Measuring Solar Abundances with Seismology

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Abstract. The revision of the photospheric abundances proposed by Asplund et al has rendered opacity theory inconsistent with the seismologically determined opacity through the Sun. This highlights the need for a direct seismological measurement of solar abundances. Here we describe the technique used to measure abundances with seismology, examine our ability to detect differences between solar models using this technique, and discuss its application in the Sun.

1. Introduction

A decade ago, solar models matched the Sun remarkably well. Sound speed profiles in models differed from the Sun’s actual sound speed by less than half a percent. Then Asplund et al. (2005) reanalyzed the solar optical spectrum using improved atomic physics and a 3D hydrodynamical model of the atmosphere (instead of a 1D hydrostatic model). Their results indicate that the abundances of the heavy elements should be lower by a substantial amount. For example, carbon was lowered by 35%, nitrogen by 27.5%, oxygen by 48%, and neon by 74%. Solar models that are evolved using the new abundance mixture give worse agreement with helioseismic constraints than models using the old abundances. Problems include a tripling in the sound-speed discrepancy below the convection zone, a convection zone that is too shallow, and a helium abundance in the convection zone that is too low.

Many attempts have been made to reconcile the new abundances with seismology. A few examples include increasing the opacities below the convection zone, increasing abundances within their uncertainties, increasing the neon abundance, enhancing diffusive settling rates, and including early accretion of lower-Z material. Various combinations of these changes have also been explored. None of the alterations has resolved the discrepancy between the revised abundances and seismology. For further discussion of the problem and the attempted solutions, see Basu (2008) and Guzik et al. (2008) in these proceedings. Since the mismatch between seismology and the new abundances remains a problem, a seismic determination of the heavy element abundances in the solar convection zone...
zone is necessary. The techniques for a seismic measurement of heavy elements are discussed in the following sections.

2. Techniques

Däppen & Gough (1984) measured the solar helium abundance through the effect of helium ionization on the adiabatic exponent \( \gamma_1 \). Ionization lowers \( \gamma_1 \) from the ideal gas value of 5/3. Since the convection zone stratification is almost purely adiabatic, the gradient of the square of the sound speed in that region can be written as

\[
\frac{d c^2}{dr} = \frac{Gm}{r^2} \left\{ 1 - \gamma_1 \left[ 1 + \left( \frac{\partial \ln \gamma_1}{\partial \ln p} \right)_s \right] \right\}.
\]

(1)

This expression can be separated into a term called \( W \) that contains the seismic variables and a term called \( \Theta \) that contains only thermodynamic variables.

\[
W = \frac{r^2}{Gm} \frac{dc^2}{dr}
\]

(2)

\[
\Theta = 1 - \gamma_1 \left[ 1 + \left( \frac{\partial \ln \gamma_1}{\partial \ln p} \right)_s \right]
\]

(3)

Since \( \Theta \) contains a derivative of \( \gamma_1 \), it shows a more pronounced variation due to the ionization of each element than \( \gamma_1 \) alone shows. For the helium abundance measurement, Däppen and Gough used seismic data to obtain the sound speed, then computed \( W \) and \( \Theta \) for their model. Figure 1 shows the helium hump that Däppen and Gough discovered in \( W \) and \( \Theta \) where He II ionization takes place in the convection zone. Here \( W \) and \( \Theta \) are plotted against the acoustic radius \( \tau = \int c^{-1} dr \). The signatures of heavy-element ionization can already be seen in the small wiggles lower in the convection zone (Gough 2006). Near \( \tau = 0.63 \), we see C V ionization. From 0.45-0.55, O VII ionization appears. Near 0.58, N VI shows up as an imperceptibly small hump that is impossible to measure here. Near 0.68, there is a double hump due to the first two K-shell ionizations of Ne. The larger humps are merged ionization zones. Since the humps from heavy element ionization are such small features, great care must be taken to obtain an accurate measurement of the abundances.

2.1. Inversion

It is common to use sound speed and density or sound speed and \( \gamma_1 \) as solar inversion variables. However, the extremely small ionization humps of the heavy elements require a more sensitive inversion. We suggest inverting the frequency differences directly for \( W \) and \( \Theta \).

\[
\frac{\delta \omega^2}{\omega^2} = \int_0^R \left( K_{W,\Theta} \delta W + K_{\Theta, W} \delta \Theta \right) dr
\]

(4)

In the adiabatically stratified convection zone, \( W = \Theta \). This allows us to rewrite the integral as

\[
\frac{\delta \omega^2}{\omega^2} = \int_0^R \left( K_{W, \Theta} + K_{\Theta, W} \right) \delta \Theta \ dr.
\]

(5)
Figure 1. \( W \) and \( \Theta \) throughout the convection zone in model S of Christensen-Dalsgaard et al. (1996). The large hump around \( \tau = 0.82 \) is due to helium ionization. The smaller wiggles lower in the convection zone are due to heavy element ionization.

Since this formulation has the added advantage of reducing the problem to a single-variable inversion, we no longer need to worry about contamination from a second inversion variable.

2.2. Isothermal Sound Speed

The conversion from the familiar sound speed and density kernels (see, for example, Gough & Thompson 1991) to the desired \( W \) and \( \Theta \) kernels requires quite a bit of algebra. One possible trick to simplify the calculation is to reconfigure \( W \) and \( \Theta \) in terms of the square of the isothermal sound speed \( u = p/\rho \). The gradient of \( u \) in an adiabatically stratified region is

\[
\frac{du}{dr} = \frac{Gm}{r^2} \left( \frac{1}{\gamma_1} - 1 \right).
\] (6)

This leads to a set of definitions analogous to equations (2) and (3) for the separation of seismic and thermodynamic variables:

\[
\tilde{W} = \frac{r^2}{Gm} \frac{du}{dr},
\] (7)

\[
\tilde{\Theta} = \frac{1}{\gamma_1} - 1.
\] (8)
The derivative of $\gamma_1$ in the definition of $\Theta$ does not appear in the new $\tilde{\Theta}$. This greatly simplifies the kernel conversion. However, without the $\gamma_1$ derivative, $\tilde{\Theta}$ is less sensitive to the effects of ionization than $\Theta$ is. As shown in figure 2, $\tilde{\Theta}$ does not have visible heavy-element ionization humps. Since the effect of ionization is so small to begin with, reducing it even further by switching to $\tilde{\Theta}$ is not worth the gain in algebraic simplicity in the kernel conversion. The $\gamma_1$ derivative is essential to highlight the effect of heavy-element ionization. We therefore choose to use the original definitions of $W$ and $\Theta$.

![Figure 2](image.png)

Figure 2. $\Theta$ and $\tilde{\Theta}$ throughout the convection zone in model S of Christensen-Dalsgaard et al. (1996). The heavy-element ionization humps are clearly visible in $\Theta$, yet they do not appear in $\tilde{\Theta}$.

### 2.3. Pressure Ionization

One source of uncertainty in solar equations of state is the effect of pressure ionization. Pressure ionization occurs when matter is so dense that electron orbits overlap. The electrons in the overlapping orbits no longer belong to a particular atomic nucleus. The pressure effectively frees these electrons, increasing the ionization of the plasma. Baturin et al. (2000) developed a technique to account for pressure ionization in an equation of state by applying constraints to the effective atomic and ionic sizes. They used a single pressure-ionization parameter $\zeta$ for all species of ions. They then implemented a model in which they calibrated $\zeta$ to fit solar data. The calibrated pressure-ionization model fit the data better than the parameter-free equations of state, highlighting the importance of pressure ionization for an accurate equation of state.
Changing the effective atomic size in the pressure-ionization model leads to a vertical shift in $\gamma_1$. Since $\gamma_1$ appears as a factor in $\Theta$, $\Theta$ also suffers from this shift due to the atomic size parameterization. Therefore, we cannot just measure the average value of $\Theta$ to obtain the heavy-element abundances. In order to obtain an accurate seismological measurement, we must consider the effective atomic size by using a pressure-ionization model for the equation of state or measure the humps relative to the background level of $\Theta$.

3. Conclusions

In order to obtain a seismic measurement of the heavy-element abundances in the Sun the following techniques should be used: the frequency data should be inverted directly for $W$ or $\Theta$ instead of sound speed, the original definitions for $W$ and $\Theta$ should be used instead of the $\tilde{W}$ and $\tilde{\Theta}$ derived from the isothermal sound speed formulation, and the effect of atomic size calibration on the background level of $\Theta$ should be taken into account. These choices are crucial for accurately measuring the small effect of heavy-element ionization in order to determine solar abundances seismically.

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