Dynamic analysis and regulation of the flexible pipe conveying fluid with a hard-magnetic soft segment

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Abstract The recently developed hard-magnetic soft (HMS) materials can play a significant role in the actuation and control of medical devices, soft robots, flexible electronics, etc. To regulate the mechanical behaviors of the cantilevered pipe conveying fluid, the present work introduces a segment made of the HMS material located somewhere along the pipe length. Based on the absolute node coordinate formulation (ANCF), the governing equations of the pipe conveying fluid with an HMS segment are derived by the generalized Lagrange equation. By solving the derived equations with numerical methods, the static deformation, linear vibration characteristic, and nonlinear dynamic response of the pipe are analyzed. The result of the static deformation of the pipe shows that when the HMS segment is located in the middle of the pipe, the downstream portion of the pipe centerline will keep a straight shape, providing that the pipe is stable with a relatively low flow velocity. Therefore, it is possible to precisely regulate the ejection direction of the fluid flow by changing the magnetic and fluid parameters. It is also found that the intensity and direction of the external magnetic field greatly affect the stability and dynamic response of the pipe with an HMS segment. In most cases, the magnetic actuation increases the critical flow velocity for the flutter instability of the pipe system and suppresses the vibration amplitude of the pipe.

Key words hard-magnetic soft (HMS) material, pipe conveying fluid, absolute node coordinate formulation (ANCF), stability, dynamic response, regulation

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1 Introduction

Pipes conveying fluid have important applications in engineering, and their vibration and stability have received much research attention in the past decades. As a typical non-conservative fluid-structure interaction (FSI) system, the pipe conveying fluid with clamped-free boundary conditions loses stability and vibrates when the flow velocity becomes sufficiently high. When the flow velocity exceeds a certain critical value, oscillation of the pipe occurs and may induce fatigue damage to the pipe\cite{1–3}. Therefore, it is necessary to improve the stability of fluid-conveying pipe systems. As early as 1993, Païdoussis and Semler\cite{4} derived the dynamical governing equation of a cantilevered pipe supported by a linear spring, and studied the stability of the system. Since then, various methods have been proposed and used to control/regulate the stability and dynamics of cantilevered pipes conveying fluid. The control methods can be divided into two types, i.e., passive control methods\cite{5–6} and active control methods\cite{7–8}. Tuned mass damper\cite{9}, functionally graded material\cite{10–11}, nonlinear energy sink\cite{12–13}, and additional outside tube\cite{14} are commonly used in passive control methods, while piezoelectric control\cite{15–16}, boundary control\cite{17}, and time-delayed feedback control\cite{18} methods have been recently developed to realize active control.

The magnetism-based method was also used to control the stability and vibration characteristics of pipes conveying fluid. Tang and Dowell\cite{19} studied the oscillations of a cantilevered pipe with an embedded steel plate subject to a magnetic force, which was generated by two permanent magnets placed near the free end of the pipe. Their results showed that chaotic motions of the pipe might occur under a magnetic force. Szmidt and Przybyłowicz\cite{20} installed an electromagnetic actuator on the pipe, and found that this actuator could improve the stability of the cantilevered pipe conveying fluid. It should be pointed out that in all the mentioned previous studies, no matter passive or active control methods, additional components (such as mass block, spring, and damper) are generally required to be added to the pipe. Indeed, these control methods bring two obvious disadvantages. One is that the additional components could somehow change the vibration characteristics of the pipe itself (e.g., the natural frequency), and the other is that the additional components would increase the space occupied by the whole pipe system.

In 2018, Kim et al.\cite{21} manufactured a hard-magnetic soft (HMS) material by embedding hard-magnetic particles (e.g., NdFeB) into a soft matrix material. During the three-dimensional (3D) printing process, the soft structures were endowed with magnetic properties with the aid of an external magnetic field. It was shown that the HMS elastomer can rapidly respond to the actuation of an external magnetic field. Recently, Chen et al.\cite{22} introduced this novel magnetically responsive material to control the mechanical behaviors of a flexible pipe conveying fluid. It was found that the stability of a fluid-conveying pipe could be effectively improved under the action of a magnetic force. However, the designed pipe was embedded with hard-magnetic particles along the whole length, without attention to the distribution adjustment of hard-magnetic particles.

To improve the regulation efficiency of HMS pipes, the present work proposes a new kind of pipe structure with a local segment embedded with hard-magnetic particles, and analyzes the static deformation and dynamical behavior of this pipe system in detail. The research motivation of this study mainly comes from two aspects. On the one hand, the mechanism of stability enhancement by using a local magnetic segment will be revealed, and the vibration amplitude of the pipe system will be regulated. On the other hand, the static deformation of the pipe under the combined action of magnetic force and internal fluid flow is examined, and the local magnetic force will be used to adjust the ejection direction of the fluid flow. The first
research motivation is related to the stability and vibration of the system, which is the major concern of the dynamics of pipes conveying fluid. The second research motivation is inspired by recent advances in biomedicine\cite{23} and soft robotics\cite{24-25}. If the ejection direction of the fluid flow in the pipe can be effectively controlled, wound cleaning, targeted drug delivery, and biological printing can be precisely achieved in minimally invasive surgeries for cardiovascular disease\cite{26}, renal surgery\cite{27}, skin adhesion\cite{28}, etc.

The structure of this paper is organized as follows. In Section 2, based on the absolute node coordinate formulation (ANCF), the governing equations of the HMS cantilevered pipe conveying fluid are derived through the generalized Lagrange equation. In Section 3, the static equilibrium, linear stability, and nonlinear dynamic response of the HMS pipe are analyzed. The effects of the strength and direction of the external magnetic field on the mechanical behaviors of the system are studied. The regulation method for the static deformation of the pipe and the ejection direction of the fluid is explored. In Section 4, some conclusions are summarized.

2 Theoretical model of the HMS pipe conveying fluid

The system under consideration is a fluid-conveying cantilever composed of two different materials, with a total length $L$. As shown in Fig. 1(a), Parts I and II are the rubber segment and the HMS segment, respectively. Young’s modulus and the mass per unit length of Part I (or Part II) of the pipe are denoted as $E_I$ (or $E_{II}$) and $m_I$ (or $m_{II}$), respectively. The cross-sectional area and moment of inertia of the whole pipe are $A_p$ and $I$, respectively. The velocity of the fluid flow is $U$, and the mass per unit length of the fluid is $M$. The magnetic flux density of the external magnetic field is $B_{a}$, and the angle between the magnetic field direction and the positive direction of the $x$-axis is denoted as $\alpha$. The residual magnetic flux density of Part II after deformation is denoted as $B_{r}$.

Before deriving the governing equations of the system, some reasonable assumptions are made. (i) The internal fluid in the pipe is steady and is a plug-like flow. (ii) The influence of gravity is slight and can be neglected. (iii) The pipe behaves as an Euler-Bernoulli beam, and hence the rotary inertia and shear deformation of the pipe are ignored. (iv) The HMS material is uniformly magnetized along the axial direction.

2.1 Governing equations based on the ANCF

In this subsection, closely following the work by Stangl et al.\cite{29} and Zhou et al.\cite{30}, the governing equations of the pipe conveying fluid are derived based on the ANCF. Although the pipe consists of two different materials, the pipe element in the ANCF mesh is uniform. Therefore, for the convenience of expression, $m$ and $E$ without giving the subscript associated with Part I or Part II are used in the following derivation of governing equations.

The two-node plane beam element\cite{31} shown in Fig. 1(b) is used. Each node of the element has four degrees of freedom, and the local coordinates of any point $P_0$ in the initial configuration.
are expressed as \((x, y)\). The global coordinates after deformation are expressed as \((X, Y)\). Therefore, the position vector \(r\) of the global coordinates of the point \(P\) is given by\(^{[32]}\):

\[
r = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = Sq,
\]

where \(q\) is the nodal coordinate vector of the two nodes at both ends of the pipe element, and \(S\) is the shape function of the global element.

Due to the longitudinal elongation of the pipe centerline, the axial deformation between the current configuration of the pipe and the initial configuration would occur. The axial deformation gradient of the element can be expressed as\(^{[30]}\):

\[
f = \frac{ds}{dx} = \frac{\sqrt{r'^T r'}}{\sqrt{r'_0^T r'_0}},
\]

where \(r_0\) is the position vector of the initial configuration of the pipe. Then, the velocity of the pipe can be determined by the first-order derivative of the position vector with respect to the time \(t\).

The coordinate vector of the element node is defined by\(^{[32]}\):

\[
q = \begin{bmatrix} r_{11} \\ r_{21} \end{bmatrix} = \begin{bmatrix} \partial r_{11} / \partial x \\ \partial r_{12} / \partial x \\ \partial r_{j1} / \partial x \\ \partial r_{j2} / \partial x \end{bmatrix}^T.
\]

The global shape function \(S\) of the two-node plane beam element can be expressed as\(^{[32]}\):

\[
S = \begin{bmatrix} S_1 & 0 & lS_2 & 0 & S_3 & 0 & lS_4 & 0 \\ 0 & S_1 & 0 & lS_2 & 0 & S_3 & 0 & lS_4 \end{bmatrix},
\]

where the expressions of \(S_k(\xi)\) are given as follows:

\[
S_1 = 1 - 3\xi^2 + 2\xi^3, \quad S_2 = \xi - 2\xi^2 + \xi^3, \quad S_3 = 3\xi^2 - 2\xi^3, \quad S_4 = \xi^3 - \xi^2,
\]

in which \(\xi = x/l\), and \(l\) is the length of the element in the undeformed configuration. The motion state of any point on the pipe can be described by multiplying the global shape function with the nodal coordinates.

For non-material volume, the generalized Lagrange equation is given by\(^{[33]}\):

\[
\frac{d}{dt} \frac{\partial T}{\partial q} - \frac{\partial T}{\partial q} \frac{\partial T'}{\partial q} + \int_S da \cdot (v_t - v_p) \frac{\partial T'}{\partial q} - \int_S da \cdot \left( \frac{\partial v_t}{\partial q} - \frac{\partial v_p}{\partial q} \right) T'_f = Q,
\]

where \(T\) is the total kinetic energy of both the pipe and the fluid, and \(T'_f\) represents the kinetic energy of the fluid per unit volume. \(v_p\) and \(v_t\) are the velocities of the pipe and the fluid, respectively. In addition, \(Q\) is the generalized force term of the pipe system. The two integral terms in Eq. (6) represent the effects of the fluid inflow and outflow on the pipe. The integral region \(S\) represents the inlet and outlet surfaces of the fluid of each element. \(da\) is the oriented surface element of the end surface, and the directions of \(da\) on the inflow and outflow surfaces are opposite. Therefore, when the third term in Eq. (6) is calculated, it will be equal to the values at \(x = 0\) and \(x = L\), and can be obtained as follows:

\[
\int_S da \cdot (v_t - v_p) \frac{\partial T'}{\partial q} = aM U \left( S^T S q + U \frac{S^T S'}{\sqrt{q_0^T S^T S q_0}} \right) \bigg|_{0,L},
\]

where the scalar \(a\) represents the sign of the integral term, and the values of \(a\) at the inlet \((x = 0)\) and outlet \((x = L)\) are taken as 1 and \(-1\), respectively. As for the second integral
term in Eq. (6), the difference between the velocity vectors of the fluid and the pipe \((v_f - v_p)\) is independent of \(\dot{q}\). Therefore, the second integral term is equal to 0.

The total kinetic energy of the pipe element includes the kinetic energy of the pipe and the fluid and can be written as

\[
T = T_p + T_f = \frac{1}{2}m \int_0^l v^T_p v_p \, dx + \frac{1}{2}M \int_0^l v^T_i v_i \, dx.
\] (8)

The generalized force of the element consists of the elastic force and the magnetic force. The elastic force induced by the deformation of the pipe includes two parts, associated with the stretching stress and the bending stress, respectively. The magnetic force is generated by the HMS segment under the external magnetic field. The axial strain of the element can be obtained from the Green-Lagrange strain tensor, i.e.,

\[
\varepsilon = \frac{1}{2}(f^2 - 1) = \frac{1}{2} \left( \frac{r^T \dot{r}}{r_0^T \dot{r}_0} - 1 \right).
\] (9)

In addition, the bending potential energy can be calculated through the curvature of the pipe during the bending deformation of the element. The curvature can be derived from the position vector as

\[
\kappa = \frac{\|r' \times r''\|}{\|r'\|^3} = \sqrt{q^T S'' \tilde{I} S' q q^T S' T^T S'' q}.
\] (10)

where

\[
\tilde{I} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.
\] (11)

Therefore, the generalized elastic force on the element can be obtained by the derivative of the total potential energy with respect to \(q\) and is given by

\[
Q_{el} = -\int_0^l E A_p \varepsilon \frac{\partial \varepsilon}{\partial q} \, dx - \int_0^l EI(\kappa - \kappa_0) \frac{\partial \kappa}{\partial q} \, dx,
\] (12)

where \(\kappa_0\) is the curvature of the pipe in the initial configuration. According to the description at the beginning of this section, the magnetic flux density of the external magnetic field and the residual magnetic flux density can be, respectively, expressed as

\[
B^a = B^a_{\text{ex}}[\cos \alpha \quad \sin \alpha] = B^a \tau_a,
\] (13)

\[
B^r = B^r \frac{r'}{\sqrt{r'^T r'}}.
\] (14)

Therefore, the magnetic potential energy density can be written as

\[
\phi_m = -\frac{1}{\mu_0} B^a \cdot B^r,
\] (15)

where \(\mu_0\) is the vacuum permeability. Finally, the generalized force of the pipe generated by the external magnetic field is obtained as

\[
Q_m = \frac{A B^b B^a}{\mu_0} \int_0^l S^T \cdot \tau_a q^T S' S' q - S'^T S' q \cdot \tau_a S' q (q^T S'^T S')^2 \, dx.
\] (16)
Since only straight pipes conveying fluid are studied in this paper, the initial curvature \( \kappa_0 = 0 \), and the deformation gradient of the element can be simply expressed as

\[
f = \frac{ds}{dx} = \sqrt{r' T r'}.
\]  
(17)

As shown in Eqs. (6), (7), (8), (12), and (16), the governing equations of the motion of the pipe element can be obtained by sorting and simplifying. However, it is worth mentioning that the generalized force of Eq. (16) is only generated in Part II and does not affect Part I of the pipe. Therefore, the governing equations of the two parts will be different in this term, and the expressions are

\[
M_e \ddot{q} + C_e \dot{q} + K_e q + N_e(q) + N_m(q) = 0,
\]

where \( M_e, C_e \), and \( K_e \), respectively, represent the linear mass, damping, and stiffness matrices of the element, \( N_e(q) \) is the nonlinear term, and \( N_m(q) \) exists only in Part II.

2.2 Dimensionless governing equations

In order to render the dimensionless form of the governing equations, the following quantities are introduced:

\[
\tau = \left( \frac{E I}{M + m} \right)^{\frac{1}{2}} \frac{t}{L^2}, \quad q^* = \frac{q}{L}, \quad q_0^* = \frac{q_0}{L}, \quad u = \left( \frac{M}{E I} \right)^{\frac{1}{2}} U L,
\]

\[
P = \frac{A_p L^2 B^3 B^a}{E_1 I_0}, \quad \beta = \frac{M}{M + m}, \quad \Pi_0 = \frac{A_p L^2}{M}, \quad \alpha_1 = \frac{M + m_1}{M + m}, \quad \alpha_2 = \frac{E_1}{E_1}.
\]

By substituting Eq. (19) into Eq. (18), the dimensionless governing equations of the pipe element can be written as

\[
\begin{align*}
M_e^* \ddot{q}^* + C_e^* \dot{q}^* + K_e^* q^* + N_e^*(q^*) &= 0, \\
M_0^* \ddot{q}^* + C_0^* \dot{q}^* + K_0^* q^* + N_m^*(q^*) &= 0.
\end{align*}
\]  
(20)

By assembling the coefficient matrices of all elements, the generalized coordinate vector \( \mathbf{e} \), the mass matrix \( M \), the damping matrix \( C \), the linear stiffness matrix \( K \), and the nonlinear terms \( N(e) \) of the whole pipe can be obtained. The final dynamic equations are given by

\[
M \ddot{\mathbf{e}} + C \dot{\mathbf{e}} + K \mathbf{e} + N(\mathbf{e}) = 0.
\]

(21)

3 Numerical results and discussion

The results of the static deformation, linear vibration characteristics, and nonlinear responses of the pipe system will be calculated and analyzed in this section. Two key system parameters of Part I of the pipe are chosen as \( \beta = 0.2 \) and \( \Pi_0 = 16792 \). For Part II of the pipe, the rubber material is used as the matrix, and the magnetic particles (NdFeB) are mixed in the matrix with a certain proportion. When the volume fraction of the NdFeB is taken as 20.7%, the pipe has a better magnetic actuation effect\(^{[35]} \). According to Refs. [35] and [36], the mass ratio \( \alpha_1 \) and Young’s modulus ratio \( \alpha_2 \) between Parts I and II of the pipe are 1.54 and 2.05, respectively.

In most cases, it is assumed that the HMS segment is located in the middle of the pipe with a fixed length \( L_\Pi = L/9 \). Even if the location of the magnetic segment is varied, the length \( L_\Pi \) remains unchanged. Based on extensive calculations, the results of the static deformations of the pipe obtained using 9 elements (\( N = 9 \)) are convergent. Therefore, to reduce calculation costs, \( N = 9 \) is selected for all later calculations.
3.1 Static equilibrium configuration

Although the pipe considered in this study is shaped with an initially straight configuration, due to the existence of the HMS segment, the static deformation of the pipe might occur when the external magnetic field is applied. No matter whether the pipe is stable or unstable, there is a static equilibrium position of the pipe. Bearing in mind this fact, the pipe would vibrate around the static equilibrium position rather than the initially straight configuration once the pipe loses stability. The analysis of the stability about the pipe’s static equilibrium is beneficial before calculating the nonlinear dynamic responses of the pipe. With this consideration, the generalized coordinate vector is divided into two parts,

\[ e = e_s + e_d, \]  

(22)

where \( e_s \) is the static (steady) part of the nodal displacement, and \( e_d \) is a perturbation (dynamic) part around the static one. Because \( e_s \) is independent of time, the equations governing the static equilibrium can be obtained by substituting Eq. (22) into Eq. (21) and are given by

\[ K e_s + N(e_s) = 0. \]

(23)

The linearized equations governing the dynamic part can be obtained by using the Taylor expansion and are given by

\[ M \ddot{e}_d + C \dot{e}_d + K e_d + \frac{\partial N(e)}{\partial e} \bigg|_{e = e_s} = 0. \]

(24)

The common Newton-Raphson method is utilized to solve the nonlinear equation (23). First, when the magnetic actuation angle \( \alpha \) is chosen as 0, \( \pi/3 \), \( \pi/2 \), and \( 2\pi/3 \), the static equilibrium of the pipe with \( P = 20 \) is calculated. The results are shown in Fig. 2, where the blue and red solid lines represent the static deformations of the pipe as the pipe system is stable. Obviously, these static deformations are realistic since they are stable. It is noted that the red solid line in Fig. 2 denotes the pipe’s static deformation when the flow velocity is slightly below the critical value for flutter instability. The black dotted line in Fig. 2 represents the pipe’s static equilibrium for the flow velocities beyond the critical value. Since the pipe would lose stability with flutter behavior when the flow velocity exceeds a critical value, the configurations shown with black dotted lines are not real. Although these unstable static equilibrium positions are not realistic, they may provide remarkable values in revealing the nonlinear vibration mechanism of the system.

As shown in Fig. 2(a), when the excitation direction of the external magnetic field is consistent with the abscissa (i.e., \( \alpha = 0 \)), the pipe could not produce any static deformation in the transverse direction, but will generate a small elongation of the pipe. This can be easily understood because the external magnetic force is acting along the axial direction of the pipe in this case. The results for \( \alpha = \pi/3 \) are shown in Fig. 2(b), where it is seen that the static deformation of the pipe with \( u = 0 \) reaches the maximum. With the continuous increase in the flow velocity, the transverse displacement of the pipe gradually decreases and tends to change from positive to negative. When the flow velocity is larger than 9, the static equilibrium position of the pipe will move upward. The results shown in Figs. 2(c) and 2(d) display a similar trend as that shown in Fig. 2(b). Also, the amplitude range of the static deformation increases with the increase in \( \alpha \). This can be understood by looking at the governing equations, from which it is noted that a distributed magnetic force is applied to the middle segment of the pipe. This magnetic force deflects the pipe and tends to make the pipe axis consistent with the direction of the external magnetic force.

The results given in Fig. 3 show the static equilibrium configurations of the pipe with \( \alpha = \pi/3 \) under different values of the magnetic strength. It can be seen that the displacement of the pipe
Fig. 2 Static configurations of the pipe with $P = 20$ at different flow velocities when the magnetic actuation angle is chosen to be (a) $\alpha = 0$, (b) $\alpha = \pi/3$, (c) $\alpha = \pi/2$, and (d) $\alpha = 2\pi/3$ (color online)

Fig. 3 Static configurations of the pipe with $\alpha = \pi/3$ at different flow velocities when the external magnetic strength is (a) $P = 10$, (b) $P = 20$, and (c) $P = 30$ (color online)

increases with increasing $P$, indicating that the magnetic strength has significant influence on the deformation amplitude of the pipe. It should be mentioned that when the HMS segment is in the middle of the pipe, the pipe portion near the free end remains straight, and the bending deformation of the pipe mainly occurs in the range of $x = [0, 0.5]$. The new feature shown in Fig. 3 that the portion of the pipe centerline near the free end maintains a straight shape seems interesting. More importantly, the direction of the straight portion of the pipe centerline can be regulated by changing the flow velocity and magnetic actuation angle. This new feature brings a possibility to the application of the HMS pipe in the field of minimally invasive surgery. As a possible way, we can make use of the printing technology proposed in Ref. [22] to prepare the
HMS pipe as shown in Fig. 1. Then, based on the bio-printing technology proposed in Ref. [28], we can realize the application of the HMS pipe in minimally invasive surgery. For example, by using the HMS pipe under an external magnetic field, we could print biological tissue directly onto an animal’s gut by precisely adjusting the flow velocity inside the pipe. Because the bio-printing technique is less invasive, it can reduce the risk of wound infection and shorten the recovery time from surgery.

3.2 Linear vibration characteristics and stability

As mentioned above, instability is one of the key scientific problems in the dynamical system of pipes conveying fluid. In this subsection, therefore, we turn our attention to the stability of the pipe system based on the linearized governing equation (24) obtained in Subsection 3.1. The complex frequency $\omega$ of the pipe system can be obtained by solving a generalized eigenvalue problem. The imaginary part $\text{Im}(\omega)$ of the complex frequency represents the natural frequency of the pipe system. The stability of the dynamical system, indeed, can be determined by analyzing the real part $\text{Re}(\omega)$, which is related to the damping of the pipe conveying fluid. If $\text{Re}(\omega) < 0$, the pipe will continuously absorb energy from the internal fluid flow and may be subject to flutter instability due to the negative damping effect.

To show the evolution of the lowest several complex frequencies of the pipe with an increasing flow velocity, several Argand diagrams are produced and plotted in Fig. 4 for $\alpha = 0$, $P = 20$, and various locations of the HMS segment. Each black dot in the figures represents the complex eigenfrequencies of the pipe system when the flow velocity changes continuously. The following location parameters indicate that the midpoint of the HMS segment is located at different locations of the pipe. From Fig. 4(a), it is seen that when the middle of the HMS segment is

![Argand diagrams](image_url)
located at $0.25L$, the damping of the dynamical system becomes negative as the flow velocity exceeds 5.6, indicating that the pipe becomes unstable via flutter at this critical flow velocity.

As the location of the magnetic segment is changed from the fixed end to the free end, the critical flow velocity of the pipe system increases first and then decreases. Among the four locations of the HMS segment, it is found that when the magnetic segment is located in the middle of the pipe, the critical flow velocity of the pipe approximately reaches the maximum, indicating that $x = 0.5L$ is one optimized position for improving the stability of the fluid-conveying pipe system. In the following analysis, therefore, all results will be calculated for $x = 0.5L$.

The magnetic actuation angle also has an effect on the stability of the pipe system. To show this effect, several Argand diagrams are plotted in Fig. 5, for $x = 0.5L$ and various values of the magnetic actuation angle $\alpha = 0, \pi/3, \pi/2, \text{and } 2\pi/3$. It can be seen from Fig. 5(a) that when $\alpha = 0$, the critical flow velocity of the pipe is about 6.9. Compared with the critical flow velocity of a uniform pipe without the HMS segment, the critical flow velocity of the HMS pipe increases from 5.5 to 6.9 when the external magnetic field acts on the HMS segment. Although the magnetic actuation does not generate transverse static deformation of the pipe in the case of $\alpha = 0$, the stability of the pipe system can be most effectively improved with this magnetic actuation angle. When $\alpha = \pi/3$, as shown in Fig. 5(b), the critical flow velocity of the pipe is about 6.6. When $\alpha = \pi/2$ or $2\pi/3$, the critical flow velocity becomes 6.4 or 6.1 approximately, as can be seen in Figs. 5(c) or 5(d). Thus, with the increase in $\alpha$, the critical flow velocity of the pipe system tends to decrease. Even so, the stability of the pipe system is much better than

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig5.png}
\caption{Argand diagrams of the pipe with $P = 20$ when Part II is located at the middle of the pipe and the magnetic actuation angle is (a) $\alpha = 0$, (b) $\alpha = \pi/3$, (c) $\alpha = \pi/2$, and (d) $\alpha = 2\pi/3$ (color online)}
\end{figure}
that of a uniform pipe without the HMS segment.

In order to analyze the influence of the magnetic strength \( P \) on the critical flow velocity of the pipe system, the curves of the critical flow velocity varying with \( P \) for different magnetic actuation angles are shown in Fig. 6. When \( \alpha = 0 \), the critical flow velocity increases with the increase in \( P \). When \( \alpha = \pi/3 \) and \( \alpha = \pi/2 \), the critical flow velocity of the pipe system will also increase with the increase in \( P \), but the increase is not as significant as that for \( \alpha = 0 \). In fact, as \( \alpha \) increases, the magnetic force acting on the pipe will have a component in the negative direction of the \( x \)-axis. This component behaves as an axial compressive load and would reduce the critical flow velocity of the pipe system. Because of this, the stability of the pipe system will decrease as the value of \( \alpha \) increases. Indeed, when \( \alpha = 2\pi/3 \), the critical flow velocity even decreases slightly in a wide range of \( P \).

From Fig. 6, one might have found that there is a ‘jump’ in the evolution curve of the critical flow velocity at about \( P = 26 \) for \( \alpha = 0 \). In order to further explore the reason for this jump phenomenon, two values of \( P \) in the vicinity of 26 are chosen for further calculations of Argand diagrams. As shown in Fig. 7, it can be seen that when \( P = 25 \), the second mode of the pipe becomes unstable at about \( u = 7.3 \). When \( P = 30 \), however, the instability occurs in the third mode of the pipe at about 8.65. Thus, it is due to the transition (or shift) of the unstable mode that leads to the jump phenomenon of the critical flow velocity. In addition, it can be seen from Fig. 7(a) that although the pipe is unstable in the flow velocity range of [7.3, 7.9], the system will regain stability in the flow velocity range of [7.9, 8.5], and finally become unstable again.

**Fig. 6** Critical flow velocity of the pipe system varying with increasing \( P \) for various \( \alpha \) (color online)

**Fig. 7** Argand diagrams of the pipe with \( \alpha = 0 \) when the external magnetic strength is chosen as (a) \( P = 25 \) and (b) \( P = 30 \) (color online)
when the flow velocity is higher than 8.5.

3.3 Nonlinear dynamic response

The nonlinear dynamic responses of the cantilevered pipe after instability will be discussed in this subsection. For this purpose, Eq. (21) is solved by using the fourth-order Runge-Kutta method. The bifurcation diagrams of the lateral displacement of the tip end of the pipe are plotted in Fig. 8 for \( P = 20 \) and various \( \alpha \). In these bifurcation diagrams, when the vibration velocity of the free end of the pipe becomes zero, the corresponding displacement is recorded. The blue dots in this figure represent the lateral displacement of the pipe at \( x = L \). To better understand the relationship between the static equilibrium position and the dynamic response of the pipe, the static equilibrium positions at the end of the pipe varying with different flow velocities are also given in each subfigure with a red dotted line.

When \( \alpha = 0 \), it can be seen from Fig. 8(a) that the Hopf bifurcation occurs at about \( u = 6.9 \), indicating that the pipe system changes from a stable state to an unstable state. The bifurcation point for flutter instability shown in Fig. 8(a) is consistent with the critical flow velocity shown in Fig. 5(a). This good agreement demonstrates the correctness of both linear and nonlinear results. When the flow velocity is beyond the critical value, interestingly, the lateral displacement of the tip end of the pipe does not always increase with the increase in the flow velocity but decreases slightly in the flow velocity range of \([7.8, 8.5]\). By comparing this result with that of a uniform pipe without the HMS segment, it can be found that in addition to improving the stability of the pipe, the overall amplitude of the lateral displacement of the pipe is suppressed to a certain extent. As shown in Figs. 8(c)–8(d), when the flow velocity is below the critical value, the static deformations (and hence the bifurcation routes) are consistent with

\begin{figure}[h]
\centering
\begin{subfigure}{0.45\textwidth}
  \includegraphics[width=\textwidth]{figure8a.png}
  \caption{}
\end{subfigure}\hfill
\begin{subfigure}{0.45\textwidth}
  \includegraphics[width=\textwidth]{figure8b.png}
  \caption{}
\end{subfigure}
\begin{subfigure}{0.45\textwidth}
  \includegraphics[width=\textwidth]{figure8c.png}
  \caption{}
\end{subfigure}\hfill
\begin{subfigure}{0.45\textwidth}
  \includegraphics[width=\textwidth]{figure8d.png}
  \caption{}
\end{subfigure}
\caption{Amplitudes of the tip-end displacements of the pipe varying with \( u \) for \( P = 20 \) and (a) \( \alpha = 0 \), (b) \( \alpha = \pi/3 \), (c) \( \alpha = \pi/2 \), and (d) \( \alpha = 2\pi/3 \) (color online) }
\end{figure}
Dynamic analysis and regulation of the flexible pipe conveying fluid with an HMS segment

To further understand the dynamic responses of the pipe, some typical results of the time history, phase portrait, Poincare map, and oscillating shapes of the pipe for \( P = 20, \alpha = \pi/3, \) and \( u = 8 \) are given in Fig. 9. It can be seen from Fig. 9(a) that the amplitude of the lateral displacement (red line) of the free end of the pipe with Part II is relatively reduced than that (blue line) without Part II, and the pipe vibrates around the static equilibrium position rather than the initially straight configuration. As shown in Fig. 9(c), the oscillating shapes of the pipe are mainly dominated by the second mode of a cantilevered beam. In this figure, the red line represents the static deformation of the pipe for a given flow velocity. It is noted that the oscillation trajectory of the free end of the pipe is shaped like an Arabic number ‘8’. In addition, it can be easily seen from Figs. 9(b) and 9(d) that the pipe undergoes a period-1 motion in this case.

![Graphs and diagrams showing time history, phase portrait, oscillating shapes, and Poincare map of pipe oscillations.](image_url)

**Fig. 9** (a) Time history, (b) phase portrait, (c) oscillating shapes, and (d) Poincare map of the oscillation of the pipe with \( P = 20, \alpha = \pi/3, \) and \( u = 8 \) (color online)

It is recalled that in Fig. 6(a) a jump phenomenon is found in the curve of the critical flow velocity versus \( P \) in the case of \( \alpha = 0 \). Then, we are interested in how the oscillating shapes of the pipe differ before and after the jump point. Figures 10(a) and 10(b) show the oscillating shapes and bifurcation diagrams of the pipe with \( \alpha = 0, P = 25, \) and \( u = 7.6 \). It can be seen that in this case, the oscillating shapes of the pipe are mainly in the form of the second mode of a cantilevered beam. However, when \( \alpha = 0, P = 30, \) and \( u = 9 \), the oscillating shapes of the pipe are mainly dominated by the third mode of a cantilevered beam, as shown in Fig. 10(c). Therefore, the oscillating shapes of the pipe may be linked to the unstable mode of the pipe system.
In this paper, a novel method for regulating the mechanical behaviors of cantilevered pipes conveying fluid is proposed by introducing an HMS segment located somewhere along the pipe length. With the help of the ANCF method, the governing equations of the pipe with an HMS segment are derived by the generalized Lagrange equation. The static equilibrium position, stability, and dynamic response of the pipe are calculated, respectively. Based on static and dynamic analyses, the following conclusions are obtained.

(i) The designed magnetic segment of the pipe can generate a static deformation due to the actuation of the external magnetic field. When the HMS segment is located in the middle of the pipe, the static bending deformation mainly occurs in the upstream portion of the pipe.

(ii) In many cases, the pipe with a magnetic segment has better stability than that without magnetic particles. The strength and direction of the magnetic actuation have great influence on the critical flow velocity of the pipe system.

(iii) When the flow velocity exceeds the critical value, flutter instability is induced and dynamic responses around the static equilibrium position would occur with a period-1 motion. The vibration amplitude of the pipe can be suppressed by the action of the magnetic field force. It is found that the transition of unstable modes might occur in some cases.

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