Type I superconductivity in Dirac materials

B Ya Shapiro1, I Shapiro1, Dingping Li2,3,5 and Baruch Rosenstein4

1 Department of Physics, Institute of Superconductivity, Bar-Ilan University, Ramat-Gan 52900, Israel
2 Collaborative Innovation Center of Quantum Matter, Beijing, People’s Republic of China
3 School of Physics, Peking University, Beijing 100871, People’s Republic of China
4 Department of Electrophysics, National Chiao Tung University, Hsinchu, Taiwan, Republic of China

E-mail: vortexbar@yahoo.com

Received 13 May 2018, revised 3 July 2018
Accepted for publication 12 July 2018
Published 26 July 2018

Abstract
Superconductivity of the second kind was observed in many 3D Weyl and Dirac semi-metals while in the PdTe2, superconductivity is clearly of the first kind. This is very rare in Dirac semi—metals, but is expected in clean conventional metallic superconductors with 3D parabolic dispersion relation. The conduction bands in this material exhibit the linear (Dirac) dispersion only along two directions, while in the third direction the dispersion is parabolic. Therefore the ‘hybrid’ Dirac-parabolic material is intermediate between the two extremes. A microscopic pairing theory is derived for arbitrary tilt parameter of the 2D cone and used to determine anisotropic coherence lengths, the penetration depths and applied to recent extensive experiments. Magnetic properties of these superconductors are then studied in the parallel to the layers magnetic field on the basis of microscopically derived Ginzburg–Landau effective theory for the order parameter.

Keywords: Dirac semi-metals, magnetic properties of superconductors, Ginzburg-Landau theory

(Some figures may appear in colour only in the online journal)

1. Introduction

Dispersion relation near Fermi surface in recently synthesized two and three dimensional Dirac (Weyl) semi-metals [1–3] is linear, qualitatively distinct from conventional metals, semi—metals or semiconductors in which it is parabolic. In type I Dirac semi-metals (DSM), the band inversion results in Dirac points in low-energy excitations being anisotropic massless ‘relativistic’ fermions. More recently type-II DSM with Dirac cone strongly tilted, so that they can be characterized by a nearly flat band at Fermi surface were discovered [4].

The type-II DSM also exhibit exotic properties different from the type-I ones. Many Dirac materials are known to be superconducting. A detailed study of superconductivity in DSM under hydrostatic pressure revealed a curious dependence of critical temperature of the superconducting transition on pressure. The critical temperature $T_c$ in some of these systems like HfTe$_5$ shows [5] a maximum as a function of pressure. Superconductivity happens to be of the second kind with penetration depth $\lambda$ much larger than the coherence length $\xi$. However in recently studied material [6] PdTe$_2$ it was demonstrated that superconductivity is of the first kind.

Although various pairing mechanisms in DSM turned superconductors have been considered [7–9], experiments indicate the conventional phonon mediated one. If the Fermi level is not situated too close to the Dirac point, the BCS type pairing occurs, otherwise a more delicate formalism should be employed [10]. A theory predicted possibility of superconductivity in the type II Weyl semimetals was developed recently in the framework of Eliashberg model [11, 12]. In particular the case strongly layered 2D Dirac materials in clean limit like MoTe$_2$ [13] was considered in [14]. The critical fields, coherence lengths magnetic penetration depths and the Ginzburg number characterizing the strength of fluctuations were found. It turned out that in most cases the superconductivity was of the second kind. Moreover the thermal fluctuations were shown to be strong enough to qualitatively affect the Abrikosov vortex phase diagram. The vortex lattice ‘melts’ into the vortex liquid. This is reminiscent of a well known (possibly non—Dirac semi-metal) layered dichalcogenides

Author to whom any correspondence should be addressed.
superconductor NbSe$_2$ that is perhaps the only low $T_c$ material with fluctuations strong enough to exhibit vortex lattice melting [15].

The layered superconductor is similar to the present case in that the Dirac spectrum is two dimensional and parabolic in the third direction. However it is qualitatively different in that the anisotropy in the third direction is extremely strong. In PdTe$_2$ the dispersion relation is parabolic, but anisotropy is mild. Thus the ‘hybrid’ Dirac-parabolic materials can be viewed as an intermediate between the two extremes, 3D DSM and conventional metals in the ‘clean limit’ [16].

Superconductivity in PdTe$_2$ with a transition temperature $T_c$ of 1.5 K was discovered in 1961 by Guggenheim et al [17]. The material was revisited recently when the type II Dirac dispersion relation was observed by ARPES [18]. In this material, the pair of type-II Dirac points disappears at 4.7 GPa. It was recently predicted by theories and confirmed in experiments, making PdTe$_2$ [6] the first material that processes both superconductivity and type-II Dirac fermions under proper choice is that the material PdTe$_2$ in many aspects behaves as a 3D anisotropic material. 

In the present paper we extend the study of superconductivity in the ‘hybrid’ Dirac-parabolic clean semimetals. The phenomenological Ginzburg–Landau theory for superconducting DSM of the arbitrary type is microscopically derived and used to establish magnetic phase diagram. In particular the Abrikosov parameter $\kappa^2$ used to distinguish between the superconductivity of the first from the second kind is determined. We applied our theory to explanation of the recent studied material, PdTe$_2$ as a representative example of hybrid layered DSM. A major reason is that magnetic properties of this superconductor were investigated in a wide range of temperatures and magnetic fields with the magnetic field directed parallel to the layers. An additional advantage of this choice is that the material PdTe$_2$ in many aspects behaves as a 3D anisotropic material.

This paper is motivated by very recent experiment [20] where the magnetic and transport measurements of the single crystals Dirac semimetal (DSM) PdTe$_2$ unambiguously show that the 3D Dirac layered semimetal PdTe$_2$ is a first kind superconductor while the rest of the DSM are usually of kind two. In this experiment the magnetic field was directed parallel to the layers [20] while the layers were strongly coupled. Since in our recent theory [12] the magnetic field was directed perpendicular to the weakly interacting layers it cannot be used to explain this experiment.

In the present paper a microscopic pairing theory is constructed and used to determine anisotropic coherence lengths, the penetration depths, thermodynamic critical field. The results are applied to recent extensive experiments on PdTe$_2$.
potential \( \mu = 0 \). We restrict ourself to the case of just one left handed and one right handed Dirac points, typically but not always separated in the Brillouin zone. Generalization to include the opposite chirality and several ‘cones’ is straightforward. We assume that different valleys are paired independently and drop the valley indices (multiplying the density of states by number of valleys).

Dispersion relations of normal electrons are different in Type I and Type II DSM as it’s presented in figures 2 and 3 (see \([12]\)).

The effective electron–electron attraction due to the electron–phonon interaction opposed by Coulomb repulsion (pseudopotential) mechanism creates pairing below \( T \) and the Fermi energy crossing the dispersion relation surface. Here the two Weyl operators are, (tilt vector \( \mathbf{w} \) is assumed to be directed along \( x \)-axes):

\[
L_{1\gamma\beta}^{\dagger} = [(i\omega + \mu' + i\mathbf{w} \cdot \nabla_x) \delta_{\gamma\beta} - i\sigma^i_{\gamma\beta} \nabla_i] ;
\]

\[
L_{2\gamma\beta}^{\dagger} = \left[-i\omega + \mu' + i\mathbf{w} \cdot \nabla_x\right] \delta_{\gamma\beta} - i\sigma^i_{\gamma\beta} \nabla_i] .
\]

Figure 2. Spectrum of normal Weyl semimetal (a) type I \((w/v = 0.4)\). (b) Strongly tilted Dirac cone for the type II semimetals \((w/v = 1.3)\) and the Fermi energy crossing the dispersion relation surface.

Figure 3. Equipotential surfaces inside the Brillouin zone in \( p_x, p_y \) plane for type I \((w/v = 0.4)\) and type II \((w/v = 1.3)\) DSM.
The effective 2D chemical potential was denoted by \( \mu' \equiv \mu - \frac{g^2}{\varepsilon} \).

The gap function defined as

\[
\Delta_{\beta\alpha}(r) = g^2 T \sum_{\omega} f_{\beta\alpha}^+(r,\omega).
\]  

(6)

The gap function in the s-wave channel is \( \Delta_{\alpha\beta}(r) = \sigma_{\alpha\beta} \Delta(r) \). This is the starting point for derivation of the GL free energy functional of \( \Delta(r) \).

3. Derivation of the GL equations

In this section, the Ginzburg–Landau equations in a homogeneous material (including the gradient terms) is derived. Magnetic field and fluctuations effects will be discussed in the next two sections by generalizing the basic formalism.

\[
b(p) = \left\{ \begin{array}{l}
 g_{21}^1 (p) g_{12}^1 (-p) g_{11}^2 (-p) g_{21}^1 (p) + g_{21}^2 (p) g_{12}^1 (-p) g_{21}^2 (-p) g_{21}^1 (p) + \\
 + g_{22}^2 (p) g_{12}^1 (-p) g_{11}^1 (-p) g_{21}^2 (p) + g_{11}^2 (p) g_{22}^1 (-p) g_{11}^1 (-p) g_{21}^2 (p) + \\
 + g_{12}^1 (p) g_{12}^1 (-p) g_{12}^2 (p) + g_{12}^2 (p) g_{12}^1 (p) +
\end{array} \right\}.
\]

(13)

Normal Green function are obtained [14] from equations equation (8):

\[
\begin{align*}
g_{12}^2(p) &= z^{-1}(i\omega + \mu - \mathbf{w}p); \quad g_{12}^1(p) = -z^{-1}v_p e^{-i\theta}; \\
g_{11}^1(p) &= z^{-1}(i\omega + \mu - \mathbf{w}p); \quad g_{21}^1(p) = -z^{-1}v_p e^{i\theta}; \\
g_{11}^2(p) &= z^{-1}(i\omega + \mu - \mathbf{w}p); \quad g_{12}^2(p) = -z^{-1}v_p e^{-i\theta}; \\
g_{22}^2(p) &= z^{-1}(i\omega + \mu - \mathbf{w}p); \quad g_{21}^2(p) = -z^{-1}v_p e^{i\theta}. 
\end{align*}
\]

(14)

To derive the GL equations including the derivative term one needs the integral form of the Gor’kov equations (see appendix A), equation (4):

\[
g_{\beta\alpha}(r,r',\omega) = g_{\beta\alpha}^1(r-r',\omega) - \int_{r''} g_{\beta\alpha}^1(r-r'',\omega) \Delta_{\alpha\beta}(r'') d^2r''
\]

\[
\Delta_{\alpha\beta}(r'') f_{\beta\alpha}^+(r'',\omega); \\
f_{\beta\alpha}^+(r,r',\omega) = \int_{r''} g_{\beta\alpha}^2(r-r'',\omega) \Delta_{\alpha\beta}(r'') d^2r''
\]

\[
\times \left\{ g_{\beta\alpha}^1(r''-r',\omega) - \int_{r''} g_{\beta\alpha}^1(r''-r'',\omega) \Delta_{\alpha\beta}(r'') d^2r'' \right\}
\]

\[
\Delta_{\alpha\beta}(r'') f_{\beta\alpha}^+(r'',\omega).
\]

(7)

Here \( g_{\beta\alpha}^1(r,r') \) and \( g_{\beta\alpha}^2(r,r') \) are GF of operators \( L_{\gamma\beta}^1 \) and \( L_{\gamma\beta}^2 \):

\[
L_{\gamma\beta}^1 g_{\beta\alpha}^1(r,r') = \delta^{\gamma\delta}(r-r'); L_{\gamma\beta}^2 g_{\beta\alpha}^2(r,r') = \delta^{\gamma\delta}(r-r').
\]

(8)

This will be enough to derive the GL expansion to the third order in the gap function \( \Delta(r) \) that will be used as an order parameter [23].

Using the first and the second iteration of equations equation (7) and specializing on the case \( r = r' \), and specify the Fourier the Fourier transformation for the GF,

\[
g_{\beta\alpha}^{2,1}(r) = \sum_p g_{\beta\alpha}^{1,1}(p) e^{ipr}, \Delta(r) = \sum_q \Delta(q) e^{iqr}
\]

(9)

one rewrites the Gorkov’s equation equation (4) as [see details in [14]],

\[
\Delta(r) = \frac{g^2 T}{2} \sum_{\omega,p} \left\{ a(p) \Delta(r) + C_{\beta\alpha}^{1,1}(p) \frac{\partial^2 \Delta(r)}{\partial r_{\beta} \partial r_{\alpha}} - b(p) \Delta^3(r) \right\}.
\]

(10)

The function appearing in an expression for the coefficient \( a \) is:

\[
a(p) = g_{21}^1 (p) g_{12}^1 (p) + g_{11}^1 (p) g_{22}^1 (p) + g_{12}^2 (p) g_{12}^1 (p) + g_{11}^1 (p) g_{22}^1 (p).
\]

(11)

while the gradient term coefficients take a form:

\[
C_{\beta\alpha}^{1,1}(p) = \frac{1}{2} \left\{ \frac{\partial g_{\beta\alpha}^{1,1}(p)}{\partial p_\beta} \frac{\partial g_{\beta\alpha}^{1,1}(p)}{\partial p_\alpha} - \frac{\partial g_{\beta\alpha}^{1,1}(p)}{\partial p_\alpha} \frac{\partial g_{\beta\alpha}^{1,1}(p)}{\partial p_\beta} \right\}.
\]

(12)

The cubic term’s coefficient is given by

\[
\Delta(r) = \frac{g^2 T}{2} \sum_{\omega,p} \left\{ a(p) \Delta(r) + C_{\beta\alpha}^{1,1}(p) \frac{\partial^2 \Delta(r)}{\partial r_{\beta} \partial r_{\alpha}} - b(p) \Delta^3(r) \right\}.
\]

4. Critical temperature and the linear term in GL expansion.

4.1. Critical temperature

The linear terms in the GL expansion read:

\[
a(T) = T \sum_{\omega,p} a(p) - \frac{1}{g^2},
\]

(15)

while the critical temperature \( T_c \) is defined by the condition \( a(T_c) = 0 \). Substituting GF of equations (14) into equation (11), one obtains in dimensionless variables, \( \varepsilon = (2n+1), \sigma = vp/T, \varepsilon_z = p_z^2/2mT, \pi = \mu/T \),
\[ a(T) = \frac{1}{\lambda} - \frac{3}{16\pi^2} \frac{1}{\bar{p}^{3/2}} \sum_{n} \int_{\epsilon_{\text{shell}}} \frac{\epsilon d\epsilon}{\sqrt{\epsilon_{\text{z}}}} d\theta \frac{\epsilon_{\text{z}}}{(\epsilon_{\text{z}} + \epsilon_{\text{z}} + \Phi_{\text{z}}^2)^2} \left( \epsilon_{\text{z}}^2 + (\Phi_{\text{z}}^2) \right) \left( \epsilon_{\text{z}}^2 + (\Phi_{\text{z}}^2)^2 \right). \]  

(16) 

Here \( \Phi = (\bar{p} - \epsilon_{\text{z}} - \kappa \epsilon \cos \theta) \), \( \lambda = g^2 D(\mu) = \lambda_{gf} \), and the density of states of the normal state electrons (per spin and valley) is 

\[ D(\mu) = \frac{\sqrt{2m\mu}}{12\pi^2} f(\kappa). \]  

(17) 

Dimensionless constant \( \lambda_0 \) is the electron–electron strength for zero tilt parameter \( \kappa \).

The function (see details in appendix C) 

\[ f(\kappa) = \frac{1}{2\pi} \int_{\phi=0}^{\pi} \frac{\text{sign}(\kappa \cos \theta + 1)}{(\kappa \cos \theta + 1)^2} d\theta \]  

(18) 

is different for Type I and Type II DSM, but has the same form as in 2D DSM [12, 14]. For the type I WSM, \( \kappa < 1 \), in which the Fermi surface is a closed ellipsoid, it is given by: 

\[ f = \frac{1}{(1 - \kappa^2)^{3/2}}. \]  

(19) 

In the type II phase, \( \kappa > 1 \), the Fermi surface becomes open, extending beyond the Brillouin zone, and the corresponding expression is: 

\[ f = \frac{\kappa^2}{\pi (\kappa^2 - 1)^{3/2}} \left\{ 2\sqrt{1 + \kappa - 1 + \log \left[ \frac{2(\kappa^2 - 1)}{\kappa (1 + \sqrt{1 + \kappa})^2} \delta \right] \right\}. \]  

(20) 

Here \( \delta \) is an ultraviolet cut off parameter \( \delta = a\Omega / \pi w \) (see appendix C), where \( a \) is an interatomic spacing.

The integration in equation (16) is performed in the BCS shell around the chemical potential:

\[ \epsilon + \epsilon_{\text{z}} + \bar{\Omega} > \bar{\mu} > \epsilon + \epsilon_{\text{z}} - \bar{\Omega}. \]  

(21) 

After Matsubara frequencies summation one obtains, 

\[ a(T) = \frac{1}{\lambda} - \frac{3}{8\pi^2} \frac{1}{\bar{p}^{3/2}} \int_{\epsilon_{\text{z}}}^{\epsilon_{\text{z}} + \epsilon_{\text{z}} + \Phi_{\text{z}}^2} \frac{\text{sign}(\kappa \cos \theta + 1)}{(\kappa \cos \theta + 1)^2} d\theta \left[ \tanh \frac{\epsilon_{\text{z}} + E - \bar{\mu}}{2} + \tanh \frac{\epsilon_{\text{z}} - E - \bar{\mu}}{2} \right] \cdot \]  

\[ a(T) = \frac{1}{\lambda} - \frac{3}{8\pi^2} \frac{1}{\bar{p}^{3/2}} \int_{\epsilon_{\text{z}}}^{\epsilon_{\text{z}} + \epsilon_{\text{z}} + \Phi_{\text{z}}^2} \frac{d\epsilon}{\sqrt{\epsilon_{\text{z}}}} \cdot \]  

\[ \tanh \frac{\epsilon_{\text{z}} + E - \bar{\mu}}{2} + \tanh \frac{\epsilon_{\text{z}} - E - \bar{\mu}}{2} \right], \]  

(22) 

where \( E = \kappa \epsilon \cos \varphi + \epsilon \). Performing integration in equation (22), one obtains for \( T_c \) and \( a(T) \) usual BCS expressions,

\[ T_c = 1.14 \Omega \exp \left( -\frac{1}{f(\kappa)} \lambda_0 \right). \]  

(23) 

The critical temperature as a function of the cone tilt parameter \( \kappa \) is presented in figure 4.

This behavior in 2D case was discussed in details in our previous paper [12]. In fact a Weyl semimetal with tilted dispersion relation Dirac cones undergoes a topological (‘2.5’) phase transition from type-I to type-II phases at \( \kappa = 1 \). At this point the Fermi surface of the WSM changes dramatically, developing a flat band (see for example figures 3 and 4 of the [12]. As a result the density of states indeed diverges and BCS—like methods for weak electron–phonon coupling become inapplicable. Far enough, on the Type II side of the transition \( \kappa > 1 \) DOS does not diverge. It is still dominated by the contribution of the Dirac points since the divergence is just logarithmic and parts of the contribution in which the dispersion should be modified be order \( nh/\alpha \) away from the Dirac points, where \( \alpha \) is lattice spacing (see details in appendix C).

4.1.1. The gradient and the cubic terms of the GL equation. Non-diagonal components of the second derivative tensor \( C_{ik} \) are zero due to the reflection symmetry in \( p_t \) direction, when the cone tilt vector \( w \) is directed along the \( x \) axis. Using equations (12) and (14), we obtain for diagonal components.

\[ C_{zz} = \frac{3h^2 \sqrt{T_c}}{4\pi^2 m \mu^{1/2}} \sum_{\omega} \int d\epsilon d\lambda d\bar{\mu} \left[ \epsilon d\epsilon + \lambda d\lambda d\bar{\mu} \right] Z \cdot \]  

\[ \left\{ 4\epsilon^2 (\varphi^2 + \Phi^2) + (\varphi^2 + \Phi^2)^2 + 2\Phi^2 \varphi^2 + \epsilon^4 \right\}, \]  

(24) 

where \( Z = (\varphi^2 + (\Phi - \varphi)^2)^2 \left( \varphi^2 + (\Phi + \varphi)^2 \right)^2 \). In the Dirac directions,
\[ C_{xx} = \frac{3\hbar^2}{8\pi T^2 P^2} \sum_{\omega} \int \varepsilon d\varepsilon d\theta \frac{d\varepsilon}{\varepsilon} \left( \frac{\varepsilon^2}{\sqrt{\varepsilon}} \right) \left( + (2\Phi_\varepsilon \cos \theta + \varepsilon^2 \cos 2\theta + \Phi^2 - \omega^2)^2 + 8 (\varepsilon \sin \theta)^2 (\kappa \Phi + \varepsilon \cos \theta)^2 + \omega^2 (\pi - \varepsilon)^2 + 8 \omega^2 (\kappa \Phi + \varepsilon \cos \theta)^2 + 2 (\kappa \Phi^2 + 2\Phi \cos \theta + \kappa \varepsilon^2) + 2 \omega^4 \kappa^2 - 4\omega^2 (\kappa^2 \Phi^2 + 2\kappa \Phi \varepsilon \cos \theta + \kappa^2 \varepsilon^2) \right) \] \[ C_{yy} = \frac{3\hbar^2}{4\pi T^2 P^2} \sum_{\omega} \int \varepsilon d\varepsilon d\theta \frac{d\varepsilon}{\varepsilon} \left( \frac{\varepsilon^2}{\sqrt{\varepsilon}} \right) \left[ (\varepsilon^2 - 2\varepsilon^2 \sin^2 \theta - \Phi^2 + \varepsilon^2)^2 + 4 \left( (\varepsilon^2 \sin \theta \cos \theta)^2 + \omega^2 \Phi^2 \right) \right] \] \[ \xi_x^2 = C_{xx}, \xi_y^2 = C_{yy}, \xi_z^2 = C_{zz}. \] 

After after summation over Matsubara frequencies the integration over momenta were performed numerically in a wide range of tilt parameter \( \kappa \) and presented in figure 5. Figures 5(a) and (b) defined the coherence and penetration length in-plane \((x)\) and extra-plane \((z)\) directions correspondingly.

Substituting GF from equations (14) into (13), one can express the cubic term in the form

\[ \beta = \frac{3}{4\pi T^2 P^2} \sum_{\omega} \int \frac{d\varepsilon}{\sqrt{\varepsilon}} d\theta d\varepsilon \frac{\varepsilon^2}{(\pi - \varepsilon)^2} \left[ \varepsilon^2 + (\Phi - \varepsilon)^2 \right] \left[ \varepsilon^2 + \omega^2 + \Phi^2 \right] \left[ \varepsilon^2 + \omega^2 + \Phi^2 + \Phi' \right] \left[ \varepsilon^2 + \omega^2 + \Phi' \right] \left[ \varepsilon^2 + \omega^2 + \Phi^2 \right] \left[ \varepsilon^2 + \omega^2 + \Phi^2 + \Phi' \right] \left[ \varepsilon^2 + \omega^2 + \Phi^2 \right]. \] 

Figure 5. Coherence length \( \xi \) (bold lines) and penetration lengths (dashed lines) \( \lambda \sqrt{Z} \) as functions on the Dirac cone tilt parameter \( \kappa \). Same parameters as in figure 4.

where \( \Phi = \pi - \varepsilon - \kappa \varepsilon \cos \theta, \Phi' = \pi - \varepsilon + \kappa \varepsilon \cos \theta. \)

4.2. Free GL energy for 3D DSM superconductor and penetration depths in London limit

Effects of the external magnetic field are accounted for by the minimal substitution, \( \nabla \rightarrow D = \nabla - \frac{\Phi}{c} A \) in the GL equation equation (10) due to gauge invariance. The GL equation in the presence of magnetic field allows the description of the magnetic response to homogeneous external field. We start from the strong field that destroys superconductivity.

Density of superconducting currents is obtained by the variation of the free energy functional including the magnetic energy,

\[ F = \int d^3 r \left( \frac{\kappa}{2} |\nabla \Delta|^2 - \tau |\Delta|^2 + \frac{\beta}{2} |\Delta|^4 \right) + \frac{(\nabla \times A)^2}{8\pi} \] 

with respect to components of the vector potential:

\[ J_i = D(\mu) \frac{2ei\varepsilon}{h} \Delta(\mathbf{r}) D_i \Delta^* (\mathbf{r}) + c.c.. \] 

Here \( D(\kappa, \mu) = D_0 (0, \mu) f(\kappa) \) is the effective 3D density of states.

Within the London approximation, with magnetic field parallel to the \( x \) direction (see figure 1), taking the order parameter in the form \( \Delta(\mathbf{r}) = \Delta e^{i\theta} \), one obtains,
\[ J_i = \frac{4e}{h} D(\mu) \xi^2 \Delta^2 \left( \partial \varphi - \frac{2e}{c} A_i \right). \]  

The London penetration lengths in our case of DSM with parabolic dispersion relation along z axis are:

\[ \lambda_z^2(T) = \frac{c^2 h^2}{32 \pi^2 D(\mu) \xi^2 \Delta^2}, \]

\[ \lambda_0(T) = \lambda_c(T) \xi_0 \xi_c, \]  

(33)

where \( \Delta^2 = \frac{\tau}{\beta} \). Substituting DOS from equation (17) one obtains,

\[ \lambda_z^2(T) = \frac{3\pi^2 h^3 \xi^2 \Delta^2 \beta}{8\pi^2 \xi^2 \sqrt{2m^* \mu^3/3f}}, \]

and presented in figure 5.

The Abrikosov parameter is isotropic despite large anisotropies:

\[ \kappa^2 = \kappa^2_z = \frac{\lambda}{\xi} = \frac{ch}{e \xi^2 \Delta} \sqrt{\frac{1}{32\pi D(\mu)}}. \]

4.3. Critical magnetic fields

Thermodynamic critical field for kind I superconductors is given by

\[ H^2_c(0) = 8\pi F_s = 4\pi D_0(\mu) f \Delta^2 = \frac{4\sqrt{2m^* \mu^3/3f^2}}{3\pi h^3 \xi^2 \beta}. \]

(36)

The upper critical magnetic field \( H_{c2} \) in kind I superconductors is defined as the overcooled critical field [24]. It can be calculated as usual from the linear part of the GL equation,

\[ (-\xi_0^2 D_1 - \tau) \Delta = 0, \]

(37)

as the lowest eigenvalue of the linear operator (including the magnetic field). Representing the homogeneous magnetic field in the x axis direction in the Landau gauge, \( A = H(0,0,y) \), one obtain near \( T_{c} \) as

\[ H_{c2}(T) = H_c(0) \tau, \]

(38)

where the zero temperature intercept magnetic field is \( H_c(0) = \Phi_0/2\pi \xi \xi_0 \).

5. Discussion and conclusions

Magnetic properties of Dirac (Weyl) semi—metals superconductors with ‘hybrid’ dispersion relation of the electrons (Dirac in \( x-y \) plane and parabolic in \( z \) direction) at low temperatures were derived from a microscopic phonon mediated two—band pairing model via the Gorkov approach for the (singlet) order parameter. Microscopically derived Ginzburg–Landau effective theory was used to determine microscopically anisotropic coherence length, the penetration depth, figure 5, determining the Abrikosov parameter for a such materials. It is found that generally strongly second kind superconductivity in Dirac semimetals becomes first kind especially in type II WSM. It was shown that relatively large Fermi energy is crucial for existence of the kind one superconductivity effectively reducing the Abrikosov parameter \( \kappa^2 \) separating superconductors in two groups with different magnetic properties. In DSM superconductors of first kind both the thermodynamic field \( H_c(T) \) and upper critical field \( H_{c2}(T) \) that takes a role of the supercooling field is calculated.

Main results of the paper are presented in figures 4 and 5 where solid and dashed curves related to the coherence superconducting length and magnetic penetration depths \( \lambda^2 \) correspondingly. Figures 5(a) and (b) defined the coherence \( \xi_{\perp} \) and penetration length \( \lambda_{\perp} \) in-plane and extra-plane directions correspondingly. Figure 5 demonstrate that in the Type I phase of the DSM superconductivity is of the kind two \((\kappa^2 > 1/\sqrt{2})\) while in the phase Type II the DSM it overcomes to the kind II superconducting state. Our results applied to the DSM superconductor PdTe2 and related systems. In particular, the superconductor PdTe2 was recently classified as a Type II Dirac semimetal with magnetic measurements confirmed that PdTe2 was a first kind superconductor with \( T_c = 1.64 \) K and the thermodynamic critical field of \( H_c(0) \) = 13.6 mT (intermediate state under magnetic field is typical to a first kind superconductor, as demonstrated by the differential paramagnetic effect [20]). Experimentally measured effective Abrikosov parameter \( \kappa^2 \approx 0.1 \) takes place at the magnitude of cone tilt parameter \( \kappa = 2 \) where \( T_c = 1.64 \) K (see figure 5 and equation (36)). The temperature dependence of the thermodynamic magnetic field is in agreement with results of our theory.

Acknowledgments

We are grateful to NL Wang for valuable discussions and Anne de Visser for useful remarks and information. BR was supported by NSC of ROC Grants No. 103-2112-M-009-014-MY3 and is grateful to School of Physics of Peking University and Bar Ilan Center for Superconductivity for hospitality. The work of DL also is supported by National Natural Science Foundation of China (No. 11674007 and No. 91736208).

Appendix A. Gorkov equations in integral form

Gorkov equations equation (4) can be presented in an integral form:

\[ g^{+}_{\kappa \kappa}(r, r', \omega) = g^{+}_{\kappa \kappa}(r-r', \omega) - \int g^{+}_{\kappa \theta}(r-r''', \omega) \Delta_{\theta \kappa}(r''', r', \omega); \]

(A.1)
Expanding in small order parameter $\Delta$, one obtains:

$$\Delta(r) = \frac{e^2 T}{2} \sum \int d^2 r' \left[ \frac{g_{11}^2}{2} (r - r') \frac{1}{2} \right] \Delta(r') \left[ \frac{g_{11}^2}{2} (r - r') \frac{1}{2} \right] \Delta(r')$$

$$- \int d^2 r'' d^2 r' \langle \Pi(r, r'', r', r_3) \rangle \Delta(r') \Delta(r'') \Delta(r_3)$$

where $\Pi(r, r'', r', r_3) = \frac{g_{11}^2}{2} (r - r') \frac{1}{2} (r' - r'') \frac{1}{2} (r'' - r_3) \Delta(r') \Delta(r'') \Delta(r_3)$.

\section*{Appendix B. Calculation of the normal GF}

Normal Green function obeyed the equations (5) and (8). First four GF are calculated from the equation

$$L_{\gamma \beta}^1 g_{1 \kappa}^1 (r - r') = \delta^{\gamma \kappa} \delta (r - r'),$$

where

$$L_{\gamma \beta}^1 = \left[ (\omega + \mu + i w \nabla_{z}) \delta_{\gamma \beta} + (-i v \sigma_{\gamma \beta} \nabla_{r}) \right]$$

by performing Fourier transform for different pseudo-spin indexes. In particular for $\gamma = 1, \kappa = 1$ it reads in momentum representation

$$(\omega + \mu - w p) g_{11}^1 (p) + \nu (p^r - i p^t) g_{21}^1 (p) = 1;$$

$$(\omega + \mu - w p) g_{11}^1 (p) + \nu (\cos \varphi - i \sin \varphi) g_{21}^1 (p) = 1.$$

The rest of the normal GF may be obtained by the same method. The second group of the normal Green functions obey the equations $L_{\gamma \beta}^2 g_{2 \kappa}^2 (r - r') = \delta^{\gamma \kappa} \delta (r - r')$ with $L_{\gamma \beta}^2$ defined in equation (5) are obtained by the same method. The GF obtained after solution of these equations are:

$$g_{22}^1 (p) = z^* (\omega + \mu - w p); g_{12}^1 (p) = -z^* (\omega - \mu - w p); g_{21}^1 (p) = -z (\omega - \mu - w p); g_{11}^1 (p) = z (\omega + \mu - w p); g_{21}^2 (p) = -z (\omega - \mu - w p); g_{12}^2 (p) = -z^* (\omega - \mu - w p); g_{22}^2 (p) = z^* (\omega + \mu - w p);$$

$$z = (-\omega + \mu - w p)^2 - (\nu p)^2.$$

where $p$ is the 2D momentum and $\theta$ is the azimuthal angle in the $p_x, p_y$ plane.

\section*{Appendix C. Density states of WSM. Calculation of the f(κ) function}

In this appendix we calculate the DOS for the normal electrons described by the Hamiltonian (1). Using the dispersion relation for a single electron,

$$E = \varepsilon + \varepsilon_z + \varepsilon_{KCO} \cos \theta,$$

one obtains for electron density (for two sublattices and two spins)

$$n = \frac{4}{(2\pi)^2} \int d\varepsilon d\varphi dp \langle E [\varepsilon, p] - \mu \rangle.$$

The DOS is

$$\frac{dn}{d\mu} = \frac{4}{(2\pi)^2 \hbar^2} \int \frac{d\varepsilon d\varphi}{\sqrt{\varepsilon^2 - \mu}} \Big[ \frac{1}{2} \Big] \frac{1}{\sqrt{\varepsilon^2 - \mu}}$$

where new variables were defined as $\varepsilon_z = \frac{p_z^2}{2m}$. Performing integration over $\varepsilon_z$, one obtains

$$\frac{dn}{d\mu} = \frac{4}{(2\pi)^2 \hbar^2} \frac{m}{2} \int \frac{d\varepsilon d\varphi}{\sqrt{\varepsilon^2 - \mu}} \frac{1}{\sqrt{\varepsilon^2 - \mu - \varepsilon_{KCO} \cos \theta}} = \frac{\mu}{2} \frac{\sqrt{2m}}{3\pi^2 \hbar^2 \varepsilon} f,$$

where the angle integral gives the $f(\kappa)$ function.

Integration over azimuthal angle $\theta$ was obtained by replacing variables $\theta$ by $x = v / w + \cos \theta$. In the case $w / v < 1$ (type I semimetal phase), $x > 0$ and integral over the azimuthal angle reads

$$f(\kappa) = \frac{1}{\sqrt{1 + \kappa^2}} \int_0^1 \frac{1}{\sqrt{1 + \kappa^2}} = \frac{1}{\sqrt{1 + \kappa^2}}.$$
\[ \delta = \kappa^{-1} (1 + \kappa \cos \theta_0) = \frac{c\kappa^{-1}}{\wp} < \frac{\Omega}{\wp \pi} = \frac{a\Omega}{\pi \wp} \quad \text{(C.9)} \]

**ORCID iDs**

Dingping Li  @ https://orcid.org/0000-0001-6948-4945

**References**

[1] Weng H, Dai X and Fang Z 2016 *J. Phys.: Condens. Matter* **28** 303001

Bansil A, Lin H and Das T 2016 *Rev. Mod. Phys.* **88** 021004

Weng H, Fang C, Fang Z, Bernevig B A and Dai X 2015 *Phys. Rev. X* **5** 011029

Lv B Q et al 2015 *Phys. Rev. X* **5** 031013

Xu S-Y et al 2015 *Science* **349** 613

[2] Huang L et al 2016 *Nat. Mater.* **15** 1155–60

Wang Y et al 2016 *Nat. Commun.* **7** 13142

Deng K et al 2016 *Nat. Phys.* **12** 1105

[3] Cao J et al 2015 *Nat. Commun.* **6** 7779

Yu W, Jiang Y, Yang J, Dun Z L, Zhou H D, Jiang Z, Lu P and Pan W 2016 *Sci. Rep.* **6** 35357

[4] Soluyanov A A, Gresch D, Wang Z, Wu Q, Troyer M, Dai X and Bernevig B A 2015 *Nature* **527** 495

[5] Qi Y et al 2016 *Phys. Rev. B* **94** 054517

[6] Xiao R C et al 2017 *Phys. Rev. B* **96** 075101

[7] Li D, Rosenstein B, Shapiro B Ya and Shapiro I 2018 *Phys. Rev. B* **99** 144510

Brydon P M R, Das Sarma S and Li Q 2013 *Phys. Rev. X* **3** 011029

He L P et al 2014 *Phys. Rev. Lett.* **113** 246402

[10] Li D, Rosenstein B, Shapiro B Ya and Shapiro I 2014 *Phys. Rev. B* **90** 054517

[11] Alidoust M, Halterman K and Zyuzin A A 2017 *Phys. Rev. B* **95** 155124

[12] Shi L, Rosenberg B, Shapiro B Ya and Shapiro I 2017 *Phys. Rev. B* **95** 094513

[13] Qi Y et al 2016 *Nat. Commun.* **7** 11038

[14] Li D, Rosenberg B, Shapiro B Ya and Shapiro I 2018 *Phys. Rev. B* **97** 144510

Ghosh K et al 1996 *Phys. Rev. Lett.* **76** 4600

[16] Ketterson J B and Song S N 1999 *Superconductivity* (Cambridge: Cambridge University Press)

[17] Guggenheim J, Hulliger F and Muller J 1961 *Helv. Phys. Acta* **34** 408

[18] Noh H-J, Jeong J, Chom E-J, Kim K, Min B I and Park B-G 2017 *Phys. Rev. Lett.* **119** 016401

[19] Fei F et al 2017 *Phys. Rev. B* **96** 041201

[20] Leng H, Paulsen C, Huang Y K and de Visser A 2017 *Phys. Rev. B* **96** 220506

[21] Wang Z et al 2012 *Phys. Rev. B* **85** 195320

[22] Wang Z et al 2013 *Phys. Rev. B* **88** 125427

[23] Borisenko S et al 2014 *Phys. Rev. Lett.* **113** 027603

[24] Saint-James D, Sarma G and Thomas E J 1969 *Type II Superconductivity* (Oxford: Pergamon)