Gravitational Contributions to Running of Gauge Couplings

TANG Yong (汤勇) and WU Yue-Liang (吴岳良)
Kavli Institute for Theoretical Physics China (KITPC), Key Laboratory of Frontiers in Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Science, Beijing 100190, China
(Received November 2, 2010)

Abstract Gravitational contributions to the running of gauge couplings are calculated by using different regularization schemes. As the $\beta$ function concerns counter-terms of dimension four, only quadratic divergences from the gravitational contributions need to be investigated. A consistent result is obtained by using a symmetry-preserving loop regularization with string-mode regulators which can appropriately treat the quadratic divergences and preserve non-abelian gauge symmetry. The harmonic gauge condition for gravity is used in both diagrammatical and background field calculations, the resulting gravitational corrections to the $\beta$ function are found to be nonzero, which is different from previous results presented in the existing literatures.

PACS numbers: 04.60.-m, 11.10.Hi
Key words: quantum gravity, unification, asymptotic

1 Introduction
Enclosing general relativity into the framework of quantum field theory is one of the most interesting and frustrating questions. Since its coupling constant $\kappa$ is of negative mass dimension, general relativity has isolated itself from the renormalizable theories. Renormalization is tightly connected with symmetry and divergence of loop diagrams. It was until the invention of dimensional regularization$^{[1]}$ that divergences of gravitational were not investigated systematically. Although, pure gravity at one-loop is free of physically relevant divergences,$^{[2]}$ if higher-loop diagrams are considered, higher dimensional counter-terms are needed to add to the original lagrangian. With the hope that quantum gravity, coupled with other fields, might show some miraculous cancelations, Einstein–Scalar system was investigated by using dimensional regularization which makes all divergences reduced to logarithmic ones, and found to be non-renormalizable.$^{[3]}$ Later, Einstein–Maxwell, Einstein–Dirac, and Einstein–Yang–Mills systems were studied and shown to be also non-renormalizable.$^{[4]}$ In spite of this theoretical inconsistency of general relativity and quantum field theory, we cannot deny that gravity has effect on ordinary fields and it contributes to the corrections of physical results since all matter has gravitational interaction. Treating general relativity as an effective field theory provides a practical way to look into quantum gravity’s effects.$^{[5]}$

Recently, Robinson and Wilczek$^{[6]}$ calculated gravitational corrections on gauge theory and arrived at an interesting observation that at or beyond Planck scale, all gauge theories, abelian or non-abelian, are asymptotically free due to gravitational corrections to the running of gauge couplings. The result obtained in Ref. [6] has attracted one’s attention. After reconsidering gravitational effects on gauge theories, different conclusions were reached by several groups. The result in [6] was derived in the framework of background field method, as such a method has off-shell and gauge-dependent problem, it was shown in [7] that the result obtained in [6] is gauge dependent, and the gravitational correction to $\beta$ function at one-loop order is absent in the harmonic gauge or general $\xi$-gauge. Using Vilkovisky–DeWitt method, it was found in [8] that the gravitational corrections to the $\beta$ function also vanishes in dimensional regularization. Late on, a diagrammatical calculation for two and three point functions was performed in [9] to obtain the $\beta$ function by using cut-off and dimensional regularization schemes, as a consequence, the same conclusion was yielded that quadratic divergences are absent.

In all these calculations, the crucial question concerns how to appropriately treat quadratic divergences. If gravitational corrections have no quadratic divergences, there would be no change to the $\beta$ function of gauge couplings. Since different regularization schemes treat quadratic divergences differently, we would like to apply in this paper the symmetry-preserving Loop Regularization (LR) with string-mode regulators$^{[10]}$ to evaluate the gravitational contributions to the running of gauge couplings. This is because the LR method has been shown to preserve both the non-abelian gauge symmetry and the divergent behavior of original integrals, it has to be applied to consistently obtain all one-loop renormalization constants of non-abelian gauge theory and QCD $\beta$ function$^{[11]}$ to derive the dynamically generated spontaneous chiral symmetry breaking in chiral effective field theory,$^{[12]}$ to clarify the ambiguities of quantum chiral anomalous$^{[13]}$ and the topological Lorentz/CPT violating Chern–Simons term,$^{[14]}$ and to verify the supersymmetric Ward identities and non-renormalization theorem in supersymmetric field theories.$^{[15]}$
In the following, after briefly introducing the symmetry-preserving Loop Regularization (LR) method proposed in [10], and presenting the general formalism needed for considering gravitational effects, we first make a careful check to recover the results obtained in [7–9] by using dimensional and cut-off regularization schemes. Then by using the LR method with both the diagrammatic and background field method calculations, we present new results. Finally, we arrive at a conclusion which differs from the ones presented in the existing literatures.

2 Loop Regularization

All loop calculations of Feynman diagrams can be reduced, after Feynman parametrization and momentum translation, into some simple scalar and tensor type loop integrals. For divergent integrals, a regularization is needed to make them physically meaningful. There are many kinds of regularization schemes in literature. For the reason mentioned above, we shall adopt the Loop Regularization method. In the LR method, the crucial concept is the introduction of the irreducible loop integrals (ILIs) which are defined, for example, at one-loop level as

\[ I_{-2\alpha}(M^2) = \int d^4k \frac{1}{(k^2 - M^2)^{2+\alpha}}, \]

\[ I_{-2\alpha \mu \nu}(M^2) = \int d^4k \frac{k^\mu k^\nu}{(k^2 - M^2)^{3+\alpha}}, \]

(1)

and higher rank of tensor integrals, with \( \alpha = -1, 0, 1, \ldots \). Here \( I_2 \) and \( I_0 \) denote the quadratic and logarithmic divergent ILIs respectively. In general, the divergent integrals are meaningless. To see that, let us examine tensor and scale type quadratically divergent ILIs \( I_{2\mu \nu}(0) \) and \( I_2(0) \), one can always write down, from the Lorentz decomposition, the general relation that \( I_{2\mu \nu}(0) = a g_{\mu \nu} I_2(0) \) with \( a \) to be determined appropriately. When naively multiplying \( g_{\mu \nu} \) on both sides of the relation, one yields \( a = 1/4 \), which is actually no longer valid due to divergent integrals. To demonstrate that, considering the zero component \( I_{200} \) of tensor ILIs \( I_{2\mu \nu} \), and performing an integration over the momentum \( k^0 \) for both \( I_{200} \) and \( I_2 \), it is then not difficult to find after comparing both sides of the relation that \( a = 1/2 \) which should hold as the integrations over \( k^0 \) for \( I_{200} \) and \( I_2 \) are convergent. To check its consistence, considering the vacuum polarization of QED. In terms of the ILIs, the vacuum polarization is given by

\[ \Pi_{\mu \nu} = -4e^2 \int d^4x [2I_{2\mu \nu}(m) - I_2(m)g_{\mu \nu} + 2x(1-x)(p^2 g_{\mu \nu} - p_\mu p_\nu)I_0(m)], \]

(2)

which shows that only quadratic divergences violate gauge invariance.

Note that the cut-off regularization together with the imposition of the Ward–Takahashi identities could also give the gauge invariant result of vacuum polarization in QED by introducing a photon mass renormalization as shown in [11], but the cut-off regularization, itself, cannot respect Ward–Takahashi identities which are crucial for unitarity of S-matrix. Therefore, the cut-off regularization alone violates gauge invariance.

However, if an explicit regularization has a property that the regularized quadratic divergences satisfy the consistency condition

\[ I_{2\mu \nu}^R = \frac{1}{2} g_{\mu \nu} I_2^R, \]

(3)

namely \( a = 1/2 \), then the vacuum polarization becomes gauge invariant. Here the superscript “R” denotes the regularized ILIs. Though dimensional regularization preserves gauge invariance with the above consistency condition and can handle with quadratic divergences, nevertheless, it is not originally intended to maintain the divergent behavior of original integrals. The LR method has been shown to satisfy the consistency conditions and meanwhile maintain the divergent behavior of original integrals. The regularized divergent ILIs in loop regularization have the following explicit results

\[ I_{2\mu \nu}^R = \frac{1}{2} g_{\mu \nu} I_2^R, \quad I_0^R = \frac{1}{4} g_{\mu \nu} I_0^R, \]

\[ I_2^R = \frac{-i}{16\pi^2} \left[ M_c^2 - \mu^2 \left[ \ln \frac{M_c^2}{\mu^2} - \gamma_w + 1 + y_2 \left( \frac{\mu^2}{M_c^2} \right) \right] \right], \]

\[ I_0^R = \frac{i}{16\pi^2} \left[ \ln \frac{M_c^2}{\mu^2} - \gamma_w + y_0 \left( \frac{\mu^2}{M_c^2} \right) \right], \]

(4)

with \( \mu^2 = \mu_s^2 + M_c^2 \), \( \gamma_w = \gamma_E = 0.5772 \cdots \), and

\[ y_0(x) = \int_0^x d\sigma \frac{1 - e^{-\sigma}}{\sigma}, \quad y_1(x) = e^{-x} - 1 + x, \]

\[ y_2(x) = y_0(x) - y_1(x), \quad \lim_{x \to 0} y_1(x) = 0. \]

Here the scales \( M_c \) and \( \mu_c \) play the role of characteristic energy scale and sliding energy scale. A detailed derivation is referred to Ref. [10].

It is interesting to notice that to understand the above consistency condition for the quadratic divergences in dimensional regularization, one may simply base on the observation that the quadratic divergences for \( D = 4 \) show up as logarithmic divergences for \( D = 2 \), thus the quadratic divergences may be dealt with by naively taking \( D = 2 \) as discussed in [12] and further developed in [13]. Nevertheless, such a treatment on the quadratic divergences in dimensional regularization can only be regarded as a special alternative and incidental explanation on the relation Eq. (3) rather than a systematical approach.

3 General Formalism

The action of Einstein–Yang–Mills theory is

\[ S = \int d^4x \sqrt{-g} \left[ \frac{1}{\kappa^2} R - \frac{1}{4} g^{\alpha \beta} F_{\mu \nu}^a F_{\alpha \beta}^a \right], \]

(6)

where \( R \) is Ricci scalar and \( F_{\mu \nu}^a \) is the Yang–Mills fields strength \( F_{\mu \nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu - ig[A_\mu, A_\nu] \). It is hard to quantize this lagrangian because of gravity-part’s non-linearity and minus-dimension coupling constant \( \kappa = \sqrt{16\pi G} \). Usually, one expands the metric tensor around a background metric \( \bar{g}_{\mu \nu} \) and treats graviton field as quantam fluctuation \( h_{\mu \nu} \) propagating on the background spacetime determined by \( \bar{g}_{\mu \nu} \).

\[ g_{\mu \nu} = \bar{g}_{\mu \nu} + \kappa h_{\mu \nu}. \]

(7)
The above expansion is exact, but the expansions of inverse metric and determinant are approximate with ignoring higher-order terms. To the second order in \( \kappa \),

\[
\begin{align*}
g^{\mu\nu} &= \bar{g}^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu}_{\rho}h^{\rho\nu} + \cdots, \\
\sqrt{-g} &= \sqrt{-\bar{g}} \left[ 1 + \frac{1}{2} \kappa h - \frac{1}{4} \kappa^2 \left( h^{\mu\nu}h_{\mu\nu} - \frac{1}{2} h^2 \right) \right].
\end{align*}
\]

(8)

In fact, the above expansion is an infinite series and the truncation is up to the question considered. These infinite series partly indicate that gravity is not renormalizable. From this perspective, it is more accurate to say that the system is treated as an effective field theory. After assembling the same order terms in \( \kappa \), we could get the lagrangian for usual quantization.

4 Diagrammatical Calculation

Let us set \( \bar{g}_{\mu\nu} = \eta_{\mu\nu} \), where \( \eta_{\mu\nu} \) is the Minkowski metric. \( h_{\mu\nu} \) is interpreted as graviton field, fluctuating in flat space-time. The lagrangian can be arranged to different orders of \( h_{\mu\nu} \) or \( \kappa \). The free part of gravition is of order unit and gives the graviton propagator

\[
P_G^{\mu\nu\rho\sigma}(k) = \frac{i}{k^2} [g^{\rho\sigma}g^{\mu\nu} + g^{\mu\rho}g^{\nu\sigma} - g^{\mu\nu}g^{\rho\sigma}],
\]

(10)
in the de Donder harmonic gauge

\[
C^{\mu} = \partial_{\mu}h^{\alpha\nu} - \frac{1}{2}\partial^{\mu}h^{\nu}_{\nu} = 0.
\]

For simplicity, in the following, the metric \( g^{\mu\nu} \) is understood as \( \eta^{\mu\nu} \). The interactions of gauge field and gravity field are determined by expanding the second term of the lagrangian (6). And various vertices could be derived. Details of these Feynman rules are referred to [9]. Using the traditional Feynman diagram calculations, we can compute the \( \beta \) function by evaluating two and three point functions of gauge fields. These green functions are general divergent, so counter-terms are needed to cancel these divergences. The relevant counter-terms to the \( \beta \) function are

\[
T^{\mu\nu} = i\delta_{ab}Q^{\mu\nu}\delta_{2}, \quad T^{\mu\nu\rho} = gf^{abc}V_{\mu\nu\rho},
\]

\[
Q^{\mu\nu} = q^{\mu\nu} - g^{\mu\nu}Q^{\mu\nu}, \quad V_{\mu\nu\rho} = g^{\mu\nu}(q - k)^{\rho} + g^{\mu\nu}(k - p)^{\rho} + g^{\mu\nu}(p - q)^{\rho}.
\]

(11)

The \( \beta \) function is defined as

\[
\beta(g) = g\mu\frac{\partial}{\partial\mu} \left( \frac{3}{2}\delta_2 - \delta_1 \right).
\]

In gauge theories without gravity, the counter-terms are logarithmically divergent as the quadratic divergences cancel each other due to the gauge symmetry. However, if gravitational corrections are taken into account, divergent behavior becomes different. On dimensional ground, it is known that quadratic divergences can appear and will contribute to the counter-terms defined above, so that they will also lead to the corrections to the \( \beta \) function. Although gravitational corrections contain logarithmic divergences, these divergences are multiplied by high order momentum. So logarithmic divergences will lead us to introduce counter-terms of six dimension, like \( \text{Tr}[(D_{\mu}F_{\nu\rho})^2], \quad \text{Tr}[(D_{\mu}F_{\nu\rho})^2], \quad \text{Tr}[F_{\mu\nu}F_{\mu\nu}] \). These terms do not affect \( \beta \) function of gauge coupling constant. So, in later calculations, we will omit the logarithmic divergences and only pay our attention to the quadratic divergences. For convenience, the gravitational contributions are labeled with a superscript \( \kappa \).

\[
\Delta\beta^{\kappa} = g\mu\frac{\partial}{\partial\mu} \left( \frac{3}{2}\delta_2^{\kappa} - \delta_1^{\kappa} \right).
\]

(12)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.pdf}
\caption{Feynman diagrams that gravitation contributes at one loop order to gauge two and three point functions.}
\end{figure}

It can be shown that at one-loop level, gravity will contribute to two and three point functions from five diagrams (see Fig. 1). The first two diagrams are corresponding to two point function, and the other three diagrams are for three point function. One can easily show that Fig. 1(c) has no quadratic divergences and only logarithmic divergences are left. For Fig. 1(d), as one of the end of graviton propagator could be attached to any gauge external leg, there are two similar contributions which can be obtained with a cycle for the momenta and their corresponding Lorentz index. In harmonic gauge, the two point functions from Fig. 1(a) and Fig. 1(b) are found in terms of ILIs to be

\[
T^{(a)\mu\nu} = 2\kappa^2 \int d\mathbf{x} \left[ Q^{\mu\nu}[I_2 + q^2(3x^2 - x)]I_0 + q^{\mu\nu}q_\rho I_2^{\rho\nu} + q^{\mu\nu}q_\rho I_2^{\rho\nu} - g^{\mu\nu}q_\rho q_\sigma I_2^{\rho\sigma} - q^2 I_2^{\mu\nu} \right](M_q^2),
\]

\[
T^{(b)\mu\nu} = -3\kappa^2 Q^{\mu\nu}I_2(0),
\]

(13)

and the three point functions from Figs. 1(d) and 1(e) are found, when keeping only the quadratically divergent terms, to be

\[
T^{(d)\mu\nu\rho} = ig\kappa^2 \left\{ -V_{\mu\nu\rho}I_2(0) + \int d\mathbf{x} \left[ (g^{\mu\nu}q_\sigma I_2^{\rho\sigma} - g^{\mu\rho}q_\sigma I_2^{\nu\sigma} + q^{\rho\sigma}I_2^{\mu\nu} - q^{\mu\nu}I_2^{\rho\sigma})(M_q^2) + (g^{\sigma\rho}k_\sigma I_2^{\mu\nu} - g^{\mu\rho}k_\sigma I_2^{\nu\sigma} + k^{\sigma\rho}I_2^{\mu\nu})(M_k^2) + (g^{\sigma\nu}p_\sigma I_2^{\mu\rho} - g^{\mu\rho}p_\sigma I_2^{\nu\sigma} + p^{\sigma\rho}I_2^{\mu\nu} - p^{\mu\nu}I_2^{\rho\sigma})(M_p^2) \right] \right\},
\]
\[ T^{(c)}_{\mu \nu \rho} = 3i g k^2 V_{g_{\mu \nu \rho}} I_2(0), \]

with \( M_2^2 = x(x - 1)q^2 \). Contraction is performed by using FeynCalc package.\(^{[19]}\)

Now we shall apply the different regularization schemes to the divergent ILIs. In cut-off regularization, when keeping only quadratically divergent terms, one has

\[ I_{2 R}^{\mu \nu} = \frac{1}{4} g^{\mu \nu} I_2^{R}, \quad I_2^{R} \simeq \frac{i}{16 \pi^2} g^{\mu \nu} \Lambda^2. \tag{15} \]

The resulting two and three point functions are

\[ T^{(a+b)}_{\text{cutoff}} \equiv T^{(a)}_{\text{cutoff}} + T^{(b)}_{\text{cutoff}} \approx 2 Q^{\mu \nu} k^2 \int dx \left[ \frac{i}{2} \frac{1}{16 \pi^2} \Lambda^2 + \left[ \ln \frac{\Lambda^2}{2 \pi} - \frac{3}{2} \frac{i}{16 \pi^2} \Lambda^2 \right] \right] = 0, \tag{16} \]

\[ T^{(d+c)}_{\text{cutoff}} \equiv T^{(d)}_{\text{cutoff}} + T^{(c)}_{\text{cutoff}} \approx 0, \tag{17} \]

which agrees with the result obtained in \([9]\).

In dimensional regularization, where \( I_2^R(0) = 0 \) and \( I_{2 \mu \nu}^{R} = g_{\mu \nu} I_2^{R}/2 \), the two and three point functions are found to be

\[ T_{\text{DR}}^{(a+b)} \approx 4 \kappa^2 Q^{\mu \nu} \int dx I_2^{R}(M_2^2), \tag{18} \]

\[ T_{\text{DR}}^{(d+c)} = 2i g k^2 \int dx \left[ (g^{\mu \nu} q^\rho - q^\mu g^{\rho \nu}) I_2^R(M_2^2) + (g^{\nu \rho} k^\mu - k^\nu g^{\rho \mu}) I_2^R(M_2^2) + (g^{\rho \mu} p^\nu - p^\rho g^{\mu \nu}) I_2^R(M_2^2) \right], \tag{19} \]

where the regularized quadratic divergence in dimensional regularization behaves as the logarithmic one

\[ I_2^R(M_2^2)|_{\text{DR}} = -\frac{i}{16 \pi^2} M_2^2 \left[ \frac{1}{2} \gamma_E - 1 \right] + O(\varepsilon). \tag{20} \]

So far, we have shown that in both cut-off regularization and dimensional regularization, there are no gravitational corrections to the gauge \( \beta \) function, which agrees with the ones yielded in \([7,9]\).

We now make a calculation by using loop regularization. With the consistency condition \( I_{2 \mu \nu}^{R} = g_{\mu \nu} I_2^{R}/2 \), we obtain for two and three point functions

\[ T_{\text{LR}}^{(a+b)} = 2 \kappa^2 Q^{\mu \nu} \int dx \left[ \frac{3}{2} I_2^{R}(0) + 2 I_2^R(M_2^2) + q^2 (3x^2 - x) I_2^R(M_2^2) \right], \tag{21} \]

\[ T_{\text{LR}}^{(d+c)} = 2i g k^2 \int dx \left[ \frac{1}{2} V_{g_{\mu \nu \rho}} I_2^R(0) + (g^{\mu \nu} q^\rho - q^\mu g^{\rho \nu}) I_2^R(M_2^2) + (g^{\nu \rho} k^\mu - k^\nu g^{\rho \mu}) I_2^R(M_2^2) + (g^{\rho \mu} p^\nu - p^\rho g^{\mu \nu}) I_2^R(M_2^2) \right], \tag{22} \]

from which we can directly read off the two-point and three-point counter-terms \( \delta_2^2 \) and \( \delta_1^2 \) respectively

\[ \delta_2^2 = \kappa^2 \frac{1}{16 \pi^2} \left[ M_2^2 - \mu^2 \left[ \ln \frac{M_2^2}{\mu^2} - \gamma_E + 1 + y_2 \left( \frac{\mu^2}{M_2^2} \right) \right] \right], \]

\[ \delta_1^2 = \kappa^2 \frac{1}{16 \pi^2} \left[ M_2^2 - \mu^2 \left[ \ln \frac{M_2^2}{\mu^2} - \gamma_E + 1 + y_2 \left( \frac{\mu^2}{M_2^2} \right) \right] \right]. \tag{23} \]

Putting the leading quadratically divergent part of \( \delta_1^2 \) and \( \delta_2^2 \) into Eq. (12), we obtain the gravitational corrections to the gauge \( \beta \) function

\[ \Delta \beta^c = - \kappa^2 \frac{2}{16 \pi^2}, \tag{24} \]

which shows that there are gravitational quadratic corrections to the gauge \( \beta \) function when loop regularization method is adopted to evaluate the quadratic divergent integrals, which is different from the results yielded by using the cut-off and dimensional regularization schemes.

## 5 Background Field Method

We have also performed a calculation by using the background field formalism. By using the loop regularization method and taking the harmonic gauge condition, we then obtain the same \( \beta \) function correction as Eq. (24).

In general, the total \( \beta \) function of gauge field theories including the gravitational effects may be written as

\[ \beta^c = \mu \frac{dg}{d \mu} = - \frac{b_0}{16 \pi^2} g^3 - \frac{\mu^2}{16 \pi^2} g \kappa^2. \tag{25} \]

The interesting feature of gauge theory interactions is the possible gauge couplings unification\(^{[20,21]}\) at ultra-high energy scale when the gravitational effects are absent. To illustrate the effects of gravitational contributions, we plot in Fig. 2 the running of gauge couplings based on the minimal supersymmetric standard model (MSSM) in which the three gauge couplings are unified at the energy scale \( 10^{18} \) GeV.

The running of gauge couplings in the MSSM without gravitational contributions is known to be \( (\alpha_i = g_i^2/4\pi) \)

\[ \alpha_i^{-1} = \alpha_i^{-1}(M) + \frac{33}{10 \pi} \ln \frac{M}{\mu}, \]

\[ \alpha_i^{-1} = \alpha_i^{-1}(M) + \frac{1}{2 \pi} \ln \frac{M}{\mu}, \]

\[ \alpha_i^{-1} = \alpha_i^{-1}(M) + \frac{9}{4 \pi} \ln \frac{M}{\mu}, \]
\[ \alpha^{-1}_s(\mu) = \alpha^{-1}_s(M) - \frac{3}{2\pi} \ln \frac{M}{\mu}, \] (26)

with experimental input at \( M_Z \)

\[ \alpha^{-1}_e(M_Z) = 58.97 \pm 0.05, \]
\[ \alpha^{-1}_\mu(M_Z) = 29.61 \pm 0.05, \]
\[ \alpha^{-1}_s(M_Z) = 8.47 \pm 0.22. \] (27)

It is noticed from Fig. 2 that only in the energy scale near and above Planck energy scale (~\( 10^{19} \) GeV) the gravitational effects become significant.

**Fig. 2** An illustration of gravitational contributions to the running of gauge couplings in the MSSM.

### 6 Conclusions

We have investigated the gravitational contributions to the running of gauge couplings by adopting different regularization schemes. From the above explicit calculations, it is not difficult to see that the different conclusions resulted from different regularization schemes mainly arise from the treatment for the quadratic divergent integrals. Since LR takes the consistency condition for the regularized quadratic divergent ILIs that \( I_{2\mu\nu}^{IL} = g_{\mu\nu} I_2^{R}/2 \), and maintains the quadratically divergent behavior for the regularized ILIs \( I_2^{R} \), it then gives a nonzero gravitational quadratic corrections to the gauge \( \beta \) function with the de Donder harmonic gauge condition. If there is an other regularization scheme, which has the same properties as above, it will give the same result. Also it is of interest to note that the quadratic corrections in pure gauge theory are known to cancel each other due to the consistency conditions of gauge symmetry,\(^{[10]}\) while the gravitational quadratic corrections to the running of gauge couplings are all gauge invariant and there is no symmetry to forbid their existence. In the de Donder harmonic gauge, ghost fields for the gravity are not coupled with gauge fields at one-loop order, which simplifies the calculation. We then come to the conclusion that at one-loop order in de Donder harmonic all gauge theories at or beyond Planck scale become asymptotically free as the resulting \( \beta \) function from the gravitational corrections are negative in this case. Note that the result may have gauge condition dependence, we\(^{[22]}\) are also looking in gauge condition independent formalism.\(^{[23-24]}\)

### References

[1] G.’t Hooft and M. Veltman, Nucl. Phys. B 44 (1972) 189.
[2] G.’t Hooft and M. Veltman, Ann. Poincare Phys. Theor. A 69 (1974) 69; M. Veltman, Lecture Notes in Method in Field Theory, Les Houches (1975).
[3] G.’t Hooft, Nucl. Phys. B 62 (1973) 444.
[4] S. Deser and P.van Nieuwenhuizen, Phys. Rev. D 10 (1974) 401; S. Deser, H.S. Tsao, and P.van Nieuwenhuizen, Phys. Rev. D 10 (1974) 3337.
[5] J.F. Donoghue, Phys. Rev. Lett. 72 (1994) 2996; Phys. Rev. D 50 (1994) 3874.
[6] S.P. Robinson and F. Wilczek, Phys. Rev. Lett. 96 (2006) 231601.
[7] A.R. Pietrykowski, Phys. Rev. Lett. 98 (2007) 061801.
[8] D.J. Toms, Phys. Rev. D 76 (2007) 045015.
[9] D. Ebert, J. Plefka, and A. Rodigast, Phys. Lett. B 660 (2008) 579.
[10] Y.L. Wu, Int. J. Mod. Phys. A 18 (2003) 5363, [arXiv:hep-th/0209021]; Y.L. Wu, Mod. Phys. Lett. A 19 (2004) 2191, [arXiv:hep-th/0311082].
[11] J.M. Jauch and F. Rohrlich, The Theory of Photons and Electrons, Springer-Verlag, New York, Heidelberg, Berlin (1976).
[12] M. Veltman, Acta Physica Polonica B 12 (1981) 437.
[13] I. Jack and D.R.T. Jones, Nucl. Phys. B 342 (1990) 127.
[14] J.W. Cui and Y.L. Wu, Int. J. Mod. Phys. A 23 (2008) 2861 [arXiv:0801.2199 [hep-ph]].
[15] Y.B. Dai and Y.L. Wu, Eur. Phys. J. C 39 (2004) 1.
[16] Y.L. Ma and Y.L. Wu, Int. J. Mod. Phys. A 21 (2006) 6383, [arXiv:hep-ph/0509083].
[17] Y.L. Ma and Y.L. Wu, Phys. Lett. B 647 (2007) 427, [arXiv:hep-ph/0611199].
[18] J.W. Cui, Y. Tang, and Y.L. Wu, Phys. Rev. D 79 (2009) 125008.
[19] R. Mertig, M. Böhm, and A. Denner, Comput. Phys. Commun. 64 (1991) 345.
[20] H. Georgi, H. Quinn, and S. Weinberg, Phys. Rev. Lett. 33 (1974) 451.
[21] S. Dimopoulos, S. Raby, and F. Wilczek, Phys. Rev. D 24 (1981) 1681.
[22] Yong Tang, Yue-Liang Wu, in preparation.
[23] G.A. Vilkovisky, Nucl. Phys. B 234 (1984) 125, The Quantum Theory of Gravity, ed. S.M. Christensen and Adam Hilger, Bristol (1984).
[24] B.S. DeWitt, in Quantum Field Theory and Quantum Statistics, Volume I, ed. I.A. Batalin, C.J. Isham, and G.A. Vilkovisky, Adam Hilger, Bristol (1987).