The bread production process using application of the Hotelling $T^2$ control chart

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Abstract. Quality Control is an activity to maintain and direct to quality company product and services which can maintain the existence of the product in the long term period. Statistical Quality Control is a method of using the control charts of the cause-effect techniques and diagrams in quality control to minimize the rejected products. In this study, we used one of Statistical Quality Control methods, which is Hotelling $T^2$ Control Chart to control the quality of bread production. Hotelling $T^2$ Control chart is the type of multivariate control chart that is used to monitor the process of identifying the causes of variance and improve the process. In the application of the Hotelling $T^2$ Control Chart, we used two major characteristics of quality, those are size and net. Based on the identification the number of products that fail is still within reasonable limits located between UCL and LCL. The main cause of the signal out of control is the packaging of the product. As the endeavour improvement from the company, the production process is increasing in the second phase.

1. Introduction

In the manufacturing process, quality is an important factor. To maintain the quality of a product, there is an activity in controlling each process of a product from the beginning of how the product was made until the product delivered to the customers. Quality control is essential to build and achieve a product that meets the customers’ satisfaction. As well as in bread production, quality control becomes an essential thing to achieve the customer’s expectation of the product. Recently, there are many situations in which the continuous or control is necessary in the Industry development [1]. Therefore, the manufacturer looks for enhancing their product and quality. In the manufacturing process, especially in bread production, the result of the product may still generate errors. The bread products can be damaged or defected. The substandard quality of the products can make the customers' satisfaction not fulfilled which will cause a decreasing in the demand of the products. Based on Saniga that employees who have high levels of trust in management are likely to provide higher levels of support for change and lower levels of resistance to change, it shown that in process production need some improvement in the process [2].

According to Montgomery, Statistical Quality Control (SQC) is a technique used to control and improve the quality of a product in the manufacturing process [3]. This method is very essential in the manufacturing process since the quality of a product needs to be maintained as the aspect to achieve the customers' satisfaction. Therefore, product sustainability is assured as the quality of the product itself is controlled. As the part of the technique used to maintain the product quality, Statistical Quality Control
comes up with the control charts of the cause-effect technique and diagram in order to minimize the rejected product in the manufacturing process [4].

As a way to decrease the several sources of error which can damage the quality of the bread production process, there needs a selection of the products with the standards set by the company. In order to establish the standards, the various data from variables of the quality are needed. Based on the regulations of the Indonesian republic government regarding food labels and advertisements, companies must put a net weight on product packaging as food information to consumers. By seeing how those variables are affecting the consumers’ satisfaction, size and net are considered as the variables of the bread production quality. As the variables of the quality are various, the Multivariate statistical process control can be used. One of the multivariate control procedures can be applied in this process is the Hotelling T\(^2\) Control Chart. This process is a scalar that combines the information from the dispersed data and the mean value from several variables examined [5].

Since there are two variables used in this experiment, those control chart cannot be implemented. Therefore the method used is the Hotelling T\(^2\) Control chart. This method is the type of multivariate control chart that is used to monitor the process of identifying the causes of variance and improve the process. It can be used to monitor the mean vector of the process and detect the subtle change in the entire system. The change of one variable can cause a rippling effect in the entire system. This method considers the interrelationships between the variables. The version of the Hotelling T\(^2\) control chart that will be implemented in this research is subgroup data. In this version, the characteristics of the data are jointly distributed due to the normal distribution of the multivariate data.

2. Method

Improving the quality of a product is not only through the process of selecting quality, but also conducting an evaluation of the production process. One effort to evaluate the production process is to find the source of problems that cause defects in the product. The method that can be used is with Statistical Quality Control (SQC). Statistical Quality Control (SQC) is the term used to describe the set of statistical tools used by quality professionals [6]. The implementation of SQC is divided into three parts. The first part is descriptive statistics, which is used to describe the characteristics of quality and relationship of the quality of the product. The second is statistical process control, a method that uses a random sampling of a process and deciding whether the process is producing a product that satisfies the range of the quality. Then the last implementation is acceptance sampling, a determining process of the sample of goods for an entire lot of the productions. This method is used because those three categories are supporting the process of measurement and evaluation product’s quality. This method aims to find the largest percentage of several problem factors in the production process which then looks for the best solution for the problem.

Control charts are the primary tools of statistical process control [7]. These charts may be designed by using a simple rule suggested by Shewhart, by a statistical criterion, or by an economic criterion.

In the book titled Introduction to Statistical Quality Control, Montgomery states that \(\bar{X}\) and R control chart is the control where it implements the means or average data. During this process, the variability of the process can be monitored using the standard deviation, called by \(\bar{X}\) and S control chart. While MA-MR chart is a set of control charts for variables data. The moving average chart monitors the process location over time, based on the average of the current subgroup and one or more prior subgroups. The moving range chart monitors the variation between the subgroups over time.

As the object of the research, we observed the bread production process in a company named S Bread Factory located in West Bandung. The measurements of quality products are difficult to measure. Facilitating limitation of the quality observed, we use the variables which are: size and net. A product is said to be successful if it meets standards on all variables. On the contrary, a product is said to fail if it cannot meet the minimum standards on each variable.

In this research, there are two types of data processed. They are primary and secondary data. The primary data is obtained by observing the processes from the raw material to the final product directly.
Therefore, the secondary data is primary data which includes the chemical and physical resulting data and the quality control process during the production process.

The quality control chart used in this research at Bandung Bakery has carried out an attribute of the measurement of the quality of the product characteristics that are difficult to measure. The qualities that are intended are good or bad quality and success or failure the products are. Therefore, this control chart is used to control whether the defect products are still within tolerance or not.

S Bread Factory produces 2,000 packs per day with eight different types of bread. The bakery production generates 250 packages per type, which results in 62,000 packages in one month. In this study, the writer takes two types of products, those are chocolate bread and cheese bread. From the total population of the products, the writer took a month of production results. So the population of this study is taken from a production capacity of 500 packs from two different types of bread. The sampling technique used is by withdrawing the bread types of samples based on random sampling and judgment sampling.

The resulting data is multivariate data so that an advanced method is needed to sort out the data. The most familiar multivariate process monitor and control procedure is the Hotelling T² control chart which used for monitoring the mean vector of process. It is a direct analog of the univariate Shewhart \( \bar{x} \) chart [3]. There are two versions of the Hotelling T² Control chart, subgroup data, and individual observations.

In this study, the writer used a subgroup data variation:

\[
X_0^2 = \frac{n}{\sigma_1^2 \sigma_2^2 - \sigma_{12}^2} \left[ \sigma_1^2 (X_1 - \mu_1)^2 + \sigma_2^2 (X_2 - \mu_2)^2 - 2\sigma_{12}(X_1 - \mu_1)(X_2 - \mu_2) \right] \tag{1}
\]

In subgroup data, this equation is applied as the control chart of the process \( \mu_1, \mu_2, \) and \( \mu_3 \). Whenever the values of the processes remain the same, the value of \( X_0^2 \) should be below the UCL. With three predetermined variables, on Hotelling T² Chart the writer creates a control chart, UCL and LCL limit. So that data that exceeds or is less than the standard on established statistical data will be considered a failed product. There are two Phase in Hotelling T², in a phase I operation, where the focus is on detecting and removing outliers, consideration is given to batch processes where the batch observations are taken from either a common multivariate normal distribution or a series of multivariate normal distribution with different vectors. In Phase II operation, where the monitoring of future observations is of primary concern, emphasis is placed on the application of the T² statistic using a known or estimated in- control mean vector [2].

The T² control chart is based on the multivariate normal distribution and on the Mahalanobis distance (MD) [8]. The T² statistics follows a beta distribution in the population and constructed from two phases. The unknown values of mean vector \( m \) and the covariance matrix will be approximated from the historical data set of an individual specimens, the mean vector is represented by \( \bar{x} \) and the sample covariance matrix by S. The T² statistics is indicated by the equation:

\[
T^2 = n(\bar{x} - \bar{\bar{x}})'S^{-1}(\bar{x} - \bar{\bar{x}}) \tag{2}
\]

Phase I is to specify the mean vector and the sample covariance when the process is in control. This phase also observe the in-control set and set up the control limit for the second phase. By implementing this phase, the Upper Control Limit (UCL) and Lower Control Limit (LCL) are given by:

\[
UCL = \frac{p(m + 1)(n - 1)}{mn - m - p + 1} F_{a,p,mn-m-p+1} \tag{3}
\]

\[
LCL = 0 \tag{4}
\]

Then in the Phase II, the chart monitors the future production, and the control limits are given by:

\[
UCL = \frac{p(m + 1)(n - 1)}{mn - m - p + 1} F_{a,p,mn-m-p+1} \tag{5}
\]
The type of data population data in this study is attribute data. Attribute data is qualitative data from quality characteristics that cannot be measured. The values of attribute data are obtained. Attribute values are acquired through the product examination of product characteristics whose results are declared production.

The type of quality measurement that used to calculate the proportion of the defect product which exists in production is an np-chart attribute [1]. An np-chart is an attributes control chart used with data collected in subgroups that are the same size. Np-chart shows how the process, measured by the number of nonconforming items it produces, changes over time. The process attribute (or characteristic) is always described in a yes/no, pass/fail, go/no go form. For example, the number of incomplete accident reports in a constant daily sample of five would be plotted on an np-chart. Np-chart is used to determine if the process is stable and predictable, as well as to monitor the effects of process improvement theories. These control charts are used in attribute quality control that is to control the damaged product part of the production and to find out if it is still within tolerance. In this study, the data collection techniques that will be used are observation and documentation. Observation is a technique of acquisition of data from the production process by viewing and recording the process from raw materials until the final products. In addition, this method determines the enforcement of quality control in the production process. Documentation is a data collection technique by studying the document of the production process formed in history, production activity reports, and also the production quantity including the defect products and employment documents.

3. Data results and analysis

In this observation, we analyze the data based on the data containing two variable set. They are net and size. As the sample, there are two kinds of bread observed, which each of the kind includes 250 pcs of breads. The data used in this paper is processed using Minitab. The Analysis for each variable observed of bread product produced by S Bread Factory located in West Bandung is as follow:

![Normality test variable net](image)

**Figure 1.** Normality test variable net.
Based on the resulted data from the normal test, what can be interpreted from the Figure 1 and 2 is that the data is categorized as normal data. In Kolmogorov-Smirnov normality data test, the mean resulted for the net variable is also 200,1 with a standard deviation 2,159. In these two parts, the value between Anderson-Darling method and Kolmogorov-Smirnov are the same. However, in P-Value there is a difference. In Kolmogorov-Smirnov, the P-Value is >0,150. The difference in the value is as the result of the data distribution focused in each method. In Anderson-Darling, the data focused is in the tail distribution while in Kolmogorov-Smirnov method, the data focused in the centre distribution. Similarly, in the size variable, the difference is located in the P-Value. In this variable, the normal test of Kolmogorov-Smirnov is at the value of 0,076.

Based on the T² control chart graph, the data shown is can be concluded as normal. The data is located in between UCL and LCL. The UCL for the data resulted is 12,02 and the LCL is 0. According to this data, the conclusion can be made is that from both of the variable examined in this research, the bread production is still in control. This may be resulted from the minimum human error.
Table 1. Test of normality for two variables.

|           | Anderson-Darling | Kolmogorov-Smirnov |
|-----------|------------------|--------------------|
|           | Mean  | StDev | N  | AD   | P-Value | Mean  | StDev | N  | KS  | P-Value |
| Net       | 200.1 | 2.159 | 500 | 0.293 | 0.603    | 200.1 | 2.159 | 500 | 0.030 | >0.150  |
| Size      | 9,508 | 0.5050 | 500 | 0.394 | 0.374    | 9,508 | 0.5050 | 500 | 0.038 | 0.076   |

Table 2. UCL and LCL.

|       | UCL | LCL |
|-------|-----|-----|
|       | 12.02 | 0   |

4. Conclusion

In this research, the data resulted from several tests shows that the data is normally distributed and categorized as in control data. However, in this research the applied statistical method is Hotelling T2 Control Chart method. The implementation of this method is can be helpful to the bread company to control their production as the way to increase the productivity of the company.

Since the data resulted is normally distributed an in control, the solution can be offered to the company is that to increase the production. Since the variable examined in this research shows that it contributes a very small number of error which affects in the normally distributed data. The error in the production can be sourced from the human error from the employee, not the variables tested in this experiment which are net and size.

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