COOLING OF YOUNG STARS GROWING BY DISK ACCRETION

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ABSTRACT

In the initial formation stages young stars must acquire a significant fraction of their mass by accretion from a circumstellar disk that forms in the center of a collapsing protostellar cloud. Throughout this period mass accretion rates reach $10^{-6}$ to $10^{-5} M_\odot \text{yr}^{-1}$, allowing the disk to extend all the way to the stellar surface unimpeded by the protostellar magnetic field. We study the effect of irradiation of the stellar surface produced by the hot inner disk on properties of accreting fully convective low-mass stars and also look at objects such as young brown dwarfs and giant planets. At high $M$ irradiation raises the surface temperature of the equatorial region above the photospheric temperature $T_\odot$ that a star would have in the absence of accretion. In these regions an almost isothermal outer radiative zone forms on top of the fully convective interior, leading to a suppression of the local internal cooling flux derived from stellar contraction (similar suppression occurs in irradiated “hot Jupiters”). The high-latitude (polar) parts of the stellar surface, where disk irradiation is weak, preserve their temperature at the level of $T_\odot$. The total intrinsic luminosity integrated over the whole stellar surface is reduced compared to the nonaccreting case, by up to a factor of several for some objects (young brown dwarfs, stars in quasar disks, and forming giant planets), potentially leading to the retardation of stellar contraction.

Subject headings: planets and satellites: formation — solar system: formation

1. INTRODUCTION

Our understanding of advanced stages of star formation (T Tauri and later phases) has been significantly improved with the advent of infrared, submillimeter, and high-resolution optical observatories such as the Hubble Space Telescope (HST) and Spitzer. However, a great deal of uncertainty still remains regarding the earliest, so-called Class 0 and Class I, stages of the star formation process. In the conventional nomenclature, Class 0 stars are the protostellar cloud cores in the very beginning of their collapse, while Class I are the protostars embedded within an envelope of circumstellar material that is infalling, accumulating in the centrifugally supported disk, and being accreted by the protostars.

At present, our knowledge exhibits a significant gap when it comes to describing the actual buildup of the stellar mass, from $M_* = 0$ $M_\odot$ in the Class 0 phase to $M_* \sim 1$ $M_\odot$ in the end of the Class I phase. From the observational point of view the major reasons for this are (1) the heavy obscuration provided by the increased densities in the central part of the infalling protostellar core and the molecular cloud as a whole and (2) the difficulty in deriving the spectra of the central objects, namely, distinguishing between the intrinsic protostellar and accretion luminosities. At the same time, our ignorance concerns not only the history of mass accumulation by the protostars. The thermodynamical state of the accumulated gas is also an important ingredient of the picture. Stars that form out of material with high entropy, in particular that processed through the accretion shock, tend to have large sizes, while objects formed out of lower entropy gas are more compact. The uncertainty in the initial thermodynamical state of protostellar objects precludes us from getting a good handle on the evolutionary tracks of the fully assembled (in terms of mass) protostars in the first 1–10 Myr after their formation. Beyond about 10 Myr, when the initial conditions become largely forgotten, the evolution tracks calculated under different assumptions about the initial conditions typically converge (Baraffe et al. 2002). However, prior to this stage there are significant discrepancies between the results of different groups, and the uncertainty in the initial conditions for such calculations is the prime suspect for the difference.

Early attempts to address the problem of stellar buildup have assumed spherical symmetry of the accretion flow onto a growing protostar. In this simplified framework many important features of the protostellar growth process have been elucidated. In particular, the works of Winkler & Newman (1980), Stahler et al. (1980a, 1980b), and Stahler (1988) have clarified the role of the radiative accretion shock that forms when the material falling in at almost free-fall velocity suddenly comes to rest at the protostellar surface. It was demonstrated that shock suppresses convection in the surface layers of the protostar and regulates the entropy of its inner convective regions. Additional (although not very significant) heating of the protostar is provided by the radiation coming from the dust destruction front (the backwarming effect). Stahler et al. (1980a) and Stahler (1988) have also clarified an important role of the deuterium burning in driving the convection inside the protostar and setting the protoplanetary mass-radius relation. Current spherically symmetric models of protostar formation take into account effects such as line cooling, chemical reactions in shocked gas, detailed radiative transfer, and so on (see, e.g., Omukai 2007).

Nowadays it is generally accepted that conservation of angular momentum in the collapsing protostellar cloud should result in accumulation of the collapsed gas in a rotationally supported disk in the cloud center. Only a small fraction of the cloud mass has low enough angular momentum to collapse directly into the protostellar core. The majority of stellar mass most likely first accumulates in the circumstellar disk and only then accretes onto the star. According to observations, Class I stars acquire most of their mass on timescale of several times $10^5$ yr, which implies...
that disks around their precursors must exhibit time-averaged mass accretion rates of \( \dot{M} \sim 10^{-6} \) to \( 10^{-5} \, M_\odot \, \text{yr}^{-1} \). This rate exceeds typical \( \dot{M} \) observed in disks of more mature classical T Tauri stars by 2–4 orders of magnitude (Gullbring et al. 1998), which turns out to be very important as we demonstrate below.

Gas accretion onto classical T Tauri stars is thought to be mediated by the stellar magnetic field, as one can easily show that a dipole magnetic field with a kG surface strength (as typically measured in the mature T Tauri systems; see Johns-Krull et al. 1999; Bouvier et al. 2007) would disrupt accretion flow with \( \dot{M} \sim 10^{-8} \, M_\odot \, \text{yr}^{-1} \) at a distance of several stellar radii\(^4\) from the star (Königl 1991). At this distance accretion flow gets channelled by the stellar magnetic field toward the magnetic poles where the inflowing material first accumulates in accretion columns and then joins the star. Disk accretion mediated by the action of the stellar magnetic field has been considered by Königl (1991), Shu et al. (1994a, 1994b), and others.

The radius of disk truncation by the stellar magnetic field shrinks as \( \dot{M} \) increases (as \( M^{2/7} \) in the model of Königl 1991) so that at high \( \dot{M} \sim 10^{-6} \) to \( 10^{-5} \, M_\odot \, \text{yr}^{-1} \) typical of the mass accumulation stage of the protostars even a kG magnetic field\(^5\) would not be able to disrupt accretion flow outside the forming protostar. As a result, a picture of accretion in these early, high \( \dot{M} \) systems should be quite different from that adopted for the classical T Tauri stars: instead of being disrupted by the stellar magnetic field, the accretion disk should extend all the way to the stellar surface. This is an important distinction of the setup under consideration from the conventional picture of T Tauri accretion.

The accretion luminosity released in processing \( \dot{M} \sim 10^{-6} \) to \( 10^{-5} \, M_\odot \, \text{yr}^{-1} \) through the disk exceeds the intrinsic luminosity of the protostar. A significant fraction of this energy is liberated in the immediate vicinity of the star, and this tendency is only exacerbated by the presence of a highly dissipative boundary layer that must form at the interface between the disk and the stellar surface (Lynden-Bell & Pringle 1974; Popham et al. 1993). This immediately raises the issue of possible nontrivial radiative coupling between the protostar and its circumstellar disk.

The effects of direct disk accretion on the structure of young stars have been investigated by Mercer-Smith et al. (1984), Palla & Stahler (1992), Siess & Forestini (1996), Hartmann et al. (1997), and Siess et al. (1997, 1999). Some of these authors studied how the heat advected into the star by the freshly accreted material affects protostellar properties. However, none of these investigations looked at the effect of heat deposited at the stellar surface by radiation originating in the inner parts of the circumstellar disk, where most of the accretion energy is released (see Fig. 1 for a schematic representation). Given that accretion luminosity may easily exceed the intrinsic stellar luminosity (luminosity derived from gravitational contraction, cooling, and, possibly, deuterium burning in the stellar interior), omission of this effect may not be justified.

In this paper we investigate stellar irradiation by the circumstellar disk and address the importance of this effect in determining the intrinsic luminosity of young stars. We calculate the spatial distribution of the disk flux on the stellar surface and determine when irradiation is important in \( \S \) 2. The effect of irradiation on stellar cooling is investigated locally in \( \S \) 3 and globally in \( \S \) 4. Finally, in \( \S \) 5 we discuss the applications of this study to some real objects and its possible limitations.

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\(^4\) This estimate assumes a dipolar magnetic field. In the case of nondipolar field geometry, for which there is growing evidence (Johns-Krull 2007; Lamzin et al. 2007), truncation occurs even closer to the stellar surface than in the dipolar case.

\(^5\) Measurements of \( B \) field strength are not available for Class 0 objects. The magnetic fields of these very young objects (and of very young brown dwarfs and giant planets) may easily be weaker than kG, thus further facilitating direct disk accretion onto a protostar.

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**Fig. 1.**—Schematic representation of stellar illumination by the disk (slant hashed region). The filled region at the point where the disk joins the star marks the boundary layer where intense energy dissipation takes place. Part of the radiation emitted by the disk (arrows) gets intercepted by the star and heats it to temperature \( T_0 \), higher than the temperature \( T_0 \) that a star would have had in the absence of irradiation. The photospheric temperature is preserved at the level of \( T_0 \) only in the polar regions of the star (indicated with dashed lines) where disk illumination is weak. An external radiative zone (horizontally hashed region) forms in the strongly irradiated equatorial parts of the stellar surface.
2. TEMPERATURE DISTRIBUTION DUE TO DISK IRRADIATION

We start by calculating the distribution on the stellar surface of the radiative flux $F_{irr}$ produced by the disk. We consider an axisymmetric geometrically thin disk accreting onto a star with radius $R_*$ and mass $M_*$ and extending all the way to the stellar surface.

Radiative flux $F_{irr}$ intercepted by the star is a function of $\theta$—the angle between the normal to the stellar surface and the normal to the disk (coincident with the polar axis of the star, assuming that the disk lies in the stellar equatorial plane). Polar regions of the star are exposed to the radiation of only the distant, cool parts of the disk, while the equatorial regions have a direct view to the innermost parts of the disk where most of the energy is dissipated. One can easily show that a disk extending all the way to the stellar surface gives rise to irradiation flux $F_{irr}$ (given by Adams & Shu 1986; Popham 1997),

$$F_{irr}(\theta) = 2 \frac{R_* \cos \theta}{\pi} \int_{R_a}^{\infty} F_d(R) R dR \times \int_{0}^{\phi_0} d\phi \left( \frac{R \sin \theta \cos \phi - R_*}{(R^2 + R_*^2 - 2R_*R \sin \theta \cos \phi)^2} \right), \quad (1)$$

where $R$ is the cylindrical radius, $\cos \phi_0 = R_*/(r \sin \theta)$, $R_a = R_*/\cos \theta$, and $F_d(R)$ is the energy radiated by the unit surface area of the disk per unit of time. In Appendix A we demonstrate that this expression can be reduced to a one-dimensional integral, which is easier to analyze than equation (1).

To find the explicit dependence of $F_{irr}$ on $\theta$ one needs to know $F_d(R)$, which is determined by the viscous dissipation in the disk. Studies of steady state thin accretion disks have generally found that

$$F_d(R) = \frac{3}{8\pi} \frac{GM_* \dot{M}}{R^3} f(R), \quad (2)$$

where $\dot{M}$ is the mass accretion rate and the function $f(R)$, embodying the details of the disk emissivity near the stellar surface, behaves as $f \to 1$ when $R \gg R_*$. With $F_d$ given by equation (2) one finds

$$F_{irr}(\theta) = \frac{GM_* \dot{M}}{R_*^3} g(\theta), \quad (3)$$

where the dimensionless function $g(\theta)$ is given by equation (A3).

A standard disk with zero torque at the stellar surface (situation appropriate for accretion onto black holes) has (Shakura & Sunyaev 1973) $f(R) = 1 - (R_*/R)^{1/2}$. The total viscous dissipation in such a disk is $\dot{E}_d = \frac{3}{4} GM_* \dot{M} R_*$, and the gas at the inner edge of the disk rotates at the local Keplerian velocity. This is inappropriate in our case, since the gas speed has to match the velocity of the stellar surface at $R = R_*$, for simplicity assumed to be zero. As a result a boundary layer must form near the stellar surface in which the azimuthal velocity of the gas is lowered by the viscous torque from the local Keplerian value to the stellar rotation speed. Viscous dissipation dramatically increases gas temperature in this layer, creating an additional source of radiative flux very close to the stellar surface. Irradiation by the boundary layer emission boosts the stellar surface temperature in a narrow belt at the equator (with thickness in the $\theta$-direction comparable to the thickness of the boundary layer) above that expected from the irradiation by the more distant parts of the disk, outside the boundary layer. Thus, the existence of the boundary layer significantly modifies disk structure and emissivity near the stellar surface (Popham et al. 1993; Popham & Narayan 1995), complicating the calculation of $f(R)$.

Fortunately, it is shown later in § 4 that cooling of irradiated stars depends only weakly on the behavior of $f(R)$ at $R \sim R_*$ and is thus relatively insensitive to the structure of the boundary layer. For the mass accretion rates considered in this work ($10^{-6}$ to $10^{-5} M_\odot \text{yr}^{-1}$) the geometric thickness of the boundary layer is $\lesssim 0.15 R_*$ (Popham et al. 1993), so the fraction of the stellar surface covered by the boundary layer and affected by the energy dissipation in it is rather small. For this reason we further assume for simplicity that $f(R) \approx 1$. In this case $F_d$ keeps increasing all the way to the stellar surface [unlike the zero inner torque case in which $F_d(R) \to 0$], thus roughly mimicking the contribution of the boundary layer to the disk flux. We plot the behavior of function $g(\theta)$ in Figure 2 for both $f(R) = 1$ and $f(R) = 1 - (R_*/R)^{1/2}$.

Irrespective of the complications related to the existence of the boundary layer, one can derive useful results for $F_{irr}(\theta)$ in two asymptotic regimes. In particular, using equation (A1) one finds that as $\theta \to \pi/2$

$$F_{irr} \to \frac{F_d(R_*)}{2}, \quad g \approx \frac{3}{16 \pi}, \quad (4)$$

a result that is easy to understand, since any point at the stellar equator receives disk radiation with uniform temperature corresponding to local disk flux $F_d(R_*)$ from $\pi$ sr and reemits it into $2\pi$ sr. In the other limit of $\theta \to 0$ one finds from equation (A1) that

$$g \approx I_1 \sin^2 \theta, \quad (5)$$

Such an assumption results in $\dot{E}_d = (3/2)GM_* \dot{M} R_*$, implying a rate of energy release larger than what can be provided by the total change of the potential energy of accreting disk material, but this inconsistency does not strongly affect our results.
where constant $I_i$ is given by equation (A4). One can see from Figure 2 that approximation (5) works quite well (better than 22% accuracy) for $\theta \leq 0.5$. This asymptotic behavior is insensitive to the details of the disk emissivity at $R \sim R_\ast$, since polar regions are irradiated only by parts of the disk at $R \gg R_\ast$, where $f(R) \approx 1$.

Let us denote $T_0$ and $L_0 = 4\pi R_\ast^2 \sigma T_0^4$ the temperature and luminosity that a star with mass $M_\ast$ and radius $R_\ast$ would have in the absence of irradiation ($\sigma$ is the Stefan-Boltzmann constant). To characterize the importance of irradiation we introduce an irradiation parameter $\Lambda$:

$$\Lambda \equiv \frac{G M \dot{M}}{\sigma T_0^4 R_\ast^4} \approx 1.6 M_\ast \dot{M}_\ast R_{113}^{-3} T_3^{-4},$$

where $T_\ast \equiv T_0/10^n K$, $R_\ast \equiv R_/10^n cm$, $M_\ast \equiv M_/M_\odot$, and $\dot{M}_\ast \equiv \dot{M}/(10^n M_/ yr^{-1})$. By construction, $\Lambda$ is roughly the ratio of the accretion luminosity to the stellar luminosity $L_\ast$ in the absence of irradiation.

When irradiation is allowed for the photospheric temperature of the star $T_{ph}$ is a function of $\theta$, since energy balance in steady state requires

$$\sigma T_{ph}^4(\theta) = \sigma T_{irr}^4(\theta) + F_{in}(\theta),$$

at each point on the stellar surface, where $F_{in}$ is the intrinsic energy flux coming from the stellar interior (derived from cooling of the stellar interior, gravitational contraction, and D burning) and

$$T_{irr}(\theta) \equiv \left[ \frac{F_{in}(\theta)}{\sigma} \right]^{1/4} = T_0 (\Lambda g)^{1/4} \approx 3.5 \times 10^4 K (M_\ast \dot{M}_\ast R_{113})^{1/4} g^{1/4}.$$  \hspace{1cm} (8)

Equation (7) assumes that all radiation intercepted by the star gets fully absorbed by its surface and reflection is negligible. Our discussion can be easily extended for the case of nonzero stellar albedo.

In the absence of irradiation $F_{in} = \sigma T_{ph}^4$. With irradiation the local flux emitted by the photosphere $\sigma T_{ph}^4$ exceeds $\sigma T_{irr}^4$, but the intrinsic stellar flux $F_{in}$ derived from the gravitational contraction and cooling of the stellar interior actually becomes smaller than $\sigma T_{irr}^4$ as we demonstrate in § 3.1.

We define the regime of weak irradiation as that corresponding to low $\dot{M}$ such that

$$T_{irr}(\theta) \lesssim T_0$$

for any $\theta$ (i.e., $\Lambda g \ll 1$). As the irradiation is most intense near the stellar equator, weak irradiation at any point on the stellar surface requires $T_{irr}(\pi/2) \lesssim T_0$ (or $\Lambda \lesssim 1$), or accretion rates lower than

$$\dot{M}_\ast \approx \frac{16 \pi}{3} \frac{R_\ast^3 \sigma T_0^4}{G M_\ast} \approx 10^{-8} T_3^4 R_{113}^{-3} M_\odot^{-1} \dot{M}_\odot^{-1} \text{ yr}^{-1}.$$  \hspace{1cm} (10)

Energy dissipation in the equatorial boundary layer (which we do not account for here) can heat the equatorial region above $T_0$ even at $\dot{M} \leq \dot{M}_\ast$, but this heating does not spread very far from the equator and does little to affect the large-scale stellar structure.

A regime of strong irradiation is defined as that corresponding to $\dot{M} \geq \dot{M}_\ast$ ($\Lambda \gg 1$) so that at least some parts of the stellar surface have

$$T_{irr}(\theta) \gtrsim T_0$$

(or $\Lambda g \gtrsim 1$). When $\dot{M} \geq \dot{M}_\ast$, this condition is satisfied only near the stellar equator where an irradiated belt with $T_{irr}(\theta) \approx T_0$ forms. As $\dot{M}$ increases this belt expands in the $\theta$-direction, although rather slowly, since $F_{in}$ is a rapidly decreasing function of $\theta$ (see Fig. 2). As is shown in § 3.1, in irradiated regions the intrinsic energy flux $F_{in}$ coming from the stellar interior is suppressed compared to $\sigma T_0^4$ so that the effective temperature of the irradiated part of the star can be well approximated by

$$T_{ph} \approx T_{irr}(\theta).$$  \hspace{1cm} (12)

The transition between the low-latitude irradiated region and the high-latitude part of the stellar surface where $T_{ph} \approx T_0$ takes place at $\theta_{irr}$ given by (see eq. [5])

$$\sin \theta_{irr} \approx \left( \frac{R_\ast^4 \sigma T_0^4}{I_3 \frac{G M_\ast}{\dot{M}_\ast}} \right)^{1/5} = (I_3 \Lambda)^{-1/5} \approx 0.5 T_3^{-4/5} R_{113}^{-3/5} M_\odot^{-1/5} \dot{M}_\odot^{-1/5}.$$  \hspace{1cm} (13)

According to this formula, at $\dot{M} = 10^{-5} M_/ yr^{-1}$ polar regions having temperature $T_0 \approx 3000 K$ occupy about 15% of the stellar surface. The rest of the surface has $T_{ph}$ significantly modified by intense radiation coming from the disk. At this $\dot{M}$ the equatorial temperature reaches $T_{ph}(\pi/2) \approx 1.8 \times 10^4 K$, much higher than $T_0 \approx 3000 K$ corresponding to the typical Hayashi track of a young star.

3. COOLING OF IRRADIATED STELLAR SURFACE

Young stars, brown dwarfs, and giant planets are fully convective objects. It is well known (Kippenhahn & Weigert 1990) that the luminosity and effective temperature of such objects are determined mainly by the properties (opacity behavior, ratio of the specific heats of the gas) of their outermost, near-photospheric layers and are rather insensitive to the processes occurring in the convective interior. Given that irradiation changes the boundary conditions on the surface of an accreting fully convective object we may also expect that it should affect the luminosity of such objects (Arras & Bildsten 2006).

Intense heating of some parts of the stellar surface suppresses convection in the subsurface layers and gives rise to a convectively stable radiative zone sandwiched between the photosphere and convective interior (see Fig. 1 for illustration). The appearance of this zone is analogous to the formation of a roughly isothermal radiative layer in the outer parts of the close-in extrasolar giant planets caused by the intense radiation of their parent stars (Guillot et al. 1996; Burrows et al. 2000). It is the structure of this zone that we want to investigate in order to assess the effect of irradiation on stellar cooling. Here we assume that the radiative zone is

1. optically thick, as measured from its bottom (convective-radiative boundary) to the photosphere, and
2. geometrically thin compared to $R_\ast$.

The validity of these assumptions is verified in § 3.1.
In the absence of internal energy sources radiation transport in the optically thick radiative layer is governed by
\[ \nabla \cdot \mathbf{F} = 0, \quad \mathbf{F} = -\frac{16 \sigma T^3}{3 \kappa \rho} \nabla T, \quad (14) \]
where \( F \) is the radiative flux density, \( \kappa \) is the opacity, and \( \rho \) is the gas density. The equation of hydrostatic equilibrium reads \( \nabla P = -\rho g \), where \( P \) is the gas pressure and \( g \) is the local gravitational acceleration. These two equations describe the structure of the radiative zone subject to the boundary condition
\[ T \bigg|_{\tau=2/3} = T_{\text{ps}}(\theta), \quad (15) \]
where \( \tau \) is the optical depth. The radiative boundary condition (15) coupled with equation (7) is appropriate here because most of the stellar surface is not obscured by the accreting gas and is free to radiate energy into space.

### 3.1. 1D Approximation for the Structure of the Radiative Layer

In general, equation (14), together with the equation of hydrostatic equilibrium, must be solved in two dimensions, \( r \) and \( \theta \). However, under the circumstances clarified in § 3.4 the \( r \)-component of the radiative flux \( F_r \) is much larger than its \( \theta \)-component \( F_\theta \), so that the latitudinal transport of energy can be neglected. This leaves \( r \) as the only independent variable in equation (14), effectively making it one-dimensional. A dependence on \( \theta \) then appears only through the external boundary condition, namely, \( T_{\text{ps}}(\theta) \). This is the limit that we focus on in this work.

Because of the thinness of the radiative zone, \( r \) varies only weakly through the radiative zone, so one can neglect the divergence of the radial component of \( F_r \), thereby reducing equation (14) to simply \( \partial F_r / \partial r = 0 \). With these simplifications equation (14) can be integrated once to find
\[ F_{\text{in}} = -\frac{16 \sigma T^3}{3 \kappa \rho} \frac{\partial T}{\partial r}, \quad (16) \]
where the integration constant on the left-hand side is independent of \( r \) and as such has to coincide with the intrinsic flux \( F_{\text{in}} \) coming from the convective interior of the star. The determination of \( F_{\text{in}} \) is the goal of our calculation.

Radial pressure gradients in the radiative zone are much larger than the latitudinal gradients, so the equation of hydrostatic equilibrium can be written as
\[ \frac{\partial P}{\partial r} = -\rho g, \quad (17) \]
where \( g = |g| \approx GM_s/R^2 \) is the gravitational acceleration, which is roughly constant within the thin radiative layer (stellar rotation is neglected throughout this work).

Subsequent consideration is very similar to the calculation of the atmospheric structure of the protoplanetary core immersed in a protostellar nebula, which can be found in Rafikov (2006). We assume that \( \kappa \) depends on the gas pressure and temperature as
\[ \kappa = \tilde{\kappa} P^\alpha T^\beta, \quad (18) \]
where \( \alpha > 0 \) and \( \beta \) are constants. Equation (18) is a reasonable approximation to the opacity behavior in some density and temperature intervals typical for young stars. In particular, at 2500 K < \( T \leq 5000 \) K opacity is mainly due to H$^+$ absorption with electrons supplied by elements heavier than H with low ionization potentials. Bell & Lin (1994) have demonstrated that in this regime \( \kappa \) can be well fit by
\[ \kappa \approx 6 \times 10^{-14} p^{2/3} T^{7/3}. \quad (19) \]
At \( T \gtrsim 5000 \) K electrons from partial hydrogen ionization enhance H$^+$ opacity and hydrogenic absorption dominates.

Here we also assume that the equation of state (EOS) of gas in the whole star, including the external radiative zone, can be characterized by a single ratio of specific heats \( \gamma \). In other words, we assume that under adiabatic conditions gas behaves as \( P = K \rho \gamma \), where \( K \) is a constant set by the entropy of the gas and \( \gamma \) is fixed throughout the star. We adopt \( \gamma = 5/3 \), which should work fine in fully ionized, convective interiors of young low-mass stars, although this approximation is not very accurate at the transition between the outer radiative zone and the convective interior, since gas is only partly atomic there. Continuing dissociation and ionization cause variations of \( \gamma \) in this region that may be quite important (see § 5.2). Nevertheless, to get a qualitative picture of the effect of irradiation on stellar cooling and for making rough numerical estimates this constant-\( \gamma \) approximation should be sufficient.

With \( \kappa \) in the form of equation (18), equation (16) can be integrated using equations (17) and (18) and the ideal gas law:
\[ \left( \frac{P}{P_{\text{ps}}} \right)^{1+\alpha} - 1 = \frac{\nabla_0}{\nabla_{\text{ps}}} \left[ \left( \frac{T}{T_{\text{ps}}} \right)^{4-\beta} - 1 \right], \quad (21) \]
where
\[ \nabla_0 = 1 + \frac{\alpha}{4 - \beta}, \quad (22) \]
\[ \nabla_{\text{ps}} = \frac{3}{16} \frac{F_{\text{ps}}} {P_{\text{ps}} P_{\text{ps}}} \Gamma_{\text{ps}}, \quad (23) \]
and \( P_{\text{ps}} \) and \( \kappa_{\text{ps}} = \tilde{\kappa} P_{\text{ps}}^\alpha T_{\text{ps}}^\beta \) are the values of pressure and opacity at the photosphere.

Solution (21) allows us to calculate the temperature gradient
\[ \nabla(T) = \frac{\partial \ln T}{\partial \ln P} = \nabla_0 \left[ 1 - \frac{T_{\text{ps}}^{4-\beta}}{T} \left( 1 - \frac{\nabla_{\text{ps}}}{\nabla_0} \right) \right], \quad (24) \]
which determines whether the gas is stable against convection. Note that at the photosphere \( \nabla(T_{\text{ps}}) = \nabla_{\text{ps}} \). Everywhere inside the radiative zone
\[ \nabla < \nabla_{\text{ad}} \equiv \frac{\gamma - 1}{\gamma}, \quad (25) \]
where \( \nabla_{\text{ad}} \) is the adiabatic temperature gradient. In convective regions \( \nabla > \nabla_{\text{ad}} \). For our adopted \( \gamma = 5/3 \) one finds \( \nabla_{\text{ad}} = \frac{5}{3} \).

We also assume that at some depth an object under consideration does become convective and determine what is necessary
for this transition to occur. If $\beta < 4$, then equations (24) and (25) demonstrate that convection sets in only provided that
\[
\nabla_0 > \nabla_{ad}.
\]

The situation described by equation (26) is realized, e.g., for opacity given by equation (19), when $\nabla_0 = 1$ exceeds $\nabla_{ad} = \frac{\tilde{\alpha}}{\tilde{\kappa}}$, implying that radiative energy transport does indeed change to convective at some depth, as we have assumed.

On the other hand, when opacity is characterized by $\beta > 4$ equation (24) guarantees that a transition to convection occurs at some depth, irrespective of the exact value of either $\beta$ or $\nabla_0$. This situation is appropriate for $\kappa$ given by equation (20), since in that case $\beta \approx 10$.

Despite this difference, in both cases the temperature $T_{cb}$ and pressure $P_{cb}$ at the convective-radiative boundary are given by
\[
T_{cb} = T_{ph} \left( \frac{\nabla_0 - \nabla_{ph}}{\nabla_0 - \nabla_{ad}} \right)^{1/(4-\beta)} ,
\]
\[
P_{cb} = P_{ph} \left( \frac{\nabla_{ad} - \nabla_0 - \nabla_{ph}}{\nabla_{ph} - \nabla_0 - \nabla_{ad}} \right)^{1/(1+\alpha)} ,
\]
which can be derived by setting $\nabla(T_{cb}) = \nabla_{ad}$ and using equation (21).

The value of $P_{ph}$ can be fixed in the following way. Above the photosphere gas is roughly isothermal with temperature $T_{ph}$—an approximation that is good enough for our purposes. Then at height $z$ above the photosphere $\rho(z) = \rho_{ph} \exp(-z/T_{ph})$, where $\rho_{ph}$ is the photospheric gas density. Using this result and equation (18) we find the photospheric optical depth
\[
\frac{2}{3} = \int_0^{\infty} \kappa \rho \, dz = \kappa_{ph} \rho_{ph} \frac{T_{ph}}{(\alpha + 1) g} ,
\]
from which it follows that
\[
P_{ph} = \left[ \frac{2(\alpha + 1)}{3} \frac{g}{\kappa T_{ph}^\beta} \right]^{1/(1+\alpha)} .
\]

As a by-product of relation (29) one can rewrite equation (23) as
\[
\nabla_{ph} = \frac{\alpha + 1}{8} \frac{F_{in}}{\sigma T_{ph}^4} ,
\]
\[
\nabla_{ad} ,
\]
where a result that is verified in § 3.3 (see eq. [45]). It then follows from equations (31) and (32) that $F_{in} \ll \sigma T_{ph}^4$.

Using equations (17), (21), (28), and (30) one can find that the optical depth at the convective-radiative boundary
\[
\tau_{cb} \sim \nabla_{ph}^{-1} ,
\]
while the radial extent of the outer radiative zone is
\[
\Delta R_{r} \sim H_{ph} \ln \nabla_{ph}^{-1} ,
\]
where $H_{ph} = k_B T_{ph}/(\mu g)$ is the photospheric scale height. Given that irradiation cannot heat the star to a temperature comparable to its central temperature (otherwise outer layers would be unbound) $H_{ph}$ should be much smaller than $R_r$ even under rather extreme irradiation.

According to equation (33) the smallness of $\nabla_{ph}$ results in $\tau_{cb} \gg 1$, thus justifying our assumption 1 about the radiative zone properties. At the same time, because of the rather weak (logarithmic) dependence of $\Delta R_r$ on $\nabla_{ph}$, the thickness of the outer radiative zone should not be much different from $H_{ph} \ll R_r$. As a result, $\Delta R_r \ll R_r$, verifying our assumption 2. Thus, condition (32) can be viewed as a prerequisite for the formation of a geometrically thin, optically thick radiative zone with roughly isothermal temperature profile under the action of intense external irradiation. Outer radiative layers with similar near-isothermal structure are expected to exist in the envelopes of irradiated hot Jupiters (Guillot et al. 1996; Baraffe et al. 2003; Chabrier et al. 2004) and in the outer parts of the low-luminosity atmospheres of protoplanetary cores immersed in the protoplanetary nebulae (Rafikov 2006).

3.2. Calculation of $F_{in}(\theta)$

We are now in position to evaluate $F_{in}$ and see how irradiation affects the cooling of convective objects. To do this we note that the inner boundary of the radiative zone is also the outer boundary of the convective interior. We assume that convective transport is so efficient that entropy is constant throughout the inner convective zone, so that the EOS can be well represented by $P = K \rho^\gamma$, where $K$ is the adiabatic constant. As a result, $P_{cb}$ and $T_{cb}$ must be related via $(K T_{cb}/\mu g)^\gamma = K P_{cb}^{-\gamma-1}$, which, coupled with equations (23), (27), (28), and condition (32), yields the following expression for $F_{in}$:
\[
F_{in}(\theta) = \frac{16 \nabla_{ad}}{3} \left( \frac{\nabla_0 - \nabla_{ad}}{\nabla_0} \right)^{\nabla_0/\nabla_{ad} - 1} \frac{\sigma g}{\kappa} \times \left( \frac{\mu K^{1/\gamma}(\gamma+1)/\nabla_{ad}}{k_B} \right) \left[ T_{ph}(\theta) \right]^{4-\xi} \]
\[
\sim \sigma T_{ph}^4 \left( \frac{P_{ph}}{P_e} \frac{T_c}{T_{ph}} \right)^{\nabla_{ad}^\gamma} \]
$P_e$ and $T_e = \mu P_e^{1/3} K^{1/7}/k_b$ are the pressure and temperature at the stellar center, and in deriving estimate (36) we have used equation (30). The intrinsic stellar flux $F_{in}$ exhibits an explicit latitudinal dependence because it is a function of $T_{ph}(\theta)$.

As discussed before, when $\beta < 4$ and $\nabla_0 > 0$ a transition to convection at some depth requires $\nabla_0 > \nabla_{ad}$. As a result,

$$4 - \xi = (4 - \beta) \left(1 - \frac{\nabla_0}{\nabla_{ad}}\right) < 0. \quad (38)$$

On the other hand, when $\beta > 4$ one also finds $4 - \xi < 0$ because $\nabla_0 < 0$ in this case. Thus, in both situations $F_{in}$ decreases as $T_{ph}$ increases. In other words, irrespective of the opacity behavior external irradiation of the stellar surface suppresses stellar cooling, a result known from the studies of irradiated giant planets (Guillot et al. 1996; Burrows et al. 2000).

Since the external radiative zone is rather thin compared to $R_*$, it must contain a negligible amount of mass compared with $M_*$. Then the structure of the fully convective inner region of the star should be well described by the classical theory of polytropic spheres (Landau & Lifshitz 1984; Kippenhahn & Weigert 1990). In particular, the adiabatic constant $K$ can be related to the stellar mass and radius as

$$K = \zeta(\gamma)GM_e^{2/\gamma}R_e^{2(1-\gamma)}, \quad (39)$$

where $\zeta(\gamma) \sim 1$ is a parameter set by the equation of state of the gas. In the particular case of convective young stars with fully ionized interior characterized by $\gamma = 5/3$ one has $\zeta(5/3) = 0.1286$ and

$$K = 1.081 \times 10^{14} M_1^{1/3} R_{11}^{11/3}. \quad (40)$$

Equations (35) and (39) unambiguously determine cooling of the star as a function of stellar parameters $R_*$ and $M_*$, temperature distribution at the photosphere $T_{ph}(\theta)$, and the opacity behavior in the outer radiative zone.

3.3. Comparison with the Case of an Isolated Star

We now compare stellar cooling in the irradiated case with that occurring in isolated stars, in the absence of external illumination. In the latter case $T_{ph} = T_0$, $F_{ph} = F_0 = \sigma T_0^4$, and equation (34) gives $\nabla_{ph} = \nabla_{eff} = (\alpha + 1)/8 \sim 1$. Substituting this result into equations (27) and (28) and using the adiabatic relation at the convective-radiative boundary and equation (33) we find

$$T_0 = \left[\frac{16\nabla_{ad}}{3} \left(\frac{\nabla_0 - \nabla_{eff}}{\nabla_0 - \nabla_{ad}}\right)^{1-\nabla_0/\nabla_{ad}} \times \frac{g}{\kappa} \left(\frac{\mu K^{1/\gamma}}{k_b}\right)^{(1-\alpha)/\nabla_{eff}}\right]^{1/\xi}. \quad (41)$$

This expression sets the effective temperature of the star and its cooling rate $F_0$ as functions of $M_*$, $R_*$, and the opacity behavior. In particular, for $\kappa$ typical at temperatures below 5000 K one finds using equation (40) that

$$T_0 \approx 1200 \ K \ M_1^{1/39} R_{11}^{11/3}. \quad (42)$$

This is considerably smaller than $T_{eff} \approx 3000$–4000 K, which one obtains with the detailed numerical stellar structure calculation of an isolated fully convective star on the Hayashi track (Siess et al. 2000). We ascribe this difference to our adoption of fixed $\gamma$ throughout the whole star and the neglect of superadiabaticity in the outer parts of the convective region (see $\S$ 5.2). At the same time equation (42) still captures the main property of the Hayashi track—extremely weak sensitivity of $T_0$ to $R_*$ and, consequently, stellar luminosity.

If we now go back to equation (35) one can easily see that it can be rewritten as

$$F_{in} \approx F_0 \left[\frac{T_{ph}(\theta)}{T_0}\right]^{4-\xi}, \quad (43)$$

or, with equation (7), as

$$\left(\frac{F_{in}}{F_0}\right)^{4/(4-\xi)} = \left(\frac{T_{ph}}{T_0}\right)^4. \quad (44)$$

This result, together with equation (38), once again vividly illustrates the inhibition of stellar cooling by external irradiation and specifies the magnitude of this effect.

Using equations (34) and (43) we can also write

$$\nabla_{ph} \approx \left(\frac{\alpha + 1}{8}\right) \left(\frac{T_{ph}}{T_0}\right)^{-\xi}, \quad (45)$$

which shows that $\nabla_{ph} \ll 1$ when the stellar surface is strongly irradiated ($T_{ph} \approx T_0$), thus confirming our conjecture (31). Note the strong dependence of $\nabla_{ph}$ on $T_{ph}/T_0$, with our power-law Anzatz for opacity $\xi \approx 13/2$ and $\approx 13$ below and above 5000 K, respectively (see eqs. [19] and [20]).

3.4. Conditions for Validity of 1D Approximation

In Appendix B we determine the circumstances under which the results of $\S$ 3.1 hold true. There we show that the 1D approximation works well whenever

$$\left(\frac{H_{ph}}{T_0}\right)^2 \lesssim \nabla_{ph}, \quad (46)$$

where $L_\theta$ is a characteristic scale in the $\theta$-direction over which the external boundary condition [in our case $T_{ph}(\theta)$] experiences variation. Equation (1) and Figure 2 demonstrate that the case of irradiation by accretion disk $L_\theta \sim R_*$. Then equations (34) and (43) allow us to rewrite condition (46) as (assuming that $T_0$ and $T_{ph}$ are in the same opacity regime)

$$T_{ph} \lesssim T_0 \left(\frac{R_*}{H_0}\right)^{2/(2+\xi)}, \quad (47)$$

where $H_0 = k_b T_0/(\mu \rho)$ is the photospheric scale height in the absence of irradiation. Given that

$$\frac{R_*}{H_0} \approx 5 \times 10^3 M_1 R_{11} T_{30}^{-1} R_{35}^{-1} \quad (48)$$

we conclude that 1D approximation should be rather accurate even if $T_{ph}$ exceeds $T_0$ by a factor of several (e.g., $T_{ph} \approx 7 T_0$ for $T \lesssim 5000$ K).

Whenever condition (46) is violated the redistribution of energy in the $\theta$-direction within the radiative layer becomes important. In this case one needs to solve the full two-dimensional equation (14) without assuming that radiative flux in the $\theta$-direction is small. A similar situation arises at the stellar equator where a lot of energy is released in a boundary layer that is not very extended in the $\theta$-direction (Popham et al. 1993). As a result, at the equator
\( L_0 \ll R_* \), and condition (46) can be violated there even though at all other latitudes a 1D approximation works fine.

### 4. INTEGRATED STELLAR COOLING

We are now in a position to calculate the integrated intrinsic luminosity \( L \) (due to the stellar contraction and interior cooling) of a convective star that is irradiated by a circumstellar disk:

\[
L = 4\pi R_*^2 \int_0^{\pi/2} F_{\text{irr}}(\theta) \sin \theta d\theta,
\]

(49)

where \( F_{\text{irr}}(\theta) \) is given by expression (35) in the irradiated part of the stellar surface, for \( \theta \geq \theta_{\text{irr}} \), while \( F_{\text{irr}}(\theta) \approx \sigma T_0^4 \) in the weakly irradiated polar regions, for \( \theta \approx \theta_{\text{irr}} \).

Convective objects can exhibit different modes of cooling, which is best illustrated by considering the strong irradiation limit \( \theta_{\text{irr}} \ll 1 \) (\( \Lambda \gg 1 \)). In this limit the contribution of polar caps to the total luminosity is

\[
L_{\text{pc}} \approx 4\pi F_0 R_*^2 (1 - \cos \theta_{\text{irr}}) \approx 4\pi F_0 R_*^2 \theta_{\text{irr}}^2,
\]

(50)

while the irradiated equatorial regions contribute

\[
L_{\text{ct}} \approx 4\pi CF_0 R_*^2 \left( I \sin^2 \theta_{\text{irr}} \right)^{(4-\xi)/4} \int_{\theta_{\text{ct}}}^{\pi/2} [g(\theta)]^{(4-\xi)/4} \sin \theta d\theta,
\]

(51)

(see eq. [43]).

Using equations (5) and (13) it is easy to see that for \( \theta_{\text{irr}} \ll 1 \) the integral in equation (51) is dominated by \( \theta \approx \theta_{\text{irr}} \) if

\[
\xi > \frac{28}{5}.
\]

(52)

In this case, according to equation (5), one may approximate \( g(\theta) \approx I \sin^2 \theta \) and find that

\[
L_{\text{ct}} \approx \frac{16\pi}{5(\xi - 28/5)} F_0 R_*^2 \theta_{\text{irr}}^2 \sim L_{\text{pc}}.
\]

(53)

This result leads us to the interesting conclusion that as long as condition (52) is fulfilled, an object cools predominantly through its polar caps and its integrated luminosity \( L \) is almost independent of the details of the opacity behavior in its outer layers. The latter point is easy to understand, since in this case \( L \sim F_0 S \), where \( S \) is the surface area of the polar caps. But according to equation (13) the value of \( S \) is determined only by irradiation and \( T_0 \). As a result, \( L \) depends on \( \kappa \) only weakly, through the \( L_{\text{ct}} \) contribution.

We call the regime of stellar cooling realized under condition (52) high-latitude cooling. This regime naturally occurs in irradiated young stars, since \( \xi > 28/5 \) for \( \kappa \) given by either equation (19) or (20). Equations (50) and (53) demonstrate that in this regime \( L \) is suppressed with respect to \( L_0 \) roughly by \( \sim \theta_{\text{irr}}^2 \), which may be as low as \( \sim 0.2 - 0.4 \) according to expression (13). Thus, disk irradiation can substantially slow down cooling of young stars.

In the opposite case of \( \xi < 28/5 \) cooling is in the low-latitude regime, so that the star loses most of its internal energy through the equatorial regions even though they are strongly irradiated. In this case one should use the full expression (51) to evaluate \( L \approx L_{\text{ct}} \). Stellar luminosity suppression for \( \theta_{\text{irr}} \ll 1 \) is given by

\[
L/L_0 \approx CI_2 \left( \sin \theta_{\text{irr}} \right)^{(4-\xi)/4},
\]

(54)

\[
I_2(\xi) = I_2^{(4-\xi)/4} \int_0^{\pi/2} [g(\theta)]^{(4-\xi)/4} \sin \theta d\theta.
\]

(55)

Function \( I_2(\xi) \sim 1 \) is shown in Figure 3. Knowing that \( 4 < \xi < 28/5 \) in the low-latitude case one can easily see that the degree of luminosity suppression is smaller than in the high-latitude cooling regime.

In Figure 4 we plot \( L/L_0 \)—the ratio of stellar luminosities in the irradiated and isolated cases—as a function of irradiation parameter \( \Lambda \), for different values of \( \xi \). This calculation does not
explicitly make the assumption $\theta_{\text{ir}} \ll 1$ (or $\Lambda \gg 1$), although it
covers this regime as well. Here $L/L_0$ is computed by the straight-
forward integration of $F_{\text{in}}$ over the stellar surface (including the
polar caps where $F_{\text{in}} = F_0$), with the distribution of $T_{\text{ir}}(\theta)$ found
from equation (3) and $g(\theta)$ displayed in Figure 2. We also indicate
the asymptotic behavior of $L/L_0$ as given by equations (50) and (53) for $\xi > 28/5$ and equation (54) for $\xi < 28/5$.

One can easily see that, as expected, $L/L_0 \propto \theta_{\text{ir}}^2 \propto \Lambda^{-2/5}$ as
$\Lambda \gg 1$ for $\xi = 13$ and 6.5 independent of the actual value of $\xi$
(only the normalizations of the curves are different because of the
different contributions produced by the near-polar cap regions),
since for both $\xi > 28/5$. Significant suppression of the stellar flux
(by $\sim 2$) is already found in this case at $\Lambda \sim 10^{-2} - 10^3$. In the case
$\xi < 28/5$ asymptotic behavior for $\Lambda \gg 1$ agrees well with equa-
tion (54), $L/L_0 \propto \Lambda^{4-\xi/4}$, and the degree of stellar flux suppres-
sion is weaker than in the high-latitude regime: $L/L_0 \approx 0.5$ only
at $\Lambda \approx 5 \times 10^3$ for $\xi = 5.3$ and at $\Lambda \approx 2 \times 10^3$ for $\xi = 4.5$.

Note that the results presented in Figure 3 are calculated neg-
lecting any additional heating that can be produced near the
equator by the boundary layer dissipation. We address this point
in more detail in § 5.2.

5. DISCUSSION

Luminosity suppression by disk irradiation is one of the most
important results of this work. An analogous phenomenon has
been previously found in studies of extrasolar giant planets in
short-period orbits, where stellar irradiation is quite severe (Guillot
et al. 1996; Burrows et al. 2000). In that case irradiation affects
only one side of the planet, which always faces the star. Some
heat from the day side gets redistributed to the night side by
atmospheric circulation (Menou et al. 2003; Dobbs-Dixon & Lin
2008), which complicates the calculation of the photospheric
boundary conditions across the whole planetary surface. In our
case irradiation is azimuthally symmetric, which makes our cal-
culation more robust. Analogous to the case of extrasolar giant
planets we expect that luminosity suppression by irradiation would
tend to retain the heat inside the star and increase stellar radius
above the value found in the absence of irradiation (Baraffe et al.
2003). Whether this radius increase is significant is investigated in
Rafikov (2008).

Luminosity suppression may have some effect on the strength of
the magnetic field that can be generated by the dynamo action
in the convective interior of the star. Since in the irradiated case
convective eddies transport smaller energy flux to the stellar sur-
fase than in the case of an isolated star, the speed of convective
motions is expected to be smaller. This results in a less vigorous
dynamo action and likely weaker magnetic field generated inside
the star.

The formation of an optically thick radiative zone underneath
the irradiated parts of the stellar surface is another result of this
work that has important implications. Spherically symmetric mod-
els of protostellar formation developed by Stahler et al. (1980a,
1980b) have predicted that convection is going to be suppressed
in the outer layers of the protostar by the energy release taking
place in the accretion shock. In our case accretional energy
release occurs in the disk, and the surface of the star is affected by
this energy source only because of irradiation. However, despite
the difference in geometry, the external radiative zone predicted
in this work bears some similarity to the radiative postshock
region described in Stahler et al. (1980a, 1980b).

As the star grows the hot gas in the vicinity of the boundary
layer (where disk meets the star) gets advected into the convec-
tive interior, thereby raising stellar entropy. This provides another
way of slowing down stellar contraction, in addition to the lu-
minosity suppression discussed above. Hartmann et al. (1997)
argued that the effect of the heat advection on stellar structure
is not significant as long as the temperature of the advected gas
is much smaller than the central temperature of the star. In the
irradiated case it is the properties of the external radiative zone
that determine the temperature of the gas at the convective-
radiative boundary and thus the amount of thermal energy ad-
verted into the stellar interior. Indeed, the hot gas sinking through
the radiative zone will lose a significant fraction of its thermal
energy by radiative diffusion in the latitudinal direction, so the tem-
perature at the convective-radiative boundary is lower than in the
center of the boundary layer. It is important to build a detailed
2D model of the radiative transport in the vicinity of the bound-
ary layer to quantify this effect and to verify the significance of
heat advection (see also § 3.4).

The presence of the radiative zone may also affect atmospheric
opacity in accreting brown dwarfs and giant planets. Accreted gas
brings a significant amount of dust into the object’s atmosphere
(dust can also form out of the gas phase under the low-temperature
conditions, which changes $\kappa$ and the radiative properties of
the star (Chabrier et al. 2000). However, if the object is fully convec-
tive all the way to its photosphere, vertical fluid motions quickly
advect dust grains into the hot interior where grains get easily
destroyed. This is not the case in irradiated regions of accretion
objects, since dust grains can hover in the radiative zone for a
long time, as long as their gravitational settling is not too fast.
Of course, for the grains to exist in the outer radiative layer
$T_{\text{irr}}$ must be lower than the sublimation temperature of dust
material, which is possible only in accreting brown dwarfs and giant
planets.

We expect that in the case of young stars it would be difficult
to obtain a direct observational confirmation of the luminosity
suppression by irradiation. The major reason for this is that $L/L_0$
starts to deviate from unity only when $M$ and, correspondingly,
accretion luminosity are very large (see Fig. 4). At this stage the
luminosity of a star+disk system is completely dominated by the
direct emission from the disk and the disk flux intercepted and
reradiated by the stellar surface. Intrinsic stellar luminosity pro-
vides a negligible contribution that would be almost impossible to
distinguish. Besides, a forming protostar should still be en-
shrouded in the dense veil of the residual gas collapsing onto the
circumstellar disk. Reprocessing of the star+disk emission in this
infalling envelope would complicate things even more. Another
potential way of detecting the luminosity suppression is indirect,
through its effect on the stellar radius and luminosity as the star
emerges as an almost fully formed Class I object at the end of the
active accretion phase.

The effects of disk accretion in star formation have been pre-
viously investigated by a number of authors. Adams & Shu (1986)
and Popham (1997) have calculated the amount of energy that is
emitted by the disk and is intercepted by the star. Unlike these
authors we were not primarily concerned with the details of the dis-
bution of irradiation flux over the stellar surface. This is a cru-
cial point of our study, allowing us to identify the two different
regimes of stellar cooling, high and low latitude.

Mercer-Smith et al. (1984) were the first to explore the effect of
disk accretion on the stellar structure. They handled disk ac-
cretion by specifying mass addition rate and accretion luminos-
ity as external boundary conditions. They find that the stars formed
by disk accretion have larger radii than nonaccreting stars of the
same mass. As mentioned in Hartmann et al. (1997) this outcome
most likely results from allowing the accreted material to have
very high entropy, which leads to stellar swelling (see Prialnik &
Livio 1985). This approximation is unlikely to be valid in reality,
since the disk material joining the stellar surface should have enough time to radiate most of its thermal energy before being fully incorporated into the star. Palla & Stahler (1992) and Hartmann et al. (1997) allowed the accreted material to have low entropy in their studies of intermediate- and low-mass stars. They found that accretion reduces stellar size compared to the nonaccreting case, since in this case the addition of mass leads only to an increase of the gravitational energy of the star and is not accompanied by an increase of the thermal energy.

All these studies have either ignored irradiation of the star by the disk or accounted for it in the averaged sense, which may not be acceptable as our study demonstrates. To get a complete picture of a protostellar evolution one needs to take the luminosity suppression by disk irradiation into account. Such a calculation must necessarily allow for the spatial distribution of irradiation flux on the stellar surface, since only in this way can a proper estimate of the luminosity suppression be obtained.

5.1. Applications to Real Systems

Here we apply our results to several different classes of fully convective objects that may accrete through the disk at high $M$. In doing our estimates, which require the knowledge of $R_e$ and $T_0$, we use the radii and photospheric temperatures of corresponding objects determined in the absence of irradiation and mass inflow by accretion as proxies for the $R_e$ and $T_0$ that these objects would have if irradiation and accretion were properly accounted for. Needless to say, a truly accurate estimate of the effect of irradiation can be obtained only if $R_e$ and $T_0$ are calculated self-consistently accounting for the effects of accretion and irradiation (Rafikov 2008).

5.1.1. Young Stars

Young low-mass stars transitioning from Class 0 to Class I phase are fully convective and should be assembled by accretion from a circumstellar disk within several times $10^5$ yr. This implies a very high accretion rate, and we adopt $M = 5 \times 10^{-6} M_\odot$ yr$^{-1}$ for a simple estimate. An isolated $1 M_\odot$ star at an age of $10^5$ yr has a radius $R = 3.9 R_\odot$ and effective temperature $T_0 = 3760$ K (Siess et al. 2000). These parameters yield $\Lambda = 10^5$ and $T_{\text{in}}(\pi/2) \approx 6000$ K, although the latter is likely to be higher because of the boundary layer dissipation. At this $\Lambda$ cooling in the equatorial region is suppressed but the size of the polar caps is reduced only weakly: according to Figure 2 $\theta_{\text{in}} \approx 73^\circ$ [$\theta_{\text{in}}$ is given by the implicit relation $g(\theta_{\text{in}}) = \Lambda^{-1}$]. In Figure 5 we present more general results for $\Lambda$ calculated for stars of different masses using stellar parameters from Siess et al. (2000) and assuming constant $M = M_e/\tau_{\text{acc}}$, where $\tau_{\text{acc}}$ is the accretion time (assumed equal to the stellar age). One can see that the low-mass stars assembled within $3 \times 10^5$ yr generally have $\Lambda$ in the range of $30 - 10^2$, agreeing with our simple estimate.

Since $\xi \approx 6$ for $T \lesssim 5000$ K, young stars cool mainly through the polar regions, and we find from Figure 4 that stellar luminosity is reduced by irradiation only by about 10% for $\Lambda \approx 10^2$. On the other hand, if $M$ is not constant but increases as $M_e$ grows, one may expect values of $\Lambda$ larger by a factor of several. Detailed investigation (Rafikov 2008) demonstrates that the intrinsic luminosity of a strongly irradiated star can go down by a factor of several compared to the nonirradiated case, although this does not affect stellar radius very much.

5.1.2. Young Stars in Quasar Disks

A very interesting mode of star formation is possible in the accretion disks around the supermassive black holes (SMBHs) in the centers of galaxies (Illarionov & Romanova 1988; Goodman & Tan 2004; Nayakshin 2006). It is currently known that our own Galactic center harboring a SMBH of mass $M_{\text{BH}} \approx 3.7 \times 10^6 M_\odot$ (Ghez et al. 2005) contains a number of young ($\lesssim 6$ Myr) massive ($M \gtrsim 10 M_\odot$) stars that form two misaligned disks around the SMBH (Paumard et al. 2006). One of the most likely scenarios for the origin of these stars is the fragmentation of a gravitationally unstable gaseous disk (or disks) followed by the growth of fragments to their present masses by gas accretion from the residual disk (Levin 2007; Nayakshin 2006). Assuming that the disk temperature is kept at the level of 50 K by the radiation of the nearby stars (Levin 2007), one finds that at $a = 0.1$ pc from the SMBH (which is the typical dimension of the observed disks) a surface mass density $\Sigma \approx 27$ g cm$^{-2}$ is required for the disk to be Toomre unstable.

Fragments formed as a result of instability at 0.1 pc have a typical mass $M_* \sim \Sigma a^2 \approx 10^{-3} M_\odot$ (approximately 1 Jupiter mass), where $h$ is a disk scale height. At formation the Hill radius of such an object, $R_{\text{Hill}} = a(M_*/M_{\text{BH}})^{1/3}$, is already comparable to $h$, and as $M_*$ grows by accretion $R_{\text{Hill}}$ becomes larger than $h$. As a result, accretion onto the fragment proceeds through the subdisk that forms within the fragment’s Hill sphere, presenting us with the setting investigated in this paper. The rate at which gas flows into the fragment’s Hill sphere is the Hill accretion rate $\dot{M}_{\text{Hill}} \approx \Sigma a \dot{M} = 2 \times 10^{-6} \dot{M}_\odot/(a/0.1 \text{ pc})^2 M_\odot$ yr$^{-1}$. Note that $\dot{M}_{\text{Hill}}$ is smaller than the Eddington mass accretion rate $\dot{M}_{\text{Edd}} = 4\pi c R_\odot /\kappa_{\text{scat}} = 1.4 \times 10^{-11} R_{\odot} M_\odot$ yr$^{-1}$ (here $c$ is the speed of light and $\kappa_{\text{scat}}$ is the electron scattering opacity) at $a = 0.1$ pc but may become comparable to $\dot{M}_{\text{Edd}}$ farther out from the SMBH provided that $R_*$ is not much larger than $R_{\odot}$.

Such high $M$ is also typical for FU Orioni objects. As demonstrated by Popham et al. (1993) in the high-$M$ regime the boundary layer is so thick that it covers a significant ($\gtrsim 0.5$) fraction of the stellar surface, slowing down interior cooling. Heat advection into the stellar interior may become an issue in this case (Popham 1997).

![Fig. 5](image-url)
If gas in the disk is able to accrete at the same high rate \( \dot{M}_{\text{HI}} \) onto the stellar surface, then

\[
\Lambda \approx 3 \times 10^5 M_{\odot}^{5/3} T_{\text{3.5} M_{\text{HI}}^{-3}}. \tag{56}
\]

At present we do not have a theory for the structure of stars formed by fragmentation of the gravitationally unstable disks, so the value of \( \Lambda \) is highly uncertain. If \( R \leq 10 R_\odot \), then \( \Lambda \approx 10^5 \) and the luminosity suppression by irradiation should be quite important, reducing \( L \) by a factor of 2–3 compared to \( L_0 \), as Figure 4 demonstrates for \( \xi = 6.5 \).

5.1.3. Young Brown Dwarfs

Brown dwarf (BD) formation is likely to be a scaled down version of low-mass star formation: one again expects the formation of a centrifugally supported disk around a fully convective object that grows by disk accretion. The biggest uncertainty in determining \( \Lambda \) is again \( R_\odot \): 0.1 Gyr old BDs have radii of 0.1–0.2 \( R_\odot \) (Baraffe et al. 2003), but accumulation of their mass (poorly investigated at present) likely takes less than 10\(^5\) yr, during which time their entropy is still quite high, resulting in considerably larger \( R_\odot \). Assuming that an object with \( M_* = 0.03 M_\odot \) grows at constant \( M \) in time \( t_{\text{sec}} = 5 \times 10^5 \) yr and has \( R_* = 0.5 R_\odot \) and \( T_0 \approx 3000 \) K we find \( \Lambda \approx 800 \). Provided that opacity can still be characterized by expression (19) (which is a somewhat questionable assumption), we conclude that irradiation may lead to order unity reduction in \( L \). As the brown dwarf cools and contracts \( L \) increases, bringing \( L/L_0 \) down even more, provided that \( M \) could still be maintained at a high level.

5.1.4. Young Giant Planets

Finally, we consider the situation arising during the late stages of giant planet formation. A conventional core instability scenario assumes the buildup of a \( \sim 10 M_\oplus \) refractory core in the protoplanetary nebula by planetesimal agglomeration. The self-gravity of the core triggers an instability and leads to rapid gas accumulation (Mizuno 1980). While the initial stages of this process can be adequately described in the spherically symmetric approximation, the later epoch of unstable gas accretion must have distinctly non-spherical morphology. Indeed, as mentioned in Rafikov (2006), as soon as the mass of a rapidly growing planet exceeds the so-called transitional mass \( M_\text{tr} = c_3^2 10^4 G \approx 40 M_\odot a_5^{3/2} \) (here \( c_3 \) is the gas sound speed in the nebula and \( a_5 \equiv a/5 \) AU is the planetary semimajor axis scaled by 5 AU) the Hill radius of the planet \( R_{\text{HI}} \) becomes larger than the scale height of the disk \( h \). As a result, the protoplanet starts accreting gas from the surrounding nebula through the subdisk that forms within its Hill sphere, thereby presenting a situation analogous to the star formation in the Galactic center described in § 5.1.2 (except that now the collapsing fragment of a gravitationally unstable disk is replaced by a growing planet). Here we assess how important irradiation can be for planetary cooling when \( M_p \geq M_\text{tr} \).

The maximum \( M \) available to the planet is still likely given by the Hill rate \( \dot{M}_{\text{HI}} \approx 2 \times 10^{-3} M_{\odot}^{2.5} a_5^{-1} M_2 \text{yr}^{-1} \), where \( M_{p,2} \equiv M_p/10^2 M_\odot \) and we have adopted a surface density profile \( \Sigma = 270 a_5^{-3} \text{g cm}^{-2} \) typical for the minimum-mass solar nebula. This allows us to compute

\[
\Lambda \approx 10^4 M_2^{5/3} a_5^{-4} \left( \frac{R_p}{5 R_1} \right)^{-3} \left( \frac{T_0}{10^3 \text{K}} \right)^{-4}. \tag{57}
\]

This estimate is rather uncertain because of poorly constrained \( R_p \) and \( T_0 \) during the stage of active gas accretion by the planet. Here we adopt \( R_p = 5 R_1 \) and \( T_0 = 10^3 \) K mainly for illustrative purposes.

Dust is the major source of opacity in the outer layers of forming giant planets. It scales as \( \kappa \propto T^3 \) with \( \beta \approx 0.5–2 \) depending on dust grain composition, spectrum of grain sizes, etc. Here we adopt \( \beta = 1 \) in which case \( \xi = 4.5 \). This corresponds to cooling dominated by the equatorial regions, which is different from the stellar case. As a result, \( L/L_0 \) should be more sensitive to the structure of the boundary layer through which disk material accretes onto the planet; namely, \( L/L_0 \) should be lower than Figure 4 implies. Forgetting about this complication for the moment and using \( \xi = 4.5 \) and \( \Lambda \) from equation (57), we find from Figure 4 that in the planetary case \( L \) can be suppressed compared to \( L_0 \) by several tens of percent. Given the existing uncertainties in modeling the late stages of planet formation this degree of luminosity suppression by irradiation is likely not a serious issue. However, more massive, compact and cooler planets can easily have \( \Lambda \approx 10^{-3}–10^6 \), in which case irradiation would reduce \( L \) by a factor of several.

5.2. Additional Complications

Here we address various complications that may arise when the results of this work are applied to real objects. All our derivations and estimates explicitly assumed opacity in the form of equation (18). While this representation can be quite accurate within some temperature intervals one has to bear in mind that on the surface of a star irradiated by the disk temperature can vary appreciably between the equator and the poles. Indeed, the equatorial regions of a \( M_* = 1 M_\odot \), \( R_* = 2 R_\odot \), star accreting at \( M = 10^{-3} M_\odot \text{yr}^{-1} \) are heated to 1.1 \( \times 10^4 \) K, so equation (20) applies, while polar caps still have \( T \approx 3 \times 10^3 \) K, so equation (19) is more appropriate. Thus, in different regions of stellar surface \( \kappa \) has different dependence on \( P \) and \( T \). In this case equation (54) becomes invalid, and to properly compute the luminosity suppression one would need to take into account the latitudinal variation of not only \( T_{\text{in}} \) but also \( \kappa(P, T) \).

Even at a fixed latitude opacity can switch from one regime to another within the outer radiative zone. Although \( \kappa \) is much more sensitive to \( T \) than to \( P \) and the radiative zone is roughly isothermal, pressure at its bottom \( P_{\text{ch}} \approx \sum a_{\text{ph}} 1/(1+a_{\text{ph}}) \gg 1 \) of the photospheric pressure \( P_{\text{ph}} \), so even a weak dependence of \( \kappa \) on \( P \) can lead to opacity transition within the outer radiative zone. In particular, this situation is likely to occur at latitudes where \( T_{\text{in}} \approx 5000 \) K and \( \kappa \) switches from equation (19) to (20).

In this case calculation of the local radiative flux \( F_{\text{in}} \) gets more complicated as the external radiative region splits into two layers characterized by different opacity behaviors.

Our numerical estimates of the luminosity suppression strongly rely on the assumption of fixed \( \gamma \) within the radiative zone. Equation (42), which assumes \( \gamma = 5/3 \) throughout the whole star, fails to predict the correct photospheric temperature of an isolated star. The reason for this is the variation of \( \gamma \) at low \( T \) caused by molecular dissociation. This results in \( \nabla_{\text{ad}} \approx 0.1 \) and leads to a smaller drop of temperature in the outer convective parts of the star. Superadiabaticity in the outer layers of the convective zone may also be an issue, although in the irradiated case the outer boundary of the convective zone is pushed toward higher densities where convective transport is more efficient (see Appendix C). Our simple estimate (42) underpredicts \( T_{\text{in}} \) by a factor of 2–3, which is primarily a consequence of our assumption of fixed \( \gamma = 5/3 \). A proper calculation of \( F_{\text{in}}(\theta) \) and \( L/L_0 \) must be able to account for the variation of \( \gamma \) with \( P \) and \( T \) inside the outer radiative zone.

Our analysis is affected to some extent by the presence of the boundary layer through which disk material joins the star. Viscous dissipation in this layer heats accreting gas to very high
temperature. Since this energy release takes place very close to the stellar surface, most of the heat is likely to leak out and not get carried into the star with the accreted gas. However, some residual heat may still be accreted. Moreover, in addition to advective there could also be a radiative energy transfer from the boundary layer into the outer layers of the star (Popham 1997). The increase of $T$ in the external radiative zone driven by these processes acts to additionally slow down stellar cooling in the equatorial region, as equation (35) demonstrates. Thus, the presence of the boundary layer reduces $L/L_0$ even more than our analysis predicts, and the results presented in Figure 4 should be viewed as upper limits on $L/L_0$. This effect is likely not very important for young stars that cool predominantly through their polar regions, largely unaffected by the additional heat deposition at the equator. However, in the case of young giant planets that lose a fair amount of energy through the low-latitude part of the surface (see § 5.1.4) the reduction of intrinsic flux in the equatorial region may produce a quite noticeable decrease of $L/L_0$ compared to the idealized case considered in this work.

When discussing the external radiative zone throughout this work we have been concerned only with the radiative energy transport. At the same time, it is well known that in the case of hot Jupiters advective transport in the form of atmospheric jets and winds should be very important in redistributing heat across the planetary surface (Menou et al. 2003; Dobbs-Dixon & Lin 2008). In our azimuthally symmetric setting only meridional atmospheric motions can lead to energy exchange between the hot equatorial and cold polar regions. Fluid motions occurring on surfaces of constant effective potential (gravitational plus centrifugal) are unlikely to produce efficient equator-pole energy exchange (compared to the radiative transfer) because of rather fast rotation typical for objects formed by disk accretion. Rotation forces angular momentum conservation and prevents significant fluid motions in the $\theta$-direction, suppressing this mode of advective transport. On the other hand, rotation tends to promote meridional circulation within the radiative layer (Kippenhahn & Weigert 1990), whose impact on the energy transport should be investigated in more detail.

Finally, our basic assumption of direct mass accretion from the disk will be challenged if the star possesses a magnetic field strong enough to disrupt the accretion flow outside of $R_*$ (Königl 1991). This becomes an issue for stars older (T Tauri stars) than the objects that we have considered here. T Tauri stars have low enough $M$ for the stellar magnetic field to be able to channel accreting gas toward stellar magnetic poles. This completely changes the topology of the accretion flow, but our major conclusions about the effect of irradiation on stellar cooling still hold. Indeed, the magnetically channeled gas travels toward the stellar surface at a good fraction of the free-fall velocity, and at some point it must pass through the radiative shock, after which it accumulates at the top of the magnetospheric column of accreted material, as schematically indicated in Figure 6. The total energy release within the shock and magnetospheric column is comparable to that occurring if the accretion disk were extending all the way to the stellar surface. This hot column of accreted material illuminates the surface of the star, leading to the same suppression of intrinsic stellar flux as we discussed in this work. In this case, however, irradiation is strongest near the magnetic poles while the magnetic equator is likely to be the coolest part of the stellar surface. The calculation of stellar irradiation and integrated luminosity in this case would involve constructing a model for the magnetospheric column structure and its radiative properties. The impact of these details on the structure and evolution of young stars should be addressed by future work.

6. SUMMARY

The luminosity of young stars actively accreting from the circumstellar disk can be significantly affected by the radiation that is produced in the inner parts of the disk and is intercepted by the stellar surface. We showed that if a star gains its mass via disk accretion on a timescale of several $10^5$ yr, then the radiative flux caused by viscous dissipation in the disk is more than sufficient to increase the surface temperature of the star above the photospheric temperature that an isolated star with the same mass and radius would have. Irradiation by the disk is strongest in the equatorial regions and is almost negligible near the poles. An outer radiative zone of almost constant temperature forms above the fully convective interior in the strongly irradiated parts of the stellar surface. This leads to the local suppression of intrinsic energy flux escaping from the stellar interior.

We have demonstrated that there are two distinct modes in which a fully convective object can cool: mainly through the cool high-latitude polar regions or predominantly through the low-latitude parts of the stellar surface. A particular regime of cooling in a given object is set by the opacity behavior and the adiabatic temperature gradient $\nabla_{ad}$ in the outer radiative zone. Accreting young stars and brown dwarfs cool mainly through the polar regions, while forming giant planets cool through the whole surface.

Integrated stellar luminosity in the accreting case is suppressed compared to the case of an isolated object, by up to a factor of several in some classes of objects (actively accreting brown dwarfs and planets, stars forming in gravitationally unstable disks in the galactic nuclei). This may affect the initial conditions that are used to calculate the evolution of the low-mass objects on timescales of $\sim 10$ Myr after their formation. The existence of an external radiative zone may facilitate the retention of dust in the atmospheres of brown dwarfs and planets and may affect the
strength of the magnetic field generated by the internal dynamo in convective objects. Some of the results obtained in this work may be applicable to accreting white dwarf and neutron star systems.

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APPENDIX A
IRRADIATION FLUX

Performing an integral over $\phi$ in equation (1) we find

$$ F_{in}(\theta) = \frac{2 \cos \theta}{\pi} \int_{x_m}^{\infty} \frac{F_d(xR_*)q(x, \theta) x dx}{D^2}, \quad (A1) $$

$$ q(x, \theta) = \sqrt{x^2 \sin^2 \theta - 1} - 2 \frac{x^2 (1 - 2 \sin^2 \theta) + 1}{D} \arctan \left( \frac{x^2 + 2x \sin \theta + 1}{x \sin \theta + 1} \sqrt{\frac{x^2 \sin^2 \theta - 1}{D^2}} \right), \quad (A2) $$

where $D^2 = (x^2 + 1)^2 - 4x^2 \sin^2 \theta$ and $x_m = 1/\sin \theta$. For $F_d(R)$ obeying equation (2), equation (A1) can be rewritten as equation (3) with

$$ g(\theta) = \frac{3 \cos \theta}{4\pi^2} \int_{x_m}^{\infty} \frac{f(xR_*)q(x, \theta) dx}{x^2 D^2}. \quad (A3) $$

The integral in equation (A3) is dominated by $x \sim x_m$. Near the equator, where $\theta \to \pi/2$ one can expand the integrand in equation (A1) in terms of $\pi/2 - \theta \ll 1$ and $x - 1 \ll 1$, which results in $F_d(\pi/2) = \frac{F_d(R_*)}{2}$. The polar regions of the star ($\theta \to 0$) are illuminated only by distant parts of the disk, $R \sim R_*/\sin \theta \gg R_*$, so that $x \gg 1$ (while $x \sin \theta \sim 1$) in equation (A1). Also, far from the star one can safely use equation (2) with $f = 1$ to finally arrive at equation (5) with

$$ I_1 = \frac{3}{4\pi^2} \int_{1}^{\infty} dt \left( \sqrt{t^2 - 1} - 2 \arctan \frac{\sqrt{t^2 - 1}}{1 + t} \right) = \frac{1}{50\pi^2}. \quad (A4) $$

This result is independent of the structure of the boundary layer near the stellar surface, since the polar regions of the star do not have direct sight lines to the boundary layer. This is evidenced by the convergence in Figure 2 at $\theta \to 0$ of the two curves calculated assuming $f(R) = 1$ and $f(R) = 1 - \left( R_/R_* \right)^2$.

APPENDIX B
VALIDITY OF 1D APPROXIMATION

To determine the validity limits of the 1D solution for the structure of the radiative zone found in § 3.1 we evaluate the magnitude of the corrections arising when the latitudinal radiative transfer is accounted for. Considering 1D solution (21) as a zeroth-order approximation we plug it into the full equation (14) and carefully expand all $\theta$-derivatives, remembering that $P$ is almost independent of $\theta$ (latitudinal pressure gradients are small). Integrating the resulting expression once over $r$ we again arrive at equation (16) but with $F_{in} \to F_{in} + \delta F_{in}$ in the left-hand side, where

$$ \delta F_{in} = \left( \frac{k_B T_{ph}^{4-3}}{\mu L_\theta^2} \right)^2 \int_{P_*}^{T_{ps}} \frac{T^{3/2} - T_{ps}^{3/2}}{P^{3/2}} Z(P) dP. \quad (B1) $$

Here $Z(P) \sim 1$ is a weak function of pressure (varying by at most a factor $\sim 1$) and $L_\theta$ is a characteristic scale of the latitudinal variation of $T_{ph}, L_\theta = R_*(\partial \ln T_{ph}/\partial \theta)^{-1}$. Our 1D approximation is justified if the correction to the 1D result $\delta F_{in}$ is small compared to $F_{in}$ given by equation (35).

The integral in equation (B1) is dominated by $P \sim P_{ph}$ (latitudinal radiation transfer is easiest in the upper, low-density layers of the star just below the photosphere), and one can easily find using equations (33), (34), and (35) that

$$ \frac{\delta F_{in}}{F_{in}} \sim \left( \frac{H_{ph}}{L_\theta} \right)^2 \nabla_{ph}^{-1}, \quad (B2) $$

where $H_{ph}$ is the photospheric scale height. This result makes it clear that the 1D solution for the structure of the radiative zone should be reasonable as long as condition (46) is fulfilled.
APPENDIX C

SUPERADIABATICITY OF CONVECTION

To estimate the degree of superadiabaticity near the outer edge of the convective zone we introduce, following Kippenhahn & Weigert (1990),

\[ x = \nabla - \nabla_{\text{ad}}, \quad W = \nabla_{\text{rad}} - \nabla_{\text{ad}}, \quad U = \frac{6\sqrt{2\nabla_{\text{ad}}} \sigma T^4}{\eta^2 P_{\kappa}\rho} \left( \frac{\alpha^2}{GM_cH_{\rho}^2} \right)^{1/2}, \quad \nabla_{\text{rad}} = \frac{3}{16\pi G M_c T^4}, \quad H_{\rho} = \frac{\partial r}{\partial \ln P} = \frac{kT}{g\mu}, \]

(C1)

where \( \eta \approx 1 \) is the mixing-length parameter, \( H_{\rho} \) is the pressure scale height, and \( x \) is the deviation of the temperature gradient from \( \nabla_{\text{ad}} \). The value of \( x \) can be found from the following equation (Kippenhahn & Weigert 1990):

\[ \left( \sqrt{x + U^2} - U \right)^3 = \frac{8}{9} U(W - x). \]

(C2)

From the functional form of our opacity law (18) and definitions (C1) it is obvious that

\[ W = \nabla_{\text{ad}} \left[ \left( \frac{P}{P_{\text{cb}}} \right)^{1+\alpha} \left( \frac{T}{T_{\text{rb}}} \right)^{-\beta - 4} - 1 \right]. \]

(C3)

Near the convective radiative boundary \( W \approx 1 \), but as \( P \) and \( T \) increase deeper down the convective zone one finds \( W \approx \nabla_{\text{rad}} \).

On the other hand, using equations (18), (27), and (28) and neglecting constant factors \( U \) can be written as

\[ U \approx \frac{\sigma T_{\text{ph}}^4 - \beta g}{\kappa P_{\text{ph}}^{2+\alpha}} \left( \frac{\mu}{kT_{\text{ph}}} \right)^{1/2} \left( \frac{T_{\text{ph}}}{T_{\text{cb}}} \right)^{3/2 - \beta}. \]

(C4)

For a 1 M\(_\odot\) star with \( R_\odot \approx 10^{11} \) cm, \( L \approx 0.2 L_\odot \) irradiated at \( T_{\text{ph}} = 10^4 \) K and opacity given by equation (20) one finds that \( U \leq 0.1 \) at the convective-radiative boundary.

Equation (C4) clearly demonstrates that \( U \) rapidly decreases as \( T \) and \( P \) increase inside the convective zone. Thus, it would be typical to find \( U \ll 1 \) in the convective zone of an irradiated star. As shown in Kippenhahn & Weigert (1990) under such conditions equation (C2) has a solution \( x \approx (8UW/9)^{2/3} \). In the aforementioned numerical example \( W \sim 1 \) and \( U \leq 1 \) just below the convective-radiative boundary, resulting in mild superadiabaticity, \( x \sim 0.1 \). In addition, \( x \) rapidly decreases with depth, since \( W \approx \nabla_{\text{rad}} \) at large enough depth so that \( UW \propto r^{-1}T^{-3/2} \). Thus, at \( T_{\text{ph}} \approx 10^4 \) K superadiabaticity should be rather minor throughout the convective regions of the star underlying the outer radiative layer.

Using equations (33) and (23) one can also obtain that

\[ U \propto T_{\text{ph}}^{4 - \xi (1/\nabla_{\text{ad}}) - 1/2}, \]

(C5)

i.e., \( U \) decreases as the irradiation intensity increases and \( T_{\text{ph}} \) grows. Thus, we conclude that the stronger the irradiation the better our assumption of weak superadiabaticity in the convective zone. This is a natural result, since higher \( T_{\text{ph}} \) pushes the convective-radiative boundary to greater depth where the gas density is higher and radiative losses of convective eddies are lower, ensuring more efficient convective energy transport.

REFERENCES

Adams, F. C., & Shu, F. H. 1986, ApJ, 308, 836
Arias, P., & Bildsten, L. 2006, ApJ, 650, 394
———, 2002, A&A, 382, 563
Baraffe, I., Chabrier, G., Barman, T. S., Allard, F., & Hauschildt, P. H. 2003, A&A, 402, 701
Bell, K. R., & Lin, D. N. C. 1994, ApJ, 427, 987
Bouvier, J., Alencar, S. H. P., Harries, T. J., Johns-Krull, C. M., & Romanova, M. M. 2007, in Protostars and Planets V, ed. B. Reipurth, D. Jewitt, & T. Keil (Tucson: Univ. Arizona Press), 479
Burrows, A., Guilbert, T., Hubbard, W. B., Marley, M. S., Saumon, D., Lunine, J. I., & Sudarsky, D. 2000, ApJ, 534, L97
Chabrier, G., Baraffe, I., Allard, F., & Hauschildt, P. H. 2000, ApJ, 542, 464
Chabrier, G., Barman, T., Baraffe, I., Allard, F., & Hauschildt, P. H. 2004, ApJ, 603, L53
Dobbs-Dixon, I., & Lin, D. N. C. 2008, ApJ, 673, 513
Ghez, A. M., Salim, S., Hornstein, S. D., Tanner, A., Lu, J. R., Morris, M., Becklin, E. E., & Duchêne, G. 2005, ApJ, 620, 744
Goodman, J., & Tan, J. C. 2004, ApJ, 608, 108
Guillot, T., Burrows, A., Hubbard, W. B., Lunine, J. I., & Saumon, D. 1996, ApJ, 459, L35
Guillot, E., Hartmann, L., Briceno, C., & Calvet, N. 1998, ApJ, 492, 323
Hartmann, L., Cessen, P., & Kenyon, S. J. 1997, ApJ, 475, 770
Illarionov, A. F., & Romanova, M. M. 1988, Soviet Astron., 32, 148
Johns-Krull, C. M. 2007, in IAU Symp. 243, Star-Disk Interaction in Young Stars, ed. J. Bouvier (Cambridge: Cambridge Univ. Press), 31
Johns-Krull, C. M., Valenti, J. A., & Koresko, C. 1999, ApJ, 516, 900
Kippenhahn, R., & Weigert, A. 1990, Stellar Structure and Evolution (Berlin: Springer)
Königl, A. 1991, ApJ, 370, L39
Lamzin, S. A., Romanova, M. M., & Kravtsova, A. S. 2007, in IAU Symp. 243, Star-Disk Interaction in Young Stars, ed. J. Bouvier (Cambridge: Cambridge Univ. Press), 115
Landau, L. D., & Lifshitz, E. M. 1984, Statistical Physics (Oxford: Butterworth-Heinemann)
Levin, Y. 2007, MNRAS, 374, 515
Lynden-Bell, D., & Pringle, J. E. 1974, MNRAS, 168, 603
Menou, K., Cho, J.-Y., Seager, S., & Hansen, B. M. S. 2003, ApJ, 587, L113
Mercer-Smith, J. A., Cameron, A. G. W., & Epstein, R. I. 1984, ApJ, 279, 363
Mizuno, H. 1980, Progr. Theor. Phys., 64, 544
Nayakshin, S. 2006, MNRAS, 372, 143
Omukai, K. 2007, PASJ, 59, 589
Palla, F., & Stahler, S. W. 1992, ApJ, 392, 667
Paumard, T., et al. 2006, ApJ, 643, 1011
Popham, R. 1997, ApJ, 478, 734
Popham, R., & Narayan, R. 1995, ApJ, 442, 337
Popham, R., Narayan, R., Hartmann, L., & Kenyon, S. 1993, ApJ, 415, L127
Prialnik, D., & Livio, M. 1985, MNRAS, 216, 37
Rafikov, R. R. 2006, ApJ, 648, 666
———. 2008, ApJ, 682, 542
Shakura, N. I., & Sunyaev, R. A. 1973, A&A, 24, 337
Shu, F., Najita, J., Ostriker, E., Wilkin, F., Ruden, S., & Lizano, S. 1994a, ApJ, 429, 781
Shu, F., Najita, J., Ruden, S., & Lizano, S. 1994b, ApJ, 429, 797
Siess, L., Dufour, E., & Forestini, M. 2000, A&A, 358, 593
Siess, L., & Forestini, M. 1996, A&A, 308, 472
Siess, L., Forestini, M., & Bertout, C. 1997, A&A, 326, 1001
———. 1999, A&A, 342, 480
Stahler, S. W. 1988, ApJ, 332, 804
Stahler, S. W., Shu, F. H., & Taam, R. E. 1980a, ApJ, 241, 637
———. 1980b, ApJ, 242, 226
Winkler, K.-H. A., & Newman, M. J. 1980, ApJ, 236, 201