Analytical Study of $^{90}$Sr Betavoltaic Nuclear Battery Performance Based on p-n Junction Silicon

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Abstract. Previously, an analytical calculation of $^{63}$Ni p-n junction betavoltaic battery has been published. As the basic approach, we reproduced the analytical simulation of $^{63}$Ni betavoltaic battery and then compared it to previous results using the same design of the battery. Furthermore, we calculated its maximum power output and radiation-electricity conversion efficiency using semiconductor analysis method. Then, the same method were applied to calculate and analyse the performance of $^{90}$Sr betavoltaic battery. The aim of this project is to compare the analytical performance results of $^{90}$Sr betavoltaic battery to $^{63}$Ni betavoltaic battery and the source activity influences to performance. Since it has a higher power density, $^{90}$Sr betavoltaic battery yields more power than $^{63}$Ni betavoltaic battery but less radiation-electricity conversion efficiency. However, beta particles emitted from $^{90}$Sr source could travel further inside the silicon corresponding to stopping range of beta particles, thus the $^{90}$Sr betavoltaic battery could be designed thicker than $^{63}$Ni betavoltaic battery to achieve higher conversion efficiency.

Keywords: betavoltaic, battery, $^{90}$Sr, $^{63}$Ni, p-n junction, efficiency

1. Introduction

Nuclear battery has been recently investigated to fulfill the next generation energy source. Betavoltaic is one of the non-thermal converter type nuclear battery which uses semiconductor devices to convert radiation decay energy into electricity. Due to the higher energy densities and longer lifetimes of radioisotopes compared with chemical fuels, micro betavoltaic batteries are very attractive for MEMS...
applications, particularly for long-term applications, such as space and undersea explorations, implanted biomedical microdevices, and sensor networks for environmental monitoring [2].

In this paper, a p-n junction silicon semiconductor was used as the radiation-electricity converter. Generation of electron-hole pairs (EHPs) are created by the scattering process of beta particle and atom. When EHP’s diffuse into the depletion region of the p-n junction, an electric field sweeps them across the depletion region, thus generating a radiation-induced current [1]. Selection of radioisotopes is a critical aspect for betavoltaic conversion. $^{63}$Ni is one of the radioactive isotopes which has been widely used for micro-betavoltaic battery experiments in recent years, due to characteristic of the radiation which has low specific activity, low decay energy, long half-life, and pure beta emitter. An analytical simulation of $^{63}$Ni betavoltaic battery has been done by Guoping Zuo et al. (2013) [1] whose also derived the equations for p-n junction betavoltaic. The beta spectrum dependence was also involved in calculation to have a clear performance of the battery. Another pure beta emitting radioisotope is $^{90}$Sr which emits low energy beta with maximum decay energy only 0.546 MeV. $^{90}$Sr is known to be produced in nuclear reactor easily rather than $^{63}$Ni as one of uranium fission product.

Using the same method as the previous analytical simulation of $^{63}$Ni betavoltaic battery, the simulation for $^{90}$Sr betavoltaic battery had been produced. Furthermore, the analytical J-V curve and power density curve had been made to analyse the performance of $^{90}$Sr betavoltaic battery. However, the radiation damage effect to semiconductor from gamma ray produced via Bremsstrahlung’s effect and via decay product of $^{90}$Y was neglected in this study, which probably could give another conclusion to its capability.

2. Beta energy dependence to performance

In contrast of alpha particle, beta particles from a given radioactive nuclide are not emitted in discrete energy group, but with a continuous energy distribution extending from zero to maximum value [3]. Several theoretical study over the last decade only focused on maximum and mean kinetic energy to determine the performance of betavoltaic battery. However, some studies showed that the spectrum of sources affects the deposition distribution of beta particles in absorbers, thus influencing the design and performance of the device [1]. In order to maximize the energy conversion, it is important to approximate the maximum penetration depth, which also affects the design of battery, using Eq. (1) as known as Katz-Penfold semi-empirical formula [3][4].

$$R_{max} = \begin{cases} 0.412 E^{-1.265-0.954 \ln(E)}; & 0.01 \leq E \leq 2.5 \text{ MeV} \\ 0.530 E^{-0.106}; & E > 2.5 \text{ MeV} \end{cases}$$

There are two major deviations from the photovoltaic theory. The first is that each beta particle produces thousands of EHPs as it inelastically scatter through the silicon substrate. This point leads into thesecond major deviation. The range of the beta particles, and therefore the location of their deposited energy is deep within the silicon substrate [5]. Since the average kinetic energy of typical beta particles used for betavoltaic devices is in the Kilo Electron Volt (keV) range, a single beta particle can be responsible for generating multiple EHPs [6]. According to the Klein formula, the average kinetic energy required to generate an electron-hole pair equal to the semiconductor band-gap ($E_g$). In addition, during the conversion process, there are some energy lost by emission of acoustic and optical phonons, which in scale of eV and doesn’t give a significant effects to performance [6].
Figure 1. Beta energy spectrum of $^{63}$Ni and $^{90}$Sr source.

Figure 2. Stopping range of beta particle in silicon ($\Delta =$ beta particle of $^{63}$Ni; $\blacksquare =$ beta particle of $^{90}$Sr).

Figure 1 (a) and (b) shown the beta spectrum of $^{63}$Ni and $^{90}$Sr source, respectively. Both mostly decays low energy beta particles. Although the mean beta particle energy of $^{90}$Sr is equal to 0.196 MeV, far beyond $^{63}$Ni with just 0.017 MeV. Corresponding to maximum penetration depth from Figure 2, beta particle with kinetic energy equal to 0.017 MeV and 0.196 MeV can travel with maximum range inside silicon to 2.1 $\mu$m and 174.7 $\mu$m, respectively.

Beta particles emitted from source can interact with both electrons and nuclei inside the material, causing them to be slowing down, scattered, and lose some sort of their kinetic energy. The analytical expression of energy loss per depth of penetration is determined by the sum of two process of beta interaction-the ionization process and the bremsstrahlung radiation process- described by this following equation [1]:

$$(-dE/dx)_{total} = (-dE/dx)_{ion} + (-dE/dx)_{rad}$$

At relativistic energies, the ratio of energy loss via bremsstrahlung radiation to energy loss via ionization process is relatively small, thus the effect of photon interaction in the medium caused by bremsstrahlung radiation to EHP’s generation can be neglected.

$$(-dE/dx)_{ion} = \frac{4\pi e^4}{m_0 v^2} ZN \left[ \ln \frac{2m_0 \text{v}^2}{l} + 1.2329 \right]$$
\( (-dE/dx)_{ion} \) is the expression which explain the energy loss caused by interaction of beta particles with electrons whether hard or soft collisions occurred in the process, where \( N \) is the atomic density of the absorber, \( E \) is the kinetic energy of particle, \( Z \) is the atomic number of absorber, \( m_0 \) is rest mass of particle, \( v \) is the velocity of electron in the medium, and \( I \) is mean ionization and excitation potential of absorbing atoms.

### 3. Electron-hole pairs generation

Generation of EHPs was calculated by analysing energy deposition across all regions of semiconductor. The relationship between the energy deposition and penetration depth approximately follows an exponential decay law, thus analytical calculation for energy deposition along semiconductor can be described as [1]:

\[
(-dE/dx)_{tot} = \int f(E) (-dE/dx)_{ion} dE = a_1 \exp(-a_2x)
\]  

(4)

This 1-D approximation is needed in order to simplify case of solving minority carrier diffusion equation, which could give influence total EHPs generation. The generation probability of EHPs is determined by the energy of incident beta particle along the depth of semiconductor, which follows this equation[1]:

\[
G(x) = A \frac{(-dE/dx)_{tot}}{E_{pair}} = A \frac{a_1 \exp(-a_2x)}{E_{pair}}
\]  

(5)

where \( f(E) \) is the emission probability of beta particle with energy \( E \), \( A \) is activity of radioisotope source, \( a_1 \) and \( a_2 \) are fitting coefficients that depend on the physical properties and geometric configurations of the device, and \( E_{pair} \) is minimum energy to create electron-hole pair. Similar to photovoltaic, EHPs that are beta-generated inside of or within a minority carrier diffusion length of the depletion region are separated by the built-in electric field and drifted apart[6].

![Figure 3. A simple 1-D schematic diagram of betavoltaic battery.](image-url)

The theoretical model of betavoltaic battery can be seen in **Figure 3**, where beta particles isotropically emitted from radioisotopes initially interact with N-type surface and penetrate into certain depths of semiconductor along the axis, depending on their initial kinetic energy and direction. The beta particles then interact and deposit some of their energy in the semiconductor. Some of the energy is transferred to electrons, making it possible for electrons to leave their orbitals and leaving holes in the atoms.

As shown in Fig. 3, the p-n junction semiconductor region is divided to three main active regions along axis. \( W_N \) is total width of N-type region, \( W_D \) is total width of depletion region, and \( W_P \) is total width of P-type region. \( H \) is total width of whole semiconductor. However, \( W_D \) depends on the doping concentration of donor and acceptor giving to the semiconductor. The total width of semiconductor was
determined using stopping range calculation. The simulation of energy deposition along the thickness of silicon was done by Monte Carlo N-Particle transport code. The results for each simulations and the exponential fitting can be seen in Figure 4 (a) and (b).

Figure 4. Energy deposition along the width of silicon using different radioisotope sources: (a) $^{63}$Ni, and (b) $^{90}$Sr.

4. Analytical performance calculation

In order to compare the analytical calculation results between $^{63}$Ni and $^{90}$Sr betavoltaic battery, we used the same semiconductor structure parameters from previous calculation by Guoping Zuo, et al. (2013). The impurity and geometric semiconductor parameters used in this study listed in Table 1. The calculation of battery performance was derived from minority carrier diffusion equation, had also been described [1]. The schematic diagram of the battery as shown in Figure 3 is used as the boundary condition to solve the equation.

| Table 1. Structure parameters of semiconductor |
|-----------------------------------------------|
| Parameters                  | Value                  |
| Intrinsic concentration $n_i$ (cm$^{-3}$) | $1.45 \times 10^{-10}$ |
| Acceptor doping $N_A$ (cm$^{-3}$)       | $1 \times 10^{17}$    |
| Donor doping $N_D$ (cm$^{-3}$)          | $1 \times 10^{20}$    |
| Area (cm$^2$)                      | 0.16                   |
| p-Type width (cm)                  | $1 \times 10^{-6}$    |
| n-Type width (cm)                  | $0.8 \times 10^{-6}$  |
| Total width (cm)                   | $1.8 \times 10^{-6}$  |

By solving the minority carrier equation, the total current density generated by beta radiation or known as radiation-induced current density is the sum of generated current in p-type, n-type, and depletion region of semiconductor. It is assumed that all EHPs created in the junction are collected as current. In addition, EHPs created within a minority carrier diffusionlength of the depletion region also have a chance to be collected as a function of their distance from the junction edge [5]. The radiation-induced current density which is in ideal situation is equal to short-circuit current density [1].

$$J_{rad} = J_{sc} = J_n + J_p + J_d$$
where \( J_n \), \( J_p \), and \( J_d \) are generated current density in the n-type, p-type, and depletion region, respectively.

The analysis of battery current can be easily explained by the betavoltaic circuit model. The definition of basic betavoltaic cell figure of merit is the open circuit voltage \( V_{oc} \); the short-circuit current density \( J_{sc} \); the fill factor (FF); the power output density (\( P_{out} \)); and the power conversion efficiency. The expression to obtain analytical J-V curve can be derived as[8]:

\[
J_{tot} (V) = J_{sc} - J_0 \left[ \exp \left( \frac{qV}{nRT} \right) - 1 \right]
\]

and open circuit voltage, which is the highest voltage can be obtained from betavoltaic battery when the net current is equal to zero can be expressed by this following equation[8]:

\[
V_{oc} = \frac{nRT}{q} \ln \left[ \frac{J_{sc}}{J_0} + 1 \right]
\]

where \( n \) is ideality factor, \( k \) is Boltzmann constant, \( T \) is absolute temperature, and \( q \) is charge of an electron.

The maximum power generated and the radiation-electricity conversion efficiency can be calculated by this following equation[1]:

\[
P_{out} = V_{mp} J_{mp} = V_{oc} J_{sc} FF
\]

\[
\eta = \frac{P_{out}}{P_{in}} = \frac{V_{mp} J_{mp} FF}{3.7 \times 10^{10} qAE_{avg}}
\]

where \( V_{mp} \) is maximum point voltage, \( J_{mp} \) is maximum point current density, \( FF \) is fill-factor, \( P_{in} \) is the incident power of the beta particles, and \( E_{avg} \) is average energy of beta particles.

5. Results and Discussion

| Parameter | Activity (mCi) | Experimental reference | Analytical reference | This study | Relative error (%) |
|-----------|----------------|------------------------|---------------------|------------|-------------------|
| \( I_{sc} \) (nA) | 0.25 | 0.71 | 0.73 | 0.714 | 0.58 | 2.17 |
| | 1 | 2.41 | 2.44 | 2.856 | 18.52 | 17.06 |
| \( V_{oc} \) (volt) | 0.25 | 0.064 | 0.130 | 0.121 | 89.81 | 6.55 |
| | 1 | 0.115 | 0.161 | 0.157 | 36.70 | 2.36 |

From each selected radioisotope sources, two different activities were used in order to analyse isotope activity influence to betavoltaic. We validated our result with references from previous analytical result [1] and experimental result [7] as listed in Table 2, the value of relative error varies with minimum value at 0.58% and maximum value at 89.81%. The difference in the open-circuit voltage from experimental results, which is most likely caused by the defects created by the heavily doped p-type glass-source, thus resulting a big reverse saturation current [1]. However, self-absorption by radioisotope source also occurred in the process, resulting difference between actual activity and apparent activity. The significant difference in short-circuit current result to analytical reference is likely caused by the deviation of fitting coefficient results, and different semiconductor theory to investigate minority carrier behavior which
probably used by reference such as and minority carrier mobility empirical formula [9,10], surface recombination velocity assumptions [11], and minority carrier lifetime [9,10].

![Figure 5. Analytical J-V curves of betavoltaic devices using different radioisotope sources: (a) $^{63}$Ni, and (b) $^{90}$Sr.](image)

Using Eq. (7), the analytical J-V curves of betavoltaic battery with two different activities from $^{63}$Ni and $^{90}$Sr were obtained as shown in Fig. 5 (a) and (b), respectively. From Fig. 5 (a) and (b), $V_{mp}$ and $J_{mp}$ value from each devices were determined. For $^{63}$Ni betavoltaic battery, increasing the activity of radioisotope from 0.25 mCi to 1 mCi will increase $V_{mp}$ to 1.35 times and $J_{mp}$ equal to 4.23 times. For $^{90}$Sr betavoltaic battery, increasing the activity of radioisotope from 0.25 mCi to 1 mCi will increase $V_{mp}$ equal to 1.29 times and $J_{mp}$ equal to 4.17 times. In comparison, the activity increment to raise battery performance for $^{63}$Ni source is more effective than $^{90}$Sr source.

**Table 3. Comparison between current density and voltage generated by using $^{63}$Ni and $^{90}$Sr.**

| Radioisotope Source | $A$ (mCi) | $V_{oc}$ (volt) | $J_{sc}$ (nA/cm$^2$) | $V_{mp}$ (volt) | $J_{mp}$ (nA/cm$^2$) |
|---------------------|-----------|-----------------|----------------------|----------------|----------------------|
| $^{63}$Ni           | 0.25      | 0.121           | 4.55                 | 0.084          | 3.44                 |
|                     | 1         | 0.157           | 18.03                | 0.114          | 14.57                |
| $^{90}$Sr           | 0.25      | 0.149           | 13.13                | 0.107          | 10.59                |
|                     | 1         | 0.185           | 52.50                | 0.138          | 44.21                |

Despite of the activity increment will also increase the performance of battery, it must has to tolerate the radiation damage to silicon. Those indicate by increasing the activity also reducing the durability of battery and increasing the production costs. As listed in Table 3, the off-circuit voltage generated by $^{63}$Ni source is higher than $^{90}$Sr, as well as the short-circuit current density.

**Table 4. Comparison between power density, fill factor, and efficiency by using $^{63}$Ni and $^{90}$Sr.**

| Radioisotope Source | $A$ (mCi) | $P_{in}$ (nW/cm$^2$) | $P_{out}$ (nW/cm$^2$) | FF | $\eta$ (%) |
|---------------------|-----------|----------------------|-----------------------|----|------------|
| $^{63}$Ni           | 0.25      | 25.308               | 0.289                 | 0.534          | 1.15       |
|                     | 1         | 101.232              | 1.658                 | 0.590          | 1.64       |
| $^{90}$Sr           | 0.25      | 290.08               | 1.135                 | 0.579          | 0.39       |
|                     | 1         | 1160.32              | 6.083                 | 0.625          | 0.53       |
The fill factor (FF) value is a measure of the squareness of the J-V characteristic[8] which theoretically achieved its maximum value at maximum power point. From our calculation, FF value of $^{90}$Sr battery is higher than $^{63}$Ni in every activity used. This can be seen at Fig. 5 (b) that $^{90}$Sr J-V curve looks more identical to ideal J-V curve than $^{65}$Ni J-V curve in Fig. 5 (a). This indicates that the more energy deposited to semiconductor, the more recombination current could be reduced, due to recombination current not influenced by the radioisotope but structure parameters of semiconductor itself.

The incident power $P_{in}$ is determined by the property of the radiation source upon the betavoltaic cell. It is known that the power density of $^{90}$Sr source is larger than $^{63}$Ni source, but resulting in higher output power density $P_{out}$. However, the analytical power conversion efficiency of $^{90}$Sr battery is lower than $^{63}$Ni battery. In order to maximize the performance of $^{90}$Sr battery, the design of p-n junction should be thicker near its $E_{avg}$ stopping range. However with increasing the thickness of battery compensates leakage current value, thus the defect of semiconductor would also increase and further analytical calculation would be needed in order to fit experimental results.

6. Conclusion

A theoretical model of betavoltaic battery based on p-n junction silicon diode using $^{63}$Ni and $^{90}$Sr radioisotope sources was described. The minority carrier current coefficients were calculated using theoretical approach from several reports. The analytical calculation results between previous report, our study, and also with experimental results were compared and validated, where the values of relative error are varies. From our study, we conclude that increasing the activity of radioisotopes of $^{63}$Ni is more effective than $^{90}$Sr to raise the performance. In order to maximize power conversion of $^{90}$Sr betavoltaic battery, the design of p-n junction plays main role to the performance, which could be designed thicker than $^{63}$Ni betavoltaic battery. Despite of its radiation damage, $^{90}$Sr betavoltaic battery could give better performance than $^{63}$Ni betavoltaic battery for a short period. Since the comparative results of power output density is higher than $^{63}$Ni betavoltaic battery.

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