A Model for Neutrino Masses and Dark Matter

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We propose a model for neutrino masses that simultaneously results in a new dark matter candidate, the right-handed neutrino. We derive the dark matter abundance in this model, show how the hierarchy of neutrino masses is obtained, and verify that the model is compatible with existing experimental results. The model provides an economical method of unifying two seemingly separate puzzles in contemporary particle physics and cosmology.

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I. INTRODUCTION

The discovery of very small, but non-zero neutrino masses via the observation of oscillations in atmospheric and solar neutrino experiments\textsuperscript{1} is one of the most exciting results in particle physics in recent years, and may provide important new information to help guide us in the search for new physics beyond the standard model. As we describe here, efforts to uncover a mechanism for generating such small mass differences may also shed light on the nature of non-baryonic dark matter.

The simplest way to introduce neutrino masses into the standard model is to postulate a new right-handed fermion, a singlet under the SM gauge group, that mixes with the SM fermions via a Dirac mass term. Unfortunately, in order to obtain neutrino masses below an eV, it is necessary to fine-tune the corresponding Yukawa coupling to one part in $10^{11}$, which is unacceptable. Thus, more elaborate models are needed.

Currently the most popular way to generate small neutrino masses is the seesaw mechanism\textsuperscript{2}. In this model one introduces a right handed neutrino $N_R$, and allows both Majorana and Dirac mass terms. Since the right handed neutrino is a singlet under the SM gauge group $SU(2)_L \times U(1)_Y$, its mass $M_R$ can be much larger than the electroweak scale without contributing large quantum corrections to the other couplings in the theory. One may integrate out such a heavy field leaving a Majorana neutrino with mass $m_\nu \sim \frac{m_D^2}{M_R}$, where $m_D$ is the Dirac neutrino mass. The seesaw mechanism arises naturally in many extensions of the standard model gauge group, for example in left-right (L-R) models with group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, the Pati-Salam model with group $SU(4)_C \times SU(2)_L \times SU(2)_R$ or in $SO(10)$ grand unified theories (GUTs).

Another possible way of achieving small neutrino masses is to extend the Higgs sector of the theory by adding a very heavy triplet scalar field $\Delta$, with mass $M_\Delta$, which couples to the SM fields via

$$\mathcal{L}_\Delta = f_{\alpha\beta} L^\alpha L^\beta c i \tau_2 \Delta \tau_2 + \mu \Phi^T i \tau_2 \Delta^T H \Phi + \text{h.c.} ,$$

where, $L_\alpha$ is the left-handed lepton doublet, $\Phi$ is the standard model Higgs doublet and $C$ is the charge-conjugation matrix. Here the Yukawa couplings $f_{\alpha\beta}$ are antisymmetric in the generation indices $\alpha$ and $\beta$.

Integrating out our heavy triplet $\Delta$ one obtains an effective theory whose Lagrangian density contains the term

$$\mathcal{L}_{LLH} = \frac{1}{\Lambda} LLHH ,$$

where the scale $\Lambda \sim M_\Delta^{-2}$. Due to this term, after electroweak symmetry breaking the neutrino gets a mass $m_\nu \sim \frac{V_{\text{EW}}^2}{\Lambda^2}$, where $V_{\text{EW}}$ is the vacuum expectation value (VEV) of the standard model higgs field.

Clearly, in both of these sets of models for generating neutrino masses, it is crucial that the masses of the right-handed neutrino and the scalar triplet respectively are much higher than the electroweak scale. As a consequence, these models are difficult to test in collider experiments. In addition, the existence of a large hierarchy between the electroweak scale and the new physics scale generates its own set of naturalness problems.

Here we reconsider the idea that SM neutrinos are massless at tree level, but acquire a nonzero mass at the one-loop level. A particularly simple example of this idea is provided by the Zee model\textsuperscript{2} in which one augments the SM higgs doublet $\Phi_1$ with a second higgs doublet $\Phi_2$, and a charged field $S$ which transforms as a singlet under $SU(2)$. The Lagrangian density then contains the following relevant pieces

$$\mathcal{L}_{\text{Zee}} = f_{\alpha\beta} L^\alpha L^\beta c i \tau_2 \Phi_2 S^+ + \mu \Phi^T_1 i \tau_2 \Phi_2 S^- + \text{h.c.} .$$

Other extensions of the Zee model where small neutrino masses arise at loop levels have also been proposed\textsuperscript{3}.
that may have interesting implications for the generation of the baryon asymmetry of the universe [3, 10].

In this letter we will demonstrate how a variant of the Zee model allows the generation of small phenomenologically acceptable neutrino masses and at the same time produces a new viable dark matter candidate near the electroweak scale.

II. THE MODEL

We consider a model with sufficient symmetry that neutrino masses appear only at the three loop level. To achieve this we supplement the SM fields with two charged scalar singlet scalars $S_1$ and $S_2$, with masses $M_{S_1}$ and $M_{S_2}$ respectively and one right handed neutrino $N_R$, with mass $M_R$. We break lepton number explicitly by including a Majorana mass term for the right-handed neutrino, and impose a discrete $Z_2$ symmetry under which the SM fields and $S$ transform as

$$Z_2: \{S_2, N_R\} \rightarrow \{-S_2, -N_R\}, \quad (4)$$

forbidding Dirac masses for the neutrinos. Given this symmetry, the most general renormalizable terms that may be added to the SM Lagrangian density

$$\mathcal{L}_{\text{new}} = f_{\alpha\beta} L^T_{\alpha} C i \tau_2 L_{\beta} S_1^2 + g_{\alpha} N_R S_2^T l_{\alpha R}$$
$$+ M_R N_R^T C N_R + V(S_1, S_2) + \text{h.c.}, \quad (5)$$

in which the potential $V(S_1, S_2)$ contains a $(S_1 S_2^* )^2$ coupling $\lambda$. Let us assume a mild hierarchy of masses $M_R < M_{S_1} < M_{S_2} \sim \text{TeV}$ and that the Yukawa couplings $f_{\alpha\beta}$, $g_{\alpha}$ are of order one. This implies that $N_R$ is stable if the above discrete symmetry is unbroken. Also, since lepton number is broken, we expect a left handed Majorana neutrino mass at the quantum level, and indeed this arises from the three loop diagram in figure 1.

Here the scale $\Lambda$ is given in terms of the heaviest singlet scalar mass $M_{S_2}$ and the Yukawa couplings $\lambda_l = \frac{\mu_l}{M_{\nu_{\text{EW}}}}$, $f_{\alpha\beta}$, $g_{\alpha}$ and $\lambda_l$ as

$$\Lambda \sim \frac{(4\pi^2)^{\frac{3}{2}}}{f^2 \lambda_{\alpha l} \lambda_{l}^* g_{\alpha}^2} M_{S_2}, \quad (7)$$

where a subscript $l$ denotes a leptonic quantity. For $M_{S_2} \sim \text{TeV}$, we obtain $\Lambda > 10^9 \text{ GeV}$ and so the model provides a hierarchy between the electron and neutrino masses and yields neutrino masses at the $0.1 \text{ eV}$ scale without involving mass scales significantly larger than a TeV.

III. DARK MATTER

As we commented above, in our model the right-handed neutrino is stable, since its mass is smaller than $M_{S_1}$, and the discrete symmetry $Z_2$ is unbroken. It is natural to wonder whether this stable, neutral, weakly-interacting particle might be a reasonable cold dark matter (CDM) candidate. To determine this we need to calculate the relic abundance of right-handed neutrinos produced as the universe cools.

We begin by noting that the right-handed neutrino annihilates into right-handed charged leptons with cross-section given by

$$\langle \sigma v \rangle \sim \frac{g^4 M_R^2}{\pi M_{S_2}^2}. \quad (8)$$

We also need the standard result that the number density of right-handed neutrinos at the decoupling temperature $T_D$ is

$$n_{N_R+\bar{N}_R} = \frac{2T_D^3}{(2\pi)^2} \left( \frac{M_R}{T_D} \right)^{3/2} \exp \left( - \frac{M_R}{T_D} \right). \quad (9)$$

Freeze-out occurs when

$$n_{N_R} \sigma \simeq \sqrt{g_\ast} \frac{T_D^2}{M_{\text{pl}}}, \quad (10)$$

where $g_\ast$ is the effective number of massless degrees of freedom of at decoupling. Therefore, using our expressions for $n_{N_R+\bar{N}_R}$ and the cross-section we solve for $\frac{M_R}{T_D}$ to obtain

$$\frac{M_R}{T_D} = - \ln \left( \frac{1.66(2\pi)^{\frac{4}{3}} M_{\text{pl}}^{\frac{4}{3}}}{M_{\text{pl}} g_\ast^4 \text{GeV}^4} \right) - \frac{1}{2} \ln \left( \frac{M_R}{T_D} \right)$$
$$+ 3 \ln \left( \frac{M_R}{\text{GeV}} \right) - \ln \left( \frac{\sqrt{\sigma}}{g_\ast} \right), \quad (11)$$

where the minus sign in front of the second term arises because the annihilation of $N_R$ is a p-wave process, and where we can ignore the masses of the annihilation.
products with masses less than $M_R$. For typical values $g^2 \sim 0.1$ and $M_{S_2} \sim \text{TeV}$ we obtain

$$\frac{M_R}{T_D} \simeq 20 . \quad (12)$$

The ratio of the energy density in right-handed neutrinos to the critical density is approximately given by

$$\Omega_{N_R} \simeq \frac{10^{-37} \text{cm}^2}{\langle \sigma v \rangle} . \quad (13)$$

Note that if we take typical values $g^2 \sim 0.1$, $M_R \sim \text{TeV}$ then $\langle \sigma v \rangle \sim 10^{-36} \text{cm}^2$ and so one quite easily finds

$$\Omega_{N_R} \simeq 0.1 - 1 , \quad (14)$$
as required for a viable CDM candidate.

Note also that because the right handed neutrino has no direct neutral current couplings to quarks, it would not be detected in existing WIMP detection experiments sensitive to elastic scattering off of nuclei.

IV. CURRENT EXPERIMENTAL CONSTRAINTS

We next consider whether what current phenomenological constraints on lepton number violation imply for this model.

First note that stringent constraint on the parameters $f_{\alpha \beta}$ can be derived from the new contribution to the muon decay effective lagrangian mediated by $S_1$. We find

$$\frac{f_{e\mu}}{M_{S_1}} \leq 10^{-4} \text{GeV}^{-2} \quad (15)$$

The Lagrangian (6) can also lead to the flavor violating process $\mu \to e + \gamma$. Using the experimental bound on its branching ratio we find:

$$\frac{f_{e\alpha} f_{\alpha\mu}^*}{M_{S_1}} \leq 2.8 \cdot 10^{-9} \text{GeV}^{-2} \quad (16)$$

The above constraints can be satisfied for the $f_{e\alpha} \sim 0.1$ and $M_{S_2} \sim \text{few TeV}$, and are also consistent with limits from neutrinoless double $\beta$-decay experiments, and are thus comparable with the requirements needed to generate neutrino masses and dark matter, as described above.

Although the right-handed neutrino is a singlet under the SM gauge group, it may be produced in our model through the scatterings of SM particles and the subsequent decay of $S_2$.

If the center of mass energy of the next generation linear collider exceeds twice the mass of $S_2$ then these particles would be produced in, for example, $e^+ e^-$ annihilation. In this case, the dominant decay channel for the $S_2$ particle would be into a charged lepton and the lightest right-handed neutrino, providing a possible collider signature of the model.

V. CONCLUSIONS

It would be particularly exciting if a single mechanism might resolve two of the most important outstanding puzzles in particle physics and cosmology: the nature of dark matter, and the origin of neutrino masses. As we demonstrate here, it is possible to extend standard model of particle physics in a way so that both puzzles may find a common resolution. The symmetry structure of our model is such that neutrino masses occur only at three-loop level, and so are naturally small. As a consequence, the right-handed neutrino is not constrained to be have a large mass, as in other examples of neutrino mass generation, such as the seesaw model. Such a light right-handed neutrino is a possible weakly interacting massive particle (WIMP) dark matter candidate.

We have explored the experimental constraints on our model and it currently passes all standard model tests. More exciting perhaps is the fact that the TeV-scale scale energies required should be accessible at colliders in the near future. The model may also be tested by future neutrinoless double $\beta$-decay experiments. A new sort of direct detection WIMP technology would be required to directly detect the dark matter candidate we propose.

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