The effects of a recess on co-flowing planar jets under supercritical pressure are numerically studied. Two-dimensional hybrid LES/RANS simulations are performed in a wide range of recess lengths and injection momentum flux ratios, which are important design parameters for liquid rocket engine injectors. The present study showed that confinement effects of the nearinjector flowfield by applying a recess suppress outer jet spreading and thus enhances the penetration of the outer jet flow into the inner jet. The enhanced penetration of the outer jet flow results in the appearance of flapping motions in the inner jet. Low-frequency oscillations corresponding to the flapping motions clearly appear in the case of recessed injectors. Moreover, the confinement effects promote interactions between vortex structures resulting in vortex breakdown. Consequently, the inner-jet length is shortened, indicating an improvement in mixing when a recess is applied. The present results also show that the inner-jet length deceases as the recess length increases, and the effects of a recess remarkably appear at higher momentum flux ratios. This is explained by the vortices generated behind the post lip that is strengthened as the result of increased velocity ratio.

Key Words: Jet, Supercritical Fluid, Computational Fluid Dynamics, Propulsion

Nomenclature

| Symbol | Description |
|--------|-------------|
| $a$ | sound speed |
| $C_p$ | constant-pressure specific heat |
| $C_v$ | constant-volume specific heat |
| $d_{wi}$ | distance to wall |
| $e$ | internal energy |
| $E$ | total energy |
| $H$ | inner jet height |
| $h$ | outer jet height |
| $J$ | momentum flux ratio, $(\rho U^2)_{out}/(\rho U^2)_{in}$ |
| $k$ | thermal conductivity |
| $l$ | local length scale in turbulence model |
| $L$ | mean inner-jet length |
| $p$ | pressure |
| $R$ | gas constant |
| $Re_H$ | Reynolds number of inner jet, $U_{in}H/v_{in}$ |
| $Re_h$ | Reynolds number of outer jet, $U_{out}h/v_{out}$ |
| $Rn$ | recess length |
| $\mathbf{S}$ | strain rate tensor |
| $t$ | time |
| $T$ | temperature |
| $\mathbf{u}$ | velocity vector |
| $U$ | bulk streamwise velocity |
| $V$ | volume |
| $x, y$ | Cartesian coordinates |
| $\kappa$ | Karman constant |
| $\mu$ | viscosity |
| $\nu$ | kinetic viscosity |

$\omega_a$: acentric factor  
$|\omega|$: absolute vorticity  
$\rho$: density  
$\tau$: viscous stress tensor

Subscripts

| Symbol | Description |
|--------|-------------|
| $ch$ | chamber |
| $cr$ | critical point |
| $in, out$ | inner, outer |
| $t$ | turbulent |

Superscripts

| Symbol | Description |
|--------|-------------|
| $x^*$ | normalized value |
| $x'$ | fluctuation |

1. Introduction

The dynamics and mixing of injected propellants in liquid rocket engines have an impact on combustion characteristics such as flame holdings, combustion stability, and consequently engine performance. Pressure in the rocket combustion chamber reaches over 10 MPa, and injected fluids are under supercritical pressure, i.e., the pressure exceeding the critical point of the cryogenic propellants. The fluid injected at a supercritical pressure shows unique characteristics due to its specific thermodynamic nature, for example, surface tension decreases at supercritical pressures, and then the jet mixing can be considered as gas-like mixing with large density gradients rather than the breakup of droplets.

For the effective mixing of propellants, coaxial injectors have been widely applied to practical liquid rocket engines. Cold-flow coaxial injection under rocket operating conditions has been investigated experimentally and numerically. One important parameter for coaxial mixing is the outer-to-inner jet momentum flux ratio. The inner jet
length decreases as the momentum flux ratio increases, and a scaling rule for the inner-jet length has been proposed. The geometric configuration of the injector exit is also a critical factor for coaxial injections, and its design is important for obtaining efficient mixing and stable combustion. Behind the inner post, burnt gases are entrained in a recirculation region and provide an ignition source, which stabilizes the flame. To understand flame stabilization, the flowfield and combustion behind the bluff body or step have been studied extensively.

In particular, recessing the inner post is known to be an important geometric parameter for both mixing and combustion. Practical rocket coaxial injectors typically apply a recess having a length equivalent to the inner injector diameter. Kendrick et al. and Lux and Haidn have reported that recessing enhances flame expansion in hot-fire coaxial flame experiments. Atomization and mixing of the inner jet are well improved in recessed injectors; consequently, the shortening of inner-jet-core length is obtained when compared to non-recessed injectors. In addition to improving mixing, past studies have reported that recessing largely changes the dynamics and structures of the jet. An experimental study of coaxial LOX/GH2 flames showed that large-scale sinusous structures in the inner jet are introduced by applying a recess. In recessed coaxial water/air injections, the jet disintegration pattern was observed to change from a fiber-type to a pulsating-type as the outer jet velocity increased. Some numerical studies of cold-flow coaxial jets have also reported improved mixing and the appearance of large-scale structures in recessed injectors. Kim et al. showed the appearance of strong hydrodynamic instability in the recessed region and suggested that such instability may trigger combustion instability in practical liquid rocket engines.

In recessed injectors, the near-injector flowfield is “confined” by walls. The influence of confinement on the hydrodynamic instability of a channelled-wake and jet has been investigated by a series of studies on instability analysis. Juniper and Candel showed that confinement enhances the unstable nature of the flow. They concluded that confinement-induced global instability leads to large-scale sinusous or helical motion in the coaxial jet. The numerical simulation of a confined-wake performed by Biancofo et al. also confirmed that moderate confinement induces oscillation in the wake. Moreover, Biancofo et al. showed that the transition to turbulence moves upstream when flow confinement is strengthened, and interactions between the boundary layer and shear layer increase turbulent intensity.

Although past studies have demonstrated the importance of recesses for mixing, the mechanisms changing jet structures and dynamics have not been well explained. Moreover, the effects of recess length on jet structures need to be understood and assessed to improve mixing and ensure effective mixing control. The present study performs numerical simulations of cold co-flowing planar jets under supercritical pressure in non-recessed and recessed injectors. Two-dimensional simulations are carried out to understand the fundamental effects of recesses, with a parametric study covering a wide range of recess lengths and injection conditions. The recess length is varied from zero to a length of four times the inner injector height, and the inner-to-outer momentum flux ratio is varied from 1 to 6. The non-recessed and recessed co-flowing jets are compared in terms of jet structures, inner-jet-core lengths, and velocity spectra. Then, the effects of the recess length on the inner jet length are evaluated by varying the momentum flux ratio.

2. Numerical Method

Compressible Navier-Stokes equations are used as the governing equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0,$$

$$\frac{\partial \rho u}{\partial t} + \nabla \cdot (\rho u \otimes u + p I - \tau) = 0,$$

and

$$\frac{\partial E}{\partial t} + \nabla \cdot ((E + p) u - \tau \cdot u - k \nabla T) = 0.$$ (3)

Here, $I$ is the unit tensor. The viscous stress tensor $\tau$ is

$$\tau = \mu(2S) - \frac{2}{3} \mu(\nabla \cdot u)I, \quad S = \frac{1}{2}(\nabla u + (\nabla u)^T).$$ (4)

In thermodynamic relationships of real fluids, the internal energy $e$ is expressed as

$$e(T, V) = \int \left( \frac{\partial e}{\partial T} \right)_V dT + \int \left( \frac{\partial e}{\partial V} \right)_T dV$$ (5)

$$= e_0(T) + \int \left( \frac{\partial p}{\partial T} \right)_V - p \right] dV.$$ (6)

where $V = 1/\rho$ is the volume per unit mass and $e_0(T)$ is the reference internal energy for a thermally perfect gas. The sound speed and specific heats are written as

$$a^2 = \frac{C_p}{C_v} \left( \frac{\partial p}{\partial \rho} \right)_T,$$ (7)

$$C_v = \left( \frac{\partial e}{\partial T} \right)_\rho, \quad C_p = C_v + \frac{T}{\rho^2} \left( \frac{\partial p}{\partial T} \right)_\rho \left[ \left( \frac{\partial p}{\partial \rho} \right)_T \right].$$ (8)

As an equation of state (EOS), the Soave-Redlich-Kwong (SRK) EOS is used to evaluate the thermodynamic properties of cryogenic fluids at supercritical conditions. The SRK EOS is written as

$$p = \frac{RT}{V - b} - \frac{a(T)}{V^2 - bV - b^2},$$ (9)

$$a(T) = \left[ 1 + (0.48 + 1.574) \omega_a - 0.176 \omega_a^2 \right] (1 - (T/T_c)^{\frac{1}{2}})^2.$$ (10)

Here, the coefficient $a$ is 0.42748 $R^2 T_c^2 / p_c$, and $b$ is 0.08664 $R T_c / p_c$. A detailed description of the thermodynamic relationships and implementation of the SRK EOS can be found in Meng and Yang and Terashima et al.
Figure 1 shows the density and constant-pressure specific heat of nitrogen at 5 MPa calculated from the SRK EOS as compared to the Peng-Robinson (PR) EOS and the reference data from NIST. The critical pressure, density, and temperature of nitrogen are $p_{cr} = 3.4$ MPa, $\rho_{cr} = 313.3$ kg/m$^3$, and $T_{cr} = 126.2$ K, respectively. The acentric factor of nitrogen is 0.0372. The SRK EOS does well to predict the real-gas thermodynamics near the critical point, and is more accurate than the PR EOS in the low-temperature region. The dynamic viscosity and thermal conductivity are estimated using the model of Zeberg-Mikkelsen et al. and Vasserman and Nedostup, respectively. Spatial derivative terms in the governing equations are evaluated using the SLAU scheme of AUSM family upwind-biased schemes. The MUSCL approach with the van-Albada limiter is applied to obtain third-order accurate discretization. The present MUSCL method is corrected using Thormer's method to reduce numerical dissipation at low-Mach number flows. The third-order TVD Runge-Kutta method is used for time integration.

The present study applies the hybrid LES/RANS approach. To predict near-wall eddy viscosity, the Spalart-Allmaras turbulence model (SA-model) is used. The present computational setup and grids

| Case | J1 | J2 | J4 | J6 |
|------|----|----|----|----|
| $p$, MPa | 5.0 | 522.0 | 120 | 30.0 |
| $\rho_{\text{in}}, \text{kg/m}^3$ | 30.0 | 56.0 | 56.0 |
| $U_{\text{in}}, \text{m/s}$ | 300 | 300 |
| $T_{\text{in}}, \text{K}$ | 92.0 | 130.0 | 183.0 | 225.0 |
| $J$ | 1.0 | 2.1 | 4.0 | 6.0 |
| $U_{\text{in}}/U_{\text{m}}$ | 3.07 | 4.33 | 6.1 | 7.5 |
| $R_{\text{ref}}(\times 10^5)$ | 1.3 | 1.9 | 2.7 | 3.3 |

3. Results and Discussion

3.1. Computational setup and grids

The two-dimensional co-planar jet simulations are performed under supercritical pressure. Figure 2 shows a schematic of the present co-flowing planar injector. The inner injector height $H$ is 0.5 mm, the outer height is 0.25 mm, and the post wall thickness is 0.125 mm. Table 1 shows the simulation conditions. Nitrogen is used as the working fluid in the present simulations. Cryogenic nitrogen is injected from the inner injector, and gas-like nitrogen is injected from the two outer injectors. The pressure is set to 5 MPa, which is a supercritical pressure for nitrogen. We perform the simulations varying two parameters: outer-to-inner jet momentum flux ratio $J$ and recess length $Rn$. The pressure is set to 5 MPa, which is a supercritical pressure for nitrogen. We perform the simulations varying two parameters: outer-to-inner jet momentum flux ratio $J$ and recess length $Rn$. In the present study, the recess length $Rn$ is set to 0 (non-recessed), 1, 2, and 4, where $Rn$ is normalized by $H$. The momentum flux ratio varies from 1 to 6 by varying the outer-jet injection velocity. Note that the density ratio is fixed in all cases. Hereafter, case names are defined as, for example, the case with $Rn = 2$ and $J = 2$ is denoted as $Rn2J2$.

Figure 3 shows the computational grids. The computational domain consists of an inner injector region, two outer injector regions, a chamber region, and a recessed region (for recessed cases). The chamber domain size is $300H \times 100H$. The domain lengths on the inner and outer injectors are $50H$ and $50h$, respectively. The adiabatic non-slip wall boundary condition is applied on the injector walls. Slip-wall boundary conditions are imposed at the top and bottom boundaries of the chamber. At the outflow boundary, the pressure is fixed.
the inlet. The simulations are performed at a CFL number of more than 18 times. The data is recorded every 10 time steps, and fast Fourier transform (FFT) is used together with a Hanning window. The results obtained for the non-recessed domain show a discrepancy in the downstream region.

Table 2. Number of grid points in the streamwise and transverse directions, and total grid points.

|       | Chamber | Inner injector | Outer injector |
|-------|---------|----------------|----------------|
| Coarse | 351 x 507 | 201 x 131 | 141 x 55 |
| Middle | 501 x 661 | 231 x 181 | 171 x 81 |
| Fine   | 621 x 811 | 251 x 211 | 191 x 101 |

to the chamber pressure, and the other variables are extrapolated. For the present simulations, no perturbation is added at the inlet. The simulations are performed at a CFL number of approximately 0.6. The statistics are estimated by averaging the flow at least 20 flow-through times after a transient stage of 10 flow-through times for all cases. For frequency analysis, fast Fourier transform (FFT) is used together with a Hanning window. The data is recorded every 10 time steps, and FFT is performed with 65536 data points and is averaged more than 18 times.

A grid convergence study is conducted for the 2H-recessed domain (Rn2) at \( J = 2 \). Three different grids are used. Table 2 summarizes the number of grid points. The grid is clustered near the injector wall and injector plate. The first grid point normal to the injector wall is located at approximately \( y^+ = 1.0 \) for all grids, where \( y^+ \) is the wall unit.

Figure 4 shows the mean profiles of normalized density \( \rho^+ = \left( \rho - \rho_{\infty} \right) / \left( \rho_{m} - \rho_{\infty} \right) \) on the inner jet centerline for both domains. The results obtained for the non-recessed domain from our previous study are also shown.\(^{21} \) The Coarse grid results show a discrepancy in the downstream region. The results of the Middle grid agree reasonably well with the Fine grid results; therefore, we consider that the Middle grid has an adequate grid resolution. In the following simulations, the numerical grids for each recess length have an identical grid resolution with the Middle grid.

### 3.2. Jet structures in recessed injectors

Figure 5 shows the instantaneous fields of the normalized density and absolute vorticity at \( J = 2 \) for each recess length. Note that \( x = 0 \) is located at the exit of the inner injector in the following results. The results show that vortex structures are periodically generated behind the post lip. These vortex structures entrain the surrounding low-density fluid of the outer flow into the inner dense jet. Then, they are gradually collapsed and disintegrated, and these “vortex breakdown” events generate the fine structures. Compared with the non-recessed case, the density contours of the recessed cases show that the outer jets penetrate deeply into the inner jet, resulting in the appearance of flapping-like structures in the inner jet. The results of the recessed cases also show that generation of the fine structures seems to be promoted. Figure 6 shows the mean density on the inner jet centerline at \( J = 2 \). The region having the initial injection density is called the “inner core region.” Behind the core regions, the density gradually decays along the streamwise direction due to the entrainment of surrounding fluid and the mixing. The results of the recessed cases show an abrupt shortening of the inner-jet cores in the near-injector regions, and the appearance of “plateaus” at approximately \( x/\text{H} = 1.0–1.8 \). This is due to the flapping structures in the inner jet. The inner-jet dense core sporadically flaps across the centerline in the near-injector regions, and as a result, a plateau appears for time-averaged centerline density. In the far-injector region, the centerline density decays faster as the recess length increases.

Figure 7 shows the mean streamwise velocity profiles in the transverse direction for \( J = 2 \). Two velocity peaks in the near-injector region at \( x/\text{H} = 1 \) correspond to the centers of the inner and outer jets. The results for the non-recessed case show that the velocity peak of the outer jet spreads out laterally. Conversely, in the recessed cases, the outer-jet centers are observed to approach the centerline. With the recessed injector, the near-injector flowfield is confined by the walls. This confinement prevents the outer jet from spreading laterally and causes the near-injector entrainment and penetration of the outer jet to become stronger. As a result, the inner jet is bent by the outer-jet penetration, which generates the flapping structures. Figure 8 shows the contours of the root-mean-square streamwise velocity fluctuations and the fluctuation intensities for \( J = 2 \). The fluctuation intensity is defined as \( 0.5\left( \overline{u'w'} \right) / \overline{U_{\text{out}}} \). Large fluctuations in velocity and intensity are shown in the inner mixing layer. This is due to the motions of large-scale vortex structures generated behind the post lip and their subsequent breakdown. The fluctuation intensity is more widely distribu-
uted in the recessed cases. As shown in the velocity profiles in Fig. 7, recessing prevents the outer jets from moving toward in the lateral direction. This results in the interaction of vortex structures in the upper and lower mixing layers. Additionally, in Fig. 8(a), large velocity fluctuations are observed along the wall in the recessed region, indicating the interaction between the vortex structures behind the post lip and the boundary layer. These observations of the boundary layer/vortex interactions agree with the results presented in past studies of turbulent confined wake\(^\text{34}\) and a turbulent flow past a square cylinder.\(^\text{49}\) Figure 9 shows the effects of the recess length on the centerline velocity fluctuations for \(J = 2\). The fluctuation intensity is maximized as the recess length increases. As the recess length increases, the interactions between the large-scale vortex structures and a boundary layer occur over a long distance inside the injector. Consequently, applying a recess with a longer length further shortens the inner jet length and increases the fluctuations.

Figure 10 schematically represents the flow structures in the recessed injector. The confinement of the near-injector flowfield causes the inner-jet flapping and vortex interactions. The recess-induced vortex interactions promote vortex breakdown and the generation of fine structures. Conse-
main usually well predicts experimental results. Therefore, it is expected that two-dimensional simulations lose three-dimensional turbulent structures, but captures the large-scale vortex structures that have a dominant role in near-injector mixing and recess effects. Consequently, we believe that the present results reproduce the fundamental mechanisms of the recess effects, and the influence of the parameters of the recess length and the momentum flux ratio (discussed in Section 3.3) can be qualitatively extended for three-dimensional jets.

Figures 11 and 12 show the power spectra of the transverse velocity fluctuations for $J = 2$ and 6. The fluctuations are obtained at three-probe positions: probe A in the inner mixing layer (Figs. 11(a) and 12(a)), probe B at the near-injector position on the centerline (Figs. 11(b) and 12(b)), and probe C at the far-injector position on the centerline (Fig. 11(c) and 12(c)). At position A in the inner mixing layer, dominant frequency peaks of approximately 75 kHz and 109 kHz are observed for $J = 2$ and 6, respectively. These frequencies correspond to the frequency of vortex shedding behind the post lip. In the case of recessed injectors, additional low-frequency fluctuations clearly appear in the spectra: approximately 45 kHz and 60 kHz for $J = 2$ and 6, respectively. These low-frequency fluctuations correspond to the inner-jet flapping structures introduced by applying the recess. As discussed previously, the strengthened outer-jet penetration in the recessed injectors due to the near-injector confinement effect causes the flapping inner-jet structures. Moreover, past instability analyses have shown that confined wakes and jets are globally unstable for sinuous perturbation, and the induced global instability may lead to large-scale sinuous motion in co-flowing jets and helical motions in coaxial jets. Thus, according to this theoretical prediction, the confined near-injector flow in the recessed region is expected to be unstable for sinuous perturbation, which may trigger the generation of flapping structures.

3.3. Effects of recess lengths on different momentum flux ratios

Figure 13 shows the centerline mean density for $J = 1$, 4, and 6. At $J = 1$ and 2 (in Fig. 6), the plateau appears in the case of recessed injectors. On the other hand, no plateau is observed at $J = 4$ and 6, and the density decay starts just after injection. This means that in the cases of $J = 4$ and 6, recess-induced inner-jet flapping is promoted, and large flapping appears just behind the inner injector exit. In the far-injector region, the centerline density decay is well enhanced by recessing at $J = 4$ and $J = 6$, but not at $J = 1$. This is because the strength of the vortex structures behind the post has a large impact on recess effects. Increasing the $J$ value also increases the outer-to-inner jet velocity ratio. Consequently, increasing the $J$ value reinforces the generation of vortex structures behind the post lip, and further enhances the recess-induced outer-jet entrainment and vortex interactions.

Figure 14 shows the change in mean inner-jet length $L$ relative to the momentum flux ratio and recess length. Here, the
inner-jet length is defined as the streamwise position where the mean normalized density $\bar{\rho}^*$ is 0.5. As shown in Fig. 14(a), the inner-jet length is shortened by increasing the $J$ value. A similar trend can be found in the cases of both non-recessed and recessed injectors. Figure 14(b) shows the effect of the recess length on the mean inner-jet length at fixed $J$ values. Here, $L_{\text{inj}}^{R=0}$ is the mean inner-jet length at $Rn = 0$. The inner-jet length decreases as the recess length increases. It is noteworthy that the effect of the recess on shortening the inner-jet length becomes greater as the $J$ value increases. When the recess length of $Rn = 4$ is applied, the inner-jet length is shortened about 2% at $J = 1$, but 22% at $J = 6$. Moreover, changing the recess length from $Rn = 2$ to 4 further shortens the jet length obtained at $J = 4$ and 6, but not at $J = 1$ and 2. Assuming that the recess length is further increased, the outer jet gradually loses its streamwise momentum due to the mixing, and eventually the mixing between the inner and outer jets is completed inside the injector, resulting in a flow that becomes close to being a fully developed channel flow. In this situation, the effects of increasing the recess length are no longer obtained. Therefore, it is expected that the longer recess length effectively works in the case of higher outer-flow momentum flux. This is consistent with the present results; when applying a longer recess, the shortening of the inner-jet length is only obtained when $J$ is higher.

4. Conclusion

The effects of a recess on cryogenic co-planar jets under a supercritical pressure were studied using two-dimensional hybrid LES/RANS simulations for a wide range of recess lengths and momentum flux ratios. In recessed injectors, con-
finement of the near-injector fieldflow plays a role in suppressing outer-jet spreading and enhances outer-jet penetration into the inner jet. This confinement effect promotes interactions between the vortex structures, resulting in vortex breakdown. Moreover, the enhanced penetration of the outer jet causes the appearance of flapping structures in the inner jet. In the velocity fluctuation spectra, in addition to the shedding frequency of vortices behind the post lip, low-frequency peaks clearly appeared in the case of recessed injectors, corresponding to the recess-induced flapping motions in the inner jet. Due to these recess-induced flow structures (i.e., enhanced vortex breakdown, inner-jet flapping), centerline density decay increases, which suggests that mixing is improved when using recessed injectors.

The results also show that the inner-jet length decreases as the recess length increases. It was found that, in the cases of higher momentum flux ratios, the recess effects on the shortening of the inner-jet length is effectively obtained. This is because increasing the velocity ratio strengthens the generation of vortex structures behind the post lip and enhances the recess effect.

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To prevent numerical oscillations at the initial time, a relaxation factor is set to 0.85. The injection density and velocity are obtained at the beginning of the simulation through an extrapolation. The simulation is performed using a CFL number of approximately 0.6. The injection density and velocity are obtained at the beginning of the simulation through an extrapolation. The simulation is performed using a CFL number of approximately 0.6.

Note that, for computational efficiency, the injection velocity is four times as high as the experimental value, while the inlet diameter of the jet is one-quarter of that used in the experiment to have the same Reynolds number as the experiment. A rectangular domain of $400D \times 60D \times 60D$ is used. The number of grid points is $441 \times 301 \times 301$. The initial profiles of the primitive variables $q = (\rho, u, p)$ are smoothly provided using the error function as described in Kawai and Terashima to prevent numerical oscillations at the initial stage. No perturbations are introduced at the inlet. A slip wall boundary is applied on the chamber walls. At the outflow boundary, the pressure is fixed to the chamber pressure, and the other variables are extrapolated. The simulation is performed using a CFL number of approximately 0.6.

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files along the jet centerline. In addition to the experimental results obtained by Mayer et al.,\textsuperscript{2} the LES results of Schmitt et al.,\textsuperscript{5} and Terashima and Koshi\textsuperscript{6} are also plotted. The present results agree well with the experimental results. Slight differences in jet-core lengths can be found between the LES results. A longer core length is obtained in the present work and the LES by Terashima and Koshi\textsuperscript{6} compared to the LES by Schmitt et al.,\textsuperscript{5} This may be because the difference of applied inlet perturbation levels (where no perturbation is introduced in the present LES and the LES by Terashima and Koshi\textsuperscript{6}) affects the growth of shear-layer instability and the transition to turbulence.

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Fig. 15. Mean density profiles on the axis for a round jet simulation at a supercritical pressure.