Conservative estimates of the mass of the neutrino from cosmology

C Zunckel and P G Ferreira

Astrophysics Physics, University of Oxford, Denys Wilkinson Building, Keble Road, Oxford OX1 3RH, UK
E-mail: clz@astro.ox.ac.uk and p.ferreira1@physics.ox.ac.uk

Received 10 April 2007
Accepted 11 July 2007
Published 3 August 2007

Abstract. A range of experimental results point to the existence of a massive neutrino. The recent high precision measurements of the cosmic microwave background and the large scale surveys of galaxies can be used to place an upper bound on this mass. In this paper we perform a thorough analysis of all assumptions that go into obtaining a credible limit on $\sum m_\nu$. In particular we explore the impact of extending parameter space beyond the current standard cosmological model, the importance of priors and the uncertainties due to biasing in large scale structure. We find that the mass constraints are independent of the choice of parametrization as well as the inclusion of spatial curvature. The results of including the possibility of dark energy and tensor perturbations are shown to depend critically on the datasets used. The difference between an upper bound of 2.2 eV, assuming generic initial conditions, and an upper bound of 0.63 eV, assuming adiabaticity and a galaxy bias of 1, demonstrate the dependence of such a constraint on the assumptions in the analysis.

Keywords: dark matter, cosmological neutrinos

ArXiv ePrint: astro-ph/0610597
1. Introduction

The neutrino is an integral component of the standard model of particle physics. Until recently it was assumed to be massless. With the new advances in non-accelerator particle physics there is now definitive evidence that this cannot be so: neutrinos must have mass. The first signatures of their masses were observed flavour oscillations in atmospheric and solar neutrinos and have now been verified in accelerator and nuclear reactor sources [1]. When these observations are interpreted within the three-neutrino-type scenario of the standard model, their masses are required to be of the order of the measured mass differences, leading to two possible mass scales; $\delta m_{\text{atm}}^2 \simeq 3 \times 10^{-3} \text{eV}^2$ and, in conjunction with the nuclear reactor experiment results from KamLAND, $\delta m_{\text{sol}}^2 \simeq 5 \times 10^{-5} \text{eV}^2$. These mass scales can be accommodated by a model in which at least two eigenstates have mass. Alternatively, if their true masses exceed 0.1 eV then these results indicate that all three species are nearly degenerate. Tritium decay measurements have also been able to place an upper limit on the electron neutrino mass of 2.3 eV at the 95% confidence level (CL) [2]. Although difficult to reconcile with the rest of the data, it is interesting to note the result from the Los Alamos Liquid Scintillator Neutrino Detector (LSND) which implies a lower limit of $m_\nu > 0.4$ eV [3]. Given our certainty that they exist, these results make the neutrino one of the most convincing dark matter candidates [4].

Although it is unlikely that these massive neutrinos are the dark matter it appears conceivable that they may still affect the growth of density perturbations in a measurable way. Neutrinos with a mass less than 2 eV are still relativistic when entering the horizon for scales of $k = 0.1 \text{ h Mpc}^{-1}$ and are quasi-relativistic at recombination. Therefore...
they cannot be treated as a non-relativistic component of the CMB and are not entirely
degenerate with the other relativistic components [5]. In a matter-dominated universe,
a modicum of massive neutrinos will free-stream on scales of clusters and galaxies and
therefore suppress the rate of growth of density perturbations from being proportional to
the scale factor $a$ to being proportional to $a^{1-\epsilon}$ where $\epsilon = \frac{5}{4}[1-(1-24\Omega_{\nu}/25)^{1/2}]$, where the
neutrino mass, $\sum m_{\nu}$, is related through the expression: $\Omega_{\nu} = \sum m_{\nu}/(93.15h^2)$. Here the
Hubble constant today is $H_0 = 100 h$ km s$^{-1}$ Mpc$^{-1}$. This relation can be applied even in
the event that the neutrino species are found to be non-degenerate [4]. This effect on the
growth of structure supplies us with a useful method for constraining $\Omega_{\nu}$ and, as a result,
$\sum m_{\nu}$. By measuring the amplitude of clustering on large scales (above the free-streaming
scale) and comparing it to the level of clustering on small scales (below the free-streaming
scale) it is possible to tease out the level of damping due to the neutrinos. The amount
of clustering on large scales is well constrained by measurements of fluctuations in the
cosmic microwave background (CMB) which map out the density perturbations on scales
up to the horizon. Surveys of galaxies allow us to pin down the amount of clustering on
small scales. Combining the two allows us to place a constraint on $\Omega_{\nu}$.

This approach has been applied extensively over the last decade. In [6],
using a combination of data from the Wilkinson Microwave Anisotropy Probe 3-year
(WMAP3) [7], 2dF Galaxy Redshift Survey [8], Sloan Digital Sky Survey (SDSS) [9] and
the supernova (SN) [10,11], the WMAP team placed an upper limit of $\sum m_{\nu} < 0.66$ eV
on the neutrino mass when the SDSS bias constraint [5] is included. Seljak [12] recently
reported an huge improvement on this when the same dataset is supplemented with the
baryon acoustic oscillation (BAO) [13] and Lyman-\(\alpha\) [14] constraints. The Lyman-\(\alpha\) forest
provides information about the matter power spectrum on small scales (where neutrinos
suppress power) and at high redshift ($z = 2-4$) where the nonlinear evolution is less
significant. The Lyman-\(\alpha\) data prefers a higher normalization and is the primary source of
the strong upper bound on the neutrino mass of $\sum m_{\nu} < 0.17$ eV at 95% CL in the absence
of the SDSS bias constraint. This constraint is found using the recent measurement of the
position of a peak in the galaxy correlation function at $\sim 140$ Mpc $h^{-1}$ arising from the
baryon acoustic oscillations once they have decoupled from photons. Given its dependence
on $\Omega_m$ and the Hubble parameter $H(z)$ and the degeneracy of neutrino mass with both
these parameters the use of the SN data to fix $\Omega_m$ in conjunction with the information
about $\Omega_{\Lambda}$ provided from the BAO result proves to be powerful.

The use of cosmological observations can, in principle, supply us with limits on the
neutrino mass which are comparable with experimental bounds. Yet there is a valid
concern that cosmological constraints depend largely on the chosen cosmological model
and accompanying priors. The impact of certain assumptions that go into such an analysis
has been considered by other authors [15,16]. However, a detailed review of all relevant
factors that may be getting in the way of a truly robust constraint seems necessary. In this
paper we wish to elucidate how robust the neutrino constraints are to the assumed model.
The structure of this paper is as follows. We first establish the details of our analysis
and determine the way in which choices of prior probability distributions for the neutrino
content affect our results within the context of a $\Lambda$CDM universe. We then extend the
model to include the possibilities of dark energy, curvature, tensors, spectral running
and more general initial conditions. We then consider how knowledge of cosmological
parameters such as the galaxy bias can be harnessed to improve the neutrino mass limits.
2. The approach

Neutrinos will impact cosmological observables in a variety of ways. Relativistic neutrinos have a marginal effect by increasing the radiation density before decoupling, thereby impinging on the acoustic peak morphology. Massive neutrinos that become non-relativistic prior to recombination suppress the early integrated Sachs–Wolf effect, with their signature being a modification of the height and position of the first peak. Furthermore, the free-streaming of massive neutrinos induces a more rapid decay of the gravitational potential, fueling the acoustic oscillations within this free-streaming scale. This modifies the heights of the second and third peaks relative to the first which cannot be reconciled in the context of the shift in peak position and facilitates the constraint. As shown in [17] it is thus possible to derive limits on neutrino mass from the CMB alone. However, these above-mentioned degeneracies weaken the power of the data and the constraints come out to be $\sum m_\nu < 2.0$ eV for flat power-law $\Lambda$CDM. The merits of this are that the limit suffers less from systematic errors and is robust since it is derived from the single experiment with one set of systematics. The most effective use of the CMB data in pinning down $\sum m_\nu$ is, however, in the normalization of the large scale galaxy clustering power. The primary effect of the neutrino mass on the matter power spectrum $P(k)$ is to reduce power on scales smaller than the free-streaming scale. Using both LSS and CMB data concurrently has been identified as the means of arriving at reliable and competitive constraints.

The empirical relationship between the morphology of the supernova type Ia light curves and their intrinsic luminosities facilitates their use as standard candles. These datasets [10, 11] have become key in determining the expansion rate of the universe and thus in testing models of dark energy. However, the physical mechanism that forms the basis for this relation has yet to be established and given the enormous impact of these measurements we choose not to include them in our standard dataset but will incorporate the SN results taken from [11] at points for the sake of comparison.

We approach the Lyman-α measurements [14] with even more caution given the as yet still unresolved systematics that still plague these datasets. When re-analysed by [12] and [18] discrepancies arose, with the authors of [18] finding a lower normalization than the primary result in [14]. Since the Lyman-α data has been shown to tighten the bound on neutrino mass considerably we choose not to include it in our analysis. The impact of the inclusion of these different datasets on the neutrino mass limits has been comprehensively reviewed in [19] and found to be significant.

The practicalities of the method are as follows. CMB polarization and temperature power spectra as well as the matter power spectra are computed using the CAMB package [20]. We compare the spectra computed from the sample model to our fiducial dataset, comprising of the CMB temperature anisotropy measurements from WMAP 3-year [7] and a combination of small scale (high $\ell$) CMB data from ACBAR, BOOMERANG, CBI and VSA [21] which probe the power spectrum on smaller angular scales than WMAP. There is no overlap between these datasets and WMAP to ensure that there are no cross-correlations [7]. The CMB data is complemented with the galaxy power spectrum derived from the SDSS [9]. We additionally use the constraint on the baryon density today of $\Omega_b h^2$ from Big Bang nucleosynthesis (BBN) [22, 23]. In [24] tension between the WMAP 3-year data and the BBN measurement was identified. However, this
Conservative estimates of the mass of the neutrino from cosmology

prior is necessary to exclude wild high $\Omega_b h^2$ cosmological models which are favoured when isocurvature is allowed.

A likelihood analysis using the likelihood function in [25] is then performed in order to compare the spectra generated from the models with the data. We use a Monte Carlo Markov chain method that invokes a Metropolis algorithm as described in [26,27] to explore the resulting likelihood distribution in parameter space efficiently.

We take the concordance model as our starting point: a spatially flat universe with nearly scale-invariant adiabatic fluctuations dominated by a cosmological constant. A Fisher matrix analysis [15] reveals strong degeneracies between the neutrino density $\Omega_\nu$, the galaxy bias $b$ and the matter density $\Omega_m = \Omega_b + \Omega_d$. These parameters impact the matter power spectrum strongly on scales smaller than the free-streaming scale, while their effects on the CMB are similar in their subtlety (in the case of the bias, none at all). Neutrinos with masses of the order of eV become non-relativistic only after decoupling, when the evolution determining the shape of the CMB spectrum has already taken place [4].

The bias parameter shares an effect on the normalization of the power spectrum on all scales so an increase in $\sum m_\nu$ could be partly compensated for by a decrease in $b$. Increasing the matter content $\Omega_m$ brings matter domination forward in time, resulting in less suppression on small scales (i.e. shallower gradient at larger $k$ [28]) and a smaller horizon at matter–radiation equality [29]. It then makes sense that the power suppression at larger $k$ caused by the massive neutrinos must be corrected for this opposing effect; $\Delta P/P \simeq -8(\Omega_\nu/\Omega_m)$. A change in $\Omega_m$ is equivalent to a change in $\Omega_\Lambda$ and the degeneracy between this parameter and the neutrino mass.

Adding in three types of massive neutrinos degenerate in mass (we in fact use $N = 3.04$ as predicted by theory [5]), the cosmological parameters that sufficiently describe this scenario are the physical baryon density $\omega_b$, the physical cold dark matter (CDM) density $\omega_d$, the neutrino density relative to CDM $f_\nu = \omega_\nu/\omega_d$, the fractional density of cosmological constant $\Omega_\Lambda$, the galaxy bias $b$ (assumed to be a constant based on theoretical reasoning in [30]), and the amplitude and scalar spectral index of the primordial fluctuation spectrum $A_s$ and $n_s$, respectively.

The parametrization used by the WMAP team $f_\nu$ is the ratio of the density of this hot dark matter component to the CDM. Given the degeneracy between $\omega_\nu$ and $\Omega_\nu$ the ratio of the two is likely to be a sensitive parameter. There are, however, alternatives; in other work the parametrization is chosen to be $F_\nu$, the fraction of the total matter density $\Omega_M = \Omega_b + \Omega_d + \Omega_\nu$ that the neutrinos comprise. In other cases one simply works with the total mass, $\sum m_\nu$, of all three species. It seems worthwhile to check the effect of the neutrino parametrization by repeating the same analysis using $\sum m_\nu$ and $F_\nu = \Omega_\nu/\Omega_M$ as the variables. If seemingly benign flat priors are imposed different parametrizations the effective priors on the quantity being constrained, in this case $\sum m_\nu$, will diverge. Taking the priors shown in table 1 which are typically placed on the various neutrino variables in such work, we find that the upper limits on $\sum m_\nu$ at the 95% CL (using the CMB and LSS data alone) are in agreement for all three parametrizations. In the region of $\sum m_\nu$ space in which the data falls (<4 eV), these priors are all effectively equivalent top-hat and thus have no impact. If we add the SN data [11], which amounts to imposing the constraint $\Omega_m = 0.263 \pm 0.042$, the effective posterior probabilities look notably different in this region of $\sum m_\nu$ space, but again do not lead to major discrepancies in the mass...
Table 1. Constraints on neutrino mass when different parametrizations are used.

| Parametrization | Prior | \(\sum m_\nu\) for \(N_\nu = 3.04\) 95% CL |
|-----------------|-------|---------------------------------|
| \(f_\nu = \Omega_\nu/\Omega_d\) | 0–1   | 1.26 eV                         |
| \(F_\nu = \Omega_\nu/\Omega_m\) | 0–1   | 1.27 eV                         |
| \(\sum m_\nu\) | 0–5 eV | 1.29 eV                         |

Table 2. Constraints on neutrino mass for different extensions to parameter space for the fiducial CMB + LSS dataset.

| Additional parameters | \(\sum m_\nu\) for \(N_\nu = 3.04\) 95% CL |
|-----------------------|---------------------------------|
| Spatial curvature, \(\Omega_k\) | 1.17 eV                         |
| Dark energy (\(w = \) constant) | 1.18 eV                         |
| Tensors, \(r\)         | 1.38 eV                         |
| Running spectral index, \(\alpha_s\) | 1.66 eV                         |
| Isocurvature (all modes) | 2.2 eV                          |

constraints. The situation does not change when \(\sigma(\Omega_m)\) is reduced to 0.01 as predicted for future experiments such as SNAP. This is reasonable given that \(\Omega_m\) and \(\Omega_b\) are well constrained by the current data. Given that the addition of cosmological parameters to a \(\Lambda\)CDM model has the potential to alter the posterior distributions of the parameters, we explored the effect of parametrization on the neutrino mass constraints when we extended the cosmological model in section 3 and found it to be minimal. We conclude the use of different subsets of the current data does not affect the independence of parametrization on the neutrino mass limits and thus continue using \(f_\nu\) throughout the rest of the analysis.

We now explore deviations from the standard model, specifically spatial curvature characterized by \(\Omega_k\), the presence of dark energy, the presence of tensors, the running of the spectral index and isocurvature contributions to the initial fluctuation spectrum.

3. Extending parameter space beyond \(\Lambda\)CDM

Despite the vast amount of data now in support of the simple concordance picture, more complicated scenarios are possible in certain paradigms. In order to be certain that the least restrictive mass limit is obtained, the model dependence of these constraints should be explored. The upper bounds on neutrino mass when the model space is extended are summarized in table 2.

3.1. Dark energy

In recent work [6, 31, 32] a degeneracy between the equation of state of dark energy \(w\) and the neutrino mass was identified. Massive neutrinos contribute to the energy density of the matter component in the universe and thus can potentially alter the epoch of matter–radiation equality as well as the diameter angular distance to the last scattering surface. We wish to clarify this result. If \(w\) is allowed to vary, the effect of larger neutrino masses can be mimicked in the power spectrum by more negative values of \(w\) instead of larger \(\Omega_m\).
values which become incompatible with the SN data [31]. If we make no assumption about the cosmological constant and include a constant $w$ in the hypothesis space, the resulting confidence contours for the CMB and LSS dataset (disregard the SN results) show no signs of a relationship between the neutrino mass and this new parameter. The inclusion of the SN data [11] tightens the correlation between $w$ and $\Omega_\Lambda$ and hence links the equation of state to $\sum m_\nu$ more strongly, giving rise to the degeneracy between that has been pointed out. This contrast is demonstrated in figure 1. The investigation of the correlation between neutrino mass and dark energy has been extended in [33] to include dynamical models in which the equation of state is characterized by a smooth time-varying function. Dynamical dark energy will impact the time evolution of the gravitational potential wells and thus contribute to a late time integrated Sachs–Wolf (ISW) effect which will in turn manifest in the CMB power spectrum. A strong anti-correlation between $\sum m_\nu$ and the time-dependent part of $w(z)$ was found which accounts for the substantial weakening of the upper bound to $\sum m_\nu < 2.8$ (depending on inclusion/exclusion of dark energy perturbations) for the CMB + LSS + SN dataset. As our understanding of dark energy improves, we may have more reason to use more complex models of this energy density component in neutrino mass analyses.

3.2. Spatial curvature

The analysis so far has been performed under the assumption of flat space. We now consider the possibility of non-flat geometry and include the curvature density $\Omega_k$ as a
Conservative estimates of the mass of the neutrino from cosmology

parameter. The CMB feels the impact of a non-zero value of Ω_k; the position of the first acoustic peak corresponds to the angular size at which the largest CMB hot spots should appear at very early times and is found to depend primarily on the amount of spatial curvature from the current epoch and the surface of last scattering. The matter power spectrum however, remains largely unaffected by the introduction of this parameter. When we admit spatial curvature, a slightly open universe is favoured with a best-fit model of Ω_k = −0.05. Evidently only small deviations from flatness are allowed and the results remain consistent with flat space within 2σ. This is, however, accompanied by a marginally tighter constraint on the neutrino mass of 1.17 eV. Given that the range of non-flat models that can be tolerated by the data is small, the posterior probability distribution is more sharply peaked around the allowed open models. If one then integrates the area below this distribution to include 68% of the sampled points in hypothesis space, the models which lie on this boundary are likely to have smaller neutrino masses even if the values of ∑m_ν are generally higher than when flatness is imposed. The confidence contour in figure 1 rules out any degeneracy between the neutrino mass and Ω_k. Including the possibility of non-flat models does not have a significant impact on the neutrino mass that can be tolerated by the data and ignored from the parameter space.

3.3. Running spectral index

Inflationary cosmological scenarios seem to provide the most convincing explanations for the initial conditions for structure formation and a scale-invariant spectrum of primordial fluctuations is regarded as an intrinsic accompaniment of these models [37]. However, this picture of the universe undergoing this period of exponential expansion is only valid within certain specific cosmologies, such as a pure de Sitter universe which fails to describe our cosmological history [34] and should thus not necessarily dictate the features we expect to see in the matter power spectrum. There have been claims of strong observational indications from the CMB, LSS and Ly-α data that the spectrum is a function of k (e.g. see [35]). However, the fit of combinations of other datasets have been shown to deteriorate when running is allowed [5]. Even though the presence of significant running of the spectral index is still under debate, the existence of a strong degeneracy between Ω_m and the spectral slope on the scales probed by the CMB [4,17,36] has been confirmed and could have implications for the neutrino mass constraints. A spectral index which decreases on small scales (negative running) would limit the amount of small-scale damping that could be tolerated from massive neutrinos and hence it seems to worthwhile to explore this possibility. In recent work [38] the neutrino mass constraints were found to be lowered in the presence of a scale-variant n_s when the CMB, LSS and SN data was used. The SN data prefers a lower value of Ω_m which means that the effect of non-zero α_s may not be absorbed entirely by an increase in the matter density, which demands a smaller neutrino mass in order to fit the data. We consider a running spectral index in the presence of massive neutrinos without the admission of tensors using our fiducial dataset which does not include the SN results. In this analysis we describe the power spectrum in terms of the perturbations in the spatial curvature of a comoving slicing of space-time Δ_R:

$$1 + \frac{\text{d} \ln(\Delta_R^2(k))}{\text{d} \ln(k)} = n_s(k_0) + \alpha_s \ln \left( \frac{k}{k_0} \right)$$

(1)
and thus parametrize the running of the spectral using $\alpha_s = \frac{dn_s}{d\ln k}$. Here the pivot scale is $k_0 = 0.05$ Mpc$^{-1}$. We find the constraint on $\sum m_\nu$ opens up to 1.66 eV (95% CL) while the amount of running allowed comes out as $\alpha_s = -0.062 \pm 0.02$ in agreement with the findings of the WMAP team using the 3-year data. This result, although at odds with the findings in [38], is reasonable given the significantly higher values of $\Omega_m$ that are preferred in the presence of running than in the case of a scale-invariant $n_s$. In addition to small-scale damping in the matter power spectrum a negative $\alpha_s$ induces suppression of the acoustic peaks in the CMB power spectrum. Since a change in the neutrino mass has a negligible effect on the CMB, models with negative running must have higher matter densities in order to produce the same CMB and LSS power spectra. Without the SN data to throw out high matter density models, the net small-scale structure suppression (minus the damping coming from the negative $\alpha_s$) must be offset by an increase in the neutrino mass. Figure 1 shows the absence of a degeneracy between $\alpha_s$ and $\sum m_\nu$.

### 3.4. Primordial tensor perturbations

Temperature anisotropies are sourced not only by density fluctuations (scalar modes) but by tensor modes on small angular scales. Inflationary models predict the production of tensor modes by gravitational waves and their evolution independently of scalar modes, leading to an uncorrelated power spectrum [39]. On scales of the Hubble radius such modes interfere with the photon propagation along the line of sight and, in so doing, induce extra anisotropy predominantly on large angular scales. In the CMB, the presence of tensor perturbations is registered by increased low-$\ell$ power as well as a lower amplitude of density fluctuations (the matter power spectrum is suppressed on all scales). Since $A_s$ affects the height of the first acoustic peak we can use the LSS data to normalize the CMB power spectrum, finding the best fit to the peak as well as the large angular scale part of the spectrum and, in so doing, can constrain the amplitude of the tensor and scalar contributions simultaneously. Different models predict varying amounts of tensor perturbations and hence it is parametrized in terms of its ratio to the scalar mode anisotropies $r$. Given the degeneracy between the amplitude $A_t$ of these metric perturbations with $\omega_m$, $\omega_b$ and $n_s$ it is important to check whether a higher contribution $r$ is tolerated in the presence of massive neutrinos. We use the same convention as given in [40] where $r$ is the ratio of the primordial amplitudes of the tensor and scalar fluctuations

$$r \equiv \frac{P_{\text{tens}}(k_*)}{P_{\text{scalar}}(k_*)}. \tag{2}$$

Here the tensor pivot scale is taken to be $k_* = 0.002$ Mpc$^{-1}$. This parameter relates to the spectral index of the tensor modes in the following way;

$$n_t = -r/8. \tag{3}$$

We do not allow for the running of the spectral indices which would reduce the amount of tensors that can fit the data. The upper limit on the neutrino mass degrades slightly to $\sum m_\nu = 1.38$ eV with the introduction of tensors with their contribution found to be $r < 0.32$ (95% CL). Bearing in mind that the current theoretical models which support the presence of tensors are few, it is reasonable to conclude from this result that excluding tensor perturbations from the cosmological model will not affect the robustness of neutrino mass constraints.
3.5. Mixed initial conditions

Up to this point we have assumed that the initial conditions in the universe are purely adiabatic (AD). When this assumption is relaxed, four isocurvature regular modes are assumed to arise, namely CDM (CI), baryon (BI), neutrino density (NID) and neutrino velocity (NIV) (where the momentum densities of the neutrino and photon–baryon fluids are assumed to cancel exactly [41]). It was proposed in [42] that the CMB and baryon isocurvature modes cannot be differentiated observationally and confirmed by the identical (up to an unimportant multiplicative constant) correlation matrix elements [43]. We consider the simultaneous mixing of all distinct modes so there will be 10 components of the initial power spectrum matrix:

$$\langle A_i(\vec{k}) A_i^*(\vec{k}') \rangle = P_{ij}(k) \delta^3(\vec{k} - \vec{k}')$$

(4)

where \((i, j = 1, 2, 3, 4, 5)\) label the modes and their amplitudes \(A_i\). Assuming that the initial power spectrum of each mode is a smoothly varying function of \(k\) and given by

$$P_{ij}(k) = A_{ij} k^{n_s},$$

where each correlation is characterized by an amplitude \(A_{ij}\) and a single scalar spectral index (the auto-correlated adiabatic mode is already characterized by the parameter \(A_s\)). Following [45] we characterize all distinct isocurvature modes and their correlations with each other and the adiabatic mode by the introduction of nine additional parameters, namely the coefficients \(z_{ij}\) which give the relative contributions of each spectra to the normalization. It then holds that

$$A_{ij} \propto z_{ij},$$

(5)

where

$$\text{trace} \left( z z^T \right) = \sum_{i,j=1}^{N} z_{ij}^2 = 1.$$

(6)

The choice to parametrize the contribution of each correlation in terms of the parameter \(z_{ij}\) is useful because in a case where \(n_{ad} = n_{iso}\) (as in ours), the results are independent of the choice of the value of the pivot scale \(k_0\), which tends to vary from one analysis to another.

3.5.1. AD + CDM isocurvature. We initially consider the simplest extension of the adiabatic regime, which is a single isocurvature mode mixed with the adiabatic mode [44]. Including only the CI and AD modes we arrive at the same constraints as in the adiabatic case of 1.27 eV. We do not find any interdependence between the parameters characterizing the CI modes and the neutrino mass.

3.5.2. AD + CI + NID. The neutrino isocurvature modes are generally difficult to motivate theoretically. Vanishing neutrino chemical potentials mean that there are the same numbers of neutrinos as anti-neutrinos produced in each generation. The implication is that spatial differences in the densities of neutrinos and photons will be naturally erased via the processes involving \(\nu \bar{\nu}\) annihilation. The neutrino modes can thus only be generated at temperatures as low as a few MeV where these processes have ceased. Avoiding these issues requires the introduction of non-zero chemical potentials. We find that, when the NID isocurvature mode is admitted, significantly higher neutrino masses with an upper limit of 2.4 eV at the 95% CL are permissible when compared to our adiabatic constraint of 1.3 eV.
3.5.3. AD + all isocurvature modes. To generate the neutrino velocity mode, a spatially varying relative velocity is needed between the neutrinos and photons but must be constructed initially such that the overall perturbation in total momentum density is zero [41]. Such modes therefore require even more exotic generation mechanisms and are met with more skepticism. Adding in the neutrino velocity mode and permitting all four distinct isocurvature modes to mix, leads to a slightly lower neutrino mass constraint of 2.2 eV. The distribution of the galaxy bias as shown in figure 2 exhibits a shift to higher values than in the pure adiabatic case. Since the matter power spectrum is dominated by the adiabatic mode [27], including an isocurvature mode reduces the overall power $P(k)$, requiring a higher biasing of the galaxy spectrum to that of the underlying matter distribution. But because our value for $b$ agrees with the $1.3 \pm 0.2$ obtained for all four correlated isocurvature modes in the presence of massless neutrinos [45], we infer that the bias is not sufficiently high to require suppression of the matter power by such high mass neutrinos. A lower value of $\Omega_\Lambda$ than that reported in [45] must then compensate for the higher allowed value of neutrino mass. This is a strong indication of a degeneracy between $\sum m_\nu$ and a combination of isocurvature parameters. Note that the neutrino mass distribution in figure 2 seems to exclude the model with $\sum m_\nu = 0$. However this feature can be attributed to a larger prior space than the massless case in which NIV and NID modes are 0. In other words, there are effectively more ways of fitting the data with massive neutrinos.

4. The path to tighter constraints

If we consider the degradation of the neutrino mass constraints which results as the hypothesis space enlarges, it seems important to determine how we can improve our knowledge of $\sum m_\nu$ in the presence of these additional parameters. We have identified two high strong degeneracies between $\sum m_\nu$ and $\Omega_m$ and $b$ which become important when seeking stronger constraints. Any supplementary information that reduces the freedom in these degenerate parameters will impact on the neutrino constraints. This information must come from independent experiments which measure phenomena on which the neutrino mass has negligible effect.

Stringent neutrino mass constraints can be obtained by limiting the allowable values of $b$. The upper bound of $\sum m_\nu < 3$ eV (95% CL) found by Hannestad [29] using the
Table 3. Constraints on neutrino mass for different information about the bias $b$ for the standard eight-dimensional parameter space.

| Dataset | $\sum m_\nu$ for $N_\nu = 3.04$ 95% confidence level (CL) |
|---------|-------------------------------------------------|
| CMB     | 2.0 eV                                          |
| CMB + LSS | 1.3 eV                                         |
| CMB + LSS + 2dF bias [25] | 1.05 eV                                      |
| CMB + LSS + SDSS bias [5] | 0.79 eV                                      |
| CMB + LSS + $b = 1$ | 0.63 eV                                      |

WMAP 1st-year [46] and 2dFGRS [47] datasets, leaving $b$ and the normalization as free parameters, was lowered to 0.7 eV by the WMAP team using the same data by assuming $b \approx 1$. This assumption was based on the bi-spectrum analysis of the 2dFGRS [48]. Using the most recent CMB + LSS data, we update this bound to 0.63 eV. The effects of introducing different measurements of the bias for a $\Lambda$CDM cosmology are illustrated in table 3. We wish to dissect the effect of including bias measurements on neutrino mass constraints in order to establish how they need to be improved to better pin down $\sum m_\nu$.

When the analysis is performed using the SDSS data its natural distribution is found to be normal with mean $\mu_b = 1.09$ and width $\sigma_b = 0.11$. If we impose different Gaussian priors on $b$ keeping the mean $\mu_b$ fixed at its natural value and reducing the standard deviation to $0.75\sigma_b$ and $0.5\sigma_b$, the resulting 1D $\sum m_\nu$ distributions tighten slightly. Turning to the allowed regions in ($\sum m_\nu$, $b$) space, the effect of the imposed standard deviation of $b$ on the mass limit progressively weakens, indicative of a threshold value of $\sigma_b$ beyond which it has no effect at all. These results are shown in the right panel of figure 3. If we now lower the mean value $\mu_b$ of the Gaussian priors and assume uncertainties of 10% on these peak values (i.e. $\sigma_b = 0.1 \times \mu_b$), the allowable region in ($\sum m_\nu$, $b$) space is reduced in a relatively linear fashion. This is seen in the left panel of figure 3. Given the positive correlation between $b$ and $\sum m_\nu$, a decrease in the mean of the prior $\mu_b$ effectively weights models with a lower neutrino mass with a higher probability. From this analysis we infer that reducing the error bars on the measurement of the bias parameter will help constrain neutrino mass to a limited extent. In order to improve our knowledge of $\sum m_\nu$ a more precise measurement of the mean $b$ from future analyses, such as weak lensing, will be necessary.

In light of recent results [49] the assumption that the power spectrum of the galaxy distribution is linearly biased relative to the matter power spectrum may be short-sighted, with implications for current neutrino constraints. The lower matter density favoured by the SDSS DR5 galaxy data could be explained in terms of a scale-dependent bias. This has the potential to degrade neutrino constraints because the effect of the bias will no longer be confined to the amplitude but will now be felt by the shape of the linear power spectrum, making it harder to break the degeneracy with $\sum m_\nu$. In the work [19] in which the authors assess the effect of different datasets on the neutrino mass limits, discrepancies were pointed out between upper bounds on $\sum m_\nu$ obtained using the CMB data and either the LSS measurements from 2DF [8] or SDSS [9]. Normalizing the total matter power spectrum without having to make such assumptions can be achieved from weak lensing.
Figure 3. Top panel: the 1D marginalized distribution for $\sum m_\nu$ when different Gaussian priors are imposed on the bias parameter. On the left, the peak value of the bias distribution, denoted by $\mu_b$, is taken to be 1.1 (solid), 1.0 (dotted) and 0.92 (dashed). On the right, $\mu_b$ is kept as 1.1 and the Gaussian uncertainty denoted by $\sigma_b$ is fixed at 0.11 (solid), 0.0825 (dotted) and 0.055 (dashed). Bottom panel: the 95% constraints in the ($\sum m_\nu, b$) plane for the same three cases described above.

experiments. The limit on the neutrino mass found using the measurement of the cluster mass function in [50] and the CMB data was considerably weaker.

5. Conclusions

In this paper, the dependence of constrains on the neutrino mass using cosmological observations (CMB + LSS) on the underlying model has been assessed. Before proceeding with the analysis we systematically reviewed most decisions that go into such an investigation and are satisfied that analysis with current datasets is unaffected when different neutrino parametrizations are employed. The BAO [13] and SN [11] results are shown to impact the mass limits significantly and are thus incorporated with caution.

When parameter space is extended beyond our current standard cosmological model ($\Lambda$CDM), we rule out any degeneracy between neutrino mass and spatial curvature. We also find that the relationship between dark energy equation of state $w$ and $\sum m_\nu$ depends on the datasets used. A decrease in $w$ away from $-1$ can be accommodated by an increase in the neutrino mass or an increase in $\Omega_m$ which rapidly becomes incompatible with the SN data. The inclusion of a running spectral index no longer tightens the upper bound on $\sum m_\nu$ if the SN measurements are removed from the dataset. Both of these mentioned results disagree with former findings in analyses which include the SN data and highlight issues regarding the dependence of the results not only on the cosmological model but the cosmological observations used (see [19]). Allowing for tensor perturbations is found to have minimal effect: however, the analysis excluded the possibility of a running spectral
index. The substantial degradation of the upper mass bound to 2.2 eV with more general initial conditions points to a degeneracy with the isocurvature parameters, namely the neutrino density mode. These high-$\sum m_{\nu}$ models are, however, ruled out by the SN constraints on $\Omega_m$ [11] which bring the limit down to 1.17 eV. Constraining the value of bias impacts significantly on the upper bound, with the most stringent constraint of $\sum m_{\nu} < 0.63$ eV being obtained when taking $b = 1$. Assuming a known prior distribution for the bias which is based on a measurement of this parameter amounts to effectively incorporating a dataset and all accompanying systematic uncertainties. For this reason the same discernment should be exercised. We find that improvements in the accuracy of the measurements of the galaxy bias beyond a point will not dramatically aid our constraints on $\sum m_{\nu}$. In conclusion it has been shown that the cosmological constraint on neutrino mass is sensitive to many factors and it is only once all assumptions have been evaluated can we regard the resulting limit as robust.

Acknowledgments

We are extremely grateful to J Dunkley for guidance in this project. We thank S Biller and K Moodley for discussions. CZ acknowledges support from a Domus A scholarship awarded by Merton College.

References

[1] Fukuda S et al, 2000 Phys. Rev. Lett. 85 3999 [SPIRES]
Fukuda S et al, 2002 Phys. Lett. B 539 179 [SPIRES]
Smy M B et al, 2004 Phys. Rev. D 69 011104 [SPIRES]
Ahmed S N et al, 2004 Phys. Rev. Lett. 92 181301 [SPIRES]
Eguchi K et al, 2003 Phys. Rev. Lett. 90 021802 [SPIRES]

[2] Bonn J et al, 2001 Nucl. Phys. (Proc. Suppl.) 91 273 [SPIRES]
Kraus Ch et al, 2005 Eur. Phys. J. C 40 447 [SPIRES]

[3] Athanassopoulous C et al, 1996 Phys. Rev. Lett. 77 3082 [SPIRES]
Aguilar A et al, 2001 Phys. Rev. D 64 112007 [SPIRES]

[4] Lesgourges J and Pastor S, 2006 Phys. Rep. 429 307 [SPIRES]

[5] Seljak U et al, 2005 Phys. Rev. D 71 103515 [SPIRES]

[6] Spergel D et al, Wilkinson Microwave Anisotropy Probe (WMAP) three year results: implications for cosmology, 2006 Preprint astro-ph/0603449

[7] Hinshaw G et al, Three-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: temperature analysis, 2006 Preprint astro-ph/0603451

Page L et al, Three year Wilkinson Microwave Anisotropy Probe (WMAP) observations: polarization analysis, 2006 Preprint astro-ph/0603450

[8] Cole S et al, 2005 Mon. Not. R. Astron. Soc. 362 505

[9] Tegmark M et al, 2004 Astrophys. J. 606 702 [SPIRES]

[10] Riess A G et al, 2004 Astrophys. J. 607 665 [SPIRES]

[11] Astier P et al, 2006 Astron. Astrophys. 447 31 [SPIRES]

[12] Seljak U, Slosar A and McDonald P, 2006 J. Cosmol. Astropart. Phys. JCAP10(2006)014 [SPIRES]

[13] Eisenstein D J et al, 2005 Astrophys. J. 633 560 [SPIRES]

[14] McDonald P et al, 2005 Astrophys. J. 635 761 [SPIRES]

[15] Meiksin A and White M, 2004 Mon. Not. R. Astron. Soc. 350 1107

[16] Hannestad S, 2002 Phys. Rev. D 66 125011 [SPIRES]

[17] Elgarøy Ø and Lahav Ø, 2003 J. Cosmol. Astropart. Phys. JCAP04(2003)004 [SPIRES]

[18] Ichikawa K, Fukugita M and Kawasaki M, 2005 Phys. Rev. D 71 043001 [SPIRES]

[19] Viel M, Haehnelt M G and Springel V, 2006 Mon. Not. R. Astron. Soc. 367 1655

[20] Viel M and Haehnelt M G, 2006 Mon. Not. R. Astron. Soc. 365 231

[21] Viel M, Haehnelt M G and Lewis A, 2006 Mon. Not. R. Astron. Soc. Lett. 370 L51
Conservative estimates of the mass of the neutrino from cosmology

[19] Kristiansen J R, Elgaroy O and Dahle H, ‘Cosmological neutrino mass limits: variations with choice of data sets and a new, bias-free limit, 2006 Preprint astro-ph/0611761
[20] Lewis A, Challinor A and Lasenby A, 2000 Astrophys. J. 538 473 [SPIRES]
[21] Kuo C L et al, 2004 Astrophys. J. 600 32 [SPIRES]
MacTavish C J et al, 2006 Astrophys. J. 647 799 [SPIRES]
Readhead A C S et al, 2004 Astrophys. J. 609 498 [SPIRES]
Dahle H, 2006 Astrophys. J. 653 954 [SPIRES]