Specific heat of classical disordered elastic systems

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We study the thermodynamics of disordered elastic systems, applied to vortex lattices in the Bragg glass phase. Using the replica variational method we compute the specific heat of pinned vortons in the classical limit. We find that the contribution of disorder is positive, linear at low temperature, and exhibits a maximum. It is found to be important compared to other contributions, e.g. core electrons, mean field and non linear elasticity that we evaluate. The contribution of droplets is subdominant at weak disorder in d = 3.

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Understanding the temperature dependence of the specific heat in glasses remains puzzling, e.g. the linear low T behaviour observed in structural glasses [11] and spin glasses [2]. In some systems the crossover temperature from quantum to classical behaviour may be quite low. The phenomenological two levels system model [8] yields a linear behaviour both in the classical and quantum regime. Although the classical problem appears simpler there are only few, and mostly mean field, solvable models of glassy systems where one can actually compute the specific heat [2, 4]. Ordered systems with continuous symmetry admit spin wave type excitations which yield a T-independent specific heat \( C_v = C_{eq} \) from the equipartition of the energy. Non linearities such as quenched disorder will cause a deviation which is interesting to characterize and compare with the linear contribution from the two level system arguments.

A class of glassy systems recently much studied are disordered elastic systems, ranging from vortex lattices [2, 3, 5, 6, 7, 8], electron crystals [4], charge and spin density waves [14] to disordered liquid crystals [11]. In all of these systems the competition between disorder and elasticity leads to pinning and glassy behaviour. Specific heat measurements in density waves gapped systems showed intermediate linear and sublinear regimes with non-equilibrium effects [12, 13]. In superconductors in a field, the H and T dependence of \( C_v \) relates both to the symmetry of the order parameter and to the thermodynamics of the vortex lattice. If the contribution of the normal electrons in the vortex cores dominate, the standard expectation is that \( C_v \) is linear in T with a linear in H dependence for s wave [14] and \( H^{1/2} \) for d-wave [15]. A specific heat linear in temperature has indeed been measured in various materials. A \( H^{1/2} \)-dependence has been observed and argued for d-wave superconductivity in YBaCuO [16, 17, 18]. However the nonlinear H dependence observed in other, a priori non d-wave materials, is a well known puzzle, as discussed in [14, 20]. On the other hand, the contribution of the phonons of the vortex lattice VL, the so-called “vortons”, seems to be within experimental resolution [21], and may lead, within the full temperature range below VL melting, to more complex behaviors. These were analyzed in absence of disorder, assuming a dissipative quantum dynamics with friction \( \eta \) arising from interactions between vortons and vortex core electrons [22, 23]. It yields again \( C_v \propto \eta T^2 \) with different H dependence but only for \( T < T_v^D \), the vortex Debye-like temperature which is poorly known (estimates in YBaCuO range from well below 1K up to 10K in the superclean limit [22]). Above \( T_v^D \) one recovers the equipartition value to which the specific heat anomaly was compared at melting [24, 25, 26, 27]. These analysis however neglect disorder and other non linearities which for \( T > T_v^D \) can be treated classically.

In this Letter we compute the specific heat of an elastic system in presence of pinning disorder in the classical regime. We show that disorder produces a substantial rise above equilibrium, linear at low temperature and exhibiting a maximum at a characteristic depinning temperature. We show that in \( d > 2 \) the contribution from the two well droplet arguments is subdominant at weak disorder. We find that the disorder contribution is quite sizable compared to other contributions, e.g. of the non linear elasticity that we also evaluate. These results hold for a periodic object, i.e. a Bragg glass, or for interfaces with continuous degrees of freedom. In a companion paper [23] an analysis of the quantum regime revealed, in the absence of dissipation, a \( C_v \sim T^3 \) behaviour.

An elastic system, such as the vortex lattice (VL) with external field aligned with z axis, is described by a N-component vector displacement field \( u_\alpha(R_z, z) \) \((N = 2)\) for the VL in \( d = 3 \). The equilibrium positions \( R_z \) form a perfect N-dimensional lattice of spacing a. Interactions result in an elastic energy \( H_{el} \) associated to the phonons of the vortex lattices \( H_{el}[u] = \frac{1}{2} \int_\Omega u_\alpha(q)\Phi_{\alpha\beta}(q)u_\beta(-q) \). Here \( \int_\Omega = \int_{BZ} \frac{d^d q}{(2\pi)^d} \int_{-\pi/2}^{\pi/2} dq_z/(2\pi) \) denotes integration on the first Brillouin zone, \( q = (q_1, q_2, q_3) \) a d dim-vector, and s is the distance between layers. For the triangular VL:

\[
\Phi(q) = (c_{66}q_1^2 + c_{44}q_2^2)P_T(q_\perp) + (c_{11}q_1^2 + c_{44}q_2^2)P_L(q_\perp) \tag{1}
\]

with \( P_L^{\alpha\beta}(k) = k_\alpha k_\beta/k^2 \) and \( P_T = 1 - P_L \). The disper-

\[\Phi(q) = (c_{66}q_1^2 + c_{44}q_2^2)P_T(q_\perp) + (c_{11}q_1^2 + c_{44}q_2^2)P_L(q_\perp) \]
sion of elastic moduli is implicit whenever needed. Impurity disorder is modeled by a short range gaussian random potential with in plane correlator $\Delta(r) = 6\delta\kappa^2 e^{-r^2/(4\kappa^2)}$ interacting with the local vortex density. Here $2r_\xi = \xi$ the superconducting coherence length, $\delta = (\Phi_0/4\pi\lambda)^2$ is the vortex energy scale per unit length along $z$ and $\delta a$ (small) dimensionless disorder parameter $[33]$. The equilibrium Bragg glass phase (absence of dislocations, $a/R_0 \ll 1$, $R_0$ being the translational correlation length) is described by the replicated partition function $Z \equiv T \exp(-\beta E_{\text{dis}}) = \beta = 1/T$ and $\equiv$ denotes disorder average. After standard manipulations $[29]$ the replicated Hamiltonian becomes $H_{\text{dis}}[u] = \sum a H_{\text{el}}[u^a] + H_{\text{dis}}[u]$ with:

$$H_{\text{dis}} = -\beta \int d^2 r d z \sum_{ab} R(u_a(r,z) - u_b(r,z)) \tag{2}$$

Here $R(u) = \rho_0^2 \sum K \Delta_K \cos(K \cdot u)$ in terms of $\rho_0$ the average vortex density and the disorder harmonics $\Delta_K = \int d^N u \Phi_0^K \Delta(u)$ at the reciprocal lattice vectors. More generally, an elastic manifold (such as a directed polymer $d = 1$) in a $N$-dimensional embedding space in presence of a random potential $W(u,x)$ is described by a similar model with $W(u,x)W(u',x') = \delta^{(d)}(x-x') R(u-u')$, $R(u) = -N V(u^2/N)$. We compute the specific heat per unit volume $C_u(T) = -\frac{\partial^2}{\partial T^2} F_u$ where $F_u$ is the free energy, $\Omega = L^d$ the volume. We present the description using an isotropic disorder and elasticity tensor $\Phi_{\alpha\beta}(q) = cq^2 \delta_{\alpha\beta}$, and generalize to the vortex lattice later.

To obtain the low $T$ behavior, a first approach would be to assume a single minimum of the energy $H[u]$ and expand around it. One then finds:

$$C_v(T) = C_{\text{eq}} + AT + O(T^2) \tag{3}$$

The exact expression for the linear term $A = A_{1\text{min}}$, given in $[34]$, involves cubic and quartic anharmonicity in $H[u]$ in a given disorder realization. Disorder averaging is only easy to perform perturbatively in disorder, yielding:

$$A = A_{1\text{min, pert}} = \frac{1}{6} J_1^2 (\nabla_a^2)^3 R(u)|_{u=0} \tag{4}$$

with $J_1 = \int d^2 r (e^q u^2)$ (for the manifold problem it gives $A = 4J_1^2 (N^2 + 6N + 8) V''''(0)/3N$). Although such a single minimum low $T$ expansion is useful for pure systems, such as the Sine-Gordon model, for disordered systems more than one minimum typically exists beyond the Larkin length $R_c$. The resulting contribution to the specific heat can then be estimated combining the droplet picture with the two levels argument $[3, 31]$. At each length scale $l$ each of the subsystems $i \in (L/l)^d$ may have a low lying secondary minimum (droplet) at excitation energy $E_i$ independently distributed with probability $P(E) \propto e^{-\beta E_i} F_{\text{eff}}(R/c)$ where $F_{\text{eff}}$ is the typical pinning energy $E_c = c \pi^2 R_c^2/4$, $\theta$ the free energy exponent. Approximating the contribution from scale $l$ to the specific heat as:

$$C_l = L^{-d} \sum_{i=1}^{(L/l)^d} \left( \frac{E_i}{T} \right)^2 \frac{e^{-\beta E_i}}{(1 + e^{-\beta E_i})^2} \approx \frac{\pi^2 T F_0}{6} \left( \frac{R_c}{l} \right)^\theta$$

treating as independent two level systems, $C_v \equiv \int_{l \approx R_c} \frac{4}{T} C_l$ is dominated by the smallest scales, yielding:

$$A_{\text{drop}} \approx \frac{\pi^2 F_0}{6(d + \theta)} E_c^{-1} R_c^{-d} \tag{5}$$

While the two wells-droplet argument estimates as $A_{\text{drop}}$ the contribution only of scales larger than $R_c$, one can only hope to use $A_{1\text{min}}$ to estimate the contributions of scales smaller than $R_c$. It is thus instructive to compare them. The perturbative expression $[34]$ is infrared divergent for $d \leq 2$ as thermal fluctuations diverge. If one restricts by hand the integral in $[34]$ to $q > 1/R_c$ one finds a contribution of the same order than the droplet one $A_{\text{drop}}$. In $d > 2$ the integral is instead controlled by small scales and the droplet contribution is then sub-dominant. The two levels droplet model of $[33]$ can be improved by including anharmonicity in each well, for identical wells it simply adds $A = A_{1\text{min}} + A_{\text{drop}}$.

The variational method $[29, 32]$ extends this phenomenological considerations into a first principle quantitative calculation in which the Larkin length naturally appears in a self consistent way. We introduce a Gaussian trial Hamiltonian $H_0 = \frac{1}{2} \int q G_{\alpha\beta}(q) u^a(q) u^b(q)$ which minimizes the variational free energy $F_{\text{var}} = F_0 + \langle H_{\text{eff}} - H_0 \rangle/\theta$, where $F_0$ denotes the free energy calculated with $H_0$. The specific heat $C_v = \lim_{\theta \to 0} -\partial_T \partial_\beta \int \int \varphi$ can be reexpressed as:

$$C_v = \partial_T \lim_{\theta \to 0} \frac{1}{\theta} \int \int \left( \sum_a H_{\text{el}}[u^a] \right) H_{\text{dis}} + 2 \langle H_{\text{dis}}[u] \rangle H_{\text{dis}} \tag{6}$$

Note the factor of 2 due to the $\beta$ dependence of the disorder term. Here we evaluate these averages using the variational hamiltonian $H_0$ instead of the exact one $H_{\text{eff}}$. Thanks to the variational equations this is equivalent to $C_v(T) = -(T/\Omega) \partial^2 F_{\text{var}}/\partial T^2$.

We applied this variational approach to some pure models and checked that it is quite accurate $[30]$. At low $T$ it exactly matches the expansion around the minimum $A_{\text{var}} = A_{1\text{min}}$ and we checked, for the Sine-Gordon model in $d = 2$, that it is identical to the low $T$ expansion of the exact result $[33]$. In the disordered case the solution of the variational equations requires Replica Symmetry Breaking (RSB). One denotes $G_\alpha(q) = G_{\alpha\beta}(q)$ and parametrizes $G_{\alpha\beta}(q)$ by $G(q, u)$, where $0 < u < 1$ and similarly for $B_{\alpha\beta}(x = 0) = B_{\alpha\beta} = \langle (u^\alpha(x) - u^\beta(x))^2 \rangle / N$ with $\tilde{B} = 0$ and $B(u)$. For $d > 2$ one finds $[29, 32]$ a continuous RSB with a breakpoint $u_c$ and

$$B(u > u_c) = B = 2\gamma T J_1(\Sigma) \tag{7}$$
where \( \gamma = 1 \), \( \hat{V}(B) = -\frac{1}{2}\sum K \Delta_K \exp(-BK^2/2) \) and \( R_c \) is the Larkin length \([30]\). Eq. (8) is the so called marginality condition (MC) which also holds for the one step solution in \( d = 2 \). Starting from the expression:

\[
\frac{1}{N\Omega}(\Omega) = \int q^2 \hat{G}(q) + \frac{1}{T} \int_0^1 du[\hat{V}(0) - \hat{V}(B(u))] \tag{10}
\]

which, as \( B(w) = B(u) \) (setting \( w = u/T \)) and \( w_c = u_c/T \) \([31]\), turns out to depend implicitly on \( T \) only through \( \Sigma \) and \( B \), and using (7) and (8), one obtains for the specific heat

\[
C_v(T) = C_{eq} + \frac{N}{T^2} F(B) \tag{11}
\]

\[
F(B) = \hat{V}(B) - \hat{V}(0) - B\hat{V}'(B) + \frac{1}{2}B^2\hat{V}''(B)
\]

where \( C_{eq} = 1/4N/\Omega = \frac{N}{T} \int q_0 \) is the equipartition value, i.e. half the total number of modes per unit volume \([33]\). Eq. (11) is valid for any \( T \), for periodic objects (Bragg glass) as well as manifolds, and independently of the replica structure of the solution (if broken, provided MC holds). One thus finds that disorder increases the specific heat which has now a maximum and decreases back to equipartition \( C_{eq} \) at high \( T \). Expanding (11) at low \( T \) one finds again \([3]\) with an amplitude \( A_{var} = \frac{8}{37}NV''(0)\Omega^2 \Sigma^3 \). For weak disorder \( R_c \approx a \), dispersionless elasticity and \( r_f \ll a \), one finds from (11) by neglecting (i.e. replica symmetric) perturbation theory are cured, the Larkin length being the natural scale. A plot of \( C_v(T) \) is shown on Fig. 1.

The generalization to the vortex lattice using (1) and \( (\Sigma) = \frac{1}{2} \) yields several regimes analyzed in \([30]\) and for simplicity we present here only results for the weak disorder regime \( R_c > a \). The fluctuation of a vortex position is then measured from \( B = T\Omega(\Sigma) \approx T\Omega(0) \approx c_6^2 a^2 T/T_m \), where \( T_m \) is the melting temperature and \( c_6 \approx 0.1 - 0.2 \) is the Lindemann number. Inserting (11) this yields \( C_v(T) \) which has a maximum for \( T = T^* \) i.e. \( B = B^* \) determined by solving \( 4F(B^*) = (B^*)^3\hat{V}''(B^*) \). These fluctuations are estimated as \( J_1(\Sigma) \approx 1/(4a_\Delta c_6 c_{14}) \) where here and below we denote \( c_{14} = c_{14}(q=\pi/a) \) (below the dimensional crossover field, \( c_{66}/(c_{44} a^2) \approx 1/s^2 \), the integral over \( q_z \) can be extended up to infinity and the integral is dominated by \( q_\perp = \pi/a \).

We can now discuss in details the behaviour of \( C_v(T) \) for \( r_f < a \). In this limit \( \hat{V}(B) \) takes the dependence \( \hat{V}(B) \approx -D/(2r_f^2 + B) \), with \( D = \delta^2 \alpha_0 c_6^2/a^2 \), for \( B \ll a^2 \) which holds up to melting. One finds:

\[
C_v(T) = C_{eq} + AT/(1 + T/(2T_*)^3)
\]

\[
A \approx \frac{128^{\frac{1}{3}}}{c_6^2} \frac{a^2}{s^2}(c_4/c_{14} a^2)(r_fT_m)^{\frac{1}{3}}
\]

The maximum occurs for \( B^* = r_f^2 \) and thus \( T = T^* = T_m r_f^2/(c_4^2 a^2) \), i.e. the so called depinning temperature \([3]\), which can be below melting. The amplitude \( A \) (and large \( T \) tail) is independent of the detailed form of \( \hat{V}(B) \) and is strongly enhanced by the anisotropy and dispersion of \( c_{44} \). The value at the maximum is \( C_v^* - C_{eq} = 8AT^*/27 \). Using \( c_{44} = c_6^2/a^2, c_{66} = c_6^2/(4a^2) \) the calculation of \( J_2(\Sigma) \) shows \([30]\) that \( R_c > a \) holds for \( \delta < \delta_c = 4\pi^2 c_6^2 a^2 \). At this value of the disorder one obtains \( A \approx c_6^2/(T_m r_f^2) \) and \( C_v^* - C_{eq} \approx 1/\alpha^2 \) ( \( \epsilon < 1 \) the anisotropy parameter \([3]\)). \( C_v^* - C_{eq} \) should be compared with the equipartition value, estimated as \( C_{eq} = 1/(sa^2) \). These become comparable around the dimensional crossover field \( B = B_{cr} \) such that \( s \sim a \).

The above classical contribution \([12]\) will hold only above the vorton Debye temperature \( T_v^D \), below which quantum effects become important. \( T_v^D \) depends on the vortex mass, the Hall force and the friction force, which arise from the coupling of moving vortices with the normal electrons bath in presence of scattering. Estimates for \( T_v^D \) range \([22]\) from \( 10^{-3}T_c \) in dirty (friction dominated) materials to \( 10^{-1}T_c \) in superclean limit (Hall dominated). There should thus exist a broad regime of temperature and field where the Bragg glass is stable and the result \([12]\) holds.

To assess whether this contribution \([12]\) is observable, let us compare it with other terms linear in \( T \) in \( C_v \). First
the normal electrons in the vortex core lead to [14]:

\[ C_{\text{core}}(T) \approx \frac{T}{T_f k_f^2 s} \left( \frac{H}{H_{c2}} \right)^\alpha \] (14)

where \( T_f \) is the Fermi temperature of the normal metal, \( \alpha = 1 \) for s-wave superconductor [23] and \( \alpha = 1/2 \) for lines of nodes in the gap [13]. Given the large ratio \( T_f/T_m = O(10^6) \) for e.g. YBaCuO, one finds, comparing \( A_{\text{core}} \) and \( A_{\text{dis}} \) that the contribution from the cores can be comparable or smaller than the one from the disorder.

There are other contributions from the vortex lattice as well. The mean field specific heat [23] being \( C_{mf} \approx \epsilon_0 T/(T_f^2 a^2) \), the ratio \( A_{\text{dis}}/A_{mf} \approx (c_L^2 a T_m/(r_f T_m))^2 \) can be large. We have also computed the contribution from non linear elasticity of the VL. Performing, as in [34], an expansion of the vortex interaction energy \( \int dz \sum_{ij} \epsilon_0 K_0((R_i - R_j + u_i(z) - u_j(z))/\lambda) \), we obtain:

\[ A_{nl} = \rho_0 \epsilon_0 a^{-4} \left( \frac{c_L^2 a^2}{T_m} \gamma_4 + \frac{\gamma_3 \sqrt{c_4} 460}{c_L a} \left( \frac{c_L^2 a^2}{T_m} \right)^\alpha \right) \] (15)

up to \( O((a/\lambda)^2) \) where the numerical prefactors \( \gamma_4 \) and \( \gamma_3 \) are complicated lattice sums, given in [30]. One can then estimate \( A_{nl}/A_{\text{dis}} \sim r_f^2/a^2 \) and this contribution is likely to remain small until melting. None of these contributions [30] is expected to exhibit a maximum at scale distinct from \( T_m \).

To conclude, we found by explicit calculation that the vortex lattice classical contribution to \( C_v \) due to disorder can be important at the very least in the range \( 10^{-1}T_c \) to \( T_m \), and possibly more. The coefficient \( A_{\text{dis}} \) of the low \( T \) linear behavior is magnetic field dependent, \( A_{\text{dis}} \propto 1/(T_m a) \). It is convenient to express results using the melting temperature \( T_m \), which is experimentally measured [24, 25, 27] and can also be estimated from the standard elastic expression of \( T_m = 4a^2 \sqrt{c_6} c_4 c_L^2 \) allowing to extract directly the magnetic field dependence. \( T_m \sim 1/\sqrt{B} \) leads to \( T^* \propto \sqrt{B} \) and \( A_{\text{dis}} \propto B \). \( C_v(T) \) exhibits, as compared to other contributions, a distinct maximum around the depinning temperature scale, whenever smaller than melting. A crude estimate of \( T^* \) is \( T^*_m \sim T_m B/(c_L^2 H_{c2}) \), which for a typical \( c_L \sim 0.12 \) gives a \( T^* \) which is a fraction of \( T_m \) for fields of about a Tesla (a fraction of order unity for YBaCuO around \( B \approx 10 T \) the so-called tricritical point [27]). It would be very interesting to perform precise measurements of \( C_v \) to check the present proposal.

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[35] δ = δ_0ξ/s in the notations of [7].
[36] for the manifold \( \tilde{V}(B) \) is defined from \( V(B) \) in [32].
[37] one finds \( w_2 = 4V''''(B)J_3(\Sigma) \) for fields of about a Tesla (a fraction of order unity for YBaCuO around \( B \approx 10 T \) the so-called tricritical point [27]).
[38] \( C_{eq} \) doubles upon adding the kinetic energy for \( T > \hbar \omega_0 \) the so-called mini-gap.
[39] we can also neglect \( T \) dependence of \( V \) from \( \xi \) and \( \lambda \) of order unity for YBaCuO around \( B \approx 10 T \) the so-called tricritical point [27].