The role of conjecturing via analogical reasoning in solving problem based on Piaget’s theory

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Abstract. This study aims to reveal the role of conjecture through analogical reasoning in learning. Uncover the role of conjecture via analogical reasoning; Students are given open analogy problem. Researcher exploration to 52 students of seventh grade; who can to conjecture via analogical reasoning correctly. To unravel the thought process; Researchers use Think out loud method. Results of problem solving studied based on Piaget’s theory and level of thinking of Krulik. The results are obtained; the level of student thinking is variation among others; critical thinking, approaching creative thinking and creative thinking.

1. Introduction
Learning is an interaction between students and the environment. The environment in question can be a problem, so students adapt to the problem. Students construct knowledge through problem solving [1,2]. The cause of concern in Indonesia is still much happening in teaching; teachers transfer knowledge by providing formulas and procedures to solve problems. Not surprisingly, if the teacher gives a modified problem, the student has difficulty in solving the problem [3]. In addition, students increase learning time through the course to get a shortcut in troubleshooting. So there is no construction as anticipated by Anthony [1]. Based on that, the teacher is required to be expert in presenting teaching materials to students. So the teacher must prepare a learning plan that really involves cognitive students. Although temporarily discovered by Supratman [4] that in general the ability of the pedagogic analogy of teachers is very low, involving teachers can not touch the cognitive realm in learning. Though we realize that students at the junior high school level already has a basic knowledge that can be developed. Because basically in problem solving; students will think of analogies [5,6,7,8,9,10].

In addition, students solving problems will do conjecture, hunch, and guess [11]. But in learning we hope; students solve problems through conjecturing, because Fischbein [12], argues, conjecturing is the expression of mental activity to solve problems based on knowledge that has been owned before as the truth needs to be proven. Furthermore, Van de Walle [13] explains that when learning mathematics in the classroom, students are encouraged to make conjecturing and test conjecturing. Moreover Stacey, K, at al. [14] said that basically the competence of students in mathematics includes 2 things namely conjecturing and convincing. According to Canadas et al. [15] There are 5 types of conjecturing based on how the conjecturing is obtained, such as conjecturing based on analogies. Lee, K.H. and Sriraman [16] conjecturing mathematical problem solving in learning is directed at constructing new knowledge for students from existing knowledge.

The phenomenon of mathematical discoveries, mechanisms and thought processes of students remain central to the attention of educational research [17,18]. In addition, Supratman [19] found that learning using conjecturing via analogical reasoning can construct ordinary students into gifted students. As is
known, the concept of mathematical discovery has many common features with the learning process to be considered together. Researchers understand conjecturing via analogy when learning in the classroom is an active learning process aimed at developing students' ability to assimilate and accommodate new knowledge through the use and interpretation of existing knowledge structures (Piaget). The main question of this research is as follows: What is the student's level of thinking according to Krulik et al in using conjecturing via analogy reasoning in solving problems based on Piaget's theory?

We try to answer this question by giving analogy problems, whose basic analogy is assumed to have been mastered beforehand by the students.

2. Experimental Method
This research is qualitative-explorative. The purpose of qualitative research in taking the subject is done explorations of 52 new students entering junior high school in Ciamis District. Namely students are given alternating opportunities to solve problems until found a subject that suits the purpose of research. To uncover the thought process the method of think alouds [21] is supplemented by interviews. So there is no structured interview instrument depending on the information provided by the students. The subjects are selected from students who are new to junior high school in order to happen natural conjecturing, because the students have not got a lesson of arithmetic ranks. As for the problem presented: "specify the result of \(1 - 2 + 3 - 4 + 5 - 6 + 7 - 8 + \ldots - 100\). Adopted from A. Rahman A et al [22]

3. Result and Discussion
3.1. Student Answers
Some conjectures via analogical reasoning are done by the students in solving the problem. The basic analogy used by students on the sum of successive numbers. Another basic analogy is the sum of similar numbers. Problem solving both on the analogy basis and on the analogy target is the sum of the sequence of the initial number with the last number, the second beginning with the second end and so on. In the end divided by 2 considering the numbered rows there are 2. Although it is not shown the basis of the analogy but in general in the presentation of the problem, students solve the problem on the target analogies based on known knowledge. Otherwise natural conjecture occurs in the students, because it has not been obtained before.

Details of conjecture results from students as shown in Figure 1 to Figure 4.

Figure 1. Results conjecturing from S2.

Figure 1 shows that S2 is incapable of resolving the next conjecture resulting in an imperfect answer, since S2 does not know how many sections -1. S2 does not make use of the existing knowledge base of the analogy to solve the problem on the target analogy.

Original Results
(1–2) + (3–4) + (5–6) + \ldots + (95–96) + (97–98) + (99–100) as a target analogy

\[-1 + -1 + -1 + \ldots + -1 + -1 + -1\]

there are 50 pairs -1, because in case of 10 tribes there will be 5 pairs like \((1–2) + (3–4) + (5–6) + (7–8) + (9–10)\) as basic analogy

\[-1 + -1 + -1 + -1 + -1\]

So 1–2 + 3–4 + 5–6 + \ldots + 95–96 + 97–98 + 99–100 = 50 \times -1 = -50

Figure 2. Results of conjecturing via analogical reasoning of S4
Figure 2 shows that S4 to conjecture the target of the analogy; he made use of the knowledge already possessed as the basis of the analogy. So S4 is able to determine the number of values from -1 based on the analogy basis.
analogy to the target analogy. Sequence number and the second

Figure 4 shows that S7 is able to conjecture perfectly in 2 ways. I.e. first using the sum of the nearest sequence number and the second using the sum of similar numbers. This is done either on the basis of analogy to the target analogy.

Figure 3. Results of conjecturing via analogical reasoning of S5

Figure 3 shows S5 conjecture the target of the analogy, he made use of the knowledge already possessed as the basis of the analogy. S5 uses the basic analogy of grouping the sum of similar numbers.

Figure 4a. Results of conjecturing via analogical reasoning of S7 through the first way

Figure 4b. Results of conjecturing via analogical reasoning of S7 through the second way

Figure 4 shows that S7 is able to conjecture perfectly in 2 ways. I.e. first using the sum of the nearest sequence number and the second using the sum of similar numbers. This is done either on the basis of analogy to the target analogy.
3.2. Subject Group by Answer

Results of student answers are divided into 7 groups of subjects as follows: Subject 1 (S1) as much as 1 student does not do conjecture. S2 as many as 7 student’s result of conjecture is wrong and not via analogical reasoning, S3 as many as 6 students not conjecture not through analogical reasoning but the answer is correct, S4 as many as 8 students and S5 as many as 7 results of the guess is true via analogy reasoning in one way, but S4 and S5 using different way. S6 as many as 12 students of the alleged results via analogical reasoning approaching two ways, and S7 as many as 11 students conjecturing via analogy reasoning in 2 ways. More clearly in Table 1.

| Table 1. The results of conjectures solution of the tasks, via analogy reasoning. |
|---|---|---|---|---|---|---|
| No Answer | Result (n=52) | Non Analogy | via analogical reasoning |
| | | Incorrect | Correct | 1 ways | Approaching 2 ways | 2 ways |
| S1 | S2 | S3 | S4 | S5 | S6 | S7 |
| 1 | 7 | 6 | 8 | 7 | 12 | 11 |

Table 1 shows that S1 does not give an answer at all. After the researchers asked; why not answer? S1 answer dizzy sir. S2 performs the right start step by adding up the two nearest numbers like \((1-2 = -1), (3-4 = -1), (5-6 = -1), \ldots, (99-100 = -1)\). With respect to S2 it does not use the analogy base to solve the problem (target analogies), such as the sum of the 10 initial numbers \((1-2 + 3-4 + 5-6 + 7-8 + 9-10)\) so S2 finds the concept of \((1-2 = -1), (3-4 = -1), (5-6 = -1), (7-8 = -1)\) and \((9-10 = -1)\). There are 5 groups -1. Thus S2 assumes also applies to \(1-2 + 3-4 + 5-6 + 7-8 + \ldots + 100\). There are 50 pairs -1. However S2 does not do that so the result of conjecture is wrong.

S3 can solve problems such as S4 and S5 but after the researchers asked S3. They generally read the book of teachers, so know the problem solving. S3 does not conjecture naturally and does not experience analogy reasoning. According to Piaget; S3 experienced assimilation of problems, assimilation of strategies and assimilation of relations [2,3]. As for Krulik et al. [23] S3 is at the level of recall.

S4 uses basic analogies in the sum \((1-2 + 3-4 + 5-6 + 7-8 + 9-10)\). So S4 finds the concept \((1-2 = -1), (3-4 = -1), (5-6 = -1), (7-8 = -1)\) and \((9-10 = -1)\). There are 50 pairs -1. Thus S4 assumes the concept applies also to \((1-2 + 3-4 + 5-6 + 7-8 + \ldots + 100)\). I.e. add the nearest number of \((1-2 = -1), (3-4 = -1), (5-6 = -1), \ldots, (95-96 = -1), (97-98 = -1)\) and \((99-100 = -1)\). So there are 50 pairs -1. Finally obtained \(50 \times -1 = -50\) According to Piaget S4 experiencing assimilation problems, accommodation strategy and assimilation relations [2,3].

S5 using basic analogy begins to partition from \((1-2 + 3-4 + 5-6 + 7-8 + 9-10)\) become \((1 + 3 + 5 + 7 + 9) + (-2-4-6-8-10)\). S5 pair \((1 + 9 = 10), (5 + 5 = 10), (7 + 3 = 10), (9 + 1 = 10)\). So it has 5 pairs of 10 values. Finally obtained \(5 \times 10 = 50\). Since 50 is constructed by two lines \((1 + 3 + 5 + 7 + 9)\) and \((9 + 7 + 5 + 3 + 1)\), whereas only one line \((1 + 3 + 5 + 7 + 9)\) is \(50 + 2 = 25\). Next S5 pair \((-2-10 = -12), (-4-8 = 12), (-6-6 = -12), (-8-4 = 12), (-10-4 = -12)\) so it has 5 pairs of values -12 so \(5 \times 12 = 60\). Because -60 is built by two lines \((-2-4-6-8-10)\) and \((-10-8-6-4-2)\) whereas it takes only one line \((-2-4-6-8-10)\) then \(-60 \div 2 = -30\). Then S5 sums the result \((1 + 3 + 5 + 7 + 9)\) with \((-2-4-6-8-10)\) i.e. \(25-30 = -5\). S5 utilizes the concept as a base of analogy to solve \((1-2 + 3-4 + 5-6 + 7-8 + \ldots -100)\) (target analogy). S5 partition \((1-2 + 3-4 + 5 \ldots + 99-100)\) becomes \((1 + 3 + 5 + \ldots + 95 + 97 + 99) + (-2-4-6\ldots -96-98-100\)\). Then S5 sets up \((1 + 99 = 100), (3 + 97 = 100), \ldots, (95 + 5 = 100), (93 + 7 = 100)\) and \((99 + 1 = 100)\) and pairing \((-2-100 = -102), (-4-98 = -102), (-6-96 = 102), (-96-6 = 102)\), \((-98-4 = 102\)\), \((-100-2 = 102)\). So obtained \(50 \times 100 = 5000\) and \(50 \times 102 = -5100\). Since \(5000\) and \(-5100\) are obtained from each \((1 + 3 + 5 + \ldots + 95 + 97 + 99)\) with \((99 + 97 + 95 + \ldots + 5 + 3 + 1)\) and \((-2-4-6\ldots -96-98-100\)\) with \((-100-98-96\ldots -6-4-2)\), then S5 does the following division of \(5000 + 2 = 2500\) and \(-500 \div 2 = -2550\). So the result of conjecturing S5 from \((1-2 + 3-4 + 5 \ldots + 99-100) = 2500-2550\) gained.
-50. S5 experienced assimilation problems, accommodation strategies and assimilation of relationships [2,3]. Thus S4 and S5 according to Krulik et al are at the critical thinking level.

S6 in solving the problem (1+2+3+4+5+6+7+8+...-100) the first way as done by S4 and the second way is almost the same as S5. S6, however, takes an error in perceiving (-2+4-6-⋯-96-98-100), ie (-2-98 = -100), (- 4-96 = -100), ( 6-94 = -100), ⋯, (- 94-6 = -100), ( -96-4 = -100), (- 98-2 = -100), (- 100-0 = -100) to (51 × -100) = - 2550. So the result of conjecturing S6 from (1+2+3+4+5⋯+99-100) = - 50. If we notice that there is a mistake because the resulting pair is 50 pairs -100 at a glance as true but after traced it was wrong. S6 takes the basis of the analogy of (-2+4-6-8-10) is (-2-8 = -10), (- 4-6 = -10), (- 6-4 = -10), (- 8-2 = -10) and (-10-0 = -10). So it can be 5 pairs -10. S6 experienced 2 complete assimilation problems, once true strategy accommodation and one incorrect strategy accommodation, as well as a true assimilation of the problem and one of incorrect assimilation of relationships. S6 according to the researchers led to the level of creative thinking from Krulik et al.

S7 performs conjecturing via analogical reasoning perfectly. S7 able to solve the problem through 2 ways. S7 solves the problem as S4 and S5 do. So S7 experiences 2 assimilation problems, 2 accommodation strategies and twice assimilated relationships correctly [2,3]. Thus, S7 according to Krulik et al [23] is at the level of creative thinking.

4. Conclusion

The first junior high school student in solving the problem based on Piaget and Krulik et al combination theory found the following level of thinking: a) Students in recall level are Students in solving problem/knowledge construction tends to be dominated by assimilation process b) Students in the level of critical thinking are Students in solving the problem / construction of knowledge there are variations of the assimilation process and each accommodation at least once c) Students in the level toward creative thinking is a variation between the assimilation process and each accommodation at least 2 times, will occur one of the imperfect accommodations. d) Students who are at the creative thinking level tend to perform at least 2 assimilation variations and perfect accommodation

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