Polydisperse bubble flows: a study of the bubble cloud evolution by CFD methods

A S Chernyshev and A A Schmidt
Computational Physics Laboratory, Ioffe Institute, Politekhicheskaya 26, Saint Petersburg, 194021, Russia

E-mail: alexander.tchernyshev@mail.ioffe.ru

Abstract. The evolution of a polydisperse bubble cloud is analyzed by means of computational fluid dynamics. Gravity-driven bubble flows in a column with a circular cross-section as well as a downward flow in a long vertical pipe are numerically investigated. A method employed in the simulations is based on the Euler-Euler description of the multiphase medium. An inhomogeneous multiple-size group (MUSIG) model is used to describe the bubble size distribution. The formulated approach provides detailed description of the bubble flow structure including variation of the bubble size distribution.

1. Introduction
The study of multiphase bubbly flows is of great importance due to their frequent appearance in nature and industrial applications. Bubble clouds, or plumes, are groups of a large number of bubbles with relatively small sizes compared to the scale of the problem (examples are cloud cavitation under pressure drop, aeration, etc.) [1-2]. Investigations by experimental methods are not always possible due to multiscale and multiphysical nature of such flows. Numerical methods can serve as a robust and effective choice in that situation.

Direct numerical simulation with interphase boundary resolution is resource-demanding and not possible for real scale problems. Investigations of the multiphase media can be carried out in the framework of two frequently used approaches: the Euler-Lagrange [3] and the Euler-Euler [4]. The carrier liquid is treated in the framework of the Euler-Lagrange approach as continuum, while bubbles are described as a set of dispersed particles. This approach can be resource demanding at a high concentration of particles. Both carrier and dispersed phases are treated in the Euler-Euler approach as continuous ones, therefore same continual equations are applied for all phases. This approach is suitable for almost any concentration of dispersed particles and is used in the paper presented.

Polydispersity plays a significant role in the formation of the flow regime and its structure and is a characteristic feature of a multiphase bubbly media [5]. Correct description of the bubble distribution
by bubble sizes over time and space can significantly improve predictive features of the mathematical
model applied and provide additional information about the flow structure.

Simulations of polydisperse bubble clouds in different flow domains are presented in the paper. Two bubble flow types are described: airlift flow in a vertical bubble column with a circular cross-
section and downward flow in a long vertical cylindrical pipe. The evolution of the bubble size
distribution occurs due to the impact of interphase forces, which leads to the modification of the flow
pattern. Distinctive bubble plume shapes are obtained in each case, with bubble redistribution and
plume smearing across the column in the airlift flow and bubble localization close to the pipe axis for
the downward flow.

2. Mathematical model. The Euler-Euler description

The study was based on the Euler-Euler approach to the description of multiphase flows, which was
used in the development of the model described in [6]. Within this approach, the concept of the phase
volume fraction $\alpha$ is introduced, densities of each continuum are calculated as $\alpha \cdot \rho$, where $\rho$ is the
material density of the corresponding phase (sub-index $l$ corresponds to the carrier fluid, $b$ is for
bubbles). The proposed model takes into account the compressibility of the medium, turbulence and
polydispersity of bubbles and the interphase momentum transfer.

The description of polydispersity is carried out within the framework of a multi-class model of the
bubble size distribution with different velocities per each class of bubbles (inhomogeneous MUSIG
model, [5]). $M$ classes of bubbles are introduced, which corresponds to a piecewise size distribution.
For each class of bubbles $i$ with a radius $R_{ib}$, the corresponding system of conservation equations of
mass and momentum is written with additional terms describing the interphase momentum exchange.
The initial bubble size distribution is selected on the basis of experimental data.

Taking into account that the total void fraction of all phases is equal to 1, it can be written:

$$\alpha_l = 1 - \sum_{i=1}^{M} \alpha_{ib}. \quad (1)$$

Here $\alpha_i$ – the void fraction of the liquid phase, $\alpha_{ib}$ – the void fraction of bubbles of class $i$.

The Navier-Stokes equations for the dispersed and carrier phases will have the following form:

$$\frac{\partial \alpha_{ib} \rho_{ib}^0}{\partial t} + \text{div} \left( \alpha_{ib} \rho_{ib}^0 \vec{V}_{ib} \right) = D_{ib},$$

$$\frac{\partial \alpha_{ib} \rho_{ib}^0 \vec{V}_{ib}}{\partial t} + \text{div} \left( \alpha_{ib} \rho_{ib}^0 \vec{V}_{ib} \vec{V}_{ib} + p \vec{E} \right) = S_{ib},$$

$$\frac{\partial \alpha_i \rho_i^0}{\partial t} + \text{div} \left( \alpha_i \rho_i^0 \vec{V}_i \right) = 0,$$

$$\frac{\partial \alpha_i \rho_i^0 \vec{V}_i}{\partial t} + \text{div} \left( \alpha_i \rho_i^0 \vec{V}_i \vec{V}_i + p \vec{E} - \vec{\tau}_i \right) = S_i. \quad (2)$$

In this set of equations: $\vec{V}$ – the velocity vector of the carrier or dispersed phases, $p$ – pressure, $\vec{E}$ –
the unity tensor, $S$ – source terms responsible for the interphase momentum exchange, $D$ – source
terms responsible for the diffusion due to turbulence, $\vec{\tau}$ – the strain rate tensor.
\[ \tau_{kl} = \mu_{\text{eff}} \left( \frac{\partial v_k}{\partial x_l} + \frac{\partial v_l}{\partial x_k} \right), \mu_{\text{eff}} = \mu_{\text{lam}} + \mu_{\text{turb}}. \] (3)

Here \( \mu_{\text{eff}} \) is an effective dynamic viscosity coefficient of the carrier phase which is equal to the material viscosity plus turbulent one.

This system should be accomplished by conservation equations of the number density per each class since the bubble radius is not constant and can change with the variation of the bubble gas density:

\[ \frac{\partial N_{ib}}{\partial t} + \text{div} \left( N_{ib} \vec{V}_{ib} \right) = D_i N. \] (4)

The bubble size distribution at inlet boundaries can be described by either the Log-Normal or the Rosin-Rammler distributions:

- LN: \[ \frac{1}{R \sigma \sqrt{2\pi}} \cdot \exp \left( - \left( \ln (R) - \mu \right)^2 \cdot 0.5 \sigma^{-2} \right), \]
- RR: \[ \frac{k}{\lambda} \left( \frac{R}{\lambda} \right)^{k-1} \cdot \exp \left( - \left( R/\lambda \right)^k \right), R \geq 0. \] (5)

Turbulence is taken into account using the widespread and proven k-\( \omega \) SST Menter model [7], supplemented by members responsible for the generation and dissipation of turbulence due to the relative motion of bubbles and fluid [8]:

\[ S_{ibk} = \frac{3}{8} C_{id} \alpha_{ib} \rho_l |\vec{V}_{irel}|^3, S_{ib\omega} = 0.8 \frac{k}{\mu_{\text{turb}}} S_{ibk}, \vec{V}_{rel} = \vec{V}_i - \vec{V}_{ib}. \] (6)

Here \( k \) is the kinetic energy of turbulence, \( C_{id} \) – drag coefficient.

Interphase momentum exchange term includes the buoyancy, Stokesian drag, Saffman lift, virtual mass and wall lubrication forces, which can be written in the form:

\[ S_{ib} = \vec{F}_{ib} + \vec{F}_{id} + \vec{F}_{il} + \vec{F}_{iw} + \vec{F}_{iVM}, S_i = - \sum_{i=1}^{M} \left( \vec{F}_{ib} + \vec{F}_{il} + \vec{F}_{iw} + \vec{F}_{iVM} \right), \] (7)

\[ \vec{F}_{ib} = \alpha_{ib} \left( \rho_{ib} - \rho_l \right) \cdot \vec{g}, \] (8)

\[ \vec{F}_{id} = \frac{3 \rho_l}{8 R_{ib}} C_{id} \vec{V}_{irel} |\vec{V}_{irel}|, \vec{V}_{irel} = \vec{V}_i - \vec{V}_{ib}, \] (9)

\[ \vec{F}_{il} = C_{il} \alpha_{ib} \rho_l \vec{V}_{irel} \times \text{rot} \vec{V}_i, \] (10)

\[ \vec{F}_{iVM} = 0.5 \alpha_{ib} \rho_l \left( \frac{D_b \vec{V}_{ib}}{Dt} - \frac{D_l \vec{V}_i}{Dt} \right), \] (11)

\[ \vec{F}_{iw} = - C_{iw} \alpha_{ib} \rho_l |\vec{V}_{irel} - (\vec{V}_{irel} \cdot \vec{n}_W) \cdot \vec{n}_W|^2 \cdot \vec{n}_W. \] (12)

Expressions for \( C_{id}, C_{il}, C_{iw} \) were taken from [9], [10] and [11], respectively.

In addition to taking into account the effect of bubbles on the intensity of turbulence in the carrier medium, it is necessary to take into account the effect of turbulence on the motion of bubbles due to
the intensification of diffusion transport mechanisms. An additional diffusion term is included in the equations for the volume fraction and number density of bubbles, which is responsible for the dispersion of bubbles [12]:

\[
D_{\text{ia}} = \frac{1}{Sc} \nabla \left( \frac{\mu_{\text{eff}}}{\rho_l} \nabla \alpha_{ib} \right), \quad D_{\text{IN}} = \frac{1}{Sc} \nabla \left( \frac{\mu_{\text{eff}}}{\rho_l} \nabla N_{ib} \right),
\]

(13)

Here Sc is the Schmidt number.

3. Numerical method

Based on the mathematical model described above a modeling algorithm and its software implementation have been developed. The algorithm is based on the finite volume method and unstructured meshes. The spatial variable approximation has the second order of accuracy and is implemented using upwind schemes that satisfy the TVD criterion. The SIMPLE algorithm with corrections taking into account the presence of the second phase is used to resolve pressure-velocity coupling. The model and the code were tested on the available experimental data [6].

4. Results

All numerical experiments are carried out at normal conditions with ambient pressure is equal to 101350 Pa and temperature is equal to 297 K. Selection of the number of classes M per each case is based on two criteria: (a) detailed description of the flow and (b) minimization of computational time. A normalized bubble volume fraction is introduced to analyze the evolution of the bubble size distribution inside a cloud:

\[
\alpha_{ib}^{\text{norm}} = \frac{\alpha_{ib}}{\sum_{j=1}^{M} \alpha_{jb}},
\]

(14)

4.1. Axisymmetric air-lift reactor

The first case is an axisymmetric air-lift flow in a circular bubble column with diameter \( D = 0.143 \) m and height \( h = 0.65 \) m. The column is filled with water. Gas in the form of polydisperse bubbles enters the column through a coaxial axisymmetric aerator of constant diameter \( d = 0.1 \) m mounted into the bottom of the column. The bubble volume fraction at the inlet is 0.37%, superficial gas velocity is \( V_G = 0.53 \) mm/s. Top boundary is assumed as a free surface which lets bubbles leave the domain. The buoyancy force causes bubbles to move towards the top boundary which in turn accelerates fluid in the column. The initial size distribution of bubbles is the log-normal one with bubble radius \( R_{ib} \) varies from 0.1 to 0.45 mm and the most probable bubble radius is equal to 0.25 mm [13]. 7 classes of bubbles are introduced.
The evolution of the volume fraction per each bubble class along the axis of symmetry is presented in figure 1. Volume fraction decays towards the free surface due to acceleration of the liquid and bubbles and impact of lateral forces. Smaller bubbles are more susceptible to the impact of interphase forces because of their low inertia, which helps to achieve the equilibrium with carried liquid faster. That leads to the stabilization of the bubble volume fraction and reaching the plateau at the outlet.

Radial distributions of the volume fraction are shown in figure 2 and 3. It can be seen from comparison of distributions at 50 mm above the aerator (figure 2) and in the middle of the column (figure 3) that smallest bubbles of $R_{ib}=0.1$ mm have reached the uniform distribution across the column at the half of the column height. Some decay close to the wall is due to the impact of the wall lubrication force, which pushes bubbles towards the axis of symmetry, opposite to the lift force.

4.2. Downward flow in an axisymmetric long pipe
Simulations of the downward bubbly flow inside a vertical axisymmetric pipe were carried out according to conditions mentioned in [14]. Water with immersed air bubbles is supplied to the top end of the straight vertical pipe with diameter $D=25.4$ mm and length $L=2$ m. At the inlet, the superficial liquid velocity $V_L=1.25$ m/s, superficial gas velocity $V_G=0.087$ m/s and gas void fraction $\alpha=0.067$. The Rosin-Rammler distribution was used with 5 classes of bubbles with $R_{ib}$ variation from 0.5 to 1 mm and the most probable bubbles are at $R_{ib}=0.75$ mm.
Figure 4. Bubble volume fraction distribution along the axis of the symmetry of the pipe from the inlet at the top to the outlet at the bottom

Bubble volume fraction along the axis of the pipe is presented in figure 4. Slight drop of the volume fraction can be seen at the beginning of the pipe for all bubble classes. That can be explained by the velocity profile formation of the carrier phase and acceleration of the liquid velocity in the middle of the pipe. Just after that volume concentration starts to grow due to the lateral motion of the bubbles driven by drag and lift forces. Slow decay towards the outlet can be expressed by the impact of turbulent dispersion which became more pronounced with the flow development.

Figure 5. Bubble volume fraction distribution along the pipe radius close to the outlet location (solid lines). Dashed lines – distribution of the same classes at the inlet. Same colour key for class numbers.

Radial distribution at the outlet (figure 5) looks as dome-shaped per each class with the low bubble concentration zone close to the wall. In that case both lift and wall lubrication forces are co-directional towards the axis. Smaller bubbles have wider distribution with the less inclined slope at the axis which can be addressed to the more intense smearing of bubbles due to the turbulence. That effect can be seen from the distribution of normalized volume fraction on figure 6. Each band on the graph corresponds to the specific class, the width of the band is equal to the normalized fraction at each point on the radius. At the axis of the symmetry classes 1 and 2 bubbles are occupying ~55% of total bubble volume fraction, while close to the wall this value increases up to 70%.

Figure 6. Normalized bubble volume fraction distribution along the pipe radius close to the outlet location. Width of the each band represents the fraction of the occupied volume of the corresponding class.

5. Conclusion

Simulations of the gravity-driven polydisperse bubbly flow in the cylindrical bubble column and the downward polydisperse bubbly flow in the long vertical pipe have been carried out. The impact of lateral forces resulted in smearing of the bubble plume in the case of bubble column and localization of bubbles close to the pipe center in the case with downward flow.
It was shown that smallest bubbles have reached the uniform distribution across the column at the half of the column height. Still, bubbles of all classes fill the entire column at the middle cross-section.

The non-monotonic behavior of the volume fraction evolution was captured for the downward bubbly flow. Such behavior can be addressed to the carrier phase flow field development at the initial stage (drop of the volume fraction close to the inlet) with the subsequent increase of the fraction due to drag and lateral forces. Intensification of turbulence leads to smearing of the profile and decrease of the volume fraction towards the outlet. Smaller bubbles have more flattened profile with increased relative concentration towards pipe walls.

References

[1] Kitanin E L, Kumzerova E Yu, Chernyshev A S and Schmidt A A 2007 *Tech. Phys. Lett.* **33**(8) 704–7
[2] Talvy S, Debaste F, Martinelli L, Chauveheid E and Haut B 2011 *Chem. Eng. Sci.* **66** 3185–94
[3] Lain S, Broder D and Sommerfeld M 1999 *Chem. Eng. Sci.* **54** 4913-20
[4] Bannari R, Kerdouss F, Selma B, Bannari A and Proulx P 2008 *Comp. and Chem. Eng.* **32** 3224–37
[5] Krepper E, Lucas D, Frank T, Prasser H-M and Zwart P J 2008 *Nucl. Eng. and Design* **238** 1690–702
[6] Chernyshev A S and Schmidt A A 2016 *J. of Phys.: Conf. Series* **754**(3), 032005
[7] Menter F R, Kuntz M and Langtry R 2003 *Turbulence, Heat and Mass Transfer* **4** 625–32
[8] Troshko A A and Hassan Y A 2001 *Int. J. Mult. Flow* **27** 1965-2000
[9] Lain S, Sommerfeld M, Broder D and Goz M 2002 *Int. J. Multipl. Flow* **28** 1381–407
[10] Tomiyama A, Tamai H, Zun I and Hosokawa S 2002 *Chem. Eng. Sci.* **57** 1849–58
[11] Frank Th, Zwart P J, Krepper E, Prasser H-M and Lucas D 2008 *Nucl. Eng. and Design* **238** 647-59
[12] Sokolichin A and Eigenberger G 1999 *Chem. Eng. Sci.* **54** 2273-84
[13] Lain S, Broder D and Sommerfeld M 1999 *Chem. Eng. Sci.* **54** 4913-20
[14] Ishii M, Paranjape S, Kim S and Sun X 2004 *Int. J. of Mult. Flow* **30** 779–801