Turbulence in the Interstellar Medium

Energetics and Driving Mechanisms

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Abstract. Interstellar turbulence is expected to dissipate quickly in the absence of continuous energy input. I examine the energy available for driving the turbulence from six likely mechanisms: magnetorotational instability, gravitational instability, protostellar outflows, H ii region expansion, stellar winds, and supernovae. I conclude that supernovae contribute far more energy than the other mechanisms, and so form the most likely driving mechanism.

Keywords: turbulence, interstellar medium

1. Introduction

Turbulent flows appear more and more central to our understanding of the interstellar medium. The distribution of pressures and densities are probably determined as much by turbulent ram pressures as by thermal phase transitions (Vázquez-Semadeni, Gazol, & Scalo 2000; Mac Low et al. 2002). Furthermore, compression by large-scale turbulent flows may form molecular clouds (Ballesteros-Paredes, Hartmann, & Vázquez-Semadeni 1999), and drive turbulence within those clouds (Ossenkopf & Mac Low 2002) to support them against gravitational collapse (Klessen, Heitsch, & Mac Low 2000).

Both support against gravity and maintenance of observed motions appear to depend on continued driving of the turbulence, which has kinetic energy density $e = (1/2)\rho v_{\text{rms}}^2$. Mac Low (1999, 2002) estimates that the dissipation rate for isothermal, supersonic turbulence is

$$\dot{e} \simeq -(1/2)\rho v_{\text{rms}}^3/L_d$$

$$= -(3 \times 10^{-27} \text{ erg cm}^{-3} \text{ s}^{-1}) n \left( \frac{v_{\text{rms}}}{10 \text{ km s}^{-1}} \right)^3 \left( \frac{L_d}{100 \text{ pc}} \right)^{-1},$$

where $n$ is the number density in units of cm$^{-3}$, $L_d$ is the driving scale, which we have somewhat arbitrarily taken to be 100 pc (though it could well be smaller), and we have assumed a mean mass per particle $\mu = 2.11 \times 10^{-24}$ g. The dissipation time for turbulent kinetic energy

$$\tau_d = e/\dot{e} \simeq L/v_{\text{rms}} = (9.8 \text{ Myr}) \left( \frac{L_d}{100 \text{ pc}} \right) \left( \frac{v_{\text{rms}}}{10 \text{ km s}^{-1}} \right)^{-1},$$

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which is just the crossing time for the turbulent flow across the driving scale (Elmegreen 2000b). What then is the energy source for this driving? We here review the energy input rates for a number of possible mechanisms.

2. Magnetorotational Instabilities

One energy source for interstellar turbulence that has long been considered is shear from galactic rotation (Fleck 1981). However, the question of how to couple from the large scales of galactic rotation to smaller scales remained open. Work by Sellwood & Balbus (1999) has shown that the magnetorotational instability (Balbus & Hawley 1991, 1998) could couple the large-scale motions to small scales efficiently. The instability generates Maxwell stresses (a positive correlation between radial $B_R$ and azimuthal $B_\phi$ magnetic field components) that transfer energy from shear into turbulent motions at a rate $\dot{\epsilon} = T_{R\phi} \Omega$ (Sellwood & Balbus 1999). Numerical models suggest that the Maxwell stress tensor $T_{R\phi} \simeq 0.6 B^2/(8\pi)$ (Hawley, Gammie & Balbus 1995). For the Milky Way, the value of the rotation rate recommended by the IAU is $\Omega = (220 \text{ Myr})^{-1} = 1.4 \times 10^{-16} \text{ rad s}^{-1}$, though this may be as much as 15% below the true value (Olling & Merrifield 1998, 2000). The magnetorotational instability may thus contribute energy at a rate

$$\dot{\epsilon} = (3 \times 10^{-29} \text{ erg cm}^{-3} \text{ s}^{-1}) \left( \frac{B}{3 \mu \text{G}} \right)^2 \left( \frac{\Omega}{(220 \text{ Myr})^{-1}} \right). \quad (3)$$

For parameters appropriate to the HI disk of a sample small galaxy, NGC 1058, Sellwood & Balbus (1999) find that the magnetic field required to produce the observed velocity dispersion of 6 km s$^{-1}$ is roughly 3 $\mu$G, a reasonable value for such a galaxy. This instability may provide a base value for the velocity dispersion below which no galaxy will fall. If that is sufficient to prevent collapse, little or no star formation will occur, producing something like a low surface brightness galaxy with large amounts of HI and few stars.

3. Gravitational Instabilities

Motions coming from gravitational collapse have often been suggested as a local driving mechanism in molecular clouds, but fail due to the quick decay of the turbulence (Klessen, Burkert, & Bate 1998). If the turbulence decays in less than a free-fall time, as suggested by equation 1, then it cannot delay collapse for substantially longer than a free-fall time.
On the galactic scale, spiral structure can drive turbulence in gas disks. Roberts (1969) first demonstrated that shocks would form in gas flowing through spiral arms formed by gravitational instabilities in the stellar disk (Lin & Shu 1964, Lin, Yuan, & Shu 1969). These shocks were studied in thin disks by Tubbs (1980) and Soukup & Yuan (1981), who found few vertical motions. More recently, it has been realized that in a more realistic thick disk, the spiral shock will take on some properties of a hydraulic bore, with gas passing through a sudden vertical jump at the position of the shock (Martos & Cox 1998, Gómez & Cox 2002). Behind the shock, downward flows of as much as 20 km s$^{-1}$ appear (Gómez & Cox 2002). Some portion of this flow will contribute to interstellar turbulence. However, the observed presence of interstellar turbulence in irregular galaxies without spiral arms, as well as in the outer regions of spiral galaxies beyond the regions where the arms extend suggest that this cannot be the only mechanism driving turbulence. A more quantitative estimate of the energy density contributed by spiral arm driving has not yet been done.

The interaction between rotational shear and gravitation can, at least briefly, drive turbulence in a galactic disk, even in the absence of spiral arms. This process has been numerically modeled at high resolution (sub-parsec zones) in two dimensions in a series of papers by Wada & Norman (1999, 2001), Wada, Spaans, & Kim (2000), and Wada, Meurer & Norman (2002). However, these models all share two limitations: they do not include the dominant stellar component, and gravitational collapse cannot occur beneath the grid scale. The computed filaments of dense gas are thus artificially supported, and would actually continue to collapse to form stars, rather than driving turbulence in dense disks (see Sánchez-Salcedo [2001] for a detailed critique). In very low density disks, where even the dense filaments remained Toomre stable, this mechanism might operate, however.

Wada et al. (2002) estimated the energy input from this mechanism following the lead of Sellwood & Balbus (1999), but substituting Newton stresses (Lynden-Bell & Kalnajs 1972) for Maxwell stresses. The Newton stresses will only add energy if a positive correlation between radial and azimuthal gravitational forces exists, however, which is not demonstrated by Wada et al. (2002). Nevertheless, they estimate the order of magnitude of the energy input from Newton stresses as

\[
\dot{e} \simeq G(\Sigma_g/H)\lambda^2\Omega \\
\simeq (4 \times 10^{-20} \text{ erg cm}^{-3} \text{ s}^{-1}) \left(\frac{\Sigma_g}{10 \text{ M}_\odot \text{ pc}^{-2}}\right)^2 \times
\]
\[ \times \left( \frac{H}{100 \text{ pc}} \right)^{-2} \left( \frac{\lambda}{100 \text{ pc}} \right)^2 \left( \frac{\Omega}{(220 \text{ Myr})^{-1}} \right), \]  

where \( G \) is the gravitational constant, \( \Sigma_g \) the density of gas, \( H \), the scale height of the gas, \( \lambda \) a length scale of turbulence, and \( \Omega \) the angular velocity of the disk. Values chosen are appropriate for the Milky Way. This is two orders of magnitude below the value required to maintain interstellar turbulence (eq. [1]).

### 4. Protostellar outflows

Protostellar jets and outflows are a popular suspect for the energy source of the observed turbulence. We can estimate their average energy input rate, following McKee (1989), by assuming that some fraction \( f_{w} \) of the mass accreted onto a star during its formation is expelled in a wind travelling at roughly the escape velocity. Shu et al. (1988) argue that \( f_{w} \simeq 0.4 \), and that most of the mass is ejected from close to the stellar surface, where the escape velocity

\[ v_{\text{esc}} = (2GM/R)^{1/2} = (200 \text{ km s}^{-1})(M/\text{M}_\odot)^{1/2}(R/10 \text{ R}_\odot)^{-1/2}, \]  

where the scaling is appropriate for a protostar with mass \( M = 1 \text{ M}_\odot \) and radius \( R = 10 \text{ R}_\odot \). Observations of neutral atomic winds from protostars suggest outflow velocities of roughly this value (Lizano et al. 1988, Giovanardi et al. 2000).

The total energy input from protostellar winds will substantially exceed the amount that can be transferred to the turbulence due to radiative cooling at the wind termination shock. We represent the fraction of energy lost there by \( \eta_{w} \). A reasonable upper limit to the energy loss is offered by assuming fully effective radiation and momentum conservation, so that

\[ \eta_{w} < \frac{v_{\text{rms}}}{v_{w}} = 0.05 \left( \frac{v_{\text{rms}}}{10 \text{ km s}^{-1}} \right) \left( \frac{200 \text{ km s}^{-1}}{v_{w}} \right), \]  

where \( v_{\text{rms}} \) is the rms velocity of the turbulence, and we have assumed that the flow is coupled to the turbulence at typical velocities for the diffuse ISM. If we assumed that most of the energy went into driving dense gas, the efficiency would be lower, as typical rms velocities for CO outflows are 1\nobreakdash–\nobreakdash2 km s\(^{-1}\). The energy injection rate

\[ \dot{e} = \frac{1}{2} f_{w} \eta_{w} \frac{\Sigma_{g}}{H} v_{w}^2 \]
\[ \simeq (2 \times 10^{-28} \text{ erg cm}^{-3} \text{ s}^{-1}) \left( \frac{H}{200 \text{ pc}} \right)^{-1} \left( \frac{f_w}{0.4} \right) \left( \frac{v_{\text{rms}}}{10 \text{ km s}^{-1}} \right) \times \left( \frac{\dot{\Sigma}_*}{200 \text{ km s}^{-1}} \right) \left( \frac{v_{\text{rms}}}{10 \text{ km s}^{-1}} \right) \left( \frac{\dot{\Sigma}_*}{4.5 \times 10^{-9} \text{ M}_\odot \text{ pc}^{-2} \text{ yr}^{-1}} \right), \tag{7} \]

where \( \dot{\Sigma}_* \) is the surface density of star formation, and \( H \) is the scale height of the star-forming disk. The scaling value used for \( \dot{\Sigma}_* \) is the solar neighborhood value (McKee 1989).

Although protostellar jets and winds are indeed quite energetic, they deposit most of their energy into low density gas (Henning 1989), as is shown by the observation of multi-parsec long jets extending completely out of molecular clouds (Bally & Devine 1994). Furthermore, observed motions of molecular gas show increasing power on scales all the way up to and perhaps beyond the largest scale of molecular cloud complexes (Ossenkopf & Mac Low 2002). It is hard to see how such large scales could be driven by protostars embedded in the clouds.

5. Massive Stars

In active star-forming galaxies, however, massive stars appear likely to dominate the driving. They do so through ionizing radiation and stellar winds from O stars, and clustered and field supernova explosions, predominantly from B stars no longer associated with their parent gas. The supernovae appear likely to dominate, as we now show.

5.1. Stellar Winds

First, we consider stellar winds. The total energy input from a line-driven stellar wind over the main-sequence lifetime of an early O star can equal the energy from its supernova explosion, and the Wolf-Rayet wind can be even more powerful. However, the mass-loss rate from stellar winds drops as roughly the sixth power of the star’s luminosity if we take into account that stellar luminosity varies as the fourth power of stellar mass (Vink, de Koter & Lamers 2000), and the powerful Wolf-Rayet winds (Nugis & Lamers 2000) last only \( 10^5 \) years or so, so only the very most massive stars contribute substantial energy from stellar winds. The energy from supernova explosions, on the other hand, remains nearly constant down to the least massive star that can explode. As there are far more lower-mass stars than massive stars, with a Salpeter (1955) IMF giving a power-law in mass of \( \alpha = -2.35 \), supernova explosions will inevitably dominate over stellar winds after
the first few million years of the lifetime of an OB association, until the lifetime of the least massive star to explode of around 40–50 Myr.

5.2. HII Region Expansion

Next, we consider ionizing radiation from OB stars. The total amount of energy contained in ionizing radiation is vast. Abbott (1982) estimates the total luminosity of ionizing radiation in the disk to be

\[ \dot{e} = 1.5 \times 10^{-24} \text{ erg s}^{-1} \text{ cm}^{-3}. \]  

(8)

However, only a very small fraction of this total energy goes to driving interstellar motions, as we now show.

Ionizing radiation primarily contributes to interstellar turbulence by ionizing HII regions, heating them to 7000–10,000 K, and raising their pressures above that of surrounding neutral gas, so that they expand supersonically. Matzner (2002) computes the momentum input from the expansion of an individual HII region into a surrounding molecular cloud, as a function of the cloud mass and the ionizing luminosity of the central OB association. By integrating over the HII region luminosity function derived by McKee & Williams (1997), he finds that the average momentum input from a Galactic region is

\[ \langle \delta p \rangle \simeq (260 \text{ km s}^{-1}) \left( \frac{N_H}{1.5 \times 10^{22} \text{ cm}^{-2}} \right)^{-3/14} \left( \frac{M_{cl}}{10^6 \text{ M}_\odot} \right)^{1/14} \langle M_* \rangle. \]  

(9)

The column density \( N_H \) is scaled to the mean value for Galactic molecular clouds (Solomon et al. 1987), which varies little as cloud mass \( M_{cl} \) varies. The mean stellar mass per cluster in the Galaxy \( \langle M_* \rangle = 440 \text{ M}_\odot \) (Matzner 2002).

The number of OB associations contributing substantial amounts of energy can be drawn from the McKee & Williams (1997) cluster luminosity function

\[ N(> S_{49}) = 6.1 \left( \frac{108}{S_{49}} - 1 \right), \]  

(10)

where \( N \) is the number of associations with ionizing photon luminosity exceeding \( S_{49} = S/(10^{49} \text{ s}^{-1}) \). The luminosity function is rather flat below \( S_{49} = 2.4 \), the luminosity of a single star of 120 M_\odot, which was the highest mass star considered, so taking its value at \( S_{49} = 1 \) is about right, giving \( N(> 1) = 650 \) clusters.

To derive an energy input rate per unit volume \( \dot{e} \) from the mean momentum input per cluster \( \langle \delta p \rangle \), we need to estimate the average velocity of momentum input \( v_i \), the time over which it occurs \( t_i \), and
the volume $V$ under consideration. Typically expansion will not occur supersonically with respect to the interior, so $v_i < c_{s,i}$, where $c_{s,i} \approx 10 \text{ km s}^{-1}$ is the sound speed of the ionized gas. McKee & Williams (1997) argue that clusters typically last for about five generations of massive star formation, where each generation lasts $\langle t_\ast \rangle = 3.7 \text{ Myr}$. The scale height for massive clusters is $H_c \sim 100 \text{ pc}$ (e.g. Bronfman et al. 2000), and the radius of the star-forming disk is roughly $R_{sf} \sim 15 \text{ kpc}$, so the relevant volume $V = 2\pi R_{sf}^2 H_c$. The energy input rate from HII regions is then

$$\dot{e} = \frac{(\delta p) N(>1)v_i}{V t_i}$$

$$= \left(3 \times 10^{-30} \text{ erg s}^{-1} \text{ cm}^{-3}\right) \left(\frac{N_H}{1.5 \times 10^{22} \text{ cm}^{-2}}\right)^{-3/14} \times$$

$$\times \left(\frac{M_{cl}}{10^6 M_\odot}\right)^{1/4} \left(\frac{\langle M_\ast \rangle}{440 M_\odot}\right) \left(\frac{N(>1)}{650}\right) \times$$

$$\times \left(\frac{v_i}{10 \text{ km s}^{-1}}\right) \left(\frac{H_c}{100 \text{ pc}}\right)^{-1} \left(\frac{R_{sf}}{15 \text{ kpc}}\right)^{-2} \left(\frac{t_\ast}{18.5 \text{ Myr}}\right)^{-1}, \quad (11)$$

where all the scalings are appropriate for the Milky Way as discussed above. Nearly all of the energy in ionizing radiation goes towards maintaining the ionization of the diffuse ionized medium, and hardly any towards driving turbulence. Flows of ionized gas may be important very close to young clusters, but do not appear to contribute significantly on a global scale.

5.3. Supernovae

The largest contribution from massive stars to interstellar turbulence comes from supernova explosions. To estimate their energy input rate, we begin by finding the supernova rate in the Galaxy $\sigma_{SN}$. Cappellaro et al. (1999) estimate the total supernova rate in supernova units to be $0.72 \pm 0.21 \text{ SNu}$ for galaxies of type S0a-b and $1.21 \pm 0.37 \text{ SNu}$ for galaxies of type Sbc-d, where $1 \text{ SNu} = 1 \text{ SN (100 yr)}^{-1} (10^{10} L_B/L_\odot)^{-1}$, and $L_B$ is the blue luminosity of the galaxy. Taking the Milky Way as lying between Sb and Sbc, we estimate $\sigma_{SN} = 1 \text{ SNu}$. Using a Galactic luminosity of $L_B = 2 \times 10^{10} L_\odot$, we find a supernova rate of $(50 \text{ yr})^{-1}$, which agrees well with the estimate in equation (A4) of McKee (1989). If we use the same scale height $H_c$ and star-forming radius $R_{sf}$ as above, we can compute the energy input rate from supernova explosions with energy $E_{SN} = 10^{51} \text{ erg}$ to be

$$\dot{e} = \frac{\sigma_{SN} \eta_{SN} E_{SN}}{\pi R_{sf}^2 H_c}$$
The efficiency of energy transfer from supernova blast waves to the interstellar gas $\eta_{SN}$ depends on the strength of radiative cooling in the initial shock, which will be much stronger in the absence of a surrounding superbubble (e.g. Heiles 1990). Substantial amounts of energy can escape in the vertical direction in superbubbles as well, however. (Norman & Ferrara [1996] make an analytic estimate of the effectiveness of driving by SN remnants and superbubbles.) The scaling factor $\eta_{SN} \approx 0.1$ used here was derived by Thornton et al. (1998) from one-dimensional numerical simulations of SNe expanding in a uniform ISM, or can alternatively be drawn from momentum conservation arguments comparing a typical expansion velocity of $100 \text{ km s}^{-1}$ to typical interstellar turbulence velocity of $10 \text{ km s}^{-1}$. Detailed multidimensional modeling of the interactions of multiple SN remnants (e.g. Avillez 2000) will be required to better determine it.

Supernova driving appears to be powerful enough to maintain the turbulence even with the dissipation rates estimated in equation (1). It provides a large-scale self-regulation mechanism for star formation in disks with sufficient gas density to collapse despite the velocity dispersion produced by the magnetorotational instability. As star formation increases in such galaxies, the number of OB stars increases, ultimately increasing the supernova rate and thus the velocity dispersion, which restrains further star formation.

Supernova driving not only determines the velocity dispersion, but may actually form molecular clouds by sweeping gas up in a turbulent flow. Clouds that are turbulently supported will experience inefficient, low-rate star formation, while clouds that are too massive to be supported will collapse (e.g. Kim & Ostriker 2001), undergoing efficient star formation to form OB associations or even starburst knots.

6. Conclusions

Interstellar gas is only quiescent on the very smallest scales ($< 0.1 \text{ pc}$) in dense cores. The transsonic and supersonic flows observed elsewhere must be driven, as turbulent motions otherwise decay away on timescales of order the crossing time of their outer scale. The largest energy source in star-forming regions of galaxies is supernova explosions. Even turbulence within dense molecular clouds may derive primarily from field
supernovae (not the few supernovae that may be associated with the cloud itself before its disappearance, but the background explosions from B stars formed within the previous 50 Myr or so.) Outside of star-forming parts of galactic disks, turbulence may be driven by magnetorotational instabilities, or even interaction between gravitation and rotation.

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