MODEL-INDEPENDENT EVIDENCE FOR DARK ENERGY EVOLUTION FROM BARYON ACOUSTIC OSCILLATIONS

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ABSTRACT

Baryon acoustic oscillations (BAOs) allow us to determine the expansion history of the universe, thereby shedding light on the nature of dark energy. Recent observations of BAOs in the Sloan Digital Sky Survey (SDSS) DR9 and DR11 have provided us with statistically independent measurements of $H(z)$ at redshifts of 0.57 and 2.34, respectively. We show that these measurements can be used to test the cosmological constant hypothesis in a model-independent manner by means of an improved version of the $Om$ diagnostic. Our results indicate that the SDSS DR11 measurement of $H(z) = 222 \pm 7$ km s$^{-1}$ Mpc$^{-1}$ at $z = 2.34$, when taken in tandem with measurements of $H(z)$ at lower redshifts, imply considerable tension with the standard ΛCDM model. Our estimation of the new diagnostic $Om h^2$ from SDSS DR9 and DR11 data, namely, $Om h^2 \approx 0.122 \pm 0.01$, which is equivalent to $\Omega_{m0} h^2$ for the spatially flat ΛCDM model, is in tension with the value $\Omega_{m0} h^2 = 0.1426 \pm 0.0025$ determined for ΛCDM from Planck+WP. This tension is alleviated in models in which the cosmological constant was dynamically screened (compensated) in the past. Such evolving dark energy models display a pole in the effective equation of state of dark energy at high redshifts, which emerges as a smoking gun test for these theories.

Key words: cosmology; observations – dark energy – methods: statistical

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1. INTRODUCTION

There is ample observational evidence to suggest that the expansion of the universe is accelerating, fuelled perhaps by dark energy (DE) which violates the strong energy condition, so that $\rho + 3P < 0$. While the cosmological constant with $8\pi G T_{ik} = \Lambda g_{ik}$ and $P = -\rho \equiv -\Lambda/8\pi G$, envisioned by Einstein almost a century ago, fulfills this requirement, the tiny value associated with $\Lambda$ has prompted theorists to look for alternatives in which dark energy evolves with time, including modified gravity (Sahni & Starobinsky 2000; Carroll 2001; Peebles & Ratra 2003; Padmanabhan 2003; Sahni 2004; Copeland et al. 2006; Sahni & Starobinsky 2006; Clifton et al. 2012; Shafieloo 2014).

Meanwhile, the very simplicity of the cosmological constant has prompted the search for null-diagnostics, which can inform us, on the basis of observations, whether or not DE is the cosmological constant.

One such diagnostic is the Statefinder $r = \bar{a}/a H^3$ (also called the jerk) whose value remains at unity only in ΛCDM (Sahni et al. 2003; Alam et al. 2003) (also see Chiba & Nakamura 1998; Visser 2004). Thus, if observations were to inform us that $r \neq 1$, then this would imply a falsification of the cosmological constant hypothesis.

A second null diagnostic, $Om(z)$, is defined as (Sahni et al. 2008; Zunckel & Clarkson 2008)

$$Om(z) = \frac{\text{H}^2(z) - 1}{(1 + z)^3} = \frac{1}{1 + z}, \quad \text{H} = H(z)/H_0.$$  (1)

A remarkable feature of $Om$ is that its value remains at $\Omega_{m0}$ in ΛCDM. In all other DE models, the value of $Om(z)$ evolves with time.

While the Statefinder has proven to be exceedingly versatile in differentiating between rival DE models, a distinguishing feature of $Om$ is that it depends only upon the expansion rate, $H(z)$, and is therefore easier to determine from observations than $r$ (see also Shafieloo et al. 2012; Visser 2004; Chiba & Nakamura 2000; Arab salvarmi & Sahni 2011). $Om$ can also be written as a two-point diagnostic (Shafieloo et al. 2012)

$$Om(z_i; z_j) = \frac{\text{H}^2(z_i) - \text{H}^2(z_j)}{(1 + z_i)^3 - (1 + z_j)^3}. \quad (2)$$

with $Om(z; 0)$ defined in Equation (1). Consequently, if the Hubble parameter is known at two or more redshifts, then $Om(z_i; z_j)$ can be reconstructed and one can address the issue of whether or not DE is the cosmological constant. Recent observations of baryon acoustic oscillations (BAOs) in the Sloan Digital Sky Survey (SDSS) catalog have paved the way for reconstructing $Om$ by determining statistically independent values of $H(z)$ at several redshifts (Delubac et al. 2014). Using their determination of $H(z = 2.34) = 222 \pm 7$ km s$^{-1}$ Mpc$^{-1}$, Delubac et al. (2014) reported a surprising 2-2.5σ tension with the predictions of standard ΛCDM with best-fit Planck parameters. In this Letter, we revisit this inconsistency using a null diagnostic approach involving an improved version of $Om$. We affirm the results of Delubac et al. (2014) and also demonstrate that screened models of dark energy provide a better fit to the BAO data than ΛCDM. (Note that Delubac et al. (2014) is a preprint, and it is possible that their results may be revised prior to publication.)

2. DATA, METHOD, AND RESULTS

An advantage of using BAOs to deduce the nature of DE is that the former are measured on large scales, and hence determined...
primarily by the linear regime of gravitational instability, a theory that has been meticulously developed and studied over the past several decades. In this Letter, we reconstruct \(Om\) using recent determinations of \(H(z)\) and attempt to answer the question as to whether DE behaves like the cosmological constant. Consider first the following small improvement of \(Om\), which yields large dividends. Multiplying both sides of Equation (2) by \(h^2\), where \(h = H_0/100\text{ km s}^{-1}\text{Mpc}^{-1}\), results in the improved \(Om\) diagnostic

\[
Om h^2(z_i; z_j) = \frac{h^2(z_i) - h^2(z_j)}{(1 + z_i)^3 - (1 + z_j)^3},
\]

where \(h(z) = H(z)/100\text{ km s}^{-1}\text{Mpc}^{-1}\). A significant advantage of \(Om h^2\) is that, for \(\Lambda CDM\),

\[
Om h^2 = \Omega_{\text{dm}} h^2.
\]

Since observations of the cosmic microwave background (CMB) inform us that (Ade et al. 2013) \(\Omega_{\text{dm}} h^2 = 0.1426 \pm 0.0025\), it follows that for the cosmological constant \(\Lambda\)

\[
Om h^2 = 0.1426 \pm 0.0025.
\]

Consequently, a departure of \(Om h^2\) from the above value would signal that DE is not \(\Lambda\). As we shall show, this is precisely what is suggested by the recent measurement of \(H(z) = 222 \pm 7\text{ km s}^{-1}\text{Mpc}^{-1}\) at \(z = 2.34\) made on the basis of BAOs in the Ly\(\alpha\) forest of BOSS DR11 quasars (Delubac et al. 2014).

Note that for \(n\) independent measurements of \(H(z_i), z_i \in z_1 \cdots z_n\), the pairwise diagnostic \(Om h^2(z_i; z_j)\) can be determined in \(n(n-1)/2\) different ways. In the present case, \(n = 3\), which leads to three independent measurements of \(Om h^2(z_i; z_j)\), namely,

\[
\begin{align*}
Om h^2(z_1; z_2) & = 0.124 \pm 0.045 \\
Om h^2(z_1; z_3) & = 0.122 \pm 0.010 \\
Om h^2(z_2; z_3) & = 0.122 \pm 0.012,
\end{align*}
\]

where \(z_1 = 0, z_2 = 0.57\), and \(z_3 = 2.34\), and the Hubble parameters at these redshifts are \(H(z = 0) = 70.6 \pm 3.3\text{ km s}^{-1}\text{Mpc}^{-1}\) (Efstathiou 2014), \(H(z = 0.57) = 92.4 \pm 4.5\text{ km s}^{-1}\text{Mpc}^{-1}\) (Samushia et al. 2013), and \(H(z = 2.34) = 222 \pm 7\text{ km s}^{-1}\text{Mpc}^{-1}\) (Delubac et al. 2014).

Note from Equation (6) that the model-independent value of \(Om h^2\) is quite stable and is in tension with the \(\Lambda CDM\)-based value \(Om h^2_{\Lambda CDM} = 0.14\). For the pair \(Om h^2(z_1; z_3)\) and \(Om h^2(z_2; z_3)\), the tension with \(\Lambda\) is at over 2\(\sigma\).

We note here that these results are quite robust and not unduly sensitive to the value of \(H(z = 0)\). Assuming \(H(z = 0) = 73.8 \pm 2.4\text{ km s}^{-1}\text{Mpc}^{-1}\), which is the best estimated value by Riess et al. (2011), results in \(Om h^2 = 0.234 \pm 0.009\). While using \(H(z = 0) = 67.1 \pm 1.2\), which is the best-fit value for the Hubble parameter from Planck concordance \(\Lambda\)CDM model, results in \(Om h^2 = 0.234 \pm 0.009\). Hence, it is clear that the ‘final’ value of \(H(z = 0)\) should not significantly affect the derived value of \(Om h^2\), which suggests that our results for this quantity are robust. Likewise, using the more recent SDSS galaxy BAO DR10 and DR11 result of \(H(z = 0.57) = 96.8 \pm 3.4\text{ km s}^{-1}\text{Mpc}^{-1}\) (Anderson et al. 2014), we get \(Om h^2(z_2; z_3) = 0.120 \pm 0.010\), which is in agreement with our earlier estimations of \(Om h^2\). This is mainly due to the high-precision measurement of \(H(z = 2.34)\), which makes the determination of \(Om h^2\) less sensitive to the value of \(H(z)\) at lower redshifts.

Thus far, our treatment has been model-independent and we have refrained from commenting on the physical implications of the SDSS measurements of \(H(z)\). However, as already noted in Delubac et al. (2014), these implications can be quite serious. Indeed, the expansion rate at \(z = 2.34\), namely, \(H(z = 2.34) = 222 \pm 7\text{ km s}^{-1}\text{Mpc}^{-1}\) (Delubac et al. 2014), could be in tension not only with \(\Lambda CDM\), but also with DE models based on the general relativistic equation \(\kappa = 8\pi G/3\)

\[
H^2(z) = \kappa [\rho_{DE}(z) + \rho_{\text{dm}}(1 + z)^3] \geq \kappa \rho_{DE}(z) \geq 0.
\]

Note that by setting \(\rho_{DE} = 0\) in Equation (7), one finds

\[
H^2(z) = \frac{\Lambda}{3} + \kappa \rho_{\text{dm}}(1 + z)^3 - f(z), \quad f(z) > 0.
\]

Examples of this behavior may be found in: (1) theories in which \(\Lambda\) relaxes from a large initial value via an adjustment mechanism (Dolgov 1985; Bauer et al. 2010); (2) cosmological models based on Gauss-Bonnet gravity (Zhou et al. 2009); and (3) braneworld models (Sahni & Shtanov, 2003), etc. More generally, this behavior occurs in modified gravity (e.g., in scalar-tensor gravity) when the effective gravitational constant \(G_{eff}(z) < G_{eff}(0) \equiv G\), if we define \(\rho_{DE}(z)\) using the present value of \(\kappa\) in Equation (7) following Boisseau et al. (2000); Sahni; Starobinsky (2006).

A key feature of such models is that if \(f(z)\) grows monotonically with redshift (but at a slower rate than \((1 + z)^3\)) in order to preserve the matter-dominated regime), then a time will come when \(\Lambda/3\) is exactly balanced by \(f(z)\), resulting in \(H^2(z) \simeq \kappa \rho_{\text{dm}}(1 + z_e)^3\). At \(z_e\), the effective equation of state of dark energy, \(w(z_e)\), develops a pole at which \(|w(z_e)| \rightarrow \infty\). This is easily seen from the expression (Sahni & Starobinsky 2006)

\[
w(x) = \frac{2q(x) - 1}{3(1 - \Omega_m(x))} \equiv \frac{(2\varsigma/3) \ln H / dx - 1}{1 - (H_0/H)^2 \Omega_{m0} x^3} > 0,
\]

where \(x = 1 + z, \Omega_m(x) = \Omega_{m0} x^3 H_0^2 / H^2(x), \) and \(q\) is the deceleration parameter. One finds from Equations (9) and (10) that at \(f(z_e) = \Lambda/3\)

\[
w(z_e) = - \frac{(1 + z_e) f'(z_e)}{H^2(z_e) - \kappa \rho_{\text{dm}}(1 + z_e)^3}.
\]
In other words, \( w(z) \) diverges when \( f(z_*) = \Lambda / 3 \) and \( H^2(z_*) \simeq \kappa \rho_0 m (1 + z_*)^3 \), provided \( f'(z_*) \neq 0 \).

As a specific example of a model with this behavior, consider the Braneworld model proposed in Sahni & Shtanov (2003) and described, in a spatially flat universe, by the equations:

\[
\frac{H^2(z)}{H_0^2} = \Omega_\Lambda + \Omega_{\text{om}} (1 + z)^3 + 2 \Omega_\Lambda - \sqrt{\Omega_{\text{om}} (1 + z)^3 + \Omega_\Lambda + \Omega_\Lambda},
\]

\[
\Omega_\Lambda = 1 - \Omega_{\text{om}} + 2 \sqrt{\Omega_\Lambda},
\]

(12)

where the densities \( \Omega \) are defined as

\[
\Omega_{\text{om}} = \frac{\rho_0}{3 m^2 H_0^2}, \quad \Omega_\Lambda = \frac{\Lambda}{3 m^2 H_0^2}, \quad \Omega_\Lambda = \frac{1}{l_c^2 H_0^2}.
\]

(13)

\( l_c = m^2 / M^3 \) is a new length scale (\( m \) and \( M \) refer to the four and five dimensional Planck masses, respectively), and \( \Lambda \) is the brane tension associated with a three-dimensional brane embedded in a 4+1-dimensional bulk space-time.

As shown in Figure 1, the expansion rate in this model can drop below that in \( \Lambda \)CDM at high \( z \). It can therefore better account for the lower than anticipated value for \( H(z = 2.34) \) discussed in Delubac et al. (2014). Note also the pole in \( w(z) \) at \( z \approx 2.4 \). Note that the presence of the pole in this model does not signal any pathologies since \( w(z) \) is an effective equation of state. This is also true for the other theoretical models in which \( w(z) \) exhibits a pole (Bauer et al. 2010; Zhou et al. 2009). Note that a pole in the equation of state may be possible to determine in future Type Ia supernova (SNIa) data sets using model-independent reconstruction, as demonstrated in Shafieloo et al. (2006). Finally, one might point out that although dark energy in the Braneworld behaves like a phantom, it does not share the latter’s pathologies (Sahni & Shtanov 2003; Sahni 2005). The model also agrees with SNIa observations Alam & Sahni (2006).

A detailed analysis of models with screened/compensated dark energy will be the subject of future work.

There is another important issue that requires elaboration. The derived value of \( H(z = 2.34) \) given by Delubac et al. (2014) is scaled at \( r_d = 147.4 \) Mpc from the Planck+WP fitting of concordance cosmology, where \( r_d \) is the sound horizon at the drag epoch. One may argue that playing with the parameter, \( r_d \), may help reconcile the concordance model with data. However, this cannot be true since the value of \( r_d \) used to derive \( H(z = 2.34) \) has been obtained assuming \( \Lambda \)CDM and the discrepancy between \( Omh^2 \) and \( \Omega_{\text{om}} h^2 \) obtained by us is also based on \( \Lambda \)CDM cosmology—see Equation (4). One should, however, note that it is possible to lower the value of \( r_d \) by increasing the expansion rate in the early universe through the inclusion of an extra relativistic species. However, this would imply a departure from the minimal standard \( \Lambda \)CDM model (though not in its dark energy sector).

It is also important to point out that a lower (than in \( \Lambda \)CDM) value of \( H(z) \) at high \( z \) would affect the growth of matter density perturbations, perhaps speeding them up relative to \( \Lambda \)CDM. Indeed, on scales much smaller than the horizon and within the framework of general relativity, linearized perturbations are described by the equation (Peebles 1980)

\[
\ddot{\delta} + 2H \dot{\delta} - 4\pi G \bar{\rho} \delta = 0.
\]

(14)

Clearly, a lower value of \( H(z) \) results in a suppression of the damping term \( 2H \dot{\delta} \) (relative to \( \Lambda \)CDM) and therefore to a faster growth in \( \delta \). This could have important implications for structure formation which will soon be probed to great depth and accuracy by SKA, LSST, etc. However, Equation (14) generically does not hold in modified gravity theories. In particular, in scalar-tensor gravity this equation has formally the same form at sufficiently small scales but with the effective gravitational constant \( G_{\text{eff}}(t) \) instead of \( G \) (Boisseau et al. 2000). Therefore, a detailed analysis of perturbation growth in such models needs to be carried out before firm predictions can be made about \( \delta(z) \).
3. SUMMARY

To summarize, this Letter demonstrates that the recent estimation of $H(z = 2.34)$ from BAO observations in the SDSS DR11 data is in tension with CMB observations assuming standard ΛCDM. This tension is independent of the current value of the Hubble parameter $H(z = 0)$. In our analysis, we have implemented an improved version of the Om diagnostic, called $Omh^2$, which can be derived by having independent measurements of $H(z)$ at two redshifts. $Omh^2$ should be equal to $Ω_0 mh^2$ if the universe corresponds to spatially flat ΛCDM. Our estimated value of $Omh^2 ≈ 0.122 ± 0.01$ (which should also be the value of $Ω_0 mh^2$ for ΛCDM) is robust against variations of the Hubble parameter $H_0$ and is in strong tension with $Ω_0 mh^2 = 0.1426 ± 0.0025$ given by Planck+WP.

In the absence of systematics in the CMB and SDSS data sets, our results suggest a strong tension between concordance cosmology and observational data. Since resolving this discrepancy by changing initial conditions and/or the form of the primordial spectrum might be difficult (note that $Ω_0 mh^2$ does not change much if one deviates smoothly from the power-law form of the primordial spectrum (Hazra et al. 2013; Hazra & Shafieloo 2014; Hazra et al. 2014), allowing dark energy to evolve seems to be the most plausible approach to this problem. Evolving dark energy models which might accommodate the SDSS data better than ΛCDM include those in which the cosmological constant was screened in the past. The effective equation of state in such models develops a pole at high $z$, which emerges as a smoking gun test for such scenarios.

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