STRONG-FIELD GENERAL RELATIVITY AND QUASI-PERIODIC OSCILLATIONS IN X-RAY BINARIES

PHILIP KAARET, ERIC C. FORD, AND KAIYOU CHEN

Department of Physics and Columbia Astrophysics Laboratory, Columbia University, 538 West 120th Street, New York, NY 10027; kaaret@astro.columbia.edu

Received 1996 November 18; accepted 1997 February 10

ABSTRACT

Quasi-periodic oscillations (QPOs) at frequencies near 1000 Hz were recently discovered in several X-ray binaries containing neutron stars. Two sources show no correlation between QPO frequency and source count rate. We suggest that the QPO frequency is determined by the Keplerian orbital frequency near the marginally stable orbit predicted by general relativity in strong gravitational fields. The radius of the marginally stable orbit does not depend on the luminosity or mass accretion rate of the source; therefore the QPO frequency does not depend on the source count rate. The QPO frequencies observed from 4U 1636–536 imply that the mass of the neutron star is 2.02 ± 0.12 Msun. Interpretation of the 4.1 keV absorption line observed from 4U 1636–536 as due to Fe XXV ions then implies a neutron star radius of 9.6 ± 0.6 km.

Subject headings: accretion, accretion disks — gravitation — relativity — stars: individual (4U 1636–536, 4U 1608–52) — stars: neutron — X-rays: stars

1. INTRODUCTION

Observations of the low-mass X-ray binaries 4U 1608–52 (Berger et al. 1996) and 4U 1636–536 (Zhang et al. 1996) with the Rossi X-Ray Timing Explorer (Bradt, Rothschild, & Swank 1993) have revealed quasi-periodic oscillations (QPOs) at 800–900 Hz that show no or little dependence of QPO frequency on source count rate. In 4U 1608–52, the QPO frequency changes by less than 10% as the count rate changes by a factor of 2 (Berger et al. 1996; van der Klis 1997). This is in sharp contrast to the strong correlation of QPO frequency and source count rate seen in the low-mass X-ray binaries 4U 1728–34 (Strohmayer et al. 1996) and 4U 0614+091 (Ford et al. 1997); in both of these sources, the QPO frequency changes by more than a factor of 2 as the count rate changes by a factor of 2. In 4U 1608–52 and 4U 1636–536, the QPO frequency is constant to within 10% and is, at most, only weakly influenced by the luminosity or mass accretion rate of the source. We suggest that the QPO frequency is determined by the Keplerian orbital frequency near the marginally stable orbit predicted by general relativity in strong gravitational fields (Muchotrzeb-Czerny 1986; Paczyński 1987; Kluzniak & Wagoner 1985; Kluzniak, Michelson, & Wagoner 1990). The radius of the marginally stable orbit depends only on the mass and rotation of the neutron star and not on the luminosity. This offers a possible means to produce QPO frequencies that are independent of source count rate.

2. ACCRETION FLOW AT THE MARGINALLY STABLE ORBIT

In general relativity, no stable orbits exist close to a sufficiently compact massive object. The radius of the marginally stable orbit r_{ms} is determined by the mass and rotation of the star and is independent of the mass accretion rate or radiation flux in the accretion disk. In a Schwarzschild spacetime, r_{ms} = \frac{6GM}{c^2}, where M is the mass of the star. For “soft” nuclear matter equations of state, the marginally stable orbit can lie outside the neutron star (Klüüniak & Wagoner 1985; Cook, Shapiro, & Teukolsky 1994). If the neutron star’s magnetic field is sufficiently small, then the accretion disk will pass through r_{ms}. Keplerian orbits near r_{ms} have frequencies near 1000 Hz and could generate QPOs that are independent of source count rate.

An analysis of thin accretion disks by Muchotrzeb-Czerny (Muchotrzeb 1983; Muchotrzeb-Czerny 1986) showed that a stationary flow cannot be supported near r_{ms} for viscosities greater than a critical value. Stationary flow breaks down at the sonic radius, where the radial velocity of matter in the accretion disk changes from subsonic to supersonic. This instability may lead to observable oscillations via either Doppler beaming or eclipses and provides a mechanism to generate QPOs at Keplerian orbital frequencies near r_{ms} (Paczynski 1987; Kluzniak et al. 1990; Miller, Lamb, & Psaltis 1996).

The sonic radius is located close to r_{ms} but may move as a result of variations in surface density in the inner disk (Muchotrzeb-Czerny 1986). The analysis of Muchotrzeb-Czerny (1986) suggests that the sonic radius will oscillate over a relatively narrow range near r_{ms}. If the surface density variations are irregular, then changes in the QPO frequency will be irregular. The QPO frequencies in 4U 1608–52 and 4U 1636–536 show an irregular time evolution, wandering over a small range in frequency (50 Hz) on a timescale of 1000 s (Berger et al. 1996; Zhang et al. 1996). The observed range of QPO frequency variations implies variations in the orbital radius of ∆r/r = 2Δν/3ν ~ 0.03. This is consistent with the allowed range for sonic radius variations calculated by Muchotrzeb-Czerny (1986) for α ~ 0.2–0.3, where α is a dimensionless viscosity parameter (Shakura & Sunyaev 1973). Small surface density variations in the disk need not be correlated with the total mass accretion rate onto the neutron star, particularly if the disk is not axisymmetric, and QPO frequency variations may arise independent of variations in the source count rate.

Motion of the sonic radius should occur on the viscous timescale t_v \sim R^2/ν, where R is the disk radius and ν is the kinematic viscosity. Using an α-prescription for the viscosity, we find that t_v \sim α^{-1/2}t_\nu, where \nu is the Mach number of the flow defined as the ratio of the circular velocity to the sound speed, and t_\nu is the orbital timescale (Pringle 1981). If the QPOs are associated with Keplerian orbits, then t_\nu \sim 10^{-3} s. For the inner edge of the disk at a temperature kT \sim 1 keV,
we estimate $M \sim 300$. For this model to hold, we must have a viscosity greater than the critical value discussed above, which implies $\alpha > \alpha_c \approx 0.03$. Estimates of $\alpha$ for the inner accretion disk are in the range 0.1–1 (Eardley & Lightman 1975). Viscosities in this range lead to viscous timescales of $10^2$–$10^3$ s. This matches the observed $10^3$ s timescale for QPO frequency variations in 4U 1608–52 and 4U 1636–536.

We conclude that motion of the sonic point near the marginally stable orbit produces small QPO frequency variations that are consistent in timescale and frequency range with those observed in 4U 1608–52 and 4U 1636–536. Our calculation of the frequency range and timescale for the variations shows that small irregular variations are consistent with QPO production near the marginally stable orbit. However, the magnitude of these variations is small, less than 10%. The crucial point is that the radius of the marginally stable orbit does not depend on the source count rate; therefore the QPO frequency does not depend on the source count rate.

In one observation of 4U 1636–536, two QPOs are present at frequencies of $1171 \pm 11$ Hz and $989.7 \pm 1.4$ Hz (van der Klis et al. 1996). This is similar to the high-frequency QPOs in 4U 1728–34 (Strohmayer et al. 1996) and 4U 0614+091 (Ford et al. 1997), which have been interpreted with models in which the higher frequency is the Keplerian frequency in the inner region of the accretion disk, while the lower frequency “beat” signal results from differential rotation of the inner disk and the spinning neutron star (Alpar & Shaham 1985; Lamb et al. 1985). We suggest that the QPOs near $800–900$ Hz in 4U 1608–52 and 4U 1636–536 are beat frequency signals. Comparison with 4U 1728–34 and 4U 0614+091 supports this identification. In these two sources the fractional rms amplitude of the higher frequency QPO decreases with count rate, while the amplitude of the lower frequency peak increases. Extrapolation of these trends would lead one to anticipate that only the beat frequency QPO should be visible in higher luminosity sources such as 4U 1608–52 and 4U 1636–536.

A possible explanation of why only the beat frequency signal is visible is that the instability at the sonic point produces a modulation in the beaming pattern of the radiation but not in the total luminosity of the source, while a luminosity variation at the beat frequency can result from radiation feedback from the neutron star onto the disk (Miller et al. 1996). If the optical depth near the neutron star is large, then the observed amplitude of beaming modulations will be reduced relative to luminosity modulations (Kylafis & Phinney 1989), and, for the high optical depths expected at high mass accretion rates, only the beat frequency QPO would be visible. If the optical depth between the star and the disk is sufficiently large, the radiation feedback, and therefore the beat frequency QPO, may be suppressed (Miller et al. 1996).

The strong dependence of QPO frequency in 4U 1728–34 and 4U 0614+091 on source intensity implies that the accretion disks in these sources must be disrupted before reaching $r_{\text{ms}}$. The qualitatively different behavior of the QPOs in 4U 1728–34 and 4U 0614+091 compared with those in 4U 1636–536 and 4U 1608–52 is a natural consequence of where the inner disk is disrupted relative to $r_{\text{ms}}$.

3. DISCUSSION

If, indeed, the QPOs in 4U 1608–52 and 4U 1636–536 are associated with Keplerian orbits near the marginally stable orbit, then they provide us with a tool for determination of the masses of the neutron stars in these systems (Kluźniak et al. 1990). The 1171 Hz QPO from 4U 1636–536, which we associate with a Keplerian orbital frequency, is detected simultaneously with an 899 Hz beat frequency QPO. This beat frequency is near the maximum beat frequency observed from this source. Since the analysis of Muchotrzeb-Czerny (1986) indicates that the sonic radius is always equal to or larger than $r_{\text{ms}}$ for viscosities above the critical value, this suggests that the Keplerian orbital frequency of 1171 Hz corresponds to an orbit near $r_{\text{ms}}$. The analysis of Chen & Taam (1993) also shows that the sonic radius lies very close to, although perhaps inside, $r_{\text{ms}}$ for accretion rates below the Eddington rate.

In a Kerr spacetime, the relation between the mass of the neutron star and the Keplerian orbital frequency is $M = 2.198 M_\odot \left(\nu_\nu / 1000 \text{ Hz}\right)^2 \left(1 - 0.748j\right)^{-3}$, to first order in $j$, where $j = c I / G M^2$ is a dimensionless measure of the angular momentum of the star (Bardeen, Press, & Teukolsky 1972; Kluźniak et al. 1990). The Keplerian orbital frequency $\nu_\nu = 1171$ Hz, the stellar rotation frequency $\nu_\nu / 2 \sigma = 272$ Hz, and an assumed moment of inertia $I = 2.0 \times 10^{34} \text{ g cm}^2$, imply a neutron star mass of $2.02 M_\odot$. Uncertainty in the Keplerian frequency corresponding to $r_{\text{ms}}$ arises both from theoretical uncertainty about the relative locations of the sonic radius and $r_{\text{ms}}$ and from observational limitations in that only a portion of the complete frequency range of the QPO may have been sampled during the limited observing time. We allow an uncertainty of $50$ Hz in the Keplerian frequency, which contributes an uncertainty of $5\%$ to the mass estimate. Allowing moments of inertia in the range 1–3 $\times 10^{34} \text{ g cm}^2$ contributes an uncertainty of $4\%$ to the mass estimate. We conclude that the mass of the neutron star in 4U 1636–536 is $2.02 \pm 0.12 M_\odot$. This mass is within the allowed range of neutron star masses (Kalogera & Baym 1996) and is not unreasonable given that accretion onto the neutron star has occurred.

It is interesting to note that detection of an absorption feature at $4.1 \pm 0.1 \text{ keV}$ in thermonuclear X-ray bursts from 4U 1636–536 has been reported by Waki et al. (1984). Similar absorption features have been reported for 4U 1608–52 (Nakamura, Inoue, & Tanaka 1988) and EXO 1747–214 (Magnier et al. 1989) by Tenma and EXOSAT. The most plausible interpretation of the line-like absorption feature is absorption by Fe xxv ions in matter accreted onto the neutron star surface after the outburst (Waki et al. 1984; Inoue 1988). The transverse Doppler effect is negligible for a neutron star spin rate of $272$ Hz. In this case, the redshift of the line and the neutron star mass quoted above implies a neutron star radius of $9.6 \pm 0.6 \text{ km}$. This result is consistent with the mass-radius relation for the AV14+UVII equation of state (Wiringa, Fiks, & Fabrocini 1988) and allowing for rotation (Cook et al. 1994). We note that this equation of state produces a moment of inertia of $2.0 \times 10^{34} \text{ cm}^2$ for a $2.0 M_\odot$ star, consistent with our assumed $I$ above. Confirmation of the absorption feature at $4.1 \text{ keV}$ using a higher resolution X-ray detector, such as the SIS on ASCA, would be of great interest.

If confirmed, the relation between high-frequency QPOs in X-ray binaries and the marginally stable orbit in general relativity may open a new experimental venue for the study of strong gravitational fields and will permit the determination of neutron star masses as described above. Further work is required to improve our theoretical understanding of accretion flows near $r_{\text{ms}}$ and to obtain more detailed observations of high-frequency QPOs and the X-ray binary systems that produce QPOs. Independent determination of the mass of the
neutron star in 4U 1636−536, via measurement of the binary orbital parameters, would be of great interest. Comparison of the mass determined from the orbital parameters with the mass determined from the QPO frequency would permit a test of strong-field and inertial frame dragging effects in general relativity (Kluźniak et al. 1990).

Note added in manuscript.—Recently, Zhang et al. (1997) reported 581 Hz oscillations in X-ray bursts from 4U 1636−536 and detection of two simultaneous QPOs with a frequency difference of 255 ± 25 Hz between bursts. The frequency difference is consistent with their proposed neutron star spin frequency of 290 Hz (half of 581 Hz) within 1.4 σ. Also, Wijnands et al. (1997) reported detection of two simultaneous QPOs at frequencies of 890–920 Hz and 1150–1193 Hz and single QPOs at frequencies 853–896 Hz in 4U 1636−536. The simultaneous QPOs have a constant frequency difference of 276 ± 10 Hz. The frequency difference is, again, consistent with 290 Hz within 1.4 σ.

The single QPO has a frequency which is not correlated with the inferred mass accretion rate. The properties of the single QPO are quite similar to those of the single QPO observed previously by Zhang et al. (1996) and discussed above. The simultaneous QPOs have frequencies that are correlated with inferred mass accretion rate.

The transition from a QPO with frequency correlated with mass accretion rate to a QPO with frequency independent of mass accretion rate is a natural consequence of where the inner disk is disrupted relative to $r_{m}$: a single mechanism can suffice to produce QPOs with both behaviors. We note that the single QPO frequencies observed are somewhat lower (3%–7%) than the maximum beat frequency observed in the simultaneous QPOs. This can be understood if the analysis of Muchotrzeb-Czerny (1986) is correct and the outermost critical point of modal type moves outward as the accretion rate increases. The sonic radius would then tend toward larger radii at higher mass accretion rates, producing lower QPO frequencies. The maximum-frequency QPOs would be produced when the sonic radius makes its closest approach to the marginally stable orbit, and, in this case, would arise at the transition between mass accretion rate–dependent and mass accretion rate–dependent QPOs. Adopting 1193 Hz as the Keplerian orbital frequency at $r_{m}$ and a neutron star spin frequency of 290 Hz gives a mass of $1.98 \pm 0.12 M_{\odot}$, consistent with that quoted above.

We thank M. C. Miller, F. K. Lamb, and M. Ruderman for useful discussions, and M. van der Klis and R. A. D. Wijnands for providing data for 4U 1636−536 before publication and for pointing out an error in Berger et al. (1996). We also thank the referee for comments that improved the clarity of the paper.

REFERENCES

Alpar, M. A., & Shaham, J. 1985, Nature, 316, 239
Bardeen, J. M., Press, W. H., & Teukolsky, S. A. 1972, ApJ, 178, 347
Berger, M., et al. 1996, ApJ, 469, L13
Bradt, H. V., Rothschild, R. E., & Swank, J. H. 1993, A&AS, 97, 355
Chen, X., & Taam, R. E. 1993, ApJ, 412, 254
Cook, G. B., Shapiro, S. L., & Teukolsky, S. A. 1994, ApJ, 424, 823
Eardley, D. M., & Lightman, A. P. 1975, ApJ, 200, 187
Ford, E., et al. 1997, ApJ, in press
Inoue, H. 1988 in Physics of Neutron Stars and Black Holes, ed. Y. Tanaka (Tokyo: Universal Academy Press), 235
Kalogera, V., & Baym, G. 1996, ApJ, 470, L61
Kluźniak, W., Michelson, P., & Wagoner, R. V. 1990, ApJ, 358, 538
Kluźniak, W., & Wagoner, R. V. 1985, ApJ, 297, 548
Kylafis, N., & Phinney, E. S. 1989, in Timing Neutron Stars, ed. H. Ögelman & E. P. J. van den Heuvel (Dordrecht: Kluwer)
Lamb, F. K., Shibazaki, N., Alpar, M. A., & Shaham, J. 1985, Nature, 317, 681
Magnier, E., et al. 1989, MNRAS, 237, 729
Miller, M. C., Lamb, F., & Psaltis, D. 1996, ApJ, submitted
Muchotrzeb, B. 1983, Acta Astron., 33, 79
Muchotrzeb-Czerny, B. 1986, Acta Astron., 36, 1
Nakamura, N., Inoue, H., & Tanaka, Y. 1988, PASJ, 40, 209
Paczyński, B. 1987, Nature, 327, 303
Pringle, J. E. 1981, ARA&A, 19, 137
Shakura, N. I., & Sunyaev, R. A. 1973, A&A, 24, 337
Strohmayer, T., et al. 1996, ApJ, 469, L9
van der Klis, M. 1997, private communication
van der Klis, M., et al. 1996, IAU Circ. 6428
Waki, I., et al. 1984, PASJ, 36, 819
Wijnands, R. A. D., et al. 1997, ApJ, submitted
Wiringa, R. B., Fiks, V., & Fabrocini, A. 1988, Phys. Rev. C, 38, 1010
Zhang, W., Lapidus, I., White, N. E., & Titarchuk, L. 1996, ApJ, 469, L17
Zhang, W., et al. 1997, IAU Circ. 6541