ANGULAR DISTRIBUTION OF PHOTONS IN COHERENT BREMSSTRAHLUNG IN DEFORMED CRYSTALS

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Abstract

We investigate the angular distribution of photons in the coherent bremsstrahlung process by high-energy electrons in a periodically deformed single crystal with a complex base. The formula for the corresponding differential cross-section is derived for an arbitrary deformation field. The case is considered in detail when the electron enters into the crystal at small angles with respect to a crystallographic axis. The results of the numerical calculations are presented for SiO$_2$ single crystal and Moliere parameterization of the screened atomic potentials in the case of the deformation field generated by the acoustic wave of S-type.

Keywords: Interaction of particles with matter; coherent bremsstrahlung; physical effects of ultrasonics.

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1 Introduction

The investigation of high-energy electromagnetic processes in crystals is of interest not only from the viewpoint of underlying physics but also from the viewpoint of practical applications. From the point of view of controlling the parameters of various processes in a medium, it is of interest to investigate the influence of external fields, such as acoustic waves, temperature gradient etc., on the corresponding characteristics. The considerations of concrete processes, such as diffraction radiation [1], transition radiation [2], parametric X-radiation [3], channelling radiation [4], bremsstrahlung by high-energy electrons [5], have shown that the external fields can essentially change the angular-frequency characteristics of the radiation intensities. Recently there has been broad interest to compact crystalline undulators with periodically deformed crystallographic planes an efficient source of high energy photons [6] (for a review with more complete list of references see [7]). The coherent bremsstrahlung of high-energy electrons moving in a crystal is one of the most effective methods to produce intense beams of highly polarized and monochromatic photons. Such radiation has a number of remarkable properties and at present it has found many important applications. This motivates the importance of investigations for various mechanisms of controlling the radiation parameters. In [5] we have discussed the influence of hypersonic waves excited in a crystal on the process of the bremsstrahlung of high-energy electrons. To have an essential influence of the acoustic wave high-frequency hypersonics is needed. Usually this type of waves is excited by high-frequency electromagnetic field through
the piezoelectric effect in crystals with a complex base. In the paper [5], we have generalized the results of [5] for crystals with a complex base and for acoustic waves with an arbitrary profile. For the experimental detection of final particles in the process of coherent bremsstrahlung it is important to know their angular distribution. In the present paper the angular distribution of photons in the coherent bremsstrahlung in crystals is investigated in the presence of hypersonic wave. The numerical calculations are carried out for the quartz single crystal and for the electrons of energy 20 GeV.

The paper is organized as follows. In the next section we derive the general formula for the coherent part of the bremsstrahlung cross-section averaged on thermal fluctuations and the conditions are specified under which the influence of the deformation field can be considerable. In Sec. 3 the analysis of the general formula is presented in the cases when the electron enters into the crystal at small angles with respect to crystallographic axes or planes and the results of the numerical calculations for the cross-section as a function of the angle between the momenta of electron and photon. Sec. 4 summarizes the main results of the paper.

2 Angular dependence of the cross-section

We consider the bremsstrahlung by high-energy electrons in a single crystal. The corresponding cross section can be obtained from the cross section for the bremsstrahlung on a single atom derived in [9]. The formulae [10] practically cannot be used for the investigation of bremsstrahlung in case of potentials of a complex form, for example in a crystal, as errors of approximations, admissible in the case of a single scattering center, can become inadmissible greater for many centers. By making use of the results derived in [9], the differential cross section for the bremsstrahlung on a single atom is presented in the form (the system of units $\hbar = c = 1$ is used)

$$
\frac{d^5\sigma_{\text{cob}}}{d\omega dq \| dq \perp dy} = \frac{e^2 q_1^2}{8\pi^4 e^2 q_0^2} \left[ 1 + \frac{\omega^2}{2\epsilon_1\epsilon_2} - 4y^2 \frac{\delta}{q_\|} \left( 1 - \frac{\delta}{q_\|} \right) \right] \frac{|u(q)|^2}{\sqrt{1 - y^2}}
$$

where $e$ is the electron charge, $\omega$, $\epsilon_1$, $\epsilon_2$, are the energies of photon, initial and final electrons respectively, $\delta = m_e^2 \omega / (2\epsilon_1\epsilon_2)$, $m_e$ is the mass of electron, $q_\|$ and $q_\perp$ are the components of the vector of momentum transfer $q$, $q = p_1 - p_2 - k$ ($k$, $p_1$, $p_2$ are the momenta of photon, initial and final electrons respectively), parallel and perpendicular to the direction of the electron momentum, $u(q)$ is the Fourier transform of the atom potential. The variable $y$ is expressed in terms of the angle $\theta_\gamma$ between the momenta $p_1$ and $k$ by the following relation:

$$
\left( \frac{e_1\theta_\gamma}{m_e} \right)^2 = \frac{1}{\delta} \left( q_\| - \delta + \frac{q_\perp^2}{2\epsilon_1} + \frac{q_\perp^2 \delta}{m_e^2} \right) + y \frac{2q_\perp}{m_e} \left( \frac{q_\|}{\delta} - 1 - \frac{q_\perp^2}{2\epsilon_1\delta} \right)^{\frac{1}{2}}.
$$

The regions of variables $q_\|, q_\perp, y$ in cross-section [11] are as follows [9]:

$$
q_\| \geq \delta + \frac{q_\perp^2}{2\omega}, \quad -1 \leq y \leq 1, \quad q_\perp \geq 0.
$$

The differential cross-section for the bremsstrahlung in a crystal can be written in the form

$$
\sigma(q, y) \equiv \frac{d^5\sigma_{\text{cob}}}{d\omega dq \| dq \perp dy} = \frac{1}{\sum_{n,j} u_n^{(j)} e^{iq_n^{(j)}}} \sigma_0(q, y),
$$

2
where \( \mathbf{r}_i^{(j)} \) is the position of an atom in the crystal. In the discussion that follows, the collective index \( n \) enumerates the elementary cell and the subscript \( j \) enumerates the atoms in a given cell of a crystal. Here \( \mathbf{q} \) is the momentum transferred to the crystal, \( \mathbf{q} = \mathbf{p}_1 - \mathbf{p}_2 - \mathbf{k} \) and the differential cross-section in a crystal given by \( (1) \), differs from the cross-section on an isolated atom by the interference factor which is responsible for coherent effects arising due to the periodical arrangement of the atoms in the crystal.

After averaging on thermal fluctuations, the cross-section is written in the form (see, for instance \( [5] \) for the case of a crystal with a simple cell)

\[
\sigma (\mathbf{q}, y) = \left\{ N \sum_{j} |u_0^{(j)}|^2 \left( 1 - e^{-q^2u_0^{(j)}r_i^{(j)}} \right) + \left| \sum_{n,j} u_0^{(j)} e^{i\mathbf{q}\cdot\mathbf{r}_i^{(j)}} e^{-\frac{1}{2}q^2u_0^{(j)2}} \right|^2 \right\} \sigma_0 (\mathbf{q}, y),
\]

where \( N \) is the number of cells, \( \mathbf{r}_{n0}^{(j)} \) determine the equilibrium positions of the atoms, \( u_0^{(j)}r_i^{(j)} \) is the temperature-dependent mean-squared amplitude of the thermal vibrations of the \( j \)th atom, \( e^{-q^2u_0^{(j)2}} \) is the corresponding Debye-Waller factor. In formula \( (5) \) the first term in figure braces does not depend on the direction of the vector \( \mathbf{k} \) and determines the contribution of incoherent effects. The contribution of coherent effects is presented by the second term. By taking into account the formula \( (1) \) for the cross-section on a single atom, in the region \( \epsilon_1 q^2 / \omega m_c^2 \ll 1 \) the corresponding part of the cross-section in a crystal can be presented in the form

\[
\sigma_c = \frac{\epsilon^2}{8\pi \epsilon_1} \frac{g^2}{q^2} \left[ 1 + \frac{\omega^2}{2\epsilon_1 \epsilon_2} - 4y^2 \frac{\delta}{q} \left( 1 - \frac{\delta}{q} \right) \right] \frac{1}{1 - y^2} \left| \sum_{n,j} u_0^{(j)} e^{i\mathbf{q}\cdot\mathbf{r}_i^{(j)}} e^{-\frac{1}{2}q^2u_0^{(j)2}} \right|^2 \]

When external influences are present (for example, in the form of acoustic waves) the positions of atoms in the crystal can be written as \( \mathbf{r}_{n0}^{(j)} = \mathbf{r}_{ne}^{(j)} + \mathbf{u}_0^{(j)} \), where \( \mathbf{r}_{ne}^{(j)} \) determines the equilibrium position of an atom in the situation without deformation, \( \mathbf{u}_0^{(j)} \) is the displacement of the atom caused by the external influence. We consider deformations with the periodical structure:

\[
\mathbf{u}_n^{(j)} = \mathbf{u}_0 f \left( \mathbf{k}_s, \mathbf{r}_{ne}^{(j)} \right)
\]

where \( \mathbf{u}_0 \) and \( \mathbf{k}_s \) are the amplitude and wave vector corresponding to the deformation field, \( f (x) \) is an arbitrary function with the period \( 2\pi \), \( \max f (x) = 1 \). In the discussion that follows, we assume that \( f (x) \in C^\infty (R) \). Further following \( [11] \) and having made similar steps we receive the following formula for the differential cross section of coherent bremsstrahlung:

\[
\frac{d^2 \sigma_c^s}{d\omega dy} = \frac{\epsilon^2 N}{\pi \epsilon_1 N_0 \Delta} \sum_{m,g} \frac{g_{m\perp}^2}{g_{m\parallel}^2} \left[ 1 + \frac{\omega^2}{2\epsilon_1 \epsilon_2} - 4y^2 \frac{\delta}{g_{m\parallel}} \left( 1 - \frac{\delta}{g_{m\parallel}} \right) \right] \frac{|F_m (\mathbf{g}_m \mathbf{u}_0)|^2 |S (\mathbf{g}_m, \mathbf{g})|^2}{\sqrt{1 - y^2}},
\]

where \( N_0 \) is the number of atoms in the crystal \( \mathbf{g}_m = \mathbf{g} - mk_s \), \( \mathbf{g} \) is the reciprocal lattice vector, \( -mk_s \) stands for the momentum transfer to the external field, \( g_{\parallel} \) and \( g_{\perp} \) are the components of the vector \( \mathbf{g}_m \). The function \( F_m (x) \) is defined by relation

\[
F_m (x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{ixf(t)-imt} dt, S (\mathbf{g}_m, \mathbf{g}) = \sum_j u_{q}^{(j)} e^{i\mathbf{g}_m \mathbf{r}_i^{(j)}} e^{-\frac{1}{2}g^2u_0^{(j)2}}
\]

is the factor determined by the structure of the elementary cell. For a lattice with a complex cell the coordinates of the atoms are written as \( r_{ne}^{(j)} = R_n + \rho_j \), with \( R_n \) being the positions of the atoms for one of primitive lattices, and \( \rho_j \) are the equilibrium positions for other atoms
inside $n$-th elementary cell with respect to $\mathbf{R}_n$. Now the relation between the variables $y$ and $\theta_\gamma$ is written in the form:

$$
y = \frac{m_e}{2g_{m\perp}} \left( \frac{\epsilon_1 \theta_\gamma/m_e}{\epsilon_1} \right)^2 - 1/\delta \left( g_{m\parallel} - \delta - g_{m\perp}/(2\epsilon_1) + g_{m\perp}^2/\delta^2 \right)
\left( g_{m\parallel}/\delta - 1 - g_{m\perp}/(2\epsilon_1\delta) \right)^{1/2} \quad (10)
$$

The regions of variables in cross-section [1] are

$$
g_{m\parallel} \geq \delta + \frac{g_{m\perp}^2}{2\epsilon_1}, \quad -1 \leq y \leq 1, \quad g_{m\perp} \geq 0. \quad (11)
$$

For sinusoidal deformation field, $f(z) = \sin(z + \varphi_0)$, one has the Fourier-transform

$$
F_m(x) = e^{im\varphi_0} J_m(x), \quad (12)
$$

with the Bessel function $J_m(x)$.

The formula for the pair creation in an undeformed crystal is obtained from [8] taking $u_0 = 0$. In this limit, the contribution of the term with $m = 0$ remains only with $F_0(0) = 1$. Now we see that formula [8] differs from the formula in an undeformed crystal by the replacement $g \to g_m$, and by the additional summation over $m$ with the weights $|F_m(g_mu_0)|^2$. This corresponds to the presence of an additional one-dimensional superlattice with the period $\lambda_s$ and the reciprocal lattice vector $m\mathbf{k}_s$, $m = 0, \pm 1, \pm 2, \ldots$. As the main contribution into the cross-section comes from the terms with $g_{m\parallel} \sim \delta$, the influence of the deformation field may be considerable if $|mk_{s\parallel}| \gtrsim \delta$. Combining this with the previous estimates, we find the following condition: $u_0/\lambda_s \gtrsim a/4\pi^2c$. At high energies one has $a/l_c \ll 1$ and this condition can be consistent with the condition $u_0/\lambda_s \ll 1$.

In the presence of the deformation field the number of possibilities to satisfy the condition $g_{m\parallel} \geq \delta + g_{m\perp}^2/(2\epsilon_1)$ in the summation of formula [8] increases due to the term $mk_{s\parallel}$ in the expression for $g_{m\parallel}$. This leads to the appearance of additional peaks in the angular distribution of the radiated positrons. After the integration of [8] over $y$, due to these additional peaks, there can be an enhancement of the cross-section of the process [8].

### 3 Limiting cases and numerical results

Now, we consider the most interesting case when the electron enters into the crystal at small angle $\theta$ with respect to the crystallographic $z$-axis of the orthogonal lattice. The corresponding reciprocal lattice vector components are $g_i = 2\pi n_i/a_i$, $n_i = 0, \pm 1, \pm 2, \ldots$, where $a_i$, $i = 1, 2, 3$, are the lattice constants in the corresponding directions. For the longitudinal component we can write

$$
g_{m\parallel} = g_{mz} \cos \theta + (g_{my} \cos \alpha + g_{mx} \sin \alpha) \sin \theta, \quad (13)
$$

where $\alpha$ is the angle between the projection of the vector $\mathbf{p}_1$ on the plane $(x, y)$ and axis $y$. For small angles $\theta$ the main contribution into the cross-section comes from the summands with $g_z = 0$. Having made the replacement of variable $y \to \epsilon_1 \theta_\gamma/m_e$ using the formula [10] from formula [8] one finds

$$
\frac{d^2\sigma_k^e}{d\omega d(\epsilon_1 \theta_\gamma/m_e)} \approx \frac{e^2 N}{\pi \epsilon_1^2 N_0} \sum_{m,g_m,g_y} \frac{g_{m\parallel}^2}{g_{m\perp}^2} \left[ 1 + \frac{\omega^2}{2\epsilon_1^2} - 4y^2(\theta_\gamma) \frac{\delta}{g_{m\parallel}} \left( 1 - \frac{\delta}{g_{m\parallel}} \right) \right]
\times |F_m(g_mu_0)|^2 |S(g_m, g)|^2 \frac{\epsilon_1 \theta_\gamma/m_e}{\sqrt{1 - y^2(\theta_\gamma)}} \frac{\epsilon_1 \theta_\gamma/m_e}{(g_{m\parallel}/m_e) \left( g_{m\parallel}/\delta - 1 - g_{m\perp}^2/(2\epsilon_1\delta) \right)^{1/2}}, \quad (14)
$$

4
where the notation \( y^2(\theta, \gamma) \) is introduced in accordance with:

\[
y^2(\theta, \gamma) = \frac{m^2_e}{4g^2} \left[ (\varepsilon_1\varepsilon_k/m_e)^2 - (1/\delta) \left( \frac{g_m^|| - \delta - g_m^2}{2\varepsilon_1} + \frac{g_m^2\delta}{m_e} \right) \right].
\]

(15)

In (14) \( g_m^2 = g_x^2 + g_y^2 \), and the summation goes over the region \( g_m^|| \geq \delta + g_m^2/(2\varepsilon_1) \), \( 0 \leq y^2(\theta, \gamma) \leq 1 \) with

\[
g_m^|| \approx -mk_z + (g_{mx} \sin \alpha + g_{my} \cos \alpha) \theta.
\]

(16)

Note that in the argument of the functions \( F_m \) and \( S \) we have \( g_m \approx (g_x, g_y, 0) \).

We now assume that the electron enters into the crystal at small angle \( \theta \) with respect to the crystallographic axis \( z \) and near the crystallographic plane \((y, z)\) (the angle \( \alpha \) is small). In this case with the change of \( \delta \), the sum over \( g_x \) and \( g_y \) will drop sets of terms which lead to the abrupt change of the corresponding cross-section. Two cases have to distinguish. Under the condition \( \delta \sim 2\pi\theta/a_2 \), in Eq. (14) for the longitudinal component, one has

\[
g_m^|| \approx -mk_z + \theta g_y \geq \delta + \frac{g^2}{2\varepsilon_1}.
\]

(17)

The formula (14) can be further simplified under the assumption \( u_0 \perp a_1 \). In this case, in the argument of the function \( F_m \), one has \( g_m u_0 \approx g_y u_0 y \) and we obtain the formula

\[
\frac{d^2\sigma^c}{d\omega d(\varepsilon_1\theta, \gamma/m_e)} \approx \frac{e^2N}{\pi^2\varepsilon_k N_0} \sum_{m_g, g_x, g_y} \frac{g_m^2}{g_m^||} \left[ 1 + \frac{\omega^2}{2\varepsilon_1\varepsilon_2} - 4y^2(\theta, \gamma) \frac{\delta}{g_m^||} \left( 1 - \frac{\delta}{g_m^||} \right) \right] \frac{|F_m(g_y u_0)|^2 |S(g_m, g)|^2}{\varepsilon_1\theta, \gamma/m_e} \frac{\varepsilon_1\theta, \gamma/m_e}{g_m^||} \left[ \frac{g_m^||}{\delta - 1 - g_m^2/(2\varepsilon_1\delta)} \right]^\frac{3}{2}.
\]

(18)

In the second case, we assume that \( \delta \sim 2\pi\theta_0/a_1 \). Now the main contribution into the sum in Eq. (14) comes from terms with \( g_y = 0 \) and summations remain over \( m \) and \( n_1 \), \( g_x = 2\pi n_1/a_1 \).

The formula for the cross-section takes the form

\[
\frac{d^2\sigma^c}{d\omega d(\varepsilon_1\theta, \gamma/m_e)} \approx \frac{e^2N}{\pi^2\varepsilon_k N_0} \sum_{m_n, g_y} \frac{g_m^2}{g_m^||} \left[ 1 + \frac{\omega^2}{2\varepsilon_1\varepsilon_2} - 4y^2(\theta, \gamma) \frac{\delta}{g_m^||} \left( 1 - \frac{\delta}{g_m^||} \right) \right] \frac{|F_m(g_m u_0)|^2 |S(g_m, g)|^2}{\varepsilon_1\theta, \gamma/m_e} \frac{\varepsilon_1\theta, \gamma/m_e}{g_m^||} \left[ \frac{g_m^||}{\delta - 1 - g_m^2/(2\varepsilon_1\delta)} \right]^\frac{3}{2},
\]

(19)

where

\[
g_m^|| \approx -mk_z + g_x \psi, \quad \psi = \alpha \theta.
\]

(20)

and the summation goes over the values \( m \) and \( n_1 \) satisfying the condition \( g_m^|| \geq \delta + g_x^2/(2\varepsilon_1) \).

We have carried out numerical calculations for the coherent bremsstrahlung cross-section for various values of parameters in the case of SiO\(_2\) single crystal at zero temperature. To deal with an orthogonal lattice, we choose as an elementary cell the cell including 6 atoms of silicon and 12 atoms of oxygen (Shrauf elementary cell [12]). For this choice the \( y \) and \( z \) axes of the orthogonal coordinate system \((x, y, z)\) coincide with the standard \( Y \) and \( Z \) -axes of the quartz crystal, whereas the angle between the axes \( x \) and \( X \) is equal to \( \pi/6 \). For the potentials of atoms we take Moliere parametrization with

\[
u^{(j)} = \sum_{i=1}^{3} \frac{4\pi Z_i e^2 \alpha_i}{q^2 + (\chi_i/R_j)^2}
\]

(21)
Figure 1: Coherent bremsstrahlung cross-section, $10^{-4}(m_e^2\omega/e^2)d\sigma_b^c/d\omega$, evaluated by the formula from ref. [8], as a function of $\omega/\epsilon_1$ for $2\pi u_0/a_1 = 0$ (dashed curve), $2\pi u_0/a_1 = 0.82$ (full curve), and as function of $2\pi u_0/a_1$ (right panel) for the photon energy corresponding to $\omega/\epsilon_1 = 0.055$. The values for the other parameters are as follows: $\psi = 0.00037$, $\epsilon_1 = 20 GeV$, $\nu_s = 5 \cdot 10^9$ Hz.

where $\alpha_i = \{0.1, 0.55, 0.35\}$, $\chi_i = \{6.0, 1.2, 0.3\}$, $R_j$ is the screening radius for the $j$-th atom in the elementary cell.

The calculations are carried out for the sinusoidal transversal acoustic wave of the S-type (the corresponding parameters can be found in Ref. [13]) for which the vector of the amplitude of the displacement is directed along $X$ direction of quartz single crystal, $u_0 = (u_0, 0, 0)$, and the velocity is $4.687 \cdot 10^5 cm/sec$. The vector determining the direction of the hypersound propagation lies in the plane $YZ$ and has the angle with the axis $Z$ equal to 0.295 rad. As the axis $z$ we choose the axis $Z$ of the quartz crystal. The corresponding function $F(x)$ is determined by formula [8]. We have numerically evaluated the pair creation cross-section by making use of formulae (19) for values of parameters $\epsilon_1, \psi, u_0$ taken from [8] when one has an enhancement of the cross-section.

Numerical calculation show, that in dependence of the values for parameters, the external excitation can either enhance or reduce the cross-section of the bremsstrahlung process. As an illustration of the enhancement in the cross-section integrated over the angle $\theta_\gamma$, on the left panel of Fig. 1 we have plotted the quantity $10^{-4}(m_e^2\omega/e^2)d\sigma_b^c/d\omega$, evaluated by using the formula from ref. [8], as a function of the ratio $\omega/\epsilon_1$ in the case of SiO$_2$ mono crystal and Moliere parametrization of the screened atomic potential for $2\pi u_0/a_1 = 0$ (dashed curve), $2\pi u_0/a_1 = 0.82$ (full curve). On the right panel the same quantity is plotted as a function of $2\pi u_0/a_1$ for the photon energy corresponding to $\omega/\epsilon_1 = 0.055$. The values for the other parameters are taken as follows: $\epsilon_1 = 20 GeV$, $\psi = 0.00037$, $\nu_s = 5 \cdot 10^9$ Hz for the frequency of acoustic waves. For the amplitude of the deformation field corresponding to the numerical data of Fig. 1 the relative displacement of the neighboring atoms is of the order $10^{-3} \AA$, which is much smaller than the interatomic distance ($\sim 5 \AA$). For these values of parameters, when one has an enhancement of the cross-section integrated over the angle $\theta_\gamma$, we have numerically analyzed the angular dependence of the bremsstrahlung cross-section by making use of formula (19). In Fig. 2 the quantity $10^{-4}(m_e^2\omega/e^2)d^2\sigma_b^c/d\omega d(\epsilon_1 \theta_\gamma/m_e)$ is depicted as a function of $\epsilon_1 \theta_\gamma/m_e$ in the case of SiO$_2$ monocystal for $u_0 = 0$ (dashed curve) and $2\pi u_0/a_1 = 0.82$ (full curve). The values for the other parameters are taken as follows: $\omega/\epsilon_1 = 0.055$, $\epsilon_1 = 20 GeV$, $\nu_s = 5 \cdot 10^9$ Hz, $\psi = 0.00037$. 
In order to see the dependence of the results on the energy of the incoming electron, in Fig. 3 we have presented the quantity $10^{-4}(m_e^2\omega/e^2)d\sigma^c_{\|}/d\omega$, evaluated by using the formula from ref. [8], as a function of the ratio $\omega/\epsilon_1$ in the case of SiO$_2$ mono crystal for $2\pi u_0/a_1 = 0$ (dashed curve), $2\pi u_0/a_1 = 0.85$ (full curve). On the right panel the same quantity is plotted as a function of $2\pi u_0/a_1$ for the photon energy corresponding to $\omega/\epsilon_1 = 0.0266$. The values for the other parameters are taken as follows: $\epsilon_1 = 10$ GeV, $\psi = 0.000364$, $\nu_s = 5 \cdot 10^9$ Hz for the frequency of acoustic waves.

In Fig 3 we have depicted the quantity $10^{-4}(m_e^2\omega/e^2)d^2\sigma^c_{\|}/d\omega d(\epsilon_1\theta_\gamma/m_e)$ as a function of $\epsilon_1\theta_\gamma/m_e$ in the case of SiO$_2$ mono crystal for $u_0 = 0$ (dashed curves), $2\pi u_0/a_1 = 0.85$ and for $\psi = 0.000364$. The values for the other parameters are as follows: $\omega/\epsilon_1 = 0.0266$, $\epsilon_1 = 10$ GeV, $\nu_s = 5 \cdot 10^9$ Hz for the frequency of acoustic waves.

As we see from the presented example, the presence of the deformation field leads to the appearance of additional peaks in the angular distribution of the emitted photon even for such ranges of values of an angle of photon momentum, where due to the requirement $-1 \leq y \leq 1$ the cross-section of process is zero when the deformation is absent. As we have already mentioned before, this is related to that in the presence of the deformation field the number of possibilities to satisfy the condition $g_{m\|} \geq \delta + g_{m\perp}^2/(2\epsilon_1)$ in the summation in formula (8) increases due to the presence of the additional term $mk_{s\|}$ in the expression for $g_{m\|}$.

Figure 2: Coherent bremsstrahlung cross-section, $10^{-4}(m_e^2\omega/e^2)d^2\sigma^c_{\|}/d\omega d(\epsilon_1\theta_\gamma/m_e)$, evaluated by formula (19), as a function of $\omega\theta_\gamma/m_e$ for $2\pi u_0/a_1 = 0$ (dashed curve), $2\pi u_0/a_1 = 0.82$ (full curve), $\psi = 0.00037$. The values for the other parameters are as follows: $\omega/\epsilon_1 = 0.055$, $\epsilon_1 = 20$ GeV, $\nu_s = 5 \cdot 10^9$ Hz for the frequency of acoustic waves.

4 Conclusion

The present paper is devoted to the investigation of the angular distribution of the electron in the bremsstrahlung process by high-energy electrons in a crystal with a complex lattice base in the presence of deformation field of an arbitrary periodic profile. The latter can be induced, for example, by acoustic waves. The influence of the deformation field can serve as a possible mechanism to control the angular-energetic characteristics of the created particles. The importance of this is motivated by that the coherent bremsstrahlung of high-energy electrons moving in a crystal is one of the most effective methods to produce intense beams of highly...
Figure 3: Coherent bremsstrahlung cross-section, $10^{-4}(m_e^2\omega/e^6)d\sigma_c^*/d\omega$, evaluated by the formula from ref. [8], as a function of $\omega/\epsilon_1$ for $2\pi u_0/a_1 = 0$ (dashed curve), $2\pi u_0/a_1 = 0.85$ (full curve), and as function of $2\pi u_0/a_1$ (right panel) for the photon energy corresponding to $\omega/\epsilon_1 = 0.0266$. The values for the other parameters are as follows: $\psi = 0.000364$, $\epsilon_1 = 10 GeV$, $\nu_s = 5 \cdot 10^9$ Hz.

Figure 4: Coherent bremsstrahlung cross-section, $10^{-4}(m_e^2\epsilon_1/e^6)d^2\sigma_c^*/d\omega d(\epsilon_1 \theta_\gamma/m_e)$, evaluated by formula (19), as a function of $\omega\theta_\gamma/m_e$ for $2\pi u_0/a_1 = 0$ (dashed curve), $2\pi u_0/a_1 = 0.85$ (full curve), $\psi = 0.000364$. The values for the other parameters are as follows: $\omega/\epsilon_1 = 0.0266$, $\epsilon_1 = 10 GeV$, $\nu_s = 5 \cdot 10^9$ Hz for the frequency of acoustic waves.
polarized and monochromatic photons. In a crystal the cross-section is a sum of coherent and incoherent parts. The coherent part of the cross-section per single atom, averaged on thermal fluctuations, is given by formula (8). In this formula the factor $|F_m(g m u_0)|^2$ is determined by the function describing the displacement of the atoms due to the deformation field, and the factor $|S(g m, g)|^2$ is determined by the structure of the crystal elementary cell. Compared with the cross-section in an undeformed crystal, formula (8) contains an additional summation over the reciprocal lattice vector of the one-dimensional superlattice induced by the deformation field. We have argued that the influence of the deformation field on the cross-section can be remarkable under the condition $4\pi^2 u_0/a \gtrsim \lambda_s/l_c$. Note that for the deformation with $4\pi^2 u_0/a > 1$ this condition is less restrictive than the naively expected one $\lambda_s \leq l_c$. The role of coherence effects in the pair creation cross-section is essential when the photon enters into the crystal at small angles with respect to a crystallographic axis. In this case the main contribution into the coherent part of the cross-section comes from the crystallographic planes, parallel to the chosen axis (axis $z$ in our consideration). The behavior of this cross-section as a function on the positron energy essentially depends on the angle $\theta$ between the projection of the electron momentum on the plane $(x, y)$ and $y$-axis. When the electron enters into the crystal near a crystallographic plane, two cases have to be distinguished. For the first one $\theta \sim a_2/2\pi l_c$ the formula (14) is further simplified to the form (18) under the assumption $u_0 \perp a_1$. In the second case one has $\psi = \alpha \theta \sim a_1/2\pi l_c$, and the main contribution into the cross-section comes from the crystallographic planes parallel to the incidence plane. The corresponding formula for the cross-section takes the form (19).

The numerical calculations for the cross-section are carried out in the case of SiO$_2$ single crystal with the Moliere parametrization of the screened atomic potentials and for the deformation field generated by the transversal acoustic wave of $S$-type with frequency 5 GHz. Examples of numerical results are depicted in figures. The numerical calculations for values of the parameters in the problem when one has an enhancement of the cross-section show that the presence of the deformation field leads to the appearance of additional peaks in the angular distribution of the radiated photon even for such ranges of values of an angle of a photon, where due to the requirement $-1 \leq y \leq 1$ the cross-section is zero when deformation is absent. This can be used to control the parameters of the photon sources.

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