Black holes and singularities in string theory

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Abstract: This is a summary of a lecture I gave at the workshop on dynamics and thermodynamics of black holes and naked singularities at Politecnico Milano. It is directed to a non-expert audience and reviews several ways in which string theory accounts for black hole microstates. In particular, I give an elementary introduction to the correspondence principle by Horowitz/Polchinski, to the state counting for the three-charge black hole by Strominger and Vafa, and to the recent proposal by Mathur et al. concerning the gravity description of black hole microstates. The second part of the lecture is dedicated to naked singularities and reviews an argument by Horowitz and Myers why naked singularities are not necessarily bad. Finally, I comment on a possible resolution of singularities in Born-Infeld type gravity theories.

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1. Black holes

1.1 Black holes as thermodynamical objects

In 1971, Hawking proved the so-called "area theorem" of black hole physics [1]. It says that under reasonable conditions, e. g. no "exotic matter" with negative energy density or the like, the total area $A$ of the event horizons of any collection of black holes can never decrease,

$$\delta A \geq 0. \tag{1.1}$$

This sounds curiously similar to the second law of thermodynamics,

$$\delta S \geq 0, \tag{1.2}$$

with the area of the black hole playing the role of entropy. It might appear that this similarity is of a very superficial nature, because the area theorem is a mathematically rigorous consequence of general relativity, whereas the second law of thermodynamics is believed not to be a rigorous consequence of the laws of nature but rather a law that holds with overwhelming probability for systems with a large number of degrees of freedom. Nevertheless, this analogy extends to all the laws of black hole mechanics,
Law nr. | Thermodynamics | Black holes
--- | --- | ---
0 | $T$ constant in thermal equilibrium | Surface gravity $\kappa$ constant on horizon
1 | $dE = TdS$ + work terms | $dM = \frac{\kappa^2}{8\pi}dA + \Omega_H dJ$
2 | $\delta S \geq 0$ | $\delta A \geq 0$
3 | Impossible to achieve $T = 0$ by a physical process | Impossible to achieve $\kappa = 0$ by a physical process

Table 1: Analogy between the four laws of black hole mechanics and the laws of thermodynamics. $M$ and $J$ denote the mass and angular momentum of the black hole, and $\Omega_H$ is the angular velocity of the horizon.

derived by Bardeen, Carter and Hawking in 1973 [2], and thermodynamic principles. This is summarized in table [1].

Physicists were thus led to ask the question if this analogy is only formal, or if there is some deeper meaning behind. Note in this context that classically a black hole has zero temperature, because nothing can escape from the region behind the horizon. Hawking himself tended not to believe in a profound meaning behind this analogy, but he had to correct his opinion when he discovered [3] that, due to quantum effects, black holes radiate like a black body with temperature

\[ T = \frac{\hbar \kappa}{2\pi} . \]  

At this point it became clear that the surface gravity $\kappa$ indeed represents the thermodynamical temperature of the black hole, and that the resemblance of the four laws of black hole mechanics and the thermodynamical laws is more than just a formal analogy: The laws of black hole mechanics are the laws of thermodynamics, applied to black holes. In particular this implies that we should assign the entropy

\[ S_{BH} = \frac{A_{Hor}}{4G}, \]  

called Bekenstein-Hawking entropy, to a black hole. This identification raises the question what the black hole microstates are, and where they are located. In other words, we would like to know what the statistical mechanics of black holes is, and write (1.4) as the logarithm of a number of microstates that are compatible with a given macrostate, i. e., with a given set $(M, J, Q)$, where $Q$ stands for the charges that can be carried by the hole.

In the remainder of this section, I will try to argue that string theory can provide an answer to this question.
1.2 The correspondence principle

The correspondence principle, formulated by Horowitz and Polchinski in 1996 [4], is essentially based on Susskind’s idea that Schwarzschild black holes are in one-to-one correspondence with fundamental string states [5]. If one starts at week string coupling \( g_s \) with a highly excited string state, and raises \( g_s \), then also the Newton constant \( G \) increases, because in four dimensions \( G \) is related to \( g_s \) and the string scale \( \ell_s \) by \( G \sim g_s^2 \ell_s^2 \). At a certain point, the Schwarzschild radius of the string, \( m_{\text{str}} G \), becomes larger than the string length \( \ell_s \), and the string turns into a black hole. Conversely, as one decreases the coupling, the size of a black hole eventually becomes less than the string scale. At this point, the metric is no longer well-defined near the horizon, so it can no longer be interpreted as a black hole. Susskind proposed that it should be described in terms of some string state. The point where the black hole turns into a string is called the correspondence point. The mechanism is represented graphically in figure 1.

\[
\begin{align*}
\text{highly excited} & \quad \text{correspondence point} \\
\text{string state} & \quad \text{black hole}
\end{align*}
\]

\[
m_{\text{str}} G \sim \ell_s
\]

Figure 1: The Susskind-Horowitz-Polchinski correspondence principle: A highly excited string state at low string coupling \( g_s \) turns into a black hole when one increases \( g_s \), because at a certain point the Schwarzschild radius \( m_{\text{str}} G \sim m_{\text{str}} g_s^2 \ell_s^2 \) of the string becomes larger than the string length \( \ell_s \).

At the correspondence point one sets the string mass equal to the black hole mass, \( m_{\text{str}} = m_{\text{bh}} \). The string spectrum in flat space is given by

\[
m_{\text{str}}^2 \sim \frac{n}{\alpha'}, \quad n = 0, 1, 2, \ldots
\]  

(1.5)

In (1.5), \( \alpha' = \ell_s^2 \) denotes the inverse of the string tension. Combined with \( m_{\text{str}}^2 \sim \ell_s^2 / G^2 \), this yields

\[
\frac{\ell_s^2}{G} \sim \sqrt{n}.
\]  

(1.6)
The entropy of the four-dimensional Schwarzschild black hole is given by

\[ S_{bh} \sim \frac{r_{hor}^2}{G} \sim \frac{\ell^2}{G} \sim \sqrt{n}, \]  

(1.7)

where \( r_{hor} \) is the location of the event horizon, and we used (1.6) and the fact that \( r_{hor} \sim \ell_s \) at the correspondence point. On the other hand, it is well-known that the string entropy has the same behaviour,

\[ S_{str} \sim \sqrt{n}, \]  

(1.8)

so that the Bekenstein-Hawking entropy is comparable to the string entropy. In other words, an excited string provides the correct number of degrees of freedom to account for the entropy of the Schwarzschild black hole. This approach works also in other than four dimensions and for charged black holes [4]. The general idea is that, when the size of the horizon drops below the size of a string, the black hole state becomes a state of strings and D-branes (cf. next subsection) with the same charges.

Note that this calculation does not yield the correct numerical coefficient of the Bekenstein-Hawking entropy, that’s why we did not even try to put the correct prefactors in the above equations. It gives however the correct dependence on \( n \).

1.3 The three-charge black hole

The first microstate counting for black holes in string theory that reproduced also the right numerical coefficient of the Bekenstein-Hawking entropy was done by Strominger and Vafa [6]. They considered supersymmetric (and thus necessarily extremal and charged) black holes. In the presence of enough supersymmetry, there exist so-called non-renormalization theorems, which essentially say that weak coupling results are protected from quantum corrections. This means that the number of states one counts at weak coupling cannot change as one increases the coupling, i. e., when a black hole forms.

In order to understand the results of [6], we need an additional input with respect to the previous subsection, namely the concept of D-branes (where D stands for Dirichlet). Let us therefore open a parenthesis on D-branes. String theory is not a theory of strings alone, but it contains also other extended objects, among these the so-called D-branes [7], whose existence is required by string theory dualities. Dp-branes are p-dimensional hyperplanes on which open strings can end (cf. figure 2).

\[ ^1 \text{For an introduction to D-branes see e. g. [8].} \]
The open strings that are attached to D-branes satisfy Dirichlet boundary conditions in directions transverse to the branes (that’s where the name comes from), and Neumann boundary conditions in directions tangential to the branes, so that they are free to move along the branes. The open strings ending on a Dp-brane represent the excitations of the branes. At low energies $E \ll 1/\ell_s$, these excitations are described by a U(1) supersymmetric Yang-Mills (SYM) theory on the brane worldvolume, i.e., in $p+1$ dimensions. If we have $N$ coincident branes, the gauge group of the SYM theory is enhanced to U($N$). This comes from the fact that the end of each string has now $N$ possible D-branes on which to attach.

Let us now consider the system of branes from a different point of view: Dp-branes carry so-called Ramond-Ramond (RR) charges (they couple to a $(p+1)$-form RR vector potential $A^{(p+1)}$, just like a particle (which we can imagine as a ”0-brane”) couples to a one-form potential $A^{(1)}$ in electromagnetism), and they have a mass. This means that D-branes represent also sources for the gravitational field (and for the other supergravity fields). Now we are interested in the gravitational field of a particular configuration of D-branes, that describes a black hole. To this end, we consider a number $Q_5$ of coincident D5-branes along the directions $x^5, \ldots, x^9$ and $Q_1$ coincident D1-branes along $x^5$, so that their common transverse directions are $x^1, \ldots, x^4$ (superstring theory can be formulated consistently only in ten dimensions). Furthermore, we add a gravitational wave that moves along the $x^5$-direction. The resulting brane plus wave configuration
is shown in figure 3.

Figure 3: Configuration of $Q_5$ D5-branes along $x^5, \ldots, x^9$ and $Q_1$ D1-branes along $x^5$, with a wave propagating along the common direction.

The gravitational field created by this source is given by (cf. the nice review [9] for details of the construction)

$$ds_{10}^2 = f_1^{-1/2}f_5^{-1/2}[-dt^2 + (dx^5)^2 + (f_n - 1)(dt + dx^5)^2]$$

$$+ f_1^{1/2}f_5^{1/2}dx_i dx^i + f_1^{1/2}f_5^{-1/2}dx_a dx^a,$$

where $i = 1, \ldots, 4$, $a = 6, \ldots, 9$ and

$$f_{1,5,n} = 1 + \frac{r_{1,5,n}^2}{r^2}, \quad r^2 = (x^1)^2 + (x^2)^2 + (x^3)^2 + (x^4)^2.$$  \hspace{1cm} (1.9)

In (1.10), $r_{1,5}^2$ are proportional to the numbers $Q_{1,5}$ of D1- and D5-branes, and $r_n^2$ is proportional to the momentum $N$ along the direction $x^5$, carried by the wave (for the correct prefactors cf. [9]). Apart from the metric, also the RR 3-form field strength and the dilaton of type IIB supergravity are turned on. (Recall that the 2-form RR gauge potential couples to the D1-branes, and its dual, which is a 6-form gauge potential, couples to the D5-branes). This geometry preserves four of the 32 supercharges of type IIB supergravity.
Now perform a Kaluza-Klein compactification to five dimensions along the directions $x^5$ and $x^6, \ldots, x^9$, which we assume to be wrapped on a circle $S^1$ and a four-torus $T^4$ respectively. This results in the five-dimensional metric

$$ds_5^2 = -f^{-2/3}(r)dt^2 + f^{1/3}(r)(dr^2 + r^2d\Omega_3^2), \quad (1.11)$$

where $d\Omega_3^2$ denotes the round metric on the unit three-sphere and

$$f(r) = f_1(r)f_5(r)f_n(r). \quad (1.12)$$

The geometry (1.11) describes the so-called three-charge black hole (with charges $Q_1, Q_5$ and $N$). In the case $r_1^2 = r_5^2 = r_n^2$, it reduces to the extremal five-dimensional Reissner-Nordström solution. (1.11) has a horizon at $r = 0$ and zero Hawking temperature. The entropy of the three-charge black hole is given by

$$S = \frac{A_{Hor}}{4G_5} = \frac{2\pi^2r_1r_5r_n}{4G_5} = 2\pi\sqrt{Q_1Q_5N}, \quad (1.13)$$

where $G_5$ denotes the Newton constant in five dimensions and in the last step we used the correct proportionality factors between $r_{1,5,n}^2$ and $Q_{1,5}, N$.

Our aim is now to reproduce the entropy (1.13) by counting excitations of the D1-D5 system. In doing so, we will follow [10] rather than [6], which is based on a sophisticated analysis of the cohomology of instanton moduli spaces. Due to limitations of space, the state counting will be presented only schematically. For the details we refer e. g. to [9].

The various types of D1-D5 excitations are shown in figure 4. One can show that the low energy effective field theory describing these excitations is a $(1+1)$-dimensional $(4,4)$ supersymmetric gauge theory with gauge group $U(Q_1) \times U(Q_5)$ [11] (see also [9] for a review). The strings of (1,5) or (5,1) type are described by charged matter fields (corresponding to hypermultiplets) in the fundamental representation of $U(Q_1) \times U(Q_5)$. Now the presence of many open (1,5) or (5,1) strings effectively gives an expectation value to these matter fields, which therefore act as Higgs fields [10].

The D1-D5 system is thus described by the Higgs branch of the gauge theory. The Higgs fields make the vector multiplets (which describe the (1,1) and the (5,5) strings) massive, so that they can be dropped from the state counting. One now counts the total number of bosonic degrees of freedom from the hypermultiplets and subtracts both the number of conditions coming from the vanishing of the superpotential (which is necessary in order to have a supersymmetric vacuum) and the number of pure gauge degrees of freedom. This leaves $4Q_1Q_5$ gauge invariant bosonic degrees of freedom [11]. Supersymmetry implies then that there are also $4Q_1Q_5$ fermionic degrees of freedom.
Figure 4: Various types of excitations of the D1-D5 system: (1,1)-strings that stretch between two D1-branes, (5,5)-strings that stretch between two D5-branes, and (1,5)- and (5,1)-strings with one end attached to a D1-brane and the other to a D5-brane. Due to the orientation carried by the strings (denoted by an arrow in the figure), one has to distinguish between (1,5) and (5,1). The rightmost arrow indicates that the strings move along the $x^5$-direction, corresponding to the momentum $N$ along $x^5$.

Now we are interested in low energy black hole processes. For low energies (i.e., in the infrared), the above gauge theory flows to a (super-)conformal field theory. (At very low energies, we are far below any scale in the system, so that these scales effectively disappear and the theory becomes conformally invariant). Two-dimensional conformal field theories are characterized by a so-called central charge $c$ that appears in the Virasoro algebra, and represents the number of degrees of freedom. Every (free) boson contributes the value 1 to the central charge, whereas a (free) fermion contributes $1/2$. As we have $4Q_1Q_5$ bosonic and $4Q_1Q_5$ fermionic degrees of freedom, the central charge reads

$$c = 4Q_1Q_5 \cdot 1 + 4Q_1Q_5 \cdot \frac{1}{2} = 6Q_1Q_2.$$  \hspace{1cm} (1.14)

The microstates corresponding to the D1-D5 black hole are states with eigenvalues $l_0 = N$ and $\tilde{l}_0 = 0$ of the Virasoro generators $L_0$ and $\tilde{L}_0$ respectively. This comes from
the fact that all strings attached to the D-branes move in the direction $x^5$ (momentum $N$), and there are no strings moving in the opposite direction. The asymptotic number $\Omega$ of distinct states with $l_0 = N$, $\tilde{l}_0 = 0$ is given by the Cardy formula [12] (see also [13] for a nice derivation)

$$\Omega = \exp \left( 2\pi \sqrt{\frac{c l_0}{6}} \right) + \exp \left( 2\pi \sqrt{\frac{c \tilde{l}_0}{6}} \right) = \exp(2\pi \sqrt{Q_1 Q_5 N}).$$

(1.15)

This yields the entropy

$$S = \ln \Omega = 2\pi \sqrt{Q_1 Q_5 N},$$

(1.16)

which is in exact agreement with the Bekenstein-Hawking entropy (1.13).

Callan and Maldacena showed [10] that, remarkably enough, the same state counting works also for near-extremal five-dimensional black holes. In the D-brane picture, going away from extremality means exciting also left-moving momentum (i. e. , strings moving in $-x^5$-direction), and having also antibranes. ("Anti" means that these branes carry the opposite RR charge, in the same way in which an antiparticle carries the opposite charge of the corresponding particle).

Let us finally see what Hawking radiation corresponds to in the D-brane/string picture. Figure 5 shows a typical process that leads to Hawking radiation: A right-moving open string excitation collides with a left-moving open string excitation to give a closed string that leaves the brane. In [10], the rate for this process was computed, and it was shown that the radiation has a thermal spectrum at the Hawking temperature.

As we said, the geometry (1.11) has a horizon at $r = 0$, and that’s where the branes are located. If we invert the above picture of Hawking radiation, and consider a closed string infalling towards the horizon, i. e. , towards the branes, once it arrives at $r = 0$, it can split into open strings that move along the horizon. Note that in this picture there is no information loss: Quantum states falling into the horizon from the outside would cause a unitary evolution in the Hilbert space of horizon states (i. e. , states of the D-brane system) that "records" the infalling quantum information [10].

1.4 Gravity description of black hole microstates

In the previous subsection we saw how to identify black hole microstates in the D-brane picture, namely as D-brane excitations. As one can consider the system of branes also from a different perspective, namely as a source for the gravitational and the other supergravity fields, we can ask the question if one can see these microstates also in that picture (which we call the gravity side). In other words: Can we see the black hole microstates in general relativity (or, more generally, in supergravity)?
Figure 5: Hawking radiation in the D-brane/string picture: A right moving excitation and a left-moving excitation annihilate to give a closed string that leaves the brane.

Early attempts to find "hair" on black holes were based on looking for small perturbations in the metric while demanding smoothness at the horizon. However, one found no such perturbations. It was argued convincingly in [14] that, if we had found such hair, we would be faced with a curious difficulty: The microstates would look like in figure 6b), i.e., like black holes with a horizon.

![Figure 6](image)

**Figure 6:** a) The usual picture of a black hole. b) If the microstates represent small deformations of a) then each would itself have a horizon and an entropy.

This implies that we must associate an entropy $\approx S$ to each microstate, so we have
$e^S$ configurations, each having an entropy $\approx S$. But this makes no sense, because we wanted the microstates to *explain* the entropy, not to have further entropy themselves. Thus, if we do find the microstates in the gravity description, then they should turn out to have *no horizon* themselves. This argument led Mathur et al. [14] to the formulation of the following requirements for black hole "hair":

1. There must be $e^S$ states of the hole.

2. These individual states should have no horizon and no singularity.

3. "Coarse graining" (to be explained below) over these states should give the notion of entropy for the black hole.

In addition, the states should carry the same charges and preserve the same amount of supersymmetry as the hole. In [15], various gravity microstates for the D1-D5 system were constructed\(^2\). The resulting metrics are rather complicated, but they share the common feature that they have no horizon and no singularity. Furthermore, they all look essentially the same if the radial coordinate $r$ is larger than some value $r_0$, but differ from each other for $r < r_0$. This is illustrated in figure 7.

![Figure 7](image_url)

**Figure 7:** The metrics representing the black hole microstates have no horizon and no curvature singularity. They all look essentially the same if $r > r_0$, but differ from each other for $r < r_0$.

The terminology "coarse graining" in point 3 above means now the following: If we compute the area $A$ of the surface $r = r_0$ beyond which the geometries look different,

\(^2\)The geometries corresponding to the ground states of the D1-D5 system without momentum along $x^5$ (two-charge black hole) were obtained earlier in [16].
this should satisfy

\[ S = \frac{A}{4G}, \tag{1.17} \]

where \( S \) is the entropy (1.13) of the D1-D5 black hole. It is in this way that the notion of black hole entropy arises in this picture.

2. Naked singularities

2.1 . . . are not necessarily bad!

The second part of this lecture is dedicated to naked singularities. I will first explain an argument by Horowitz and Myers [17], that essentially states the following:

- Spacetime singularities play a useful role in gravitational theories by eliminating unphysical solutions.

- Any modification of general relativity which is completely nonsingular cannot have a stable ground state.

To understand this, let us start from the action

\[ S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} + F(g_{\mu\nu}, \nabla_\mu, R_{\mu\nu\rho\sigma}) \right], \tag{2.1} \]

where \( F \) denotes an arbitrary scalar function of the metric, the curvature and its derivatives. (For instance in type IIB supergravity, there is a term \( F \sim \alpha'^3 R_{\mu\nu\rho\sigma}^4 \), which comes from integrating out massive string modes). The claim is now that the theory described by (2.1) has always solutions with unbounded curvature. In order to see this, consider the gravitational wave

\[ ds^2 = -du dv + dx_i dx_i + h_{ij}(u) x^i x^j du^2. \tag{2.2} \]

This metric admits a covariantly constant null vector \( l = \partial_v \). The only nonzero component of the Ricci tensor is given by \( R_{uu} = -h_{i}(u) \), so (2.2) is a solution to general relativity if \( h_{i} = 0 \). (Physically, the two independent components of \( h_{ij} \) represent the amplitudes corresponding to the two polarizations of the gravitational wave). Furthermore, it is also a solution to the general theory (2.1), because the Riemann tensor of (2.2) is proportional to two powers of \( l_\mu \), and any contraction of \( l_\mu \) vanishes, so all second rank tensors constructed from the curvature and its derivatives vanish as well. We can now consider the case where \( h_{ij}(u) \) diverges, which leads to a curvature singularity (in the sense of unbounded gravitational tidal forces as the singularity is
approached). This means that all extensions of general relativity have solutions with null singularities.

Actually they must have timelike singularities as well: If, for instance, the considered extension of general relativity regulated the negative mass Schwarzschild solution, then the theory would have a regular negative energy solution, and thus Minkowski spacetime would not be stable! In conclusion, if we want the theory to have stable lowest energy solutions, it must have singularities in order that one may discard what would otherwise be pathological solutions [17].

In general, one can interpret the appearance of a naked singularity in a gravity solution as indicating the presence of an external source. Hence one should not necessarily rule out such solutions as unphysical, but rather ask if it has a reasonable physical source. Indeed, in certain cases, one finds that the source has a reasonable physical interpretation, in particular in string theory, where many extended sources are present.

2.2 Born-Infeld type gravity

It has been proposed [18] that curvature singularities might be regulated in gravity theories of Born-Infeld type, in the same way in which the divergent Coulomb potential of a point charge is regulated in Born-Infeld theory. In order to see how this could work, let us start from a very simple example, namely a free particle, with Newtonian Lagrangian \( \frac{1}{2}mv^2 \). Replacing this by the relativity expression

\[ L = mc^2 \left( 1 - \sqrt{1 - \frac{v^2}{c^2}} \right) \]  

(2.3)

yields an upper bound on the velocity \( v \), \( v \leq c \). This idea was used by Born and Infeld in 1934, who felt disturbed by the infinite self-energy of a point charge [19]. In order to regularize this, they introduced an upper bound on the electromagnetic field strength, replacing \( L_0 = \frac{1}{2}F_{\mu\nu}F^{\mu\nu} \) by

\[ L = b^2 \left( \sqrt{1 + \frac{1}{2b^4}F_{\mu\nu}F^{\mu\nu}} - 1 \right), \]  

(2.4)

where \( b \) denotes a constant\(^3\). The Lagrangian (2.4) reduces to \( L_0 \) for small fields \( (|F_{\mu\nu}| \ll b) \).

\(^3\)Born and Infeld called the constant \( b \), that has dimension of a field strength, *absolute field*. In the spirit of a *unitarian standpoint*, in which particles of matter are considered as singularities of fields and in which mass is a derived notion to be expressed by field energy, they determined the value of \( b \) by equating the electromagnetic field energy of a point charge with \( m_0c^2 \), where \( m_0 \) denotes the electron mass. This yields \( b \approx 10^{16} \) electrostatic units [19], which is an enormous magnitude.
In the new theory (2.4), the potential of a point charge $e$ is now given by \[ \phi(r) = e \frac{f \left( \frac{r}{r_0} \right)}{r_0}, \quad f(x) = \int_x^\infty \frac{dy}{\sqrt{1 + y^4}} \] so $r_0 = \sqrt{\frac{e}{b}}$. \hspace{1cm} (2.5)

The behaviour of $\phi(r)$ is shown in figure 8. Both the potential and the electric field $E_r = -d\phi/dr$ are finite in $r = 0$.

\[ ~ \frac{e}{r} \]

Figure 8: The potential of a point charge $e$ in Born-Infeld theory. There is no divergence for $r \to 0$. At large distances, $\phi(r)$ goes like $e/r$.

This example raises the question if we can do something similar for gravity in order to eliminate curvature singularities. A possible Born-Infeld type generalization of the Einstein-Hilbert action would be \[ 4S \sim \int d^4x \sqrt{-\det(a g_{\mu\nu} + b R_{\mu\nu} + c X_{\mu\nu})}, \] \hspace{1cm} (2.6)

where $a, b, c$ are constants and the tensor $X_{\mu\nu}$ is quadratic or higher order in the curvature. Deser and Gibbons formulated criteria that such a theory should satisfy \[ 18] 4:

1. Reduction to the Einstein-Hilbert action for small curvatures
2. Freedom of ghosts

\[^4\text{Cf. also [20].}\]
3. Supersymmetrizability

4. Regularization of (some) singularities. (Note that, according to what was said in the previous subsection, the negative mass Schwarzschild solution should not be regularized).

Notice that the Schwarzschild singularity can only be recognized from invariants constructed with the Riemann tensor, but not from those constructed with the Ricci tensor. This means that a gravity theory that cures this singularity should include the full Riemann tensor in the action.

It is at present an unsettled question if a Born-Infeld type gravity that meets the above criteria exists.

3. Conclusions

In this lecture I reviewed some ways in which string theory provides the correct black hole microstates. The general idea is that one has a configuration of D-branes at weak string coupling that turns into a black hole at strong coupling. The D-brane excitations are given by open strings attached to the branes, and we saw in the particular example of the D1-D5 system that the low energy degrees of freedom of these open strings represent the black hole microstates and reproduce exactly the Bekenstein-Hawking entropy. In this picture, Hawking radiation comes from collision of left- and right-moving open string excitations on the brane, which annihilate to give a closed string that leaves the brane [10].

I also gave a brief introduction to the recent proposal of Mathur et al. concerning a gravity description of black hole microstates. We saw that these gravity microstates cannot have an event horizon, and that coarse graining gives the notion of black hole entropy: The geometries of the microstates differ essentially from each other only if the radial coordinate $r$ is smaller than some value $r_0$, where the area $A(r_0)$ of the surface $r = r_0$ satisfies $S = A(r_0)/4G$. This implies a drastic modification of our picture of the interior of a black hole, which is not just empty space with a singularity, but rather one has a nontrivial interior exhibiting the degrees of freedom contributing to the entropy [14].

Finally, I commented on naked singularities and explained that they can play a useful role by eliminating unphysical solutions, so that one obtains a stable ground state [17]. I tried to motivate how a gravitational theory of Born-Infeld type might regulate bad curvature singularities, a question which is at present unsettled.
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