Recent observations have revealed that our universe is flat and in a phase of acceleration (Riess et al. 1998; Perlmutter et al. 1999; Spergel et al. 2003). It implies that some mysterious dark energy dominantly fills the universe at the present epoch, exerting antibigravity. The nature of this mysterious dark energy, which holds a key to understanding the ultimate fate of the universe, is often specified by its equation of state, i.e., the ratio of its pressure to density: \( w \equiv P_{de}/\rho_{de} \). The antibigravity corresponds to the negative value of \( w \). The simplest candidate for the dark energy is the vacuum energy \( (\Lambda) \) with \( w = -1 \) that is constant at all times (Einstein 1917). Although all current data are consistent with the vacuum energy model (e.g., Wang & Tegmark 2004; Jassal et al. 2004; Percival 2005; Guzzo et al. 2008), the notorious failure of the theoretical estimate of the vacuum energy density (see Caroll et al. 1992, for a review) has led a dynamic dark energy model to emerge as an alternative. In this dynamic dark energy model, which is often collectively called quintessence, the dark energy is described as a slowly rolling scalar field with a time-varying equation of state.

The following observables have so far been suggested to discriminate the dark energy models: the luminosity-distance measure of a type Ia supernova (Riess et al. 2004, 2007; Davis et al. 2007; Kowalski et al. 2008); the abundance of galaxy clusters as a function of mass (Wang & Steinhardt 1998; Haiman et al. 2001; Wellers et al. 2002); the baryonic acoustic oscillations in the galaxy power spectrum (Blake & Glazebrook 2003; Hu & Haiman 2003; Cooray 2004; Seo & Eisenstein 2005), and the weak gravitational lensing effect (Hu 1999; Huterer 2001; Takada & Jain 2004; Song & Knox 2004). True as it is that these observables can powerfully constrain the value of \( w \), it is still quite necessary and important to find out as many different observables as possible for consistency tests.

Another possible observable for a dark energy constraint may be the shapes of the cosmic voids. As the voids behave like bubbles due to their extremely low densities, their shapes determined by the spatial distribution of the void galaxies tend to change sensitively according to the competition between the tidal distortion and the gravitational rarefaction effect. Therefore, the shape evolution of the voids must depend sensitively on the background cosmology. In this Letter, we study the ellipticity evolution of cosmic voids in the quintessence + cold dark matter (QCDM) model with the help of the analytic formalism developed by Park & Lee (2007) and explore the possibility of using it as a complementary probe of the dark energy equation of state.

According to Park & Lee (2007), the shape of a void region is related to the eigenvalues of the local tidal shear tensor as

\[
\lambda_1(\mu, v) = \frac{1 + (\delta_v - 2)v^2 + \mu^2}{(\mu^2 + v^2 + 1)},
\]

\[
\lambda_2(\mu, v) = \frac{1 + (\delta_v - 2)\mu^2 + v^2}{(\mu^2 + v^2 + 1)},
\]

where \( \{\lambda_i\}_{i=1}^3 \) (with \( \lambda_1 > \lambda_2 > \lambda_3 \)) are the three eigenvalues of the local tidal field smoothed on the void scale, \( \delta_v \) is the density contrast threshold for the formation of a void: \( \delta_v = \sum_{i=1}^3 \lambda_i \), and \( \{\mu, v\} \) (with \( v < \mu \)) represents a set of the two parameters that quantify the anisotropic distribution of the void galaxies. They defined the void ellipticity as \( \varepsilon \equiv 1 - v \) and evaluated its probability density distribution as

\[
p(1 - \varepsilon; z) = p(v; z, R_L) = \int_v^1 p(\mu, v|\delta = \delta_v; \sigma(z, R_L))d\mu
\]

\[
= \frac{3375\sqrt{2}}{10\pi\sigma^2(z, R_L)} \exp \left[ -\frac{5\delta_v^2}{2\sigma^2(z, R_L)} + \frac{15\delta_v(\lambda_1 + \lambda_2)}{2\sigma^2(z, R_L)} \right] \times \exp \left[ -\frac{15(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3\lambda_1\lambda_2 - \lambda_1\lambda_3 - \lambda_2\lambda_3)}{2\sigma^2(z, R_L)} \right] \times \frac{(\lambda_1 - \lambda_2)(\lambda_1 + 2\lambda_2 - \delta_v)}{(\mu^2 + v^2 + 1)^3}. \tag{3}
\]

Here, \( \sigma(z, R_L) \equiv D^2(z) \int_{-\infty}^\infty \Delta^2(k)W^2(kR_L)d\ln k \) is the linear rms fluctuation of the matter density field smoothed on a Lagrangian void scale of \( R_L \) at redshift \( z \) where \( D(z) \) is the linear growth factor, \( W(kR_L) \) is a top-hat window function, and \( \Delta^2(k) \) is the dimensionless linear power spectrum. Throughout this study, we adopt the linear power spectrum of the cold dark matter (CDM) cosmology that does not depend explicitly on \( w \) (Bardeen et al. 1986).

Equation (3) was originally derived under the assumption of a ΛCDM model (\( w = -1 \)). We propose here that it also holds for the case of a QCDM model where the dark energy equation of state changes with time as \( w(z) = w_0 + \omega_a(z)/(1 + z) \).
(Chevallier & Polarski 2001; Linder 2003) where \( w_0 \) is the value of \( w \) at present epoch and \( w_a \) quantifies how the dark energy equation of state changes with time. Then, we employ the following approximation formula for the linear growth factor, \( D(z) \), for a QCDM model (Basilakos 2003; Percival 2005):

\[
D(z) = \frac{5\Omega_m}{2(z + 1)} \left[ \Omega_m^w - \Omega_Q + \left( 1 + \frac{\Omega_m}{2} \right)(1 + A\Omega_Q) \right]^{-1},
\]

where

\[
E^2(z) = \Omega_m(1 + z)^3 + \Omega_Q(1 + z)^{−\frac{f(z)}{w}},
\]

\[
f(z) = -3(1 + w_0) - \frac{3w_a}{2\ln(1 + z)},
\]

\[
\alpha = \frac{3}{5 - 2/(1 - w)} + \frac{3}{125} \frac{(1 - w)(1 - 3w/2)}{(1 - 6w/5)^3}(1 - \Omega_m),
\]

\[
A = -\frac{0.28}{w + 0.08} - 0.3.
\]

The CDM density parameter \( \Omega_m \) and the dark energy density parameter \( \Omega_Q \) evolve with \( z \) respectively as

\[
\Omega_m(z) = \frac{\Omega_m(1 + z)^3}{E^2(z)}, \quad \Omega_Q(z) = \frac{\Omega_Q0}{E^2(z)(1 + z)^{\frac{f(z)}{w}}},
\]

where \( \Omega_m0 \) and \( \Omega_Q0 \) represent the present values. Equation (3) implies that the mean ellipticity of voids decreases with \( z \). A key question is how the rate of the decrease changes with the dark energy equation of state. Since most of the recent observations indicate that the dark energy equation of state at present epoch is consistent with \( w = −1 \) (e.g., see Guzzo et al. 2008, and references therein) we focus on how the mean void ellipticity depends on the value of \( w_a \). Even in case that \( w_0 = −1 \), if \( w_a \) is found to deviate from zero, it would imply the dynamic dark energy, disproving the simple ΛCDM model.

To explore how the void ellipticity evolution depends on \( w_a \), we evaluate the mean ellipticity of voids as

\[
\bar{\epsilon}(z) = \int_0^1 \epsilon(z) \, d\epsilon \text{ for different values of } w_a \text{ through Equations (3)–(9)\). The other key cosmological parameters are set at } \Omega_m = 0.75, \Omega_Q = 0.75, h = 0.73, \sigma_8 = 0.9, \text{ and } w_0 = -1. \text{ When the abundance of evolution of galaxy clusters is used to constrain the dark energy equation of state, the cluster mass is usually set at a certain threshold, } M_E, \text{ defined as the mass within a certain comoving radius (Wang & Steinhardt 1998). Likewise, we set the Lagrangian scale of a void, } R_L, \text{ at } 4h^{-1} \text{ Mpc, which is related to the mean effective radius of a void as } R_E = (1 + \delta_c)^{-1/3} R_L/(1 + z). \text{ The Lagrangian scale } R_L = 4h^{-1} \text{ Mpc corresponds to the mean effective size of a void at present epoch, } R_E \sim 8.5h^{-1} \text{ Mpc.}\)

Figure 1 plots \( \bar{\epsilon}(z) \) for the four different cases: \( w_a = -1/3, \ 0, \ 1/3, \text{ and } 2/3 \) (long-dashed, solid, dashed, and dotted line, respectively). As can be seen, the higher the value of \( w_a \), the more rapidly \( \bar{\epsilon}(z) \) decreases. It also suggests that \( \bar{\epsilon}(z) \) is well approximated as a linear function of \( z \) in recent epochs \((0 < z < 0.2)\). Therefore, we fit \( \epsilon(z) \) to a straight line as

\[
\bar{\epsilon}(z) \approx A_v z + B_v.
\]

Varying the value of \( w_a \) in the range of \([0, 2/3]\), we compute the best-fit slope \( A_v \). The range, \( 0 \leq w_a \leq 2/3 \), corresponds to the dark energy equation of state range, \(-1 \leq w \leq -0.9\). The result is plotted in Figure 2. As can be seen, the void ellipticity evolves more rapidly as the value of \( w_a \) increases. That is, the void ellipticity undergoes a stronger evolution when the antigravitational effect is less strong in recent epochs. Note that \( A_v \) shows a noticeable 30% difference as the dark energy equation of state changes \( w \) from \(-1 \) to \(-0.9\).

We have so far neglected the parameter degeneracy between \( w \) and the other key parameters. However, as the dependence of the void ellipticity distribution on the dark energy equation of state comes from its dependence on \( \Delta^2(k; \Omega_m0, \sigma_8, h, w) \), it...
is naturally expected that there should be a strong parameter degeneracy. Here, we focus on the degeneracy between $\Omega_{m0}$ and $w_0$. First, we recompute $A_v$, varying the values of $\Omega_{m0}$ and $w_0$ with setting $w_{a} = 1/3$. The left panel of Figure 3 plots a family of the degeneracy curves in the $\Omega_{m0}$–$w_0$ plane for the three different values of $A_v$. As can be seen, there is a strong degeneracy between the two parameters. For a given value of $A_v$, the value of $w_0$ increases as the value of $\Omega_{m0}$ decreases. A similar trend is also found in the $\Omega_{m0}$–$w_{a}$ degeneracy curves that are plotted in the right panel of Figure 3 for which the value of $w_0$ is set at $-1$. It is worth noting that this degeneracy trend is orthogonal to that found from the cluster abundance evolution (see Figure 3 in Wang & Steinhardt 1998). Thus, when combined with the cluster analysis, the void ellipticity analysis may be useful to break the degeneracy between $\Omega_{m0}$ and $w$.

We have shown that the void ellipticity evolution is in principle a useful constraint of the dark energy equation of state. We have also shown that it provides a new degeneracy curve for $\Omega_{m0}$ and $w$. When combined with the cluster abundance analysis, it should be useful to break the degeneracy. Furthermore, unlike the mass measurement of high-z clusters which suffers from considerable scatters, the void ellipticities are readily measured from the positions of the void galaxies without requiring any additional information.

To use our analytic tool in practice to constrain the dark energy equation of state, however, it will require to account for the redshift distortion effect since the positions of the void galaxies are measured in redshift space. In our companion paper (D. Park & J. Lee 2009, in preparation), we have analyzed the Millennium Run Redshift-Space catalog (Springel et al. 2005) and determined the ellipticity distribution of the galaxy voids. From this analysis, it is somewhat unexpectedly found that the void ellipticity distribution measured in redshift space is hardly changed from the one in real space. In fact, this result is consistent with the recent claims of Hoyle & Vogeley (2002) and that of R. van de Weygaert (2008, private communication) who have already pointed out that the redshift distortion effect has only negligible, if any, effect on the shapes of voids. We hope to constrain the dark energy equation of state by applying our theoretical tool to real observational data and report the result elsewhere in the near future.

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REFERENCES

Bardeen, J. M., Bond, J. R., Kaiser, N., & Szalay, A. S. 1986, ApJ, 304, 15
Basilakos, S. 2003, ApJ, 590, 636
Blake, C., & Glazebrook, K. 2003, ApJ, 594, 665
Caldwell, R. R., Dave, R., & Steinhardt, P. J. 1998, Phys. Rev. Lett., 80, 1582
Carroll, S., Press, W. H., & Turner, E. C. 1992, Ann. Rev., 30, 499
Chevallier, M., & Polarski, D. 2001, Int. J. Mod. Phys. D, 10, 213
Cooray, A. 2004, MNRAS, 348, 250
Davis, T. M., et al. 2007, ApJ, 666, 716
Einstein, A. 1917, Sitz. Preuss. Akad. Wiss., 142, 121
Guazzo, L., et al. 2008, Nature, 451, 31
Haiman, Z., Mohr, J. J., & Holder, G. P. 2001, ApJ, 553, 545
Hoyle, F., & Vogeley, M. S. 2002, ApJ, 566, 641
Hu, W. 1999, ApJ, 522, L21
Hu, W., & Haiman, Z. 2003, Phys. Rev. D, 68, 063004
Huterer, D. 2001, Phys. Rev. D, 65, 63001
Jassal, H. K., Bagla, J. S., & Padmanabhan, T. 2004, MNRAS, 356, L11
Kowalski, M., et al. 2008, ApJ, 686, 749
Linder, E. 2003, Phys. Rev. Lett., 90, 091301
Park, D., & Lee, J. 2007, Phys. Rev. Lett., 98, 081301
Percival, W. J. 2005, A&A, 819, 830
Perlmutter, S., et al. 1999, ApJ, 517, 565
Riess, A. G., et al. 1998, ApJ, 116, 1009
Riess, A. G., et al. 2004, ApJ, 607, 665
Riess, A. G., et al. 2007, ApJ, 659, 98
Seo, H., & Eisenstein, D. J. 2005, ApJ, 633, 575
Song, Y. S., & Knox, L. 2004, Phys. Rev. D, 70, 063510
Spergel, D. N., et al. 2003, ApJS, 148, 175
Springel, V., et al. 2005, Nature, 435, 629
Takada, M., & Jain, B. 2004, MNRAS, 348, 897
Wang, L., & Steinhardt, P. J. 1998, ApJ, 508, 483
Wang, Y., & Tegmark, M. 2004, Phys. Rev. Lett., 92, 241302
Weller, J., Battye, R. A., & Kneissl, R. 2002, Phys. Rev. Lett., 88, 231301