Interplay between $\mu - \tau$ reflection symmetry, four-zero texture and universal texture

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Abstract

In this paper, we consider exact $\mu - \tau$ reflection symmetries for quarks and leptons. By a bi-maximal transformation, the four-zero textures lead to a $\mu - \tau$ reflection symmetric form in a particular basis. In this basis, up- and down-type mass matrices from four-zero texture separately satisfy exact $\mu - \tau$ reflection symmetries, and it predicts the maximal CP violation in the Fritzsch – Xing parameterization.

In the lepton sector, in order to reconcile the $\mu - \tau$ reflection symmetries and observed $\sin \theta_{13}$, the maximal CP violation is discarded and it predicts $\delta^{PDG} \simeq \sin^{-1}\sqrt{\frac{\sin^{2} \theta_{12} \sin^{2} \theta_{23}}{\sin \theta_{13} \sin \theta_{23}}}$ \simeq 203^\circ$.

Moreover, if the universal texture $(m_f)_{11} = 0$ for $f = u, d, \nu, e$ is imposed, it predicts the lightest neutrino mass $|m_1| = 6.26$ or $2.54$ meV in the case of the normal hierarchy (NH), because the $\mu - \tau$ reflection symmetry restricts the Majorana phases to be $\alpha_i/2 = n\pi/2$. 
1 Introduction

Recently the T2K experiment indicates the CP violating Dirac phase $\delta_{CP}$ \cite{1,2}. Then CP violation (CPV) in the lepton sector draws strong attention. Among studies of flavor structures, the $\mu - \tau$ reflection symmetry \cite{3,4} is widely studied \cite{5,6,7,8,9,10,11} because it predicts bi-maximal mixing $\theta_{23} = 45^\circ$ and the maximal CPV Dirac phase $\delta_{CP} = \pm \pi/2$.

In this context, the universal texture \cite{12,13} that imposes $\mu - \tau$ permutation symmetry or $2 - 3$ symmetry for all the SM fermions is appealing. However, universal texture with $\mu - \tau$ reflection symmetry is naively fails because it predicts zero CP phase $\delta_{CP} = 0$.

In this paper, we consider exact $\mu - \tau$ reflection symmetries for quarks and leptons. By a bi-maximal transformation, the four-zero textures \cite{16,17,18,19,20,21} lead to a $\mu - \tau$ reflection symmetric form in a particular basis. In this basis, up- and down-type mass matrices from four-zero texture separately satisfy exact $\mu - \tau$ reflection symmetries, and it predicts the maximal CP violation in the Fritzsch – Xing parameterization \cite{23}.

In the lepton sector, in order to reconcile the $\mu - \tau$ reflection symmetries and observed $\sin \theta_{13}$, the maximal CP violation is discarded and it predicts $\delta_{PDG}^{CP} \simeq \sin^{-1} \left[ \sqrt{\frac{\sin^2 \theta_{13}}{m_\mu m_\tau}} \right] \simeq 203^\circ$. This value is rather close to the best fit and in the 1$\sigma$ region $\delta_{CP}/^\circ = 217^{+40}_{-28}$ \cite{24}.

Moreover, if the universal texture $(m_f)_{11} = 0$ for $f = u, d, \nu, e$ is imposed, it predicts the lightest neutrino mass $|m_1| = 6.26$ or 2.54 meV in the case of the normal hierarchy (NH), because the $\mu - \tau$ reflection symmetry restricts the Majorana phases to be $\alpha_i/2 = n\pi/2$ \cite{25,26}.

This paper is organized as follows. In the next section, we review the four-zero texture \cite{16} and $\mu - \tau$ reflection symmetry. In Sec. 3, $\mu - \tau$ symmetries are imposed on the lepton sector. The final section is devoted to conclusions.

2 Four-zero texture and $\mu - \tau$ reflection symmetry

In this section, we review the four-zero texture and its interplay of the $\mu - \tau$ reflection symmetry \cite{16,17}. First of all, the phenomenological mass matrices of the SM fermions $f = u, d, e$ and neutrinos $\nu_L$ are defined by

$$\mathcal{L} \supset \sum_f - \bar{f}_L m_{fij} f_R^j - \bar{\nu}_L m_{\nu ij} \nu^c_L j + \text{h.c.}.$$  \hspace{1cm} (1)

Diagonalization of the mass matrices $m_f = U_{Lf} m^{\text{diag}}_f U_{Rf}^\dagger$ leads to the CKM and MNS mixing matrices

$$V_{\text{CKM}} = U_{Ld}^\dagger U_{Lu}, \quad U_{\text{MNS}} = U_{Le}^\dagger U_{L\nu}.$$  \hspace{1cm} (2)

Since both the matrix have large Dirac phase, the maximal CP violation have been discussed \cite{16}. The CP phase depends on the basis of the fermions. In particular, the phase
of the CKM matrix becomes almost maximal in the Fritzsch–Xing parameterization [23]:

\[ V_{\text{CKM}} = \begin{pmatrix} c_u & s_u & 0 \\ -s_u & c_u & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-i\delta_{FZ}} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & c & s \end{pmatrix} \begin{pmatrix} c_d & -s_d & 0 \\ s_d & c_d & 0 \\ 0 & 0 & 1 \end{pmatrix}. \]  

(3)

The best fit values are found to be [27]

\[ s_u = 0.0863, \quad s_d = 0.212, \quad s = 0.0423, \quad \delta_{FZ} = 87.9^\circ. \]  

(4)

Although the original Kobayashi–Maskawa parameterization [28] has similar result \( \delta_{KM} \approx \pi/2 \) [29], the Fritzsch–Xing parameterization has reasonable physical view because it factorizes the large mixing 1-2 generations and the small one of 2-3 generations.

If we assume matrices have "four-zero texture" or "modified Fritzsch texture" [16],

\[ m_u = \begin{pmatrix} i & 0 & 0 \\ 0 & A_u & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & A_d & 0 \\ r_u B_u & r_u B_d & C_u \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -i & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad m_d = \begin{pmatrix} 0 & A_d & 0 \\ A_d & B_d & r_d B_d \\ 0 & 0 & 1 \end{pmatrix}. \]  

(5)

For later convenience, the relative phase is pressed on the \( m_u \). In this case, the rotation matrices \( U_{u,d} \) at leading order is written by the mass eigenvalues and a parameter \( r_{u,d} \).

\[ V_u \approx \begin{pmatrix} 1 & \sqrt{m_u/m_c} & 0 \\ -\sqrt{m_u/m_c} & 1 & r_u \frac{m_c}{m_t} \\ r_u \frac{m_c}{m_t} & -r_u \frac{m_c}{m_t} & 1 \end{pmatrix}, \quad V_d \approx \begin{pmatrix} 1 & \sqrt{m_d/m_s} & 0 \\ -\sqrt{m_d/m_s} & 1 & r_d \frac{m_s}{m_b} \\ r_d \frac{m_s}{m_b} & -r_d \frac{m_s}{m_b} & 1 \end{pmatrix}. \]  

(6)

Then, the CKM matrix \( V_{\text{CKM}} = U_u^T U_d \) is found to be

\[ V_{\text{CKM}} = V_u^T \begin{pmatrix} -i & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} V_d \]  

(7)

\[ \approx \begin{pmatrix} -i & -i \sqrt{m_d/m_s} & -i \sqrt{m_u/m_c} \\ -i \sqrt{m_d/m_s} & 1 & r_u \frac{m_c}{m_t} \\ r_u \frac{m_c}{m_t} & r_u \frac{m_c}{m_t} & 1 \end{pmatrix} \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_u & 0 \\ 0 & 0 & m_s \end{pmatrix} \begin{pmatrix} -i & -i \sqrt{m_d/m_s} & -i \sqrt{m_u/m_c} \\ -i \sqrt{m_d/m_s} & 1 & r_u \frac{m_c}{m_t} \\ r_u \frac{m_c}{m_t} & r_u \frac{m_c}{m_t} & 1 \end{pmatrix}. \]  

(8)

It predicts \( V_{cb} \) and \( V_{ts} \) at leading order as follows

\[ \lvert V_{cb} \rvert \approx \lvert V_{ts} \rvert \approx r_d \frac{m_s}{m_b} - r_u \frac{m_c}{m_t}. \]  

(9)
$r_u \approx r_d = \sqrt{81/32} \approx 1.59$ gives nice agreement between the prediction and the observation [31].

A bi-maximal transformation of the mass matrices by the following $U_{BM}$,

$$m_f^{BM} \equiv U_{BM}^\dagger m_f U_{BM}, \quad U_{BM} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & i \sqrt{\frac{2}{3}} & i \sqrt{\frac{2}{3}} \\ 0 & -i \sqrt{\frac{2}{3}} & i \sqrt{\frac{2}{3}} \end{pmatrix}, \quad (10)$$

leads to

$$m_u^{BM} = \begin{pmatrix} 0 & -\frac{A_u}{\sqrt{2}} \frac{B_u}{2} + C_u \frac{C_u}{2} & -\frac{A_u}{\sqrt{2}} \frac{B_u}{2} + C_u \frac{C_u}{2} - ir_u B_u \\ -\frac{A_u}{\sqrt{2}} \frac{B_u}{2} - C_u \frac{C_u}{2} + i r_u B_u & \frac{B_u}{2} - C_u \frac{C_u}{2} & \frac{B_u}{2} - C_u \frac{C_u}{2} + ir_u B_u \\ 0 & -\frac{A_d}{\sqrt{2}} \frac{B_d}{2} + C_d \frac{C_d}{2} & \frac{B_d}{2} - C_d \frac{C_d}{2} + ir_d B_d \\ \frac{A_d}{\sqrt{2}} \frac{B_d}{2} - C_d \frac{C_d}{2} + i r_d B_d & \frac{B_d}{2} - C_d \frac{C_d}{2} & \frac{B_d}{2} + C_d \frac{C_d}{2} \end{pmatrix}, \quad (11)$$

$$m_d^{BM} = \begin{pmatrix} 0 & i A_d \frac{B_d}{2} + C_d \frac{C_d}{2} & i A_d \frac{B_d}{2} - C_d \frac{C_d}{2} - i r_u B_d \\ -i A_d \frac{B_d}{2} \frac{C_d}{2} + i r_d B_d & \frac{B_d}{2} - C_d \frac{C_d}{2} & \frac{B_d}{2} + C_d \frac{C_d}{2} \\ 0 & i A_d \frac{B_d}{2} + C_d \frac{C_d}{2} \frac{A_d}{\sqrt{2}} \frac{B_d}{2} - C_d \frac{C_d}{2} - i r_u B_u \\ -i A_d \frac{B_d}{2} - C_d \frac{C_d}{2} + i r_d B_d & \frac{B_d}{2} - C_d \frac{C_d}{2} & \frac{B_d}{2} + C_d \frac{C_d}{2} \end{pmatrix}. \quad (12)$$

These matrices (11), (12) separately satisfy exact $\mu - \tau$ reflection symmetries:

$$T_u(m_u^{BM})^* T_u = m_u^{BM}, \quad T_d(m_d^{BM})^* T_d = m_d^{BM}, \quad (13)$$

where

$$T_u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad T_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}. \quad (14)$$

The form of matrices (11), (12) has been indicated a study of universal texture [12]. However, they considered no symmetry and these forms were phenomenological description.

### 3 $\mu - \tau$ reflection symmetry in the lepton sector

In this section, we impose the symmetry (13) on the lepton sector and research some predictions. If we anticipate the lepton mass matrices $m_{\nu,e}$ have the four-zero textures, they are approximately diagonalized by the rotation matrices:

$$V_e \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & r e \frac{m_\mu}{m_\tau} \\ 0 & -r e \frac{m_\mu}{m_\tau} & 1 \end{pmatrix} \begin{pmatrix} 1 & \sqrt{\frac{m_\mu}{m_\tau}} & 0 \\ \frac{m_\mu}{m_\tau} \sqrt{\frac{m_\mu}{m_\tau}} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (15)$$

and similar holds for the neutrinos. Then the MNS matrix has the similar form to the CKM matrix in the Fritzsch–Xing parameterization

$$U_0 = V_e^\dagger \begin{pmatrix} +i & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (16)$$
with

$$\sin \theta_{23}^{\text{PDG}} \simeq 1/\sqrt{2}, \quad \sin \delta_{13}^{\text{PDG}} \simeq -\pi/2, \quad \sin \theta_{13}^{\text{PDG}} \simeq \sin \theta_{23}^{\text{PDG}} \sqrt{m_e/m_\mu} \simeq 0.05, \quad (17)$$

in the standard PDG parameterization. It predicts the maximal CP violation \(\delta_{13}^{\text{PDG}} = -\pi/2\). However, small \(\sin \theta_{13}\) disagree with the observation. These are well known results in the previous studies [12, 13].

In order to derive a proper \(\sin \theta_{13}\), we change the mixing matrix Eq. (16) in the following way.

$$U = V_e^\dagger \begin{pmatrix} +i & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} V_\nu, \quad (18)$$

$$V_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (19)$$

The Majorana phases are omitted here and discussed later. Small 2-3 mixing of the \(V_e\) can be absorbed to that of the \(V_\nu\). Then,

$$U = \begin{pmatrix} 1 & \sqrt{m_e/m_\mu} & 0 \\ -\sqrt{m_e/m_\mu} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} +i & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (20)$$

It leads to

$$|U_{e3}| = |\sin \theta_{13}^{\text{PDG}}| = |i s_{13} - \sqrt{m_e/m_\mu} c_{13} s_{23}|. \quad (21)$$

Considering the parameter \(\sqrt{m_e/m_\mu} \simeq 0.07\) as a perturbation, one obtains \(c_{13} \simeq \cos \theta_{13}^{\text{PDG}}, \quad s_{23} \simeq \sin \theta_{23}^{\text{PDG}}\) and then

$$s_{13} \simeq \pm \sqrt{\sin^2 \theta_{13}^{\text{PDG}} - \frac{m_e}{m_\mu} \cos^2 \theta_{13}^{\text{PDG}} \sin^2 \theta_{23}^{\text{PDG}}} = \pm 0.140. \quad (22)$$

Here, we use \(\sin \theta_{13}^{\text{PDG}} = 0.150\) from the latest global fit [24] and the following approximate values

$$\sin \theta_{23}^{\text{PDG}} = \cos \theta_{23}^{\text{PDG}} = 1/\sqrt{2}, \quad \sin \theta_{12}^{\text{PDG}} = 1/\sqrt{3}, \quad \cos \theta_{12}^{\text{PDG}} = \sqrt{2}/\sqrt{3}. \quad (23)$$

The sign \(\pm\) corresponds to the sign of \(\cos \delta_{13}^{\text{PDG}}\). We adopt \(s_{13} = -0.140\) because the latest global fit found \(\cos \delta_{13}^{\text{PDG}} < 0\) [24].
The absolute values of reconstructed $U_{MNS}$ are found to be

$$|U_{MNS}| = \begin{pmatrix} 0.821 & 0.551 & 0.149 \\ 0.277 & 0.597 & 0.753 \\ 0.500 & 0.583 & 0.640 \end{pmatrix}. \quad (24)$$

All of the components is in the range of $3 \sigma$.

On the analogy of quark masses, the lepton mass matrix $m_{\nu,e}$ which predict the mixing matrix \cite{19} will be the following forms

$$m_\nu = \begin{pmatrix} -i & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a_\nu & b_\nu & c_\nu \\ b_\nu & d_\nu & e_\nu \\ c_\nu & e_\nu & f_\nu \end{pmatrix} \begin{pmatrix} -i & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad m_e = \begin{pmatrix} 0 & A_e & 0 \\ A_e & B_e & r_eB_e \\ 0 & r_eB_e & C_e \end{pmatrix}. \quad (25)$$

These matrices have no symmetry at first glance. However, the bi-maximal transformation in Eq. \cite{10} leads to

$$m_{\nu}^{BM} \equiv U_{BM}^{\dagger}m_\nu U_{BM} = \begin{pmatrix} -a_\nu & \frac{1}{\sqrt{2}}(b_\nu - ic_\nu) & \frac{1}{\sqrt{2}}(b_\nu + ic_\nu) \\ \frac{1}{\sqrt{2}}(b_\nu - ic_\nu) & -\frac{f}{2} - \frac{d}{2} + ie & \frac{f}{2} - \frac{d}{2} - ie \\ \frac{1}{\sqrt{2}}(b_\nu + ic_\nu) & \frac{f}{2} - \frac{d}{2} + ie & -\frac{f}{2} - \frac{d}{2} - ie \end{pmatrix}, \quad (26)$$

$$m_{e}^{BM} \equiv U_{BM}^{\dagger}m_e U_{BM} = \begin{pmatrix} 0 & \frac{iA_e}{\sqrt{2}} & \frac{iB_e}{\sqrt{2}} \\ \frac{iA_e}{\sqrt{2}} & \frac{B_e}{2} + \frac{C_e}{2} & \frac{B_e}{2} - \frac{C_e}{2} - ir_eB_e \\ \frac{iB_e}{\sqrt{2}} & \frac{C_e}{2} + ir_eB_e & \frac{B_e}{2} + \frac{C_e}{2} \end{pmatrix}. \quad (27)$$

These matrices \cite{20}, \cite{21} also separately satisfy exact $\mu - \tau$ reflection symmetries \cite{13}:

$$T_u(m_\nu^{BM})^*T_u = m_\nu^{BM}, \quad T_d(m_e^{BM})^*T_d = m_e^{BM}. \quad (28)$$

Therefore, in this basis, we found quarks and leptons satisfy some universal $\mu - \tau$ reflection symmetries. Note that the $\mu - \tau$ symmetry is not imposed on $m_\nu$ in the basis of four-zero texture \cite{23}.

### 3.1 Dirac phase $\delta^{PDG}$

In order to show the Dirac phase $\delta^{PDG}$, we evaluate the Jarskog invariant \cite{31},

$$J^{PDG} = \sin \delta^{PDG} \sin \theta_{13}^{PDG} \cos \theta_{12}^{PDG} \sin \theta_{13}^{PDG} \cos^2 \theta_{23}^{PDG} \sin \theta_{23}^{PDG} \cos \theta_{23}^{PDG}. \quad (29)$$

The phase $\delta^{PDG}$ vanishes in a limit of $s_e \equiv \sqrt{m_e/m_\mu} \rightarrow 0$. Then, the invariant can be evaluated from Eq. \cite{25} treating $\sqrt{m_e/m_\mu} \simeq 0.07$ as a perturbation.

$$J = \text{Im} [U_{\mu 3}U_{\tau 2}U_{\mu 2}^*U_{\tau 3}^*]$$

$$= \frac{c_{13}c_{12}s_{23}}{2}s_{12}s_{23}^2 + s_{13}c_{23}s_{23}(c_{12}^2 - s_{12}^2) + s_{13}c_{12}s_{12}c_{23}^2 = -0.0130 \quad (31)$$

$$\simeq -s_{13}c_{12}s_{12}s_{23}^2 = (-0.0120). \quad (32)$$
The value with (without) parentheses is induced from full calculation (only leading order).

Since $s_{13} = -0.14$ is small,

$$
\sin \delta_{PDG} \simeq \sqrt{\frac{m_e c_{13} s_{23}}{m_\mu s_{13}}} \simeq -0.390 \ (-0.360).
$$

(33)

$\cos \delta_{PDG}$ is obtained as

$$
\cos \delta_{PDG} = \frac{|U_{22}^{PDG}|^2 - (s_{12}^{PDG} s_{13}^{PDG} s_{23}^{PDG})^2 - (c_{12}^{PDG} c_{23}^{PDG})^2}{-2 s_{12}^{PDG} s_{13}^{PDG} s_{23}^{PDG} c_{12}^{PDG} c_{23}^{PDG}}
$$

(34)

$$
\left|U_{22}\right|^2(1 - \left|U_{13}\right|^2)^2 - \left|U_{13}\right|^2 |U_{23}|^2 - |U_{11}|^2 |U_{33}|^2
- 2|U_{13}||U_{12}||U_{23}||U_{33}|
= -0.920,
$$

(35)

Therefore

$$
\delta_{PDG} \simeq 203^\circ \ (201^\circ).
$$

(36)

This value is rather close to the best fit and in the 1σ region $\delta_{CP}/^\circ = 217^{+40}_{-28} \ [24]$.

### 3.2 Majorana phases, universal zero texture and mass

The standard PDG convention of the Majorana phases are

$$
U_{MNS} = UP, \ P \equiv \text{diag}(1, e^{i\alpha_2/2}, e^{i\alpha_3/2}).
$$

(37)

The neutrino mass matrix $m_\nu$ reconstructed in the four-zero basis is obtained as

$$
m_\nu = \begin{pmatrix}
 i & 0 & 0 \\
 0 & 1 & 0 \\
 0 & 0 & 1
\end{pmatrix} V_\nu P \begin{pmatrix}
 m_1 & 0 & 0 \\
 0 & m_2 & 0 \\
 0 & 0 & m_3
\end{pmatrix} P^\dagger V_\nu = \begin{pmatrix}
 i & 0 & 0 \\
 0 & 1 & 0 \\
 0 & 0 & 1
\end{pmatrix}.
$$

(38)

If this mass matrix with Majorana phases satisfies the symmetry Eq. [13], $\alpha_{2,3}/2 = n \pi/2$ ($n = 1, 2, 3, ...$) should be hold. This result agrees the previous studies by Xing [25, 26].

Moreover, if the universal texture $(m_f)_{11} = 0$ for $f = u, d, \nu, e$ [12] is imposed on these mass matrices (and assuming the normal ordering), the lightest neutrino mass $m_1$ is determined as

$$
(m_\nu)_{11} = 0 \Rightarrow |m_1| = 6.26 \text{ meV} \text{ for } (\alpha_2, \alpha_3) = (0, 0) \text{ or } (\pi, \pi)
$$

(39)

$$
= 2.54 \text{ meV} \text{ for } (\alpha_2, \alpha_3) = (0, \pi) \text{ or } (\pi, 0).
$$

(40)

Finally, the effective mass $m_{ee}$ of the double beta decay is obtained as [32]

$$
|m_{ee}| = \sum_{i=1}^{3} \left| m_i U_{ei}^2 \right|
$$

(41)

$$
= |c_{13}^2 (m_1 c_{12}^2 + m_2 s_{12}^2 e^{i\alpha_2}) + m_3 s_{13}^2 e^{i(\alpha_3 - 2\delta)}| 
$$

(42)

$$
= 0.17 \text{ meV} \text{ for } (\alpha_2, \alpha_3) = (0, 0) \text{ or } (\pi, \pi)
$$

(43)

$$
= 1.25 \text{ meV} \text{ for } (\alpha_2, \alpha_3) = (0, \pi) \text{ or } (\pi, 0).
$$

(44)

These values are rather small than other models because it vanishes in a limit of $(m_\nu)_{11} = \sqrt{m_e/m_\mu} = 0$. In particular, the phase factor $+i$ in Eq. [33] generate destructive interference for $\alpha_2 = \alpha_3$. 

7
4 Conclusions

In this paper, we consider exact $\mu - \tau$ reflection symmetries for quarks and leptons. By a bi-maximal transformation, the four-zero textures lead to a $\mu - \tau$ reflection symmetric form in a particular basis. In this basis, up- and down-type mass matrices from four-zero texture separately satisfy exact $\mu - \tau$ reflection symmetries, and it predicts the maximal CP violation in the Fritzsch – Xing parameterization.

In the lepton sector, in order to reconcile the $\mu - \tau$ reflection symmetries and observed $\sin \theta_{13}$, the maximal CP violation is discarded and it predicts $\delta^{PDG} \simeq \sin^{-1}\left[\sqrt{\frac{m_\mu}{m_\mu m_e}} s_{13} s_{23} s_{13}\right] \simeq 203^\circ$.

Moreover, if the universal texture $(m_f)_{11} = 0$ for $f = u, d, \nu, e$ is imposed, it predicts the lightest neutrino mass $|m_1| = 6.26$ or 2.54 meV in the case of the normal hierarchy (NH), because the $\mu - \tau$ reflection symmetry restricts the Majorana phases to be $\alpha_i/2 = n\pi/2$.

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