Transonic solutions of isothermal galactic winds in a cold dark matter halo

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ABSTRACT
We study fundamental properties of steady, spherically symmetric, isothermal galactic outflows in appropriate gravitational potential models. We aim at constructing a universal scale-free theory not only for galactic winds, but also for winds from clusters/groups of galaxies. In particular, we consider effects of mass–density distribution on the formation of transonic galactic outflows under several models of the density distribution profile predicted by cosmological simulations of structure formation based on the cold dark matter (CDM) scenario. In this study, we have clarified that there exist two types of transonic solutions: outflows from the central region and from a distant region with a finite radius, depending upon the density distribution of the system. The system with a sufficiently steep density gradient at the centre is allowed to have the transonic outflows from the centre. The resultant criterion intriguingly indicates that the density gradient at the centre must be steeper than the prediction of conventional CDM models including Navarro, Frenk & White and Moore et al. This result suggests that an additional steeper density distribution originated by baryonic systems such as the stellar component and/or the central massive black hole is required to realize transonic outflows from the central region. On the other hand, we predict the outflow, which is started at the outskirts of the galactic centre and is slowly accelerated without any drastic energy injection like starburst events. These transonic outflows may contribute secularly to the metal enrichment of the intergalactic medium.

Key words: ISM: jets and outflows – galaxies: evolution – intergalactic medium – galaxies: ISM – dark matter.

1 INTRODUCTION
Galactic winds are widely thought to be an essential ingredient in the evolution of galaxies. These play a key role by which energy and heavy elements are recycled in galaxies and are deposited into the intergalactic space. In the modern paradigm of galaxy formation based on the cold dark matter (CDM) hypothesis, it is deduced that galaxies formed hierarchically in a bottom-up fashion, where a larger system results from the assembly of smaller dark matter haloes. Baryonic gas falls into the gravitational potential well of dark matter haloes, and condenses rapidly as a result of the radiative cooling for atoms or molecules. The galactic winds seem to be inevitably influenced by the dark matter gravitational field. We here study the fundamental nature of the galactic winds in the dark matter gravitational potential.

The existence of heavy elements in the intrachannel medium of galaxy clusters and in the low-density intergalactic medium (IGM) at $z \sim 3$ is clear evidence of galactic outflows (Songaila 1997; Ellison et al. 2000). A large number of large-scale outflows from star-forming galaxies have been observed in local galaxies (Lehnert & Heckman 1996; Martin 1999). In the spectra of high-$z$ galaxies, there is evidence for large-scale outflows. For instance, Ly$\alpha$ emission with a red asymmetric or P Cygni type profile is commonly seen in $z > 5$ Ly$\alpha$ emitters (Dey et al. 1998; Ellis et al. 2001; Ajiki et al. 2002; Dawson et al. 2002). In addition, extended Ly$\alpha$ nebulae, the so-called Ly$\alpha$ nebulae, have been possibly interpreted as spatially resolved large-scale winds driven by starbursts (Francis et al. 2001; Ohyama & Taniguchi 2004; Mori & Umemura 2006). Lyman break galaxies at $z > 3$ also show signs of outflowing gas with a velocity of several hundred km s$^{-1}$. Furthermore, Weiner
et al. (2009) and Rubin et al. (2010) recently reported that galactic outflows are ubiquitous in the optically selected star-forming galaxies at a medium redshift $z \sim 1–2$ using stacked spectra from the DEEP2 survey. From the theoretical point of view, so far, a large number of studies about the galactic winds have been produced by numerical simulations. The recent developments in computer technology and numerical methods have made it possible to simulate the multi-dimensional dynamical or chemodynamical evolution of galaxies, including the effect of star formation and supernova (SN) feedback. For example, the simulation of galactic winds from elliptical galaxies is motivated by the observation that elliptical galaxies have very little interstellar medium (ISM). While the numerical simulation has become a powerful tool to explore the formation and evolution of galaxies, a simple analytical approach plays an indispensable role to reveal the essential nature of the acceleration process of winds and to interpret observational data.

The fundamental nature of thermally driven spherically symmetric transonic outflows was examined by Parker (1958) in relation to the solar wind, and then it has been discussed for more than 50 years. This model was based on a hydrodynamic description of the Sun’s atmosphere, and proposed that the solar wind is a smooth, spherically symmetric, time-steady transonic outflow of hot gas. We must note that the transonic solution is very special and plausible one, because it is the entropy maximum solution connecting the Sun with infinity. Thus, such transonic solutions are widely accepted as the most plausible universal solution of the outflows from any object. Parker’s solution shows that the flow inside the transonic point resembles the hydrostatic equilibrium closely, whereas the flow tends to freely expand outside the transonic point. Then, Holzer & Axford (1970) provided the generalization of Parker’s approach to galactic outflows (see also Burke 1968; Chevalier & Clegg 1985). Wang (1995a,b) studied for a radial, steady solution of galactic winds, including the effect of the galactic gravitational potential and the efficiency of radiative cooling. Assuming a single-power-law distribution of mass density, they focused on the outflow that is supersonic everywhere from less massive galaxies. Recently, Everett & Murray (2007) investigated the Parker-type acceleration of galactic winds under a realistic but complicated situation with gravity from a point mass, cloud/ISM drag force, adiabatic cooling process and photoionization/absorption process. They concluded that the outflow accelerated insufficiently under adiabatic cooling. Moreover, Sharma & Nath (2012) emphasized the importance of an extra energy source such as the photoionization/absorption process by active galactic nucleus (AGN) radiation or SN energy/momentum injection in order to attain sufficient acceleration.

However, recent cosmological N-body simulations based on a collisionless CDM scenario no longer indicated such a single-power-law distribution, but always predicted a double-power-law density distribution. Navarro, Frenk & White (1997) pointed out that their structure can be approximated as $\rho_{\text{DM}}(r) \propto r^{-1}(r + r_0)^{-2}$, where $r$ is the radius from the centre of the galaxy and $r_0$ is the scale radius at which the density profile agrees with the isothermal profile [i.e. $\rho_{\text{DM}}(r) \propto r^{-1}$]. This is related to the formation epoch of the CDM halo (see also Navarro et al. 1997). Then, Fukushige & Makino (1997) and Moore et al. (1999) used high-resolution simulations and showed that the CDM haloes have a steeper central cusp than quoted above. The resulting structure of the CDM haloes depends on the number of particles used in the simulation and is still an open question (see Navarro et al. 2010; Ishiyama et al. 2013). In contrast, recent observations of nearby dwarf galaxies and low surface brightness galaxies have revealed that the density profile of the dark matter halo is constant at the centre of such galaxies. For instance, Burkert (1995) proposes that the density profile, $\rho_{\text{DM}}(r) \propto (r + r_0)^{-1}(r^2 + r_0^2)^{-1}$, nicely reproduces the rotation curves of nearby dwarf galaxies and the central density is correlated with $r_0$ through a simple scaling relation (e.g., Moore 1994; Burkert 1995; de Blok et al. 2001; Swaters et al. 2003; Gen- tile et al. 2004; Spekkens, Giovanelli & Haynes 2005). This is well known as an unsolved problem in the CDM scenario, the so-called ‘core–cusp problem’.

Under these inconclusive circumstances for determining the CDM distribution, no model considers facing the transonic galactic winds in a realistic constellation of the CDM distribution of galaxies. These situations motivate us to explore the series of the solutions for the transonic galactic winds in appropriate CDM halo models. In this paper, we focus on the analytical steady-state solution for a spherically symmetric isothermal outflow in various CDM halo models. We aim at clarifying the influence of the dark matter gravitational field on the galactic winds. We are interested especially in realizability of the transonic winds. This research is conducted to be a scale-free universal wind theory and does aim not only at winds from galaxies, but also at winds from groups/clusters of galaxies because the CDM halo density distribution model seems to be rather universal for these hierarchies of objects (Navarro et al. 1997). For analytical convenience, we assume an isothermal flow without any mass injection along the flow, because the gas temperature in actual galaxies seems to be constant (Li et al. 2011) and the locus of the resultant transonic point in this study is far distant from the star-forming region (Fig. 2). The adequacy of these assumptions is discussed later.

The structure of this paper is as follows. In Section 2, we describe the basic equations for the analysis of the transonic galactic winds. In Section 3, we show the critical condition for the existence of the transonic winds and their series of solutions. Finally, we discuss the results in Section 4.

## 2 Transonic Flow in a CDM Halo

### 2.1 Basic Equations

We consider steady, spherically symmetric, isothermal gas outflows from galaxies. The basic equations for this problem are the mass and momentum conservation laws:

\[
4\pi r^2 \rho v = \dot{M} = \text{constant} 
\]  

(1)

and

\[
\frac{dv}{dr} = -\frac{c_s^2}{\rho} \frac{d\rho}{dr} - \frac{d\Phi}{dr},
\]

(2)

where $\rho$, $v$, $c_s$, $\dot{M}$ and $\Phi$ are the density, velocity, sound speed of the gas, mass-loss rate and gravitational potential of the galaxy, respectively. We introduce the non-dimensional distance $x = r/r_0$ from the galactic centre, where $r_0$ is the scale radius of the galaxy. Substituting $\rho$ from equation (1) into equation (2), we obtain

\[
\frac{c_s^2}{\dot{M}} \frac{d\dot{M}}{dx} = \frac{(2c_s^2/x) - (d\Phi/dx)}{\dot{M}^2 - 1},
\]

(3)

where $\dot{M} = v/c_s$ is the Mach number. The numerator on the right-hand side denotes a change of the effective cross-section of a Laval nozzle. Thus, we can see that the gravitational force plays a role...
of choking the cross-section (hereafter, the gravitational choking). Integrating equation (3), we obtain the relation

$$M^2 - \ln M^2 = 4 \ln x - \frac{2\phi}{\alpha} + C,$$

(4)

where $C$ is the integration constant. A series of wind solutions are obtained from equation (4) as $M^2(x)$ parametrized by $C$.

The feature of the solutions of equation (3) is similar to Parker’s solution of the solar wind (Parker 1958). The denominator on the right-hand side of equation (3) is negative for subsonic ($M < 1$) and positive for supersonic ($M > 1$). If the gravitational choking is effective ($2c_s^2/x < \phi/\alpha$), the denominator is negative, while if the gravitational choking is not effective ($2c_s^2/x > \phi/\alpha$), the numerator is positive. It is equivalent to the cross-section of a Laval nozzle which is decreasing (increasing) for negative (positive) numerator.

Subsonic flows accelerate in the region where the numerator is negative and supersonic flows accelerate in the region where the numerator is positive. The singular point at which both the denominator and the numerator equal to zero ($M = 1, 2c_s^2/x = \phi/\alpha$), and the sign of the numerator changes from negative to positive, is called the transonic point (or the critical point, see Bondi 1952; Parker 1958). Only the transonic flow can continuously accelerate from subsonic to supersonic by passing through the transonic point.

2.2 Density distributions

The Navarro–Frenk–White (NFW) model, which is a widely accepted density profile of CDM haloes, was empirically derived from cosmological N-body simulations (Navarro et al. 1997). Fukushige & Makino (1997) and Moore et al. (1999) derived a steeper distribution from N-body simulations with higher resolution. The resultant empirical profiles of density distributions are approximately fitted by

$$\rho_{DM}(r; \alpha) = \frac{\rho_0 r_0^3}{r^{\alpha}(r + r_0)^{\alpha - 3}},$$

(5)

where $\rho_0$ is the scale density. In this equation, $\alpha = 1$ is the NFW model and $\alpha = 1.5$ corresponds to the model advocated in Fukushige & Makino (1997) and Moore et al. (1999). On the other hand, the observed density profile derived by Moore (1994) and Burkert (1995) is represented by $\alpha = 0$. The plausible value of the index $\alpha$ remains an open question. Thus, we treat $\alpha$ as a parameter and study the variation of solutions depending on $\alpha$. By choosing $r_0$ as the unit of length, equation (5) is rewritten as

$$\rho_{DM}(x; \alpha) = \frac{\rho_0}{x^\alpha(x + 1)^{\alpha - 3}},$$

(6)

where $x = r/r_0$. In the limit of $x \to 0$, $\rho_{DM}(x; \alpha) \propto x^{-\alpha}$ and $\rho_{DM}(x; \alpha) \propto x^{-3}$ for $x \to \infty$. Note that the functional form of the gravitational potential near the centre crucially depends on the index $\alpha$.

3 SERIES OF GALACTIC WIND SOLUTIONS

3.1 Condition for the transonic flow from the centre

The acceleration of outflows obviously depends on the shape of the gravitational potential, from the discussion in Section 2.1, and the existence of transonic solutions starting from the centre therefore relates closely to the power-law index $\alpha$ in equation (5). Here, we consider the critical condition upon $\alpha$ for the transonic solution from the centre.

The total mass $M(x)$ within $x$ under the density distribution (6) can be described analytically using Gauss’ hypergeometric function (see Appendix A) as

$$M(x; \alpha) = \frac{4\pi\rho_0 r_0^3}{3 - \alpha} \alpha F_1 [3 - \alpha, 3 - \alpha, 4 - \alpha; -x],$$

(7)

if $0 \leq \alpha < 3$. Substituting $d\phi(x; \alpha)/dx = GM(x; \alpha)/(r_0c^2)$ into the numerator on the right-hand side of equation (3), we obtain

$$\frac{2c_s^2}{x} = \frac{d\phi(x; \alpha)}{dx} x = \frac{2c_s^2}{x} \left[ 1 - \sum_{n=0}^\infty A_n (-1)^n x^{\alpha - 2 - \alpha} \right].$$

(8)

where $\{A_n\}$ are positive constants (see Appendix A). In the limit of $x \to 0$, equation (8) approaches

$$\frac{2c_s^2}{x} = \frac{d\phi(x; \alpha)}{dx} = \frac{2c_s^2}{x} (1 - A_0 x^{\alpha - 2}).$$

(9)

If the choking by the gravitational force is effective in the innermost region, the numerator is negative. In the case of $0 \leq \alpha < 2$, the numerator is positive because $1 - A_0 x^{\alpha - 2} \to 1$ as $x \to 0$. In the case of $\alpha = 2, 1 - A_0 x^{\alpha - 2} = 1 - A_0$. Therefore, the numerator is negative if $A_0 > 1$. In the case of $2 < \alpha < 3$, the numerator is negative because $1 - A_0 x^{\alpha - 2} \to -\infty$ as $x \to 0$. Finally, we conclude that the condition for realizing the transonic flow from the central region is $\alpha \geq 2$.

We note that this is rather a steep density profile in the innermost region. Widely accepted values of $\alpha$ for the density distribution profile of CDM haloes are $\alpha = 1$ and $1.5$, but they do not fulfill the critical condition $\alpha \geq 2$. The galactic wind solutions for the case of $0 \leq \alpha < 2$ (Case 1) and the case of $2 \leq \alpha \leq 3$ (Case 2) are shown in the following sections. Here, we focus only on the cases $\alpha = 0, 1$ and $1.5$ in Case 1, and $\alpha = 2$ and 2.5 in Case 2 because of analytical simplicity.

3.2 Case 1: $0 \leq \alpha < 2$

3.2.1 Galaxy mass, gravitational force and gravitational potential

The total mass $M(x; \alpha)$ within $x$ is given for specified values of $\alpha$ as follows:

$$M(x; \alpha = 0) = 4\pi\rho_0 r_0^3 \left[ \ln(x + 1) - \frac{x(2 + 3x)}{2(x + 1)^2} \right],$$

(10)

$$M(x; \alpha = 1) = 4\pi\rho_0 r_0^3 \left[ \ln(x + 1) - \frac{x}{x + 1} \right]$$

(11)

and

$$M(x; \alpha = 1.5) = 8\pi\rho_0 r_0^3 \left[ \ln(\sqrt{x} + \sqrt{x + 1}) - \sqrt{\frac{x}{x + 1}} \right].$$

(12)

Therefore, the corresponding gravitational forces are given by

$$\frac{d\phi(x; \alpha = 0)}{dx} = 4\pi\rho_0 r_0^2 G x \left[ \ln(x + 1) - \frac{x(2 + 3x)}{2(x + 1)^2} \right],$$

(13)

$$\frac{d\phi(x; \alpha = 1)}{dx} = 4\pi\rho_0 r_0^2 G \frac{1}{x^2} \left[ \ln(x + 1) - \frac{x}{x + 1} \right].$$

(14)
and
\[
\frac{d\phi(x; \alpha = 1.5)}{dx} = 8\pi\rho_0 r_0^2 G
\times \frac{1}{x^2} \left[ \ln(\sqrt{x} + \sqrt{x + 1}) - \ln \left( \frac{x}{x + 1} \right) \right],
\]
respectively. By integrating equations (13), (14) and (15), the corresponding gravitational potentials are given by
\[
\phi(x; \alpha = 0) = -4\pi\rho_0 r_0^2 G \left( \frac{1}{x} \ln(x + 1) - \frac{1}{2(x + 1)} \right),
\]
and
\[
\phi(x; \alpha = 1.5) = -8\pi\rho_0 r_0^2 G \left( \frac{1}{x} \left[ 1 + \ln(\sqrt{x} + \sqrt{x + 1} - \sqrt{x} \sqrt{x + 1}) \right] \right),
\]
respectively.

### 3.2.2 Mechanism of acceleration

From the numerator on the right-hand side of equation (3), we define
\[
2c_s^2 = \frac{d\phi(x; \alpha)}{dx} = \frac{2c_s^2}{x} N(x; K, \alpha),
\]
where
\[
N(x; K, \alpha) = 1 - Kx^{2-\alpha} \left( 3 - 4\alpha - 3x^{-\alpha} \right).
\]
We substitute equations (16), (17) and (18) into equation (4) successively and draw solutions in the plane \((x, M^2)\) for each case of \(\alpha\). Fig. 2 shows a family of solutions for \(\alpha = 1\), for example. Fig. 2 is the so-called 'phase diagram' of the solutions of equation (4). We can see that the topological property of the phase diagram strongly depends on \(K\) as shown in Fig. 1. Figs (2a), (b) and (c) are for \(K = 3.8 < K_c\), \(K = K_c \approx 4.6\) and \(K = 7.5 > K_c\), respectively.

If \(K \leq K_c\), the subsonic flow decelerates and the supersonic flow accelerates monotonically (Figs 2a and b). Consequently, the transonic solution does not appear in these cases. If \(K > K_c\), the subsonic flow accelerates and the supersonic flow decelerates in \(x_a < x < x_b\) (Fig. 2c). The point at \((x, M^2) = (x_a, 1)\) is the so-called 'O-point' from its topological property in Fig. 2(c). Similarly, the point at \((x, M^2) = (x_b, 1)\) is the so-called 'X-point'. This is the transonic point. In Fig. 2(c), the sign of the velocity gradient changes at \(x_a \approx 0.43\) and \(x_b \approx 12.7\). The transonic point forms at \((x, M^2) \approx (12.7, 1)\). Therefore, a transonic solution which passes \((x, M^2) \approx (12.7, 1)\) exists.

### 3.3 Case 2: \(2 \leq \alpha < 3\)

#### 3.3.1 Galaxy mass, gravitational force and gravitational potential

The total mass \(M(x; \alpha)\) within \(x\) for \(\alpha = 2\) and 2.5 is given by
\[
M(x; \alpha = 2) = 4\pi\rho_0 r_0^3 \ln(x + 1)
\]
and
\[
M(x; \alpha = 2.5) = 8\pi\rho_0 r_0^3 \ln (\sqrt{x} + \sqrt{x + 1}),
\]
respectively. The gravitational force for each case therefore is
\[
\frac{d\phi(x; \alpha = 2)}{dx} = 8\pi\rho_0 r_0^2 G \left( x^2 \right) \ln(x + 1).
\]
Transonic solutions of galactic winds

Figure 2. Family of solutions for $\alpha = 1$: the upper panel is for $K = 3.8(<K_c)$, and the middle and lower panels are for $K = K_c \simeq 4.6$ and $K = 7.5(>K_c)$, respectively. In the lower panel, the sign of the velocity gradient changes at $x_A \simeq 0.43$ and $x_B \simeq 12.7$, and the transonic solution which passes the transonic point $(x, M^2) \simeq (12.7, 1)$ appears. The patterns of solution curves for $0 \leq \alpha < 2$ do not topologically change.

Figure 3. $N(x; K, \alpha)$ with $\alpha = 2$ (upper panel) and with $\alpha = 2.5$ (lower panel). The lower panel is for $K = 0.50$. The pattern of the lower panel does not topologically change in $2 \leq \alpha < 3$.

### 3.3.2 Mechanism of acceleration

Substituting equation (27) or (28) into the numerator on the right-hand side of (3), we obtain

$$N(x; K, \alpha = 2) = K \frac{x}{x + 1} \ln \left( \sqrt{x} + \sqrt{x + 1} \right), \quad (31)$$

and

$$N(x; K, \alpha = 2.5) = \frac{2K}{x} \ln \left( \sqrt{x} + \sqrt{x + 1} \right). \quad (32)$$

Fig. 3(a) shows $N(x; K, \alpha)$ for $\alpha = 2$ and Fig. 3(b) shows $\alpha = 2.5$ as a function of $x$. We can see that the function $N(x; K, \alpha)$ monotonically increases for $\alpha \geq 2$; it is critically different from Case 1. In Case 2, one X-point without an O-point appears. In this case, we define $B$ as shown in Fig. 3. In the case of $\alpha = 2$, the value of $N(x; K, \alpha)$ is finite at $x = 0$ and there is a critical value $K = K_c = 1$. Therefore, if $K > 1$, $N(x; K, \alpha)$ does not become negative, while if $K < 1$, $N(x; K, \alpha)$ becomes negative in $x < x_B$ ($x_B$ is the locus of B) in Fig. 3(a). Thus, the point where $(x, M^2) = (x_B, 1)$ becomes the transonic point. In the case of $\alpha = 2.5$, the value of the function is negative infinity in the limit of $x \to 0$. Therefore, a transonic point always exists at $(x, M^2) = (x_B, 1)$ in Fig. 3(b).

### 3.3.3 Topology of the solution

By substituting equation (29) or (30) into equation (4), we can draw a phase diagram for each case. Figs 4 and 5 are the phase diagrams for $\alpha = 2$ and 2.5, respectively. Figs 4(a), (b) and (c) are for $K = 0.75(<K_c)$, $K = 1(=K_c)$ and $K = 1.25(>K_c)$, respectively, and Fig. 5 is for $K = 0.50$. In the case of $\alpha = 2$ (Fig. 4), if $K > K_c$, $x_B$ at which the sign of the velocity gradient changes appears and

$$\frac{d\phi(x; \alpha = 2.5)}{dx} = -8\pi \rho_0 \rho_0 \frac{x^2}{x} \ln \left( \sqrt{x} + \sqrt{x + 1} \right), \quad (28)$$

respectively. By integrating equations (27) and (28), the gravitational potential for each case is given by

$$\phi(x; \alpha = 2) = -4\pi \rho_0 \rho_0 \frac{G}{x} \left[ \frac{1}{x} \ln(x + 1) - \frac{x}{x + 1} \right], \quad (29)$$

and

$$\phi(x; \alpha = 2.5) = -8\pi \rho_0 \rho_0 \frac{G}{x} \left[ \frac{\sqrt{x} + \sqrt{x + 1}}{x} + \ln \left( \sqrt{x} + \sqrt{x + 1} \right) - 1 \right], \quad (30)$$
there is a transonic solution which passes the transonic point at \((x, M^2) = (x_B, 1)\). In the case of \(\alpha = 2.5\) (Fig. 5), there is also a transonic solution which passes the transonic point at \((x, M^2) = (x_B, 1)\). In this case, according to Section 3.3.2, we find such critical solution for any value of \(K\). According to Figs 4(c) and 5, the transonic wind can start from the centre \(x = 0\) in Case 2.

4 SUMMARY AND DISCUSSION

The property of a stationary, spherically symmetric, isothermal galactic wind in a CDM halo is examined in this paper. Depending on the mass–density profile of the CDM halo, three types of outflows can exist: (i) supersonic or subsonic flows everywhere, (ii) flows with an X-point, or (iii) flows with a pair of an O-point and an X-point. The condition for existence of a transonic solution is sensitive to the mass–density profile of the CDM halo. Especially, the transonic outflow from the galactic centre is realized under the necessary condition that the power-law index of the mass–density distribution near the galactic centre must be steeper than 2 \((\alpha \geq 2\) in equation 5).

4.1 The locus of the transonic point

Our analysis shows that the locus of the transonic point is critically related to both the power-law index \(\alpha\) of the mass–density distribution and the coefficient \(K\) (see equations 6 and 21). In Fig. 6, we summarize the locus \(x_B\) of the transonic point obtained by solving the equation \(N(x_B; K, \alpha) = 0\) for \(0 \leq \alpha < 3\) and \(1 \leq K \leq 19\). Each solid curve shows the locus \(x_B\) as a function of \(\alpha\) for a given value of \(K\) in the range from \(K = 1\) (bottom) to 19 (top). Using equation (7) and \(c_s^2 = k_b T / \mu m_p\), where \(\mu\) is the mean molecular weight, \(m_p\) is the proton mass, \(k_b\) is Boltzmann’s constant and \(T\) is the gas temperature, equation (21) becomes

\[
K = \frac{GM(r_0; \alpha) \mu m_p}{2\rho_0 k_b T} f(\alpha),
\]

where \(M(r_0; \alpha)\) is the enclosed mass within the radius \(r = r_0\) for a given \(\alpha\), and \(f(\alpha)\) is defined by

\[
f(\alpha) = \frac{3 - \alpha}{2 F_1[3 - \alpha, 3 - \alpha, 4 - \alpha; -1]},
\]

where \(F_1[\cdot, \cdot, \cdot; \cdot]\) is the hypergeometric function.

Figure 6. Locus of the outer X-point \(x_B\) obtained by solving the equation \(N(x_B; K, \alpha) = 0\) for \(0 \leq \alpha < 3\) and \(1 \leq K \leq 19\). Each solid curve corresponds to \(K = 1, 3, 5, 7, 9, 11, 13, 15, 17\) and 19 from the bottom to top. Each dashed curve corresponds to \(K = f(\alpha)/\gamma\) with \(\gamma = 1, 0.5\) and 0.1 from the bottom to top.
The numerical values of $f(\alpha)$ for typical $\alpha$ are summarized in Table 1. Accordingly, the coefficient $K$ is roughly interpreted as the ratio of the gravitational potential energy of the galaxy to the thermal energy of the gas.

Fig. 6 indicates the allowable range of the transonic point for a given value of $K$. For instance, the transonic solution can exist only in the range $2 < \alpha < 3$ for $K = 1$ and $0 < \alpha < 3$ for $K > 8.15$. Especially, the transonic solution starting from the centre $x = 0$ can exist only for $\alpha \geq 2$. This is consistent with the discussion in Section 3.1. In addition, we can see that the locus of the transonic point is an increasing function of $K$ for given $\alpha$. For the NFW model ($\alpha = 1$), the transonic point locates at $x_B = 4.22$ for $K = 5$ and at $x_B = 21.6$ for $K = 10$. Incidentally, the virial radius of the CDM halo is usually defined by $r_{\text{vir}} = c r_B$ for the NFW model, where $c$ is the so-called concentration parameter. Using the concentration–mass relation

$$\log_{10} c = 0.971 - 0.094 \log_{10}(M/10^{12}\, \text{h}^{-1} \, M_\odot),$$

(35)
derived by Maccio, Dutton & van den Bosch (2008), we obtain $c$ as a function of the virial mass. This equation gives $7 \lesssim c \lesssim 30$ with respect to a reasonable range of the virial mass $7 \lesssim \log_{10}(M/M_\odot) \lesssim 13$ for the NFW model. In other words, the virial radius lies in the range $7 \lesssim r_{\text{vir}} \lesssim 30 r_B$. Note that the virial radius is comparable to the locus of the transonic point for $\alpha = 1$ in Fig. 6. Depending on the coefficient $K$ and the virial mass of the galaxies therefore the wind is accelerated mainly in the region far outside the optically visible scale of the galaxies in some cases. Moreover, we must note that the density structure of the transonic wind in the subsonic region $x < x_B$ is crucially similar to that of the hydrostatic equilibrium except in the vicinity of the transonic point. Thus, we may observationally confound the slowly accelerated wind structure having $x_B r_B \gg r_{\text{vir}}$ with the hydrostatic one.

There possibly exist such slowly accelerated outflows in the outskirts of galaxies with no drastic heating by, for example, starburst events, but by quasi-stationary heating from the heat reservoir of the shock-heated CDM halo (see the next subsection). Such slowly accelerated galactic winds may play an important role of the metal enrichment of the IGM though it resembles closely the hydrostatic gas in the optically observable region.

4.2 Availability of our simplified model

4.2.1 Isothermal approximation

In the standard CDM picture of galaxy formation, gas falling into CDM haloes is shock-heated approximately to the halo virial temperature; thus, the CDM material maintains quasi-hydrostatic equilibrium (Binney 1977; Rees & Ostriker 1977; Silk 1977; White & Rees 1978). We assume that the wind gas is in thermal equilibrium with such heated CDM haloes. In this model, the CDM halo plays a role of a heat reservoir to keep the wind gas temperature close to the halo virial temperature. In this case, the gas could be treated as isothermal and its temperature $T$ is expected to be close to the virial temperature in a wide range up to the virial radius,

$$T_{\text{vir}} = \gamma \frac{G \mu m_p M(r_B; \alpha)}{2 k_B r_B},$$

(36)

where $\gamma$ is a fudge factor of the order of unity which should be determined by the efficiency of the shock-heating. A typical value of $T_{\text{vir}}$ is approximately several times $10^6$ K for galaxies with mass $\sim 10^{11} M_\odot$ and scale radius $\sim 10$ kpc.

Indeed, in the case of the Sombrero galaxy, the gas temperature seems to be isothermal in a considerably wide spatial range $\lesssim 25$ kpc from the galactic centre (Li et al. 2011). We note that the temperature of the gas in the Sombrero galaxy ($\sim 0.6$ keV $\sim 7 \times 10^6$ K) is consistently recognized as the virial temperature (see also Mathews & Brighenti 2003). On the other hand, recent X-ray observations reveal a variety of temperature profiles of the hot ISM in early-type galaxies. Diehl & Statler (2008) categorized the observed temperature profiles into four major groups: isothermal (flat), positive gradient (outwardly rising), negative gradient (falling) and hybrid (falling at small radii and rising at larger radii) (see also Fukazawa et al. 2006). We have, however, a poor understanding of the origin and evolution of these temperature gradients. In addition, since X-ray emissivity in the outskirts of the galaxies is too low to be detected with current X-ray satellites, the maximum radius of the observed X-ray emission is usually smaller by a factor of 10 than the virial radius of the CDM halo (Fukazawa et al. 2006). Thus, there is a significant ambiguity keeping the temperature gradient in the outskirts of the galaxies. Under these complicated situations, we focused on the flat temperature profile in this paper.

4.2.2 Is mass injection along the wind necessary?

One may think that mass injection along the wind flow due to SNe is essential and cannot be neglected for plausible galactic wind models. Actually, it may play the intrinsic role of a material/energy source of the wind for the central star-forming region of galaxies that is in a locus of smaller radius than the transonic point. In addition, mass injection results in an effective braking force on the wind-like gravity. These make essential influences on the wind nature in the central region.

However, in the resultant wind solution of our analysis for actual CDM halo models, the transonic point forms at a far distant region from the galactic centre ($\sim 10$ times the scale radius of the CDM halo density distribution which is much larger than the radius of the typical locus of the star-forming region, see Fig. 2). In such a distant region, we can neglect any effect from star formation; thus, we can neglect mass injection by SNe along the wind flow when we discuss the acceleration process of the wind. Moreover, when we apply our result to the wind from groups/clusters of galaxies, the mass injection along the flow also can be neglected. Any effect from mass injection is meaningful only at the region in the vicinity of the starting point of the flow, and thus, it does not play any important role in the actual acceleration process of the wind if the transonic point forms sufficiently outside the scale radius like in our case.

We can conclude that the mass injection along the wind is not necessarily important for the galactic wind theory except for the case that the transonic point is very close to the galactic centre. This simple but analytically effective approximation we adopted in this paper may be allowed as the first step to explore the transonic nature of the galactic winds.

4.2.3 Range of the coefficient $K$

We here discuss the range of expected values of the coefficient $K$ defined in equation (21). As discussed in Section 3, the value of $K$ is critical to determine the topology of the solution curves in the
$M^2$–$x$ phase diagrams (see Figs 1–5). However, note that the value of $K$ is still uncertain both observationally and numerically.

Substituting $T = T_{\text{vir}}$ into equation (33), we obtain

$$K = f(\alpha)/\gamma.$$  

(37)

Eke, Navarro & Frenk (1998) adopted $\gamma = 2.3$ and Kitayama & Suto (1997) adopted $\gamma = 1.8$ as their canonical value in the analysis of galaxy cluster number counts. For instance, assuming $\gamma = 1$, the NFW model ($\alpha = 1$) yields $K = 5.2$ and the Fukushige–Makino–Moore model ($\alpha = 1.5$) yields $K = 2.9$. According to Table 1, a reasonable range is $0 < K < 15$ in this case. Each dashed curve corresponds to $K = f(\alpha)/\gamma$ with $\gamma = 1, 0.5$ and 0.1 from the bottom to top in Fig. 6.

In addition, a combination of the extra heating mechanisms such as SNe and ultraviolet (UV) background radiation and the radiative cooling process could keep the temperature constant in many situations (see Dekel & Silk 1986; Yoshii & Arimoto 1987; Babul & Rees 1992; Efstathiou 1992; Mori, Yoshii & Nomoto 1999). In this case, $K$ is no longer a function of the one-parameter family of $\alpha$ quoted above.

4.3 Implication of the critical condition $\alpha \geq 2$

We have revealed that the transonic solution starting from the galactic centre needs rather a steep density distribution (see Section 3.3). This is a natural consequence that the solution must approach asymptotically to Parker’s solution in the limit of a steep density gradient. It is striking that none of the cosmological N-body simulations based on the collisionless CDM predicts such a steep power-law index ($\alpha \geq 2$) around the centre of CDM haloes. Therefore, the density enhancement of the baryon at the central region of the CDM halo may be essential for the transonic galactic wind from the galactic centre. In other words, the gravitational potential induced by a stellar system and/or a central massive black hole plays a crucial role for the acceleration of the galactic wind from the galactic centre.

Though we studied only the effect from the CDM halo as a source of the gravitational potential in this paper, it is very interesting to examine the case, including the gravitational potential induced by the stellar system and the central massive black hole. We would then also need to consider the effects of the star formation and subsequent feedback process such as stellar winds and SN heating from massive stars. These feedback processes will supply thermal energy into the gas and will accelerate the outflows.

In a series of forthcoming studies, we plan to report the results, taking into account the multiphase states of the gas with cool components in a galaxy, including the effect of the radiative cooling of the gas, energy input from stars and AGNs and the UV background radiation. In this case, the efficiency of the acceleration of the outflow may be quite different.

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APPENDIX A: GAUSS’ HYPERGEOMETRIC FUNCTION

A hypergeometric function is defined as follows:

$$\genfrac{}{}{0pt}{}{p}{q}[a_1, \ldots, a_p; b_1, \ldots, b_q; z] = \sum_{n=0}^{\infty} \frac{(a_1)_n \cdots (a_p)_n z^n}{(b_1)_n \cdots (b_q)_n n!},$$  

(A1)

$$(a)_n = a(a + 1) \cdots (a + n - 1) = \frac{\Gamma(a + n)}{\Gamma(a)},$$  

(A2)

$$(a)_0 = 1.$$  

(A3)
Especially, in the case of $p = 2$, $q = 1$, it is called Gauss' hypergeometric function, given by

$$\textstyle _2F_1[a, b; c; z] = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)} \frac{z^n}{n!}. \quad (A4)$$

When using $\alpha$ directly, without substituting a number for it, the total mass of the galaxy within $x$, $M(x)$, is given only by a series expansion, using Gauss' hypergeometric function:

$$M(x) = \frac{4\pi\rho_0 r_0^3}{3-\alpha} x^{1-\alpha} \textstyle _2F_1 [3-\alpha, 3-\alpha; 4-\alpha; -x]. \quad (A5)$$

Substituting equation (A5) in the numerator on the right-hand side of equation (3), we obtain

$$\frac{(2c_s^2/x) - (d\phi/dx)}{x} = \frac{2c_s^2}{x} - \frac{4\pi\rho_0 r_0^2 G}{3-\alpha} x^{1-\alpha} \textstyle _2F_1 [3-\alpha, 3-\alpha; 4-\alpha; -x], \quad (A6)$$

$$= \frac{2c_s^2}{x} - \frac{4\pi\rho_0 r_0^2 G}{3-\alpha} \frac{\Gamma(4-\alpha)}{\Gamma(3-\alpha)\Gamma(3-\alpha)} \frac{1}{\Gamma(4-\alpha-n)} \sum_{n=0}^{\infty} \frac{\Gamma(3-\alpha-n)\Gamma(3-\alpha-n)}{n!} (-1)^n x^{n+1-\alpha}, \quad (A7)$$

$$= \frac{2c_s^2}{x} \left[ 1 - \sum_{n=0}^{\infty} A_n (-1)^n x^{n+2-\alpha} \right], \quad (A8)$$

$$A_n = \frac{2\pi\rho_0 r_0^2 G}{c_s^2(3-\alpha)} \frac{\Gamma(4-\alpha)}{\Gamma(3-\alpha)\Gamma(3-\alpha)} \frac{\Gamma(3-\alpha-n)\Gamma(3-\alpha-n)}{\Gamma(4-\alpha-n)}, \quad (A9)$$

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