Nonclassicality of quantum excitation of classical coherent fields in thermal environments

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The nonclassicality of photon-added coherent fields in the thermal channel is investigated by exploring the volume of the negative part of the Wigner function which reduces with the dissipative time. The Wigner functions become positive when the decay time exceeds a threshold value. For the case of the single photon-added coherent state, we derive the exact threshold values of decay time in the thermal channel. For arbitrary partial negative Wigner distribution function, a generic analytical relation between the mean photon number of heat bath and the threshold value of decay time is presented. Finally, the possible application of SPACSs in quantum computation has been briefly discussed.

I. INTRODUCTION

Nonclassical optical fields play a crucial role in understanding fundamentals of quantum physics and have many applications in quantum information processing \cite{1}. Recently, the preparations of nonclassical non-gaussian optical fields have attracted much attention. Usually, the nonclassicality manifests itself in specific properties of quantum statistics, such as the antibunching \cite{2}, sub-poissonian photon statistics \cite{3}, squeezing in one of the quadratures of the field \cite{4}, partial negative Wigner distribution \cite{5}, etc.. The interaction between the nonclassical optical fields and their surrounding environment causes the dissipation or dephasing \cite{6}, which ordinarily deteriorates the degree of nonclassicality. The Wigner function is a quasi-probability distribution, which fully describes the state of a quantum system in phase space. The partial negativity of the Wigner function is indeed a good indication of the highly nonclassical character of the optical fields. Reconstruction of the Wigner distribution in experiments with quantum tomography \cite{7,8,9} demonstrates appearance of its negative values. This peculiarity can not be explained in the framework of the probability theory, hence it does not have any classical counterparts. Thus, to seek certain relations between the partial negativity of the Wigner distribution and the intensity of thermal noise may be very desirable for experimentally quantifying the variation of nonclassicality of optical fields in thermal channels. The volume of negative part of the Wigner function has been proposed as a useful tool for characterizing the nonclassical optical fields \cite{10,11,12}.

The photon-added coherent states (PACSs) were introduced by Agarwal and Tara \cite{13}. The single photon-added coherent state (SPACS) has been experimentally prepared by Zavatta et al. and its nonclassical properties have been detected by homodyne tomography technique \cite{14,15}. Such a state represents the intermediate non-Gaussian state between quantum Fock state and classical coherent state (with well-defined amplitude and phase) \cite{16}. For the SPACS, a quantum to classical transition has been explicitly demonstrated by ultrafast time-domain quantum homodyne tomography technique. Previous numerical study has indicated that the partial negativity of the Wigner function of photon-added coherent states in photon loss channel will completely disappear as the decay time exceeds a threshold value \cite{17}. However, the exact threshold value of the decay time has not been explicitly given. In this paper, we further investigate the nonclassicality of photon-added coherent states in the thermal channel by exploring the partial negative Wigner distribution. The volume of the negative part of the Wigner function is analyzed. The threshold values of the decay time corresponding to the transition of the Wigner distribution from partial negative to completely positive are derived. For SPACS in thermal channel, it is shown that the threshold value of the decay time is independent of the initial seed beam intensity. In addition, for any initial partial negative Wigner functions in thermal channel, we also obtain a general formula about the threshold value of decay time which is given by

$$\gamma_{t_c}(n) = \ln \left( \frac{\gamma_{t_c}(0) + 2n}{\gamma_{t_c}(0)} \right),$$

where $\gamma_{t_c}(n)$ denotes the threshold value of the decay time for the case of thermal channel with mean thermal photon number $n$. From this generic expression, we can clarify how the thermal noise shortens the threshold value of the decay time compared with the case of photon-loss.

This paper is organized as follows: In Sec.II, we briefly outline the basic contents of the photon-added coherent states. The explicit analytical expression of time evolution Wigner function of the SPACS in the thermal channel is derived. In Sec.III, the dynamical behaviors of the volume of the negative part of the Wigner distribution for
the SPACS and the two-photon added coherent states (TPACSs) is numerically calculated. It is shown that the volume of the negative part decreases with the decay time. The volume of the negative part of Wigner function of the TPACS more rapidly decreases than the one of the SPACS, though their Wigner functions become non-negative at the same threshold decay time. A generic relation between the threshold decay time of arbitrary partial negative Wigner functions in the thermal channel and the mean thermal photon number of the thermal reservoir is presented. Moreover, the possible application of SPACSs in quantum computation has been briefly discussed. In Sec. IV, there are some concluding remarks.

II. WIGNER FUNCTIONS OF PHOTO-ADDED COHERENT STATES IN THERMAL ENVIRONMENT

Let us first briefly recall the definition of the photon-added coherent states (PACSs) \[13\]. The PACSs are defined by

\[ a^\dagger |a\rangle = \sqrt{N(a,m)}|a\rangle, \]

where \(|a\rangle\) is the coherent state with the amplitude \(a\) and \(a^\dagger\) (\(a\)) is the creation (annihilation) operator of the optical mode. \(N(a,m) = m!L_m(-|a|^2)\), where \(L_m(x)\) is the \(m\)th-order Laguerre polynomial. When the PACS evolves in the thermal channel, the evolution of the density matrix can be described by \[6\]

\[
\frac{d\rho}{dt} = -\frac{\gamma}{2}(2a^\dagger a\rho - a^\dagger a^\dagger a\rho - a\rho - a^\dagger a\rho a^\dagger) + \frac{\gamma n}{2}(2a^\dagger a\rho - aa^\dagger a\rho - a\rho a^\dagger),
\]

(1)

where \(\gamma\) represents dissipative coefficient and \(n\) denotes the mean thermal photon number of the heat bath. When \(n = 0\), the Eq. (1) reduces to the master equation describing the photonic loss channel.

The presence of negativity in the Wigner function of the optical field is the indicator of nonclassicality. For an optical field in the state \(\rho\), the Wigner function, the Fourier transformation of characteristics function \[18\] of the state \(\alpha, m\)

\[
W(\beta) = \frac{2}{\pi} \text{Tr}[(\hat{O}_e - \hat{O}_o) \hat{D}(\beta) \rho \hat{D}^\dagger(\beta)],
\]

(2)

where \(\hat{O}_e \equiv \sum_{n=0}^{\infty} |2n\rangle \langle 2n|\) and \(\hat{O}_o \equiv \sum_{n=0}^{\infty} |2n+1\rangle \langle 2n+1|\) are the even and odd parity operators respectively. In the thermal channel described by the master equation (1), the time evolution Wigner function satisfies the following Fokker-Planck equation \[21\]

\[
\frac{\partial}{\partial t} W(q, p, t) = \frac{\gamma}{2} \left( \frac{\partial}{\partial q}(q^2) + \frac{\partial}{\partial p}(p^2) \right) W(q, p, t)
\]

(3)

where \(q\) and \(p\) represent the real part and imaginary part of \(\beta\), respectively. The time evolution Wigner function can be derived as following:

\[
W(q, p, \gamma t) = \exp(\gamma t) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W_T(x, y) W(q - \sqrt{1 - e^{-\gamma t}} x, p - \sqrt{1 - e^{-\gamma t}} y, 0) dx dy
\]

(4)

where

\[
W_T(x, y) = \frac{2}{\pi (1 + 2n)} \exp\left(-\frac{2(x^2 + y^2)}{1 + 2n}\right)
\]

(5)

is the Wigner function of the thermal state with mean photon number \(n\). Substituting the initial Wigner function of the pure SPACS \[13\]

\[
W_S(q, p, 0) = \frac{-2L_1(|q + 2ip - \alpha|^2)}{\pi L_1(|\alpha|^2)} e^{-2|q + ip - \alpha|^2}
\]

(6)

and the Wigner function of the pure TPACS \[13\]

\[
W_T(q, p, 0) = \frac{2L_2(|q + 2ip - \alpha|^2)}{\pi L_2(|\alpha|^2)} e^{-2|q + ip - \alpha|^2}
\]

(7)
into the Eq.(4), we can obtain the corresponding time evolution Wigner functions. For the case of SPACS, the analytical time evolution Wigner function can be obtained

\[
W^S(q, p, \gamma t) = \frac{2e^{\gamma t}[(\xi - c^2 \Re \alpha)^2 + (\zeta - c^2 \Im \alpha)^2 + c^4 - 1]\exp[-2(\mu^2 + \nu^2)/(1 + c^2)]}{\pi(1 + |\alpha|^2)(1 + c^2)^3},
\]

\[
c = [(\exp(\gamma t) - 1)(1 + 2n)]^{1/2},
\]

\[
\mu = \Re(\alpha) - q \exp(\gamma t/2),
\]

\[
\nu = \Im(\alpha) - p \exp(\gamma t/2),
\]

\[
\xi = \Re(\alpha) - 2q \exp(\gamma t/2),
\]

\[
\zeta = \Im(\alpha) - 2p \exp(\gamma t/2).
\]

In Fig.1, the Wigner function of the SPACS with \(\alpha = 0.5\) in the thermal channel with \(n = 1\) at two different values of decay time are plotted. It is shown that the phase space Wigner distribution of the pure SPACS with \(\alpha = 0.5\) loses its circular symmetry and moves away from the origin due to the appearance of a definite phase. The partial negativity of the Wigner function indicates the nonclassical nature of the single quantum excitation of the classical coherent field. The thermal noise causes the disappearance of the partial negativity of the Wigner function if the decay time exceeds a threshold value. The tilted ringlike wings in the distribution gradually start to disappear and the distribution becomes more and more similar to the Gaussian typical of a thermal state. Eq.(8) also indicates that the negative region of the Wigner function of the slightly decayed SPACS is always a circle. Fig.2 illustrates the phase space Wigner distributions of the TPACS with \(\alpha = 0.5\) in the thermal channel with \(n = 1\). The upper figure of Fig.2 shows the region of the negative part of the Wigner function of the pure TPACS is a ring belt which is different from the case of SPACSs. The lower figure of Fig.2 shows that the absolute value of negative minimum of the Wigner distribution decreases as \(\gamma t\) increases. When \(\gamma t\) exceeds a threshold value, the partial negativity of the Wigner distribution of the TPACS also completely disappears.

### III. INFLUENCE OF THERMAL NOISE ON THE NONCLASSICALITY OF QUANTUM EXCITATION OF CLASSICAL COHERENT FIELDS

Recently, the volume \(P_{NW}\) of the negative part of the Wigner function has been suggested as a good choice for quantifying the nonclassicality [10, 11, 12]. \(P_{NW}\) is defined by

\[
P_{NW} = |\int_{\Omega} W(q, p)dqdp|,
\]

where \(\Omega\) is the negative Wigner distribution region. In Ref. [17], we have investigated \(P_{NW}\) of SPACS and TPACS in the photon-loss channel. It was shown that \(P_{NW}\) and entanglement potential defined in Ref. [22] exhibit the consistent behaviors in short decay time.

In this section, we bring our attention to the influence of thermal noise on the nonclassicality of the quantum excitation of classical coherent optical fields by calculating \(P_{NW}\). In Eq.(8), we have obtained the analytical solution of the time evolving Wigner function of the SPACS in the thermal channel. Following similar lines, we can derive the one of the TPACS too. But the expressions are lengthy and do not exhibit a simple structure.

Based on the analytical expressions of the Wigner functions, we have calculated the volume of the negativity in Wigner functions for both the SPACSs and the TPACSs. In Fig.3, \(P_{NW}\) of SPACSs and TPACSs with the parameter \(\alpha = 1.5\) is plotted as the function of \(\gamma t\) for nine different values of the mean thermal photon number \(n\) of the thermal channel. It is shown that the thermal noise deteriorates the partial negativity, and \(P_{NW}\) monotonically decreases with the decay time. The larger the thermal noise, the more rapidly \(P_{NW}\) decreases, which implies that the nonclassicality of the optical fields are very fragile against the thermal noise. From Fig.4, it can be observed that, the volume of the negative part of the TPACS’s Wigner distribution is more fragile than the one of the SPACS’s Wigner distribution against the thermal noise if the seed beam intensity \(|\alpha|^2\) is not very large, though the initial pure TPACSs have larger values of \(P_{NW}\) than the pure SPACSs with the same value of \(\alpha\).

The above results also indicate that \(P_{NW}\) becomes zero at a threshold decay time \(\gamma t_e\) which depends on the value of \(n\). For the case of the SPACS in thermal channel, we can directly derive the threshold decay time from the Eq.(8). \(\gamma t_e\) can be obtained as follows:

\[
\gamma t_e = \ln\left(\frac{2 + 2n}{1 + 2n}\right),
\]
which shows the threshold decay time is independent of the seed beam intensity \(|\alpha|^2\) of the SPACSs. For the case of the TPACSs, the threshold decay times also satisfy the Eq. (10). In Fig. 5, we have plotted both the exact analytical and the numerical results of the threshold decay time as the function of \(n\) for the case of SPACSs with \(\alpha = 0.5\). Well consistent between the analytical results and the numerical solutions is found.

For arbitrary nonclassical optical fields which have the partial negative Wigner distribution function, there exists a relation between the mean photon number \(n\) of the thermal reservoir and the threshold decay time \(\gamma_t(n)\) beyond which their Wigner function become positive.

\[
\gamma_t(n) = \ln \frac{e^{\gamma_t(0)} + 2n}{1 + 2n}, \tag{11}
\]

where \(\gamma_t(0)\) is the threshold decay time in the photon loss channel. In the derivation of the Eq. (11), we have reformulated Eq. (4) as

\[
W(q, p, \gamma_t) = \exp(\gamma_t) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W_0(x', y') W(e^{\gamma_t/2} q' - \sqrt{\sqrt{1 + 2n}(e^{\gamma_t} - 1)} x', e^{\gamma_t/2} p' - \sqrt{\sqrt{1 + 2n}(e^{\gamma_t} - 1)} y', 0) dx' dy', \tag{12}
\]

where

\[
W_0(x', y') = \frac{2}{\pi} \exp(-2(x'^2 + y'^2))
\]

\[
x' = \frac{x}{\sqrt{1 + 2n}}
\]

\[
y' = \frac{y}{\sqrt{1 + 2n}}
\]

\[
q' = \frac{e^{\gamma_t/2}}{\sqrt{1 + (1 + 2n)(e^{\gamma_t} - 1)}} q
\]

\[
p' = \frac{e^{\gamma_t/2}}{\sqrt{1 + (1 + 2n)(e^{\gamma_t} - 1)}} p
\]

\[
\gamma_t' = \ln[1 + (1 + 2n)(e^{\gamma_t} - 1)]. \tag{13}
\]

Obviously, the right side of the Eq. (12) represents a scaled time evolution Wigner function in the photon loss channel, i.e. \(n = 0\). Therefore we have

\[
W(q, p, \gamma_t)/e^{\gamma_t} = W^{(0)}(q', p', \gamma_t')/e^{\gamma_t'}, \tag{14}
\]

where \(W^{(0)}(q, p, \gamma_t)\) is the time evolution Wigner function of the optical field in the photon loss channel. Based on Eq. (14) and the definition of \(P_{NW}\) in Eq. (9), we can prove

\[
P_{NW}(\gamma_t) = P^{(0)}_{NW}(\gamma_t'), \tag{15}
\]

where \(P^{(0)}_{NW}(\gamma_t')\) denotes the volume of the negative part of the photon-loss-induced time evolving Wigner function at the time \(\gamma_t'\) of the optical field initially described by the wigner function \(W(q, p, 0)\). Assuming we have known the threshold decay time of arbitrary partial negative Wigner function in the photon loss channel, we can derive the corresponding threshold decay time of that Wigner function in the thermal channel via the Eq. (15). If \(P^{(0)}_{NW}(\gamma_t')\) becomes zero at the decay time \(\gamma_t' = \gamma_t(0)\), it can be found that \(P_{NW}(\gamma_t)\) becomes zero at \(\gamma_t = \ln \frac{e^{\gamma_t(0)} + 2n}{1 + 2n}\) according to the relation \(\gamma_t' = \ln[1 + (1 + 2n)(e^{\gamma_t} - 1)]\). Thus, we finish proving the relation in Eq. (11) for arbitrary partial negative Wigner function. This relation may have some potential applications in the measurement of the temperature of the surrounding heat bath. In Ref. [17], we have numerically investigated the entanglement potential and partial negativity of the Wigner function of SPACSs in photon-loss channel. The nonclassical properties of single-photon subtracted squeezed vacuum states in amplitude decay channel or in the phase diffusion channel have also been studied by Biswas and Agarwal [23]. The photon loss channel is only a specific example of the thermal channel at which the thermal reservoir is at zero-temperature. Present results not only generalize the results in Ref. [17], and give out an analytical expression of the evolving Wigner function of SPACSs in thermal channel, but also find out the exact analytical relation between the mean thermal photon number of the thermal channel and the threshold time concerning the complete disappearance of partial negativity of the Wigner function of SPACSs. Furthermore, for any initial partial negative Wigner distribution function in thermal channel, we derive a general relation between the
threshold decay time and the mean thermal photon number of the thermal channel. Roughly speaking, the formula in Eq.\((11)\) implies that the higher the temperature of thermal channel, the more rapidly the nonclassicality of the optical fields with partial negative Wigner distribution decays. From Eq.\((11)\), it is also easy to know that \(\gamma t_c(n) \approx e^{\gamma t_c(0) - 1/2n}\) when \(n \gg 1\), and \(\gamma t_c(n) \approx \gamma t_c(0) - 2(1 - e^{-\gamma t_c(0)n})n\) when \(n \ll 1\).

In the past five years, much attention has been focused on the application of coherent states in quantum computation \[24,25]\; SPACSs can also find many applications in quantum information processes such as the quantum computation. Similar to the schemes in the literatures, one can regard the vacuum state as two orthogonal states of an optical qubit \(|0\rangle_L\) and \(|1\rangle_L\), respectively. As an illustration, in this state encoding, a nontrivial two qubit gate can be implemented using only a beam splitter. Consider the beam splitter interaction described by the unitary transformation

\[
U_{BS} = \exp[i\phi (ab^\dagger + a^\dagger b)],
\]

where \(a\) and \(b\) represent the annihilation operators corresponding to two qubits \(|\kappa\rangle_a\) and \(|\chi\rangle_b\) which take the states \(|0\rangle\) or \(|\sqrt{\alpha}\rangle\) respectively. It is easy to derive that the output state produced by such interaction is

\[
U_{BS}\langle 0|_a\rangle_b = \langle 0|_a\rangle_b,
\]

\[
U_{BS}\sqrt{N(\alpha,1)}a^\dagger|\alpha\rangle_0\rangle_b = \sqrt{N(\alpha,1)}(\cos(\phi)a^\dagger + i\sin(\phi)b^\dagger)|\cos(\phi)\alpha\rangle_a|\sin(\phi)\alpha\rangle_b,
\]

\[
U_{BS}\sqrt{N(\alpha,1)}b^\dagger|\alpha\rangle_0\rangle_b = \sqrt{N(\alpha,1)}(\cos(\phi)b^\dagger + i\sin(\phi)a^\dagger)|\sin(\phi)\alpha\rangle_a|\cos(\phi)\alpha\rangle_b,
\]

\[
U_{BS}\sqrt{N(\alpha,1)}a^\dagger b^\dagger|\alpha\rangle_0\rangle_b = \sqrt{N(\alpha,1)}(\cos(\phi)a^\dagger + i\sin(\phi)b^\dagger)(\cos(\phi)b^\dagger + i\sin(\phi)a^\dagger)|e^{i\phi}\alpha\rangle_a|e^{i\phi}\alpha\rangle_b,
\]

The overlap between the output and input states can be expressed as follows:

\[
\langle a(0)_b(0)U_{BS}\langle 0|_a\rangle_b = 1,
\]

\[
\frac{1}{N(\alpha,1)}\langle a|_b\rangle_0\langle aU_{BS}a^\dagger|\alpha\rangle_a\rangle_0\rangle_b b = \frac{1}{N(\alpha,1)}\langle a|_b\rangle_0\langle aU_{BS}b^\dagger|\alpha\rangle_a\rangle_b,
\]

\[
\frac{1}{N^2(\alpha,1)}\langle a|_b\rangle_0\langle aU_{BS}a^\dagger b^\dagger|\alpha\rangle_a\rangle_b = \frac{1}{N^2(\alpha,1)}|\alpha|^4 e^{4i\phi} + 2|\alpha|^2 e^{2i\phi} + \cos(2\phi)|e^{2i\phi} - 1|\]

If \(\phi\) is assumed to be sufficiently small such that \(\phi^2|\alpha|^2 \ll 1\) but \(|\alpha|\) is sufficiently large that \(\phi|\alpha|^2\) is of order one. Then Eq.\((18)\) can approximately become

\[
\langle a|_b\rangle_0\langle aU_{BS}\rangle_0\langle a|_b\rangle_0 = 1,
\]

\[
\frac{1}{N(\alpha,1)}\langle a|_b\rangle_0\langle aU_{BS}a^\dagger|\alpha\rangle_a\rangle_0\rangle_b b \approx \frac{1}{N(\alpha,1)}\langle a|_b\rangle_0\langle aU_{BS}b^\dagger|\alpha\rangle_a\rangle_b \approx 1,
\]

\[
\frac{1}{N^2(\alpha,1)}\langle a|_b\rangle_0\langle aU_{BS}a^\dagger b^\dagger|\alpha\rangle_a\rangle_b \approx \frac{1}{N^2(\alpha,1)}|\alpha|^2 e^{2i\phi|\alpha|^2}.
\]

Furthermore, the condition \(2\phi|\alpha|^2 = \pi\) will guarantee the realization of a controlled phase gate by the beam splitter acted on the above encoding space.

The results in this paper may be useful for investigating the influence of the thermal noise on the quantum information processes in which the photon added coherent state is used as a resource. It is also interesting to investigate the fidelity of the above controlled phase gates in thermal channel. The details will be discussed elsewhere.
IV. CONCLUSIONS

In summary, we have investigated the non-classicality of photon excitation of classical coherent field in the thermal channel by exploring the partial negativity of the Wigner function. The total volume of the negative part defined by the absolute value of the integral of the Wigner function over the negative distribution region is calculated and the partial negativity of the Wigner function can not be observed when the decay time exceeds a threshold value which depends on the mean thermal photon number. For the cases in which the seed beam intensity $|\alpha|^2$ is not very large, it is found that the TPACSs more rapidly lose their partial negativity of the Wigner distribution function than the SPACSs with the same seed beam intensity. For the case of SPACSs in thermal channel, the exact threshold value of the decay time beyond which the evolving Wigner function becomes positive is given as $\gamma t_c = \ln\left(\frac{\gamma t_c}{1+\gamma t_c}\right)$. For the arbitrary nonclassical optical fields with partial negative Wigner function, we also present a generic relation between the threshold decay time of the thermal channel and the mean thermal photon number of the thermal reservoir under the assumption of the known threshold decay time of the photon-loss channel. Finally, the possible application of SPACSs in quantum computation has been briefly discussed. Present results in this paper may be useful for checking the influence of the thermal noise on the quantum information processes in which the photon added coherent state is used as a resource.

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Fig.1. The Wigner functions of the SPACS with $\alpha = 0.5$ in thermal channel with $n = 1$ are depicted for two different values of decay time $\gamma t$.

Fig.2. The Wigner functions of the TPACS with $\alpha = 0.5$ in thermal channel with $n = 1$ are depicted for two different values of decay time $\gamma t$.

Fig.3. The $P_{NW}$ of the SPACS and TPACS with $\alpha = 1.5$ in thermal channel are depicted as the function of $\gamma t$ for different values of mean thermal photon number $n$. From top to bottom, $n = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$.

Fig.4. $P_{NW}$ is plotted as the function of the decay time $\gamma t$ for SPACS and TPACS with different values of $\alpha$. (Solid line) from top to bottom, SPACS with $\alpha = 0.1, 0.5, 1.0, 1.5$, respectively; (Dash line) from top to bottom, TPACS with $\alpha = 0.1, 0.5, 1.0, 1.5$, respectively. $n = 0.5$.

Fig.5. The threshold decay time $\gamma t_c$ beyond which $P_{NW} = 0$ is plotted as the function of the mean thermal photon number $n$ for the case of the SPACS. (Solid square) numerical results; (Solid line) $\gamma t_c = \ln \frac{2+2n}{1+2n}$. It is shown that $\gamma t_c \simeq 0.5/n$ when $n \gg 1$. The numerical calculations are based on the SPACS with $\alpha = 0.5$. 
FIG. 1: The Wigner functions of the SPACS with $\alpha = 0.5$ in thermal channel with $n = 1$ are depicted for two different values of decay time $\gamma t$. 
FIG. 2: The Wigner functions of the TPACS with $\alpha = 0.5$ in thermal channel with $n = 1$ are depicted for two different values of decay time $\gamma t$. 
FIG. 3: The $P_{NW}$ of the SPACS and TPACS with $\alpha = 1.5$ in thermal channel are depicted as the function of $\gamma t$ for different values of mean thermal photon number $n$. From top to bottom, $n = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$. 
FIG. 4: $P_N W$ is plotted as the function of the decay time $\gamma t$ for SPACS and TPACS with different values of $\alpha$. (Solid line) from top to bottom, SPACS with $\alpha = 0.1, 0.5, 1.0, 1.5$, respectively; (Dash line) from top to bottom, TPACS with $\alpha = 0.1, 0.5, 1.0, 1.5$, respectively. $n = 0.5$.

FIG. 5: The threshold decay time $\gamma t_c$ beyond which $P_N W = 0$ is plotted as the function of the mean thermal photon number $n$ for the case of the SPACS. (Solid square) numerical results; (Solid line) $\gamma t_c = \ln \frac{2 + 2n}{1 + 2n}$. It is shown that $\gamma t_c \simeq 0.5/n$ when $n \gg 1$. The numerical calculations are based on the SPACS with $\alpha = 0.5$. 