Precision electro-weak parameters from $AdS_5$, localized kinetic terms and anomalous dimensions.

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Abstract

I compare the tree level estimate of the electro-weak precision parameters in two (exactly solvable) toy models of dynamical symmetry breaking in which the strong dynamics is assumed to be described by a five-dimensional (weakly coupled) gravity dual. I discuss the effect of brane-localized kinetic terms, their use as regulators for the couplings of otherwise non-normalizable modes, and the impact of a large deviation from its natural value for the scaling dimension of the background field responsible for spontaneous symmetry breaking. The latter is assumed to model the effects of walking dynamics, i.e. of a large anomalous dimension of the chiral condensate, it has a strong impact of the spectrum of spin-1 fields and, as a consequence, on the electro-weak precision parameters. The main conclusion is that models of dynamical symmetry breaking based on a large-$N_c$ strongly interacting $SU(N_c)$ gauge theory are compatible with precision electro-weak constraints, and produce a very distinctive signature testable at the LHC. Some of the considerations discussed are directly relevant for analogous models in the context of $AdS – QCD$.

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INTRODUCTION

Some special super-Yang-Mills conformal field theories are known to admit a dual description in terms of a (weakly interacting) higher-dimensional gravity theory \[1\] in a negative-curvature background (\(AdS_5\) space). This suggests the speculative idea that a much larger class of strongly interacting gauge theories, in which not only conformal invariance, but also supersymmetry are (softly) broken, might admit such a dual description. It is hence interesting to explore the space of the five-dimensional models that can be obtained with simple and controllable deformations of the pure \(AdS_5\) background, looking for the gravity-dual of a wider class of strongly interacting four-dimensional theories, with the hope of learning something about phenomenologically relevant strongly interacting theories that would otherwise be difficult to study.

The starting point of the simplest such construction consists of writing the effective action of a gauge theory in a five-dimensional space-time containing a warped gravity background described by the metric:

\[
ds^2 = \left(\frac{L}{z}\right)^2 \left(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\right),
\]

where \(x^\mu\) are four-dimensional coordinates, \(\eta_{\mu\nu}\) the Minkoski metric with signature (+, −, −, −), and \(z\) is the extra (warped) dimension. The dimensionful parameter \(L\) is the \(AdS_5\) curvature, and sets the overall scale of the model. Schematically, the interpretation in terms of four dimensional conformal theory relates the rescaling in the fifth dimension \(z\) to conformal transformations in the four-dimensional dual description. Conformal symmetry is broken by the boundaries

\[
L_0 < z < L_1,
\]

with \(L_0 > L\), where \(L_0\) and \(L_1\) correspond to the UV and IR cut-offs of the conformal theory. The gauge symmetry of the five-dimensional bulk is related to the global symmetries of the dual CFT. More details about the general construction and interpretation of these models can be found elsewhere in the rich literature on the subject (see for instance \[2\] for a simple, clear and general summary of the basic elements of these constructions).

Very recently, this approach has been use in order to formulate an Effective Field Theory (EFT) description of QCD at the energies above the range of validity of the chiral
Lagrangian [3][4][5], in order to give a simple description of the physics of (strongly interacting) mesonic resonances (see also [6][7]). Besides possible modifications of the gravity background and of the field content of the EFT, these models differ by the assumed bulk profile of the chiral symmetry breaking background, by the introduction of dilaton-type backgrounds, and by how the UV and IR cut-offs are introduce and regulated.

Besides QCD, another class of strongly interacting theories relevant for phenomenology contains the models of dynamical electro-weak symmetry breaking, generically referred to as technicolor (TC) [8]. Apart from the very different energy scale, the non-linear sigma model description of TC is very similar to the chiral Lagrangian of QCD, the main difference being given by the fact that in the former a subset of the chiral symmetry is (weakly) gauged and corresponds to the Standard Model $SU(2)_L \times U(1)_Y$ gauge group. It is hence natural to construct a five-dimensional weakly coupled gravity model with the appropriate symmetry content so as to describe the physics responsible for electro-weak symmetry breaking, aimed at the study of the EFT in the energy window ranging from the $W$-boson mass to a few orders of magnitude above the electro-weak scale itself. Part of this range will be soon explored at the LHC, and it is hence crucial to have models that are compatible with all present data, but produce new distinctive signatures accessible at these higher energies.

Models in this class are strongly constrained by experimental data on precision electro-weak observables [9][10], in particular on the parameters $\hat{S}$ and $\hat{T}$ to be defined later on. Based on dimensional analysis, one expects them to scale as the ratio of the electro-weak gauge boson masses and mass differences to the mass of the lightest spin-1 excited state of the strong sector (techni-$\rho$) as $\hat{S} \propto M^2_W/M^2_\rho$ and $\hat{T} \propto (M^2_Z - M^2_W)/M^2_\rho$ up to multiplicative model-dependent factors determined by the strong TC dynamics. These estimates are, generically speaking, too large in comparison with experimental data, unless the mass of the techni-$\rho$ is pushed unnaturally far above the electro-weak scale, in the multi-TeV range. It would be impossible to detect these new states directly even at the LHC.

However, a non-trivial departure from QCD-like behavior of the underlying TC-dynamics might stabilize a substantial hierarchy between different scales, and even produce significant suppression factors in the computation of $\hat{S}$ and $\hat{T}$. Such a possibility is exemplified by TC models with walking behaviour [11]: the presence of a regime in which the theory is quasi conformal, together with the large anomalous dimension of the chiral condensate, can change in a substantial way the dependence of the masses of fermions and gauge bosons
as a function of the (dynamically generated) scales in the underlying theory, and affect the precision electro-weak parameters (see for instance \[12\]).

The effect of walking behavior on the phenomenology of SM fermions has been studied extensively, and is known to be very big. In TC, fermion masses are introduced by coupling two techni-quark fields and two SM fermions via a four-fermion operator, with a (dimensionful) coefficient $1/\Lambda_{ETC}^2$. $\Lambda_{ETC}$ is the scale, much higher than the electro-weak scale, at which the (global) flavor symmetries of the SM are broken. Walking modifies the dependence on this scale of the mass of the SM fermions $m_f$. With no big anomalous dimensions, $m_f \propto \Lambda_{TC}^3/\Lambda_{ETC}^2$, where $\Lambda_{TC}$ is in the range of the electro-weak scale. In full generality, this scaling is dictated by the dimension $d$ of the condensate, and given by

$$m_f \propto \Lambda_{TC} \left( \frac{\Lambda_{TC}}{\Lambda_{ETC}} \right)^{d-1},$$

in such a way as to gain an enhancement factor of the order of a power of $\Lambda_{ETC}/\Lambda_{TC}$ if $d < 3$. This enhancement factor is important for obtaining large enough masses for the fermions while at the same time suppressing Flavor Changing Neutral Currents (FCNC).

Walking behavior is also expected to affect the phenomenology of the spin-1 states of the theory. Many studies have been carried on in the literature in order to gauge the magnitude of such effect, which is generally believed to reduce via, non-perturbative effects, the perturbative estimates of precision electro-weak parameters. The major obstacle to a precise refinement of this statements is the fact that it is very difficult to compute reliably the precision parameters $\hat{S}$ and $\hat{T}$ in the context of a four-dimensional $SU(N_c)$ strongly interacting gauge theory [14].

In this paper, inspired by the works on $AdS - QCD$, I discuss some examples of the use of the techniques developed there for the construction of models of dynamical electro-weak symmetry breaking. The main interest of my analysis is to compute the precision observables $\hat{S}$ and $\hat{T}$, and to study how the results depend on the assumptions used in constructing the effective Lagrangian in five-dimensions. Previous studies can be found in the literature of the Higgsless models [15], in the context of composite Higgs models [16] and deconstruction [17]. Most of the results on the strong-dynamics rely ultimately on the idea of vector-dominance and hidden local symmetry [18], and could be rewritten in term of four-dimensional deconstructed models [19], but the number of free parameters is far smaller in the $AdS - CFT$ context, and manifest conformal symmetry plays a crucial role in the
present study.

I study a set of five-dimensional models with a $SU(2)_L \times U(1)_Y$ gauge symmetry in the bulk, the lightest modes of which correspond to the photon, and to the standard-model $W$ and $Z$ gauge bosons. Electro-weak symmetry breaking is induced by the background vacuum expectation value (VEV) of a bulk scalar field, (the dual description of the chiral condensate). The (four-dimensional) physical mass of the scalar excitations is assumed to be large, so that the non-linear sigma-model description applies.

I do not include the standard-model fermions. Ordinary quarks and leptons are fundamental fields, because they are not supposed to carry TC interactions, and hence are confined to live on the UV brane. The effect of their existence is reflected here in the introduction of (divergent) localized kinetic terms. These break explicitly the conformal invariance, and provide a natural regulator for the theory. In the absence of such boundary terms, in the limit in which one sends to infinity the UV cut-off, the zero-modes of the gauge fields become non-normalizable and decouple from the spectrum. The regulator allows to take the limit of infinite cut-off while keeping the gauge coupling of the zero modes finite. In this way, the final EFT depends only on quantities that are physically well defined at low energy: the scale of confinement (position of the IR boundary) and the value of the couplings of the heavy resonances (gauge coupling in the bulk) and of the electro-weak gauge bosons (gauge couplings on the UV-brane), with no explicit dependence on the UV cut-off of the model.

The present study aims primarily to illustrate the effect on the spectrum and on the precision parameters of the conformal symmetry and of the large anomalous dimension of the chiral symmetry breaking condensate, and hence a semi-realistic toy-model is enough to capture the main interesting dynamical features. For instance, I do not consider the effect of introducing a custodial symmetry in the bulk to suppress $\hat{T}$, since this would just add to the technical difficulties, without substantial changes to the phenomenology. For the same reason, I do not include different boundary terms for (nor a dilaton background with different coupling to) the axial and vector components of the gauge fields.

The main part of the analysis consists of the computation of the polarization tensors for the SM gauge bosons in the $AdS_5$ background in the cases in which the chiral symmetry

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1 A possible exception could be represented by the top quark. Models in which the dynamics of the third generation has a special role have been studied at length in order to explain the largeness of the top mass. See however [21].
breaking condensate has two different (large) anomalous dimensions. The spectrum of spin-1 states, as well as the estimate of the precision observables, are significantly modified by the anomalous dimensions. This can be a useful tool for the construction of more realistic, testable models of dynamical electro-weak symmetry breaking.

The paper is organized as follows. I first review the definitions and experimental bounds on precision electro-weak parameters. Then I devote two sections to the definition of the models under consideration and to the algebraic manipulations that lead to the polarizations. These two sections are very detailed, and intended for the reader who is not familiar with the AdS – CFT language. The following two sections are devoted to the explicit derivation of the electro-weak precision parameters in the two models. Finally, the phenomenology is presented, with comparison to the experimental limits, and I conclude with a critical discussion and interpretation of the whole procedure.

**PRECISION PARAMETERS.**

Before entering the specific discussion of the five-dimensional models, I briefly recall here the basic formulae and experimental constraints from precision electro-weak physics. The (bilinear part of the) gauge boson sector of the Standard Model Lagrangian in 4 dimensions can be written as (after integrating out all heavy states)

\[
\mathcal{L} = \frac{\eta_{\mu\nu}}{2} A_i^\mu \pi_{ij} (q^2) A_j^\nu + g_4^a J_{a\mu} A_a^\mu ,
\]

where the index \(i\) runs over the \(SU(2)_L \times U(1)_Y\) generators, \(P_{\mu\nu} = \eta_{\mu\nu} - q_{\mu} q_{\nu} / q^2\), the \(g_4^a = g_4, g_4'\) are the SM gauge couplings of \(SU(2)_L \times U(1)_Y\) and \(\pi_{ij}\) is the polarization tensor of the SM gauge bosons. The precision electro-weak parameters of interest here are defined by:

\[
\hat{S} \equiv \frac{g_4}{g_4'} \pi_{WB}(0) ,
\]

\[
\hat{T} \equiv \frac{1}{M_W^2} (\pi_{WW}(0) - \pi_+(0)) ,
\]

where \(M_W^2\) is the mass of the \(W\)-boson, and \(\pi' = d\pi / dq^2\), and where I call \(\pi_{WW}\) and \(\pi_{WB}\) and \(\pi_{BB}\) the entries of the \(2 \times 2\) polarization tensor in the \(T_3\) direction, while \(\pi_+\) is the polarization in the \(T_{1,2}\) direction of \(SU(2)_L\).
I take as indicative of the experimentally allowed ranges (at the 3\(\sigma\) level):

\[
\hat{S}_{\text{exp}} = (-0.9 \pm 3.9) \times 10^{-3}, \quad (7)
\]

\[
\hat{T}_{\text{exp}} = (2.0 \pm 3.0) \times 10^{-3}, \quad (8)
\]

from [10], with the caution that these are bounds extrapolated to the case of a Higgs boson with mass of 800 GeV\(^2\).

These bounds, in particular the one on \(\hat{S}\), are much more stringent than the ones obtained including Z-pole observables only [10], often referred to in the literature. The main message is that both the observables have to lie in the few \(\times 10^{-3}\) range, and that a positive \(\hat{T}\) is actually favored (at the 1\(\sigma\) level) in the case in which there is no light Higgs in the spectrum. However, the fact that the comparison is done here in a non rigorous way suggests that probably the error bars are under-estimated, and that hence the results of the study carried on in this paper have to be understood as conservative.

The polarizations \(\pi_i\) defined above can be rewritten in terms of the propagator of the SM gauge fields, defined as the boundary values of the five-dimensional gauge bosons, i.e. as the fields that couple to the localized SM currents. The propagators associated with charged currents \(ip_+\) and vectorial and axial-vectorial neutral currents \(ip_v\) and \(ip_a\) are related to the \(\pi_i\) tensors by:

\[
\pi_+ = \frac{1}{p_+}, \quad (9)
\]

and

\[
\pi = \frac{1}{(g_4^2 + g_6^2)p_a p_v} \begin{pmatrix} g_4^2 p_v + g_6^2 p_a & -g_4 g'_4 (p_v - p_a) \\ -g_4 g'_4 (p_v - p_a) & g_4^2 p_a + g_6^2 p_v \end{pmatrix}, \quad (10)
\]

from which

\(^2\)Some of the approximations used in the extraction of these limits from the experimental data do not rigorously apply in this range, since the comparison is done with the Standard Model at the one-loop level, and for large masses of the Higgs boson, i.e. large effective quartic coupling, the loop expansion is not well behaved. Also, the strongly interacting sector I am describing here does not contain a Higgs field at all, since this is supposed to be very heavy, and is integrated out constructing a non-linear sigma-model description of symmetry breaking. The dependence on the Higgs mass should hence be replaced with the dependence on the UV cut-off of the effective theory, which is not a physical quantity, and hence introduces uncertainty in the procedure.
\[ \hat{S} = \frac{M_{W}^{2}}{M_{Z}^{2}} (\pi_{v} - \pi_{a})' (0), \]  
\[ \hat{T} = \frac{1}{M_{W}^{2}} \left( \frac{M_{W}^{2}}{M_{Z}^{2}} (\pi_{a} - \pi_{+}) \right) (0). \]  

Here, due to the fact that this is going to be a tree-level analysis of the new-physics contributions to the precision observables, I traded the (weak) gauge couplings for the masses of the gauge bosons, using the (tree-level) relations of the Standard Model. The error introduced in this way is negligible at the present level of precision.

The information about the heavy modes in the theory will be contained in the corrections to the propagators of the photon, \( W \) and \( Z \) gauge bosons (denoted as \( p_{v}, p_{+} \) and \( p_{a} \)), after integrating out the heavy excitations, and will depend on the masses and coupling constants of the heavy modes. A crucial assumption I am working with throughout this whole study is that, at the EFT level at which none of the heavy resonances has been yet integrated out, the only mixing between light and heavy states is present in the mass matrices for the gauge bosons, neglecting the more general case of a non-trivial kinetic mixing.

It is worth recalling that the combination of all the precision measurements, besides those performed at the \( Z \) pole, yields significant constraints on a large set of universal and non-universal parameters. The latter are mainly affected by the coupling to fermions, and hence are not discussed here. Of the former, only \( \hat{S} \) and \( \hat{T} \) are directly relevant in the present context, since the others are produced only by higher-derivative terms in the Taylor expansion of the polarization tensors, and hence can be neglected.

Finally, before turning to the models, it is useful to look more in details at the expression of \( \hat{S} \) in Eq. (11). This can be recast, using dispersion relations, as

\[ \hat{S} \propto g_{4}^{2} \sum_{n} \left( \frac{f_{\rho n}^{2}}{M_{\rho n}^{2}} - \frac{f_{a1 n}^{2}}{M_{a1 n}^{2}} \right), \]

where \( f_{\rho,a1 n} \) are the decay constants of the heavy resonances, and \( M_{\rho,a1 n} \) their masses. This expression will be derived and discussed better later on, and for the moment neglects the presence of possible suppression factors.

It is not difficult to see how to reduce, in a generic phenomenological model, the contribution to \( \hat{S} \), playing with three different possibilities. The most naif one is to make the masses of the resonances bigger. This could render them too heavy for observation. A second way is to suppress the decay constants. This can be done by tuning the coupling of the
heavy resonances to the electro-weak currents, in respect to the coupling of the (lightest) SM gauge bosons. This tuning is unnatural, because it requires to assume the presence of very a very strong coupling in the strong sector, which makes impossible to perform any kind of computation. It also would make the resonances more difficult to observe, since at LHC the proton-proton initial scattering process can be described in terms of Standard Model currents, and hence this strategy would reduce the production probability. Finally, a more appealing possibility is to arrange for a cancellation between the axial-axial and vector-vector contributions to $\hat{S}$. This approach [12] could lead to light enough new states (with large enough couplings to the SM currents) as to allow for their detection at the LHC.

This third mechanism requires to tune the masses and decay constants in order for the cancellation to take place. This is exact in the limit in which there is no isospin violation. The goal is hence to construct a model in which the lightest states are directly sensitive to electro-weak symmetry breaking, while the heavy resonances are less sensitive to it. This is what is expected to happen in models with walking dynamics, in which the (strong) running gauge coupling approaches an IR fixed-point, and hence heavy modes are only marginally affected by the blowing off of the coupling itself in the IR and the consequent formation of a symmetry breaking condensate.

This paper aims at a more quantitative discussion of the size and model dependence of the aforementioned three mechanisms, and at the study of their feasibility within models that try to minimize fine-tuning without loosing testability at the LHC and predictive power.

**THE MODEL(S).**

I adopt the conventions for the metric defined in the Introduction. In particular, the determinant of the metric is $\sqrt{G} = (L/z)^5$ for the $AdS_5$ background.

The field content consists of a single complex scalar $\Phi$ transforming as a $(2, 2)$ of $SU(2)_L \times SU(2)_R$. I gauge the $SU(2)_L \times U(1)_Y$ subgroup, in which the generator of $U(1)_Y \subset SU(2)_R$ is the $T_3$, with $T_i = \tau_i/2$ and $\tau_i$ the Pauli matrices.

The bulk action for $\Phi$ and the gauge bosons $L = L_i T_i$ of $SU(2)_L$ and $R = R_3 T_3$ of $U(1)_Y$ is given by:

$$S_5 = \int d^4 x \int_{L_0}^{L_1} dz \sqrt{G} \left[ \text{Tr} \left( G^{MN} D_M \Phi D_N \Phi - M^2 |\Phi|^2 \right) \right]$$
\[-\frac{1}{2} \text{Tr} (L_{MN}L_{RS} + R_{MN}R_{RS}) G^{M}R G^{NS} \] (14)

and the boundary terms are given by

\[ S_4 = \int d^4x \int_{L_0}^{L_1} dz \sqrt{G} \left[ -\frac{1}{2} D \delta(z - L_0) \right. \]
\[ \left. \text{Tr} [L_{\mu \nu} L_{\rho \sigma} + R_{\mu \nu} R_{\rho \sigma}] G^{\mu \rho} G^{\nu \sigma} \right. \]
\[ + C \delta(z - L_0) \text{Tr} [G^{\mu \nu} D_\mu \Phi D_\nu \Phi] \]
\[ - \delta(z - L_0) 2 \lambda_0 \left( \text{Tr} |\Phi|^2 - \frac{1}{2} v_0^2 \right)^2 \]
\[ - \delta(z - L_1) 2 \lambda_1 \left( \text{Tr} |\Phi|^2 - \frac{1}{2} v_1^2 \right)^2 \],

where the covariant derivative is given by

\[ D_M \Phi = \partial_M \Phi + i(g L_M \Phi - g' \Phi R_M) , \]

and where the Yang-Mills action is written in terms of the antisymmetric field-strength tensors \( L_{\mu \nu} \) and \( R_{\mu \nu} \) (for most of the following, these tensors are approximated by neglecting the quadratic terms). In the action, \( M^2 \) is a bulk mass term for the scalar, and \( g \) and \( g' \) are the (dimensionful) gauge couplings in five-dimensions.

The choice of boundary terms is dictated by the rules of holographic renormalization \([22]\): the presence of the UV brane, and the assumption that SM fermions are localized on it, introduces an explicit breaking of conformal invariance, that would manifest itself with (localized and divergent) radiative corrections to the kinetic terms of the bulk fields, and hence require the presence of \( C \) and \( D \). I am not going to discuss the complete spectrum in this paper, but just focus on the spin-1 modes of the 4-dimensional action, treating the vacuum expectation value (VEV) of \( \Phi \) as a background.

The first step consists of solving the equations of motion for the lowest mode of the scalar field \([24]\). In the limit \( \lambda_i \rightarrow +\infty \), the physical scalar mass diverges. Hence I consider the non-linear realization, in which the transverse degrees of freedom are set to zero and decoupled. I use the same notation \( \Phi \) also after these massive fluctuations around the VEV are integrated out, and hence I write \( \Phi \) in terms of its background value as:

\[ \Phi(x, z) = \frac{1}{2} v(z) e^{2i\phi(x,z)/\sqrt{v(z)}} \] (16)

in which \( \phi = \phi_i T_i \) are the (would-be) Goldstone bosons, and the VEV is assumed to be constant in each Minkoski slice of the space. The boundary terms for the scalar potential
reduce to the constraints on the (classical) background \( v(z) \):

\[
\begin{align*}
  v(L_0) &= v_0, \\
  v(L_1) &= v_1,
\end{align*}
\]

(17) (18)

I consider two distinct cases in the following, so defined

- AdS background with bulk mass term \( M^2 = -3/L^2 \), condensate of dimension \( d = 1 \),
- AdS background with bulk mass term \( M^2 = -4/L^2 \), condensate of dimension \( d = 2 \).

With AdS metric the scalar background satisfies:

\[
\partial_z \left( \frac{L^3}{z^3} \partial_z v \right) - \frac{L^5}{z^5} M^2 v = 0,
\]

(19) (20)

the general solutions of which depend on the choice of \( M^2 \) in the following way:

\[
\begin{align*}
  v(z) &= Az + Bz^3, \text{ for } M^2 = -3/L^2, \\
  v(z) &= Az^2 + Bz^2 \log(z/L), \text{ for } M^2 = -4/L^2,
\end{align*}
\]

(21) (22)

with \( A \) and \( B \) determined by the boundary conditions. The special choices for the value of \( M^2 \) discussed here are dictated by the dictionary of AdS/CFT. The symmetry breaking VEV of the scalar field corresponds to the condensate of the operator breaking the chiral symmetry in the four-dimensional dual, and its scaling in the fifth dimension corresponds to the scaling dimension of the condensate.

I choose the boundary terms for \( \Phi \) in such a way as to set \( B = 0 \) at finite \( L_0 > L > 0 \). This choice reduces to

\[
\begin{align*}
  \frac{v_0}{L_0} &= \frac{v_1}{L_1}, \text{ for } M^2 = -\frac{3}{L^2}, \\
  \frac{v_0}{L_0^2} &= \frac{v_1}{L_1}, \text{ for } M^2 = -\frac{4}{L^2},
\end{align*}
\]

(23) (24)

which yields

\[
\begin{align*}
  v(z) &= \frac{v_1}{L_1} z, \text{ for } M^2 = -\frac{3}{L^2}, \\
  v(z) &= \frac{v_1}{L_1^2} z^2, \text{ for } M^2 = -\frac{4}{L^2},
\end{align*}
\]

(25) (26)
At this point, I take the limits (after proper regularization) \( L_0 \to L \to 0 \).

As a result, the two cases I discuss correspond to the assumption of having a chiral condensate with dimensions \( d = 1 \) and \( d = 2 \) respectively. The first is somewhat equivalent (from the electro-weak scale EFT point of view) to a four-dimensional model in which symmetry breaking is triggered by a physical scalar Higgs (composite), the second to the case of walking TC, in which symmetry breaking is induced by a \( \langle \bar{\psi}\psi \rangle \) techni-quark condensate with large anomalous dimension \( (d = 2) \). Both are relevant phenomenological choices in the dynamical electro-weak symmetry breaking context.

Notice how the choice of setting \( B = 0 \) does not violate unitarity bounds in these two cases \[25\]. The choice \( M^2 = -4/L^2 \) saturates the lowest bound on the possible mass for the bulk scalar \[26\]. In this case, the choice \( B = 0 \) is the only one compatible with conformal symmetry. For \( M^2 = -3/L^2 \), in principle both choices of \( B = 0 \) or \( A = 0 \) give rise to consistent theories \[27\]. The \( d = 3 \) case has been studied at length in the literature, being the natural choice in the \( AdS/QCD \), because the QCD condensate \( \langle \bar{q}q \rangle \) is represented by a solution to the bulk equations with scaling dimension \( d = 3 \). This has the disadvantage that the bulk equations for the axial spin-1 fields can be solved only numerically. The choice \( d = 1 \) is, instead, exactly solvable, but requires some attention in the regularization procedure, in order to make the zero-modes normalizable (and hence allowing for non-vanishing couplings with the tower of excited states) without introducing tachyonic degrees of freedom.

**PRELIMINARIES.**

Proper quantization of the gauge theory requires the introduction of gauge-fixing terms:

\[
\mathcal{L}_{GF} = \frac{1}{\xi_V z} \text{Tr} \left[ \partial_\mu V^\mu - \frac{z}{L} \xi_V \partial_z \frac{L}{z} V_5 \right]
\]

\[
+ \frac{1}{\xi_A z} \text{Tr} \left[ \partial_\mu A^\mu - \frac{z}{L} \xi_A \partial_z \frac{L}{z} A_5 - \xi_A \frac{L^2}{z^2} \sqrt{g^2 + g'^2} v \phi_0 \right],
\]

\[
+ \frac{1}{\xi_+ z} \text{Tr} \left[ \partial_\mu W^\mu - \frac{z}{L} \xi_+ \partial_z \frac{L}{z} W_5 - \xi_+ \frac{L^2}{z^2} g v \phi_W \right].
\]

Here and in the following, \( V = V_3 T_3 \), \( A = A_3 T_3 \) and \( W = W_1 T_1 + W_2 T_2 \) are defined in terms of the corresponding components of the original fields as:

\[
V = \frac{g' L + g R}{\sqrt{g^2 + g'^2}}, \quad (28)
\]
Analogously, $\phi_0 \equiv \phi_3 T_3$ and $\phi_W \equiv \phi_1 T_1 + \phi_2 T_2$.

These choices of gauge fixing allow to cancel the bilinear mixing between spin-1 and spin-0 fields arising from the bulk action. With this, the action can be written as the sum over vectorial part $S_V$ and axial part $S_A$ (the $S_+$ part is identical to the $S_A$ after replacing $A \rightarrow W$, $g^2 + g'^2 \rightarrow g^2$, $\xi_A \rightarrow \xi_+$ and $\phi_0 \rightarrow \phi_W$):

$$S_V = \int d^4x \int_{L_0}^{L_1} dz \left[ 1 + D \delta(z - L_0) \right] \left[ -\frac{1}{2} \text{Tr} V_{\mu\nu} V^{\mu\nu} \right] + \frac{L}{z} \text{Tr} \partial_z V_{\mu} \partial_z V^\mu - \frac{1}{\xi_V} \frac{L}{z} \text{Tr} \left[ \partial_\mu V^{\mu} \right]^2 + \frac{L}{z} \text{Tr} \left[ \partial_\mu \phi V^\mu \right] - \xi_V \frac{L}{z} \text{Tr} \left[ \partial_z \frac{L}{z} \phi \right]^2 + 2 \partial_z \left\{ \frac{L}{z} \text{Tr} \left[ \partial_\mu V^{\mu} \phi \right] \right\},$$

$$S_A = \int d^4x \int_{L_0}^{L_1} dz \left[ 1 + D \delta(z - L_0) \right] \left[ -\frac{1}{2} \text{Tr} A_{\mu\nu} A^{\mu\nu} \right] + \frac{L}{z} \text{Tr} \partial_z A_{\mu} \partial_z A^\mu - \frac{1}{\xi_A} \frac{L}{z} \text{Tr} \left[ \partial_\mu A^{\mu} \right]^2 + \left( \frac{L}{z} \right)^3 \left[ 1 + C \delta(z - L_0) \right] \frac{g^2 + g'^2}{4} v^2 \text{Tr} \left[ A_{\mu} A^{\mu} \right] + \frac{L}{z} \text{Tr} \left[ \partial_\mu A_5 \partial^\mu A_5 \right] - \xi_A \frac{L}{z} \text{Tr} \left[ \partial_z \frac{L}{z} A_5 + \frac{\sqrt{g^2 + g'^2}}{2} \left( \frac{L}{z} \right)^3 v \phi \right]^2 + 2 \partial_z \left\{ \frac{L}{z} \text{Tr} \left[ \partial_\mu A_5 A^{\mu} \right] \right\} + \left( \frac{L}{z} \right)^3 C \delta(z - L_0) v \sqrt{g^2 + g'^2} \text{Tr} \left[ A_{\mu} \partial_\mu \phi_0 \right] + \left( \frac{L}{z} \right)^3 \left[ 1 + C \delta(z - L_0) \right] \text{Tr} \left[ \partial_\mu \phi_0 \partial^\mu \phi_0 \right] - \left( \frac{L}{z} \right)^3 \frac{v^2}{g^2 + g'^2} \text{Tr} \left[ \frac{g^2 + g'^2}{2} A_5 + \sqrt{g^2 + g'^2} \partial_z \phi_0 \right]^2.$$

The unitary gauge is defined by $\xi_i = +\infty$.

The wave equations deduced from $S_V$ for the pseudo-scalar in unitary gauge, together with the boundary terms, are satisfied by:

$$V_5 = 0.$$
Hence one finds the action for the spin-1 states to reduce to
\[
S_V = \int d^4x \int_{L_0}^{L_1} dz \left[ 1 + D \delta(z - L_0) \right] \left[ -\frac{1}{2} \text{Tr} V_{\mu\nu} V^{\mu\nu} \right] \\
+ \frac{L}{z} \text{Tr} \partial_z V_{\mu} \partial_z V^{\mu},
\]
\[(34)\]
\[
= \int d^4q \int_{L_0}^{L_1} dz \frac{L}{z} \text{Tr} V_{\mu} \left[ (1 + D \delta(z - L_0)) q^2 P_{\mu\nu} + \eta_{\mu\nu} \frac{z}{L} \partial_z \frac{L}{z} \partial_z \right] V^{\nu} \\
+ \int d^4q \int_{L_0}^{L_1} dz \partial_z \left( \frac{L}{z} \text{Tr} V_{\mu} \partial_z V^{\mu} \right).
\]
\[(35)\]
After Fourier transforming in the four-dimensional coordinates, one can factorize the dependence on the fifth coordinate, and write: \( V_{\mu}(q, z) = V_{\mu}(q) v_{\nu}(z, q) \). Imposing the bulk equations
\[
\frac{z}{L} \partial_z \frac{L}{z} \partial_z v_{\nu}(z, q) = -q^2 v_{\nu}(z, q),
\]
\[(36)\]
and using Neumann boundary conditions in the IR
\[
\partial_z v_{\nu}(L_1, q) = 0,
\]
\[(37)\]
the action reduces to the action at the UV boundary:
\[
S_{\partial V} = \int d^4q \text{Tr} V_{\mu} \left[ \int dz \frac{L}{z} \delta(z - L_0) v_{\nu}(z, q) \left( (D q^2 P_{\mu\nu} v_{\nu}(z, q) + \eta_{\mu\nu} \partial_z v_{\nu}(z, q)) \right) \right] V^{\nu},
\]
\[(38)\]
from which, introducing appropriate localized currents for the SM gauge fields, the polarization appears to be:
\[
\pi_{\nu} = \frac{1}{p_{\nu}} = \mathcal{N} \left( D q^2 + \frac{\partial_z v_{\nu}}{v_{\nu}} \right)(L_0, q),
\]
\[(39)\]
with the overall normalization
\[
\mathcal{N}^{-1} \equiv \int_{L_0}^{L_1} dz \frac{L}{z} \left[ 1 + D \delta(z - L_0) \right] \\
= L \left( \frac{D}{L_0} + \ln \frac{L_1}{L_0} \right).
\]
\[(40)\]
From \( S_A \), the unitary gauge implies:
\[
\phi_0 = -\left( \frac{z}{L} \right)^3 \frac{2}{\sqrt{g^2 + g'^2}} \partial_z \frac{L}{z} A_5,
\]
\[(41)\]
and the boundary conditions for \( A_5 \) are derived from the condition of not having mixing at the boundaries:
\[
A_5(L_1) = 0,
\]
\[(42)\]
\[
\left( \frac{L}{z} A_5 - C \partial_z \frac{L}{z} A_5 \right) (L_0) = 0.
\]
\[(43)\]
The action for the spin-1 fields is hence, in unitary gauge:

\[
S_A = \int d^4x \int_{L_0}^{L_1} dz \frac{L}{z} \left[ 1 + D \delta(z - L_0) \right] \left[ -\frac{1}{2} \text{Tr} A_{\mu\nu} A^{\mu\nu} \right] \\
+ \frac{L}{z} \text{Tr} \partial_z A_\mu \partial_z A^\mu \\
+ \left( \frac{L}{z} \right)^3 \left[ 1 + C \delta(z - L_0) \right] \frac{g^2 + g'^2}{4} v^2 \text{Tr} [A_\mu A^\mu]'
\]

(44)

very similar to the previous case, but for the presence of the symmetry breaking terms induced by the non-vanishing \( v(z) \). Defining \( A_\mu(q, z) \equiv A_\mu(p) v_a(z, q) \), and redoing the same procedure as above, the polarization is:

\[
\pi_a = \frac{1}{p_a} = N \left( D q^2 + \frac{\partial_z v_a}{v_a} - C \left( \frac{L}{z} \right)^2 \frac{g^2 + g'^2}{4} v_a^2 \right) (L_0, q),
\]

(46)

where \( v_a \) satisfies

\[
\frac{z}{L} \partial_z \frac{L}{z} \partial_z v_a(z, q) - \left( \frac{L}{z} \right)^2 \frac{g^2 + g'^2}{4} v^2 v_a(z, q) = -q^2 v_a(z, q),
\]

(47)

\[
\partial_z v_a(L_1, q) = 0,
\]

(48)

and where the normalization constant \( N \) is the same as above. The analysis of the \( S_+ \) is the same, with the due substitutions.

The boundary terms \( C \) and \( D \) have a very important role, that is well illustrated by looking at the wave functions for the zero modes of the vector bosons and of the pions.

Focusing of the \( S_A \) sector in unitary gauge, and writing \( A_5 \equiv A_5(q) f(z, q) \), the zero-modes satisfy the equation

\[
\partial_z \left( \frac{z}{L} \right)^3 \frac{1}{v^2} \partial_z \frac{L}{z} f(z, 0) = \frac{1}{4} (g^2 + g'^2) f(z, 0),
\]

(49)

with the boundary conditions

\[
f(L_1, 0) = 0,
\]

(50)

\[
\left( \frac{L}{z} f(z, 0) - C \partial_z \frac{L}{z} f(z, 0) \right) (z = L_0) = 0,
\]

(51)

and the normalization given by

\[
1 = \int_{L_0}^{L_1} dz \left[ \frac{L}{z} f(z, 0)^2 + \left( \frac{z}{L} \right)^3 \frac{4(1 + C \delta(z - L_0))}{(g^2 + g'^2)^2 v^2} (\partial_z f(z, 0))^2 \right].
\]

(52)
In the two cases discussed here, it is convenient to write

\[ v(z) = a z^d, \]

and then rewrite \( M_Z^2 = (g^2 + g'^2) a^2 L^2 / 4 \). The solutions, having imposed the boundary conditions in the IR, can be written as

\[ f(z, 0) = c_0 z \left( K_0(M_Z L_1) I_0(M_Z z) - I_0(M_Z L_1) K_0(M_Z z) \right), \text{ for } d = 1, \]
\[ f(z, 0) = c_0 z e^{-M_Z z^2 / 2} \left( 1 - e^{M_Z (z^2 - L_1^2)} \right), \text{ for } d = 2, \]

with the constant \( c_0 \) determined by the normalization conditions.

Taking the limit \( L_0 \to L \to 0 \) is somewhat problematic, and will be discussed explicitly in the two examples later on, showing that this procedure dictates the form of the counter-terms \( C \) and \( D \). From Eq.\( (32) \), one can see that the counter-term \( C \) enters in the overall normalization of the pion state. Unitarity requires such normalization to be positive and finiteness of the coupling to other states requires it to be finite, and hence constraints the value of \( C \). Similarly, \( D \) enters the normalization of the photon field, and hence finiteness of the gauge coupling requires a finite normalization, while unitarity requires positive normalization.

It is useful to consider the limit in which in the dual strongly interacting theory the chiral symmetry is global, and the model becomes the effective description of a QCD-like theory (i.e., in which the weak gauging of the global symmetry of the CFT is set to zero). In order to do this, remind that the propagator of the photon can be expressed in terms of the generating functional for Green functions \( \Sigma(q) \), computed by treating the value of the gauge bosons at the UV-boundary as a (non-dynamical) source coupled to the currents \( J_\mu \):

\[ \langle J_\mu J_\nu \rangle = i P_{\mu\nu} \Sigma(q), \]

in the form

\[ \pi_v(q^2) = q^2 - g_4^2 \Sigma(q). \]

Hence, the current-current correlators of the QCD-like limit for these models can be computed as

\[ \Sigma_{VV}(q^2) = \frac{1}{g_4^2} \left( q^2 - \pi_v(q^2) \right), \]
\[ \Sigma_{AA}(q^2) = \frac{1}{g_4^2} \left( q^2 - \pi_a(q^2) \right), \]
with $g_4$ the effective gauge coupling in four dimensions of the zero-modes.

These correlators can be written (up to divergent constants) as:

$$\Sigma_{VV}(q^2) = q^2 \sum_n \frac{f_{\rho n}^2}{q^2 - M_{\rho n}^2},$$  \hspace{1cm} (60)

$$\Sigma_{AA}(q^2) = f_\pi^2 + q^2 \sum_n \frac{f_{a_1 n}^2}{q^2 - M_{a_1 n}^2},$$  \hspace{1cm} (61)

where the first sum runs over the vector mesons, and the second over the axial-vector mesons. The poles of the correlators give the masses of $\rho$ mesons and $a_1$ mesons, with all their excited states, and the residues the decay constants, while $f_\pi^2$ is the pion decay constant.

**AdS$_5$ BACKGROUND, $d = 1$**

I consider first the case with $d = 1$, i.e.:

$$v(z) = \frac{V_1}{L_1} z.$$  \hspace{1cm} (62)

The solution of the bulk equations, after imposing the Neumann boundary conditions in the IR, reads:

$$v_i(z, q) = c_i z \left( J_0(k_i L_1) Y_1(k_i z) - Y_0(k_i L_1) J_1(k_i z) \right),$$  \hspace{1cm} (63)

where $i = v, a, +$, $c_i$ are normalization constants, and where $k_v = q$, $k_+ = \sqrt{q^2 - M_W^2}$ and $k_a = \sqrt{q^2 - M_Z^2}$.

Consider first the vectorial sector. The polarization tensor, using the formulae discussed in the previous section, is given by:

$$\pi_v(q^2) = \frac{Dq^2 + \partial_z v_v(z, q)/v_v(z, q)}{L(D/L + \ln L_1/L_0)} \bigg|_{z=L_0}.$$  \hspace{1cm} (64)

Taking $L_0 \to L$, defining $D \equiv LD'$, and expanding for $L \to 0$:

$$\pi_v(q^2) = \frac{q^2}{D' + \ln \frac{L}{L_1}} \left( \frac{\pi Y_0(qL_1)}{2J_0(qL_1)} - \left( \gamma_E + \ln \frac{qL}{2} - D' \right) \right),$$  \hspace{1cm} (65)

which contains divergent terms for $L \to 0$. These can be reabsorbed in the localized boundary terms, by defining:

$$D' \equiv \ln \frac{L}{L_1} + \frac{1}{\varepsilon^2},$$  \hspace{1cm} (66)
and hence concluding that
\[ \pi_v(q^2) = q^2 \left( 1 - \varepsilon^2 \left( \gamma_E + \ln \frac{q L_1}{2} - \frac{\pi Y_0(q L_1)}{2 J_0(q L_1)} \right) \right). \tag{67} \]

The meaning of this procedure can be understood by looking at the explicit expression of the wave function of the zero mode of the vectorial part (to be identified with the photon), which with these definitions is given by
\[ v_v^{(0)} = \frac{\varepsilon}{\sqrt{L}}. \tag{68} \]

By contrast, for a constant \( v_v^{(0)}(z) = C^{(0)} \):
\[ \int_{L_0}^{L_1} \frac{L}{z} C^{(0)^2} = L \log \frac{L_1}{L_0} C^{(0)^2}. \tag{69} \]

Without the introduction of \( D \), the normalization condition would introduce a spurious logarithmic dependence on the unphysical UV cut-off scale \( L_0 \). It would not be possible to take the limit \( L_0 \to L \to 0 \) without effectively decoupling the photon from the spectrum (this been a non normalizable mode). To retain non-vanishing couplings of the photon, it would be necessary to work with finite \( L_0 \). This would affect all the physical quantities with an explicit, divergent, dependence on the precise choice of the cut-off (and of the regularization procedure itself, since there is no reason to think that a hard-wall cut-off in the UV has any physical meaning). The presence of the first term in Eq. (66) cancels this spurious dependence, trading this with a physical quantity \( \varepsilon \), which encodes the difference of the couplings of the zero modes and of the heavy modes to the brane.

The (five-dimensional) gauge coupling vanishes as \( g \sim \sqrt{L} \) for \( L \to 0 \). The four-dimensional, standard-model weak couplings \( g_4 \) and \( g'_4 \) can be read off the tree-level cubic and quartic interactions among the zero-modes and are given by
\[ g_4 = \frac{g \varepsilon}{\sqrt{L}}, \tag{70} \]
\[ g'_4 = \frac{g' \varepsilon}{\sqrt{L}}, \tag{71} \]
so that, at fixed \( \varepsilon \), they are finite for \( L \to 0 \). This means that the procedure adopted here is equivalent to taking the limit in which the UV cut-off goes to infinity by keeping the electro-weak gauge couplings fixed.

The procedure for the other sectors is similar,
\[ \pi_a(q^2) = \frac{D q^2 - C M_a^2 + \partial_z v_a(z, q) / v_a(z, q)}{L(D/L + \ln L_1/L_0)} \bigg|_{z=L_0}, \tag{72} \]
but for the fact that new divergences arise, in connection with the symmetry breaking term, that can be reabsorbed in $C$, 

$$C \equiv L \left( \ln \frac{L}{L_1} + \frac{1}{\rho^2} \right),$$  \hspace{1cm} (73)

yielding 

$$\pi_a(q^2) = q^2 - \varepsilon^2 \left( \frac{M_a^2}{\rho^2} + (q^2 - M_a^2) \left( \gamma_E + \ln \frac{L_1 \sqrt{q^2 - M_a^2}}{2} - \frac{\pi Y_0(L_1 \sqrt{q^2 - M_a^2})}{2 J_0(L_1 \sqrt{q^2 - M_a^2})} \right) \right)^2, \hspace{1cm} (74)$$

The expressions for the charged sector are identical, up to the replacement $M_a \rightarrow M_+$. The spectrum can be read off the poles of the propagators. Assuming the existence of a hierarchy between confinement scale and masses of the gauge bosons, i.e. $M_i L_1 \ll 1$: 

$$M_{W,Z}^2 \simeq \frac{\varepsilon^2}{\rho^2} M_{+,a}^2.$$

(75)

The mass of the neutral techni-\(\rho\)'s is controlled by the confinement scale $L_1$ and depends on $\varepsilon$. The mass of the lightest such state can be approximated by:

$$M_{\rho} \simeq \begin{cases} 4 \frac{L_1}{\varepsilon}, & \text{if } \varepsilon \ll 1 \\ 4 \frac{L_1}{\varepsilon}, & \text{if } \varepsilon = 1 \\ 4 \frac{L_1}{\varepsilon}, & \text{if } \varepsilon \gg 1. \end{cases} \hspace{1cm} (76)$$

The precision observables are computed starting from the series expansion of the polarization tensors in the momenta:

$$\pi_v(q^2) \simeq q^2 + \mathcal{O}(q^4), \hspace{1cm} (77)$$

$$\pi_a(0) = \varepsilon^2 M_a^2 \left( \gamma_E + \ln \left( \frac{M_a L_1}{2} \right) - \frac{1}{\rho^2} + \frac{K_0(M_a L_1)}{I_0(M_a L_1)} \right), \hspace{1cm} (78)$$

$$\pi'_a(0) = 1 - \varepsilon^2 \frac{1}{2} \left( 1 + 2 \gamma_E + \ln \frac{L_1^2 M_a^2}{4} - \frac{1 - 2 K_0(M_a L_1) I_0(M_a L_1)}{I_0(M_a L_1)^2} \right), \hspace{1cm} (79)$$

$$= 1 - \varepsilon^2 \frac{1}{2} \left( 1 + 2 \gamma_E - \frac{1}{I_0(M_a L_1)^2} \right) - \frac{\pi_a(0)}{M_a^2} \hspace{1cm} (80)$$

$$\hat{S} = \frac{\varepsilon^2 M_W^2}{2 M_Z^2} \left( \frac{2 I_0(L_1 M_a) K_0(L_1 M_a)}{I_0(L_1 M_a)^2} - 1 \right) + \log \left( \frac{L_1^2 M_a^2}{4} \right) + 2 \gamma_E + 1 \hspace{1cm} (81)$$
\[ \approx \frac{1}{2} \varepsilon^2 \frac{M_W^2}{M_Z^2} M_W^2 L_1^2 = \frac{\rho^2}{2} M_W^2 L_1^2, \]  
(82)

\[ \hat{T} \approx \frac{\beta^4}{4 \varepsilon^2} L_1^2 \left( M_Z^2 - M_W^2 \right), \]  
(83)

where the approximations are valid for \( M_{W,Z} \ll 1/L_1 \). The ratio of the two is independent of the confinement scale:

\[ \frac{\hat{T}}{S} \approx \frac{M_Z^2 - M_W^2 \rho^2}{2 M_W^2} \frac{1}{\varepsilon^2} \approx 0.15 \frac{\rho^2}{\varepsilon^2}. \]  
(84)

In order to have a better understanding of what the boundary terms mean, and also in order to illustrate why the spectrum depends on \( \varepsilon \), I turn the attention to the limit in which the strongly and weakly coupled sectors decouple from each other. The spectrum of the strongly interacting sector of the model reduces to a QCD-like spectrum, containing a set of pions and two towers of heavy \( \rho \) and \( a_1 \) states, decoupled from the electro-weak gauge bosons (photon, \( W \) and \( Z \) are non normalizable in this limit). This is obtained in the limit \( \varepsilon \to 0 \).

The vector-vector correlator is:

\[ \Sigma_{VV}(q^2) = \frac{1}{g_4^2} \left( q^2 - \pi_v(q^2) \right) \]  
(85)

\[ = \frac{1}{g_\rho^2} q^2 \left( \gamma_E + \ln \frac{qL_1}{2} - \frac{\pi}{2} \frac{Y_0(qL_1)}{J_0(qL_1)} \right) \]  
(86)

\[ = q^2 \left( \sum_n \frac{f_{\rho n}^2}{M_{\rho n}^2 (q^2 - M_{\rho n}^2)} \right), \]  
(87)

\[ = q^2 \left( \sum_n \frac{f_{\rho n}^2}{M_{\rho n}^2} + \sum_n \frac{f_{\rho n}^2}{q^2 - M_{\rho n}^2} \right), \]  
(88)

from which one derives the spectrum of the strong sector. The coupling \( g_\rho = g/\sqrt{L} \) is the effective coupling of the vector mesons in the four-dimensional language. The masses of the vector mesons are given by the zeros of \( J_0(qL_1) \)

\[ M_{\rho n} = \frac{1}{L_1} \left( 2.4, 5.5, 8.6, 11.8, \ldots \right), \]  
(89)

\[ \approx \frac{1}{L_1} (\pi (n - 1/4)) \]  
(90)

and the residues of \( \Sigma_{VV} \) give the decay constants:

\[ f_{\rho n} = \frac{1}{g_\rho L_1} \left( 2.7, 4.1, 5.2, 6.1, \ldots \right). \]  
(91)

\[ f_{\rho n}^2 \approx \frac{1}{g_\rho^2 L_1^2} (-2.45 + \pi^2 n). \]  
(92)
Notice how a parametric suppression of the decay constants could be achieved by tuning by hand the coupling $g_\rho$ to very large values.

In the limit in which $q \gg 1/L_1$, the correlator behaves as:

$$\Sigma_{VV}(q^2) \to \frac{1}{2g_\rho^2}q^2 \ln q^2,$$

which, compared with the OPE results, suggests that

$$\frac{1}{g_\rho^2} \approx \frac{N_c}{12\pi^2},$$

with $N_c$ the number of colors of the $SU(N_c)$ QCD-like dynamics. In all what done here, the basic assumption is that the five-dimensional gauge coupling $g$ be small. This expression shows that this corresponds to the limit of large $N_c$ of the dual description, and hence indicates that the computations performed here are accurate only in this regime. In particular, this means that one cannot take $g_\rho$ to arbitrarily large values, because this would invalidate the five-dimensional perturbative expansion used here, or equivalently, in the dual description, this would be equivalent to the study of a small $N_c$ strongly coupled model, for which no parametric suppression of the loop effects is present.

For the axial-axial correlator:

$$\Sigma_{AA}(q^2) = \frac{1}{g_\rho^2} \left( \frac{M_\rho^2}{\rho^2} + (q^2 - M^2) \left( \gamma_E + \ln \frac{L_1 \sqrt{q^2 - M^2}}{2} - \frac{\pi Y_0(L_1 \sqrt{q^2 - M^2})}{J_0(L_1 \sqrt{q^2 - M^2})} \right) \right)$$

$$= \frac{M^2}{g_\rho^2 \rho^2} + \Sigma_{VV}(q^2 - M^2)$$

$$= \frac{M^2}{g_\rho^2 \rho^2} + (q^2 - M^2) \left( \sum_n \frac{(q^2 - M^2)f_{\rho n}^2}{M_{\rho n}^2(q^2 - M^2 - M_{\rho n}^2)} \right)$$

$$= \frac{M^2}{g_\rho^2 \rho^2} - M^2 \sum_n \frac{M_{\rho n}^2 f_{\rho n}^2}{M_{\rho n}^2(M^2 + M_{\rho n}^2)} + q^2 \sum_n \frac{f_{\rho n}^2}{M_{\rho n}^2}$$

$$+ q^2 \sum_n \frac{f_{\rho n}^2 M_{\rho n}^2}{(M^2 + M_{\rho n}^2)(q^2 - M^2 - M_{\rho n}^2)},$$

where $M^2 = g^2 L_1^2 \nu_0^2/2$ (notice that in looking at the strong sector by itself, it is convenient to look at the $g = g'$ limit, directly comparable to QCD-like theories). From here, the masses and decay constants of the axial excitations can be derived to be:

$$M_{a_{1n}}^2 = M_{\rho n}^2 + M^2,$$

$$f_{a_{1n}}^2 = f_{\rho n}^2 \frac{M_{\rho n}^2}{M^2 + M_{\rho n}^2},$$

$$f_{a_{1n}}^2 = f_{\rho n}^2 \frac{M_{\rho n}^2}{M^2 + M_{\rho n}^2},$$
which automatically implies that one of the Weinberg sum rules is satisfied:

$$\sum_n (f^2_{\rho n} M^2_{\rho n} - f^2_{a1n} M^2_{a1n}) = 0.$$  \tag{101}

The pion decay constant is

$$f^2_\pi = \Sigma_{AA}(0)$$  \tag{102}

$$= \frac{M^2}{g^2\rho^2} - M^4 \sum_n \frac{f^2_{\rho n}}{M^2_{\rho n} M^2_{a1n}} - \frac{g^2\rho^2}{M^2 - M^4 \sum_n f^2_{\rho n} / (M^2_{\rho n} + M^2)}$$  \tag{103}

$$= \frac{M^2}{g^2\rho^2} - M^2 \left(\gamma_E + \ln \frac{ML^1}{2} + \frac{K_0(ML^1)}{I_0(ML^1)}\right)$$  \tag{104}

which allows to replace \(\rho\) with the decay constant:

$$\frac{1}{\rho^2} = g^2\rho \left(\frac{f^2_\pi}{M^2} + \gamma_E + \ln \frac{ML^1}{2} + \frac{K_0(ML^1)}{I_0(ML^1)}\right).$$  \tag{105}

The other Weinberg sum rule is not satisfied:

$$\sum_n (f^2_{\rho n} - f^2_{a1n}) - f^2_\pi = \sum_n f^2_{\rho n} \left(1 - \frac{M^2_{\rho n}}{M^2_{\rho n} + M^2} - \frac{M^4}{M^2_{\rho n}(M^2_{\rho n} + M^2)}\right) - \frac{M^2}{g^2\rho^2},$$  \tag{106}

$$= M^2 \left(- \frac{1}{g^2\rho^2} + \sum_n f^2_{\rho n} \frac{M^2_{\rho n} - M^2}{M^2_{\rho n}(M^2_{\rho n} + M^2)}\right)$$  \tag{107}

$$\neq 0,$$  \tag{108}

unless \(M = 0\). This is a test illustrating the fact that the interpretation of the setting given is the one anticipated, namely that the condensate responsible for symmetry breaking has dimension \(d = 1\).

Going back to the precision observables, it is possible to trade now the unknown parameter \(\rho\) for \(f_\pi\), and explicitly verify that

$$\hat{S} = \varepsilon^2 \frac{M^2_W}{f^2_\pi} \sum \left(\frac{f^2_{\rho n}}{M^2_{\rho n}} - \frac{f^2_{a1n}}{M^2_{a1n}}\right),$$  \tag{109}

This result comes from the regularization on the UV-brane. Removing completely the boundary term \(C\), and taking the \(L_0 \to 0\) limit naively, the normalization of the pion field diverges. Analogously, if \(D = 0\) the normalizations of the photon, \(W\) and \(Z\) gauge bosons diverge. This theory would consist of a set of massless, free gauge bosons and pseudo-scalars, with vanishing couplings to the SM currents. The regularization and renormalization procedure adopted here requires a choice of \(C\) and \(D\) such as to make these normalizations both finite and positive, so as to restore finite interactions while preserving unitarity. As such, it should not come as a surprise that both the gauge coupling of the photon as well as the pion decay constant are free parameters in the \(d = 1\) case.
which relates the precision observables to the masses and decay constants of the strong sector of the theory. In Eq. (109), masses and decay constants are not those of the techni-mesons, but the position of the poles in $\Sigma_{VV}$, giving the spectrum of the strong sector alone, decoupled from the SM gauge bosons. The quantity $\hat{S}$ is positive definite, as a result of the fact that $\varepsilon^2$ has to be chosen to be positive in order to have a positive normalization of the photon wave function, and hence positivity of the spin-1 boson contribution to $\hat{S}$ is a consequence of unitarity requirements.

Looking at the expression for $f_\pi$, or equivalently to $\pi_a(0)$, one sees that setting $1/\rho^2 \to 0$ results in the model being pathological, since Eq. (108) is not positive in this case. Nevertheless, the fact that $C$ is needed as a divergent counter-term also implies that $1/\rho^2$ is a parameter in the model, that has to be chosen as to make these quantities positive. Setting $1/\rho \to 0$ would be just a wrong choice for the regulator. The model with $d = 1$ has, hence, two free parameters more than the naive counting of the bulk interaction terms would suggest. One controls the relative strength of the effective coupling of the SM gauge bosons in respect to the one of the excited states ($\varepsilon$). The other controls the relative size of the symmetry breaking effects experienced by the lowest modes in respect to those experienced by the excited states ($\rho$).

**AdS\textsubscript{5} BACKGROUND, \(d = 2\).**

If the scalar background is

$$v = \frac{V_1}{L_1^2} z^2 = \frac{v_0}{L_0^2} z^2,$$

this has the effect of modifying the equations of motion in the AdS background for the tower of excitations of $W$ and $Z$ gauge bosons:

$$\partial_z \frac{L}{z} \partial_z v_v = -q^2 \frac{L}{z} v_v,$$

$$\partial_z \frac{L}{z} \partial_z v_a - \mu_Z^4 L z v_a = -q^2 \frac{L}{z} v_a,$$

$$\partial_z \frac{L}{z} \partial_z v_+ - \mu_W^4 L z v_+ = -q^2 \frac{L}{z} v_+,$$

where $\mu_W^4 = 1/4g^2 v_0^2/L^2$ and $\mu_Z^4 = 1/4(g^2 + g'^2)v_0^2/L^2$. Notice that the term responsible for symmetry-breaking has now a different $z$-dependence with respect to the mass term.
The general solution to the differential equation can be written in terms of generalized Laguerre and Hypergeometric functions:

\[ v_a(z, q) = e^{-\frac{\mu^2 Z z^2}{2}} \left[ c_1 U \left( -\frac{q^2}{4\mu Z}, 0, \mu_Z^2 z^2 \right) + c_2 L \left( \frac{q^2}{4\mu Z}, -1, \mu_Z^2 z^2 \right) \right], \]  

(114)

with \( c_1 \) and \( c_2 \) integration constants. Their ratio is fixed by the IR-boundary conditions, and the overall normalization by the normalization of the states. I choose to write them as:

\[ c_1 = 2L \left( 1 + \frac{q^2}{4\mu^2 Z}, \mu_Z^2 L_1^2 \right) + L \left( \frac{q^2}{4\mu^2 Z}, -1, \mu_Z^2 L_1^2 \right), \]  

(115)

\[ c_2 = -U \left( -\frac{q^2}{4\mu^2 Z}, 0, \mu_Z^2 L_1^2 \right) + \frac{q^2}{2\mu_Z} U \left( 1 - \frac{q^2}{4\mu^2 Z}, 1, \mu_Z^2 L_1^2 \right). \]  

(116)

On the UV boundary, for \( L_0 \to 0 \):

\[ \frac{\partial_z a}{a} \to L_0 \left\{ \mu_Z^2 - q^2 \left[ \gamma_E + \ln(\mu_Z L_0) + \frac{1}{2} \psi \left( -\frac{q^2}{4\mu_Z} \right) - \frac{c_2}{2c_1} \Gamma \left( -\frac{q^2}{4\mu_Z} \right) \right] \right\}, \]  

(117)

with \( \psi \) the digamma function.

The counter-term \( D \) has been fixed by the normalization of the photon, and hence:

\[ D = L_0 \left( \ln \frac{L_0}{L_1} + \frac{1}{\varepsilon^2} \right), \]  

(118)

\[ \mathcal{N} = \frac{\varepsilon^2}{L}. \]  

(119)

Substituting this in the expression for the polarization, with the redefinition \( C \equiv y^2/L_0 \):

\[ \pi_a(q^2) = \mathcal{N} \left( Dq^2 - C\mu_Z^4 L_0^2 + \frac{\partial_z a}{a}(q^2, L_0) \right) \]  

(120)

\[ = \frac{L_0}{L} \left\{ q^2 - \varepsilon^2 \left[ -\mu_Z^2 + y^2 \mu_Z^4 \right] + q^2 \left[ \ln \mu_Z L_1 + \gamma_E + \frac{1}{2} \psi \left( -\frac{q^2}{4\mu_Z^2} \right) - \frac{c_2}{2c_1} \Gamma \left( -\frac{q^2}{4\mu_Z^2} \right) \right] \right\}, \]  

(121)

which shows how the regulator needed for the vector fields also regularizes the axial fields in this case. Notice that, as expected, in the \( \varepsilon \to 0 \) limit the polarization reduces to that of a single, massless spin-1 field. Finally, it is now possible to take the limit \( L_0 \to L \to 0 \), which has been regularized, along as \( \mu_Z \) and \( y \) are kept fixed.

It is easier to discuss the properties of the strong interacting sector of this model by looking at \( \Sigma_{AA}(q^2) \):

\[ \Sigma_{AA}(q^2) = \frac{1}{g_\rho^2} \left[ -\mu_Z^2 + y^2 \mu_Z^4 + q^2 \left[ \gamma_E + \ln \mu_Z L_1 + \psi \left( -\frac{q^2}{4\mu_Z^2} \right) - \frac{c_2}{c_1} \Gamma \left( -\frac{q^2}{4\mu_Z^2} \right) \right] \right] \]  

(122)
Taking the zero-momentum limit:

\[ f_\pi^2 = \Sigma_{AA}(0) = \frac{\mu_Z^2}{g_\rho^2} \left( y^2 \mu_Z^2 + \tanh \frac{\mu_Z^2 L_1^2}{2} \right). \]  

(123)

Notice how \( f_\pi^2 \) is a positive-definite quantity in absence of the counter-term \( C \) \((y = 0)\), which proves the model being automatically unitary, in contrast with what found in the \( d = 1 \) case.

The function \( \Sigma_{AA}(q^2) \) can be plotted and studied easily, in spite of its non-inspiring analytical expression. The explicit computation of residues and poles is non trivial, though, and can be done only numerically. It is hence worth discussing some special limits, in which the expressions simplify.

In the limit in which \( \mu_Z^2 L_1 \to 0 \), the bulk equations reduce to those of the vectorial sector. Hence, for very small values of \( \mu_Z^2 L_1 \), \( \Sigma_{AA} \sim \Sigma_{VV} \). In particular, the spectrum of the axial sector is given by the zeros of the Bessel function \( J_0(qL_1) \), up to small corrections due to symmetry breaking. The position of the poles can be approximated by a quadratic sequence \( M_{a_1}^2 \propto n^2 \). The main deviation from \( \Sigma_{VV}(q^2) \) is in the region \( q^2 \ll \mu^4 L_1^2 \), where the bulk equations admit a simple solution in terms of hyperbolic functions, which is the origin of the simple expression for \( f_\pi \).

In the opposite limit, in which \( \mu_Z L_1^2 \gg 1 \), the spectrum is severely modified. Below \( \mu_Z \), the position of the poles grows linearly \( M_{a_1}^2 \propto n \), and then goes back to the quadratic behavior for very large masses \( M_{a_1} \gg \mu_Z \). To understand why, one can look explicitly at the expression for the ratio \( c_2/c_1 \): in the limit of small \( q^2 \), and for asymptotically large values of \( \mu_Z^2 L_1^2 \), this vanishes, so that the correlator reduces to

\[ \Sigma_{AA} \sim \frac{1}{g_\rho^2} \left( -\mu_Z^2 + y^2 \mu_Z^4 + q^2 \left( \gamma_E + \ln \mu_Z L_1 + \psi \left( -\frac{q^2}{4\mu_Z^2} \right) \right) \right). \]

(124)

For small momenta, the poles are those of the \( \psi(-q^2/(4\mu_Z^2)) \), i.e. the poles of the \( \Gamma \) function. This property emerged in a similar model discussed in [6], and was used there to reproduces the Regge trajectories. Incidentally, the Veneziano amplitude reproduces the Regge trajectories for the same reason, its non-analytic structure being dictated by the Euler Gamma function. Of course, here this property applies only to the axial sector, and hence this interpretation is not viable.

The actual computation of the decay constants is not simple in this model, and not very illuminating. It is hence difficult to verify the Weinberg sum rules. However, as long as the AdS-CFT interpretation holds, one expects that the second such rule be violated, due to
the presence of a dimension-2 condensate. This is analogous to what was studied in [12] in the model-building effort to reduce $\hat{S}$ using the dispersion relations in order to quantify the non-perturbative corrections to the perturbative estimates. As for the first sum rule, if the counter-term $C$ is assumed to scale as $C \sim L_0$ (which is the natural choice), it disappears from the polarizations, and hence the only symmetry breaking term would be the bulk VEV. In this way the first Weinberg sum rule must hold. Not so if one assumes the scaling $C = y^2/L_0$, with $y^2$ kept fixed when taking the limit $L_0 \to L \to 0$. This would imply the presence of an additional, dimension-1, symmetry breaking term localized on the UV-brane, possibly to be interpreted as the VEV of an additional Higgs field, localized on the brane and not connected to the strong sector. This might be a useful tool for the construction of realistic mass matrices for the SM fermions. But the phenomenology would be determined by the interplay between these two terms, one of which is not dictated by any specific reason, since $C$ is here neither required by the regularization procedure nor by unitarity arguments, and hence from now on I will set $y = 0 = C$ in the $d = 2$ case.

For the actual computation of the precision observables, it is useful to expand the polarization functions in powers of the momentum, obtaining:

$$\pi_a(0) = -\epsilon^2 \mu_Z^2 \left( \tanh \frac{\mu_Z^2 L_1^2}{2} \right).$$

(125)

In the limit of large $\mu^4 L_1^4$, the $q^2$ coefficient of the expansion can be approximated as:

$$\pi'_a(0) \sim \left( 1 - \frac{\epsilon^2}{2} \left( \gamma_E + \ln \mu_Z^2 L_1^2 \right) \right),$$

(126)

while in the more interesting regime in which $\mu_Z L_1 \ll 1$, it is approximated by:

$$\pi'_a(0) \approx 1 - \frac{\epsilon^2}{2e} \mu_Z^4 L_1^4,$$

(127)

with $e \approx 2.7$.

If $\mu_Z L_1$ is large, then one obtains for $\hat{S}$:

$$\hat{S} = \frac{\epsilon^2}{2} \frac{M_W^2}{M_Z^2} \left( \gamma_E + \ln \mu_Z^2 L_1^2 \right),$$

(128)

which is an $O(1)$ number, and hence totally incompatible with the experimental constraints. This case is hence excluded by experimental data on precision electro-weak observables.

In the limit in which the symmetry breaking term is small ($\mu_Z L_1^2 \ll 1$) and in which the coupling between weak and strong sector is small ($\epsilon \ll 1$) the spectrum consists of a tower
of heavy spin-1 fields, in which the mass splitting between $Z$, $W$ and photon excited states is negligibly small, and the lightest of which has a mass:

$$M_{\rho^0} = \frac{2.4}{L_1},$$

while the only light states are the SM gauge bosons, whose masses are given by:

$$M_{\gamma}^2 = 0$$

$$M_Z^2 \simeq \frac{\varepsilon^2}{2} \mu_2^4 L_1^2,$$

$$M_W^2 \simeq \frac{g^2}{g^2 + g'^2} M_Z^2.$$  

Finally, the precision parameters are given by

$$\hat{S} \simeq \frac{\varepsilon^2 M_W^2}{2e M_Z^2} \mu_2^4 L_1^4$$

$$\simeq \frac{1}{e} M_W^2 L_1^2,$$

$$\hat{T} = \varepsilon^2 \left( -\frac{\mu_2^2}{M_Z^2} \tanh \frac{\mu_2^2 L_1^2}{2} + \frac{\mu_W^2}{M_W^2} \tanh \frac{\mu_W^2 L_1^2}{2} \right)$$

$$\simeq \frac{M_Z^2 - M_W^2}{6\varepsilon^2} L_1^2,$$

so that the ratio

$$\frac{\hat{T}}{\hat{S}} \simeq \frac{e M_Z^2 - M_W^2}{2M_W^2},$$

does not depend on the confinement scale, as in the previous case.

**ELECTRO-WEAK PRECISION PARAMETERS AND SPIN-1 EXCITATIONS.**

The precision parameter $\hat{T}$ is a very model-dependent quantity, and in particular can be set to zero by modifying the model so that the excited states do not violate custodial symmetry, for by gauging a full $SU(2)_L \times SU(2)_R$ in the bulk, and adding some additional symmetry breaking term on the UV-brane. Furthermore, the experimental bounds on $\hat{T}$ are easy to satisfy, because less stringent than the bounds on $\hat{S}$, and because in these models the ratio $\hat{T}/\hat{S}$ is always suppressed by the ratio $(M_Z^2 - M_W^2)/2M_W^2 \simeq 0.15$. The only problematic regime would be the one in which $\varepsilon$ is very small, in which case all the
computations performed here would not be reliable anyhow. For these reasons, I focus the
discussion here on $\hat{S}$.

For the case $d = 1$, I derived the approximate expression

$$\hat{S} \simeq \frac{k^2}{2} M_W^2 L_1^2,$$

$$= \frac{k^2 \rho^2 M_W^2}{2 M_{\rho}^2},$$

(138)

(139)

where the constant $k \in [2.4, 4.7]$ is determined by the value of $\varepsilon$. The experimental bound
on $\hat{S}$ implies that

$$M_{\rho} > k \rho (1 \text{TeV}).$$

(140)

For small values of $\varepsilon$ and $\rho$, this is obviously satisfied even for very light masses $M_{\rho} \sim 1$
TeV. For all practical purposes, $\hat{S}$ in this model is a free parameter, only constrained to be positive by unitarity requirements. The mechanism that makes $\hat{S}$ small by choosing small
values for $\rho$ essentially corresponds to a localization of the symmetry breaking effects to the
UV-brane, so that the SM gauge bosons are directly affected by it, while the excited modes experience symmetry breaking with an additional suppression factor.

The spectrum of the model contains, besides the usual SM gauge fields, with SM-like
phenomenology, a tower of spin-1 states, with tiny splitting between the excitations of the
photon, the $W$ and the $Z$ bosons. The lightest such states have masses in the LHC energy
range.

The production (and decay) rates at the LHC are well illustrated by looking at the $g_{\rho \pi \pi}$
coupling of the strong sector. In models in which the tower of excited spin-1 states is
described by a (local) extra-dimension theory, the KSRF relation is modified to

$$g_{\rho \pi \pi}^2 \simeq c_g \frac{M_{\rho}^2}{f_{\pi}^2},$$

(141)

where $c_g > 3$ is a model dependent numerical constant. The choice of a small $\rho$ is equivalent
to the choice of an enhanced value for $f_{\pi}$, and hence a parametrical suppression of the
coupling of the techni-mesons to the $W$ and $Z$ gauge bosons. As a result, the feature allowing
for a very light spectrum of excitations, would also result in a suppression of the production

\footnote{A precise computation of the value of $g_{\rho \pi \pi}$ requires to explicitly compute the 3-point functions, which goes beyond the aims of this work.}
and decay rate of these states. But moderately small values of $\rho \sim 1/2$ would be sufficient anyhow to evade the bounds, without incurring in this problem. A more accurate study of the decay rate and production mechanism at the LHC is needed in order to determine how easy (or difficult) it could be to directly detect these states, but this is not a parametrically big problem.

There is a substantial difference in the $d = 2$ case. Here, the boundary term $C$ is not required. In principle, it is certainly possible to add by hand such a term, and hence suppress the contribution to $\hat{S}$ in the very same way as done in the $d = 1$ case. But this would be an ad hoc, unnecessary additional ingredient, essentially equivalent to adding a higher-dimensional operator in the four-dimensional dual, which would produce a tree-level contribution to $\hat{S}$ fine-tuned in such a way as to cancel the contribution coming from the heavy states. It is hence more interesting to study the model without this parameter.

As explained at length in the previous section, the limit in which $\mu^2 W^L \gg 1$ leads to a big modification of the polarizations at small momenta, and as a consequence to a big modification of the spectrum of the axial states. In particular, the mass-squared of the standard-model $Z$ and $W$ gauge bosons are found to scale linearly with the gauge coupling in this regime. This is obviously not compatible with the standard-model predictions, as seen from the precision parameters, that turn out to be $O(1)$ quantities (see for instance Eq. (128)). This regime is hence clearly incompatible with experimental data, implying that $\mu^2 W^L \ll 1$.

The expression for $\hat{S}$ in the phenomenologically acceptable range for the parameters of the model is hence

$$\hat{S} \simeq \frac{1}{\epsilon} M^2 W^L_1,$$

which can be translated in a bound for the lightest techni-$\rho$ mass

$$M_\rho > k (880 \text{GeV}),$$

where $k$ is the same, $\varepsilon$-dependent constant of the $d = 1$ case. Depending on $\varepsilon$ this means

$$M_\rho > (2 - 4) \text{TeV}.$$

This result is in the upper limit of reach at the LHC. If the most pessimist bound is assumed, it is difficult to believe that the LHC signal for the new states could be the appearance of
a resonance in the spectrum, and much more elaborate data analysis strategies would be needed. The lowest end of the limit however is within LHC reach and is obtained for moderately small values of $\varepsilon \sim 1/3$. Again, it is instructive to look at the $g_{\rho\pi\pi}$ coupling:

$$g_{\rho\pi\pi}^2 \simeq c_g \frac{M_\rho^2}{f_\pi^2} \frac{\varepsilon^2 g_\rho^2 M_\rho^2}{M_Z^2} \simeq c_g (g_4^2 + g_4'^2) \frac{M_\rho^2}{M_Z^2},$$

where here $M_\rho = 2.4/L_1$. This is certainly a strong coupling, though a more accurate estimate is needed. The experimental signal of this model is hence expected to be quite clear: there is no parametric suppression neither of the production nor decay rate of the spin-1 techni-meson excitations, and it should be possible to detect them at the LHC. In particular, even if the most pessimistic bound is assumed, it should be possible to collect a large number of techni-$\rho$ decay events from the tails of its (broad) resonance.

The bound on the mass of the lightest techni-$\rho$ of few TeV is in substantial agreement with analogous estimates done by simple re-scaling of QCD. One might wonder what has been gained here with this long exercise. At first sight, it seems that $\hat{S}$ is suppressed by the same naif idea of pushing $M_\rho$ to large values. This fact deserves a comment.

If an interpretation of the present results in terms of a four-dimensional theory holds, it does so only as long as loop corrections are parametrically suppressed, i.e. only at large-$N_c$. At large $N_c$, in a QCD-like theory the parametric separation between $f_\pi$ and $M_\rho$ disappears, and at the same time the decay constants $f_\rho$ become bigger, because controlled by the effective coupling itself. Hence, the large-$N_c$ expectation, based on the results from dispersion relations, is that $\hat{S}$ has to be very big, because none of the mechanisms for its suppression is available. By contrast, what found here is that the parametric separation between $M_\rho$ and $f_\pi$ can be obtained through walking dynamics, even at large-$N_c$. On top of that, walking makes the excited states less sensitive to symmetry-breaking effects, so that, while it is still true that the decay constants $f_\rho$ are big, this effect is compensated by a high degree of cancellation between axial-vector and vector contribution, which is absent in a QCD-like model. This high degree of degeneracy among the towers of axial and vectorial excitations and their broadness are the most striking signature distinguishing the present models from traditional small-$N_c$ QCD-inspired models.
Concluding this section, in both models with $d = 1$ and $d = 2$ there are significant regions of parameter space that could be tested at the LHC and are compatible with precision electro-weak data. In both cases, the signature would be the existence of a set of strongly interacting spin-1 states with masses in the TeV region, and mass splitting far too tiny for direct resolution. The decay modes of the resulting resonance would hence comprise both even and odd parity channels.

**THE EFFECT OF WALKING BEHAVIOR.**

In the models discussed here, experimental bounds are evaded by assuming the existence of a substantial hierarchy between the masses of the SM gauge bosons and those of their excited states. This hierarchy might, in general, result from two complementary effects.

One is the hierarchy between the weak couplings of the SM and the strong effective coupling of the new states. This effect is parameterized by $\varepsilon$. The computations developed here are reliable only for values of $\varepsilon$ not too small, because tiny values would translate into a large coupling $g$, and the perturbative expansion in the five-dimensional gauge theory would not hold. Large or $\mathcal{O}(1)$ values of $\varepsilon$ correspond to large-$N_c$, small values to small-$N_c$. Hence, this cannot be the main reason why $\hat{S}$ is small enough to evade the experimental constraints.

The major effect is produced by the parametric separation between the symmetry breaking scale and the confinement scale. At large $N_c$ one would expect this to be just an $\mathcal{O}(1)$ effect. The claim here is that walking can explain the presence of such a hierarchy at large $N_c$, and as a consequence the bounds on $\hat{S}$ are evaded by having a large enough mass of the techni-$\rho$, while at the same time walking produces also a quasi-degenerate spectrum of techni-$\rho$ and techni-$a_1$ states, that compensates for the largeness of the decay constants for the excited states.

In order to assess the meaning of this assumption, I focus on the $d = 2$ case, which is the cleanest and less model-dependent. The whole study performed here is supposed to describe the effect of conformal symmetry (and walking) in modifying the behavior of a QCD-like model, and as a tool for the computation of non-perturbative corrections to the tree-level estimates of the precision parameter $\hat{S}$. It is hence instructive to compare the $d = 2$ results to those of a perturbative $SU(N_c)$ QCD-like technicolor model, and to the non-perturbative estimates obtained in absence of large anomalous dimension.
The perturbative estimate of \( \hat{S} \) in a \( SU(N_c) \) technicolor model with \( N_d \) fermions in the fundamental of both \( SU(N_c) \) and \( SU(2)_L \) can be written as

\[
\hat{S}_p = \frac{\alpha}{4 \sin^2 \theta_W} \frac{N_c N_d}{6\pi}.
\] (148)

For comparison, in the models discussed here, \( N_c \) can be extracted from the high-energy behavior of the vector-vector correlator, and is given by:

\[
N_c N_d = \frac{12\pi^2}{g_\rho^2}
= \frac{12\pi^2 \varepsilon^2}{g_4^2}
= \frac{3\pi \varepsilon^2 \sin^2 \theta_W}{\alpha},
\] (149)\( \) (150)\( \) (151)

and hence the perturbative estimate would be

\[
\hat{S}_p = \frac{\varepsilon^2}{8}.
\] (152)

This estimate is in general agreement with what expected in a QCD-like technicolor model, since in this case \( \varepsilon \sim g_4/g_\rho \sim 1/8 \) can be estimated from the ratio of the mass of the \( \rho \) meson and the decay constant of the pion, though in this range of \( \varepsilon \) the computations performed here are not reliable and hence this comparison should not be taken too literally.

The estimate in Eq. (152) is, obviously, a gigantic departure from what computed here, namely

\[
\hat{S} \simeq \frac{\varepsilon^2}{2\epsilon} \mu_W^4 L_1^4,
\] (153)

in which an additional suppression factor is coming from the parametric separation between the symmetry breaking mass term for the spin-1 excitations and the confinement scale \( L_1 \).

Remember that, for \( L_0 \simeq L \),

\[
\mu_W^4 L_1^4 = \frac{g^2 v_0^2}{4 L^2} L_1^4 = \frac{g^2 v_1^2 L_0^4}{4 L^2} \simeq \frac{g^2}{4} v_1^2 L_0^2,
\] (154)

is the (dimensionless) product of the coefficient of the symmetry-breaking term in the bulk equations for the charged gauge bosons, times the fourth power of the confinement scale.

The claim of this paper is that this suppression factor is natural, and is precisely the effect of walking, which makes the non-perturbative estimate of \( \hat{S} \) deviate by orders of magnitude
in respect to the perturbative estimates. This claim seems to contradict analogous studies in the literature, and hence it is worth comparing to the $d = 3$ case.

In [3], the $d = 3$ case is solved numerically. Imposing the experimental constraints that $M_\rho \simeq 770$ MeV, $f_\rho \simeq 140$ MeV and $M_{a_1} \simeq 1230$ MeV, the authors of [3] find the predictions $f_\pi \simeq 87$ MeV and $f_{a_1} \simeq 160$ MeV. These results are obtained for a value of their symmetry-breaking parameter $\xi \simeq 4$, while for small values of $\xi$, $f_\pi$ turns out to be parametrically small, with $M_\rho \sim M_{a_1}$. For such large values of $\xi$, the authors estimate $\hat{S}$, finding a modest dependence on the scaling dimension $d$.

For comparison, using of the exact expression for $\Sigma_{AA}$ in the $d = 2$ case discussed here, after imposing the same constraints on $M_\rho$ and $f_\rho$, and requiring $f_\pi = 87$ MeV, I obtain the predictions for $M_{a_1} \simeq 1060$ MeV and $f_{a_1} \simeq 135$ MeV. In particular, the ratio $f_{a_1}^2/M_{a_1}^2$ is substantially unchanged, and accordingly $d$ has no significant effect on the estimate of $\hat{S}$. The effect of the anomalous dimension is a reduction of the sensitivity to symmetry breaking of the excited modes of the theory, in respect to the heavy modes. However, this effect is not dramatic in this case, consistently with what pointed out in [3].

While the results in [3] are consistent, up to $O(1)$ factors, with the perturbative effects, this does not mean that non-perturbative effects are always small. The reason for the smallness of the effect of changing $d$ observed in [3] is that the choice of parameters giving the QCD-like spectrum corresponds to the choice $\mu_4^4 L_1^4 \simeq 10$, and hence no additional parametric suppression on $\hat{S}$ is present, but, as also computed here in Eq.(128), $\hat{S}$ is just proportional to $\varepsilon^2$ via a $O(1)$ parameter.

The real question is then: are small values of $\mu_4^4 L_1^4$ to be considered as natural, or is this just another way of recasting in a different language an old fine-tuning problem? In order to answer this question, one has to associate the parameter $\mu_4^4 L_1^4$ with the physical scales relevant for the system discussed here.

Looking back at the definitions given in the setup, working at finite $L_0$, before regularization and renormalization. The SM fields, and hence the SM currents, are localized at the UV brane, where the symmetry breaking effects are controlled by $v_0$. This is the parameter that controls the magnitude of $G_F$, the Fermi constant. The symmetry breaking effects experienced on the IR-brane, i.e. for modes localized near the IR brane, such as the first
techni-\(\rho\) excitations, are rescaled as

\[ v_1 \sim \left( \frac{L_1}{L_0} \right)^d v_0. \]  

(155)

This is the parameter that controls \(\hat{S}\). If the technicolor dynamics consists of a large-\(N_c\) theory, with a quasi-conformal energy window between \(L_0\) and \(L_1\), but no anomalous dimensions, the perturbative estimate of \(\hat{S}\) should hold, and hence the perturbative result should be consistent with a scaling \(v_1 \sim (L_1/L_0)^3v_0\). If, on the contrary, there is an anomalous dimension, for the same value of \(G_F\) and confinement scale \(L_1\) one expects the symmetry breaking effects at the IR brane to be suppressed in respect to the perturbative estimate, due to the different scaling of \(v(z)\). As a result

\[ \hat{S} \sim \left( \frac{L_0}{L_1} \right)^{6-2d} \hat{S}_p, \]  

(156)

and in particular for the \(d = 2\) case one expects a suppression

\[ \hat{S} \sim \frac{L_0^2}{L_1^2} \hat{S}_p, \]  

(157)

in respect to the perturbative estimate.

If these scaling arguments are correct, by comparing Eq. (152) and Eq. (153) with Eq. (157) one is lead to conclude that

\[ \mu_4^4L_1^4 \sim \left( \frac{L_0}{L_1} \right)^2. \]  

(158)

It remains to be proven that the very existence of a substantial hierarchy between \(L_0\) and \(L_1\) implies the smallness of \(\mu_4^4L_1^4\). The UV cut-off \(L_0\) corresponds to the \(\Lambda_{ETC}\) scale, at which the behavior of the theory changes, and up to which the theory is (quasi) conformal. This is the scale at which higher-order interactions become important. The IR cut-off is the technicolor scale \(\Lambda_{TC} \sim 1/L_1\), at which the theory confines and a symmetry-breaking condensate is formed. A hypothetical particle whose life be confined on the IR brane, cannot experience other energy scales than the one fixed by \(L_1\), because conformal symmetry is screening the effect of the UV cut-off. It is unreasonable to expect the symmetry breaking condensate to form at a scale parametrically higher than the confinement scale itself. It is hence reasonable to write

\[ g v_1 \propto \frac{1}{L_1}, \]  

(159)
with some proportionality factor that grows with the effective coupling, and hence cannot be more than a $O(1)$ coefficient. For this reason, in the limit in which $L_0 \sim L$:

$$\mu^4_W L_1^4 \propto \left(\frac{L_0}{L_1}\right)^2,$$

(160)

up to a coefficient that grows with the effective coupling $g^2/L$. This is what desired.

Again, one could argue that I just transferred the fine-tuning problem of the smallness of $\hat{S}$, first into the fine-tuning problem of the smallness of $\mu_W L_1$, and then again in the fine-tuning problem of the hierarchy between $L_0$ and $L_1$, which rather than a solution might look like a mere rewriting of the problem in a different language. The point is that this hierarchy is natural, because the scales $L_0$ and $L_1$, together with their separation, are generated dynamically, and stabilized by conformal symmetry. This deserves some more comments, in order to highlight the intrinsic limitations of this idea.

From the five-dimensional point of view, the existence of a hierarchy between $L_0$ and $L_1$ demands for a five-dimensional stabilization mechanism. The presence of a bulk scalar field with non-trivial profile can itself provide such stabilization, in the spirit of the Goldberger-Wise mechanism. In order to produce an exponential separation of scales, this mechanism would require a choice of $M^2 L^2 \simeq 0$, very different from the choices discussed here. Yet, for $d \simeq 2$, the modest factor of $L_0/L_1 \sim 1/10$, required by electro-weak precision measurements, does not seem to pose severe problems. This framework hence provides a natural suppression mechanism for $\hat{S}$.

If a four-dimensional interpretation holds, a generic $SU(N_c)$ gauge theory, such as TC is supposed to be, is not going to exhibit conformal behavior at large coupling in the IR energy region. Nevertheless, an accurate choice of the fermionic field content can suppress the beta-function coefficient, and the running of the gauge coupling, below some symmetry breaking scale $L_0$. Conformal, strong-interacting, gauge symmetries of this type have been studied at length, both in the context of supersymmetric and non-supersymmetric models. The fact that at the UV-scale $L_0$ the coupling is already strong, together with the fact that in real-world models exact conformality is an unreasonable assumption, implies that the scale $L_1$ at which the theory finally confines cannot be exponentially far away from $L_0$. But it is perfectly reasonable to think that models with $L_0/L_1 \sim 1/10$ exist and are natural. And this is the size required by the phenomenology discussed in the present paper. As a result, walking behavior can naturally suppress the electro-weak precision parameters, that
hence do not constrain the number $N_c$ of techni-colors to be very small, as opposed to what indicated by perturbative estimates.

By contrast, using the arguments developed here, one sees that the choice of large value of $\mu^4 W^4 L_1^4 \sim 10$, dictated by the attempt to reproduce low-energy QCD data, corresponds to a regime in which $L_0 \sim L_1$, with a largish five-dimensional coupling $g$. The effective coupling is large because QCD is a small-$N_c$ theory, and in this regime the computations performed here have large systematic uncertainties. More important is the observation that in QCD a conformal energy window in the IR (at large coupling) does not exist, and the QCD coupling actually runs fast in the energy region around $0.5 - 3$ GeV, instead of walking. This enforces $L_0 \simeq L_1$, and explains why changing $d$ does not affect significantly the results for $\hat{S}$ in this class of models of QCD-like theories. As a result, these models do not provide a good description of QCD in the interesting intermediate energy region just above the confinement scale, as is well illustrated by the fact that the Regge trajectories are not reproduced. General features of low-energy QCD require a substantial departure of the background from the pure $AdS_5$ (i.e. from conformal symmetry) in the IR in order for a model to reproduce the data.

The toy models discussed here show that the presence of a sizable energy window in which the theory is conformal, and in which the condensates have large, non-perturbative, anomalous dimensions, provide a suppression mechanism for $\hat{S}$ which is a viable alternative to those discussed for instance in [16] and [28]. In the latter class of models, the suppression is achieved by enhancing the scale of strong dynamics above the electro-weak scale, and by explaining electro-weak symmetry breaking with the VEV of a light composite scalar emerging from the higher scale condensation, in the spirit of composite Higgs and little Higgs models. The main phenomenological distinction is that in the models discussed in the present paper there is no light scalar, while in this other class of models there are in general several very light physical scalars, whose masses tend to be even too small to satisfy the present experimental bounds from direct searches.

CONCLUSIONS.

The two examples of models discussed here share the advantages of being exactly solvable, and of assuming the presence of large anomalous dimensions for the chiral condensate. The
accuracy of the computations performed is limited by the fact that only the bilinear terms in the action have been retained, and hence they can be used to describe a $SU(N_c)$ model only in the large $N_c$ limit.

From the model-building perspective. At large $N_c$, the results show a huge parametric departure from the perturbative estimate, in the form of a suppression of $\hat{S}$ proportional to powers of the ratio between the upper and lower energy scales between which the theory is approximately conformal ($walks$) in the IR. This suppression relies on the existence of a stabilization mechanism between these two scales and on the existence of condensates with large anomalous dimensions, both of which are natural in the context of the $AdS - CFT$ correspondence. Walking is anyhow required in the construction of semi-realistic models of dynamical electro-weak symmetry breaking by the requirement of providing large enough masses for the SM fermions, while at the same time suppressing flavor-changing neutral current processes. This indicates that electro-weak precision measurements do not constrain these models to have a small number $N_c$ of techni-colors, but rather they confirm the fact that a non-perturbative, large anomalous dimension for the chiral condensate and a significant regime of quasi-conformal behavior in the IR are necessary requirements in the construction of viable models.

On the phenomenological aspects. This study indicates that models of this class are compatible with electro-weak precision data for a range of masses of the excited spin-1 states that is testable at the LHC. This result is, for all practical purposes, independent of the number of techni-colors of the underlying strong dynamics. The level of degeneracy between the spectrum of techni-$\rho$ and techni-$a_1$ resonances provides a distinction between QCD-like technicolor models with small $N_c$ and walking technicolor theories with large $N_c$. The practical feasibility of such a search is subject to the precise determination of the strength of the coupling of the new states to the SM currents and gauge bosons, which requires further investigation, but there is no reason to expect these couplings to be suppressed. These resonances should be quite broad, so that a significant number of events should be detectable even for masses in the upper limit of energy reached at the LHC.

In the end, LHC experimental data will tell us whether these models have anything to do with nature or not. What this study shows is that large-$N_c$ dynamical electro-weak symmetry breaking models are not ruled out by present indirect constraints from electro-weak precision measurements, and have a very distinctive and clear experimental signature.
that is testable at the LHC.

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