Observation of two dual electromagnetically induced transparencies in cold $^{87}$Rb atoms

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We demonstrate the observation of two dual electromagnetically induced transparency (EIT) peaks simultaneously in $D_2$ line transition of $^{87}$Rb atom in a sample of cold $^{87}$Rb atoms trapped in a magneto-optical trap (MOT). These dual EIT peaks are generated from two different N-systems in $D_2$ line transition of $^{87}$Rb atom. The dependence of this dual EIT spectral feature on driving beams (i.e., coupling and control beams) intensity and detuning has been investigated in detail for both the N-systems. The results show that the dual EIT spectral features in both the N-systems can be manipulated by varying the intensity and detuning of the driving beams. The two N-systems have been modelled theoretically using a density matrix formalism and results have shown a good agreement with the experimental observations.

Keywords: Dual electromagnetically induced transparency; N-systems; Density matrix formalism

I. INTRODUCTION

The interaction of two coherent electromagnetic fields with a three-level atomic system gives rise to an interesting phenomenon known as Electromagnetically Induced Transparency (EIT). EIT is an effect of quantum interference that modifies the optical property of an atomic medium for a weak probe field in presence of another strong coupling field [1]. The fascinating applications of EIT includes slow light propagation [2-4], optical switching [5-6], precise atomic clocks [7], tight laser frequency locking [8-9], non-linear optical process [10-11] etc. The three-level atomic system giving rise to EIT effect are A-, 1215, ladder-1618 and vee-system 1920.

The ideal three-level EIT system creates a dark state which does not interact with the probe field. This dark state was then perturbed using a microwave field [2122], radio-frequency field [23] and an additional coupling field [2427], in and all cases a dual EIT structure has been observed. Niu [28] had earlier reported that the four-level EIT (dual structure) shows greatly enhanced non-linear effects than an ideal EIT obtained through three-level system. The first proposal of quantum interference effect in four-level N-system was given by Harris and Yamamoto [29], lateron, experimental implementation was done by Braje [30] and Kang [31]. In literature, enhancement in non-linear effects in N-system has also been reported [32-37].

In this work, we have implemented an experimental technique to attain two simultaneous N-systems in $D_2$ line transition of cold $^{87}$Rb atoms which has resulted in two dual EIT peaks in the probe transmission spectrum. The experiments have been performed on cold $^{87}$Rb atoms trapped in a magneto-optical trap (MOT). In order to realize two N-systems, two driving beams with fixed frequency and one probe beam in scan mode was used in our experiments. The driving beams intensity and detuning dependent probe transmission spectrum has been investigated in detail in this work. The obtained results show that the dual EIT spectral features in both the N-systems can be controlled by varying the driving beams intensity and detuning. The two N-systems have also been modelled theoretically using a density matrix formalism. The theoretical results explain adequately the experimental observations.

II. EXPERIMENTAL SETUP

The experiments have been performed on a double magneto-optical trap setup, the schematic of which has been illustrated in figure 1 (a). The vacuum chamber comprises of a stainless steel chamber (vapor chamber) kept at a pressure $\sim 5 \times 10^{-8}$ Torr and a quartz glass cell (ultra high vacuum cell) kept at a pressure $\sim 5 \times 10^{-11}$ Torr. The two parts of the chamber were connected through a differential pumping tube with varying inner diameter to maintain the pressure gradient. Two sputter ion pumps on both parts of the vacuum chamber along with a nonevaporable getter pump and a Titanium sublimation pump on the ultrahigh vacuum cell facilitated the required vacuum conditions. The vapor of Rb atoms are ejected in vapor chamber using a Rb getter connected with the vapor chamber.

Two magneto-optical traps (MOTs) are prepared in the setup, in which first MOT is prepared in vapor chamber (called VC- MOT) and second MOT is prepared in UHV glass cell (called as UHV-MOT). The MOTs were realised using a set of frequency locked laser beams along with a magnetic field gradient generated by pairs of antihelmholtz coils. For both the MOTs, the cooling beams were derived from an external cavity diode laser (Toptica, DL 100, Germany) feeding an amplifier (TABoosta, Toptica, Germany) providing a total output power $\sim 550$ mW. The transition frequency of the cooling beams were locked at $\sim 15$ MHz red detuned from cooling transition frequency $^5S_{1/2} F = 2 \rightarrow ^5P_{3/2} F' = 3$ of $^{87}$Rb atom (figure 2 (b)). Similarly, another external cavity diode...
using a CCD camera (Pixelfly model USB) along with an appropriate 2f-imaging optics. The collected fluorescence from the UHV-MOT was utilized to calculate the number of trapped atoms using the count of the CCD camera and the experimental parameters. The estimated number of atoms trapped in the atom cloud was $\sim 5 \times 10^7$ with a temperature of $\sim 300 \mu K$.

The probe beam for the EIT experiments was derived from another external cavity diode laser (Toptica, DL 100, Germany) having laser linewidth less than 1 MHz and $1/e^2$-radius $\sim 3$ mm was aligned to traverse through the UHV-MOT as shown in figure 1 (a). In order to obtain the drive beams for the EIT experiments, a part of the laser beam was extracted from the laser amplifier that drives the repumping transitions in both the MOTs and passed through an acousto optic modulator (AOM) in double-pass configuration. The frequency of the output beam corresponds to the transition $5^2S_{1/2} F = 1 \rightarrow 5^2P_{3/2} F' = 2$. This output beam was split using a half-waveplate and a polarizing beam splitter. One part, hereafter denoted as drive beam ‘$C_1$’, was copropagated with the probe beam through the UHV-MOT. The other part was again passed though another AOM in double pass configuration to obtain the transition frequency $5^2S_{1/2} F = 1 \rightarrow 5^2P_{3/2} F' = 1$ with independent frequency control. This beam, denoted as drive beam ‘$C_2$’ and was also made to pass through the UHV-MOT copropagating to the probe beam but with a relative angle of $\sim 0.5$ degree. The probe and the two drive beams (‘$C_1$’ and ‘$C_2$’) were linear but orthogonal polarised, the probe beam was separated from the drive beams using a half-waveplate and a polarizing beam splitter and collected on a photodiode. The output of this photodiode was then recorded using a digital oscilloscope. The fixed frequencies of two drive beams (‘$C_1$’ and ‘$C_2$’) and scan of probe beam (P) frequency forms two different N-systems in the $D_2$ line transition of $^{87}Rb$ atom simultaneously. The description of these N-systems are given in section IV.

### III. DENSITY MATRIX FORMALISM

In order to understand the experimental observations, the composite atom-field system has been studied using a density matrix formalism. This method involves solving a partial differential equation, known as the Liouville equation, numerically. The Liouville equation can be written under the electric dipole approximation in terms of the total Hamiltonian $H$, density matrix ($\rho$) and decay operator $R$ as,

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar} [H, \rho] - \frac{1}{\hbar} [R, \rho], \quad (1)$$

The first term in right-hand side of equation 1 describes the unitary evolution and second term describes the decay of the atomic system. Under the rotating wave ap-
proximation the complete density matrix evolution equations for atomic system \(N_B\) can be written as,

\[
\frac{\partial \rho_{ij}}{\partial t} = \gamma_i \rho_{44} + \gamma_3 \rho_{33} + \frac{i}{2} \Omega_{c2} (\rho_{11} - \rho_{13}) \\
+ \frac{i}{2} \Omega_{c1} (\rho_{41} - \rho_{14}) \\
\frac{\partial \rho_{12}}{\partial t} = \gamma_2 \rho_{44} + \frac{i}{2} \Omega_p (\rho_{24} - \rho_{22}) \\
\frac{\partial \rho_{33}}{\partial t} = \gamma_3 \rho_{33} + \frac{i}{2} \Omega_{c2} (\rho_{13} - \rho_{31}) \\
\frac{\partial \rho_{14}}{\partial t} = - (\gamma_1 + \gamma_2) \rho_{44} - 2 \gamma_3 \rho_{33} \\
+ \frac{i}{2} \Omega_p (\rho_{24} - \rho_{42}) - \frac{i}{2} \Omega_{c1} (\rho_{41} - \rho_{14}) \\
\frac{\partial \rho_{12}}{\partial t} = - i (\Delta_p - \Delta_c) \rho_{12} - \frac{i}{2} \Omega_{c2} \rho_{23} \\
- \frac{i}{2} \Omega_p \rho_{12} - \frac{i}{2} \Omega_{c1} \rho_{24} \\
\frac{\partial \rho_{23}}{\partial t} = \left( -i \Delta_c + \frac{1}{2} \sigma_3 \right) \rho_{13} - \frac{i}{2} \Omega_{c1} \rho_{43} \\
- \frac{i}{2} \Omega_{c2} (\rho_{11} - \rho_{33}) \\
\frac{\partial \rho_{14}}{\partial t} = \left( -i \Delta_p + \Delta_c \right) \rho_{14} - \frac{i}{2} \Omega_p \rho_{21} \\
+ \frac{i}{2} \Omega_{c2} \rho_{34} - \frac{i}{2} \Omega_{c1} (\rho_{11} - \rho_{44}) \\
\frac{\partial \rho_{24}}{\partial t} = \left( -i \Delta_p + \frac{1}{2} \sigma_3 \right) \rho_{24} \\
- \frac{i}{2} \Omega_p (\rho_{22} - \rho_{44}) - \frac{i}{2} \Omega_{c1} \rho_{12} \\
\frac{\partial \rho_{34}}{\partial t} = \left( -i \Delta_c - \Delta_c \right) \rho_{34} \\
- \frac{i}{2} \Omega_p \rho_{32} + \frac{i}{2} \Omega_{c1} \rho_{31} - \frac{i}{2} \Omega_{c2} \rho_{14} \\
\frac{\partial \rho_{ij}}{\partial t} = \frac{\partial \rho^*_{ji}}{\partial t}
\]

Similarly, set of density matrix equations for atomic system \(N_A\) can also be derived. In order to solve the above set of equations in the steady state, the matrix method are used which can be described as,

\[
\dot{\rho} = \mathcal{L} \rho = 0.
\]

The steady state value of the density matrix \(\rho\) are determined by evaluating the eigenvector of the Liouvil-lian super-operator \(\mathcal{L}\) corresponding to the ‘zero’ eigenvalue.

**IV. RESULTS AND DISCUSSION**

The N-type atomic system is an extension of the basic \(A\)-system where an additional laser beam, called as the control beam, couples one of the ground state with another excited state. The studied N-type systems are shown in figure 2. The first \(A\)-system denoted as \(A_1\), formed using the transitions \(5^2S_{1/2} F = 1 \rightarrow 5^2P_{3/2} F' = 1 \leftarrow 5^2S_{1/2} F = 2\), was converted to an N-type atomic system \(N_A\) by adding another drive beam ‘\(C_1\)’ resonant to transition \(5^2S_{1/2} F = 1 \rightarrow 5^2P_{3/2} F' = 2\). Likewise, the atomic system \(A_B\), formed using the transitions \(5^2S_{1/2} F = 1 \rightarrow 5^2P_{3/2} F' = 2 \leftarrow 5^2S_{1/2} F = 2\), was converted to another N-type atomic system \(N_B\) in presence of drive beam ‘\(C_2\)’ resonant to transition \(5^2S_{1/2} F = 1 \rightarrow 5^2P_{3/2} F' = 1\). The coupling, probe and control beams frequencies for different systems \((A_1, A_B, N_A, N_B)\) are shown in table 1. We note that the drive beam ‘\(C_1\)’ acts as a control beam for system \(N_A\), whereas drive beam ‘\(C_2\)’ acts as a control beam for system \(N_B\).

As the strength of a transition is governed by the Clebsch-Gordan (CG) coefficient connecting two states, the probe beam strength in atomic system \(N_B\) is higher than the strength in system \(N_A\). However, the CG coefficients of both the driving beams ‘\(C_1\)’ and ‘\(C_2\)’ are same giving same strength for equal powers in these beams. In the experiments, probe power was kept fixed at \(\sim 30\) \(\mu\) W. In all the studies, the transmitted probe signal for both the atomic systems \((A\) and \(B)\) were captured in a single scan of probe detuning using a digital storage oscilloscope, but the results of two systems are presented in separate plots. The experimental data shown in the figures has been processed for better visibility using a convolution algorithm.

The transmitted probe spectrum for both the \(A\)-systems and corresponding N-systems were measured. In these measurements, both the driving beam frequencies were kept near resonance with equal power. The ob-
TABLE I: Transitions involved in various systems

| system | coupling | probe | control |
|--------|----------|-------|---------|
| \( \Lambda_A \) | \( 5^2S_{1/2} F' = 1 \rightarrow 5^2P_{3/2} F' = 1 \) | \( 5^2S_{1/2} F = 2 \rightarrow 5^2P_{3/2} F' = 1 \) | - |
| \( \Lambda_B \) | \( 5^2S_{1/2} F' = 1 \rightarrow 5^2P_{3/2} F' = 1 \) | \( 5^2S_{1/2} F = 2 \rightarrow 5^2P_{3/2} F' = 2 \) | - |
| \( N_A \) | \( 5^2S_{1/2} F = 1 \rightarrow 5^2P_{3/2} F' = 1 \) | \( 5^2S_{1/2} F = 2 \rightarrow 5^2P_{3/2} F' = 1 \) | \( 5^2S_{1/2} F = 1 \rightarrow 5^2P_{3/2} F' = 2 \) |
| \( N_B \) | \( 5^2S_{1/2} F = 1 \rightarrow 5^2P_{3/2} F' = 2 \) | \( 5^2S_{1/2} F = 2 \rightarrow 5^2P_{3/2} F' = 2 \) | \( 5^2S_{1/2} F = 1 \rightarrow 5^2P_{3/2} F' = 1 \) |

FIG. 3: The transmitted probe signal as a function of probe beam detuning for \( \Lambda \) (shown by red curves) and \( N \) (shown by black curve) systems. Here plots (i) and (ii) correspond to systems (A) and (B) respectively, with both driving beams \( C_1 \) and \( C_2 \) are of power 12 mW and probe beam of power \( \sim 30 \) µW.

The absorption at line center due to the presence of an ent for the system \( N \) has been reported earlier [39, 40], which is more prominent than that in system \( \Lambda \). In the presence of control beam, both the systems (\( N_A \) and \( N_B \)) show a transmission dip at the center of the existing EIT peak, resulting in splitting of the single EIT peak into two separate EIT peaks (dual EIT), shown by black curves in figure 3 (i) and (ii). This type of enhanced absorption has been reported earlier [39, 40], which is more prominent for the system \( N_A \) than the system \( N_B \) in our case. The absorption at line center due to the presence of additional control field opens the path for the system to be useful in optical switching devices.

To investigate the atomic systems \( N_A \) and \( N_B \) further, the power of one of the driving beams was varied while keeping the other one fixed. The other experimental parameters were kept unchanged during these measurements. First, we varied power in drive beam ‘\( C_1 \)’, which acts as a control beam for system \( N_A \) and coupling beam for system \( N_B \). Figure 4 (a) and (c) show the transmitted probe beam spectrum for different powers of \( P_{c1} \) and fixed value of \( P_{c2} = 12 \) mW. The power values in \( C_1 \) beam \( P_{c1} \) are: (i) 12 mW, (ii) 10 mW, (iii) 8 mW, and (iv) 6 mW. Plots (b) and (d) show the calculated transmission as a function of the probe beam detuning \( \Delta_p \) with \( \Omega_{c1} \) as: (i) \( 2\pi \times 8 \) MHz, (ii) \( 2\pi \times 7 \) MHz, (iii) \( 2\pi \times 6 \) MHz, and (iv) \( 2\pi \times 5 \) MHz, for a fixed \( \Omega_{p} = 2\pi \times 8.0 \) MHz, for both the systems \( N_A \) and \( N_B \). The other parameters used in the numerical simulations for plots (b) are \( \Omega_{p} = 2\pi \times 0.01 \) MHz, \( \Delta_{c1} = \Delta_{c2} = 2\pi \times 2.0 \) MHz, and for plot (d) are \( \Omega_{p} = 2\pi \times 0.1 \) MHz, \( \Delta_{c1} = \Delta_{c2} = 2\pi \times 1 \) MHz.
FIG. 5: (a) and (c) show the transmitted probe beam signal as a function of the probe beam detuning $\Delta_p$ in systems $N_A$ and $N_B$ for different values of $P_{c2}$ and fixed value of $P_{c1} = 12$ mW. The power values in $C_2$ beam $P_{c2}$ are: (i) 12 mW, (ii) 10 mW, (iii) 8 mW, and (iv) 6 mW. Plots (b) and (d) show the calculated transmission as a function of the probe beam detuning $\Delta_p$ with $\Omega_{c2}$ as: (i) $2\pi \times 8$ MHz, (ii) $2\pi \times 7$ MHz, (iii) $2\pi \times 6$ MHz, and (iv) $2\pi \times 5$ MHz, for a fixed $\Omega_{c1} = 2\pi \times 8.0$ MHz, for both the systems $N_A$ and $N_B$. The other parameters used in the numerical simulations for plot (b) are $\Omega_p = 2\pi \times 0.01$ MHz, $\Delta_{c1} = \Delta_{c2} = 2\pi \times 1.0$ MHz, and for plot (d) are $\Omega_p = 2\pi \times 0.1$ MHz, $\Delta_{c1} = \Delta_{c2} = 2\pi \times 1$ MHz.

FIG. 6: (a) and (c) show the measured probe transmission spectrum and (b) and (d) show the calculated probe transmission spectrum for systems $N_A$ and $N_B$, for $\Delta_{c1}$ and $\Delta_{c2}$: (i) $\sim - 2\pi \times 9$ MHz, (ii) $\sim - 2\pi \times 3$ MHz, (iii) $\sim 2\pi \times 10$ MHz. Powers of both the driving beams are 12 mW and corresponding Rabi strength in simulation was considered $\Omega_{c1} = \Omega_{c2} = 2\pi \times 8$ MHz. The other parameters used in simulations are $\Omega_p = 2\pi \times 0.01$ MHz and $2\pi \times 0.1$ MHz for systems $N_A$ (plot (b)) and $N_B$ (plot (d)) respectively.

an observation consistent with the results obtained with the variation in power of driving beam ‘$C_1’$ for system $N_B$ discussed before. For the system $N_B$, the driving beam ‘$C_2’$ acts as a control field, therefore, the decrease in the strength of only one EIT peak with the decrease in $P_{c2}$ in figure 5(c) is an expected result as obtained with variation in ‘$C_1’$ beam power in figure 4(a). The corresponding calculated spectra for both the systems $N_A$ and $N_B$ are shown in figure 5(b) and (d) respectively, which have shown an agreement with the experimental observations.

From the above observations, it is evident that among two EIT peaks of dual EIT observed in a N-system, one peak is due to presence of control beam while another peak is due to coupling beam required for $\Lambda$-system.

The dependence of probe transmission spectrum on detuning of both the driving beams for both the atomic systems $N_A$ and $N_B$ has also been studied. For this study, the driving beams frequency detunings were kept equal ($\Delta_{c1} = \Delta_{c2}$), but varied from $-2\pi \times 9$ MHz to $2\pi \times 10$ MHz. The power of both the beams during this measurement was 12 mW. The corresponding experimental and numerical results are shown in figure 6. It has already been observed that when the drive beams are resonant ($\Delta_{c1} = \Delta_{c2} = 0$) with equal powers, three transmission

originated due to presence of control beam, whereas the unaffected peak is mainly due to ‘$C_2’$ driving beam (i.e. coupling beam for $\Lambda_A$ system). The corresponding numerical results have also been obtained by solving equation 2 using equation 3. We note here from figure 4(b) that the numerical simulations results are in good agreement with the experimental observations in figure 4(a). For system $N_B$, the beam ‘$C_1’$ acts as a coupling field. The decrease in $P_{c1}$ for this system results in reduction in strength of both the split EIT peaks, as shown in figure 4(c). This shows that both the EIT peaks are dependent on the coupling field of N-system. The numerical results for this N-system are shown in figure 4(d) which have shown agreement with these experimental observations.

The effect of variation in the power of driving beam ‘$C_2’$ was also studied for a fixed power of driving beam ‘$C_1’’ and the corresponding results are shown in figure 5(a) and (c). The power $P_{c2}$ was varied from 12 mW to 6 mW, while keeping $P_{c1}$ fixed at 12 mW. Since the driving beam ‘$C_2’$ is the coupling field for atomic system $\Lambda_A$ and $N_A$, the reduction in strength of both the EIT peaks is
dips appeared in the spectrum due to emergence of two EIT peaks, shown previously in figure 4 curve (i). For near resonant case (curve (iii) in figure 7 (a) and (b)), the spectral features recovered to three transmission dips (dual EIT). For system $N_B$, the drive beam ‘$C_2$’ acted as a control field and its effect on probe transmission is shown in figure 7 (c) and (d). The far detuned control field has resulted in a single EIT peak at resonance (shown by curve (i), (ii) and (iv)). This signifies that far detuned control field has negligible effect on the spectral feature of corresponding $\Lambda$-system (i.e. $\Lambda_B$). In summary, the above studies suggest that the detuning of drive beams controls the spectral features of the probe transmission considerably. The single EIT peak can be made to dual EIT peak, and vice-versa, by adjusting the detuning of the control fields.

V. CONCLUSION

The two dual EIT peaks have been observed in $D_2$ line transition of $^{87}\text{Rb}$ atoms trapped in a magneto-optical trap. These dual EIT peaks are due to the formation of two simultaneous $N$-type systems, in which each $N$-system contributes to a dual EIT peak. These two $N$-systems in $D_2$ line transition of $^{87}\text{Rb}$ atom have been investigated in detail for dependence of probe transmission spectrum on intensity and detuning of driving beams (i.e. coupling and control beams). The results have shown that one peak of dual EIT observed in each $N$-system is due to presence of both the coupling and control beams while another peak of dual EIT is due to only coupling beam for $\Lambda$-system. Using simultaneously far detuned coupling and control beams, dual EIT feature gets destroyed and two transmission dips with unequal magnitudes are observed for each $N$-system. The spectral positions of these transmission dips in probe spectrum depend on the sign and magnitude of the detuning values.

The effect of variation in detuning of one of the drive beams (i.e. ‘$C_2$’) on the probe transmission spectrum has also been studied while keeping the detuning of other drive beam ‘$C_1$’ fixed at zero. The results of this study are presented in figure 7 (a) and (c). As observed in figure 7 (a) curves (i) and (ii), when the coupling field is red detuned, the two transmission dips appear at far red detuned positions and one transmission dip appears at resonance. The numerical results have also shown similar features in probe transmission spectrum which are shown in figure 7 (b) curve (i) and (ii). For blue detuned coupling field ‘$C_2$’, similar spectral features appear in probe transmission spectrum with positions of dips swapped towards blue side (curve (iv) in figure 7 (a) and (b)). For near resonant case (curve (iii) in figure 7 (a) and (b)), the spectral features recovered to three transmission dips (dual EIT). For system $N_B$, the drive beam ‘$C_2$’ acted as a control field and its effect on probe transmission is shown in figure 7 (c) and (d). The far detuned control field has resulted in a single EIT peak at resonance (shown by curve (i), (ii) and (iv)). This signifies that far detuned control field has negligible effect on the spectral feature of corresponding $\Lambda$-system (i.e. $\Lambda_B$). In summary, the above studies suggest that the detuning of drive beams controls the spectral features of the probe transmission considerably. The single EIT peak can be made to dual EIT peak, and vice-versa, by adjusting the detuning of the control fields.

VI. ACKNOWLEDGMENTS

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