Measuring $|V_{ts}|$ directly using strange-quark tagging at the LHC

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Abstract
The Cabibbo–Kobayashi–Maskawa (CKM) element $V_{ts}$, representing the coupling between the top and strange quarks, is currently best determined through fits based on the unitarity of the CKM matrix, and measured indirectly through box-diagram oscillations, and loop-mediated rare decays of the $B$ or $K$ mesons. It has been previously proposed to use the tree level decay of the $t$ quark to the $s$ quark to determine $|V_{ts}|$ at the LHC, which has become a top factory. In this paper, we extend the proposal by performing a detailed analysis of measuring $t \to sW$ in dileptonic $tt$ events. In particular, we perform detector response simulation, including the reconstruction of $K^0_S$, which are used for tagging jets produced by $s$ quarks against the dominant $t \to bW$ decay. We show that it should be possible to exclude $|V_{ts}| = 0$ at $5.5\sigma$ with the expected High Luminosity LHC luminosity of 3000 fb$^{-1}$, considering only the statistical uncertainties, and not the systematic uncertainties which will play a role in setting the final analysis limits.

Keywords CKM matrix · $S$-jet tagging · Top quark

1 Introduction
The Cabibbo–Kobayashi–Maskawa (CKM) matrix [1] gives the strength of the cross-generational weak couplings between the up and down type quarks, and is currently the only known source of charge-parity violation in the Standard Model (SM), which is required for understanding the observed matter–anti-matter asymmetry in the universe [2]. Measurements of the CKM matrix aim to overconstrain the matrix, testing the unitarity assumption. Non-unitarity would indicate the existence of an additional, as yet unknown, coupling from Beyond the Standard Model (BSM) physics. The CKM matrix element $V_{ts}$ determines the relative strength of the $t$ quark’s weak decay to the $s$ quark compared to other down-type quarks. The magnitude of $|V_{ts}|$ determined through fits based on the unitarity of the CKM matrix is $39.78^{+0.82}_{-0.60} \times 10^{-3}$. It can be measured indirectly through box diagram oscillations or rare decays involving loops. The $B \to \bar{B}$ oscillations were first measured at ARGUS [3] and the mass difference is now well measured in $B_d$ of $\Delta M_d = (0.5065 \pm 0.0019)$ ps$^{-1}$ [4] and $B_s$ of $\Delta M_s = (17.749 \pm 0.020)$ ps$^{-1}$, which is the world average dominated by an LHCb measurement [5]. Using inputs from QCD lattice results [6], the value for $|V_{ts}|$ from these results is $38.8 \pm 1.1 \times 10^{-3}$ [7]. Also, a recent study of the theoretical prediction of $\Delta M_s$ using Heavy Quark Effective Theory sum rules gives a theoretical value of $|V_{ts}| = 40.91^{+0.67}_{-0.64} \times 10^{-3}$ [8]. However, a reanalysis of Tevatron and 8 TeV LHC data has shown that after relaxing the unitarity constraints, $|V_{ts}|$ can be as large as 0.1 [9]. Additionally, a recent CMS analysis of 13 TeV data with the Single Top channel has given the constraint $|V_{ts}| + |V_{td}| < 0.057$ at the 95% CL under the SM assumption of CKM unitarity, and $|V_{ts}| + |V_{td}| = 0.06 \pm 0.06$ after relaxing the unitarity constraints and allowing BSM contributions to the top width [10]. Further measurements are, therefore, required to constrain $|V_{ts}|$ in the most general scenario, and in particular, the decay of $t \to sW$ has not yet been observed.

A direct measurement of $|V_{ts}|^2 = \frac{B(t \to sW)}{B(t \to qW)}$ at the LHC using the properties of strange hadrons to tag $s$ jets was proposed in [11]. That proposal made a generator level analysis...
to argue that the measurement would be feasible at the LHC, showing that under the assumption of perfect non-\(\bar{t}t\) background rejection and perfect top and hadron reconstruction, \(10 \text{ fb}^{-1}\) is sufficient to observe the \(t \rightarrow sW\) decay. Now that the LHC has collected more than an order of magnitude more luminosity than the proposal considered, we extend that study. In particular, we perform a full reconstruction analysis using the delphes fast detector simulation package, to make a more realistic estimate of the data luminosity required to observe the decay, and analyze the difficulties that would arise in such a measurement. With our more realistic simulation setup, we investigate the prospects of measuring \(|V_{ub}|\) in several scenarios, including the future High Luminosity LHC (HL-LHC) [12].

2 Simulation settings

We used \textsc{mg5\_amc@nlo} 2.6.5 to generate \(\bar{t}t\) events in the dilepton decay channel with up to 2 additional jets (\(tt\), \(tt + j\), \(\bar{t}t + j\)) at next to leading order [13]. We use the next-to-next-to-leading order top pair production cross-section \(\sigma(pp \rightarrow \bar{t}t) = 831.76 \text{ pb}\) for a collision energy of 13 TeV, which was calculated using the \textsc{top++} program [14]. We generated about 10 million signal \(tt\) events where one of the \(t\) quarks is forced to decay to a \(s\) quark and about 10 million background \(\bar{t}t\) events are generated where both \(t\) quarks decay to \(b\) quarks. Drell–Yan events with 2 additional partons and Single Top events are the dominant non-\(\bar{t}t\) backgrounds for dilepton \(\bar{t}t\) events [15]. We generated 20 million Drell–Yan plus two parton events for each of the 4 jet flavour categories: \(bb\), \(cc\), \(ss\) and \(qq\) (\(q = u, d, g\)). We use the leading order cross-section reported by \textsc{mg5\_amc@nlo} for each of the categories, these are: \(bb = 42.9\) pb, \(cc = 4.31\) pb, \(ss = 4.37\) pb and \(qq = 23.8\) pb. We have also generated 30 million Single Top events (\(t + j\) and \(t + W\)) and use the leading order cross-section from \textsc{mg5\_amc@nlo}, which is 241 pb. In addition to the non-\(\bar{t}t\) backgrounds, we generated 10 million \(\bar{t}t\) plus one \(s\) quark and two \(s\) quarks, respectively, with the leading order cross-section of 9.41 pb and 1.57 pb.

\textsc{pythia8} 8.240 was used to simulate parton showering and hadronization [16, 17] with the FxFx merging scheme [18] and CP5 tuning parameters [19, 20]. We modified the \(K^0_S\) decay in \textsc{pythia8} to allow the \(K^0_S\) to decay inside a fiducial volume of a cylinder centered at the proton collision point with a radius of 860 mm and a length of 4400 mm. This is equivalent to the region of the CMS tracking detector where the pions from the decay may still pass through three silicon detectors, and would allow us to reconstruct \(K^0_S\) using reconstructed tracks when it decays to a charged pion pair.

We used \textsc{delphes} 3.4.2 to simulate the response of a CMS-like detector with particle flow (PF) outputs [21]. For jet clustering, we use the anti-\(k_T\) algorithm with jet radius \(R = 0.4\) using FastJet 3.3.2 [22]. We used the default CMS card included in \textsc{delphes} but updated it to match the CMS setup used of Run 2. The jet radius was decreased from 0.5 to 0.4. The \(\Delta R\) cone used to calculate lepton isolation was reduced from 0.5 to 0.3 for electrons and 0.5–0.4 for muons. The track transverse momentum resolution formula was updated using the function given in [23]. The \(b\)-tagging efficiency was updated to closely match the Run 2 response of CMS [24] and a \(b\)-tagging based on track counting module was added. Smearing of the track impact parameter in the transverse plane was also added to emulate a more realistic \(K^0_S\) reconstruction using the associated module and parameters provided with \textsc{delphes} to replicate the performance of the CMS tracker (Fig. 1) [23].

3 Event selection

In this study, we use dilepton events in order to remove the additional jet activity from the \(W\) decay, which additionally suppresses the background contribution from other processes, especially the multi-jet QCD background. The \(tt\) event selection criteria are based on the CMS measurement of the top pair cross-section with the dilepton channel [25]. First, we select events with two isolated leptons satisfying \(I_{\text{rel}} < 0.0588(0.0571)\) for an electron in the barrel (endcap) region and \(I_{\text{rel}} < 0.15\) for a muon, each of which has \(p_T > 25(20)\) GeV for a leading (sub-leading) lepton and pseudorapidity \(|\eta| < 2.4\) and, in addition, the electron in 1.44 < \(|\eta| < 1.57\) is excluded. Then an invariant mass of a lepton pair is more than 20 GeV. After reconstructing the pair, we veto \(Z\) boson production by excluding events in the dilepton invariant mass range of \(|M_Z - M_{\ell\ell}| < 15\) GeV, where \(M_Z = 91.1876\) GeV [7]. We require events to have missing energy \(\not{E}_T > 40\) GeV and at least two reconstructed jets with \(p_T > 30\) GeV and \(|\eta| < 2.4\).

We define primary jets as reconstructed jets which are matched to a generator level quark \(q\) from the \(t \rightarrow qW\) decay by finding the highest \(p_T\) jet within \(\Delta R < 0.4\) of the quark. 97% of top pair production events have one primary jet, while 74% of events have two primary jets, and therefore fully match the dilepton decay topology after reconstruction. These primary jets will be used to train two Boosted Decision Trees (BDT). The Toolkit for Multivariate Data Analysis in ROOT (TMVA) is used to train the BDT using the adaptive boosting algorithm [26]. The first BDT is trained to select the two primary jets out of all the reconstructed jets in the events. The second BDT is trained to discriminate between \(s\)-quark initiated jets from other jets. Once the first BDT selects the two primary jets, the second BDT is applied on these primary jets to look for \(t \rightarrow sW\) decay. This process is described in further detail.
below. Both BDTs are trained using the signal $\bar{t}t \rightarrow sWbW$ and background $\bar{t}t \rightarrow bWbW$ samples.

Additional jets in $tt$ events create ambiguities in the selection of the primary jets, so we employ the first BDT to improve the efficiency of the primary jet selection. We constructed a BDT model with the inputs of two jet and two lepton four vectors, the missing transverse momentum, and the $\Delta R$ between the two jets. Each jet pair in the event is evaluated by the BDT. We use the signal $\bar{t}t$ sample to train this BDT, defining the signal to be the jet pair made from the two primary jets and the background is when one or more of the jets are not the primary jet. The output of the first BDT is shown in Fig. 2a. For each event, the jet pair with the highest BDT output is selected as the $\bar{t}t$ primary jet candidates. Figure 3c shows the top jet pair selection output for the signal, $\bar{t}t$, and Drell–Yan backgrounds. Figure 4 shows the distribution of the most discriminating variables that are input into the BDT.

Next, to distinguish $s$ jets from the background, predominantly $b$ jets from the dominant $\bar{t}t \rightarrow bWbW$, we reconstruct $K_S^0$ candidates inside the primary jet candidates. In $s$ jets, $K_S^0$ can be produced directly from the initiating $s$ quark, whereas in $b$ jets they will be produced after a cascade of decays of the $b$ hadron or from the quarks produced in the parton shower. This means that $K_S^0$ should be harder (relative to the jet energy) and more collimated in the case of $s$ quark initiated jets. We reconstruct $K_S^0$ using its decay into oppositely charged pion pairs, and due to the long lifetime of the $K_S^0$, we require the tracks to come from a displaced vertex within the tracker volume. Using the charged hadron objects from the delphes particle flow reconstruction, we consider all oppositely charged hadron pairs. Since general purpose detectors like CMS do not distinguish between pions from other charged hadrons, we assume all charged hadrons to be pions. We require the charged hadron pair to have $p_T > 0.95$ GeV and $|\eta| < 2.4$ and the significance of the transverse

Fig. 1 Plots for the efficiency (left) and the occupancy (right) of reconstructed $K_S^0$ in the tracker volume. From the occupancy plot, about 74% of total reconstructed $K_S^0$ is within $R = 400$ mm and $|z| = 1000$ mm. The detectable area in the plot means the region where the particles can leave at least 3 hits in the CMS tracking detector

Fig. 2 BDT output distribution of signal (blue) and background (red) on the BDT for the primary jet pair selection (a) and $s$ jet tagging (b)
impact parameter of each track to be greater than two, to ensure the tracks are not from the primary vertex. Then, we select reconstructed $K^0_S$ candidates with an invariant mass of $M_{K^0_S} < 0.1$, where $M_{K^0_S} = 497.611$ MeV [7]. We check that the reconstructed $K^0_S$ candidate is from a primary jet candidate by requiring the angle between the candidate momentum and the jet axis to satisfy $\Delta R < 0.4$. If there are more than one reconstructed $K^0_S$ candidates, we select the $K^0_S$ with the highest $p_T$.

After matching the $K^0_S$ candidates to the primary jet candidates, we use both hadron and jet information to discriminate $s$ jets from all other jets. First, from the jet information, we use jet’s $p_T$, mass, its minor and major axes and their substructure-related quantities such as charged jet multiplicity, charged daughter’s $p_T$ fraction in a jet, lepton constituent’s $p_T$ fraction in a jet $x = \sum p_T^{lep} / p_T^{jet}$, and $p_T D$. Also called the jet energy sharing, defined as $\sqrt{\sum p_T^{lep}} / \sum p_T$. To simulate $b$-tagging, we used a simple track counting method, which tags a jet as a $b$ if more than two tracks are found with a high impact parameter. From the $K^0_S$ kinematics, we use the hadron’s $p_T$ fraction relative to the jet $x = p_T^{had} / p_T^{jet}$, the $\Delta R$ between the jet axis and the hadron momentum, the Distance of Closest Approach (DCA) between the two tracks, the cosine of the 2D pointing angle between momentum vector and position vector of hadron and decay length calculated by assuming the $K^0_S$ vertex is the midpoint of DCA between the two charged tracks. In addition, the following information from both the charged pion daughters are included: the pT fraction compared to the $K^0_S$, the significance of the transverse impact parameter, and of the longitudinal impact parameter. For training the second BDT, the signal is defined as jets matched to an $s$-quark from the $t \rightarrow sW$ decay with a

Fig. 3 BDT output distribution of jet pair selection for signal and background events normalized (a) and scaled to an integrated luminosity of 137.6 fb$^{-1}$ (c) and BDT output of $s$ jet tagging on the primary jets normalized (b) and scaled to an integrated luminosity of 137.6 fb$^{-1}$ (d)
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Matching $K^0_S$ whose momentum fraction, $x > 0.15$ and the background is all other jets with $K^0_S$ with $x > 0.15$. Figure 5 shows the distribution of the most discriminating variables that are input into the BDT. The result from the s-jet discriminating BDT training is shown in Figs. 2b and 3d shows the s-jet discriminating BDT for the primary jets on the signal and background processes.

For the final s jet selection, we first reject primary jets that are $b$ tagged in each event. If both the primary jets are $b$ tagged, the event is vetoed from the analysis. Of the remaining primary jets in each event, we select the jet with the highest s tagging BDT output. Figure 3d shows the final s tagging BDT output for signal and background events.
4 Results

The expected number of signal $\bar{t}t$ and background $\bar{t}t \to bWbW$ events is given by

$$N_{\text{sig}} = \sigma(\bar{t}t) \times \mathcal{L} \times B(\bar{t}t \to q\ell^+ \bar{q}\ell^- \nu) \times 2|V_{ts}|^2|V_{tb}|^2 \times \epsilon_{\text{sig}}$$

(1)

$$N_{\text{bkg}} = \sigma(\bar{t}t) \times \mathcal{L} \times B(\bar{t}t \to q\ell^+ \bar{q}\ell^- \nu) \times |V_{tb}|^4 \times \epsilon_{\text{bkg}} + N_{\text{DY}} + N_{\text{ST}}$$

(2)

where $\sigma(\bar{t}t)$ is the cross-section, $\mathcal{L}$ is a integrated luminosity, $B(\bar{t}t \to q\ell^+ \bar{q}\ell^- \nu)$ is the branching ratio of dileptonic decay mode, $\epsilon_{\text{sig}}$ ($\epsilon_{\text{bkg}}$) is the selection efficiency after the selections described in the previous section for the signal
sWbW (background bWbW) \(\bar{t}t\) sample, and \(N_{DY} \) (\(N_{ST}\)) is the expected number of Drell–Yan (Single Top) background events.

To study the feasibility of a direct measurement of \(|V_{ts}|\), we perform an analysis to find the expected significance to reject the hypothesis \(H_0\) of no \(t \rightarrow sW\) decays, the expected upper limit on \(|V_{ts}|\) and also an expected confidence interval \(|V_{ts}|\). We use RooFit [27] and RooStats [28] to perform the statistical analysis using the RooStats asymptotic calculator based on the asymptotic properties of likelihood function [29]. We fit the \(s\) jet tagging BDT distributions from Fig. 3d to obtain approximations of the probability density functions (PDFs) for further study. The PDFs for the signal and background are separately modeled by a sum of four Gaussian distributions and the fitted PDFs are shown in Fig. 6. The BDT distribution in this figure has a different definition of signal and background from the BDT distribution of jet discriminator. The BDT distribution shown in the previous section (Fig. 2b) defines the signal (background) as jets matched to generator level \(s\) (\(b\)) quarks, while the distribution here defines the signal as the jet with the highest BDT output in an event from \(\bar{t}t \rightarrow sWbW\) signal events and the background as the jet with the highest output from \(\bar{t}t \rightarrow bWbW\), Drell–Yan and Single Top events among the jet pair selected through the primary jet pair selection. The total model is thus

\[
P_{s+b} = \mu \times N_{\text{sig}} \times P_{\text{sig}} + \nu \times N_{\text{bkg}} \times P_{\text{bkg}}
\]

where \(N_{\text{sig}}\) is the number of \(\bar{t}t \rightarrow sWbW\) events expected by the SM, corrected for the selection efficiency and \(\mu\) is the signal strength relative to the SM \(|V_{ts}|\), \(|V_{ts}^{\text{SM}}| = 39.78 \times 10^{-3}\) [7], and \(\nu\) is a nuisance parameter to control the background level relative to the expected background. From the model PDF, we generate an Asimov dataset which we use as the observed dataset for the following studies.

Figure 7 and 8 show the results of one-sided and two-sided scanning using the gaussian fitting for several integrated luminosities. The left plot in Fig. 7 shows a median expected local \(p_0\) under assumption of \(H_0\) (\(|V_{ts}| = 0\)) versus the true signal strength \(\mu\). For \(\mu = 1\), corresponding to the current SM expectation, the significance of rejecting \(H_0\) is expected to be more than 4\(\sigma\) when the integrated line is on \(\mu = 1\). Black dots in the right plot are \(\mu\) corresponding to CL of 95% at 137 (Run 2), 300 (expected for Run3), 600, 1200, 2000 and 3000 (expected for HL-HLC) fb\(^{-1}\).
The expected significance to exclude $|V_{ts}| = 0$, the expected 95% CL median upper limit under the assumption of $|V_{ts}| = 0$ and the expected 95% CL$_{s+b}$ median confidence interval on $\mu = \frac{|V_{ts}|}{|V_{ts}|_{	ext{exp}}^2}$ for several integrated luminosities ($\mathcal{L}$) and the corresponding expected number of the signal ($N_{s|0}$) and the background ($N_{b|0}$).

| $\mathcal{L}$ (fb$^{-1}$) | Expected significance ($\alpha$) for $|V_{ts}| = 0$ exclusion | Expected 95% CL median upper limit ($\mu$) | Expected 95% CL$_{s+b}$ median interval ($\mu$) | ($N_{s|0}$, $N_{b|0}$) |
|-------------------------|---------------------------------|---------------------------------|---------------------------------|-----------------|
| 137.6                   | 1.2                             | $< 1.41$                        | $[0.000, 2.45]$                 | (1076, 321,118) |
| 300                     | 1.7                             | $< 0.957$                       | $[0.000, 2.11]$                 | (2344, 699,908) |
| 600                     | 2.4                             | $< 0.677$                       | $[0.196, 1.81]$                 | (4689, 1,399,817) |
| 1200                    | 3.5                             | $< 0.479$                       | $[0.431, 1.57]$                 | (9377, 2,799,633) |
| 2000                    | 4.5                             | $< 0.371$                       | $[0.559, 1.44]$                 | (15,629, 4,666,056) |
| 3000                    | 5.5                             | $< 0.305$                       | $[0.640, 1.36]$                 | (23,443, 6,999,083) |

Table 1 summarizes several results shown above: the expected significance to exclude $|V_{ts}| = 0$, the median 95% CL upper limit if $\mu = 0$ and the 95% CL for each luminosity. Under the assumption of $\mu = 1$, the expected significance from the hypothesis test calculation shows a 1.2$\sigma$ significance to reject the background-only hypothesis for the integrated luminosity of the Run 2. The value becomes 1.7$\sigma$ when the integrated luminosity is 300 fb$^{-1}$.
as is expected to be collected during Run 3 of the LHC, and $5.5 \sigma$ for the full HL-LHC integrated luminosity of 3000 fb$^{-1}$. Conversely, if the decay is suppressed, and $\mu = 0$, then Run 3 of the LHC will be able to exclude $\mu = 1$ at the 95% CL level.

For this study, we have only considered the statistical uncertainties and have not included any systematics, which will certainly impact the final exclusion limits. For instance, the jet fragmentation model and QCD radiation settings (ISR/FSR) could affect the jet shape and $K_S^0$ variables which are among the most powerful separation variables in the $s$-tagging BDT. Jet energy uncertainties will also affect the $p_T$ ratios used in the BDT and will be an important systematic uncertainty which the full analysis will need to consider.

5 Conclusion

We have studied the feasibility of a direct measurement of $|V_{ts}|$ from the dileptonic $t\bar{t}$ production process, using hadronization by PYTHIA8 and DELPHES detector simulation to produce a more realistic expectation of the results of the LHC experiments. With an integrated luminosity of 3000 fb$^{-1}$, which is expected to be achieved at the HL-LHC period, $|V_{ts}| = 0$ can be excluded above the 5$\sigma$ significance level, considering only the statistical uncertainties, and not the systematic uncertainties which will play a role in setting the final analysis limits.

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