Conservative Dynamics of Binary Systems at Fourth Post-Minkowskian Order in the Large-eccentricity Expansion

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We compute the conservative dynamics of non-spinning binaries at fourth Post-Minkowskian order in the large-eccentricity limit, including both potential and radiation-reaction tail effects. This is achieved by obtaining the scattering angle in the worldline effective field theory approach and deriving the bound radial action via analytic continuation. The associated integrals are bootstrapped to all orders in velocities through differential equations, with boundary conditions in the potential and radiation regions. The large angular momentum expansion captures all the local-in-time effects as well as the trademark logarithmic corrections for generic bound orbits. Agreement is found in the overlap with the state-of-the-art in Post-Newtonian theory.

Introduction. The era of gravitational wave (GW) science began in spectacular fashion with several detections already reported by the LIGO-Virgo-KAGRA collaboration [1], and many more yet to come with the future planned observatories such as LISA [2] and the Einstein Telescope [3]. Motivated by the initial breakthroughs and the expected scientific output [4–8], a community effort has been established toward constructing high-accurate waveform models for the emission of GWs from binary systems. This includes numerical simulations for the merger phase [9,11] as well as analytic techniques to tackle the inspiral in the Post-Newtonian (PN) regime, using both traditional [12,13] and modern methodologies such as the effective field theory (EFT) approach [14–17].

These developments, in particular the use of tools from particle physics pioneered in [18], have impacted our understanding of the two-body problem in PN theory, e.g. [19–54], leading to the knowledge of the conservative spin-independent gravitational potential at fourth PN (4PN) order [52,56], in parallel with independent derivations using traditional methods [55–59]. The current state-of-the-art includes reports of contributions at 5PN [58,59], and partial results at 6PN [55,56,61,63].

Inspired by the EFT framework in the PN regime [14–17], novel ideas from the theory of scattering amplitudes [64], and the existence of a correspondence for observables in hyperbolic-like and elliptic-like orbits via analytic continuation in the binding energy and angular momenta [64], the Boundary-to-Bound (B2B) correspondence [55,56]—significant efforts have been invested in recent years towards studying scattering processes within the Post-Minkowskian (PM) expansion, both with amplitude-based [55,59] and EFT-based [90–101] methodologies. The PM expansion naturally encapsulates an infinite (resumed) series of velocity corrections at each order in Newton’s constant, which may result in improved waveform models, e.g. [102].

After the seminal result at third PM (3PM) order for non-spinning binary systems [75,76,93], partial (potential-only) corrections at 4PM have been obtained within both approaches [83,99]. Here we extend the knowledge of the two-body dynamics at $\mathcal{O}(G^4)$, by including both potential and radiation-reaction tail effects—the latter being due to the back-scattering of the outgoing GWs on the background geometry—thus removing spurious divergent terms in previous potential-only computations. Similarly to the Lamb shift [34], yet in a classical setting, mode-factorization into potential (off-shell) and radiation (on-shell) regions led to infrared (IR)/ultraviolet (UV) divergences in PN computations [53,59], which ultimately cancel out in physical observables [32,33,36]. As we demonstrate here, the explicit cancelation is also manifest in the PM regime, yielding (ambiguity-free) finite results.

Our derivation proceeds through the scattering angle computed in the EFT approach [92,93], in conjunction with the B2B map [55,56] between unbound and bound motion extended to radiative effects [103]. Using multiloop integration tools from particle physics [104–132], the calculation is reduced to a series of ‘three-loop’ (massless) integrals which are computed through differential equations. The resulting deflection angle features logarithm, dilogarithm and complete elliptic integrals of the first and second kind, and agrees in the overlap with the state-of-the-art in PN theory [40,43,61,62]. For completeness, we reconstruct the center-of-mass momentum.

The EFT formalism. Following the procedure discussed in [92,93,99], the effective action $(S_{\text{eff}})$ is obtained by integrating out the metric perturbation, $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$, using the (classical) saddle-point approximation of the path integral, schematically

$$\int Dh \, e^{i(S_{\text{EH}} + S_{\text{pp}})} \to e^{iS_{\text{eff}}},$$

(1)

in Einstein’s gravity $(S_{\text{EH}})$ coupled to point-like worldline sources $(S_{\text{pp}})$, ignoring quantum effects. The computation is reduced to a series of (‘tree-level’) Feynman diagrams. The full set of topologies at 4PM are shown in Fig. 1. As before [99], we must include mirror images and iterations from lower order solutions to the trajectories.
The computation of the following set of (three-loop) integrals, the total mass. As expected, the combined result is de-
tion scale (with $\gamma^2$ restricted by Dirac-
where $p^2 = -p^2 + i0$, as long as we consider the real part of the
effective action [33], ignoring dissipative effects. This allows us to retain the integration machinery intact.

From the resulting effective action we can then compute the scattering angle, $\chi$, through the impulse, $\Delta P_a = 1, 2, 3$, evaluated on the classical trajectory [92–94], as long as we consider the real part of the differential equations [105–111], with boundary conditions computed in the near-static limit $x \approx 1$. We make extensive use of the combination-by-parts (IBP) relations [112–113], via FIRE6 [114–115] and LiteRed [116–117], as well as the initial algorithm [130]. The final result for the deflection angle involves logarithms, dilogarithms $(\text{Li}_2(x))$, as well as complete elliptic integrals of the first $(K(x))$ and second $(E(x))$ kind $[83]$. (See [133] for more details in the integration procedure.)

**Potential Region.** As a check, we re-evaluate the boundary conditions for the differential equations at $x = 1$ with potential modes. As discussed in [99], these may be reduced into a basis containing only seven integrals via additional IBP identities, which we compute by direct integration. As expected, the self-energy diagrams in Fig. 1 turn into (scaleless) integrals which vanish in dimensional regularization, and therefore do not contribute in the potential region. Adding the pieces together we recover the result in $[83]$.

$$\frac{\chi_b(\text{pot})}{\pi \Gamma} = \chi_s(x) + \nu \left( \frac{\chi_2(x)}{2\epsilon} + \chi_p(x) \right)$$

for the potential contribution to the scattering angle at $O(G^0)$, where $\Gamma \equiv E/M$, with $E$ the total energy, and $\nu = m_1 m_2 / M^2$ the symmetric mass-ratio. Expressions for the $(\chi_s, \chi_2, \chi_p)$ coefficients are given in [99] and the ancillary file, see also [8] and the supplemental material.

**Tail Region.** The boundary conditions including radiation-reaction effects is more challenging, mainly due to the interplay between potential and radiation modes. We use the asy2.m code in the FIESTA package to identify the relevant regions of integration [119–120]. We find several contributions featuring one, two and up to three radiation modes. We keep only regions consistent with conservative radiation-reaction tail effects.

After performing a Laurent expansion around $x \approx 1$, yielding the anticipated pole in $(1 - x)^{-4\epsilon}$ [99], we computed the associated boundary integrals using various consistency relations [132]. Collecting the terms we find

$$\frac{\chi_b(\text{tail})}{\pi \Gamma} = \nu \left( -\frac{\chi_2(x)}{2\epsilon} (1 - x)^{-4\epsilon} + \chi_l(x) \right)$$

for the (conservative) contribution to the deflection angle due to tail effects at 4PM. The value of $\chi_l(x)$ is given in the supplemental material and ancillary file.
Combined Result. As expected, the divergence and $\mu$-dependence cancel out and the combined result becomes

$$\chi^{(4)}_{\text{comb}} = \chi_s + v \left( \chi_c(x) + 2\chi_2(x) \log(1 - x) \right),$$

where

$$\chi_c(x) = \frac{105h_1(x)}{128(x^2 - 1)^4}, \quad \chi_2(x) = -\frac{3h_2(x) \log(x) + 3h_3(x) \log \left( \frac{x+1}{x-1} \right) + h_4(x)}{32r(x^2 - 1)^4} + \frac{h_4(x)}{64r(x^2 - 1)^7},$$

$$\chi_\nu(x) = -\frac{21h_6(x)E^2(1 - x^2)}{8(x^2 - 1)^4} + \frac{3h_7(x)K(1 - x^2)E(1 - x^2)}{8(x^2 - 1)^4} - \frac{15h_8(x)K^2(1 - x^2)}{16(x^2 - 1)^4} - \frac{h_{16}(x) \log(x^2 + 1)}{32r(x^2 - 1)^4}.$$

with the explicit value of the $h_i(x)$ polynomials displayed in the ancillary file. The contribution to gauge-invariant observables for generic bound orbits (also for aligned-spin effects). Remarkably, the same is true for the trademark (non-local) logarithmic tail corrections, which may be obtained via

$$i^{\text{4PM}} = \frac{2v}{3} \left[ \frac{1 - \gamma^2}{(\Gamma r)^3} \chi_2(\gamma) \log |\mathcal{E}| \right],$$

to all orders in velocity, with $\mathcal{E} \equiv \frac{E - M}{M}$ the (reduced) binding energy. This is not the case, however, with other non-local-in-time effects for generic orbits, which do not transition smoothly between hyperbolic- and elliptic-like motion and therefore cannot be derived from the knowledge of the scattering angle.

From the deflection angle we can also reconstruct the 4PM coefficient, $f_4(\mathcal{E})$, of the PM expansion of an effective (local-in-time) center-of-mass momentum

$$p^2 = p^2_\infty \left[ 1 + \sum_{n=1}^{\infty} f_n(\mathcal{E}) \left( \frac{GM}{r} \right)^n \right],$$

in an isotropic gauge, such that $i_r \propto \int p_r dr$, using the relationship and, together with previous results to 3PM. Explicit values are given in the ancillary file. The Hamiltonian can be also reconstructed using the algebraic relations in Notice, as advertised in that in all cases the factors of $\log r$ in the intermediate results drop out of the final expressions.

\footnote{There is a mismatch at $\mathcal{O}(\nu^2)$ between the results in and those in , which can be traced to the definition of conservative terms in .}


Conclusions. We have computed the contribution from potential and radiation-reaction tail effects to the conservative dynamics of binary compact objects to 4PM order in the large-eccentricity limit. Our (ambiguity-free) result for the deflection angle at 4PM agrees in the overlap with state-of-the-art computations in PN theory for the combined potential and tail contributions [40,43,61,62]. As it was already the case in previous derivations in the EFT approach [93,99], the PM result can be entirely bootstrapped from PN data to all orders in velocities through differential equations—at this order including a sector involving elliptic integrals—without resorting to PN resummations. The boundary conditions (in the near-static limit) were obtained via the method of regions with potential and radiation modes.

There are, however, some important caveats that need to be addressed in order to complete the knowledge of the conservative 4PM dynamics for generic orbits, notably the mapping between unbound and bound motion for all the non-local-in-time effects. Moreover, there is also the issue of conservative non-linear memory terms. The latter arise from the interaction between the outgoing GW radiation and the waves emitted by the binary system at an earlier time. The derivation of memory effects at 4PM, the extension of the B2B map to generic non-local-in-time terms, as well as the computation of higher PM orders, is underway in the EFT approach.

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Supplemental Material

The coefficients of the scattering angle for the potential (beyond the test-body limit) and tail contributions to 4PM order displayed in the text are given by:

\[
\chi_p(x) = -\frac{\pi^2 h_5(x)}{1024 x^4 (x^2 - 1)^5} - \frac{21 h_6(x) E^2 (1 - x^2)}{8 (x^2 - 1)^4} + \frac{3 h_7(x) K (1 - x^2) E (1 - x^2)}{8 (x^2 - 1)^4} - \frac{15 h_8(x) K^2 (1 - x^2)}{16 (x^2 - 1)^4} + \frac{3 h_9(x) \log^2 \left(\frac{x + 1}{x - 1}\right)}{16 x^2 (x^2 - 1)^2} \\
+ \frac{3 h_{10}(x) \log^2 \log(x)}{8 (x^2 - 1)^3} + \frac{h_{11}(x) \log^2 \log(x)}{128 x^3 (x^2 - 1)^3} + \frac{3 h_{12}(x) \log^2 \log(x)}{512 x^4 (x^2 - 1)^5} - \frac{3 h_{13}(x) \log(x) \log(x + 1)}{256 x^4 (x^2 - 1)^3} + \frac{h_{14}(x) \log(x)}{128 x^3 (x^2 - 1)^7} \\
- \frac{h_{15}(x) \log(x + 1)}{128 x^3 (x^2 - 1)^5} \frac{h_{16}(x) \log(x + 2)}{64 x^2 (x^2 - 1)^4} + \frac{3 h_{17}(x) \Li_2 \left(-\frac{x + 1}{x - 1}\right)}{256 x^4 (x^2 - 1)^5} - \frac{3 h_{18}(x) \Li_2 \left(-\frac{x + 1}{x - 1}\right)}{256 x^4 (x^2 - 1)^5} + \frac{h_{19}(x) \Li_2 \left(-\frac{x + 1}{x - 1}\right)}{256 x^4 (x^2 - 1)^5} \\
- \frac{h_{20}(x) \Li_2 \left(-\frac{x + 1}{x - 1}\right)}{128 x^4 (x^2 - 1)^5} + \frac{h_{21}(x)}{1536 x^3 (x^2 - 1)^9 (x^2 + 1)^7},
\]

\[
\chi_t(x) = -\frac{h_{16}(x) \log(x + 1)}{64 x^3 (x^2 - 1)^4} + \frac{3 h_{19}(x) \Li_2 \left(-\frac{x + 1}{x + 1}\right)}{256 x^4 (x^2 - 1)^2} + \frac{h_{22}(x) \left(12 \Li_2 (-x) - \pi^2\right)}{1024 x^4 (x^2 - 1)^2} - \frac{24 h_{23}(x) \log^2 \log(x)}{(x^2 - 1)^2} + \frac{6 h_{24}(x) \log \log(x)}{(x^2 - 1)^5} \\
+ \frac{3 h_{26}(x) \log^2 \log(x + 1)}{128 x^3 (x^2 - 1)^4} - \frac{h_{26}(x) \log(x + 1)}{128 x^3 (x^2 - 1)^4} - \frac{3 h_{27}(x) \log^2 \log(x)}{512 x^4 (x^2 - 1)^8} + \frac{3 h_{28}(x) \log(x + 1) \log(x)}{256 x^4 (x^2 - 1)^3} - \frac{h_{29}(x) \log(x)}{128 x^4 (x^2 - 1)^7} \\
- \frac{3 h_{30}(x) \log^2 \log(x + 1)}{128 x^3 (x^2 - 1)^8} + \frac{h_{31}(x) \log(x + 1)}{128 x^3 (x^2 - 1)^8} - \frac{3 h_{32}(x) \Li_2 \left(-\frac{x + 1}{x - 1}\right)}{256 x^4 (x^2 - 1)^5} + \frac{3 h_{33}(x) \Li_2 \left(-\frac{x + 1}{x - 1}\right)}{256 x^4 (x^2 - 1)^5} + \frac{h_{34}(x)}{1536 x^3 (x^2 - 1)^9 (x^2 + 1)^7},
\]

\(^2\) The result in [43] suggests the appearance of a conservative memory term in the overlap between 4PM and 5PN orders at $\mathcal{O}(\nu^2)$.  

where the $h_i(x)$’s are polynomials in $x$ up to degree 32, collected in the ancillary file. We use the following conventions

\[
\text{Li}_2(z) \equiv \int_0^z \frac{\log(1-t)}{t} \, dt,
\]

\[
K(z) \equiv \int_0^1 \frac{dt}{\sqrt{(1-t^2)(1-z^2)}},
\]

\[
E(z) \equiv \int_0^1 \frac{dt}{\sqrt{1-z^2}},
\]

for the dilogarithm, and complete elliptic integral of the first and second kind, respectively.

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