Noise-created bistability and stochastic resonance of impurities diffusing in a semiconductor layer

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Abstract

We investigate the dynamics of impurities walking along a semiconductor layer assisted by thermal noise of strength $D$ and external harmonic potential $V(x)$. Applying a nonhomogeneous hot temperature in the vicinity of the potential minimum may modify the external potential into a bistable effective potential. We propose the ways of mobilizing and eradicating the unwanted impurities along the semiconductor layer. Furthermore, the thermally activated rate of hopping for the impurities as a function of the model parameters is studied in high barrier limit. Via two state approximation, we also study the stochastic resonance (SR) of the impurities dynamics where the same noise source that induces the dynamics also induces the transition from mono-stable to bistable state which leads to SR in the presence of time varying field.

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1 Introduction

Recently the physics of semiconductors has received considerable attentions as devices made from semiconductors are vital in the construction of modern electronic equipment. One of the beneficial features of these semiconductors is the possibility to adjust their conductivities by introducing impurities (dopants) into their crystal lattice. The level of conductivity can be controlled by the amount and type of impurities. Unlike conductors, their conductivity increase with temperature; this property makes them even more important.

Since the conductivity of the semiconductor relies not only on the concentration of the dopants but also on the thermal background strength of the medium, the physics of thermal diffusion of the impurities along the semiconductor layer has attracted considerable attentions [1, 2, 3, 4]. Recently, the proposal by [1] unveiled the method of eliminating unwanted impurities from the region of the semiconductor via movable external harmonic potential along the semiconductor. The same group has investigated how to control the diffusion of impurities via an external potential which has an advantage of preserving the crystal structure as diffusion can take place at low temperature [2]. Their numerical results exhibit that the diffusion of the dopant increases when the strength of the external potential decreases. At high doping density, the internal field becomes significant and it renormalizes the effect of the external potential. Thus the diffusivity of the dopant increases as the dopant density increases.

Because manipulating the diffusion of impurities (either acceptors or donors) to a desired region of the semiconductor layer is vital, in this work we propose different ways of mobilizing the impurities along the semiconductor. We consider the impurities walking from one lattice trap to the other lattice trap with a trap depth potential along the semiconductor layer assisted by thermal noise of strength $D$ and the external harmonic potential. Furthermore, a nonhomogeneous hot temperature is applied in the vicinity of the potential minimum which may modify the external potential into a bistable effective potential. Neglecting the interaction between the dopants at low impurity density, we show that the diffusion of impurities can be controlled not only by varying the external potential but also by altering the intensity of the temperature of the hot spot and trap depth of the impurities. We also propose the way of eradicating the unwanted impurities from the certain region of the semiconductor by varying the different model parameters.
without considering movable external potential. Furthermore, we investigate the dependence of the rate on different model parameters.

One very crucial but unexplored issue is the way of enhancing the mobility of the impurities along the semiconductor. This can be achieved by applying symmetry breaking fields such as time varying signal which may lead the system into stochastic resonance. Stochastic resonance has been widely studied over the last few decades \cite{6,7,8,9,10} and has been adopted to describe an interesting statistical property of periodically modulated and noise driven multi-stable dynamical system. It exhibits that under proper condition an increase in the input noise level results in an increase in the output signal-to-noise ratio.

The phenomenon of SR was first introduced by Benzi \cite{5}. Later, the idea of stochastic resonance has been implemented in many model systems. Notable examples include: SR for confined systems \cite{11}, SR for complex systems such as polymers \cite{7,8,12,13} and SR for a Brownian particle moving across a porous membrane \cite{14}. The vast majority of studies on stochastic resonance have been focused on analyzing the dynamics of a bistable system. Among such models, the two state model \cite{6,16} has been proven to be extremely useful in the understanding of the stochastic phenomenon offering a simple framework able to provide analytical results.

In this paper, via two state approximation, we study the stochastic resonance (SR) of a particular noise sustained dynamics where the same noise source that induces the dynamics also induces the transition from mono-stable to bistable state which leads to SR in the presence of time varying field. We find that the spectral amplification $\eta$ attains an optimal value at a certain finite value of noise strength $D_{opt}$. As the temperature of the hot spot increases, the peak of $\eta$ decreases. On the other hand, the value of thermal strength $D_{opt}$ increases as the intensity of the nonhomogenous hot temperature increases. The strength of trap depth also considerably affects the $\eta$. We find that the peak of the $\eta$ decreases as the strength of the potential trap increases. Furthermore, we explore the dependence of $\eta$ on other model parameters.

It is important to note that the results and model presented in this work are not specific to the impurities dynamics. Rather the present study serves as a basic paradigm in which to understand diffusion and noise induced nonequilibrium phase transition in discrete systems. Thus the present model is of broad interest in various fields.

The rest of the paper is organized as follows: in Section 2, we present
the model. In Section 3, we study the diffusion of the impurities in the presence of a nonhomogeneous temperature. In section 4, we study the SR of impurities and explore how the $\eta$ for the impurities behaves as a function of the model parameters. Section 5 deals with summary and conclusion.

2 The model

We consider a one-dimensional system where non-interacting impurities jump from one lattice trap to the other lattice trap assisted by thermal noise and external potential energy which is monostable

$$V(x) = V_0 x^2$$

where $V_0$ and $x$ denote the potential energy and the position of the impurities, respectively. When background temperature is homogeneous, particles concentrate around the potential minimum. On the other hand, in the presence of a nonhomogeneous temperature background,

$$T(x) = T_c + T_h \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

which is hot around the potential minimum and decays to a lower temperature as one goes away on both sides from the potential minimum, the system may undergo a phase transition in which the particles pile up around two points of the potential minima. Here the parameters $T_h$ and $T_c$ designate the temperature of the hot and cold reservoirs, respectively, while $\sigma$ denotes the standard deviation.

At low impurity density, the impurity dynamics is governed by

$$\frac{\partial P(x,t)}{\partial t} = \frac{\partial}{\partial x}\left\{ \frac{V'(x)}{k_B T(x)} \right\} e^{\frac{-\Phi}{k_B T(x)}} P(x,t) - \frac{\Phi}{k_B T(x)} e^{\frac{-\Phi}{k_B T(x)}} P(x,t)$$

where $P(x,t)$ is the impurity density at a position $x$ and time $t$. The impurities jump from one lattice trap, which has an internal potential depth (trap depth) $\Phi$, to the next lattice trap along the one dimensional lattice. The steady state probability distribution of the impurity $P_{ss}(x)$ is given by

$$P_{ss}(x) = C e^{\frac{-\Phi}{k_B T(x)}} e^{\int_0^x \frac{V'(y)}{k_B T(y)} dy}$$
where $C$ is a normalizing constant. Rearranging Eq. 4, one gets the probability distribution

$$P_{ss}(x) = C_0 e^{-\frac{V_{eff}(x)}{k_B T_c}}$$  \tag{5}$$

with an effective potential energy $V_{eff}(x)$ which is given by

$$V_{eff}(x) = V_0 x^2 + 2V_0 \sigma^2 \ln \left[ \frac{1 + \alpha e^{-\frac{x^2}{2\sigma^2}}}{1 + \alpha} \right] - \frac{\Phi}{\left[1 + \alpha e^{-\frac{x^2}{2\sigma^2}}\right]}$$  \tag{6}$$

where dimensionless parameter $\alpha$ relates $T_h$ and $T_c$ as $T_h = \alpha T_c$. The effective potential energy is bistable when the condition

$$\Phi > 2V_0 \sigma^2 \left( \frac{\alpha + 1}{\alpha} \right)$$  \tag{7}$$

is met. In this regime, the location of the saddle point is at $x = 0$ while the two symmetric stable points are located at

$$x_m = \pm \sqrt{2} \sigma \sqrt{\ln[\alpha(\frac{\Phi}{2\sigma^2 V_0} - 1)]}.$$  \tag{8}$$

The effective potential energies at the saddle and stable points are given as

$$V_{eff}(0) = \frac{-\Phi}{1 + \alpha}$$  \tag{9}$$

and

$$V_{eff}(x_m) = -\Phi + 2\sigma^2 V_0 \left[ 1 + \ln \left[ \frac{\Phi \alpha}{2\sigma^2 V_0} - \alpha \right] + \ln \left[ \frac{\Phi}{(1 + \alpha)(\Phi - 2\sigma^2 V_0)} \right] \right],$$  \tag{10}$$

respectively. The two minima of the potential are separated by a barrier of height $\Delta V_{eff} = V_{eff}(0) - V_{eff}(x_m)$. The curvatures at the barrier top $\omega_0$ and the well minima $\omega_{x_m}$ take simple forms

$$\omega_0 = \frac{(-\alpha \Phi/\sigma^2 - 2\alpha(1 + \alpha)V_0 + 2(1 + \alpha)^2 V_0)}{(1 + \alpha)^2},$$  \tag{11}$$

and

$$\omega_{x_m} = \frac{4V_0}{\Phi^2}(\Phi - 2\sigma^2 V_0)^2 \ln[\alpha(\frac{\Phi}{2\sigma^2 V_0} - 1)],$$  \tag{12}$$
respectively.

Transforming Eq. (4) into Eq. (5) is equivalent to converting the non-equilibrium problem to an equilibrium one where the usual Boltzmann statistics holds. Hereafter we consider impurities undergoing a random walk motion along the effective potential assisted by the thermal kicks \( D = k_B T_c \) where \( k_B \) is the Boltzmann constant. One can note that the information about the non-equilibrium features of the system is stored in the effective potential. In the next section we explore how the dynamics of the impurities under the effective potential \( V^{eff} \) behaves as a function of the model parameters.

3 Impurity diffusion in a nonhomogeneous temperature

We consider non-interacting impurities (either acceptors or donors \(^2\)) of low density hopping from one lattice site to the other site within the semiconductor layer. It is assumed that only the donors are sensitive to the external potential and hereafter the term impurity refers to the donor only. Exposing the impurities to the external harmonic potential compel the particles to accumulate around the potential minimum. Furthermore, applying a nonhomogeneous temperature profile in the vicinity of the potential minimum, may modify the external potential into a bistable effective potential as long as the condition given in Eq. (7) is obeyed. Otherwise the effective potential is a monostable potential.

Figure 1a gives a plot of the effective potential \( V^{eff} \) versus \( x \) for two different values of \( \alpha \). The effective potential has two stable potential minima at \( \pm x_m \). Exploiting Eq. (6), one can see that the barrier height \( \Delta V^{eff} \) and the width of the effective potential \( 2x_m \) increase as \( \alpha \) and the trap depth \( \Phi \) increase. On the contrary, as the strength of the external potential \( V_0 \) and \( \sigma \) decrease, the barrier height \( \Delta V^{eff} \) and the width \( 2x_m \) increase.

We plot the dependence of the probability distribution \( P_{ss}(x) \) as a function of \( x \) in Fig. 1b. The figure depicts that the impurities accumulate around the two potential wells. As \( \Phi \) increases, the probability of finding the impurities around the two stable points increases. When \( \Phi \) increases, not only the peak of \( P_{ss}(x) \) intensifies but also the distance between the two peaks \( 2x_m \) increases.
Let us now discuss how one can translocate the dopants to a desired location along the semiconductor layer. By looking at how $V_{\text{eff}}$ and $P_{ss}(x)$ behave, when the different model parameters vary, one can infer the position of the impurities in the medium. As discussed before when $\alpha$ and $\Phi$ monotonously increase or as $V_0$ and $\sigma$ constantly decrease, the particles which are accumulated around the two potential minima leave the central region $x = 0$ and migrate to the peripheral part of the semiconductor layer creating a depletion zone around $x = 0$. This suggests ways of controlling the diffusivity of the particles by tuning these parameters. Further increase in $\alpha$ or as one further decrease $V_0$ and $\sigma$, the impurities may become eliminated from a certain region of the semiconductor layer.

The impurities that are exposed to the effective potential exhibit noise-induced nonequilibrium phase transition [15] where the noise plays a counterintuitive role by inducing ordering phenomena. Exploiting Eq. (7), one can see that the regimes of noise induced ordering phase transition are demarcated by the critical values of the different model parameters. The critical potential energy $V_0^*$ and the critical trap depth $\Phi^*$ have a simple form: $V_0^* = (\Phi \alpha/(2\sigma^2(\alpha + 1)))$ and $\Phi^* = 2V_0\sigma^2((\alpha + 1)/\alpha)$ while the other critical points are given by $\sigma^* = \sqrt{(\alpha \Phi)/(2V_0(\alpha + 1)))$ and $\alpha^* = (2V_0\sigma^2)/(\Phi - 2V_0\alpha^2)$. When $0 < \Phi < \Phi^*$, $\alpha < \alpha^*$, $\sigma > \sigma^*$ or $V_0 > V_0^*$, the effective potential is a monostable potential and the dopants concentrate around the $x = 0$ position. In such a case the impurity dynamics is similar to the previously
exposed works [1, 2]. On the other hand when Φ > Φ∗, α > α∗, σ < σ∗ or
V_{0} < V_{0}^{*}, the effective potential has two stable minima and the impurities
split into two impurity-rich regions which coexist with impurity-poor regions.
In this regime, the donors pile up towards the two potential minima while
the oppositely charged particles hop in the opposite direction showing that a
series of p−n−p or n−p−n junctions can be fabricated by manipulating the
diffusion of the impurities. Note that the effect of internal field is neglected
which is appropriate at low doping level.

Now let us study how the thermally activated barrier crossing rate of
the impurities behaves. Let us consider non-interacting impurities initially
situated at one of the potential minima. Due to thermal fluctuation, the par-
ticles cross the potential barrier assisted by the thermal kicks they encounter
along the reaction coordinate. The crossing rate for the impurities in a high
barrier limit ∆V_{eff} ≫ k_{B}T is approximated [19, 20] as

$$R = \sqrt{\frac{|\omega_{0}| \omega_{x_{m}}}{2\pi}} e^{-\Delta V_{eff}/k_{B}T}.$$  \hspace{1cm} (13)

Figure 2: (a) (Color online) Escape rate $R$ versus $T_c$ in a unit of Kelvin
for parameters choice $\alpha = 1.5$ (black line), $\alpha = 3$ (red line), $\Phi = 2eV$ and
$\sigma = 1nm$. (b) The crossing rate $R$ versus $T_c$ in a unit of Kelvin for parameters
choice $V_{0} = 0.4eV/nm^{2}$ (black line) and $V_{0} = 0.3eV/nm^{2}$ (red line). Other
parameters are fixed as $\alpha = 2$, $\Phi = 2eV$ and $\sigma = 1nm$.

Figure 2a plots the dependence of the rate $R$ on $T_c$. The particles cross
the potential barrier at the expense of the thermal background kicks. Thus
the rate increases with $T_c$. When $\alpha$ steps up, $\Delta V_{eff}$ and $2x_{m}$ increase and
the particles attain difficulty in crossing the high barrier. Hence the rate
becomes lower with $\alpha$. Figure 2b exhibits that when $V_{0}$ rises, the escape
rate increases due to the fact that increasing in $V_0$ results in a lower barrier height. Exploiting Eq. (13), one can also see that the rate intensifies when $\Phi$ decreases or as $\sigma$ increases.

4 Stochastic resonance of the impurities

Let us now study the noise-assisted dynamics for the bistable potential in the presence of time varying signal. The interplay between noise and time varying force in the bistable system may lead the system into stochastic resonance as long as the random kicks are adjusted in an optimal way to the recurring external force. Next we study the dependence of the SR on the model parameters employing two state approximation.

Via two state model approach \[6, 16\], two discrete states $x(t) = \pm x_m$ are considered. Let us denote $n_+$ and $n_-$ to be the probability to find the impurity in the right ($x_m$) and in the left ($-x_m$) sides of the potential wells, respectively. In the presence of time varying weak periodic force (AC field) of type $v(t) = A_0 \cos(\Omega t)$, the master equation that governs the time evolution of $n_\pm$ is given by

$$\dot{n}_\pm(t) = -W_\pm(t)n_\pm + W_\mp(t)n_\mp$$

where $W_+(t)$ and $W_-(t)$ correspond to the time dependent transition probabilities towards the right ($x_m$) and the left ($-x_m$) sides of the potential wells, respectively. The time dependent rate \[6, 16\] takes a simple form

$$W_\pm = R \exp \left[ \pm \frac{x_{\text{min}}}{D} A_0 \cos(\Omega t) \right],$$  \hspace{1cm} (15)

where $R$ is the Kramers rate for the particles in the absence of periodic force $A_0 = 0$. For sufficiently small amplitude, one finds the spectral amplification, $\eta$ to be \[6, 21\]

$$\eta = \left( \frac{x_m}{D} \right)^2 \frac{4R^2}{4R^2 + \Omega^2}.$$  \hspace{1cm} (16)

The input signal that modulates the symmetric bistable system makes one stable state less stable than the other, alternatively, over half forcing period. When the random switching frequency matches the forcing angular frequency by tuning the noise intensity, the system attains the maximum probability of escaping out of the less stable state into the more stable one, before a random back switching event takes place. Due to this reason, the
particle is more likely to be in the more stable state. Consequently η attains a maximum value at a particular noise level \(D_{opt}\). Let us now explore how the \(\eta\) behaves as a function of the trap depth \(\Phi\), standard deviation \(\sigma\), applied potential energy \(V_0\) and \(\alpha\).

![Figure 3: (a) (Color online) Spectral amplification \(\eta\) versus \(T_c\) in a unit of Kelvin for parameters choice \(V_0 = 0.4eV/\text{nm}^2\) (black line) and \(V_0 = 0.3eV/\text{nm}^2\) (red line). Other parameters are fixed as \(\alpha = 2\), \(\Phi = 2eV\) and \(\sigma = 1\text{nm}\). (b) Spectral amplification \(\eta\) versus \(T_c\) in a unit of Kelvin for parameters choice \(\alpha = 1.5\) (black line), \(\alpha = 3\) (red line), \(\Phi = 2eV\) and \(\sigma = 1\text{nm}\).](image)

Figure 3a plots the dependence of the \(\eta\) on the noise strength \(T_c\) for parameter choice \(V_0 = 0.4eV/\text{nm}^2\) for black line and \(V_0 = 0.3eV/\text{nm}^2\) for red line. Other parameters are fixed as \(\alpha = 2\), \(\Phi = 2eV\) and \(\sigma = 1\text{nm}\). In the limit \(T_c \to 0\) and \(T_c \to \infty\), \(\eta \to 0\). This is because when the noise intensity \(T_c\) is too small, the intrawell crossing rate is too small while for large \(T_c\) a similar loss of synchronization occurs since the particles flip too many times between the two stable points. In between \(\eta\) attains an optimal value at optimal \(T_{c, opt}\). The peaks of \(\eta\) increases with \(V_0\). On the other hand, \(T_{c, opt}\) increases as \(V_0\) decreases. Figure 3b exhibits the dependence of \(\eta\) on \(T_c\) for parameters choice \(\alpha = 1.5\) for black line, \(\alpha = 3\) for red line, \(\Phi = 2eV\), \(\sigma = 1\text{nm}\) and \(V_0 = 0.4eV/\text{nm}^2\). When \(\alpha\) increases, \(T_{c, opt}\) increases; the peak of the \(\eta\) rises when \(\alpha\) diminishes. This loss of synchronization with the increase in \(\alpha\) is plausible because as \(\alpha\) increases, \(\Delta V_{eff}\) and the width \(2x_m\) increase; the particle crosses the high potential barrier at the expense of higher thermal strength \(T_c\).

In Fig. 4a, we show how the \(\eta\) behaves as \(\Phi\) varies. When \(\Phi\) increases, the peak of \(\eta\) becomes smaller. \(T_{c, opt}\) shifts to the right as \(\Phi\) increases showing the response of signal to the background noise strength is significant at lower
Figure 4: (a) (Color online) Spectral amplification $\eta$ versus $T_c$ in a unit of Kelvin for parameters choice $\Phi = 2eV$ (black line) and $\Phi = 2.5eV$ (red line). Other parameters are fixed as $\alpha = 1.5, V_0 = 0.4eV/nm^2$ and $\sigma = 1nm$. (b) Spectral amplification $\eta$ versus $T_c$ in a unit of Kelvin for parameters choice $\sigma = 1$ (black line), $\sigma = 1.1$ (red line), $\Phi = 2eV$, $\alpha = 2$ and $V_0 = 0.4eV/nm^2$.

values of $\Phi$. On the other hand, as $\sigma$ monotonously decreases, the resonance becomes more significant. When $\sigma$ diminishes, $T_{c}^{opt}$ increases (see Fig. 4b).

These results reveal the weak signal passing through the semiconductor layer can be amplified and detected by tuning the temperature of the hot locality since the noise level can be directly affected by the hot temperature. By tuning the angular frequency $\Omega$, one can control the mobility of the impurities to a desired location. At the resonance temperature $T_{c}^{opt}$, the impurities undergo a fast unidirectional drift from the less stable potential minima to the more stable one over half forcing period revealing novel way of achieving a fast transportation of impurities along the semiconductor layer without exposing the impurities to a higher temperature. Thus the present study is crucial in the designing of artificial semiconductor.

At this point we stress that this theoretical work can be realized experimentally. As shown in the schematic diagram (Fig. 5), one can model the harmonic potential via the metallic gates which are kept at a certain voltage [1, 2]. The metallic gates are situated at the top of the sample semiconductor. Similar to the work [1, 2], the harmonic potential is extended along the $x$ direction. A tiny region around the potential minimum $x = 0$ is heated up and this creates a depletion zone of the impurities around $x = 0$; the hot locality forces the impurities to migrate towards the peripheral regions. The diffusion of the particles can be controlled by tuning the different model parameters. Applying periodic signals such as AC field along the $x$ direction, modulates the effective potential. With the proper adjustment of the
Figure 5: (Color online) Schematic diagram showing a semiconductor which is exposed to an external potential (in $x$ direction) and a nonhomogeneous temperature background, which is hot around the potential minimum ($x = 0$) and decays to a lower temperature as one goes away on both sides from the potential minimum. The cross section along the $z$ direction is not shown. The impurities are assumed to be uniformly distributed along the $y$ and $z$ directions. Furthermore, a time varying field (AC field) is applied along the $x$ direction.

different parameters, the system may show the stochastic resonance and this arrangement could be used to amplify very weak signals.

5 Summary and conclusion

This theoretical work exposes the way of manipulating the disfusiblity of the impurities along the semiconductor layer by locally heating the semiconductor layer around the potential minimum of the exerted external harmonic potential. The theoretical results obtained in this work depicts that the dopant mobilizes to the peripheral regions when the trap depth $\Phi$ and $\alpha$ increase or when $\sigma$ and the external potential $V_0$ decrease. The thermally activated rate for the impurities is also studied at high barrier limit $\Delta V^{\text{eff}} \gg k_B T$. It is shown that the rate increases with $V_0$ and $\sigma$, and it grows smaller when $\alpha$ and $\Phi$ increase.
In the presence of periodic signals, the dependence of the spectral amplification $\eta$ on the different control parameters is explored. The peak of $\eta$ rises when $\Phi$ and $\alpha$ monotonously fall. When $\sigma$ and $V_0$ increase, the response of signal to the background temperature is significant. The magnitude of optimum noise intensity $T_c^{opt}$ becomes considerable with $\Phi$ and $\alpha$, and when $V_0$ and $\sigma$ grow smaller.

The model presented in this work is not limited to the impurities dynamics. Rather the present model is of broader interest in various fields and serves as a basic paradigm in which to understand diffusion and noise induced nonequilibrium phase transition in discrete systems. In conclusion, the proposed model is crucial in designing artificial semiconductors. We believe that a semiconductor device under the suggested model can be designed to detect weak signals of extremely small modulation.

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