Approaching the Dark Sector through a bounding curvature criterion

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ABSTRACT

Understanding the observations of dynamical tracers and the trajectories of lensed photons at galactic scales within the context of General Relativity (GR), requires the introduction of a hypothetical dark matter dominant component. The onset of these gravitational anomalies, where the Schwarzschild solution no longer describes observations, closely corresponds to regions where accelerations drop below the characteristic $a_0$ acceleration of MOND, which occur at a well established mass-dependent radial distance, $R_M$. At cosmological scales, inferred dynamics are also inconsistent with GR and the observed distribution of mass. The current accelerated expansion rate requires the introduction of a hypothetical dark energy dominant component. We here show that for a Schwarzschild metric at galactic scales, the scalar curvature, $K$, multiplied by the area function, both at the critical MOND transition radius, has an invariant value of $\kappa_B = K \times A = 192 \pi a_0^2/c^4$. Further, assuming this condition holds for $r > R_M$, is consistent with the full spacetime which under GR corresponds to a dominant isothermal dark matter halo, to within observational precision at galactic level. For a FLRW metric, this same constant bounding curvature condition yields for a flat spacetime a cosmic expansion history which agrees with the $\Lambda$CDM concordance model for recent epochs, and which similarly tend to a de Sitter solution having a Hubble constant consistent with current inferred values. Thus, a simple covariant purely geometric condition identifies the low acceleration regime of observed gravitational anomalies, and can be used to guide the development of modified gravity theories at both galactic and cosmological scales.

Key words: gravitation — galaxies: kinematics and dynamics — cosmology: theory

1 INTRODUCTION

Within GR, the gravitational anomalies detected at galactic scales are viewed as indirect evidence for a dominant dark matter halo. Such an approach is generally consistent with observations and leads to the $\Lambda$CDM paradigm at galactic and cosmological scales. However, over the last year alone, LHC results eliminating simple super-symmetric candidates (CMS collaboration 2016), the astrophysical searches for dark matter annihilation signals being consistent with no dark matter signal (e.g. Fermi-LAT and DES collaborations 2014 searching for dark matter annihilation signals in local dwarf galaxies reporting results consistent with expected backgrounds), and various recent direct detection experiments returning only ever stricter exclusion limits (e.g. Yang et al. 2016 yielding no dark matter signal from the PANDAX-II experiment, ruling out previous claims, and Szydagis et al. 2016 for the LUX and LZ collaborations returning also zero dark matter signal), encourage the sustained search for alternative gravity theories where the need for dark, undetected, hypothetical components might be eliminated.

The first such attempt to identify that ‘dark matter’ gravitational anomalies occur only (and always) at acceleration scales below a critical value was MOND, Milgrom (1983). Under MOND and MONDian theories, the principal features are a change in gravity for acceleration scales below $a_0 = 1.2 \times 10^{-10} m/s$, where centrifugal equilibrium velocities (and local velocity dispersion for pressure supported systems) about a point mass $M$, become constant at a value of $V_{TF} = (GM_0)^{1/4}$.

MONDian dynamics have been reported at scales of wide binary stars (Hernandez et al. 2012), Galactic globular clusters (e.g. Scarpa et al. 2003, Hernandez et al. 2017),
local dwarf spheroidal galaxies (e.g. Hernandez et al. 2010, Lüghausen et al. 2014), elliptical galaxies (e.g. Jiménez et al. 2013, Tian & Ko 2016), spiral galaxies (e.g. McGaugh & de Blok 1998, Lelli 2016) and the overall velocity dispersion profiles and radial acceleration relation across over 7 orders of magnitude in mass for galactic systems (e.g. Dabringhausen et al. 2016, Lelli et al. 2017, Durazo et al. 2017).

In MOND and similar theories (e.g. Bekenstein 2004, Moffat & Toth 2008, Zhao & Famaey 2010, Capozziello & De Laurentis 2011, Mendoza et al. 2013) the transition in regimes for gravity is generally introduced by hand, with no clear fundamental physical principle identified as the source of this regime transition. Further, the identification at low velocities of a critical transition acceleration as the MOND $a_0$ constant seems problematic in terms of constructing fundamental underlying covariant theories, where acceleration always appears as frame dependent. At cosmological scales, the numerical coincidence of $a_0 \approx cH_0$ also remains unexplained, with a relation between $a_0$ and the cosmological constant sometimes being suggested (e.g. Verlinde 2016). We here show that a covariant and purely geometric condition, the product of an area, sometimes being suggested (e.g. Verlinde 2016). W e explained, with a relation between $a_0$ and scalar curvature, $K$, being constant, unequivocally identifies the transition to the low acceleration MONDian regime. Moreover, keeping this condition fixed beyond this transition is consistent with the empirically determined metric coefficients identified with a dominant dark matter halo under GR and Newtonian physics.

Lastly, it is remarkable that the same geometric condition imposed upon a FLRW flat universe yields an asymptotically de Sitter universe with an expansion rate accurately tracing the LCDM concordance fit for recent epochs. It is therefore suggested that the geometric condition presented here provides important clues as to the origin of the regime transition for gravity towards low accelerations, and must be satisfied by any extended theories of gravity attempting to explain observed gravitational anomalies in the absence of dark ad hoc entities.

Section (2) shows the invariance of the product of the Kretschmann scalar times the area function when evaluated at the MOND transition radius. Extending this $\kappa_B = K \times A = cte.$ condition to $r > R_M$ under a spherically symmetric and static metric is shown to be consistent with the metric potentials required by rotation curve and lensing inferences, this time in the absence of any dark matter, in section (3). In section (4) we explore the cosmological implications of the constant bounding curvature criterion by applying it to a flat FLRW universe, and obtain an asymptotic de Sitter solution with an expansion factor compatible with current observations. Our conclusions appear in section (6).

2 BOUNDING CURVATURE INVARIANCE AT THE MOND TRANSITION RADIUS FOR SCHWARZSCHILD SPACETIME

As is well known, the gravitational anomalies often attributed to the presence of a dominant dark matter halo at galactic scales, appear always (and only), in the low acceleration regime, where the acceleration drops below the critical MOND acceleration constant of $a_0 = 1.2 \times 10^{-10} m/s$ e.g. Milgrom (1984), Famaey & McGaugh (2012). This occurs at radial distances larger than a corresponding MOND radius of:

$$R_M = (GM/a_0)^{1/2},$$

(1)

where $M$ is the total baryonic mass of a galactic system, which in turn, has essentially converged by $R_M$, with any remaining baryonic matter distribution beyond being a very minor component usually treated as constituting test particles. In the high acceleration regime, where $a > a_0$, no gravitational anomalies appear beyond observational errors and uncertainties in inferred baryonic mass to light ratios. Hence, whenever $r < R_M$ the validity of GR is unchallenged. Thus, we can assume the validity of the Schwarzschild metric:

$$ds^2 = dr^2 \left(1 - \frac{2GM}{c^2 r}\right)^{-1} + r^2 \left(d\theta^2 + \sin^2 \theta d\phi^2\right) - c^2 dt^2 \left(1 - \frac{2GM}{c^2 r}\right),$$

(2)

for the radial range $r < R_M$. Beyond this point, covariant extended theories of gravity attempting to explain observed dynamics of massive and massless particles in the absence of dark matter, impose a change in regime to some modified version of GR, e.g. TeVeS of Bekenstein (2004), (f(R) varieties in Capozziello et al. (2007), f(X) in Mendoza et al. (2013), conformal gravity in Mannheim & Kazanas (1994). Generally, this change in regime is introduced by hand, no fundamental physical explanation for a change in regime for gravity at this mass-dependent radius of $R_M$ has been proposed. Relative exceptions are the recent works of McCulloch (2017) and Verlinde (2016) where however, it is the effect of a cosmological constant resulting from a quantum vacuum which leads to $R_M$. It is our interest to attempt a geometric explanation for the entirety of the dark sector, so explaining galactic dark matter as a result of cosmological dark energy appears unsatisfactory. It is interesting that a purely geometric invariant appears for all masses at $R_M$, as shown below.

We begin by noting that the Kretschmann curvature scalar, given in general by:

$$K = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta},$$

(3)

a full contraction of the $R^{\mu\nu\alpha\beta}$ Riemann tensor, has under a Schwarzschild metric a value given by:

$$K_S = 48 \frac{G^2 M^2}{c^4 r^6}.$$ 

(4)

Notice that multiplying $K_S$ times an area function $4\pi r^2$, and evaluating the resulting quantity at the critical transition radius of $R_M$, we obtain a constant value for all masses:

$$K_S 4\pi r^2|_{r=R_M} = 192\pi \left(\frac{a_0^3}{c^4}\right).$$

(5)

We see that a quantity which we shall henceforth refer to as bounding curvature, $\kappa_B = 4\pi K_S r^2$, has a fixed value for all masses at the transition point beyond which gravitational anomalies appear at galactic systems, the region where extended theories of gravity introduce modifications to GR. We see from equation (4) that $\kappa_B$ decreases radially with $r^{-2}$ within a Schwarzschild spacetime, until reaching always a fixed value at $r = R_M$, for all masses.

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3 CONSISTENCY OF CONSTANT BOUNDING CURVATURE WITH INFERRRED SPACETIME BEYOND RM AT GALACTIC SCALES

We now explore the extension of the $\kappa_B = 4\pi K r^2$ condition to the radial range $r > R_M$, where the isothermal dark matter halos are invoked to explain observed dynamics and lensing observations under GR. Working under a general spherically symmetric and static metric,

$$ds^2 = B(r)dr^2 + r^2 \left( d\theta^2 + \sin^2\theta d\varphi^2 \right) - A(r)c^2dt^2.$$  

(6)

The Kretschmann scalar curvature is now given by the following expression:

$$K_{SSS} = \frac{A_{r,r}}{(AB)^3} \left[ A_{r,r}AB - A_rAB_r - A^2r \right] + \frac{A_r^2}{4(AB)^4} \left[ B_r^2A^2 + 2A_rB_rAB + (A_rB_r^2 + 8 \left( \frac{AB}{r} \right)^2 \right] + \frac{2}{(Br)^4} \left[ (Br)^2 + 2B^4 - 4B^3 + 2B^2 \right].$$  

(7)

The consistency of the constant bounding curvature condition in the 'dark halo' region can be tested by introducing the metric potentials of the isothermal dark matter halo into the above expression. These are:

$$A = 1 + 2\beta^2 ln(r/R_M) \simeq (r/R_M)^{2\beta^2}, \quad B = 1 + 2\beta^2,$$  

where $\beta = V_{TF}/c, V_{TF} = (GMho)^{1/4}$ the Tully-Fisher flat asymptotic galactic rotation curve of a system of baryonic mass $M$ e.g. Misner, Thorne & Wheeler (1973) section (25.7), Campigoti et al. (2016). These metric coefficients $g_{\alpha\alpha} = A, g_{\mu
u} = B$ are accurate to order $\beta^2$, which given observational uncertainties in galactic rotation curves, lensing observations and inferences of baryonic mass to light ratios, defines $\beta^2$ as the highest order to which the potentials in eqation (8) are relevant. The above empirical metric potentials reduce equation(7) above to:

$$r^4 K_{SSS} = O(\beta^4).$$  

(9)

Hence, multiplying the above by $4\pi r^2$, the constant bounding curvature condition $\kappa_B = 192\pi a_0^2/c^4$ reads:

$$r^4 K_{SSS} = 48\pi a_0^2 r^2/c^4.$$  

(10)

Since the right hand side of equation (9) vanishes to order $\beta^2$, left hand side of the above equation contains only terms of order higher than $\beta^2$, and hence the empirically determined spacetime of equation (8) will be compatible with the $\kappa_B = 192\pi a_0^2/c^4$ condition only to a maximum critical radius $R_K$ at which point the right hand side of the preceding equation becomes equal to $\beta^2$. For $r < R_K$, the right hand side of equation (10) is smaller than $\beta^2$, and hence the metric potentials of equation (8) will satisfy the constant bounding curvature condition $\kappa_B = 192\pi a_0^2/c^4$. The mass dependant $R_K$ is given by:

$$R_K = \frac{1}{\sqrt{18}} \left( \frac{cV_{TF}}{a_0} \right).$$  

(11)

The compatibility of the constant bounding curvature criterion with the empirical metric inferred at galactic scales will only be relevant if $R_K$ is much larger than $R_M$, to ensure a significant 'dark matter halo' region. This can be probed by evaluating $R_K/R_M$, which by using equation (1) and $V_{TF} = (GMho)^{1/4}$ yields,

$$\frac{R_K}{R_M} = \frac{1}{6.93} \left( \frac{c}{V_{TF}} \right).$$  

(12)

We see that at galactic scales, were $5km/s < V_{TF} < 300km/s$, the outer validity range of the flat rotation curve region is predicted by the above equation to reach $200R_M < R_K < 8600R_M$ respectively. Given the large value of $c$ and the small value of $a_0$, from equation (11) it is already obvious that this outer validity radius will extend far beyond the onset of the MONDian region at $R_M$, and hence shows no inconsistency with any observational 'dark matter' inferences, which typically extend only to about $10R_M$, both for rotation curves and lensing data e.g. Famaey & McGaugh (2012), Brimioule et al. (2013).

4 CONSISTENCY OF CONSTANT BOUNDING CURVATURE WITH INFERRRED SPACETIME AT COSMOLOGICAL LEVEL

We now test the bounding curvature criterion presented above in a cosmological context, where matter-energy sources at sufficiently large scales are described as perturbations on a homogeneous and isotropic FLRW background whose metric is given by:

$$ds^2 = dr^2 \left( \frac{a^2(t)}{1 - Kr^2} \right) + a^2(t)r^2(\alpha^2 + \sin^2\theta d\varphi^2) - c^2dt^2.$$

(13)

The Kretschmann invariant now becomes:

$$K_F = \frac{12}{(ca)^2} \left[ (a\ddot{a})^2 + (k - \dot{a}^2)^2 + \dot{H}^2 + H^2 \right].$$  

(14)

where dot denotes temporal derivatives. Taking a flat universe $k = 0$ yields,

$$K_F = \frac{12}{c^2} \left[ (\dot{H} + H^2)^2 + H^4 \right]$$  

(15)

where we have introduced the Hubble scalar, $H = \dot{a}/a$. We now multiply by an area function, which for the cosmological system becomes $4\pi$ times the square of the natural characteristic length-scale furnished by the Hubble radius, $4\pi(c/H)^2$, to give,

$$\kappa_B = \frac{48\pi}{c^2} \left[ \left( \frac{\dot{H}}{H} \right)^2 + 2\dot{H} + 2H^2 \right].$$  

(16)

The constant bounding curvature condition $\kappa_B = 192\pi a_0^2/c^4$ in the cosmological context becomes:

$$\left[ \left( \frac{\dot{H}}{H} \right)^2 + 2\dot{H} + 2H^2 \right] = \frac{4}{33.64}H^2_{70},$$  

(17)

where we have used the well known numerical relation $cH_{70} = 5.83a_0$ (e.g. Milgrom 2002), with $H_{70}$ being a value of $70km/sMpc^{-1}$. A particular solution to the above equation is $\dot{H} = 0$ and,

$$H = 0.244H_{70}.$$  

(18)

We see that a de Sitter universe is consistent with imposing upon a FLRW spacetime the constant bounding curvature condition developed for spherically symmetric

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static spacetimes at galactic scales. Given the $r^{-4}$ scaling of $\kappa_B$ under a Schwarzschild spacetime, c.f. equation (4), a slight adjustment within empirical uncertainty bounds of $R_M = 0.5(GM/a_0)^{1/2}$ would lead to $\kappa_B \rightarrow 16\kappa_B$ and hence yield a Hubble constant of 70 km/s/Mpc $^{-1}$ for the de Sitter solution which follows from $\kappa_B = \text{cte.}$ under the assumption of a flat spacetime. We can go beyond fitting the asymptote de Sitter universe, to comparing the full analytical solution of equation (17) to the inferred expansion history of the concordance $\Lambda$CDM universe, this last given by:

$$a(t) = \left( \frac{\Omega_M}{\Omega_\Lambda} \right)^{1/3} \left[ \sinh \left( \frac{3}{2} \sqrt{\Omega_\Lambda} H_0 t \right) \right]^{2/3},$$

(19)

for parameters $\Omega_M = 0.3$ and $\Omega_\Lambda = 0.7$. The corresponding expansion history is plotted in figure (1) as a function of time in units of $\tau = H_0(t - t_0)$, where $t_0$ is today, lower curve to the right of the plot. Equation (17) is separable, and taking the $'+'$ sign in the square root determining $H$ and again the right hand side of equation (17) as $4/3H_0$, yields the following implicit exact solution, which appears as the upper curve to the right in figure 1:

$$\tau + T_0 = \left( \frac{3}{32} \right)^{1/2} \ln \left( \frac{\sqrt{8 - 6y^2 + \sqrt{6y^2 + 4}}}{6 - 9y^2} \right) + \sqrt{\frac{3}{8}} \ln \left( \frac{81}{16} \right)$$

$$+ \left( \frac{3}{2} \right)^{1/2} \ln \left( \frac{y}{\sqrt{4 - 3y^2} + 2} \right) + \left( \frac{3}{8} \right)^{1/2} \tan^{-1} \left( \sqrt{\frac{3}{2y}} \right),$$

(20)

where $y = H/H_0$ and $T_0$ is a constant of integration which can be chosen to yield $H(\tau = 0) = H_0$. From figure 1 we see that the solution to imposing the $\kappa_B = \text{cte.}$ condition upon a flat FLRW universe, quite accurately (to within a fraction of a percent) traces the optimal $\Lambda$CDM concordance fit over the $0.79 < a < 1.59$ scale factor range displayed. In this case the asymptotic value for $H$ will not match $H_0$, but it is after all the recent expansion history and not the asymptotic de Sitter phase over which the $\Lambda$CDM parameters are fitted.

Having fixed the right hand side of equation (17) to $(4/3)H_0$ requires taking $R_M = 0.547(GM/a_0)^{1/2}$, again, well within observational inferences, which since $V_{pF}$ is unchanged, does not alter the validity of our results of sections 2 and 3.

As happens in the galactic case explored in sections 2 and 3, before the need for hypothetical dark components arise, in the cosmological instance at high redshift, $\kappa_B$ has a large value (c.f. eq.(16) decreases as $\propto t^{-2}$ in a matter dominated phase) at earlier times, which falls until reaching a critical value, which if forced to remain fixed at this minimum, leads to dynamics compatible with empirical inferences.

An equivalent comparison of $\Lambda$CDM and the cosmological consequences of imposing the constant bounding curvature criterion to that of figure (2), comes from calculating the equation of state parameter $w = p/\rho$ that would result from a GR solution for which $H$ is given by (20), which is a solution of the geometric condition (17). Assuming a spatially flat FLRW metric whose sources are cold dark matter and dark energy: $\Omega_M + \Omega_\Lambda = 1$, so that $p = w\rho_\Lambda$ with $w = w(a)$, the Raychaudhuri equation becomes:

$$H = -H^2 \left[ 1 + \frac{1 + 3w(1 - \Omega_M)}{2} \right].$$

(21)

Substituting (21) into (17) yields after some algebraic manipulation the following relation,

$$3w(1 - \Omega_M) = \pm 2 \left( \frac{4 H_0^2}{3 H^2} - 1 \right)^{1/2} - 1,$$

(22)

which provides (under GR) a link between $w$, $H$ and $\Omega_M$. For the present cosmic time $\Omega_M/0 \sim 0.3$, $H = H_0$ and we obtain the following values for the present day dark energy equation of state parameter: $w_0 = 0.0736$ and $w_0 = -1.0260$. Evidently, the second root $w_0 = -1.026$ is very close to the $w_0$ parameter of a $\Lambda$CDM model and is thus consistent with current observations. In fact, it fits very well the various attempts to estimate empirically a dynamical dark energy distinct from a cosmological constant e.g. Jassal et al. (2005), de Felice et al. (2012), Postnikov et al. (2014). However, an alternative gravity theory that incorporates the geometric condition (17) would yield a conceptually different approach to cosmic dynamics and thus would not require such empirical fitting of a dynamical dark energy source.

It might appear counter intuitive that the same condition which allows to explain observed rotation curves in the absence of dark matter should also yield cosmic accelerated expansion in the absence of dark energy, since at a naive zero order level, galactic dark matter provides "extra attractive" gravity, while cosmological dark energy leads to "repulsive" gravity. The answer lies in considering the very different boundary and symmetry conditions of both limits; galactic rotation curves arise in bound systems that are approximately isolated and well described by spherically symmetric and static metrics that are approximately asymptotically flat, and therefore where one can identify a well defined centre towards which gravity pulls, extra gravity leads to more pull. At cosmological scales matter–energy can no longer be described as stationary near asymptotically flat localised sources, but rather as close to homogeneous patches that ex-
pand isotropically and are all statistically equivalent, thus together approximating an FLRW model. All observers see others receding, being pulled away faster in all directions by the same geometric modification to gravity, yielding a different effect for the case of a very distinct assumed symmetry.

This is in fact what has been shown to occur with some $f(\chi)$ modified gravity options tailored to reproduce observed galactic dynamics in the absence of dark matter (Mendoza et al. 2013), which also yield agreement with the observed SN Hubble plot (Carranza et al. 2013). Both limits are idealisations, we have shown that the proposed geometric condition is consistent with empirical inferences for spacetimes at both limits. Indeed, it is tempting to speculate that the covariant geometric $\kappa_B = cte.$ condition might arise from some more fundamental holographic or entropic criterion, and perhaps form the basis of a more complete extended gravity.

5 CONCLUDING REMARKS

We have shown that for a Schwarzschild metric, the mass dependant MOND transition radius corresponds to an invariant purely geometric condition given by the bounding curvature $\kappa_B = K \times A = 192\pi a_0^2/c^2$, with $\kappa_B$ having units of $m^{-2}$. Thus, the oft noted incompatibility of the MOND 'acceleration scale' with covariant frameworks is removed.

Assuming a spherically symmetric and static metric for $r > R_M$ and imposing $\kappa_B = cte.$, is compatible with the empirically determined metric potentials usually interpreted as evidence for a dominant dark matter halo, out to an outer validity radius of $R_K$, which is between hundreds and thousands of times larger than $R_M$, ensuring an extensive radial validity range for the 'flat rotation curve' galactic region, without the need of any dark matter.

Imposing the $\kappa_B = cte.$ condition on a flat FLRW metric results in an asymptotic de Sitter solution with a Hubble constant compatible with observations without the need of any dark energy, which for recent epochs ($a > 0.79$) accurately traces the concordance $\Lambda$CDM fit to better than 0.6%.

The condition found can be used as a necessary restriction on modified theories of gravity seeking to explain the observed gravitational anomalies in absence of hypothetical dark matter and dark energy components. Indeed one can speculate that a fixed minimum $\kappa_B$ principle could constitute the basis of a covariant modified theory of gravity.

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