Numerical methods for time-domain and frequency-domain analysis: applications in engineering

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Abstract. Numerical methods are widely used for modeling different physical phenomena in engineering, especially when an analytic approach is not possible. Time-domain or frequency-domain type variations are generally investigated, depending on the nature of the process under consideration. Some methods originate from mechanics, although most of their applications belong to other fields, such as electromagnetism. Conversely, other methods were firstly developed for electromagnetism, but their field of application was extended to other fields. This paper presents some results that we have obtained by using a general purpose method for solving linear equations, i.e., the method of moments (MoM), and a time-domain method derived for electromagnetism, i.e., the Transmission Line Matrix method (TLM).

1. Introduction

Characteristic equations modeling different phenomena in engineering do not generally lead to analytic solutions. Numerical methods are therefore widely used in order to compute solutions to most equations with linear operators e.g., differential and integral equations.

Most phenomena are time-dependent, so they can be modeled as functions of time. A time-domain approach would better model a transient behavior; however, when an oscillatory or repetitive process is investigated, a frequency-domain representation might be preferred. This actually is the case of many problems to be solved in mechanics, electromagnetism, heat transfer, etc. A frequency-domain approach saves time provided that the response of a system is needed for a given frequency of oscillation and/or for a small number of harmonics. Nevertheless, a time-domain analysis with a pulsed excitation would provide thorough information on the oscillatory response for a myriad of frequencies by means of an inverse Fourier transform, and that despite of a long computing time. In this paper, we present and compare two numerical methods primarily employed in electromagnetism, but potentially applicable to other fields, as similar equations are to be solved.

Although some of our examples belong to the field of electromagnetism, they are far from being typical applications and we show therefore the high flexibility and the potential of application of the two methods.

The Method of Moments (MoM) [1, 2] is a general purpose method for solving equations with a linear operator applied to a function to be found. It is based on an approximate expansion of the unknown function, by using known basis functions. Basically, applications of the MoM in electromagnetism deal with functions of a coordinate (e.g., current distributions); though, we show that time-domain equations can also be accurately solved, and further a frequency-domain approach is absolutely feasible.
The Transmission Line Matrix method (TLM) [3,4] is a more specific method that could solely be applied to wave-type differential equations, as that method is inspired from the way in which a pulse propagates within a transmission line mesh. It is therefore more appropriated to the time-domain analysis of the propagation of electromagnetic waves, sound, heat, etc. An insight of how the two methods could be applied to other fields of engineering is eventually provided.

2. Method of moments

The general form of an equation to be solved with the MoM is:

\[ L_z [I(z')] = h(z) \]  

(1)

with \( L_z \) a linear operator. In (1), \( L_z \) and \( h(z) \) are known, and \( I(z') \) is an unknown distribution.

The method of moments technique is based on a finite expansion of \( I(z') \), by using of a function basis, \( \Phi_n(z') \). The resulting approximate form of (1) can be written as:

\[ I(z') \approx \sum_{n=1}^{N} \mathcal{Z}_n \Phi_n(z') \]  

(2)

where \( \mathcal{Z}_n \) are weights to be calculated. Since the operator is linear, the equation to be solved becomes:

\[ \sum_{n=1}^{N} \mathcal{Z}_n L_z [\Phi_n(z')] = h(z) \]  

(3)

By sampling the \( z \) variable, equation (3) can be put in the following discrete form:

\[ \sum_{n=1}^{N} \mathcal{Z}_n L_z [\Phi_n(z')]_{z=z_m} = h(z_m) \]  

(4)

with \( m=1,2,\ldots,M \) and further in the following matrix form:

\[ [A] [\mathcal{Z}] = [B]. \]  

(5)

The resolution of the equation actually reduces to a matrix inversion leading to the weights \( \mathcal{Z}_n \).

3. Transmission line matrix method

The transmission line matrix (TLM) method [3,4] has originally been developed to model propagation phenomena, based on the analogy between the space and time variation of the characteristic parameters of the process under consideration, and the voltage and/or current variation within a 2D or 3D transmission line mesh, respectively. Examples of 2D and 3D nodes are given in figures 1 and 2, respectively [5].

The resolution of the equations of propagation is performed iteratively, including pulse scattering in each node of the mesh, propagation between two nodes, and boundary conditions. As an example, when characteristic figures are modeled as incident and reflected voltages in the mesh, scattering and propagation can be expressed as

\[ k+1 [V]^r = [S] k [V] \]  

(6)

and

\[ k+1 [V]^f = [C] k+1 [V] \]  

(7)

where \([S]\) is the scattering matrix, \([C]\) is the propagation matrix, and \(k\) denotes the time step.
3.1. Example of application of the MoM in kinematics: decomposition of a complex displacement

Let us consider a mobile platform moving along the x-axis with the instantaneous acceleration \( a_x(t) \).

The relationship between displacement and acceleration is given by a second order differential equation,

\[
d^2 x(t) \over dt^2 = a_x(t)
\]  

Equation (8) is one with a linear operator, namely:

\[
L(I) = d^2 \over dt^2
\]  

where the displacement \( x(t) \) is still unknown.

In order to compute the unknown distribution \( x(t) \), an alternative to the double integration of equation (8) would be to expand the movement into a finite series by means of appropriately chosen basis functions, and to apply the method of moments, in order to calculate the coefficients of the series.

In that case the finite series is:

\[
x(t) \approx \sum_{n=1}^{N_b} A_n \cdot \varphi_n(t)
\]  

where \( A_n \) are the unknown weights, \( \varphi_n(t) \) are the basis functions, and \( N_b \) is the total number of basic functions.

In order to assess the stresses to which the mobile platform is subjected and to assess the effects of movement on the crew comfort, the basis function should be chosen such as to correspond either to different orders of accelerated movements, or to oscillations of different frequencies. In practice, a model with accelerations of order of no more than three (throttle acceleration) can be used. The most important parameter for assessing the crew comfort is the second order acceleration (speed of acceleration), the third order acceleration being particularly important for space crafts. Thus, the model includes three basis time-domain polynomial functions.

The basic functions of order more than 4 are oscillations of different frequencies. As for a Fourier series, half of them will be sine functions and half cosine functions.

We used the point matching technique, that is, the test functions used for discretization are delayed Dirac pulses [6,7].

For validation purposes, we measured the instantaneous acceleration onboard a small boat. We used the method of moments with \( N=107 \) basis functions, in order to extract the x-coordinate. Then, we calculated the second derivative of the x-coordinate resulting from the method of moment
approach, and we compared it to the measured, instantaneous acceleration [6, 7]. Both results are given on figure 3.

**Figure 3.** Approximate, instantaneous acceleration resulting from the MoM approach versus measured, instantaneous acceleration.

3.2. Example of application of the TLM method: characterization of a short dipole antenna

Typical applications of the TLM method in electromagnetism include antenna characterization in the time-domain, and particularly evaluation of the antenna response to a pulsed excitation [8]. Figure 4 gives the waveform of the normalized, electric field radiated by a short dipole antenna; simulated and analytical results are given. A good agreement can be noted, although the TLM approach is based on some approximations.

**Figure 4.** Response of a short dipole to a Gauss pulse

3.3. Example of joint application of the MoM and TLM method: characterization of ultra-wide band antennas

Let us consider an antenna with a pulsed excitation. In practice, the impulse response of the antenna must be calculated when excitation and radiated field waveforms are known. Referring to the notations in figure 5, the equation to be solved is
where $V_g(t)$ is the antenna input voltage, $h(t)$ is the impulse response, and $E(r, t+r/c_0)$ is the electric far-field. Both the potential signature i.e., the right-hand side member of (11), and the impulse response depend on the direction alone.

The unknown impulse response can be approximated as follows:

$$h_t(\mathbf{r}, t) \equiv h_{t,\text{app}}(\mathbf{r}, t) = \sum_{n=1}^{N} h_n \Phi_n(t)$$

(12)

where $\{\Phi_{n=1,N}\}$ are the basis functions. By discretizing the time variable $t$, the coefficients $h_n$ can be computed by solving a linear system of equations that can be written in matrix form as follows:

$$[V] [h] = [E]$$

(13)

where $[h]$ is the vector of the weighting coefficients $h_n$, $[E]$ is the vector of samples of the potential signature, and $[V]$ is a matrix whose elements are $V_{mn} = (V_g \ast \Phi_n)(t_m)$. $m, n = 1...N$.

Let $t_1$ be the duration of the input signal and $t_2$ the duration of the potential signature. Prior to calculate the coefficients $h_n$ time-shifted Gauss pulses were used as basic functions:

$$\Phi_n(t) = \exp \left[-\left(\frac{b - t - (2n-1)t_1/2}{2T} \right)^2\right] p_n(t, t_1)$$

(14)

with $n = 1, 2, ... N$, $b = Nt_1/(t_2-t_1)$, and $p_n(t, t_1)=1$ if $t \in [(n-1)t_1/b, nt_1/b]$ and 0, otherwise.

As an example, we consider a rectangular notched monopole antenna on a square ground plane (figure 6). The excitation is a voltage pulse proportional to the first derivative of a Gauss function. The maximum of the spectrum magnitude of the input waveform occurs around 100MHz. We used the TLM method to compute the time-domain near-field on a closed surface including the antenna, and then the electric far field $E_\theta(r, t)$ on the direction ($\theta=\pi/2, \varphi=0$) was calculated by a time-domain near-field to far-field transformation. Subsequently, the approximate impulse response $h_{t,\text{app}}$ was computed (figure 7) by deconvolution using the time-domain approach that we propose.

Figure 5. Electric far-field of an antenna: notations.

Figure 6. Rectangular notched monopole antenna

The accuracy of the method is evaluated by reconstructing the potential signature from the resulting approximate impulse response and from the excitation, i.e., $h_{t,\text{app}}(t) \ast V_g(t)$. This result is compared to the original potential signature, computed by using the TLM method and a time-domain, near-field to far-field transformation (figure 8).
4. Conclusions

In our first example, we compared the instantaneous, measured acceleration provided by an inertial navigation system to the approximate, instantaneous acceleration resulting from the second derivative of the approximate coordinate, calculated with the MoM. Then, we compared the impulse response of a short dipole computed by using the TLM method to the analytic result.

Finally, we showed how an approximate form of the impulse response of an ultra-wide band antenna can be found, by using the method of moments. We compared the time-domain response of an antenna, issued from simulation by using the TLM method, for the first derivative of a Gauss function as an excitation, to the response calculated by convolving the approximate impulse response with that excitation waveform.

As a conclusion, the MoM and TLM approaches yield accurate results for every one of the applications we have presented; nevertheless, the TLM method can be applied for modeling other propagation phenomena (sound, heat, etc.), and the MoM can also be used to solve other linear operator equations, in order to model electrical circuits, wave propagation, current distribution on high-frequency devices etc.

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