Tracing, Ranking and Pricing DER Flexibility in Active Distribution Networks

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Abstract—This paper presents a framework for analysing the aggregated flexibility of active distribution networks (ADNs) with distributed energy resources (DER). The analysis takes a different perspective than existing studies, which focus on characterising flexibility as the limits of the flexible power provision, i.e., the set of the network feasible operating points in the P-Q space. Instead, this work aims to estimate the contributions of different flexible units to the aggregated flexibility, which is essential for flexible power ranking and pricing. The proposed framework exploits cost-minimising OPF models complemented with cooperative game formulations that are able to capture the combinatorial nature of activating multiple flexible units. Moreover, in contrast to existing studies that imply perfect coordination of units, the proposed framework specifies the actions needed to reach feasible operating points, reflecting the nonlinearities of the network flexibility model. Extensive simulations are performed for different flexibility metrics to demonstrate the applicability of the framework. Depending on the metric selected (capacity, cost, or economic surplus of flexibility), distribution system operators (DSOs) can identify the most critical flexible units or remunerate units for participating in flexibility services provision.

Index Terms—Active distribution network (ADN), Cooperative Game Theory, distributed energy resources (DER), flexibility services, Shapley value, TSO-DSO coordination.

I. INTRODUCTION

The increasing integration of distributed energy resources (DER) and flexible consumers makes active distribution networks (ADNs) natural providers of flexibility services [1], [2]. Such services can be utilised within distribution networks or traded between distribution system operators (DSOs) and transmission system operators (TSOs). In this regard, multiple TSO-DSO coordination schemes and flexibility market designs have been proposed to enable flexible power trading between distribution and transmission systems [3]–[7]. It is pivotal for the TSO-DSO coordination to estimate the range of feasible flexible power exchanges through the TSO/DSO interface. Therefore, in recent years, much research effort has been devoted to modeling the aggregated flexibility at the TSO/DSO interface as sets of the network feasible operating points in the P-Q space. Such sets are known as flexibility P-Q areas. Several recent works, such as [18]–[21], address the problem of flexibility aggregation and disaggregation in ADNs. However, existing flexibility disaggregation algorithms are designed primarily for network secure operation needs, i.e., they allocate flexible power between units so that the feasibility of network operation is guaranteed. Such algorithms do not consider the economics of flexible power provision and cannot be used for ranking and pricing flexible power. Another drawback of existing studies is the widespread assumption of flexible unit perfect coordination. It is usually assumed that units act together to maximise network flexibility, taking any control actions needed to reach feasible operating points. However, this assumption may not be realistic. It is necessary to analyse optimal control actions for each flexibility request using accurate AC power flow models that capture nonlinearities and nonconvexities of flexible power provision by multiple units. As will be demonstrated in this work, operation of multiple flexible units in ADN exhibits nonlinear complex behaviour. For example, there can be rapid shifts in the flexible power output of units between close operating points. Some units provide active network management, i.e., alleviate network constraints or regulate voltages via reactive power control. Moreover, units can exchange (swap) flexible power to maximise network flexibility, i.e., some units can produce flexible power while other units consume it. This nonlinear complex operation of flexible units poses additional challenges to ranking and pricing flexible power in ADN.

A relevant attempt to trace flexibility and allocate the cost of its procurement was recently made in [22], where Sanjab et al. formulated the TSO-DSO coordination problem in the joint flexibility market as a cooperative game among system operators. The early studies, such as [8]–[11], focused on the flexibility area boundary approximation by sequentially solving optimal power flow (OPF) optimisation models. A thorough comparison of such models and approximation methods is given in [12]–[14]. Later research, e.g., [15]–[17], introduced more advanced models for ADNs flexibility area estimation, including stochastic and robust optimisation models.

Flexible units are resources with the technical ability to regulate their power exchange with the grid, e.g., controllable DER, such as battery energy storage systems, prosumers, electric vehicle aggregators, etc.
operators. Solution concepts from Cooperative Game Theory were used to allocate the cost of flexibility among system operators and analyse the stability of such cooperation. However, the approach presented in [22] is only applicable to the TSO-DSO coordination level: only the aggregated contributions of transmission and distribution networks can be considered, but not individual flexible units. No extension has been made to explore flexibility pricing within ADNs. Moreover, the authors relied on a linearised power flow model, which cannot accurately capture the physics of flexible power provision in ADNs, neglecting possible issues of unit coordination.

**But why is adequate tracing, ranking and pricing of flexibility needed?**

**Motivating example:** Imagine two identical flexible units (with the same flexible active power regulation capabilities) located in a distribution network, as shown in Fig. [1]. Suppose that one of the units can provide flexible power at a slightly lower cost than the other. Finally, assume that due to congestion issues, voltage constraints, grid codes, or TSO-DSO coordination mechanism, both units cannot produce their maximum flexible power regulation simultaneously. Under such conditions, how to estimate the contributions of units to the aggregated network flexibility? How to rank the units to identify the most critical ones or remunerate them for participating in the flexibility services provision? If ranking units only by the maximum flexible power that they can provide (capacity-based ranking), these units can be classified as equally critical. However, this ranking does not account for the cost assumptions and locational differences in the placement of units and, therefore, cannot be used in flexibility pricing mechanisms. If ranking units using a cost-minimisation model, the cheaper unit is allocated most of the flexible power and service payments. But, the cost-minimising ranking does not consider potential contributions by the more expensive unit and may not incentivise it to participate in the flexibility service provision. It follows that comprehensive flexibility tracing, ranking, and pricing mechanisms should consider both capacities and costs of flexible units, as well as the effects of network constraints. In view of new challenges in DER pricing [23], it becomes necessary to develop adequate mechanisms that provide the right coordination and incentives for flexible resources in distribution networks.

In this regard, the paper proposes a framework for tracing, ranking, and pricing flexible power in ADNs with multiple flexible units. The framework exploits cost-minimising OPF models to estimate several metrics of flexibility, such as the limits of flexible power provision, its cost, economic surplus, and output of flexible units. To deal with the combinatorial nature of multiple units activation, a cooperative game formulation is introduced that captures possible contributions of units to flexibility requests. Solution concepts from Cooperative Game Theory, such as the Shapley value, have been found useful for solving numerous allocation and ranking problems in power systems research [24–27]. Recently, cooperative game formulations have been successfully used in pricing energy communities and prosumers [28–30], as well as for TSO-DSO flexibility cost allocation [22]. In this work, Cooperative Game Theory is leveraged to estimate the contributions of flexible units to the aggregated flexible capacity of ADN (capacity-based ranking) and to the economic surplus of flexibility provision (surplus-based pricing). The applicability of the proposed framework is demonstrated through extensive simulations for a well-known 33-bus radial test system.

Specifically, the paper makes the following contributions:

- A novel framework is introduced for tracing, ranking, and pricing flexible power in distribution networks. It enables characterising flexibility with several metrics derived from the cost-minimising OPF models, while the combinatorial nature of the flexible power provision is reflected through the cooperative game formulation. The framework can be used by DSOs to identify the most critical flexible units or remunerate units for participating in the flexibility services provision.

- It is demonstrated that flexible units exhibit nonlinear complex behaviour when providing aggregated flexibility in ADN. Some units can be required to shift their power output rapidly to perform active network management and voltage control. The flexible power swap phenomenon is discovered, which happens when different units simultaneously produce and consume flexible power to alleviate network constraints and maximise network flexibility. This behaviour poses challenges for both operation and pricing of flexible units.

- The simulations illustrate that the game-theoretic approach to flexible unit pricing (economic surplus allocation according to the Shapley value) can be advantageous over the classical cost-minimising approach: units get additional incentives to participate in the flexibility market and declare their maximum capability at a lower cost.

**II. Modelling Framework: Approaches and Metrics to Characterise Network Flexibility**

**A. Network Flexibility as a Set of Feasible Operating Points**

The currently established approach to analysing the flexibility of an ADN lies in estimating the set of feasible operating points at the reference bus where the flexible power from different units is aggregated, e.g., at the TSO-DSO interface [8–11]. The boundary of the feasible set can be approximated by solving a series of optimisation models that impose network constraints and DER limits on the flexible power provision. A generalised formulation of optimisation models for feasibility boundary estimation can be described as given in [1a–1d].

The variables of the model are the active and reactive power flows, $p_i$, $q_i$, bus voltages, $V_k$, and flexible power produced or consumed by flexible units, $p_{k,f}$, $q_{k,f}$, where...
MODEL 1 Boundary of the network feasibility set

\[
\begin{align*}
\text{min} & \quad u_p \varepsilon_{\text{ref}} P_{\text{ref}} + u_q \varepsilon_{\text{ref}} Q_{\text{ref}} \\
\text{s.t.} & \quad \text{power balance constraints} \\
& \quad \forall k \in \mathcal{K} \\
& \quad \text{network constraints} \\
& \quad \forall k \in \mathcal{K}, \forall l \in \mathcal{L} \\
& \quad p_{k,f}, q_{k,f} \in \mathcal{S}_{k,f} \\
& \quad \forall k \in \mathcal{K}, \forall f \in \mathcal{F}
\end{align*}
\]

\(k \in \mathcal{K}\) and \(l \in \mathcal{L}\) are the sets of network buses and lines, and \(f \in \mathcal{F}\) denotes the set of flexible units located in the network. The objective function (1a) minimises or maximises network’s power consumption at a selected reference bus. The coefficients \(u_p \varepsilon_{\text{ref}}\) and \(u_q \varepsilon_{\text{ref}}\) are the weights introduced to control the optimisation directions by setting the ratio between network’s active and reactive power consumption, \(P_{\text{ref}}, Q_{\text{ref}}\). Equations (1b) and (1c) impose power balance and network constraints, which depend on the selected power flow formulation. Several different power flow formulations have been used in the literature to describe the flexibility of distribution networks, such as linearised OPF models \([10]\), nonlinear AC OPF models in polar and rectangular voltage coordinates \([9], [11]\), the DistFlow OPF formulation for radial networks \([12]\). The latter formulation will be used in Section III to perform tracing, ranking, and pricing of flexibility in a radial distribution network, where problem (1a)-(1d) is cast as a quadratically constrained linear program (QCLP). Finally, the active and reactive power of flexible units is limited by their P-Q capability sets in (1d).

The optimisation model (1a)-(1d) can be solved iteratively to approximate the boundary of the network feasibility area at the reference bus with the desired level of granularity \(M\). However, this approach does not provide additional information on the operating conditions within the area, e.g., what units have to be activated, what level of coordination is required to reach certain operating points, and how to remunerate units for providing different flexibility requests. Moreover, the boundary-estimation model cannot incorporate additional metrics of flexibility, such as the cost and economic surplus of flexible power provision. These issues are addressed with more advanced models and tools in the following subsections.

B. Network Flexibility as a Cost-minimising OPF Problem

The flexibility of a distribution network can also be characterised by the cost of flexible units activation \([9], [31]\). A cost-minimisation model can identify the cheapest units to be activated for providing a specific flexible power request, naturally capturing their contributions to the network aggregated flexibility. A generalised formulation of such models can be described as given in (2a)-(2d).

To estimate the cost of flexible power provision, the model contains variables representing upward and downward regulations of flexible units, \(p_{k,f}^\uparrow, q_{k,f}^\uparrow, q_{k,f}^\downarrow\). These regulations are the shifts in active and reactive power of the units compared to the initial states, as defined in (2b)-(2c). The initial state of each unit, \(p_{k,f}^0, q_{k,f}^0\), corresponds to its power production or consumption before the control actions required to meet a flexibility request. For example, the initial state might correspond to the regimes where DER and demand response programs are not activated. Equations (2d)-(2f) forbid simultaneous consumption and production of flexible power for each unit by constraining the corresponding integer decision variables. The objective function (2a) minimises the costs of upward and downward regulations for all flexible units. Note that the units’ cost functions \(C_p^k, C_q^k\) can differentiate the cost of producing and consuming flexible active or reactive power. The feasibility constraints in (2g) correspond to Model (1a)-(1d) and define a feasible operating point.

The cost-minimising model (2a)-(2g) enables to analyse network flexibility at any feasible operating point, both at the flexibility area boundary and within the boundary. A solution to (2a)-(2g) provides information on the components of flexible power for a given flexibility request, i.e., the flexible power outputs of the units, \(p_{k,f}^\uparrow, p_{k,f}^\downarrow, q_{k,f}^\uparrow, q_{k,f}^\downarrow\). Therefore, this model can serve as a tool for tracing, ranking, and pricing network flexibility, where the cheapest flexible units get activated subject to network constraints. Moreover, the cost-based formulation enables estimating additional metrics to characterise network flexibility. For example, the optimised objective function (2a) provides the total minimum cost of meeting a flexibility request:

\[
F_{\text{cost}} = \sum_{k \in \mathcal{K}} \sum_{f \in \mathcal{F}} \left( C_p^k (p_{k,f}^\uparrow, p_{k,f}^\downarrow) + C_q^k (q_{k,f}^\uparrow, q_{k,f}^\downarrow) \right)
\]

\(k \in \mathcal{K}, f \in \mathcal{F}\) are the sets of network buses and lines, and \(f \in \mathcal{F}\) denotes the set of flexible units located in the network. The objective function (2a) minimises or maximises network’s power consumption at a selected reference bus. The coefficients \(u_p \varepsilon_{\text{ref}}\) and \(u_q \varepsilon_{\text{ref}}\) are the weights introduced to control the optimisation directions by setting the ratio between network’s active and reactive power consumption, \(P_{\text{ref}}, Q_{\text{ref}}\). Equations (1b) and (1c) impose power balance and network constraints, which depend on the selected power flow formulation. Several different power flow formulations have been used in the literature to describe the flexibility of distribution networks, such as linearised OPF models \([10]\), nonlinear AC OPF models in polar and rectangular voltage coordinates \([9], [11]\), the DistFlow OPF formulation for radial networks \([12]\). The latter formulation will be used in Section III to perform tracing, ranking, and pricing of flexibility in a radial distribution network, where problem (1a)-(1d) is cast as a quadratically constrained linear program (QCLP). Finally, the active and reactive power of flexible units is limited by their P-Q capability sets in (1d).

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\[
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\]

(3)

Assume that there exists a TSO-DSO flexibility market in which TSO pays DSO for active and reactive power control at the reference bus, \(\Delta P_{\text{ref}}, \Delta Q_{\text{ref}}\). If these payments can be described by a market price function \(\pi_{\text{ref}}\), then the economic surplus of flexibility provision is the difference between the payment and the optimised cost of flexible power:

\[
F_{\text{surplus}} = \pi_{\text{ref}} (\Delta P_{\text{ref}}, \Delta Q_{\text{ref}}) - F_{\text{cost}}
\]

(4)

This market assumption, used to estimate the economic surplus of flexibility provision, may not be true for some cases and countries. Nevertheless, the presented ideas of flexibility ranking and pricing are general and can be adapted to different TSO-DSO coordination schemes.
These metrics are useful for interpreting network flexibility since they capture both the flexible power outputs of the units and their economic impacts, such as the cost and the economic surplus of power provision. However, the cost-minimising OPF problem formulation is biased as it selects only the cheapest flexible units and does not consider all possible contributions of other units to network flexibility. In this regard, the following subsection introduces the cooperative game formulation of network flexibility, where different possible combinations of flexible units and their contributions are included in the tracing, ranking, and pricing mechanisms.

C. Network Flexibility as a Cooperative Game among Flexible Units

The flexibility of a distribution network with multiple flexible units has an inherent combinatorial nature: some flexible power requests can be ensured by any of the available flexible units, whereas other requests require different combinations of units (even all the units) to be activated. Therefore, a comprehensive analysis of network flexibility should account for possible combinations of units and their contributions to the flexible power provision. Such an analysis can be performed using the well-established tools from Cooperative Game Theory. This section introduces the main game-theoretic concepts that can be used for tracing, ranking, and pricing flexibility in distribution networks. A more thorough description of cooperative games and Cooperative Game Theory solution concepts can be found in [32], [33].

Assuming that simultaneous activation of flexible units located in a distribution network brings some value of cooperation, which can be divided and transferred between the units, a cooperative game \( (N; v) \) with transferable utility can be defined as follows:\(^4\)

- \( N \) is a finite set of players (flexible units available in the network for flexible power provision). A subset of \( N \) is called a coalition. The largest possible coalition containing all players is called the grand coalition. As further demonstrated in this work, the grand coalition provides the greatest amount of available flexible power. The collection of all coalitions is denoted by \( 2^N \).
- \( v : 2^N \rightarrow \mathbb{R} \) is the characteristic function associating each coalition \( S \) with a real number \( v(S) \), a metric describing the value of a coalition. In this work, various metrics will be used to describe the value of coalitions, such as the limits of the aggregated network flexibility, the cost of flexible power provision, and its economic surplus.

Any coalition of flexible units, \( S \), can be described by the values obtained from the network flexibility models \([13, 14]\) and \([23, 24]\). For example, the flexible power provided by each coalition can be estimated as the optimised network power consumption at the reference bus, \( P_{\text{ref}}, Q_{\text{ref}} \), in \([1a]\). The cost of a coalition, \( F_{\text{cost}}(S) \), and the economic surplus, \( F_{\text{surplus}}(S) \), can be estimated as defined in \([3] \) and \([4]\) after solving the cost-minimising OPF problem \([2a] - [2b]. Then, the cooperative game formulation can be used to allocate the value of the grand coalition, \( v(N) \), among the units, thus ranking them according to the selected metric.

The crucial measure of a player’s impact in a cooperative game is the marginal contributions to the coalitions the player can join. The marginal contribution to coalition \( S \) by player \( i \) is estimated as the difference in the value of the coalition with and without the player:

\[
MC(S)_i = v(S \cup \{i\}) - v(S) \quad \forall i \in S \quad \forall S \subseteq N
\]

Then, the allocation of the grand coalition value, \( v(N) \), to player \( i \) can be found as the weighted average of player’s marginal contributions to all possible coalitions, as defined by the Shapley value formula:

\[
Sh_i = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(|N| - |S| - 1)!}{|N|!} MC(S)_i
\]

The weight of each coalition depends on the number of players in the coalition, \(|S|\), and the total number of players in a cooperative game, \(|N|\). The Shapley value has been acknowledged as a useful tool in cost allocation mechanisms, data analysis, and power systems research \([22], [24] - [30]\). Several desirable properties of the Shapley value make it suitable for tracing, ranking, and pricing flexibility in distribution networks. The symmetry property guarantees that two identical flexible units that bring equal contributions to coalitions will always be allocated the same value. The null player property states that a unit that contributes nothing to any coalition will not receive a share of the grand coalition value. The efficiency property requires that the sum of the values allocated to all players is equal to the value of the grand coalition.

In this work, the Shapley value is exploited to interpret components of network flexibility according to different metrics. It is also demonstrated that the Shapley value can be used as an allocation and remuneration mechanism for flexible units.

III. Simulation Results

A. Case Study: 33-bus Radial Distribution Network

The proposed methodology is demonstrated with the IEEE 33-bus test system, which is a 12.66kV radial distribution network. The total power demand of the network consumers is 3.7 MW and 2.3 MVar. The 33-bus system is visualised in Fig. 2 with the force-directed graph layout algorithm ForceAtlas2 \([34]\). The figure illustrates the network topology, power demands (as circles of different sizes), network voltage profile (nodes colouring), and electrical distances between the buses (lengths of the arcs). Note that the voltage levels at buses 18 and 33 are close to the lower limit of 0.9 p.u., which creates additional constraints on the network power consumption increase\(^5\). As further demonstrated by the simulations, such constraints can vastly affect the flexible power provided by units located in different parts of the network.

\(^4\)Note that the representation of flexible units as players joining coalitions is an abstract notion used to describe the combinatorial problem of joint flexible power provision by multiple units.

\(^5\)In the original 33-bus distribution system, the voltage limits are set to 0.9 p.u. and 1.1 p.u. However, more realistic cases can have tighter voltage constraints. For example, in the UK distribution networks, voltage deviations are limited to ±6% of the nominal voltage. Regardless, the selection of the voltage limits does not alter the findings of this paper.
To explore the constraints that voltage limits can have on the provision of flexibility, four flexible units are placed in the network (one at the end of each feeder). The four units are assumed to have identical P-Q capabilities: \( p_{i,f} \in [-500, 500] \) kW, \( q_{i,f} \in [-500, 500] \) kVAR, where \( i \in \{18, 22, 25, 33\} \) are the buses that the units are connected to. To ease the notations, the flexible units will be referred to as A, B, C, and D, as indicated in Fig. [2]. The initial states of the units, \( p^0_{i,f}, q^0_{i,f} \), correspond to their not activated condition, i.e., units neither produce nor consume flexible power. Bus 1, the TSO/DSO interface, is considered as the reference bus, where the flexibility from the units is aggregated. To investigate the applicability of pricing mechanisms, it is assumed that the units have different costs of providing flexible active power: 375, 350, 325, and 300 $/MWh for units A, B, C, and D, respectively. Such costs are comparable to balancing market prices and are aligned with the cost assumptions in recent studies [31], [35]. The cost of flexible reactive power is assumed to be 50% of the flexible active power cost for each unit. The price that TSO pays for flexible power in the TSO-DSO flexibility market is set at 400 $/MWh and 200 $/MVArh. Thus, the cost and the economic surplus of providing flexibility can be estimated as defined in (3) and (4).

### B. Structure of the Aggregated Network Flexibility

First, the feasibility boundary estimation model (1a)-(1d) is applied to analyse the aggregated network flexibility, which is the set of network feasible operating points when all flexible units are activated and fully coordinated. This flexibility is displayed in Fig. 2 as the area reached by the set of units \( \{A,B,C,D\} \), the grand coalition. Note that network flexibility at the TSO/DSO interface is more complex than a linear combination of units’ P-Q capabilities (Minkowski addition).

The resulting flexibility area has a nonlinear boundary due to the nonlinearities of the power flow model, such as the presence of power losses and voltage constraints. The aggregated network flexibility can be decomposed into the P-Q capabilities of individual flexible units and their combinations. This decomposition corresponds to the coalitional structure of the cooperative game among units. A cooperative game with 4 players (units) consists of 15 possible coalitions, as illustrated in Fig. 3. Model (1a)-(1d) was iteratively solved for each coalition to approximate the flexibility areas’ boundaries. Synergy can be observed in the flexible unit coordination: units provide much more flexibility in large coalitions than in smaller coalitions or when being activated individually.

Note that even though being identical, the flexible units offer different individual flexibility areas. This is caused by locational effects, such as power losses and voltage constraints. For example, units C and D are located at the edges of feeders with low voltage profiles (buses 33 and 18). These units cannot increase their power consumption significantly due to the voltage limits, which results in reduced flexibility areas of coalitions \( \{C\} \) and \( \{D\} \). It can also be seen how these constraints propagate in the coalitional structure once more players join the coalitions with units C and D.

The presented coalitional structure illustrates the maximum P-Q capacities of the units and the impact of network constraints. However, the flexibility areas estimated by model (1a)-(1d) do not provide additional information on the operating conditions within the area, e.g., what units have to be activated, what level of coordination is required to reach certain operating points, and how to remunerate units for providing different flexibility requests. In this regard, in the rest of this section, the cost-minimising OPF model and the game-theoretic model are implemented to estimate the contributions of flexible units to specific flexible power requests.

### C. Allocation of Flexible Power Requests: a Cost-minimising OPF Approach

The allocation of flexible power among the units can be explicitly derived from the cost-minimising OPF model (2a)-(2g) solved for any feasible flexible power request. The model indicates which units provide flexible power to meet the request while trying to activate the cheapest units first. Such allocations can differ significantly depending on the requested operating point in the P-Q space. Therefore, model (2a)-(2g) was solved 14,520 times for different feasible flexibility requests (the feasible P-Q space was discretised by a grid with step 0.03 MVA). The resulting allocations are displayed in Fig. 4 as a percentage of the total apparent flexible power provided (in MVA). Note that many low-magnitude flexible power requests (close to the initial operating point) can be fully covered by unit D, which provides flexible power at the lowest cost. Therefore, for such requests, unit D is allocated up to 100% of the requested power and should be paid much more than the other units. On the contrary, unit A has the highest

\[ \text{Note that unit D is allocated less power than other units for the flexibility requests that increase the active and reactive power consumption of the network (the upper right side of the area). These differences stem from the voltage limitation at bus 18, which reduces the P-Q capability of unit D.} \]
The grand coalition:

Coalitions of three flexible units:

Coalitions of two flexible units:

Coalitions of one flexible unit:

Fig. 3. Coordinational structure of the cooperative game among four flexible units. Each coalition is characterised by the aggregated network flexibility area in the P-Q space at the TSO/DSO interface. The markers correspond to the initial operating point of the network, while the coordinates represent the network’s power consumption.

cost and is not activated for many low-magnitude flexible power requests. It is activated and paid only for requests close to the flexibility area limits. Thus, under the cost-minimising OPF approach, the cheapest units get activated more often and are allocated more power and payments.

It can be observed from the cost-minimising allocation results that flexible power provision from several units has a highly nonlinear behaviour. There exist multiple shifts in the flexible power output of the units. These shifts occur due to both technical and economic reasons amplified by the nonlinearity of the network power flow model. For example, due to power losses and reactive power management, providing flexible power only by the cheapest units may not be the optimal solution for some operating points. Thus, for such points, model (2a)-(2g) shifts a share of flexible power between the units to offer the least-cost solution. Moreover, the analysis of flexible power allocations reveals the power swap phenomenon that happens when multiple units provide flexible power under network constraints. Specifically, some units can be producing flexible power, whereas other units consume flexible power. Such power swaps enable alleviating network constraints and reaching the limits of the network flexibility area. For example, in the 33-bus test system, a significant increase in the network power consumption cannot be achieved by increasing the power consumption of all flexible units since units C and D already operate close to the lower voltage limit of 0.9 p.u. Therefore, to reach the operating points close to the flexibility area boundary, model (2a)-(2g) suggests solutions where units C and D produce power. This enables alleviating the voltage limits at buses 33 and 18, whereas the remaining units consume flexible power to increase the network power consumption and meet the flexibility request.

The power swap phenomenon has not been analysed in the existing literature on the flexibility of distribution networks. Most of the studies imply perfect coordination of flexible units and focus on estimating the limits of the aggregated network flexibility, without specifying what actions are needed to reach the boundaries. However, the assumption of perfect
unit coordination may not be realistic and requires further research, e.g., rapid shifts in flexible power allocation between close operating points impose additional ramp constraints on the unit operation. Using power swaps between flexible units to provide certain flexibility requests can also be controversial:

- From the operating standpoint, it can be more complex and less reliable to control flexible units working in different directions (producing and consuming power). For example, a failure of unit C or D to produce enough power while the overall network consumption is increased can result in voltage collapse of the distribution network.
- From the economic standpoint, power swaps lead to inconsistent solutions and issues with flexible units remuneration. For example, units can use much more flexible power due to simultaneous consumption and production than the total flexible power requested from DSO.

It is possible to forbid the power swap between flexible units by adding constraints similar to (2b)-(2i) to the OPF model. The no-swap constraints contain global binary variables that restrict all units to either produce or consume power. The flexibility area with no flexible power swap is indicated in Fig. 4 by the solid line, while the entire flexibility area is denoted by the dashed line. It appears that network flexibility can be overestimated if not considering the issues related to flexible power swap. Moreover, imposing the no-swap constraints makes the aggregated flexibility areas nonconvex. Such nonconvexity could cause additional problems with distribution networks operation. For instance, a path between sequential operating points can contain infeasible points that require swapping power between flexible units.

It follows that a purely cost-based analysis of network flexibility has several disadvantages. First, cost-minimising models consider only a single outcome of flexibility provision for each operating point. Other possible combinations of units and their contributions are neglected. Therefore, such models cannot be used to comprehensively rank units and assess their criticality. Second, pricing mechanisms based on cost-minimising models always favour the cheapest units and do not give incentives for more expensive units to participate in the flexibility market. In this regard, the next subsection illustrates the advantages of the proposed game-theoretic approach, which captures possible contributions of units and enables including additional metrics of flexibility provision.

D. Allocation of Flexible Power Requests: a Game-Theoretic Approach

Any feasible flexible power request can be analysed using the cooperative game framework, where the grand coalition fully provides the required flexible power, and the subcoalitions maximise their flexible power provision in the required direction in the P-Q space. As discussed in Section II-C, different metrics can be used to characterise coalitions, such as the flexible power provided, its cost, or its economic surplus. Then, the Shapley value can be applied to estimate the contributions of units and perform their ranking and pricing. This subsection presents simulations for multiple flexible power requests based on the game-theoretic approach and analyses the allocation of flexibility using different metrics.

First, the aggregated apparent flexible power (in MVA) is selected as the metric to characterise coalitions, i.e., each coalition is represented by the P-Q limits that flexible units can reach for a given operating point and power factor. This capacity-based metric enables ranking units by their contributions to flexibility requests and identifying the most critical units. The cooperative game is solved using the Shapley value for multiple feasible flexibility requests, as displayed in Fig. 5. The results indicate that for low-magnitude requests, the units are equally useful and get allocated 25% of the requested flexible power. This happens because the units can provide the same flexible power for such requests and bring equal contributions to the coalitions. Thus, they are symmetric players in the cooperative game. However, a significant divergence in the contributions happens for flexibility requests that increase the power consumption of the network. For such requests, units C and D cannot contribute much since their ability to increase power consumption is limited due to the voltage constraints. These limitations are captured by the cooperative game formulation and result in the lower ranking for units C and D. Note that such differences stem from the individual P-Q capabilities of the units, as previously illustrated by the coalitional structure in Fig. 3.

In this context, the criticality of a flexible unit can be seen as the priority ranking of its contributions to a given flexibility request. More critical units contribute the most to flexible power provision and therefore have the greatest impact on the flexibility service reliability. Inaccurate forecasting or unavailability of such units can result in the infeasibility of the related operating points.
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Fig. 5. Allocation of the total apparent flexible power among the units for different flexibility requests according to the Shapley value, in %.

Fig. 6. Allocation of the flexibility service economic surplus among the units for different flexibility requests according to the Shapley value, in %.

Second, the economic surplus of providing flexible power (in $/h) is selected to characterise coalitions. This is a complex metric that captures both the P-Q capabilities of the units and their economic impact, i.e., the potential profits that they bring when providing flexible power. The resulting economic surplus allocations for multiple flexibility requests obtained by the Shapley value are displayed in Fig. 6. These simulations reflect the cost causality in flexibility pricing and indicate which units contribute to the surplus of the flexible power provision. For instance, unit D can produce flexible power at the lowest cost and contributes the most to the surplus of the flexible power provision, except in cases of the network consumption increase, where unit D cannot provide much flexible power.

The allocation solutions estimated by the game-theoretic approach for different metrics provide a variety of options to rank flexible units and remunerate them. The capacity-based allocation estimates the criticality of units for a specific flexibility request. The surplus-based game-theoretic approach explicitly considers the cost of flexible power provision and estimates the economic impact of flexible units. This approach can serve as the remuneration mechanism: it allocates more economic surplus to units that provide more flexible power at lower costs and get activated in many possible coalitions. Note that unlike the allocations obtained by the cost-minimising OPF model, the solutions provided by the game-theoretic approach have no operating points where one of the units is allocated 100% of the cooperation value. This happens since the cooperative game formulation considers not only the flexible power of units in the grand coalition but also their contributions to all possible subcoalitions. The advantages and the applicability of the proposed game-theoretic framework are further discussed in Section IV.

IV. DISCUSSION

A. Potential Applications

The above simulations demonstrate how the proposed framework can be used to meet various needs of DSOs. The capacity-based analysis of network aggregated flexibility enables DSOs to identify the most critical units for certain flexibility requests. The cost-based and surplus-based approaches incorporate more complex metrics and capture both the outputs of flexible units activated and their contributions to the value of cooperation. Such approaches can be used not only for ranking the usefulness of units but also for flexibility pricing mechanisms. The cost-minimising OPF model (2a)-(2g) offers straightforward solutions where the cheapest units are activated and paid first. Despite its simplicity, this model does not account for possible contributions of more expensive units and does not consider scenarios where the cheapest units may not be available. In this regard, the allocation of the flexibility service economic surplus among flexible units according to the Shapley value can constitute a more promising pricing mechanism.
To support the Shapley-based pricing mechanism, Fig. 7 presents the comparison of the payments allocated to flexible units with the payments obtained from the cost-minimising OPF model. A thousand randomly generated flexibility requests were simulated. A normal probability distribution was assumed with the mean set at the initial operating point of the network and the standard deviation of 0.6 MVA. The simulations reflect realistic flexibility requests, where low-magnitude flexible power deviations are requested more often than high-magnitude ones. According to the cost-minimising OPF model, the cheapest unit D is activated and paid for each of the simulated flexibility requests. Other units provide flexibility less frequently and receive far fewer payments. Unit A, the most expensive one, is called in only about 1/10 of all flexibility requests. Such payments might not incentives unit A to participate in the flexibility market. However, this unit is still valuable for the provision of flexibility. Its contributions are considered in the Shapley-based pricing mechanism, which always allocates to unit A a share of the flexibility service economic surplus. In this way, more expensive units get additional incentives to participate in the market and declare their real P-Q capabilities and costs. A thorough analysis of the Shapley-based flexibility pricing, its incentive compatibility, and manipulability is the subject of future research.

B. Scalability and Applicability Issues

Unfortunately, the cooperative game formulation is prone to scalability issues. The number of possible coalitions in a cooperative game, $2^N$, increases exponentially with the number of players $N$ (flexible units available in a network). To describe each coalition, model (2a)-(2g) has to be solved, and the selected metric should be derived. Therefore, for games with hundreds or thousands of units, it becomes intractable to consider all coalitions and implement the Shapley value formula (6) directly. In this regard, recent advances in clustering and decomposition techniques can be used to approximate the Shapley value [36], [37]. The idea behind such approximations is to truncate a cooperative game by considering a limited number of coalitions. However, the coalitions should be selected wisely so the approximation error is minimal. For example, in [38], compressed sensing was used to approximate the Shapley value for solving data valuation problems. Additionally, clustering techniques can be deployed to group flexible units with similar contributions and reduce the number of players in a cooperative game.

The practical limit of exact enumeration of possible coalitions and implementation of the Shapley value formula (6) is 10 flexible units, which requires solving 1023 OPF problems. Considering the performance of modern algorithms and solvers, each OPF can be solved in seconds or even fractions of a second for an average distribution network. Thus, with a single computer, the proposed game-theoretic flexibility pricing mechanism can take up to 20-30 minutes to allocate payments for flexible units. For larger numbers of units, the mentioned approximation and clustering techniques should be applied to estimate the allocations by solving only a few thousand OPF problems. Moreover, since the OPF problems for different coalitions of units are independent, the allocation process can be sped up using parallel computing. Therefore, applying the proposed pricing mechanism to flexibility pricing in intraday 30-minute and hourly markets should be realistic.

V. CONCLUSION

This work investigates the formation of network aggregated flexibility and proposes a framework for tracing, ranking, and pricing flexible power within ADNs. The framework exploits cost-minimising OPF models to estimate the flexible power of units activated for a flexibility service request, as well as the associated cost and economic surplus. To deal with the combinatorial nature of multiple units activation, the cooperative game formulation is proposed that captures possible contributions of units to flexibility requests. The extensive simulations performed for numerous feasible operating points of the 33-bus radial test system demonstrate the effectiveness of the proposed game-theoretic approach. The simulations also illustrate the principles of aggregated flexibility formation and the effects of network constraints. It is found that major differences in the contributions of flexible units stem from their individual P-Q capabilities, which could be reduced by voltage limits and other technical constraints. The flexible
power swap effect is discovered, which happens when different units simultaneously produce and consume flexible power to alleviate network constraints and maximize the flexibility service provided by a distribution network.

The proposed framework incorporates different metrics of flexibility and can be used by DSOs in the following applications. First, ranking flexible units by their contributions to the aggregated network flexibility identifies the most critical units in the network and provides information on the structure and diversification of flexible resources. Second, cost-based and surplus-based ranking can serve as a remuneration mechanism for flexible units. As discussed in the paper, the surplus-based allocation mechanism can give flexible units incentives to declare their maximum capability at a lower cost. However, the combinatorial nature of flexible power aggregation makes the cooperative game formulation intractable for cases with hundreds or thousands of flexible units. Future research will try to overcome these limitations by using advanced clustering and compression techniques, Shapley value approximations, and cooperative game decompositions.

REFERENCES

[1] C. Eid, P. Codani, Y. Perez, J. Reneses, and R. Hakvoort, “Managing electric flexibility from Distributed Energy Resources: A review of incentives for market design,” Renewable Sustain. Energy Rev., vol. 64, 2016.
[2] S. Chowdhury, S. P. Chowdhury, and P. Crossley, Microgrids and active distribution networks. IET, 2009.
[3] H. Gerard, E. I. Rivero Puente, and D. Six, “Coordination between transmission and distribution system operators in the electricity sector: A conceptual framework,” Utilities Policy, vol. 50, 2018.
[4] A. Vicente-Pastor, J. Nieto-Martin, D. W. Bunn, and A. Laur, “Evaluation of flexibility markets for retailer-DSO-TSO coordination,” IEEE Trans. Power Syst., vol. 34, no. 3, 2019.
[5] T. Schittekatte and L. Meeus, “Flexibility markets: Q&A with project pioneers,” Utilities Policy, vol. 63, 2020.
[6] H. Le Cadre, I. Mezghani, and A. Papavasiliou, “A game-theoretic analysis of transmission-distribution system operator coordination,” Eur. J. Oper. Res., vol. 274, no. 1, 2019.
[7] A. G. Givisiez, K. Petrov, and L. F. Ochoa, “A Review on TSO-DSO Coordination Models and Solution Techniques,” Electr. Power Syst. Res., vol. 189, 2020.
[8] M. heleno, R. Soares, J. Sumaili, R. J. Bessa, L. Seca, and M. A. Matos, “Estimation of the flexibility range in the transmission-distribution boundary,” in Proc. IEEE Eindhoven PowerTech, 2015.
[9] J. Silva, J. Sumaili, R. J. Bessa, L. Seca, M. A. Matos, V. Miranda, M. Caujolle, B. Goncer, and M. Sebastian-Viana, “Estimating the Active and Reactive Power Flexibility Area at the TSO-DSO Interface,” IEEE Trans. Power Syst., vol. 33, no. 5, 2018.
[10] D. A. Contreras and K. Rudion, “Improved assessment of the flexibility range of distribution grids using linear optimization,” in Proc. 20th Power Systems Computation Conference, PSCC, 2018.
[11] F. Capitanescu, “TSO-DSO interaction: Active distribution network power chart for TSO ancillary services provision,” in Proc. 20th Power Systems Computation Conference, PSCC, 2018.
[12] L. Lopez, A. Gonzalez-Castellanos, D. Pozo, M. Rozobehani, and M. Dahleh, “QuickFlex: a Fast Algorithm for Flexible Region Construction for the TSO-DSO Coordination,” in 2021 International Conference on Smart Energy Systems and Technologies (SEST), 2021.
[13] M. Bolek and T. Capuder, “An analysis of optimal power flow based formulations regarding DSO-TSO flexibility provision,” Int. J. Electr. Power Syst., vol. 131, 2021.
[14] D. A. Contreras and K. Rudion, “Computing the feasible operating region of active distribution networks: Comparison and validation of random sampling and optimal power flow based methods,” IET Generation, Transmission and Distribution, vol. 15, no. 10, 2021.
[15] S. Stankovic and L. Soder, “Probabilistic Reactive Power Capability Charts at DSOTSO Interface,” IEEE Trans. Smart Grid, vol. 11, no. 5, 2020.