Flavor changing neutral current in rare top quark decay $t \rightarrow c\ell\bar{\ell}$ in generic $Z'$ and left-right symmetric models

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Abstract. We study the rare top decay $t \rightarrow c\ell\bar{\ell}$, which involves flavor violation, as a possible probe of new physics. This decay is analyzed with the simplest Standard Model (SM) extensions with additional gauge symmetry $U(1)$ or $SU(2)$, known as generic $Z'$ model or left-right symmetric model, respectively. The considered models allow to obtain this decay at tree level through Flavor Changing Neutral Currents (FCNC). We have considered diagonal non-universal couplings for up-type quark sector and found that $\text{BR}(t \rightarrow c\ell\bar{\ell})$ for $1 \text{ TeV} < M_{Z'} < 3 \text{ TeV}$ can be of order $\sim 10^{-11}$ in generic $Z'$ model or $\sim 10^{-12}$ in left-right symmetric model.

1. Introduction
Processes with Flavor Changing Neutral Current (FCNC) give an important signal for physics beyond Standard Model (SM). One of them is the top quark goes to charm quark and a lepton-antilepton pair, $t \rightarrow c\ell\bar{\ell}$. Rare top decay is important not only as a test of FCNC but also as a means to identify an extension of the SM. FCNC are absent in the SM at the tree-level due to the Glashow-Iliopoulos-Maiani (GIM) mechanism. However, new FCNC states can appear in top decays if there is physics beyond the SM. In this context, rare top quark decays are interesting because they might be a source of possible new physics effects [1]. In some particular models beyond the SM, rare top decays may be significantly enhanced to reach detectable levels [2]. In the Large Hadron Collider (LHC) or future International Linear Collider (ILC) an eventual signal of FCNC in the top quark decay will have to be ascribed to new physics.
One of the puzzle of the SM has to do with the understanding of the origin of parity violation in low-energy weak interaction processes. Within the framework of left-right symmetric models, based on the gauge group $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$, this problem finds a natural answer [3].

No study is complete without including the generic $Z'$ model based on the gauge group $SU(2)_L \otimes U(1)_Y \otimes U(1)$ which is defined to have identical couplings as for usual $Z$ SM boson [4]. In this paper we consider both extensions and the non universal family couplings in order to induce the FCNC for the rare top decay [5], [6].

This paper is organized as follows. In Sec. 2 we introduce the family non-universal couplings and define some notation. In Sec 3 we analyze the rare top decay and in Sec. 4 we discuss and summarize our result.

2. FCNC and family non-universal couplings

The generic $Z'$ and left-right symmetric models introduce two neutral gauge bosons, one of them comes from SM gauge group and the other one comes from extra gauge group. If there is no mixing between the neutral gauge bosons $Z_1$ and $Z_2$, then the neutral current terms can be written as

$$\mathcal{L}_{NC} = -e J^\mu_{EM} A_\mu - g J^\mu_i Z_{1\mu} - g J^\mu_2 Z_{2\mu},$$

(1)

where $Z_{1\mu}$ is the usual electroweak neutral gauge boson, $Z_{2\mu}$ is the neutral gauge boson associated with extra gauge symmetry and $g_{1,2}$ are the respective gauge couplings. The current associated with the additional gauge symmetry is

$$J_{2\mu} = \sum_{i,j} \bar{\psi}_i^{0j} \gamma_\mu \left[ \epsilon_{\psi Li j} P_L + \epsilon_{\psi Ri j} P_R \right] \psi_j^0,$$

(2)

where $i, j$ running over all quarks and leptons, $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$ and $\epsilon_{\psi L,R}$ are chiral couplings between fermions and neutral gauge boson $Z_{2\mu}$. The superscript zero denotes the interaction basis.

We shall assume that couplings of the $Z_2$ with leptons and quarks are diagonal family non-universal. The simplest case is obtained when only the left-handed top quark coupling is diagonal family non-universal [5]. Under last assumption the up quark sector chiral couplings are written as $\epsilon_{uRij} = Q^u_R \delta_{ij}$ and

$$\epsilon_{uL} = Q^u_L \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & x \end{pmatrix}.$$

(3)

The $Q^u_L$ and $Q^u_R$ are chiral couplings of the up-type quarks in the additional gauge symmetry and $x$ is a parameter which can be of order 1 but not equal to 1. In generic $Z'$ model the chiral couplings are the same as the SM $Z$ boson. The chiral couplings in LRSM are obtained when neutral gauge bosons are written in mass basis through the orthogonal transformation

$$\begin{pmatrix} W^i_{L3} \\ W^i_{R3} \\ B \end{pmatrix} = \begin{pmatrix} -s_W & c_W & 0 \\ -s_W & -s_W & \epsilon_W \sqrt{\frac{c_W^2 - s_W^2}{c_W}} \\ \epsilon_W \sqrt{\frac{c_W^2 - s_W^2}{c_W}} & \epsilon_W \sqrt{\frac{c_W^2 - s_W^2}{c_W}} & \frac{c_W}{s_W} \end{pmatrix} \begin{pmatrix} A \\ Z_1 \\ Z_2 \end{pmatrix},$$

(4)

where $s_W = \sin \theta_W$, $c_W = \cos \theta_W$ and $\theta_W$ is the Weinberg angle. Therefore, the chiral couplings in LRSM are given by

$$Q^i_L = -\sqrt{\frac{3}{5}} \left( \frac{1}{2\alpha} \right) (B - L)_i,$$

(5)

and

$$Q^i_R = \sqrt{\frac{3}{5}} \left( \alpha T^i_{3R} - \frac{1}{2\alpha} (B - L)_i \right),$$

(6)
### Table 1. Numerical values of the chiral couplings.

| Chiral coupling | $U(1)$ | $SU(2)_R$ |
|-----------------|--------|-----------|
| $Q^u_L$         | 0.3456 | -0.0849   |
| $Q^u_R$         | -1.544 | 0.5038    |
| $Q^d_L$         | -0.4228 | -0.0849 |
| $Q^d_R$         | 0.0772 | -0.6736   |
| $Q^e_L$         | -0.2684 | 0.2548   |
| $Q^e_R$         | 0.2316 | -0.3339   |
| $Q^\nu_L$       | 0.5    | 0.2548    |

where $B$ and $L$ denote the baryon and lepton numbers of fermion $i$, respectively. $T_{3R}$ is the third component of its right-handed isospin in the $SU(2)_R$ group and the parameter $\alpha = \sqrt{(1 - 2s_W^2)/s_W^2}$. Numerical values of the chiral couplings for generic $Z'$ and left-right symmetric models are shown in table 1. Therefore the neutral current associated with extra neutral gauge boson and up-type quarks in the mass basis is

\[
\mathcal{L}^Z_{NC} = -gZ^\mu (\pi, \tau, \tau^c) \gamma^\mu \left[ B^u_L P_L + B^u_R P_R \right] \left( \begin{array}{c} u \\ c \\ t \end{array} \right),
\]

where $B^u_L = V_L \epsilon u_L V_L^\dagger$ and $B^u_R = \epsilon u_R$. The assumption of the down quark sector has no mixing and the unitary conditions of CKM matrix give

\[
B^u_L \approx Q^L \left( \begin{array}{ccc} 1 & (1 - x) V_{ub} V_{cb}^* & (x - 1) V_{ub} V_{tb}^* \\ (x - 1) V_{cb} V_{ub}^* & 1 & (x - 1) V_{cb} V_{tb}^* \\ (x - 1) V_{tb} V_{ub}^* & (x - 1) V_{tb} V_{cb}^* & x \end{array} \right)
\]

(8)

If the $Z_2$ couplings are diagonal but family non-universal, FCNC are induced by fermion mixing.

### 3. Rare top decay

We now proceed to calculate the branching ratio for the rare top decay $t \rightarrow cll$ using the formalism introduced in the previous section. The diagram for this decay are shown in figure 1.
Table 2. Numerical values of the integrals for each charged lepton.

| Lepton   | $I_1$    | $I_2$    | $I_3$    | $I_4$    | $I_5$    |
|----------|----------|----------|----------|----------|----------|
| Tau      | 0.083207 | 0.08327  | 0.33240  | 0.16647  | 0.49901  |
| Muon     | 0.083314 | 0.083314 | 0.33332  | 0.16663  | 0.49999  |
| Electron | 0.083314 | 0.083314 | 0.33333  | 0.16663  | 0.5      |

If we consider that $q^2 < < M_2^{Z'}$, then the average of amplitude and width decay can be written as

$$|\mathcal{M}_{t \rightarrow cl}|^2 = \frac{4g^2}{M_2^{Z'}} [x_1(q_1 \cdot k_1)(q_2 \cdot k_2) + x_2(q_1 \cdot k_2)(q_2 \cdot k_1) + x_3m_t^2(q_1 \cdot q_2) - x_4m_cm_c(k_1 \cdot k_2) - m_tm_cm_t^2x_5],$$

and

$$\Gamma_{t \rightarrow cl} = \frac{g^2m_t^5}{256\pi^3M_2^{Z'}} \left(x_1I_1 + x_2I_2 + \frac{2m_l^2}{m_t^2}x_3I_3 - \frac{2m_e}{m_t}x_4I_4 - \frac{8m_cm_e^2}{m_t^2}x_5I_5\right),$$

respectively, where $I_i$, $i = 1, ..., 5$, are integrals obtained in the kinematic development and their numerical values are shown in table 2. The parameters $x_{1,2,3,4,5}$ are defined as

$$x_1 = 2 \left(|(B_L^L)_{23}|^2 \left|(B_R^U)_{33}\right|^2 + |(B_R^U)_{23}|^2 \left|(B_R^L)_{33}\right|^2 \right),$$

$$x_2 = 2 \left(|(B_L^L)_{23}|^2 \left|(B_R^U)_{33}\right|^2 + |(B_R^L)_{23}|^2 \left|(B_R^L)_{33}\right|^2 \right),$$

$$x_3 = \text{Re} \left[(B_L^L)_{33}(B_R^L)_{23}\right] \left(|(B_L^L)_{23}|^2 + |(B_R^U)_{23}|^2 \right),$$

$$x_4 = \text{Re} \left[(B_L^L)_{33}(B_R^L)_{23}\right] \left(|(B_L^L)_{23}|^2 + |(B_R^R)_{23}|^2 \right),$$

$$x_5 = \text{Re} \left[(B_L^L)_{33}(B_R^L)_{23}(B_L^U)_{33} + \text{Re} \left[(B_L^L)_{23}(B_R^L)_{33}(B_R^L)_{23}\right] \left|(B_R^R)_{33}\right|^2 \right].$$

Therefore, the branching ratio as function of the model parameters and $Z'$ mass is

$$BR(t \rightarrow cl^+l^-) = \frac{g^2m_t^5}{128\pi^3M_2^{Z'}\Gamma_{top}} |Q_L^{2L}Q_U^{2U}|V_{tb}|V_{cb}|^2 I_1(x - 1)^2 + \frac{m_c}{m_t} \text{Re} \left(V_{tb}V_{cb}\right) I_4Q_R^{2L}Q_L^{2L}(Q_R^{2L} + Q_R^{2L})(x - 1).$$

The SM parameters used in these calculations are $m_t = 172.9$ GeV, $m_c = 1.29$ GeV, $V_{tb} = 0.99$, $V_{cb} = 0.041$, $\Gamma_{top} = 1.2$ GeV and $\sin\theta_W = 0.2316$ [7]. The figures 2, 3, 4 and 5 show the behavior of the branching ratio of the rare top decay $t \rightarrow cl$ in both models.

4. Conclusion
The family non-universal couplings is important in order to induce the FCNC from gauge bosons in extra gauge groups models. The branching ratio of the rare top decay $t \rightarrow cl$ can be of the order of $10^{-12}$ for $1$ TeV $\leq m_{Z'} \leq 3$ TeV and $0.1 \leq x \leq 0.9$ in the framework of $Z'$ models that we have discussed. Note that the branching ratio in LRSM is approximately 10 times smaller than the branching ratio in generic $Z'$ model. It is found that rare top decay is extremely suppressed due to the $Z'$ boson mass and the family non-universal parameter $x$. 


Figure 2. Branching ratio as function of the $Z'$ mass in generic $Z'$ model.

Figure 3. Branching ratio as function of the family non-universal parameter $x$ in generic $Z'$ model.

Figure 4. Branching ratio as function of the $Z'$ mass in LRSM.

Figure 5. Branching ratio as function of the family non-universal parameter $x$ in LRSM.

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