Energy Dissipation of Axionic Boson Stars
in Magnetized Conducting Media

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Abstract

Axions are possible candidates of dark matter in the present Universe. They have been argued to form axionic boson stars with small masses $\sim 10^{-12} M_\odot$. Since they possess oscillating electric fields in a magnetic field, they dissipate their energies in magnetized conducting media. We show that colliding with a magnetized white dwarf, the axionic boson stars dissipate their energies and heat the white dwarf. Consequently the white dwarf cooled sufficiently can emit detectable amount of radiations with the collision. Using a recent evaluation of the population of the white dwarfs as candidates of MACHOs, we estimate the event rate of the collisions and obtain a result that the rate is large to be detectable.

The axion is the Goldstone boson associated with Peccei-Quinn symmetry [1], which was introduced to solve naturally the strong CP problem. In the early Universe some of the axions condense and form topological objects [2,3], i.e. strings and domain walls, although they decay below the temperature of QCD phase transition. After their decay, however, they have been shown to leave a magnetic field [4] as well as cold axion gas as relics in the present Universe; the field is a candidate of a primordial magnetic field leading to galactic
magnetic fields observed in the present Universe.

In addition to these topological objects, the existence of axionic boson stars has been argued \cite{5,6}. It has been shown numerically \cite{6} that in the early Universe, axion clumps are formed around the period of 1 GeV owing to both the nonlinearity of an axion potential and the inhomogeneity of coherent axion oscillations on the scale beyond the horizon. These clumps are called axitons since they are similar to solitons in a sense that its energy is localized. Then, the axitons contract gravitationally to axion miniclusters \cite{7} after separating out from the cosmological expansion. They are incoherent axions bound loosely, while the axion gases are distributed uniformly; they are generated by the decay of the axion strings or the coherent axion oscillations. Furthermore, depending on energy densities, some of these miniclusters may contract gravitationally to coherent boson stars \cite{8,6,9}. Their masses have been estimated roughly to be order of $\sim 10^{-12}M_\odot$. Eventually we expect that in the present Universe, there exist the axion miniclusters and the axion boson stars as well as the incoherent axion gas as dark matter candidates. It has been estimated \cite{10} that a fairly amount of the fraction of the axion dark matter is composed of the axion miniclusters and the axion boson stars. A way of the observation of the axion miniclusters has been discussed \cite{10}.

In this letter we wish to point out an intriguing observable effect associated with the coherent axionic boson stars; we call them axion stars. Namely, they dissipate their energies in magnetized conducting media so that the temperature of the media increases and strong radiations are expected. The phenomena are caused by electric fields generated by the coherent axion stars under external magnetic fields. The electric fields induce electric currents in the conducting media and loose their energies owing to the existence of resistances. Consequently the axion stars dissipate their energies in the magnetized conducting media. Although the electric fields themselves are small, the total amount of the energy dissipation is very large because the dissipation arises all over the volume of the axion stars: Radii of the axion stars of our concern are such as $10^8$ cm $\sim 10^{10}$ cm. Consequently detectable amount of radiations are expected from the media heated in this way. Because the strength
of the electric fields is proportional to the strength of the magnetic field, the phenomena are revealed especially in strongly magnetized media such as neutron stars, white dwarfs e.t.c.. We show that the amount of the energy dissipated in white dwarfs, for instance, with mass $\sim 0.5 M_\odot$ and with magnetic field larger than $10^5$ Gauss is approximately given by $10^{35}$ erg/s $\left(\frac{M}{10^{-14} M_\odot}\right)^4 \left(\frac{m}{10^{-5} \text{eV}}\right)^6$ where $M (M_\odot)$ is the mass of the axion star (the sun) and $m$ is the mass of the axion. In our discussions we assume that the axion stars, when they collide with the media, are not destructed and that they pass the media. Later we discuss on this point.

Let us first sketch to review our solutions of the axionic boson stars. Originally Seidel and Suen [12] have found solutions of a real scalar axion field, $a$, coupled with gravity. Their solutions represent spherical oscillating axion stars with masses of the order of $10^{-5} M_\odot$; the solution $a$ possesses oscillation modes with various frequencies. On the other hand, axion stars of our concern are ones with much smaller masses, $\sim 10^{-12} M_\odot$. So, in order to find the existence of such solutions and explicit relations among the parameters, e.g. radius, $R$, mass, $M$, e.t.c., of these axion stars, we have numerically obtained solutions of the spherical axionic boson stars [11,12] in a limit of a weak gravitational field. Relevant equations are a free field equation of the axion and Einstein equations. It means that our solutions represent the axion stars with small masses, e.g. $10^{-12} M_\odot$; their gravitational fields are sufficiently weak and amplitudes $a$ are much small. Thus nonlinearity of the axion potential is irrelevant; we have found that the nonlinearity arises only for the axion stars with masses larger than $\sim 10^{-9} M_\odot$. The field $a$ of our solutions possesses only one oscillation mode with its frequency approximately given by $m$. Other oscillation modes, which exist in general solutions representing the axion stars with larger masses, appear gradually as the masses increase. We have confirmed that our numerical solutions may be approximated by the explicit formula,

$$a = f_{PQ} a_0 \sin(mt) \exp(-r/R),$$  \hspace{1cm} (1)$$

where $t$ ( $r$ ) is time (radial) coordinate and $f_{PQ}$ is the decay constant of the axion. The
value of $f_{PQ}$ is constrained from cosmological and astrophysical considerations \[3\] such as $10^{10}\text{GeV} < f < 10^{12}\text{GeV}$.

In the limit of the small mass of the axion star we have found a simple relation \[11\] between the mass, $M$ and the radius, $R$ of the axion star,

$$M = 6.4 \frac{m_{pl}^2}{m^2 R},$$

with Planck mass $m_{pl}$. Numerically, for example, $R = 1.6 \times 10^5 m_5^{-2}\text{cm}$ for $M = 10^{-9} M_\odot$, $R = 1.6 \times 10^8 m_5^{-2}\text{cm}$ for $M = 10^{-12} M_\odot$, e.t.c. with $m_5 \equiv m/10^{-5}\text{eV}$. A similar formula has been obtained in the case of boson stars of complex scalar fields. We have also found an explicit relation \[11\] between the radius and the dimensionless amplitude $a_0$ in eq(1),

$$a_0 = 1.73 \times 10^{-8} \left(10^8\text{cm}\right)^2 \frac{10^{-5}\text{eV}}{R^2}.$$

These explicit formulae are used for the evaluation of the dissipation energy of the axion stars in the magnetized conducting media.

We now proceed to explain how the axion field representing these axion stars generates an electric field in an external magnetic field. The point is that the axion couples with the electromagnetic fields in the following way,

$$L_{a\gamma\gamma} = c\alpha a \vec{E} \cdot \vec{B}/f_{PQ\pi},$$

with $\alpha = 1/137$, where $\vec{E}$ and $\vec{B}$ are electric and magnetic fields respectively. The value of $c$ depends on the axion models \[13,14\]; typically it is the order of one.

It follows from this interaction that Gauss law is

$$\vec{\partial} \vec{E} = -c\alpha \vec{\partial} (a\vec{B})/f_{PQ\pi} + \text{“matter”}$$

where the last term “matter” denotes contributions from ordinary matters. The first term in the right hand side represents a contribution from the axion. Thus it turns out that the axion field has an electric charge density, $-c\alpha \vec{\partial} a \cdot \vec{B}/f_{PQ\pi}$, under the magnetic field $\vec{B}$ \[13\]. We assume that the field $\vec{B} = (0, 0, B)$ is spatially uniform and that the field $a$ representing
the axion stars is given by our solutions. Then we understand that this axion star has a charge distribution such that it has negative charges on a hemisphere \((z > 0)\) and positive charges on the other hemisphere \((z < 0)\). Net charge is zero. Therefore the star possesses the electric field, \(\vec{E}\), parallel to the magnetic field associated with the charge distribution; the electric field is given such that \(\vec{E} = -\alpha a \vec{B}/f_{PQ} \pi\). This field induces an electric current in conducting media and the energy of the field is dissipated.

Denoting the conductivity of the media by \(\sigma\) and assuming the Ohm law, we find that the axion star with it’s radius \(R\) dissipates an energy \(W\) per unit time,

\[
W = \sigma \alpha^2 c^2 B^2 a_0^2 R^3 / \pi = \alpha^2 c^2 B^2 a_0^2 R^3 / 4\pi^2 \nu_m
\]

\[
= 2.2 \times 10^{21} \text{erg/s} \frac{c^2}{\nu_m/\text{cm}^2\text{s}^{-1}} \frac{M}{10^{-14} M_\odot} \frac{B^2}{(1 \text{G})^2},
\]

with magnetic diffusivity, \(\nu_m = 1/4\pi \sigma\). We have used the explicit formulae eq(1), eq(2) and eq(3). Since the field \(a\) oscillates \([11,12]\) with a frequency given approximately by the mass of the axion \(m\), we have taken an average in time over the period, \(m^{-1}\). Here, length scales of the media have been assumed to be larger than the radius of the axion star \(R\); the whole of the axion star is included in the media. On the contrary, when the scale \(L\) of the medium is smaller than \(R\), we need to put a volume factor of \((L/R)^3\) on \(W\); only the fraction \((L/R)^3\) of the volume of the axion star is relevant for the dissipation.

We comment that the formula may be applied to the conducting media where the Ohmic law is hold even for oscillating electric fields with their frequencies \(m = 10^{10} \sim 10^{12} \text{Hz}\). The law is hold in the media where electrons interact sufficiently many times in a period of \(m^{-1}\) with each others or other charged particles and diffuse their energies acquired from the electric field. Actually the law is hold in the convection zone of the sun, white dwarfs, neutron stars e.t.c..

We would like to point out that although the electric field \(\vec{E} = -\alpha a \vec{B}/f_{PQ} \pi\) is much small owing to the large factor of \(f_{PQ}\), the amount of the dissipation energy \(W\) becomes large. The reason is that such a large value of the energy is resulted from the dissipation arising over the large volume; \(W\) is proportional to \(R^3\).
In order to evaluate the value of $W$, we need to know the mass of the axion star realized in the Universe. According to a creation mechanism of the axion star by Kolb and Tkachev [6–8], the axionic boson stars are formed by some of miniclusters contracting gravitationally. They have shown that the mass of the axion stars is typically $10^{-12}M_\odot\Omega_a h^2$; $\Omega_a$ is the ratio of the axion energy density to the critical density in the Universe and $h$ is Hubble constant in the unit of 100 km s$^{-1}$ Mpc$^{-1}$. Actual values of the mass may range from $10^{-12}M_\odot$ to $10^{-14}M_\odot$ corresponding to the value, $0.01 \leq \Omega_a h^2 \leq 1$. Hence in this paper we discuss the axion stars with such a range of the masses. Correspondingly the radius of the axion stars ranges from $10^8$ cm to $10^{10}$ cm.

Now we discuss what amount of the axion energy is dissipated in a real magnetized conducting medium. As a first example we take the sun which is a typical star with a strong magnetic field $10^3 \sim 10^4$ G in its convection zone of the sun. Assuming the depth of the convection zone being $\sim 2 \times 10^{10}$ cm, and the magnetic diffusivity $\nu_m \sim 10^7$ cm$^2$s$^{-1}$ [16], it follows that

$$W = 5.5 \times 10^{22} \text{erg/s} \frac{B^2}{(5 \times 10^3 G)^2} \frac{M}{10^{-13}M_\odot},$$

(8)

where we have set $c^2 = 1$. Hence the total energy dissipated, when the axion star passes the sun, is given such that

$$W_t = 4 \times 10^{10} \text{cm} \times W/v = 7.3 \times 10^{25} \text{erg} \frac{B^2}{(5 \times 10^3 G)^2} \frac{M}{10^{-13}M_\odot},$$

(9)

where the velocity, $v$, of the axion star is assumed to be $3 \times 10^7$ cm/s; this is a value obtained by equating a kinetic energy with a gravitational energy of the axion star in our galaxy. We have assumed the absence of the magnetic field in the radiation zone. Therefore, the effect of the energy dissipation of the axion star in the sun is difficult to be observed; the luminosity of the sun is the order of $10^{33}$ erg/s. Similarly, as far as we are concerned with such stars with the same physical parameters as those of the sun, it is difficult to detect the effect of the energy dissipation.

Next we go on to discuss the case of white dwarfs which may possess strong magnetic field $\sim 10^6$ G with their typical radius $L \sim 10^6$ cm. The radius is larger than that ($R \sim 10^8$
cm) of the axion star with mass $\sim 10^{-12} M_\odot$, but smaller than that ($R \sim 10^{10} \text{ cm}$) of the axion star with mass $\sim 10^{-14} M_\odot$. So we need to treat separately the two cases. But since the recent observations indicate much smaller values of $\Omega h^2$ than 1 ($\Omega$ is the ratio of the energy density to the critical density), here we consider the case of the axion star with mass $\sim 10^{-14} M_\odot$, which corresponds to the case, $\Omega_a h^2 \sim 0.01$. Thus the radius of the axion star is 10 times larger than that of the white dwarf. (It will be turned out that with such a choice of small masses $\sim 10^{-14} M_\odot$, the rate of the white dwarfs colliding with the axion stars is large to be observed in our galaxy.)

According to recent observations of gravitational microlensing [17], the white dwarfs are most plausible candidates of the phenomena. Most of them seem to be dark enough not to be observed. Probably, they have reached a stage of a fast Debye cooling and have lost almost of all their internal thermal energies [18]. Although this conjecture might be not correct, we assume it with an additional assumption that the dark white dwarfs have strong magnetic field $\sim 10^6 \text{ G}$.

Then, the observation indicates that the population of the white dwarfs is $2 \times 10^{11} M_\odot/0.5 M_\odot \sim 4 \times 10^{11}$ in the halo of our galaxy when their typical mass is $0.5 M_\odot$. Since the number is so large, the collision with the axion stars is expected to occur frequently in our galaxy. Furthermore, as we will show soon, the axion star dissipates the energy in the white dwarf so much that the effect of the dissipation in the white dwarfs with very low temperature may be observed in a search like MACHO searches.

When we apply naively the formula eq(7), the amount of the energy dissipated in the white dwarf is given such that

$$W \sim 10^{30} \text{ erg/s} \frac{c^2}{\nu_m \text{ cm}^2 \text{s}^{-1}} \frac{M^4}{(10^{-14} M_\odot)^4} \frac{B^2}{(10^6 \text{G})^2} \frac{m^6}{(10^{-5} \text{eV})^6}.$$  \hspace{1cm} (10)

Here we have taken account of the volume factor, $(L/R)^3 \sim 10^{-3} (M/10^{-14} M_\odot)^3 m_5^6$, because the radius $R(\sim 10^{10} \text{cm})$ of the axion star is larger than the radius $L(\sim 10^9 \text{cm})$ of the white dwarf. We expect that $\nu_m$ is much smaller (or conductivity is much larger) than ones of normal metals because the number density of degenerate electrons in the white dwarf is
much larger than that of the metals. According to theoretical evaluations [19] it follows that 
\( \nu_m \sim O(10^{-5}) \text{cm}^2/\text{s} \) in the case of a crystallized white dwarfs with temperature \( \sim 10^4 \text{K} \) and density \( 10^6 \text{g/cm}^3 \). Note that lower temperature leads to smaller \( \nu_m \) (larger conductivity).

Since our concern is dark white dwarfs with very low temperature as we mentioned before, actual values of \( \nu_m \) are expected to be much lower than the above one.

Then \( W \) reaches a value more than \( 10^{35} \text{erg/s} \). But the dissipation of such a large amount of the energy more than \( W \sim 10^{35} \text{erg/s} \), is impossible owing to the energy conservation.

Namely, the maximum energy, which a part of the axion star swept by the white dwarf can dissipate, is the energy stored in the part; we note that the white dwarf is smaller than the axion star. We find that the energy stored in the part is given by 
\[
3L^2vM/4R^3 \sim 10^{35} \text{erg/s} (M/10^{-14}M_\odot)^4m_5^6.
\]
This is smaller than \( W \) estimated naively. Thus real amount of the energy dissipated is at most given by 
\[
W_{\text{real}} \sim 10^{35} \text{ erg/s} (M/10^{-14}M_\odot)^4m_5^6.
\]
The dissipation of the energy continues until the white dwarf passes the axion star. Thus total energy dissipated by the axion star is 
\[
2R/v \times W_{\text{real}} \sim 10^{38} \text{erg} (M/10^{-14}M_\odot)^3m_5^4.
\]

Since we suppose that the white dwarfs of our concern have lost most of their internal energies by cooling and that their temperature, \( T \), is sufficiently low, their specific heat, \( c_v \), per ion is given approximately by [20] 
\[
c_v \sim 16\pi^4(T/\theta_D)^3/5,
\]
where \( \theta_D \) is the Debye temperature, typically being \( 10^7 \text{K} \). Hence the injection of the energy, \( 10^{38} \text{ erg} (M/10^{-14}M_\odot)^3m_5^4 \), increases the temperature of the white dwarfs to \( \sim 10^4 \text{K} (M/10^{-14}M_\odot)^{3/4}m_5 \). If surface temperature is the same as this temperature, the luminosity is roughly \( 10^{-3}L_\odot (M/10^{-14}M_\odot)^3m_5^4 \). Although this estimation is too naive, the result is interesting because the luminosity is large enough to be observed.

To analyze more precisely the phenomena, however, we need to solve several questions. Among them, we have to make clear whether or not a gravitational attraction of the white dwarfs destabilizes the axion stars, which are composites of axions bound gravitationally. We expect that since the axion stars are coherent objects, their stability is not affected by the attraction, although the original spherical distribution of the field \( a \) is fairly distorted. Namely the coherent axions will not be transformed into incoherent axions by the slowly
changing gravitational perturbations. Therefore even if we include the gravitational effects of the white dwarfs, similar amount of the dissipation energy is expected to be released from the axion star although the geometrical form of the star is fairly deformed. More difficult problem is to analyze the back reaction of the energy dissipation on the axion stars and to ask the question whether or not they are destabilized or trapped by the white dwarfs with the effects of the dissipation. It seems that both effects of the gravitational attraction and of the dissipation make the axion stars be trapped owing to the dissipation of their kinetic energies. In this case more energies are released from the axion stars, compared with the case of they passing the white dwarfs. Consequently, stronger radiations are expected. The analysis of these problems will be presented in future publication.

We have shown that the collision between the axion star and the invisible white dwarf makes the white dwarf become visible. Thus it is important to see how large the rate of the collision is. We estimate the rate of such a collision in our galaxy. Especially, we are concerned with the event rate observed in a solid angle, $5\degree \times 5\degree$, for example. We assume that as indicated by the recent observations of gravitational microlensing, the half of the halo is composed of the white dwarfs with mass $0.5 \times M_\odot$. The other half is assumed to be composed of the axion stars. Total mass of the halo is supposed to be $\sim 4 \times 10^{11} M_\odot$. Furthermore, the distribution $[21]$ of the halo is taken such that its density $\propto (r^2 + 3R_c^2)/(r^2 + R_c^2)^2$ with $R_c = 4$ kpc where $r$ denotes a radial coordinate with the origin being the center of the galaxy (the final result does not depend practically on the value of $R_c = 2 \sim 8$ kpc). Then it is easy to evaluate the event rate of the collisions,

$$0.5\text{/year} \times \frac{(10^{-14} M_\odot)^3}{M^3} \frac{(10^{-5}\text{eV})^4}{m^4} \frac{\Omega}{5\degree \times 5\degree}$$

where $\Omega$ is a solid angle. We have taken into account the fact that the earth is located at about 8 kpc from the center of our galaxy, simply by counting the number of the collisions arising in the region from 8 kpc to 50 kpc. Therefore, it is possible to observe the phenomena associated with the energy dissipation of the axion stars in the white dwarfs, although the rate depends heavily on both of the mass of the axion stars and the axion mass.
Similarly as the white dwarf, neutron stars possess strong magnetic fields and high electric conductivities. Thus the energy dissipation of the axion star is expected to be large in the neutron stars. But since the radius of the neutron star is about $\sim 10^6$ cm, the actual amount of the dissipation energy is much smaller than that in the white dwarf. The reason is that the maximum energy, which can be dissipated, is the energy stored in the part of the axion star swept by the neutron star and that the volume of the part is smaller than that in the case of the white dwarf. Therefore, the rate of the energy dissipated is approximately given by $10^{35}(10^6/10^9)^2 \text{erg/s (M/10}^{-14}M_\odot)^4m_5^6 \sim 10^{29} \text{erg/s (M/10}^{-14}M_\odot)^4m_5^6$. Total amount of the energy is $10^{32} \text{erg (M/10}^{-14}M_\odot)^3m_5^4$. This is too small for the resultant radiation to be detectable.

As explained in above examples, the axion stars are possible sources for generating energies in the magnetized conducting media. We may apply the idea to systems such as accretion disks with strong magnetic fields around black holes e.t.c.. Probably the existence of the axion will be confirmed indirectly by observing the phenomena associated with the energy dissipation of the axion star.

In summary, we have shown that the coherent axion stars dissipate their energies in the magnetized conducting media such as the sun, or the white dwarfs. Among them, the white dwarfs with much low temperature are heated in this mechanism and emit the radiations with detectable luminosities. Furthermore, we have shown that the rate of the white dwarfs colliding with the axion stars is not small in our galaxy. Therefore the collision may be detected in a research of the phenomena like a noba. Although the possibility of these phenomena being observable depends sensitively on the physical parameters such as the axion mass, the mass of the axion star, or the strength of the magnetic field, it is interesting that there is an allowed range of the parameters making the phenomena detectable.

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