Modal parameters identification of light rail bridges

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Abstract. One of important aspect in Structural Health Monitoring System (SHMS) is the initial condition of the structure. This paper studies the initial condition of light rail train (LRT) bridges before officially operated. There are four bridges to be evaluated, consists of three simple supported bridges and one continuous bridge. Data measurement was using tri-axial accelerometer EZDYN with a train-set as the dynamic loading source. The modal parameters are obtained by means of operating modal analysis (OMA) in time domain. Three methods are simulated, that are ARMA, ITD and SSI. The data processing applied shows that all three methods give satisfactory results that confirm each other, for both natural frequencies and vibration modes.

1. Introduction
To assure the best performance of structure during its life service time, the health of structure should be monitored continuously. Structure can undergo low performance due to natural aging, lack of maintenance or changing of load. Structural health monitoring (SHM) is intended to detect the damage of material or geometric properties of structures.

One of important aspect in SHM is the initial condition of the structure. The initial condition is taken as a datum of considered health structure. Take an advantage of evaluation of modal parameters as the structure constructed, the structural design can be evaluated, as well. It was done by comparing the actual modal parameter from field measurements to modal parameters from structural design.

This paper discusses the investigation of modal parameters of a new constructed light rail bridges by means of dynamic vibration. The investigation is carried out to yield the initial modal parameter of the bridge before it officially operates. Periodic evaluation of modal parameter will give an early detection of damage and prevent a bigger loss. For completeness, numerical model using finite element method is performed and compared to the field measurement results.

The modal parameters identification of bridge has been reported in some papers. Vu et al. [1] have reported the identification of modal parameters of a bridge before and after rehabilitation. Modal parameters were extracted using Operating Modal Analysis (OMA) in four methods, that are the pick picking method (PP), the Least Square Complex Exponential method (LSCE), the Autoregressive method (AR) and the Autoregressive Moving Average method (ARMA). Cabboi et al. [2] conducted FDD and covariance-based SSI schemes to identify the modal parameters of a centenary iron-arch bridge. The paper discussed natural frequencies, vibration modes and furthermore the possible damage or structural anomaly. OMA method has been applied in general structure as reported in Ediansjah et al. [3], that evaluate modal parameters in platform structure.
2. Operating modal analysis

Operating Modal Analysis (OMA) is one of promising methods to obtain structural modal parameters. The method requires no input excitation and is applicable to wide structures. OMA is highly applicable in evaluating the real structure compared to FRF methods that need certain and measurable laboratory excitation. Some methods of OMA are based on time domain and others on frequency domain. This paper focuses on time domain OMA, that are Auto regressive moving average (ARMA), Ibrahim Time Domain (ITD) and Stochastic Subspace Identification (SSI). Every method intends to gather information related to physical behavior of structural systems from correlation functions from field measurements [4,5].

2.1. Auto regressive moving average (ARMA).

ARMA method identifies a system and predicts the output based on previous input and output. The complete discussion about ARMA can be found in Modal Analysis book by He and Fu [5]. Only short explanation is presented below.

The ARMA(n,m) model is characterized by

\[ x_t = \sum_{r=1}^{n} \phi_r x_{t-r} - \sum_{s=1}^{m} \theta_s a_{t-s} + a_t, \quad n > m \]  

The order of auto regressive model number of previous responses denoted by n. m denotes the order of moving average. \( x_t \) is the response that correlates with previous response \( x_{t-1}, x_{t-2}, \cdots x_{t-n} \), input of time \( t \), \( a_t \), and its previous input \( a_{t-1}, a_{t-2}, \cdots, a_{t-m} \). At the time \( t \), the response \( x_t \) is decomposed into two parts: the deterministic and dependent part \( \sum_{r=1}^{n} \phi_r x_{t-r} - \sum_{s=1}^{m} \theta_s a_{t-s} \) that happened in the past; and random and independent part, \( a_t \), that represent a current input. \( \phi_r \) denotes auto regressive coefficients and \( \theta_s \) denotes moving average coefficients.

The ARMA(n,m) model for N DOF system follows

\[ x_t = \{ -x_{t-1}, \cdots, -x_{t-2N}, a_{t-1}, \cdots, a_{t-2N+1} \} \begin{bmatrix} \phi_1 \\ \vdots \\ \phi_{2N} \\ \theta_1 \\ \vdots \\ \theta_{2N-1} \end{bmatrix} \]  

The transfer function for N-DOF system is

\[ \alpha_j(s) = \frac{x_j(s)}{a_j(s)} = \frac{b_0 + b_1 s + \cdots + b_{2N-2N-2} s^{2N-2}}{a_0 + a_1 s + \cdots + a_{2N} s^N} \]  

Applying z-transform to transfer function results

\[ \alpha_j(z) = \frac{x_j(z)}{a_j(z)} = \frac{\theta_0 + \theta_1 z^{-1} + \cdots + \theta_{2N-1} z^{-2N+1}}{1 + \phi_1 z^{-1} + \cdots + \phi_{2N} z^{-2N}} \]  

Then, solving the characteristic equation of form

\[ 1 + \phi_1 z^{-1} + \cdots + \phi_{2N} z^{-2N} = 0 \]  

And taking relationship between z-transform and Laplace transform results

\[ z_r = e^{s_L} \]
Then the natural frequency, \( \omega \) and damping ratio, \( \xi \), of the \( r \)-th mode can be obtained from

\[
\omega_r = \frac{1}{\Delta} \sqrt{z_r^* z_r^r} \quad \text{and} \quad \xi_r = \frac{\ln(z_r^* z_r^r)}{2\omega_r \Delta}
\]

where \( \Delta \) is sampling time resolution. Order of ARMA model takes a significant role in estimating the modal parameter [5].

### 2.2. Ibrahim Time Domain (ITD)

ITD method obtains modal parameter using free vibration of displacement, velocity or acceleration responses. The decaying free vibration is obtained by sampling the displacement data with a selected time interval. Brief description of the scheme is displayed here, the comprehensive explanation can be found in He and Fu [5] and some papers [6,7,8].

From the dynamic equation

\[
M \ddot{x} + C \dot{x} + K x = F(t),
\]

define a new vector

\[
y = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}.
\]

Rewrite Equation (8)

\[
\dot{Y} = AY \quad \text{and} \quad A = \dot{Y} Y^{-1}
\]

where

\[
Y = \begin{bmatrix} x(t_1) & x(t_2) & \ldots & x(t_{2N}) \\ \dot{x}(t_1) & \dot{x}(t_2) & \ldots & \dot{x}(t_{2N}) \end{bmatrix} ; \quad \dot{Y} = \begin{bmatrix} \dot{x}(t_1) & \dot{x}(t_2) & \ldots & \dot{x}(t_{2N}) \\ \ddot{x}(t_1) & \ddot{x}(t_2) & \ldots & \ddot{x}(t_{2N}) \end{bmatrix}.
\]

And modal parameters can be found from Equation (11) and (12)

\[
A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}
\]

### 2.3. Stochastic Subspace Identification (SSI)

Simple description of SSI method is displayed here. Complete explanation can be found in Overschee and De Moor [9]. Consider a dynamic system under the stochastic random excitation or random noises in state space form

\[
z(k + 1) = A z(k) + w(k)
\]

\[
y(k + 1) = C z(k) + v(k)
\]

\( w(k) \) and \( v(k) \) are white noises vector representing the process and measured noises, respectively. \( w(k) \) and \( v(k) \) are statistically uncorrelated. The cross-correlation function is determined using Equation (13)

\[
R(k) = E[y(k + m)y(m)^T] = CA^{k+m}E[z(m + 1)y(m)^T] = CA^{k+m}G
\]

where \( G = E[z(m + 1)y(m)^T] \) Then, block Hankel matrix can be constructed using cross correlation matrix
\[ H_{n_1,n_2} = \begin{bmatrix} R_1 & \cdots & R_{n_2} \\ \vdots & \ddots & \vdots \\ R_{n_1} & \cdots & R_{n_1+n_2-1} \end{bmatrix} \begin{bmatrix} CG & \cdots & CA^{n_2-1}G \\ \vdots & \ddots & \vdots \\ CA^{n_1-1}G & \cdots & CA^{n_1+n_2-2}G \end{bmatrix} = O_{n_1}B_{n_2} \quad (15) \]

where \( O_{n_1} = \begin{bmatrix} C \\ \vdots \\ CA^{n_1-1} \end{bmatrix} \) is observability matrix and \( B_{n_2} = \begin{bmatrix} G & \cdots & A^{n_2-1}G \end{bmatrix} \) is extended controllability matrix.

Multiplying weighting matrices, \( W_1 \) and \( W_2 \), to Hankel matrix and decomposing it will form

\[ W_1H_{n_1,n_2}W_2 = [U_1 \quad U_2] \begin{bmatrix} S_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} = U_1^T S_1 V_1^T \quad (16) \]

Equilibrating (3) and (4) and decomposing into \( W_1 O_{n_1} \) and \( B_{n_2} W_2 \) results possible solution

\[ O_{n_1} = W_1^{-1} U_1 S_1^{1/2} \quad (17) \]

Then the matrices A and C can be obtained using (3) and known observability matrix in (5)

\[ O_{n_1}^{-1} = O_{n_1}^{-1}A \quad (18) \]

where \( O_{n_1}^{-1} = \begin{bmatrix} CA \\ \vdots \\ CA^{n_1-1} \end{bmatrix} \) and \( O_{n_1} = \begin{bmatrix} C \\ \vdots \\ CA^{n_1-2} \end{bmatrix} \)

The system matrix \( C \) is the first row of \( O_{n_1}^{-1} \).

SSI can be determined as balanced realization (BR) if no weighting matrices are employed. Otherwise, by maximizing the correlation between the measured time history data the weighting matrices governs

\[ W_1 = (L^+)^{-1} \quad \text{and} \quad W_2 = (L^-)^{-1} \quad (19) \]

where

\[ R^+ = \begin{bmatrix} R_0 & \cdots & R_{n_1-1} \\ \vdots & \ddots & \vdots \\ R_{n_1-1} & \cdots & R_0 \end{bmatrix} = L^+ L^{+T} \quad \text{and} \quad R^- = \begin{bmatrix} R_0 & \cdots & R_{n_2-1} \\ \vdots & \ddots & \vdots \\ R_{n_2-1} & \cdots & R_0 \end{bmatrix} = L^- L^{-T} \quad (20) \]

3. Case studies
Four light rail bridges are taken as study cases. Three bridges are simply supported bridges and the other is continuous and curved bridge. The length of bridges is displayed at Table 1.

| Type of bridge | Section | Length [m] |
|----------------|---------|------------|
| PC30M          | Simple support | PC-I Girder | 30 |
| PC40M          | Simple support | PC-I Girder | 40 |
| SB60M          | Simple support | Composite Steel Box Girder | 60 |
| SB60L          | Curved-continuous | Composite Steel Box Girder | 60 |
Data acquisition was using 6 units digital accelerometer EZDYN as shown in Figure 1, each unit consists of tri-axial accelerometer transducer and built-in data logger. Acceleration data was time history that has been recorded offline with time synchronized to all sensors. Time synchronizing is important to guarantee that certain responses from 18 sensors relate to the same certain time. Sampling rate is set to 100Hz with vary time duration depends on the situation on site.

Figure 1. Tri-axial accelerometer EDZYN

Accelerometer is placed in ¼, ½ and ¾ bridge span, as displayed in Figure 2, at the bottom side of bridge deck as shown in Figure 3.

Figure 2. Typical accelerometer placements along longitudinal span.

Figure 3. Typical bridge section of I-girder and composite steel girder
Dynamic vibration of bridge sourced from running train of 51860mm length, 2650 width, 12 ton axle load, 100ton of net weight (without payload) and 25 km/h speed. Dimension and loading set of train is displayed in Figure 4.

![Train dimension and loading set](image)

**Figure 4.** Train dimension and loading set

### 4. Discussion

Raw measurement data from accelerometer is processed to obtain responses: acceleration, velocity and displacement from each channel. Typical responses in time domain and frequency domain are shown in Figure 5 - Figure 8, for four bridges being evaluated. As an example, the accelerometer BD records acceleration in z-direction (vertical). The figure informs the maximum responses (MAX), peak to peak (P2P) and root mean square (RMS) of each time domain responses; while frequency domain displays maximum responses and its corresponding frequency.

Figure 5 shows time history response of 30m bridge with simple support (PC30M). The maximum acceleration, velocity and displacement are 959mm/s², 21.73mm/s and 3.033mm, respectively.

![Figure 5](image)

**Figure 5.** Typical output of structural responses of bridge PC30M: displacement, velocity and acceleration in time domain and frequency domain

The dynamic responses, for example the Figure 5, are further processed using Operational Modal Analysis to extract the modal parameters. For comparison, OMA is done in 3-time domain methods that are ARMA, ITD and SSI. Parameters to be obtained are structural natural frequencies, mode shapes and mass participation ratio.

Table 2 shows the natural frequency estimation using several data sets of 30m simply supported bridge (PC30M). Three methods of OMA (ARMA, ITS and SSI) is employed to obtain estimated natural frequency from each data set. In P30M bridge, the ARMA method gives lower value compare to two
other methods. The estimated natural frequency is taken from average natural frequency or three analysis methods.

### Table 2. Natural frequency estimation of PC-I girder bridge PC30M

| Data set | ARMA | ITD  | SSI  |
|----------|------|------|------|
| 0        | 3.621| 3.775| 3.708|
| 1        | 3.519| 3.798| 3.725|
| 2        | 3.541| 3.723| 3.784|
| 3        | 3.546| 3.722| 3.785|
| 4        | 3.245| 3.22  | 3.297|
| 5        | 3.499| 3.88  | 3.625|

| Average Natural Frequency [Hz] | 3.495 | 3.686 | 3.654 |
| Estimated Natural Frequency [Hz] | ± 3.61 |

Measurement of 40m simply supported bridge gives maximum acceleration 450mm/s², velocity 9.8mm/s and displacement 1.5313mm, as shown in Figure 6. The estimated natural frequency is 2.66Hz, evaluated from 2 data sets as shown in Table 3. In this case, the SSI method gives the lowest natural frequency.

![Figure 6](image.png)

**Figure 6.** Typical output of structural responses of bridge girder PC40M: displacement, velocity and acceleration in time domain and frequency domain

### Table 3. Natural frequency estimation of bridge girder PC40M

| Data set | ARMA | ITD  | SSI  |
|----------|------|------|------|
| 1        | 2.813| 2.818| 2.378|
| 2        | 2.822| 2.826| 2.329|

| Average Natural Frequency [Hz] | 2.818 | 2.822 | 2.354 |
| Estimated Natural Frequency [Hz] | ± 2.66 |

Estimated natural frequency of SB60M (simply supported, 60m length composite steel box) bridge is 2.14Hz, as shown in Table 4. The SSI method results the lowest natural frequency, compared to other method. Time history responses in Figure 7 displays the maximum acceleration, velocity and displacement are 1813 mm/s², 60.061m/s and 7.3577mm respectively.
Figure 7. Typical output of structural responses of bridge girder SB60M: displacement, velocity and acceleration in time domain and frequency domain.

Table 4. Natural frequency estimation of bridge girder SB60M

| Data set | ARMA   | ITD    | SSI    |
|----------|--------|--------|--------|
| 1        | 2.178  | 2.081  | 1.9911 |
| 2        | 2.162  | 2.164  | 2.1076 |
| 3        | 2.150  | 2.1507 | 2.2718 |

Average Natural Frequency [Hz]: 2.163
Estimated Natural Frequency [Hz]: ±2.14

Structural response of 60m steel composite with continuous support (SB60L) bridge is shown in Figure 8. Maximum responses are 1415mm/s² acceleration, 51.6719mm/s velocity and 7.3268mm displacement. The estimated natural frequency of SB60L is 1.72Hz, as shown in Table 5.

Figure 8. Typical output of structural responses of bridge girder SB60L: displacement, velocity and acceleration in time domain and frequency domain.
Table 5. Natural frequency estimation of bridge girder SB60L

| Data set | ARMA | ITD | SSI |
|----------|------|-----|-----|
| 1        | 1.718| 1.718| 1.715|

Estimated Natural Frequency [Hz] ± 1.72

Mode shape extracted from measurement data meets the mode shape of finite element analysis. For example, the PC30M case shown in Figure 9. At natural frequency 3.52Hz, finite element model gives torsional mode (Figure 9a) and the measurement with ARMA (Figure 9b) yields the torsional mode, as well.

![Mode shape from finite element model](image1)

![Mode shape from measurement](image2)

**Figure 9. Mode shape of simple supported bridge 30m**

5. Conclusions

Some remarks in this contribution are:

- The three OMA methods, namely ARMA, ITD and SSI gives convergence results one to another,
- The mode shape from measurement meet the mode shape from finite element analysis,
- The estimated natural frequencies meet the natural frequencies from finite element analysis

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