Hadronic $B_c$ decays as a test of $B_c$ cross-section

Alexander Rakitin
Lauritsen Laboratory, California Institute of Technology, Pasadena, California 91125

Sergey Koshkarev
Instituto de Física de Cantabria, CSIC-University of Cantabria, Santander, Cantabria E-39005, Spain
(Dated: January 12, 2010)

This paper focuses on disagreement between theoretical predictions and experimental results of the production properties of $B_c$ meson. Hadronic decays of $B_c$ are used to separate predictions of production cross-section and predictions of branching ratio. The branching ratios of $B_c$ decays to $J/\psi + \pi$ and to $J/\psi + 3\pi$ are also presented.

PACS numbers: 13.25.Hw, 13.20.He, 13.85.Ni

INTRODUCTION

Study of $B_c$ meson is important because it stands out of the crowd of other heavy-quark mesons. This is the only meson consisting of two different heavy quarks. Also, the lighter $c$ quark has a decay rate (∼65%) [1] larger than heavier $b$ quark, which is uncommon for heavy-quark mesons. The mass and lifetime of $B_c$ meson have been measured by CDF [2, 3] and DØ [4, 5] in decays $B_c \to J/\psi \pi$ and $B_c \to J/\psi \ell$. They are in pretty good agreement with theory [1] (see Table I). Also, the production properties of $B_c$ meson have been measured and compared to that of $B$ meson [6]:

\[
R_{e} = \frac{\sigma(B_c) \cdot Br(B_c \to J/\psi e^+\nu)}{\sigma(B) \cdot Br(B \to J/\psi K^\pm)} = 0.282 \pm 0.038 \pm 0.074
\]

and

\[
R_{\mu} = \frac{\sigma(B_c) \cdot Br(B_c \to J/\psi \mu^+\nu)}{\sigma(B) \cdot Br(B \to J/\psi K^\pm)} = 0.249 \pm 0.045^{+0.107}_{-0.076}
\]

in the kinematic region $p_T(B_c) > 4.0$ GeV and $|y(B_c)| < 1.0$. Using the theoretical predictions for the branching fraction $Br(B_c \to J/\psi e^+\nu) \approx 2 \cdot 10^{-2}$ [1, 8] and taking into account well-measured branching $Br(B^+ \to J/\psi K^+) = (1.007 \pm 0.0035) \cdot 10^{-3}$ [9], one can obtain the ratio of the production cross-sections:

\[
\frac{\sigma(B_c)}{\sigma(B)} = R_{e} \cdot \frac{Br(B \to J/\psi K^\pm)}{Br(B_c \to J/\psi K^\pm)} \approx 1.4 \cdot 10^{-2}.
\]

Comparing this result with theoretical predictions of $B_c$ cross-section [10, 13] and of the ratio of production cross-section $\sim 10^{-3}$ we see that $B_c$ semileptonic branching fraction has to be an order of magnitude larger than theoretical prediction, about 20%. This is a significant discrepancy between theory and experiment. Another discrepancy comes from the measurement of the production properties of $B_c$ in CDF data collected in Run I [14]. CDF presented a 95% C.L. on $\sigma(B_c^+) \cdot Br(B_c^+ \to J/\psi \pi^+)/\sigma(B^+) \cdot Br(B^+ \to J/\psi K^+)$ as a function of $B_c$ lifetime (see Fig. 1). Using known $B_c$ lifetime (0.46 ± 0.07) ps we clearly see an order of magnitude disagreement between the theoretical prediction and data. Either our theoretical estimate of $B_c$ semileptonic branching fraction is incorrect or we do not understand the production cross-section of $B_c$. To clarify this issue we suggest to measure the ratio of the production cross-sections using hadronic decay modes of $B_c$, namely $B_c \to J/\psi \pi$ and $B_c \to J/\psi 3\pi$. If the experimental branching fraction ratio coincides with theoretical predictions, the problem is in the production cross-section, otherwise the prediction for $B_c$ semileptonic branching fractions is incorrect. This measurement can be done in.
TABLE I. This table shows good agreement of theoretical predictions of $B_c$ properties with experimental results from Tevatron.

| Source | $B_c$ mass (MeV/c^2) | $B_c$ lifetime (ps) |
|--------|----------------------|---------------------|
| CDF [2, 3] | 6285 ± 5.3(stat) ± 1.2(sys) | 0.463^{+0.042}_{-0.038} (stat) ± 0.036(sys) |
| DO [4, 5] | 6300 ± 14(stat) ± 5(sys) | 0.448^{+0.026}_{-0.023} (stat) ± 0.032(sys) |
| Theory [1, 6] | | 0.48 ± 0.05 |

CDF or DO, where $B_c$ mesons are produced and already were observed.

THEORETICAL BASEMENT

In this paper we will use the fact that a hadronic matrix element of heavy-quark current might be written in a simple form if expressed in terms of the velocities of heavy particles [3, 15, 16]. Also, we will base on definition of nonrecoil form factor. The validity of using the nonrecoil approximation is strongly supported by the fact that the kinematic variable $\omega = v_1 \cdot v_2$ is restricted to values close to unity (indexes 1 and 2 do mean initial and final hadrons, respectively). Let heavy-quark $Q_i$ undergo a weak decay to $Q_f$ with a spectator quark $Q_s$. At $s = (p_1 - p_2)^2 = 0$, we have

$$(v_1 \cdot v_2)_{max} = 1 + \frac{(m_1 - m_2)^2}{2m_1m_2} \approx 1 + \frac{(m_{Q_i} - m_{Q_s})^2}{2(m_{Q_i} + m_{Q_s})(m_{Q_i} + m_{Q_s})},$$

(1)

In our case the initial state $(bc)$ decays into $(cc)$, therefore $(v_{B_c} \cdot v_{J/\psi})_{max} \approx 1.29$ for the mass values $m_b = 4.8$ GeV/c^2 and $m_c = 1.5$ GeV/c^2.

Following [8] the hadronic matrix elements in the nonrecoil approximation ($v_i = v_f = v$) for the weak process $Q_i \rightarrow Q_f W^+$ can be presented as

$$
\begin{align*}
&<0^P, \epsilon_2|V^\mu|0^P> \simeq \pm 2\eta_2 \sqrt{m_1m_2}\epsilon_2^\mu \\
&<1^P, \epsilon_2|A^\mu|0^P> \simeq \pm 2\eta_2 \sqrt{m_1m_2}\epsilon_2^\mu \\
&<0^P, A^\mu|1^P, \epsilon_1 > \simeq \pm 2\eta_2 \sqrt{m_1m_2}\epsilon_1^\mu , \\
&<1^P, A^\mu|1^P, \epsilon_1 > \simeq \pm 2\eta_2 \sqrt{m_1m_2}\epsilon_1^\mu
\end{align*}
$$

(2)

where the vector and axial-vector currents are $V^\mu = Q_f\gamma^\mu Q_i$ and $A^\mu = \bar{Q}_f\gamma^\mu\gamma_5 Q_i$ and $\eta_2$ is a form factor playing the role of Isgur-Wise functions for transition between initial and final states of hadrons. Here, $\eta_2$ can be parametrized as

$$\eta_2 = \left(\frac{2\beta_1\beta_2}{\beta_1^2 + \beta_2^2}\right)^{3/2}.$$

For the case of $B_c$ decays to $J/\psi$, the parameters $\beta_1$ and $\beta_2$ are equal to 0.82 and 0.66, respectively.

DECAY $B_c^+ \rightarrow J/\psi + \pi^+$

The amplitude of this decay includes two factors, one of them is a pionic decay amplitude, and the other is the formfactor appearing in semileptonic decay. This gives us a direct relation between pionic and semileptonic decays. In the case $s = m_\pi^2 \simeq 0$, the width of pionic decay may be given as [17]

$$
\Gamma(B_c \rightarrow J/\psi + \pi) = 6\pi^2 f^2_\pi |V_{ud}|^2 \simeq 1 \text{ GeV}^2.
$$

(3)

Upon contracting Eq. (2) with leptonic current $\bar{\epsilon}\gamma^\mu(1 - \gamma_5)\nu$, the width of $B_c \rightarrow J/\psi + \ell\nu$ is

$$
\frac{d\Gamma}{ds} \simeq 3 \cdot \frac{G_F^2(\lambda^{3/2} + 12m_B^2\lambda^{1/2}) m_{J/\psi}\eta_{B_c}}{576\pi^3m_{B_c}^2} |V_{ud}|^2 |V_{cb}|^2
$$

(4)

where $\lambda = \lambda(m_{J/\psi}^2, m_{B_c}^2, s)$ is "triangle" Källen function denoted as

$$
\lambda(x, y, z) = (x^2 + y^2 + z^2 - 2xy - 2yz - 2zx)^{1/2}.
$$

Combining Eqs. (8) and (4) and using $B_c$ lifetime $\tau = 0.46$ ps, we may expect the pionic decay branching ratio to be

$$
Br(B_c^+ \rightarrow J/\psi + \pi^+) \simeq 0.2%.
$$

This result is in good agreement with other results (see [1, 8] and references therein).

DECAY $B_c^+ \rightarrow J/\psi + \pi^+\pi^-$

Axial current

The amplitude of $B_c^+ \rightarrow J/\psi + \pi^+\pi^-\pi^+$ is

$$A \sim <J/\psi|A^\mu|B_c> <\pi^+\pi^-\pi^+|J^\mu_{axial}(0)|0>,$$

(5)

where $<J/\psi|A^\mu|B_c> = 2\eta_{B_c,J/\psi}\sqrt{m_{B_c}m_{J/\psi}}\epsilon_{\mu J/\psi}^*, A^\mu = \bar{c}\gamma^\mu\gamma_5 b$ is the axial-vector current and $\epsilon_{\mu J/\psi}^*$ presents the polarization four-vector of $J/\psi$. Let us remind the reader that the phase space can be represented as

$$dPS(B_c \rightarrow J/\psi + 3\pi) = \frac{ds}{2\pi}dPS(B_c \rightarrow J/\psi W^*)dPS(W^* \rightarrow 3\pi),$$

where $dPS(B_c \rightarrow J/\psi W^*)dPS(W^* \rightarrow 3\pi)$ is the phase space for the weak process with two kaons and one pion.
where $q$ the four momentum vector of the three-pion state and $s = q^2$. It is easy to show that $\rho_0 = 0$.

We have the same situation in $\tau^- \to \nu_\tau + \pi^-\pi^+\pi^-$ decay, therefore we will follow $a_1$ meson domination model of Ref. [18] ($a_1$ dominance is also discussed in [19, 20], angular distributions of $\tau \to \nu + 3\pi$ are discussed in [21]). The spectral function $\rho_1(s)$ can be cast into the form:

$$\rho_1(s) = \frac{1}{6} \frac{1}{(4\pi)^{4/3} f^2_\pi} |BW_{a_1}(s)|^2 \frac{g(s)}{s}. \tag{7}$$

The Breit-Wigner function $BW_{a_1}$ is parametrized including energy dependent width $\Gamma_{a_1}(s)$:

$$BW_{a_1} = \frac{m_{a_1}^2}{m_{a_1}^2 - s - i\sqrt{s}\Gamma_{a_1}(s)}, \quad \Gamma_{a_1}(s) = \frac{m_{a_1}}{\sqrt{s}} \Gamma_{a_1} \frac{g(s)}{g(m_{a_1})} \tag{8}$$

where $m_{a_1} = 1251 \pm 13$ GeV, $\Gamma_{a_1} = 599 \pm 44$ MeV, and the function $g(s)$ has been calculated in Ref. [18] and is derived from the observation that the axial-vector resonance $a_1$ decays predominately into tree pions. In this way, the branching is

$$Br(B_c^+ \to J/\psi + \pi^+\pi^-\pi^+) \approx 0.3\%.$$