New physics contributions to $\bar{B}_s \to \pi^0(\rho^0)\eta^{(')}$ decays

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Abstract

We study the decay modes $\bar{B}_s \to \pi^0(\rho^0)\eta^{(')}$ within the frameworks of several new physics models. These models include two-Higgs doublet models type-II and type-III and a model with an additional $U(1)'$ gauge symmetry. We adopt in our study Soft Collinear Effective Theory as a framework for the calculation of the amplitudes. We find that, within two-Higgs doublet models type-II and type-III, the enhancement in the branching ratios does not exceed 24% with respect to the total predictions including the standard model contributions. Within a model with an additional $U(1)'$ gauge symmetry, we find that sizeable enhancement can be obtained in both scenarios of the model. The new contributions in this case enhances the branching ratio of $\bar{B}_s \to \eta\rho^0$ from its predicted value $3 \times 10^{-8}$ in the standard model to a total value $1.2 \times 10^{-7}$. For the decay channel $\bar{B}_s \to \eta\pi^0$ this enhancement increases the branching ratio from $3 \times 10^{-8}$ within standard model to $1 \times 10^{-7}$ and from $6 \times 10^{-8}$ within standard model to $1.4 \times 10^{-7}$ for the branching ratio of $\bar{B}_s \to \eta'\rho^0$ decay channel.

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I. INTRODUCTION

Flavor-changing neutral current (FCNC) decays can be generated at the one loop level in the Standard Model (SM). As a consequence they are highly suppressed and thus they are best candidate to probe New Physics (NP) beyond the SM. These decays include the purely isospin-violating decays $\bar{B}_s \to \phi \pi (\rho^0)$, $\bar{B}_s \to \pi \eta (\eta')$ and $\bar{B}_s \to \rho^0 \eta (\eta')$ that are dominated by electroweak penguins\cite{1,4}.

The decay modes $\bar{B}_s \to \phi \pi (\rho^0)$ have been studied within SM in different frameworks such as QCD factorization as in Refs.\cite{5,6}, in PQCD as in Ref.\cite{7} and using Soft Collinear Effective Theory (SCET) as in Refs.\cite{8,9}. The study has been extended to include NP models namely, a modified $Z^0$ penguin, a model with an additional $U(1)'$ gauge symmetry and the MSSM using QCDF\cite{6}. In addition, the investigation of NP in these decay modes has been recently extended to include supersymmetric models with non-universal A-term\cite{9} and two Higgs doublet models (2HDMs)\cite{10} using SCET. The results of these studies showed that the additional $Z'$ boson of the $U(1)'$ gauge symmetry with couplings to leptons switched off can lead to an enhancement in their branching ratios up to an order of magnitude making these decays are very interesting for LHCb and future $B$ factories searches\cite{6}.

Motivated by this finding we study NP effects to the other decay modes $\bar{B}_s \to \pi \eta (\eta')$ and $\bar{B}_s \to \rho^0 \eta (\eta')$. These decay modes have been studied within SM using different frameworks such as Naive Factorization (NF)\cite{2}, generalized factorization \cite{3}, POCD\cite{7,11} and QCDF\cite{4,12}. On the other hand, using SCET, an investigation of $\bar{B}_s \to \pi \eta (\eta')$ has been carried out in Ref.\cite{13} while the decay modes $\bar{B}_s \to \rho^0 \eta (\eta')$ has been studied in Ref.\cite{8}. NP effects namely 2HDMs has been investigated in these decay modes in Ref.\cite{2} using NF and using generalized factorization in Ref.\cite{14}. In the present work we will extend the study to include other NP models such as a model with an additional $U(1)'$ gauge symmetry and two Higgs doublet models (2HDMs) with general Yukawa couplings that has been recently introduced in Refs.\cite{15,16}. In our study we will adopt SCET as a framework for the calculation of the amplitudes\cite{17,20}.

SCET provides a systematic and rigorous way to deals with the processes in which energetic quarks and gluons have different momenta modes such as hard, soft and collinear modes. The power counting in SCET reduces the complexity of the calculations. In addition, the factorization formula given by SCET is perturbative to all powers in $\alpha_s$ expansion.
This paper is organized as follows. In Sec. II we review the decay amplitude for $B \to M_1M_2$ within SCET framework. Accordingly, we give the SM predictions to the branching ratios of the decay modes under the study. Then we proceed to analyze NP contributions in the case of Two Higgs-doublets models type II, type III and non universal $Z'$ model in section III. Finally, we give our conclusion in Sec. IV.

II. $B \to M_1M_2$ IN SCET

At leading order in $\alpha_S(m_b)$ expansion, the amplitude of $B \to M_1M_2$ where $M_1$ and $M_2$ are light mesons can be written as [13]

$$A_{B\to M_1M_2} = \frac{G_F}{\sqrt{2}} m_B^2 \left\{ f_{M_1} \zeta_{BM_2} \int du \phi_{M_1}(u) T_{1J}(u) + f_{M_1} \zeta_{BM_2} T_{1\zeta} + 1 \leftrightarrow 2 + \lambda^{(f)} A_{cc}^{M_1M_2} \right\}$$

(1)

where $A_{cc}^{M_1M_2}$ represents the non-perturbative long distance charm contribution which does not contribute to our decay modes. The expressions of the hard kernels $T_{iJ}(u)$ and $T_{i\zeta}$ in terms of the Wilson coefficients, for $i = 1, 2$, depend on the final states mesons $M_1$ and $M_2$. Their explicit expressions, for the decay modes under consideration, are listed in Table II in Refs. [13].

The hadronic parameters $\zeta_{BM_1}$ and $\zeta_{BM_2}$ are treated as non-perturbative parameters to be fit using the experimental data of the branching fractions and CP asymmetries of the non leptonic $B$ and $B_s$ decays [8, 13, 21, 22]. In our analysis, we use the values given in ref. [13] for $\zeta_{BM_1}$ and $\zeta_{BM_2}$ corresponding to the two solutions obtained from the $\chi^2$ fit and assume a 20% error in their values due to the SU(3) symmetry breaking. For the light cone distribution amplitudes we use the same input values given in ref. [22]. The amplitudes
corresponding to solution 1 of the SCET parameters are given as

\[ \mathcal{A}(\bar{B}_s^0 \to \eta \pi^0) \times 10^6 \simeq (4.3 C_{10} + 2.6 \tilde{C}_{10} - 5.2 C_7 + 5.2 \tilde{C}_7 + 0.8 C_8 - 0.8 \tilde{C}_8 + 5.2 C_9 + 0.1 \tilde{C}_9) \lambda_t^s + (-2.9 C_1 - 1.7 \tilde{C}_1 - 3.5 C_2 - 0.1 \tilde{C}_2) \lambda_u^s \]

\[ \mathcal{A}(\bar{B}_s^0 \to \eta' \pi^0) \times 10^6 \simeq (-0.5 C_{10} - 1.7 \tilde{C}_{10} - 1.0 C_7 + 1.0 \tilde{C}_7 - 1.2 C_8 + 1.2 \tilde{C}_8 + 1.0 C_9 - 2.7 \tilde{C}_9) \lambda_t^s + (0.3 C_1 + 1.2 \tilde{C}_1 - 0.6 C_2 + 1.8 \tilde{C}_2) \lambda_u^s \]

\[ \mathcal{A}(\bar{B}_s \to \eta \rho^0) \times 10^6 \simeq (4.6 C_{10} + 4.4 \tilde{C}_{10} + 12.1 C_7 - 12.1 \tilde{C}_7 + 1.0 C_8 - 1.0 \tilde{C}_8 + 4.4 C_9 + 4.0 \tilde{C}_9) \lambda_t^s + (-3.0 C_1 - 2.9 \tilde{C}_1 - 2.9 C_2 - 2.7 \tilde{C}_2) \lambda_u^s \]

\[ \mathcal{A}(\bar{B}_s \to \eta' \rho^0) \times 10^6 \simeq (-1.6 C_{10} - 1.4 \tilde{C}_{10} + 4.7 C_7 - 4.7 \tilde{C}_7 + 2.6 C_8 - 2.6 \tilde{C}_8 - 1.6 C_9 - 1.1 \tilde{C}_9) \lambda_t^s + (1.0 C_1 + 1.0 \tilde{C}_1 + 1.1 C_2 + 0.8 \tilde{C}_2) \lambda_u^s \]  

while for solution 2 of the SCET parameters we have

\[ \mathcal{A}(\bar{B}_s^0 \to \eta \pi^0) \times 10^6 \simeq (3.3 C_{10} + 1.4 \tilde{C}_{10} - 4.7 C_7 + 4.7 \tilde{C}_7 + 0.2 C_8 - 0.2 \tilde{C}_8 + 4.7 C_9 - 1.1 \tilde{C}_9) \lambda_t^s + (-2.2 C_1 - 0.9 \tilde{C}_1 - 3.1 C_2 + 0.7 \tilde{C}_2) \lambda_u^s \]

\[ \mathcal{A}(\bar{B}_s^0 \to \eta' \pi^0) \times 10^6 \simeq (-4.1 C_{10} - 6.1 \tilde{C}_{10} + 0.9 C_7 - 0.9 \tilde{C}_7 - 3.5 C_8 + 3.5 \tilde{C}_8 - 0.9 C_9 - 6.9 \tilde{C}_9) \lambda_t^s + (2.7 C_1 + 4.1 \tilde{C}_1 + 0.6 C_2 + 4.6 \tilde{C}_2) \lambda_u^s \]

\[ \mathcal{A}(\bar{B}_s \to \eta \rho^0) \times 10^6 \simeq (2.9 C_{10} + 3.2 \tilde{C}_{10} + 12.6 C_7 - 12.6 \tilde{C}_7 + 2.1 C_8 - 2.1 \tilde{C}_8 + 2.3 C_9 + 3.4 \tilde{C}_9) \lambda_t^s + (-1.9 C_1 - 2.2 \tilde{C}_1 - 1.6 C_2 - 2.3 \tilde{C}_2) \lambda_u^s \]

\[ \mathcal{A}(\bar{B}_s \to \eta' \rho^0) \times 10^6 \simeq (-6.4 C_{10} - 6.8 \tilde{C}_{10} + 2.8 C_7 - 2.8 \tilde{C}_7 + 5.5 C_8 - 5.5 \tilde{C}_8 - 5.6 C_9 - 6.9 \tilde{C}_9) \lambda_t^s + (4.3 C_1 + 4.6 \tilde{C}_1 + 3.7 C_2 + 4.6 \tilde{C}_2) \lambda_u^s \]  

where \( C_i \) and \( \tilde{C}_i \) are the Wilson coefficients that can be expressed as

\[ C_i = C_i^{SM} + C_i^{NP}, \quad \tilde{C}_i = \tilde{C}_i^{NP} \]  

\( C_i \) are the Wilson coefficients corresponding to four-quark operators in the weak effective Hamiltonian that can be obtained by flipping the chirality from left to right and so in the SM \( \tilde{C}_i^{SM} = 0 \).

The predictions for the branching ratios of \( \bar{B}_s^0 \to \eta^{(')} \pi^0 \) and \( \bar{B}_s \to \eta^{(')} \rho^0 \) within SM are presented in Table II. As can be seen from Table II, the SCET predictions for the branching ratios are smaller than PQCD and QCDF predictions. This can be explained as the predicted form factors in SCET are smaller than those used in PQCD and QCDF.
TABLE I. Branching ratios in units $10^{-6}$ of $\bar{B}_s \to \eta \pi^0$ and $\bar{B}_s \to \eta' \pi^0$ decays. The last two columns give the predictions corresponding to the amplitudes in Eqs. (24). On the SCET predictions the errors are due to the CKM matrix elements and $SU(3)$ breaking effects respectively. For a comparison with previous studies in the literature, we list the results evaluated in QCDF [5], PQCD [7].

| Decay channel | QCDF | PQCD | SCET solution 1 | SCET solution 2 |
|---------------|------|------|----------------|----------------|
| $\bar{B}_s \to \eta \pi^0$ | $(0.075^{+0.013+0.030+0.008+0.010}_{-0.012-0.025-0.010-0.007})$ | $(0.05^{+0.02+0.01+0.00}_{-0.02-0.00})$ | $(0.03+0.004+0.004_{-0.004-0.004})$ | $(0.02+0.007+0.005_{-0.007-0.005})$ |
| $\bar{B}_s \to \eta' \pi^0$ | $(0.11^{+0.02+0.04+0.01+0.01}_{-0.02-0.04-0.01-0.01})$ | $(0.11^{+0.05+0.02+0.00}_{-0.03-0.01-0.00})$ | $(0.005^{+0.002+0.003}_{-0.002-0.003})$ | $(0.04^{+0.005+0.01}_{-0.005-0.01})$ |
| $\bar{B}_s \to \eta \rho^0$ | $(0.17^{+0.03+0.07+0.02+0.02}_{-0.03-0.06-0.02-0.01})$ | $(0.06^{+0.03+0.01+0.00}_{-0.02-0.01-0.00})$ | $(0.005^{+0.01+0.007}_{-0.01-0.007})$ | $(0.01^{+0.008+0.004}_{-0.008-0.004})$ |
| $\bar{B}_s \to \eta' \rho^0$ | $(0.25^{+0.06+0.10+0.02+0.02}_{-0.05-0.08-0.02-0.02})$ | $(0.13^{+0.06+0.02+0.00}_{-0.04-0.02-0.00})$ | $(0.004^{+0.001+0.0}_{-0.001-0.0})$ | $(0.06^{+0.02+0.01}_{-0.02-0.01})$ |

III. NP CONTRIBUTIONS TO $\bar{B}_s \to \pi^0(\rho^0)\eta(\eta')$ DECAYS

In this section we consider NP contributions to the decay modes under study. We will consider model with charged Higgs and a model with additional $U(1)'$ gauge symmetry.

Within SM the Wilson coefficients of the electroweak penguins follow the hierarchy $|C_9^SM(M_W)| \gg |C_7^SM(M_W)| \gg |C_8^SM(M_W)|, |C_10^SM(M_W)|$. Thus for NP in order to compete SM contribution to the branching ratios $C_{7,9}^{NP}$ should be comparable to $C_{7,9}^SM$. In order to estimate the enhancements in the full Wilson coefficients $C_7$ and $C_9$ due to the contributions from NP we define the ratios: $R_i^{NP} = |C_i^{NP}| / |C_i|$ and $\bar{R}_i^{NP} = |\tilde{C}_i^{NP}| / |C_i|$ for $i = 7, 9$ where $C_i$ are the total Wilson coefficients. We also define the ratios $R_{b_i}^{M_1M_2} = (BR_i^{SM+NP}(\bar{B}_s \to M_1M_2) - BR_i^{SM}(\bar{B}_s \to M_1M_2)) / BR_i^{SM+NP}(\bar{B}_s \to M_1M_2)$ where $i = 1, 2$ refers to solutions 1, 2 for the SCET parameter space for which the corresponding amplitudes are given in Eqs. (24) and $BR_i^{SM+NP}(\bar{B}_s \to M_1M_2)$ and $BR_i^{SM}(\bar{B}_s \to M_1M_2)$ are the branching ratios obtained when we consider the total contributions including NP and the SM contributions alone respectively. The numerical values of $R_{b_i}^{M_1M_2}$ will give us an estimation of the size of the enhancement or reduction to the branching ratios of our decay processes compared to the total branching ratios including SM and NP contributions.
A. Two Higgs doublets model type-II

Any possible extension of the Higgs sector of the SM such as two Higgs doublet models can result in charged Higgs as one of the physical new Higgs mass states. Generally in the two Higgs doublet models both Higgs can couple to up and down type quarks. This is referred in the literature as two Higgs doublet models type-III and upon taking some limits, setting some parameters to zeros, we restore back two Higgs doublet model type-II [15, 16].

Within the framework of 2HDMs type-III the Yukawa Lagrangian can be written as [15, 16]:

\[ L_{\text{eff}}^{Y} = \bar{Q}_i^a \left[ Y_{fi}^d \epsilon_{ab} H_{d}^{b*} - \epsilon_{fi}^d H_{u}^a \right] d_i R - \bar{Q}_i^a \left[ Y_{fi}^u \epsilon_{ab} H_{u}^{b*} + \epsilon_{fi}^u H_{d}^a \right] u_i R + \text{h.c.}, \]

(5)

here \( \epsilon_{ab} \) is the totally antisymmetric tensor, and \( \epsilon_{ij}^q \) parameterizes the non-holomorphic corrections which couple down (up) quarks to the up (down) type Higgs doublet. Up on electroweak symmetry breaking the two Higgs doublets \( H_d \) and \( H_u \) result in the five physical Higgs mass eigenstates \( A^0 \) (CP-odd Higgs), \( H^0 \) (heavy CP-even Higgs), \( h^0 \) (light CP-even Higgs) and \( H^\pm \). Following Refs. [15, 16] and assume a MSSM-like Higgs potential the charged Higgs mass can be expressed as

\[ m_{H^\pm}^2 = m_{A^0}^2 + m_W^2 \]

(6)

where \( m_W \), the W boson mass, is related to the the vacuum expectation values of the neutral component of the Higgs doublets, \( v_u \) and \( v_d \), through

\[ m_W^2 = \frac{1}{2} g^2 (v_u^2 + v_d^2) = \frac{1}{2} g^2 v^2 \]

(7)

and the mass \( m_{A^0} \) is treated as a free parameter. In the limit \( v << m_{A^0} \) one finds that all heavy Higgs masses (\( m_{H^0}, m_{A^0} \) and \( m_{H^\pm} \)) are approximately equal [23].

Using the effective Lagrangian \( L_{\text{eff}}^{Y} \) we can deduce the following charged Higgs-quarks interaction Lagrangian:

\[ L_{H^\pm}^{\text{eff}} = \bar{u}_f \Gamma_{u_f d_i}^{H^\pm LR_{\text{eff}}} P_R d_i + \bar{u}_f \Gamma_{u_f d_i}^{H^\pm RL_{\text{eff}}} P_L d_i, \]

(8)
with \[15\]

\[
\begin{align*}
\Gamma_{u_f d_i}^{H^\pm LR_{\text{eff}}} &= \sum_{j=1}^{3} \sin \beta V_{fj} \left( \frac{m_d}{v_d} \delta_{ji} - \epsilon_{ji}^d \tan \beta \right), \\
\Gamma_{u_f d_i}^{H^\pm RL_{\text{eff}}} &= \sum_{j=1}^{3} \cos \beta \left( \frac{m_u}{v_u} \delta_{jf} - \epsilon_{jf}^u \tan \beta \right) V_{ji}
\end{align*}
\] (9)

where \(\tan \beta = v_u/v_d\) and \(V\) is the CKM matrix.

Using the Feynman-rules given in Eq.(8) one can derive the contributions of the charged Higgs mediation to the weak effective Hamiltonian governs the \(b \rightarrow s\) transition. This has been done in our previous work in Ref. [10]. Thus we can proceed to an analysis charged Higgs contributions to the decay channels of our interest.

We first consider two Higgs doublets models type II. The contribution of the charged Higgs in the two Higgs doublet models type-III to the Wilson coefficients are given in Ref. [10]. Setting \(\epsilon_{33}^u = \epsilon_{33}^d = 0\) in the expressions of the Wilson coefficients results in the Wilson coefficients in two Higgs doublets models type II.

We discuss now the relevant constraints on the parameter space of the two Higgs doublets models type II. The requirement for the top and bottom Yukawa interaction to be perturbative yields a constraint on \(\tan \beta\) to be in the range, \(0.4 \lesssim \tan \beta \lesssim 91\) [24]. Direct searches for a charged Higgs in 2HDM type-II at LEP have set a lower limit on the mass of the charged Higgs boson of 80 GeV at 95\% C.L., with the process \(e^+ e^- \rightarrow H^+ H^-\) upon the assumption \(BR(H^+ \rightarrow \tau^+\nu) + BR(H^+ \rightarrow c\bar{s}) + BR(H^+ \rightarrow AW^+) = 1\) [25]. If \(BR(H^+ \rightarrow \tau^+\nu) = 1\) the bound on the mass of the charged Higgs is 94 GeV [25]. On the other hand, recent results on \(B \rightarrow \tau\nu\) obtained by BELLE [26] and BABAR [27] have strongly improved the indirect constraints on the charged Higgs mass in type II 2HDM [28]:

\[
m_{H^+} > 240 \text{GeV at 95\%CL}
\]

(10)

Other bounds can be applied on the \((\tan \beta, m_{H^\pm})\) plane such as the ones from \(B \rightarrow X_s\gamma\) [23, 29], \(B_s \rightarrow \mu^+\mu^-\), \(B \rightarrow \tau\nu\), \(K \rightarrow \mu\nu/\pi \rightarrow \mu\nu\) [23] and the bounds from ATLAS [30] and CMS [31] collaborations coming from \(pp \rightarrow tt \rightarrow b\bar{b}W^\mp H^\pm(\rightarrow \tau\nu)\).

The dependency of the Wilson coefficients \(C_{7,9}^{(H^\pm)}\) are on \(\cos^2 \beta/v_u^2 = 1/(v \tan \beta)^2\). As a consequence, for small values of \(\tan \beta\) the values of \(C_{7,9}^{(H^\pm)}\) will blow up and can enhance sizeably the branching ratios of the decay processes under consideration. On the other hand the situation is reversed for \(\tilde{C}_{7,9}^{(H^\pm)}\) as the dependency in this case is on \(\cot^2 \beta\) allowing large
values of tan β to enhance the branching ratios. In both cases small values of charged Higgs mass are required. This behavior of the Wilson coefficient leads to a similar behavior in the ratios $R_i^{H\pm}$ and $\tilde{R}_i^{H\pm}$ for $i = 7, 9$ that is clear from their graphs in Fig. (1) as functions of tan β for a value of the charged Higgs mass $m_{H^\pm} = 380 \text{ GeV}$. The charged Higgs mass considered here is the lower limit of the charged Higgs mass allowed by $B \to X_s \gamma$ constraints [29].

As discussed above the Wilson coefficients $C_{7,9}^{(H\pm)}$ vary inversely with tan$^2$ β which can is clear from Fig. (1). Therfore larger values of $C_{7,9}^{(H\pm)}$ can be obtained for smaller values of tan β and for a value of tan β = 0.4 allowed by the perturbativity of the top and bottom Yukawa interaction we obtain the maximum value $R_7^{H\pm} \approx 81\%$. Recall that tan β < 0.4 is excluded by the perturbativity of the top and bottom Yukawa interaction constraints. Regarding the Wilson coefficients $C_9^{(H\pm)}$ we find that $R_9^{H\pm}$ is few percent which is expected as SM contribution to $C_9$ is so large compared to its contribution to $C_7$. As clear from the plots in Fig. (1), with the increase of the values of tan β the ratios $R_7^{H\pm}$ become so small and close to zero indicating that charged Higgs contributions to the total Wilson coefficients are tiny.

Turning now to the ratios $\tilde{R}_i^{H\pm}$ where the dependency in this case will be directly on tan$^2$ β. As a consequence, larger values of $\tilde{R}_i^{H\pm}$ can be obtained for larger values of tan β which is clear from Fig. (1) where they are represented by the red curves. Clearly form
FIG. 2. $\mathcal{R}_{b_1}^{M_1M_2}$ ($\mathcal{R}_{b_2}^{M_1M_2}$) in units of $10^{-2}$ blue (red) curve as a function of $\tan\beta$ for $m_{H^\pm} = 380\,\text{GeV}$.

the figure that $\tilde{R}_t^{H^\pm}$ will not exceed 10% while $\tilde{R}_t^{H^\pm}$ is less than 1% indicating that the contributions from Wilson coefficients from flipping the chirality from left to right are still small.

Having given an estimation of the enhancement in the Wilson coefficients due to the contribution from the charged Higgs mediation we turn now to give predictions for the enhancements or reductions in the branching ratios of the decay processes under study. In Figs. (2,3) we plot $\mathcal{R}_{b_1}^{M_1M_2}$ ($\mathcal{R}_{b_2}^{M_1M_2}$), blue(red) curve, as a function of $\tan\beta$ for $m_{H^\pm} = 380\,\text{GeV}$ and $m_{H^\pm} = 700\,\text{GeV}$ respectively. For the lower bound on $\tan\beta = 0.4$ and for $m_{H^\pm} = 380\,\text{GeV}$ we find that $\mathcal{R}_{b_1}^{\rho\eta_1} \simeq 1\%$, $\mathcal{R}_{b_2}^{\rho\eta_2} \simeq 24\%$, $\mathcal{R}_{b_1}^{\rho\eta_1'} \simeq 10\%$ and $\mathcal{R}_{b_2}^{\rho\eta_1'} \simeq 6\%$. On the
other hand we find that $\mathcal{R}_{b_1}^{\pi\eta} \simeq 10\%$, $\mathcal{R}_{b_2}^{\pi\eta} \simeq 17\%$, $\mathcal{R}_{b_1}^{\pi\eta'} \simeq 2\%$ and $\mathcal{R}_{b_2}^{\pi\eta'} \simeq 9\%$. This shows that the maximum enhancement due to charged Higgs contributions can reach a maximum value 24\% of the total branching ratio. For $m_{H^\pm} = 700\, GeV$ and for $\tan \beta = 0.4$ we find that $\mathcal{R}_{b_1}^{\rho\eta} \simeq 4\%$, $\mathcal{R}_{b_2}^{\rho\eta} \simeq 23\%$, $\mathcal{R}_{b_1}^{\rho\eta'} \simeq -2\%$ and $\mathcal{R}_{b_2}^{\rho\eta'} \simeq 4\%$. On the other hand we find that $\mathcal{R}_{b_1}^{\pi\eta} \simeq 5\%$, $\mathcal{R}_{b_2}^{\pi\eta} \simeq 10\%$, $\mathcal{R}_{b_1}^{\pi\eta'} \simeq -5\%$ and $\mathcal{R}_{b_2}^{\pi\eta'} \simeq 8\%$. The negative values such as $\mathcal{R}_{b_1}^{\rho\eta'} \simeq -2\%$ indicate that charged Higgs contribution can be a destructive to the SM and thus decreases the total branching ratio than the SM one.

Turning now to Figs.(2,3) we see that $\mathcal{R}_{b_i}^{M_1M_2}$ varies with $\tan \beta$ and in some cases $\mathcal{R}_{b_i}^{M_1M_2}$ can have positive, zero and negative values. The reason can be explained as follows: for
\( \tan \beta < 5 \) we see from Fig.(1) that \( C_7^{(H^\pm)} \gg \tilde{C}_7^{(H^\pm)} \). In addition, \( C_7^{(H^\pm)} \) has a similar sign to \( C_{SM}^7 \) and thus it leads to instructive effect and enhance the amplitude. For \( 5 < \tan \beta < 20 \) we find that the term in the amplitude proportional to \( \tilde{C}_7^{(H^\pm)} \) starts to be non zero and have opposite sign to the total Wilson coefficient \( C_7 \) leading to a destructive effect and approximately Higgs contributions become negligible and thus leads to \( R_{b_1 M_2}^{M_1 M_2} = 0 \). For larger values, \( \tan \beta \geq 20 \), we find that \( \tilde{C}_7^{(H^\pm)} > C_7^{(H^\pm)} \) and thus it reduces the amplitude leading to the negative values in the plot.

As can be seen from Figs.(2,3) \( R_{b_2}^{M_1 M_2} \) is always positive. This can be explained as the effect caused by the relative size of \( \tilde{C}_7^{(H^\pm)} \) and \( C_7^{(H^\pm)} \) is small. This is because the coefficients of the \( \tilde{C}_7^{(H^\pm)} \) terms in the amplitudes corresponding to solution 2 in most cases are smaller than their corresponding ones in solution 1 and thus their effects will be not effective as in the case of \( R_{b_1}^{M_1 M_2} \).

**B. Two Higgs doublet model type-III**

In the case of two Higgs doublet models type III the Wilson coefficients are those with non vanishing couplings \( \epsilon_{ij}^q \) where \( q = u, d \).

We start our analysis by discussing the constraints on the parameter space which includes the parameters \( \epsilon_{33}^u \), \( \epsilon_{22}^d \) and \( \epsilon_{33}^d \) relevant to our decay modes. According to the naturalness criterion of 't Hooft the smallness of a quantity is only natural if a symmetry is gained in the limit in which this quantity is zero [15]. Hence applying this criterion to the quark masses in the 2HDM of type III we find that \( \epsilon_{22}^d \) will be severely constrained by the small mass of the strange quark. On the other hand, since the bottom quark mass is very large compared to the mass of the strange quark, we find that \( \epsilon_{33}^d \) is less constrained compared to \( \epsilon_{22}^d \). Regarding \( \epsilon_{33}^u \) we find that the constraints due to the naturalness criterion of 't Hooft are loose due to the so large top quark mass. For a detailed discussion about the other relevant constraints on \( \epsilon_{ij}^q \) and the other parameters in the the two Higgs doublet model type-III we refer to Ref.[23] where an extensive study of the flavor physics has been performed to constrain the model both from tree-level processes and from loop observables. In our analysis we take into account these constraints especially those imposed on \( \epsilon_{33}^d \) and \( \epsilon_{33}^u \) which are relevant to our decay processes.

The presence of the \( \epsilon_{23}^u \) parameter can relax the constraints on the charged Higgs mass
from $B \to X_s \gamma$ and make it becomes weaker comparing with the case in 2HDMs type-II. This is basically because $\epsilon_{23}^u$ can lead to a destructive interference with the SM (depending on its phase) and thus reduces the amplitude. As a consequence the lower limit on the charged Higgs mass in 2HDMs type-II can be pushed down in 2HDMs type-III.

In our previous work in Ref.[10] we showed that the presence of the $\epsilon_{33}^d$ terms can enhance or reduce $\tilde{C}^{(H^\pm)}_{7,9}$ by few percent for $m_{H^\pm} = 300 \, GeV$ and $\tan \beta \leq 50$. Thus It is expected that they can not affect much the branching ratios and so we can safely neglect these terms in our analysis.

Turning now to the $\epsilon_{33}^u$, it is shown that the important constraints can be imposed from $B \to X_s \gamma$ decay process [23]. For $m_{H^\pm} = 500 \, GeV$ and $\tan \beta = 50$ the coupling $\epsilon_{33}^u$ should satisfy $|\epsilon_{33}^u| \leq 0.55$ and the constrains become more strong for large values of $\tan \beta$ and smaller values of $m_{H^\pm}$. As $\epsilon_{33}^u$ is generally complex we expect that these terms can enhance or reduce $C^{(H^\pm)}_{7,9}$. In figures 17 and 18 in Ref.[23] it is shown that the allowed values for $\epsilon_{33}^u$ exclude negative values of the real part of $\epsilon_{33}^u$. Thus $\epsilon_{33}^u$ terms will always enhance the Wilson coefficients $C^{(H^\pm)}_{7,9}$. However, we expect the enhancements to be larger for the Wilson coefficient $C^{(H^\pm)}_7$ as $C^{SM}_9$ is so large to be competed by NP effects. Thus we only focus on $R_{H^\pm}^i$ in the following discussion. For $\tan \beta = 50$ and $m_{H^\pm} = 500 \, GeV$ we find that $R_{7}^{H^\pm}$ can reach 8%. For $m_{H^\pm} = 300 \, GeV$ we find that $R_{7}^{H^\pm}$ can be around 12%. This is expected as Wilson coefficients vary inversely with the square of charged Higgs mass. For $m_{H^\pm} = 300 \, GeV$ and $\tan \beta = 30$ we find that $R_{7}^{H^\pm}$ can reach 30%. For lower values of $\tan \beta$ we find that $R_{7}^{H^\pm} \leq 30\%$. This is because, in the Wilson coefficients, $\epsilon_{33}^u$ is multiplied by $\tan \beta$ and thus although the constraints on $\epsilon_{33}^u$ become weaker for small values of $\tan \beta$ however their relatively larger allowed values will not be significant when they are multiplied by small value of $\tan \beta$.

Turning now to the branching ratios of our decay processes we expect that the largest enhancement will be for the case $m_{H^\pm} = 300 \, GeV$ and $\tan \beta = 30$ as discussed above. We find that $R_{\pi^\eta} \simeq 5\%$, $R_{\pi^\eta}^\eta \simeq 10\%$, $R_{\rho^\eta}^{\eta^\prime} \simeq -6\%$ and $R_{\rho^\eta}^{\eta^\prime} \simeq 9\%$. On the other hand we find that $R_{\rho^\eta} \simeq 4\%$, $R_{\rho^\eta}^\eta \simeq 23\%$, $R_{\rho^\eta}^{\eta^\prime} \simeq -4\%$ and $R_{\rho^\eta}^{\eta^\prime} \simeq 4\%$. This shows that the maximum enhancement due to charged Higgs contributions can reach a maximum value 23\% of the total branching ratio which is around its value in two Higgs doublet model type-II.
C. $Z'$ model

One of the possible extension of the SM is to enlarge the SM gauge group to include additional $U(1)'$ gauge group. This possibility is well-motivated in several beyond SM theories such as theories with large extra dimensions [32] and grand unified theories (GUTs), for instance $E_6$ models [33]. As a consequence of the $U(1)'$ gauge group a new gauge boson arise namely, $Z'$. Basically $Z'$ can have either family universal couplings or family non-universal couplings to the SM fermions. In the case that the $Z'$ gauge couplings are family universal they remain diagonal even in the presence of fermion flavor mixing by the GIM mechanism [34]. On the other hand and in some models like string models it is possible to have family-non universal $Z'$ couplings, due to the different constructions of the different families [34–37]. This scenario with family-non universal couplings has theoretical and phenomenological motivations. For instance, possible anomalies in the $Z$-pole $b\bar{b}$ asymmetries suggest that the data are better fitted with a non-universal $Z'$ [38].

In our analysis we will consider non-universal $Z'$ couplings in a way independent to specific $Z'$ model following Refs. [38–41]. Thus the Wilson coefficients at the $M_W$ scale are given by [38]

\begin{align}
C_{7}^{Z'} &= -\frac{4}{3V_{ts}^*V_{tb}} B_{sb}^L D_{ud}^R, \\
C_{9}^{Z'} &= -\frac{4}{3V_{ts}^*V_{tb}} B_{sb}^L D_{ad}^L, \\
\tilde{C}_{7}^{Z'} &= 0, \\
\tilde{C}_{9}^{Z'} &= 0,
\end{align}

where the off-diagonal coupling $B_{sb}^L = |B_{sb}^L|e^{i\phi_s^L}$ is generally complex. In Ref. [38] the allowed values of the parameters $\phi_s^L$, $D_{ad}^{L,R}$, after taking into account the constraints from the branching ratios of $B \to \pi K, \rho K$ and the direct CP asymmetries $A_{CP}(B \to \pi K, \rho K)$, are presented in Table 6 for two scenarios. Scenario I: without any simplification for the flavour-conserving $Z'$ couplings. Scenario II: assuming that the right-handed flavour-conserving $Z'$ couplings vanish. In Table II we present our predictions for $R_{7,9}^{H^\pm}$ and $R_{7,9}^{M_1M_2}$ within these two scenarios. We note from Eq.(14) that $\tilde{R}_{7,9}^{H^\pm}$ will vanish. For $R_{7}^{H^\pm}$ we see from Table II it can reach 99% in Scenario I and it is vanishing in Scenario II as in this scenario the assumption that the right-handed flavour-conserving $Z'$ couplings vanish has been taken. Turning now to $R_{9}^{H^\pm}$ we see from Table II it can reach approximately 80% in the two scenarios.
TABLE II. Predictions for $R^H_{7,9}$ and $R^M_{b_1,b_2}$ in non-universal $Z'$ within Scenario I and Scenario II.

| ratio       | Scenario I | Scenario II |
|-------------|------------|-------------|
| $R^H_{7}$   | 99%        | 0           |
| $R^H_{9}$   | 78%        | 76%         |
| $R^{\pi\eta}_{b_1}$ | 70%        | 71%         |
| $R^{\pi\eta}_{b_2}$ | 73%        | 74%         |
| $R^{\pi\eta'}_{b_1}$ | -51%       | -49%        |
| $R^{\pi\eta'}_{b_2}$ | -9%        | -13%        |
| $R^{\eta\rho}_{b_1}$ | 75%        | 65%         |
| $R^{\eta\rho}_{b_2}$ | 81%        | 66%         |
| $R^{\eta\rho'}_{b_1}$ | 45%        | 64%         |
| $R^{\eta\rho'}_{b_2}$ | 57%        | 60%         |

With the enhancements in $R^H_{7,9}$ one expects that the branching ratios of our decay modes can be enhanced or reduced according the instructive or destructive effects of the total Wilson coefficients $C_{7,9}$. For Scenario I we deduce from Table II that the branching ratios of $\bar{B}_s \to \eta' \pi^0$ corresponding to solutions 1 and 2 of the SCET parameters will be reduced in comparing to their values within SM. However since $R^{\pi\eta'}_{b_2} = -9\%$ and $R^{\pi\eta}_{b_1} = -51\%$ we find that the reduction in the branching ratios of $\bar{B}_s \to \eta' \pi^0$ are not sizeable. For the other channels we find that the branching ratio of $\bar{B}_s \to \eta \rho^0$ increases from $3 \times 10^{-8}$ to $1.2 \times 10^{-7}$, the branching ratio of $\bar{B}_s \to \eta \pi^0$ increases from $3 \times 10^{-8}$ to $1 \times 10^{-7}$ and for $\bar{B}_s \to \eta' \rho^0$ we find that it increases from $6 \times 10^{-8}$ to $1.4 \times 10^{-7}$. Regarding Scenario II as can be seen from Table II that the enhancements and reductions in the branching ratios will be close to their values in Scenario I.

IV. CONCLUSION

In this work we have studied the decay modes $\bar{B}_s \to \pi^0(\rho^0) \eta^{(')}$ within the frameworks of some NP models including the two-Higgs doublet models type-II and type-III and a model with an additional $U(1)'$ gauge symmetry. We adopt in our study SCET as a framework
for the calculation of the amplitudes. Within the framework of two-Higgs doublet models type-II and typ-III we find that the Wilson coefficients $C_7$ and $C_9$ can be enhanced due to the contributions from the charged Higgs mediation. However the enhancement in the branching ratios in the two models does not exceed 24\% of their total values including SM contribution.

Turning now to our predictions for a model with an additional $U(1)^\prime$ gauge symmetry, we find that sizeable enhancement can be obtained in both scenarios of the model. In particular we find that the branching ratio of $\bar{B}_s \to \eta \rho^0$ increases from $3 \times 10^{-8}$ to $1.2 \times 10^{-7}$. In addition the branching ratio of $\bar{B}_s \to \eta \pi^0$ increases from $3 \times 10^{-8}$ to $1 \times 10^{-7}$ and the branching ratio of $\bar{B}_s \to \eta^\prime \rho^0$ increases from $6 \times 10^{-8}$ to $1.4 \times 10^{-7}$. This enhancements is a signature of the model and thus can be tested in upcoming B factories.

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