**WD+RG systems as the progenitors of type Ia supernovae**

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**Abstract** Type Ia supernovae (SNe Ia) play an important role in the study of cosmic evolution, especially in cosmology. There are several progenitor models for SNe Ia proposed in the past years. By considering the effect of accretion disk instability on the evolution of white dwarf (WD) binaries, we performed detailed binary evolution calculations for the WD + red-giant (RG) channel of SNe Ia, in which a carbon–oxygen WD accretes material from a RG star to increase its mass to the Chandrasekhar mass limit. According to these calculations, we mapped out the initial and final parameters for SNe Ia in the orbital period–secondary mass \((\log P - M_2)\) plane for various WD masses for this channel. We discussed the influence of the variation of the duty cycle value on the regions for producing SNe Ia. Similar to previous studies, this work also indicates that the long-period dwarf novae offer possible ways for producing SNe Ia. Meanwhile, we find that the surviving companion stars from this channel have a low mass after the SN explosion, which may provide a means for the formation of the population of single low-mass WDs (<0.45 \(M_\odot\)).

**Key words:** stars: evolution — supernovae: general — binaries: close — white dwarfs

**1 INTRODUCTION**

Type Ia Supernovae (SNe Ia) are excellent cosmological distance indicators due to their high luminosities and remarkable uniformity. They have been applied successfully in determining cosmological parameters (e.g. \(\Omega\) and \(\Lambda\); Riess et al. 1998; Perlmutter et al. 1999). However, several key issues related to the nature of their progenitors and the physics of the explosion mechanisms are still not properly understood (Hillebrandt & Niemeyer 2000; Podsialowski et al. 2008; Wang et al. 2008a), and no SN Ia progenitor system has been conclusively identified before the explosion. It is generally believed that SNe Ia are thermonuclear explosions of carbon–oxygen white dwarfs (CO WDs) in binaries (for the review see Nomoto et al. 1997). Over the past few decades, two families of SN Ia progenitor models have been proposed, i.e. the double-degenerate (DD) and single-degenerate (SD) models. Of these two models, the SD model is widely accepted at present. It is suggested that the DD model, which involves the merger of two CO WDs (Iben & Tutukov 1984; Webbink 1984; Han 1998), likely leads to an accretion-induced collapse rather than to an SN Ia (Nomoto &
For the SD model, the companion is probably a main-sequence (MS) star or a slightly evolved subgiant star (WD + MS channel), a red-giant star (WD + RG channel), or an He star (WD + He star channel) (e.g. Hachisu et al. 1996; Li & van den Heuvel 1997; Langer et al. 2000; Han & Podsiedowski 2004, 2006; Chen & Li 2007, 2009; Meng et al. 2009; Lü et al. 2009; Wang et al. 2009a,b; Wang, Li & Han 2009). Note that, some recent observations have indirectly suggested that at least some SNe Ia can be produced by a variety of different progenitor systems (e.g. Patat et al. 2007; Voss & Nelemans 2008; Wang et al. 2008b; Justham et al. 2009).

An explosion following the merger of two WDs would leave no remnant, while the companion star in the SD model would survive and potentially be identifiable. Tycho’s supernova is a Galactic SN Ia. Ruiz-Lapuente et al. (2004) found a star similar to the sun but with a lower gravity in the remnant region called Tycho G which moves at more than three times the mean velocity of the stars there. They argued that Tycho G could be the surviving companion of the supernova. Note that there has been no conclusive proof yet that any individual object is the surviving companion star of a SN Ia. It will be a promising method to test SN Ia progenitor models by identifying their surviving companions (e.g. Wang & Han 2009).

The WD + RG channel is a possible way to produce SNe Ia, and is supported by some observations. It is suggested that RS Oph and T CrB, both recurrent novae, are probable SN Ia progenitors and belong to the WD + RG channel (e.g. Belczyński & Mikolajewska 1998; Hachisu et al. 1999a, 2007; Sokoloski et al. 2006). Meanwhile, by detecting Na I absorption lines with low expansion velocities, Patat et al. (2007) suggested that the companion of the progenitor of SN 2006X may be an early RG star. Additionally, Voss & Nelemans (2008) studied the pre-explosion archival X-ray images at the position of the recent SN 2007on, and they considered its progenitor to be a WD + RG system.

Xu & Li (2009) recently emphasized that the mass-transfer through the Roche lobe overflow (RLOF) in the evolution of WD binaries may become unstable (at least during part of the mass-transfer lifetime). This important feature has been ignored in nearly all of the previous theoretical work on SN Ia progenitors except for King et al. (2003)¹ and Xu & Li (2009), who inferred that the mass-accretion rate onto the WD during dwarf nova outbursts can be sufficiently high to allow steady nuclear burning of the accreted matter and growth of the WD mass. Following the work of Xu & Li (2009), Wang, Li & Han (2009) systematically studied the WD binaries towards SNe Ia. However, the interest of Wang, Li & Han (2009) mainly focused on the WD + MS channel of SNe Ia.

The purpose of this paper is to study the WD + RG channel towards SNe Ia in a comprehensive manner, and to show the final parameter spaces of companions at the moment of an SN Ia explosion. In Section 2, we simply describe the numerical code for the binary evolution calculations. The binary evolutionary results are shown in Section 3. Finally, a discussion is given in Section 4.

2 BINARY EVOLUTION CALCULATIONS

In our WD binary models, the lobe-filling star is a RG. The star transfers some of its material onto the surface of the WD, which increases the mass of the WD as a consequence. If the WD grows to $1.378 M_\odot$, we assume that it explodes as an SN Ia. We use Eggleton’s stellar evolution code (Eggleton 1971, 1972, 1973) to calculate the WD binary evolution. The code has been updated with the latest input physics over the past three decades (Han et al. 1994; Pols et al. 1995, 1998). RLOF is treated within the code described by Han et al. (2000). We set the ratio of mixing length to local pressure scale height, $\alpha = l/H_P$, to be 2.0 and set the convective overshooting parameter, $\delta_{OV}$, to be 0.12 (Pols et al. 1997; Schröder et al. 1997), which roughly corresponds to an overshooting length of $\sim 0.25$ pressure scaleheights ($H_P$). The opacity tables are compiled by Chen & Tout (2007).

¹ King et al. (2003) adopted a similar method in Li & Wang (1998) to produce SNe Ia with long period dwarf novae in a semi-analytic approach.
our calculations, we use a typical Pop I composition with H abundance \( X = 0.70 \), He abundance \( Y = 0.28 \) and metallicity \( Z = 0.02 \).

During the mass-transfer through the RLOF, the accreting material can form an accretion disk surrounding the WD, which may become thermally unstable when the effective temperature in the disk falls below the H ionization temperature \( \sim 6500 \) K (e.g. van Paradijs 1996; King et al. 1997; Lasota 2001). This also corresponds to a critical mass-transfer rate below which the disk is unstable. Similar to the work of Xu & Li (2009), we also set the critical mass-transfer rate for a stable accretion disk to be

\[
\dot{M}_{\text{cr, disk}} \simeq 4.3 \times 10^{-9} \left( \frac{P_{\text{orb}}}{4 \text{ hr}} \right)^{1.7} M_{\odot} \text{ yr}^{-1},
\]

for WD accretors (van Paradijs 1996), where \( P_{\text{orb}} \) is the orbital period. The locations of various types of cataclysmic variable stars (e.g. the UX UMa, U Gem, SU UMa, and Z Cam systems) in a \((P_{\text{orb}}, \dot{M}_{\text{cr, disk}})\) diagram are well described by this expression (Smak 1983; Osaki 1996; van Paradijs 1996). If the mass-transfer rate, \( \dot{M}_2 \), is higher than the critical value \( \dot{M}_{\text{cr, disk}} \), we assume that the accretion disk is stable and the WD accretes smoothly at a rate \( \dot{M}_{\text{acc}} = |\dot{M}_2| \); otherwise the WD accretes only during outbursts and the mass-accretion rate is \( \dot{M}_{\text{acc}} = |\dot{M}_2|/d \), where \( d \) is the duty cycle. The mass-accretion rate is \( \dot{M}_{\text{acc}} = 0 \) during quiescence. King et al. (2003) showed that for typical values of the duty cycle, \( \sim 0.1 \) to a few \( 10^{-3} \), the accretion rates onto the WD during dwarf nova outbursts can be sufficiently high to allow steady nuclear burning of the accreted matter. The limits on the duty cycles of dwarf nova outbursts come from observations (Warner 1995): (1) The outburst intervals for each object are quasi-periodic, but within the dwarf nova family, the intervals can range from days to decades. (2) The lifetime of an outburst is typically from 2 to 20 d and is correlated with the outburst interval. The quasi-periodicity of the dwarf nova outbursts allows the use of a single duty cycle to roughly describe the change in the mass-transfer rate, though we note that this is a simplification of the real, complicated processes. Similar to previous studies (e.g. Xu & Li 2009), we also set the duty cycle to be 0.01. Meanwhile, we also do some tests for a higher or lower value of the duty cycle in our calculations.

Instead of solving stellar structure equations of a WD, we adopt the prescription of Hachisu et al. (1999b) for the mass-growth of a CO WD by accretion of H-rich material from its companion. The prescription is given below. If the mass-accretion rate of the WD, \( \dot{M}_{\text{acc}} \), is above a critical rate, \( \dot{M}_{\text{cr, WD}} \), we assume that the accreted H steadily burns on the surface of the WD and that the H-rich material is converted into He at a rate \( \dot{M}_{\text{cr, WD}} \). The unprocessed matter is assumed to be lost from the system as an optically thick wind at a mass-loss rate \( \dot{M}_{\text{wind}} = |\dot{M}_2| - \dot{M}_{\text{cr, WD}} \). The critical mass-accretion rate is

\[
\dot{M}_{\text{cr, WD}} = 5.3 \times 10^{-7} \frac{(1.7 - X)}{X} \left( \frac{M_{\text{WD}}}{M_{\odot}} - 0.4 \right) M_{\odot} \text{ yr}^{-1},
\]

where \( X \) is the H mass fraction and \( M_{\text{WD}} \) is the mass of the accreting WD. The following assumptions are adopted when \( |\dot{M}_{\text{acc}}| \) is smaller than \( \dot{M}_{\text{cr, WD}} \): (1) When \( |\dot{M}_{\text{acc}}| \) is less than \( \dot{M}_{\text{cr, WD}} \) but higher than \( \frac{1}{3} \dot{M}_{\text{cr, WD}} \), the H-shell burning is steady and no mass is lost from the system. (2) When \( |\dot{M}_{\text{acc}}| \) is lower than \( \frac{1}{3} \dot{M}_{\text{cr, WD}} \) but higher than \( \frac{1}{4} \dot{M}_{\text{cr, WD}} \), a very weak H-shell flash is triggered but no mass is lost from the system. (3) When \( |\dot{M}_{\text{acc}}| \) is lower than \( \frac{1}{4} \dot{M}_{\text{cr, WD}} \), the H-shell flash is so strong that no material is accumulated onto the surface of the WD.

We define the mass-growth rate of the He layer under the H-shell burning as

\[
\dot{M}_{\text{He}} = \eta_{\text{fl}} |\dot{M}_{\text{acc}}|,
\]

where \( \eta_{\text{fl}} \) is the efficiency of the H-shell flash.
where $\eta_H$ is the mass-accumulation efficiency for H-shell burning. According to the assumptions above, the values of $\eta_H$ are:

$$
\eta_H = \begin{cases} 
\dot{M}_{\text{cr,WD}}/|\dot{M}_{\text{acc}}|, & |\dot{M}_{\text{acc}}| > \dot{M}_{\text{cr,WD}}, \\
1, & \dot{M}_{\text{cr,WD}} \geq |\dot{M}_{\text{acc}}| \geq \frac{1}{8} \dot{M}_{\text{cr,WD}}, \\
0, & |\dot{M}_{\text{acc}}| < \frac{1}{8} \dot{M}_{\text{cr,WD}}.
\end{cases}
$$

When the mass of the He layer reaches a certain value, He is assumed to be ignited. If He-shell flashes occur, a part of the envelope mass is assumed to be blown off. The mass-growth rate of WDs in this case is linearly interpolated from a grid computed by Kato & Hachisu (2004), where a wide range of WD masses and mass-accretion rates was calculated in the He-shell flashes. We define the mass-growth rate of the CO WD, $\dot{M}_{\text{CO}}$, as

$$
\dot{M}_{\text{CO}} = \eta_{\text{He}} \dot{M}_{\text{He}} = \eta_{\text{He}} \eta_H |\dot{M}_{\text{acc}}|,
$$

where $\eta_{\text{He}}$ is the mass-accumulation efficiency for He-shell flashes.

The evolution of these WD binaries is driven by the nuclear evolution of the donor stars, and the change of the orbital angular momentum of the binaries is mainly caused by wind mass-loss from the WD. We assume that the mass lost from these binaries carries away the same specific orbital angular momentum of the WD (the mass-loss in the donor’s wind is supposed to be negligible, but its effect on the change of the orbital angular momentum, i.e. magnetic braking, is included). We incorporate the prescriptions above into Eggleton’s stellar evolution code and follow the evolution of the WD + RG systems. We have calculated a large, dense model grid, in which the lobe-filling star is an RG star.

3 BINARY EVOLUTION RESULTS

3.1 An Example of Binary Evolution Calculations

In Figure 1, we present an example of binary evolution calculations. Panel (a) shows the $M_2$, $\dot{M}_{\text{CO}}$ and $M_{\text{WD}}$ varying with time, while panel (b) is the evolutionary track of the donor star in the Hertzsprung-Russell diagram, where the evolution of the orbital period is also shown. The binary shown in this case is $(M_2^i, M_{\text{WD}}^i, \log(P^i/d)) = (1.2, 1.0, 0.2)$, where $M_2^i$, $M_{\text{WD}}^i$ and $P^i$ are the initial mass of the donor star and the CO WD in solar masses, and the initial orbital period in days, respectively. The donor star fills its Roche lobe during the RG stage which results in case B mass-transfer. During the whole evolution, the mass-transfer rate is lower than the critical value given by Equation (1). Thus, the accretion disk experiences instability. The mass-accretion rate $\dot{M}_{\text{acc}}$ of the WD exceeds $\frac{1}{8} \dot{M}_{\text{cr,WD}}$ after the onset of RLOF, leading to the mass-growth of the WD during outbursts. After about $2.6 \times 10^8$ yr, the WD grows to $1.378 M_\odot$, which explodes as an SN Ia. At the moment of the SN explosion, the mass of the donor star is $M_{\text{SN}}^2 = 0.4418 M_\odot$ and the orbital period is $\log(P_{\text{SN}}^i/d) = 0.7992$.

3.2 Initial and Final Parameters for SNe Ia

In Figure 2, we show the initial contours for SNe Ia and the final state of binary evolution in the $(\log P^i, M_2^i)$ plane at the moment of an SN Ia explosion with the duty cycle $d = 0.01$. The initial WD masses are 1.0, 1.1 and 1.2 $M_\odot$. The final contour (solid line in Fig. 2) is much lower than that of the initial contours, which results from the mass-transfer from the secondary to the WD and the mass-loss from the system. Note that, the enclosed region almost vanishes for $M_{\text{WD}}^i = 1.0 M_\odot$, which is then assumed to be the minimum WD mass for producing SNe Ia from this channel. If the initial parameters of a WD + RG system are located in the initial contours, an SN Ia is then assumed
Fig. 1 Example of binary evolution calculations. In panel (a), the solid and dash-dotted curves show $M_2$ and $M_{WD}$ varying with time, respectively. The open circles represent the evolution of $M_{CO}$ during outbursts. In panel (b), the evolutionary track of the donor star is shown as a solid curve and the evolution of the orbital period is shown as a dash-dotted curve. Dotted vertical lines in both panels and asterisks in panel (b) indicate the position where the WD is expected to explode as an SN Ia. The initial binary parameters and the parameters at the moment of the SN Ia explosion are also given in these two panels.
Fig. 2 Parameter regions producing SNe Ia in the orbital period–secondary mass plane ($\log P_i, M_{i2}$) for the WD + RG systems with the duty cycle $d = 0.01$. The initial WD masses are 1.0, 1.1 and 1.2 $M_\odot$. The final state of the WD + RG systems in the plane is encircled by the solid line, where filled stars, circles and triangles denote that the WD explodes as an SN Ia with initial WD masses of 1.0, 1.1 and 1.2 $M_\odot$, respectively.

to be produced. These initial contours for the WD + RG channel can be expediently used in the binary population synthesis studies. The data points and the interpolation FORTRAN code for these contours can be supplied on request by contacting BW.

4 DISCUSSION

Compared with most of the previous investigations on the WD + RG channel of SNe Ia, the main difference is that we considered the effect of the accretion disk instability on the evolution of WD binaries. In our results, there is no WD + RG system with a period as long as $\sim 10^3$ d as indicated in Li & van den Heuvel (1997) and Hachisu et al. (1999a). This is because if the initial period of a WD binary is too long, the mass-transfer rate between the RG donor star and the WD will be too high and the optically thick wind will blow much of the H-rich material away from the binary system. Finally, the RG donor star does not have enough material that can be accumulated onto the WD.

Similar to previous studies (e.g. King et al. 2003; Xu & Li 2009), our study also indicates that the long-period dwarf novae offer possible ways for producing SNe Ia. In particular, there is an advantage for this work, i.e. the SN Ia explosion always occurs in a WD binary of small secondary/primary mass ratio ($<1$), and with very little of the H envelope of the secondary remaining in our investigations. This feature will greatly reduce the possibility of H contamination of the SN Ia ejecta (see also King et al. 2003), which is consistent with the defining characteristic of most SNe Ia having no detectable H (Branch et al. 1995). This also has broad implications towards the work that is currently placing very tight limits on the amount of H entrained in the SN Ia ejecta, determined through the analysis of late-time spectra (e.g. Mattila et al. 2005; Leonard 2007).

However, the results in this paper depend on many uncertain input parameters, in particular for the duty cycle which is poorly known. The main uncertainties lie in the facts that it varies from one
binary system to another and may evolve with the orbital periods and mass-transfer rates (e.g. Lasota 2001; Xu & Li 2009). This is the reason why we choose an intermediate value (0.01) rather than other extreme values (e.g. 0.1 or $10^{-3}$). Furthermore, we also did some tests for a higher or lower value of the duty cycle. In Figure 3, we show the influence of the variation of the duty cycle value on the regions for producing SNe Ia with initial WD mass of $1.2 \, M_\odot$. We see that, for a high value (0.1), the right boundaries of the regions will be shifted to higher period, while, for a low value ($10^{-3}$), the right boundaries of the regions will be shifted to lower period. For the low value of the duty cycle, it will have a high mass-accretion rate of WDs during outbursts, so that the accreting WDs will lose too much mass via the optically thick wind, preventing them from increasing their masses to the Chandrasekhar mass. Thus, a low value of the duty cycle will reduce the size of regions for producing SNe Ia.

In this paper, we set the metallicity $Z = 0.02$. For the WD + RG channel, varying the metallicity would have strong influence on the regions for producing SNe Ia (Fig. 2), e.g. high metallicity leads to larger radii of zero-age MS (ZAMS) stars, then the left boundaries of the regions will be shifted to longer period. Meanwhile, stars with high metallicity evolve in a way similar to those with low metallicity but less mass (Umeda et al. 1999; Chen & Tout 2007; Meng et al. 2009). Thus, for the WD binary systems with particular orbital periods, the companion mass increases with metallicity.

At present, the existence of a population of single low-mass ($<0.45 \, M_\odot$) WDs (LMWDs) is supported by some observations (e.g. Maxted et al. 2000; Kilic et al. 2007). The formation of single LMWDs is still unclear. It is suggested that single LMWDs could result from single old metal-rich stars which experience severe mass loss prior to the He flash (Kalirai et al. 2007; Kilic et al. 2007). However, the study of the initial-final mass relation for stars by Han et al. (1994) implies that only LMWDs with masses $\gtrsim 0.4 \, M_\odot$ might be produced through such a single-star channel, even at high metallicity (Menget al. 2008). Thus, it would be difficult to conclude that single stars can produce the LMWDs.

The companion in the WD + RG channel would survive and finally evolve to a WD. In Figure 2, we see that the companion stars have a low mass at the moment of the SN explosion. The companion stars will be stripped of some mass due to the impact of SN ejecta. Marietta et al. (2000) presented
several high-resolution two-dimensional numerical simulations of the impact of SN Ia explosions with companions. They find that a RG donor star will lose almost its entire envelope (96%–98%) owing to the impact of the SN Ia explosion and leave only the core of the star. Thus, the surviving companion stars from this channel will have a relatively low mass after the SN explosion and finally evolve to a WD, which provides a possible pathway for the formation of the population of single LMWDs ($<0.45M_\odot$). Meanwhile, we also suggest that the observed, apparently single LMWDs may provide evidence that at least some SN Ia explosions have occurred with non-degenerate donor stars (such as RG donor stars).

The CO WD + RG systems can be formed by binary evolution. Wang, Li & Han (2009) find that there is one channel which can form CO WD + RG systems and then produce SNe Ia. In the detailed binary evolution procedure, the primordial primary first fills its Roche lobe at the thermal pulsing asymptotic giant branch stage. A common envelope is then easily formed owing to dynamically unstable mass-transfer during the RLOF stage. After the common envelope ejection, the primordial primary becomes a CO WD, then a CO WD + MS system is produced. The MS companion star continues to evolve until the RG stage, i.e. a CO WD + RG system is formed. For the CO WD + RG systems, SN Ia explosions occur for the ranges $M_{1,i} \sim 5.0-6.5M_\odot$, $M_{2,i} \sim 1.0-1.5M_\odot$, and $P_i \gtrsim 1500d$, where $M_{1,i}$, $M_{2,i}$ and $P_i$ are the initial mass of the primary and the secondary at ZAMS, and the initial orbital period of a binary system.

The WD + RG channel has a long delay time from the star formation to SN explosion due to the RG donor star with low initial masses ($<\sim 1.5M_\odot$). Thus, this channel can contribute to the old population of SNe Ia implied by recent observations (Mannucci et al. 2006; Totani et al. 2008; Schawinski 2009). The old population of SNe Ia may have an effect on models of galactic chemical evolution, since they would return large amounts of iron to the interstellar medium much later than previously thought. It may also have an impact on cosmology, as they are used as cosmological distance indicators.

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