ANALYSIS OF STRENGTH CRITERIA IN THE DESIGN OF PRODUCTS FROM COMPOSITE MATERIALS

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Technological progress gives rise to the continuous expansion of the class of structural materials and the improvement of their properties. The appearance of new materials is due to the natural desire to increase the efficiency of the structures under development. One of the most striking manifestations of progress in the development of materials, structures and technology is associated with the development and application of composite materials. Composites have a number of obvious advantages over other materials, in particular over metals. Such advantages are high specific strength and rigidity, high corrosion resistance, good ability to withstand alternating loads and others. It should be noted another, perhaps the most important feature of composites - is the ability to change the properties of the material in accordance with the purpose of the structure and the nature of its load during operation.

Under the influence of loads on the structure, its strength is estimated by the ultimate state of the materials of the structural elements. When a boundary state arises in a material, its transition to another mechanical state — elastic, plastic, or fracture state — occurs. This article aims to determine the optimal criterion for the strength of composite material that takes into account different values of ultimate stresses not only in different directions of the coordinate axes, but also to stretch and compress and further calculate the maximum allowable load for single-layer unidirectional composite material

During the research the main properties of composite materials, methods of manufacturing parts from composite material, their main properties and methods of destruction were considered. The characteristics of the strength criteria of composite materials are given, the most suitable for calculating the maximum value of the allowable load for a single-layer unidirectional composite material is determined. The proposed approach to the optimal design of elements of single-layer composite structures may be of interest to developers of numerous and analytical methods for solving problems of optimal design of more complex structures.

Keywords: composite materials; polymer composite materials; prepregs; polymer composite materials’ formation; single-layer unidirectional composite material.

Introduction. Formulation of the problem

Every day there are new and more stringent requirements for modern products and their characteristics, which necessitates the use of different types of materials in industry. Composite materials (CM) are a good alternative, because even at the stage of product development it is possible to provide the necessary physical and mechanical characteristics of future products. Increasingly, the ratio of composite material to total weight is used, for example in the latest aircraft. Up to 53 % of modern aircraft are composite [1]. The body of some new machines is made of 90 % of this type of material, and for the space industry, it plays an important role in that it provides weight reduction without losing the physical and mechanical properties of the structure [2].

Composite materials are made by bonding two or more interconnected components, one of which is a phase (fibrous material) and the other is a filler in Figure 1 [3].
istics of the input components. Most of the properties of the obtained CM are higher than the properties of the input components. With the advent of such materials, it became possible to selectively select the properties of composites required for the needs of each specific application [4].

**Analysis of recent research and publications**

When designing new engineering objects, as structural, it is mainly composite materials that are used, due to the possibility of purposeful change of their physical characteristics during creation, by controlling the structural parameters of the composition [5]. The latter usually include the shape and size of the particles of the dispersed phase, the distance between them, the orientation and distribution of the dispersed phase by the volume of the dispersion medium. The possibility of changing the physical characteristics of the material is realized by setting and solving optimization problems, where the optimality criteria are the required characteristics, and the optimization parameters are the structural parameters of the composite model.

Ukrainian and foreign scientists have made a significant contribution to the development of methods for finding effective characteristics in the problems of optimal design of composite materials on the basis of numerical modeling of relevant physical processes [6] Torquato S., Roberts A., Shevchenko V., Yavorsky N., Zinoviev P., Smerdov A., Vasilev V., Ostoj-Starzewski M., Narusberg V., Teters G., Hashin Z., Lubin G., Christinsen R., Zarichnyak Y., Farmaga I., Maistrenko A., Gerega A., Virovy V., Cherkashina A., Klovanych S., Szabo B., Babuska I., Zienkiewicz O.

The purpose of the study described in this study article is to analyze the strength criteria of composite materials and determine the most suitable for calculating the maximum value of the allowable load for a single-layer unidirectional composite material.

**Analysis of the properties of composite materials**

The composite material is heterogeneous. Advantages of heterogeneous polymeric materials in comparison with homogeneous polymers:
- increased rigidity, strength, dimensional stability;
- increased destructive resistance and impact strength;
- increased heat resistance;
- reduced gas and vapor permeability;
- adjustable electrical properties;
- reduced cost.

Wide variation of material properties can be achieved only by changing the morphology and strength of adhesion between the phases. For uniform transmission of external influence through the matrix and its distribution on all particles of the filler requires a strong adhesion at the boundary of the matrix - the filler, which is achieved through adsorption or chemical interaction [7].

Forms of destruction of fibrous material in different types of destruction are presented in Figure 2.

![Fig. 2. Forms of destruction of fibrous material during stretching (a) and compression (b) of fibers; when stretching (c), compression (d) and shear of the matrix (d)](image)

The simplest method of manufacturing parts from a composite material is contact molding in open forms. This method is used for large light-duty parts of complex configuration: box casings of mechanisms, tanks, housings, etc.

Contact molding of products in open forms is carried out mainly by two methods - manual laying and spraying.

Methods of forming composite materials using an elastic diaphragm include: vacuum forming, injection molding and autoclaving.

**Strength characteristics of composite materials**

Parts made of composite materials are formed by superimposing sheets of material of different weaving and directing the fibers at different angles, to ensure
the optimal allowable load in different directions from the application of load force.

To determine the total allowable load, you need to consider the angle of the fibers, the fiber material, its shape, filler, etc. For a single-layer sheet of material, various methods are used to determine the maximum force required to break this layer during stretching.

The properties of polymer composite materials (PCM) depend on the choice of source components and their ratio, the interaction between them, the method and technological conditions of manufacturing the product (pressure, temperature, time), additional processing of the product and a number of other factors [8].

Determinant in the creation of PCM is the interaction and interaction of components at the fiber-matrix boundary (connector). The higher the required properties of the created PCM design purpose, the more complex set of requirements must be maintained when choosing the input components.

Thus, the creation of composite materials is to combine similar or different components to obtain a material with new predetermined properties and characteristics different from the properties and characteristics of the input components. Most of the properties of the obtained CM are higher than the properties of the input components.

With the advent of such materials, it became possible to selectively select the properties of composites required for the needs of each specific application.

**Strength criteria for composite materials**

Under the influence of loads on the structure, its strength is estimated by the ultimate state of the materials of the structural elements. When a boundary state arises in a material, its transition to another mechanical state — elastic, plastic, or fracture state — occurs. To analyze the strength of the structure requires:

- know the ultimate stresses (or deformations) for a given structure material (determined experimentally);
- use strength criteria to assess the resistance of materials under load.

When studying composite materials, it is necessary to take into account the different resistance of layered and fibrous materials of compression and tension, which is determined by the properties of the binder and the reinforcing phase.

Therefore, to study the strength of structures made of composite materials, it is necessary to apply strength criteria that take into account different values of ultimate stresses not only in different directions of the coordinate axes, but also in tension and compression [9].

**Strength criteria based on the ultimate stresses of the monolayer**

Estimation of the strength of the composite layer of the material is based on experimental studies of its tensile, compressive and shear characteristics. In the General case, the stress-strain state of the layers and the laying of the composite material is estimated. To study the ultimate strength, strength criteria based on the maximum stresses of the monolayer are mostly used [10].

The Tsai-Hill (Hill-Mises) strength criterion is a quadratic criterion based on the fourth (energy) theory of strength and based on stress, which can be used to identify fracture, but it is impossible to distinguish the forms of fracture. This criterion can be applied in most cases to composite shells, and it is best to apply it to laminated materials when the tensile and compressive forces are equal. The main disadvantage of the criterion is the inability to determine the cause of the destruction of the monolayer: there was a destruction of the matrix or fiber. This may be an obstacle in the subsequent assessment of the strength of the composite package, as the destruction of the matrix of a single monolayer, the strength of the package may not be exhausted [11].

Hoffman’s strength test is an extended version of the Tsai-Hill test and takes into account the properties of tensile or compressive in one criterion. This criterion is based on the sum of linear and quadratic stress invariants.

The Tsai-Wu strength criterion is a modification of the Hoffman criterion and the phenomenological material theory of fracture, which is widely used for anisotropic composite materials having different tensile and compressive strengths. The disadvantage of this criterion is that it, like the Tsai Hill criterion, does not predict various forms of destruction, including the destruction of fibers and matrices. The criterion is best applied when the tensile and compressive forces are not equal. The criteria for the maximum stresses of Tsai-Hill, Hoffman and Tsai-Wu do not carry information about what happened in the monolayer - the matrix or fiber collapsed. The destruction of the matrix of a single monolayer does not always lead to the depletion of its load-bearing capacity, and the material package may continue to carry an increasing load. Therefore, the criteria in which the strength reserves of both the matrix and the fiber are analyzed separately, such as the Hashin or Pak criteria, are becoming more widespread.

Hashin’s strength criterion identifies four different ways of breaking the composite material: fibers - in tension or compression; matrix - when stretching or compressing. The equations according to the Hashin test take into account the interlayer tangential stresses, so it is necessary to determine the additional components of the tensile stress tensor, which complicates the calculation and requires additional testing of samples [12].

The Pak strength criterion identifies fiber fracture and interfiber fracture in a one-sided composite material. The criterion describes two different forms of fiber destruction. The first is tensile fracture, the second is torsional compression fracture. The equations of the Pak criterion take into account the forma-
tion of cracks in the matrix. This criterion looks the same as the criterion of maximum stresses. The difference between them is that the elastic characteristics of the matrix can be nonlinear and the deformation criteria make it possible to take into account this factor to some extent [12].

**Tsai-Wu criterion**

The Tsai-Wu test is used to determine the margin of safety of composite orthotropic materials [13].

The criterion takes into account the total deformation energy (deformation energy and expansion energy) to predict fractures.

For the stress state of the 2D (two-dimensional) plane \((\sigma_1 = 0, \tau_{13} = 0, \tau_{23} = 0)\) Tsai-Wu destruction criterion is expressed as follows:

\[
F_1\sigma_1 + F_2\sigma_2 + 2F_1\tau_{12}\sigma_1\sigma_2 + F_2\tau_{22}\sigma_2^2 + F_6\tau_{66}\sigma_2^3 = 1.
\]

Coefficients \(F_{ij}\) in orthotropic criteria, Tsai-Wu fractures are related to the strength parameters of the layered material and are determined experimentally.

They are calculated as follows:

\[
F_1 = \left(\frac{1}{X_1^C} - \frac{1}{X_1^T}\right), \quad F_2 = \left(\frac{1}{X_2^T} - \frac{1}{X_2^C}\right),
\]

\[
F_{12} = -\frac{1}{2}\sqrt{\frac{1}{X_1^C} \cdot \frac{1}{X_1^T} \cdot \frac{1}{X_2^C} \cdot \frac{1}{X_2^T}},
\]

\[
F_{11} = \frac{1}{X_1^C} \cdot \frac{1}{X_1^T}, \quad F_{22} = \frac{1}{X_2^T} \cdot \frac{1}{X_2^C},
\]

\[
F_6 = \left(\frac{1}{X_1^T} - \frac{1}{X_1^C}\right), \quad F_{66} = \frac{1}{X_1^C} \cdot \frac{1}{X_1^T} \cdot \frac{1}{X_2^C} \cdot \frac{1}{X_2^T}.
\]

Where, \(X_1^T\) is the tensile strength of the layered material along the direction of the fibers;

\(X_1^C\) is the tensile strength of the layered material when stretched across the direction of the fibers;

\(X_2^T\) is the tensile strength of the layered material when stretched across the direction of the fibers;

\(X_2^C\) is the strength limit of the layered material when compressed across the direction of the fibers;

\(X_{12}^T\) is positive shear strength of the layered material;

\(X_{12}^C\) is negative tensile strength of the layered material.

The stress state of the layered material is described by the following components:

\(\sigma_1\) is the tension of the layered material along the direction of the fibers;

\(\sigma_2\) is the tension of the layered material across the direction of the fibers;

\(\tau_{12}\) is shear stress of the layered material.

**Example of calculating the maximum allowable load by the Tsai-Wu method**

As a result of the review and analysis of the existing methods of calculating the maximum value of the allowable load for a single-layer unidirectional composite material, it was found that the most suitable for determination is the Tsai-Wu method.

Consider an example of calculating the maximum value of the allowable load for a composite layer of graphite, if \(S > 0\), \(\sigma_1 = 2S\), \(\sigma_2 = -3S\) and \(\tau_{12} = 4S\).

The main values of stresses for this layer are taken from the table.

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix}
= 
\begin{bmatrix}
0.2500 \\
0.7500 \\
-0.4330
\end{bmatrix}
= 
\begin{bmatrix}
2S \\
3S \\
3S
\end{bmatrix}
= 
\begin{bmatrix}
0.1714 \times 10^4 \\
0.2714 \times 10^4 \\
0.4165 \times 10^4
\end{bmatrix}

S.
\]

Next, the coefficients for the Tsai-Wu formula are calculated:

\[
H_1 = \frac{1}{1500 \times 10^6} - \frac{1}{1500 \times 10^6} = 0 Pa,
\]

\[
H_2 = \frac{1}{40 \times 10^6} - \frac{1}{246 \times 10^6} = 2.093 \times 10^{-8} Pa,
\]

\[
H_6 = 0 Pa
\]

\[
H_{11} = \frac{1}{(1500 \times 10^6)(1500 \times 10^6)} = 4.4444 \times 10^{-19} Pa,
\]

\[
H_{22} = \frac{1}{(40 \times 10^6)(256 \times 10^6)} = 1.0162 \times 10^{-16} Pa,
\]

\[
H_{66} = \frac{1}{(68 \times 10^6)^2} = 2.1626 \times 10^{-16} Pa,
\]

\[
H_{12} = -0.5 \left[ (4.4444 \times 10^{-19})(1.0162 \times 10^{-16}) \right]^{\frac{1}{2}} = -3.360 \times 10^{-18} Pa.
\]

Substituting the value into the formula we obtain:

\[
(0)(1.714S) + (2.093 \times 10^{-8})(-2.714S) +
\]

\[
(0)(-4.165S) + (4.444 \times 10^{-19})(1.714S)^2 +
\]

\[
(1.0162 \times 10^{-16})(-2.714S) + (2.1626 \times 10^{-16}) \times
\]

\[
(4.165S)^2 + 2(-3.360 \times 10^{-18})(1.714S)(-2.714S) < 1
\]

or

\[
S < 22.39 Pa.
\]

Therefore, the maximum allowable load for this plate is calculated by the method Tsai-Wu is 22.34 MPa.
Conclusions

The range of application of composite materials is very wide: from ornaments and decorative architectural panels to high-quality bearing designs of a difficult form. In terms of physical and mechanical characteristics, parts made of composite materials are competitive, but their manufacture is still expensive, and performance needs to be improved and researched.

The classification of composite materials was analyzed, their description was given, and their characteristics were analyzed. The types of manufacturing parts based on composite materials, the main advantages and disadvantages of these types and their differences are analyzed. Also, the strength characteristics of composite materials were given, the strength criteria for this type of material were considered. Methods for determining the maximum value of material stress, including the criterion of maximum stresses, Hill's criterion, Hashin's criterion, Tsai-Wu criterion, are considered for analysis. Their strengths and weaknesses are given, the corresponding formulas for calculation are given.

After analyzing various strength criteria for a single-layer material, the Tsai-Wu method was determined to be the most optimal for calculating the maximum allowable load value for a single-layer unidirectional composite material and, accordingly, an example of calculating a composite material for an allowable load was given.

In the future, further research is planned to derive the mathematical dependences of the stress of the composite material on the angles of laying layers in it, as well as on the basis of the proposed mathematical models software development to determine the maximum allowable stress on the product.

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технології пов’язаний з розробкою і застосуванням композитних матеріалів. Композити мають ряд очевидних переваг перед іншими матеріалами, зокрема перед металами. Такими перевагами є висока питома міцність і жорсткість, висока корозійна стійкість, хороша здатність витримувати знакомітні навантаження та інші. Слід зазначити ще одну, можливо, найважливішу особливість композитів – ще здатність до спрямованої зміни властивостей матеріалу відповідно до призначення конструкції і характеру її навантаження під час експлуатації.

При впливі навантажень на конструкцію її міцність оцінюється за граничним станом матеріалів елементів конструкції. Коли в матеріалі виникає граничний стан, то відбувається його перехід в інший механічний стан – пружний, пластичний або стан руйнування. Дана стаття направлена на визначення оптимального критерію містності композитного матеріалу, що враховує різну величину граничних напружень не тільки за різними напрямками осей координат, а й на розтягування і сніщення та подальшого обчислення максимально-відповідного допустимого навантаження для одношарового однонаправленого композитного матеріалу. Під час дослідження було розглянуто основні властивості композитних матеріалів, методи виготовлення деталей із композитного матеріалу, розглянуто основні її властивості та методи руйнування. Наведено характеристикку критеріїв міщності композитних матеріалів, визначено найбільш придатний для обчислення максимально-відповідного допустимого навантаження для одношарового однонаправленого композитного матеріалу. Запропонований підхід до оптимального проектування елементів одношарових композитних конструкцій може становити інтерес для розробників чисельних і аналітичних методів вирішення завдань оптимального проектування більших складових систем.

Ключові слова: композитні матеріали; полімерний композитний матеріал; препрепги; формування полімерного композитного матеріалу; одношаровий однонаправлений композитний матеріал.

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АНАЛІЗ КРИТЕРІЄВ ПРОЧНОСТІ ПРИ ПРОЕКТИРОВАНИИ ИЗДЕЛИЙ ИЗ КОМПОЗИТНЫХ МАТЕРИАЛОВ

Технический прогресс порождает непрерывное расширение класса конструкционных материалов и совершенствование их свойств. Появление новых материалов обусловлено естественным стремлением повысить эффективность разрабатываемых конструкций. Одно из самых ярких проявлений прогресса в развитии материалов, конструкций и технологии связано с разработкой и применением композитных материалов. Композиты имеют ряд очевидных преимуществ перед другими материалами, в частности перед металлами. Такими преимуществами являются высокая удельная прочность и жесткость, высокая коррозионная стойкость, хорошая способность выдерживать знакопеременные нагрузки и другие. Следует отметить еще одну, возможность, самую главную особенность композитов — способность к направленному изменению свойств материала в соответствии с назначением конструкции и характера ее нагрузки во время эксплуатации. При воздействии нагрузок на конструкцию ее прочность оценивается по предельному состоянию материалов элементов конструкции. Когда в материале возникает предельное состояние, то происходит его переход в другое механическое состояние — упругое, пластическое или состояние разрушения. Данные подходы определяют оптимального критерия прочности композитного материала, учитывающего разную величину предельных напряжений не только по разным направлениям осей координат, но и на растяжение и сжатие в последующем вычисления максимального значения допустимой нагрузки для однослойного однонаправленного композитного материала. В ходе исследования были рассмотрены основные свойства композитных материалов, методы изготовления деталей из композитного материала, рассмотрены основные их свойства и методы разрушения. Приведена характеристика критерия прочности композитных материалов, определен наиболее подходящий для вычисления максимального значения допустимой нагрузки для однослойного однонаправленного композитного материала. Предложенный подход к оптимальному проектированию элементов однослойных композитных конструкций может представлять интерес для разработчиков численных и аналитических методов решения задач оптимального проектирования более сложных структур.

Ключевые слова: композитные материалы; полимерный композитный материал; препрепги; формирование полимерного композитного материала; однослойный однонаправленный композитный материал.

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