Atom interferometry using temporal Talbot effect on a Bose-Einstein condensate

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Obtaining a large momentum difference between matter wave-packets traversing the two paths in an atom interferometer while preserving the phase coherence is a challenge. We experimentally investigate a pulse sequence in which this momentum difference is obtained by using the matter-wave Talbot effect as an alternative to the conventional light-pulse atom interferometers. An ensemble of coherent atoms (Bose-Einstein condensate) is exposed to a periodic sequence of $2N$ standing wave pulses of far-detuned light. The first set of $N$ pulses in the sequence produces multiple wavepackets with momentum difference up to $28\ hk$ (at $N = 5$), by virtue of Talbot resonance. The second set of $N$ pulses reverses the momentum transfer and allows the wavepackets to retrace the path and interfere to retrieve the initial wavepacket. We observe that the degree of reversal and thus the practical performance of the interferometer is sensitive to the initial momentum width of the atomic ensemble.

I. INTRODUCTION

Atom interferometry has proved to be a promising approach for both commercial use and fundamental research. In addition to applications in gravimetry [1–4] and inertial sensing [5–7], atom interferometry has also been used in probing fundamental physics with experiments to test general relativity [8], measuring fundamental constants with high precision [9, 10], studying interaction of matter with gravity [11], searching for dark matter candidates [12], etc. Despite these achievements, light-pulse based atom interferometers are yet to realize their full-potential in sensitivity due to the restrictions imposed on achieving large area enclosed by the paths of interfering wavepackets [13]. There have been several efforts to increase the enclosed area by introducing large momentum difference between the interfering arms using sequential two-photon Raman transitions [14], single multiphoton Bragg diffraction [15] and Bloch oscillations in an optical lattice [16, 17]. However, the scalability of these schemes has some limitations. In high-order Bragg diffraction, the fidelity of populating the target states is very sensitive to the width of the momentum spread along the beam axis [18]. Additionally the significant atom laser interaction time for all of these schemes introduces finite amount of spontaneous emission which causes dephasing [19, 20].

To produce the desired momentum difference, one can exploit the temporal matter-wave Talbot effect [21] in an ensemble of cold atoms subjected to pulses of a sinusoidal optical potential. The system under consideration is the atom optic realization of the quantum kicked rotator [22, 23]. When this rotor is driven at the resonant period, there is a ballistic growth in the momentum imparted to the atomic ensemble [24]. This resonance phenomenon is based on the matter wave analog of the optical Talbot effect [25] and the drive period at which it occurs is called the Talbot time [21, 26]. The typical light-atom interaction time for the pulses used here ($\sim 0.5\ \mu s - 1\ \mu s$) is orders of magnitude less in comparison to the Bragg pulses ($\sim 100\ \mu s$) [13] and hence can lead to decrease in the laser induced noise and decoherence effects [19]. The first demonstration of temporal the Talbot effect in matter waves using a Bose-Einstein Condensate (BEC) was given by L. Deng et al. (1999) [21]. In their two kick sequence, the effect of first kick is negated by the second one at the anti-resonance Talbot condition, resulting in initial state recovery. This constitutes an interferometer, where any deviation from the Talbot time results in an imperfect overlap with the initial state. One can also use the resonant Talbot effect to generate large momentum difference between the involved states [27] and make them interfere destructively by the action of appropriate operators. Ref. [28] shows an example of such a scheme where the effect of multiple resonant kicks was canceled out by a final high order kick. The sub-Fourier scaling of fractional resolution observed in this scheme was the basis of it being proposed as a platform to perform precision metrology [29, 30]. A similar reversal was also demonstrated in a quantum random walk realization in momentum space using a delta-kicked BEC [31] and may find applications in interferometry.

In the previous realization of the BEC based Talbot interferometer [28], the final kicks’ strength needed to be dynamically ramped up over few tens of $\mu s$. This ramp scales as $N$ (number of kicks) and hence the required laser intensity ramp with both high dynamic range and peak amplitude can be a detriment in applicability. As an alternative to this scheme, we study a Talbot effect based atom interferometer which is similar in principle. As suggested in Ref. [32], this interferometer can be realized in a symmetric fashion by replacing the high order kick in Ref. [28] with a series of low order kicks. We can quantify this interferometer’s performance by the precision to which it can measure the Talbot time $T_T$, which is in turn related to the recoil frequency ($\omega_r$) [33] by the

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relation $T_T = \pi/2\omega_s$. The fractional resolution in measurement of $T_T$ at resonance is defined as $S = \Delta T/T_T$, where $\Delta T$ is the width of the resonance. We measure $S$ by monitoring the overlap of the final state w.r.t. the initial one, at deviations from the resonant period. The minimum value of $S$ we achieve here, limited by BEC temperature, lies close to that predicted for a similar sequence in Ref. [30]. It is also observed that the peak overlap value is drastically affected by the momentum spread of the initial state, as predicted in Ref. [34]. Numerical simulations of the evolution of the system are in agreement with the experimental results.

II. THEORY

![Diagram of kick sequence and standing wave](image)

FIG. 1: Billiard ball representation of kick sequence: The evolution of population distribution in different momentum orders at resonance is shown for $\delta \phi = 0.7$. $k_1$ and $k_2$ denote the grating vectors of the two beams forming the standing wave that is pulsed according to the sequence shown. The field circles represent the population of atoms in each diffracted order after being kicked by the pulse shown below it. The dashed line marks the time at which the phase of the standing wave is reversed for the subsequent kicks. $g\bar{y}$ denotes the direction of gravity.

The dynamics of an atom with mass $m$ subjected to a periodically pulsed standing wave of far detuned light of wavelength $\lambda$ is governed by the following Hamiltonian (in scaled units):

$$H(t) = \hat{p}^2 + \phi_d \cos(\hat{x}) \sum_{n=0}^{\infty} \delta(t - nl)$$  \hspace{1cm} (1)

This is the standard kicked rotor Hamiltonian, which has been extensively studied in the context of quantum chaos [22, 35–37]. The momentum $\hat{p}$ and position $\hat{x}$ are scaled in terms of the recoil momentum ($hK$) and lattice spacing ($K^{-1}$) respectively, where $K = 4\pi/\lambda$ is the grating vector. The scaled period is $l = 2T/T_T$, where $T$ is the pulse period and $T_T = 4\pi m/hK^2$ is the Talbot time. $T_T$ is the temporal analog of the spatial Talbot effect which is seen in light [25]. A pulse period of $T_T$ at a constant lattice phase leads to a ballistic increase in the energy due to resonance [38]. The kick strength is given by $\phi_d = \Omega^2 \Delta t/8\delta$, where $\Delta t$ is the pulse duration and $\Omega$ is the effective Rabi frequency experienced by the atoms at a detuning of $\delta$ from the resonant transition of the atoms. The momentum $\hat{p}$ can be expressed in terms of a discrete ($\hat{k}$) and a continuous component ($\beta$), such that $\hat{p} = \hat{k} + \beta$, where $-1/2 < \beta < 1/2$. The interaction with the periodically pulsed lattice changes the momentum state of the atoms in discrete units of $K$ only; $\beta$ for any initial momentum state is conserved.

The time evolution of any initial state $|k + \beta\rangle$ can be obtained by application of a transformed Floquet operator [39]. This operator $\tilde{F}$ can be broken down into two parts: the free evolution operator acting between two sequential kicks and the kick operator. Under the assumption that the kick period $T \gg \delta t$ (Raman-Nath regime) [40], it can be written as:

$$\tilde{F}(\beta) = [\exp(-i\pi l(\hat{k} + \beta)^2)] \times [\exp(i\phi_d \cos(\hat{x}))]$$  \hspace{1cm} (2)

The wavefunction after $N$ kicks is then determined by consecutive application of $\tilde{F}(\beta)$. For the kick sequence consisting of $2N$ kicks acting on initial state $|k + \beta\rangle$, where the phase of the standing wave is shifted by $\pi$ radians after application of $N$ kicks, the wavefunction of the final state can be written as:

$$|\psi(t = 2N)\rangle = (\tilde{F}_N(\beta))^N (\tilde{F}_0(\beta))^N |k + \beta\rangle$$  \hspace{1cm} (3)

Where, $\tilde{F}_0(\beta)$ is the Floquet operator for each first $N$ kicks and $\tilde{F}_N(\beta)$ is that for the subsequent phase-shifted kicks. We are interested in the fidelity ($I$) of the kick sequence with respect to the initial state which is given by: $I = \langle |k + \beta| \tilde{F}_N^N(\beta) \tilde{F}_0^N(\beta) |k + \beta\rangle^2$. In principle (neglecting the noise present in the experiment controls), at the resonance condition ($l = 2$), $I = 1$ for an initial state $|k + \beta = 0\rangle$. Because of the phase factor containing $\beta$ in eqn.2, a finite value of $\beta$ will compromise the echo process and will result in $I < 1$. Therefore obtaining the maximum value of $I$ requires the initial distribution of atoms to have very narrow momentum distribution ($\Delta \beta$). A BEC is typically used for such experiments, but even it possesses a finite spread in momentum ($\sim 1\% \ hK$) [41]. As we will show later, even such a small momentum distribution width can affect the peak $I$ value drastically with increasing $N$. Other major factors which affect the fidelity are noise in the phase and the amplitude of the standing wave lattice and in the timing of the kick sequence [37, 42].
The quantity of interest for utility of this interferometry sequence is the fractional resolution of the fidelity near resonance. The fractional resolution is defined as, \( S = \frac{\Delta T}{T_R} \), where \( \Delta T \) is the width of the fidelity peak \( I(T) \) at the resonance condition \( (l = 2, T = T_R) \) and \( T_R \) is the Talbot time. \( S \) of the BEC based kicked rotor interferometry schemes has been predicted to have a cubic scaling with the number of kicks \([29, 32]\) as demonstrated in Ref. [28]. The scaling can be attributed to the significant relative phase change induced between the participating momentum orders at deviations from resonant period [29].

This type of scaling behavior (sub-Fourier) may be advantageous for precision measurements of photon-recoil frequency and acceleration [28, 43]. In addition to number of kicks, \( S \) also scales as the inverse square of \( \phi_d \) so that \( S \sim 1/N^3 \phi_d^2 \). To improve fractional resolution, more diffraction orders should participate in the process, which requires either \( \phi_d \) or \( N \) to increase. However, the finite momentum spread is found to limit the maximum \( \phi_d \) and \( N \) that can be used without loss in resolution. R. A. Horne et. al. [30] had investigated performance limits of a similar kick scheme and had suggested a requirement of \( \phi_d < (0.7/N^2)(\hbar K/\Delta \beta) \) which leads to a lower limit of fractional resolution:

\[
S = \frac{\Delta T}{T_R} > 4.3N \left( \frac{\Delta \beta}{\hbar K} \right)^2 \tag{4}
\]

We observe that a finite \( \Delta \beta \) limits achievable \( \phi_d \) and \( N \) and thus \( S \), in agreement with the above prediction.

### III. EXPERIMENTAL SEQUENCE

We briefly present the experimental sequence here. A detailed description of the experimental set-up used to carry out this experiment is provided in [37, 44]. We obtain a rubidium (\(^{87}\)Rb) Bose-Einstein condensate (BEC) of \( \approx 3 \times 10^4 \) atoms after laser cooling in a standard Magneto-Optical Trap (MOT) and forced evaporative cooling in a hybrid 1064 nm crossed dipole trap. The BEC is prepared in the \( |5^2 S_{1/2}, F = 1, m_F = -1 \rangle \) manifold. The beams which are used to realize the standing wave are 6.8 GHz red detuned from the \( |5^2 S_{1/2}, F = 2 \rangle \rightarrow |5^2 P_{3/2}, F = 2 \rangle \) 780 nm D2 transition. A schematic of the experimental kick sequence is shown in FIG. 1.

The two beams are derived from the same laser but passed through different AOMs (acousto-optic modulator). The first diffracted orders from the AOMs are then delivered to the experiment via polarization maintaining optical fibers. The kick strength (\( \phi_d \)) for the sequence can be controlled by changing the duration and amplitude of the of the RF signal applied to the AOMs. The sequence requires the phase of the standing wave to rapidly

![FIG. 2: Fidelity (I) as a function of pulse period (T) for different number of kicks N at constant \( \phi_d \). The dots are the experimental data and the dashed line is the numerical simulation. The deviations in the experimental fidelity values are about ±0.01. The kick strength and fluctuations in it are kept constant for all the simulations at \( \phi_d = 0.8 \) and \( \Delta \phi_d = 0.08 \) respectively. The initial quasimomentum distribution and the phase noise were kept between \( w = 0.025 - 0.029 \) and \( \Delta \theta = 0.35 - 0.52 \) respectively.](image)

![FIG. 3: Fidelity width(\( \Delta T \)) vs. number of kicks(N). The width is determined by fitting a Gaussian to the fidelity data as shown in FIG. 2. From fitting the data to power law we find that, \( \Delta T \propto N^{-2.33} \).](image)
switch by $\pi$ radians after $N$ pulses, typically over $\approx 10$ μs. To do this reliably, we supply the RF signal to one of the AOMs via a high frequency switch, which can be toggled between two RF frequency signals [42]. These signals are provided by two function generators which are phase locked with a relative phase shift of $\pi$ radians between them. The pulse duration was kept to be $\approx 550$ ns for this experiment and falls well within the criteria for the kicks to be in the Raman-Nath regime [40]. In order for the diffracted orders to be resolved spatially for time-of-flight imaging, the atomic cloud is allowed to fall freely under gravity for 7 ms. An on resonant probe pulse is then applied to get an absorption image that is used to deduce the amount of atoms populated in the different diffracted orders. The fidelity is then calculated as $I = \frac{f(0)}{\sum f(n)}$, where $f(n)$ denotes the number of atoms in the $n^{th}$ momentum state.

IV. RESULTS

FIG. 2 shows the variation of fidelity $I$ vs. the kick period $T$, after application of kick a sequence consisting of $N$ kicks where $N$ varies from $2-5$. The kick strength for all sequences was kept $\approx 0.8$. The fidelity plot has a maxima at the resonance condition $l = 2$ ($T = 65.5$ μs) and the width of this resonance feature decreases with $N$. To validate the results, numerical simulations were performed in MATLAB with parameter values fitted to data but bounded within the estimated uncertainty of the experimental values. The numerical recipe was taken from Ref. [39] and modified to accommodate phase reversal, phase noise and amplitude noise for our sequence.

As mentioned earlier, the initial state is a distribution of atoms with finite spread in momentum ($\Delta \beta$). In the simulation, we take the initial condition to be 1000 non interacting plane waves in $k$-space which spans from $-20 \ hK$ to $20 \ hK$. The momentum values of the initial condition are distributed according to normal distribution $D$. The initial momentum state values $p = k + \beta$ are drawn from this distribution $D = \frac{1}{\Delta \beta \sqrt{2\pi}} \exp\left(\frac{-(k+\beta)^2}{2\Delta \beta^2}\right)$, where $T_{\Delta \beta} = \frac{\Delta \beta^2 (\hbar K)^2}{m_{\text{K}}}$ is the temperature of the initial state. The Floquet operators mentioned in eqn.3 are then applied to evolve each state vector sample. The value of fidelity thus obtained is averaged over the number of samples and over 5 simulation runs. We also incorporate phase and amplitude noise by adding shifts to the phase and kick strength at each kick, which are randomly picked from normal distributions with widths $\Delta \theta$ and $\Delta \phi_d$ respectively. The best fit from the numerical simulations for each experimental $I(T)$ data is used to calculate the resonance width $\Delta T$. A Gaussian is fitted between the first minima on either side of the central maxima of this simulated $I(T)$ curve for each $N$ and the width thus obtained is $\Delta T$. FIG.3 shows the width of this fitted Gaussian ($\Delta T$) as a function of $N$. As shown in FIG. 3, we observe a scaling of $\Delta T \propto N^{-2.33±0.16}$ which is less than the scaling of $N^{-3}$ expected from analytical calculations in Ref. [32]. Previous Talbot interferometry scheme with BEC had reported the exponent of this
scaling to be $-2.73 \pm 0.19$ [28]. We suspect that this deviation is due to the low values of $N$ ($N < 5$) for the $\phi_d \sim 0.8$ that we use here. The validity of $\Delta T / T_T$ being small made in the prediction of the cubic scaling in Ref. [32] may not hold in this case. The minimum fractional resolution $S$ we achieve here is $1.4 \times 10^{-2}$ for $N = 5$ and $\phi_d = 0.8$. We can estimate the theoretical lower bound on $S$ from eqn.4. The optimal $\phi_d$ for $N = 5$ and $\Delta \beta = 0.03 \hbar K$ is $\sim 0.93$ for which $S = 1.9 \times 10^{-2}$, which is close to the experimentally obtained value for $N = 5$, confirming the limits on achievable $S$ that are estimated in Ref. [30]. This shows that the finite value of $\Delta \beta$ leads to suppression of the peak fidelity value ($\sim 80\%$ for $N = 5, \phi_d = 0.8$) and the limits the practical resolution can be achieved. This suppression of peak fidelity can be observed in the evolution of momentum distribution for a BEC as a function of kicks is shown in FIG. 4. In the left section of the figure, each momentum order is a depicted by a Gaussian whose area is proportional to the simulated population occupied by that order for the simulated population occupied by that order for $N = 4, \phi_d = 0.8$ and $\Delta \beta = 0.03 \hbar K$. The right section shows the absorption images taken after each kick in the experiment. The population distribution observed experimentally resembles the distribution obtained with simulations.

As suggested by the simulations, the performance of this scheme can be improved by using a narrower momentum ensemble as initial state for e.g. ensembles with $\Delta \beta$ an order of magnitude less than the one we utilize here have already been reported ($\Delta \beta \leq 0.004 \hbar K$) [24, 45, 46]. To reduce $\Delta \beta$ one can also use a velocity filtering kicked rotor sequence to operate on the BEC before application of the interferometry sequence [47].

V. CONCLUSION

In conclusion we have probed the fractional resolution ($S = \Delta T / T_T$) in measuring Talbot time ($T_T$) of the kicked rotor atom interferometry pulse scheme as suggested in [32]. The width of the resonance ($\Delta T$) was obtained by measuring fidelity ($I$) of the final state w.r.t the initial one as a function of pulse period ($T$). The scaling of $\Delta T$ with the number of kicks was observed to follow a power law trend with exponent $= -2.33 \pm 0.16$, which is lesser than the previously reported scaling for a similar sequence (exponent $= -2.73 \pm 0.19$) and the cubic scaling expected from analytical calculations [32].

We also observe that the best fractional resolution achieved here : $S = 1.4 \times 10^{-2}$ (at $N = 5$ and $\phi_d = 0.8$) lies close to the theoretically calculated lower bound : $1.9 \times 10^{-2}$ for a pulse scheme which is similar to the one we study in this work [30]. This result confirms the suppression of peak $I$ ($\sim 80\%$ reduction for $N = 5, \phi_d = 0.8$) and the achievable resolution because of high sensitivity to $\Delta \beta$ in accordance with the theoretical calculations in Ref. [30]. Simulations suggest that using BECs having narrower momentum distributions ($< 0.01 \hbar K$) [45, 46] or velocity filtering [47] to achieve better performance.

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