Multi-damage localization in plate structure using frequency response function-based indices

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Abstract. Vibration signal and its derivative have shown some promise in structural damage detection in previous research. However, the theoretical and practical difficulties of multi-damage detection in plate structures based on dynamic responses remain. In this paper, an efficient damage localization index based on frequency response function (FRF) is presented. The imaginary part of FRF (IFRF) is extracted to derive the new localization index due to its relation to modal flexibility. For avoiding the finite element model error, two-dimensional gapped smoothing method (GSM) is employed without the need for baseline data from a presumably undamaged structure. Experimental studies on a steel plate with two localized defects in different boundary conditions are performed. The results are compared with some typical damage indices in the literature, such as mode shapes, uniform load surface and IFRF. In order to mitigate the inherent disadvantages of GSM in anti-noise ability, a simple statistical treatment based on Thompson outlier analysis is finally used for noise suppression. The effect of damage level and boundary condition on the detection results is also investigated.

1. Introduction

A great deal of numerical methods for structural damage detection have been proposed and developed during recent years. However, few methods can successfully solve this problem with satisfactory accuracy in practical engineering. In this sense, a promising method, not requiring measurements on the healthy structure, for use in practice would have the ability to identify slight damage with limited sensor placement, more importantly, to have strong noise immunity.

Mode shapes are widely used for damage localization, which can be easily extracted from modal analysis, and possible to describe the local features of structures [1]. Then their derivatives (e.g. modal
flexibility and uniform load surface) were found to have better performance in high-noise cases. Some studies indicated that only a few pieces of information can be obtained from mode shapes [2]. To construct a direct damage localization index without modal identification, FRF was increasingly used by many researchers [3-5] which can be obtained directly from vibration measurement. Another advantage is that FRF provides a wide range of frequencies and gives more information related to structural dynamical characteristics.

Liu et al. [6] proposed a non-modal scheme including some significant modifications such as using the imaginary parts of FRF shapes (IFS) and normalized FRF shapes (NFS) before comparison to detect the fault in a single damaged beam. The damage can be easily found by using IFS index. To validate the proposed method, they demonstrated the close relation between residual imaginary FRF shapes and residual mode shapes at the same modal frequencies. To the authors’ knowledge, only this work has explored the feasibility of IFS for damage detection. This motivated the authors to further explore the potential of IFS and its derivatives for identifying damage in plate-like structures.

On the base of the authors’ previous research [7], a novel efficient damage localization index from FRFs is formulated in this paper. For carrying out multi-damage localization in plate structures, two-dimensional gapped smoothing method (GSM) is employed. To further obtain a more explicit detection result, a simple statistical treatment based on outlier analysis is finally performed. By comparing with some traditional indices (e.g. modal shape, uniform load surface, and IFS), the effectiveness of this proposed method is verified by the results of experimental studies on a multi-damage steel plate.

2. Principle of Two-dimensional GSM

The basic concept of GSM for damage detection in a two-dimensional structure was proposed by Yoon [8], and demonstrated to be robust by many researchers. Here a brief description of the fundamental theory is given. Firstly, a two-dimensional grid is meshed on the plate structure in the x and y directions. The curvature of the damage localization index $\psi$ can be calculated at grid point $(s, t)$ by

$$\nabla^2 \psi_{s,t} = \psi_{x}^{s,t} + \psi_{y}^{s,t}$$

(1)

in which

$$\psi_{x}^{s,t} = (\psi_{s+1,t} + \psi_{s-1,t} - 2\psi_{s,t})/h_x^2$$

(2a)

$$\psi_{y}^{s,t} = (\psi_{s,t+1} + \psi_{s,t-1} - 2\psi_{s,t})/h_y^2$$

(2b)

where $h_x$ and $h_y$ are the distances between a point and its neighbors in the x and y directions, respectively. Assume that the healthy structure has a smooth deformed curvature, it can be approximated by the polynomial function constructed as

$$C_{s,t} = a_0 + a_1x + a_2y$$

(3)

where the coefficients $a_i$ can be determined explicitly by the curvatures of neighboring points from the damaged plate structure.
The least squares method is employed to solve this curve-fitting problem. More details can be found in [8]. Consequently, the value of function $C_{s,t}$ at grid point $(s, t)$ can be calculated by using Eq. 3. It should be noted that there are three cases for interpolation: (a) The value of the inner point can be estimated by using the neighboring eight points around it, and hence Eq. 3 can be extended as $C_{s,t} = a_0 + a_1 x + a_2 y + a_3 x y + a_4 x^2 + a_5 y^2 + a_6 x y^2 + a_7 x^2 y$ to obtain a higher-order surface; (b) if the point is on a boundary line, its value should be evaluated by using the neighboring five points. In this case, Eq. 3 can be extended as $C_{s,t} = a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 y^2$; (c) particularly, only three points can be used when the point is at a borderline intersection, as shown in figure 1. The damage index map is then calculated by using the absolute difference between the estimated value and the curvature

$$\delta_{s,t} = |\nabla^2 \psi_{s,t} - C_{s,t}|$$

(4)

Finally the outlier of the damage localization index indicates the damage region.

3. Construction of damage localization index

In this section, some previous indices (e.g. modal shape, uniform load surface, and IFS) are introduced here for performance comparison by making an experimental study reported in the next section. Mode shape index was first used in GSM by Ratcliffe [9] for locating structural damage in beams, which can be written as

$$\text{MS} = \{\phi_k\} \quad i = 1.2...N, \quad k = 1.2...m$$

(5)

where $\phi_k$ is the $k$th mode shape obtained at the $i$th point when $m$ lower modes are available for an $N$-DOFs structure. Since the MS index may be easily contaminated by measurement noise, uniform load surface index (ULS) [10] was proposed as

$$\text{ULS} = \{u(i)\} = \sum_{k=1}^{m} \phi_k \sum_{i=1}^{N} \phi_k$$

(6)

where $u(i)$ represents the modal deflection at point $i$ under uniform unit load all over the structure. Some researchers extended mode shape-based index to broadband frequency ranges using FRF shape information. Liu et al [6] presented what was possibly the first use of imaginary part of FRF shape (IFS) because of the close connection between residual curvatures of IFS and MS. The IFS can be easily extracted out of FRF. Each element in FRF matrix can be written as
\[ \alpha_i(\omega) = \sum_{k=1}^{m} \frac{\phi_{ik} \phi_{ik}}{\omega_i^2 (\omega_k^2 - \omega^2) + 2i \omega_i \omega} = \sum_{k=1}^{m} \frac{\phi_{ik} \phi_{jk} (\omega_k^2 - \omega^2)}{(\omega_k^2 - \omega^2)^2 + (2 \xi_k \omega_i \omega)^2} - \sum_{k=1}^{m} \frac{\phi_{ik} \phi_{jk} 2 \xi_k \omega_i \omega}{(\omega_k^2 - \omega^2)^2 + (2 \xi_k \omega_i \omega)^2} \]  \hfill (7)

where \( \alpha_i \) is the receptance FRF measured at location \( i \) for a force input at location \( j \), \( \phi_{ik} \) and \( \phi_{jk} \) are the \( i \)th and \( j \)th elements of the \( k \)th mode shape. \( \omega_k \) and \( \xi_k \) denote the \( k \)th frequency and damping ratio, respectively. The \( l \)th column of the IFS is denoted as

\[
IFS_i(\omega) = \begin{bmatrix}
-\sum_{k=1}^{m} \frac{2 \phi_{ik} \phi_{jk} \xi_k \omega_i \omega}{(\omega_k^2 - \omega^2)^2 + (2 \xi_k \omega_i \omega)^2}
-\sum_{k=1}^{m} \frac{2 \phi_{ik} \phi_{jk} \xi_k \omega_i \omega}{(\omega_k^2 - \omega^2)^2 + (2 \xi_k \omega_i \omega)^2}
\end{bmatrix}^T, \quad \sum_{k=1}^{m} \frac{\phi_{ik} \phi_{jk} 2 \xi_k \omega_i \omega}{(\omega_k^2 - \omega^2)^2 + (2 \xi_k \omega_i \omega)^2}
\hfill (8)
\]

It can be seen that when \( \omega = \omega_k \), each element in Eq.8 can be reduced to

\[
\sum_{k=1}^{m} \frac{\phi_{ik} \phi_{jk} \xi_k \omega_i \omega}{(\omega_k^2 - \omega^2)^2 + (2 \xi_k \omega_i \omega)^2} = \sum_{k=1}^{m} \frac{1}{2 \xi_k \omega_i \omega} \hfill (9)
\]

where \( \phi_{ik} \phi_{jk} / \omega_k^2 \) is the \( k \)th modal flexibility. This means that the IFS index at each natural frequency is closely related to its modal flexibility form. Therefore the damage location can be described by IFS index, which in turn supports the validity of the method in [6]. Inspired by ULS and IFS index, here a novel index is proposed as

\[
ULS - IFS = \{ w(i) \} \sum_{\omega=\omega_1}^{\omega_2} \frac{IFS_i(\omega) \sum_{k=1}^{N} IFS_i(\omega)}{\omega^2} \hfill (10)
\]

in which \( \omega_1 \) and \( \omega_2 \) limited the frequency range, consisting of \( m \) natural frequencies. It is noteworthy that the definition of ULS-IFS here includes more shape-based information in considered frequency broadband, compared with conventional ULS. Meanwhile, it inherits the merit of IFS index that the random error can be averaged out in each measuring point.

4. Experimental study on Multi-damage detection of a steel plate

To assess the detection performance and the advantage of the proposed index, a rectangular plate with an area of 300×300mm and 5mm thickness is considered. To simulate the damage, two defects \( a \) and \( b \) were manufactured both with area of 30×30 mm. Their position is shown as dotted box in figure 4. The material properties of the steel plate are as follows: Young’s modulus \( E=208.5 \)GPa, Poisson’s ratio \( \mu=0.27 \), density \( \rho=7830 \)kg/m\(^3\). For studying the effects of damage magnitude and structural boundary condition on the detection results, three damage cases are studied, as defined in table 1.

![Figure 2. Rectangle steel plate for experiment](image1)

![Figure 3. The measured FRF waterfall map](image2)
Table 1. Damage cases of the steel plate

| Case | Damage level (mm) | Boundary condition | Measuring point |
|------|-------------------|--------------------|-----------------|
| 1    | $a$: 1.0 $b$:1.2 (slight) | resilient polyurethane foam | 121             |
| 2    | $a$: 2.0 $b$:3.0 (severe) | resilient polyurethane foam | 121             |
| 3    | $a$: 2.0 $b$:3.0 (severe) | One edge clamped | 110             |

The grid map consists of 121 points as shown in figure 2. An acceleration transducer is mounted at the position marked by a red solid dot. All points are excited by roving hammer impact. Test data are collected and processed using a 6-channel intelligent signal acquisition and analysis system B&K 3050. The FRFs of the damaged plate are shown in figure 3. Then the natural frequencies and corresponding mode shapes of the damaged plate can be easily identified to calculate MS index and ULS index for the performance comparison.

Table 2. First three modal frequencies for all cases

| Case | 1st frequency (Hz) | 2nd frequency (Hz) | 3rd frequency (Hz) |
|------|---------------------|--------------------|--------------------|
| 1    | 231                 | 336                | 359                |
| 2    | 228                 | 336                | 356                |
| 3    | 59.5                | 128                | 313                |

Figure 4. Damage localization results of Case 1: (a) MS; (b) ULS; (c) IFS; (d) ULS-IFS

Figure 5. Damage localization results of Case 2: (a) MS; (b) ULS; (c) IFS; (d) ULS-IFS

Figures 4(a)-(d) compare the normalized values of each kind of indices at the same damage level. It is found that MS and ULS indices can approximately detect defect $b$. However, a shortcoming is that they give a poor prediction of the relatively slightly damaged region $a$. It can be observed from figures 4(c) and 4(d) that both damaged regions are successfully detected and the peak values explicitly show the relative levels of two damaged regions. And one can find large gradient of the IFS index surface in
undamaged area, while the ULS-IFS index surface is relatively smooth.

Case 1 and Case 2 shows the effects of damage magnitude on detection performance of all the indices. It can be noted that the values of all the indices in the undamaged region are lower than the damaged ones in Case 1 if the damage magnitude increases.

To assess the effectiveness and reliability of the proposed index when changing structural boundary conditions, the right edge of the steel plate is clamped so that only 110 measuring points on the plate are available for Case 2. The detection results of all four indices can be calculated in a similar way, as shown in figure 6. It is found that MS and ULS indices in the undamaged region are somewhat sensitive to the boundary change, while IFS and ULS-IFS indices still show stable performance. It can be seen that ULS-IFS index provides a clearer damage localization result of both damaged regions than IFS index based on the measured points, as shown in figures 6(c) and (d). In the respect of detection performance in the undamaged area, most of ULS-IFS index values are less than 0.1, which is exactly what we expect.

5. Noise suppression method based on outlier analysis

GSM is based on curvature which can amplify outliers as a result of the presence of damage. However, GSM makes outliers caused by noise more obvious on the smooth curve simultaneously. In this aspect, GSM seems to be less capable in noise suppression. To obtain more explicit results of damage detection, a statistical treatment is performed in this section. The basic idea is from [8] but some refinement is made.

Figure 6. Damage localization results of Case 2: (a) MS; (b) ULS; (c) IFS; (d) ULS-IFS

Figure 7. Process of damage detection with noise suppression method based on outlier detection
It is assumed that damage region is very small compared with the entire structure, and more importantly, the mean and standard deviation of damage indices of the undamaged area of a damage structure are similar to those from the repeated tests of an undamaged structure. Therefore, outliers of averaged damage indices correspond to the irregularity caused by damage. The process of damage detection with noise suppression method using Thompson outlier detection [11] is shown as following steps. To show the effectiveness of proposed noise suppression method, ULS-IFS index is used to recalculate all damage cases. The considered frequency points was randomly ordered and divided into eight intervals to carry out the outlier detection. The damage detection results are shown in figures 8 and 9.

![Figure 8](image1.png)

**Figure 8.** Localization results based on ULS-IFS index in: (a) Case 1; (b) Case 2; (c) Case 3

![Figure 9](image2.png)

**Figure 9.** Final outlier analysis based on ULS-IFS index in: (a) Case 1; (b) Case 2; (c) Case 3. The red circles are outliers, and the blue dots between red parallel lines represent values caused by noise.

It can be seen that the irregular values in the undamaged area have been filtered so that the damaged area is highlighted. But the identified damage area is somewhat larger than the true area because a plate is a continuum on which the measurement data at different points are probably not independent, that is, the identified boundary between slightly damage areas and undamaged areas seems vague. Figure 9(b) shows that the bandwidth of non-outliers decreases with increasing damage severity. Also one can see that different boundary conditions can affect the identified results. It is important to note that, the accuracy of the GSM depends on density of the measurement mesh on a structure. Laser vibrometers can be used to obtain more measurement points and give a relatively smooth interpolating surface for damage detection.
6. Conclusions

In this paper, a novel damage localization index from FRFs is firstly formulated for damage detection. To avoid using a finite element model, two-dimensional gapped smoothing method with a statistical noise suppression technique is employed. An experimental study on a steel plate demonstrates the effectiveness of the proposed index. Some conclusions can be obtained as follows:

1. For the same boundary conditions, FRF-based index has a better anti-noise ability than mode shape-based index, since it can be averaged over the frequency bandwidth. Also it can be seen that the ULS-based index shows superior anti-noise ability over those basic indices (MS and IFS index) because the summations $\sum_{i=1}^{N} \phi_i$ and $\sum_{i=1}^{N} IFS_i(\omega)$ average out the random error at each measuring point.

2. ULS-IFS index gives a better performance than other three indices, which combines the advantages of IFS and ULS in noise suppression.

3. Boundary conditions and damage level do affect the damage localization results.

4. Outlier detection is an easy and effective technique for noise suppression, which makes damage localization results of GSM more explicit.

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