Observation of Ultrahigh Energy Neutrino Interactions by Orbiting Detectors

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Orbiting detectors will be able to observe showers initiated by neutrinos penetrating the Earth and interacting close to their exit point. There is a correlation between the impact parameter of the incident neutrino and its energy. We study the development of upward going, neutrino induced showers in the atmosphere.

1 Introduction

The study of extraterrestrial, ultrahigh energy neutrinos is important from at least two points of view. First, they probably carry astrophysical information about point sources which are optically thick at (almost) all wavelengths, for instance, an active galactic nucleus (AGN). Second, incoming neutrinos of laboratory energies of the order of $10^{18}$eV (or even higher) are expected to be emitted by the highest energy sources. In a collision with a nucleon, the CMS energy is, therefore, of the order of 40 TeV, far exceeding the CMS energies available in accelerator generated neutrino beams. Although such events are expected to be rare ones, they provide a unique window on physics beyond the Standard Model. For both reasons it is important to refine the observation of ultrahigh energy (UHE) neutrinos. The main purpose of this paper is to sketch a novel way of observing such interactions.

The basic principle is a very simple one. Neutrino–nucleon interactions have a cross section which is a well known, monotonically increasing function of the incident neutrino energy; for a recent calculation, up to neutrino energies of $10^{12}$GeV, cf. ref. [1]. Consequently, on the average, a neutrino penetrating the Earth will interact after a well defined
distance beneath the surface. If the interaction occurs reasonably close to the exit point of the neutrino, the ensuing shower develops largely in the atmosphere and it can be detected by means of an orbiting detector, such as OWL (Orbiting Wide angle Light collector) or AIR WATCH. Such detectors are suitable for a study of the longitudinal development of showers. Thus, a shower developing upwards gives a unique signature of a neutrino interaction. Further, one obtains some information about the primary energy of the incident neutrino: the Earth acts as an energy filter. Indeed, given the impact parameter of the incident neutrino, its (energy dependent) interaction mean free path has to be approximately equal to the length of the chord of the trajectory inside the Earth. If it is considerably shorter (high energies), the interaction and the development of the shower takes place largely inside the Earth: no atmospheric shower is observed. Conversely, at low energies, the neutrino penetrates the Earth without interacting. Once it leaves the Earth, its chance of interacting within the atmosphere is rather slim.

This paper is organized as follows. In Section 2 we summarize the calculation used in order to obtain the shower development. The results are given in Section 3. Finally, in Section 4 the results are discussed. A preliminary account of these results was given in ref. [2].

2. The Cascade Equations

We use one dimensional cascade theory in Approximation A; due to the fact that we are mainly interested in ultrahigh energies, this should be adequate in order to obtain the first estimate of the behavior of the showers induced by neutrino interactions.

The incoming neutrino interacts with a quark in the target nucleus and generates a hadronic jet. In the present work, we assume that the hadrons generated share the primary energy equally; hence we start with showers induced by a single hadron.

In a hadronic interaction, most of the secondaries produced are pions. Fast nucleons are relatively rare, with the possible exception of the leading nucleon in a nucleon–nucleus interaction. At this point, we neglect the production of particles other than pions.

We assume that pions of either charge are produced in equal numbers. Since the average multiplicity in an interaction is large, this is a permissible simplification. We neglect photoproduction of pions: hence, neutral pions act as the source of the electromagnetic component without the latter affecting the hadronic one. As a consequence, the equations governing the hadronic component are decoupled from the ones governing the electromagnetic one.

We assume that Feynman scaling is valid and parametrize the single particle inclusive distributions by means of a simple expression,

\[ \frac{1}{\sigma} \frac{d\sigma}{dE} = F(z) = K z^\alpha (1 - z)^\beta \Theta (z - z_0). \]

(1)

Here, \( z = E/E' \) is the Feynman parameter, \( E' \) being the primary energy. The quantities...
$K, \alpha, \beta$ are to be determined from the normalization conditions,

$$\int dE \frac{d\sigma}{dE} = < n > \sigma_{\text{inel}}$$  \hspace{1cm} (2)

and

$$\int dEE \frac{d\sigma}{dE} = E'$$  \hspace{1cm} (3)

and from fits to the experimental data. An infrared (IR) cutoff, $z_0$ has been introduced. Equation (2) follows from the definition of a single particle inclusive cross section, whereas (3) expresses energy conservation.

Due to the fact that we treat all light hadrons together, some compromises are necessary: in fact, the parameter $\beta$ governs the leading particle behavior which is different for the various secondaries. Nevertheless, the available data \cite{3} suggest that taking $\alpha = 0$ and $\beta = 5$ gives a reasonable fit to all data. (By choosing $\alpha = 0$ one gets an inclusive distribution $\propto 1/z$ for small values of the Feynman parameter. This is consistent with the fact that at low momenta the inclusive distribution is dominated by soft gluon emission.) The infrared cutoff has been chosen at a fixed Feynman parameter rather than at a fixed energy. This is consistent with the fact that in the following, we use an energy independent average multiplicity, $< n > = 50$ — again a compromise. By doing so, one underestimates the number of soft secondaries. However, we found that these approximations have virtually no effect on the integral distributions with a threshold energy $E_{\text{th}} \gtrsim 100\text{GeV}$. Likewise, the shower development is rather insensitive to the value of $\beta$: even a distribution $F(z) \propto \Theta(1-z)$ gives reasonable showers. Likewise, the integral distributions at the threshold energies indicated above are quite insensitive to the choice of the IR cutoff. For this reason, we put $z_0 = 0$ whenever this choice does not give rise to IR divergences.

We are now ready to write down the transport equations for the hadronic component of the shower. There are two distinct equations. The first one is for the stable hadrons: nucleons and charged pions; at the relevant energies the charged pions may be considered stable. The second equation characterizes the behavior of neutral pions: in that case, decay is essential\footnote{One recalls that in air at ground level, the decay and interaction mean free paths of a neutral pion become equal at an energy of about $10^{18}\text{eV}$.}. We denote the differential distribution of charged hadrons and neutral pions by $h$ and $p$, respectively. The depth of the medium is measured in units of the hadronic interaction mean free path, $\tau = t/\lambda$, where $t$ is the target thickness measured in $\text{g/cm}^2$. The approximate transport equations read as follows.

$$\frac{\partial h}{\partial \tau} = -h + \frac{2}{3} \int \frac{dE'}{E} F \left( \frac{E}{E'} \right) h \left( E' \right)$$

$$\frac{\partial p}{\partial \tau} = -\frac{\lambda}{\rho} p + \frac{1}{3} \int \frac{dE'}{E} F \left( \frac{E}{E'} \right) h \left( E' \right).$$  \hspace{1cm} (4)

In writing down equation (4) we neglected the interaction of neutral pions. The loss term contains merely their decay. The factor $\frac{\lambda}{\rho}$ serves to convert the real space decay...
length, $D$, to the units used in writing down the transport equation. The decay length is, of course, given by the expression:

$$D = \frac{m_0}{T \rho},$$

where $T$ is the lifetime in the rest frame and $m_0$ is the mass of the neutral pion. The density of the medium, $\rho$, is expressed as a function of $\tau$. The mass of the neutral pion is denoted by $m_0$. The factors $2/3$ and $1/3$ reflect the fact that approximately two out of three hadrons produced are charged pions and one out of three is a neutral pion.

In a similar fashion, we can write down the equation governing the electromagnetic component. To the accuracy desired for the purposes of the present calculation, the (common) inclusive distribution of Bremsstrahlung and pair production can be approximated by a step function, $F(E/E') = \Theta(1 - E/E')$. (This gives, of course, a pure $1/E$ inclusive spectrum for both the charged leptons and the Bremsstrahlung photons, a reasonably accurate approximation.) In the same approximation, we put the cross sections of Bremsstrahlung and pair production equal to each other (i.e. $7/9 \approx 1$). We denote the number of particles interacting only electromagnetically by $l$, i.e.

$$n_{e^+} + n_{e^-} + n_{\gamma} \approx l.$$ 

The latter approximate equality is valid after a few radiation lengths; initially, the number of charged leptons is slightly overestimated.

We write down the the evolution equation for $l$ by measuring distances in units of the radiation length, $X_0$. Assuming again that all neutral pions decay instantaneously and that the energy of the $\pi^0$ is shared equally between the decay photons, we have with $\sigma = t/X_0$:

$$\frac{\partial l(E, \sigma)}{\partial \sigma} = \frac{X_0}{D} p \left(2E, \frac{X_0 \sigma}{\lambda} \right) + \int \frac{dE'}{E} \Theta(E' - E) l(E', \sigma) - l(E, \sigma)$$

(5)

The evolution equations given by (4) and (5) form a hierarchical structure. Due to the (very reasonable) approximation made in writing down these equations, namely that neutral pions decay instantly and do not interact, one has to solve only the first one of equations (4); the evolution equation for neutral pions can be solved by quadrature and equation (5) is solved in terms of a Green function.

In fact, the evolution equation for neutral pions is solved by the expression:

$$p = \int d\tau' e^{-\lambda/\tau'} \Theta(\tau - \tau') g(\tau'),$$

(6)

where $g(\tau)$ is the gain term as one can read it off from the second of equations (4). Due to the fact that $\lambda/D \gg 1$, one can pull out $g(\tau')$ from under the integral in eq. (5) at $\tau' = \tau$. Upon substituting this into the evolution equation of leptons, eq. (3), one realizes that in the rapid decay approximation, that equation also scales as the hadronic evolution equations are supposed to do. Hence we conclude that Feynman scaling holds for a mixed hadron - lepton cascade if:
• The energies are sufficiently high so that effects of the rest masses of the particles are negligibly small,

• the energies are sufficiently low so that the reinteraction of decaying neutral particles is negligible together with the QCD loop corrections containing logarithms of the energy. (If the decay length does not cancel, there is a scaling violation due to the Lorentz factor present in the expression of the decay length.)

In order to find the retarded Green function of eq. (5), one uses Mellin transform techniques. The result is:

\[
G(q, \tau - \tau') = \exp (\tau - \tau') \Theta (\tau - \tau') \Theta (1 - q) \left( \frac{\ln |q|}{2(\tau - \tau')} \right)^{-1/2} 
\times I_1 \left( 2^{3/2} (\tau - \tau')^{1/2} (\ln |q|)^{1/2} \right). \tag{7}
\]

In eq. (7), \( q \) stands for the scaling variable, \( q = E/E' \) and \( I \) is a modified Bessel function of order one, cf. [4].

To summarize: assuming the validity of Feynman scaling and the rapid decay of neutral pions, one has to solve one evolution equation only, viz. the one governing the evolution of charged hadrons. Other components of the mixed hadronic and leptonic cascade are obtained by quadratures, which is rapidly accomplished by using standard numerical integration techniques.

The solution of the first of the equations (4) is best accomplished by iteration. In fact, one notices that by converting that equation into a Volterra integral equation, an iterative procedure rapidly converges. Roughly speaking, the number of iterations needed is about the number of collision mean free paths at which the distribution is to be computed. (This has been known for a very long time: Bhaba’s method of “successive collisions” for obtaining the evolution of an electromagnetic cascade is, in essence, based on this observation, see, e.g. [5].) Equations for the integral distributions can be derived in a straightforward manner; the calculation is particularly easy in the Feynman scaling limit.

3 Results

We calculated the integral distributions of electrons and positrons for a number of hadron energies and nadir angles. In each case, we assumed that the incident neutrino interacts in the Earth, just below its exit point. (In practical terms, this means that the interaction has to take place within a few hadronic interaction mfp below the surface.) For each nadir angle, the target thickness penetrated by a neutrino before interaction was determined on the basis of the results given in ref. [1]. The profile of the electromagnetic component of the shower is shown in the following Figures. In each calculation, the integral spectrum of the hadrons was cut off at \( E_0 = 100\text{GeV} \), corresponding to a CMS energy of \( \sqrt{s} \approx 14\text{GeV} \). Below such energies, particle production becomes insignificant; moreover, the particles produced tend to decay into soft muons and gammas instead of contributing to the
cascade. Likewise, the integral distribution of the electromagnetic component was cut off at $E_c = 100\text{MeV}$, roughly corresponding to the critical energy in air. In estimating the primary neutrino energies, we assumed an average inelasticity $< y >= 0.5$ and an average hadronic multiplicity, $< n >= 50$ in the first interaction. Thus, on the average, $E_{\text{hadron}} = E_{\nu}/100$. The variation of the nadir angle around the horizontal direction corresponds to a slight variation of the target thickness around one neutrino mfp.

The following figures display the electron–positron component generated by a single hadron. Due to the linearity of the evolution equations, the curves displayed here can be easily scaled to accommodate other inelasticities and/or neutrino primary energies. The shower profile is presented in “real space”. For purposes of converting target depth to length an exponential atmosphere was used, with scale height, $h = 16.8\text{km}$ and ground level density, $\rho_0 = 1.3 \times 10^{-3}\text{g/cm}^3$. The notation used in the Figures is summarized in Table 1.

| $E_h$ (GeV) | diamonds | triangles | asterisks |
|------------|-----------|-----------|-----------|
| 10$^7$     | 80°       | 85°       | 88°       |
| 10$^8$     | 80°       | 84°       | 87°       |
| 10$^9$     | 80°       | 84°       | 88°       |

Table 1. Summary of hadron energies and symbols used for showers developing at different nadir angles. The average single hadron energy, $E_h$ is assumed to be 1% of the incident neutrino energy, see text.
Fig. 1: Integral distribution of electrons and positrons, \( E > 100 \text{MeV} \). \( E_h = 10^7 \text{GeV} \)
Fig. 2: Same, $E_h = 10^8\text{GeV}$
4 Discussion

The figures displayed in the preceding section reveal a characteristic feature of the showers induced by a neutrino interaction near the exit point from the Earth. Those showers are “upside–down” — they start in the dense portion of the atmosphere and evolve upwards. As a consequence, such showers reach their maximum rather soon, typically at distances of the order of 5km. Thereafter, they have an unusually long tail, i.e. a very slowly decreasing particle number as the shower evolves. This is an effect which is easily understood: the “upside–down” shower starts near the surface of the Earth and proceeds upwards. As far as the hadronic (charged pions and nucleons) and the purely leptonic ($\gamma, e^\pm$) components are concerned, the patterns of a “normal” or of an “upside–down” shower are the same if plotted against the target depth. However, the distribution of neutral pions (the main source of photons responsible for initiating the leptonic component) is asymmetric: upward and downward going beams of $\pi^0$-s can be easily distinguished.

Fig.3: Same, $E_h = 10^9\text{GeV}$
from each other. As a consequence, any orbiting detector which is able to follow the longitudinal development of a shower is, \textit{a fortiori} able to pick up upward going showers.

It remains to be seen whether sufficiently powerful extraterrestrial neutrino sources exist so that one can collect a significant number of neutrino events of the type described here. Besides the obvious astrophysical interest of such events, one would be able to look for signatures of “new physics”, assuming that it affects the longitudinal development of the neutrino induced showers. For instance, it was proposed that at some higher energy scale, neutrinos begin to interact more strongly than it would be expected on the basis of the standard electroweak theory see refs. \cite{6, 7}. Should this be the case, the correlation between the impact parameter of the incident neutrino and the development of the shower would change. Assuming a sufficiently large statistical sample of “anomalous” events, one could deduce the onset of a new physical phenomenon.\footnote{It was pointed out that, taken literally, such schemes violate unitarity, see \cite{8}. This is due to the fact that in refs. \cite{6, 7} the onset of “new physics” was modelled by adding a piece to the cross section which is proportional to a step function. As a consequence, the real part of the corresponding amplitude develops a logarithmic singularity. Almost any smooth function approximating a step function is free of such a singularity and can be reconciled with unitarity.}

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