Deuteron Electromagnetic Form Factors in the Intermediate Energy Region

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Abstract

Based on a Perturbative QCD analysis of the deuteron form factor, a model for the reduced form factor is suggested. The numerical result is consistent with the data in the intermediate energy region.

PACS number(s): 13.40.Gp, 12.38.Bx, 24.85.+p, 27.10.+h

I. INTRODUCTION

Exclusive processes involving the hadron at large momentum transfer were first studied in perturbative QCD (pQCD) by Brodsky and Lepage \[^{[1]}\]. Analysis of the deuteron form factor in the intermediate energy region \[^{[2]}\] revealed that QCD could strongly affect the behavior of the deuteron electromagnetic form factors when \(Q^2\) is the order of several GeV\(^2\). It was pointed out in Refs. \[^{[3,4]}\] that the domain for leading-power pQCD predictions for the deuteron form factors is \(Q^2 \gg 2M_d \Lambda_{QCD} \sim 0.8\text{GeV}^2\) where a calculation with the Paris potential \[^{[5]}\] shows explicit deviation, although it can explain the data well for \(Q^2 < 1\text{GeV}^2\) (See Fig. 1). In this domain the deuteron form factor can be written to the leading order in \(1/Q^2\) as a convolution:

\[
F_d(Q^2) = \int_0^1 [dx][dy]\Phi_d(y_j, Q)T_H^{6q+\gamma\rightarrow 6q}(x_i, y_j, Q)\Phi_d(x_i, Q),
\]  

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where the distribution amplitude $\Phi_d(x_i, Q)$ is defined as

$$
\Phi_d(x_i, Q) = \int_Q [d^2k_{\perp}] \Psi_{6q/d}(x_i, k_{\perp}) \ .
$$

The notation $[dx]$ and $[d^2k_{\perp}]$ is denoted by

$$
[dx] \equiv \delta(\sum_{i=1}^{n} x_i) \prod_{i=1}^{n} dx_i \ ,
$$

and

$$
[d^2k_{\perp}] \equiv 16\pi^3\delta(\sum_{i=1}^{n} k_{\perp i}) \prod_{i=1}^{n} \frac{d^2k_{\perp i}}{16\pi^3} \ .
$$

However, the calculation of the normalized $T_{6q+\gamma^*\rightarrow 6q}^{\text{HF}}$ to leading order in $\alpha_s(Q^2)$ would require the evaluation of over 300,000 Feynman diagrams involving five gluons. Farrar et al. [7] have done perturbative calculations on the helicity zero to zero deuteron form factor and found that it is much smaller than the deuteron form factor data at experimentally accessible momentum transfer.

In order to make more detailed and experimentally accessible predictions, it was suggested in Ref. [3] to define a reduced nuclear form factor by removing the nucleon compositeness,

$$
f_d(Q^2) \equiv \frac{F_d(Q^2)}{F_N^2(Q^2/4)} \ .
$$

The argument for each of the nucleon form factors, $F_N$, is $Q^2/4$ since, in the limit of zero binding energy, each nucleon must change its momentum from $P/2$ to $(P+q)/2$.

For the reduced form factor of the deuteron one finds the asymptotic scaling behavior

$$
Q^2 f_d(Q^2) \sim \left(\ln\frac{Q^2}{\Lambda^2}\right)^{-1-\frac{2\alpha_F}{\pi}} \ ,
$$

which can be compared with the available data in the large $Q^2$ region, although this prediction is only for asymptotic momentum transfer. Equation (6) reminds us the reduced form factor of deuteron may be derived in a way which is similar to the meson case in a perturbative QCD calculation.

The aim of this paper is to build a model to calculate the reduced form factor of the deuteron in the intermediate energy region. The remainder of this paper is organized as follows: In Sec. II we analyze the reduced form factor, $f_d(Q^2)$, and the deuteron wave function. A QCD inspired model is built in Sec. III. In Sec. IV the numerical results for $f_d(Q^2)$ is given from our model. The final section is reserved for summary and discussion.

II. THE REDUCED FORM FACTOR AND THE WAVE FUNCTION OF THE DEUTERON

In the case of electron-deuteron elastic scattering, the standard Rosenbluth cross section [4] is written (in the laboratory frame) as

$$
\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}}[A(Q^2) + B(Q^2)\tan^2(\frac{\theta}{2})] \ ,
$$

where

$$
A(Q^2) = \frac{-\frac{1}{2}G_F^2}{\pi^2} \left(\frac{2m}{Q^2}\right)^2 \ ,
$$

and

$$
B(Q^2) = \frac{G_F^2}{\pi^2} \left(\frac{2m}{Q^2}\right)^2 \ .
$$
where $A(Q^2)$ and $B(Q^2)$ are determined by $G_C$, $G_M$ and $G_Q$:

$$A(Q^2) = G_C^2 + \frac{2}{3} \eta G_M^2 + \frac{8}{9} \eta^2 G_Q^2 ,$$ (8)

$$B(Q^2) = \frac{4}{3} \eta (1 + \eta) G_M^2 ,$$ (9)

with $\eta = Q^2/4M^2$. The deuteron form factor $F_d(Q^2)$ is defined as $F_d(Q^2) \equiv \sqrt{A(Q^2)}$.

As mentioned in Sec. I, it is helpful to study the reduced form factor since the effects of nucleon compositeness (represented by $F_N$) have been removed from it. Equation (8) means that the deuteron form factor $F_d(Q^2)$ can be factorized into two parts, and reduced form factor $f_d(Q^2)$ can be regarded as the form factor of a composite of two point-like nucleons. This factorization was obtained by assuming

$$\Psi_d = \psi_d^{body} \times \psi_N \times \psi_N$$ (10)

in a simple covariant model [11]. $\psi_N$ is the nucleon wave function and $\psi_d^{body}$ is the usual two-body wave function of the deuteron. The equation of motion for $\Psi_d(x_i, k_{\perp i})$ in the light-cone frame (LCF) is given by

$$[M^2 - \sum_{i=1}^{6} \frac{k_{\perp i}^2 + m_i^2}{x_i}] \Psi_d(x_i, k_{\perp i}) = \int [dy][d^2 j_{\perp i}] V(x_i, k_{\perp i}; y_j, j_{\perp j}) \Psi_d(y_j, j_{\perp j}) .$$ (11)

The factorized form of the deuteron form factor can be got by substituting Eq. (11) into Drell-Yan formula [3]:

$$F_d(Q^2) = \sum_{a=1}^{6} e_a \int [dx][d^2 k_{\perp i}] \Psi_d^*(x_i, k_{\perp i} + (\delta_{ia} - x_i) q_{\perp}) \Psi_d(x_i, k_{\perp i}) ,$$ (12)

where $q_{\perp}$ is absorbed by the $a$-th quark, $q = (0, q, q_{\perp})$ and $Q^2 = q_{\perp}^2$. Noting that the gluon is a color octet in the SU(3) color group, the single-gluon exchange between two color-singlet nucleons is forbidden. Thus the real kernel calculation requires the inclusion of other components rather than two-nucleons. Brodsky and Ji [11] suggested a simple covariant model to incorporate the quark structure of the nucleon. The hard kernel at large $Q^2$ was assumed to be the perturbative amplitude for the six quarks to scatter from collinear to the initial two-nucleon configuration to collinear to the final two-nucleon configuration, where each nucleon has roughly equal momentum. They argued that the dominant configuration for this recombination is the quark-interchange plus one-gluon exchange between two nucleons. Thus, roughly speaking, we can divide the kernel into two parts. One represents the interchange of quarks and the gluon exchange between two nucleons, which transfer about half of the transverse momentum of the virtual photon from the struck nucleon to the spectator nucleon. Another part is the inner evolution of two nucleons. The first part leads to the reduced form factor of the deuteron and the latter leads to the form factors of two nucleons together with the factorized wave function mentioned above.

The body wave function in Eq. (10) can be written as

$$\Psi_d^{body}(y, l_{\perp}) = Aexp[-\frac{1}{2\alpha^2} \frac{l_{\perp}^2 + m_{N_1}^2}{4y_1y_2}]$$ (13)
by using Brodsky-Huang-Lepage prescription \[12\] from a harmonic oscillator wave function, 
\[ A' \exp\left(-\frac{1}{2}a^2r^2\right) \], in the rest frame. As mentioned above, the reduced form factor can be obtained

\[
f_d(Q^2) = D \int [dx][dy] \phi_d^\dagger(x,Q)t_H(x,y,Q)\phi_d(y,Q) ,
\]

where D is a kinematic factor. The body distribution amplitude \( \phi_d(x,Q) \) is defined by

\[
\phi_d(x,Q) = \int [dk] \Psi_d^{body}(x_i,k_{\perp i})
\]

with \( x_1 + x_2 = 1 \) and \( k_{\perp 1} + k_{\perp 2} = 0 \). The kernel \( t_H(x,y,Q) \) is dominated by the quark-interchange plus one-gluon-exchange diagrams. Since the binding energy of the deuteron is very small, the lowest Fock state (NN configuration) is dominant.

### III. QCD INSPIRED MODEL

For the deuteron case, the matrix elements of the electromagnetic current \( J^\mu \) can be written in terms of three form factors as

\[
G^\mu_{\lambda'\lambda} = \langle P'\lambda'|J^\mu|P\lambda\rangle = -\{G_1(Q^2)\epsilon^{\lambda'\lambda}\cdot\epsilon [P^\mu + P'^\mu] + G_2(Q^2)\epsilon^{\lambda'\lambda}\cdot\epsilon' - \epsilon^\mu\epsilon\cdot q - \epsilon^\mu\epsilon\cdot q\}
\]

\[
- G_3(Q^2)\epsilon\cdot q\epsilon^{\lambda'\lambda}\cdot q(P^\mu + P'^\mu)/(2M^2) \}
\]

with \( Q^2 = -q^2 \), \( q = P' - P \), and \( \epsilon \equiv \epsilon_\lambda, \epsilon' \equiv \epsilon_{\lambda'} \) are the initial and final polarization vectors, respectively. \( |P\lambda\rangle \) is an eigenstate of momentum \( P \) and helicity \( \lambda \). The Lorentz invariant form factors \( G_i \) are related to the charge, magnetic and quadrupole form factors \[10\]:

\[
G_E = G_1 + \frac{2}{3}\eta G_Q ,
\]

\[
G_M = G_2 ,
\]

\[
G_Q = G_1 - G_2 + (1 + \eta)G_3 .
\]

Perturbative QCD predicts \[1\] that the helicity-zero to zero matrix element \( G_{00}^+ \) dominates helicity amplitude at large \( Q^2 \) for lepton scattering on the deuteron. In the standard LCF, defined by \( q^+ = 0, q_y = 0 \) and \( q_x = Q \), we have the following relations approximately:

\[
G_2 = 2G_1 = \frac{2}{2P+(2\eta + 1)}G_{00}^+ ,
\]

\[
G_3 = 0
\]

and

\[
G_E : G_M : G_Q = (1 - \frac{2}{3}\eta) : 2 : -1
\]

while \( Q \gg \Lambda_{QCD} \). Also, Calson and Gross \[13\] have shown that the LCF helicity-flip amplitudes \( G_{+0}^+ \) and \( G_{+-}^+ \) are suppressed by factors of \( \Lambda_{QCD}/Q \) and \( (\Lambda_{QCD}/Q)^2 \), respectively.
Equation (14) shows that the reduced form factor \( f_d(Q^2) \) is determined by the body distribution amplitude \( \phi_d(x, Q) \) and the kernel which can be obtained from the quark-interchange plus one-gluon-exchange diagrams between two nucleons. To represent these diagrams, a model can be built by introducing a vector boson (color singlet) with an effective mass \( M_b \). The picture of this model is described by Eq. (14), which implies that the formulation is similar to that of meson form factor and \( t_H \) can be computed in the one-boson exchange approximation, replacing the gluon by a massive vector boson and coupling constant \( g_s \) by effective coupling constant \( g_{\text{eff}} \). We suggest that the effective mass \( M_b \) can be determined by the empirical scaling law [3, 15]:

\[
(1 + \frac{Q^2}{m_0^2}) f_d(Q^2) = \text{constant} \quad (21)
\]

with \( m_0^2 = M_b^2 = 0.28 \text{ GeV}^2 \).

The hard scattering amplitude can be obtained by calculating the diagrams shown in Fig. 2:

\[
t_H(x, y, Q) = \frac{4 M^2 g_{\text{eff}}^2}{xyQ^2 + M_b^2 - (x - y)^2 M^2} \cdot \frac{1}{xQ^2 + (\frac{1}{4} - (1 - x)^2 M^2)} \quad (22)
\]

where \( M \) is the deuteron mass, and we have taken the nucleon mass to be half of \( M \). The kinematic factor \( D \) is

\[
D = \sqrt{1 + \frac{4}{3} \eta + \frac{4}{3} \eta^2} \quad (23)
\]

**IV. NUMERICAL RESULTS**

Substituting Eq. (22) into Eq. (14), numerical analysis can be done with several parameters, \( \alpha, M_b \) and \( \alpha_{\text{eff}} = g_{\text{eff}}^2/4\pi \), in the expression. From the empirical scaling law, \( M_b \) is around 0.5 GeV. Instead of inputting the parameter \( \alpha_{\text{eff}} \), we normalize the amplitude at \( Q^2 = 2.5 \text{ GeV}^2 \) data point. The variation of the reduced form factor \( Q^2 f_d(Q^2) \) vs \( Q^2 \), with different values of \( M_b \), is displayed in Fig. 3. The effective coupling constant \( \alpha_{\text{eff}} \) increases as \( M_b \) become larger. By varying the parameter \( \alpha \) in the wave function we can get different behaviors of \( Q^2 f_d(Q^2) \) in the intermediate energy region, \( Q^2 \geq 1 \text{ GeV}^2 \). The corresponding effective coupling constant, which decreases as \( \alpha \) become larger, is shown in Fig. 4.

It is shown from fitting the data that \( M_b = 0.5 \text{ GeV}, \alpha = 0.21 \text{ GeV} \) and \( \alpha_{\text{eff}} = 0.15 \). The result, comparing with the calculations with the Paris potential and the experimental data [15, 14], is shown in Fig. 1. Our results reveal that our model with the above parameters can explain the deuteron form factor well for \( Q^2 \geq 1 \text{ GeV}^2 \).

**V. SUMMARY AND DISCUSSION**

The scaling law of the reduced form factor suggest [3] that the dominance of \( G_{00}^+ \) begins at \( Q^2 \sim 1 \text{ GeV}^2 \). Thus one can calculate \( G_{00}^+ \) to predict the reduced form factor in the intermediate energy region.
However, it is a very complicated problem to directly calculate $G^{+0}_{00}$, since there are over 300,000 diagrams and the evolution of the deuteron wave function leads to the dominance of hidden-color state contributions in the very large $Q^2$ region due to the gluon exchange in the kernel. In fact, Farrar, Huleihel and Zhang [7] found that hidden-color degrees of freedom in the deuteron wave function might be important in order to fit the data. In this paper we have tried to build a model to calculate the reduced form factor in the intermediate energy region, instead of doing a full QCD analysis. The point of this model is that the reduced form factor $f_d(Q^2)$ can be evaluated in a way similar to the meson form factor. It is determined by the body wave function $\phi_d(x, Q^2)$ and a kernel with a massive boson exchange. Our results show that our prediction can fit the data well for $Q^2 > 1$ GeV$^2$. To fit the data we have chosen:

(i) the effective gluon mass $M_b \sim 0.5$ GeV, which is consistent with the empirical law, (ii) the parameter in the deuteron wave function $\alpha = 0.21$ GeV, and (iii) the normalization at $Q^2 = 2.5$ GeV$^2$ data point, which corresponds to an effective coupling constant $\alpha_{eff} = 0.15$.

In addition, we restrict ourselves to calculate the reduced form factor only in the intermediate energy region. One can’t expect that $G^{+0}_{00}$ dominates the helicity amplitude in the low $Q^2$ region, say, $Q^2 < 1$ GeV$^2$. On the other hand this picture can’t apply to the form factor at very large $Q^2$ since the hidden-color state contributions may be important in that region, where the full evolution of the six-quark wave function is involved.

This model could be improved by taking into account the contributions of $G^{+0}_{00}$ and $G^{+ -}_{1}$ in the low $Q^2$ region and the hidden-color contributions in the very large $Q^2$ region. We believe this model can be generalized to other light nuclei.

**Acknowledgement**

The authors would like to thank Professor T. Huang for his valuable discussions.

This work was supported by National Science Foundation of China (NSFC) and Grant No. LWTZ-1298 of Academia Sinica.
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**Figure Captions**

**Fig. 1** Structure function $A(Q^2)$ of the elastic $ed$-scattering from our model (the solid line) with $M_b = 0.5$ GeV, $\alpha = 0.21$ GeV, and the effective coupling constant $\alpha_{\text{eff}} = 0.15$. The dashed line corresponds to the Paris potential calculation.

**Fig. 2** The hard scattering diagrams.

**Fig. 3** Comparison of the $Q^2 f_d(Q^2)$ data with our calculations by using the different effective mass of the vector boson, $M_b$, while fixing $\alpha$ at 0.21 GeV and normalized at the $Q^2 = 2.5$ GeV$^2$ data point.

**Fig. 4** Comparison of the $Q^2 f_d(Q^2)$ data with our calculations by using the different parameter in the wave function, $\alpha$, while fixing $M_b$ at 0.5 GeV and normalized at the $Q^2 = 2.5$ GeV$^2$ data point.
Fig. 1
Fig. 2
$Q^2 f_p(Q^2)$

- Arnold

$\alpha = 0.16 \text{ GeV} \quad \alpha_{\text{eff}} = 0.32$

$\alpha = 0.21 \text{ GeV} \quad \alpha_{\text{eff}} = 0.15$

$\alpha = 0.26 \text{ GeV} \quad \alpha_{\text{eff}} = 0.09$

$M_b = 0.5 \text{ GeV}$

Fig. 4