Endogenous versus Exogenous Origins of Crises

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Abstract

Are large biological extinctions such as the Cretaceous/Tertiary KT boundary due to a meteorite, extreme volcanic activity or self-organized critical extinction cascades? Are commercial successes due to a progressive reputation cascade or the result of a well orchestrated advertisement? Determining the chain of causality for extreme events in complex systems requires disentangling interwoven exogenous and endogenous contributions with either no clear or too many signatures. Here, I review several efforts carried out with collaborators, which suggest a general strategy for understanding the organization of several complex systems under the dual effect of endogenous and exogenous fluctuations. The studied examples are: Internet download shocks, book sale shocks, social shocks, financial volatility shocks, and financial crashes. Simple models are offered to quantitatively relate the endogenous organization to the exogenous response of the system. Suggestions for applications of these ideas to many other systems are offered.

1 Introduction

Extreme events are pervasive in all natural and social systems: earthquakes, volcanic eruptions, hurricanes and tornadoes, landslides and avalanches, lightning strikes, magnetic storms, catastrophic events of environment degradation, failure of engineering structures, crashes in the financial stock markets, social unrests leading to large-scale strikes and upheaval and perhaps to revolutions, economic drawdowns on national and global scales, regional and national power blackouts, traffic gridlocks, diseases and epidemics and so on.

Can we forecast them, manage, mitigate or prevent them? The answer to these questions requires us to investigate their origin(s).

Self-organized criticality, and more generally, complex system theory contend that out-of-equilibrium slowly driven systems with threshold dynamics relax through a hierarchy of avalanches of all sizes. Accordingly, extreme events are seen to be endogenous [6, 5], in contrast with previous prevailing views. In addition, the preparation processes before large avalanches are almost undistinguishable from those before small avalanches, making the prediction of the former basically impossible (see [54] for a discussion). But, how can one assert with 100% confidence that a given extreme event is really due to an endogenous self-organization of the system, rather than to the response to an external shock? Most natural and social systems are indeed continuously subjected to external stimulations, noises, shocks, sollications, forcing, which can widely vary in amplitude. It is thus not clear...
a priori if a given large event is due to a strong exogenous shock, to the internal dynamics of the system organizing in response to the continuous flow of small solicitations, or maybe to a combination of both. Addressing this question is fundamental for understanding the relative importance of self-organization versus external forcing in complex systems and for the understanding and prediction of crises.

This leads to two questions:

1. Are there distinguishing properties that characterize endogenous versus exogenous shocks?

2. What are the relationships between endogenous and exogenous shocks?

Actually, the second question has a long tradition in physics. It is at the basis of the interrogations that scientists perform on the enormously varied systems they study. The idea is simple: subject the system to a perturbation, a “kick” of some sort, and measure its response as a function of time, of the nature of the solicitations and of the various environmental factors that can be controlled. In physical systems at the thermodynamic equilibrium, the answer is known under the name of the theorem of fluctuation-dissipation, sometimes also referred to as the theorem of fluctuation-susceptibility [64]. In a nutshell, this theorem relates quantitatively in a very precise way the response of the system to an instantaneous kick (exogeneous) to the correlation function of its spontaneous fluctuations (endogenous). An early example of this relationship is found in Einstein’s relation between the diffusion coefficient \( D \) of a particle in a fluid subjected to the chaotic collisions of the fluid molecules and the coefficient \( \eta \) of viscosity of the fluid [18, 19]. The coefficient \( \eta \) controls the drag, i.e., response of the particle velocity when subjected to an exogenous force impulse. The coefficient \( D \) can be shown to be a direct measure of the (integral of the) correlation function of the spontaneous (endogenous) fluctuations of the particle velocity.

In out-of-equilibrium systems, the existence of a relationship between the response function to external kicks and spontaneous internal fluctuations is not settled [51]. In many complex systems, this question amounts to distinguishing between endogeneity and exogeneity and is important for understanding the relative effects of self-organization versus external impacts. This is difficult in most physical systems because externally imposed perturbations may lie outside the complex attractor which itself may exhibit bifurcations. Therefore, observable perturbations are often misclassified.

It is thus interesting to study other systems, in which the dividing line between endogenous and exogenous shocks may be clearer in the hope that it would lead to insight about complex physical systems. The investigations of the two questions above may also bring new understanding of these systems. The systems to which the endogenous-exogenous question (which we will refer to as “endo-exo” for short) is relevant include the following:

- Biological extinctions such as the Cretaceous/Tertiary KT boundary (meteorite versus extreme volcanic activity (Deccan traps) versus self-organized critical extinction cascades),
- immune system deficiencies (external viral/bacterial infections versus internal cascades of regulatory breakdowns),
• cognition and brain learning processes (role of external inputs versus internal self-organization and reinforcements),
• discoveries (serendipity versus the outcome of slow endogenous maturation processes),
• commercial successes (progressive reputation cascade versus the result of a well orchestrated advertisement),
• financial crashes (external shocks versus self-organized instability),
• intermittent bursts of financial volatility (external shocks versus cumulative effects of news in a long-memory system),
• the aviation industry recession (9/11/2001 terrorist attack versus structural endogenous problems),
• social unrests (triggering factor or rotting of social tissue),
• recovery after wars (internally generated (civil wars) versus imported from the outside) and so on.

It is interesting to mention that the question of exogenous versus endogenous forcing has been hotly debated in economics for decades. A prominent example is the theory of Schumpeter on the importance of technological discontinuities in economic history. Schumpeter argued that “evolution is lopsided, discontinuous, disharmonious by nature... studded with violent outbursts and catastrophes... more like a series of explosions than a gentle, though incessant, transformation” [52]. Endogeneity versus exogeneity is also paramount in economic growth theory [50]. Our analyses reviewed below suggest a subtle interplay between exogenous and endogenous shocks which may cast a new light on this debate.

In the sequel, we review the works of the author with his collaborators, which have investigated the endo-exo question in a variety of systems.

2 Exogenous and endogenous shocks in social networks

One defining characteristics of humans is their organization in social networks. It is probable that the large brain, which makes what we are, has been shaped by social interactions, and may have co-evolved with the size and complexity of social groups [16, 68]. A single individual may belong to several intertwined social networks, associated with their different activities (work colleagues, college alumni societies, friends, family members, etc.). The formation and the evolution of social networks and their mutual entanglements control the hierarchy of interactions between humans, from the individual level to society and to culture. In this section, we review a few original probes of several social networks which unearth a remarkable universality: the distribution of human decision times in social networks seem to be described by a power law $1/t^{1+\theta}$ with $\theta = 0.3 \pm 0.1$. This constitutes an essential ingredient in models describing how the cascade of agent decisions leads to the bottom-up organization of the response of social systems. We first present such a model in terms of a simple epidemic process of word-of-mouth effects [25, 58, 57] and then discuss the different data sets.
2.1 A simple epidemic cascade model of social interactions

Let us consider an observable characterizing the activity of humans within a given social network of interactions. This activity can be the rate of visits or downloads on an internet website, the sales of a book or the number of newspaper articles on a given subject.

We envision that the instantaneous activity results from a combination of external forces such as news and advertisement, and of social influences in which each past active individual may impregnate other individuals in her network of acquaintances with the desire to act. This impact of an active individual onto other humans is not instantaneous as people react at a variety of time scales. The time delays capture the time interval between social encounters, the maturation of the decision process which can be influenced by mood, sentiments, and many other factors and the availability and capacity to implement the decision. We postulate that this latency can be described by a memory kernel \( \phi(t - t_i) \) giving the probability that an action at time \( t_i \) leads to another action at a later time \( t \) by another person in direct contact with the first active individual. We consider the memory function \( \phi(t - t_i) \) as a fundamental macroscopic description of how long it takes for a human to be triggered into action, following the interaction with an already active human.

Then, starting from an initial active individual (the "mother") who first acts (either from exogenous news or by chance), she may trigger action by first-generation "daughters," which themselves propagate the drive to act to their own friends, who become second-generation active individuals, and so on. This cascade of generations can be shown to renormalize the memory kernel \( \phi(t - t_i) \) into a dressed or renormalized memory kernel \( K(t - t_i) \), giving the probability that an action at time \( t_i \) leads to another action by another person at a later time \( t \) through any possible generation lineage. In physical terminology, the renormalized memory kernel \( K(t) \) is nothing but the response function of the system to an impulse. This is captured by the following equations

\[
A(t) = s(t) + \int_{-\infty}^{t} d\tau A(\tau) \phi(t - \tau) = \int_{-\infty}^{t} d\tau s(\tau) K(t - \tau) .
\]  

The meaning of these two equivalent formulations is the following. The \( s(t) \)'s are the spontaneous exogenous activations. The integral \( \int_{-\infty}^{t} d\tau A(\tau) \phi(t - \tau) \) gives the additional contribution due to past activities \( A(\tau) \) whose influences to the present are mediated by the direct influence kernel \( \phi \) of first generation. The last integral \( \int_{-\infty}^{t} d\tau s(\tau) K(t - \tau) \) expresses the fact that the present activity \( A(t) \) can also be seen as resulting from all past exogenous sources \( s(\tau) \) mediated to the present by the renormalized kernel \( K \), which takes into account all generations of cascades of influences.

The following functional dependence is found to provide an accurate description, as we shall discuss below:

\[
K(t) \sim 1/(t - t_c)^p , \quad \text{with} \quad p = 1 - \theta .
\]  

The dependence \( p \) implies \( \phi(t) \sim 1/(t - t_c)^{1+\theta} \).

We should stress that the renormalization from the usually (but not always) unobservable "bare" response function \( \phi(t) \) with exponent \( 1 + \theta \) in \( [3] \) to the observable "renormalized" response function \( K(t) \) in \( [2] \) with exponent \( 1 - \theta \) is obtained if the network is close
to critical, i.e., if the average branching ratio \( n \) is close to 1 (\( n \) is defined as the average number of daughters of first generation per mother). In other words, there is on average approximately one triggered daughter per active mother. This condition of criticality ensures, in the language of branching processes, that avalanches of active people triggered by a given mother are self-similar (power law distributed). In contrast, for \( n < 1 \), the cascade of triggered actions is “sub-critical” and avalanches die off more rapidly. It can be shown that there is in this case a characteristic time scale

\[
t^* \sim \frac{1}{(1-n)^{1/\theta}}
\]  
(4)

acting like a correlation time, which separates two regimes:

- for \( t < t^* \), the renormalized response function \( K(t) \) is indeed of the form (2);
- for \( t > t^* \), the renormalized response function \( K(t) \) crosses over to an asymptotic decay with exponent \( 1 + \theta \), of the form of \( \phi(t) \) in (3).

For \( n > 1 \), the epidemic process is super-critical and has a finite probability of growing exponentially fast. We will not be concerned with this last regime which does not seem relevant in the data discussed below.

In the absence of strong external influences, a peak in social activity can occur spontaneously due to the interplay between a continuous stochastic flow of small external news and the amplifying impact of the epidemic cascade of social influences. It can then be shown that, for \( n \) close to 1 or equivalently for \( |t - t_c| < t^* \), the average growth of the social activity before such an “endogenous” peak and the relaxation after the peak are proportional to

\[
\int_0^{+\infty} K(t - t_c + u)K(u)du \sim 1/|t - t_c|^{1-2\theta},
\]  
(5)

where the right-hand-side expression holds for \( K(t) \) of the form (2). The prediction that the relaxation following an exogeneous shock should happen faster (larger exponent \( 1 - \theta \)) than for an endogeneous shock (with exponent \( 1 - 2\theta \)) agrees with the intuition that an endogeneous shock should have impregnated the network much more and thus have a longer lived influence. In a nutshell, the mechanism at the origin of the endogenous response function (5) is the constructive interference of accumulated small news cascading through the social influence network. In other words, the presence of a hierarchy of nested relaxations \( K(t) \) given by (2), each one associated with each small news, creates the effective endogenous response (5).

Dodds and Watts have recently introduce a general model of contagion which, by explicitly incorporating memory of past exposures to, for example, an infectious agent, rumor, or new product, includes the main features of existing contagion models and interpolates between them [15].

### 2.2 Internet download shocks

In Ref. [35], Johansen and Sornette report the following experiment. The authors were interviewed by a journalist from the leading Danish newspaper JyllandsPosten on a subject
Figure 1: Cumulative number of downloads $N$ as a function of time $t$ from the appearance of the interview on Wednesday the 14 April 1999. The fit is $N(t) = a t^{1-p} + ct$ with $b \approx 0.58 \pm 0.03$. Reproduced from [35].

of rather broad and catchy interest, namely stock market crashes. The interview was published on April 14, 1999 in both the paper version of the newspaper as well as in the electronic version (with access restricted to subscribers) and included the URLs where the authors’ research papers on the subject could be retrieved. It was hence possible to monitor the number of downloads of papers as a function of time since the publication date of the interview. The rate of downloads of the authors’ papers as a function of time was found to obey a $1/t^p$ power law with exponent $b = 0.58 \pm 0.03$, as shown in figure 1.

Within the model of epidemic word-of-mouth effect summarized in section 2.1 the relaxation of the rate of downloads after the publication of the interview characterizes the response function $K(t)$ given by (2) with respect to an exogenous peak: prior to the publication of the interview, the rate of downloads was slightly less than one per day; it suddenly jumped to several tens of downloads per day in the first few days after the publication and then relaxed slowly according to (2). The reported power law with exponent $p \simeq 0.6$ is compatible with the form (2) with $\theta = 0.4$, which is within the range of other values: $\theta = 0.3 \pm 0.1$.

Johansen [29] has reported another similar observation following another web-interview on stock market crashes, which contained the URL of his articles on the subject. He found again a power law dependence (2), but with an exponent $p$ close to 1, leading in the terminology of the model of epidemic word-of-mouth effect to $\theta \simeq 0$. Two interpretations are possible: (i) the exponent $\theta$ is non-universal; (ii) the social network is not always close to criticality ($n \simeq 1$) and the observable response function $K(t)$ is then expected to cross-over smoothly from a power law with exponent $1 - \theta$ to another asymptotic power law with exponent $1 + \theta$. According to this second hypothesis, the exponent $p$ of the relaxation kernel $K(t)$ may be found in the range $1 - \theta$ to $1 + \theta$, depending upon the range of investigated time scales and the proximity $1 - n$ to criticality. We find hypothesis (ii) more attractive as it puts the blame on the non-universal parameter $n$, which embodies
the connectivity structure, static and dynamics, of social interactions at a given moment. It does not seem unrealistic to think that \( n \) may not be always at its critical value 1, due to many possible other social influences. In contrast, one could postulate that the power law \( (3) \) for the direct influence function \( \phi(t) \) between two directly linked humans may reflect a more universal character. But, of course, only more empirical investigations will allow to put more light on this issue.

Eckmann, Moses and Sergi [17] also report an original investigation probing the temporal dynamics of social networks using email networks in their universities. They find a distribution of response times till a message is answered which seems to be a power law with exponent less than 1 at early times (1 hour) to another power law with exponent larger than 1 at long times (days), which could be a direct evidence of the direct response function \( \phi(t) \) defined in \( (3) \). The relationship between their investigation and the previous works using web downloads [35, 29] has been noted by Johansen [31].

### 2.3 Book sale shocks

Sornette, Deschatres, Gilbert and Ageon have used a database of sales from Amazon.com as a proxy for commercial growth and successes [57]. Fig. 2 shows about 1.5 years of data for two books, Book A (“Strong Women Stay Young” by Dr. M. Nelson) and Book B (“Heaven and Earth (Three Sisters Island Trilogy)” by N. Roberts), which are illustrative of the two classes found in this study. Book A jumped on June 5, 2002, from rank in the 2,000s to rank 6 in less than 12 hours. On June 4, 2002, the New York Times published an article crediting the “groundbreaking research done by Dr. Miriam Nelson” and advising the female reader, interested in having a youthful postmenopausal body, to buy the book and consult it directly [8]. This case is the archetype of an “exogenous” shock. In contrast, Book B culminated at the end of June 2002 after a slow and continuous growth, with no such newspaper article, followed by a similar almost symmetrical decay, the entire process taking about 4 months. We will show below that the peak for Book B belongs to the class of endogenous shocks. Qualitatively, such endogeneous growth is well explained in Ref. [22] by taking the example of the book “Divine Secrets of the Ya-Ya Sisterhood” by R. Wells, which became a bestseller two years after publication, with no major advertising campaign. Following the reading of this originally small budget book, “Women began forming Ya-Ya Sisterhood groups of their own [...] The word about Ya-Ya was spreading [...] from reading group to reading group, from Ya-Ya group to Ya-Ya group” [22]. Generally, the popularity of a book is based on whether the information regarding that book will be able to propagate far and long enough into the network of potential buyers.

Another dramatic example of exogenous shocks is shown in figure 3, here the personal trainer of Oprah Winfrey had his book presented 7 or 8 times during the Oprah Winfrey Show, leading to dramatic overnight jumps in sales.

Each relaxation of sales for about 140 books that reached the top 50 in the Amazon.com ranking system have been analyzed and shown to fall into two categories: the relaxations described by a power law with an exponent close to \( 0.7 = 1 - \theta \) and the relaxations described by a power law with an exponent close to \( 0.4 = 1 - 2\theta \), for \( \theta \simeq 0.3 \). An example of such fits for the two books shown in figure 2 is presented in figure 4. In addition, Sornette et al. [57] checked that an overwhelming majority of those sale peaks classified as exogenous from the value of their exponent \( \simeq 0.7 = 1 - \theta \) were preceded by an abrupt jump, in agreement with
Figure 2: Time evolution over a year and a half of the sales per day of two books: Book A (bottom, blue, left scale) is “Strong Women Stay Young” by Dr. M. Nelson and Book B (top, green, right scale) is “Heaven and Earth (Three Sisters Island Trilogy)” by N. Roberts. The difference in the patterns is striking, Book A undergoing an exogenous peak on June 5, 2002, and Book B endogenously reaching a maximum on June 29, 2002. Reproduced from [57].

Figure 3: Time evolution of the book entitled “Get with the Program.” Each time the book appeared on Oprah Winfrey Show (B. Greene is O. Winfrey’s trainer), the sales jumped overnight.
Figure 4: The bottom curve (blue) shows the relaxation of the sales of Book A after the peak of $t_c = June 5, 2002$ as a function of the time $t - t_c$ from the time of the peak. The least squares best fit with a power law gives a slope $\approx -0.7$. Since this peak is identified as exogenous with theoretical slope $1 - \theta$, we get the estimate $\theta = 0.3 \pm 0.1$. The curve in the middle (green shifted up by a factor 6) shows the relaxation of sales of Book B after the peak of $t_c = June 29, 2002$ as a function of the time $t - t_c$ from the time of the peak. The least squares fit gives a slope of $\approx -0.4$, which provides the independent estimate $\theta = 0.3 \pm 0.1$ from the theoretical endogenous exponent $1 - 2\theta$. The top curve (red shifted up by a factor 25 with respect to the bottom curve) shows the acceleration of the sales of Book B leading to the same peak at $t_c = June 29, 2002$ as a function of the time $t_c - t$ to the time of the peak. The time on the x-axis has been reversed to compare the precursory acceleration with the aftershock relaxation. The least squares slope is $\approx -0.3$ not far from the prediction $1 - 2\theta$ of the cascade model, with $\theta = 0.3 \pm 0.1$.

the epidemic cascade model of social interactions described in section 2.1. In contrast, those sale peaks falling in the endogenous class according to their exponent $\simeq 0.4 = 1 - 2\theta$ of their relaxation after the peak were found to be preceded by an approximately symmetry growth described by a power law with the same exponent, as predicted by [5]. An example is shown also for book B in figure 4.

The small values of the exponents (close to $1 - \theta$ and $1 - 2\theta$) both for exogenous and endogenous relaxations imply that the sales dynamics is dominated by cascades involving high-order generations rather than by interactions stopping after first-generation buy triggering. Indeed, if buys were initiated mostly by the direct effects of news or advertisements, and not much by triggering cascades in the acquaintance network, the cascade model predicts that we should then measure an exponent $1 + \theta$ given by the “bare” memory kernel $\phi(t)$, as already said. This implies that the average number $n$ (the average branching ratio in the language of branching models) of impregnated buyers per initial buyer in the social epidemic model is on average very close to the critical value 1, because the renormaliza-
tion from $\phi(t)$ to $K(t)$ given by (2) only operates close to criticality characterized by the occurrence of large cascades of buys. Reciprocally, a value of the exponent $p$ larger than 1 suggests that the associated social network is far from critical. Such instances can actually be observed. Examples of cross-overs from the renormalized response function $K(t)$ (2) to $\phi(t)$ in (3) with an asymptotic decay with exponent $1 + \theta$ has been documented [57, 14]. Note that it is possible to give an analytical description of this cross-over exhibited by $K(t)$ as a function $n$ [24], thus allowing in principle to invert for $n$ for a given data set. This opens the tantalizing possibility of measuring the dynamical connectivity of the social network, and possibly to monitor it as a function of time.

This findings open other interesting avenues of research. While this first investigation has emphasized the distinction between exogenous and endogenous peaks to set the fundamentals for a general study, repeating peaks as well as peaks that may not be pure members of a single class are also frequent. In a sense, there are no real “endogenous” peaks, one could argue, because there is always a source or a string of news impacting on the network of buyers. What Sornette et al. [57] have done is to distinguish between two extremes, the very large news impact and the structureless flow of small news amplified by the cascade effect within the network. One can imagine and actually observe a continuum between these two extremes, with feedbacks between the development of endogeneous peaks and the attraction of interest of the media as a consequence, feeding back and providing a kind of exogenous boost, and so on. In those and in more complicated cases, the epidemic model of word-of-mouth effects should provide a starting platform to predict the sales dynamics as a function of an arbitrary set of external sources. Tracking dynamically the connectivity $n(t)$ of each social network relevant to a given product, it should also be possible to target the most favorable times, corresponding to the largest $n(t)$, for promoting or sustaining the sales of a given product, with obvious consequences for marketing and advertisement strategies. An additional extension includes the possible feedback of the marketing strategy on the control parameter $n(t)$ which could be manipulated so as to keep the system critical, an ideal situation from the point of view of marketers and firms. Quantifying this effect requires to extend the simple epidemic model in the spirit of mechanisms leading to self-organized criticality by positive feedbacks of the order parameter onto the control parameter [60, 21]. Sornette et al.’s results suggest that social networks have evolved to converge very close to criticality. As Andreas S. Weigend, chief scientist of Amazon.com (2002-2004) wrote on his webpage: “Amazon.com might be the world’s largest laboratory to study human behavior and decision making.” I share this view point.

Actually, I envision that an extension of Sornette et al.’s study to a broad database of sales from all products sold by e-retailers like Amazon.com could give access to the equivalent of the “social climate” of a country such as the USA and its evolution as a function of time under the various exogenous and endogenous factors at work. Indeed, Amazon.com categorizes its products in different (tradable) dimensions of possible interest to a human being, such as

- Books, Music, DVD,
- Electronics (audio and video, camera and photo, software, computer and video games, cell phones...),
- Office,
• Kids and Baby,
• Home and Garden (which includes pets),
• Gifts, Registries, Jewelry and watches,
• Apparel and Accessories,
• Food,
• Health, Personal Care, Beauty,
• Sports and Outdoors,
• Services (movies, restaurants, travel, cars, ...),
• Arts and Hobbies,
• Friend and Favorites,

with many sub-categories. Monitoring and analyzing the sales as a function of time in these different categories is like getting the temperature, wind velocity, humidity in meteorology in many different locations. The flow of interest of society at large and of sub-groups could in principle tells us how society is responding in its spending habits to large scale influence. As an illustrative example, it has been shown that, during bullish periods characterized by strong stock market gains (bubble regimes), the number of books written and sold related to financial investments soared [48, 47].

Another potentially fruitful application is the music industry and the impact on sales versus Internet piracy of the quality of performers (endogenous effect on the network of potential buyers who can promote a CD by word-and-mouth in the network of potential buyers) versus the promotion campaigns of short-lived performers and their one-only-hit wonders [41]. Indeed, according to an internal study done by one of the big companies that dominate the production and distribution of music, the drop in sales in America may have less to do with internet piracy than with other factors, among them the decreasing quality of music itself. The days of watching a band develop slowly over time with live performances are over, according to some professionals. Even Wall Street analysts are questioning quality. If CD sales have shrunk, one reason could be that people are less excited by the industry’s product. A poll by Rolling Stone magazine found that fans, at least, believe that relatively few “great” albums have been produced recently [41]. This is clearly an endo-exo question that can be analyzed with databases available on the Internet.

2.4 Social shocks

Roehner, Sornette and Andersen [49] have used the concept of exogeneous shocks to propose a general method for quantifying the response function in order to advance the social sciences. By using a database of newspaper articles called Lexis-Nexis, which is available in many departments of political science or sociology, they have quantified the response to shocks, such as the following:
Figure 5: Relaxation of three different social variables after the shock of September 11, 2001. The solid line curve is the number of articles writing on the destruction of mosques after the event; the broken line (scale on the right-hand side) shows the number of anti-arab aggressions in California in the three months after September 11; the dotted line shows the changes in the level of the Dow Jones Index with respect to its pre-Sep.11 level as given by the difference DJI(pre-9/11)-DJI(current). Source: California's Attorney General Office, published in the San Jose Mercury News, 11 March 2002. Reproduced from [49].

- On October, 31, 1984, the Prime minister of India, Indira Gandhi, was assassinated by two of her Sikh bodyguards. This event triggered a wave of retaliations against Sikh people and Sikh property, not only in India (particularly in New Delhi), but in many other countries as well.

- In the early hours of December, 6, 1992, thousands of Hindus converged toward the holy city of Ayodhya in northern India and began to destroy the Babri mosque which was said to be built on the birthplace of Lord Rama. The old brick walls came down fairly easily and soon the three domes of the mosque crashed to the ground. This event triggered a burst of protestations and retaliations which swept the whole world from Bangladesh to Pakistan, to England or the Netherlands. In all these countries, Hindu people were assaulted, Hindu temples were firebombed, damaged or destroyed.

- On September, 11, 2001, two planes crashed into the twin towers of the World Trade Center in New York. This event triggered a wave of reactions against Islamic people and property not only in the United States but also in other countries.

For these different events, Roehner et al. [49] show that different quantitative measures of social responses exhibit an approximate universal behavior, again characterized by a power law, as shown in figure 5. This figure gives the time evolution after September 11, 2001 of newspaper articles, anti-arab aggressions and the Dow Jones Industrial Average, which are approximated by a power law $\sim 1/t^p$. Due to the coarseness of the measures, the exponent $p$ is not well-constrained: $p = -1.8 \pm 0.7$ (newspaper articles), $p = -1.4 \pm 0.5$ (anti-arab aggressions) and $p = -2.2 \pm 1.6$ (DJI). Comparing the reaction to September 11,
2001 in different countries such as Canada, Great Britain and the Netherlands, Roehner et al. [49] have suggested that the response function actually expresses an information on “cracks” pre-existing in the social networks of the corresponding countries. For instance, the number of attacks to Mosques has been larger in the Netherlands, which is in line with other information on the concerns at high political levels (private communication to the authors) about the integrity of the social tissue in the Netherlands, a fact illustrated more recently on the political scene by the rapid rise and then assassination of the rightist politician Fortuyn in May 2002. This line of evidence could be quantified within the epidemic model of social influence by different values of the connectivity parameter $n$ in different countries.

Burch, Emery and Fuerst [9] have used also the unique opportunity offered by the “nine-eleven” terrorist attack to confirm clearly the hypothesis that closed-end mutual fund discounts from fund net asset values reflect small investor sentiment. Carter and Simkins [10] investigated the reaction of airline stock prices to the 9/11 terrorist attack and found that the market was concerned about the increased likelihood of bankruptcy in the wake of the attacks and distinguished between airlines based on their ability to cover short-term obligations (i.e., liquidity).

3 Exogenous and endogenous shocks in financial markets

3.1 Volatility shocks

Standard economic theory holds that the complex trajectory of stock market prices is the faithful reflection of the continuous flow of news that are interpreted and digested by an army of analysts and traders. Accordingly, large shocks should result from really bad surprises. It is a fact that exogenous shocks exist, as epitomized by the recent events of Sept. 11, 2001, and there is no doubt about the existence of utterly exogenous bad news that move stock market prices and create strong bursts of volatility. A case that cannot be refuted is the the market turmoil in Japan following the Kobe earthquake of Jan. 17, 1995 that led to a total cost estimated around $200 billion dollars. Indeed, as long as the science of earthquake prediction is still in its infancy, destructive earthquakes are not endogeneized in advance in stock market prices by rational agents ignorant of seismological processes. One may also argue that the invasion of Koweit by Iraq on Aug. 2, 1990 and the coup against Gorbachev on Aug., 19, 1991 were strong exogenous shocks. However, some could argue that precursory fingerprints of these events were known to some insiders, suggesting the possibility that the action of these informed agents may have been reflected in part in stock markets prices. Even more difficult is the classification (endogenous versus exogenous) of the hierarchy of volatility bursts that continuously shake stock markets. While it is a common practice to associate the large market moves and strong bursts of volatility with external economic, political or natural events [66], there is not convincing evidence supporting it.

Perhaps the most robust observation in financial stock markets is that volatility is serially correlated with long-term dependence (approximately power law like). Volatility autocorrelation is typically modeled using autoregressive conditional heteroskedasticity
(ARCH) \[20\], generalized ARCH \[7\], stochastic volatility \[3\], Markov switching \[23\] \[24\], nonparametric \[45\] and extensions of these models (see \[44\] for comparisons). Recent powerful extensions include the Multifractal Random Walk model (MRW) introduced by Muzy, Bacri and Delour \[42\] \[4\], which belongs to the class of stochastic volatility models. Using the MRW, Sornette, Malevergne and Muzy \[61\] have shown that it is possible to distinguish between an endogenous and an exogenous origin to a volatility shock. Tests on the Oct. 1987 crash, on a hierarchy of volatility shocks and on a few of the obvious exogenous shocks have validated the concept. This study shows that the relaxation with time of a burst of volatility is distinctly different after a strong exogenous shock compared with the relaxation of volatility after a peak with no identifiable exogenous sources. This study does not explain the origin of volatility correlation. But it identifies the “natural” response function of the system to an external shock, from which the stationary long-term dependence structure of the volatility and its intermittent bursts derive automatically. In other words, Sornette et al.’s study leads to the view that the properties of the volatility can be in large part understood from a single characteristic, which is the response of the agents to a new piece of news. This response function must ultimately be derived from the behavior of financial agents, for instance taking into account their sensitivity to wealth changes, their loss aversion as well as their finite-time memory of past losses that may impact their future decision \[40\].

The multifractal random walk is an autoregressive process with a long-range memory decaying as $t^{-1/2}$, which is defined on the logarithm of the volatility. Using the MRW model for the dependence structure of the volatility, Sornette et al. predict that exogenous volatility shocks will be followed by a universal relaxation

$$\simeq \lambda / t^{1/2} ,$$

where $\lambda$ is the multifractal parameter, while endogenous volatility shocks relax according to a power law

$$\simeq 1/t^{p(V_0)} , \quad \text{with} \quad p(V_0) \simeq \lambda^2 \ln(V_0) ,$$

with an exponent $p(V_0)$ which is a linear function of the logarithm $\ln(V_0)$ of the shock of volatility $V_0$. The difference between these behaviors and those reported above modeled by the epidemic process with long-term memory stems from the fact that the stock market returns $r_{\Delta t}(t)$ at time scale $\Delta t$ at a given time $t$ can be accurately described by the following process \[42\] \[4\]:

$$r_{\Delta t}(t) = \epsilon(t) \cdot \sigma_{\Delta t}(t) = \epsilon(t) \cdot e^{\omega_{\Delta t}(t)} ,$$

where $\epsilon(t)$ is a standardized Gaussian white noise independent of $\omega_{\Delta t}(t)$ and $\omega_{\Delta t}(t)$ is a nearly Gaussian process with mean and covariance:

$$\mu_{\Delta t} = \frac{1}{2} \ln(\sigma^2_{\Delta t}) - C_{\Delta t}(0)$$

$$C_{\Delta t}(\tau) = \text{Cov}[\omega_{\Delta t}(t), \omega_{\Delta t}(t+\tau)] = \lambda^2 \ln \left( \frac{T}{|\tau| + e^{-3/2\Delta t}} \right) .$$

where $\sigma^2_{\Delta t}$ is the return variance at scale $\Delta t$ and $T$ represents an “integral” (correlation) time scale. $\lambda$ is called the multifractal parameter: when it vanishes, the MRW reduces to a standard Wiener process (standard continuous random walk). Such logarithmic decay of
log-volatility covariance at different time scales has been evidenced empirically in \[4, 42\]. Typical values for \(T\) and \(\lambda^2\) are respectively 1 year and 0.04.

The MRW model can be expressed in a more familiar form, in which the log-volatility \(\omega_\Delta(t)\) obeys an auto-regressive equation whose solution reads

\[
\omega_\Delta(t) = \mu_\Delta + \int_{-\infty}^{t} d\tau \, \eta(\tau) \, K_\Delta(t - \tau) ,
\]

where \(\eta(t)\) denotes a standardized Gaussian white noise and the memory kernel \(K_\Delta(\cdot)\) is a causal function, ensuring that the system is not anticipative. The process \(\eta(t)\) can be seen as the information flow. Thus \(\omega(t)\) represents the response of the market to incoming information up to the date \(t\). At time \(t\), the distribution of \(\omega_\Delta(t)\) is Gaussian with mean \(\mu_\Delta\) and variance \(V_\Delta = \int_0^\infty d\tau \, K_\Delta^2(\tau) = \lambda^2 \ln \left( \frac{Te^{3/2}}{2\Delta t} \right)\). Its covariance, which entirely specifies the random process, is given by

\[
C_\Delta(\tau) = \int_0^\infty dt \, K_\Delta(t)K_\Delta(t + |\tau|) .
\]

Performing a Fourier transform, we obtain

\[
\hat{K}_\Delta(f)^2 = \hat{C}_\Delta(f) = 2\lambda^2 \, f^{-1} \left[ \int_0^T f(t) \sin(t) \frac{dt}{t} \right] + O(f \Delta t \ln(f \Delta t)) ,
\]

which shows, using (10), that for \(\tau\) small enough,

\[
K_\Delta(\tau) \sim K_0 \sqrt{\frac{\lambda^2 T}{\tau}} \quad \text{for} \quad \Delta t << \tau << T ,
\]

which is the above stated exogenous response function \(6\). This slow power law decay \(14\) of the memory kernel in \(11\) ensures the long-range dependence and multifractality of the stochastic volatility process \(8\).

The main difference between the MRW model and the previous class of epidemic process is that the long-term memory appear in the logarithm of the variable in the former, as shown from equation \(11\). As a consequence, the MRW basically describes a variable with is the exponential of a long-memory process. It is the interplay between this strongly nonlinear exponentiation and the long-memory which gives the multifractal properties to the MRW and, as a consequence, the shock amplitude dependence of the exponents \(p(r)\) of the relaxation of the volatility following endogenous shocks. In contrast, the linear long-term memory structure \(1\) of the epidemic processes of section \(2.1\) ensures universal exponents which are independent of the shock amplitudes (but not of the endo-versus-exo nature). In the epidemic process \(1\), the relationship between exogenous and endogenous relaxations is expressed by the exponents of the power laws \(\sim 1/t^{1-\theta} (\text{exo}) \) versus \(\sim 1/t^{1-2\theta} (\text{endo})\). In the MRW, notice that the relationship between exogenous \(6\) and endogenous relaxations \(7\) is through the multifractal parameter \(\lambda\): the fact that an amplitude of the exogenous response function impacts the power law exponent of the endogenous relaxation is again a signature of the exponential structure of the multifractal model. The MRW extends the realm of possible relationships between endogenous and exogenous responses discussed until now.
3.2 Financial crashes

The endo-exo question also appears to be crucial for understanding financial crashes. In contrast with the previous examples, the distinction is not that much in the relaxation or recovery after the shock but rather in the precursory behavior before the crash. An endogenous crash might be expected to end a period of strong price gains due for instance to speculative herding. In contrast, an exogenous crash would be the response of the financial system to a very strong adverse piece of information.

Indeed, according to standard economic theory, the complex trajectory of stock market prices is the faithful reflection of the continuous flow of news that are interpreted and digested by an army of analysts and traders. Accordingly, large market losses should result from really bad surprises only. It is indeed a fact that exogenous shocks exist, as epitomized by the recent events of Sept. 11, 2001 and the coup in the Soviet Union on Aug. 19, 1991, which move stock market prices and create strong bursts of volatility, as discussed above. However, it is always the case? A key question is whether large losses and gains are indeed slaved to exogenous shocks or on the contrary may result from an endogenous origin in the dynamics of that particular stock market. The former possibility requires the risk manager to closely monitor the world of economics, business, political, social, environmental news for possible instabilities. This approach is associated with standard “fundamental” analysis. The later endogenous scenario requires the investigation of signs of instabilities to be found in the market dynamics itself and could rationalize in part so-called “technical” analysis (see references therein).

Johansen and Sornette have carried out a systematic investigation of crashes to clarify this question. They have proceeded in several steps.

1. They have developed a methodology to identify crashes as objectively and unambiguously as possible. Specifically, they have studied the distribution of drawdowns (runs of losses) in several markets: the two leading exchange markets (US dollar against the Deutsch and against the Yen), the major world stock markets, the U.S. and Japanese bond market and in the gold market. By introducing and varying a certain degree of fuzziness in the definition of drawdowns, they have tested the robustness of the empirical distributions of drawdowns.

2. By a careful analysis of these distributions, they have shown that the extreme tail belongs to a different population than the bulk (typically the 1% most extreme drawdowns occur 10 to 100 times more often than would be predicted by an extrapolation of the distribution of the 99% remaining drawdowns).

3. These extreme events which seem to belong to a different population have been called “outliers”. Others have referred to such events as “kings” or “black swans.” Johansen and Sornette have taken these kings as the crashes that need to be explained. Note that this procedure ensures that the definition of a crash is relative to the specific market rather than obeying so arbitrary absolute rule.

4. Then, for each identified king, Johansen and Sornette have checked whether a specific market structure, called log-periodic power law (LPPL), is present in the price trajectory preceding the occurrence of the drawdown king. The rational for this
Figure 6: The Hang-Seng composite index of the Hong Kong stock market from Nov. 1969 to Sept. 1999. Logarithmic scale in the vertical axis. The culmination of the bubbles followed by strong corrections of crashes are indicated by the arrows and correspond to the times Oct. 1971, Feb. 1973, Sept. 1978, Oct. 1980, Oct. 1987, April 1989, Jan. 1994 and Oct. 1997. This figure shows that the Hang-Sing index has grown exponentially on average at the rate of $\approx 13.6\%$ per year represented by the straight line corresponding to the best exponential fit to the data. Eight large bubbles (among them 5 are very large) can be observed as upward accelerating deviations from the average exponential growth characterized by LPPL signatures ending in a crash, here defined as a more than 15% drop in less than two weeks. The eight small panels at the bottom are given to show the LPPL price trajectory over a period of six months preceding each of these 8 crashes. Constructed from [59] and other papers from the author.
approach was based on their previous works \cite{34, 38, 32, 59}, in which they documented that the existence of such log-periodic power law signatures associated with speculative bubbles before crashes. The work \cite{37} is in this respect an out-of-sample test of the LPPL bubble-crash hypothesis applied to a population of financial time series selected according to a criterion (outlier test in the distribution of drawdowns) which is unrelated to the LPPL structure itself.

5. In this test, Johansen and Sornette \cite{37} take the existence of a LPPL as the qualifying signature for an endogenous crash: a drawdown outlier is seen as the end of a speculative unsustainable accelerating bubble generated endogenously.

6. With these criteria fixed, Johansen and Sornette \cite{37} identify two classes of crashes. Those which are not preceded by a LPPL price trajectory are classified as exogenous. It turns out that for those, it was possible to identify what seems to have been the relevant historical event, i.e., a new piece of information of such magnitude and impact that it is reasonable to attribute the crash to it, following the standard view of the efficient market hypothesis. Such drawdown outliers are classified as having an exogenous origin.

7. The second class, characterized by LPPL price trajectories, is called endogenous. Figure \ref{fig:6} illustrate a series of endogenous crashes preceded by LPPL bubble trajectories on the Heng-Seng composite index of the Hong-Kong stock market, perhaps one of the most speculative markets in the world. All the events shown belong to the endogenous class.

8. Globally over all the markets analyzed, Johansen and Sornette \cite{37} identified 49 outliers, of which 25 were classified as endogenous, 22 as exogenous and 2 as associated with the Japanese “anti-bubble” starting in Jan. 1990. Restricting to the world market indices, they found 31 outliers, of which 19 are endogenous, 10 are exogenous and 2 are associated with the Japanese anti-bubble.

The combination of the two proposed detection techniques, one for drawdown outliers and the second for LPPL signatures, provides a novel and systematic taxonomy of crashes further substantiating the importance of LPPL (see also \cite{55, 56, 67, 62, 63} for reviews and extensions).

A more microscopic approach formulated in terms of agent-based models has also allowed to identify some mechanisms for the occurrence of extreme events, such as the excess bias on nodes in the de Bruijn diagram of active agent strategies \cite{39}, or the decoupling of strategies which become transiently independent from the recent past \cite{2}.

4 Concluding remarks

Let us end by a discussion of other domains of applications.

While the idea is not yet developed, I think that beyond the products sold by e-retailers discussed above, which are proxies of reputation and commercial successes, the endo-exo question is relevant to understanding the characteristics of Initial Public Offerings (IPO) \cite{28} and the movie industry \cite{12}. In the later, the mechanism of information cascade derives
from the fact that agents can observe box office revenues and communicate word of mouth about the quality of the movies they have seen.

Earthquakes are now thought to be due to a mixture of spontaneous occurrences driven by plate tectonics and triggering by other previous earthquakes. Within such a picture which rationalize many of the phenomenology of seismic catalogs, Helmstetter and Sornette have shown that there is a fundamental limit to earthquake predictability resulting from the "exogenous" class of earthquakes which are not triggered by other earthquakes. Furthermore, the rate of foreshocks preceding mainshocks can be understood from the idea that mainshocks may result from endogenous triggering by previous events, as developed above in section 2.4. The dependence with time of the seismic rate of foreshocks is predicted and observed to follow (5). The memory kernels \( \phi(t) \) given by (3) and \( K(t) \) given by (2) correspond in the present case respectively to the bare and renormalized Omori law for triggered aftershocks.

The weather and the climate also involve extremely complex processes, which are often too difficult to disentangle. This leads to major uncertainties in what are the important mechanisms that need to be taken into account, for instance, to forecast the future global warming of the earth due to anthropogenic activity coupled with natural variability. 9/11 has again offered a unique window. Travis and Carleton noted the following: "Three days after suicide airplane hijackers toppled the World Trade Center in New York and slammed into the Pentagon in Washington, D.C., the station crew noted an obvious absence of airborne jetliners from their perch 240 miles (384 kilometers) above Earth. I'll tell you one thing that's really strange: Normally when we go over the U.S., the sky is like a spider web of contrails, U.S. astronaut and outpost commander Frank Culbertson told flight controllers at NASA’s Mission Control Center in Houston. ‘And now the sky is just about completely empty. There are no contrails in the sky,’ he added. ‘It’s very, very weird.’ ‘I hadn’t thought of that perspective,’ fellow astronaut Cady Coleman replied.” Travis and Carleton showed that a significant elevation of the average diurnal temperature of the US in the three days following 9/11 when most jetliners were grounded and no contrails were present. This is the archetype of an exogenous response. It remains to be seen if the endo-exo viewpoint turns out to offer new fruitful perspectives to make progress in understanding and in forecasting the weather and the climate.

Finally, from a theoretical viewpoint, another potentially interesting domain of research is to extend the concept of the response function to nonlinear systems and to study its relationship with the internal fluctuations.

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