Constraining the Equation of State of Neutron Stars from Binary Mergers

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Abstract. Determining the equation of state of matter at nuclear density and hence the structure of neutron stars has been a riddle for decades. We show how the imminent detection of gravitational waves from merging neutron star binaries can be used to solve this riddle. Using a large number of accurate numerical-relativity simulations of binaries with nuclear equations of state, we have found that the postmerger emission is characterized by two distinct and robust spectral features. While the high-frequency peak has already been associated with the oscillations of the hypermassive neutron star produced by the merger and depends on the equation of state, a new correlation emerges between the low-frequency peak, related to the merger process, and the compactness of the progenitor stars. More importantly, such a correlation is essentially universal, thus providing a powerful tool to set tight constraints on the equation of state. If the mass of the binary is known from the inspiral signal, the combined use of the two frequency peaks sets four simultaneous constraints to be satisfied. Ideally, even a single detection would be sufficient to select one equation of state over the others. We have tested our approach with simulated data and verified it works well for all the equations of state considered.

1. Introduction

The first direct detection of gravitational waves (GWs) will be made by a series of advanced detectors such as LIGO, Virgo, and KAGRA which will become operational in the next five years. The inspiral and postmerger of neutron-star binaries or neutron-star–black-hole binaries, and binary black holes are expected as the sources of GWs. Population-synthesis models suggest that binary neutron star mergers (BNSs) may be the most promising source and an expected detection rate is $\sim 40 \text{yr}^{-1}$ [1].

The first evidence that the information of the postmerger signal could be extracted from the corresponding spectrum was provided by Bauswein and Janka [2], who performed a large number of simulations using a smoothed particle hydrodynamics code solving the Einstein field equations assuming conformal flatness, and employing a GW backreaction scheme within a post-Newtonian approximation. In particular, they pointed out the presence of a peak at high-frequency in the spectrum, which corresponds to a fundamental fluid mode with $m = 2$ of the hypermassive neutron star (HMNS) [3], and showed it correlated with the properties of the equation of state (EOS).

By performing a large number of accurate simulations in full general relativity of equal-mass and unequal-mass BNSs with a number of different nuclear EOSs, we have examined the spectral
properties of the postmerger GW signal. In this proceeding, we summarize our recent results in our papers [4, 5].

2. Numerical Setup

Our results have been obtained in full general relativity solving the Einstein equations with the McLachlan code [7, 8]. The solution of the relativistic hydrodynamics equations is obtained using the Whisky code [9, 10]. The stars are modeled by using a nuclear EOS and we have considered five different models, i.e., APR4, ALF2, SLy, H4, GNH3. As the corresponding TOV sequences are shown in Fig. 1, all of these EOSs satisfy the current observational constraint on the observed maximum mass in neutron stars, i.e., $2.01 \pm 0.04 M_{\odot}$ obtained for the pulsar PSR J0348+0432 [6]. Rather than using tables, it is more convenient to use $n$ piecewise polytropic approximations to these EOSs [11], expressing the "cold" contribution to pressure and specific internal energy as

$$p_c = K_i \rho^{\Gamma_i}, \quad \epsilon_c = \epsilon_i + \frac{K_i \rho^{\Gamma_i - 1}}{(\Gamma_i - 1)},$$

where $\rho$ and $K$ are the rest-mass density and the polytropic constant, respectively. We adapt $n = 4$ for all cases, because it is sufficient to obtain an accurate representation of the different EOSs.

In addition, to model the thermal effects arising from the merger, the cold pressure is augmented through an ideal-fluid EOS, so that the total pressure and specific internal energy are $p = p_c + p_{th}$, $\epsilon = \epsilon_c + \epsilon_{th}$, with $p_{th} = \rho \epsilon_{th} (\Gamma_{th} - 1)$ [12]. Following Ref. [13], we use $\Gamma_{th} = 2$, because we have verified that our results are not sensitive to this choice [5]. Finally, to span a larger range in stellar compactness and go beyond the one covered by the nuclear EOSs above, we have considered a sixth EOS given by a pure polytrope with $\Gamma = 2$ and $K = 123.6$ in units where $c = G = M_{\odot} = 1$.

For each EOS, we have considered five equal-mass binaries with average gravitational mass...
Figure 2. Snapshots of the rest-mass density on the $(x,y)$ plane for the binary with H4 EOS and $\bar{M} = 1.3 M_\odot$. The panels refer to four characteristic times: the initial time, the time of the merger, the time right after the merger (i.e., at $t = 1.7 \text{ ms}$), and the time with a quasi-stationary core (i.e., at $t = 10.0 \text{ ms}$).

at infinite separation in the range $\bar{M} \equiv (M_1 + M_2)/2 = (1.275 - 1.375)M_\odot$ for the APR4 EOS, $(1.225 - 1.325)M_\odot$ for the ALF2 EOS, $(1.250 - 1.350)M_\odot$ for the GNH3, H4, and SLy EOSs, and $(1.350 - 1.450)M_\odot$ for the $\Gamma = 2$ polytrope. We have also considered two unequal-mass binaries for the GNH3 and SLy EOSs having $\bar{M} = 1.300 M_\odot$ and mass ratio $\simeq 0.92$. The binaries are modeled as irrotational in quasicircular orbits and computed with the LORENE code [14], assuming a conformally flat metric. To increase resolution we have employed a reflection symmetry across the $z = 0$ plane, a $\pi$-symmetry condition across the $x = 0$ plane, and a moving-mesh refinement via the Carpet driver [15]. We have used six refinement levels, the finest having a resolution of $0.15 M_\odot \simeq 0.221 \text{ km}$, and extracted the GWs near the outer boundary at a distance $R_0 = 500 M_\odot \simeq 738 \text{ km}$.

3. Results

First of all, we show a dynamics of a binary with H4 EOS and $\bar{M} = 1.3 M_\odot$ in Fig. 2, in which the two stellar cores collide and bounce repeatedly as a result of the strong rotation and very high densities. The figure contains four different panels for the rest-mass density on the $(x,y)$ plane at four characteristic times: the initial time, the time of the merger, the time right after the merger, and the time with a quasi-stationary core.

As pointed out by several authors (e.g., [2, 16]), the power spectral density (PSD) of the postmerger GW signal has three clear peaks. As examples, two binaries with gravitational masses $\bar{M}/M_\odot = 1.325$, and APR4 and GNH3 EOSs are presented in Fig. 3. The top panel shows the evolution of the $\ell = m = 2$ plus polarization of the strain aligned at the merger for sources at a polar distance of 50 Mpc. On the other hand, the bottom panel shows the spectral densities $2\tilde{h}(f)f^{1/2}$ for the two EOSs, comparing them with the sensitivity curves of Advanced LIGO and of the Einstein Telescope. The dotted lines refer to the whole time series, while the
Figure 3. Top panel: evolution of $h_+$ for representative binaries with the APR4 and GNH3 EOSs (dark-red and blue lines, respectively) for sources at a polar distance of 50 Mpc. Bottom panel: spectral density $2\tilde{h}(f)f^{1/2}$ the merger for the two EOSs and sensitivity curves of Advanced LIGO (green line) and ET (light-blue line); the dotted lines show the power in the inspiral, while the circles mark the “contact frequency”.

The most interesting and important result of our spectral analysis is that there is a very clear correlation between the low-frequency peak $f_1$ and the compactness $C$. The results of the fitting procedure for the low-frequency peak are collected in the left panel of Fig. 4. Also shown as a shaded gray band is the estimate of the total error, which is effectively dominated by the fitting procedure of the PSD. The behavior of the low-frequency peak is remarkably consistent with a cubic polynomial function. The essentially universal behavior of the $f_1$ frequency with compactness provides a powerful tool to constrain the EOS. On the other hand, the right panel of Fig. 4 shows the behavior of the high-frequency peak $f_2$ as a function of the average rest-mass density $(\bar{M}/\bar{R}^3)^{1/2}$. We perform a linear fit for each EOS.

Finally, we discuss how to use them to constrain the EOS. Let us assume that the GW signal from a BNS has been detected and all of the spectral features are clearly identifiable. Using the measured values of $f_1$ and $f_2$ we can draw on the $(\bar{M}, \bar{R})$ plane a series of curves given by the relations $\bar{M} = \bar{M}(\bar{R}, f_1)$ (solid gray line) and $\bar{M} = \bar{M}(\bar{R}, f_2; \text{EOS})$ (solid colored lines). This is shown in Fig. 5 for the ALF2 EOS. We can see that the $\bar{M} = \bar{M}(\bar{R}, f_1)$ relation intersects each of the various equilibrium curves (colored dashed lines) at one point (e.g., at $\bar{M} \simeq 1.325 M_\odot$, $\bar{R} \simeq 12.3$ km for the ALF2 EOS), but also other crossings take place for the other EOSs. However, when using also the relations $\bar{M} = \bar{M}(\bar{R}, f_2; \text{EOS})$, some EOSs can be readily excluded (e.g., APR4, SLy, and GNH3, in our example) and only the ALF2 and H4 EOSs have “near crossings” between the equilibrium-models curves and the frequency-correlations curves. Fortunately, the uncertainty can be removed if the mass of the binary is known from the inspiral signal. In this case, in fact, there will be a horizontal line in the $(\bar{M}, \bar{R})$ plane that will
**Figure 4.** Left panel: the low-frequency peaks as a function of the stellar compactness for the six different EOSs considered, where crosses and filled squares refer to equal-mass and unequal-mass binaries, respectively. The cubic fit is shown by a solid black line, while the gray band is the estimate of the total errors. Right panel: the high-frequency peaks as a function of the average rest-mass density.

**Figure 5.** Examples of use of the spectral features to constrain the EOS. Once a detection is made, the relations $M = M(R, f_1)$ and $M = M(R, f_2; \text{EOS})$ (colored solid lines) will cross at one point the curves of equilibrium configurations (colored dashed lines). Information of the mass of the system (horizontal line) will provide a fourth constraint, removing possible degeneracies. The panel refer to the ALF2, but all other EOSs behave in the same way.
break the degeneracy imposing four simultaneous constraints. This is shown with the horizontal light-blue line, which clearly intersects the three curves relative to the ALF2 EOS at one point only (green solid circle). Despite the simplifying assumptions, this method shows that even a single detection of a GW signal with high SNR and from which the mass of the binary can be calculated, would be sufficient to set tight constraints on the EOS. This approach works well for all of other binaries considered.

4. Conclusions
We have carried out a large number of accurate and fully general-relativistic simulations of the inspiral and postmerger of BNSs with nuclear EOSs. This has allowed us to have a comprehensive view of the spectral properties of the complex postmerger GW signal and to highlight the presence of two robust frequency peaks. We have shown for the first time that the low-frequency peaks exhibit a correlation with the stellar compactness that is essentially EOS independent and that can be used to constrain the EOS once the peak is measured. In addition, the combined use of other EOS-dependent correlations from the high-frequency peaks can further constrain the EOS. In principle, if the mass is known from the inspiral and the peaks are clearly measurable, a single detection would be sufficient to set constraints on the EOS.

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