Factorization and Scaling in Hard Diffraction

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ABSTRACT

We compare results on diffractive W-boson production at the Tevatron with predictions based on the diffractive structure function measured in deep inelastic scattering at HERA assuming (a) conventional factorization or (b) hard factorization combined with a rapidity gap distribution scaled to the total gap probability. We find that conventional factorization fails, while the scaling prediction agrees with the data.

1 Introduction

Hard diffraction is defined as the class of hadronic diffractive processes that incorporate a hard scattering. Recently, CDF reported results on diffractive W-boson production in $\bar{p}p$ collisions at $\sqrt{s} = 1.8$ TeV at the Tevatron. In this paper, we compare the CDF results with predictions based on the diffractive structure function (SF) of the proton measured in $e^+p \rightarrow e^+ [\gamma^*p \rightarrow Xp]$ deep inelastic scattering (DIS) at HERA [3, 4]. Two such comparisons are made, one assuming conventional factorization and the other assuming that the rapidity gap (RG) dependence of the diffractive SF scales to the total RG probability.

Our predictions for the Tevatron can be obtained directly from the HERA diffractive structure function without reference to a “pomeron flux” or even to the pomeron. However, to relate them to predictions based on the standard and renormalized pomeron flux factors, it is useful to first introduce the pomeron flux language [5].

In both the CDF and HERA cases, the hard scattering involves a parton from the pomeron, $\mathcal{P}$, which is presumed to be “emitted” by the proton. In $\bar{p}p \rightarrow Xp$, the $\mathcal{P}$-parton interacts with a $p$-parton producing a W or a dijet, while in DIS the $\gamma^*$ is absorbed by a quark in the $\mathcal{P}$. The proton emitting the pomeron remains intact, carrying a fraction $x$ of its initial momentum; the remaining fraction, $\xi = 1 - x$, is carried by the pomeron. Because of the colorless nature of the $\mathcal{P}$ ($\mathcal{P}$ has the quantum numbers of the vacuum), a rapidity gap (absence of particles) occurs between the final

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(surviving) proton and the particles in the system \( X \). The gap, whose nominal width is \( \Delta y = \ln \frac{1}{\xi} \), provides a characteristic signature for diffraction and is used to identify (“tag”) diffractive events. The CDF and HERA data discussed here were selected using a rapidity gap tag and are integrated over the transverse momentum squared, \( t \), of the leading proton.

The differential hard diffraction cross section \( d^4\sigma_{\text{hard}}/d\xi dt dQ^2 d\beta \), where \( \beta \) is the momentum fraction of the parton in the pomeron that participates in the hard scattering, is the product of the hard \( pp \) (or \( \gamma^* - pp \) in DIS) cross section, which depends on the pomeron structure function, and a “pomeron flux factor”, \( f_{\text{IP}/p}(\xi, t) \), which is the probability density of pomerons “carried” by the proton. In Regge theory, the flux factor has the form

\[
f_{\text{IP}/p}(\xi, t) \equiv \frac{\beta^2_{\text{IP}}(t)}{16\pi} \xi^{1-2\alpha(t)} = \frac{\beta^2_{\text{IP}}(0)}{16\pi} \xi^{1-2\alpha(t)} F^2(t) = K \xi^{1-2\alpha(t)} F^2(t)
\]

where \( \alpha(t) = 1 + \epsilon + \alpha' t \) is the pomeron trajectory, \( \beta_{\text{IP}}(t) \) is the coupling of the pomeron to the proton, and \( F(t) \) the proton form factor. Following Ref. [5], we will use \( \epsilon = 0.115 \), \( \alpha' = 0.26 \text{ GeV}^{-2} \), \( K = 0.73 \text{ GeV}^{-2} \) and \( F^2(t) \approx e^{4.6t} \) (valid at small-\(|t|\)).

Assuming Regge factorization, the diffractive SF is expected to be the product of the pomeron flux times the pomeron SF:

\[
F_2^{D(4)}(\xi, t, Q^2, \beta) \equiv f_{\text{IP}/p}(\xi, t) \cdot F^2_{\text{IP}}(Q^2, \beta) = \frac{K e^{b(\xi)t}}{\xi^{1+2\epsilon}} \cdot F_2^p(Q^2, \beta)
\]

where \( b(\xi) = 4.6 + 2\alpha' \ln \frac{1}{\xi} \). Below, after reviewing the CDF results and standard pomeron flux predictions, we present the diffractive structure function measured at HERA, use it to predict the CDF results assuming factorization or RG scaling, relate the RG scaling to the renormalized pomeron flux of Ref. [5], and draw conclusions on factorization and scaling in diffraction.

## 2 Diffractive W and dijet production

The CDF Collaboration reported diffractive to non-diffractive ratios for \( W \) [1] and dijet [2] production, as well as Monte Carlo predictions based on POMPYT for diffractive and PYTHIA for non-diffractive events. The diffractive events were generated using the standard pomeron flux and a hard pomeron structure of the form \( f_{\text{IP}}(\beta) = (f_q + f_g) \cdot [6\beta(1 - \beta)] \). The predicted rates depend on the product of the quark (gluon) fraction of the pomeron, \( f_q \) (\( f_g \)), and on the normalization of the pomeron flux factor. The gluon fraction of the pomeron can be determined from the ratio of the \( W \) to the dijet measured rates independent of the flux normalization or of the validity of the momentum sum rule for the pomeron. This is possible due to the different sensitivity of the \( W \) and dijet production rates to the quark and gluon content of the pomeron. Thus, any deviation from unity found in the ratio of measured to predicted rates, \( D \), can be attributed either to a discrepancy in the flux normalization or to a failure of the
pomeron momentum sum rule (defined as $f_q + f_g = 1$). Comparison of the measured $W$ rate with the rate predicted from the diffractive structure function measured in DIS provides a direct test of conventional factorization.

The diffractive to non-diffractive ratios measured by CDF are:

$$R_W = [1.15 \pm 0.51\,(stat) \pm 0.20\,(syst)]\% = (0.115 \pm 0.55)\% \quad (\xi < 0.1)$$

$$R_{JJ} = [0.75 \pm 0.05\,(stat) \pm 0.09\,(syst)]\% = (0.75 \pm 0.10)\% \quad (E^\text{jet}_T > 20\ GeV, |\eta^\text{jet}| > 1.8, \eta_1 \eta_2 > 0, \xi < 0.1)$$

The POMPYT standard flux predictions are $R^{MC}_W = 16\% \ (1.1\%)$ for a three quark-flavor (full-gluon) pomeron, and $R^{MC}_{JJ} = 2\% \ (5\%)$ for a full-quark (full-gluon) pomeron structure. From these predictions and the measured rates, CDF derived the gluon fraction, $f_g$, and the pomeron flux/momentum discrepancy factor, $D$:

$$f_g = 0.7 \pm 0.02 \quad D = 0.18 \pm 0.04$$

3 The diffractive structure function

Both the ZEUS $^3$ and H1 $^4$ Collaborations find that in the region $8.5 < Q^2 < 65$ GeV$^2$ the integral of the $F^{D(4)}_2$ structure function over $t$ has a form similar to that of Eq. 2, namely

$$F^{D(3)}_2(\xi, Q^2, \beta) = \int_{t_{\min}}^{t_{\max}} F^{D(4)}_2(\xi, t, Q^2, \beta) \, dt = \frac{1}{\xi^{1+n}} \cdot A(Q^2, \beta)$$

where $n \sim 0.2 - 0.3$. To simplify numerical comparisons with Eq. 2 we will use $n = 2\epsilon = 0.23$. The term $A(Q^2, \beta)$ is rather flat in $\beta$ and increases slowly with $Q^2$. Its average value is represented well by the value at $Q^2 \sim 20$ GeV$^2$, $A(Q^2, \beta)|_{Q^2=20} \approx 0.009$. To facilitate comparison with the CDF predictions based on the pomeron flux we will use $A(Q^2, \beta)|_{Q^2=20} = 0.009[6\beta(1 - \beta)]$.

3.1 Factorization

To calculate $R_W$ from $F^{D(3)}_2$, we first establish the correspondence with the pomeron flux, so that we may make use of the MC results of CDF. Following CDF, we set

$$F^{P}_{2}(Q^2, \beta) = \frac{2}{9} f_q \cdot 6\beta(1 - \beta)$$

where the factor $2/9$ is the average quark charge for three quark flavors (3f) in the pomeron and $f_q$ is the quark fraction of the pomeron. Using this form for $F^{P}_{2}$ and
equating the integral over $t$ of Eq. 2 with the structure function (SF) measured at HERA, we obtain

$$\frac{K}{\xi^{1+2\epsilon}} \cdot \frac{1}{7.5} \cdot \left[ \frac{2}{9} f_q \cdot 6\beta(1-\beta) \right] = \frac{0.009}{\xi^{1.23}} [6\beta(1-\beta)]$$

(4)

where we have used the average $t$-slope of 7.5 GeV$^{-2}$ in carrying out the integration over $t$. With the standard flux normalization, $K = 0.73$ GeV$^{-2}$, the quark fraction turns out to be $f_q = 0.42$, which multiplied by the MC prediction of 16% for a full 3f-quark pomeron yields $R_{SF}^{W} = 6.7\%$. This value is 5.8 times larger than the measured value of $R_{W} = 1.15\%$. Thus, conventional factorization breaks down.

3.2 Scaling

Let us now assume that the RG distribution scales to the total gap probability and rewrite the $F_D^{D(3)}$ to reflect such scaling. We note that for fixed $Q^2$ and $\beta$ the kinematically allowed $\xi$-limits are $\xi_{\text{min}} = Q^2/\beta s$ and $\xi_{\text{max}} \approx 0.1$ (the coherence limit). Integrating the factor $\xi^{-1.23}$ between these limits yields

$$N(Q^2, \beta, s) \equiv N(\xi_{\text{min}}) = \int_{\xi_{\text{min}}}^{0.1} \frac{1}{\xi^{1.23}} d\xi = \frac{1}{0.23} \left[ \left( \frac{\beta s}{Q^2} \right)^{0.23} - 1.7 \right]$$

(5)

The scaled SF can now be written as

$$F_D^{D(3)}(\xi, Q^2, \beta) \vert_{\text{scaled}} \approx \frac{0.009}{\xi^{1.23}} \cdot \left[ 6\beta(1-\beta) \right] \cdot \frac{C}{N(Q^2, \beta, s)}$$

(6)

where the constant $C$ is chosen to be $C = N(20, 0.5, 300^2) = 18.3$ so that the scaled SF for $Q^2 = 20$ and $\beta = 0.5$ is the same as the unscaled SF. Through $N(Q^2, \beta, s)$, the scaled SF acquires an additional $\beta$ dependence and a $Q^2$ dependence. The extra $\beta$ dependence helps flatten the $\beta(1-\beta)$ distribution at small $\beta$ and make it more similar to that measured at HERA. The acquired $Q^2$ dependence is very close to that observed at HERA. Thus, in the pomeron flux language, it is the flux at fixed $\xi$ that increases with $Q^2$ while the pomeron structure remains relatively unchanged. For a more detailed discussion see Ref. [7].

To use the SF of Eq. 6 at the Tevatron, one simply has to evaluate $N(Q^2, \beta, s)$ for $\sqrt{s} = 1800$ GeV and $Q^2 = M_0^2 = 1.5$ GeV$^2$, where $M_0$ is the effective diffractive threshold for $\bar{p}p \rightarrow X p$. For $\beta = 0.5$, $N(M_0^2, \beta, s)_{\text{TEV}} = 98.8$. This value is larger than the corresponding HERA value of 18.3 by a factor of 5.4. Thus, the $R_{W}^{SF} = 6.7\%$ has to be multiplied by the scaling factor $D_{\text{scale}} = 1/5.4 = 0.19$, yielding $1.24\%$. The value of 0.19 agrees with the standard flux discrepancy factor of $0.18 \pm 0.04$ reported by CDF. The scaled SF prediction for the diffractive to non-diffractive $W$ production ratio, $1.24\%$, is in excellent agreement with the measured value of $(1.15 \pm 0.55)\%$. Note that using the measured SF at a $Q^2$ value other than $Q^2 = 20$ would yield a slightly different prediction for $R_{W}$, but the prediction obtained with the scaled flux would remain the same.
The scaling of the diffractive SF to the total gap probability is equivalent to the renormalized flux hypothesis of Ref. [5]. When correctly applied to hard processes, i.e. using $\xi_{min} = M_0^2/\beta s$ in evaluating the normalization factor $N(\xi_{min})$ (where $M_0$ is the effective diffractive mass threshold), the renormalized flux gives the same results as the scaled structure function. Thus, the factorization breakdown observed between DIS and hard diffractive production at the Tevatron is traced back to the breakdown of factorization in soft diffraction.

The pomeron flux scaling implies that the soft diffractive cross section $d^2\sigma/dM^2dt|_{t=0}$ is approximately independent of $s$, contrary to the expectation of an $\sim s^{2\epsilon}$ dependence from the triple-pomeron amplitude. Figure 1, which shows $d^2\sigma/dM^2dt|_{t=-0.05}$ for $pp/\bar{p}p$ data at different $s$-values [9, 10], confirms the scaling of the $M^2$-distribution with $s$ [8].
5 Conclusions

We have compared the CDF results on diffractive $W$ production with predictions using the structure function measured at HERA. We find that conventional factorization fails, but a SF in which the gap probability distribution is scaled to the total gap probability yields predictions which are in excellent agreement with the data. Through the connection between the scaled SF and the renormalized pomeron flux, we conclude that the breakdown of factorization of the diffractive SF of the proton is due to the breakdown of the Regge factorization already observed in soft diffraction and, therefore, that factorization of the pomeron structure function in hard processes still holds. Finally, we have shown that the approximate scaling of $d^2\sigma/dtdM^2|_{t=0}$ with $s$ implied by flux renormalization is confirmed by the data.

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