QUARK-ANTIQUARK CONTRIBUTION TO THE BFKL KERNEL *

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Abstract

The quark-antiquark contribution to the BFKL kernel is calculated. Using the effective vertex for the $q\bar{q}$ pair production in the Reggeon-Reggeon collision we find this contribution by integrating the square of this vertex over relative transverse momenta and fractions of longitudinal momenta of produced particles.

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1. Introduction

The investigation of parton distributions in the region of small values of the Bjorken variable \( x \) is nowadays one of the most important problems of perturbative QCD [1], especially in connection with results of recent experiments on deep inelastic scattering [2]. The calculation of these distributions in the leading logarithmic approximation (LLA), where only terms of the type \( \alpha_s^n \ln^n(1/x) \) are summed up to all orders in \( \alpha_s \), can be performed using the BFKL evolution equation [3]. Now the results of the LLA are widely known and used for the analysis of the experiments on semihard processes [4]. But to make the analysis reliable one has to know the radiative corrections to the leading logarithmic results.

The BFKL equation can be presented in the form

\[
\frac{\partial}{\partial \ln(1/x)} F(x, \vec{q}^2_1) = \int d^{D-2}q_2 K(\vec{q}_1, \vec{q}_2) F(x, \vec{q}^2_2) ,
\]

where \( D = 4+2\epsilon \) is the space-time dimension, different from 4 to have on each step of calculations well defined expressions free from infrared and collinear divergencies, and the function \( F(x, \vec{k}^2) \) is the unintegrated gluon density, connected with the gluon distribution \( g(x, Q^2) \) as follows:

\[
xg(x, Q^2) = \int_0^{Q^2} d \left( \vec{k}^2 \right) F(x, \vec{k}^2) .
\]

Here \( \vec{k} \) denotes the projection of the parton 4-momenta \( k \) into the plane orthogonal to the initial particle momenta. The kernel \( K(\vec{q}_1, \vec{q}_2) \) is expressed in terms of the gluon Regge trajectory \( \omega(t) \) and the contribution of the real particle production

\[
K(\vec{q}_1, \vec{q}_2) = 2\omega(-\vec{q}_1^2)\delta(\vec{q}_1 - \vec{q}_2) + K_{\text{real}}(\vec{q}_1, \vec{q}_2) .
\]

It was argued in Ref. [5] that in the next-to-leading logarithmic approximation (NLLA) the form (1) of the BFKL equation, as well as the representation (3) of the kernel remain unchanged, but the gluon trajectory has to be taken in the two-loop approximation, whereas the contribution of the real particle production is the sum...
of the contributions coming from one-gluon, two-gluon and quark-antiquark pair productions in the Reggeon-Reggeon collision:

\[
\mathcal{K}_{\text{real}}(\vec{q}_1, \vec{q}_2) = \mathcal{K}_{\text{one-loop}}^{\text{RRG}}(\vec{q}_1, \vec{q}_2) + \mathcal{K}_{\text{Born}}^{\text{RRGG}}(\vec{q}_1, \vec{q}_2) + \mathcal{K}_{\text{Born}}^{\text{RRQQ}}(\vec{q}_1, \vec{q}_2).
\] (4)

Let us remind that in the LLA only the first term in Eq. (4) taken in the Born approximation does contribute. In the NLLA this term has to be calculated with the one-loop accuracy, in contrast with the second two terms, which are pure next-to-leading corrections.

The gluon Regge trajectory \(\omega(t)\) was calculated in the two-loop approximation in Ref. [6]. One-gluon production amplitudes were found with the required accuracy in Ref. [7] and the contribution \(\mathcal{K}_{\text{Born}}^{\text{RRGG}}\) for two-gluon production was obtained in Refs. [8, 9]. The quark-antiquark production in the central rapidity region was considered in Refs. [10, 8, 11]. The singular at \((\vec{q}_1 - \vec{q}_2)^2 \to 0\) part of \(\mathcal{K}_{\text{Born}}^{\text{RRQQ}}(\vec{q}_1, \vec{q}_2)\) was found for the case of massless quarks in Ref. [8]; for this case in Ref. [11] the azimuthal averaged regular part of this piece of the kernel is shown.

In this paper we calculate a complete expression for the massless quark-antiquark contribution \(\mathcal{K}_{\text{Born}}^{\text{RRQQ}}\) to the BFKL kernel. We use the representation of the amplitude of the \(q\bar{q}\) pair production in the central rapidity region in terms of the effective vertex for the \(q\bar{q}\) pair production in the Reggeon-Reggeon collision. The expression for \(\mathcal{K}_{\text{Born}}^{\text{RRQQ}}(\vec{q}_1, \vec{q}_2)\) is obtained by integrating the square of this vertex over relative momenta of produced particles.

2. The quark-antiquark production in Reggeon-Reggeon collision

The production of a quark-antiquark pair contributes to cross sections of high energy processes in the NLLA if the pair is produced in the quasi multi-Regge kinematics (QMRK), i.e. if it has a limited invariant mass and is divided in the
rapidity space by large, growing with the energy intervals from other produced particles \[3\]. In this case the amplitudes of the processes contain the effective vertex \(\gamma_{i_1i_2}^{Q\overline{Q}}(q_1, q_2)\) for the quark-antiquark production in the collision of two Reggeized gluons, having momenta \(q_1 = \beta p_A + q_{1\perp}\) and \(-q_2 = \alpha p_B - q_{2\perp}\) with \(\alpha, \beta \ll 1\) and colour indices \(i_1, i_2\). Here and below \(p_A\) and \(p_B\) are light cone vectors such that the momenta of the initial particles \(A\) and \(B\) are equal to \(p_A + (m_A^2/s)p_B\) and \(p_B + (m_B^2/s)p_A\), with \(s = (p_A + p_B)^2\). The effective vertex \(\gamma_{i_1i_2}^{Q\overline{Q}}(q_1, q_2)\) can be extracted from any you like amplitude of the \(q\overline{q}\) production in the QMRK. In the simplest case of the process \(A + B \rightarrow A' + B' + q\overline{q}\) we have

\[
A_{AB}^{A'Q\overline{Q}B'} = 2s \frac{1}{q_{1\perp}^2} \Gamma_{i_1}^{i_1} \gamma_{i_1i_2}^{Q\overline{Q}}(q_1, q_2) \frac{1}{q_{2\perp}^2} \Gamma_{i_2}^{i_2} \Gamma_{BB'}, \tag{5}
\]

where \(q_1 = p_A - p_{A'}\), \(q_2 = p_{B'} - p_B\), \(\Gamma_{i_1}^{i_1}\) are the particle-particle-Reggeon vertices at the lowest order,

\[
\Gamma_{i_1}^{i_1} = g \langle P'|T^i|P \rangle \delta_{\lambda_P, \lambda_{P'}}; \tag{6}
\]

here \(\langle P'|T^i|P \rangle\) represents matrix elements of the group generators, \(\lambda_P\) are helicities of corresponding particles, \(g\) is the gauge coupling constant \((\alpha_s = g^2/(4\pi))\). Using the results of Ref. \[3\] we obtain

\[
\gamma_{i_1i_2}^{Q\overline{Q}}(q_1, q_2) = \frac{1}{2} g^2 \bar{u}(k_1) \left[t^{i_1} t^{i_2} b(k_1, k_2) - t^{i_2} t^{i_1} \bar{b}(k_2, k_1)\right] v(k_2), \tag{7}
\]

where \(t^i\) are the colour group generators in the fundamental representation. We will use the Sudakov parametrization

\[
k_i = \beta_i p_A + \alpha_i p_B + k_{i\perp}, \quad s\alpha_i\beta_i = -k_{i\perp}^2 = \vec{k}_{i\perp}^2, \quad i = 1, 2,
\]

\[
\beta_1 + \beta_2 = \beta \ll 1, \quad \alpha_1 + \alpha_2 = \alpha \ll 1, \tag{8}
\]

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and the denotations
\[ x = \frac{\beta_1}{\beta_1 + \beta_2}, \quad \Delta = k_1 + k_2 = q_1 - q_2, \]
\[ \Lambda = k_1 - x\Delta, \quad Z = -\bar{\Lambda}^2 - x(1 - x)\bar{\Delta}^2. \] (9)
The expressions for \( b(k_1, k_2) \) and \( \bar{b}(k_2, k_1) \) can be presented in the following way:
\[ b(k_1, k_2) = \frac{4p_A Q_1 \bar{p}_B}{st} - \frac{1}{\Delta^2} \Gamma \] (10)
and
\[ \bar{b}(k_2, k_1) = \frac{4p_B Q_2 \bar{p}_A}{st} - \frac{1}{\Delta^2} \Gamma, \] (11)
where
\[ t = (q_1 - k_1)^2, \quad \bar{t} = (q_1 - k_2)^2, \]
\[ Q_1 = q_{1\perp} - k_{1\perp}, \quad Q_2 = q_{1\perp} - k_{2\perp}, \]
\[ \Gamma = 2 \left[ (q_1 + q_2)_{\perp} - \beta p_A \left( 1 + 2x(1 - x) \frac{\bar{q}_1^2}{Z} \right) - \frac{p_B}{\beta s} \left( 2\bar{q}_2^2 + \frac{Z}{x(1 - x)} \right) \right]. \] (12)
The quark-antiquark contribution to the BFKL kernel can be put in the form
\[ K_{RRQ\bar{Q}}^{Born}(\vec{q}_1, \vec{q}_2) = \frac{1}{2q_1^2 q_2^2 (N^2 - 1)} \sum_{i_1, i_2, f} \int d\kappa d\rho_f \]
\[ \times \delta^{(D)}(q_1 - q_2 - k_1 - k_2)|\gamma_{i_1 i_2}^{Q\bar{Q}}(q_1, q_2)|^2, \] (13)
where \( k_1 \) and \( k_2 \) are the quark and antiquark momenta, the sum is taken over the colour indices \( i_1, i_2 \) and over spin, colour and flavour states of the produced quark-antiquark pair, \( \kappa = (q_1 - q_2)^2 \) is the squared invariant mass of the two Reggeons and the element \( d\rho_f \) of the phase space is
\[ d\rho_f = \prod_{n=1,2} \frac{d^{D-1}k_n}{(2\pi)^{D-1}2\omega_n}. \] (14)
From the representation (7) we obtain

\[ \sum_{i_1,i_2,f} |\gamma^{Q_i_2}_i(q_1,q_2)|^2 = \frac{g^4(N^2 - 1)n_f}{16N}[(N^2 - 1)A + B + (k_1 \leftrightarrow k_2)] , \tag{15} \]

where \( n_f \) is the number of light quark flavours and

\[ A = tr \left( \bar{k}_1 b(k_1,k_2) \bar{k}_2 b(k_1,k_2) \right) \tag{16} \]

and

\[ B = tr \left( \bar{k}_1 b(k_1,k_2) \bar{k}_2 b(k_2,k_1) \right) . \tag{17} \]

The calculation of the traces gives us

\[ A = 32x(1-x) \left\{ - (1-x)(1-2x)^2 \left( \bar{q}_1^2 \right)^2 \left( \frac{1}{xt^2} + \frac{x}{Z^2} \right) - x(1-x) \frac{\bar{q}_1^2 \bar{q}_2^2}{\Lambda^2 Z} \right. \]

\[ -4x^2(1-x)^2 \frac{\bar{q}_1^2 (\bar{\Lambda} \Delta)}{\Lambda^2 Z} \left[ 2 \frac{(\bar{\Lambda} \bar{q}_1)}{\Lambda^2} + x(1-x) \frac{(\bar{\Lambda} \Delta) \bar{q}_1^2}{\Lambda^2 Z} + (1-2x) \frac{\bar{q}_1^2}{Z} \right] \]

\[ -4x(1-x) \left[ \frac{\left( (\bar{\Lambda} \bar{q}_1) + (1-2x)(\bar{k}_1 \bar{\Lambda}) + x \bar{\Lambda}^2 - (\bar{q}_1 \bar{\Delta}) \right)}{xt} \right]^2 \]

\[ + \frac{\bar{q}_1^2 (\bar{\Lambda} \Delta)}{\Lambda^2 Z t} \left[ (\bar{k}_1 \bar{q}_1) + (1-2x)(\bar{k}_1 \bar{\Lambda}) + x \bar{\Lambda}^2 - (\bar{q}_1 \bar{\Delta}) \right] \]

\[ - (1-2x) \frac{\bar{q}_1^2}{t} \left[ \frac{\bar{q}_2^2}{Z} + 2(1-x) \frac{2(\bar{k}_1 \bar{q}_1) - \bar{q}_1^2}{Z} - 4(1-x) \frac{\bar{q}_1(\bar{k}_1 - x \bar{q}_1)}{xt} \right] \]

\[ + 2(1-x)(1-2x) \frac{\bar{q}_1^2 \left( (\bar{\Lambda} \Delta) + 2(\bar{\Lambda} \bar{q}_1) \right)}{\Lambda^2} \left( \frac{1}{t} - \frac{x}{Z} \right) \right\} , \tag{18} \]

and

\[ B - A + (k_1 \leftrightarrow k_2) = 64x(1-x) \left\{ \frac{1}{2x(1-x)tt} \left[ -\bar{q}_1^2 \bar{q}_2^2 \right. \right. \]

\[ + 2x(1-x)\bar{q}_1^2(\bar{q}_2^2 - \bar{\Delta}^2) + 8x(1-x)(\bar{k}_1 \bar{q}_1)(\bar{k}_2 \bar{q}_1) \]
\[(1 - x) \frac{\left( \vec{q}_1^2 - 2(k_1 \vec{q}_1) \right)^2}{2x t^2} + x \frac{\left( \vec{q}_2^2 - 2(k_2 \vec{q}_1) \right)^2}{2(1 - x) t^2} \right\}.

(19)

3. The quark-antiquark contribution to the BFKL kernel

Going in Eq. (13) to the variables \( x, \vec{k}_1 \), we get

\[
\int d\kappa d\rho f(D)(q_1 - q_2 - k_1 - k_2) = \int_0^1 \frac{dx}{2x(1 - x)} \int \frac{d^{D-2}k_1}{(2\pi)^{D-1}}
\]

and using Eq. (15) we obtain

\[
K_{RRQQ}^{Born}(\vec{q}_1, \vec{q}_2) = \frac{g^4 \mu^{2\epsilon}}{64 \vec{q}_1^2 \vec{q}_2^2 (2\pi)^{D-1}} n_f \times \int_0^1 \frac{dx}{x(1 - x)} \int \mu^{-2\epsilon} \frac{d^{D-2}k_1}{(2\pi)^{D-1}} [(N^2 - 1)A + B + (k_1 \leftrightarrow k_2)] ,
\]

(21)

where \( \mu \) is the renormalization scale. In order to obtain the contribution to the BFKL kernel we should perform the integration in Eq. (21). The integrals appearing here have a similar structure as the integrals considered in Ref. [11] for the calculation of the two gluon contribution to the BFKL kernel. The details of the calculation of integrals for the quark-antiquark case will be given elsewhere [12]. Here we present only the results:

\[
\int_0^1 \frac{dx}{x(1 - x)} \int \mu^{-2\epsilon} \frac{d^{D-2}k_1}{(2\pi)^{D-1}} A =
\]

\[
\frac{64 \Gamma(1 - \epsilon)}{(4\pi)^{2+\epsilon}} \left\{ \frac{\vec{q}_1^2 \vec{q}_2^2}{\bar{\Delta}^2} \left( \frac{\bar{\Delta}^2}{\mu^2} \right)^{\sigma + \epsilon} \frac{4 \Gamma^2(2 + \epsilon)}{\epsilon \Gamma(4 + 2\epsilon)} \right. \]

\[
+ \frac{\vec{q}_1^2 \vec{q}_2^2}{\bar{\Delta}^2} \left[ 1 - \frac{\bar{\Delta}^2 (\vec{q}_1^2 + \vec{q}_2^2 + 4 \vec{q}_1 \vec{q}_2)}{3(\vec{q}_1^2 - \vec{q}_2^2)^2} \right] \ln \left( \frac{\vec{q}_1^2}{\vec{q}_2^2} \right) + \frac{\bar{\Delta}^2}{(\vec{q}_1^2 - \vec{q}_2^2)^2} \left( 2 \vec{q}_1^2 \vec{q}_2^2 - \frac{3}{3} \bar{\Delta}^2 (\vec{q}_1^2 + \vec{q}_2^2) + \frac{1}{3} \left( 2 \bar{\Delta}^2 - \vec{q}_1^2 - \vec{q}_2^2 \right) \right) \}
\]

(22)

\[
\int_0^1 \frac{dx}{x(1 - x)} \int \mu^{-2\epsilon} \frac{d^{D-2}k_1}{(2\pi)^{D-1}} (B - A + (k_1 \leftrightarrow k_2)) =
\]

(23)
\[
\frac{128\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \left\{ -\bar{q}_1^2 \bar{q}_2^2 - \frac{(\bar{q}_1^2 - \bar{q}_2^2)^2}{4} + \left( (\bar{q}_1 \bar{q}_2)^2 - 2\bar{q}_1^2 \bar{q}_2^2 \right) \right. \\
\times \left. \frac{(2\bar{q}_1^2 \bar{q}_2^2 - 3\bar{q}_1^4 - 3\bar{q}_2^4)}{216\bar{q}_1^2 \bar{q}_2^2} \int_0^\infty \frac{dx}{(\bar{q}_1^2 + x^2 \bar{q}_2^2)} \ln \left| \frac{1 + x}{1 - x} \right| \right. \\
\left. + \frac{3(\bar{q}_1 \bar{q}_2)^2 - 2\bar{q}_1^2 \bar{q}_2^2}{16\bar{q}_1^2 \bar{q}_2^2} \left[ (\bar{q}_1^2 - \bar{q}_2^2) \ln \left( \frac{\bar{q}_1^2}{\bar{q}_2^2} \right) + 2(\bar{q}_1^2 + \bar{q}_2^2) \right] \right\}. 
\] (23)

Using these last equations, from Eq. (21) we get the final result for the quark-antiquark contribution to the BFKL kernel:

\[
K^{Born}_{RRQ}(\bar{q}_1, \bar{q}_2) = \frac{4\bar{q}_2^2\mu^{-2\epsilon}N_f}{\pi^{1+\epsilon}(\Gamma(1-\epsilon))^N} \left\{ N^2 \left[ \frac{1}{\Delta^2} \left( \frac{\Delta^2}{\mu^2} \right) \right] \frac{\epsilon}{3} \left( \frac{1}{\epsilon} - \frac{5}{3} + \epsilon \left( \frac{28}{9} - \frac{\pi^2}{6} \right) \right) \right. \\
\left. + \frac{1}{\bar{q}_1^2 - \bar{q}_2^2} \left( 1 - \frac{\Delta^2(\bar{q}_1^2 + \bar{q}_2^2 + 4\bar{q}_1 \bar{q}_2)}{3(\bar{q}_1^2 - \bar{q}_2^2)^2} \right) \ln \left( \frac{\bar{q}_1^2}{\bar{q}_2^2} \right) \right. \\
\left. + \frac{\Delta^2}{(\bar{q}_1^2 - \bar{q}_2^2)^2} \left( 2 - \frac{\Delta^2(\bar{q}_1^2 + \bar{q}_2^2)}{3\bar{q}_1^2 \bar{q}_2^2} \right) + \frac{2\Delta^2 - \bar{q}_1^2 - \bar{q}_2^2}{3\bar{q}_1^2 \bar{q}_2^2} \right\} \\
\times \int_0^\infty \frac{dx}{(\bar{q}_1^2 + x^2 \bar{q}_2^2)} \ln \left| \frac{1 + x}{1 - x} \right| + \frac{3(\bar{q}_1 \bar{q}_2)^2 - 2\bar{q}_1^2 \bar{q}_2^2}{16\bar{q}_1^2 \bar{q}_2^2} \\
\times \left[ (\bar{q}_1^2 - \bar{q}_2^2) \ln \left( \frac{\bar{q}_1^2}{\bar{q}_2^2} \right) + 2(\bar{q}_1^2 + \bar{q}_2^2) \right] \right\}, 
\] (24)

where

\[
g_\mu^2 = \frac{g_\mu^2 N\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}, \quad g_\mu = g\mu^\epsilon. 
\] (25)

4. Discussion

The expression given in Eq. (24) represents the correction to the BFKL kernel connected with the quark-antiquark production. It includes all terms which give nonvanishing contributions after integration over \( \Delta \) when \( \epsilon = (D - 4)/2 \) tends to its physical value \( \epsilon = 0 \). This correction contains two types of infrared singularities:
the explicit pole in $\epsilon$ at fixed $\Delta$ and the singularity at $\Delta^2 = 0$, which leads to a new pole in $\epsilon$ after integration over $\Delta$. All singularities of $K_{\text{Born}}^{\text{RRQQ}}$ are contained in the first line of Eq. (24) and agree with the corresponding result of Ref. [8]. The first singularity is cancelled with the corresponding one in the virtual correction to the one-gluon production contribution; the singularities appearing after integration over $\Delta$ cancel the corresponding singularities in the gluon Regge trajectory.

Let us define the singular part of $K_{\text{Born}}^{\text{RRQQ}}$ as given by the first line of Eq. (24). It can be rewritten with the required accuracy as

$$K_{\text{Born}}^{\text{RRQQ}}(\vec{q}_1, \vec{q}_2)_{\text{sing}} = \frac{16g_4^4\mu^{-2\epsilon}N_f}{\pi^{1+\epsilon}(1-\epsilon)N} \frac{1}{\Delta^2} \left( \frac{\Delta^2}{\mu^2} \right)^\epsilon \frac{\Gamma^2(2+\epsilon)}{\epsilon\Gamma(4+2\epsilon)}. \tag{26}$$

In the remaining part of Eq. (24) we can put $\epsilon = 0$. Various terms in this part have singularities at $\vec{q}_1^2 = 0$, $\vec{q}_2^2 = 0$ and $\vec{q}_1^2 = \vec{q}_2^2$. But all these singularities are spurious and cancel one another. The singularities at $\vec{q}_1^2 = \vec{q}_2^2$ explicitly cancel after azimuthal averaging:

$$< K_{\text{RRQQ}}^{\text{Born}}(\vec{q}_1, \vec{q}_2)_{\text{nonsing}} > = \frac{\alpha_s^2N_f}{4\pi^3N} \left\{ \frac{2N^2}{3(\vec{q}_1^2 - \vec{q}_2^2)} \ln \left( \frac{\vec{q}_1^2}{\vec{q}_2^2} \right) - \frac{1}{32\vec{q}_1^2\vec{q}_2^2} \left[ (\vec{q}_1^2 - \vec{q}_2^2) \ln \left( \frac{\vec{q}_1^2}{\vec{q}_2^2} \right) + 2(\vec{q}_1^2 + \vec{q}_2^2) 
\right. $$

$$
\left. + (22\vec{q}_1^2\vec{q}_2^2 - \vec{q}_1^4 - \vec{q}_2^4) \frac{1}{\sqrt{\vec{q}_1^2\vec{q}_2^2}} \left( \ln \left( \frac{\vec{q}_1^2}{\vec{q}_2^2} \right) \arctg \left( \frac{\vec{q}_2}{|\vec{q}_1|} \right) \right) + 2\text{Im Li}_2 \left( i\frac{\vec{q}_2}{|\vec{q}_1|} \right) \right]\right\}, \tag{27}$$

where we have taken into account that

$$\int_0^\infty \frac{dx}{(\vec{q}_1^2 + x^2\vec{q}_2^2)} \ln \left| \frac{1+x}{1-x} \right| = $$

$$= \frac{1}{\sqrt{\vec{q}_1^2\vec{q}_2^2}} \left( \ln \left( \frac{\vec{q}_1^2}{\vec{q}_2^2} \right) \arctg \left( \frac{\vec{q}_2}{|\vec{q}_1|} \right) + 2\text{Im Li}_2 \left( i\frac{\vec{q}_2}{|\vec{q}_1|} \right) \right). \tag{28}$$
The expression (27) should be compared with the result given in Eq. (3.17) of Ref. [1], but there are some misprints in this equation, as it was indicated in Ref. [13]. Unfortunately, corresponding expression of Ref. [13] is not free from missprints also. To agree Eq. (3.17) of Ref. [1] with our expression the first colour structure there should have an overall 1/2 factor and the factor 2 of the last term in square brackets should multiply only the dilogarithm and not the \( \ln \frac{1}{\rho} \) piece; in the second term \( C_\Lambda \) has to be changed for \( n_f \). We thank M.Ciafaloni, who informed us that these are just the misprints in Eq. (3.17) of Ref. [1].

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