Spin waves in diluted magnetic quantum wells

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We study collective spin excitations in two-dimensional diluted magnetic semiconductors, placed into external magnetic field. Two coupled modes of the spin waves (the electron and ion modes) are found to exist in the system along with a number of the ion spin excitations decoupled from the electron system. We calculate analytically the spectrum of the waves taking into account the exchange interaction of itinerant electrons both with each other and with electrons localized on the magnetic ions. The interplay of these interactions leads to a number of intriguing phenomena including tunable anticrossing of the modes and a field-induced change in a sign of the group velocity of the ion mode.

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Diluted magnetic semiconductors (DMS) have recently been the subject of great interest \cite{1,2} due to their potential in combining magnetic and semiconductor properties in a single material. The DMS are formed by replacing of cations in ordinary semiconductors with magnetic ions, typically Mn ions. Strong exchange interaction between the itinerant electrons and the electrons localized on d-shells of the magnetic ions leads to a number of remarkable features of the DMS. In particular, it results in the effective indirect interaction between the ion spins thus promising for creating room-temperature ferromagnetic systems that may offer advantages of semiconductors. It also dramatically enhances the effective coupling of the itinerant electrons with the external magnetic field. In contrast to conventional GaAs/GaAlAs systems, where small values of g-factor prevents manipulation of the spin degree of freedom, the giant electron Zeeman splitting arising in the DMS as a manifestation of the exchange interaction can be on the order of the Fermi energy \cite{3,4}, offering a wide range of spintronics applications.

Here we discuss spin excitations in the two-dimensional DMS. Our studies are motivated by recent experiments \cite{5,6} and a theoretical discussion \cite{7,8} focused on the spin dynamics in diluted magnetic Cd\textsubscript{1−x}Mn\textsubscript{x}Te quantum wells placed into the magnetic field \cite{9}. In Ref. \cite{9}, the spectrum of the spin waves, \(\omega(k)\), was measured. Only one excitation mode was observed. It was demonstrated that the excitations exist in a finite range of wavelengths, \(k < k_m\), and their group velocity is negative: \(d\omega(k)/dk < 0\). The experimental data were interpreted \cite{5,8} in terms of conventional spin waves in the Fermi liquids \cite{11}, while \(k_m\) was attributed to the edge of the Stoner continuum of the single-particle spin excitations. Such interpretation implies that the only effect of the magnetic ions on the electron spin waves is the strong renormalization of the electron Zeeman splitting. However, more recent experimental observations \cite{6,7} supported by theoretical studies \cite{8,10} appear to be in disagreement with this conclusion. Indeed, in Refs. \cite{6,7}, two modes of the collective homogeneous \((k = 0)\) spin excitations were observed in Cd\textsubscript{1−x}Mn\textsubscript{x}Te wells. The modes were identified \cite{6,7} as the spin excitations of delocalized electrons (the electron mode) and the electrons on d-shells of Mn ions (the ion mode). The dependencies of the frequencies \(\omega_{1,2}(0)\) of observed modes on \(B\) are shown schematically in Fig. \(\text{(1)}\). The most important observation is the anticrossing of the modes which occurs at a certain “resonant” value of magnetic field \(B = B_{res} \approx 6.5 K\). As it was also shown in Ref. \cite{7}, other types of the homogeneous spin modes may exist in the system corresponding to excitations of the ion spins decoupled from the spins of the itinerant electrons.

In this paper, we develop a theory of the spin waves in diluted magnetic quantum wells placed into magnetic field. We study analytically two collective modes which correspond to coupled propagation of the electron and ion spin excitations. We also discuss the ion modes decoupled from the electron system. To describe the homogeneous spin oscillations \((k = 0)\), it is sufficient to take into account only one type of exchange interaction: the interaction of the itinerant electrons with electrons localized on the Mn ions. The thus
obtained results coincide with those presented in Refs. [4, 5]. For $k \neq 0$, the exchange interaction between delocalized electrons comes into play. Our main purpose is to demonstrate that the simultaneous presence of two types of exchange interaction gives rise to interesting phenomena the most remarkable one is the magnetic-field-driven anticrossing of the spin modes. In contrast to the case $k = 0$, the anticrossing can take place in a wide range of the magnetic fields and may be tuned by the field to occur at a certain value of $k$ (see Fig. 2).

We consider the 2D degenerate electron gas interacting with randomly placed magnetic ions. The electrons are located in the $r = (x, y)$ plane and occupy the lowest level in the well. The ions are distributed homogeneously with the 2D concentration $n_j$, which is assumed to be much higher than the electron concentration $n_e$. The magnetic field is applied parallel to the well plane ($B \parallel e_x$). The field leads to the Zeeman splitting of the electron and ion spin levels with energies $\hbar \omega_e$ and $\hbar \Omega_j$, respectively, while the orbital motion remains intact. The Hamiltonian of electron-ion exchange interaction reads $H_{\text{ex}} = -\alpha/2 \sum_k \hat{\sigma} \hat{J}_k \delta(r - r_k)|\Psi(z_k)|^2$, where $\hat{\sigma}$ is the Pauli matrix vector, $\hat{J}_k$ are the spin operators of the ions located at the points $r_k = (r_k, z_k)$, and $\Psi(z) = \sqrt{2/\pi} \sin(\pi z/a)$ is a wave function of the lowest level in a rectangular well of width $a$. Since $n_j \gg n_e$, the distance between the ions is much smaller than the electron wave length and the mean-field approximation is applicable. In this approximation, we first replace $\hat{\sigma} \hat{J}_k$ with $\langle \hat{\sigma} \rangle \hat{J}_k + \hat{\sigma} \langle \hat{J}_k \rangle$, where averaging is taken over density matrix of the system $\hat{\rho}$. After such decoupling one may search the solution of the quantum Liouville equation for $\hat{\rho}$ as a product of the electron and ion density matrices: $\hat{\rho} = \hat{\rho}_e(r, r', t) \prod_k \hat{\rho}_i(t)$. The averaged spins of the electrons and ions are given by $\langle s_0(r, t) = n_e^{-1} \int s(r, p, t) d^2p/(2\pi\hbar)^2, \hat{J}_k(t) = \text{Tr}(\hat{\rho}_k(t)\hat{J}_k)$, where $s(r, p, t) = \text{Tr}(\hat{\sigma} \hat{f})/2$ is the electron spin density and $\hat{f} = \hat{f}(r, p, t)$ is the Wigner function corresponding to $\hat{\rho}_e$. As a next step, we replace $\hat{J}_k(t)$ with a smooth function $\hat{J}(r, z, t)$. Doing so, one finds the electron spin precession frequency in the ion-induced exchange field

$$\omega_{ee}(r, t) = \alpha n_j \hat{J}(r, z, t)/\hbar,$$  \quad (1)  

[here $\hat{J}(r, t) = \int d(z)|\Psi(z)|^2 \hat{J}(r, z, t)$], and the local frequency of the ion spin precession

$$\omega_{ij}(r, z, t) = \alpha n_e s_0(r, t)|\Psi(z)|^2/\hbar.$$  \quad (2)  

In addition to the electron-ion exchange interaction, we take into account the isotropic ferromagnetic electron-electron exchange interaction by adding the following term [11]

$$\omega_{ee}(r, t) = -2G n_e \nu^{-1} s_0(r, t),$$  \quad (3)  

to the electron spin precession frequency. Here $G < 0$ is the interaction constant (we assume $|G| < 1$), $\nu = m/2\pi\hbar^2$, and $m$ is the electron effective mass.

The equilibrium electron spin density, $s_{\text{eq}}(p) = [n_+(\epsilon) - n_-(\epsilon)] e_x/2$ (here $n_{\pm}(\epsilon) = \{\exp[\epsilon \mp \hbar \Omega_{ij}^0/2 - E_F]/T + 1\}^{-1}$ and $\epsilon = p^2/2m$, is expressed via the effective Zeeman splitting $\hbar \Omega_{ij}^0$. The frequency $\Omega_{ij}^0 = \omega_{ee} + \omega_{ee} + \omega_{ee}$ is found self-consistently from Eqs. (1) and (3):

$$\Omega_{ij}^0 = \Omega_e/(1 + G),$$  \quad (4)  

where

$$\Omega_e = \omega_e + \alpha n_j J_x^{\text{eq}}/a h,$$  \quad (5)  

is the effective electron Zeeman splitting renormalized by exchange interaction with the ions. In deriving these equations, $\hat{J}(r, z)$ was substituted with the equilibrium ion polarization, $\hat{J}^{\text{eq}} = J_{x}^{\text{eq}} e_x = \frac{B}{2} B_{ij}(\hbar \Omega_{ij}/T) e_x$, where $B_j(x)$ is the Brillouin function. We also assumed that the equilibrium exchange field acting on the ions is small, $\alpha n_e s_{\text{eq}}^0/a = \alpha h \Omega_{ij}^0 n_e/4E_F a \ll \hbar \Omega_j$, which implies that the equilibrium ion polarization is not affected by exchange interaction. In contrast, the electron Zeeman splitting is strongly
enhanced due to high ion concentration, so that \( \Omega_e > |\omega_e| (\omega_e < 0) \), because of the negative electron \( g \)-factor, which explains non-monotonic dependence of \( \Omega_e \) on \( B \) shown in Fig. 1 \[\text{[6, 11]}\).

The out-of-equilibrium spin dynamics can be described by the Landau-Silin equation \[\text{[12]}\] for the electrons and the Bloch equation for the ions:

\[
\frac{\partial \hat{f}}{\partial t} + \frac{\mathbf{p}}{m} \nabla \hat{f} - \frac{1}{\hbar} \left\{ \frac{\partial \hat{f}}{\partial \mathbf{p}} , \hat{\mathbf{e}} \right\} + \frac{i}{\hbar} [\hat{\mathbf{e}} , \hat{f}] = 0 ,
\]

\[
\frac{\partial \mathbf{J}}{\partial t} + [\Omega_e \mathbf{e} + \omega_J \mathbf{e}_J] \times \mathbf{J} = 0 .
\]

Here \([\cdots]\) and \(\{\cdots\}\) stand for the commutator and the anticommutator, respectively, and \(\hat{\mathbf{e}} = -\hbar [\omega_e \mathbf{e} + \omega_J \mathbf{e}_J, \hat{\mathbf{r}}]/2\). For \(\hbar \Omega_0^2 \ll E_F\), Eqs. (6) and (7) give the following system of equations for the perpendicular (with respect to \(\mathbf{e}\)) components of the electron and ion spins

\[
\frac{\partial \mathbf{s}}{\partial t} + (v_F n \nabla + i\Omega_0^2) (s + G s_0) = \delta (\mathbf{i} \mathbf{J} + \mathbf{e} \nabla \mathbf{J} ,)
\]

\[
\frac{\partial \mathbf{J}}{\partial t} + i \Omega_e \mathbf{J} - i \omega_J \mathbf{J} = i \delta_2 s_0 .
\]

Here \(s = s_y + i s_z, \mathbf{J} = \mathbf{J}_y + i \mathbf{J}_z, v_F\) is Fermi velocity, \(\mathbf{n} = (\cos \varphi, \sin \varphi)\), \(\varphi\) is the velocity angle in the well plane, \(\delta_1 = \alpha a_0 J_s^2/\hbar a, \delta_2 = 3\pi a_0 J_s^2/2h a, \xi = v_F/\Omega_e^0\) and \(s_0 = \beta_{\pi - \varphi} / 2 \pi\) \[\text{[13]}\] After Fourier transform of Eqs. (8), one can find the dispersion equation for the collective modes

\[
\sqrt{1 - \frac{v_F^2 k^2}{(\omega - \Omega_e^0)^2}} = \frac{\omega}{\omega - \Omega_e^0} \frac{\delta_1^2 + G \Omega_0^2 (\omega - \Omega_J)}{\delta_1^2 + \Omega_e (\omega - \Omega_J)} ,
\]

where \(\delta = \sqrt{\delta_1 \delta_2}\). For \(k = 0\), we get \[\text{[6, 11]}\]

\(\omega_{1,2}(0) = (\Omega_e + \Omega_J)/2 \pm \sqrt{(\Omega_e - \Omega_J)^2/4 + \delta^2}\). The anticrossing occurs when \(\Omega_e(B) = \Omega_J(B)\).

We see that the constant \(G\) drops out from \(\omega_{1,2}(0)\). In contrast, the dispersion of the collective modes strongly depends on the relation between \(|G|\) and the dimensionless parameter \(\delta/\Omega_e\). For \(\delta/\Omega_e \ll |G|\) (this was the case in the experiment \[\text{[6]}\]), the anti-crossing occurs for \(B < B_{res}\), when \(\Omega_e > \Omega_J\) (see Fig. 2a). To see this, one may consider the case \(\delta = 0\) (coupling between transverse components of electron and ion spins is turned off) as a first approximation. This approximation was implicitly used in Ref. \[\text{[6]}\]. For \(\delta = 0\), there are two branches of the spectrum (dashed lines in Fig. 2a), corresponding respectively to the Fermi-liquid spin waves with negative dispersion and the dispersionless excitations of the ion spins. Importantly, for \(B < B_{res}\) these two branches intersect each other. Turning on a finite coupling between modes, \(\delta \neq 0\), results in the anticrossing, which, for \(B\) close to \(B_{res}\), occurs at the point

\[
B_{res} = \sqrt{2|G| \Omega_e (\Omega_e - \Omega_J)} / v_F (1 + G) .
\]

Remarkably, \(B_{res}\) depends on \(B\), so that the anticrossing position may be tuned by the external field. The splitting between the modes for \(k \approx k_{res}\) is given by \(\omega_1(k_{res}) - \omega_2(k_{res}) \approx 2 \delta\).

As seen from Fig. 2a the upper branch of the spectrum at a certain wave vector \(k_m\) reaches the Stoner continuum (single-particle excitations), which is defined by inequality \(|\omega - \Omega_e^0| \leq v_F k\). For \(k > k_m\), the corresponding ion-type excitations slowly decay in time due to weak exchange coupling with the system of itinerant electrons.

This decay is similar to the well-known Landau damping in plasma \[\text{[12]}\], so that the decay rate \(\gamma\) is calculated in a quite analogous way, yielding

\[
\gamma \approx \frac{\delta^2 \Omega_e v_F \sqrt{k^2 - k_m^2}}{\Omega_e^0 (1 + G)^2 v_F^2 (k^2 - k_m^2) + G^2 \Omega_J^2} .
\]

As a function of \(k, \gamma\) has a maximum. The maximal value is given by \(\gamma_{max} = \delta^2/2 |G| \Omega_e \approx 3 \pi n e^2 s_{\pi} / 4 h |G| a\). Using data of Ref. \[\text{[2]}\] \((n_e = 0.7 \cdot 10^{11} \text{ cm}^{-2}, a = 80 \AA, \alpha = 1.5 \cdot 10^{-23} \text{ eV cm}^3\), \(|G| \approx 0.2 s_{\pi} \approx 0.2\) we find \(\gamma_{max} \approx 2 \cdot 10^9 \text{ s}^{-1}\).

Another interesting phenomenon arising due to simultaneous presence of two types of interaction is a change in a sign of the group velocity of the ion mode. It can be understood by analyzing the spectrum in the limit \(k \rightarrow 0\), when \(\omega_{1,2}(k) \approx \omega_{1,2}(0) + \beta k^2/2 \beta\). Here \(\beta = \Omega_e^0 |\omega_{1,2}(0) - \Omega_e^0| \approx (|\omega_{1,2}(0) - \Omega_e^0|)^2 + 2 \delta^2/|\omega_{1,2}(0)| \delta^2\), so that the dispersions of the modes are controlled
by signs of $\omega_1(0) - \Omega_e^0$ and $\omega_2(0) - \Omega_e^0$, respectively. As seen from Fig 1, there is a critical field $B = B_0$, at which $\omega_2(0) = \Omega_e^0$. For $B < B_0$, both spin-wave branches are below the Stoner continuum and have negative dispersions. While $B$ increases, approaching $B_0$, the ion spin-wave branch becomes shorter ($k_m \to 0$) and disappears when $B = B_0$. For $B > B_0$, this branch appears again above the Stoner continuum and has a positive dispersion as shown in Fig. 2b. The dispersion of one of the modes can also change sign for $\Omega_e > \Omega_J$, provided that $\delta/\Omega_e \gg |G|$.

Above we discussed the coupled collective modes. Now we notice that Eqs. 8 have also a solution $s = 0$ and $J = 0$. Importantly, the latter equation apart from the trivial solution $\mathbf{J}(r, z, t) = 0$ has also non-zero solutions obeying the constraint $\int dz \frac{d}{dz} |\Psi(z)|^2 = 0$. Such solutions were called "decoupled" modes 10. To find a number of such modes one should go beyond continuous approximation and replace integration over $dz$ in all above equations with the sum over $N$ atomic layers. This yields $N - 1$ decoupled modes, corresponding to independent solutions of the equation $\sum_m J_m |\Psi(z_m)|^2 = 0$, where $m$ numerates layers 10. All these modes are, indeed, decoupled from electron system provided that one neglects the equilibrium electron exchange field acting on the ion spins. In this approximation, we find from Eq. 7 that the modes have no dispersion and their frequencies coincide and are equal to $\Omega_J$. In fact, weak interaction with the electrons gives rise to a small splitting of the ion Zeeman energies, which become dependent on the atomic layer number $m$: $\hbar \omega_{Jm} = \hbar \Omega_J + \alpha n_e \sigma^z_m |\Psi(z_m)|^2$. Taking into account this splitting, one finds that in a symmetric quantum well, which we consider here, the decoupled modes with antisymmetric distribution of ion spins $J_m \propto \delta_{m,m_0} - \delta_{m,-m_0}$ still obey the condition $J = 0$, thus having no dispersion. For $m_0$–th mode, the ion spin precession frequency is equal to $\Omega_{Jm_0}$. As for the modes with a symmetric distribution, they become weakly coupled to the electron collective mode. One can show, however, that the corresponding dispersion is very weak provided that $n_e \approx n_J \ll 1$. Symmetric modes also become weakly coupled to the single electron excitations and, consequently, slowly decay in the region of the Stoner continuum with the characteristic rate $\delta_v \nu = \sqrt{k^2 - k_{m_0}^2}/\Omega_J N$.

To conclude, we have developed a theory of the spin waves in the 2D diluted magnetic semiconductors. We have described analytically two collective modes corresponding to the coupled excitations of the electron and ion spins, and a large number of decoupled excitations of the ion spins. Our main finding is the tunable anticrossing of the collective modes. We have also predicted a field-induced change in a sign of the group velocity of the ion mode and have calculated the decay of the waves in the Stoner continuum.

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