ν − K⁰ Analogy, Dirac-Majorana Neutrino Duality
and the Neutrino Oscillations

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Abstract

The intent of this paper is to convey a new primary physical idea of a Dirac-Majorana neutrino duality in relation to the topical problem of neutrino oscillations. In view of the new atmospheric, solar and the LSND neutrino oscillation data, the Pontecorvo ν − K⁰ oscillation analogy is generalized to the notion of neutrino duality with substantially different physical meaning ascribed to the long-baseline and the short-baseline neutrino oscillations. At the level of CP-invariance, the suggestion of dual neutrino properties defines the symmetric two-mixing-angle form of the widely discussed four-neutrino (2 + 2)-mixing scheme, as a result of the lepton charge conservation selection rule and a minimum of two Dirac neutrino fields. With neutrino duality, the two-doublet structure of the Majorana neutrino mass spectrum is a vestige of the two-Dirac-neutrino origin. The fine neutrino mass doublet structure is natural because it is produced by a lepton charge symmetry violating perturbation on a zero-approximation system of two twofold mass-degenerate Dirac neutrino-antineutrino pairs. A set of inferences related to the neutrino oscillation phenomenology in vacuum is considered.

1 Introduction

In view of the neutrino oscillation data, a simple (2 + 2)-form of the four-neutrino mixing phenomenology [1] is considered in [2] which is restricted to CP-conservation and based on an, initiated by Pontecorvo [3], extended neutrino analogy with the duality behavior of the hadronic system of the neutral kaons [4]. Unlike the kaons, the neutrinos are elementary particles, leptons, and this lepton-hadron oscillation analogy prompted the suggestion of a new physical notion, the Dirac-Majorana neutrino duality. As it is known, a carrying lepton charge four-component Dirac field can be represented as a maximal mixing superposition of two two-component Majorana fields with equal masses and opposite CP-parities, e.g. [5]. “Duality” means that massive neutrinos behave either as Dirac particles, or antiparticles, or as pairs of Majorana ones depending on the specific physical
phenomena in which they are observed, and therefore the Majorana neutrinos come only in pairs with opposite CP-parities.

Beyond the formal differences, the suggested Dirac-Majorana duality properties of the neutrinos and the known duality properties of the neutral kaons have certain similar roots in particle physics. A massive charged field, ($K^0$-meson with strangeness, or Dirac neutrino fields with lepton charge) is a maximal mixing superposition of two “neutral” fields with equal masses, the unification is protected by symmetry. The “long-baseline” particle oscillations (oscillations of strangeness, or lepton charge) afford a universal mechanism for the transformation of very small perturbation effects of a real short-range charge symmetry violating interaction into certain large observable physical effects. It is because the charge symmetry violation renders the mentioned above charged field superposition phase-unlocked what leads via the long-baseline oscillations to large observable effects at large distances $L$ from the particle production vertex with a relativistic dimensional relation $L \sim E/\Delta m^2$ where $\Delta m^2$ is the mass-squared difference of the emerging pair of real neutral fields ($K^0_1$, $K^0_2$-mesons, or a pair of Majorana neutrino fields) and $E$ is their energy. Hence the origin of the suggested Dirac-Majorana neutrino duality can be analogous to the known origin of the neutral kaon duality.

Just because particle-antiparticle oscillation phenomena are well known in physics, they cannot be easily dismissed in the neutrino oscillation phenomenology. With such oscillations, the necessary condition that the charge of a particle maintains its explicit physical meaning reads $\Delta m^2 \ll m^2$, i.e. the charge violating interaction must be a small perturbation effect (particle oscillations with $\Delta m^2 \gtrsim m^2$, if possible, should be charge conserving “short-baseline” oscillations, what is a different kind of particle oscillations). The long-baseline oscillations generate maximal charge violating effects no matter how weak are the new charge violating interactions. If the latter vanish, the long-baseline oscillation length would just grow infinitely, with the relevant particles being definitely Dirac fermions. Otherwise, they have dual properties. The neutrinos can be described as Dirac neutrino or antineutrino states in the semiweak interactions while being described as pairs of Majorana neutrino states in the long-baseline neutrino oscillations. No consistent theory of such a fundamental Dirac-Majorana neutrino duality is known as yet, also [5]. But the neutrino oscillation data seem to point out new physics, and this notion is not excluded as yet. The notion of Dirac-Majorana neutrino duality at the approximation level of CP-invariance leads below to a physically motivated minimal neutrino mixing ansatz (the simplest symmetric form of the so-called (2 + 2)-scheme) with not more than two mixing angles, it conforms naturally to a number of well known neutrino oscillation data. Indications that may be in apparent disagreement with the minimal model of Dirac-Majorana neutrino duality, Sec. 2, come from the recent Super-Kamiokande solar neutrino oscillation data, Sec. 3.
2 Dirac-Majorana neutrino duality ansatz

The three weak interaction neutrino eigenstates \( \nu_e, \nu_\mu, \nu_\tau \) in the known charged and neutral weak interaction lepton currents of the SM, plus one sterile neutrino \( \nu_s \), and also the weak interaction eigenstates of the corresponding antineutrinos \( \bar{\nu}_e \equiv (\nu_e)^c, \bar{\nu}_\mu \equiv (\nu_\mu)^c, \bar{\nu}_\tau \equiv (\nu_\tau)^c, \) and \( \bar{\nu}_s \equiv (\nu_s)^c \), are represented by the following superposition ansatz \( \mathbb{1} \):

\[
\begin{align*}
\nu_e &= [\nu_1^D \cos \vartheta + \nu_1^D \sin \vartheta]_L, & \bar{\nu}_e &= [(\nu_1^D)^c \cos \vartheta + (\nu_1^D)^c \sin \vartheta]_R, \\
\nu_\mu &= [-\nu_1^D \sin \vartheta + \nu_2^D \cos \vartheta]_L, & \bar{\nu}_\mu &= [-(\nu_1^D)^c \sin \vartheta + (\nu_2^D)^c \cos \vartheta]_R, \\
\nu_\tau &= [(\nu_1^D)^c \sin \varphi + (\nu_2^D)^c \cos \varphi]_L, & \bar{\nu}_\tau &= [\nu_1^D \sin \varphi + \nu_2^D \cos \varphi]_R, \\
\nu_s &= [(\nu_1^D)^c \cos \varphi - (\nu_2^D)^c \sin \varphi]_L, & \bar{\nu}_s &= [\nu_1^D \cos \varphi - \nu_2^D \sin \varphi]_R.
\end{align*}
\]

Here, \( \nu_1^D \) and \( \nu_2^D \) are two carrying lepton charge four-component Dirac neutrino fields with masses \( m_1^D \) and \( m_2^D \) and with the mass Lagrangian term

\[
\Delta L^D = -\frac{1}{2} \sum_i m_i^D \left[ \nu_i^D \nu_i^D + (\nu_i^D)^c (\nu_i^D)^c \right], \quad i = I, II.
\]

The symbols \((\nu_i^D)^c\) denote the Dirac antineutrino fields and the superscript \(c\) means charge conjugation. The L and R subscripts denote left and right chirality states of the neutrinos in the weak interaction currents, which are defined by the projection operators \((1 - \gamma_5)/2\) and \((1 + \gamma_5)/2\), respectively.

With the ansatz in Eqs. \( \mathbb{1} \)–\( \mathbb{4} \), the lepton charge is conserved in the semiweak interactions of the SM with the definition of the seven “leptons” \((l_k = +1, k = 1 \div 7)\):

\[
\nu_e, \nu_\mu, \bar{\nu}_\tau, \bar{\nu}_s, e^-, \mu^-, \tau^+,
\]

and the seven “antileptons” \((l_k = -1, k = 1 \div 7)\):

\[
\bar{\nu}_e, \bar{\nu}_\mu, \nu_\tau, \nu_s, e^+, \mu^+, \tau^-.
\]

The selection rule of lepton charge conservation in the semiweak interactions with the lepton charge definition in Eq. \( \mathbb{1} \) leads to the important no-mixing of the Dirac neutrino mass eigenstates \( \nu_i^D \) with the Dirac antineutrino mass eigenstates \((\nu_i^D)^c\) in the ansatz \( \mathbb{1} \)–\( \mathbb{4} \) and therefore to the physically motivated restriction to only two mixing angles \( \vartheta \) and \( \varphi \).

In contrast to the conservation of the separate electron, muon and tauon numbers, the lepton charge conservation with the definition \( \mathbb{3} \) is not violated by the Dirac neutrino masses, Eq. \( \mathbb{6} \), in the electroweak perturbation theory.

The definition of the lepton charge in Eq. \( \mathbb{6} \) is rather uncommon. Usually all three lepton flavor pairs \((\nu_e, e), (\nu_\mu, \mu)\) and \((\nu_\tau, \tau)\) are implied to have the same lepton charge, e.g. \( l = +1 \). The suggestion that the three neutrinos are represented (beyond the SM) by three independent four-component Dirac fields would lead then to three sterile neutrinos as the right components of the Dirac neutrino fields with zero weak isospin and hypercharge.
With the definition of the lepton charge (6) and the ansatz (1)–(4) and (5), we introduced not three but only two four-component Dirac neutrinos and so there is here only one sterile neutrino. The new \( SU_L(2) \times U(1) \) electroweak symmetry representation and lepton charge content of the neutrinos in the ansatz (1)–(4) get more transparent in the limit of equal mixing angles \( \varphi = -\vartheta \). In this case we can define two four-component Dirac neutrinos \( \nu_e \) and \( \nu_\mu \) in a one-parameter economical version of the ansatz (1)–(4),

\[
\begin{align*}
\nu_e &= \nu_L^e \left( \bar{\nu}_e = \bar{\nu}_R^e \right), \quad \bar{\nu}_e = \nu_R^e \left( \nu_s = \bar{\nu}_L^e \right), \\
\nu_\mu &= \nu_L^\mu \left( \bar{\nu}_\mu = \bar{\nu}_R^\mu \right), \quad \bar{\nu}_\mu = \nu_R^\mu \left( \nu_\tau = \bar{\nu}_L^\mu \right), \\
\nu^e &= \nu_L^D \cos \vartheta + \nu_R^D \sin \vartheta, \quad \nu^\mu = -\nu_L^D \sin \vartheta + \nu_R^D \cos \vartheta.
\end{align*}
\]

(7)

Both the left and right components of the four-component Dirac “muon” neutrino \( \nu^\mu \) are active in the weak interactions and represent the up- and down-components of the \( SU_L(2) \)-doublets \( (\nu^\mu, \mu^-)_L \) and \( (\tau^+, \nu^\mu)_R \), respectively. On the other hand, only the left component of the four-component Dirac “electron” neutrino \( \nu^e \) in the electroweak doublet \( (\nu^e, e^-)_L \) is active in the weak interactions, whereas its right component \( \nu_R^e \) remains a sterile neutrino in the SM. In contrast to the case of massless neutrinos in the SM, the Dirac mass Lagrangian term (5) generates transitions \( \nu_L^\mu \leftrightarrow \nu_R^\mu \) and \( \nu_L^e \leftrightarrow \nu_R^e \), i.e. it makes possible the transitions of the muon neutrino into the tau antineutrino, \( \nu_\mu \leftrightarrow \bar{\nu}_\tau \), and also transitions of the electron neutrino into sterile antineutrino, which will be realized by the involvement of the Dirac masses in the weak interactions. In the more general mixing with \( \vartheta \not= -\varphi \), as in Eqs. (1)–(4), the Dirac neutrino masses generate all four possible types of the lepton charge conserving transitions in the weak interactions \( (\nu^e, \nu^\mu) \leftrightarrow (\bar{\nu}_\tau, \bar{\nu}_s) \). With the definition in Eq. (6), interesting though extremely suppressed (factor \( m^2_\nu/E^2_\nu \)) lepton charge conserving reactions, e.g.

\[
\nu_\mu + (A, Z) \rightarrow (A, Z - 1) + \tau^+,
\]

are possible through the involvement of the Dirac neutrino masses in the lepton weak interactions. It is appropriate to note here that massive neutrinos are produced in the weak interactions with not complete longitudinal polarization. In our neutrino mixing pattern (1)–(4), it means that the incoming \( \nu_\mu \) in the reaction above is an effective state including the muon neutrino plus a very small admixture of the tau and sterile antineutrinos. Because of the suppression by the very small neutrino masses, the violation of the \( (e - \mu) - \tau \) universality in the definition of the lepton charge in Eq. (6) does not disagree with any known data.

The data of the atmospheric, solar and the LSND experiments can be explained with the following values of the neutrino mass-squared differences (1)

\[
\begin{align*}
\Delta m^2_{11} &= m^2_2 - m^2_1 = \Delta m^2_{\text{atm}} \sim 10^{-3} \div 10^{-2} \text{ eV}^2, \\
\Delta m^2_{21} &= m^2_2 - m^2_1 = \Delta m^2_{\text{atm}} \sim 10^{-3} \div 10^{-2} \text{ eV}^2, \\
\Delta m^2_{12} &\sim m^2_2 - m^2_1 \sim 1 \text{ eV}^2.
\end{align*}
\]

(8)
with a possible scheme of the neutrino mass spectrum

\[ m_1 < m'_1 \ll m_2 < m'_2. \] (9A)

and another possible scheme (9B) where the positions of the “solar” and the “atm” doublet splittings of Eq. (9A) are interchanged.

To describe the neutrino mass data of Eq. (8) in accordance with the single phenomenological approach of the \( \nu - K^0 \) analogy, we should introduce an effective mass Lagrangian term \( \Delta L^M \), called Majorana term, with maximal lepton charge violation \( \Delta l = \pm 2 \),

\[ \Delta L^M = \frac{1}{2} \sum_i m_i^M \left[ \bar{\nu}_i^D (\nu_i^D)^c + (\nu_i^D) c \nu_i^D \right], \quad 0 < m_i^M \ll m_i^D, \quad i = I, II. \] (10)

It has the physical meaning of a very small perturbation of the two-fold mass-degenerate Dirac neutrino-antineutrino zero order approximation system, Eq. (5). This type of exact neutrino mass-degeneracy is certainly a natural zero-order approximation because it is protected by the CPT-invariance. To consider the lepton charge violating interaction as a small perturbation effect is necessary for modelling the Dirac-Majorana neutrino duality, see Sec. 1.

The simplest admissible texture of the neutrino interactions, used here, reads: the sole location of lepton charge violation in the lepton interactions is in the left-right symmetric effective Majorana mass term (10) with physical meaning of a small perturbation, whereas the sole location of the left-right asymmetry is in the lepton charge conserving weak interactions. The left-right symmetry of the Majorana term (10) leads below to the maximal mixing of the Majorana neutrinos. The perturbative status of the Majorana term (10) leads to the fine doublet structure of the neutrino mass spectrum.

It should be noted that the Dirac and Majorana neutrino mass Lagrangian terms (5) and (10) are not compatible with the minimal Higgs mechanism of the SM electroweak theory. They are introduced here as a continuation of the neutrino mixing ansatz (1)–(4) in an attempt to scheme the new physics beyond the SM, which could be the source of the suggested dual Dirac-Majorana neutrino properties; the Majorana mass term (10) being analogous to the \( (K_1^0, K_2^0) \) mass Lagrangian term which schemes the relevant perturbative \( K_0 \leftrightarrow \bar{K}_0 \) effect of the weak interactions. As follows, the neutrino mixing ansatz in (1)–(4), (5) and (10) leads uniquely to the symmetric form [2] of the widely discussed, e.g. [6], four-neutrino \((2 + 2)\)-mixing scheme.

The total neutrino mass Lagrangian term can be rewritten in another form

\[ \Delta L' = \Delta L^D + \Delta L^M = -\frac{1}{2} \sum_k (m_k \bar{\nu}_k \nu_k + m'_k \bar{\nu}'_k \nu'_k), \quad k = 1, 2, \] (11)

\[ m_k = (m_k^D - m_k^M), \quad m'_k = (m_k^D + m_k^M), \quad m'_k - m_k \ll m_k. \]

Here, \( (\nu_k, \nu'_k), \quad k = 1, 2 \), are two pairs of truly neutral Majorana neutrino mass eigenstates (with CP-invariance they are stationary states). The Majorana neutrino states are related
to the primary neutrino and antineutrino Dirac mass eigenstates $\nu_D^I$, $\nu_D^{II}$ and $(\nu_D^I)^c$, $(\nu_D^{II})^c$:

$$\nu_k = \frac{1}{\sqrt{2}} [\nu_k^D + (\nu_k^D)^c], \quad \nu'_k = \frac{1}{\sqrt{2}} [\nu_k^D - (\nu_k^D)^c]. \quad (12)$$

The CP-parities of these Majorana neutrinos are opposite in each of the two doublets, see also [7].

After the small perturbation by the Majorana mass term (10) is switched on, the Majorana neutrino states $\nu_k$ and $\nu'_k$ are the new stationary states, while $\nu_k^D$ and $(\nu_k^D)^c$ in Eq. (12) are the new nonstationary neutrino states, which are not Dirac states with lepton charge any more, comp. Sec. 1. It is because of phase-unlocking effect due to the different masses of the Majorana neutrinos $\nu_k$ and $\nu'_k$ in the superpositions

$$\nu^s_k = \frac{1}{\sqrt{2}}(\nu_k + \nu'_k), \quad \nu^a_k = \frac{1}{\sqrt{2}}(\nu_k - \nu'_k), \quad k = 1, 2. \quad (12')$$

The superscripts $s$ and $a$ here denote symmetric and antisymmetric states under the interchange of the Majorana neutrinos $\nu_k$ and $\nu'_k$ [2]. At distances $L$ from the neutrino production vertex in the semiweak interactions, which are much shorter than the long-baseline oscillation length, $\nu^s_k$ and $\nu^a_k$ remain nearly Dirac neutrino and antineutrino states. It is mainly because of two factors: one is the maximal mixing of the Majorana neutrinos $\nu_k$ and $\nu'_k$ in (12') as a vestige of their primary Dirac neutrino origin in the semiweak interactions, and the other is negligible physical effects of lepton charge violation owing to the small differences of the Majorana neutrino phases in the neutrino mass doublets at the stated distances $L$. The Majorana neutrino states in Eq. (12) correspond to the truly neutral meson states $K^0_1$ and $K^0_2$ in the used here analogy, while the Dirac neutrinos $\nu^D_i$ and antineutrinos $(\nu^D_i)^c$ correspond to the primary neutral kaons which are produced in the strong interactions with definite strangeness as $K^0$ or $\bar{K}^0$ mesons and become the symmetric and antisymmetric superpositions, i.e. $K^0 = (K^0_1 + K^0_2)/\sqrt{2}$ and $\bar{K}^0 = (K^0_1 - K^0_2)/\sqrt{2}$, at finite distances $L$ from the production vertex, comp. Eq. (12').

Unlike the formal description in the review [1], where the charge parities of the Majorana neutrinos are always chosen to be $C = +1$, the choice here of opposite charge parities $C = +1$ and $C = -1$ for the Majorana neutrinos $\nu_k$ and $\nu'_k$ respectively proved to be more convenient for the purpose of modelling the Dirac-Majorana neutrino duality.

Lepton charge violating reactions, e.g. neutrinoless double $\beta$-decay, are possible because of the Majorana neutrino mass doublet splittings, they are naturally suppressed by factors much smaller than in the mentioned above possible lepton charge conserving $\nu_\mu \leftrightarrow \bar{\nu}_\tau$ transformations because of the relation $\Delta m^2 \ll m_1^2$.

As a rule, the doublet structure of a quantum system reveals an underlying broken symmetry. There is no exception here: the doublet structure of the physical Majorana neutrino mass spectrum reveals the broken, by the perturbative effective Majorana term $\Delta L^M$, lepton charge symmetry of the semiweak interactions. The stabilizing condition for the fine two-doublet structure of the Majorana neutrino mass spectrum here is the protected by CPT-invariance mass-degeneracy of the zero-order approximation Dirac
neutrino-antineutrino system. This fine structure of the Majorana neutrino mass spectrum is natural: if $\Delta m^2_1$ and $\Delta m^2_2$ were zero, the lepton charge symmetry would be restored.

The formal definition of the nearly Dirac neutrino $\nu^D_1$ and antineutrino $\bar{\nu}^D_1$ fields is close to the definition of the “pseudo-Dirac neutrinos” in Ref. [8]. The pseudo-Dirac neutrino field approximation is analogous to the well known effective approximation of carrying strangeness $K^0$ and $\bar{K}^0$ meson states of the strong interactions so far as we count the weak interactions as very small perturbations, $m(K^0_2) - m(K^0_1) \ll m(K^0)$. Just as the $K^0$ and the $\bar{K}^0$ meson states are produced in the strong interactions with definite strangeness (charge) whereas the dual truly neutral meson states $K^0_1$ and $K^0_2$ show up in the $K^0$-oscillations, the $\nu^D_1$ and $(\nu^D_1)^c$ neutrino states are produced in the standard weak interactions with definite lepton charge as Dirac neutrinos whereas the dual truly neutral Majorana neutrino states $\nu_k$ and $\nu'_k$ show up in the long-baseline neutrino oscillations. The significant difference is that with three lepton flavors we need two massive Dirac neutrino fields $\nu^D_1$ and $\nu^D_2$ (and Dirac antineutrino fields $(\nu^D_1)^c$ and $(\nu^D_2)^c$) and unlike the $K^0$-case there is a possibility of their mixing. Neutrino mixing in Eqs. (1)–(4), with different mixing angles $\theta$ and $\varphi$ for the two chiral neutrino states $\nu^D_{IL} [(\nu^D_{cL}^D)^c]$, $i = I, II$, and the two chiral antineutrino states $(\nu^D_{cL})^c$, $i = I, II$, respectively, is considered below. The condition of different mixings for the left and right components of the two four-component neutrinos $\nu^D_1$ and $\nu^D_2$ is here a part of the left-right asymmetry of the lepton weak interactions. In the one-parameter version (5) of the present model with the relation $\varphi = -\theta$ this additional left-right asymmetry disappears.

The sterile neutrino occurrence is here a necessary result of two conditions: 1) there is the lepton charge conservation in the semiweak interactions with three lepton flavors and 2) the neutrinos represented by the minimum of primary two massive four-component Dirac fields. Because of these conditions and the maximal parity nonconservation in the neutrino weak interactions, there must be one two-component nonactive neutrino degree of freedom, which must be imparted as a sterile neutrino in both the short-baseline and the long-baseline neutrino oscillations, see equations below.

The long-baseline neutrino oscillations, including the atmospheric and solar ones, are effective four-component Dirac neutrino-antineutrino lepton charge oscillations (i.e. left Dirac neutrino $\leftrightarrow$ left Dirac antineutrino, or right Dirac neutrino $\leftrightarrow$ right Dirac antineutrino, e.g. (7)), as a result of neutrino duality; beyond the formal differences, these lepton charge oscillations have the same physical meaning as the oscillations of strangeness in the $K^0$-meson oscillations: $K^0 \leftrightarrow (K^0_1 + K^0_2)/\sqrt{2}, \Delta m \neq 0 \leftrightarrow <\text{oscillations}, \Delta t \neq 0 > \leftrightarrow (K^0_1 - K^0_2)/\sqrt{2} \leftrightarrow \bar{K}^0$, as a result of neutral kaon duality. On the other hand, the short-baseline neutrino oscillations are mainly lepton charge conserving effective two-Dirac-neutrino oscillations resulting in the transformations $\nu_\mu \leftrightarrow \nu_e$ with the oscillation amplitude $\sin^2 2\theta$, and $\nu_\tau \leftrightarrow \nu_s$ with the oscillation amplitude $\sin^2 2\varphi$. These short-baseline oscillations have no $K^0$-analogy because of the absence of a relevant second $K$-generation, they would remain mainly unchanged even if the neutrino mass matrix were of the regular Dirac type with no neutrino mass doublet splittings. The main inference here is that only the two mentioned types of the short-baseline neutrino oscillations are possible: because of lepton charge conservation in the oscillations of this type, there are
no short-baseline $\nu_\mu, \nu_e \leftrightarrow \nu_\tau, \nu_s$, oscillation transformations. This inference is supported by the positive LSND indications [9] and also by the negative data of the CHORUS and NOMAD experiments [10].

3 Fitting to the neutrino oscillation data

The equations for the neutrino oscillation probabilities below resulted from Eqs. (1), (2), (3) and (4) after revealing there (and also in Eq. (7)) the Dirac-Majorana neutrino duality condition:

$$\nu^D_i \rightarrow \nu^s_i = \frac{1}{\sqrt{2}} (\nu_i + \nu'_i), \quad (\nu^D_i)^c \rightarrow \nu^p_i = \frac{1}{\sqrt{2}} (\nu_i - \nu'_i),$$

in accordance with Eq. (12). The probabilities are written in terms of the neutrino mass scheme (9A), but the conclusions do not depend on this choice.

The probability of the $\nu_\mu \leftrightarrow \nu_e$ oscillations reads

$$W(\nu_\mu \leftrightarrow \nu_e) = \sin^2 2\vartheta \left[ < \sin^2(\delta m^2_{12}) > - \frac{1}{4} \sin^2(\delta m^2_1) - \frac{1}{4} \sin^2(\delta m^2_2) \right],$$

where the symbol $< >$ in the first term denotes the arithmetic mean value of the appropriate four terms related to the four “large” mass-squared differences among the two different neutrino mass doublets. The notations in Eq. (14) and below are

$$\delta m^2_{ij} \equiv \frac{\Delta m^2_{ij} L}{4E}, \quad \Delta m^2_{ij} \equiv m^2_j - m^2_i, \quad (\delta m^2_1) \equiv (\delta m^2_{11'}), \quad (\delta m^2_2) \equiv (\delta m^2_{22'}),$$

with $i, j = 1, 1', 2, 2'$. $E$ is the neutrino energy and $L$ is the distance from the neutrino source to the detector. The probabilities of the $\nu_\mu \leftrightarrow \nu_e$ transformations are determined by one factor $\sin^2 2\vartheta$, which is the effective LSND “two-neutrino” oscillation amplitude with an estimation [4]

$$\sin^2 2\vartheta \approx 2 \times 10^{-3} \div 4 \times 10^{-2}. \quad (16)$$

The second and third terms in Eq. (14) give small contributions to the total $\nu_\mu$- and $\nu_e$-disappearance atmospheric and solar oscillation probabilities (comp. Eqs. (20) and (23)) from the channels $\nu_\mu \rightarrow \nu_e$ and $\nu_e \rightarrow \nu_\mu$, respectively. These negative sign terms in the probability $W(\nu_\mu \leftrightarrow \nu_e)$ indicate subtraction of the long-baseline lepton charge disappearance probabilities, what is a necessary result of the lepton charge conservation in the short-baseline oscillations in the present model.

The probability of the short-baseline $\nu_\tau \leftrightarrow \nu_s$ oscillation transformations follows from Eq. (14) after the substitution $\vartheta \rightarrow \varphi$,

$$W(\nu_\tau \leftrightarrow \nu_s) = \frac{\sin^2 2\varphi}{\sin^2 2\vartheta} W(\nu_\mu \leftrightarrow \nu_e). \quad (17)$$
A direct estimation of the mixing angle $\varphi$ in Eq. (17) could be by the measurement of the $\nu_\tau$-survival probability in the short-baseline tau-neutrino oscillation experiment. It is not available at present, but the recent detection of the tau-neutrino at Fermilab [11] should eventually make possible such an experiment.

The probability of the $\nu_\mu \leftrightarrow \nu_\tau$ oscillations reads

$$W(\nu_\mu \leftrightarrow \nu_\tau) = \cos^2 \vartheta \cos^2 \varphi \sin^2(\delta m^2_{12}) + \sin^2 \vartheta \sin^2 \varphi \sin^2(\delta m^2_1) \quad (18)$$

\[+ \frac{1}{4} \sin 2 \vartheta \sin 2 \varphi \left[ \sin^2(\delta m^2_{12}) + \sin^2(\delta m^2_{12}) - \sin^2(\delta m^2_{12}) - \sin^2(\delta m^2_{12}) \right].\]

The probability of the $\nu_\mu \leftrightarrow \nu_s$ oscillations can be obtained from Eq. (18) by the substitution $\varphi \rightarrow \varphi + \pi/2$, and so the probability of the $\nu_\mu \rightarrow \nu_\tau + \nu_s$ transformations,

$$W(\nu_\mu \rightarrow \nu_\tau + \nu_s) = \cos^2 \vartheta \sin^2(\delta m^2_2) + \sin^2 \vartheta \sin^2(\delta m^2_1), \quad (19)$$

is independent of the second mixing angle $\varphi$ and of the neutrino mass doublet separation $\Delta m_{12}$. Eq. (19) describes the probability of the long-baseline $\nu_\mu$ oscillation transformations. With the contribution from Eq. (14), the total probability of the $\nu_\mu$ oscillation transformations reads

$$W(\nu_\mu \rightarrow \nu_\tau + \nu_s + \nu_e) = \cos^4 \vartheta \sin^2(\delta m^2_2) + \sin^4 \vartheta \sin^2(\delta m^2_1) + \sin^4 \vartheta < \sin^2(\delta m^2_1) \geq 1. \quad (20)$$

The first term in Eq. (20) is the dominant one, it gives the main part of the muon neutrino disappearance probability in the atmospheric long-baseline $\nu_\mu$-neutrino oscillations with the large amplitude

$$A_{\text{atm}} \cong \cos^4 \vartheta \cong 1. \quad (21)$$

It comes mainly from the transformations $\nu_\mu \rightarrow \nu_\tau$ and $\nu_\mu \rightarrow \nu_s$, their partial contributions carry the factors $\cos^2 \varphi$ and $\sin^2 \varphi$, respectively. At $\varphi = 45^\circ$ the ratio of these contributions $\cot^2 \varphi$ is near to one. The contribution from the transformations $\nu_\mu \rightarrow \nu_e$ in Eq. (20) is a small short-baseline effect. The recent data indications in favor of the $\nu_\mu \rightarrow \nu_\tau$ transitions in the atmospheric $\nu_\mu$-oscillations [12] seem to exclude a dominant contribution from the $\nu_\mu \rightarrow \nu_s$ mode in these oscillations.

The probability of the $\nu_\mu$-survival oscillations and the $\nu_\mu \rightarrow \nu_\tau$ appearance transformations in the coming long-baseline accelerator $\nu_\mu$-oscillation experiments should be

$$W(\nu_\mu \rightarrow \nu_\mu) \cong \cos^2 \frac{\Delta m^2_2 L}{4E}, \quad W(\nu_\mu \rightarrow \nu_\tau) \cong \cos^2 \varphi \sin^2 \frac{\Delta m^2_2 L}{4E}. \quad (22)$$

where $L$ is the distance from the neutrino source to the detector. The results of these experiments should lead to a clear estimation of the mixing angle $\varphi$ through the term $\cos^2 \varphi$ in Eq. (22).

The probability of the $\nu_\tau$ oscillation transformations $\nu_\tau \rightarrow \nu_\mu + \nu_e + \nu_s$ follows from Eq. (20) after the substitution $\vartheta \rightarrow \varphi$.  

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The probability of the $\nu_e \to \nu_s$ and $\nu_e \to \nu_s + \nu_\tau$ long-baseline oscillation transformations can be obtained from Eqs. (18) and (19), respectively, after the substitution $\Delta m_2^2 \leftrightarrow \Delta m_1^2$. The total probability of the $\nu_e$ oscillation transformations reads

$$W(\nu_e \to \nu_s + \nu_\tau + \nu_\mu) = \cos^4 \vartheta \sin^2(\delta m_1^2) + \sin^4 \vartheta \sin^2(\delta m_2^2) + \sin^2 2\vartheta < \sin^2(\delta m_{12}^2) > .$$

(23)

The first term in this equation is the dominant one with nearly maximal amplitude. It comes from the transformations $\nu_e \to \nu_s$ and $\nu_e \to \nu_\tau$, their partial contributions are with factors $\cos^2 \varphi$ and $\sin^2 \varphi$, respectively. Again, the contribution from the transformations $\nu_e \to \nu_\mu$ is a small short-base-line effect.

The phenomenology above leads to a complementary relation between the possible dominant modes in the atmospheric and the solar oscillations: if the tau neutrino dominates the atmospheric $\nu_\mu$-oscillations, then the sterile neutrino should dominate the solar $\nu_e$-oscillations, and conversely.

If the second mixing angle in the ansatz (1)–(4) is very small, $\varphi \ll 45^\circ$, as in the economical one-parameter version (3), the tau neutrino will strongly dominate the atmospheric $\nu_\mu$-oscillations, and the sterile neutrino will strongly dominate the solar $\nu_e$-oscillations. This inference is in good agreement with the recent atmospheric Super-Kamiokande muon neutrino oscillation data [12], but it does not agree with the solar neutrino data [13, 14], which seem to exclude the $\nu_e \to \nu_s$ oscillation mode as the dominant one in the solar neutrino oscillations.

In the other extreme case with a large mixing angle $\varphi = 45^\circ$, the dominant disappearance neutrino transformations in both the atmospheric and solar neutrino oscillations should be the same, $\nu_\mu \to (\nu_\tau + \nu_s)$ and $\nu_e \to (\nu_\tau + \nu_s)$, respectively, with equal $\nu_\tau$ and $\nu_s$ modes in each case, while the $\nu_\mu \leftrightarrow \nu_e$ modes are strongly suppressed in these long-baseline neutrino oscillations in accordance at least with the most confident atmospheric Super-Kamiokande data [13].

Note, that none of the solar neutrino oscillation data are comparable with the Super-Kamiokande atmospheric neutrino oscillation data [15] by their confidence as yet [13, 14]. With the new atmospheric neutrino indication [12], the mixing angle region $\varphi \lesssim 45^\circ$ is certainly the least disfavored by the recent neutrino oscillation data.

### 4 Conclusion

To conclude, an attempt is made to explain relations between well known different neutrino data by the suggestion of a new unifying physical notion, the Dirac-Majorana neutrino duality. The nearly maximal neutrino transformation $\nu_\mu \to (\nu_\tau + \nu_x)$, $x \neq e$, oscillation amplitude in the atmospheric Super-Kamiokande neutrino experiment [13], together with lepton charge conservation in all the known weak interaction reactions, and the data dictated doublet character of the neutrino mass spectrum, Eq. (8), with the LSND indications accepted, e.g. [1], prompts an extended analogy between the neutrino properties and the duality properties of the neutral kaons. Duality means an indication
of new physics: the neutrinos are carrying lepton charge Dirac fields in the semiweak interactions, and they are pairs of dual truly neutral Majorana fields in the long-baseline neutrino oscillations. The massive neutrinos are produced in the semiweak interactions as Dirac particles with not complete longitudinal polarization, and they remain nearly Dirac neutrinos (see the discussion of Eq. (12)) at finite distances from the production vertex until these distances remain much shorter than the long-baseline oscillation lengths.

The necessary restriction to three active in the weak interactions lepton flavors determines a minimum of two four-component massive Dirac neutrinos, two associated pairs of Majorana neutrinos, and it leads to the involvement of one sterile neutrino which has to be imparted in the neutrino oscillations.

The suggested dual Dirac-Majorana properties of the neutrinos are modelled in a minimal four-neutrino mixing ansatz for the three flavor and one sterile neutrinos (1)–(4) and the Dirac neutrino mass term (3) which describe lepton charge conservation in the semiweak interactions with a special lepton charge pattern (5), plus a Majorana neutrino mass term (4) which is regarded by definition as a small perturbation effect. As a result, we get a simple unifying neutrino oscillation phenomenology with two nearly mass-degenerate Majorana neutrino doublets in Eq. (11), the maximal neutrino mixing condition in Eq. (13) and the physically motivated important restriction to not more than two mixing angles \( \theta \) and \( \varphi \) in the CP-invariant lepton semiweak interactions with a symmetric \((2 + 2)\) neutrino mixing pattern (1)–(4) plus (13). The fine neutrino mass doublet structure is natural here because it is produced by a lepton charge symmetry violating perturbation on a zero-approximation system of two exactly twofold mass-degenerate Dirac neutrino-antineutrino pairs.

The model implies violation of the \((e, \mu) - \tau\) universality and allows a group of suppressed by the small Dirac neutrino masses reactions, which involve basically the helicity-flip lepton charge conserving Dirac neutrino transformations \(\nu_\mu(\nu_e) \leftrightarrow \bar{\nu}_\tau\) (in contrast to the lepton charge violating, \(\Delta l = \pm 2\), helicity conserving Majorana neutrino transformations \(\nu_\mu(\nu_e) \leftrightarrow \nu_\tau\) in the long-baseline neutrino oscillations), which will be testable in the next generations of neutrino reaction experiments. If the effective Dirac mass of the muon neutrino is not in the eV, but in the 100 keV region indeed [16], the production of the antitauons \(\tau^+\) by the high energy muon neutrinos \(\nu_\mu\) on nuclei is more interesting, it is singled out in Sec. 2.

The necessary involvements of the sterile neutrino in the solar and atmospheric neutrino oscillations will be tested in many coming neutrino oscillation experiments as SNO, Super-Kamiokande, MiniBooNE, KARMEN, the accelerator long-baseline K2K and MINOS experiments and other neutrino oscillation experiments [17].

A crucial test of the ansatz (1)–(4) will be the coincidence of the two seemingly independent experimental values of the mixing angle \(\varphi\) from the long-baseline accelerator neutrino oscillation transformations \(\nu_\mu \rightarrow \nu_\tau\), Eq. (22), and the short-baseline \(\nu_\tau\)-survival oscillations, Eq. (17). With Dirac-Majorana neutrino duality, the restriction to not more than two mixing angles is feasible in the phenomenology of neutrino oscillations at the approximation of CP-invariance.

Beyond the formal differences, the suggested duality properties of the neutrinos are
analogous to the well known duality properties of the neutral kaons. This physical idea
was initiated by Pontecorvo and has gained new support \cite{2,4} from the leading neutrino
oscillation data since the main Super-Kamiokande atmospheric neutrino oscillation results
in 1998 \cite{15}, and also the LSND indications \cite{9}. The new versatile developments in
experimental neutrino physics will eventually prove or disprove this physical idea finally,
e.g. \cite{6}.

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