Gravitation interaction with extra dimension and periodic structure of the hadron scattering amplitude

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Abstract

The behavior of the hadron scattering amplitude determined by the gravitation interaction of hadron at high energies with impact of the KK-modes in d-brane models of gravity is examined. The possible periodic structure of the scattering amplitude and its dependence on the number of additional dimensions are analyzed. The effects of the gravitational hadron form factors obtained from the hadron generalized parton distributions (GPDs) on the behavior of the interaction potential and the scattering amplitude are analyzed. It is shown that in most part the periodic structure comes from the approximation of our calculations.

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1 Introduction

Research of a structure of elastic hadron scattering amplitude at super-high energies and small momentum transfer - t can gives a connection between an experimental knowledge and the basic asymptotic theorems, which are based on the first principles. It gives an information about hadron interaction at large distances where the perturbative QCD does not work, and a new theory, as, for example, instantons or string theories, must be developed. In the early 70s there were many works in which the consequences of breaking the Pomeranchuk theorem were investigated [1]. It was shown [2] that if the Pomeranchuk theorem is broken and the scattering amplitude grows to a maximal possible extant, but not breaks the Froissart boundary, many zeros in the scattering amplitude should be available in the nearest domain of t → 0 in the limit s → ∞. Hence, with increase energy of colliding beams some new effect in a form of small oscillations in differential cross sections can be discovered at very small t.

The differential cross section are defined as follows:

\[ \frac{d\sigma}{dt} = \frac{2\pi}{s^2} (|\Phi_1|^2 + |\Phi_2|^2 + |\Phi_3|^2 + |\Phi_4|^2 + 4|\Phi_5|^2) . \]  (1)
Every amplitudes contain the hadronic and electromagnetic parts.

$$\Phi_i = \Phi_{i}^{em} + \Phi_{i}^{h} \exp[-i\alpha \varphi]$$

(2)

where

$$\varphi(s, t) = \pm \left[ \gamma + \log(B(s, t)|t|/2) + \nu_1 + \nu_2 \right]$$

(3)

is the Coulomb-hadron interference phase with $\nu_1$ connect with taking into account the second Born diagram (2 photon approximation) [3] and $\nu_2$ is determined by the hadron form factor [4]. At high energy and small angles of scattering it is usually neglect the spin-flip amplitudes. The scattering at small momentum transfer is determined by the interference of the electromagnetic and hadronic parts.

If there is some unknown additional amplitude with non-small real part the differential cross sections is

$$d\sigma/dt(s, t) \sim |\text{Re} F_C(t) + \text{Re} F_h(s, t) + \text{Re} F_{ad}(s, t)|^2$$

$$+ |\text{Im} F_C(t) + \text{Im} F_h(s, t) + \text{Im} F_{ad}(s, t)|^2,$$

(4)

and main contribution at small momentum transfer is determined by the Coulomb-hadron interference term

$$\Delta(d\sigma/dt)_{ad}(s, t) \sim [2\text{Re} C_{\text{Im}} F_h(\rho(s, t) + \sin[\alpha_{em}(\varphi_C(t) + \varphi_{Ch}(s, t))])$$

$$+ 2\text{Re}_ad(s, t)[\text{Re} F_C(t) + \rho \text{Im} F_h(s, t)].$$

(5)

where $F_C = \mp 2\alpha G^2/|t|$ is the Coulomb amplitude; $\alpha$ is the fine-structure constant and $G^2(t)$ is the proton electromagnetic form factor squared; $\text{Re} F_N(s, t)$ and $\text{Im} F_N(s, t)$ are the real and imaginary parts of the nuclear amplitude; $\rho(s, t) = \text{Re} F(s, t)/\text{Im} F(s, t)$.

It is clear that the size and energy dependence of the basic parameters of the elastic scattering amplitude at small $t$ are mostly determined by a potential of the hadron interaction at large distances. However, as the parameters of these potential in the $r$-representation are connected with the real physical effects at small $q$ by the integral transformation and on the background of the leading contributions. The possibilities of experimental research are limited, though there are some proposals, for example [?].

Standard Regge representation shows that the energy dependence of the scattering amplitude depends on the spin of the exchanged $t$-channel particles. So the exchange of a graviton of spin 2 leads to a growth of the scattering amplitude with energy proportional to $s$ [5]. As the usual gravitational interaction is very small, this growth can be seen at the Plank scale. However, a modern development of the fundamental theory, introduced long ago [6, 7], is connected with the fruitful idea that spacetime has a dimension higher than $D = 4$.

Now there are many different paths in the development of these ideas [8]. Especially it is connect with the different supersymmetry and string models. For example in [9], was analized the proposal for using large (TeV) extra dimensions in the Standard Model, motivated from the problem of supersymmetry breaking in string theory, and [10], where it gave the string realization of low scale gravity and braneworld models, and pointed out the motivation of TeV strings from the stabilization of mass hierarchy.
A number of studies of higher-dimensional (Kaluza-Klein) field theories were carried out \cite{11, 12, 13, 14, 15}. In a modern context, the Kaluza-Klein (KK) theories arise naturally from (super)string theories in the limit where the relevant energies $E$ are much smaller than the string mass scale $M_s \sim (\alpha')^{-1/2}$, $\alpha'$ being the slope parameter. Since field theories of gravity behave badly in the ultraviolet limit, Kaluza-Klein formulations should in general be regarded as effective actions, with an implicit or explicit ultraviolet cutoff $\Lambda$ \cite{16}. As a first approximation, we may suppose that all Standard Model fields are confined to a four-dimensional brane world-volume. In the Arkani-Hamed, Dimopoulos and Dvali approach (ADD) \cite{17} a large number $d$ of extra dimensions is responsible for a lower Planck scale, down to a TeV and only the graviton propagates in the $4 + d$ dimensions. This propagation manifests itself in the standard 4 dimensions as a tower of massive KK-modes. The effective coupling is obtained after summing over all the KK modes and, due to the high multiplicity of the KK modes, the effective interaction has strength $1/M_d$ \cite{18, 19}. Setting $M_{4+d}^{d+2} = (2\pi)^d \hat{M}_{4+d}^{d+2}$, as in Ref. \cite{17} (motivated by toroidal compactification, in which the volume of the compactified space is $V_d = (2\pi r_d)^d$) and applying Gauss's law at $r << r_d$ and $r >> r_d$, one finds that $M_{Pl}^2 = r_d^d M_{4+d}^{2+d}$. In the higher-dimensional models with a warped extra dimension \cite{14, 15}, the first KK mode of the graviton can have a mass of the order of 1 TeV and the coupling with matter on the visible brane is of the order $1 \text{TeV}^{-1}$. There are also ”intermediate” models with a small warp which consider the brane as almost flat. Such models remove some cosmological bounds on the number of additional dimensions. All these models provide some experimental possibilities to check (or discover) the impact of the extra dimensions on our 4-dimensional world. Now in many papers new effects are examined which in principle can be seen at future colliders. In \cite{20}, we show that these effects can also be discovered in experiments on elastic polarized hadron scattering. We explore in this case the sensitivity of interference spin effects to small corrections to the scattering amplitude, linear rather than quadratic (as in the case of cross sections) functions of a small parameter.

In this paper we examined the behavior of the gravitation amplitude of the hadron hadron scattering in the case of the one and two additional dimensions with taking into account the Kaluza-Kline states. At high energies such amplitude can give the additional real part to the standard hadron scattering amplitude. Especially we examine the possible oscillation in this additional part of the scattering amplitude. It can be important effect as it can be reveal in the future experimental data at LHC.

2 The graviton contribution with KK-modes

Assuming that the higher-dimensional theory at short distances is a string theory, one expects that the fundamental string scale $M_s$ and the Planck mass $M_{4+d}$ are not too different (a perturbative expectation is that $M_s \sim g_s M_{4+d}$). As of now, only known framework that allows a self-consistent description of quantum gravity is string theory \cite{21}. Thus, a compactification radius $r_d < < M_{Pl}^{-1}$ corresponds to a short-distance Planck scale and string mass $M_s$ which are $<< M_{Pl}$. So, we apply constrain on the quantum gravity scale $M_D$. to the string scale $M_s$.

Following \cite{18, 19}, the amplitude taking into account the KK-modes can be written as
\[ A_{\text{grav.}} \sim \int_{0}^{\infty} \frac{d^{d-1}q_{T}}{q^{2} + q_{T}^{2}} = \frac{\pi}{2} \frac{1}{q^{2}} \left( \frac{1}{q^{2}} \right)^{d/2} \csc(d\frac{\pi}{2}) \]  

(6)

This solution has the poles at \( d = 2, 4, 6, \ldots \). However such answer corresponds the bound of \( d < 2 \). In the case of \( d = 1 \) the scattering amplitude have the \( 1/q \) behavior. When \( 2 \leq d \) the integral divergent. If we take upper bound of the integral - \( M_{s} \) we obtain

\[ A_{\text{grav.}} \sim \int_{0}^{M_{s}} \frac{d^{d-1}q_{T}}{q^{2} + q_{T}^{2}} = \frac{M_{s}^{d}}{d} \frac{1}{q^{2}} F_{1}[1, d/2, 1 + d/2, -M_{s}^{2}m^{2}/q^{2})]. \]  

(7)

The hypergeometric function \( _{2}F_{1} \) has the smooth behavior but the upper integral limit income as multiple coefficient which leads to divergence of the Born amplitude if the upper limit of integral \( \to \infty \) and \( 2 \leq d \) For more high additional dimensions we obtain from eq.(7):

\[ A_{\text{grav.}}^{\text{Born}} = \sum_{\ell_{1}, \ldots, \ell_{n}} A_{1g+KK} = \frac{\pi s^{2}}{M_{4+2}^{4}} \ln \left( 1 + \frac{M_{s}^{2}}{q^{2}} \right) \]  

(8)

and for \( n = 3 \),

\[ A_{\text{grav.}}^{\text{Born}} = \frac{S_{d} s^{2}}{M_{4+3}^{4+3}} \left( \frac{M_{s}}{M_{4+3}} \right)^{n-2} \left[ 1 - \frac{q}{M_{s}} \text{ArcTh} \left( \frac{M_{s}}{q} \right) \right] ; \]  

(9)

and for \( n = 4 \):

\[ A_{\text{grav.}}^{\text{Born}} = \frac{S_{d} s^{2}}{M_{4+4}^{4+4}} \left( \frac{M_{s}}{M_{4+4}} \right)^{2} \left[ 1 - \frac{q^{2}}{M_{s}^{2}} \text{Ln} \left( \frac{M_{s}^{2}}{q^{2}} \right) \right] ; \]  

(10)

where \( S_{d} \)

\[ S_{d} = \frac{2\pi^{d/2}}{\Gamma(d/2)} \]  

(11)

is the area of the unit sphere in \( \mathbb{R}^{d} \). In this work we bound himself by the case \( d = 2 \). The higher dimensional lead to some additional \( q \) dependence of amplitude, but with the same problems of the excluded the large constant term. which, according [19], can be removed.

If our particles live on the 3-dimensional brane, we can obtain the amplitude in an impact parameter representation, where the Born amplitude corresponds to the eikonal of the scattering amplitude [?]

\[ \chi(s, b) = \frac{1}{2\pi} \int_{0}^{\infty} b J_{0}(qb) A^{\text{Born}}(q^{2}) \, db. \]  

(12)

An exact calculation of \([19]\) for \( d = 2 \) gives (see fig.1a)

\[ \chi(b) = \frac{s}{M_{D}} \left( 1 - bM_{D} \, K_{1}(b \, M_{D}) \right) / b^{2}. \]  

(13)

Some detailed calculations can be found in our work [22].
Figure 1: a) [left] $F(q) = -F_C(q) \ast q^2$ with artificial $\alpha_e = 1$ and $\lambda = 10^{-5}$; b) [right] $F(q) = -F_C(q) \ast q^2$ with real $\alpha_e$ and $\lambda = 10^{-5}$.

3 Oscillations of the amplitude

The Coulomb force, falling as $1/r^2$ lead to the scattering amplitude is proportional $1/t$, where the $t$ is the momentum transfer. In the second Born approximation there appear the additional phase and the amplitude will be

$$F(s, t) = \frac{2\alpha}{-i e^{i\alpha e \varphi_C}},$$

(14)

where $\alpha_e$ is electromagnetic constant, and $\varphi_C = Log(\lambda/q^2)$ with the effective photon mass. This phase divergence when we put the effective photon mass equal zero. Obviously, this phase do not impact on the differential cross sections. In the case of the hadron electromagnetic interaction it is need take into account the hadron form-factor and the phase will be more complicated

$$\varphi(s, t) = \varphi_C(t) + \varphi(s, t),$$

(15)

The $\varphi(s, t)$ was calculated in [4]. If take $\alpha_e \sim 1$ and there is some small photon mass such amplitude will be oscillate at small momentum transfer (Fig.1a). Of course, the real situation leads to the disappear such periodic structure - Fig.1b.

The Veniziano [21] amplitude was obtained also for gravitation force but with taking into account the sum of the diagrams using the standard eikonal representation

$$F(s, t) = \frac{8\pi\alpha_G}{q^2} \frac{4}{q^2} = i\alpha_G,$$

(16)

where $\alpha_G = Gs$. The additional phase is practically the same as in the t’Hoft amplitude. If $\alpha_G = 5$ there also will be some oscillations at small momentum transfer (Fig.2a). However already $\alpha_G = 1$ such oscillation disappear - Fig.2b.

The standard t’Hoft pole [5] and also [23, 24], obtained for the gravitation force,

$$F(s, t) = \frac{\Gamma(1 - iGs)}{4\pi \Gamma(iGs)} \frac{4}{-i}$$

(17)
with $G$ - the standard gravitation constant. It also contain the additional phase but it leads in this case to the some oscillation of the amplitude (Fig.2b) and also disappear in the differential cross sections.

Taking into account the additional dimension leads to the renormalization of the gravitation constant and the effective gravitation constant $\alpha_G$ can be near the unity or above already at LHC energies. The scattering amplitude in the case for $n = 4 + 2$ after the eikonalization has also the oscillation behavior [19] and [25]. In that cases the eikonalized amplitude calculated by some approximation using the method of stationary point. In the case of the eikonal phase (13) for $n = 4 + 2$ gave the amplitude which can be have some periodic structure. If we will be calculate the amplitude by numerically it is need carefully check up the result, which can be dependence from the accuracy and the integral limits. In the standard hadron scattering with Gaussian potentials there is not such problems as the Gaussian form of phase has good behavior at zero impact parameter and faster decreasing at large $b$. Let us see how the result will be dependence from the upper bound of integral. In the case of the one addition dimension and take the integral in the form as in [25]

$$A(s, b)_{\text{eik}} = 4\pi s b_0^2 F_n(b_n q),\quad (18)$$

$$F_n(y) = -i \int_0^A b J_0(qb) \left(e^{ix^n} - 1\right),\quad (19)$$

where input $x = b/b_c$ and take [19] [25]

$$\chi(b) = (b_c/b)^n, \quad \text{with} \quad b_c = \left(\frac{(4\pi)^{n/2} - 1}{2M_D^{n+2}}\right)^{1/n}\quad (20)$$

In this case for $n = 1$ the amplitude will be has the same periodic structure as in [25] (Fig.3) if the upper bound of the integral $B = 25$ (Fig.3a - upper). The period of this oscillation will be inverse proportional to the size of upper integral value $A$. If the $A = 50$ the period decrease in to time. If the upper bound of the integral sufficients large $a = 500$ the visible oscillations is disappear (Fig.3b - upper). However, if it is take a small interval ind made
the magnification it then the periodic structure appear again (Fig3.b - low). In most part such effect connect with the slowly decreasing the eikonal phase in the case $n = 1$. Such calculation for the $n = 2$, using the phase (13) show the practically disappear the periodic structure.

4 Conclusion

Taking the cut of the integration of the contribution of the the Kaluza-Kline states, which is proportional string mass $M_s$ the gravitation Born amplitude in the n-dimensions was obtained. The corresponding eikonal phase for the two additional dimension show the change the behavior of phase at small impact parameter from the standard $1/b^2$ behavior. The numerical calculation of the eikonalized amplitude show the some periodic structure in the case of one additional dimension. However such structure has the period which is inverse proportional to the upper bound of the eikonal integration. It means that if the such gravitation potential has some saturation regime at large, but not huge, distances (for example, order $50 fm$) it can leads to the some small oscillation behavior of the diffraction differential cross sections. Of course, in the real case we need take into account the form factors of the scattering particles.
Our calculations of the impact of two additional dimensions on the gravitational potential at small distances show that it differs from a simple power at \[ r \leq 3 \text{ GeV}^{-1} \] and it changes the profile function of proton-proton scattering. Including the effects of Kaluza-Klein modes of graviton scattering amplitudes, with two extra dimensions and taking into account the gravitational form factor, which we calculated from the GPDs of the nucleons \( A(x, t) \), it was shown that the impact-parameter dependence of the gravitational eikonal heavily changes from the standard \( 1/b^2 \) dependence. This is the main result of our paper. We think that this effect has to be taken into account in the calculation of the production of Black Holes at super-high energy accelerators.

We have shown that the gravitational interaction additional dimensions and the possible small spin-flip amplitude, proportional to the gravitational form factor \( B(t) \), lead to large spin correlation effects at small angles and \(-t \sim 10 - 30 \text{ GeV}^2\). However, the inclusion of the gravitational form factors \( A(b) \) decreases this effect and drastically changes its form.

**Acknowledgments**

The authors express their thanks to Ir. Arefieva and O. Teryaev for fruitful discussions.

**References**

[1] S.M. Roy, *Phys. Lett.* B 34, 407 (1971).
[2] G. Auberson, T. Kinoshita, A. Martin, *Phys. Rev.* D3, 3185 (1971).
[3] O.V. Selyugin, Int. Jour. Mod. Phys. A 12 1379 (1997);
[4] O.V. Selyugin, Phys. Rev. D 61, 074028 1-9 (1999).
[5] G.’t Hooft, Phys.Lett. B 198, 61-63 (1987).
[6] T. Kaluza, Sitzungsber.Preuss.Akad.Wiss.Berlin (Math.Phys.) 1921:966-972,1921.
[7] O. Klein, Zeit. f. Phys. 37, 895 (1926).
[8] John Ellis, N. E. Mavromatos, D.V. Nanopoulos Phys.Lett. B 665 412-417(2008).
[9] I. Antoniadis, Phys.Lett. B 246 377 (1990).
[10] I. Antoniadis, et all., Phys.Lett. B 436 257 (1996).
[11] See, e.g., T. Appelquist, A. Chodos, and P. G. O. Freund, *Modern Kaluza-Klein Theories* (Addison-Wesley, New York, 1987).
[12] V.A. Rubakov, M.E. Shaposhnikov, Phys. Lett. B 125, 136–138 (1983).
[13] A. Perez-Lorenzana, J. Phys. Conf. Ser. 18, 224 (2005).
[14] L. Randall, and R. Sundrum, Phys. Rev. Lett, 83, 3370;
[15] L. Randall, and R. Sundrum, Phys. Rev. Lett, 83, 4690 (1999).
[16] T. Han, J. D. Lykken, and R.-J. Zhang, Phys. Rev. 59, 105006 (1999).
[17] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Rev. D 59, 086004 (1999).
[18] S. Nussinov, and R. Shock, Phys. Rev. D 59, 105002 (1999).
[19] G. F. Giudice, R. Rattazzi, and J. D. Wells,
[20] O. V. Selyugin, O. V. Teryaev, Foundation Physics (2010)
[21] D. Amati, M. Ciafaloni, and G. Veneziano, International Jour. of Mod. Phys. A 3, 1615-1661 (1988).
[22] O. V. Selyugin, O. V. Teryaev, Czech. J. Phys. (Suppl. C) 56, 249-256 (2006).
[23] D. Amati, M. Ciafaloni, and G. Veneziano, Phys. Lett. 197B, 81 (1987).
[24] I. J. Muzinich, and M. Soldate, Phys. Rev. D 37 (1988) 359.
[25] I. Yu. Aref’eva, arXiv:1007.4777 (2010).