A STUDY OF THE ORIENTATION AND 
ENERGY PARTITION OF THREE-JET EVENTS 
IN HADRONIC $Z^0$ DECAYS*

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ABSTRACT

We have measured the distributions of the jet energies in $e^+e^- \rightarrow q\bar{q}g$ events, and of the three orientation angles of the event plane, using hadronic $Z^0$ decays collected in the SLD experiment at SLAC. We find that the data are well described by perturbative QCD incorporating vector gluons. We have also compared our data with models of scalar and tensor gluon production, and discuss limits on the relative contributions of these particles to three-jet production in $e^+e^-$ annihilation.

1. Introduction

The observation of $e^+e^-$ annihilation into final states containing three hadronic jets [1], and their interpretation in terms of the process $e^+e^- \rightarrow q\bar{q}g$, provided the first direct evidence for the existence of the gluon, the gauge boson of the theory of strong interactions, Quantum Chromodynamics (QCD) [2]. Following these initial observations studies of the partition of energy among the three jets were performed at the PETRA and PEP storage rings. Comparison of the data with leading-order QCD predictions, and with a model incorporating the radiation of spin-0 (scalar) gluons, provided qualitative evidence [3] for the spin-1 (vector) nature of the gluon, which is a fundamental element of QCD. Similar studies have since been performed at LEP [4, 5].

An additional interesting observable in three-jet events is the orientation of the event plane w.r.t. the beam direction, which can be described by three Euler angles. These angular distributions were studied first by TASSO [6], and more recently by L3 [4] and DELPHI [7]. Again, the data were compared with the predictions of perturbative QCD and a scalar gluon model, but the Euler angles are less sensitive than the jet energy distributions to the differences between the two cases [4].

Here we present measurements of the jet energy and event plane orientation angle distributions from hadronic decays of $Z^0$ bosons produced by $e^+e^-$ annihilations at the SLAC Linear Collider (SLC) and recorded in the SLC Large Detector (SLD). We used particle energy deposits measured in the SLD Liquid Argon Calorimeter, which covers 98% of the solid angle, for jet reconstruction. We compare our measured distributions with the predictions of perturbative QCD and a scalar gluon model. In addition, we make the first comparison [8] with a model which comprises spin-2 (tensor) gluons, and discuss limits on the possible relative contributions of scalar and tensor gluons to three-jet production in $e^+e^-$ annihilation.

In Section 2 the observables are defined, and the predictions of perturbative QCD
and of the scalar and tensor gluon models are discussed. We describe the detector, the event trigger, and the selection criteria applied to the data, in Section 3. The three-jet analysis is described in Section 4, and a summary and conclusions are presented in Section 5.

2. Observables and Theoretical Predictions

A. Scaled Jet Energy Distributions

Ordering the three jets in $e^+e^- \rightarrow q\bar{q}g$ according to their energies, $E_1 > E_2 > E_3$, and normalising by the c.m. energy $\sqrt{s}$, we obtain the scaled jet energies:

$$x_i = \frac{2E_i}{\sqrt{s}} \quad (i = 1, 2, 3),$$

(1)

where $x_1 + x_2 + x_3 = 2$. Making a Lorentz boost of the event into the rest frame of jets 2 and 3 the Ellis-Karliner angle $\theta_{EK}$ is defined [9] to be the angle between jets 1 and 2 in this frame. For massless partons at tree-level:

$$\cos \theta_{EK} = \frac{x_2 - x_3}{x_1}.$$  

(2)

The inclusive differential cross section can be calculated to $O(\alpha_s)$ in perturbative QCD incorporating spin-1 (vector) gluons and assuming massless partons [10]:

$$\frac{1}{\sigma} \frac{d^2\sigma^V}{dx_1 dx_2} \propto \frac{x_1^3 + x_2^3 + (2 - x_1 - x_2)^3}{(1 - x_1)(1 - x_2)(x_1 + x_2 - 1)}.$$  

(3)

One can also consider alternative 'toy' models of strong interactions. For a model incorporating spin-0 (scalar) gluons one obtains at leading order at the $Z^0$ resonance [11]:

$$\frac{1}{\sigma} \frac{d^2\sigma^S}{dx_1 dx_2} \propto \frac{x_1^2(1 - x_1) + x_2^2(1 - x_2) + (2 - x_1 - x_2)^2(x_1 + x_2 - 1)}{(1 - x_1)(1 - x_2)(x_1 + x_2 - 1)} - R.$$  

(4)

where

$$R = \frac{10}{\Sigma_j a_j^2} \frac{\Sigma_j v_j^2}{\Sigma_j (v_j^2 + a_j^2)}.$$  

(5)

and $a_j$ and $v_j$ are the axial and vector couplings, respectively, of quark flavor $j$ to the $Z^0$. For a model of strong interactions incorporating spin-2 (tensor) gluons (see Appendix) one obtains at leading order:

$$\frac{1}{\sigma} \frac{d^2\sigma^T}{dx_1 dx_2} \propto \frac{(x_1 + x_2 - 1)^3 + (1 - x_1)^3 + (1 - x_2)^3}{(1 - x_1)(1 - x_2)(x_1 + x_2 - 1)}.$$  

(6)
Singly-differential cross sections for \( x_1, x_2, x_3 \) or \( \cos \theta_{EK} \) were obtained by numerical integrations of Eqs. 3, 4 and 5. These cross sections are shown in Fig. 1; the shapes are different for the vector, scalar and tensor gluon cases.

It is well known that vector particles coupling to quarks in either Abelian or non-Abelian theories allow consistent and renormalizable calculations to all orders in perturbation theory. However, the scalar and tensor gluon models have limited applicability beyond leading order. In the scalar model no symmetry, such as gauge invariance, exists to prevent the gluons from acquiring mass. In the tensor case the model is non-renormalizable (see Appendix), so that higher order predictions are not physically meaningful. Given these difficulties we limit ourselves to the leading-order expressions for 3-jet event production in these two cases. In the vector case we do consider the influence of higher-order corrections to the leading-order predictions. We also assume that the transformation of the partons in 3-jet events into the observed hadrons is independent of the gluon spin.

B. Event Plane Orientation

The orientation of the three-jet event plane can be described by the angles \( \theta, \theta_N \) and \( \chi \) illustrated in Fig. 2. When no explicit quark, antiquark or gluon jet identification is made, \( \theta \) is the polar angle of the most energetic jet w.r.t. the electron beam direction, \( \theta_N \) is the polar angle of the normal to the event plane w.r.t. the electron beam direction, and \( \chi \) is the angle between the event plane and the plane containing the electron beam and the most energetic jet. The distributions of these angles may be written [11]:

\[
\frac{d\sigma}{d\cos \theta} \propto 1 + \alpha(T)\cos^2 \theta \tag{7}
\]

\[
\frac{d\sigma}{d\cos \theta_N} \propto 1 + \alpha_N(T)\cos^2 \theta_N \tag{8}
\]

\[
\frac{d\sigma}{d\chi} \propto 1 + \beta(T)\cos 2\chi \tag{9}
\]

where \( T \) is the thrust value [12] of the event. The coefficients \( \alpha(T), \alpha_N(T) \) and \( \beta(T) \) depend on the gluon spin; they are shown in Fig. 16 for leading-order calculations incorporating vector, scalar and tensor gluons. In perturbative QCD \( O(\alpha_s^4) \) corrections to the leading-order result have been calculated and are small [13].

In \( Z^0 \) decay events produced with longitudinally-polarized electrons an additional term \( \beta_N S_{Z \cos \theta_N} \), representing a correlation between the event-plane orientation and the \( Z^0 \) spin direction, should be added to eq. (8). For Standard Model processes the
correlation parameter \( \beta_N \) is expected [14] to be of order \( 10^{-5} \), which is well below our current experimental sensitivity [15]. In this analysis we have ignored information on the helicity of the electron beam and are hence insensitive to a term in eq. (8) linear in \( \cos \theta_N \).

3. Apparatus and Hadronic Event Selection

The \( e^+e^- \) annihilation events produced at the \( Z^0 \) resonance by the SLC in the 1993 run were recorded using the SLD. A general description of the SLD can be found elsewhere [16]. The analysis presented here used particle energy deposits measured in the Liquid Argon Calorimeter (LAC) [17], which contains both electromagnetic and hadronic sections, and in the Warm Iron Calorimeter [18]. The trigger for hadronic events required a total LAC electromagnetic energy greater than 12 GeV.

Clusters were formed from the localized energy depositions in the LAC; energy depositions consistent with background muons produced upstream in the accelerator were identified and removed [19]. The measured cluster energies were then corrected [8] for the response of the LAC, which varies with polar angle \( \theta \) due to the material of the inner detector components as well as the thinner calorimeter coverage at the endcap-barrel interface, using a detailed Monte Carlo simulation of the detector. We first verified that the measured energy of clusters in each polar-angle bin, integrated over all selected clusters in all selected hadronic events, was well described by the simulation. Next, the ratio of simulated cluster energy to generated particle energy was calculated for each cluster. This ratio was averaged over all clusters in each polar-angle bin to yield the response function \( r(\theta) \). Finally, the measured energy of each cluster in the data was weighted by \( 1/r(\theta) \). The normalised r.m.s. deviation of the distribution of the total cluster energy in hadronic events was 21% before, and 16.5% after, application of this procedure [8].

Corrected clusters were then required to have a non-zero electromagnetic energy component and a total energy \( E_{cl} \) of at least 100 MeV. For each event the total cluster energy \( E_{tot} \), energy imbalance \( \Sigma|E_{cl}|/E_{tot} \), and thrust axis polar angle \( \theta_T \) [12] were calculated from the selected corrected clusters. Events with \( |\cos \theta_T| \leq 0.8 \) (\( |\cos \theta_T| \geq 0.8 \)) were then required to contain at least 8 (11) such clusters, to have \( E_{tot} > 15 \) GeV, and to have \( \Sigma|E_{cl}|/E_{tot} < 0.6 \). From our 1993 data sample approximately 51,000 events passed these cuts. The efficiency for selecting hadronic events was estimated to be \( 92 \pm 2\% \), with an estimated background in the selected sample of \( 0.4 \pm 0.2\% \) [20], dominated by \( Z^0 \rightarrow \tau^+\tau^- \) and \( Z^0 \rightarrow e^+e^- \) events.
4. Data Analysis

Jets were reconstructed from selected LAC clusters in selected hadronic events. The JADE jet-finding algorithm \cite{21} was used, with a scaled invariant mass cutoff value \( y_c = 0.02 \), to identify a sample of 22,114 3-jet events. This \( y_c \) value maximises the rate of events classified as 3-jet final states; other values of \( y_c \) were also considered and found not to affect the conclusions of this study. A non-zero sum of the three jet momenta can be induced in the selected events by particle losses due to the acceptance and inefficiency of the detector, and by jet energy resolution effects. This was corrected by rescaling the measured jet momenta \( \vec{P}_i \) \((i = 1,2,3)\) according to:

\[
P_i' = P_i - R^i |P_i|
\]

where \( P_i^j \) is the \( j \)-th momentum component of jet \( i \), \( j = x, y, z \), and \( R^i = \frac{\sum_{j=1}^{3} P_i^j}{\sum_{j=1}^{3} |P_i^j|} \).

The jet energy components were then rescaled according to:

\[
E_i' = \frac{|\vec{P}_i'|}{|\vec{P}_i|} E_i
\]

This procedure resulted in a slight improvement in the experimental resolution of the scaled jet energies \( x_i \) \cite{8}.

A. Scaled Jet Energy Distributions

The measured distributions of the three scaled jet energies \( x_1, x_2, x_3 \), and the Ellis-Karliner angle \( \theta_{EK} \), are shown in Fig. 3. Also shown in Fig. 3 are the predictions of the HERWIG 5.7 \cite{22} Monte Carlo program for the simulation of hadronic decays of \( Z^0 \) bosons, combined with a simulation of the SLD and the same selection and analysis cuts as applied to the real data. The simulation describes the data well.

For each observable \( X \), the experimental distribution \( D_{SLD}^{data}(X) \) was then corrected for the effects of selection cuts, detector acceptance, efficiency, resolution, particle decays and interactions within the detector, and for initial state photon radiation, using bin-by-bin correction factors \( C_D(X) \):

\[
C_D(X)_m = \frac{D_{hadron}^{MC}(X)_m}{D_{SLD}^{MC}(X)_m},
\]

where: \( m \) is the bin index; \( D_{SLD}^{MC}(X)_m \) is the content of bin \( m \) of the distribution obtained from reconstructed clusters in Monte Carlo events after simulation of the
detector; and $D_{\text{hadron}}^{MC}(X)_i$ is that from all generated particles with lifetimes greater than $3 \times 10^{-10}$ s in Monte Carlo events with no SLD simulation and no initial state radiation. The bin widths were chosen from the estimated experimental resolution so as to minimize bin-to-bin migration effects. The $C_D(X)$ were calculated from events generated with HERWIG 5.7 using default parameter values \cite{22}. The hadron level distributions are then given by

$$D_{\text{hadron}}^{\text{data}}(X)_m = C_D(X)_m \cdot D_{\text{SLD}}^{\text{data}}(X)_m. \quad (14)$$

Experimental systematic errors arising from uncertainties in modelling the detector were estimated by varying the event selection criteria over wide ranges, and by varying the cluster energy response corrections in the detector simulation \cite{8}. In each case the correction factors $C_D(X)$, and hence the corrected data distributions $D_{\text{hadron}}^{\text{data}}(X)$, were rederived. The correction factors $C_D(X)$ are shown in Figs. 4(b)–7(b); the errors comprise the sum in quadrature of the statistical component from the finite size of the Monte Carlo event sample, and the systematic uncertainty. It can be seen that the $C_D(X)$ are close to unity and slowly-varying, except near the boundaries of phase-space. The hadron level data are listed in Tables I–IV, together with statistical and systematic errors; the central values represent the data corrected by the central values of the correction factors.

Before they can be compared with parton-level predictions the data must be corrected for the effects of hadronization. In the absence of a complete theoretical calculation, the phenomenological models implemented in JETSET 7.4 \cite{23} and HERWIG 5.7 represent our best description of the hadronization process, and are not based upon a particular choice of the gluon spin. These models have been compared extensively with, and tuned to, $e^+e^- \rightarrow$ hadrons data at the $Z^0$ resonance \cite{24}, as well as data at $W \sim 35$ GeV from the PETRA and PEP storage rings \cite{25}. We find that they provide a good description of our data in terms of the observables presented here (Fig. 3) and other hadronic event shape observables \cite{26}, and hence employ them to calculate hadronization correction factors. The HERWIG parameters were left at their default values. Several of the JETSET parameters were set to values determined from our own optimisation to hadronic $Z^0$ data; these are given in Table V.

The hadronization correction procedure is similar to that described above for the detector effects. Bin-by-bin correction factors

$$C_H(X)_m = \frac{D_{\text{parton}}^{MC}(X)_m}{D_{\text{hadron}}^{MC}(X)_m}, \quad (15)$$

where $D_{\text{parton}}^{MC}(X)_m$ is the content of bin $m$ of the distribution obtained from Monte Carlo events generated at the parton level, were calculated and applied to the hadron
level data distributions $D_{\text{hadron}}^\text{data}(X)_m$ to obtain the parton level corrected data:

$$D_{\text{parton}}^\text{data}(X)_m = C_H(X)_m \cdot D_{\text{hadron}}^\text{data}(X)_m.$$  \hspace{1cm} (16)

For each bin the average of the JETSET- and HERWIG-derived values was used as the central value of the correction factor, and the difference between this value and the extrema was assigned as a symmetric hadronization uncertainty. The correction factors $C_H(X)$ are shown in Figs. 4(c)–7(c); the errors comprise the sum in quadrature of the statistical component from the finite size of the Monte Carlo event sample, and the systematic uncertainty. It can be seen that the $C_H(X)$ are within 10% of unity and are slowly-varying, except near the boundaries of phase space. The fully-corrected data are shown in Figs. 4(a)–7(a); the data points correspond to the central values of the correction factors, and the errors shown comprise the statistical and total systematic components added in quadrature. These results are in agreement with an analysis of our 1992 data sample using charged tracks for jet reconstruction [27].

We first compare the data with QCD predictions from $O(\alpha_s)$ and $O(\alpha_s^2)$ perturbation theory, and from parton shower (PS) models. For this purpose we used the JETSET 7.4 $O(\alpha_s)$ matrix element, $O(\alpha_s^2)$ matrix element, and PS options, and the HERWIG 5.7 PS, and generated events at the parton level. In each case all parameters were left at their default values [22, 23], with the exception of the JETSET parton shower parameters listed in Table V. The QCD scale parameter values used were $\Lambda = 1.0$ GeV ($O(\alpha_s)$), 0.25 GeV ($O(\alpha_s^2)$), 0.26 GeV (JETSET PS) and 0.18 GeV (HERWIG PS). The shapes of the $x_1$, $x_2$, $x_3$ and $\cos\theta_{EK}$ distributions do not depend on $\Lambda$ at $O(\alpha_s)$, and only weakly so at higher order. The resulting predictions for $x_1$, $x_2$, $x_3$ and $\cos\theta_{EK}$ are shown in Figs. 4(a)–7(a). These results represent Monte Carlo integrations of the respective QCD formulae and are hence equivalent to analytic or numerical QCD results based on the same formulae; in the $O(\alpha_s)$ case we have checked explicitly that JETSET reproduces the numerical results of the analytic calculation described in Section 2.

The $O(\alpha_s)$ calculation describes the data reasonably well, although small discrepancies in the details of the shapes of the distributions are apparent and the $\chi^2$ for the comparison between data and MC is poor (Table VI). The $O(\alpha_s^2)$ calculation describes the $x_1$, $x_2$ and $x_3$ data distributions better, but the description of the $\cos\theta_{EK}$ distribution is slightly worse; this is difficult to see directly in Figs. 4(a)–7(a), but is evident from the $\chi^2$ values for the data–MC comparisons (Table VI). Both parton shower calculations describe the data better than either the $O(\alpha_s)$ or $O(\alpha_s^2)$ calculations and yield relatively good $\chi^2$ values (Table VI). This improvement in the quality of description of the data between the $O(\alpha_s)$ and parton shower calculations can be interpreted as an indication of the contribution of multiple soft gluon emission to the fine details of
the shapes of the distributions. In fact for all calculations the largest discrepancies, at the level of at most 10%, arise in the regions $x_1 > 0.98$, $x_2 > 0.93$, $x_3 < 0.09$ and $\cos \theta_{E\!K} > 0.9$, near the boundaries of phase space where soft and collinear divergences are expected to be large and to require resummation in QCD perturbation theory \[28\]; such resummation has not been performed for the observables considered here.

For each observable we chose a range such that the detector and hadronization correction factors are close to unity, $0.8 < C_D(X), C_H(X) < 1.2$, have small uncertainty, $\Delta C_D(X), \Delta C_H(X) < 0.2$, and are slowly-varying (see Figs. 4-7). The ranges are: $0.688 < x_1 < 0.976$, $x_2 < 0.93$, $x_3 > 0.09$ and $\cos \theta_{E\!K} < 0.9$; they exclude the phase-space boundary regions. Within these ranges the comparison between data and calculations yields significantly improved $\chi^2$ values (values in parentheses in Table VI); the $O(\alpha_s^2)$ calculation has acceptable $\chi^2$ values and those for both parton shower models are typically slightly better. These results support the notion that QCD, incorporating vector gluons, is the correct theory of strong interactions.

We now consider alternative models of strong interactions, incorporating scalar and tensor gluons, discussed in Section 2. Since these model calculations are at leading order in perturbation theory we also consider first the vector gluon (QCD) case at the same order. The data within the selected ranges are shown in Fig. 8; from comparison with the raw data (Fig. 3) it is apparent that the shapes of the distributions are barely affected by the detector and hadronization corrections. The leading-order scalar, vector and tensor gluon predictions, normalised to the data within the same ranges, are also shown in Fig. 8. The vector calculation clearly provides the best description of the data; neither the scalar nor tensor cases predicts the correct shape for any of the observables. The $\chi^2$ values for the comparisons are given in Table VII. This represents the first comparison of a tensor gluon calculation with experimental data.

It is interesting to consider whether the data allow an admixture of contributions from the different gluon spin hypotheses. For this purpose we performed simultaneous fits to a linear combination of the vector ($V$) + scalar ($S$) + tensor ($T$) predictions, allowing the relative normalisations to vary according to:

\begin{equation}
(1 - a - b) V + a S + b T
\end{equation}

where $a$ and $b$ are free parameters determined from the fit. For the vector contribution we used in turn the $O(\alpha_s)$, $O(\alpha_s^2)$, JETSET PS and HERWIG PS calculations. In all cases the fit to the distribution of each observable yielded a slightly lower $\chi^2$ value than the vector-only fit. We found that the allowed contributions of scalar and tensor gluons depend upon the order of the vector calculation used, as well as on the observable. The largest allowed scalar contribution was $a = 0.11$ from the fit to $\cos \theta_{E\!K}$ using the $O(\alpha_s^2)$ calculation. The largest allowed tensor contribution was $b = 0.31$ from the fit
to $x_1$ using the $O(\alpha_s)$ calculation. The smallest allowed contributions were $a$ and $b < 0.001$ from the fit to $x_1$ using the HERWIG PS.

Any pair of the observables $x_1, x_2, x_3$ and $\cos\theta_{EK}$ may be taken to be independent variables, subject to the overall constraint $x_1 + x_2 + x_3 = 2$. Therefore, in order to utilise more information, we also performed fits of Eq. 17 simultaneously to the $x_2$ and $x_3$ distributions. We found the relative S, V and T contributions and the $\chi^2/d.o.f.$ values to be comparable with those from the fits to $x_2$ alone.

### B. Event Plane Orientation

We now consider the three Euler angles that describe the orientation of the event plane: $\theta$, $\theta_N$, and $\chi$ (Fig. 2). The analysis procedure is similar to that described in the previous section. The measured distributions of these angles are shown in Fig. 9, together with the predictions of HERWIG 5.7, combined with a simulation of the SLD and the same selection and analysis cuts as applied to the data. The simulations describe the data reasonably well. The data distributions were then corrected for the effects of selection cuts, detector acceptance, efficiency, and resolution, particle decays and interactions within the detector, and for initial state photon radiation using bin-by-bin correction factors determined from the Monte Carlo simulation. The correction factors $C_D$ are shown in Figs. 10(b)–12(b); the errors comprise the sum in quadrature of the statistical component from the finite size of the Monte Carlo event sample, and the systematic uncertainty derived as described in the previous section. The hadron level data are listed in Tables VIII–X, together with statistical and systematic errors; the central values represent the data corrected by the central values of the correction factors.

The data were further corrected bin-by-bin for the effects of hadronisation. The hadronisation correction factors are shown in Figs. 10(c)–12(c); the errors comprise the sum in quadrature of the statistical component from the finite size of the Monte Carlo event sample, and the systematic uncertainty. The fully-corrected data are shown in Figs. 10(a)–12(a); the data points correspond to the central values of the correction factors, and the errors shown comprise the statistical and total systematic components added in quadrature. Also shown in Figs. 10(a)–12(a) are the parton-level predictions of the JETSET 7.4 $O(\alpha_s)$ matrix element, $O(\alpha_s^2)$ matrix element, and parton shower options, and the HERWIG 5.7 parton shower. All calculations describe the data well, and higher-order corrections to the $O(\alpha_s)$ predictions are seen to be small.

The data were divided into four samples according to the thrust values of the events: (i) $0.70 < T < 0.80$, (ii) $0.80 < T < 0.85$, (iii) $0.85 < T < 0.90$ and (iv) $0.90 < T < 0.95$. The distributions of $\cos\theta$, $\cos\theta_N$ and $\chi$ are shown for these four ranges in Figs. 13, 14 and 15 respectively. Also shown in these figures are fits of
Eqs. (7), (8) and (9) (Section 2), where the parameters $\alpha(T)$, $\alpha_N(T)$ and $\beta(T)$ were determined, respectively, from the fits. The fitted values of these parameters are listed in Table XI, and are shown in Fig. 16, where they are compared with the leading-order QCD predictions and with the predictions of the scalar and tensor gluon models. Values of $\chi^2$ for these comparisons are given in Table XII. The data are in agreement with the QCD predictions, and the scalar and tensor gluon predictions are disfavoured. It should be noted, however, that the event plane orientation angle distributions are less sensitive to the different gluon spin cases than are the jet energy distributions discussed in the previous section.

5. Conclusions

We have measured distributions of the jet energies, and of the orientation angles of the event plane, in $e^+e^- \rightarrow Z^0 \rightarrow$ three-jet events recorded in the SLD experiment at SLAC. Our measurements of these quantities are consistent with those from other experiments [4, 5, 7] at the $Z^0$ resonance. We have compared our measurements with QCD predictions and with models of strong interactions incorporating scalar or tensor gluons; this represents the first comparison with a tensor gluon calculation.

The leading-order vector gluon (QCD) calculation describes the basic shape of the scaled jet energy distributions, and addition of higher-order perturbative contributions leads to a reasonable description of the finer details of these distributions, provided the regions of phase space are avoided where soft and collinear singularities need to be resummed. One may speculate that the addition of as yet uncalculated higher-order QCD contributions may yield further improvement. The shapes of the jet energy distributions cannot be described by leading-order models incorporating either scalar or tensor gluons alone. However, the ad hoc addition of leading-order contributions from scalar and tensor gluons, each with arbitrary relative weight, to the QCD predictions can also improve the description of the data; even for the QCD parton shower calculations slightly better fit qualities are obtained with such contributions included. The allowed relative contributions of scalar and tensor gluons depend upon the order of the vector calculation, as well as the observable; the smallest allowed contribution of 0.1% for both scalar and tensor gluons is obtained with the HERWIG parton shower fit to the scaled energy of the most energetic jet.

The event plane orientation angles are well described by $O(\alpha_s)$ QCD and higher-order corrections are small. These quantities are less sensitive to the gluon spin than the jet energies, but the data disfavor the scalar and tensor hypotheses.
6. Acknowledgements

We thank Lance Dixon for contributions to the tensor gluon model. We thank the personnel of the SLAC accelerator department and the technical staffs of our collaborating institutions for their efforts which resulted in the successful operation of the SLC and the SLD.

Appendix: Tensor Gluon Model

Since the tensor gluon toy model is new, whereas the vector and scalar cases have been studied in detail in the literature, we discuss briefly how Eq. 6 was obtained.

The only well-known theory involving the exchange of massless, spin-2 gauge fields is the quantized version of General Relativity, which is both highly non-linear and non-renormalizable. To obtain a simple parallel model for tensor gluons, which couple only to color non-singlet sources, we begin by linearizing the theory of quantum gravity based on General Relativity by keeping only the lowest order terms in the coupling and by ignoring the tensor field self-interactions \[29\]. Although now linear, the theory remains non-renormalizable, as will be the tensor gluon model, which should be viewed only as a toy model against which to test the predictions of QCD.

If tensor gluons behaved in the same way as gravitons one could write down the complete gauge-invariant amplitude for the tree-level process \(Z^0 \rightarrow q \bar{q} g\). The various contributions arise from a set of four Feynman diagrams: the usual two which involve gluon bremsstrahlung from the \(q\) or \(\bar{q}\) in the final state, the bremsstrahlung of a tensor gluon from the \(Z^0\) in the initial state, producing an off-shell \(Z^0\) which ‘decays’ to \(q \bar{q}\), and finally a new \(Z^0 q \bar{q} g\) contact interaction. We need to remove or modify the \(Z^0 Z^0 g\) piece of the amplitude as the \(Z^0\) is known phenomenologically not to carry a color charge.

We consider two possible approaches to this problem. In the first instance we surrender the possibility of a gauge symmetry for the tensor gluon theory and omit the diagram involving the \(Z^0 Z^0 g\) vertex. (We note that the scalar gluon model is also not a gauge theory.) In this case, using the Feynman gauge for the tensor gluon, we arrive at the distribution given in Eq. 6. A second possibility is to mimic the quantum gravity theory as far as possible and include the \(Z^0 Z^0 g\) diagram in a modified form. To do this we extend the particle spectrum of the Standard Model by introducing a color-octet partner to the \(Z^0\), \(Z^0_8\), which is degenerate with the \(Z^0\) and couples to quarks in exactly the same way as does the \(Z^0\), except for the presence of color generators. The problematic \(Z^0 Z^0 g\) vertex is now replaced by the \(Z^0 Z^0_8 g\) coupling. In this case we
arrive at a form for the tensor distribution given by [30]:

\[
\frac{1}{\sigma} \frac{d^2 \sigma^T}{dx_1 dx_2} \propto \frac{(x_1 + x_2 - 1)(x_1^2 + x_2^2)}{(2 - x_1 - x_2)^2} + \\
\frac{(1 - x_2)(x_1^2 + (2 - x_1 - x_2)^2)}{x_2^2} + \frac{(1 - x_1)(x_2^2 + (2 - x_1 - x_2)^2)}{x_1^2},
\]

which, apart from the overall normalisation, is the same as that for graviton radiation in \(Z^0\) decays. Although algebraically different, this form yields numerically similar results to Eq. 6 (Fig. 17).

In the analysis presented in the text the comparison of the tensor model with the data is based on Eq. 6. It is clear from Fig. 17, however, that our conclusions would not differ if Eq. 18 had been chosen instead.
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Table I. The measured scaled jet energy of the highest-energy jet in 3-jet events. The data were corrected for detector effects and for initial state photon radiation. The first error is statistical, and the second represents the experimental systematic uncertainty.

|   | $\frac{1}{\sigma_{3\text{-}jet}} \frac{d\sigma}{dx_1}$ | stat. | exp. syst. |
|---|---|---|---|
| 0.676 | 0.025 | 0.007 | 0.008 |
| 0.700 | 0.072 | 0.016 | 0.018 |
| 0.724 | 0.133 | 0.018 | 0.022 |
| 0.748 | 0.260 | 0.025 | 0.033 |
| 0.772 | 0.423 | 0.028 | 0.044 |
| 0.796 | 0.530 | 0.032 | 0.044 |
| 0.820 | 0.749 | 0.039 | 0.048 |
| 0.844 | 1.065 | 0.048 | 0.061 |
| 0.868 | 1.603 | 0.056 | 0.071 |
| 0.892 | 2.351 | 0.069 | 0.088 |
| 0.916 | 3.83 | 0.09 | 0.11 |
| 0.940 | 6.74 | 0.11 | 0.14 |
| 0.964 | 13.8 | 0.17 | 0.27 |
| 0.988 | 9.08 | 0.13 | 0.17 |

Table II. The measured scaled jet energy of the second highest-energy jet in 3-jet events. The data were corrected for detector effects and for initial state photon radiation. The first error is statistical, and the second represents the experimental systematic uncertainty.

|   | $\frac{1}{\sigma_{3\text{-}jet}} \frac{d\sigma}{dx_2}$ | stat. | exp. syst. |
|---|---|---|---|
| 0.5275 | 0.490 | 0.024 | 0.031 |
| 0.5625 | 1.031 | 0.039 | 0.050 |
| 0.5975 | 1.267 | 0.043 | 0.050 |
| 0.6325 | 1.356 | 0.044 | 0.051 |
| 0.6675 | 1.546 | 0.048 | 0.058 |
| 0.7025 | 1.689 | 0.048 | 0.057 |
| 0.7375 | 1.815 | 0.051 | 0.068 |
| 0.7725 | 1.938 | 0.053 | 0.061 |
| 0.8075 | 2.089 | 0.055 | 0.063 |
| 0.8425 | 2.619 | 0.060 | 0.071 |
| 0.8775 | 2.966 | 0.063 | 0.074 |
| 0.9125 | 3.391 | 0.064 | 0.082 |
| 0.9475 | 3.813 | 0.062 | 0.079 |
| 0.9825 | 2.205 | 0.056 | 0.075 |
Table III. The measured scaled jet energy of the lowest-energy jet in 3-jet events. The data were corrected for detector effects and for initial state photon radiation. The first error is statistical, and the second represents the experimental systematic uncertainty.
Table IV. The measured Ellis-Karliner angle distribution in 3-jet events. The data were corrected for detector effects and for initial state photon radiation. The first error is statistical, and the second represents the experimental systematic uncertainty.
| Parameter     | Variable Name | Default    | Optimised     |
|---------------|---------------|------------|---------------|
| $\Lambda_{QCD}$ | PARJ(81)      | 0.29 GeV  | 0.26 GeV     |
| $\sigma_q$    | PARJ(21)      | 0.36 GeV/c| 0.39 GeV/c   |
| $a$           | PARJ(41)      | 0.3        | 0.18         |
| $b$           | PARJ(42)      | 0.58 GeV$^{-2}$ | 0.34 GeV$^{-2}$ |
| $\epsilon_c$ | PARJ(54)      | −0.05      | −0.06        |
| $\epsilon_b$ | PARJ(55)      | −0.005     | −0.006       |
| diquark prob. | PARJ(1)       | 0.10       | 0.08         |
| s quark prob. | PARJ(2)       | 0.30       | 0.28         |
| s diquark prob.| PARJ(3)       | 0.40       | 0.60         |
| V meson prob. (u,d) | PARJ(11) | 0.50     | 0.50     |
| V meson prob. (s) | PARJ(12)   | 0.60     | 0.45     |
| V meson prob. (c,b) | PARJ(13)  | 0.75     | 0.53     |
| $\eta'$ prob. | PARJ(26)      | 0.40       | 0.20         |

Table V. Parameters in JETSET 7.4 that were changed from default values (see text).

| Distribution | # bins | JETSET $O(\alpha_s)$ | JETSET $O(\alpha_s^2)$ | JETSET PS | HERWIG PS |
|--------------|--------|-----------------------|-------------------------|-----------|-----------|
| $x_1$        | 14 (12)| 88.2 (72.6)           | 38.5 (26.3)             | 13.5 (6.3)| 11.2 (10.6)|
| $x_2$        | 14 (12)| 37.8 (20.0)           | 36.8 (12.2)             | 34.9 (21.0)| 15.2 (6.5)|
| $x_3$        | 15 (13)| 92.9 (49.8)           | 86.5 (29.6)             | 22.3 (17.5)| 25.7 (11.8)|
| $\cos\theta_{EK}$ | 20 (18) | 60.6 (26.3) | 86.2 (44.6) | 15.8 (9.0) | 48.2 (30.2) |

Table VI. Numbers of bins and $\chi^2$ values for comparison between fully corrected data and parton-level QCD Monte Carlo calculations. Values in parentheses are for the restricted ranges which exclude the regions where soft and collinear contributions are expected to be large.

| Distribution | # bins | Vector | Scalar | Tensor |
|--------------|--------|--------|--------|--------|
| $x_1$        | 12     | 45.2   | 1116.4 | 141.9  |
| $x_2$        | 12     | 33.5   | 1321.7 | 490.6  |
| $x_3$        | 13     | 39.9   | 2011.4 | 546.9  |
| $\cos\theta_{EK}$ | 18     | 19.5   | 1684.0 | 772.1  |

Table VII. Numbers of bins and $\chi^2$ values for comparison between fully corrected data and leading-order vector (QCD), scalar, and tensor gluon calculations.
Table VIII. The measured polar angle w.r.t. the electron beam of the highest-energy jet in 3-jet events. The data were corrected for detector effects and for initial state photon radiation. The first error is statistical, and the second represents the experimental systematic uncertainty.

| \( \cos \theta \) | \( \frac{1}{\sigma_{3\text{-jet}}} \frac{d\sigma}{d\cos \theta} \) | stat. | exp. syst. |
|-----------------|--------------------------------|-------|------------|
| 0.071           | 0.792                         | 0.021 | 0.031      |
| 0.214           | 0.822                         | 0.023 | 0.031      |
| 0.357           | 0.853                         | 0.023 | 0.030      |
| 0.500           | 0.982                         | 0.024 | 0.033      |
| 0.643           | 1.088                         | 0.026 | 0.031      |
| 0.786           | 1.135                         | 0.028 | 0.035      |
| 0.929           | 1.306                         | 0.035 | 0.090      |

Table IX. The measured polar angle w.r.t. the electron beam of the normal to the three-jet plane. The data were corrected for detector effects and for initial state photon radiation. The first error is statistical, and the second represents the experimental systematic uncertainty.

| \( \cos \theta_N \) | \( \frac{1}{\sigma_{3\text{-jet}}} \frac{d\sigma}{d\cos \theta_N} \) | stat. | exp. syst. |
|---------------------|--------------------------------|-------|------------|
| 0.071               | 1.159                         | 0.034 | 0.076      |
| 0.214               | 1.079                         | 0.029 | 0.046      |
| 0.357               | 1.110                         | 0.026 | 0.029      |
| 0.500               | 0.969                         | 0.025 | 0.028      |
| 0.643               | 0.967                         | 0.025 | 0.035      |
| 0.786               | 0.917                         | 0.023 | 0.036      |
| 0.929               | 0.804                         | 0.020 | 0.030      |

Table X. The measured angle between the event plane and the plane containing the highest-energy jet and the electron beam. The data were corrected for detector effects and for initial state photon radiation. The first error is statistical, and the second represents the experimental systematic uncertainty.

| \( \chi \) (rad.) | \( \frac{1}{\sigma_{3\text{-jet}}} \frac{d\sigma}{d\chi} \) | stat. | exp. syst. |
|-------------------|--------------------------------|-------|------------|
| 0.112             | 0.671                         | 0.025 | 0.034      |
| 0.336             | 0.644                         | 0.025 | 0.027      |
| 0.561             | 0.633                         | 0.025 | 0.026      |
| 0.785             | 0.642                         | 0.024 | 0.025      |
| 1.009             | 0.635                         | 0.023 | 0.025      |
| 1.234             | 0.592                         | 0.021 | 0.023      |
| 1.458             | 0.645                         | 0.021 | 0.023      |
Table XI. Thrust ranges, values and errors of the fit parameters $\alpha$, $\alpha_N$ and $\beta$, and $\chi^2$ values for the fits. For each fitted observable there are 7 bins.

| Thrust range | $\alpha(T)$ | $\chi^2$ | $\alpha_N(T)$ | $\chi^2$ | $\beta(T)$ | $\chi^2$ |
|--------------|-------------|-----------|---------------|-----------|------------|-----------|
| $0.7 < T < 0.8$ | $0.61 \pm 0.18$ | 6.1 | $-0.42 \pm 0.10$ | 1.9 | $0.090 \pm 0.069$ | 5.4 |
| $0.8 < T < 0.85$ | $0.83 \pm 0.19$ | 3.6 | $-0.31 \pm 0.11$ | 0.6 | $0.034 \pm 0.071$ | 3.3 |
| $0.85 < T < 0.9$ | $0.82 \pm 0.12$ | 8.3 | $-0.33 \pm 0.07$ | 7.8 | $0.004 \pm 0.041$ | 4.4 |
| $0.9 < T < 0.95$ | $0.81 \pm 0.09$ | 2.6 | $-0.26 \pm 0.06$ | 6.8 | $-0.033 \pm 0.030$ | 0.5 |

Table XII. Values of $\chi^2$ for comparisons between the predictions including vector, scalar or tensor gluons for the coefficients $\alpha(T)$, $\alpha_N(T)$ and $\beta(T)$ and the measured values (Fig. 16).

| Gluon spin | $\alpha(T)$ | $\alpha_N(T)$ | $\beta(T)$ |
|-----------|-------------|---------------|------------|
| Vector    | 3.0         | 2.8           | 2.4        |
| Scalar    | 17.4        | 38.0          | 8.8        |
| Tensor    | 7.3         | 5.7           | 4.4        |
Figure captions

Figure 1. Leading-order calculations, incorporating vector (solid), scalar (long dashed), and tensor (short dashed) gluons, of distributions of: (a) scaled energy of the highest-energy jet; (b) scaled energy of the second highest-energy jet; (c) scaled energy of the lowest-energy jet; (d) the Ellis-Karliner angle.

Figure 2. Definition of the Euler angles $\theta$, $\theta_N$ and $\chi$ that describe the orientation of the event plane.

Figure 3. Measured distributions (dots) of: (a) scaled energy of the highest-energy jet; (b) scaled energy of the second highest-energy jet; (c) scaled energy of the lowest-energy jet; (d) the Ellis-Karliner angle. The errors are statistical only. The predictions of a Monte Carlo simulation are shown as solid histograms.

Figure 4. (a) The measured distribution (dots) of the scaled energy of the highest-energy jet, fully-corrected to the parton level, compared with QCD Monte Carlo calculations. The errors comprise the total statistical and systematic components added in quadrature. The correction factors for detector effects and initial-state radiation (b) and for hadronisation effects (c); the inner error bars show the statistical component and the outer error bars the total uncertainty.

Figure 5. (a) The measured distribution (dots) of the scaled energy of the second highest-energy jet, fully-corrected to the parton level, compared with QCD Monte Carlo calculations. The errors comprise the total statistical and systematic components added in quadrature. The correction factors for detector effects and initial-state radiation (b) and for hadronisation effects (c); the inner error bars show the statistical component and the outer error bars the total uncertainty.

Figure 6. (a) The measured distribution (dots) of the scaled energy of the lowest-energy jet, fully-corrected to the parton level, compared with QCD Monte Carlo calculations. The errors comprise the total statistical and systematic components added in quadrature. The correction factors for detector effects and initial-state radiation (b) and for hadronisation effects (c); the inner error bars show the statistical component and the outer error bars the total uncertainty.

Figure 7. (a) The measured distribution (dots) of the Ellis-Karliner angle, fully-corrected to the parton level, compared with QCD Monte Carlo calculations. The errors comprise the total statistical and systematic components added in quadrature. The correction factors for detector effects and initial-state radiation (b) and for hadronisation effects (c); the inner error bars show the statistical component and the outer error bars the total uncertainty.

Figure 8. Measured distributions, fully corrected to the parton level (dots), of: (a) scaled energy of the highest-energy jet; (b) scaled energy of the second highest-energy jet; (c) scaled energy of the lowest-energy jet; (d) the Ellis-Karliner angle. The errors
comprise the total statistical and systematic components added in quadrature. The
leading-order predictions described in Section 2 are shown as lines: vector (solid), scalar
(long dashed), and tensor (short dashed).

Figure 9. Measured distributions (dots) of the event plane orientation angles: (a)
$\cos \theta$, (b) $\cos \theta_N$, (c) $\chi$. The errors are statistical only. The predictions of a Monte
Carlo simulation are shown as solid histograms.

Figure 10. (a) The measured distribution (dots) of $\cos \theta$, fully-corrected to the parton
level, compared with QCD Monte Carlo calculations. The errors comprise the total
statistical and systematic components added in quadrature. The correction factors
for detector effects and initial-state radiation (b) and for hadronisation effects (c); the
inner error bars show the statistical component and the outer error bars the total
uncertainty.

Figure 11. (a) The measured distribution (dots) of $\cos \theta_N$, fully-corrected to the parton
level, compared with QCD Monte Carlo calculations. The errors comprise the total
statistical and systematic components added in quadrature. The correction factors
for detector effects and initial-state radiation (b) and for hadronisation effects (c); the
inner error bars show the statistical component and the outer error bars the total
uncertainty.

Figure 12. (a) The measured distribution (dots) of $\chi$, fully-corrected to the parton
level, compared with QCD Monte Carlo calculations. The errors comprise the total
statistical and systematic components added in quadrature. The correction factors
for detector effects and initial-state radiation (b) and for hadronisation effects (c); the
inner error bars show the statistical component and the outer error bars the total
uncertainty.

Figure 13. The measured distributions (dots) of $\cos \theta$, fully-corrected to the parton
level, in the event thrust ranges: (a) $0.70 < T < 0.80$, (b) $0.80 < T < 0.85$, (c)
$0.85 < T < 0.90$, (d) $0.90 < T < 0.95$. The errors comprise the total statistical and
systematic components added in quadrature. Fits of Eq. 7 are shown as solid lines.

Figure 14. The measured distributions (dots) of $\cos \theta_N$, fully-corrected to the parton
level, in the event thrust ranges: (a) $0.70 < T < 0.80$, (b) $0.80 < T < 0.85$, (c)
$0.85 < T < 0.90$, (d) $0.90 < T < 0.95$. The errors comprise the total statistical and
systematic components added in quadrature. Fits of Eq. 8 are shown as solid lines.

Figure 15. The measured distributions (dots) of $\chi$, fully-corrected to the parton
level, in the event thrust ranges: (a) $0.70 < T < 0.80$, (b) $0.80 < T < 0.85$, (c)
$0.85 < T < 0.90$, (d) $0.90 < T < 0.95$. The errors comprise the total statistical and
systematic components added in quadrature. Fits of Eq. 9 are shown as solid lines.

Figure 16. Coefficients (a) $\alpha(T)$, (b) $\alpha_N(T)$, (c) $\beta(T)$ from the fits shown in Figs. 13,
14, 15 respectively. Also shown are the leading-order vector (solid), scalar (long dashed)
and tensor (short dashed) gluon predictions.

**Figure 17.** Leading-order tensor gluon model calculations, based on Eq. 6 (short dashed) and Eq. 18 (dash-dotted), of distributions of: (a) scaled energy of the highest-energy jet; (b) scaled energy of the second highest-energy jet; (c) scaled energy of the lowest-energy jet; (d) the Ellis-Karliner angle.
Fig. 1

SLD         Vector         Scalar         Tensor

(a) \(1/N \frac{dn}{dx_1}\) for different x_1 values
(b) \(1/N \frac{dn}{dx_2}\) for different x_2 values
(c) \(1/N \frac{dn}{dx_3}\) for different x_3 values
(d) \(1/N \frac{dn}{d\cos\theta_{EK}}\) for different \(\cos\theta_{EK}\) values
Fig. 2
Fig. 3
Fig. 4
Fig. 5

- SLD
- JETSET 7.4 O($\alpha_s$)
- JETSET 7.4 O($\alpha_s^2$)
- JETSET 7.4 PS
- HERWIG 5.7 PS

1/N $d^2N/dx_2^2$

$C_D(x_2)$

$C_H(x_2)$

$x_2$ range: 0.5 to 1.0
Fig. 6

(a) $1/N \frac{dn}{dx_3}$

(b) $C_D(x_3)$

(c) $C_H(x_3)$
Fig. 7
Fig. 8
Fig. 9

(a) 

(b) 

(c) 

$\frac{1}{N} \frac{dn}{d\cos\theta}$

$\frac{1}{N} \frac{dn}{d\cos\theta_N}$

$\frac{1}{N} \frac{dn}{d\chi}$

$\chi$ (rad)
Fig. 10
Fig. 12
Fig. 13
Fig. 14

(a) $0.70 \leq T < 0.80$

(b) $0.80 \leq T < 0.85$

(c) $0.85 \leq T < 0.90$

(d) $0.90 \leq T < 0.95$
Fig. 15

\(\frac{1}{N} \frac{dN}{d\chi}\)

(a) \(0.70 \leq T < 0.80\)

(b) \(0.80 \leq T < 0.85\)

(c) \(0.85 \leq T < 0.90\)

(d) \(0.90 \leq T < 0.95\)

SLD

Eq. 9
Fig. 16
Fig. 17