Diffractive Contribution to the Elasticity and the Nucleonic Flux in the Atmosphere

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Abstract

We calculate the average elasticity considering non-diffractive and single diffractive interactions and perform an analysis of the cosmic-ray flux by means of an analytical solution for the nucleonic diffusion equation. We show that the diffractive contribution is important for the adequate description of the nucleonic and hadronic fluxes in the atmosphere.
It is well known that the evolution of the nucleonic cosmic ray component is controlled by two physical quantities related to high energy hadron interactions: the interaction mean-free-path $\lambda_{\text{p-air}}$, which is inversely proportional to the inelastic proton-air cross section, $\sigma_{\text{in-p-air}}$, and the average elasticity $\langle x \rangle_{\text{p-air}}$, the fraction of energy retained by the incident particle after a collision. It was shown in Refs. [1, 2] that, when one supposes the interaction mean-free-path and the mean elasticity as energy-dependent quantities, the analytical solution for the nucleonic diffusion equation in the atmosphere is given by

$$F_N(E, t) = N_0 E^{-(\gamma+1)} \exp \left[ -\frac{t(1 - (\langle x \rangle_{\text{p-air}})\gamma)}{\lambda_{\text{p-air}}(E)} \right]$$

(1)

where $t$ is the atmospheric depth and $N_0 E^{-(\gamma+1)}$ is the primary differential spectrum.

In previous papers [1, 2, 3] we have discussed the importance of the energy dependence of the leading particle spectrum, which was done through the mean inelasticity. As a first approach, we have neglected the diffractive interactions in the proton-air collisions. By diffractive interactions we mean processes like

$$a + b \rightarrow a + X$$

where particle $b$ is excited to a system $X$ with the same quantum numbers and characterized by an invariant mass $M$. This kind of process is called single diffraction (for details see, for instance, Ref. [4]). In general, diffractive interactions are neglected in cosmic-ray physics (mostly in analytical calculation of cascades), because their contributions are a priori considered to be very small.

In this paper, we consider that the leading particle distribution has two contributions,
namely non-diffractive (ND) and single-diffractive (SD), and we show that, although the latter is really small, its effect is relevant for an accurate description of cosmic-ray fluxes.

In spite of its simplicity, Eq. (1) has two parameters, \( \langle x \rangle^{p \rightarrow \text{air}} \) and \( \lambda_{\text{in}}^{p \rightarrow \text{air}} \), which must contain all dynamic aspects of the hadron collisions occurring in the atmosphere. Therefore, in the following we shall describe how these aspects can be taken into account and, mainly, how to compute diffractive effects in the leading particle distribution.

In Eq. (1), the interaction mean-free-path is given by

\[
\lambda_{\text{in}}^{p \rightarrow \text{air}}(E) = \frac{2.4 \times 10^4}{\sigma_{\text{in}}^{p \rightarrow \text{air}}(\text{mb})} \text{ (g/cm}^2) ,
\]

with the \( p \)-air inelastic cross section calculated here by means of the Glauber model [5]

\[
\sigma_{\text{in}}^{p \rightarrow \text{air}} = \int d^2 b \left\{ 1 - \exp \left[ -\sigma_{\text{tot}}^{pp} T(b) \right] \right\} ,
\]

where \( b \) is the impact parameter and \( T(b) \) is the nuclear thickness

\[
T(b) = \int_{-\infty}^{+\infty} \rho(b, z) dz
\]

given in terms of the nuclear distribution \( \rho(b, z) \) (see Ref. [3]). For \( \sigma_{\text{tot}}^{pp} \), we use in Eq. (3) the best fit of UA4/2 Collaboration [4].

Now we need to establish \( \langle x \rangle^{p \rightarrow \text{air}} \) in order to calculate the nucleonic flux. We begin by writting the partial average elasticities, SD and ND, as

\[
\langle x \rangle_{\text{SD}} = \frac{\int_0^1 x \frac{d\sigma_{SD}}{dx} dx}{\sigma_{SD}} ; \quad \langle x \rangle_{\text{ND}} = \frac{\int_0^1 x \frac{d\sigma_{ND}}{dx} dx}{\sigma_{ND}} ,
\]

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which are correctly normalized to compound the average elasticity in $pp$ collisions by

$$
\langle x \rangle_{pp} = \frac{\sigma_{SD}^{pp}}{\sigma_{in}^{pp}(x)_{SD}} + \frac{(\sigma_{in}^{pp} - \sigma_{SD}^{pp})}{\sigma_{in}^{pp}} \langle x \rangle_{ND}^{pp}.
$$

(6)

In the above expression, we are assuming that $\sigma_{in}^{pp} = \sigma_{SD}^{pp} + \sigma_{ND}^{pp}$, where $\sigma_{in}^{pp}$ is given by the Landshoff parametrization, $\sigma_{in}^{pp} = 56s^{-0.56} + 18.16s^{0.08}$.[7]

In the whole calculation, whose results we shall show farther on, we have used two models from which we borrowed the ND and SD distributions. For the non-diffractive elasticity ($\langle x \rangle_{ND}^{pp}$), we use the Interacting Gluon Model (IGM), revised by Durães et al.[8] with the purpose of including semi-hard interactions, responsible for mini-jets events. In Ref. [3], we have shown that the correspondent non-diffractive leading particle distribution produces reasonable results for $pp$ total cross section and for $p$-air inelastic cross section.

In order to include the diffractive contribution, we use the leading particle distribution of the Covolan-Montanha model [9], which reads

$$
\frac{d\sigma_{SD}}{dx} = \int \frac{d^2\sigma_{SD}}{dtdM^2} dt
$$

(7)

where, for $pp$ interactions, the invariant cross section is given by

$$
\frac{d^2\sigma_{SD}}{dtdM^2} = \frac{(3\beta_p G_p(t))^2}{16\pi} s^{2\alpha_{IP}(t)-1} \sigma_{IP_p}(M^2),
$$

(8)

$$
\sigma_{IP_p}(M^2) = 3\beta_p \xi \langle r_p^2(M^2) \rangle,
$$

(9)

with $\beta_p = 2.502 \text{GeV}^{-1}$, $\xi = 0.0764 \text{GeV}$, $G_p(t)$ is the electric form factor of the proton,
\( \alpha_p(t) = 1.08 + 0.25t \), where \( t \) is the squared four-momentum transfer, and \( \langle r^2_p(M^2) \rangle = 12.75 + 0.84 \ln M^2 \). We remind the reader that, in diffractive processes, we have \( x = 1 - M^2/s \).

As we intend to calculate hadronic interactions in the atmosphere, \( \langle x \rangle^{pp} \) must be corrected to include air effect. This is done by the procedure given in Ref. [10],

\[
\langle x \rangle_c^{p-\text{air}} = \sum_{n=1}^{n_{\text{max}}} P_n \langle \langle x \rangle^{pp} \rangle^n, \tag{10}
\]

where

\[
P_n = \frac{\int d^2b P_n(b)}{\sigma^{p-\text{air}}_{\text{in}}} \tag{11}
\]

and

\[
P_n(b) = \frac{1}{n!} \left[ \sigma^{pp}_{\text{tot}}(b) \right]^n \exp \left[ -\sigma^{pp}_{\text{tot}}(b) \right]. \tag{12}
\]

Here \( P_n \) is the probability of \( n \)-fold collisions of the primary nucleon inside the nucleus and \( n_{\text{max}} \), the maximum number of collisions, is roughly given by \( 2.3 A^{1/3} \) [10].

The diffractive contribution included in Eq. (6) comes from inelastic interactions among the incident hadron and the nucleons inside the struck nucleus. In addition to this contribution, it is necessary to consider the diffractive dissociation of the nucleus as a whole. For this reason, we add to \( \langle x \rangle_c^{p-\text{air}} \) (once again, in a weighted way) a second component coming from nuclear diffractive processes, so that our final expression is

\[
\langle x \rangle^{p-\text{air}} = \frac{\sigma^{p-\text{air}}_{\text{SD}}}{\sigma^{p-\text{air}}_{\text{in}}} \langle x \rangle^{p-\text{air}}_{\text{SD}} + \frac{\sigma^{p-\text{air}}_{\text{in}} - \sigma^{p-\text{air}}_{\text{SD}}}{\sigma^{p-\text{air}}_{\text{in}}} \langle x \rangle_c^{p-\text{air}}. \tag{13}
\]

In order to calculate \( \langle x \rangle^{p-\text{air}}_{\text{SD}} \) and \( \sigma^{p-\text{air}}_{\text{SD}} \), we use an extension of the Covolan-Montanha
model to nuclear diffractive interactions which is made by a radial scaling \( r \), \( \text{i.e.} \) multiplying Eq. (9) by

\[
\frac{\langle r^2_N(A) \rangle^{\frac{1}{2}}}{\langle r^2_p \rangle^{\frac{1}{2}}},
\]

where \( \langle r^2_N(A) \rangle^{\frac{1}{2}} = 1.096 A^{\frac{1}{3}} - 0.41 A^{-\frac{1}{3}} \) \( \text{[2]} \) corresponds to the atomic radius as a function of the atomic mass \( A \), and \( \langle r^2_p \rangle^{\frac{1}{2}} = 0.197\sqrt{12.75 + 0.84 \ln s} \) is the hadronic radius of the proton, both given in fermis.

As experimental data for the nucleonic flux cover a large range of energy, \( 1 < E (\text{GeV}) < 10^3 \), we need to establish a \textit{cutoff} for our elasticity at laboratory energy \( E = 50 \text{ GeV} \); for lower energies, the \( \langle x \rangle_{pp} \) is kept constant and the \( \langle x \rangle_{SD} \) is turned off. This is necessary basically because of two reasons: 1) for energies below 50 GeV, the elasticity given by the IGM drops steeply to very low values; 2) at these low energies the Covolan-Montanha model, which is based on the Triple-Pomeron formalism, is out of its validity range and does not hold. In order to avoid a discontinuity of the calculated flux due to this abrupt change in the elasticity, we use its value at 50 GeV as a constraint and calculate the correspondent \( \langle x \rangle_{pp} \). Then, this value (\( \langle x \rangle_{pp} = 0.6125 \)) is kept constant for lower energies, only being corrected to include air effect.

In the Fig. 1, the calculated nucleonic flux is compared with experimental data measured at sea level \([11, 12]\), using the Ryan primary spectrum \([13]\), \( N_0 = 2(\text{cm}^2.\text{s.sr.GeV})^{-1} \) and \( \gamma = 1.5 \). With only ND contribution, our solution underestimates the nucleonic flux, as shown by the dashed line in Fig. 1. The final result, including both contributions (ND and SD) and free of parameters, is shown by the solid line. One can see a significant improvement of our theoretical calculation, when diffractive effects are taken into account.
Recently, measurements of hadronic fluxes at sea level, in the energy range $1 < E \text{(GeV)} < 10^5$, were performed by the hadronic calorimeter of the KASCADE experiment [14]. In order to compare the hadronic flux calculated by means of the formalism developed in this paper with those data, we borrow the KASCADE parametrization for the pion to nucleon ratio

$$R = \frac{\pi^+ + \pi^-}{n + p} = 0.04 + 0.27 \log(E/\text{GeV}),$$

(15)

and we correct with this factor our nucleonic flux to obtain the hadronic flux

$$F_H(E, t) = (1 + 2R)F_N(E, t).$$

(16)

In the Fig. 2, the analytical solution of the hadronic flux is compared with KASCADE’s experimental data, showing a good agreement between them.

In conclusion, the diffractive contribution to the mean elasticity is taken into account by two mechanisms. The first one comes from the inelastic interactions among the incident hadron and the nucleons of the target, while the second considers the diffractive dissociation of the nucleus as a whole. The net effect of these two corrections is to increase the value of the mean elasticity. This enables us to perform a satisfactory description of the nucleonic and hadronic fluxes at sea level, without further parameters.

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Figure Captions

FIGURE 1: Nucleonic flux at sea level. Dashed line: only ND contribution. Solid line: ND and SD contributions. Open circles, Ref. [11]. Full circles, Ref. [12].

FIGURE 2: Hadronic flux at sea level. The solid line is our calculated hadronic flux. Stars, Ref. [14]. Open diamonds, Ref. [15].
$F_N(E,t)(\text{cm}^2 \cdot \text{s} \cdot \text{sr} \cdot \text{GeV})^{-1}$
$F_H(E,t)(\text{cm}^2 \cdot \text{s} \cdot \text{sr} \cdot \text{GeV})^{-1}$

$E$ (GeV)