Image Denoising Algorithm based on Improved NCSR Model

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Abstract. The Nonlocally Centralized Sparse Representation (NCSR) algorithm aims to reduce sparse coding noise and utilizes the Nonlocal Self-similarity of the image to improve the effectiveness and accuracy of the image denoising algorithm, which has achieved good denoising effect. However, in the local dictionary learning and sparse coefficient estimation, the algorithm directly uses the Euclidean distance of the noise image to measure the similarity, which will have a certain impact on the final denoising effect. Aiming at this deficiency, it is proposed to use the Gaussian Mixed Model (GMM) to train nonlocal self-similar image patch of the external clean image in the local dictionary learning, and use the learned GMM to guide the internal image clustering; when performing the sparse coefficient estimation, the structural similarity index (SSIM) is introduced into the Euclidean distance and the improved Euclidean distance is introduced into the weight calculation. The experimental results show that the improved algorithm proposed in this paper has improved the PSNR and the visual effect of the image.

1. Introduction

With the continuous development of computer science, image processing technology has become a vital part of the field of computer vision. However, the image may be influenced by external environmental conditions or human factors, etc., and noise is inevitably introduced, resulting in a significant impact on subsequent processing of images. Therefore, how to effectively remove the noise in the image is a hot topic that lasts for a long time. In recent years, the image denoising algorithm has been gradually improved. The existing image denoising methods mainly include the following types: methods based on wavelet, methods based on total variation, methods based on nonlocal self-similarity [1, 2, 3, 6, 7] and methods based on sparse representation [4, 5] and so on. The method based on nonlocal self-similarity was first proposed by BuadesAntoni et al. [1] in 2005, and achieved excellent results in image denoising. Then there are many excellent denoising algorithms based on nonlocal self-similarity, such as: Block Matching with 3D filtering (BM3D) algorithm [2], Weighted Nuclear Norm Minimization (WNNM) method [3] and so on. In 2006, Micheal Elad [4] et al. proposed a K-singular value decomposition (KSVD) algorithm through effective dictionary construction method. In 2013, Dong et al. [5] proposed a nonlocal centralized sparse representation (NCSR) image denoising algorithm. In addition to the above algorithms, some researchers have proposed some image denoising algorithms, which use external clean image to guide the internal image patch, such as the PGPD algorithm proposed by Xu [6]and the PCLR algorithm proposed by Chen [7]. The above algorithms can achieve excellent denoising effect, especially the NCSR algorithm, which not only utilizes the advantages of sparse representation, but also utilizes the nonlocal self-similarity of the image to achieve the purpose of denoising. However, the algorithm uses the Euclidean distance of the noise image to measure the similarity when learning the PCA sub-dictionary and estimating the sparse coefficient, using the Euclidean distance of the degraded noise image will affect the similarity of the image patches and the accurate estimation of the sparse coefficient. Based on the above considerations, this paper proposes an improved
NCSR algorithm, which is improved as follows: (1) In the dictionary learning stage, the NCSR algorithm directly uses the Euclidean distance of the degraded noise image as a measure to K-means clustering, which affect the similarity of image patches. In this paper, the Gaussian Mixed Model (GMM) is used to train the external clean image nonlocal self-similar image patches, and the learned GMM is used to guide the internal clustering of the noise image. (2) In the sparse coefficient estimation stage, this paper introduces the SSIM into the Euclidean distance, taking into account the three factors of brightness, contrast and structure, and the improved Euclidean distance is introduced into the calculation of the weight coefficient.

2. NCSR Model

For an image \( x \in R^n \), given a dictionary \( \Phi \in R^{n \times M} \), each patch can be sparsely represented as \( x_i \approx \Phi \alpha_i \), and the sparse decomposition can be obtained by solving an \( l_1 \)-norm minimization problem:

\[
\alpha_{i,j} = \arg \min_{\alpha_{i,j}} \left( \| x_i - \Phi \alpha_{i,j} \|_2^2 + \lambda \| \alpha_{i,j} \|_1 \right)
\]  

(1)

The entire image can then be represented as a sparsely encoded set \( \{ \alpha_{i,j} \} \), which can be reconstructed by solving a least-square solution:

\[
x \approx \Phi \cdot \alpha = \left( \sum_{i=1}^{N} \alpha_i R_i^T \right) \left( \sum_{i=1}^{N} \alpha_i \Phi_i \right)
\]

(2)

The method based on sparse representation is to solve the following minimization problem:

\[
\alpha_y = \arg \min_{\alpha} \left( \| y - H \Phi \cdot \alpha \|_2^2 + \lambda \| \alpha \|_1 \right)
\]

(3)

and the image \( x \) can be refactored to: \( \hat{x} = \Phi \cdot \alpha_y \).

Since the image \( x \) is disturbed by noise and other factors, \( \alpha_x \) and \( \alpha_y \) are different. Based on this consideration, Dong defines the difference between the sparse coding coefficient: \( v_{\alpha} = \alpha_y - \alpha_x \), which is called sparse coding noise (SCN). It can be seen that the key to reconstructing high quality images is to reduce sparse coding noise. However, \( \alpha_x \) is unknown, so replace \( \alpha_x \) with the good estimation \( \beta \), the sparse coding model is:

\[
\alpha_y = \arg \min_{\alpha} \left( \| y - H \Phi \cdot \alpha \|_2^2 + \lambda \sum_i \| \alpha_i - \beta_i \|_p \right)
\]

(4)

Solving this model is to get the dictionary and the estimate. The dictionary is obtained by the K-means method and the PCA principle. First, the image is divided into patches, and all the patches are divided into K clusters by the K-means method, and then a PCA sub-dictionary is learned for each cluster. The estimation of \( \beta \) is to utilize the nonlocal self-similarity of the image. Assuming that a set of similar patches for a given image patch \( x \), is \( \Omega \), and \( \alpha_{\omega} \) denotes the sparse coding of the image patch \( x_{\omega} \) in \( \Omega \), so that \( \beta \) can be calculated by weighted averaging of \( \alpha_{\omega} \):

\[
\beta_i = \sum_{\omega \in \Omega} \omega_{\omega} \cdot \alpha_{\omega}
\]

(5)

Where \( \omega_{\omega} \) is the weight, similar to the idea of non-local mean. Setting the weight is inversely proportional to the distance between image patches \( x \) and \( x_{\omega} \):

\[
\omega_{\omega} = \frac{1}{W} \exp\left(-\frac{\| \hat{x} - \hat{x}_{\omega} \|}{h}\right)
\]

(6)

Where \( W \) is the normalization factor, \( h \) is the default constant, \( \hat{x} = \Phi \hat{\alpha} \) and \( \hat{x}_{\omega} = \Phi \hat{\alpha}_{\omega} \) are estimates for patch \( x \) and patch \( x_{\omega} \).

The NCSR algorithm obtains the final result by iterative execution, then we can change the formula (4) to:

\[
\alpha_y = \arg \min_{\alpha} \left( \| y - H \Phi \cdot \alpha \|_2^2 + \sum_j \sum_i \| \alpha_i - \beta_i (j) \|_p \right)
\]

(7)
This problem is a convex optimization problem and the alternative algorithm can be used to solve the problem effectively. In the \((l+1)\)th iteration:

\[
\alpha_i^{(l+1)}(j) = S_j \left( v_i^{(l)} - \beta_j(j) \right) + \beta_j(j)
\]

(8)

Where \(S_j(\cdot)\) is the classic soft threshold operator. \(v_i^{(l)} = K^T \left( y - K \cdot \alpha_i^{(l)} \right) / c + \alpha_i^{(l)}\), and \(K = H\Phi\), \(K^T = \Phi^T \cdot H^T\), \(\tau = \lambda_i / c\).

3. Improved NCSR Algorithm

As can be seen from Section 2, the NCSR model:

\[
\alpha_i = \arg \min_{a} \left( \|y - H\Phi \cdot a\|^2 + \lambda \sum_{i} \| a_i - \beta_i \|_2 \right)
\]

(9)

To solve this model, we need to solve two problems: the first one is the learning of dictionary \(\Phi\), and the second is the estimation of unknown sparse coding coefficients. The common point of these two problems is that the Euclidean distance is used to measure the similarity. However, it is problematic to directly use the Euclidean distance of the noise image as the measurement, as described in the first section. This paper proposes an improved NCSR algorithm, which is improved as follows: (1) In the dictionary training stage, to make full use of the rich prior knowledge of external clean images, and use the GMM to train external noise-free clean image nonlocal self-similar patches, then use the learned GMM to guide the internal clustering of the noise image, in place of the K-means clustering which use the Euclidean distance of the degraded image as the measurement. (2) In the sparse coefficient estimation stage, the SSIM is introduced into the Euclidean distance, taking into account structural similarity, contrast similarity and brightness similarity, and the improved Euclidean distance is used in the weighted average calculation of nonlocal image patches, for the purpose of obtaining a more accurate estimate of the unknown sparse coding coefficients.

3.1 Improved PCA sub-dictionaries learning

Assuming that the potential structures of the image patches form \(K\) low-dimensional subspaces, the probability of a given image patch can be defined by the weighted sum of \(K\) Gaussian:

\[
p(x_i | \Theta) = \sum_{i=1}^{K} \omega_k p_k(x_i | \theta_k)
\]

(10)

where \(\Theta = (\theta_1, \ldots, \theta_k, \ldots, \theta_{K})\) and \(\sum_{k=1}^{K} \omega_k = 1\). Each \(\theta_k\) describes a Gaussian density function, the mean is \(\mu_k\) and covariance matrix is \(\Sigma_k\):

\[
p_k(x_i | \theta_k) = c \cdot \exp \left( -\frac{1}{2} (x_i - \mu_k) \Sigma_k^{-1} (x_i - \mu_k) \right)
\]

(11)

The negative exponent here denotes the Mahalanobis distance between \(x_i\) and the center \(\mu_k\), and \(c\) is the normalization constant. \(\Theta\) can be obtained by Expectation-Maximization algorithm.

Introducing class label \(C \in \{1, 2, \ldots, K\}\). For each image patch \(x_i\), using the Probability Density Function (PDF) of the learned GMM to calculate the likelihood of each cluster, and find the cluster whose Gaussian component produces the maximum likelihood, for the purpose of judging the class \(C\) to which \(x_i\) belongs.

\[
p(k | x_i; \Theta) = \frac{\omega_k p_k(x_i | \theta_k)}{\sum_{j=1}^{K} \omega_j p_j(x_i | \theta_j)}, k = 1, 2, \ldots, K
\]

(12)

By solving equation (16), the maximum value is assigned to \(C\), that is, the image patch is classified into the category. Next, for the image patch, the Mahalanobis distance is used as the patch similarity measurement to find nonlocal image patches, forming a set of nonlocal self-similar image patches.

After using the above GMM to guide the noise image clustering into \(K\) clusters, then using the PCA
principle to learn the dictionary for each cluster. This not only makes full use of the prior knowledge of the external clean image, but also avoids the influence of the Euclidean distance of the degraded image on the similarity of the image patch and even the quality of the learning dictionary.

3.2 Improved estimation of unknown sparse coding coefficient

The Euclidean distance is not enough to measure the similarity of the image patches, thus the structural similarity measurement is introduced. The structural similarity is defined as:

\[
SSIM(x, y) = \left( \alpha \gamma \beta \right)^T \left( \beta^T \right) \left( \alpha \gamma \beta \right) \left( \beta^T \right)
\]

\[
= \left( \alpha \gamma \beta \right)^T \left( \beta^T \right) \left( \alpha \gamma \beta \right) \left( \beta^T \right)
\]

(13)

Where \( l(x, y), c(x, y) \) and \( s(x, y) \) represent the brightness similarity, contrast similarity, and structural similarity of the image, respectively. The value range of SSIM is between 0 and 1, so the structural similarity of image patches can be introduced into the Euclidean distance in a weighted form, and the information such as gray scale and structure can be considered at the same time. The exponential function is monotonic and the convergence speed is fast, so the exponential function is used to calculate the weight, and the weight is:

\[
\omega = k \cdot \exp \left( \frac{-SSIM(x, y)}{h_s} \right)
\]

(14)

Where \( k \) is the correction parameter and \( h_s \) is the smoothing parameter. Introducing the SSIM into the Euclidean distance formula in a weighted form, the following improved Euclidean distance formula is obtained:

\[
D(x, y) = d(x, y) \cdot \omega
\]

(15)

The improved formula also utilizes a variety of metrics, both using mathematical statistics and utilizing the human visual system. Applying the improved distance formula to the estimate of the unknown sparse coding coefficient. If the new weight is \( W_{i, q} \), then:

\[
W_{i, q} = \frac{1}{W} \exp \left( -\omega \| \hat{x}_i - \hat{x}_{i, q} \|_2^2 / h \right)
\]

(16)

And we can get the new estimate:

\[
\beta_i = \sum_{q \in k} W_{i, q} \cdot \alpha_{i, q}
\]

(17)

The improved Euclidean distance formula can obtain a more accurate estimate of the sparse coding coefficient, thereby obtain a higher quality restored image.

3.3 Algorithm Summary

The steps of the improved NCSR algorithm are as follows:

1. Initialization:
   (1) For image denoising, set the initial estimate as \( \hat{x} = y \);
   (2) Set initial regularization parameter \( \lambda \) and \( \delta \);
2. Outer loop: iterate on \( l = 1, 2, \ldots, L \)
   (1) Update the dictionaries \( \{ \Phi_i \} \), learn GMM priors for external noise-free images, and guide the noise image to generate \( k \) clusters, then learn a PCA sub-dictionary for each cluster;
   (2) Inner loop: iterate on \( j = 1, 2, \ldots, J \)
      (a) \( \hat{x}^{(j+1)} = \hat{x}^{(j)} + \delta H^T \left( y - H \hat{x}^{(j)} \right) \), where \( \delta \) is the pre-determined constant;
      (b) Compute \( \chi^{(j)} = \left[ \Phi_k R_{\hat{x}^{(j+1)}}, \ldots, \Phi_k R_{\hat{x}^{(j+1)}} \right] \), where \( \Phi_k \) is the dictionary assigned to patch \( \hat{x}_i = R_{\hat{x}^{(j+1)}} \);
      (c) Compute \( \alpha^{(j+1)} \) using (8);
      (d) if \( \text{mod}(j, J) = 0 \), calculate its weight \( W_{i, q} \), update the parameters \( \lambda_{i, j} \) and \( \{ \beta_i \} \);
4. Experimental Results

In order to verify the effectiveness of the image denoising algorithm proposed in this paper, the algorithm is applied to the real life, and the PGPD [6], PCLR [7], SSC_GSM [8] and NCSR [5] algorithms with good denoising effect are selected for comparison. The peak signal to noise ratio (PSNR) is used as an image quality evaluation standard. The programming environment of this experiment is matlab2017b, the operating system is win7 professional version (64-bit), the computer is configured as Intel(R) Core(TM) i7-2600, and the memory size is 4G. In this paper, the GMM samples are taken from the Berkeley Image Segmentation Dataset (BSD), and 200 training images are selected for uniform sampling, and $2 \times 10^6$ image patches are extracted.

![Figure 1. Experimental image](image1)

This paper selects four standard images of House, Boat, Lena and Barbara as experimental test images, as shown in Figure 1. Gaussian noise of different intensities is added to the experimental images, and the noise image is denoised by the above four algorithms and the newly proposed algorithm. PSNR is used as the evaluation standard of image denoising effect. Table 1 records the experimental results of each algorithm after denoising under the influence of Gaussian noise of different intensities. Figure 2 records the denoising effect of the Lena image after denoising using various algorithms when the noise intensity is 30.

Table 1. Denoising results after adding Gaussian noise of different intensities to each experimental image

| Test image | Noise intensity | PGPD | PCLR | SSC_GSM | NCSR | Ours |
|------------|----------------|------|------|---------|------|------|
| House      | 10             | 36.56| 36.83| 36.70   | 36.80| **36.93** |
|            | 30             | 32.24| 32.17| 32.49   | 32.08| **32.55** |
|            | 50             | 29.93| 29.78| 30.11   | 29.62| **30.33** |
| Boat       | 10             | 34.99| 35.29| 35.28   | 35.08| **35.44** |
|            | 30             | 29.32| 29.45| 29.24   | 29.13| **29.53** |
|            | 50             | 26.82| 26.82| 26.78   | 26.41| **26.97** |
| Lena       | 10             | 35.29| 35.48| 35.47   | 35.23| **35.52** |
|            | 30             | 29.60| 29.68| 29.53   | 29.36| **29.72** |
|            | 50             | 27.15| 27.19| 27.09   | 26.95| **27.27** |
| Barbara    | 10             | 34.72| 34.96| 35.15   | 34.88| **35.42** |
|            | 30             | 28.93| 28.82| 29.21   | 28.77| **29.47** |
|            | 50             | 26.27| 26.19| 26.46   | 26.15| **26.84** |

![Figure 2. Denoising effect of Lena image under the influence of Gaussian noise with intensity 30](image2)
Table 1 records the data of denoising with different algorithms after adding Gaussian noise of different intensities to the four standard images of House, Boat, Lena and Barbara. We can see clearly that the improved algorithm has certain advantages compared with the original NCSR algorithm, the PSNR is improved by 0.13~0.71dB, and compared with the other classical image denoising methods, the advantages are equally obvious. Figure 2 depicts the effect of denoising of the Lena image with Gaussian noise intensity of 30. We can find that the improved algorithm has advantages in texture and edge information retention. According to the experimental data, the improved algorithm is better than the traditional algorithm, which proves its effectiveness.

5. Conclusion
This paper proposes an improved algorithm based on the traditional NCSR algorithm. For the original algorithm directly uses the Euclidean distance of the degraded image as the measurement in the stage of dictionary learning and estimation of sparse coding coefficient, it is proposed that in the dictionary learning stage, we use the GMM to train external noise-free clean image nonlocal self-similar patches, then use the learned GMM to guide the internal clustering of the noise image, in place of the K-means clustering which use the Euclidean distance of the degraded image as the measurement, and then learn a PCA sub-dictionary for each cluster; In the stage of estimating the sparse coding coefficient, we introduce the SSIM to the Euclidean distance and introduce the improved Euclidean distance into the weight calculation. After a series of experiments, it is found that the improved algorithm has advantages over the original algorithm in both subjective visual and objective evaluation criteria. However, we also found that although the denoising effect of the newly proposed algorithm is improved, it is not for noise of all intensities. When the intensity of the noise is very small, the denoising effect is not particularly good. When the intensity of the noise increases gradually, the denoising effect begins to become apparent. In addition, the newly proposed method is nearly equivalent in running time to the original method. Therefore, how to ensure that the algorithm can exert good denoising effect under various noise intensity conditions, and minimize the running time of the algorithm at the same time is a key point that needs to be paid attention to in subsequent research.

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