STELLAR ROTATION EFFECTS IN POLARIMETRIC MICROLENSING

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ABSTRACT

It is well known that the polarization signal in microlensing events of hot stars is larger than that of main-sequence stars. Most hot stars rotate rapidly around their stellar axes. The stellar rotation creates ellipticity and gravity-darkening effects that break the spherical symmetry of the source’s shape and the circular symmetry of the source’s surface brightness respectively. Hence, it causes a net polarization signal for the source star. This polarization signal should be considered in polarimetric microlensing of fast rotating stars. For moderately rotating stars, lensing can magnify or even characterize small polarization signals due to the stellar rotation through polarimetric observations. The gravity-darkening effect due to a rotating source star creates asymmetric perturbations in polarimetric and photometric microlensing curves whose maximum occurs when the lens trajectory crosses the projected position of the rotation pole on the sky plane. The stellar ellipticity creates a time shift (i) in the position of the second peak of the polarimetric curves in transit microlensing events and (ii) in the peak position of the polarimetric curves with respect to the photometric peak position in bypass microlensing events. By measuring this time shift via polarimetric observations of microlensing events, we can evaluate the ellipticity of the projected source surface on the sky plane. Given the characterizations of the FOcal Reducer and low dispersion Spectrograph (FORS2) polarimeter at the Very Large Telescope, the probability of observing this time shift is very small. The more accurate polarimeters of the next generation may well measure these time shifts and evaluate the ellipticity of microlensing source stars.

Key words: gravitational lensing: micro – methods: numerical – polarization – stars: luminosity function, mass function – stars: rotation – techniques: polarimetric

1. INTRODUCTION

Stellar rotation refers to the angular motion of a star around its axis. If there is no stellar rotation, the gravitational force condenses celestial bodies into perfect spheres. But if an object rotates around its axis, some portion of the gravitational attraction provides the centrifugal acceleration whose value depends on the stellar latitude, decreasing as the latitude increases from the stellar equator to the stellar pole. Therefore, this rotation creates stellar oblateness (Collins & Harrington 1966; Lebovitz 1967). For rapidly rotating stars the polar surface brightness is greater than the equatorial one, the so-called gravity-darkening effect. This effect results from von Zeipel’s (1924) theorem, i.e., the radiative flux is proportional to the local effective gravity.

For main-sequence stars the stellar angular velocity $\Omega$ decreases with the stellar age $t$: $\Omega \propto t^{-1/2}$, which is Skumanich’s relationship (Skumanich 1972; Durney & Latour 1978). According to this relation, the star age’s can be derived from the rotational rate (Barnes 2007). Another result of this relation is that the rotational velocities of pre-main-sequence stars are higher than those of main-sequence stars. Stars in spectral classes from F5 up to O5 often rotate fast, with a mean rotational velocity of the order of 100–200 km s$^{-1}$ (Peterson et al. 2004; McAlister et al. 2005). Also, their rotational velocity increases with mass and is maximal for massive B-class stars. Less massive stars have much lower rotational speeds, about a few kilometers per second after a few hundred million years (Kraft 1970, p. L385; Gallet & Bouvier 2013), since magnetized stellar winds over the surfaces of these stars transport the angular momentum (Schatzman 1962). However, Irwin et al. (2011) measured the rotational periods of stars with masses less than 0.35$M_\odot$ and found some exceptionally fast and slow rotators. Also, brown dwarfs have an average period of the order of 15 hr at young ages (Scholz & Eislöffel 2004, 2005).

The stellar rotation is determined by several methods. The projected radial velocity of rotating stars can be measured by spectroscopic observations of the Doppler broadening in the absorption lines of stars (Abney 1877). However, this method is not convenient for slowly rotating stars with projected radial velocities of less than 20 km s$^{-1}$ (Bouvier 2013). For nearby fast rotating stars, the stellar oblateness projected on the sky plane can be measured by interferometry (e.g., Kervella et al. 2004). Combining the spectroscopic and interferometric methods helps us to characterize the inclination angle of the rotational axis with respect to the sky plane (e.g., Le Bouquin et al. 2009). The rotational periods of nearby spotted stars can also be determined via photometric observations of their light curves (e.g., Affer et al. 2012; McQuillan et al. 2013). In that case, the stellar spots on rotating stars disturb the stars’ light curves periodically. This method was first used to measure the rotational period of the Sun by Galileo Galilei (Casas 2006). Generally, the methods mentioned here can identify the rotational properties of nearby or bright (and massive) stars.

Gravitational microlensing is a useful tool to highlight the rotational properties of distant stars. One channel is photometric observations of these events. In that case, the ellipticity or the radial velocity of source stars can perturb microlensing light curves. Some aspects of this subject have been studied by a number of authors. Heyrovský & Loeb (1997) introduced an efficient method for calculating the microlensing light curve of an elliptical source created by a point-mass lens and studied some properties of such light curves. The projected radial velocity of source stars or even the Einstein angular radius can be measured by spectroscopic observations of high-magnification microlensing events (Maoz & Gould 1994; Gould 1997).
Also, Gaudi & Haiman (2004) studied microlensing of elliptical sources by fold caustics and concluded that the deviation of ellipticity in microlensing light curves is qualitatively similar to that due to the limb-darkening effect. The gravity-darkening effect resulting from stellar ellipticity was not considered in these references. Recently, Giordano et al. (2015) investigated the effect of rotating spotted stars on photometric microlensing curves.

In this work, we study the possibility of detecting and characterizing the rotational properties of distant source stars by performing polarimetric observations of high-magnification microlensing events. Rotating stars are oblate objects and their projected surface brightness will not be symmetric owing to the projection process unless their rotation axis is oriented toward the observer. Consequently, a rotating star has a net polarization signal whose value is a function of the stellar angular velocity and the inclination angle of its rotation axis. Lensing can magnify these small polarization signals and cause them to be detected or even characterized through polarimetric observations.

In Section 2, we explain the formalism used for calculating the polarization signal of an elliptical source star. The properties of polarimetric microlensing events of elliptical sources will be studied in the next section. In Section 4, we first study the statistics of fast rotating stars using Kepler data. Then, we do a Monte Carlo simulation of high-magnification microlensing events of rotating stars toward the Galactic spiral arms to evaluate the efficiency of detecting the rotation-induced (polarimetric and photometric) perturbations of source stars. Finally, we conclude in the last section.

2. THE POLARIZATION SIGNAL OF AN ELLIPTICAL SOURCE

The intrinsic polarization signal of rapidly rotating early-type stars was first studied by Harrington & Collins (1968). Also, the scattering polarization generated by nonradial pulsating stars was calculated by Stamford & Watson (1980). Bjorkman & Bjorkman (1994) have analytically calculated the polarization signal due to a fast rotating star surrounded by an axisymmetric disk. Al-Malki et al. (1999) and Ignace et al. (2009) estimated the Stokes parameters due to anisotropic light sources with spherical envelopes and envelopes of arbitrary shapes by ignoring the finite size effect of the source star. Recently, several codes were written to solve the coupled problem of the non-local thermodynamic equilibrium (NLTE) and radiative equilibrium for arbitrary three-dimensional envelope geometries, using the Monte Carlo method (e.g., Cacciari & Bjorkman 2006; Cacciari et al. 2006; Whitney et al. 2013).

We use the formalism developed by Harrington & Collins (1968) to calculate the intrinsic polarization signals of rotating stars. Accordingly, we assume the following. (i) The stellar envelope is optically thin enough to use the single-scattering approximation. (ii) The rotational angular velocity is constant over the stellar surface and there is no effect of differential rotation (see, e.g., Kitchatinov 2005). (iii) The stellar angular momentum is not transported from the stellar surface during microlensing events. (iv) The star is rotating as a rigid body so that von Zeipel’s theorem is applicable. (v) The magnetic field of the source star is negligible. (vi) There is no disk around the source star. (vii) The rotational angular velocity normalized to the break-up velocity of the source star is small enough that the local polarization over the source surface can be approximated with the local polarization over a spherical surface. Note that fast rotating stars, e.g., B-class stars, are intrinsically variable and are not suitable candidates to study for microlensing observations.

We consider some parameters to describe an elliptical source: (i) the stellar equatorial and polar radii \( R_{eq} \) and \( R_p \); (ii) the inclination angle of the stellar rotation axis with respect to the sky plane \( i \); (iii) two parameters to represent the limb-darkening, coefficients \( c_1 \) and \( c_2 \).

To describe an elliptical source in the sky plane we need two coordinate systems: (i) the observer coordinate frame \((x_o, y_o, z_o)\) so that the projected source center is at its origin, the observer is on the \(z_o\)-axis at \(+\infty\), and the \(z_o\)-\(y_o\) plane contains the stellar rotation axis (i.e., its \(x_o\)-axis is parallel with the semimajor axis of the projected elliptical source); (ii) the stellar coordinate system \((x_s, y_s, z_s)\) so that the \(y_s\)-axis is along the stellar rotation axis and the \(x_s\)-axis is along the observer’s \(x_o\)-axis. We transform the second coordinate system to the first one by a rotation around the \(x_s\)-axis by the inclination angle \(-i^\circ\), so that

\[
\begin{align*}
x_o &= x_s, \\
y_o &= y_s \cos(i) - z_s \sin(i), \\
z_o &= y_s \sin(i) + z_s \cos(i).
\end{align*}
\]

We use \((R_s, \theta_s, \phi_s)\) to represent points over the stellar surface in the spherical stellar coordinate, i.e.,

\[
\begin{align*}
x_s &= R_{eq} \sin(\theta_s) \sin(\phi_s), \\
y_s &= R_p \cos(\theta_s), \\
z_s &= R_{eq} \sin(\theta_s) \cos(\phi_s).
\end{align*}
\]

Note that \(R_{eq}\) can be determined according to the stellar angular velocity \(\Omega\). Generally, \(R_s = \sqrt{x_s^2 + y_s^2 + z_s^2}\) depends on \(\theta_s\) owing to the stellar oblateness (Collins & Harrington 1966):

\[
R_s(\omega, \theta_s) = \frac{3R_p}{w \sin \theta_s} \cos \left[ \pi + \cos^{-1}(\omega \sin \theta_s) \right] \left( \frac{3}{2} \right)
\]

where \(\omega = \Omega/\Omega_{crit}\) is the ratio of the star’s angular velocity to the critical or break-up velocity \(\Omega_{crit}\) at which the centrifugal force at the stellar equator becomes equal to the gravitational attraction, given by \(\Omega_{crit}^2 = (2/3)G M_s/R_s^3\) where \(M_s\) is the stellar mass and \(\Omega_{crit}\) is measured in units of radian per second.

In our formalism, the projected position of the stellar rotation pole in the observer coordinate system is \(d_2 = (0, R_p \cos(i), 0)\). The emergent radiative flux of the source star is maximized at this point. The projection of the source star on the sky plane is an ellipse whose semimajor and semiminor axes are \(a = R_{eq}\) and \(b = \sqrt{R_{eq}^2 \sin^2(i) + R_p^2 \cos^2(i)}\) respectively. Also the \(x_o\)- and \(y_o\)-axes are toward its semimajor and semiminor axes. We use \((\rho, \phi)\) to represent points over the stellar surface projected on the sky plane in the polar observer coordinate where \(\phi \in [-\pi, \pi]\) and \(\rho \in [0, \rho_m]\). \(\rho_m = b/\sqrt{b^2 \cos \phi (i) + (a \sin \phi)^2}\) is normalized to \(a\) and in the range of \(\rho_m \in [0, a/1]\).

To calculate the polarization signal of an elliptical star, we use the Stokes intensities. There are four Stokes intensities, \(I_p, I_Q, I_U,\) and \(I_V\), which represent the total intensity, two components of linear polarized intensities, and the circular polarized intensity over the source surface, respectively.

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Note: The text contains a series of mathematical equations and expressions that are not fully rendered in the image. The equations are related to astrophysical phenomena, such as microlensing and stellar properties, and involve variables like \(R_{eq}, R_p, \Omega, \Omega_{crit},\) and \(\omega\), as well as operations like trigonometric functions and coordinate transformations. The equations are used to describe the behavior of rotating stars and their effects on microlensing events in the context of the stellar rotation and the distribution of polarization signals over the source surface.
(Tinbergen 1996). Taking into account that there is only linear polarization of light scattered on the stellar atmosphere, we set \( I_V = 0 \). The other intensities are estimated according to the limb-darkening effect, producing a local polarization owing to the light scattering in the stellar atmosphere. This local polarization depends on the species of scatterer and the nature of the source star (Ingrosso et al. 2012, 2015). For example, in late-type main-sequence stars, the polarization signal is generated mainly by Rayleigh scattering on neutral hydrogen atoms and partially due to Thompson scattering by free electrons (Fluri & Stenflo 1999). In cool giant stars Rayleigh scattering on atomic and molecular species or on dust grains generates the polarization signal. In that case, Simmons et al. (2002) introduced the appropriate Stokes parameters for giant stars with spherical circumstellar envelopes lensed by a single lens. In hot early-type stars with a free electron atmosphere, mostly Thompson scattering produces the polarization signal.

The magnitude of Stokes (total and polarized) intensities over the surface of these stars can be written in the form (Schneider & Wagoner 1987)

\[
I_l(\rho, \phi) = I_{l,0}(\rho, \phi) [1 - c_1 (1 - \mu)],
I_p(\rho, \phi) = I_{p,0}(\rho, \phi) [c_2 (1 - \mu)],
\]

where \( I_p = \sqrt{I_Q^2 + I_U^2} \), \( c_1 = 0.64 \), \( c_2 = 0.32, \mu = \sqrt{1 - \rho^2/\rho_m^2} \), and \( I_{l,0}(\rho, \phi) \) is the emergent radiative flux of the source star in the observer coordinate system. Note that Chandrasekhar (1960) considered a spherically symmetric, isotropically scattering atmosphere to calculate the Stokes intensities. Indeed, we assume that the effect of the stellar rotation is small enough to adopt for rotating stars the local Stokes intensities in Equation (4). We calculate the radiative flux \( I_{l,0} \) in the observer coordinate system starting from the flux in the stellar coordinate system \( I_{l,0} \) and using the coordinate transformations in Equation (1). The gravity-darkening effect, which is a result of the stellar ellipticity, causes the intrinsic radiative flux of the source star \( I_{l,0} \) to change over the source surface. According to von Zeipel’s (1924) theorem, we can express the stellar radiative flux as a function of the effective gravity \( g_{eff} \) over the surface of a uniformly rotating star (in the stellar coordinate), and it is given by

\[
I_{l,0}(\Omega, \theta_s) = -\frac{L_s(\beta)}{4\pi GM_s(\beta)} |g_{eff}(\Omega, \theta_s)|,
\]

where the stellar luminosity \( L_s \) and the stellar mass \( M_s \) are evaluated on a surface of a constant pressure \( \beta \). If there is no angular momentum transport on the star’s surface, the vector of the effective gravity in the Roche model and in hydrostatic equilibrium is given by (see, e.g., Maeder & Meynet 2011)

\[
geff(\Omega, \theta_s) = \left[ -\frac{GM_s}{R_s^2(\theta_s)} + \Omega^2 R_s(\theta_s) \sin^2 \theta_s \right] e_r + \left[ \Omega^2 R_s(\theta_s) \sin \theta_s \cos \theta_s \right] e_\theta,
\]

where \( e_r \) and \( e_\theta \) are the unit vectors in the radial and latitudinal directions. Accordingly, \( I_{l,0} \) for an elliptical source is proportional to \( |g_{eff}(\Omega, \theta_s)| \) and a function of \((\Omega, \theta_s)\).

Also we assume that the stellar atmosphere has an elliptical shape due to the stellar rotation, so that polarization vectors are tangential to concentric ellipses whose centers coincide with the stellar center. The normal vector to the elliptical source surface at each point \((x_o, y_o)\) from the source center is given by \( \mathbf{n} = (y_o/b^2, -x_o/a^2) \).

Now by integrating these Stokes intensities over the source surface, the corresponding Stokes parameters \( S_I, S_Q, \) and \( S_U \) are obtained. If the source light is magnified by a microlens, we should add a weight function for each source surface element, i.e., the magnification factor \( A \). In that case, the Stokes parameters are given by

\[
S_l = \rho^2 \int_0^{\pi} d\phi \int_0^{\phi+\omega_0} \rho \ d\rho I_l(\rho, \phi) A(u),
S_Q = \rho^2 \int_0^{\pi} d\phi \int_0^{\phi+\omega_0} \rho \ d\rho I_p(\rho, \phi) A(u) \left( -\cos 2\varphi \right) \sin 2\varphi,
\]

where \( \rho_s = R_\text{eq} x_{\text{rel}}/R_\odot \) is the stellar equatorial radius projected on the lens plane and normalized to the Einstein radius \( R_\odot \), \( x_{\text{rel}} = D_{\text{Ly}}/D_s \) is the ratio of the lens distance to the source distance from the observer’s position, \( u = |\mathbf{u}_\text{cm} - \rho_{p_s}| \) is the distance of each projected element over the source surface with respect to the lens position, \( \mathbf{u}_\text{cm} \) is the vector of the lens position from the source center, and \( \varphi \) is the angle of the normal vector to the concentric ellipse passing the point \((\rho, \phi)\) (i.e., \( \mathbf{n} \)) with respect to the \( x_o \)-axis. In Equation (7), we have aligned the signs of two components of polarized Stokes intensities so that polarization vectors become tangential to the concentric ellipses.

Finally, the degree of polarization \( P \) and the angle of polarization \( \theta_p \) as functions of total Stokes parameters are (Chandrasekhar 1960)

\[
P = \frac{S_Q^2 + S_U^2}{S_I},
\theta_p = \frac{1}{2} \tan^{-1} \frac{S_U}{S_Q}.
\]

If there is no lensing effect, the elliptical source has a net polarization signal whose magnitude depends on the inclination angle and its rotational speed. In Figure 1 we plot the net polarization signal of an elliptical source versus the inclination angle for different values of \( \omega \); i.e., the ratio of the stellar angular velocity to the critical velocity. The used parameters are \( M_s = 5 M_\odot \) and \( R_\odot = 3.6 R_\odot \) (Chandrasekhar 1960).
of Harrington & Collins (1968), representing the polarization signal of rotating stars. The inclination angle of the rotational axis in our formalism is the complementary angle to the one introduced by Harrington & Collins. If \( i = 90^\circ \), the projected shape of the source star on the sky plane is a perfect circle with radius equal to \( R_{\text{eq}} \). If \( i = 0^\circ \), the projected shape will be an ellipse whose semimajor and semiminor axes are \( R_{\text{eq}} \) and \( R_p \). Therefore, by decreasing the inclination angle the ellipticity of the source surface projected on the sky plane increases. On the other hand, faster rotating stars are more oblate. When \( \omega = 0 \) the source star is a perfect sphere without any gravity-darkening effect. In that case, the inclination angle does not alter the net polarization signal. Note that the stellar polar radius is determined using the mass–radius relation while its equatorial radius depends on \( \omega \) (see Equation (3)). Also, the polarization angle of an elliptical source is zero with respect to its semimajor axis and it does not depend on the inclination angle or the stellar angular velocity.

According to Figure 1, the intrinsic polarization signal produced by an elliptical source is generally less than 0.2\%, and in the case of very fast rotating stars (\( \omega > 0.6 \)) it is comparable to the polarization signal in the transit microlensing events, for which \( P \approx 0.6\%–0.7\% \). Polarization signals of this amplitude can be measured directly. Therefore, the effects of stellar rotation should be considered in the polarimetric microlensing calculations of fast rotating stars, allowing the elliptical properties of the source stars to be characterized. On the other hand, the intrinsic polarization signals for slowly or moderately rotating stars with \( \omega < 0.6 \), which are too small to be measured by themselves, may be magnified during a microlensing event. In the next section, we add the lensing effect and study the polarimetric microlensing of elliptical sources. Our aim is to investigate whether these signals can be detected in polarimetric and photometric observations of high-magnification microlensing events.

### 3. Characteristics of Polarimetric Microlensing of Elliptical Sources

Stellar rotation causes (i) the ellipticity effect and (ii) the gravity-darkening effect, both of which make perturbations in the light and polarimetric curves of microlensing events. Here, we aim to study these perturbations, which are classified in the following subsections.

#### 3.1. Time Shift in the Location of the Polarimetric Peak

In the case of spherically symmetric (non-rotating) source stars, photometric and polarimetric light curves in single-lens microlensing events have symmetric shapes with respect to the time \( t_0 \) of the closest approach. In the case of rotating source stars, the ellipticity effect breaks the symmetry of these curves around \( t_0 \).

In particular, (i) the location of the peak(s) in the transit and bypass polarimetric microlensing curves changes from the points \( u \approx 0.96r_\alpha \) and \( u = u_0 \) respectively, and (ii) the location of the peak in light curves shifts from the time of closest approach. However, we cannot detect the shift in the position of the photometric peak from its true value (i.e., the time of closest approach) by making only photometric observations of microlensing events. By performing polarimetric observations of transit microlensing events, we can discern the time shift in the position of the second peak with respect to the symmetric (with regard to \( t_0 \)) position of the first peak, since the symmetry in positions of polarimetric peaks breaks due to the stellar ellipticity. Also, in transit microlensing events the ellipticity can shift the relative location of the minimum between two polarimetric peaks with respect to the photometric peak position. In bypass microlensing events, the time shift between the position of the polarimetric peak and the position of the photometric peak helps to distinguish the ellipticity effect. However, in bypass polarimetric microlensing events with \( u_0 \) much larger than \( \rho_e \), the rotation-induced perturbations becomes too small, because of the averaging process.

As we shall see in Section 3.2, gravity-darkening breaks the symmetry of the source surface brightness and affects the peak position of polarimetric and light curves, unless \( i = 90^\circ \). However, if the inclination angle is large enough that the projected position of the stellar pole on the sky plane (i.e., \( d_R \)) is not at the stellar limb, we can ignore the gravity-darkening effect when calculating the time shift in peak positions of polarimetric and light curves. In that case, by considering only the ellipticity effect we assess the time shift in the position of the second polarimetric peak. In transit microlensing events, the lens crosses the source edges at two moments, which are given by

\[
\begin{align*}
\&t_{1,2}[\tilde{t}_0] \\
&= -u_0 \sin(2\xi)(\alpha^2 - 1) \pm 2\alpha \sqrt{b^2 \cos^2 \xi + a^2 \sin^2 \xi - u_0^2} \\
&\quad \frac{2(\cos^2 \xi + \alpha^2 \sin^2 \xi)}{2},
\end{align*}
\]

(9)

where \( t_{1,2}[\tilde{t}_0] \) are in units of the Einstein crossing time \( T_E \). \( \xi \) is the angle between the lens trajectory and the semimajor axis of the source, and \( \alpha = a/b \) is the ratio of the semimajor axis to semiminor axis of the source. Hence, the second peak in polarimetric curves with respect to the symmetric position of the first peak shifts due to the ellipticity by

\[
\delta_t = \frac{u_0[\rho_e] \sin(2\xi)(\alpha^2 - 1)}{(\cos^2 \xi + \alpha^2 \sin^2 \xi)} - \tilde{t}_0,
\]

(10)

where \( u_0[\rho_e] \) is the lens impact parameter in units of \( \rho_e \) and \( \tilde{t}_0 = \rho_e/T_E \) is the timescale for crossing the source radius. The
time shift $\delta t$ is a maximum when $\cos \xi = \alpha/\sqrt{\alpha^2 + 1}$. In Figure 2, we plot the contour lines of $\delta t$ in the plane containing $u_0$ and $\alpha$, for values of the lens trajectory angle $\xi$ that offer the maximum values of $\delta t$. This time shift is less than $t_*$ by one or two orders of magnitude.

To estimate the magnitude of $t_*$ and $\delta t$, we consider early-type stars as the microlensing sources that rotate fastest. These stars have an effective temperature in the range $T_{\text{eff}} \in [7500 : 30000]$ K and stellar polar radius in the range $R_* \in [1.4 : 6.6] R_\odot$. It is well known that the abundance of early-type stars in the Galactic spiral arms is greater than that in the Galactic bulge. Therefore, we consider hypothetical microlensing events toward the Galactic spiral arm and in the direction of $l = 300^\circ$ and $b = -1^\circ$, i.e., the Carina–Sagittarius arm. This arm mostly contains young stellar objects (e.g., Churchwell et al. 2009). By using a Monte Carlo simulation we estimate the mean lens mass $M_l = 0.3 M_\odot$ from the Kroupa mass function (Kroupa et al. 1993; Kroupa 2001) and the average distances $D_l = 4.0$ kpc and $D_s = 8.5$ kpc of lenses and sources from the observer by using the angular distribution of stars in the Galactic bulge and disk. In this way the Einstein radius is found to be $R_E = 2.3$ AU. Moreover, by adopting the synthetic Besançon model (Robin et al. 2003) we estimate the radius of the source star $\rho_s$, which is in the range $[0.001 : 0.006]$ (in units of $R_\odot$). The average Einstein crossing time toward this Galactic spiral arm is longer than that toward the Galactic bulge and is about $t_{E} \simeq 97$ days (Rahal et al. 2009). Consequently, for microlensing events toward this Galactic spiral arm the value of $t_*$ will be in the range $t_* \in [2.3 : 14.0]$ hr. Considering a common value for the time shift $\delta t \sim 0.1 t_{E}$, it will be in the range $\delta t \in [14.0 : 83.8]$ minutes.

To evaluate how many polarimetric data points can potentially be taken during this time shift $\delta t$, we assume that these observations are made by the FOCal Reducer and low dispersion Spectrograph (FORS2) polarimeter at the Very Large Telescope (VLT). The necessary exposure time for FORS2 to achieve a polarimetric accuracy of 0.1% for a magnified star with an apparent magnitude of about $m_l = 14.5$ mag is about $8$ s. In addition to the exposure time, there are two extra wasted times due to the rotation of the retarder waveplate and CCD readout. Indeed, to accurately determine the polarization signal, the source flux should be measured in 16 directions from $0^\circ$ to $337^\circ$, in steps of $22.5^\circ$. The signal-to-noise ratio (S/N) of the accumulated flux from the total exposure time in all retarder waveplate positions should reach 1000 to yield a polarimetric accuracy of 0.1% (Ejeta et al. 2012). Rotating the retarder waveplate of FORS2 takes some time, about $\sim 1$ minute. On the other hand, the FORS2 CCD will be saturated after an exposure time of $2$ s by a bright star with $m_l = 14.5$ mag. Therefore, the CCD detector should be read four times for each polarimetric data point taken with polarimetric accuracy 0.1%. The FORS2 CCD readout takes 30 s. Accordingly, we should add to the exposure time about 18 minutes as overhead time to account for the rotation of the retarder waveplate and CCD readout. This overhead time does not depend on the magnification factor or the source brightness and is constant for each data point taken by FORS2 with the highest polarimetric accuracy. Thus, the total observational time for each polarimetric data point of a magnified source star with $m_l = 14.5$ mag is about $T_{\text{obs}} = 18.13$ minutes.

Consequently, if FORS2 uninterrupted observers a transit microlensing event of an early-type star, it will on average take 1–5 data points during this time shift, where we estimate the number of possible data points from the factor $\delta t/T_{\text{obs}}$. This number is not sufficient to correctly detect $\delta t$. However, the factor $\delta t/T_{\text{obs}}$ increases in some specific microlensing events, e.g., when (i) the Einstein crossing time is very long, (ii) the lens crosses the source surface with a large impact parameter (see Figure 2), and (iii) the projected radius of the source star normalized to the Einstein radius is large, which happens, e.g., when the lens and source stars are very close to each other, i.e., $x_{\text{eq}} \sim 1$. Otherwise, the possibility of discerning this time shift is almost beyond the present technology (FORS2 polarimeter at VLT). However, this time shift can likely be detected by high-quality instruments of the next generation.

If the time shift $\delta t$ is measured from the polarimetric observations of microlensing events, we can estimate the parameter $\alpha = R_{\text{eq}} \sqrt{\left(\frac{R_{\text{eq}}^2 \sin^2(i) + R_s^2 \cos^2(i)}{2}\right)}$, which is a degenerate function of the inclination angle and the intrinsic ellipticity of the source star, assuming that the lens impact parameter and $\xi$ are carefully measured from photometric observations. The parameter $\alpha$ shows the ellipticity of the source surface projected on the sky plane. Note that the uncertainties in the parameters $u_0$, $\xi$, and $\delta t$ cause an uncertainty in the parameter $\alpha$. The uncertainty in $\delta t$ is due to time intervals between consecutive polarimetric data points hypothetically taken during this time shift and their uncertainties.

In Figure 3(a) we plot the photometric and polarimetric light curves of a microlensing event of an elliptical source for three different values of the lens impact parameter. In this figure, the light and polarimetric curves are shown in the left and right panels respectively. The projected surface of the source star on the lens plane and the lens trajectories are shown in the inset in the left-hand panel. The map over the source surface represents the surface brightness considering the gravity-darkening effect. The simple models without stellar rotation are shown by red solid lines. The thinner straight lines in the right panel represent the intrinsic polarization signal of the elliptical source. The photometric and polarimetric residuals with respect to the simple models are plotted in the bottom panels. The top polarimetric residual is the residual in the degree of polarization $\Delta P = P' - P$ and the bottom one is the absolute value of the residual in the polarization vector $|\Delta P| = \sqrt{P'^2 + P^2 - 2 P P' \cos 2(\theta_p' - \theta_p)}$, where the prime symbol refers to the related quantity considering the stellar rotation. The parameters used to create this figure can be found in Table 1. We also set $M_l = 0.3 M_\odot$, $D_l = 6.5$ kpc, $D_s = 8$ kpc, $c_1 = 0.64$, and $c_2 = 0.032$. For each microlensing event, the radius of the spherical source of the simple model is equal to $\rho_s = \sqrt{u_{\text{eq}}^2 + t_{\text{eq}}^2}$, i.e., the radius of the elliptical source where the lens enters the source surface. Accordingly, the first peaks of polarimetric curves coincide with the first peaks of simple models. The sharp peaks in polarimetric and photometric residuals while the lens leaves the source surface represent the mentioned time shift and are due to the elliptical shape of the source surface, which increases with increasing lens impact parameter.

If $u_0 > \rho_s$ and for large inclination angles, the peak position of microlensing light curves of elliptical source stars does not
respectively. We also set the lens mass $M = 0.6 \times 10^5$ M$_\odot$, $D_l = 6.5$ kpc, $D_s = 8$ kpc, and the limb-darkening coefficients over the source surface $c_1 = 0.64$ and $c_2 = 0.032$.

![Figure 3](image_url1)

**Figure 3.** Example polarimetric microlensing events of elliptical source stars. In every subfigure, the light and polarimetric curves are shown in left and right panels. The source surface projected on the lens plane and the lens trajectory are shown in the inset in the left-hand panel. The map over the source surface represents stellar surface brightness. However, gravity-darkening does not break the axial symmetry of the projected source surface with respect to its semiminor axis ($x_0$-axis). This effect causes asymmetric perturbations in photometric and polarimetric light curves of microlensing events, unless $\xi = 0^\circ$ or $i = 90^\circ$. The approach due to the ellipticity of the source surface, since the maximum value of the polarized Stokes intensity $I_p$ occurs at the stellar limb (see Equation (4)) and the stellar ellipticity also affects these points. We expect that the peak position in polarimetric curves occurs where the distance between the lens and the source edge is a minimum. Accordingly, the time shift between polarimetric and photometric peaks is given by

$$\delta_t = \left(\alpha - \frac{1}{\alpha}\right) \frac{\cos \xi}{\sqrt{\alpha^2 + \cot^2 \xi}} r_s. \quad (11)$$

The maximum value of this time shift occurs when $\cot \xi = \sqrt{\alpha}$, which is equal to 0.33x when $\alpha = 1.5$. However, the gravity-darkening affects this time shift when the inclination angle is small.

### 3.2. Asymmetric Perturbations Owing to the Gravity-darkening Effect

Gravity-darkening breaks the circular symmetry of the source surface brightness. However, gravity-darkening does not break the axial symmetry of the projected source surface with respect to its semiminor axis ($x_0$-axis). This effect causes asymmetric perturbations in photometric and polarimetric light curves of microlensing events, unless $\xi = 0^\circ$ or $i = 90^\circ$. The

| Figure | $M_\odot$ ($M_\odot$) | $R_\odot$ ($R_\odot$) | $\omega$ | $i$ (deg) | $w_0(p_\rho)$ | $\xi$ (deg) |
|--------|----------------------|----------------------|----------|------------|----------------|-------------|
| 3(a)   | 7.0                  | 4.7                  | 0.55     | 30.0       | ...            | 43.9        |
| 3(b)   | 5.0                  | 3.6                  | 0.5      | 25.0       | 0.9 cos($\xi$) | ...         |
| 3(c)   | 8.0                  | 5.3                  | ...      | ...        | 0.35           | ...         |
| 3(d)   | 9.0                  | 5.8                  | 0.55     | 30.0       | 0.0            | ...         |
| 7      | 3.8                  | 2.9                  | 0.50     | 15.0       | 0.43           | 42.0        |

Note. The columns contain (i) the figure number, (ii) the mass of the source star $M_\odot$ ($M_\odot$), (iii) the polar source radius $R_\odot$ ($R_\odot$), (iv) the angular speed of the source normalized to the break-up velocity $\omega$, (v) the inclination angle of the stellar rotational axis with respect to the sky plane $i$, (vi) the impact parameter of the lens trajectory with respect to the source center normalized to $p_\rho$, $w_0(p_\rho)$, and (vii) the angle of the lens trajectory with respect to the semimajor axis of the source $\xi$ respectively. We also set the lens mass $M_\odot = 0.3 M_\odot$, the lens and source distances from the observer $D_l = 6.5$ kpc and $D_s = 8.0$ kpc for Figures 3(a)–(d), and $M_\odot = 0.7 M_\odot$, $D_l = 4.1$ kpc, and $D_s = 8.2$ kpc for Figure 7. The limb-darkening coefficients are fixed at $c_1 = 0.64$ and $c_2 = 0.032$. The shift significantly from the time of closest approach, since the maximum value of the Stokes intensity $I_p$ occurs in the stellar center (see Equation (4)) and the ellipticity affects stellar limb points. In contrast, in these bypass microlensing events the polarimetric peak positions vary from the time of closest...
The maximum relative deviations in the stellar surface brightness due to the gravity-darkening effect (red solid line) and to the limb-darkening effect (green dashed line) vs. $\omega$.

Figure 4.

The gravity-darkening effect can be evaluated from the relative maximum deviation in the source surface brightness, i.e., $\delta_g$, which is given by

$$
\delta_g = \left| \frac{g_{\text{eff}}(\Omega, 0) - g_{\text{eff}}(\Omega, 90^\circ)}{g_{\text{eff}}(\Omega, 0)} \right| = 1 - \frac{R_p^2}{R_{\text{eq}}^2} + \omega^2 \left( \frac{2}{3} \right)^3 \frac{R_{\infty}}{R_p}.
$$

which depends only on $\omega$, i.e., the stellar angular velocity normalized to the break-up velocity. On the other hand, the relative maximum deviation in the stellar surface brightness due to the limb-darkening effect is $\delta_l = c_1 (=0.64)$. To compare the gravity-darkening and limb-darkening effects, we plot these relative maximum deviations in Figure 4, in which the red solid line represents $\delta_g$ and the horizontal green dashed line shows $\delta_l$. Thus, the gravity-darkening effect generally is much smaller than the limb-darkening effect. It mostly makes small perturbations in microlensing curves, unless the lens crosses the projected position of the stellar rotation pole, i.e., $d_R$. In these cases in which the lens impact parameter is equal to $u_0 = R_p \cos(i) \cos(\xi)$, the gravity-darkening effect can make detectable perturbations. The maximum deviation in microlensing light curves due to the gravity-darkening effect happens when $\xi = 90^\circ$, since the maximum Stokes intensity $I_9$, which occurs at the source center (due to the limb-darkening effect), is significantly perturbed by the gravity-darkening effect. The maximum polarimetric perturbation takes place when $\tan(\xi) = R_p \cos(i)/a$, because in that case the lens trajectory crosses the stellar edge points that have the maximum value of $\delta_g$ while these points also have the largest polarized Stokes intensity. These asymmetric perturbations in polarimetric and photometric curves of microlensing events can be identified by comparing the left and right sides of these curves.

We show the maximum asymmetric perturbations in the light and polarimetric curves of a microlensing event due to the gravity-darkening effect in Figure 3(b). The characterizations of this plot are the same as those of Figure 3(a). We align the source trajectory so that it crosses the point $d_R$ for three different values of $\xi$. When $\xi = 90^\circ$ the photometric residual due to the gravity-darkening effect is maximum. When $\xi = 42^\circ$, the polarimetric residual becomes maximum. Asymmetry in microlensing curves due to the gravity-darkening effect is obvious, because the polarimetric and photometric residuals are not symmetric with respect to the time of maximum magnification. Note that the sharp peaks when the lens enters the source surface are due to the ellipticity effect and the mentioned time shift. However, the polarimetric deviations due to the gravity-darkening in this figure are less than the polarimetric precision of FORS2, which means that higher quality polarimeters can detect these rotation-induced perturbations.

There is a problem in microlensing observations, which is degeneracy. Even if microlensing observers correctly discern the type of anomaly, it is not possible to uniquely derive all parameters of this anomaly from the observed light or polarimetric curves. This degeneracy exists in polarimetric or photometric microlensing events of elliptical source stars. The intrinsic polarization signal of an elliptical source is a degenerate function of the inclination angle and the stellar angular velocity (see Figure 1). This degeneracy cannot be resolved in microlensing observations. Figure 3(c) represents two different microlensing events of elliptical sources with different parameters, but the same polarimetric and photometric curves. These two microlensing events are degenerate. However, microlensing degeneracy can even exist between different models with different kinds of anomalies. For example, the microlensing curves and the intrinsic polarization signal of a rotating source star can be the same as those of two close binary source stars. This point is not studied in this work.

The polarization angle of an elliptical source (in our formalism) is zero with respect to its semimajor axis. The polarization angle of a lensed source star is $90^\circ$ with respect to the line connecting the source center and the lens position (see, e.g., Sajadian & Rahvar 2015). Hence, when the lens enters the source surface with $\xi = 0^\circ$ these polarization vectors are normal and after some time cancel each other out, so that the total polarization signal tends to zero at that time. Whereas, when $\xi = 90^\circ$ these two polarization vectors are parallel and always magnify each other, so that the total polarization signal increases as the lens enters the source surface. These points are shown in Figure 3(d). Detecting these features in the polarimetric curves of microlensing events helps to discern the angle between the lens trajectory and the semimajor axis of the elliptical source, which breaks the microlensing degeneracy. In this figure we set $u_0 = 0$, so the peak positions of polarimetric and photometric curves have no time shift. However, there are some asymmetric perturbations due to the gravity-darkening effect. The photometric perturbation due to the ellipticity is a maximum when $\xi = 90^\circ$.

According to the different panels of Figure 3, it seems that most rotation-induced perturbations in polarimetric curves of microlensing events are less than the FORS2 accuracy, i.e., 0.1%. Hence, detection of the rotation-induced perturbations in the polarimetric microlensing curves can probably be done by the next-generation polarimeters with higher precision than that of FORS2.

4. OBSERVATIONAL REMARKS

In the previous section we studied some aspects of polarimetric and photometric microlensing events of elliptical sources. In this section, we first investigate what percentage of
the magnetically active stars observed by the Kepler satellite rotate fast and have considerable values of \( \omega \). The rotational periods of these stars were evaluated from their light curves by some groups (e.g., Reinhold et al. 2013; McQuillan et al. 2014; Reinhold & Gizon 2015). Then, we study whether the polarimetric observations by FORS2 of high-magnification microlensing events of fast rotating stars toward the Galactic spiral arms can give information on rotation-induced perturbations.

4.1. Statistic of Fast Rotating Stars Based on the Kepler Data

Accurate statistics of fast rotating stars cannot be fully determined owing to several limitations in observational methods. For example, the photometric method for measuring stellar rotational periods is sensitive to magnetically active stars with stellar spots. Also, the interferometric method evaluates the stellar oblateness of just nearby stars.

The Kepler satellite has provided light curves of a very large sample of stars for more than four years by making uninterrupted high-resolution photometric observations (Borucki et al. 2010; Koch et al. 2010). These data were analyzed to derive stellar rotation periods and differential rotation effects by several authors; e.g., Reinhold et al. (2013) and Reinhold & Gizon (2015) analyzed a large sample of Kepler stars to determine their rotation periods using different approaches based on the Lomb–Scargle periodogram. They first selected magnetically active stars that exhibit stellar spots on their surface, i.e., often main-sequence stars with \( \log g_0(g) > 3.5 \). They noticed that 24.6% of stars in their sample were active. Using the stellar rotation periods given by Reinhold et al., the angular velocities of these active stars are inferred. We also estimate their critical velocities \( \Omega_{\text{crit}} \), and as a result the ratio of the stellar angular velocity to the critical velocity \( \omega \), according to the mass and radius of these stars given by Huber et al. (2014). The distribution of \( \log g_0(\omega) \) for this sample of stars is plotted in Figure 5. About 3.7% and 6.6% of these stars have \( \omega \) larger than 0.2 and 0.1 respectively. This sample contains just active main-sequence stars with \( \log g_0(g) > 3.5 \), but not all unbiased stars.

The color–magnitude (CM) diagram of these Kepler stars is shown in Figure 6 (red points). In this figure, the stars with \( \omega \) larger than 0.1 are shown with black stars. Most fast rotating stars are early-type and hot stars. The green triangles represent stars with intrinsic polarization signals larger than 0.2%, i.e., their polarization signals are measurable by FORS2 even without the lensing effect. For stars of this sample, we estimate their absolute magnitude using a synthetic CM diagram. We first generate a large ensemble of stars using the isochrones of Padova (Marigo et al. 2008) and according to the strategy explained in Section 3 of Sajadian & Rahvar (2012). We compare the temperature–luminosity diagram of stars in this sample with the related diagram of the generated synthetic sample of stars. For each star in our sample, we pick the characteristics of the most similar synthetic star in the generated ensemble.

In the next subsection, we simulate high-magnification microlensing of the stars specified by the black stars in Figure 6 to study whether rotation-induced perturbations are discernable through hypothetical polarimetric and photometric observations of these events.

4.2. Monte Carlo Simulation

It is well known that massive and hot stars rotate very fast, whereas most main-sequence or red giant stars have small or moderate rotational speeds (e.g., Bouvier 2013). The Galactic bulge often contains old and cold stars while the Galactic disk stars have a wide range of ages (e.g., Ortolani et al. 1995; Russeil 2003). Indeed, our Galaxy seems to have two spiral arms containing old stars and four spiral arms that include gas and young stars (Urquhart et al. 2014). Although the microlensing optical depth toward the Galactic spiral arms is less than that toward the Galactic bulge by one order of magnitude (Rahal et al. 2009), the mean duration of microlensing events in these directions is \( \sim 60 \) days (Rahal et al. 2009), which is longer than that toward the Galactic bulge, i.e., \( \sim 27 \) days (Wyrzykowski et al. 2015). Thus, the timescale for the lens to cross the source radius, i.e., \( t_{\text{s}} \), toward the spiral arms is about twice that toward the Galactic bulge. We note that \( t_{\text{s}} \) is the polarimetric timescale during a microlensing event. On the whole, the Galactic spiral arms are more suitable to be probed for finding stellar rotation effects. Hence, we simulate high-magnification microlensing.
events of those source stars whose rotational properties were studied by Reinhold et al. toward the Galactic spiral arms. We choose the Carina–Sagittarius arm and in the direction $l = 300^\circ$ and $b = -1^\circ$. Indeed, we assume that the local stellar population probed by the Kepler satellite is representative of the Galactic population. This hypothesis is most probably justified for stars in the Galactic disk.

We have two criteria for selecting these stars as source stars: (a) stars with $\omega > 0.1$ and (b) those brighter than 21 mag in the $I$-band after being located in that Galactic arm. These criteria decrease the number of possible source stars to 773. We finally investigate the possibility of discerning polarimetric and photometric rotation-induced perturbations in these high-magnification microlensing events.

The generic procedure for a Monte Carlo simulation as well as the distribution functions used to determine the mass of lenses, the velocities of both sources and lenses, and the distribution of matter in the Galaxy to determine the lens and source positions toward the Galactic arm were described in our previous works (Sajadian 2014, 2015b; M. Moniez et al. 2016, in preparation) and we do not repeat them here. Also, we use the Galactic extinction model in three dimensions developed by Marshall et al. (2006). We consider only high-magnification events with lens impact parameter less than the threshold value of 0.001. The inclination angle of the rotational axis for each source star is uniformly chosen in the range $[0, 90^\circ]$.

We assume these events are observed by FORS2, which reaches the highest polarimetric precision $\sigma_p = 0.1\%$. The necessary $S/N$ to achieve this precision is $1/\sigma_p = 1000$ for the imaging polarimetry mode (IPOL). We generate synthetic data points hypothetically taken by FORS2 over every polarimetric curve. In this regard, we calculate the necessary exposure time to achieve the highest polarimetric accuracy. The definition of $S/N$ can be found in Sajadian (2015b). In addition to the exposure time, there are two extra wasted times due to the rotation of the retarder waveplate and CCD readout. This overhead time lasts about 18 minutes for each polarimetric data point taken by FORS2. In this regard, all details are explained in Section 3.1. The start time of observation by FORS2 is chosen randomly in the range $[-3.0; 3.0] t_s$, and the observation is interrupted after about 6 hr until the next night (after 18 hr). For all simulated events, we set $t_0 = 0$.

To generate photometric data points, we use the sampling and photometric uncertainties (i.e., $\sigma_i$) taken by some archived high-magnification events around the peak of their light curves. We use the high-magnification microlensing events given by Choi et al. (2012). The simulated (photometric and polarimetric) data points are shifted with respect to the model light and polarimetric curves according to their photometric and polarimetric uncertainties by Gaussian functions. One of the simulated light and polarimetric curves of high-magnification microlensing events and its synthetic data points is shown in the left panel of Figure 8. Six source stars have net polarization signals greater than 0.2%. These stars are indicated in Figure 6 by green triangles.

b. As discussed in the previous section, the stellar rotation breaks the symmetry of polarimetric and photometric microlensing curves with respect to $t_0$. This anomaly shifts (i) the position of the second polarimetric peak with respect to the symmetric position of the first polarimetric peak in transit microlensing events and (ii) the time position of the polarimetric peak with respect to the photometric peak position in bypass cases. For simulated microlensing events, we calculate these time shifts, i.e., $\delta t$. The ratio of this time shift to the mean value of the time interval between two consecutive polariometric data points gives the possible number of polarimetric data points that can be taken during this time shift. The histogram of this quantity on a the logarithmic scale, $\log_{10} [\delta t/T_{obs}]$, is plotted in the right panel of Figure 8. The number of events that have $\delta t/T_{obs} > 3$ is 221, i.e., about 28.6% of the total number of simulated events. Detecting this time shift requires there to be enough polarimetric data points at the polarimetric peak position (s). We assume that the real positions of photometric peaks can be inferred from the fitting process. The number of simulated events for which FORS2 has observed the polarimetric peak(s) is 188, about 24.3% of events. On the whole, 4.7% of the simulated events have $\delta t/T_{obs} > 3$ as well as having their polarimetric peaks covered by synthetic data points, which means that during the time shift $\delta t$ more than three polarimetric data points are taken by FORS2. However, to discern these perturbations, the polarimetric residual during this time shift with respect to the nonrotating (spherical) star model should be more than at least $2 \sigma_p$, which is probable according to the rapid decrease in polarimetric microlensing curves around their peaks.

c. Finally, we can discern the effects of stellar rotation on polarimetric and photometric microlensing curves by detecting asymmetric perturbations that are mostly due to gravity-darkening. In that case, for each simulated event we calculate $\Delta \chi^2 = \chi^2_1 - \chi^2_2$, in which $\chi^2_1$ and $\chi^2_2$ result respectively from fitting the real model with the elliptical source star and the simple model with the nonrotating
Figure 8. The results of the Monte Carlo simulation. Left panel: histogram of the intrinsic polarization signals of source stars. Right panel: histogram of the time shifts induced by the stellar ellipticity in the polarimetric peak positions \( \delta t \) normalized to the averaged observational time for taking each polarimetric data point \( T_{\text{obs}} \). This quantity shows the possible number of polarimetric data points during the rotation-induced time shifts. The blue dashed line shows the quantity \( \delta t / T_{\text{obs}} = 3 \). Both of them are plotted on a logarithmic scale.

Figure 9. The distributions of \( \Delta \chi^2 = \chi^2_e - \chi^2_s \) on a logarithmic scale, where \( \chi^2_e \) with the indices e and s result from fitting the real models of elliptical source stars and the simple models of spherical source stars respectively, to photometric (red dashed histogram) and polarimetric (blue histogram) data points. For each simulated event, the parameters used to generate the simple model are similar to the parameters used to create the real model without considering the stellar rotation.

(spherical) source star to the simulated data points. The parameters of the simple model are similar to the parameters used to create the real model, except that the source star does not rotate. Also, the radius of the spherical source star in simple models is equal to the radius of the source star where the lens enters the source surface, i.e., \( \rho_{s,i} = \sqrt{u^2_0 + t_i^2} \) in transit microlensing events and \( \rho_{s,i} = \sqrt{ab} / R_E \) in bypass events. The distributions of \( \Delta \chi^2 \) on a logarithmic scale from fitting to photometric data points (red dashed histogram) and to polarimetric data points (blue histogram) are plotted in Figure 9. In about 83.1% and 0.1% of simulated light and polarimetric microlensing curves (respectively), the values of \( \Delta \chi^2 \) are higher than 150. The small number of events for which \( \Delta \chi^2 > 150 \) means that (i) FORS2 by itself probably cannot cover polarimetric curves of high-magnification microlensing events, and (ii) its polarimetric precision is too low to distinguish rotation-induced perturbations.

Considering all of the mentioned tests for discerning rotation-induced perturbations, we conclude that in 37 and 642 simulated events (which contribute 4.8% and 83.1% of all simulated events) the polarimetric and photometric perturbations induced by stellar rotations are distinguishable respectively. Although the photometric observation is more efficient than the polarimetric one in detecting the effects of stellar rotation, by making only photometric observations we cannot detect the time shift in the photometric peak position, yet this time shift is very helpful in discerning the kind of anomaly. However, the small polarimetric efficiency for detecting stellar rotation effects is rather due to the lack of sufficient polarimetric data points to cover the polarimetric peak(s), if we assume that these observations are made by FORS2 by itself.

5. SUMMARY AND CONCLUSIONS

Stellar rotation causes the ellipticity and gravity-darkening effects that break the spherical symmetry of the source surface and the circular symmetry of its surface brightness respectively.

Accordingly, a rotating star has a net polarization signal whose intensity depends on the inclination angle of the rotational axis as well as the stellar angular velocity. For fast rotating stars \( (\omega > 0.6) \) the magnitude of this polarization signal is comparable with the polarization signal in transit microlensing events. This anomaly should be considered in calculations of the polarimetric microlensing of these stars, allowing the elliptical properties of the source stars to be characterized. The intrinsic polarization signals for slowly or moderately rotating stars with \( \omega < 0.6 \), which are too small to be measured by themselves, may be highlighted during a high-magnification microlensing event.

Polarimetric and photometric curves in single-lens microlensing events of spherical (nonrotating) source stars have symmetric shapes with respect to the time \( t_0 \) of closest approach. The stellar ellipticity breaks this symmetry and causes time shifts \( \delta t \) in (i) the position of the second polarimetric peak with respect to the symmetric (with regard
to $t_0$ position of the first peak in transit microlensing events and (ii) the position of the polarimetric peak with respect to the photometric peak position in bypass cases. However, when $\xi = 0^\circ$, $\xi = 90^\circ$, or $u_0 = 0$ there is no time shift, because of the symmetric shape of the elliptical source surface projected on the sky plane with respect to its semimajor and semiminor axes.

Gravity-darkening produces asymmetric perturbations in polarimetric and photometric curves of microlensing events. These perturbations are maximized whenever the lens trajectory crosses the projected position of the stellar rotation pole on the sky plane. In that case, if its angle with respect to the semimajor axis of the source is equal to $\xi = 90^\circ$, the photometric perturbation is maximized, and if $\xi = \arctan(R_p \cos(i)/R_{eq})$ the polarimetric perturbation becomes maximum.

The intrinsic polarization signal of an elliptical source is a degenerate function of the inclination angle and the stellar angular velocity (see Figure 1). Hence, different elliptical source stars with the same intrinsic polarization signals can have the same polarimetric and photometric microlensing curves.

In order to study and compare the photometric and polarimetric efficiencies for detecting the effects of stellar rotation in high-magnification microlensing events toward the Galactic spiral arms, we simulated them and considered the fast rotating stars observed by the Kepler satellite ($\omega > 0.1$) as the source stars. We generated synthetic (polarimetric and photometric) data points hypothetically taken by FORS2 and by survey and follow-up telescopes (respectively) for each microlensing event. About 0.5% of the source stars had intrinsic polarization signals greater than 0.2%, which can potentially be measured by FORS2 directly. In 4.7% of the total simulated events more than three polarimetric data points were taken during the time shift $\delta_t$ by FORS2. In these events, the rotation-induced time shift, and hence the ellipticity of the source surface projected on the sky plane, can likely be measured. We also investigated whether asymmetric perturbations in simulated polarimetric and photometric curves due to the gravity-darkening effect can be inferred, by calculating the difference between the values of $\chi^2_{xx}$ from fitting (a) the real microlensing models of elliptical source stars and (b) simple microlensing models of nonrotating (spherical) source stars to synthetic data points. Almost 83.1% and 0.1% of photometric and polarimetric curves had $\Delta \chi^2$ greater than 150 respectively.

On the whole, polarimetric and photometric perturbations due to stellar rotation were detectable in 4.8% and 83.1% of all simulated events respectively. Therefore, the stellar rotation signatures in high-magnification microlensing events of early-type stars are mostly detectable through photometric observations with the technology currently available. Although photometric observation is more efficient than polarimetric observation for detecting stellar-induced anomalies, by making only photometric observations we cannot discern the time shift in polarimetric peak positions that results from the stellar ellipticity. This time shift is very helpful in determining the nature of the anomaly and obtaining some information about the ellipticity of the projected source surface. However, the small polarimetric efficiency for detecting stellar rotation effects is due to (i) the low polarimetric precision of FORS2 and (ii) the long observational time necessary for taking each polarimetric data point by FORS2 (in comparison with the polarimetric timescales of microlensing events), which results in an insufficient number of data points to cover the polarimetric peak(s). This time shift can likely be distinguished by high-quality polarimeters of the next generation.

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