Semi-Supervised Learning with Meta-Gradient

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Abstract

In this work, we propose a simple yet effective meta-learning algorithm in the semi-supervised settings. We notice that existing consistency-based approaches mostly do not consider the essential role of the label information for consistency regularization. To alleviate this issue, we bridge the relationship between the consistency loss and label information by unfolding and differentiating through one optimization step. Specifically, we exploit the pseudo labels of the unlabeled examples which are guided by the meta-gradients of the labeled data loss so that the model can generalize well on the labeled examples. In addition, we introduce a simple first-order approximation to avoid computing higher-order derivatives and guarantee scalability. Extensive evaluations on the SVHN, CIFAR, and ImageNet datasets demonstrate that the proposed algorithm performs favorably against the state-of-the-art methods.

1 Introduction

The rapid advances of deep neural networks can be in part attributed to the availability of large-scale datasets with extensive annotations, which require considerable human labor. However, a typical real-world scenario is that only a small amount of data has the corresponding annotations while the majority of training examples are unlabeled. Numerous semi-supervised learning (SSL) methods have since been developed in which the unlabeled data are exploited to facilitate generalizing the learned models.

Existing SSL algorithms include co-training [1,2], label propagation [3], graph regularization [4], and the consistency-enforcing approaches [5,6,7,8,9,10,11]. Notably, the consistency-based approaches treat SSL as a generalization problem and enforce consistent predictions against small perturbations of the input data or model parameters. The basic assumption is that similar training examples are more likely to belong to the same category, and that the predictions of the network in multiple passes should be consistent [6]. As such, the consistency-based approaches are essentially designing pseudo labels from the predictions of the same input signals, while the incorrect predictions may misguide the training process. To improve the quality of the pseudo labels, two orthogonal directions, i.e., dedicating to carefully designed perturbations [8,11] and delving into better role models [7,9], have been introduced. Aside from the aforementioned approaches, Athiwaratkun et al. [10] analyze the training dynamics of the models trained with the consistency regularization and propose a variant of the stochastic weight averaging (SWA) [12], i.e., fastSWA, to improve performance and accelerate convergence.
Viewed from another perspective, the consistency-based algorithms can be broadly formulated as generating pseudo labels for the training examples, especially for the unlabeled data, in a way that the model can generalize well on the unseen examples. However, the label information, which is the only task-specific information for SSL, is not considered in the computation of the previous consistency losses. As a result, these consistency losses are somewhat task-agnostic and not guaranteed to improve the generalization ability on the specific task. To alleviate this issue, we propose a meta-learning algorithm in which the pseudo labels are designed explicitly for generalization on the task of interest. Specifically, we treat the labeled data as a validation set, and infer the pseudo labels from the meta-gradients of the validation loss in an end-to-end manner. In this way, the label information is included in the consistency loss and the generalization ability on the specific task is guaranteed to be improved, as indicated by the decrease of validation loss (see Theorem 1). We further introduce a simple first-order approximation to alleviate the issue of computing higher-order derivatives, and an improved training protocol to address the sample bias problem. Under mild conditions, the proposed meta-learning algorithm enjoys a convergence rate of $O(1/\epsilon^2)$, which is identical to that of the stochastic gradient descent (SGD) algorithm. Extensive experimental results demonstrate that our method performs favorably against the state-of-the-art approaches on the SVHN [13], CIFAR [14], and ImageNet [15] datasets, and the ablation studies validate the effectiveness of each component of our approach.

2 Related Work

Consistency-based Semi-Supervised Learning. The consistency regularization term measures the discrepancy between the predictions and the pseudo labels, which are typically generated by the same data with small perturbations on the input signals [6,8] or model parameters [9]. For the II-model [6], the predictions and pseudo labels are generated by the same model with different data augmentations through different forward passes. Laine et al. [7] propose the temporal ensembling approach to improve the quality of pseudo labels by keeping an exponential moving average (EMA) of the history predictions of each training example. However, the scalability of this method is limited since the memory footprint grows linearly with the number of training examples. Instead, Tarvainen et al. [8] present the mean teacher method to track the model parameters and generate pseudo labels using the teacher model parameterized by the EMA of the history model parameters. On the other hand, Miyato et al. [8] present the virtual adversarial training scheme to focus on disturbing the input data in an adversarial direction, and Yu et al. [11] decouple the adversarial direction into the tangent and normal directions of the embedded training data manifold. With the dedicated perturbation directions, the robustness of the learned model can be significantly improved. Aside from the above-mentioned methods, Athiwaratkun et al. [10] introduce the fastSWA method to average the model parameters along the training timeline.

We notice that existing SSL methods do not exploit the label information when computing the consistency regularization. To alleviate this, we relate the consistency loss with the label information by unfolding and differentiating through one optimization step. In this way, the update of the pseudo labels is guided by the meta-gradients of the labeled data loss, and the consistency loss is designed to improve the generalization ability specially for the underlying task. We also experimentally verify the important role of the label information in the effectiveness of consistency regularization.

Optimization-based Meta-Learning. Numerous optimization-based meta-learning algorithms [18,19,20,21,22] have been developed in recent years. Notably, Finn et al. [18] formulate the meta-learning problem in a nested optimization format, where the inner loop imitates the process of adaptation, while the outer loop focuses on optimizing the meta-objective. The inner-optimization is further replaced by a single SGD step so that the meta-objective can be optimized in an end-to-end manner. Thanks to its simplicity and effectiveness, optimization-based meta-learning algorithms have been applied to a wide range of vision and learning problems including example re-weighting [23], neural architecture search [24], and unrolled generative models [25]. In this work, we develop a meta-learning algorithm in the semi-supervised settings and demonstrate the potential of meta-learning for these tasks. In addition, we present the theoretical convergence analysis of the proposed algorithm.

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1 Due to the existence of randomized operations, such as dropout [16] and shake-shake regularization [17], the outputs of the same input signal may be different in multiple forward passes.
3 Proposed Algorithm

In a typical semi-supervised setting, we are given a few labeled data \( D_l = \{(x_i, y_i) : k = 1, \ldots, N_l^l\} \) and a large amount of unlabeled data \( D_u = \{x_i^u : i = 1, \ldots, N_u^u\} \), where \( N_l \ll N_u \).

The goal is to train a classifier that generalizes well on the unseen test data drawn from the same distribution. In the following, we present the algorithmic details. For presentation clarity, a table of notations is provided in the supplementary material.

3.1 Learning to Generalize

Let \( f(x; \theta) \) be a generic classifier parameterized by \( \theta \) and \( \Phi(p, y) \) be a non-negative function that measures the discrepancy between distributions \( p \) and \( y \). We further assume \( \Phi(p, y) = 0 \) if and only if \( p = y \), and thus if \( y \) is fixed, then \( p = y \) is the (global) minima of the function \( \Phi(., y) \). We formulate the loss of \((x, y)\) as \( \mathcal{L}(x, y; \theta) = \Phi(f(x; \theta), y) \). The learning-to-generalize problem is then formulated as

\[
\begin{align*}
\min_{\theta} & \quad \sum_{k=1}^{N_l^l} \mathcal{L}(x_k^l, y_k; \theta^*(\mathcal{Y})) \\
\text{s.t.} & \quad \theta^*(\mathcal{Y}) = \arg \min_{\theta} \sum_{i=1}^{N_u^u} \mathcal{L}(x_i^u, \tilde{y}_i; \theta),
\end{align*}
\]

where \( \mathcal{Y} = \{\tilde{y}_i : i = 1, \ldots, N_u^u\} \) denotes the pseudo labels of the unlabeled data.

Solving the nested minimization problem exactly is computationally prohibitive because calculating the gradients of the outer loop requires an entire inner optimization. Thus, we online approximate the outer loop gradients in a way similar to [23]. Specifically, we adapt the generated pseudo labels based on the current mini-batch and replace the inner optimization with a single SGD step. As such, the descent direction of the pseudo labels is guided by the back-propagated signals of the labeled data loss.

Consider the gradient-based deep learning framework in which gradients are calculated at the mini-batch level and the SGD-like optimizer is used to update model parameters. With a little bit abuse of notations, at the \( t \)th training step, a mini-batch of labeled data \( \{(x_k^l, y_k) : k = 1, \ldots, B_l^l\} \) and a mini-batch of unlabeled data \( \{x_i^u : i = 1, \ldots, B_u^u\} \) are sampled, where \( B_l^l \) and \( B_u^u \) denote the batch sizes of the labeled data and unlabeled data, respectively. The pseudo labels of the unlabeled data are initialized as the current predictions of the classifier:

\[
\tilde{y}_i = f(x_i^u; \theta_t).
\]

We then compute the unlabeled data loss and back-propagate the gradients:

\[
\begin{align*}
\mathcal{L}(x_i^u, \tilde{y}_i; \theta_t) &= \Phi(f(x_i^u; \theta_t), \tilde{y}_i), \\
\nabla \theta_t &= \frac{1}{B_u^u} \sum_{i=1}^{B_u^u} \nabla_{\theta} \mathcal{L}(x_i^u, \tilde{y}_i; \theta_t).
\end{align*}
\]

Note that since the initialized pseudo labels are precisely the predictions of the classifier, the unlabeled loss is zero, which gives zero gradients as well. However, the Jacobian matrix of \( \nabla \theta_t \) w.r.t. the pseudo labels is not necessarily a zero matrix, thus making optimization via differentiating \( \nabla \theta_t \) possible.

We apply one SGD step on the model parameters:

\[
\tilde{\theta}_{t+1} = \theta_t - \alpha_t \nabla \theta_t,
\]

where \( \alpha_t \) is the learning rate of the inner loop. The SGD step is then evaluated on the labeled data and the labeled data loss is treated as the meta-objective. We differentiate the meta-objective through the SGD step and compute gradients w.r.t. the pseudo labels:

\[
\begin{align*}
\mathcal{L}(x_k^l, y_k; \tilde{\theta}_{t+1}) &= \Phi(f(x_k^l; \tilde{\theta}_{t+1}), y_k), \\
\nabla \tilde{y}_k &= \frac{1}{B_l^l} \sum_{k=1}^{B_l^l} \nabla_{\tilde{y}_k} \mathcal{L}(x_k^l, y_k; \tilde{\theta}_{t+1}).
\end{align*}
\]
Note that by unfolding one SGD step, the labeled data loss is related to the pseudo labels of the unlabeled data. Moreover, since the labeled data loss serves as the meta-objective to be differentiated, the update of the pseudo labels is guided by the label information, \( i.e. \), the meta-gradients, and thus concerns the specific task on interest. Similar techniques are developed in the optimization-based meta-learning literature [13] and employed in a wide range of applications [23, 24, 25, 26].

### Algorithm 1 Meta-Learning Algorithm.
**Input:** regular learning rates \( \{\alpha_t\} \), meta learning rates \( \{\beta_t\} \)

```plaintext
for \( t := 1 \) to \#iters do
  \{ \( x_i^t, y_i \) \}_{k=1}^{B_t} \leftarrow \text{BatchSampler}(D^t)
  \{ x_i^t \}_{k=1}^{B_t} \leftarrow \text{BatchSampler}(D^n)
  \bar{y}_i = f(x_i^t; \theta_t)
  \mathcal{L}(x_i^t, \bar{y}_i; \theta_t) = \Phi(f(x_i^t; \theta_t), \bar{y}_i)
  \nabla \theta_t = \frac{1}{B_t} \sum_{k=1}^{B_t} \nabla_{\theta} \mathcal{L}(x_i^t, \bar{y}_i; \theta_t)
  \theta_{t+1} = \theta_t - \alpha_t \nabla \theta_t
  \mathcal{L}(x_i^t, \bar{y}_i; \theta_{t+1}) = \Phi(f(x_i^t; \theta_{t+1}), y_i)
  \nabla \bar{y}_i = \frac{1}{B_t} \sum_{k=1}^{B_t} \nabla_{\bar{y}_i} \mathcal{L}(x_i^t, y_i; \theta_{t+1})
  \bar{y}_i = \bar{y}_i - \beta_t \nabla \bar{y}_i
  \mathcal{L}(x_i^t, \bar{y}_i; \theta_{t+1}) = \Phi(f(x_i^t; \theta_{t+1}), \bar{y}_i)
  \nabla \theta_{t+1} = \text{Optimizer}(\theta_t, \nabla \theta_t, \alpha_t)
end
```

Finally, we perform one SGD step on the pseudo labels,
\[
\hat{y}_i = \bar{y}_i - \beta_t \nabla \bar{y}_i,
\] (6)

where \( \beta_t \) is the meta learning rate, and compute the consistency loss from the unlabeled data and the updated pseudo labels. The meta-learning algorithm is summarized in Algorithm 1.

### 3.2 First-Order Approximation

In Section 3.1 the most computationally expensive operation is differentiation through the SGD step in Eq. (5), as the second-order derivative is involved. To avoid this, we apply the chain rule to the second-order derivative:
\[
\frac{\partial}{\partial y_{i,j}} (\mathcal{L}(x_i^t, y_i; \theta_{t+1})) = \frac{\alpha_t}{B_t} \nabla_{\theta} \frac{\partial}{\partial y_{i,j}} (\mathcal{L}(x_i^t, \bar{y}_i; \theta_t) \cdot \nabla_{\theta} \mathcal{L}(x_i^t, y_i; \theta_t)).
\] (7)

The gradients of the validation loss w.r.t. the pseudo labels can thus be formulated as
\[
\nabla y_{i,j} = \frac{1}{B_t} \sum_{k=1}^{B_t} \frac{\partial}{\partial y_{i,j}} (\mathcal{L}(x_i^t, y_i; \theta_t)) = -\frac{\alpha_t}{B_t} \nabla_{\theta} \frac{\partial}{\partial y_{i,j}} (\mathcal{L}(x_i^t, \bar{y}_i; \theta_t) \cdot \left( \frac{1}{B_t} \sum_{k=1}^{B_t} \nabla_{\theta} \mathcal{L}(x_i^t, y_i; \theta_t) \right)).
\] (8)

Let
\[
\nabla \theta_t = \frac{1}{B_t} \sum_{i=1}^{B_t} \nabla_{\theta} \mathcal{L}(x_i^t, y_i; \theta_t),
\] (9)

and then it can be easily shown with Taylor expansion that
\[
\nabla y_{i,j} = -\frac{\alpha_t}{2B_t^2} \left( \frac{\partial}{\partial y_{i,j}} (\mathcal{L}(x_i^t, \bar{y}_i; \theta_t + \epsilon \nabla \theta_t) - \frac{\partial}{\partial y_{i,j}} (\mathcal{L}(x_i^t, \bar{y}_i; \theta_t - \epsilon \nabla \theta_t)) \right), \quad \epsilon \to 0.
\] (10)

Thus, we adopt the first-order approximation and use a sufficiently small \( \epsilon \) to approximate \( \nabla y_{i,j} \). As suggested in [24], we use \( \epsilon = 0.01/\|\nabla \theta_t\|_2 \) in this work.
Furthermore, the gradients w.r.t. the pseudo labels can be calculated in the closed form. Here, following the common practice of consistency-based SSL [9], we adopt the Kullback–Leibler divergence loss $\Phi_{\text{KL}}(\mathbf{p}, \mathbf{y}) = \sum_{n} y_{n} \log(y_{n}/p_{n})$ as the regular labeled data loss, and the mean squared error (MSE) loss $\Phi_{\text{MSE}}(\mathbf{p}, \mathbf{y}) = \|\mathbf{p} - \mathbf{y}\|_{2}^{2}$ for the consistency loss. For the MSE loss, the gradients w.r.t. the pseudo labels are approximated by:

$$\nabla \tilde{y}_{i} \approx \frac{\alpha_{t}}{B u_{i}} \left( f(x_{i}^{u}; \theta_{t} + \epsilon \nabla \theta_{t}) - f(x_{i}^{u}; \theta_{t} - \epsilon \nabla \theta_{t}) \right).$$

(11)

### 3.3 Improved Training Protocol

The above-discussed meta-learning algorithm utilizes the unlabeled examples to improve the generalization ability. However, the sampling bias issue still remains in SSL. Motivated by the success of the mixup augmentation [27] in SSL [28, 29], we incorporate the cross-domain mixup augmentation in the proposed meta-learning algorithm. We assume the mini-batches of labeled and unlabeled data are of the same batch size, i.e., $B^{l} = B^{u} = B$. Next, we interpolate between each pair of labeled and unlabeled examples to generate new training data. Note that when generating the corresponding labels, we interpolate between the actual labels $y_{i}$ of the labeled examples and the updated pseudo labels $\tilde{y}_{i}$ of the unlabeled examples,

$$x_{i}^{\text{in}} = \lambda_{i} x_{i}^{l} + (1 - \lambda_{i}) x_{i}^{u}, \quad i = 1, \ldots, B,$$

$$y_{i}^{\text{in}} = \lambda_{i} y_{i} + (1 - \lambda_{i}) \tilde{y}_{i}, \quad i = 1, \ldots, B,$$

(12)

where $\lambda_{1}, \ldots, \lambda_{B}$ are i.i.d. samples drawn from the Beta($\gamma, \gamma$) distribution. Finally, the total loss is formulated as

$$L = \sum_{i=1}^{B} \mathcal{L}_{\text{KL}}(x_{i}^{\text{in}}, y_{i}^{\text{in}}; \theta_{t}) + \sum_{i=1}^{B} \mathcal{L}_{\text{MSE}}(x_{i}^{u}, \tilde{y}_{i}; \theta_{t}),$$

(13)

and the algorithm with first-order approximation and mixup augmentation is illustrated in Algorithm 2.

### 3.4 Convergence Analysis

In this section, we present the convergence analysis of Algorithm 1. Due to the scarcity of labeled examples, we assume all labeled data are sampled at each step, i.e., $B^{l} = N^{l}$, and that the MSE loss is used in the unlabeled consistency loss (Eq. (3)). Under mild conditions, we show that Algorithm 1 is guaranteed to converge to a critical point of the meta-objective (Theorem 1), and enjoys a convergence rate of $O(1/t^{2})$ (Theorem 2), which is the same as the regular SGD. The proofs are presented in the supplementary material.

**Theorem 1.** Let

$$G(\theta; \mathcal{D}^{l}) = \frac{1}{N^{l}} \sum_{k=1}^{N^{l}} \mathcal{L}(x_{k}^{l}, y_{k}; \theta_{t})$$

(14)

be the loss function of the labeled examples. Assume

(i) the gradient function $\nabla_{\theta}G$ is Lipschitz-continuous with a Lipschitz constant $L_{0}$; and

(ii) the norm of the Jacobian matrix of $f$ w.r.t. $\theta$ is upper-bounded by a constant $M$, i.e.,

$$\|J_{\theta}f(x_{i}^{u}; \theta)\| \leq M, \quad \forall i \in \{1, \ldots, N^{u}\}.$$  

(15)

If the regular learning rate $\alpha_t$ and meta learning rate $\beta_t$ satisfy $\alpha_{t}^{2} \beta_{t} < (4M^{2}L_{0})^{-1}$, then each SGD step of Algorithm 2 will decrease the validation loss $G(\theta)$, regardless of the selected unlabeled examples, i.e.,

$$G(\theta_{t+1}) \leq G(\theta_{t}), \quad \text{for each } t.$$ 

(16)

Furthermore, the equality holds if and only if $\nabla \tilde{y} = 0$ for the selected unlabeled batch at the $t^{th}$ step.
Table 1: Semi-supervised classification error rates of the Conv-Large [9] architecture on the SVHN, CIFAR-10, and CIFAR-100 datasets. The numbers of labeled data are 1k, 4k, and 10k for these three datasets, respectively.

| Method          | SVHN  | CIFAR-10 | CIFAR-100 |
|-----------------|-------|----------|-----------|
| Π-Model [7]     | 4.82% | 12.36%   | 39.19%    |
| TE [7]          | 4.42% | 12.16%   | 38.65%    |
| MT [9]          | 3.95% | 12.31%   | -         |
| MT+SNTG [31]    | 3.86% | 10.93%   | -         |
| VAT [8]         | 5.42% | 11.36%   | -         |
| VAT+Ent [8]     | 3.86% | 10.55%   | -         |
| VAT+Ent+SNTG [31]| 3.83% | 9.89%    | -         |
| VAT+VAdD [32]   | 3.55% | 9.22%    | -         |
| MA-DNN [33]     | 4.21% | 11.91%   | 34.51%    |
| Co-training [34]| 3.29% | 8.35%    | 34.63%    |
| MT+fastSW A [10]| -     | 9.05%    | 33.62%    |
| TNAR-VAE [11]   | 3.74% | 8.85%    | -         |
| ADA-Net [28]    | 4.62% | 10.30%   | -         |
| ADA-Net+fastSW A [28]| -     | 8.72%    | -         |
| DualStudent [35]| -     | 8.89%    | 32.77%    |
| Ours            | 3.15% | 7.78%    | 30.74%    |
| Fully-Supervised| 2.67% | 4.88%    | 22.10%    |

Theorem 2. Assume the same conditions as in Theorem 1 and
\[
\inf \beta_t - 4 \alpha_t^2 \beta_t^2 M^2 L_0 = D_1 > 0 \quad \text{and} \quad \inf \alpha_t = D_2 > 0.
\]  
We further assume that the unlabeled dataset contains the labeled dataset, i.e., \( D^l \subseteq D^u \). Then, Algorithm 1 achieves \( \mathbb{E} [\|\nabla \theta G(\theta_t)\|^2] \leq \epsilon \) in \( O(1/\epsilon^2) \) steps, i.e.,
\[
\min_{1 \leq t \leq T} \mathbb{E} [\|\nabla \theta G(\theta_t)\|^2] \leq \frac{C}{\sqrt{T}},
\]
where \( C \) is a constant independent of the training process.

Remarks. (i) The assumption in (15) is realistic. Here, we assume the neural network \( f \) is continuously differentiable w.r.t. \( \theta \). Due to the existence of norm-based regularization, i.e., weight decay, we can assume \( \theta \) is optimized within a compact set in the parameter space. The Jacobian function \( J_\theta f \) is thus bounded within the compact set due to its continuity. Furthermore, since there are finite training examples, the bound in (15) is plausible. (ii) The conditions in (17) specify that the learning rates \( \alpha_t \) and \( \beta_t \) can neither grow too large nor decay to zero too rapidly. The step learning rate annealing strategy can satisfy this condition as long as the initial learning rate is sufficiently small. (iii) The condition \( D^l \subseteq D^u \) can be satisfied by incorporating the labeled data into the unlabeled set.

4 Experiments

We evaluate the proposed algorithm on the SVHN [13], CIFAR [14], and ImageNet [15] datasets. The 13-layer Conv-Large [9] and 26-layer ResNet [30] with the shake-shake regularization [17] are used as the backbone models. More implementation details can be found in the supplementary material.

Table 2: Semi-supervised classification error rates of the 26-layer ResNet [30] architecture with the shake-shake regularization [17].

| Dataset | CIFAR-10 | CIFAR-100 |
|---------|----------|-----------|
| #Images |          |           |
| #Labels |          |           |
| 50k     | 50k      | 50k       |
| 1k      | 2k       | 4k        |
| fastSW A [10]| 6.6% | 5.7% | 5.1% |
| Ours    | 6.3%     | 5.2%      | 4.1% |
| 50k     | 50k      | 50k       |
| 1k      | 2k       | 4k        | 6k    |
| 8k      | 1k       | 10k       |
| 66%     | 57%      | 51%       |
| 26.7%   | 25.1%    | 22.9%     |

Table 3: Semi-supervised classification error rates on the ImageNet [15] dataset. 10% training images are used as the labeled data.

| Method          | Top-1 | Top-5 |
|-----------------|-------|-------|
| Labeled-Only    | 53.65%| 31.01%|
| MT [9]          | 49.07%| 23.59%|
| Co-training [34]| 46.50%| 22.73%|
| ADA-Net [28]    | 44.91%| 21.18%|
| Ours            | 44.87%| 18.88%|
| Fully-Supervised| 29.75%| 10.32%|
4.1 Results on the SVHN and CIFAR Datasets

In Table 1, we report the semi-supervised classification error rates of the proposed algorithm and state-of-the-art methods on the SVHN, CIFAR-10, and CIFAR-100 datasets. The proposed meta-learning algorithm performs favorably against the previous approaches on all three datasets. In addition, we explore the effectiveness of the proposed algorithm on different backbone architectures and evaluate on the 26-layer ResNet [30] with the shake-shake regularization [17]. Since only a few previous papers include experiments on this backbone, we just compare the performance with the “fastSWA” method [10] which gives quite complete results and achieves the state-of-the-art accuracy. Table 2 shows that the proposed algorithm performs favorably under all different experimental configurations, even with fewer labeled examples, indicating the efficacy of the consistency loss guided by the meta-gradients.

4.2 Results on the ImageNet Dataset

The evaluation results with the ResNet-18 [30] backbone on the ImageNet dataset [15] are summarized in Table 3. The proposed algorithm performs well against the ADA-Net [28] in terms of top-5 accuracy. In addition, we demonstrate the accuracy curves of the baseline setting and our approach, where the baseline setting means only 10% of the total training examples are used as the labeled data used during training. Figure 1 shows that merely involving 10% training examples will lead to severe overfitting, while the problem is alleviated in our approach thanks to the explicit learning-to-generalize training scheme. These results suggest that the consistency loss can effectively regularize the training and the proposed algorithm is beneficial to the generalization ability of the learned model.

4.3 Ablation Studies

Effectiveness of Components. We analyze the contributions of the meta-learning and mixup augmentation components of the proposed algorithm. The experimental settings are the same as those in Table 1. Table 4 shows both components can significantly improve the classification accuracy in the semi-supervised settings. Moreover, the meta-learning component can further improve performance with the presence of the mix-up augmentation, indicating that meta-learning is orthogonal to the existing data augmentation techniques as a research direction.

Table 4: Ablation study of the meta-learning component and the mixup augmentation. The same number of labeled data is used as in Table 1.

| No. | Meta | Mix-Up | SVHN  | CIFAR-10 | CIFAR-100 |
|-----|------|--------|-------|----------|----------|
| 1   |      |        | 9.76% | 15.43%   | 38.74%   |
| 2   | ✓    |        | 3.68% | 11.63%   | 35.40%   |
| 3   |      | ✓      | 5.60% | 11.10%   | 32.67%   |
| 4   | ✓    | ✓      | 3.15% | 7.78%    | 30.74%   |
Impact of #Labels. As shown in Figure 2, we evaluate the robustness of our method against the variation of the number of labels on the SVHN and CIFAR-10 datasets. The same experimental settings, except the number of labels, are used as in Table 1. We can find that the accuracy of the labeled-only baseline degrades heavily when reducing the number of labels. However, the proposed SSL method can retain a relatively high performance under each setting. In general, the scarcity of labeled data may lead to a worse generalization ability and more severe overfitting. Therefore, the proposed method can effectively improve the generalization ability even with fewer labeled examples.

Feature Visualization. To further analyse the efficacy of our method, we visualize the SVHN features by projecting 128-dimensional features onto a two-dimensional space using the t-SNE technique. For comparison, we also present the feature visualization of the labeled-only baseline method. As displayed in Figure 3, there is a considerable empirical distribution discrepancy between labeled and unlabeled examples of the baseline method. In contrast, such discrepancy is reduced and the feature distributions of different domains are aligned to some extent by our approach, indicating the generalization ability is improved. Furthermore, considering the unlabeled and test samples, we observe the margin among features of different categories is obviously clearer under our approach, which results in more discriminative features and better classification accuracy.
5 Conclusion

In this work, we propose a simple yet effective meta-learning method for semi-supervised learning tasks. While existing methods do not consider the essential role of the labeled data loss when enforcing the consistency regularization terms, we bridge the relationship between the consistency regularization and the labeled data loss by the meta-gradients. As such, the pseudo labels of the unlabeled examples are learned explicitly for improving the generalization ability on the specific task. We further introduce a first-order approximation and an improved training protocol to alleviate the heavy computational load and boost the performance. In addition, we analyze the convergence conditions of the proposed algorithm. Extensive experimental results demonstrate that the proposed algorithm performs favorably against the state-of-the-art methods on the SVHN, CIFAR, and ImageNet datasets, indicating the meta-learning approach can be effectively applied to semi-supervised learning tasks.

Broader Impact

Despite proposing yet another competitive semi-supervised classification method, this paper could have a broad potential impact in the following aspects. First, the idea of this paper may pave a new path for other semi-supervised tasks. By viewing the labeled data as the validation set and resorting to the training labels, we propose a better consistency loss, which is more task-specific and thus leads to better generalization ability on the task of interest. This could inspire different approaches to other semi-supervised tasks and applications. Second, we address the nested optimization in a meta-learning fashion with theoretical convergence guarantees, which could bring more rigorous and insightful mathematical perspectives and understanding. These are beneficial to the theoretical research communities. Third, the proposed semi-supervised method could have broader large-scale industrial applications. It could save much human labor for annotation and lots of training costs by exploiting the abundant unlabeled data. Moreover, the learned models by our method have better generalization ability, which are more suitable for the real-life scenarios applications.

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A Table of Notations

The notations in this work are summarized in Table 5.

| Symbol | Description |
|--------|-------------|
| $x^l_k$ | The $k$th labeled training example |
| $x^u_i$ | The $i$th unlabeled training example |
| $y_k$ | Actual label of $x^l_k$ |
| $\tilde{y}_k$ | Initialized proximal label of $x^u_i$ |
| $\hat{y}_k$ | Updated proximal label of $x^u_i$ |
| $\tilde{y}_{i,j}$ | The $j$th entry of $\tilde{y}_i$ (Same for $\hat{y}_{i,j}$) |
| $\tilde{y}$ | Proximal labels of the unlabeled mini-batch $\tilde{y} = \{\tilde{y}_i : i = 1, \cdots, B^u\}$ (Seen as a vector) |
| $\hat{y}$ | Proximal labels of the unlabeled mini-batch at the $t$th step |
| $\nabla \theta_{l,t}$ | Gradients of labeled mini-batch at the $t$th step |
| $\nabla \tilde{y}_{i}$ | Gradients of the proximal label $\tilde{y}_i$ |
| $\nabla \hat{y}$ | Gradients of proximal labels $\nabla \hat{y} = \{\nabla \hat{y}_i : i = 1, \cdots, B^u\}$ (Seen as a vector) |
| $\alpha_t, \beta_t$ | Regular learning rate and meta learning rate at the $t$th step |
| $N^l, N^u$ | Numbers of labeled examples and unlabeled examples |
| $B^l, B^u$ | Batch sizes for labeled examples and unlabeled examples |

B Convergence Analysis of Semi-Supervised Learning with Meta-Gradient

Lemma 1. Let

\[
G(\theta; D^l) = \frac{1}{N^l} \sum_{k=1}^{N^l} \mathcal{L}(x^l_k, y_k; \theta_t) \tag{19}
\]

be the loss function of the labeled examples. Assume

(i) the gradient function $\nabla \theta G$ is Lipschitz-continuous with a Lipschitz constant $L_0$; and

(ii) the norm of the Jacobian matrix of $f$ w.r.t. $\theta$ is upper-bounded by a constant $M$, i.e.,

\[
\|J_{\theta} f(x^i; \theta)\| \leq M, \quad \forall i \in \{1, \cdots, N^u\} \tag{20}
\]

2 For a bit abuse of notations, the subscript $\tau$ of $\tilde{y}$ specify the current step number, while subscript $(i,j)$ of indicates the $j$th entry of the i-th proximal label. The step subscript is omitted when there is no ambiguity.
If the labeled data loss is considered as a function of the pseudo-targets $\tilde{y} = \{\tilde{y}_i : i = 1, \cdots, B^n\}$, i.e., $H(\tilde{y}) = G(\theta_{t+1}(\tilde{y}))$, then the gradient function $\nabla_{\tilde{y}} H$ is also Lipschitz-continuous and its Lipschitz constant is upper-bounded by $4\alpha_t^2 M^2 L_0$.

**Proof.** Recall the SGD update formula

$$
\tilde{\theta}_{t+1} = \theta_t - \frac{\alpha_t}{B^n} \sum_{i=1}^{B^n} \nabla_{\theta} L(x^u_i, \tilde{y}_i; \theta_t),
$$

and we have

$$
\frac{\partial \tilde{\theta}_{t+1}}{\partial \tilde{y}_{i,j}} = -\frac{\alpha_t}{B^n} \frac{\partial^2 L}{\partial \tilde{y}_{i,j} \partial \theta_t}(x^u_i, \tilde{y}_i; \theta_t).
$$

Then, we expand the partial derivative of each entry $\tilde{y}_{i,j}$:

$$
\frac{\partial H}{\partial \tilde{y}_{i,j}} = \frac{1}{N^1} \sum_{k=1}^{N^1} \sum_{l} \frac{\partial L}{\partial \theta_t} (x^l_k, y^i_k; \tilde{\theta}_{t+1}) \frac{\partial \tilde{\theta}_{t+1}}{\partial \tilde{y}_{i,j}}
$$

$$
= -\frac{\alpha_t}{B^n N^1} \sum_{k=1}^{N^1} \sum_{l} \frac{\partial L}{\partial \theta_t} (x^l_k, y^i_k; \tilde{\theta}_{t+1}) \frac{\partial^2 L}{\partial \tilde{y}_{i,j} \partial \theta_t}(x^u_i, \tilde{y}_i; \theta_t)
$$

$$
= -\frac{\alpha_t}{B^n} \sum_{k=1}^{N^1} \sum_{l} \nabla_{\theta} L(x^l_k, y^i_k; \tilde{\theta}_{t+1}) \cdot \nabla_{\theta} \frac{\partial L}{\partial \tilde{y}_{i,j}}(x^u_i, \tilde{y}_i; \theta_t)
$$

Then, for arbitrary $\tilde{y}^1$ and $\tilde{y}^2$,

$$
\frac{\partial H}{\partial \tilde{y}_{i,j}} \bigg|_{\tilde{y} = \tilde{y}^1} - \frac{\partial H}{\partial \tilde{y}_{i,j}} \bigg|_{\tilde{y} = \tilde{y}^2}
$$

$$
= \frac{\alpha_t}{B^n} \left( \nabla_{\theta} G(\tilde{\theta}^2_{t+1}) \cdot \nabla_{\theta} \frac{\partial L}{\partial \tilde{y}_{i,j}}(x^u_i, \tilde{y}_i^2; \theta_t) - \nabla_{\theta} G(\tilde{\theta}^1_{t+1}) \cdot \nabla_{\theta} \frac{\partial L}{\partial \tilde{y}_{i,j}}(x^u_i, \tilde{y}_i^1; \theta_t) \right)
$$

$$
= \frac{\alpha_t}{B^n} \left( \nabla_{\theta} \frac{\partial L}{\partial \tilde{y}_{i,j}}(x^u_i, \tilde{y}_i^2; \theta_t) \cdot \left( \nabla_{\theta} G(\tilde{\theta}^2_{t+1}) - \nabla_{\theta} G(\tilde{\theta}^1_{t+1}) \right) + \nabla_{\theta} G(\tilde{\theta}^1_{t+1}) \cdot \left( \nabla_{\theta} \frac{\partial L}{\partial \tilde{y}_{i,j}}(x^u_i, \tilde{y}_i^1; \theta_t) - \nabla_{\theta} \frac{\partial L}{\partial \tilde{y}_{i,j}}(x^u_i, \tilde{y}_i^2; \theta_t) \right) \right),
$$

where $\tilde{\theta}^r_{t+1} = \tilde{\theta}_{t+1}(\tilde{y}^r)$. As the MSE loss is used for unlabeled data, we have $\frac{\partial L}{\partial \tilde{y}_{i,j}}(x^u_i, \tilde{y}_i; \theta_t) = -2(f_j(x^u_i; \theta_t) - \tilde{y}_i)$. Here, $f_j$ denotes the $j^{th}$ entry of $f$. Therefore,

$$
\frac{\partial H}{\partial \tilde{y}_{i,j}} \bigg|_{\tilde{y} = \tilde{y}^1} - \frac{\partial H}{\partial \tilde{y}_{i,j}} \bigg|_{\tilde{y} = \tilde{y}^2}
$$

$$
= \frac{2\alpha_t}{B^n} \nabla_{\theta} \frac{\partial L}{\partial \tilde{y}_{i,j}} f_j(x^u_i; \theta_t) \cdot \left( \nabla_{\theta} G(\tilde{\theta}^2_{t+1}) - \nabla_{\theta} G(\tilde{\theta}^1_{t+1}) \right), \quad \text{and}
$$

$$
\nabla_{\tilde{y}} H(\tilde{y}^1) - \nabla_{\tilde{y}} H(\tilde{y}^2)
$$

$$
= \frac{2\alpha_t}{B^n} J_{\theta} f(x^u_i; \theta_t) \cdot \left( \nabla_{\theta} G(\tilde{\theta}^2_{t+1}) - \nabla_{\theta} G(\tilde{\theta}^1_{t+1}) \right).
$$

By taking the norm, we have

$$
\left\| \nabla_{\tilde{y}} H(\tilde{y}^1) - \nabla_{\tilde{y}} H(\tilde{y}^2) \right\| \leq \frac{2\alpha_t}{B^n} \left\| J_{\theta} f(x^u_i; \theta_t) \right\| \left\| \nabla_{\theta} G(\tilde{\theta}^2_{t+1}) - \nabla_{\theta} G(\tilde{\theta}^1_{t+1}) \right\|.
$$

By assumptions, we have

$$
\left\| J_{\theta} f(x^u_i; \theta_t) \right\| \leq M, \quad \text{and}
$$

$$
\left\| \nabla_{\theta} G(\tilde{\theta}^2_{t+1}) - \nabla_{\theta} G(\tilde{\theta}^1_{t+1}) \right\| \leq L_0 \left\| \tilde{\theta}^2_{t+1} - \tilde{\theta}^1_{t+1} \right\|.
$$
We further assume that the unlabeled dataset contains the labeled dataset, i.e., $E_1$ achieves

Moreover, the equality holds if and only if

Theorem 3. Assume the same conditions as in lemma 1. If the regular learning rate $\alpha_t$ and meta learning rate $\beta_t$ satisfy $\alpha_t^2 \beta_t < (4M^2L_0)^{-1}$, then each SGD step of Alg. I will lead to the decrease of the validation loss $G(\theta)$, regardless of the selected unlabeled examples, i.e.,

$$ G(\theta_{t+1}) \leq G(\theta_t), \text{ for each } t. $$

Moreover, the equality holds if and only if $\nabla \tilde{y} = 0$ for the selected unlabeled batch at the $t^{th}$ step.

Proof. According to the Lagrange Mean Value Theorem,

$$ \exists \xi \in (0, 1), \text{ s.t. } H(\tilde{y}) = H(\tilde{y} + \nabla_{\tilde{y}} H(\tilde{y} + \xi(\tilde{y} - \tilde{y})) \cdot (\tilde{y} - \tilde{y}). $$

Recall the update formula of the pseudo-targets, i.e., $\tilde{y} = \tilde{y} - \beta_t \nabla \tilde{y}$. Then, by the Lipschitz-continuity of $\nabla_{\tilde{y}} H$, we have

$$ H(\tilde{y}) = H(\tilde{y} - \beta_t \nabla_{\tilde{y}} H(\tilde{y} - \xi \beta_t \nabla \tilde{y}) \cdot \nabla \tilde{y} $$

$$ = H(\tilde{y}) - \beta_t \nabla_{\tilde{y}} H(\tilde{y}) \cdot \nabla \tilde{y} - \beta_t (\nabla_{\tilde{y}} H(\tilde{y} - \xi \beta_t \nabla \tilde{y}) - \nabla_{\tilde{y}} H(\tilde{y})) \cdot \nabla \tilde{y} $$

$$ \leq H(\tilde{y}) - \beta_t \nabla_{\tilde{y}} H(\tilde{y}) \cdot \nabla \tilde{y} + \beta_t^2 L_t \|\nabla \tilde{y}\|^2 $$

(According to (29))

$$ = H(\tilde{y}) - (\beta_t - \beta_t^2 L_t)\|\nabla \tilde{y}\|^2 $$

(By definition, $\nabla \tilde{y} = \nabla \tilde{y} H(\tilde{y})$)

$$ \leq H(\tilde{y}). $$

(Since $\beta_t < L_t^{-1}$)

Therefore, $G(\theta_{t+1}) = H(\tilde{y}) \leq H(\tilde{y}) = G(\theta_t)$.

Moreover, as long as $\alpha_t^2 \beta_t < (4M^2L_0)^{-1}$ is satisfied, the equality holds if and only if $\nabla \tilde{y} = 0$. 

Theorem 4. Assume the same conditions as in lemma 7 and

$$ \inf_t (\beta_t - 4\alpha_t^2 \beta_t^2 M^2L_0) = D_1 > 0 \text{ and } \inf_\alpha \alpha_t = D_2 > 0. $$

We further assume that the unlabeled dataset contains the labeled dataset, i.e., $D^u \subseteq D^u$. Then, Alg. I achieves $\mathbb{E} [\|\nabla G(\theta_t)\|^2] \leq \epsilon$ in $O(1/\epsilon^2)$ steps, i.e.,

$$ \min_{1 \leq t \leq T} \mathbb{E} [\|\nabla G(\theta_t)\|^2] \leq \frac{C}{\sqrt{T}}, $$

where $C$ is a constant independent of the training process.
Proof. According to (32) in the proof of Theorem 3 we have
\[
G(\theta_{t+1}) \leq G(\theta_t) - (\beta_t - \beta_t^2 L_t) \| \nabla \tilde{y}_t \|^2 \leq G(\theta_t) - (\beta_t - 4\alpha_t^2 \beta_t^2 M^2 L_0) \| \nabla \tilde{y}_t \|^2.
\] (35)

Therefore,
\[
G(\theta_t) - G(\theta_{t+1}) \geq (\beta_t - 4\alpha_t^2 \beta_t^2 M^2 L_0) \| \nabla \tilde{y}_t \|^2 \geq D_1 \| \nabla \tilde{y}_t \|^2.
\] (36)

By taking the expectation, we have
\[
E_{1 \sim t} [G(\theta_t)] - E_{1 \sim t} [G(\theta_{t+1})] \geq D_1 \| \nabla \tilde{y}_t \|^2.
\] (37)

Here, \( E_{1 \sim t} \) indicates the expectation is taken over the selected mini-batches of the first \( t \) steps. Next, we show \( E_{1 \sim t} [G(\theta_t)] = E_{1 \sim t-1} [G(\theta_t)] \), which is intuitive as the value of \( \theta_t \) only relies on the selected batches of the first \( t - 1 \) steps. We rigorously prove it with conditional expectation:
\[
E_{1 \sim t} [G(\theta_t)] = E_{1 \sim t-1} [E_{t} [G(\theta_t)|1 \sim t-1]] = E_{1 \sim t-1} [G(\theta_t)].
\] (38)

Here, the first equality comes from the law of total expectation, while the second one comes from the fact that \( G(\theta_t) \) is deterministic given the selected batches of the first \( t - 1 \) steps. Besides, when \( t = 1 \), (37) is adapted to
\[
G(\theta_1) - E_1 [G(\theta_2)] \geq D_1 E_1 \left[ \| \nabla \tilde{y}_1 \|^2 \right],
\] (39)

where \( G(\theta_1) \) is the loss of the initialized model parameters so the expectation is omitted. Then, by taking a summation over the first \( T \) steps, we have
\[
D_1 \sum_{t=1}^{T} E_{1 \sim t} \left[ \| \nabla \tilde{y}_t \|^2 \right] \leq G(\theta_1) - E_{1 \sim T} [G(\theta_{T+1})] \leq G(\theta_1).
\] (40)

Therefore, there exists \( \tau \in \{1, \cdots, T\} \), s.t.
\[
E_{1 \sim \tau} \left[ \| \nabla \tilde{y}_\tau \|^2 \right] \leq \frac{G(\theta_1)}{D_1 T}.
\] (41)

Then, we attempt to build a relationship between \( \nabla \tilde{y}_\tau \) and \( \nabla \theta G(\theta_\tau) \). Similar to Eq. (23), we have
\[
\nabla \tilde{y}_{i,\tau} = -\frac{\alpha_\tau}{B_u} \nabla^2 \tilde{y}_{i,\tau} \cdot \theta(\theta_t; \theta_\tau) \cdot \nabla \theta G(\theta_\tau) = \frac{2\alpha_\tau}{B_u} J_\theta f(x_{i,\tau}; \theta_\tau) \cdot \nabla \theta G(\theta_\tau).
\] (42)

Therefore,
\[
\| \nabla \tilde{y}_{i,\tau} \|^2 = \sum_{i=1}^{B_u} (\nabla \tilde{y}_{i,\tau} \cdot \nabla \tilde{y}_{i,\tau}) = \frac{4\alpha_\tau^2}{(B_u)^2} \nabla \theta^T G(\theta_\tau) \cdot \left( \sum_{i=1}^{B_u} J_\theta^T f(x_{i,\tau}; \theta_\tau) \cdot J_\theta f(x_{i,\tau}; \theta_\tau) \right) \cdot \nabla \theta G(\theta_\tau).
\] (43)

Now consider the potential unlabeled batches \( \{B_k : k = 1, \cdots, N^l\} \) of the \( \tau \)th step. Since, \( D^l \subseteq D^u \), we can assume \( x_{i,\tau} \in B_k, k = 1, \cdots, N^l \) and these batches are sampled with non-zero probabilities \( \{p_k : k = 1, \cdots, N^l\} \). Let \( p = \min_k p_k > 0 \), and we have
\[
E_{1 \sim \tau} \left[ \| \nabla \tilde{y}_\tau \|^2 \right] = E_{1 \sim \tau-1} \left[ E_\tau \left[ \| \nabla \tilde{y}_\tau \|^2 \right] \right]
= E_{1 \sim \tau-1} \left[ \frac{4\alpha_\tau^2}{(B_u)^2} \nabla \theta^T G(\theta_\tau) \cdot \left( \sum_{i=1}^{B_u} J_\theta^T f(x_{i,\tau}; \theta_\tau) \cdot J_\theta f(x_{i,\tau}; \theta_\tau) \right) \cdot \nabla \theta G(\theta_\tau) \right]
\geq E_{1 \sim \tau-1} \left[ \frac{4\alpha_\tau^2}{(B_u)^2} \nabla \theta^T G(\theta_\tau) \cdot \left( \sum_{k=1}^{N^l} p_k J_\theta^T f(x_{k,\tau}; \theta_\tau) \cdot J_\theta f(x_{k,\tau}; \theta_\tau) \right) \cdot \nabla \theta G(\theta_\tau) \right]
\geq \frac{4pD^l}{(B_u)^2} E_{1 \sim \tau-1} \left[ \nabla \theta^T G(\theta_\tau) \cdot \left( \sum_{k=1}^{N^l} p_k J_\theta^T f(x_{k,\tau}; \theta_\tau) \cdot J_\theta f(x_{k,\tau}; \theta_\tau) \right) \cdot \nabla \theta G(\theta_\tau) \right].
\] (44)

Note that similar to Eq. (38), the inner expectation is also conditioned on the selected batches of the first \( \tau - 1 \) steps, which is equivalent to that conditioned on \( \theta_1 \).
By applying the chain rule, we have
\[ \nabla_\theta G(\theta) = \frac{2}{N_l} \sum_{k=1}^{N_l} J^\top_\theta f(x_k^i; \theta) \cdot (f(x_k^i; \theta) - y_k). \] (45)

Since both \( f(x_k^i; \theta) \) and \( y_k \) are distributions on the category space, there exists a constant \( R > 0 \), s.t. \( \| f(x_k^i; \theta) - y_k \| \leq R \). Therefore,
\[
\sum_{k=1}^{N_l} J^\top_\theta f(x_k^i; \theta) \cdot J_\theta f(x_k^i; \theta) \\
\geq \frac{1}{R^2} \sum_{k=1}^{N_l} J^\top_\theta f(x_k^i; \theta) \cdot (f(x_k^i; \theta) - y_k) \cdot (f(x_k^i; \theta) - y_k) \cdot J_\theta f(x_k^i; \theta) \\
\geq \frac{1}{N_l R^2} \left( \sum_{k=1}^{N_l} J^\top_\theta f(x_k^i; \theta) \cdot (f(x_k^i; \theta) - y_k) \right) \cdot \left( \sum_{k=1}^{N_l} J_\theta f(x_k^i; \theta) \cdot (f(x_k^i; \theta) - y_k) \right) \cdot \left( \sum_{k=1}^{N_l} J^\top_\theta f(x_k^i; \theta) \cdot (f(x_k^i; \theta) - y_k) \right) \\
= \frac{N_l}{4R^2} \nabla_\theta G(\theta) \cdot \nabla_\theta G(\theta). \] (46)

Here, the symbol \( \succeq \) indicates certain matrix relationship where \( A \succeq B \) means \( A - B \) is a positive semidefinite matrix.

We prove the first inequality in (46) with simplified notations. Suppose \( v \) is a vector and \( A \) is a matrix of proper dimension. Then, we show that if \( \| v \| \leq R \), then \( R^2 A \succeq A^\top v v^\top A \). For an arbitrary vector \( u \) of proper dimension, we have
\[ u^\top A^\top v v^\top A u = \| v^\top A u \|^2 \leq \| v \|^2 \| A u \|^2 \leq R^2 \| A u \|^2 = R^2 u^\top A^\top A u. \] (47)

By definition, \( R^2 A^\top A - A^\top v v^\top A \) is positive semidefinite. The second inequality in (46) comes from the Cauchy-Schwartz inequality that \( \mathbb{E} [A^\top A] \succeq \mathbb{E} [A^\top] \mathbb{E} [A] \) for any random matrix \( A \).

With (44) and (46), it is easy to show that
\[
E_{1 \sim \tau} \left[ \| \nabla y_\tau \|^2 \right] \geq \frac{p D^2_2 N_l}{(B^n)^2 R^2} E_{1 \sim \tau - 1} \left[ \| \nabla_\theta G(\theta_\tau) \|^4 \right] \geq \frac{p D^2_2 N_l}{(B^n)^2 R^2} \left( E_{1 \sim \tau - 1} \left[ \| \nabla_\theta G(\theta_\tau) \|^2 \right] \right)^2. \] (48)

Again, the second inequality comes from the Cauchy-Schwartz inequality. Incorporating with (41), we have
\[
E_{1 \sim \tau - 1} \left[ \| \nabla_\theta G(\theta_\tau) \|^2 \right] \leq \frac{C}{\sqrt{T}}, \quad \text{and} \quad C = \frac{B^n R}{D_2} \sqrt{\frac{G(\theta_1)}{p N_l D_1}}. \] (49)
which concludes this proof.

\section{Implementation Details}

Our implementation is based on the PyTorch\footnote{PyTorch} library and the proposed algorithm is evaluated on the SVHN\footnote{SVHN}, CIFAR\footnote{CIFAR}, and ImageNet\footnote{ImageNet} datasets.

\textbf{Evaluation on the SVHN and CIFAR datasets.} As the standard evaluation protocol, 1k category-balanced labels are used for supervision out of the 73,257 training examples of the SVHN dataset. For the CIFAR-10 (resp. CIFAR-100) dataset, the number of labeled examples is 4k (resp. 10k) out of the 50k training examples. For the backbone architectures, the Conv-Large architecture is the same as the one in previous work\footnote{Previous work}. The detailed configurations are summarized in Table\footnote{Table}. For the ResNet architecture, we adopt the ResNet-26-2x96d Shake-Shake regularized architecture with 12 residual blocks as in\footnote{Shake-Shake regularization}. The same architecture is used in prior SSL methods\footnote{Prior SSL methods}. We follow a common practice of data augmentation, i.e., zero-padding of 4 pixels on each side of the

image, random crop of a $32 \times 32$ patch, and random horizontal flip, for the CIFAR datasets, and omit
the random horizontal flip for SVHN. The meta learning rate $\beta_t$ is always set equal to the regular
learning rate $\alpha_t$. We train from scratch for 400k iterations with an initial learning rate of 0.1, and
decay the learning rate by a factor of 10 at the end of 300k and 350k iterations. We use the SGD
optimizer with a momentum of 0.9, and the weight decay is set to $10^{-4}$ for the CIFAR datasets, and
$5 \times 10^{-5}$ for SVHN. The batch size is 128 for both labeled and unlabeled data. The shape parameter
$\gamma$ of the Beta distribution is set to 1.0 for the CIFAR datasets, and 0.1 for SVHN, as suggested by [28].

**Evaluation on the ImageNet dataset.** The large-scale ImageNet benchmark contains 1.28M
training images of 1k fine-grained classes. We evaluate on the ResNet-18 [30] backbone with
10% labels. The standard data augmentation strategy [38, 30, 39] is adopted: image resize such
that the shortest edge is of 256 pixels, random crop of a $224 \times 224$ patch, and random horizontal
flip. The overall batch size is 512, and the same optimizer as the aforementioned one is employed
with a weight decay of $10^{-4}$. We train for 600 epochs in total, and decay the learning rate from 0.1
according to the cosine annealing strategy [40]. The shape parameter $\gamma$ is set to 1.0.

| Layer       | Configurations | Output Size |
|-------------|----------------|-------------|
| Convolution | 128, 3, 1, 1, 1 | 32 × 32     |
| Convolution | 128, 3, 1, 1, 1 | 32 × 32     |
| Convolution | 128, 3, 1, 1, 1 | 32 × 32     |
| MaxPooling  | 128, 2, 2, 0    | 16 × 16     |
| Dropout     | Drop probability = 0.5 | 16 × 16     |
| Convolution | 256, 3, 1, 1, 1 | 16 × 16     |
| Convolution | 256, 3, 1, 1, 1 | 16 × 16     |
| Convolution | 256, 3, 1, 1, 1 | 16 × 16     |
| MaxPooling  | 128, 2, 2, 0    | 8 × 8       |
| Dropout     | Drop probability = 0.5 | 8 × 8       |
| Convolution | 512, 3, 1, 0    | 6 × 6       |
| Convolution | 256, 1, 1, 0    | 6 × 6       |
| Convolution | 128, 1, 1, 0    | 6 × 6       |
| AvgPooling  | 128, 6, 1, 0    | 1 × 1       |
| Linear      | 128 → 10, 1     | 1 × 1       |