Spin correlations in the Drell-Yan process, parton entanglement, and other unconventional QCD effects

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Abstract

We review ideas on the structure of the QCD vacuum which had served as motivation for the discussion of various non-standard QCD effects in high-energy reactions in articles from 1984 to 1995. These effects include, in particular, transverse-momentum and spin correlations in the Drell-Yan process and soft photon production in hadron-hadron collisions. We discuss the relation of the approach introduced in the above-mentioned articles to the approach, developed later, using transverse-momentum-dependent parton distributions (TDMs). The latter approach is a special case of our more general one which allows for parton entanglement in high-energy reactions. We discuss signatures of parton entanglement in the Drell-Yan reaction. Also for Higgs-boson production in \( pp \) collisions via gluon-gluon annihilation effects of entanglement of the two gluons are discussed and are found to be potentially important. These effects can be looked for in the current LHC experiments. In our opinion studying parton-entanglement effects in high-energy reactions is, on the one hand, very worthwhile by itself and, on the other hand, it allows to perform quantitative tests of standard factorisation assumptions.

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1 Introduction

In this article we want to give a synopsis and an update of the results of [1–5] concerning some unconventional QCD effects in the Drell-Yan process and in soft-photon production in hadron-hadron collisions. In addition we shall investigate possible effects of parton entanglement for Higgs boson production in hadron-hadron collisions. We think that our study is quite timely. On the one hand there are the current LHC experiments. On the other hand there is an experimental program under way to investigate over a large c.m. energy range the Drell-Yan process and the related $Z$-production reaction

$$h_1 + h_2 \rightarrow V + X, \quad l^+ + l^-,$$

$$V = \gamma^*, Z.$$  \hspace{1cm} (1.1)

Here $h_{1,2}$ are hadrons, $X$ stands for the final hadronic state, and $l = e, \mu$ for the leptons. We shall be interested in particular in the angular distribution of the leptons where “anomalies” have first been seen by the NA10 experiment at CERN [6, 7] and then confirmed by the E615 experiment at FNAL [8, 9]. The interesting findings of these experiments have only recently led to great further experimental efforts. In table 1 we list the original experiments and some recent ones which are either completed or planned. This list is not intended to be exhaustive, it is only meant to indicate the wide range of ongoing studies, concerning both the incoming hadrons $h_{1,2}$ in (1.1) and the c.m. energy $\sqrt{s}$. All these experiments should be very suitable for studying the unconventional QCD effects discussed in [1–5].

Our paper is organised as follows. In section 2 we recall some ideas on the QCD vacuum structure which were developed in the 1970s and 1980s. We sketch the motivation which led to the introduction of spin correlations in the Drell-Yan process in [1, 2]. In section 3 we discuss the framework developed in [2] for treating the reaction (1.1). The relation of our framework to the one using transverse-momentum-dependent-parton distributions (TMDs) is given. We emphasise that our framework allows to investigate effects from parton entanglement which may occur, for instance, due to instantons. In section 4 we investigate possible effects of parton entanglement - in this case for gluons - on the production of Higgs bosons in hadron-hadron collisions. Section 5 contains our conclusions. In appendices we discuss the Drell-Yan reaction with general quark-antiquark density matrix, conventions for kinematic variables, and an example of a non-trivial two-gluon density matrix for Higgs-boson production via gluon-gluon annihilation for entangled gluons.

2 The QCD vacuum structure as a possible source of unconventional effects

In the 1970s and 1980s many interesting ideas on the QCD vacuum structure were developed. Instantons were introduced and shown to have important effects in [22–24]. Savvidy [25] showed that a colour-magnetic field will lower the energy of the vacuum state. Shifman, Vainshtein and Zakharov (SVZ) introduced the gluon condensate of the vacuum [26–28]. A particularly nice
Table 1: The parameters of the experiments NA10 and E615 from the late 1980s and a partial list of recently completed and planned experiments for reaction (1.1). The approximate c.m. energies $\sqrt{s}$ and years of running are also indicated. This table is in part based on material presented in [21].

| Experiment       | $h_1$ | $h_2$ | $V$    | $\sqrt{s}$ [GeV] | years          |
|------------------|-------|-------|--------|------------------|----------------|
| NA10 (CERN)      | $\pi^-$ | $W, d$ | $\gamma^*$ | 16 to 23        | 1986-1988      |
| [6, 7]           |       |       |        |                  |                |
| E615 (FNAL)      | $\pi^-$ | $W$   | $\gamma^*$ | 22               | 1989-1991      |
| [8, 9]           |       |       |        |                  |                |
| PANDA (GSI)      | $\bar{p}$ | $p$   | $\gamma^*$ | 5.5             | $> 2016$       |
| [10]             |       |       |        |                  |                |
| PAX (GSI)        | $\bar{p}$ | $p$   | $\gamma^*$ | 14               | $> 2017$       |
| [11]             |       |       |        |                  |                |
| E906 (FNAL)      | $p$   | $p$   | $\gamma^*$ | 15               | 2011           |
| [12]             |       |       |        |                  |                |
| COMPASS II (CERN)| $\pi^\pm$ | $p$   | $\gamma^*$ | 17.4             | 2014           |
| [13]             |       |       |        |                  |                |
| E866 (FNAL)      | $p$   | $p, d$| $\gamma^*$ | 39               | 2007-2009      |
| [14, 15]         |       |       |        |                  |                |
| PHENIX (BNL)     | $p$   | $p$   | $\gamma^*$ | 200              | $> 2018$       |
| [16]             |       |       |        |                  |                |
| CDF (FNAL)       | $\bar{p}$ | $p$   | $\gamma^*, Z$ | $1.96 \times 10^3$ | 2011         |
| [17]             |       |       |        |                  |                |
| ALICE ATLAS CMS  | $p$   | $p$   | $\gamma^*, Z$ | $2 \times 10^3$ to $14 \times 10^3$ | 2010-2014 |

picture of the QCD vacuum was developed by Ambjørn and Olesen [29,30]; see figure 1a. Note the hexagonal structure of the chromomagnetic flux tubes. For comparison we show in figure 1b the hexagonal structure of the ether of electrodynamics as envisaged by Maxwell [31] in 1861. Thus, some ideas on the QCD vacuum structure resemble strikingly the dielectric ether of the 19th century which was discarded by Einstein. Maybe, some time in the future we shall also have a deeper and simpler understanding of the QCD vacuum structure. For the present
we shall take these ideas on the QCD vacuum as working hypothesis; for reviews see [32] and chapters 6 and 8 of [33].

But let us come to the Drell-Yan (DY) reaction (1.1) with $V = \gamma^*$. In leading order we have the annihilation of a quark-antiquark pair into a virtual photon $\gamma^*$ which then decays into a lepton pair; see figure 2. The standard description of the process is well known [34–36]. For polarised hadrons $h_1$ and $h_2$ the quark $q$ and antiquark $\bar{q}$ are supposed to be also polarised and completely uncorrelated.

In the paper [1] of 1984 this assumption was questioned. The argument was that the $q\bar{q}$ annihilation takes place in a non-trivial background full of colour fields if we believe the ideas on the QCD vacuum. It was argued that this could give rise to spin and even colour-spin correlations of $q$ and $\bar{q}$, and possibly to entanglement of the two partons. Before we recall these arguments we want to mention that already in [37] it was shown that instanton effects in the DY process will not go away at high energies. But these authors only considered the total rates and were not concerned with spin effects. In fact, they only considered spinless partons.

Let us start with the gluon condensate of SVZ [26–28]. From Lorentz and parity invariance of the strong interactions we find for the vacuum expectation value of the product of two gluon field strengths at the same space-time point

$$\langle 0 \mid \frac{g_s^2}{4\pi^2} : G_{\mu\nu}^a(x)G_{\rho\sigma}^b(x) : \mid 0 \rangle = \frac{1}{96} \delta^{ab} (g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho})G_2.$$  (2.1)
Here $g_s$ is the QCD coupling constant and $a, b, \in \{1, \ldots, 8\}$ are the colour indices. A typical phenomenological value for $G_2$ is

$$G_2 = \frac{1}{4\pi^2}(0.95 \pm 0.45) \text{ GeV}^4; \quad (2.2)$$

see [38]. For further discussion of the value of $G_2$ see for instance [39,40]. For the chromoelectric and chromomagnetic fields we get from (2.1)

$$-\langle 0| g_s^2 : E^a(x) E^a(x) : |0 \rangle = \langle 0| g_s^2 : B^a(x) B^a(x) : |0 \rangle = \pi^2 G_2 \cong (700 \text{ MeV})^4. \quad (2.3)$$

One assumes that the products of the fields in (2.1) and (2.3) are normal ordered with respect to the “perturbative vacuum state”, whatever this is. The interpretation of (2.3) should, therefore, be that in the physical vacuum the chromomagnetic field fluctuates with larger amplitude, the chromoelectric field with smaller amplitude than in the perturbative vacuum. In [1–4] possible effects from these strong chromomagnetic vacuum fields were discussed.

Let us envisage a light quark $u$ or $d$ with mass of a few MeV moving in a constant chromomagnetic background field of the strength given in (2.3). What will happen? The quark will be deflected due to the chromomagnetic Lorentz force and perform a cyclotron motion much like an electron in a storage ring; see figure 3. The quark will emit synchrotron gluons, and since it also carries electric charge, also synchrotron photons. The quark will also emit spin-flip synchrotron gluons and by this get a transverse polarisation. This is the analogon of the well known Sokolov-Ternov effect in storage rings [41–43].

If now in addition an antiquark sails through the same background field it also will get deflected, emit synchrotron gluons and photons and will get a polarisation. If all this happens for $q$ and $\bar{q}$ in the same background field they will develop a correlation, both, in transverse momenta and spins. The background field may thus be a source of parton entanglement.
Figure 3: A quark and an antiquark with momenta $k_1$ and $k_2$, respectively, moving through a chromomagnetic background field.

Of course, there can be no constant background chromomagnetic field in the vacuum. But there can be correlations of fields at different space-time points. Indeed, in Euclidean QCD one finds from lattice calculations, see for instance [40,44], that such correlations fall off exponentially with a typical correlation length

$$a \simeq 0.2 \text{ to } 0.3 \text{ fm.}$$

(2.4)

That is, the gluon field strengths are highly correlated for points of Euclidean separation $x_{\text{Eucl.}}$ satisfying

$$x_{\text{Eucl.}}^2 \lesssim a^2.$$  

(2.5)

An instanton enthusiast may think of $a$ as the typical size of instantons contributing to our effects. For the size distribution of instantons see for instance [45, 46]. Translating this in the most straightforward way to Minkowski space we find that (2.5) implies a strong correlation of fields separated by a Minkowskian distance $x_{\text{Mink.}}$ with

$$|x_{\text{Mink.}}^2| \lesssim a^2.$$  

(2.6)

A sketch of such a region, centered at $x_{\text{Mink.}} = 0$ is shown in figure 4.

What can we say from these consideration for the Drell-Yan process shown in figure 2? Suppose that the fast quark $q$ and antiquark $\bar{q}$ from hadrons $h_1$ and $h_2$, respectively, annihilate at point $x = 0$ in Minkowski space; see figure 5. The $q$ and $\bar{q}$ will then have the chance to spend a long time in a correlated domain. Explicit estimates show that, indeed, there is enough time for transverse-momentum, spin-transverse-momentum, and spin-spin correlations of hard partons $q$ and $\bar{q}$ to develop; see [1–3].

With these remarks we shall end our short review of the ideas on the QCD vacuum structure and how this gave motivations in [1–4] to discuss various unconventional QCD effects for high-energy processes. The picture emerging from these considerations can be summarised in the following points.
Summary of section 2:

(i) The quarks, and similarly hard gluons, of a fast hadron $h$ feel a background chromomagnetic field of typical strength $g_s B_c$. The chromomagnetic Lorentz force causes the coloured partons to “wiggle” and to emit ordinary and spin-flip synchrotron gluons. Quarks, being charged, also emit synchrotron photons. Of course, for an isolated hadron $h$ there can be no real radiation. The synchrotron gluons and photons are part of the cloud of virtual particles around the hadron.

(ii) In [3] it was estimated that different hard partons of the hadron $h$ travel generally in uncorrelated field domains. This implies, for instance, that we should add the synchrotron photons emitted from various hard partons in the hadron incoherently.

(iii) In a hadron-hadron collision the cloud of synchrotron photons may be shaken off and should give rise to soft photons with a characteristic energy spectrum, see [1, 3]. The main parameter governing this soft-photon yield turned out to be an effective length $l_{\text{eff}}$ defined as the distance a fast quark has to travel in the chromomagnetic field of strength $g_s B_c$ for obtaining the typical transverse momentum $\bar{p}_T$ which quarks have in the hadron. The ordinary cyclotron formulae lead to the estimate

$$l_{\text{eff}} \approx \frac{\bar{p}_T}{g_s B_c}.$$  \hfill (2.7)
Figure 5: Annihilation of a quark $q$ and antiquark $\bar{q}$ from two hadrons $h_1$ and $h_2$, respectively, with production of a virtual photon $\gamma^*$. Before the annihilation hard partons $q$ and $\bar{q}$ spend a long time in a correlated field region (from figure 2 of [3]).

From a comparison of the theory with the experimental results on soft photons in $p - Be$ collisions of [47] a value of

$$ l_{\text{eff}} \simeq 20 \text{ to } 40 \text{ fm} $$

was extracted in [3]. With $\hat{p}_T \simeq 300 \text{ MeV}$, a typical value for the mean transverse momentum of quarks in hadrons, we obtain from (2.7) for the effective chromomagnetic background field in a hadron

$$ g_s B_c \simeq (44 \text{ MeV})^2, \text{ for } l_{\text{eff}} = 30 \text{ fm}. $$

This is much smaller than the strength of the vacuum fields in (2.3):

$$ g_s B_{\text{vac}} \simeq (700 \text{ MeV})^2. $$

A possible resolution of this puzzle was suggested in [3, 4]: the vacuum fields must be shielded by gluons, maybe those from the synchrotron effects, otherwise quarks in a fast hadron could not get far without being strongly bent. Indeed, inserting $g_s B_{\text{vac}}$ instead of $g_s B_c$ in (2.7) we find $l_{\text{eff}} \simeq 0.1 \text{ fm}$ which, in our opinion, is ridiculously small. Thus, in this view gluons in a fast hadron play an important dynamical role: they have to shield the vacuum chromomagnetic fields in order to allow quarks to travel more or less on straight paths.

(iv) The gluonic spin-flip-synchrotron radiation of quarks in the chromomagnetic background field may also have some relevance for the proton spin puzzle; see [48] for a recent review.
Indeed, consider a fast proton with helicity $+1/2$. This angular momentum must be built up by the spin and orbital angular momenta of the constituents, the partons,

$$\frac{1}{2} \Delta \Sigma + L_q + \Delta G + L_g = \frac{1}{2}. \quad (2.11)$$

Here $\Delta \Sigma/2$ and $L_q$ are the contributions from the spin and orbital angular momenta, respectively, of quarks and antiquarks. The corresponding contributions from the gluons are denoted by $\Delta G$ and $L_g$. It is known for about 25 years that

$$\Delta \Sigma \approx 0.25 < 1; \quad (2.12)$$

see [48]. The puzzle posed by (2.12) is why the spin of the quarks contributes so little to the proton helicity. In our framework we can argue that in the chromomagnetic background field an original longitudinal polarization of a quark in the fast proton will be degraded and be partly turned into a transverse one by gluon spin-flip synchrotron radiation. This is in complete analogy to what happens in $e^+e^-$ storage rings due to the Sokolov-Ternov effect [41–43]. The expectation is then that rather soft gluons will guarantee the angular momentum balance in these spin-flip processes and thus in (2.11). In experiments on the proton tomography being currently discussed [49,50] one may be able to check these ideas, originally put forward in [3,4] and summarised here as points (iii) and (iv).

(v) For the Drell-Yan process, see (1.1) and figure 2, transverse-momentum, spin-transverse-momentum, and spin-spin correlations of the annihilating quark-antiquark pair were predicted in [1,2]. Such correlations were indeed observed by experiments; see [6–8]. We shall discuss all this in some detail in section 3 below. In section 4 we will discuss gluon-gluon spin correlations and their effects on Higgs-boson production via gluon-gluon fusion.

### 3 Transverse-momentum, spin-transverse-momentum, and spin-spin correlations in the Drell-Yan process

In this section we consider the general Drell-Yan process, that is, the production of a virtual photon $\gamma^*$ or $Z$ boson in a hadron-hadron collision

$$h_1(p_1) + h_2(p_2) \rightarrow V(k, \epsilon) + X, \quad V = \gamma^*, Z. \quad (3.1)$$

Completely analogous is $W^\pm$ production

$$h_1(p_1) + h_2(p_2) \rightarrow W^\pm(k, \epsilon) + X. \quad (3.2)$$

Here we shall consider explicitly only $\gamma^*$ and $Z$ production. For a discussion of $W^\pm$ production from our point of view we refer to [2]. In (3.1) and (3.2) the momenta are indicated in brackets and $\epsilon$ is the polarisation vector of the vector boson. We will only discuss the case of polarised hadrons $h_1, h_2$ and the leading order process where a quark-antiquark pair annihilates to give $V$ in (3.1)

$$q(k_1) + \bar{q}(k_2) \rightarrow V(k, \epsilon). \quad (3.3)$$
For massless quarks their momenta in the overall c.m. system are

$$\begin{align*}
k_1 &= \left( \sqrt{x_1^2 p_1^2 + k_{1T}^2}, \frac{x_1 p_1 + k_{1T}}{x_1 p_1 + k_{1T}} \right), \\
k_2 &= \left( \sqrt{x_2^2 p_2^2 + k_{2T}^2}, \frac{x_2 p_2 + k_{2T}}{x_2 p_2 + k_{2T}} \right)
\end{align*}$$

(3.4)

where \( x_1(x_2) \) is the fraction of longitudinal momentum of \( h_1(h_2) \) carried by quark \( q \) (antiquark \( \bar{q} \). Of course, in the following it is always understood that one also has to consider the case where \( \bar{q} \) comes from \( h_1 \) and \( q \) from \( h_2 \).

In the rest system of the produced vector boson we have then the situation shown in figure 6 for \( V = \gamma^* \) or \( Z \), a head-on collision of \( q \) and \( \bar{q} \), where

$$k'_1 + k'_2 = 0.$$  

(3.5)

Here and in the following momenta with a prime refer to the rest system of the \( V \) boson. In leading order, this also is the c.m. system of the \( q\bar{q} \) collision.

![Figure 6: The leading order Drell-Yan process (3.1) in the rest system of the produced vector boson \( V = \gamma^* \) or \( Z \)](image)

### 3.1 The general \( q\bar{q} \) spin density matrix

In [1, 2] the ideas on the QCD vacuum structure, as sketched in section 2, were taken as a motivation to propose a more general framework for treating the Drell-Yan process than used at the time.

The proposal was and is to allow for a general spin-density matrix for the \( q\bar{q} \) system which includes all possible spin-momentum correlations. In addition, it was and is proposed to allow for correlations of the transverse momenta of \( q \) and \( \bar{q} \).

The latter correlations were motivated by effects of the chromomagnetic Lorentz force deflecting quark and antiquark in a correlated way in the background field. We should stress that this proposal was motivated by the QCD vacuum structure but was clearly supposed to be considered in its own right. We shall discuss below that the framework using transverse momentum dependent parton distribution functions (TMDs) is included as a special case in our more general framework which was proposed earlier.

Thus, in [1, 2] the general two-particle density matrix for the \( q\bar{q} \) system in the reaction (3.3) was analysed. In [1] only partons \( q \) and \( \bar{q} \) collinear with \( h_1 \) and \( h_2 \), respectively, were considered
and their spin and colour correlations analysed. It was shown in [51] that colour correlations lead in general to infrared divergences in higher order calculations. This is not dramatic since the confinement radius provides, anyway, a natural infrared cutoff. Nonetheless, in [2–4], colour correlations were assumed to be absent. In [2] the most general correlated $q \bar{q}$ spin density matrix for (3.3) was written down allowing also for arbitrary, even correlated, transverse momenta of $q$ and $\bar{q}$. For this purpose a three-dimensional vector basis in the $q \bar{q}$ c.m. system was constructed:

$$e'_1 = \frac{(p'_1 + p'_2) \times k'_1}{|(p'_1 + p'_2) \times k'_1|}, \quad e'_3 = k'_1/|k'_1|, \quad e'_2 = e'_3 \times e'_1. \quad (3.6)$$

The ansatz for the most general $q \bar{q}$ density matrix of [2] reads then

$$\rho^{(q, \bar{q})}(k'_1, p'_1, p'_2) = (\rho^{(q, \bar{q})}_{\alpha \beta, \alpha' \beta'}(k'_1, p'_1, p'_2))$$

$$= \frac{1}{4}\{1 \otimes 1 + F_j(\sigma \cdot e'_j) \otimes 1 + G_j 1 \otimes (\sigma \cdot e'_j) + H_{ij}(\sigma \cdot e'_i) \otimes (\sigma \cdot e'_j)\}. \quad (3.7)$$

Here

$$1 \otimes 1 = (\delta_{\alpha \alpha'} \delta_{\beta \beta'}), \quad \sigma \otimes 1 = (\sigma_{\alpha \alpha'} \delta_{\beta \beta'}), \text{ etc.} \quad (3.8)$$

with $\alpha, \alpha'$ and $\beta, \beta'$ the quark and antiquark spin indices, respectively. The parameters of $\rho^{(q, \bar{q})}$ are two vectors, $F$ and $G$, and a second rank tensor $(H_{ij})$ which all may depend on the momenta $k'_1, p'_1$, and $p'_2$,

$$F = F_j e'_j = F(k'_1, p'_1, p'_2),$$
$$G = G_j e'_j = G(k'_1, p'_1, p'_2),$$
$$H = H_{ij} e'_i \otimes e'_j = H(k'_1, p'_1, p'_2). \quad (3.9)$$

Parity (P) invariance of the strong interactions implies

$$\rho^{(q, \bar{q})}(k'_1, p'_1, p'_2) = \rho^{(q, \bar{q})}(-k'_1, -p'_1, -p'_2). \quad (3.10)$$

Therefore, $F$ and $G$ must be pseudovectors, $H$ a P-even tensor:

$$F(-k'_1, -p'_1, -p'_2) = F(k'_1, p'_1, p'_2),$$
$$G(-k'_1, -p'_1, -p'_2) = G(k'_1, p'_1, p'_2),$$
$$H(-k'_1, -p'_1, -p'_2) = H(k'_1, p'_1, p'_2). \quad (3.11)$$

The standard assumption of unpolarised quarks and antiquarks corresponds to

$$F = 0, \quad G = 0, \quad H = 0,$$
$$\rho^{(q, \bar{q})}_{\text{standard}} = \frac{1}{4}1 \otimes 1. \quad (3.12)$$

At this point it is appropriate to discuss the dependencies of the functions $F_j$, $G_j$, and $H_{ij}$ of (3.7) on other parameters than the momenta indicated explicitly in (3.9). Already in [1] it was written that the general density-matrix framework does not require the same non-perturbative effects for muon pair production in proton-proton and in other hadron-hadron collisions. Thus, in the general density-matrix formalism $\rho^{(q, \bar{q})}$ should be allowed to depend on the nature of
the parent hadrons $h_1$ and $h_2$. Let us also recall point (iii) of the summary in section 2. The shielding of the chromomagnetic vacuum fields may be different in different hadrons and, thus, lead to a dependence of $\rho^{(q\bar{q})}$ on the hadrons $h_1$ and $h_2$. For given hadrons $h_1$ and $h_2$ $\rho^{(q\bar{q})}$ certainly should depend on the quark flavour $q$. This is clear from the whole discussion of the “synchrotron” effects of the chromomagnetic background fields in [1–4] and section 2. “Synchrotron” effects clearly should be quite different for very light $u$ and $d$ quarks compared to heavier $s, c$, or $b$ quarks. Thus, in the general density-matrix approach $\rho^{(q\bar{q})}$ should be allowed to depend on the hadrons $h_1$ and $h_2$ as well as on the quark flavour $q$.

In order to calculate in our general framework the cross section for the Drell-Yan processes (3.1) one has to evaluate the production matrix for $q + \bar{q} \rightarrow V(k, \epsilon)$ using the density matrix (3.7). Then, the vector-boson production matrix from $q\bar{q}$ annihilation has to be summed over quark flavours and integrated over the momentum distributions of $q$ and $\bar{q}$, which in [2] were also allowed to be correlated, to get the overall vector-boson production matrix for $h_1 + h_2 \rightarrow V + X$. Finally, this production matrix has to be contracted with the decay matrix for the decay of $V$ into the channel one wants to observe. Typically one considers leptonic decays ($l = e, \mu$):

$$\gamma^* \rightarrow l^+l^-, \quad Z \rightarrow l^+l^-$$ (3.13)

and similarly for $W$ production (3.2) the decays

$$W^+ \rightarrow l^+\nu_l, \quad W^- \rightarrow l^-\bar{\nu}_l. \quad \text{(3.14)}$$

All formulae for performing this program are given in [2]; see also appendix A of the present paper where a misprint of [2] is corrected. Furthermore, in [2] various predictions for the process (3.1) with $V = \gamma^*$ and $Z$ were worked out. At this point we want to point out that the work of [2] was started as a natural continuation of [1] and was half way completed without the authors knowing about the relevant experiments. Only then, in a discussion, H. J. Pirner kindly pointed out to the present author and his collaborators that in the NA10 experiment [6, 7] an extensive study of the lepton-pair angular distribution in the Drell-Yan process had been done. Of course, this was an exciting moment for us. Was everything in the experimental distributions according to the standard $q\bar{q}$ density matrix, (3.12), or was there room for some non-standard effects? It turned out that there was indeed a large deviation from the standard expectation for the lepton-pair angular distribution as we shall recall in the next section.

### 3.2 Comparison with the NA10 data

In the NA10 experiment [6, 7] the Drell-Yan reaction in $\pi^-$ nucleon collisions was studied:

$$\pi^- + N \rightarrow \gamma^* + X, \quad \leftrightarrow \mu^+ + \mu^- \quad \text{(3.15)}$$

The momenta of the incident $\pi^-$ were 140 GeV/c, 194 GeV/c, and 286 GeV/c, the targets were deuterium and tungsten. The experiment collected enough statistics to make a detailed investigation of the muon’s angular distributions.
The lepton-pair distribution can be analysed in the so-called Collins-Soper (CS) frame where the following basis vectors are introduced in the $\gamma^*$ rest frame

$$e'_{1,CS} = \frac{\hat{p}'_1 + \hat{p}'_2}{|\hat{p}'_1 + \hat{p}'_2|},$$

$$e'_{2,CS} = \frac{\hat{p}'_1 \times \hat{p}'_2}{|\hat{p}'_1 \times \hat{p}'_2|},$$

$$e'_{3,CS} = \frac{\hat{p}'_1 - \hat{p}'_2}{|\hat{p}'_1 - \hat{p}'_2|}. \quad (3.16)$$

Here $p'_{1,2}$ are the momenta of $h_{1,2}$ in the $\gamma^*$ rest frame and $\hat{p}'_i = p'_i/|p'_i|$, $i = 1, 2$. The general formula for the angular distribution of the lepton $l'$ in the Drell-Yan reaction (3.15) reads

$$\frac{1}{\sigma} \frac{d\sigma}{d\Omega'} = \frac{3}{4\pi} \frac{1}{\lambda + 3} \left( 1 + \lambda \cos^2 \theta + \mu \sin(2\theta) \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos(2\phi) \right). \quad (3.17)$$

Here $\theta, \phi$ are the polar and azimuthal angles, respectively, of the $l'$ momentum in the CS frame. The functions $\lambda, \mu, \nu$ depend on the other kinematic variables, notably the pseudorapidity $\eta$, the absolute value of the transverse momentum, $|k_T|$, and the mass of the virtual photon $\gamma^*$. We remark that there exists now a “Trento convention 2012” for defining the transformation from the c.m. system to the vector-boson rest system and for the angles $\theta, \phi$; see [21]. For obvious reasons we stick here to the original conventions used in [2], but we give the relations to the new Trento convention in appendix B.

We return to the discussion of the NA10 experiment. With the standard $q\bar{q}$ density matrix (3.12) one finds the Lam-Tung relation

$$1 - \lambda - 2\nu = 0 \quad (3.18)$$

which is valid also to order $\alpha_s$; see [52]. In [53] it was found that even to order $\alpha_s^2$ the relation (3.18) is hardly changed with the standard density matrix. But in the experiment [6, 7] a large violation of the Lam-Tung relation (3.18) is found; see figure 7 which is taken from figure 8 of [2]. We see that in the $|k_T|$ interval explored by the experiment $\lambda \approx 1, \mu \approx 0$. Then, the Lam-Tung relation (3.18) predicts:

$$\nu \approx 0 \quad (3.19)$$

in violent disagreement with experiment. The theory with the standard $q\bar{q}$ density matrix (3.12) gives the dashed lines in figure 7, in accordance with (3.19). Also soft gluon resummations do not change this result appreciably; see [54]. What can one say on the data from figure 7 assuming a non-trivial $q\bar{q}$ density matrix (3.7)?

To answer this question it is useful to discuss first which elements of the density matrix (3.7) can be probed in the Drell-Yan reaction (3.1) with $V = \gamma^*$ and $Z$ at leading order. We consider the $q\bar{q}$ annihilation (3.3) for massless quarks. Then $\gamma_5$ invariance of the $Vq\bar{q}$ vertex tells us that the annihilation can only occur in the following helicity configurations

$$q_L \bar{q}_R \rightarrow V, \quad q_R \bar{q}_L \rightarrow V \quad (3.20)$$

where $L$ and $R$ stand for left- and right-handed polarisations, respectively. With the standard representation of the Pauli matrices and using the coordinate system (3.6) the 2-spinors of $q$
Figure 7: The $|k_T|$ dependence of the structure functions $\lambda, \mu, \nu$ for $\pi^- N$ collisions at 194 GeV/$c$ incident $\pi^-$ momentum. The pseudorapidity and mass of $\gamma^*$ are $\eta = 0$ and $m_{\gamma^*} = 8$ GeV, respectively. The data are from Table 4 of [7]. The dashed lines are for the standard $q\bar{q}$ density matrix (3.12), the solid lines for the spin-correlated $q\bar{q}$ density matrix (3.7) with $\kappa$ from (3.23),(3.24). See the discussion there for the details. This figure is reproduced from figure 8 of [2].

and $\bar{q}$ of definite helicity are as follows:

quark $q$: \[\chi_R = \begin{pmatrix} 1 \\ \hline 0 \end{pmatrix}, \quad \chi_L = \begin{pmatrix} 0 \\ 1 \end{pmatrix},\] (3.21)

antiquark $\bar{q}$: \[\chi'_R = \begin{pmatrix} 0 \\ \hline 1 \end{pmatrix}, \quad \chi'_L = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.\] (3.22)

From this we get the density matrix $\rho^{(q,\bar{q})}$ (3.7) in the helicity basis as shown in table 2. With (3.20) we see that only the matrix elements of the $2 \times 2$ submatrix corresponding to the entries $RL$ and $LR$ enter in the Drell-Yan reaction (3.1) with $V = \gamma^*$ or $Z$. Furthermore, for the ordinary Drell-Yan reaction, $V = \gamma^*$, with $\gamma^* \to l^+l^-$ and no observation of the lepton polarisations only $H_{33}, H_{11} - H_{22}$, and $H_{12} + H_{21}$ remain as relevant parameters; see appendix A. It turns out that $1 + H_{33}$ enters in the calculation of the overall rate. In [2] $H_{12} + H_{21}$ was
Table 2: The matrix elements \( \langle q_{A}, \bar{q}_{B} | \rho^{(q, \bar{q})} | q_{A'}, \bar{q}_{B'} \rangle \) where \( \rho^{(q, \bar{q})} \) is given in (3.7) and \( A, B, A', B' \in \{ R, L \} \). Only the upper-left \( 2 \times 2 \) submatrix is relevant for the Drell-Yan reaction (3.1).

| \( A'B' \) | \( RL \) | \( LR \) | \( RR \) | \( LL \) |
|---|---|---|---|---|
| \( RL \) | \( 1 + F_{3} + G_{3} + H_{33} \) | \( H_{11} - H_{22} - iH_{12} - iH_{21} \) | \( G_{1} - iG_{2} + H_{31} - iH_{32} \) | \( F_{1} - iF_{2} + H_{13} - iH_{23} \) |
| \( LR \) | \( H_{11} - H_{22} + iH_{12} + iH_{21} \) | \( 1 - F_{3} + G_{3} + H_{33} \) | \( F_{1} + iF_{2} - H_{13} - iH_{23} \) | \( G_{1} + iG_{2} - H_{31} - iH_{32} \) |
| \( RR \) | \( G_{1} + iG_{2} + H_{31} + iH_{32} \) | \( F_{1} - iF_{2} - H_{13} + iH_{23} \) | \( 1 + F_{3} - G_{3} - H_{33} \) | \( H_{11} + H_{22} - iH_{12} - iH_{21} \) |
| \( LL \) | \( F_{1} + iF_{2} + H_{13} + iH_{23} \) | \( G_{1} - iG_{2} - H_{31} + iH_{32} \) | \( H_{11} + H_{22} - iH_{12} + iH_{21} \) | \( 1 - F_{3} + G_{3} - H_{33} \) |

set to zero and it was found that then the parameter

\[
\kappa = \frac{H_{22} - H_{11}}{1 + H_{33}} \tag{3.23}
\]
determines the lepton angular distribution. In [2] an ansatz was made for \( \kappa \) at pseudorapidity \( \eta = 0 \):

\[
\kappa = \kappa_{0} \frac{|k_{T}|^{4}}{|k_{T}|^{4} + m_{T}^{4}} \tag{3.24}
\]

with \( \kappa_{0} \) and \( m_{T} \) as parameters. It was found that instead of the Lam-Tung relation (3.18) one has now

\[
1 - \lambda - 2\nu \approx -4\kappa. \tag{3.25}
\]

The complete calculation for \( \lambda, \mu, \) and \( \nu \) has to take into account the integration over the parton distributions. The results are shown in figure 7 as the solid lines for the choice

\[
\begin{align*}
\kappa_{0} & = 0.17, \\
\m_{T} & = 1.5 \text{ GeV}
\end{align*} \tag{3.26}
\]

in (3.24). The agreement with experiment is quite satisfactory. In [2] it was also shown that the sign of \( \kappa \) is precisely as expected from the chromomagnetic Sokolov-Ternov effect. Indeed, consider figure 3 with \( q \) and \( \bar{q} \) annihilating in the common background field \( B^{a} \). Then, \( q \) and \( \bar{q} \) must come with corresponding colour and anticolour, and thus, will get opposite transverse polarisation due to the chromomagnetic Sokolov-Ternov effect. Taking the reaction plane as spanned by \( k_{1}^{1} \) and \( p_{1}^{1} + p_{2}^{1} \), see figure 6 and (3.6), the transverse direction is spanned by \( e_{1}^{1} \).

In [2] such a correlated transverse polarisation of \( q \) and \( \bar{q} \) was found to lead to a density matrix (3.7) with

\[
F_{1} = s, \ G_{1} = -s, \ H_{11} = -s^{2}
\]

and all other elements \( F_{j}, G_{j} \) and \( H_{ij} \) equal to zero. Here \( s \), with \( |s| \leq 1 \), is the degree of transverse polarisation of the quark. Clearly, from (3.23) and (3.27) we get \( \kappa \geq 0 \). We
emphasise that (3.27) is only supposed to give an example of parameters leading to a density matrix with \( \kappa \geq 0 \). It is not to be cited as “the prediction of our model”. We should also emphasise that the simple ansatz for \( \kappa \) in (3.24) made in comparison with the data shown in figure 7, was never supposed to be universally valid. On the contrary, after (3.13) of [2] it was written that the parameters \( F, G \) and \( H \) of the density matrix \( \rho(q, \bar{q}) \) are real functions of the invariants of the problem. We shall come back to this point in section 3.5 and appendix A below.

3.3 Relation to the TMD approach

When we discussed at the time the results of the paper [2], as outlined in section 3.2 here, with theorists and experimentalists there was little resonance. One reason may be that at the time no further experiments studying the Drell-Yan reaction were on the horizon. Thus, the present author went on to work on other topics. His interest in the Drell-Yan problem was rekindled at a physics meeting in 2003 where he met D. Boer. The latter told him about the work on this problem he had done [55]. In the common paper [5] the relation of the respective approaches was discussed. The conclusions were as follows. In the TMD approach of [55] a factorizing density matrix is assumed for the \( q \bar{q} \) pair:

\[
\rho^{(q, \bar{q})}(k_1', p_1', p_2') = \rho^{(q)}(k_1', p_1') \otimes \rho^{(\bar{q})}(-k_1', p_2').
\] (3.28)

Here the density matrix for the quark from hadron \( h_1 \) is allowed to depend only on the quark momentum \( k_1' \) and the \( h_1 \) momentum \( p_1' \). Similarly, \( \rho^{(\bar{q})} \) depends here only on the momenta of the antiquark \((-k_1')\) and of hadron \( h_2 \left( p_2' \right) \). Thus, we get

\[
\rho^{(q)}(k_1', p_1') = \frac{1}{2} \left\{ 1 + F^B(k_1', p_1') \cdot \sigma \right\},
\]

\[
\rho^{(\bar{q})}(-k_1', p_2') = \frac{1}{2} \left\{ 1 + G^B(-k_1', p_2') \cdot \sigma \right\}.
\] (3.29)

From P invariance of the strong interactions \( F^B \) and \( G^B \) must be pseudovectors, see (3.11), and therefore we must have

\[
F^B(k_1', p_1') \propto k_1' \times p_1' \propto e_3' \times p_1',
\]

\[
G^B(-k_1', p_2') \propto (-k_1') \times p_2' \propto -e_3' \times p_2'.
\] (3.30)

With the functions \( f_1, h^+_1, f^+_1, \tilde{h}^+_1 \) as defined in [5,55] the ansatz for \( \rho^{(q, \bar{q})} \) using TMDs finally reads as in (3.7), (3.9) with the replacements

\[
F(k_1', p_1', p_2') \rightarrow F^B(k_1', p_1') = \frac{h^+_1 x_1}{f_1 M_1} (e_3' \times p_1'),
\] (3.31)

\[
G(k_1', p_1', p_2') \rightarrow G^B(-k_1', p_2') = \frac{-\tilde{h}^+_1 x_2}{f_1 M_2} (e_3' \times p_2'),
\] (3.32)

\[
H_{ij}(k_1', p_1', p_2') \rightarrow H^B_{ij}(k_1', p_1', p_2') = F^B_i(k_1', p_1')G^B_j(-k_1', p_2').
\] (3.33)
Here \( f_1, \ldots, \bar{h}_1 \) have a functional dependence as follows:

\[
\begin{align*}
    f_1 &= f_1(x_1, k_{1T}^2), \\
    h_1^+ &= h_1^+(x_1, k_{1T}^2), \\
    \tilde{f}_1 &= \tilde{f}_1(x_2, k_{2T}^2), \\
    \tilde{h}_1^+ &= \tilde{h}_1^+(x_2, k_{2T}^2)
\end{align*}
\]

where \( x_i \) and \( k_{2T}^2 \) are defined in (3.4).

We see from (3.30) to (3.33) that in the TMD approach we have, in particular,

\[
\begin{align*}
    F_3^B &= C_3^B = 0, \\
    H_{33}^B &= 0.
\end{align*}
\]

Clearly, the TMD framework where only factorizing \( q\bar{q} \) density matrices are considered is included as a special case in our framework of (3.7) to (3.11). Thus, it makes no sense to ask for an effect which is describable by the TMD approach and not in our more general framework. But is makes a lot of sense to ask if there are observable effects which cannot be described by a factorizing \( q\bar{q} \) density matrix, (3.28) to (3.33), but require a general \( q\bar{q} \) density matrix. Such effects would point to the phenomenon of parton entanglement.

### 3.4 Signatures of parton entanglement in the Drell-Yan process

Here we discuss some effects which, if observed, would point towards a general, non-factorising, \( q\bar{q} \) density matrix (3.7).

**Correlations of transverse momenta of quark and antiquark**

In our leading order calculation we get for the mean transverse momentum of the vector boson \( V = \gamma^*, Z \) in the reaction (1.1)

\[
\langle k_T^2 \rangle = \langle (k_{1T} + k_{2T})^2 \rangle = \langle k_{1T}^2 \rangle + \langle k_{2T}^2 \rangle + 2\langle k_{1T} \cdot k_{2T} \rangle;
\]

see (3.3), (3.4). Here \( k_{1T} \) and \( k_{2T} \) refer to the quark \( q \) and antiquark \( \bar{q} \) transverse momenta, respectively, and the average is also over quark from \( h_1 \), antiquark from \( h_2 \) and vice versa. If now the \( q \) and \( \bar{q} \) transverse momenta are uncorrelated we get:

\[
\begin{align*}
    \langle k_{1T} \cdot k_{2T} \rangle &= 0, \\
    \langle k_T^2 \rangle &= \langle k_{1T}^2 \rangle + \langle k_{2T}^2 \rangle.
\end{align*}
\]

On the other hand, maximal positive \( k_T \) correlations imply

\[
\begin{align*}
    \langle k_{1T} \cdot k_{2T} \rangle &= \sqrt{\langle k_{1T}^2 \rangle} \sqrt{\langle k_{2T}^2 \rangle}, \\
    \langle k_T^2 \rangle &= \left( \sqrt{\langle k_{1T}^2 \rangle} + \sqrt{\langle k_{2T}^2 \rangle} \right)^2.
\end{align*}
\]
Suppose, as an example, that
\[
\langle k_1^2 \rangle = \langle k_{2T}^2 \rangle. \tag{3.39}
\]
Then, assuming no $k_T$ correlations, we get from (3.37)
\[
\langle k_1^2 \rangle = \frac{1}{2} \langle k_{2T}^2 \rangle. \tag{3.40}
\]
But with maximal $k_T$ correlations we find from (3.38)
\[
\langle k_1^2 \rangle = \frac{1}{4} \langle k_{2T}^2 \rangle. \tag{3.41}
\]
Already in [2] an explicit distribution of the $q$ and $\bar{q}$ transverse momenta was given which interpolates between the two extreme cases above. This is recalled and discussed in detail in appendix A.

The message from these considerations is the following. If positive $k_T$ correlations are present in the DY reaction these must be taken into account when estimating the $q$ and $\bar{q}$ transverse momenta from the $k_T$ of the vector boson. Neglecting the correlations the estimates of $\langle k_1^2 \rangle$, $\langle k_{2T}^2 \rangle$ from DY will, for positive correlations, be too large compared, for instance, to estimates from semi-inclusive-deep-inelastic scattering (SIDIS).

It is interesting to note that, indeed, there is a tension between the $k_T$ determinations of $q$ and $\bar{q}$ from DY and SIDIS; see [21]. This could point to parton entanglement as discussed here. But one must be careful since, in reality, the influence of higher order QCD effects, both for DY and SIDIS, has to be investigated before one can draw more firm conclusions.

**Absolute normalisation of the DY cross section**

We see from the discussion in [2, 5] and from (A.25), (A.35) and (A.45), (A.46) that the absolute normalisation of the DY cross section is sensitive to $1 + H_{33}$. In the TMD approach one has strictly $H_{33} = 0$; see (3.35). Thus, a good measurement of the absolute normalisation in the DY reaction could reveal parton entanglement. Here the proton-antiproton reaction ($h_1 = p, h_2 = \bar{p}$ in (1.1)) would be most suitable since there the parton distribution functions are well known. The effect from $1 + H_{33}$ is in essence the effect already discussed in [37] on the basis of an instanton calculation.

**Angular distribution of the lepton pair in the reaction (1.1)**

Here the task is, in principle, to determine all parameters $F, G$ and $H_{ij}$ in the general density matrix (3.7). However, we see from (A.24) and (A.25) that from (1.1) with $V = \gamma^*$ and $Z$ only $(1 + H_{33})$, $(F_3 + G_3)$, $(H_{11} - H_{22})$ and $(H_{12} + H_{21})$ can be determined. Nonzero values for $(H_{11} - H_{22})$ and $(H_{12} + H_{21})$ are also allowed in the TMD approach, but there $(F_3 + G_3)$ and $H_{33}$ must be zero; see (3.35). Thus, an experimental result of $F_3 + G_3 \neq 0$ would point to parton entanglement. But $(F_3 + G_3)$ is hard to observe; see appendix A. One may have a chance in $Z$ production; see (A.24), (A.41). For the DY reaction (1.1) with $V = \gamma^*$ one would need observation of the lepton polarisation. This seems very difficult. Maybe, the DY reaction with $\tau$ leptons could be suitable. Instanton effects for the angular distributions in the DY reaction were discussed in [5, 56].
3.5 Remarks on some recent experiments

In this section we want to make comments on the findings of some recent experiments.

In the experiments [14, 15] on the DY reactions

\[ p + d \rightarrow \gamma^* + X, \]
\[ p + p \rightarrow \gamma^* + X, \]  \hspace{1cm} (3.42)

only very small, if any, non-standard QCD effects as discussed here were observed. Does this present a problem to the views and the ansätze discussed in [1–4] and the present article? Certainly not! In our general approach to the \( q\bar{q} \) density matrix its parameters are left free; see section 3.1 and appendix A. In our view, in a phenomenological approach, these parameters from non-perturbative QCD are to be determined from experimental data as is the case for the parton distribution functions. We also recall point (iii) of the physical picture explained in the summary of section 2. It is quite probable that the shielding of vacuum effects due to soft gluons is more important for sea quarks in the nucleons than for valence quarks and antiquarks in pions and valence quarks in nucleons. That is, the parameters of \( \rho^{(q\bar{q})} \) (3.7) must be allowed to depend on the quark flavour \( q \), on the type of hadrons \( h_1, h_2 \) in (1.1), and on the kinematic variables; see the discussion in appendix A.

In the experiment [17] \( e^+e^- \) pairs in the \( Z \) mass region were studied in \( p\bar{p} \) collisions at \( \sqrt{s} = 1.96 \text{ TeV} \). Again, no significant non-standard QCD effects as discussed by us here were found. Here we have to remark the following.

1. In the experiment [17] the transverse momenta of the \( e^+e^- \) pair are on average quite large. Thus, our considerations, which here are restricted to the leading order process (3.3), are not directly applicable. A careful study of higher order QCD effects together with our non-standard ansatz would be necessary.

2. The experiment [17] does not distinguish \( \gamma^* \) and \( Z \) production. As already emphasised in [2] the non-standard effects from the parameters \( (H_{11} - H_{22}) \) and \( (H_{12} + H_{21}) \) in the \( q\bar{q} \) density matrix (3.7) enter with opposite sign for \( \gamma^* \) and \( Z \) production. We see this by comparing (A.24) and (A.25). For \( Z \) production, (A.24), the term with \( (H_{11} - H_{22}) \) and \( (H_{12} + H_{21}) \) is multiplied by \( -(g_{Vq}^2 - g_{Aq}^2) \), for \( \gamma^* \) production, (A.25), by \(-1\). We note that for \( \sin^2 \theta_W \approx 0.23 \) we have from (A.12), (A.13)

\[ -(g_{Vq}^2 - g_{Aq}^2) > 0, \quad \text{for } q = u, d. \]  \hspace{1cm} (3.43)

Thus, in a mixture of \( \gamma^* \) and \( Z \) events the effects of \( (H_{11} - H_{22}) \) and \( (H_{12} + H_{21}) \) will be reduced.

3. At these high energies, \( \sqrt{s} = 1.96 \text{ TeV} \), \( q\bar{q} \) annihilation for quarks other than \( q = u, d \) will be non-negligible. Already for \( s \) quarks we expect smaller and for \( c \) and \( b \) quarks even much smaller non-standard QCD effects than for \( u \) and \( d \) quarks; see the discussion in section 2 and in [1–4].

To summarise: the interesting findings of the experiments [14, 15] put constraints on the parameters of \( \rho^{(q\bar{q})} \) for \( pp \) and \( pd \) collisions. For experiment [17] at least an analysis of \( \gamma^* \) and \( Z \) production separately – and also of the \( \gamma^*-Z \) interference term – would be necessary before one could draw further conclusions on \( \rho^{(q\bar{q})} \).
4 Higgs-boson production and parton entanglement

Recently, at the LHC a boson was discovered [57, 58] which, presumably, is the Higgs-boson $H_{SM}$ of the standard model of particle physics. The main production mechanism for $H_{SM}$ in $pp$ collisions at LHC is gluon-gluon fusion; see figure 8. In this section we want to discuss the question if entanglement of the spins of the two gluons in figure 8 may influence the production rate of $H_{SM}$ or other scalar bosons.

![Figure 8: Higgs-boson production in $pp$ collisions via gluon-gluon annihilation](image)

We consider, thus, the reaction

$$p(p_1) + p(p_2) \rightarrow H(k) + X$$

(4.1)

via gluon-gluon annihilation

$$G(k_1) + G(k_2) \rightarrow H(k), \quad k_1 + k_2 = k.$$  

(4.2)

Here $H$ stands generically for a scalar boson. We shall restrict ourselves to the collinear approximation and work in the $pp$ c.m. system. We choose Cartesian unit vectors $e_1, e_2, e_3$ with

$$e_3 = p_1/|p_1|,$$

$$e_i \cdot e_j = \delta_{ij},$$

$$(e_1 \times e_2) \cdot e_3 = 1.$$  

(4.3)

We have then

$$p_1 = |p_1|e_3, \quad p_2 = -|p_1|e_3,$$

$$k_1 = x_1p_1, \quad k_2 = x_2p_2.$$  

(4.4)

We set

$$s = (p_1 + p_2)^2$$  

(4.5)
and assume high energies, that is,

$$s \gg m_p^2.$$  \hfill (4.6)

We have then, neglecting terms of relative order \(m_p^2/s,\)

\[
\begin{align*}
p_1^0 & = p_0^0 = \frac{1}{2} \sqrt{s}, \\
|p_1| & = |p_2| = \frac{1}{2} \sqrt{s}, \\
x_1 & = \frac{k^0 + k^3}{\sqrt{s}}, \\
x_2 & = \frac{k^0 - k^3}{\sqrt{s}}, \\
m_{H}^2 & = x_1 x_2 s.
\end{align*}
\hfill (4.7)

4.1 Gluonic spin density matrices

Consider now an unpolarised proton \(p(p_1)\) with a collinear gluon \(G(k_1)\) in it. Its spin and colour-spin density matrix must be of the form

\[
\rho^{(G)}_{a,i,a',i'}(x_1) = \frac{1}{8} \delta_{aa'} \left[ a(x_1) \delta_{i'i} + b(x_1) \varepsilon_{i'i} \right],
\hfill (4.8)
\]

\(i, i' \in \{1, 2\}, \quad a, a' \in \{1, \ldots, 8\}.

Here we use rotational invariance around the axis \(e_3, a, a'\) are the colour-spin indices, \(i, i'\) the spin indices in the linear polarisation basis, and

\[(\varepsilon_{ii'}) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \hfill (4.9)

Parity (P) invariance requires

\[b(x_1) = 0. \hfill (4.10)\]

Then, (4.8) together with the normalisation condition

\[
\rho^{(G)}_{a,i,a',i'}(x_1) \delta_{aa'} \delta_{ii'} = 1
\hfill (4.11)

implies

\[
\rho^{(G)}_{a,i,a',i'}(x_1) = \frac{1}{8} \delta_{aa'} \frac{1}{2} \delta_{ii'}. \hfill (4.12)
\]

Thus, the spin density matrix for a collinear gluon in an unpolarised proton is the trivial one.

Now we come to the general spin and colour-spin density matrix for the two gluons in (4.2). The two protons in (4.1) are supposed to be unpolarised and the gluons collinear to them. We have rotational invariance around the \(e_3\) axis, P invariance, and we assume absence of colour correlations of the two gluons. The most general density matrix reads then

\[
\rho^{(G,G)}_{a,b,i,j,a',b',i',j'}(x_1, x_2) = \frac{1}{64} \delta_{aa'} \delta_{bb'} \rho^{(G,G)}_{i,j,i',j'}(x_1, x_2),
\hfill (4.13)
\]

\[a, b, a', b' \in \{1, \ldots, 8\}, \quad i, j, i', j' \in \{1, 2\}.\]
with
\[ \hat{\rho}_{i,j;i',j'}^{(G,G)}(x_1, x_2) = c_1(x_1, x_2)\delta_{ii'}\delta_{jj'} + c_2(x_1, x_2)\delta_{ij}\delta_{i'j'} + c_3(x_1, x_2)\delta_{ij'}\delta_{i'j}. \]

(4.14)

Here \(a, i, i'\) and \(b, j, j'\) refer to the colour-spin and spin indices of gluon \(G(k_1)\) and \(G(k_2)\), respectively. The parameters of the density matrix are the real functions \(c_l(x_1, x_2) (l = 1, 2, 3)\).

For the further discussion it is useful to introduce the helicity basis. We define for the gluon \(G(k_1)\)
\[ e_+^{(1)} = -\frac{1}{\sqrt{2}}(e_1 + ie_2), \]
\[ e_-^{(1)} = \frac{1}{\sqrt{2}}(e_1 - ie_2) \]
and for gluon \(G(k_2)\)
\[ e_+^{(2)} = -\frac{1}{\sqrt{2}}(e_1 - ie_2), \]
\[ e_-^{(2)} = \frac{1}{\sqrt{2}}(e_1 + ie_2) \]
The density matrix \(\rho^{(G,G)}\), \(4.14\), reads then
\[ \hat{\rho}^{(G,G)}(x_1, x_2) = e_i \otimes e_j \hat{\rho}_{i,j;i',j'}^{(G,G)}(x_1, x_2)e_{i'}^{\dag} \otimes e_{j'}^{\dag}, \]
\[ = e_m^{(1)} \otimes e_n^{(2)} \hat{\rho}_{m,m';n,n'}^{(G,G)}(x_1, x_2)e_{m'}^{\dag} \otimes e_{n'}^{\dag}, \]
(4.17)
where
\[ m, n, m', n' \in \{+, -\}. \]

(4.18)

From (4.14) to (4.17) we get \(\hat{\rho}_{m,m';n,n'}^{(G,G)}\) as follows
\[ \hat{\rho}_{++;++}^{(G,G)}(x_1, x_2) = \hat{\rho}_{--;--;}^{(G,G)}(x_1, x_2) = c_1(x_1, x_2) \]
\[ \hat{\rho}_{++;--;}^{(G,G)}(x_1, x_2) = \hat{\rho}_{--;++}^{(G,G)}(x_1, x_2) = c_2(x_1, x_2) \]
\[ \hat{\rho}_{++;--}^{(G,G)}(x_1, x_2) = \hat{\rho}_{--;++}^{(G,G)}(x_1, x_2) = c_3(x_1, x_2), \]
(4.19)
and all other matrix elements \(\hat{\rho}_{m,m';n,n'}^{(G,G)}(x_1, x_2) = 0\). We still have the normalisation condition
\[ \text{Tr}\hat{\rho}^{(G,G)}(x_1, x_2) = 1 \]
(4.20)
which implies
\[ 4c_1(x_1, x_2) + 2c_2(x_1, x_2) + 2c_3(x_1, x_2) = 1. \]
(4.21)
Thus, one of the functions \(c_l(x_1, x_2)\) can be eliminated and we choose as independent parameters of \(\hat{\rho}^{(G,G)}\)
\[ \xi(x_1, x_2) = 2c_1(x_1, x_2) + 2c_2(x_1, x_2), \]
\[ \zeta(x_1, x_2) = 2c_2(x_1, x_2) + 2c_3(x_1, x_2). \]
(4.22)
Table 3: The matrix elements $\hat{\rho}^{(G,G)}_{m,n;m',n'}(x_1, x_2)$ parametrised by two real functions $\xi(x_1, x_2)$ and $\zeta(x_1, x_2)$.

| $m', n'$ | ++ | -- | +- | -- |
|---------|----|----|----|----|
| $m, n$  | +  |    |    |    |
| ++      | $\frac{1}{2}\xi(x_1, x_2)$ | $\frac{1}{2}\xi(x_1, x_2)$ | 0  | 0  |
| --      | $\frac{1}{2}\zeta(x_1, x_2)$ | $\frac{1}{2}\xi(x_1, x_2)$ | 0  | 0  |
| +-      | 0  | 0  | $\frac{1}{2}(1 - \xi(x_1, x_2))$ | 0  |
| --      | 0  | 0  | 0  | $\frac{1}{2}(1 - \xi(x_1, x_2))$ |

Together with (4.21) this gives

$$
c_1(x_1, x_2) = +\frac{1}{4} - \frac{1}{4}\zeta(x_1, x_2),
c_2(x_1, x_2) = -\frac{1}{4} + \frac{1}{2}\xi(x_1, x_2) + \frac{1}{4}\zeta(x_1, x_2),
c_3(x_1, x_2) = +\frac{1}{4} - \frac{1}{2}\xi(x_1, x_2) + \frac{1}{4}\zeta(x_1, x_2).$$

(4.23)

With all this the matrix $\hat{\rho}^{(G,G)}$ reads in the helicity basis as shown in Table 3.

We still have the constraint that $\hat{\rho}^{(G,G)}$ must be a positive semi-definite matrix

$$
\hat{\rho}^{(G,G)}(x_1, x_2) \geq 0.
$$

(4.24)

This implies

$$
0 \leq \xi(x_1, x_2) \leq 1,
-\xi(x_1, x_2) \leq \zeta(x_1, x_2) \leq \xi(x_1, x_2).
$$

(4.25)

Finally, for a reaction with identical parent hadrons, as is the case in (4.1), we have

$$
\xi(x_1, x_2) = \xi(x_2, x_1),
\zeta(x_1, x_2) = \zeta(x_2, x_1).
$$

(4.26)

This is all we can say on general grounds about the density matrix $\hat{\rho}^{(G,G)}$. The trivial
matrix corresponding to uncorrelated gluon spins is, of course, given by

\[
\hat{\rho}^{(G,G)}(x_1, x_2)_{\text{standard}} = \frac{1}{4} \mathbb{1}_{4},
\]

\[
\xi(x_1, x_2)_{\text{standard}} = \frac{1}{2},
\]

\[
\zeta(x_1, x_2)_{\text{standard}} = 0.
\]

(4.27)

An example of a non-standard density matrix is discussed in appendix C. We also note that here a factorising two-gluon density matrix, with the one-gluon density matrices satisfying rotational and P invariance, must be of the standard form, (4.27), due to (4.12).

4.2 Production of a \( CP = +1 \) Higgs boson in \( pp \) collisions

We consider now the reaction (4.1), (4.2) for a \( CP = +1 \) Higgs boson \( H \). An example of \( H \) is, of course, the SM Higgs boson. From colour, Lorentz, CPT, and CP invariance we have

\[
\langle H(k) \mid T \mid G(k_1, \varepsilon^{(1)}, a), G(k_2, \varepsilon^{(2)}, b) \rangle \\
= \langle G(k_1, -\varepsilon^{(1)*}, a), G(k_2, -\varepsilon^{(2)*}, b) \mid T \rangle H(k) \rangle \\
= -\delta_{ab} \frac{A}{m_H^2} \left[ (\varepsilon^{(2)} \cdot \varepsilon^{(1)})(k_1 \cdot k_2) - (\varepsilon^{(2)} \cdot k_1)(\varepsilon^{(1)} \cdot k_2) \right], \quad a, b \in \{1, \ldots, 8\}. \quad (4.28)
\]

Here \( k_{1,2} \) and \( \varepsilon^{(1,2)} \) are the momenta and polarisation vectors of the gluons 1 and 2, respectively, and we have

\[
k_1 + k_2 = k, \\
k_1^2 = m_H^2. \quad (4.29)
\]

Furthermore, \( A \) is a dimensionless complex constant.

A standard calculation gives for the decay rate of \( H \) into two gluons

\[
\Gamma(H \rightarrow GG) = \frac{1}{8\pi} m_H |A|^2. \quad (4.30)
\]

Another standard calculation gives for the transition rate of \( GG \rightarrow H \) with the two gluons correlated according to the spin and colour spin density matrix (4.13), (4.14)

\[
d\Gamma(G(k_1) + G(k_2) \rightarrow H(k)) = \frac{1}{V^2} \frac{1}{2k_1^0 2k_2^0} \frac{d^3k}{2\pi^2} \delta^{(4)}(k - k_1 - k_2) R, \quad (4.31)
\]

\[
R = \langle H(k) \mid T \mid G(k_1, \varepsilon^{(1)}, a), G(k_2, \varepsilon^{(2)}, b) \rangle \\
\times \hat{\rho}^{(G,G)}_{a,b,i,j,a',b',i',j'}(x_1, x_2) \\
\times \langle H(k) \mid T \mid G(k_1, \varepsilon^{(1)}_{a'}, a'), G(k_2, \varepsilon^{(2)}_{b'}, b') \rangle^*. \quad (4.32)
\]

23
Here $V$ is the normalisation volume and we work in the $pp$ c.m. system, see (4.3), (4.4), where
\[
\varepsilon^{(1)}_i = \begin{pmatrix} 0 \\ e_i \end{pmatrix}, \quad \varepsilon^{(2)}_j = \begin{pmatrix} 0 \\ e_j \end{pmatrix}, \quad i, j \in \{1, 2\}. \tag{4.33}
\]
From (4.28) we have
\[
\langle H(k) | T | G(k_1, \varepsilon^{(1)}_i, a), G(k_2, \varepsilon^{(2)}_j, b) \rangle = \delta_{ab} \delta_{ij} \frac{1}{2} m_H A. \tag{4.34}
\]
Inserting (4.13), (4.14), and (4.34) in (4.32) we get
\[
R = \frac{1}{6} m_H^2 A^2 \left[ c_1(x_1, x_2) + 2 c_2(x_1, x_2) + c_3(x_1, x_2) \right]. \tag{4.35}
\]
With (4.23) and (4.30) we obtain
\[
R = \frac{\pi}{4} m_H \Gamma(H \rightarrow GG) \left[ \xi(x_1, x_2) + \zeta(x_1, x_2) \right]. \tag{4.36}
\]
Inserting this in (4.31) and using standard formulae from the parton model, see for instance chapter 18.5 of [59], we get the differential and total cross sections for reaction (4.1) as follows
\[
\frac{d\sigma}{k^2}(p(p_1) + p(p_2) \rightarrow H(k) + X) = \frac{\pi^2 \Gamma(H \rightarrow GG)}{4s} \frac{m_H^p}{m_H k^0} N^p_G(x_1) N^p_G(x_2) \left[ \xi(x_1, x_2) + \zeta(x_1, x_2) \right], \tag{4.37}
\]
\[
\sigma(p(p_1) + p(p_2) \rightarrow H + X) = \frac{\pi^2 \Gamma(H \rightarrow GG)}{4s} \frac{m_H}{m_H} \times \int_0^1 dx_1 \int_0^1 dx_2 \delta(x_1 x_2 - \frac{m_H^2}{s}) \times N^p_G(x_1) N^p_G(x_2) \left[ \xi(x_1, x_2) + \zeta(x_1, x_2) \right]. \tag{4.38}
\]
Note that in our collinear approximation the $k_T$ distribution of the $H$ boson is proportional to $\delta^2(k_T)$ and has been integrated over in (4.37). Furthermore, $N^p_G(x)$ are the gluon distribution functions of the proton. In (4.37) $x_1$ and $x_2$ are to be inserted from (4.7).
The standard results at leading-order QCD are obtained from (4.37) and (4.38) by setting $\xi(x_1, x_2) = \frac{1}{2}$ and $\zeta(x_1, x_2) = 0$; see (4.27). For the example of a correlated density matrix discussed in appendix C we have $\xi(x_1, x_2) = 1/2$, $\zeta(x_1, x_2) = 1/2$. Inserting this in (4.37) and (4.38) we get twice the standard results. Note that the positivity constraints (4.25) allow from zero to four times the standard results.

### 4.3 Production of a $CP = -1$ Higgs boson in $pp$ collisions

In many models with an extended scalar sector there are both, $CP = +1$ scalar bosons $H$ and $CP = -1$ scalar bosons which we shall denote generically by $\tilde{H}$. Of course, also scalar
bosons with no definite $CP$ quantum number exist in various models. The extension of our considerations to this case is straightforward.

We consider, thus, in this section the production of a $CP = -1$ scalar boson $\tilde{H}$ in $pp$ collisions via gluon-gluon fusion; see (4.1), (4.2) with $H$ replaced by $\tilde{H}$. From colour, Lorentz, CPT and CP invariance we find here (compare (4.28)):

$$
\langle \tilde{H}(k) | \mathcal{T} | G(k_1, \varepsilon^{(1)}_i, a), G(k_2, \varepsilon^{(2)}_j, b) \rangle
= \langle G(k_1, -\varepsilon^{(1)*}_i, a), G(k_2, -\varepsilon^{(2)*}_j, b) | \mathcal{T} | \tilde{H}(k) \rangle
= -i \delta_{ab} \frac{\tilde{A}}{m_{\tilde{H}}} \varepsilon_{\mu\nu\rho\sigma} \varepsilon^{(1)\mu} \varepsilon^{(2)\nu} k_1^\rho k_2^\sigma,
$$

$$
a, b \in \{1, \ldots, 8\}.
$$

(4.39)

Here $\tilde{A}$ is a dimensionless complex constant and $\varepsilon_{\mu\nu\rho\sigma}$ ($\varepsilon_{0123} = +1$) is the totally antisymmetric tensor. From this we find for the decay and production rates

$$
\Gamma(\tilde{H} \to GG) = \frac{1}{8\pi} m_{\tilde{H}} |\tilde{A}|^2,
$$

$$
d\Gamma (G(k_1) + G(k_2) \to H(k)) |_\rho = \frac{1}{V} \frac{1}{2k_1^0 k_2^0} \frac{d^3 k}{2\pi^2} \delta^{(4)}(k - k_1 - k_2) \tilde{R},
$$

(4.40)

$$
\tilde{R} = \langle \tilde{H}(k) | \mathcal{T} | G(k_1, \varepsilon^{(1)}_i, a), G(k_2, \varepsilon^{(2)}_j, b) \rangle
\times \rho_{a, b, i, j}^{G, G} \delta^{(4)}(k_1 - k_2)
\times \langle \tilde{H}(k) | \mathcal{T} | G(k_1, \varepsilon^{(1)}_i, a'), G(k_2, \varepsilon^{(2)}_j, b') \rangle^*.
$$

(4.42)

We find from (4.39) in the $pp$ c.m. system with (4.3), (4.4), (4.9)

$$
\langle \tilde{H}(k) | \mathcal{T} | G(k_1, \varepsilon^{(1)}_i, a), G(k_2, \varepsilon^{(2)}_j, b) \rangle = i \delta_{ab} \frac{1}{2} m_{\tilde{H}} \tilde{A} \varepsilon_{ij},
$$

(4.43)

$$
\tilde{R} = \frac{\pi}{4} m_{\tilde{H}} \Gamma(\tilde{H} \to GG) [2c_1(x_1, x_2) - 2c_3(x_1, x_2)]
= \frac{\pi}{4} m_{\tilde{H}} \Gamma(\tilde{H} \to GG) [\xi(x_1, x_2) - \zeta(x_1, x_2)].
$$

(4.44)

The differential and total cross sections read here as follows (compare (4.37), (4.38)):

$$
\frac{d\sigma}{dk^2}(p(p_1) + p(p_2) \to \tilde{H}(k) + X)
= \frac{\pi^2}{4s} \Gamma(\tilde{H} \to GG) \frac{m_{\tilde{H}}}{k^0} N_G^p(x_1) N_G^p(x_2) \left[ \xi(x_1, x_2) - \zeta(x_1, x_2) \right],
$$

(4.45)

$$
\sigma(p(p_1) + p(p_2) \to \tilde{H} + X) = \frac{\pi^2}{4s} \Gamma(\tilde{H} \to GG) \frac{m_{\tilde{H}}}{k^0} \left[ \int_0^1 dx_1 \int_0^1 dx_2 \delta(x_1 x_2 - \frac{m_{\tilde{H}}^2}{s}) \right] N_G^p(x_1) N_G^p(x_2) \left[ \xi(x_1, x_2) - \zeta(x_1, x_2) \right].
$$

(4.46)
In (4.45) $x_1$ and $x_2$ are to be inserted from (4.7). Using (4.27) we obtain the standard leading-order QCD results from (4.45), (4.46) for $\xi(x_1, x_2) = 1/2$, $\zeta(x_1, x_2) = 0$. The example of the correlated density matrix of appendix C gives here zero cross sections.

To summarise: in this chapter we have shown that entanglement of the gluon spins can have a drastic influence on the production of $CP = +1$ and $CP = -1$ scalar bosons in $pp$ collisions via gluon-gluon annihilation. This happens already for the collinear case. A two-gluon density matrix factorising into single-gluon density matrices gives, in the collinear case, the standard results. Thus, scalar-boson production via gluon-gluon annihilation is a sensitive probe of two-gluon entanglement effects.

### 5 Conclusions

In this paper we have first reviewed ideas on the QCD vacuum and how non-perturbative QCD effects may influence for instance the Drell-Yan process and soft photon production in hadron-hadron collisions. In chapter 2 this was taken as motivation to discuss the Drell-Yan process with the ansatz of a general quark-antiquark density matrix, as suggested in [1, 2]. We emphasise that this ansatz allows for both a factorising and a non-factorising, that is, entangled density matrix. The TMD approach working with transverse momentum dependent parton distributions, see for instance [5, 49, 55, 60, 61], only allows a factorising density matrix. We emphasise that our approach is more general than the TMD approach and was proposed earlier. Our suggestion to experimentalists working on the Drell-Yan process is to use our general approach for the analysis. In this way they may discover signs of parton entanglement as was discussed in chapter 3.4. Already a study of parton transverse momenta as extracted from the Drell-Yan reaction compared to, for instance, semi-inclusive-deep-inelastic scattering could be very interesting in this respect. Even for enthusiasts of the TMD approach it should be very interesting to have a more general framework which allows to test experimentally the basic assumption of a factorising density matrix made there. The theoretical proofs of factorisation rely on QCD perturbation theory; see [62, 63] for reviews. These proofs clearly do not exclude violations of the factorisation hypothesis due to non-perturbative QCD effects as discussed in [1–4, 37] and in the present article.

In chapter 4 we have discussed the production of $CP = +1$ and $CP = -1$ scalar bosons $H$ and $\tilde{H}$, respectively, via gluon-gluon annihilation in $pp$ collisions. We have shown that already in the collinear case gluon-gluon entanglement may drastically influence the differential and total cross sections for $H$ and $\tilde{H}$ production. In this collinear case a factorising two-gluon density matrix must be the trivial one. Thus, a careful study of Higgs-boson production at the LHC should allow to discover – or at least to set limits on – the gluon-gluon entanglement effects discussed here.

In this paper we have only been concerned with the Drell-Yan reaction and the Higgs-boson production for unpolarised hadrons in the initial state. The generalisation of the discussions to polarised hadrons would be straightforward. For the Drell-Yan case the parameters $F, G, H$ of the $qq$ density matrix would then also depend on the spin parameters of the initial hadrons; compare (3.7) and (A.29), (A.30). Clearly, also higher order QCD effects should be investigated. But all this is beyond the scope of the present paper.

To summarise: we think that a search for effects of parton entanglement in high-energy...
hadron-hadron collisions should be a very worth-while goal for experimentalists. We have dis-
cussed the Drell-Yan reaction and Higgs-boson production in \( pp \) collisions. But, if entanglement
effects exist, they should also show up in other reactions. An example may be the production
of quarkonium states in hadronic collisions. We note that \( 0^+ \) and \( 2^+ \) quarkonium states can
be produced via gluon-gluon annihilation. Thus, effects similar to those discussed for scalar
bosons in section 4 may be important also there.

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**A The Drell-Yan reaction with general \( q\bar{q} \) density matrix**

Here we give the detailed formulae for the Drell-Yan reaction

\[
h_1(p_1) + h_2(p_2) \rightarrow V(k) + X
\]

\[
\downarrow l^+(q_1) + l^-(q_2)
\]

with \( V = \gamma^* \) or \( Z \) for a general density matrix (3.7) of the annihilating \( q\bar{q} \) pair. We use the
definitions and notations of [2] except for the replacements \( e_i^* \rightarrow e_i' \) \((i = 1, 2, 3), \ p_i^* \rightarrow p_i' \)
\((i = 1, 2)\). etc.

We note first that the general density matrix \( \rho^{(q\bar{q})} \) (3.7) must be positive semi definite

\[
\rho^{(q\bar{q})} \geq 0.
\]

(A.2)

For the elements of the upper right \( 2 \times 2 \) part of \( \rho^{(q\bar{q})} \) which is relevant for the Drell-Yan
reaction (see table 2) this implies in particular

\[
1 \geq H_{33} \geq -1,
\]

\[
(1 + H_{33})^2 - (F_3 + G_3)^2 - (H_{11} - H_{22})^2 - (H_{12} + H_{21})^2 \geq 0.
\]

(A.3)

In (3.25) to (3.33) of [2] the \( V \) production matrix at parton level is defined and its general
expansion is given:

\[
r_{ij}^V(k'; \rho; q\bar{q}) = r_{ij}^{V,ij}(k'; \rho; q\bar{q})
\]

\[
= \langle V(i) | T | q(k'_1, \alpha'; A'), \bar{q}(-k'_1, \beta; B') \rangle
\]

\[
\times \rho^{(q\bar{q})}_{\alpha\beta, \alpha'\beta'} \delta_{AA'} \delta_{BB'}
\]

\[
\times \langle V(j) | T | q(k_1, \alpha; A), \bar{q}(-k_1, \beta; B) \rangle^*
\]

\[
= c^V \left\{ \frac{1}{3} \delta_{ij} \hat{a}^V + \frac{1}{2l} \varepsilon_{ijl} \mathcal{B}_{ijl} - \hat{C}_{ijl} \right\}.
\]

(A.4)
Here $A, A', B, B' \in \{1, 2, 3\}$ are the colour indices of $q$ and $\bar{q}$. The Cartesian polarisation indices, $i, j \in \{1, 2, 3\}$, of the vector boson $V$ refer to the coordinate axes (3.6) as defined in the $qq$ c.m. system which is also the rest system of $V$ in the leading order calculations considered here. The coefficients $\tilde{a}^V, \tilde{B}^{V,i}$ and $\tilde{C}^{V,ij}$ occuring in (A.4) are given for $V = Z$ in (3.27) to (3.30) of [2] for an arbitrary factorising $qq\bar{q}$ density matrix (3.22) of [2]. Note that there is a misprint in (3.30) of [2]. The correct equation for $\tilde{C}^Z$ reads

$$\tilde{C}^{Z,ij}(e'_3, s, r, q) = \tilde{C}^{Z,ij}(e'_3, s, r, q)$$
$$= (e'_3 e'_3 - \frac{1}{3} \delta_{ij}) \left\{ \frac{1}{2} (g_{Vq}^2 + g_{Aq}^2) [1 + (s \cdot e'_3) (r \cdot e'_3)] \right.$$  
$$- g_{Vq} g_{Aq} e'_3 \cdot (s + r) + \frac{1}{2} (g_{Vq}^2 - g_{Aq}^2) \times [s \cdot r - (s \cdot e'_3) (r \cdot e'_3)] \right\}$$  
$$+ [(s^i - e'^{ij}_3 (s \cdot e'_3)) (r^j - e'^{ij}_3 (r \cdot e'_3))$$  
$$+ (s^j - e'^{ij}_3 (s \cdot e'_3)) (r^i - e'^{ij}_3 (r \cdot e'_3))$$  
$$- \frac{2}{3} \delta_{ij} (s \cdot r - (s \cdot e'_3) (r \cdot e'_3)) \frac{1}{2} (g_{Vq}^2 - g_{Aq}^2).$$  

(A.5)

From (3.25) to (3.29) of [2] and (A.5) we get the $Z$ production matrix at parton level for the $qq\bar{q}$ density matrix (3.7) by making the following replacements. In the terms linear in $s$ and $r$ we have

$$s \rightarrow F,$$
$$r \rightarrow G.$$  

(A.6)

In the bilinear terms we have

$$s \otimes r \rightarrow H_{ij} e'_i \otimes e'_j.$$  

(A.7)

In this way we obtain for quark-flavour $q$

$$e^2 = \frac{e^2 m_Z^2}{12 \sin^2 \theta_W \cos^2 \theta_W},$$  

(A.8)

$$\tilde{a}_q^Z = (g_{Vq}^2 + g_{Aq}^2) (1 + H_{33}) - 2 g_{Vq} g_{Aq} (F_3 + G_3),$$  

(A.9)

$$\tilde{B}_q^Z = - e'_3 [2 g_{Vq} g_{Aq} (1 + H_{33}) - (g_{Vq}^2 + g_{Aq}^2) (F_3 + G_3)],$$  

(A.10)

$$\tilde{C}^{Z,ij} = (e'^n_3 e'^{n}_3 - \frac{1}{3} \delta_{ij}) \left\{ \frac{1}{2} (g_{Vq}^2 + g_{Aq}^2) (1 + H_{33}) - g_{Vq} g_{Aq} (F_3 + G_3) \right\}$$  
$$+ \frac{1}{2} (g_{Vq}^2 - g_{Aq}^2) \left[ (e'^i_1 e'^{ij}_1 - e'^n_2 e'^{ij}_2) (H_{11} - H_{22}) + (e'^i_1 e'^{ij}_2 + e'^n_2 e'^{ij}_2) (H_{12} + H_{21}) \right].$$  

(A.11)

Here we have

$$g_{Vq} = T_{3q} - 2 Q_q \sin^2 \theta_W,$$
$$g_{Aq} = T_{3q},$$  

(A.12)
with $\theta_W$ the weak mixing angle and

\[
\begin{align*}
T_{3q} &= \frac{1}{2}, \quad Q_q = \frac{2}{3} \quad \text{for } u\text{-type quarks}, \\
T_{3q} &= -\frac{1}{2}, \quad Q_q = -\frac{1}{3} \quad \text{for } d\text{-type quarks}.
\end{align*}
\] (A.13)

Comparing with table 2 we find that for the massless quark flavours $q$ considered here only the matrix elements of $\rho^{(q,\bar{q})}$ with $RL$ and $LR$ enter in (A.9) to (A.11) as it must be due to (3.20). We have

\[
\begin{align*}
1 + H_{33} &= 2(\rho_{RL,RL}^{(q,\bar{q})} + \rho_{LR,LR}^{(q,\bar{q})}), \\
F_3 + G_3 &= 2(\rho_{RL,RL}^{(q,\bar{q})} - \rho_{LR,LR}^{(q,\bar{q})}), \\
H_{11} - H_{22} &= 2(\rho_{RL,LR}^{(q,\bar{q})} + \rho_{LR,RL}^{(q,\bar{q})}), \\
H_{12} + H_{21} &= 2i(\rho_{RL,LR}^{(q,\bar{q})} + \rho_{LR,RL}^{(q,\bar{q})}).
\end{align*}
\] (A.14)

For the ordinary Drell-Yan process, $V = \gamma^\ast$, we have to make the following replacements in (A.8) to (A.11):

\[
\begin{align*}
m_Z &\rightarrow m_{\gamma^\ast}, \\
\frac{e}{\sin \theta_W \cos \theta_W} &\rightarrow e, \\
g_{\nu q} &\rightarrow 2Q_q, \\
g_{A q} &\rightarrow 0.
\end{align*}
\] (A.15)

This gives

\[
\begin{align*}
c_{\gamma^\ast} &= \frac{1}{12} e^2 m_{\gamma^\ast}^2, \\
\tilde{a}_q^{\gamma^\ast} &= 4Q_q^2(1 + H_{33}), \\
\tilde{B}_q^{\gamma^\ast} &= 4Q_q^2 e_3^2(F_3 + G_3), \\
\tilde{C}_q^{\gamma^\ast,ij} &= 2Q_q^2 \left[ (e_3^i e_3^j - \frac{1}{3} \delta_{ij}) (1 + H_{33}) \\
&\quad + (e_1^i e_1^j - e_2^i e_2^j) (H_{11} - H_{22}) \\
&\quad + (e_1^i e_2^j + e_2^i e_1^j) (H_{12} + H_{21}) \right].
\end{align*}
\] (A.16)

It is easy to write the partonic production matrix (A.4) and (A.8) to (A.11) in a covariant way. We consider here massless quarks where $k_1^2 = k_2^2 = 0$. We set

\[
\begin{align*}
K &= k_1 + k_2, \\
\hat{s} &= K \cdot K = 2k_1 \cdot k_2, \\
s &= (p_1 + p_2)^2
\end{align*}
\] (A.17)

and we have then

\[
\begin{align*}
k_1 \cdot K &= k_2 \cdot K = \frac{1}{2} \hat{s}.
\end{align*}
\] (A.18)
We define now
\[ \varepsilon_0^\mu = K^\mu \frac{1}{\sqrt{s}}, \]
\[ \varepsilon_1^\mu = (k_1^\mu - k_2^\mu) \frac{1}{\sqrt{s}}, \]
\[ \varepsilon_2^\mu = \varepsilon_{\mu\rho\sigma} \varepsilon_{3\rho\varepsilon_{1\rho}K_\sigma} \frac{1}{\sqrt{s}}, \]
\[ \varepsilon_1 = \varepsilon_{\mu\rho\sigma} (p_1 + p_2) \cdot k_{1\rho} k_{2\sigma} \xi, \]
\[ \xi = \left[ \hat{s}((p_1 + p_2) \cdot k_1)((p_1 + p_2) \cdot k_2) - \frac{ss_2^2}{4} \right]^{-1/2}, \]
\[ \varepsilon_2 = \varepsilon_{\mu\rho\sigma} \varepsilon_{3\rho\varepsilon_{1\rho}K_\sigma} \frac{1}{\sqrt{s}} \]
where we use the normalisation \( \varepsilon_{0123} = +1 \) for the totally antisymmetric \( \varepsilon_{\mu\rho\sigma} \) symbol. In the c.m. system of the \( q\bar{q} \) collision we have
\[ (\varepsilon_0^\mu) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \]
\[ (\varepsilon_0^a) = \begin{pmatrix} 0 \\ e_a^a \end{pmatrix}, \quad \text{for } a = 1, 2, 3; \]
see (3.6).

The partonic production matrix reads now in covariant form, using (A.4) and (A.8) to (A.11), as follows
\[ r^{V,\mu\nu}(k_1, k_2; \rho; q\bar{q}) = \langle V^\mu(k) \mid T \mid q(k_1, \alpha, A), \bar{q}(k_2, \beta, B) \rangle \]
\[ \times \rho_{\alpha\beta, \alpha'\beta'} \frac{1}{9} \delta_{AA'} \delta_{BB'} \]
\[ \times \langle V^\nu(k) \mid T \mid q(k_1, \alpha', A'), \bar{q}(k_2, \beta', B') \rangle^*, \]
\[ V = \gamma^*, Z. \]

For \( Z \) production we have
\[ r^{Z,\mu\nu}(k_1, k_2; \rho; q\bar{q}) = \frac{1}{2} e^Z \left\{ \left[ (g_{Vq}^2 + g_{Aq}^2)(1 + H_{33}) - 2g_{Vq}g_{Aq}(F_3 + G_3) \right] - g^{\mu\nu} + \varepsilon_0^\mu \varepsilon_0^\nu - \varepsilon_3^\mu \varepsilon_3^\nu \right\} \]
\[ + \left[ -2g_{Vq}g_{Aq}(1 + H_{33}) + (g_{Vq}^2 + g_{Aq}^2)(F_3 + G_3) \right] \frac{2i}{\hat{s}} \varepsilon^{\mu\rho\sigma} k_{1\rho} k_{2\sigma} \]
\[ - (g_{Vq}^2 - g_{Aq}^2) \left[ (H_{11} - H_{22}) (\varepsilon_1^\mu \varepsilon_1^\nu - \varepsilon_2^\mu \varepsilon_2^\nu) \right] \]
\[ + (H_{12} + H_{21}) (\varepsilon_1^\mu \varepsilon_2^\nu + \varepsilon_2^\mu \varepsilon_1^\nu) \right\}. \]
\[ (A.24) \]

The partonic production matrix for \( \gamma^* \) in covariant form is obtained from (A.24) by making
the replacements (A.15):

$$r^{\gamma*,\mu\nu}(k_1, k_2; \rho; q\bar{q}) = \frac{2\pi}{3} \alpha m_\gamma^2 Q_4^2 \left\{ (1 + H_{33})(-g^{\mu\nu} + \varepsilon_0^{\mu} \varepsilon_0^{\nu} - \varepsilon_3^{\mu} \varepsilon_3^{\nu}) \right. \\
+ (F_3 + G_3) \frac{2i}{s} \varepsilon^{\mu\nu\rho\sigma} k_1^\rho k_2^\sigma \\
- \left[ (H_{11} - H_{22})(\varepsilon_1^{\mu} \varepsilon_1^{\nu} - \varepsilon_2^{\mu} \varepsilon_2^{\nu}) \\
+ (H_{12} + H_{21})(\varepsilon_1^{\mu} \varepsilon_2^{\nu} + \varepsilon_2^{\mu} \varepsilon_1^{\nu}) \right] \right\}. \tag{A.25}$$

In the $q\bar{q}$ c.m. system we have

$$\left( r^{V,\mu\nu}(k_1, k_2; \rho; q\bar{q}) \right) = \left( \begin{array}{c|c} 0 & 0 \\ \hline 0 & r^{V,ij}(k'_1; \rho; q\bar{q}) \end{array} \right) \tag{A.26}$$

with $r^{V,ij}(k'_1; \rho; q\bar{q})$ from (A.4).

Here it is appropriate to discuss again the functional dependences of the “unconventional” parameters $F_3, G_3, H_{ij}$ occurring in (A.24) and (A.25). These parameters are discussed here for massless quarks and, in general, will depend on the quark flavour $q$. We have given the parity properties of $F, G$ and $H$ in (3.11). From this we find that $F_3$ and $G_3$ must be P-odd, that is, proportional to the only P-odd invariant we can form from the four vectors $p_1, p_2, k_1$ and $k_2$ available:

$$I = \varepsilon^{\mu\nu\rho\sigma} p_1^\mu p_2^\nu k_1^\rho k_2^\sigma. \tag{A.27}$$

Note that all four vectors are needed to form $I$. The P-even parameters available are (see (3.1) and (3.4))

$$J = \{ s, x_1, x_2, k_{1T}^2, k_{2T}^2, k_{1T} \cdot k_{2T} \}. \tag{A.28}$$

Thus, in general, we get

$$F_3 = I f_3^{(q)}(J),$$
$$G_3 = I g_3^{(q)}(J), \tag{A.29}$$

and

$$H_{33} = H_{33}^{(q)}(J),$$
$$H_{11} - H_{22} = (H_{11} - H_{22})^{(q)}(J),$$
$$H_{12} + H_{21} = I (h_{12} + h_{21})^{(q)}(J). \tag{A.30}$$

Note that we have factored out the P-odd invariant $I$ in $(H_{12} + H_{21})$ since $\varepsilon_1$ is P-odd, $\varepsilon_2$ is P-even, but the tensor

$$(H_{12} + H_{21})(\varepsilon_1^{\mu} \varepsilon_2^{\nu} + \varepsilon_2^{\mu} \varepsilon_1^{\nu}) \tag{A.31}$$

occurring in (A.24), (A.25) must be P-even; see also (3.9), (3.11). In the collinear case,

$$k_{1T} = k_{2T} = 0, \tag{A.32}$$

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we have rotational symmetry around the $q\bar{q}$ collision axis in the $q\bar{q}$ c.m. system. This implies together with P invariance,
\begin{align}
F_3 = G_3 &= 0, \\
H_{11} - H_{22} &= 0, \\
H_{12} + H_{21} &= 0,
\end{align}
(A.33)
for $k_{1T} = k_{2T} = 0$. Note that only $H_{33}$ survives in this case which corresponds to the situation already discussed in [1].

From $r^{V,\mu\nu}$ (A.23) we get the overall production matrix of the $V$ boson in the $h_1$-$h_2$ collision by integration over the quark and antiquark momentum distributions in the hadrons. As in [2] we allow for $k_T$ correlations of $q$ and $\bar{q}$. We define the $V$ production matrix as follows
\begin{align}
R^{V,\mu\nu}(h_1, h_2; k) &= \sum_X (2\pi)^4 \delta^{(4)}(p_1 + p_2 - k - k_X) \\
&\quad \times \sum'_{h_1, h_2 \text{ spins}} \langle V^{\mu}(k), X(k_X) \rangle \langle T | h_1(p_1), h_2(p_2) \rangle \langle V^{\nu}(k), X(k_X) \rangle \langle T | h_1(p_1), h_2(p_2) \rangle^* \\
&= \sum_{h_1, h_2} \int \frac{d^2k_{1T} d^2k_{2T}}{2\pi} P_{12}(k_{1T}, k_{2T}) (2\pi)^4 \delta^{(4)}(k - k_1 - k_2) r^{V,\mu\nu}(k_1, k_2; \rho; q\bar{q}) \\
&\quad + \text{an analogous term with } \bar{q} \text{ from } h_1 \text{ and } q \text{ from } h_2
\end{align}
(A.34)
where $\sum'$ means the average over the spins of $h_1$ and $h_2$. See also (2.3) of [2]. We get then
\begin{align}
R^{V,\mu\nu}(h_1, h_2; k) &= \sum_q \left\{ \int_0^1 dx_1 N_q^{h_1}(x_1) \int_0^1 dx_2 N_{\bar{q}}^{h_2}(x_2) \\
&\quad \times \int d^2k_{1T} d^2k_{2T} P_{12}(k_{1T}, k_{2T}) (2\pi)^4 \delta^{(4)}(k - k_1 - k_2) r^{V,\mu\nu}(k_1, k_2; \rho; q\bar{q}) \\
&\quad + \text{an analogous term with } \bar{q} \text{ from } h_1 \text{ and } q \text{ from } h_2 \right\},
\end{align}
(A.35)
Here $k_{1,2}$ are the quark and antiquark momenta as given in the overall c.m. system in (3.4) and $N_q^{h_1}(x_1), N_{\bar{q}}^{h_2}(x_2)$ are the standard parton distribution functions. In (A.35) $P_{12}(k_{1T}, k_{2T})$ is the $q\bar{q}$ transverse-momentum distribution. In [2] a simple ansatz was made, allowing for transverse momentum correlations:
\begin{align}
P_{12}(k_{1T}, k_{2T}) &= \frac{\alpha_T(\alpha_T + 2\beta_T)}{\pi^2} \exp \left[ -\alpha_T(k_{1T}^2 + k_{2T}^2) - \beta_T(k_{1T} - k_{2T})^2 \right], \\
\alpha_T > 0, \quad \beta_T > -\frac{1}{2} \alpha_T.
\end{align}
(A.36)
Here $\alpha_T$ and $\beta_T$ are parameters which could depend on $s$, $x_1$ and $x_2$. For $\beta_T \neq 0$ the transverse momenta of $q$ and $\bar{q}$ are correlated. From the ansatz (A.36) we get the mean squares of the $q, \bar{q}$ and the vector boson’s transverse momenta, $k_{1T}, k_{2T}$ and $k_T = k_{1T} + k_{2T}$, respectively, as
follows:

$$\langle k_T^2 \rangle = \frac{2}{\alpha_T},$$

$$\langle k_{1T}^2 \rangle = \langle k_{2T}^2 \rangle = \frac{\alpha_T + \beta_T}{\alpha_T(\alpha_T + 2\beta_T)} = \frac{1}{2} \langle k_T^2 \rangle \frac{1 + \beta_T/\alpha_T}{1 + 2\beta_T/\alpha_T}.$$  \hspace{1cm} (A.37)

For no correlation, $\beta_T = 0$, this gives

$$\langle k_{1T}^2 \rangle = \langle k_{2T}^2 \rangle = \frac{1}{4} \langle k_T^2 \rangle.$$  \hspace{1cm} (A.38)

For maximal positive correlation, $\beta_T/\alpha_T \to \infty$, we get, however,

$$\langle k_{1T}^2 \rangle = \langle k_{2T}^2 \rangle = \frac{1}{4} \langle k_T^2 \rangle.$$  \hspace{1cm} (A.39)

Let us suppose now that in nature the transverse momenta of $q$ and $\bar{q}$ are indeed highly correlated. Then, an estimate of $\langle k_{1T}^2 \rangle$ and $\langle k_{2T}^2 \rangle$ from the observed $\langle k_T^2 \rangle$ of the vector boson and assuming no correlation, that is using (A.38), will give a value of the partonic transverse momenta which is too large by a factor up to 2; see (A.37) and (A.39).

In the ansatz (A.36) we have chosen a function symmetric under the exchange $k_{1T} \leftrightarrow k_{2T}$. Clearly, this could easily be made more general allowing for different mean squared transverse momenta of quarks and antiquarks in $h_1$ and $h_2$.

To write down the cross section for the whole reaction (A.1) of $V$ production with subsequent leptonic decay we still need the decay matrices for $V \rightarrow l^+l^-$. These are defined as

$$D_{\mu\nu}^V(q_1, q_2) = \sum_{\alpha, \beta} \langle l^+(q_1, \alpha), l^-(q_2, \beta) \mid T \mid V_\mu(k) \rangle^* \langle l^+(q_1, \alpha), l^-(q_2, \beta) \mid T \mid V_\nu(k) \rangle$$  \hspace{1cm} (A.40)

where $\alpha$ and $\beta$ are the spin indices of $l^+$ and $l^-$, respectively, and we assume no observation of the lepton polarisations. These matrices are given in the $V$ rest system in appendix A of [2]. From (A.5) of [2] we find, in covariant notation, for $V = Z$ setting $m_t = 0$:

$$D_{\mu\nu}^Z(q_1, q_2) = 48\pi m_Z \Gamma(Z \rightarrow l^+l^-)$$

$$\times \left\{ -\frac{1}{2} g_{\mu\nu} + \frac{1}{m_Z^2} (q_1q_{2\nu} + q_2q_{1\nu}) + \frac{i}{m_Z} \varepsilon_{\mu\nu\rho\sigma} q_1^\rho q_2^\sigma \frac{2 g_{Vl} g_{Al}}{g_{Vl}^2 + g_{Al}^2} \right\}. \hspace{1cm} (A.41)$$

Here

$$g_{Vl} = -\frac{1}{2} + 2 \sin^2 \theta_W,$$

$$g_{Al} = -\frac{1}{2},$$  \hspace{1cm} (A.42)

$$\Gamma(Z \rightarrow l^+l^-) = \frac{\alpha m_Z (g_{Vl}^2 + g_{Al}^2)}{12 \sin^2 \theta_W \cos^2 \theta_W}.$$  \hspace{1cm} (A.43)
The cross section for the reaction (A.1) with $V = Z$ is then
\[
d\sigma(h_1 + h_2 \to Z + X \to l^+ + l^- + X) = \frac{1}{2w(s, m_1^2, m_2^2)} \frac{d^3q_1 d^3q_2}{(2\pi)^6 2q_1^0 q_2^0} \times \left[ (k^2 - m_Z^2)^2 + m_Z^4 \Gamma_Z^2 \right]^{-1} D_{\nu\mu}^Z(q_1, q_2) R_{\nu\mu}^{Z,\mu}(h_1, h_2; k),
\]
\[k = q_1 + q_2.\]  
(A.43)

Here $R_{\nu\mu}^{Z,\mu}$ is defined in (A.34), (A.35), $m_{1,2}$ are the masses of $h_{1,2}$, and
\[w(x, y, z) = (x^2 + y^2 + z^2 - 2xy - 2yz - 2xz)^{1/2}.\]  
(A.44)

We have described the $Z$ lineshape by a simple Breit-Wigner formula. For $V = \gamma^*$ in (A.1) we get for the decay matrix from (A.8) of [2] and (A.41)
\[D_{\nu\mu}^{\gamma^*}(q_1, q_2) = 8\pi\alpha \left[ -(q_1 + q_2)^2 g_{\nu\mu} + 2(q_{1\mu} q_{2\nu} + q_{2\mu} q_{1\nu}) \right].\]  
(A.45)

The cross section reads
\[
d\sigma(h_1 + h_2 \to \gamma^* + X \to l^+ + l^- + X) = \frac{1}{2w(s, m_1^2, m_2^2)} \frac{d^3q_1 d^3q_2}{(2\pi)^6 2q_1^0 q_2^0} \times \left( \frac{1}{k^2} \right)^2 D_{\nu\mu}^{\gamma^*}(q_1, q_2) R_{\nu\mu}^{\gamma^*,\mu}(h_1, h_2; k),
\]
\[k = q_1 + q_2, \quad k^2 = m_{\gamma^*}^2.\]  
(A.46)

We note that from (A.45) we have
\[D_{\nu\mu}^{\gamma^*}(q_1, q_2) = D_{\nu\mu}^{\gamma^*}(q_1, q_2).\]  
(A.47)

Therefore, in the contraction with $R_{\nu\mu}^{\gamma^*,\mu}$ in (A.46) the antisymmetric part of the latter drops out. Looking at (A.35) and (A.25) we see that from the correlation effects ($F_3 + G_3$) will drop out and the Drell-Yan reaction with $V = \gamma^*$ and without observation of the lepton polarisations is only sensitive to $(1 + H_{33})$, $(H_{11} - H_{22})$, and $(H_{12} + H_{21})$.

Finally we remark that the lepton angular distributions following from (A.43) and (A.46) are guaranteed to satisfy all general positivity constraints of [64] since our $q\bar{q}$ density matrix (3.7) is required to be positive semi definite; see (A.2), (A.3).

If the lepton polarisations could be observed one would get access also to $(F_3 + G_3)$ in the ordinary Drell-Yan process. Indeed, suppose that we could select exclusively leptons $l^+$ with longitudinal polarisation $r_1/2$ where $r_1 \in \{-1, 1\}$. Then the corresponding decay matrix of the $\gamma^*$ reads, neglecting the lepton mass,
\[D_{\nu\mu}^{\gamma^*}(q_1, r_1; q_2) = 4\pi\alpha \left\{ -g_{\nu\mu}(q_1 + q_2)^2 + 2(q_{1\mu} q_{2\nu} + q_{2\mu} q_{1\nu}) + 2ir_1 \varepsilon_{\nu\mu\rho\sigma} q_1^\rho q_2^\sigma \right\}.\]  
(A.48)

This has an antisymmetric part giving a non-zero result when contracted with the antisymmetric part in $r_{\gamma^*,\mu}$ which is proportional to $(F_3 + G_3)$; see (A.25).

With this we close our review of the kinematic formulae for the reaction (A.1). These formulae are in essence from [2] but are written here in covariant form. We note that there also is a $\gamma^*-Z$ interference term which could be easily written down for our general $q\bar{q}$ density matrix with the methods presented here.
B The angular distribution of the lepton pair in the New Trento Convention.

In [21] a new convention for the notation of momenta and the definition of angular variables is given. We discuss here how this influences the angular distribution (3.17). In the New Trento Convention (TR) the angles $\theta_{TR}$ and $\phi_{TR}$ refer to the $l^-$ momentum in the Collins-Soper frame, whereas we used in [2] and in (3.17) the angles $\theta$ and $\phi$ of the $l^+$ momentum. Thus, we have

$$\theta = \pi - \theta_{TR},$$
$$\phi = \pi + \phi_{TR}. \tag{B.1}$$

This gives

$$\sin \theta = \sin \theta_{TR},$$
$$\cos \theta = - \cos \theta_{TR},$$
$$\sin(2\theta) = - \sin(2\theta_{TR}),$$
$$\cos \phi = - \cos \phi_{TR},$$
$$\cos(2\phi) = \cos(2\phi_{TR}). \tag{B.2}$$

Inserting this in (3.17) we see that the angular distribution looks exactly the same in our convention from [2] and in the New Trento Convention.

C An example of a non-standard two-gluon spin density matrix

Let us assume that the two gluons annihilating to give the boson $H$ in figure 8 have correlated transverse polarisation. As an example we first consider completely correlated transverse polarisation of the two gluons with the three-dimensional polarisation vectors of both gluons given by

$$e(\phi) = \cos \phi \, e_1 + \sin \phi \, e_2. \tag{C.1}$$

Here we use the coordinate system (4.3), (4.4). The spin part of the two-gluon density matrix (4.13), (4.14), (4.17) is then constructed by integrating over $\phi$

$$\hat{\rho}^{(G,G)}(x_1, x_2) = \frac{1}{2\pi} \int_0^{2\pi} d\phi \, e(\phi) \otimes e(\phi) \, e^\dagger(\phi) \otimes e^\dagger(\phi). \tag{C.2}$$

The integral in (C.2) is easily performed and gives a density matrix as shown in table 3 with

$$\xi(x_1, x_2) = \frac{1}{2},$$
$$\zeta(x_1, x_2) = \frac{1}{2}. \tag{C.3}$$

We emphasise that this exercise is only meant to give an example how a non-trivial two-gluon density matrix could be built up. Background fields as discussed in section 2, for instance instantons, may do such a job. But this remains to be investigated in detail. The density matrix with the parameters as in (C.3) is certainly not to be cited as “the prediction of our model”.

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