Disk formation in the collapse of supramassive neutron stars

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ABSTRACT
Short gamma-ray bursts (sGRBs) show a large diversity in their properties. This suggests that the observed phenomenon can be caused by different “central engines” or that the engine produces a variety of outcomes depending on its parameters, or possibly both. The most popular engine scenario, the merger of two neutron stars, has received support from the recent Fermi and INTEGRAL detection of a burst of gamma rays (GRB170817A) following the neutron star merger GW170817, but at the moment it is not clear how peculiar this event potentially was. Several sGRBs engine models involve the collapse of a supramassive neutron star that produces a black hole plus an accretion disk. We study this scenario for a variety of equations of states both via angular momentum considerations based on equilibrium models and via fully dynamical Numerical Relativity simulations. We obtain a broader range of disk forming configurations than earlier studies but we agree with the latter that none of these configurations is likely to produce a phenomenon that would be classified as an sGRB.

Key words: accretion discs, hydrodynamics, methods: numerical, stars: gamma-ray burst: general, stars: neutron, stars: rotation.

1 INTRODUCTION
With the first detection of a neutron star merger in both gravitational (Abbott et al. 2017a) and electromagnetic waves (Abbott et al. 2017c; Coulter et al. 2017) the era of multi-messenger astrophysics has begun in earnest. This single event brought a major leap forward for a number of areas: it allowed for a new, independent measurement of the Hubble constant (Abbott et al. 2017b), it conclusively established that neutron star mergers are a major cosmic source of r-process elements (Lattimer & Schramm 1974; Eichler et al. 1989; Rosswog et al. 1999; Freiburghaus et al. 1999; Cowperthwaite et al. 2017; Smartt et al. 2017; Kasliwal et al. 2017; Kasen et al. 2017; Tanvir et al. 2017; Rosswog et al. 2017), and, with precise limits on the propagation speed of gravitational waves (Abbott et al. 2017c), it placed strict constraints on alternative theories of gravity. Moreover, the triggering of the Fermi and INTEGRAL satellites on a short gamma-ray burst (sGRB) 1.7 seconds after the gravitational wave (GW) peak lends support to the long-held conjecture that neutron star mergers produce GRBs (Paczynski 1986; Eichler et al. 1989). It has, however, been debated whether this GRB event was an intrinsically sub-luminous one with $E_{\gamma,iso} \sim 6 \times 10^{46}$ erg (Kasliwal et al. 2017; Mooley et al. 2018; Nakar et al. 2018) or a typical short GRB with $10^{50} \sim 10^{52}$ erg (Berger 2014; Fong et al. 2015), but seen off axis, see for example Margutti et al. (2018); Lyman et al. (2018).
In general, sGRBs exhibit a large variety of properties and it is not well understood how this diversity relates to the central engine(s). A particularly puzzling property is late-time X-ray activity on time scales that exceed the dynamical time scales of a compact engine ($\sim 1$ ms) by many orders of magnitude, see e.g. Villasenor et al. (2005); Barthelmy et al. (2005); Rowlinson et al. (2013); Gompertz et al. (2014). One possibility would be that the sGRB is produced by a magnetar (Metzger et al. 2011; Bucciantini et al. 2012), provided that excessive “baryonic pollution” e.g. due to a neutrino-driven wind (Dessart et al. 2009; Perego et al. 2014) can be avoided, otherwise the outflow will be choked (Murguia-Berthier et al. 2017). Also models involving quark stars have been suggested (Drago et al. 2016; Pili et al. 2016). Alternatively, MacFadyen et al. (2005) proposed that such bursts could be caused by neutron stars (NSs) accreting from a non-degenerate companion star. Upon collapse, a black hole (BH) plus accretion disk system would form and launch the relativistic outflow that produces the GRB. The late X-ray activity would result from the interaction of the outflow with the extended companion star. In a black hole accretion flow, a fraction of the accreted rest mass energy is released as ra-
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diation (Frank et al. 2002). Therefore, to produce $E_{\gamma,iso}$ as GRB energy, the accretion disk would need to have a mass of the order of

$$M_{\text{disk}} \sim 2 \times 10^{-4} M_\odot \left( \frac{E_{\gamma,iso}}{10^{51} \text{erg}} \right) \left( \frac{f_b}{1/50} \right) \left( \frac{0.05}{\epsilon} \right),$$

(1)

where $M_{\text{disk}}$ is the disk mass, $\epsilon$ is the accretion efficiency and $f_b = \Delta \Omega/4\pi$ is the beaming fraction. The late X-ray activity is also addressed in so-called “time-reversal scenarios” (Ciolfi & Siegel 2015; Rezzolla & Kumar 2015) where a long-lived supramassive neutron star produces the long-lasting X-ray emission which initially is trapped in an optically thick nebula. As in the scenario proposed by MacFadyen et al. (2005), also here it is crucial for the model that at some point the supramassive neutron star collapses to black hole plus torus system to launch the GRB.

The question whether such a collapse really produces an accretion torus that is massive enough for launching a typical sGRB, has recently been addressed by Margalit et al. (2015). They constructed rapidly rotating neutron stars using the RNS code (Stergioulas & Friedman 1995) and studied the corresponding angular momentum distribution. The authors came to the conclusion that it is unlikely that an accretion disk massive enough to launch an energetic GRB can be formed. In this paper, we revisit this problem. We construct our initial conditions with the XNS code (Bucciantini & Del Zanna 2011; Pili et al. 2014), that makes use of the extended conformal flatness approximation of Cordero-Carrión et al. (2009) and we study the angular momentum spectrum to estimate the resulting disk mass after collapse. We scrutinize our conclusions by simulating for a selected set of configurations the collapse directly with fully dynamical Numerical Relativity simulations. We summarize our numerical methods in Sec. 2, discuss the rotating equilibrium configurations in Sec. 3 and describe the dynamical collapse simulations in Sec. 4. Our results are summarized in Sec. 5. Comparisons between XNS and RNS results are provided in Appendix A.

2 NUMERICAL METHODS

2.1 Governing Equations

Our goal is to construct rigidly rotating neutron stars as pre-collapse initial conditions. We assume stationarity and axisymmetry and therefore can write the metric in quasi-isotropic coordinates $(t, r, \theta, \phi)$ (Gourgoulhon 2010) as

$$ds^2 = -N^2 dt^2 + A^2 (dr^2 + r^2 d\theta^2) + r^2 B^2 \sin^2 \theta (d\phi - \omega dt)^2.$$  (2)

Here the cylindrical radius is defined as $R = Br \sin(\theta)$. The metric functions $N, A, B, \omega$ depend purely on $r$ and $\theta$, where $N$ denotes the lapse and $\omega$ is the intrinsic angular velocity of the zero angular momentum observer (ZAMO) relative to infinity.\(^1\) The first integral (Gourgoulhon 2010) for a cold

\(^1\) $\omega$ is non-zero because of the frame dragging effect due to the rotation of the neutron star.

![Figure 1. Example of the six-piece polytropic EOS employed in this work. The light blue line refers to the SLy crust employed at low densities, while the dark blue line refers to the high density part of the EOS. We mark the transition between the different polytropic pieces with dashed orange lines. The thin black vertical line corresponds to $\rho_0 = 10^{14.3} \text{g cm}^{-3}$, namely the density at which the crust is attached to the high density EOS.](image)
In this paper, density range \( \rho \), polytropic exponent \( \Gamma \), and polytropic constant \( K \) are reported in each column. The horizontal line separates the low-density SLy crust from the high-density core EOS. The quantities denoted with an asterisk ‘*’ depend on the choice of \( p_1 \) and \( \Gamma_2 \). Beware that the dimensions of \( K \) depend on \( \Gamma \).

The first polytrope \( (\rho_0 < \rho < \rho_1) = 10^{14.7} \text{g cm}^{-3} \) is determined on one side by the SLy EOS that fixes the value of the pressure \( p(\rho_0) \) on the other side by the free parameter \( p_1 \). The second polytrope \( (\rho > \rho_1) \) is determined by \( p_1 \) and by the polytropic exponent \( \Gamma_2 \). Therefore, the whole EOS is fixed by only two parameters, \( p_1 \) and \( \Gamma_2 \), cf. Tab. 1.

In the computation of the equilibrium configurations we neglect thermal effects, but (apart from one test) we include them for the dynamical simulations, see Sec. 4, by adding a thermal pressure component to the barotropic pressure \( p(\rho) \)

\[
p(\rho, \epsilon) = p(\rho) + \epsilon \rho (\Gamma_{th} - 1).
\]  

In accordance with previous work and following the discussion in Bauswein et al. (2010), where full tabulated EOSs are compared against the approximate description of Eq. (8), we employ \( \Gamma_{th} = 1.75 \). Additionally, we also perform a dynamical simulation without additional thermal component to allow for an assessment of systematic uncertainties.

### 2.2 Equilibrium configurations

Our investigation of the equilibrium configurations for different EOSs is based on the XNS code (Bucciantini & Del Zanna 2011; Pili et al. 2014), which determines the rotating stellar configuration in quasi-isotropic coordinates under the extended conformal flatness approximation (Cordero-Carrion et al. 2009). In the extended conformal flatness approximation, all elliptic equations that characterize the spacetime metric are hierarchically decoupled, which leads to a simplified metric with

\[
A(r, \theta) \equiv B(r, \theta) \equiv \psi^2(r, \theta),
\]

where \( \psi \) denotes the conformal factor. The approximation is justified since the metric functions \( A \) and \( B \) typically differ at most by about 0.1% (Gourgoulhon 2010).

The XNS code originally descends from the X-ECHO code (Bucciantini & Del Zanna 2011) and therefore inherits some features not needed for our purposes. Its main focus is on the interplay between rigid or differential rotation and poloidal and/or magnetic fields (Pili et al. 2017). For this study we have modified the publicly available version of XNS to the following workflow:

(i) set the target stellar parameters central rest mass density, \( \rho_c \), and angular speed seen by an observer at infinity, \( \Omega \).

(ii) determine the initial configuration from the TOV solution ( Tolman-Oppenheimer-Volkoff, namely the one that describe a spherical neutron star) with central density \( \rho_c \), or if available, load a previously relaxed configuration from a sequence, obtained for example in the search for the Keplerian configuration.

(iii) repeat until \( \max_{r,\theta} |p_{old}(r, \theta) - p_{new}(r, \theta)| < 10^{-9} M_{\odot}^{-2} \)

\[ \text{(in units } c = G = 1) \]

(a) using the old metric and matter quantities, solve the hierarchically decoupled equations of the extended conformal flatness approximation and update the metric fields.

(b) update the matter fields solving the first integral, Eq. (3), with central density \( \rho_c \) and angular velocity \( \Omega \).

The main differences between our workflow and the original one are that we update the matter fields only through the first integral inversion avoiding conservative-to-primitive variable inversion. We directly set the central density in the first integral instead of using an external root-finding cycle, and we allow for an initial configuration other than the TOV one. Additional major technical modifications are the adoption of an inner (uniformly spaced) and an outer (increasingly spaced) radial grid, an angular grid defined on the Gauss-Legendre quadrature points, and the use of a true vacuum outside the neutron star instead of an artificial atmosphere.

Our modified XNS version is ten times faster than RNS\(^2\). This is in part due to the hierarchical decoupling of the equations for the spacetime metric in the extended conformal flatness approximation. In Appendix A we present a detailed convergence study and compare the results of our modified XNS version with the publicly available RNS code (Stergioulas & Friedman 1995). In particular, we find that XNS recovers the stellar properties within the precision of the extended conformal flatness approximation and yield practically identical results of RNS. For the exploration of the parameters space we employ XNS, while the initial configurations which we evolve dynamically with the BAM code are constructed with RNS, since the interface between RNS and BAM has been implemented and tested in detail in a previous work (Dietrich & Bernuzzi 2015).

### 2.3 Dynamical evolution

For the dynamical evolution we solve Einstein’s field equations in their 3+1 form recast in the Zc4 evolution system (Bernuzzi & Hilditch 2010; Hilditch et al. 2013). The gauge sector employs the 1+log and gamma-driver equations developed for black holes in the moving puncture approach (Bona et al. 1996; Alcubierre et al. 2003; van Meter et al. 2006; Campanelli et al. 2006; Baker et al. 2006). This particular gauge choice, often called ‘puncture gauge’, handles automatically the gravitational collapse of a neutron star to a black hole as discussed in Baiotti et al. (2007); Thierfelder

\[ ^2 \text{On a 1.40GHz CPU (Intel(R) Core(TM) i3-2365M) and 4GB RAM laptop with -O2 optimization.} \]
et al. (2011a); Dietrich & Bernuzzi (2015) and is therefore particularly well-suited for our study.

The simulations are performed with the BAM code (Brigmann et al. 2008; Thierfelder et al. 2011b). BAM employs the method of lines approximating spatial derivatives of the metric variables by 4th order finite differences. Time integration is performed with an explicit 4th order Runge-Kutta scheme. The grid used in this work consists of a hierarchy of 7 cell-centered nested Cartesian boxes. Every box \( l = 0, \ldots, 6 \) employs a constant grid spacing \( h_l \) and \( n \) points per direction. Boxes use a 2 : 1 refinement strategy, i.e., each coarser box employs a grid spacing \( h_{l-1} = 2 h_l \). For the time stepping of the mesh refinement, we employ the Berger-Oliger algorithm (Berger & Oliger 1984) extended by a refluxing step that enforces energy and momentum conservation across refinement boundaries (Berger & Colella 1989; East et al. 2012; Reisswig et al. 2013; Dietrich et al. 2015). The equations of GRHD are solved with a standard 2nd-order Godunov flux (Thierfelder et al. 2011a; Dietrich & Bernuzzi 2015) and is therefore called WENOZ. Other limiters are used for comparison to (WENO) scheme (Borges et al. 2008; Bernuzzi et al. 2012), high-resolution-shock-capturing (HRSC) scheme based on the method of lines approximating spatial derivatives by 4th order finite differences. Time integration is performed with an explicit 4th order Runge-Kutta scheme. The grid used in this work consists of a hierarchy of 7 cell-centered nested Cartesian boxes. Throughout this work we employ for the threshold \( f_{\text{thr}} \), we set a grid point to the atmosphere values if the density falls below the threshold

\[
\rho_{\text{atm}} = f_{\text{atm}} \cdot \max[\rho(t = 0)].
\]

During the inversion from conservative to primitive variables we set a grid point to the atmosphere values if the density falls below the threshold

\[
\rho_{\text{thr}} = f_{\text{thr}} \cdot \rho_{\text{atm}}.
\]

Table 2. Configurations employed for the dynamical evolutions. The columns refer to: configuration name, number of points in the Cartesian boxes, grid spacing in the refinement level covering the neutron star, atmosphere factor [see Eq. (10)], and flux limiter: WENOZ (Borges et al. 2008; Bernuzzi et al. 2012), linear total variation diminishing (LINTVD) (Shu & Osher 1989), 3rd order Essentially-Non-Oscillatory 3rd-order method (CENO3) (Liu & Osher 1998; Del Zanna et al. 2003). In addition to the listed setups, we also employ for one physical configuration the setup Res2WENOZ but with zero thermal contribution, Eq. (8). This setup is labeled as Res2WENOZ atm19cold.

#### Table 2

| Name                     | \( n \) | \( h_{\text{thr}} [M_{\odot}] \) | \( f_{\text{atm}} \) | Limiter       |
|--------------------------|--------|----------------------------------|---------------------|--------------|
| Res1WENOZ atm19          | 120    | 0.1250                           | 10^{-19}            | WENOZ        |
| Res2WENOZ atm19          | 180    | 0.0833                           | 10^{-19}            | WENOZ        |
| Res3WENOZ atm19          | 240    | 0.0625                           | 10^{-19}            | WENOZ        |
| Res4WENOZ atm19          | 360    | 0.0417                           | 10^{-19}            | WENOZ        |
| Res2WENOZ atm18          | 180    | 0.0833                           | 10^{-18}            | WENOZ        |
| Res2WENOZ atm20          | 180    | 0.0833                           | 10^{-20}            | WENOZ        |
| Res2LINTVD atm18         | 180    | 0.0833                           | 10^{-18}            | LINTVD       |
| Res2CENO3 atm20          | 180    | 0.0833                           | 10^{-20}            | CENO3        |

3 EQUILIBRIUM CONFIGURATIONS

In an axisymmetric dynamical system the spectrum of angular momentum, i.e., the integrated baryon rest mass of all fluid elements with a specific angular momentum, is strictly conserved in the absence of viscosity (Stark & Piran 1987). Even if some viscosity is present in either Nature or a numerical simulation, the (dynamical) collapse timescales are too short for viscosity effects to become important. This suggests to use as a necessary condition for the formation of a debris disk that the specific angular momentum of a matter element at the stellar equator before the collapse is greater than the specific angular momentum of the innermost stable circular orbit (ISCO) of a Kerr BH with the same mass \( M \) and angular momentum \( J \) of the progenitor neutron star, see Fig. 2. This is also the criterion that has been applied in the study of Margalit et al. (2015). Since the specific angular momentum increases with the rotational frequency of the star, one expects that for a given EOS and central density \( \rho_c \), the collapse of a maximally rotating star (that rotates at the Keplerian frequency) will produce the largest debris disk mass. Furthermore, the configuration should be unstable to collapse to a BH.

In this section we state the stability and disk formation criteria, delineate our procedure to find the maximally rotating configurations, describe our results comparing them with Margalit et al. (2015), choose the configurations that we further analyze with dynamical simulations in the next section, and compute possible debris disk masses for a set of realistic EOSs constructed in Read et al. (2009).

#### 3.1 Stability condition

If the star is non-rotating, the marginally stable configuration obeys (Sec. 10.11 of Zeldovich & Novikov 1971)

\[
\frac{dM}{d\rho_c} = 0.
\]

Increasing the central density beyond this point the star becomes unstable and collapses to a BH. In the non-rotating case, this configuration also has the maximal gravitational mass.

For rotating stars the marginally stable criterion has to be modified as follows (Friedman et al. 1988)

\[
\left. \frac{dM}{d\rho_c} \right|_J = 0,
\]

where \( J \) is the total angular momentum; this condition is
sufficient for instability (Takami et al. 2011). However, J is difficult to access during our computation with the XNS code since it uses $\rho_c$ and $\Omega$ as input variables. One can circumvent this problem with a root finding cycle on $J$ code since it uses $\rho_c$ and $\Omega$ as input variables. One can circumvent this problem with a root finding cycle on $J$ code since it uses $\rho_c$ and $\Omega$ as input variables.

\[
\frac{\partial M}{\partial \rho_c} = \frac{\partial M}{\partial J} \bigg|_{\Omega} \cdot \frac{\partial J}{\partial \Omega} \bigg|_{\rho_c} \cdot \left( \frac{\partial J}{\partial \rho_c} \bigg|_{\rho_c} \right)^{-1}. 
\]  
(14)

We have obtained Eq. (14) from

\[
dM(\rho_c, J) = \frac{\partial M}{\partial \rho_c} \bigg|_{J} \ d\rho_c + \frac{\partial M}{\partial J} \bigg|_{\rho_c} \ dJ,
\]

(15)

\[
dM(\rho_c, \Omega) = \frac{\partial M}{\partial \rho_c} \bigg|_{\Omega} \ d\rho_c + \frac{\partial M}{\partial \Omega} \bigg|_{\rho_c} \ d\Omega,
\]

(16)

\[
dJ(\rho_c, \Omega) = \frac{\partial J}{\partial \rho_c} \bigg|_{\Omega} \ d\rho_c + \frac{\partial J}{\partial \Omega} \bigg|_{\rho_c} \ d\Omega.
\]

(17)

One first equates Eqs. (15) and (16) (the total variation of the mass should be the same no matter which are the independent variables), then substitutes Eq. (17) and equates the terms that multiply $\rho_c$. Eq. (14) is finally obtained using

\[
\frac{\partial M}{\partial \rho_c} = \frac{\partial M}{\partial J} \bigg|_{\Omega} \cdot \frac{\partial \Omega}{\partial \rho_c} \bigg|_{\rho_c} \cdot \frac{\partial J}{\partial \Omega} \bigg|_{\rho_c} \cdot \left( \frac{\partial J}{\partial \rho_c} \bigg|_{\rho_c} \right)^{-1}, 
\]  
(18)

where the equalities follow from the fact that, fixing $\rho_c$, the quantities $M, J, \Omega$ are functions of just one variable. Using Eq. (14) we can test the stability condition with only 3 configurations: those corresponding to ($\rho_c, \Omega$), ($\rho_c + \rho_c, \Omega$), and ($\rho_c, \Omega + d\Omega$).\footnote{One can use Eqs. (14)–(18) also with the RNS code (Stergioulas & Friedman 1995), using $e_c = \epsilon(\rho_c)$ (energy density at the center) and $a \equiv r_{\text{surf}}(\theta = 0)/r_{\text{surf}}(\theta = \pi/2)$ (stellar quasi-isotropic radii ratio) as independent variables instead of $\rho_c$ and $\Omega$, respectively.}

3 Up to our knowledge this is the first time that Eq. (14) is discussed and used; we check it in the third panel of Fig. 3, where the solid red line refers to the value of $\partial M/\partial \rho_c$ obtained through Eq. (14) and the dashed light blue line is obtained with a root-finding cycle on $J$. The two approaches agree within numerical uncertainties.

3.2 Disk formation condition

We use the specific angular momentum $j$ of a fluid element (i.e., per baryon rest mass; e.g. Eq. (3.85) in Gourgoulhon 2010)

\[
j = h \Omega R^2 U^\theta. 
\]  
(19)

The co-rotating ISCO specific angular momentum for a Kerr black hole with total gravitational mass $\mathcal{M}_{\text{BH}}$ and angular momentum $j_{\text{ISCO}}$ is given by (Eqs. (2.12), (2.13), and (2.21) of Bardeen et al. 1972)

\[
j_{\text{ISCO}} = \sqrt{\mathcal{M}_{\text{BH}}^2 - 2\chi \mathcal{M}_{\text{BH}}^2 r_{\text{ISCO}}^2 + \chi^2 \mathcal{M}_{\text{BH}}^2}, 
\]  
with

\[
\chi = \frac{j_{\text{ISCO}}}{\mathcal{M}_{\text{BH}}^2},
\]

\[
Z_1 = \sqrt{1 - \chi^2} \left( \sqrt{1 - \chi + \sqrt{\chi^2 + 1}} \right) + 1, 
\]

\[
Z_2 = \sqrt{3\chi^2 + Z_1^2},
\]

\[
d = r_{\text{ISCO}}^2 \sqrt{\frac{4}{Z_1} - 3\mathcal{M}_{\text{BH}}^2 r_{\text{ISCO}}^2 + 2\chi \mathcal{M}_{\text{BH}}^2}, 
\]  
(24)

\[r_{\text{ISCO}} = \mathcal{M}_{\text{BH}} \left( 3 + Z_2 - \sqrt{3 - 3(1/Z_1 + 2Z_2) + 3} \right). 
\]

This allows to write the condition for disk formation as (Shapiro 2004; Margalit et al. 2015)

\[j(r_{\text{surf}}, \pi/2) > j_{\text{ISCO}} (\mathcal{M}_{\text{BH}} = M, J_{\text{BH}} = J), 
\]  
with $M$ and $J$ are the neutron star gravitational mass and total angular momentum, and $r_{\text{surf}}$ is the stellar radius. In Fig. 2 we plot the specific angular momentum distribution in a neutron star. The black line corresponds to the ISCO angular momentum and divides the material which will collapse into the forming black hole and that will form a debris disk. We remark that, to be fully consistent, we should have taken a black hole with a total mass and angular momentum equal to that of the pre-collapse neutron star without the contribution from the debris disk. This could be accomplished within an iterative procedure (Shapiro 2004). However, such a procedure is not well defined since the local energy is not well defined in General Relativity and hence in a neutron star. In any case we have checked that the results obtained with the iterative procedure are indistinguishable from those obtained without it because the debris disk has very little mass and angular momentum (see discussion below).

3.3 Parameter space exploration

Following the above discussion we estimate, for each choice of the EOS (specified by parameters $p_1$ and $\Gamma_2$), the bary-

\[&\text{and Friedman Stergioulas & Friedman, 1995},\text{ using } e_c = \epsilon(\rho_c)\text{ (energy density at the center)} \text{ and } a \equiv r_{\text{surf}}(\theta = 0)/r_{\text{surf}}(\theta = \pi/2)\text{ (stellar quasi-isotropic radii ratio) as independent variables instead of } \rho_c \text{ and } \Omega, \text{ respectively.} \]
The code settings adopted in the parameter space exploration are the same of the “baseline” configuration described in Appendix A, apart for the radius of the inner grid which is set to 20 (in code units, see Appendix A for details). Our procedure to find the interesting stellar models, for each choice of the EOS parameters $p_1$ and $\Gamma_2$, is:

1. start from the maximal TOV (spherical) mass configuration, with $\Omega = 0$ and central density $\rho_c$.
2. keep $\rho_c$ fixed and increase $\Omega$ until the Keplerian (i.e., maximally rotating) configuration is reached. We will refer to this configuration as “step 2”\(^4\). The Keplerian angular speed $\Omega_K$ is determined by evaluating the co-rotating case of Eq. (4.93) of Gourgoulhon (2010) at the equator,

$$\Omega_K = \omega + \frac{\omega R}{2 R^2} \left( \frac{N^2 N}{R} \right) + \left( \frac{\omega R}{2 R^2} \right)^2,$$

where all quantities are evaluated at the equator and primes denote derivatives along the radial direction $r$.

3. repeat:

   a. compute $\partial M/\partial \rho_c |_J$ [Eq. (14)] varying $\rho_c$ and $\Omega$.
   b. if $\partial M/\partial \rho_c |_J \geq 0$, the configuration is maximally rotating and at the verge of collapse (i.e., it is the marginally stable Keplerian configuration).
   c. if the mass is lower than that of the previous configuration, the previous is the maximal mass one.
   d. once the maximal mass configuration and the marginally stable Keplerian configuration are found, exit the cycle.
   e. reduce $\Omega$ by $d\Omega = 10^{-4} M_\odot^{-1}$ (c = $G = 1$ units).
   f. reduce $\rho_c$ until you reach the Keplerian configuration.

In general, the step 2 and the maximal mass configurations can be stable or unstable (Stergioulas & Friedman 1995). As an example, in Fig. 3 we report the search for a case in which the maximal mass configuration is stable and the step 2 configuration is unstable, which is the most common case in our analysis.

We estimate the mass of the debris disk from a given equilibrium configuration integrating the baryon mass in the neutron star that fulfills the condition $j(r, \theta) > j_{\text{ISCO}}$ (lower plot of Fig. 4). This estimated disk mass increases as we step to lower central densities along the Keplerian curve (see lower plot in Fig. 3). However, these lower density configurations would not give rise to a debris disk because they are stable.

We have explored the same EOS parameter space as Margalit et al. (2015), namely $p_1 \in [10^{13.8}; 10^{35.2}]$ dyne cm$^{-3}$ and $\Gamma_2 \in [1.1; 4]$, our results are shown in Fig. 4. Most of the EOS parameter choices result in disk formation; moreover the general trend is that the greater the maximal mass (stiffer EOSs, i.e., greater $p_1$ and $\Gamma_2$), the smaller the disk mass of the marginally stable Keplerian configuration.

We select a few configurations that we also study with

\(^4\) For most of the EOS parameter space considered in this paper, the step 2 configuration is unstable ($\partial M/\partial \rho_c |_J < 0$) and has a central density greater than that of the maximal mass configuration (see Fig. 3). When this does not hold, for very low $p_1$ and $\Gamma_2$, we have re-started the search from a greater central density.

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**Figure 3.** Initial configuration search (extended to lower densities on the Keplerian line) for the piecewise polytropic EOS corresponding to Case B (see Tab. 3). The configurations discussed in the text are marked. From top to bottom we show the angular frequency $\Omega$, the gravitational mass $M$, the partial derivative at constant $J$: $M' = \partial M/\partial \rho_c |_J$, and the baryonic mass of the disk estimated from the equilibrium configuration $M_{\text{disk}}$ [i.e., the baryon mass of the material that fulfill Eq. (26)], all plotted against the central density $\rho_c$. The solid red lines are obtained using Eq. (14) and the dashed light blue lines are obtained with a root-finding cycle on $J$. The results of the two approaches differ only in $M'$, in which case are consistent within the numerical uncertainties.

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Figure 4. Stellar properties dependence on the EOS parameters $p_1$ and $\Gamma_2$. From top to bottom: maximal TOV gravitational mass, maximal gravitational mass (at the Keplerian frequency), and estimated disk baryon mass of the marginally stable Keplerian configuration. The black area in the lower plot represents the parameter space for which no disk formation is possible ($M_{\text{disk}} = 0$). We mark with diamonds the configurations that we have further studied with dynamical simulations. The white dashed line denotes the disk/no-disk separation curve previously found by Margalit et al. (2015).

Fully dynamic Numerical Relativity simulations (Cases A, $\tilde{A}$, B, and C, see marks in Fig. 4). These configurations are reported in Tab. 3. The rationale behind our choices is the following:

A marginally stable Keplerian configuration with the greatest disk mass compatible with the request to have a maximal TOV mass greater than $1.5M_\odot$, see upper plot in Fig. 4. The corresponding value of $\Gamma_2 = 1.3$ is substantially smaller than what is considered realistic ($\geq 2.5$; see e.g. Fig. 5 in Rosswog & Davies 2002 for an illustration).

$\tilde{A}$ maximal mass configuration equivalent to another Case studied with a dynamical simulation that results in a col-
lapse to test our prediction on the stability of the maximal mass configuration (we picked a stable case).

**B** marginally stable Keplerian configuration with the maximal TOV mass equal to $2M_\odot$ and $\Gamma_2 = 2$. This choice of the high density polytropic exponent is still smaller than current predictions but it is a common choice in many numerical applications.

**C** marginally stable Keplerian configuration with no predicted disk and a maximal TOV mass smaller than $3M_\odot$.

### 3.4 Comparison with Margalit et al. (2015)

While in good qualitative agreement, our results differ quantitatively from those of Margalit et al. (2015). In fact, we find that disk formation is possible also for $\Gamma_2 \gtrsim 2$ and $\log_{10}(p_i/\text{dyn cm}^{-2}) \gtrsim 34.9$, cf. lower panel of Fig. 4. None of the cases considered in our study, however, is a candidate for producing an sGRB, because the disk mass is too small, cf. Eq. (1).

In principle, the main difference in the employed methods between our work and Margalit et al. (2015) are:

- Margalit et al. (2015) uses the RNS code, while we use XNS. As we show in Appendix A, the configurations found by both codes are in very good agreement.
- Margalit et al. (2015) searches for the maximal mass configuration instead of the marginally stable configuration. As argued in Sec. 3.3 and in Stergioulas & Friedman (1995), the maximal mass configuration is not necessarily unstable, because the stability condition should be checked at constant $J$, and in any case it is not on the verge of instability. However, we checked that the maximal mass configuration generates a disk similar to that of the marginally stable Keplerian configuration.

We note that the step 2 configuration actually reproduces Fig. 2 of Margalit et al. (2015). However, this configuration is unstable for most of the choices of the EOS parameters (apart for very small $p_1$ and $\Gamma_2$) and was therefore not considered in our analysis.

### 3.5 Fit to realistic EOSs of Read et al. (2009)

In addition to the general consideration of neutron stars described by a 2-piece polytropic core, we have applied the outlined procedure to some more realistic multi-piecewise polytropic EOSs. Those fits have been constructed in Read et al. (2009) and model EOSs describing full tabulated EOSs for different nuclear physical models. The results for these EOSs are given in Tab. 4. We have chosen this subset of EOSs since it is in agreement with current observations: (i) maximum supported masses are above $2.0M_\odot$ (Antoniadis et al. 2013); (ii) maximum supported masses are below $\sim 2.3M_\odot$ (Rezzolla et al. 2017; Shibata et al. 2017; Ruiz et al. 2017; Margalit & Metzger 2017); and (ii) the compactness and tidal deformability are in agreement with the measurements obtained from GW170817 (Abbott et al. 2017a; Abbott et al. 2018). The results for realistic EOSs confirm the conclusions for the EOS parameter search made in this section, namely that even if a debris disk can form, its mass is too small to generate an energetic GRB.

A run with $\rho = 9.3 \times 10^3 \text{g/cm}^3$ (Case A, red line). We mark the formation of the apparent horizon for Case A with a vertical purple dashed line.

An important point to stress here is that we are discussing the mass and the extractable GRB energy of a debris disk formed by material of the pre-collapse neutron star. This means that we are not addressing the possibility that the GRB is caused by a pre-existing debris disk (Michel & Dessler 1981), for example due to fallback from the original supernova event. These disks may potentially be more massive than the disks we predict in our analysis (e.g., Wang et al. 2006 found observational evidence of a fallback disk of $\sim 10^{-5}M_\odot$, see also Wang 2014 for a recent review).

### 4 DYNAMICAL EVOLUTIONS

In the following we study the configurations marked in Fig. 4 and described in Tab. 3 and Sec. 3.3 to determine whether dynamical effects can facilitate the debris disk formation.

#### 4.1 Cases A and Ā

Cases A and Ā employ an EOS with $\log(p_1) = 34.7$ and $\Gamma_2 = 1.3$, see Tab. 3. The Case A (marginally stable Keplerian configuration) is characterized by a central density of $\rho_c = 1.212 \times 10^{15} \text{g/cm}^3$ and an angular speed of $\Omega = 7.016 \text{rad/ms}$. The Case Ā (maximal mass configuration) has a central density $\sim 15\%$ lower than Case A and an angular speed of $\Omega = 6.834 \text{rad/ms}$. Within our simulations we trigger the gravitational collapse by introducing a small artificial pressure perturbation. This is a common approach for the study of gravitational collapse. Generally, large perturbations lead to a faster collapse which reduces the computational cost of the individual simulations, but on the other hand it might affect the dynamical evolution. We employ a small perturbation of $0.05\%$ to reduce nonphysical effects\(^5\), reminding the reader that the introduced pressure perturbation leads to Hamiltonian constraint violations at $t = 0$. Previous studies have used larger perturbations, see e.g. Giacomazzo & Perna (2012) where a $0.1\%$ perturbation,
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Table 3. Properties of the configurations marked in Fig. 4 and described in Sec. 3.3. The columns contain, in order: the configuration name (“Case”); the EOS parameters ($\rho_1$ in dyne cm$^{-3}$ and $\Gamma_2$); the configuration parameters (central density $\rho_c$, gravitational mass $M$); the angular velocity and disk baryonic mass estimated from the equilibrium model of the marginally stable Keplerian configuration (3rd and 4th columns respectively). We report the disk mass estimated as average of the material surrounding the central BH. The density dropped several orders of magnitude and the bound mass (namely, the debris disk mass) has decreased to $10^{-13} M_\odot$. Note that Case A does not collapse, as expected.

| Case | $\log_{10}(\rho_1)$ | $\Gamma_2$ | $\rho_c$ [$10^{15}$ g cm$^{-3}$] | $\Omega$ [rad/ms] | $\alpha$ | $M$ [$M_\odot$] | $M_{\text{disk}}^\text{eq}$ [$M_\odot$] | $M_{\text{disk}}^\text{dyn}$ [$M_\odot$] |
|------|-------------------|-----------|-------------------------------|-----------------|--------|---------------|------------------|------------------|
| A    | 34.7              | 1.3       | 1.212                         | 7.016           | 0.5667 | 1.924         | $7 \times 10^{-8}$ | $\lesssim 10^{-7}$ |
| A    | 34.7              | 1.3       | 1.033                         | 6.834           | 0.5524 | 1.933         | -                | -                |
| B    | 34.8              | 2.0       | 1.238                         | 8.511           | 0.5599 | 2.679         | $3 \times 10^{-10}$ | $\lesssim 10^{-7}$ |
| C    | 34.9              | 3.3       | 0.9634                        | 9.573           | 0.5630 | 3.614         | $4 \times 10^{-13}$ | $\lesssim 10^{-8}$ |

Dietrich & Bernuzzi (2015) where a 0.5% perturbation, and Baiotti et al. (2005, 2007); Reisswig et al. (2013) where a 2% perturbation were applied.

Comparison of maximal mass and marginally stable Keplerian configurations. Before discussing the gravitational collapse in detail, we compare the simulations of the maximal mass and the marginally stable Keplerian configurations. Figure 5 shows the maximum density evolution for both cases, where the maximal mass configuration is shown in blue (Case A) and the marginally stable Keplerian configuration in red (Case A). We find that, as outlined in our previous discussion (Sec. 3.1), the maximal mass configuration is stable and does not undergo gravitational collapse while the marginally stable Keplerian configuration is characterized by a rapid increase of the central density until a BH forms at $\sim 1$ ms after the begin of the simulation. The vertical dashed line in Fig. 5 denotes the horizon formation. For the maximal mass model we find small density oscillations introduced by the pressure perturbation (not visible at the density scale of Fig. 5). Whereas these oscillations are too small to cause a gravitational collapse, imposing a larger pressure perturbation would have led to BH formation also for the maximal mass configuration.

Collapse morphology. In the following we discuss the dynamics during the gravitational collapse of the marginally stable Keplerian model. For this purpose we show for different instants of time the density within the x-z-plane (corresponding to a slice with constant $\phi$ in the equilibrium case) in Fig. 6. The shown time snapshots are marked in Fig. 7, where we report the disk mass estimated as average of the two highest resolutions (the shaded region shows the difference between these simulations). The top panel of Fig. 6 shows the initial equilibrium configuration at $t = 0$ ms. The stellar shape is characterized by its oblate form due to the large intrinsic rotation.

At $t = 0.5$ ms (second panel of Fig. 6) the stellar surface is less sharp compared to the initial configuration. This is typically observed in all Numerical Relativity simulations of neutron star spacetimes using grid-based codes, see e.g. Guercilena et al. (2017) for further discussions. It is introduced by the fact that the numerical scheme is unable to resolve the sharp, step-like surface of the star. This effect becomes even more pronounced due to artificial shock heating at the stellar surface. While an increased resolution and less dissipative schemes for the numerical fluxes reduce the effect, there are currently no full 3D Numerical Relativity simulations of dynamical spacetimes which retain the exact shape of the surface. We suggest that due to this effect Numerical Relativity simulations are likely to overestimate the material surrounding the NS.

At $t = 1.0$ ms (third panel of Fig. 6) the star has further contracted and the central density has increased. Most notably some low density material leaves the star with high velocity along the z-axis. This matter becomes unbound and is ejected from the system. We mark material as unbound/ejected once the geodesic criterion is fulfilled, i.e., when the time-component of the four-velocity is $u_t < -1$ and when the radial component of the velocity is positive. The ejection of material is caused by shocks at the stellar surface. Since these shocks might be associated to the artificial atmosphere employed in the dynamical evolution, we assess the error of the numerical method by simulating configurations with different resolutions and atmosphere values, as well as flux limiting schemes, see Fig. 9 and the discussion below.

At $t = 2$ ms (forth panel of Fig. 6), the star has collapsed to a BH; we mark the apparent horizon with a black solid line. The density dropped several orders of magnitude and reaches new maximum values of $\rho \sim 10^8$ g/cm$^3$. At this time the bound mass (namely, the debris disk mass) has decreased to $M_{\text{disk}} \approx 10^{-12} M_\odot$.

At $t = 4$ ms (fifth panel of Fig. 6), the density decreases further to $\rho \sim 10^6$ g/cm$^3$ and finally at $t = 8$ ms (last panel) the density surrounding the central BH has dropped to $\rho \lesssim$...
Figure 6. Snapshots of the density profile in the x-z-plane at $t = 0.0, 0.5, 1.0, 2.0, 4.0, 8.0$ ms for the Case A. The snapshot times are marked in Fig. 7 with black diamonds. The black solid line marks the BH horizon.

Figure 7. Evolution of the bound (disk) mass for Case A. Black diamonds refer to the times shown in Fig. 6 and the vertical dashed purple line to the formation time of the apparent horizon. The shaded region shows the difference between the two higher resolution simulations.

$10^5$ g/cm$^3$. The final disk mass at this time has settled at about $\sim 10^{-7} M_\odot$.

4.2 Case B

For Case B we employ an EOS characterized by $\log(p_1) = 34.8$ and $\Gamma_2 = 2.0$. The marginally stable Keplerian configuration has a central density of $\rho_c = 1.238 \times 10^{15}$ g/cm$^3$ and an angular velocity of $\Omega = 8.511$ rad/ms. Since the collapse dynamics follows the same qualitative steps outlines for Case A in Fig. 6, we restrict our considerations to quantitative statements. The debris disk mass is slightly smaller than for Case A, as expected from our findings for the equilibrium configuration, but overall also of the order of $\sim 10^{-7} M_\odot$. Similarly to Case A we obtain an ejecta mass of the order of $10^{-3} M_\odot$; see discussion below for more details.

4.3 Case C

Case C employs an EOS determined by $\log(p_1) = 34.9$ and $\Gamma_2 = 3.3$. The marginally stable Keplerian configuration has a central density of $\rho_c = 9.634 \times 10^{14}$ g/cm$^3$ and an angular speed of $\Omega = 9.573$ rad/ms. We find for this setup that the mass of the disk (bound material) falls below $10^{-10} M_\odot$ about 2 ms after BH formation. The increase of bound mass after this time might be caused either by material which is initially marked as unbound and later falls back onto the remnant or simply by inaccuracies of the numerical scheme. However, overall Case C produces the smallest amount of bound material as expected from the equilibrium configuration analysis.
Figure 8. Evolution of the bound (disk) mass for Case B (left panel) and Case C (right panel). The vertical dashed, purple lines mark the formation time of an apparent horizon and the shaded region represents the differences between the two higher resolution simulations.

Figure 9. Evolution of the bound (top panel) and the unbound (lower panel) baryonic mass for different resolutions, atmosphere settings, and numerical flux limiters. The vertical dashed line refers to the apparent horizon formation for setup Res\textsuperscript{WENOZ} atm19. Note that the main reason for the decrease in the ejecta mass after about $\sim 5$ ms (see Fig. 7) is the material which leaves the numerical domain covered by our simulation.

4.4 Accessing the numerical uncertainty

Fig. 9 gives an overview of the bound (disk) mass and the unbound (ejecta) mass for all simulations of Case A.

Artificial atmosphere. Let us start by considering the imprint of the artificial atmosphere, cf. green lines in top and bottom panels. Although the artificial atmosphere threshold has been varied by a factor of 100, we find that the disk and ejecta masses are almost unchanged. Therefore, although the artificial atmosphere introduces errors, the previous conclusions remain valid.

Resolution. We continue the discussion by focusing on the simulations with different resolutions. We have varied the resolution by a factor of three, which generally is a very large range for full Numerical Relativity simulations where computational costs scale with the forth power of the number of grid points. We do find that the results are not monotonically converging with increasing resolution. This behavior is unfortunately often seen in full 3D Numerical Relativity simulations estimating disk and ejecta masses, see e.g. Hotokezaka et al. (2013); Dietrich & Ujevic (2017); Fujibayashi et al. (2017). However, although precise statement about the bound/unbound mass can not be made, the fact that the mass estimates change only about one order of magnitude for the large range of resolutions employed leads to the conclusion that the order of magnitude estimates necessary for our study are indeed valid.

Numerical flux limiter. We also discuss the imprint of the flux limiter used in the GRHD scheme. For this purpose we employ 3 different flux reconstruction schemes: LINTVD (Shu & Osher 1989), CENO3 (Liu & Osher 1998; Del Zanna et al. 2003), and WENOZ (Borges et al. 2008; Bernuzzi et al. 2012). As expected we find that less sophisticated, lower order schemes as LINTVD and CENO3 predict smaller bound and unbound masses. In particular the ejecta mass drops to zero for these two schemes. This analysis shows that high order flux limiters as WENOZ seem to be required for a proper modeling of the system. Although we can not exclude that with even more improved HRSC methods larger disk masses might be observed, we do expect that the results are robust and allow order of magnitude estimates. This statement is based on investigations of binary systems that show that the WENOZ reconstruction scheme is among the state-of-the-art methods and allows accurate and reliable simulations of neutron star spacetimes, see e.g. Bernuzzi et al. (2012); Bernuzzi & Dietrich (2016).

Thermal effects. Finally, we consider the imprint of the thermal effects added through Eq. (8). For this purpose we compare the setups Res\textsuperscript{WENOZ} atm19 and Res\textsuperscript{WENOZ} atm19cold. We find

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6 It is possible that material which is first marked as unbound falls back onto the remnant, since the geodesic criterion used to characterize fluid elements assumes that fluid elements follow a geodesic motion, which is only approximately correct for a dynamical spacetimes as the one considered in this article.
that while the disk mass is compatible with simulations including thermal effects, the ejecta mass is reduced. This supports our suggestion that most of the ejecta is caused by shock heating. Consequently, although we found that the ejecta mass is not affected by resolution and is robustly around $10^{-4} M_\odot$, we can not rule out that the shock heating is artificially caused by the numerical scheme and not caused by a physical mechanism.

5 SUMMARY

We have performed a detailed analysis of the conditions under which a supramassive neutron star can collapse to a Kerr black hole surrounded by an accretion disk. Our approach has been two-fold: we first analyzed the angular momentum spectrum of the collapsing configurations and subsequently performed dynamical 3D collapse simulations to confirm our findings. We constructed rigidly rotating initial neutron stars using the XNS code (Bucciantini & Del Zanna 2011; Pili et al. 2014). These initial configurations were analyzed for the mass that has enough angular momentum to remain outside of the ISCO of the forming black hole. A similar study has recently been performed by Margalit et al. (2015) who used the RNS code (Stergioulas & Friedman 1995). We argue here that, contrary to what has been done in their work, the configuration that can collapse to a BH is not the maximal mass configuration, but instead the marginally stable Keplerian configuration, for which $\partial M/\partial \rho_c |_J = 0$. Moreover, we find that if a disk can form for a larger volume of the parameter space, albeit its mass is very small. Despite these small differences we confirm their main result that it is very difficult to form a massive disk from a collapsing neutron star and all the cases that were investigated fall short by orders of magnitude to produce an energetic sGRB. These conclusions were subsequently confirmed by fully dynamical Numerical Relativity simulations performed with the BAM code (Brügmann et al. 2008; Thierfelder et al. 2011b).

In this work, we have assumed uniform rotation and a cold EOS for the initial configurations of the collapsing stars. The uniform rotation is justified for the sGRB models that motivate this study. If the supramassive NS is formed by accretion from a non-degenerate companion star (MacFadyen et al. 2005), there is no reason to expect differential rotation. At the moment of collapse, however, the NS — while being essentially cold throughout the bulk of the high-density matter — may be engulfed by a high-temperature envelope, which is not modelled in this work.

If, in contrast, the supramassive neutron star is formed as a result of a neutron star merger, as invoked by “time reversal models” (Ciolfi & Siegel 2015; Rezzolla & Kumar 2015), it is expected to be both hot ($\gtrsim 10$ MeV) and differentially rotating, at least initially. Such differentially rotating, “hypermassive” neutron stars can support a substantially larger mass than rigidly rotating ones, but magnetic braking and viscosity will drive the stars to collapse on a short time scale even if the initial seed magnetic field is low and viscosity is small (Shapiro 2000). Therefore, neutron stars that remain stable for long enough to explain the long term X-ray emission ($\sim 10^8$ s), have likely dissipated their differential rotation and have cooled to temperatures where thermal effects in the high-density matter are small, since the Kelvin-Helmholtz neutrino cooling time is of the order of only seconds (Radice et al. 2018). Therefore, we consider also in this case our assumption of essentially cold EOS and rigid rotation as valid.

For the equations of state expected in neutron stars ($\Gamma > 2$), the resulting disk masses after the collapse are orders of magnitude lower ($\lesssim 10^{-7} M_\odot$) than what is needed for a typical sGRB. Therefore, we interpret this result as disfavoring those sGRB models that require the collapse of a supramassive NS into a BH plus disk configuration.

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Figure A1. Comparison between XNS (black lines) and RNS (red crosses) of the rest mass density radial profiles along the equatorial and polar axes.

Table A1. Comparison in the model parameters in output from RNS and XNS. The columns contain: the stellar quantity, the RNS results, the XNS results, and the relative difference. The stellar quantities read from top to bottom: the gravitational mass $M$, the total angular momentum $J$, the angular speed $\Omega$, the Keplerian angular speed $\Omega_K$, the circumferential radius $R$, and the specific angular momentum at the equator $j$.

| quantity | RNS  | XNS  | difference |
|----------|------|------|------------|
| $M$      | 2.6394 | 2.6382 | 0.05%      |
| $J$      | 4.6745 | 4.6373 | 0.8%       |
| $\Omega$| 0.040811 | 0.041000 | 0.5%       |
| $\Omega_K$| 0.046103 | 0.046194 | 0.2%       |
| $R$      | 10.668 | 10.596 | 0.7%       |
| $j$      | 6.066  | 6.018  | 0.8%       |

For this purpose, we adopt the same piecewise polytrope EOS of Case B, namely $p_1 = 10^{14.8} \text{ dyne/cm}^3$ and $\Gamma_2 = 2.0$, and choose a model very close to the marginally stable Keplerian configuration. This is a quite demanding test, since the EOS is not a simple 1-piece polytrope and the model is very close to mass shedding.

The settings of the RNS benchmark are:

- radial grid points $SDIV = 3601$,
- angular grid points $MDIV = 1201$,
- polynomial expansion $LMAX = 36$,
- relative accuracy $= 10^{-11}$,
- EOS recovered from interpolation of a 2000 point table.

The setting of the XNS configuration, which we will call “baseline” model, are the following:

- radial grid points $NR = 2000$,
- angular grid points $NTH = 51$,
- harmonic expansion $MLS = 30$,
- absolute convergence $= 10^{-9}$ (cf. condition (iii) in Sec. 2.2),
- radius of the inner grid (which encompass half of the radial points) = 8,
- radius of the outer grid = 200.

The input parameters are the central density $\rho_c = 2.004 \times 10^{-3}$, and the angular speed $\Omega = 4.1 \times 10^{-2}$ for XNS and the aspect ratio $a = 0.61764727$ for RNS. We remark that the aspect ratio in output from the baseline XNS model is an input for the benchmark RNS model, and should therefore considered as “exact” for both codes.

In Tab. A1 we compare the output of the two codes and in Fig. A1 we compare the equatorial and polar density profiles. The range of the relative differences in the model quantities between XNS and RNS is $\sim 0.1$–0.8%, except for the gravitational mass that is recovered within 0.05%. We remark that the extended conformal flatness approximation, on which XNS is based, neglects differences between the metric functions $A$ and $B$ of the order of $\sim 0.1$. Since the specific angular momentum at the ISCO of the equivalent Kerr black hole is $j_{\text{ISCO}} \approx 7.0$ both for RNS and XNS, no debris disk is expected. This should not surprise since the configuration is close but not equal to the “Case B” described in the paper, for which we expect disk formation instead.

In Fig. A2 we plot the relative differences of the Keplerian angular velocity between the RNS benchmark and those obtained from the XNS model with the same settings of the XNS baseline model, apart for the setting that is varied in each plot. XNS shows a good convergence for each of the setting varied.

In conclusion, the two codes are in very good agreement.

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Figure A2. Convergence of the Keplerian angular velocity $\Omega_K$ obtained with XNS. We plot the relative error in percentage between the RNS benchmark and the XNS result, varying one XNS setting every time with respect to the baseline XNS model. From left to right and from top to bottom, we vary respectively the dimension of the radial grid NR, the dimension of the angular grid NTH, the number of components in the spherical harmonics expansion MLS, and the absolute tolerance criterion for convergence. The dashed, purple, vertical lines mark the settings of the XNS baseline model.