Quadrotor Formation Flying Resilient to Abrupt Vehicle Failures via a Fluid Flow Navigation Function

Matthew Romano\textsuperscript{a,\dagger}, Harshvardhan Uppaluru\textsuperscript{b,\dagger}, Hossein Rastgoftar\textsuperscript{b}, Ella Atkins\textsuperscript{a}

Abstract—This paper develops and experimentally evaluates a navigation function for quadrotor formation flight that is resilient to abrupt quadrotor failures and other obstacles. The navigation function is based on modeling healthy quadrotors as particles in an ideal fluid flow. We provide three key contributions: (i) A Containment Exclusion Mode (CEM) safety theorem and proof which guarantees safety and formally specifies a minimum safe distance between quadrotors in formation, (ii) A real-time, computationally efficient CEM navigation algorithm, (iii) Simulation and experimental algorithm validation. Simulations were first performed with a team of six virtual quadrotors to demonstrate velocity tracking via dynamic slide speed, maintaining sufficient inter-agent distances, and operating in real-time. Flight tests with a team of two custom quadrotors were performed in an indoor motion capture flight facility, successfully validating that the navigation algorithm can handle non-trivial bounded tracking errors while guaranteeing safety.

I. INTRODUCTION

Quadrotors are becoming increasingly popular due to advances in sensing, actuation, and computational power. A quadrotor is cheap and highly maneuverable with a wide range of useful applications. A single quadrotor has limited processing and payload carriage capabilities. However, a group of cooperatively controlled quadrotors can achieve difficult tasks with considerable advantages in terms of resilience to failure, mission complexity, and scalability.

Several multi-agent coordination approaches have been investigated with applications ranging from surveillance [1], [2] to formation flying [3], [4], rescue missions [5], wildlife monitoring and exploration [6], precision agriculture [7], cooperative payload transport [8], [9] and hazardous environment sensing [10]. Virtual structure (VS) is a centralized technique in which a group of agents, operating as particles of a virtual rigid body, preserve a strict geometric relationship to one another and a frame of reference [11], [12]. Desired agent trajectories are calculated in a centralized manner through rigid body rotations and translations of the virtual body in 3-D motion space. However, due to the rigid body requirement and a single point of failure, the range of applications using VS is limited. Consensus is one of the most extensively studied cooperative control approaches in which a team of agents reach agreement/consensus by communicating only with their neighbours. It is a decentralized coordination approach and is broadly divided into two categories: consensus without a leader (i.e., leaderless consensus) [13], [14] and consensus with a leader (i.e., leader-follower consensus) [15], [16]. Containment control [17]–[20] is a decentralized leader-follower method where collective motion of all agents is achieved with multiple leaders. Defined by geometric constraints i.e., all agents are contained within a particular area, the follower agents obtain the desired positions through local communication with in-neighbor agents. Based on the principles of continuum mechanics, continuum deformation is a recent multi-agent coordination approach for which agents in a multi-agent system (MAS) are treated as particles of a body deforming under a homogeneous transformation [21]–[24].

The idea of using potential fields for real-time path planning was originally proposed for robot arms [25]. An attractive potential is placed at the goal, repulsive potentials on obstacles, and gradient descent is used to plan a path. This can be called a navigation function. This is a useful approach, however it has issues of local minima. The randomized potential field approach attempts to get around this by executing a random walk whenever the planner gets stuck in a local minimum, but still provides no guarantee of planning completeness [26]. For harmonic functions subject to Laplace’s equations, a navigation function can be constructed

\dagger Both authors contributed equally to this work.

\textsuperscript{a} Authors are with the Robotics Department at the University of Michigan

\texttt{mmroma, ematkins@umich.edu}

\textsuperscript{b} Authors are with the Aerospace and Mechanical Engineering department at the University of Arizona

\texttt{huppaluru, hrastgoftar@arizona.edu}
that only has saddle points as local minima such that they can be easily escaped [27]. This model has been applied to robot formations, but with explicit models of repulsive functions between robots, one goal sink node, and computation time on the order of one second. However, the scenario considered in this paper needs an algorithm to quickly respond to an abrupt failure on the order of milliseconds.

This paper proposes a novel safety recovery algorithm for a multi-quadrotor system (MQS) and experimentally validates the approach in simulation and quadrotor flight experiments. We consider each quadrotor in a MQS as a finite number of particles in an ideal fluid flow pattern. By classifying quadrotors as healthy and failed agents, we consider failed quadrotors as singularity points of the fluid flow. We then define desired trajectories of the healthy agents along flow streamlines so that the failed quadrotors are safely partitioned outside the motion space. This approach is called containment exclusion mode (CEM) throughout this paper.

Compared to the existing literature, this paper offers the following contributions:

1) We propose a real-time, computationally efficient CEM navigation algorithm that dynamically modifies quadrotor sliding speed to maintain a desired maximum speed for all quadrotors.

2) We formally specify safety and collision avoidance by providing two CEM safety theorems. Theorem 1 applies to a large quadrotor system with multiple obstacles whereas Theorem 2 has been developed for experiments for a single obstacle.

3) We experimentally evaluate the CEM navigation algorithm using a team of quadrotors in formation while validating our CEM safety theorem.

This paper is organized as follows. A problem statement (Section II) is followed by a description of our proposed CEM approach (Section III). Section IV presents our experimental setup, followed by experimental results in Section V. Section VI concludes the paper.

II. PROBLEM STATEMENT

Consider a system of quadrotors, flying in formation, defined by set $S = \{1, \cdots, N_q\}$, where $N_q$ is the number of quadrotors. Let $N_f$ be the number of quadrotors that fail to follow the desired formation and are assumed to hover in this work. The failed quadrotors are identified by set $\mathcal{F} \subseteq S$ and $N_f < N_q$. The remaining quadrotors are termed as healthy quadrotors because they follow the desired formation, and is defined by the set $\mathcal{H} = S \setminus \mathcal{F}$.

At reference time $t_0$, $N_f$ quadrotors fail to follow the desired formation. The failed quadrotors are enclosed by virtual obstacles as defined below in Assumption 1. The healthy quadrotors should now follow the safety-recovery approach called CEM where the quadrotors in $\mathcal{H}$ should avoid the failed quadrotors and safely pass them before continuing with their individual trajectories.

Given the problem setup defined above, we first develop a navigation function where the healthy quadrotors are treated as particles in ideal fluid flow combining uniform flow and doublet flow in the $x-y$ plane.

III. APPROACH

The set $\mathcal{F}$ satisfies the following assumption:

\textbf{Assumption 1:} We assume failed quadrotor is enclosed by a virtual obstacle defined as no-fly zone that is completely contained within an enclosed cylinder of radius $a_f$, centered at $z_f = (x_f, y_f)$, elongated in the $z$-direction (e.g. Figure 1).

This ideal fluid flow pattern is generated by the complex function

$$f(z) = \sum_{f \in \mathcal{F}} \left( z - z_f + \frac{a_f^2}{z - z_f} \right),$$

where $z = x + jy$ is the complex variable, $z_f = x_f + jy_f$ is the position of singularity point $f \in \mathcal{F}$ with finite set $\mathcal{F}$ identifying all singularities in the $x-y$ plane, and $a_f = a_f + \delta + \epsilon$ is the planned exclusion radius. $a_p$ is larger than the actual exclusion radius ($a_f$) by a bound on healthy agent controller error ($\delta$) and healthy agent radius ($\epsilon$). Eq. (1) can be rewritten as

$$f(z) = \phi(x, y) + j\psi(x, y)$$

where

$$\phi(x, y) = \sum_{f \in \mathcal{F}} \frac{(x - x_f) \left( (x - x_f)^2 + (y - y_f)^2 + a_p^2 \right)}{(x - x_f)^2 + (y - y_f)^2}$$

and

$$\psi(x, y) = \sum_{f \in \mathcal{F}} \frac{(y - y_f) \left( (x - x_f)^2 + (y - y_f)^2 - a_p^2 \right)}{(x - x_f)^2 + (y - y_f)^2}$$

are called potential and stream functions, respectively. Note that Eq. (2) provides a conformal mapping between $x-y$ and $\phi-\psi$ planes, where Cauchy-Reimann and Laplace equations are satisfied by $\phi(x, y)$ and $\psi(x, y)$.

$$\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}, \quad \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}$$

$$\nabla^2 \psi = 0, \quad \nabla^2 \phi = 0$$

Using the ideal fluid flow model on healthy quadrotors $\mathcal{H}$ at any time $t \geq t_0$, $x$ and $y$ components are constrained to slide along the stream curve

$$\psi((x_i(t_0), y_i(t_0))) = \psi_{i,0}, \forall i \in \mathcal{H}$$
Assumption 2: Velocity in the z-direction is 0 i.e., the altitude remains constant for each healthy quadrotor recovery trajectory.

Assumption 3: As the recovery approach is active, we assume that the failed quadrotors do not leave the enclosing no-fly cylinders of radius \(a_f\).

Using the approach defined above, the main goal of this paper is to present a real-time, safe and efficient CEM algorithm that can plan safe recovery trajectories for all healthy quadrotors. To this end, we first present CEM as a navigation algorithm in Section III-A. Then, we formally specify safety for CEM by providing inter-agent collision avoidance conditions (Section III-B).

A. CEM Navigation Algorithm

Algorithm 1 guides agents along constant \(\psi\) streamlines in the fluid flow pattern. This is performed in real-time such that the “\(t \text{ for loop}\)” (line 12) is run every \(\Delta T\) seconds a total of \(m\) times. \(\Delta T = 10ms\) (100Hz) in this paper. To minimize computational overhead, this method acts as a navigation function, calculating the immediate command for each agent to avoid collisions and then incrementing the trajectory in \(\phi\) on each subsequent loop.

**Algorithm 1 CEM Navigation**

1: procedure CEM(\(F\), \(m\), \(v_{des}\), \(r\)) 
2: Inputs: Failed agents \(F\), number of timesteps \(m\), desired speed \(v_{des}\), healthy vehicle positions \(r\) 
3: Output: Trajectory for each vehicle via sendCommands() 
4: \(r_d \leftarrow r\) 
5: \(\Delta \phi \leftarrow v_{des} \times \Delta T\) 
6: for \(i \leftarrow 1, N_h\) do 
7: \(\phi[i] \leftarrow \phi(r[i], F)\) \> Eq. 3a
8: \(\psi[i] \leftarrow \psi(r[i], F)\) \> Eq. 3b
9: end for 
10: for \(t \leftarrow 1, m\) do 
11: \(\phi_{last} \leftarrow \phi\) 
12: for \(k \leftarrow 1, K\) do 
13: for \(i \leftarrow 1, N_h\) do 
14: \(\phi[i] \leftarrow \phi_{last}[i] + \Delta \phi\)
15: \(r_d[i], r_d[i] \leftarrow \text{CalcXY}(\phi[i], \psi[i], r_d[i])\)
16: end for 
17: \(v_{max} \leftarrow \max_{i \in N_h} ||r_d[i]||\)
18: \(\Delta \phi \leftarrow \Delta \phi \times v_{des}/v_{max}\)
19: end for 
20: sendCommands(r_d, \(r_d\)) 
21: sleep(\(\Delta T\)) 
22: end for 
23: end procedure

\(\phi\) and \(\psi\) values for each agent are initialized to their starting positions. Also, sliding speed \(\Delta \phi\) is set to match the desired vehicle velocities for the non-distorted case. At every time step, \(\phi\) is increased by \(\Delta \phi\) and the resulting position and velocity for each vehicle is calculated using “CalcXY” (Alg. 2). Maximum velocity \(v_{max}\) is then used to update \(\Delta \phi\) for the next iteration to more closely match \(v_{des}\) (Line 20). This update step assumes \(v_{max} = C \Delta \phi_{last}\) and wants to calculate \(\Delta \phi_{next}\) such that \(v_{des} = C \Delta \phi_{next}\) where \(C\) is a constant assumed to encapsulate the gradient for the small step sizes. Therefore, \(\Delta \phi_{next} = \Delta \phi_{last} \times \frac{v_{max}}{v_{des}}\) should calculate the proper \(\Delta \phi\) to obtain \(v_{des}\). This is executed \(K\) times. If the assumption that \(C\) is constant is reasonable, then \(K = 2\) is a good selection which effectively plans twice per time step, first to calculate \(\Delta \phi\) and second to calculate the trajectories using \(\Delta \phi\). \(K = 2\) was used in this paper for the main results and varied for additional results to see its effects. After the \(K\) loop, commands are output for that time step and the process repeats for the next time step.

Alg. 2 calculates the position given \(\psi\), \(\phi\) using a gradient descent inspired approach. On each iteration (\(n = 20\) in this paper), the current estimate is used to calculate the associated \(\phi\) and \(\psi\) and then the estimate is updated in the direction of decreasing error via the inverse of the Jacobian matrix, defined as:

\[
J(x, y, F) = \begin{bmatrix}
\frac{\partial \phi}{\partial x} & \frac{\partial \phi}{\partial y} \\
\frac{\partial \psi}{\partial x} & \frac{\partial \psi}{\partial y}
\end{bmatrix}.
\] (5)

\(r_{noise}\) is added to the update step to escape saddle points on the edge of the exclusion zone as:

\[
r_{noise} = \begin{cases}
|N(0, \sigma)||\hat{r}_a + N(0, \sigma)||\hat{r}_b\quad &\text{if } ||r - r_f|| \leq a_p \\
0, &\text{else}
\end{cases}
\] (6)

where \(N(0, \sigma)\) generates Gaussian noise with zero-mean and standard deviation \(\sigma\), \(\hat{r}_a = \frac{r - r_f}{||r - r_f||}\) is the direction away from the exclusion zone, and \(\hat{r}_b = \begin{bmatrix}0 & 1 \end{bmatrix} \hat{r}_a\) is perpendicular to \(\hat{r}_a\). \(\sigma = 1mm\) was used in this paper. This procedure ensures that commanded positions never violate the planned exclusion radius \(a_p\). Lastly, the first-order difference equation is used to calculate the velocity.

**Algorithm 2 CalcXY**

1: procedure CALCXY(\(\phi, \psi, r\)) 
2: \(r_{-1} \leftarrow r\) 
3: for \(i \leftarrow 1, n\) do 
4: \(\phi[i] \leftarrow \phi(r, F)\) \> Eq. 3a
5: \(\psi[i] \leftarrow \psi(r, F)\) \> Eq. 3b
6: \(J_h \leftarrow J_h(r, F)\) \> Eq. 5
7: \(r \leftarrow r - J_h^1 \left[\phi[i] - \phi\right] + r_{noise}\) \> Eq. 6
8: end for 
9: \(\hat{r} \leftarrow (r - r_{-1})/\Delta T\) 
10: return \(r, \hat{r}\) 
11: end procedure
B. Safety Analysis

Collision avoidance between healthy agents and obstacles is assured via the construction of the ideal fluid flow pattern. The planned exclusion radius for obstacles is \( \delta + \epsilon \) larger than the actual exclusion radius. Therefore, if no agents are initialized inside the planned exclusion radius, the navigation function will never take them closer than \( \delta + \epsilon \) which guarantees safety.

Inter-agent collision avoidance is more complex as it involves distortion in the \( x \)-\( y \) plane along constant \( \psi \) streamlines. Theorem 1 provides a general, conservative, inter-agent collision avoidance condition while Theorem 2 provides a tighter, more useful condition for the single obstacle case.

**Theorem 1:** Let \( r_{i,0} \) be the position of healthy agent \( i \in \mathcal{H} \) at the time of failure and is defined as

\[
r_{i,0} = [x_{i,0}, y_{i,0}]^T
\]

Define \( \phi_{i,0} = \phi(x_{i,0}, y_{i,0}), \psi_{i,0} = \psi(x_{i,0}, y_{i,0}), \)

\[
P_{\min,0} = \min_{i,j} \left( \phi_{i,0} - \phi_{j,0} \right)^2 + \left( \psi_{i,0} - \psi_{j,0} \right)^2, \quad (7)
\]

with \( P_{\min,0} \) the minimum separation distance between agents in the \( \phi \)-\( \psi \) plane, and

\[
\lambda_{\max} = \max_{i,j} \left( \phi_i^2 + \phi_j^2 \right) .
\]

Assume that the trajectory control error of each individual quadcopter does not exceed \( \delta \) and every quadcopter can be enclosed by a ball of radius \( \epsilon \). Inter-agent collision avoidance during CEM mode is guaranteed, if the sliding speed \( \dot{\phi}_i = \dot{\phi} \) is the same for every healthy quadcopter, and

\[
P_{\min,0}^2 \geq \frac{4(\delta + \epsilon)^2}{\lambda_{\max}} . \quad (8)
\]

**Proof:** Under conformal mapping (1), there exists a one-to-one mapping between infinitesimal element \( dx \times dy \) in the \( x \)-\( y \) plane and infinitesimal element \( d\phi \times d\psi \) in the \( \phi \)-\( \psi \) plane, where \( (dx, dy) \) and \( (d\phi, d\psi) \) can be related by

\[
\frac{d\phi}{d\psi} = J \begin{bmatrix} dx \\ dy \end{bmatrix} ;
\]

and \( J \) is the Jacobian matrix in (5). Therefore,

\[
d\phi^2 + d\psi^2 = \begin{bmatrix} dx & dy \end{bmatrix} \begin{bmatrix} dx \\ dy \end{bmatrix} . \quad (9)
\]

Using Cauchy-Reimann Relations, we can show that \( J^T J = \text{diag} \left( \phi_x^2 + \phi_y^2, \phi_x^2 + \phi_y^2 \right) \in \mathbb{R}^{2 \times 2} \) is diagonal, and thus, we can write

\[
d\phi^2 + d\psi^2 = \begin{bmatrix} dx & dy \end{bmatrix} \begin{bmatrix} \phi_x^2 + \phi_y^2 \\ 0 \\ 0 \phi_x^2 + \phi_y^2 \end{bmatrix} \begin{bmatrix} dx \\ dy \end{bmatrix} = \begin{bmatrix} \phi_x^2 + \phi_y^2 \end{bmatrix} \begin{bmatrix} dx^2 + dy^2 \end{bmatrix} . \quad (10)
\]

This also implies that

\[
dx^2 + dy^2 \geq \frac{d\phi^2 + d\psi^2}{\lambda_{\max}}, \quad x \neq x_f, \ y \neq y_f . \quad (11)
\]

Eq. [12] implies that

\[
(d_{\min}(t))^2 \geq \left( \frac{p_{\min}(t)}{\lambda_{\max}} \right)^2, \quad \forall t
\]

where

\[
p_{\min}(t) = \min_{i,j} \sqrt{(\phi_i(t) - \phi_j(t))^2 + (\psi_i(t) - \psi_j(t))^2}, \quad \forall t.
\]

When the sliding speed \( \dot{\phi}_i = \dot{\phi} \) is the same for every healthy quadcopter, the healthy agent team moves as particles of a rigid-body in the \( \phi \)-\( \psi \) plane, and thus, inter-agent distances in the \( \phi \)-\( \psi \) plane are time-invariant. As a result, the minimum separation distance of the desired formation \( \phi \)-\( \psi \) plane can be assigned at reference time \( t_0 \), when the failed agent no-fly zone first appears. Therefore, \( P_{\min,0} = p_{\min}(t) \) and Eq. (15) simplifies to

\[
(d_{\min}(t))^2 \geq \left( \frac{p_{\min,0}}{\lambda_{\max}} \right)^2, \quad \forall t.
\]

Since \( d_{\min}(t) \geq 2(\delta + \epsilon) \forall t \) is the collision avoidance condition, this implies that the inter-agent collision avoidance is assured, if Eq. (8) is satisfied.

**Theorem 2:** Inter-agent collision between every agent is avoided if the minimum separation distance \( d_{\min,0} \) at reference time \( t_0 \), when the failed agent appears, satisfies the following condition:

\[
d_{\min,0} = \min_{i,j} \sqrt{(x_{i,0} - x_{j,0})^2 + (y_{i,0} - y_{j,0})^2} \geq 2(\delta + \epsilon) + a_p .
\]

**Proof:** Inter-agent collision between agents is avoided if \( d_{\min}(t) \), defined by (14b), satisfies the following condition:

\[
d_{\min}(t) \geq 2(\delta + \epsilon) , \quad \forall t.
\]

When a single failure exists in the \( x \)-\( y \) plane, \( \psi(x, y) = 0 \) wraps the failed agent by a circle of radius \( a_p \) with the center positioned at \( (x_f, y_f) \) (see Eq. (35), Figure 2). The maximum contraction of the distance between two arbitrary recovery paths \( \psi(x, y) = \psi_{i,0} \) and \( \psi(x, y) = \psi_{j,0} \) is less than \( a_p \), and it occurs at \( x = x_f \). So, \( d_{\min}(t) \geq d_{\min,0} - a_p = 2(\delta + \epsilon) \forall t \). Therefore, inter-agent collision between every
two vehicles is $d_{\text{min}} = 2(\epsilon + \delta) + a_p$. Additionally, when a failure occurs and CEM mode is activated we assume that the failed agent will remain within $\delta$ of its setpoint so $a_f = \epsilon + \delta$. Therefore, $a_p = 1.36m$ and $d_{\text{min}} = 2.72m$.

**V. EXPERIMENTAL RESULTS**

**A. CEM Navigation Validation with Six Virtual Vehicles**

For this test, the ground control station computer was configured to run exactly as described in the experimental setup section, except that only the setpoints were logged and no vehicles were flying. This was done to validate that the navigation algorithm was able to generate safe trajectories while also maintaining a desired velocity along constant streamlines and operating within runtime constraints.

Figure 3 shows the formation and trajectory followed. The formation is similar to [28] with a leading triangle and interior agents. The initial formation has agents with a minimum separation of 2.72m. The vehicles begin in a six agent formation on the bottom before Q1 fails. This activates the CEM navigation algorithm. An initial $(\phi, \psi)$ coordinate is calculated for each vehicle and then on each iteration $\phi$ is advanced in a dynamic fashion to track a 1.0 m/s desired maximum velocity. Figure 6 shows the commanded velocities. This shows each component of 2D velocity while the norm at each step was the quantity being tracked. Figure 7 shows the values of $\phi$ advancing while Figure 8 shows how $\Delta \phi$ is changed to maintain the specified velocity. Notice how at about 6s and 13s $\Delta \phi$ gets very small. This coincides with Q2 traversing saddle points at each end of the exclusion zone. The Jacobian is large here which causes small changes in $\phi$ to produce a large change in the $x$-$y$ plane (i.e. Q2 moves fast while all other vehicles move slowly). Additionally, Figure 9 shows the distance between nearest agents. Every agent pair maintains at least $2(\delta + \epsilon)$ separation.

To explore the effectiveness of the velocity tracking aspect of our navigation algorithm the six vehicle flight was repeated while the value for $K$ was varied in the range $\{1, 2, 5\}$. Figure 10 shows the maximum velocity of any agent for the various values of $K$ during one of the two saddle points that cause the most difficulty. $K=1$ reached a 2D speed of 2.4 m/s, much higher than the 1.0 m/s setpoint. Speed error with $K=2$ was significant lower, while $K=5$ performed a bit better. However, increasing the value of $K$ essentially replans with a new $\Delta \phi$ $K$ times each iteration. This has a runtime cost which...
can be seen in Figure 11. Runtime is proportional to the value of K and the number of agents. Runtimes of roughly 0.2ms, 0.4ms, and 1ms were measured for the values of 1, 2, and 5 respectively. 10ms is the hard upper limit since the navigation algorithm is running at 100Hz. A value of K=2 follows the velocity profile well while offering a reasonably low runtime.

B. Two Vehicle Flight Test

The same formation from the previous section in a reduced configuration (only Q1 and Q2) was flight tested to validate the navigation algorithm working safely to avoid collisions with non-zero controller error bounded by $\delta = 40cm$ for each vehicle. Figure 1 shows this test. Figure 12 shows the trajectories for failed agent Q1 within its exclusion zone (solid red circle) and Q2 which tracked its desired trajectory computed in real-time (dotted green) closely by its actual trajectory (solid green). The controller error of both agents remained within the $\delta$ bound as seen in Figure 13. Figure 14 shows that the actual trajectory of Q2 was a safe distance...
VI. CONCLUSION

This paper has presented an algorithm for quadrotor formation flight that allows healthy quadrotors to safely continue their mission in the presence of quadrotor failure(s). We presented a novel real-time fluid-flow based CEM navigation algorithm where recovery trajectories of healthy quadrotors are calculated as streamlines of an ideal fluid flow. We analyzed two safety conditions that specify the minimum separation distance required to assure safety. Experimental results validated the safety condition provided.

Experiments were conducted with quadrotors flying in the $x$ direction of the $x-y$ plane i.e., constant $\psi$ direction of the $\phi-\psi$ plane. Using a rotation matrix, the CEM navigation algorithm can be improved further to accommodate quadrotor formation flight in any direction of the $x-y$ plane. Although this paper demonstrated experiments with a two quadrotor team and a single failure as well as a six virtual
quadrotor team with a single failure, using Theorem 1, the CEM navigation algorithm can be extended to large quadrotor groups and multiple quadrotor failures. We will extend simulation and experiments to larger teams with multiple failed quadrotors in future work.

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