Calculation the total resistance and squat of underwater towed vehicle based on CFD

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Abstract. CFD method is used to calculate drag and sinking force of underwater towed body. Using cubic spline curve interpolation and finite difference method are used to calculate the tangential force and normal force of towing cable. Finally, the squat of underwater towed vehicle in the water and the total force at the end of towing cable are obtained.

1. Introduction
Underwater towed vehicle is widely used in various underwater products. Underwater towed vehicle are carried with sonar, flowmeter, salinometer and other types of sensors, which are used to measure various hydrologic parameters. Underwater towed vehicle is connected with the ship and then moves together. It is an interesting thing to calculate the pulling force at the end of the towing cable at different speeds and the sinking of the underwater towed vehicle.

2. Introduction of underwater towed vehicle and cable finite element
2.1. Underwater towed vehicle shape
The shape of the underwater towed vehicle is shown in Figure 1. The diameter of the underwater towed vehicle is 200mm, and the length is 1200mm.

![Figure 1. Three-dimensional graph of underwater vehicle](image)

When towing cable moved at a certain speed, its shape is unknown, so it is impossible to calculate the resistance performance directly by CFD method. In addition, even knowing its shape, because its length diameter ratio is very large, up to 600, therefore, the towing cable must be discrete.
2.2. Discretization of towing cable

Because the towing cable is flexible, it will be deformed by the force. Therefore, the angle of attack of the towing cable at different place is different. The finite difference method is used to process the towing cable into finite element, and the stress analysis is carried out for each element. The force, rotation angle and coordinate point of the unknown position are obtained from the initial point.

In order to meet the needs of the project and basically meet the actual situation, the following assumptions are made:

a) Telescopic overlook of cable finite element under tension;

b) The towing cable is flexible and cannot transfer bending moment

c) The flow field is only considered as a two-dimensional flow.

d) The hydrodynamic coefficient on the finite element of the towing cable is equivalent to a towing cable that acts on an infinite length.

Force analysis of finite element of the towing cable is shown in Figure 2.

![Figure 2. Force analysis of finite element of towing cable](image)

According to the decomposition of the tangential force and the normal force, get:

\[(T + dT) \sin(d\phi) + pds \cos(\phi) = Dds\]

\[(T + dT) \cos(d\phi) = pds \sin(\phi) + Fds + T\]

There, \(T\) is force, \(D\) is the normal force of finite element of the towing cable, \(F\) is the tangential force of finite element of the towing cable, \(p\) is the weight of finite element of the towing cable.

Neglecting the small amount of the two orders in equality (1), get:

\[ds = \frac{T}{D - p \cos \phi} d\phi\]

The equation (3) is replaced by the equation (2), get

\[dT = \frac{F + p \sin \phi}{D - p \cos \phi} Td\phi\]

By equation (3) and (4), the towing force and rotation angle of the towing cable in different positions can be obtained.

In addition:

\[dx = ds \times \cos \phi\]
\[ dy = ds \times \sin \phi \]  

(6)

The coordinates of the corresponding position points can be obtained by equation (5) and (6). The length of the towing cable is discretized, and the iterative calculation is carried out by equation (3) and (4).

Computational fluid dynamics and finite difference method are used to calculate \( D, F \).

It is necessary to explain that the empirical formula calculation of \( D \) and \( F \) do not consider the mutual interference between the tangential velocity and the normal velocity, and the calculation results are not correct enough. At present, CFD technology has been developed more mature, and can be directly calculated. In addition, CFD can only calculate tangential forces and normal forces under limited attack angles. In order to meet the need of actual attack angle calculation, the three-spline interpolation method is applied to solve them.

In this study, the diameter of the towing cable is 9mm and the length is 50m.

3. Principle of curve interpolation

Curve interpolation usually has exponential function interpolation, Lagrange interpolation, Cubic spline interpolation, B spline curve, NURBS curve and so on.

3.1. Cubic spline curve interpolation

There, \( a = x_0 < x_1 < \cdots < x_n = b \), if function \( y(x) \) satisfying two conditions:

(a) \( y(x) \) is Cubic Polynomial on each interval \([x_{i-1}, x_i], i = 1, 2, \cdots, n\);

(b) \( y(x) \) is twice continuously differentiable function on interval \([a, b]\).

Then, \( y(x) \) is Cubic spline on interval \([a, b]\). There, \( x_i (i = 0, 1, \cdots, n) \) is the node. The interpolation points are shown in Table 1.

| \( x \) | \( x_0 \) | \( x_1 \) | \cdots | \( x_n \) |
|---|---|---|---|---|
| \( y \) | \( y_0 \) | \( y_1 \) | \cdots | \( y_n \) |

Cubic spline interpolation function:

\[
s(x) = \frac{h_k}{h_k^3} + \frac{2(x-x_k)}{h_k^4} (x-x_{k+1})^2 y_k + \frac{h_k^2 - 2(x-x_{k+1})}{h_k^4} (x-x_k)^2 y_{k+1}
+ \frac{(x-x_k)(x-x_{k+1})^2}{h_k^2} m_k + \frac{(x-x_{k+1})(x-x_k)^2}{h_k^2} m_{k+1}
\]  

(7)

In equality (1),

\[
h_k = x_{k+1} - x_k (k = 0, 1, \cdots, n - 1)
\]  

(8)

\[
\lambda_k m_{k-1} + 2m_k + \mu_k m_{k+1} = g_k (k = 1, 2, \cdots, n - 1)
\]  

(9)

Where,
\[ \lambda_k = \frac{h_k}{h_k + h_{k-1}}, \mu_k = \frac{h_{k-1}}{h_k + h_{k-1}}, g_k = 3(\mu_k \frac{y_{k+1} - y_k}{h_k} + \lambda_k \frac{y_k - y_{k-1}}{h_{k-1}})(k = 1, 2, \ldots, n - 1) \]

4. Basic theory of fluid mechanics

4.1. Control equation
The continuity equation and momentum equation for incompressible viscous fluid are:

\[ \frac{\partial \bar{u}_i}{\partial x_i} = 0 \]  
(10)

\[ \rho \frac{\partial \bar{u}_i}{\partial t} + \rho \frac{\partial \bar{u}_j}{\partial x_j} \bar{u}_i = \rho F_i - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}(\mu \frac{\partial \bar{u}_i}{\partial x_j} - \rho u_i u_j) \]  
(11)

Where, \( \rho \) is density, \( \mu \) is Viscosity Coefficient, \( \bar{p} \) is average pressure, \( \bar{F}_i \) is external force, \( \bar{u}_i \) is mean velocity, \( u'_i \) is fluctuating velocity, \( -\rho u'_i u'_j \) is reynolds stress.

4.2. Turbulence model
In order to make the equation closed, a new turbulent model equation must be introduced to link the fluctuating values with the time average in the stress terms. There, we chose the RNGk – \( \varepsilon \) equations.

The transport equations of turbulent kinetic energy and turbulent fluctuation intensity in the RNGk – \( \varepsilon \) equation is:

\[ \frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_j}(\rho k u_j) = \frac{\partial}{\partial x_j}[\alpha_k \mu_{\text{eff}} \frac{\partial k}{\partial x_j}] + G_k - \rho \varepsilon + S_k \]  
(12)

\[ \frac{\partial}{\partial t}(\rho \varepsilon) + \frac{\partial}{\partial x_j}(\rho \varepsilon u_j) = \frac{\partial}{\partial x_j}[\alpha_k \mu_{\text{eff}} \frac{\partial \varepsilon}{\partial x_j}] + G_{\varepsilon} \frac{\varepsilon}{k} G_k - C_{2\varepsilon} \rho \varepsilon^2 / k - R_k + S_{\varepsilon} \]  
(13)

Where, \( \mu_{\text{eff}} = \mu + \mu' \), \( \mu_i = \rho C \mu \frac{k^2}{\varepsilon} \), \( G_k = -\rho \bar{u}_i u_j \frac{\partial u_j}{\partial x_i} \), \( R_k = \frac{C_{\mu} \rho \eta^3 (1 - \frac{\eta}{\eta_0}) \varepsilon^2}{1 + \beta \eta^2} \), \( \eta = \frac{S_k}{\varepsilon} \), \( \mathbf{S} = \sqrt{2 S_{ij} S_{ij}} \), \( S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \), \( S_k \) and \( S_{\varepsilon} \) are user-defined source terms.

Constant, \( G_{\varepsilon} = 1.42 \), \( G_{2\varepsilon} = 1.68 \), \( C_\mu = 0.0845 \), \( \sigma_k = 1.0 \), \( \sigma_{\varepsilon} = 1.3 \), \( \eta_0 = 4.38 \), \( \beta = 0.012 \)

5. CFD Simulation
The grid diagram of the underwater towed vehicle is shown in Figure 3.
In order to obtain high-quality meshes, structured grids are used for the entire computational domain. The governing equations and turbulence models are discretized by the finite volume method and solved by the coupled implicit solver.

The pressure-velocity coupling iterative algorithm using SIMPLEC algorithm, pressure by the standard discretization, momentum, turbulent kinetic energy and turbulent dissipation rate by two order upwind scheme, relaxation factors are the default values. Wall function method is adopted in the near wall region.

The drag and sinking force of the underwater towed vehicle at different speeds is shown in Table 2.

| v/kn | 0.5 | 3   | 5   | 7   | 9   | 12  |
|------|-----|-----|-----|-----|-----|-----|
| R/N  | 0.61| 17.79| 46.95| 89.14| 144.01| 249.61|
| F/N  | 1.28| 37.36| 98.60| 187.19| 302.41| 524.17|

By using cubic spline interpolation mentioned above, the tangential force and normal force at different speeds and angles of attack can be obtained. The tangential force and normal force of streamline microelement at different positions are solved by finite difference method. It should be noted that this calculation method was started by underwater towing body.

In this way, the squat and terminal resistance values of the underwater towed vehicle are shown in Table 3.

| v/kn | 0.5 | 3   | 5   | 7   | 9   | 12  |
|------|-----|-----|-----|-----|-----|-----|
| s/m  | 18.2| 12.3| 9.9 | 9.3 | 9.1 | 8.6 |
| F/N  | 8.1 | 52.5| 135.5| 252.9| 404.5| 695.9|

6. Conclusion
In this paper, CFD method is used to calculate drag and sinking force of underwater towed vehicle in water. In order to continue to calculate its squat in water and drag value at the end of towing cable. The finite-difference method is used for the microelement of the towing cable. Using cubic spline interpolation, the tangential force and normal force of streamline microelement at different angles of attack are obtained. Finally, the squat of the underwater towed body and the total force at the end of the towing cable are calculated.

The method is simple and reliable.
1) Compared with the traditional empirical formula, it considers the mutual interference between tangential force and normal force, and the calculation is more accurate.
2) It is simpler and more efficient to solve the tangential force and normal force of microelement in different direction of attack than CFD.
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