A simple plan for the $\Delta$ resonance

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We construct the $\Delta$ resonance as a superposition of a bare $\Delta$ state and the $\pi N$ continuum. It is parametrized by three coupling constants for local $\pi N \Delta$ and $\pi \pi NN$ couplings and the $\Delta$ mass. The latter incorporates the mass renormalization due to the $\pi N \Delta$ interaction, while the results depend only weakly, if at all, on its wave-function renormalization. Three more renormalization constants are needed for the derivative contact interaction. They allow one to generate the $\Delta$ resonance dynamically. A large number of fits test the quality of different model assumptions in the $P_{33}$, $P_{31}$ and $P_{13}$ $\pi$-wave $\pi N$ scattering channels.

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I. INTRODUCTION

The $\Delta$ resonance is present in most of baryonic physics, due to its low energy and its strong coupling to the pion-nucleon systems. Therefore it is crucial to have good and simple methods to handle the $\Delta$ resonance and its effects on other hadronic physics. Such method should be based on a local theory where the renormalization is handled properly, and divergences can be re-absorbed consistently in the interactions. Otherwise, the method is nothing more than a particular fit of the data, where the $\pi N \Delta$ interactions have no universal meaning. Especially this interaction should be handled properly, due to strength, which makes the application of perturbation theory dubious. In this paper we propose such a method with local interactions, which can be solved in closed form, without resorting to perturbation theory. The $\Delta$ resonance is a spin 3/2 state, which is hard to handle in field theory, as it is generally part of a non-renormalizable field theory, due to the high-order derivatives in the coupling and the propagator, which yield high-order divergences in the self-energy corrections. Furthermore, there are at least two other issues that blur our image of the $\Delta$ resonance. First, the propagator has some unknown off-shell dependence, reflected by a free parameter. Second, field transformations allow different representations for the interactions and the propagators which, however, lead to identical observable results.

In this short paper all the problems raised above are circumvented. The propagator and the interaction are lumped together, so there are no issues concerning field transformations that shift terms around. Furthermore, dimensional analysis, combined with local field theory, implies that the interaction depends, in lowest order, on four parameters only: two $\pi N \Delta$ couplings, one $\pi \pi NN$ couplings, and the $\Delta$ mass. The finite renormalization constants are fixed by the $\Delta$ mass and the wave function renormalization.

II. THEORY

The self-energy diagram of the $\pi N$ loop correction to the $\Delta$, which has an imaginary part because of the $\pi N$ decay, has only two types of local momentum dependence of the loop integral in the rest frame:

$$\Sigma(E) = \frac{1}{4\pi} \int \frac{d^3p}{\omega_N \omega_\pi} \frac{[1; p^2]}{E - \omega_\pi - \omega_N},$$

due to the different terms in the traces over polarizations and spins. The attention focuses on the real part. The same diagram occurs in other spin-isospin channels, with, possibly, different masses and coupling constants, however, the momentum and energy dependence will be the same. All the singularities are treated a principal value integrals. The imaginary part, associated with the homogeneous solution of the wave equation will follow from the eigenvalue equation below. The two different momentum dependencies are associated with two different vertices (see Fig. 1). They serve as the two $\pi N \Delta$ couplings. Furthermore, there is a third coupling for the $\pi \pi NN$ contact interaction, which will yield the phase shift in the absence of the $\Delta$ resonance in other spin-isospin channels. In lowest order, the contact interaction is dominated by the pionic current $[\pi^*, \partial_\mu, \pi]$ which acts on the side of the triangle in Figure 1. The two different time-orderings will be associated with two different terms in the Hamiltonian.
FIG. 1: The four types of interactions of the Hamiltonian. The dot represents a derivative coupling. The triangle indicates the pion line on which the derivative acts.

For each channel and partial wave, in the rest frame, we can write down a general Hamiltonian for low-energy \( \pi N \) scattering based on the observations above:

\[
H = |\Delta|m_\Delta \langle \Delta | + \int_{m + m_N}^{\infty} d\epsilon |\pi N(\epsilon)\rangle \langle \pi N(\epsilon) | \\
+ \sum_{n=0}^{1} \epsilon_{nk} \int_{m + m_N}^{\infty} d\epsilon' \sqrt{V_\lambda(\epsilon')} |\pi N(\epsilon') \rangle \int_{m + m_N}^{\infty} d\epsilon |\pi N(\epsilon) \rangle \sqrt{V_\lambda(\epsilon)} \\
+ \sum_{n=0}^{1} \int_{m + m_N}^{\infty} d\epsilon |\Delta \rangle \sqrt{W_n(\epsilon)} |\pi N(\epsilon) \rangle + |\pi N(\epsilon) \rangle \sqrt{W_n(\epsilon)} |\Delta |, 
\]

(2)

where \( \epsilon_{nk} = 1 \) if \( n \neq k \) and zero otherwise. The momentum dependence of the contact interaction is based on a single \( \pi \pi NN \) coupling through the pionic current \( \sim [\pi, \partial_\mu \pi] \) with the nucleon current \( \sim N^\gamma_\lambda \gamma_\mu N \), where the single derivative acts either on the incoming or the outgoing pion state, which leads to the asymmetric form.

The coupling functions \( V \) and \( W \), which correspond to local interactions, contain both two terms with the same functional form, however, different coupling constants \( (V_n = \frac{1}{g_n}W_n) \):

\[
W_n(\epsilon) = g_n \frac{k^{2n+2}}{\omega_N \omega_{\pi}} \frac{\partial k}{\partial \epsilon} = \frac{g_n}{\epsilon} \left( \frac{\epsilon^4 - 2\epsilon^2 (m_{\pi}^2 + m_N^2) + (m_{\pi}^2 - m_N^2)^2}{2\epsilon} \right)^{2n+1},
\]

(3)

The kinematical factors \( (\omega_\pi \omega_N)^{-1} \) make the interaction relativistic invariant, and the factor \( k^2 \) for \( n = 1 \) is due to the local, derivative coupling, in the rest frame. The coupling constants \( g_n \) have a dimension \( [\text{energy}^{2-2n}] \)(GeV), and \( \lambda \) has a dimension of \( [\text{energy}^{1-2n}] \)(GeV). All the numerical factors from angular integrations, etc. are lumped in with the coupling constants \( g_n \) and \( \lambda \). The self-energy diagram, Eq. (4), is equivalent to \( (g_n = 1) \):

\[
\Sigma(E) = \int_{m + m_N}^{\infty} d\epsilon \frac{|W_0(\epsilon):W_1(\epsilon)|}{E - \epsilon},
\]

(4)

which is again equivalent with the first order perturbation theory results for the Hamiltonian Eq. (2).

This single channel problem with separable, but local interactions can be solved exactly, apart from the divergencies for which the finite renormalization has to be determined. The coupling functions \( W \) and \( V \) have the same functional form, therefore the ansatz wave function for the energy \( \omega \) yields:

\[
|\Psi(\omega)\rangle = \alpha(\omega) |\Delta \rangle \\
+ \int d\epsilon \left( \frac{1}{\omega - \epsilon} - z(\omega) \delta(\omega - \epsilon) \right) \left( \sqrt{W_0(\epsilon)} \beta_0(\omega) + \sqrt{W_1(\epsilon)} \beta_1(\omega) \right) |\pi N(\epsilon) \rangle \\
\equiv \alpha(\omega) |\Delta \rangle + \beta_0(\omega) |\omega(\pi N) \rangle_0 + \beta_1(\omega) |\omega(\pi N) \rangle_1.
\]

(5)

The function \( z(\omega) \) models the ratio of the homogeneous and the inhomogeneous part of the \( \pi N \) scattering state. For this single channel scattering state it is the only observable, which is directly related to the phase
shift. When this wave function is inserted into the continuum eigenvalue equation $0 = (\omega - H)\Psi(\omega)$ it yields the relations between the spectroscopic densities $\beta_i$ and the occupation number $\alpha$:

$$
0 = \beta_0|\omega(\pi N)_0\rangle + \beta_1|\omega(\pi N)_1\rangle + (\omega - m_\Delta)\alpha|\Delta\rangle
- (\tilde{W}_0(\omega) - zW_0(\omega))\beta_0|\Delta\rangle
- (\tilde{W}_1(\omega) - zW_1(\omega))\beta_1|\Delta\rangle
- \alpha(|\omega(\pi N)_0\rangle + |\omega(\pi N)_1\rangle)
- (\tilde{V}_0(\omega) - zV_0(\omega))\beta_0|\omega(\pi N)_1\rangle
- (\tilde{V}_1(\omega) - zV_1(\omega))\beta_1|\omega(\pi N)_0\rangle ,
$$

(7)

where the tilde refers to the Hilbert transform, without a factor of $\pi^{-1}$:

$$
\frac{g_n}{\lambda} \tilde{V}_n(\omega) = \tilde{W}_n(\omega) = \int d\epsilon \frac{W_n(\epsilon)}{\omega - \epsilon} .
$$

(8)

Projecting on the states $|\omega(\pi N)_0\rangle$ and $|\omega(\pi N)_1\rangle$ yields: $(i,j = 0,1)$

$$
\beta_i = \epsilon_{ij} \frac{1 + \tilde{V}_j(\omega) - zV_j(\omega)}{1 - [\tilde{V}_0(\omega) - zV_0(\omega)][\tilde{V}_1(\omega) - zV_1(\omega)]} \omega ,
$$

(9)

inserting this back into Eq. (7) and projecting onto $|\Delta\rangle$ yields a relation for $z$:

$$
\omega - m_\Delta = \sum_{i=0}^1 \epsilon_{ij} [\tilde{W}_j(\omega) - zW_j(\omega)][1 + \tilde{V}_j(\omega) - zV_j(\omega)]
- \sum_{i=0}^1 [\tilde{V}_0(\omega) - zV_0(\omega)][\tilde{V}_1(\omega) - zV_1(\omega)] ,
$$

(10)

which yields a straightforward but lengthy quadratic equation for $z$. In the case of weak contact interactions $\lambda \sim 0$, the solution can be approximated by:

$$
z = -\frac{1}{W_0 + W_1}(\omega - m_\Delta - \tilde{W}_0 - \tilde{W}_1) ,
$$

(11)

which can be cast in the form of an analyticity corrected energy-dependent Breit-Wigner resonance.

The general phase shift is given by:

$$
\tan \delta = \frac{\pi}{z} .
$$

(12)

Since there is only one channel, the scattering process is fully characterized by $z$ through the phase shift. Only the Hilbert transforms of $W_0 = \frac{g_0}{\lambda}V_0$ and $W_1 = \frac{g_1}{\lambda}V_1$ still need to be determined.

The backward or $Z$-diagram contributions are included, to make the results covariant and the expressions simpler. The singularity is treated as a principal value singularity:

$$
\tilde{W}_n(\omega) = g_n \int ds \sqrt{s} \left( \sqrt{s^2 - 2s(m_\Delta^2 + m_N^2) + (m_\Delta^2 - m_N^2)^2} \right)^{2n+1} \frac{1}{\omega^2 - s} .
$$

(13)

The integral is divergent. Subtractions make the integrals finite:

$$
\tilde{W}^r_n(\omega) = \int \frac{\omega^{2n+2}}{s^{n+1}} ds W_n(\sqrt{s}) \frac{1}{\omega^2 - s} ,
$$

(14)

which yield an analytical results for the Hilbert transform for an arbitrary integer value of $n$:

$$
\tilde{W}^r_n(\omega) = -\frac{g_n(1 - T_{2n+1})}{\omega(2\omega)^{2n+1}} \Re \left[ \left( \sqrt{A} \right)^{2n+1} \log \left( \frac{-\omega^2 + m_\Delta^2 + m_N^2 + \sqrt{A}}{m_\pi m_N} \right) \right] ,
$$

(15)

where $T_m$ stands for the $m$-th order Taylor expansion in $\omega^2$ around $\omega^2 = 0$, and

$$
\sqrt{A} \equiv \sqrt{\omega^4 - 2\omega^2 (m_\Delta^2 + m_N^2) + (m_\Delta^2 - m_N^2)^2} .
$$

(16)
FIG. 2: The phase shift in the $P_{33}$ scattering channel which contains the $\Delta$ resonance. The dashed-double-dotted line is a fit with a dynamically generated mass through the contact interaction with $\lambda$. The solid line is a fit with $g_0$ and $g_1$. The dashed line is a fit with $g_0$, $g_1$, and $\lambda$. The long-dashed line is a fit with $g_0^2$, which has a much better $\chi$-square value due to a better threshold behavior. The dot-dashed line is a fit with two $g_0^2$ couplings to two different resonances.

It should noted that the real part of the function above arises through the real part of the logarithm and the real part of the square root for $\omega^2 > (m_N + m_\pi)^2$ and $\omega^2 < (m_N - m_\pi)^2$ and through the product of the imaginary parts of both for $(m_N - m_\pi)^2 < \omega^2 < (m_N + m_\pi)^2$.

The subtractions correspond to two divergent constants that should be reabsorbed in the mass and the wave function of the bare $\Delta$ state:

\[ \tilde{W}_0(\omega) = \tilde{W}_r^0(\omega) + c_0^{(0)} , \]
\[ \tilde{W}_1(\omega) = \tilde{W}_r^1(\omega) + c_0^{(1)} + c_1^{(1)} \omega^2 , \]

where the renormalization constants $c$ are fixed through: $\tilde{W}_n(m_\Delta) = (\omega^2 - m_\Delta^2)^n + O(n + 1)$. Arguably, $W_0$ could have a finite wave function renormalization. However, as one can see from Eq. (11) this renormalization factor $c_1^{(0)}$ can be reabsorbed in an overall shift of the coupling constants $g_n \to g_n / c_1$ for the only observable quantity $z$, where $\omega^2 - m_\Delta^2$ is approximated by $2m_\Delta(\omega - m_\Delta)$. The coupling functions $W$ can clearly be identified with the bare $\Delta$ state; the constant part of $\tilde{W}$ is indistinguishable from the mass $m_\Delta$. For the contact interaction there is no such identification, therefore these renormalization constants will be determined from the data.

III. WITHOUT BARE RESONANCE STATE

In the case of only contact interactions and no resonance state the expressions simplify somewhat. It corresponds to the Hamiltonian, Eq. (3), with $W_0 = 0$ and $W_1 = 0$, which decouples the resonance from the scattering states. The ratio of the homogeneous to inhomogeneous solution $\pi / z(\omega)$ that leads to the phase shift is given by:

\[ z = \frac{V_0 \tilde{V}_1 + V_1 \tilde{V}_0 + \sqrt{(V_0 \tilde{V}_1 + V_1 \tilde{V}_0)^2 + 4V_0 V_1 (1 - \tilde{V}_0 \tilde{V}_1)}}{2V_0 \tilde{V}_1} . \]

Depending on the coupling constant $\lambda$, the phase shift can be positive or negative. In the physical range $m_N + m_\pi < \omega < 2.0$ GeV the sign change in the phase shift occurs for $\lambda$ varying between 1.0 and 3.0. In that case the finite renormalization is set to zero, such that $\tilde{V}_i$ vanishes at $\omega = 0$.

It is possible to mimic part of the resonance with a mass $m_V$ through a non-trivial renormalization of $\tilde{V}_0$ and $\tilde{V}_1$:

\[ \tilde{V}_0 = \tilde{V}_0^\tau + c_0^{(\lambda,0)} \]
FIG. 3: The quality of the fit and the variation of the coupling constants for the $P_{33}$ channel. The fit with $g_0$ and $g_1$ includes all the data points from threshold to the energy indicated. The circles are the $\chi^2$/#, the squares the values of $g_0$ in 100 GeV, and the diamonds are the values of $g_1$ in 2 GeV. The lines only guide the eye. The $\chi^2$/# drops for large energies due to the poorer quality of the data at higher energies. It is clear from the fits that $g_1$ dominates at threshold, however, cannot fit the data alone at higher energies. There are three parameters fitted: the $\Delta$ mass and the coupling constants $g_0$ and $g_1$, the smallest number of data points is nine.

\[ \tilde{V}_1 = \tilde{V}_1^r + c_0^{(\lambda,1)} + c_1^{(\lambda,1)} \omega^2, \tag{21} \]

where $m_\nu^2 = -(\tilde{V}_1^r(m_\nu) + c_0^{(\lambda,1)})/(c_1^{(\lambda,1)} \omega^2 + \partial_\omega \tilde{V}_1^r(m_\nu))$. This is normally referred to as a dynamically generated resonance.

IV. RESULTS

In order to test the quality of the model a whole set of fits were performed, excluding certain parts of the interaction, and including others. The least-square fit is determined by minimizing the $\chi^2$-square value. However, since there is a definite non-linear dependence of the scattering data on the coupling constants some care is required. As expected already above, fitting with both the contact interaction and the bare resonance is indeterminate, as both yield a similar effect and a flat minimum in parameter space for $\chi^2$-square.

However, as it turns out all the possible fits for the $\Delta$ resonance in the $P_{33}$ channel do much worse than a fit with a quasi-local interaction $n = \frac{1}{2}$. This interaction has the wrong dimensionality, however, produces the best fit with two parameters: the coupling constant and the renormalized resonance mass. This is also the only way to yield the right threshold behavior, if the fit includes data points beyond the resonance.

The various results are listed below and shown in Fig. 3. In the cases that the bare resonance is presence, i.e., $g_0 \neq 0$ or $g_1 \neq 0$, its mass is set to zero, i.e., $m_\Delta = 0$, the renormalization constants $c_0$ will account for the mass, with a finite renormalization. The energy for which $z = 0$ and the phase shift crosses 90° is for every fit within a few MeV of the $\Delta$ resonance at 1.232 GeV. The number of data points for the fits ranges between 30 and 50 depending on the range.

- For the range 1.1 GeV to 1.6 GeV, fitting only with nonzero $g_0$ yields $\chi^2$/# = 120 and $g_0 = 0.082$ and $c_0^{(0)} = 1.294$.
- For the range 1.1 GeV to 1.6 GeV, fitting only with nonzero $g_1$ yields $\chi^2$/# = 208 and $g_1 = 76000$, and $c_0^{(1)} = 10100$. The results do not depend significantly on $c_1^{(1)}$, which is subsequently set to zero. However, if we set $c_1^{(1)} = 1$, we find more appropriate numbers for $g_1 = 3.78$ and $c_0^{(1)} = 0.493$, since for this particular energy $\partial_\omega \tilde{W}_1^r(m_\Delta)$ gives a finite wave-function renormalization near unity, which makes the energy dependence of $z$ very weak.
- For the range 1.1 GeV to 1.6 GeV, fitting only with nonzero $\lambda$ yields $\chi^2$/# = 140, $\lambda = 2.31$, $c_0^{(\lambda,1)} =$
2.26, $c_1^{(\lambda,1)} = -1.47$, and $c_0^{(\lambda,0)} = 8.69$. The fit yields a better threshold behavior but a worse high-energy behavior compared to the $g_0 + g_1$ fit.

- For the range 1.1 GeV to 1.6 GeV, fitting with nonzero $g_0$ and $g_1$ yields $\chi^2/# = 82$, $g_0 = 0.060$, $g_1 = 0.79$, $c_1^{(0)} = 0.69$, and $c_1^{(1)} = 0.69$. Both renormalization constants take about half the resonance mass, changing this fraction does not change the $\chi^2$ value. Changing the range of the data set changes the results substantially, as can be seen in Figure 3. At threshold till 1.25 GeV the $g_1$ coupling fits the data well.

- For the range 1.1 GeV to 1.6 GeV, fitting only with nonzero $g_1\lambda$ yields $\chi^2/# = 56$, $g_1\lambda = 0.44$, and $c_0^{(\lambda)} = 1.227$. It is by far the best fit achieved.

- For the extended range 1.1 GeV to 1.8 GeV, the fit even improves, due to poorer quality data: $\chi^2/# = 45$ and $g_1\lambda = 0.44$, and $c_0^{(\lambda)} = 1.227$. The actual values of the coupling constant and the renormalization constant do not change. For the whole range the two parameters $g_1\lambda$ and $m_\Delta$ fit the data with a consistent coupling strength of $g_1\lambda = 47 \pm 2$, as can be seen in Figure 3. Fits with data below the inelastic threshold the $\chi^2/#$ ranges between 16 and 23.

- A further dramatic improvement is achieved with an additional resonance. For the range 1.1 GeV to 2.0 GeV, the fit yields: $\chi^2/# = 13$, $g_{a1\lambda} = 0.48$, $c_0^{(a,\lambda)} = 1.231$, $g_{a1\lambda} = 1.75$, and $c_0^{(b,\lambda)} = 3.74$. The second resonance is around 2.5-4.0 GeV, however, its position cannot be determined accurately from the data, since it lies outside the range.

Although, consistency with non-relativistic treatment, where the power $n$ in $W_n$ is directly related to the partial wave, of the $\Delta$ resonance would require that $g_0$ is zero, it is impossible to fit the $P_{33}$ data properly with that assumption. Attempts to make sensible fits with $g_1$ and two bare states failed. The accurate data at low energies forces a $g_1\lambda$ interaction in the case of two bare states. However, the $P_{31}$ and the $P_{13}$ channels do give results consistent with $g_0 = 0$. A number of fits were made, which are listed below, and show respectively in Fig. 3 and Fig. 6.

- For the $P_{31}$ a fit with $g_1$ only gives excellent results from the threshold up to the first structure in the data at 1.55 GeV. The $\chi$-square is comparable to unity.

- Similarly for the $P_{31}$ channel, a fit with $\lambda$ only gives good results too. However, the second solution of the quadratic equation starts to interfere with the physical solution at 1.6 GeV.

- Combining $g_1$ and $\lambda$ interactions does not give a significant improvement with respect to either fit.
FIG. 5: The phase shift in the $P_{31}$ scattering channel. The long-dashed line is a fit with $g_1$ only. The dot-dashed line is a fit with $\lambda$ only. The solid line is a fit with $g_0$, $g_1$, and $\lambda$, which does not yield a significant improvement at low energies.

- For the $P_{13}$ the data is rather coarse. Some structure appears at 1.4 GeV. Both $g_1$ on its own, as $\lambda$ on its own give a fit with a $\chi^2$-square comparable to unity. The shape of the fit depends on the range of the data points, however, as the $\chi^2$-square is comparable to unity, no conclusions can be drawn.
- Combining both fits for the $P_{13}$ data leads to an improvement as some of the structure at 1.7 GeV is reproduced, however, as the threshold behavior fails, it is considered an artificial improvement.

Since in the case of the $P_{31}$ and the $P_{13}$ channels there is no resonance that fixed a scale, there is considerable freedom to scale the coupling constants and renormalization constants and still produce the same results. However, especially for a non-zero $\lambda$, there seems to be no obvious redundant parameters in the model, which can be fixed a priori. The different coupling constants and renormalization constants are not quoted as they, on their own, have lost their significance.

V. CONCLUSIONS

A local field theory without cut-offs or form factors should fit the data at all energies. Deviations can only occur if other decay channels open, or higher-order terms in the interaction are important. Therefore we expect the data to be fitted well with our model below the two-pion production threshold, which is the case for the $P_{31}$ and the $P_{13}$ channels. In the same theory resonance states can be pushed out of the physical energy region, and resonances can be generated dynamically through a contact interaction. Eventually it is the quality of the fit that determines which is the likely process. For the $P_{33}$ channel the bare state gives a better results, for the $P_{31}$ and the $P_{13}$ channels both the contact interaction and the bare resonance state whose pole is pushed away from the real axis through a strong interaction do a good job fitting the data below 1.5 GeV.

From the results it is clear that one cannot simply add a resonance structure to a contact interaction incoherently. The effects of the contact interaction are not like what one would expect from perturbation theory. When the interaction is summed to all orders, the resonance dominate the results. Any other, perturbative, effects will be irrelevant for the phase shift around the resonance peak. In all the different versions of the model, where some interactions are switched off, it is impossible to create the effect of a rising phase shift beyond 180° after the $\Delta$ resonance, usually associated with a background, without distorting the resonance itself. It can only consistently be explained by a second resonance, which is found around 2.5-4.0 GeV.

Furthermore, from the data the $W_{\frac{1}{2}}$ coupling function prevails over the $W_0$ and the $W_1$ coupling functions, or even a combination of both with five fitting parameters, compared to the two of the $W_{\frac{1}{2}}$ coupling alone. The same model, with different values for the coupling constants and the renormalization constants, fit the low-energy $P_{13}$ and $P_{31}$ data well, which have no resonance structure. The fits give a model in with the
FIG. 6: The phase shift in the $P_{13}$ scattering channel. The dashed line is a fit with $g_1$ only. The long-dashed line is a fit with $\lambda$ only. The solid line is a fit with $g_0$, $g_1$, $\lambda$.

derivative coupling dominates, associated with the $g_1$ coupling constant.

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