A methodology to predict residual life in LCF regime

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Abstract. In this paper a methodology to calibrate a nonlinear constitutive model, implemented in a finite element commercial code and able to correctly predict the cyclic behaviour of material, is proposed. This model allows to take into account all the main phenomena that occur when a metallic material is subjected to cyclic loadings, in particular cyclic hardening or softening, Bauschinger effect, ratcheting and shakedown. High Cycle Fatigue and Low Cycle Fatigue tests under different strain levels have been performed on an aluminum alloy for high temperature applications, and the obtained experimental data have been used to determine the four constitutive parameters of the isotropic and kinematic hardening parts of the model. The comparison between the number of stabilized cycles predicted by numerical simulations and experimental data has been very satisfying. Using a self-made post-processing software, the Basquin-Manson-Coffin, Neu-Sehitoglu, Chaboche and Skelton damage models have been applied to determine the residual life of the specimen and to compare the results.

1. Introduction

Today the continuous improvement in performances demanded from machine components obliges engineers to adopt more and more accurate design methodologies in order to predict, as realistically as possible, the components response under working conditions. This necessity is fundamental especially when the stress state to which the material is subjected, is such as to cause the overcoming of yield stress and, consequently, the plastic strain is not negligible.

In this paper a nonlinear constitutive model implemented in a finite element (FE) commercial code is proposed. The model allows to predict the response of metallic materials under cyclic loadings in plastic field, the typical working conditions of Low Cycle Fatigue (LCF) regime [1, 2]. Using the data obtained from experimental High Cycle Fatigue (HCF) and LCF tests campaigns under different strain levels on a commercial aluminum alloy (2024-T351), the constitutive model has been calibrated and used to reply the fatigue tests on specimens. The results in terms of number of cycles to reach the stabilized conditions and the shape of stabilized cycles, compared with experimentation, have been very satisfactory.

The stress and the strain states calculated by FE simulations have been elaborated in order to predict the residual life of the specimen. By means of a self-made post-processing numerical code, four literature damage models have been applied and, as it has been done for the calibration of the constitutive model, the results have been compared with the actual number of cycles to failure registered on specimens during experimental tests. This activity allows to understand which between

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The constitutive model here presented predicts the nonlinear cyclic response of the material combining two parts. The first one, which takes into account the isotropic hardening, allows to predict the cyclic hardening or softening, while the second one, which considers the nonlinear kinematic hardening, allows to model the Bauschinger effect, the ratcheting and the shakedown [3-6].

The analytical formulation adopted to implement this model in the FE commercial code is derived from the combined hardening model of Chaboche [7], where the final relation related to the isotropic hardening is:

\[
dR = b(Q - R)dp,
\]

where \( R \) is a generic isotropic variable that describes the size of elastic domain, \( b \) and \( Q \) are material constants that represent respectively the speed of the stabilization and the asymptotic value of \( R \) corresponding to hysteresis loop in stabilized conditions, and \( dp \) is the increment of the accumulative plastic strain.

The nonlinear kinematic hardening formulation is represented instead by:

\[
dX = \frac{2}{3}C\varepsilon_p - \gamma X dp,
\]

where \( X \) is the back stress tensor that permits to take into account the translation of the elastic field in the stress-strain plane, \( C \) and \( \gamma \) are material constants representing respectively the kinematic hardening modulus and the speed of the decrease of \( C \) values with the increase of plastic strain, and \( \varepsilon_p \) is the plastic strain tensor. The derivative form of equations (1) and (2) is justified because they represent a time evolution of the respective variables.

The formulation used to implement the constitutive model in the FE commercial code derives from equations (1) and (2). In details, the isotropic part computes the evolution of the yield stress by:

\[
\sigma_Y = \sigma|_0 + Q_o\left(1 - e^{-be_{pl}}\right).
\]

Equation (3) is obtained integrating the equation (1) with respect to time [8]: \( \sigma_Y \) is the yield stress, \( \sigma|_0 \) is the value of yield stress resulting from the first application of the load, \( Q_o \) is equal to \( Q \), \( b \) is the same of equation (1), and \( e_{pl} \) is the plastic strain.

The nonlinear kinematic hardening part presents instead the following form:

\[
\dot{\varepsilon} = C(\sigma_Y)^{-1}(\sigma - \alpha)\dot{e}_{pl} - \gamma \alpha \dot{e}_{pl}.
\]

Equation (4) is obtained elaborating the equation (2) and the analogies can be appreciated considering that \( \alpha \) is the back stress tensor named \( X \) in equation (2) and \( \dot{e}_{pl} = d\varepsilon_p \).

3. Experimental tests campaign

In order to characterize the fatigue behaviour of the 2024-T351 aluminum alloy, a complete experimental test campaign has been carried out. A first sequence of tensile tests and HCF tests, both at five temperature levels, followed by LCF tests at two temperature levels and three imposed strain levels for each temperature have been performed. In this way, the Young Modulus, the yield stress, the elongation to fracture, the fatigue limit at \( 10^7 \) cycles, the parameters of Basquin curve and the parameters of LCF strain-endurance curve, according to the Manson-Coffin relation, have been...
determined, as it will be explained in paragraph 5.1. In the following, the y axis scale of graphs has been normalized because the data are protected by industrial confidentiality restrictions.

3.1. High Cycle Fatigue tests
The tests have been performed using the 100 kN Amsler Vibrofore of the Department of Mechanics of Politecnico di Torino and the staircase methodology has been followed. The cylindrical geometry of the specimens adopted has been imposed by the user manual of the testing machine and it is reported in figure 1, while in figure 2 the FE model of the specimen is represented.

Figure 1. Geometry of the specimen adopted in High Cycle Fatigue tests (dimensions in mm).

For each temperature level a minimum of 15 meaningful specimens has been tested with amplitude ratio $R = 0.1$ to avoid problems in the phase of load reversal. The results have been then converted in $R = –1$ condition using the Goodman relation. The tests have been carried out with a frequency of about 160 Hz and they have been automatically interrupted when $10^7$ cycles have been reached. Figure 3 reports an example of results: the line is the best fitting obtained using the Basquin curve reported in equation (5) where $\sigma_a$ is the alternate stress, $\sigma'_f$ and $b$ are material constants, and $N_f$ is the number of cycles. The best fitting has been obtained using the least squares method: on a log-log plot the equation (5) is a straight line, so it is easy to determine the parameters $\sigma'_f$ and $b$. This law models the material behaviour in the HCF regime and its parameters are used in the Basquin-Manson-Coffin damage model as it is explained in paragraph 5.1.

$$\sigma_a = \sigma'_f \cdot (2 \cdot N_f)^b.$$ (5)

Figure 2. FE model of the specimen adopted in the numerical simulations.

Figure 3. Example of HCF experimental results.

3.2. Low Cycle Fatigue tests
The tests have been performed using, for each temperature and imposed strain level, three specimens of the same geometry of HCF tests but without threads in the grip sections, with a narrow section of 30 mm in length and a total length of 95 mm. This characterization has been done in a laboratory outside the Politecnico di Torino using a 250 kN servohydraulic testing machine which works with a
frequency of about 2 Hz. All tests have been conducted with an amplitude ratio $R = -1$ and the data have been acquired in terms of hysteresis loops and number of cycles to failure. An example is reported in figure 4, where are plotted the first ten cycles and then one cycle every ten. As in paragraph 3.1, figure 5 represents the results gathered in terms of plastic strain and number of cycles to failure testing two specimens for each of the three different strain ranges. Now the line is the best fitting obtained using the Manson-Coffin relation reported in equation (6), where $\varepsilon_{pl}$ is the plastic strain, $\varepsilon'_f$ and $c$ are material constants, and $N_f$ is the number of cycles. This law models the material behaviour in the field of LCF and its parameters are used by the Basquin-Manson-Coffin damage model, as it is explained in paragraph 5.1.

$$\varepsilon_{pl} = \varepsilon'_f \cdot (2 \cdot N_f)^c.$$

(6)

**Figure 4.** Example of experimental hysteresis loops.

**Figure 5.** Experimental data and best fitting with the Manson-Coffin relation of equation (6).

4. Constitutive model calibration and comparison between numerical and experimental results

4.1. Model calibration

The nonlinear constitutive model has been calibrated using the LCF experimental data [9] shown, as an example, in figure 4. For each temperature level the experimental stabilized cycle related to the widest strain range applied has been isolated: the couples stress-plastic strain extrapolated from the half stabilized cycle in tension have been used for the calibration of the isotropic part. From the first one to the stabilized one, the hysteresis loops have been used to calibrate the kinematic part of the model. For each cycle have been determined both the amplitude of the elastic field as half the difference between the yield stress in tension and compression, and an equivalent plastic strain range. All these data have been specified in the commercial FE code in tabular form.

4.2. Numerical simulations

The numerical simulations have been realized with the aim to reply the LCF test conditions on the specimen. The FE model used is reported in figure 2 and its geometry is explained in paragraph 3.2.

One of the grip sections has been constrained with a joint on all the nodes, while on the other grip section a symmetric cyclic displacement has been imposed, whose magnitude produces the same strain of the tests. The simulations cover the whole time spent by the material to reach the stabilized conditions, that is when the experimental hysteresis loops do not change anymore their shape and their position on the stress-strain plane. The simulations have been performed on a Linux pc with a Pentium IV processor with 2 GB of RAM, and they take about 45 minutes to be completed.
4.3. Comparison between numerical results and experimental data
In figure 6 a comparison between experimental data and numerical results for one of the imposed
strain level investigated is reported. For a better clearance in the plot, only the stabilized cycles are
presented, but the simulations cover the whole time in which the cycles come to stabilization and the
transient behaviour of the material is correctly predicted. It is possible to appreciate the goodness of
numerical results obtained: the two cycles are very close in terms of shape and dimensions, so the
material response under cyclic plastic loadings is correctly predicted by the constitutive model.

![Figure 6. Comparison between the numerical stabilized cycle and the experimental one.](image)

5. Residual life estimation
The results in terms of cyclic stress and strain, calculated through the FE simulation illustrated, have
been elaborated to obtain the life prediction under LCF working conditions. In this study four
literature damage models have been used: Basquin-Manson-Coffin, Neu-Sehitoglu, Chaboche and
Skelton model.

For using these models in a fast and simply way, a numerical code has been developed: this
software allows to choose the damage model to use and to import the stress-strain histories of the
material, giving back the component life in tabular or graphic form [10]. The number of cycles
predicted has been then compared with the actual life registered during the tests.

5.1. Basquin-Manson-Coffin damage model
This is a strain-based model [11] that links the number of cycles \( N_f \) with the total strain range \( \Delta \varepsilon_{\text{tot}} \) which is the sum of an elastic component \( \Delta \varepsilon_e \) described by the Basquin law presented in equation (5),
and a plastic component \( \Delta \varepsilon_p \) governed by the Manson-Coffin relation of equation (6). The whole
model is:

\[
\frac{\Delta \varepsilon_{\text{tot}}}{2} = \frac{\Delta \varepsilon_e}{2} + \frac{\Delta \varepsilon_p}{2} = \frac{\sigma'_{f}}{E} \cdot \left( 2 \cdot N_f \right)^b + \varepsilon'_{f} \cdot \left( 2 \cdot N_f \right)^c,
\]

(7)

where \( E \) is the Young Modulus, \( \sigma'_{f} \) and \( b \) are material constants related to the elastic part of the total
strain, \( \varepsilon'_{f} \) and \( c \) are material constants related to the plastic part of the total strain.

The determination of the four constants can be done, as in figures 3 and 5, interpolating separately
the experimental data related to HCF and LCF tests. If the data are reported in log-log plots, the
Basquin and the Manson-Coffin interpolating relations become linear, so it is easy to determine the \( \sigma'_{f}, \)
\( b \), \( \varepsilon'_{f} \) and \( c \) parameters. The four constants can be also determined analytically [11] from tests at only
two fixed strain range levels, for which the measured quantities are stress range and life.

5.2. Neu-Sehitoglu damage model
This model predicts the total damage \( D_{\text{tot}} \) summing the damage caused by mechanical fatigue \( D_{\text{mech}} \),
the part caused by creep \( D_{\text{ creep}} \) and the one caused by oxidation \( D_{\text{ox}} \) [12], as reported:
The term $D_{\text{mech}}$ is calculated by the Basquin-Manson-Coffin relation from which it is possible to determine the number of cycles $N_{\text{mech}}$. Assuming that the fracture happens when the total damage is equal to one, the damage $D_{\text{mech}}$ can be calculated as:

$$\frac{\Delta \varepsilon_{\text{mech}}}{2} = \frac{\sigma_f'}{E} \cdot (2N_{\text{mech}})^{b} + \varepsilon_f' (2N_{\text{mech}})^{c} \rightarrow D_{\text{mech}} = (N_{\text{mech}})^{-1}$$

(9)

The four parameters $\sigma_f'$, $b$, $\varepsilon_f'$ and $c$ are the same determined for Basquin-Manson-Coffin model presented in paragraph 5.1. The creep damage $D_{\text{creep}}$ can be calculated directly:

$$D_{\text{creep}} = \Phi_{\text{creep}} \cdot \int_{0}^{t_c} A \cdot e^{-\left(\frac{\Delta H}{RT}\right)} \left(\frac{\alpha_1 \sigma + \alpha_2 \sigma_H}{K}\right)^{m} dt \rightarrow D_{\text{creep}} = \frac{1}{N_{\text{creep}}}$$

(10)

where $t_c$ is the whole working time, $\sigma$ the equivalent Von Mises stress, $\sigma_H$ the hydrostatic stress, $K$ the maximum stress in tension that acts on the direction of maximum stress, $A$ and $m$ material parameters depending on temperature, $\Delta H$ the activation energy, $R$ the universal gas constant and $T$ the working temperature. The material parameters have been found in literature [13, 14] for a similar aluminum alloy and calibrated for a better description of the material of this study. $\Phi_{\text{creep}}$ is a phase factor between mechanical and thermal strain and it is represented as:

$$\Phi_{\text{creep}} = \exp[{-0.5 \left(\frac{\dot{\varepsilon}_{\text{th}}}{\dot{\varepsilon}_{\text{mech}}} - 1\right)^{-1} \left(\varepsilon_{\text{creep}}\right)^{-1}]}].$$

(11)

The phase factor $\Phi_{\text{creep}}$ is an exponential function depending on the ratio $\dot{\varepsilon}_{\text{th}}/\dot{\varepsilon}_{\text{mech}}$ between the thermal and the mechanical strain rate and on the factor of creep sensitivity $\varepsilon_{\text{creep}}$. This function has a bell shape distribution which assumes different values in isothermal fatigue ($\Phi_{\text{creep}} = 4.3937 \cdot 10^{-2}$), in in-phase thermo-mechanical fatigue ($\Phi_{\text{creep}} = 1$) and in out of phase thermo-mechanical fatigue ($\Phi_{\text{creep}} = 3.7267 \cdot 10^{-6}$). The oxidation damage term has not been considered because the constitutive parameters required could not been determined with the available experimental data.

5.3. Chaboche damage model
This model calculates the total damage taking into account stresses [15, 16]. The general formulation is similar to Neu-Sehitoglu, but the contributions to the total damage $dD_{\text{tot}}$ are only due to a mechanical damage $dD_{\text{mech}}$ and to a creep damage $dD_{\text{creep}}$, without a term linked to oxidation. In equation (12) is presented the general form of the model:

$$dD_{\text{tot}} = dD_{\text{mech}} + dD_{\text{creep}}.$$  

Equation (12) is written in an incremental form and this allows to avoid the superposition of effects, formally incorrect in nonlinear problems, and to consider the interactions between mechanical and creep damage, which can accumulate independently one from the other. The components $dD_{\text{mech}}$ and $dD_{\text{creep}}$ are modelled as:

$$dD_{\text{fat}} = \left[1 - (1 - D)^{-b+1}\right] a \left[\frac{\sigma_{\text{max}} - \sigma_m}{A(1-D)}\right]^{-b} dN$$

$$dD_{\text{creep}} = \left(\frac{\sigma}{M}\right)^{r} (1-D)^{-h} dt$$

(13)

where $\sigma_{\text{max}}$ and $\sigma_m$ are respectively the maximum and the mean stress, $D$ the instantaneous damage, $M$, $a$, and $b$ material coefficients, $A$, $r$ and $h$ parameters depending on temperature. All these six parameters have been deducted from literature [7] for a similar aluminum alloy.
5.4. Skelton damage model
This model calculates the total damage through an energetic approach [17, 18]. The energy density that the material dissipates during hysteresis cycles can be linked to the fatigue crack propagation. Skelton and al. [17] have demonstrated that the fatigue life of materials can be also related to the total dissipated energy density \( \Delta U_{\text{tot}} \), corresponding to the area circumscribed by hysteresis loops in stress-strain plane, as:

\[
\Delta U_{\text{tot}} = \Delta U_{\text{mech}} + \Delta U_{\text{th}} = \oint \sigma \cdot d\epsilon_{\text{mech}} + \oint \sigma \cdot d\epsilon_{\text{th}}.
\]

(15)

Moreover it has been verified that the amount of energy that a material can dissipate before fracture is almost independent from the loading conditions and it is influenced by temperature only. The sum of all the energies \( U_i \) calculated for each stabilized cycle [19] is therefore equal to a constant \( K_s \) related to the material, as:

\[
\sum_{i=1}^{N_f} U_i = \text{constant} = K_s.
\]

(16)

The value of \( K_s \) is computable summing the areas of all the stabilized hysteresis cycles obtained from an LCF test performed until fracture (figure 2). The number of cycles \( N_f \) can be then predicted by equation (17), where \( U_{\text{stab}} \) is the energy density of the generic stabilized cycle:

\[
N_f = K_s / U_{\text{stab}}.
\]

(17)

5.5. Analysis of results
In table 1 the results in terms of number of cycles before fracture are reported and compared with the actual life of specimens tested; with B-M-C it has been indicated the Basquin-Manson-Coffin model.

| Strain range | 1st specimen | 2nd specimen | Mean | B-M-C | Neu-Sehitoglu | Chaboche | Skelton |
|-------------|--------------|--------------|------|-------|---------------|----------|---------|
| \( 0.8(\Delta \epsilon_{\text{max}}/2) \) | 2050         | 2340         | 2195 | 2420  | 2315          | 2296     | 2230    |
| \( 0.6(\Delta \epsilon_{\text{max}}/2) \) | 6890         | 7510         | 7200 | 7615  | 7480          | 7390     | 7110    |

The results show that the Basquin-Manson-Coffin model is the one that gives the greatest overestimation of the number of cycles. This fact can be justified because its formulation takes into account only the damage related to mechanical fatigue, omitting creep and oxidation. Moreover, the calibration of the four parameters could have been better if more LCF experimental data would have been available. The predictions are however close to the actual life of the first specimen. The Neu-Sehitoglu model is more severe than the Basquin-Manson-Coffin model because also the creep damage is counted for and the predictions are more reliable. Chaboche model is quite similar to Neu-Sehitoglu model in the results and, in general, it is more severe. Unfortunately, the determination of its constitutive parameters has not been rigorous because of the lack of appropriate experimental data; the parameters used have been taken from literature and they refer to an aluminum alloy similar to the one investigated in this work. Finally, the Skelton model permits to obtain the best predictions because it works directly on the actual hysteresis loops of the material.

6. Conclusions
In this paper a methodology to predict life in LCF regime, consisting in the calibration of a combined hardening model to simulate the material cyclic behaviour and in the estimation of the number of cycles to failure, has been proposed. The numerical results of FE simulations are satisfactory in terms of shape and dimensions of the stabilized hysteresis cycle: the areas of the experimental and the numerical cycles are almost the same. The life estimations show that, starting from the most
overestimating one, the investigated models can be ordered as it follows: Basquin-Manson-Coffin, Neu-Sehitoglu, Chaboche and Skelton. This is justified considering that the level of detail in the analytical formulation increases following this sequence and all the main phenomena which contribute to the damage are modelled. Consequently, Skelton model seems to be the most appropriate to investigate LCF conditions: working directly on the actual hysteresis loops of the material, all the contributions to the damage are taken into account. The determination of material parameters (omitted as they are protected by industrial confidentiality restrictions), based on the available experimental data, has been rigorous for Basquin-Manson-Coffin, Neu-Sehitoglu and Skelton models, while for Chaboche they have been deducted from literature, so the confidence on the results obtained from this model are a little bit lower with respect to the others. The parameters estimation is the most critical point when damage models have to be used. When appropriate experimental data are available it is possible to perform a rigorous model calibration and the possible mismatch between experiments and predictions can be ascribe to the model formulation. Consequently, it is not obvious that a damage model calibrated through experimental data gives the exact number of cycles to failure.

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