Class Specific or Shared? A Hybrid Dictionary Learning Network for Image Classification

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Abstract Dictionary learning methods can be split into two categories: i) class specific dictionary learning ii) class shared dictionary learning. The difference between the two categories is how to use the discriminative information. With the first category, samples of different classes are mapped to different subspaces which leads to some redundancy in the base vectors. For the second category, the samples in each specific class can not be described well. Moreover, most class shared dictionary learning methods use the $\ell_0$-norm regularization term as the sparse constraint. In this paper, we first propose a novel class shared dictionary learning method named label embedded dictionary learning (LEDL) by introducing the $\ell_1$-norm sparse constraint to replace the conventional $\ell_0$-norm regularization term in LC-KSVD method. Then we propose a novel network named hybrid dictionary learning network (HDLN) to combine the class specific dictionary learning with class shared dictionary learning together to fully describe the feature to boost the performance of classification. Extensive experimental results on six benchmark datasets illustrate that our methods are capable of achieving superior performance compared to several conventional classification algorithms.

Keywords Class Specific Dictionary Learning · Class Shared Dictionary Learning · Label Embedded

Fig. 1 Illustration of the variation of sample distribution. Different circles represent different subspaces.

Dictionary Learning · Hybrid Dictionary Learning Network · Image Classification

1 Introduction

Recent years, image classification has been a classical issue in pattern recognition. With advancements in theory, many image classification methods have been proposed [1–21]. In these methods, there is one category that contributes a lot for image classification which is the dictionary learning (DL) based method. DL is a generative model which the concept was firstly proposed by Mallat et al. [22]. A few years later, Olshausen et al. [23, 24] proposed the application of DL on natural images and then it has been widely used in many fields such as image denoising [25, 26], image superresolution [27, 28] and image classification [29, 30]. According to different ways of utilizing the discriminative information, DL methods can be split into two categories: i) class specific dictionary learning ii) class shared dictionary learning.

Class specific dictionary learning method utilises the discriminative information by adding discrimination ability into dictionary. The learned dictionary is for each class. This category can gain the representative feature information of a class. The feature information
that most samples of the class have are focused on, while the feature information that only a few samples of the class have is ignored to some extent. That is to say, the learned dictionary has higher weight on the feature information which samples close to the distribution center, and lower weight on the feature information that samples off the center. With this method, some abnormal sample points are ignored so that the robustness of the learned dictionary can be improved. There are many classical class specific dictionary learning algorithms have been reported recent years such as [1, 8, 31]. However, the dictionary learned by this approach has a drawback: Due to the learned dictionary is for each class, the training samples of each class are mapped to a separate subspace. It leads to some redundancy in the base vectors among different subspaces. For example, in face datasets, the features of eyes are similar in different classes. In other words, we may obtain similar base vectors with different classes. During testing stage, it is hard to opt the base vector which belongs to the same class of the testing sample to fit the testing sample for eyes. Thus, despite this way can describe the training samples well, it is not conducive to representing the testing samples while the dictionaries of all classes are cascaded together.

For class shared dictionary learning method, the discriminative information is directly embedded into the objective function to learn a dictionary for all classes. With this method, the training samples from all classes are mapped into one subspace. Hence, the representative feature information of all classes can be adopted. However, it can not describe the samples in each specific class well. Moreover, most class shared dictionary learning methods use the $\ell_0$-norm regularization term as the sparse constraint which leads to the NP-hard problem. Despite some greedy methods such as orthogonal matching pursuit (OMP) can help solve this problem to some extent, it is usually to find the suboptimum sparse solution instead of the optimal sparse solution.

In comparison to class specific dictionary learning and class shared dictionary learning, it is clear that the two methods have complementary advantages. It can help to get significant boost in classification accuracy if the advantages of the two dictionary learning methods can be properly combined. In this paper, we first propose a novel class shared dictionary learning algorithm named label embedded dictionary learning (LEDL). This method introduces the $\ell_1$-norm regularization term to replace the $\ell_0$-norm regularization of LC-KSVD [14]. Then we propose a novel network named hybrid dictionary learning network (HDLN) to combine a class specific dictionary learning method with a class shared dictionary learning method together.

Our network contains two layers. Specifically, the first layer is consisted of the class specific dictionary learning for sparse representation (CSDL-SRC) method, it is used to extract the crucial feature information of a class to wipe off singular points and improve robustness. The second layer is composed of LEDL which pulled the feature information belongs to different subspaces back into the same subspace to obtain the relationship among different classes. Figure 1 shows the variation of sample distribution. Figure 1a shows the random distribution samples belong to three classes; Figure 1b shows that the samples belongs to the same
class are clustered while the samples of three classes are in different subspaces; Figure 1 shows that the samples in different subspaces are pulled back into the same subspace. A schematic description of our proposed HDLN is given in Figure 2. We adopt the alternating direction method of multipliers (ADMM) [33] algorithm and blockwise coordinate descent (BCD) [35] algorithm to optimize HDLN. The contributions of this work are four-fold:

1) We propose a novel class shared dictionary learning method named label embedded dictionary learning (LEDL) that introduces the $\ell_1$-norm regularization term as the sparse constraint. The $\ell_1$-norm sparse constraint can easily find the optimal sparse solution.

2) We propose a novel dictionary learning network named hybrid dictionary learning network (HDLN) that discriminative information is used in different ways to fully describe the feature while completely maintain the discriminative information. The HDLN can be considered as the extension of conventional dictionary learning algorithms.

3) We propose to utilize the alternating direction method of multipliers (ADMM) [33] algorithm and blockwise coordinate descent (BCD) [35] algorithm to optimize each layer of dictionary learning task.

4) The proposed LEDL and HDLN methods are evaluated on six benchmark datasets and verifies the superior performance of our methods.

The rest of this paper is organized as follows. Section 2 briefly reviews related work on CSDL-SRC and LEDL. Section 3 presents LEDL and HDLN methods for image classification. The optimization approach is elaborated in Section 4. Section 5 shows experimental results on six well-known datasets. And finally Section 6 is the conclusion.

2 Related work

In this section, we overview two related dictionary learning methods, including class specific dictionary learning for sparse representation (CSDL-SRC) and label consistent K-SVD (LC-KSVD).

2.1 Class specific dictionary learning for sparse representation (CSDL-SRC)

Liu et al. [8] proposed CSDL-SRC to reduce the high residual error and instability of SRC. The authors consider the weight of each sample feature when generating the dictionary. Assume that $X = [x^1, x^2, \ldots, x^C] \in \mathbb{R}^{d \times N}$ is the training sample matrix, where $d$ represents the dimensions of the sample features, $N$ and $C$ are the number of training samples and the class number of training samples, respectively. The $c_{th}$ class of training sample matrix is denoted as $X^c \in \mathbb{R}^{d \times N^c}$, where $c = 1, 2, \ldots, C$ and $N^c$ is the $c_{th}$ class of $N = \sum_{c=1}^{C} N^c$. Liu et al. build a weight coefficient matrix $P^c \in \mathbb{R}^{N^c \times K^c}$ for $X^c$, where $K$ is the dictionary size of CSDL-SRC and $K^c$ is the $c_{th}$ class of $K = \sum_{k=1}^{K} K^c$. The objective function of CSDL-SRC is as follows:

$$<P^c, U^c> = \arg \min_{P^c, U^c} \|X^c - P^c U^c\|_F^2 + 2\zeta \|U^c\|_{\ell_1}$$

s.t. $\|P^c u^*_c\|_2 \leq 1 \quad (k = 1, 2, \ldots, K)$

where $U^c \in \mathbb{R}^{K^c \times N^c}$ is the sparse codes of $X^c$, the $\ell_1$-norm regularization term is utilized to enforce the sparsity, $\zeta$ is the regularization parameter to control the tradeoff between fitting goodness and sparseness. The $(u^*_c)_c$ denote the $k_{th}$ column vector of matrix $(u)$.

2.2 Label consistent K-SVD (LC-KSVD)

Jiang et al. [14] proposed LC-KSVD to combine the discriminative sparse codes error with the reconstruction error and the classification error to form a unified objective function which is defined as follows:

$$<B, W, A, V> = \arg \min_{B, W, A, V} \|X - BV\|_F^2 + \lambda \|H - WV\|_F^2,$$

$$+ \omega \|Q - AV\|_F^2,$$

s.t. $\|v_i\|_2 < T \quad (i = 1, 2, \ldots, N)$(2)

where $T$ is the sparsity constraint factor, $B \in \mathbb{R}^{d \times K}$ is the dictionary matrix of $X$, $V \in \mathbb{R}^{K \times N}$ is the sparse codes matrix of $X$. $W \in \mathbb{R}^{C \times K}$ is a classifier matrix learned from the given label matrix $H \in \mathbb{R}^{C \times N}$. We hope $W$ can return the most probable class this sample belongs to, $Q \in \mathbb{R}^{K \times N}$ represents the discriminative sparse codes matrix and $A = [a_1, a_2, \ldots, a_K] \in \mathbb{R}^{K \times K}$ is a linear transformation matrix relies on $Q$, $\lambda$ and $\omega$ are the regularization parameters balancing the discriminative sparse codes errors and the classification contribution to the overall objective function, respectively.

Here, CSDL-SRC is a class specific dictionary learning method, while LC-KSVD is a class shared dictionary method. The difference of the two methods is shown in Figure 3. $0$ represents the zero matrix.

3 Proposed hybrid dictionary learning network (HDLN)

In this section, we elaborate the construction of hybrid dictionary learning network (HDLN). Specifically, in subsection 3.1 we introduce the first layer of the network which is composed of CSDL-SRC. In subsection 3.2 we propose LEDL and let it be the second layer of the network.
where $D, H$ the input of the next layer. In addition, the label matrix $S$ above, we explicitly construct a sparse codes matrix and discriminative sparse codes matrix of the first layer in our proposed HDLN, respectively. Based on the computation above, we explicitly construct a sparse codes matrix $S$ from the first layer and make it to be one of the input of the next layer. In addition, the label matrix $H \in \mathbb{R}^{C \times N}$ and discriminative sparse codes matrix $Q \in \mathbb{R}^{K_2 \times N}$ are also introduced to the second layer. After giving a reasonable dictionary size $K_2$ of LEDL, the objective function can be written as follows:

$$<D_{2I}, W, A, S_{1I} > = \arg \min_{D_{2I}, W, A, S_{1I}} \left\| S_{1I} - D_{2I} S_{2} \right\|_{F}^{2} + \lambda \left( \left\| H - WS_{1I} \right\|_{F}^{2} + \omega \left\| Q - AS_{1I} \right\|_{F}^{2} + 2\zeta \left\| S_{2} \right\|_{\ell_{1}} \right).$$

(4)

where $D_{1I} \in \mathbb{R}^{d \times K_1}$ and $S_{1I} \in \mathbb{R}^{K_1 \times N}$ are the dictionary matrix and sparse codes matrix of the first layer in our proposed HDLN, respectively.

3.2 The second layer

We propose a novel class shared dictionary method named label embedded dictionary learning (LEDL) which introduces the $\ell_{0}$-norm regularization term to replace the $\ell_{0}$-norm regularization of LC-KSVD. And the second layer is consisted of LEDL. Based on the computation above, we explicitly construct a sparse codes matrix $S$ from the first layer and make it to be one of the input of the next layer. In addition, the label matrix $H \in \mathbb{R}^{C \times N}$ and discriminative sparse codes matrix $Q \in \mathbb{R}^{K_2 \times N}$ are also introduced to the second layer. After giving a reasonable dictionary size $K_2$ of LEDL, the objective function can be written as follows:

$$<D_{2I}, W, A, S_{1I} > = \arg \min_{D_{2I}, W, A, S_{1I}} \left\| S_{1I} - D_{2I} S_{2} \right\|_{F}^{2}$$

$$+ \lambda \left( \left\| H - WS_{1I} \right\|_{F}^{2} + \omega \left\| Q - AS_{1I} \right\|_{F}^{2} + 2\zeta \left\| S_{2} \right\|_{\ell_{1}} \right).$$

(4)

where $D_{1I} \in \mathbb{R}^{d \times K_1}$ and $S_{1I} \in \mathbb{R}^{K_1 \times N}$ are the dictionary of the first layer and make it to be one of the input of the next layer. In addition, the label matrix $H \in \mathbb{R}^{C \times N}$ and discriminative sparse codes matrix $Q \in \mathbb{R}^{K_2 \times N}$ are also introduced to the second layer. After giving a reasonable dictionary size $K_2$ of LEDL, the objective function can be written as follows:

$$<D_{2I}, W, A, S_{1I} > = \arg \min_{D_{2I}, W, A, S_{1I}} \left\| S_{1I} - D_{2I} S_{2} \right\|_{F}^{2}$$

$$+ \lambda \left( \left\| H - WS_{1I} \right\|_{F}^{2} + \omega \left\| Q - AS_{1I} \right\|_{F}^{2} + 2\zeta \left\| S_{2} \right\|_{\ell_{1}} \right).$$

(4)

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$$+ \lambda \left( \left\| H - WS_{1I} \right\|_{F}^{2} + \omega \left\| Q - AS_{1I} \right\|_{F}^{2} + 2\zeta \left\| S_{2} \right\|_{\ell_{1}} \right).$$

(4)

where $D_{1I} \in \mathbb{R}^{d \times K_1}$ and $S_{1I} \in \mathbb{R}^{K_1 \times N}$ are the dictionary of the first layer and make it to be one of the input of the next layer. In addition, the label matrix $H \in \mathbb{R}^{C \times N}$ and discriminative sparse codes matrix $Q \in \mathbb{R}^{K_2 \times N}$ are also introduced to the second layer. After giving a reasonable dictionary size $K_2$ of LEDL, the objective function can be written as follows:

$$<D_{2I}, W, A, S_{1I} > = \arg \min_{D_{2I}, W, A, S_{1I}} \left\| S_{1I} - D_{2I} S_{2} \right\|_{F}^{2}$$

$$+ \lambda \left( \left\| H - WS_{1I} \right\|_{F}^{2} + \omega \left\| Q - AS_{1I} \right\|_{F}^{2} + 2\zeta \left\| S_{2} \right\|_{\ell_{1}} \right).$$

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where $D_{1I} \in \mathbb{R}^{d \times K_1}$ and $S_{1I} \in \mathbb{R}^{K_1 \times N}$ are the dictionary of the first layer and make it to be one of the input of the next layer. In addition, the label matrix $H \in \mathbb{R}^{C \times N}$ and discriminative sparse codes matrix $Q \in \mathbb{R}^{K_2 \times N}$ are also introduced to the second layer. After giving a reasonable dictionary size $K_2$ of LEDL, the objective function can be written as follows:

$$<D_{2I}, W, A, S_{1I} > = \arg \min_{D_{2I}, W, A, S_{1I}} \left\| S_{1I} - D_{2I} S_{2} \right\|_{F}^{2}$$

$$+ \lambda \left( \left\| H - WS_{1I} \right\|_{F}^{2} + \omega \left\| Q - AS_{1I} \right\|_{F}^{2} + 2\zeta \left\| S_{2} \right\|_{\ell_{1}} \right).$$

(4)
to solve the first subproblem while BCD [35] method offers the key to addressing the other subproblems.

4.1 Optimization of the first layer

ADMM is usually used to solve the equality-constrained problem while the objective function of CSDL-SRC is unconstrained. Thus the core idea of imposing ADMM framework here is to introduce an auxiliary variable to reformulate the original function into a linear equality-constrained problem. By introducing the auxiliary variable $z_i^c$, the $s_i$ in Equation [3] can be substituted by $c_{i1}$ and $z_i^c$, thus we can rewritten Equation [3] as follows:

$$<D_i^c, C_{i1}^c, Z_{i1}^c> = \arg \min \frac{1}{\varphi} \|X^c - D_i^c C_{i1}^c\|_F^2 + 2\|Z_{i1}^c\|_F$$

s.t. $C_{i1}^c = Z_{i1}^c$, $\|\left(D_i^c\right)_\bullet\|_F \leq 1 \ (k = 1, 2, \cdots, K_1)$

(5)

Then the lagrangian function of the problem [5] with fixed $D_i^c$ can be rewritten as:

$$<C_{i1}^c, Z_{i1}^c, L_i^c> = \arg \min \frac{1}{\varphi} \|X^c - D_i^c C_{i1}^c\|_F^2 + 2\|Z_{i1}^c\|_F$$

$$+ 2\left(\left(D_i^c\right)_\bullet\right)^T (C_{i1}^c - Z_{i1}^c)$$

$$+ \varphi \|C_{i1}^c - Z_{i1}^c\|_F^2$$

where $L_i^c \in \mathbb{R}^{K_1 \times N_c}$ is the augmented lagrangian multiplier and $\varphi > 0$ is the penalty parameter. We can gain the closed-form solution with respect to each iteration by follows:

(1) **Updating $C_{i1}^c$ while fixing $D_i^c$, $Z_{i1}^c$, and $L_i^c$**:

$$C_{i1}^c \_m = \left(\left(D_i^c\right)_\bullet \right)^T \left(\left(D_i^c\right)_\bullet\right)^{-1} \left(D_i^c\right)_\bullet$$

where $m (m = 0, 1, 2, \cdots)$ is the iteration number and $\bullet$ means the value of matrix $\bullet$ after $m$th iteration, the closed form solution of $C_{i1}^c$ is:

$$C_{i1}^c \_m + 1 = \left(\left(D_i^c\right)_\bullet \right)^T \left(\left(D_i^c\right)_\bullet\right)^{-1} \left(D_i^c\right)_\bullet$$

the closed form solution of $Z_{i1}^c$ is:

$$Z_{i1}^c \_m + 1 = Z_{i1}^c + Z_{i1}$$

(6)

(2) **Updating $Z_{i1}^c$ while fixing $D_i^c$, $C_{i1}^c$, and $L_i^c$**:

$$Z_{i1}^c \_m = \left(\left(D_i^c\right)_\bullet \right)^T \left(\left(D_i^c\right)_\bullet\right)^{-1} \left(D_i^c\right)_\bullet$$

$$+ \varphi \|C_{i1}^c - Z_{i1}^c\|_F^2$$

(7)

$$Z_{i1}^c \_m + 1 = Z_{i1}^c + Z_{i1}$$

(8)

To this end, we can solve the closed-form solution with respect to the single column by follows:

(4) **Updating $D_i^c$ while fixing $C_{i1}^c$, $Z_{i1}^c$, and $L_i^c$**:

$$D_{i1}^c \_m = \left(\left(C_{i1}^c\right)_\bullet \right)^T \left(\left(C_{i1}^c\right)_\bullet\right)^{-1} \left(C_{i1}^c\right)_\bullet$$

$$+ \varphi \|C_{i1}^c - Z_{i1}^c\|_F^2$$

where $\left(D_i^c\right)_\bullet$ denote the $k$th row vector of matrix $\bullet$.

$$D_{i1}^c \_m + 1 = \left(\left(D_i^c\right)_\bullet\right)_{p \neq k}$$

(9)

(10)

Based on the above ADMM steps, we obtain the closed form solution of $C_{i1}^c$, $Z_{i1}^c$, and $L_i^c$. Then we utilise BCD method with fixed $C_{i1}^c$, $Z_{i1}^c$, and $L_i^c$ to solve the constrained minimization problem of Equation [5]. The objective function can be rewritten as follows:

$$\frac{1}{\varphi} \|X^c - D_i^c C_{i1}^c\|_F^2 + 2\|Z_{i1}^c\|_F$$

$$+ 2\left(\left(D_i^c\right)_\bullet\right)^T (C_{i1}^c - Z_{i1}^c)$$

$$+ \varphi \|C_{i1}^c - Z_{i1}^c\|_F^2$$

s.t. $\|\left(D_i^c\right)_\bullet\|_F \leq 1 \ (k = 1, 2, \cdots, K_1)$

(11)

(12)

(13)

(14)

(15)
4.2 Optimization of the second layer

Similar to the above procedure, LEDL problem can be decomposed into two subproblems which are the same with the ones of CSDL-SRC that can be optimized by ADMM and BCD methods, respectively.

For finding sparse codes subproblem, we utilise ADMM method to optimize the objective function, hence the Equation (20) with $D_{l_2}$, $W$, $A$ fixed can be written as follows:

$$ < C_{l_2}, Z_{l_2}, L_{l_2} >= \arg \min_{C_{l_2}, Z_{l_2}, L_{l_2}} \left\| C_{l_2} - D_{l_2} C_{l_2} \right\|_F^2 $$

$$ + \lambda \left\| H - W C_{l_2} \right\|_F^2 + \omega \left\| Q - A C_{l_2} \right\|_F^2 $$

$$ + 2 \left( L_{l_2} \right)^T \left( C_{l_2} - Z_{l_2} \right) + \rho \left\| C_{l_2} - Z_{l_2} \right\|_F^2 $$

$$ + 2 \left( L_{l_2} \right)^T \left( C_{l_2} - Z_{l_2} \right) + \rho \left\| C_{l_2} - Z_{l_2} \right\|_F^2 $$

$$ + 2 \left( L_{l_2} \right)^T \left( C_{l_2} - Z_{l_2} \right) + \rho \left\| C_{l_2} - Z_{l_2} \right\|_F^2 $$

(20)

where the definitions and applications of $C_{l_2}$, $Z_{l_2}$, $L_{l_2}$ and $\rho$ in Equation (20) are similar with the $C_1$, $Z_1$, $L_1$ and $\varphi$ in Equation (4). Thus, we can obtain the closed-form solution with respect to each iteration by follows:

1) Updating $C_{l_2}$ while fixing $D_{l_2}$, $W$, $A$, $Z_{l_2}$ and $L_{l_2}$, the closed-form solution of $C_{l_2}$ is:

$$ (C_{l_2})_{m+1} = (C_{l_2})^{-1} C_{l_2} $$

(21)

where

$$ C_{l_2} = \left( (D_{l_2})_m \right)^T (D_{l_2})_m + \lambda W_m^T W_m + \omega A_m^T A_m + \rho I $$

(22)

2) Updating $Z_{l_2}$ while fixing $D_{l_2}$, $W$, $A$, $C_{l_2}$ and $L_{l_2}$, the closed-form solution of $Z_{l_2}$ is:

$$ (Z_{l_2})_{m+1} = Z_{l_2} + \hat{Z}_{l_2} $$

(24)

where

$$ Z_{l_2} = \max \left\{ (C_{l_2})_{m+1} + \frac{(L_{l_2})_m}{\rho} - \frac{\varepsilon}{\rho} I, 0 \right\} $$

(25)

$$ \hat{Z}_{l_2} = \min \left\{ (C_{l_2})_{m+1} + \frac{(L_{l_2})_m}{\rho} + \frac{\varepsilon}{\rho} I, 0 \right\} $$

(26)

3) Updating $L_{l_2}$ while fixing $D_{l_2}$, $W$, $A$, $C_{l_2}$ and $Z_{l_2}$, the closed-form solution of $L_{l_2}$ is:

$$ (L_{l_2})_{m+1} = (L_{l_2})_m + \rho \left( (C_{l_2})_{m+1} - (Z_{l_2})_{m+1} \right) $$

(27)

For learning bases subproblem, BCD method is used to optimize the objective function, thus the Equation (20) with $C_{l_2}$, $Z_{l_2}$ and $L_{l_2}$ fixed can be rewritten as follows:

$$ < D_{l_2}, W, A > = \arg \min_{D_{l_2}, W, A} \left\| C_{l_2} - D_{l_2} C_{l_2} \right\|_F^2 + 2 \left( L_{l_2} \right)^T \left( C_{l_2} - Z_{l_2} \right) + \rho \left\| C_{l_2} - Z_{l_2} \right\|_F^2 $$

$$ + 2 \left( L_{l_2} \right)^T \left( C_{l_2} - Z_{l_2} \right) + \rho \left\| C_{l_2} - Z_{l_2} \right\|_F^2 $$

$$ + 2 \left( L_{l_2} \right)^T \left( C_{l_2} - Z_{l_2} \right) + \rho \left\| C_{l_2} - Z_{l_2} \right\|_F^2 $$

$$ + 2 \left( L_{l_2} \right)^T \left( C_{l_2} - Z_{l_2} \right) + \rho \left\| C_{l_2} - Z_{l_2} \right\|_F^2 $$

(28)

s.t. \( \left\| (D_{l_2})_k \right\|_2 \leq 1 \), \( \left\| W_k \right\|_2 \leq 1 \),

$$ \left\| A_k \right\|_2 \leq 1 \) (k = 1, 2, \ldots, K_2)

To this end, we can solve the closed-form solution with respect to the single column by follows:

4) Updating $D_{l_2}$ while fixing $W$, $A$, $C_{l_2}$, $Z_{l_2}$ and $L_{l_2}$, the closed-form solution of $D_{l_2}$ is:

$$ (D_{l_2})_{m+1} = \frac{D_{l_2}}{\left\| D_{l_2} \right\|_2} $$

(29)

the $D_{l_2}$ here can be written as:

$$ D_{l_2} = C_{l_2} \left[ (C_{l_2})_{m+1} \right]^T $$

(30)

5) Updating $w$ while fixing $D_{l_2}$, $A$, $C_{l_2}$, $Z_{l_2}$ and $L_{l_2}$, the closed-form solution of $W$ is:

$$ (W_k)_{m+1} = \frac{W}{\left\| W \right\|_2} $$

(31)

the $W$ here can be rewritten as:

$$ W = H \left[ (C_{l_2})_{m+1} \right] - (W_k)_{m+1} \left[ (C_{l_2})_{m+1} \right] $$

(32)

where $W_k = \left\{ W_k, p \neq k \right\}$.

6) Updating $A$ while fixing $D_{l_2}$, $W$, $C_{l_2}$, $Z_{l_2}$ and $L_{l_2}$, the closed-form solution of $A$ is:

$$ (A_k)_{m+1} = \frac{\hat{A}}{\left\| A \right\|_2} $$

(33)

the $A$ here can be rewritten as:

$$ \hat{A} = Q \left[ (C_{l_2})_{m+1} \right] - (\hat{A}_k)_{m+1} \left[ (C_{l_2})_{m+1} \right] $$

(34)

where $\hat{A}_k = \left\{ A_k, p \neq k \right\}$. 
4.3 Convergence analysis

The convergence of CSDL-SRC has been demonstrated in [5].

Assume that the result of the objective function after \( m_{it} \) iteration is defined as \( f(C_m, Z_m, L_m, B_m, W_m, A_m) \). Since the minimum point is obtained by ADMM and BCD methods, each method will monotonically decrease the corresponding objective function. Considering that the objective function is obviously bounded below and satisfies the Equation 35, it converges.

\[
\begin{align*}
& f(\{(C_m)_m, (Z_m)_m, (L_m)_m, B_m, W_m, A_m\}) \\
& \geq f(\{(C_m)_{m+1}, (Z_m)_{m+1}, (L_m)_{m+1}, B_m, W_m, A_m\}) \\
& \geq f(\{(C_m)_{m+1}, (Z_m)_{m+1}, (L_m)_{m+1}, B_{m+1}, W_{m+1}, A_{m+1}\}) \\
& \geq \frac{\rho}{2} \left\| (C_m)_{m+1} - (C_m)_m \right\|^2 + \frac{\rho}{2} \left\| (Z_m)_{m+1} - (Z_m)_m \right\|^2 + \frac{\rho}{2} \left\| (L_m)_{m+1} - (L_m)_m \right\|^2, \\
& \text{for } k = 1, 2, \ldots, K_1
\end{align*}
\]

4.4 Overall algorithm

The overall updating procedures of our proposed network is summarized in Algorithm 1. Here, \( \text{maxiter} \) is the maximum number of iterations, \( 1 \in \mathbb{R}^{K_1 \times K_1} \) is a square matrix with all elements 1 and \( \circ \) indicates element dot product. In the algorithm 1, we first update the parameters of first layer to get the sparse codes \( s_1 \) and dictionary \( D_1 \), then \( s_1 \) is treated as one of the inputs of second layer to obtain the corresponding bases \( D_2 \), \( W \).

In testing stage, the constraint terms are based on \( \ell_1 \)-norm sparse constraint. Here, we first exploit the learned dictionary \( D_1 \) to fit the testing sample \( y \) and the output is the sparse codes \( r_1 \). Then the learned dictionary \( D_2 \) are utilised to fit \( r_1 \) and we can obtain the sparse codes \( r_2 \). At last, we use the trained classifier \( W \) to predict the label of \( y \) by calculating \( \max \{W_{rk} \} \).

5 Experimental results

In this section, we evaluate the performance of our approach on several benchmark datasets, including two face datasets (Extended YaleB [33] dataset, CMU PIE [57] dataset), two handwritten digit datasets (MNIST [38] dataset and USPS [39] dataset) and two remote sensing datasets (RSSCB7 dataset [40] and UC Merced Land Use dataset [41]), then compare it with other state-of-the-art methods such as SVM [42], SRC [2], CRC [4], SLRC [43], LC-KSVD [44] and CSDL-SRC [8].

For all the experiments, we evaluate our methods by randomly selecting 5 samples per class for training. In addition, to eliminate the randomness, we carry out every experiment 8 times and the mean of the classification rates is reported. For convenience, the dictio-
5.1 Extended YaleB dataset

The Extended YaleB dataset is consists of 2,432 face images from 38 individuals, each having around 64 nearly frontal images under varying illumination conditions. Here, we resize each image to 32 × 32 pixels and then pull them into column vectors, after that, we normalize the images to form the raw $\ell_2$ normalized features. Figure 4 shows some images of the dataset.

In addition, we set $\lambda = 2^{-3}$, $\omega = 2^{-11}$ and $\varepsilon = 2^{-8}$ for LEDL algorithm and set $\zeta = 2^{-10}$, $\lambda = 2^{-6}$, $\omega = 2^{-10}$ and $\varepsilon = 2^{-8}$ in our experiment to achieve highest accuracy for both algorithms, respectively. The experimental results are summarized in Table 1. From Table 1 we can see that our proposed LEDL and HDLN algorithms achieve superior performance to other methods. Compared with some conventional algorithms which the DL method is not involved in such as SVM, SRC, CRC and SLRC, the classification performance is improved by 2.1% and 2.9% with our proposed LEDL algorithm and HDLN algorithm, respectively. Compared with two classical DL based algorithms, including LC-KSVD and CSDL-SRC, our proposed LEDL algorithm and HDLN algorithm exceeds 1.1% and 1.9%, respectively. Additionally, the classification performance of HDLN algorithm exceeds that of LEDL algorithm by 0.8%

To further illustrate the superiority of our proposed HDLN, we choose the first 20 classes samples of Extended YaleB dataset as a subdataset to build a confusion matrix. The confusion matrices of different methods are shown in Figure 5. As can be seen that, our method achieve higher rate in most of the chosen 20 classes.

5.2 CMU PIE dataset

The CMU PIE dataset contains 41,368 images of 68 individuals with 43 different illumination conditions. Each human is under 13 different poses and with 4 different expressions. Similar with Extended YaleB dataset, each face image is cropped to 32 × 32 pixels, pulled into column vectors and normalized to have unit $\ell_2$ norm. Several samples from this data set are listed in Figure 6.

The results are shown in Table 1 as can be seen that our methods outperforms all the competing approaches by setting $\lambda = 2^{-3}$, $\omega = 2^{-11}$, $\varepsilon = 2^{-8}$ for LEDL algorithm and $\zeta = 2^{-12}$, $\lambda = 2^{-5}$, $\omega = 2^{-11}$, $\varepsilon = 2^{-3}$ for HDLN algorithm. Specifically, our proposed method achieves an improvement of at least 5.0% and 4.0% over some traditional methods such as SVM, SRC, CRC and SLRC for LEDL and HDLN algorithm, respectively. Compared with DL based methods, our proposed LEDL algorithm and HDLN algorithm exceed the CSDL-SRC algorithm 0.3% and 1.3%, respectively.

5.3 MNIST dataset

The MNIST dataset includes 70,000 images for digit numbers from 0 to 9. Here, we pull the original images which the size are 28 × 28 into column vectors. There are some samples from the dataset are given in Figure 7.

In Table 2 we can see that the classification rates of some conventional methods such as SVM, SRC, CRC and SLRC can achieve the similar ones of DL based methods (e.g. the classification rates between SRC and CSDL-SRC are similar). However, our proposed HDLN can achieve the highest accuracy by an improvement of at least 0.5% compared with all the methods in Table 2. The optimal parameter for LEDL algorithm are $\lambda = 2^{-8}$, $\omega = 2^{-14}$, $\varepsilon = 2^{-4}$ and the optimal parameters for HDLN algorithm are $\zeta = 2^{-8}$, $\lambda = 2^{-6}$, $\omega = 2^{-6}$, $\varepsilon = 2^{-2}$.

### Table 1 Classification rates (%) on face datasets

| Methods       | Extended YaleB | CMU PIE |
|---------------|----------------|---------|
| SVM           | 73.6           | 71.8    |
| SRC           | 79.1           | 73.7    |
| CRC           | 79.2           | 73.3    |
| SLRC          | 76.7           | 76.1    |
| LC-KSVD       | 73.5           | 67.1    |
| CSDL-SRC      | 80.2           | 77.4    |
| Our LEDL      | 81.3           | 77.7    |
| Our HDLN      | 82.1           | 78.7    |
5.4 USPS dataset

The USPS dataset consists of 9,298 handwritten digit images from 0 to 9 which come from the U.S. Postal System. For USPS dataset, the images are resized into $16 \times 16$ and pulled into column vectors. Several samples from this dataset are listed in Figure 8.

The results are showed in Table 2. For LEDL algorithm, we adjust $\lambda = 2^{-4}$, $\omega = 2^{-8}$, $\varepsilon = 2^{-5}$. For HDLN algorithm, we adjust $\zeta = 2^{-11}$, $\lambda = 2^{-10}$, $\omega = 2^{-14}$, $\varepsilon = 2^{-8}$ to achieve the highest accuracy. Compared with the methods (SVM, SRC, CRC and SLRC) which the DL is not added into the classifiers, HDLN algorithm achieves an improvement of at least 3.1% and LEDL algorithm achieves an improvement of at least 2.3%. Compared with the DL based method, LEDL algorithm achieves an improvement of 3.1%.

5.5 RSSCN7 dataset

The RSSCN7 dataset consists of seven different RS scene categories of 2800 aerial-scene images in total, which are grassland, forest, farmland, industry, parking lot, residential, river and lake region. Each class included 400 images and all images are of the same size of $400 \times 400$ pixels. Here, we use resnet model [21] to extract the features. Specifically, the layer pool5 is utilized to extract 2048-dimensional vectors for them. Figure 9 shows several samples belongs to this dataset.

The experimental results are showed in Table 3. It is clearly to see that all the methods in Table 3 achieve similar classification rates except HDLN algorithm. The
Table 3 Classification rates (%) on remote sensing datasets

| Methods       | UC Merced | RSSCB7 |
|---------------|-----------|--------|
| SVM [42]      | 67.5      | 80.5   |
| SRC [42]      | 67.1      | 80.4   |
| CRC [42]      | 67.7      | 80.7   |
| SLRC [42]     | 66.4      | 80.9   |
| LC-KSVD [14]  | 68.0      | 79.4   |
| CSDL-SRC [8]  | 66.6      | 80.5   |
| Our LEDL [43] | 67.9      | 80.7   |
| Our HDLN      | 69.6      | 81.0   |

5.6 UC Merced Land Use dataset

The UC Merced Land Use Dataset contains totally 2100 land-use images. The dataset is collected from the United States Geological Survey National Map of 20 U.S. regions. The size of each original images is $256 \times 256$ pixels. Here, we also uses resnet model to obtain 2048-dimensional vectors. Some samples are listed in Figure 10.

Table 3 shows the classification rates of different methods. It is hard to say that the DL method contributes a lot for image classification in this dataset if the discriminative information is utilised only by one way. Whether LC-KSVD, CSDL-SRC or LEDL can not get better performance than traditional methods such as SVM, SRC, CRC and SLRC. However, our proposed HDLN algorithm which adopt two ways can achieve an improvement of at least 0.1% over all other methods in Table 3.

5.7 Analysis and discussion

From the experimental results, we can obtain the following conclusions.

(1) Our proposed HDLN algorithm achieves superior performance to other state-of-the-art methods on these six benchmark datasets. That is to say, Hybrid Dictionary Learning Network is an effective and general classifier on various datasets, including face datasets, handwritten datasets and remote sensing datasets.

(2) Confusion matrices on Extended YaleB dataset in Figure 5 illustrate the superiority of our method. More specifically, for some classes such as class3, class 4, class 11, class 15, class 17, we get poor classification rates by utilising CSDL-SRC and LEDL separately. However, there are notable gains while using HDLN. And for some classes(class 1, class 6, class 8, class 9, class 10) which the accuracies have large differences between CSDL-SRC and LEDL, the classification rate of HDLN is similar with the result of the optimal one of CSDL-SRC and LEDL.

6 Conclusion

In this paper, we first propose a novel class shared dictionary learning method named label embedded dictionary learning (LEDL). This method introduces the $\ell_1$-norm regularization term to replace the $\ell_0$-norm regularization of LC-KSVD. Then we propose a novel network named hybrid dictionary learning network (HDLN) to combine a class specific dictionary learning method with a class shared dictionary learning method together to fully describe the feature to boost the performance of classification. In addition, we adopt ADMM algorithm to solve $\ell_1$-norm optimization problem and BCD algorithm to update the corresponding dictionaries. Finally, extensive experiments on six well-known benchmark datasets have proved the superiority of our proposed LEDL and HDLN methods.

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