The Simulation of the Inelastic Impact

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Abstract. The coefficient of normal restitution (COR) in an oblique impact is theoretically studied. Using a two-dimensional lattice models for an elastic disk and an elastic wall, we investigate the dependency of COR on an incident angle and demonstrate that COR can exceed one and have a peak against an incident angle in our simulation. Finally, we explain these phenomena based upon the phenomenological theory of elasticity.

INTRODUCTION

The coefficient of restitution (COR) $e$ is introduced to determine the post-collisional velocity in the normal collision of two materials and defined by $u' \cdot n = -eu \cdot n$, where $u$ and $u'$ are respectively the velocity of the contact point of two colliding materials before and after the impact, and $n$ is the normal unit vector of the tangential plane of them. In the oblique impact, it has become clear that COR depends on the incident angle as well as on the impact velocity. Louge and Adams[1] recently reported that COR increases as a linear function of the tangent of the incident angle in the oblique impact of a hard aluminum oxide sphere on a thick elasto-plastic plate. They also suggested that COR can exceed 1 in most grazing impacts. In this proceeding, we carry out the two-dimensional simulation of the oblique impact and investigate the dependency of COR on the incident angle based upon the theory of elasticity.

MODEL

Our numerical model is a two-dimensional model which is composed of an elastic disk and an elastic wall[2]. Each of them is composed of randomly placed mass particles connected by nonlinear springs each other. Numbers of mass particles are 400 for the disk and 2000 for the wall. The width and the height of the wall are $8R$ and $2R$ respectively where $R$ is the radius of the disk. Both sides and bottom of the wall are fixed. Spring potential is described as $V(x) = \frac{1}{2}k_a x^2 + \frac{1}{4}k_b x^4$, where $x$ is a stretch from natural length. The values of $k_a$ are $1.0 \times mc^2/R^2$ for the disk and $k_a = 1.0 \times 10^{-2}mc^2/R^2$ for the wall, where $c$ is the one-dimensional sound of velocity. And we adopt $k_b = k_a \times 10^{-3}/R^2$ for each of them. Poisson’s ratio $\nu$ can be evaluated from the strains of the band of random lattice in vertical and horizontal directions to the applied force. We obtain Poisson’s ratio $\nu$ of the model as $\nu = (7.50 \pm 0.11) \times 10^{-2}$.

In our simulation, we define the incident angle $\gamma$ by the angle between the normal vector of the wall and the initial velocity vector of the disk (Fig.1). We fix the initial colliding velocity of the disk as $|v(0)| = 0.1c$ to control the normal and tangential components of the initial colliding velocity as $v_n(0) = |v(0)| \sin \gamma$ and $v_t(0) = |v(0)| \cos \gamma$, respectively. From the normal components of the contact point velocities before and after the collision, we calculate COR for each $\gamma$. We use the fourth order symplectic numerical method for the numerical scheme of integration with the time step $\Delta t = 10^{-3}R/c$.

RESULTS AND DISCUSSION

Figure[2] is the COR against the tangent of the incident angle $\Psi_1 = \tan \gamma$ in our simulation. The cross points are
the average and the error bars are the standard deviation of 100 samples for each incident angle. This result shows that the COR increases as $\Psi_1$ increases to exceed 1, and has a peak around $\Psi_1 = 5.0$. This behavior is contrast to that in the experiment by Louge and Adams[1].

To explain this result, we consider the correction of COR by the local deformation of the wall. We assume that the normal unit vector $\mathbf{n}$ to the surface of the wall rotates toward the incoming disk by an angle $\alpha$ to become $\mathbf{n}'$. If we define $e = -(\mathbf{u} \cdot \mathbf{n})/(\mathbf{u} \cdot \mathbf{n})$ and $e' = -(\mathbf{u}' \cdot \mathbf{n}')/(\mathbf{u} \cdot \mathbf{n}')$, the relation between $e$ and $e'$ becomes

$$e = (e'' + \Psi_2 \tan \alpha)/(1 - \Psi_1 \tan \alpha),$$ \hspace{1cm} (1)

where $\Psi_1 = (\Psi_1 - \tan \alpha)/(1 + \Psi_1 \tan \alpha)$ and $\Psi_2 = (\Psi_1 - \tan \alpha)/(1 + \Psi_1 \tan \alpha) - 3(1 + e'')(\mu + \tan \alpha)/(1 - \mu \tan \alpha)$. As for $\Psi_2$, we use the phenomenological theory for the oblique impact by Walton and Braun[3]. The correction angle $\alpha$ can be estimated by the theory of elasticity. If we express the contact area by a parabola, $\tan \alpha$ equals to $|x_c - x_n|(1 - 2\theta)/R(2 - 2\theta)$ with $\theta = (\pi/2)\arctan(1 - 2\nu) / (\mu + 2\nu)$, where $\mu$ is the coefficient of friction and $x_c$ and $x_n$ are the x coordinates of both ends of the contact area. $\mu$ can be calculated from the simulation data through the definition $\mu = |J_1|/|J_n|$, where $J_n$ and $J_1$ are the normal and tangential components of the impulse. The relation between $\mu$ and $\Psi_1$ are cross points in Fig. [1].

The solid curve in Fig. [2] is Eq. (1) with $e'' = 0.95$ which is COR in the normal impact. Our numerical results can be reproduced by our phenomenological theory.

The relation between $\mu$ and $\Psi_1$ can be explained as follows. We assume that jags are uniformly placed on the surface of the wall with the density $\rho$ per unit length and the tangential velocity of the disk is decreased by $\eta$ when the disk interacts with the one jag. The tangential and normal impulses can be calculated by calculating the number of jags the disk interacts during collision time $t$ as $J_t = -m\eta p|v(0)|\sin \gamma \pi R/c \sqrt{\ln(4R|v(0)|\cos \gamma)}/(e + 1)$ and $J_n = -m(e + 1)|v(0)|\cos \gamma$. We also assume that the tangential impulse decreases by $J_t' = -m\zeta|v(0)|\sin \gamma$ which is proportional to the initial tangential velocity with the proportionality constant $\zeta$. Thus, $\mu$ can be calculated by the ratio of $J_t - J_t'$ to $J_n$ as

$$\mu = \zeta \tan \gamma - \eta \rho \tan \gamma \pi R/c \sqrt{\ln(4R|v(0)|\cos \gamma)}/(e + 1).$$ \hspace{1cm} (2)

The solid curve in Fig. [3] is Eq. (1) with $\zeta = 0.317$ and $\eta \rho = 0.0416c/R$. Our numerical results can be well reproduced by our phenomenological theory of the coefficient of friction.

**SUMMARY**

In the present study, we have shown that COR can exceed 1 also in our two-dimensional simulation and depends on $\mu$ in the oblique impact. Our results can be explained by our simple phenomenological theory.

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