Solution of the Quark Gap Equation by using Gluon Propagator Models inspired by Lattice QCD

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Abstract. In this work we present the solution of the Schwinger-Dyson equation (SDE) for the quark propagator with gluon propagator models inspired by Lattice Quantum Chromodynamics (QCD) and a model of running coupling. Mass function $M(p^2)$ of the quarks and wave function renormalization $F(p^2)$ are obtained for the different models respectively. We also calculated the chiral quark condensate for each model and test for confinement through violation of positivity of the dynamical fermion propagator.

1. Gap equation

In this proceeding we explore the QCD gap equation

$$S(p)^{-1} = \gamma \cdot p + m_d + \Sigma(p)$$

where

$$\Sigma(p) = \int \frac{d^4 k}{(2\pi)^4} g^2 D_{\mu\nu}(k-p) \lambda^a \frac{\lambda^a}{2} \gamma^\mu S(k) \frac{\lambda^a}{2} \Gamma_\nu(k, p).$$

Our truncation is defined such that $\Gamma_\nu(k, p) = \gamma_\nu, g^2 D_{\mu\nu}(k-p) = D(q^2) (\delta_{\mu\nu} - q_\mu q_\nu / q^2)$ is the dressed gluon propagator, which is modeled or takes a form given by solving the coupled equation between gluon and ghost or by parameterizing the data of lattice QCD simulations, $m_d$ is the current quark mass and $\lambda^a$ are the Gell-Mann’s matrices of the color group $SU(3)_c$.

The quark propagator $S(k)$ is decomposed as

$$S(k) = \frac{F(k^2)}{\nu \gamma \cdot k + M(k^2)} = \nu \gamma \cdot k \sigma_v(k^2) + \sigma_s(k^2),$$

where $F(k^2)$ and $M(k^2)$ are the renormalized wave function and mass function of the quark respectively, and

$$\sigma_s(k^2) = \frac{F(k^2) M(k^2)}{k^2 + M^2(k^2)}, \quad \sigma_v(k^2) = \frac{F(k^2)}{k^2 + M^2(k^2)}.$$

The order parameter of the dynamical chiral symmetry breaking is the chiral condensate,

$$-\langle \bar{q}q \rangle = \frac{3}{4\pi^2} \int_0^{\Lambda^2} dp^2 \frac{p^2 F(p^2) M(p^2)}{p^2 + M(p^2)^2},$$

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which we explore below for different models of the gap kernel. From Eq.(2) we have

\[
\frac{1}{F(p^2)} = 1 + \frac{1}{3\pi^2 p^2} \int d^4 k D(q^2) \sigma_v(k^2) \left[ (k \cdot p) + \frac{2(k \cdot q)(p \cdot q)}{q^2} \right],
\]

\[
M(p^2) = m_q + \frac{1}{\pi^2} \int d^4 k D(q^2) \sigma_s(k^2).
\]

We take \( D(q^2) = A_2 \alpha(q^2) D(q^2) \), and consider the model for the strong coupling [1]

\[
\alpha_s(q^2) = \frac{a + b q^2}{1 + c q^2 + d q^4 + \alpha q^6} + \frac{\pi \gamma}{\log(e + q^2)}
\]

where \( a = 1.47; b = 0.881; c = 0.314; d = 0.00986; \alpha = 0.00168; \gamma = 12/25; \Lambda = 0.234 \), all quantities in the appropriate units of GeV, and \( D(q^2) \) is the model of the gluon propagator obtained from lattice simulations. We have surveyed different types of models found in the literature [2] and select the best suitable models from Lattice QCD.

- The Lienweber model (LINA) [3] is of the form:

\[
D(q^2) = Z \left\{ \frac{A \Omega^{2 \alpha}}{(q^2 + \Omega^2)^{1+\alpha}} + \frac{L(q^2, \Omega)^2}{q^2 + \Omega^2} \right\}
\]

where \( Z = 2.01, \Omega = 0.5, A = 9.84, \alpha = 2.17 \). Here

\[
L(q^2, \Omega^2) = [0.5 \log(q^2 + \Omega^2)(q^{-2} + \Omega^{-2})]^{-\frac{13}{N_f}}
\]

is the infrared regulated version of the one loop logarithmic correction in order to ensure that this model have a correct leading ultraviolet behavior.

- The Refined Gribov-Zwanziger (RGZ) is of the form [4]:

\[
D(q^2) = \frac{q^2 + M_2}{q^4 + \mu_2 q^2 + \lambda_4}
\]

where \( M_2 = 2.15; \mu_2 = 0.34; \lambda_4 = 0.2685 \).

For comparison, we consider the well known Maris-Tandy model (MT) of the gap kernel [5]

\[
4\pi \alpha D(q^2) = \frac{4\pi^2 Dq^2 \exp\left(\frac{-q^2}{\Omega^2}\right)}{\Omega^6} + \frac{8\pi^2 \gamma \left(1 - \exp\left\{-\frac{q^2}{4m_t^2}\right\}\right)}{q^2 \log\left(\tau + 1 + \frac{\gamma}{\lambda_4}\right)^2}
\]

where \( m_t = 0.5 GeV; \gamma = 12/(33 - 2N_f); N_f = 4; \tau = e^2 - 1; D = (0.96)^2; \Omega = 0.4 \). This model has been widely used in SDE studies of hadron phenomenology. Finally, the \( A_2 \) parameter is selected such that the height of the mass functions for the three models are the same. The three models are compared in Fig. 1.

2. Numerical results

The mass function and wave function renormalization for the lattice inspired gluon models are shown in Figs. 2 and 3. For comparison, the results for the chiral condensate are tabulated in Table 1 for different current quark masses.

For the quark confinement test, we use the spatially averaged Schwinger function

\[
\Delta(T) = \frac{1}{\pi} \int_0^\infty dp \cos(pT) \frac{F(p^2)M(p^2)}{p^2 + M(p^2)^2}.
\]

We check for positiveness of this function. Logarithm of \( \Delta(T) \) for various values of the current quark mass are shown in Fig. 4. The dips in these curves signal confinement.
Figure 1. The gluon dressing function: The solid curve represents the RGZ model, the dashed curve represent the MT model and the dotted dashed curve represent the LINA model.

Figure 2. Mass function. Left panel: LINA model. Right panel: RGZ model. Solid curves represent the chiral limit \( m_q = 0 \)GeV, long dashed curve represent \( m_q = 0.0037 \)GeV, short dashed curve \( m_q = 0.082 \)GeV, dotted curve \( m_q = 0.59 \)GeV and dotted-dashed curve \( m_q = 2.0 \)GeV.

| Current Quark Masses(GeV) | MT          | LINA        | RGZ         |
|---------------------------|-------------|-------------|-------------|
| \( m_q = 0 \)             | 0.278536    | 0.281368    | 0.343       |
| \( m_u/d = 0.0037 \)      | 0.537689    | 0.544319    | 0.563984    |
| \( m_s = 0.082 \)         | 1.45734     | 1.45496     | 1.47338     |
| \( m_c = 0.59 \)          | 2.793       | 2.78651     | 2.81001     |
| \( m_b = 2.0 \)           | 4.06249     | 4.11062     | 4.13086     |

Table 1. Chiral condensate (in units of GeV\(^3\)) for different truncations of the gap equation.

3. Discussion and Conclusions
In this work we have presented the solution of QCD gap equation with models for gluon propagator inspired by Lattice QCD. The Lienweber model and Refined Gribov Zwinzager (RGZ) model are compared against the MT model which is known for last fifteen years in the SDE literature. The Lienweber and RGZ models with coupling model and with an appropriate weighting parameter \( A_2 \) parameter (\( A_2=1.3 \) for RGZ and \( A_2=0.8 \) for LinA model) yield good agreement with the well established MT results. All the models have shown enhancement in the
Wave function renormalization. **Left panel:** LINA model. **Right panel:** RGZ model. Solid curves represent the chiral limit $m_q = 0$ GeV, long dashed curve represent $m_q = 0.0037$ GeV, short dashed curve $m_q = 0.082$ GeV, dotted curve $m_q = 0.59$ GeV and dotted-dashed curve $m_q = 2.0$ GeV.

Confinement test. **Left panel:** LINA model. **Right panel:** RGZ model. Circles correspond to the chiral limit $m_q = 0$ GeV, squares $m_q = 0.0037$ GeV, diamonds $m_q = 0.082$ GeV, upper-triangles $m_q = 0.59$ GeV and lower-triangles $m_q = 2.0$ GeV.

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