FROM CONSTITUENT QUARKS TO HADRONS
IN COURSE OF NUCLEAR MATTER EXPANSION

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Abstract

The updated three-phase concept of nuclear matter evolution in course of cooling down - from the phase of quark-gluon plasma (QGP) through the intermediate phase allowing for massive constituent quarks $Q$ (valons), pions and kaons ($Q\pi K$) to the phase of hadronic matter (H) - is exploited for the treatment of relative hadronic yields in the central region of heavy ion collisions. The most attention is paid to the description of the $Q\pi K$-phase which is argued to be a gaseous one and lasts until the valonic spacing approaches the confinement radius (at the temperature $T_H \approx (110 \pm 5)$ MeV), when the valons start fusing to be locked, in the end, within the hadrons. The hadronic yields emerged from thermal treatment of $Q\pi K$-phase and simple combinatorial approach to the hadronization process are shown to fit the available experimental data from AGS, SPS and RHIC quite well. This provides an alternative insight into the real origin of the observed relative hadronic yields which is (to a considerable extent) free of the well known puzzle inherent in some conventional models where the early chemical freeze-out is assumed: namely, why the gaseous thermal approach to actually tightly packed (even overlapping) hadrons seems workable? Many predictions for the other hadronic yields which could be observed at these machines as well as at LHC are given.

*This work is supported in part by the Russian Foundation for Basic Researches, grant 00-02-17250 and by Scientific School grant 00-15-96696
Introduction

The conventional way of treatment of hadron production in the central rapidity region of heavy ion collisions at high energies incorporates a QGP as the short initial phase of nuclear matter (just after the nuclei encounter each other) which then undergoes the chiral breaking transition into hot and dense hadronic matter, the latter evolving somehow into experimentally observed free hadrons. The relative yields of different species of these hadrons depend on the equation of state (EoS) of the nuclear matter they are originated from. It has been noticed \[2\] that the SPS data on the hadronic yields are quite compatible with the assumption about the early chemical freeze-out in an ideal gas of crucially modified in-medium hadrons (their effective radii has being assumed to be as small as \(\simeq 0.3\) fm only!) at the temperature \(T_{ch} \simeq 170\) MeV \[\dagger\] (which is rather close to the chiral breaking temperature itself, \(T_c \simeq 200\) MeV). The theoretical observation has been also made \[3\] that thermal description of such a type could be spread to elucidate the AGS and SIS data as well under the assumption that chemical freeze-out temperature \(T_{ch}\) was varied properly along with baryonic chemical potential of fireball (expanding nuclear medium), being about 110 MeV and 60 MeV for AGS and SIS, respectively. As a result, a suggestion about possible existence of low temperature QGP has been put forward and then discussed in some more detail \[3, 4\]. Both attributes of this approach - crucially modified in-medium hadrons (but still survived!) and low temperature chiral transition - seem rather controversial and mysterious. That is why a feeling of dissatisfaction remains and looking for some more consistent approach seems quite reasonable.

In this connection, the attention seems to be called reasonably to the rather old-fashioned notion of valons \(Q\) linked properly to the corresponding current quarks - first of all, to the valons \(Q(q), q \equiv u, d\ (m_{Q(u,d)} \equiv m_{Q(q)} \simeq 330\) MeV), and \(Q(s)\ (m_{Q(s)} \simeq 480\) MeV) - which was widely and fruitfully exploited before QCD was developed as a consistent field theory that had no valons as its inalienable entities. However, till now, QCD remains actually a quite workable theory of hard processes, but it suffers from the lack of even qualitative results for the processes at low and intermediate energies, i.e., just for those ones which proceed within nuclear matter below the chiral breaking temperature \(T_c\). Thus, maybe, embedding the valons unambiguously into the body of QCD is still

\[\dagger\]The straightforward estimate shows that no nearly ideal gas of slightly modified hadrons could exist at the relevant particle (hadron) densities, since the normal hadron wave functions would overlap substantially.
waiting for its turn, although the known endeavors met no success \[1\]. Qualitative physical motivation in favor of possibly essential role of valonic mass scale and of an intermediate phase, allowing for valons and separating QGP and hadronic ones, can be found in ref.s \[3, 4, 5\]. It is worthy to mention that experimentally observed (in AA collisions) significant excess in low-mass dilepton \((e^+e^-)\) yield (as compared to pA ones) could be described quite well \[9\] in the framework of an approach allowing for Q\(\pi\)K-phase.

Recently the general idea has been put forward \[10\] which could provide an insight into the physical reasons underlying the unified thermal description of yields of different hadron species observed in heavy ion collisions. In the present paper, we show in more detail how could valons help in treatment of relative hadronic rates, getting rid of the above curiosity inherent in the conventional approaches. The theory is confronted to the available results of AGS, SPS and RHIC, and many other ratios of hadronic yields are predicted (for LHC as well).

**General Description of the Approach**

Most probably, just after chiral symmetry breaking, the nuclear matter is subjected for dual description \[9\]: it can be treated either as highly compressed ”hadronic liquid” or as a state, in which the most of hadron species can not survive and the dominant degrees of freedom are not still hadrons but valons instead \[7\]. Of course, one can choose either. However, for taking the first way, one is supposed to know, at least, the corresponding EoS. Who is aware of it? The basic point of the suggested approach is that the valons which would be produced in course of chiral symmetry breaking are to be treated reasonably as a gas from the very beginning (in contrast to the hadrons!) due to their it really small size \[11\], \(r \simeq 0.3\) fm. Indeed, even at \(T = 170\) MeV the particle density within the ”ideal” Boltzmann valonic gas would be about

\[
\frac{12 T^3}{\pi^2} [2 \left( \frac{m_{Q(q)}}{T} \right)^2 K_2 \left( \frac{m_{Q(q)}}{T} \right) \cosh \left( \frac{\mu_{Q(q)}}{T} \right) + \left( \frac{m_{Q(s)}}{T} \right)^2 K_2 \left( \frac{m_{Q(s)}}{T} \right) \cosh \left( \frac{\mu_{Q(s)}}{T} \right) ] \simeq 1 \text{ fm}^{-3}
\]

where \(\mu_{Q(q)}\) and \(\mu_{Q(s)}\) are typical for heavy ion collisions \(Q(q)\)- and \(Q(s)\)-valon chemical potentials, respectively \[3\], and thus the ”valonic bodies” themselves occupy about 10% of the total volume only. This rather optimistic estimate noticeably worsens after taking into

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\[2\] Presence of massive valons is just indicative for chiral symmetry breaking.

\[3\] We do not distinguish here between chemical potentials of \(Q(u)\)- and \(Q(d)\)-valons caused by difference in proton and neutron content of heavy colliding nuclei, although at low energies and large values of \(\mu_q/T\) (SIS, AGS) it may result in a noticeable effect.
account some balancing fraction of the "large-sized" pions and kaons which are produced inevitably as the chemical equilibrium sets in \[4\], see Fig. 1. Fortunately, this hadronic fraction never exceeds 25\% (see below), and thus this three-component $Q\pi K$ state is still to be treated reasonably as a gas \[5\], although quasi-ideality of this gas becomes somewhat more questionable. That is why below we proceed as far as possible with a more general consideration based on the chemical kinetics only, and then confront the results to what one can obtain in the selfconsistent ideal gas approximation. We will see \textit{a posteriori} that the results emerged from these two ways of doing are quite compatible.

Now, the general pattern of evolution of the hot nuclear fireball produced in course of heavy ion collisions looks as follows. While expanding and cooling down from $T_c$ to $T_H$, the nuclear matter is gradually enriched with pions and kaons and impoverished with valons. Situation changes dramatically when mean spacing of valons scales up to become about the confinement radius. As a result, gradually increasing color screening length approaches the critical value and the dominant degrees of freedom become no longer valons (since they are getting not free) but the hadrons themselves, i.e., the bulk of hadrons is to build up. This is a phase transition, if the hadronization proceeds near the same temperature $T \simeq T_H$; otherwise, this is a phase crossover. The lattice calculations suggest that peculiarities of phase transition and even its occurrence itself is regulated by the value of chemical potential (or net baryonic density) within nuclear matter. We do not touch here this delicate question and concentrate on the pattern linked to a certain phase transition. This pattern is shown below to face no obvious contradictions with the available observables within the nowday accuracy of the data (see the Table). The relevant diagrams of hadron production are depicted in Fig. 2.

\textbf{Calculations and Results}

At the hadronization phase transition ($T = T_H$), all hadron species are produced, generally, in the same manner. Unlike the $Q\pi K$-phase, the correlation length remains large comparing to the mean particle spacing (even "infinite", if it is the second order one) under the whole this phase transition long. Therefore, multiparticle interactions...
dominate at this stage. One can assume reasonably also that now, once being created, all
the hadrons survive because the confinement starts working.

Thus, the final (observable) pions (kaons) are produced in the two-fold way: first,
there are equilibrium pions (kaons) mentioned above (those, which are "stored up" in
course of $Q\pi K$-phase evolution, before overall hadronization, see Fig. 1), and second,
there are pions coming from $Q(q)\bar{Q}(q) \to \pi + X$ reactions (kaons coming similarly from
$Q(q)\bar{Q}(s)$ and $Q(s)\bar{Q}(q)$ reactions) just at $T = T_H$. The latter are produced at the stage
of hadronization only, just like the other hadron species, see Fig. 2. The number of the
former pions reads obviously

$$n_\pi \simeq \frac{bn}{1 - b}$$

(1)

where $n$ is the total number of color particles within fireball and $b$ is the pionic fraction
which is calculated in the Appendix (see eq.s A(1)-A(6)). The numerical value of $b$
turns out to be rather stable for SPS, RHIC and LHC: $b \simeq 0.22$, whereas for AGS it is
considerably lower: $b \simeq 0.13$. Making use of the diagrams shown in Fig. 1(b), 1(c), the
number of former kaons is estimated quite similarly (see eq. A(7)) to be

$$n_{K^+} \simeq n_{K^{0}} \simeq \frac{4n_\pi + 3n_{Q(q)}}{6(4n_\pi + 3n_{\bar{Q}(q)})} n_{Q(s)}$$

(2)

where $n_{\bar{Q}(s)}$ is the number of ($\bar{Q}(s)$ valons. The number of $K^-$ ($\bar{K}^{0}$) is obtained by
replacement here valons $\leftrightarrow$ antivalons.

As for the latter pions, one can express their number by tracing a valon $Q(q)$ (antivalon
$\bar{Q}(q)$) within hadronizing nuclear medium. While moving along the mean free path, it
coalesces with an antivalon (valon) with the probability $n_{Q(q)}/n$ ($n_{\bar{Q}(q)}/n$), the total
rate of such collisions thus being $n_{Q(q)}n_{\bar{Q}(q)}/n$. In each collision, $\pi$-meson plus $X$ are
produced, "$X" allowing possibly for a number $j$ of pions too, see Fig. 2 (the latter is
also true for the processes of production of other hadrons). Thus, accounting eq. (1), the
total negative pion yield (which is assumed to be equal to 1/3 of the total rate of pions)
reads

$$N_{\pi^-} \simeq \frac{1}{3} \left[ \frac{bn}{1 - b} + \frac{(1 + \langle j \rangle)n_{Q(q)}n_{\bar{Q}(q)}}{n} \right] + \frac{\langle j \rangle}{3} [N_B + N_{\bar{B}} + (N_K - n_K) + ...]$$

(3)

$^6$Of course, at the hadronization stage, any valonic interaction which could result in producing a
hadron is reasonably assumed to do produce it. Besides, the valon-hadronic interaction is insignificant
at this stage.
where $N_B (N\bar{B})$ and $N_K$ are the baryon (antibaryon) and total $K$-meson yields, respectively, and $\langle j \rangle$ is the mean value of $j$ which is easily estimated to be $0 \leq j \leq 1$ because of the phase space limitation\(^7\) (points stand for a number of small terms which are meaningless within the accuracy of both the theory and the data). Proceeding similarly and making use of eq. (2), one obtains for $K$-meson yield:

$$N_{K^+} \approx N_{K^0} \approx \frac{4n_\pi + 3n_{Q(q)}}{6(4n_\pi + 3n_{Q(q)})} n_{Q(s)} + \frac{n_{Q(q)}n_{Q(s)}}{2n},$$

$$N_{K^0_S} \approx N_{K^0_L} \approx 0.5(N_{K^0} + N_{\bar{K}^0}),$$

$N_{K^-}$ and $N_{\bar{K}^0}$ being obtained as before (see eq. (2)). The yields of some other mesons are easily estimated by means of the similar combinatorial and/or relevant cross-section consideration. For example,

$$N_{\phi} \approx \frac{n_{Q(s)}n_{Q(s)}}{n}$$

As for $\eta$-meson, the relative rate $N_\eta/N_{\pi^0}$ can be immediately calculated by comparing the corresponding cross sections, since two channels involved ($Q(q)\bar{Q}(q) \rightarrow \pi\pi^0$ and $Q(q)\bar{Q}(q) \rightarrow \pi\eta$, see Fig. 2(b)) directly compete with each other. In addition to distinction in masses of these mesons which results in about (2÷3)-time contraction (at the relevant energies/temperatures) of the $\pi\eta$ final phase space (as compared to the $\pi\pi^0$ one), one should take into account that no $\rho$-meson intermediate state is allowed in $\pi\pm\eta$ channel. Being integrated over the energy, the corresponding excess in the $\pi\pi$ yield results in an extra factor about 4÷5, thus making finally $N_\eta/N_{\pi^0} \simeq (0.08\div0.1)$. Unfortunately, the flexibility of this estimate is rather large.

Some additional peculiarities are to be accounted for getting the formulae for the baryon rates. First, the factor $(1 + 2n_{Q(q)}^2/n^2)^{-1}$ (or $(1 + 2n_{\bar{Q}(q)}^2/n^2)^{-1}$) should enter the formulae to eliminate the double counting of a ”propagating” (anti)valon and a pair of the identical (anti)valons it encounters (actually, this factor may be significant for (anti)nucleon production only); and second, one should keep in mind that what is taken experimentally as $\Lambda(\bar{\Lambda})$ is actually $\Lambda(\bar{\Lambda}) + \Sigma^0(\bar{\Sigma})$ (since $\Sigma^0(\bar{\Sigma})$ decays into $\Lambda(\bar{\Lambda}) + \gamma$ in $\approx 10^{-19}\text{s}$), and that the final momentum phase spaces for $\Lambda$ and $\Sigma$ production differ from each other (compare to the above remark on $\eta/\pi^0$-yield ratio) as well as the combinatorial factors for production of the neutral and charged hyperons (see the diagram h in Fig. 2).\(^8\)

\(^7\)The results appear to be rather insensitive to the value of $\langle j \rangle$ within this domain.

\(^8\)As a result, the rate of $\Lambda(\bar{\Lambda})$ production turns out to be almost twice as large as that of both $\Sigma^\pm(\Sigma^\pm)$-meson species.
Thus, we arrive at

\[ N_p \simeq N_n \simeq \frac{n^3_Q(q)}{2n^2(1 + 2n^2_Q/q^2/n^2)}, \quad N_{\bar{p}} \simeq N_{\bar{n}} \simeq \frac{n^3_{Q(\bar{q})}}{2n^2(1 + 2n^2_{Q(\bar{q})}/n^2)} \]  

(7)

\[ N_\Lambda + N_{\Sigma^+} + N_{\Sigma^-} \simeq \frac{n_Q(q)n^2_Q(q)}{n^2} \simeq 1,6N_\Lambda \]  

(8)

\[ N\bar{\Lambda} + N\bar{\Sigma}^+ + N\bar{\Sigma}^- \simeq \frac{n_{\bar{Q}(\bar{q})}n^2_{\bar{Q}(\bar{q})}}{n^2} \simeq 1,6N_{\bar{\Lambda}} \]  

(9)

\[ N_{\Xi^0} \simeq N_{\Xi^-} \simeq \frac{n_Q(q)n^2_Q(s)}{2n^2}, \quad N_{\bar{\Xi}^0} \simeq N_{\bar{\Xi}^+} \simeq \frac{n_{\bar{Q}(\bar{q})}n^2_{\bar{Q}(\bar{q})}}{2n^2} \]  

(10)

\[ N_{\Omega^-} \simeq \frac{n^2_{Q(s)}}{n^2}, \quad N_{\Omega^+} \simeq \frac{n^2_{\bar{Q}(\bar{q})}}{n^2}, \]  

(11)

and so on.

The comparison of the above predictions for baryon and antibaryon rates with the experimental data asks for a more careful discussion (especially, for antibaryons at AGS). Theoretical consideration of antibaryon production is a subtle point of any model because it implies the details of the evolution dynamics of hadronizing nuclear matter. In contrast to mesons, which, once being produced in course of hadronization, preserve their identity (undergo predominantly elastic scattering only), the (anti)baryons start to annihilate right away after production; the lower is \( T_H \), the larger is the annihilation cross section. A rather high density of surrounding baryons (large chemical potential at AGS) and very large annihilation cross section, \( \sigma_{p\bar{p}} \simeq 100 \text{ mb} \), favor to very strong absorption which crucially diminish the antibaryon number before hadrons scatter away and stop interacting. Then, what remains passes through the detectors. Of course, this effect is missed in the above formulae. Since any theoretical estimate of this absorption could be only qualitative and is far from being reliable, we try to draw it directly from the experimental data. We call the attention to the fact [13] that measured at AGS \( \bar{p}/\pi^+ \) yield in low participant number (\( \simeq 20 \)) collisions is about twice as large as it is in the high participant number (\( \simeq 80 \div 100 \)) ones. Meanwhile, the antiproton production is a rare process and thus it is expected to rise up proportionally to the number of participants squared. If so, then one would expect that relative \( \bar{p}/\pi^+ \) yield should increase linearly. That is why one can deduce that annihilation eats at AGS about up to 90% of the initially produced antiprotons. At SPS, the relevant ratio of yields is [14] about 0.5 and thus annihilation eats (if any) not more than 50% of antiprotons. This is due to both substantial increase of the pionic fraction amongst the producing hadrons and decrease of baryonic chemical potential (both "dilute"
the baryonic content and make lower the probability of baryon-antibaryon collisions). At RHIC and LHC, absorption of (anti)baryons seems getting, most probably, out of the game.

Below, the results are presented in the Table (see the column $th_1$) and Fig. 3. The ratios $\frac{n_{Q(q)}}{n_{Q(q)}}$, $\frac{n_{Q(s)}}{n_{Q(s)}}$, $\frac{n_{Q(q)}}{n_{Q(s)}}$ and value of $\langle j \rangle$ were found by minimization of the sum of squared deviations:

$$\chi^2 \simeq \sum_{i=0}^{i=k} \left(1 - \frac{a_{ih}}{a_{exp}}\right)^2$$

were $k$ is the number of different processes under consideration.

Note, that till now no specific assumptions about the properties of $Q\pi K$ gas were made. The assumption of its quasi-ideality becomes unavoidable, when we try to extract the hadronization temperature $T_H$ from the ideal-gas formulae:

$$\frac{n_{Q(q)}}{n_{Q(q)}} = \exp(2\mu_{Q(q)}/T), \quad \frac{n_{Q(s)}}{n_{Q(s)}} = \exp(2\mu_{Q(s)}/T)$$

and

$$\frac{n_{Q(q)}}{n_{s}} \simeq \left(\frac{m_{Q(q)}}{m_{Q(s)}}\right)^{3/2} \exp\left[\frac{(m_{Q(s)} - m_{Q(q)})}{T}\right] \exp\left[\frac{(\mu_{Q(q)} - \mu_{Q(s)})}{T}\right]$$

This way of doing results in nearly the same temperature, $T_H \simeq (110 \pm 5)$ MeV for AGS, SPS and RHIC. Predictions for LHC given in the Table are obtained for the same value of $T_H$ at the point $\mu_i = \mu = 0$, see Fig. 3.

Discussion and Concluding Remarks

It is worth mentioning that the above results for SPS are quite similar to those which one can carelessly obtain in the framework of the ideal gas approximation for description of $Q\pi K$-phase instead using the detailed balancing equations (compare the columns $th_1$ and $th_2$ in the Table. This suggests that either the valon-pion-kaon gas is really quasi-ideal or the relative content of different components in this gas is a crude characteristic (this seems quite plausible) which is rather insensitive to its fine tuning. Anyway, the compatibility of the results seems indicative for the validity of the approach itself.

The value of hadronization temperature, $T_H \simeq 110$ Mev, we have found is quite close to the value of thermal freeze-out temperature which has been estimated \[15\] for SPS and RHIC to be $T_f \simeq (100 \div 110)$ MeV by fitting the transverse momentum spectra within the framework of the hydrodynamical model. If so, then fireball evolution from $T_H$ to $T_f$ takes a rather short time (of some few Fermi), and one can qualitatively understand, why
the antiproton absorption in course and just after hadronization is crucially significant for AGS only (when the relative content of the nucleons is very high). In particular, that is why we do not need to look for a special mechanism of levelling the absorption in contrast to what should be necessarily invented \[16\] in the schemes with early chemical freeze-out.

Still one point which deserves mentioning is the approximate coincidence of the hadronization temperatures for AGS, SPS and RHIC, although the relevant chemical potentials differ substantially and thus the relevant EoS’s are expected to be different as well. Actually, we can not insist on the validity of this result: $\chi^2$ minimization is not the meaningful test of the theory for AGS and RHIC, since the number of parameters to be varied is only slightly less, than the number of the necessary data from these accelerators available at present. In addition, one should not take seriously too low values of $\chi^2$ because of a rather insufficient accuracy of the data themselves.

Possible variation of hadronization temperature along with valonic chemical potential (and thus with colliding ion energy) is closely related to the very interesting hypothesis \[7\] that color, namely valonic, deconfinement is reasonably expected to take place already at very low interaction energies (ion kinetic energy being about 300 MeV only!) because the nucleonic density is only about twice as large as the nucleus one, and one should make a rather weak effort for getting valons unable to ”recognize their original nucleons”. If it is the case, then, in particular, a thermal treatment of hadron production should be applicable even at such a low energies. This would provide the direct indication on validity of the very notion of valon itself. Of course, a high-luminosity machine equipped with high-precision detectors is needed to try looking for the relevant manifestations.

Thus, our conclusion is that the suggested approach provides quite successive treatment of the available data for the hadron yields as well as for the low-mass dilepton production\[9\] in heavy ion collisions. At the same time, it is free of the crucial inconsistency inherent in some conventional approaches, although some significant questions still remain which are unanswered.

This is our pleasure to express our deep gratitude to E.L. Feinberg for the invaluable discussions and permanent interest to the work.
APPENDIX

1. Being averaged over the particle distributions, the detailed balancing equation reads:

\[ \nu_{Q(q)}(T) \overline{\Omega}_\pi(T) \simeq \nu_\pi(T) \overline{\Omega}_{Q(q)}(T) \quad (A.1) \]

where \( \overline{\Omega}_i \) are the mean values of the corresponding final state phase spaces. Since each antivalon \( \bar{Q}(q) \) of a certain color and flavor encounters a valon \( Q(q) \) with the probability \( \frac{n_{Q(q)}}{n + n_\pi} \), the rate of \( Q(q) \bar{Q}(q) \) collisions is

\[ d\nu_{Q(q)} = \frac{(1 - b)n_{Q(q)}n_{\bar{Q}(q)}}{n} \frac{dt}{\langle t \rangle}, \quad (A.2) \]

\( \langle t \rangle \) being the mean free time between the successive collisions. Quite similarly, a \( \pi^0 \)-meson encounters another \( \pi^- \)-meson with the probability \( b \), the total rate of \( \pi^0 \pi^- \) collisions being, therefore, \( \frac{7}{18}bn_\pi \frac{dt}{\langle t \rangle} \) (\( \pi^0 \pi^\pm \) collisions) plus \( \frac{bn_\pi}{n} \frac{dt}{\langle t \rangle} \) (\( \pi^0 \pi^0 \) collisions); the rate of \( \pi^+ \pi^- \) collisions is obviously \( \frac{bn_\pi}{n} \frac{dt}{\langle t \rangle} \). Of course, \( \pi^+ \pi^+ \) and \( \pi^- \pi^- \) collisions are out of the game in the detailed balancing principle equation (within the above approximation), since they never result in a two-valonic final state. Thus, for the total rate of \( \pi \pi \) collisions to be accounted one gets

\[ d\nu_\pi = \frac{7}{18}bn_\pi \frac{dt}{\langle t \rangle} \quad (A.3) \]

The averaged valonic and pionic phase spaces are

\[ \overline{\Omega}_{Q(q)} \simeq N_c^2(2S_Q + 1)^2N_c \overline{p^2_{Q(q)}} \quad \text{and} \quad \overline{\Omega}_\pi \simeq (2I_\pi + 1)^2 \overline{p^2_\pi} \quad (A.4) \]

respectively, where \( S_Q \) is the valonic spin and \( I_\pi \) is the pionic isospin, \( p_i \) is the valonic or pionic momentum in the CMS of two interacting valons or pions, \( N_c \) and \( N_f \) are the color and flavor numbers (\( N_c \) stands in eq. (A3) instead of \( N_c^2 \), since only the color-singlet sector of the total two valon phase space is allowed for). The straightforward averaging over the Boltzmann distribution gives for the mean energy of a particle with mass \( m \):

\[ E^2(m, T) = T^2 \left[ 3 \frac{\overline{p^2_\pi}}{K_2(\frac{m}{T})} + 12 + \frac{m^2}{T^2} \right] \quad (A.5) \]

where \( K_{1,2} \) are the corresponding Bessel functions. The CMS value of \( p^2_\pi \) (or \( \overline{p^2_{Q(q)}} \)) of each particle in the pionic (valonic) final state is obtained obviously by insertion into this.

9The binary reactions are to be considered only because \( 2 \to 4 \) reactions are substantially suppressed by scarcity of the typical thermal final state phase space at \( T \simeq T_H \).
expression \( m = m_{Q(q)} (m = m_q) \), subtraction \( m^2_\pi \) \( (m^2_{Q(q)}) \) and taking a half of this difference. Combining eq.s (A1)-(A5), we easily obtain

\[
b \simeq [1 + 2, 5 \left( \frac{n_q n_{\bar{q}}}{\bar{p}_q} \right) p^2_{Q(q)})^{-1/2}]^{-1} \quad (A.6)
\]

where the values of \( \bar{p}_i \) are to be taken in the accordance with eq. (A5).

Now, consider the equilibrium kaon fraction and derive eq. (2) for the in-Q\(\pi\)K-medium \(K^{0,+}\)-meson yields. For doing that, one should take into account the diagram shown in Fig. 1(b), 1(c). It is easy to check that, at the relevant temperatures, the averaged CMS momenta squared are nearly equal for all four states under consideration, and thus the corresponding detailed balancing equation reads:

\[
\frac{(2I_K + 1)}{(2I_\pi + 1)} n_\pi n_{Q(s)} + \frac{(2I_K + 1)(2I_\pi + 1)}{(2S_Q + 1)^2} \frac{n_{Q(s)} n_{Q(q)}}{N_c} \simeq \frac{(2I_\pi + 1)}{(2I_K + 1)} n_{Q(q)} n_K \quad (A.7)
\]

where \( K \) stands for \( K^0 \) or \( K^+ \) and \( I_i \) is the relevant isospin. This gives eq. (2).

Making use of these formulae, one can easily estimate that the \( K \)-meson induced correction to the value of \( b \) is really quite small: say, for SPS, \( \frac{\Delta b}{b} \simeq + (4 \div 5) \times 10^{-2} \) and it is undoubtedly absorbed into inaccuracy of the approach itself.

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Figure captions

Fig. 1. The processes taken into account for getting the detailed balancing equations (see Appendix) which regulates the relative content of different components in QπK gas. Diagrams (b) and (c) refer, actually, to the same diagram linked to the two cross channels.

Fig. 2. The typical processes which proceed at the hadronization phase transition ($T \simeq T_H$). The dotted and wave lines refer to a number of the attendant pions and gluons, the latter being absorbed by medium.

Fig. 3. The $(\mu T)$ phase diagram allowing for the QπK phase. The solid line (1) refers to the QGP → QπK phase transition, the strip (2) between two dashed lines marks predicted in the present work temperature interval around $T = 110$ MeV for the expected value of QπK → H phase transition temperature $T_H$, and two lines (3) are quoted from the ref. [3] where they refer to the strip found by the authors for the suggested unified description of the hadronic chemical freeze-out after the direct QGP → H phase transition.
This figure "fig1.gif" is available in "gif" format from:

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### Table

Comparison of experimental and theoretical data

| N_i/N_j | AGS Au+Au, E_L=11 GeV | SPS Pb+Pb, E_L=160 GeV | RHIC Au+Au, \(\sqrt{s}=130\) GeV | LHC \(\sqrt{s} \approx 5.5\) TeV |
|---------|---------------------|----------------------|------------------------|----------------------|
|         | exp | th_1 | th_2 | exp | th* | th_1 | th_2 | exp | th_1 | th_2 | th_1 | th_2 |
| \(p/\pi\) | 1   | 0.86  | 0.76 | 0.228 | 0.238 | 0.271 | 0.260 | 0.12 | 0.12 | 0.1 | 0.07 |
| \(\bar{p}/p\) | 5*10^{-4} | 4.7*10^{-4} | 0.067 | 0.055 | 0.084 | 0.08 | 0.67 | 0.67 | 0.67 | 1 | 1 |
| \(K^+/\pi\) | 0.175 | 0.175 | 0.173 | 0.17 | 0.15 | 0.14 | 0.13 | 0.09 | 0.09 | 0.095 |
| \(K^-/\pi\) | 0.034 | 0.035 | 0.035 | 0.09 | 0.08 | 0.16 | 0.14 | 0.12 | 0.09 | 0.095 |
| \(\bar{K}^0/\pi\) | 0.15 | 0.148 | 0.125 | 0.137 | 0.13 | 0.12 | 0.2 | 0.2 | 0.15 | 0.15 |
| \(\eta/\pi\) | 0.097 | 0.097 | 0.081 | 0.087 | 0.09 | 0.09 | 0.097 | 0.097 | 0.097 |
| \(A/\pi\) | 0.24 | 0.25 | 0.077 | 0.096 | 0.097 | 0.089 | 0.043 | 0.04 | 0.028 | 0.018 |
| \(A/K^0_s\) | 0.45 | 0.48 | 0.63 | 0.76 | 0.35 | 0.38 | 0.23 | 0.21 | 0.21 | 0.22 |
| \(K^+/K^-\) | 5.14 | 5 | 5 | 1.85 | 1.9 | 1.91 | 1.93 | 1.1 | 1.17 | 1.12 | 1 | 1 |
| \(\Lambda/\Lambda\) | 0.001 | 0.001 | 0.131 | 0.1 | 0.1 | 0.1 | 0.8 | 0.725 | 0.725 | 1 | 1 |
| \(\bar{\Lambda}/\Lambda\) | 0.072 | 0.072 | 0.101 | 0.11 | 0.12 | 0.012 | 0.168 | 0.148 | 0.121 | 0.121 |
| \(\Xi^-/\Lambda\) | 0.375 | 0.375 | 0.188 | 0.185 | 0.24 | 0.24 | 0.197 | 0.174 | 0.121 | 0.121 |
| \(\Sigma^-/\Xi^-\) | 0.005 | 0.005 | 0.232 | 0.228 | 0.2 | 0.2 | 0.85 | 0.852 | 0.852 | 1 | 1 |
| \(\phi/\pi\) | 0.0061 | 0.008 | 0.021 | 0.019 | 0.014 | 0.013 | 0.016 | 0.014 | 0.008 | 0.007 |
| \(\chi^2\) | 0.03 | 0.06 | 0.34 | 0.57 | 0.5 | 0.035 | 0.06 |

\(\mu_q/T_H\) = 1.4, \(\mu_s/T_H\) = 0.59

\(T_H=110\pm5\) MeV

\(\mu_q/T_{ch}=1.47\), \(\mu_s/T_{ch}=0.6\)

\(T_{ch}=110\) MeV

\(\mu_q/T_H=0.47\), \(\mu_s/T_H=0.16\)

\(T_H=110\pm5\) MeV

\(\mu_q/T_{ch}=0.45\), \(\mu_s/T_{ch}=0.16\)

\(T_{ch}=170\) MeV

\(\mu_q/T_H=0.026\), \(\mu_s/T_H\approx0\)

\(T_H=110\pm5\) MeV

\(\mu_q/T_{ch}=0.04\), \(\mu_s\approx0\)

\(T_{ch}=190\) MeV

\(\mu_q/T_H\approx0\), \(\mu_s/T_H\approx0\)

\(T_{ch}=?\)

\(\chi^2\): model of refs. [2] and [3];

\(\chi^2\): two versions of our model, \(\langle j \rangle = 0.5, 0.7 < j_2 < 1\); (see in the text);

\(\chi^2\): temperature of thermal freeze-out from thermal hydrodynamical models.