The interaction between mathematics and architecture

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Abstract. Mathematics and architecture have always been closely interrelated, the main methods of design in architecture has always belonged to mathematics. At the present stage in connection with the development of computer technology and the Internet began a new round in the relationship between mathematics and architecture. A competent architect should have excellent skills in painting and sculpture, as well as a good knowledge of mathematics. We came to the conclusion that the purpose of mathematical training in the University of students of construction specialties is the development of their mathematical competence. This requires:
1) students should be able to solve applied mathematical problems;
2) students should carry out projects related to their future professional activities;
3) students should be able to create mathematical models.

We have developed criteria, indicators and levels of development of mathematical competence of students of the University, which allows you to organize control and measuring materials to assess the level of mathematical training of students.

1. Introduction

Since Ancient times, mathematics and architecture have been closely interrelated, the main methods of design have always belonged to mathematics. Even in Ancient Greece, mathematics, with its geometry of Euclid, was one of the sections of architecture. This idyll was broken in the 18th century, when the first Engineering school was established in Paris in 1747. Since then, mathematics and architecture have evolved each on their own. A new stage in the relationship between mathematics and architecture began with the development of computer technology and the Internet. For successful work architects should be able to solve applied mathematical problems. These applied mathematical problems include, for example, the problem of determining the strength of structures, structural optimization, stability and control modes of their operation. A competent architect should have excellent skills in painting and sculpture, but also good knowledge of mathematics. The ideas of the architect will remain a dream on paper, if he cannot calculate the design of the designed structure. Therefore, while studying at the University, the future architect should study well many fundamental sections of higher mathematics, including the theory of probability and mathematical statistics, and get the skills of mathematical modeling.

We studied various sources of information concerning the application of mathematics in architecture and construction. Some of these works are devoted to mathematical methods in architecture, others-to architectural creativity and form formation, there are works devoted to the methodology of design, as well as the history of architecture.

Publications concerning mathematical methods in architecture can be divided into two groups. The first such group includes works that consider the interaction of mathematics and architecture in a certain historical era. To this group can be attributed, for example, "Treatise on architecture" Antonio Filaret [1], which is a guide of his time on the design of civil architecture. This may also include V.S. Lesovik [2], S. Lysov, M. Balzannikov, V. Evtropov, M. Lysov [3], A. V. Voloshinov [4] and others. Another group includes works that study the variety of applications of modern methods of mathematics in architectural design, considering the modern view of the proportions of the "Golden section" in the architecture of ancestors. Such works are the works of V. Erofeev [5], T.V. Nazmeeva, N. I. Vatin [6], V. I. Sazonov [7], Yu. V. Nemirovsky, N. A. Fedorova [8] and others. The authors
have the latest modern research on this issue: W. Wu, A. Tesei, S. Ayer [9], S. I. Rifaat [10], E. I. Saqan, A. K. Ashmawy [11], Choudhary A, Dogne N [12]. We studied these and other works and drew conclusions about the great importance of the study of mathematics for future architects, and identified the problem of mathematical training of students of construction specialties of the University. The analysis of methodical literature showed that the question of development of mathematical competence of students of construction specialties insufficiently studied. It is necessary to develop the conditions for the development of mathematical competence of students of construction specialties.

For the successful implementation of these tasks it is important to develop a culture of logical thinking, the ability to creatively non-standard approach to solving professional issues, the ability to find the necessary information in books, reference books, the Internet. That is, to be a competent architect, a student must have an average, or better, a high level of development of mathematical competence. Which, in turn, is an integral part of the professional competence of the architect.

We consider the mathematical competence of students as an integral quality of a specialist, manifested in the free possession of a system of professionally significant mathematical knowledge and skills, in the ability to independently carry out meaningful multidirectional cognitive activity and creatively solve professional problems of various levels of complexity [13].

We have developed criteria, indicators and levels of development of mathematical competence of University students, which makes it possible to organize control and measuring materials to assess the level of mathematical training of students [13].

Students—architects having studied the course of mathematics should be able:
1) formulate the problem in mathematical language,
2) build a mathematical model from the available data,
3) choose the necessary mathematical apparatus for the solution,
4) develop an algorithm to solve the problem,
5) if necessary, use numerical methods to solve the problem using a computer,
6) using the methods of mathematical statistics to draw conclusions.

Thus, the aim of mathematical training of students of architects is to develop their mathematical competence.

2. Materials and methods
Mathematics should not be separated from the future professional activity of students, from their daily lives. If students feel the importance of mathematics for themselves, they will study it with enthusiasm. We try to use this in our work, so when studying each topic, we focus on solving applied problems, performing applied tasks and projects. Let us consider some of these problems on the topic "Surfaces".

1) What is the amount of material required for covering three columns of cylindrical shape with a height of 5 m and a diameter of 50 cm? Take into account the consumption of material for waste and seams – 15%.

2) A wooden ball with a mass of 630 g and a diameter of 20 cm. Find the density of the wood used.

3) It is necessary to make two pipes with a length of 7 m and a diameter of 30 cm. How many square meters of sheet metal will be needed if the seams need to add 3% of the area of its side surface?

4) Reinforced Concrete panel has dimensions 600x120x22 cm. Along its entire length there are six cylindrical holes whose diameter is 14 cm. Find the mass of the panel if the density of the material is 2500 kg/m³

Studying the topic "Integration of functions" we consider the following tasks:
1) calculate the coordinates of the center of mass of the figure, bounded by the arc of the ellipse $x = 2\cos t$, $y = 3\sin t$, and coordinate axes (Consider 1 quarter).

2) calculate the volume of the surface of the ellipse at $a=2$, $b = 3$, $c = 1$.

In mathematics classes, we also use the technique of self-preparation of tasks by students according to known formulas. Students receive a table with formulas and for a certain allotted time, for example,
10-15 minutes should come up with the condition of the problems that will be solved by these formulas.

For example, given table 1, it is necessary to create a problem in the study of the topic "A certain integral".

| Values               | Derivative calculation | Integral calculation |
|----------------------|------------------------|----------------------|
| $m$ – thin rod weight| $\rho(x) = m'(x)$      | $m = \int_{a}^{b} \rho(x) dx$ |
| $\rho$ – linear density|                        |                      |
| $Q$ – amount of heat | $\rho(t) = Q'(t)$      | $Q = \int_{a}^{b} c(t) dt$ |
| $C$ – heat capacity |                        |                      |
| $S$ – movement       | $v(t) = S'(t)$         | $S = \int_{a}^{b} v(t) dt$ |
| $v$ – speed          |                        |                      |

Another technique that we use in mathematics, is the technique of "its support." Students make their own reference notes on the topics studied. Abstracts can be made both in handwriting and on the computer. To teach to make a basic abstract we teach students directly, in the study of the very first topics in high school, as a rule, the “Matrix”. The teacher introduces students to the algorithm of drawing up a reference abstract on a given topic and work with him. Students learn to identify the basic concepts on this topic, to find their essential properties, to make algorithms for solving typical tasks. Be sure to reference the syllabus must reflect the practical application of the topic. The algorithm of drawing up a reference abstract includes the following stages: the study of theoretical material on a given topic; drawing up a plan; the development of symbols; determining the logic of the presentation of the material and its layout in blocks; color design of the reference abstract. After mastering the algorithm, students at home make their reference notes, they can use them in further practical classes. Making the basic abstract on the studied subject, the student fixes the received knowledge, systematizes them, learns to isolate the main points in a subject, to represent the studied material visually. And most importantly, he is using a variety of information sources, looking for the use of this topic in other subjects and their future professional activities.

Any future architect should be able to build a mathematical model from the available source data. Since in the future it may face the tasks of organization, planning and management of activities, and they are characterized by a plurality of possible solutions, often great uncertainty and dynamic processes. In the process of working on a plan for the construction of any construction object, it is necessary to compare different options and choose the most suitable one. To solve all these problems, the student must learn modeling.

Under the model, we understand the generalized, idealized, visual-logical image of the investigated object (process), more accessible to study than the process itself [13].

Modeling allows us to identify the essential features of the real object for the researcher, helps to make experiments. The results of calculations and experiments obtained on the model are analyzed and conclusions about the real object are made.

The process of mathematical modeling includes three stages: 1) formalization, 2) solving the problem within the model, 3) interpretation. Teaching students mathematical modeling, we pay special attention to the work on the stages of formalization and interpretation, because these two stages cause the greatest difficulties for students [13].

In mathematics classes there is a comparison of real objects and their mathematical models. For example, when studying a linear function, an analogy is drawn between the linear function $y = kx + b$ and

a) the length of the circle and its radius $C = 2\pi r$;

b) the dependence between the length of the rod and the heating temperature $l = (1 + at)$;

c) the speed of sound in the air depending on the temperature $v = 331 + 0.6t$. 

For students, there are several problems to solve which it is necessary to make a linear function and explore it. Consider such tasks.

1) A resident of our city put in the Bank 20 thousand rubles, which are charged monthly 7%. The deposit amount is calculated using the simple interest formula. It is necessary to express the dependence of the Deposit amount on the number of months that this deposit will lie in the Bank and answer the questions:
   a) what is the deposit amount in 3 years?
   b) in how many years the deposit amount can be increased by 2 times?
   According to the data of the problem with students the function \( y = 20000 + 1400 \cdot n \), then answer the questions of the problem.

2) Two mines A and B are located at a distance of 150 km on the highway. Mine A produces 200 tons of ore per day, and mine B produces 100 tons of ore per day. Where do you need to build an ore processing plant, so that the number of tons-kilometers for its transportation is the least?
   Make a function and explore it: \( y = 200x + 100(150 - x) = 100x + 15000 \).

Homework for students give examples of linear function related to future profession or life.

In order to teach students mathematical modeling, we consider the subject problems in the classroom, translating them into mathematical language.

Under the topic "Sequence. The ranks of" students are offered problem about two shooters (masters and novice) on the shooting range. The sequence of hits for each arrow can be represented as a series of numbers, but with the proviso that the competition will continue indefinitely (since the sequence is an infinite set of numbered elements). Watching the master, we will see that no matter how small a circle we take, starting with a certain shot, all subsequent hits will be inside this circle. Hence, the sequence of hits is committed (converge) to the center of the target, thus the target is the limit of a sequence of hits. Watching the newcomer on the shooting range, you can outline around the center of the target, a circle of a certain radius, such that whatever number we make come true, there is a hit with a large number that falls outside this circle. This means that the sequence of hits does not tend (does not converge) to the center of the circle. Then, in this example, we consider the Cauchy criterion (convergence criterion of the sequence). Think of any distance. Then find a number so that the distance between any two hits with large numbers would be less than the intended distance. If, no matter how small a distance we plan, we always succeed, then the sequence satisfies the Cauchy criterion. From this it follows that it converges.

As another example, you can consider the problem of candy, as in the song: "You half—and I half." Candy is divided between the girls in half, the resulting share one of them again, bisects, etc. From a share of the candy you can get to the numbers. A decreasing sequence of numbers is formed. Further, the convergence of this sequence can be solved.

The illustration of the theorem "every monotonically decreasing and bounded from below numerical sequence has a limit" We build on the identification of the answer to the question: "Is there a limit of sports capabilities of a person?". The question of records of freestyle swimming for 100 meters is considered. Not so long ago, the record was to overcome a hundred meters in 50 seconds, and now he is beaten. There are no eternal records, there is always someone who improves the previous record, reducing the time of the race. On the scale of the results of swimming freestyle for a hundred meters there is a point to which the sequence of record achievements tends. No matter how small a neighborhood of this point we take, starting with some member of the sequence, all the other members will lie in this neighborhood. A similar example of the theorem that every monotonically increasing and bounded from above numerical sequence has a limit is the example with the records of weightlifters.

We give another theorem as an example: "A Series is called convergent if the sequence of its partial sums has a limit; this limit is called the sum of the series. If the sequence of partial sums has no limit, the series is called divergent". Students are offered the subject problem connected with family life: "In some family every year on birthday of the daughter on a door jamb a notch growth of the
birthday girl is noted. Over the years, on the jamb there is a whole ladder marks. In this example, we can observe two sequences: the first-a sequence of values of sprouts from year to year, the second – a sequence of values of growth. These two sequences are related to each other. The second sequence is obtained from the first addition. Growth is the sum of all previous years’ gains. The members of the second sequence are supposed to be summarized, they are called members of the series. The sum of several n first terms of the series is called the n-th sum of the series (partial sum of the series). Over time, the marks on the door jamb converge, touching, more closely to each other, that is, the sequence of growth values displayed on the door jamb, has a limit (the series converges).

The presented examples illustrate the process of studying the topic: "Sequences; Ranges». Similarly, you can study all the topics of mathematics, as well as to consider the concepts and theorems in the study of the "Function", "Differential and integral calculus", "Functions of several variables", "Functional series".

Using the considered methods of actualization, we change the students' understanding of mathematics, creativity in it, increase interest in its study, increase the importance of new knowledge in the field of mathematics, which ultimately stimulates the development of students' motivation to study mathematics, as evidenced by the results of a survey conducted at the end of the experimental work [13].

Mandatory sections of mathematics for students of construction specialties is the theory of probability and mathematical statistics. In these areas of mathematics, many problems are of applied importance. Here are examples of tasks performed by students in mathematical statistics in Microsoft Excel.

Target 1. Find the average value and the standard error percentage of compressive strength (random value X) 100 test grades of concrete (or the same – for the percentage of resistance of the same grades of concrete to rupture (random value Y)). To estimate the degree of deviation of the empirical distribution of this measurable trait from the normal distribution. (Data are taken from laboratory work)

Target 2. According to task 1, check the statistical hypothesis of coincidence with the normal distribution of one measurable feature of the population.

To get acquainted with the main tasks of statistical testing of hypotheses, with frequently used methods of testing the hypothesis of normal distribution. To study the solution of the problem of consistency of theoretical and statistical distributions.

In order to make students aware of the importance of mathematics for themselves, they perform creative project tasks in large sections of mathematics, which study inter-subject projects. All creative project tasks performed by students meet the following requirements:

1) existence of an open problem;
2) in the process of the assignment students go through all stages of the project;
3) creative project task is aimed at achieving personal and socially significant results;
4) the task reflects the applied importance of mathematics [14].

Before the first topic of the research is presented to the students, a methodological seminar is held, which explains the theoretical foundations of the design, demonstrates the completed projects on various topics, discusses the pros and cons of these projects. When carrying out the first projects on the basis of the principle of cooperation by the teacher together with students the main directions of work are considered, important characteristics of future projects are noted. In the future, in the process of working on subsequent projects gradually increases the level of independence of students, and the teacher is enough only to voice the topic. After each voiced research problem, students are given at least three weeks to develop the project.

We give examples of projects performed by students related to the solution of professional problems in which mathematics performs an applied function. Thus, in the study of the topic "Curves of the second order and spatial bodies" students architects carried out projects on the theme "the Existence of unusual figures in our world." They made models of figures, geometric bodies, prepared multimedia presentations, reports. The most interesting topics of the projects: "Impossible figures", "Hyperboloids", "Cylinders", "Spheres".
"Penrose Triangle", "Impossible Reutersvard’s triangle", "Mobius Strip". These projects are carried out with a creative approach on the part of students, a huge share of imagination and a fairly close connection with their profile orientation [15].

A mandatory theme of group projects is "the Golden section in architecture and life". With this mathematical formula, the architect calculates the proportions of his creations. Great success among the students produce projects that show the relationship of mathematics with the future profession, with everyday life. For example, carried out projects of the future giving or future house, connected mathematics with the specialty and others. We have considered examples of such projects in our previous articles [16].

According to the results of work on the projects, student conferences were held. Students made presentations, demonstrated and defended their projects. The best papers were then published in the journal of Nosov Magnitogorsk State Technical University "Youth. Science. Future".

The main research methods in our work were: analysis, comparison, synthesis, generalization, specification, abstraction and modeling.

3. Results
The aim of our work, as we noted above, was the development of mathematical competence of students. The experimental work was carried out, which was attended by 72 students studying in the field of "Architecture" in the Nosov Magnitogorsk State Technical University. Two groups of students-architects, one experimental and the other control, were chosen for the experiment. The control group of students studied the fundamentals of higher mathematics, classes in it were standard. In the experimental group, in addition to standard training, students solved applied tasks and tasks on each topic, carried out projects, including interdisciplinary ones, and spoke at conferences. That is, we have implemented a methodology developed by us, different from the methods of other authors.

To justify the reliability of the results obtained in the experiment, two problems were solved. Firstly, the methods of mathematical processing of the experimental results were selected, and secondly, the representativeness of the sample was proved.

To process the results of the experiment, the statistical Pearson criterion $\chi^2$ was used to justify the reasons for the changes in the results. The null statistical hypothesis reads as follows: "The levels of mathematical competence of students of the experimental and control groups will not differ as a result of the experiment."

At the beginning of the experimental work, criteria and indicators of the level of development of mathematical competence of students were developed [16]. Processing the experimental data, each indicator of the level of development of mathematical competence of students we evaluated on a three-point scale.

At the beginning of the first semester, primary data were determined, which showed a fairly low level of development of mathematical competence of students.

According to the selected criteria and indicators, the level of development of mathematical competence in each of the selected groups was checked at the beginning of the study of mathematics at the University (the beginning of the first semester) and at the end of the study of mathematics at the University (the end of the third semester). The results are shown in table 2.

At the end of the course of mathematics at the University was repeated diagnosis.

4. Discussion
The experiment showed an increase in the number of students at the average and high level of development of mathematical competence. As the survey of students showed at the end of the experiment, many students realized the importance of mathematics for themselves, their future professional activity.
### Table 2. Results of development of mathematical competence of students in the process of experimental work.

| Group | Number of persons | Stage | low Number | low % | medium Number | medium % | high Number | high % | $\chi^2$ obsr. |
|-------|-------------------|-------|-----------|------|---------------|----------|-------------|------|---------------|
| EG    | 38.0              | Initial | 23.0      | 60.5 | 11.0          | 29.0     | 4.0         | 10.5 | 0.1           |
|       |                   | End    | 3.0       | 8.0  | 24.0          | 63.0     | 29.0        | 10.5 | -             |
| KG    | 34.0              | Initial | 20.0      | 58.8 | 11.0          | 32.4     | 3.0         | 8.8  | -             |
|       |                   | End    | 13.0      | 38.0 | 17.0          | 50.0     | 4.0         | 12.0 | -             |

In determining the dynamics of the level of development of mathematical competence of students will use the following statistical indicators of time series:

1. The average ($C$), we measured according to the formula $C = \frac{a + 2b + 3c}{100}$, where $a$, $b$, $c$ – expressed as a percentage of the number of students who are respectively at low, medium and high levels of mathematical competence.

2. The efficiency coefficient ($K$), which reflects the effectiveness of the experimental study, is determined by the formula $K = \frac{C(EG)}{C(KG)}$, where $C(EG)$ – the average value of the experimental group, $C(KG)$ – the average value of the control group.

3. The index of absolute growth ($G$), shows the difference between the final and initial values of any studied indicator, it was calculated by the formula $G = II(final) - II(initial)$, where $II$ (final) – final value of the indicator, $II$ (initial) – initial value of the indicator [16].

The data obtained during the experiment are shown in table 3.

### Table 3. Table of results of development of mathematical competence of students during the forming experiment.

| Group | Stage | $C$  | $K$  | $G$  |
|-------|-------|------|------|------|
| EG    | Initial | 1.50 | 1.27 | 0.71 |
|       | End    | 2.21 |      |      |
| KG    | Initial | 1.51 | –    | 0.23 |
|       | End    | 1.74 |      |      |

According to the results of table 2, we see that at the level of significance $\alpha = 0.05$ in the experimental group $\chi^2$ obsr. $> \chi^2$ critical ($\chi^2$ critical $= 5.99$), that is, the condition of application of applied problems, tasks and implementation of projects in the process of mathematical training is statistically significant for the development of mathematical competence of students. Based on the results of the work, we conclude that the successful development of mathematical competence of students of architects of the experimental group is not accidental, but is the result of the application of applied problems, tasks and creative projects in mathematics classes.

### Conclusions

All of these design decisions in architecture are mathematically justified, proved. According to the results of our study, we can draw the following conclusions, a student, who studying in the field of "Architecture", will not take place in the future as an architect, if he do not know the theoretical foundations of mathematics, will not be able to build a mathematical model on the condition of the problem, will not be able to apply the methods of mathematical statistics for the processing of experimental data. That is, a student at the end of the University should be well developed mathematical competence.

This requires:
1) students should be able to solve applied mathematical problems;
2) students should carry out projects related to their future professional activities;
3) students should be able to create mathematical models.

We also made sure that the best results in the experimental group suggest that the application of applied problems, tasks and projects in the classroom of mathematics allows more qualitative development of mathematical competence of students of architects.

The purpose of our research is carried out, problems have been solved.

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