Study on influence factors of submerged floating tunnel tether-tube parametric resonance

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Abstract. Considering the effect of tether sag of situation, submerged floating tunnel tether-tube parameter resonance mathematical model and the nonlinear equations of motion are built. Then the equation method is simplified by the Galerkin method, meanwhile, MATLAB is used to solve this equation. The parameter resonance which can occur between the constraints is analyzed. The results show that frequency ratio between submerged floating tunnel tether and tube is in an area. Parameter resonance phenomenon occurs among them. Under the condition of tether damping ratio unchanged, if the damping ratio of tube is increased, their resonant parameters of probability may be reduced to some extent.

1. Introduction
Submerged floating tunnel is construction combined by attachment and communion with dual revetment artifacts. For the attachment, which is suspended in the tunnel tube, subaqueous foundation, anchorage system on the basis of anchorage and tube in certain depth water. In the meantime, it is also a new traffic construction after subsea tunnel, crossing channel, rivers and lakes and waterway. Depending on the difference of submerged floating tunnel anchorage, it can be divided into tether form, buoy form and fixed form [1-2].

Tether is the main load-bearing structure or component of submerged floating tunnel. It is fixed at one end of the tunnel on the tube, the other side on the basis of anchorage [3-4]. Tether vibration is roughly divided into two types. The first is referred to as a forced vibration of anchor rope, such as vortex-induced vibration, etc.; the second is interaction occurred between submerged floating tunnel tether and tube, including parametrically excited and vibration, which is caused by submerged floating tunnel tube and tether mutual coupling [5-6]. When submerged floating tunnel tether is hit, the tube will be damaged. At the moment, parametric excitation frequency and natural frequency of tether will be as part of the multiple relations. Thus parameter vibration phenomenon will be happened. So in the paper, parametric resonance of the limit condition is discussed.

2. Establishment of tether-tunnel parameter resonance equation
As shown in figure 1, it is assumed that tether is considered a tilt rod. It is an angle of α with underwater, and tether anchorage of the bottom and the bottom are secured by anchorage. The top of one is connected by submerged floating tunnel. The top of tether movement is simplified as sine excitation vibration $A_0 \sin(\omega t)$.
Through analysis and calculation is derived from the stress and energy conservation. Assuming that motion control equation solution of tether is given as follows by using the theory of Hamiltonian (Hamilton) \((7)\).

\[ y_i = \sum_{n=1}^{N} y_{in}(t) \sin \left( \frac{n\pi z}{L} \right) + \frac{z}{L} A_0 \sin(\omega t) \sin \alpha \]  

Since tether will be droopy under the effect of itself. Elastic modulus of tether sag effect must be considered for reduction in the stress calculation and analysis \((8)\).

\[
E_{eq} = \frac{E}{1 + \frac{r^2 L_e^2}{12T_0^3}}
\]  

So tether equation of motion can be considered as follows.

\[
(\rho_iA + m_i) y_{in} = (\rho_i + \rho_s) Ag \sin \alpha + \frac{1}{L} \frac{\partial}{\partial z} \left[ \left( T_0 + \frac{E_{eq} A}{L} \frac{\partial y_i}{\partial z} \right) \left( 1 + \frac{1}{L} \frac{\partial y_i}{\partial z} \right) \right] - \frac{1}{2} C_d \rho_2 D y_{in} \left| y_{in} \right|
\]  

\[
(\rho_i A + m_i) y_{2n} = (\rho_i + \rho_s) Ag \cos \alpha + \frac{1}{L} \frac{\partial}{\partial z} \left[ \left( T_0 + \frac{E_{eq} A}{L} \frac{\partial y_i}{\partial z} \right) \left( 1 + \frac{1}{L} \frac{\partial y_i}{\partial z} \right) \right] - \frac{1}{2} C_d \rho_2 D y_{2n} \left| y_{2n} \right|
\]  

\[
\ddot{y}_{1n} = \frac{2}{\pi} \omega^2 A_0 \sin \omega t \sin \alpha - \frac{T_0 + E_{eq} A}{(\rho_i A + m_i)L} \pi^2 y_{1n} - \frac{2E_{eq} A}{(\rho_i A + m_i)L} A_0 \sin \omega t \sin \alpha \pi^2 y_{1n} + \frac{4(\rho_i + \rho_s)Ag}{(\rho_i A + m_i)} \sin \alpha
\]  

\[
\ddot{y}_{2n} = -\frac{1}{\pi} \omega^2 A_0 \sin \omega t \sin \alpha - \frac{T_0 + E_{eq} A}{(\rho_i A + m_i)L} (2\pi)^2 y_{2n} - \frac{2E_{eq} A}{(\rho_i A + m_i)L} A_0 \sin \omega t \sin \alpha (2\pi)^2 y_{2n}
\]  

Submerged floating tunnel tube is used as a quality piece inlaid at the tether end. Tube stiffness is modeled by spring and damping by the damper. Tether-tube parameter vibration model is set up as figure 2 \([2]\).

![Figure 1. Force diagram of tether](image1)

![Figure 2. Tether-tube parameter vibration model](image2)
As is shown in the graph, origin of coordinates is located in the bottom of the sea side. x and z axis is assumed as above.

Mass movement equation can be represented as:

\[ M\ddot{Z} + C\dot{Z} + KZ + \frac{E_eqA}{L} \int_0^L \varepsilon dZ = 0 \]  

(7)

Since the dynamic strain of tether is expressed as formula (8), formula (7) is simplified as formula (9).

\[ \varepsilon = \frac{\partial y_i}{\partial z} = \frac{L}{\pi} y_{ii} \cos \frac{\pi z}{L} + \frac{L}{2\pi} y_{iz} \cos \frac{2\pi z}{L} + \frac{A_i}{L} \sin \omega t \sin \alpha \]  

(8)

\[ \ddot{Z} + 2\omega_z \xi \dot{Z} + \omega_z^2 Z + 0.866 \frac{E_eqA}{LM} A_i \sin \omega t = 0 \]  

(9)

In which, \( \omega_z \) is a quality piece of excitation frequency, \( \xi \) is the mass of the damping ratio.

Parameter vibration equations are given as follows.

\[
\begin{align*}
\ddot{y}_{ii} = & \frac{2}{\pi} \omega_i^2 A_i \sin \omega t \sin \alpha - \frac{T_i + E_eqA}{(\rho_i A + m_i) L} \pi^2 y_{ii} - \frac{2E_eqA}{(\rho_i A + m_i) L} A_i \sin \omega t \sin \alpha \pi^2 y_{ii} \\
& + \frac{4(\rho_i + \rho_j)Ag}{(\rho_i A + m_i) \pi} \sin \alpha \\
& + \frac{2\omega_z \xi \dot{Z} + \omega_z^2 Z + 0.866 \frac{E_eqA}{LM} A_i \sin \omega t = 0}{4(\rho_i + \rho_j)Ag / (\rho_i A + m_i)}
\end{align*}
\]  

(10)

3. **Analysis of constraints examples and results**

The selection of basic parameters is as follows [7][9-10].

**Table1. Basic parameters**

| Parameter                        | Value  |
|----------------------------------|--------|
| depth of water (m)              | 140    |
| length (m)                      | 161.66 |
| diameter (m)                    | 0.489  |
| linear mass (kg/m)              | 1474.23|
| linear mass additionally (kg/m) | 193.06 |
| density (kg/m³)                 | 7850   |
| elasticity modulus (Pa)         | 2.1×10⁵|
| initial tension (N)             | 1.045×10⁶|
| damping ratio                   | 0.0018 |
| inclination (°)                 | 60°    |
| mass coefficient (C_m)          | 1.0    |
| drag coefficient (C_d)          | 0.7    |
| water density (kg/m³)           | 1000   |

3.1. **Frequency ratio**

When the frequency ratio of submerged floating tunnel tube and tether is from 1.5 to 2.2, time-displacement of tether and tube are shown as in figures (3-8).
As it can be seen from figures above, while frequency ratio of tube and tether is between 1.5 to 2.0, maximum amplitude of tether is increased respectively in this area and parameter resonance will be generated. However, when their proportion is 2.2, resonance phenomenon will not be generated.

3.2. Damping ratio

When the frequency ratio of tube and tether is 2.0, displacement process diagram is discussed when the damping ratio is 0 to 0.5.
From the above pictures, in the case that the damping ratio of tether is 0, as the damping ratio of tube increases gradually, maximum amplitude reduces gradually. At a word, when the damping ratio is as soon as possible, tether and tube have opportunity to be occurred resonance effect.

4. Conclusions
In this paper, submerged floating tunnel tether-tube parameter resonance mathematical model and the nonlinear equations of motion are built. Some restrictive conditions were analyzed briefly, which will lead to tether- tube parameter resonance. Conclusions are drawn as follows.

1) When the frequency ratio of submerged floating tunnel tether and tube is in a certain area, parameter resonance phenomenon will be generated. However, when the ratio of is outside this area, resonance phenomenon would not be happened.

2) Under the condition of tether damping ratio unchanged, if the damping ratio of tube increases, probability of their parameter resonance can be reduced in some degree.

3) When tether damping or tube damping is increased to a certain value, sudden increase of value generated by parameter resonance will be weak. So the parameter resonance can be reduced through increasing the damping properly.
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