Role of graphene on the enhancement of near-field radiative heat transfer between two homogeneous lossy media plates

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Abstract. In this paper, the near-field radiative heat transfer between two semi-infinite plates with different temperatures, which are composed of homogeneous lossy media, has been studied firstly. Attributed to the evanescent wave generated by total internal reflection, the near-field radiative heat transfer is much larger than the far-field radiative heat transfer. And, the closer the distance between the two plates is, the greater the heat transfer is. Then, a graphene layer has been used to cover on the surface of the plate with lower temperature to study the effect of graphene on the near-field radiative heat transfer. The results show that the existence of graphene can promote the near-field radiative heat transfer due to the excitation of surface plasmon polaritons on the surface of graphene. In addition, the heat transfer varies with chemical potential of graphene, which indicates that the near-field radiative heat transfer can be controlled by an external gate circuit. In the presence of graphene, the thermal resistance between the two plates ranges from \(6.7 \times 10^{-4}\) to \(0.18\) K/(W/m²) and increases with the increase of the thickness of vacuum gap between the plates.

1. Introduction

When the gap between two surfaces is comparable to or smaller than the characteristic wavelength of thermal radiation, the radiative heat transfer between the two surfaces can be several orders of magnitude larger than that of Planck’s law predictions between two blackbodies at the corresponding temperature [1-5]. In this case, the evanescent wave produced by total internal reflection (TIR) plays an important role, and the thermal radiation is called near-field thermal radiation (NFTR) [1-4]. In addition, if the surface polaritons can be excited on the material surfaces, the evanescent waves caused by surface polaritons can also enhance the near-field thermal radiative heat transfer [6].

Graphene is a two-dimensional (2D) layered material consisting of a single layer of carbon atoms arranged in a hexagonal lattice. Since Novoselov et al. [7] successfully separated graphene from graphite in the laboratory in 2004, the preparation and application of graphene have become hot research topics. Graphene exhibits unique optical, electrical and mechanical properties due to its unique electronic structure. Electrons in 2D graphene can move at very high speeds, behaving like relativistic particles (Dirac particles) without static mass [8]. The unique properties of graphene include unusually high electron mobility [9], quantum Hall effect at room temperature [10], high light transmittance [11], super...
mechanical properties [12], adjustable plasma frequency from near infrared to THz [13], etc. On graphene surface the surface polaritons can be supported in the infrared region, and can be adjusted by chemical doping or external gate circuits [14,15]. Based on these characteristics, graphene has potential applications in transform optics [16], plasma waveguide [17], tunable terahertz optical antenna [18], mid-far infrared photonic devices [19] and so on. Because the raw material for preparing graphene is inexpensive graphite, the application prospect of graphene is very broad.

In this paper, the promotion effect of graphene on near-field thermal radiative heat transfer has been investigated. Firstly, the near-field radiative heat transfer between two semi-infinite plates with different temperatures, which are made up of homogeneous lossy media (HLM, ε = 4 + i0.4), has been studied to clarify the mechanisms of near-field radiative heat transfer. Also, the difference between near-field radiative heat transfer and far-field radiative heat transfer has been shown. Then, a graphene layer is introduced into the structure to study the effect of graphene on radiative heat transfer. And, the thermal resistance of graphene system is calculated, which is compared with the thermal resistance of interface thermal conductive materials commonly used in industry. It is pointed out that near-field heat transfer has potential application prospects in the cooling of electronic devices and other equipment.

2. Calculation model and fluctuating-dissipation theory

2.1. Calculation Model
As shown in Fig. 1 (a), the temperatures of two semi-infinite HLM plates are set to 300 K and 350 K, respectively. In order to study whether graphene can promote near-field thermal radiation, a layer of graphene is covered on the surface of the 300K plate. The temperature of graphene is also set to 300 K, which is shown in Fig. 1 (b). In this paper, the fluctuating-dissipation theory is used to calculate the near-field radiative heat transfer.

2.2. Optical parameters of graphene
Graphene is a planar thin film of a single-layered sheet structure composed of carbon atoms and is a two-dimensional material with only one carbon atom thickness. Graphene exhibits unique electrical and optical properties compared with conventional conductors. The optical parameters of graphene have been studied by many researchers [13-23]. It is found that the conductivity of graphene can be written as the sum of two parts:

$$\sigma(\omega) = \sigma^{\text{intra}}(\omega) + \sigma^{\text{inter}}(\omega)$$  (1)

Figure 1. Schematics of calculation model. (a) Two semi-infinite HLM plates, (b) Graphene attached to a plate with lower temperature.
Where, the first term $\sigma_{\text{intra}}(\omega)$ corresponds to the scattering process of intraband electrons and phonons in graphene, and the second term $\sigma_{\text{inter}}(\omega)$ corresponds to the interband electron transition. $\sigma_{\text{intra}}(\omega)$ can be obtained by the following formula [13,20]:

$$
\sigma_{\text{intra}}(\omega) = \frac{i e^2}{\omega + i/\tau} \frac{e^2}{\pi \hbar^2} 2k_B T \ln \left[ 2 \cosh \left( \frac{\mu}{2k_B T} \right) \right]
$$

Where, $i$ is imaginary unit, $e = 1.602 \times 10^{-19}$ C is the electron charge, $\hbar = 1.05457 \times 10^{-34}$ Js/rad is the Planck constant divided by $2 \pi$, $\omega$ is angular frequency in rad/s, $\tau = 10^{-13}$ s/rad is relaxation time for Drude model, $\mu$ is the chemical potential in the unit of J, it can be adjusted by chemical doping or by an external gate circuit, the unit of chemical potential $\mu$ is usually represented by eV, and the relationship between eV and J is $1\text{ eV} = 1.602 \times 10^{-19}$ J. Boltzmann constant $k_B = 1.381 \times 10^{-23}$ J/K, $T$ is temperature in K.

$\sigma_{\text{inter}}(\omega)$ can be obtained by the following formula [13,20].

$$
\sigma_{\text{inter}}(\omega) = \frac{e^2}{4\hbar} \left[ G\left( \frac{\hbar \omega}{2} \right) + i \frac{4\hbar \omega}{\pi} \int_0^{\infty} \frac{G(\eta) - G(\hbar \omega/2)}{(\hbar \omega)^2 - 4\eta^2} d\eta \right]
$$

Where, $G(\eta) = \frac{\sinh(\eta/k_B T)}{\cosh(\eta/k_B T) + \cosh(\mu/k_B T)}$, the unit of integral variable $\eta$ is J.

After the conductivity is obtained by using (1) ~ (3), the relative dielectric constant of graphene can be obtained according to the following formula [13]:

$$
\varepsilon_r(\omega) = 1 + i\sigma(\omega)/(\omega\varepsilon_0\delta_g)
$$

Where, $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m is the dielectric constant of vacuum, $\delta_g = 0.34$nm is the thickness of single graphene layer, the relative dielectric constant of graphene at different chemical potentials ($\mu=0.1eV$, $0.3eV$, $0.5eV$ and $0.7eV$) is shown in Fig. 2.

![Figure 2. The relative dielectric constant of graphene at different chemical potentials (μ=0.1eV, 0.3eV, 0.5eV and 0.7eV)](image-url)
3. Evanescent waves and surface polaritons

3.1. Evanescent waves

The process of radiating electromagnetic thermal radiation from the medium can be viewed as that there are an infinite number of radiation sources radiating electromagnetic wave in all direction from the medium, which is shown in Fig. 3. When the incident angle is larger than the critical angle $\theta_c$, TIR will occur at the interface of medium and vacuum. The relationship between $\theta_c$ and refractive index $n$ of the medium is as follows

$$\sin \theta_c = 1/n \quad (5)$$

When the incident angle is less than the critical angle, refraction and reflection occur at the interface of medium and vacuum. When the incident angle is larger than the critical angle, evanescent waves are generated on the surface of the medium.

At the interface of medium and vacuum, the parallel component of the wave vector on the medium side is:

$$k_\parallel = k_{\parallel,0} \sin \theta \quad (6)$$

Where, $k_{\parallel,0} = 2\pi / \lambda$ is the wave vector of vacuum. According to the phase matching condition, the parallel component of the wave vector on the vacuum side should be equal to that of medium side. Thus, the vertical component of the wave vector on the vacuum side should be $\gamma = \sqrt{k_{\parallel,0}^2 - \beta^2}$, at the condition of $\beta > k_{\parallel,0}$, the electromagnetic wave decays exponentially in the direction perpendicular to the interface, which resulting in the formation of evanescent wave.

According to Eqs. (5) and (6), it can be obtained that when the incident angle $\theta$ is larger than the critical angle $\theta_c$, $\beta > k_{\parallel,0}$, and the evanescent wave will be generated on the vacuum side of the medium-vacuum interface. When $\theta$ is less than $\theta_c$, thermal radiation electromagnetic wave will be reflected and refracted at the medium-vacuum interface.

Attenuation length is defined as the distance into a material when the intensity of the electromagnetic wave has dropped to $1/e$. When TIR occur, the attenuation length $l_a$ of evanescent wave is [24]:

$$l_a = 1/\left| \text{Im} \left( k_{\parallel,0} \sqrt{1 - (n \sin \theta)^2} \right) \right| \quad (7)$$

Where, $\text{Im}(\ldots)$ represents imaginary part of a number, $\left| \ldots \right|$ denotes absolute value. It can be obtained from Eq. (7) that $l_a$ is relevant to the wavelength, refractive index and incident angle. Its value is about of magnitude of sub-wavelength. If another medium is placed in the distance of attenuation length, the
energy will be transferred between the mediums by the evanescent wave. This phenomenon is called photon tunnelling effect which is play an important role in the near-field thermal radiation.

3.2. Surface polaritons
Surface polaritons are electromagnetic surface wave. They exist on the surface of material, propagate along the surface, and are attenuated perpendicular to the surface. Thus, surface polaritons are evanescent waves. For different materials, there are two types of surface polaritons which can be stimulated on the material surface, one is surface plasmon polaritons (SPPs), which are generated by the coupling between electromagnetic waves and resonant free electrons in metallic materials, another is surface phonon polaritons (SPhPs), which are generated by the coupling between electromagnetic waves and resonant phonon in polar materials. To stimulate surface polaritons on the surface of material, the optical properties of the materials should satisfy the following formula [3,25,26]:

\[ \varepsilon_1 \gamma_2 + \varepsilon_2 \gamma_1 = 0 \]  

Where, \( \varepsilon_1 \) and \( \varepsilon_2 \) are dielectric constant of the two materials at the interface, respectively. \( \gamma_1 \) and \( \gamma_2 \) are vertical components of wave vectors in the two materials, respectively. \( \gamma_1^2 = \varepsilon_1 k_0^2 \beta^2 \), \( \gamma_2^2 = \varepsilon_2 k_0^2 \beta^2 \), \( k_0 \) is wave vector of vacuum, \( \beta \) is parallel component. When one of the materials at the interface is vacuum, it can be obtained that:

\[ \beta = k_0 \sqrt{\varepsilon_1 / (\varepsilon_1 + 1)} \]  

In order to stimulate surface polaritons, \( \beta \) should be larger than \( k_0 \). It can be obtained that the real part of the refractive index \( \varepsilon_1 \) of material should be negative.

4. Results and discussions

4.1. The heat transfer between two HLM plates without graphene layer
Firstly, the thermal radiative heat transfer of two semi-infinite HLM plates have been investigated, the temperatures of the two plates are 300K and 350K, respectively. Fig. 4 shows the spectral heat flux \( q(\lambda) \) between the two HLM plates (red dashed line), and that of Blackbody (black solid line) for comparison. It can be obviously found that the spectral heat transfer of HLM plates is larger than that of Blackbody.

Figure 4. Comparison between the near-field spectral heat transfer (red dashed line) of two semi-infinite HLM plates and spectral heat transfer (black solid line) of Blackbody, the distance \( d \) between the two HLM plates is 100 nm

Figure 5. Radiative heat flux \( q(\omega,\beta) \) with the angular frequency \( \omega \) and the parallel wave vector component \( \beta \) as variables. The dash and dot lines represent the light lines in vacuum and HLM, respectively.
To figure out what causes the improved heat transfer between HLM plates, the radiative heat flux $q(\omega, \beta)$ with the angular frequency $\omega$ and the parallel wave vector component $\beta$ as variables has been calculated, which is shown in Fig. 5. $q(\lambda)$ can be obtained by integrating the variable $\beta$ of the function $q(\omega, \beta)$ and converting angular frequency $\omega$ to wavelength $\lambda$. In Fig. 5, the dash and dot lines correspond to light lines in vacuum and HLM, respectively, which are plotted based on the formula $k = \text{Re}(n)\omega / c$ by using the wave vector $k$ as the horizontal coordinate and the angular frequency $\omega$ as the longitudinal coordinate, where $n$ is refractive index.

It can be figured out from Fig.5 that the radiative heat flux $q(\omega, \beta)$ is almost entirely on the left side of light line of HLM. This phenomenon is reasonable. According to Eq. (6), the horizontal wave vector in vacuum is $\beta = k_0 n \sin \theta$, it is obviously smaller than the wave vector $k_0 n$ in HLM. When $\theta < \theta_c$, horizontal wave vector $\beta = k_0 n \sin \theta < k_0$, the evanescent wave cannot be produced in vacuum gap, and the thermal radiative heat transfer between HLM plates is achieved by propagation wave. This part of the energy transferred is located on the left side of the light line in vacuum. When $\theta > \theta_c$, horizontal wave vector $\beta > k_0$, the evanescent wave can be produced at the interface between vacuum and HLM and is attenuated along the vertical direction of the interface. In this case, the thermal radiative heat transfer between HLM plates is achieved by both of the evanescent wave and propagation wave. This part of the energy transferred is located between the light lines in vacuum and HLM. It is shown in Fig.5 that the heat flux is mainly located between the two light lines, which illustrates that the enhanced near-field heat transfer mainly attributed to the evanescent wave (photon tunnelling effect).

\[
Q = \int_{d_{2\mu m}}^{d_{50\mu m}} q(\lambda) d\lambda
\]  

In Eq. 10, the integration interval should be $0$ to $+\infty$ theoretically, actually, the quantity of radiative heat flux in the wavelength range of $2$–$50\mu m$ is amount of $95\%$ of total quantity. Thus, Considering the limitation of numerical integration, the interval $[2, 50] \mu m$ is used. Fig. 6 shows the variation of the total radiative heat flux $Q$ with the thickness $d$ of vacuum gap. It can be intuitively found that $Q$ decreases
with the increase of $d$, which results from the exponentially attenuation of the evanescent wave with the increase of $d$. When $d$ is larger than $2 \times 10^4$ nm, $Q$ is almost no more changed with $d$. This is because the evanescent wave has been attenuated at all for this vacuum gap thickness, and the heat transfer completely attributes to the propagating wave.

Thermal resistance $R$ and the total radiative heat flux $Q$ have the relationship of

$$ R = \frac{\Delta T}{Q} \quad (11) $$

Where, $\Delta T$ represents the temperature difference of the two HLM plates. Therefore, the thermal resistance $R$ with different $d$ can be calculated using Eq. (11) which is shown in Fig. 7. As the figure shown that $R$ is in the range of $1.77 \times 10^{-3}$ to 0.18 K/(W/m$^2$) and increase with the increase of $d$.

4.2. The heat transfer between two HLM plates with graphene layer
In order to study whether graphene can promote near-field thermal radiation, a layer of graphene is attached to the HLM plate with temperature of 300 K. The temperature of graphene is also considered to be equal to 300 K.

The chemical potential of graphene can be altered by introducing chemical doping or gate circuit, thus the optical and electrical properties of graphene can be changed. Generally, the chemical potential can be changed in the range of 0.1 to 1 eV [13]. With the different chemical potential $\mu$, the spectral quantity $q(\lambda)$ of the structure with graphene layer has been shown in Fig. 8. As shown in the figure, when $\mu=0.1$ eV, $q(\lambda)$ has a peak value near the wavelength of 30 μm. This peak shifts in the direction of the short wavelength with the increase of $\mu$, and in long wavelength range, $q(\lambda)$ is decreasing as a whole.

![Figure 8. The spectral heat flux $q(\lambda)$ with different chemical potential ($\mu=0.1$, 0.3, 0.5 and 0.7) (Image)](image)

![Figure 9. Distribution of radiative heat flux $q(\omega, \beta)$. The dot, dash and dash dotted lines represent light lines of HLM, vacuum and graphene, respectively. (Image)](image)

In order to understand the reason for the greater amount of radiative heat transfer after the addition of graphene layer, Fig. 9 shows the radiative heat flux $q(\omega, \beta)$, where the chemical potential $\mu$ equal to 0.7 eV. The dot, dash and dash dotted lines denote the light lines in HLM, vacuum and graphene, respectively. Similarly, the three light lines are plotted based on the formula $k = \text{Re}(n)\omega / c$, where $n$ is refractive index.

It is the same as shown in Fig. 5, the radiative heat flux $q(\omega, \beta)$ is mainly located on the left side of light line of HLM, which proves that the evanescent wave in TIR play an important role in the heat transfer. Another band of strong heat flux can be found in the lower frequency range. This enhanced
heat transfer is attributed to the stimulation of the SPPs on the graphene surface, a part of energy is transmitted by SPPs. SPPs promotes the radiative heat transfer between the two HLM plates.

Similarly, the total heat flux $Q$ of heat transfer and the thermal resistance $R$ have been calculated, whose variations with thickness $d$ of vacuum gap have been shown in Fig. 10 and 11, respectively. For comparison, the heat transfer without graphene layer is also shown.

From the figure, it can be found intuitively that the total heat flux $Q$ and the thermal resistance $R$ vary with the thickness $d$ of the vacuum gap, which is consistent with that without graphene. This is due to the exponential attenuation of evanescent waves (originating from TIR and SPPs) along the vertical direction of interface.

![Figure 10](image1.png)  
**Figure 10.** The relationship between the total heat flux $Q$ and the gap thickness $d$. The chemical potential of graphene is 0.7eV.  

![Figure 11](image2.png)  
**Figure 11.** The relationship between thermal resistance $R$ and the gap thickness $d$. The chemical potential of graphene is 0.7eV.

As can be seen from the figure, when the gap thickness $d$ is less than 400 nm, the total heat flux $Q$ with graphene is significantly larger than that without graphene. Correspondingly, the thermal resistance $R$ of HLM with graphene is significantly smaller than that without graphene. In the value of distance $d$ shown in figure, the thermal resistance $R$ ranges from $6.7 \times 10^{-4}$ to 0.18 K/(W/m²). It is obvious that the addition of graphene can significantly reduce the thermal resistance of near-field radiative heat transfer. It has been reported that the thermal resistance of suspended graphene can be between $0.3 \times 10^{-5}$ to $0.2 \times 10^{-5}$ K/(W/m²) [27]. In other words, the thermal resistance of near-field thermal radiation is obviously larger than that of suspended graphene. However, when the heat conduction is not applicable, it may be a good way to realize heat transfer by near-field thermal radiation.

5. Conclusion

In this paper, the effect of graphene on near-field thermal radiative heat transfer is studied. Firstly, the thermal radiative heat transfer between two semi-infinite HLM ($\varepsilon = 4 + i0.4$) plates without graphene is discussed. The results show that the near-field thermal radiative heat transfer between the two semi-infinite plates is greater than the radiative heat transfer of blackbody. By calculating and analyzing the distribution of radiative heat flux $q(\alpha, \beta)$, it is found that the energy is mainly between the light lines of vacuum and HLM, which indicates that the energy is mainly transmitted by evanescent wave produced by TIR in the near-field range. The thermal resistance $R$ between two semi-infinite plates ranges from $1.77 \times 10^{-3}$ to 0.18 K/(W/m²) and increases with the thickness $d$ of the vacuum gap.

Then, it has been investigated whether graphene can promote near-field thermal radiative heat transfer. The results show that the presence of graphene can indeed promote the near-field radiative heat transfer, and the chemical potential of graphene has significant effect on the near-field radiative heat transfer, which indicates that the near-field radiative heat transfer can be controlled by an external gate.
circuit. When the chemical potential of graphene is 0.7eV, the distribution of heat flux $q(\omega, \beta)$ indicates that it is the SPPs on the graphene surface that promotes the near-field radiative heat transfer. When the vacuum gap thickness $d$ is less than 400 nm, the thermal resistance $R$ with graphene is significantly smaller than that without graphene. It is obvious that the addition of graphene can significantly reduce the thermal resistance of near-field radiative heat transfer. This research in the paper has certain guiding significance for the application of near-field radiative heat transfer in heat dissipation of electronic devices and other equipment.

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