Heavy flavor decays and OPE in two-dimensional
ˈt Hooft model[*]

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Abstract

The ’t Hooft model (two-dimensional QCD in the limit of large number of colors) is used as a testground for calculations of nonleptonic and semileptonic inclusive widths of heavy flavors based on the operator product expansion (OPE). The OPE-based predictions up to terms $O(1/m_Q^4)$, inclusively, are confronted with the “phenomenological” results, obtained by summation of all open exclusive decay channels, one by one, a perfect match is found.

The issue of duality violations is discussed, the amplitude of oscillating terms is estimated. The method is applied to the realistic case of hadronic $\tau$ decays.

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1 Overview

In this talk I will present recent results obtained in collaboration with Ikaros Bigi, Mikhail Shifman and Nikolai Uraltsev [1]. They refer to the applications of Wilson’s operator product expansion (OPE) [2] to inclusive decays of heavy flavor hadrons. The theory of such decays is at a rather advanced stage now (see [3] and references therein).

The decays of heavy flavor hadrons $H_Q$ are shaped by nonperturbative dynamics. While QCD at large distances is not yet solved, considerable progress has been achieved in this problem. The inclusive widths are expressed through OPE. The nonperturbative effects are then parameterized through expectation values of various local operators $O_i$ built from the quark and/or gluon fields. Observable quantities, such as total semileptonic and nonleptonic widths of heavy hadrons $H_Q$, are then given by

$$\Gamma_{H_Q} = \frac{1}{M_{H_Q}} \sum_i \text{Im} c_i(\mu) \langle H_Q|O_i(\mu)|H_Q\rangle$$

where $c_i$ are the OPE coefficients, and $\mu$ stands for a normalization point separating out soft contributions (which are lumped into the matrix elements $\langle H_Q|O_i(\mu)|H_Q\rangle$) from the hard ones (which belong to the coefficient functions $c_i$).

There are a number of issues, both conceptual and technical, associated with the operator product expansion in QCD: nonperturbative contributions to the OPE coefficients, dependence on normalization point, violations of local duality. These questions are circumvented in the so-called practical version of OPE [4] routinely used so far in all instances when there is need in numerical predictions. This version is admittedly approximate, however. The questions formulated above are legitimate, and they deserve to attract theorists’ attention, as they continue to cause confusion in the literature.

We find it useful and instructive to study these subtle issues in framework of the ’t Hooft model [5] (see also Refs. [6]). The model is 1+1 dimensional QCD in the limit $N_c \to \infty$. While retaining basic features of QCD – most notably quark confinement – this mode is simpler without being trivial and can be solved dynamically. The theory is superrenormalizable, i.e. very simple in the ultraviolet domain. As a result, the book-keeping of OPE becomes simple, and all subtle aspects in the construction of the OPE can be studied in a transparent environment.

Our study was motivated also by Ref. [7] where heavy flavor inclusive widths were calculated numerically, by adding the exclusive channels one by one. It was found that the inclusive width $\Gamma_{H_Q}$ approaches its asymptotic (partonic) value, and the sum over the exclusive hadronic states converges rapidly. At the same time, small deviations from the asymptotic value observed in the numerical analysis [7] were claimed to be a signal of $1/m_Q$ corrections in the total width, in contradiction with the OPE-based result.

We treated the very same problem analytically. The simplification which allows the analytical solution comes from taking fermions $\psi$ (leptons or quarks) emitted in
transition to be massless. Calculating OPE coefficients and comparing
the OPE representation (1) with the sum over exclusive channels which can be found
from the ’t Hooft equation we observe a perfect match through order \(1/m_Q^4\).

After testing the validity of OPE, we discuss the issue of violation of local duality.
Oscillating contributions to the total width are estimated. They are suppressed by
the high power of \(1/m_Q\) which we determined. Then we apply the same method in
the real 1+3 QCD to estimate duality violations in hadronic \(\tau\) decays.

2 Setup of the problem

2.1 ’t Hooft model

In two-dimensional QCD the Lagrangian looks superficially the same as in four di-

\[
\mathcal{L}_{1+1} = -\frac{1}{4g^2} G_{\mu\nu} G^{\mu\nu} + \sum \bar{\psi}_i (i \not{D} - m_i) \psi_i , \quad iD_\mu = i\partial_\mu + A_\mu^a T^a ;
\]

\(T^a\) denote generators of \(SU(N_c)\) in the fundamental representation and \(\psi_i\) the quark
field \((i\) is a flavor index) with a mass \(m_i\); \(g\) the gauge coupling constant.

One has to keep the following peculiarities in mind: \(g\) carries dimension of mass
as does \(\bar{\psi}\psi\). With the theory being superrenormalizable no (infinite) renormaliza-
tion is needed; observables like the total width \(\Gamma_{HQ}\) can be expressed in terms of the bare
masses \(m_i\) and bare coupling \(g\) appearing in the Lagrangian.

The ’t Hooft model is the \(N_c \to \infty\) limit of 1+1 QCD. A parameter fixed in this
limit is

\[
\beta^2 = \frac{g^2}{2\pi} \left( N_c - \frac{1}{N_c} \right) \quad \text{with} \quad \lim_{N_c \to \infty} \beta^2 = \text{finite} .
\]

This dimensionful quantity \(\beta\) which provides an intrinsic mass unit for the ’t Hooft
model, can be seen as the analog of \(\Lambda_{QCD}\) of four-dimensional QCD.

The heavy hadron \(H_Q\) is the bound state of a heavy quark \(Q\) and a light spec-
tator antiquark \(q_{sp}\). The bound state is described by the light-cone wave function
\(\phi(x), \ (x \in [0,1])\) which is the eigenfunction of the ’t Hooft equation,

\[
M_H^2 \varphi_{HQ}(x) = \left[ \frac{m_Q^2 - \beta^2}{x} + \frac{m_{sp}^2 - \beta^2}{1-x} \right] \varphi_{HQ}(x) - \beta^2 \int_0^1 dy \frac{\varphi_{HQ}(y)}{(y-x)^2}
\]

with boundary conditions \(\varphi_{HQ}(x = 0) = \varphi_{HQ}(x = 1) = 0\).

2.2 Weak interaction

Next we need to introduce a flavor-changing weak interaction; we choose it to be of
the current-current form:

\[
\mathcal{L}_{\text{weak}} = -\frac{G}{\sqrt{2}} (\bar{q} \gamma_\mu Q) (\bar{\psi} \gamma^\mu \psi) .
\]
The field $\psi$ can be either the light quark or the lepton field to describe nonleptonic or semileptonic decays, respectively.

For $N_c \to \infty$ factorization holds; i.e., the transition amplitude can be written as the product of the matrix elements of the currents $\bar{q}_\mu Q$ and $\bar{\psi}_\mu \psi$. For the inclusive widths which are discussed below the property of factorization can be expressed as follows:

$$M_{H_Q} \Gamma_{H_Q} = \text{Im} \int d^2 x \, i \langle H_Q | T \left\{ \mathcal{L}_{\text{weak}}(x) \mathcal{L}_{\text{weak}}^\dagger(0) \right\} | H_Q \rangle$$

$$= G^2 \int d^2 x \, \text{Im} \Pi_{\mu\nu}(x) \text{Im} T^{\mu\nu}(x),$$

$$\Pi_{\mu\nu}(x) = i \langle 0 | T \left\{ \bar{\psi}(x) \gamma_\mu \psi(x) \bar{\psi}(0) \gamma_\nu \psi(0) \right\} | 0 \rangle,$$

$$T^{\mu\nu}(x) = i \langle H_Q | T \left\{ \bar{q}(x) \gamma^\mu Q(x) \bar{Q}(0) \gamma^\nu q(0) \right\} | H_Q \rangle.$$

This factorization follows from the fact that at $N_c \to \infty$ there is no communication between $\bar{\psi}_a \gamma_\mu \psi_b$ and $\bar{q}_\gamma Q$ currents: any gluon exchange brings in a suppression factor $1/N_c^2$.

The only difference between the semileptonic and nonleptonic widths resides in $\Pi_{\mu\nu}(x)$, in the first case $\psi$ is the leptonic field while in the second case it is the quark field. At $m_\psi = 0$, we get same $\Pi_{\mu\nu}(x)$ (up to the overall normalization factor $N_c$). Indeed, for massless fermions $\Pi_{\mu\nu}$ is given by the well-known expression,

$$\Pi_{\mu\nu}(q) = \int d^2 x \, e^{i q x} \Pi_{\mu\nu}(x) = -\frac{1}{\pi} \left( \frac{q_\mu q_\nu}{q^2} - g_{\mu\nu} \right).$$

This expression obtained from a one-loop graph is known to be exact. For this reason at $m_\psi = 0$ the distinction between the nonleptonic and semileptonic cases is actually immaterial.

A remarkable feature of Eq. (6) is the occurrence of the pole at $q^2 = 0$, which is specific for the vector interaction. This means that a pair of massless leptons or quarks produced by the vector current is equivalent to one massless boson, whose coupling is proportional to its momentum $q_\mu$. In the case of the quark fields, it is known from the early days of the 't Hooft model that the current $\bar{\psi}_a \gamma^\mu \psi_b$ produces from the vacuum only one massless meson, the pion.

It means that we can use the substitution $\bar{\psi}_a \gamma^\mu \psi_b \to \epsilon^{\mu\nu} \partial_\nu \phi/\sqrt{\pi}$ where the field $\phi$ describes a massless boson. For the $Q$ quark decay width we have

$$\Gamma_Q = \Gamma(Q \to \psi \bar{\psi} q) = \Gamma(Q \to \phi q) = \frac{G^2}{4\pi} \frac{m_Q^2 - m_\phi^2}{m_Q}.$$
3 Operator product expansion for inclusive widths

The $1/m_Q$ expansion for inclusive widths of heavy flavor hadrons is based on OPE for the weak transition operator \[ 8 \]

\[
\int d^2 x \ i T \left\{ \mathcal{L}_{\text{weak}}(x) \mathcal{L}_{\text{weak}}^\dagger(0) \right\} = \sum c_i(\mu) \mathcal{O}_i(\mu). \tag{9}
\]

The local operators $\mathcal{O}_i$ are ordered according to their dimensions. The leading one is $\bar{Q}Q$ with dimension $d_{\bar{Q}Q} = 1$. Higher operators have dimensions $d_i > 1$. By dimensional counting the corresponding coefficients are proportional to $(1/m_Q)^{(d_i - 2)}$.

The coefficients $c_i$ (actually, we need $\text{Im} c_i$) are determined in perturbation theory as a series in $\beta^2/m_Q^2$. These coefficients are saturated by the domain of virtual momenta $\sim m_Q$ and are infrared stable by construction. All infrared contributions reside in the matrix elements of the operators $\mathcal{O}_i$.

At this point we see a drastic distinction between four- and two-dimensional QCD. In four dimensions the expansion parameter for the coefficients is the running coupling $\alpha_s(m_Q)$; nonperturbative contributions to the coefficients coming from distances $\sim 1/m_Q$ could, in principle, show up in the form $\exp(-C/\alpha_s(m_Q)) \sim (\Lambda_{\text{QCD}}/m_Q)^\delta$ where $\delta$ is some positive index. In two-dimensional QCD such terms cannot appear: an analog of the exponential term above would be $\exp(-Cm_Q^2/\beta^2)$.

For the leading operator $\bar{Q}Q$ its coefficient in the zero order in $\beta$ is $\text{Im} c_{\bar{Q}Q}^0 = \Gamma_Q/2$ where $\Gamma_Q$ is given by Eq. \[ 9 \]. We found $\text{Im} c_{\bar{Q}Q}$ in all orders: the only effect of high orders is the substitution $m_Q^2 \rightarrow m_Q^2 - \beta^2$ and $m_q^2 \rightarrow m_q^2 - \beta^2$. This result is based on the nonrenormalization theorem we proved for the $\bar{q}^\mu Q$ current at $q^2 = 0$. For the next by dimension four-fermion operators $\bar{Q}\Gamma Qq\Gamma q$ we show that they will appear only in the $\beta^4$ order. By dimensional counting it means that their contribution to the total widths is of order of $1/m_q^5$.

Putting everything together we get the OPE representation for the inclusive width:

\[
\Gamma_{H_Q} = \Gamma_Q \left[ \frac{m_Q}{\sqrt{m_Q^2 - \beta^2}} \frac{\langle H_Q | \bar{Q}Q | H_Q \rangle}{2M_{H_Q}} + \mathcal{O} \left( \frac{1}{m_Q^5} \right) \right]. \tag{10}
\]

Note that dependence on $m_Q$ resides not only in the explicit OPE coefficient but also in the matrix element. The relevant matrix element is

\[
\frac{\langle H_Q | \bar{Q}Q | H_Q \rangle}{2M_{H_Q}} = 1 + \mathcal{O} \left( \frac{1}{m_Q^2} \right), \tag{11}
\]

thus the absence of linear in $1/m_Q$ corrections is clear. We will prove a much stronger statement in the next section: with the accuracy of $1/m_Q^5$ the OPE representation \[ 10 \] matches the summation of exclusive widths based on the 't Hooft equation.
4 Match between OPE-based expressions and hadronic saturation

The exclusive width $\Gamma_n$ of two body decay $H_Q \to h_n + \phi$ where $h_n$ is the mesonic $[q\bar{q}_{sp}]$ state with the mass $M_n$ is expressed via the overlap of the initial and final ’t Hooft wave functions,

$$\Gamma_n = \frac{G^2}{4\pi} \frac{M_{H_Q}^2 - M_n^2}{M_{H_Q}} \left| \int_0^1 dx \varphi_n(x) \varphi_{H_Q}(x) \right|^2. \quad (12)$$

The wave function $\varphi_n(x)$ satisfies the same ’t Hooft equation (4) with evident substitutions, $Q \to q$, $M_{H_Q} \to M_n$.

Using this equation together with the completeness,

$$\sum \varphi_n(x) \varphi_n(y) = \delta(x-y),$$

we get a set of three sum rules for $\Gamma_n$:

$$\frac{4\pi M_{H_Q}}{G^2} \sum_{n=0}^{\infty} \frac{\Gamma_n}{M_{H_Q}^2 - M_n^2} = \int_0^1 dx \varphi_{H_Q}^2(x) = 1, \quad (13)$$

$$\frac{4\pi M_{H_Q}}{G^2} \sum_{n=0}^{\infty} \Gamma_n (M_{H_Q}^2 - M_n^2) = (m_Q^2 - m_q^2) \int_0^1 \frac{dx}{x} \varphi_{H_Q}^2(x), \quad (14)$$

$$\frac{4\pi M_{H_Q}}{G^2} \sum_{n=0}^{\infty} \Gamma_n (M_{H_Q}^2 - M_n^2)^2 = (m_Q^2 - m_q^2)^2 \int_0^1 \frac{dx}{x^2} \varphi_{H_Q}^2(x). \quad (15)$$

Note that the sums runs over all states $h_n$, including those unaccessible in the real decays of $H_Q$, i.e. with masses $M_n > M_{H_Q}$. These transitions are still measurable by the process of inelastic lepton scattering off the $H_Q$ meson. The first two sum rules were introduced in Ref. [9].

It is not difficult to see that the next moment, $\sum_{n=0}^{\infty} \Gamma_n (M_{H_Q}^2 - M_n^2)^2$, is a divergent sum because $\varphi_{H_Q}^2(x)/x^3$ would no longer be integrable. It defines the asymptotics, $\Gamma_n \sim 1/M_n^6$, at large $n$. For this reason the “unphysical” part of the sum (14) gives only $1/m_Q^2$ corrections and we derive from the sum rule (14)

$$\Gamma_{H_Q} = \Gamma_Q \left[ \frac{m_Q}{M_{H_Q}} \int_0^1 \frac{dx}{x} \varphi_{H_Q}^2(x) + \mathcal{O} \left( \frac{1}{m_Q^2} \right) \right]. \quad (16)$$

This expression coincides with the OPE result of Eq. (10) as seen by rewriting the matrix element in Eq. (10) in terms of the ground state wave function $\varphi_{H_Q}(x)$. (The factor $m_Q/(m_Q^2 - \beta^2)^{1/2}$ accounts for nonlogarithmic dependence of $QQ$ operator on normalization point).

Thus, we have demonstrated the perfect match between OPE and the hadronic saturation for the inclusive width.
5 Deviations from local duality

Having established a perfect match between the OPE prediction for the total width and the result of the saturation by exclusive decay modes, through $O(1/m_Q^4)$, we must now turn to the issue of where the OPE-based prediction is supposed to fail. The failure usually goes under the name of “duality violations”, a topic under intense scrutiny in the current literature.

We must precisely define what is meant by duality and its violations. Sum rules relating certain moments of the imaginary part of transition amplitude to matrix elements of consecutive terms in the OPE series will be referred to as global duality. Taken at their face value, they are exact, to the extent of our knowledge of the coefficient functions and matrix elements of the operators involved in OPE. Therefore, it does not make any sense to speak about violations of the global duality.

The notion of local duality on the other hand requires further assumptions. The difference between the OPE-based smooth result and the experimental hadronic measurement is referred to as the duality violation meaning the violation of local duality. Thus, the duality violation is something we do not see in the (truncated) OPE series. The duality-violating terms are exponential in the Euclidean domain and oscillating (like $\sin(E/\Lambda_{QCD})^k$) in the Minkowski domain.

The appearance of duality violations in the form of oscillating terms is evident in the 't Hooft model where the spectral density is formed by zero-width discrete states. Indeed, each time a new decay channel opens $d\Gamma_{H_Q}/dm_Q$ experiences a jump ($\Gamma_{H_Q}$ is continuous) so that immediately above the threshold, $d\Gamma_{H_Q}/dm_Q$ is larger than the smooth OPE curve. In the middle between two successive thresholds it crosses the smooth prediction, and immediately below the next threshold $d\Gamma_{H_Q}/dm_Q$ is lower than the OPE-based expectation. To estimate the amplitude of oscillations we use the following model for exclusive widths $\Gamma_n$ at the range of $M_n$ in the vicinity of $M_{H_Q}$:

$$\Gamma_n = 3\pi G^2 \beta^7 \frac{M_{H_Q}^2 - M_n^2}{M_n^8}. \quad (17)$$

Then for the oscillating part $\Delta\Gamma_{osc}$ of the total width $\Gamma_{H_Q}$ as a function of $m_Q$ we get

$$\frac{\Delta\Gamma_{osc}}{\Gamma_Q} = 6\pi^4 \left(\frac{\beta}{M_{H_Q}}\right)^9 \left[x(1-x) - \frac{1}{6}\right] \quad (18)$$

where

$$x = \text{Fractional Part of } \left(\frac{M_{H_Q}^2}{\pi^2 \beta^2}\right), \quad x \in [0, 1) .$$

The plot of $\Delta\Gamma_{osc}$ is presented in Fig. 1. The amplitude of oscillations is $(3\pi^4/2)(\beta/M_{H_Q})^9$. Thus, we see that violation of local duality is present but suppressed as $1/m_Q^9$. If we could average over $m_Q$ in a sufficiently large interval we would get an exponential suppression. Then it would be possible to consider the OPE-based predictions beyond $1/m_Q^9$. 

6
6 Duality violations in $\tau$ decays

Let us discuss along similar lines a quantity of practical interest in 1+3 dimensions, namely the normalized hadronic $\tau$ width $R_\tau$:

$$R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} = \int_0^{M_\tau^2} \frac{ds}{M_\tau^2} \left( 1 - \frac{s}{M_\tau^2} \right)^2 \left( 1 + 2 \frac{s}{M_\tau^2} \right) \rho(s) \tag{19}$$

where $\rho(s)$ is the sum of spectral densities in the vector and axial-vector channels.

To estimate the oscillating part of $R_\tau$ we consider the limit of large $N_c$ and $M_\tau$. For large $N_c$ the spectrum of 1+3 QCD is expected to consist of an infinite comb of narrow resonances – in complete analogy to the ’t Hooft model. To keep the closest parallel to it we further assume that the high excitations are equally spaced in $m^2$. This agrees with the general expectation of a string-like realization of confinement leading to asymptotically linear Regge trajectories. The masses of the excited states in, say, the vector channel are then given by $m_n^2 = m_0^2 + 2n/\alpha'$, with $\alpha'$ being the slope of the Regge trajectory. Experimentally one finds $2/\alpha' \simeq 2 \text{ GeV}^2$.

Then spectral density $\rho(s)$ at large values of $s$ approaches the form:

$$\rho(s) = 2N_c \cdot \sum_{n=1}^{\infty} \delta \left( \frac{s}{\sigma^2} - n \right) ; \quad \sigma^2 = \frac{2}{\alpha'} , \tag{20}$$

Equation (20) is clearly not expected to hold at moderate and small values of $s$ where the vector and axial-vector channels are drastically different and the resonances are not equidistant. However, details of the spectral densities at small $s$ play no role in duality violation, they change only the regular terms of the $1/M_\tau^2$ expansion. The spectral density in Eq. (20) is dual to the parton model result; i.e., it coincides with it after averaging over energy, $\langle \rho(s) \rangle = 2N_c$, and $R_\tau$ approaches $N_c$ asymptotically.
The sum over resonances in $R_\tau$ is calculated analytically:

$$R_\tau = R_{\tau}^{\text{OPE}} + \delta R_{\text{osc}}, \quad \tag{21}$$

$$\frac{R_{\tau}^{\text{OPE}}}{N_c} = 1 - \frac{\sigma^2}{M_{\tau}^2} + \frac{1}{30} \left( \frac{\sigma^2}{M_{\tau}^2} \right)^4,$$

$$\frac{\delta R_{\text{osc}}}{N_c} = -x(1-x)(1-2x) \left( \frac{\sigma^2}{M_{\tau}^2} \right)^3 + \left[ x^2(1-x)^2 - \frac{1}{30} \right] \left( \frac{\sigma^2}{M_{\tau}^2} \right)^4,$$

where

$$x = \text{Fractional Part of } \left( \frac{M_{\tau}^2}{\sigma^2} \right), \quad x \in [0,1).$$

The result is a sum of two functions of $M_{\tau}^2$, the first one, $R_{\tau}^{\text{OPE}}$, is a smooth function expandable in $1/M_{\tau}^2$, it coincides with the OPE prediction in the model. The second one, $\delta R_{\text{osc}}$, oscillates with the period $\sigma^2$, see the plot of $\delta R_{\text{osc}}/N_c$ in Fig. 2. The dominant component of $\delta R_{\text{osc}}/N_c$ scales as $1/M_{\tau}^6$. This oscillating component vanishes at values of $M_{\tau}$ corresponding to the new thresholds, and at one point in the middle between the successive resonances; its second derivative has a jump at the thresholds, so we deal with a nonanalytical dependence on $M_{\tau}$. The average of $\delta R_{\text{osc}}$ vanishes while the amplitude of oscillations amounts to

$$\left| \frac{\delta R_{\text{osc}}}{R_{\tau}} \right|_{\text{max}} = \frac{1}{3\sqrt{12}} \left( \frac{\sigma^2}{M_{\tau}^2} \right)^3. \quad \tag{22}$$

Taking our estimate of the oscillation amplitude at its face value and using the actual value of the $\tau$ mass in Eq. (22) we find $\delta R_{\text{osc}}/R_{\tau} \sim 3\%$. One should not take too literally for many reasons: first of all the $\tau$ mass is not much
larger than the spacing between the resonances, $\sigma^2/M^2_\tau \sim 2/3$; second, $N_c$ is not large enough to warrant the zero width approximation, the nonvanishing widths lead to a further suppression of deviations from duality. Still we believe that the consideration is instructive in a qualitative aspect.

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