Statistical properties of bipolar magnetic regions

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Abstract Using observations from the Michelson Doppler Imager (MDI) onboard Solar and Heliospheric Observatory (SOHO), we develop a computational algorithm to automatically identify bipolar magnetic regions (BMRs) in active regions (ARs), and then study their statistical properties. The individual magnetic (positive or negative) pole of a BMR is determined from the region with an absolute strength above 55 G and with an area larger than 250 pixel² (~495 Mm²), while a BMR is identified as a pair of positive and negative poles with the shortest area-weight distance between them. Based on this method, 2234 BMRs are identified from MDI synoptic magnetograms between Carrington Rotations 1909 (1996 May 06) and 2104 (2010 December 10). 1005 of them are located in the northern hemisphere, while the other 1229 are in the southern hemisphere. We find that the BMR parameters (e.g., latitude, separation, fragment number and strength) are similar to those of ARs. Moreover, based on the maximum likelihood estimation (MLE) method, the frequency distributions representing the occurrence of these BMRs as functions of area and magnetic flux exhibit a power-law behavior, i.e., \( dN/dx \propto x^{-\alpha} \), with indices of \( \alpha_A = 1.98 \pm 0.06 \) and \( \alpha_F = 1.93 \pm 0.05 \) respectively. We also find that their orientation angles (\( \theta \)) follow “Hale’s Polarity Law” and deviate slightly toward the direction of the solar equator. Consistent with previous findings, we obtain the dependence of orientation angles on latitudes for normal BMRs during the 23rd solar cycle. The north-south asymmetry of these BMRs is also detected here.

Key words: methods: statistical — Sun: activity — Sun: magnetic fields

1 INTRODUCTION

Magnetic fields are believed to be the dominant reasons for evolution of solar activity, and stronger and larger magnetic fields on the solar surface are mainly in active regions (ARs). Moreover, most energetic and geoeffective events take place at the ARs, such as solar flares, coronal mass ejections (CMEs), solar energetic particle events and eruptive prominences. Therefore, quantitative study of magnetic fields in ARs is important to basic solar physics. Since as early as more than 350 years ago, ARs have been studied as sunspots in white light images (e.g., Wolf 1861; Maunder 1904; McKinnon & Waldmeier 1987; Hathaway et al. 1999; Li et al. 2001; Hathaway et al. 2003; Zhang et al. 2010; Jiang et al. 2011; Hathaway 2010). It is well known that the number of sunspots on the solar disk displays a periodic behavior, which has an average period of about 11 years. The positions of sunspots exhibit butterfly shapes, which is well-known as the “Butterfly Diagram.” This suggests that the behavior of sunspots follows the “Spörer’s Law of Zones.” Sunspots are regions with stronger magnetic fields on the Sun, and their magnetic nature follows the famous “Hale’s Polarity Law.” For both unipolar spots and preceding members of bipolar spots, magnetic polarity is negative before the last sunspot minimum and positive after the solar minimum in the northern hemisphere, but in the southern hemisphere, their magnetic polarity is positive before the last sunspot minimum and negative after the solar minimum (Hale et al. 1919). That is to say, the signs of a sunspot could be reversed at solar minimum, thus the period of the solar magnetic field is about 22 years, which is called the solar magnetic cycle.

Characteristics of magnetic fields at ARs have been reported by many authors (e.g., Howard 1989; Wang & Sheeley 1989; Harvey & Zwaan 1993; Zhang et al.
2010). Using the Mount Wilson daily magnetogram data set, Howard (1989) described various properties of magnetic fields at solar ARs. For example, the average separation of magnetic polarity was about 7 deg (~86 Mm), and the distribution of magnetic flux per AR showed a peak of about $2 \times 10^{21}$ Mx. This was similar to the value of $4 \times 10^{21}$ Mx obtained by Wang & Sheeley (1989), who used data from National Solar Observatory/Kitt Peak (NSO/KP) during 1976—1986. Later, Harvey & Zwaan (1993) studied the properties of ARs from NSO/KP full-disk magnetograms taken during 1975—1986 throughout Solar Cycle 21, and their conclusion was that the shape of the characteristic size distribution for ARs was a fundamental invariant property of solar magnetic activity. Recently, using high-resolution synoptic magnetograms constructed from Solar and Heliospheric Observatory (SOHO)/Michelson Doppler Imager (MDI) images during 1996—2008, Zhang et al. (2010) identified 1730 ARs and quantified their physical properties. The mean and maximum magnetic flux of individual ARs were $1.67 \times 10^{22}$ Mx and $1.97 \times 10^{22}$ Mx, while those of each Carrington Rotation (CR) were $1.83 \times 10^{23}$ Mx and $6.96 \times 10^{23}$ Mx, respectively.

A number of literatures (e.g., Bogdan et al. 1988; Abramenko & Longcope 2005; Canfield & Russell 2007; Zhang et al. 2010) have investigated the distribution of magnetic flux or area of ARs and sunspots. These papers reported that the distribution related to ARs was usually log-normal. For example, Zhang et al. (2010) analyzed the frequency distribution of ARs, and found that the distribution was a function of area and magnetic flux following a log-normal function. This was consistent with results obtained by Abramenko & Longcope (2005) and Canfield & Russell (2007). Bogdan et al. (1988) found the sunspot umbral area was distributed log-normally with Mount Wilson white light data in the interval between 1917 and 1982. However, Parnell et al. (2009) analyzed magnetograms from SOHO/MDI and Hinode/SOT, and they found that all feature fluxes followed a power law distribution with a slope of $1.85 \pm 0.14$. However, Tang et al. (1984) studied 15 years of AR data using Mount Wilson daily magnetograms from 1967 to 1981, and their results revealed that the number of ARs decreased exponentially with increasing AR size. This was also demonstrated by Zharkov et al. (2005), who found that the number of sunspots grew nearly exponentially with their area decreasing. Meanwhile, Harvey & Zwaan (1993) successfully used a polynomial function to fit NSO/KP data. These different fitting functions were caused by the different observational data, and they may be related to the physical mechanism of AR emergences. The resulting log-normal distribution has been regarded as a result of magnetic fragmentation in the solar envelope (Bogdan et al. 1988), while the log-normal distribution of AR flux may also suggest that the process of fragmentation dominates over the process of concentration in the formation of magnetic structure in ARs (Abramenko & Longcope 2005; Canfield & Russell 2007). On the other hand, the power-law distribution of the AR's flux possibly suggests a self-similar nature for all ARs (McAteer et al. 2005). If considering the magnetic flux between $2 \times 10^{17}$ Mx and $10^{23}$ Mx, then the power-law distribution also suggests that the mechanisms of surface magnetic features are scale-free (Parnell et al. 2009). Finally, Schrijver et al. (1997) proposed that the exponential distribution was led by frequent fragmentation and collision (or merging) of magnetic features.

The orientation angles of the magnetic fields at ARs are important for understanding solar physics. They not only represent an important quantity that is related to the large-scale properties of magnetic field distribution, but may be also related to the dynamo process which is believed to lead the solar activity cycle (Babcock 1961; Leighton 1964, 1969; Sheeley et al. 1985). Howard (1989) studied the orientation angles of magnetic regions at ARs with Mount Wilson magnetograms. He found that the distribution of AR orientation angles shows two broad maxima centered on the 'normal' orientation, and that reverse oriented ARs tend to be relatively evenly distributed in terms of orientation angle compared to normally oriented ones. However, most studies concentrate on tilt angles of magnetic regions, which are similar to the orientation angles but not the same. An orientation angle is defined as the angle from the positive/negative towards negative/positive fields (Howard 1989). The tilt angle is defined to be the angle between the bipolar axis line and the heliographic east-west line (Wang & Sheeley 1989). It is the angle measured in a positive sense for magnetic regions with leading fields equatorward of following fields and in a negative sense for magnetic regions with leading fields poleward of following fields (Howard 1991b; Li & Ulrich 2012). In other words, the tilt angle is very useful for studying Hale’s and Joy’s Laws (Li & Ulrich 2012), while the orientation angle only exhibits an incline of ARs on the solar disk. The first detailed study on tilt angles of magnetic regions was Hale et al. (1919), who found that the average tilt angles of sunspots increase with solar latitudes. This result was confirmed by Brunner (1930), who also found that larger, well-developed sunspots tend to have smaller tilt angles than smaller, less-developed ones. Then Wang & Sheeley (1989) used NSO magnetograms to conclude that the av-
average tilt angles of all bipolar magnetic regions (BMRs) relating to the east-west line showed a progressive increase toward high latitude, and the value was close to 9°. After that, Howard (1991b) studied the tilt angles of magnetic regions in ARs with Mount Wilson magnetograms. He found that variation of tilt angles with solar latitudes was not dependent on solar cycle phases. These ARs with larger absolute tilt angles have rapid separation of magnetic poles on average, and their sizes are smaller than those with smaller absolute tilt angles. Later, Howard (1991a) studied the tilt angles of sunspots with Mount Wilson daily white light photographs, and obtained similar results to the earlier studies about ARs (e.g., Wang & Sheeley 1989; Howard 1991b), and concluded that the average tilt angle of the sunspots was 4.2 ± 0.2°.

In this paper, we automatically identify BMRs in ARs with high-resolution CR synoptic magnetogram charts constructed from SOHO/MDI observations between 1996 and 2010, and then study their orientation angles. This paper is organized as follows: the observation and data reduction are introduced in Section 2, and the observation results are given in Section 3, then our conclusions and discussions are given in Section 4.

2 OBSERVATIONS AND DATA REDUCTION

2.1 Observations

The data used here are from SOHO/MDI magnetograms (Scherrer et al. 1995). MDI is designed to measure the velocity, intensity and magnetic fields in the photosphere, and further to study the magnetic fields in the corona. It can record a full solar disk magnetogram with a spatial resolution of ~2″ pixel⁻¹ every 96 minutes (Domingo et al. 1995). However, full disk MDI magnetograms are not used directly in this paper, but CR synoptic charts are used in the analysis, and they can be downloaded from the MDI homepages. The CR synoptic charts from MDI have two forms: magnetic field and intensity synoptic charts. Only the magnetic field charts are used in this paper. They are generated from MDI magnetograms at level 1.8. They are well re-calibrated, and several observations of every location which have been collected over the course of a solar rotation (~27 days) are averaged to make up the charts. Therefore, the strength of the magnetic field at each synoptic grid point is averaged, and they have previously been corrected for differential rotation. Through the averaging process, the effects of cosmic rays have also been reduced. The projection effect of the magnetic field has been corrected by assuming that MDI makes line-of-sight (LOS) measurements of a radial magnetic field. Finally, the correction to the pixel area has also been performed in this paper with a scale factor (see Berger & Lites 2003; Tran et al. 2005; Ulrich et al. 2009).

The final synoptic charts of the magnetic fields can be used to produce two versions of the maps: a radial and an LOS version. The synoptic charts related to the projection effect along longitude are much better than snapshot magnetograms, but they lose temporal resolution. The noise level of the synoptic charts is about 5 G, and the resolution of the synoptic charts has been changed to a 3600 × 1080 pixel synoptic map. The Carrington longitude is linear, while the Carrington latitude is sinusoidal. Noting that data at each longitude in these synoptic charts are observed at different times, the longitude in the synoptic charts also represents time information.

All the synoptic maps, whether radial or LOS, are constructed from the data, which were observed by implementing disk meridians. These maps using data that were observed near the central meridian (0°) are called ordinary synoptic charts, and additional charts incorporate data from other disk meridians, such as 60°E, 45°E, 30°E, 15°E, 15°W, 30°W, 45°W and 60°W. However, those additional charts have the disadvantage of disk longitude being offset, but the ordinarily synoptic charts do not, so only ordinarily synoptic charts are used in this paper. Therefore, only the LOS magnetic field observed near the central meridian for CR synoptic charts are used to identify BMRs. Figure 1(a) shows an example of the synoptic images which we use in this paper.

2.2 Data Reduction

Two magnetic poles of the BMRs in the ARs are defined by the threshold method, which include the strength and area thresholds. As illustrated in Figure 1, there are essentially two steps, each of which corresponds to one panel in this figure. The first step is to define the magnetic poles of the BMRs. The positive and negative magnetic fields are separated from the observational data, and the two magnetic poles are identified respectively. Then we determine the strength threshold of the positive or negative magnetic field for every magnetogram with Equation (1). However, the magnetograms used in this paper are from May 1996 to December 2010, which include the whole 23rd solar cycle and the beginning phase of the 24th solar cycle. The strength thresholds with Equation (1) for different phases of the magnetograms exhibit large differences. To rule out these differences, we take the average value (TH) of all thresholds, and this value is 55 G for the positive field and ~55 G for the negative field. This value is similar to that used by
Fig. 1 (a) The LOS magnetic field synoptic charts for CR 1960 near the central meridian, which started on 2000 February 25 and ended on 2000 March 23. (b) The magnetogram after subtracting the strength threshold from panel (a); (c) The magnetogram after subtracting the area threshold from panel (b), the blue and yellow circles are positive and negative poles, while the ‘+’ and ‘×’ represent geometric centers and flux-weighted centers, respectively; the identified BMRs are connected with red lines, and filled circles are the positions of the BMRs. The green line in each panel represents the solar equator.

Zhang et al. (2010), who identified ARs with a minimum magnetic field of 50 G using SOHO/MDI synoptic magnetograms. Figure 1(b) gives the results after subtracting $\overline{T}\overline{H}$ from the original observational data.

$$TH = \mu \pm \gamma \cdot \sigma.$$  

(1)

In Equation (1), $\mu$ is the mean value and $\sigma$ represents standard deviation, while $\gamma$ is a constant which is determined empirically based on the type of feature to be detected. In this paper, we set $\gamma = 2$, which is the same as what Colak & Qahwaji (2008) relied on to determine the magnetic polarities of ARs. ‘+’ is for a positive field and ‘−’ is for a negative field.

Next, we will identify the two poles of BMRs by an area threshold. The magnetic images have been separated into several individual unconnected regions after subtracting the strength threshold, and these regions can be marked automatically by the code LABEL_REGION.pro in Interactive Data Language (IDL). However, these isolated regions are not considered to be magnetic poles, because these isolated regions which are close to each other may be the same magnetic pole. Therefore, we need to determine the area of the magnetic poles. That is to say, it is possible that these closer regions belong to the same magnetic pole. So, the positions (geometric centers) and equivalent diameters (considering an isolated region as a circle) of these isolated regions have been calculated; and for every isolated region, we regard the geometric center as the reference point and two times the equivalent diameters as the edge length to draw a square box. If these square boxes have overlapping regions, they are regarded as one magnetic pole, otherwise they are considered to be different magnetic poles. Thus, these closer isolated regions may be one magnetic pole, but not all of the magnetic poles could be real poles of the BMRs in this paper. These poles which have a small area are possibly not real
poles of the BMRs in the ARs, because we mainly study larger and stronger BMRs here. Therefore, small magnetic poles need to be ruled out. The minimum area used here is similar to previous studies. For example, Harvey & Zwaan (1993) studied the area of BMRs larger than 2.5 square degrees (∼373 Mm$^2$); Tang et al. (1984) detected ARs as small as 450 Mm$^2$; the smallest area studied by Schrijver (1988) was 310 Mm$^2$; Wang & Sheeley (1989) even studied ARs as small as about 200 Mm$^2$. Based on these values, we define the area (A) threshold as 250 pixel$^2$ (∼495 Mm$^2$) in this paper. In other words, regions with a total area less than 250 pixel$^2$ are excluded as magnetic poles. Figure 1(c) shows positive (blue circles) and negative (yellow circles) poles in CR 1960. The plus (‘+’) and cross (‘×’) represent the geometric center and the flux-weighted center, respectively. We consider the flux-weighted centers as positions of the magnetic poles. For each magnetic pole (positive or negative), we also calculate their radius (R), fragment number (N), magnetic field (B), magnetic flux (F) and magnetic flux density (f). Here, the magnetic pole is supposed to be a circle. The fragment number is the amount of isolated regions in the magnetic pole, and the magnetic field is the maximum value of the magnetic pole. The magnetic flux is the total value of the magnetic pole, while the magnetic flux density is the mean value of the magnetic field.

The second step is to locate BMRs from the identified positive and negative poles. In this paper, we assume that BMRs are formed by the closest magnetic poles with opposite polarity, but the BMR direction in the northern hemisphere is opposite to that in the southern hemisphere. In the northern hemisphere, a BMR is defined as a negative pole linked to the closest positive pole; while in the southern hemisphere, a positive pole is linked to the closest negative one, such as what is shown by the red lines linking two magnetic poles in Figure 1(c). The distance (D) between the positive and negative poles of each BMR is measured, and the ratio of D/R is shown in Figure 2. There is a normal distribution of D/R, and the maximum value is around 3. These BMRs with a big value of D/R, such as greater than 10, indicating a large distance between positive and negative poles with a small area, are possibly not real. Therefore, these BMRs with D/R ≥ 10 (the vertical line in Fig. 2) are ruled out in the analysis used in this paper, i.e., the one marked with the rectangle. In particular, a positive pole linked to a negative one with the shortest area-weight distance is identified as a BMR in the southern hemisphere. The same rule is applied in the northern hemisphere. The red lines in Figure 1(c) trace the identified BMRs. The filled circles represent the positions of the BMRs, which are the middle positions of positive and negative poles associated with the BMRs.

3 RESULTS

Using the method mentioned in Section 2.2, we analyze SOHO/MDI data from CR 1909 (May 1996) to 2104 (December 2010). This period covers the whole 23rd solar cycle and the beginning phase of the 24th solar cycle. There are four MDI images (CR1938, CR1939, CR1940 and CR1941) missing due to SOHO malfunctioning in 1998. Some MDI images (e.g., CR1937, CR1944, CR1945, CR1956, CR2011, CR2015 and so on) are partial, but they are enough for our analysis. Finally, 2948 positive poles and 2940 negative poles are recognized.
from these MDI images, and 2234 BMRs are identified; 1005 of them are located in the northern hemisphere, while the other 1229 are located in the southern hemisphere. However, there were 3171 NOAA ARs published during the same period. Such a difference is mainly because only LOS magnetic fields observed near the central meridian are used. The criterion to identify BMRs and NOAA ARs is also different.

3.1 The Positions of BMRs

In our study, each BMR has its own latitude and time. On the MDI synoptic image, the midpoint of the positive and negative poles is marked as the BMR position, which is a function of latitude and time.

Figure 3(a) shows the distribution of BMR latitudes. Most (95.7%, 2138/2234) of the BMRs are located at low latitudes on the solar disk, i.e., between $-30^\circ$ and $30^\circ$; only 4.3% (96/2234) of BMRs exceed this range, but are not higher than $\pm 50^\circ$. Here, ‘+’ and ‘−’ represent the latitudes of the BMRs which are located in the northern and southern hemispheres, respectively. This is consistent with ARs and sunspots, which are also typically located at low latitudes on the solar surface. The mean values of the BMR latitudes are $16^\circ$ and $-16^\circ$ in the northern and southern hemispheres, respectively. Panel (b) further displays BMR latitudes in the solar disk varying with solar cycle. It appears similar to the butterfly diagram described by Maunder (1904, 1922), which was explained as the emergence positions of ARs (or sunspots) progressively drifting toward the solar equator (Hathaway et al. 2003). This further confirms that BMRs in this paper are essentially bipolar fields.

3.2 Parameters associated with the BMRs

For these identified BMRs, we statistically study their separations ($D_S$), fragment number ($N$), area ($A$), magnetic field ($B$), magnetic flux ($F$) and magnetic flux density ($f$). The separations are the distances between pos-
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Fig. 4 The solid profiles are the distributions of the BMR (a) separation distances, (b) fragment number, (c) magnetic field and (d) magnetic flux density. The dashed and dotted profiles represent the BMR parameter distributions in the northern and southern hemispheres, respectively. The other two panels are (e) the distribution of BMR flux and (f) area in log-log space.

itive and negative poles of the BMRs, and the fragment number, area and magnetic flux are the sum of absolute values for positive and negative poles of the BMRs, while the magnetic field and magnetic flux density are half of the total absolute values for positive and negative poles of the BMRs. The statistical results are shown in Figures 4 and 5. Tables 1 and 2 also list these parameters in other references that can be compared with previous studies. Here AR1 and AR2 are the ARs which are identified from the different definitions, i.e., AR1 is the AR identified by Zhang et al. (2010) based on their automated method, while AR2 is the AR defined from NOAA.

Figure 4 shows the distribution of each BMR parameter, and their typical values are listed in Table 1. The separation distances ($D_s$) range from 3.2 Mm to 448 Mm, which is shown in panel (a). The largest separation in this paper is greater than 26 deg ($\sim$320 Mm), which is obtained by Howard (1989). However, the number of such large separations is very few, for example, only one separation of a BMR exceeds 400 Mm, while only 0.9% (20/2234) of the separations of the BMRs exceed 300 Mm. The mean separation distance of these BMRs is 112 Mm, which is similar to 86 Mm (Howard 1989). The fragment number ($N$) of these BMRs is from 2 to 168, and the average number of them is about 26, which is shown in panel (b). From panels (a) and (b), we can see that the distribution is similar to that of the separation distances, and the larger number is much less, with only 1.9% (42/2234) being greater than 100. The same results are applied to the distribution of BMR flux density, as shown in panel (d). The minimum flux density is 58 Mx cm$^{-2}$, the maximum flux density is 339 Mx cm$^{-2}$ and the average flux density is 117 Mx cm$^{-2}$, while only 0.45% (10/2234) of them exceed 250 Mx cm$^{-2}$. However, the distribution of the BMR magnetic field is different. As shown in panel (c), there are two peaks in the distribution of magnetic field. One is about 600 G, the
Fig. 5 The BMR parameters per CR with time from 1996 to 2010. (a) The BMR number (black); (b) the BMR total fragment number (black), positive fragment number (blue) and negative fragment number (red); (c) the BMR total magnetic field (black), positive magnetic field (blue) and negative magnetic field (red); (d) the BMR total area (black), positive area (blue) and negative area (red); (e) the BMR total flux (black), positive flux (blue) and negative flux (red). The dotted profile shows the parameters in the northern (blue dashed) and southern (red dashed) hemispheres.

Table 1 Statistical Results on BMRs and ARs

| Parameter          | Mean  | Median | Min  | Max  | BMR  | AR1  | AR2  | BMR  | AR1  | AR2  | BMR  | AR1  | AR2  |
|--------------------|-------|--------|------|------|------|------|------|------|------|------|------|------|------|
| $D_S$ (Mm)         | 112   | –      | 100  | 3.2  | 448  | –    | –    | –    | –    | –    | –    | –    | –    |
| $N$                | 26    | 7.8    | 8    | 5    | 2    | 1    | 0    | 168  | 69   | 90   | –    | –    | –    |
| $A$ ($10^{19}$ cm$^2$) | 8.8   | 6.1    | 0.3  | 5.5  | 0.06 | 1.0  | 0.3  | 79.7 | 68   | 6.9  | –    | –    | –    |
| $B$ (G)            | 1194  | –      | 1107 | 252  | –    | 2972 | –    | –    | –    | –    | –    | –    | –    |
| $f$ (Mx cm$^2$)    | 117   | –      | 109  | 58   | –    | –    | 339  | –    | –    | –    | –    | –    | –    |
| $F$ ($10^{21}$ Mx) | 22.4  | 16.7   | 12.3 | 1.7  | 0.9  | –    | 179  | –    | –    | –    | –    | –    | –    |

Notes: $^1$ AR1: parameters associated with ARs cited from Zhang et al. (2010). $^2$ AR2: parameters associated with ARs from NOAA.

other is around 1500 G, and the magnetic strength ranges from 252 G to 2972 G.

For these 2234 BMRs, their area and magnetic flux are also measured. The area of these BMRs ranges from $1.0 \times 10^{19}$ cm$^2$ ($\sim$1000 Mm$^2$) to $7.97 \times 10^{20}$ cm$^2$ ($\sim$7.97 $\times$ 10$^4$ Mm$^2$). The smallest area of BMRs in this paper is larger than the previous results of $300 - 400$ Mm$^2$ for the ARs (Tang et al. 1984; Schrijver 1988; Harvey & Zwaan 1993; Zhang et al. 2010), and the largest area is also greater than the earlier results of about $(1 - 7) \times 10^4$ Mm$^2$ for the ARs (Tang et al. 1984; Schrijver 1988; Wang & Sheeley 1989; Harvey & Zwaan 1993;
The flux of these BMRs is from $1.7 \times 10^{21} \text{ Mx}$ to $2.43 \times 10^{23} \text{ Mx}$, which is similar to the AR flux from $8.6 \times 10^{20} \text{ Mx}$ to $1.97 \times 10^{23} \text{ Mx}$ obtained by Zhang et al. (2010), but the maximum value is larger than earlier results of about $10^{22} \text{ Mx}$ for ARs (Howard 1989; Wang & Sheeley 1989). Then the frequency distributions for the occurrence of these BMRs as functions of flux and area are shown in the two bottom panels of Figure 4. It is hard to generalize the behaviors of the frequency distributions for the whole area and flux. However, using the maximum likelihood estimation (MLE) method developed by Clauset et al. (2009), both the BMR area and flux exhibit a power-law behavior, i.e., \(dN/dx \propto x^{-\alpha_x}\). The MLE method is based on the Kolmogorov-Smirnov statistic to determine the lower cutoff \((x_{\text{min}})\) of the power-law behavior, which is marked by dashed lines in panels (e) and (f). Using the MLE method, we obtain the power-law index of \(\alpha_x = 1.93 \pm 0.05\) for the BMR flux and \(\alpha_A = 1.98 \pm 0.06\) for the BMR area. This is consistent with the power-law distribution of large solar activities, such as radio bursts, soft X-rays, hard X-rays, interplanetary type III bursts, interplanetary particle events and CMEs (Crosby et al. 1993; Aschwanden et al. 1998). Dennis (1985) and Crosby et al. (1998) further summarize \(\alpha_x\) for the distributions of different flare-related parameters and state that it varies from 1.4 to 2.4. This was also demonstrated by many authors for solar flares (Wheatland 2000; Su et al. 2006; Li et al. 2012), CMEs (Wheatland 2003), radio bursts (Ning et al. 2007; Song et al. 2012) and other small-scale magnetic fields (Parnell et al. 2009; Li et al. 2013). In our results, \(\alpha_A = 1.98 \pm 0.06\) for the area of the BMRs can be applied using fractal models (Aschwanden & Parnell 2002). In addition, \(\alpha_F = 1.93 \pm 0.05\) for the BMR flux is consistent with the index of coronal activities (i.e., solar flares, CMEs, radio bursts) and bright points (Li et al. 2013). Our findings indicate that there are not fundamental differences during their generation in the solar atmosphere, regardless of scale (small or large) or height (low or high) above the solar surface. Observations also show that both coronal activities and bright points are strongly related to the magnetic fields, highlighting common features involved in generating magnetic structures at small or large scales.

Figure 5 displays the variation of BMR parameters per CR with period from 1996 to 2010. The typical values of these BMR parameters per CR are listed in Table 2. Here \(N_0\) is the number of BMRs or ARs per CR, and the other symbols are the same as in Table 1. As shown in Figure 5, there are double peaks during solar maximum. If we carefully examine panels (c), (d) and (e), we can find that the second peak flux (in late 2001) of BMRs is mainly caused by the large area of the emerged BMRs, but not the mean strength of the magnetic field. Based on this, the 23rd solar cycle peaked in late 2001 but not in early 2000. These results are consistent with previous findings in Zhang et al. (2010). From Table 2 we can see that the BMR parameters are consistent with AR parameters except for the fragment number.

In Figure 5, we also plot the BMR parameters varying with solar cycle in the northern (blue dashed line) and southern (red dashed line) hemispheres, which clearly show the north-south asymmetry of the BMR distribution. This north-south asymmetry is also shown in panels (a) – (d) of Figure 4. Similar north-south asymmetry of ARs has been reported earlier (Temmer et al. 2002; Zharkov & Zharkova 2006; Zhang et al. 2010). However, this asymmetry is not yet well understood. We also plot the parameters of positive (blue solid line) and negative (red solid line) poles that vary with solar cycle, and the positive and negative parameters of the BMRs are almost the same during the solar cycles, although there may be some difference in one or two CR periods. In Table 2, the parameters of the BMRs and ARs are zero at solar minimum, both in our data and in other data. This is because the chance of ARs appearing is very small or even non-existent in one CR during solar minimum.

### 3.3 The Orientation of BMRs

The orientation angles of ARs are important for understanding solar cycles and have been studied in the past (e.g., Wang & Sheeley 1989; Howard 1989, 1991b). However, most of the data they used are full-disk magnetograms which are observed from the ground. In this paper, we use MDI CR charts to study the orientation angles.

Figure 6 gives the definition of the orientation angle \(\theta\) of BMRs in the northern and southern hemispheres. In the northern hemisphere, the orientation angle \(\theta\) is defined to be the angle of a vector originating in the flux-weighted center of a positive pole with clockwise rotation that terminates in the flux-weighted center of a negative pole; while it is defined to be the angle of a vector originating in the flux-weighted center of the negative pole with counterclockwise rotation that terminates in the flux-weighted center of a positive pole in the southern hemisphere. Note that we separately define the BMR orientation angle in the northern and southern hemispheres. BMR orientation angles in both the northern and southern hemispheres range from \(-90^\circ\) to \(270^\circ\). In this paper, normally oriented BMRs refer to those BMRs whose ori-
Table 2  Statistical Results on BMRs vs. ARs Per CR

| Parameter                  | Mean | Median | Min  | Max  |
|----------------------------|------|--------|------|------|
|                            | BMR  | AR1    | AR2  | BMR  | AR1    | AR2  | BMR  | AR1    | AR2  | BMR  | AR1    | AR2  |
| \( N_0 \)                  | 11   | 11     | 13.5 | 10   | 11     | 8    | 11   | 0      | 0    | 35   | 37     | 39   |
| \( N \)                    | 312  | 85     | 126  | 210  | 55     | 101  | 0    | 0      | 0    | 1265 | 327    | 413  |
| \( A \) (10^{20} \text{ cm}^2) | 10.2 | 6.7    | 0.5  | 5.4  | 4.3    | 0.3  | 0    | 0      | 0    | 44.0 | 26.7   | 1.8  |
| \( B \) (10^3 \text{ G})  | 1.39 | –      | –    | 1.07 | –      | –    | 0    | 0      | 0    | 4.19 | –      | –    |
| \( f \) (10^4 \text{ Mx cm}^2) | 1.37 | –      | –    | 1.13 | –      | –    | 0    | 0      | 0    | 4.19 | –      | –    |
| \( F \) (10^{23} \text{ Mx}) | 2.6  | 1.8    | –    | 1.4  | 1.3    | –    | 0    | 0      | –    | 10.5 | 6.96   | –    |

Notes: 1 AR1: the parameters of ARs are cited from Zhang et al. (2010). 2 AR2: the parameters of ARs are from NOAA.

Fig. 6  Schematic representation of the orientation angle (\( \theta \)) of BMRs. In the northern hemisphere, the positive pole (‘+’) is at the center of the angle, and the negative pole (‘−’) undergoes clockwise rotation; while in the southern hemisphere, the negative pole is at the center of the angle, and the positive pole undergoes counterclockwise rotation. The angle ranges from –90° to 270°.

Orientation angles range from –90° to 90°, while abnormal orientation angles are between 90° and 270° in the solar disk, no matter if they are in the northern hemisphere or southern hemisphere.

Figure 7 gives the orientation angle (\( \theta \)) distribution of these BMRs with a bin of 10° in the northern (solid lines) and southern (dashed lines) hemispheres. Panel (a) shows all the BMRs are from CR 1909 to 2104, including the whole 23rd solar cycle and the beginning phase of the 24th solar cycle. The orientation angles range from –90° to 270° and most of them are concentrated around 0° or 180°. In this paper, the median values of the whole data are 13° and 11° in the northern and southern hemispheres, respectively, indicating that in both the northern and southern hemispheres, the orientation angles of the BMRs slightly deviate toward the direction of the solar equator. In panel (a), we can see that there are two peaks in both the northern and southern hemispheres. There are many more BMRs around 0° than around 180°. We then separated the BMRs into the entire 23rd solar cycle (from CRs 1909 to 2070) and the beginning phase of the 24th solar cycle (from CRs 2071 to 2104), as shown in panels (b) and (c). From this, there is only one peak in both the northern or southern hemispheres during the entire
23rd solar cycle, and the median values of the BMR orientation angles are 11° and 10° in the northern and southern hemispheres respectively, which is similar to that of all the data. At the beginning of the 24th solar cycle, there is also one peak in both the northern and southern hemispheres, but the median values of the BMR orientation angles are 187° and 181° in the northern and southern hemispheres, respectively. These results suggest that the two peaks of the orientation angles in panel (a) are from the 23rd and 24th solar cycles, respectively.

Our results show that the orientation angles of the BMRs generally slightly deviate toward the direction of the solar equator (panel (a) in Fig. 7), that the directions of the BMRs are opposite in the northern and southern hemispheres (panel (a) in Fig. 7), and that the magnetic polarity of BMRs reverses during the minimum of the solar cycle (panel (b) and (c) in Fig. 7). These results are also shown in Figure 8, which gives the variation of BMR orientation angles with respect to the phase of solar cycles from May 1996 to December 2010. The squares and triangles represent the BMRs located in the northern and southern hemispheres, respectively. In this figure, the BMR orientation angles vary with solar cycles, and the polarities of BMRs are changing in different solar cycles. We note that the orientation angles reverse around June 2008. That is to say, the 23rd solar cycle is from 1996 to 2008, and the duration is about 12 years. This perhaps can be explained by the extended activity cycle, because the sunspot region of a new solar cycle could begin to appear as much as ~1.6 years before the defined solar minimum and continue to emerge up to ~1.8 years after the following minimum. These results are consistent with previous findings about ARs (Howard 1989; Wang & Sheeley 1989; Howard 1991b) and sunspots (Maunder 1904; Hale et al. 1919; Howard 1991a), indicating that all large scale phenomena related to bipolar magnetic fields follow “Hale’s Polarity Law.” That is to say, during the first solar cycle, the positive poles of BMRs are the leading polarity in the northern hemisphere and the negative poles of BMRs are the leading polarity in the southern hemisphere; while in the next solar cycle, the BMR polarity is reversed; the leading polarities are negative poles in the northern hemisphere and positive poles in the southern hemisphere.

Figure 9 displays the dependence of BMR orientation angle on their (a) latitude, (b) magnetic field, (c) separation, (d) area, (e) fragment number, (f) total flux (the sum of values between the absolute positive and negative flux), (g) flux density and (h) net flux (the absolute positive values minus the negative flux). It is hard to see any relationship between the orientation angles and other parameters in the BMRs, especially between the orientation angles and their latitudes. Here all of the 2234 BMRs are analyzed, including the normal and abnormal BMRs from May 1996 to December 2010. Then only the BMRs (2127) that occurred during the 23rd solar cycle are selected for analysis, regardless of whether they are from the northern or southern hemispheres.

Figure 10 gives the average parameters for various values of orientation angles for these 2127 BMRs. The BMRs with larger absolute orientation angles tend to show greater deviation in average values of parameters (e.g. (a) separation, (b) area and (c) flux) than those with smaller absolute orientation angles. Especially for these normally oriented BMRs whose orientation angles are between ~50° and 50° (dotted lines), their deviation and fluctuation are much smaller than other BMRs. However, the net flux of these normally oriented BMRs whose orientation angles are between ~50° and 50° (dotted lines) are always smooth and fluctuations are very small (panel d). But for BMRs with other orientations (the absolute orientation angles exceed 50°), their fluctuation and deviation are much larger. These results are similar to what was obtained from ARs by Howard (1989, 1991b), who found that ARs with smaller absolute orientation angles tend to show smaller deviation. Finally, the magnetic flux of BMRs in this paper is larger by about one magnitude than others, which is possibly due to the resolution and sensitivity of the different instruments. Howard (1989) stated that the inherent disadvantage of their data is poor resolution, and this is the advantage of our data as the MDI magnetic charts have high resolution and high sensitivity.

Based on the above analysis, only 1543 normal BMRs which have orientation angles ranging from ~50° to 50° in the 23rd solar cycle are selected for study, regardless of if they are in the northern or southern hemisphere.

Figure 11 shows the dependence of orientation angles on the absolute latitudes for these 1543 normally oriented BMRs during the 23rd solar cycle; the average orientation angles are taken over 2.5° latitudes, and the error bars represent the standard deviations associated with these mean values. For these normally oriented BMRs, the orientation angles increase with their latitude on the whole. This is consistent with the result that tilt angles of ARs and sunspots general increase with latitude (e.g., Hale et al. 1919; Wang & Sheeley 1989; Howard 1989, 1991b,a). At latitudes equatorward of about 5 degrees, the orientation angles show negative values on average but this is not very significant, which is also similar to results on the tilt angles of ARs (Howard 1991b).
Fig. 8 Time evolution of the BMR orientation angles ($\theta$) from May 1996 to December 2010 in the northern (squares) and southern (triangles) hemispheres.

Fig. 9 Dependence of BMR orientation angles ($\theta$) on their (a) latitude, (b) magnetic field, (c) separation, (d) area, (e) fragment number, (f) total flux, (g) flux density and (h) net flux.
Fig. 10 The average BMR parameters (a) separation, (b) area, (c) total magnetic flux and (d) net magnetic flux for $2^\circ$ intervals of BMR orientation angles. Error bars represent standard deviations associated with these mean values, and dashed lines correspond to their mean values with $0^\circ$, while dotted lines correspond to their mean values with $-50^\circ$ or $50^\circ$.

Fig. 11 Average BMR orientation angles for various intervals of absolute solar latitudes in degrees. The average values are taken over $2.5^\circ$ latitudes, and the error bars represent the standard deviations associated with these mean values.
4 CONCLUSIONS AND DISCUSSIONS

Using SOHO/MDI LOS magnetic data from CRs 1909 to 2104, we statistically study the observational features of BMRs. To obtain the BMRs, we firstly identify positive and negative poles with the strength and area thresholds, respectively; then apply the criteria of closest area-weight distance between positive and negative poles to determine the BMRs. Finally, 2234 BMRs are obtained from observational data; 1005 of them are located in the northern hemisphere and the other 1229 BMRs are located in the southern hemisphere.

For these BMRs, the average latitudes are 16° and −16° in the northern and southern hemispheres, respectively. Most (95.7%) of these BMRs are located between −30° and 30°, while only a few (4.3%) BMRs are located in high latitude but do not exceed 50°. The time variation of these BMR latitudes with solar cycles is similar to the classical butterfly diagram of sunspots (Maunder 1904, 1922). These BMRs also follow “Spörer’s Law of Zones.” The BMR separation, area and flux are calculated (see Table 1 and Fig. 4), which are similar to the results obtained by Zhang et al. (2010). However, the fragment number is much larger than others, and this is because we do not limit the area of the fragments. The variation of BMR parameters per CR with solar cycles is also studied in this paper, and we further confirm that the 23rd solar cycle peaked in late 2001 but not in early 2000, which is consistent with results obtained by Zhang et al. (2010). We also find north-south asymmetry of BMRs, which is similar to previous findings about the north-south asymmetry of ARs (e.g., Temmer et al. 2002; Zharkov & Zharkova 2006; Zhang et al. 2010; Shetye et al. 2015).

We find that the frequency distributions of these 2234 BMRs as a function of magnetic flux and area exhibit a power-law behavior, and the power-law indexes are $\alpha_F = 1.93 \pm 0.05$ for the magnetic flux and $\alpha_A = 1.98 \pm 0.06$ for the area. These are consistent with previous findings on large-scale coronal activities (e.g., solar flares, CMEs, radio bursts) and small-scale magnetic elements (Li et al. 2013). This suggests that the area of the BMRs can be represented by fractal models (Aschwanden & Parnell 2002) and the mechanisms of surface magnetic features on the Sun are free of scales, indicating that all the surface magnetic features, regardless of their scale, are generated by similar mechanisms, or indicating that surface processes (i.e., fragmentation, coalescence, cancelation and so on) can lead to a distribution that is scale-free. This is also confirmed by Parnell et al. (2009).

Using the definition introduced in Figure 6, we study the orientation angles ($\theta$) of BMRs. We find that most BMRs display a slight deviation toward the solar equator direction on the solar disk, both in the northern and southern hemispheres; their median values are 13° and 11°, respectively. The orientation angles of these BMRs follow “Hale’s Polarity Law,” and the polarity of the BMRs is reversed in different solar cycles. We do not find any clear one to one correlation between the orientation angles and other parameters for all the BMRs. However, if only considering the average values of orientation angles between −50° and 50° for normally oriented BMRs during the 23rd solar cycle, the dependence of orientation angles on latitude is found as before, which is that the orientation angles increase with latitude (see. Fig. 11). But for all the BMRs, those with smaller absolute orientation angles tend to show smaller deviation (e.g., Hale et al. 1919; Wang & Sheeley 1989; Howard 1989, 1991b,a).

In this paper, the BMRs in ARs are defined by the closest magnetic poles with opposite polarities, which could result in some wrong or missing BMRs. Therefore, estimation of errors will be given by comparing NOAA ARs with our BMRs. Here, the NOAA catalog is assumed to provide the ‘ground truth’.

Figure 12 gives the MDI synoptic charts for CR 1960. In this rotation, 24 BMRs are identified by our method, which are marked with red pluses (‘+’). For the same time intervals and meridian regions, 30 NOAA ARs are published, as indicted by turquoise crosses (‘x’). For these identified BMRs, 70% (21/30) of them are found to have a one to one correspondence with NOAA ARs, as shown with overlapping symbols. For these BMRs without corresponding NOAA ARs, only one (‘I’) might not be a real BMR because it has the same positive pole as another BMR. The other two (‘II’ and ‘III’) could be real BMRs as they have larger and stronger magnetic fields with opposite polarities. Therefore, the true BMRs that are identified from the NOAA ARs could be more than 70%. On the other hand, there are nine NOAA ARs that were missed by our method. Most of these missing NOAA ARs are possibly not real BMRs according to our method, because some of these missing NOAA ARs only display one stronger pole, such as ARs 5’, 6’ and 9’. Some others have two opposite but very disperse magnetic poles, i.e., ARs 1’, 2’, 7’, and 8’. In summary, our method is useful for these BMRs that are connected to opposite magnetic poles with stronger and more compact magnetic fields.

Based on the computational algorithm, we have automatically identified 2234 BMRs in ARs from MDI synoptic magnetograms taken during 1996 to 2010.
Moreover, there are a total of 3171 NOAA ARs published during the same interval. We could locate ~70.5% (2234/3171) of all NOAA ARs. Considering that only these LOS magnetic fields observed near the central meridian are used in this paper, some NOAA ARs may not appear in the central meridian regions. That is to say, not all the 3171 NOAA ARs could be detected by our observations. Therefore, we could find more than ~70.5% of the NOAA ARs that appeared. On the other hand, Figure 7(b) shows that the peak distribution of 180 degrees still exists but is less obvious than that in panel (a). This peak is possibly due to the wrongly identified BMRs from our computational algorithm.

Figure 8 suggests that these abnormal orientations appear mainly around solar maximum. Based on these facts, we could locate the wrongly identified BMRs around solar maximum. Finally, 52 (or 77) BMRs are wrongly identified in the northern (or southern) hemisphere, and so the accuracy rate is about 94.3%.

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