SYSTEMS & CONTROL | RESEARCH ARTICLE

A novel tube model predictive control for surge instability in compressor system including piping acoustic

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Abstract: Tube model predictive control (MPC) is one of a few suitable techniques for robust stabilization of disturbed nonlinear systems subject to constraints. This paper introduces the integration of $H_\infty$ control scheme into a robust tube MPC control algorithm for surge prevention in centrifugal compressor system in the presence of bounded disturbances. Nonlinear dynamic of compression system is considered that contains acoustic of compressor system and brings up the effects of the station’s piping system on the compressor surge. Also, close-coupled valve is selected as actuator for the surge control system. The proposed method is composed of two controllers: a nominal MPC controller which solves a standard surge problem for the nominal system, and an ancillary MPC controller from $H_\infty$ control approach to stabilize the states despite disturbances. Numerical simulation results verify the

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PUBLIC INTEREST STATEMENT

In this paper, nonlinear dynamics of compressor system with piping acoustics and close-coupled valve actuator is considered and a new tube model predictive control method is described for the optimization of closed loop process dynamics with fast and reliable real-time solution. The proposed tube model predictive control uses a robust control approach for solving an ancillary control problem, which serves to contain the trajectories of the actual system in a tube around the nominal trajectory. This tube-based robust model predictive control algorithm can deal with bounded disturbances which provides a strong guarantee of robust stability. The method maintains feasible controlled trajectories while obtaining good nominal control performance.
capability of the proposed surge control method and guaranty the asymptotic stability of compression system in the presence of disturbances.

Subjects: Engineering Mathematics; Mechanics; Systems & Control Engineering; Automation Control; Systems Engineering; Control Engineering; Dynamical Control Systems

Keywords: centrifugal compressor; closed coupled valve; $H_{\infty}$ control; piping acoustic; surge control; tube model predictive control

1. Introduction

Compressors are important machines in current industrial developments for the pressing and transportation of gases and fluids. They are used, for example, as part of a gas turbine for jet and marine propulsion or power generation, in superchargers and turbochargers for internal combustion engines, and in a wide variety of industrial processes (Cohen, Rogers, & Saravanamutto, 1996; Gravdahl & Egeland, 1999a; Whitfield & Baines, 1990). Moreover, compressors are critical components of the entire system for heating, ventilation, and air conditioning in homes and commercial buildings.

Compressors can exhibit a variety of instabilities under different operating conditions. A review of instabilities found in compression systems is given by Greitzer (1981). Surge is one of the main dynamic instability that affects the entire compression system and it is characterized by a limit cycle oscillation that results in large amplitude fluctuations of the pressure and flow rate. This event is an unstable operating mode of a compression system that occurs at low mass flows which not only limits compressor performance and efficiency but it can also damage the compressor and auxiliaries due to the large thermal and mechanical loads involved. Furthermore, the vibrations associated with surge can result in unacceptable noise levels.

Having an appropriate model from a compressor system is first step to investigate the effect of changes in the system or environment, disturbances and even the effect of a controller on the system behavior. There are several mathematical models extended over the years to describe the dynamics of the flow in compression systems. Greitzer (1976a) not only developed the equations for his successful compression system model, he also compared experimental results with theoretical predictions (Greitzer, 1976b) and discussed the underlying physics and the influence of various model parameters. However, the author paid most attention to the effect of the plenum volume on the encountered dynamics. After the pioneering work by Greitzer (1976a, 1976b), the model and the underlying principles have been exploited in numerous studies. Macdougal and Elder (1983) investigated the accuracy and parameter influence of a somewhat different dynamic model for a centrifugal compression system. The authors concluded that the accuracy of the geometry and compressor characteristic are of major importance when developing a model for an actual compression system. Elder and Gill (1985) presented a study of the various factors affecting surge in centrifugal compression systems. Willems (2000) conducted a sensitivity analysis of Greitzer model for a small laboratory scale compression system.

Also, the effect of piping on the stability of the compression system was studied in Brun and Kurz (2015), Hagino, Kashiwara, and Uda (2005), van Helvoirt and de Jager (2007), Jungowski, Weiss, and Price (1996) and Sparks (1983). According to the consequences of the findings of Brun and Kurz (2015) and Sparks (1983), the complete piping system connected to the compressor has an important role for the stability of a centrifugal compression system and it can intensify/expedite of surge occurrence and restrict the stability range of compressor but the above models cannot take these effects. In addition, as pointed out in van Helvoirt and de Jager (2007), the Greitzer model lumps the spatially distributed parameters of the compressor piping into simple constants, thus it is unable to capture the acoustic resonances in the pipeline and to predict the dynamics associated with fluid flow in distributed systems, such as acoustic waves and flow pulsations in pipelines. The
mathematical model for the compression system that takes the effect of the piping acoustics during both the stable and unstable operating situations, accompanied by the dynamics during the transition between these two states is extended in Yoon, Lin, Goyne, and Allaire (2011a).

Research efforts during the past decades have led to important advancements in the field of active and passive approaches for centrifugal compressor surge control. Various survey papers (Greitzer, 1998; Gu, Sparks, & Banda, 1999; Paduano, Greitzer, & Epstein, 2001) show that various surge control studies have been carried out on different systems. An overview of past studies is given by Willems and de Jager (1999) and more recently Arnulfi, Blanchini, Giannattasio, Micheli, and Pinamonti (2006), Nelson, Paduano, and Epstein (2000), Spakovszky (2004) and Willems (2000) presented results from their surge control investigations. A surge avoidance system is a widely used passive method, which enjoys good reliability but expenses the efficiency and operating range (Gravdahl & Egeland, 1999b; Willems & de Jager, 1999). A promising technique to cope with surge is the active suppression of aerodynamic flow instabilities. When successful, active surge control enlarges the operational envelope of the compressor towards lower mass flows. Thereby, surge control under several types of actuators as mentioned in Bartolini, Muntoni, Pisano, and Usai (2008), Gravdahl, Egeland, and Vateland (2002), Spakovszky et al. (1999), Uddin and Gravdahl (2011) and Willems and De Jager (1998) makes the compression system more versatile and it allows the machine to run at the most efficient operating points, which are usually located near the surge initiation point.

Although some attainments have been obtained by employing advanced algorithms like MPC, the backstepping method, and sliding mode variable structure control (Ghanavati, Salahshoor, Jahed Motlagh, Ramezani, & Moarefianpour, 2017; Taleb Ziabari, Jahed Motlagh, Salahshoor, Ramezani, & Moarefianpour, 2017), however, none of these controllers have not considered the effects of pipe that is a negative point in model accuracy for stability of active controller. Based on a recently presented enhanced compression system model with variable impeller tip clearance and pipeline acoustics, a surge controller is designed (Yoon, Lin, Goyne, & Allaire, 2014), but there are objections in this case including the linear controller is not capable in capturing strongly nonlinear dynamics of compression system and also common actuators are the close-coupled valve (CCV) (Gravdahl & Egeland, 1999b) and the throttle valve (Krstić, Fontaine, Kokotovic', & Paduano, 1998). Drawback of linear controllers is the limited operating region in which these controllers are valid. Hence, stabilization can only be achieved when the perturbed system or, ideally, the entire surge limit cycle is contained in the domain of attraction of the desired equilibrium. As mentioned paper is not considered close-coupled valve as an actuator, but it yields the best surge control performance according to Simon, Valavani, Epstein, and Greitzer (1993) and Van De Wal, Willems, and De Jager (2002). Considering optimization of control signal, as well as limitations on actuators and states are another important topics that should be considered in the subject of controller design. Since disturbance is always found in real-world applications, stability of the disturbed centrifugal compressor should be ensured, too.

Overcoming the problems, we present new numerical results in the compressor surge tube MPC using the CCV as an actuator in the compression system. A surge controller is designed for a centrifugal compressor system based on the enhanced compression system model recently presented (Yoon et al., 2011a), and modified to support the CCV as actuator in this study. In this paper, considering the more general condition on the centrifugal compressor dynamics model, a new algorithm is proposed for centrifugal compressor surge control based on tube-based MPC. The proposed tube MPC use $H_\infty$ control approach for solving an ancillary MPC problem, which serves to contain the trajectories of the actual system in a tube around the nominal trajectory. This tube-based robust MPC algorithm can deal with bounded disturbances and provide a strong guarantee of robust stability. Therefore, this article is categorized as follows. In the second part, the compressor system model taking into consideration the effects of pipeline and CCV actuator will be detailed. In Section 3, a novel synthesis approach for tube-based robust MPC is proposed. In Section 4, the simulation results are presented. The conclusions are then drawn in Section 5.
2. System model

The equations of system model are manipulated in this section and converted into a form that is more convenient for the design of the active surge controller. The considered dynamic model of the compression system was offered in details (Yoon et al., 2011a; Yoon, Lin, Goyne, & Allaire, 2011b). The compression system in the study of surge comprised of three basic elements. They are the compressor/plenum increasing and keeping the energy in the system, the throttle valve governing the average flow rate, and the piping transferring the pressed gas/fluid. The nondimensional pressure rise $\psi$ and mass flow rate $\Phi$ are determined as the subsequent functions of the mass flow rate $m$ and the absolute pressure $P$:

$$\Phi = \frac{m}{\rho_0 U A_c}, \quad \psi = \frac{P - P_{o1}}{\frac{1}{2} \rho_{o1} U^2}$$  \hspace{1cm} (1)

The constants in (1) are the impeller velocity $U$, the cross sectional area of the compressor duct $A_c$, the inlet absolute pressure $P_{o1}$, and the density at the inlet $\rho_{o1}$. The Greitzer model can take the surge limit cycle for a compression system, and it delivers as a pleasant basis for improvement of surge control instructions. As explained in Yoon et al. (2011a), the acoustic resonance from compressor pipeline is appended to the model. The resultant block illustration for the compression system with pipeline dynamics is presented in Figure 1.

2.1. Compressor and plenum

The foundation of the equations explaining the flow in the compressor and the plenum volume originates from the Greitzer model in Greitzer (1976a, 1976b). Assumptions for the model are low pressure rise compared with ambient pressure, low compressor inlet match number, isentropic compression process in the plenum with uniform pressure distribution, and negligible fluid velocities in the plenum. The use of a close coupled valve for surge control was exactly modelled and presented in Gravdahl and Egeland (1999a) and Uddin and Gravdahl (2015). The method is to bring forward a valve close to the plenum volume in the compressor. The CCV signifies that there is no mass storage of gas between the compressor outlet and the valve, as can be regarded from Figure 2.

Based on mentioned statement, the pressure intensification in the compressor and the pressure fall down across the valve can be combined into an equivalent compressor. This is required in order to empower the valve for controlling the characteristic of the equivalent compressor.
The outlet mass flow from the compressor through a close coupled valve is specified as follows:

$$\Phi_c = c_r u_r \sqrt{(\Psi_p - \Psi_c)}$$  \hspace{1cm} (2)

where $c_r$ is valve constant, $u_r$ is the CCV percentage, and $\Psi_p$, $\Psi_c$ are plenum pressure rise and compressor pressure rise, respectively. The variety value of $u_r$ is from 0 to 100%.

Finally, according to Gravdahl and Egeland (1999a) and Uddin and Gravdahl (2015), equations for explaining the pressure and mass flow in the constant speed centrifugal compressor can now be expressed as follows:

$$\Phi_c = B_0 \omega_{th} \left( \Psi_{c, ss} - \Psi_p \right)$$  \hspace{1cm} (3)

$$\Phi_p = \frac{c_r u_r}{B} \left( \Psi_{c, ss} - \Psi_p \right)$$

where $B$ is the Greitzer stability parameter and $\omega_{th}$ is the Helmholtz frequency (Greitzer, 1976). The state variables of the model are the compressor mass flow rate $\Phi_c$ and the plenum pressure rise $\Psi_p$, which are nondimensionalized as shown in (1). The value of the plenum mass flow rate $\Psi_p$ is dependent on the dynamics of the piping. Eventually, the steady-state compressor pressure rise $\Psi_{c, ss}$ is achieved from the compressor characteristic curve as a function of the compressor mass flow rate

$$\Psi_{c, ss}(\Phi_c) = \left\{ \begin{array}{ll} A_1 \Phi_c^3 + B_1 \Phi_c^2 + D_1, & \text{If } \Phi_c > \Phi_s \\ A_2 \Phi_c^3 + B_2 \Phi_c^2 + D_2, & \text{If } \Phi_c \leq \Phi_s \end{array} \right.$$  \hspace{1cm} (4)

The characteristic curve $\Psi_{c, ss}$ divided into the stable and unstable part by the surge point at $\Phi_s$. The coefficients $A_i$, $B_i$, and $D_i$ of the characteristic curve match to the stable flow region of the compressor, whereas the coefficients $A_i$, $B_i$, and $D_i$ meet unstable flow region, as Yoon et al. (2014).

2.2. Piping

To express the dynamics of the piping system, a modal approximation of the transmission line dynamics (Yang & Tobler, 1991) has been encompassed in the compression system equations (Yoon et al., 2011a). The single-mode state space demonstration of the piping equations can be stated in the following form:

$$\begin{bmatrix} p_{th} \\ q_p \end{bmatrix} = \begin{bmatrix} 0 & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} p_{th} \\ q_p \end{bmatrix} + \begin{bmatrix} 0 & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} p_p \\ q_{th} \end{bmatrix}$$  \hspace{1cm} (5)

for some matrix coefficients $A_{ij} \in R$ and $B_{ij} \in R$, where $P_p$ and $Q_p$ are the upstream (plenum) pressure and volumetric flow rate, respectively. Correspondingly, $P_{th}$ and $Q_{th}$ are the downstream (throttle duct) pressure and flow rate.

The above piping model signifies the system in relation to the absolute pressure and volumetric flow rate. With the assumption that the variation of density $\rho$ of gas in the pipeline because of the pressure and temperature fluctuation affected by the piping acoustics is small, the states of the piping model can be nondimensionalized as described in (1). Afterwards the coordinate transformation, the consequential piping equation with nondimensional states is expressed in this way:

$$\begin{bmatrix} \Psi_{th} \\ \Phi_p \end{bmatrix} = \begin{bmatrix} 0 & 2A_{12} \rho \omega_{th} \\ A_{21} \rho \omega_{th} & A_{22} \end{bmatrix} \begin{bmatrix} \Psi_{th} \\ \Phi_p \end{bmatrix} + \begin{bmatrix} 0 & 2B_{12} \rho \omega_{th} \\ B_{21} \rho \omega_{th} & B_{22} \end{bmatrix} \begin{bmatrix} \Psi_p \\ \Phi_{th} \end{bmatrix} + \frac{\rho p_{th}}{\rho \omega_{th} \omega_{th}} \begin{bmatrix} \Psi_{th} \\ \Phi_{th} \end{bmatrix}$$  \hspace{1cm} (6)
2.3. Throttle valve
The flow rate through the throttle valve is a function of the pressure drop across the valve. Here, it is supposed the dynamics at the throttle duct section are considerably faster than the rest of the system, and only the steady-state behavior is seized. The relation between the pressure and the mass flow rate in the throttle valve part is specified by:

\[
\Phi_{th} = c_{th} u_{th} \sqrt{\psi_p}
\]  

(7)

where \(c_{th}\) is the valve constant and \(u_{th}\) is the throttle valve opening percentage.

2.4. Equations set
The equations for the dynamics of the complete compression system are achieved by combining (3), (6), and (7). The ensuing nonlinear system has the compressor mass flow, the plenum pressure rise, the throttle section pressure rise, and the plenum mass flow rate as state variables. The state space equations of the collected system are as follows:

\[
\begin{bmatrix}
\Phi_c \\
\psi_p \\
\psi_{th} \\
\Phi_p
\end{bmatrix}
= 
\begin{bmatrix}
0 & -B_{th} & 0 & 0 \\
\frac{\mu}{\rho} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{2B_p}{A_c} \\
\rho \frac{C}{\mu} \frac{A_{th}}{2A_c} & \frac{A_{th}}{2A_c} & \frac{A_{th}}{2A_c} & 0
\end{bmatrix}
\begin{bmatrix}
\Phi_c \\
\psi_p \\
\psi_{th} \\
\Phi_p
\end{bmatrix}
+ 
\begin{bmatrix}
B_{th}(\psi_c, \text{ss}) \\
0 \\
0 \\
\mu \frac{C}{\mu} (A_{21} + B_{21})
\end{bmatrix}
\begin{bmatrix}
\Phi_r \\
\Phi_{th}
\end{bmatrix}
\]

(8)

An Euler approximation of system (8) with sampling time \(T_c\) is given by

\[
\begin{bmatrix}
\Phi_c \\
\psi_p \\
\psi_{th} \\
\Phi_p
\end{bmatrix}
(k + 1) = T_c \times 
\begin{bmatrix}
\frac{1}{T_c} & 0 & 0 & 0 \\
0 & \frac{1}{T_c} & 0 & 0 \\
0 & 0 & \frac{1}{T_c} & 0 \\
0 & 0 & 0 & \frac{2B_p}{A_c}
\end{bmatrix}
\begin{bmatrix}
\Phi_c \\
\psi_p \\
\psi_{th} \\
\Phi_p
\end{bmatrix}
+ 
T_c \times 
\begin{bmatrix}
\frac{\mu}{\rho} & 0 & 0 & 0 \\
0 & \frac{2B_p}{A_c} & 0 & 0 \\
0 & 0 & \frac{\mu}{\rho} & 0 \\
0 & 0 & 0 & \frac{2B_p}{A_c}
\end{bmatrix}
\begin{bmatrix}
\Phi_r \\
\Phi_{th}
\end{bmatrix}
\]

(9)

That is a subset of discrete-time disturbed nonlinear systems in the form

\[
x(k + 1) = Ax(k) + f(x(k)) + Bu(k) + Bw(k)
\]  

(10)

In this paper, we'll design a robust tube MPC for the particular class. The parameters of the theoretical model are summarized in Table 1. Also, the coefficients of the characteristic curve correspond to the stable and unstable operating regions of the compressor are given in this table. The corresponding matrix coefficients of the piping equation in (5) are found to be \(A_{12} = 3.7 \times 10^4\), \(A_{11} = -1.92 \times 10^{-3}\), \(A_{12} = -8\), \(B_{12} = -3.7 \times 10^5\), \(B_{21} = 1.92 \times 10^{-3}\) and \(B_{22} = 7.98\).

3. Tube MPC based \(H_\infty\) control approach
In this section a robust tube MPC based on \(H_\infty\) approach is proposed for ensuring robust stability in the face of disturbance. It is described based on the approach proposed in Magni, De Nicolao, Scattolini, and Allgöwer (2003), Raimondo, Limon, Lazar, Magni, and Camacho (2009), Yu, Maier, and Allgower (2009) with some modifications that reduces the online computational burden and achieves robustness against disturbances. Control signal has two components: a nominal controller that obtains from online optimization problem subject to nominal dynamics, and an ancillary controller which maintains the state of the disturbed system, real model of centrifugal compressor, close to the trajectory of the nominal system. The constrained discrete-time nonlinear system is described.
where \( x \in \mathbb{R}^n \) is the state of the system, \( u \in \mathbb{R}^n \) is the control input. The signal \( w \in \mathbb{R}^n \) is the exogenous disturbance or uncertainty, which is unknown but bounded, and lies in a compact set, \( \forall k \geq 0 \).

The system is subject to constraints:

\[
L \text{ diagonal matrix with positive constant known numbers.}
\]

Define a nominal system as:

\[
\bar{x}(k + 1) = A\bar{x}(k) + f(\bar{x}(k)) + B\bar{u}(k)
\]

(11)

where \( x \in \mathbb{R}^n \) is the state of the system, \( u \in \mathbb{R}^n \) is the control input. The signal \( w \in \mathbb{R}^n \) is the exogenous disturbance or uncertainty, which is unknown but bounded, and lies in a compact set,

\[
W: = \{ w \in \mathbb{R}^n \mid w \leq w_{\max} \}
\]

(12)

i.e. \( w(k) \in W \), for all \( k \geq 0 \). The system is subject to constraints:

\[
x(k) \in X, u(k) \in U \forall k > 0
\]

(13)

The \( A, B, B_w \) matrices are known and constant. \( f(x):\mathbb{R}^n \rightarrow \mathbb{R}^n \) is continuous nonlinear function over the \( x \) which satisfy Lipschitz condition.

\[
f(x_1) - f(x_2) \leq Lx_1 - x_2 \forall x_1, x_2 \in X
\]

(14)

\( L \) diagonal matrix with positive constant known numbers.

Define a nominal system as

\[
\bar{x}(k + 1) = A\bar{x}(k) + f(\bar{x}(k)) + B\bar{u}(k)
\]

(15)

The nominal optimal control problem is subject to the nominal dynamics (15), i.e. there are no disturbances. The nominal control problem for the current state \( x(k) \), which is an online solution, is formulated as follows:

\[
\begin{align*}
\text{minimize} & \quad J(k) \\
\text{subject to} & \quad \bar{u}(k + i) \quad i = 0, 1, 2, \ldots, N \\
& \quad \bar{x}(k + 1) = A\bar{x}(k) + f(\bar{x}(k)) + B\bar{u}(k)
\end{align*}
\]

(16)

Table 1. Model parameters for the compression system

| Parameter                  | Symbol | Unit | Value  |
|----------------------------|--------|------|--------|
| Comp. duct length          | \( L_c \) | m    | 1.86   |
| Comp. duct cross area      | \( A_c \) | m²   | 0.0082 |
| Char. curve \( A_1 \) coeff.| \( A_1 \) | –    | -4.717 |
| Char. curve \( A_2 \) coeff.| \( A_2 \) | –    | -172.6 |
| Char. curve \( B_1 \) coeff.| \( B_1 \) | –    | -2.859 |
| Char. curve \( B_2 \) coeff.| \( B_2 \) | –    | 36.88  |
| Char. curve \( D_1 \) coeff.| \( D_1 \) | –    | 1.193  |
| Char. curve \( D_2 \) coeff.| \( D_2 \) | –    | 1.029  |
| Gritzer Stab. parameter    | \( B \)  | –    | 0.44   |
| Helmholtz frequency        | \( w_o \) | rad/s | 80.1   |
| Impler Tip speed           | \( U \)  | m/s  | 213.24 |
| Inlet pressure             | \( P_{in} \) | Pa   | 101325 |
| Inlet gas density          | \( \rho_{in} \) | kg/m³ | 1.165  |
| Plenum volume              | \( V_p \) | m³   | 0.049  |
| Pipeline length            | \( L \)  | m    | 6.5    |
| Throttle constant          | \( C_m \) | –    | 1.8569 |
| CCV constant               | \( C_r \) | –    | 0.5    |
\[ \ddot{x}(k) \in X \]
\[ \ddot{u}(k) \in U \]

and the nominal cost functional is defined as:

\[ J(k) = \sum_{i=0}^{N} \left( \ddot{x}(k+i|k)\ddot{Q}\ddot{x}(k+i|k) + \ddot{u}(k+i|k)\ddot{R}\ddot{u}(k+i|k) \right) \tag{17} \]

Here \( N \) is the prediction horizon, \( \ddot{Q} \) and \( \ddot{R} \) are positive definite weighing matrices.

The goal of the paper is to design a control signal with both nominal controller and state feedback control (ancillary control), i.e.

\[ u(k) = \ddot{u}(k) + v(k) \tag{18} \]

Which guarantees the robustness of compressor system in presence of disturbance.

And defining error

\[ e(k) = x(k) - \ddot{x}(k) \tag{19} \]

The dynamics of the error can be achieved as follows:

\[ e(k+1) = A e(k) + f(x(k)) - f(\ddot{x}(k)) + B_w w(k) + B v(k) \tag{20} \]

Describing

\[ h(x, \ddot{x}, e) = A e(k) + f(x(k)) - f(\ddot{x}(k)) \tag{21} \]

Finally, we have

\[ e(k+1) = h(x, \ddot{x}, e) + B_w w(k) + B v(k) \tag{22} \]

**Definition 1.** A set \( s \) is a robust control invariant set for error system (22) if and only if there is an ancillary feedback control law \( v(\cdot) \) with \( v(\cdot) + \ddot{u}(\cdot) \subseteq U \in R^n \) such that \( \forall v(\cdot) \in \Omega_c \) and \( \forall w \in W \), the trajectories \( v(k) \) remain in \( \Omega_c \) for all \( k \geq 0 \).

**Definition 2.** A dynamic system is asymptotically ultimately bounded if a set of initial conditions of the system converges asymptotically to a bounded set.

In this section, \( H_\infty \) approach is used to obtain the ancillary control law. The main idea is originated from Magni et al. (2003), where \( H_\infty \) approach is used to compute an auxiliary central law with its associated robust invariant set. By some modification, the idea is used for computing ancillary control law.

**Theorem 1.** The control law \( v(k) \) can be obtained by solving \( H_\infty \) control problem for the system (22). To do this, a \( n \times n \) square matrix, \( P \) and a symmetric matrix \( M \) must be defined

\[ M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \tag{23} \]
\[ M_{11} = B'PB + R \]
\[ M_{12} = M'_{21} = B'PB_w \]
\[ M_{22} = B'PB_w - \gamma^2 I \]

where \( \gamma \) is the designing parameter. The quadratic function is also defined as
\[ V(e) = e'Pe \] 

(24)

According to Magni et al. (2003), suppose there exists a positive definite matrix \( P \) such that

(i) \( M_{22} < 0 \)

(ii) \( -P + (A + L)'P(A + L) + Q - (A + L)'[B \ B_w]M^{-1}[B \ B_w]'P(A + L) < 0. \)

Then, the control law \( v(k) = \kappa(x, \bar{x}, e) \) becomes

\[
\begin{bmatrix}
\kappa(x, \bar{x}, e) \\
\xi(x, \bar{x}, e)
\end{bmatrix} = -M^{-1}
\begin{bmatrix}
B & B_w
\end{bmatrix}
\begin{bmatrix}
Ph(x, \bar{x}, e)
\end{bmatrix}
\] 

(25)

With
\[ \Gamma_c = \{ e : e'Pe \leq \alpha \} \] 

(26)

where \( \alpha \) is a finite positive constant. The matrix \( P \) can also be derived by solving a discrete time \( H_\infty \) algebraic Riccati equation as follows

\[ P = A(A + L)'P(A + L) + Q - (A + L)'[B \ B_w]M^{-1}[B \ B_w]'P(A + L) < 0. \] 

(27)

Proof. Define
\[ H(x, \bar{x}, e, v, w) = V(h + Bv + B_w w) - V(e) < -\left(e'Qe + v'Rv - \gamma^2 \| w \|^2 \right) \] 

(28)

\[ (h + Bv + B_w w)'P(h + Bv + B_w w) - e'Pe + e'Qe + v'Rv - \gamma^2 \| w \|^2 < 0 \] 

(29)

\[ h'Ph + e'(Q - P)e + v'(B'PB + R)v + w'(B_wPB_w - \gamma^2 I)w + 2\begin{bmatrix} v' & w' \end{bmatrix}
\begin{bmatrix}
B'Ph \\
B_w'Ph
\end{bmatrix} + 2v'B'PB_w < 0 \] 

(30)

\[ h'Ph + e'(Q - P)e + v'
\begin{bmatrix}
B'Ph \\
B_w'Ph
\end{bmatrix} + v'
\begin{bmatrix}
B'Ph \\
B_w'Ph
\end{bmatrix} < 0 \] 

(31)

And computing \( H(x, \bar{x}, e, v, w) \) for \( v = \kappa(x, \bar{x}, e) \) and \( w = \xi(x, \bar{x}, e) \)

\[ H(x, \bar{x}, e, \kappa(x, \bar{x}, e), \xi(x, \bar{x}, e)) = h'Ph + e'(Q - P)e + \begin{bmatrix}
\kappa'(x, \bar{x}, e) & \xi'(x, \bar{x}, e)
\end{bmatrix}
\begin{bmatrix}
\kappa(x, \bar{x}, e) \\
\xi(x, \bar{x}, e)
\end{bmatrix} < 0 \] 

(32)
\[ h'Ph + e'(Q - P)e - h'P \begin{bmatrix} B & B_w \end{bmatrix} M^{-1} \begin{bmatrix} B' & B_w' \end{bmatrix} P h < 0 \] (33)

According to (21)

\[(Ae + f(x) - f(\hat{x})) P(Ae + f(x) - f(\hat{x})) + e'(Q - P)e - (Ae + f(x) - f(\hat{x})) P \begin{bmatrix} B & B_w \end{bmatrix} M^{-1} \begin{bmatrix} B' & B_w' \end{bmatrix} P(Ae + f(x) - f(\hat{x})) < 0\]

According to Lipschitz conditions (14)

\[e' \left( (A + L) P(A + L) + Q - P - (A + L) P \begin{bmatrix} B & B_w \end{bmatrix} M^{-1} \begin{bmatrix} B' & B_w' \end{bmatrix} P(A + L) \right) e < 0\] (35)

There exist positive constants \(\epsilon, r_2\) are followed from (ii) such that

\[H(x, \tilde{x}, e, \kappa(x, \tilde{x}, e), \xi(x, \tilde{x}, e)) \leq -\epsilon \|e\|^2 \quad \forall e \in \Omega_2 = \{e: \|e\| \leq r_2 \}\] (36)

By Taylor expansion theorem

\[H(x, \tilde{x}, e, v, w) = H(x, \tilde{x}, e, \kappa(x, \tilde{x}, e), \xi(x, \tilde{x}, e)) + \frac{1}{2} \left[ v - \kappa(x, \tilde{x}, e) \cdot w - \xi(x, \tilde{x}, e) \right] \times R \times \left[ v - \kappa(x, \tilde{x}, e) \cdot w - \xi(x, \tilde{x}, e) \right] \] (37)

where the first order term evaluated and terms of order >2 are null. If the system is controlled by \(\kappa(x, \tilde{x}, e)\) then

\[H(x, \tilde{x}, e, \kappa(x, \tilde{x}, e), w) = H(x, \tilde{x}, e, \kappa(x, \tilde{x}, e), \xi(x, \tilde{x}, e)) + \frac{1}{2} (w - \xi(x, \tilde{x}, e)) M_{22} (w - \xi(x, \tilde{x}, e)) \] (38)

Because \(M_{22} < 0\) and (36)

\[H(x, \tilde{x}, e, \kappa(x, \tilde{x}, e), w) \leq H(x, \tilde{x}, e, \kappa(x, \tilde{x}, e), \xi(x, \tilde{x}, e)) \leq -\epsilon \|e\|^2 \quad \forall e \in \Omega_2 = \{e: \|e\| \leq r_2 \} \quad \text{all } w \in W \]

From (28) and (39) follows that

\[V(h + Bv + B_w w) \leq V(e) - \left( e' Q e + v' v - \gamma^2 \|w\|^2 \right) - \epsilon \|e\|^2 \leq V(e) - \epsilon \|e\|^2 \quad \forall e \in \Omega_2 = \|e: V(e) \leq a \| \subset \Omega_2 \quad \text{all } w \in W \] (40)

And that \(\Omega_2\) is an invariant set.

Finally, the following controlling algorithm is employed to stabilize the compressor system.

**Algorithm 1**

Step 0. At time \(k = 0\) set \(\hat{X} - X(0)\) in which \(X(0)\) is the current states.

Step 1. At time \(k\) and current states \(\hat{X}(k), X(k)\), solve problem (16) to obtain nominal control action \(\hat{u}(k)\) and the actual control action \(u(k) = \hat{u}(k) + v(k)\).

Step 2. Apply the control \(u(k)\) to the system (11), during the sampling interval \([k, k + 1]\).

Step 3. Measure the states \(X(k + 1)\) at the next time instant \(k + 1\) of the system (11) and compute the successor states \(\hat{X}(k + 1)\) of the nominal system (15) under nominal control \(\hat{u}(k)\).
Step 4. Set \((\bar{X}(k), X(k)) = (\bar{X}(k+1), X(k+1))\), \(k = k + 1\) and go to the step 1.

We should notice only the nominal model is used for prediction and the scheme online computational burden is as some as standard MPC. General results of the proposed algorithm is stated by following theorem.

**Theorem 2.** Assuming \(X, v, \Omega_c\) are given and optimization problem is feasible at \(k = 1\); Then

1. Optimization problem is feasible for all \(\geq 0\).
2. According to algorithm 1, the trajectory of the system (1) under control law is asymptotically ultimately bounded.
3. The closed-loop system is input-to-state stable.

**Proof:** Since the online optimization problem only uses the “measured” state of nominal system and the nominal system dynamics, it does not depend on the uncertainties and disturbances at all.

Thus, if the optimization problem has a feasible solution at \(k = 0\), recursive feasibility is guaranteed. There is a finite quantity \(\beta(x_0), \beta(0) = 0\), such that

\[
\|x\|(k) \leq \beta\left(\|x_0\|, k\right) \forall k \geq 0
\]

Because \(e(k) \in \Omega_c\) there is a class \(K\) function such that

\[
e(k) \leq \gamma\left(\sup_i \|w(i)\|\right) \quad \forall k > 0
\]

Finally, applying to \(\hat{x}(0) = x(0)\) and \(x(k) = \hat{x}(k) + e(k)\)

\[
x(k) \leq \beta\left(\|x_0\|, k\right) + \gamma\left(\sup_i \|w(i)\|\right) \quad \forall k \geq 0 \quad 0 \leq i \leq k
\]

Therefore by using of algorithm 1, system (11) is asymptotically ultimately bounded and closed-loop system is input-to-state stable (Khalil, 2002).

4. Simulation results
In this section, the model of centrifugal compressor is used to illustrate the effectiveness and performance of the proposed tube-based MPC in stabilizing the compressor. The compressor contains four states. The control law of Algorithm 1 is applied to the model (11) of centrifugal compressor. The control objective is to avoid surge, i.e. stabilizing the system. The control constraint is

\[
u(k) \in \bigcup_{\Delta} \left\{ u| |u(k)| \leq 0.2 \right\}
\]

and the disturbance is bounded

\[
w(k) \in \mathcal{W} \triangleq \{ w|\|w(k)\|_{\infty} \leq 0.1 \}
\]

The cost function is defined by (14) with \(Q = I, R = 0.01\).

To demonstrate the various features of the proposed surge control method, a following set of simulation is performed on the compressor model with disturbance and change in throttle opening percentage.

Compressor initially operate in steady state where the throttle valve openings equals to 20%. At time \(t = 0.8s\), the throttle is closed to 10% and the disturbance is applied simultaneously as the most difficult conditions. A 10% sinusoidal variation in mean flow is considered as disturbance. This flow fluctuation simulates the type of flow unsteadiness observed from vortex shedding or other periodic flow excitation (Brun & Kurz, 2015).
If the controller will not be activated, the compressor is interred into surge as shown in the Figure 3. The states and control signal for compressor system using proposed method are shown in Figures 4–8. Also, the tube MPC is compared with NMPC design by Imani, Jahed Motlagh, Salahshoor, Ramazani, and Moarefianpur (2017) on compressor system to demonstrate the effectiveness of the proposed controller. As it can be seen, simulation results illustrate the robustness of the proposed controller to steer the centrifugal compressor against disturbance and change in throttle opening percentage, while the NMPC controller is not able to precisely control in steady state, in spite of relatively good transient response. Although, the control signal obtained of NMPC has the maximum value in its range, system states are moving to instability and disturbances will cause the surge.

Simulation results show controller capability against disturbance and comprehending the effects of pipe, throttle and whole of compressor system. Using of a tube based predictive control and taking into account nonlinear dynamics of pipe, we’ll be able to stabilize the compressor system under the most difficult conditions to prevent from the surge occurrence and also enlarging the range of compressor by the CCV as actuator.

**Figure 3. States of system when surge controller is inactivated.**

**Figure 4. Comparison of compressor flow under proposed Tube-MPC and NMPC.**
Figure 5. Comparison of plenum pressure under proposed Tube-MPC and NMPC.

Figure 6. Comparison of throttle pressure under proposed Tube-MPC and NMPC.

Figure 7. Comparison of plenum flow under proposed Tube-MPC and NMPC.
5. Conclusion
In this paper, the model of the compression system with piping acoustics, disturbances and control constraints is considered. Based on tube-based MPC, a control design methodology is presented which can steer a centrifugal compressor under surge to neighborhood of the origin in a short finite time. Control input is generated with considering their constraints and minimizing cost functions. The controller solves two optimal control problem, one which solves a standard problem for the nominal system without uncertainties and disturbances to define a central guidance path and an ancillary problem to steer the states towards the central path despite uncertainties and disturbances. Numerical simulation illustrates that the proposed controller can successfully stabilize the states to within expected neighborhood of the origin.

Funding
The authors received no direct funding for this research.

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Citation information
Cite this article as: A novel tube model predictive control for surge instability in compressor system including piping acoustic, Hashem Imani, Mohammad Reza Jahed-Motlagh, Karim Salahshoor, Amin Ramezani & Ali Moarefianpur, Cogent Engineering (2017), 4: 1409373.

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