Floquet Weyl semimetal induced by off-resonant light

RUI WANG, BAIGENG WANG, RUI SHEN, L. SHENG and D. Y. XING

National Laboratory of Solid State Microstructures and Department of Physics, Nanjing University
Nanjing 210093, China

received 14 November 2013; accepted in final form 3 January 2014
published online 31 January 2014

PACS 73.20.At – Surface states, band structure, electron density of states
PACS 73.43.Nq – Quantum phase transitions
PACS 03.65.Vf – Phases: geometric; dynamic or topological

Abstract – We propose that a Floquet Weyl semimetal state can be induced in three-dimensional
topological insulators, either non-magnetic or magnetic, by the application of off-resonant light.
The virtual photon processes play a critical role in renormalizing the Dirac mass and so resulting
in a topological semimetal with vanishing gap at Weyl points. The present mechanism via off-
resonant light is quite different from that via on-resonant light, the latter being recently suggested
to give rise to a Floquet topological state in ordinary band insulators.

Topological states of matter, such as two-dimensional (2D) quantum spin Hall insulators or 3D topological insulators (TIs) [1–8], have received great interest in recent years. Searching for such states in solid-state materials [8–10] has partly gained success; however, candidate materials for TIs are still very limited now. Inspiringly, an intriguing method was put forward to realize topologically non-trivial phases in non-equilibrium by applying time-dependent perturbations to trivial phases [11–16]. Typical examples are the optically activated anomalous Hall effect and spin Hall effect in n-doped paramagnetic semiconductors [14], and the so-called Floquet topological insulator (FTI) suggested by Lindner, Refael, and Galitski [16], whose quasi-energy spectrum exhibits a single pair of helical edge states due to the on-resonant-light–induced band inversion. In the bulk FTI spectrum there is an avoided crossing separating the reshuffled valence band from the conduction band.

Despite these interesting results, which mainly focus on the photon-dressed quasi-energy spectrum, one also needs to pay attention to the other effect of the external field, i.e., it leads to redistribution of electron occupation numbers through the absorption/emission of photons. This non-equilibrium distribution also affects the topological properties. To this end, we suggest that the off-resonant light is more favorable [11], since electrons cannot be directly excited through absorbing photons, which will be discussed later in this letter.

In parallel with the study of the photon-dressed topological phase, recently, another type of topological phase termed the Weyl semimetal (WSM) [17] has attracted much attention. Different from topological insulators, whose bulk energy spectrum is gapped, the WSM has no bulk gap but enjoys gapless nodes distributed in momentum space. If one regards the TIs as massive Dirac fermion systems with lattice regularization [18], then the WSM phase should be viewed as a system of massless Weyl fermions. Since a massless Dirac spinor can be decomposed into two copies of Weyl spinors with opposite chiralities, it is expected that if one were to generate the WSM phase from TIs, there are two conditions need to be satisfied. First, we have to make the massive fermions massless and this generates the gapless dispersion nodes. Second, certain symmetries, such as time reversal symmetry (TRS) or inversion symmetry (IS), are required to be broken [19]. This further turns the gapless nodes into non-degenerate accidental band-touchings, i.e., Weyl points. Similar to magnetic monopoles, Weyl points are sources of the Berry curvature in momentum space and they can only be annihilated in pairs with opposite chiralities. This explains the topological stability of the WSM phase, which is distinct from TIs, whose robustness is due to the bulk insulating gap. Besides, WSM possesses chiral surface states and Fermi arcs terminating at Weyl points with opposite chiralities. Moreover, since it is well known that chiral Weyl fermions are associated with the phenomenon of chiral anomaly, WSM is predicted to enjoy remarkable electromagnetic properties such as the anomalous Hall effect [20,21] and the chiral magnetic effect [22]. This can be explained by a local axion field $\theta(r,t)$ in a topological $\theta$-term [23], which is distinct from 3D TIs, where $\theta = \pi$ [24]. Due to these novel electromagnetic
properties, WSM is regarded as a promising candidate for future applications in spintronics. Unfortunately, despite several proposals have been put forward in various systems [17,21,25], the WSM has not been realized experimentally yet. As a result, theoretical proposals on the WSM state, that are feasible for experimental realization, are still highly desirable.

In this letter, we show that a non-equilibrium WSM state can be induced in 3D topological insulators by the application of off-resonant light, using the Floquet picture. In contrast to the on-resonant optical induction in ref. [16], we focus on the off-resonant effect of light on band structures, and the underlying physics is outlined as follows. Consider a two-band system with bulk energy gap \( E_g \) under the application of off-resonant light. For light with off-resonant frequency \( \omega \), the real process of photon absorption/emission cannot occur because of the limitation of the energy conservation condition. However, the off-resonant light can affect the electron system via virtual photon processes, e.g., the second-order processes shown in fig. 1(a), where electrons absorb and then emit a photon and electrons first emit and absorb a photon. Such virtual photon processes have at least two effects. First, they can break the TRS if the driven field is non-linearly polarized. Second, they can generate a self-energy \( \Sigma \) that renormalizes the Dirac mass and the so the bulk energy gap \( E_g(A) \) of the electron system. It will be shown that \( E_g(A) \) first decreases with the light field amplitude \( A \) and vanishes at threshold \( A_1 \), as shown in fig. 1(b). Once the light field amplitude is increased to another threshold \( A_2 \), the bulk gap reopens and \( E_g(A) \) increases with further increasing \( A \). For \( A_1 < A < A_2 \), the bulk energy spectrum is gapless at Weyl points, and the system will be shown to be a Floquet WSM.

We start from considering a 3D magnetic topological insulator, which can be described by a modified Dirac Hamiltonian [26]

\[
H(k) = \sum_{i=1}^{3} a_{k_i} \Gamma_i + M\Gamma_4 + m\Gamma'.
\]

The first two terms describe the non-magnetic 3D TI, where \( \Gamma_i \) (\( i = 1, \ldots, 4 \)) are the Dirac matrices that satisfy Clifford algebra in Euclidean space: \( \{ \Gamma_i, \Gamma_j \} = \delta_{ij} \). The Dirac mass is given by \( M = M_0 + b k^2 \). It is well known that \( M_0 b < 0 \) guarantees a topologically non-trivial phase while \( M_0 b > 0 \) suggests a normal insulator (NI) [18,27]. To be explicit, we choose \( M_0 > 0 \) and \( b < 0 \) to describe a topological non-trivial state. Once \( \Gamma_1, \ldots, \Gamma_4 \) are specified, the other Dirac matrices can be constructed as \( \Gamma_0 = \Gamma_4 \times \Gamma_5 = \prod_{i=0}^{5} \Gamma_i \), and \( \Gamma_{ij} = -\frac{i}{2} \Gamma_i \Gamma_j \). The last term represents a time-reversal-breaking exchange field with \( \Gamma' \) satisfying \( [\Gamma', \Gamma_i] = 0 \). A competition between \( m\Gamma' \) and \( M\Gamma_4 \) will enable the surface Dirac fermions to acquire mass, leading to a quantum anomalous Hall (QAH) effect. The choice of \( \Gamma' \) depends on the real physical problem that eq. (1) describes. In general, two different types of phases are likely to be generated under the driven field. One is the WSM phase, characterized by pairs of Weyl nodes with opposite chiralities, the other is the line nodal semimetal (LNSM) that enjoys gapless line nodes in momentum space. We prefer to focus on the more interesting WSM phase due to the following reason: contrary to the topologically stable Weyl nodes (with codimension of three), the line nodes Fermi surface (with codimension of two) generically belongs to the \( \mathbb{Z}_2 \) homotopy group so that its stability is ensured by symmetry but not topology. Therefore, we choose \( \Gamma' = \Gamma_2 \) for the purpose of clarity, which favors a WSM phase under a driven field.

An in-plane polarized electric field can be expressed by its vector potential \( \mathbf{A} = (A_x \sin \omega t, A_y \sin(\omega t + \phi), 0) \) with \( \omega \) as the frequency of the electric field and \( \phi \) describing the polarization direction. For a spatially uniform electromagnetic field, the effect on the spin degree of freedom may be negligible while the effect on the orbital degree of freedom can be included through the substitution \( H(k) \rightarrow H(k + \mathbf{A}(t)) \), yielding

\[
H(k, t) = H(k) + \mathbf{V}(t) \cdot \mathbf{\Gamma}.
\]

Here the second term is the time-dependent perturbation induced by the driven field with \( \mathbf{\Gamma} = (\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4) \) and \( \mathbf{V}(t) = (aA_x \sin \omega t, aA_y \sin(\omega t + \phi), 0, bA_x^2 \sin^2 \omega t + bA_y^2 \sin^2(\omega t + \phi) + 2bA_x A_y \sin \omega t + 2bA_x A_y \sin(\omega t + \phi)) \).

Due to the discrete time- translational invariance of \( H(k, t) \), the eigenvectors can be written as \( |\psi_{\alpha}(t)| = e^{-i\epsilon_{\alpha}(t)}|\phi_{\alpha}(t)| \), where \( |\phi_{\alpha}(t)| = |\phi_{\alpha}(t + T)| \) is the Floquet state with \( T = 2\pi/\omega \) and \( \epsilon_{\alpha} \) is the quasi-energy restricted in the range of \(-\omega/2 < \epsilon_{\alpha} < \omega/2\). In the framework of the Floquet theory, the time-dependent Hamiltonian eq. (2) can be mapped onto a time-independent Floquet operator \( F = \Omega \otimes H_{1 \times 4} \) in the Sambe space [28], with \( \Omega \) as an infinite-dimensional matrix in the Floquet band space and \( H_{1 \times 4} \) in the basis of \( H(k) \) of the undriven system. The matrix element of the Floquet operator is given by

\[
F_{m,n}(k) = H(k) + \mathbf{\Gamma} \cdot \mathbf{V}_{m,n} - n\omega\delta_{mn}.
\]
where \( \mathbf{V}_{m,n} = \frac{1}{T} \int_{-T/2}^{T/2} dt \mathbf{V}(t)e^{-i(n-m)\omega t} \), and \( m \) and \( n \) denote the indices of Floquet bands. In principle, an exact diagonalization of the Floquet operator can give all the quasi-energy spectra. In order to obtain analytic results, we perform a perturbation theory, which holds for \( a^2 A_x A_y/\hbar \omega \ll 1 \) (see footnote 1), to obtain the effective Floquet operator \([11]\): \( F_{\text{eff}} = F_{0,0} + [F_{-1,0}, F_{1,0}]/\omega \). Both terms \( F_{0,0} \) and \( [F_{-1,0}, F_{1,0}]/\omega \) arise from the second-order virtual photon processes shown in fig. 1(a), where a photon is first absorbed (released) and then released (absorbed).

As will be demonstrated below, \( [F_{-1,0}, F_{1,0}]/\omega \) breaks the TRS for a non-linearly polarized electric field, while \( F_{0,0} \) differs from the undriven Hamiltonian \( H(k) \) by a modification \( \Sigma \) that renormalizes the Dirac mass and generates Weyl points.

Using the perturbation theory, we obtain the effective Floquet operator around the \( \Gamma \) point in momentum space as

\[
F_{\text{eff}}(k) = \sum_{i=1}^{3} a_k \Gamma_i + \tilde{M} \Gamma_4 + \tilde{m} \Gamma',
\]

for the driven system, where \( \tilde{m} = m + a^2 (A_x A_y/\omega) \phi \) and \( \tilde{M} = \tilde{M}_0 + b k^2 \), with \( \tilde{M}_0 = M_0 + b (A_x^2 + A_y^2)/2 \). Comparing eq. (4) with eq. (1) for the undriven system, one finds two important differences between them. First, the renormalized exchange field is enhanced due to the term \( [F_{-1,0}, F_{1,0}]/\omega \). For a non-linearly polarized field (\( \phi \neq 0 \)), the TRS is broken even though there is no exchange field \( (m = 0) \). Second, the Dirac mass is dressed due to the term \( F_{0,0} \). Since the parameters have been chosen to be \( M_0 > 0 \) and \( b > 0 \), \( \tilde{M}_0 \) is effectively reduced. Since \( \tilde{m} > m \) and \( \tilde{M} < M \), it is expected that the competition between \( \tilde{m} \Gamma' \) and \( \tilde{M} \Gamma_4 \) is favorable to the emergence of the QAH phase and even of the WSM phase. If the field intensity is large enough to make \( \tilde{M}_0 \) change its sign from positive to negative so that \( \tilde{M}_0 b > 0 \), the system can be driven to be a topologically trivial insulator phase.

To be more explicit, we now represent the Dirac matrices as \((\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4) = (\tau^x s^x, \tau^y s^y, \tau^z s^z, -\tau^z s^y)\), where \( s (\tau) \) is the Pauli matrix in spin (orbital) space. In this representation, eq. (4) reads

\[
F_{\text{eff}}(k) = a \tau^x (s^x k_x + s^y k_y + s^z k_z) - \tilde{M} \tau^z s^0 + \tilde{m} \tau^0 s^z. \tag{5}
\]

In the absence of the last term which describes a magnetization orthogonal to the plane of the sample, eq. (5) is an isotropic version of the model in ref. [8], which describes the 3D TI Bi$_2$Se$_3$. To see the possibility of the topological phase transition, we first make a rotation in orbital space: \( \tau^x \rightarrow \tau^z \) and \( \tau^z \rightarrow \tau^x \), and then a canonical transformation: \( \tau^x \rightarrow s^x \tau^x + s^z \tau^z \) and \( \tau^z \rightarrow s^z \tau^z + s^x \tau^x \). After \( F_{\text{eff}}(k) \) in eq. (5) is diagonalized in orbital space, we have

\[
F_{\pm}(k) = a (s^x k_x + s^y k_y + s^z k_z) \pm \tilde{m} \Delta_{\pm}(k), \tag{6}
\]

with \( \Delta_{\pm}(k) = \tilde{m} \pm \sqrt{\tilde{M}^2 + a^2 k^2} \). For a fixed \( k_z \), eq. (6) is a 2D Dirac Hamiltonian. It is well known that a sign change of the 2D Dirac mass signals a topological phase transition, characterized by a change of the first Chern number, which provides a method to distinguish different phases. From \( \Delta_{\pm}(0,0,k_z = 0) = 0 \), we have a solution \( k_z = \pm k_{zc} \) with \( ak_{zc} = \sqrt{\tilde{m}^2 - \tilde{M}^2} \). For \( \tilde{m}^2 > \tilde{M}^2 \), there are two real roots \( \pm k_{zc} \) for \( k_z \), and \( (0,0, \pm k_{zc}) \) are just the two Weyl points in momentum space of the WSM. Furthermore, an expansion of Floquet operator \( F_{-} \) in eq. 6 near the two gapless nodes \( \pm k_{zc} \) yields

\[
F_{-}(k) = a \tau^x q_x - a \tau^y q_y \mp a \sqrt{\tilde{m}^2 - \tilde{M}^2} \tau^z q_z, \tag{7}
\]

which is the same as the Hamiltonian defining the Weyl fermions with chirality \( C = \pm 1 \). Here \( bk^2 \) in the Dirac mass is temporarily neglected for clarity. On the other hand, for \( \tilde{m}^2 < \tilde{M}^2 \), there exists no real root of \( k_{zc} \) for \( \Delta_{\pm}(0,0,k_z = 0) = 0 \) and so the quasi-energy spectrum is gapped throughout the momentum space. In this case, either QAH or NI phase is possible, depending on whether \( \tilde{M}_0 b < 0 \) or \( \tilde{M}_0 b > 0 \). As a result, there is a topological phase transition at \( \tilde{m}^2 = \tilde{M}^2 \) from the WSM phase to either QAH or NI phase.

From eq. (5) we have numerically calculated the bulk quasi-energy dispersion in a single Floquet band for different polarizations. We first consider the case of \( m = 0 \) and \( \phi = \pi/2 \) (circularly polarized light). It is found that with increasing the off-resonant light intensity, the energy gap \( E_g \) of the undriven TI becomes less and less, and vanishes at \( k_{zc} = 0 \). With further increasing \( A \), a pair of Weyl points \( \pm k_{zc} \) is separated from each other, as shown in fig. 2(a). A continued increase of \( A \) can make the two Weyl points merge into a single Dirac point at \( k_{zc} = 0 \), and then the gap reopens, resulting in a topologically trivial insulator. Figure 2(b) is the calculated phase diagram, where the WSM phase is in between the QAH and NI phases. Second, we consider the linearly polarized case of \( \phi = 0 \) with \( m \neq 0 \) taken. Similar topological phase transitions are obtained, as shown in fig. 3. If \( \phi = 0 \) and \( m = 0 \), the system still has the TRS. In this case, there is no WSM.

---

1This condition is not necessary in the lattice model, because the vector potential enters into \( F_{m,n} \) through \( (m-n) \)th order Bessel functions, which decay oscillatorily.
A symmetry, which can be represented as $\hat{\Theta} = \Theta_0$. Figure 4(a) shows the phase diagram, in which there is a transited phase in the phase diagram, and the TI phase is transited to the NI phase under the application of off-resonant light.

In what follows we study a more realistic diamond lattice model to support the above results. Its tight-binding Hamiltonian is given by

$$
H = t \sum_{\langle ij \rangle} c_i^\dagger c_j + i(8\lambda_{SO}/a^2) \sum_{\langle ij \rangle} c_i^\dagger \mathbf{s} \cdot (\mathbf{d}_i^x \times \mathbf{d}_j^x) c_j. \quad (8)
$$

This model was introduced to describe 3D TI [5]. Here we take $t_1 = t + \delta t$ and $t_2 = t_3 = t_4 = t$ with the distortion $\delta t$ along the (111) direction. For $\delta t > 0$, the system lies in the strong topological insulator (STI) phase with $Z_2$ index as $\nu_0 = 1$, and $\delta t < 0$ corresponds to a weak topological insulator (WTI) phase with $\nu_0 = 0$.

First, we do not consider Zeeman splitting and only focus on the linearly polarized electric field $\mathbf{A} = (A_x \sin \omega t, A_y \sin \omega t, 0)$. Using the same procedure as above, we obtain the Floquet operator in Sambe space as

$$
F_{m,n}(\mathbf{k}) = \Gamma \cdot \mathbf{D}_q(\mathbf{k}) - \omega \delta_{mn}, \quad (9)
$$

where $m$ and $n$ denote different Floquet bands and $q = m - n$. The Dirac matrices in this case are represented as $\Gamma = (\tau^2 s_x^2, \tau^2 s_y^2, \tau^2 s_z^2, \tau^2 s_0^0, \tau^2 s_0^0)$, where $\tau$ and $s$ are the Pauli matrices associated with the A-B sublattice and true spin subspace, respectively; and $\mathbf{D}_q(\mathbf{k}) = (d_{1q}(\mathbf{k}), d_{2q}(\mathbf{k}), d_{3q}(\mathbf{k}), d_{4q}(\mathbf{k}), d_{5q}(\mathbf{k}))$, where $d_{ijq}(\mathbf{k})$ are expressed via the $q$-th order Bessel functions, which can be obtained by first Fourier-transforming eq. (8) and then mapping it onto the Floquet operator in Sambe space. In the off-resonant region of $\omega > 8t$, we set $m = n = 0$ in eq. (9) to obtain $F_{0,0}$, which is the most relevant contribution from all the virtual photon processes.

In eq. (9) for $q = 0$, both the TRS and the inversion symmetry, which can be represented as $\hat{\Theta} = is^y K$ and $\hat{P} = \tau^x s^x$, respectively [7], remain unchanged. Therefore, the $Z_2$ index of the driven Hamiltonian can be conveniently calculated, which is modulated by the off-resonant light. Figure 4(a) shows the phase diagram, in which there is a topological phase transition between the STI and WTI phases.

Next, we take the Zeeman splitting into account with the term $m\tau^4 s^2$ included in eq. (9). The eigenvalues are obtained as $E_{\pm}^2 = d_1^2 + d_2^2 + (\sqrt{d_3^2 + d_4^2 + d_5^2} \pm m)^2$. Note that there are gapless nodes only for $E_{-}$, which requires $d_1 = d_2 = 0$ and $m = \sqrt{d_3^2 + d_4^2 + d_5^2}$. There are three equations that determine three variables $k_x$, $k_y$, and $k_z$. The solutions for them are exactly the Weyl nodes in momentum space. To make this argument more convincing, we expand the Floquet operator around point $X^2 = (0, 0, 2\pi)$ and then make the canonical transformation $s^\pm \rightarrow \tau^x s^\pm$ and $\tau^\pm \rightarrow \tau^\pm s^2$ to obtain the effective Hamiltonian as

$$
H_{\text{eff}} = \lambda_{SO}(s^x q_x - s^y q_y) + s^2 \Delta^\pm(\mathbf{q}; \mathbf{A}). \quad (10)
$$

Here the effect of the light field $\mathbf{A}$ has been absorbed into the Dirac mass: $\Delta^\pm(\mathbf{q}; \mathbf{A}) = m \pm [M_2^2 + h_{ab}q_a q_b]^{1/2}$ with $M_2 = \sum_{i=1}^N \text{sgn}(e_i \cdot \hat{z})$, where $e_i$ is the nearest bond vector of the diamond lattice and $h_{ab}$ is a tensor describing the anisotropy that is determined by field parameters $A_x$ and $A_y$. Equation (10) is of the same form as eq. (6), so that a similar analysis can be made here. The numerical results searching for different phases are shown in fig. 4(b), in which the WSM phase is between the QAH and magnetic WTI phases.

Now we discuss the necessary experimental conditions to realize the above theoretical results. We focus on the following three different aspects. First, in order to take advantage of the off-resonant process, the frequency of the electric field should be larger than the insulating bulk gap, that is of the order of $10^2$ THz (the wave length $\lambda = 1250$ nm) for Bi$_2$Se$_3$ and $10^3$ THz for the lattice model.

Second, the field intensity has to cross the topological phase transition line as shown in fig. 3, i.e., $A_0 > 0.1$ Å$^{-1}$. In terms of the electric field amplitude $E_0$, it should be of the order of $0.1$ V Å$^{-1}$. It needs to be stressed that the amplitude threshold will be greatly reduced for a TI material with a smaller $M_0$ and a larger $|b|$. For example, $E_0$ is reduced to $10^{-4}$ V Å$^{-1}$ for $M_0 \sim 0.01$ eV and $b \sim -100$ eV Å$^2$.

Third, the thickness (along the $z$-direction) of the bulk material has to be carefully chosen. On the one hand,
it should be less than the penetration depth so that the absorption of light can be neglected. On the other hand, the sample should not be too thin so that the finite-size quantization of $k_z$ will not hamper the observation of the WSM phase. In order to make this point clear, we suggest applying two beams of linearly polarized light, with the wave length to be around 795 nm, to the magnetically doped Bi$_2$Se$_3$ from the two opposite directions along the $z$-axis. We also demand the magnetization to be in the $x$-$y$ plane of the sample. Therefore eq. (5) turns into

$$F_{\text{eff}}(k) = ax^2(s^2k_x + s^2k_y + s^2k_z) - M\tau^2s_0 + m\tau^0s_z$$ (11)

which supports the two Weyl points separated in momentum space along $k_z$. According to recent experiments [29,30], the penetration depth $\xi$ is about 25 nm. Therefore, a sample with thickness $d \sim 50$ nm can be completely penetrated, within which at least $N \sim 50$ quintuple layers (QLs) can be grown along the trigonal axis [31]. Now due to the finite-size effect, the electrons form the standing waves in the $z$-direction, and $k_z$ is discretized to be $k_z = \pi n/2N$, where $a$ is the lattice constant and $n = \pm 1, \pm 2, \ldots, \pm N/2$. Even though the wave vector $k_z$ is coarse-grained, $N$ is large enough to preserve the general dispersion as well as the three-dimensional feature. Moreover, since the Weyl points are now split along $k_z$, their observation will not be affected by the momentum quantization along $k_z$.

In terms of observation, the most direct way may be detecting the Weyl points in the bulk spectrum using time-and-angle–resolved photoemission spectroscopy (TrARPES), which seems to be promising due to a recent experimental report [32]. Besides, we suggest performing transport experiments to measure the quantum Hall conductance. Since the photon-induced WSM is a non-equilibrium phase, not only the quasi-energy spectrum but also the non-equilibrium distribution of electrons will affect the transport properties, which complicates the calculation. However, in the case of off-resonant perturbations, where electrons cannot absorb photons directly, it has been justified that the transport properties of the non-equilibrium systems attached to the leads are consistent with those described by the static effective Hamiltonian (see eq. (4)) which incorporates the virtual photon processes [11]. Therefore, a quantum Hall conductance of the following form is expected [20,21]:

$$\sigma_{xy} = \frac{e^2}{2\pi \hbar} 2k_{zc}$$ (12)

$2k_{zc}$ is the distance between the two Weyl points in momentum space, which is modulated by external light. With increasing the off-resonant light intensity continuously, the Hall conductance will arise from zero and then increase to the maximum value, after which it will decrease to zero again.

In conclusion, we have shown that the Floquet WSM phase can be induced in 3D TIs by the use of the off-resonant light. The virtual photon absorption/emission processes play a key role in renormalizing the Dirac mass, in closing the bulk gap, as well as in breaking the TRS, resulting in a non-trivial WSM phase. From both the continuous and lattice models of the Floquet theory, very similar phase diagrams have been obtained, as shown in figs. 2(b), 3(b), and 4(b). With increasing the light intensity, the QAH phase first transits to the WSM phase and then to the NI or magnetic WTI phase. Moreover, the feasibility of the low-energy effective theory in the first part can be justified by both the consideration of the general Floquet theory and the results from the lattice model in the second part. It can be concluded that the higher-energy Floquet spectrum in real materials is irrelevant in shaping the low-energy dispersion, therefore the Floquet WSM phase is stable. Besides, according to recent experiments such as the observation of the QAH effect in magnetic TIs [33] and the realization of Floquet topological insulators in a photonic system [34], the present proposal of the Floquet WSM state can be realized in experiments. In addition, we wish to point out that the generalized Dirac Hamiltonian, eq. (1), can describe not only the 3D TIs, but also the other systems such as the polyacetylene, $p$-wave pairing superconductor, and $^3$He-A and $^3$He-B phases [26]. Therefore, it is expected that the light field may also induce topological phase transitions in those systems.

***

This work is supported by the State Key Program for Basic Research of China under Grant No. 2011CB922103, and by the National Natural Science Foundation of China under Grants No. 60825402, No. 11023002, and No. 91021003.

REFERENCES

[1] Kane C. L. and Mele E. J., Phys. Rev. Lett., 95 (2005) 226801.
[2] Kane C. L. and Mele E. J., Phys. Rev. Lett., 95 (2005) 146802.
[3] Bernevig B. A. and Zhang S. C., Phys. Rev. Lett., 96 (2006) 106802.
[4] Wu C. J., Bernevig B. A. and Zhang S. C., Phys. Rev. Lett., 96 (2006) 106401.
[5] Fu L., Kane C. L. and Mele E. J., Phys. Rev. Lett., 98 (2007) 106803.
[6] Moore J. E. and Balents L., Phys. Rev. B, 75 (2007) 121306.
[7] Fu L. and Kane C. L., Phys. Rev. B, 76 (2007) 045302.
[8] Zhang H., Liu C.-X., Qi X.-L., Dai X., Fang Z. and Zhang S.-C., Nat. Phys., 5 (2009) 438.
[9] Bernevig B. A., Hughes T. L. and Zhang S.-C., Science, 314 (2006) 1757.
[10] Hsieh D., Qian D., Wray L., Xia Y., Hor Y. S., Cava R. J. and Hasen M. Z., Nature, 452 (2008) 970.
[11] Kitagawa T., Oka T., Brataas A., Fu L. and Demler E., Phys. Rev. B, 84 (2011) 235108.
[12] Gu Z. H., Ferigo H. A., Arovas D. P. and Auerbach A., Phys. Rev. Lett., 107 (2011) 216601.
[13] **Eric Suárez Morell** and **Luis E. F. Foa Torres**, *Phys. Rev. B*, **86** (2012) 125449.

[14] **Yao W.**, **MacDonald A. H.** and **Niu Q.**, *Phys. Rev. Lett.*, **99** (2011) 047401.

[15] **Kitagawa T.**, **Berg E.**, **Rudner M.** and **Demler E.**, *Phys. Rev. B*, **82** (2010) 235114.

[16] **Lindner N. H.**, **Refael G.** and **Galitski V.**, *Nat. Phys.*, **7** (2011) 490.

[17] **Wan X. G.**, **Turner A. M.**, **Vishwanath A.** and **Savrasov S. Y.**, *Phys. Rev. B*, **83** (2011) 205101.

[18] **Qi X.-L.** and **Zhang S.-C.**, *Rev. Mod. Phys.*, **83** (2011) 1057.

[19] **Murakami S.**, *New J. Phys.*, **9** (2007) 356.

[20] **Yang K. Y.**, **Lu Y. M.** and **Ran Y.**, *Phys. Rev. B*, **84** (2011) 075129.

[21] **Burkov A. A.** and **Balents L.**, *Phys. Rev. Lett.*, **107** (2011) 127205.

[22] **Fukushima K.**, **Kharzeev D. E.** and **Warringa H. J.**, *Phys. Rev. D*, **78** (2008) 074033.

[23] **Chen Y.**, **Wu S.** and **Burkov A. A.**, *Phys. Rev. B*, **88** (2013) 125105.

[24] **Qi X.-L.**, **Hughes T. L.** and **Zhang S.-C.**, *Phys. Rev. B*, **78** (2008) 195424.

[25] **Cho G. Y.**, arXiv:1110.1939.

[26] **Shen S. Q.**, *Topological Insulators - Dirac Equation in Condensed Matters, Springer Series in Solid-State Sciences*, Vol. 174 (Springer) 2012.

[27] **Lu H.-Z.**, **Shan W.-Y.**, **Yao W.**, **Niu Q.** and **Shen S.-Q.**, *Phys. Rev. B*, **81** (2010) 115407.

[28] **Sambe H.**, *Phys. Rev. A*, **7** (1973) 2203.

[29] **McIver J. W. et al.**, *Phys. Rev. B*, **86** (2012) 035327.

[30] **Glinka Y. D. et al.**, *Appl. Phys. Lett.*, **103** (2013) 151903.

[31] **Zhang W.**, **Yu R.**, **Zhang H.-J.**, **Dai X.** and **Fang Z.**, *New J. Phys.*, **12** (2010) 065013.

[32] **Wang Y. H.**, **Steinberg H.**, **Herrero P. J.** and **Gedik N.**, *Science*, **342** (2013) 25.

[33] **Chang C. Z. et al.**, *Science*, **340** (2013) 167.

[34] **Rechtsman M. C. et al.**, *Nature*, **496** (2013) 196.