Calculation of the Observationally Small Cosmological Constant in the Model of Six-Dimensional Warped Brane-Bolt World

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Abstract

In the 6-dimensional model of warped compactification based on the euclidean AdS Reissner-Nordstrom metric it is possible to escape artificial 4-brane’s stress-energy tensor anistropy necessary for fulfillment of Israel junction conditions by introducing in 3+1 spacetime of the cosmological constant which turns out to be $\sim G^2$ ($G$ - Newton’s constant) and acquires the value compatible with observations.
1. Introduction

Recent experimental data (see e.g. review [1]) appear to confirm the acceleration rate of the universe, $H$, which is the evidence of small positive vacuum energy $\rho_{\text{vac}} \approx 9 \cdot 10^{-12} \text{(eV)}^4$ and hence of the cosmological constant $\lambda$ ($c = \hbar = 1$):

$$\lambda = H^2 = 8\pi G \rho_{\text{vac}} \approx (10^{-33} \text{eV})^2.$$  \hfill (1)

At the same time "in the standard framework of low energy physics there appears to be no natural explanation for vanishing or extreme smallness of the vacuum energy", as Witten put it in [2], defining the problem as "the mistery of the cosmological constant". "The problem is so severe, - write authors of [3], - that it seems reasonable to put aside all the other cosmological issues treating them as secondary and focus completely on the cosmological constant problem". Some scientists, including Sakharov [4], prefer the anthropic principle approach to explain zero or extremely small cosmological constant in the universe where we could develop to the level of asking questions about $\lambda$. However attempts to find a conventional scientific resolution of $\lambda$-problem, in particular advocated by Witten [2], seem to be more challenging and inspiring than anthropic "explanation". Certain review of mechanisms for generating a small current value of cosmological constant is given in Chapter 7 of [1]. Among them the idea of cosmologically slow decaying "cosmological constant" (this field is called now "quintessence") would be satisfactory if it did not result in changing with time of fundamental constants which is strongly restricted by observations. Also it is worthwhile to note the Zeldovich’s numerological observation [5] that gravitational correction to quantum vacuum energy of particle of mass $m$ is $Gm^6$; this gives the appropriate value of the cosmological vacuum energy for $m$ in between electron and proton mass (why ???). (In [6] it was shown that Brans-Dicke theory with $\phi^{-1}$ dependence of potential of the BD-field $\phi$ describes equal to $Gm^6$ and changing with time vacuum energy, i.e. unifies both ideas, giving at the same time Dirac’s theory of variable gravitational interaction unexceptable experimentally).

In this paper it is shown that Zeldovich-type "numerology" for cosmological constant follows from dynamics of certain higher dimensional models with branes. We shall not escape here fine-tuning of the brane’s tension well
known in the Randall-Sundrum type models. However their generalization to
the models with additional compact dimension provides additional dynam-
Certain justification of (3) could be received from "energy" considerations, where general expression \( E \sim \int N(K - K_0) \) was applied to calculate the energy \( E \) of AdS spacetime. Here \( K \) is the trace of extrinsic curvature of co-dimension two surface of constant time and constant asymptotically large \( r \); \( K_0 \) is the same for the background (or reference) AdS spacetime, which in our case is characterized by \( a = b = 0 \) in (5) and by the same values of \( l, \lambda \) and period \( T_{\varphi} \) of compact coordinate as in the solution (3)-(5). The energy \( E \) of this solution is found to be:

\[
E = (3/2)M^8T_{\varphi}a^3V_3. \tag{7}
\]

The most interesting feature of this result is perhaps the absence of contribution to \( E \) from the "Maxwell" \( b \)-term of \( \Delta \) (5). On the other hand the non-zero Schwarzschild term, \( a \neq 0 \) in (5), results in \( E = \infty \) because 3-volume \( V_3 \) in (8) is infinite.

Thus we consider model with \( a = 0 \) in (5). Extra coordinate \( r \) is limited from below and from above, \( r_0 < r < R \), where \( \Delta(r_0) = 0 \). The point \( r = r_0 \), which is co-dimension two surface known as "bolt" (11), is regular provided \( \varphi \) has period

\[
T_{\varphi} = 4\pi(\partial\Delta/\partial r|_{r=r_0})^{-1} = \frac{\pi l^2}{2r_0}, \tag{8}
\]

where

\[
r_0 = (b^6l^2)^{1/8}. \tag{9}
\]

(In (8), (9) it was taken into account that in (5) \( \lambda/M^2 \ll 1 \), and only higher order terms were retained).

Positive tension 4-brane located at \( r = R \) limit the space via cut and paste procedure; it possesses one large (\( \sim R \)) compact dimension \( \varphi \). We shall not seek for artificial smallest anisotropy of the brane’s stress-energy introduced e.g. in (9) to satisfy Israel junction conditions imposed upon brane’s (3+1) space-time coordinates and upon \( \varphi \)-coordinate. The simplest case of brane described by the single tension parameter is considered. Difference in \( r \)-dependence of scale factors \( M^2r^2 \) and \( \Delta \) in (3) (this difference is rather

\[\text{In (7), contrary to (3), there was received negative value of } E, \text{ which, as we suppose, was a result of mistakenly taken non-constant, dependent on } r, \text{ period of compact coordinate of the background spacetime (cf. (3.7) in (3))}\]
small for large $r$ because of AdS asymptotics of $\Delta$ in (5)) yields well known
additional condition of compatibility for stationary brane’s location (see e.g.
in [12]):

$$\frac{\partial}{\partial r} \left( \frac{\Delta}{r^2} \right) \bigg|_{r=R} = 0. \tag{10}$$

It is essential that term $r^2/l^2$ of (5) drops out in (10). We follow [8] where
cosmological constant $\lambda$ in 4 dimensions was introduced to satisfy condition
(10).

3. Effective dynamics in 4 dimensions. Weak/Planck hierarchy
and calculation of the cosmological constant

Condition (10), with account of (3), (8), (9) results in the expression for
$\lambda$:

$$\lambda = \frac{4M^2 l^6}{R^6} = \frac{4M^2 r_0^8}{l^2 R^6}. \tag{11}$$

This permits to calculate $\lambda$ as a function of Newton’s constant $G$.

We follow the traditional approach of evaluating dynamics in 4 dimen-
sions, described by the action $S_4$, by inserting ansatz (3) - (5), with account
of (6), (8), (9), into the action $S_6$ (2) and integrating out extra coordinates
$r, \varphi$:

$$S_4(\tilde{g}_{\mu \nu}(x)) = \int_{r_0}^R (\cdots) dr d\varphi = \int \left\{ \frac{1}{16\pi G}(\tilde{R} - 2\tilde{\lambda}) \right\} \sqrt{-\tilde{g}} \, dx. \tag{12}$$

wherefrom:

$$\frac{1}{16\pi G} = M^6 T_\varphi \int_{r_0}^R r^2 \, dr \approx (1/3) M^6 T_\varphi R^3 = \frac{\pi M^6 l^2 R^3}{6r_0}. \tag{13}$$

Thus $G^{-1}$ being $\sim R^3$ is "diluted" in extra space. Week/Planck hierarchy
is obtained if matter fields are trapped upon the additional "week scale"
brane placed at $r = r_0$, or if fields describing the masses of matter fields
are, contrary to the Planck scale, "concentrated" near $r = r_0$. The example
of such a "mass field" concentrated near lower values of $r$ may be vector-
potential (4). It may provide mass to the charged fields interacting with
$A_B$, or, in case $A_\varphi$ is a component of some non-abelian gauge field, presence of $A_\varphi^2$ in quartic terms of the Yang-Mills action will result in breakdown of gauge symmetry in 4 dimensions; then some components of the gauge field will acquire masses. These possibilities deserve further investigation.

Cosmological constant $\tilde{\lambda}$ in $S_4$ (12) is given by the bulk value of action $S_6$ (2) which, according to the well known consistency condition, coincide on the solution of dynamical equations with input cosmological constant $\lambda$ of metric $\bar{g}_{\mu\nu}$ in (3), (5) (the proof see in [13]):

$$\tilde{\lambda} = \lambda.$$ (14)

To receive final expression for cosmological constant in 4 dimensions we substitute $R$ by $G$ from (13) into the expression (11) for $\lambda$, it is convenient also to express $\lambda$ not by ”charge” $b$ but by more ”geometrical” constant - minimal value $r_0$ of the radial coordinate, connected with $b$ via (9):

$$\lambda = (16\pi^2/3)^2 G^2 M^{14} l^2 r_0^6.$$ (15)

If fundamental scale $M$ and ”length” $l$ which determines the bulk cosmological constant in the action (2) ($\Lambda = -10/l^2$) are supposed to be of the electroweek scale $M = l^{-1} = 1$TeV, then observable value (1) of $\lambda$ is obtained from (13) for $r_0 = 10^{-22}$cm.

If on the other hand all three low energy scales in (13) are equal:

$$M = l^{-1} = r_0^{-1} \equiv m,$$ (16)

then Zeldovich formula [5] for vacuum energy $\rho_{\text{vac}}$ follows from (13), (16):

$$\rho_{\text{vac}} = \lambda/8\pi G = \frac{32\pi^3}{9} G m^6.$$ (17)

This gives observable value of vacuum energy for $m \approx 10^7$eV. But if we take $r_0 = l$ and $M l = 10^{-3}$ then observable value of $\lambda$ (1) is obtained for the values of parameters which are rather close to the electroweek scale: $r_0^{-1} = l^{-1} = 10^{14}$eV, $M = 10^{11}$eV. The game with numbers may be continued. It does not make much sense however before the physical meaning of constants of the model and their possible connections with elementary particles masses are understood.
4. Comments

Contrary to the conventional situation in warped models with extra dimensions where $\Lambda$-term of the 4-dimensional universe is an arbitrary constant of solutions of dynamical equations, in the model considered here this constant is not an arbitrary one but, because of sophisticated Israel junction conditions, is calculable through parameters $(M, \Lambda)$ of the theory and constants $(R, r_0)$ of the solution; the same is true for Newton’s constant (see final expressions in (11), (13)). Thus any physical mechanism which defines (stabilizes) the boundaries $R, r_0$ of the extra space will simultaneously determine the mass hierarchy and value of the cosmological constant.

Essential drawback of the model is fine-tuning of the brane’s tension, which is proportional to $\sqrt{\Delta(R)/R}$ [8], and hence includes corrections depending on extremely small value of $\lambda/M^2$. It would be interesting to study the ”self-tuning” theory with additional scalar field dependence in the bulk and brane actions in (2).

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