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Probing the Spins of Supermassive Black Holes with Gravitational Waves from Surrounding Compact Binaries

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Abstract

Merging compact black hole (BH) binaries are likely to exist in the nuclear star clusters around supermassive BHs (SMBHs), such as Sgr A*. They may also form in the accretion disks of active galactic nuclei. Such compact binaries can emit gravitational waves (GWs) in the low-frequency band (0.001–1 Hz) that are detectable by several planned space-borne GW observatories. The angular momentum vector of the compact binary (\(L_{in}\)) may experience significant variation due to the frame-dragging effect associated with the spin of the SMBH. The dynamical behavior of \(L_{in}\) can be understood analytically as a resonance phenomenon. We show that the rate of change of \(L_{in}\) encodes the information on the spin of the SMBH. Therefore, detecting GWs from compact binaries around SMBHs, particularly the modulation of the waveform associated with the variation of \(L_{in}\), can provide a new probe of the spins of SMBHs.

Unified Astronomy Thesaurus concepts: Gravitational wave sources (677); Black hole physics (159); Compact binary stars (283)

1. Introduction

The spins of the supermassive black holes (SMBHs) at the centers of galaxies are poorly constrained; this is the case even for Sgr A* in the Galactic Center (e.g., Ghez et al. 1998, 2008; Genzel et al. 2010). The spin vector of an accreting SMBH could in principle be constrained by modeling the accretion/radiation processes (e.g., Mościbrodzka et al. 2009; Broderick et al. 2011; Shcherbakov et al. 2012) and comparing with observations, such as those of Sgr A* and M87 from Event Horizon Telescope (e.g., Dexter et al. 2010; Broderick et al. 2016; Event Horizon Telescope Collaboration et al. 2019). The Galactic Center hosts a population of young massive stars (e.g., Genzel et al. 2000; Merritt 2013; Alexander 2017); it has been suggested that the relativistic frame-dragging effect on their orbits could put constraints on Sgr A*’s spin (e.g., Levin & Beloborodov 2003; Fragione & Loeb 2020).

Given the fact that the S stars around Sgr A* are close to the SMBH (~0.01 pc, with the newly discovered S4714 orbit reaching a pericenter distance of 12.6 au; Gillessen et al. 2017; Gravity Collaboration et al. 2019; Peißker et al. 2020), it is likely that binaries of compact objects could be present near Sgr A* (e.g., Antonini & Perets 2012; Stephan et al. 2019). Similar compact binaries may also exist in nuclear star clusters around other SMBHs (e.g., O’Leary et al. 2009; Hopman 2009; Leigh et al. 2018) and/or form in disks of active galactic nuclei (e.g., McKernan et al. 2012; Bartos et al. 2017; Tagawa et al. 2020). These compact binaries may radiate gravitational waves (GWs) in the low-frequency band (0.001–1 Hz), which can be detectable by the planned/conceived space-borne GW observatories (e.g., Randall & Xianyu 2019b; Hoang et al. 2019; Deme et al. 2020), including LISA (e.g., Amaro-Seoane et al. 2017), TianQin (e.g., Luo et al. 2016), Taiji (e.g., Hu & Wu 2017), and other detectors.

2. Compact Binary near a Spinning SMBH

We consider a binary with masses \(m_1, m_2\), semimajor axis \(a_{in}\), and eccentricity \(e_{in}\), moving around an SMBH tertiary (\(m_3\)) on a wider orbit with \(a_{out}\) and \(e_{out}\). The angular momenta of the inner and outer binaries are denoted by \(L_{in}\) and \(L_{out}\), respectively. Gravitational perturbation from the SMBH makes the inner binary precess and may also induce Lidov–Kozai oscillations if the mutual inclination between \(L_{in}\) and \(L_{out}\) is sufficiently high. The relevant timescale is

\[
t_{LK} \approx \left( \frac{1}{\Omega_{LK}} \right) = \frac{1}{n_{in} m_3} \left( \frac{a_{out}}{a_{in}} \sqrt{1 - e_{out}^2} \right)^3 \frac{1}{m_2} \left( \frac{Gm_1}{a_{in}} \right)^{1/2},
\]

where \(n_{12} = n_1 + m_2\), and \(n_{in} = (Gm_1/a_{in})^{1/2}\) is the mean motion of the inner binary.

The first-order post-Newtonian (PN) theory introduces pericenter precession in both inner and outer binaries. In particular, the precession of the inner orbit competes with \(\Omega_{LK}\) and plays a
crucial role in determining the maximum eccentricity $e_{\text{max}}$ in LK oscillations (e.g., Fabrycky & Tremaine 2007; Liu et al. 2015).

Because the tertiary mass $m_3$ is much larger than the masses of the inner binary, $m_3 \gg m_1, m_2$, several general relativity (GR) effects involving the SMBH can generate extra precessions on the binary orbits and qualitatively change the dynamics (e.g., Naoz et al. 2013; Will 2014; Liu et al. 2019; Liu & Lai 2020). In a systematical post-Newtonian framework of triple dynamics (e.g., Will 2014; Fang et al. 2019b; Lim & Rodriguez 2020), there are numerous terms. We summarize the most essential effects below (also the leading-order effects). The related equations are either from the classical work on binaries with spinning bodies (e.g., Barker & O’Connell 1975) or can be derived (or extended to include eccentricity) “by analogy,” i.e., by recognizing that the inner binary’s orbital angular momentum $L_{in}$ behaves like a “spin” (Liu et al. 2019; Liu & Lai 2020). As we see below, the vector forms of the equations we use are much more transparent than the equations based on orbital elements (see Will 2014; Lim & Rodriguez 2020), especially when dealing with misaligned $L_{in}$, $L_{out}$, and $S_3$. Here, we consider the double-averaged (DA; averaging over both the inner and outer orbital periods) approximation and present the secular equations of $L_{in}$ in vector forms. The coupled eccentricity equations for various GR effects are summarized in Appendix A.

(i) **Effect I: Precession of $L_{out}$ around $S_3$.** If the SMBH is rotating (with the spin angular momentum $S_3 = \chi_3 G m_3^2 / c$, where $\chi_3 \leq 1$ is the Kerr parameter), $L_{out}$ experiences precession around $S_3$ due to spin–orbit coupling if the two vectors are misaligned (1.5 PN effect) (e.g., Barker & O’Connell 1975; Fang & Huang 2019a):

$$\frac{dL_{out}}{dt} \bigg|_{L_{in}S_3} = \Omega_{L_{in}S_3} \dot{S}_3 \times L_{out},$$

with

$$\Omega_{L_{in}S_3} = \frac{GS_3(4 + 3m_2/m_3)}{2c^2a^3_{out}(1 - e^2_{out})^{3/2}}.$$  

For the binary–SMBH system, $S_3$ can be easily much larger than $L_{out}$, and we can assume $\dot{S}_3 = \text{constant}$.

(ii) **Effect II: Precession of $L_{in}$ around $L_{out}$.** In addition to the Newtonian precession (driven by the tidal potential of $m_3$), $L_{in}$ experiences an additional de Sitter–like (geodesic) precession in the gravitational field of $m_3$ introduced by GR (1.5 PN effect), such that the net precession is governed by

$$\frac{dL_{in}}{dt} \bigg|_{L_{in}L_{out}} = \Omega_{L_{in}L_{out}} \dot{L}_{out} \times L_{in},$$

with

$$\Omega_{L_{in}L_{out}} = -\frac{1}{2} \Omega_{L_{in}L_{out}}^{(N)}(\dot{L}_{out} \cdot \dot{L}_{in}) + \Omega_{L_{in}L_{out}}^{(GR)},$$

where

$$\Omega_{L_{in}L_{out}}^{(N)} = \frac{3}{4} \Omega_{L_{in}L_{out}}^{(N)} + \frac{3}{4} \Omega_{L_{in}L_{out}}^{(GR)} = \frac{3}{2c^2a^3_{out}(1 - e^2_{out})^{3/2}}.$$  

Note that the Newtonian contribution to the precession rate ignores octuple and higher-order terms; this is justified because dynamical stability of the triple requires $a_{out} \gg a_{in}$ when $m_3 \gg m_{1,2}$. For the GR part, high-order contributions to $\Omega_{L_{in}L_{out}}^{(GR)}$ can be found in, e.g., Will (2014) and Lim & Rodriguez (2020).

(iii) **Effect III: Precession of $L_{in}$ around $S_3$.** Because the semimajor axis of the inner orbit ($a_{in}$) is much smaller than the outer orbit ($a_{out}$), the inner binary can be treated as a single body approximately. Thus, the angular momentum $L_{in}$ is coupled to the spin angular momentum $S_3$ of $m_3$ and experiences Lens–Thirring precession (2 PN effect):

$$\frac{dL_{in}}{dt} \bigg|_{L_{in}S_3} = \Omega_{L_{in}S_3} \dot{S}_3 \times L_{in} - 3\Omega_{L_{in}S_3}(\dot{L}_{out} \cdot \dot{S}_3)\dot{S}_3 \times L_{in},$$

where

$$\Omega_{L_{in}S_3} = \frac{GS_3}{2c^2a^3_{out}(1 - e^2_{out})^{3/2}}.$$  

Note that because the tertiary companion studied here is an SMBH, the binary embedded in the nuclear star cluster might also be influenced by the “environmental” effects, including binary evaporation (e.g., Binney & Tremaine 1987), resonant relaxation (e.g., Hamers et al. 2018), and nonspherical mass distribution (e.g., Petrovich & Antonini 2017). All these effects operate on timescales much longer than considered here and can be safely ignored.

3. Analytic Understanding of the $L_{in}$ Evolution for the Circular Inner Binary

In Liu et al. (2019), we have examined how various GR effects modify the LK eccentricity evolution/growth of the inner binary and enhance the merger rate in the LIGO band. In Liu & Lai (2020), we have studied eccentricity growth due to apsidal precession resonance in nearly coplanar triple systems. Here, we focus on the dynamics of the inner binary far from merger, which radiates GW at a low-frequency band instead. Note that the evolution of $L_{in}$ is more sensitive to the SMBH spin than the orbital eccentricity. This is because the eccentricity excitation can be completely suppressed if the mutual inclination angle lies outside the LK window or the binary is relatively far away from the SMBH.

To develop an analytic understanding of the dynamics of the binary–SMBH system, we first consider the case where the inner binary remains circular throughout the evolution. Because $L_{out}$ rotates around $\dot{S}_3$ at a constant rate, $\Omega_{L_{in}S_3}$, it is useful to consider the evolution of $\dot{L}_{in}$ in the frame corotating with $L_{out}$; combining Equations (4), (2), and (6), we have

$$\frac{dL_{in}}{dt}_{\text{rot}} = [(\Omega_{L_{in}L_{out}} - 3\Omega_{L_{in}S_3}(\dot{L}_{out} \cdot \dot{S}_3))\dot{L}_{out} + (\Omega_{L_{in}S_3} - \Omega_{L_{in}S_3}S_3) \times L_{in}].$$

The corresponding Hamiltonian is

$$\mathcal{H} = -\frac{1}{2}\Omega_{L_{in}L_{out}}^{(N)}(\dot{L}_{in} \cdot \dot{L}_{out})^2 + \Omega_{L_{in}L_{out}}^{(GR)}(\dot{L}_{in} \cdot \dot{L}_{out})$$

$$+ (\Omega_{L_{in}S_3} - \Omega_{L_{in}S_3}) (\dot{L}_{in} \cdot \dot{S}_3)$$

$$- 3\Omega_{L_{in}S_3}(\dot{L}_{out} \cdot \dot{S}_3) \times \dot{L}_{in}. $$

We set up a coordinate system with $\hat{z} = \dot{L}_{out} \times \dot{S}_3$, and let $L_{in} = \sin\alpha (\cos\phi \hat{x} + \sin\phi \hat{y}) + \cos\phi \hat{z}$, where $\alpha$ is the angle between $\dot{L}_{out}$ and $\dot{S}_3$, and $l$ is the angle...
between $\tilde{L}_{\text{in}}$ and $\tilde{L}_{\text{out}}$. The (dimensionless) Hamiltonian becomes
\[
\hat{H} = \frac{\mathcal{H}}{\Omega_{\text{in-out}}} = -\frac{1}{2} \cos^2 I + \lambda \cos I
- \frac{3}{4} \eta (2 \cos \alpha \cos I + \sin \alpha \sin I \cos \varphi),
\]
where we have introduced the dimensionless ratios
\[
\lambda = \frac{\Omega^{(\text{GR})}_{\text{in-out}}}{\Omega^{(N)}_{\text{in-out}}}, \quad \eta = \frac{\Omega_{\text{S}S_{\text{in}}}}{\Omega_{\text{L}L_{\text{out}}}},
\]
and have used $\Omega_{\text{L}_{\text{s}S_{\text{in}}}}/\Omega^{(N)}_{\text{L}L_{\text{out}}} = (4 + 3m_2/m_3)^{-1} \simeq 1/4$. Note that $\cos I$ and $\cos \varphi$ are canonical variables.

Depending on the ratio of $\lambda$ and $\eta$, we expect three possible behaviors: (i) for $|\lambda \pm 1| \gg \eta$ (“adiabatic”), $\tilde{L}_{\text{in}}$ closely follows $\tilde{L}_{\text{out}}$, maintaining an approximately constant $I$; (ii) for $|\lambda \pm 1| \ll \eta$ (“nonadiabatic”), $\tilde{L}_{\text{in}}$ effectively precesses around $\tilde{S}_I$ with constant $\theta_{\text{L}S_{\text{in}}}$ (the angle between $\tilde{L}_{\text{in}}$ and $\tilde{S}_I$); (iii) when $|\lambda \pm 1| \sim \eta$ (“transadiabatic”), a resonance behavior of $\tilde{L}_{\text{in}}$ may occur, and a large orbital inclination $I$ can be generated.

Figure 1 presents the parameter space indicating the relative importance of various GR effects for compact BH binaries (BHBs) around Sgr A*. In the left panel, we see that for BHBs in a frequency band ($10^4 - 10^7$ Hz), $\tilde{L}_{\text{in}}$ evolves irregularly on the secular timescale ($\sim 10^6$ yr). The “nonadiabatic” parameter regime ($\lambda \ll \eta$) is not allowed for realistic systems because of the stability criterion and the effect of ISCO (innermost stable circular orbit). As $\tilde{a}_{\text{out}}$ increases, $\lambda/\eta$ increases, and the dynamics of $\tilde{L}_{\text{in}}$ transitions from “transadiabatic” to “adiabatic.” Thus, the resonance behavior

\[
\lambda = \frac{\Omega^{(\text{GR})}_{\text{in-out}}}{\Omega^{(N)}_{\text{in-out}}}, \quad \eta = \frac{\Omega_{\text{S}S_{\text{in}}}}{\Omega_{\text{L}L_{\text{out}}}}.
\]

Figure 1. Parameter space in the $a_{\text{in}}$ vs. $a_{\text{out}}$ plane indicating the relative importance of various GR effects for the system parameters are $m_1 = 20 M_s$, $m_2 = 10 M_s$, $m_3 = 4 \times 10^5 M_s$, $e_{\text{in}} = 0$, $e_{\text{out}} = 0.5$, and $\chi_3 = 1$. The dark blue region corresponds to dynamically unstable triple systems (the instability limit according to Kiseleva et al. 1996), and the light blue region indicates the innermost stable circular orbits (ISCO) for the outer binary, with $a_{\text{out}} \lesssim 9 R_s = 9(Gm_3/c^2)$. In the left panel, the solid lines show different values of $\lambda$ and $\eta$ (evaluated using Equation (11)), and the dashed lines characterize the frequency of GWs emitted by the inner binary. In the right panel, the solid (dashed) lines are obtained by setting the relevant timescales (Equations (1), (3), (5), and (7)) to 10 yr (1 yr).

4. General Case

The BHB may experience eccentricity growth through LK oscillations when the inclination $I$ is sufficiently large. The precession of $\tilde{L}_{\text{out}}$ around $\tilde{S}_I$ can increase the inclination window of eccentricity excitation (e.g., Liu et al. 2019). In this situation, the finite eccentricity of the inner binary increases the GW strain, which affects the overall signal-to-noise ratio and improves the detectability (e.g., Randall & Xianyu 2019b; Hoang et al. 2019; Deme et al. 2020).

The left panels of Figure 4 show the maximum values of $|d\tilde{L}_{\text{in}}/dt|$ and $|d\tilde{L}_{\text{out}}/dt|$ and the maximum eccentricity $e_{\text{max}}$ as a function of the initial inclination $I_0$ for the same system parameters as in Figure 2(A). We see that the absence of spin effects (purple dots), the maximum rates and eccentricity are uniquely determined by $I_0$, and $|d\tilde{L}_{\text{in}}/dt|$ agrees with Equation (12) for a wide range of $I_0$ even when the inner binary develops eccentricities (this arises because $|d\tilde{L}_{\text{in}}/dt|$ is achieved when $e_{\text{in}} \simeq 0$ during the LK cycles). However, with the inclusion of the SMBH spin effects (cyan dots), the eccentricity excitation window widens (e.g., Liu et al. 2019), and there can be a finite spread of the maximum values of $|d\tilde{L}_{\text{in}}/dt|$ and $|d\tilde{L}_{\text{out}}/dt|$ for each $I_0$. Note that for systems with

\[\eta = 0.74 \text{ and } \lambda = 0.57,\]

Figure 2 (panel A) illustrates how to obtain variations of the orbit on relatively short timescales ($\lesssim 10$ yr; see the right panel of Figure 1), the BHB cannot be too far away from the SMBH (i.e., $a_{\text{out}} \lesssim 50$ au).
constant
L \parallel q

The eccentricity excitation can be more significant if the BHB is closer to the SMBH. At the same time, the precession timescales become shorter, and a wide range of variations of the BHB orbit can be potentially captured during the observational span of a few years. Panel (B) of Figure 2 shows the example with \( a_{\text{out}} = 10 \) au. We see that because of the SMBH spin effects, the normal periodic oscillations in \( e \) and \( \cos l \) transform into irregular oscillations. In this case, the
“circular” Hamiltonian (Equation (10)) no longer applies. Instead, as indicated in panel (D) of Figure 2, a large degree of scatter fills up the phase space and the variation of \( \mathbf{L}_{\text{in}} \) becomes chaotic (see Appendix B).

5. Summary and Discussion

We have studied the effects of the spin of SMBH (such as Sgr A*) on the evolution of the orbital axis (\( \mathbf{L}_{\text{in}} \)) of the surrounding compact binaries. We find that for typical BHBs \( (m_1 \sim 20M_{\odot} \text{ and } m_2 \sim 10M_{\odot}) \) that are close to the SMBH (\( a_{\text{out}} \lesssim 50 \text{ au} \)), \( \mathbf{L}_{\text{in}} \) may experience complex (and even resonant) evolution, leading to a significant variation of \( \mathbf{L}_{\text{in}} \). For the BHBs that remain circular during the evolution, this variation of \( \mathbf{L}_{\text{in}} \) can be calculated analytically. We show that a spinning SMBH can greatly influence the variation of \( \mathbf{L}_{\text{in}} \) (even for the circular BBHs), increasing \( |d\mathbf{L}_{\text{in}}/dt| \) significantly compared to the case of a nonspinning SMBH. The maximum \( |d\mathbf{L}_{\text{in}}/dt| \) can reach many tens of degrees per year for BBHs emitting GWs in the low-frequency band \( (10^{-3} - 10^{-1} \text{ Hz}) \). Such rapid variation of \( \mathbf{L}_{\text{in}} \) therefore provides a probe on the mass and spin of the SMBH.

The SMBH spin can also affect other types of compact binaries, including neutron star binaries and white dwarf binaries. Although there is no direct observational evidence for their existence near Sgr A*, massive stars with distances within ~13 au from Sgr A* are known (e.g., Gillessen et al. 2017; Gravity Collaboration et al. 2019; Peißker et al. 2020), and it is plausible to expect stellar binaries their remnants to exist at such distances. In addition, various dynamical processes can lead to the enhanced production of compact binaries around SMBHs, including gravitational bremsstrahlung (e.g., O’Leary et al. 2009), mass segregation (e.g., Antonini & Rasio 2016; Leigh et al. 2018; Fragione & Sari 2018; Sari & Fragione 2019; Arca Sedda 2020), scatters via eccentric disks (e.g., Gennaro & Madigan 2020), and tidal/GW capture (e.g., Chen & Han 2018).

For the detectability in GWs, the types of compact binaries studied here are luminous low-frequency GW sources in the Galaxy (e.g., the signal-to-noise ratio \( \geq 37 \) with LISA’s sensitivity; see also Appendices C and D). Because the orbital period of the outer binary is much shorter than the duration of GW detection, the system parameters can be well constrained through a Doppler phase shift (e.g., Inayoshi et al. 2017; Randall & Xianyu 2019a). Detecting the GW signal containing the signature of the frame-dragging effect is more challenging. A recent study (Yu & Chen 2021) (which ignores the frame-dragging effect) found that the regular precession of \( \mathbf{L}_{\text{in}} \) around \( \mathbf{L}_{\text{out}} \) due to Newtonian torques can be measurable if the precession period is less than the observation time. Similar detectability is expected to apply for the nonregular evolution of \( \mathbf{L}_{\text{in}} \) discussed in this paper. If the system happens to be observed during the time when \( |d\mathbf{L}_{\text{in}}/dt| \) is significantly enhanced, the effect would be more “visible.”

To conclude, our proof-of-concept calculations demonstrate that the SMBH spin can have a large imprint on the BHB waveforms. A joint detection of multiple compact binary systems may be necessary to reduce the degeneracy of the GW signals on various parameters and provide sufficient constraints on the SMBH spin. Future studies on detailed strategy to measure the SMBH spin using low-frequency GWs from compact binaries would be of great value.

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Appendix A

GR Effects Due to a Rotating SMBH

We summarize the most essential GR effects for the BHB–SMBH triple system below. The related equations follow from the DA (averaging over both the inner and outer orbital periods) approximation.

(i) Effect I: Precession of \( \mathbf{L}_{\text{out}} \) around \( S_3 \). In the BHB–SMBH system, the angular momentum of the outer binary \( \mathbf{L}_{\text{out}} \) and the spin angular momentum \( S_3 \) of \( m_3 \) precesses around each other due to spin–orbit coupling if the two vectors are misaligned (1.5 PN effect (e.g., Barker & O’Connell 1975; Fang & Huang 2019a):

\[
\frac{d\mathbf{L}_{\text{out}}}{dt}\bigg|_{\mathbf{L}_{\text{in}}=S_3} = \Omega_{\mathbf{L}_{\text{out}}=S_3} \mathbf{S}_3 \times \mathbf{L}_{\text{out}}, \quad (A1)
\]

\[
\frac{d\mathbf{e}_{\text{out}}}{dt}\bigg|_{\mathbf{L}_{\text{in}}=S_3} = \Omega_{\mathbf{L}_{\text{out}}=S_3} \mathbf{S}_3 \times \mathbf{e}_{\text{out}} - 3\Omega_{\mathbf{L}_{\text{out}}=S_3} (\mathbf{L}_{\text{out}} \cdot \mathbf{S}_3) \mathbf{L}_{\text{out}} \times \mathbf{e}_{\text{out}}, \quad (A2)
\]

\[
\frac{d\mathbf{S}_3}{dt}\bigg|_{\mathbf{L}_{\text{in}}=\mathbf{L}_{\text{out}}} = \Omega_{\mathbf{L}_{\text{out}}=\mathbf{L}_{\text{out}}} \mathbf{L}_{\text{out}} \times \mathbf{S}_3, \quad (A3)
\]

where the orbit-averaged precession rates are

\[
\Omega_{\mathbf{L}_{\text{out}}=S_3} = \frac{GS_3(4 + 3m_2/m_3)}{2c^2a_{\text{out}}^3(1 - e_{\text{out}}^2)^{3/2}} = \Omega_{\mathbf{L}_{\text{in}}=\mathbf{L}_{\text{out}}}. \quad (A4)
\]

Because in our case \( S_3 \) can be easily larger than \( \mathbf{L}_{\text{out}} \), the de Sitter precession (Equation (A3)) is negligible.

(ii) Effect II: Precession of \( \mathbf{L}_{\text{in}} \) around \( \mathbf{L}_{\text{out}} \). In addition to the Newtonian precession (driven by the tidal potential of \( m_3 \)), \( \mathbf{L}_{\text{in}} \) experiences an additional de Sitter–like (geodesic) precession in the gravitational field of \( m_3 \) introduced by GR. This is a 1.5 PN spin–orbit coupling effect, with \( \mathbf{L}_{\text{in}} \) behaving like a “spin.” We have

\[
\frac{d\mathbf{L}_{\text{in}}}{dt}\bigg|_{\mathbf{L}_{\text{out}}=\mathbf{L}_{\text{in}}} = \Omega_{\mathbf{L}_{\text{out}}=\mathbf{L}_{\text{in}}} \mathbf{L}_{\text{in}} \times \mathbf{L}_{\text{in}}, \quad (A5)
\]

\[
\frac{d\mathbf{e}_{\text{in}}}{dt}\bigg|_{\mathbf{L}_{\text{out}}=\mathbf{L}_{\text{in}}} = \Omega_{\mathbf{L}_{\text{out}}=\mathbf{L}_{\text{in}}} \mathbf{L}_{\text{out}} \times \mathbf{e}_{\text{in}}, \quad (A6)
\]

and the feedback from \( \mathbf{L}_{\text{in}} \), \( \mathbf{e}_{\text{in}} \) on \( \mathbf{L}_{\text{out}} \) and \( \mathbf{e}_{\text{out}} \) are given by (e.g., Barker & O’Connell 1975)

\[
\frac{d\mathbf{L}_{\text{out}}}{dt}\bigg|_{\mathbf{L}_{\text{in}}=\mathbf{L}_{\text{out}}} = \Omega_{\mathbf{L}_{\text{out}}=\mathbf{L}_{\text{in}}} \mathbf{L}_{\text{in}} \times \mathbf{L}_{\text{out}}, \quad (A7)
\]

\[
\frac{d\mathbf{e}_{\text{out}}}{dt}\bigg|_{\mathbf{L}_{\text{in}}=\mathbf{L}_{\text{out}}} = \Omega_{\mathbf{L}_{\text{out}}=\mathbf{L}_{\text{in}}} \mathbf{L}_{\text{in}} \times \mathbf{e}_{\text{out}} - 3\Omega_{\mathbf{L}_{\text{out}}=\mathbf{L}_{\text{in}}} (\mathbf{L}_{\text{out}} \cdot \mathbf{L}_{\text{in}}) \mathbf{L}_{\text{out}} \times \mathbf{e}_{\text{out}}, \quad (A8)
\]
with
\[
\Omega_{\text{L}_{\text{out-in}}}^{(\text{GR})} = \frac{3}{2} \frac{G(m_3 + \mu_{\text{out}}/3)n_{\text{out}}}{c^2 a_{\text{out}}(1 - e_{\text{out}}^2)} = \frac{\Omega_{\text{L}_{\text{out-in}}}^{(\text{L})}}{L_{\text{in}}}, \tag{A9}
\]

where \(n_{\text{out}} = (Gm_3/a_{\text{out}}^3)^{1/2}\).

(iii) Effect III: Precession of \(L_{\text{in}}\) around \(S_3\). Because the semimajor axis of the inner orbit \(a_{\text{in}}\) is much smaller than the outer orbit \(a_{\text{out}}\), the inner binary can be treated as a single body approximately. Therefore, the angular momentum \(L_{\text{in}}\) is coupled to the spin angular momentum \(S_3\) of \(m_3\), and experiences Lens–Thirring precession. This is a 2 PN spin–spin coupling effect, with \(L_{\text{in}}\) behaving like a “spin.” We have

\[
\frac{dL_{\text{in}}}{dt} \bigg|_{S_3, \text{in}} = \Omega_{S_3, \text{in}} \hat{S}_3 \times L_{\text{in}} - 3\Omega_{S_3, \text{in}} (\hat{L}_{\text{out}} \cdot \hat{S}_3) \hat{L}_{\text{out}} \times L_{\text{in}}, \tag{A10}
\]

\[
\frac{de_{\text{in}}}{dt} \bigg|_{S_3, \text{in}} = \Omega_{S_3, \text{in}} \hat{S}_3 \times e_{\text{in}} - 3\Omega_{S_3, \text{in}} (\hat{L}_{\text{out}} \cdot \hat{S}_3) \hat{L}_{\text{out}} \times e_{\text{in}}. \tag{A11}
\]

The backreaction on the outer binary gives (see Equations (64), (65), and (70) of Barker & O’Connell 1975)

\[
\frac{dL_{\text{out}}}{dt} \bigg|_{S_3, \text{in}} = -3\Omega_{S_3, \text{in}} \left[ (\hat{L}_{\text{out}} \cdot \hat{L}_{\text{in}}) \hat{S}_3 + (\hat{L}_{\text{out}} \cdot \hat{S}_3) \hat{L}_{\text{in}} \right] \times L_{\text{out}}, \tag{A12}
\]

\[
\frac{de_{\text{out}}}{dt} \bigg|_{S_3, \text{in}} = -3\Omega_{S_3, \text{in}} \left[ (\hat{L}_{\text{out}} \cdot \hat{L}_{\text{in}}) \hat{S}_3 + (\hat{L}_{\text{out}} \cdot \hat{S}_3) \hat{L}_{\text{in}} ight] \times e_{\text{out}}, \tag{A13}
\]

In the above, the orbit-averaged precession rates are

\[
\Omega_{L_{\text{in}}, S_3} = \frac{GS_3}{2c^2 a_{\text{out}}^3 (1 - e_{\text{out}}^2)^{3/2}} = \Omega_{S_3, \text{in}} \frac{L_{\text{out}}}{L_{\text{in}}}. \tag{A14}
\]

Appendix B

Modified Evolution of \(L_{\text{in}}\) Due to SMBH Spin

We explore the effect of the SMBH spin on the variation of orbital axis \(\hat{L}_{\text{in}}\) and eccentricity \(e_{\text{in}}\), taking into account the full range of misalignment angle \(\alpha\) (and \(\chi_3\)). The results are shown in Figures 5, 6, and 7.

Figure 5. Similar to Figure 4 in the main text, but we consider various values of \(\alpha\) (as indicated). The fifth row shows the ratio \(|\hat{L}_{\text{in,max}}(\chi_3 = 1)/\hat{L}_{\text{in,max}}(\chi_3 = 0)|\) as a function of \(\iota_{\text{in}}\), where the shaded region is obtained from the distribution of \(|\hat{L}_{\text{in,max}}(\chi_3 = 1)|\) shown in the fourth row, and the solid line corresponds to the mean value.
Figure 6. Similar to Figure 5, but for the outer binary with $a_{\text{out}} = 10$ au.

Figure 7. Values of the maximum rate of change of $|\dot{L}|$ in the $\chi_3 - \alpha$ plane. The system parameters are from the example in Figure 2(A) in the main text. For each combination of $(\chi_3, \alpha)$, we carry out 100 integrations with a uniform distribution of $I_0$ and plot the median values of $|d\dot{L}_{\text{in}}/d\theta|_{\text{max}}$. The two lines (gray dashed and black solid) specify $|d\dot{L}_{\text{in}}/d\theta|_{\text{max}} = 42^\circ$/yr, $45^\circ$/yr, respectively.
Appendix C
Signal-to-noise Ratio

An individual binary generates a GW strain composed of discrete harmonics

\[ h(t) = \sum_{n=1}^{\infty} h_n(f_n) e^{2\pi i f_n t}, \]  

(C1)

where \( n \) is the number of the harmonics. The frequency harmonic is \( f_n = n f_{\text{orb}}, f_{\text{orb}} = \sqrt{Gm_1m_2/\alpha^3}/2\pi \), and

\[ h_n(f_n) = \frac{2}{n} \sqrt{g(n, e_{\text{in}})} h_0, \]  

(C2)

here, the function \( g(n, e_{\text{in}}) \) is given by

\[ g(n, e_{\text{in}}) = \frac{n^4}{32} \left[ (J_{n-2} - 2e_{\text{in}}J_{n-1} + \frac{2}{n}J_n \right. \\
+ 2e_{\text{in}}J_{n+1} - J_{n+2})^2 + (1 - e_{\text{in}}^2) \times (J_{n-2} - 2J_n + J_{n+2})^2 + \left. \frac{4}{3n^2} \right] \]  

(C3)

with \( J_n \equiv J_n(x) \) the \( n \)th Bessel function evaluated at \( x = ne_{\text{in}} \).

Note that we have introduced the rms strain amplitude for the circular orbit at distance \( D \):

\[ h_0 = \frac{\sqrt{32/5}}{\sqrt{G^2 m_1m_2}} = \frac{1}{\sqrt{5 D_{\text{in}}}}. \]  

(C4)

The prefactor \( \sqrt{32/5} \) accounts for rms averaging the GW strain over inclination. Because we only consider the BHB mergers in our Milky Way, Equation (C4) has no redshift dependence.

The signal-to-noise ratio \( (S/N)^2 \) is evaluated by

\[ (S/N)^2 = \int_{0}^{\infty} \frac{4|\tilde{h}(f)|^2}{S_n(f)} df, \]  

(C5)

here

\[ \tilde{h}(f) = \sum_{n=1}^{\infty} h_n(f_n) T_{\text{obs}} \sin[\pi (f - f_n) T_{\text{obs}}]. \]  

(C6)

\( T_{\text{obs}} \) is the observation time, and \( S_n(f) \) is the full strain spectral sensitivity density. Here, we consider the LISA instrumental noise and confusion noise from the unresolved galactic binaries (e.g., Robson et al. 2019).

If the system has an orbital decay timescale much longer than the LISA mission time, we have

\[ (S/N)^2 = \sum_{n=1}^{\infty} \frac{4|\tilde{h}_n(f_n)|^2 T_{\text{obs}}}{f_n S_n(f_n)}. \]  

(C7)

Also, we can define the characteristic strain as

\[ h_{c,n} = 2h_n(f_n) \sqrt{T_{\text{obs}}}. \]  

(C8)

Equations (C7)–(C8) suggest that \( S/N \) can be enhanced by a factor of \( \sqrt{T_{\text{obs}}} \).

We find that for the GW sources \( (m_1 = 20 \, M_\odot, m_2 = 10 \, M_\odot, \text{and } d_{\text{in}} = 0.01 \, \text{au}) \), the total \( S/N \) is greater than 37 if we assume the observation time \( T_{\text{obs}} = 5 \, \text{yr} \) and the distance \( D = 8 \, \text{kpc} \). An elliptic inner orbit can enhance the detectability, increasing the overall \( S/N \) by a factor of 10 (or even more), as shown in Figure 8.

Figure 8. The GW strain curve of BH binary \( (m_1 = 20 \, M_\odot, m_2 = 10 \, M_\odot, \text{and } d_{\text{in}} = 0.01 \, \text{au}) \). Different orbital eccentricities are taken into account (as labeled). The red curves are obtained by Equation (C8) with a series of \( n \), and the spectral sensitivity density of LISA is from Robson et al. (2019).

Appendix D
Modified GW Waveforms

As the compact binary precesses, both amplitude and phase of the GW waveform can be modified. The signature of the time-varying orientation \( \hat{L}_{\text{in}} \) can be extracted by projecting the GW radiation onto the antenna (detector) coordinates (e.g., Yu & Chen 2021).

Considering the circular inner orbit, the waveform is expressed in terms of frequency:

\[ \tilde{W}(f) = \tilde{\Lambda}(f) \tilde{W}_C(f), \]  

(D1)

where \( \tilde{W}_C(f) \) is the antenna-independent “carrier,” which is a function of the chirp mass, distance, time and phase of coalescence, and

\[ \tilde{\Lambda}(f) = \sqrt{\Lambda_1^2(t) \Lambda_2^2(t) + \Lambda_3^2(t) \Lambda_4^2(t)} \times \exp \{ -i[\Phi_p(t) + 2\Phi_T(t) + \Phi_D(t)] \}. \]  

(D2)

Here, we have introduced the amplitude terms

\[ A_+(t) = 1 + \hat{L}_{\text{in}}(t) \cdot \hat{N}, \]  

(D3)

\[ A_-(t) = -2\hat{L}_{\text{in}}(t) \cdot \hat{N}, \]  

(D4)

where \( \hat{N} \) is the direction of the line of sight, and \( F_{\pm}(\chi)(t) \) is the antenna pattern coefficient. For the phase terms, \( \Phi_p \) shows the polarization phase

\[ \Phi_p(t) = \arctan \left[ \frac{A_-(t) \hat{L}^*_2(t)}{A_+(t) \hat{L}^*_1(t)} \right], \]  

(D5)

\( \Phi_T \) characterizes the Thomas precession:

\[ \Phi_T(t) = -\int dt \left[ \frac{\hat{L}^*_1(t) \cdot \hat{N}}{1 - (\hat{L}^*_1(t) \cdot \hat{N})^2} (\hat{L}^*_1(t) \times \hat{N}) \cdot \frac{d\hat{L}^*_1(t)}{dt} \right], \]  

(D6)

and \( \Phi_D \) is the Doppler phase induced by the motions of the outer orbit (and/or the detector orbiting around the Sun).

Therefore, the change of the \( \hat{L}_{\text{in}} \) orientation can be tracked for the GW source with sufficient \( S/N \). Yu & Chen (2021)
explored the detectability of the precessing $\hat{L}_m$ around $\hat{L}_m$. In
their study, the Newtonian precession and the de Sitter–like
precession (Effect II here) are taken into account and the
 evolution of $\hat{L}_m$ is regular. It is suggested that the orientation
change is measurable if the precession timescale is less than the
observation time. Similarly, because the spin-induced precess-
sion timescale we considered is comparable to or less than 10
yr, the frame-dragging effect in the orbital plane might also be
detected through the modified GW waveform—the only thing
is $\hat{L}_m$ evolves in a nonregular way.

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