Composite scattering from complex conductor-medium assembly object above a rough surface

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Abstract. To solve the scattering from complex object combining conductor and medium, this paper proposes a hybrid iterated method combining single integral equation (SIE) and Kirchhoff approximation (KA). Firstly, SIE is used to calculate the electromagnetic current in the facet of the conductor-medium assembly object. Then, the electric and magnetic currents on the facet of a rough surface are calculated by KA. The electromagnetic current on the facet of object and a rough surface are updated by mutual iteration. Finally, some examples for composite scattering from object above a rough surface are analysed and discussed in detailed.

1. Introduction
Composite scattering from object above a rough surface has been attracted the interest of many researchers in recent years. It has been widely applied in object detection and remote sensing [1, 2]. In the study of composite scattering from object above a rough surface, most of the articles are about the perfect electric conductive object above a rough surface or medium object above a rough surface [3, 4], and some papers only study the conductor-medium assembly object regardless of the rough surface [5-7]. To solve the scattering from conductor-medium assembly object, the coupled integral equation (CIE) are mostly used with the surface of the conductor analyzed by electric field integral equation (EFIE), magnetic integral field equation (MFIE) or the combined field integral equation (CFIE), while the medium surface analyzed by EFIE [8] or Poggio-Miller-Chang-Harrington-Wu-Tsai (PMCHWT) [9]. However, with the coupled integral equation, the equivalent electric and magnetic currents are needed to be computed, which is very time-consuming. To reduce the computation complexity, the single integral equation (SIE) [10, 11] is proposed by substituting the equivalent electric and magnetic currents with the effective current inner the medium, which reduce the unknowns by half. However, there are rare articles about scattering from conductor-medium object above a rough surface, which is widely used in the practical application.

In this paper, a hybrid method combining the single integral equation (SIE) and the Kirchhoff approximation (KA) is proposed for calculating the conductor-medium assembly object above a rough surface. Firstly, the composite model from conductor-medium assembly object above a rough surface is constructed. Next, the electric and magnetic currents on object are calculated by SIE and the electric and magnetic current on the rough surface are calculated by KA. The electric and magnetic currents on the facets of object and a rough surface are updated by mutual iteration. Finally, the total scattering field is obtained.

2. Composite model for conductor-medium assembly object above a rough surface
The geometry of conductor-medium assembly object above a rough surface is shown in figure 1. The object is the combination of conductor and medium, the surface of the conductor is signed as $S_1$; the surface of the medium is signed as $S_2$; the interface of the conductors and the medium is $S_3$; the surface of the rough surface is $S_4$. The permittivity of the medium part of the object is $\varepsilon_r$; the magnetic conductivity of the medium part of the object is $\mu_r$; the permittivity of the free space is $\varepsilon$; the magnetic conductivity of the free space is $\mu$. $k_i$ and $k_s$ are the incident electric field direction and scattering electric field direction, respectively.

![Figure 1. Geometry of composite scattering from conductor-medium assembly object above a rough surface.](image)

In composite model, both the object and the rough surface are discrete by RWG function. The surface of the conductive part of the object is divided into $N_1$ faces. The surface of the medium part of the object is divided into $N_2$ faces. The interface between the conductive part and the medium part is divided into $N_3$ faces. The electric current on the facet of conductive part of objects $\eta_0 J_{e_1}(r)$, the electric and magnetic current on the facet of medium part of object are $\eta_0 J_{e_2}(r)$ and $J_{m_2}(r)$, respectively. The electric and magnetic current on the facet of the rough surface are $\eta_0 J_{e_4}(r)$ and $\eta_0 J_{m_4}(r)$, respectively.

\[
\eta_0 J_{e_1}(r) = \sum_{j=1}^{N_1} x_{1j} f_{1j}(r), \quad r \in S_1
\]

\[
\eta_0 J_{e_2}(r) = \sum_{j=1}^{N_2} c_{2j} f_{2j}(r), \quad r \in S_2
\]

\[-\eta_0 J_{m_2}(r) = \sum_{j=1}^{N_3} d_{2j} f_{2j}(r), \quad r \in S_2\]

\[-\eta_0 J_{e_4}(r) = \sum_{j=1}^{N_4} a_{4j} f_{4j}(r), \quad r \in S_3\]

\[
\eta_0 J_{m_4}(r) = \sum_{j=1}^{N_4} b_{4j} f_{4j}(r), \quad r \in S_5
\]
\[ \eta_0 J_{se}(r) = \sum_{j=1}^{N} k_j f_j(r), \quad r \in S_S \]  
(6)

Where, \( J_{se} \) denotes the equivalent electric current on the facet of the conductive part of the object, \( J_{2e} \) denotes the electric current on the facet of the medium part of the object, \( J_{2m} \) denotes the magnetic current on the facet of medium part of the object, \( J_{an} \) denotes the electric current on the facet of the interference between the conduct and the medium. \( J_{sm} \) denotes the equivalent current on the facet of a rough surface \( S_S \), \( J_{sw} \) denotes the equivalent magnetic current on the facet of surface \( S_S \), \( f_{1j}, f_{2j}, f_{3j}, f_{sj} \) denote the RWG function on the facet of \( S_1, S_2, S_4 \) and \( S_s \), respectively.

The scattering in the free space is generated by the electromagnetic current on the facet of the object, as well as the rough surface:

\[ E' = -\eta_0 L_{o1}(J_{se}) - \eta_0 L_{o2}(J_{2e}) + K_{o2}(J_{2m}) - \eta_0 L_{o3}(J_{sm}) + K_{o3}(J_{sw}) \]  
(7)

Where, \( L \) and \( K \) denotes the electric and magnetic field integral operator, respectively. The first subscript denotes the integration domain in free space or medium area, the second subscript denotes that integration on which faces. The specific definition of \( L \) and \( K \) are:

\[ L_{0q}(X) = jk \int_{S_q} [G_0 \cdot X(r') + \frac{1}{k} \nabla G_0 \cdot \nabla' \cdot X(r')] dS' \]
\[ K_{0q}(X) = \int_{S_q} [X(r') \times \nabla G_0(r,r')] dS' \]  
(q = 1, 2)  
(8)

Where, \( G_0 = e^{-jkr}/4\pi R \) is the green function in the free space, \( \eta_0 = \sqrt{\mu_0/\varepsilon_0} \) is the wave impedance in free space.

2.1. Coupled integral equations for scattering from conductor-medium assembly object

As we see, the geometry of conductor-medium assembly object is shown in figure 2. The medium of object is isotropic media.

![Figure 2. Geometry of 3D arbitrary shape conductor-medium assembly object.](image)

The electric field integral equation on the surface \( S_1 \) and \( S_2 \) can be expressed as:
\[ S_1: \mathbf{n}_1 \times \mathbf{E}' = \mathbf{n}_1 \times \mathbf{L}_{e1}(\eta_1 \mathbf{J}_{1w}) + \mathbf{n}_2 \times \mathbf{L}_{e2}(\eta_2 \mathbf{J}_{2e}) - \mathbf{n}_1 \times \tilde{K}_{i0}(\mathbf{J}_{2m}) \]  
\[ S_2: \mathbf{n}_2 \times \mathbf{E}' = \mathbf{n}_2 \times \mathbf{L}_{e1}(\eta_1 \mathbf{J}_{1w}) + \mathbf{n}_2 \times \mathbf{L}_{e2}(\eta_2 \mathbf{J}_{2e}) - \frac{1}{2} \mathbf{J}_{2m} - \mathbf{n}_2 \times \tilde{K}_{i0}(\mathbf{J}_{2m}) \]

Where, \( \tilde{K} \) is the principal value except the singular integral at point \( r' = r \). The field in the region of medium is expressed by the electric current \( \mathbf{J}_{cd} \) on the interface of conductor-medium and the electromagnetic current \( -\mathbf{J}_{2e} \) and \( -\mathbf{J}_{2m} \) in the medium. Thus, the inner field in the medium can be expressed as:

\[ \mathbf{E}' = -\eta_1 \mathbf{L}_{e3}(\mathbf{J}_{cd}) - \eta_1 \mathbf{L}_{i1}(\mathbf{J}_{2e}) + K_{is}(\mathbf{J}_{2m}) \]  

Where, \( \eta_1 = \sqrt{\mu_1 / \varepsilon_1}, \ k_1 = \omega \sqrt{\mu_1 \varepsilon_1} \); when the observation point is internally close to the object surface, the electric field integral equation on the inner surface of \( S_i \) and \( S_j \) is:

\[ S_2: 0 = \mathbf{n}_2 \times \mathbf{L}_{e3}(\eta_1 \mathbf{J}_{1w}) + \frac{\eta_1}{\eta_0} \mathbf{n}_2 \times \mathbf{L}_{e2}(\eta_2 \mathbf{J}_{2e}) + \frac{1}{2} \mathbf{J}_{2m} + \mathbf{n}_2 \times \tilde{K}_{i12}(\mathbf{J}_{2m}) \]  
\[ S_3: 0 = \mathbf{n}_1 \times \mathbf{L}_{e3}(\eta_1 \mathbf{J}_{1w}) + \frac{\eta_1}{\eta_0} \mathbf{n}_1 \times \mathbf{L}_{e2}(\eta_2 \mathbf{J}_{2e}) + \mathbf{n}_1 \times \tilde{K}_{i12}(\mathbf{J}_{2m}) \]

By substituting the equation (1)-(4) into equations (9), (10), (12) and (13), and the \( n \times \text{RWG} \) function \( g_{i1} = \mathbf{n}_1 \times f_{i1} \) and \( g_{i1} = \mathbf{n}_2 \times f_{i2} \) are used to equations (9) and (10), respectively. The function \( g_{i2} = \mathbf{n}_2 \times f_{i2} \) and \( g_{i3} = \mathbf{n}_3 \times f_{i3} \) are used to test equations (12) and (13). Finally, the matrix equation is obtained:

\[
\begin{bmatrix}
Q_{11} & Q_{12} & P_{12} & 0 & 0 \\
Q_{21} & Q_{22} & U_{22} + P_{22} & 0 & 0 \\
0 & Q_{32} & -U_{32} + P_{32} & Q_{33} & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
x_1 \\
c \\
x_3 \\
\end{bmatrix}
= \begin{bmatrix}
b_1 \\
c \\
x_3 \\
\end{bmatrix}
\]

Where

\[ Q_{pq}(i, j) = \int_{T_p} f_{p(i)}(r) \cdot L_{qj}(f_{qj}) dS, p = 1, 2; q = 1, 2 \]  
\[ P_{pq}(i, j) = \int_{T_p} f_{p(i)}(r) \cdot \tilde{K}_{qj}(f_{qj}) dS, \quad p = 1, 2; q = 2 \]  
\[ U_{pq}(i, j) = \frac{1}{2} \int_{T_p} g_{p(i)}(r) \cdot f_{qj}(r) dS, \quad p = 2; q = 2 \]  
\[ b_p(i) = \int_{T_p} f_{p(i)}(r) \cdot \mathbf{E}'(r) dS, \quad p = 1, 2 \]  
\[ \tilde{Q}_{pq}(i, j) = \int_{T_p} f_{p(i)}(r) \cdot \mathbf{L}_{qj}(f_{qj}) dS, \quad p = 2, 3 \]  
\[ \tilde{Q}_{pq}(i, j) = \eta_1 \int_{T_p} f_{p(i)}(r) \cdot \mathbf{L}_{i1}(f_{qj}) dS, \quad p = 2, 3 \]  
\[ \tilde{P}_{pq}(i, j) = \int_{T_p} f_{p(i)}(r) \cdot \tilde{K}_{i1}(f_{qj}) dS, \quad p = 2, 3 \]

Where, \( \{x_i\} \) denotes expansion coefficient of the electric current \( \mathbf{J}_{ew} \) on the surface \( S_i \), \( \{c\} \) denotes expansion coefficient of electric current on the surface \( S_i \), \( \{d\} \) denotes the expansion coefficient of
magnetic current \( J_{2m} \) on the surface \( S_3 \), \( \{x_j\} \) denotes the expansion coefficient of electric current \( J_{2e} \) on the surface \( S_1 \).

2.2. Single integral equations for scattering from conductor-dielectric assembly object

To reduce the unknowns on the surface of conductor-dielectric assembly object, the effective electricity \( J_{2e}^e \) is introduced to expressed the field inner the dielectric part instead of the electric \( J_{2e} \) and the magnetic \( J_{2m} \), as well as the electric \( J_{ed} \). Thus, the electric and magnetic field can be expressed as

\[
E_x = -\eta J_{2e}^e (J_{2e}^e) \quad H_z = -K_{1d} (J_{2e}^e)
\]

(16)

where, the operator \( L_{1d} \) and \( K_{1d} \) work inner the whole surface of the dielectric part \( S_d = S_2 + S_3 \).

The equivalent electromagnetic current on the surface of dielectric part can be expressed by the effective electric \( J_{2e}^e \) and magnetic \( J_{2m} \). Thus, the electric and magnetic field can be expressed as

\[
J_{2e} = n_2 \times H_z = -\frac{1}{2} J_{2e}^e - n_2 \times K_{1d} (J_{2e}^e)
\]

(17)

\[
-J_{2m} = n_2 \times E_z = -n_2 \times \eta L_{1d} (J_{2e}^e)
\]

(18)

On the conductor-medium connection surface \( S_1 \), the tangential electric field is zero, that is

\[
0 = n_3 \times E_z = -n_3 \times \eta L_{1d} (J_{2e}^e)
\]

(19)

The effective electric currents can be approximated with RWG basic function

\[
-\eta J_{2e}^e (r) = \sum_{j=1}^{N_2} x_j f_j (r) + \sum_{j=1}^{N_2} x_j f_j (r)
\]

(20)

\[
x_p = \frac{1}{I_{pp}^{1d}} \int (\hat{l}_p \times n_p) \cdot (-\eta J_{2e}^e) dl, \quad p = 2, 3
\]

(21)

where, \( (\hat{l}_p \times n_p) \) denotes the electric direction on the edge that is from \( T_{2i}^+ \) to \( T_{2j}^- \). The expansion coefficient \( \{c\} \) and \( \{d\} \) can be denoted as the average value of the equivalent electric current and magnetic current flowing through the edge, and the equations (17) and (18) can be expresses as the matrix form.

\[
\{c\} = \left( \frac{1}{2} [I_{22}] + [P_{22}] \right) \{x_j\} + \{\tilde{P}_{23}\} \{x_j\}
\]

(22)

\[
\{d\} = \left[Q_{22}\right] \{x_j\} + \{\tilde{Q}_{23}\} \{x_j\}
\]

(23)

where, \( [I_{22}] \) is the unit matrix with the dimension of \( N_2 \times N_2 \), the other matrix elements are

\[
P_{2i}(i,j) = -\frac{1}{I_{2i}^{1d}} \int \hat{l}_{2i} \cdot \tilde{K}_{1d} (f_j) dl, \quad i = 1, 2, \ldots, N_2; \quad j = 1, 2, \ldots, N_q
\]

(24)

\[
Q_{2i}(i,j) = -\frac{1}{\eta_0} \int \hat{\eta} \tilde{L}_{1d} (f_j) dl, \quad i = 1, 2, \ldots, N_2; \quad j = 1, 2, \ldots, N_q
\]

(25)

Testing the equation (19) with \( g_q = n_q \times f_q \), we can obtain that
\[
\{0\} = [\hat{Q}_{i1}][x_1] + [\hat{Q}_{i2}][x_2]
\]
(26)

\[
\hat{Q}_{ni}(i, j) = \int_{S_1} f_n(r) \frac{\eta_n}{\eta_0} L_n(f_s) dS, \ i = 1, 2, \cdots, N_i; \ j = 1, 2, \cdots, N_n
\]
(27)

By substituting the equations (22) and (23) into equations (9) and (10), and along with the equation (26), we can obtain that

\[
\begin{bmatrix}
A_{i1} & A_{i2} & A_{i3} \\
A_{i2} & A_{i2} & A_{i3} \\
0 & A_{i2} & A_{i3}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
= \begin{bmatrix}
h_1 \\
h_2 \\
0
\end{bmatrix}
\]
(28)

Where

\[
[A_{i1}] = [\hat{Q}_{i1}]
\]
\[
[A_{i2}] = [\hat{Q}_{i1}] \cdot \left( \frac{1}{2} [U_{22}] + [\hat{P}_{22}] \right) + [P_{12}] \cdot [\hat{Q}_{22}]
\]
\[
[A_{i3}] = [\hat{Q}_{i1}] \cdot [P_{23}] + [P_{12}] \cdot [\hat{Q}_{23}]
\]
\[
[A_{i2}] = [\hat{Q}_{31}]
\]
\[
[A_{i2}] = [\hat{Q}_{21}] \cdot \left( \frac{1}{2} [U_{22}] + [\hat{P}_{22}] \right) + ([U_{22}] + [P_{22}]) \cdot [\hat{Q}_{22}]
\]
\[
[A_{i3}] = [\hat{Q}_{31} \cdot [\hat{P}_{23}] + ([U_{22}] + [P_{22}]) \cdot [\hat{Q}_{23}]
\]
\[
[A_{i2}] = [\hat{Q}_{33}]
\]
\[
[A_{i3}] = [\hat{Q}_{33}]
\]
(29)

2.3. Scattering from a rough surface
The Kirchhoff approximation (KA) is used to compute the electrical large rough surface. With KA, the rough surface is supposed to consist of numerous little patches, and the mutual interaction inner the rough surface and the edge diffraction as along as the shape point diffraction are neglected, which greatly increase the computational speed. For the isotropic rough surface, the correlation length \( l_c \), the root mean square \( h_{rms} \) should satisfy the condition that:

\[
k_i l_c > 6.1 \frac{l_c^2}{2} > 2.76 h_{rms} \lambda, \quad \frac{l_c^2}{2 \sqrt{3} h_{rms}} \left( 1 + \frac{2 h_{rms}^2}{l_c^2} \right) > \lambda
\]
(30)

Where, \( k_i = \frac{2 \pi}{\lambda} \sqrt{\varepsilon_r} \). Supposing that \( \hat{n} \) is the unit normal vector in the rough surface, its local orthogonal system is \( (\hat{h}, \hat{v}, \hat{k}) \), the horizontal and vertical polarization vector are defined as:

\[
\hat{h} = \hat{n} \times \hat{k}, \quad \hat{v} = \hat{n} \times \hat{h}
\]

The local incident angle \( \theta_i \) is expressed as:

\[
\cos \theta_i = -\hat{n} \cdot \hat{k}_{inc}
\]
(31)

And the local reflection coefficient \( R_v \) and \( R_h \) on the rough surface are expressed as:

\[
R_v(\theta_i) = \frac{\varepsilon_r \cos \theta_i - \sqrt{\varepsilon_r - \sin^2 \theta_i}}{\varepsilon_r \cos \theta_i + \sqrt{\varepsilon_r - \sin^2 \theta_i}} \quad R_h(\theta_i) = \frac{\cos \theta_i - \sqrt{\varepsilon_r - \sin^2 \theta_i}}{\cos \theta_i + \sqrt{\varepsilon_r - \sin^2 \theta_i}}
\]
(32)

Thus, the induced currents on the rough surface can be expressed as:
\[ J = \hat{n} \times \mathbf{H} = \frac{1}{\eta_0} \left\{ (\hat{e}_i \cdot \hat{h})(\hat{n} \cdot \hat{k}_i)(1 - R_H)\hat{h} + (\hat{e}_i \cdot \hat{v})(\hat{n} \times \hat{h})(1 + R_V) \right\} E_0 p(x, y)e^{-jkr}, \]

\[ M = E \times \hat{n} = \left\{ (\hat{e}_i \cdot \hat{h})(\hat{n} \times \hat{h})(1 + R_V) + (\hat{e}_i \cdot \hat{v})(\hat{n} \cdot \hat{k}_i)(1 - R_H)\hat{h} \right\} E_0 p(x, y)e^{-jkr}, \]

Where, the impedance matrix in free space is \( \eta_0 = \sqrt{\mu_0 / \varepsilon_0} = 120\pi \), the incident electric field is \( E^i(r) \hat{e}_i E_0 p(x, y)e^{-jkr} \), \( H^i(r) \hat{e}_i E_0 p(x, y) \) is the incident wave modulation function. When the incident wave is plane wave: \( p(x, y) = 1 \).

### 2.4. Interaction between object and a rough surface

The interaction process between object and a rough surface at \( n\)-th step can be expressed as:

\[ E^{s(n)}_s(r) = E^i(r) + E^{s(n-1)}_s(r), r \in S_s \]

\[ E^{a(n)}_a(r) = E^i(r) + E^{a(n-1)}_a(r), r \in S_a \quad (n = 1, 2, \ldots) \]

Where, \( E^i \) is the incident field, \( E^{s\_s}_s \) and \( E^{a\_a}_a \) are the interaction field. The interaction field \( E^{s\_s}_s \) and \( E^{a\_a}_a \) can be expressed as:

\[ E^{s(n-1)}_s(r) = \iint_{S_s} \left\{ -j \omega \mu_0 [J^{(n-1)}_s(r)g(r, r') + \frac{1}{k^2} \nabla \cdot J^{(n-1)}_s(r)\nabla g(r, r')] \right\} dS' \]

\[ + \iint_{S_s} \left\{ -j \omega \mu_0 [J^{(n-1)}_a(r)g(r, r') + \frac{1}{k^2} \nabla \cdot J^{(n-1)}_a(r)\nabla g(r, r')] + J^{a(n-1)}_a(r) \times \nabla g(r, r') \right\} dS' \]

\[ E^{a(n)}_a(r) = \iint_{S_a} \left\{ -j \omega \mu_0 [J^{(n)}_a(r)g(r, r') + \frac{1}{k^2} \nabla \cdot J^{(n)}_a(r)\nabla g(r, r')] + J^{a(n)}_a(r) \times \nabla g(r, r') \right\} dS' \]

By updating the excitation item at the left part of equations (36) and (37), and then updating the induced electromagnetic current on the surface of a rough surface and object, the electromagnetic current on the surface of a rough surface and object are obtained.

The iteration error at \( n\)-th step is:

\[ ||z^{(k)}_s||_2 = \frac{||J^{k+1} - J^{k}||_2}{||J^{k}||_2} \quad ||z^{(k)}_M||_2 = \frac{||M^{k+1} - M^{k}||_2}{||M^{k}||_2} \]

Where, \( ||\cdot||_2 \) denotes norm-2. The convergence threshold of iteration error is set as \( 10^{-2} \), at which point the induced electric current and magnetic current are supposed to be stable and the iteration process will stop. Finally, the electric current and magnetic current are obtained.

### 3. Numerical results

In the example, incident frequency is 300MHz, the polarization mode is vertical to vertical polarization (VV polarization), size of the rough surface is \( L_x \times L_y = 20 \times 20 (m^2) \); root mean square (rms) is \( h = 0.1m \); the correlation of a rough surface is \( l_i = l_s = 1m \); size of the sphere is \( r = 2m \); height of sphere above underlying a rough surface is \( H = 4m \); the properties of the rough surface is medium; the properties of the sphere is perfect electric conductor, conduct-medium combination sphere and medium sphere, respectively. The permittivity and the magnetic conductivity of the medium part of conductor-media combination sphere is \( \varepsilon_r = 4 + 0.3j \) and \( \mu_r = 1.2 + 0.6j \), respectively. The permittivity and
the magnetic conductivity of the medium sphere is \( \varepsilon_r = 4 + 0.3j \) and \( \mu_r = 1.2 + 0.6j \), respectively. The comparing result is shown in figure 3.

As we see, the scattering from medium object above a rough surface is the strongest and the scattering from conduct sphere above a rough surface is the weakest. What’s more, for three examples, the mirror scattering is the strongest. The reason is that the scattering from media is stronger than conductor.

4. Conclusion
In this paper, a hybrid method combining the single integral equation (SIE) and the Kirchhoff approximation (KA)are proposed for computing the conductor- medium assembly object above a rough surface. First, the composite model from conductor-medium assembly object above a rough surface is constructed. Then, the scattering from the object is calculated by SIE and the scattering from a rough surface is calculated by KA. Next, the electromagnetic current on the object and the rough surface is obtained by iteration and the total scattering field is obtained. Finally, the scattering field from conduct- medium assembly object above a rough surface is compared to the scattering field from a conducting object and a medium object above a rough surface, respectively.

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