Tomographic and statistical properties of superposition states for two-mode systems

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Abstract

The two-mode even and odd coherent states and two-mode squeezed correlated state are discussed. Photon distribution functions, means, dispersions, Fano factor for even and odd coherent states and squeezed correlated state are calculated. The photon distribution function for two-mode squeezed correlated state is obtained. The tomograms of two-mode even and odd coherent states and squeezed correlated states are investigated within the framework of a symplectic tomography scheme. It is shown that the phenomenon of entanglement appears in the system. Two different measures of entanglement are employed.

Keywords: squeezed correlated state, odd and even coherent states, Schrödinger cat states, Wigner function, photon number distribution function, Fano factor, entanglement, symplectic tomography scheme

1 Introduction

The odd and even coherent states were introduced in [1, 2]. In this works, the name ”even and odd” coherent states has been given to even and odd superpositions of two Gaussian packets describing coherent states. In [3] these states were discussed as a
subclass of some generic set of nonclassical states. A scheme of generation of even and odd coherent states of a trapped ion have been proposed in [4]. This scheme gives the possibility of studying quantum interference phenomena with essentially higher stability than realization of the even and odd coherent states in quantum optics [5]. The importance of the even and odd coherent states of the electromagnetic fields is also related to the possibilities of reducing the noise influence on the signal in the process of quantum-state signal transmission used in optical communication [6]. Also, the even and odd coherent states might be used as alternatives to squeezed states of light to improve the sensitivity of interferomagnetic gravitational wave detectors [7]. For large amplitudes of the partners of the superposition of two coherent states, these states of light and their slight modifications were interpreted as ”Schrödinger cat states” in [8] where their generation due to propagation of initially coherent light in Kerr medium was suggested. One can use generalized correlated states [9] as the partners of the superposition to take into account the influence of mode quadratures correlations on the nonclassical properties of light. Last years quantum statistical properties of odd and coherent states have been the subject of intensive experimental and theoretical investigations and still remain the subject of study [10]-[19].

Generic systems with quadratic Hamiltonians (multivariable parametric oscillators) have statistical properties of their Fock states (number eigenstates) described by multivariable Hermite polynomials [20]-[23]. General formulas for matrix elements of the Gaussian density operator for a multimode oscillator in Fock basis were calculated explicitly in [20]. The photon distribution for an $N$-mode mixed state of light described by a Wigner function of the generic Gaussian form was calculated explicitly in terms of Hermite polynomials of $2N$ variables in [21] and the parameters of the photon distribution function were determined through the dispersion matrix and mean values of quadrature components of the light. The photon distribution for the two-mode squeezed vacuum was investigated in [22] where its dependence on four parameters (two squeezing coefficients, relative phase between the two oscillators, and their spatial orientation) was shown. In [23] the case of generic two-mode squeezed coherent states was consid-
ered, and the photon distribution function for the states was expressed through both four-variable and two-variable Hermite polynomials dependent on two squeezing coefficients, the relative phase between the two oscillators, their spatial orientation, and four-dimensional shift in the phase space of the electromagnetic-field oscillator. In [24] the multimode even and odd coherent states were considered. The explicit formulae for the photon distribution, Wigner function were derived. In [25] the multimode Schrödinger cat state were constructed for polymode parametric oscillators of the electromagnetic field. The evolution of the photon distribution function was evaluated explicitly, the distribution function were expressed in terms of multivariable Hermite polynomials.

In two-mode odd and even coherent states and two-mode squeezed correlated states, one has the interaction of the photon modes, which creates different correlations of the mode properties. Such correlations can be considered as phenomenon of the mode entanglement. The entanglement of subsystems of composite systems is important ingredient of quantum information processing and quantum computing. It is worthy to clarify the properties of entanglement between the modes in the states under study. We investigate two measures of entanglement introduced in [26, 27] for two-mode squeezed correlated state and two-mode even and odd coherent states.

Recently the tomographic method of state reconstructing was introduced for generic quantum systems in [28]-[30]. One measures the state tomogram which is the standard probability distribution function. The reconstruction formula gives the possibility to obtain the density matrix of the state. For optical tomography, this method was realized for measuring photon states (see, for example, [31, 32]). The tomographic approach was used to present the new formulation of quantum mechanics in which quantum states are associated with standard probability distributions [33]-[35]. The symplectic tomograms of Schrödinger cat states of a trapped ion were investigated in [36, 37]. The two-mode even and odd coherent states and two-mode squeezed correlated states can be associated with the tomograms which, being the standard probability distributions, contain the same information on the states as the wave function or density matrix does.

The aim of the paper is to consider two-mode even and odd coherent states and
two-mode squeezed correlated states, to study their photon distribution functions, to consider the phenomenon of entanglement between the modes in both states and to obtain in explicit form their tomograms within the framework of symplectic tomography scheme.

The paper is organized as follows. In Sec. 1 we discuss the wave function, Wigner function and dispersions of quadratures in pure two-mode squeezed correlated states. In Sec.2, we concentrate on the generic case of the photon statistics of a pure two-mode squeezed coherent state and obtain for it two explicit formulae through Hermite polynomials of four variables and through Hermite polynomials of two variables. In Sec. 3 we discuss the averaging of photon distribution function of two-mode squeezed correlated states over one mode and obtain the probability to have $n$ photon in other mode. In Sec. 4 we investigate the phenomenon of the entanglement in the two-mode squeezed correlated states. We present in Sec. 5 the exact formulae for tomogram of two-mode squeezed correlated states within the framework of symplectic tomography schemes. In Sec. 6 we discuss the wave function, Wigner function, photon number probability distribution function and dispersions of quadratures and photon numbers in the modes, concentrate on the phenomenon of entanglement and obtain the tomogram within the framework of symplectic tomography for two-mode even and odd coherent states.

2 Two-mode squeezed correlated states

In this section, we discuss the wave function, Wigner function and dispersion of quadratures in two-mode squeezed correlated states. The wave function $\Psi_{sq}(x_1, x_2)$ that described the generic form of the photon statistics of a pure two-mode squeezed coherent state was found in [23] and is of the form

$$\Psi_{sq}(x_1, x_2) = \mathcal{N} \exp \left( -Ax_1^2 - Bx_2^2 + 2Cx_1x_2 + Dx_1 + Ex_2 \right),$$  

(1)
with the normalization constant
\[ N = \sqrt{\frac{2}{\pi}} \left( A_1 B_1 - C_1^2 \right)^{1/4} \exp \left[ -\frac{1}{4(A_1 B_1 - C_1^2)} (B_1 D_1 D + A_1 E_1 E + C_1 [E_1 D + D_1 E]) \right]. \] (2)

The wave function is a shifted Gaussian function described by the five complex numbers
\[ A = A_1 + iA_2, \quad B = B_1 + iB_2, \quad C = C_1 + iC_2, \quad D = D_1 + iD_2, \quad E = E_1 + iE_2. \]

The Wigner function of squeezed correlated states can be represented in the form
\[ W(\vec{Q}) = (2\pi)^{-2} (\det \sigma)^{-1/2} \exp \left[ -\frac{1}{2} \left[ \langle \vec{Q} - \langle \vec{Q} \rangle \rangle \right] \sigma^{-1} \left( \vec{Q} - \langle \vec{Q} \rangle \right) \right], \] (3)

where \( \sigma \) is the quadrature dispersion matrix and the four-dimensional vector \( \vec{Q} \) has the components \((p_1, p_2, x_1, x_2)\). The four-dimensional vector \( \langle \vec{Q} \rangle \) has the quadratures means as its components:
\[ \langle p_1 \rangle = \frac{E_1 (A_1 C_2 - A_2 C_1) - D_1 (A_2 B_1 - C_1 C_2) + D_2 (A_1 B_1 - C_1^2)}{A_1 B_1 - C_1^2}, \]
\[ \langle p_2 \rangle = \frac{D_1 (B_1 C_2 - B_2 C_1) - E_1 (A_1 B_2 - C_1 C_2) + E_2 (A_1 B_1 - C_1^2)}{A_1 B_1 - C_1^2}, \]
\[ \langle x_1 \rangle = \frac{B_1 D_1 + C_1 E_1}{2 (A_1 B_1 - C_1^2)}, \quad \langle x_2 \rangle = \frac{A_1 E_1 + C_1 D_1}{2 (A_1 B_1 - C_1^2)}. \] (4)

The dispersion matrix \( \sigma \) with the matrix elements:
\[ \sigma_{x_1,x_1} = \frac{B_1}{4 (A_1 B_1 - C_1^2)}, \quad \sigma_{x_1,x_2} = \frac{C_1}{4 (A_1 B_1 - C_1^2)}, \quad \sigma_{x_1,p_1} = \frac{C_1 C_2 - B_1 A_2}{2 (A_1 B_1 - C_1^2)}, \]
\[ \sigma_{x_1,p_2} = \frac{C_2 B_1 - B_2 C_1}{2 (A_1 B_1 - C_1^2)}, \quad \sigma_{x_2,x_2} = \frac{A_1}{4 (A_1 B_1 - C_1^2)}, \quad \sigma_{x_2,p_1} = \frac{C_2 A_1 - A_2 C_1}{2 (A_1 B_1 - C_1^2)}, \]
\[ \sigma_{x_2,p_2} = \frac{C_1 C_2 - A_1 B_2}{2 (A_1 B_1 - C_1^2)}, \quad \sigma_{p_1,p_1} = \frac{A_1^2 B_1 - A_1 C_1^2 + A_1 C_2^2 + A_2^2 B_1 - 2 A_2 C_1 C_2}{A_1 B_1 - C_1^2}, \]
\[ \sigma_{p_1,p_2} = \frac{-A_1 B_1 C_1 + A_2 B_2 C_1 + C_1^3 + C_1 C_2^2 - A_1 B_2 C_2 - A_2 B_1 C_2}{A_1 B_1 - C_1^2}, \]
\[ \sigma_{p_2,p_2} = \frac{A_1 B_2^2 - B_1 C_1^2 + B_1 C_2^2 + B_2^2 A_1 - 2 B_2 C_1 C_2}{A_1 B_1 - C_1^2}. \] (5)

The matrix elements of the dispersion matrix \( M \) can also be calculated by using the wave function (1). The determinant of the dispersion matrix \( M \) can be checked to
be equal to $1/16$, which means that our squeezed state (1) minimizes the generalized Schrödinger uncertainty relation [38]

$$\det M \geq \frac{1}{16}. $$

In this sense, the wave function $\Psi_{sq}$ is called a minimum uncertainty state.

## 3 Photon Distribution Function for two-mode squeezed correlated states

In this section, we concentrate on the generic case of the photon statistics of a pure two-mode squeezed coherent state. In order to calculate the distribution function $W(n_1, n_2)$, we construct the two-mode squeezed coherent wave function (1) with the probability amplitude of the photon-energy states

$$\Psi_{n_1,n_2}(x_1, x_2) = \prod_{i=1}^{2} \pi^{-1/4} \frac{1}{\sqrt{2^{n_i} n_i!}} e^{-x_i^2/2} H_{n_i}(x_i),$$

where $H_n$ denotes the $n$th Hermite polynomial, and arrive at

$$P(n_1, n_2) = \frac{|\mathcal{N}|^2}{\pi^{2n_1+n_2} n_1! n_2!} \int_{-\infty}^{\infty} \exp \left[ -\left( A + \frac{1}{2} \right) x_1^2 - \left( B + \frac{1}{2} \right) x_2^2, +2C x_1 x_2 + D x_1 + E x_2 \right] H_{n_1}(x_1) H_{n_2}(x_2) \, dx_1 \, dx_2.$$ (7)

We calculate this integral and obtain

$$P(n_1, n_2) = W(0, 0) \frac{1}{n_1! n_2!} \left| H^{(R)}_{n_1,n_2}(y_1, y_2) \right|^2,$$ (8)

with the probability to have no photon in any of the modes

$$P(0, 0) = \frac{2 \sqrt{A_1 B_1 - C_1^2}}{|(A+1/2)(B+1/2) - C_2|} \exp \left[ -\frac{A_1 E_1^2 + B_1 D_1^2 + 2C_1 D_1 E_1}{2(A_1 B_1 - C_1^2)} \right]$$

$$\times \exp \left[ \frac{(A + 1/2) E^2 + (B + 1/2) D^2 - 2 C D E}{2((A + 1/2)(B + 1/2) - C^2)} \right].$$ (9)
In Eq. (8), $H_{n_1,n_2}^{\{R\}}(y_1,y_2)$ denotes the two-dimensional Hermite polynomial. We use the generating function for the Hermite polynomials
\[
\exp \left( 2tx - x^2 \right) = \sum_{n=0}^{\infty} \frac{x^n}{n!} H_n(t),
\]
(10)
The matrix $R$ and the vector $\vec{y}$ in Eq. (8) are given by the relations
\[
R = \frac{1}{(A + 1/2)(B + 1/2) - C^2} \begin{pmatrix} (A - 1/2)(B + 1/2) - C^2 & -C \\ -C & (A + 1/2)(B - 1/2) - C^2 \end{pmatrix}
\]
(11)
and
\[
\vec{y} = \frac{1}{\sqrt{2}(A - 1/2)(B - 1/2) - C^2} \begin{pmatrix} D(B - 1/2) + CE \\ E(A - 1/2) + CD \end{pmatrix}.
\]
(12)
Equation (8) describes the generic distribution function of a pure two-mode squeezed coherent state.

It is worth noting that the two-dimensional Hermite polynomial $H_{n_1,n_2}^{\{R\}}(y_1,y_2)$ can be expressed in terms of usual Hermite polynomials or generalized Laguerre polynomials depending on the structure of the matrix $R$ [38].

Now we show that the photon distribution of a pure two-mode squeezed coherent light obtained in terms of the modulus squared of a two-dimensional Hermite polynomial can be expressed linearly in terms of Hermite polynomials of four variables. For this, we use the expression for the photon distribution that was obtained in [21] and is given in our notation by
\[
P(n_1, n_2) = P(0,0) \frac{H_{n_1,n_2,n_3,n_4}^{\{\tilde{R}\}}(y_1,y_2,y_3,y_4)}{n_1! n_2!}.
\]
(13)
Here the probability to have no photons is
\[
P(0,0) = \left[ \text{det} \left( \sigma + \frac{1}{2} E_4 \right) \right]^{-1/2} \exp \left[ -\langle \vec{Q} \rangle \left( 2\sigma + E_4 \right)^{-1} \langle \vec{Q} \rangle \right],
\]
(14)
where the elements of the quadrature dispersion $4 \times 4$ matrix $\sigma$ are given by formula (5). The matrix $\tilde{R}$ and the vector $\vec{y} = (y_1,y_2,y_3,y_4)$ are given by the expressions
\[
\tilde{R} = U^\dagger (E_4 - 2\sigma)^{-1} U^*,
\]
\[\vec{y} = 2U^T(E_4 - 2\sigma)^{-1}\langle \vec{Q} \rangle,\]  

with

\[U = \frac{1}{\sqrt{2}} \begin{pmatrix} -iE_2 & iE_2 \\ E_2 & E_2 \end{pmatrix}.\]  

If we insert the dispersion matrix \(\sigma\) (5), we arrive at the matrix \(\tilde{R}\)

\[\tilde{R} = \begin{pmatrix} r & 0 \\ 0 & r^* \end{pmatrix},\]  

with the 2 \(\times\) 2 matrix \(r\)

\[r = \frac{1}{(A + 1/2)(B + 1/2) - C^2} \begin{pmatrix} (A - 1/2)(B + 1/2) - C^2 & -C \\ -C & (A + 1/2)(B - 1/2) - C^2 \end{pmatrix}.\]  

We note that the matrix \(r\) is identical to the matrix \(R\) in Eq. (11). In this case, the argument \(\vec{y}\) of the four-dimensional Hermite polynomial may be split in

\[\vec{y} = \begin{pmatrix} \vec{Y} \\ \vec{Y}^* \end{pmatrix},\]  

where

\[\vec{Y} = \frac{1}{\sqrt{2}} \frac{1}{(A - 1/2)(B - 1/2) - C^2} \begin{pmatrix} D(B - 1/2) + CE \\ E(A - 1/2) + CD \end{pmatrix},\]

which is the same vector as in Eq. (12). Thus, the photon distribution function expressed in terms of modulus squared of Hermite polynomials of two variables may be also expressed linearly in terms of Hermite polynomials of four variables with equal pairs of indexes.
4 Photon Distribution Function for squeezed correlated states averaged over one mode

In experiments, it is the photon number in the one mode that is usually measured. In this section we will average the two-mode photon distribution function over one mode and obtain the probability distribution function for having $n$ photon in other mode.

For this purpose, first we write the density matrix in coordinate representation

$$\rho(x_1, x_2, x_1', x_2') = |N|^2 \exp \left(-Ax_1^2 + Dx_1 + 2Cx_1x_2 + EX_2 - A^*x_1'^2 - B^*x_2'^2 + 2C^*x_1'x_2' + D^*x_1' + E^*x_2'\right)$$

(21)

Taking $x_2$ equal to $x_2'$ and integrate over $x_2'$ we obtain the density matrix of one mode averaged over other mode

$$\rho(x_1, x_1') = \sqrt{\frac{(A_1B_1 - C_1^2)}{\pi B_1}} \exp \left(- \frac{B_1D_1^2 + A_1E_1^2 + 2C_1E_1D_1}{2(A_1B_1 - C_1^2)} + \frac{E_1^2}{2B_1}\right)$$

$$\times \exp \left(- \left(A_1 - \frac{C_1^2}{2B_1} + \frac{1}{2}\right)x_1^2 + - \left(A^*_1 - \frac{C^*_1}{2B_1} + \frac{1}{2}\right)x_1'^2 + 2x_1x_1'\frac{|C|}{2B_1} \right.$$

$$\left.+x_1 \left(D + \frac{CE_1}{B_1}\right) + x_1' \left(D^* + \frac{CE^*_1}{B_1}\right)\right).$$

(22)

In order to obtain averaged photon statistics we take the integral

$$P_n = \int \psi_n(x_1')\psi_n^*(x_1)\rho(x_1, x_1')d x_1 \ d x_1',$$

(23)

where $\psi_n(x_1, x_1')$ is wave function of Fock states and arrive at

$$P_n = \frac{P_0}{n!} \mathcal{H}_{nn}^{(R)}(y_1, y_2).$$

(24)

where

$$P_0 = 2\sqrt{\frac{A_1B_1 - C_1^2}{B_1(|a^2 - b^2|)}} \exp \left(- \frac{B_1D_1^2 + A_1E_1^2 + 2C_1E_1D_1}{2(A_1B_1 - C_1^2)} + \frac{E_1^2}{2B_1} + \frac{\text{Re}(a^*d) + b^2|d|^2}{2(|a|^2 - b^2)}\right),$$

$$a = A + \frac{1}{2} - \frac{C^2}{2B_1}, \quad b = \frac{|C|}{2B_1}, \quad d = D + \frac{CE_1}{B_1}, \quad f = A - \frac{1}{2} - \frac{C^2}{2B_1}.$$ 
We see that averaged over one mode photon number distribution function is expressed through Hermite polynomials of two variables with $R$ matrix equal to

$$R = \frac{1}{|a_1|^2 - b^2} \begin{pmatrix} |a|^2 - a^* - b^2 & -b \\ -b & |a|^2 - a - b^2 \end{pmatrix}$$

and arguments of Hermite polynomials equal to

$$y_1 = y_2^* = \frac{df^* + d^*b}{\sqrt{2(|a|^2 - b^2)}}.$$  

The photon number mean in the mode is determined by the quadrature dispersions and is equal to

$$\langle n_1 \rangle = \frac{1}{2} (\sigma_{p_1,p_1} + \sigma_{q_1,q_2} - 1)$$

$$= \frac{((1 - 2A_1)^2 + 4A_2^2)B_1 + 4(1 - A_1)C_1^2 - 8A_2C_1C_2 + 4A_1C_2^2}{8(A_1B_1 - C_1^2)}.$$  

The photon number dispersion is determined by the formula

$$\sigma_{n_1^2} = \frac{1}{2} \left( \sigma_{q_1,q_1}^2 + \sigma_{p_1,p_1}^2 + 2\sigma_{q_1,p_1}^2 - \frac{1}{2} \right).$$

It takes the following value for the squeezed correlated states of the photon modes

$$\sigma_{n_1^2} = \frac{1}{(A_1B_1 - C_1^2)^2} \left[ -\frac{1}{4} + \frac{1}{32}B_1^2 + \frac{1}{4}(A_1B_2 - C_1C_2)^2 
+ \frac{1}{2}(A_1^2B_1 + A_2(A_2B_1 - 2C_1C_2) + A_1(C_2^2 - C_1^2))^2 \right]$$

The Fano factor is the ratio of the photon dispersion and the photon number mean

$$F_{Fano} = \frac{\sigma_{n_1^2}}{\langle n_1 \rangle}.$$  

The Fano factor for the state under study reads

$$F_{Fano} = \frac{-8(A_1B_1 - C_1^2) + B_2^2 + 8A_1B_2 - C_1C_2 + 32(A_1^2B_1 + A_2(A_2B_1 - 2C_1C_2) + A_1(C_2^2 - C_1^2))^2}{4(A_1B_1 - C_1^2)[((1 - 2A_1)^2 + 4A_2^2)B_1 + 4(1 - A_1)C_1^2 - 8A_2C_1C_2 + 4A_1C_2^2]}.$$  

We see that the arguments of Hermite polynomial, matrix $R$, photon number mean, photon number dispersion and Fano factor are the functions of coefficients in formulae (1), determining the wave function of squeezed correlated states.
5 Entanglement in the two-mode squeezed correlated states

In this section we discuss entanglement phenomena in the two-mode squeezed correlated states. Entangled states are the states which are constructed as a superposition of states each of which has the wave function expressed as a product of wave functions depending on the different degrees of freedom. We will employ two different simple measures of entanglement appropriate for the Gaussian states. In [26] the following measure of entanglement was suggested

\[ E = \sigma_{q_1q_2}^2 + \sigma_{p_1p_2}^2 + \sigma_{q_1p_2}^2 + \sigma_{p_1q_2}^2. \]  

(31)

We employ the measure of entanglement (31) for evaluating the correlations between the photon modes appearing in the two-mode squeezed correlated states. Thus, one has the measure of entanglement of the modes in the form

\[ E_1 = \frac{C_1^2 + 4(A_2 C_1 - A_1 C_2)^2 + 4(B_2 C_1 - B_1 C_2)^2}{16(A_1 B_1 - C_1^2)^2} \]

\[ + \left[ A_2(B_2 C_1 - B_1 C_2) + C_1(C_1^2 + C_2^2) - A_1(B_1 C_1 + C_2(B_2 + C_2)) \right]^2 \]

\[ \frac{2}{(A_1 B_1 - C_1^2)^2}. \]  

(32)

In [27] another measure of entanglement was introduced as the distance between the system density matrix and the tensor product of the matrix partial traces over the subsystem degrees of freedom. For the Gaussian states, the measure of entanglement reads

\[ e_G = \frac{1}{4\sqrt{\det \sigma(t)}} + \frac{1}{4\sqrt{\det \tilde{\sigma}}} - \frac{2}{\sqrt{\det (\sigma(t) + \tilde{\sigma})}}, \]  

(33)

where we use the block notation for the quadrature dispersion matrix of the two-mode system

\[ \sigma(t)^{-1} = \begin{pmatrix} B & A \\ \alpha & \gamma \end{pmatrix}, \quad \tilde{\sigma} = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix}, \]

\[ \sigma_1^{-1} = \beta - \alpha \gamma^{-1} \alpha^T, \quad \sigma_2^{-1} = \gamma - \alpha^T \beta^{-1} \alpha. \]
Inserting the expressions for quadrature dispersions (5) into (33) one obtains the explicit formulas for the measure of entanglement. The expressions obtained for both measures of entanglement demonstrate that the intermode interaction creates nonzero mode correlations in the two-mode squeezed correlated states.

6 Symplectic tomogram of two-mode squeezed correlated states

In this section, we apply the symplectic tomography scheme to consideration of two-mode squeezed correlated states. In the tomographic representation the quantum state is reconstructed employing a probability distribution function called tomographic probability (or marginal distribution). Such a representation was introduced [28] in signal analysis. In [29] the Wigner function of a quantum state was expressed in terms of measurable experimentally tomographic probability. In [31] this idea was realised experimentally and the method of measuring quantum state of photon was called the optical tomography method. The optical tomography was used in [32] to measure squeezed mixed states of photon. In our work we discuss the case of tomographic probability distribution for two-mode system. We use symplectic tomography scheme [35, 36], which is the generalization of optical tomography scheme, and take into account the results of [39, 40]. The tomogram of the quantum state of photons is nonnegative probability distribution of the two quadratures $X_1$ and $X_2$ (tomographic quadratures). It is measured in the reference frame in the phase space of the quadrature components of the photons which is labeled by four real parameters. The quadrature $X_1$ can be interpreted as the eigenvalue of rotated and scaled quadrature operator of the first photon mode. The quadrature $X_2$ can be interpreted as the eigenvalue of rotated and scaled quadrature operator of the other photon mode. The two operators read

$$\hat{X}_1 = \mu_1 q_1 + \nu_1 p_1, \quad \hat{X}_2 = \mu_2 q_2 + \nu_2 p_2. \quad (34)$$
For $\mu = \cos \theta$, $\nu = \sin \theta$ (in the one-mode case), the tomogram $w(X_1, \cos \theta, \sin \theta)$ is the measurable probability distribution of the optical tomography procedure [29, 31], in which the measurable observable $\hat{X}_1$ takes the form of rotated photon quadrature operator

$$\hat{X}_{1\theta} = q_1 \cos \theta + p_1 \sin \theta. \quad (35)$$

Such photon quadrature can be measured by means of the homodyne photon detection scheme [31]. The density matrix of both photon modes determines the Wigner function of the system

$$W_{12}(q_1, q_2, p_1, p_2) = \int \rho_{12} \left( q_1 + \frac{u_1}{2}, q_2 + \frac{u_2}{2}, q_1 - \frac{u_1}{2}, q_2 - \frac{u_2}{2} \right) e^{-ip_1 u_1 - ip_2 u_2} du_1 du_2, \quad (36)$$

where the density matrix $\rho(x_1, x_2, x'_1, x'_2)$ is considered in the position representation. The coordinates $x_1, x'_1$ are used for the one photon mode and the coordinates $x_2, x'_2$ are used for other photon mode. The tomogram of the two-mode system is given in terms of the Wigner function by the relation

$$w_{12}(X_1, X_2, \mu_1, \mu_2, \nu_1, \nu_2) = \frac{1}{4\pi^2} \int W_{12}(q_1, q_2, p_1, p_2) \times \delta(X_1 - \mu_1 q_1 - \nu_1 p_1) \delta(X_2 - \mu_2 q_2 - \nu_2 p_2) dq_1 dq_2 dp_1 dp_2. \quad (37)$$

Here we use the following notation. The parameters $\mu_1$ and $\nu_1$ describe the rotated and scaled reference frame in the phase space of the photon quadratures of first mode. The parameters $\mu_2$ and $\nu_2$ describe the rotated and scaled reference frame in the phase space of the photon quadratures of second mode. The Wigner function of the two-mode system is determined by the tomogram due to relation, which is inverse of (37)

$$W_{12}(q_1, q_2, p_1, p_2) = \frac{1}{4\pi^2} \int w_{12}(X_1, X_2, \mu_1, \mu_2, \nu_1, \nu_2) \times \exp \left[ i(X_1 - \mu_1 q_1 - \nu_1 p_1 + X_2 - \mu_2 q_2 - \nu_2 p_2) \right] dX_1 dX_2 d\mu_1 d\mu_2 d\nu_1 d\nu_2. \quad (38)$$

One can show that the tomogram of the photon subsystem is related to the tomogram of complete system by the relation

$$w_1(X_1, \mu_1, \nu_1) = \int w_{12}(X_1, X_2, \mu_1, \mu_2, \nu_1, \nu_2) dX_2, \quad (39)$$
The tomogram of the two-mode system for Gaussian density matrix has the form of the standard two-dimensional Gaussian distribution

\[
w_G(X_1, X_2, \mu_1, \mu_2, \nu_1, \nu_2) = \frac{1}{2\pi \sqrt{\text{det} \sigma_X}} \exp \left[ -\frac{1}{2} (X - \langle X \rangle) \sigma_X^{-1} (X - \langle X \rangle) \right],
\]

(40)
determined by means and dispersions of the random variables \(X = (X_1, X_2)\). In (40) one has

\[
\langle X \rangle = \begin{pmatrix} \mu_1 \langle q_1 \rangle + \nu_1 \langle p_1 \rangle \\ \mu_2 \langle q_2 \rangle + \nu_2 \langle p_2 \rangle \end{pmatrix},
\]

(41)
where \(\langle q_1 \rangle, \langle p_1 \rangle\) are quadrature means of the first photon mode and \(\langle q_2 \rangle, \langle p_2 \rangle\) are quadrature means of other photon mode. Inserting the expressions for quadrature means (4) into (41) we obtain

\[
\langle X \rangle_1 = \frac{(B_1 D_1 + C_1 E_1)(\mu_1 - 2\nu_1 A_2)}{2(A_1 B_1 - C_1^2)} + \nu_1 \left( \frac{D_2 + \frac{C_2(E_1 A_1 + D_1 C_1)}{2(A_1 B_1 - C_1^2)}}{2(A_1 B_1 - C_1^2)} \right),
\]

\[
\langle X \rangle_2 = \frac{(A_1 E_1 + C_1 D_1)(\mu_2 - 2\nu_2 B_2)}{2(A_1 B_1 - C_1^2)} + \nu_2 \left( \frac{E_2 + \frac{C_2(D_1 B_1 + E_1 C_1)}{2(A_1 B_1 - C_1^2)}}{2(A_1 B_1 - C_1^2)} \right)
\]

(42)
The matrix elements of the symmetric dispersion matrix

\[
\sigma_X = \begin{pmatrix} \sigma_{X_1 X_1} & \sigma_{X_1 X_2} \\ \sigma_{X_1 X_2} & \sigma_{X_2 X_2} \end{pmatrix}
\]

(43)
are variances

\[
\sigma_{X_1}^2 = \mu_1^2 \sigma_{q_1}^2 + \nu_1^2 \sigma_{p_1}^2 + 2\mu_1 \nu_1 \sigma_{q_1 p_1}, \quad \sigma_{X_2}^2 = \mu_2^2 \sigma_{q_2}^2 + \nu_2^2 \sigma_{p_2}^2 + 2\mu_2 \nu_2 \sigma_{q_2 p_2}
\]

(44)
and covariance

\[
\sigma_{X_1 X_2} = \mu_1 \mu_2 \sigma_{q_1 q_2} + \nu_1 \nu_2 \sigma_{p_1 p_2} + \mu_1 \nu_2 \sigma_{q_1 p_2} + \mu_2 \nu_1 \sigma_{q_2 p_1}
\]

(45)
of the photon and phonon tomographic quadratures. Inserting the expressions for quadrature dispersions (5) into (44,45) we obtain

\[
\sigma_{X_1 X_2} = \frac{B_1(\mu_1 \mu_2 - 4\nu_1 \nu_2 A_2 C_2 + 2\mu_1 \nu_2 C_2)}{4(A_1 B_1 - C_1^2)} - \frac{C_1(\mu_1 \nu_2 B_2 - \nu_1 \nu_2 C_2^2)}{2(A_1 B_1 - C_1^2)}
\]

\[-\nu_1 \nu_2 \left( C_1 + \frac{A_1 C_2^2}{A_1 B_1 - C_1^2} \right)
\]

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\begin{align*}
\sigma_{X_1^2} &= \frac{B_1(\mu_1 - 2\nu_1 A_2)^2}{4(A_1 B_1 - C_1^2)} + \frac{\nu_1 C_1 C_2(\mu_1 - 2\nu_1 A_2)}{A_1 B_1 - C_1^2} + \nu_1^2 A_1 \left(1 + \frac{C_2}{A_1 B_1 - C_1^2}\right) \\
\sigma_{X_2^2} &= \frac{A_1(\mu_2 - 2\nu_2 B_2)^2}{4(A_1 B_1 - C_1^2)} + \frac{\nu_2 C_1 C_2(\mu_2 - 2\nu_2 B_2)}{A_1 B_1 - C_1^2} + \nu_2^2 B_1 \left(1 + \frac{C_2}{A_1 B_1 - C_1^2}\right)
\end{align*}
(46)

Inserting (42,46) into (40) the tomogram of two-mode squeezed correlated states can be obtained in explicit form.

\section{Two-mode even and odd coherent states}

In this section we will consider the two-mode even and odd coherent states. We define the two-mode even and odd coherent states as the simplest superposition of two-mode coherent states

\[ |\alpha_1,\alpha_2\rangle_\pm = N_\pm (|\alpha_1,\alpha_2\rangle \pm |-\alpha_1,-\alpha_2\rangle), \]
(47)

here \( \alpha_1 \) and \( \alpha_2 \) are complex numbers. For even coherent states one has to take plus in superposition, and for odd coherent states one has to take minus. The normalization constants for two-mode even and odd coherent states are

\[ N_+ = \frac{\exp \left(\frac{|\alpha_1|^2}{2} + \frac{|\alpha_2|^2}{2}\right)}{2\sqrt{\cosh (|\alpha_1|^2 + |\alpha_2|^2)}}, \quad N_- = \frac{\exp \left(\frac{|\alpha_1|^2}{2} + \frac{|\alpha_2|^2}{2}\right)}{2\sqrt{\sinh (|\alpha_1|^2 + |\alpha_2|^2)}}. \]
(48)

We can write the wave functions of even and odd coherent states in following explicit forms

\[ |\alpha_1,\alpha_2\rangle_+ = \frac{N_+}{\sqrt{\pi}} \exp \left(-\frac{q_1^2}{2} - \frac{q_2^2}{2} - \frac{|\alpha_1|^2}{2} - \frac{|\alpha_2|^2}{2} - \frac{\alpha_1^2}{2} - \frac{\alpha_2^2}{2}\right) \cosh \left(\sqrt{2} x_1 \alpha_1 + \sqrt{2} x_2 \alpha_2\right) \]
\[ |\alpha_1,\alpha_2\rangle_- = \frac{N_-}{\sqrt{\pi}} \exp \left(-\frac{q_1^2}{2} - \frac{q_2^2}{2} - \frac{|\alpha_1|^2}{2} - \frac{|\alpha_2|^2}{2} - \frac{\alpha_1^2}{2} - \frac{\alpha_2^2}{2}\right) \sinh \left(\sqrt{2} x_1 \alpha_1 + \sqrt{2} x_2 \alpha_2\right). \]
(49)

The Wigner functions for even and odd coherent states are

\[ W_{\alpha_1,\alpha_2\pm} = 4|N_\pm|^2 \exp \left(-q_1^2 - q_2^2 - p_1^2 - p_2^2\right) \]
\[
\times \left\{ \exp \left( -2|\alpha_1|^2 - 2|\alpha_2|^2 \right) \cosh \left[ 2\sqrt{2} \left( \text{Re}[\alpha_1]q_1 + \text{Re}[\alpha_2]q_2 + \text{Im}[\alpha_1]p_1 + \text{Im}[\alpha_2]p_2 \right) \right] \\
\pm \cos \left[ 2\sqrt{2} \left( \text{Im}[\alpha_1]q_1 + \text{Im}[\alpha_2]q_2 - \text{Re}[\alpha_1]p_1 - \text{Re}[\alpha_2]p_2 \right) \right] \right\}.
\]

One can see that the Wigner functions for even and odd coherent states are a sum of four gaussians. The probabilities of finding \( n \) photons in the first mode and \( n \) photons in the second mode are

\[
P_+(n_1, n_2) = \frac{|\alpha_1|^{2n_1}|\alpha_2|^{2n_2}}{n_1!n_2! \cosh (|\alpha_1|^2 + |\alpha_2|^2)}, \quad n_1 + n_2 = 2k,
\]

\[
P_-(n_1, n_2) = \frac{|\alpha_1|^{2n_1}|\alpha_2|^{2n_2}}{n_1!n_2! \sinh (|\alpha_1|^2 + |\alpha_2|^2)}, \quad n_1 + n_2 = 2k + 1
\]

We see that in the present case of two-mode even and odd coherent states we cannot factorize their photon distribution functions due to the presence of the nonfactorizable \( \cosh(|\alpha_1|^2 + |\alpha_2|^2) \) and \( \sinh(|\alpha_1|^2 + |\alpha_2|^2) \). This fact implies the phenomenon of statistical dependences of different modes of these states one on each other. These probabilities are equal to zero, if the sum of \( n_1 \) and \( n_2 \) is an odd number for even states, or if this sum is an even number in the case of odd states. After the averaging over one mode we obtain the probability to have \( n \) photon in other mode in the form

\[
\tilde{P}_+(n_1) = \sum_{n_2=0}^{\infty} P_+(n_1, n_2) = \frac{|\alpha_1|^{2n_1}e^{\alpha_2^2}}{n_1! \cosh (|\alpha_1|^2 + |\alpha_2|^2)}
\]

\[
\tilde{P}_-(n_1) = \sum_{n_2=0}^{\infty} P_+(n_1, n_2) = \frac{|\alpha_1|^{2n_1}e^{\alpha_2^2}}{n_1! \sinh (|\alpha_1|^2 + |\alpha_2|^2)}
\]

The dispersions of quadrature components in the two-mode even coherent states are

\[
\sigma_{q_1^2} = |\alpha_1|^2 \tanh(|\alpha_1|^2 + |\alpha_2|^2) + \text{Re}[\alpha_1^2] + \frac{1}{2},
\]

\[
\sigma_{q_2^2} = |\alpha_2|^2 \tanh(|\alpha_1|^2 + |\alpha_2|^2) + \text{Re}[\alpha_2^2] + \frac{1}{2},
\]

\[
\sigma_{q_1q_2} = \left( \tanh(|\alpha_1|^2 + |\alpha_2|^2) + 1 \right) \text{Re}[\alpha_1\alpha_2],
\]

\[
\sigma_{p_1^2} = |\alpha_1|^2 \tanh(|\alpha_1|^2 + |\alpha_2|^2) - \text{Re}[\alpha_1^2] + \frac{1}{2},
\]

\[
\sigma_{p_2^2} = |\alpha_2|^2 \tanh(|\alpha_1|^2 + |\alpha_2|^2) - \text{Re}[\alpha_2^2] + \frac{1}{2}.
\]
\[
\sigma_{p_1, p_2} = \left( \tanh(|\alpha_1|^2 + |\alpha_2|^2) - 1 \right) \text{Re}[\alpha_1 \alpha_2], \\
\sigma_{q_1, p_1} = \text{Im}[\alpha_1^2], \\
\sigma_{q_2, p_2} = \text{Im}[\alpha_2^2], \\
\sigma_{q_1, p_2} = \sigma_{q_2, p_1} = \text{Im}[\alpha_1 \alpha_2].
\]

The dispersions of quadrature components in the two-mode odd coherent states are

\[
\sigma_{q_1}^2 = |\alpha_1|^2 \coth(|\alpha_1|^2 + |\alpha_2|^2) + \text{Re}[\alpha_1^2] + \frac{1}{2}, \\
\sigma_{q_2}^2 = |\alpha_2|^2 \coth(|\alpha_1|^2 + |\alpha_2|^2) - \text{Re}[\alpha_2^2] + \frac{1}{2}, \\
\sigma_{q_1, q_2} = \left( \coth(|\alpha_1|^2 + |\alpha_2|^2) + 1 \right) \text{Re}[\alpha_1 \alpha_2], \\
\sigma_{p_1}^2 = |\alpha_1|^2 \coth(|\alpha_1|^2 + |\alpha_2|^2) - \text{Re}[\alpha_1^2] + \frac{1}{2}, \\
\sigma_{p_2}^2 = |\alpha_2|^2 \coth(|\alpha_1|^2 + |\alpha_2|^2) - \text{Re}[\alpha_2^2] + \frac{1}{2}, \\
\sigma_{p_1, p_2} = \left( \coth(|\alpha_1|^2 + |\alpha_2|^2) - 1 \right) \text{Re}[\alpha_1 \alpha_2], \\
\sigma_{q_1, p_1} = \text{Im}[\alpha_1^2], \\
\sigma_{q_2, p_2} = \text{Im}[\alpha_2^2], \\
\sigma_{q_1, p_2} = \sigma_{q_2, p_1} = \text{Im}[\alpha_1 \alpha_2].
\]

We see that the formulae for dispersions of quadratures in odd coherent states can be obtained from formulae for dispersions of quadratures in even coherent states by changing the function tanh by the function coth. The photon number means for even and coherent state are

\[
\langle n_1 \rangle_+ = |\alpha_1|^2 \tanh(|\alpha_1|^2 + |\alpha_2|^2), \\
\langle n_1 \rangle_- = |\alpha_1|^2 \coth(|\alpha_1|^2 + |\alpha_2|^2).
\]

The photon number dispersions in the two-mode odd and even coherent states are

\[
\sigma_{n_1}^2 = |\alpha_1|^4 \text{sech}^2(|\alpha_2|^2 + |\alpha_1|^2) + |\alpha_1|^2 \tanh(|\alpha_1|^2 + |\alpha_2|^2),
\]
\[\sigma_{n^2} = |\alpha_1|^2 \coth(|\alpha_1|^2 + |\alpha_2|^2) - |\alpha_1|^4 \coth^2(|\alpha_2|^2 + |\alpha_1|^2).\]

We can calculate the Fano factor in even and odd coherent states

\[F_+ = 1 + \frac{|\alpha_1|^2 \text{sech}^2(|\alpha_1|^2 + |\alpha_2|^2)}{\tanh(|\alpha_1|^2 + |\alpha_2|^2)},\]

\[F_- = 1 - \frac{|\alpha_1|^2 \text{cosech}^2(|\alpha_1|^2 + |\alpha_2|^2)}{\coth(|\alpha_1|^2 + |\alpha_2|^2)}.\]

The Fano factor for even coherent states is more than unity for all values of \(\alpha_1, \alpha_2\), so the photon distribution function in even coherent states is always super-Poissonian. The Fano factor for odd coherent states is less than unity for all values of \(\alpha_1, \alpha_2\), so the photon distribution function in odd coherent states is always sub-Poissonian. The measures of entanglement in even and odd coherent states exhibited the appearance of correlations between the modes and are of the form

\[E_+ = 2\text{Re}^2|\alpha_1, \alpha_2|\text{sech}^2(|\alpha_1|^2 + |\alpha_2|^2) + 2\text{Im}^2|\alpha_1, \alpha_2|,\]

\[E_- = 2\text{Re}^2|\alpha_1, \alpha_2|\text{cosech}^2(|\alpha_1|^2 + |\alpha_2|^2) + 2\text{Im}^2|\alpha_1, \alpha_2|.\]

Let us apply symplectic tomography scheme to the even and odd coherent states and obtain the tomogram of two-mode even and odd coherent states in explicit form

\[\omega_{0\pm} = 8|N_{\pm}|^2 \exp\left(-|\alpha_1|^2 - |\alpha_2|^2\right) \frac{\pi \mu_1 \mu_2}{\sqrt{\mu_1^2 + \nu_1^2} \sqrt{\mu_2^2 + \nu_2^2}} \exp \left(\frac{x_1^2}{\mu_1^2} - \frac{x_2^2}{\mu_2^2}\right) \times\]

\[\times \exp \left[\frac{1}{\mu_1^2 + \nu_1^2} \left(\frac{\nu_1^2 x_1^2}{\mu_1^2} + 2\nu_1^2 \text{Re}^2 \alpha_1 + 2\mu_1^2 \text{Im}^2 \alpha_1 - 4\mu_1 \nu_1 \text{Re} \alpha_1 \text{Im} \alpha_1\right) + \right.\]

\[+ \frac{1}{\mu_2^2 + \nu_2^2} \left(\frac{\nu_2^2 x_2^2}{\mu_2^2} + 2\nu_2^2 \text{Re}^2 \alpha_2 + 2\mu_2^2 \text{Im}^2 \alpha_2 - 4\mu_2 \nu_2 \text{Re} \alpha_2 \text{Im} \alpha_2\right)\] \times

\[\times \cosh \left[2\sqrt{2} \left(\frac{x_1}{\mu_1} \text{Re} \alpha_1 + \frac{x_2}{\mu_2} \text{Re} \alpha_2 + \frac{x_1 \nu_1}{\mu_1} \frac{1}{\mu_1^2 + \nu_1^2} (\mu_1 \text{Im} \alpha_1 - \nu_1 \text{Re} \alpha_1) \right.\right.\]

\[+ \frac{x_2 \nu_2}{\mu_2^2 + \nu_2^2} \left(\mu_2 \text{Im} \alpha_2 - \nu_2 \text{Re} \alpha_2\right)\] \pm


\[ \pm 8 |N_\pm|^2 \frac{\pi \mu_1 \mu_2}{\sqrt{\mu_1^2 + \nu_1^2}} \sqrt{\mu_2^2 + \nu_2^2} \exp \left( -\frac{x_1^2}{\mu_1^2} - \frac{x_2^2}{\mu_2^2} \right) \times \]

\[ \times \exp \left[ \frac{1}{\mu_1^2 + \nu_1^2} \left( \frac{\nu_1^2 x_1^2}{\mu_1^2} - 2\mu_1^2 \alpha_1 - 2\nu_1^2 \alpha_1 - 4\mu_1 \nu_1 \alpha_1 \alpha_1 \right) + \right. \]

\[ + \frac{1}{\mu_2^2 + \nu_2^2} \left( \frac{\nu_2^2 x_2^2}{\mu_2^2} - 2\mu_2^2 \alpha_2 - 2\nu_2^2 \alpha_2 - 4\mu_2 \nu_2 \alpha_2 \alpha_2 \right) \times \]

\[ \times \cos \left[ 2\sqrt{2} \left( \frac{x_1 \alpha_1}{\mu_1} + \frac{x_2 \alpha_2}{\mu_2} - \frac{x_1 \nu_1}{\mu_1^2 + \nu_1^2} (\mu_1 \alpha_1 + \nu_1 \alpha_1) \right. \]

\[ - \frac{x_2 \nu_2}{\mu_2} \left. \left( \mu_2 \alpha_2 + \nu_2 \alpha_2 \right) \right] \]

These tomograms are the images of the nonclassical even and odd coherent states in the probability representation of quantum mechanics. We can use tomograms for describing even and odd coherent states instead of using Wigner functions or wave function of the states.

### 8 Conclusions

We discuss photon-number probability distribution function for two-mode even and odd coherent states and two-mode squeezed correlated states and after averaging over one mode we obtain the probability of having \( n \) photons in other mode for the states under study. We calculated means, dispersions of quadrature components and of photon numbers in the modes, Fano factors and tomograms within the framework of symplectic tomography scheme for two-mode even and odd coherent states and two-mode squeezed correlated state. We saw that for the two-mode even coherent states the photon statistics is always super-poissonian and for the two-mode odd coherent states it is always sub-poissonian. We evaluated the measure of entanglement employing two different methods. The nonzero measure of entanglement shows the statistical dependence between the modes in the states under study.
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