Buckling of Rock-Socketed Piles Embedded in Soft Soils

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Abstract. Buckling of rock-socketed piles embedded in soft soils is intensively explored by utilizing three dimensional Finite-Element model in this paper. For pile-soil systems with a wide range of values of geometric parameters and material properties, the critical buckling load and buckling mode are obtained, and compared with the results based on Winkler foundation model. The simulation results show that the exact critical buckling load is a little bigger than that based on Winkler foundation model. The effect of rock-socketed ratio on the critical buckling load with three types of constrains is also investigated. With increasing rock-socketed ratio, the critical buckling load increases dramatically. The obtained results are useful to optimize the design of the pile-soil system against buckling in geotechnical applications.

1. Introduction

Elastic fibers subjected to compressive loads can buckle, even when embedded inside an elastomeric matrix[1-4]. In recent years, buckling behaviors have drawn considerable attention in microtubules[5], fiber-reinforced composites[6], pipelines on seabeds[7], plant roots growing in soil[8], packaged DNA in viruses[9], silicon nano-wire on substrate[10], coil tubing in oil-field operations[11] and so on. The buckling instability of long slender piles particularly in soft soils is a key consideration in geotechnical design. Potential buckling failure also exists when slender piles are embedded in soft soil, erodible soil and liquefiable soil[12]. Furthermore, with the ongoing evolution of piles with higher capacity (i.e., higher allowable stress), the common design practices that fully embedded piles will not buckle before yielding of the pile cross section are no longer valid.

Extensive studies have been performed on the buckling response of axially loaded piles[13-22]. The interaction between the compressed pile and the soil can be modelled as a pile embedded in an elastic foundation with a series of springs acting along the length of the pile[23]. Winkler foundation model by Hetenyi has been widely used and extended to the buckling analysis of piles constrained laterally by elastic matrix[23]. Based on Winkler foundation model, Davisson and his coworker investigated the effects of partial embedment and different degrees of freedom at the ends of the pile on the buckling behavior[24, 25]. Reddy and Valsangkar used the energy method to compute the buckling capacity of piles[26]. An elastic Winkler foundation with spring stiffness varying linearly with depth was established to investigate the stability of end-bearing piles, and the buckling loads and the buckling modes were presented[27]. Based on cusp catastrophe theory, Chen et al. investigated the buckling of a top-free bottom-fixed pile and a top-hinged bottom-hinged pile, and obtained the critical load for the pile buckling[28]. An analytical method was presented based on the modified Vlasov foundation model for the buckling of fully embedded single piles in an elastic medium subjected to
axial load, and the numerical results show that the medium stiffness has significant influence on the buckling behaviors[29]. A general power distribution of the soil modulus along the length was proposed to investigate the critical buckling load of partially embedded piles in elastic foundations[30]. A nonlinear Winkler foundation model has been proposed to investigate the importance of the bending-buckling interaction in seismic design of piles in liquefiable soils by using numerical techniques[31]. The effect of the friction force on the buckling load of embedded piles was analyzed based on Winkler foundation model, and found to be insignificant[32].

In the above mentioned analyses, however, it was assumed that the supporting elastic matrix can be modeled as Winkler foundation, which ignores the interaction between different contact areas[23]. Generally, the Winkler model does provide a reasonably accurate method of representing the soil in the estimation of the lateral response of piles. However, the spring stiffness of the Winkler foundation cannot be easily and exactly given by the geometrical and material properties of the matrix[33-35]. Therefore, the buckling behaviors of rock-socketed piles are difficult to be analytically studied by Winkler foundation model. In this paper, we will establish three dimensional Finite Element model to explore the buckling behaviors of rock-socketed piles. The aim of this study is to introduce a three dimensional finite element model to deal with the buckling of rock-socketed piles by taking into account both the lateral stiffness and rock constrain. The buckling behaviors of fully embedded rock-socketed piles with different boundary conditions are simulated numerically. We propose a mount of numerical examples of the pile-soil system for a wide range of values of geometric parameters and material properties. The obtained results for simplified cases of the current problem are compared with available analytical solutions based on Winker foundation model.

2. Finite element model for rock-socketed pile-soil system

2.1. Finite element model

![Figure 1. Schematics and finite element model of rock-socketed piles.](image)

In order to make clear the effect of the elastomeric matrix on the instability mode of the pile, we have performed finite element (FE) simulations of the pile-soil system using the commercial package Abaqus. Figure 1 shows the schematics and finite element model of the pile-soil system. In these analyses, the pile and the matrix are both discretized using brick elements (Abaqus element type C3D8R). The accuracy of the mesh is found out through a mesh refinement study, resulting in 16000 elements for the elastomeric matrix and 2400 elements for the pile. In all FE simulations, unless otherwise specified, we consider a pile model with the elastic modulus $E_p=59$ GPa and the Poisson’s ratio 0.35 under three boundary conditions of both ends. The pile contacts the surrounding soil in normal direction, neglecting the tangential friction in main consideration of the response of rock-socketed pile.
2.2. Buckling analysis

Based on the three-dimensional FE model, we utilize the BUCKLE module to analyze the stability of the embedded piles. Through the buckling analysis, buckling mode and critical load can be obtained. As an example, Figure 2 shows the five leading-order buckling modes and critical loads. In the following of this paper, we will focus on the first-order buckling mode and critical load, which is the minimum critical load and characterizes the loading ability of the embedded piles. It is noted that the wavenumber of 1st buckling mode is not one, which is different from the result of Euler buckling. The buckling mode of the embedded piles results from the competition between bending energy of the piles and elastic energy of the matrix. The bending energy of the piles prefers the buckling mode with long wavelength, while the elastic energy prefers the buckling mode with short wavelength.

![Figure 2](image)

Figure 2. First five destabilizing eigenmodes, and the corresponding critical load values can be obtained. Wavenumber results from energy competition between the pile and the soil matrix.

2.3. Validation of FEM method

To validate our method of buckling analysis, we will compare our results with that based on Winkler foundation, for the case of piles embedded in elastic matrix. For the convenience of comparison, we define the non-dimensional critical load as,

\[ \mu = \frac{F_{\text{crit}}}{F_{E}} \]

where \( F_{\text{crit}} \) is the critical load, and \( F_{E} = \frac{\pi^2 E_p I_p}{L_p^2} \) is Euler buckling load for a hinged-end beam, with \( L_p \) being the length of the pile, and \( E_p I_p \) being the bending stiffness of the pile. Through theoretical analysis, the non-dimensional critical load can be derived as[36],

\[ \mu = \min_{n=1,2,\ldots} \left( n^2 + \frac{k}{n^2} \right), \]

with \( k = \frac{E_{m}}{E_{p}} \), and \( E_{m} \) being the elastic modulus of the matrix.

![Figure 3](image)

Figure 3. Comparison of the results based on FEM and Winkler foundation. Dependence of non-dimensional critical load on the modulus ratio of the soil matrix and the pile. In the calculation, \( L_p/r_p=200 \) and \( \nu_m=0.35 \).
where \( k = K L_p^4 / \pi^4 E_p I_p \), with \( K \) being the spring stiffness of the Winkler foundation model. When the pile inside a matrix with pinned-pinned boundary buckles into mode \( n \), the spring stiffness can be given as\([33,34]\)

\[
K = \frac{16 \pi G_m (1 - v_m)}{2 (3 - 4 v_m) K_0(n \pi r_p / L_p) + n \pi r_p K_1(n \pi r_p / L_p) / L_p},
\]

where \( r_p \) is the radius of the pile, \( G_m \) is the shear modulus of the elastic matrix, \( v_m \) is the Poisson’s ratio of the matrix, and \( K_0 \) and \( K_1 \) are the modified Bessel functions.

Figure 3 shows the non-dimensional critical load of the pile embedded in an elastic matrix, from both the 3D FEM model and Winkler foundation model. The results indicate that the two results are almost identical, which validates our method. The simulation result of FEM model is larger than that of Winkler model, which is due to the stronger constraint from the surrounding 3D soil than from the 1D simplified Winkler spring model. Unlike the Winkler model ignoring the interaction of springs, our FEM model considers the elastic effects of the soil particles. However, the buckling of piles with the other boundary conditions can not easily been studied, for in these cases the spring stiffness is hard to be determined. In the following, we will use the 3D FEM model to analyze the buckling behaviors of the rock-socketed pile embedded in an elastic matrix, which is difficult to be analytically studied by the simple Winkler foundation model.

3. Results and discussions

The buckling behaviors of the piles should depend on the geometrical and physical parameters. We will explore the influence of the rock-socketed ratio, matrix modulus, Poisson’s ratio, and length-radius-ratio on the critical buckling load and buckling mode. In all the calculations, three boundary conditions are considered, such as fixed-fixed, pinned-pinned, and free-free tip boundaries.

3.1. Effects of rock-socketed ratio

Figure 4(a) plots the dependence of the critical buckling force on the rock-socketed ratios, for three different boundary conditions. In the simulation, the radius of the pile is 100mm, the length is 20m, the elastic modulus of the rock is 20GPa, and the Poisson’s ratio of the rock is 0.2. The depth of rock-socketed pile ranges from 0m to 19m. The results show that the rock-socketed ratio has little effect on the critical buckling mode for small rock-socketed ratio, while the critical buckling load extremely increases with increasing rock-socketed ratio for larger rock-socketed ratio. This can be understood from the lateral constraints of the rock and elastic matrix. With the increasing rock-socketed ratio, the lateral constraints from the rock and elastic matrix become stronger, which stabilize the compressed pile and increase the critical buckling load.

Figure 4(b) plots the eigenmode number of buckling for different rock-socketed ratios. With increasing rock-socketed ratio, the number of eigenmode of buckling decreases. For smaller rock-socketed ratio, there appears more wavenumbers for fixed-fixed boundary than for pinned-pinned boundary, and for pinned-pinned boundary than for free-free boundary. It can be explained that the stronger constraint of two end sections of the pile hinders the release of bending energy of the pile, the more eigenmode numbers arise to lower the total strain energy of the system.

The effect of boundary condition on the buckling behaviors is also illustrated in Figure 4. The results show that the critical buckling load of piles with fixed-fixed boundary is larger than that of piles with pinned-pinned and free-free boundary. This indicates that the boundary has significant effect on the buckling behaviors, and the stronger lateral constraint leads to larger critical buckling load.
3.2. Effects of matrix stiffness and Poisson’s ratio

The matrix constrains the embedded pile, and therefore the properties of the matrix should have impact on the buckling behaviors. In this section, we will explore the influence of the matrix stiffness and Poisson’s ratio on the buckling behaviors of the rock-socketed piles.

Figure 5(a) plots the dependence of the buckling behaviors on the matrix stiffness, for three different boundary conditions. In the simulation, the radius of pile is 100mm, the length is 10m, and the elastic modulus is 59GPa. With increasing matrix modulus, the critical buckling force increases. Figure 5(b) plots the dependence of the eigenmode number of the pile on the matrix modulus. The wavenumber increases with increasing matrix modulus. The result indicates that the matrix with larger modulus has stronger lateral constraint on the piles, which leads to larger critical buckling load. The effect of boundary conditions on the buckling behaviors is the same as the previous discussions.
Figure 5. Effects of matrix stiffness on the critical buckling load. (a) The critical loads of the buckling with different modulus ratios of the soil to the pile. (b) The eigenmode number of the buckling with different modulus ratios of the soil to the pile. (c) Buckling profile of different modulus ratios of the soil to the pile (0.0001, 0.0002, 0.0005, 0.001).

Figure 6(a) plots the effect of the Poisson’s ratio on the critical buckling load, for different boundary conditions. In the simulation, the radius of the pile is 100mm, the length is 10m, and the modulus of the pile is 59GPa. The result shows that the Poisson’s ratio has little effect on the critical buckling load. This implies that in practical engineering, we can ignore the effect of the Poisson’s ratio on the critical buckling load of the compressed piles embedded in the matrix. Figure 6(b) plots the wavenumber of the buckled piles, which also indicates that the effect of Poisson’s ratio can be ignored in buckling analysis. This result will simplify the buckling analysis in practical engineering.

3.3. Effect of the length-radius-ratio of the pile
The geometry of the pile also influences the buckling behaviors. Figure 7 plots the dependence of the critical buckling load on the length-diameter, for different boundary conditions. In the simulation of Figure 7, the length of the pile is 10m. The result shows that for the given length of the pile, the critical buckling load decreases with the decrease of the radius of the pile. In other words, as the pile becomes slenderer, the pile is more liable to buckle. Figure 7(b) indicates that for a certain pile of fixed length, the thinner the pile, the more eigenmode waves appear to lower the total energy of the pile-soil system.
Figure 6. Effects of Poisson’s ratio on the critical buckling load. (a) The critical loads of the buckling with different Poisson’s ratios of the soil. (b) The eigenmode numbers of the buckling with different Poisson’s ratios of the soil. (c) Buckling profile of different Poisson’s ratios of the soil (0.05, 0.2, 0.4, 0.495).

Figure 7. Effect of the length-radius-ratio of the pile on the critical buckling load (the length of the pile is 10m). (a) The critical loads of the buckling with different length-radius-ratios of the pile. (b) The eigenmode numbers of the buckling with different length-radius-ratios of the pile.

3.4. Buckling mode
In the practical engineering, the critical buckling load determines the load bearing ability, while the buckling mode may provide some implications in stabilizing the piles. The linear stability analysis above shows that the wavenumber of the buckling mode varies with the geometrical and material parameters, as well as the boundary conditions. The predicted buckling modes based on linear stability analysis are given under the condition of three rock-socketed ratios(RSR) in Figure 8, which cannot provide the amplitude of the deflection of the pile. Post-buckling analysis is performed through nonlinear simulations, which can calculate the deflection profile of the pile in Figure 8. In the simulation, the length of pile is 10m and the radius is 100mm. Figures 8(a) and (b) show the buckling mode and post-buckling profile with RSR=10%, respectively. Similarly, Figures 8(c) and (d) do with RSR=40% while Figures 8(e) and (f) do with RSR=70%. From the given profiles, both the number of the destabilizing eigenmode and the amplitude of the pile decrease with the increasing rock-socketed
ratios in post-buckling analysis. The critical loads are basically the same for linear stability analysis and post-buckling analysis, which indicates and proves the reasonability and validity of the prediction of the simulation. This prediction of the buckling shape will have potential applications in designing rock-socketed piles.

Figure 8. Buckling modes (a, c, e) and post-buckling profiles (b, d, f) under three rock-socketed ratios (RSR).

4. Conclusions
Buckling of rock-socketed piles embedded in soft soils is intensively explored by utilizing three dimensional finite element model. For pile-soil systems with a wide range of values of geometric parameters and material properties, the critical buckling loads and buckling modes are obtained, and compared with the results based on Winkler foundation model. The simulation results show that the exact critical buckling load is a little bigger than that based on Winkler foundation model, which can be explained in that Winkler model neglects the shearing interaction and the diffusion of pressing among soil particles. The effect of rock-socketed ratio on the critical buckling load with three types of constraints is also investigated. With increasing rock-socketed ratio, the critical buckling load increases dramatically. The obtained results are useful to optimize the design of the pile-soil system against buckling in geoengineering applications.

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