X-ray coherent diffraction interpreted through the fractional Fourier transform

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Abstract. Diffraction of coherent X-ray beams is treated through the fractional Fourier transform. The transformation allow us to deal with coherent diffraction experiments from the Fresnel to the Fraunhofer regime. The analogy with the Huygens-Fresnel theory is first discussed, a generalized uncertainty principle is introduced and the successive diffraction of two objects is interpreted through the fractional Fourier transform.

1 Introduction: diffraction of a rectangular aperture

Almost two centuries ago, Fresnel has shown a beautiful consequence of the wave character of light. If an opaque disc is located in front of a monochromatic light source, a bright spot is observed at a given distance \( z_b \) at the center of the shadow of the disc [1]. This phenomenon is known as the Poisson bright spot. In the opposite situation, the diffraction pattern of an aperture gives rise to a minimum of intensity if the detector is located at a very specific distance. This minimum of intensity will be called the dark spot in the following.

The dark spot results from a destructive interference. It is located in the Fresnel regime where the beam remains almost parallel and where the amplitude oscillates rapidly. The diffraction pattern of a squared aperture simulated from the Fresnel’s integral is displayed in Figure 1 where the dark spot is clearly visible.

The Fraunhofer regime is reached for distances \( z \), between the detector and the diffracted object [2], larger than \( z \gg \frac{\lambda a^2}{\pi} \). In this regime, the diffracted amplitude is proportional to the Fourier transform (FT) of the amplitude at the diffracted object position:

\[
F(q) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)e^{-iyx} dx,
\]

with \( q = \frac{2\pi}{\lambda} \). In the case of a rectangular function \( s(x) \) which is only non-zero in the interval \( [-\frac{a}{2}, \frac{a}{2}] \), we obtain:

\[
I(q) = F^*(q)F(q) \propto \left[ \sin \left( \frac{q a^2}{2} \right) \right]^2.
\]

The diffracted intensity by a rectangular function is proportional to the well-known cardinal sinus function squared, in perfect agreement with measurements. The measurement of the diffraction pattern of a 2 \( \mu \)m \( \times \) 2 \( \mu \)m slit by a X-ray beam in the Fraunhofer regime is displayed in Figure 2. In this regime, the amplitude varies very smoothly and the beam width \( \Delta \) is proportional to \( z \): \( \Delta \propto \frac{\lambda z}{a} \).

This old optical problem has recently recover a great interest by the scientific community using coherent X-ray diffraction. Indeed, coherent X-ray beams are obtained from weakly coherent synchrotron sources by cleaning and collimating the beam thanks to rectangular slits [3,4].

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Concretely, two sets of rectangular slits are usually used along the optical path. The downstream one is located at few centimeters from the sample and is opened at few tens of micrometers. Within this setup, the knowledge of the dark spot position is important because it is usually located at few tens of centimeters downstream the slit. As a consequence, the sample may be located either in the Fresnel or in the Fraunhofer regime as respect to aperture and wavelength, which may strongly influence diffracted patterns.

The dark spot position is not obvious to calculate analytically. In first approximation, $z_m$ can be estimated by assuming that it corresponds to the distance $z$ where the beam width in the Fraunhofer regime ($\Delta \approx \lambda^2/a$) is equal to the beam width in the Fresnel regime ($\Delta \approx a$), that is:

$$z_m = \frac{a^2}{\lambda}.$$

From numerical simulations, the dark spot position is located close to $z_b \approx \frac{a^2}{2\pi\lambda}$ and the location where the beam width is minimum at $z_m \approx \frac{a^2}{\lambda}$ (see Fig. 1).

2 Fractional Fourier transform

Diffraction and Fourier transform are usually associated, since a long time, and the mathematics of Fourier transform are used implicitly in experimental studies with X-rays. Nevertheless the behaviour of a X-ray beam interacting with slit or matter is not always described by Fourier transform or oppositely within the geometrical approximation of light beam. In the generic case it is possible to use equations of wave propagation, but calculations are often not “straight forward” and do not allow an intuitive view of the phenomena. If the experimental situation is that of diffraction the elegance of the Fourier transform makes thinks very clear. The purpose presented here is to show that there is a mathematical tool which plays this role when experimental conditions are not always that of diffraction, the fractional Fourier transform.

We first discuss in this section the relation between the Huygens-Fresnel theory and the fractional Fourier transform, introduced by Namias [6]. The previous slit diffraction in Figure 1 can be obtained from the fractional Fourier transform. Within this framework, the resolution of the Fresnel’s integral is nothing else but the resolution of the quantum harmonic oscillator.

2.1 Fractional Fourier transform: an operator

The most natural way to introduce the fractional Fourier transform is to note that the Fourier transform, defined in equation (1), can be written as an operator $\mathcal{F}$ acting on function:

$$F = \mathcal{F}[f].$$

If the same operator $\mathcal{F}$ is applied two times, we obtain:

$$\mathcal{F}^2[f](x) = f(-x).$$

The operator $\mathcal{F}$ has to be applied four times to recover the original function:

$$\mathcal{F}^4[f](x) = f(x)$$

as illustrated in Figure 3 from a 2D picture. Within this framework, the standard Fourier transform (FT) is an angular transformation of $\pi/2$ in the $(x,q)$ plane. A generalized Fourier transform can thus be developed for any angle of rotation $\alpha$ from zero to $\frac{\pi}{2}$. The integral form of the fractional Fourier transform (in 1D case) can be written as [6,7]:

$$\mathcal{F}^\alpha[f](x) = \int_{-\infty}^{\infty} K(x,u)f(u)du$$

(3)
with
\[ K(x, u) = \sqrt{\frac{1 - i \cot \alpha}{2\pi}} e^{i \frac{ux}{\sin \alpha} + \frac{i}{2} (x^2 + u^2) \cot \alpha} \]

For \( \alpha = 0 \), it can be shown that this expression is equivalent to the identity [8]. The standard Fourier transform (Eq. (1)) is easily obtained for \( \alpha = \pi/2 \).

### 2.2 The fractional Fourier transform and slit diffraction

The fractional Fourier transform (FrFT) in 2D case, so with appropriate factor in front of integral is:
\[ F_{\alpha} f(q) = \frac{-i\alpha}{2\pi \sin \alpha} e^{i \frac{q^2 \cot \alpha}{2}} \ldots \int_{R^2} e^{-i \frac{q^2 \cot \alpha}{2}} e^{-i q \cdot r} f(r)dr \]  

It has to be compared to the diffracted field amplitude. We consider a diffracting plane \( \Sigma \) (containing the slit) and we observe the field amplitude at distance \( z \) of \( \Sigma \) on a plane screen \( \Pi \) orthogonal to the axis. The field amplitude at a position on \( \Pi \) defined by the variable \( \xi \) is:
\[ A_{\Pi}(q) = \frac{1}{\lambda z} e^{-i \frac{\xi^2 q^2}{\lambda z}} \ldots \int_{R^2} e^{-i \frac{\xi^2 q^2}{\lambda z}} e^{-2i \pi \frac{\xi}{\lambda z} r} A_{\Sigma}(r)dr. \]  
The similarity between equations (4) and (5) suggests to write diffraction in terms of FrFT. Papers by Pellat-Finet [9,10] give the relation between the two expressions.

The method is based on the use of intermediate spherical surface on which the amplitude of the field is considered, then within conditions on curvature and position of these surfaces, field on it can be related by Fourier transform or fractional Fourier transform.

#### 2.2.1 The Fraunhofer regime

The simplest example is the case of the Fraunhofer regime. Consider two spherical surfaces (see Fig. 4a). The first one is the object surface \( S \) of radius \( z \) passing through the origin \( \Omega_S \) (the middle of the slit if the object is a slit) and with a center \( \Omega_E \) on the optical axis at positive distance \( z \) of \( \Omega_S \). The other spherical surface \( E \), a screen where the field is observed, contains \( \Omega_E \) and has center \( \Omega_S \), so his radius is \( -z \) if oriented from surface to center along the axis. Points on these surfaces are defined by the coordinates of their projection parallel to the axis on the planes orthogonal to the axis in \( \Omega_S \) and \( \Omega_E \). In this case the field \( U_S \) on \( S \) and \( U_E \) on \( E \) are related by a Fourier transform:
\[ U_E(r') = \frac{i}{\lambda z} \int_{R^2} e^{2i \pi \frac{r' \cdot r}{\lambda z}} U_S(r)dr. \]  

#### Fig. 4. Fraunhofer diffraction (a): the field on \( E \) is the Fourier transform of the field on \( S \). Fresnel and fractional Fourier transform (b): the field on \( S \) is change into the field on \( D \) by a fractional Fourier transform defined by \( \alpha \) with cot(\( \alpha \)) = (1 - \( \mu \))/\( \mu \). The field on \( S \) can be transform into the field on \( E \) combining two fractional Fourier transform defined by angles \( \alpha \) and \( \beta \) such that \( \alpha + \beta = \pi/2 \).

Using reduced variables \( \rho = \left(\frac{2\pi}{\lambda z}\right)^{1/2} r \) and \( \rho' = \left(\frac{2\pi}{\lambda z}\right)^{1/2} r' \) as arguments of scaled functions \( V_S(\rho) \) and \( V_E(\rho') \):
\[ V_E(\rho') = i F_{\pi/2}[V_S](\rho') \]  

The field on \( E \) is the Fourier transform of the field on \( S \). If \( z \to \infty \) this gives the limit of the Fraunhofer regime: a plane object is Fourier transform into a plane diffraction at infinity. Spherical surfaces in place of planes have to be considered for finite \( z \). The Fraunhofer regime corresponds to the large \( z \) domain in which phases shift between planes and spherical surfaces remain small.

#### 2.2.2 The Fresnel regime

In the FrFT, \( \alpha = 0 \) corresponds to \( z = 0 \) and \( \alpha = \pi/2 \) corresponds to \( z \to \infty \). It is tempting to get a relationship between \( z \) and \( \alpha \). Comparison of two terms \( \exp(-\frac{iq \cdot r}{\sin \alpha}) \) in equation (4) and \( \exp(-\frac{2i \pi \xi \cdot r}{\lambda z}) \) in equation (5) suggests
\[ \sin(\alpha) \propto \frac{\lambda z}{2\pi}. \]  

but other terms in these equations contradict this expression mainly for small \( \alpha \), in the Fresnel regime. There is a difficulty comparing variable \( q \) which defines a wave vector in the reciprocal space with \( \xi \) in real space.

The solution to this problem has been given by Pellat-Finet (see Fig. 4). In place of the spherical surface through \( \Omega_S \) with radius \( z \) a surface \( C \) with a smaller radius \( \mu z \), also through \( \Omega_S \) is considered. Call \( \Omega_C \) its center. We consider also a spherical surface \( B \) through \( \Omega_C \) and center \( \Omega_S \), so with oriented radius \( -\mu z \). Field transfer from \( S \) to \( C \) implies a quadratic phase factor, and transfer from \( C \) to \( B \) is again a Fourier transform. Consider a reduced field function \( V_B \) of a reduced variable \( \rho'' \):
\[ V_B(\rho'') = U_B \left(\frac{\lambda z}{2\pi}\right)^{1/2} \frac{\rho''}{\cos(\alpha) + \sin(\alpha)} \]  

The reduced variable are defined by \( \rho = \left(\frac{2\pi}{\lambda z}\right)^{1/2} r \) and \( \rho'' = \left(\frac{2\pi}{\lambda z}\right)^{1/2} \rho' \). Notice that variables \( r \) and \( r'' \) give position of points on spherical surfaces \( C \) and \( B \). Such points are defined by coordinates of their orthogonal projection on a plane orthogonal to the optical axis.
The $\mu$ parameter smaller than one, is related to $\alpha$ by
\[
\cot(\alpha) = \frac{1 - \mu}{\mu} \text{ or } \mu = \frac{\sin(\alpha)}{\cos(\alpha) + \sin(\alpha)}.
\] (10)

In order to have a fractional Fourier transform with the correct quadratic phase factor, an other spherical surface $D$ is needed. A sphere through $\Omega_C$ with positive oriented radius $R$ such that $R = (\mu^2 + (1 - \mu)^2)z/(1 - 2\mu)$. Note that surface $D$ corresponds to a plane when $\mu = \frac{1}{2}$.

Then we have:
\[
V_D(\rho''') = e^{i\alpha}(\cos(\alpha) + \sin(\alpha)) F_{\alpha} [V_S] (\rho''').
\] (11)

It can be shown that transfer from the field on $D$ to the field on $E$, using intermediated spherical surfaces $D'$ and $E'$ to have correct phase factors, is given by a fractional Fourier transform $F_{\beta}$ where $\beta$ defined by the coefficient $\mu' = 1 - \mu$ is $\beta = \pi/2 - \alpha$.

This prove the continuity of the transformation following that of the fractional Fourier transform, $F_{\pi/2} = F_{\alpha} + F_{\beta}$. Notice that this continuity is false for the Fresnel transformation.

2.3 Relation with the quantum harmonic oscillator

The most remarkable property of the fractional Fourier transform is that a possible choice for the eigenfunctions of the operator $F_{\alpha}$ is given by the set of normalized Hermite-Gauss functions $[6, 11, 12]$, similar to the orthogonal harmonic oscillator basis:
\[
\Phi_n(x) = \frac{1}{\pi^{n/2}n!} \exp \left[-\frac{x^2}{2}\right] H_n(x),
\]
where $H_n(x) = (-1)^n \exp[\nu^2/2] \sum_{m=0}^{\infty} \frac{n^m}{m!} \exp[-\nu^2/2]$. The eigenvalues of $F_{\alpha}$ are $e^{i\alpha}$:
\[
F_{\alpha} \Phi_n(x) = e^{i\alpha} \Phi_n(x).
\]

Extension to two dimensions is straight forward. Obviously,
\[
F_{\alpha} F_{\beta} \Phi_n(x) = F_{\alpha + \beta} \Phi_n(x).
\] (12)

Consequently the diffraction problem can be mapped on that of the harmonic oscillator. The Fresnel’s integral or the fractional Fourier transform applied to the diffraction of a squared aperture is equivalent to the resolution of the Schrodinger equation with time in a harmonic potential, with a rectangular function as initial time condition [13].

Then time evolution is related to the $\alpha$ value. In terms of quantum mechanics, the fractional Fourier transform offers the way to continuously switch from the position space to the impulse space.

2.3.1 Gaussian beams

It is interesting to consider the behavior of a Gaussian beam propagating from the Fresnel to the Fraunhofer regime (see Fig. 5a). Since the first eigenfunction of the basis is a Gaussian, it could be expected that a Gaussian beam will remain Gaussian if its width correspond to that of this eigenfunction. Following the Pellat-Finet approach, we have introduced reduced variables $\rho = (\frac{\sqrt{2}}{2}) \frac{\sqrt{\sigma}}{r}$. It is with such variables that the width of a beam have to be given. Then we have to consider, as given in Figure 4, a Gaussian beam modulating a spherical wave $S$ with center $\Omega_E$ at distance $z$. If the Gaussian amplitude on $S$ is proportional to $\exp(-\frac{r^2}{2\sigma^2})$ with $\sigma = 1$ for the first eigenvalue of the harmonic oscillator, this width will be kept propagating from $S$ to $E$ through $D$. Expressed using $r, r'$ or $r''$ variables the width is $\frac{1}{2\sigma^2}$. With numerical values $A = 1$ Å and $z = 1$ m, the constant width is $4 \mu$m. The physical meaning of this value is that a beam ($A = 1$ Å) falling on a curved mirror designed to focalized at 1 m, with a width of 1 $\mu$m just after the mirror, have in fact a constant width.

What is the behavior of narrower or wider beams ($\sigma \neq 1$) than the first eigenfunction? The 1-D fractional Fourier transform $F_{\alpha} f(x)$ of a Gaussian function $f(x)$,
\[
f(x) = \frac{1}{\pi^{1/4} \sqrt{\sigma}} \exp \left(-\frac{x^2}{2\sigma^2}\right),
\] (13)
is:
\[
F_{\alpha}(q) = \frac{\sqrt{\sigma} \sin \alpha - i \cos \alpha}{\sqrt{(\sin \alpha - i \sin 2\alpha \cos \alpha)}} \exp \left(-\frac{q^2}{2\sigma^2} \cot \alpha + i\alpha \right).
\] (14)

Note that the amplitude $F_{\alpha}(q)$ remains a real function and is invariant with $\alpha$ when $\sigma = 1$. In the general case, it appears that the transform of a narrow function is wide and reverse, but it appears also that the $F_{\alpha}(q)$ function is
not a Gaussian for $\alpha \in [0, \pi/2]$. Nevertheless the module
of this function remains a Gaussian in agreement with
the experiment. The amplitude of the beam has a width
$\delta = \sigma \sqrt{2 / \sqrt{1 - \cos(2\alpha) + \sigma^4(1 + \cos(2\alpha))}}$.

2.4 Fractional Fourier transform and the uncertainty principle

In terms of quantum mechanics, every function $f(x)$ with
a normalized probability density function ($\int_{-\infty}^{\infty} |f(x)|^2 dx = 1$) and its Fourier transform $F(q)$ fulfilled the
inequality:

$$\text{Var}[f(x)] \times \text{Var}[F(q)] \geq \frac{1}{4}, \quad (15)$$

with $\text{Var}[f(x)] = \int_{-\infty}^{\infty} (x - \bar{x})^2 |f(x)|^2 dx$ and $\bar{x} = \int_{-\infty}^{\infty} x |f(x)|^2 dx$. This principle is a direct property of the
standard Fourier transform. As respect to the FrFT, the
uncertainty principle appears to be a peculiar case for
$\alpha = \frac{\pi}{2}$ and can be generalized for any $\alpha$:

$$\text{Var}[f(x)] \times \text{Var}[F^\alpha f(x)] \geq \frac{1}{4}, \quad (16)$$

2.4.1 The generalized uncertainty principle for a Gaussian beam

Let’s first consider the case of a Gaussian beam as given
in equation (13) which fulfilled $\int_{-\infty}^{\infty} |f(x)|^2 dx = 1$ and
$\text{Var}[f(x)] = \frac{\sigma_x^2}{2}$. The Fourier transform $F(q)$ gives:

$$F(q) = F_{\hat{x}} f(x) = \frac{\sqrt{\sigma}}{\pi^\frac{1}{2}} \exp\left(-\frac{\sigma^2 q^2}{2}\right) \quad (17)$$

with $\text{Var}[F(q)] = \frac{1}{4\sqrt{\pi}}$. The minimum value is obtained for
the Gaussian probability function:

$$\text{Var}[f(x)] \times \text{Var}[F_{\hat{x}} f(x)] = \frac{1}{4}. \quad (18)$$

The variance of the Gaussian beam along the propagation
for any angle $\alpha$ is given by equation (14):

$$\text{Var}[F_{\hat{x}} f(x)] = \frac{\sigma_x^2 \cos^2 \alpha}{2} + \frac{\sin^2 \alpha}{2\sigma_x^2}. \quad (19)$$

Let’s consider the relation which gives the generalized in-
certainty principle from two applications of the operator
on $f(x)$ (with first $F_{\alpha}$ and then $F_{\beta}$) in the case of a
Gaussian function and displayed in Figure 5b:

$$\text{Var}[f(x)] \times \text{Var}[F_{\alpha} f(x)] = \frac{\sigma_x^4 \cos^2 \alpha}{4} + \frac{\sin^2 \alpha}{4} \quad (20)$$

and compare to that given by Shen [14] in the general case
of any function $\phi$:

$$\text{Var}[\phi] \times \text{Var}[F_{\alpha} \phi] \geq \frac{\sin^2 \alpha}{4}. \quad (21)$$

2.4.2 The generalized uncertainty principle for a rectangular function

The rectangular function $s(x)$ is a peculiar case. Because
of the abrupt discontinuity, the variance of $F_{\alpha} s(x)$ is
infinite whatever $\alpha \neq 0$. The intensity profile in Fig-
ure 1 never vanishes completely, from the Fresnel to the
Fraunhofer regime. In terms of quantum mechanics, the
probability for finding the particle anywhere until the first
moment is not zero.

To treat diffraction of slit by considering a rectangular
function is not completely right from an experimental
point of view, since absorption through the blades of the
slit induces a not abrupt truncation of the incident wave
front [3,4] and thus a finite variance. To take into account
this effect, we could apply the FrFT to a rectangular func-
tion convoluted by a Gaussian function $g(x)$,

$$F_{\alpha}[s(x) \otimes g(x)], \quad (22)$$

which would lead to intensity profiles with finite extension.
Too large Gaussian functions smooth the diffraction pat-
ttern and reduce the wave’s extension but may make the
dark spot disappear. It is difficult to sufficiently reduce the
wave’s extension without vanishing the dark spot, which
is observed experimentally. We thus measure the variance
over a limited area centered at the maximum intensity. For
each $\alpha$, the full width at half maximum is measured and
the rms variance is calculated over 2.35 times the FHWM.
This is justify from an experimental point of view since,
in most cases, the lack of intensity or finite sizes of detec-
tors do not allow us to measure the diffraction pattern of
slit far from the direct beam. The result is displayed in
Figure 5d. A clear minimum is obtained for $\alpha = \beta = \frac{\pi}{2}$
which shows that a successive coherent diffraction of two
apertures focuses beams in the Fresnel regime. This is the
basic idea of Fresnel zone plates used in X-rays.

2.5 Fractional Fourier transform and diffraction of a periodic modulation

Let’s consider a 2D periodic modulation defined by a sin-
gle wave vector $q_0 = \frac{2\pi}{\lambda}$, such as:

$$\rho(x, y) = \rho_0 \cos(q_0 x), \quad (23)$$

which gives rise to two Bragg reflections at $\pm q_0$. By diffrac-
tion. By continuously varying $\alpha$ from 0 to $\frac{\pi}{2}$, the FrFT
simply allows us to calculate the continuous evolution of
the diffraction pattern from the real space to the recipro-
cal space (see Fig. 6).

2.5.1 Successive diffraction of two objects

To deal with successive diffraction of several objets by
using the FrFT is particularly well suited thanks to the
property of continuity (12).

\footnote{1}{In the Fraunhofer regime, the variance of the cardinal sine
squared is infinite: $x^2 = \int_{-\infty}^{\infty} x^2 f(x)^2 dx = \frac{\sigma_x^2}{2} \int_{-\infty}^{\infty} \sin^2(x \frac{\pi}{2}) dx.$}
As discussed in the introduction, the use of rectangular slits is necessary to obtain coherent X-ray beams from synchrotron sources and their location relative to the diffracted object may influence diffraction patterns. To quantify this effect, the diffraction of two successive objects has to be taken into account: diffraction of a rectangular function \( s(x) \) followed by the modulation \( \rho(x) \), as respect to \( \alpha, \beta \in [0, \frac{\pi}{2}] \):

\[
\mathcal{F}_\alpha [\mathcal{F}_\beta [s(x) \times \rho(x)]].
\]

If the sample is located in the Fraunhofer regime of the aperture \( \beta = \frac{\pi}{2} \) and the detector in the Fraunhofer regime of the sample \( \alpha = \frac{\pi}{2} \):

\[
\mathcal{F}_\alpha [\mathcal{F}_\beta [s(x) \times \rho(x)]] = s(x) \otimes \mathcal{F}_\alpha [\rho(x)].
\]

The Bragg peak will mainly display the diffraction pattern of the rectangular function in the Fresnel regime. If now the sample is located in the Fresnel regime of the aperture \( \beta \approx 0 \) and the detector in the Fraunhofer regime \( \alpha = \frac{\pi}{2} \), equation (24) gives:

\[
\mathcal{F}_\alpha [\mathcal{F}_\beta [s(x) \times \rho(x)]] = \mathcal{F}_{\alpha \beta} [s(x)] \otimes \mathcal{F}_{\alpha \beta} [\rho(x)].
\]

In that case, the reflection profile is a convolution of the FT of the aperture with the FT of the periodic modulation. The profile versus the distance between the aperture and the sample is summarized Figure 7. It is worthwhile to note that Figure 7d corresponds to the inverse of Figure 1. By observing Bragg reflection, the double diffraction is similar to a time reversal operator.

The product of variances

\[
\text{Var} [\mathcal{F}_\alpha [s(x)] \times \mathcal{F}_{\alpha \beta} [\mathcal{F}_\alpha [s(x)] \times \rho(x)]
\]

is displayed in Figure 8 in the case of a gaussian beam and a rectangular aperture. It is clear that to increase the width of Bragg reflection, the slit has to be as close as possible from the sample.

To conclude, the fractional Fourier transform appears to be appropriate to treat coherent diffraction. Especially, the property of continuity of the FrFT could be useful for iterative reconstruction algorithms as for instance ptychography [15] or Coherent X-ray diffraction microscopy which needs a method of image reconstruction from the coherent X-ray beam behaviour [16].

Even if all the physics of waves in included in the wave equation, some approximations are very useful and help to go deeper in the phenomena. The fractional Fourier transform is a newly introduced concept in physics of waves which allows going from the geometrical (Fresnel) to the diffraction (Fraunhofer) regime continuously. In the first regime the geometrical description is done in the real space when it is done in the reciprocal space, using Fourier transform in the Fraunhofer regime. This continuity is well understood if we consider the fractional Fourier transform as a rotation acting in a space mixing real and reciprocal space, defined by an angle \( \alpha \) which vary from \( \alpha = 0 \) to \( \alpha = \frac{\pi}{2} \). Coherent X-ray beams are often defined by their spatial and spectral coherence. It is the same idea mixing real and reciprocal spaces and associated to the uncertainty principle. This new point of view open the field for stimulating discussions. The fractional Fourier transform creates a clear relationship between different domain of physics.
physics including wave propagation, quantum physics and
diffraction.

References

1. A. Fresnel, *La diffraction de la lumière* (Académie des sciences, 1819)
2. M. Born, E. Wolf, *Principles of Optics* (Cambridge University Press, 1999)
3. D. Le Bolloc’h, F. Livet, F. Bley, T. Schulli, M. Veron, T.H. Metzger, J. Synchrotron Radiat. 9, 258 (2002)
4. D. Le Bolloc’h, J.P. Itié, A. Polian, S. Ravy, High Pressure Res. 29, 635 (2009)
5. V.L.R. Jacques, D. Le Bolloc’h, S. Ravy, C. Giles, F. Livet, S.B. Wilkins, Eur. Phys. J. B 70, 317 (2009)
6. V. Namias, J. Inst. Math. Appl. 25, 241 (1980)
7. A. Bultheel, H. Martinez, Report TW337, Department of Computer Sciences Katholieke Universiteit, Leuven, 2002
8. A.C. McBride, F.H. Kerr, IMA J. Appl. Math. 39, 159 (1987)
9. P. Pellat-Finet, Opt. Lett. 19, 1388 (1994)
10. P. Pellat-Finet (C.R. Acad. Sci., Paris, 1995), t 320, Serie IIb, p. 91
11. J.H. McCellan, T.W. Parks. IEEE Trans. Audio Electroacoust. 20, 66 (1972)
12. G. Dattoli, A. Torre, G. Mazzacurati, IMA J. Appl. Math. 60, 215 (1998)
13. C. Cohen-Tannoudji, B. Diu, F. Laloe, *Quantum Mechanics* (John Wiley & Sons, New York, 1977)
14. J. Shen, *Wavelet Analysis: Twenty Year’s Developments*, edited by Z.-X. Zhou (World Scientific Press, Singapore, 2002)
15. M. Dierolf, A. Menzel, P. Thibault, P. Schneider, C.M. Kewish, R. Wepf, O. Bunk, F. Pfeiffer, Nature 467, 436 (2010)
16. M.A. Pfeifer, G.J. Williams, I.A. Vartanyants, R. Harder, I.K. Robinson, Nature 442, 63 (2006)