Hadronic Phases and Isospin Amplitudes in \( D(B) \to \pi\pi \) and \( D(B) \to K\bar{K} \) Decays

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Abstract

Hadronic phases in \( \pi\pi \) and \( K\bar{K} \) channels are calculated à la Regge. At the \( D \) mass one finds \( \delta_{\pi\pi} \simeq \frac{\pi}{3} \) and \( \delta_{K\bar{K}} \simeq -\frac{\pi}{6} \) in good agreement with the CLEO data while at the \( B \) mass these angles are predicted to be, respectively, 11° and -7°. With the hadronic phase \( e^{i\delta_{K\bar{K}}} \) taken into account, a quark diagram decomposition of the isospin invariant amplitudes in \( D \to K\bar{K} \) decays fits the data provided the exchange diagram contribution is about 1/3 of the tree level one.
1 Introduction

To extract information on weak interaction parameters from non leptonic two body decays of the $D$ and $B$ mesons, it is crucial to understand the final hadronic effects which are at work in these decays. For $(K\pi)$, $(\pi\pi)$ and $(K\bar{K})$ decay modes, the important hadronic parameter is an angle $\delta$ which is the difference between s-wave phase shifts in the appropriate isospin invariant amplitudes.

In a previous paper [1] we used a Regge model to determine $\delta_{K\pi}$ as a function of energy. Good agreement with data at the $D$ mass [2] was obtained and $\delta_{K\pi}$ is predicted to be around $20^\circ$ at the $B$ mass.

In this letter we extend this Regge analysis to $(\pi\pi)$ and $(K\bar{K})$ channels and determine $\delta_{\pi\pi}(s)$ and $\delta_{K\bar{K}}(s)$. At the $D$ mass, we find $\delta_{\pi\pi}(m_D^2) \simeq \frac{\pi}{3}$ and $\delta_{K\bar{K}}(m_D^2) \simeq -\frac{\pi}{6}$ once again in agreement with the data[3][2]. At the $B$ mass these angles are predicted to be of the order of $11^\circ$ and $-7^\circ$ respectively, implying that hadronic effects in $B$ decays remain important.

Details of the derivation of these results are given in Section 2.

With hadronic phases thus determined it becomes interesting to compare a quark diagram decomposition of isospin invariant amplitudes with the data. In Section 3, we argue that for $D \to K\bar{K}$ decays a fit to the data [2] implies that the contribution of exchange quark diagrams be of the order of $1/3$ of the tree-level one.
2 $\delta_{\pi\pi}(s)$ and $\delta_{K\bar{K}}(s)$ in a Regge Model

In $\pi\pi \rightarrow \pi\pi$ scattering, isospin eigenamplitudes ($I = 0, 1, 2$) in the $s, t, u$-channels are related by the crossing matrices

\[
\begin{pmatrix}
A_0^s \\
A_1^t \\
A_2^u
\end{pmatrix} =
\begin{pmatrix}
1/3 & 1 & 5/3 \\
1/3 & 1/2 & -5/6 \\
1/3 & -1/2 & 1/6
\end{pmatrix}
\begin{pmatrix}
A_0^t \\
A_1^s \\
A_2^u
\end{pmatrix} =
\begin{pmatrix}
1/3 & 1 & 5/3 \\
-1/3 & -1/2 & 5/6 \\
1/3 & -1/2 & 1/6
\end{pmatrix}
\begin{pmatrix}
A_0^s \\
A_1^t \\
A_2^u
\end{pmatrix}
\]

while for $K\bar{K} \rightarrow K\bar{K}$ scattering ($I = 0, 1$) the $s - t$ crossing matrix reads

\[
\begin{pmatrix}
\tilde{A}_0^s \\
\tilde{A}_1^t
\end{pmatrix} =
\begin{pmatrix}
1/2 & 3/2 \\
1/2 & -1/2
\end{pmatrix}
\begin{pmatrix}
\tilde{A}_0^t \\
\tilde{A}_1^s
\end{pmatrix}.
\]

The basic physical idea of a Regge model is that the high energy behaviour of $s$-channel amplitudes is determined by "exchanges" in the crossed channels. For $\pi\pi$ scattering, the dominant exchanges in the $t$-channel are the Pomeron ($P$) and the exchange degenerate $\rho - f_2$ trajectories while in $K\bar{K}$ scattering one must also add the exchange degenerate $\omega - a_2$ trajectories. The $u$-channel exchanges in $\pi\pi$ scattering are identical to the $t$-channel ones ($P, \rho, f_2$) while in $K\bar{K}$ scattering there are no exchanges in the (exotic) $u$ channel ($KK \rightarrow KK$).

In the energy range ($3 \text{ GeV}^2 < s \lesssim 35 \text{ GeV}^2$) which is of interest to us, the Pomeron ($P$) contribution to the isoscalar $t$-channel amplitude is phenomenologically well described by the formula

\[
A_P = i\beta_P(0)e^{ib_P t}s
\]

where the residue $\beta_P(0)$ and slope $b_P$ depend on the scattering process considered. The $\rho, f_2, \omega, a_2$ Regge trajectories are all degenerate i.e.

\[
\alpha_\rho(t) = \alpha_{f_2}(t) = \alpha_\omega(t) = \alpha_{a_2}(t) = \frac{1}{2} + t
\]
The $\rho$ and $\omega$ trajectories have negative signatures while the $f_2$ and $a_2$ trajectories are of positive signature.

The effective $\rho$ trajectory contribution to the isovector $t$-channel amplitude is written as

$$A_\rho = \frac{\beta_\rho}{\sqrt{\pi}}(1 + ie^{-i\pi t})s^{0.5+t}$$

while the $a_2$ contribution to $\bar{A}_1^t$ reads

$$A_{a_2} = \frac{\beta_{a_2}}{\sqrt{\pi}}(-1 + ie^{-i\pi t})s^{0.5+t}.$$ 

Similar expressions are used for the effective $\omega$ and $f_2$ trajectory contributions to the isoscalar $t$-channel amplitude.

The residues of these trajectories are related as follows:

a) in $\pi\pi$ scattering

$$\bar{\beta}_{f_2} = \frac{3}{2}\bar{\beta}_\rho$$

b) in $KK$ scattering

$$\bar{\beta}_{f_2} = \bar{\beta}_\rho$$

$$\bar{\beta}_{a_2} = \bar{\beta}_\omega.$$ 

Furthermore, $SU(3)$ symmetry and ideal mixing give the additional relation

$$\bar{\beta}_\rho = \bar{\beta}_\omega.$$ 

Eqs (7)-(9) follow from “duality” : the scattering processes $(\pi^+\pi^+ \rightarrow \pi^+\pi^+)(A_2^s)$ and $(KK \rightarrow KK)$ are purely diffractive, hence the imaginary part of the Regge trajectory contributions to these processes must cancel [4].

Using Eqs (1) (3)-(5) and (7) our Regge model for $\pi\pi$ scattering near the forward direction ($t$ small) reads:

$$A_0^s(s \text{ large, small } t) = \frac{i}{3}\beta_p(0)e^{b_pt}s + \frac{1}{2\sqrt{\pi}}\beta_\rho e^{0.5+t}s + \frac{3i}{2\sqrt{\pi}}\bar{\beta}_\rho e^{-i\pi t}s^{0.5+t}.$$ 

(11)
\[ A_2(s \text{ large, small } t) = \frac{i}{3} \beta_P(0) e^{b_P s} - \frac{\bar{\beta}_P}{\sqrt{\pi}} s^{0.5 + t}. \] (12)

In the backward direction (u small) exactly the same formulae hold with \( t \) replaced by \( u \).

Similarly, for \( K\bar{K} \to K\bar{K} \) scattering one obtains from Eq.(2), using the relations Eqs (8)-(10), that

\[ \tilde{A}_0(s \text{ large, small } t) = i \frac{1}{2} \beta_P(0) e^{\bar{\beta}_P s} + \frac{4i \bar{\beta}_P}{\sqrt{\pi}} e^{-i\pi t} s^{0.5 + t} \] (13)

\[ \tilde{A}_1(s \text{ large, small } t) = \frac{i}{2} \beta_P(0) e^{\bar{\beta}_P s}. \] (14)

From Eqs (11)-(14) we compute the \( l = 0 \) partial wave amplitudes and find, up to irrelevant overall real factors

\[ a_0(s) = \frac{i}{3} \frac{\beta_P(0)}{b_P} s + \frac{\bar{\beta}_P}{2\sqrt{\pi}} s^{1/2} + \frac{3i}{2\sqrt{\pi}} \bar{\beta}_P (\ln s) + i\pi s^{1/2} \] (15)

\[ a_2(s) = \frac{i}{3} \frac{\beta_P(0)}{b_P} s - \frac{\bar{\beta}_P}{\sqrt{\pi}} s^{1/2} \] (16)

\[ \tilde{a}_0(s) = \frac{i}{2} \frac{\bar{\beta}_P(0)}{b_P} s + \frac{4i \bar{\beta}_P}{\sqrt{\pi}} (\ln s) + i\pi s^{1/2} \] (17)

\[ \tilde{a}_1(s) = \frac{i}{2} \frac{\bar{\beta}_P(0)}{b_P} s. \] (18)

The \( u \)-channel contributions in Eqs. (15)-(16) are identical to the \( t \)-channel ones and we have dropped a (common) factor of 2 in these equations.

Clearly the phases \( e^{i\delta_0} \) and \( e^{i\delta_2} \) of \( a_0(s) \) and \( a_2(s) \) depend on the phenomenological parameter

\[ x_{\pi\pi} = \sqrt{\frac{\pi \beta_P(0)}{\beta_P(0)}} \frac{1}{b_P} \] (19)

and similarly \( e^{i\tilde{\delta}_0} \) and \( e^{i\tilde{\delta}_1} \) depend on

\[ x_{KK} = \sqrt{\frac{\pi \bar{\beta}_P(0)}{\bar{\beta}_P(0)}} \frac{1}{\bar{b}_P}. \] (20)

From the fits given in references [5] and [6] we extract the values

\[ x_{\pi\pi} = 0.69 \pm 0.10 \] (21)

\[ x_{KK} = 1.72 \pm 0.30. \] (22)
With these values we obtain respectively

\[
\delta_{\pi\pi}(m_D^2) = \delta_2(m_D^2) - \delta_0(m_D^2) = 60^\circ \pm 4^\circ
\]  

(23)

\[
\delta_{K\bar{K}}(m_D^2) = \tilde{\delta}_1(m_D^2) - \tilde{\delta}_0(m_D^2) = -29^\circ \pm 4^\circ
\]  

(24)

and predict

\[
\delta_{\pi\pi}(m_B^2) = 11^\circ \pm 2^\circ
\]  

(25)

\[
\delta_{K\bar{K}}(m_B^2) = -7^\circ \pm 1^\circ.
\]  

(26)

At the D mass, the experimental values given by the CLEO collaboration are,

\[
\delta_{\pi\pi}(m_D^2) = 82^\circ \pm 10^\circ
\]  

(27)

\[
\delta_{K\bar{K}}(m_D^2) = \pm(24^\circ \pm 13^\circ).
\]  

(28)

Clearly our Regge model calculation of these phases is in good agreement with the data as announced previously.

In summary, for (ππ) and (K\bar{K}) decay channels as well as for (Kπ) channels, hadronic angles are correctly predicted at the D mass by a Regge model and are found to be quite sizeable at the B mass: hadronic effects simply cannot be ignored in B decays.

3 Isospin Amplitudes and Quark Diagrams in (D → K\bar{K}) Decays

Having determined the hadronic phase \(\delta_{K\bar{K}}\), we now illustrate the strategy advocated in Ref.3 to analyze the D → K\bar{K} data.

In lowest order the weak Hamiltonian responsible for (D → K\bar{K}) decays contains an isodoublet \(H_{W}^{1/2}\) and an isoquadruplet \(H_{W}^{3/2}\) part. With the reduced matrix elements

\[
w_1 = \langle D | H_{W}^{1/2} | (K\bar{K})I = 1 \rangle
\]  

(29)
\[ w_0 = \ll D \mid H_{W^-}^{1/2} \mid (K \bar{K}) I = 0 \gg \]  
(30)

\[ v_1 = \ll D \mid H_{W^-}^{3/2} \mid (K \bar{K}) I = 1 \gg \]  
(31)

and the hadronic angle \( \delta_{K \bar{K}} = \delta_1 - \delta_0 \) one readily obtains, up to an overall phase factor

\[ A(D^+ \to K^+ \bar{K}^0) = -\frac{v_1}{2} + w_1 \]  
(32)

\[ A(D^0 \to K^+ \bar{K}^-) = \frac{v_1}{2} + \frac{w_1}{2} + \frac{w_0}{2} e^{-i\delta_{K \bar{K}}} \]  
(33)

\[ A(D^0 \to K^0 \bar{K}^0) = -\frac{v_1}{2} - \frac{w_1}{2} + \frac{w_0}{2} e^{-i\delta_{K \bar{K}}} . \]  
(34)

We assume again all reduced matrix elements to be real and expressed in terms of quark diagrams classified following their \((1/N\)-inspired) topology.

In this “phenomenological” picture where we keep the explicit \((V - A)\) times \((V - A)\) \(W^\pm\) propagations, the annihilation diagram \((A)\) is helicity-suppressed and the \(b, s\) and \(d\) quarks are exchanged in the penguin diagrams \((P_q)\). On the contrary, in the “formal” language the effects of \(W^\pm\) and \(b\) would be hidden in the short-distance Wilson coefficients of local operators built out of the \(u, d, s\) and \(c\) quarks only.

The contributions from the tree-level \((T)\), annihilation \((A)\), penguins \((\Delta P)\) exchanges with either a \(u\bar{u}\) or a \(d\bar{d}\) pair created \((E)\) and, finally, exchanges with a \(s\bar{s}\) pair created \((E_s)\) lead to the relations

\[ w_0 = T + \Delta P + 2E - E_s \]  
(35)

\[ w_1 = T + \Delta P + \frac{1}{3}E_s - \frac{2}{3}A \]  
(36)

\[ v_1 = \frac{2}{3}E_s + \frac{2}{3}A . \]  
(37)

If one assumes

\[ E = E_s \]  
(38)

\(^1\)If we neglect the (multi-) Cabibbo-suppressed \(b\) quark contribution, there are two diagrams to consider with opposite signs: the “chin” of the penguin is either a \(d\) quark or a \(s\) quark. In the limit where \(m_d = m_s\), \(\Delta P \equiv P_s - P_d = 0\)
which is what one expects in the $SU(3)$ limit, then Eqs (35)-(37) imply
\[ w_0 = w_1 + v_1 \]  

and Eqs (32)-(34) now read
\begin{align*}
A(D^+ \to K^+ K^0) &= w_0 - \frac{3v_1}{2} \quad (40) \\
A(D^0 \to K^+ K^-) &= \frac{w_0}{2}(1 + e^{-i\delta_{KK}}) \quad (41) \\
A(D^0 \to K^0 \bar{K}^0) &= -\frac{w_0}{2}(1 - e^{-i\delta_{KK}}). \quad (42)
\end{align*}

Eqs (41)-(42) are in good agreement with experimental data when Eq.(24) is used.

It is difficult to imagine a more spectacular illustration of final state hadronic effects than Eqs (41)-(42). Furthermore, from the experimental value
\[ \frac{\Gamma(D^0 \to K^+ K^-)}{\Gamma(D^+ \to K^+ K^0)} \simeq 1.6 \quad (43) \]

one deduces $v_1 \simeq \frac{1}{6}w_0$ or, in terms of quark diagrams,
\[ \frac{E + A}{T + \Delta P + E} \simeq \frac{1}{4}. \quad (44) \]

Since $A$ and $\Delta P$ are expected to be quite small in our phenomenological picture, Eq.(44) entails
\[ \frac{E}{T} \simeq \frac{1}{3}. \quad (45) \]

This result may be at odds with some theoretical prejudices but is required by the data: with $\delta_{KK}$ given by Eq.(24), all the $D \to K\bar{K}$ data are indeed very nicely fitted by Eqs (40)-(42) provided Eq.(44) holds.

The detailed analysis of $D \to K\bar{K}$ decays presented in this section can be repeated for $(D \to \pi\pi)$ and $(D \to K\pi)$ channels. In these channels, sizeable color-suppressed quark diagrams (C) operate and nothing as striking as Eq.(42) or as unexpected as Eq.(45) emerges from such an analysis.
To summarize our earlier work on $(K\pi)$ channels as well as the results of the present paper on $(\pi\pi)$ and $(K\bar{K})$ decays let us insist on the following points:

- at the $D$ mass, hadronic phases are rather well estimated in the context of a Regge model. Note that the hierarchy

$$\delta_{K\pi} \simeq \frac{\pi}{2}, \ \delta_{\pi\pi} \simeq \frac{\pi}{3}, \ \delta_{K\bar{K}} \simeq -\frac{\pi}{6}$$

follows from the difference in $u$-channel exchanges for the corresponding scattering processes combined with different Clebsch Gordan coefficients weighing the relative contributions of the Pomeron and the $I = 1$ Regge trajectories. In this paper, we have ignored inelastic channels such as $\{K\eta\} \rightarrow \{K\pi\}$ or $\{\pi\eta\} \rightarrow \{K\bar{K}\}$. In fact, our Regge analysis shows that they have little effect on phases at least at the $D$ mass.

- at the $B$ mass, hadronic phases are predicted to be non negligible in the three channels considered so far:

$$\delta_{K\pi} \approx 17^\circ, \ \delta_{\pi\pi} \approx 11^\circ, \ \delta_{K\bar{K}} \approx -7^\circ.$$  

We have assumed that inelastic channels have a small overall effect on these hadronic phases \cite{11}. Whether this is true or not is an experimental question. But clearly final state hadronic phases remain large in $B$ decays and the prospect for CP-asymmetries looks particularly promising in the $K\pi$ channel.

- the parametrization suggested in Ref \cite{8} works very nicely as exemplified by our analysis of $(D \rightarrow K\bar{K})$ decays. When hadronic phases are important, quark-diagram absorptive parts and inelastic effects on the phases seem to be negligible. These latter conclusions may not hold for decay processes where hadronic phases are quite small.

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