NUCLEAR AXIAL-CHARGE TRANSITIONS
IN CHIRAL PERTURBATION THEORY

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ABSTRACT

We develop a systematic chiral perturbation expansion for the calculation of meson-exchange currents in nuclei and apply the formalism to nuclear axial currents. We summarize the principal results of such a calculation to one loop order on nuclear axial-charge transitions which provides a strong support to the conjecture of “chiral filter phenomenon” in nuclear medium. The use of heavy baryon chiral perturbation theory enables us to obtain a remarkably simple result valid next to the leading order in chiral counting. The dominant role of a soft-pion exchange in axial-charge transitions in heavy nuclei is confirmed. An important, albeit indirect, consequence of our result on the empirically observed enhancement in axial-charge transitions in heavy nuclei is pointed out.
While meson-exchange currents in nuclei are fairly well understood in low-energy and low-momentum regime both experimentally and theoretically [1], there remains the issue of understanding them from the point of view of QCD. This is not an academic issue since the purported high-energy electron machines in construction or in project are to probe nuclei for signals of direct QCD degrees of freedom in the deviation from exchange-current effects and for this, a contact with QCD at lower energy and highly nonperturbative regime will be clearly needed. At present the only applicable method to address this issue in nonperturbative regime is chiral perturbation theory (ChPT) [2]. Chiral perturbation theory has had much success in Goldstone boson (π, K) interactions [2], making nonperturbative QCD accessible to laboratory phenomena. However incorporating baryons in the scheme has proven to be quite difficult, the main reason being that the standard power counting used in ChPT does not apply when baryons are involved. Even for the elementary π-N interaction, the calculation becomes horrendously complicated, requiring approximations that are hard to control. Some progress has recently been made in describing by chiral perturbation expansion such processes as π-N scattering [3], threshold pion production from nucleon [4] and nucleon’s electromagnetic polarizabilities [5]. Nonetheless a systematic chiral perturbation calculation of many-body nuclear properties remains a formidable, if not hopeless, task at the present stage of development. This technical difficulty explains in part the paucity up to date of work of this nature in the literature. Recently, however, this task was greatly facilitated by the formulation of heavy-baryon chiral perturbation theory (which we shall call HBChPT in short) by Weinberg [6], Jenkins and Manohar [7] and others [8] along the line developed for heavy-quark effective theory [9].

The purpose of this letter is to summarize the principal results of a complete chiral perturbation calculation of the meson-exchange axial-charge operator to one-loop order that establishes rigorously the “chiral filter phenomenon” in nuclei conjectured a long time ago [10] and given a partial justification recently by one of the authors [11] which states that whenever kinematically unsuppressed, soft-pion exchanges should dominate in electroweak processes in nuclei. As discussed in [10], the chiral filter phenomenon occurs in nuclear axial-charge transitions and in nuclear M1 transitions (such as threshold radiative np capture). In this paper, we will focus on the former. The latter will be discussed in a separate paper in preparation [12]. Towner [13] and Riska et al. [14] have recently addressed a similar issue within, however, a phenomenological framework. We shall make comparison with their results at the end of the paper.

As in [3, 11], we shall take the most general chiral Lagrangian consisting of nucleons and pions with all other degrees of freedom integrated out. (One can alternatively take a Lagrangian that also contains vector mesons and Δ’s. We have satisfied ourselves that these elements do not alter our result significantly. This matter will be discussed in detail in a later publication.) The key observation made by the authors in [3, 11] is that the standard derivative expansion successful in low energy pion dynamics breaks down when baryons are involved, for the reason that time derivatives on the baryon field, typically of the order
Baryon momentum-dependent terms appear as higher order interaction terms in the chiral counting. The pair terms are suppressed at the leading order, appearing as “1/m” corrections subsumed in the ellipsis in (1).

Our task is to compute higher-order corrections in the chiral expansion parameter \(Q\) – where \(Q\) is the momentum or energy scale parameter probed by an external field, assumed to be small compared with the chiral scale \(\Lambda\) – to the leading soft-pion amplitude, denoted \(\mathcal{M}_{soft}\), that contributes in two-body effective currents responding to a slowly varying electroweak field \(J_\mu\).

\[
\mathcal{M} = \mathcal{M}_{soft}(1 + \delta + O(Q^n)), \quad \delta \sim O(Q^2), \quad n \geq 3.
\]  

There are no corrections of \(O(Q)\) to \(\delta\) for the same reason that they are absent in nucleon-nucleon potentials \[11\]. This chiral counting was established in \[11\] for both electromagnetic and axial exchange currents. In what follows we will focus on the axial-charge operator

\* We can equally well rewrite this form in terms of the Sugawara field \(U = e^{i\phi/F_\pi}\) frequently used in the literature \[6\].

\† The HBChPT as formulated is essentially equivalent to making static approximation on baryons fields, with the baryon velocity effectively conserved and with typical off-shell momentum of the baryon counted as of the same order as the pion momentum.
as it is currently receiving considerable phenomenological attention \[\text{[15]}\]. We will use the axial current $A^\mu_a$ obtained from the Lagrangian (1) by Noether construction, \textit{i.e.}, $A^\mu_a = \sum_{\phi=\pi,\pi_b} \frac{\partial L}{\partial (\partial_\mu \phi)} [X_a, \phi]$.

As emphasized recently by Weinberg \[\text{[6, 16]}\], one should restrict the use of chiral perturbation strictly to \textit{irreducible} graphs that are free of small energy denominators responsible for binding. Exchange currents belong to this class of graphs. Now corrections to the soft-pion result can come from two sources: from contact four-fermion interactions involving derivatives and from one-loop graphs. We shall argue below that there should be no contribution from the former to the order we are considering. As for the latter, there are three classes of one-loop graphs to be calculated: The first (denoted as A) consists of one-pion-exchange current with one-loop renormalization of either the internal pion line or an external nucleon line or the $\pi NN$ vertex or the $J_\mu \pi NN$ vertex; the second (denoted as B) consists of one-loop two-pion-exchange current with or without nucleon intermediate states; and the third (denoted as C) involves the four-fermion contact interaction $\Gamma$ in (1). Most of the diagrams in A can be absorbed into the standard perturbative renormalizations of the pion mass, the nucleon mass and wavefunction and of the constants $F_\pi$ and $g_A$. Since to one loop in the HBChPT there are no momentum-dependent corrections to these quantities, they are trivial and do not directly figure in our result. The only nontrivial contribution in this class comes from the $J_\mu \pi NN$ vertex, the relevant diagrams of which are given in Fig.A(a-n). The class-B graphs are given in Fig.B(a-h) and the class-C in Fig.C(a-b).

At first sight, the number of graphs may look daunting but in the HBChPT with the Lagrangian (1), many of these graphs do not contribute to the chiral order we are interested in. Some graphs vanish identically due to isospin symmetry (\textit{viz.}, Fig.Be). Other graphs such as Figs.Ca, Ai, Aj, Ak, Al, Am and An vanish because they are proportional to $v \cdot S_v$, which is zero. The graphs Cb, Ag, Ah, Bf, Bg and Bh are proportional to $S^\mu_v$, hence do not contribute to the time component since $S^\mu_v \sim O(Q/m_B)$. Thus we are left with only four graphs B(a, b, c, d) in the classes B and C and six graphs A(a-f) in the class A. We should note that while contact four-fermion interactions are very important in nuclear forces \[\text{[1, 3]}\], they are suppressed not only at the leading chiral order as shown in \[\text{[1]}\] but also at higher orders. This feature is manifest in Weinberg’s form of nonlinear chiral Lagrangian, the consequence of which constitutes one of the key elements in this work. This is in stark contrast with nuclear forces where numerous counter terms that are not readily available from experiments make systematic loop corrections highly problematic \[\text{[16]}\].

We shall take the momentum carried by the current to be zero and evaluate the two-body amplitudes corresponding to the exchange axial-charge operator. The HBChPT renders the loop integrals doable analytically. We note that even after the usual (wavefunction, mass and coupling constant) renormalization, there are additional (logarithmic) divergences left over (\textit{i.e.}, $L$ defined below). This is expected as we are dealing with a nonrenormalizable theory. This however poses no difficulty in our case. As is the standard practice \[\text{[2]}\], these divergences can be absorbed in the coefficients of the counter terms
that are next order in chiral expansion. The counter terms that absorb all the one-loop divergences in our calculation – that are subsumed in the ellipsis in (3) – are (in Weinberg representation) of the following form involving higher derivatives:

\[
\mathcal{L}_{ct} = -i \frac{d_2}{F_\pi^2} \bar{B}_v[D^{\mu}, [v \cdot D, D_\mu]] B_v \\
- \frac{4g_A}{F_\pi^2} v^\mu D_\mu \vec{\pi} \cdot \left\{ d_4^{(1)}(\bar{B}_v \tau D_B v) \times (\bar{B}_v \tau S^\mu_B B_v) \right. \\
+ i d_4^{(2)} \left( (\bar{B}_v \left[ S^\alpha S^\beta \right] \tau D_B v) (\bar{B}_v S_{\alpha B} v) + (B_v \left[ S^\alpha S^\beta \right] D_B v) (\bar{B}_v \tau S_{\alpha B} v) \right) \\
\left. + h.c. \right\}
\]

with \( d_2 = \kappa_2 + \frac{1}{24\pi^2} (1 + 5g_A^2) \eta, \) \( d_4^{(1)} = \kappa_4^{(1)} + \frac{1}{16\pi^2} (3g_A^2 - 2) \eta \) and \( d_4^{(2)} = \kappa_4^{(2)} - \frac{1}{2\pi^2} g_A^2 \eta \) where \( \eta = \frac{1}{2\pi^2} + \Gamma(1) + \ln(4\pi) - \ln(m_\pi^2) \), \( \Gamma(1) \approx -0.577215 \) and \( \kappa \)'s are finite constant counter terms.\(^4\) Now performing the calculation with dimensional regularization, we find

\[
\delta(1\pi) = \frac{Q^2}{F_\pi} \left[ \kappa_2 + \frac{1 + 3g_A^2}{8\pi^2} K_0(Q^2) - \frac{1 + 2g_A^2}{2\pi^2} K_2(Q^2) \right],
\]

\[
\delta(2\pi) = \frac{Q^2 + m_\pi^2}{F_\pi^2} \left[ \kappa_4^{(1)} + \kappa_4^{(2)} \xi + \frac{3g_A^2 - 2 - 8g_A^2 \xi}{16\pi^2} K_0(Q^2) + \frac{g_A^2}{8\pi^2} K_1(Q^2) \right]
\]

where \( q_\mu \) the four-momentum transferred from nucleon ‘1’ to nucleon ‘2’ carried by pions, \( K_0(Q^2) = -2 + \sigma y, \) \( K_1(Q^2) = 1 - \frac{\sigma^2 - 1}{2\sigma} y \) and \( K_2(Q^2) = -\frac{4}{9} + \frac{\sigma^2}{6} + \frac{\sigma(3-\sigma^2)}{12} y \) with \( y \equiv \ln \frac{\sigma + 1}{\sigma}, \)

\( \sigma \equiv \sqrt{\frac{4m_\pi^2 + Q^2}{Q^2}}. \) Note that all the constants appearing here are physical ones which should be identified with experimental values. We have replaced at appropriate places the momentum \( q_\mu^2 \) by the expansion scale parameter \(-Q^2\). The ratio of the spin-isospin matrix elements \( \xi \equiv \frac{(\langle \vec{S}_1 \times \vec{S}_2 \rangle \cdot \vec{S}_1 \times \vec{S}_2)}{(\langle \vec{S}_1 \times \vec{S}_2 \rangle^2 (\vec{S}_1 \cdot \vec{S}_2))} \) figuring in the two-pion contribution is introduced for convenience. This ratio can be estimated in various nuclear models: In simple jj shell model, in Wigner supermultiplet model as well as in Fermi gas model of the nucleus, the ratio comes out to be \( \frac{1}{3} \) so we shall set \( \xi = 1 \) in the numerical estimates made below. As defined in (4), the quantity \( \delta(1\pi) \) comes from \( J_\pi \pi NN \) vertex renormalization for which only the six graphs Figs.Aa-Af survive to the chiral order we are calculating and \( \delta(2\pi) \) from the surviving two-pion exchange graphs.

A close inspection shows that the terms involving \( d_4 \) in (3) cannot arise from single vector-meson exchanges or other excitations that are lower than the chiral scale \( \Lambda_\chi \sim 1 \)

\(^4\) \( \eta \) and \( d \)'s are singular and contain logarithms of mass. Here we briefly sketch our renormalization prescription. In calculating loop graphs, we use dimensional regularization. We encounter singular quantities in the form of \( \eta \) as given above. To remove them, we write a counter-term Lagrangian which is formally of the same chiral order as the one-loop graphs we compute. We adjust the coefficients of this counter-term so as to obtain a regular expression. The constants \( d_i \) so introduced contain two parts: one is proportional to \( \eta \) and removes divergences and the other, the finite constants \( \kappa_i \) which are to be determined (in principle) from experiments. The renormalization will be done at on-shell point for the nucleon and at zero-momentum for the pion. To one loop, the pion mass \( m_\pi \) and the decay constant \( F_\pi \) are independent of the renormalization point and can be taken from experiments.

\(^5\) We would like to thank K. Kubodera for an invaluable help on this ratio.
GeV. Furthermore a constant term proportional to \( \kappa_4^{(1,2)} \) (plus other counter terms that are in principle present even though no regularization is required) implies a zero-range interaction depicting the exchange of very massive degrees of freedom. Since we are to apply chiral perturbation expansion to only irreducible graphs while all reducible graphs are to be taken into account in calculating nuclear wave functions from a Schrödinger equation (or a relativistic generalization thereof) with a potential defined with the irreducible graphs and consequently the nuclear wave functions so obtained must contain short-range correlations, the consistency with the scheme requires that when embedded in nuclear matter, such a contact term be suppressed by nucleon-nucleon correlations. This invites us to drop the constant terms (or \( \delta \) function terms in coordinate space) \( \partial Q \). The remaining constant \( \kappa_2 \), which figures in the calculation of \( A_{\mu} \pi NN \) vertex, can be fixed by the isovector charge radius of the nucleon,

\[
\kappa_2 = -\frac{1}{6} F_\pi^2 (r^2)^V_1 \simeq -0.0856. \tag{5}
\]

This results because the \( A_{\mu} \pi NN \) vertex is related to the isovector Dirac form factor of the nucleon, \( F_V^1(t) \). This can be understood also by current algebra or in terms of vector-meson exchange \([12]\).

One can have a rough idea of how large the chiral loop corrections can be by taking, in eq.(4), \( Q \sim m_\pi \) as befits the scale involved in the chiral expansion. It comes out to be

\[
|\delta(Q \approx m_\pi)| \leq 0.05. \tag{6}
\]

To make a more quantitative estimate in nuclear matter, we have to go to coordinate space. It is in this space that short-range correlations mentioned above are most straightforwardly taken into account.\footnote{Purists might object to this procedure by arguing that one has to calculate both nuclear forces and current matrix elements to the same order of chiral expansion. This we believe is not the right way of using chiral perturbation theory in nuclei. In fact, it is not a fruitful way of doing physics as a little thought would reveal that such a so-called “consistent” approach is doomed to fail. This “failure” should however not be construed as a failure of ChPT in nuclear physics as some people seem to argue.}

Let us write the two-body axial-charge operator as

\[
\mathcal{M} = (1 + \delta_{soft}) \mathcal{M}_{soft} + \mathcal{M}_{loop} \tag{7}
\]

where

\[
\delta_{soft} = -\kappa_2 \frac{m_\pi^2}{F_\pi^2} + \frac{m_\pi^2}{4\pi^2 F_\pi^2} \left[ \frac{1}{2} \left( 2 - \frac{\pi}{\sqrt{3}} \right) - (2 + 4g_A^2) \left( \frac{13}{9} - \frac{\pi \sqrt{3}}{4} \right) \right] \simeq 0.0455. \tag{8}
\]

In (7), we have separated out the long-range \( (O(Q^2)) \) contribution, denoted \( \delta_{soft} \), from one-loop corrections to the one-soft-pion exchange. The remainder is shorter-ranged and

\footnote{In momentum space, this procedure roughly corresponds to subtracting constant terms from the expressions of (4). This is analogous to the constant subtraction one does to incorporate the Lorentz-Lorenz effect in \( \pi \)-nucleus scattering. However in the present case, because of non-analytic terms, such simple prescriptions are not reliable. We have no choice but to go to coordinate space.}
hence combined with two-pion-exchange contribution into $\mathcal{M}_{\text{loop}}$, representing the nontrivial part of one-loop corrections. The expression for $\mathcal{M}_{\text{loop}}$ is rather involved and so will not be written down explicitly in this paper. See [12] for details. In Fig. 2 is plotted the quantity $4\pi r^2 \mathcal{M}(r)$ where $r = |\vec{r}_1 - \vec{r}_2|$. The axial-charge matrix element in nuclear medium calculated in fermi-gas model with a hard-core cut-off of $d = 0.4 - 0.7$ fm is given in Fig. 3 as a function of matter density. Two key features to note in these results are: First, the loop corrections are very small for $r \geq 0.6$ fm compared with the soft-pion term consistent with the chiral filter argument and second the density dependence of the loop correction is weak. More precisely, for the reasonable hard-core cut-off of $d = 0.5$ fm, the ratio of loop correction over soft-pion $R \approx 0.067$ for $\rho = 0.5\rho_0$ and $\approx 0.089$ for $\rho = \rho_0$. Even at nuclear matter density, the loop correction represents less than 10% of the soft pion result.

To summarize, we have shown that the loop corrections to the soft-pion exchange axial-charge operator can be easily calculated in the HBChPT that provides a consistent chiral expansion. In this formalism, nucleon-antinucleon pairs are suppressed to next to the leading order. They can only contribute at higher chiral orders. Here we focused on the axial-charge operator but the same calculation can be done with no greater difficulty for the space component of the vector current, e.g., the $M_1$ operator relevant for the electrodisintegration of the deuteron where the soft-pion effect is even more spectacular [12]. An important outcome of our calculation to one loop is that the chiral filter mechanism is robust, the soft-pion term playing a predominant role with one-loop chiral corrections remaining in the noise at small momentum transfers. It is hopeful that it will survive higher loop corrections. It is pleasing that our result eq.(7) is totally free of unknown parameters inherited from more massive degrees of freedom once it is accepted that short-range correlations are operative in the transition matrix elements.

We should point out one important, though indirect, consequence of our result. Since the suppression of higher-order chiral corrections established in this paper is likely to persist independently of the environment into which the operator is embedded, it is natural to conclude that higher-order chiral corrections cannot generate the apparently significant density dependence of the meson-current enhancement in heavier nuclei observed in nature [13]. The present calculation strongly suggests that the origin of the apparent density dependence of the axial charge transitions observed in Nature lies outside of higher-order chiral corrections, thus pointing to the possibility that the basic mechanism for the enhanced axial charge in heavy nuclei is indeed, as proposed in [17], the scaling of hadron masses and quark condensate in dense medium, which is an intrinsic vacuum property [18]. Since the argument is based simultaneously on chiral symmetry and scale anomaly of QCD, it is perfectly consistent with the chiral expansion. We note that an empirical support for the scaling notion comes from a recent experiment on $^{10}$B($\vec{p}, \vec{p}'$) at 200 MeV [19].

This conclusion raises the question as to how the phenomenological Lagrangian approach of [13, 14] can be understood in terms of the result we have obtained here. In particular, since the HBChPT relies on the suppression of pairs, it is natural to ask whether
or if so, how the pair terms involving heavy mesons of \cite{13,14} can be interpreted. The answer to this question that we propose is that at least the main pair term that figures in \cite{13,14}, namely the $\sigma$ exchange, corresponds to replacing the mass of the nucleon $m_B$ by the medium quantity $m_B^*$ in the sense suggested in \cite{18}, with the vector meson mediated pair terms suitably suppressed by short-range correlations. A similar suggestion has been made in \cite{20}. This means that while two-body pair term in \cite{13,14} renormalizes the mass in the single-particle operator, three-body operators involving pairs will be needed to renormalize the two-body operator in the way it figures in \cite{17}. It would be interesting to verify this by an explicit calculation with phenomenological Lagrangians.

Acknowledgments

We are grateful for useful discussions with Gerry Brown and Kuniharu Kubodera. We would also like to acknowledge helpful comments on our work from Ulf Meissner. This work was initiated while one of the authors (M.R.) was visiting the Center for Theoretical Physics, Seoul National University. He wishes to thank the members of the Center for their hospitality. It was supported in part by the KOSEF through the Center for Theoretical Physics.
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FIGURE CAPTIONS

Fig. 1

- A: The class-A one-loop graphs that renormalize the $J_{\mu}^\pi NN$ vertex contributing to one-pion exchange axial-charge operator. The cross represents the axial current, the solid line the nucleon and the dotted line the pion. Only the graphs (a), (b), (c), (d), (e) and (f) survive to contribute.

- B: The class-B one-loop graphs (a)-(h) for two-pion-exchange axial-charge operator. Only the graphs a, b, c and d survive.

- C: The class-C one-loop graphs (a)-(b) involving four-fermion interaction. Both graphs do not contribute to the chiral order considered.

Fig. 2

- Two-body axial-charge operator as a function of the separation distance obtained by putting $\vec{r}_1 \times \vec{r}_2 (\sigma_1 + \sigma_2) \cdot \hat{r} = (\vec{r}_1 + \vec{r}_2) \sigma_1 \times \sigma_2 \cdot \hat{r} = 1$. Solid line is $4\pi r^2(1 + \delta_{soft})M_{soft}$ and dotted line is $4\pi r^2M_{loop}$.

Fig. 3

- The ratio of the matrix elements $R = \langle M_{loop} \rangle / \langle (1 + \delta_{soft})M_{soft} \rangle$ as a function of $\rho/\rho_0$ for various hard-core cut-off $d$ in the cut-off function $\theta(r - d)$. 
