Solitons in the false vacuum

Yutaka Hosotani

Department of Physics, Osaka University, Toyonaka, Osaka 560-0043

Abstract

When a potential for a scalar field has two local minima, there arises structure of spherical shells due to gravitational interactions.

Gravitational interactions, inherently attractive for ordinary matter, can produce soliton-like objects even when such things are strictly forbidden in flat space. They become possible as a consequence of the balance between repulsive and attractive forces. One such example is a monopole or dyon solution in the pure Einstein-Yang-Mills theory in the asymptotically anti-de Sitter space.[1] In this talk I report a new shell structure in a simple real scalar field theory.

In a scalar field theory given by

$$\mathcal{L} = \frac{1}{16\pi G} R + \frac{1}{2} \phi^{\mu} \phi_{\mu} - V[\phi]\quad(1)$$

we seek a static, spherically symmetric spacetime for which the metric can be written as

$$ds^2 = \frac{H(r)}{p^2(r)} dt^2 - \frac{dr^2}{H(r)} - r^2 d\Omega^2.\quad(2)$$

We suppose that the potential $V(\phi)$ has two minima at $f_1$ and $f_2$ separated by a barrier. Einstein equations and matter equation reduce to

$$\phi''(r) + \Gamma_{\text{eff}}(r) \phi'(r) = \frac{1}{H} V'[\phi], \quad \Gamma_{\text{eff}} = \frac{2}{r} + \frac{4\pi G r \phi'^2}{H} + \frac{H'}{H},\quad(3)$$

$$H = 1 - \frac{2GM}{r}, \quad M(r) = \int_0^r 4\pi r^2 dr \left\{ \frac{1}{2} H \phi'^2 + V[\phi] \right\} .\quad(4)$$

I. False vacuum black hole

Suppose that $V(f_1) = \epsilon > 0$ and $V(f_2) = 0$. See figure 1(a). $\phi = f_1$ and $\phi = f_2$ correspond to the false and true vacuum, respectively. If the universe is in the false vacuum, a bubble of the true vacuum is created by quantum tunneling which expands with accelerated velocity. The configuration is called a bounce.[2] The bounce is a Minkowski bubble in de Sitter.

Let us flip the configuration.[3] The universe is in the true vacuum and the inside of a sphere is excited to the false vacuum. Is such a de Sitter lump in Minkowski possible? If the lump is too small, then it would be totally unstable. The energy localized inside the lump can dissipate to space infinity. If the lump is big enough, the Schwarzschild radius

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becomes larger than the lump radius, i.e. the lump is inside a black hole. The energy cannot escape to infinity. It looks like a soliton in the Minkowski space. However, as being a black hole, it is a dynamic object. The configuration is essentially time-dependent.

The critical configuration of size $R$ carries interesting numbers. The transition region located around $R$ is approximated by a thin wall. The inside is a de Sitter space with $H = 1 - (r/a)^2$, $a^2 = 3/8\pi G\epsilon$. The outside is described by the Schwarzschild metric with $H = 1 - (r_S/r)$, $r_S = 2GM = R^3/a^2$. At the critical radius $R_c = r_S = a = (3/8\pi G\epsilon)^{1/2}$; the Schwarzschild radius and cosmological horizon coincide. If the energy scale, $\epsilon^{1/4}$, is at the GUT scale ($10^{15}$ GeV), then $R_c \sim 10^{27}$ m and $M_c \sim 1$ kg. If $\epsilon^{1/4} \sim 1$ GeV, then $R_c \sim 1$ km and $M_c \sim 10^5 \cdot M_{\text{sun}}$, a typical mass of a black hole located near the center of each galaxy. If $\epsilon^{1/4} = 2.4$ meV as suggested from the estimated value of the present cosmological constant, then $R_c \sim 5.5$ Gpc.

II. Spherical shells

The above false-vacuum-black-hole configuration does not solve Eq. (3) at the horizon. Let us look for nontrivial static solutions to Eqs. (3) and (4) which are regular everywhere. I show that such solutions exist, if the energy scale determined by the scalar potential is sufficiently small compared with the Planck scale.[4, 5] An explicit example is given when $V(f_1) = \epsilon > 0$ and $V(f_2) = 0$. The latter condition $V(f_2) = 0$ can be relaxed.

![Figure 1: Spherical shells in the de Sitter space. (a) Scalar potential. (b) Potential, $U[\phi] = -V[\phi]$, in the particle analogy. (c) $H(r)$ in the metric. (d) Schematic behavior of $\phi(r)$. In the solutions $w/R \ll 1$.](image)

To see how such solutions become possible, we interpret Eq. (3) as an equation for a particle with a coordinate $\phi$ and time $r$. Except for a factor $1/H$ a particle is in a potential
\[ U[\phi] = -V[\phi]. \] The coefficient \( \Gamma_{\text{eff}}(r) \) represents time-dependent friction. We are looking for a solution which starts at \( \phi \sim f_1 \), moves to \( \sim f_2 \), and comes back to \( f_1 \) at \( r = \infty \). It is impossible in flat space, as \( \Gamma_{\text{eff}} \) is positive definite so that the particle loses an energy and cannot climb back to the original starting point.

In the presence of gravity the situation changes. The non-vanishing energy density can make \( H \) decrease as \( r \), and \( \Gamma_{\text{eff}} \) can become negative. The lost energy of the particle during the initial rolling can be regained on the return path by ‘negative friction’, or by thrust. Indeed, this happens.

The schematic behavior of a solution is displayed in figure 1. Take, as an example, a quartic potential \( V[\phi] \) with \( |f_1|, |f_2| \sim 10^{-3} M_{\text{pl}} \), and \( \epsilon/V_0 \sim 10^{-3} \) where \( V_0 \) is the barrier height. In the solution \( \phi \) starts at \( \sim f_1 \) and makes a transition to \( \sim f_2 \) at \( R \sim 10^{7} l_{\text{pl}} \). The transition width is about \( w/R \sim 10^{-3} \). [figure 1(d)] \( H(r) \) in the metric drops at \( \sim R \). [figure 1(c)]

Why or how can such a configuration become possible? First of all I remark that there is no static regular solution which starts at \( \phi(0) \sim f_1 \) and ends at \( \phi(\infty) = f_2 \). In this configuration there is a single transition from one minimum, \( f_1 \), to the other, \( f_2 \), or one shell (wall). Such a configuration is unstable. The shell collapses.

In the configuration depicted in figure 1, however, \( \phi \) makes two transitions: \( f_1 \to f_2 \to f_1 \). It defines two shells. The energy density between the two shells is higher than \( \epsilon \), which gives rise to an attractive force between the shells. There also is an intrinsic repulsive force between two domain walls. [6] Apparently these two forces balance each other out to form static structure. Yet, it is not clear how the configuration is stabilized against shrinkage.

The shell structure disappears as the energy scale of the scalar potential becomes larger and approaches the Planck scale. When the energy scale, instead, becomes smaller, a new type of structure with four or six shells emerges.

The global structure of the configuration is noteworthy. In the metric of the \( R^1 \times S^3 \) type, a mirror of shells appear in the other hemisphere of \( S^3 \). As the universe shrinks and expands, the intrinsic scale of the shells remain constant.

Detailed analysis will be given in ref. [4].

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