Research of one interpolation formula at creation of the finite-difference scheme for functions with internal zones of the bigger gradients

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Abstract. In work the method creation of the interpolation formula which is bringing closer function of the smoothing gap, meeting in solutions of one-dimensional tasks with a heatmass transfer is investigated. Interpolation on the basis of the decision for the smoothing gap is used for creation of finite-difference schemes to multidimensional tasks with ruptures of the first sort in boundary and initial and regional conditions. Use of additional knots with the constructed interpolation gives considerable reduction of approximating viscosity in finite-difference schemes for the convective and diffusive equations of transfer. For the constructed interpolation in case of existence of an internal interface which thickness less step of a regular grid is conducted an accuracy research at the asymptotic approximations in small parameters corresponding to changes of required function in the neighborhood of a thin interface. The carried-out numerical calculations on a heat and mass transfer for a test task in two-dimensional rectangular area on the basis of the developed method show increase in an asymptotic order of accuracy of numerical decisions to the second from a grid step at application of the known schemes in the wide range of small parameter of diffusion.

Keywords: numerical modeling, interpolation formula, final and differential approximation, regular grids, diffusion parameter

1. Introduction
In work the method creation of qualitative schemes for the multidimensional convective and diffusive equation of transfer when using additional settlement templates on regular grids and application for this purpose of the interpolation modeling areas with internal interfaces is investigated. Construction and application of such schemes was considered in work [1]. Creation of such schemes with approach in the differential equations of the first and second private derivatives final and differential derivatives is caused by need of use of economic methods on compact templates with reduction of settlement errors and circuit viscosity. On the one hand use of big strides of a grid leads to increase in errors of method with increase of circuit viscosity which can exceed physical viscosity in problem definition, and on the other hand strong crushing leads to increase of errors from roundings on a limited numerical template. Creation of the considered method is connected with reduction of errors of approximation, including circuit viscosity that allows to receive more exact numerical decisions on large grids and to leave from the errors of the second sort connected with roundings of numbers. Such works [2,3,4] in which creation of the schemes having uniform convergence of rather small parameter at the senior derivative with the first order on h (a step on a grid) and is offered above are devoted to researches and development of the improved schemes for problems of convective and
diffusive transfer, for example. Application of the received algorithm with the known schemes for multidimensional problems of transfer gives in the decision higher precision, than application of schemes separately for each spatial direction (with the known methods of splitting) generally because of reduction of size of additional approximating viscosity [2].

Carrying out the analysis of emergence of approximating viscosity of difference schemes with construction on a two-dimensional template for the final and differential derivatives describing convective and diffusive transfer in a spatial case was considered. Application of various interpolation for approximation of the first and second derivatives on templates with a turn of checkouts on the set nodal lines was investigated. At creation of additional knots the best interpolation is determined by nodal lines that gives considerable reduction of approximating viscosity in final and difference schemes for the convective and diffusive equations of transfer. The carried-out numerical calculations on a warm mass transfer for a test task in two-dimensional rectangular area on the basis of the developed method showed increase in an order of accuracy of decisions to the second with application of the known schemes [3,4] in the wide range of small parameter [1,5]. In view of complexity of the analytical analysis of an order of accuracy and approximations in the developed complex algorithm (use of uncommon interpolation and difficult schemes in itself), conclusions of these data generally are difficult. But for some considered cases the carried-out analysis confirms numerical results.

On the developed algorithm with application of different schemes for achievement of a certain accuracy several orders less checkouts can be required that can generally belong to numerical calculation of convective and diffusive transfer with internal interfaces.

At the same time with use of the set grid and identical number of knots on one iteration of calculations it is required to this algorithm in comparison with other methods only 10 times more computing memory and 10 times more the spent time. But as a result the final prize on several orders (3-5) can exceed these increases by 10 times of memory and time of calculation at small numbers of diffusion.

2. Problem definition

For numerical realization of delimitation with strong changes of the decision in problems of convective and diffusive transfer with free borders or limited in multidimensional areas the test problem of a current of liquid or a heat and mass transfer in the current liquid will be considered by firm walls. The interpolation formulas constructed for this purpose on the basis of approximations to analytical decisions can be used at construction and increase in accuracy of final and difference schemes on multidimensional grids for problems of convective and diffusive transfer.

Let’s consider finding of interpolation formulas at creation of the settlement scheme in two coordinate directions \(x\) and \(y\) on the basis of analytical solutions of the non-stationary equation of diffusion with change on \(t\) time in a look:

\[
\frac{\partial c}{\partial t} = \nu \left( \frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} \right),
\]

where \(\nu\) – diffusion coefficient, \(c\) – required function of transfer.

For tasks with this equation there are two characteristic decisions [6, 7]:

1. the decision at pointed emission of size \(A\) (masses, energy) in a point \((x_0, y_0)\) at the initial moment –

\[
g(x, y, t) = \frac{A}{4\pi\nu t} \left[ \exp\left(-\frac{(x-x_0)^2}{4\nu t}\right) + \exp\left(-\frac{(y-y_0)^2}{4\nu t}\right) \right];
\]

2. the decision with smoothing of a gap –
\[ f(x,t) = \frac{A}{\sqrt{4\pi vt}} \int_{x_0}^{x} \exp \left( -\frac{(x-x_0)^2}{4vt} \right) dx = A \cdot \text{erf} \left( \frac{(x-x_0)}{\sqrt{4vt}} \right), \]

with entry conditions of \( f(x, 0) = -A \) at \( x < x_0 \), \( f(x, 0) = A \) at \( x \geq x_0 \).

Considering these decisions for tasks with convection existence by the equations of a look (1) in the neighborhood of any checkout \((i, j)\) in direction \(x\) or at in the field of speeds \((u, v)\) and considering one-dimensional self-similarity of transfer on a formula \( t = t_j + (y-y_j)/v_i \) or \( t = t_i + (x-x_i)/u_j \), it is possible to receive interpolation formulas. For nodal lines in the \( \text{Oy} \) direction these formulas can be presented in the following form (points indexes \(i, j\) to a binding of values to knot \((x_i, y_j)\)):

\[ g^y(x, y, x_i, y_{0j}, t_i, v_i, A) = \frac{A_i}{4\pi v t_i + \frac{x-x_i}{u_i}} \exp \left( -\frac{(y-y_{0j})^2}{4v_t (t_i + \frac{x-x_i}{u_i})} \right) + g_{0j}, \]

\[ f^y(x, y, x_i, y_{0j}, t_i, v_i, A) = \frac{A_i}{\sqrt{4\pi vt} t_i + \frac{x-x_i}{u_i}} \int_{y_i}^{y} \exp \left( -\frac{(y-y_{0j})^2}{4v_t (t_i + \frac{x-x_i}{u_i})} \right) dy + g_{0j}, \]

where \( A_i, t_i^* = t_i + \frac{x-x_i}{u_i}, y_{0j}, g_{0j} \) will be considered as unknown sizes for each knot at creation of interpolation on an interval from \((i, j)\) to \((i, j+1)\) and which are required to be determined by the known values of function \( f(x, y) \) in four hubs. When using values of the discrete decision these four sizes can be found in four next knots somehow, but it can be badly solvable task with use of numerical methods. On the other hand it is possible to construct interpolation with approximations of an order of \( O(h^3) \) or \( O(h^4) \) with four required coefficients which can be defined analytically.

3. Creation of interpolation formulas for functions of the smoothing gap

We investigate creation of interpolation on the basis of the decision for the smoothing gap in the form of (5) that can be applied to approximations in multidimensional tasks with ruptures of the first sort in boundary and initial and regional conditions. Their application is more important for dimensions 2 and 3, and not so - for one-dimensional cases. For this purpose originally we will make transformation of coordinates and we will make decomposition in ranks on the meeting functions, leaving main composed and rejecting elements of bigger trifle. For (1) we will consider such decomposition separately for \( y < y_{0j} \) and for \( y > y_{0j} \). At designation \( t_j^* = t_i + \frac{x-x_i}{u_i} \) in the first case we will enter a variable \( z_i = 1 - \exp \left( \frac{(y-y_{0j})}{\sqrt{4vt_j^*}} \right) \), and in the second \( -z_2 = 1 - \exp \left( -\frac{(y-y_{0j})}{\sqrt{4vt_j^*}} \right) \).

According to the conditions of application of \( z_l \) and \( z_2 \), their use in the form of power series in the neighborhood of \( z \) point = 0, the corresponding \( y = y_{0j} \), leads to decrease like power series from \((y-y_{0j})\) with small values that is required at decomposition in Taylor's ranks.

For \( z_i \) at \( y < y_{0j} \) expression (5) we will present in the form:
\[ f_i^y(y, y_{0j}, t_j^*, A_j, g_{0j}) = \frac{2A_i}{\sqrt{\pi}} \int_{y_{0j}}^{y} \exp\left(-\frac{(y - y_{0j})^2}{4vt_j^*}\right) d\left(\frac{y - y_{0j}}{\sqrt{4vt_j^*}}\right) + g_{0j} = \]

\[ = f_i^z(z_i, y_{0j}, t_j^*, A_j, g_{0j}) = \frac{2A_i}{\sqrt{\pi}} \int_{0}^{\ln(1-z_i)} \exp\left(-\ln^2(1-z_i)\right) d\left(\ln(1-z_i)\right) + g_{0j} = \]

\[ = -\frac{2A_i}{\sqrt{\pi}} \int_{0}^{\ln(1-z_i)} \exp\left(-\ln^2(1-z_i)\right) (1-z_i)^{-1} \, dz_i + g_{0j}. \]  

Further we will make decomposition (6) in a row on \( z_i \) degrees in the neighborhood of \( z_i \) point = 0:

\[ f_i^z(z_i, t_j^*, A_j) = f_i^z(0, t_j^*, A_j) + \frac{z_i}{1!} \frac{\partial f_i^z(y_{0j}, t_j^*, A_j)}{\partial y} + \]

\[ + \frac{z_i^2}{2!} \left[ -\frac{\partial^2 z_i}{\partial y^2} + \frac{1}{\partial y^2} \frac{\partial f_i^z(y_{0j}, t_j^*, A_j)}{\partial y} \right] + O(z_i^3) = \]

\[ = g_{0j} + \frac{z_i}{1!} \frac{A_i}{\sqrt{\pi vt_j^*}} + \frac{z_i^2}{2!} \left( \frac{-1}{1/(4vt_j^*)^3} \right) \frac{A_i}{\sqrt{\pi vt_j^*}} + 0 + O(z_i^3) = \]

\[ = g_{0j} - z_i - \frac{2A_i}{\sqrt{\pi vt_j^*}} + O(z_i^3). \]

It is possible to consider this decomposition to the second order in a look:

\[ f_i^z(z_i, t_j^*, A_j) = g_{0j} - z_i \frac{2A_i}{\sqrt{\pi}} + O(z_i^3), \]  

what corresponds \( f_i^y(y, y_{0j}, t_j^*, A_j, g_{0j}) = g_{0j} - \frac{2A_i}{\sqrt{\pi}} \left( 1 - \exp\left(\frac{y - y_{0j}}{\sqrt{4vt_j^*}}\right) \right) + O(z_i^3), \)

also it can be presented approximately in the form of an interpolation formula

\[ \tilde{f}_i^y(y, a_{0j}, a_{2j}, a_{3j}, a_{4j}) = a_{0j} + a_{2j} \exp\left(\frac{y - a_{3j}}{a_{4j}}\right), \]  

where \( a_{0j} = g_{0j} - \frac{2A_i}{\sqrt{\pi}}, a_{2j} = \frac{2A_i}{\sqrt{\pi}}, a_{3j} = y_{0j}, a_{4j} = 1/\sqrt{4vt_j^*}. \)

For a case \( y > y_{0j} \) the second branch interpolants is defined similarly in a look:

\[ \tilde{f}_i^y(y, a_{1j}, a_{2j}, a_{3j}, a_{4j}) = a_{1j} - a_{2j} \exp\left(\frac{-(y - a_{3j})}{a_{4j}}\right), \]  

where \( a_{1j} = g_{0j} + \frac{2A_i}{\sqrt{\pi}} \) with an interface condition

\[ a_{2j} = (a_{1j} - a_{0j})/2. \]  

Thus, to an interpolation with two branches (9), (10), having five unknown coefficients, it is possible to determine on an interval from knot \((i, j)\) to knot \((i, j+1)\) by a condition (11) and four values of function \(f\) in the next knots \((i, j-1), (i, j), (i, j+1), (i, j+2)\). For unambiguous determination of such five coefficients of rather monotonous change of function on these knots. Otherwise this function will not belong to the smoothing function for a gap on these knots, and then there will be no need for use of such interpolation and it is possible to use simple smooth a spline interpolation of the second or fourth of orders of approach.
Determination of coefficients for (9), (10) is the irrational task demanding application of numerical methods. Therefore these interpolating formulas can be replaced with the following approximations after decomposition with rational functions:

\[ F_1^y(y, a_{0j}^*, a_{2j}^*, a_{3j}^*, a_{4j}^*) = a_{0j}^* + \frac{a_{2j}^*}{y - a_{3j} - a_{4j}^*}, y < a_{3j}; \]  
\[ F_2^y(y, a_{1j}^*, a_{2j}^*, a_{3j}^*, a_{4j}^*) = a_{1j}^* + \frac{a_{2j}^*}{y - a_{3j} + a_{4j}^*}, y \geq a_{3j}. \]  

(12)  

(13)

For them similar coefficients can be already determined by obvious formulas.

Interpolation (12)–(13) can be generalized on some tasks with linear changes of functions by introduction of additional coefficient \( a_3 \) and to present in the following form:

\[ f_3^y(y) = \begin{cases} 
    a_0 + \frac{a_2}{y - a_3 - a_4}, & y < a_3; \\
    a_1 + \frac{a_2}{y - a_3 + a_4}, & y \geq a_3.
\end{cases} \]  

(14)

Such interpolation will correspond to the function changing on coordinate with smoothing of a gap, and the coefficient of \( a_4 \) will correspond to background change of linear function on the coordinate or approach to such function in the neighborhood of checkouts with \( O(h^2) \).

Let's consider receiving approach (12) to (9), and (13) to (10) will be similar. For this purpose we will increase (9) by size \( (y - a_{3j} - a_{4j}^*) \) and we will make decomposition in a row on degrees from \( y - a_{3j} \), considering that \( a_{4j}^* \) is small and \( (y - a_{3j})(y - a_{3j} - a_{4j}^*) < 1 \). At the same time it follows from the choice (12) as the left branch of interpolation \( a_{4j}^* > 0 \) and \( y < a_{3j} \).

As a result we receive:

\[ f_1^y(y, a_{0j}, a_{2j}^*, a_{3j}, a_{4j})(y - a_{3j} - a_{4j}^*) = a_{0j}(y - a_{3j} - a_{4j}^*) + a_{2j}^*(y - a_{3j} - a_{4j}^*)\exp((y - a_{3j})a_{4j}^*) = \]  
\[ = (a_{0j} - a_{2j})a_{4j}^* + \frac{(y - a_{3j})}{1!}(a_{0j} + a_{2j} - a_{3j} - a_{4j}^*) + \frac{(y - a_{3j})^2}{2!}(2a_{2j}a_{4j}^* - a_{2j}^2 + a_{3j}^*) + O((y - a_{3j})^3). \]

And after division into size \( (y - a_{3j} - a_{4j}^*) \) we receive

\[ f_1^y(y, a_{0j}, a_{2j}^*, a_{3j}, a_{4j}) = a_{0j} + a_{2j}^* - \frac{a_{2j}a_{4j}^*a_{4j}^*}{(y - a_{3j} - a_{4j}^*)} + \]  
\[ + \frac{(y - a_{3j})^2}{2(y - a_{3j} - a_{4j}^*)}(2a_{2j}a_{4j}^* - a_{2j}a_{4j}^* + a_{3j}^*) + O((y - a_{3j})^2), \]  
\[ = a_{0j}^* + \frac{a_{2j}^*}{(y - a_{3j} - a_{4j}^*)} + O((y - a_{3j})^2) \approx a_{0j}^* + \frac{a_{2j}^*}{(y - a_{3j} - a_{4j}^*)} = F_1^y(y, a_{0j}^*, a_{2j}^*, a_{3j}^*, a_{4j}^*), \]

(15)

where \( a_{0j}^* = a_{0j} + a_{2j} - a_{3j}a_{4j}^* \), \( a_{2j}^* = -a_{2j}a_{4j}^* \).

Similarly, it is possible to receive expression (9) for the second branch interpolations with approach \( O((y - a_{3j})^2) \) and additional coefficient \( a_{1j}^* = a_{1j} - a_{2j} + a_{2j}a_{4j}^* \) for which like (11) the condition will be satisfied.
\[ a_{2j}^* = (a_{i+j}^* - a_{i-1}^*) / 2. \]

From (15) - (16) on values of coefficients for (9) - (10) it is possible to present coefficients for an interpolation (12), (13) through initial parameters of the smoothing gap (7) \( y_{0j}, t_j^*, A_j, g_{0j} \) in a look:

\[ a_{0j}^* = \frac{g_{0j}}{\sqrt{\pi vt_j}}, a_{1j}^* = g_{0j} + \frac{A_j a_{i,j}^*}{\sqrt{\pi vt_j}}, a_{2j}^* = \frac{A_j a_{i-1,j}^*}{\sqrt{\pi vt_j}}, a_{3j}^* = a_{3j} = y_{0j}, a_{i,j}^* \approx 4\sqrt{vt_j}. \tag{17} \]

At these values the first and second derivatives an \( i \) interpolation (9)-(10) and (12) - (13) at \( y = y_0 \) coincides:

\[ \frac{\partial^2 F_1^y}{\partial y^2}(y_{0j}, a_{0j}^*, a_{2j}^*, a_{3j}^*, a_{i,j}^*) = \frac{\partial^2 F_1^y}{\partial y^2}(y_{0j}, a_{0j}, a_{2j}, a_{3j}, a_{i,j}) = \frac{A_i}{\sqrt{2\pi vt_j}}, \tag{18} \]

\[ \frac{\partial^2 F_1^y}{\partial y^2}(y_{0j}, a_{0j}^*, a_{2j}^*, a_{3j}^*, a_{i,j}^*) = \frac{\partial^2 F_1^y}{\partial y^2}(y_{0j}, a_{0j}, a_{2j}, a_{3j}, a_{i,j}) = \frac{A_i}{2vt_j}\sqrt{\pi}. \tag{19} \]

Derivatives from (6) coincide also with these values.

Similar equalities and for the second branches an interpolation that is important for approximations when values of derivatives (18) and (19) are high at perhaps small values of sizes \( v \) and \( t_j^* \). Also there is a coincidence of derivatives from (6) with values (18) that confirms the good approach of interpolation to the decision having great values of derivatives in the neighborhood of \( y = y_0 \).

Still it is possible to consider approximations of interpolation (14) after decomposition of all coefficients of rather small sizes on one of branches and to estimate an order of approximation of values of required function between knots.

We will for this purpose be limited to a coefficient task case = ah where a from an interval \([0, 1]\). Also without restriction of community we will put that \( a_0 < a_1, y < a_3, a_3 < 0, a_4 > 0 \).

Thus, interpolation formulas (12), (13) it agrees (8), (15) have the second order of approach to (9), (10) and also to (6) on h grid step taking into account that \( h \geq |y - a_{3j}| \).

For these formulas coefficients \( a_i^* (a_{0i}^*, a_{1i}^*, a_{3i}^*, a_{4i}^*) \) decide after some transformations from the solution of the algebraic equation of the fourth degree on the known formulas on their unambiguous definition. Application conditions and uniqueness of coefficients for formulas is existence of monotony of change of the decision on the considered interval. At a task \( a_{3j}^* = a_{3j} \) or definition from the sedate equation of the fourth a lock following from equating of interpolation (12), (13) values of required function \( f_1, f_2, f_3, f_4 \) in knots \((i, j-l, (i, j), (i, j+1), (i, j+2)\), other coefficients are defined as follows:

\[
\begin{align*}
a_4^* &= (-a_3^* c_1 + a_3^* c_3 - c_3)/(a_3^* c_2 + c_4), \\
a_1^* &= f_1 + (-a_3^* - a_4^*)(f_2 - f_1)/h, \\
a_0^* &= f_4 + (h - a_3^* + a_4^*)(f_4 - f_3)/h, \\
a_2^* &= a_4^*(a_3^* - a_4^*)/2,
\end{align*}
\]

where \( c_1 = (f_4 - f_3 + f_2 - f_1)/h, c_2 = (f_4 - f_3 - f_2 + f_1)/h, c_3 = (3f_4 - 3f_3 - f_2 + f_1)/h, c_4 = (-f_4 + 2f_3 - f_2)/h, c_5 = 2h(-f_4 + f_3). \)
Formulas (12) - (13) have enough third order of approximation for further creation of the general settlement scheme with the second order that is characteristic in case of application of symmetric difference schemes. But generally at application of several interpolation functions and difficult difference schemes the second order of accuracy can be carried out on interpolation formulas of the fourth order of accuracy that is checked by the harmonious analysis.

Use of interpolation formulas of a look (12)-(13), but not any, at the solution of tasks with transfer on the basis of the equations of a convective and diffusive look can increase the accuracy of calculations in a type of the maximum approximations to behavior of the decisions arising in problems of transfer (with gaps or without gaps in boundary conditions). Results of the made numerical experiments confirm preservation of accuracy of the applied schemes at application of the developed interpolation on test tasks with such features.

Thus, on the basis of an algorithm of allocation of the spatial direction of transfer in two-dimensional or three-dimensional tasks it is possible to use the known schemes with $O(h^2)$ in this spatial direction as one-dimensional with maintaining their qualities and other usual schemes on three-point templates for calculation of convective and diffusive transfer for other spatial directions with $O(h^3)$ too. At the same time the grid remains tied to the set rectangular borders, and the direction of transfer can change from knot to knot. Advantage of use of this algorithm is considered on test calculations.

4. Results of numerical calculations

We will consider the carried-out test calculations for distribution of heat with a gap on border below (in a point (0.0)) at small values of the set diffusion and the direction of convective transfer incoincident with nodal lines of a settlement grid. For calculations finite-difference schemes were applied to convective and diffusive transfer on a three-point compact template in each spatial direction with a turn of checkouts on nodal lines, speeds, consistent with the directions. For determination of values of required function in additional knots interpolation was used (14). Conditions of a test task are shown in Fig. 1.

![Figure 1. Area with a settlement grid and a template](image1)

![Figure 2. Calculations with diffusion $10^{-2}$ at x=0.5](image2)

Calculations were carried out at coefficient of diffusion $v$ from 0.1 to $10^7$, speed components $(u, v) = (1, 0.66)$ on regular grids 1, 2, 3 ((10x10), (20x20), (40x40)) to areas 1.0 x 1.0 with boundary conditions of $C(0, y) = 1$, $C(x, 0) = C(1, y) = C(x, 1) = 0$.

In Fig. 2 - 4 calculations in an area section at $x = 0.5$ at coefficient of diffusion are shown $10^2$, $10^3$, $10^4$, respectively, for the exponential scheme of the second order of accuracy with use of the developed algorithm [4] (curves 1,2,3 on three grids 1, 2, 3), for the scheme of the directed differences
(curves 4,5,6 on the corresponding grids 1, 2, 3) and the exponential scheme of the first order [3] (curves 7,8,9) and also for the Samarsky A.A. scheme (a curve 10 counting upon a grid 3).

Figure 3. Calculations with diffusion $10^{-3}$ at $x=0.5$

Figure 4. Calculations with diffusion $10^{-4}$ at $x=0.5$

From these results it is visible that payment under usual schemes gives approximating diffusion about $10^{-2}$ and there is no big difference of curves with Fig. 3-4 ($v = 10^{-3}$ and $10^{-4}$) from the decision in Fig.2 ($v = 10^{-2}$) while curves with application of the developed algorithm correspond to the decision with the set coefficients of diffusion ($10^{-2}$, $10^{-3}$, $10^{-4}$).

The research of process of convergence of the received numerical decisions on the developed technique with stationary boundary conditions on an asymptotics on time with multiple reduction of steps of a grid showed the following orders of accuracy of P at diffusion of $v$ from $10^{-1}$ to $10^{-7}$ (see Fig. 5-7) for three schemes (exponential with $O(h^2)$ - a curve 1, with the directed differences - 2, exponential with $O(h)$ - 3).

For Fig. 5 calculations are carried out at constant components of speed ($u$, $v$), and for Fig. 6 c change of $u$ from 1 to 2 and for Fig. 7 with change of $u$ from 1 to 3. Entering of such nonlinearity into coefficients of the settlement equations shows preference for use in the developed scheme algorithm with $O(h^2)$ for which the settlement order of accuracy on this test remains closer to the second, than to the first. Other schemes correspond to the first order.

Figure 5. Change of an order of $P$ from $k$, $v = 10^k$

Figure 6. Change of an order of $P$ from $k$, $v = 10^k$
On the basis of the developed algorithm of reduction of artificial diffusion in multidimensional tasks the way of creation of the finite-difference scheme of the second order of accuracy for space-time problems of transfer is offered. For a class of tasks with explosive boundary entry conditions special interpolation formulas are constructed. Creation of the finite-difference scheme for the transfer equations in multidimensional area on a usual rectangular grid is carried out. For some problems of transfer of the developed methods the test calculations showing a high order of accuracy are carried out.

5. Conclusions
On the basis of the conducted research results are received:
1). The interpolation formula for approach to an analytical formula of the smoothing gap is offered and investigated.
2). Coefficients of interpolation are defined on the basis of the known analytical solution of the equation of the fourth degree that eliminates finding of coefficients with numerical methods.
3). Application of the constructed interpolation in final and differential schemes for the multidimensional equations of convective and diffusive transfer allows to increase the accuracy of calculations, including the second order of uniform convergence in the wide range of small values of diffusion parameter.

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