Sliding mode control with a neural network compensation scheme for electro-hydraulic systems

Josiane Maria de Macedo Fernandes, Marcelo Costa Tanaka, Wallace Moreira Bessa

Abstract

Electro-hydraulic servo-systems are widely employed in industrial applications such as robotic manipulators, active suspensions, precision machine tools and aerospace systems. They provide many advantages over electric motors, including high force to weight ratio, fast response time and compact size. However, precise control of electro-hydraulic systems, due to their inherent nonlinear characteristics, cannot be easily obtained with conventional linear controllers. Most flow control valves can also exhibit some hard nonlinearities such as dead-zone due to valve spool overlap. This work describes the development of a sliding mode controller with a neural network compensation scheme for electro-hydraulic systems subject to an unknown dead-zone input. The boundedness and convergence properties of the closed-loop signals are proven using Lyapunov stability theory. Numerical results are presented in order to demonstrate the control system performance.

INTRODUCTION

Electro-hydraulic actuators play an essential role in several branches of industrial activity and are frequently the most suitable choice for systems that require large forces at high speeds. Their application scope ranges from robotic manipulators to aerospace systems. Another great advantage of hydraulic systems is the ability to keep up the load capacity, which in the case of electric actuators is limited due to excessive heat generation.

However, the dynamic behavior of electro-hydraulic systems is highly nonlinear, which in fact makes the design of controllers for such systems a challenge for the conventional and well established linear control methodologies. The increasing number of works dealing with control approaches based on modern techniques shows the great interest of the engineering community, both in academia and industry, in this particular field. The most common approaches are the adaptive (Guan and Pan, 2008a, b) and variable structure (Bessa et al., 2006) methodologies, but nonlinear controllers based on quantitative feedback theory (Mihailov et al., 2002), optimal tuning PID (Liu and Daley, 2000), adaptive neural network (Knoll and Unbehauen, 2000) and adaptive fuzzy system (Bessa et al., 2010a) were also presented.

In addition to the common nonlinearities that originate from the compressibility of the hydraulic fluid and valve flow-pressure properties, most electro-hydraulic systems are also subjected to hard nonlinearities such as dead-zone due to valve spool overlap. It is well-known that the presence of a dead-zone can lead to performance degradation of the controller and limit cycles or even instability in the closed-loop system. To overcome the negative effects of the dead-zone nonlinearity, many works (Tao and Kokotovic, 1994; Kim et al., 1994; Oh and Park, 1998; Selmi and Lewis, 2000; Ibrir and Chang, 2004; Zhou et al., 2006) use an inverse function even though this approach leads to a discontinuous control law and requires instantaneous switching, which in practice can not be accomplished with mechanical actuators. An alternative scheme, without using the dead-zone inverse, was originally proposed by Lewis et al. (1999) and also adopted by Wang et al. (2004). In both works, the dead-zone is treated as a combination of a linear and a saturation function. This approach was further extended by Ibrir et al. (2007) and Zhang and Ge (2007), in order to accommodate non-symmetric and unknown dead-zones, respectively.

Intelligent control, on the other hand, has proven to be a very attractive approach to cope with uncertain nonlinear systems (Bessa, 2005; Bessa and Barreto, 2010; Bessa et al., 2008, 2012, 2014, 2017, 2018, 2019; Dos Santos and Bessa, 2019; Lima et al., 2018, 2020, 2021; Tanaka et al., 2019). By combining nonlinear control techniques, such as feedback linearization or sliding modes, with adaptive intelligent algorithms, for example fuzzy logic or artificial neural networks, the resulting intelligent control strategies can deal with the nonlinear characteristics as well as with modeling imprecisions and external disturbances that can arise.

In this work, a sliding mode controller with a neural network compensation scheme is proposed for electro-hydraulic systems subject to an unknown dead-zone input. The adopted approach does not require previous knowledge of dead-zone parameters nor the construction of an inverse function. On this basis, a smooth sliding mode controller is considered to confer robustness against modeling imprecisions and a Radial Basis Function (RBF) neural network is embedded in the boundary layer to cope with dead-zone effects. The boundedness and convergence properties of the closed-loop system are analytically proven using Lyapunov stability theory. Numerical simulations are carried out in order to demonstrate the control system performance.

ELECTRO-HYDRAULIC SYSTEM MODEL

In order to design the adaptive fuzzy controller, a mathematical model that represents the hydraulic system dynamics is needed. Dynamic models for such systems are well documented in the literature (Merritt, 1967; Walters, 1967).

The electro-hydraulic system considered in this work consists of a four-way proportional valve, a hydraulic cylinder and variable load force. The variable load force is represented by a mass–spring–damper system. The schematic diagram of the system under study is presented in Fig. 1.

The balance of forces on the piston leads to the following equation of motion:

$$ F_q = A_1 P_1 - A_2 P_2 = M \ddot{x} + B \dot{x} + K x $$

(1)
where $F_g$ is the force generated by the piston, $P_1$ and $P_2$ are the pressures at each side of cylinder chamber, $A_1$ and $A_2$ are the ram areas of the two chambers, $M_t$ is the total mass of piston and load referred to piston, $B_t$ is the viscous damping coefficient of piston and load, $K_s$ is the load spring constant and $x$ is the piston displacement.

Defining the pressure drop across the load as $P_l = P_1 - P_2$ and considering that for a symmetrical cylinder $A_p = A_1 = A_2$, Eq. (1) can be rewritten as

$$M_t\ddot{x} + B_t\dot{x} + K_s x = A_p P_l$$  \hspace{1cm} (2)

Applying continuity equation to the fluid flow, the following equation is obtained:

$$Q_l = A_p \dot{x} + \frac{V_t}{4\rho} \dot{P}_l$$  \hspace{1cm} (3)

where $Q_l = (Q_1 + Q_2)/2$ is the load flow, $C_{tp}$ the total leakage coefficient of piston, $V_t$ the total volume under compression in both chambers and $\dot{B}_t$ the effective bulk modulus.

Considering that the return line pressure is usually much smaller than the other pressures involved ($P_0 \approx 0$) and assuming a closed center spool valve with matched and symmetrical orifices, the relationship between load pressure $P_l$ and load flow $Q_l$ can be described as follows

$$Q_l = C_d w x_{sp} \sqrt{\frac{1}{\rho} (P_s - \text{sgn}(x_{sp}) P_l)}$$  \hspace{1cm} (4)

where $C_d$ is the discharge coefficient, $w$ the valve orifice area gradient, $\dot{x}_{sp}$ the effective spool displacement from neutral, $\rho$ the hydraulic fluid density, $P_s$ the supply pressure and $\text{sgn}(\cdot)$ is defined by

$$\text{sgn}(z) = \begin{cases} -1 & \text{if } z < 0 \\ 0 & \text{if } z = 0 \\ 1 & \text{if } z > 0 \end{cases}$$

Assuming that the dynamics of the valve are fast enough to be neglected, the valve spool displacement can be considered as proportional to the control voltage ($u$). For closed center valves, or even in the case of the so-called critical valves, the spool presents some overlap. This overlap prevents from leakage losses but leads to a dead-zone nonlinearity within the control voltage, as shown in Fig. 2.

![Fig. 1. Schematic diagram of the electro-hydraulic servo-system.](image)

![Fig. 2. Dead-zone nonlinearity.](image)

The adopted dead-zone model is a slightly modified version of that proposed by [Zhang and Ge, 2007], which can be mathematically described by

$$\dot{x}_{sp} = \begin{cases} g_l(u) & \text{if } u \leq \delta_l \\ 0 & \text{if } \delta_l < u < \delta_r \\ g_r(u) & \text{if } u \geq \delta_r \end{cases}$$  \hspace{1cm} (5)

where $g_l$ and $g_r$ are functions of control voltage and the dead-band parameters $\delta_l$ and $\delta_r$ depends on the size of the overlap region.
In respect of the dead-zone model presented in Eq. (5), the following assumptions can be made:

**Assumption 1:** The dead-zone output $\bar{\delta}_\nu$ is not available to be measured.

**Assumption 2:** The dead-band parameters $\bar{\delta}_\ell$ and $\bar{\delta}_r$ are unknown but bounded and with known signs, i.e., $\bar{\delta}_{\text{min}} \leq \bar{\delta}_\ell < 0$ and $0 < \bar{\delta}_{\text{min}} \leq \bar{\delta}_r \leq \bar{\delta}_{\text{max}}$.

**Assumption 3:** The functions $g_l : (-\infty, \bar{\delta}_\ell]$ and $g_r : [\bar{\delta}_r, +\infty]$ are $C^1$ and with bounded positive-valued derivatives, i.e.,

\[
0 < k_{l_{\text{min}}} \leq g'_l(u) \leq k_{l_{\text{max}}}, \quad \forall u \in (-\infty, \bar{\delta}_\ell],
\]

\[
0 < k_{r_{\text{min}}} \leq g'_r(u) \leq k_{r_{\text{max}}}, \quad \forall u \in [\bar{\delta}_r, +\infty),
\]

where $g'_l(u) = dg_l(z)/dz|_{z=-u}$ and $g'_r(u) = dg_r(z)/dz|_{z=-u}$.

**Remark 1:** Assumption 3 means that both $g_l$ and $g_r$ are Lipschitz functions.

From the mean value theorem and noting that $g_l(\bar{\delta}_\ell) = g_r(\bar{\delta}_r) = 0$, it follows that there exist $\tilde{\xi}_l : \mathbb{R} \to (-\infty, \bar{\delta}_\ell]$ and $\tilde{\xi}_r : \mathbb{R} \to (\bar{\delta}_r, +\infty)$ such that

\[
g_l(u) = g'_l(\tilde{\xi}_l(u))[u - \bar{\delta}_\ell]
\]

\[
g_r(u) = g'_r(\tilde{\xi}_r(u))[u - \bar{\delta}_r]
\]

In this way, Eq. (5) can be rewritten as follows:

\[
\dot{x}_{\nu} = \begin{cases} 
  g'_l(\tilde{\xi}_l(u))[u - \bar{\delta}_\ell] & \text{if } u \leq \bar{\delta}_\ell \\
  0 & \text{if } \bar{\delta}_\ell < u < \bar{\delta}_r \\
  g'_r(\tilde{\xi}_r(u))[u - \bar{\delta}_r] & \text{if } u \geq \bar{\delta}_r
\end{cases}
\]

or in a more appropriate form:

\[
\dot{x}_{\nu} = k_e(u)[u - d(u)]
\]

where

\[
k_e(u) = \begin{cases} 
  g'_l(\tilde{\xi}_l(u)) & \text{if } u \leq 0 \\
  g'_r(\tilde{\xi}_r(u)) & \text{if } u > 0
\end{cases}
\]

and

\[
d(u) = \begin{cases} 
  \bar{\delta}_\ell & \text{if } u \leq \bar{\delta}_\ell \\
  u & \text{if } \bar{\delta}_\ell < u < \bar{\delta}_r \\
  \bar{\delta}_r & \text{if } u \geq \bar{\delta}_r
\end{cases}
\]

**Remark 2:** Considering Assumption 2 and Eq. (9), it can be easily verified that $d(u)$ is bounded: $|d(u)| \leq \bar{\delta}$, where $\bar{\delta} = \max\{-\bar{\delta}_{\text{min}}, \bar{\delta}_{\text{max}}\}$.

Combining equations (2), (3), (4) and (7) leads to a third-order differential equation that represents the dynamic behavior of the electro-hydraulic system:

\[
\ddot{x} = -a^T x + b(x, u)[a - b(x, u)]d(u)
\]

where $\mathbf{x} = [x, \dot{x}, \ddot{x}]$ is the state vector with an associated coefficient vector $\mathbf{a} = [a_0, a_1, a_2]$ defined according to

\[
a_0 = \frac{4\beta_e C_P K_t}{V_t M_t}; \quad a_1 = K_t \frac{4\beta_e A_p^2}{V_t M_t} + 4\beta_e C_P B_t; \quad a_2 = B_t \frac{4\beta_e C_P}{V_t}
\]

and

\[
b(x, u) = \frac{4\beta_e A_p C_d w_k}{V_t M_t} \sqrt{\frac{1}{p} \left[ p_{\text{e}} - \text{sgn}(u)(M_t \ddot{x} + B_t x + K_t x)/A_p \right]}
\]

In respect of the dynamic system presented in Eq. (10), the following assumptions will also be made:

**Assumption 4:** The coefficients $a_0, a_1$ and $a_2$ are unknown but bounded: $|\dot{\mathbf{a}} - \mathbf{a}| \leq \alpha$, where $\alpha$ is an estimate of $\mathbf{a}$.

**Assumption 5:** The input gain $b(x, u)$ is unknown but positive and bounded: $0 < b_{\text{min}} \leq b(x, u) \leq b_{\text{max}}$.

Based on the dynamic model presented in Eq. (10), a neural network sliding mode controller is developed.
NEURAL NETWORK BASED SLIDING MODE CONTROLLER

The proposed control problem is to ensure that, even in the presence of parametric uncertainties, unmodeled dynamics and an unknown dead-zone input, the state vector \( \mathbf{x} \) will follow a desired trajectory \( \mathbf{x}_d = [\dot{x}_d, \ddot{x}_d, x_d] \) in the state space.

Regarding the development of the control law, the following assumptions should also be made:

Assumption 6: The state vector \( \mathbf{x} \) is available.

Assumption 7: The desired trajectory \( \mathbf{x}_d \) is once differentiable in time. Furthermore, every element of vector \( \mathbf{x}_d \), as well as \( \dot{x}_d \), is available and with known bounds.

Let \( \ddot{x} = x - x_d \) be defined as the tracking error in the variable \( x \), \( \dot{x} = \ddot{x} + [\ddot{x}, \dot{x}] \) as the tracking error vector and consider a sliding surface \( S \) defined in the state space by the equation \( s(\ddot{x}) = 0 \), with the function \( s : \mathbb{R} \rightarrow \mathbb{R} \) satisfying

\[
s(\ddot{x}) = \ddot{x} + 2\lambda \dot{x} + \lambda^2 x
\]

(11)

where \( \lambda \) is a strictly positive constant.

Now, the problem of controlling the system dynamics \( \ddot{x} \) can be treated according to the sliding mode methodology, by defining a control law composed by an equivalent control \( \hat{u} \) where 

\[
\lambda_b = \sqrt{\gamma}
\]

as the tracking error vector and consider a sliding surface \( S \) defined in the state space by the equation \( s(\mathbf{x}) = 0 \), with the function \( s : \mathbb{R} \rightarrow \mathbb{R} \) satisfying

\[
\eta(\ddot{x}) = \ddot{x} + 2\lambda \dot{x} + \lambda^2 x
\]

(11)

where \( \lambda \) is a strictly positive constant.

Now, the problem of controlling the system dynamics \( \ddot{x} \) can be treated according to the sliding mode methodology, by defining a control law composed by an equivalent control \( \hat{u} = \ddot{b}^{-1}(\dddot{a}^T \mathbf{x} + \ddot{x}_d - 2\lambda \dot{x} - \lambda^2 x) \), an estimate \( \ddot{d} \) and a discontinuous term \(-K \text{sgn}(s)\):

\[
u = \ddot{b}^{-1}(\dddot{a}^T \mathbf{x} + \ddot{x}_d - 2\lambda \dot{x} - \lambda^2 x) + \ddot{d} - K \text{sgn}(s)
\]

(12)

where \( K \) is the control gain.

Based on Assumption 6 and considering that the estimate \( \ddot{b} = \sqrt{\gamma} \) could be chosen according to the geometric mean \( \ddot{b} = \sqrt{b_{\text{max}}/b_{\text{min}}} \), the bounds of \( b \) may be expressed as \( \gamma^{-1} \leq \ddot{b} \leq \gamma \), where \( \gamma = \sqrt{b_{\text{max}}/b_{\text{min}}} \).

Under this condition, the gain \( K \) should be chosen according to:

\[
K \geq \gamma \ddot{b}^{-1}(\eta + \gamma) + \delta + |\ddot{d}| + (\gamma - 1)|\ddot{u}|
\]

(13)

where \( \eta \) is a strictly positive constant related to the reaching time.

At this point, it should be highlighted that the control law (12), together with (13), is sufficient to impose the sliding condition

\[
\frac{1}{2} \frac{d}{dt} \phi^2 \leq -\eta|\dot{s}|
\]

and, consequently, the finite time convergence to the sliding surface \( S \).

In spite of the demonstrated properties of the controller, the presence of a discontinuous term in the control law leads to the well known chattering phenomenon. In order to overcome the undesirable chattering effects, a thin boundary layer, \( S_\phi \), in the neighborhood of the switching surface can be adopted (Slotine, 1984):

\[
S_\phi = \{ \ddot{x} \in \mathbb{R}^3 \mid |s(\ddot{x})| \leq \phi \}
\]

where \( \phi \) is a strictly positive constant that represents the boundary layer thickness.

The boundary layer is achieved by replacing the sign function by a continuous interpolation inside \( S_\phi \). It should be noted that this smooth approximation must behave exactly like the sign function outside the boundary layer. There are several options to smooth out the ideal relay but the most common choice is the saturation function:

\[
\text{sat}(s/\phi) = \begin{cases} 
\text{sgn}(s) & \text{if } |s/\phi| \geq 1 \\
0 & \text{if } |s/\phi| < 1 
\end{cases}
\]

In this way, to avoid chattering, a smooth version of Eq. (12) is defined:

\[
u = \ddot{b}^{-1}(\ddot{a}^T \mathbf{x} + \ddot{x}_d - 2\lambda \dot{x} - \lambda^2 x) + \ddot{d} - K \text{sgn}(s/\phi)
\]

(14)

In order to establish the attractiveness and invariant properties of the defined boundary layer, let a new Lyapunov function candidate \( V \) be defined as

\[
V(t) = \frac{1}{2} s^2
\]

where \( s_\phi \) is a measure of the distance of the current state to the boundary layer, and can be computed as follows

\[
s_\phi = s - \phi \text{ sat}(s/\phi)
\]

(15)

Noting that \( s_\phi = 0 \) inside the boundary layer and \( s_\phi = \ddot{s} \), one has \( V(t) = 0 \) inside \( S_\phi \), and outside

\[
V(t) = s_\phi \ddot{s}_\phi = s_\phi \ddot{s} = (\ddot{x} + 2\lambda \dot{x} + \lambda^2 x)s_\phi = (\ddot{a}^T \mathbf{x} + \ddot{b} \dot{x} - \ddot{b} \dot{x} + 2\lambda \dot{x} + \lambda^2 x)s_\phi
\]

It can be easily verified that outside the boundary layer the control law (14) takes the following form:

\[
u = \ddot{b}^{-1}(\dddot{a}^T \mathbf{x} + \ddot{x}_d - 2\lambda \dot{x} - \lambda^2 x) + \ddot{d} - K \text{sgn}(s_\phi)
\]

Thus, the time derivative \( \dot{V} \) can be written as

\[
\dot{V}(t) = -(|K \text{sgn}(s_\phi) - (\ddot{a} - a)^T \mathbf{x} + \ddot{b} \dot{u} - \ddot{b} \dot{u} + \dot{b} \ddot{u}) |s_\phi
\]

Therefore, by considering Assumptions 2-5 and defining \( K \) according to (13), \( V(t) \) becomes:

\[
\dot{V}(t) \leq -\eta|s_\phi|
\]

(16)
which implies $V(t) \leq V(0)$ and that $s$ is bounded. The definitions of $s$ and $s_0$, respectively Eqs. (11) and (15), imply that $\bar{x}$ is bounded. From the definition of $\bar{x}$ and Assumption 7, it can be verified that $\bar{x}$ is also bounded.

The finite-time convergence of the states to the boundary layer can be shown by integrating both sides of (16) over the interval $0 \leq t \leq t_{reach}$, where $t_{reach}$ is the time required to hit $S_0$. In this way, noting that $|s_0(t_{reach})| = 0$, one has:

$$t_{reach} \leq \frac{|s_0(0)|}{\eta}$$

(17)

which guarantees the convergence of the tracking error vector to the boundary layer in a time interval smaller than $|s_0(0)|/\eta$.

Nevertheless, it should be emphasized that the substitution of the discontinuous term by a smooth approximation actually means that a steady-state error will always remain. However, it can be verified that, once inside the boundary layer, the tracking error vector will exponentially converge to a closed region $\Phi$.

Considering that $|s(\bar{x})| \leq \phi$ may be rewritten as $-\phi \leq s(\bar{x}) \leq \phi$ and from the definition of $s(\bar{x})$, Eq. (11), one has:

$$-\phi \leq \bar{x} + 2\bar{x} + \lambda^2 \bar{x} \leq \phi$$

(18)

Multiplying (18) by $e^{\lambda t}$ and integrating between 0 and $t$:

$$-\phi e^{\lambda t} \leq (\bar{x} + 2\bar{x} + \lambda^2 \bar{x}) e^{\lambda t} \leq \phi e^{\lambda t}$$

(19)

or conveniently rewritten as

$$-\phi e^{\lambda t} - \left( \frac{d}{dt}(\bar{x}e^{\lambda t}) \right)_{t=0} + \phi \lambda e^{\lambda t} \leq \phi e^{\lambda t} - \left( \frac{d}{dt}(\bar{x}e^{\lambda t}) \right)_{t=0} + \phi \lambda e^{\lambda t}$$

(20)

Furthermore, dividing (20) by $e^{\lambda t}$, it can be easily verified that for $t \to \infty$:

$$-\frac{\phi}{\lambda^2} \leq \bar{x} \leq \frac{\phi}{\lambda^2}$$

(21)

By imposing the bounds (21) to (19), noting that $d(\bar{x}e^{\lambda t})/dt = \bar{x}e^{\lambda t} + \bar{x}\lambda e^{\lambda t}$ and dividing again by $e^{\lambda t}$, it follows that, for $t \to \infty$,

$$-2\frac{\phi}{\lambda} \leq \dot{\bar{x}} \leq 2\frac{\phi}{\lambda}$$

(22)

Finally, applying (21) and (22) to (18), one has:

$$-6\phi \leq \dot{\bar{x}} \leq 6\phi$$

(23)

In this way, the tracking error will be confined within the limits $|\bar{x}| \leq \phi/\lambda^2$, $|\dot{\bar{x}}| \leq 2\phi/\lambda$ and $|\bar{x}| \leq 6\phi$. However, these bounds define a box that is not completely inside the boundary layer. Considering the demonstrated attractiveness and invariant properties of $S_0$, the region of convergence can be stated as the intersection of the boundary layer and the box defined by the preceding bounds. Therefore, it follows that the tracking error vector will exponentially converge to a closed region $\Phi = \{ \bar{x} \in \mathbb{R}^3 \mid |s(\bar{x})| \leq \phi \ and \ |\bar{x}| \leq \phi/\lambda^2 \ and \ |\dot{\bar{x}}| \leq 2\phi/\lambda \ and \ |\bar{x}| \leq 6\phi \}$. It should be highlighted that the convergence region $\Phi$ is in perfect accordance with the bounds proposed by Bessa (2009) for $n^{th}$-order nonlinear systems subject to smooth sliding mode controllers.

Now, in order to obtain a good approximation to the disturbance $d$ and to enhance the tracking performance inside the convergence region $\Phi$, the estimate $\hat{d}$ will be computed directly by a neural network. Due to its simplicity and fast convergence feature, radial basis functions (RBF) are used as activation functions and the related tracking error as input. In this case, the output of the network is defined as:

$$\hat{d}(\bar{x}) = \sum_{i=1}^{M} w_i \cdot \varphi_i(||\bar{x} - \mathbf{t}||)$$

(24)

where $\varphi_i(\cdot)$ are the activation functions and $\mathbf{t}$ a vector containing the coordinates of the center of each activation function.

Now, the signal $v = \hat{d}(\bar{x}) + k_{i-1} \hat{d}(\bar{x}) + \ldots + k_1 \bar{x} + k_0 \bar{x} - \hat{d}$ is used to train the neural network and the weights of the output layer are adjusted using the pseudo-inverse matrix.

Considering a training set $T = \{(\bar{x}, d_1), (\bar{x}, d_2), \ldots, (\bar{x}, d_p)\}$ and
the RBF weights are computed with the pseudo-inverse \( \hat{\phi}^+ \)

\[
\{w\} = \hat{\phi}^+ \{d\}
\]

and approximation error, \( E \), by the euclidean norm

\[
E = ||\{d\} - \hat{\phi}\{w\}||
\]

In the following section some numerical simulations are presented in order to evaluate the performance of the neural network based sliding mode controller.

SIMULATION RESULTS

The simulation studies were performed with sampling rates of 500 Hz for control system and 1 kHz for dynamic model. The differential equations of the dynamic model were numerically solved with the fourth order Runge-Kutta method.

It was assumed in the simulation studies that the model parameters were not exactly known and nonlinear functions were considered for \( g_l(u) = k_l(u + 0.2\sin u - \delta_l) \) and \( g_r(u) = k_r(u - 0.2\cos u - \delta_r) \), with \( k_l = k_r = 2 \times 10^{-6} \text{ m/V} \). On this basis, considering a maximal uncertainty of \( \pm 10\% \) in the supply pressure, \( P_s = 7[1 + 0.2\sin(x)] \text{ MPa} \), the estimates \( k_l = 2 \times 10^{-6} \text{ m/V} \) and \( P_s = 7 \text{ MPa} \) were chosen for the computation of \( \delta \) in the control law. The other model and controller parameters were \( \rho = 850 \text{ kg/m}^3 \), \( C_f = 0.6 \), \( w = 2.5 \times 10^{-4} \text{ m} \), \( A_p = 3 \times 10^{-4} \text{ m}^2 \), \( C_{pf} = 2 \times 10^{-12} \text{ m}^3/(\text{s Pa}) \), \( B_s = 700 \text{ MPa} \), \( V_s = 6 \times 10^{-3} \text{ m}^3 \), \( M_f = 250 \text{ kg} \), \( B_t = 100 \text{ Ns/m} \), \( K_f = 75 \text{ N/m} \), \( \delta_p = -1.1 \text{ V} \), \( \delta_r = 0.9 \text{ V} \), \( \lambda = 8 \), \( \phi = 4 \), \( \gamma = 1.2 \), \( \delta = 1.1 \), \( \phi = 1 \) and \( \eta = 0.1 \). Figure 3 shows the obtained results for the tracking of \( x_d = 0.5\sin(0.1t) \text{ m} \). It should be highlighted that the first 50 seconds of each simulation study were used to train the neural network.

![Tracking performance](image1)

![Control voltage](image2)

(a) Tracking performance.  
(b) Control voltage.

Fig. 3. Tracking performance with \( x_d = 0.5\sin(0.1t) \text{ m} \).

As observed in Fig. 3, even in the presence of uncertainties with respect to model parameters and an unknown dead-zone input, the neural network based sliding mode controller is able to provide trajectory tracking with no chattering at all.

The improved performance of the proposed controller can be clearly ascertained if the evolution in time of the related tracking error measure, \( s \), is compared with the result obtained with a conventional smooth sliding mode controller. For simulation purposes, the proposed controller can be easily converted to the classical one by setting \( \hat{d} = 0 \). Figure 4 shows the obtained results. It can be easily verified in Fig. 4(b) that after 50 seconds, when the compensation scheme is enabled, the magnitude of the tracking error measure \( s \) is significantly reduced.

CONCLUDING REMARKS

The present work addressed the problem of controlling electro-hydraulic systems subject to an unknown dead-zone. A neural network based sliding mode controller was implemented to deal with the trajectory tracking problem. The boundedness and convergence properties of the closed-loop systems was proven using Lyapunov stability theory. The control system performance was also confirmed by means of numerical simulations. The neural network algorithm could automatically recognize the dead-zone nonlinearity and previously compensate for its undesirable effects.
Fig. 4. Comparison of the tracking error measure with and without neural network compensation.

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