On the generality of the Cohen and Glashow constraints on the neutrino velocity

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Abstract

We discuss the kinematic limits for the process $\nu_\mu \rightarrow \nu_\mu + e^+ + e^-$ in the assumption that neutrinos are superluminal. We derive our results by assuming that: \textit{i)} it exists one reference frame in which energy and momentum are conserved; \textit{ii)} the Hamilton-Jacobi equation $v = dE/dp$ is valid; \textit{iii)} the present experimental information on the neutrino velocity at different energies are correct. We show that the considered process cannot be avoided unless \textit{very} peculiar neutrino dispersion laws are assumed.
**Introduction** The OPERA experiment recently presented a $\sim 6\sigma$ evidence in favor of superluminal neutrinos [1]. By determining the time delay between $\sim 10^4$ neutrino events recorded in Gran Sasso (GS) and the proton collisions in CERN through a statistical comparison, they concluded that:

$$\delta c_\nu = v_\nu - 1 = (2.48 \pm 0.28 \pm 0.30) \times 10^{-5}$$  \hspace{1cm} (1)$$

The OPERA neutrino data sample covers a large energy range with an average energy $\langle E_\nu \rangle \approx 17$ GeV. The experimental results do not show evidence for an energy dependence of neutrino velocity. By performing a two bin analysis, the experimental collaboration reported:

$$\langle \delta c_\nu \rangle_1 = (2.16 \pm 0.77) \times 10^{-5} \quad \langle E_\nu \rangle_1 = 13.9 \text{ GeV}$$

$$\langle \delta c_\nu \rangle_2 = (2.74 \pm 0.74) \times 10^{-5} \quad \langle E_\nu \rangle_2 = 42.9 \text{ GeV}$$

If the OPERA results will be confirmed, then they will have very profound consequences on our understanding of Nature. In the meanwhile it is important to try to falsify them, by using theoretical, phenomenological and experimental argument. In ref.[2] it was argued that the existence of superluminal neutrinos may be in conflict with the fact that high energy neutrinos propagate over the CERN-GS baseline without suffering energy losses. In particular, if the neutrino limiting velocity is assumed to be $(1 + \delta c_\nu)$, the process:

$$\nu_\mu \rightarrow \nu_\mu + e^- + e^+$$  \hspace{1cm} (2)$$

become possible for $E_\nu \geq 2 m_e / \sqrt{2 \delta c_\nu} \approx 140$ MeV, with the results that OPERA neutrinos should lose a substantial part of their energy. In view of the importance of this result, we would like to comment on the generality of the underlying assumptions. Namely, we calculate the kinematic limits of the process (2) assuming that: i) it exists at least one reference frame in which space and time translations and spatial rotations are exact symmetries (and, thus, energy, spatial momentum and angular momentum are conserved); ii) the neutrino velocity is related to its energy by Hamilton-Jacobi equation; iii) the present experimental information on the neutrino velocity at different energies are correct.

**The basic assumptions** Let us assume that it exists at least one reference frame in which space and time translations and spatial rotations are exact symmetries. This implies that energy and momentum in that reference frame are conserved, i.e. we can write for a generic physical process:

$$E_{\text{ini}} = E_{\text{fin}}$$

$$p_{\text{ini}} = p_{\text{fin}}$$  \hspace{1cm} (3)$$

where $E_{\text{ini}}$ ($E_{\text{fin}}$) and $p_{\text{ini}}$ ($p_{\text{fin}}$) are the total initial (final) energy and momentum in the process.

In the above assumptions, the energy cannot depend on space and time coordinates. We can, thus, safely assume that the energy of a given particle $i$ is a function of the modulus $p_i$ of its spatial momentum, according to:

$$E_i = E_i(p_i)$$  \hspace{1cm} (4)$$

The specific form of the dispersion relation $E_i(p_i)$ may depend on the particle type $i$. We assume, however, that electrons and positrons satisfy the dispersion relation provided by special relativity:

$$E_e = \sqrt{p_e^2 + m_e^2}$$  \hspace{1cm} (5)$$

since there are very strong constraints on possible deviations, as it is e.g. reviewed in [3].
To infer the dispersion relation of neutrinos, we consider that the velocity of a given particle \( v_i \) is related to its energy \( E_i \) by the Hamilton-Jacobi equation\(^1\). We have, in fact:

\[
v_i = \frac{dE_i}{dp_i}
\]

This means that if we know the neutrino velocity \( v_\nu(p_\nu) \) as a function of its momentum we can determine the function \( E_\nu(p_\nu) \) by performing a simple integration:

\[
E_\nu(p_\nu) = \int_0^{p_\nu} dq_\nu \, v_\nu(q_\nu) + E_0
\]

This information can then be used to study the kinematic limit of the process \(^2\).

**The neutrino dispersion relation**  The observation of neutrino flavor oscillations puts very strong bounds on the possibility that neutrino dispersion relations depend on their flavor \(^4\). We can thus assume that neutrino and anti-neutrino of different flavors have all the same velocities. In this assumption, the bounds obtained from SN1987A can be used to conclude:

\[
v_\nu(p_\nu) \equiv 1
\]

with accuracy at the level of \( 10^{-9} \) or more, for a neutrino momentum \( p_\nu \leq 40 \text{ MeV} \), see \(^5\). This gives automatically:

\[
E(p_\nu) = p_\nu
\]

at low energy, where we assumed \( E_0 = 0 \) and neglected the neutrino mass\(^5\).

The OPERA results suggest, however, that neutrino velocity deviates from 1 at high energy. At \( p_\nu \sim 17 \text{ GeV} \), one has:

\[
v_\nu = 1 + \delta c_\nu
\]

where \( \delta c_\nu = (2.48 \pm 0.28 \pm 0.30) \times 10^{-5} \). Since there is no evidence for momentum dependence in the range \( E_\nu \approx 1 \div 40 \text{ GeV} \), we assume that \( \delta c_\nu \) is approximately constant above a given momentum \( \bar{p}_\nu \) which marks the transition between the low and high energy regimes. We can, thus, integrate eq.(6) obtaining:

\[
E_\nu(p_\nu) = (1 + \delta c_\nu)p_\nu - \delta c_\nu \bar{p}_\nu
\]

for \( p_\nu \geq \bar{p}_\nu \). The transition region between the low and high energy regime should be approximately at \( \bar{p}_\nu \sim 0.1 \div 1 \text{ GeV} \), between the energy region probed by SN1987A and that explored by OPERA.

**The kinematic of the process**  By using energy-momentum conservation and the dispersion relations \((5)\), \((9)\) and \((11)\), we can show that the process \(^2\) is allowed by kinematic constraints. We assume that the particles in the initial and final state are collinear. We also assume that the electron and the positron have the same momentum \( p_e \). In this assumption, we obtain:

\[
\begin{align*}
E_\nu - E'_\nu &= 2 \sqrt{m_e^2 + p_e^2} \\
p_\nu - p'_\nu &= 2p_e
\end{align*}
\]

\(^1\)As it is well known, in wave mechanics the Hamilton-Jacobi equation gives the group velocity of the particle wave packet.

\(^2\)The 95% CL bound on neutrino mass is 2.3 eV from Mainz \(^6\) and 2.5 eV from Troitsk \(^7\) beta decay experiments respectively, and 5.7 eV from SN1987A itself \(^8\). The constant in eq.(4) is bound to be smaller than 5 eV in modulus again from beta decay studies \(^9\). All these quantities are absolutely negligible in the present context.
We now consider the case in which the initial neutrino momentum is larger than $\nu$ while the final neutrino momentum is negligible $\nu' \approx 0$. We thus have:

$$ (1 + \delta c_\nu)p_\nu - \delta c_\nu \overline{p}_\nu = 2 \sqrt{m_e^2 + p_e^2} $$

$$ p_\nu = 2p_e $$

By solving the above equations, we obtain a condition:

$$ p_\nu \geq p_{\text{thr}} = \frac{\overline{p}_\nu + \sqrt{p_\nu^2 + 8m_e^2/\delta c_\nu}}{2} $$

where we neglected terms proportional to $\delta c_\nu^2$. By using $\overline{p}_\nu = 100$ MeV and $\delta c_\nu = 2.5 \times 10^{-5}$, we obtain $p_{\text{thr}} \approx 200$ MeV. Assuming transition at larger energies, e.g. choosing $p_\nu \geq 1$ GeV, the above equation gives $p_{\text{thr}} \sim \overline{p}_\nu$. All this shows that neutrinos produced in OPERA should be affected by pair production energy losses.

The above conclusion has been obtained by using eqs. (9) and (11) which corresponds to assuming a relatively fast transition between the low and high energy regime, as it is done e.g. in [10]. It continues to hold with small changes in all scenarios in which velocity is a monotonic growing function of momentum that has $\delta c_\nu(p_{\text{OPERA}}) \sim 2.5 \times 10^{-5}$, like those developed before OPERA results [11].

In order to avoid pair production, we need that:

$$ E_\nu(p_\nu)^2 - p_\nu^2 \leq (2m_e)^2 $$

at OPERA energies. This can be obtained only if we assume that neutrinos moves slower than light in the intermediate region $p_\nu \sim 0.1 - 1$ GeV, so that the contribution from a negative $\delta c_\nu$ in integral (7) compensates the effect of the $\delta c_\nu > 0$ reported by OPERA, for instance

$$ \delta c_\nu = \begin{cases} 
-2.728 \times 10^{-3} & \text{when } p_\nu = 0.1 - 1 \text{ GeV} \\
+2.480 \times 10^{-5} & \text{when } p_\nu = 1 - 100 \text{ GeV} \\
0 & \text{otherwise}
\end{cases} $$

where we set $\delta c_\nu$ to zero below 100 MeV in order not to contradict SN1987A findings but also above 100 GeV in order to account for the observation of high energy muon neutrinos [2]. The constant are arranged so that $E_\nu < p_\nu$ for $p_\nu = 0.1 - 100$ GeV, and $E_\nu = p_\nu$ elsewhere. Such a possibility seems implausible, but can be possibly further studied testing neutrino velocity in a different range of momenta and in principle considering processes, as the emission of neutrino pairs $e^- \rightarrow e^-\gamma\nu\bar{\nu}$ in vacuum. In short, it seems that the very special possibilities where the constraints of Cohen and Glashow can be evaded will be directly constrained by present and future experimental data (e.g. by K2K, MINOS, and T2K that have experimental data in the intermediate energy region).

**Conclusion** We have shown that the conclusion of [2] who argued that the existence of superluminal neutrinos may be in conflict with the fact that high energy neutrinos propagate from CERN to Gran Sasso without suffering severe energy losses follows from very general assumptions.

Namely, the kinematic limit for the process [2] can be derived by only assuming that: i) it exist one reference frame in which space and time translations and spatial rotations are exact symmetries (and, thus, energy, spatial momentum and angular momentum are conserved); ii) the neutrino velocity is related to its energy by Hamilton-Jacobi equation; iii) the present experimental information on the neutrino velocity at different energies are correct.
By considering the SN1987A bounds on neutrino velocity at low energy and assuming that the neutrino velocity above a certain momentum $\vec{P}_\nu \sim 100$ MeV (up to the OPERA neutrino energy range) is approximately constant, we derived the condition (15) that shows that neutrinos produced in OPERA should be affected by pair production energy losses. In conclusion, it appears difficult to evade the argument of [2] unless energy-momentum conservation is broken in any reference frame and/or very peculiar neutrino dispersion laws are considered.

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