of a cable-driven snake-like robot with coupled joints. The experimental results were verified on a fully integrated prototype and compared with the simulation results [16]. Jeremy Begey et al. proposed a standard computational torque substitution method using feedback linearization to control highly flexible CDPRs. The proposed control scheme was experimentally validated on a planar three-degree-of-freedom (3-DOF) CDPR and compared with a simple motion control law [17]. Tej Dallej et al. studied the modeling and visual control of a large dimension CDPR. In order to control the motion of the parallel robot, a visual servo control method based on position is proposed, and the position and pose of the moving platform are measured by vision. The simulation is carried out when the cable-driven parallel robot is larger than the prototype CoGiGiRo [18].

Based on the analysis of the above research results, it can be concluded that the proposed control schemes for the wire-driven parallel robots are different due to different application purposes. However, the above research results do not take into account the influence of system uncertainties and external disturbance on the control error. Therefore, in order to further improve the control effect of the WDPR in the wind-tunnel test, in this paper, a sliding mode robust control law based on Hamilton-Jacobi inequality (HJI) theory and a disturbance observer is proposed by referring to the design advantages of the above research results and combining with the work tasks and characteristics of the WDPR used in a wind-tunnel test. The innovation of this paper is to design a sliding mode robust control law for a WDPR combined with HJI theory and a disturbance observer. The stability of the closed-loop system is verified by stability analysis. This paper lays a theoretical foundation and provides technical support and data support for the practical application of the wire-driven parallel robot prototype in a wind-tunnel test.

The outline of the paper is as follows. Sections II and III present the structure principle, kinematics model, and dynamics model of the wire-driven parallel robot respectively. Section IV proposes the HJI theory and the design of the disturbance observer. In Section V, the design and analysis of the sliding mode robust control law are proposed based on the HJI theory and the disturbance observer. In Section VI, the simulation results of compound motion and 6-DOF motion are given, and the advantages of the designed control law are further verified through the simulation comparative analysis. Finally, Section VII concludes the paper.

II. DESCRIPTION OF THE WIRE-DRIVEN PARALLEL ROBOT
The research object of this paper is a wire-driven parallel robot driven by eight wires for a wind-tunnel test. The wire-driven parallel robot is mainly composed of a frame, an end-effector (a standard aircraft model-SDM), a towing wire, a drive system, a motion control system, a vision system, a control cabinet, and so on. The wire-driven parallel robot is a redundancy driven parallel robot that increases the stiffness of the mechanism, improves the flexibility of the end-effector, solves the problem of singular configuration, and makes the mechanism performance fault-tolerant. In this paper, the change of three translational motions and three rotations of the end-effector with six degrees of freedom can be realized by controlling the wire length variation. The physical image of the prototype is shown in Fig. 1.

The structural schematic diagram of the WDPR is shown in Fig. 2, which indicates the kinematic geometric relationship of the WDPR. Pxyz is the moving coordinate system, OXYZ is the static coordinate system, P is the mass center of the end-effector, Bi (i = 1~8) is the pulley hinge point, and \( P_i (i = 1~8) \) is the connection point of the end-effector. Meanwhile, wires 1 and 2 are symmetric, wires 3 and 4 are symmetric, wires 5 and 8 are symmetric, and wires 6 and 7 are symmetric. Let \( P_i = OB_i, P_i = \overrightarrow{O\mathbf{B}_i} \) be the connection point coordinates in the static coordinate system OXYZ, the coordinates of the connection point \( P_i(X_{P_i}, Y_{P_i}, Z_{P_i})\) of towing wires and aircraft
WDPR can be written as \( x \) origin \( P \) of the moving coordinate system \( Pxyz \) in the static coordinate system \( X \) and \( Z \) in the moving coordinate system to the static coordinate system.

In equation (1), the wire length vector \( L \) is the coordinate of point \( P \) in the moving coordinate system \( Pxyz \); and \( R \) is the rotation transformation matrix from the moving coordinate system to the static coordinate system.

According to the structure schematic diagram 2 and combined with Equation (1), the wire length vector \( L_i \) of the WDPR can be written as:

\[
P_i = X_P + R x_P \tag{1}
\]

In equation (1), \( X_P = (X_P, Y_P, Z_P)^T \) is the coordinate of the origin \( P \) of the moving coordinate system \( Pxyz \) in the static coordinate system \( OXYZ; x_P(x_{Pi}, y_{Pi}, z_{Pi})^T \) is the coordinate of point \( P_i \) in the moving coordinate system \( Pxyz \); and \( R \) is the rotation transformation matrix from the moving coordinate system to the static coordinate system.

Then the length of the \( i \)th wire can be expressed as:

\[
L_i = \sqrt{(X_P + R x_P - B_i)^T(X_P + R x_P - B_i)} \tag{2}
\]

In Equations (2) and (3), \( B_i(X_{Bi}, Y_{Bi}, Z_{Bi})^T \) is the coordinate of \( B_i \) point of each fixed pulley in the static coordinate system.

Set \( X = (X_P, Y_P, Z_P, \phi, \theta, \psi)^T \) as the pose of the end-effector; \( \dot{X}_w = (\dot{X}_P, \dot{Y}_P, \dot{Z}_P, \omega_X, \omega_Y, \omega_Z)^T \) as the motion velocity vector of the end-effector; and \( \Phi, \theta, \) and \( \psi \) as the roll angle, pitch angle, and yaw angle of the end-effector, respectively. The linear velocity vector is \( v = (\dot{X}_P, \dot{Y}_P, \dot{Z}_P)^T \), the angular velocity vector of the end-effector is \( \omega = (\omega_X, \omega_Y, \omega_Z)^T \), and the wire vector is \( L = (L_1, L_2, L_3, L_4, L_5, L_6, L_7, L_8)^T \), while, \( \dot{Q} = (\phi, \theta, \psi)^T \) is the angular velocity vector of the attitude angle. According to the principle of a rigid body rotating around a fixed point, it can be obtained as follows:

\[
\omega = \begin{bmatrix}
\omega_x \\
\omega_y \\
\omega_z
\end{bmatrix} = \begin{bmatrix}
cos \theta \cos \psi & -\sin \psi & 0 \\
\cos \theta \sin \psi & \cos \psi & 0 \\
-\sin \theta & 0 & 1
\end{bmatrix} \cdot \begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix} = H \dot{Q}
\tag{4}
\]

Let

\[
H = \begin{bmatrix}
cos \theta \cos \psi & -\sin \psi & 0 \\
\cos \theta \sin \psi & \cos \psi & 0 \\
-\sin \theta & 0 & 1
\end{bmatrix},
\]

the relationship between the motion velocity vector \( \dot{X}_w \) of the end-effector and the pose \( X \) of the end-effector can be obtained:

\[
\dot{X}_w = \begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} I_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & H \end{bmatrix} \begin{bmatrix} v \\ \dot{Q} \end{bmatrix} = G \dot{X}
\tag{5}
\]

\[
\dot{X}_w = \dot{G} \dot{X} + G \ddot{X}
\tag{6}
\]

where \( G \) is the transformation matrix based on the attitude angle.

Since \( \dot{L} = [\dot{L}_1 \dot{L}_2 \cdots \dot{L}_8]^T, u_i = L_i/L_i \) it can be concluded that:

\[
\dot{L}_i = J \dot{X}_w
\tag{7}
\]

According to the differential kinematics of the parallel robot, the kinematic model of the wire length vector can be obtained:

\[
\dot{L} = J G \ddot{X}
\tag{8}
\]

In Equations (7) and (8), \( J \) is the Jacobian matrix.

### III. SYSTEM DYNAMIC MODELING

Dynamic modeling plays an important role in the dynamic analysis and control method design of the WDPR. The accuracy of dynamic modeling directly affects the design of the control law and the performance analysis of dynamics. Therefore, considering the characteristics of system uncertainty, external disturbance, and redundant drive, the dynamic models of the end-effector, driving system and whole system, respectively, are established [19].

#### A. DYNAMIC MODELING OF THE END-EFFECTOR

The Newton-Euler method is used to establish the dynamic equation of the end-effector.

\[
M(X) \ddot{X} + N(X, \dot{X}) - w_g - w_e + \Delta(X, \dot{X}) + \tau_d = -J^T T
\tag{9}
\]
In Equation (9), \( M(X) \) is the inertia matrix of the end-effector, as shown in Equation (10); \( N(X, \dot{X}) \) is the nonlinear Coriolis centrifugal force matrix, as shown in Equation (11); \( w_g \) is the gravity vector of the end-effector and is expressed as \( w_g = (0, 0, mg, 0, 0, 0)^T \); \( w_e \) is the aerodynamic load on the end-effector and is expressed as \( w_e = \left[ f_e, \tau_e \right]^T \); \( \Delta(X, \dot{X}) \) is the uncertain part of modeling; \( \tau_d \) is the external disturbance; and \( T \) is the wire tension vector.

\[
M(X) = \begin{bmatrix}
(mI)^{3 \times 3} & 0_{3 \times 3} \\
0_{3 \times 3} & A_G G^{3 \times 3} H
\end{bmatrix}
\tag{10}
\]

\[
N(X, \dot{X}) = \begin{bmatrix}
0_{3 \times 1} \\
A_G H + (H \dot{Q}) \times (A_G H \dot{Q})
\end{bmatrix}
\tag{11}
\]

In Equations (10) and (11), \( m \) is the mass of the end-effector; \( A_G = RA_G^T R^T \), and \( A_G^* \) is the inertia matrix of the end-effector with respect to the center of gravity.

**B. DYNAMIC MODELING OF THE DRIVING SYSTEM**

The drive system of WDPR is composed of eight AC servo motors and servo drivers. The wire length is adjusted by a ball screw and slider. Therefore, the dynamic equation of the drive system is established based on the dynamic torque balance equation of AC servo motor, as shown in Equation (12).

\[
M_0 \ddot{\theta}_m + C_0 \dot{\theta}_m + \tau_l = \tau
\tag{12}
\]

where \( M_0 \) is the inertia matrix equivalent to the driver, \( C_0 \) is the viscous friction coefficient matrix equivalent to the driver, \( \theta_m \) is the rotation angle of the servo motor, \( \tau_l = \mu T \) is the load torque generated by the wire tension, \( \mu \) is the transmission coefficient of the ball screw, and \( \tau \) is the output torque vector of the driver.

**C. DYNAMIC MODELING OF THE WHOLE SYSTEM**

Based on Equations (9) and (12), the dynamic model of the whole system is constructed, as shown in Equation (13).

\[
(M(X) - \frac{1}{\mu^2} \cdot J^T M(0) JG) \ddot{X} + \frac{1}{\mu^2} (J^T M(0) G + J^T M(0) \dot{G}) \dot{X} + J^T C(0) \dot{J} \dot{X} + \Delta(X, \dot{X}) + \tau_d = -\frac{1}{\mu} \cdot J^T \tau \ c w_g c w_e - N(X, \dot{X})
\tag{13}
\]

In order to simplify the designed control law, it is necessary to simplify the whole dynamic equation. Let,

\[
H_1 = -\mu (M(X) - \frac{1}{\mu^2} \cdot J^T M(0) G)
\tag{14}
\]

\[
H_2 = \frac{1}{\mu} (J^T M(0) G + J^T M(0) \dot{G}) + J^T C(0) \dot{J} \dot{X}
\tag{15}
\]

\[
H_3 = -\mu w_g - \mu w_e + \mu N(X, \dot{X})
\tag{16}
\]

Then Equation (13) can be simplified as

\[
H_1 \ddot{X} + H_2 \dot{X} - \mu \Delta(X, \dot{X}) - \mu \tau_d = J^T \tau + H_3
\tag{17}
\]

Let \( \tau_{d1} = \mu \Delta(X, \dot{X}) + \mu \tau_d \), then Equation (17) is simplified as

\[
H_1 \ddot{X} + H_2 \dot{X} - \tau_{d1} = J^T \tau + H_3
\tag{18}
\]

Through verification, \( H_1 \) and \( H_2 \) in the above equation are positive definite matrices.

**IV. HJI THEORY AND THE DESIGN OF THE DISTURBANCE OBSERVER**

**A. HJI THEORY**

In the wind-tunnel test, the state of the WDPR is disturbed by the outside. Therefore, based on the established mathematical model of WDPR uncertainty and external disturbance, by adopting HJI theory, the model of the closed-loop control error system of the WDPR is designed as follows:

\[
\begin{align*}
\dot{x} &= e(x) + \xi(x) \tau_{d1}(t) \\
\lambda &= \gamma(x)
\end{align*}
\tag{19}
\]

In Equation (19), \( \tau_{d1}(t) \) is external disturbance and \( \lambda \) is the evaluation index of the system. The sliding mode function is defined as the evaluation index, that is, \( \lambda = \dot{e} + \delta \xi \), \( \delta > 0 \) when \( \lambda \rightarrow 0, e \rightarrow 0, \dot{e} \rightarrow 0 \). The \( L_2 \) index of interference signal \( \tau_{d1}(t) \) is set as \( \| \tau_{d1}(t) \|_2 = \left\{ \int_{0}^{\infty} \tau_{d1}(t) \tau_{d1}(t)^T dt \right\}^{1/2} \); its \( L_2 \) norm can measure the energy of \( \tau_{d1}(t) \).

In order to express the anti-interference ability of the WDPR, the following performance indexes are set:

\[
\eta = \sup_{\| \tau_{d1}(t) \|_2 \neq 0} \frac{\| \lambda \|_2}{\| \tau_{d1}(t) \|_2}
\tag{20}
\]

where \( \eta \) is the \( L_2 \) gain of the WDPR nonlinear system, representing the robust performance of the WDPR system. The smaller the \( \eta \) value is, the better the robust performance of the WDPR system will be. Therefore, the influence of external interference \( \tau_{d1}(t) \) on control error can be suppressed by reducing \( L_2 \) gain of the WDPR system [20].

According to the theory in [21]–[23] and the model of the closed-loop control error system, the \( L_2 \) gain condition of the WDPR system with less than or equal to a positive number \( \chi \) is given. The HJI theory is defined as follows: given any positive real number \( \chi \), if there is a positive definite and differentiable function \( V(x) \geq 0 \), then

\[
\dot{V}(x) \leq -\frac{1}{2} \left\{ \chi^2 \| \tau_{d1}(t) \|_2^2 - \| \lambda \|^2 \right\} (\forall \tau_{d1}(t))
\tag{21}
\]

In this case, \( \eta \leq \chi \), which shows that the WDPR system is not only stable with bounded input and bounded output, but also asymptotically stable.

**B. DESIGN OF THE DISTURBANCE OBSERVER**

In practical engineering applications, external disturbances and modeling uncertainties affect the control accuracy of the system. Therefore, in order to avoid these problems,
an exponential convergence disturbance observer is designed in this paper and used to compensate for the disturbance in the designed sliding mode robust control law.

Take \( \hat{\tau}_{d1} = K(\tau_{d1} - \hat{\tau}_{d1}) \), define the auxiliary parameter vector as \( z = \hat{\tau}_{d1} - A(X, \dot{X}) \), and let \( A(X, \dot{X}) = KH_1 \dot{X} \) \((K > 0)\), then \( \hat{z} = \hat{\tau}_{d1} - KH_1 \dot{X} \).

Due to
\[
\dot{\hat{\tau}}_{d1} = K(\tau_{d1} - \hat{\tau}_{d1}) = K(H_1 \ddot{X} + H_2 \dot{X} - J^T \tau - H_3) - K \hat{\tau}_{d1}
\]
Then
\[
\dot{z} = K(H_1 \ddot{X} + H_2 \dot{X} - J^T \tau - H_3)
- K \dot{\hat{\tau}}_{d1} - KH_1 \ddot{X}
= K(H_2 \ddot{X} - J^T \tau - H_3) - K \hat{\tau}_{d1}
\]
(23)

Based on the above conditions, the following disturbance observer is designed:
\[
\begin{align*}
\dot{z} &= K(H_2 \ddot{X} - J^T \tau - H_3) - K \hat{\tau}_{d1} \\
\ddot{\hat{\tau}}_{d1} &= z + A(X, \dot{X})
\end{align*}
\]
(24)

Then
\[
\dot{z} = K(H_2 \ddot{X} - J^T \tau - H_3 - A(X, \dot{X})) - Kz
\]
(25)

For slow disturbance, assume \( \dot{\hat{\tau}}_{d1} = 0 \), then
\[
\dot{\hat{\tau}}_{d1} = \ddot{\hat{\tau}}_{d1} - \hat{\tau}_{d1} = -\dot{z} - KH_1 \dot{X}
\]
(26)

When \( \ddot{z} \) is substituted into Equation (26), then
\[
\dot{\hat{\tau}}_{d1} = -K(H_2 \ddot{X} - J^T \tau - H_3 - A(X, \dot{X}))
+ Kz - KH_1 \ddot{X}
= -K(H_1 \ddot{X} + H_2 \dot{X} - J^T \tau - H_3)
+ K(\ddot{z} + A(X, \dot{X}))
= K \ddot{\hat{\tau}}_{d1} - K \tau_{d1} = K(\ddot{\hat{\tau}}_{d1} - \tau_{d1})
= -K \hat{\tau}_{d1}
\]
(27)

Therefore, the error equation of the disturbance observer can be obtained
\[
\ddot{\hat{\tau}}_{d1} + KH_1 \ddot{X} = 0
\]
(28)

Equation (28) indicates that \( \ddot{\hat{\tau}}_{d1} = \ddot{\tau}_{d1}(t_0)e^{-Kt} \). The convergence accuracy of the disturbance observer mainly depends on the parameter \( K \). By designing the parameter \( K \), the estimated value \( \hat{\tau}_{d1} \) can approach the disturbance \( \tau_{d1} \) exponentially. At the same time, the disturbance observer Equation (24) indicates that the observer does not need \( \dot{X} \).

V. DESIGN AND ANALYSIS OF A SLIDING MODE CONTROL LAW

A. DESIGN OF THE CONTROL LAW

Since the WDPR is a redundant driving robot, the mechanism has complex characteristics, and the control environment is complex. In order to meet the conditions of the wind-tunnel test of WDPR system and improve the robustness of the system, the HJI theory and disturbance observer are applied to the designed control law. The dynamic equation of the controlled object is considered to be Equation (13). The ideal pose is set as \( X_d \), and the tracking error, velocity error, and acceleration error are defined as \( e = X - X_d, \dot{e} = \dot{X} - \dot{X}_d \) and \( \ddot{e} = \ddot{X} - \ddot{X}_d \), respectively. Since the pose is a column vector with six rows and one column, \( e, \dot{e} \) and \( \ddot{e} \) are all column vectors with six rows and one column.

According to Equation (18), the sliding mode function is designed
\[
s = \dot{e} + \delta e
\]
(29)

Since \( \dot{X} = H_1^{-1}(J^T \tau + H_3 - \dot{\hat{\tau}}_{d1}) \), then
\[
\dot{\hat{e}} = \dot{\hat{X}}_d - \dot{\hat{X}}
= \dot{\hat{X}}_d - H_1^{-1}(J^T \tau + H_3 - H_2 \dot{X} + \tau_{d1})
\]
(30)

\[
\dot{s} = \dot{e} + \delta \dot{e}
= -H_1^{-1}(J^T \tau + H_3 - H_2 \dot{X} + \tau_{d1}) + \delta \dot{e} + \dot{\hat{X}}_d
\]
(31)

Combining Equation (31) and referring to the design advantages in [24], a sliding mode robust control law based on HJI theory and the disturbance observer is designed, as shown in Equation (32):
\[
\tau = (J^T)^+(H_2 \dot{X} - H_3 - \ddot{\hat{\tau}}_{d1} - H_1 \tau_1)
+ H_1 \dot{\hat{X}}_d + H_1 \dot{X}_d - H_1 \dot{X}
\]
(32)

where \((J^T)^+\) is the generalized inverse matrix of the Jacobian matrix transposition and \( J^T(J^T)^+ = I_{6 \times 6} \), \( \tau_1 \) is the feedback control law.

Substituting Equation (32) into equation (18), the following results can be obtained:
\[
\dot{\hat{e}} + \dot{s} + H_1^{-1} \ddot{\hat{\tau}}_{d1} = \tau_1
\]
(33)

The sliding mode function is defined as the evaluation index \( \lambda \):
\[
\lambda = s = \dot{e} + \delta e
\]
(34)

Then,
\[
\begin{align*}
\dot{\hat{e}} &= s - \delta e \\
\dot{\hat{H}_1} \ddot{s} &= -\hat{H}_1 s + \hat{H}_1 \omega - \ddot{\hat{\tau}}_{d1} + H_1 \tau_1
\end{align*}
\]
(35)

In Equation (35), \( \omega = \delta \ddot{e} + \delta e \).

Using Equation (21) in Section 4.1, Equation (35) is written in the form of Equation (19), where
\[
\ddot{s}(x) = \begin{bmatrix} s - \delta e \\ -s + \omega + H_1^{-1} \ddot{\hat{\tau}}_{d1} + \tau_1 \end{bmatrix}, \quad \ddot{\hat{x}}(x) = \begin{bmatrix} 0 \\ -H_1^{-1} \end{bmatrix}
\]

In order to make the error closed-loop system satisfy \( \eta \leq \chi \), that is, make the errors \( \hat{e} \) and \( s \) approach zero, the feedback control law is designed as follows:
\[
\tau_1 = -\Gamma \text{sgn} s - \omega - \frac{1}{2\chi^2} \hat{s} - \frac{1}{2} \hat{s}
\]
(36)

where \( \Gamma \) is a column vector with six rows and one column.
By substituting Equation (36) and Equation (32) into Equation (31), it can be concluded that:

\[ \dot{s} = -H_1^{-1} \ddot{\tau}_{d1} + \tau_1 - \dot{e} + \delta \dot{e} \]

\[ = -H_1^{-1} \ddot{\tau}_{d1} - \Gamma \text{sgn} s \]

\[ - \left( \frac{1}{2} \dot{\varepsilon}^2 + \frac{3}{2} \right) \dot{e} \]

\[ + H_1^{-1} \ddot{\tau}_{d1} + \Gamma \text{sgn} s = 0 \quad (37) \]

By substituting the designed feedback control law into Equation (33), the following conclusions can be obtained:

\[ \dot{\epsilon} + \left( \frac{1}{2} \dot{\varepsilon}^2 + \frac{3}{2} \right) \dot{e} + \left( \frac{1}{2} \dot{\varepsilon}^2 + \frac{3}{2} \right) \delta = -H_1^{-1} \ddot{\tau}_{d1} + \Gamma \text{sgn} s \]

\[ + H_1^{-1} \ddot{\tau}_{d1} + \Gamma \text{sgn} s = 0 \quad (38) \]

\[ \dot{V} = \dot{V}_1 + \dot{V}_2 \]

\[ = -\frac{1}{2} s^T \dot{H}_1 s + \frac{1}{2} s^T \dot{\tau}_{d1} \dot{\tau}_{d1} \]

By substituting Equation (33) into Equation (31), it can be concluded that:

\[ \dot{\tau} = H_1^{-1} \ddot{\tau}_{d1} - \dot{\tau}_1 \]

\[ = -H_1^{-1} \ddot{\tau}_{d1} - \Gamma \text{sgn} s \]

\[ - \left( \frac{1}{2} \dot{\varepsilon}^2 + \frac{3}{2} \right) \dot{e} \]

\[ + H_1^{-1} \ddot{\tau}_{d1} + \Gamma \text{sgn} s = 0 \quad (37) \]

\[ \dot{\tau} + \delta \dot{\epsilon} = -H_1^{-1} \ddot{\tau}_{d1} - \Gamma \text{sgn} s \]

\[ + H_1^{-1} \ddot{\tau}_{d1} + \Gamma \text{sgn} s = 0 \quad (38) \]

\[ B. \text{ SYSTEM STABILITY ANALYSIS} \]

To verify the stability of the closed-loop system, according to Equations (37) and (38), the Lyapunov function of the closed-loop system is set as follows:

\[ V = \frac{1}{2} s^T H_1 s + \frac{1}{2} s^T \dot{\tau}_{d1} \dot{\tau}_{d1} \quad (39) \]

The derivative of the Lyapunov function \( V \) shown in Equation (39) can be obtained:

\[ \dot{V} = s^T \dot{H}_1 s + \frac{1}{2} s^T \dot{\tau}_{d1} \dot{\tau}_{d1} \quad (40) \]

In order to make \( \dot{V} < 0 \), let \( \dot{V}_1 = s^T \dot{H}_1 s + \frac{1}{2} s^T \dot{\tau}_{d1} \dot{\tau}_{d1} \), \( \dot{V}_2 = \dot{\tau}_{d1} \dot{\tau}_{d1} \). Further analysis is carried out on \( \dot{V}_1 \) and \( \dot{V}_2 \) respectively.

\[ \dot{V}_1 = s^T \dot{H}_1 s + \frac{1}{2} s^T \dot{\tau}_{d1} \dot{\tau}_{d1} \]

\[ = s^T (-H_1 s + H_1 \omega - \ddot{\tau}_{d1} + H_1 \tau_1) + \frac{1}{2} s^T \dot{\tau}_{d1} \dot{\tau}_{d1} \]

\[ = s^T (-H_1 s + H_1 \omega - \ddot{\tau}_{d1}) \]

\[ + H_1 (-\Gamma \text{sgn} s - \omega - \frac{1}{2} \dot{\varepsilon}^2 s) \]

\[ - \left( \frac{1}{2} \dot{\varepsilon}^2 s \right) + \frac{1}{2} s^T \dot{\tau}_{d1} \dot{\tau}_{d1} \]

\[ = s^T (-H_1 s - \ddot{\tau}_{d1}) \]

\[ - \left( \frac{1}{2} \dot{\varepsilon}^2 s \right) + \frac{1}{2} s^T \dot{\tau}_{d1} \dot{\tau}_{d1} \]

\[ = -s^T (-H_1 s - \ddot{\tau}_{d1}) \]

\[ - \left( \frac{1}{2} \dot{\varepsilon}^2 s \right) + \frac{1}{2} s^T \dot{\tau}_{d1} \dot{\tau}_{d1} \]

\[ = -s^T (-H_1 s - \ddot{\tau}_{d1}) \]

\[ - \left( \frac{1}{2} \dot{\varepsilon}^2 s \right) + \frac{1}{2} s^T \dot{\tau}_{d1} \dot{\tau}_{d1} \]

\[ = -s^T (-H_1 s - \ddot{\tau}_{d1}) \]

\[ - \left( \frac{1}{2} \dot{\varepsilon}^2 s \right) + \frac{1}{2} s^T \dot{\tau}_{d1} \dot{\tau}_{d1} \]

(41)

Since \( \ddot{\tau}_{d1} = -\dot{\tau}_{d1} = -K (\tau_{d1} - \ddot{\tau}_{d1}) = -K \ddot{\tau}_{d1} \),

\[ \dot{V}_2 = \dot{\tau}_{d1} = -K \ddot{\tau}_{d1} \quad (42) \]

Then,

\[ \dot{V} = \dot{V}_1 + \dot{V}_2 \]
TABLE 1. Coordinates of \( P_i \) point and \( B_i \) point.

| Symbol | Coordinate point (mm) | Symbol | Coordinate point (mm) |
|--------|-----------------------|--------|-----------------------|
| \( P_1 \) | \((-208,78, -1)^T\) | \( B_1 \) | \((200, 415, -1280)^T\) |
| \( P_2 \) | \((-208, -78, -1)^T\) | \( B_2 \) | \((200, -415, -1280)^T\) |
| \( P_3 \) | \((0, -27.7, -10.1)^T\) | \( B_3 \) | \((-300, -308, -1250)^T\) |
| \( P_4 \) | \((0.277, -10.1)^T\) | \( B_4 \) | \((-300, 308, -1250)^T\) |
| \( P_5 \) | \((0.277, 10.1)^T\) | \( B_5 \) | \((-315, 315, -70)^T\) |
| \( P_6 \) | \((-208, 78, 1)^T\) | \( B_6 \) | \((160, 415, -70)^T\) |
| \( P_7 \) | \((-208, -78, 1)^T\) | \( B_7 \) | \((160, -415, -70)^T\) |
| \( P_8 \) | \((0.277, 10.1)^T\) | \( B_8 \) | \((-315, -315, -70)^T\) |

\[ X_d = [0; 0; -0.582; \frac{\pi}{18}\sin(\frac{\pi}{2}t); \frac{\pi}{18}\sin(\frac{\pi}{2}t); \frac{\pi}{18}\sin(\frac{\pi}{2}t)]^T \] (46)

The simulation results are shown in Fig. 3-7. Fig. 3 and Fig. 4 indicate that the angle tracking error and angular velocity tracking error tend to be zero basically and that the control effect is good. Fig. 5 and Fig. 6 indicate that the coincidence degree of estimated disturbance and external disturbance is very good, and the disturbance observation error tends to be zero basically. The control input of the eight motors shown in Fig. 7 has certain regularity and is within the rated torque range (1.27 N.m); the variation of wire length shown in Fig. 8 conforms to the law of compound motion.

**B. 6-DOF MOTION SIMULATION**

The simulation analysis of the system with six degrees of freedom is carried out. The initial pose of the system is set as
The initial velocity is \([0.05 \, 0.05 \, 0.05 \, \frac{\pi}{36} \, \frac{\pi}{36} \, \frac{\pi}{36}]\), and the theoretical pose is:

\[
X_d = [0.05 \times \sin t; 0.05 \times \sin t; 0.05 \times \sin t - 0.582; \frac{\pi}{18} \sin(\frac{\pi}{2} t); \frac{\pi}{18} \sin(\frac{\pi}{2} t); \frac{\pi}{18} \sin(\frac{\pi}{2} t)]
\]  

(47)

The simulation results are shown in Fig.9-13. Fig.9 and Fig.10 indicate that the pose tracking error with six degrees of freedom movement basically tends to be zero. Fig.11 to Fig.12 indicate that the position velocity tracking error is within the range of \(\pm 0.2 \times 10^{-5} \text{m/s}\), and the angular velocity tracking error is basically within the range of \(\pm 0.002 \degree/s\). Fig.13 to Fig.14 indicate that the coincidence degree between the estimated disturbance and the external disturbance of 6-DOF motion is also very good, and the disturbance observation error basically tends to be zero. Fig.15 indicates that the control input of eight motors is within the rated torque (1.27N.m). Fig.16 indicates that due to the 6-DOF movement, there is no certain regularity in the variation of wire length, but the wire length of the eight wires varies within a certain range (900-1200mm).
C. COMPARATIVE ANALYSIS OF SIMULATION

When the disturbance observer is not added to the control law, the control input curves of compound motion and 6-DOF motion are shown in Fig. 17-18.

Fig. 17 and 18 indicate that there will be noticeable chatter in the control input once there is no disturbance observer in the control law. And it will cause the vibration of the driving device and affect the life and reliability of the system. (Compared with the control inputs of the sliding mode robust control law based on HJI theory and the disturbance observer (Fig. 7 and Fig. 15))

The disturbance observer is used to observe the disturbance and compensate for the disturbance in the sliding mode robust control law, which can effectively reduce the switching gain and reduce the chattering.

D. ANALYSIS OF SIMULATION RESULTS

The following results can be obtained through the simulation analysis of compound motion and 6-DOF motion: the
tracking error is within the range set by the wind-tunnel test, which meets the requirements of the wind-tunnel test. At the same time, the control input value of the compound motion is relatively small compared with the control input value of the 6-DOF motion. This indicates that the control input value increases with the increase of the DOF of the system’s motion. Compared with the variation of wire length of the 6-DOF, the variation of wire length of compound motion has symmetrical regularity. The above results show that the designed sliding mode robust control law based on HJI theory and disturbance observer can effectively suppress the influence of uncertain factors and external disturbances on the control accuracy, which lays a theoretical foundation and technical support for the practical application of the prototype in the wind-tunnel test. 4) Finally, the chattering is proven to be able to be effectively reduced using the disturbance observer through the comparative simulation analysis.

In the future, the elastic deformation and vibration of the wire should be considered in the design of the control algorithm, since they can practically limit the implementation of the method proposed in this paper. In addition, the prototype will be further optimized so as to provide better adherence to the mathematical model used for planning. In the end, a comprehensive solution will be provided for the dynamic operation of the wire driven parallel robot by considering the above problems.

VII. CONCLUSION
In this paper, based on HJI theory and a disturbance observer, a sliding mode robust control law is designed for a wire driven parallel robot (WDPR) used in wind tunnel test to suppress the influence of external disturbances on the control accuracy. The relevant conclusions are as follows: 1) Aiming at the uncertain factors and external disturbances of the system, the dynamic models of the end effector and the driving system are established respectively, and the overall dynamic model of the system is also established. 2) A sliding mode robust control law is designed based on HJI theory and disturbance observer, and the stability of the closed-loop system is proved by Lyapunov’s second method. 3) According to the simulation analysis of compound motion and 6-DOF motion, the designed control law can effectively suppress the influence of uncertain factors and external disturbances on the control accuracy, which lays a theoretical foundation and technical support for the practical application of the prototype in the wind-tunnel test. 4) Finally, the chattering is proven to be able to be effectively reduced using the disturbance observer through the comparative simulation analysis.

In the future, the elastic deformation and vibration of the wire should be considered in the design of the control algorithm, since they can practically limit the implementation of the method proposed in this paper. In addition, the prototype will be further optimized so as to provide better adherence to the mathematical model used for planning. In the end, a comprehensive solution will be provided for the dynamic operation of the wire driven parallel robot by considering the above problems.

AUTHOR CONTRIBUTIONS
All authors contributed to this study presented in the manuscript. Methodology: WANG Yuqi and LIN Qi; investigation: HUANG Jiacai, ZHOU Lei and CAO Jinjiang; writing—original draft preparation: WANG Yuqi; writing—review and editing: WANG Yuqi, LIN Qi and HUANG Jiacai; supervision: QIAO Guifang and SHI Xinxin.

CONFLICTS OF INTEREST
The authors declare no conflicts of interest.

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