2D Rashba system in AC magnetic field

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The response of an electron system to a DC measurement electric field has been investigated in the case when the system is driven out of the equilibrium by the magnetic ultra-high frequency field that leads to combined transitions. The discussed model includes contributions from Landau quantization and from microwave irradiation. Impurity centers are considered as sources of scattering. It has been shown that the perturbation of the electron system by the ultra-high frequency magnetic field leads to oscillations of the diagonal components of the conductivity tensor.

I. INTRODUCTION

The interest in theoretical studies of transport phenomena in 2D electron systems has raised substantially after the discovery of oscillations of the diagonal components of the conductivity tensor in “ultraclean” GaAs/AlGaAs samples in the classical interval of the magnetic field intensity, where the Shubnikov–de Haas (SdH) oscillations don’t manifest themselves. Together with the oscillations of the diagonal components of the conductivity tensor caused by absorption by charge carriers of the energy of the microwave radiation field and transitions between Landau levels, “beats” have been found experimentally in the interval of more weak magnetic fields. Such beats are usually related with the manifestations of the interaction between kinetic and spin degrees of freedom of the conductivity electrons. Such interaction is the spin-orbit interaction (SOI) that is known to be the origin of numerous effects in transport phenomena observed in such systems. Among them there are, e.g., beats in SdH oscillations, spin accumulation, magneto-electric effect, etc. SOI also leads to the possibility of electron transitions between Landau levels in the magnetic field at the combined resonance frequencies, thus transitions being possible both in antinodes of electric and magnetic fields. Finally, operation of a spin transistor (schemes of which have been considered in [1]) is based upon spin degrees of freedom. All of the above has determined elevated interest to investigations of the SOI in 2D semiconductor structures.

For the purpose of studying the SOI, it appears interesting to investigate a model in which the role of the SOI should manifest itself the strongest. Since the SOI depends upon both translational and spin degrees of freedom, then it is a channel over which energy (both electric and magnetic) can be absorbed from the ultra-high frequency field, thus causing transitions between Landau levels. Because of that, it is interesting to investigate the response of a non-equilibrium electron system to a DC weak (“measurement”) electric field for the case when the initial non-equilibrium state is created with a high-frequency AC magnetic field that leads to combined transitions. The question is how this perturbation affects transport coefficients, in particular, the conductivity tensor.

The discussed model includes the contributions from Landau quantization and (in the long-wavelength limit) from the microwave radiation exactly, without use of the perturbation theory. We consider impurity centers for the role of scatterers, treating the scattering process perturbatively.

II. EFFECTIVE HAMILTONIAN

The Hamiltonian of the system under consideration is:

\[ H(t) = H_k + H_s + H_{ks} + H_{ch}(t) + H_{0f} + H_v + H_{cv}. \]  \tag{1}

Here \( H \) and \( H_s \) are kinetic and Zeeman energies, respectively, in the magnetic field \( H = (0, 0, H) \):

\[ H_k = \sum_i \frac{(p_i - (e/c)A(x_i))^2}{2m}, \]

\[ H_s = \hbar \omega_s \sum_i S_i^z, \quad \hbar \omega_s = g\mu_B H. \] \tag{2}

\( S_i^z \) and \( p_i^z \) are operators of the components of the spin and kinetic momentum of the ith electron, where \( [p_i^z, p_j^z] = -\delta_{ij}i\hbar\omega_c\varepsilon_{\alpha\beta}\gamma_\beta \), \( \omega_c = |e|H/\hbar m \) is the cyclotron frequency, and \( \mu_B \) is Bohr magneton. \( H_{0f} \) is the hamiltonial of the electrons’ interaction with the electric field \( E = (E_x, 0, 0) \):

\[ H_{0f} = -eE \sum_i r_i. \] \tag{3}

\( H_{ch}(t) \) is the interaction of electrons with the AC magnetic field:

\[ H_{ch}(t) = g\mu_B H(t) \sum_i S_i. \] \tag{4}

\( H(t) = (H_x(t), H_y(t), H_z(t)) \). \( H_{cv} \) and \( H_x \) are Hamiltonians of the electron-lattice interaction and of the lattice itself, respectively. \( H_{ks}(p) \) is the interaction between translational and spin degrees of freedom. Its most general form is:

\[ H_{ks}(p) = \sum_j f(p_j) S_j. \] \tag{5}
Here $f(p_j)$ is a pseudo-vector, components of which can be represented as a form of order $s$ in the components of the kinetic momentum $p_j^s$.

The spin-orbit interaction leads to correlation of spatial and spin motion of electrons, thus, the translational and spin-related subsystems are not well-defined. Since the SOI is in some sense small, then one can perform a momentum-dependent canonical transformation that decouples kinetic and spin degrees of freedom. Naturally, all other terms in the Hamiltonian, describing the interaction of electrons with the lattice and external fields (if any) also undergo the transformation. In this case, the effective interaction of electrons in the system with external fields appears, which leads to resonant absorption of the field energy not only at the frequency of the paramagnetic resonance $\omega_s$ or cyclotron resonance $\omega_c$, but also at their linear combinations, i.e. the combined resonance. The gauge-invariant theory of the combined resonance has been developed in [10].

Assuming the SOI to be small, we perform the canonical transformation of the Hamiltonian. Up to the terms linear in $T(p)$, we have:

$$\tilde{H} = e^{T(p)H_0}e^{-T(p)} \approx H + [T(p), H].$$

The operator of the canonical transformation $T(p)$ has to be determined from the requirement that, after the transformation, the $k$ and $s$ subsystems become independent. This requirement can be written as the following condition:

$$H_{ks}(p) + [T(p), H_k + H_s] = 0.$$  \hspace{1cm} (7)

Note that, after the canonical transformation, the $H_k$ and $H_s$ operators are the integrals of motion if $H_{ev} = 0$ and there is no interaction with external fields.

We assume the specific form of the SOI term, namely, Rashba interaction, which is non-zero even in the linear order in momentum:

$$H_{ks}(p) = \alpha \varepsilon z ik \sum_j S_j^z p_j^k = \frac{i\alpha}{2} \sum_j (S_j^+ p_j^- - S_j^- p_j^+),$$

$$S^\pm = S^x \pm i S^y, \quad p^\pm = p^x \pm ip^y.$$  \hspace{1cm} (8)

Here $\alpha$ is the constant characterizing the SOI, $\varepsilon$ is the fully-antisymmetric Levi–Civita tensor.

Now we find the explicit expression for the operator $T(p)$. Inserting the operator $T(p)$ into the general solution for Eq. (7) and integrating, we obtain:

$$T(p) = \frac{i}{\hbar} \lim_{\varepsilon \to +0} \int_0^{-\infty} dt e^{i\varepsilon t} e^{itH_s/\hbar} H_{ks}(p) e^{-itH_s/\hbar} =$$

$$= \frac{i \alpha}{2\hbar(\omega_s - \omega_k)} \sum_j (S_j^+ p_j^- - S_j^- p_j^+).$$  \hspace{1cm} (9)

Obviously, the criteria for the applicability of this theory is that, for characteristic values of the electron momentum $p$, the inequality $\alpha p \ll \hbar(\omega_s - \omega_k)$ should hold.

One can write the transformed Hamiltonian in the following form:

$$\tilde{H}(t) = H_0 + H_{ef}^0 + H_{ch}(t) + [T(p), H_{ch}(t) + H_{ef}^0 + H_{ev}].$$  \hspace{1cm} (10)

Using the explicit expression for the operator $T(p), we find:

$$g\mu_0[T(p), S^\alpha]H^\alpha(t) = \frac{ig\alpha \mu_0}{2\hbar(\omega_s - \omega_k)} \{(T^{\pm\dagger} H^\dagger)(t) - (T^{-\dagger} H^-(t)) + (T^{\pm\dagger} - T^{\dagger\pm})H^z(t)\}.$$  \hspace{1cm} (11)

$$T^{\alpha\beta} = \sum_i S_i^\alpha p_i^\beta.$$  \hspace{1cm} (12)

Now we find the operators of power $\tilde{H}_{k(h)}(t) = (i\hbar)^{-1}[H, H_{ch}(t) + [T(p), H_{ch}(t)]]$ absorbed by kinetic ($i = k$) and spin ($i = s$) subsystems due to the interaction of electrons with the AC magnetic field. We have:

$$\tilde{H}_{k(h)}(t) = \frac{g\alpha \mu_0 \omega_c}{2\hbar(\omega_s - \omega_k)} \{(T^{\pm\dagger} H^z)(t) - (T^{-\dagger} H^+(t)) + (T^{\pm\dagger} - T^{\dagger\pm})H^z(t)\}.$$  \hspace{1cm} (13)

The total absorbed power can be written as:

$$\tilde{H}_{k(h)}(t) + \tilde{H}_{s(h)}(t) = J^\beta_m H^\beta(t),$$  \hspace{1cm} (14)

$$J^\beta_m = \frac{g\mu_0}{i\hbar} [S^\beta, [T(p), S^\beta], H_k(p) + H_s].$$

The interaction of the spin degrees of freedom of the conductivity electrons with the AC magnetic field $H_{ch}(t)$ leads to resonant transitions at the frequency $\omega_s$. However, as one can see from the expressions above, the effective interaction $[T(p), H_{ch}(t)]$ leads to combined transitions at frequencies $\omega_s \pm \omega_s$ and the cyclotron frequency $\omega_c$. Since, for our further calculations, the response of the non-equilibrium system to the measurement electric field is interesting, in which the contribution from the translational degrees of freedom dominates, we will restrict our consideration to the effective interaction solely. Besides that, we limit the consideration to the case when the DC and AC magnetic fields are perpendicular to each
other: \( \mathbf{H}(t) = (H_x(t), H_y(t), 0) \). In this case, the effective interaction responsible for the combined transitions has the following form:

\[
H_{\text{eff},1}(t) = [T(p), H_{\text{eff}}(t)] = \frac{i\omega_1 \hbar}{2} \sum_j \left( p_j^+ e^{-\omega_1 t} - p_j^- e^{i\omega_1 t} \right).
\] (15)

\( \omega_1 = geH_1/(2m_0c) \). \( H_1 \) is the intensity of the circularly polarized magnetic field, rotating with the frequency \( \omega \).

The dependence of the effective interaction \( H_{\text{eff},1}(t) \) upon time causes certain difficulties while calculating the non-equilibrium response of the electron system to the measurement electric field. Thus, it is expedient to carry out one more canonical transformation (Appendix A), that removes the interaction \( H_{\text{eff},1}(t) \) and renormalizes the electron-impurity interaction Hamiltonian (Appendix B), which acquires the time dependence then. In the canonically transformed system, impurities act as a coherent oscillating field that leads to resonant transitions.

### III. NON-EQUILIBRIUM RESPONSE

We assume that the initial non-equilibrium state of the system under consideration is created by the ultra-high frequency magnetic field and can be described with the distribution \( \tilde{\rho}(t) \). Obviously, if some additional perturbation acts upon the system, then a new non-equilibrium state is formed in the system, that requires an extended set of basis operators for its description. The new non-equilibrium distribution is described with the operator \( \rho(t,0) \). The task is to find the response of a non-equilibrium system to a weak measurement field.

We write the operator \( \rho(t) \) using the integral representation for the non-equilibrium statistical operator. In the linear approximation in the external field \( \mathbf{E} \), we have:

\[
\rho(t) = \tilde{\rho}(t) - i \int_{-\infty}^{0} dt_1 \varepsilon_t U(t + t_1) \times \times iL_{0f} \rho(t + t_1) U^+(t + t_1). \] (16)

\[
iL_{0f} A = \frac{1}{\hbar} [A, H_{0f}].
\]

The operator \( \tilde{\rho}(t) \) satisfies the equation

\[
\frac{\partial \tilde{\rho}(t)}{\partial t} + \frac{1}{\hbar} [\tilde{\rho}(t), H(t)] = -\varepsilon(\tilde{\rho}(t) - \rho_q(t)),
\] (17)

which has the following solution:

\[
\tilde{\rho}(t) = \rho_q(t) - \int_{-\infty}^{0} dt_1 \varepsilon_t U(t + t_1) \times \times \left\{ \frac{\partial}{\partial t} + iL(t) \right\} \rho_q(t + t_1) U^+(t + t_1),
\] (18)

\[
iL(t)A = \frac{1}{\hbar} [A, H(t)],
\]

where \( U(t_1) \) is the evolution operator that is in fact a chronologically ordered exponent and satisfies the differential equation

\[
\frac{\partial U(t)}{\partial t} = \frac{1}{\hbar} H(t)U(t).
\]

The quasiequilibrium statistical operator \( \rho_q(t) \) can be expressed as:

\[
\rho_q(t) = e^{-S(t)}, \quad S(t) = \Phi(t) + \sum_n P_n^+ F_n(t), \quad (19)
\]

where \( S(t) \) is the entropy operator, \( \Phi \) is the Massieu—Planck functional. \( P_n, F_n \) are sets of the basis operators and their conjugate functions, that describe the electron system.

Characterizing the state of the system by the mean values of the operators \( H_k, p, H_s, N, H_v \) (\( N \) being the operator of the number of electrons), we obtain for the entropy operator:

\[
S(t) = \Phi(t) + \beta_k (H_k - V(t) p - \mu' N) + \beta_s H_s + + \beta H_v = S_0 + \Delta S(t). \quad (20)
\]

\[
\Delta S(t) = -\beta_k V(t)p.
\]

Here \( \beta_k, \beta_s, \mu' = \mu - mV^2/2, V, \beta \) are parameters thermodynamically conjugate to the average values of the introduced operators. They have the meaning of inverse effective temperatures of the kinetic and spin electron subsystems, chemical potential, drift velocity and the inverse lattice temperature. Introduction of the effective temperatures allows one to treat effects related to “heating” of the electron and spin subsystems by external fields.

A linear admittance corresponding to an arbitrary operator \( B \) in the case when the external force oscillates harmonically in time with the frequency \( \omega_1 \) can be expressed as:

\[
\chi_{BA}(t,\omega_1) = -\int_{-\infty}^{0} dt_1 e^{i\omega_1 t_1} \frac{1}{\hbar} \text{Sp}\{B \times \
\times e^{i\varepsilon_t L[A, \tilde{\rho}(t + t_1,0)]}\} \] (21)

Within the framework described above, the task of the admittance calculation is reduced to obtaining the transport matrix \( T_{BA}(t,\omega_1) \), which plays in the non-equilibrium case the same role as in the case of the equilibrium response:

\[
\chi_{BA}(t,\omega_1) = \chi_{BA}(t,0) \frac{T_{BA}(t,\omega_1) + \varepsilon}{T_{BA}(t,\omega_1) + \varepsilon - i\omega_1}. \quad (22)
\]

\[
\chi_{BA}(t,0) = \langle B, A \rangle, \quad T_{BA} = \frac{1}{\langle B, A \rangle \omega_1} \langle B, A \rangle. \quad (23)
\]
Here
\begin{equation}
(B, A) = -\frac{1}{\hbar} \int_{-\infty}^{0} dt_1 e^{\varepsilon t_1} \text{Sp}\{B e^{it_1 L} [A, \bar{\rho}(t + t_1, 0)]\},
\end{equation}
(24)
\begin{equation}
(B, A)^{\omega_1} = \frac{1}{i \hbar} \int_{-\infty}^{0} dt_1 e^{(-i \omega_1) t_1} dt_2 e^{\varepsilon t_2} \text{Sp}\{B \times \nonumber \}
\end{equation}
\begin{equation}
\times e^{i(t_1 + t_2) L} [A, \bar{\rho}(t + t_1 + t_2, 0)]\}.
\end{equation}
(25)

The real part of the transport matrix determines the relaxation frequency of the non-equilibrium electrons' momentum.

**IV. MOMENTUM RELAXATION RATE**

Assuming the temperatures of the translational and spin subsystems to be equal (that corresponds to neglecting any heating effects), in the Born approximation upon the electron-scatterer interaction we obtain for the relaxation frequency:
\begin{equation}
\frac{1}{\tau} = \frac{\beta}{2mn} \Re \frac{1}{\hbar} \int_{-\infty}^{0} dt_1 e^{(-i \omega_1) t_1} dt_2 e^{\varepsilon t_2} \int_{0}^{1} d\lambda \times \nonumber 
\end{equation}
\begin{equation}
\times \text{Sp}\{\bar{p}_0^+(t_1) e^{iL_0(t_1 + t_2)} \rho_0^1 \bar{p}_0^-(t_1 + t_2), H_k + H_s \rho_0^1 \}, \rho_0^1 = \frac{1}{i \hbar} [p^0, \bar{H}_c] \text{.}
\end{equation}
(26)

Now we expand the formula using the explicit expression for the renormalized electron-impurity interaction $H_{\text{imp}}$ from Appendix B.

It is convenient to introduce the following notation:
\begin{equation}
A(\lambda, t_1 + t_2) = \text{Sp}\{\bar{p}_0^+(t_1) e^{iL_0(t_1 + t_2)} \times \nonumber \}
\end{equation}
\begin{equation}
\times \rho_0^1 \bar{p}_0^-(t_1 + t_2), H_k + H_s \rho_0^1 \lambda \}. (27)
\end{equation}

Inserting the explicit expression for the electron-impurity interaction and averaging over the system of scatterers, we obtain:
\begin{equation}
A(\lambda, t_1 + t_2) = \sum_{q, j, l} |V(q)|^2 N_i e^{-i \omega(t_1 + t_2) j^2 (|K| q)^2} \times \nonumber \}
\end{equation}
\begin{equation}
\times \text{Sp}\{2 S_j^e e^{i q r_j} e^{iL_0(t_1 + t_2)} \rho_0^1 \times \nonumber \}
\end{equation}
\begin{equation}
\times \rho_0^1 \bar{p}_0^-(t_1 + t_2), H_k + H_s \rho_0^1 \lambda \}. (28)
\end{equation}

Here $N_i$ is the impurity concentration. Rewriting this expression using secondary quantization and averaging the electron operators using Wick's theorem, we have:
\begin{equation}
A(\lambda, t_1 + t_2) = \sum_{q, \nu, \mu} |V(q)|^2 N_i e^{-i \omega(t_1 + t_2)} \times \nonumber \}
\end{equation}
\begin{equation}
\times \text{Sp}\{2 S_j^e e^{i q r_j} e^{iL_0(t_1 + t_2)} \rho_0^1 \times \nonumber \}
\end{equation}
\begin{equation}
\times \rho_0^1 \bar{p}_0^-(t_1 + t_2), H_k + H_s \rho_0^1 \lambda \times \nonumber \}
\end{equation}
\begin{equation}
\times J_0^2 (|K| q)^2 (\varepsilon_\nu - \varepsilon_\mu) |2 S_j^e e^{i q r_j} \nu, \mu \nu^2 f_\nu (1 - f_\mu), (29)
\end{equation}

where $f$ is the Fermi–Dirac distribution.

After integration over $\lambda$, $t_1$ and $t_2$, this produces:
\begin{equation}
\int_{0}^{1} d\lambda A(\lambda, t_1 + t_2) = \sum_{q, \nu, \mu} |V(q)|^2 N_i J_0^2 (|K| q)^2 |2 S_j^e e^{i q r_j} \nu, \mu \nu^2 (f_\nu - f_\mu) \times \nonumber 
\end{equation}
\begin{equation}
\times (\varepsilon - i \lambda \omega - (i/\hbar) (\varepsilon_\nu - \varepsilon_\mu))^{-1} \varepsilon - i \lambda \omega - (i/\hbar) (\varepsilon_\nu - \varepsilon_\mu) \}. (30)
\end{equation}

In the $\varepsilon \rightarrow 0$ limit, we obtain:
\begin{equation}
\frac{1}{\hbar^2} = \left( \frac{P}{\hbar \omega + (\varepsilon_\mu - \varepsilon_\nu)} - i \pi \delta(\hbar \omega + (\varepsilon_\mu - \varepsilon_\nu)) \right), (31)
\end{equation}

where $P$ denotes the Cauchy principal value. Taking the $\omega_1 \rightarrow 0$ limit (because calculating a zero-frequency response is our goal), we have:
\begin{equation}
\Delta(\frac{1}{\tau}) = -\frac{\pi \hbar}{2 mn} \sum_{q, \nu, \mu} \int d\varepsilon |V(q)|^2 N_i J_0^2 (|K| q)^2 \times 
\end{equation}
\begin{equation}
\times |(2 S_j^e e^{i q r_j}) \nu, \mu \nu^2 (f(\varepsilon + \hbar \omega) - f(\varepsilon)) \times \nonumber \}
\end{equation}
\begin{equation}
\times \delta(\varepsilon - \varepsilon_\mu) \frac{\partial}{\partial \varepsilon} \delta(\hbar \omega + \varepsilon - \varepsilon_\nu) \}. (32)
\end{equation}

The equation contains a singularity in its right hand side, which is removed, as usual, due to broadening of the Landau levels by scattering electrons on impurities:
\begin{equation}
\delta(\varepsilon - \varepsilon_\mu) \rightarrow D_\mu(\varepsilon) = \frac{\sqrt{\pi/2}}{\Gamma} \exp \left( -\frac{(\varepsilon - \varepsilon_\mu)}{2\Gamma} \right). (33)
\end{equation}

The Landau level width $\Gamma$ can be expressed via the electron mobility $\mu$ in zero magnetic field:
\begin{equation}
\Gamma = \hbar \sqrt{\frac{2\gamma \omega_c}{\pi \tau_{tr}}} = \frac{m\mu}{|e|} (34)
\end{equation}

Note that, if $T > \Gamma$, one can pull $f(\varepsilon \pm \hbar \omega) - f(\varepsilon)$ out of the $\varepsilon$ integral as a slowly-changing factor. This yields:
\begin{equation}
\int d\varepsilon \frac{\partial}{\partial \varepsilon} D_\mu(\varepsilon \pm \hbar \omega) D_\mu(\varepsilon) = 
= -\frac{\pi^{3/2}(\varepsilon_\mu - \varepsilon_\nu \pm \hbar \omega)}{4\Gamma^3} \exp \left( -\frac{(\varepsilon_\mu - \varepsilon_\nu \pm \hbar \omega)^2}{4\Gamma^2} \right) (35)
\end{equation}

The wave functions upon which the matrix elements in are calculated, have the form:
\begin{equation}
\psi_\nu \equiv \psi_{nk^x s^z} = \frac{1}{\sqrt{2^n n! \pi^{1/2} \ell}} \exp(ik^x x) \times \nonumber \}
\end{equation}
\begin{equation}
\times \exp \left( -\frac{(y - y_0)^2}{2\ell^2} \right) H_n \frac{y - y_0}{\ell} x s^z (36)
\end{equation}
Calculation of the matrix element in (32) produces:

\[ |\langle \nu | 2S^z \exp (i q r) | \mu \rangle |^2 = \exp \left( -\frac{\ell^2 q^2}{2} \right) \times \]
\[ \times \left( \frac{\min(n_\nu, n_\mu)}{\max(n_\nu, n_\mu)} \right) ! \left( \frac{\ell^2 q^2}{2} \right)^{|n_\nu - n_\mu|} \times \]
\[ \times \left( L_{\min(n_\nu, n_\mu)} \left( \frac{\ell^2 q^2}{2} \right) \right)^2 \delta_{x_\nu, x_\mu} \delta_{y_\nu, y_\mu} \delta_{\nu, \mu} \] (37)

In the case of sufficiently weak AC magnetic field one can neglect terms with \(|l| > 1\) and use the approximation 
\[ J_{\pm 1}(x) = \pm x / 2. \]
As a result of such simplifications, we obtain:
\[ \Delta(\frac{1}{\tau}) = \frac{\hbar}{8 \pi n^2} \sum_{q, n_\nu, n_\mu = \pm 1} |V(q)|^2 N_i |K_q|^2 q^2 \times \]
\[ \times \exp \left( -\frac{\ell^2 q^2}{2} \right) \left( \frac{\min(n_\nu, n_\mu)}{\max(n_\nu, n_\mu)} \right) ! \left( \frac{\ell^2 q^2}{2} \right)^{|n_\nu - n_\mu|} \times \]
\[ \times \left( L_{\min(n_\nu, n_\mu)} \left( \frac{\ell^2 q^2}{2} \right) \right)^2 \left( f(\varepsilon_\nu) - f(\varepsilon_\mu) \right) \times \]
\[ \times \frac{\pi^{3/2}(\varepsilon_\mu - \varepsilon_\nu + \hbar \omega)^2}{4 \Gamma^3} \exp \left( -\frac{\varepsilon_\mu - \varepsilon_\nu + \hbar \omega)^2}{4 \Gamma^2} \right) \] (38)

Integrating over \( q \), we have:
\[ \int_0^{\infty} d(q^2)q^4 \exp \left( -\frac{\ell^2 q^2}{2} \right) \left( \frac{\ell^2 q^2}{2} \right)^{|n_\nu - n_\mu|} \times \]
\[ \times \left( L_{\min(n_\nu, n_\mu)} \left( \frac{\ell^2 q^2}{2} \right) \right)^2 = \frac{8}{\pi^6} \left( \frac{\max(n_\nu, n_\mu)}{\min(n_\nu, n_\mu)} \right) ! \times \]
\[ \times \left( n_\nu^2 + n_\mu^2 + 3(n_\nu + n_\mu) + 4n_\nu n_\mu + 2 \right) \] (39)

Thus, for the case of point scatterers, where \( V(q) \) does not depend on \( q \), the radiation-induced correction to the inverse relaxation time is:
\[ \Delta(\frac{1}{\tau}) = \frac{\hbar}{4 \pi n^2} \sum_{n_\nu, n_\mu = \pm 1} |V(q)|^2 N_i \sigma^2 \omega_\perp \times \]
\[ \times (n_\nu^2 + n_\mu^2 + 3(n_\nu + n_\mu) + 4n_\nu n_\mu + 2) \times \]
\[ \times (f(\varepsilon_\nu) - f(\varepsilon_\mu)) \frac{\pi^{1/2}(\varepsilon_\mu - \varepsilon_\nu + \hbar \omega)^2}{4 \Gamma^3} \times \]
\[ \times \exp \left( -\frac{(\varepsilon_\mu - \varepsilon_\nu + \hbar \omega)^2}{4 \Gamma^2} \right) \] (40)

Using the expression for the momentum relaxation rate, one can also write the formula for the diagonal components of the conductivity tensor \( \sigma_{xx} \), according to which the numerical calculations in the next section are carried out:
\[ \sigma_{xx} = \frac{n e^2}{m} \frac{\tau^{-1}}{\omega_\perp^2 + \tau^{-2}}. \] (41)

V. NUMERICAL ANALYSIS

Numerical calculations of the diagonal components of the conductivity tensor according to Eq. (41) have been carried out with the following parameters: \( m = 0.067 m_0 \) (\( m_0 \) is the free electron mass), the Fermi energy is \( E_F = 10 \text{ meV} \), the mobility of the 2D electrons varies as \( \mu \approx 0.1 - 1.0 \times 10^7 \text{ cm}^2/\text{Vs}, \) the electron density \( n = 3 \times 10^{11} \text{ cm}^{-2} \). The microwave radiation frequency is \( f = 50 \text{ GHz}, \) the temperature is \( T \approx 2.4 \text{ K} \). The magnetic field varied as \( 0.02 - 0.3 \text{ T} \).

The dependence of the 2D electron gas photoconductivity on the \( \omega/\omega_\perp \) ratio is presented in Fig. 1. One can see that the dependence of electron mobility upon the magnetic field has the oscillating character. In the region of low magnetic field, the oscillation amplitude drops significantly when zero-magnetic-field mobility is decreased.

In Fig. 2 the photoconductivity dependence upon the magnetic field is presented for different values of \( \gamma = 2 \) and the same microwave radiation frequency 50 GHz and electron mobility \( \mu = 0.6 \times 10^7 \text{ cm}^2/\text{Vs}. \) As one can see, the oscillation amplitude is very sensitive to the width of Landau levels.

VI. CONCLUSION

The response of a non-equilibrium electron system to the DC electric measurement field has been studied for the case when the initial non-equilibrium state of the system is created by an ultra-high frequency magnetic field that leads to combined transitions. Within the proposed theory, it has been shown that such perturbation of the electron system essentially influences the transport coefficients and leads to the oscillations of the diagonal components of the conductivity tensor. The discussed effect is analogous to the phenomenon observed in GaAs/AlGaAs heterostructures with ultra-high elec-
tron mobility $\mathcal{g}$. However, unlike that phenomenon, the manifestation of the oscillatory pattern is dictated by the spin-orbit interaction existing in the crystals under consideration.

**APPENDIX A**

In this Appendix, a canonical transformation $W_2(t)$ is built, that excludes the renormalized interaction with the AC magnetic field from the effective Hamiltonian. The canonical transformation operator is searched from the equation:

$$iW_2^\dagger(t)(-i\hbar \frac{\partial}{\partial t} + H_k + H_s + H_{ch,1}(t))W_2(t) =$$

$$=-i\hbar \frac{\partial}{\partial t} + H_k + H_s.$$  \hspace{1cm} (A1)

The operator $W_2(t)$ is searched in the following form:

$$W_2(t) = \exp(i \sum_j (\eta^+ (t) p_j^+ S_j^z + \eta^- (t) p_j^- S_j^z)) \times$$

$$\times \exp(i \theta(t)), \hspace{1cm} (A2)$$

where one has to determine the parameters $\theta(t)$ and $\eta^\pm(t)$.

In order to determine these parameters, the canonical transformation is applied to all terms in the left-hand side of Eq. (A1):

$$W_2^\dagger(t)(-i\hbar \frac{\partial}{\partial t})W_2(t) = -i\hbar \frac{\partial}{\partial t} + \hbar \theta(t) +$$

$$+ \hbar \sum_j (\dot{\eta}^- (t) p_j^+ + \dot{\eta}^+ (t) p_j^-) \hspace{1cm} (A3)$$

$$W_2^\dagger(t)H_kW_2(t) = \frac{1}{4m} \sum_j (p_j^+ p_j^-)$$

$$- 4im\hbar \omega_c \eta^+ (t) S_j^z p_j^- + 4im\hbar \omega_c \eta^- (t) S_j^z p_j^+$$

$$+ m^2 \hbar^2 \omega_c^2 \eta^+ (t) \eta^- (t) \hspace{1cm} (A4)$$

$$W_2^\dagger(t)H_sW_2(t) = (A5)$$

$$W_2^\dagger(t)H_{ch,1}(t)W_2(t) = \frac{i\alpha \omega_{1s}}{2(\omega_c - \omega_s)} \sum_j S_j^z \tilde{\eta}_j^+(t)$$

$$- 2im\hbar \omega_c \eta^+ (t) S_j^z e^{-i\omega t} -$$

$$\tilde{\eta}_j^- (t) S_j^z e^{i\omega t} \hspace{1cm} (A6)$$

Thus one can write down the explicit expressions for $\eta^\pm(t)$, $\theta(t)$:

$$\eta^\pm (t) = \frac{\alpha \omega_{1s}}{2\hbar (\omega_c - \omega_s)} e^{\pm i\omega t}, \hspace{1cm} (A7)$$

$$\theta(t) = \frac{Nm \omega_{1s} (3\omega_c - 4\omega) t}{16\hbar (\omega_c - \omega_s)^2 (\omega_c - \omega)^2} \hspace{1cm} (A8)$$

Therefore, the explicit form of the canonical transformation $W_2$ is known. The physical meaning of this transformation is the change to two non-uniformly translationally moving reference frames, different for electrons with opposite spin directions.

**APPENDIX B**

Since we consider only elastic scattering, the electron-impurity interaction Hamiltonian has the form:

$$H_{ev} = \sum_{q} V(q) \rho(q) e^{i qr}, \hspace{1cm} (B1)$$

where $V(q)$ is a Fourier component of the potential created by a single impurity corresponding to the wave vector $q$, $\rho(q)$ is a Fourier component of the impurity number density.
As a result of the canonical transformation \( W_2(t) \), renormalization of the electron-impurity interaction happens. For obtaining the renormalized Hamiltonian of the electron-impurity interaction, it is sufficient to calculate \( W_2^\dagger(t) \exp(i{\bf qr}_j)W_2(t) \). Using the explicit form of the operator \( W_2^\dagger(t) \), we obtain:

\[
W_2^\dagger(t) e^{i{\bf qr}_j} W_2(t) = \exp(i{\bf qr}_j + \Delta{\bf r}_j), 
\]

where

\[
\Delta{\bf r}_j = -\frac{\omega_1s}{(\omega_c - \omega_s)(\omega - \omega_c)} S_j^z \times (\cos \omega t, \sin \omega t, 0). \tag{B3}
\]

Expansion of the exponent in (B2) in terms of Bessel functions \( J_l(x) \) yields:

\[
e^{-i \text{Re}(2K_q S_j^z e^{i \omega t})} = \sum_{l=-\infty}^{\infty} \left( 2S_j^z \frac{K_q}{i|K_q|} e^{i \omega t} \right)^l J_l(|K_q|), \tag{B4}
\]

Thus, the renormalized Hamiltonian of the electron-impurity interaction is:

\[
\tilde{H}_{ee}(t) = \sum_{\bf q} \sum_{l=-\infty}^{\infty} V(\bf q) \rho(\bf q) e^{i{\bf qr}_j} \times \left( 2S_j^z \frac{K_q}{i|K_q|} e^{i \omega t} \right)^l J_l(|K_q|). \tag{B5}
\]

[1] M. A. Zudov and R. R. Du, L. N. Pfeiffer and K. W. West, [arXiv:cond-mat/0210034] Phys. Rev. Lett. 90, 046807 (2003); EP2S-15, Nara, Japan. 2003
[2] R. G. Mani, J. H. Smet, K. von Klitzing, V. Narayananmurti, W. B. Johnson, and V. Umansky, Nature, 420, 646 (2002); [arXiv:cond-mat/0306388] 26-Inter. Conf. Phys. of Semicond. Edinburg. 2002; Ep2S-15, Nara, Japan. 2003
[3] B.Das, D. C. Miller, S. Datta, R. Reifenberger, W.P. Hong P.K. Bhattacharya, M. Jaffe, Phys. Rev B bf 39, 1411, (1989).
[4] P.R. Hammar, M. Johnson, Phys. Rev. B 61 11, 7207 (2000).
[5] L.S. Levitov, Yu.V. Nazarov, G.M. Eliashberg, Sov.Phys. JETP 61 1333, (1985).
[6] E.I. Rashba, Uspehi Fizichekikh Nauk, 84 557, (1964).
[7] V.P. Kalashnikov, I.I. Lyapilin, Teoreticheckaya i Matematicheskaya Fizika 18, 273,(1974).
[8] V.P. Kalashnikov, Teoreticheckaya i Matematicheskaya Fizika 5, 2293 (1970).