MEASUREMENT OF $\sin^2 2\theta_{13}$ BY REACTOR EXPERIMENTS AND ITS SENSITIVITY

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Reactor experiments offer a promising way to determine $\theta_{13}$ and are free from parameter degeneracies in neutrino oscillations. It is described how reactor measurements of $\sin^2 2\theta_{13}$ can be improved by a near-far detector complex. The experimental lower bound is derived on the sensitivity to $\sin^2 2\theta_{13} \gtrsim 0.02$ based on the rate analysis, and an idea is given which may enable us to circumvent this bound. It is shown that in the Kashiwazaki-Kariwa plan (KASKA) the sensitivity to $\sin^2 2\theta_{13}$ is approximately 0.02.

1. Introduction

The recent experiments on atmospheric and solar neutrinos and KamLAND have been so successful that we now know the approximate values of the mixing angles and the mass squared differences of the atmospheric and solar neutrino oscillations: $(\sin^2 2\theta_{12}, \Delta m^2_{21} \equiv m^2_2 - m^2_1) \simeq (0.8, 7 \times 10^{-5}\text{eV}^2)$ for the solar neutrino and $(\sin^2 2\theta_{23}, |\Delta m^2_{31}| \equiv |m^2_3 - m^2_2|) \simeq (1.0, 2 \times 10^{-3}\text{eV}^2)$ for the atmospheric neutrino, where the three flavor framework of neutrino oscillations is assumed. The quantities which are still unknown are the third mixing angle $\theta_{13}$, the sign of the mass squared difference $\Delta m^2_{31}$ which indicates whether the mass pattern of neutrinos is of normal hierarchy or of inverted one, and the CP phase $\delta$. Among these, $\theta_{13}$ is the most important quantity in the near future neutrino experiments.

It has been known that the oscillation parameters $\theta_{jk}, \Delta m^2_{jk}, \delta$ cannot be determined uniquely even if the appearance probabilities $P(\nu_\mu \rightarrow \nu_e)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ are measured precisely from a long baseline accelerator.

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experiment due to so-called parameter degeneracies, and this problem has to be solved to determine the CP phase in the future long baseline experiments. Among the ideas which have been proposed to solve the problem, combination of a reactor measurement and a long baseline experiment offers a promising possibility (See Ref.\(^1\) and references therein). This combination works because reactor experiments measure the disappearance oscillation probability which depends only on \(\theta_{13}\) to a good approximation:

\[
P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \simeq 1 - \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta m_{13}^2 L}{4E}\right),
\]

where \(E\) and \(L\) stand for the neutrino energy and the baseline length.

### 2. Reactor measurements of \(\sin^2 2\theta_{13}\) and its sensitivity

To illustrate how a near-far detector complex improves the sensitivity to \(\sin^2 2\theta_{13}\) of reactor neutrino experiments, let me discuss the case with a single reactor, one near and one far detectors. To discuss the sensitivity, let me introduce \(\chi^2\), which basically indicates whether the difference between the number of events with oscillations and that without oscillations is large enough compared to the total error whose square is the sum of the statistical and systematic errors squared. For simplicity I will mainly discuss the sensitivity in the limit of infinite statistics, i.e., with the systematic errors only. Throughout my talk I will perform the rate analysis only.

Let \(m_n\) and \(m_f\) be the number of events measured at the near and far detectors, \(t_n\) and \(t_f\) be the theoretical predictions. Then \(\chi^2\) is given by

\[
\chi^2 = \min_{\alpha's} \left\{ \left[ \frac{m_n - t_n(1 + \alpha_c + \alpha_c^{(r)} + \alpha_u^{(r)})}{t_n \sigma_u} \right]^2 + \left[ \frac{m_f - t_f(1 + \alpha_c + \alpha_c^{(r)} + \alpha_u^{(r)})}{t_f \sigma_u} \right]^2 + \left( \frac{\alpha_c}{\sigma_c} \right)^2 + \left( \frac{\alpha_c^{(r)}}{\sigma_c^{(r)}} \right)^2 + \left( \frac{\alpha_u^{(r)}}{\sigma_u^{(r)}} \right)^2 \right\},
\]

where \(\alpha_u\), \(\alpha_c\), \(\alpha_c^{(r)}\) and \(\sigma_u^{(r)}\) are the variables to introduce the uncorrelated systematic error \(\sigma_u\) of the detectors, the correlated systematic error \(\sigma_c\) of the detector, the correlated systematic error \(\sigma_c^{(r)}\) of the flux and the uncorrelated systematic error \(\sigma_u^{(r)}\) of the flux, and it is assumed that the uncorrelated errors for the two detectors are the same and are equal to \(\sigma_u\).
After some calculations\(^2\), I obtain

$$\chi^2 = \left( \frac{m_n}{t_n} - 1, \frac{m_f}{t_f} - 1 \right) V^{-1} \left( \begin{array}{c} \frac{m_n}{t_n} - 1 \\ \frac{m_f}{t_f} - 1 \end{array} \right),$$

where

$$V \equiv \begin{pmatrix} \sigma_u^2 + \sigma_c^2 + (\sigma_{u}^{(r)})^2 + (\sigma_{c}^{(r)})^2 & \sigma_c^2 + (\sigma_{u}^{(r)})^2 + (\sigma_{c}^{(r)})^2 \\ \sigma_c^2 + (\sigma_{u}^{(r)})^2 + (\sigma_{c}^{(r)})^2 & \sigma_u^2 + \sigma_c^2 + (\sigma_{u}^{(r)})^2 + (\sigma_{c}^{(r)})^2 \end{pmatrix}$$

is the covariance matrix. After diagonalizing \(V\), I have

$$\chi^2 = \left( \frac{m_n}{t_n} - 1 \right) + \left( \frac{m_f}{t_f} - 1 \right) \left( \frac{\sigma_u^2}{4} + 4(\sigma_{u}^{(r)})^2 + 4(\sigma_{c}^{(r)})^2 + 2\sigma_u^2 \right).$$

(2)

The strategy in this talk is to assume no neutrino oscillation for the theoretical predictions \(t_j\) \((j=n,f)\) and assume the number of events with oscillations for the measured values \(m_j\) \((j=n,f)\) and to examine if a hypothesis with no oscillation is excluded, say at the 90\%CL, from the value of \(\chi^2\). Hence I have

$$\frac{m_j}{t_j} - 1 = -\sin^2 2\theta_{13} \left\langle \sin^2 \left( \frac{\Delta m_{13}^2 L_j}{4E} \right) \right\rangle, \quad (3)$$

where \(L_j\) is the distance between the reactor and the near or far \((j=n,f)\) detector, and

$$\left\langle \sin^2 \left( \frac{\Delta m_{13}^2 L_j}{4E} \right) \right\rangle = \frac{\int \limits dE \epsilon(E) f(E) \sigma(E) \sin^2 \left( \frac{\Delta m_{13}^2 L_j}{4E} \right)}{\int \limits dE \epsilon(E) f(E) \sigma(E)}.$$  

\(\epsilon(E), f(E), \sigma(E)\) stand for the detection efficiency, the neutrino flux, and the cross section, respectively. Thus Eq. (2) becomes

$$\chi^2 = \sin^4 2\theta_{13} \left\{ \frac{[D(L_f) + D(L_n)]^2}{4\sigma_u^2 + 4(\sigma_{u}^{(r)})^2 + 4(\sigma_{c}^{(r)})^2 + 2\sigma_u^2} + \frac{[D(L_f) - D(L_n)]^2}{2\sigma_u^2} \right\}$$

(4)

where (3) was used, and

$$D(L) \equiv \left\langle \sin^2 \left( \frac{\Delta m_{13}^2 L}{4E} \right) \right\rangle$$

was defined. The numerical value of \(D(L)\) is plotted in Fig.1 as a function of \(L\). Here I adopt the reference values \(\sigma_c = 0.8\%/\sqrt{2} = 0.6\%\) and
$D(L) \equiv \langle \sin^2 \left( \frac{\Delta m^2_{13} L}{4E} \right) \rangle$ as a function of $L$. $D(L)$ has its maximum value 0.82 at $L=1.8$ km, and approaches to its asymptotic value 0.5 as $L \to \infty$. $|\Delta m^2_{13}| = 2.5 \times 10^{-3}$ eV$^2$ is used as the reference value.

$\sigma_u = \sqrt{(2.7\%)^2 - (2.1\%)^2 - (0.8\% / \sqrt{2})^2} = 1.6\%$ used in Ref.\(^1\), where basically the reference values were deduced from extrapolation of the previous reactor experiments Bugey and CHOOZ. In the estimation of $\sigma_c$, I used 2.7\% total error and 2.1\% error of the flux which are the reference values in the CHOOZ experiment. As for the correlated and uncorrelated errors of the the flux from the reactors, I adopt the same reference values as those used by the KamLAND experiment: $\sigma_c^{(r)} = 2.5\%$, $\sigma_u^{(r)} = 2.3\%$. Putting the present reference values together, I have

$$2\sigma_u^2 = (0.8\%)^2,$$

$$4\sigma_c^2 + 4(\sigma_u^{(r)})^2 + 4(\sigma_c^{(r)})^2 + 2\sigma_u^2 = (7.6\%)^2.$$  

The contribution from $(4\sigma_c^2 + 4(\sigma_u^{(r)})^2 + 4(\sigma_c^{(r)})^2 + 2\sigma_u)^{-1}$ in Eq. (4) is only 1\% compared to that from $(2\sigma_u^2)^{-1}$, so virtually this term can be ignored in Eq. (4). Hence $\chi^2$ is given approximately by

$$\chi^2 \simeq \sin^4 2\theta_{13} \frac{[D(L_i) - D(L_n)]^2}{2\sigma_u^2}. \tag{5}$$

Since no oscillation is assumed for the theoretical predictions, the best fit value in $\sin^2 2\theta_{13}$ for the measurement is $\sin^2 2\theta_{13} = 0$. If $\chi^2$ is larger than 2.7, which corresponds to the value at the 90\% CL for one degree of freedom, then the hypothesis of no oscillation is excluded at the 90\% CL. This implies that the systematic limit on $\sin^2 2\theta_{13}$ at the 90\% CL, or the sensitivity in
the limit of infinite statistics, is given by

$$\left(\sin^2 2\theta_{13}\right)_{\text{sys only}} \simeq \sqrt{2.7} \frac{\sqrt{2}\sigma_u}{D(L_f) - D(L_n)}.$$  

(6)

Eq. (6) tells us that, in order to optimize $\left(\sin^2 2\theta_{13}\right)_{\text{sys only}}$, I have to minimize $D(L_n) \equiv \langle \sin^2 (\Delta m^2_{13} L_n/4E) \rangle$ and maximize $D(L_f) \equiv \langle \sin^2 (\Delta m^2_{13} L_f/4E) \rangle$. Since the possible maximum value of $D(L_f) - D(L_n)$ is 0.82, which is attained for $L_f = 1.8$km and $L_n = 0$, the lower bound of $\sin^2 2\theta_{13}$ at a single reactor experiment can be estimated as:

$$\text{lower bound of } \sin^2 2\theta_{13} \simeq \sqrt{2.7} \sqrt{2}\frac{\sigma_u}{0.82} = 2.8 \sigma_u.$$  

(7)

Eq. (7) indicates that, unless one develops a technology to improve $\sigma_u$ significantly compared to the reference value $\sigma_u = 0.6%$ assumed in Ref.1, it is difficult to achieve the sensitivity below $\sin^2 2\theta_{13} = 0.016$ in an experiment with one reactor and two detectors. It should be stressed here that the correlated systematic error $\sigma_c$ and the errors $\sigma_u^{(r)}$, $\sigma_c^{(r)}$ of the flux do not appear in the dominant contribution to $\chi^2$, and that it is the uncorrelated systematic error $\sigma_u$ divided by the factor $D(L_f) - D(L_n)$ that determines the systematic limit on $\sin^2 2\theta_{13}$.

In the ideal case with $N$ pairs of the reactors and the near detectors and one far detector, one can show\(^2\) that Eq. (7) becomes

$$\text{lower bound of } \sin^2 2\theta_{13} \simeq \sqrt{2.7} \sqrt{2}\frac{\sigma_u}{0.82} > 2.0 \sigma_u.$$  

(8)

Eq. (8) indicates that even in the ideal case the sensitivity can be approximately 0.012 at best, as far as the rate analysis is concerned, no matter how many reactors and detectors there may be, if one adopts $\sigma_u = 0.6%$.

In principle there is a way to improve this bound. In the case with one reactor, if one puts $M$ identical detectors at the near site and $M$ identical detectors at the far site, where all these detectors are assumed to have the same uncorrelated systematic error $\sigma_u$, then the value of $\chi^2$ is simply multiplied by $M$. Eqs. (5) and (7) imply that the sensitivity becomes

$$\text{lower bound of } \sin^2 2\theta_{13} \simeq \sqrt{2.7} \sqrt{2}\frac{\sigma_u}{0.82\sqrt{M}} = \frac{2.8}{\sqrt{M}} \sigma_u.$$  

Therefore, it follows theoretically that the more identical detectors one puts, the better sensitivity one gets. Notice that this conclusion is based crucially on the assumption that the uncorrelated systematic error $\sigma_u$ of the detectors is independent of the number of the detectors $2M$. This assumption may
not be satisfied in general, but if the dependence of \( \sigma_u \) on \( M \) is weaker than \( \sqrt{M} \), \( \sigma_u/\sqrt{M} \) decreases as \( M \) increases, and this possibility may give us a way to improve the sensitivity. It is therefore important to examine experimentally the dependence of \( \sigma_u \) on \( M \).

3. The KASKA plan

The Kashiwazaki-Kariwa nuclear plant consists of two clusters of reactors, and one cluster consists of four reactors while the other consists of three. In the Kashiwazaki-Kariwa (KASKA) plan\(^3\) one near detector is placed near one cluster of the reactors while the other near detector is placed near another cluster. In addition, a far detector is located 1.3 km away from the reactors. In the presence of seven reactors, the total number \( m_j \) of the events measured at the \( j \)-th detector (\( j = 1 \) (the first near), 2 (the second near), 3 (the far detector)) is a sum of contributions \( m_{aj} \) (\( a = 1, \cdots, 7 \)) from each reactor, and this is also the case for the theoretical predictions \( t_j \) and \( t_{aj} \) (\( a = 1, \cdots, 7 \)) at the \( j \)-th detector. So I have

\[
m_j = \sum_{a=1}^{7} m_{aj}, \quad t_j = \sum_{a=1}^{7} t_{aj}.
\]

In the limit of infinite statistics \( \chi^2 \) is defined as

\[
\chi^2 = \min_{\alpha^s} \left\{ \sum_{j=1}^{3} \frac{1}{\sigma_j^2} \left[ m_j - t_j \left( 1 + \alpha_c + \alpha_c^{(r)} + \sum_{a=1}^{7} \frac{t_{aj}}{t_j} \alpha^{(r)}_{ua} \right) \right]^2 + \left( \frac{\alpha_c}{\sigma_c} \right)^2 + \left( \frac{\alpha_c^{(r)}}{\sigma_c^{(r)}} \right)^2 + \sum_{a=1}^{7} \left( \frac{\alpha^{(r)}_{ua}}{\sigma^{(r)}_{ua}} \right)^2 \right\}.
\]

After some calculations, I get

\[
\chi^2 = \left( \frac{m_1}{t_1} - 1, \frac{m_2}{t_2} - 1, \frac{m_3}{t_3} - 1 \right) V^{-1} \left( \frac{m_1}{t_1} - 1, \frac{m_2}{t_2} - 1, \frac{m_3}{t_3} - 1 \right)^T,
\]

where

\[
V_{jk} = \delta_{jk} \sigma_u^2 + \sigma_c^2 + (\sigma_c^{(r)})^2 + (\sigma^{(r)}_a)^2 \sum_{a=1}^{7} \frac{t_{aj}}{t_j} \frac{t_{aj}}{t_k}.
\]

As in the case with one reactor (3), I have

\[
\frac{m_j}{t_j} - 1 = \sin^2 2\theta_{13} \sum_{a=1}^{7} \frac{t_{aj}}{t_j} D(L_{aj}),
\]

where \( L_{aj} \) is the distance between the \( a \)-th reactor (\( a = 1, \cdots, 7 \)) and the \( j \)-th detector (\( j = 1,2,3 \)). Therefore \( \chi^2 \) is proportional to \( \sin^4 2\theta_{13} \):

\[
\chi^2 = C \sin^4 2\theta_{13},
\]
Figure 2. The contour plot of the sensitivity to $\sin^2 2\theta_{13}$ in the limit of infinite statistics (a) and for 20 ton-yr (b) in the KASKA experiment. The optimized and currently planned positions of the detectors are also depicted. When the contour for each detector is plotted, it is assumed that other detectors are located in the optimized positions. The position of the far detector is varied within the campus of the Kashiwazaki-Kariwa nuclear plant. For the data size 20 ton-yr, the value $\sin^2 2\theta_{13} = 0.025$ of the sensitivity seems to be inconsistent with the contour plot in Fig.2(b), which suggests that it is better than $\sin^2 2\theta_{13} = 0.022$ for the planned location of the far detector. However, the contour for the far detector is plotted here on the assumption that the near detectors are both in the optimized position, and therefore this calculation gives the sensitivity better than that for the locations currently planned.
where
\[
C \equiv \sum_{j,k=1}^{3} \sum_{a=1}^{7} \frac{t_{a,j}}{t_j} D(L_{aj})(V^{-1})_{jk} \sum_{b=1}^{7} \frac{t_{b,k}}{t_k} D(L_{bk}).
\]

Hence the sensitivity to \(\sin^2 2\theta_{13}\) at 90\%CL is given by
\[
\sin^2 2\theta_{13} = \left( \frac{\chi^2 |_{90\%CL}}{C} \right)^{1/2} = \left( \frac{2.7}{C} \right)^{1/2},
\]
where it is assumed that the value of \(|\Delta m^2_{13}|\) is precisely known and therefore degree of freedom in this analysis is one. The covariance matrix (9) cannot be inverted analytically, so \(C\) has to be evaluated numerically. Fig. 2 shows the contour plot of the sensitivity to \(\sin^2 2\theta_{13}\) in the limit of infinite statistics (a) and for 20 ton·yr (b). Fig. 2 indicates that optimization forces the distances between each near detector and the reactors in each cluster be approximately (300±130)m. This results in slightly poorer sensitivity to \(\sin^2 2\theta_{13}\) than the hypothetical single reactor case with near and far detectors. This is because in the case with a single reactor, the near detector can be theoretically arbitrarily close to the reactor and the factor \(D(L_f) - D(L_n) \rightarrow D(L_i)\) can in principle be the maximum value 0.82, whereas in the KASKA case, the sensitivity is given by
\[
\lim_{\text{sys only}} \frac{\sqrt{2.7} \sqrt{1.04}}{\sigma_u} \approx \frac{\sqrt{2.7} \sqrt{1.04}}{\sigma_u},
\]
and the second and third term in the denominator give small contribution to spoil the sensitivity. From the numerical calculations, one can show that the KASKA experiment has the sensitivity \(\sin^2 2\theta_{13} \approx 0.025\) (0.019) with the data size of 20 ton·yr (\(\infty\) ton·yr). Since the sensitivity to \(\sin^2 2\theta_{13}\) at KASKA is close to the lower bound 0.016 (cf. (7) with \(\sigma_u = 0.6\%\)), the setup of the KASKA plan is not far from the optimum.

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