Boundary Node Identification in Three Dimensional Wireless Sensor Networks for Surface Coverage

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SUMMARY With the existing of coverage holes, the Quality of Service (such as event response, package delay, and the life time et al.) of a Wireless Sensor Network (WSN) may become weaker. In order to recover the holes, one can locate them by identifying the boundary nodes on their edges. Little effort has been made to distinguish the boundary nodes in a model where wireless sensors are randomly deployed on a three-dimensional surface. In this paper, we propose a distributed method which contains three steps in succession. It first projects the 1-hop neighborhood of a sensor to the plane. Then, it sorts the projected nodes according to their angles and finds out if there exists any ring formed by them. At last, the algorithm validates a circle to confirm that it is a ring surrounding the node. Our solution simulates the behavior of rotating a semicircle plate around a sensor under the guidance of its neighbors. Different from the existing results, our method transforms a three-dimensional problem into a two-dimensional one and maintaining its original topology, and it does not rely on any complex Hamiltonian Cycle finding to test the existence of a circle in the neighborhood of a sensor. Simulation results show our method outperforms others at the correctness and effectiveness in identifying the nodes on the edges of a three-dimensional WSN.

key words: boundary node, distributed, surface coverage, three-dimensional network, coverage problem

1. Introduction

Wireless sensor networks have been widely applied in a great number of areas in human life, such as environment monitoring, industry manufacturing, agricultural irrigation, battlefield surveillance, et al. They are responsible for collecting information from several Regions of Interest (RoI) and transmitting data to end users for analysis [1], [2]. The accuracy of the data not only depends on the hardware of sensors but also on the topology of networks. A coverage hole, which is a blank area not been covered by any node in the RoI, of a network could vitiate the accumulated data and further misdirect the end users [3]. It may also help to increase the traffic load on the sensors around the holes. Therefore, it is crucial to eliminate coverage holes to ensure the QoS of networks [4]. As the holes are always surrounded by nodes, we can locate them by finding out the boundary nodes on the edges of them. Thus, it is a key step to firstly identify the boundary nodes before locating the coverage holes.

Generally, there are three types of boundary nodes in a WSN: the nodes on the border of the entire network, the sensors on the edges of the coverage holes, and the points on the boundary of an event. For convenience, we mention the former two as network-boundary nodes and the third one as event-boundary sensors. The later one can be distinguished out by reporting different statistical data through various measurements according to events [5]. However, these methods are almost incapable of identifying network-boundary sensors as there could be no active event in a RoI. In an environment where sensors are deterministically deployed, boundary nodes are the ones surrounding the disabled sensors (which are usually the main reason for the existence of coverage holes) and the ones on the RoI’s frontier. However, the shape of the frontier, the position and the number of the coverage holes are often unpredictable in a non-deterministic network where sensors are randomly arranged (such as been dropped out by aircrafts) [6]. Thus, there is a need for an algorithm to identify the network-boundary nodes.

Researchers have proposed numerous solutions to solve the problem in two-dimensional WSNs. They are roughly categorized into three kinds: the algorithms based on geometry [7]–[9], the statistical methods [10], [11], and the techniques using topology [12]–[14]. The first one usually provides accurate results but needs every sensor’s coordinate. The second makes decisions according to the statistical results of a sensor’s local environment and it needs an even distribution of the nodes. The third kind does not need sensors’ positions and is adaptive to various deployments, but it requires to be operated in a centralized way or demands a comprehensive overview of the local area of a sensor.

Several works investigated the problem in a three-dimensional volume WSN. In this model, sensors are arranged both in the body and on the surface of a three-dimensional entity [15] where the boundary nodes are the ones on the surface, and there is no coverage hole in it. But sometimes people are interest in transmitting data between nodes on the surface of an entity, eg. monitoring the exterior of a volcano. Such work requires a three-dimensional surface
WSN [16] and little effort has been put to solve the problem in it. Imagine a fire alarm WSN deployed on a mountain with curved surface and severe environments. Due to the outdoor bad conditions, sensors in the network are dropped out by airplanes and the economical static nodes in it could not move away from their initial positions. In order to fill in the blank areas in the network caused by sensors’ random deployment, we may deploy a few mobile sensors together with the static ones. These mobile nodes can relocate themselves to the holes [17]. The reposition of them requires the locations of the holes and this work needs guidance from boundary sensors.

In this paper, our algorithm picks out the network-boundary nodes in a nondeterministic three-dimensional surface WSN. In Sect. 2, we discussed the related works. Section 3 gives the network model we used for our algorithm. In Sect. 4, we explained the details of our method. Section 5 contains the simulation results. The advantages of our algorithm are the following:

- It is a distributed algorithm that only needs knowledge of the nodes in the 1-hop neighborhood of a sensor. After the communication procedure in the first step, a sensor executes the subsequent steps alone and is capable to make independent decisions on its role.
- It can solve problems in random networks with uneven density of nodes. It is also capable to identify boundary nodes in sparse three-dimensional WSNs.

2. Related Work

2.1 Boundary Detection in Planar Networks

The algorithms based on geometry often require the usage of Voronoi Diagrams or Delaunay Triangles [7], [8]. Li et al. [7] distinguished boundary nodes out from the inner ones by searching for all the empty circles after a Delaunay Triangulation of the network. Fang et al. [9] proposed an algorithm that picking out a hole in a network through communication among the sensors. A hole in it is defined as a circle built by the nodes with the local minia of a phenomena. This kind of methods are usually able to provide precise results but they need a prerequisite of the sensors’ coordinates. The traditional ones of them is hard to be directly applied to a three-dimensional model as the missing of vertical coordinates would introduce errors. We will discuss this problem more specifically in Sect. 4.1.

The statistical methods tend to assume a particular distribution of sensors and need to carefully set one or more thresholds. Fekete et al. suggested a notion of ‘Restricted Stress Centrality’ in [10] to pick out the boundary nodes in a social network. Li et al. proposed an algorithm named as CDBD that uses the criteria of ‘betweenness’ and ‘closeness’ in the neighborhood of a node to decide its role [11]. In the method, the higher value of a sensor’s betweenness or the lower value of its closeness indicates the higher possibility of it been a boundary node. Though this kind of algorithms do not require sensors’ locations, they need an even distribution of the nodes and this is difficult to be realized in a three-dimensional surface WSN. Besides, people also need to carefully set the thresholds for the vital criteria and vary them while changing the parameters of the networks.

The techniques using topology are adaptive to the networks with different models of sensor coverage and communication capability, they are mostly based on the connectivity between nodes [12]. Kroller et al. [13] presented a method that searches the 8-hop neighborhood of a node to find out if there is any structure of a ‘flower’ and uses augmenting cycles to identify the boundary sensors. Wang et al. [14] proposed a distributed algorithm in which there is a message flooding initiated from a random root sensor before the decision making on each node. The flooding brings in an extra time delay and communication load to the network. This kind of method either tries to find out a certain topological pattern in the entire network [18] or needs collaboration from a large number of nodes for preparation. When a sensor only has the information from its 1-hop or 2-hop neighbors, limited knowledge often forces it to search for a Hamiltonian Cycle in its local area, which means that it needs to solve a NP problem [19], not to mention the difficulties in determining the position and shape of the circle.

Beghdad et al. suggested an algorithm called BD-CIS [20] that does not depend on the finding of any Hamiltonian Cycle. It looks for a circle in the 1-hop neighborhood of a sensor by obtaining some closed paths of connected nodes from several independent sets. It seems that this method could be directly applied to a three-dimensional model but the core step of searching for all the independent sets with a threshold cardinality limits its usage. In a randomly deployed sensor network, a node may only have three or four 1-hop neighbors so it could be wrongly identified as a boundary point.

2.2 Boundary Detection in Volume Networks

Feng Li et al. proposed an on-line algorithm called UNFOLD to detect boundary nodes in a dynamic volume WSN [21]. The environment for using UNFOLD is a three-dimensional cloud in which each point is a sensor node. In the first step of it, each node exchanges information with its neighbors to obtain a local coordinate for a common viewpoint. Then, the generated coordinates are transformed to perform a Convexity Test. In step three, a decision is made to set the role of a given node based on whether the image of it is on a specific convex hull. UNFOLD can hardly be applied to the surface coverage model as each sensor in it stands on a convex hull formed by other nodes.

Zhou et al. suggested another method to solve the boundary node identification problem in this model [22]. It first constructs a tetrahedral structure to divide the entire surface of a network into some coarse triangles. Then, it seals them to exclude the inner sensors. The raw results obtained from the former two steps are then refined by a series of processes as the following: boundary landmark expansion,
boundary face split, and boundary surface thinning. Though it is a distributed method that only uses sensors’ connectivity, it requires in each step the cooperation through communication between the nodes.

Zhou et al. also proposed a two-step localized algorithm for precise boundary detection [23]. In the first step, an Unit Ball Fitting (UBF) process is used to coarsely identify the boundary nodes. It is based on the idea that a boundary node should stand on the surface of an empty unit ball. In the second phase, an Isolated Fragment Filtering (IFF) step triggers the formerly picked out boundary nodes to send out local flooding packets to refine the results. When sensor density is high, the local message flooding would introduce heavy communication load into the network.

2.3 Surface Coverage

Zhao et al. proposed a WSN model in which nodes are deployed on a curved surface [16]. It focuses on the partial exterior of a three-dimensional entity rather than its inner part. They studied the problem of sensor deployment in it and calculated the number of nodes in need for a full coverage. By using the theories coming from Discrete Surface Ricci Flow and Centroidal Voronoi Partition, Jin et al. [24] provided an optimal node deployment strategy in this model.

Little work has been put to find out the boundary nodes in the WSNs for surface coverage. An algorithm suggested by Dhanapala et al. [25] uses Topology Preserving Map (TPM) to transform the connectivity between sensors into a virtual map and uses it to identify the boundary nodes. Although, the authors extended the usage of TPM into a couple of three-dimensional surface networks, it requires a number of anchor nodes and an uniform distribution of the sensors. As the virtual coordinate system is built based on hop-distance between sensors, it highly depends on the even distribution of nodes and could hardly deal with the problem in which nodes are randomly placed with points of different curvature. Not to mention that it needs to firstly broadcast messages in the entire network from several anchors to build the coordinate system, and there still remains problems related to the placement of the anchors.

3. Network Model

The RoI in our model is a curved surface and it is randomly deployed with wireless sensors. Each sensor obtains its three-dimensional Euclidean coordinate \((x, y, z)\) through calculations of the surface localization algorithms [26], [27]. The sensors in the network are homogeneous ones with the same sensing range and communication radius. There is no strictly determined pattern for deploying them in our model. As our algorithm focuses on finding out the nodes on the edges on a curved surface and an edge could either be the frontier of a coverage hole or the boundary of the entire WSN, a few isolated nodes could be tolerated in the network. All of the nodes are static so they could not leave their initial positions after deployment. Each of them has an unique ID. It could either be configured by the administrator or represented by its coordinate.

A network in our model is represented by a graph \(G = (V, E)\). Each sensor \(s\) in the set of nodes \(S\) is a vertex \(v \in V\). If two sensors \(u\) and \(v\) are located within each other’s communication range, there is an undirected edge \(e \in E\) between them in \(G\). In the following part of this paper, we will represent a sensor simply by \(v\). A sensor \(v\) collects data from the nodes in its 1-hop neighborhood \(N_1(v)\). These data are stored by \(v\) in a set \(C_0(v)\) containing the nodes’ IDs and their coordinates.

With our main purpose as boundary node identification, we would not discuss the aspects like sensors’ scheduling or the implementation of communication between nodes in this paper.

Some of the terms in this paper are defined as the following:

**Definition 1. (Inner Sensor)** Given a node \(v\) and its sensing area \(A_v\) on a curved surface, \(c_v\) is the circumference of \(A_v\), if \(c_v\) is fully covered by a number of nodes that can directly communicate with \(v\), \(v\) is an inner sensor.

**Definition 2. (Boundary Node)** If a sensor \(v\) is not an inner one, it is a boundary node.

4. Identifying Boundary Nodes in Surface Coverage

Our solution is inspired by a simple idea, a sensor in our model is an inner node if it is fully surrounded by a couple of other ones. Otherwise, it is a boundary sensor. The sensors surrounding an inner node are the 1-hop neighbors of it that are located on the surface or in the body of a ball that using the inner sensor as its center and the sensor’s communication range as its ball radius. Different from an unit ball been used in UBF in [23], the ball does not need to be fully filled with nodes, and the 1-hop neighbors of an inner sensor \(v\) roughly shape a latitude-circle-liked closed path \(r_v\) in the ball. The routes on the path are ensured by the connections between nodes.

Conversely, if we start from a 1-hop neighbor of \(v\) and move to the next stop by finding out another 1-hop neighbor of \(v\) that connects with the former one, the travel process should form a closed path. In the view of a ball \(B_v\), if we use \(v\) as its center and move a semicircle plate along the \(z\)-axis from a random node \(u \in r_v\), the trajectory of the plate should build a complete ball after the travel mentioned above. During the rotation of the plate, each 1-hop neighbor of \(v\), even though it could be out of \(r_v\), should be met with the plate. On the contrary, if we start from a 1-hop neighbor of a boundary sensor, we may stop the process halfway due to the impossibility of finding out a following node or discovering the final path to be an open one.

According to this idea, we suggest a distributed algorithm called “Circle Sweeping” that containing three steps in succession. It first projects the 1-hop neighborhood of a sensor to the plane. Then, it sorts the projected nodes according to their angles and finds out if there exists any ring formed by them. At last, the algorithm validates a circle to
Fig. 1 Three sensors on a curve and their projections.

Fig. 2 A planar view and the 3D one of an example network. All sensors are boundary nodes.

Fig. 3 A curve on the longitudinal profile of a RoI. The blue dot represents a sensor.

confirm that it is a ring surrounding the node.

4.1 Local Surface Planarization

As described in [16], if we only focus on the projected plane of a three-dimensional surface WSN, some holes on the curve would disappear (Fig. 1). When this happens, some sensors on the edges of the holes would be categorized as inner nodes. In an extreme case like the one depicted in Fig. 2, a few sensors are boundary nodes in the projected image, but in fact, all the nodes are boundary ones on the surface.

The controversial result in the above example rises from the reason that a line on the longitudinal profile of the RoI in our model is a curve (Fig. 3). As there exists an angle between the curve and the horizontal line, the length of any line segment on the curve is shortened after projection.

Therefore, the interval distances between sensors on the surface are cut down on the plane. When an angle grows large enough, the space between two apart nodes on the surface disappears on the projected plane and it is viewed as if they were connected.

From the above discussion, we learn that the behavior of “eliminating the heights of nodes” modifies the connections between sensors. Therefore, if we preserve the original topology of a WSN, we could find a way to solve our problem in a two-dimensional fashion. In order to minimize potential communication loads, we limit our algorithm to run in the 1-hop neighborhood of a sensor. The process of shadowing the 1-hop neighborhood of a sensor to the plane (Fig. 4) is named as Local Surface Planarization. In this step, the connections between sensors are kept as their original ones on the curved surface and we still name the projected nodes as their original ones.

4.2 Circle Finding

After the first step, the local area of a sensor is flattened on the plane. This area contains the sensor itself and the nodes in its 1-hop neighborhood. Based on our intuitive idea, if there exists a closed path built by the points around the sensor, it is an inner node, otherwise it stands on a boundary line. Thus, our next step is to find out a circle in the planar local area.

**Lemma 1.** Starting from a node $u$ in the 1-hop neighborhood $N^1_u$ of an inner sensor $v$, we rotate a semicircle plate $P$ along the z-axis with $v$ as the center, if $P$ meets a point $q \in N^1_u$ and $q$ connects with $u$, we draw a line segment $s_{u,q}$ between $u, q$. After the rotation, we project the set of the line segments to the plane. Then there exists a sectionalized ring $R_v$ around $v$.

**Proof.** We assume $v$ is an inner node without any closed ring on the planar graph. This situation happens when the rotation of $P$ stops at a node $l \in N^1_u$ and $l, u$ are not connected. The plate $P$ stops before turning $360^\circ$ and its trajectory can not build a complete ball. Therefore, $v$ is a boundary...
node.

An immediate idea on searching for a closed path in a graph is by finding out a Hamiltonian Cycle in it. However, this means that we need to solve a NP problem. Though we can use an approximation algorithm to solve this problem, as the distribution of sensors are random in the network, it is time consuming when there appears dense nodes in some areas. Another thought would be looking for a circle by performing Depth-First-Search in the graph. But in this way, there would be too many useless closed paths interfere with our work so that we have to distinguish a qualified one out from a number of circles.

As we only need to locate one circle that is around an inner node and it does not need to pass all its 1-hop neighbors, points that do not stand on any closed path should not participate in the process of circle finding. We build a graph \( G^1 \) to represent the local area of \( v \) after local surface planarization. A point in \( G^1 \) is either a node in \( N^1 \) or \( v \). By collecting \( N^1_u \) from each \( u \in N^1 \), there is an edge \( e_{pq} \) connecting two nodes \( p \) and \( q \) in \( G^1 \) if they are connected. Then we prune \( G^1 \) by removing any node with less than 2 neighbors together with any edge connected to it.

**Lemma 2.** After pruning, only the sensors on loops stay in \( G^1 \).

**Proof.** It is obvious that any node with only one neighbor could not stand on a loop. If there is any sensor with a degree of 1 exists in \( G^1 \), it should stand on the end of an open path and it would be removed together with the edge connected with it in the pruning process. Assuming there exists an open path in \( G^1 \) after pruning, as the sensor on the end of the path has a degree of less than 2, the pruning process should not have stopped.

At this point, our work is to find out a circle from the remaining part in \( G^1 \). We solve it by simulating the process of rotating a semicircle plate \( P \) in the local area of \( v \). Starting from a node \( u \) in \( N^1 \), the first sensor \( q \) met by \( P \) is the closest one to it and the angle between \( u \) and \( q \) that using \( v \) as the vertex is the smallest. Using \( q \) as the new starting point, another node \( r \in N^1 \) should be met by \( P \) after rotating for an angle of \( a_v^1 \) and \( r \) is the closest sensor next to \( q \). The plate \( P \) keeps moving along with the \( z \) axis around \( v \) until it reaches to the last sensor in \( N^1 \) that has not met before. Using \( v \) as the apex and \( u, r \) as the other two vertexes, the angle \( \angle vur \) keeps growing if we fix the initial starting point \( u \). According to this, we transform the coordinate system of \( G^1 \) into a local one by occupying \( v \) as the origin. In this system, we calculate an angle for each node in \( N^1 \). An angle \( a_v^1 \) of sensor \( v \neq u \) equals to the value of \( \angle vfr \), \( p \) is a random point on the positive \( x \) axis. Then, we record the value of \( a_v^1 \) of \( v \) with its ID in a set \( M_{ur} \). Next, we sort the items in \( M_{ur} \) according to the size of the angle of each sensor. Assuming \( j \) is the last node in \( C^1 \) that is excluded from \( M_{ur} \), calculate the number of neighbors of \( u \) in \( G^1 \), obtain a set \( M_e \). Then, we move the connection window \( C^1 \) that starts from \( u \) to \( j \) toward a new position that starts from \( j \) to \( g \) and rename it as \( C^1_w \), \( g \) is the last node in \( C^1 \). The size of the connection window varies during the movement. If it shrinks to a small one that contains only one node, the movement stops.

**Definition 3.** (Connection Window) A connection window \( C^1_w \) is an unfixed-sized sliding window without any predetermined starting point. It contains a subset of the sensors in \( N^1 \). For the first node \( s \) in \( C^1_w \), it is connected with any other node \( t \in C^1_w, t \neq s \).

We find a connection window \( C^1_w \) of \( v \) from the first node \( i \) in the sorted set \( M_r \). For any node in \( C^1_w \), it is connected with \( i \); but for any one in \( M_r \) that is excluded from \( C^1_w \), it is disconnected with \( i \). Assuming \( j \) is the last node in \( C^1_w \), the sensor \( k \) that is indexed behind \( j \) in \( M_r \) is disconnected with \( i \). The mutual connection between two sensors in \( C^1_w \) is ensured by the original topology of the WSN. Then, we move the connection window \( C^1_w \) that starts from \( i \) to \( j \) toward a new position that starts from \( j \) to \( g \) and rename it as \( C^1_w \), \( g \) is the last node in \( C^1 \). The size of the connection window varies during the movement. If it shrinks to a small one that contains only one node, the movement stops.

**Algorithm 1 Circle Finding**

1. At each node \( v \):
2. Set \( visBoundary = false \).
3. Set \( FindCircle = false \).
4. Send ("Hello") to the 1-hop neighbors of \( v \).
5. Create \( N^1_v \).
6. if \( |N^1_v| < 3 \) then
7. \( visBoundary = true \).
8. end if
9. Send ("1-hop neighbor") to the 1-hop neighbors of \( v \).
10. Receive \( N^1_v \) from the neighbors, create \( G^1_v \).
11. Prune \( G^1_v \).
12. For each node \( u \in G^1_v \), calculate the number of neighbors of \( u \) in \( G^1_v \),
13. if Less than 2 then
14. Remove \( u \) from \( G^1_v \).
15. end if
16. end for
17. if Number of points in \( G^1_v \) is less than 4 then
18. \( visBoundary = true \).
19. end if
20. Calculate an angle \( a_v^1 \) for each node \( u \) in \( G^1_v \) except for \( v \), obtain a set \( M_e \).
21. Sort \( M_e \) according to the size of the angle of each sensor.
22. Set the first node \( i \) in \( M_e \) as the starting point.
23. while \( FindCircle = false \) and \( i \) is not the last one indexed in \( M_e \) do
24. Find the connection window \( C^1_w \) of \( v \) from \( i \) in the sorted set \( M_e \).
25. Use the last node \( j \) in \( C^1_w \) as the new starting point.
26. Slide the connection window \( C^1 \) on \( M_e \).
27. if \( CW \) can not move forward then
28. Get the last element \( e \) in \( CW \).
29. if \( i \) and \( e \) are connected then
30. \( FindCircle = true \).
31. Record the sensors encountered by \( CW \) during its movement.
32. else
33. \( i = \) the next node of \( i \) in \( M_e \).
34. end if
35. end if
36. end while
37. if \( FindCircle = false \) then
38. \( visBoundary = true \).
39. end if
connected, we obtain a ring $R_n$. Otherwise, we find another connection window $CW_v^t$ of $v$ from the second node $t$ in $M_v$ and slide it. This process is repeated again and again until we successfully find out a circle or we start the sliding of a connection window from the last node in $M_v$.

**Theorem 1.** If the position of a connection window on $M_v$ changes for at least once during its movement from a start point $s$ to an end point $t$ and $s,t$ are connected, there exists a head (or tail) node $u$ from one $M_v$.

Proof. As the position of the connection window $CW_v^t$ changes, there exists at least a middle sensor $m$ between $s$ and $t$ on $M_v$. From the definition of a connection window we know both of the pairs of sensors $s - m$ and $m - t$ are connected, $m$ is the last node in $CW_v^s$ and the first one in $CW_v^t$. After the connection window sliding from the position of $CW_v^s$ to the one of $CW_v^t$ the movement stops at node $t$, $t$ is the last sensor in $CW_v^t$. Since $s - m$, $m - t$ and $t - s$ are connected, there exists a close path built by the sensors $s, m$ and $t$ in $G_v^1$. □

After obtaining $R_n$, we can not guarantee that $v$ is an inner sensor as the searching of $R_n$ is guided by the structure of $G_v^1$. Therefore, we need a further step to confirm the validity of $R_n$.

4.3 Circle Verification

In this step, we validate if $R_n$ is a circle surrounding $v$. With the help of coordinates, we divide the nodes on $R_n$ into four quadrants in the local coordinate system of $v$. If the sensors are gathered together in a single quadrant or scattered in two consecutive ones, we obtain the result that $R_n$ stands at one side of $v$. Next, we focus on situations in the remaining cases. As $M_v$ is a sorted set, the sensors in each quadrant $Q_v^i$, $i \in \{1, 2, 3, 4\}$ are also placed in order.

**Definition 4. (Quadrant Head)** A quadrant head is a sensor $u$ in $Q_v^i$ that has the minimum angle $a_{u}^{vi}$.

**Definition 5. (Quadrant Tail)** A quadrant tail is a sensor $u$ in $Q_v^i$ that has the maximum angle $a_{u}^{vi}$.

**Definition 6. (Consecutive Quadrants)** For any two quadrants in the local coordinate system of $v$, they are consecutive ones if none of them is lack of node.

**Lemma 3.** Given two consecutive quadrants $Q_v^j$ and $Q_v^j$ with nodes ($j = i+1$ or $j = i-1$ is not required as a quadrant may be empty), we obtain a head (or tail) node $u$ from one quadrant and a tail (or head) sensor $p$ from the other. If $u$ and $p$ are not connected, $R_n$ locates beside $v$. Otherwise, if the angle difference between $a_{u}^{vi}$ and $a_{u}^{vi}$ is greater than $180^\circ$, $R_n$ locates beside $v$.

Proof. We use Fig. 5 to illustrate. A sensor pair $u, p$ or $u, q$ is obtained from a ring $R_n$. If the circle is large, we use the pair of $u, p$. If it is small, we use the pair of $u, q$. Sensor $u$ is connected with node $q$, not $p$. It is clear in the figure that if $R_n$ is large and locates beside $v$, $u$ and $p$ are not connected. Otherwise, $v$ stands inside of $R_n$. If $R_n$ is small and $u, q$ are connected, as $R_n$ locates at one side of $v$, the angle difference between $a_{u}^{vi}$ and $a_{u}^{vi}$ is greater than $180^\circ$. Otherwise, $v$ should be surrounded by $R_n$. □

**Algorithm 2 Circle Validation**

1. At each node $v$ that has a $R_n$:
2. Set ValidCircle = true.
3. Divide the nodes on $R_n$ into four sets that each one belongs to a quadrant.
4. for Each quadrant $Q_v^i$ with at least one node: do
5. Obtain a quadrant head $q_{hi}$ and a quadrant tail $q_{ti}$ of $Q_v^i$.
6. for Any pair of two consecutive nodes $i$ and $j$ on $M_v$ in $Q_v^i$, do
7. if $i$ and $j$ are disconnected: then
8. ValidCircle = false.
9. end if
10. end for
11. end for
12. if ValidCircle = true: then
13. for Any two consecutive quadrants $Q_v^j$ and $Q_v^j$: do
14. Get the quadrant head $q_{hi}$ and the quadrant tail $q_{ti}$ (or the quadrant tail $q_{ti}$ and the quadrant head $q_{hi}$), $i, j$ are two consecutive sensors on $M_v$.
15. if $q_{hi}$ and $q_{ti}$ (or $q_{ti}$ and $q_{hi}$) are disconnected: then
16. ValidCircle = false.
17. end if
18. Calculate the angle difference $a$ between $a_{u}^{vi}$ and $a_{u}^{vi}$.
19. if $a > 180^\circ$: then
20. ValidCircle = false.
21. end if
22. end for
23. end if
24. if ValidCircle = false then
25. $v$.isBoundary = true.
26. end if

**Theorem 2.** If $R_n$ passes circle validation, it is a ring surrounding $v$.

Proof. We assume there is a ring $R$ stands next to the sensor $v$, but it passes circle validation. From Fig. 5 we know that a ring locates next to $v$ could have a shape like the letter ‘U’. We pick the two nodes $i$ and $j$ on the two ends of the ‘U’ shaped circle. The two sensors $i$ and $j$ are disconnected, otherwise $R$ is a circle that surrounding $v$. If $i$ and $j$ appear in the same quadrant, the ring could not pass the validation as the two nodes are disconnected. If $i$ and $j$ are sensors at two different quadrants, it would also fail at the test due to the same reason. If the ring $R$ does not look like ‘U’ and simply locates next to $v$, we could find a line $l$ that separates
the plane of $G^1_1$ into two half, one contains $v$ and the other has $R$. We find two nodes $i$ and $j$ on $R$ that are nearest to $l$. The sensors $i$ and $j$ should fall into two distinct quadrants. As $i$ and $j$ are both on the other side of $l$, the angle difference between them is greater than $180^\circ$, so that $R$ should fail at the test.

In Circle Sweeping, any sensor $v$ only needs to communicate with its 1-hop neighbors in local surface planarization. The communication load in this step is at most $O(n^2)$, so that $R$ should fail at the test. We find two nodes $i$ and $j$ on $R$ that are nearest to $l$. The sensors $i$ and $j$ should fall into two distinct quadrants. As $i$ and $j$ are both on the other side of $l$, the angle difference between them is greater than $180^\circ$, so that $R$ should fail at the test.

5. Simulation Results

We generate several networks in Matlab to perform our simulations. We obtain three-dimensional networks by modifying the parameters in the “Peak” function and deploying sensors randomly on the smoothly curved surfaces. All the networks are perturbed ones managed by the grids on the projected plane. In each area on the curved surface that corresponds to a grid on the plane, a number of $k$ nodes are randomly deployed with a distance of at most $\sigma$ away from the grid center, $k$ varies according to the settings of the networks.

5.1 Visual Results

We first get two groups of visual results to show the effectiveness of Circle Sweeping.

5.1.1 Single Algorithm

We generate a 40 * 40 three-dimensional surface network with 3143 sensors (Fig. 6(a)) and run Circle Sweeping on it. The shape of the testing network is a relatively complex figure that contains sharp corners and inward and outward angles which is designed to show the effectiveness of our algorithm. Figure 6(b) shows the result. From it we see that Circle Sweeping successfully depicts the nodes on the boundary of the entire network and the ones on the edge of the inner hole. We notice that there are some boundary nodes gathered together at the bottom in Fig. 6(b), it seems that these sensors are wrongly identified inner ones. But another look at the same position in Fig. 6(a) ensures us that these nodes are the boundary sensors in the network caused by a relatively sparse node density on the three-dimensional local surface.

5.1.2 Multiple Algorithms’ Comparison

Next, we generate a 20 * 20 three-dimensional surface network with 938 sensors and run Circle Sweeping, the statistical based algorithm CDBD [11], and the topology based method BDCIS [20] on it. We compare these three algorithms as they are all running on the 1-hop neighborhood of a single sensor, output decisions by each node individually, and all of them do not require any behavior of Hamiltonian Cycle finding. The testing network has a structured outer boundary and a major inner coverage hole with curved edge. The boundary of the coverage hole has one inward and two outward obtuse angles, and one outward acute angle. Each grid has 5 sensors so the average node degree is 16.38 and the density of network is neither sparse nor too dense. Thus, we could obtain an overview of the performance of the three algorithms.

Figure 7(a) shows a three-dimensional visual output of Circle Sweeping. We can see that the three-dimensional view could not well present the results, so we show them in the two-dimensional view (Fig. 7(b) to Fig. 7(d)). Figure 7(b) displays several clear boundary lines of the network and the inner coverage holes. Circle Sweeping has successfully identified the corners and the boundary lines. Figure 7(c) shows that the boundary nodes picked out by CDBD are not enough. The edge lines are break in several areas which means that the nodes on the edges are wrongly identified as inner ones. Figure 7(d) indicates that BDCIS selected too many boundary nodes, the edge lines in it are thick and many of them are intersecting, which means that a lot of inner nodes are wrongly identified as boundary sensors.

5.2 Statistical Results

The above two groups of simulations give us a visual impression of our algorithm. For the statistical results we run another two groups of tests to obtain the correctness and effectiveness of Circle Sweeping. We add the algorithm based on Delaunay Triangulation in two-dimensional networks [7] (represented by PDT) to further support our opinion that the geometric strategies used in two dimensional networks are no longer effective in the three-dimensional ones.

5.2.1 Algorithms’ Comparison

We generate ten 20 * 20 three-dimensional surface networks

Fig. 6 Boundary node identification of Circle Sweep on a random network.
Fig. 7  Visual comparison of the outputs from three algorithms. The magenta line segments show the connectivity between boundary sensors.

Fig. 8  Average node degree of the 10 simulation networks with the number of nodes per grid ranges from 1 to 10.

with their shapes the same as the one used in Sect. 5.1.2. In these networks, each area on the curved surface that corresponds to a grid on the plane, a number of \( k \) nodes are randomly deployed, \( k \) ranges from 1 to 10. Accordingly, the average node degree in each of the network ranges from 2.71 to 33.65 (Fig. 8). We test the four algorithms on these random networks with diverse sensor density to obtain their performances.

Figure 9 shows the correctness of the four algorithms. The Y axis of it represents the percentage of the boundary nodes been correctly identified by each of the method. We see from the figure that correctness of Circle Sweeping is above 85 percent when \( k \) is no more than 4. This value lowered as the sensors became denser and denser but still higher than the results of PDT and CDBD. It rises as we sort the nodes according to the angles we defined in Sect. 4.2. Due to the increasing of the number of nodes in the network, there may exist multiple sensors with the same angle but different connection with their neighbors. Therefore, the sliding of Connection Windows are disturbed in Circle Finding. We also see that PDT output a good result when \( k = 1 \). It is because the three-dimensional network is a rather sparse one when \( k = 1 \), the connectivity between nodes are well kept when the network is projected to the two-dimensional plane. However, when sensor density increases its correctness dropped sharply. The line of CDBD in Fig. 9 well explains its character, it is an algorithm useful for the networks with high sensor density. It only picked out 20 percent of the boundary nodes of the test networks when they are sparse and performed better when they became denser.

It seems that BDCIS is the best one for identifying boundary nodes among the four algorithms in Fig. 9 as the correctness of it is always higher than 90 percent. However, Fig. 10 indicates that the effectiveness of the method is low. It obtained the result by overly picking out boundary nodes. The Y axis of Fig. 10 represents the percentage of nodes that are real boundary ones which have been effectively identified by each of the method. We can see that CDBD also has the same problem in the three-dimensional environments. In networks where sensors are not evenly deployed, the high dependence on the statistical inter-relation between nodes made its performance getting lower and lower. The statistical results support our visual ones in Sect. 5.1.2, Circle Sweeping outperforms the other three in identifying boundary nodes in nondeterministic three-dimensional surface WSNs.
5.2.2 Algorithm Performance in the Presence of Positioning Errors

As Circle Sweeping is a geometric algorithm that uses the coordinates of nodes, we test the performance of it in the presence of positioning errors. We use a 20 × 20 three-dimensional surface network with 370 sensors, a number of 2 nodes are randomly deployed in each grid. We use a sparse network as the results in Sect. 5.2.1 already showed that we need to improve the performance of Circle Sweeping in dense sensor networks. The random location errors of each sensor ranges from 1% to 20%. The red line in Fig. 11 presents the effectiveness of Circle Sweeping and the blue line shows the correctness of it. We see that the correctness of the method is relatively steady when sacrificing its effectiveness. Thus, Circle Sweeping is tolerable with minor positioning errors of the sensors.

6. Conclusion

In this paper, we study the problem of identifying boundary nodes in a three-dimensional surface WSN. We use an algorithm that solve the problem in the 1-hop neighborhood of a sensor. The method that we named as Circle Sweeping contains three steps: local surface planarization, circle finding, and circle validation. This distributed algorithm projects the 1-hop neighborhood of a sensor to the plane, finds out if there is any circle on the plane and then checks whether it is surrounding the sensor. Simulation results show the correctness and effectiveness of it. Our future work will focus on improving its performance in dense sensor networks with high average node degree.

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