REGGE TRAJECTORIES IN QCD

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We discuss some problems concerning the application of perturbative QCD to high energy soft processes. We show that summing the contributions of the lowest twist operators for non-singlet $t$-channel leads to a Regge-like amplitude. Singlet case is also discussed.

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The famous asymptotic freedom of quantum chromodynamics (QCD) [1] enables one to use perturbation theory (PT) for large momenta. However, any physical process also involves small momentum scales $p^2$ (e.g., quark and hadron masses). As a rule, this results in logarithmic contributions $\ln(Q^2/p^2)$ singular for $p^2 = 0$ (mass singularities) [2]. In such a situation $p^2$ cannot be neglected. Within PT it is possible to show that for inclusive [2–4] and some exclusive hard processes (see, e.g., [4, 5]) the $Q^2$-dependence of the corresponding amplitude $T(Q^2,p^2)$ can be factorized from the $p^2$-dependence

$$T(Q^2,p^2) = Q^N \{ E(Q^2/\mu^2, \alpha_s(\mu)) \otimes f(\mu^2, p^2) + R(Q,p) \} ,$$

(1)

where $R$ is sum of power suppressed contributions. Note, that $E \otimes f$ does not depend on a particular choice of $\mu$, the boundary between large and small momenta.

Our aim here is to recall [6] that a similar approach is also possible for soft binary processes $12 \to 1'2'$ in the region $S = (s-u)/2 \gg |t|$, $m_{\text{hadr}}$ and attract attention to some dangerous points in widespread constructions. Just like in our derivation of factorization formula (1) in Refs. [3–6], we start with the $\alpha$-representation [8]

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*This talk was motivated by papers [6], [7].
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Fig. 1. Illustration of Equation (3).

\[ F(S, t) \sim \int_0^\infty \prod_\sigma d\alpha_\sigma D^{-2}(\alpha) G(S, t, \alpha) \exp \left[ iS\alpha + itI(\alpha) - i \sum_\sigma \alpha_\sigma m_\sigma^2 \right], \quad (2) \]

which has many advantages for analysis of the large \( S \) behaviour of \( F \).

In particular, according to (2), integration over a region where \( A(\alpha) > \rho \) gives for \( S \to \infty \) an exponentially damped contribution \( O[\exp(-S\rho)] \). Hence, contributions having a power \( O(S^{-N}) \) behaviour for \( S \to \infty \) are due to integration over regions where \( A(\alpha) \) vanishes. Three main possibilities to get \( A(\alpha) = 0 \) are: 1) short-distance (SD, or small-\( \alpha \)) regime, when \( \alpha_\sigma^1 = \alpha_\sigma^2 = \ldots = \alpha_\sigma^n = 0 \) for some lines \( \sigma_1, \sigma_2, \ldots, \sigma_n \); 2) infrared regime, when \( \alpha_\sigma^1 = \alpha_\sigma^2 = \ldots = \alpha_\sigma^n = \infty \) for a set of lines \( \{\sigma_1, \sigma_2, \ldots, \sigma_n\} \); 3) pinch regime, when \( A(\alpha) = 0 \) for nonzero finite \( \alpha \)'s because \( A(\alpha) \) is a difference of two positive terms.

In a wide class of processes, the pinch regime does not work (see Refs. [3], [9]). Since for \( A(\alpha) = 0 \) the amplitude \( F \) lacks its \( S \)-dependence, one must find the subgraphs \( V, L \) with the property that, when lines of \( V \) subgraphs are contracted into point \( (\alpha_\sigma = 0) \) and/or lines of \( L \) subgraphs are removed \( (\alpha_\sigma = \infty) \), the resulting diagram does not depend on \( S \). The power \( N \) of such a \( O(S^{-N}) \) contribution may be easily estimated by “twist counting rules”: \( t_{SD}^V \lesssim S^{4 - \sum_i t_i} ; t_{IR}^L \lesssim S^{4 - \sum_j t_j} ; t_{SD;IR}^{V, L} \lesssim S^{4 - \sum i t_i - \sum j t_j} \), where \( t_i (t_j) \) is twist of the \( i \)-th \( (j \)-th) external line of the subgraph \( V (L) \). Since \( t_{i,j} = 1 \) for \( \psi, \bar{\psi} \)-fields and the field strength \( G_{\mu\nu} \), whereas \( t_{i,j} = 0 \) for the vector potential \( A_\mu \), it is necessary in QCD (in covariant gauges) to sum up over external gluon lines of the subgraphs \( V, L \).

In a Yukawa-type theories it has been shown 40 years ago [9] that, for amplitudes with positive signature, the logarithmic terms \( (\log S)^N \) appear only from the SD integration, and their summation gives the following representation (see Fig. 1)

\[ f^+(j, t) = C^T(j, t)[1 - B(j, t)v(j)]^{-1} v(j)C(j, t) \quad (3) \]

for the Mellin transform of the scattering amplitude \( F^\pm(S, t) \):

\[ F^\pm(S, t) = \frac{1}{2t} \int_{-\infty}^\infty dj \left| \frac{S^j(e^{i\pi j} + 1)}{\Gamma(j + 1)\sin(\pi j)} \right| f^\pm(j, t), \quad (4) \]

where \( \pm \) stands for signature, \( C, v \) and \( B \) are matrices \( 2 \times 2 \) or \( 3 \times 3 \). According to the representation [3], the Mellin transform \( f^\pm(j, t) \) possesses moving \( (t\)-dependent)
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Fig. 2. The leading terms of a particular box diagram.

Regge poles due to zeros of \( \text{Det}[1 - B(j, t)v(j)] \). It has also fixed \((t\text{-independent})\) singularities in the complex \( j \)-plane accumulated in the function \( v(j) \). The type of fixed singularities depends on the ultraviolet behavior of the effective coupling constant. Note that \( v(j) \) has the form of LLA result, corrected by next approximations, while Regge pole structure cannot be seen from NLLA, NNLLA, etc.

The Mellin transform of Eq. (4) has the following structure in the \( \alpha \)-representation

\[
f^\pm(j, t) \propto \int \prod_\sigma d\omega \sigma^{\mathcal{D}-2}(\alpha)g(j, t, \alpha)|A(\alpha)|^j[\theta(A) \pm \theta(-A)] \exp[iJ(\alpha, t, m^2)], \tag{5}
\]

where \( g(j, t, \alpha) \) is a polynomial in \( j \) (it corresponds to the function \( G \) in Eq. (2)) and \( A \) is the coefficient in front of the large variable \( S \). The asymptotic behavior of \( F(S, t) \) for large \( S \) is determined by the rightmost singularities of its Mellin transform \( f(j, t) \). These are the poles \( 1/(j - N) \) generated by integrations corresponding to the regimes discussed earlier. By the twist counting rules given above, the leading poles (at \( j = 0 \)) are due to the SD subgraphs \( V_i \) with 4 external lines, since the IR-regime in a Yukawa theory gives only non-leading poles at \( j = -1, -2, \ldots \).

Furthermore it was proven \([10]\) that for even \( j \) the pinch regime contributes only to the negative signature amplitude \( F^-(S, t) \), while for the odd \( j \) – only to positive signature amplitude \( F^+(S, t) \). That is the reason why for \( F^+(S, t) \) it is sufficient to consider only the SD poles at \( j = 0 \) (Eq. (3)).

In general, the SD-subgraphs \( V_i \) may contain smaller \( V \)-subgraphs with 4 external lines, and the total singularity due to the SD-regime of \( V_i \) may be a multiple pole \( j^{-N_i} \). Treating a particular diagram as a ladder composed by 2-particle irreducible blocks \( k_j \), we see that the maximal value of \( N_i \) is determined by the number of \( k_j \)'s inside \( V_i \) (and also by the number of the UV-divergent subgraphs inside \( V_i \)). In Fig. 2, the pole parts are circled by the thin (red) line and the regular ones are marked by the blue lines. It immediately gives equation \( \mu \frac{d\mu}{d\mu} = jv = r + bv^2 \), where \( b \) and \( r \) are some functions regular at \( j = 0 \). Using this equation, one can sum up all leading poles at \( j = 0 \) due to the SD-regime of all possible \( V \)-subgraphs (for summation of all poles see Ref. [5]), i.e. to sum all leading \( \log^N(S/p^2) \) contributions. The solution has square root branch points in the complex \( j \)-plane \([9]\) (see also Ref. [11]). However, \( v(j) \) has also poles due to divergent subgraphs. These poles
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Fig. 3. The same as Fig. 11, but for the case of QCD.

(i.e. \( \log^N(S/\mu_R^2) \)-contributions) are summed by the renormalization group equation

\[
\left( \mu_R \partial / \partial \mu_R + \beta(g) \partial / \partial g - 4 \gamma_{\psi} \right) v(j, \mu_R, g, \mu) = 0.
\]

Generally \( \mu_R \) and \( \mu \) are quite different scales but if we take \( \mu = \mu_R \), we obtain the combined equation

\[
\beta(g) \partial v / \partial g = \left( j - 2 \gamma - 4 \gamma_{\psi} \right) v - b v^2 - r.
\]

In the lowest order of PT
\( b = 1 \), \( r \sim \gamma \sim \gamma_{\psi} \sim g^2 \), \( \beta(g) \sim g^3 \), and the solution of this equation has condensing poles at \( j = 0 \) [9, 11, 12].

Summarizing, in addition to a fixed singularity near \( j = 0 \), the amplitude \( F(S,t) \) has a Regge-type behaviour

\[
F(S,t) \sim C^2(t)S^{\alpha(t)}
\]

for large \( S \). To find the function \( \alpha(t) \) explicitly, one must solve the equation

\[
\text{Det}[1 - B(j, t, \mu_R, g, \mu, m)] v(j, \mu_R, g, \mu) = 0.
\]

In fact [6], \( \alpha(t) \) does not depend on \( \mu \) and \( \mu_R \): \( \alpha(t) = \phi(m_0^2/t, t/\mu^2, \tilde{g}(\mu^2)) = \phi(m_0^2/t, 1, \tilde{g}(t)) \). Hence, one may try to calculate the Regge trajectories in the region where \( g(t) \) is small, e.g. in QCD for sufficiently large \( t \).

In QCD, for non-singlet \( t \)-channel one faces complications discussed earlier. First, SD-subgraphs \( V_i \) may have an arbitrary number of external gluon lines. Still, if the \( t \)-channel is color singlet, the only change is (see e.g. Ref. [3]) a path-ordered exponential between the \( x \) and \( y \) points for all bilocal operators \( \bar{\psi}(x)\Gamma\psi(y) \) entering into \( B \)- and \( C \)-functions. For local operators, this corresponds to the change \( \partial_\mu \rightarrow D_\mu = \partial_\mu - ig\hat{A}_\mu \). The second complication is due to the IR-regime (soft exchanges, see Fig. 3). However, if the \( t \)-channel is color singlet, then the sum of all soft exchanges gives only power corrections in each order of perturbation theory. Thus, all terms responsible for the leading power contribution have the structure of Fig. 1 and as a result, we get Eq. 3.

Note, that the function \( B(t) \) describes the long-distance dynamics, and one must take into account nonperturbative effects, e.g., using QCD Sum Rules approach [13] that assumes non-zero vacuum expectation values of some products of field operators (vacuum condensates). Then only equations for \( B \) and \( C \) are changed. However, one cannot tell what kind of contributions dominates: fixed singularity in \( j \)-plane (LLA type) or a Regge pole. As it is known the Regge poles are dominant for nonsinglet \( t \)-channel. In this sense, improvements of LLA, like including next-to-leading logs,
could be misleading!

We have discussed above the flavor nonsinglet, positive signature amplitude $F_{NS}^+$ only. For $F_{NS}^-$, the pinch regime also gives leading $j$-poles for non-planar diagrams. However, in QCD the non-planar diagrams have an additional color factor $(1/N_c)^2 = (1/3)^2$. This suggests that the pinch contributions in QCD may be suppressed.

For flavor singlet amplitudes $F_S$ in QCD, the poles generated by the pinch regime are at $j = 1$ rather than at $j = 0$ due to the 2-gluon intermediate states. As a result, the asymptotic behavior of Pomeron amplitude $F_S^+$ is determined by a complicated mixture of SD and pinch singularities. The LL results were obtained starting with Ref. [14]. Usefulness of recent NLL results is not very clear for us due to reasons discussed above (namely, what is eventually the leading contribution).

On the other hand, the asymptotic behavior of the odderon amplitude $F_S^-$ has a much simpler structure: it is determined by small distance singularities at $j = 1$ only. The corresponding direct analysis, starting from diagrams, was not done till now. Nevertheless, the representation odderon = colored Pomeron + gluon looks misleading for us.

In conclusion, a lot of important and interesting things are yet not solved in QCD of soft high energy processes, which asks for new young forces and new ideas.

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