Chapter 1

Microscopic origin of pairing

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A brief review of recent progress in the ab initio theory of nuclear pairing is given. Nowadays several successful solutions of the ab initio BCS theory gap equation were published which show that it is a promising first step in the problem. However, the role of many-body correlations that go beyond the BCS scheme remains uncertain and requires further investigations. As an alternative, the semi-microscopic model is discussed in which the effective pairing interaction calculated from the first principles is supplemented with a small phenomenological addendum containing one phenomenological parameter universal for all medium and heavy atomic nuclei.

1. Introduction

Recently, the fifty years anniversary of Cooper pairing in nuclei\(^1\) took place. However, only in the last few years some progress has been made in the microscopic theory of nuclear pairing, first, by the Milan group,\(^2\)\(^-\)\(^4\) a little later by Duguet et al.\(^5\)\(^-\)\(^6\) and finally by the Moscow-Catania group.\(^7\)\(^-\)\(^11\) In the first seminal paper of the Milan series, the BCS gap equation for neutrons with the Argonne \(v_{14}\) potential was solved for the nucleus \(^{120}\)Sn. The Saxon-Woods Shell-Model basis with the bare neutron mass \(m^* = m\) was used, and the discretization method in a spherical box was applied to simulate the continuum states restricted by the limiting energy \(E_{\max} = 600\) MeV. Rather optimistic result was obtained for the gap value, \(\Delta_{\text{BCS}} = 2.2\) MeV. Although it is bigger of the experimental one, \(\Delta_{\exp} \simeq 1.3\) MeV, the difference is not so dramatic and left the hope to achieve a good agreement by developing corrections to the scheme. In Refs.\(^3\)\(^,\)\(^4\) the basis was enlarged to \(E_{\max} = 800\) MeV, and, what is more important, the effective mass
$m^* \neq m$ was introduced into the gap equation. The new basis was calculated within the Skyrme–Hartree–Fock (SHF) method with the Sly4 force,\textsuperscript{12} that makes the effective mass $m^*(r)$ coordinate dependent and essentially different from the bare one $m$. E.g., in nuclear matter the Sly4 effective mass is equal to $m^* = 0.7m$. As it is well known, in the weak coupling limit of the BCS theory, the gap is exponentially dependent, i.e. $\Delta \propto \exp(1/g)$, on the inverse dimensionless pairing strength $g = m^* V_{\text{eff}} k_F / \pi^2$, where $V_{\text{eff}}$ is the effective pairing interaction. Therefore, a strong suppression of the gap takes place in the case of $m^* < m$. The value of $\Delta_{\text{BCS}} = 0.7 \text{ MeV}$ was obtained in Ref.\textsuperscript{3} and $\Delta_{\text{BCS}} = 1.04 \text{ MeV}$, in Ref.\textsuperscript{4} In both cases, the too small value of the gap was explained by invoking various many-body corrections to the BCS approximation. The main correction is due to the exchange of low-lying surface vibrations ("phonons"), contributing to the gap around 0.7 MeV,\textsuperscript{3} so that the sum turns out to be $\Delta = 1.4 \text{ MeV}$ very close to the experimental value. In Ref.\textsuperscript{4} the contribution of the induced interaction caused by exchange of the high-lying in-volume excitations was added either, the sum again is equal to $\Delta \simeq 1.4 \text{ MeV}$. Thus, the calculations of Refs.\textsuperscript{3,4} showed that the effects of $m^* \neq m$ and of many-body corrections to the BCS theory are necessary to explain the difference of $(\Delta_{\text{BCS}} - \Delta_{\text{exp}})$. In addition, they are of different sign and partially compensate each other. Unfortunately, both effects contain large uncertainties. This point is discussed in Section 3.

In 2008 Duguet and Losinsky\textsuperscript{5} made a fresh insight to the problem by solving the \textit{ab initio} BCS gap equation for a lot of nuclei on the same footing. It should be noticed that the main difficulty of the direct method to solve the nuclear pairing problem comes from the rather slow convergence of the sums over intermediate states $\lambda$ in the gap equation, because of the short-range of the free $NN$-force. To avoid the slow convergence, the authors of Refs.\textsuperscript{5,6} used the "low-k" force $V_{\text{low-k}}$\textsuperscript{13,14} which is in fact very soft. It is defined in such a way that it describes correctly the $NN$-scattering phase shifts at momenta $k<\Lambda$, where $\Lambda$ is a parameter corresponding to the limiting energy $\simeq 300 \text{ MeV}$. The force $V_{\text{low-k}}$ vanishes for $k>\Lambda$, so that in the gap equation one can restrict the energy range to $E_{\text{max}} \simeq 300 \text{ MeV}$. In addition, a separable version of this force was constructed that made it possible to calculate neutron and proton pairing gaps for a lot of nuclei. Usually the low-k force is found starting from some realistic $NN$-potential $V$ with the help of the Renormalization Group method, and the result does not practically depend on the particular choice of $V$.\textsuperscript{13} In addition, in Ref.\textsuperscript{5} $V_{\text{low-k}}$ was found starting from the Argonne potential $v_{18}$, that is different
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only a little bit from Argonne v14, used in Ref.4. Finally, in Ref.5 the same SLy4 self-consistent basis was used as in Ref.4. Thus, the inputs of the two calculations look very similar, but the results turned out to be strongly different. In fact, in Ref.5 the value $\Delta_{\text{BCS}} \simeq 1.6$ MeV was obtained for the same nucleus $^{120}$Sn which is already bigger than the experimental one by $\simeq 0.3$ MeV. In Refs.7,8 the reasons of these contradictions were analyzed. It turned out that, in fact, these two calculations differ in the way they take into account the effective mass. It implies that the gap $\Delta$ depends not only on the value of the effective mass at the Fermi surface, as it follows from the above exponential formula for the gap, but also on the behavior of the function $m^*(k)$ in a wide momentum range. However, this quantity is not known sufficiently well,8 which makes rather uncertain the predictions of such calculations.

To avoid such uncertainties, a semi-microscopic model for nuclear pairing was suggested by the Moscow-Catania group.9–11 It starts from the ab initio BCS gap equation with the Argonne force v18 treated with the two-step method. The complete Hilbert space $S$ of the problem is split into the model subspace $S_0$ of low-energy states and the complementary one $S'$. The gap equation is solved in the model space with the effective interaction $V_{\text{eff}}$ which is found by projecting out the complementary subspace. A new version of the local approximation, the so-called Local Potential Approximation (LPA),15 is used in the subspace $S'$. This ab-initio term of $V_{\text{eff}}$ is supplemented by a small addendum proportional to the phenomenological parameter $\gamma$ that should hopefully embody all corrections to the simplest BCS scheme with $m^* = m$. Smallness of the correction term is demonstrated in Fig. 1 where a localized “Fermi average” form of $V_{\text{eff}}$ is displayed without ($\gamma = 0$) and with ($\gamma = 0.06$) the phenomenological correction. Non-negligible effect of so small change of $V_{\text{eff}}$ to the gap value is owing to the above mentioned exponential enhancement effect. Explicit definition of the functions displayed in Fig. 1 is given in Section 4 where some results of the semi-microscopic model are presented.

Fig. 1 demonstrates also the surface nature of nuclear pairing, the effective pairing interaction at the surface is ten times stronger than inside. This explains the relative success of the BCS approximation. Indeed, at the surface the main corrections to the Brueckner-like theory which leads to the BCS scheme are small.
2. The microscopic BCS equation. LPA approximation

The general many-body form of the equation for the pairing gap is as follows,\(^1\)

\[
\Delta_\tau = U^\tau G_\tau G^*_\tau \Delta_\tau, \tag{1}
\]

where \(\tau = (n, p)\) is the isotopic index, \(U^\tau\) is the \(nn\) interaction block irreducible in the two-particle \(\tau\)-channel, and \(G_\tau\) (\(G^*_\tau\)) is the one-particle Green function without (with) pairing. A symbolic multiplication denotes the integration over energy and intermediate coordinates and summation over spin variables as well. The BCS approximation in Eq. (1) means, first, the change of the block \(U\) of irreducible interaction diagrams with the free \(nn\)-potential \(\mathcal{V}\) in Eq. (1), and, second, the use of simple quasi-particle Green functions \(G\) and \(G^*\), i.e. those without phonon corrections and so on. In this case, Eq. (1) turns greatly simplified and can be reduced to the form usual in the Bogolyubov method,

\[
\Delta_\tau = -\mathcal{V}^\tau \kappa_\tau, \tag{2}
\]

where

\[
\kappa_\tau = \int \frac{d\varepsilon}{2\pi i} G_\tau G^*_{\tau} \Delta_\tau \tag{3}
\]
is the anomalous density matrix which can be expressed explicitly in terms of the Bogolyubov functions $u$ and $v$,

$$\xi_{\tau}(r_1, r_2) = \sum_i u^\dagger_i(r_1) v_i^\dagger(r_2).$$ (4)

Summation in Eq. (4) scans the complete set of Bogolyubov functions with eigen-energies $E_i > 0$.

To overcome the slow convergence problem, a two-step renormalization method of solving the gap equation in nuclei was used in Refs. 9–11. The complete Hilbert space of the pairing problem $S$ is split in the model subspace $S_0$, including the single-particle states with energies less than a separation energy $E_0$, and the complementary one, $S'$. The gap equation is solved in the model space:

$$\Delta_{\tau} = V_{\text{eff}}^\tau G_{\tau}^S G_{\tau}^S \Delta_{\tau}|_{S_0},$$ (5)

with the effective pairing interaction $V_{\text{eff}}^\tau$ instead of the block $V^\tau$ in the BCS version of the original gap equation (1). It obeys the Bethe–Goldstone type equation in the subsidiary space,

$$V_{\text{eff}}^\tau = V^\tau + V_{\text{eff}}^\tau G_{\tau} G_{\tau} V_{\text{eff}}^\tau|_{S'},$$ (6)

In this equation, the pairing effects can be neglected provided the model space is sufficiently large, $E_0 \gg \Delta$. That is why we replaced the Green function $G^S_{\tau}$ for the superfluid system with its counterpart $G_{\tau}$ for the normal system. To solve Eq. (6) in non-homogeneous systems a new form of the local approximation, the Local Potential Approximation (LPA), was developed by the Moscow–Catania group. Originally, it was found for semi-infinite nuclear matter, then for the slab of nuclear matter (see review article 15) and, finally, for finite nuclei.\textsuperscript{7,8} It turned out that, with a very high accuracy, at each value of the average c.m. coordinate $R = (r_1 + r_2 + r_3 + r_4)/4$, one can use in Eq. (6) the formulae of the infinite system embedded into the constant potential well $U = U(R)$. This significantly simplifies the equation for $V_{\text{eff}}^\tau$, in comparison with the initial equation for $\Delta$. As a result, the subspace $S'$ can be chosen as large as necessary to achieve the convergence. Accuracy of LPA depends on the separation energy $E_0$. For finite nuclei, the value of $E_0 = 40$ MeV guarantees the accuracy better than 0.01 MeV for the gap $\Delta$.

Let us notice that the use of the low-$k$ force $V_{\text{low}}-k$ could be also interpreted in terms of the two-step renormalization scheme of solving the BCS version ($U \rightarrow V$) of the gap equation (1), with $E_0 \simeq 300$ MeV and with free nucleon Green functions $G$ in Eq. (6) (i.e. $U(R) = 0$). Then, one obtains
\( V_{\text{eff}} \rightarrow V_{\text{low-k}} \) (see Ref. 14 where the usual renormalization scheme is used to find \( V_{\text{low-k}} \) instead of the Renormalization Group equation).

3. Corrections to the BCS scheme

As it was mentioned in the Introduction, there are mainly three types of corrections to the plain \textit{ab initio} BCS gap equation with bare nucleon mass \( m \). The first one is the effect of the effective mass \( m^* \neq m \) considered in Refs. 3,4 and Refs. 5,6 as well. The second one is the contributions of low-lying surface phonons3,4 and the third one, the induced pairing interaction due to high-lying in-volume excitations.4

Let us begin from the effective mass. In the analysis7,8 of the difference between the BCS gap values of Ref. 4 and Refs., 5,6 it was found that the gap \( \Delta \) depends on the behavior of the function \( m^*(k) \) in a wide momentum range. However, this quantity is not known sufficiently well even in nuclear matter. In Fig. 2 the effective mass \( m^*(k) \) of symmetric nuclear matter at different Fermi momentum values \( k_F \) are displayed. They were found8 with the Brueckner–Hartree–Fock method which is also, of course, an approximation. These functions behave rather non-regular in vicinity of \( k_F \) and tend to the bare mass value very slowly. This makes rather doubtful the way to account for the \( m^* \) effect in Refs. 3,4 and in Refs. 5,6 as well. In the first case, the effective mass was taken \( k \)-independent (equal to \( m^*_{\text{Sly4}} \)) till the cut-off \( k_{\text{max}} \approx 6 \text{ fm}^{-1} \). In the second case, the ansatz was used of
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$m^* = m^*_{\text{SLy4}}$ for $k<\Lambda \simeq 3\text{ fm}^{-1}$ and $m^* = m$ for $k>\Lambda$.

Similar problems appear if one tries to use an explicit form of the $Z$-factor, which is an additional ingredient of the gap equation, not only in the combination yielding the complete effective mass $m^*(k)$. In this Section, speaking for brevity about the effective mass we mean in fact both the $k$-mass and $E$-mass.

Corrections to the BCS scheme owing to phonon contributions are of primary importance. To our knowledge, the most advanced calculation of these corrections was carried out in Ref. It includes as the phonon-exchange term of the block $\mathcal{U}$ in Eq. (1), the so-called induced interaction, and corrections to the Green functions $G, G^*$ as well. However, it also possesses some deficiency connected with omitting the so-called tadpole diagrams. Up to now, they were consistently taken into account within the self-consistent Finite Fermi Systems (FFS) theory only for magic nuclei where pairing is absent. It turned out that their contribution is usually important and often is of the opposite sign to the usual diagrams diminishing the total value of the effect under consideration. This formalism was generalized for superfluid nuclei in Ref., but numerical applications are still absent.

Let us finally discuss corrections to the BCS version of Eq. (1) due to the induced interaction from high-lying particle-hole in-volume excitations. The attempt in Ref. to determine the latter from the SLy4 force together with the nuclear mean field looks questionable. Indeed, the SLy4 parameters were fitted to the nuclear mass table data mainly related to the scalar Landau–Migdal (LM) amplitudes $f, f'$. As to the spin amplitudes $g, g'$, they remain practically undetermined in the SHF method. But the contribution of the spin channel to the induced interaction is not smaller than that of the scalar one. The LM parameters $g, g'$ are well known from the calculations of nuclear magnetic moments within the FFS theory but, as for the Skyrme parameters, only at the Fermi surface. However, the states distant from the Fermi surface are important to calculate the induced interaction. The induced interaction for such states has been determined only in nuclear matter within the microscopic Brueckner theory.

4. The semi-microscopic model for nuclear pairing

To avoid uncertainties of explicit consideration of corrections to the BCS scheme discussed above, the semi-microscopic model was suggested in Refs. In this model, a small phenomenological addendum to the ef-
effective pairing interaction is introduced which embodies approximately all these corrections. The simplest ansatz for it is as follows:

$$V_{\tau \text{eff}}(r_1, r_2, r_3, r_4) = V_{\tau \text{BCS} \text{eff}}(r_1, r_2, r_3, r_4) + \gamma^\tau C_0 \frac{\rho(r_1)}{\bar{\rho}(0)} \prod_{i=2}^{4} \delta(r_1 - r_i).$$

(7)

Here $\rho(r)$ is the density of nucleons of the kind under consideration, and $\gamma^\tau$ are dimensionless phenomenological parameters. To avoid any influence of the shell fluctuations in the value of $\rho(0)$, the average central density $\bar{\rho}(0)$ is used in the denominator of the additional term. It is averaged over the interval of $r<2$ fm. The first, \textit{ab initio}, term in the r.h.s. of Eq. (7) is the solution of Eq. (6) in the framework of the LPA method described above, with $m^* = m$ in the subspace $S'$.

![Fig. 3. Neutron gap in Sn isotopes](image)

We will see below that a rather small value of the phenomenological parameter $\gamma_n = \gamma_p \simeq 0.06$ is sufficient to produce the necessary effect of suppressing theoretical gaps predicted by the \textit{ab initio} calculation. The smallness of the phenomenological addendum to the effective interaction itself is demonstrated in Fig. 1 where the localized “Fermi average” effective interaction is drawn for $\gamma = 0$ and $\gamma = 0.06$ values for two heavy nuclei. In the mixed coordinate-momentum representation, this quantity is defined as follows: $V_{\text{eff}}(k_1, k_2, r_1, r_2) \rightarrow V_{\text{eff}}^F(R = r_1) \prod_{i=2}^{4} \delta(r_1 - r_i)$, where

$$V_{\text{eff}}^F(R) = \int d^3t V_{\text{eff}}(k_1 = k_2 = k_F(R), \mathbf{R} - \mathbf{t}/2, \mathbf{R} + \mathbf{t}/2),$$

(8)
with \( k_F(R) = \sqrt{2m(\mu - U(R))} \), provided \( \mu - U(R) \geq 0 \), and \( k_F(R) = 0 \) otherwise. Here \( \mu \) and \( U(R) \) are the chemical potential and the potential well, respectively, of the kind of nucleons under consideration. A similar quantity was considered before in the nuclear slab to visualize the effective interaction properties.\(^{15}\)

![Fig. 4. Neutron gap in Pb isotopes.](image)

In Ref.\(^{10}\), the above equations were solved in the self-consistent \( \lambda \)-basis of the Energy Density Functional (EDF) of Fayans et al.\(^{22,23}\). Two sets of the functional were used, the original one DF3\(^{23}\) and its modification DF3-a.\(^{24}\) In the latter, the spin-orbit and effective tensor terms of the initial functional were modified. The results for the pairing gap in three chains of semi-magic nuclei are displayed in Figs. 2–4.

In accordance with the recipe of Ref.\(^{4}\), we represent the theoretical gap with the “Fermi average” combination

\[
\Delta_F = \sum_\lambda \frac{(2j+1)\Delta_{\lambda \lambda}}{\sum_\lambda (2j+1)},
\]

where the summation is carried out over the states \( \lambda \) in the interval of \( |\varepsilon_\lambda - \mu| < 3 \text{ MeV} \). The “experimental” gap is determined by the symmetric 5-term odd-even mass difference. As it is argued in Ref.\(^{10}\), the relevance of the mass difference to the gap has an accuracy of \( \simeq (0.1 \div 0.2) \text{ MeV} \).
Therefore, it is reasonable to try to achieve the agreement of the gap within such limits. It should be noted also that the theoretical accuracy of the approach based on the “developed pairing” approximation with particle number conservation on average only is also about 0.1 MeV.25

Let us begin from neutron pairing and consider first the tin isotopes, Fig. 2. We see that the BCS gap (\(\gamma = 0\)) is approximately 30% greater than the experimental one. Switching on of the phenomenological addendum with \(\gamma = 0.06\) makes theoretical gap values closer to experiment. However, predictions of two versions of the functional used are significantly different, being much better for the DF3-a functional. In particular, the \(A\)-dependence of the experimental gap is reproduced with a pronounced minimum in the center of the chain. As the analysis in Ref.10 has shown, this strong difference between results for two functionals is ought to the strong influence to the gap of the high \(j\) intruder state \(1h_{11/2}\). Its position depends essentially on the spin-orbit parameters and is noticeably different for DF3 and DF3-a functionals. It explains the effect under discussion.

In the lead chain, see Fig. 3, the overall pattern is quite similar. Again the BCS gap is approximately 30% bigger of the experimental one and again inclusion of the phenomenological term with \(\gamma = 0.06\) gives a qualitative agreement. Now, the difference between two functionals is much less. In
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this case, the agreement is quite perfect for the DF3 functional and a little worse for the DF3-a one, but also within limits for the accuracy discussed above.

Let us go to proton pairing, \( N = 82 \) chain, see Fig. 4. In this case, the Coulomb interaction should be included into the pairing effective interaction,

\[
\mathcal{V}_{\text{eff}}^{p} = \mathcal{V}_{\text{eff}}^{n} + \mathcal{V}_{C}.
\]  

(10)

As it is argued in Ref.,\(^{10}\) the bare Coulomb potential could be with high accuracy used in this equation. The strong Coulomb effect in the gap is demonstrated in Fig. 4. It is also explained with the exponential dependence of the gap on the pairing interaction. It should be mentioned that Duguet and co-authors\(^{6}\) were the first who inserted the Coulomb interaction into the pairing force for protons. Only after inclusion of the Coulomb interaction into \( \mathcal{V}_{\text{eff}} \), we can use the same value of \( \gamma = 0.06 \) for protons and neutrons. As for tin isotopes, the difference between DF3 and DF3-a results is rather strong, now in favor of the DF3 functional. This effect is again due to different positions of the \( 1h_{11/2} \) level, but now for protons. Overall agreement with experiment is for protons worse, maybe, because of closeness of some nuclei to the region of the phase transition to deformed state.

5. Conclusions

We reviewed briefly the recent progress in the microscopic theory of pairing in nuclei involving contributions of the Milan group\(^{2-4}\) of Duguet with coauthors\(^{5,6}\) and of Moscow-Catania group\(^{9-11}\). It became clear that the plain \textit{ab initio} BCS gap equation with bare mass \( m^* = m_2^{10} \) is a good starting point for such theory. As the analysis in Refs.\(^{3,4}\) showed, the effect of \( m^* \neq m \) and that of many-body corrections to the BCS theory are of different sign and partially compensate each other. Qualitatively, they can explain the difference between \( \Delta_{\text{BCS}} \) and \( \Delta_{\text{exp}} \) but both effects contain large uncertainties and hardly can be took into account definitely at the modern level of nuclear theory.

As an alternative, the semi-microscopic model was suggested in Refs.\(^{7,9,10}\) starting from the plain \textit{ab initio} BCS theory with the use of the self-consistent EDF basis characterized by the bare nucleon mass. The gap equation is recast in the model space \( S_0 \), replacing the bare interaction with the effective pairing interaction \( \mathcal{V}_{\text{eff}} \) determined in the complementary
subspace $S'$. The Argonne $v_{18}$ potential was adopted to find $V_{\text{eff}}$ along with the LPA method. A small phenomenological term is added to this effective interaction that contains one parameter, common to neutrons and protons, which should embody approximately the effective mass and other corrections to the pure BCS theory. Calculations were carried out with two versions of the EDF, the initial DF3 functional and its version DF3-a with modified spin-orbit and effective tensor terms. They involve semi-magic lead and tin isotopic chains and the $N = 82$ isotonic chain as well. The Coulomb interaction is explicitly included in the proton gap equation. It was found that the model reproduces reasonably well the experimental values of the neutron and proton gaps for both functionals. However, the results depend essentially on the single-particle spectrum of the self-consistent basis used, especially on the position of high $j$-levels. Thus, for tin isotopes agreement is much better for the DF3-a functional which reproduces better the position of the “intruder” $1h_{11/2}$ neutron level. On the contrary, for $N = 82$ isotones agreement for this functional is worse. In this case, the DF3 functional describes better the position of the same intruder state but for protons. The overall disagreement is $\sqrt{(\Delta_{\text{th}} - \Delta_{\text{exp}})^2} \approx 0.13$ MeV for the DF3 functional and $\approx 0.14$ MeV, for the DF3-a one.

The model under discussion exhibits a week point by including to the “universal” phenomenological addendum all corrections to the BCS scheme. The effective mass correction and the one due to the induced interaction from high-lying excitations are mainly in-volume. Therefore, indeed, they should be universal for medium and heavy nuclei. On the contrary, the phonon correction is surface and may vary from one nucleus to another as low-lying phonon characteristics do. A more consistent scheme should, evidently, include the explicit consideration of the low-lying phonons, as e.g. in, but taking into account the tadpole diagrams. In this case, the phenomenological constant $\gamma$ should, of course, change. Such extended model is much more complicated but should be more accurate in reproducing experimental data.

**Acknowledgments**

We are thankful to U. Lombardo, S. S. Pankratov and M. V. Zverev which are coauthors of articles which are the basis of the present review. We thank also G. L. Colo, T. Duguet, V. A. Khodel and S. V. Tolokonnikov for valuable discussions. The work was partly supported by the DFG and
RFBR Grants Nos.436RUS113/994/0-1 and 09-02-91352NIO-a, by Grant NSh-7235.2010.2 of the Russian Ministry for Science and Education, and by the RFBR grant 11-02-00467-a.

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