Multiple and virtual photon processes in radiation-induced magneto-resistance oscillations in two-dimensional electron systems

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Recently discovered new structures and zero-resistance states outside the well-known oscillations are demonstrated to arise from multiphoton processes, by a detailed analysis of microwave photoresistance in two-dimensional electron systems under enhanced radiation. The concommitant resistance dropping and the peak narrowing observed in the experiments are also reproduced. We show that the radiation-induced suppression of average resistance comes from virtual photon effect and exists throughout the whole magnetic field range.

The recent interest in radiation related magneto-transport in two-dimensional (2D) electron gas (EG) has been stimulated by the discovery of microwave induced magnetoresistance oscillations (MIMO) and zero-resistance states (ZRS) in ultra-high mobility systems at low temperatures. Tremendous experimental and theoretical progress has been made to study this exciting phenomenon and a general understanding of it has been reached. MIMOs emerge as the magnetoresistance $R_{xx}$ of the 2D system subject to a microwave radiation of frequency $f = \omega/2\pi$, exhibiting periodic oscillation as a function of the inverse magnetic field $1/B$. It features the periodical appearance of peak-valley pairs around $\omega/\omega_c = 1, 2, 3, 4, \cdots$, i.e. a maximum at $\omega/\omega_c = j - \delta_j^{-}$ and a minimum at $\omega/\omega_c = j + \delta_j^{+}$, with $j = 1, 2, 3, 4, \cdots$ and $0 < \delta_j^{+} \leq 1/4$. Here $\omega_c$ is the cyclotron frequency and $\omega/\omega_c = j$ are the node points of the oscillation. With increasing the radiation intensity the minimum value of $R_{xx}$ drops down until a vanishing resistance is measured, i.e. ZRS.

In addition to these well-established features, a clear secondary structure between the first and second main peaks was observed experimentally and predicted theoretically with reference to two-photon process. A distinct minimum and/or maximum outside the first main peak was also observed experimentally at 45, 52 and 58 GHz, at 45 and 35 GHz, and at 30 GHz which was referred to two-photon or the second harmonic effect. Theoretically a similar structure due to two-photon process was anticipated at 60 and 40 GHz, followed by further experimental observations at lower frequency (10-50 GHz). A more prominent experimental result was recently reported by Zudov et al. at 27 GHz, where, with intensified microwave radiation the above-mentioned minima evolve into ZRS. Despite the consensus on the existence of these structures the assignment of their locations has so far been different among different groups. Physically these peak-valley structures can be due to multi-photon process or to higher harmonics of the radiation-induced high-frequency current. To ascertain these structures as arising from multiphoton effect and to help identifying their positions, it is imperative to reproduce them convincingly from a careful theoretical calculation extended to higher radiation intensity.

Another feature is the descent of the average dissipative resistance under microwave irradiation, which was experimentally observed on the high magnetic field side where MIMO shows up relatively weak. Theoretically the magnetoresistance drop was also anticipated and referred to virtual photon effect, or to microwave-induced dynamic localization. Nevertheless, the range and the degree of this resistance descent remain unclear, and a unified theoretical treatment covering higher radiation intensity to demonstrate it together with multiphoton structures is desirable.

In this Letter we report on a detailed analysis of MIMOs with enhanced radiation intensity, based on a theoretical model which covers all orders of real and virtual photon processes of the base frequency, but excludes higher harmonic current response.

The model considers that a dc electric field $E_0$ and a high frequency (HF) field $E(t) = E_0 \sin(\omega t) + E_c \cos(\omega t)$ are applied in a quasi-2D system consisting of $N_e$ interacting electrons in a unit area of the $x$-$y$ plane, together with a magnetic field $B = (0, 0, B)$ along the $z$ direction. The approach is based on the separation of the center-of-mass motion from the relative electron motion of the electrons and describes the transport state of a high-carrier-density many-electron system under a radiation field in terms of a time-dependent electron drift velocity oscillating with base radiation frequency, $v(t) = v_c \cos(\omega t) + v_s \sin(\omega t)$, and another part $v_0$, describing the slowly varying electron drift motion, with an electron temperature $T_e$ characterizing the electron heating. In the case of ultra-clean electron gas at low temperatures, $v_0$ and $T_e$ satisfy the following force- and energy-balance equations:

\begin{equation}
\frac{m dv_0}{dt} = eE_0 + e(v_0 \times B) + \frac{F_0}{N_e},
\end{equation}

\begin{equation}
N_e E_0 \cdot v_0 + S_p - W = 0,
\end{equation}

with $v_c$ and $v_s$ determined by

\begin{equation}
-m \omega v_c = eE_c + e(v_c \times B),
\end{equation}

\begin{equation}
-m \omega v_s = eE_c + e(v_c \times B).
\end{equation}
Here $e$ and $m$ are the electron charge and effective mass,

$$F_0 = \sum_{\mathbf{q}_\parallel} |U(\mathbf{q}_\parallel)|^2 \sum_{n=-\infty}^{\infty} q_\parallel J_n^2(\xi)\Pi_2(\omega - n\omega)$$

(5)

is the damping force of the moving center of mass, and

$$S_p = \sum_{\mathbf{q}_\parallel} |U(\mathbf{q}_\parallel)|^2 \sum_{n=-\infty}^{\infty} n\omega J_n^2(\xi)\Pi_2(\omega - n\omega)$$

(6)

is the averaged rate of the electron energy absorption from the HF field, and

$$W = \sum_{\mathbf{q}_\parallel} |M(\mathbf{q}_\parallel)|^2 \sum_{n=-\infty}^{\infty} \Omega_n J_n^2(\xi)\Pi_2(\Omega_n + \Omega q - n\omega)$$

(7)

is the average rate of the electron energy loss to the lattice. In the above equations, $J_n(\xi)$ is the Bessel function of order $n$, $\Omega_n(\xi) = \sqrt{(q_\parallel \cdot \mathbf{v}_0)^2 + (\mathbf{q}_\parallel \cdot \mathbf{v}_0)^2}/\omega$; $\omega_0 \equiv q_\parallel \cdot \mathbf{v}_0$, $U(\mathbf{q}_\parallel)$ and $M(\mathbf{q}_\parallel)$ are effective impurity and phonon scattering potentials, $\Pi_2(\mathbf{q}_\parallel, \Omega)$ and $\Pi_2(\mathbf{q}_\parallel, \Omega) = 2\Pi_2(\mathbf{q}_\parallel, \Omega)[n(\Omega q/T) - n(\Omega q/T)]$ (with $n(x) \equiv 1/(e^x - 1)$) are the imaginary parts of the electron density correlation function and electron-phonon correlation function of the system in the magnetic field. The $\Pi_2(\mathbf{q}_\parallel, \Omega)$ function can be expressed in the Landau representation:

$$\Pi_2(\mathbf{q}_\parallel, \Omega) = \frac{1}{2\pi^2} \sum_{n, n'} C_{n, n'}(l_0^2 q_\parallel^2/2)\Pi_2(n, n', \Omega),$$

(8)

$$\Pi_2(n, n', \Omega) = \frac{2}{\pi} \int d\varepsilon \left[ f(\varepsilon) - f(\varepsilon + \Omega) \right] \times \text{Im}G_n(\varepsilon + \Omega)\text{Im}G_{n'}(\varepsilon),$$

(9)

where $C_{n, n'+1}(Y) \equiv \ln ![n!(n + t)! - 1]^{1/2}e^{-Y[L_n(Y)]^2}$ with $L_n(Y)$ the associate Laguerre polynomial, $l_0 \equiv \sqrt{1/|eB|}$, $f(\varepsilon) = (\exp(\varepsilon - \mu) / T_e + 1)^{-1}$ is the Fermi function at electron temperature $T_e$. The density of states of the $n$-th Landau level is modeled with a Gaussian form:

$$\text{Im}G_n(\varepsilon) = -\sqrt{2\pi/\Gamma} \exp[-2(\varepsilon - \varepsilon_n)^2/\Gamma^2].$$

(10)

having a half-width $\Gamma = (8\omega_c\alpha/\pi m\mu_0)^{1/2}$ around the level center $\varepsilon_n$. Here $\mu_0$ is the linear mobility at lattice temperature $T$ in the absence of magnetic field, $\omega_c = eB/m$ and $\alpha$ is a seminemisotropic broadening parameter.

For time-independent $v_0$, we obtain the transverse and longitudinal dc resistivities from Eq. (11):

$$R_{xx} = -F_0 \cdot v_0/(N_e^2 e^2 v_0^2).$$

(11)

The (linear) magnetoresistivity is its $v_0 \to 0$ limit. Within certain field range, $R_{xx}$ can be negative at small $v_0$, but increases with increasing $v_0$ and passes through zero at a finite $v_0 \equiv v_{0f}$ implying that the time-independent small-current solution is unstable and a spatially nonuniform or a time-dependent solution may develop, which exhibits measured zero resistance. Therefore we identify the region where a negative dissipative magnetoresistance develops as that of the ZRS.

Assume that the 2DEG is contained in a thin sample suspended in vacuum at plane $z = 0$. When an electromagnetic wave illuminates the plane perpendicularly with the incident electric field $E_i(t) = E_{ix} \sin(\omega t) + E_{iy} \cos(\omega t)$, the HF electric field in the 2DEG is

$$E(t) = \frac{N_e e v(t)}{2\alpha c} + E_i(t).$$

(12)

Using this $E(t)$ in Eqs. (5) and (6), $v_1$ and $v_2$ are explicitly expressed in terms of incident field $E_{ix}$ and $E_{iy}$.

Note that all orders of real $|n| > 0$ and virtual $n = 0$ photon processes are included in the summations over $n$ in Eqs. (5), (6) and (7). The phonon contributions to $F_0$ and $S_p$ have been neglected because of the low temperature setup in the experiments. The short-range scatterers are considered to give the dominant contribution to the resistance and energy absorption for the ultra-clean samples used. The numerical calculations are performed for $x$-direction (parallel to $E_0$) linearly polarized incident microwave fields [$E_{ix} = (E_i, 0)$, $E_{iy} = 0$], using the material parameters of GaAs.

FIG. 1: The magnetoresistivity $R_{xx}$ of a GaAs-based 2DEG with $N_e = 3.0 \times 10^{15}$ m$^{-2}$, $\mu_0 = 2000$ m$^2$/Vs and $\alpha = 10$, subjected to 50 GHz radiations of incident amplitudes $E_{ix} = 4.5, 6.5$ and 8 V/cm at lattice temperature $T = 1$ K.

Careful analysis shows that the appearance of oscillatory magnetoresistance comes from real photon-assisted electron transitions between different Landau levels as
indicated in the summation of the electron density-correlation function in Eq. (5). We denote a real-photon assisted process in which an electron jumps across Landau-level spacings with the assistance (emission or absorption) of n photons as \( n\omega_c/\omega \), or \( n\omega \). This process contributes, in the \( R_{xx} \) vs. \( \omega_c/\omega \) curve, a pair structure consisting of a minimum and a maximum on both sides of \( \omega_c/\omega = n/l \). The location of its minimum or maximum may change somewhat depending on the strength of the incident microwave, but the node point, which is roughly in the center, essentially keeps at the position \( \omega_c/\omega = n/l \). Therefore we use its node position, rather than its minimum or the maximum, to identify a pair structure. Thus, the single-phonon process 1:1, the two-phonon process 2:2, the three-photon process 3:3, etc., all contribute to the minimum–maximum pair around \( \omega_c/\omega = 1 \); the single-phonon process 1:2, the two-phonon process 2:4, the three-photon process 3:6, etc., all contribute to the minimum–maximum pair around \( \omega_c/\omega = 1/2 \); etc. These are indicated in Fig. 1, as well as in Fig. 2, where we plot the evolution of magnetoresistivity \( R_{xx} \) with increasing radiation intensity \( E_i \) = 1.5, 2, 3 and 4 V/cm vs \( \omega_c/\omega \) for a GaAs-based system with \( N_c = 3.0 \times 10^{15} \text{ m}^{-2} \), \( \mu_0 = 2000 \text{ m}^2/\text{Vs} \), and \( \alpha = 5 \), irradiated by 27 GHz microwaves. Several other valley-peak pairs associated with 2-, 3-, and 4-photon assisted processes are also identified.

The predicted two-photon structure 2:1 centered at \( \omega_c/\omega = 2 \) with a minimum around \( \omega_c/\omega \approx 1.73 \) (Fig. 2(a) and (b)) reasonably compares with the second harmonic minimum around \( \omega_c/\omega = 8/5 \) observed in Refs. [1, 7]. The two-photon structures 2:3 centered at \( \omega_c/\omega = 2/3 \) and 2:5 centered at \( \omega_c/\omega = 2/5 \) are in agreement with experimental findings [1, 2, 7, 11].

Note that, with the progressive emergence of new multiphoton-related pairs when increasing radiation power, the peaks (valleys) of the low-order photon related pairs become narrower, as is clearly seen for the single-photon related pairs at \( \omega/\omega_c = 1, 2, 3 \) in both figures. This feature and the anticipated positions of the peaks and possible ZRSs, agree with the recent experimental observations [2, 7].

Another interesting aspect is that, concomitantly with enhanced \( R_{xx} \) oscillation, the average magnetoresistance descends down significantly with increasing radiation power. This resistance drop is due to effect of virtual photon processes, i.e. intra-Landau-level electron scattering by impurities with simultaneous emission and absorption of an arbitrary number of photons. To show this we plot the resistivity contributed from the virtual photon processes alone, i.e. the \( n = 0 \) term (\( J_0 \)) in Eq. (6), in both figures. The resistance suppression appears almost throughout the whole magnetic field range. It may not be previously noticed in the region where exhibits strong \( R_{xx} \) oscillation and ZRS.

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