Control and generation of localized pulses in passively mode-locked semiconductor lasers

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Abstract—We show experimentally and theoretically that localized pulses can be generated from an electrically biased 200 μm multi-transverse mode Vertical-Cavity Surface-Emitting Laser. The device is passively mode-locked using optical feedback from a distant Resonant Saturable Absorber Mirror and it is operated below threshold. We observe multistability between the off solution and a large variety of pulsating solutions with different number and arrangements of pulses per round-trip, thus indicating that the mode-locked pulses are localized, i.e. mutually independent. We show that a modulation of the bias current allows controlling the number of the pulses travelling within the cavity, thus suggesting that our system can be operated as an arbitrary pattern generator of 10 ps pulses and 1 W peak power.

Index Terms—Mode-Locking, Broad-Area Lasers, VCSELs

I. INTRODUCTION

Mode-locking (ML) is a fascinating phenomenon that allows the generation of ultrashort pulses from a laser [1] and that is still a subject of intense research. Passive ML (PML) is arguably the most successful approach and it is achieved by combining two elements, a laser amplifier which provides gain and a saturable absorber (SA) acting as a pulse shortening element, see [1] for a review. Pulsed emission with a fundamental period corresponding to the cavity round-trip time arises from the different dynamical properties of the SA and the amplifier, which open a short window for amplification around the pulse [2], [3]. ML has led to the shortest and most intense optical pulses ever generated and pulses in the femtosecond range are produced by dye [4] and solid state lasers [5]. Large output powers in the Watt range are commonly achieved from coupled VECSELs-SESAM [6] configurations. On the other hand, the large gain of semiconductor materials allows building sub-millimeter Monolithic PML lasers. The round-trip of such short devices is typically of the order of 10 ps and they can therefore reach repetition frequencies of several tens of GHz. They have also the advantage of being compact, low cost and adaptable to many cavity geometries [7], yet their power is in the milli-Watt range.

In spite of the research efforts dedicated to the general understanding of the basic issues, several aspects of PML still represent a scientific challenge. For instance, while the breadth of the active medium gain curve is well known to govern the pulse-width, it was only very recently understood that the SA also provides a strongly asymmetric and non linear filtering. Such an usually overlooked effect was recently proven to be the key mechanism explaining wavelength instabilities in PML lasers [8], [9]. In addition, recent studies indicated that the optimal position of the SA in the cavity was not following intuitive rules [10]. Similarly, it was commonly thought until recently that PML semiconductor lasers can not operate at low repetition rates due to the fast recovery time of their material gain (τg ∼ 1 ns) which should limit them to high repetition frequencies (≥ 1 GHz). Too long cavities result in the so-called regime of harmonic mode-locking in which several pulses circulate in the cavity, according to the background stability criterion [1]. Until recently, the record of the lowest frequency PML was experimentally attained around 300 MHz [11], [12], [13].

We have recently shown that, in the limit of cavity round-trip much longer than the gain recovering time, mode-locked pulses may coexist with the zero intensity background and can be interpreted as temporal Localized Structures (LS) [14]. Localized pulses can be independently addressed and used as elementary bits of information, hence the cavity can be used as an all-optical buffer, as shown in [15]. A single localized pulse can be activated within the cavity, independently from the cavity size, thus leading to arbitrary low repetition rates which allowed establishing a new low frequency record of 65.5 MHz [14]. The possibility of addressing individually localized pulses opens interesting possibilities for the optical generation of arbitrary trains of narrow pulses, which has a large number of potential applications in different domains, e.g. time-resolved spectroscopy, pump-probe sensing of material properties, generation of frequency combs, Optical Code Division Multiple Access Communication Networks [16] and LIDAR [17], [18].

In this manuscript, we show how localized pulses form in a passively mode-locked semiconductor broad-area VCSEL coupled to a resonant saturable absorber mirror (RSAM). The VCSEL bias current is used as a control parameter for addressing localized pulses and for generating patterns of closely packed narrow pulses. The experimental evidences and theoretical analysis pave the path towards an optical arbitrary pattern generator of short light pulses.
The second records the VCSEL’s far-field profile. The external first one records the near-field profile of the VCSEL, while a detector. Part of the light is sent to two CCD cameras; the imaging condition does not induce an appropriate ratio of aperture (0.68) aspheric lens and a similar lens is placed in front of the RSAM. A 10% reflection beam splitter allows for light extraction from the external cavity and to monitor both the VCSEL and the RSAM outputs. Intensity output is monitored by a 33 GHz oscilloscope coupled with fast 10 GHz detector. Part of the light is sent to two CCD cameras; the first one records the near-field profile of the VCSEL, while the second records the VCSEL’s far-field profile. The external cavity length round-trip ($\tau$) is fixed at $\tau = 15$ ns which corresponds to a 66.6 MHz fundamental repetition rate.

B. From conventional mode-locking to localized pulses

The combination of the VCSEL and the RSAM in self-imaging condition does not induce an appropriate ratio of the saturation parameters for obtaining PML. Notwithstanding, mode-locking was obtained when placing the RSAM surface in the exact Fourier transform plane of the VCSEL near-field profile, i.e. when imaging the VCSEL far-field profile onto the RSAM surface [20]. As a consequence, the VCSEL profile was imaged onto itself after a single external cavity round-trip, but inverted (i.e. a magnification factor of −1). As shown in [20], such configuration leads to the generation of two opposed tilted plane waves, i.e. waves that have a propagation wave-vector slightly out of the cavity axis $z$ and which travel in the external cavity with an opposite transverse component and alternating each other at every round-trip. Each one of these plane waves gives birth to a train of mode-locked pulses separated by twice the external cavity round-trip ($2\tau$), while the two trains are time shifted of $\tau$. Intuitively, one understands that injecting the Fourier transform of the VCSEL near-field profile into the RSAM strongly favors the emission of a tilted wave from all points of the VCSEL, since the Fourier transform of such an emission consists in a tight focused single spot that saturates easily the RSAM. While this scheme leads to conventional mode-locking and harmonic mode-locking for cavity lengths and round-trips shorter than the gain recovery time ($\tau < \tau_g$), we operate the system in the regime of long cavities, i.e. $\tau \gg \tau_g$, and for bias currents below the laser threshold. In these conditions, as shown in [14], several emission states, including the zero intensity (off) solution, coexist for the same values of bias current.

These emission states are characterized by a different number of pulses circulating in the cavity, and for the same number of pulses, a large variety of pulse arrangement can be observed. In Fig. 2a-d) few examples between the large number of pulses case, we show two situations where they appear either grouped, see Fig. 2d), or equally separated Fig. 2e). The case of $N = 19$ pulses circulating within the cavity represents the largest number of pulses that can be obtained for the size of the external cavity chosen. All the above listed pulsing...
solutions coexist in the parameter space for a wide range of VCSEL pumping current $J$. The pulse width cannot be determined precisely from the oscilloscope traces shown in Fig. 2, which are limited by our real-time detection system (10 GHz effective bandwidth). However, an estimation of the pulse width can be obtained from the optical spectrum of the output, which exhibits a broad spectral peak whose FWHM is around 0.12 nm that corresponds, assuming a time-bandwidth product of 0.4, to a pulse width of 10 ps FWHM. The pulse was also detected by a 36 GHz detector, which confirms a pulse width of less than 12 ps FWHM considering the oscilloscope bandwidth limit. Finally, the pulse peak power has been measured to be 1 Watt.

The multistability between a large number of different solutions is shown in Fig. 2g), where we plot the bifurcation diagram of these solutions as a function of the pumping current. Figure 2g) is obtained increasing the parameter $J$ from $J = 210$ mA, where only the steady off solution is stable, up to the value where this solution loses its stability ($J < 350$ mA) and then sweeping it down until a periodic emission with $N = 19$ pulses per round-trip appears, see Fig. 2f). In analogy to spatial localized structures [21], [22], this state corresponds to the fully developed temporal pattern which is, together with the coexisting stable off solution, at the origin of the localized structures. As $J$ is decreased, the state with $N = 19$ pulses per round-trip loses its stability and the system bifurcates progressively towards states with smaller numbers of LS, until a single one is present in the cavity. Each state is spontaneously appearing as $J$ is scanned downward and, once a new state appears, we increase $J$ to explore its stability up to $J = 290$ mA. As far as the system remains on the same branch there is no change in the pulse arrangement, thus showing that, even if several arrangements are possible, once one is chosen, it is stable versus parameter changes. Figure 2 indicates that in this regime, mode-locked pulses in our system can be interpreted as temporal localized structures, i.e. any pulse is independent from the other and it can be individually addressed. It is worthwhile noting that the localized character of the pulse implies that the pulse is localized not only in the intensity but also in all the variables describing the system. This means that, after the short intensity pulse emission, the gain and saturable loss variables recover their steady state value on a longer time scale thereby defining the effective temporal extent of the LS. This is the main difference between localized pulses and conventional mode-locked pulses. In the latter case, which is obtained for pumping levels above laser threshold, the gain does not recover to its steady state value between pulses. This is well explained by the background stability criterion [1] which states that a pulsating solution is stable if losses are larger the gain between pulses. Since the laser is pumped above threshold, the only stable pulsating solution is the one having a number of pulses in the cavity large enough such that, in-between pulses, gain cannot recover sufficiently to overcome the unsaturated loss level. This implies that fundamental mode-locking is possible only for $\tau < \tau_p$. In the case of localized pulses, we are in the opposite limit and the gain can recover fully between pulses because the maximal gain level remains below the unsaturated loss level, even for an infinitely long interval between pulses. Thus, localized PML pulses have no lower limit in their repetition rate, as shown in Fig. 2a) where a repetition rate of 66.6 MHz is obtained. On the other hand, the recovery time of the slowest variable, in our case the gain recovery time, determines the effective temporal extent of the localized pulse, since no other pulse can occur before the recovery of all the variables. Another important difference between localized pulses and conventional mode-locked pulses is that, for a given set of parameters, the energy of the localized pulses does not depend significantly on the number of pulses circulating inside the cavity. Instead in conventional mode-locking the energy of the system is shared between the pulses which circulate inside the cavity and fundamental mode-locking is preferred over harmonic mode-locking for maximum pulse energy.

In order to represent the evolution of the localized pulses travelling within the cavity over many round-trips, we use the so-called space-time diagram representation where the temporal trace is folded onto itself at the cavity round-trip period. Accordingly, the round-trip number $n$ becomes a pseudo-time variable while the pseudo-space variable corresponds to the timing of the pulse modulo $\tau$. This representation is similar to the one used for following the evolution of pulses along optical fibers where fast and slow time scales are well separated and it has been proposed also for delayed system [23], [24], [25]. In Fig. 3a-d) we show some space-time like diagrams obtained from the same time traces used for Fig. 2 where all the situations depicted coexist for the same parameter values. Panel a) shows the evolution of a single LS in the external cavity, while panel b), c) and d) represent the evolution of respectively $N = 3$, 4 and 19 pulses. In all the situations, the pulses do not visibly jitter over the considered timescale of $6.5 \times 10^4 \tau \approx 100 \mu s$ and they seems to be insensitive to the surrounding noise. This can be attributed to the presence of the saturable absorber which acts as an effective noise eater. While the RSAM opens a very short window of net gain around the pulse, the remaining low intensity emission from the VCSEL is absorbed by the RSAM. Accordingly, the background shown in Fig. 3 is homogeneous and corresponds to zero emission.

In presence of localized structures, as the ones shown in Fig. 3, the possibility of forming bounded states of localized structures, also called molecules, naturally arises.
erased in molecules, the pulses are not independent anymore, while the molecule itself becomes a compound localized structure which is independent from other localized structures that may travelling within the cavity. Such molecules have been widely observed in fiber lasers [26] and in general only a few if not an unique equilibrium distance separates the “atoms” of a molecule. The signature of such bound states of LS in a space-time like diagram is the presence of preferred distances between two neighbor pulses. We have never found any preferred distances between close pulses in the space-time like diagrams that we have analyzed. Instead we have observed a continuum of temporal separations. One can understand the absence of molecules in our system by the specificity of the interaction between neighbor LS. Indeed, it is known that the gain depletion induced by each LS produces a repulsive interaction [27] that decays over a time scale $\tau_g$. The potentially attractive interaction mediated by the oscillating tails of the pulses vanishes after a few times the effective pulse width, i.e. a time scale of $\sim 30$ ps. As such, this clear scale separation between the attractive and repulsive forces explain why LS molecules would be hard to obtain with semiconductor active materials.

It may be useful to compare our results with the extensive literature on mode-locking. Localized pulses in mode locked laser might have been observed in the past. For example, several papers mention PML regimes obtained at exceedingly low frequencies with respect to the limits of the background stability criterion; repetition rates of the order of few hundreds MHz were reported in [11], [12], [13]. We believe that these regimes could be explained in terms of LS. While it is difficult to know exactly the experimental conditions of these works, in [11] it is clearly mentioned that the off solution coexists with the PML solution. The experimental result reported recently in [28], claiming mode-locking at a repetition rate of 85.7 MHz, can also be explained in terms of LS. In this work, neighboring pulses of 50 ps FWHM, separated of 1 ns, are interpreted theoretically as bound states. While the experimentally observed distance corresponds to the vanishing of the repulsive interaction induced by the gain depletion [27], the theoretical analysis is performed after adiabatically eliminating the carrier dynamics, thus neglecting such repulsive interactions. We believe that this approximation in [28], correct for fiber lasers but not for semiconductor devices, has led to a wrong interpretation of independent neighbor pulses in terms of bound states resulting of the locking of the oscillating tails of the pulses.

C. Localized pulse addressing

The most important property of localized structures is their mutual independence which allows for their individual addressing. The localized pulses that we have obtained can be individually triggered by shining light pulses inside the cavity, as we have shown numerically in [14]. By encoding an information bit in the form of a localized pulse, our system can be operated as an all-optical buffer, similar to the one proposed using fiber Kerr resonator with a driving field [15], but taking advantage of the compactness and fast response of semiconductor materials as in [25]. The buffer bit-rate depends on the minimal time interval between two independent pulses, i.e. the temporal extent of the LS which is, in our case, fixed by the gain recovery time. The experimental results shown in Fig. 2f) indicate that a separation of 1 ns is certainly sufficient to avoid interaction between pulses, thus leading to buffer bit rate of $\sim 1$ GHz. While the all-optical addressing of the mode-lock localized pulses is the topic of ongoing research, the nucleation of a single pulse can be observed when perturbing mechanically the system, as shown in Fig. 4. Yet this kind of perturbation does not allow any control of the switching; it may lead to the nucleation of several pulses at the same time with random positions within the cavity.

Notwithstanding, a higher degree of control can be implemented when perturbing the bias current of the VCSEL. The main limitation using this parameter comes from the frequency response of the VCSEL to a modulated signal coupled onto the pumping current via a bias T. The electrical characteristics of the broad-area VCSEL we used, together with the bandwidth of the bias-T used, limit the frequency modulation below 0.3 GHz. Accordingly, the individual addressing of localized pulses via the emission of electrical pulses suffers from the too long rise time of the perturbation. Nevertheless a sinusoidal modulation of the pumping current with a period corresponding to the cavity round-trip $\tau$ allows for the excitation of localized pulses in a narrow time interval within the round-trip time, i.e. a well-defined vertical band of the space-like variable, as plotted in Fig. 3. The idea would be to modulate the current between a lowest value where the off solution is the only possible solution up to a highest value where the fully developed temporal pattern shown in Fig. 2f) is also the unique possible situation. Since this modulation is synchronous with the cavity round-trip, a portion of this monostable temporal pattern will appear only in correspondence with the modulation signal peak. Once the sinusoidal perturbation of the pumping current is removed, the portion of the monostable pattern will give way to a solution with one, two or more localized pulse depending on the continuous bias current value of the VCSEL, see Fig. 2g), and on the size of the the temporal pattern excited. The result of this experiment is shown in Fig. 5 where the VCSEL is biased at $J = 215$ mA, a current value for which for the sake of simplicity only the state with a single localized
pulse per round-trip coexists with the off state. We prepare the system in the off state and then we apply a modulation having a peak to peak amplitude of 460 mA. In correspondence with the modulation peak (where \( J = 445 \text{ mA} \)), six localized pulses appear at a distance of 800 ps, which corresponds to the interpulse distance of Fig. 2f), i.e. \( \tau/19 \). Once the modulation signal is removed, the continuous value of pumping current does not support the stable existence of six localized pulses and the system bifurcates towards the state with a single pulse per period. This regime is stable and it persists indefinitely.

![Figure 5. Time traces showing the experimental process for LS generation via current modulation. First the laser is in the 0-LS state without modulation (red, bottom line) and \( J = 215 \text{ mA} \). Second, modulation is applied with a peak-to-peak amplitude of 460 mA and 6 LS appear close to the peak of the modulation (green, middle line). When we switch-off current modulation, only a single LS remains in the cavity (blue, top line).](image)

**D. Multi-Pulse pattern generation**

The experiment reported above suggests that a current modulation can be used to obtain a pattern of closely packed localized pulses. This functionality is appealing for many applications like e.g. OCDMA [16] and LIDAR [17], [18], since narrow optical pulses (<10 ps) are typically obtained using conventional PML systems which deliver a periodic train of pulses with no versatility in the pulse arrangement. Pulse patterning can be of course obtained by adding optical gates based on Electro-Optical Modulators, but these solutions are expensive, energetically inefficient, and their implementation requires synchronization between the pulse source and the gating. We implement a pulse pattern generator by driving the system pumping current with a forcing at a period equal to the cavity round-trip and which brings the system into the monostable region where only a single pulse per period. This regime is stable and it persists indefinitely.

The results obtained are shown in Fig. 6 where we control the number of pulse emitted from one to five by changing the amplitude of the sinusoidal forcing. The distance between pulses is fixed by the effective temporal extent of the pulses which, as explained above, depends on the carrier recombination time. The close inspection of panel e) and f) reveals that the distance between pulses can change slightly within a pattern. This can be explained by the use of a sinusoidal signal for bringing the system in the monostable region of Fig. 2f), where a periodic pattern of pulses is emitted. In the portion of the sinusoidal signal where this occurs, the current is still increasing, thus leading to an effectively faster carrier recovery rate and therefore to closer pulses. For the same reason, pulse intensities vary slightly inside the pattern. The appearance of an increasing number of pulses for larger amplitudes of the forcing, as well as the reversed phenomenon while decreasing the modulation strength, occurs with an high degree of hysteresis as seen in Fig. 6f), thus revealing multistability between the different situations depicted in Fig. 6(a)-e)

While Fig. 6 discloses the proof-of-principle of a pulse pattern generator based on localized pulses, more complicated pulse pattern structures can be envisioned using other kind of modulation signals, as square or pulsed current modulation. Even if these modes of operation require to solve the problem of the electrical coupling with the VCSEL, we have tried to use a sinusoidal signal at a period corresponding to \( \tau/2 \) for addressing two patterns of pulses within the cavity. The result is shown in Fig. 7, where two localized pulses are separated by half of the round-trip time, in correspondence.

![Figure 6. Modulation of the bias current can be used to address temporal LS in the cavity. \( J = 210 \text{ mA}, \tau = 14.8 \text{ ns}, \nu_{\text{mod}} = 1/\tau \). As the modulation peak-to-peak amplitude is increased a) 297 mA, b) 306 mA, c) 349 mA, d) 405 mA and e) 454 mA, the number of pulses close to the peak of the modulation is incremented by one. f) The reverse sequence appears when decreasing the modulation amplitude. The modulation value at which transition from \( N - 1 \) pulses to \( N \) pulses happens is slightly different with respect the modulation value at which the inverse transition from \( N \) pulses to \( N - 1 \) occurs, denoting a strong multistability.](image)
to the modulation peaks.

III. THEORETICAL RESULTS

The proper study and the simulation of PML lasers is a demanding problem from the computational point of view: while pulses may form on a relatively short time scale of a few tens of round-trips, the pulse characteristics only settle on a much longer time scale [29]. If anything, the complex transverse dynamics [20] also present in our case shall slow down the dynamics even further. In particular, we consider here extremely long cavities and therefore very long delays for which with the pulse to round-trip aspect ratio is \( \sim 10^3 \). The necessary presence of noise prevents the use of adaptive step-size algorithms, which would be particularly suitable for such strongly nonlinear problems.

Hence, for the sake of simplicity, we use a purely temporal model as for instance the generic delay differential equation model of [30] which generalizes Haus’ model as it encompasses both the pulsating and the steady regimes. While more detailed results could also be obtained with a traveling-Wave approach [31], we found that such a simple model was sufficient to give a good qualitative agreement with the experimental results. Denoting by \( A \) the amplitude of the optical field, \( G \) the gain, and \( Q \) the saturable absorber losses, the model reads

\[
\frac{\dot{A}}{\gamma} = \sqrt{\kappa} \exp \left[ \frac{(1 - i \alpha) G \tau - (1 - i \beta) Q \tau}{2} \right] A - A, \quad (1)
\]

\[
\dot{G} = g_0 + \Delta g \sin \left( \frac{2 \pi t}{\tau_m} \right) - \Gamma G - e^{-Q} (e^{G} - 1) |A|^2, \quad (2)
\]

\[
\dot{Q} = q_0 - Q - q \left( 1 - e^{-Q} \right) |A|^2, \quad (3)
\]

where time has been normalized to the SA recovery time, \( \alpha \) and \( \beta \) are the linewidth enhancement factors of the gain and absorber sections respectively, \( \kappa \) is the fraction of the power remaining in the cavity after each round-trip, \( g_0 \) is the pumping rate, \( \Delta g \) and \( \tau_m \) the amplitude and the period of the modulation of the gain, \( \tau \) is the gain recovery rate, \( q_0 \) is the value of the unsaturated losses which determines the modulation depth of the SA, \( q \) is the ratio of the saturation energy of the SA and of the gain sections and \( \gamma \) is the bandwidth of the spectral filter. In Eq. 1 the subscript \( \tau \) denotes a delayed value of the variable, \( x_{\tau} = x (t - \tau) \). This delay renders the dynamical system infinitely-dimensional and it describes the spatial boundary conditions of a cavity closing onto itself. As such it governs the fundamental repetition rate of the PML laser. We use standard parameter values that closely represents the experimental situation: \( \kappa = 0.8 \), \( s = 15 \), \( q_0 = 0.3 \), \( \Gamma^{-1} = 66.66 \), \( \gamma = 3 \) and \( \tau = 3000 \).

Assuming a SA recovery time of 5 ps, this corresponds to a gain recovery of 333 ps, a bandwidth FWHM of 400 GHz, i.e. 1.3 nm around 980 nm in good agreement with the VCSEL resonance linewidth measurement with a tunable laser and a time delay of 15 ns. Importantly, we stress that the existence of PML regimes below the lasing threshold is mostly independent of the phase-amplitude couplings which explains while in the past we choose \( \alpha = \beta = 0 \) for the sake of simplicity. However, we set here the more realistic values of \( \alpha = 1.5 \) and \( \beta = 1 \). The lasing threshold is determined by the pump level \( g_0 \) for which the off solution \( (A, G, Q) = (0, \Gamma^{-1} g_0, q_0) \) becomes linearly unstable; in our case it is given by \( g_{th} = \Gamma (q_0 - \ln \kappa) \).

Above threshold, the continuous wave (CW) solution bifurcates supercritically from the off state, but PML is found below threshold where the pulsating branches emerge as saddle-node bifurcation of limit cycles, see [14] for more details.

A. Multi-pulse pattern

![Figure 8. Different co-existing time traces of the temporal intensity over several and two round-trips for \( \Delta g = 0.23 \) and \( \tau_m = \tau + 0.2 \). The dashed green lines (not to scale) represent the modulation of the gain.](image-url)
via the gain dynamics which is apparent in the fact that they do not have exactly the same heights. Gain dynamics is known to induce repulsive interaction [27] between pulses. Intuitively, pulses will move toward higher gain, which corresponds to maximizing the distance between them, hence explaining the repulsive nature of the interaction. However, here they cannot move away from each other since the gain varies in time and go below the minimal value at which they can exists. As such, the equilibrium distance between pulses comes for the complex interplay between the repulsive interaction between pulses and the presence of the modulation that acts as an external potential.

1 and 4 pulses, say for instance at $\Delta g = 0.22g_{th}$, this certainly implies that we can also have 2 and 3 pulses. As such the diagram of Fig. 10 only gives the extreme envelope of a more complex multi-stable diagram.

A rich multistability diagram for the number of pulse as a function of the gain modulation $\Delta g$ can be appreciated in Fig. 10, in agreement with experimental results of Fig. 6f). For increasing modulation starting from the off solution, we find in Fig. 10 that small amplitude broad peaks appear for $\Delta g = 0.2g_{th}$. This corresponds to the laser reaching exactly threshold at some instant since $g_0 = 0.8g_{th}$ and such low and broad intensity pulses (not shown) can be interpreted as locally amplified spontaneous emission. The first real LS appears at $\Delta g = 0.23g_{th}$. Upon increasing further the modulation amplitude, the pulses energy slowly increases which explains the slope of the plateau (the experiment considering only the pulse number) while the jumps correspond to the apparition of another pulse. Notice in addition that the pulses within a group pulse number) while the jumps correspond to the apparition of another pulse. Notice in addition that the pulses within a group

Experimental results also shown a strong sensitivity of dynamics with respect to the modulation period. We notice that the period of the PML regimes scales as $T \sim \tau + \gamma^{-1}$, see [30] for instance. The pulsewidth is proportional to the inverse of the filter bandwidth and there is a deep relation between pulsewidth and deviation of the pulse train period from the value given by the time of flight. One can intuitively consider that $\gamma$ is the “inertia” of the filter. As such, if a short pulse, say a Dirac delta, is re-injected after a time of flight $\tau$, the filter needs a typical time $\gamma^{-1}$ to filter and re-emit it. In our case, we found that for $g_0 = 0.8g_{th}$ the fundamental period is $\tau_m = \tau + 0.26$ which explains why we choose this value in our analysis. We depict in Fig. 11 the sensitivity to the modulation period for a low amplitude modulation of $\Delta g = 0.23g_{th}$. One can verify from Fig. 10 that in this case we can have either 1 or 4 pulses. In order to have a simple picture, we start from an initial condition with as many pulses as possible, i.e. the harmonic mode-locking of maximal order found just threshold at $g_0 = 1.2g_{th}$ in the absence of modulation $\Delta g = 0$. Then we suddenly decrease $g_0$ toward $g_0 = 0.8g_{th}$ and set the modulation to $\Delta g = 0.23g_{th}$. Such a method allows us
to find the maximal number of packed pulse that the system can support as a function of the detuning of the modulation frequency. We see in Fig. 11 that indeed a precise tuning is necessary as the full width of the resonance tongue is 0.5, i.e. of the same order of magnitude than the pulselength. Saying that the resonance tongue is of the order of the pulselength yields a bandwidth of $\Delta \nu \sim 10 \text{ ps}/\text{rad} \sim 40 \text{ kHz}$ in good qualitative agreement with the experimental results.

IV. CONCLUSIONS

We have shown that electrically biased broad-area VCSELs with optical feedback from a RSAM can be operated in a regime where the passively mode-locked pulses can be addressed and controlled individually when the compound system is operated below threshold. The strong multistability between the off solution and a large variety of pulsating solutions with different number and arrangements of pulses per round-trip, demonstrate that the mode-locked pulses are mutually independent. We show how a modulation of the bias current allows controlling the number of the pulses travelling within the cavity, paving the way an arbitrary pattern generator of picosecond pulses.

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