Time Graph Pattern Mining for Network Analysis and Information Retrieval

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SUMMARY Graph pattern mining has played important roles in network analysis and information retrieval. However, temporal characteristics of networks have not been estimated sufficiently. We propose time graph pattern mining as a new concept of graph mining reflecting the temporal information of a network. We conduct two case studies of time graph pattern mining: extensively discussed topics on blog sites and a book recommendation network. Through examination of case studies, we ascertain that time graph pattern mining has numerous possibilities as a novel means for information retrieval and network analysis reflecting both structural and temporal characteristics.

key words: graph mining, time-series data, network analysis, information retrieval

1. Introduction

A graph pattern on a given set of graphs represents a subgraph that appears frequently on graphs in the set. Therefore, the graph pattern can be regarded as a structural characteristic of the set of graphs. Mining graph patterns has become one method of network analysis and information retrieval, especially on a web graph in which the nodes are web pages and where edges are links among them. The fundamental idea of trawling proposed by Kumar et al. [1] is that complete bipartite subgraphs are graph patterns appearing frequently on the link structures of web communities. Several studies have suggested that a clique, a subgraph of which every two nodes are connected by an edge, is also a graph pattern representing a community. Leskovec et al. [2] found that a specified type of graph patterns, designated as cascades, is observed for the growth of web graphs.

Previously proposed graph pattern mining applications reflect only the structural information of a network. However, several actual networks, such as web and social networks, are changing dynamically. Therefore, temporal information of a network is also expected to be important for analysis of the network and information retrieval using the network. In other words, graph pattern mining based solely on the structure cannot distinguish two actual graphs of the same structure, but with different growth rates, as shown in Fig. 1. For example, the speed of information propagation might be important, although existing graph pattern-based analysis cannot estimate how rapid that propagation is.

To use both structural and temporal characteristics fully, we propose time graph pattern mining as a novel framework of graph mining. A time graph pattern is a graph pattern on a set of time label graphs in which each node has a label that abstracts temporal information of the node. Consequently, we can analyze both structural and temporal information using time graph patterns. Once a set of time label graphs is available, our framework enables a previously proposed graph pattern mining algorithm, such as gSpan [3], [4] or GraphSig [5], to mine time graph patterns on the set without modification of the algorithm. Therefore, existing graph mining applications can be extended easily to time graph mining.

To construct a time label graph from a given graph, our framework requires the following two phases: (1) acquiring time-series data for the nodes of the given graph, and (2) labeling the nodes by simplifying the time-series data.

Phase (1) binds time-series data to nodes of the given graph. For example, we use the creation time of each node as a part of the time-series data for the nodes. We also present two approaches in Sect. 4.2 for obtaining additional time-series data. One uses the increase of the number of nodes of the graph. The other uses the counts of searches as data obtained from outside the graph. These approaches are developed for our two case studies conducted in Sects. 5 and 6, but they are applicable to various cases. We only treat time-series data for nodes throughout this paper. Using data for edges is anticipated as an interesting topic for future study.

We propose three labeling methods for the phase (2),...
including naive ones and a method using “inflection points” in the time-series data. The labeling method is important for time graph pattern mining. For example, one might consider the simplest labeling, which uses the node creation time itself as the node label. The labeling generates too many labels because each page usually has a different creation time. If the kinds of labels are too numerous, then a graph mining algorithm cannot enumerate graph patterns in practical time. However, if the kinds of labels of nodes are too few, labels cannot convey sufficient temporal information of the original graph. Our method using inflection points is specialized for the cases studied in Sects. 5 and 6, although ideas for the method are expected to be useful for establishing a labeling method for another case.

To examine how useful time graph pattern mining is for network analysis and information retrieval, we implement our framework for two practical cases by exploiting the approaches for obtaining time-series data and the labeling methods explained above. The first case deals with topics discussed extensively on only blog sites because they fairly represent the temporal characteristics of the web. The second case deals with a network representing recommendations of books in an on-line book store because the trend of books about a topic changes as time passes. Results of the case studies give evidence that time graph pattern mining has plenty of possibilities to serve as a novel means for information retrieval and network analysis reflecting both structural and temporal characteristics.

The remainder of this paper is organized as follows. Section 2 surveys related works reported in the literature. Section 3 explains the concept of time graph patterns, and describes our mining process. Sections 5 and 6 present two case studies of time graph pattern mining for network analysis and information retrieval. Section 7 presents important conclusions. This work builds on and significantly extends our preliminary work [6], [7].

2. Related Work

2.1 Graph Pattern Mining

The definition of a graph pattern is the following.

**Definition 1:** Let $S$ be a given set of graphs having a common label set $L$. Every node and edge of the graphs is assigned to at most one label in $L$. A graph pattern of which the frequency is at least $\min S up$ is defined as graph $G_p$ with the same label set $L$ such that there are at least $\min S up$ graphs in $S$ having a subgraph that is isomorphic to $G_p$.

Therefore, enumerating graph patterns includes the subgraph isomorphism problem, which is known as an NP-complete problem.

Several algorithms have been proposed for mining graph patterns. Yan and Han [3], [4], [8] proposed the gSpan algorithm, which enumerates graph patterns. The algorithm represents a subgraph by a code using a depth-first search order to enumerate patterns efficiently. A single subgraph might have several codes. Therefore, they define the minimum code for a subgraph as a canonical representation. The time complexity of gSpan is $O(kFS + rF)$, where $k$ stands for the number of occurrences of a frequent subgraph in a graph in the set, $F$ is the number of frequent subgraphs, $S$ represents the set size, and $r$ is the maximum number of duplicated codes of a frequent subgraph that grow from other minimum codes [8]. Sayan et al. [5] proposed the GraphSig algorithm. They introduced a $p$-value to measure how featureless a graph pattern is. GraphSig enumerates every graph pattern of which the frequency is at least $\min S up$ and the $p$-value is, at most, the user’s specified value. Han et al. [9] implemented a framework for comparing the efficiency of algorithms for the subgraph isomorphism problem. They conducted several experiments using the framework and several algorithms including gSpan.

Many proposals have used graph patterns for information retrieval and network analysis. We first explain information retrieval using graph pattern mining [1], [10]–[14]. Trawling proposed by Kumar et al. [1] for retrieving web communities used the fact that complete bipartite subgraphs are graph patterns appearing frequently in the link structures of web communities. Several studies [12] have produced the assertion that a clique, a subgraph of which every two nodes are connected by an edge, is also a graph pattern representing a community. Makino and Uno [10] proposed a method for enumerating maximal cliques efficiently. Uno [11] proposed a method for enumerating pseudo-cliques, which is a subgraph obtained by removing a few edges from a clique.

We turn to a discussion of network analysis using graph pattern mining [2], [15]. Leskovec et al. [2] reported that a specified type of graph pattern, named cascades, can be observed on the growth of web graphs. McGlohon et al. [15] classified blog sites using cascades. They convert each site into a feature vector representing how many subgraphs in the site match to each of cascades of different types. They conducted principal component analysis (PCA) on the matrix of which the rows are the feature vectors of sites. They ascertained that blog sites are classifiable into conservative ones and humorous ones using the first and the second principal components (PCs). Cascade graph patterns use the order of the creation times of nodes, but do not reflect the interval of the creation times. Therefore, cascades were unable to estimate temporal characteristics of the web sufficiently.

2.2 Network Analysis Using Time-Series Data

In recent years, several studies have examined temporal properties of a network. Nakajima et al. [16] identify influential bloggers by analyzing the time stamps of blog entries. The forest fire model proposed by Leskovec et al. [17] can simulate properties of the web: its degree distribution follows a power law [18] and its diameter shrinks over time. Kumar et al. [19] introduced a web time graph to track the burst of growth of communities. The studies explained above analyze the temporal variation of the number of pages statistically, whereas our study analyzes the time variation...
of graph structures.

Dynamic graph mining has been studied for dealing with temporal changes of graphs [20]. The number of edges might change in a dynamic graph, although the number of nodes does not change. A dynamic graph is suitable for analyzing a graph such that each edge is created and deleted repeatedly. However, on actual huge networks including the web, the number of nodes increases exponentially, whereas a number of edges usually remain unchanged long periods after they are created or deleted once. Consequently, few studies have analyzed the web using dynamic graphs.

3. Time Graph Pattern Mining Framework

We propose our framework of time graph pattern mining as follows.

I Collect a set of graphs.
II Construct time label graphs from the graphs using the following procedures.
   (1) Acquire time-series data for the nodes of each graph.
   (2) Assign labels to the nodes of each graph by simplifying the time-series data.
III Mine time graph patterns on the set of time label graphs using a frequent graph pattern mining algorithm such as gSpan or GraphSig.

Our framework begins from Step (I) for collecting a set of graphs. Implementing the collection of graphs depends on a case for network analysis or information retrieval. In the next section, we will introduce the two practical cases examined herein, and explain how we implement our framework for the two cases. The collection for the cases is explained in Sect. 4.1.

Definition 2: A time label graph is defined as a graph of which every node \( v \) has an integer label \( \ell(v) \) such that the labels \( \ell(u) \) and \( \ell(v) \) of nodes \( u \) and \( v \) satisfy \( \ell(u) < \ell(v) \) if and only if \( u \) was created before \( v \).

The construction of time label graphs, Step (II), consists of the two phases (1) and (2) as explained in Sect. 1. Details of our proposal for the phases (1) and (2) are explained respectively in Sect. 4.2 and Sect. 4.3.

Definition 3: A time graph pattern is defined as a graph pattern on a set of time label graphs.

Therefore, we can use a previously proposed graph pattern mining algorithm for our Step (III) with no modification. This is a benefit of our framework in terms of implementation. It is possible to implement our framework by modifying only phases (1) and (2) for the target case.

4. Implementation of the Proposed Framework

4.1 Our Two Cases and Collection of Graphs

We herein introduce our two cases and explain how we collect graph structures for each case.

Extensively Discussed Topics on Blog Sites. The first case is analysis of web graphs corresponding to topics discussed extensively in a short period on blog sites only. We implement Step (1) for the first case as follows. We first prepare some extensively discussed topics. We then obtain the top-\( k \) (we set \( k = 300 \)), web pages by inputting each topic to a search engine\(^1\). Consequently, we can construct a web graph with nodes representing the obtained pages. Edges are links among them.

Book-Recommendation Network. The second case is information retrieval of technical trends on a book-recommendation network of Amazon, the largest online book store in the world. Nodes on a book-recommendation network represent books for sale. A directed edge represents recommendations between a book and another book. That is, if many customers who bought book \( b_1 \) also buy book \( b_2 \), then a directed edge exists \((b_1, b_2)\) from book \( b_1 \) to book \( b_2 \). Edge \((b_2, b_1)\) might not exist even if edge \((b_1, b_2)\) exists.

Such a recommendation network is widely used in online shops and e-commerce services including Amazon. We implement Step (1) of our framework for the second case as follows. We first choose sample terms representing particular technologies. We then collect the top 100 books related to each term, the recommendations among the books, and the publication dates of books, using the Product Advertising API\(^{11}\) of Amazon. Consequently, we obtain a book-recommendation network for each term.

4.2 Acquisition Phase

Creation Time Data. We first consider using the time or date naturally corresponding to each node, if it is available. For example, the creation time of each node is regarded as a candidate of fundamental time-series data. For the first case dealing with web graphs, we use the indexed time of a web page by a web search engine as an approximate creation time of the page. For the second case dealing with book-recommendation networks, we regard the publication date of a book as the creation time of the node corresponding to the book. We define “the whole period” for both cases, which will be used below.

Definition 4: The whole period on a graph is the period from the date on which the first node was created to the date on which the last node was created.

Node Count Data. The additional time-series data for the first case are node count data, which represent the increase of the number of nodes during the whole period. In other words, these data clarify how many nodes are present at a specified date. These data can be computed from the creation time data explained above. Consequently, these data would be available not only for web graphs but also for various networks.

\(^1\)We use Google http://www.google.co.jp as a search engine throughout the paper.

\(^{11}\)affiliate.amazon.co.jp/gp/advertising/api/detail/main.html
The additional time-series data for the second case are the counts of searches of the keyword representing a topic. We use Google Trends as a resource for the search count. Google Trends tells us the degree to which the number of times of searching the keyword has changed over time. In other words, we can obtain data representing how many times the keyword has been sought up to a specified date.

Google Trends has sufficient data only for well-known topics that can be represented as a keyword and which have continued for long periods. Therefore, these data are available for a book-recommendation networks that are related to a single technical trend. However, Google Trends returns insufficient data for most topics that are discussed extensively on blog sites during a short period. Therefore, search count data are not available for the first case.

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4.3 The Labeling Phase

The labeling phase first divides the whole period on each graph into several sub-periods. Then it assigns a distinct label to the nodes created at each sub-period. We herein introduce two naive labeling methods and propose an inflection point method.

4.3.1 Naive Methods

We introduce the following two naive labeling methods: naive (A) and (B).

Naive (A) Dividing the whole period equally into \( n \) sub-periods, the nodes created during the \( i \)-th sub-period are labeled as \( i \), where \( i \in \{0, 1, \ldots, n - 1\} \).

Naive (B) Dividing the whole period into \( n \) sub-periods so that the number of nodes created during each period is equal, then the nodes created during the \( i \)-th sub-period are labeled as \( i \), where \( i \in \{0, 1, \ldots, n - 1\} \).

If \( n = 4 \) for the example portrayed in Fig. 2 in which the whole period is April 1–8, then naive (A) regards the period of Apr. 1–2 as the first period; the second period is Apr. 3–4. These naive methods are applicable only if the creation time data explained above are available.

Both naive (A) and (b) have a readily apparent deficit: the meaning of the \( i \)-th sub-period might differ among graphs in the given set. For example, the first period depicted in Fig. 2 corresponds to a period during which the topic spreads gradually, although the same period might correspond to an explosive spread for another topic. In other words, this method might be incapable of reflecting temporal characteristics in labels sufficiently.

4.3.2 Inflection Point Method

To dissolve the deficit of the naive labelings, we intend to assign the same label to periods of plural graphs if the periods have similar temporal characteristics such as how the topics spread. Therefore, we propose a labeling method, named “inflection point method”, which uses inflection points in time-series data as labeling boundaries.

The inflection point method is applicable both to the first and second cases. The method uses creation time data and data representing an increase of accumulated counts in the whole period on the graph, such as the node count data for the first case and search count data for the second case. Therefore, the method is applicable to another case in which similar data are available.

The inflection point method first detects the time points at which the number of accumulated counts for each graph changes drastically. It treats these points as inflection points for the graph. Then, the whole period on the graph is divided into sub-periods of which the boundaries are these inflection points. Subsequently, the method assigns label \( i \) to a node for a graph if the node is created during \( i \)-th sub-period for the graph. Figure 2 presents a graph labeled using the inflection point method. Two inflection points exist. Therefore, the whole period is divided into three periods. We explain details of inflection point calculation below.

We first divide the whole period on a graph into \( n \) virtual “time points.” We set \( n \) to 10 throughout the studies. We assume here that the minimum unit of a period is a day. Let the first date of the \( i \)-th time point be \( t_i \) for \( i = 0, 1, \ldots, n - 1 \), and let the next date of the last date of the whole period be \( t_n \). Consequently, the \( i \)-th time point consists of dates in \( [t_i, t_{i+1}) \) for \( i = 0, 1, \ldots, n - 1 \). Let \( v_i \) be the number of counts in the \( i \)-th time point. This value corresponds to the velocity of the increase of accumulated counts. The acceleration of the increase at the \( i \)-th time point is defined as \( a_i = v_i - v_{i-1} \) for \( i = 1, 2, \ldots, n - 1 \), where \( a_0 \) is not defined.

We then determine inflection points using \( a_i \). The increase of accumulated counts changes considerably if the absolute value of the acceleration is greater than a certain threshold. In the first case, we can use the number \( N \) of nodes in the graph to ascertain the threshold. We regard every \( i \)-th time point that meets \( |a_i| > \frac{N}{n} \) as an inflection point. In the second case, the search counts are independent of the graph size. Therefore, we use the mean \( \mu \) and standard deviation \( \sigma \) of all \( a_i \) to determine the threshold.

\(^{1}\)www.google.com/trends
\[ \mu = \frac{1}{n-1} \sum_{i=1}^{n-1} a_i, \quad \sigma^2 = \frac{1}{n-1} \sum_{i=1}^{n-1} (a_i - \mu)^2 \] (1)

Then, the \( i \)-th time point that meets \( \frac{|a_i - \mu|}{\sigma} > \theta \) is regarded as an inflection point. In other words, we regard the \( i \)-th time point as an inflection point when the ratio of its deviation to the standard deviation is greater than \( \theta \). The distribution of \( a_i \) approaches the standard distribution. A value in the standard distribution is regarded as uncommon if the difference between the value and the mean is greater than twice the standard deviation. Therefore, we set \( \theta \) to 2.

After the inflection points are obtained, we assign a label to each sub-period of which the boundaries are the first dates of the points, according to Definition 2. Let the first dates of the obtained \( m \) inflection points be \( t_{p1}, \cdots, t_{pm} \), for \( p_m < n \). Label 0 is assigned to nodes created during sub-period \([t_0, t_{p1})\), and label \( i \) is assigned to nodes created during sub-period \([t_{pi}, t_{pi+1})\) for \( 1 \leq i \leq m - 1 \); label \( m \) is assigned to the nodes created during sub-period \([t_{pm}, t_n)\).

In the example depicted in Fig. 2, two inflection points are obtained: Apr. 4 and Apr. 6. Consequently, the nodes created during Apr. 1–3 are labeled as 0. Similarly, the nodes created during Apr. 4–5 and Apr. 6–8 are labeled, respectively, as 1 and 2.

5. Case Study (1)

Using our time graph pattern mining framework, we analyze the spread of topics that are extensively discussed in a short period on blog sites. After preliminary investigation, we observed as explained in Sect. 5.1 that three types, named the primary source, trigger and summarizer, of blog sites play important roles in the spread. Therefore, we first mine time graph patterns on the set of time label graphs representing the propagation of such topics in Sect. 5.2; then we try to classify blog sites into the three types in Sect. 5.3 using the mined patterns.

5.1 Graph Set and Observation

We prepared the following seven topics.

- Discussion of the average cost for producing a web site.
- Discussion of the relation between a person’s academic background and intelligence.
- Discussion about hakenmura—makeshift shacks for temporary workers whose contracts have been terminated—in a Japanese park.
- Discussion related to a problem by which some people post illustrations tracing other illustrations in pixiv, which is a Japanese SNS specializing in posting illustrations.
- Discussion about yominige which means that a person reads another person’s diary in Mixi without commenting on the diary; Mixi is a popular SNS in Japan.
- Rumors about the release date of a software game “Final Fantasy 13.”

Finding these topics from the web was a difficult task because these topics were not discussed on major news sites. However, constructing a method of searching for such topics is anticipated as interesting future work.

The topics were collected during November 2008 – April 2010, and were selected from the fields of computer software, web and social networks, electronics, science, politics, and Japanese culture.

We investigated the selected topics and a number of similar topics, and observed that an interesting feature: “primary source,” “trigger,” and “summarizer” play the most important roles for discussion and propagation of the topics. We explain the feature using an example topic corresponding to a rumor about the announcement date of iPhone 4. Figure 3 presents some web pages reporting the rumor. First, page (a) reporting the topic appeared on a news site. Then, some sites began to create pages linking to page (a). Therefore, we call page (a) a primary source page for the topic. Subsequently, someone created page (b), which links to the primary source and which introduces the news of iPhone 4 with Japanese translations. Page (b) was linked by many Japanese blog sites and news sites. In other words, page (b) became a trigger of extensive discussion. Nakajima et al. [16] regard an important blogger who posts many such trigger pages as an agitator. After the topic was fairly discussed, someone created page (c), which links to many pages related to the topic, including the primary source page and the trigger page, to summarize the topic. We designate page (c) as a summarizer page.

Fig. 3 Some web pages about the rumor.

- Discussion related to the coding style of HTML on a blog community.
- Discussion about the coding style of HTML on a blog community.
- Discussion of the relation between a person’s academic background and intelligence.
- Discussion of the relation between a person’s academic background and intelligence.
- Discussion about hakenmura—makeshift shacks for temporary workers whose contracts have been terminated—in a Japanese park.
- Discussion related to a problem by which some people post illustrations tracing other illustrations in pixiv, which is a Japanese SNS specializing in posting illustrations.
- Discussion about yominige which means that a person reads another person’s diary in Mixi without commenting on the diary; Mixi is a popular SNS in Japan.
- Discussion about the release date of a software game “Final Fantasy 13.”

It is desired that the labeling of nodes reflect temporal characteristics of extensively discussed topics. The propagation process of such topics often describes a trajectory similar to the chart depicted in Fig. 2. The three roles explained above are characteristic of the process. Typically, such a topic spreads gradually for a while after the primary source is created. That is, few pages related to the topic are created during this period. An example of this period is the early period depicted in Fig. 2. The trigger page usually causes an explosive spread: numerous pages related to the topic are created in a few days. The growth period depicted in Fig. 2 corresponds to the days. After the days, new pages referring to the topic do not increase to any great degree:

- Discussion of the average cost for producing a web site.
- Discussion of the relation between a person’s academic background and intelligence.
- Discussion about hakenmura—makeshift shacks for temporary workers whose contracts have been terminated—in a Japanese park.
- Discussion related to a problem by which some people post illustrations tracing other illustrations in pixiv, which is a Japanese SNS specializing in posting illustrations.
- Discussion about yominige which means that a person reads another person’s diary in Mixi without commenting on the diary; Mixi is a popular SNS in Japan.
- Discussion about the release date of a software game “Final Fantasy 13.”
the propagation enters a stable period for which an example is also depicted in Fig. 2. Summarizer pages often appear in the stable period. Thereafter, if some progress or change is made on this topic, then a second or third explosion of activity occurs. In such a manner, the temporal characteristics of such topics can be represented by the increase of the number of nodes related to a topic. Our inflection point method reflects changes of the increase to labels. Therefore, the method can capture such temporal characteristics well.

5.2 Mined Time Graph Patterns

If the minimum frequency $\text{minSup}$ of mined patterns in Definition 1 is too large, then each mined maximal pattern becomes extremely small. That is, the nodes in each pattern become extremely few. Small patterns are usually found not only on the sample graphs but also almost all web graphs. Therefore, they are not useful for finding interesting characteristics. In contrast, if $\text{minSup}$ is too small, then mined patterns can depend too much on the sample graphs. Consequently, the patterns would not be sufficiently versatile to extensively discussed topics other than the sample topics. In addition, if $\text{minSup}$ is too small, then the number of patterns would be too large to compute them in practical time. We conducted preliminary experiments to mine patterns by setting $\text{minSup}$ to various values. It was difficult to discover versatile patterns when $\text{minSup}$ was 5 or greater, although we were unable to obtain results in a particular time when $\text{minSup}$ was 3 or fewer. We also tried varying the number of sample graphs used for mining. It was impossible to obtain results in a particular time if the number was greater than seven. Therefore, we set $\text{minSup}$ to 4 for both gSpan and GraphSig.

We obtain 107 patterns using gSpan, and 55 patterns using GraphSig setting $p$-value to 0.1, from the time label graphs labeled by the inflection point method. The respective computation times of gSpan and GraphSig are shorter than ten minutes on our Windows Vista PC with Intel Core i7 920 and 4GB memory, which proves that our labeling method enables existing graph mining algorithms to obtain time graph patterns in practical time.

5.3 Network Analysis Using the Patterns

We attempt to classify automatically blog sites dealing with extensively discussed topics into the three types introduced into Sect. 5.1: primary sources, triggers, and summarizers. A single site can include two or three types of pages. Therefore, we regard a site as a primary source if the site contains more primary source pages than the pages of each of the other two types. Similarly, we can determine whether a site is a trigger or a summarizer.

The automatic classification process comprises the following three steps.

1. Converting each site into a feature vector; our dataset consists of these feature vectors.

2. Partitioning the dataset half-and-half randomly: a training dataset and a test dataset.

3. Generating a classifier by learning the training dataset using SVM. This learning process involves five-fold cross-validation, which is conducting an analysis of one subset of the training dataset and validating the analysis of the other subset.

4. Predicting the test dataset using the classifier.

Our dataset comprises 164 total sites. We manually classified the sites into 44 primary source sites, 55 trigger sites, and 62 summarizer sites. We determined the class of a page in a blog site by a majority decision of three voluntary students who majors in informatics. If the class answered by each of the three was different from the other two, such a case occupies only less than 4% of all, then one of the author made the final decision. The Fleiss’s Kappa of the classification of the three students is 0.635.

The first step of the automatic classification process is conversion of each site into a feature vector. For this purpose, we extend a technique using cascades introduced by McGlohon et al. [15]. Their technique matches a subgraph to a cascade employing structural information only. However, we must match a subgraph of a site to a time graph pattern. Therefore, our technique becomes more complicated than theirs.

Each graph in the graph set used in the mining process corresponds to a distinct topic. The labels of a graph reflect temporal information that is characteristic to the topic. Therefore, an integer label has a distinct meaning on graph $G_1$ from that on graph $G_2$. That is, our labelings might assign the same label to nodes $v_1 \in G_1$ and $v_2 \in G_2$ even if they were created in different periods. However, our dataset for analysis consists of sites including several topics. Therefore, we must determine which label matches a page in a site to convert the site into a feature vector.

We explain our matching technique using the example depicted in Fig. 4. A web graph includes six pages $a, b, c, d, e, f, g$ of three web sites 1, 2 and 3, and two mined time graph patterns $p_1$ and $p_2$. We first find subgraphs with structures matching those of respective patterns. Here we refer neither to labels of patterns nor to the creation time of
nodes in the web graph. We specifically examine the link structures of them only. For example, we can find two subgraphs matching the structure of \( p_2 \), say \([a, d, e]\) and \([c, f, g]\). We then check whether the creation time of nodes of each subgraph is consistent with the labels of the pattern. Label \( \ell_1 \) corresponds to an older period than that corresponding to label \( \ell_2 \) if \( \ell_1 < \ell_2 \). For example, the order of the creation time of \([a, d, e]\) is \((e, a, d)\), which is consistent with labels of \( p_2 \) by assuming that \( e \) corresponds to label 0, \( a \) corresponds label 1, and \( d \) corresponds to label 2. Similarly, \((g, q, c)\) is consistent with the labels of \( p_2 \) by assuming that \( q \) corresponds to label 0, \( e \) corresponds to label 1, and \( f \) corresponds to label 2. If two subgraphs share a node, then the label of the node in one of the subgraphs might be different from that in the other subgraph. Using such an assumption, which enables the nodes of a subgraph to be labeled, we can count the labels matching pages in each site. For subgraphs \([a, d, e]\) and \([c, f, g]\) for pattern \( p_2 \), site 1 has two pages labeled as 1; site 2 has a page labeled 0 and two pages labeled as 2; site 3 has only a page labeled as 0. We repeat this operation for each pattern.

We use the number of labels matching pages in each site to convert the site into a feature vector. Letting \( L \) be the set of used integer labels \( \{0, 1, \ldots, |L| - 1\} \), \( P \) be the set of mined patterns, and \( N \) be the set of sites, then \( |N| \) feature vectors are obtained, each with \(|L| \times |P| \) dimensions. Letting \( \mathbf{e}' = (e'_1, e'_2, \ldots, e'_{|L|\times|P|}) \) be the feature vector of the \( i \)-th site, for \( 1 \leq i \leq |N| \), then the pair of the \( j \)-th pattern and label \( \ell \), for \( 1 \leq j \leq |P| \) and \( 0 \leq \ell < |L| \), corresponds to element \( e'_{(j-1)|L|+\ell} \). The value of \( e'_{(j-1)|L|+\ell} \) is defined as the number of pages in site \( i \) matching a node with label \( \ell \) in pattern \( j \). If \(|L| = 3 \) in the example explained above, then \( e'_{(j-1)|L|+\ell} = e'_3 \) for the pair of \( j = 2 \) and \( \ell = 0 \) is set to 0. Similarly, \( e'_3 \) for \( j = 2 \) and \( \ell = 1 \) is 2, and \( e'_2 \) for \( j = 2 \) and \( \ell = 2 \) is 0; \( e'_3 \) for \( j = 2 \) and \( \ell = 0 \) is 1, \( e'_2 \) for \( j = 2 \) and \( \ell = 1 \) is 0, and \( e'_3 \) for \( j = 2 \) and \( \ell = 3 \) is 1.

We calculate how accurately the classifier predicts the test data. A classifier judges which of two roles a vector the test data belong to. We obtain a classifier for each of the three possible pairs of two roles. We conduct prediction using each classifier. The precision and recall of the classification are shown in the following three tables: 1, 2, and 3. Similarly, the precision and recall of the remaining two results are presented in Tables 2 and 3.

### Table 1 SVM on primary sources (A) and triggers (B).

|          | precision | recall |
|----------|-----------|--------|
| (A) sources | 68.2%     | 62.5%  |
| (B) triggers | 83.3%     | 74.1%  |

### Table 2 SVM on primary sources (A) and summarizers (C).

|          | precision | recall |
|----------|-----------|--------|
| (A) sources | 64.0%     | 76.2%  |
| (C) summarizers | 83.3%     | 73.5%  |

### Table 3 SVM on triggers (B) and summarizers (C).

|          | precision | recall |
|----------|-----------|--------|
| (B) triggers | 66.7%     | 76.9%  |
| (C) summarizers | 79.3%     | 69.7%  |

We compare the precision of the prediction using the inflection point method and the two naive methods. The results are presented in Table 4. The precision using the inflection point method is higher than that using the naive methods. Therefore, our idea using inflection points as labeling boundaries is effective for estimating temporal information, as compared with the simple ideas used in the naive methods.

The results explained above confirmed that time graph pattern mining can reflect both temporal and structural characteristics of the three classes of blog sites. Therefore, our framework of the time graph pattern mining can be an effective means for conducting network analysis.

### 6. Case Study (2)

We here study books about technical trends by application of our time graph pattern mining framework to book-recommendation networks on Amazon. We first mine time graph patterns on a set of book-recommendation networks of books about technologies described in Sect. 6.1. We then apply principal component analysis (PCA) to a book-recommendation network using the mined patterns in Sect. 6.1.1. Finally, we try to retrieve books of introduction to technologies employing the result of PCA in Sect. 6.2.

#### 6.1 Graph Set, Observation, and Mined Patterns

We explain how we construct our dataset of book-recommendation networks. We first choose sample terms representing particular technologies of the following two groups: group (A) technologies have grown in popularity in April 2010; group (B) technologies which are growing at April 2010. The group (A) comprises eight technologies: Android, Business Intelligence, Business Process Management, Cloud computing, e-Book, International Financial Reporting Standards (IFRS), Twitter, and Smart grid. These groups are expected to be useful for analyzing types of the growth of technologies. We identify the group of a technol-

### Table 4 Comparison of labeling methods: Precision of the classifications.

| method          | primary source | trigger | summarizer |
|-----------------|----------------|---------|-----------|
| inflection       | 66.1%          | 67.9%   | 81.3%     |
| naive (A)       | 58.4%          | 67.8%   | 50.0%     |
| naive (B)       | 61.9%          | 55.0%   | 58.4%     |
ogy according to some technology reports including Gartner’s hype cycles [21]. They specifically describe current technical trends and the future. We then construct a sample graph set for each group.

We assign a label to every node using the inflection point method. This method uses the number of queries on a search engine. Therefore, it reflects people’s interests and trendy words well. Applying the inflection point method to these time-series data including the publication dates of books, we obtain a time label graph for each term. We observe that books published on around the peak time of a technology are usually labeled as 2 or 3; larger number labels are assigned to books published during later periods of the technology, whereas small number labels correspond to early periods. We apply GraphSig to the sets of the time label graphs for the terms belonging to each group by setting \( \minSup \) to 4 and \( p\text{-value} \) to 0.5. Consequently, we obtain a set of time graph patterns from each group. Mined patterns from the set of group (A) include a number of nodes labeled with large numbers such as 3, 4, or larger. However, mined patterns from group (B) include a number of nodes labeled with small numbers such as 0, 1, or 2.

### 6.1.1 Network Analysis

We analyze a book-recommendation network for term “data mining” using the obtained time graph patterns. We first convert books into feature vectors in a way similar to but simpler than the one presented in Sect. 5.3. Then, we conduct principal component analysis (PCA) of the matrix of which the rows are the feature vectors of books.

Let \( G \) be the time label graph of a book-recommendation network for analysis. Let \( P \) be the set of all patterns mined from both groups in Sect. 6.1, \( L \) be the set of used integer labels \([0, 1, \ldots, |L|] \) for \( G \) and \( P \), and \( V \) be the set of books corresponding to the nodes of \( G \). Then, \( |V| \) feature vectors are obtained, each with \(|L| \times |P| \) dimensions. Let \( e^j_i \) be the feature vector of the \( i \)-th book, for \( 1 \leq i \leq |V| \), and \( label(i) \) be the label of the book on \( G \). The pair of the \( j \)-th pattern and label \( \ell \), for \( 1 \leq j \leq |P| \) and \( 0 \leq \ell < |L| \), corresponds to element \( e^j_i (j-1)|L|+\ell \). If \( \ell \neq label(i) \), then the value of \( e^j_i (j-1)|L|+\ell \) is set to 0. Otherwise, the value of \( e^j_i (j-1)|L|+\ell \) is set to the number of subgraphs that contain the \( i \)-th book and which match the \( j \)-th pattern.

We apply PCA to the matrix with rows consisting of feature vectors obtained from the book-recommendation network for “data mining.” We plot the result of PCA on a scatter diagram using the first and second PCs in Fig. 5. We classify almost all books into the following six classes representing the content: introduction, Excel (data mining using Excel), theoretical data mining, practical data mining (e.g. for marketing), statistics, and machine learning. These classes are also depicted in Fig. 5. They are apparently clustered to some degree. The books of introduction, Excel and machine learning have scores that are close to zero with respect to the second PC. However, the books of theoretical data mining and statistics have positive scores with respect to the first PC, but negative scores with respect to the second PC. The practical books have positive scores with respect to both PCs.

The following scenario might explain why the six classes are clustered by PCA. In the early period, mainly books of statistics and theoretical data mining were published. In the middle period, the number of books related to introduction, Excel and machine learning increased. In the most recent period, practical books have been more popular. Actually, a positive correlation is found between the score with respect to the second PC and the integer of the label. The result of PCA presented here provides evidence that the time graph pattern mining can be an interesting tool for network analysis, reflecting temporal information.

We also compare the result of PCA employing patterns obtained by the naive methods and GraphSig without any time label. Figure 6 depicts a scatter diagram for the naive method (B). This figure plots several books classified into “Excel” and “Introduction” according to the first PC and the second PC. We observe that these two classes of books are mingled, not separated as the result depicted in Fig. 5 for the inflection point method. We here plot these two classes of books only, because the plot of all the six classes is far mingled and hard to distinguish points. We note that the result for the naive method (A) is worse than the naive method (B). Similarly, Fig. 7 depicts a scatter diagram obtained without any time label. The two classes of books are not separated as well as the naive methods. Consequently, we can apply only the result of PCA using the inflection point method to information retrieval explained below.
6.2 Information Retrieval Using the Patterns

We consider applying the result of the analysis above to retrieval of useful information from book-recommendation networks. As an example, we attempt to retrieve books of introduction to several technical trends. We first investigate time graph patterns having characteristics of books of introduction to “data mining.” We then enumerate every subgraph matching one of the patterns on the time label graphs constructed for the technical trends in Sect. 6.1. Results show that books of introduction to a part of the trends are obtained with high precision.

We obtain two characteristics for use in retrieval. First, it is apparent that the books presenting an introduction to “data mining” have scores that are close to zero with respect to the second PC. In addition, all books having such scores have integer label 2 or 3. Second, Fig. 5 indicates that these books have large negative scores with respect to the first PC. Third, the books are “initial points” of some of the time graph pattern mined in this Section. Figure 8 presents the patterns.

We designate a node on a pattern is an initial node if the node has no outgoing edges but many incoming edges. We mark the initial nodes with the symbol “∗” on the patterns in Fig. 8. The following scenario might explain why introductory books are initial nodes. A user who bought a difficult book about a technology would have bought a book of introduction to the technology with high probability. Therefore, the introductory book can collect many incoming edges on a book-recommendation network. However, few users who bought the introductory book would buy a more difficult book. Consequently, the introductory book has no outgoing edges on the pattern.

We consider that we can retrieve book of introduction to a technology from the book-recommendation network for the term corresponding to the technology using the following approach. We first enumerate every book with label 2 or 3, with feature vectors having a score of at most -10 with respect to the first PC. We then regard the book as an introductory book if it is contained in a subgraph matched to a mined pattern and it is an initial node of the pattern. We apply this approach to all technologies described in Sect. 6.1. Results show seven books for three technologies. All seven books are actually introductory books for beginners: we introduce the books below.

1. Ajax

We find two books\textsuperscript{†}: one is an introduction to Ajax for Java programmers. The other, written in plain expressions, includes many illustrations.

2. Cloud Computing

We find only a single book\textsuperscript{††}. Many reviewers on Amazon recommend this book for beginners because it is understandable.

3. IFRS (International Financial Reporting Standards)

Four books\textsuperscript{†††} are mined. Two books are actually introductions to IFRS, one of which uses simple sentences and illustrations. Another is written for beginner accountants.

While we have retrieved several introductory books with high precision, many introductory books were not found using our approach. One reason why the approach has low recall might be strictness of the matching of patterns to subgraphs. A subgraph is not regarded as matching a specified pattern even if the subgraph is quite similar but not equal to the pattern. The results of our retrieval indicate that the time graph pattern mining can reflect several temporal and structural characteristics of introductory books. Therefore, our framework of the time graph pattern mining can become a useful tool for information retrieval.

7. Conclusion

We proposed “time graph pattern mining,” a novel framework of graph mining that can estimate both temporal and structural information of a network. We proposed several

\textsuperscript{†}“Java kaihatsusha no tame no Ajax jissen kaihatsu nyumon (an introduction to practical development using Ajax for Java developers)”, “Zukai de wakaru Ajax no subete (Illustrated explanation for understanding all about Ajax)”

\textsuperscript{††}“Kuraudo no shogeki – IT shijou saidai no souzouteki hakai ga hajimatta (The impact of the cloud – it begins the largest creative destruction in the history of IT)”

\textsuperscript{†††}“Sugu wakaru, kuwashiku wakaru IFRS (quick and detailed understanding of IFRS)”, “Kanzen hikaku: kokusai kaikei kijun to nihon kijun (complete comparison: IFRS and Japanese standard)”, “Zukai irasuto ni yoru IFRS kokusai kaikei kijun nyumon (An introduction to IFRS with illustrative explanation)”, “IFRS no keiri nyumon (an introduction to accounting of IFRS)”
ideas for applying our framework to practical cases: we presented time-series data of three kinds. Then we established two naive methods and the inflection time point method for constructing time label graphs. Based on results of the two case studies, we confirmed that the time graph pattern mining can be a promising tool for network analysis and information retrieval.

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