A Local Realistic Model for Two-Particle Einstein-Podolsky-Rosen Pairs

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The 20th century saw the birth, growth, and prosperity of quantum mechanics, and its influence in every aspect of science and technology. The prediction of quantum theory has been found in excellent agreement with experimental results in an extremely wide range. However, the interpretation of quantum mechanics has been widely disputed over a long time [1-16], notably coming from Einstein in one side and Bohr in the other. The central point of the controversy is whether or not the wave function is a complete description of an individual system at the quantum level. In a historic paper [1], Einstein, Podolsky, and Rosen (EPR) analyzed a system consisting of two spatially-separated but quantum-mechanically correlated particles, and argued that both particles could simultaneously have predetermined values of non-commuting operators, such as position and momentum. Thus, they concluded that quantum mechanics is not complete. In regard to this kind of entangled system, Bell proved an inequality [11] based on which it is possible to test quantum mechanics and the opponent deterministic hidden-variable theory [9-14] in a quantitative manner. Most experiments to date have favored quantum mechanics [12,17-20]. Today, it has been well-established in the orthodox formulation of quantum mechanics that spooky nonlocal characteristics are indeed present in these EPR pairs. This, together with the abandonment of realism (enforced in the wave-packet reduction hypothesis in old quantum measurement problem) represents the central viewpoint of orthodox quantum mechanics on physics in microscopic world. Both viewpoints will drastically change our concepts and philosophy of nature.

Is it possible to avoid such a radical revolution of concepts about nature by using some local realistic viewpoints to understand microscopic world while pertaining agreement with experimental observations? In this paper we will give a positive answer to this question. We will propose a local realistic model which directly comes from standard quantum mechanics for many-particle systems.

To this end, we first consider in general the motion of two independent particles under the law of quantum mechanics. One typical case is that these particles are separated in space, and noninteracting with each other. The Schrödinger equation for this two-particle system is written as

$$i\hbar \frac{\partial}{\partial t} \Psi = H \Psi = (H_1 + H_2)\Psi. \quad (1)$$

where $H_1(r_1)$ and $H_2(r_2)$ are the Hamiltonian for particles 1 and 2, respectively. From physical intuition and simple argument one can find that the system wave function $\Psi$ is just the direct product of the wave function of each particle, namely

$$\Psi(r_1, r_2) = \psi_1(r_1)\psi_2(r_2). \quad (2)$$

$$\psi_1$$ and $\psi_2$ satisfy

$$i\hbar \frac{\partial}{\partial t} \psi_1 = H_1 \psi_1; \quad i\hbar \frac{\partial}{\partial t} \psi_2 = H_2 \psi_2. \quad (3)$$

The general solution of Eq. (3) is

$$\psi_1 = \sum_{\lambda_1} c_{1,\lambda_1} \psi_1(r_1, \lambda_1), \quad \psi_2 = \sum_{\lambda_2} c_{2,\lambda_2} \psi_2(r_2, \lambda_2), \quad (4)$$

where $\psi_1(r_1, \lambda_1)$ and $\psi_2(r_2, \lambda_2)$ are eigenstates of particles 1 and 2 under Hamiltonian $H_1$ and $H_2$, respectively, with $\lambda_1$ ($\lambda_2$) being their eigenvalues, and $c_1$ and $c_2$ are the superposition coefficients. Eqs. (2)-(4) verify that there is no mechanical correlation between particles 1 and 2, as is exactly what we have supposed that these two particles are independent. Any external influence on one particle will not affect the wave function of the other particle.

The above analysis is also applicable to a special two-particle system, the free EPR pair, since what we only require in Eqs. (1)-(4) is that two particles are far-away separated and non-interacting, no matter these two particles are either uncorrelated or correlated. To be more
specific, we consider Bohm’s scheme with a pair of spatially separated spin-1/2 atoms, which is produced from the dissociation of a molecule in the spin singlet (total spin 0) state (e.g., Hg molecule)\[5,6\]. Up to some time \( t = 0 \), the wave function of this combined system is

\[
\Psi = \frac{1}{\sqrt{2}}[\psi_{+}(1)\psi_{-}(2) - \psi_{-}(1)\psi_{+}(2)],
\]

where \( \psi_{+}(1) \) means that atom 1 has spin +\( \hbar/2 \), and so on. As \( \Psi \) is rotationally invariant, the spin axis “\(+\)” can be selected along any direction in space. This antisymmetric spin wave function is very common in molecular bound states. A good example is the spin singlet state of a hydrogenic molecule, which is formed from the group motion of two electrons under nuclear attraction. It should be emphasized that two free hydrogenic atoms cannot form a spin singlet state.

What happens if the two atoms are separated by a method that does not influence the total spin? In the orthodox formulation, the wave function in Eq. (5) keeps unchanged even when the two atoms have separated so far away that they cease to interact, namely, atoms in both the bound state and free state have the same spin-singlet state. And it is the nonlocality (something of action-at-a-distance) that uphold the perfect correlation of the spins of the two atoms. Once one measures one spin and finds it is along \( \hat{n} \) direction, the other spin is immediately known to be along \(-\hat{n} \) even if no measurement is performed. Although spooky, this assumption has found strong experimental supports for a long time. These experiments are closely related to the test of Bell’s inequality, and found to favor quantum mechanics \[10,15-18\]. The joint outcome of the spin of particle 1 along the direction \( \mathbf{n}_1 \) and that of particle 2 along direction \( \mathbf{n}_2 \) in orthodox quantum mechanics is \[12\]

\[
E_{\Psi}(\mathbf{n}_1, \mathbf{n}_2) = \langle \psi(\mathbf{n}_1) | \sigma_1 \cdot \mathbf{n}_1 | \psi(\mathbf{n}_1) \rangle \times \langle \psi(\mathbf{n}_2) | \sigma_2 \cdot \mathbf{n}_2 | \psi(\mathbf{n}_2) \rangle = -\mathbf{n}_1 \cdot \mathbf{n}_2,
\]

where \( \sigma \) is the vector of Pauli matrices. The result in Eq. (6) clearly violates Bell’s inequality [11-14]. It is such kinds of violation that validate orthodox quantum mechanics and refutes any hidden-variable theory.

In our model, the system wave function \( \Psi \) has the form of Eqs. (2) and (4), each atom having its own wave function independent of the other. Then, because the total spin is conserved in every process of molecular dissociation, the two particles must have a specific spin vector antiparallel to each other, and thus both are in a pure spin state each time. This yields to

\[
\Psi = \psi_1 \psi_2,
\]

\[
\psi_1 = \psi_+(1, \mathbf{n}), \quad \psi_2 = \psi_-(2, \mathbf{n}),
\]

where \( \mathbf{n} \) is a random unit vector of direction uniformly distributed in space, in accordance with the rotational invariance of original spin-singlet state in the molecule. The wave function Eq. (7) changes its form under axis rotation if it refers to a single spin pair. However, since here it actually represents the whole ensemble of spin pairs, it is rotationally invariant, as is clear from the fact that \( \mathbf{n} \) is uniformly distributed in space. This ensemble is composed of many spin pairs each of which is composed of two spins with definite but antiparallel directions.

The physical picture involved in Eq. (7) is quite simple. If we assume the spin-singlet molecule as an atom source, then this source emits at each time a pair of atoms who have definite spin vector antiparallel to each other, but what direction the spins are along is unknown. This is similar to spontaneous emission of photon from excited atoms. Each time the emitted photon has a definite polarization, but the direction is unspecified, all are possible. One can utilize this similarity to calculate the quantum-mechanical expectation value (statistical average) of all components of each spin, \( \hat{S}_x \), \( \hat{S}_y \), and so on, which are of course all zero, the same as predicted by the orthodox formulation. The joint outcome can be found according to Fig. 1, which is

\[
E(\mathbf{n}_1, \mathbf{n}_2) = \langle \psi_1(\mathbf{n}_1) | \sigma_1 \cdot \mathbf{n}_1 | \psi_1(\mathbf{n}_1) \rangle \times \langle \psi_2(\mathbf{n}_2) | \sigma_2 \cdot \mathbf{n}_2 | \psi_2(\mathbf{n}_2) \rangle = -\mathbf{n}_1 \cdot \mathbf{n}_2,
\]
direction the spin is along, then neither our model nor the orthodox formulation can give definite results, only statistical results can be predicted definitely.

Several important differences can be found between our model and the orthodox formulation. One significant consequence of Eq. (7) is that all components of each spin can be measured simultaneously, since now the two spins are noninteracting while in perfect correlation and thus allow measuring one particle without affecting the other. This can be accomplished by means of two spin-measuring apparatuses (Stern-Gerlach magnets) oriented perpendicular to each other, say, along the x-axis and y-axis, respectively. Particles 1 and 2 passing through apparatuses 1 and 2 will have their spin components measured to be $S_{1x}$ and $S_{2y}$. Because of perfect antiparallel correlation of the two spins, we at once know $S_{1y} = -S_{2y}$, and $S_{2x} = -S_{1x}$. This is in contradiction with Heisenberg’s uncertainty principle in the orthodox formulation, which asserts that $S_x$ and $S_y$ of a particle cannot be measured simultaneously because they are non-commuting operators.

Another significant point of Eq. (7) is that the conventional concept of wave-packet reduction [5,6] is not present in our model, because now each atom is in a pure state with a definite spin vector. The measurement does not determine what spin state the atom is in, but just tells us what it is in. From this viewpoint, the measurement is not an inseparable part combined with the quantum system considered, a manner the orthodox formulation strongly stresses. If one wishes to uphold the concept of wave-packet reduction, then this process must have happened during the process of molecular dissociation. This also has nothing to do with external measurements by the observer.

In our model the perfect antiparallel correlation of the two spins is maintained by the conservation law, while in the orthodox formulation it is preserved by the non-local entanglement of spin wave function and realized in measurement by the mechanism of wave-packet reduction. In experiments to test Bell’s inequalities [17-20], two loopholes are generally supposed to leave the conclusion uncertain. One is that only a small subset of all pairs are detected due to low detection/collection efficiency of the apparatus, the other is that the Einstein locality condition might not be maintained strictly. These two loopholes are, however, not present in our model. The measurement process involved is always local, and a small subset always represents fairly the whole ensemble in a statistical manner. It is not the communication between the two particles, but the strict maintenance of conservation law in nature that upholds the perfect correlation. From this point of view, our model agrees with experimental observations in a more natural manner than the orthodox formulation does.

Now we can see that our proposed model is realistic and local, and at the same time it is in as good agreement with experimental observations as the orthodox formulation, when two-particle EPR pairs are concerned. From the viewpoint of the orthodox formulation, our model essentially lies inside the framework of hidden-variable theory. Then, since the hidden-variable theory constructed here is local, realistic, and makes same predictions as orthodox quantum mechanics does, one is led to conclude that Bell’s theorem is not always true in regard to two-particle EPR pairs.

Following the concept implied in above arguments, we turn to analyze the scheme of entangled pair originally proposed by EPR [1]. The system they studied consists of two particles, and lies in the state $\Psi(x_1, x_2) = \delta(x_1 - x_2 - a)$, which is the eigenfunction of the operator $x_1 - x_2$ with eigenvalue $a$, and of the operator $p_1 + p_2$ with eigenvalue $0$. We emphasize that this $\delta$-type wave function implies very strong interaction between the two particles. This pair as a whole can not be subject to a simultaneous measurement of the position and momentum without affecting each other. But the situation changes completely when the pair is dissociated at time $t_0$ and becomes noninteracting by a process that does not influence the total momentum. Each particle is now in a free state, with the momentum ($p_1$ and $p_2$) specifying its motion feature (a pure state). The conservation of total momentum satisfied at each event leads to a perfect correlation of the momentum $p_1 + p_2 = 0$, from which we derive

$$x_1(t) - x_2(t) = a + p_1\left(\frac{1}{m_1} + \frac{1}{m_2}\right)(t - t_0)$$

under assumption of free propagation of particles. $m_1$ ($m_2$) is the mass of particle 1 (2), $t_0$ is the time when the particle dissociation occurs, and $t$ is the time at measurement.

Whether or not the position of the two particles persists a perfect correlation depends on the accuracy of determining at what time the dissociation of the molecule takes place. If this can be accomplished, we immediately obtain a perfect correlation in both momentum and position, and conclude that both the position and momentum of each particle can be measured simultaneously, and that Heisenberg’s uncertainty principle is violated. Even if we can not acquire a perfect correlation in position, we can still simultaneously measure the position and momentum of one of the two particles. For example, we now decide to see particle 1. We first measure its position and get a value of $x_1$, at the same time we measure the momentum of particle 2 and get a value of $p_2$. Due to the perfect correlation of the momentums, we at once know $p_1 = -p_2$, and arrive at a position that the uncertainty principle is violated. Note that our argument is different from original EPR’s, where the two particles are supposed to be always in a perfect entangled state (for both position and momentum) when they are subject to measurements. In this entangled state, the momentum and position of a particle can not be measured simultaneously, because of the strong entanglement between the two composites. We can further argue that in Bohm’s scheme the violation of uncertainty principle remains even if we only
have a system with a perfect correlation in one of the spin components instead of all components.

In the orthodox quantum theory, it is assumed that the wave function completely specifies the physical state of an individual system. According to our local realistic model, it is possible to obtain through conservation laws in nature (which hold true in all mechanical theories) to obtain simultaneous knowledge of non-commuting variables such as different components of a spin, and the position and momentum of a particle. This violates Heisenberg’s uncertainty principle, one of the basic foundations of orthodox quantum mechanics. Therefore, it is possible to obtain more complete information about individual system beyond that allowed by the wave function in the orthodox formulation, while at the same time arrive at the same statistical information on an ensemble of individual systems as that predicted by the orthodox formulation. This suggests that the wave function in orthodox quantum theory is only a complete description of an ensemble of individual particles, but not a complete description of individual particles. Let us look at the wave functions Eq. (5) and Eq. (7) once again. They are equivalent to each other when they are referred to an ensemble of EPR pairs, while totally different when referred to an individual pair. In the orthodox formulation, the wave function is always assumed to describe an individual system. Now one finds it is more likely that the wave function [like Eq. (5)] only describes an ensemble of individual systems. It is this difference in the wave function for an individual system that cause significant results of the proposed model beyond the orthodox formulation.

In summary, we have proposed a local realistic model for quantum mechanics of two-particle EPR pairs in the framework of standard quantum mechanics for many-body systems. In this model, the spooky nonlocal characteristics present in the space-separated free EPR pairs as assumed in the orthodox formulation is abandoned. Also abandoned is the concept of wave-packet reduction in the orthodox formulation which is assumed to happen when external measurement occurs. It is the strict obedience of conservation laws in each event at the quantum level that maintain the perfect correlation of two spatially-separated particles. With a different starting viewpoint from orthodox quantum mechanics, the proposed model allows more information on individual particles beyond the wave function in the orthodox formulation to be obtained. At the same time it leads to the same statistical information on an ensemble of individual particles as predicted by the orthodox formulation. It may be expected that the proposed local realistic model will help people to understand quantum mechanics in a conceptually easy way, as has been the case for classical physics.

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FIG. 1. Schematic configuration of an entangled spin-1/2 pair with antiparallel spins $s_1$ and $s_2$ passed through two Stern-Gerlach apparatuses along the $n_1$ and $n_2$ directions with an inclination of $\theta$. Due to rotational invariance and completely unpolared nature of the spin-pair beam, $s_1$ can be set to be parallel to $n_1$. 
Fig. 1  Li