On the Yang-Mills wave functional in Coulomb gauge

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Abstract

We investigate the dependence of the Yang-Mills wave functional in Coulomb gauge on the Faddeev-Popov determinant. We use a Gaussian wave functional multiplied by an arbitrary power of the Faddeev-Popov determinant. We show, that within the resummation of one-loop diagrams the stationary vacuum energy is independent of the power of the Faddeev-Popov determinant and, furthermore, the wave functional becomes field-independent in the infrared, describing a stochastic vacuum. Our investigations show, that the infrared limit is rather robust against details of the variational ansätze for the Yang-Mills wave functional. The infrared limit is exclusively determined by the divergence of the Faddeev-Popov determinant at the Gribov horizon.

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1 Introduction

Recently Yang-Mills theory in Coulomb gauge has become the subject of intensive studies both on the lattice, refs. [1], [2] and in the continuum, ref. [3], [4], [6], [7], [8]. The Coulomb gauge is a physical gauge and in this gauge confinement is realized by the statistical dominance of the field configurations near the Gribov horizon, which gives rise to an infrared enhanced static color charge potential.

For the calculation of static properties of continuum Yang-Mills theory, the Schrödinger equation approach seems to be most convenient. In refs. [6], [7], [8] the Yang-Mills Schrödinger equation was approximately solved in Coulomb gauge, using the variational principle. Different ansätze for the vacuum wave functional and different renormalization conditions have been used, and different infrared behaviours of the gluon and ghost propagators were obtained. One might wonder, whether the different results are a consequence of the different ansätze for the wave functional. To answer this question, in this paper we consider a more general class of
wave functionals, which includes, in particular, the wave functionals previously used in refs. [6], [7] and [8]. We will show, that the different ansätze used so far, have to yield the same unique infrared behaviour of the vacuum wave functional (at least to the order considered). We will also show, that in the infrared the wave functional becomes field independent, describing a stochastic vacuum. Furthermore, the infrared limit of the wave functional agrees with the exact vacuum wave functional in $D = 1 + 1$.

2 The variational ansatz

For the Yang-Mills vacuum we consider trial wave functionals of the form

$$\Psi[A^\perp] = J^{-\alpha}[A^\perp] \phi[A^\perp],$$

(1)

where

$$J[A^\perp] = \frac{\text{Det}(-\hat{D}_i \partial_i)}{\text{Det}(-\partial^2)}$$

(2)

is the Faddeev-Popov determinant, which for later convenience has been normalized to $J[A^\perp = 0] = 1$. Here $\hat{D} = \partial + \hat{A}^\perp$ is the covariant derivative and $\hat{A}^\perp = A^{\perp a} T^a$ denotes the gauge field in the adjoint representation. Furthermore $\phi[A^\perp]$ is a Gaussian wave functional defined by

$$\phi[A^\perp] = \mathcal{N} \exp(-S[A^\perp])$$

$$S[A^\perp] = \frac{1}{2} \int d^3 x \int d^3 x' A_i^{\perp a}(x) \omega^{ab}_{ij}(x, x') A_j^{\perp b}(x')$$

(3)

with $\mathcal{N} = \mathcal{N}(\alpha, \omega)$ being a normalization constant to ensure $\langle \psi | \psi \rangle = 1$. The Faddeev-Popov determinant arises as Jacobian in the transformation from “cartesian” coordinates $A_i^a(x)$ to the “curvilinear coordinates” $A_i^{\perp a}(x)$ satisfying the Coulomb gauge $\partial_i A_i^\perp = 0$, and defines the metric in the space of transversal gauge orbits $A_i^\perp(x)$. Accordingly the scalar product in the space of transversal gauge orbits is defined by

$$\langle \Psi | \Phi \rangle = \int D A^\perp J[A^\perp] \Psi^*[A^\perp] \Phi[A^\perp],$$

(4)

where the integration should in principal be restricted to the fundamental modular region [4], [14]. The choice $\omega^{ab}_{ij}(x, x') = \delta_{ij} \delta^{ab} \omega(x, x')$ and $\alpha = 0$ was used in refs. [6], [7], while $\alpha = \frac{1}{2}$ was chosen in ref. [8]. From eq. (1) it is seen, that for the latter choice $\phi[A^\perp]$ represents just the “radial” wave functional1.

We wish to study the dependence of the vacuum Yang-Mills wave functional (1) on the power of the Faddeev-Popov determinant $\alpha$, which can take, in principle, any real value as long as

$^1$For a point particle in a s-state the wave function is of the form $\Psi(r) = \frac{\phi(r)}{r}$, where $\phi(r)$ is the radial wave function and the Jacobian is given by $J = 1/r^2$
$\Psi[A^\perp]$ is normalizable. The integral kernel $\omega$ as well as the parameter $\alpha$ have to be determined by minimizing the expectation value of the energy

$$\langle H \rangle = \int DA^\perp J[A^\perp] \Psi^*[A^\perp] H \Psi[A^\perp].$$

(5)

3 Minimization of the energy

Inserting the explicit form of the wave functional eq. (1) into eq. (5) variation of the energy yields the “gap equation”:

$$\frac{\delta \langle H \rangle}{\delta \omega} = 2 \left\langle \frac{\delta S}{\delta \omega} \right\rangle \langle H \rangle - \left\langle \left\{ \frac{\delta S}{\delta \omega}, H \right\} \right\rangle = 0.$$ 

(6)

Here the first term arises from the variation of the normalization constant $(\delta N / \delta \omega = N \langle \frac{\delta S}{\delta \omega} \rangle)$ and $(\{,\})$ denotes the anti-commutator. Minimization of the energy $\langle H \rangle$ (5) with respect to the power $\alpha$ yields the condition

$$\frac{d\langle H \rangle}{d\alpha} = 2 \langle \ln J[A^\perp] \rangle \langle H \rangle - \left\langle \left\{ \ln J[A^\perp], H \right\} \right\rangle = 0,$$

(7)

where we have used $dN/d\alpha = N \langle \ln J[A^\perp] \rangle$.

Consider now the structure of the Faddeev-Popov determinant (2), which obviously satisfies $\ln J[A^\perp] = 0$. Furthermore, by definition (2) we have

$$\frac{\delta \ln J[A^\perp]}{\delta A^\perp_{i,a}(x)} = -Tr \left( G \Gamma^0_{\gamma,a}(x) \right),$$

(8)

where

$$G = (-\hat{D}_i \partial_i)^{-1}$$

(9)

is the inverse Faddeev-Popov operator and $\Gamma^0_{\gamma,a}(x) = \delta G^{-1}/\delta A^\perp_{i,a}(x)$ is the bare ghost gluon vertex. Since $\Gamma^0_{\gamma,a} \sim \hat{T}^a$ (group generator in the adjoint representation) and $\hat{T}^a$ occurs in $G$ only in the combination $A^\perp = A^{1a}_{i} \hat{T}_a$ it is clear, that the quantity (8) has to be proportional to $A^\perp$, since $tr \hat{T}^a = 0$. Therefore we find the representation

$$\ln J[A^\perp] = \int d^3xd^3x' C^{ab}_{ij}[A^\perp](x, x') A^\perp_{i,a}(x) A^\perp_{j,b}(x')$$

(10)

with some, not explicitly known functional $C^{ab}_{ij}[A^\perp]$. In one-loop approximation the expectation value of eq. (10) is given by

$$\langle \ln J[A^\perp] \rangle \approx \int d^3xd^3x' \left\langle C^{ab}_{ij}[A^\perp](x, x') \right\rangle \cdot \left\langle A^\perp_{i,a}(x) A^\perp_{j,b}(x') \right\rangle.$$ 

(11)
Furthermore, to this order we can neglect terms of the form \(\delta C \delta A \perp A \perp \rangle\) and find from (10) for the curvature in orbit space [8]

\[
\chi_{ik}^{ab}(x, x') = -\frac{1}{2} \left< \frac{\delta^2 \ln J}{\delta A_i^{\perp a}(x) \delta A_j^{\perp b}(x')} \right> = -\left< C_{ij}^{ab}[A^{\perp}](x, x') \right>.
\] (12)

To this order we can also replace \(C[A]\) in (11) by its expectation value \((-\chi)\) yielding

\[
\ln J[A^{\perp}] = -\int d^3x d^3x' \chi_{ij}^{ab}(x, x') A_i^{\perp a}(x) A_j^{\perp b}(x').
\] (13)

Inserting eq. (13) into eq. (7) we find

\[
\frac{d\langle H \rangle}{d\alpha} = -\int d^3x d^3x' \chi_{ij}^{ab}(x, x') \left[ 2 \left< A_i^{\perp a}(x) A_j^{\perp b}(x') \right> \langle H \rangle - \left< \left\{ A_i^{\perp a}(x) A_j^{\perp b}(x'), H \right\} \right> \right]
\] (14)

On the other hand for the Gaussian wave functional (1), (3) we have

\[
\frac{\delta S}{\delta \omega_{ij}^{ab}(x, x')} = \frac{1}{2} A_i^{\perp a}(x) A_j^{\perp b}(x'),
\] (15)

so that the equation (6) becomes

\[
2 \frac{\delta \langle H \rangle}{\delta \omega_{ij}^{ab}(x, x')} = 2 \left< A_i^{\perp a}(x) A_j^{\perp b}(x') \right> \langle H \rangle - \left< \left\{ A_i^{\perp a}(x) A_j^{\perp b}(x'), H \right\} \right>.
\] (16)

Comparison of eqs. (14) and (16) yields

\[
\frac{d\langle H \rangle}{d\alpha} = -2 \int d^3x d^3x' \chi_{ij}^{ab}(x, x') \frac{\delta \langle H \rangle}{\delta \omega_{ij}^{ab}(x, x')}.
\] (17)

Thus stationarity of the energy with respect to \(\omega_{ij}^{ab}(x)\), \(\delta \langle H \rangle/\delta \omega = 0\) implies also stationarity with respect to \(\alpha\), \(d\langle H \rangle/d\alpha = 0\). Let us emphasize, that eq. (17) is exact to one-loop order in the equation of motion (i.e. to two-loop order in \(\langle H \rangle\)).

### 4 The energy functional

The above obtained result can be also immediately inferred from the explicit expression of the expectation value of the Yang-Mills Hamiltonian in the state (1), which is given by

\[
\langle H \rangle = E_k + E_B + E_C
\]

\[
E_k = \delta^3(0) \frac{N_C^2 - 1}{2} \int d^3k \frac{[\Omega(k) - \chi(k)]^2}{\Omega(k)}
\]
\[ E_B = \delta^3(0) \frac{N_C^2 - 1}{2} \int d^3k \left( \frac{k^2}{\Omega(k)} + \frac{N_C g^2}{8} \int \frac{d^3k'}{(2\pi)^3} \left[ 3 - (kk')^2 \right] \frac{1}{\Omega(k)\Omega(k')} \right) \]

\[ E_C = \delta^3(0) \frac{N_C(N_C - 1)}{16} \int d^3kd^3k' \frac{1}{(2\pi)^3} \left[ 1 + (kk')^2 \right] \cdot \frac{d^2(k + k')f(k + k')}{(k + k')^2} \cdot \frac{[\Omega(k) - \chi(k)] - [\Omega(k') - \chi(k')]}{\Omega(k)\Omega(k')} \]

where \( d(k) \) and \( f(k) \) are the ghost and Coulomb form factors defined in ref. \[8\] and \( \chi \) is the scalar curvature defined in terms of the curvature tensor \([12]\) by

\[ t_{kn}(x)\chi_{\alpha\beta}^{ab}(x, y) = \delta^{ab}t_{kl}(x)\chi(x, y) \]  

\[ \chi(k) = \frac{N_C}{4} \int \frac{d^3q}{(2\pi)^3} \left[ 1 - (kq)^2 \right] \frac{d(k - q)d(q)}{(k - q)^2} \]

with \( t_{kl}(x) = \delta_{kl} - \partial_k \partial_l/\partial^2 \) being the transversal projector. \(^2\) Furthermore

\[ \Omega(k) = \omega(k) - (2\alpha - 1)\chi(k) \]  

is the inverse of the gluon propagator

\[ \langle A_{i}^{+a}(k)A_{j}^{-b}(-k) \rangle = \frac{1}{2}\delta^{ab}t_{ij}(k)\Omega^{-1}(k). \]  

Note, that the curvature \( \chi(k) \) \([20]\) is entirely determined by the ghost form factor \( d(k) \) and does not depend on \( \omega(k) \). The energy \([18]\) depends on \( \alpha \) and \( \omega(k) \) only through the combination \( \Omega(k) = \omega(k) - (2\alpha - 1)\chi(k) \). From this fact immediately follows, that eq. \([11]\) implies eq. \([7]\), so we find again, that the wave functional \([11]\) which minimizes the energy is independent of \( \alpha \). In fact, since \( \langle H \rangle \) \([18]\) depends on \( \omega \) and \( \alpha \) only through the combination \( \Omega = \omega - (2\alpha - 1)\chi \) it suffice to minimize the energy with respect to \( \Omega \). The resulting gap equation \(^3\) also depends only on \( \Omega \) and its solution is independent of \( \alpha \). This shows, that the infrared behaviour of the gluon propagator \( \langle A_{i}^{+a}A_{j}^{-b} \rangle \) \([22]\) is independent of the power \( \alpha \) of the Faddeev-Popov determinant assumed in the wave functional \([11]\). Therefore we are free to choose \( \alpha \) for our convenience, for example, \( \alpha = \frac{1}{2} \). This choice has the technical advantage, that \( \Omega(k) = \omega(k) \), which allows a straightforward application of Wick’s theorem in the calculation of expectation values.

In this context let us also mention, that the choice \( \alpha = \frac{1}{2} \) in eq. \([11]\) yields the wave function used by the present authors in ref. \([8]\), while the wave function used in refs. \([6, 7]\) corresponds to the choice \( \alpha = 0 \). Inspite of the different wave functions chosen in refs. \([6, 7]\) and ref. \([8]\) the same infrared behaviour of the gluon propagator should be obtained in one-loop order, as

\(^2\)The ghost and Coulomb form factor, \( d(k) \) and \( f(k) \), satisfy the Schwinger-Dyson equation derived in ref. \([8]\) with \( \omega \) replaced by \( \Omega \).

\(^3\)Although the curvature \([20]\) does not explicitly depend on \( \omega \) it depends implicitly on \( \omega \) via the ghost form factor \( d(k) \). However, also the ghost form factor \( d(k) \) depends on \( \omega \) only through \( \Omega \). This dependence is, however, a higher order effect and to one-loop order \( \frac{\delta \omega}{\delta k} \) or \( \frac{\delta \chi}{\delta k} \) can be neglected. Then the gap equation \( \frac{\delta \langle H \rangle}{\delta \chi} = 0 \) is exactly the one obtained in ref. \([8]\) with \( \omega \) replaced by \( \Omega \).
shown above, provided the same renormalization condition is used. However, while refs. \[6\], \[7\] finds an infrared finite gluon propagator, we find an infrared vanishing gluon propagator \[8\]. Two sources of the different behaviours obtained in ref. \[8\], and refs. \[6\], \[7\] come to mind: i) different choices of the renormalization condition and ii) different treatments of the curvature of orbit space. In ref. \[8\] we choose the so-called horizon condition \[14\]

\[d^{-1}(k \to 0) = 0 \quad (23)\]
as renormalization condition while refs. \[6\], \[7\] require the kernel \(\omega\) in the Gaussian wave functional to be infrared finite \(\omega(k \to 0) = \text{const.}\). Furthermore, while the curvature of orbit space \(\chi\) \[12\] was fully included in ref. \[8\], it was completely neglected in \[6\] and ignored \[5\] in the Coulomb energy in the numerical calculations of ref. \[7\]. As will be shown in the next section ignoring the curvature in the Coulomb energy will change the infrared behaviour of the wave functional. It was already observed in ref. \[8\], that the full inclusion of the curvature is vital for the infrared limit of the theory. This is consistent with the observation in Landau gauge, that in the Schwinger-Dyson equations the ghost loop is by far more important than the gluon loop \[9\].

## 5 The Yang-Mills wave functional in the infrared

With the relation \[13\] the wave functional \[11\] becomes

\[
\Psi[A^\perp] \simeq e^{\alpha \int A^\perp \chi A^\perp - \frac{1}{2} \int A^\perp \omega A^\perp}.
\]

(24)

Furthermore, the solution of the gap equation eq. \[6\] is such, that in the infrared

\[
\chi(k \to 0) = \Omega(k \to 0) \quad , \quad 2\alpha \chi(k \to 0) = \omega(k \to 0)
\]

holds. This is an extension of the relation \(\chi(k \to 0) = \omega(k \to 0)\) found in ref. \[8\] for \(\alpha = \frac{1}{2}\). With the relation \[25\] the vacuum Yang-Mills wave functional becomes in the infrared

\[
\Psi[A^\perp] = 1.
\]

(26)

In ref. \[10\] this wave functional was assumed in the infrared regime, for sake of simplicity. We have thus shown, that, to one-loop order, eq. \[26\] is the correct wave function in the infrared. The infrared wave functional \(\Psi[A^\perp] = 1\) means, that gauge fields at distant positions \(x, x', |x - x'| \to \infty\) are completely uncorrelated. Accordingly, the gluon propagator \(\langle A^\perp(x) A^\perp(x')\rangle\) has to vanish rapidly in the infrared \(|x - x'| \to \infty\), which is in agreement with the findings of ref. \[8\]. Thus, the wave functional \(\Psi[A^\perp] = 1\) describes a stochastic vacuum, in which color (correlation) cannot propagate over large distances. This is nothing but color confinement. In

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\[4\]In \(D = 2 + 1\) we find a self-consistent solution to the coupled Schwinger-Dyson equations only when we impose the horizon condition \[23\].

\[5\]In the formal part of \[7\] the curvature was fully included.
this sense, the infrared wave functional \( \mathcal{W} \) supports the picture of a stochastic Yang-Mills vacuum \( \mathcal{W} \).

Let us also stress, that in view of the relation (25) the infrared behaviour of the gluon propagator \( \Pi_{ab} \) is exclusively determined by the curvature \( \chi \) \((\mathcal{W})\) in orbit space. Furthermore from eqs. (26) and (13) follows that in the infrared limit the vacuum expectation values are described by an Gaussian ensemble

\[
\langle \cdots \rangle = \int \mathcal{D}A^\perp J[A^\perp] \cdots = \int \mathcal{D}A^\perp \cdots e^{-\int A^\perp \chi A^\perp}.
\]

Finally, let us also note that the relation (25) holds independent of the employed renormalization condition as long as the curvature \( \chi(k) \) is infrared divergent. Since the Faddeev-Popov determinant vanishes on the Gribov horizon, which contains the infrared dominant field configurations, from eq. (13) follows that the curvature has, indeed, to be infrared divergent. The condition (25) is, however, lost when the curvature is neglected in the Coulomb energy as done in ref. \([6], [7]\). Given the infrared singular behaviour of \( \chi(k \to 0) \) the condition (25) implies that for \( \alpha \neq 0 \) (in particular for \( \alpha = \frac{1}{2} \) \([8]\)) the variational kernal \( \omega(k) \) in the Gaussian ansatz \((\mathcal{W})\) has to be infrared singular, while the choice \( \alpha = 0 \) \([6], [7]\) can tolerate an infrared finite \( \omega(k) \).

6 Yang-Mills theory in \( D = 1 + 1 \)

Let us test the above result in \( 1 + 1 \) dimension, where Yang-Mills theory can be solved exactly on a torus and reduces to quantum mechanics in curved space.

Implementing the Coulomb gauge \( \partial_1 A_1 = 0 \), there is only a constant gauge field \( A_1(x_1) = \text{const.} \), which can be diagonalized in color space by exploiting the residual global gauge freedom \( U \), not fixed by \( \partial_1 A_1 = 0 \). Defining the remaining quantum mechanical degree of freedom, \( a \), by

\[
g A_1 L \equiv gLA_1^a \tau^a = U^a \frac{L}{2} \tau_3 U^\dagger,
\]

where \( L \) is spatial extension of the torus the Faddeev-Popov determinant becomes \([12]\)

\[
J(a) = \sin^2 a.
\]

The Gribov horizon occurs at \( a = n\pi \) and the fundamental modular region is obviously given by \( 0 \leq a \leq \pi \). Furthermore the Yang-Mills Hamiltonian in the variable \( a \) is given by

\[
H_{\text{kin}} = -g^2 L \frac{1}{8} \frac{d}{\sin^2 a da} \sin^2 a \frac{da}{d a}.
\]

In one spatial dimension there is no magnetic field and no dynamical gluon charge \( -\dot{A}_{ab}^a \Pi_1^a = 0 \), since the gauge field has only one (non-zero) color degree of freedom. Accordingly, the Coulomb term of the Yang-Mills Hamiltonian \([5]\) vanishes in the absence of external color
charges.

With the ansatz

\[ \Psi_k(a) = \frac{1}{\sqrt{J(a)}} \phi_k(a) = \frac{1}{\sin a} \phi_k(a), \quad (31) \]

which corresponds to the choice \( \alpha = \frac{1}{2} \) in eq. (11), the Schrödinger equation \( H \Psi_k = E_k \Psi_k \) reduces to

\[ -\frac{g^2 L}{8} \phi_k''(a) = \left( E_k + \frac{g^2 L}{8} \right) \phi_k(a), \quad (32) \]

whose solution is given by\(^6\)

\[ \phi_k(a) = \sin(ka), \quad E_k = \frac{g^2 L}{8} (k^2 - 1). \quad (33) \]

In the continuum limit \( L \to \infty \) only the vacuum state \( k = 1 \) survives \( (E_1 = 0) \), while all excited states \( k > 1 \) acquire an infinite energy and are thus frozen out. The vacuum wave function is given by (31), (33)

\[ \Psi_{k=1}(a) = 1, \quad (34) \]

which is precisely the infrared limit of the vacuum Yang-Mills wave functional in \( D = 3 + 1 \) found above (see eq. (26)). Note also that the radial wave function \( \phi_k(a) \) (33) vanishes on the Gribov horizon \( a = n\pi \) to compensate for the vanishing of the Faddeev-Popov determinant \( J(a) \) (29), just like in the \( D = 3 + 1 \) dimensional case where (for \( \alpha = \frac{1}{2} \)), the radial wave functional \( \phi[A^\perp] \) (3) vanishes in the infrared due to the infrared divergence of \( \omega(k \to 0) \).

### 7 Summary and Conclusions

We have studied the variational solution of the Yang-Mills Schrödinger equation in Coulomb gauge for a class of wave functionals (1) consisting of a Gaussian and an arbitrary power \((-\alpha)\) of the Faddeev-Popov determinant. We have found, that up to one-loop in the gap equation (i.e. two loops in the energy) the stationary solution is independent of this power \( \alpha \). The same is true for the transversal gluon propagator (22) which is exclusively determined by the self-consistent solution \( \Omega \) of the gap equation \( \frac{\delta(H)}{\delta\omega} = 0 \). This solution \( \Omega \) is independent of the choice of \( \alpha \). Different choices of \( \alpha \) will lead to different kernels \( \omega \) (with possibly different infrared behaviours) in the wave functional (3). But this will not affect the gluon propagator (22). Furthermore in the infrared the Yang-Mills vacuum wave-functional becomes field-independent describing a stochastic vacuum, in which color cannot propagate over large distances. The infrared limit of the wave functional becomes exact in \( D = 1 + 1 \).

Our investigations show, that the infrared behaviour of Yang-Mills theory in Coulomb gauge is rather robust with respect to changes in the variational ansätze for the wave functional as long as the curvature in orbit space induced by the Faddeev-Popov determinant is properly included.

\(^6\)This solution was previously found in ref. 13.
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