Joint Source and Relay Design for MIMO Relaying Broadcast Channels

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Abstract—In this letter, we address the optimal source and relay matrices design for the multiple-input multiple-output (MIMO) relaying broadcast channels (BC) with direct links (DLs) based on weighted sum-rate criterion. This problem is nonlinear nonconvex and its optimal solution remains open. To develop an efficient way to solve this problem, we first set up an equivalent problem, and solve it by decoupling it into two tractable subproblems. Finally, we propose a general linear iterative design algorithm based on alternative optimization. This iterative design algorithm is convergent since the solution of each subproblem is optimal and unique. The advantage of the proposed iterative design scheme is demonstrated by numerical experiments.

Index Terms—Source and relay matrices design; sum rate; MIMO ; relaying broadcast channels; direct links.

I. INTRODUCTION

RECENTLY, MIMO relaying broadcast channel (BC) has attracted much research interest. For a MIMO relaying BC, there are two independent channel links between source and receivers; i.e., source-relay-receivers links, and source-receivers direct links (DLs). Many works have investigated the linear strategy for MIMO relaying BC. In [1], an implementable multiuser precoding strategy that combines Tomlinson-Harashima precoding at source and linear signal processing at relay is presented. In [2], a joint optimization of linear beamforming at source and relay to minimize the weighted sum-power consumption under the minimum SINR-constraints is presented. In [3], the singular value decomposition (SVD) and zero forcing (ZF) precoder are respectively used to the source-relay channel and relay-receiver channel to optimize the joint precoding. The authors use an iterative method to show that the optimal precoding matrix always diagonalizes the compound channel of the system. In [4], the authors use the quadratic programming to joint precoding optimization to maximize the system capacity. In [5], the authors propose a scheme based on duality of MIMO MAC and BC to maximize the system capacity. All these works did not consider the DLs and each receiver is assumed to be single antenna.

In practice, the DLs provide valuable spatial diversity to the MIMO relay system and should not be ignored, especially to MIMO relaying BC. Recently, Phuyal et al. in [6] has considered the DLs in design to deal with the power control problem, but only use the ZF scheme with single-antenna receivers. In our previous work [7], we also consider the DLs in design but only consider the RZF scheme to avoid complexity. Both schemes will cause severe noise amplification when a part of the DLs are near to zero. In this letter, we consider a general design strategy to deal with the source precoding matrix (PM) and relay beamforming matrix (BM) by using an alternative optimization algorithm which is inspired by the earlier works in [8, 9]. But [8, 9] only focus on the classical broadcast channels without relay BM design. To incorporate relay BM with source PM will make the analysis more complicated. Finally, simulation results demonstrate that the proposed strategy outperforms the existing methods.

Notations: $E(\cdot)$, $\text{Tr}(\cdot)$, $(\cdot)^{-1}$, $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^\dagger$, and $\det(\cdot)$ denote expectation, trace, inverse, transpose, conjugate, conjugate transpose, and determinant, respectively. i.i.d. stands for independent and identically distributed. I is the identity matrix with appropriate dimensions. $\text{diag}(\cdot)$ is a diagonal matrix. $\log$ is of base 2. $\mathbb{C}^{M \times N}$ represents the set of $M \times N$ matrices over complex field, and $\sim \mathbb{C}N(x,y)$ means satisfying a circularly symmetric complex Gaussian distribution with mean $x$ and covariance $y$. $\mathcal{U} = \{1, 2, \cdots, K\}$.

II. SYSTEM MODEL

Consider a MIMO relaying BC where a base station (source) is equipped with $M$ transmit antennas to serve $K$ multi-antenna users with the help of an $M$ antennas relay in a cell as depicted in Fig 1. It is also assumed that the $k$th user is with $N_k$ ($\forall k \in \mathcal{U}$) receive antennas and satisfied $\sum_{k=1}^{K} N_k \leq M$ to support $N_k$ independent streams for the $k$th user simulta-
neously, We consider a two phase transmission scheme with a non-regenerative and half-duplex relay.

Let $P \triangleq \{P_1, \cdots, P_K\}$ denote the source PM where $P_k \in \mathbb{C}^{M \times N_k}$ is a PM acting on signal vector $s_k \sim \mathcal{C} \mathcal{N}(0, I)$ for user $k$, and the signal vectors for different users be independent from each other. During the first phase, source broadcasts the precoded data streams to relay and users by applying a linear PM $P$. During the second phase, relay forwards the received signal vector to users after a linear BM $F$. Then, the received signal vector at user $k$ can be expressed as

$$
\begin{bmatrix}
y_{1k} \\
y_{2k}
\end{bmatrix} = \begin{bmatrix}
H_{kb} \\
H_k \cdot F_{rb}
\end{bmatrix} P_k s_k + \begin{bmatrix}
n_{1k} \\
H_{kr} F_{nr} + n_{2k}
\end{bmatrix} G_k
$$

where $y_{1k}$ is the received signal during the $i$th phase at user $k$, matrix $H_{ij}$ represents the channel from the transmitter $j$ to receiver $i$, and $n_i \sim \mathcal{C} \mathcal{N}(0, I)$ $(i = 1k, 2k$ and $r)$ are the Gaussian noise vectors at user $k$ during the first and second phase, and at relay, respectively. The power constraints at source and relay can be expressed, respectively, as

$$
\begin{align}
\sum_{k=1}^{K} \text{Tr}(P_k P_k^\dagger) & \leq P_s \quad \text{(Source)}, \\
\text{Tr}(F F^\dagger) P_k P_k^\dagger (H_{rb}^\dagger F^\dagger + F F^\dagger) F_{rb}^\dagger P_k & \leq P_r \quad \text{(Relay)}.
\end{align}
$$

In this paper, we treat the multi-user interference as noise and consider an MMSE receiver at each user to deal with the received signal so that the estimated signal at user $k$ can be written as $\hat{s}_k = A_k y[k]$. According to the MMSE-filter principle \[10\] and the received signal at user $k$ in \[1\], the MMSE receive filter for user $k$ can be written as:

$$
A_k = P_k^\dagger H_k^\dagger \left( H_k P_k P_k^\dagger H_k + G_k G_k^\dagger \right)^{-1} = [A_{1k} A_{2k}],
$$

where $G_k^\dagger = \text{diag}(I, I + H_{kr} F F^\dagger)$, and $A_{nk}$ denotes the receive matrix at user $k$ during the $i$th phase.

Then, the MSE-matrix for the $k$th user can be expressed as:

$$
E_k = \mathbb{E} \left[ \| s_k - \hat{s}_k \|^2 \right] = (I - A_k H_k P_k)(I - A_k H_k P_k)^\dagger + \sum_{i \neq k} A_i H_i P_i P_i^\dagger H_i^\dagger A_k^\dagger + A_k G_k G_k^\dagger A_k^\dagger.
$$

Hence, assuming Gaussian signaling for source, the achievable rate for the $k$th user during the two phases is given as

$$
R_k = \log \left| I + P_k H_k R_{rk}^{-1} H_k^\dagger P_k \right| = \log |E_k^{-1}|,
$$

where $R_{rk} = G_k G_k^\dagger + \sum_{i=1, i \neq k}^{K} H_i P_i P_i^\dagger H_k^\dagger$.

The main objective of this paper is to find the PM $P$ and BM $F$ to maximize the weighted sum-rate. This problem can be expressed as the following:

$$
\begin{align}
[P,F] = \arg \max_{P,F} \sum_{k=1}^{K} w_k R_k, \quad \text{s.t.:} \quad (2),
\end{align}
$$

where the weighted factor $w_k$ usually expresses the priority of the $k$th user in the system. However, it is difficult to directly obtain the optimum closed-form solution because this optimization problem is shown to be non-linear and non-convex.

In the following sections, we first set up an equivalent problem and then propose a general iterative algorithm to design the PM $P$ and BM $F$ based on alternative optimization that updates one precoder at a time while fixing the others.

III. SOURCE AND RELAY MATRICES DESIGN

To find the optimal source and relay matrices for the aforementioned problem, we first set up a new equivalent problem by introducing a lemma, and then we solve this new problem to replace the original problem. Furthermore, we also show two simple relay beamforming schemes with given source PM. Finally, we summarize a general iterative algorithm for source PM and relay BM.

**Lemma 1:** Let $A \triangleq \{A_k\}_{k=1}^{K}$ and $W \triangleq \{W_k\}_{k=1}^{K}$, where $W_k \succeq 0$ is a weight matrix for the $k$th receiver. Then, the optimal solution for the following problem is also the solution for the original problem formulated in \[6\]:

$$
\begin{align}
\min_{P,F,W,A} \sum_{k=1}^{K} w_k \left( \text{Tr}(W_k E_k) - \log \text{det}(W_k) \right), \quad \text{s.t.:} \quad (2),
\end{align}
$$

**Proof:** The proof is similar to Theorem 1 in \[8\] and omitted to save space.

In fact, this Weighted MMSE (WMMSE) problem is also non-linear and non-convex. But, if fixing three of the four variables, the problem is convex with respect to (w.r.t.) the remaining variable, and the solution has a closed-form. In particular, with given $P$, $F$ and $A$, the optimal weight matrix $W_k$ can be expressed in a closed-form as

$$
W_k = E_k^{-1},
$$

and the solution for MMSE-receiver $A_k$ is given in \[3\].

A. Source Precoding Matrix Design

If the relay BM $F$ is given, the MIMO relaying BC becomes a general MIMO BC \[8, 9\]. Thus, for given $F$, $W$ and $A$, the problem in \[7\] w.r.t. $P$ can be reformulated as

$$
\begin{align}
\min_{P} \sum_{k=1}^{K} w_k \left( \text{Tr}(W_k E_k) \right), \quad \text{s.t.:} \quad (2)a
\end{align}
$$

which is a convex quadratic optimization problem and can be solved by KKT conditions. Thus, we can readily obtain the Lagrangian function of \[9\] as

$$
L(a,P_k) = \sum_{k=1}^{K} \left( w_k \left( \text{Tr}(W_k (I - H_k P_k)(I - H_k P_k)^\dagger) \right) + \sum_{j \neq k} w_j \left( \text{Tr}(W_j H_j P_k P_j H_j^\dagger) \right) \right) + \lambda \left( \text{Tr}(PP^\dagger) - P_s \right),
$$

\[1\] Here, the relay power constraint is ignored which does not affect the final result since it will be dealt with the iterative algorithm.
where $\bar{H}_k = A_k H_k$. Then, the first-order necessary condition of $\mathcal{L}$ w.r.t. each $P_k$ yields

$$P_k(\lambda) = w_k \left( \sum_{j=1}^{K} w_j \bar{H}_j^H W_j \bar{H}_j + \lambda I \right)^{-1} \bar{H}_k^H W_k, \quad (10)$$

where $\lambda \geq 0$ is the Lagrangian multiplier which should satisfy the KKT complementarity conditions for power budget constraint, i.e., $\lambda (\text{Tr} (P PP^H) - P_k) = 0$ and $\text{Tr} (P PP^H) \leq P_k$. The $\lambda$ can be found by a 1-D search method such as the bisection method since $\text{Tr}(P \lambda |P(\lambda)|) \text{Tr}(\lambda)$ is monotonically decreasing function of $\lambda$.

### B. Relay Beamforming Matrix Design

To find an optimal relay BM $F$ for the original problem, we also consider dealing with the new problem in (11). Thus, with given $P$, $W$ and $A$, the problem in (11) w.r.t. relay BM $F$ can be recast as following

$$\min_F \sum_{k=1}^{K} w_k \left( \text{Tr} (W_k E_k) \right), \quad \text{s.t. : } (20). \quad (11)$$

This WMMSE problem w.r.t. $F$ can be proven to be convex by the following lemma.

**Lemma 2:** With given $P$, $A$ and $W$, the problem of relay BM design to minimize the total weighted-MSE in the considered relaying BC formulated in (11) is convex.

**Proof:** The proof is similar to Appendix A in [11] and omitted to save space.

**Remark 1:** Here, we have a similar argument as in [11] which is to deal with the two-way relaying channel with only two transceivers and a relay node based on MSE criterion.

Thus, the Lagrangian function of (11) for $F$ is given as

$$\mathcal{L}(F) = \sum_{k=1}^{K} \left( \text{Tr} \left( W_k (I - A_k H_k P_k)^\dagger (I - A_k H_k P_k)^\dagger \right) \right) + w_k \text{Tr} \left( W_k \left( \sum_{i \neq k} A_k H_k P_i P_i^\dagger H_i^\dagger A_i^\dagger + A_k G_k G_k^\dagger A_i^\dagger \right) \right) + \mu \left( \text{Tr} \left( F H_{r b} P P^H H_{r b}^\dagger F + F F^\dagger \right) - P_r \right). \quad (12)$$

Before dealing with the KKT conditions, we first substitute $A_k = \begin{bmatrix} A_{1k} & A_{2k} \end{bmatrix}$ and $H_k = \begin{bmatrix} H_{kk} \\ H_{kr} F_{r b} \end{bmatrix}$ into (12) to get a function w.r.t. $F$. Then, the KKT conditions can be expressed as following

$$\frac{\partial f}{\partial F} = \sum_{k=1}^{K} \left( w_k \Theta_k + \sum_{k=1}^{K} w_k \Theta_k + \mu I \right) F (\Pi + I) = 0,$$

$$\mu \left( \text{Tr} \left( F (\Pi + I) F^\dagger \right) - P_r \right) = 0,$$

$$\text{Tr} \left( F (\Pi + I) F^\dagger \right) \leq P_r,$$

where $\Pi = H_{r b} P P^H H_{r b}^\dagger$, $\Theta_k = H_{kk} A_{2k}^\dagger W_k A_{2k} H_{kr}$, and $\Delta_k = H_{kr} A_{2k}^\dagger W_k A_{2k} H_{kr} F_{r b}^\dagger - H_{kr} A_{2k}^\dagger W_k A_{2k} H_{kr} F_{r b}^\dagger$. Based on (13), we can obtain

$$F = \left( \sum_{k=1}^{K} w_k \Theta_k + \mu I \right)^{-1} \left( \sum_{k=1}^{K} -w_k \Delta_k \right) \left( \Pi + I \right)^{-1}, \quad (16)$$

where $\mu$ is the Lagrangian multiplier which can also be solved by a 1-D search method since $\text{Tr}(F(\mu)(\Pi + I)F(\mu)^\dagger)$ is monotonically decreasing function of $\mu$.

### C. Relay Beamforming Matrix Design by Other Schemes

If the source PM $P$ is given, the relay BM can be also constructed by following schemes.

1) **MRC and MRT Beamforming**: According to the principles of maximum ratio combining (MRC) and maximum ratio transmission (MRT), we can formulate the MRC-MRT relay beamforming scheme for given source PM $P$ as

$$F = \rho_1 H_{ur}^\dagger \left( H_{ur} H_{ur}^\dagger + \frac{M}{P_r} I \right)^{-1} P^\dagger H_{r b}^\dagger, \quad (17)$$

where the factor $\rho_1$ can be derived from (26).

2) **MRC and RZF Beamforming**: According to the principles of MRC and regularized zero-forcing transmission (RZF) [12], we can formulate the MRC-RZF relay beamforming scheme for given source PM $P$ as

$$F = \rho_2 H_{ur}^\dagger \left( H_{ur} H_{ur}^\dagger + \frac{M}{P_r} I \right)^{-1} P^\dagger H_{r b}^\dagger, \quad (18)$$

where the factor $\rho_2$ can also be derived from (26).

### D. A General Iterative Design Algorithm

In summary, a general design algorithm for source PM $P$ and relay BM $F$ can be summarized as following:

**Algorithm 1**: A General Iterative Design Algorithm

1. **Initialize**: $P = \sqrt{\frac{P_r}{M}} I$, $F = \rho I$, $A_k (\forall k \in U)$ uses (3) with $P = \sqrt{\frac{P_r}{M}} I$ and $F = \rho I$, where $\rho$ satisfies the power constraints.

2. **Repeat**:
   3. Update $P_k$ using (10) for fixed $W_k$, $A_k$ and $F$, $\forall k \in U$.
   4. Update $F$ using (16) for fixed $W_k$, $A_k$ and $P$, $\forall k \in U$.
   5. Update $A_k$ using (3) for fixed $P$ and $F$, $\forall k \in U$.
   6. Update $W_k$ using (4) and (3) for fixed $P$ and $F$, $\forall k \in U$.

7. **Until**: The termination condition is satisfied.

This algorithm is always convergent to a stationary point. The convergence analysis of this algorithm is similar to Theorem 2 in [11] or referred to the block coordinate descent algorithm in [13]. Simulation part will show the convergence.

### IV. Numerical Result

This section presents numerical results to evaluate the proposed precoding designs over 2000 random channel realizations. For fair comparison, the other schemes for comparison are also considering the DLs contributions, which are: 1) BZF-BZF&BZF in [6], 2) BRZF-BZF&BRZF in [7], and 3) SVD-RZF in [14]. All these schemes are adjusted to suitable for the multi-antenna users case as in (12) for fair comparison. The channel gains are set to be the combination of large scale fading and small scale fading, i.e., all channel matrices...
receivers DLs based on sum-rate criterion. To deal with the design for the MIMO relaying BC with considering the source-scheme can better deal with the multi-user interferences and amplify noise signal, especially at the case that the DLs gains of SVD-RZF scheme are approximately equal to zero, and all relay’s position. This is because that the DLs contributions at BS and relay are fixed. From Fig. 3 and Fig. 4, we can see nodes positions are fixed. Fig. 4 shows the average sum-rate of the network versus the transmitting power, when all point.

have i.i.d. \( \mathcal{CN}(0, \frac{1}{\ell}) \) entries, where \( \ell \) is the distance between two nodes, and \( \tau = 3 \) is the path loss exponent. In these simulations, we consider that BS and relay are deployed in a line with users, where all the users are deployed at the same point.

We first show the convergence properties of the proposed precoding strategy in Fig. 2. Fig. 3 shows the average sum-rate of the network versus the transmitting power, when all nodes positions are fixed. Fig. 4 shows the average sum-rate of the network versus the relay’s position, when the powers at BS and relay are fixed. From Fig. 3 and Fig. 4 we can see that the average sum-rate of the proposed strategy is higher than those of the other linear schemes at all SNR regime and all relay’s position. This is because that the DLs contributions of SVD-RZF scheme are approximately equal to zero, and the BZF-BZF&BZF and BRZF-BZF&BRZF schemes will amplify noise signal, especially at the case that the DLs gains are close to zero. However, the proposed WMMSE-WMMSE scheme can better deal with the multi-user interferences and noise, and the relation between DLs gains and source-relay-users channel gains.

V. CONCLUSION

In this letter, we consider the joint source and relay matrices design for the MIMO relaying BC with considering the source-receivers DLs based on sum-rate criterion. To deal with the source-relay-receivers channels and DLs gains, and the multi-user interference, we propose a general joint design strategy based on alternative optimization algorithm in which we solve an equivalent problem to replace the original problem by a WMMSE method. We also present other two linear relay beamforming schemes with less complexity. Numerical results show that the proposed strategy outperforms the other linear schemes with or without considering DLs in design.

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