On the distributional properties of size, profit and growth of Icelandic firms

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Abstract In this paper, we analyze the distributional properties of the balance sheets of Icelandic firms by performing an empirical analysis of total assets, profit rates and growth rates using a data set of 2,818 Icelandic firms during the period 2000–2009. We find that the firms size measure, i.e. total assets, have the same heavy tail characteristics as various studies have shown, e.g. for US and Italian firms. The heavy tail nature of the total assets distribution seems to be robust w.r.t. a boom-bust cycle of the economy as well as special characteristics of Icelandic firms, e.g. their relatively small size and private ownership. Another important finding is that the profit rates, or return on assets, of Icelandic firms follow a Laplace like distribution similar to US firms. Additionally, we identified deviations from the distributional regularities, namely the power law behavior of firms’ size and Laplacian distributions of growth and profit rates, during the booming period of the economy 2005–2007.

Keywords Firm size · Profit rates · Growth rates · Power law · Laplace distribution · Icelandic firms

JEL Classification C12 · D01 · D22
1 Introduction

Firms drive economic growth through investment and innovation and are an important part of any economy. In this paper, we analyze the distributional properties of the balance sheets of Icelandic firms during the period 2000–2009. First we analyze a measure of firms’ size, namely the level of total assets, then we consider the growth rates of firms’ total assets, and finally we look at profit rates, defined as earnings before interest and taxes divided by total assets. It is worth noting that the Iceland economy, for its small dimension and the recent boom-bust cycle, is a peculiar and interesting case study for testing the presence of the distributional properties that have been recently described in the literature, both for US (Axtell 2001; Podobnik et al. 2010; Alfarano et al. 2012) and Italian (Bottazzi and Secchi 2006; Cirillo 2010) firms, namely power law behavior of firm size and Laplacian distribution of firms growth and profit rates. In the last decade, the Icelandic economy witnessed a dramatic increase and decrease in asset prices as well as a housing bubble. Twelve month inflation rates reached 20%, the currency, the Icelandic Kronur (ISK), lost about half of its value w.r.t. the Euro in 2008, and almost the entire banking system collapsed in the space of a week in early October 2008. Furthermore, most of the firms in Iceland are privately owned and are not traded on any stock exchange, unlike data sets used in most studies of firms’ dynamics known to the authors. Therefore, it is interesting to analyze whether distributional properties of firms size, profit and growth rates are robust w.r.t. a boom-bust cycle of a small open economy inhabited mostly by privately owned firms.

Analysis of the measures of firms’ size have a long history in the literature. Since the seminal work of Gibrat (1931) it has been known that the distribution of firms’ size, proxied by the number of employees, follows a right skew distribution. According to Gibrat’s law, the growth rate of firms is independent of its size, giving rise to a log-normal distribution. It seems that the log-normal distribution does not account for the power law behavior in the tail of the firm size distribution. From a historical perspective, the model introduced by Champernowne (1953) with the presence of a minimum firm size\(^1\) can account for the power law behavior in the tail. The heavy-tailed distribution found for firm size has been modeled in various ways, see De Wit (2005) for an overview. Perhaps the best known of these methods is the multiplicative process (Simon and Bonini 1958; Ijiri and Simon 1974 and more recently Scalas et al. 2006), summarized by the saying “the rich get richer”. Recent models of heavy-tailed distributions of various size related phenomena include the model of Atalay et al. (2011) of a buyer-seller network for US production firms and the random group formation model of Back et al. (2011). The power-law behavior of the upper tail of firm’s size distribution has been verified for various periods and economies, e.g. for US firms by Axtell (2001), Podobnik et al. (2010) and Simon and Bonini (1958), and for Italian firms by Cirillo (2010) and Cirillo and Hüüsler (2009). The recent contribution of Gabaix (2009) reviews both the empirical literature of power law behavior in economic and social systems as well as several classes of stochastic models to generate such a scaling behavior.

\(^{1}\) The original model by Champernowne has been constructed to account for the distribution of income. However the basic mechanism can also be adapted to the firms growth process.
In a recent paper, Alfarano et al. (2012) show that the profit rate distribution of large publicly traded US companies is well described by a Laplace distribution. Using the statistical equilibrium approach, they relate the emergence of the Laplace distribution to the notion of classical competition. The same tent-like distribution has been found to fit reasonably well growth rates of firms, see e.g. Bottazzi and Secchi (2006) and Alfarano and Milakovic (2008).

A further objective of this work has been to gather important stylized facts and descriptive statistics of the Icelandic economy to be used for the validation and calibration of agent-based macroeconomic models, which recently have been recognized (Farmer and Foley 2009) as an alternative to traditional dynamic stochastic general equilibrium (DSGE) models, see e.g. Smets et al. (2002), that have been used by economic policy makers during the past decade. These new agent-based models, e.g. EURACE (Cincotti et al. 2010; Raberto et al. 2012), rely on stylized facts of the economy for calibration and validation.

In Sect. 2 we will give a detailed description of our data set, while in Sects. 3 and 4 we will analyze the distributional properties of total assets, used as a measure of firm size, and profit and growth rates of Icelandic firms respectively. In Sect. 5 we present our results.

2 Data set

The data set used in this study is public information attained from annual reports of Icelandic firms collected by the Icelandic Internal Revenue Directorate (RSK). The RSK does not give access to digital (computerized) data. Therefore the authors turned to a third party data provider, Creditinfo hf.2

Data from 2,818 Icelandic firms were collected for the years 2000–2009 giving a total of 11,084 observations. The firms in the data set have minimum assets of 100 million ISK3 and belong to various industries such as manufacturing, service, retail and fishing. The data set does not contain banks, insurance firms or other financial institutions. In Iceland only a very limited number of firms are listed on the Icelandic stock exchange, so the vast majority of firms in the data set are privately owned firms.

When analyzing data of firms’ profit and growth rates, we are interested in relatively large, long-lived firms4 for the sake of consistency with previous work (Alfarano et al. 2012; Bottazzi and Secchi 2006). To get this subset of firms, we select the firms present in the year 2000 and gather all observations for these firms in the years 2001–2009. This choice will leave out any firms established after the year 2000. Moreover, since all firms in the data set were active at the end of the period 2000–2009 considered, this choice give us a good subset of long-lived firms in Iceland.

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2 Creditinfo hf. specializes in providing credit information about Icelandic firms., http://www.creditinfo.is.
3 The average exchange rate of the ISK has been approx. 162 ISK for 1 EUR in April 2011.
4 See the paper of Wagner et al. (2010) for a discussion on statistical properties of the profit rates of short lived firms as compared to long-lived firms.
Table 1  Columns two and three represent the whole data set while columns four and five represent long-lived firms

| Year  | No. obs. | Mean TA   | No. obs. long-lived | Mean TA, long-lived |
|-------|----------|-----------|---------------------|---------------------|
| 2000  | 518      | 1.272.575 | 518                 | 1.272.575           |
| 2001  | 598      | 1.309.588 | 421                 | 1.635.462           |
| 2002  | 726      | 1.143.999 | 431                 | 1.601.477           |
| 2003  | 870      | 1.057.349 | 441                 | 1.513.737           |
| 2004  | 1,007    | 1.160.869 | 430                 | 1.977.509           |
| 2005  | 1,263    | 1.385.675 | 424                 | 2.590.019           |
| 2006  | 1,618    | 1.296.920 | 415                 | 2.737.960           |
| 2007  | 1,971    | 1.643.282 | 426                 | 3.961.210           |
| 2008  | 1,889    | 1.744.903 | 382                 | 3.981.384           |
| 2009  | 624      | 1.315.863 | 136                 | 1.836.308           |
| Aggregated | 11,084    | 1.404.404 | 4,024               | 2.298.303           |

All monetary values given are in thousands of ISK

Our data set is comprised of 5–6% of all firms in Iceland, representing about half of the total assets of firms in Iceland. When considering long-lived firms, they represent about 1.3% of the total number of firms in Iceland with about one quarter of the total assets. Table 1 shows some descriptive statistics of the data set, both w.r.t all firms as well as long lived firms.

There are some peculiarities in the Icelandic data that need to be noted. Firstly, according to Icelandic law (no. 696/1996) revenues are defined as the sum of revenues from core operations and other revenues. No further details are given for revenues, e.g. sales are not given explicitly. Secondly, firms in Iceland can turn in contracted forms of the annual report (law no. 694/1996) if they fulfill certain conditions. If firms hand in contracted forms of the annual report no information is given about revenues and costs, while the difference of the two, defined as earnings before interest and taxes, EBIT, is provided together with the value of total assets. Unfortunately it seems that firms in Iceland have a tendency to turn in the contracted form of the annual report if they possibly can. Therefore, we focus our paper on the available data, namely EBIT and total assets, which allow us to study the distribution of total assets, their growth rates and profit rates (see Sect. 4). Thirdly, although the RSK can enforce fines for not handing in annual reports it has not done so in the past. In some cases firms have taken advantage of the lack of proper control of the authorities not providing financial annual reports with the RSK. For several firms, therefore, we have missing observations in the data set of EBIT and total assets.

5 All Icelandic law can be found on-line at: http://www.althingi.is.
6 These conditions are mostly related to size measures, small firms with little turnover can file contracted forms of the annual report. The law also allows firms to file contracted forms of the annual report for reasons of competition, i.e. if the firms can argue that reporting full annual report might reversely affect competition.
These missing observations might possibly introduce a bias into our data set. Firms might systematically not hand in annual reports if their performance for the year has been poor. The main performance measure of firms is \( EBIT \), so it is reasonable to assume that if there is a bias in the data set, due to firms not handing in annual reports, it stems from a low value of \( EBIT \). The distribution of \( TA \) should not be affected much as changes in \( TA \) should not increase the likelihood that firms do not hand in annual reports. On the other hand, the vast majority of these firms are not traded on any stock exchange and therefore disclosing poor results has a very limited negative effect on the firm. In the “Appendix” of the paper we discuss in details the impact of the missing values in the results of our statistical analysis.

These peculiarities aside, the data set gives the opportunity to study the distributional properties of firms’ balance sheet entries during a boom-bust cycle of a small open economy.

3 Total assets

One of the benchmarks for any measure of firm size is that it is well described by a power-law distribution. Firm size is often measured by the number of employees it has, by the amount of sales it generates or, as in our case, by the book value of total assets (\( TA \)). Note that when analyzing \( TA \) we use the entire data set, not only long lived firms as when analyzing profit and growth rates. The lower bound\(^7\) of the data set is given by the fact that we only have data for firms with assets greater than 100 million ISK, this gives us a value for \( x_{min} = 100m \).

For Pareto distributed variables \( s \) we consider a survival function of the form:

\[
Prob(s > x) = \begin{cases} 
  \left( \frac{x_{min}}{x} \right)^\gamma & \text{if } x \geq x_{min} \\
  1 & \text{if } x < x_{min}
\end{cases}
\]  

(1)

where \( x \) are our observations of firms size, i.e. \( TA \), and \( \gamma \) is the exponent of the Pareto distribution. The variable can be plotted on a Zipf plot. In order to do that the observations of \( TA \) are ordered from the highest to the lowest and plotted against their rank, \( r \). The Zipf plot of \( TA \) versus \( r \) then has an exponent, \( \hat{\beta} = 1/\gamma \), that can be estimated using ordinary least squares regression by:

\[
\hat{\beta} = \frac{\sum_{i=1}^{N}(\log r_i \log TA_i) - (\log r)(\log TA)}{\sum_{i=1}^{N}(\log r_i)^2 - (\log r)^2},
\]

(2)

where \( r \) is the ordered rank of firms from the largest to the smallest, \( N \) is the total number of samples, \( \log r \) and \( \log TA \) are the means of the log rank and log \( TA \), respectively. The error can be estimated in the standard way.

From Fig. 1, it seems that there is a power-law regime in the Zipf plot of \( TA \) of Icelandic firms. Table 2 shows the OLS estimated exponent, \( \hat{\beta} \), for each year present

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\(^7\) Our estimator of the index of the tail might be sensitive to the minimum value \( x_{min} \). We have checked the Hill plots of the yearly data for \( TA \) in order to evaluate the impact of the choice of \( x_{min} \). The value of the index turns out to be fairly stable up to roughly 90% of the sample (material upon request).
Fig. 1 Zipf plot of normalized total assets, $TA/\bar{TA}$, versus rank, $r$, disaggregated for each year. The variables exhibit classic power-law behavior, seen as a straight line on the log–log Zipf plot. Inline figure shows normalized total assets versus normalized rank, $r/\bar{r}$.

Table 2 The exponent, $\hat{\beta}$, is estimated for each year in the data set using Eq. 2, the standard error is shown in parentheses.

| Year | Exponent, $\hat{\beta}$ | Exponent, $\hat{\beta}_{KS}$ | KS statistic |
|------|--------------------------|-------------------------------|--------------|
| 2000 | 1.25 (0.04)              | 1.27 (0.04)                   | 0.03         |
| 2001 | 1.27 (0.04)              | 1.30 (0.04)                   | 0.02         |
| 2002 | 1.22 (0.04)              | 1.26 (0.04)                   | 0.02         |
| 2003 | 1.19 (0.03)              | 1.24 (0.03)                   | 0.03         |
| 2004 | 1.21 (0.03)              | 1.27 (0.03)                   | 0.02         |
| 2005 | 1.24 (0.03)              | 1.28 (0.03)                   | 0.03         |
| 2006 | 1.22 (0.03)              | 1.28 (0.03)                   | 0.03         |
| 2007 | 1.27 (0.02)              | 1.35 (0.02)                   | 0.03         |
| 2008 | 1.27 (0.02)              | 1.35 (0.02)                   | 0.04         |
| 2009 | 1.30 (0.04)              | 1.32 (0.04)                   | 0.04         |
| Aggregated | 1.23 (0.04) | 1.31 (0.04) | 0.02 |

We also provide an estimate of the exponent minimizing the Kolmogorov–Smirnov (hereby KS) statistic, $\hat{\beta}_{KS}$. The last column shows the KS statistic calculated using $\hat{\beta}_{KS}$ for the exponent. Bolded values of the exponents, $\hat{\beta}$ and $\hat{\beta}_{KS}$, in the table represent instances where we cannot reject the hypothesis that the data comes from the power-law distribution, given a 1% significance level.

in the data set as well as the estimated standard error shown in parentheses. The exponent, $\hat{\beta}$, is relatively stable from one year to the next, ranging from 1.19 to 1.30 when using the standard OLS estimation. We also provide an estimation of the exponent found by minimizing the Kolmogorov–Smirnov (KS) statistic (for more information on the technique see Clauset et al. 2009), denoted $\hat{\beta}_{KS}$. The fluctuation is similar.
as before, between 1.24 and 1.35. Values in bold of the exponents, $\hat{\beta}$ and $\hat{\beta}_{KS}$, in Table 2, represent instances where we cannot reject the hypothesis that the data come from the power-law distribution, given a 1% significance level. \(^8\) Unfortunately, we do not have data for number of employees or sales, previously used as measures of firm size by Axtell (2001). Our results can though be compared with what Podobnik et al. (2010) found for NYSE firms; they report an exponent of about 1.4 for book value of assets for the years 2007–2009, which is consistent with our findings for $\hat{\beta}$ and $\hat{\beta}_{KS}$.

According to the KS statistic, we can only reject the null hypothesis that $TA$ follows a power-law distribution, using the exponent $\hat{\beta}_{KS}$, during the peak of the economic boom in Iceland, 2006–2008. One possible explanation for this is the fact that an influx of relatively small firms, 0.1% of $TA$ w.r.t. the maximum $TA$ of each year, is seen in the data set of Icelandic firms during this period. This influx starts in the early years of the boom, 2003–2005, but is most apparent during the peak of the boom. These relatively small firms do not seem to be captured by the statistical equilibrium of the power-law distribution, thus rejecting the null hypothesis that the data is well described by a power-law distribution. In fact, if these very small firms are removed from the data set we cannot reject the null hypothesis that $TA$ follows a power-law distribution, using the exponent $\beta_{KS}$ and 1% significance level, for any year in the data set. \(^9\)

### 4 Statistical analysis of profit rates and growth rates

Profit rates, also named as return on assets or $ROA$, are computed by dividing the earnings before interest and taxes ($EBIT$) by total assets ($TA$). Profit rates are an important measure of firms performance and are inherently linked to growth rates because many firms use internal financing to finance growth opportunities, i.e. they use part of their profits to finance investment instead of issuing new debt or equity (external financing), as rationalized by the Pecking Order Theory (Myers 1984). Notice, however, that profit rates are defined as the ratio of a flow (yearly operating gross earnings) over a stock (total asset), while growth rates are usually defined as log differences over a given time interval of a firm size measure (typically yearly sales, total assets or number of employees). Profit rates and growth rates have been shown to be well described by a Laplace distribution, see e.g. Alfarano et al. (2012) and Bottazzi and Secchi (2006). Even if profit and growth rates retain the same functional form of their empirical distributions, they might not be linked by a trivial relationship. In this section, we want to explore whether the empirically identified distributional regularity of Laplacian profit and growth rates hold for Icelandic firms.

\(^8\) The critical values are obtained from Goldstein (2004). The critical values depends on the number of observations. Given a 1% significance level the critical value is 0.04 for 500 observations, 0.02 for 2,000 observations and 0.01 for the aggregated data of approx. 11,000 observations.

\(^9\) Analysis not included in Table 2 but available in additional material upon request.
4.1 Theoretical framework

In the literature on industrial dynamics several theoretical models have been introduced to reproduce the Laplacian functional form of firms growth rates, for instance Bottazzi and Secchi (2006), Stanley et al. (1996) and Delli Gatti et al. (2005). Bottazzi and Secchi (2006), in particular, empirically tested the Laplacian hypothesis by employing a more general distributional class, the Subbotin distribution, also known as the generalized exponential-power distribution (see Eq. 4 below). They show that, in the majority of cases, the nested Laplace distribution cannot be rejected in favor of the more general Subbotin distribution for Italian manufacturing firms.

A very general framework for the Laplacian hypothesis of profit rates distribution comes out of a statistical equilibrium model introduced by Alfarano et al. (2012) and its subsequent empirical applications. More precisely, they encode the “tendency for equalization” of profit rates, due to the competitive process among firms, in a moment constraint of the underlying statistical distribution of profit rates. They propose the following very general measure of dispersion:

\[ \sigma = \sqrt{\mathbb{E} \left[ |x - m|^\alpha \right]} , \]  

(3)

where \( m \) is a measure of central tendency of the underlying distribution.\(^{10}\) Notice that for \( \alpha = 2 \), the previous definition is the standard deviation, while for the case \( \alpha = 1 \), it coincides with the absolute mean deviation. Employing the Maximum Entropy Principle, it is possible to link up the previous moment constraint to an equilibrium distribution, which turns out to be the Subbotin distribution:

\[ f(x; m, \sigma, \alpha) = \frac{1}{2\sigma \alpha^{1/\alpha} \Gamma(1 + 1/\alpha)} \exp \left( -\frac{1}{\alpha} \frac{|x - m|}{\sigma}^{\alpha} \right) \]  

(4)

where \( m \) is the location parameter, \( \sigma \) is the scale parameter and \( \alpha \) is the shape parameter. Since the Laplace distribution is nested in Eq. (4) for a value of the shape parameter \( \alpha = 1 \), the Subbotin distribution can be employed to test for the Laplace distribution of profit and growth rates at various aggregate levels. In various papers in the literature on industrial dynamics (see for instance Bottazzi and Secchi 2006), the estimation of the shape parameter of \( f(x; m, \sigma, \alpha) \) turned out to be “close” to one in the vast majority of empirical distributions of firms’ profit (see Alfarano et al. 2012) and growth rates (see Bottazzi and Secchi 2006).

Summarizing this empirical literature, we consider the Laplace distribution as a sort of first approximation model alternative to the Gaussian. The Gaussian distribution can be, in fact, employed as a reference distributional hypothesis of growth or profit rates under the absence of economic interactions among firms. For instance, Bottazzi and Secchi (2006) have derived the Laplacian distribution of firm growth rates out of a simple model of competition among firms in an increasing return to scale environment.

\(^{10}\) Note that the measure of central tendency is the general expression to characterize a focal value of a distribution. For instance, mean, median and mode are possible alternatives of measures of central tendency.
Table 3 The table shows the estimation of the shape, and scale $[\sigma_S$ from Eq. (3)] parameter of the Subbotin distribution for profit rates

| Year | $\alpha$  | $\sigma_S$  | $\sigma$    |
|------|-----------|-------------|-------------|
| 2000 | 0.95 ± 0.14 | 0.068 ± 0.008 | 0.070 ± 0.008 |
| 2001 | 1.02 ± 0.16 | 0.072 ± 0.008 | 0.072 ± 0.008 |
| 2002 | 0.92 ± 0.14 | 0.069 ± 0.008 | 0.072 ± 0.008 |
| 2003 | 0.75 ± 0.12 | 0.068 ± 0.008 | 0.078 ± 0.008 |
| 2004 | 0.96 ± 0.16 | 0.073 ± 0.008 | 0.075 ± 0.008 |
| 2005 | 0.89 ± 0.14 | 0.088 ± 0.012 | 0.093 ± 0.012 |
| 2006 | 0.98 ± 0.16 | 0.087 ± 0.010 | 0.088 ± 0.010 |
| 2007 | 0.88 ± 0.14 | 0.085 ± 0.010 | 0.090 ± 0.010 |
| 2008 | 0.87 ± 0.14 | 0.103 ± 0.014 | 0.110 ± 0.014 |
| 2009 | 0.83 ± 0.24 | 0.081 ± 0.018 | 0.088 ± 0.018 |
| Aggregated | 0.91 ± 0.04 | 0.078 ± 0.003 | 0.082 ± 0.003 |

The associated errors refer to the 95% confidence level. The parameter $\sigma$ is the computed from Eq. (5). In almost all cases $\sigma$ and $\sigma_S$ are not statistically different. The last row shows the estimates considering the entire sample of firms, pooling all years together.

4.2 Empirical analysis of profit rates

Contrary to the previously analyzed data set of profit rates mainly based on publicly traded US firms (see Alfarano et al. 2012 and references therein), our data set consists of mostly privately owned firms in what can be considered a prototypical example of a small open economy. In this section, we want to analyze to which extent the Laplace distribution can do a good job in fitting the profit rates of Icelandic firms.

As a first step in the statistical analysis of the profit rate distribution, we start estimating the main parameters of the Subbotin distribution, namely shape and scale parameters. Table 3 shows the estimation of the Subbotin distribution parameters for the yearly profit rate distribution. In almost all years, the Laplace distribution is not rejected since the case $\alpha = 1$ is included in the 95% confidence level in the estimation of the shape parameter of the distribution, and, additionally, the scale parameter is not significantly different from the mean absolute deviation around its median value. The pooled data exhibit a distribution close to a Laplacian, which is in line with the American long-lived firm sample analyzed in Alfarano et al. (2012).

The maximum likelihood estimation, however, does not provide an absolute criterion to assess the goodness-of-fit of the Laplace distribution, but it eventually provides a measure of relative performance of two nested models. We compute, therefore, the goodness-of-fit of the Laplace distribution using a KS statistics. In order to estimate the parameter $m$, we use three different measures of central tendencies: the mean, $m_{\text{Mean}}$, the median, $m_{\text{Med}}$, and the value which minimizes the KS statistics, $m_{KS}$. As a first approximation, we have associated to $m_{\text{Med}}$ and $m_{KS}$ the same error as for $m_{\text{Mean}}$, which is computed with the standard method. The scale parameter $\sigma$ is computed using the formula of the maximum likelihood for a Laplace distribution:
Table 4 The table shows the estimation of the measure of central tendency for the profit rate distribution, performed with three different estimators: the mean, the median and by minimizing the KS statistics.

| Year | $m_{\text{Mean}}$ | $m_{\text{Med}}$ | $m_{\text{KS}}$ | KS statistic |
|------|------------------|---------------|----------------|--------------|
| 2000 | 0.058 (0.010)    | 0.051         | 0.057          | 0.93         |
| 2001 | 0.062 (0.010)    | 0.060         | 0.059          | 0.74         |
| 2002 | 0.057 (0.010)    | 0.053         | 0.053          | 0.48         |
| 2003 | 0.065 (0.012)    | 0.055         | 0.062          | 0.99         |
| 2004 | 0.071 (0.010)    | 0.058         | 0.065          | 0.97         |
| 2005 | 0.085 (0.014)    | 0.061         | 0.073          | 1.24         |
| 2006 | 0.103 (0.012)    | 0.083         | 0.091          | 1.09         |
| 2007 | 0.097 (0.014)    | 0.087         | 0.081          | 1.48         |
| 2008 | 0.09 (0.01)      | 0.086         | 0.093          | 0.71         |
| 2009 | 0.092 (0.020)    | 0.079         | 0.062          | 0.76         |

The associated errors for all three methods is the error of the mean computed with the usual standard method. The bold estimates indicate that the Laplace benchmark cannot be rejected at 1% significance level. The critical value at 1% significance level is 1.065, according to Puig and Stephens (2000).

\[
\hat{\sigma} = \frac{1}{N} \sum_{i=1}^{N} |x_i - \hat{m}|, \tag{5}
\]

providing for $\hat{m}$ the pertinent value, namely $m_{\text{Mean}}$, $m_{\text{Med}}$ or $m_{\text{KS}}$. Table 4 summarizes the results of the estimation for each year in our sample. The bold value of the estimates of $m$ indicates that the Laplace distribution cannot be rejected following the KS statistic at a 1% significance level. To have an idea of the values of the KS statistics, we show in the last column its value for $m_{\text{KS}}$.

A clear-cut conclusion of our analysis is that the Laplace constitutes a good benchmark for the distribution of profit rates of Icelandic firms. In order to have an idea of the performance of the Laplace in fitting the empirical data, Fig. 3 shows the profit rate distributions disaggregated per year. The Laplacian hypothesis for the profit rate distribution of long-lived firms is, then, robust if we consider a large economy like the US one, see Alfarano et al. (2012), or a prototypical small open economy like the one of Iceland. Notice, however, that the rejection of the Laplace benchmark seems to be related the years of the economic boom (2005–2007), which coincides with an emergence of an underlying asymmetric distribution of profit rates. Table 4 and Fig. 2 show in fact a significant positive difference between mean and median during the bubble period. If we make the hypothesis that the symmetric Laplace distribution is a kind of statistical equilibrium distribution which characterized a phase of ‘equilibrated’ growth of an economy, our tentative hypothesis is that in periods of high growth (bubble), the distribution of profit rates is characterized by a significant asymmetry. Interestingly and somehow surprisingly, in the period immediately after the bubble (2008–2009), the Laplace distribution cannot

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11 The critical value at 1% significance level is 1.065, according to Puig and Stephens (2000). The typical critical value for the KS test cannot be used here since the parameters of the Laplace distribution are estimated from the data sample.
be rejected, showing a fast realignment with the symmetric Laplace benchmark. Our hypothesis should be tested in different economies that in the last decade experienced a similar behavior of the Icelandic economy, namely high growth and subsequent crisis period, for instance the Spanish and Irish economies.

4.3 Empirical analysis of growth rates

In this section we present the empirical analysis of growth rates of Icelandic firms. The definition of annual growth rate of a firm $i$ is given by:

$$g_{i,t} = \ln \left( \frac{T A_{i,t+1}}{T A_{i,t}} \right),$$

where $T A_{i,t}$ are the total assets of the firm $i$ at time $t$. It has been repeatedly shown that the distribution of firm growth rates can be conveniently described by a Laplace distribution (for instance Bottazzi and Secchi 2006 and reference therein). Table 5 shows the estimation of the Subbotin distribution used to fit the growth rates; the results are in line with other analog studies like Bottazzi and Secchi (2006), namely in the majority of cases you cannot reject the Laplace distribution as the parsimonious version of the Subbotin family. Table 6 shows the results of the estimation exercise of the location parameter of the Laplace distribution using the annual growth rates of

![Image of a graph showing two different measures of central tendency estimated for each year in the data set. Note the growing asymmetry of $m_{\text{Mean}}$ and $m_{\text{Med}}$ during the boom in Iceland (2005–2007), which coincides with the rejection of the Laplace distribution benchmark for profit rates of Icelandic firms.](image-url)
Table 5 The table shows the estimation of the shape, and scale [\( \sigma_S \) from Eq. (3)] parameter of the Subbotin distribution for growth rates

| Year     | \( \alpha \)       | \( \sigma_S \)   | \( \sigma \)   |
|----------|---------------------|------------------|----------------|
| 2000     | 0.82 ± 0.14         | 0.16 ± 0.02      | 0.16 ± 0.02    |
| 2001     | 0.88 ± 0.16         | 0.17 ± 0.02      | 0.17 ± 0.02    |
| 2002     | 0.84 ± 0.14         | 0.16 ± 0.02      | 0.16 ± 0.02    |
| 2003     | 1.00 ± 0.18         | 0.19 ± 0.02      | 0.19 ± 0.02    |
| 2004     | 1.05 ± 0.20         | 0.20 ± 0.02      | 0.20 ± 0.02    |
| 2005     | 1.17 ± 0.20         | 0.19 ± 0.02      | 0.19 ± 0.02    |
| 2006     | 1.08 ± 0.2          | 0.20 ± 0.02      | 0.21 ± 0.02    |
| 2007     | 1.00 ± 0.2          | 0.20 ± 0.03      | 0.20 ± 0.03    |
| 2008     | 0.83 ± 0.24         | 0.12 ± 0.03      | 0.12 ± 0.03    |
| Aggregated | 0.95 ± 0.06        | 0.18 ± 0.008    | 0.19 ± 0.008   |

The associated errors refer to the 95% confidence level. The parameter \( \sigma \) is the computed from Eq. (5). The last row shows the estimates considering the entire sample.

Table 6 The table shows the estimation of the measure of central tendency for the growth rates distribution, performed with three different estimators: the mean, the median and by minimizing the KS statistics

| Year     | \( g_{\text{Mean}} \) | \( g_{\text{Med}} \) | \( g_{\text{KS}} \) | KS statistics | \( \pi \) |
|----------|------------------------|-----------------------|---------------------|--------------|---------|
| 2000–2001| 0.11 (0.01)            | 0.08                  | 0.09                | 1.21         | 0.06    |
| 2001–2002| 0.00 (0.02)            | 0.02                  | 0.00                | 0.85         | 0.05    |
| 2002–2003| 0.05 (0.02)            | 0.02                  | 0.06                | 1.36         | 0.02    |
| 2003–2004| 0.11 (0.02)            | 0.07                  | 0.10                | 0.85         | 0.03    |
| 2004–2005| 0.18 (0.02)            | 0.10                  | 0.15                | 1.51         | 0.04    |
| 2005–2006| 0.14 (0.02)            | 0.11                  | 0.14                | 1.35         | 0.06    |
| 2006–2007| 0.16 (0.02)            | 0.10                  | 0.13                | 1.16         | 0.05    |
| 2007–2008| 0.06 (0.02)            | 0.05                  | 0.05                | 0.48         | 0.13    |
| 2008–2009| 0.01 (0.02)            | 0.02                  | 0.02                | 0.44         | 0.12    |

The associated errors for all three methods is the error of the mean computed with the usual standard method. The bold estimates indicate that the Laplace benchmark cannot be rejected at 1% significance level.

Icelandic firms. Moreover, Fig. 4 shows the yearly disaggregated growth rates distributions and their fitted Laplace. Contrary to the profit rates, the Laplace distribution shows a worst performance in describing the empirical probability density of growth rates. The Laplacian null hypothesis is, in fact, rejected in almost all cases according to the KS test. Following our analysis of profit rates, we employ different methods for the estimation of the measure of central tendency. Interestingly, its statistics seems to have the same pattern as observed for the profit rates, namely during the booming period the value of the KS statistics for the different definitions\(^12\) of \( m \) increases, which

\(^12\) Notice that in the Table 6, we have shown just the KS statistics for \( m_{KS} \). The other values are in additional material upon request.

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coincides with the evident asymmetry of the distribution, signaled by the significant difference between mean and median.

Table 6 includes the yearly inflation rate so to let the reader compute the real growth rate. One can then observe a negative real growth rate in the period 2001–2002 (related to the crash after the dot-com bubble) and in the crash of 2008 and 2009. To a negative real growth corresponds a significantly positive “average” profit rate, which can probably be linked to a shrinking phase of firms in order to increase profitability.

All in all, even if the two unconditional distribution of profit and growth rates might be more or less satisfactorily described by the underlying benchmark distribution, namely the Laplace, they exhibits some remarkable differences. The dispersion of the profit rates is much smaller than the growth rates (which is apparent from a simple comparison between Figs. 3 and 4), and, more interestingly, it seems that the average profit rate is much less volatile than the average growth rate (see Tables 4, 6). It seems that there is a much higher persistence in the profitability of firms than in the way they manage their assets. It might be an indication that the driving force of the dynamics of firms can be found in their profitability and not in the way they grow or shrink over time. In future work we will analyze more in details how far profit and growth rates can be considered proxies and in how far they exhibit different statistical properties.

![Fig. 3](image-url) The figure shows the empirical PDF of profit rates (dots) and fitted Laplace distribution (dashed line), using $m_{Med}$ as the measure of central tendency, of long-lived firms disaggregated by year shown on a semi-log scale for the years 2000–2008. The dominance of the tent shape, characteristic of the Laplace distribution, is clear.
5 Concluding remarks

In this paper we have verified that a measure of size of Icelandic firms, namely total assets, follows a power-law distribution, with an exponent above unity, during the period 2000–2009, in line with previous contribution from the pertinent literature. Although, we find a period within the data set, 2006–2008, which coincides with the peak of the economic boom in Iceland where we reject the null hypothesis that the data follows a power-law distribution. This deviation seems to be related to an inflow of small firms during the booming period of the economy.

Using a very general framework for the Laplacian hypothesis of profit rates distribution introduced by Alfarano et al. (2012), we have shown that the Laplace distribution is a good benchmark for the distribution of profit rates, even in a very small economy where most firms are privately owned as is the case in Iceland. The exception is, as with the power-law distribution of firms size, during the peak of the economic boom. The symmetric Laplace distribution can be thought of as a kind of statistical equilibrium distribution which characterizes a phase of equilibrated growth of an economy. Our tentative hypothesis is that in periods of high growth (bubble) of the economy, this equilibrium is broken in favor of a distribution characterized by significant asymmetry.
Contrary to the profit rates, the Laplacian benchmark for growth rates seems not to adequately describe the empirical probability density of growth rates of Icelandic firms. The Laplacian null hypothesis is, in fact, rejected in almost all cases according to the KS test. Following our analysis of profit rates, we employ different methods for the estimation of the measure of central tendency. Interestingly, its statistics seems to have the same pattern as observed for the profit rates, namely during the booming period the value of the KS statistics for the different definitions of $m$ increases, which coincides with the evident asymmetry of the distribution, signaled by the significant difference between mean and median.

Even if the two unconditional distributions of profit and growth rates might be more or less satisfactorily described by the underlying benchmark distribution, namely the Laplace, they exhibit some remarkable differences. It seems that there is a much higher persistence in the profitability of firms than in the way they manage their assets. It might be an indication that the driving force of the dynamics of firms can be found in their profitability and not in the way they grow or shrink.

In future work, we will analyze in more detail how far profit and growth rates can be considered proxies and how far they exhibit different statistical properties. Additionally, since we observe deviations from the typically identified distributional properties during a high growth of an economy, we can tentatively state that, in period of booming, some distributional stylized facts might not hold. Our hypothesis should be confirmed analyzing other European economies that faced a boom-bust period in recent years, like Spain and Ireland.

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Appendix

In this appendix we present a complementary analysis of the data in order to evaluate whether and how the presence of missing values in the time series of growth and profit rates biases our estimations.

Starting with analysis of profit rates, let us consider individual firms with six or less observations. In other words, we consider the sub-sample of firms with four or more missing values in their individual time series of profit rates. There are 103 firms in this sub-group with a median number of observations per firm equal to 4, with an average number of observations per year approximatively equal to 40. Given the few observations per year, it is statistically meaningless to characterize the yearly distributional properties of this sub-group, and then comparing with the analog quantities of the entire sample of firms to find some systematic differences. In order to increase the statistics (following the suggestion by one of the referees), we pool the data of this sub-group from year 2000 to 2004, given that the median of their yearly profit rates and their absolute mean deviation are fairly stable over time.
Taking into account the pooled sub-sample, we estimate the Subbotin distribution parameters: $\alpha = 0.85 \pm 0.17$, $\sigma = 0.09 \pm 0.01$, the median $0.039 \pm 0.015$ and the mean $0.050 \pm 0.015$. The associated errors refer to the 95% confidence interval. We have repeated the same estimation procedure for a sub-sample of firms with seven or more observations. The parameters of this pooled sub-sample during the years 2000–2004 are: $\alpha = 0.92 \pm 0.07$, $\sigma = 0.070 \pm 0.004$, the median $0.057 \pm 0.005$ and the mean $0.065 \pm 0.005$. Comparing the estimates of the two sub-groups of firms, the sub-sample composed by firms with a significant fraction of missing values tend to have a lower profit rate on average and a higher variability in profitability. Additionally, they tend to be of a smaller size as compared to the pool of firms with a few missing data. All in all, it is hard to draw sharp conclusions based on our data-set whether it exists some systematic characterization of firms releasing infrequently their accounting data.

We have repeated the same procedure for the growth rates. The number of firms with five or less number of observations is 149, with a median number of observation per firm of 3. Therefore we get around 40 number of observations per year, which would not generate a sufficient statistics for any meaningful comparison. Additionally, we cannot aggregate the data like we have done for profit rates, since the large fluctuations in their yearly median.

As an alternative method, we have checked whether the presence of missing values in the profit rates and growth rates affect our estimations of the Subbotin location, scale and shape parameter. If we estimate those parameters at yearly bases, we do not observe any significant difference (at 95% confidence interval) when comparing the estimates based on the entire sample of firms to the sample based just on firms with seven or more observations (in the case of growth rates six or more observations). In Tables 7 and 8 we show the estimates of the relevant parameters of the Subbotin distribution for these sub-samples of our dataset that can be compared with the estimates of Tables 3, 4, 5 and 6, which are instead based on the entire sample. We can conclude that the presence of the missing observations in our sample of Icelandic firms does not affect the estimates of the Subbotin parameters.

### Table 7

| Year | $\alpha$ | $\sigma$ | $m$ |
|------|----------|----------|-----|
| 2000 | 0.93 ± 0.14 | 0.066 ± 0.008 | 0.051 ± 0.008 |
| 2001 | 1.04 ± 0.17 | 0.067 ± 0.008 | 0.059 ± 0.008 |
| 2002 | 0.96 ± 0.16 | 0.069 ± 0.008 | 0.052 ± 0.008 |
| 2003 | 0.74 ± 0.12 | 0.077 ± 0.009 | 0.055 ± 0.008 |
| 2004 | 1.01 ± 0.18 | 0.072 ± 0.009 | 0.058 ± 0.008 |
| 2005 | 0.91 ± 0.15 | 0.091 ± 0.011 | 0.060 ± 0.012 |
| 2006 | 1.01 ± 0.16 | 0.086 ± 0.010 | 0.082 ± 0.010 |
| 2007 | 0.87 ± 0.15 | 0.087 ± 0.010 | 0.066 ± 0.010 |
| 2008 | 0.87 ± 0.15 | 0.109 ± 0.014 | 0.086 ± 0.014 |
| 2009 | 0.85 ± 0.24 | 0.087 ± 0.018 | 0.080 ± 0.018 |
Table 8  The table shows the estimates of the shape, scale and location parameter (median) of the Subbotin distribution based on the sub-sample of firms having six or more growth rates

| Year | $\alpha$   | $\sigma$   | $m_{Med}$ |
|------|------------|------------|-----------|
| 2000 | 0.78 ± 0.16| 0.16 ± 0.02| 0.08 ± 0.02|
| 2001 | 0.93 ± 0.18| 0.16 ± 0.02| 0.00 ± 0.02|
| 2002 | 0.85 ± 0.15| 0.16 ± 0.02| 0.02 ± 0.02|
| 2003 | 1.02 ± 0.20| 0.18 ± 0.02| 0.08 ± 0.02|
| 2004 | 1.06 ± 0.20| 0.19 ± 0.02| 0.10 ± 0.03|
| 2005 | 1.14 ± 0.20| 0.18 ± 0.02| 0.10 ± 0.03|
| 2006 | 1.07 ± 0.20| 0.20 ± 0.03| 0.09 ± 0.03|
| 2007 | 1.00 ± 0.20| 0.19 ± 0.03| 0.05 ± 0.03|
| 2008 | 0.78 ± 0.24| 0.12 ± 0.03| 0.02 ± 0.03|

not significantly impact our estimates for the distributional properties of growth and profit rates.

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