Torsional vibration of irregular single-walled carbon nanotube incorporating compressive initial stress effects

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ABSTRACT

This study is an attempt to show the impacts of surface irregularity and compressive initial stresses on the torsional vibration of a single-walled carbon nanotube (SWCNT). The governing equation and corresponding closed-form solutions were derived with the aid of Hamilton's principle. Then, the natural frequencies were obtained analytically and the influences of surface irregularity and compressive initial stresses on the torsional vibration were studied in detail. Numerical results analyzing the torsional vibration incorporating compressive initial stress effects were discussed and presented graphically. The effects of surface irregularity on the natural frequency of torsional vibrations of nanomaterials, especially for SWCNTs, have not been investigated before, and most of the previous research works have been carried for a regular carbon nanotube. Therefore, it must be emphasized that the torsional vibrations of irregular SWCNTs are novel and applicable for the design of nano-oscillators and nanodevices, in which SWCNTs act as the most prevalent nanocomposite structural element. The analytical solutions and numerical results revealed that the surface irregularity and compressive initial stress have notable effects on the natural frequency of torsional vibrations. It has been observed that, as the surface irregularity and compressive initial stress parameters increase, the torsional natural frequency of vibrations of SWCNTs also increases. Since SWCNTs have very small size, they are always subject to initial stresses from different resources; therefore, understanding the influences of compressive initial stresses on the torsional frequency of nanotubes helps the engineers and researchers to design proper nanodevices for different applications with irregular shapes.

KEYWORDS: torsional vibration, surface irregularity, compressive initial stresses, single-walled carbon nanotubes

1. INTRODUCTION

Carbon nanotubes (CNTs) have been produced as a part of microelectromechanical and nanoelectromechanical systems. Owing to their superior electrical and physical properties, single-walled CNTs (SWCNTs) are used for applications such as oscillators, charge detectors, clocks, field emission devices and sensors. For such applications, the effects of surface irregularity on free torsional vibration of SWCNTs should be realized. During the last three decades, there have been numerous studies on the vibration of SWCNTs [1–17].

In particular, to understand the dynamic behavior of torsional vibration of CNTs, numerous researchers have conducted computational simulations to study the torsional vibrations of the nanotubes. Gheshlaghi and Hasheminjed [18] have achieved the free torsional responses of the nanotube and assessed the effect of the size dependence on the response of the nanotube using the modified couple stress theory. Adeli et al. [19] employed the strain gradient theory and the generalized differential quadrature method to achieve the free torsional vibration of a nonlinear homogeneous and isotropic nanocone. The static and dynamic behaviors of the CNTs embedded in an elastic medium have been investigated by Arda and Aydogdu [20] using the nonlocal elasticity theory. Based on the Kelvin–Voigt model, El-Borgi et al. [21] have reported a novel model, including the combination of strain and velocity gradient theories to investigate the vibrational behavior of the viscoelastic nanorods. Using nonlocal elasticity theory and the Maxwell model, Zarezadeh et al. [22] have studied the torsional vibration of simply supported functionally graded nanorods embedded in an elastic bulk medium under the effect of magnetic field. Gheshlaghi et al. [23] have studied the size-dependent torsional vibration of nanotubes. Rani et al. [24] studied the torsional vibrations in initially stressed composite poroelastic cylinders. Based on nonlocal elasticity, Aydogdu and Arda [25] have analyzed the torsional vibration of double-walled CNTs. Guo et al. [26] have investigated torsional vibration of CNTs with axial velocity and velocity gradient effect. Zhu and Li [27] have used the nonlocal integral elasticity to study longitudinal and torsional vibrations of size-dependent rods. Nazemnezhad and Fahimi [28] have discussed the effects of surface energy on the free torsional vibration of cracked nanobeams at different boundary conditions based on the surface elasticity theory. Li and Hu [29] have analyzed the torsional vibration of bidirectional functionally graded nanotubes based on nonlocal elasticity theory. Based on modified couple stress theory, Yayli [30] has investigated the torsional vibrations of restrained nanotubes. Torsional wave and vibration subjected to constraint of surface elasticity have been investigated by Huang [31]. Using Eringen's...
nonlocal differential model, Yayli et al. [32] have studied the torsional vibration of cracked CNTs with torsional restraints. Khosravi et al. [33] have discussed the torsional vibrations of a triangular nanowire.

The nature of vibration analysis of SWCNTs is influenced by the change of the surface irregularity structures of the nanotube. Irregularities in the constructions of CNTs may occur as consequences of mismatch between the material properties of nanoplates and initial external load due to any other physical causes. In this manner, understanding the effects of surface irregularity on the torsional vibration of CNTs helps the engineers and researchers to design proper nanodevices for different applications with irregular shapes.

On the other hand, SWCNTs often suffer from compressive initial stresses due to residual stress, thermal effect, surface effect, mismatch between the material properties of SWCNTs and a surrounding medium, and initial external load, and may be due to any other physical causes.

It is obvious that the varying of the surface structures of the nanotube as well as the presence of the compressive initial stresses influences the nature of torsional vibration of SWCNTs. Hence, it is of great importance to deal with various constructions to study the torsional vibration of the SWCNTs. The author has investigated the torsional vibration of an irregular SWCNT using Hamilton’s principle [34]. However, the previous models could not be applied for predicting torsional vibrational behavior of pre-stressed irregular SWCNTs, due to technical limitations. On the other hand, using nanotubes for the design of nanodevices requires proper theoretical frameworks for their mechanical behaviors.

Since the SWCNTs have very small size, they are always subject to external actuations from different resources. In this manner, understanding the influences of surface irregularity as well as compressive initial stresses on the torsional frequency of nanotubes helps the engineers and researchers to design proper nanodevices for different applications with irregular shapes. As can be seen from the above literature summary, and from our best knowledge, the impacts of surface irregularity as well as initial stresses on torsional vibrations of SWCNTs have not yet been analyzed.

This work focuses on irregularity and initial compressive stress-dependent torsional vibration of clamped–clamped SWCNTs. Hamilton’s principle is used to derive governing and frequency equations of torsional vibration. The obtained results are compared with the torsional vibration of initial stress-free and uniform SWCNTs. This analysis is very important in terms of both predicting the results of the experiments and providing useful information for accurate design of nanodevices. In this sense, this study is unique and may serve as useful reference for the design of nano-oscillators and nanodevices, in which SWCNTs act as the most prevalent nanocomposite structural element.

2. FORMULATION OF THE PROBLEM

The schematic of an irregular SWCNT under compressive initial stresses is depicted in Fig. 1. The x coordinate is taken in the axial direction of the nanotube, whereas the z and y coordinates are in the radial and circumferential directions, respectively.

The displacements of the nanotube are defined by \( u_r, u_y, u_z, u_x \) and \( u_x \) in the directions of \( x, y \) and \( z \) axes, respectively. Consider an irregular single-walled carbon nanotube (ISWCNT), which undergoes torsional vibration, has a thickness \( h \), radius \( R \) and length \( L \).

The surface irregularity is considered in the form of parabola at the lower boundary of the SWCNT as shown in Fig. 1. We denote the span of the surface irregularity and maximum depth of the irregularity by \( s \) and \( H' \), respectively. In case of parabolic surface irregularity, the boundary surface may be described by [35]

\[
\delta(x) = \begin{cases} 
2s \left( 1 - \frac{x^2}{r^2} \right), & \text{for } |x| < \frac{r}{2}, \\
0, & \text{for } x \leq -\frac{r}{2} \text{ and } x \geq \frac{r}{2},
\end{cases}
\]

(1)

where \( \varepsilon = H'/s \ll 1 \) is the maximum amplitude of irregular boundary (a perturbation parameter), which is assumed to be small, and this assumption fits those naturally or artificially constituted scenarios where the depth \( H' \) of the irregularity is typically very small with respect to the span of the irregularity \( s \) and \( x = r \cos \phi \).

The stress–strain relation of the SWCNT can be written as

\[
\sigma_{\theta z} = G\varepsilon_{\theta z},
\]

(2)

\[
\sigma_{rz} = G\varepsilon_{rz},
\]

(3)

where \( G \) represents the shear modulus of the irregular SWCNT; \( \sigma_{\theta z}, \sigma_{rz} \) and \( \varepsilon_{\theta z}, \varepsilon_{rz} \) are shear stresses and strains of the ISWCNT, respectively.

The torque resultant of the ISWCNT can be written as [34]

\[
T_x = T_{\theta z} + T_{\theta \theta r},
\]

(4)

\[
T_{\theta z} = \int_A \left( y \tau_{xz} - z \tau_{yz} \right) dA,
\]

(5)

\[
T_{\theta \theta r} = \int_A \left( y \tau_{xz} + (R + h + \varepsilon \delta(x)) \tau_{xy} \right) dA.
\]

(6)
The displacement components at any point of the nanotube can be expressed as

\[ u_r(x, t) = 0, \]
\[ u_\phi(x, t) = (R + h + \varepsilon \delta(x)) \Phi(x, t), \]
\[ u_z(x, t) = y\Phi(x, t), \]

where \( \Phi(x, t) \) is the angular displacement about the center of the twist. The strains of the ISWCNT are defined as

\[ \varepsilon_{r\phi} = \frac{\partial u_r}{\partial \phi} + \frac{\partial u_\phi}{\partial x} = (R + h + \varepsilon \delta(x)) \frac{\partial \Phi}{\partial x}, \]
\[ \varepsilon_{rz} = \frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial x} = y \frac{\partial \Phi}{\partial x}, \]
\[ \varepsilon_{\phi z} = \frac{\partial u_\phi}{\partial z} + \frac{\partial u_z}{\partial \phi} = 0, \]
\[ \varepsilon_{rr} = \varepsilon_{\phi\phi} = \varepsilon_{zz} = 0. \]
By substituting the first derivatives of Eq. (8) into Eq. (10), the following equation can be derived:

\[
\kappa = \frac{1}{2} \int_V \rho \left[ \left( -z \frac{\partial \Phi}{\partial t} \right)^2 + \left( y \frac{\partial \Phi}{\partial t} \right)^2 \right] \, dV
\]

\[
= \frac{1}{2} \rho I_p \int_0^L \left( \frac{\partial \Phi}{\partial t} \right)^2 \, dx + \frac{1}{2} \rho I_{p_v} \int_{-\pi/2}^{\pi/2} \left( \frac{\partial \Phi}{\partial t} \right)^2 \, dx
\]

\[
= \frac{1}{2} \rho I_0 \int_0^L \left( \frac{\partial \Phi}{\partial t} \right)^2 \, dx + \frac{1}{2} \rho I_{p_v} \int_{-\pi/2}^{\pi/2} \left( \frac{\partial \Phi}{\partial t} \right)^2 \, dx,
\]

where \( \rho \) is the density of the nanotube material, and \( I_p, I_0 \) and \( I_{p_v} \) represent the polar moment of inertia and mass moment of inertia of uniform SWCNT and polar moment of inertia of ISWCNT, respectively, and are equivalent to the following:

\[
I_p = \int_A (y^2 + z^2) \, dA, \tag{12}
\]

\[
I_0 = \rho I_p, \tag{13}
\]

\[
I_{p_v} = \int_{-\pi/2}^{\pi/2} \int_0^{R+h} \left( y^2 + (R + h + \varepsilon \delta(x))^2 \right) \, dx \, dy. \tag{14}
\]

For the uniform SWCNT, the average radius distance from the origin of coordinates \( R^2 = y^2 + z^2 \). Thus, the polar moment of inertia of the uniform SWCNT can be expressed as

\[
I_p = \int_A r^2 \, dA = \pi \int_0^{R+h} r^2 (2r \, dr) = \frac{\pi}{2} [ (R+h)^4 - (R)^4 ]. \tag{15}
\]

From Eq. (11), it can be observed that if the surface irregularity is ignored (\( \varepsilon = 0 \), \( s = 0 \)), the kinetic energy for the SWCNT can be defined as

\[
\kappa = \frac{1}{2} \rho I_p \int_0^L \left( \frac{\partial \Phi}{\partial t} \right)^2 \, dx = \frac{1}{2} I_0 \int_0^L \left( \frac{\partial \Phi}{\partial t} \right)^2 \, dx, \tag{16}
\]

which agrees with the result of Khosravi et al. [33].

Then, the virtual form of the kinetic energy of the ISWCNT can be expressed as

\[
\delta \kappa = I_0 \int_0^L \left( \frac{\partial \Phi}{\partial t} \right) \left( \frac{\partial \delta \Phi}{\partial t} \right) \, dx + \rho I_{p_v} \int_{-\pi/2}^{\pi/2} \left( \frac{\partial \Phi}{\partial t} \right) \left( \frac{\partial \delta \Phi}{\partial t} \right) \, dx. \tag{17}
\]

In the case of a linearized deformable SWCNT, the variation of the surface irregularity-dependent strain energy \( \delta U \) can be defined as

\[
\delta U = \int_V \left( \tau_{vw} \delta \varepsilon_{vw} + \tau_{rz} \delta \varepsilon_{rz} \right) \, dV. \tag{18}
\]

Substituting Eq. (8) into Eq. (18), we get

\[
\delta U = \int_V \left( -z \tau_{vw} \frac{\partial \delta \Phi}{\partial x} + y \tau_{rz} \frac{\partial \delta \Phi}{\partial x} \right) \, dV. \tag{19}
\]

By substituting Eqs (4), (5) and (6) into Eq. (19), the virtual strain energy can be written as

\[
\delta U = \int_0^{L-s} T_{sym} \frac{\partial \delta \Phi}{\partial x} \, dx + \int_{-\pi/2}^{\pi/2} T_{sym} \frac{\partial \delta \Phi}{\partial x} \, dx. \tag{20}
\]

Because of the variation of surface irregularity-dependent kinetic and strain energies, Eqs (17) and (20) have been obtained. By substituting Eqs (17) and (20) into Eq. (9), we will get

\[
\int_0^L \left[ \int_0^{L-s} T_{sym} \frac{\partial \delta \Phi}{\partial x} \, dx - I_0 \int_0^L \left( \frac{\partial \Phi}{\partial t} \right) \left( \frac{\partial \delta \Phi}{\partial t} \right) \, dx \right] + \int_{-\pi/2}^{\pi/2} \left[ \int_{-\pi/2}^{\pi/2} T_{sym} \frac{\partial \delta \Phi}{\partial x} \, dx - \rho I_{p_v} \int_{-\pi/2}^{\pi/2} \left( \frac{\partial \Phi}{\partial x} \right) \left( \frac{\partial \delta \Phi}{\partial x} \right) \, dx \right]. \tag{21}
\]

With clamped–clamped boundary conditions (at \( x = 0 \) or \( L \)), specify \( \Phi = 0 \) or \( T_{sym} = 0 \). Here, \( \Phi(0) = 0 \) is known as the essential boundary condition, and \( T_{sym}(L), T_{sym}(L) = 0 \) are known as the natural boundary conditions.
Based on Hamilton’s principle (9), the torsional governing equation, considering the surface irregularity of the SWCNT under compressive initial stresses, can be determined as

\[
\frac{\partial T_{sd}}{\partial x} + \frac{\partial T_{si}}{\partial x} = \rho I_p \frac{\partial^2 \Phi}{\partial t^2} + \rho I_{Pu} \frac{\partial^2 \Phi}{\partial t^2} + \zeta(x), \tag{22}
\]

where

\[
\zeta(x) = G \xi_x \frac{\partial^2 v}{\partial x^2} = G \xi_x \frac{\partial^2 (-x\Phi)}{\partial x^2}
= \xi_s \left[ \frac{4G\varepsilon}{s} \Phi - (R + h + \varepsilon \delta(x)) \frac{\partial^2 \Phi}{\partial x^2} + \frac{4G\varepsilon x \partial \Phi}{s} \right]. \tag{23}
\]

and \( \xi_s = P/2G \) is the compressive initial stress parameter.

Ultimately, using stress–strain relations (2) and (3), one can obtain

\[
T_{sd} = G I_p \frac{\partial \Phi}{\partial x}, \tag{24}
\]

\[
T_{si} = G I_{Pu} \frac{\partial \Phi}{\partial x}. \tag{25}
\]

Substituting the moment of torques (23), (24) and (25) into the governing equation (22) yields the equation of torsional motion for irregular SWCNTs as follows:

\[
\frac{\partial^2 \Phi}{\partial x^2} + \frac{I_{Pu}}{I_p} \frac{\partial^2 \Phi}{\partial x^2} + \xi_s \frac{4G\varepsilon}{s} \Phi - \xi_s G (R + h + \varepsilon \delta(x)) \frac{\partial^2 \Phi}{\partial x^2} - \xi_s \frac{4G\varepsilon x \partial \Phi}{s} \frac{\partial^2 \Phi}{\partial x^2} - \frac{\rho}{G} \frac{\partial^2 \Phi}{\partial t^2} - \frac{\rho}{G} \frac{I_{Pu}}{I_p} \frac{\partial^2 \Phi}{\partial t^2} = \frac{1}{I_p} \frac{\partial I_{Pu}}{\partial x} \frac{\partial \Phi}{\partial x}. \tag{26}
\]

3. ANALYTICAL SOLUTION FOR TORSIONAL VIBRATION OF IRREGULAR SWCNT

The problem, in which mathematical formulation is expressed in the previous subsection, is a governing equation for torsional vibration of irregular SWCNTs. For the simply supported boundary condition of ISWCNTs, the general solution of equation of torsional motion (26) can be given as

\[
\Phi(x, t) = \phi_m(x) e^{i\omega t}, \tag{27}
\]

where \( i = \sqrt{-1} \) and \( \phi_m \) stands for the \( m \)th mode shape, which can be expressed as

\[
\phi_m = A_m \sin k_m x. \tag{28}
\]

The torsional angular frequency is designated by \( \omega_m \) and the torsional natural frequency of the nanotube is given by the relation \( f_m = \omega_m/2\pi \), \( k_m \) denoting the axial wave number.

We suppose that the nanotube is simply supported at its ends, so the axial wave number can be written as [36]

\[
k_m = \frac{m\pi}{L}. \tag{29}
\]

Using the expressions for \( \Phi(x, t) \) and \( \phi_m(x) \) given in Eqs (27) and (28), Eq. (26) is transformed into the torsional frequency equation for the irregular SWCNT in the following form:

\[
\omega_m = \sqrt{\xi_s \frac{G^2 I_p}{\rho (I_p + I_{Pu})} \left[ k_m^2 (R + h + \varepsilon \delta(x)) + \frac{4\varepsilon x}{s} \cot(k_m x) - \frac{4\varepsilon}{s} \right] + \frac{G}{\rho} \left( k_m^2 + \frac{\Omega_m(x)}{(I_p + I_{Pu})} \right)}, \tag{30}
\]

where

\[
\Omega_m(x) = \frac{\partial I_{Pu}}{\partial x} \cos(k_m x) \tag{31}
\]

represents the surface irregularity term.

4. PARTICULAR CASE

When the effect of compressive initial stresses is absent (i.e. \( \xi_s = 0 \)), then Eqs (26) and (30) become

\[
\frac{\partial^2 \Phi}{\partial x^2} + \frac{I_{Pu}}{I_p} \frac{\partial^2 \Phi}{\partial x^2} - \frac{\rho}{G} \frac{\partial^2 \Phi}{\partial t^2} - \frac{\rho}{G} \frac{I_{Pu}}{I_p} \frac{\partial^2 \Phi}{\partial t^2} = \frac{1}{I_p} \frac{\partial I_{Pu}}{\partial x} \frac{\partial \Phi}{\partial x}. \tag{32}
\]
In this section, numerical results for torsional vibration of the clamped–clamped ISWCNT under the effects of compressive initial stresses are obtained using the simulation parameters presented in Table 1 [33]. The torsional frequency (THz) as a function of wave number \( k_m \), surface irregularity parameter \( \varepsilon \) and initial stress parameter \( (\xi, \zeta) \) has been evaluated using Eqs (29) and (30). The results obtained are compared with the cases of uniform \( (\varepsilon = 0.0) \) and initial stress-free \( (\xi, \zeta = 0.0) \) nanotubes.

An examination of the studies concerned with fabrication procedures of CNTs reveals that perfectly straight CNTs without any deviation in geometry and orientation are very difficult to achieve [37]. Therefore, understanding the influences of surface irregularity as well as initial stresses on the torsional vibration of SWCNTs may serve as useful reference for the applications and designs of nanoscanners and nanodevices, in which SWCNTs act as the most prevalent nanocomposite structural element.

In order to show the effects of compressive initial stresses as well as surface irregularity on the torsional natural frequency of SWCNTs, the relation between the torsional frequency and axial wave number is calculated. Numerical computations are carried out for different values of compressive initial stress parameter \( (\xi, \zeta = 0.2, 0.4) \) and surface irregularity parameter \( (\varepsilon = 0.28, 0.56) \). The results of computations are plotted in Figs 2 and 3, for various values of wave number \( k_m \) at first and second modes.

Figure 2a shows the variations of torsional natural frequency of the uniform \( (\varepsilon = 0.0) \) SWCNT against axial wave number at first and second modes. It is clearly seen from the figure that the values of the torsional natural frequency of the uniform SWCNT change approximately in the range of 4–25 THz for the first mode \((m = 1)\), while the range is different and takes almost the values of 8–50 THz for the second mode \((m = 2)\). In addition, it is seen that, as the axial wave number gets higher \((k > 70 \times 10^9 \text{ nm})\), the torsional natural frequency of the SWCNT tends to decrease.

Figure 2b shows the variations of torsional natural frequency of the SWCNT with respect to axial wave number \( k \) at first and second modes for different values of surface irregularity parameter \( (\varepsilon = 0.28) \) at \( \xi, \zeta = 0.0 \). From the numerical results obtained in Fig. 2b, it is observed that, as the surface irregularity parameter gets higher, the torsional natural frequency of the SWCNT tends to increase. In addition, it is clearly seen that the torsional natural frequency of the SWCNT takes, approximately, the values between 4 and 24 THz for the first mode \((m = 1)\) and between 10 and 44 THz for the second mode \((m = 2)\). On the other hand, the effect of surface irregularity is much higher for smaller axial wave number \((k \leq 70 \times 10^9 \text{ nm})\), while the curves get closer to each other at higher axial wave number \((k > 70 \times 10^9 \text{ nm})\). This means that the torsional frequency of vibration is more sensitive to surface irregularity parameter at lower axial wave numbers.

Figure 2c shows the variations of torsional natural frequency of the SWCNT with respect to axial wave number \( k \) for the first and second modes at \( \varepsilon = 0.56 \). From Fig. 2c, it can be observed that the values of the torsional frequency of the SWCNT lie in the intervals of 5–42 and 10–85 THz for the first and second modes, respectively. It is also mentioned that the torsional frequency of the SWCNT increases considerably as the surface irregularity parameter increases. In addition, it can be seen that, for the lower axial wave number \((k \leq 70 \times 10^9 \text{ nm})\), the effects of surface irregularity on torsional natural frequency of the SWCNT are notable for the first and second modes, while these effects are not seen at higher axial wave number \((k > 70 \times 10^9 \text{ nm})\), where the corresponding curves converge to each other for both first and second modes.

The effects of compressive initial stresses on torsional vibration of an irregular SWCNT are shown in Fig. 3a–c. Figure 3a shows the effect of compressive initial stress \( (\xi, \zeta = 0.2, 0.4) \) on the torsional natural frequency of the uniform \( (\varepsilon = 0.0) \) SWCNT for the first and second modes. It is clearly seen that the values of torsional natural frequency are quite different, compared with the case of initial stress-free SWCNT, as shown in Fig. 2a. The torsional natural frequency takes the values between 20 and 200 THz and between 100 and 400 THz for the first and second modes, respectively, at \( \xi, \zeta = 0.2 \), while the range is different and takes almost the values of 50–280 and 100–520 THz for the first and second modes, respectively, at \( \xi, \zeta = 0.4 \). From Fig. 3a, it is also observed that the torsional natural frequency of the uniform SWCNT is affected by the compressive initial stresses present in the nanotube.

For more clarification of the concept of compressive initial stress-dependent behavior of torsional natural frequency of the ISWCNT, other complementary numerical examples are given in Fig. 3b and c. Figure 3b shows the variations of torsional natural frequency of the SWCNT versus the axial wave number for first and second modes for \( \xi, \zeta = 0.2 \) and \( 0.4 \) at \( \varepsilon = 0.28 \). From Fig. 3b, it is observed that the values of the torsional frequency of vibration are quite different from the values obtained in the previous results shown in Fig. 3a.
Figure 2 Variations of first and second modes of torsional frequency of the SWCNT as a function of axial wave number at $\zeta_x = 0.0$: (a) $\varepsilon = 0.0$; (b) $\varepsilon = 0.28$; and (c) $\varepsilon = 0.56$.

for the case of uniform ($\varepsilon = 0.0$) SWCNT. The reason is very clear that the change is due to the effects of both surface irregularity and compressive initial stresses.

The effects of compressive initial stress ($\zeta_x = 0.2, 0.4$) on the torsional vibration of the ISWCNT at $\varepsilon = 0.56$ are shown in Fig. 3c. Figure 3c shows the variations of torsional natural frequency of the SWCNT versus the axial wave number for first and second modes. From this figure, it is observed that the values of the torsional natural frequency of vibration are quite different from the values obtained in the previous results shown in Fig. 3a and b. The reason is very clear that the change is due to the increase of surface irregularity ($\varepsilon = 0.56$) and initial stress parameters ($\zeta_x = 0.2, 0.4$).
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6. CONCLUSION

In this article, we have reported a novel equation of motion for pre-stressed irregular SWCNTs based on Hamilton’s principle. The worth mentioning component of the present exploration is the impacts of compressive initial stresses and surface irregularity as well...
as axial wave number on the torsional natural frequency of vibration of SWCNTs. The numerical results have been shown through graphs to illustrate the dependence of torsional vibration characteristics of SWCNTs on the compressive initial stresses and surface irregularity. It has been detected that the presence of compressive initial stresses and surface irregularity in the SWCNTs considerably affects the torsional natural frequency of vibrations. The obtained numerical results show that the increase of compressive initial stress and surface irregularity parameters increases the torsional frequency of vibrations. To our best knowledge, effects of compressive initial stresses on torsional vibrations, especially for irregular SWCNTs, have not been investigated before. The previous studies were devoted to the uniform and initial stress-free torsional vibrations of SWCNTs. It should be emphasized that the torsional vibration of irregular SWCNTs under compressive initial stresses is novel, workable, and at the beginning of the path. The obtained results of this work are expected to be useful to design and analyze the torsional vibration properties of CNTs and nanostructures.

The author will extend this work using the developed nonlocal elasticity theory containing additional strain gradient scale parameter representing the size effect of high-order strain gradient CNTs. The nonlocal scale parameter depicts interactions of neighboring particles. In view of surface irregularity and initial stress effects, an additional kinematic component that represents material particles in micro-/nanoscale distinguishing those in macroscale is involved in the governing equations. The impacts of nanoscale parameters on torsional frequencies will be discussed in detail; meanwhile, comparisons of the classical model with other high-order nonclassical models, and the relations between surface irregularity and initial stresses and torsional frequencies will be shown in the future work. The relations between surface irregularity and the initial stress parameters and nanoscale parameters will be illustrated and the influences of these effects on torsional frequencies will be presented.

ACKNOWLEDGMENTS

This project was supported by the Deanship of Scientific Research at Prince Sattam bin Abdulaziz University under the research project no. 2020/01/16579.

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