Photon production in high energy proton-nucleus collisions

François Gelis\(^{(1)}\) and Jamal Jalilian-Marian\(^{(2)}\)

October 29, 2018

1. Laboratoire de Physique Théorique, Bât. 210, Université Paris XI, 91405 Orsay Cedex, France
2. Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA

Abstract

We calculate the photon production cross-section in \(pA\) collisions under the assumption that the nucleus has reached the saturation regime, while the proton can be described by the standard parton distribution functions. We show that due to the strong classical field \(O(1/g)\) of the nucleus, bremsstrahlung diagrams become dominant over the direct photon diagrams. In particular, we show that \(\gamma-\text{jet}\) transverse momentum spectrum and correlations are very sensitive to gluon saturation effects in the nucleus.

BNL-NT-02/9, LPT-ORSAY-02/40

1 Introduction

An outstanding question in the description of hadronic interactions is the problem of the parton distribution functions at very small values of the parton momentum fraction \(x\) (see \([1, 2]\) for a pedagogical introduction). Indeed, it is well known that the solution of the (linear) BFKL equation \([3, 4, 5]\), extended to very small values of \(x\), leads to cross-sections that increase asymptotically like a power of the center of mass energy, in contradiction with bounds coming from unitarity considerations. It is expected that saturation effects must limit the growth of parton distributions at small values of \(x\) \([6, 7, 8]\). It has been argued that this description becomes inadequate since non-linear effects should become important at small \(x\) when the corresponding occupation numbers reach a value comparable to \(1/\alpha_S\) \([9, 10, 11]\).
A theoretical description of this phenomenon is provided by the Colored Glass Condensate model \cite{12, 13, 14}. In this model, small-$x$ gluons are described by a classical color field, due to the fact that these modes have a large occupation number. This color field is driven by a classical Yang-Mills equation whose source term is provided by faster partons. As one varies the separation scale between the soft and hard modes, one obtains a renormalization group equation that provides a non-linear generalization of the BFKL evolution equation \cite{15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28}. An important parameter of the model is the saturation momentum scale, usually denoted $Q_s$ and defined as the transverse momentum scale below which saturation effects become important. This scale increases with energy and with the size of the nucleus \cite{1, 9, 10}.

More recently, some tests have been proposed in order to observe saturation effects in high energy hadronic or nuclear collisions. Among them are the re-analysis of HERA deep inelastic results under the hypothesis that the $\gamma^* - p$ cross-section is affected by saturation effects \cite{29, 30, 31}. In the field of heavy ion collisions, it has been suggested that one could predict the multiplicity by describing the two nuclei with classical color fields \cite{32, 33, 34, 35, 36}. This idea lead to a successful description of the observed multiplicities at RHIC \cite{37, 38, 39} which is however still under debate \cite{40}. It has also been proposed recently that saturation effects could be observed in the cleaner environment provided by ultra-peripheral heavy ion collisions \cite{41, 42} since the production of heavy quark pairs is also very sensitive to the saturation scale.

Various studies also proposed to study saturation effects in the context of $pA$ collisions \cite{43, 44, 45}. In this paper, we calculate the cross-section for photon production in $pA$ collisions and show that the result is also sensitive to saturation effects. We assume that the nucleus is large enough so that it can be described in terms of classical color fields. Concerning the proton, we do not make such an assumption and describe it as in ordinary perturbative calculations, in terms of parton distribution functions. In this framework, the leading order process for photon production involves a quark from the proton that scatters off the color field of the nucleus and emits a photon by bremsstrahlung. Therefore, the elementary process we consider in this paper is $q \rightarrow q\gamma$ in the background of a classical color field.

The paper is organized as follows. In section 2, we formulate the amplitude for the bremsstrahlung of a photon by a quark (or antiquark) in the presence of a classical color field. This amplitude is then evaluated. The average over the configurations of the hard sources is performed in section 3. In section 4, we explain how to obtain the differential cross-section from the semi-classical amplitude derived in section 2. In section 5, we show that the diffractive (i.e. with the additional constraint that the exchange between the quark and the nucleus is color singlet and carries a zero total transverse momentum) cross-section for this process is vanishing. Finally, in section 6, we derive the inclusive photon radiation differential cross-section and show that the correlations between the outgoing quark and the photon are affected by saturation effects.
2 Bremsstrahlung amplitude

We want to calculate the following processes in $pA$ collisions

\[ q(p) + A \rightarrow q(q) \gamma(k) X \]  
\[ q(p) + A \rightarrow q(q) \gamma(k) A \]

where the quark can become either a jet or a hadron.

The starting point is the amplitude:

\[ \langle q(q)\gamma(k)_{\text{out}}|q(p)_{\text{in}}\rangle = \langle 0_{\text{out}}|a_{\text{out}}(k)b_{\text{out}}(q)b_{\text{in}}^{\dagger}(p)|0_{\text{in}}\rangle \]

which, using the LSZ formalism \[46\] can be written as

\[ \langle 0_{\text{out}}|a_{\text{out}}(k)b_{\text{out}}(q)b_{\text{in}}^{\dagger}(p)|0_{\text{in}}\rangle = \frac{e}{Z_{2}\sqrt{Z_{3}}} \int d^{4}x\, d^{4}y\, d^{4}z\, e^{i(k\cdot x + q\cdot z - p\cdot y)} \times \bar{\psi}(q)(i\not\partial_{z} - m)\langle 0_{\text{out}}|T\psi(z)\epsilon\cdot J(x)\bar{\psi}(y)|0_{\text{in}}\rangle(i\not\partial_{y} + m)u(p) \]

where \( J_{\mu}(x) \equiv \bar{\psi}(x)\gamma_{\mu}\psi(x) \) is the quark component of the electromagnetic current, \( Z_{2} \) and \( Z_{3} \) are the fermion and photon wave function renormalization factors\[47\], and \( e \) is the fractional electric charge of the quark under consideration. So what we basically need is

\[ \langle 0_{\text{out}}|T\psi(z)\bar{\psi}(x)\not\partial\psi(x)\bar{\psi}(y)|0_{\text{in}}\rangle \]

which, using Wick’s theorem, can be written as

\[ \langle 0_{\text{out}}|T\psi(z)\bar{\psi}(x)\not\partial\psi(x)\bar{\psi}(y)|0_{\text{in}}\rangle = -G_{F}(z,x)\not\partial G_{F}(x,y) \]

where the fermion Feynman propagator \( G_{F} \) is defined as\[48\]

\[ G_{F}(x,y) \equiv \langle 0_{\text{out}}|T\bar{\psi}(y)\psi(x)|0_{\text{in}}\rangle \]

It should be noted that \( G_{F} \) is the fermion propagator in the background of the strong classical color field of the nucleus. Our amplitude then becomes

\[ \langle q(q)\gamma(k)_{\text{out}}|q(p)_{\text{in}}\rangle = e \int d^{4}x\, d^{4}y\, d^{4}z\, e^{i(k\cdot x + q\cdot z - p\cdot y)} \times \bar{\psi}(q)(i\not\partial_{z} - m)G_{F}(z,x)\not\partial G_{F}(x,y)(i\not\partial_{y} - m)u(p) \]

1Since we are not including loop corrections in this calculation, we will simply set these factors to 1 in the following.
2Compared to the definition of the fermion propagator in \[41, 47\], we are omitting the denominator \( \langle 0_{\text{out}}|0_{\text{in}}\rangle \) due to the fact that this factor is a pure phase in the case where the background contains the classical field of a single nucleus. This phase indeed drops out of physical quantities obtained after squaring amplitudes.
The fermion propagator $G_\xi$ is already known from [41, 42, 48, 49]. In coordinate space and in the “singular” gauge, it is given by the following formula

$$G_\xi(x, y) = G^\xi_0(x - y) + \int d^4z \delta(z^-) \left[ \theta(x^-)\theta(-y^-)(U^\dagger(z_{\perp}) - 1) - \theta(-x^-)\theta(y^-)(U(z_{\perp}) - 1) \right] G^\xi_0(x - z) \gamma^- G^\xi_0(z - y),$$  \hspace{1cm} (9)

where $G^\xi_0$ is the free Feynman propagator of a quark, and where $U(z_{\perp})$ is a unitary matrix containing the interactions between the quark and the colored glass condensate. It is convenient to separate out the interaction part of the propagator and write it in momentum space as

$$G_\xi(q, p) = (2\pi)^4 \delta^4(q - p)G^\xi_0(p) + G^\xi_0(q)T_\xi(q, p)G^\xi_0(p)$$ \hspace{1cm} (10)

where

$$T_\xi(q, p) = 2\pi \delta(q^- - p^-)\gamma^- \text{sign}(p^-) \int d^2z_{\perp} \left[ U^{\text{sign}(p^-)}(z_{\perp}) - 1 \right] e^{i(q_{\perp} - p_{\perp}) \cdot z_{\perp}}$$  \hspace{1cm} (11)

while for a nucleus moving in the positive $z$ direction we have

$$U(x_{\perp}) \equiv T \exp \left\{ -ig^2 \int_{-\infty}^{+\infty} dz^- \frac{1}{\sqrt{T}} \rho_a(z^-, z_{\perp}) t^a \right\}$$ \hspace{1cm} (12)

with $t^a$ in the fundamental representation, and where $\rho_a(z^-, z_{\perp})$ is the density of color sources in the nucleus.

Inserting this propagator in the expression for the photon production amplitude (8), we find the following expression in momentum space

$$\langle q(q)\gamma(k)_{\text{out}} | q(p)_{\text{in}} \rangle = -e \frac{\pi(q)\pi(k - p)}{(2\pi)^4 \delta^4(k + q - p)}$$

$$+ T_\xi(q, p - k) G^\xi_0(p - k) + \delta G^\xi_0(q + k) T_\xi(q + k, p)$$

$$+ \int \frac{d^4l}{(2\pi)^4} T_\xi(q, l) G^\xi_0(l) \delta G^\xi_0(k + l) T_\xi(k + l, p) \ u(p).$$  \hspace{1cm} (13)

The first term in (13) corresponds to emission of a photon by the quark with no scattering from the Color Glass Condensate. This term cannot contribute to the cross section for real photons and indeed vanishes (the delta function $\delta(p - q - k)$ has no support for on-shell particles).

The second and third term, respectively, correspond to the case when the quark multiply scatters from the Color Glass Condensate before and after emission of the photon. The last term describes the case when the quark multiply
scatters from the Color Glass Condensate, then emits a photon and again, multiply scatters from the Color Glass Condensate. Those processes are illustrated in the figure 1. Note that Eq. (13) is an exact formula for the bremsstrahlung of a quark in a colored glass condensate. Indeed, it resums the interactions to all orders with the classical background field, as required by the large gluon density in the nucleus.

Figure 1: The physical processes contributing to the bremsstrahlung of a quark in a colored glass condensate. The black dot denotes the interaction of a quark to all orders with the background field, i.e. the scattering matrix $T_p$ defined in Eq. (11).

Now we will show that the last term also vanishes. To show this, it is sufficient to consider the structure of the poles in the propagators in the last term. One has the following structure

$$\delta(q^--l^-)\delta(k^-+l^--p^-) \int_{-\infty}^{+\infty} dt^+ \frac{l^+}{l^+ - \frac{l^2+m^2-i\epsilon}{2l^-}} \frac{1}{l^+ + k^+ - \frac{(l_+ k_2)^2 + m^2 - i\epsilon}{2(k^-+l^-)}}$$

(14)

Since both $p^- > 0$ and $q^- > 0$, then $l^- > 0$ as well as $k^- + l^- > 0$. This means that both $l^+$ poles are below the real axis. Therefore, one can close the integration contour above the real axis and get a vanishing contribution. The only exception to this argument would occur if there were terms proportional to $l^+$ in the numerator which can compensate the $l^+$ in the denominator, a situation in which the theorem of residues would give a nonzero contribution from the half circle at infinity used to close the contour. To investigate this case, we write explicitly the terms which contain $l^+$ in the numerator of the last term. We have

$$\overline{u}(q)\gamma^- l \not{l} \gamma^- u(p) = l^+ l^+ \overline{u}(q)\gamma^- \gamma^- l \not{l} \gamma^- \gamma^- u(p) .$$

(15)

Since $\gamma^-\gamma^- = 0$, equation (15) identically vanishes. Therefore, the last term in the amplitude (13) is zero. That this term vanishes has a simple physical explanation. Indeed, in this term the quark first scatters off the nucleus, then propagates for a while and emits the photon, then propagates again and finally reinteracts with the nucleus. Note that the intermediate free propagations are off-shell and take some nonzero amount of time. But since the nucleus moves at the speed of light in the $+z$ direction in our model, it is already far away behind the quark by the time the photon has been emitted, and a second scattering of the quark off the nucleus cannot happen.\footnote{One can notice that the argument used to prove that Eq. (14) vanishes would not be
The amplitude can then be written as
\[ \langle q(q)\gamma(k)_{\text{out}}|g(p)_{\text{in}} \rangle = \]
\[ = -e\pi(q) \left[ T_F(q,p-k) G^0_F(p-k) \gamma + \gamma G^0_F(q+k) T_F(q+k,p) \right] u(p) \]
\[ = -ie\pi(q) \left[ \frac{\gamma^- (q-k+m) \gamma^- + \gamma^- (q+k+m) \gamma^-}{(p-k)^2 - m^2} \right] u(p) \]
\[ \times 2\pi \delta(q^- + k^- - p^-) \int d^2x_\perp e^{i(q_\perp + k_\perp - p_\perp) \cdot x_\perp} \left( U(x_\perp) - 1 \right) . \] (16)

Depending on whether one wants to calculate a diffractive or inclusive quantity, one would need to color average and then square the amplitude or square the amplitude and then color average. In either case, the color averaging does not affect the spinor structure and one can evaluate the spinor dependence and color averaging independently.

The spinor dependence of the amplitude (16) squared is given by
\[ \langle \text{tr}(L^\dagger L) \rangle_{\text{spin}} = \frac{1}{2} \text{tr} \left\{ \gamma \left[ \gamma^- (q-k+m) \gamma^- + \gamma^- (q+k+m) \gamma^- \right] \right\} \]
\[ \times (q+m) \left[ \frac{\gamma^- (q-k+m) \gamma^- + \gamma^- (q+k+m) \gamma^-}{(p-k)^2 - m^2} \right] \] (17)
where the factor 1/2 comes from averaging of the spin of the incoming quark.

Summing also over the spin of the final quark and over the polarization of the photon, a straightforward calculation gives:
\[ \langle \text{tr}(L^\dagger L) \rangle_{\text{spin}} = -4m^2 \left[ \frac{p^{-2}}{(q \cdot k)^2} + \frac{q^{-2}}{(p \cdot k)^2} + \frac{k^{-2}}{(p \cdot k)(q \cdot k)} \right] \]
\[ + 8(p^{-2} + q^{-2}) \left[ \frac{p \cdot q}{(p \cdot k)(q \cdot k)} + \frac{1}{q \cdot k} - \frac{1}{p \cdot k} \right] . \] (18)

3 Color averages

In order to perform the color averaging of this amplitude with the weight
\[ W[\rho] \equiv \exp \left\{ - \int dz^- dz_\perp \frac{\rho_0(z^-, z_\perp) \rho_0(z^-, z_\perp)}{2\mu^2(z^-)} \right\} \] (19)
we will need to evaluate expressions like \( U(x_\perp) \) and \( U(x_\perp) U(y_\perp) \) where \( U(x_\perp) \) is given by (12). These are already evaluated in [41, 42]. Following the notations of these papers, we obtain
\[ \langle U(x_\perp) - 1 \rangle_\rho = \mathcal{P}(x_\perp)(e^{-B_1} - 1) \] (20)
applicable if the nucleus was moving at a speed \( v < 1 \). Indeed, in that case we would not have the delta functions implying the conservation of the \(-\) component of the quark momentum, and we could have poles on both sides of the real \( t^+ \) axis.
where \( \mathcal{P}(x) \) is a function that describes the transverse profile of the nucleus. It can be thought of as a function whose value is 0 outside the nucleus and 1 inside the nucleus. The object \( B_1 \) appearing in this expression is given by

\[
B_1(x) \equiv Q_s^2 \int d^2 z \mathcal{P}(x - z) \sim \frac{Q_s^2}{\Lambda_{QCD}^2},
\]

with \( Q_s^2 \equiv g^4(t_a t^a) \int d^- \mu^2(z^-)/2 \) the saturation scale (the integral of \( \mu^2 \) over \( z^- \) is the number density of color sources per unit of transverse area in the target nucleus). Similarly, we have

\[
\langle (U^\dagger(x) - 1)(U(y) - 1) \rangle = \mathcal{P}(x) \mathcal{P}(y) \left[ 1 + e^{-B_2(x - y)} - 2e^{-B_1} \right]
\]

with the definition

\[
B_2(x - y) = Q_s^2 \int d^2 z \mathcal{P}(x - z) - \mathcal{P}(y - z))^2 \approx \frac{Q_s^2(x - y)^2}{4\pi} \ln \left( \frac{1}{|x - y| \Lambda_{QCD}} \right).
\]

In the above equations, \( G_0(z - y) \) is the free propagator in two dimensions, defined by

\[
\frac{\partial^2}{\partial z^2} G_0(z - y) = \delta(z - y)
\]

and given explicitly by

\[
G_0(z - y) = -\int \frac{d^2 k}{(2\pi)^2} e^{ik \cdot (z - y)} \frac{1}{k^2}.
\]

Note that the objects evaluated in Eqs. (20) and (22) are matrices in the fundamental representation of \( SU(N_c) \) that are proportional to the unit matrix. In the calculation of cross-sections, one must sum over the color of the outgoing quark and average over the color of the incoming quark, which amounts to taking the color trace of this matrix and dividing by \( N_c \). Therefore, Eqs. (21) and (22) can be seen as scalars giving directly the result of this procedure.

### 4 Cross-Section

At first sight, the square of the delta function \( \delta(p - k - q) \) that appears when we square the amplitude might seem a little worrisome. However, this is

\[\text{The saturation momentum would acquire a dependence on the rapidity of the quark via quantum evolution effects not included explicitly here. Indeed, the quark is sensitive to all of the nucleus constituents that have a rapidity between the quark rapidity and the nucleus rapidity.}\]
just a manifestation of Fermi’s Golden rule, adapted to the symmetries of the
present problem. Indeed, the target nucleus being invariant under (light-cone)
time $x^+$, the $-$ component of the projectile momentum is conserved, hence the
$\delta(p^- - k^- - q^-)$ at the amplitude level. Then, the $2\pi\delta(0^-)$ we have in the
amplitude squared is just an artifact of shooting a plane wave at this target
instead of a properly normalized wave packet. It should be interpreted as a
large but finite time $\Delta x^+$ (see [46], pages 96–97 and [50], pages 100–102 for
a nice discussion of this). It is in fact simpler to follow [51] (pages 99–107) and
introduce a wave packet instead of the incoming plane wave
\[ \langle \phi_{\text{in}} | \phi_{\text{in}} \rangle \equiv \int \frac{d^3 l}{(2\pi)^3} \frac{e^{ib \cdot l}}{\sqrt{2E_i}} |q(l)_{\text{in}}\rangle , \] (6)
where $\phi(l)$ is some wave packet that is peaked around the central value $p$. The
phase $\exp(ib \cdot l_\perp)$ is here to account for a finite impact parameter between
the wave packet of the quark and the trajectory of the center of the nucleus.
Normalization is chosen in such a way that:
\[ \langle \phi_{\text{in}} | \phi_{\text{in}} \rangle = 1 , \quad \text{i.e.} \quad \int \frac{d^3 l}{(2\pi)^3} |\phi(l)|^2 = 1 . \] (7)

Then, the differential (per unit of volume in the invariant phase space of the
final state particles) interaction probability between this wave packet and the
nucleus is simply
\[ dP(b) \equiv \frac{d^3 k}{(2\pi)^3 2k_0} \frac{d^3 q}{(2\pi)^3 2q_0} \frac{d^3 q'}{(2\pi)^3 2q'_0} \frac{d^3 l}{(2\pi)^3} \frac{d^3 l'}{(2\pi)^3} \frac{\phi(l) \phi(l')^*}{\sqrt{2E_i \sqrt{2E_i'}}} \times \langle q(q') \gamma(k)_{\text{out}} | q(l)_{\text{in}}\rangle \langle q(l')_{\text{in}} | q(q') \gamma(k)_{\text{out}} \rangle . \] (8)
Recalling now the relation between the differential cross-section and the proba-
bility defined in the previous equation
\[ d\sigma = \int d^2 b \ dP(b) , \] (9)
we have for the cross-section:
\[ d\sigma = \frac{d^3 k}{(2\pi)^3 2k_0} \frac{d^3 q}{(2\pi)^3 2q_0} \int d^2 b \ \frac{d^3 l}{(2\pi)^3} \frac{d^3 l'}{(2\pi)^3} \frac{\phi(l) \phi(l')^*}{\sqrt{2E_i \sqrt{2E_i'}}} \times \langle q(q') \gamma(k)_{\text{out}} | q(l)_{\text{in}}\rangle \langle q(l')_{\text{in}} | q(q') \gamma(k)_{\text{out}} \rangle . \] (10)
Performing the integration over the impact parameter $b$ brings a $(2\pi)^2 \delta(l_\perp - l'_\perp)$. If we factor out the delta function contained in the amplitude in the following way
\[ \langle q(q') \gamma(k)_{\text{out}} | q(l)_{\text{in}}\rangle \equiv 2\pi \delta(l^- - q^- - k^-) \mathcal{M}(l|qqk) , \] (11)
we can also perform the integral over $d^3l'$ by using one of those delta functions
and get a factor $\sqrt{2E_p/2p^+}$, where we have taken advantage of the fact that
the wave packets are peaked around $p$ to replace by $p$ the dummy variables $l$
and \( l' \) whenever they are in slowly varying quantities (i.e. everywhere except in the wave-packets themselves). At this stage, the differential cross-section reads

\[
d\sigma = \frac{d^3k}{(2\pi)^32k_0} \frac{d^3q}{(2\pi)^32q_0} \frac{1}{2p^-} |M(p|qk)|^2 2\pi \delta(p^- - q^- - k^-) . \tag{32}
\]

This is the formula we are going to use in the rest of this paper.

5 Diffraction

In order to calculate the diffractive cross section, we average the amplitude \([16]\) over color charge and then square it. This involves evaluating expressions like \( \langle U(x_\perp) - 1 \rangle_\rho \) which is given by Eq. \((20)\). Performing also the integrals with respect to the transverse positions, we obtain

\[
|M(p|qk)|^2_{\text{diff}} = e^2 \left( \langle \text{tr}(L^\dagger L) \rangle_{\text{spin}} [1 - e^{-B_1}] \right)^2 \\
\times \tilde{P}(p_\perp - q_\perp - k_\perp)\tilde{P}(-p_\perp + q_\perp + k_\perp) , \tag{33}
\]

where the function \( \tilde{P}(l_\perp) \) is the Fourier transform of the profile function \( P(x_\perp) \). Note that for a large nucleus, this Fourier transform is very sharply peaked around \( l_\perp = 0 \), with a typical width of \( 1/R \) where \( R \) is the radius of the nucleus. Since this momentum scale is much smaller than any other typical momentum scale in the problem, we can approximate

\[
\tilde{P}(p_\perp - q_\perp - k_\perp)\tilde{P}(-p_\perp + q_\perp + k_\perp) \approx \tilde{P}(0)(2\pi)^2\delta(p_\perp - q_\perp - k_\perp) \\
= \pi R^2(2\pi)^2\delta(p_\perp - q_\perp - k_\perp) . \tag{34}
\]

Therefore, we can rewrite the square of the diffractive amplitude as

\[
|M(p|qk)|^2_{\text{diff}} = e^2\pi R^2 \left( \langle \text{tr}(L^\dagger L) \rangle_{\text{spin}} [1 - e^{-B_1}] \right)^2 (2\pi)^2\delta(p_\perp - q_\perp - k_\perp) , \tag{35}
\]

so that the diffractive cross-section for bremsstrahlung becomes:

\[
d\sigma_{\text{diff}}^{q\to q\gamma} = \frac{d^3k}{(2\pi)^32k_0} \frac{d^3q}{(2\pi)^32q_0} \frac{e^2\pi R^2}{2p^-} \left( \langle \text{tr}(L^\dagger L) \rangle_{\text{spin}} [1 - e^{-B_1}] \right)^2 \\
\times (2\pi)^3\delta(p_\perp - q_\perp - k_\perp)\delta(p^- - q^- - k^-) . \tag{36}
\]

To proceed further, we assume that the incoming quark is massless and that its transverse momentum \( p_\perp \) is zero. This is reasonable if we have in mind an application of this calculation to photon production in \( pA \) collisions. Indeed, there is no significant amount of heavy quarks in the wave function of the proton and partons are collinear to the proton. Under these assumptions, it is a matter of straightforward algebra to show that

\[
\langle \text{tr}(L^\dagger L) \rangle_{\text{spin}}^{\text{diff}} = 0 \tag{37}
\]
leading to
\[
\frac{d\sigma_{\text{diff}}^{q\rightarrow q\gamma}}{d^2k_{\perp}} = 0. \quad (38)
\]

This result is in fact rather intuitive: asking for a diffractive process implies that no net transverse momentum is exchanged between the classical color field and the quark, which inhibits the radiation of a photon by bremsstrahlung.

### 6 Inclusive

In order to calculate the inclusive cross section, we square the amplitude and then average over color charge. Using (22), we obtain for the inclusive amplitude squared
\[
|M(p|qk)|^2_{\text{incl}} = e^2 \langle \text{tr}(L^\dagger L) \rangle_{\text{spin}} \int d^2x_{\perp} d^2y_{\perp} e^{i(q_{\perp}+k_{\perp}-p_{\perp}) \cdot (x_{\perp}-y_{\perp})} \times \mathcal{P}(x_{\perp})\mathcal{P}(y_{\perp}) [1 + e^{-B_2(x_{\perp}-y_{\perp})} - 2e^{-B_1}] . \quad (39)
\]

By inspecting the formula (23), we see that \( \exp(-B_2) \) becomes very small if \( |x_{\perp} - y_{\perp}| \) becomes larger than \( 1/Q_s \). Therefore, this term contributes only if the separation between \( x_{\perp} \) and \( y_{\perp} \) is much smaller than the radius of the nucleus (assuming a large nucleus such that \( R \gg Q_s^{-1} \)). Therefore, for this term we can approximate
\[
\mathcal{P}(x_{\perp})\mathcal{P}(y_{\perp}) \approx \mathcal{P}^2(x_{\perp}) \approx \mathcal{P}(x_{\perp}) , \quad (40)
\]

where the last equality is valid for a nucleus with rather sharp edges (i.e. if \( \mathcal{P}(x_{\perp}) \) goes from 0 to 1 rather fast). Using Eq. (34) and performing the Fourier transforms, we can rewrite the inclusive amplitude squared as follows
\[
|M(p|qk)|^2_{\text{incl}} = e^2 \pi R^2 \langle \text{tr}(L^\dagger L) \rangle_{\text{spin}} \times [D(p_{\perp} - q_{\perp} - k_{\perp}) + (1 - 2e^{-B_1})(2\pi)^2 \delta(p_{\perp} - q_{\perp} - k_{\perp})] , \quad (41)
\]

where, following [42], we denote
\[
D(I_{\perp}) \equiv \int d^2x_{\perp} e^{i_{\perp} \cdot x_{\perp}} (e^{-B_2(x_{\perp})} - 1) = \int d^2x_{\perp} e^{i_{\perp} \cdot x_{\perp}} \langle U(0)U^\dagger(x_{\perp}) - 1 \rangle_{\rho} . \quad (42)
\]

Inserting this in the formula (32), we obtain the following formula for the inclusive cross-section\(^5\)
\[
\frac{d\sigma_{\text{incl}}^{q\rightarrow q\gamma}}{d^3k} = \frac{d^3q}{(2\pi)^32k_0} \frac{e^2\pi R^2}{2p^-} \langle \text{tr}(L^\dagger L) \rangle_{\text{spin}} 2\pi\delta(p^- - q^- - k^-)
\]

\(^5\)The invariant phase space of a massless particle can be rewritten in terms of the light-cone variables as follows:
\[
\frac{d^3k}{(2\pi)^32k_0} = \frac{dk^+dk^-d^2k_{\perp}}{(2\pi)^3} 2\pi\theta(k^+)\delta(2k^- - k_+^2) . \quad (43)
\]
\[
\times \left[ D(p_\perp - q_\perp - k_\perp) + (1 - 2e^{-B_1})(2\pi)^2 \delta(p_\perp - q_\perp - k_\perp) \right].
\]

(44)

Another simplification can be achieved by neglecting the term in \( \exp(-B_1) \) since \( B_1 \sim Q_s^2/\Lambda_{QCD}^2 \gg 1 \) appears in the exponential with a negative sign. If one introduces \[ \text{(41)} \]

\[
C(l_\perp) \equiv \int d^2x_\perp e^{i l_\perp \cdot x_\perp} e^{-B_2(x_\perp)} = \int d^2x_\perp e^{i l_\perp \cdot x_\perp} \langle U(0)U^\dagger(x_\perp) \rangle_\rho,
\]

(45)

the inclusive cross section can be rewritten as

\[
d\sigma^{q\rightarrow q\gamma}_{\text{incl}} = \frac{d^3k}{(2\pi)^3 2k_0} \frac{d^3q}{(2\pi)^3 2q_0} \frac{e^2 \pi R^2}{2 p^-} \frac{\langle \text{tr}(L^\dagger L) \rangle_{\text{spin}}}{z} \times 2\pi \delta(p^- - q^- - k^-) C(p_\perp - q_\perp - k_\perp).
\]

(46)

Assuming again that the incoming quark transverse momentum \( p_\perp \) is zero and neglecting the quark mass, one can perform the integrals over \( q^+, k^+, q^- \) using the delta functions. There is however a complication due to collinear singularities, i.e. singularities that show up when the emitted photon is parallel to the outgoing quark. It is convenient to trade the transverse momentum of the final quark for the total transverse momentum of the final state, i.e. \( l_\perp \equiv q_\perp + k_\perp \).

In terms of this new variable, we have

\[
\frac{1}{\pi R^2} \frac{d\sigma^{q\rightarrow q\gamma}_{\text{incl}}}{d^2 k_\perp} = \frac{e^2}{(2\pi)^5 k_\perp^4} \int_0^1 dz \frac{[1 + (1 - z)^2]}{z} \left( \int d^2 l_\perp \frac{l_\perp^2 C(l_\perp)}{[l_\perp - k_\perp/z]^2} \right)
\]

(47)

where \( z \equiv k^-/p^- \) and \( [1 + (1 - z)^2]/z \) is the standard leading order photon splitting function. Eq. (47) is our main result. Note that \( C(l_\perp) \) behaves like \( 1/l_\perp^4 \) at large \( l_\perp \) which ensures that the integral converges at large momentum transfer. In this formula, \( C(l_\perp) \) is the only object that depends on the structure of the color sources describing the target nucleus. In particular, all the quantum evolution effects would go into this object via the averaging procedure in Eq. (45). One can also note that this result exhibits the standard collinear denominator \( [l_\perp - k_\perp/z]^2 \) that vanishes if the photon is emitted collinearly to the quark. This aspect of the result is of course not affected by the description of the target nucleus as a color glass condensate.

In the soft photon limit, one can see the decoupling of the photon emission subprocess from the quark scattering part. The latter agrees with the quark-nucleus scattering cross-section calculated in \[ \text{(44)} \].

It is instructive to perform the “perturbative limit” of this result. This regime is reached when the transverse momentum \( l_\perp \) transferred between the nucleus and the quark is large compared to the saturation momentum \( Q_s \). In this limit, we have \[ \text{(41)} \]

\[
C(l_\perp) \approx \frac{2Q_s^2}{l_\perp^4}.
\]

(48)
Using this result, we have:

\[
\frac{d\sigma_{q\rightarrow q\gamma}^{\text{incl}}}{d^2k_{\perp}}\bigg|_{\text{pert.}} = \frac{2N_h e_q^2 \alpha_{\text{em}} \alpha_s^2 C_F}{\pi^2} \int_0^1 dz \frac{1 + (1 - z)^2}{z} \int \frac{d^2l_{\perp}}{l_{\perp}^2 \left[l_{\perp} - k_{\perp}/z\right]^2} \quad (49)
\]

where \( e_q \) is the quark electric charge in units of the electron charge, and where \( N_h = \pi R^2 \int dz \mu^2(z^-) \) is the total number of hard color sources in the target nucleus. Therefore, this expression has all the features of the bremsstrahlung of a photon by a quark scattering off a parton inside the nucleus with the exchange of a gluon in the t-channel (this term is the dominant one at large center of mass energy).

In Eq. (47), the only factor that depends crucially on the saturation hypothesis for the nucleus is the factor \( C(l_{\perp}) \). Indeed, this term contains all the dependence on the saturation scale, as well as the modifications of the transverse momentum spectrum at scales below \( Q_s \). The transverse momentum dependence of this object is illustrated in figure 2. In order to observe effects due to this factor, it would be useful to measure both the radiated photon and the jet induced by the outgoing quark. The photon-jet correlations, and in particular the distribution of their total transverse momentum \( l_{\perp} = q_{\perp} + k_{\perp} \), would indeed enable one to extract in a rather direct way the function \( C(l_{\perp}) \) itself. On the contrary, if one measures only the photon spectrum, one can access only a given moment of this function.
7 Conclusions and perspectives

In this paper, we have studied the photon production in \( pA \) collisions by bremsstrahlung of a quark in the saturated color field of the nucleus. We have shown that the cross-section for this process depends sensitively on saturation effects. Furthermore, we have shown that photon bremsstrahlung diagrams which are, in the standard perturbation theory, higher order compared to the direct photon diagrams, become leading order. This is due to the presence of a strong color field \( O(1/g) \) in the target nucleus which compensates for the factor of \( g \) from quark-quark-gluon vertex.

Experimentally, it may be an easier task to look for a photon and a jet pair rather than a single jet. This was studied in the context of heavy ion collisions in [52]. Therefore one can directly measure the correlator \( C(l_\perp) \) that describes the interactions of a high energy probe with the target nucleus. It should be noted that saturation effects become larger in the forward region and therefore easier to measure experimentally. It is straightforward to extend this calculation to the case of production of dileptons and di-photons. This is currently under investigation.

Another interesting extension of this work would be to study the radiation of a gluon by a quark as it moves through the classical field of the nucleus. In that case, the emitted gluon can also interact with the background classical field, which makes the calculation of this process more involved.

Acknowledgment

We would like to thank D. Bodeker, A. Dumitru, E. Iancu, L. McLerran, R. Pisarski, R. Venugopalan and W. Vogelsang for useful discussions. F.G. is supported by CNRS. J.J-M. is supported in part by a PDF from BSA and by U.S. Department of Energy under Contract No. DE-AC02-98CH10886.

References

[1] A.H. Mueller, Lectures given at the International Summer School on Particle Production Spanning MeV and TeV Energies (Nijmegen 99), Nijmegen, Netherlands, 8-20, Aug 1999, [hep-ph/9911289].

[2] L. McLerran, Lectures given at the 40'th Schladming Winter School: Dense Matter, March 3-10 2001, [hep-ph/0104285].

[3] L.N. Lipatov, Sov. J. Nucl. Phys. 23, 338 (1976).

[4] E.A. Kuraev, L.N. Lipatov, V.S. Fadin, Sov. Phys. JETP 45, 199 (1977).

[5] I. Balitsky, L.N. Lipatov, Sov. J. Nucl. Phys. 28, 822 (1978).

[6] L.V. Gribov, E.M. Levin, M.G. Ryskin, Phys. Rept. 100, 1 (1983).
[7] A.H. Mueller, J-W. Qiu, Nucl. Phys. B 268, 427 (1986).
[8] L.L. Frankfurt, M.I. Strikman, Phys. Rept. 160, 235 (1988).
[9] J. Jalilian-Marian, A. Kovner, L. McLerran, H. Weigert, Phys. Rev. D 55, 5414 (1997).
[10] Yu.V. Kovchegov, L. McLerran, Phys. Rev. D 60, 054025 (1999).
[11] Yu.V. Kovchegov, L. McLerran, Erratum Phys. Rev. D 62, 019901 (2000).
[12] L. McLerran, R. Venugopalan, Phys. Rev. D 49, 2233 (1994).
[13] L. McLerran, R. Venugopalan, Phys. Rev. D 49, 3352 (1994).
[14] L. McLerran, R. Venugopalan, Phys. Rev. D 50, 2225 (1994).
[15] A. Ayala, J. Jalilian-Marian, L. McLerran, R. Venugopalan, Phys. Rev. D 52, 2935 (1995).
[16] A. Ayala, J. Jalilian-Marian, L. McLerran, R. Venugopalan, Phys. Rev. D 53, 458 (1996).
[17] J. Jalilian-Marian, A. Kovner, A. Leonidov, H. Weigert, Nucl. Phys. B 504, 415 (1997).
[18] J. Jalilian-Marian, A. Kovner, A. Leonidov, H. Weigert, Phys. Rev. D 59, 014014 (1999).
[19] J. Jalilian-Marian, A. Kovner, A. Leonidov, H. Weigert, Phys. Rev. D 59, 034007 (1999).
[20] J. Jalilian-Marian, A. Kovner, A. Leonidov, H. Weigert, Erratum Phys. Rev. D 59, 099903 (1999).
[21] J. Jalilian-Marian, A. Kovner, H. Weigert, Phys. Rev. D 59, 014015 (1999).
[22] A. Kovner, G. Milhano, Phys. Rev. D 61, 014012 (2000).
[23] A. Kovner, G. Milhano, H. Weigert, Phys. Rev. D 62, 114005 (2000).
[24] I. Balitsky, Nucl. Phys. B 463, 99 (1996).
[25] Yu.V. Kovchegov, Phys. Rev. D 61, 074018 (2000).
[26] E. Iancu, A. Leonidov, L. McLerran, [hep-ph/0011241].
[27] E. Iancu, A. Leonidov, L. McLerran, Phys. Lett. B 510, 133 (2001).
[28] A.H. Mueller, [hep-ph/0110168].
[29] K. Golec-Biernat, M. Wusthoff, Phys. Rev. D 59, 014017 (1999).
[30] K. Golec-Biernat, M. Wusthoff, Phys. Rev. D 60, 114023 (1999).
[31] K. Golec-Biernat, M. Wusthoff, Eur. Phys. J. C 20, 313 (2001).
[32] A. Kovner, L. McLerran, H. Weigert, Phys. Rev. D 52, 3809 (1995).
[33] A. Kovner, L. McLerran, H. Weigert, Phys. Rev. D 52, 6231 (1995).
[34] Yu.V. Kovchegov, hep-ph/0011252.
[35] A. Krasnitz, R. Venugopalan, Phys. Rev. Lett. 84, 4309 (2000).
[36] A. Krasnitz, R. Venugopalan, Phys. Rev. Lett. 86, 1717 (2001).
[37] D. Kharzeev, M. Nardi, Phys. Lett. B 507, 121 (2001).
[38] D. Kharzeev, E. Levin, Phys. Lett. B 523, 79 (2001).
[39] D. Kharzeev, E. Levin, M. Nardi, hep-ph/0111315.
[40] R. Baier, A.H. Mueller, D. Schiff, D. Son, hep-ph/0204211.
[41] F. Gelis, A. Peshier, Nucl. Phys. A 697, 879 (2002).
[42] F. Gelis, A. Peshier, hep-ph/0111227, to appear in Nucl. Phys. A.
[43] A. Dumitru, L. McLerran, Nucl. Phys. A 700, 492 (2002).
[44] A. Dumitru, J. Jalilian-Marian, hep-ph/0204028.
[45] A. Dumitru, J. Jalilian-Marian, hep-ph/0111357.
[46] C. Itzykson, J.B. Zuber, Quantum field theory, McGraw-Hill (1980).
[47] A.J. Baltz, F. Gelis, L. McLerran, A. Peshier, Nucl. Phys. A 695, 395 (2001).
[48] L. McLerran, R. Venugopalan, Phys. Rev. D 59, 094002 (1999).
[49] A. Hebecker, H. Weigert, Phys. Lett. B 432, 215 (1998).
[50] J.D. Bjorken, S.D. Drell, Relativistic quantum mechanics, McGraw-Hill (1964).
[51] M.E. Peskin, D.V. Schroeder, An introduction to quantum field theory, Addison-Wesley, New-York (1995).
[52] X-N. Wang and Z. Huang, Phys. Rev. C 55, 3047 (1997).