The realization of the wave function collapse in the linguistic interpretation of quantum mechanics

Shiro Ishikawa

Department of Mathematics, Faculty of Science and Technology, Keio University, 3-14-1, Hiyoshi, Kouhoku-ku Yokohama, Japan. E-mail: ishikawa@math.keio.ac.jp

Abstract

Recently I proposed the linguistic interpretation of quantum mechanics, which is characterized as the linguistic turn of the Copenhagen interpretation of quantum mechanics. This turn from physics to language does not only extend quantum theory to classical theory but also yield the quantum mechanical world view. Although the wave function collapse is prohibited in the linguistic interpretation, in this paper I show that the phenomenon like wave function collapse can be realized in the linguistic interpretation. And furthermore, I propose the justification of the von Neumann-Lüders projection postulate. After all, I conclude that the wave function collapse should not be adopted in the Copenhagen interpretation.

Key phrases: Copenhagen interpretation, Wave function collapse, von Neumann-Lüders projection postulate

1 Preparations

Recently in [3]-[6], I proposed measurement theory (i.e., quantum language, or the linguistic interpretation of quantum mechanics), which is characterized as the linguistic turn of the Copenhagen interpretation of quantum mechanics. This turn from physics to language does not only extend quantum theory to classical theory but also yield the quantum mechanical world view. The linguistic interpretation says that

(A) “Only one measurement is permitted”, and thus, we are not concerned with anything after measurement since it can not be measured any longer. Also, the Heisenberg picture should be adopted, that is, the Schrödinger picture should be prohibited. (For details, see [4-6].)

Therefore, the wave function collapse is meaningless in the linguistic interpretation. In this sense, the linguistic interpretation and the Copenhagen interpretation are different.

Although my idea proposed in this paper was discovered in the investigation of quantum language, it may be understood without the knowledge of quantum language. Hence, the readers are not required to have the usual knowledge of quantum language, but that of quantum mechanics.
1.1 Hilbert space

According to ref. [8], we briefly introduce the mathematical formulation of quantum mechanics as follows.
Consider an operator algebra $B(H)$ (i.e., an operator algebra composed of all bounded linear operators on a Hilbert space $H$ with the norm $\|F\|_{B(H)} = \sup_{\|u\|=1} \|Fu\|_H$), in which quantum mechanics is formulated. Define $Tr(H)$, the trace class, by $Tr(H) = B(H)_*$ (i.e., pre-dual space). For any $u,v \in H$, define $|u\rangle\langle v| \in B(H)$ such that
\[ (|u\rangle\langle v|)w = \langle v, w \rangle u \quad (\forall w \in H). \] (1)
The trace map $tr : Tr(H) \to \mathbb{C} (= \text{the complex field})$ is defined by
\[ tr(T) = \sum_{k=1}^{\infty} \langle e_k, Te_k \rangle \quad (\forall T \in Tr(H)) \]
where it does not depend on the choice of the complete orthonormal system $\{e_k\}_{k=1}^\infty$. The mixed state space $Tr_{+1}(H)$ is defined by $\{\rho \in Tr(H) \mid \rho \geq 0, \quad tr(\rho) = 1\}$.

1.2 Observables, state, Markov operator

We define the observable $O = (X, F, F)$ in $B(H)$ (or, POVM, cf [1]) such that

\[ (C_1) \quad X \text{ is set, } F (\subseteq 2^X: \text{the power set of } X ) \text{ is a } \sigma\text{-field.} \]
\[ (C_2) \quad F : F \to B(H) \text{ is a map such that } 0 = F(\emptyset) \leq F(\Xi) \leq F(X) = I (= \text{the identity}) \quad (\forall \Xi \in F), \]
\[ (C_3) \quad \text{for any countable decomposition } \{\Xi_1, \Xi_2, \ldots, \Xi_n, \ldots\} \text{ of } \Xi \quad \text{ (i.e., } \Xi = \bigcup_{n=1}^{\infty} \Xi_n, \Xi_n \in F, (n = 1, 2, \ldots), \Xi_m \cap \Xi_n = \emptyset \quad (m \neq n) \text{), it holds that} \]
\[ \langle u, F(\Xi)u \rangle = \lim_{n \to \infty} \sum_{k=1}^{n} \langle u, F(\Xi_k)u \rangle \quad (\forall u \in H) \] (2)
Also, a pure state is represented by $\rho = |u\rangle\langle u|$ (where $u \in H$, $\|u\| = 1$).

Let $H_1$ and $H_2$ be Hilbert spaces. A continuous linear operator $\Phi : B(H_2) \to B(H_1)$ is said to be a Markov operator, if the pre-dual operator $\Phi_* : Tr(H_1) \to Tr(H_2)$ satisfies that $\Phi_*(Tr_{+1}(H_1)) \subseteq Tr_{+1}(H_2)$.

1.3 Axioms

A measurement of an observable $O = (X, F, F)$ for a state $\rho(= |\rho\rangle\langle u|)$ is denoted by $M_{B(H)}(O :=(X, F, F), S_{|\rho|})$.
Now we introduce two axioms as follows.
Axiom 1 [Measurement]. The probability that a measured value \( x (\in X) \) obtained by the measurement \( M_{B(H)}(O := (X, F, F), S_{[\rho]} \) belongs to a set \( \Xi (\in F) \) is given by
\[
\rho(F(\Xi)) \left( = \text{tr}(\rho F(\Xi)) = \langle u, F(\Xi)u \rangle \right)
\]

Axiom 2 is presented as follows:

Axiom 2 [Causality]. Let \( t_1 \leq t_2 \). The causality is represented by a Markov operator \( \Phi_{t_1,t_2} : B(H_{t_1}) \to B(H_{t_2}) \).

2 The wave function collapse

2.1 The von Neumann-Lüders projection postulate in the Copenhagen interpretation

Let \( H \) be a Hilbert space. Let \( \mathbb{P} = [P_k]_{k=1}^{\infty} \) be a spectral decomposition in \( B(H) \), that is, \( P_k (\in B(H)) \) is a projection (\( \forall k = 1, 2, ... \)) such that
\[
\sum_{k=1}^{\infty} \langle u, P_k u \rangle = \| u \|^2 \quad (\forall u \in H)
\]

Put \( N = \{1, 2, ...\} \). Define the observable \( O_P = (N, 2^N, P) \) in \( B(H) \) such that
\[
P(k) = P_k \quad (\forall k = 1, 2, ...)
\]

Axiom 1 says:
\[
\text{(D}_1\text{)} \text{ The probability that a measured value } n (\in N) \text{ is obtained by a measurement } M_{B(H)}(O_P := (N, 2^N, P), S_{[\rho]} \) is given by
\[
\text{tr}(\rho P_n) = \langle u, P_n u \rangle, \quad (\text{where } \rho = |u \rangle \langle u|)
\]

Also, the von Neumann-Lüders projection postulate (in the Copenhagen interpretation, cf. [7]) says:
\[
\text{(D}_2\text{)} \text{ When a measured value } n (\in N) \text{ is obtained by the measurement } M_{B(H)}(O_P := (N, 2^N, P), S_{[\rho]}), \text{ the state } \rho_n \text{ after the measurement is given by}
\[
\rho_n = \frac{P_n |u \rangle \langle u| P_n}{\| P_n u \|^2} \left( = \frac{|P_n u \rangle \langle P_n u|}{\| P_n u \|^2} \right)
\]

And furthermore, when a measurement \( M_{B(H)}(O_F := (X, F, F), S_{[\rho_n]} \) is taken, the probability that a measured value belongs to \( \Xi (\in F) \) is given by
\[
\text{tr}(\rho_n F(\Xi)) \left( = \langle \frac{P_n u}{\| P_n u \|}, F(\Xi) \frac{P_n u}{\| P_n u \|} \rangle \right)
\]

Note that the von Neumann-Lüders projection postulate (D_2) is not adopted in our situation since the linguistic interpretation (A) says that the state after a measurement is meaningless.
2.2 The von Neumann-Lüders projection postulate in the linguistic interpretation

Consider a Hilbert space $H$ and a tensor Hilbert space $K \otimes H$. Let $\mathbb{P} = [P_{k}]_{k=1}^{\infty}$ be a spectral decomposition in $B(H)$, and let $\{e_{k}\}_{k=1}^{\infty}$ be a complete orthonormal system in a Hilbert space $K$. Define the pre-dual Markov operator $\Psi : Tr(H) \rightarrow Tr(K \otimes H)$ by, for any $u \in H$,

$$\Psi_{*}(|u\rangle\langle u|) = \sum_{k=1}^{\infty} |e_{k} \otimes P_{k}u\rangle\langle e_{k} \otimes P_{k}u|$$

or

$$\Psi_{*}(|u\rangle\langle u|) = \{|\sum_{k=1}^{\infty} (e_{k} \otimes P_{k}u)\rangle\langle \sum_{k=1}^{\infty} (e_{k} \otimes P_{k}u)|\}$$

Thus the Markov operator $\Psi : B(K \otimes H) \rightarrow B(H)$ is defined by $\Psi = (\Psi_{*})^{*}$.

Define the observable $O_{G} = (\mathbb{N}, 2^{\mathbb{N}}, G)$ in $B(K)$ such that

$$G(\{k\}) = |e_{k}\rangle\langle e_{k}| \quad (k \in \mathbb{N} = \{1, 2, \ldots\})$$

Let $O_{F} = (X, \mathcal{F}, F)$ be arbitrary observable in $B(H)$. Thus, we have the tensor observable $O_{G} \otimes O_{F} = (\mathbb{N} \times X, 2^{\mathbb{N}} \boxtimes \mathcal{F}, G \otimes F)$ in $B(K \otimes H)$, where $2^{\mathbb{N}} \boxtimes \mathcal{F}$ is the product $\sigma$-field.

Fix a pure state $\rho = |u\rangle\langle u| \quad (u \in H, \|u\|_{H} = 1)$. Consider a measurement $M_{B(H)}(\Psi(O_{G} \otimes O_{F}), S_{[\rho]})$. Then, Axiom 1 says that

(E) the probability that a measured value $(k, x)$ obtained by the measurement $M_{B(H)}(\Psi(O_{G} \otimes O_{F}), S_{[\rho]})$ belongs to $\{n\} \times \Xi$ is given by

$$\text{tr}[(|u\rangle\langle u|)\Psi(G(\{n\}) \otimes F(\Xi))] = \text{tr}[(\Psi_{*}(|u\rangle\langle u|))(G(\{n\}) \otimes F(\Xi))]$$

$$= \text{tr}[\sum_{k=1}^{\infty} |e_{k} \otimes P_{k}u\rangle\langle e_{k} \otimes P_{k}u|](|e_{n}\rangle\langle e_{n}| \otimes F(\Xi))] = \langle P_{n}u, F(\Xi)P_{n}u \rangle \quad (\forall \Xi \in \mathcal{F})$$

( In a similar way, the same result is easily obtained in the case of (7)).

Thus, we see:

(F1) if $\Xi = X$, then we see:

$$\text{tr}[(|u\rangle\langle u|)\Psi(G(\{n\}) \otimes F(X))] = \langle u, P_{n}u \rangle$$

(F2) when a measured value $(k, x)$ belongs to $\{n\} \times X$, the conditional probability such that $x \in \Xi$ is given by

$$\langle \frac{P_{n}u}{\|P_{n}u\|}, F(\Xi) \frac{P_{n}u}{\|P_{n}u\|} \rangle \quad (\forall \Xi \in \mathcal{F})$$

(8)

This is a direct consequence of Axioms 1 and 2.

Considering the correspondence: (D) $\leftrightarrow$ (F), that is,

$$M_{B(H)}(O_{P}, S_{[\rho]}) \leftrightarrow M_{B(H)}(\Psi(O_{G} \otimes O_{F}), S_{[\rho]}) \quad (\text{D}_1) \leftrightarrow (\text{F}_1), \quad (\text{D}_2) \leftrightarrow (\text{F}_2)$$

there is a reason to consider that the true meaning of the (5) is just the (8).
3 Conclusion

In this paper, I assert:

\( (G) \) Although the von Neumann-Lüders projection postulate \((D_2)\) concerning the measurement \( M_{B(H)}(O_F, S_{[\rho]} \) cannot be derived from Axioms 1 and 2, the similar result \((F_2)\) concerning \( M_{B(H)}(\Psi(O_G \otimes O_F), S_{[\rho]} \) holds in the linguistic interpretation.

Hence, I assert that the \((D_2)\) (i.e., the wave function collapse) should not be adopted in the Copenhagen interpretation. Although there are a lot of opinions about the Copenhagen interpretation \((cf. [2]\), I want to conclude, as mentioned in \([6]\), that the linguistic interpretation is the true colors of the Copenhagen interpretation. Also, if this is true, other interpretations \(e.g., \) the many-worlds, etc.) should be reconsidered.

I hope that my assertion will be examined from various points of view.

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