Nonreciprocity in microwave optomechanical circuits

N. R. Bernier,1 L. D. Tóth,1 A. K. Feofanov,1,∗ and T. J. Kippenberg1,†

1Institute of Physics, École Polytechnique Fédérale de Lausanne, Lausanne 1015, Switzerland

(Dated: April 26, 2018)

Nonreciprocal devices such as isolators and circulators are necessary to protect sensitive apparatus from unwanted noise. Recently, a variety of alternatives were proposed to replace ferrite-based commercial technologies, with the motivation to be integrated with microwave superconducting quantum circuits. Here, we review isolators realized with microwave optomechanical circuits and present a gyrorator-based picture to develop an intuition on the origin of nonreciprocity in these systems. Such nonreciprocal optomechanical schemes show promise as they can be extended to circulators and directional amplifiers, with perspectives to reach the quantum limit in terms of added noise.

I. INTRODUCTION

Nonreciprocal devices, such as isolators, circulators, and directional amplifiers, are pivotal components for quantum information processing with superconducting circuits because they protect sensitive quantum states from readout electronics backaction. The commercial Faraday-effect devices typically used in these applications have a large size and weight that require bigger and more powerful dilution refrigerators, contain ferromagnetic material that does not allow them to be placed close to the quantum processor, and cause losses that limit the readout fidelity of the measurement chain. In recent years, with major advances in superconducting quantum circuits [1] and attempts to scale-up quantum processors, these shortcomings have called for chip-scale lossless nonreciprocal devices without ferromagnetic materials [2].

There have been numerous demonstrations of parametric nonreciprocal devices harnessing three-wave mixing in dc-SQUIDs or Josephson parametric converters (JPCs). A three-mode nonlinear superconducting circuit using a dc-SQUID and two linear LC-resonators was engineered to implement a reconfigurable frequency-converting three-port circulator or directional amplifier [3, 4]. Similar functionality was achieved using a single-stage JPC [5]. Meanwhile, two coupled JPCs enabled implementation of frequency-preserving Josephson directional amplifiers [6–8] and a gyror [9], which can easily be converted into a four-port circulator. A four-port on-chip superconducting circulator was also demonstrated using a combination of frequency conversion and delay [10], which can be viewed as a “synthetic rotation” [11].

Another class of superconducting devices that exhibit directionality is travelling-wave parametric amplifiers (TWPAs) [12]. Near-quantum-limited TWPAs were realized employing nonlinearity of Josephson junctions [13, 14] and kinetic inductance [15, 16]. The directionality of the TWPAs gain arises from phase matching of the parametric interaction, since amplification occurs only when the signal is copropagating with the pump. Therefore, the signal travelling in the reverse direction stays unmodified: the reverse gain of an ideal TWPAs is 0 dB. Another intrinsically directional amplifier utilizes a superconducting low-inductance undulatory galvanometer (SLUG, a version of a dc-SQUID) [17] in a finite voltage state. In contrast to TWPAs, SLUG microwave amplifiers feature reverse isolation comparable to commercial isolators [18].

Similarly, nonreciprocal devices can be engineered using the nonlinearity of the optomechanical coupling [19] both in the optical and microwave domains. The first proposal of an optomechanical nonreciprocal device considered a travelling-wave geometry [20] and was implemented in the optical domain [21, 22].

A general approach to engineer nonreciprocal photon transport compatible with optomechanical systems was outlined by Metelmann and Clerk [23]. The framework was formulated in terms of interference between coherent and dissipative coupling channels linking two electromagnetic modes. Closely following this proposal, an optical isolator and directional amplifier was demonstrated [24]. An alternative scheme that does not employ direct coherent coupling between electromagnetic modes but only optomechanical interactions was implemented with superconducting circuits resulting in electromechanical isolators and circulators [25–27]. Near-quantum-limited phase-preserving and phase-sensitive electromechanical amplifiers can be engineered in a similar way [28].

Here we explain the origin of nonreciprocity in electromechanical circuits by using the gyrorator-based isolator as a conceptual starting point. We then discuss the limitations of the current experimental implementations of electromechanical nonreciprocal devices as well as the future prospects.

II. THE GYRATOR-BASED ISOLATOR

Any linear two-port device can be described by its scattering matrix \( S \), linking the incoming modes \( \hat{a}_{i,in} \) to the...
outgoing modes $\hat{a}_{1,\text{out}}$ by

$$
\begin{pmatrix}
\hat{a}_{1,\text{out}} \\
\hat{a}_{2,\text{out}}
\end{pmatrix} =
\begin{pmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{pmatrix}
\begin{pmatrix}
\hat{a}_{1,\text{in}} \\
\hat{a}_{2,\text{in}}
\end{pmatrix}.
$$

A starting definition for nonreciprocity is that a device is nonreciprocal when $S_{21} \neq S_{12}$. Physically this corresponds to a modification of the scattering process when input and output modes are interchanged.

In order to develop an intuitive picture for nonreciprocity, we introduce the gyrator as a canonical nonreciprocal element. The gyrator is a 2-port device which provides a nonreciprocal phase shift [29], as illustrated in fig. 1a. In one direction, it imparts a phase shift of $\pi$, while in the other direction it leaves the signal unchanged. The scattering matrix of a lossless, matched gyrator is given by $S_{\text{gyr}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. Recently, new implementations of microwave gyrators relying on the non-commutativity of frequency and time translations [30] have been demonstrated with Josephson junctions [9, 10]. The gyrator in itself is not commonly used in technological applications, but rather as a fundamental nonreciprocal building block to construct other nonreciprocal devices.

An isolator is a useful nonreciprocal device that can be assembled from a gyrator and additional reciprocal elements [29]. A beam splitter can divide a signal in two; one port goes through a gyrator while the other propagates with no phase shift. Recombining the signals with a second beam splitter results in a 4-port device that interferes the signal nonreciprocally, as illustrated in fig. 2a. For a signal injected in port 1, the recombined signal after the second beam splitter interferes destructively in port 4, but constructively in port 2. In contrast, a signal injected from port 2, reaches port 3 instead of port 1, since one arm of the signal is in this case subjected to a $\pi$ shift. Overall, the device is a four-port circulator that redirects each port to the next. To obtain a two-port isolator, two of the four ports are terminated by matched loads to absorb the unwanted signal. The scattering matrix for the remaining two ports is that of an ideal isolator, $S_{\text{in}} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$.

The gyrator-based scheme helps to summarize three sufficient ingredients to realize an isolator. Firstly, an element breaks reciprocity by inducing a nonreciprocal phase shift. Second, an additional path is introduced for signals to interfere such that the scattering matrix becomes asymmetric in the amplitude with $|S_{21}| \neq |S_{12}|$. Finally, dissipation is required for an isolator, since its scattering matrix between the two ports is non-unitary and some signals must necessarily be redirected to an external degree of freedom. The gyrator-based isolator provides a framework to understand how nonreciprocity arises in microwave optomechanical implementations.

III. NONRECIPROCITY IN OPTOMECHANICAL SYSTEMS

Microwave optomechanical schemes for nonreciprocity rely on scattering between coupled modes [3, 12]. As a first step towards optomechanical isolators, we introduce optomechanical frequency conversion [31] and how it relates to the gyrator.

The simplest optomechanical scheme to couple two electromagnetic modes $\hat{a}_1$ and $\hat{a}_2$ involves coupling them both to the same mechanical oscillator $\hat{b}$ (fig. 1b). The optomechanical coupling terms $\hbar g_{0,i} \hat{a}_i^\dagger \hat{b} + \hbar g_{i} \hat{a}_i \hat{b}$ ($i = 1, 2$), where $g_{0,i}$ is the vacuum coupling rate of $\hat{a}_i$ and $\hat{b}$, can be linearized by two applied tones, detuned by $\Delta_i$ with respect to each cavity resonance [19]. In a frame rotating at the pump frequencies, and keeping only the linear terms and taking the rotating-wave approximation, the effective Hamiltonian becomes [19]

$$
H = -\hbar \Delta_1 \hat{a}_1^\dagger \hat{a}_1 - \hbar \Delta_2 \hat{a}_2^\dagger \hat{a}_2 + \hbar \Omega_m \hat{b}^\dagger \hat{b} + \hbar g_1 \left( e^{i\phi_1} \hat{a}_1^\dagger \hat{b} + e^{-i\phi_1} \hat{a}_1 \hat{b}^\dagger \right) + \hbar g_2 \left( e^{i\phi_2} \hat{a}_2^\dagger \hat{b} + e^{-i\phi_2} \hat{a}_2 \hat{b}^\dagger \right),
$$

where $\Omega_m$ is the mechanical frequency, $g_i = g_0,i \sqrt{n_{e,i}}$ is the coupling rate enhanced by the photon number $n_{e,i}$ due to the pump field, and $\phi_i$ is the phase of each pump field. For the resonant case $\Delta_1 = \Delta_2 = -\Omega_m$, the frequency conversion between the two modes through mechanical motion is characterized by the scattering matrix elements (at the center of the frequency conversion window) [32]

$$
S_{21} = \frac{2\sqrt{C_1 C_2}}{1 + C_1 + C_2} e^{i(\phi_1 - \phi_2)} \quad \text{and} \quad S_{12} = S_{21}^*,
$$

where $\kappa_i$ is the energy decay rate of mode $\hat{a}_i$, and $C_i = 4g_i^2/(\kappa_i \Omega_m)$ is the cooperativity with $\Omega_m$ the energy
Ranzani and Aumentado [3] pose the stricter requirement \(|S_{12}| \neq |S_{21}|\) for nonreciprocity in coupled-modes systems. The pump tones that linearize the optomechanical coupling break reciprocity, as they are held fixed when input and output are interchanged and impose a fixed phase \(\phi_1 - \phi_2\) for the coupling. Nevertheless, there is always a frame with a different origin of time in which the two pumps have the same phase and \(\phi_1' - \phi_2' = 0\). In that frame, the symmetry between the two ports is apparently restored.

While a change of the origin of time turns a phase-nonreciprocal system reciprocal for coupled-mode systems, it simultaneously turns reciprocal systems phase-nonreciprocal. The isolator in fig. 1B provides an example (adding frequency conversion as a thought experiment). With \((\omega_2 - \omega_1) t_0 = \pi/2\), the gyator scattering matrix transforms from \(S_{\text{gyr}} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\) to \(S_{\text{gyr}}' = e^{i\pi/2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\) while the transmission line scattering matrix transforms from \(S_{\text{tl}} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\) to \(S_{\text{tl}}' = e^{i\pi/2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\). In effect, gyators and transmission lines are mapped to each other. Importantly, the combination of a gyator and a transmission line is preserved. One interpretation is that the frame-dependent nonreciprocal phase acts as a gauge symmetry, that can realize the Aharonov-Bohm effect when a loop is created [33–35]. To build an optomechanical isolator, one method is to realize two paths between \(\hat{a}_1\) and \(\hat{a}_2\), one similar to a gyator and the other to a transmission line, which can be done following one of two proposed schemes.

The first scheme, shown in fig. 2b, combines an optomechanical link between two electromagnetic modes \(\hat{a}_1\) and \(\hat{a}_2\) and a direct coherent coupling of strength \(J\). The interaction term of the latter, given by \(H_{\text{coh}} = hJ (e^{i\theta} \hat{a}_3 \hat{a}_2^\dagger + e^{-i\theta} \hat{a}_1^\dagger \hat{a}_2)\) induces by itself conversion between the two modes as

\[
S_{21} = \frac{2\sqrt{C_{\text{coh}}}}{1 + C_{\text{coh}}} e^{i\theta} \quad \text{and} \quad S_{12} = \frac{2\sqrt{C_{\text{coh}}}}{1 + C_{\text{coh}}} e^{-i\theta} \tag{4}
\]

where \(C_{\text{coh}} = 4J^2/(\kappa_1 \kappa_2)\). Compared to eq. (3), there is an intrinsic reciprocal phase shift of \(i = e^{i\pi/2}\). By choosing the phases \(\phi_1 = \phi_2\) and \(\theta = \pi/2\), the optomechanical link realizes a reciprocal transmission line while the coherent link breaks the symmetry and realizes a gyator. The combination, with matching coupling rates, functions as an isolator between the modes \(\hat{a}_1\) and \(\hat{a}_2\). One cannot tell which of the optomechanical or direct link breaks reciprocity, as it depends on the chosen frame. Nonetheless, there is a gauge-invariant phase \(\phi = \phi_1 - \phi_2 + \theta\) that globally characterizes the broken symmetry and can be seen as a synthetic magnetic flux [24, 36].

The second scheme to realize isolation, shown in fig. 2c, uses two optomechanical conversion links with two different mechanical modes. From eq. (3), one cannot realize the scattering matrices of a gyator and a transmission line with the same reciprocal phase factor. Dissipation of the mechanical modes is the key to tune the overall reciprocal phase factor of conversion. By detuning the two decay rate of the mechanical oscillator (for simplicity, the cavities are assumed to be overcoupled).

While eq. (3) apparently fulfills the condition \(S_{21} \neq S_{12}\) for nonreciprocity, the situation is more subtle due to the fact that the two modes are in fact at different frequencies in the laboratory frame. The time-dependence for the two modes can be written explicitly to understand where the issue lies. The first incoming mode \(\hat{a}_{1,\text{in}}(t) = e^{-i\omega_1 t} A_1\) results in \(\hat{a}_{2,\text{out}}(t) = S_{21} e^{-i\omega_2 t} A_1\) and reciprocally the other mode \(\hat{a}_{2,\text{in}}(t) = e^{-i\omega_2 t} A_2\) results in \(\hat{a}_{1,\text{out}}(t) = S_{12} e^{-i\omega_1 t} A_2\), where \(\omega_i\) are the mode angular frequencies and \(A_i\) are constant amplitudes. If a new origin of time \(t_0\) is chosen, the fields transform as \(\hat{a}_i'(t) = e^{-i\omega_i t_0} \hat{a}_i(t)\), equivalent to a frequency-dependent phase shift for each mode. In this new reference frame, the scattering matrix transforms as \(S_{21}' = S_{21} e^{-i(\omega_2 - \omega_1) t_0}\) and \(S_{12}' = S_{12} e^{i(\omega_2 - \omega_1) t_0}\). For different frequencies \(\omega_1 \neq \omega_2\), there always exists a \(t_0\) for which the phases of \(S_{21}\) and \(S_{12}\) are the same. A nonreciprocal phase shift is therefore unphysical for frequency conversion, as the phase depends on the chosen reference frame. For this reason,
mechanical modes in frequency, the mechanical susceptibilities induce different phases. Advantageously over the first scheme, no direct coherent coupling must be engineered and the modes $\hat{a}_1$ and $\hat{a}_2$ do not need to have the same frequency.

The first scheme (fig. 2b) derives from a proposal by Metelmann and Clerk [23], who describe the optomechanical link as an effective dissipative interaction between the modes $\hat{a}_1$ and $\hat{a}_2$. This coupling is equivalent to a non-Hermitian Hamiltonian term $H_{\text{dis}} = -i\hbar\Gamma_{\text{dis}}(e^{i(\phi_1-\phi_2)}\hat{a}_1\hat{a}_2^\dagger + e^{-i(\phi_1-\phi_2)}\hat{a}_1^\dagger\hat{a}_2)$ with $\Gamma_{\text{dis}} = 2g_{12}/T_m$. With suitable parameters, the total interaction $H_{\text{coh}} + H_{\text{dis}}$ can be made unidirectional, with for instance a term proportional to $\hat{a}_1^\dagger\hat{a}_2$ but not $\hat{a}_1\hat{a}_2^\dagger$. We note that while only the case of resonant modes is considered here, a direct coherent $J$-coupling can also be realized between modes at different frequencies using parametric interactions. The second scheme (fig. 2c) can also be understood in that framework, as each detuned mechanical conversion link is equivalent to an effective interaction between the modes $\hat{a}_1$ and $\hat{a}_2$. Here, a direct coherent $J$-coupling can also be realized that while only the case of resonant modes is considered here, a direct coherent $J$-coupling can also be realized.

Both methods to achieve nonreciprocal isolation can be decomposed in terms of the three ingredients identified in the gyration-based isolator. At the heart is the breaking of reciprocity that occurs through the time-dependent drives applied to the system that impart a complex phase to the interaction. Then a loop constituted by two arms allow the unwanted signal to be canceled in the backward direction through destructive interference while preserving it in the forward direction. The mechanical dissipative baths are used to eliminate the backward signal. As a consequence, the nonreciprocal bandwidth (where $S_{12}$ and $S_{21}$ differ) is limited by the mechanical dissipation rates, as illustrated in fig. 2d and e. Dissipation plays a double role here, since it also gives the different reciprocal phase shift in each arms necessary to implement both a gyration and a transmission line.

IV. EXPERIMENTAL REALIZATIONS

Microwave optomechanics is uniquely suited to implement the mode structure presented in fig. 2b,c and achieve the nonreciprocal frequency conversion described above. The electromagnetic (EM) modes are here modes of a superconductive LC resonator and the mechanical modes are modes of the vibrating top plate of a vacuum-gap capacitor.

It turns out that the first, seemingly simplest, scheme shown in fig. 2c, in which two EM modes are coupled to one mechanical element as well as to each other through a direct coherent coupling, poses engineering challenges. The two EM modes must have the same resonance frequencies (within their linewidth) and the fixed direct $J$-coupling must be strong enough to achieve $C_{\text{coh}} \approx 1$. Despite these difficulties, an analogous scheme has been implemented in the optical domain, in which two photonic-crystal modes are each coupled to a phononic-crystal mode and two direct coherent optical-optical and mechanical-mechanical link are realized through waveguide buses [24]. The scheme is equivalent to the mode structure in fig. 2b as the effective interaction through the two mechanical modes is similar to that through a single one.

By contrast, the simpler plaquette of the second scheme in fig. 2c, that requires only optomechanical interactions and no frequency tuning, has been realized in microwave optomechanical circuits [25–27]. Two microwave modes of a superconducting circuit are both coupled to two vibrational modes of the top plate of a capacitor. An
example circuit is shown in fig. 3a [25]. These systems are inherently frequency converting as the two microwave modes have different resonance frequencies. The optomechanical couplings are established through four phase-locked microwave pump tones slightly detuned from the lower motional sidebands of each mode. Two frequency conversion windows open, one through each mechanical mode, and interfere with each other. By varying the relative phases of each pump tone, the gauge-independent phase $\phi$ can be tuned, realizing the isolator scheme of fig. 2c,e. An example of the measured nonreciprocal transmission is shown in fig. 3b for two values of $\phi$ for which the system isolates in each direction. There is no limitation on the achievable depth of isolation in these systems. However, since the scheme relies on the mechanical dissipation to absorb the unwanted backward signal, the nonreciprocal bandwidth is limited the bare energy decay rate of the mechanical modes, typically in the 10 - 100 Hz range, although it can be effectively increased by external damping.

An important aspect of the nonreciprocal devices based on the optomechanical interaction is their noise performance. It is apparent from the gyrator scheme presented in section II and shown in fig. 2a that the dissipative elements terminating the unwanted ports modes in turn emit back their respective thermal noise into the system. In the forward direction (fig. 3c), the emitted noise, corresponding to the added noise to the signal, is spread over a wide bandwidth and can be made quantum-limited in the high-cooperativity regime. In the backward direction however (fig. 3d), noise commensurate with the high thermal occupancy of the mechanical baths is directly emitted back from the input port, within the isolation bandwidth. External cooling of the mechanical modes is therefore required for the optomechanical isolator to protect sensitive quantum devices from back-propagating noise.

V. CONCLUSION AND OUTLOOK

Microwave optomechanical circuits form a new platform to design nonreciprocal devices that could be integrated with superconducting quantum circuits. Current realizations for isolators suffer from a narrow bandwidth, limited by the mechanical dissipation rates, as well as from high back-propagating noise. External cooling of the mechanical oscillators can mitigate both issues. Microwave circulators realized with the same technology [27] provide a better solution leading to quantum-limited noise and a nonreciprocal bandwidth only limited by the dissipation rates of the cavities [25]. Furthermore, directional amplification can be implemented with the same structures as isolators, both phase-preserving and quadrature dependent, with prospects to reach the quantum limit in terms of added noise [28]. Optomechanical nonreciprocal devices have intrinsic limits to their bandwidth, but their frequencies can be made adjustable [37] and certain applications might benefit from the versatility of such tunable narrow-band nonreciprocal devices.

Comment This manuscript was submitted as a contribution to a special issue titled “Magnet-less Nonreciprocity in Electromagnetics” to appear in IEEE Antennas and Wireless Propagation Letters.

ACKNOWLEDGMENTS

This work was supported by the SNF, the NCCR QSIT, and the EU H2020 programme under grant agreement No 732894 (FET Proactive HOT). T.J.K. acknowledges financial support from an ERC AdG (QuREM).

[1] M. H. Devoret and R. J. Schoelkopf, “Superconducting circuits for quantum information: An outlook,” Science, vol. 339, p. 1169, 2013.
[2] A. Kamal, J. Clarke, and M. H. Devoret, “Noiseless non-reciprocity in a parametric active device,” Nature Physics, vol. 7, no. 4, pp. 311-315, Apr. 2011. [Online]. Available: http://www.nature.com/nphys/journal/v7/n4/abs/nphys1893.html
[3] L. Ranzani and J. Aumentado, “Graph-based analysis of nonreciprocity in coupled-mode systems,” New Journal of Physics, vol. 17, no. 2, p. 023024, 2015. [Online]. Available: http://stacks.iop.org/1367-2630/17/i=2/a=023024
[4] F. Lecocq, L. Ranzani, G. A. Peterson, K. Cicak, R. W. Simmonds, J. D. Teufel, and J. Aumentado, “Nonreciprocal microwave signal processing with a field-programmable josephson amplifier,” Phys. Rev. Applied, vol. 7, p. 024028, Feb 2017. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevApplied.7.024028
[5] K. Sliwa, M. Hatridge, A. Narla, S. Shankar, L. Frunzio, R. Schoelkopf, and M. Devoret, “Reconfigurable Josephson Circulator/Directional Amplifier,” Physical Review X, vol. 5, no. 4, p. 041020, Nov. 2015. [Online]. Available: http://link.aps.org/doi/10.1103/PhysRevX.5.041020
[6] B. Abdo, K. Sliwa, L. Frunzio, and M. Devoret, “Directional Amplification with a Josephson Circuit,” Physical Review X, vol. 3, no. 3, p. 031001, Jul. 2013. [Online]. Available: http://link.aps.org/doi/10.1103/PhysRevX.3.031001
[7] B. Abdo, K. Sliwa, S. Shankar, M. Hatridge, L. Frunzio, R. Schoelkopf, and M. Devoret, “Josephson Directional Amplifier for Quantum Measurement of Superconducting Circuits,” Physical Review Letters, vol. 112, no. 16, p. 167701, Apr. 2014. [Online]. Available: http://link.aps.org/doi/10.1103/PhysRevLett.112.167701
[8] B. Abdo, N. T. Bronn, O. Jinke, S. Olivadese, M. Brink, and J. M. Chow, “Multi-path interferometric josephson directional amplifier for qubit readout,” Quantum Science and Technology, vol. 3, no. 2, p. 024003, 2018.
[33] K. Fang, Z. Yu, and S. Fan, “Photonic Aharonov-Bohm Effect Based on Dynamic Modulation,” Physical Review Letters, vol. 108, no. 15, p. 153901, Apr. 2012.

[34] ——, “Experimental demonstration of a photonic Aharonov-Bohm effect at radio frequencies,” Physical Review B, vol. 87, no. 6, p. 060301, Feb. 2013.

[35] L. D. Tzuang, K. Fang, P. Nussenzveig, S. Fan, and M. Lipson, “Non-reciprocal phase shift induced by an effective magnetic flux for light,” Nature Photonics, vol. 8, no. 9, pp. 701–705, Sep. 2014. [Online]. Available: http://www.nature.com/nphoton/journal/v8/n9/full/nphoton.2014.177.html#ref8

[36] V. Peano, C. Brendel, M. Schmidt, and F. Marquardt, “Topological Phases of Sound and Light,” Physical Review X, vol. 5, no. 3, p. 031011, Jul. 2015. [Online]. Available: http://link.aps.org/doi/10.1103/PhysRevX.5.031011

[37] R. W. Andrews, A. P. Reed, K. Cicak, J. D. Teufel, and K. W. Lehnert, “Quantum-enabled temporal and spectral mode conversion of microwave signals,” Nature Communications, vol. 6, p. 10021, Nov. 2015.