Thermal properties of S-gears in comparison with involute gears

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Abstract. The tests with small plastic gears, which were conducted to assess Wöhler curves for S- and E-gears indicated improved behaviour of S-gears, which is due to their geometry. However, the thermal load of the gears in contact is of a crucial importance. Therefore, the paper focusses on the differences in thermal load of cylindrical, spur, plastic involute and S-gears. Whereas there are thermal (temperature) models at disposal for the involute gears, no models are available for the S-gears. Such a model should reflect actual flank shape and difference in actual thermal behaviour deriving from there. So, frictional power along the path of contact, work of friction and deriving flash temperatures along the path of contact were calculated for both gear types, which disclosed lower thermal load of S-gears. Small E- and S-gears made of polyacetal and nylon were manufactured by milling. So, gears were of higher quality and reflected actual tooth flank geometry. The later long-term experiments were planned with smaller loads to run up to 10 or 20 million cycles; in this context they were focused on measurements of a steady state bulk temperature which helped in assessment of actual contact circumstances and contact temperatures.

1. Introduction

Involute gears, common in contemporary machines, transmit power through convex–convex contact. These were gradually developed over centuries and improvements in both, manufacturing technologies and materials. However, the intrinsic property of the involute gear is its curvature radius function in the dedendum part when approaching the base circle. The curvature radii values are small in this area and limit to zero approaching the base circle. This implies high contact loads in the dedendum area. For gears with a low number of teeth, the dedendum flank is comparatively very short, which invokes excessive sliding and friction losses and the possibility of premature damage. An additional problem is undercutting of the dedendum area. For these reasons there exists a permanent need for improved gears, with such features as a convex-concave contact, a stronger root, improved curvature radii, better lubrication conditions, etc.

S-gears have been theoretically defined and their properties have been compared to the involute gear in numerous papers [3-5]. Some relevant features with important improvements regarding the involute gearings were disclosed in these treaties. Outstanding characteristics of the S-gearings were proven in heavy industry applications [1]. And subsequently tested specimen with alloy steel were discussed, implying improved behaviour of the S-gearings [2].
S-gears feature variability of tooth flank shape which is based on a possibility of diverging parameters of a ruling parabolic function, improved rolling and sliding ratio, concave-convex contact near the meshing start and end zones, radii of curvature and deriving flank pressure, and improved velocity circumstances and oil film thickness.

The thermal load has a detrimental effect on the power transmission of a mating gear pair for all types of gears, regardless of size, material or any other parameters. However, it is of particular importance in plastic gears due to their thermal sensitivity. Heat is generated by friction on the contacting surfaces of the mating teeth flanks where the load transmitting surfaces slide with the relative velocities. Work of friction depends on the flank load, the friction coefficient and on the sliding path length of a contact, whereas thermal power additionally depends on the sliding speed. The generated heat is treated as energy loss, where surface heating is unfavourable, and the temperature rise depends on heat flow into the gear material. Thermal properties of the employed gear materials are of the crucial importance in this context.

Typical circumstances of the involute gears near the meshing start point disclose very small driving pinion radii of curvature and rather high radii of curvature of the driven gear, which imply high sliding velocities in this area. The normal force $F_N$ is transmitted through the contact, which causes the force of friction $F_{fr}$ oriented tangentially to the contact and the corresponding power of friction, $P_{fr} = F_{fr} \cdot v_g$.

The power of friction being generated in the contact representing losses transforms to the heat flow, distributed to both involved flanks. A greater part of the heat is therefore distributed to the almost standing still driving gear and the rest to the longer contacting area of the driven gear. The friction force grows to high levels already at the meshing start, which negatively influences (braking) the contact point velocity along the path of contact and induces negative sliding on the driving gear flank.

The above considerations impose a sound basis for a comparative research of S- and E-gears. Next chapters will briefly discuss some relevant properties of S gears, then discuss analytical computations and finally experimental results with polymer gears.

2. S-gear properties

S-gears are defined by the cutting rack profile, which is due to the necessity of cutting tool definition in metallic gears. So, the rack tooth flank profile is given by:

$$y_{Pi} = a_p(1 - (1 - x_{Pi})^n)$$

where $x_{Pi}$, $y_{Pi}$ stand for the Cartesian coordinates of the rack profile with the kinematic pole C as the origin, $n$ is the exponent of the parabolic function and $a_p$ the height (form) factor. It should be mentioned that the module $m$ is treated as a unit. The said parabolic function then defines a single path of contact and consequently pinions and gears with arbitrary number of teeth can be derived. The shape of the rack tooth profile is curved concavely towards both profile ends in such a way that the curved path of contact emerges, which then implies the convex-concave contact zones near the meshing start and meshing end.

Eq. 1 reveals that only two parameters govern the rack profile: $n$ and $a_p$. The bijective transformation than defines the gear flank shape. The inclination angle $\alpha_{0}$ or the subsequent initial pressure angle $\alpha_{0}$ are derived. By varying parameters root thickness can be increased, convex-concave zones can be tuned, and the initial pressure angle can be lowered (or increased). The pressure angle influences the flank load, the load capacity, the curvature radii, and the contact ratio [5].

The mating gear teeth flanks combine the pinion dedendum and the gear addendum flanks from the meshing start to the kinematic pole C and the pinion addendum and the gear dedendum from C towards the meshing end. The contact is propagating on the path of contact by rolling and sliding. The active size of the pinion dedendum is smaller than that of the gear addendum. This implies amount of sliding of the addendum on the shorter pinion dedendum, which is illustrated in Figure 1. Amount of sliding also implies thermal impact. The dedendum-addendum size difference depends on module, number of teeth, pressure angle. For S-gears the said difference also depends on forming factors – the height and the exponent. The size difference in the case of S-gears is comparatively more convenient,
so less sliding is produced along the contact propagation compared to the involute case. Figure 1 (detail) also shows the dedendum and the corresponding addendum of the E-gear. The starting pressure angle for the S-gear pair is 22° and that of the E-gear pair 20°. The contacting addendum flank length of the latter is about twice as large as that of the pinion.

The most important features of the S-gears, particularly those for plastic materials, are:

− Comparably stronger roots can be designed for gears with a large load capacity by varying forming parameters. This can be advantageous for gears made of viscoelastic, polymeric or composite materials, which then exhibit less deformation, consequently less internal friction in dynamic applications.
− S-gears have convex-concave contact zones near the meshing start and meshing end and corresponding higher reduced radii of curvature implying lower flank pressures.
− The rolling-sliding ratio (Figure 1) reveals better performance of the S-gears in comparison to the E-gears. The length difference between the pinion dedendum and the gear addendum imposes amount of sliding.

3. Gear materials, geometry and manufacturing

Many polymer materials have outstanding properties in the range of plastic gears, however, the selected material combination was:

− polyoxymethylene (acetal) (POM-C) and
− polyamide (nylon) (PA6, PA66).

This material combination is quite common for plastic gears, due to its good dynamic friction coefficient and price criterion. Both materials can be used in precision engineering, the automotive industry, electrical engineering, etc. And both materials are distinguished by their slide and wear properties, strength, toughness, etc. POM-C is also characterized by good machinability, whereas PA66 is challenging in this context. Both materials have a long-term service temperature of around 100 °C. Detailed technical characteristics can be found in the corresponding data sheets [6-7]; those needed in calculations are presented in Table 1.

Initially, S-gears were manufactured by moulding and tested ($\alpha_{w0}=27°$, $n=1.4$) vs. the E-gears of the same size ($m=1.0$ mm, $z_1=z_2=20$). It was discovered that the axis distance in the case of S-gears did not correspond to that of the involute gears, it was at least 0.2 mm above the nominal value, which
amounted to more than 20% of the module. It has been already agreed that S-gears can mate even with disrupted axis distance [5], but efficiency of such action has not been defined precisely, yet. This implies that the design of moulds should be carried out with a consideration of actual shrinkage of particular materials, which is not an easy task. Another consideration is that the quality of moulded gears is rather low which influences the precise flank geometry, particularly in the case of S-gears.

Considering past experience, it was decided to have new gears cut and assure better quality and shape conformity both for S- and E-gears. So, it was decided to use the same size gears, that is \( m=1.0 \) mm, \( z_1=z_2=20 \), \( b=6.0 \) mm. The initial pressure angle \( \alpha_w=18^\circ \) and \( \alpha_p=1.5 \) were chosen for S-gears; the pressure angle varied from 26.8\(^\circ\) at A and decreased to 18\(^\circ\) at C and raised up to 26.8\(^\circ\) at E. The pressure angle of the E-gears was 20\(^\circ\).

Koepfer 160 EMAG CNC gear hobbing machine was used to produce S- and E-gears with gear hobs showed on Figure 2 with the quality grade of 8-9, whereas moulded gears have the grade 11. Moulded and cut involute gears were measured on Wenzel GearTec according to DIN 3961, DIN 3962 [8-9]. Profile and lead inspection have been conducted, as well as pitch and runout measurements.

| Table 1. Gear material properties. |
|-----------------------------------|
| **Property** | **PA66** | **POM-C** |
| Colour       | ivory opaque | white opaque |
| Density, \( \rho \)          | 1.15 [g/cm\(^3\)] | 1.41 [g/cm\(^3\)] |
| Modulus of elasticity (tensile), \( E \)  | 3500 [MPa] | 2800 [MPa] |
| Tensile strength, \( R_m \)          | 85 [MPa] | 67 [MPa] |
| Tensile strength at yield, \( R_e \) | 84 [MPa] | 67 [MPa] |
| Elongation at yield, \( \delta \) | 7% | 9% |
| Poisson’s ratio, \( \nu \)         | 0.4 [-] | 0.35 [-] |
| Service temperature (short term), \( T_m \) | 170 [\(^\circ\)C] | 140 [\(^\circ\)C] |
| Service temperature (long term), \( T \)   | 100 [\(^\circ\)C] | 100 [\(^\circ\)C] |
| Thermal expansion (CLTE), \( \alpha \) | 11-12 \( \times 10^{-5} \) [K\(^{-1}\)] | 13-14 \( \times 10^{-5} \) [K\(^{-1}\)] |
| Thermal conductivity, \( \kappa \)      | 0.36 [W/K.m] | 0.39 [W/K.m] |
| Specific heat, \( c \)            | 1.5 [J/g.K] | 1.4 [J/g.K] |
| Thermal diffusivity*, \( \chi \) | 0.1976 [mm\(^2\)/s] | 0.2087 [mm\(^2\)/s] |

*Thermal diffusivity is calculated by \( \chi=\kappa/\rho\cdot c \) and reflects the rate at which heat flows through a material.

Figure 2. Gear hob for involute gear (left) and S-gear (right).
4. Analytical approach

A program for calculation of the rack profile, the path of contact, the pinion and inner or outer gear based on S-gear was supplemented by necessary computations of power, work, flank pressure, contact width, velocities, and flash temperatures. All the parameters can be represented along the path of contact or the active flank profile.

Power of friction is therefore given by:

$$P_{fr} = F_{fr} \cdot v_g = (\mu \cdot F_t / \cos \alpha_W) \cdot v_g$$  \hspace{1cm} (2)

The friction coefficient value for a dry POM/PA contact is 0.18, whereas that of POM/POM amounts to 0.28 [10]. The input power for e.g. the nominal input torque of $T_t=0.6 \text{ Nm}$ and the rotational frequency $\nu=1439 \text{ min}^{-1}$ amounts to $P_t=90.42 \text{ W}$.

Work of friction along the active contact from $t_A=0$ to $t_E$ is given by the sum:

$$A_{fr} = \sum_{t_A}^{t_E} P_{fri} \cdot \Delta t_i = \int_{t_A}^{t_E} P_{fr} dt$$  \hspace{1cm} (3)

So, the frictional work in a single pinion rotation is $z_G A_{fr}$, which we multiply by the rotational frequency to get frictional work accomplished in a minute $\nu z_G A_{fr}$. The average frictional power is then $\nu z_G A_{fr} / 60$. Results for S- and E- gears are collected in Table 2.

| $T_t=0.6 \text{ Nm}, P_t=90.415 \text{ W}$, $\nu=1439 \text{ min}^{-1}, l_U=3.891 \text{ mm}$ | S-gears | E-gears |
|---|---|---|
| Work in a single contact | 0.0052 J | 0.0064 J |
| Work in a single rotation | 0.1040 J | 0.1278 J |
| Work in a minute | 149.61 J | 183.87 J |
| Average frictional power | 2.4935 W | 3.0644 W |

| $T_t=0.7 \text{ Nm}, P_t=104.8 \text{ W}$, $\nu=1428 \text{ min}^{-1}, l_U=4.5976 \text{ mm}$ | S-gears | E-gears |
|---|---|---|
| Work in a single contact | 0.0061 J | 0.0075 J |
| Work in a single rotation | 0.1213 J | 0.1491 J |
| Work in a minute | 173.21 J | 212.87 J |
| Average frictional power | 2.8868 W | 3.5479 W |

The flash temperature course along the path of contact or the active flank is the next to be computed. The gear contact corresponds to the line contact with high contact velocities, i.e. discriminated by high Pecllet numbers [11]. The Pecllet number is defined by $L = v_{G/H} \cdot b_h / (4 \chi)$, where $v_{G/H}$ is a moving partner velocity, and $\chi$ thermal diffusivity. First, the contact width according to the Hertzian theory defined by Eq. 4 is calculated:

$$b_h = \sqrt{32 \cdot F_N \cdot \rho_t / (\pi \cdot b \cdot E')}$$  \hspace{1cm} (4)

Here $F_N$ stands for the normal force to a gear flank, $b$ for the gear face width, $\rho_t = (1/r_1 + 1/r_2)^{-1}$ for the reduced radius of curvature, and $E' = 2((1 - v_H^2) / E_1 + (1 - v_G^2) / E_2)^{-1}$ for the reduced Young’s modulus. Since the normal force in S-gears deviates for app. 9.5% from C to E or A and the reduced radii of curvature of S-gears are higher the resulting contact width deviates from 0.1675 to 0.2738 mm at A or E and B or D, correspondingly. E-gears have lower peaks, namely 0.1675 mm at A or E, 0.1551 mm as a maximum in dual meshing zones, and 0.2195 at B or D and 0.2265 mm at C.

One could use original maximal Hertzian pressure equation or a simplified one, based on $b_h$. Both provide equal results.

$$p_{Hmax} = 4F_N / \pi \cdot b_h \cdot b$$ and $p_{Haverage} = F_N / b_h \cdot b$  \hspace{1cm} (5)
Since $F_N$ deviates only for about 9.5% and the reduced radii of curvature are higher in S-gears compared to those in E-gears the resulting maximal (and average) pressure is noticeably lower than that of the E-gears with the single exception in C, where $p_{H_{\text{max}}}$ values are nearly the same. The numerical calculation conducted by FEM [12], presented in Figure 3, confirmed the analytical results.

The Heat flow corresponds to the power of friction and heat rate is defined by $Q = \frac{\Delta T}{b_h \cdot b}$. The average flash temperature for a participant G or H (pinion flank/gear flank) can be calculated by:

$$T_{f_{\text{fl,avg},G/H}} = 0.7524 \frac{q}{\sqrt{v_G/H}} \cdot \sqrt{\frac{b_h}{b}}$$

Here $v_G$ stands for the sliding velocity, whereas $v_{G/H}$ is a velocity of a particular moving partner, pinion flank or gear flank. The maximal flash temperature increases $T_{f_{\text{fl,max}}} = 1.499$. $T_{0,\text{max},G}$ and $T_{0,\text{max},H}$ reflect flash temperatures as if all heat generated in the contact is supplied to a single contacting partner (G or H). However, the true flash temperature rise must be the same for both participants in contact, so the maximum flash temperature rise is given by Eq. 7. By substituting the high speed $T_{f_{\text{fl}}}$ expressions in Eq. 6, one gets commonly used equations for maximum contact temperature rise.

$$T_{f_{\text{fl,max}}} = \left( \frac{1}{T_{f_{\text{fl,max},G}}} + \frac{1}{T_{f_{\text{fl,max},H}}} \right)^{-1}$$

The flash temperatures for S- and E- gear pairs of the defined geometry have been calculated and presented along the path of contact in Figure 4. A single tooth pair and double tooth pairs meshing zones are clearly denoted in both cases. Whereas the S-gear pair develop similar maximum values in A and B, and D and E, that is approximately 38 K, the maximum values for the E-gear pair are in A and E and range around 59 K for the nominal load $T=0.6$ Nm and $v=1439$ min$^{-1}$.

### 5. Experimental results

The experimental system and layout has already been described in detail e.g. in [12]. Temperatures of gear surfaces have been measured by thermographic camera ThermaCam FLIR T420 with the frame acquiring frequency 60 Hz. Since it is necessary to acquire temperatures during operation, camera positioning becomes of importance. E.g., by measuring from above, errors arise from turbulent flow caused by gear meshing [13]. Angular position can read falsely due to emission changes. So, camera was positioned co-axially with the gear axes of rotation. The actual temperature is composed of ambient, gear bulk and flash temperature. Flash temperature cannot be measured with the used camera since it is extremely short-term phenomenon; in fact, the bulk temperature was measured. All diagrams are plotted with reference to a measured spot put in gear meshing zone.
Figure 4. Flash temperature diagram for the involute gear pair and the S-gear pair.

Figure 5. Experiments with lower nominal load.
Past experimental work was oriented to acquire Wöhler’s plots. It was discovered that S-gears develop less frictional heat, so S-N curves are positioned higher than those of E-gears [12]. In the latter experiments gears were run in such conditions to achieve steady state heat flow with a balance zero and a long-term operation up to more than $10^7$ cycles. So, for material combination POM/PA66 the nominal load can be up to 0.8 Nm, whereas 0.6 and 0.7 Nm load was used in these experiments.

Some results are presented in Figure 5. One can observe a clear distinction in temperature rise between E- and S-gears in favour of S-gears, so they obviously produce less frictional heat. Temperatures also rise with increase of the nominal load. Temperature deviations can be attributed (at least to a considerable extent) to ambient temperature fluctuations.

![Figure 6.](image)

Figure 6. Optical microscope photos: above left – new S-gear; above right – S-gear after 22 million cycles; below – initial fatigue fractures.

| Gear type | Volume | POM Mass | PA66 Mass | Heat/K POM | Heat/K PA66 |
|-----------|--------|----------|-----------|------------|-------------|
| S         | 1.8598 cm$^3$ | 2.6223 g  | 2.1388 g  | 3.6712 J/K | 3.2082 J/K |
| E         | 1.8337 cm$^3$ | 2.5855 g  | 2.1088 g  | 3.6197 J/K | 3.1631 J/K |

In general, the prevailing failure types are wear, fatigue and thermal failure. The above tests were run for up to 263h43'48", that is up to 22.77 million cycles. No significant wear was observed during these experiments, which is as well due to a proper material combination (POM-PA66). No thermal failure was observed, the bulk temperature was never above app. 51°C. However, there were present some initial fatigue fractures in S-gears which run over 20 million cycles. The tested specimen had been examined by an optical microscope, which are presented in Figure 6.

The aim of the paper was to establish a sound basis for a heat and temperature related calculations in particular for S gears. VDI model [10] does not discriminate between various gear flanks, the only term that differs is the contact ratio. E.g., the root temperature rise according to the VDI is for E-gears 65°C and that for S-gears 57.5°C, whereas the measured values are app. 47 and 40°C, respectively, when the load is $T=0.7$ Nm. Table 3 shows how much frictional work is necessary to rise temperature for a single degree if there is no heat dissipation.
6. Conclusion
Thermal load caused by friction in the contact of mating gears can cause failure of polymer gears. Therefore, accurate thermal models should be developed. There are some models especially for E-gears, whereas no models are available for S-gears. The paper discusses some properties of S-gears which indicate improved behaviour in comparison to E-gears. The contact density implies considerably lower amount of sliding S-gears. The convex-concave flank contact zones near meshing start and end and higher reduced radii of curvature lower flank pressure. Pinion and gear flanks are analytically calculated, based on the path of contact and the rack profile, which is governed by two parameters. This enables calculation of curvature radii, velocities, normal force, Hertzian pressure and contact width. A line contact, characteristic for a gear contact is assumed. Next, power of friction is calculated along the contact. Work of friction is also calculated along the path from A to E, however, based on the time necessary to move along this path. The flash temperature calculation is important since it indicates thermal load of a contact. All indicative functions show improved values for S-gears compared to E-gears. S-N curves acquired during the earlier experiment set [14] showed a distinctive difference between S- and E-gears in favour of the first. That was also true with unfavourable POM-POM experiments. First experiments with steady state bulk temperature also showed a temperature difference in favour of S-gears. More experiments with improved experimentation set are required to confirm so far made results and to facilitate development of the thermal model for S-gears, based on reliable data.

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