Features of electromagnetic wave propagation in a longitudinally magnetized gyrotropic elliptic waveguide

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Abstract. In scientific literature the electromagnetic waves propagation in gyrotropic elliptic waveguides has not been studied enough due to: 1) difficulties of deriving Helmholtz equations for gyrotropic elliptic waveguides in arbitrary magnetization directions, 2) difficulties of solving Helmholtz equations and deriving the dispersion equations, 3) difficulties of solving the dispersion equations themselves. In this work, for the first time, dispersion equations for longitudinally magnetized gyrotropic elliptic waveguides were solved. Basing on the solutions results, the dependencies of the constant propagation: 1) on the strength and direction of the magnetizing field, 2) on the waveguide ellipticity at a constant ferrite magnetization were investigated. Studies have revealed previously unknown features inherent only to the electromagnetic waves in such waveguides propagation and showed that: 1) with the ferrite magnetization increase ≈3,7 times the polarization plane rotation angle increases by ≈5,4 times; 2) the gyrotropic elliptic waveguide polarization plane rotation angle is greater than that of a similar round waveguide. The results show the main advantage of gyrotropic elliptic guide systems over circular ones consists in the possibilities of varying both external and geometric parameters of gyrotropic elliptic guide systems and obtaining significant phase raids, which in turn allows developing more efficient phase shifters.

1. Introduction

To study the structure of the electromagnetic field in longitudinally magnetized gyrotropic elliptic waveguides and the various characteristics of electromagnetic waves propagating in such waveguides, it is necessary to set and solve the Dirichlet problem for the corresponding Helmholtz equations. In work [1], such a Dirichlet problem was solved and dispersion equations were obtained, but for practical application in the problems of studying the propagation of electromagnetic waves in longitudinally magnetized gyrotropic elliptic waveguides, it is necessary to solve the dispersion equations themselves. The difficulties of solving the dispersion equations are related to the calculations, at a given interval, of all eigenvalues and their corresponding roots of ordinary and modified Mathieu functions [2, 3], which are included in the dispersion equations.

The purpose of the article is to numerically solve the dispersion equations corresponding to Helmholtz equations for the longitudinally magnetized gyrotropic elliptic waveguide [1] and to analyze some characteristics of hybrid electromagnetic waves propagating in the waveguide obtained as a result of the solution.
2. Dirichlet problem for Helmholtz equations of hybrid waves

The Dirichlet problem for the Helmholtz equations of EN-ordinary and HE-unusual waves corresponding to longitudinally magnetized gyrotrropic elliptic waveguides is [1]

\[
\begin{align*}
\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial \phi^2} + e^2 d^2 \left( \omega^2 \epsilon \mu_\perp - \gamma^2 \right) E_z - j e^2 d^2 \gamma \omega \mu_\parallel k \mu E_z = 0, \\
\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial \phi^2} + e^2 d^2 \left( \omega^2 \epsilon \mu_\parallel - \mu_\perp \gamma^2 \right) H_z + j e^2 d^2 \gamma \omega \mu_\perp k \mu E_z = 0,
\end{align*}
\]

under the Dirichlet condition for an electric field at the boundary of an infinitely conductive longitudinally magnetized elliptic waveguide

\[
E_z \mid_{z=0} = E_\phi \mid_{z=0} = 0. \tag{2}
\]

Here \((\xi, \phi, z)\) – coordinates of an elliptic system, \(E_x\) and \(E_z\) – components of an electric field, \(H_z\) – longitudinal component of magnetic field, \(e\) – focal distance of an ellipse, \(d = \sqrt{c h^2 \xi - \cos^2 \phi} = \sqrt{0.5 (c h^2 \xi - \cos 2\phi)}\) – geometrical parameter, \(\omega\) – cyclic frequency, \(\epsilon\) – absolute dielectric permeability of ferrite, \(\gamma\) – propagation constant, \(j\) – imaginary number, 

\[
k = \mu_0 \frac{\omega \epsilon_\mu m}{\omega^2 - \omega_0^2}, \quad \mu = \mu_0 - \mu_0 \frac{\omega \epsilon_\mu m}{\omega^2 - \omega_0^2}, \quad \mu_\perp = \mu_\parallel, \quad \mu_\parallel = 4 \pi \cdot 10^{-7} \text{ H/m} \quad \text{– magnetic constant,}
\]

\[
\omega_0 = \mu_0 Y H_0 \quad \text{– frequency of ferromagnetic resonance,}
\]

\[
Y = 1.76 \cdot 10^{11} \text{ C/kg} \quad \text{– gyromagnetic ratio,}
\]

\[
H_0 \quad \text{– intensity of constant magnetic field,}
\]

\[
\omega_m = \mu_0 Y M_0, \quad M_0 \quad \text{– magnetization of ferrite saturation.}
\]

Note that the system of differential equations (1) describes propagation of hybrid electromagnetic waves arising from gyrotropy of the propagation region [4-6].

The solution to the Dirichlet problem (1) and (2) is the dispersion equation for even waves [1]

\[
\begin{align*}
\left[ -k_0^2 - \gamma^2 - \frac{4q_1}{e^2} \left( \frac{C_\mu^e}{e^2} \frac{C_\epsilon^e}{e^2} \right) \frac{C_\epsilon^e}{e^2} \left( \frac{C_\mu^e}{e^2} \frac{C_\epsilon^e}{e^2} \right) \right] \left[ k_0^2 - \gamma^2 - \frac{4q_1}{e^2} \frac{C_\mu^e}{e^2} \left( \frac{C_\mu^e}{e^2} \frac{C_\epsilon^e}{e^2} \right) \right] \\
+ j \frac{a^2}{\omega^2 c k} \left[ k_0^2 - \gamma^2 - \frac{4q_1}{e^2} \left( \frac{C_\mu^e}{e^2} \frac{C_\mu^e}{e^2} \right) \frac{C_\mu^e}{e^2} \left( \frac{C_\mu^e}{e^2} \frac{C_\mu^e}{e^2} \right) \right] \left[ k_0^2 - \gamma^2 - \frac{4q_1}{e^2} \frac{C_\mu^e}{e^2} \left( \frac{C_\mu^e}{e^2} \frac{C_\mu^e}{e^2} \right) \right] \\
+ \frac{\gamma^2}{\mu} \left[ \frac{C_\mu^e}{e^2} \frac{C_\epsilon^e}{e^2} \left( \frac{C_\mu^e}{e^2} \frac{C_\epsilon^e}{e^2} \right) \right] \left[ k_0^2 - \gamma^2 - \frac{4q_1}{e^2} \frac{C_\mu^e}{e^2} \left( \frac{C_\mu^e}{e^2} \frac{C_\mu^e}{e^2} \right) \right] = 0.
\end{align*}
\]

Here \(C_\mu^e(\xi_0, q_1)\), \(C_\epsilon^e(\xi_0, q_1)\) – attached (modified) Mathieu functions of the 1st kind (with integer index) and their derivatives, \(c_\epsilon(\phi, q_1)\), \(c_\epsilon(\phi, q_1)\) – ordinary Mathieu functions of the 1st kind of integer order \(m\) and their derivatives.

After the following replacement in (3)
The dispersion equation for odd waves is given by:

\[
\left[ -\left( k_e^2 - \gamma^2 - \frac{4q_e}{e^2}\right) + \frac{4q_e}{e^2} Se_m(\xi_0, q_e) + \left( k_e^2 - \gamma^2 - \frac{4q_e}{e^2}\right) \right] \frac{Se_m(\xi_0, q_e)}{Se_m(\xi_0, q_e) - \frac{4q_e}{e^2}} + \frac{\gamma^2}{\mu} \frac{Se_m(\phi, q_e)}{Se_m(\phi, q_e) - \frac{4q_e}{e^2}} = 0,
\]

where \( Se(\xi_0, q_e) \) and \( Se(\xi_0, q_e) \) are odd attached (modified) Mathieu functions of the 1st genus (with integer index) and their derivatives, 

\( se(\phi, q_e) \) and \( se(\phi, q_e) \) are odd ordinary Mathieu functions of the 1st genus of the integer order \( m \) and their derivative.

### 3. Analysis of dispersion equations solution results

When studying various characteristics of hybrid electromagnetic waves propagating in a longitudinally magnetized gyrotropic elliptic waveguide and the spatial structure of the electromagnetic field, it is necessary to solve the dispersion equations (3) and (4) with subsequent analysis of the results of decisions.

To solve the dispersion equations (3) and (4), it is necessary to find all eigenvalues and their corresponding roots of ordinary and modified Mathieu functions at a given interval. Determining the eigenvalues and their respective roots of both Mathieu functions is difficult. To overcome these difficulties, we developed a computer program for numerically calculating all the roots of ordinary, modified Mathieu functions and the roots of dispersion equations over a given interval based on the standard Maple software package [7]. Further, with known eigenvalues and corresponding roots of both Mathieu functions, formulas (3) and (4) calculate the characteristics of electromagnetic waves required for analysis and plot their dependencies \( \gamma = f(l, k, \mu, H_0, M_0) \), where \( l \) – geometric parameters of the guide system (\( e \) and \( d \)); \( k, \mu \) – components of the ferrite magnetic permeability tensor; \( M_0 \) – ferrite magnetization; \( H_0 \) – intensity of the external magnetizing magnetic field.

Usually, in the analysis of dispersion equations, the dispersion characteristics of the guide system are numerically obtained at different geometrical parameters of the waveguide, magnetizing field intensity, ferrite filling magnetization, and the cut-off frequency (in this case \( \gamma = 0 \)), the most effective modes (harmonics) are determined, which allows to build epures of field distribution inside the waveguide.

In practice, the frequency \( \omega \) is selected so that the desired number of modes exists in the absence of a magnetizing field. Often, a frequency equal to the smallest is selected so that only the main mode exists in the guide system. By fixing the frequency \( \omega \), the strength of the magnetizing field \( H_0 \) is changed within reasonable limits according to a predetermined algorithm. At a certain intensity of the longitudinal external permanent magnetizing magnetic field \( H_0 \) there is a ferrimagnetic resonance consisting in resonance absorption of energy of the electromagnetic field by ferrite (gyrotropic medium), which occurs at \( \omega = \omega_0 \). At very high values of external magnetizing field intensity \( H_0 \) constant propagation of different types of waves (modes) tend to constant propagation of corresponding types of waves for isotropic case [5].

It is also known [8] that the appearance of wave types (modes) depends on the eccentricity of the
ellipse and this is a feature of elliptical wave guides, which makes it possible to choose the mode of operation of such waveguides.

The results of solving the dispersion equations (3) and (4) of the longitudinally magnetized gyrotropic elliptical waveguide allow the following studies to be carried out:

1) dependence of constant propagation $\gamma$ on intensity of magnetizing field for different magnetizations of ferrite;

2) dependence of constant propagation $\gamma$ on degree of ellipticity of waveguide at constant magnetization of ferrite.

Note that in carrying out the above studies, the size of the guide system is selected so that in the absence of an external magnetic field, several types of waves can propagate in the guide system.

3.1. Dependence on magnetizing field strength

Let gyrotropic elliptical waveguides have the same eccentricities, but different magnetizations of ferrite saturation.

Figures 1–4 show the results of solving the dispersion equations (3), (4) at $\varphi = 45^0$ and $\omega = 6.28 \cdot 10^{10}$ Hz.

**Figure 1.** The dependence of the constant propagation of HE-waves on intensity of the magnetizing field at the length of the semi-major axis of the ellipse $a = 0.016$ m, $E = 0.5$ and $\omega_m = 0.15\omega$.

Horizontal dashed lines show modes in the absence of a magnetic field: $c\ H_{11}, s\ H_{11}, c\ H_{12}, s\ H_{12}$, where the lower indices "c," "s" mean even and odd modes. The graphs of the modes $c\ HE^\pm_{12}$ and $s\ HE^\pm_{12}$ coincide by $\approx 99.5\%$. The graphs of the modes $s\ HE^+_{12}$ and $s\ HE^-_{12}$ coincide by $\approx 99.9\%$.

Brown is the $c\ HE^+_{11}$ fashion graph. Vertical dotted line corresponds to ferromagnetic resonance.

The graphs in figures 1–4 show the dependence of the constant propagation of hybrid waves on the intensity of the magnetizing field at constant eccentricity of the ellipse $E$ and for different frequencies $\omega_M$ related to the magnetization of ferrite. In all figures, normalized magnetization of external permanent magnetic field $\omega_h / \omega$ is indicated along $X$ axis, and normalized propagation constant
$\gamma_z / k_z$ is indicated along $Y$ axis. Here $\gamma_z$ is the propagation constant (calculated numerically from the dispersion equations), $k_z$ is the wave number in the limitless non-magnetized ferrite medium.

**Figure 2.** The dependence of the constant propagation of EH-waves on intensity of the magnetizing field at the length of the semi-major axis of the ellipse $a = 0.016$ m, $E = 0.5$ and $\omega_m = 0.15\omega$.

Horizontal dashed lines show modes in the absence of a magnetic field: $c E_{11}, s E_{11}, c E_{12}, s E_{12}$ . The graphs of the modes $c EH_{12}^r$ and $c EH_{12}^l$ coincide by ≈ 99.6%. Gray color – fashion graph $s EH_{11}^r$. Green color – fashion graph $s EH_{11}^l$. Vertical dotted line corresponds to ferromagnetic resonance.

It follows from the graphs of figures 1-4 that any electromagnetic wave propagating in a gyrotropic elliptical waveguide along a constant magnetic field disintegrates into two independent waves having different constant propagations, for example, a wave $c HE_{11}^r$ disintegrates into waves $c HE_{11}^r$ and $c HE_{11}^l$. The index "+" indicates the wave of right rotation, and "-" indicates the wave of left rotation. As these waves propagate, a phase shift of [6]:

$$\varphi = \left( \frac{\gamma_z^+ - \gamma_z^-}{2} \right) Z,$$

where $\gamma_z^+$, $\gamma_z^-$ are wave propagation constants with right and left rotations respectively, $Z$ is the distance.

By deliberately changing the intensity of the external longitudinal magnetic field, it is possible to adjust the phase shifts of the left and right rotation waves.

From the results obtained in figures 1-4, it follows that the appearance of modes depends on the magnetization of the ferrite $\omega_m$. In practice, the frequency $\omega$ is selected so that, in the absence of a magnetizing field, the desired number of modes exists. Often enough, such a frequency is selected so that only the basic mode that exists at the lowest frequency exists in the guide system. By fixing,
frequency $\omega$ is varied within reasonable limits of magnetizing field strength according to a predetermined algorithm. At a certain intensity of the longitudinal external constant magnetizing magnetic field, a ferromagnetic resonance occurs, which consists in resonant absorption by ferrite (gyrotropic medium) of the energy of the electromagnetic field.

Figure 3. The dependence of constant propagation of HE-waves on intensity of magnetizing field at length of semi-major axis of ellipse $a=0.016$ m, $E=0.5$ and $\omega_m = 0.55 \omega$. Horizontal dashed lines show modes in the absence of a magnetic field: $cH_{11}, cH_{12}, cH_{11}, cH_{12}$. The graphs of the modes $sH_{11}$ and $sH_{12}$ coincide by $\approx 99.8\%$. Brown color – fashion graph $cHE_{11}$. Red color – fashion graph $cHE_{12}$. Blue color – fashion graph $cHE_{12}$. Black color – fashion graph $cHE_{12}$. Vertical dotted line corresponds to ferromagnetic resonance.

It is known that this phenomenon occurs at $\omega=\omega_0$. At very large values of intensity of external magnetizing field $H_0$, propagation constants of different types of waves (modes) tend to propagation constants of corresponding types of waves for isotropic case [6].

The analysis of the graphs in figures 1-4 shows that with an increase in the magnetization of ferrite, the phase difference for left and right rotation modes (for example, $cHE_{11}^+$ and $cHE_{11}^-$ modes) increases at the same magnetization value of the constant external magnetic field $H_0$. This is particularly noticeable in the region of weak $H_0$ outer fields corresponding to the $\omega_0 / \omega < 1$ condition, i.e., in the region of frequency change prior to resonance. This is an important practical advantage of gyrotropic elliptical guide systems over circular ones, since with the same geometric parameters of the guide system it is possible to obtain more significant phase runs, therefore, to develop more efficient phase shifters.
Figure 4. The dependence of constant propagation of EH-waves on intensity of magnetizing field at length of semi-major axis of ellipse \( a=0.016 \text{ m} \), \( E=0.5 \) and \( \omega_m = 0.55 \omega \). Horizontal dashed lines show modes in the absence of a magnetic field: \( cE_{11}, sE_{11}, cE_{12} \). Violet color – fashion graph \( sEH_{11} \). Green color – fashion graph \( sEH_{11} \). Vertical dotted line corresponds to ferromagnetic resonance.

For example, we calculate the rotation angle of the polarization plane for every 1 cm for an elliptical waveguide with a long half-axis length of 1.6 cm and \( E_i = 0.5 \). The characteristics of ferrite and electromagnetic wave are as follows: frequency of electromagnetic wave \( \omega = 2\pi \cdot 10^{10} \), magnetization of ferrite \( \omega_m / \omega = 0.15 \), relative permittivity of ferrite \( \varepsilon_{\text{ferrite}} = 5 \), magnetization of external constant magnetic field \( \omega_0 / \omega = 0.2 \).

The angle of rotation of the polarisation plane is calculated using the known equation (5). Since one mode is usually sought, consider the \( cHE_{11} \) mode using, for all \( k_x = \omega\sqrt{\varepsilon_{\text{ferrite}}} \mu_{\text{ferrite}} / \omega_0 = 468.178 \text{ m}^{-1} \) cases. Next, from these graphs in figure 1 and 2, we get the angle of rotation of the polarization plane \( \phi_1 = 0.2667 \text{ rad} = 15.29^\circ \) for every 1 cm (Z=0.01m) for the elliptical waveguide with eccentricity \( E=0.5 \).

For the same waveguide at magnetization of ferrite \( \omega_m / \omega = 0.55 \), using the data in the graphs of figures 3 and 4, the angle of rotation of the polarization plane \( \phi_2 = 1.4396 \text{ rad} = 82.5^\circ \) for every 1 cm (Z = 0.01m) is obtained. From the received results it is visible that at increase in magnetization of ferrite \( \approx 3.7 \) times the angle of rotation of the plane of polarization increases by \( \approx 5.4 \) times.

3.2. Dependence on ellipticity

Let gyrotropic elliptical waveguides have different eccentricities, but the same magnetization of ferrite saturation.

Figure 5 shows the result obtained by solving the dispersion equations (3) and (4) with eccentricity \( E = 0.02 \) (almost a circle) and \( \phi = 45^\circ \), i.e a graph of the dependence of the
The propagation constant on the strength of the magnetizing field for a circular gyrotropic waveguide during longitudinal magnetization is shown.

**Figure 5.** Dependence of constant propagation on intensity of magnetizing field at length of semi-major axis of ellipse $a = 0.016$ m, $E = 0.02$ (almost circumference) and $\omega_m = 0.15 \omega$. Horizontal dashed lines show modes in the absence of a magnetic field: $H_{11}, E_{11}, H_{12}, E_{12}$. The graphs of the modes $EH_{12}^-$ and $EH_{12}^+$ coincide by $\approx 99.8\%$. Red color – fashion graph $HE_{11}^-$. Blue color – fashion graph $HE_{11}^+$. Brown color – fashion graph $EH_{11}^-$. Violet color – fashion graph $EH_{11}^+$. Vertical dotted line corresponds to ferromagnetic resonance.

Figures 6 and 7 show the results of the solution of the dispersion equations (3) and (4) at $E = 0.75$ and $\varphi = 45^\circ$. In all the X-axis figures, the magnetization of the external permanent magnetic field $\omega_b / \omega$ is deposited, and in the Y-axis – the normalized propagation constant $\gamma_z / k_z$. Here $\gamma_z$ is the propagation constant (calculated numerically from dispersion equations), $k_z$ is the wave number in a limitless non-magnetized ferrite medium.

Thus, figures 5-7 show the dependence of the propagation constant of hybrid waves on the strength of the magnetizing field and the cross-sectional shape (circular or elliptical) of the waveguide under the condition of the frequency constancy, which is related to the magnetization of ferrite.

From the data in figures 5-7, it follows that the electromagnetic wave propagating in the gyrotropic circular and elliptic waveguides along the constant magnetic field breaks down into two independent waves having different propagation constants.

Further analysis of the results in figures 5-7 shows that the appearance of modes depends on the shape of the cross-section of the waveguide (circle $E = 0$ or ellipse $E = 0.75$), that is, with an increase in the eccentricity of the ellipse, the phase difference for modes with left and right rotation (for example, for modes $cHE_{11}^+$ and $cHE_{11}^-$) increases with constant magnetization of the external magnetic field $H_0$ and magnetization of the ferrite $M_0$. This is especially noticeable in the area of weak external $H_0$ fields, i.e. when the condition $\omega_b / \omega < 1$ is met. From here follows the main advantage of gyrotropic elliptical guide systems over round ones, since with the same external parameters it is possible to obtain in practice more significant phase runs, which can give a significant effect in the development of phase shifters. Therefore, by selecting the degree of ellipticity at the same value of the large axis of the ellipse, the range of phase runs in phase shifters can be selected within a
reasonable range. In the frequency variation region where the $\omega_0 / \omega > 1$ condition is met, the phase difference for left and right rotation modes is less noticeable for different eccentricities.

**Figure 6.** The dependence of constant propagation of HE-waves on intensity of magnetizing field at length of semi-major axis of ellipse $a=0.016 \, \text{m}$, $E=0.75$ and $\omega_m = 0.15 \omega$. Horizontal dashed lines show modes in the absence of a magnetic field: $cH_{11}, sH_{11}, cH_{12}$, where the lower indices "c," "s" mean even and odd modes. Vertical dotted line corresponds to ferromagnetic resonance.

**Figure 7.** The dependence of constant propagation of EH-waves on intensity of magnetizing field at length of semi-major axis of ellipse $a=0.016 \, \text{m}$, $E=0.75$ and $\omega_m = 0.15 \omega$. Horizontal dashed lines...
show modes in the absence of a magnetic field: $c E_{11,S} E_{11}$. Vertical dotted line corresponds to ferromagnetic resonance.

As an example, using formula (5), the rotation angles of the polarisation planes were calculated for each 1 cm: a) for a circular waveguide with a radius of 1.6 cm, b) an elliptical waveguide with a length of a large half-axis of 1.6 cm, and $E = 0.75$.

The characteristics of ferrite and electromagnetic wave were as follows: $\omega = 2\pi \cdot 10^{10} \text{Hz}$, ferrite magnetization $\omega_m/\omega = 0.15$, relative permittivity of ferrite $\varepsilon_{\text{err}} = 5$, external permanent magnetic field magnetization $\omega_h/\omega = 0.2$. The $k_2 = \omega \sqrt{\varepsilon_{\text{err}} \mu_0} = 468,178 \text{m}^{-1}$ value is the same for all cases.

Using the graphs in figures 5-7 and formula (5), the following rotation angles of the polarization plane for every 1 cm are obtained:

- for the circular waveguide, $\varphi_1 = 14.89^\circ$,
- for the elliptical waveguide, $\varphi_2 = 15.83^\circ$.

From the results obtained, it can be seen that when moving from a circular waveguide to an elliptical angle of rotation of the polarization plane increases.

4. Conclusions

The Dirichlet problem (1) and (2) for the Helmholtz equations of the longitudinally magnetized gyrotropic elliptic waveguide was solved in [1]. Solutions to this Dirichlet problem are the dispersion equations (3) and (4), which also needed to be solved. The main results of this work are as follows:

- Solved dispersion equations (3) and (4) for Helmholtz equations of hybrid waves propagating in a longitudinally magnetized gyrotropic elliptical waveguide;
- Graphs of constant propagation dependence on constant magnetizing longitudinal field intensity at constant eccentricity of ellipse and various magnetisations of ferrite are constructed;
- Analysis of phase incursions in a longitudinally magnetized gyrotropic elliptical waveguide was carried out, showing that with increasing magnetization of ferrite $=3.7$ times the angle of rotation of the polarization plane increases by $\approx 5.4$ times;
- Dependencies of constant propagation of hybrid waves on intensity of magnetizing external field and degree of ellipticity of gyrotropic waveguides at constant magnetization of ferrite are shown;
- Comparison of phase runs for longitudinally magnetized gyrotropic round and elliptical waveguides has been made and it has been shown that during the transition from a circular waveguide to an elliptical waveguide, the rotation angle of the polarization plane increases.

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