QCD and QED Corrections to Higgs Boson Production in Charged Current $ep$ Scattering

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Abstract

First order QCD and leading QED corrections to Higgs boson production in the channel $e^- p \rightarrow \nu H^0 X; H^0 \rightarrow b\bar{b}$ are calculated for the kinematical conditions at LEP $\otimes$ LHC ($\sqrt{s} = 1360$ GeV) and the interesting mass range $80 < M_H < 150$ GeV. In the DIS scheme the QCD corrections (not including the corrections to the branching ratio, which are well-known) are found to be about 1% for the total cross section and $-13\%$ to $-10\%$ for the observable cross section as defined by appropriate cuts. The latter results depend on the definition of these cuts. The QED corrections amount to about $-5\%$.

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1 Introduction

The search for the Higgs boson, or physics which replaces the Higgs boson in case it does not exist as a fundamental particle, is one of the main motivations for the construction of supercolliders. While a light Higgs boson with a mass $M_H \leq M_W$ will most likely be found or excluded at LEP, the mass range from $M_H \approx 140$ GeV up to about 1 TeV can be completely covered at LHC and SSC. However, the intermediate region from $M_H \approx 80$ GeV to 140 GeV is a very difficult one for searches in $pp$ collisions. In this mass range, the Higgs boson decays dominantly into $b\bar{b}$ quark pairs, a channel which is difficult to isolate because of the huge QCD background. The rare decay $H^0 \rightarrow \gamma\gamma$, on the other hand, provides a more favourable signature, but at the cost of a substantial reduction in rate $[1,2]$. Although it appears feasible to detect some signal in the two photon channel, this task is experimentally very demanding. Clearly, the best tool for the discovery of an intermediate mass Higgs boson would be a linear $e^+e^-$ collider at a minimum CMS energy of roughly twice the LEP 200 energy $[3]$. Unfortunately, at the time when LHC and SSC are expected to start operation, a linear collider in the required energy range will most likely not yet be available. Therefore, it is an important issue to clarify whether or not the $ep$ option which exists at LHC could help to investigate this very interesting mass window.

The $ep$ option we refer to can be realized by intersecting a 50 to 100 GeV electron beam from LEP with the 7.7 TeV proton beam from LHC $[4]$. In this mode, the expected luminosity ranges from $2 \cdot 10^{31}$ cm$^{-2}$ s$^{-1}$ at the upper end of the CMS energy range, $\sqrt{s} = 1.79$ TeV, to $2 \cdot 10^{32}$ cm$^{-2}$ s$^{-1}$ at the lower end, $\sqrt{s} = 1.26$ TeV. For definiteness, we assume $\sqrt{s} = 1.36$ TeV and an integrated luminosity of 1 fb$^{-1}$ corresponding to collisions of 60 GeV electrons on 7.7 TeV protons and one year of running time.

The basic electroweak processes in $ep$ collisions are neutral current (NC) and charged current (CC) scattering through the exchange of virtual photons, $Z$ and $W$ bosons, respectively. Whereas the couplings of the standard Higgs scalar to the fermions participating in NC and CC scattering are strongly suppressed by the light fermion masses, the couplings to the $W$ and $Z$ bosons are of ordinary electromagnetic strength. Hence, the virtual weak bosons appearing in NC and CC scattering processes can act as efficient sources for the standard Higgs boson. In fact, the $WW$ fusion process depicted in fig. 1 is the dominant Higgs production mechanism in $ep$ collisions $[5,6]$, followed by $ZZ$ fusion, which has a roughly 5 times smaller cross section. Interestingly, in $pp$ collisions the analogous mechanisms become important only for very heavy Higgs bosons with masses $M_H \geq 600$ GeV $[7]$.

As can be already expected from the above remarks, Higgs bosons are not produced very frequently in $ep$ collisions, at least in comparison to the corresponding rates in $pp$ collisions. In the interesting mass range from 80 to 140 GeV and for $\sqrt{s} = 1.36$ TeV, the total cross section varies from about 200 to 100 fb. One can therefore not afford to search in rare decay channels such as $H^0 \rightarrow \gamma\gamma$, the mode considered for hadron colliders, but one must try to detect the Higgs signal in the main decay channel $H^0 \rightarrow b\bar{b}$ (or for $M_H \geq 130$ GeV through $H^0 \rightarrow WW^*$, where one of the $W$ bosons is off-shell, this possibility is not pursued further here). The main background is expected to come from NC production of jets (including photoproduction), multi-jet CC scattering, and single $W$, $Z$ and $t$–quark production with subsequent hadronic decays. Although this background is large, it is not as overwhelming as the QCD jet background to $H^0 \rightarrow b\bar{b}$ in $pp$ collisions. In addition, in $ep$ collisions the $H^0 \rightarrow b\bar{b}$ decay provides several characteristic signatures which allow for an efficient discrimination of signal against background. For the $WW$ fusion channel $ep \rightarrow \nu H^0 q + X \rightarrow \nu b\bar{b}q + X$, the selection of signal events and the suppression of the backgrounds have been investigated in great detail in ref. $[7]$. It has been demonstrated that one should be able to observe an
intermediate mass Higgs boson, provided flavour identification capabilities are available with efficiencies similar to the expected capabilities of the DELPHI detector at LEP.

This encouraging result and the importance of the physics issue call for further studies in order to corroborate the above conclusion. Obviously, the feasibility of flavour identification has to be examined in a dedicated ep detector study. Furthermore, one has not yet exploited all possibilities to optimize the event selection and background suppression. Finally, the parton level treatment in tree approximation performed in ref. \[7\] still suffers from uncertainties due to the neglect of hadronization effects and higher order corrections. In particular, K-factors of 2 would be quite crucial since the Higgs signal considered here is defined by a set of nontrivial arguments and guesses based on existing calculations for production processes of other heavy particles \[8\] since the Higgs signal considered here is defined by a set of nontrivial arguments and guesses based on existing calculations for production processes of other heavy particles \[9\] since the Higgs signal considered here is defined by a set of nontrivial arguments and guesses based on existing calculations for production processes of other heavy particles \[9\].

The lowest order production cross section for $e^-p \rightarrow \nu H^0X$ is obtained from the diagram shown in fig. \[1\]. The differential cross section is given by

$$d\sigma^{(0)} = \frac{1}{2xs} \sum_i |M_i^{(0)}|^2 q_i(x, Q^2_{i}) dPS_3(eq \rightarrow \nu H^0q) \, dx$$

$$= \frac{1}{2xs} \left( \frac{g^6 M_W^2}{(Q^2_1 + M_W^2)^2} \sum_{i=1}^2 \left[ (p_1p_2)(p'_1p'_2)u_i(x, Q^2_{i}) + (p_1p'_2)(p'_1p_2)\bar{d}_i(x, Q^2_{i}) \right] dPS_3(eq \rightarrow \nu H^0q) \right) \, dx$$

where the matrix element squared includes an average of the spins of incoming particles and a sum over the spins of outgoing particles. In the above formula $p_1(p'_1)$ and $p_2(p'_2)$ are the incoming (outgoing) lepton and quark momenta, respectively, $x$ is the fraction of the proton momentum carried by the incoming partons, $s$ the cms energy squared and $Q^2_{i} = -(p_i - p'_i)^2$. $u_i(x, Q^2_{i})$ are the distribution functions of $u$- and $c$-quarks, $\bar{d}_i(x, Q^2_{i})$ of $d$- and $\bar{s}$- antiquarks. The contributions from bottom and top quark densities can be neglected. Also we have put off-diagonal elements of the CKM-matrix

\[1\]For $e^+p$-scattering the distributions $u_i, d_i$ have to be replaced by $d_i, \bar{u}_i$. 

\[2\]The Born Approximation

The paper is organized as follows. In section 2 we recalculate the tree level cross section for Higgs production via WW fusion and examine the variation with different sets of structure functions. We also show interesting differential distributions for the channel $H^0 \rightarrow b\bar{b}$ and briefly review the selection cuts used in ref. \[7\]. Section 3 is devoted to the QCD corrections. Results are presented for the fully integrated production cross section, including the QCD corrected branching ratio for $H^0 \rightarrow b\bar{b}$. The main point, however, is the calculation of the QCD corrections to the cross section times branching ratio in the three jet channel in the presence of the kinematical cuts of ref. \[7\]. The effects of initial state QED bremsstrahlung are then obtained in section 4 using the structure function method. In section 5 we summarize our conclusions.
involving the top quark to zero. The SU(2) gauge coupling \( g \) is given by \( g = e / \sin \theta_W \), 
\( e \) being the electromagnetic coupling constant and \( \theta_W \) being the weak mixing angle. 
Furthermore, we apply the zero-width approximation, which is a valid simplification 
for \( M_H \leq 150 \text{ GeV} \) where \( \Gamma_H / M_H \lesssim 10^{-4} \) \cite{11, 12}. The phase space element \( dP_S \) is given by 
\[
dP_S = (2\pi)^{4-3n} \delta^{(4)}(p_m - \sum_{i=1}^{n} p_i) \prod_{i=1}^{n} \frac{d^3 p_i}{2p_{i,0}}.
\]

The physical boundaries in \( x \) are \( M_H^2 / s \leq x \leq 1 \). The integrated cross section \( \sigma_{tot}^{(0)} \) was 
checked against earlier calculations \cite{4, 5, 6, 7}. Throughout this paper we have taken the 
following values for the electroweak parameters: \( M_W = 80.6 \text{ GeV}, \sin^2 \theta_W = 0.230, \) \( e^2 / 4\pi = 1 / 128.5 \). The quark distributions in eq. \cite{1} are parametrized as \( Q^2 \) dependent 
functions corresponding to the solutions of the Altarelli-Parisi equations.

In fig. \ref{fig:2} we show \( \sigma_{tot}^{(0)} \) and \( \sigma_{tot}^{(0)} \times Br(H^0 \to b\bar{b}) \). The branching ratio is taken from 
ref. \cite{11} and includes integrated QCD corrections up to \( \mathcal{O}(\alpha_s^2) \). The fast decrease of the cross section times branching ratio in the \( b\bar{b} \) channel at higher Higgs boson masses 
may be due to the decay \( H^0 \to WW^* \) which becomes dominant as \( M_H \) exceeds \( 120 \) GeV.

Concerning the dependence of the cross section on the parametrization of the quark 
distribution functions the following remarks may suffice. We use the parametrization 
by Morfin and Tung \cite{12} set 2 assuming the \( \overline{\text{MS}} \) or DIS scheme \cite{13} for the definition of the 
parton densities. The resulting \( \sigma_{tot}^{(0)} \) for the two schemes differ by less than 3\%, 
as can be seen from fig. \ref{fig:2}. We have also compared \( \sigma_{tot}^{(0)} \) for other parton parametrizations. 
The deviations for HMRS (\( \overline{\text{MS}}, \text{set 2} \)) \cite{14} are also smaller than 3\%. Larger differences of 
\( \mathcal{O}(10\%) \) are obtained for older parametrizations such as DFLM (DIS) \cite{13} and DO1 (LO) \cite{16}.

Eq. \cite{1} was parametrized in terms of the individual parton densities directly. However, 
for the understanding of the size of the QCD corrections to be discussed in the 
next section, it is interesting to know which of the usual deep inelastic structure functions, 
that is 
\[
2xF_1^{(0)}(x, Q^2) = F_2^{(0)}(x, Q^2) = 2x \sum_{i=1}^{2} [u_i(x, Q^2) + \bar{d}_i(x, Q^2)]
\]
and 
\[
xF_3^{(0)}(x, Q^2) = 2x \sum_{i=1}^{2} [u_i(x, Q^2) - \bar{d}_i(x, Q^2)],
\]
yields the dominant part of the cross section. For this purpose, eq. \cite{1} may be rewritten 
as 
\[
d\sigma^{(0)} = \frac{1}{2xs} \frac{g^2 M_W^2}{(Q_1^2 + M_W^2)^2 (Q_2^2 + M_W^2)^2} \left\{ b^+ F_2^{(0)}(x, Q^2) / x + b^- F_3^{(0)}(x, Q^2) \right\} dP_S dx
\]
with 
\[
b^\pm = \frac{1}{4} \{ (p_1 p_2)(p_1 p_2') \pm (p_1 p_2')(p_1 p_2) \}.
\]

\( F_2^{(0)}(x, Q^2) \) contributes about 98\% to \( \sigma_{tot}^{(0)} \). This has two reasons: \( b^- \ll b^+ \) in a wide 
range of the phase space \cite{1} and \( xF_3^{(0)} \leq F_2^{(0)} \).

To illustrate some details of the \( WW \)–fusion process differential distributions for 
\( x = -q_2^2 / (P \cdot q_2) \) \( (P \) being the proton momentum), \( Q_2^2 = -q_2^2 \) and the transverse 
momentum \( p_t \) and rapidity \( \eta \) of the Higgs boson are shown in fig. \ref{fig:3}. The figures refer

\footnote{For \( p_H \rightarrow 0 \), this corresponds to deep inelastic scattering at small \( y \).}
to a Higgs boson mass of 100 GeV. As one can see, the production involves large $x$ and $Q^2$, typically $x \gtrsim 0.01$ and $Q^2 \gtrsim 100 \text{GeV}^2$. We thus do not expect screening corrections to the parton distributions \([17]\) to influence the cross section significantly.

The background to the $\nu \bar{b}bX$ final state was investigated in ref. \([7]\) in detail. It receives contributions from four types of reactions: photo\((\text{and NC})\) production of multijet events, CC multijet production, single $W$ and $Z$ production with subsequent hadronic decay, and top production. This background was shown to be manageable after applying the following selection criteria:

- the final state should contain three jets, defined here as partons with transverse momentum $p_T > 20 \text{GeV}$, rapidity $|\eta| < 4.5$ and a distance $\Delta R > 1$ in the $\eta$-$\phi$ plane;
- the event should have a large missing transverse momentum, $p_T^{\text{miss}} > 20 \text{GeV}$, and a large total transverse energy, $E_T > 100 \text{GeV};$
- the Higgs signal should be contained in the invariant mass distribution of the two jets lowest in rapidity.

These cuts leave from the total cross section in the $\bar{b}b$ channel, $\sigma^{(0)}_{\text{tot}} \times Br(H^0 \rightarrow \bar{b}b)$, the fraction denoted by $\sigma^{(0)}_{\text{obs}}$ in figure 2. In this sense, $\sigma^{(0)}_{\text{obs}}$ can be considered the observable cross section in this channel. If one furthermore assumes good flavour identification and a 10 GeV mass resolution the background is reduced to a level which would enable detection of the Higgs boson between 80 and 140 GeV \([7]\).

### 3 QCD Corrections

The QCD corrections to $e^-p \rightarrow \nu H^0 X; H^0 \rightarrow \bar{b}b$ consist of three contributions: the corrections to the $W$-quark vertex $W^* q \rightarrow q'$, corrections to the decay vertex $H^0 \rightarrow \bar{b}b$, and gluon exchange between the final state $b$-quarks and the interacting parton at the hadronic side. To $O(\alpha_s)$ the latter corrections vanish because of the colour structure.

We first consider the total integrated corrections. They only need to be calculated for the production vertex, as the integrated corrections to the decay are known \([10, 11]\) and have already been taken into account in the branching ratio. Then we discuss the QCD corrections to the observable cross section, which strongly depend on the cuts applied to isolate the signal. In this case there is also a negative correction to the decay width.

#### 3.1 Corrections to the total cross section

The $O(\alpha_s)$ QCD corrections to the total production cross section are described by the diagrams of fig. 4. Generally, the differential cross section for $e p \rightarrow \nu H^0 X$ can be written as

$$d\sigma = \frac{1}{2\pi s} \frac{g^6 M_W^2}{(Q_1^2 + M_W^2)^2 (Q_2^2 + M_W^2)^2} \left\{ L_{\mu\nu}^{\text{pc}} (W_{\mu\nu}^{\text{pc,L}} + W_{\mu\nu}^{\text{pc,2}}) + L_{\mu\nu}^{\text{pv}} W_{\mu\nu}^{\text{pv,3}} \right\} dP_{3,4} d\Omega$$

where $pc$ and $pv$ denote the parity conserving and violating parts of the leptonic ($L_{\mu\nu}$) and hadronic ($W_{\mu\nu}$) tensors, respectively. The hadronic tensors $W_{\mu\nu}$ are the same as those which appear in deep inelastic charged current scattering with $Q^2 = -q_2^2$.

\(^3\)We verified this using the KMRS distributions \((\overline{\text{MS}}, B-5)\) \([18]\).
For the individual contributions to eq. (3) one obtains:

\[ L_{\mu\nu}^{\mu\nu}W_{\mu\nu}^{pc,L} = b^L F_L(x, Q^2) / x \]  
\[ L_{\mu\nu}^{\mu\nu}W_{\mu\nu}^{pc,2} = b^F F_2(x, Q^2) / x \]  
\[ L_{\mu\nu}^{\mu\nu}W_{\mu\nu}^{pc,3} = b^- F_3(x, Q^2) \]  

Formally, \( b^\pm \) can be taken from eq. (3) with \( p_2 = xP, p_2' = p_2 + q_2 \) not necessarily corresponding to particle momenta (only at lowest order). The coefficient \( b^L \) is given by

\[ b^L(p_1, p_1', p_2, p_2') = \frac{1}{8} \{(p_1 q_2)(p_1' q_2) - (p_1 p_1')(p_2 q_2)\}. \]  

Here, the structure functions are defined by

\[ F_i(x, Q^2) = F_i^{(0)}(x, Q^2) + \frac{\alpha_s(Q^2)}{2\pi} F_i^{(1)}(x, Q^2) + \mathcal{O}(\alpha_s^2) \]  

with \( F_L^{(0)}(x, Q^2) = 0 \) and \( F_{2,3}^{(0)}(x, Q^2) \) as given in eqs (3) and (4). For \( \alpha_s(Q^2) \) the leading order expression for five active flavours is used. The value of the QCD scale \( \Lambda \) is chosen in accordance with the parton distribution functions.

For the regularisation of the collinear divergences a factorisation scheme has to be chosen. Because we consider a deep inelastic process the choice of the DIS scheme appears to be most natural. Different corrections would be obtained using other schemes, e.g. the \( \overline{\text{MS}} \) scheme.

In the DIS scheme the structure function

\[ F_2(x, Q^2) = F_2^{(0)}(x, Q^2) \]  

is preserved to all orders and corrections occur to the terms \( L_{\mu\nu}^{\mu\nu}W_{\mu\nu}^{pc,L} \) and \( L_{\mu\nu}^{\mu\nu}W_{\mu\nu}^{pc,3} \) only. In lowest order, eqs (3) and (4) are seen to lead to \( d\sigma^{(0)} \) as given in eq. (3). The QCD diagrams of fig. 4 integrated over all phase space, yield

\[ \sigma^{(1)}(QCD) = \int dx \frac{dPS_8}{16\pi^2} \frac{g^6 M_W^2}{Q^4 (Q^2 + M_W^2)^2 (Q_2^4 + M_W^2)^2} \times \left\{ b^L F_L^{(1)}(x, Q_2^2) / x + b^- F_3^{(1)}(x, Q_2^2) \right\} \]  

where

\[ F_L^{(1)}(x, Q^2) = x \int_0^1 \frac{dz}{z} \left\{ f_{L}(z) F_2^{(0)}(x/z, Q^2) + 4 f_{L}^G(z) G(x/z, Q^2) \right\} \]  
\[ F_3^{(1)}(x, Q^2) = -C_F \int_0^1 \frac{dz}{1 + z} F_3^{(0)}(x/z, Q^2). \]  

Here, \( f_L^G(z) = 2C_F z \) and \( f_L(z) = 8 G_F(1-z) \), with \( C_F = 4/3 \) and \( T_R = 1/2 \). \( G(x, Q^2) \) denotes the gluon density. The scale in both the distribution functions and \( \alpha_s \) is taken to be \( Q^2 = -q_2^2 \), in accordance with the definition of the DIS scheme.

\[ \text{In the limit } p_H \to 0, \text{ eqs } (3) \text{ and } (4) \text{ yield the familiar expressions for the deep inelastic scattering cross section: } L_{\mu\nu}^{\mu\nu}W_{\mu\nu}^{pc,L} \to -(sx/4)^2 g^2 F_L(x, Q^2) / x, L_{\mu\nu}^{\mu\nu}W_{\mu\nu}^{pc,2} \to (sx/4)^2 Y_L F_2(x, Q^2) / x \text{ and } L_{\mu\nu}^{\mu\nu}W_{\mu\nu}^{pc,3} \to (sx/4)^2 Y_3 F_3(x, Q^2), \text{ with } Q^2 = Q_1^2 = Q_2^2 \text{ and } Y_\pm = 1 \pm (1 - y)^2. \]

\[ \text{Since we are calculating the finite terms of the order } \alpha_s \text{ corrections, next to leading order parton densities and } \alpha_s(Q^2) \text{ should be used. However, as the final corrections turn out to be small the difference is negligible.} \]

\[ \text{Note that for higher order corrections one can choose the schemes to renormalize the } \beta \text{ function and the collinear divergences separately.} \]
The QCD corrections to the total cross section obtained from eq. (14) are displayed in fig. 3. We show separately the quark and gluon contributions from $F_L^{(1)}$, $\sigma_L^{(1)}$ and $\sigma_G^{(1)}$, and the contribution $\sigma_3^{(1)}$ from $F_3^{(1)}$. In the mass range $50 < M_H < 150$ GeV the correction is dominated by the $F_L^{(1)}$ term, to which quarks and gluons contribute about equally. The $\sigma_3^{(1)}$ correction is negative and amounts to less than 10% of $\sigma_L^{(1)}$. The total correction $\sigma_{tot}^{(1)}(QCD)$ adds up to less than 1% of the Born cross section. Using for the parametrization of the parton densities DO1 [16] instead of MT [12] the relative correction $\sigma_{tot}^{(1)}(QCD)/\sigma_{tot}^{(0)}$ does not change significantly. This reflects the similar shape of the parton densities in the region in $(x, Q^2)$ which contributes most to the cross section.

As a consequence of the above result we remark that the integral $\mathcal{O}(\alpha_s)$ correction to Higgs production in $pp$ collisions proceeding via $WW$ fusion is of $\mathcal{O}(2\%)$.

### 3.2 Inclusion of kinematical cuts

#### 3.2.1 Corrections to the production cross section

A Higgs signal in the $b\bar{b}$ channel can only be observed after applying suitable cuts (see section 2). These cuts strongly influence the QCD corrections, as exactly three observable jets are demanded. This means that there is an upper bound on the angle and energy of the extra gluon or quark with respect to the other three jets. These bounds can only be implemented numerically. We have chosen to regulate the divergences as follows: the ultra violet divergences using the usual dimensional regularization, the infra red divergences with a small gluon mass $\lambda$, and the collinear divergences (mass singularities) with a small quark mass $m \gg \lambda$. This way all phase space integrals can be evaluated numerically in four dimensions. This mass regularisation scheme has been described in ref. [22].

The total corrections can then be written as a sum of four 4-dimensional integrals:

$$
\sigma^{(1)}(QCD) = \sigma^{(1)}_{virt+soft} + \sigma^{(1)}_{hard} + \sigma^{(1)}_{glue} + \sigma^{(1)}_{counter}. \quad (17)
$$

The first two originate in virtual and real gluon radiation. $\sigma^{(1)}_{glue}$ is the contribution from initial state gluons, while the counter terms arise from the renormalization of the distribution functions and are needed to cancel the collinear divergences.

The virtual and soft contribution can be written in the form

$$
d\sigma^{(1)}_{virt+soft} = \frac{1}{2xs} \sum_{i=1}^{2} \left\{ 2\text{Re}(\mathcal{M}_{i,virt}^{(1)} \mathcal{M}_{i,0}^{(0)*}) + |\mathcal{M}_{i,soft}^{(1)}|^2 \right\} q_i(x, Q^2) dPS_3 dx
$$

$$
= \frac{1}{2xs (Q^2_i + M_W^2)^2 (Q^2 + M_W^2)^2} \sum_{i=1}^{2} \left[ (p_1 p_2)(p_1' p_2') d_{i,virt+soft}(x, Q^2) + (p_1 p_2')(p_1' p_2) d_{i,virt+soft}^{(1)}(x, Q^2) \right] dPS_3 dx \quad (18)
$$

where the functions $q^{(1)}_{i,virt+soft}$ are given in the appendix, eq. (11). They still depend on the quark mass $m$ and the soft cutoff $\Delta = (p_2' + k)^2_{min}$, where $k$ is the gluon momentum. The expressions for $d\sigma^{(1)}_{hard}$ and $d\sigma^{(1)}_{glue}$ are also provided in the appendix by eqs (9) and (10), respectively.

Because we decided to calculate the QCD corrections in the DIS scheme, demanding eq. (13) generates a counter term to the quark distribution functions:

$$
q^{(1)}_{i,counter}(x, Q^2) = - \left\{ q^{(1)}_{i,virt}(x, Q^2) + q^{(1)}_{i,soft}(x, Q^2) + q^{(1)}_{i,hard}(x, Q^2) + q^{(1)}_{i,glue}(x, Q^2) \right\} \quad (19)
$$
The various contributions to (19) were derived in several previous calculations [19, 20, 21] using different renormalization methods. They are listed in the appendix (eqs (A.1)–(A.4)) for the regularization method used in the present calculation. The ultraviolet and infrared singularities as well as the dependence on the parameter $\Delta$ separating the soft and hard part of the gluon Bremsstrahlung terms cancel in the sum eq. (19) and only the mass singularity, that is the logarithmic dependence on $m$, remains. One obtains

$$q^{(1)}_{i,\text{counter}}(x, Q^2) = \frac{\alpha_s}{2\pi} \int_0^1 \frac{d\xi}{\xi} \left\{ \tilde{P}_{qq}(\xi, \frac{m^2}{Q^2}) \left[ \theta(\xi - x) q_i(x, Q^2) - \xi q_i(x, Q^2) \right] \right\} + \tilde{P}_{qG}(\xi, \frac{m^2}{Q^2}) \theta(\xi - x) G\left(\frac{\xi}{Q^2}\right)$$

(20)

where the functions $\tilde{P}_{qq}$ and $\tilde{P}_{qG}$ are given in eqs (A.5) and (A.6) respectively. Using eq. (20) one finally has

$$d\sigma^{(1)}_{\text{counter}} = \frac{1}{2xs} \sum_{i=1}^2 |\mathcal{M}_i^{(0)}|^2 q^{(1)}_{i,\text{counter}}(x, Q^2) dPS_3 dx$$

$$= \frac{g^6 M_W^2}{2xs (Q^2_1 + M_W^2)^2 (Q^2_3 + M_W^2)^2} \sum_{i=1}^2 \left[ (p_1 p_2) (p'_1 p'_2) q^{(1)}_{i,\text{counter}}(x, Q^2_2) + (p_1 p'_2) (p'_1 p_2) q^{(1)}_{i,\text{counter}}(x, Q^2_2) \right] dPS_3 dx. \quad (21)$$

In the practical calculation we combine the counter term given in eq. (21) with the virtual and soft contribution, eq. (18). Using eq. (19) one sees that the virtual and soft parts cancel, and only the hard and glue part of the counter terms needs to be evaluated.

It turns out that the computational procedure outlined in ref. [22] (straight evaluation of the individual integrals using Monte Carlo techniques) is not suitable here, since the corrections are very small. The problem is that we still have large logarithms that are subtracted after the numerical integration over the phase space. The unavoidable inaccuracy in this integration (usually $O(10^{-3})$) then gives a very large error in the result. This problem was avoided by performing the subtraction of the collinear singularity in the hard Bremsstrahlung and glue contributions, eqs (A.10) and (A.11) respectively, and in the counter terms before integrating over the rest of phase space. One starts by rewriting the order of these integrals:

$$\sigma^{(1)}_{\text{hard}} = \int_0^1 d\hat{s} \frac{1}{2zs} \int dPS_3(eq \rightarrow \nu H^0 qg) \theta(\hat{s} - \Delta) \sum_{i=1}^2 |\mathcal{M}_{i,\text{hard}}^{(1)}|^2 q_i(z, Q^2_2)$$

$$= \int_0^1 d\hat{s} \frac{1}{2zs} \int d\hat{s} \theta(\hat{s} - \Delta) \int dPS_3(eq \rightarrow \nu H^0 q^*) \frac{1}{2\pi} \int dPS_2(q^* \rightarrow qg)$$

$$\times \sum_{i=1}^2 |\mathcal{M}_{i,\text{hard}}^{(1)}|^2 q_i(z, Q^2_2) \quad (22)$$

$$\sigma^{(1)}_{\text{counter,hard}} = -\frac{\alpha_s}{2\pi} \int_0^1 d\xi \frac{1}{2xs} \int_1^{1 - \delta} \frac{d\xi}{\xi} \int dPS_3(eq \rightarrow \nu H^0 q) \sum_{i=1}^2 |\mathcal{M}_i^{(0)}|^2 \tilde{P}_{qq}(\xi, \frac{m^2}{Q^2_2})$$

$$\times q_i\left(\frac{x}{\xi}, Q^2_2\right)$$

This is for the quark contribution. The procedure for $\sigma^{(1)}_{\text{glue}}$ is analogous.
\begin{equation}
\int \frac{1}{s} \int \frac{d^3k}{\xi} \delta^{(3)}(k - \frac{1}{\xi} p_2) \theta(1 - \xi - \delta)
\end{equation}
with \( s = (p'_2 + k)^2 \) and \( \delta = (\Delta - m^2)/Q_2^2 \). Next, we also introduce in the counter term a gluon momentum \( k = p_2(1 - \xi)/\xi \) as follows:

\begin{equation}
\int \frac{1}{s} \int \frac{d^3k}{\xi} \delta^{(3)}(k - \frac{1}{\xi} p_2) \theta(1 - \xi - \delta)
\end{equation}

In the integration of the hard and glue contributions, eqs (23) and (24), it will be necessary to treat the single and double pole terms separately. The double pole terms are non-divergent (the numerator is proportional to \( m^2 \)), but give a finite contribution. They have a malicious \( \cos \theta \) dependence, but can be integrated numerically with a suitable mapping.

In the rest, the integral over \( \cos \theta \) gives an exact cancellation of the single pole terms in the collinear limit \( \cos \theta \rightarrow 1 \) when the radiative terms are symmetrized in \( \cos \theta \), and the collinear logarithm in the counter term (see eqs (A.10) and (A.11)) is represented by

\begin{equation}
\ln(\xi(1 - \xi)n^2/Q_2^2) = \int_0^1 \frac{d \cos \theta}{\cos \theta - (1 - \xi)(1 - \xi)n^2/Q_2^2}^{-1}
\end{equation}

with \( n = \pm 1 \) for the quark and gluon contributions, respectively. Now the sum of the radiative contribution, excluding the double pole terms, and the counterterms is finite for all values of \( \cos \theta \). As the boundaries on all integrals coincide this sum can be integrated numerically.

For clarity we repeat that as the virtual and soft contributions were seen to cancel against their respective counter terms, only the above integrals, which combine the hard Bremsstrahlung and gluonic contribution with their respective counter terms, have to be evaluated numerically. The independence of the result on the soft cut-off \( \Delta \) and the quark mass \( m \) was verified by varying the value used in the computation. The numerical accuracy now suffices: without cuts we find agreement with the analytical calculation presented in the previous section within the statistical accuracy (1–3%).

When applying the cuts one has to define the way to combine the four jets produced in \( eq \rightarrow \nu H^0qg \) and \( eq \rightarrow \nu H^0q\bar{q} \) with \( H^0 \rightarrow b\bar{b} \) into three jets. The algorithm used was to combine the two partons with minimal distance in the \( \eta - \phi \) plane if this distance was less than 1, and else to require that one of the partons did not pass the \((p_1, p)\) cuts. The resulting 3-jet event was then subjected to the cuts described in section 2.
In the collinear and soft limits this is seen to be equivalent to the cuts applied to the Born approximation.\(^8\)

With the experimental cuts applied a numerical accuracy of about 10\% of the corrections is achieved. The reduction in accuracy is caused by small regions in phase space, close to the collinear limit \(\cos \theta = 1\), where the three jet signal is cut but the four jet survives, or vice versa. Of course \textit{at} the collinear limit the kinematical cuts are the same, but for any finite angle between the quark and gluon they differ. The resulting wedge-shaped regions have the original logarithmic singularity, and hence give a finite but small contribution. Unfortunately present-day numerical integration methods \(^9\) do not handle such highly discontinuous functions in a high-dimensional phase space very well.

The results of this calculation are summarized in fig. 6. One can see that the effect of the cuts is to make the corrections negative and larger. This is to be expected, as the requirement to observe exactly three jets cuts away part of the (positive) Bremsstrahlung terms. The almost exact cancellation which keeps the integrated corrections so small is now violated, thus are the relative corrections larger with cuts taken into consideration. The gluonic contribution is much smaller as the gluonic component tends to be softer, which reduces the probability that four jets are observed.

### 3.2.2 Corrections to the decay

The \(\mathcal{O}(\alpha_s)\) correction \(\Gamma^{(1)}\) to the decay width of the Higgs boson into \(b\)-quarks has long been known \(^{24}\). The diagrams for this correction are shown in fig. 7. Recently also the \(\mathcal{O}(\alpha_s^2)\) contributions \(\Gamma^{(2)}\) were evaluated \(^{25}\). In the mass range \(80 \leq M_H \leq 150\text{ GeV}\) and adopting the \(\overline{\text{MS}}\) scheme, the relative corrections \(\Gamma^{(1+2)}/\Gamma^{(0)}\) vary from \(-28\%\) to \(-43\%\). In the applications considered so far in this paper only the branching ratio \(\text{Br}(H^0 \to b\bar{b})\) enters. This is affected less than the \(b\bar{b}\) partial width, at least as long as \(H^0 \to b\bar{b}\) is the dominant decay mode. Numerically, we derive from ref. \(^{11}\) that \(-3.4\% \gtrsim \text{Br}^{(1+2)}/\text{Br}^{(0)} \gtrsim -36\%\) in the mass range considered.

However, these corrections also change in the presence of kinematical cuts. For the moment, let us consider the \(\mathcal{O}(\alpha_s)\) corrections only. A part of these corrections involves the emission of a hard gluon, \(H^0 \to b\bar{b}g\), which can change the configuration of the bottom jets considerably. These events should not all be included in the observable cross section as we demand exactly three jets, of which two originate in the bottom quarks. The difference must be considered an \(\mathcal{O}(\alpha_s)\) correction to the results given in section \(^8\) which already include the QCD corrected branching ratio \(\text{Br}(H^0 \to b\bar{b})\). This correction is scheme independent as only the hard radiation part is involved. It is also expected to be relatively small, as the main effect of the integrated corrections is to introduce a running \(b\) quark mass in the Yukawa coupling. These terms are of order \(\ln(M_H^2/m_b^2)\); they arise from the renormalisation of the \(b\bar{b}H\) coupling and do not contribute here. Furthermore, the effect of the cuts clearly makes the \(\mathcal{O}(\alpha_s)\) contribution negative.

The \(\mathcal{O}(\alpha_s)\) correction to \(\sigma^{(0)}_{\text{obs}}\) described above is given by the difference

\[
\sigma^{(1)}_{\text{decay}} = \sigma^{(1)}_{bbg} - \sigma^{(1)}_{bb}
\]

where

\[
\sigma^{(1)}_{bbg} = \int dx \int dPS_3(eq \rightarrow \nu H^0 q) \sum_{i=1}^2 |\mathcal{M}_i^{(0)}|^2 q_i(x, Q_2^2)
\]

\(^8\)The same algorithm was used in the calculation presented in \(^7\) to estimate the contribution from four jet events.

\(^9\)The QED and electroweak corrections to \(\Gamma(H^0 \to b\bar{b})\) were found in ref. \(^{26}\) to vary between \(-0.6\%\) and \(-1.7\%\) for \(50 < M_H < 150\text{ GeV}\). They have not been included here.
\[
\times \frac{1}{2M_H} \int dPS_3(H^0 \rightarrow b\bar{b}g) |\mathcal{M}_{H^0 \rightarrow b\bar{b}g}|^2 \theta(4\text{-jet cuts}) / \Gamma_{tot} \quad (27)
\]
is the contribution from hard gluon radiation subject to the appropriate 4-jet cuts, while
\[
\sigma_{bb}^{(1)} = \int dx \int dPS_3(eq \rightarrow \nu H^0 q) \sum_{i=1}^{2} |\mathcal{M}_i^{(0)}|^2 q_i(x, Q^2) \frac{\int dPS_2(H^0 \rightarrow \bar{b}\bar{b}) / \Gamma_{tot}}{\int dPS_2(H^0 \rightarrow \bar{b}\bar{b})} \quad (28)
\]
is the corresponding integrated contribution corrected for the effect of the 3-jet cuts. \(\Gamma_{tot}\) is the total width of the Higgs boson. Obviously in the absence of cuts \(\sigma_{bb}^{(1)} = 0\).

The decay matrix element, summed over all spins, \(|\mathcal{M}_{H^0 \rightarrow b\bar{b}g}|^2\), does not depend on the orientation of the \(b\) quark in the Higgs boson cms system; only the 4-jet cuts will be influenced by it. We thus extract the integral over these angles \(\int d\Omega_b\) from the 3-body decay phase space integral. The 2-body phase space integral contains a similar angle, which enters only in the 3-jet cuts. Identifying these two angles we obtain
\[
\sigma_{decay}^{(1)} = \int dx \int dPS_3(eq \rightarrow \nu H^0 q) \sum_{i=1}^{2} |\mathcal{M}_i^{(0)}|^2 q_i(x, Q^2) \times \frac{1}{2M_H} \frac{1}{(2\pi)^5} \frac{1}{8} \int d\phi dE_b dE_{\bar{b}} |\mathcal{M}_{H^0 \rightarrow b\bar{b}g}|^2 / \Gamma_{tot} \times \int d\Omega_b \{\theta(4\text{-jet cuts}) - \theta(3\text{-jet cuts})\}. \quad (29)
\]
where \(\phi\) is the azimuthal angle of the \(\bar{b}\) momentum with respect to the \(b\) momentum. In a Monte Carlo integration this means that for each Bremsstrahlung event one constructs a decay into only \(b\bar{b}\) with the \(b\) quark momentum in the same direction (in the Higgs boson cms). One examines whether this alternative kinematical configuration passes the 3-jet cuts: the non-zero region is the one in which the 3-jet configuration passes its cuts, but the 4-jet does not, or vice versa. This will not happen in the limit of soft or collinear gluon emission, so the result is free of singularities and \(\ln(M_H^2/m_b^2)\) terms.

Finally, we can include important \(O(\alpha_s^2)\) effects by reintroducing the 2-loop integrated branching ratio as
\[
\frac{|\mathcal{M}_{H^0 \rightarrow b\bar{b}g}|^2}{\Gamma_{tot}} = \frac{|\mathcal{M}_{H^0 \rightarrow b\bar{b}g}|^2}{\Gamma^{(0+1+2)}} B_{\nu(0+1+2)}. \quad (30)
\]
The fraction on the r.h.s. of eq. (30) is now almost independent of the (running) \(b\) quark mass, the main effect of which has been absorbed in the branching ratio. Effectively, we thus use the two loop expression for the \(b\bar{b}H^0\) coupling (which is proportional to the \(b\) quark mass) everywhere, while considering the effects of kinematical cuts only up to \(O(\alpha_s)\).

The expression for \(|\mathcal{M}_{H^0 \rightarrow b\bar{b}g}|^2\) used agrees with previous calculations [24]. It is given in the appendix (eq. (A.12)). The resulting correction \(\sigma_{\text{decay}}^{(1)}\) is shown in fig. 6. The numerical result is not sensitive to the precise value of \(m_b\) in the range \(3 \leq m_b \leq 8\) GeV. The correction \(\sigma_{\text{decay}}^{(1)}\) to the decay is larger than the correction \(\sigma_{\text{prod}}^{(1)}\) to the production process as the probability to detect a fourth jet radiated from a final state \(b\)-quark is higher. The rise for low Higgs mass results from the high probability that the \(b\) and \(\bar{b}\) jet become indistinguishable when a hard gluon is emitted in a boosted system.
It should be noted that the size of these corrections critically depends on the way in which the jets are reconstructed. Here we employed a rather simple method, which could be improved upon by using more sophisticated algorithms. The Monte Carlo approach given here allows such studies to be performed in a straightforward way.

4 QED Corrections

The dominant contributions to the QED corrections for $ep$ scattering at high energies can be described in the leading logarithmic approximation \cite{27,28}. For charged current processes, only initial state bremsstrahlung (see fig. 8) from the incoming electron line has to be considered, since radiation from the quark line merely leads to a small extra contribution of $O(3\alpha_e^2/4\alpha_s)$ to the QCD evolution. In this approximation, the QED correction to the Higgs production cross section is given by

$$\sigma^{(1)}(QED) = \frac{\alpha}{2\pi} \int dz \frac{1+z^2}{1-z} \ln \left( \frac{\mu^2}{m^2_{e}} \right) \{ \sigma^{(0)}(zp_1) - \sigma^{(0)}(p_1) \} \quad (31)$$

where $\alpha = 1/137$. The factorization scale $\mu$ is related to the $k_\perp$-integral of the radiated photon, $\mu^2 \approx Q^2 \approx s$. We have chosen $\mu^2 = s$ (see ref. \cite{27}).

In fig. 8 the QED correction $\sigma^{(1)}(QED)$ to the total production cross section is shown as a function of $M_H$ using the parametrization MT (DIS, set 2) \cite{12}. In the mass range considered, it amounts to $-4\%$ to $-5\%$ for the choice $\mu^2 = s$. The QED radiative corrections are rather small because of the small ratio $M^2_H/s \lesssim 0.012$. A similar behaviour was recently observed for the correction to heavy flavour production in neutral current $ep$-scattering \cite{29}, where for small values of $4m^2_{Q}/s$ only small negative corrections are obtained. The correction to the observable cross section after cuts is also shown in fig. 8. The relative correction $\sigma_{obs}^{(1)}(QED)/\sigma_{obs}^{(0)} = -4.5\%$ to $-5.8\%$.\footnote{The leading QED corrections to the $W$ and top backgrounds were also calculated and found to be comparable to the corrections to the signal.}

These results are only weakly influenced by the choice of the parton distributions.

5 Conclusions

We calculated the $O(\alpha_s)$ QCD corrections to Higgs boson production in $e^-p$ collisions via the $WW$ fusion process followed by the decay $H^0 \to bb$ in the zero-width approximation. Results are obtained for the fully integrated cross section, as well as for the observable cross section after application of the kinematical cuts suggested in ref. \cite{7}. We find surprisingly small corrections of the order of a percent and less for the uncutted cross section, and larger and negative corrections when the cuts are included. Our results are at variance with the expectation expressed in ref. \cite{7}.

We conclude that the QCD corrections that have to be added, once the known corrections to the branching ratio are included, in order to obtain the complete $O(\alpha_s)$ corrections to $e^-p \to \nu H^0X; H^0 \to bb$, are in fact very small; the only sizeable effects originate in the selection cuts which mainly affect the hard gluon radiation. The following factors contribute to make the corrections this small:

- The cross section is dominated by $F_2$, which is not renormalized in the DIS scheme. The difference between the DIS and $\overline{MS}$ structure functions suggest that the corrections would change by a few percent when changing to the $\overline{MS}$ scheme.
- The scale $Q_0^2$ is large everywhere ($Q_0^2 \gtrsim 100$ GeV$^2$). This means that $\alpha_s$ is small ($\alpha_s/2\pi \approx 0.02$).
• No large logarithms enhance the corrections, as these have already been absorbed into the evolution of the distribution functions.

The main effect due to the cuts occurs at the decay vertex, where the extra gluon is either detected as a fourth jet or causes the $b$ jets to be too close together. These effects however depend on the algorithms used to define the jets.

In addition, we also calculated the effect due to initial state QED bremsstrahlung from the electron. This correction decreases the observable cross section by roughly 5%. The relative QCD and QED corrections to the observable cross section are summarized in fig. 10. One can conclude from our results that the analysis performed in ref. 7 is not changed significantly by higher order effects as far as the signal is concerned. What remains to be done is a corresponding calculation for the background processes.

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A Appendix

In this appendix we list explicit expressions used in the calculation of the $O(\alpha_s)$ QCD corrections to the observable cross section as discussed in section 3.2.

In the regularization scheme adopted in this calculation, the $O(\alpha_s)$ modifications of the quark distributions due to the virtual, soft and hard Bremsstrahlung terms and the gluon contribution are given by

\[ q^{(1)}_{i,virt}(x, Q^2) = \frac{\alpha_s}{2\pi} C_F \left\{ -4 + \frac{\pi^2}{3} + \ln \left( \frac{m^2 Q^2}{\Lambda^2} \right) \ln \left( \frac{m^2}{Q^2} \right) - 2 \ln \left( \frac{m^2}{m^2} \right) - 3 \ln \left( \frac{m^2}{Q^2} \right) \right\} q_i(x, Q^2) \]

(A.1)

\[ q^{(1)}_{i,soft}(x, Q^2) = \frac{\alpha_s}{2\pi} C_F \left\{ \frac{3}{2} - 2\pi^2 + \frac{1}{2} \ln \left( \frac{\Delta}{m^2} \right) + 2 \ln \left( \frac{\Delta^2 Q^2}{\Delta^2} \right) \left( 1 + \ln \left( \frac{m^2}{Q^2} \right) \right) - \ln^2 \left( \frac{\Delta}{m^2} \right) \right\} \times q_i(x, Q^2) \]

(A.2)

\[ q^{(1)}_{i,hard}(x, Q^2) = \frac{\alpha_s}{2\pi} \int_{1-\delta}^{1} d\xi \hat{P}_{qq}(\xi, \frac{m^2}{Q^2}) q_i(x/\xi, Q^2) \]

(A.3)

\[ q^{(1)}_{i,glue}(x, Q^2) = \frac{\alpha_s}{2\pi} \int_{1-\delta}^{1} d\xi \hat{P}_{qG}(\xi, \frac{m^2}{Q^2}) G(x/\xi, Q^2) \]

(A.4)

with $\Delta = (p_1^2 + k_2^2 - m^2)^2/\Delta^2$ and

\[ \hat{P}_{qq}(\xi, \frac{m^2}{Q^2}) = C_F \left\{ \frac{1 + \xi^2}{1 - \xi} \ln \left[ \frac{Q^2}{m^2 \xi (1 - \xi)} \right] - \frac{3 \xi^2 + \xi - \frac{1}{2}}{1 - \xi} \right\} \]

(A.5)

\[ \hat{P}_{qG}(\xi, \frac{m^2}{Q^2}) = \frac{1}{4} \left\{ (1 - 2\xi + 2\xi^2) \ln \left[ \frac{Q^2 (1 - \xi)}{m^2 \xi} \right] - 8\xi^2 + 8\xi - 1 \right\} \]

(A.6)

The dependence on $\lambda$ cancels in the sum of the soft and virtual parts:

\[ q^{(1)}_{i,virt+soft}(x, Q^2) = -\frac{\alpha_s}{2\pi} C_F \left\{ \frac{5}{2} + \frac{\pi^2}{3} + \frac{7}{2} \ln \left( \frac{\Delta}{Q^2} \right) + \ln \left( \frac{\Delta}{Q^2} \right) + 2 \ln \left( \frac{m^2}{Q^2} \right) \left[ \ln \left( \frac{\Delta}{Q^2} \right) + \frac{3}{4} \right] \right\} \times q_i(x, Q^2). \]

(A.7)

Furthermore, $q^{(1)}_{i,hard}(x, Q^2)$ may be rewritten as

\[ q^{(1)}_{i,hard}(x, Q^2) = \frac{\alpha_s}{2\pi} \int_{0}^{1-\delta} d\xi \left\{ \hat{P}_{qq}(\xi, \frac{m^2}{Q^2}) \left[ \theta(\xi - x) \frac{1}{\xi} q_i(x/\xi, Q^2) - q_i(x, Q^2) \right] \right\} \]

\[ + \frac{\alpha_s}{2\pi} \int_{0}^{1-\delta} d\xi \hat{P}_{qq}(\xi, \frac{m^2}{Q^2}) q_i(x, Q^2). \]

(A.8)

In the first integral the limit $\delta \rightarrow 0$ can be taken because terms of $O(\delta)$ may be neglected. The second integral can be calculated analytically yielding

\[ \frac{\alpha_s}{2\pi} C_F \left\{ \frac{5}{2} + \frac{\pi^2}{3} + \frac{7}{2} \ln \delta + \ln^2 \delta + 2 \ln \left( \frac{m^2}{Q^2} \right) \left[ \ln \delta + \frac{3}{4} \right] \right\} q_i(x, Q^2). \]

(A.9)

In the limit $m \rightarrow 0$, the contribution (A.9) just cancels the term (A.7) in the sum $q^{(1)}_{i,virt+soft} + q^{(1)}_{i,hard}$.

The differential cross sections for hard gluon Bremsstrahlung for an incoming quark or antiquark and the gluon contribution were derived using FORM [30] and are given by

\[ d\sigma^{(1)}_{hard} = \frac{1}{2z_s} \sum_{i=1}^{3} \left| M^{(1)}_{i,hard} \right|^2 q_i(z, Q^2) \theta(\hat{s} - \Delta) dPS d z \]

13
Here $k$ denotes the momentum of the extra outgoing parton and $p_3 = z P$ is the momentum of the incoming parton. The decay $H^0 \to b \bar{b}$ is again inserted when the observable cross section is calculated. The matrix element squared for the hard Bremsstrahlung part of the decay width used in section 3.2.2 is given by

$$|\mathcal{M}_{H^0 \to b \bar{b}}|^2 = \frac{3 g^2 m_W^2}{2 M_W^2} 4 \pi \alpha_s C_F \left\{ +8 + 4 (p_3 k)/(p_3 k) + 8 (p_3 p_6)/(p_3 k) + 8 (p_3 p_6)/(p_3 k) \\
+ 8 (p_3 p_6)/(p_3 k) + 4 (p_3 k)/(p_3 k) - 4 m_b^2 (p_3 k)/(p_3 k)^2 - 4 m_b^2 (p_3 p_6)/(p_3 k) \\
- 4 m_b^2 (p_3 p_6)/(p_3 k)^2 - 8 m_b^2 (p_3 p_6)/(p_3 k)/(p_3 k) - 4 m_b^2 (p_3 k)/(p_3 k)^2 \\
- 4 m_b^2 (p_3 k) + 4 m_b^2 (p_3 k)^2 + 4 m_b^2 (p_3 k)^2 \right\}. \quad (A.12)$$

The two-loop corrected decay width for $H^0 \to b \bar{b}$ is derived in ref. [24]:

$$\Gamma^{(0+1+2)} = \frac{3 g^2 m_b^2}{32 \pi} \left( \frac{M_H^2 - 4 m_b^2}{M_H^2} \right)^{3/2} \left( 1 + 1.803 \alpha_s + 2.953 \alpha_s^2 \right), \quad (A.13)$$

with the running b quark mass $\tilde{m}_b$ and coupling constant $\alpha_s$ taken at the scale $M_H^2$. As stated in the text, the magnitude of the $O(\alpha_s)$ corrections discussed in this paper does not depend significantly on the value chosen for this mass.
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B Figures

Figure 1: The Born diagram for Higgs boson production via WW fusion.

Figure 2: Born cross sections using the parton distributions by Morfin and Tung (MT2). Long-dashed line: $\sigma_{tot}^{(0)}$ for MT2($\overline{MS}$); full line: $\sigma_{tot}^{(0)}$ for MT2(DIS); dash-dotted line: $\sigma_{tot}^{(0)} \times Br(H^0 \to b\bar{b})$ for MT2(DIS); short-dashed line: $\sigma_{obs}^{(0)}$ as defined in the text.

Figure 3: Differential distributions in $x$, $Q^2$, the transverse momentum $p_\perp$ and rapidity $\eta$ of the Higgs boson for $M_H = 100$ GeV using the Born approximation. The dashed lines give the distributions after application of the selection cuts proposed in ref. 7.
Figure 4: The $\mathcal{O}(\alpha_s)$ diagrams contributing to the QCD corrections at the hadronic vertex.

Figure 5: The $\mathcal{O}(\alpha_s)$ QCD corrections to $\sigma_{\text{tot}}$ in the DIS scheme. Full line: $\sigma_{\text{tot}}^{(1)}(QCD)$, long-dashed line: $\sigma_L^q$, dash-dotted line: $\sigma_{L}^{G}$, and short-dashed line: $\sigma_{3}^{q}$.

Figure 6: The $\mathcal{O}(\alpha_s)$ QCD corrections including the cuts explained in section 2. The corrections to the production process $ep \to \nu H^0 X$ from quarks and gluons and to the decay process $H^0 \to b\bar{b}$ are shown separately. The total sum $\sigma_{\text{obs}}^{(1)}(QCD)$ is the appropriate $\mathcal{O}(\alpha_s)$ correction to the observable cross section $\sigma_{\text{obs}}^{(0)}$ (which already includes the QCD corrected $Br(H^0 \to b\bar{b})$). The error bars indicate the precision of the Monte Carlo calculation.

Figure 7: The $\mathcal{O}(\alpha_s)$ diagrams contributing to the QCD corrections to the decay vertex.

Figure 8: Diagram representing the leading QED correction to charged current $ep$ processes.

Figure 9: The leading QED corrections to the total (dashed line) and observable (full line) cross sections for $e^- p \to \nu H^0 X; H^0 \to b\bar{b}$ (using $\mu^2 = s$).

Figure 10: The relative corrections $\sigma_{\text{obs}}^{(1)}/\sigma_{\text{obs}}^{(0)}$ to the observable cross section for $e^- p \to \nu H^0 X; H^0 \to b\bar{b}$, including the cuts given in section 2. The QCD and the leading QED corrections are shown separately. The error bars indicate the statistical precision of the Monte Carlo calculation.