The N-body Problem in Tetrad Gravity: a First Step towards the Unified Description of the Four Interactions.

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Abstract

After a review of the canonical reduction to the rest-frame Wigner-covariant instant form of standard theories in Minkowski spacetime, a new formulation of tetrad gravity is introduced. Its canonical reduction, also in presence of N scalar particles, is done. The modification of the ADM formulation to solve the deparametrization problem of general relativity (how to recover the rest-frame instant form for G=0), is presented.

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1 Electromagnetic, Weak and Strong Interactions in Minkowski Spacetime.

The attempt to arrive at a unified description of the four interactions [with the matter being either Grassmann-valued Dirac fields or relativistic particles] based on Dirac-Bergmann theory of constraints, which is needed for the Hamiltonian formulation of both gauge theories and general relativity, motivated the study of the canonical reduction of a new formulation of tetrad gravity [1, 2, 3, 4]. This requires to look at general relativity from the canonical point of view generalizing to it all the results already obtained in the canonical study of gauge theories, since neither a complete reduction of gravity with an identification of the physical canonical degrees of freedom of the gravitational field nor a detailed study of its Hamiltonian group of
gauge transformations (whose infinitesimal generators are the first class constraints) has ever been pushed till the end in an explicit way.

The research program aiming to express the special relativistic strong, weak and electromagnetic interactions in terms of Dirac’s observables is in an advanced stage of development. This program is based on the Shanmugadhasan canonical transformations: if a system has 1st class constraints at the Hamiltonian level, then, at least locally, one can find a canonical basis with as many new momenta as 1st class constraints (Abelianization of 1st class constraints), with their conjugate canonical variables as Abelianized gauge variables and with the remaining pairs of canonical variables as pairs of canonically conjugate Dirac’s observables (canonical basis of physical variables adapted to the chosen Abelianization; they give a trivialization of the BRST construction of observables). Putting equal to zero the Abelianized gauge variables one defines a local gauge of the model. If a system with constraints admits one (or more) global Shanmugadhasan canonical transformations, one obtains one (or more) privileged global gauges in which the physical Dirac observables are globally defined and globally separated from the gauge degrees of freedom [for systems with a compact configuration space this is impossible]. These privileged gauges (when they exist) can be called generalized Coulomb gauges. Second class constraints, when present, are also taken into account by the Shanmugadhasan canonical transformation.

Firstly, inspired by Ref. the canonical reduction to noncovariant generalized Coulomb gauges, with the determination of the physical Hamiltonian as a function of a canonical basis of Dirac’s observables, has been achieved for the following isolated systems (for them one asks that the 10 conserved generators of the Poincaré algebra are finite so to be able to use group theory; theories with external fields can only be recovered as limits in some parameter of a subsystem of the isolated system):

a) Yang-Mills theory with Grassmann-valued fermion fields in the case of a trivial principal bundle over a fixed-$x^o \mathbb{R}^3$ slice of Minkowski spacetime with suitable Hamiltonian-oriented boundary conditions; this excludes monopole solutions and, since $\mathbb{R}^3$ is not compactified, one has only winding number and no instanton number. After a discussion of the Hamiltonian formulation of Yang-Mills theory, of its group of gauge transformations and of the Gribov ambiguity, the theory has been studied in suitable weighted Sobolev spaces where the Gribov ambiguity is absent. The global Dirac observables are the transverse quantities $\vec{A}_{a\perp}(\vec{x}, x^o), \vec{E}_{a\perp}(\vec{x}, x^o)$ and fermion fields dressed with Yang-Mills (gluonic) clouds. The nonlocal and nonpolynomial (due to the presence of classical Wilson lines along flat geodesics) physical Hamiltonian has been obtained: it is nonlocal but without any kind of singularities, it has the correct Abelian limit if the structure constants are turned off, and it contains the explicit realization of the abstract Mitter-Viallet metric.

b) The Abelian and non-Abelian SU(2) Higgs models with fermion fields, where the symplectic decoupling is a refinement of the concept of unitary gauge. There is an ambiguity in the solutions of the Gauss law constraints, which reflects the existence of disjoint sectors of solutions of the Euler-Lagrange equations of Higgs
models. The physical Hamiltonian and Lagrangian of the Higgs phase have been found; the self-energy turns out to be local and contains a local four-fermion interaction.

c) The standard SU(3)xSU(2)xU(1) model of elementary particles with Grassmann-valued fermion fields. The final reduced Hamiltonian contains nonlocal self-energies for the electromagnetic and color interactions, but "local ones" for the weak interactions implying the nonperturbative emergence of 4-fermions interactions.

The next problem is how to covariantize these results valid in Minkowski spacetime with Cartesian coordinates. Again the starting point was given by Dirac\[5\] with his reformulation of classical field theory on spacelike hypersurfaces foliating Minkowski spacetime \( M^4 \) [the foliation is defined by an embedding \( R \times \Sigma \rightarrow M^4 \), \((\tau, \vec{\sigma}) \rightarrow z^{(\mu)}(\tau, \vec{\sigma}) \in \Sigma_{\tau} \), with \( \Sigma \) an abstract 3-surface diffeomorphic to \( R^3 \), with \( \Sigma_{\tau} \) its copy embedded in \( M^4 \) labelled by the value \( \tau \) (the Minkowski flat indices are \((\mu)\); the scalar parameter \( \tau \) labels the leaves of the foliation, \( \vec{\sigma} \) are curvilinear coordinates on \( \Sigma_{\tau} \) and \((\tau, \vec{\sigma}) \) are \( \Sigma_{\tau} \)-adapted holonomic coordinates for \( M^4 \); this is the classical basis of Tomonaga-Schwinger quantum field theory]. In this way one gets a parametrized field theory with a covariant 3+1 splitting of Minkowski spacetime and already in a form suited to the coupling to general relativity in its ADM canonical formulation (see also Ref.[12], where a theoretical study of this problem is done in curved spacetimes). The price is that one has to add as new configuration variables the points \( z^{(\mu)}(\tau, \vec{\sigma}) \) of the spacelike hypersurface \( \Sigma_{\tau} \) [the only ones carrying Lorentz indices] and then to define the fields on \( \Sigma_{\tau} \) so that they know the hypersurface \( \Sigma_{\tau} \) of \( \tau \)-simultaneity [for a Klein-Gordon field \( \phi(x) \), this new field is \( \tilde{\phi}(\tau, \vec{\sigma}) = \phi(z(\tau, \vec{\sigma})) \); it contains the nonlocal information about the embedding]. Besides a Lorentz-scaler form of the constraints of the given system, from the Lagrangian rewritten on the hypersurface [function of \( z^{(\mu)} \) through the induced metric \( g_{AB} = z^{(\mu)}A \eta_{(\mu)(\nu)}z^{(\nu)}B, z^{(\mu)}A = \partial z^{(\mu)} / \partial \sigma^A, \sigma^A = (\tau, \sigma^r) \)] one gets four further first class constraints \( \mathcal{H}(\tau, \vec{\sigma}) = \rho(\mu)(\tau, \vec{\sigma}) - l(\mu)(\tau, \vec{\sigma}) T^{rs}_{\text{sys}}(\tau, \vec{\sigma}) - z_r(\mu)(\tau, \vec{\sigma}) T^{rs}_{\text{sys}}(\tau, \vec{\sigma}) \approx 0 \) [where \( T^{rs}_{\text{sys}}(\tau, \vec{\sigma}) \), \( T^{rs}_{\text{sys}}(\tau, \vec{\sigma}) \), are the components of the energy-momentum tensor of the system in the holonomic coordinate system corresponding to the energy- and momentum-density of the isolated system; one has \( \{ \mathcal{H}(\mu)(\tau, \vec{\sigma}), \mathcal{H}(\nu)(\tau, \vec{\sigma}) \} = 0 \) implying the independence of the description from the choice of the foliation with spacelike hypersurfaces. The evolution vector is given by \( z^{(\mu)} = N_{[z]}(\text{flat}) (l^{(\mu)} + N^r_{[z]}(\text{flat}) z^{(\mu)} \), where \( l^{(\mu)}(\tau, \vec{\sigma}) \) is the normal to \( \Sigma_{\tau} \) in \( z^{(\mu)}(\tau, \vec{\sigma}) \) and \( N_{[z]}(\text{flat})(\tau, \vec{\sigma}) = \sqrt{4 g_{rr} + 4 g^{rs} g_{rr} 4 g_{rs}} = \sqrt{4 g/3 \gamma} \), \( N_{[z]}(\text{flat})(\tau, \vec{\sigma}) = 3 g_{rs}(\tau, \vec{\sigma}) N_{[z]}(\text{flat})(\tau, \vec{\sigma}) = 4 g_{rr}, \) are the flat lapse and shift functions defined through the metric like in general relativity; however, they are not independent variables but functionals of \( z^{(\mu)}(\tau, \vec{\sigma}) \) in Minkowski spacetime.

The original Dirac Hamiltonian contains a piece given by \( \int d^3 \sigma \lambda(\mu)(\tau, \vec{\sigma}) \mathcal{H}(\mu)(\tau, \vec{\sigma}) \) with \( \lambda(\mu)(\tau, \vec{\sigma}) \) are Dirac multipliers. By using \( 4 \eta^{(\mu)(\nu)} = (l^{(\mu)}(\nu) - z^{(\mu)} z^{(\nu)})(\tau, \vec{\sigma}) \), we can write \( \lambda(\mu)(\tau, \vec{\sigma}) \mathcal{H}(\mu)(\tau, \vec{\sigma}) = [\lambda(\mu)(\nu)](l^{(\nu)} \mathcal{H}(\nu)) - \lambda(\nu)(z^{(\nu)}(\tau, \vec{\sigma}) (3 g^{rs} z^{(\nu)} \mathcal{H}(\nu))(\tau, \vec{\sigma}) \) \( \Rightarrow \) N-flat \( \mathcal{N}(\text{flat})(\tau, \vec{\sigma})(l^{(\nu)} \mathcal{H}(\nu))(\tau, \vec{\sigma}) - N_{\text{flat}}(\tau, \vec{\sigma})(3 g^{rs} z^{(\nu)} \mathcal{H}(\nu))(\tau, \vec{\sigma}) \) with the (nonholo-
nomic form of the) constraints \((l_{(\mu})\mathcal{H}^{(\mu)})(\tau, \vec{\sigma}) \approx 0, (3g^{rs}z_{s(\mu)}\mathcal{H}^{(\mu)})(\tau, \vec{\sigma}) \approx 0\), satisfying the universal Dirac algebra. In this way we have defined new flat lapse and shift functions

\[
N_{(\text{flat})}(\tau, \vec{\sigma}) = \lambda_{(\mu)}(\tau, \vec{\sigma})l^{(\mu)}(\tau, \vec{\sigma}),
\]

\[
N_{(\text{flat})r}(\tau, \vec{\sigma}) = \lambda_{(\mu)}(\tau, \vec{\sigma})z_{r}^{(\mu)}(\tau, \vec{\sigma}).
\] (1)

which have the same content of the arbitrary Dirac multipliers \(\lambda_{(\mu)}(\tau, \vec{\sigma})\), namely they multiply primary first class constraints satisfying the Dirac algebra. In Minkowski spacetime they are quite distinct from the previous lapse and shift functions \(N_{[\text{flat}]}, N_{[\text{flat}]r}\), defined starting from the metric. Instead in general relativity the lapse and shift functions defined starting from the 4-metric are also the coefficient (in the canonical part of the Hamiltonian) of secondary first class constraints satisfying the Dirac algebra.

In special relativity, it is convenient to restrict ourselves to arbitrary spacelike hyperplanes \(z^{(\mu)}(\tau, \vec{\sigma}) = x_{s}^{(\mu)}(\tau) + b_{r}^{(\mu)}(\tau)\sigma^{r}\). Since they are described by only 10 variables [an origin \(x_{s}^{(\mu)}(\tau)\) and, on it, three orthogonal spacelike unit vectors generating the fixed constant timelike unit normal to the hyperplane], we remain only with 10 first class constraints determining the 10 variables conjugate to the hyperplane [they are a 4-momentum \(p_{s}^{(\mu)}\) and the six independent degrees of freedom hidden in a spin tensor \(S_{s}^{(\mu)(\nu)}\); with these 20 canonical variables it is possible to build 10 Poincaré generators \(p_{s}^{(\mu)}, J_{s}^{(\mu)(\nu)} = x_{s}^{(\mu)}p_{s}^{(\nu)} - x_{s}^{(\nu)}p_{s}^{(\mu)} + S_{s}^{(\mu)(\nu)}\) in terms of the variables of the system: \(\mathcal{H}^{(\mu)}(\tau) = p_{s}^{(\mu)} - p_{(\text{sys})}^{(\mu)} \approx 0, \mathcal{H}^{(\nu)(\tau)} = S_{s}^{(\mu)(\nu)} - S_{(\text{sys})}^{(\mu)(\nu)} \approx 0\).

After the restriction to spacelike hyperplanes this piece of Dirac Hamiltonian is reduced to \(\tilde{\lambda}^{(\mu)}(\tau)\mathcal{H}_{(\mu)}(\tau) - \frac{1}{2}\tilde{\lambda}^{(\mu)(\nu)}(\tau)\mathcal{H}_{(\mu)(\nu)}(\tau)\). Since at this stage we have \(z_{r}^{(\mu)}(\tau, \vec{\sigma}) \approx b_{r}^{(\mu)}(\tau), \) so that \(z_{r}^{(\mu)}(\tau, \vec{\sigma}) \approx N_{[\text{flat}]}(\tau, \vec{\sigma})l^{(\mu)}(\tau, \vec{\sigma}) + N_{[\text{flat}]r}(\tau, \vec{\sigma})\)

\[
b_{r}^{(\mu)}(\tau, \vec{\sigma}) \approx \tilde{x}_{r}^{(\mu)}(\tau) + \tilde{b}_{r}^{(\mu)}(\tau)\sigma^{r} = -\tilde{\lambda}^{(\mu)}(\tau) - \tilde{\lambda}^{(\mu)(\nu)}(\tau)b_{r(\nu)}(\tau)\sigma^{r},
\]

it is only now that we get the coincidence of the two definitions of flat lapse and shift functions:

\[
N_{[\text{flat}]}(\tau, \vec{\sigma}) \approx N_{(\text{flat})}(\tau, \vec{\sigma}) = -\tilde{\lambda}_{(\mu)}(\tau)l^{(\mu)} - l^{(\mu)}\tilde{\lambda}_{(\mu)(\nu)}(\tau)b_{r(\nu)}(\tau)\sigma^{r},
\]

\[
N_{[\text{flat}]r}(\tau, \vec{\sigma}) \approx N_{(\text{flat})r}(\tau, \vec{\sigma}) = -\tilde{\lambda}_{(\mu)}(\tau)b_{r}^{(\mu)}(\tau) - b_{r}^{(\mu)}(\tau)\tilde{b}_{r}^{(\nu)}(\tau)b_{s}^{(\nu)}(\tau)\sigma^{r}.\] (2)

The 20 variables for the phase space description of a hyperplane are:

i) \(x_{s}^{(\mu)}(\tau), p_{s}^{(\mu)}\), parametrizing the origin of the family of spacelike hyperplanes. The four constraints \(\mathcal{H}^{(\mu)}(\tau, \vec{\sigma}) \approx 0\) say that:

a) \(p_{s}^{2} \approx M_{(\text{sys})}^{2}\) \([M_{(\text{sys})}\) is the invariant mass of the system\];

b) \(u_{s}^{(\mu)}(p_{s}) = p_{s}^{(\mu)}/p_{s}^{2} \approx [\text{the orientation of the four - momentum of the isolated system with respect to an arbitrary given external observer}]\).

The origin \(x_{s}^{(\mu)}(\tau)\) is playing the role of a kinematical center of mass for the isolated system and may be interpreted as a decoupled observer with his parametrized clock.

ii)The are other six independent pairs of degrees of freedom are contained in \(b_{A}^{(\mu)}(\tau)\)
(with \( b^\mu_s = l(\mu) \tau\)-independent and normal to the hyperplanes) and \( S^{(\mu)(\nu)}_s = -S^{(\nu)(\mu)}_s \) with the orthonormality constraints \( b^{(\mu)}_A \eta^{(\nu)(\mu)} b^{(\nu)}_B = \eta_{AB} \).

However, for each configuration of an isolated system there is a privileged family of hyperplanes (the Wigner hyperplanes orthogonal to \( p^\mu_s \), existing when it is timelike) corresponding to the intrinsic rest-frame of the isolated system. To get this result, we must boost at rest all the variables with Lorentz indices by using the standard Wigner boost \( L^{(\mu)}_{(\nu)}(p_s, \vec{p}_s) \) for timelike Poincaré orbits, and then add the gauge-fixings \( b^{(\mu)}_s(\tau) - L^{(\mu)}_{(\nu)}(p_s, \vec{p}_s) \approx 0 \). Since these gauge-fixings depend on \( p^\mu_s \), the final canonical variables, apart \( p^\mu_s \) itself, are of 3 types: i) there is a non-covariant center-of-mass variable \( \vec{x}^{(\mu)}(\tau) \) [it is only covariant under the little group of timelike Poincaré orbits like the Newton-Wigner position operator]; ii) all the 3-vector variables become Wigner spin 1 3-vectors [boosts in \( M^4 \) induce Wigner rotations on them]; iii) all the other variables are Lorentz scalars.

One obtains in this way a new kind of instant form of the dynamics (see Ref.[13]), the “Wigner-covariant 1-time rest-frame instant form” [16] with a universal breaking of Lorentz covariance. It is the special relativistic generalization of the nonrelativistic separation of the center of mass from the relative motion \([H = \frac{\vec{p}^2}{2M} + H_{rel}]\). The role of the center of mass is taken by the Wigner hyperplane, identified by the point \( \vec{x}^{(\mu)}(\tau) \) and by its normal \( p^\mu_s \).

The only surviving four 1st class constraints can be put in the following form: i) the vanishing of the total (Wigner spin 1) 3-momentum of the system \( \vec{p}_s \approx 0 \) [with \( \vec{p}_s = \text{intrinsic three - momentum of the isolated system inside the Wigner hyperplane}] \); instead of putting a restriction on \( u^{(\mu)}_s(p_s) \) they say that the Wigner hyperplane \( \Sigma_{W,\tau} \) is the intrinsic rest frame [instead, \( \vec{p}_s \) is left arbitrary, since \( p^\mu_s \) depends upon the orientation of the Wigner hyperplane with respect to arbitrary reference frames in \( M^4 \)]; ii) \( \pm \sqrt{p^2_s - M_{sys}} \approx 0 \), saying that the invariant mass \( M_{sys} \) of the system replaces the nonrelativistic Hamiltonian \( H_{rel} \) for the relative degrees of freedom, after the addition of the gauge-fixing \( T_s - \tau \approx 0 \) [identifying the time parameter \( \tau \), labelling the leaves of the foliation, with the Lorentz scalar time of the center of mass in the rest frame, \( T_s = p_s \cdot \vec{x}_s/M_{sys} ; M_{sys} \) generates the evolution in this time].

In this special gauge 3 degrees of freedom of the isolated system [a 3-center-of-mass (Wigner spin 1) variable \( \vec{x}_{sys} \) defined inside the Wigner hyperplane and conjugate to \( \vec{p}_{(sys)} \)] become gauge variables [the natural gauge fixing for \( \vec{p}_{(sys)} \approx 0 \) is \( \vec{x}_{(sys)} \approx 0 \), so that it coincides with the origin \( \vec{s} = 0 \) of the Wigner hyperplane], while \( \vec{x}^{(\mu)} \) describes a physical decoupled observer. For \( N \) free particles [16] one has \( \vec{x}_{sys} = \vec{q}_+(\tau) = \sum_{i=1}^{N} \vec{q}_i(\tau) \). After the gauge-fixing \( \vec{q}_+(\tau) \approx 0 \) we remain only with Newtonian-like degrees of freedom with rotational covariance: i) a 3-coordinate (not Lorentz covariant) \( \vec{z}_s = \sqrt{p^2_s (\vec{x}_s - \vec{p}_s/\sqrt{2})} \) and its conjugate momentum \( \vec{k}_s = \vec{p}_s/\sqrt{2} \) for the decoupled center of mass in \( M^4 \); ii) a set of relative conjugate pairs of variables with Wigner covariance inside the Wigner hyperplane. When fields are present, one needs to find a rest-frame canonical basis of center-of-mass and relative variables for
fields (in analogy to particles) to identify $\overrightarrow{x}_{\text{sys}}$. Such a basis has already been found for a real Klein-Gordon field\[^{14}\] and it is under reformulation on spacelike hypersurfaces \[^{13}\].

The isolated systems till now analyzed to get their rest-frame Wigner-covariant generalized Coulomb gauges [i.e. the subset of global Shanmugadhasan canonical bases, which, for each Poincaré stratum, are also adapted to the geometry of the corresponding Poincaré orbits with their little groups; these special bases can be named Poincaré-Shanmugadhasan bases for the given Poincaré stratum of the presymplectic constraint manifold (every stratum requires an independent canonical reduction); till now only the main stratum with $P^2$ timelike and $W^2 \neq 0$ has been investigated] are:

a) The system of $N$ scalar particles with Grassmann electric charges plus the electromagnetic field \[^{16}\]. The starting configuration variables are a 3-vector $\overrightarrow{\eta}_i(\tau)$ for each particle $[x^{(\mu)}_i(\tau) = z^{(\mu)}(\tau, \overrightarrow{\eta}_i(\tau))]$ and the electromagnetic gauge potentials $A_{\lambda}(\tau, \overrightarrow{\sigma}) = \frac{\partial z^{(\mu)}(\tau, \overrightarrow{\sigma})}{\partial \sigma} A_{(\mu)}(z^{(\tau, \overrightarrow{\sigma})})$, which know the embedding of $\Sigma_\tau$ into $M^4$. One has to choose the sign of the energy of each particle, because there are not mass-shell constraints (like $p_i^2 - m_i^2 \approx 0$) among the constraints of this formulation, due to the fact that one has only three degrees of freedom for particle, determining the intersection of a timelike trajectory and of the spacelike hypersurface $\Sigma_\tau$. For each choice of the sign of the energy of the $N$ particles, one describes only one of the branches of the mass spectrum of the manifestly covariant approach based on the coordinates $x^{(\mu)}_i(\tau), p^{(\mu)}_i(\tau), i=1,...,N$, and on the constraints $p_i^2 - m_i^2 \approx 0$ (in the free case). In this way, one gets a description of relativistic particles with a given sign of the energy with consistent couplings to fields and valid independently from the quantum effect of pair production [in the manifestly covariant approach, containing all possible branches of the particle mass spectrum, the classical counterpart of pair production is the intersection of different branches deformed by the presence of fields]. The final Dirac’s observables are: i) the transverse radiation field variables $\overrightarrow{A}_\perp, \overrightarrow{E}_\perp$; ii) the particle canonical variables $\overrightarrow{\eta}_i(\tau), \overrightarrow{\kappa}_i(\tau)$, dressed with a Coulomb cloud. The physical Hamiltonian contains the Coulomb potentials extracted from field theory and there is a regularization of the Coulomb self-energies due to the Grassmann character of the electric charges $Q_i$ [$Q_i^2 = 0$]. The no-radiation conditions $\overrightarrow{A}_\perp = \overrightarrow{E}_\perp = 0$ identify an approximate (delay is neglected) canonical subspace containing only physical charged particles with mutual instantaneous Coulomb potentials. In Ref.\[^{17}\] there is the study of the Lienard-Wiechert potentials and of Abraham-Lorentz-Dirac equations in this rest-frame Coulomb gauge and also scalar electrodynamics is reformulated in it. Also the rest-frame 1-time relativistic statistical mechanics has been developed \[^{16}\].

b) The system of $N$ scalar particles with Grassmann-valued color charges plus the color SU(3) Yang-Mills field\[^{13}\]: it gives the pseudoclassical description of the relativistic scalar-quark model, deduced from the classical QCD Lagrangian and with the color field present. The physical invariant mass of the system is given in terms
of the Dirac observables. From the reduced Hamilton equations the second order equations of motion both for the reduced transverse color field and the particles are extracted. Then, one studies the N=2 (meson) case. A special form of the requirement of having only color singlets, suited for a field-independent quark model, produces a “pseudoclassical asymptotic freedom” and a regularization of the quark self-energy. With these results one can covariantize the bosonic part of the standard model given in Ref.[11].

c) The system of N spinning particles of definite energy \([(\frac{1}{2}, 0) \text{ or } (0, \frac{1}{2})\] representations of SL(2,C)] with Grassmann electric charges plus the electromagnetic field[19] and that of a Grassmann-valued Dirac field plus the electromagnetic field (the pseudoclassical basis of QED) [20]. In both cases there are geometrical complications connected with the spacetime description of the path of electric currents and not only of their spin structure, suggesting a reinterpretation of the supersymmetric scalar multiplet as a spin fibration with the Dirac field in the fiber and the Klein-Gordon field in the base; a new canonical decomposition of the Klein-Gordon field into center-of-mass and relative variables [14, 15] will be helpful to clarify these problems. After their solution and after having obtained the description of Grassmann-valued chiral fields [this will require the transcription of the front form of the dynamics in the instant one for the Poincaré strata with \(P^2 = 0\)] the rest-frame form of the full standard \(SU(3) \times SU(2) \times U(1)\) model can be achieved.

All these new pieces of information will allow, after quantization of this new consistent relativistic mechanics without the classical problems connected with pair production, to find the asymptotic states of the covariant Tomonaga-Schwinger formulation of quantum field theory on spacelike hypersurfaces: these states are needed for the theory of quantum bound states [since Fock states do not constitute a Cauchy problem for the field equations, because an in (or out) particle can be in the absolute future of another one due to the tensor product nature of these asymptotic states, bound state equations like the Bethe-Salpeter one have spurious solutions which are excitations in relative energies, the variables conjugate to relative times (which are gauge variables [16]). Moreover, it will be possible to include bound states among the asymptotic states.

As said in Ref.[17, 18], the quantization of these rest-frame models has to overcome two problems. On the particle side, the complication is the quantization of the square roots associated with the relativistic kinetic energy terms: in the free case this has been done in Ref.[21] [see Refs.[22] for the complications induced by the Coulomb potential]. On the field side (all physical Hamiltonian are nonlocal and, with the exception of the Abelian case, nonpolynomial, but quadratic in the momenta), the obstacle is the absence (notwithstanding there is no no-go theorem) of a complete regularization and renormalization procedure of electrodynamics (to start with) in the Coulomb gauge: see Ref.[23] (and its bibliography) for the existing results for QED.

However, as shown in Refs.[16, 8], the rest-frame instant form of dynamics automatically gives a physical ultraviolet cutoff in the spirit of Dirac and Yukawa:
it is the Møller radius \[^{24} \rho = \sqrt{-W^2c/P^2} \]

(W^2 = -P^2 \vec{S}^2 ) is the Pauli-Lubanski Casimir when \( P^2 > 0 \), namely the classical intrinsic radius of the worldtube, around the covariant noncanonical Fokker-Pryce center of inertia \( Y^\mu \), inside which the noncovariance of the canonical center of mass \( \vec{x}^\mu \) is concentrated. At the quantum level \( \rho \) becomes the Compton wavelength of the isolated system multiplied its spin eigenvalue \( \sqrt{s(s+1)} \), \( \rho \mapsto \hat{\rho} = \sqrt{s(s+1)} \hbar/M = \sqrt{s(s+1)}\lambda_M \) with \( M = \sqrt{P^2} \) the invariant mass and \( \lambda_M = \hbar/M \) its Compton wavelength. Therefore, the criticism to classical relativistic physics, based on quantum pair production, concerns the testing of distances where, due to the Lorentz signature of spacetime, one has intrinsic classical covariance problems: it is impossible to localize the canonical center of mass \( \vec{x}^\mu \) adapted to the first class constraints of the system (also named Pryce center of mass and having the same covariance of the Newton-Wigner position operator) in a frame independent way.

Let us remember \[^{16} \] that \( \rho \) is also a remnant in flat Minkowski spacetime of the energy conditions of general relativity: since the Møller noncanonical, noncovariant center of energy has its noncovariance localized inside the same worldtube with radius \( \rho \) (it was discovered in this way) \[^{24} \], it turns out that for an extended relativistic system with the material radius smaller of its intrinsic radius \( \rho \) one has:

i) its peripheral rotation velocity can exceed the velocity of light; ii) its classical energy density cannot be positive definite everywhere in every frame.

Now, the real relevant point is that this ultraviolet cutoff determined by \( \rho \) exists also in Einstein’s general relativity (which is not power counting renormalizable) in the case of asymptotically flat spacetimes, taking into account the Poincaré Casimirs of its asymptotic ADM Poincaré charges (when supertranslations are eliminated with suitable boundary conditions; let us remark that Einstein and Wheeler use closed universes because they don’t want to introduce boundary conditions, but in this way they loose Poincaré charges and the possibility to make contact with particle physics and to define spin). The generalization of the worldtube of radius \( \rho \) to asymptotically flat general relativity with matter, could be connected with the unproved cosmic censorship hypothesis.

Moreover, the extended Heisenberg relations of string theory\[^{25} \], i.e. \( \Delta x = \frac{\hbar}{\Delta p} + \frac{\Delta p}{T_{cs}} = \frac{\hbar}{\Delta p} + \frac{\hbar \Delta p}{L_{cs}^2} \) implying the lower bound \( \Delta x > L_{cs} = \sqrt{\hbar/T_{cs}} \) due to the \( y+1/y \) structure, have a counterpart in the quantization of the Møller radius\[^{16} \]: if we ask that, also at the quantum level, one cannot test the inside of the worldtube, we must ask \( \Delta x > \hat{\rho} \) which is the lower bound implied by the modified uncertainty relation \( \Delta x = \frac{\hbar}{\Delta p} + \frac{\hbar \Delta p}{\rho^2} \). This could imply that the center-of-mass canonical noncovariant 3-coordinate \( \vec{z} = \sqrt{P^2}(\vec{x} - \frac{P}{\rho} \vec{x}^\mu) \)\[^{16} \] cannot become a self-adjoint operator. See Hegerfeldt’s theorems (quoted in Refs. [9, 16]) and his interpretation pointing at the impossibility of a good localization of relativistic particles (experimentally one determines only a worldtube in spacetime emerging from the interaction region). Since the eigenfunctions of the canonical center-of-mass operator are playing the role of the wave function of the universe, one could also say that the center-of-
mass variable has not to be quantized, because it lies on the classical macroscopic side of Copenhagen’s interpretation and, moreover, because, in the spirit of Mach’s principle that only relative motions can be observed, no one can observe it (it is only used to define a decoupled “point particle clock”). On the other hand, if one rejects the canonical noncovariant center of mass in favor of the covariant noncanonical Fokker-Pryce center of inertia \( Y^\mu, \{Y^\mu, Y^\nu\} \neq 0 \), one could invoke the philosophy of quantum groups to quantize \( Y^\mu \) to get some kind of quantum plane for the center-of-mass description. Let us remark that the quantization of the square root Hamiltonian done in Ref.[21] is consistent with this problematic.

In conclusion, the best set of canonical coordinates adapted to the constraints and to the geometry of Poincaré orbits in Minkowski spacetime and naturally predisposed to the coupling to canonical tetrad gravity is emerging for the electromagnetic, weak and strong interactions with matter described either by fermion fields or by relativistic particles with a definite sign of the energy.

2 Canonical Reduction of Tetrad Gravity, the De- parametrization Problem and the N-body Problem.

Tetrad gravity is the formulation of general relativity natural for the coupling to the fermion fields of the standard model. However, we need a formulation of it, which allows to solve its constraints for doing the canonical reduction and to solve the deparametrization problem of general relativity (how to recover the rest-frame instant form when the Newton constant is put equal to zero, \( G=0 \)).

To implement this program we shall restrict ourselves to the simplest class of spacetimes [time-oriented pseudo-Riemannian or Lorentzian 4-manifold \((M^4, g)\) with signature \( \epsilon (+−−−) (\epsilon = \pm 1) \) and with a choice of time orientation], assumed to be:

i) Globally hyperbolic 4-manifolds, i.e. topologically they are \( M^4 = R \times \Sigma \), so to have a well posed Cauchy problem [with \( \Sigma \) the abstract model of Cauchy surface] at least till when no singularity develops in \( M^4 \) [see the singularity theorems]. Therefore, these spacetimes admit regular foliations with orientable, complete, non-intersecting spacelike 3-manifolds \( \Sigma_\tau \) \([\tau : M^4 \to R, z^\mu \mapsto \tau(z^\mu), \text{is a global timelike future-oriented function labelling the leaves (surfaces of simultaneity)}\]. In this way, one obtains 3+1 splittings of \( M^4 \) and the possibility of a Hamiltonian formulation.

ii) Asymptotically flat at spatial infinity, so to have the possibility to define asymptotic Poincaré charges \([26, 27, 28, 29, 30, 31]\): they allow the definition of a Møller radius also in general relativity and are a bridge towards a future soldering with the theory of elementary particles in Minkowski spacetime defined as irreducible representation of its kinematical, globally implemented Poincaré group according to Wigner.
iii) Admitting a spinor (or spin) structure\textsuperscript{32} for the coupling to fermion fields. Since we consider noncompact space- and time-orientable spacetimes, spinors can be defined if and only if they are “parallelizable” \textsuperscript{33}, like in our case. This implies that the orthonormal frame principal SO(3)-bundle over $\Sigma_{\tau}$ (whose connections are the spin connections determined by the cotriads) is trivial.

iv) The noncompact parallelizable simultaneity 3-manifolds (the Cauchy surfaces) $\Sigma_{\tau}$ are assumed to be topologically trivial, geodesically complete and, finally, diffeomorphic to $R^3$. These 3-manifolds have the same manifold structure as Euclidean spaces: a) the geodesic exponential map $Exp_{p} : T_{p}\Sigma_{\tau} \rightarrow \Sigma_{\tau}$ is a diffeomorphism; b) the sectional curvature is less or equal zero everywhere; c) they have no “conjugate locus” [i.e. there are no pairs of conjugate Jacobi points (intersection points of distinct geodesics through them) on any geodesic] and no “cut locus” [i.e. no closed geodesics through any point].

v) Like in Yang-Mills case \textsuperscript{9}, the 3-spin-connection on the orthogonal frame SO(3)-bundle (and therefore cotriads) will have to be restricted to suited weighted Sobolev spaces to avoid Gribov ambiguities \textsuperscript{9, 34}. In turn, this implies the absence of isometries of the noncompact Riemannian 3-manifold $(\Sigma_{\tau}, g_{\mu\nu})$ [see for instance the review paper in Ref. \textsuperscript{35}].

Diffeomorphisms on $\Sigma_{\tau}$ ($Diff\Sigma_{\tau}$) are interpreted in the passive way, following Ref.\textsuperscript{36}, in accord with the Hamiltonian point of view that infinitesimal diffeomorphisms are generated by taking the Poisson bracket with the 1st class supermomentum constraints [passive diffeomorphisms are also named ‘pseudodiffeomorphisms’].

The new formulation of tetrad gravity [see Refs. \textsuperscript{37, 38, 29, 10, 11, 12, 13, 14, 15, 16, 17} for the existing versions of the theory] utilizes the ADM action of metric gravity with the 4-metric expressed in terms of arbitrary cotetradrs. Let us remark that both in the ADM metric and tetrad formulation one has to introduce the extra ingredient of the 3+1 splittings of $M^4$ with foliations whose leaves $\Sigma_{\tau}$ are spacelike 3-hypersurfaces. However, their points $z^\mu(\tau, \vec{\sigma})$ $[(\tau, \vec{\sigma})$ are $\Sigma_{\tau}$-adapted holonomic coordinates of $M^4]$ are not configurational variables of these theories in contrast to what happens in Minkowski parametrized theories $[\frac{\partial z^\mu}{\partial \sigma^a}]$ are not tetradrs when $M^4$ is not Minkowski spacetime with Cartesian coordinates, because $4g^{AB}\frac{\partial z^\mu}{\partial \sigma^A}\frac{\partial z^\nu}{\partial \sigma^B} = 4g^{\mu\nu} \neq 4\eta^{(\mu)(\nu)}$.

By using $\Sigma_{\tau}$-adapted holonomic coordinates for $M^4$, one has found a new parametrization of arbitrary tetrads and cotetradrs on $M^4$ in terms of cotetradrs on $\Sigma_{\tau}$ $[3e_{(a)r}(\tau, \vec{\sigma})]$, of lapse $[N(\tau, \vec{\sigma})]$ and shift $[N_{(a)}(\tau, \vec{\sigma}) = \{3e_{(a)r}, N^r\}(\tau, \vec{\sigma})]$ functions and of 3 parameters $[\varphi_{(a)}(\tau, \vec{\sigma})]$ parametrizing point-dependent Wigner boosts for timelike Poincaré orbits. Putting these variables in the ADM action for metric gravity \textsuperscript{20} (with the 3-metric on $\Sigma_{\tau}$ expressed in terms of cotetradrs: $3g_{rs} = 3e_{(a)r}\cdot 3e_{(a)s}$ with positive signature), one gets a new action depending only on lapse, shifts and cotetradrs, but not on the boost parameters (therefore, there is no need to use Schwinger’s time gauge). There are 10 primary and 4 secondary first class constraints and a vanishing canonical Hamiltonian. Besides the 3 constraints associated with the vanishing Lorentz boost momenta, there are 4 constraints saying that the momenta asso-
associated with lapse and shifts vanish, 3 constraints describing rotations, 3 constraints generating space-diffeomorphisms on the cotriads induced by those \((\text{Diff} \Sigma_\tau)\) on \(\Sigma_\tau\) (a linear combination of supermomentum constraints and of the rotation ones; a different combination of these constraints generates \(\text{SO}(3)\) Gauss law constraints for the momenta \(\tilde{\pi}_\alpha^r\) conjugated to cotriads with the covariant derivative built with the spin connection) and one superhamiltonian constraint. It turns out that with the technology developed for Yang-Mills theory, one can Abelianize the 3 rotation constraints and then also the space-diffeomorphism constraints. In the Abelianization of the rotation constraints one needs the Green function of the 3-dimensional covariant derivative containing the spin connection, well defined only if there is no Gribov ambiguity in the \(\text{SO}(3)\)-frame bundle and no isometry of the Riemannian 3-manifold \((\Sigma_\tau, g^\tau_3)\). The Green function is similar to the Yang-Mills one for a principal \(\text{SO}(3)\)-bundle [9], but, instead of the Dirac distribution for the Green function of the flat divergence, it contains the DeWitt function or bitensor [18] defining the tangent in one endpoint of the geodesic arc connecting two points (which reduces to the Dirac distribution only locally in normal coordinates). Moreover, the definition of the Green function now requires the geodesic exponential map.

In the resulting quasi-Shanmugadhasan canonical basis, the original cotriad can be expressed in closed form in terms of 3 rotation angles, 3 diffeomorphism-parameters and a reduced cotriad depending only on 3 independent variables. (they are Dirac’s observables with respect to 13 of the 14 first class constraints) and with their conjugate momenta, still subject to the reduced form of the superhamiltonian constrain: this is the phase space over the superspace of 3-geometries [49].

Till now no coordinate condition [50] has been imposed. It turns out that these conditions are hidden in the choice of how to parametrize the reduced cotriads in terms of three independent functions. The simplest parametrization (the only one studied till now) corresponds to choose a system of global 3-orthogonal coordinates on \(\Sigma_\tau\), in which the 3-metric is diagonal. With a further canonical transformation on the reduced cotriads and conjugate momenta, one arrives at a canonical basis containing the conformal factor \(\phi(\tau, \vec{\sigma}) = e^{q(\tau, \vec{\sigma})/2}\) of the 3-geometry and its conjugate momentum \(\rho(\tau, \vec{\sigma})\) plus two other pairs of conjugate canonical variables \(r_\bar{a}(\tau, \vec{\sigma}), \pi_\bar{a}(\tau, \vec{\sigma}), \bar{a} = 1, 2\). The reduced superhamiltonian constraint, expressed in terms of these variables, turns out to be an integral equation for the conformal factor (reduced Lichnerowicz equation) whose conjugate momentum is, therefore, the last gauge variable. If we replace the gauge fixing of the Lichnerowicz [51] and York [52], [53, 54] approach [namely the vanishing of the trace of the extrinsic curvature of \(\Sigma_\tau\), \(\gamma K(\tau, \vec{\sigma}) \approx 0\), also named the internal extrinsic York time [55]] with the natural one \(\rho(\tau, \vec{\sigma}) \approx 0\) and we go to Dirac brackets, we find that \(r_\bar{a}(\tau, \vec{\sigma}), \pi_\bar{a}(\tau, \vec{\sigma})\) are the canonical basis for the physical degrees of freedom or Dirac’s observables of the gravitational field.

Since we have this physical canonical basis, it is possible to define “void spacetimes” as the equivalence class of spacetimes “without gravitational field”, whose members in the 3-orthogonal gauge are obtained by adding by hand the second class
constraints \( r_\alpha (\tau, \vec{\sigma}) \approx 0 \), \( \pi_\alpha (\tau, \vec{\sigma}) \approx 0 \) [one gets \( \phi (\tau, \vec{\sigma}) = 1 \) as the relevant solution of the reduced Lichnerowicz equation]. The members of this equivalence class (the extension to general relativity of the Galilean non inertial coordinate systems with their Newtonian inertial forces) are in general “not flat” [their \( \Sigma_\tau \)'s are 3-conformally flat] but gauge equivalent to Minkowski spacetime with Cartesian coordinates (this holds in absence of matter).

The next step is to find the physical Hamiltonian for them and to solve the deparametrization problem. If we wish to arrive at the soldering of tetrad gravity with matter and parametrized Minkowski formulation for the same matter, we must require that the lapse and shift functions of tetrad gravity [which must grow linearly in \( \vec{\sigma} \), in suitable asymptotic Minkowski coordinates, according to the existing literature on asymptotic Poincaré charges at spatial infinity [26, 27, 28, 29] must agree asymptotically with the flat lapse and shift functions, which, however, are unambiguously defined only on Minkowski spacelike hyperplanes.

In metric ADM gravity the canonical Hamiltonian is

\[
H_{(c)ADM} = \int d^3\sigma [N^r \dot{H} + N_r \dot{\tilde{H}}] (\tau, \vec{\sigma}) \approx 0,
\]

where \( \dot{H} (\tau, \vec{\sigma}) \approx 0 \) and \( \dot{\tilde{H}} (\tau, \vec{\sigma}) \approx 0 \) are the superhamiltonian and supermomentum constraints. It is differentiable and finite only for suitable \( N(\tau, \vec{\sigma}) = n(\tau, \vec{\sigma}) \to |\vec{\sigma}| \to \infty 0 \), \( N_r(\tau, \vec{\sigma}) = n_r(\tau, \vec{\sigma}) \to |\vec{\sigma}| \to \infty 0 \) defined in Ref [28] in suitable asymptotic coordinate systems. For more general lapse and shift functions one must add a surface term [19] to \( H_{(c)ADM} \), which contains the “strong” Poincaré charges \( P_{ADM}^\alpha \), \( J_{ADM}^{AB} \) [they are conserved and gauge invariant surface integrals]. To have well defined asymptotic Poincaré charges at spatial infinity [24, 27, 28, 29] one needs: i) the selection of a class of coordinates systems for \( \Sigma_\tau \) asymptotic to flat coordinates; ii) the choice of a class of Hamiltonian boundary conditions in these coordinate systems [all the fields must belong to some functional space of the type of the weighted Sobolev spaces]; iii) a definition of the Hamiltonian group \( G \) of gauge transformations (and in particular of proper gauge transformations) with a well defined limit at at spatial infinity so to respect i) and ii). The scheme is the same needed to define the non-Abelian charges in Yang-Mills theory [9]. The delicate point is to be able to exclude supertranslations [22], because the presence of these extra asymptotic charges leads to the replacement of the asymptotic Poincaré group with the infinite-dimensional spin group [11], which does not allow the definition of the Poincaré spin due to the absence of the Pauli-Lubanski Casimir. This can be done with suitable boundary conditions (in particular all the fields and gauge transformations must have direction independent limits at spatial infinity) respecting the “parity conditions” of Ref. [28] [see also Ref. [29]].

Let us then remark that in Ref. [31] and in the book in Ref. [3] (see also Ref. [27]), Dirac introduced asymptotic Minkowski rectangular coordinates

\[
x^{(\mu)}_\infty (\tau) = x^{(\mu)}_\infty (\tau) + b^{(\mu)}_\infty (\tau) \sigma^r \in M^4 \text{ at spatial infinity } S_\infty = \cup_\tau S^2_{\tau, \infty}. \]

For each value of \( \tau \), the coordinates \( x^{(\mu)}_\infty (\tau) \) labels a point, near spatial infinity chosen as origin. On it there is a flat tetrad \( b^{(\mu)}_\infty A(\tau) \) = \( \{ l^{(\mu)}_\infty = b^{(\mu)}_\infty \tau = \epsilon^{(\mu)}_{} (\alpha)(\beta)(\gamma) b^{(\alpha)}_\infty 1(\tau) b^{(\beta)}_\infty 2(\tau) b^{(\gamma)}_\infty 3(\tau) \)
\(b^{(\mu)}_{(\infty)}(\tau)\), with \(b^{(\mu)}_{(\infty)}\) \(\tau\)-independent, satisfying \(b^{(\mu)}_{(\infty)}A 4\eta^{(\mu)(\nu)} b^{(\nu)}_{(\infty)B} = 4\eta_{AB}\) for every \(\tau\) [at this level we do not assume that \(b^{(\mu)}_{(\infty)}\) is tangent to \(S_{\infty}\), as the normal \(l^\mu\) to \(\Sigma_{r}\)]. There will be transformation coefficients \(b^{(\mu)}_A(\tau, \vec{\sigma})\) from the holonomic adapted coordinates \(\sigma^A = (\tau, \sigma^r)\) to coordinates \(x^\mu = z^\mu(\sigma^A)\) in an atlas of \(M^4\), such that in a chart at spatial infinity one has \(z^\mu(\tau, \vec{\sigma}) = \delta^{(\mu)}_{(\infty)}z^\mu(\tau, \vec{\sigma})\) and \(b^{(\mu)}_A(\tau, \vec{\sigma}) = \delta^{(\mu)}_{(\infty)}b^{(\mu)}_A(\tau)\) [for \(r \to \infty\) one has \(4g_{\mu\nu} \to \delta^{(\mu)}_{\nu}\delta^{(\nu)}_{\mu}4\eta^{(\mu)(\nu)}\) and \(4g_{AB} = b^{(\mu)}_A 4g_{\mu\nu}b^{(\nu)}_B \to b^{(\mu)}_{(\infty)A} 4\eta^{(\mu)(\nu)} b^{(\nu)}_{(\infty)B} = 4\eta_{AB}\)].

Dirac\[56\] and, then, Regge and Teitelboim\[27\] proposed that the asymptotic Minkowski rectangular coordinates \(z^{(\mu)}(\tau, \vec{\sigma}) = x^{(\mu)}(\tau) + b^{(\mu)}(\tau)\sigma^r\) should define 10 new independent degrees of freedom at the spatial boundary \(S_{\infty}\), as it happens for Minkowski parametrized theories\[14\] when restricted to spacelike hyperplanes [defined by \(z^{(\mu)}(\tau, \vec{\sigma}) \approx x^{(\mu)}(\tau) + b^{(\mu)}(\tau)\sigma^r\)]; then, 10 conjugate momenta should exist. These 20 extra variables of the Dirac proposal can be put in the form: \(x^{(\mu)}(\infty)(\tau), p^{(\mu)}(\infty)\), \(b^{(\mu)}_{(\infty)A}(\tau)\) [with \(b^{(\mu)}_{(\infty)A} = l^{(\mu)}_{(\infty)}\) \(\tau\)-independent], \(S^{(\mu)(\nu)}(\infty)\), with Dirac brackets implying the orthonormality constraints \(b^{(\mu)}_{(\infty)A} 4\eta^{(\mu)(\nu)} b^{(\nu)}_{(\infty)B} = 4\eta_{AB}\) [so that \(p^{(\mu)}_{(\infty)}\) and \(J^{(\mu)(\nu)}_{(\infty)} = x^{(\mu)}_{(\infty)p^{(\nu)}_{(\infty)} - x^{(\mu)}_{(\infty)}p^{(\nu)}_{(\infty)} + S^{(\mu)(\nu)}_{(\infty)}\) satisfy a Poincaré algebra]. In analogy with Minkowski parametrized theories restricted to spacelike hypersurfaces, one expects to have 10 extra first class constraints of the type \(p^{(\mu)}_{(\infty)} - P^{A}_{ADM} \approx 0\), \(S^{(\mu)(\nu)}_{(\infty)} - S^{(\mu)(\nu)}_{ADM} \approx 0\) with \(P^{A}_{ADM}, S^{(\mu)(\nu)}_{ADM}\) related to the ADM Poincaré charges \(P^{A}_{ADM}, J^{AB}_{ADM}\). The origin \(x^{(\mu)}_{(\infty)}\) is going to play the role of a decoupled observer with his parametrized clock.

Let us remark that if we replace \(p^{(\mu)}_{(\infty)}\) and \(S^{(\mu)(\nu)}_{(\infty)}\), whose Poisson algebra is the direct sum of an Abelian algebra of translations and of a Lorentz algebra, with the new variables [with holonomic indices with respect to \(\Sigma_{\tau}\)] \(P^{A}_{(\infty)} = b^{A}_{(\infty)A} p^{(\mu)}_{(\infty)}\), \(J^{AB}_{(\infty)} = b^{A}_{(\infty)(\mu)} b^{B}_{(\infty)(\nu)} S^{(\mu)(\nu)}_{(\infty)}\) [\(\neq b^{A}_{(\infty)(\mu)} b^{B}_{(\infty)(\nu)} J^{(\mu)(\nu)}_{(\infty)}\)], the Poisson brackets for \(p^{(\mu)}_{(\infty)}\), \(b^{(\mu)}_{(\infty)A}\), \(S^{(\mu)(\nu)}_{(\infty)}\) imply that \(p^{A}_{(\infty)}\), \(J^{AB}_{(\infty)}\) satisfy a Poincaré algebra. This implies that the Poincaré generators \(P^{A}_{ADM}, J^{AB}_{ADM}\) define in the asymptotic Dirac rectangular coordinates a momentum \(P^{A}_{ADM}\) and only an ADM spin tensor \(S^{(\mu)(\nu)}_{ADM}\) [to define an angular momentum tensor \(J^{(\mu)(\nu)}_{ADM}\) one should find a “center of mass of the gravitational field” \(X^{(\mu)}_{ADM}\)[\(\delta g^{3\bar{\Pi}}\)] (see Ref.[14] for the Klein-Gordon case) conjugate to \(P^{A}_{ADM}\), so that \(J^{(\mu)(\nu)}_{ADM} = X^{(\mu)}_{ADM} P^{(\nu)}_{ADM} - X^{(\nu)}_{ADM} P^{(\mu)}_{ADM} + S^{(\mu)(\nu)}_{ADM}\).

The following splitting of the lapse and shift functions and the following set of boundary conditions [consistent with the ones of Ref.[57]] fulfill all the previous requirements [soldering with the lapse and shift functions on Minkowski hyperplanes; absence of supertranslations (strictly speaking one gets \(P^{A}_{ADM} = 0\) like in Ref.\[57\]; it is an open problem how to weaken the condition on \(3\bar{\Pi}^r\) without reintroducing supertranslations); \(r = |\vec{\sigma}|\)]}

\[3g_{rs}(\tau, \vec{\sigma}) \to r \to \infty \quad (1 + \frac{M}{r})\delta_{rs} + 3h_{rs}(\tau, \vec{\sigma}) = (1 + \frac{M}{r})\delta_{rs} + o_4(r^{-3/2}),\]
$3\Pi^{rs}(\tau, \sigma) \rightarrow_{r \to \infty} 3k^{rs}(\tau, \sigma) = o_3(r^{-5/2}),$

\[
N(\tau, \sigma) = N_{(a)s}(\tau, \sigma) + n(\tau, \sigma), \quad n(\tau, \sigma) = O(r^{-3+\epsilon}),
\]

\[
N_r(\tau, \sigma) = N_{(a)sr}(\tau, \sigma) + n_r(\tau, \sigma), \quad n_r(\tau, \sigma) = O(r^{-4}),
\]

\[
N_{(a)s}(\tau, \sigma) = -\tilde{\lambda}(\mu)(\tau)l^{(\mu)}_{(\infty)} - l^{(\mu)}_{(\infty)}\tilde{\lambda}(\mu)(\nu)(\tau)b^{(\nu)}_{(\infty)}(\tau)\sigma^s =
\]

\[
-\tilde{\lambda}_r(\tau) - \tilde{\lambda}_{rs}(\tau)\sigma^s,
\]

\[
N_{(a)sr}(\tau, \sigma) = -b^{(\mu)}_{(\infty)r}(\tau)\tilde{\lambda}(\mu)(\tau) - b^{(\mu)}_{(\infty)s}(\tau)\tilde{\lambda}(\mu)(\nu)(\tau)b^{(\nu)}_{(\infty)}(\tau)\sigma^s =
\]

\[
-\tilde{\lambda}_r(\tau) - \tilde{\lambda}_{rs}(\tau)\sigma^s,
\]

\[
\Rightarrow \quad N_{(a)sА}(\tau, \sigma) \overset{\text{def}}{=} (N_{(a)s} ; N_{(a)sr})(\tau, \sigma) = -\frac{1}{2}\tilde{\lambda}_A(\tau) - \frac{1}{2}\tilde{\lambda}_{As}(\tau)\sigma^s,
\]

(3)

with $h_{rs}(\tau, -\bar{\sigma}) = 3h_{rs}(\tau, \bar{\sigma}), \quad 3k^{rs}(\tau, -\bar{\sigma}) = -3k^{rs}(\tau, \bar{\sigma});$ here $3\Pi^{rs}(\tau, \bar{\sigma})$ is the momentum conjugate to the 3-metric $3g_{rs}(\tau, \bar{\sigma})$ in ADM metric gravity.

After the addition of the surface term, the resulting canonical and Dirac Hamiltonians of ADM metric gravity are

\[
H_{(c)ADM} = \int d^3\sigma[(N_{(a)s} + n)\tilde{H} + (N_{(a)sr} + n_r)3\tilde{H}^r](\tau, \sigma) \mapsto \quad \Rightarrow \quad H'_{(c)ADM} = \int d^3\sigma[(N_{(a)s} + n)\tilde{H} + (N_{(a)sr} + n_r)3\tilde{H}^r](\tau, \sigma) +
\]

\[
+ \tilde{\lambda}_A(\tau)P_{ADM}^A + \tilde{\lambda}_{AB}(\tau)J_{ADM}^{AB} = \quad \int d^3\sigma[n\tilde{H} + n_r3\tilde{H}^r](\tau, \sigma) + \tilde{\lambda}_A(\tau)\tilde{P}_{ADM}^A + \tilde{\lambda}_{AB}(\tau)\tilde{J}_{ADM}^{AB} \approx
\]

\[
\approx \tilde{\lambda}_A(\tau)\tilde{P}_{ADM}^A + \tilde{\lambda}_{AB}(\tau)\tilde{J}_{ADM}^{AB},
\]

\[
H'_{(d)ADM} = H'_{(c)ADM} + \int d^3\sigma[\nu_{n}\hat{\pi}^n + \nu_{n}^s\hat{\pi}_{(n)}^s](\tau, \sigma) +
\]

\[
+ \zeta_A(\tau)\hat{\pi}^A(\tau) + \zeta_{AB}(\tau)\hat{\pi}^{AB}(\tau),
\]

(4)

with the “weak conserved improper charges” $\tilde{P}_{ADM}^A, \tilde{J}_{ADM}^{AB}$ [they are volume integrals differing from the weak charges by terms proportional to integrals of the constraints].

The previous splitting implies to replace the variables $N(\tau, \sigma), N_r(\tau, \sigma)$ with the ones $\tilde{\lambda}_A(\tau), \tilde{\lambda}_{AB}(\tau) = -\tilde{\lambda}_{BA}(\tau), n(\tau, \sigma), n_r(\tau, \sigma)$ [with conjugate momenta $\hat{\pi}^A(\tau), \hat{\pi}^{AB}(\tau)$] in the ADM theory.

With these assumptions one has the following form of the line element (also its form in tetrad gravity is given)

\[
ds^2 = \epsilon([N_{(a)s} + n] - [N_{(a)sr} + n_r]g_{rs}[N_{(a)s} + n_s])d\tau^2 - 2\epsilon[N_{(a)sr} + n_r]d\tau d\sigma^r - \epsilon^3 g_{rs}d\sigma^rd\sigma^s =
\]

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\[
\epsilon([N_{\text{as}} + n]^2 - [N_{\text{as}}]r + n_r]_r^s e^{r}_{(a)} 3 e^{s}_{(a)} [N_{\text{as}} + n]_s (d\tau)^2 - \\
2\epsilon[n_{\text{as}} + n_r]_r^s d\tau d\sigma^r - \epsilon^3 e^{r}_{(a)} 3 e^{s}_{(a)} d\sigma^r d\sigma^s.
\]

(5)

The final suggestion of Dirac is to modify ADM metric gravity in the following way:

i) add the 10 new primary constraints \( p^A_{(\infty)} - \tilde{P}^A_{ADM} \approx 0 \), \( J^{AB}_{(\infty)} - \tilde{J}^{AB}_{ADM} \approx 0 \), where \( p^A_{(\infty)} = b^A_{(\infty)(\mu)} p^A_{(\mu)} \), \( J^{AB}_{(\infty)} = b^A_{(\infty)(\mu)} b^B_{(\infty)(\nu)} S^{AB}_{(\mu)(\nu)} \) [remember that \( p^A_{(\infty)} \) and \( J^{AB}_{(\infty)} \) satisfy a Poincaré algebra],

ii) consider \( \hat{\lambda}_A(\tau) \), \( \hat{\lambda}_{AB}(\tau) \), as Dirac multipliers for these 10 new primary constraints, and not as configurational (arbitrary gauge) variables coming from the lapse and shift functions [so that there are no conjugate momenta \( \hat{\pi}^A(\tau), \hat{\pi}^{AB}(\tau) \) and no associated Dirac multipliers \( \zeta_A(\tau), \zeta_{AB}(\tau) \)], in the assumed Dirac Hamiltonian [it is finite and differentiable]

\[
H_{(D)ADM} = \int d^3 \sigma [n_h^A + n_r^A + \lambda n^\pi + \lambda^\pi n^\pi] (\tau, \bar{\sigma}) - \\
\hat{\lambda}_A(\tau) [p^A_{(\infty)} - \tilde{P}^A_{ADM}] - \tilde{\lambda}_{AB}(\tau) [J^{AB}_{(\infty)} - \tilde{J}^{AB}_{ADM}] \approx 0,
\]

(6)

The reduced phase space is still the ADM one: on the ADM variables there are only the secondary first class constraints \( \hat{\mathcal{H}}(\tau, \bar{\sigma}) \approx 0 \), \( \hat{\mathcal{H}}^r(\tau, \bar{\sigma}) \approx 0 \) [generators of proper gauge transformations], because the other first class constraints \( p^A_{(\infty)} - \tilde{P}^A_{ADM} \approx 0 \), \( J^{AB}_{(\infty)} - \tilde{J}^{AB}_{ADM} \approx 0 \) do not generate improper gauge transformations but eliminate 10 of the extra 20 variables.

In this modified ADM metric gravity, one has restricted the 3+1 splittings of \( M^4 \) to foliations whose leaves \( \Sigma_\tau \) tend to Minkowski spacelike hyperplanes asymptotically at spatial infinity in a direction independent way. Therefore, these \( \Sigma_\tau \) should be determined by the 10 degrees of freedom \( x^{(\mu)}_\tau(\tau), b^{(\mu)}_\tau(\tau) \), like it happens for flat spacelike hyperplanes: this means that it must be possible to define a “parallel transport” of the asymptotic tetrads \( \hat{b}^{(\mu)}_\tau(\tau) \) to get well defined tetrads in each point of \( \Sigma_\tau \). While it is not yet clear whether this can be done for \( \hat{\lambda}_{AB}(\tau) \neq 0 \) [maybe Nester’s teleparallelism may be used], there is a solution for \( \hat{\lambda}_{AB}(\tau) = 0 \). This case corresponds to go to the Wigner-like hypersurfaces [the analogue of the Minkowski Wigner hyperplanes with the asymptotic normal \( l^{(\mu)}_{(\infty)} = l^{(\mu)}_{(\infty)\Sigma} \) parallel to \( \tilde{P}_{ADM}^{(\mu)} \). Following the same procedure defined for Minkowski spacetime, one gets \( \hat{S}^{rs}_{(\infty)} = \hat{J}^{rs}_{ADM} [\text{see Ref.16 for the definition of } \hat{S}^{AB}_{(\infty)}] \), \( \hat{\lambda}_{AB}(\tau) = 0 \) and \( [\epsilon_{(\infty)} = \sqrt{p^2_{(\infty)}}] \\
-\hat{\lambda}_A(\tau) [p^A_{(\infty)} - \tilde{P}^A_{ADM}] = -\hat{\lambda}_r(\tau) [\epsilon_{(\infty)} - \tilde{P}^r_{ADM} + \hat{\lambda}_r(\tau) \tilde{P}^r_{ADM}, \text{ so that the final form of these four surviving constraints is } \epsilon_{(\infty)} - \tilde{P}^r_{ADM} \approx 0,] \\
-\hat{P}^r_{ADM} \approx 0.
\]

On this subclass of foliations [whose leaves \( \Sigma^{(WSW)}_\tau \) will be called Wigner-Sen-Witten hypersurfaces] one can introduce a parallel transport by using the interpretation of Ref.19 of the Witten spinorial method of demonstrating the positivity of the ADM energy [20]. Let us consider the Sen-Witten connection \([21, 20] \) restricted to \( \Sigma^{(WSW)}_\tau \) (it depends on the trace of the extrinsic curvature of \( \Sigma^{(WSW)}_\tau \))

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and the spinorial Sen-Witten equation associated with it. As shown in Ref. [24], this spinorial equation can be rephrased as an equation whose solution determines (in a surface dependent dynamical way) a tetrad in each point of $\Sigma^{(WSW)}_r$ once it is given at spatial infinity (again this requires a direction independent limit).

On the Wigner-Sen-Witten hypersurfaces (the intrinsic asymptotic rest frame of the gravitational field), the remaining four extra constraints are: $\hat{P}^r_{ADM} \approx 0$ (this is automatically implemented with the boundary conditions of Ref. [57]) and $\epsilon(\infty) = \sqrt{P(\infty)} \approx \hat{P}^r_{ADM} \approx M_{ADM} = \sqrt{P^2_{ADM}}$. Now the spatial indices have become spin-1 Wigner indices [they transform with Wigner rotations under asymptotic Lorentz transformations]. As said for parametrized theories in Minkowski spacetime, in this special gauge 3 degrees of freedom of the gravitational field [ a 3-center-of-mass variable $\vec{X}_{ADM}[3g, 3\Pi]$ inside the Wigner-Sen-Witten hypersurface] become gauge variables, while $\tilde{\tau}(\infty)$ [the canonical non covariant variable replacing $x^{(\mu)}(\infty)$] becomes a decoupled observer with his “point particle clock” [63, 64] near spatial infinity.

Since the positivity theorems for the ADM energy imply that one has only time-like or lightlike orbits of the asymptotic Poincaré group, the restriction to timelike ADM 4-momentum allows to define the Möller radius like or lightlike orbits of the asymptotic Poincaré group, the restriction to a decoupled observer with his “point particle clock” [63, 64] near spatial infinity.

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By going from $\vec{x}^{(\mu)}(\infty), P^{(\mu)}(\infty)$, to the canonical basis $T^{(\mu)}(\infty) = p^{(\mu)}(\infty)x^{(\mu)}(\infty)/\epsilon(\infty) = p^{(\mu)}(\infty)/\epsilon(\infty), \epsilon(\infty), z^{(i)}(\infty) = \epsilon(\infty)(\vec{x}^{(i)}(\infty) = p^{(i)}(\infty)x^{(i)}(\infty)/p^{(\mu)}(\infty)), k^{(i)}(\infty) = p^{(i)}(\infty)/\epsilon(\infty) = u^{(i)}(\vec{p}^{(\mu)}(\infty))$, like in the flat case one finds that the final reduction requires the gauge-fixings $T^{(\mu)}(\infty) - r \approx 0$ and $X^{(i)}_{ADM} \approx 0$, where $\sigma^r = X^{(i)}_{ADM}$ is a variable representing the “center of mass” of the 3-metric of the slice $\Sigma_t$ of the asymptotically flat spacetime $M^4$. Since $\{T^{(\mu)}(\infty), \epsilon(\infty)\} = -\epsilon$, with the gauge fixing $T^{(\mu)}(\infty) - r \approx 0$ one gets $\lambda^r(\tau) \approx \epsilon$, and the final Dirac Hamiltonian is $H_D = M_{ADM} + \lambda^r(\tau)\hat{P}^r_{ADM}$ with $M_{ADM}$ [the ADM mass of the universe] the natural physical Hamiltonian to reintroduce an evolution in the “mathematical” $T^{(\mu)}(\infty) \equiv \tau$: namely in the rest-frame time identified with the parameter $\tau$ labelling the leaves $\Sigma^r_{SW}$ of the foliation of $M^4$. Physical times (atomic clocks, ephemeridis time...) must be put in a local 1-1 correspondence with this “mathematical” time.

The asymptotic rest-frame instant form realization of the Poincaré generators becomes (no more reference to the boosts $J^{(i)}_{ADM}$)

$$\epsilon(\infty) = M_{ADM},$$
$$p^{(i)}(\infty),$$
$$J^{(i)}_{ADM} = \vec{x}^{(i)}(\infty)P^{(i)}(\infty) - \vec{x}^{(j)}(\infty)P^{(i)}(\infty) + \delta^{(i)}\delta^{(j)s} J^{rs}_{ADM},$$

15
\[ J_{(\infty)}^{(i)} = p_{(\infty)}^{(i)} \dot{x}_{(\infty)}^{(i)} - \sqrt{M_{ADM}^2 + \vec{p}_{(\infty)}^2} \dot{z}_{(\infty)}^{(i)} - \frac{\delta^{(i)}{r}{J}_{ADM}^{rs} \delta^{(s(j))} p_{(\infty)}^{(j)}}{M_{ADM} + \sqrt{M_{ADM}^2 + \vec{p}_{(\infty)}^2}}. \] (7)

All this construction holds also in our formulation of tetrad gravity (since it uses the ADM action) and in its canonically reduced form in the 3-orthogonal gauge. In particular the Poincaré charges of void spacetimes vanish.

Therefore, the final physical Hamiltonian for the physical gravitational field is the reduced volume form of the ADM energy \( \hat{P}_r^{ADM}[r, \pi, \phi(r, \pi)] \) with \( \phi \) solution of the reduced Lichnerowicz equation in the 3-orthogonal gauge with \( \rho(\tau, \vec{\sigma}) \approx 0 \).

Let us compare the standard generally covariant formulation of gravity based on the Hilbert action with its invariance under \( Diff M^4 \) with the ADM Hamiltonian formulation.

Regarding the 10 Einstein equations of the standard approach, the Bianchi identities imply that four equations are linearly dependent on the other six ones and their gradients. Moreover, the four combinations of Einstein’s equations projectable to phase space (where they become the secondary first class superhamiltonian and supermomentum constraints of canonical metric gravity) are independent from the accelerations being restrictions on the Cauchy data. As a consequence the Einstein equations have solutions, in which the ten components \( 4g_{\mu\nu} \) of the 4-metric depend on only two truly dynamical degrees of freedom (defining the physical gravitational field) and on eight undetermined degrees of freedom. This transition from the ten components \( 4g_{\mu\nu} \) of the tensor \( 4g \) in some atlas of \( M^4 \) to the 2 (deterministic)+8 (undetermined) degrees of freedom breaks general covariance, because these quantities are neither tensors nor invariants under diffeomorphisms (their functional form is atlas dependent).

Since the Hilbert action is invariant under \( Diff M^4 \), one usually says that a “gravitational field” is a 4-geometry over \( M^4 \), namely an equivalence class of spacetimes \((M^4, 4g)\), solution of Einstein’s equations, modulo \( Diff M^4 \). See, however, the interpretational problems about what is observable in general relativity for instance in Refs.[65, 66], in particular the facts that i) scalars under \( Diff M^4 \), like \( 4R \), are not Dirac’s observables but gauge dependent quantities; ii) the functional form of \( 4g_{\mu\nu} \) in terms of the physical gravitational field and, therefore, the angle and distance properties of material bodies and the standard procedures of defining measures of length and time based on the line element \( ds^2 \), are gauge dependent.

Instead in the ADM formalism with the extra notion of 3+1 splittings of \( M^4 \), the (tetrad) metric ADM action (differing from the Hilbert one by a surface term) is quasi-invariant under the (14) 8 types of gauge transformations which are the pull-back of the Hamiltonian group \( G \) of gauge transformations, whose generators are the first class constraints of the theory. The Hamiltonian group \( G \) has a subgroup (whose generators are the supermomentum and superhamiltonian constraints) formed by the diffeomorphisms of \( M^4 \) adapted to its 3+1 splittings, \( Diff M^{3+1} \) [it is different from \( Diff M^4 \)]. Moreover, the Poisson algebra of the supermomentum and superhamiltonian constraints reflects the embeddability in \( M^4 \) of the foliation asso-
associated with the 3+1 splitting \[67\]. Now a “gravitational field” is the equivalence class of spacetimes modulo the Hamiltonian group \(G\), and different members of the equivalence class have in general different 4-Riemann tensors [these equivalence classes are connected with the conformal 3-geometries of the Lichnerowicz-York approach and contain different gauge-related 4-geometries].

The interpretation of the 14 gauge transformations and of their gauge fixings in tetrad gravity (it is independent from the presence of matter) is the following [a tetrad in a point of \(\Sigma_\tau\) is a local observer] :

i) the gauge fixings of the gauge boost parameters associated with the 3 boost constraints are equivalent to choose the local observer either at rest or Lorentz-boosted;

ii) the gauge fixings of the gauge angles associated with the 3 rotation constraints are equivalent to the fixation of the standard of non rotation of the local observer;

iii) the gauge fixings of the 3 gauge parameters associated with the passive space diffeomorphisms [\(Diff\Sigma_\tau\); change of coordinates charts] are equivalent to a fixation of 3 standards of length by means of a choice of a coordinate system on \(\Sigma_\tau\) [the measuring apparatus (the “rods”) should be defined in terms of Dirac’s observables for some kind of matter, after its introduction into the theory];

iv) according to constraint theory the choice of 3-coordinates on \(\Sigma_\tau\) induces the gauge fixings of the 3 shift functions [i.e. of \(g_{\alpha}\)], whose gauge nature is connected with the “conventionality of simultaneity” \[68\] [therefore, the gauge fixings are equivalent to a choice of simultaneity and, as a consequence, to a statement about the isotropy or anisotropy of the velocity of light in that gauge];

v) the gauge fixing on the the momentum \(\varrho(\tau, \vec{\sigma})\) conjugate to the conformal factor of the 3-metric [this gauge variable is the source of the gauge dependence of 4-tensors and of the scalars under \(DiffM^4\), together with the gradients of the lapse and shift functions] is a nonlocal statement about the extrinsic curvature of the leaves \(\Sigma_\tau\) of the given 3+1 splitting of \(M^4\); since the superhamiltonian constraint produces normal deformations of \(\Sigma_\tau\) \[67\] and, therefore, transforms a 3+1 splitting of \(M^4\) into another one (the ADM formulation is independent from the choice of the 3+1 splitting), this gauge fixing is equivalent to the choice of a particular 3+1 splitting;

vi) the previous gauge fixing induces the gauge fixing of the lapse function (which determines the packing of the leaves \(\Sigma_\tau\) in the chosen 3+1 splitting) and, therefore, is equivalent to the fixation of a standard of proper time [again “clocks” should be built with the Dirac’s observables of some kind of matter].

The 3-orthogonal gauge of tetrad gravity is the equivalent of the Coulomb gauge in classical electrodynamics (like the harmonic gauge is the equivalent of the Lorentz gauge). Only after a complete gauge fixing the 4-tensors and the scalars under \(DiffM^4\) become measurable quantities (like the electromagnetic vector potential in the Coulomb gauge). At this stage it becomes acceptable the proposal of Bergmann\[30\] of identifying the points of a spacetime \((M^4, 4g)\), solution of the Einstein’s equations in absence of matter, in a way invariant under spacetime diffeomorphisms extended to 4-tensors (so that the rule is separately valid for each 4-geometry
contained in the equivalence class of Dirac’s observables defining a gravitational field), by using four invariants bilinear in the Weyl tensors called “individuating fields” (see also Refs. [65, 66]).

Our approach breaks the general covariance of general relativity completely by going to the special 3-orthogonal gauge. But this is done in a way naturally associated with theories with first class constraints (like all formulations of general relativity and the standard model of elementary particles with or without supersymmetry): the global Shanmugadhasan canonical transformations (when they exist) correspond to privileged Darboux charts for presymplectic manifolds defined by the first class constraints. Therefore, the gauges identified by these canonical transformations should have a special (till now unexplored) role also in generally covariant theories, in which traditionally one looks for observables invariant under diffeomorphisms and not for not generally covariant Dirac observables.

Let us remember that Bergmann [36] made the following critique of general covariance: it would be desirable to restrict the group of coordinate transformations (spacetime diffeomorphisms) in such a way that it could contain an invariant subgroup describing the coordinate transformations that change the frame of reference of an outside observer (these transformations could be called Lorentz transformations; see also the comments in Ref. [69] on the asymptotic behaviour of coordinate transformations); the remaining coordinate transformations would be like the gauge transformations of electromagnetism. This is what we have done. In this way “preferred” coordinate systems will emerge the WSW hypersurfaces), which, as said by Bergmann, are not “flat”: while the inertial coordinates are determined experimentally by the observation of trajectories of force-free bodies, these intrinsic coordinates can be determined only by much more elaborate experiments, since they depend, at least, on the inhomogeneities of the ambient gravitational fields. See also Ref. [70] for other critics to general covariance: very often to get physical results one uses preferred coordinates not merely for calculational convenience, but also for understanding. In Ref. [71] this fact has been formalized as the “principle of restricted covariance”. In our case the choice of the gauge-fixings has been dictated by the Shanmugadhasan canonical transformations, which produce generalized Coulomb gauges, in which one can put in normal form the Hamilton equations for the canonical variables of the gravitational field [and, therefore, they also produce a normal form of the two associated combinations of the Einstein equations which depend on the accelerations].

If we add to the tetrad ADM action the action for N scalar particles with positive energy in the form of Ref. [16] [where it was given on arbitrary Minkowski spacelike hypersurfaces], the only constraints which are modified are the superhamiltonian one, which gets a dependence on the matter energy density \( \mathcal{M}(\tau, \vec{\sigma}) \), and the 3 space diffeomorphism ones, which get a dependence on the matter momentum density \( \mathcal{M}_r(\tau, \vec{\sigma}) \). The canonical reduction and the determination of the Dirac observables can be done like in absence of matter. However, the reduced Lichnerowicz equation for the conformal factor of the 3-metric in the 3-orthogonal gauge and with \( \rho(\tau, \vec{\sigma}) \approx \)}
0 acquires now an extra dependence on $M(\tau, \vec{\sigma})$ and $M_r(\tau, \vec{\sigma})$.

Since, as a preliminary result, we are interested in identifying explicitly the action-at-a-distance (Newton-like and gravitomagnetic) potentials among particles hidden in tetrad gravity (like the Coulomb potential is hidden in the electromagnetic gauge potential), we shall restrict ourselves to void spacetimes without gravitational field by adding the second class constraints $r_a(\tau, \vec{\sigma}) \approx 0$, $\pi_a(\tau, \vec{\sigma}) \approx 0$. Now void spacetimes are no more gauge equivalent to Minkowski spacetime in Cartesian coordinates. Moreover, since in presence of matter the equations of motion do not imply $r_a = \pi_a = 0$, void spacetimes have now to be understood as a strong approximation identifying the instantaneous action-at-a-distance interaction among the particles contained in tetrad gravity. If we develop the conformal factor $\phi(\tau, \vec{\sigma})$ in a formal series in the Newton constant $G$ [$\phi = 1 + \sum_{n=1}^{\infty} G^n \phi_n$], one can find a solution $\phi = 1 + G\phi_1$ at order $G$ (post-Minkowskian approximation) of the reduced Lichnerowicz equation. However, due to a self-energy divergence in $\phi$ evaluated at the positions $\vec{\eta}_i(\tau)$ of the particles, one needs to rescale the bare masses to physical ones, $m_i \mapsto \phi^{-2}(\tau, \vec{\eta}_i(\tau))m_i^{(phys)}$, and to make a regularization of the type defined in Refs. [72]. Then, the regularized solution for $\phi$ can be put in the reduced form of the ADM energy, which becomes $|\vec{\kappa}_i(\tau)|$ the particle momenta conjugate to $\vec{\eta}_i(\tau)$; $\vec{\eta}_{ij} = [\vec{\eta}_i - \vec{\eta}_j]/|\vec{\eta}_i - \vec{\eta}_j|$

$$\hat{P}_{ADM}^r = \sum_{i=1}^{N} c \sqrt{m_i^{(phys)} c^2 + \vec{\kappa}_i^2(\tau)} - \frac{G}{c^2} \sum_{i \neq j}^{N} \frac{\sqrt{m_i^{(phys)} c^2 + \vec{\kappa}_i^2(\tau)}}{|\vec{\eta}_i(\tau) - \vec{\eta}_j(\tau)|} \sum_{j \neq i} + \frac{G}{8c^2} \sum_{i \neq j}^{N} 3\vec{\kappa}_i(\tau) \cdot \vec{\kappa}_j(\tau) - 5\vec{\kappa}_i(\tau) \cdot \vec{\eta}_{ij}(\tau) \vec{\kappa}_j(\tau) \cdot \vec{\eta}_{ij}(\tau) \cdot \vec{\eta}_{ij}(\tau) |\vec{\eta}_i(\tau) - \vec{\eta}_j(\tau)| + O(G^2). \quad (8)$$

One sees the Newton-like and the gravitomagnetic (in the sense of York) potentials (both of them need regularization) at the post-Minkowskian level (order $G$ but exact in $c$) emerging from the tetrad ADM version of Einstein general relativity. For $G=0$ we recover $N$ free scalar particles on the Wigner hyperplane in Minkowski spacetime, as required by deparametrization. For $c \rightarrow \infty$, we get the post-Newtonian Hamiltonian

$$H_{PN} = \sum_{i=1}^{N} \frac{\vec{\kappa}_i^2(\tau)}{2m_i^{(phys)}} (1 - \frac{2G}{c^2} \sum_{j \neq i} m_j^{(phys)} |\vec{\eta}_i(\tau) - \vec{\eta}_j(\tau)|) - \frac{G}{2} \sum_{i \neq j} m_i^{(phys)} m_j^{(phys)} \frac{\vec{\kappa}_i(\tau) \cdot \vec{\kappa}_j(\tau) - 5\vec{\kappa}_i(\tau) \cdot \vec{\eta}_{ij}(\tau) \vec{\kappa}_j(\tau) \cdot \vec{\eta}_{ij}(\tau) \cdot \vec{\eta}_{ij}(\tau) |\vec{\eta}_i(\tau) - \vec{\eta}_j(\tau)|}{|\vec{\eta}_i(\tau) - \vec{\eta}_j(\tau)|} + O(G^2), \quad (9)$$

which is of the type of the ones implied by the results of Refs. [72, 73] [the differences are probably connected with the use of different coordinate systems and with the
fact that one has essential singularities on the particle worldlines and the need of regularization].

The future research program will concentrate on the following subjects:

1) the post-Minkowskian 2-body problem in void spacetimes, to see the relevance of exact relativistic recoil effects in the motion of binaries;

2) the replacement of scalar particles with spinning ones to identify the precessional effects (like the Lense-Thirring one) of gravitomagnetism;

3) the linearization of the theory in the 3-orthogonal gauge in presence of matter: besides finding the Coulomb gauge description of gravitational waves, one expects to find a consistent (post-Minkowskian) coupling of the linearized gravitational field with matter, since the Bianchi identities have been solved, and to go beyond the strong approximation of void spacetimes;

4) perfect fluids and, then, extended relativistic bodies;

5) the coupling of tetrad gravity to the electromagnetic field, to fermion fields and then to the standard model, trying to make to reduction to Dirac’s observables in all these cases and to study their post-Minkowskian approximations;

6) quantization of tetrad gravity in the 3-orthogonal gauge with $\rho(\tau, \vec{\sigma}) \approx 0$: for each perturbative (in $G$) solution of the reduced Lichnerowicz equation one defines a Schroedinger equation in $\tau$ for a wave functional $\Psi(\tau; r_{\bar{a}})$ with the associated quantized ADM energy $\hat{P}_{ADM}[r_{\bar{a}}, \Pi_{\delta r_{\bar{a}}}]$ as Hamiltonian; no problem of physical scalar product is present, but only ordering problems in the Hamiltonian; moreover, one has the Moller radius as a ultraviolet cutoff.

References

[1] L.Lusanna and S.Russo, “Tetrad Gravity I): A New Formulation”, Firenze Univ. preprint 1998 (gr-qc/9807073).

[2] L.Lusanna and S.Russo, “Tetrad Gravity II): Dirac’s Observables”, Firenze Univ. preprint 1998 (gr-qc/9807074).

[3] R.DePietri and L.Lusanna, “Tetrad Gravity III): Asymptotic Poincaré Charges, the Physical Hamiltonian and Void Spacetimes”, in preparation.

[4] R.DePietri, L.Lusanna and M.Vallisneri, “Tetrad Gravity IV): The N-body Problem”, in preparation.

[5] P.A.M.Dirac, Can.J.Math. 2, 129 (1950); ”Lectures on Quantum Mechanics”, Belfer Graduate School of Science, Monographs Series (Yeshiva University, New York, N.Y., 1964).

[6] L.Lusanna, “Solving Gauss’ Laws and Searching Dirac Observables for the Four Interactions”, talk at the “Second Conf. on Constrained Dynamics and Quantum Gravity”, S.Margherita Ligure 1996, eds. V.De Alfaro, J.E.Nelson,
G.Bandelloni, A.Blasi, M.Cavaglià and A.T.Filippov, Nucl.Phys. (Proc.Suppl.) B57, 13 (1997) (hep-th/9702114), “Unified Description and Canonical Reduction to Dirac’s Observables of the Four Interactions”, talk at the Int.Workshop “New non Perturbative Methods and Quantization on the Light Cone’, Les Houches School 1997, eds. P.Grangé, H.C.Pauli, A.Neveu, S.Pinsky and A.Werner (Springer, Berlin, 1998) (hep-th/9705154). “The Pseudoclassical Relativistic Quark Model in the Rest-Frame Wigner-Covariant Gauge”, talk at the Euroconference QCD97, ed. S.Narison, Montpellier 1997, Nucl.Phys. (Proc. Suppl.) B64, 306 (1998).

[7] S.Shanmugadhasan, J.Math.Phys. 14, 677 (1973). L.Lusanna, Int.J.Mod.Phys. A8, 4193 (1993). M.Chaichian, D.Louis Martinez and L.Lusanna, Ann.Phys.(N.Y.)232, 40 (1994). L.Lusanna, Phys.Rep. 185, 1 (1990); Riv. Nuovo Cimento 14, n.3, 1 (1991); J.Math.Phys. 31, 2126 (1990); J.Math.Phys. 31, 428 (1990).

[8] P.A.M.Dirac, Can.J.Phys. 33, 650 (1955).

[9] L.Lusanna, Int.J.Mod.Phys. A10, 3531 and 3675 (1995).

[10] L.Lusanna and P.Valtancoli, Int.J.Mod.Phys. A12, 4769 (1997) (hep-th/9606078) and Int.J.Mod.Phys. A12, 4797 (1997) (hep-th/9606079).

[11] L.Lusanna and P.Valtancoli, Int.J.Mod.Phys. A13, 4605 (1998) (hep-th/9707072).

[12] K.Kuchar, J.Math.Phys. 17, 777, 792, 801 (1976); 18, 1589 (1977).

[13] P.A.M.Dirac, Rev.Mod.Phys. 21 (1949) 392.

[14] G.Longhi and M.Materassi, “A Canonical Realization of the BMS Algebra”, to appear in J.Math.Phys. (hep-th/9803128); “Collective and Relative Variables for a Classical Klein-Gordon Field”, Firenze Univ. preprint (hep-th/9890024), to appear in Int.J.Mod.Phys. A.

[15] L.Lusanna and M.Materassi, “The Canonical Decomposition in Center-of-Mass and Relative Variables of a Klein-Gordon Field in the Rest-Frame Wigner-Covariant Instant Form”, in preparation.

[16] L.Lusanna, Int.J.Mod.Phys. A12, 645 (1997).

[17] D.Alba and L.Lusanna, Int.J.Mod.Phys. A13, 2791 (1998) (hep-th/9705155).

[18] D.Alba and L.Lusanna, Int.J.Mod.Phys. A13, 3275 (1998) (hep-th/9705156).

[19] F.Bigazzi and L.Lusanna, “Spinning Particles on Spacelike Hypersurfaces and Their Rest-Frame Description”, Firenze Univ. preprint 1998 (hep-th/9807052), to appear in Int.J.Mod.Phys.A.
[20] F. Bigazzi and L. Lusanna, “Dirac Fields on Spacelike Hypersurfaces, Their Rest-Frame Description and Dirac Observables”, Firenze Univ. preprint 1998 (hep-th/9807054), to appear in Int. J. Mod. Phys. A.

[21] C. Lämmerzahl, J. Math. Phys. 34, 3918 (1993).

[22] I. Herbst, Commun. Math. Phys. 53, 285 (1977); 55, 316 (1997).
B. and L. Durand, Phys. Rev. D28, 396 (1983); erratum Phys. Rev. D50, 6642 (1994).
J. J. Basdevant and S. Boukraa, Z. Phys. C28, 413 (1985).
A. Martin and S. M. Roy, Phys. Lett. B233, 407 (1989).
A. LeYaouanc, L. Oliver and J. C. Raynal, Ann. Phys. (N.Y.) 239, 243 (1995).
W. Lucha and F. F. Schöberl, Phys. Rev. D50, 5443 (1994).

[23] G. Leibbrandt, “Non-Covariant Gauges”, ch.9 (World Scientific, Singapore, 1994).

[24] C. Møller, Ann. Inst. H. Poincaré 11, 251 (1949); “The Theory of Relativity” (Oxford Univ. Press, Oxford, 1957).

[25] G. Veneziano, “Quantum Strings and the Constants of Nature”, in “The Challenging Questions”, ed. A. Zichichi, the Subnuclear Series n.27 (Plenum Press, New York, 1990).

[26] R. Arnowitt, S. Deser and C. W. Misner, Phys. Rev. 117, 1595 (1960); in “Gravitation: an Introduction to Current Research”, ed. L. Witten (Wiley, New York, 1962).

[27] T. Regge and C. Teitelboim, Ann. Phys. (N.Y.) 88, 286 (1974).

[28] R. Beig and Ó Murchadha, Ann. Phys. (N.Y.) 174, 463 (1987).

[29] L. Andersson, J. Geom. Phys. 4, 289 (1987).

[30] T. Thiemann, Class. Quantum Grav. 12, 181 (1995).

[31] A. Ashtekar, “Asymptotic Structure of the Gravitational Field at Spatial Infinity”, in “General Relativity and Gravitation”, Vol. 2, ed. A. Held (Plenum, New York, 1980). A. Ashtekar and R. O. Hansen, J. Math. Phys. 19, 1542 (1978). A. Ashtekar and A. Magnon, J. Math. Phys. 25, 2682 (1984). A. Ashtekar and J. D. Romano, Class. Quantum Grav. 9, 1069 (1992).

[32] R. M. Wald, “General Relativity” (Chicago Univ. Press, Chicago, 1984).

[33] R. Geroch, J. Math. Phys. 9, 1739 (1968); 11, 343 (1970).

[34] V. Moncrief, J. Math. Phys. 16, 1556 (1975).
[35] Y. Choquet-Bruhat and J. W. York jr., “The Cauchy Problem”, in “General Relativity and Gravitation”, vol. 1, ed. A. Held (Plenum, New York, 1980).

[36] P. G. Bergmann, Rev. Mod. Phys. 33, 510 (1961).

[37] H. Weyl, Z. Physik 56, 330 (1929).

[38] P. A. M. Dirac, in “Recent Developments in General Relativity”, (Pergamon Press, Oxford, and PWN-Polish Scientific Publishers, Warsaw, 1962).

[39] J. Schwinger, Phys. Rev. 130, 1253 (1963).

[40] T. W. B. Kibble, J. Math. Phys. 4, 1433 (1963).

[41] S. Deser and C. J. Isham, Phys. Rev. D 14, 2505 (1976).

J. E. Nelson and C. Teitelboim, Ann. Phys. (N.Y.) 116, 86 (1978).

M. Pilati, Nucl. Phys. B 132, 138 (1978).

L. Castellani, P. van Nieuwenhuizen and M. Pilati, Phys. Rev. D 26, 352 (1982).

J. E. Nelson and T. Regge, Ann. Phys. (N.Y.) 166, 234 (1986); Int. J. Mod. Phys. A 4, 2021 (1989).

[42] J. M. Charap and J. E. Nelson, J. Phys. A 16, 1661 and 3355 (1983).

Class. Quantum Grav. 3, 1061 (1986).

J. M. Charap, “The Constraints in Vierbein General Relativity”, in “Constraint’s Theory and Relativistic Dynamics”, eds. G. Longhi and L. Lusanna (World Scientific, Singapore, 1987).

[43] J. W. Maluf, Class. Quantum Grav. 8, 287 (1991).

[44] M. Henneaux, Gen. Rel. Grav. 9, 1031 (1978).

J. Geheniau and M. Henneaux, Gen. Rel. Grav. 8, 611 (1977).

[45] M. Henneaux, Phys. Rev. D 27, 986 (1983).

[46] J. M. Charap, M. Henneaux and J. E. Nelson, Class. Quantum Grav. 5, 1405 (1988).

[47] M. Henneaux, J. E. Nelson and C. Schonblond, Phys. Rev. D 39, 434 (1989).

[48] B. S. De Witt, Phys. Rev. 162, 1195 (1967); “The Dynamical Theory of Groups and Fields” (Gordon and Breach, New York, 1967) and in “Relativity, Groups and Topology”, Les Houches 1963, eds. C. De Witt and B. S. De Witt (Gordon and Breach, London, 1964); “The Spacetime Approach to Quantum Field Theory”, in “Relativity, Groups and Topology II”, Les Houches 1983, eds. B. S. DeWitt and R. Stora (North-Holland, Amsterdam, 1984).

[49] B. S. De Witt, Phys. Rev. 160, 1113 (1967).
[50] C.J.Isham and K.Kuchar, Ann.Phys.(N.Y.) \textbf{164}, 288 and 316 (1984).
K.Kuchar, Found.Phys. \textbf{16}, 193 (1986).

[51] A.Lichnerowicz, J.Math.Pure Appl. \textbf{23}, 37 (1944).
Y.Choquet-Bruhat, C.R.Acad.Sci.Paris \textbf{226}, 1071 (1948); J.Rat.Mech.Anal. \textbf{5}, 951 (1956); “The Cauchy Problem” in “Gravitation: An Introduction to Current Research”, ed.L.Witten (Wiley, New York, 1962).

[52] J.W.York jr, Phys.Rev.Lett. \textbf{26}, 1656 (1971); \textbf{28}, 1082 (1972). J.Math.Phys. \textbf{13}, 125 (1972); \textbf{14}, 456 (1972). Ann.Inst.H.Poincaré \textbf{XXI}, 318 (1974).
N.O’Murchadha and J.W.York jr, J.Math.Phys. \textbf{14}, 1551 (1972). Phys.Rev. \textbf{D10}, 428 (1974).

[53] J.W.York jr., “Kinematics and Dynamics of General Relativity”, in “Sources of Gravitational Radiation”, Battelle-Seattle Workshop 1978, ed.L.L.Smarr (Cambridge Univ.Press, Cambridge, 1979).

[54] I.Ciufolini and J.A.Wheeler, “Gravitation and Inertia” (Princeton Univ.Press, Princeton, 1995).

[55] A.Qadir and J.A.Wheeler, “York’s Cosmic Time Versus Proper Time”, in “From SU(3) to Gravity”, Y.Ne’eman’s festschrift, eds. E.Gotsma and G.Tauber (Cambridge Univ.Press, Cambridge, 1985).

[56] P.A.M.Dirac, Canad.J.Math. \textbf{3}, 1 (1951).

[57] D.Christodoulou and S.Klainerman, “The Global Nonlinear Stability of the Minkowski Space” (Princeton Univ. Press, Princeton, 1993).

[58] J.M.Nester, Class.Quantum Grav. \textbf{5}, 1003 (1988).
W.H.Cheng, D.C.Chern and J.M.Nester, Phys.Rev. \textbf{D38}, 2656 (1988).
J.M.Nester, J.Math.Phys. \textbf{30}, 624 (1980) and \textbf{33}, 910 (1992).
J.M.Nester, Int.J.Mod.Phys. \textbf{A4}, 1755 (1989).
J.M.Nester, Class.Quantum Grav. \textbf{8}, L19 (1991).
A.Dimakis and F.Müller-Hoissen, Phys.Lett. \textbf{142A}, 73 (1989).

[59] A.Ashtekar and G.T.Horowitz, J.Math.Phys. \textbf{25}, 1473 (1984).

[60] E.Witten, Commun.Math.Phys. \textbf{80}, 381 (1981).

[61] A.Sen, J.Math.Phys. \textbf{22}, 1781 (1981); Phys.Lett. \textbf{119B}, 89 (1982).

[62] J.Frauendiener,Class.Quantum Grav. \textbf{8}, 1881 (1991).

[63] C.J.Isham, “Canonical Quantum Gravity and the Problem of Time”, in “Integrable Systems, Quantum Groups and Quantum Field Theories”, eds.L.A.Ibort and M.A.Rodriguez, Salamanca 1993 (Kluwer, London, 1993); “Conceptual and
Geometrical Problems in Quantum Gravity”, in “Recent Aspects of Quantum Fields”, Schladming 1991, eds. H.Mitter and H.Gausterer (Springer, Berlin, 1991); “Prima Facie Questions in Quantum Gravity” and “Canonical Quantum Gravity and the Question of Time”, in “Canonical Gravity: From Classical to Quantum”, eds. J.Ehlers and H.Friedrich (Springer, Berlin, 1994).

[64] K.Kuchar, “Time and Interpretations of Quantum Gravity”, in Proc.4th Canadian Conf. on “General Relativity and Relativistic Astrophysics”, eds. G.Kunstatter, D.Vincent and J.Williams (World Scientific, Singapore, 1992).

[65] J.Stachel, in “General Relativity and Gravitation”, GR11, Stockholm 1986, ed. M.A.H.Mac Callum (Cambridge Univ. Press, Cambridge, 1987); “The Meaning of General Covariance”, in “Philosophical Problems of the Internal and External Worlds”, Essays in the Philosophy of A.Grünbaum, eds. J.Earman, A.I.Janis, G.J.Massey and N.Rescher (Pittsburgh Univ.Press, Pittsburgh, 1993).

[66] C.Rovelli, Class.Quantum Grav. 8, 297 and 317 (1991).

[67] C.Teitelboim, “The Hamiltonian Structure of Space-Time”, in “General Relativity and Gravitation”, ed.A.Held, Vol.I (Plenum, New York, 1980). A.S.Hojman, K.Kuchar and C.Teitelboim, Ann.Phys. (N.Y.) 96, 88 (1971).

[68] P.Havas, Gen.Rel.Grav. 19, 435 (1987).
   R.Anderson, I.Vetharaniam and G.E.Stedman, Phys.Rep. 295, 93 (1998).

[69] L.Landau and E.Lifschitz, “The Classical Theory of Fields” (Addison-Wesley, Cambridge, 1951).

[70] G.F.R.Ellis and D.R.Matravers, Gen.Rel.Grav. 27, 777 (1995).

[71] R.Zalaletdinov, R.Tavakol and G.F.R.Ellis, Gen.Rel.Grav. 28, 1251 (1996).

[72] A.Einstein, B.Hoffman and L.Infeld, Ann.Math. 39, 66 (1938).
   A.Einstein and L.Infeld, Ann.Math. 41, 797 (1940); Canad.J.Math. 1, 209 (1949).
   L.Infeld, Rev.Mod.Phys. 29, 398 (1957).

[73] H.A.Lorentz and L.Droste, Amst.Akad.Versl. 26, 392 (1917).
   A.Eddington and G.L.Clarke, Proc.Roy.Soc.London A166, 465 (1938).
   V.Fock, J.Phys. (U.S.S.R.) 1, 81 (1939).
   A.Papapetrou, Proc.Phys.Soc.(London) 64, 57 (1951).