A Specialized Evolutionary Strategy Using Mean Absolute Error Random Sampling to Design Recurrent Neural Networks

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Recurrent neural networks have demonstrated to be good at solving prediction problems. However, finding a network that suits a problem is quite hard because of their high sensitivity to the hyperparameter configuration. Automatic hyperparameter optimization methods help to find the most suitable configuration, but they are not extensively adopted because of their high computational cost. In this work, we study the use of the mean absolute error random sampling to compare multiple-hidden-layer architectures and propose an evolutionary strategy-based algorithm that uses its results to optimize the configuration of a recurrent network. We empirically validate our proposal and show that it is possible to predict and compare the expected performance of a hyperparameter configuration in a low-cost way, as well as use these predictions to optimize the configuration of a recurrent network.

Index Terms—neuroevolution, metaheuristics, recurrent neural network, evolutionary strategy

I. INTRODUCTION

In the last decade, Machine Learning (ML) have gained significant interest, especially through Deep Learning (DL). Particularly, DL focuses on learning features from data using multiple layers of abstraction, i.e., Deep Neural Networks (DNNs) [1], and thanks to this characteristic, DL has been able to dramatically improve the state-of-the-art of several pattern recognition and prediction problems [2], [3].

There are several types of DNNs, where each one is suited for solving a specific problem. Among these network types, Recurrent Neural Networks (RNNs) are especially good at solving sequential modeling and prediction, e.g., natural language, image, and speech recognition and modeling [1]. Basically, RNNs are feedforward networks that include feedback connections between layers and neurons, and this recurrence allows them to capture long-term dependency in the input.

Despite of their great performance, RNNs have a drawback, they are hard to train, because of the vanishing and the exploding gradient problems [4], [5].

An alternative to mitigate the problems related to DNNs training is to optimize the hyperparameters of a network. By selecting an appropriate configuration of the parameters of the network (e.g., the activation functions, the number of hidden layers, the kernel size of a layer, etc.), it is tailored to the problem and by this mean the performance is improved [6], [7], [8]. DNN hyperparameter optimization methods can be grouped into two main groups: the manual exploration-based approaches, usually, lead by expert knowledge, and the automatic search-based methods (e.g., grid, evolutionary or random search) [9].

The number of alternatives or parameters to configure a DNN are huge. Thus the hyperparameter optimization has to deal with a high-dimensional search space. In spite of this size, most methods (manual and automatic) are based on trial-and-error, meaning that each hyperparameter configuration is trained and tested to evaluate its numerical accuracy. Thus, the high-dimensional search space and the high cost of the evaluation limit the results of this methodology.

Some authors have explored different approaches to speed up the evaluation of DNN architectures to improve the efficiency of automatic hyperparameter optimization algorithms [10], [11].

A promising approach to evaluate stacked RNN architectures is the MAE (mean absolute error) random sampling [10], [12]. The main idea behind this method, inspired by the linear time-invariant theory, is to infer the numerical accuracy of a given network without actually training it. Given an input, several sets of random weights are generated and analyzed measuring the MAE. Then, the probability of finding a set of weights whose MAE is below a predefined threshold is estimated.

In this work, we study the extension of the MAE random sampling to multiple-hidden-layer networks [12], and we put forward the use of this approach as a heuristic to optimize the architecture. Particularly, we propose to use a meta-heuristic to navigate through the architectures space and guide this search using the MAE random sampling. Specifically, we propose an evolutionary strategy (ES) based algorithm. Finally, once we have found a “final” solution, we propose to train it using a gradient descent-based method.

The remainder of this paper is organized as follows: the next section outlines the related work. Section [13] introduces the multiple-hidden-layer extension of the MAE random sampling [12]. Section [15] presents the ES-based optimization algorithm. Sections [14] and [16] present the experimental results. Finally, Section [17] discusses the conclusions drawn from this study and proposes future work.
II. RELATED WORK

This section outlines the most outstanding studies related to our present work. First, we introduce the related work regarding RNNs hyperparameter optimization and evaluation. Then, we present research studies that apply meta-heuristics to address DL optimization.

A. RNN hyperparameters Optimization

An RNN is a network that incorporates recurrent (or feedback) edges that may form cycles and self connections. This approach introduces the notion of time to the model. Thus, at a time \( t \), a node connected to a recurrent edge receives input from the current data point \( x_t \) and also from the hidden node \( h_{t-1} \) (the previous state of the network). The output \( y_t \) at each time \( t \) is computed according to the hidden node values at time \( t \) (\( y_t \)). Input at time \( t - 1 \) (\( x_{t-1} \)) can determine the output at time \( t \) (\( y_t \)) and later by way of recurrent connections [13].

The majority of DL approaches to train a network are based on gradient-based optimization procedures, e.g., using a local numerical optimization such as stochastic gradient descent or second-order methods. However, these methods are not suitable for RNNs. The main issue with gradient-based approaches is that they keep a vector of activations, which makes RNNs extremely deep and aggravates the exploding and the vanishing gradient problems [4], [14], [5].

More recently, Long Short-Term Memory (LSTM) has emerged as a specific type of RNN architecture, which contains special units called memory blocks in the recurrent hidden layer [15]. LSTM mitigates gradient problems, and therefore, they are easier to train than standard RNNs. However, not only the network architecture affects the learning process but also the weight initialization [16] as well as all specific parameters of the optimization algorithm [17].

Therefore, to cope with the learning process as a whole, some authors [6], [7], [8] have proposed to perform an hyperparameter optimization. Specifically, they propose to look for a specific architecture (the number of layers, the number of hidden unit per layer, the activation function, etc.) and a set of parameters to train the network that improve the performance of the optimized network given a dataset. In other words, instead of using a general configuration, the idea is to tailor it to the problem.

When dealing with an hyperparameter configuration, an expert can discard a configuration based on his expertise, i.e., without the need of evaluating it. However, intelligent automatic hyperparameter configuration optimization procedures search more efficiently through a high-dimensional search space.

Even though intelligent methods are more competitive than experts, they are not generally adopted because they are computationally intensive. They require to fit a model and to evaluate its performance on validation data (i.e., they are data-driven), which can be an expensive process [18], [6], [19].

Hence, few methods have been proposed to address this issue by speeding up the evaluation of the proposed hyperparameterization. For example, Domhan et al. [11] analyzed an approach that detects and finishes the neural networks evaluations that under-perform a previously computed one. This solution was able to reduce the hyperparameterization search time up to 50%. More recently, Camero et al. [10] presented the MAE random sampling, a novel low-cost method to compare one-hidden layer RNN architectures without training them. MAE random sampling evaluates an RNN architecture by generating a set of random weights and evaluating their performance.

In line with the latter approach, we propose to extend the MAE random sampling to evaluate RNNs with multiple-hidden-layers to make it suitable to evaluate deeper RNNs. Then, we propose to use this method to guide a meta-heuristic algorithm to search for the most suitable hyperparameters. Nonetheless, nothing prevents the MAE random sampling to be used by any other type of hyperparameter optimization method.

B. Deep Learning and Meta-heuristics

Meta-heuristics are well known optimization algorithms to address complex, non-linear, and non-differentiable problems [20], [9]. They efficiently combine exploration and exploitation strategies to provide good solutions requiring bounded computational resources. They have been successfully used to solve real-world problems in different fields, e.g., networks design [21], smart mobility [22], [23], and facility allocation [24].

Optimization in DL may be viewed from different perspectives: training as optimization of the DNN weights, hyperparameters selection, network topologies, learning environment, etc. These different points of view are adopted to improve the DNNs generalization capabilities.

Gradient-descent based methods, such as back-propagation, are widely used to train DNNs. However, these methods need several manual tuning schemes to make its parameters optimal and it is difficult to parallelize them taking advantage of graphics processing units (GPUs). Thus, several authors have explored DNNs training by using meta-heuristics, an idea explored long before DNN rise [25], [26]. Different authors combined convolutional neural networks with meta-heuristics to improve their accuracy and performance by optimizing the layers weights and threshold. Following this idea, You and Pu used a genetic algorithm (GA) [27]; Rosa et al. applied harmony search (HS) [28]; Rere et al. analyzed simulated annealing (SA) [29], and later the same authors evaluated SA, differential evolution (DE), and HS [30].

GA has been applied to evolve increasingly complex neural networks topologies and the connection weights simultaneously, in the NeuroEvolution of Augmenting Topologies (NEAT) method [31], [32]. However, NEAT has some limitations when it comes to evolving DNNs and RNNs [33].

Focusing on RNNs, NEAT-LSTM [34] and CoDeepNeat [35] extend NEAT to mitigate its limitations when evolving the topologies and weights of the network. Besides, particle swarm optimization (PSO) has been analyzed to train RNNs instead of SGD [36], providing comparable results. El Said et al. [37] proposed the use of ant colony optimization (ACO) to improve LSTM RNNs by refining their cellular structure.

Taking into account the optimization of the RNNs hyperparameters and architecture optimization, as it is done in this study, Camero et al. [7] applied GA to search for the most efficient ones to improve the accuracy and the performance
regarding the most commonly used RNNs configurations. In this case, the authors train the network using SGD to evaluate the performance of the configurations. Therefore, the main difference with our approach is that we propose to use the MAE random sampling instead of training each network/configuration. Thus, we expect to reduce the computational cost of the evaluation process, allowing the optimization algorithm to perform a larger number of iterations.

III. MAE RANDOM SAMPLING

Inspired by the simple fact that changing the weights of a neural network affects its output [17], Camero et al. proposed a novel approach to characterize and compare RNN architectures: the MAE random sampling [10]. First, they showed its usefulness for comparing the expected performance (i.e., the probability of a good result after training) of RNNs with a single-hidden-layer. Later, they extended their technique to multiple-hidden-layers [12] and showed that there is a strong negative correlation between the estimated probability and the MAE measured after training and that this negative correlation increases when adding more hidden layers.

MAE random sampling consists in taking a user-defined number of samples of the output (on a given input) of a specific RNN architecture. Where every time a sample is taken, the weights are normally initialized independently. Then, a truncated normal distribution is fitted to the MAE values sampled, and a probability $p_t$ of finding a set of weights whose error is below a user-defined threshold is estimated. Then, the probability $p_t$ is used as a predictor of the performance (error) of the analyzed architecture.

Figure 1 depicts the MAE random sampling originally introduced by Camero et al. [10] extended to multiple-hidden-layers RNNs. The distribution of the sampled errors is used to estimate the probability of finding a good solution.

Algorithm 1 presents the adaptation of the MAE random sampling to multiple-hidden-layers [12] and showed that there is a strong negative correlation between the estimated probability and the MAE measured after training and that this negative correlation increases when adding more hidden layers.

Algorithm 1 MAE random sampling of a given architecture.

```
1: data ← LoadData()
2: rnn ← InitializeRNN(ARCH, LB)
3: mae ← {}
4: while sample ≤ MAX_SAMPLES do
5:     weights ← GenerateNormalWeights(0,1)
6:     UpdateWeights(rnn, weights)
7:     mae[sample] ← MAE(rnn, data)
8:     sample++
9: end while
10: mean, sd ← FitTruncatedNormal(mae)
11: $p_t$ ← PTruncatedNormal(mean, sd, THRESHOLD)
```

(i) we state the architecture optimization problem and then, (ii) we present an evolutionary strategy-based algorithm to perform the optimization.

A. Architecture Optimization

The optimization of an artificial neural network consists of searching for an appropriate network structure (i.e., the architecture) and a set of weights [17]. However, in spite of this definition, it is rather common to arbitrarily define the architecture and then applied a learning rule (e.g., stochastic gradient descent) to optimize the set of weights [9]. Thus, we might say that the network is partially optimized or, in other words, we are not fully leveraging the computational model. Therefore in this study, we are interested in the optimization of the architecture aiming to improve the overall performance.

Usually, the RNN architecture optimization is stated as a minimization problem [9]. For example, Equation IV-A defines this problem as looking for an RNN architecture that minimizes the mean absolute error (MAE) of the predicted output ($z_i$) against the real one ($y_i$), subject to a minimum/maximum number of hidden layers (HL), neurons per layer (NPL), and look back [7]. Normally, it is implied in this definition the training of the candidate solution. Therefore, due to the intensive computations of the training, this optimization tends to be time demanding. Therefore, we propose to restate the optimization problem using the MAE random sampling.

\[
\text{minimize} \quad \text{Fitness} = \frac{1}{N} \sum_{i}^{N} MAE(z_i, y_i) \tag{1}
\]

subject to \quad min_LB \leq LB \leq max_LB

\quad min_HL \leq H \leq max_HL

\quad min_NPL \leq L \leq max_NPL

We propose to optimize the architecture of an RNN by maximizing $p_t$, i.e., given an input $X$ and an output $Y$, we propose to look for an RNN architecture that maximizes the estimated probability of finding a set of weights whose error is below a user-defined threshold (Algorithm 1). Equation IV-A presents the referred problem.
Algorithm 2 presents a high-level view of our proposal. In the first step of the mutation (line 6, \texttt{CellMutation}) using a binary tournament from the actual population. Then, the population is evolved until the evaluations, replace, and self-adjustment (Algorithm 2). First, the solutions. Then, the population is evolved until the \texttt{Evaluate} function has to be modified to \texttt{SelfAdapting} process takes place. In the latter process, if the new population is improving on average (i.e., the average $p_t$ of the new population is greater than the former average), then, the \texttt{cell_mut_p, max_step, and layer_mut_p} parameters are multiplied by 1.5. Otherwise, these parameters are divided by 4 [33].

After the evolutionary process ends, the best solution (i.e., the solution with the greatest $p_t$) of the population is selected (line 13), and trained using a user-defined method. Without loss of generality, we defined to use Adam [39] optimizer to train the final solution for a predefined number of \texttt{epochs}.

Finally, the algorithm returns an RNN that is optimized (structure and weights) to the given problem.

It is quite interesting to notice that the Evaluate function may be changed seamlessly by any other fitness function, e.g., the MAE after training the network for a user-defined number of times. Accordingly, the Best function has to be modified to maximize or minimize the new objective function (fitness).

V. MAE RANDOM SAMPLING RESULTS

We have implemented our proposal in Python using DLOPT [40], Keras [41], and Tensorflow [42]. And we have tested it using a standard problem: the sine wave. The selection of the problem is based on two reasons. First, Camero et al. [10], [12] studied the problem so we can compare our results to theirs, and second, any periodic waveform can be approximated by adding sine waves [43].

Equation 3 expresses a sine wave as a function of time ($t$), where $A$ is the peak amplitude, $f$ is the frequency, and $\phi$ is the phase. Particularly, we used the sine wave described by $A = 1$, $f = 1$, and $\phi = 0$, in the range $t \in [0, 100]$ seconds (s), sampled at ten samples per second.

$$y(t) = A \cdot \sin(2\pi \cdot f \cdot t + \phi)$$  \hspace{1cm} (3)

A. MAE Random Sampling as a Predictor

To study the proficiency of the MAE random sampling to predict how likely is to find a good set of weights, i.e., a set of weights that have a good error performance, we sampled one up to three-hidden-layer stacked RNN.

The method to do this study is as follows [12]. First, we defined the minimum ($\text{MIN}_NPL$) and the maximum ($\text{MAX}_NPL$) number of hidden LSTM cells of each hidden

In our proposal, a solution represents an RNN architecture, and it is encoded as a variable length integer vector, $sol = < s_0, s_1, ..., s_H >$, where $s_0$ is the LB $s_0 \in [\text{min}_LB, \text{max}_LB]$, and $s_i$ ($i \in [1, H]$) correspond to the number of LSTM cells in the $i$-th hidden layer. Thus, $s_i \in [\text{min}_NPL, \text{max}_NPL]$ and $H \in [\text{min}_HL, \text{max}_HL]$. Given this definition, the number of hidden layers is implicitly derived from the length of the solution. Then, the \texttt{population} is defined as a set of $\text{population}_size$ solutions.

First, the Initialize function randomly creates a set of solutions. Next, the \texttt{Evaluate} function computes $p_t$ (Algorithm 1) for each solution. Then, the population is evolved until the termination criteria is met (i.e., the number of evaluations is greater than $\text{max_evaluations}$).

The evolutionary process is divided into selection, mutation, evaluation, replace, and self-adjustment (Algorithm 2). First, (line 5) an offspring (of $\text{offspring}_size$ solutions) is selected using a binary tournament from the actual population. Then, each solution in the offspring is mutated by a two step process. In the first step of the mutation (line 6, \texttt{CellMutation}), for every $s_j$ ($j \in [0, H]$), with a probability $\text{cell_mut_p}$, a value in the range $[\text{min_step}, \text{max_step}]$ (excluding zero) is added. Then, in the second step of the mutation (line 7, \texttt{LayerMutation}), independently, with a probability $\text{layer_mut_p}$, the layer $s_i$ ($i \in [1, H]$) is cloned or removed (with the same probability), i.e., one layer is added or subtracted to the solution.

Once the mutation is done, the offspring is evaluated (\texttt{Evaluate}), and after, the best solutions from the population and the offspring are selected by the \texttt{Best} function, i.e., the population and the offspring are gathered together, sorted, and finally, the solutions that have a higher $p_t$ give place to the new population.

The number of evaluations is increased accordingly, and finally, a \texttt{SelfAdapting} process takes place. In the latter process, if the new population is improving on average (i.e., the average $p_t$ of the new population is greater than the former average), then, the $\text{cell_mut_p, max_step, and layer_mut_p}$ parameters are multiplied by 1.5. Otherwise, these parameters are divided by 4 [33].

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$$\text{maximize } \text{Heuristic} = p_t(X, Y)$$

subject to $\text{min}_LB \leq LB \leq \text{max}_LB$

$$\text{min}_HL \leq H \leq \text{max}_HL$$

$$\text{min}_NPL \leq L \leq \text{max}_NPL$$

B. Evolutionary Approach

To solve the RNN architecture optimization problem stated in Equation IV-A, we designed a deep neuroevolutionary algorithm based on the ($\mu + \lambda$) Evolutionary Strategy (ES) [20]. Algorithm 2 presents a high-level view of our proposal.

\begin{algorithm}
\caption{Self adapting ($\mu + \lambda$)ES-based RNN optimizer.}
\begin{algorithmic}[1]
\STATE population $\leftarrow$ Initialize($\text{population}_size$)
\STATE Evaluate(population)
\STATE evaluations $\leftarrow$ \text{population}_size
\WHILE{evaluations $\leq$ \text{max_evaluations}}
\STATE offspring $\leftarrow$ \text{BinaryTournament}(population, \text{offspring}_size)
\STATE offspring $\leftarrow$ \text{CellMutation}(offspring, \text{cell_mut_p, max_step})
\STATE offspring $\leftarrow$ \text{LayerMutation}(offspring, \text{layer_mut_p})
\STATE Evaluate(offspring)
\STATE population $\leftarrow$ Best(population + offspring, \text{population}_size)
\STATE evaluations $\leftarrow$ evaluations + offspring_size
\STATE \text{SelfAdapting}(\text{layer_mut_p}, \text{max_step, cell_mut_p})
\ENDWHILE
\STATE solution $\leftarrow$ Best(population, 1)
\STATE \text{rnn_trained} $\leftarrow$ \text{Train}(solution, \text{epochs})
\RETURN\text{rnn_trained}
\end{algorithmic}
\end{algorithm}

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Finally, the algorithm returns an RNN that is optimized (structure and weights) to the given problem.

It is quite interesting to notice that the Evaluate function may be changed seamlessly by any other fitness function, e.g., the MAE after training the network for a user-defined number of times. Accordingly, the Best function has to be modified to maximize or minimize the new objective function (fitness).
layer, and we defined the look back values to be studied. Then, we took 100 samples \((\text{MAX}_{\text{SAMPLES}})\) and estimated \(p_{0.01}\) (i.e., \(\text{THRESHOLD}=0.01\)). At last, we selected 100 architectures (i.e., a number of LSTM cells and the look back), trained them using Adam optimizer [39], and analyzed the relation between the estimated probability \(p_{0.01}\) and the observed MAE.

Table I presents the parameters used in this experiment. Two and three-hidden-layer parameters were selected upon the observation of the one-hidden-layer results. Note that the greatest variation of \(p_t\) occurs in the region described by the parameters shown in the table (Figure 2).

### TABLE I

MAE RANDOM SAMPLING STUDY PARAMETERS

| Architecture       | MIN_NPL | MAX_NPL | Look back       |
|--------------------|---------|---------|-----------------|
| One-hidden-layer   | 1       | 100     | [1, 30]         |
| Two-hidden-layer   | 7       | 31      | {1, 10, 20, 30} |
| Three-hidden-layer | 7       | 31      | {1, 10, 20, 30} |

Summarizing, we estimated \(p_{0.01}\) for the RNNs defined by the constraints presented in Table I trained 100 architectures (selected uniformly from the referred sample), and study the correlation between the predicted probability and the observed error. Table II presents the correlation between the MAE random sampling results (Mean MAE, Sd MAE, and \(\log p_{0.01}\)) and the observed MAE after training the RNNs for a predefined number of epochs.

### TABLE II

CORRELATION BETWEEN THE MAE OBSERVED AFTER TRAINING THE MODEL AND THE MAE RANDOM SAMPLING RESULTS

| Architecture   | Epochs | Correlation | \(\log p_{0.01}\) |
|----------------|--------|-------------|-------------------|
| One-hidden-layer | 1      | -0.447      | -0.317            |
|                 | 10     | -0.726      | -0.431            |
|                 | 100    | -0.790      | -0.641            |
|                 | 1000   | -0.668      | -0.458            |
| Two-hidden-layer | 1      | -0.086      | -0.135            |
|                 | 10     | -0.450      | -0.632            |
|                 | 100    | -0.709      | -0.827            |
|                 | 1000   | -0.695      | -0.843            |
| Three-hidden-layer | 1      | -0.334      | -0.447            |
|                  | 10     | -0.546      | -0.724            |
|                  | 100    | -0.720      | -0.869            |
|                  | 1000   | -0.130      | -0.873            |

Figure 2 shows the relation between the number of hidden cells and \(p_{0.01}\), each color represents a different look back. The probability rapidly increases from 1 to 25 cells (more than 6 order of magnitude). Then, i.e., when the number of cells is greater than 25, the probability \(p_{0.01}\) tends to converge or stabilize for each look back.

In the one-hidden-layer case, there is a moderate-to-strong negative correlation between \(p_{0.01}\) and the MAE observed. This insight tells us that given two RNNs, we have to select the one that has a higher probability. Nonetheless, it is important to notice that we are not predicting the error (i.e., the loss value). Instead, we are predicting how likely would be to find a set of weights that have a good error performance.

The results also show a strong negative correlation between the estimated probability and the MAE observed after training the RNNs defined by the constraints presented in Table I, trained 100 architectures (selected uniformly from the referred sample), and study the correlation between the predicted probability and the observed error. Table II presents the correlation between the MAE random sampling results (Mean MAE, Sd MAE, and \(\log p_{0.01}\)) and the observed MAE after training the RNNs for a predefined number of epochs.

### TABLE III

TIME AND MEMORY USAGE COMPARISON

|                  | Time [s] | Memory [MB] |
|------------------|----------|-------------|
|                  | Mean     | Sd          | Mean   | Sd          |
| Adam 1000 epochs | 996      | 0.006       | 127    | 6.338       |
| MAE rand samp    | 6        | 0.001       | 150    | 98.264      |

**VI. OPTIMIZATION RESULTS**

Once we have shown that the MAE random sampling is a good heuristic to the RNN training performance, we studied its actual usefulness to optimize an RNN architecture using the metaheuristic algorithm proposed (Section IV). First, (i) we optimized an RNN to predict the sine wave defined in Equation 3 using the MAE random sampling as the heuristic of the algorithm. Then, (ii) we repeated the optimization but using early training results as the heuristic. Finally, (iii) we analyzed a real-world problem. Particularly, we optimized an RNN that predicts the waste generation of a city [44], [45].
and we compared our results to the state-of-the-art of urban waste containers filling level prediction methods [46].

A. MAE Random Sampling Heuristic

First, we optimized an RNN that predicts the sine wave (Equation 3) using the evolutionary-based method proposed (Section IV) and the MAE random sampling [10] as the heuristic of the optimization algorithm.

Table IV presents the parameter values of the algorithm used to optimize the RNN. It is very important to notice that during the optimization cell_mut_p, max_step, and layer_mut_p values are self-adapted (Algorithm 2, line 11). Thus, their initial values are not critical.

| Parameter   | Value |
|-------------|-------|
| cell_mut_p  | 0.2   |
| epochs      | 100   |
| max_step    | 5     |
| layer_mut_p | 0.2   |
| population_size | 10   |
| offspring_size | 10   |
| max_eval    | 100   |

We used the MAE random sampling results as the heuristic of the algorithm, i.e., we estimated the training performance of the solutions and used the estimated $p_{\text{threshold}}$ to sort them. Specifically, we compute the sampling using the parameters presented in Table V.

| Parameter     | Value |
|---------------|-------|
| num_samples   | 100   |
| $p_{\text{threshold}}$ | 0.01  |
| truncated_range | [0,2] |

On the other hand, accordingly to the definition of the optimization problem presented in Equation [V-A] we set the constraints of the problem. Table VI presents the search space.

| Parameter   | Value |
|-------------|-------|
| min_LB      | 2     |
| max_LB      | 30    |
| min_NPL     | 1     |
| max_NPL     | 100   |
| min_HL      | 1     |
| max_HL      | 3     |

We executed 30 independent times the optimization process using the parameter values already mentioned, and we computed the statistics of the error (MAE) over the final solution. Table VII summarizes this results, where MRS stands for the optimization guided by the MAE random sampling heuristic and GDET for the gradient-descent training heuristic. The results show that MRS outperforms GDET in regards to the numerical accuracy.

|               | MRS   | GDET  |
|---------------|-------|-------|
| Mean          | 0.105 | 0.142 |
| Median        | 0.100 | 0.149 |
| Max           | 0.247 | 0.270 |
| Min           | 0.063 | 0.054 |
| Sd            | 0.035 | 0.051 |

B. Gradient-descent Early Training Heuristic

We repeated the RNN optimization using our evolutionary approach but using a different heuristic. Specifically, we used early training results to predict the performance, i.e., we trained the solutions using Adam for a short time and used the loss results as the heuristic of the optimization algorithm.

Table VIII presents the early training heuristic parameters. We train the candidates for a time (heuristic_epochs) that is much smaller than the epochs we train the final solution.

| Parameter     | Value |
|---------------|-------|
| heuristic_epochs | 1     |
| dropout       | 0.5   |

We executed 30 independent times the optimization process using the gradient-descent early training parameter values and the optimization parameters set before (Table IV and VI), and we computed the statistics over the final solution. Table VII summarizes this results, where MRS stands for the optimization guided by the MAE random sampling heuristic and GDET for the gradient-descent training heuristic. The results show that MRS is significantly better than GDET.

C. Waste Generation

To continue with the validation of our proposal, we tested our proposal in a real-world problem, the waste generation prediction. We studied the problem presented by Ferrer and Alba [44], [45], where the filling level of 217 paper containers spread out in a city in Spain is predicted.

Originally, Ferrer and Alba [44] proposed to predict the filling level of each container individually using Gaussian processes, linear regression, and SMReg. Later, Camero et al. [46] outperformed those results by predicting all containers at once using an RNN.

Particularly, in the referred study [46], the authors proposed to optimize an RNN to the problem using an ES-based algorithm. More specifically, they trained each candidate solution using a gradient descent-based method for a short time (ten epochs), and once the termination criteria were met, they trained the final solution for 1000 epochs. Table IX summarizes the results presented by Camero et al. [46] under the column Short training.
As a summary, our approach has a similar error performance to state-of-the-art RNN optimization but considerably reduces the computational time. Hence, we offer a competitive alternative to RNN architecture optimization that does not rely on training but on the MAE random sampling \cite{10, 12}.

VII. Conclusions and Future Work

In this work, we studied the MAE random sampling technique to multiple-hidden-layer architectures and presented an ES-based algorithm to optimize an RNN that uses the MAE random sampling as a search heuristic.

We studied the correlation between the MAE random sampling results (i.e., the probability of finding a set of weights whose error is below a user-defined threshold) and the error after training a network using Adam optimizer on stacked RNN architectures with up to three-hidden-layers, using a sine wave.

The results show that there is a strong negative correlation, i.e., a high estimated probability is strongly related to a low error value after training a network. Moreover, as we add hidden-layers to the RNN, this negative correlation increases. We think that this might be explained in part by the growing complexity of the training process, however further analysis is required to explain this observation.

Also, we tested our ES-based RNN optimization algorithm using as heuristic the MAE random sampling and the results of a short training process. The results show that using the MAE random sampling to guide the search outperforms its competitor in the sine wave problem.

Moreover, we compared our proposal against state-of-the-art RNN architecture optimization in a real-world problem, the waste generation prediction. The results show that our approach is as good as its competitors in terms of the error, i.e., the prediction error of our solutions are similar to the ones of the architectures optimized using state-of-the-art methods. However, our approach reduces by half the time needed to optimize the network. Therefore, we conclude that our proposal offers a competitive alternative for RNN optimization.

Overall, the results suggest that the MAE random sampling is a “low-cost, training-free, rule of thumb” method to compare deep RNN architectures and that it is a very useful heuristic for architecture optimization.

As future work, we propose to extend the MAE random sampling technique to other error functions, so this technique can be used to tackle other types of problems (classification, clustering, among others).
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