Comparison of meanfield and Monte Carlo approaches to dynamic hysteresis in Ising ferromagnets

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Abstract: The dynamical hysteresis is studied in the kinetic Ising model in the presence of a sinusoidal magnetic field both by Monte Carlo simulation and by solving the dynamical meanfield equation for the averaged magnetisation. The frequency variations of the dynamic coercive field are studied below the critical temperature. In both the cases, it shows a power law frequency variation however it becomes frequency independent in the low frequency regime for the mean field case.

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I. Introduction

The hysteresis in ferromagnetic materials is a common interest of theoretical [1-13] and experimental [14-16] research. How the hysteresis loop area depends, on frequency and amplitude of the oscillating field and the temperature of the system, is quite important to know since it has crucial technological importance (e.g., magnetic recording media). More than a decade has been spent to study theoretically [1-13] the intriguing mechanism of hysteresis in the magnetic model systems for oscillating fields. The main outcome of these research is that the loop area \( A = \oint m dh \) scales as a power law \( A \sim f^a h_0^b \) with respect to frequency \( f \) and the amplitude \( h_0 \) of the applied magnetic field. Though there are some disagreement in the values of the exponents estimated, the qualitative behaviour (power law) of the hysteresis loop area are almost same in various model systems.

The theoretical studies can be divided into two classes: (i) the mean field studies [11-13], by solving the dynamical meanfield equations where the effects of fluctuations are not taken into account. In these cases, the hysteresis loop area scales as a power law with frequency apart from a residual loop area in the zero frequency limit. Mathematically, it can be written as \( A = A_0 + Cf^a h_0^b \). (ii) The Monte Carlo and other studies [1-10], where the effects of fluctuations are taken into account gives the same type of power law variation with the absence of any residual loop area (i.e., \( A_0 = 0 \)) in the zero frequency limit.

There are some experimental efforts [14-16] to study the scaling behaviour of the hysteresis loop area. The hysteresis has been studied mostly in ultrathin ferromagnetic films by magneto-optic Kerr effect. The power law frequency variation has been observed [14], however the exponent values observed [15] are very small suggesting a possible logarithmic frequency variation. Since, the hysteresis loop area linearly scales with the coercive field (loop area is four times the coercive field times the remanent magnetisation) the frequency variation of the coercive field has also been studied [16] in ultrathin (0.8 nm) ferromagnetic films. The coercive field has been found to be linear in the logarithm of the rate of change of the magnetic field (in some sense it is equivalent to the frequency). The meanfield (without any thermal fluctuation) equations are used to analyse those experimental data. The theoretical prediction agrees well with the experimental observation. However, in the zero frequency limit, the loop area does not vanish.

Comprehending the theoretical facts, it is observed that the meanfield type models gives nonzero loop area (or coercive field) in the zero frequency limit however the models which allows the effects of thermal fluctuations gives the hysteresis loop area which vanishes in the zero frequency limit following a power law variation. There is no possible explanation why the meanfield results are giving a nonzero residual hysteresis loop area in the zero frequency limit. In this paper, to get the possible explanation, the dynamic coercivity has been studied as a function of frequency in
the kinetic Ising model both by Monte Carlo simulation (to take into account the effects of fluctuation) and by solving the dynamical meanfield equation (fluctuations are absent).

The paper is organised as follows: in section II the model and the numerical schemes are discussed, the results are given in section III, in section IV the results are analysed by simple arguments.

II. The model and simulation

(a) Monte Carlo simulation

The total energy (at time $t$) of a nearest neighbour ferromagnetic Ising model with homogeneous and unit interaction energy can be written as

$$H = -(1/2) \sum_i h_i(t) \sigma_i(t), \quad h_i(t) = \sum_j \sigma_j(t) + h(t)$$

(1)

where $\sigma_i(t) = \pm 1$ and $j$ runs over the nearest neighbour of site $i$. The local field (at site $i$) $h_i(t)$ has an external field part $h(t)$, which is oscillating sinusoidally in time

$$h(t) = h_0 \sin(2\pi ft)$$

(2)

where $h_0$ and $f$ are the amplitude and frequency of the oscillating field.

According to heat-bath dynamics, the orientation probability $p_i(t)$ for the spin $\sigma_i(t)$ at time $t$ to show against the direction of the field is given as

$$p_i(t) = \frac{e^{-h_i(t)/K_B T}}{e^{h_i(t)/K_B T} + e^{-h_i(t)/K_B T}}$$

(3)

where $K_B$ is the Boltzmann constant which has been taken equal to unity for simplicity. The spin $\sigma_i(t)$ is oriented as

$$\sigma_i(t + 1) = \text{Sign}[r_i(t) - p_i(t)]$$

(4)

where $r_i(t)$ are independent random fractions drawn from the uniform distribution between 0 and 1.

In the simulation, a square lattice ($L \times L$) is considered under periodic boundary conditions. The initial condition is all spins are up (i.e., $\sigma_i(t = 0) = 1$, for all $i$). The multispin coding technique is employed here to store 10 spins in a computer word consisting of 32 bits. 10 spins are updated simultaneously (or parallel) by a single command. All words are updated sequentially and one full scan over the entire lattice consists of one Monte Carlo step (MCS) per spin. This is the unit of time in the simulation. The instantaneous magnetisation ($m(t) = (1/L^2) \sum_i \sigma_i(t)$) is calculated easily. The dynamic hysteresis loop (or $m - h$ loop) is nothing but the plot of the
magnetisation \((m(t))\) and the magnetic field \((h(t))\). One such loop is displayed in Fig. 1. The coercive field is the minimum field required for the sign reversal of the magnetisation. In Fig.1, \(A\) is the field necessary (in the negative direction) for the sign reversal of the magnetisation. Similarly, \(B\) is the field required to push the system from negative magnetisation state to the positive magnetisation state. This field is the coercive field \((h_c)\). Theoretically, \(A\) and \(B\) should be equal in magnitude. However, in practice, due to the presence of fluctuations they can differ by a small amount. Here the coercive field is measured by taking the arithmetic mean of these two values \((A\) and \(B\)). Initially, in the transient region of time, some hysteresis loops are thrown out until a stable and symmetric loop has been obtained. For a fixed frequency \((f)\) the coercive field is calculated by averaging over 10 different random samples. This simulation is performed in a SUN workstation cluster and the computational speed recorded is 7.14 Million updates per second.

(b) Solution of dynamical meanfield equation

The meanfield dynamical equation of Ising ferromagnet in the presence of time varying magnetic field is [12]

\[
\frac{dm}{dt} = -m + \tanh\left(\frac{m(t) + h(t)}{K_B T}\right),
\]

where the external time varying field \(h(t)\) has the above form \((2)\). To study the dynamic hysteresis, this equation has been solved for \(m(t)\) by fourth order Runge-Kutta method by taking the initial condition \(m(t = 0) = 1.0\). In this case also, few transient loops were discarded to have stable and symmetric loop. From the stable and symmetric hysteresis loop the coercive field is measured as the value of the magnetic field when the magnetisation changes its sign.

III. Results

(a) Monte Carlo results

In the Monte Carlo simulation, a square lattice of linear size \(L = 1000\) has been considered. The temperature has been set at 20 percent below the critical temperature \((T_c)\) and kept fixed throughout the study. The coercive fields are measured for different frequencies ranging from \(5 \times 10^{-5}\) to \(10^{-2}\). Frequency is measured in the units of inverse of MCS. For example, for a frequency \(f = 10^{-2}\), 100 MCS are required to have a closed loop (complete cycle of the oscillating magnetic field). One such loop (for \(f = 0.01\) and \(T = 0.8T_c\)) is shown in Fig.1. The frequency \((f)\) variation of dynamic coercive field \((h_c)\) for a fixed temperature \((T = 0.8T_c)\) is plotted in a double logarithmically in Fig. 2. The variation is clearly a power law over a reasonably wide range of frequency and \(h_c(f \to 0) = 0\). The exponent estimated is \(0.33 \pm 0.02\).
(b) Meanfield results

In the meanfield study, the temperature is also kept at 20 percent below $T_c$ and the frequency ranges from $10^{-5}$ to 1. The frequency variation of the dynamic coercive field ($h_c$) is shown in a double logarithmic plot in Fig. 2. Unlike the earlier case (Monte Carlo results), the frequency variation is a power law type in the high frequency range, however it becomes frequency independent over a quite wide range in the low frequency regime and $h_c(f \to 0) \neq 0$. The exponent, of the power law (in the high frequency regime), estimated here is 0.45.

IV. Concluding remarks

In conclusion, the dynamical hysteresis is studied, in the kinetic Ising model below the critical temperature and in the presence of a sinusoidal (in time) magnetic field, both by Monte Carlo simulation (using multispin coding and heat bath algorithm) and by solving the meanfield dynamical equation for the averaged magnetisation. In both the cases, the numerical results of the frequency variations of the dynamic coercive field, show a power law frequency variation. In the meanfield case the coercive field becomes frequency independent in the low frequency regime.

The dissimilarity (MC and MF results) in the qualitative behaviour of the frequency variations of the dynamic coercivity can be explained from the Landau type double well free energy. The coercive field is the field needed to bring the system from one well to another. In the case of Monte Carlo study, where the thermal fluctuations are present, there is always a nonzero finite probability that the system can go to the other well being driven by thermal fluctuation. In this case, even a vanishingly small field can help the system to cross the free energy barrier via the nucleation process if one waits for a sufficiently long time. In this respect, the coercivity can be thought as a limit of metastability. This situation means: in the zero frequency limit, the coercive field vanishes. On the other hand, in the case of meanfield study, the thermal fluctuation is absent. As a result, below $T_c$, unless a finite amount (non zero) of field is applied, the system will not jump from one well to the other, even if one waits for an infinitely long time. This is equivalent to say that below $T_c$, in the static limit ($f \to 0$), the coercive field is not zero. The numerical results are indeed qualitatively consistent with this argument. Thermal fluctuations are playing the major role to make this kind of distinction in the frequency variations of the coercive field in the static ($f \to 0$) limit.

The theoretical observations [11,13] indirectly support the results and the argument by showing nonzero residual loop area in the zero frequency limit. The experimental observations [16] are also in agreement with this argument. The logarithmic variation [16,17] is quite slow variation, essentially independent of frequency and the coercive field does not vanish as frequency tend to zero.
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Fig. 1. A typical hysteresis loop obtained from the MC simulation.

Fig. 2. Frequency ($f$) variations of dynamic coercive fields ($h_c$). (◇) Monte Carlo results and (●) meanfield results. Solid lines are linear best fit.