Different Treatment Stages in Medical Diagnosis using Fuzzy Membership Matrix

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Abstract. The field of medicine is the most important and developing area of applications of fuzzy set theory. The nature of medical documentation and uncertain information gathered in the use of fuzzy triangular matrix. In this paper, procedures are presented for medical diagnosis and treatment-stages, patient and drug is constructed in fuzzy membership matrix. Examples are given to verify the proposed approach.

Keywords: Medical diagnosis, Triangular fuzzy number matrices, New membership function, Network, Treatment-Stages, Patient and Drug.

1. Introduction

Fuzzy set theory and fuzzy logic became a vast development in the field of medicine and its applications is used in motivating knowledge based system in medical area. Several models like fuzzy matrices lead the way to developed problems under medical diagnosis. Sanchez [1] introduced the diagnostic models for fuzzy matrices representing the medical knowledge between symptoms and diseases. Esogbue and Elder [2] formulated fuzzy cluster analysis to model medical diagnostic. Meenakshi [3] introduced fuzzy matrix theory as a basic framework for characterizing the fuzzy sets concepts and its relationship with the increasingly important concept of information and complexity in various sciences and professions. Meenakshi and Kaliraja [4] and S. Elizabeth and L. Sujatha [5] studied and extended Sanchez’s work involving medical diagnosis which represents the interval valued fuzzy matrix. Also they introduce the concept of arithmetic mean matrix of an interval valued fuzzy matrix and which applied in Sanchez’s method for medical diagnosis. Edward Samuel and Balamurugan [6] extended Sanchez’s work for medical diagnosis using Intuitionistic fuzzy set.

In this paper, some basic definitions of fuzzy set theory are given. In section 2, we described numerically the concept involving triangular fuzzy number and its new membership function under medical diagnosis. Clearly network graphs are represented here. In section 3, we concluded our paper in a more sophisticated way.
2. Pre-Requisites

Definition 2.1. Triangular fuzzy number

A triangular fuzzy number is denoted as $A = (a_1, a_2, a_3)$ where $a_1, a_2, a_3 \in \mathbb{R}$ and $a_1 < a_2 < a_3$.

Definition 2.2. Triangular fuzzy number matrix

A triangular fuzzy number matrix of order $m \times n$ is defined as $A = (a_{ij})_{m \times n}$ where $a_{ij} = (a_{ijL}, a_{ijM}, a_{ijU})$ is the $ij^{th}$ element of $A$. $a_{ijM}$ is the mean value of $a_{ijL}, a_{ijU}$ which are the left and right spreads of $a_{ij}$ respectively.

Definition 2.3. Addition and Subtraction operation on triangular fuzzy number matrix

Let $A = (a_{ij})_{n \times n}$ and $B = (b_{ij})_{n \times n}$ be two triangular fuzzy number matrices of same order. Then (i) Addition Operation

$A \oplus B = (a_{ij} + b_{ij})_{n \times n}$ where $a_{ij} + b_{ij} = (a_{ijL} + b_{ijL}, a_{ijM} + b_{ijM}, a_{ijU} + b_{ijU})$ is the $ij^{th}$ element of $A \oplus B$.

(ii) Subtraction Operation

$A \ominus B = (a_{ij} - b_{ij})_{n \times n}$ where $a_{ij} - b_{ij} = (a_{ijL} - b_{ijL}, a_{ijM} + b_{ijM}, a_{ijU} + b_{ijL})$ is the $ij^{th}$ element of $A \ominus B$.

The same condition holds for triangular fuzzy membership number.

Definition 2.4. Multiplication operation on Triangular fuzzy number matrix

Let $A = (a_{ij})_{m \times p}$ and $B = (b_{ij})_{p \times n}$ be two triangular fuzzy number matrices. Then the Multiplication Operation:

$A \odot B = (c_{ij})_{n \times n}$ where $c_{ij} = \sum_{k=1}^{p} a_{ik} \cdot b_{kj}$ for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.

Definition 2.5. Max-Min Composition on fuzzy membership value matrices

Let $F_{mn}$ denote the set of all $m \times n$ matrices over $F$. Elements of $F_{mn}$ are called as fuzzy membership value matrices. For $A = a_{ij} \in F_{mp}$ and $B = b_{ij} \in F_{pn}$ the max-min product

$A \odot B = (\sup_{k}\{\inf\{a_{ik}; b_{kj}\}\}) \in F_{mn}$.

Definition 2.6. Maximum Operation on triangular fuzzy number

Let $A = (a_{ij})_{n \times n}$ where $a_{ij} = (a_{ijL}, a_{ijM}, a_{ijU})$ and $B = (b_{ij})_{n \times n}$ where $b_{ij} = (b_{ijL}, b_{ijM}, b_{ijU})$ be two triangular fuzzy number matrices of same order. Then the maximum operation on it is given by $L_{\text{max}} = \max(A, B) = (\sup\{a_{ij}; b_{ij}\})$ where

$\sup\{a_{ij}; b_{ij}\} = (\sup(a_{ijL}; b_{ijL}), \sup(a_{ijM}; b_{ijM}), \sup(a_{ijU}; b_{ijU}))$ is the $ij^{th}$ element of $\max(A, B)$.

Definition 2.7. Arithmetic Mean (AM) for triangular fuzzy number

Let $A = (a_1, a_2, a_3)$ be a triangular fuzzy number then $AM(A) = \frac{a_1 + a_2 + a_3}{3}$.

The same condition holds for triangular fuzzy membership number.

3. Patient and Drug for Different Treatment Stages in Medical Diagnosis under Fuzzy Environment

Let $P$ be the set of Patients of some diseases, $D$ is a set of drugs and $T$ is a set of Treatment stages. The triangular fuzzy number matrix elements are defined as $A = (a_{ij})_{m \times n}$ where $a_{ij} = (a_{ijL}, a_{ijM}, a_{ijU})$ is the $ij^{th}$ element of $A$, $0 \leq a_{ijL} \leq a_{ijM} \leq a_{ijU} \leq 1$. Here $(a_{ijL})$ is the lower bound, $a_{ijM}$ is the moderate value and $a_{ijU}$ is the upper bound. Procedure 3.1:

Step 1: Let us Construct a triangular fuzzy number matrix $(P, D)$ over $P$, where $F1$ is a mapping given by $F1 : D \rightarrow F(P)$, $F(P)$ is a set of all triangular fuzzy sets of $P$. This matrix is denoted by $R_0$ which is the fuzzy occurrence matrix or patient-drug triangular fuzzy number matrix.
Step 2: Construct another triangular fuzzy number matrix \((F_2, P)\) over \(T\), where \(F_2\) is a mapping given by \(F_2 : P \rightarrow F'(T)\). This matrix is denoted by \(R_F\), which is the patient-stages triangular fuzzy number matrix.

Step 3: Also convert the elements of triangular fuzzy number matrix into its membership function as follows:

Membership function of \(a_{ij} = (a_{ijL}, a_{ijM}, a_{ijU})\) is defined as \(\mu_{a_{ij}} = \left(\frac{a_{ijL}}{10}, \frac{a_{ijM}}{10}, \frac{a_{ijU}}{10}\right)\), if

\(0 \leq a_{ijL} \leq a_{ijM} \leq a_{ijU} \leq 1\) where \(0 \leq \frac{a_{ijL}}{10} \leq \frac{a_{ijM}}{10} \leq \frac{a_{ijU}}{10} \leq 1\).

The matrix \(R_0\) and \(RT\) are are converted into triangular fuzzy membership matrices namely \((R_0)_{mem}\) and \((RT)_{mem}\).

Step 4: By using definition 2.5 we have computed the relation matrices. \(R' = (RT)_{mem} \odot (R_0)_{mem}\), \(R'' = (RT)_{mem} \odot (J \odot (R_0)_{mem})\), where \(J\) is the triangular fuzzy membership matrix in which all entries are \((1, 1, 1)\) is the complement of \((R_0)_{mem}\) and known as non patient-drug triangular fuzzy membership matrix. \(R''' = (J \odot (RT)_{mem} \odot (R_0)_{mem})\), where \(J \odot (RT)_{mem}\) is the complement of \(RT\) which is non stage-patient triangular fuzzy membership matrix. Also, \(R'\) and \(R''\) are verified from Definition 2.5 and by subtraction operation. Using Definition 2.6 we have calculated \(R'''' = \max(R', R'')\). The elements of \(R', R'', R''', R''''\) are of the form \(x_{ij} = (x_{ijL}, x_{ijM}, x_{ijU})\) where \(0 \leq x_{ijL} \leq x_{ijM} \leq x_{ijU} \leq 1\). By subtraction operation, \(R'' = R' \ominus R'''\). The elements of \(R''\) is of the form \(y_{ij} = (y_{ijL}, y_{ijM}, y_{ijU}) \in [-1, 1]\) where \(y_{ijL} \leq y_{ijM} \leq y_{ijU}\).

Step 5: Finally by calculating \(R'''' = AM(y_{ij})\) and \(Row_i' = \text{Maximum of ith row}\), it concludes that the decision maker which proved with conformity that there is a drugs for the patient.

Next we will give an example to illustrate the above diagnosis.

Numerical Example 3.1:

Let us consider three patients \(P'_1, P'_2, P'_3\) in a hospital with diseases like Common cold, Asthma, Cough and Tanciles. Let the possible diseases relating to the above symptoms be lung disease and liver.

Step 1: We consider the set \(T = \{t_1, t_2, t_3, t_4\}\) as a universal set where \(t_1, t_2, t_3, t_4\) represent the symptoms Common cold, Asthma, Cough and Tanciles respectively and the set \(D = \{d_1, d_2\}\) where \(d_1\) and \(d_2\) are are the parameters lung disease and liver respectively.

For suppose that

\[F(d_1') = \{< e_1, (1, 2, 5, 4) >, < e_2, (5, 6, 5, 8) >, < e_3, (9, 9, 5, 10) >, < e_4, (1, 2, 3) >\}\n
\[F(d_2') = \{< e_1, (2, 3, 5, 5) >, < e_2, (6, 7, 5, 9) >, < e_3, (1, 5, 5, 10) >, < e_4, (8, 9, 10) >\}\n
Let us consider \((F, D)\) a triangular fuzzy number matrix and \(F(d_1'), F(d_2')\), a parameterized family of all the triangular fuzzy number matrix over the set \(T\) and concluded by expert medical documentation. Here \((R_0)\) denotes a relation matrix over the triangular fuzzy number \((F, D)\). By the medical description the triangular fuzzy number matrix medical representation of two treatment stages and their symptoms are as follows:

\[R_0 = \begin{pmatrix}
\begin{array}{cc}
d_1' & d_2' \\
(1, 2, 5, 4) & (2, 3, 5, 5) \\
(5, 6, 5, 8) & (6, 7, 5, 9) \\
(9, 9, 5, 10) & (1, 5, 5, 10) \\
(1, 2, 3) & (8, 9, 10)
\end{array}
\end{pmatrix}
\]

Step 2: Now consider \(P = P'_1, P'_2, P'_3\) a universal set where \(P'_1, P'_2\) and \(P'_3\) are patients and \(T = \{t_1, t_2, t_3, t_4\}\) be the set of parameters respectively.

Assume that,

\[\]
On simplification the following relation matrices are:

\[ F' t_1 = [ P'_1, (2, 3.5, 5) >, P'_2, (2, 3.4) >, P'_3, (5, 6.7) >] \]
\[ F' t_2 = [ P'_1, (4, 5, 6) >, P'_2, (7, 8, 9) >, P'_3, (5, 6, 8) >] \]
\[ F' t_3 = [ P'_1, (8, 9, 10) >, P'_2, (3, 4, 5) >, P'_3, (6, 7, 8) >] \]
\[ F' t_4 = [ P'_1, (1, 2.5, 4) >, P'_2, (2, 3.4) >, P'_3, (5, 6, 7) >] \]

Suppose that \((F, T)\) another parameterized family of triangular fuzzy number matrix which represents the patient-treatment stages which are the various types of disease seen in hospital. Hence the relation matrix \(RT\) and its transpose in patient-treatment stage matrix is described below

\[
R'_T = \begin{bmatrix}
(2, 3.5, 5) & (2, 3.4) & (5, 6, 7) \\
(4, 5, 6) & (7, 8, 9) & (5, 6, 8) \\
(8, 9, 10) & (3, 4, 5) & (6, 7, 8) \\
(1, 2.5, 4) & (2, 3.4) & (5, 6, 7)
\end{bmatrix}
\]

\[
R_T = \begin{pmatrix}
R'_1 & t_1 & t_2 & t_3 & t_4 \\
R'_2 & (2, 3.5, 5) & (4, 5, 6) & (8, 9, 10) & (1, 2.5, 4) \\
R'_3 & (2, 3.4) & (7, 8, 9) & (3, 4, 5) & (2, 3.4) \\
R'_4 & (5, 6, 7) & (5, 6.8) & (6, 7, 8) & (5, 6, 7)
\end{pmatrix}
\]

**Step 3:** The relation membership function matrix is

\[
(R_0)_{mem} = \begin{pmatrix}
(0.1, 0.25, 0.4) & (0.2, 0.35, 0.5) \\
(0.5, 0.65, 0.8) & (0.6, 0.75, 0.9) \\
(0.9, 0.95, 1) & (0.1, 0.55, 1) \\
(0.1, 0.2, 0.3) & (0.8, 0.9, 1)
\end{pmatrix}
\]

\[
(R_T)_{mem} = \begin{pmatrix}
P'_1 & t_1 & t_2 & t_3 & t_4 \\
P'_2 & (0.2, 0.35, 0.5) & (0.4, 0.5, 0.6) & (0.8, 0.9, 1) & (0.1, 0.25, 0.4) \\
P'_3 & (0.2, 0.3, 0.4) & (0.7, 0.8, 0.9) & (0.3, 0.4, 0.5) & (0.2, 0.3, 0.4) \\
P'_4 & (0.5, 0.6, 0.7) & (0.5, 0.6, 0.8) & (0.6, 0.7, 0.8) & (0.5, 0.6, 0.7)
\end{pmatrix}
\]

**Step 4:** On simplification the following relation matrices are:

\[
R' = (R_T)_{mem} \odot (R_0)_{mem}
\]

\[
R' = \begin{pmatrix}
P'_1 & d'_1 & d'_2 \\
P'_2 & (0.95, 1.31, 1.80) & (0.8, 1.2175, 2.19) \\
P'_3 & (0.7, 1.125, 1.65) & (1.33, 1.6, 2.4) \\
P'_4 & (0.89, 1.335, 1.96) & (0.9, 1.57, 2.6)
\end{pmatrix}
\]

\[
R'' = (R_T)_{mem} \odot (J \odot (R_0)_{mem})
\]

\[
R'' = \begin{pmatrix}
P'_1 & d'_1 & d'_2 \\
P'_2 & (0.2, 0.35, 0.5) & (0.2, 0.35, 0.5) \\
P'_3 & (0.6, 0.75, 0.9) & (0.2, 0.3, 0.4) \\
P'_4 & (0.5, 0.6, 0.8) & (0.5, 0.6, 0.7)
\end{pmatrix}
\]
\[ R'' = (J \ominus (R_T)_{mem}) \odot (R_0)_{mem} \]

\[
R'' = \begin{pmatrix}
    d_1' & d_2' \\
    P_1' & (0.4, 0.5, 0.6) \\
    P_2' & (0.5, 0.6, 0.7) \\
    P_3' & (0.2, 0.4, 0.5)
\end{pmatrix}
\]

\[ R''' = \max(R'', R''') \]

\[
R''' = \begin{pmatrix}
    d_1' & d_2' \\
    P_1' & (0.4, 0.5, 0.6) \\
    P_2' & (0.6, 0.75, 0.9) \\
    P_3' & (0.5, 0.65, 0.8)
\end{pmatrix}
\]

\[ R^v = R' \ominus R'''' \]

\[
R^v = \begin{pmatrix}
    d_1' & d_2' \\
    P_1' & (0.35, 0.81, 1.4) \\
    P_2' & (-0.2, 0.375, 1.05) \\
    P_3' & (0.09, 0.685, 1.46)
\end{pmatrix}
\]

**Step 5:** Further we will construct the maximum of \( i^{th} \) row

\[ R^{vi} = AM(y_{ij}) = \begin{pmatrix}
    P_1' & (0.81, 1.59) \\
    P_2' & 1.05 \\
    P_3' & 2.1
\end{pmatrix} \]

**Network Representation**

Here in the above graph the nodes is the fuzzy network which implies the patient and treatment stages and length or edges shows the drugs used to patients under medical diagnosis.

**4. Conclusion**

Thus we conclude by the work that the result on triangular fuzzy number matrix and its new membership function under medical diagnosis is solved by numerical analysis. Also it is clear from the network graphical representation that dark arrow lines shows more number of drugs is needed for the patients under medical diagnosis.
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