Using of sensitivity models of the steady-state of electric power systems for account of failure of elements

Dmitry Krupenev1*

1Melentiev Energy Systems Institute, 130 Lermontov str., Irkutsk, Russia

Abstract. The article presents an algorithm for obtaining sensitivity models of the first and second orders of the steady-state regime of electric power systems (EPS). The sensitivity models are intended for express calculations of steady-state when estimating the static security of EPS. The use of sensitivity models allows one to simulate failures of EPS elements without calculating new steady-state. To verify the reliability of the sensitivity models obtained, the results of an experiment performed on a 3-node test pattern are presented.

1 Introduction

For minimize of the consequences of electrical equipment failures in electric power systems (EPS), it is necessary to make a set of specific actions, one of which is an operational estimation of the security of EPS. The security estimation of EPS is determined by the indicators that most fully characterize the current steady-state from the standpoint of opposing possible disturbances in EPS. Estimation of the security of EPS is technologically capacious and computationally expensive. There are the static and the dynamic security of EPS. In this paper, we describe static security. The estimation of a static security of EPS is based on the assumption that the transition of EPS to a new state due to an element failure is not accompanied by a violation of dynamic stability.

When we estimate of security of EPS very important parameters are accuracy and time of the estimation, since the estimation should be carried out in real time. Various simplifications when we solve this problem can lead to distortion of the result and, ultimately, acceptance of a non-optimal or incorrect set of control actions.

There are some approaches to the estimation of security of EPS. We can divide such approaches as the approach based on the application of the criterion n-i [1-3], the machine-based approach [4], the Jacobi matrix based approach [5], In [6] to estimate the security of EPS using the Monte Carlo method. In most of the present approaches (methods), one of the main stages of estimation, and the most time-consuming expense, is the stage of modeling the effect of failures of power equipment on the EPS parameters or the calculation of steady-state of EPS at failures of its elements. When we estimate of security of EPS, it is necessary to analyze single and group failures of EPS elements in a minimal time. EPS consists of a many of elements, the number of possible states is large. In this paper, we study the problems of accelerating the calculation of steady-state of such steady-states of EPS. We propose to use steady-state sensitivity models or to replace the exact model of calculating of steady-state of EPS by its approximation.

Models of first-order sensitivity of steady-state of EPS are used in solve deferent electric power problems, among them one can single out the problem of analysis of voltage stability of EPS [7]. In [8] the first-order sensitivity models (matrices) are used to calculate the steady-state of EPS when to change the reactive power at the nodes of the EPS. So, in [7], [8], using the models of the first-order sensitivities, linearly approximate the steady-state and search for the steady-state parameters of the EPS during the introduction of the perturbation. The aim of this paper is to obtain sensitivity models of not only the first, but also the second order for the rapid calculation of the steady-state of the EPS. Models of second-order sensitivity give a quadratic approximation to the new steady-state point, which increases the accuracy of calculating the steady-state and, accordingly, the accuracy of the estimation of the security of EPS.

2 Formation of sensitivity models of the steady-state of electric power systems

The steady-state of EPS can be described by a vector equation of the form [9]:

\[ W(X, V) = 0, \]

where: \( W \) – vector-function, \( W : G \rightarrow \mathbb{C}^{n+m} \), \( G \subseteq \mathbb{C}^{n+m} \); \( X \) – vector of input parameters of the EPS steady-state, \( X \in \mathbb{C}^{n} \); \( V \) – vector of output parameters of the EPS steady-state, \( V \in \mathbb{C}^{m} \), \( n \) – number of power lines of EPS, \( m \) – number of nodes of EPS.

When we deal estimate of security of EPS, we should to find the dependencies of some parameters on others,
1-mn it is necessary to find the differential of
is a diagonal O. With the aim of uniquely defined
is the diagonal matrix, where the power
is a vector that composed of the
–
, (2)
-1m
(differential of
for the simulating (modeling) of
, where
; 1-m
for defining the differential
relative to the vector of the
. The column vector
; (6)
. That column vector composed of the components
at
vector by
(5.a)
is a column vector whose
is
, will be used instead
, (vector of
; (the equation has a solution at the
order of EPS lines
; (4)
(3.b)
E3S Web of Conferences 25, 03001 (2017)
DOI: 10.1051/e3sconf/20172503001
RSES 2017
Moreover, to obtain a sensitivity model of 1\textsuperscript{st}
dimension the vector equations system (3) should be
differentiated by variable that change own value during
the process of reliability assessment. This variable
depend on current problem. It is worth to be noting that
the EPS elements failures and random deviations of
consumers demand are simulating during the process of
the system reliability assessment. Hence, it is required
to differentiate the system (3.a), (3.b) by the \( Y \) (vector
of lines conductivity) for the EPS lines failure simulating
(modeling) and by \( S \) for the simulating (modeling) of
random deviations of consumers demand and failures of
generating blocks. In this work we will focused on the
differentiation by \( Y \). With the aim of uniquely defined
the differentials \( d_I \) (differential of \( I \) vector by \( Y \)
variable) and \( d_U \) (differential of \( U \) vector by \( Y \)
variable) it is necessary to fix the diagonal matrix \( \bar{U} \)
at the solution point, before differentiation of the vector
equations system (3.a), (3.b). The values of voltages in
nodes will be located on the diagonal of this matrix.
on the right-hand side.
\[
\bar{A}d_I I = 0, \tag{5.a}
\]
\[
\bar{A}'d_U I = \bar{Y}^{-1}(d\bar{Y})\bar{Y}^{-1}I + \bar{Y}^{-1}d_I I, \tag{5.b}
\]
where: \( d_I \in \mathbb{C}^n ; \bar{d}\bar{Y} \in \mathbb{C}^n ; d_U \in \mathbb{C}^{m_l}
.
The differentiation of the inverse matrix function was
carried out according to [15]: \( d(\bar{Y})^{-1} = -\bar{Y}^{-2}d\bar{Y}^{-1} \).
Next, let us convey from equation (5.b):
on the right-hand side.
\[
d_I I = \bar{Y}\bar{A}'d_U I +(d\bar{Y})\bar{Y}^{-1}I. \tag{6}
\]
Replace the \( d_I \) from (6) to (5.a):
\[
\bar{A}\bar{Y}\bar{A}'d_U I = \bar{A}(d\bar{Y})\bar{Y}^{-1}I , \tag{7}
\]
as follows from (7) equation we will take:
\[
d_U I = -(\bar{A}\bar{Y}A')^{-1}\bar{A}(d\bar{Y})\bar{Y}^{-1}I = -(\bar{A}\bar{Y}A')^{-1}\bar{A}\bar{Y}^{-1}\bar{Y}dY , \tag{8}
\]
Where \( \bar{I} \) is the diagonal matrix, where the power
flows along the power transmission lines of EPS located
on the diagonal, \( I \in \mathbb{C}^n \).
The matrix permutation was carried out in the
mathematical expression (8) that is possible for diagonal
matrices.
The linearized dependence of the voltages in the
nodes of the EPS on the conductivities change in the
elements of the EPS can be represented in the form
\( U = U_0 + d_U U \), where \( U_0 \) is a column vector whose
components are values of the voltages in the nodes of the
EPS at the solution point \( U_0 \in \mathbb{C}^{m_l} \). The column vector
\( \Delta Y \) will be used instead \( dY \) for defining the differential
\( d_U U \). That column vector composed of the components
which are increments of the conductivities on the corresponding lines \( \Delta Y \in C^n \).

Let’s substitute \( d_i U \) (8) into (6) for to get \( d_i I \).

Then

\[
d_i I = Y' A (Y A Y')^{-1} (Y Y')^{-1} I + d Y Y' I. \tag{9}
\]

The linearized dependence of the change of power flows along the EPS power transmission lines with depending on conductivities change can be represented and look like

\[
I^Y = Y^0 + d I^Y, \tag{10}
\]

where

\[
y^Y = Y^0 + d I^Y. \tag{11}
\]

In this case the \( 2^{nd} \) order sensitivity models, namely the dependence of voltage in EPS nodes on conductivity change in EPS elements can be represent as:

\[
U = U_0 + d_i U + \frac{1}{2} d_i^2 U, \tag{12}
\]

\[
I = I_0 + d_i I + \frac{1}{2} d_i^2 I. \tag{13}
\]

In event of EPS lines failure the disturbance of voltages and streams along the power transmission lines of EPS will be determined as follows:

\[
A_{i1} U = d_i U + \frac{1}{2} d_i^2 U, \tag{12}
\]

\[
A_{i2} I = d_i I + \frac{1}{2} d_i^2 I \tag{13}
\]

where \( A_{i1} U \) is a vector of voltages change in EPS nodes, \( A U \in C^{m \times 1} \), \( A_{i2} I \) is a vector of EPS power flows change, \( A I \in C^n \).

Thus, vector \( \Delta Y \) that characterizes the values of conductivities that equivalent to switching off the transmission line in equations (8-13) remains undefined. However, the conductivity \( \Delta Y \) of power transmitting line \( i \) has a definite value during the normal functioning. In case of power transmission line \( i \) failure the value of its conductivity become equal \( 0 \), \( \Delta Y = X_i = -X_i \). In summary the value of conductivity that equivalent of power transmission line \( i \) disconnections will be defined as:

\[
\Delta Y = 0 - Y = Y_i. \tag{14}
\]

During the use sensitive models for the steady-states of EPS calculation by one iteration it is possible to get the vector of voltage changes in EPS nodes in case of single and non-single disconnections of power transmission lines. It could be done if represent the \( \Delta Y \) as a matrix \( A Y \) (instead vector) with size \( n \times n \) and values of \( -Y_i \) by the main diagonal, of course if it is necessary to get the assessment of single failures of EPS power transmission lines, and other zero elements. If the assessment of multiple failures required then it is necessary to fill matrix \( A Y \) in a certain way.

3 Case study

The sensitivity models of the first and second orders applications are demonstrated for steady state calculation of the three-node EPS test scheme shown in fig.1. The experiment includes power line I capacity changing, with its final shutdown.

![Fig. 1. EPS test scheme.](image)

As a result of the steady-state calculation of the initial model (3.a), (3.b), the following voltages:

- \( U_2 = 234,94 - i7,73 \) kV; \( U_3 = 213,67 - i71,69 \) kV, and power lines flows:
  - \( I_{12} = 0,08 + i0,36 \) kA,
  - \( I_{13} = 1,57 + i0,29 \) kA,
  - \( I_{23} = 1,49 - i0,07 \) kA were obtained at the nodes of the EPS.

Further, the voltages at nodes 2 and 3 were determined using the obtained sensitivity models of the first and second orders for taken power line I capacity perturbations increments. In table 2 calculation results are demonstrated when power line I capacity changing on the levels: 1; 5; 9; 34; 48; 67; 86; 90; 95 % of the initial power line capacity. Calculation result when power line I is disabled (capacity perturbation is 100%) is shown in the last row of table 2.

| Power line | Impedance, \( \Omega \) | Node | Load, MVA | Voltage, kV |
|------------|------------------------|------|-----------|-------------|
| I          | 12,1 + i43,5           | 1    | 0         | 220,0 + i0 |
| II         | 12,1 + i43,5           | 2    | 329,0 - i114,0 | - |
| III        | 12,1 + i43,5           | 3    | 669,0 - i173,0 | - |

As a result of the steady-state calculation of the initial model (3.a), (3.b), the following voltages: \( U_2 = 234,94 - i7,73 \) kV; \( U_3 = 213,67 - i71,69 \) kV, and power lines flows: \( I_{12} = 0,08 + i0,36 \) kA, \( I_{13} = 1,57 + i0,29 \) kA, \( I_{23} = 1,49 - i0,07 \) kA were obtained at the nodes of the EPS.
Table 2. Voltage deviation in test scheme nodes on power lines I-II capacity perturbation.

| Power line capacity perturbation | $U$ (init.mod.), kV | $\Delta U_x$, kV | $\Delta U_y$, kV | $U$ (lin.mod.), kV | $\Delta U_x$, kV | $\Delta U_y$, kV | $U$ (quad.mod.), kV | $\Delta U_x$, kV | $\Delta U_y$, kV |
|---------------------------------|----------------------|------------------|------------------|-------------------|------------------|------------------|-------------------|------------------|------------------|
| % of initial.                  | absolute value       | $U_x = U_1$     | $U_y = U_2$      | $U_x = U_3$       | $U_y = U_4$      | $U_x = U_5$       | $U_y = U_6$       | $U_x = U_7$       | $U_y = U_8$       |
| 1%                             | -5.87*10^{-5} + i2.11*10^{-4} | 235.02 - i7.78 | 0.1 - i0.05 | 235.04 - i7.78 | 0.13 - 0.06i | 235.17 - 7.81i |
| 5%                             | -2.83*10^{-4} + i0.10*10^{-2} | 235.32 - i8.08 | 0.5 - i0.05 | 235.31 - i8.97 | 0.06 - 0.03i | 235.72 - 8.13i |
| 9%                             | -5.4*10^{-5} + i0.19*10^{-2} | 235.75 - i8.22 | 0.91 - i0.07 | 235.85 - i8.2 | 1.15 - 0.59i | 236.42 - 8.9i  |
| 34%                            | -1.98*10^{-3} + i0.71*10^{-2} | 238.41 - i9.9 | 3.32 - i1.72 | 238.26 - i9.45 | 4.22 - 2.18i | 240.37 - 10.54i |
| 48%                            | -2.97*10^{-3} + i0.01 | 240.84 - i11.5 | 4.98 - i2.58 | 239.92 - i10.31 | 6.32 - 3.27i | 243.08 - 11.94i |
| 67%                            | -3.96*10^{-3} + i0.014 | 244.03 - i13.74 | 6.64 - i3.43 | 241.58 - i11.16 | 8.43 - 4.36i | 245.79 - 13.35i |
| 86%                            | -4.95*10^{-3} + i0.018 | 248.48 - i17.02 | 8.3 - i4.29 | 243.24 - i12.02 | 10.54 - 5.45i | 248.51 - 14.75i |
| 90%                            | -5.4*10^{-3} + i0.019 | 250.98 - i19.08 | 9.05 - i4.68 | 243.99 - i12.41 | 11.5 - 5.95i | 249.74 - 15.39i |
| 95%                            | -5.65*10^{-3} + i0.02 | 252.68 - i20.5 | 9.48 - i4.91 | 244.43 - i12.64 | 12.04 - 6.23i | 250.45 - 15.75i |
| Disabled                        | -5.94*10^{-3} + i0.021 | 254.79 - i22.31 | 9.96 - i5.15 | 244.9 - i12.88 | 12.65 - 6.54i | 251.22 - 16.15i |

Graphical interpretation of the results is shown on the fig.2, namely the values of the real part of the voltage in the second node. The values of the imaginary part of the voltage in the second node are shown on the fig.3. Fig. 4 and 5 show similarly values for third node.

![Fig. 2. The values of the real part of the voltage in the second node.](image)

![Fig. 3. The values of the imaginary part of the voltage in the second node.](image)
As can be seen from the results, steady state linearized model using for investigated EPS with disabled power line 1 gives voltage deviation in comparison with calculation of steady state with disabled power line in 9.89kV.

When sensitivity model of second order was used deviation was 3,57 kV (1,4%) on the active part of power and 6,16 kV (27,6%) on the reactive part. The voltage deviation by modulus in the second node was 1.6%, in the third node it was 0.8%.

4 Conclusion

In this paper we consider the approximation task of the postfault EPS steady state calculation while assess its static security. Sensitivity models based on first-order and second-order differentials of EPS steady state are suggested for doing that. When security assess, it becomes necessary to calculate the set of EPS steady states because of equipment failures and deviations of customers load. Therefore, when using sensitivity models changings of powerlines capacity, customers load and generating capacity are taken as input parameters and voltages in EPS nodes and powerlines overflows as output. Some differentials of the first and second orders of the steady-state EPS are obtained.

Within the framework of a numerical experiment, investigations were carried out on a three-node EPS scheme where one of the powerlines was disabled. The values of the voltages in the EPS nodes were obtained as a result of steady state calculating on the initial model and on the sensitivity model, which characterizes the voltage deviation in the EPS nodes from the powerlines capacity changing.

The reported study was funded by RFBR according to the research projects No. RFBR 16-38-00312 мол а

References

1. Y.N Kucherov, O.M. Kucherova, L. Kapoyi, Yu.N. Rudenko, Reliability and efficiency of large transnational power plants. Methods of analysis: European dimension (Novosibirsk: Science, Siberian Publishing Company, Russian Academy of Sciences, 1996).
2. Manov N.A., Chukreev Yu.Ya., Methods and models of investigation of reliability of electric power systems (Syktyvkar, 2010).
3. Pei-Qing Liu, Hua-Qiang Li, Yang Du, Ke Zeng Risk assessment of power system security based on component importance and operation state, 2014 International Conference on Power System Technology, (2014).
4. Panasetksy D., Tomin N., Voropaev N., Kurbatsky V., Zhukov A., Sidorov D., Proceedings of the 2015 IEEE Eindhoven PowerTech, (2015).
5. Gamm A.Z., Golub I.I., Sensors and weaknesses in electric power systems (SEI SB RAS. 1996).
6. Domyshev A.V., Krupenev D.S. Electricity, 2, (2015).
7. L.A.Ll. Zarate and C.A. Castro. IEEE Proc.-Gener. Transm. Distrib., Vol. 153, (2006).
8. S. Chen, W. Huang, W. Lai, P. Shi. Power System Fast Line Flow Calculation for Security Control by Sensitivity Factor Second International Conference on Innovative Computing, Informatio and Control (2007).
9. Idelchik V.I. Calculation of steady-state modes of electrical systems (M. Energy, 1977).
10. Zorich V.A. Mathematical analysis. Part 1 (M, 2002).
11. Trenogin V.A. Functional analysis (Moscow: Science. Main edition of physical and mathematical literature. 1980).
12. Epifanov S.P., Novitsky N.N. Proceedings of the XIIIth Baikal International School-Seminar "Optimization Methods and their Applications", Irkutsk: ISEM SB RAS, Vol. 5, (2005).
13. Epifanov S.P., Novitsky N.N., Borovin D.I. Pipeline Energy Systems: Methodological and Applied Problems of Mathematical Modeling, Novosibirsk: Science (2015).
14. Rainshke K. Models of reliability and sensitivity of systems (Moscow: "THE WORLD", 1979).
15. Magnus J.R., Neidekker H. Matrix differential calculus with applications to statistics and econometrics (Moscow, FIZMATLIT, 2002).