PWIA Extraction of the Neutron Magnetic Form Factor from Quasi-Elastic $^3\text{He}(e,e')$

A. Saha, G. Here, we report the extraction of

A dependent asymmetry

$A_{20}$

the squared four-momentum transfer,

$Q^2$

the neutron magnetic form factor.

A high precision measurement of the transverse spindependent asymmetry $A_T^\perp$ in $^3\text{He}(e,e')$ quasielastic scattering was performed in Hall A at Jefferson Lab at values of the squared four-momentum transfer, $Q^2$, between 0.1 and 0.6 (GeV/c)$^2$. $A_T^\perp$ is sensitive to the neutron magnetic form factor, $G_M^n$. Values of $G_M^n$ at $Q^2 = 0.1$ and 0.2 (GeV/c)$^2$, extracted using Faddeev calculations, were reported previously. Here, we report the extraction of $G_M^n$ for the remaining $Q^2$-values in the range from 0.3 to 0.6 (GeV/c)$^2$ using a Plane-Wave Impulse Approximation calculation. The results are in good agreement with recent precision data from experiments using a deuterium target.

The electromagnetic form factors of the nucleon have been a longstanding subject of interest in nuclear and particle physics. They describe the distribution of charge and magnetization within nucleons and allow tests of nucleon models based on Quantum Chromodynamics. Precise knowledge of the form factors advances our understanding of nucleon structure.

The proton electromagnetic form factors have been determined with good precision at low values of the squared four-momentum transfer, $Q^2$, while the neutron form factors are known with much poorer precision because of the lack of free neutron targets. Over the past decade, with the advent of high-quality polarized beams and targets, the precise determination of both the neutron electric...
form factor, \(G_E^n\), and the magnetic form factor, \(G_M^n\), has become a focus of experimental activity. Considerable attention has been devoted to the precise measurement of \(G_M^n\). While knowledge of \(G_M^n\) is interesting in itself, it is also required for the determination of \(G_E^n\), which is usually measured via the ratio \(G_E^n/G_M^n\). Furthermore, precise data for the nucleon electromagnetic form factors are essential for the analysis of parity violation experiments designed to probe the strangeness content of the nucleon.

Until recently, most data on \(G_M^n\) had been deduced from elastic and quasi-elastic electron-deuteron scattering. For inclusive measurements, this procedure requires the separation of the longitudinal and transverse cross sections and the subsequent subtraction of a large proton contribution. Thus, it suffers from large theoretical uncertainties due in part to the deuteron model employed and in part to corrections for final-state interactions (FSI) and meson-exchange currents (MEC). These complications can largely be avoided if one measures the spin contribution to the asymmetry data [1,2] with uncertainties of less than 2% at \(Q^2\) below 1 (GeV/c)\(^2\). Despite the high precision reported, however, there is considerable disagreement among some of the experiments with respect to the absolute value of \(G_M^n\). The most recent deuteron data further emphasize this discrepancy.

Thus, additional data on \(G_M^n\), preferably obtained using a complementary method, are highly desirable. Inclusive quasi-elastic \(^3\)He\((e',e' p)\) scattering provides just such an alternative approach. In comparison to deuteron experiments, this technique employs a different target and relies on polarization degrees of freedom. It is thus subject to completely different systematics. As demonstrated recently, a precision comparable to that of deuteron ratio experiments can be achieved with the \(^3\)He technique.

The sensitivity of spin-dependent \(^3\)He\((e',e')\) scattering to neutron structure originates from the cancellation of the proton spins in the dominant spatially symmetric \(S\) wave of the \(^3\)He ground state. As a result of this cancellation, the spin of the \(^3\)He nucleus is predominantly carried by the unpaired neutron alone. Hence, the spin-dependent contributions to the \(^3\)He\((e',e')\) cross section are expected to be sensitive to neutron properties. Formally, the spin-dependent part of the inclusive cross section is contained in two nuclear response functions, a transverse response \(R_T\) and a longitudinal-transverse response \(R_{TL'}\), which occur in addition to the \(R_L\) and \(R_T\) responses. \(R_{TL'}\) can be isolated experimentally by forming the spin-dependent asymmetry \(A\) defined as \(A = (\sigma_{h^+} - \sigma_{h^-})/(\sigma_{h^+} + \sigma_{h^-})\), where \(\sigma_{h^\pm}\) denotes the cross section for the two different helicities of the polarized electrons. In terms of the nuclear response functions, \(A\) can be written

\[
A = \frac{-(\cos \theta^* \nu_T R_T + 2 \sin \theta^* \cos \phi^* \nu_{TL'} R_{TL'})}{\nu_L R_L + \nu_T R_T}
\]

where the \(\nu_k\) are kinematic factors and \(\theta^*\) and \(\phi^*\) are the polar and azimuthal angles of target spin with respect to the 3-momentum transfer vector \(q\). The response functions \(R_k\) depend on \(Q^2\) and the electron energy transfer \(\omega\). By choosing \(\theta^* = 0\), i.e. by orienting the target spin parallel to the momentum transfer \(q\), one selects the transverse asymmetry \(A_T\), proportional to \(R_T\). Various detailed calculations have confirmed that \(R_T\), and thus \(A_T\), is strongly sensitive to \(G_M^n\)\(^2\).

The experiment was carried out in Hall A at the Thomas Jefferson National Accelerator Facility (JLab), using a longitudinally polarized continuous-wave electron beam incident on a high-pressure polarized \(^3\)He gas target. Six kinematic points were measured corresponding to \(Q^2\) = 0.1 to 0.6 (GeV/c)\(^2\) in steps of 0.1 (GeV/c)\(^2\). An incident electron beam energy, \(E_e\), of 0.778 GeV was employed for the two lowest \(Q^2\) values of the experiment, while the remaining points were obtained at \(E_e = 1.727\) GeV. The spectrometer settings of the six quasielastic kinematics are listed in Table I. To maximize the sensitivity to \(A_T\), the target spin was oriented at 62.5° to the right of the incident electron momentum direction. This corresponds to \(\theta^*\) from –8.5° to 6°, resulting in a contribution to the asymmetry due to \(R_{TL'}\) of less than 2% at all kinematic settings, as determined from plane-wave impulse approximation (PWIA) calculations. Further experimental details can be found in references [9,19,20].

| \(Q^2 (\text{GeV/c})^2\) | \(E (\text{GeV})\) | \(E' (\text{GeV})\) | \(\theta (\text{degree})\) |
|------------------------|-------------|-------------|----------------|
| 0.10                   | 0.778      | 0.717      | 24.44          |
| 0.193                  | 0.778      | 0.667      | 35.50          |
| 0.30                   | 1.727      | 1.559      | 19.21          |
| 0.40                   | 1.727      | 1.506      | 22.62          |
| 0.50                   | 1.727      | 1.453      | 25.80          |
| 0.60                   | 1.727      | 1.399      | 28.85          |

Table I. The spectrometer settings for the six quasielastic kinematics of the experiment, where \(E\) is the incident electron beam energy, \(E'\) and \(\theta\) are the spectrometer central momentum and scattering angle settings, respectively.

Results for \(A_T\) (Fig. 3) as a function of \(\omega\) for all six kinematics of this experiment together with the extracted \(G_M^n\) values at the two lowest \(Q^2\) kinematics of the experiment were reported previously. A state-of-the-art non-relativistic Faddeev calculation was employed in the extraction of \(G_M^n\) at these two \(Q^2\) kinematics. As discussed in this paper, this calculation, while very accurate at low \(Q^2\), is not believed to be sufficiently precise for a reliable extraction of \(G_M^n\) from the \(^3\)He asymmetry data at higher \(Q^2\) because of its non-relativistic nature. Thus, it was not used to extract \(G_M^n\) for \(Q^2\) values of 0.3 and 0.4.
FIG. 1. The transverse asymmetry $A_{T'}$ near the peak of quasielastic scattering at the six kinematics of the experiment. The data are shown with statistic uncertainties and the experimental systematic uncertainties are shown as dark bands. Also shown are the quasielastic peak positions. The dashed curve is the PWIA calculation which includes the FSI and MEC effects and FSI effect only, respectively. The solid and the dash-dotted curves are the Faddeev calculation which includes the FSI and MEC effects and FSI effect from the same experiment at the breakup region from the same experiment [22] at $Q^2$ values of 0.1 and 0.2 (GeV/c)$^2$. Thus, a fully relativistic three-body calculation is highly desirable for a reliable extraction of $G_M^n$ at higher values of $Q^2$. Unfortunately, such a calculation is not available and difficult to perform due to present time.

On the other hand, the size of FSI and MEC corrections to inclusive scattering data is well known to diminish with increasing momentum transfer, and so PWIA will likely describe the data well at higher $Q^2$. Indeed as shown in Fig. 1, the PWIA calculation provides an excellent description of the data at $Q^2$ values of 0.5 and 0.6 (GeV/c)$^2$. In light of this, we felt it was reasonable to extract $G_M^n$ from our asymmetry data using PWIA. In order to estimate the model uncertainty of this procedure, we used results from the full Faddeev calculation up to a $Q^2$ value of 0.4 (GeV/c)$^2$ to study quantitatively the size and $Q^2$-dependence of FSI and MEC corrections.

A recent PWIA calculation [14] which takes into account the relativistic kinematics and current using the AV18 NN interaction potential and the Höhler nucleon form factor parameterization [23] (for the proton form factors and $G_E^n$) was used for the extraction of $G_M^n$ at $Q^2 \geq 0.3$ (GeV/c)$^2$. In this calculation, the struck nucleon is described by a plane wave, and the interaction between the nucleons in the spectator pair is treated exactly by including the NN and the Coulomb interaction between the pp pair. The de Forest CC1 off-shell prescription [24] was adopted for the electron-nucleon cross section. Furthermore, the Urbana IX three-body forces [25] were included in the $^3$He bound state. To extract $G_M^n$, measured transverse asymmetry data from a 30 MeV region around the quasielastic peak were used. The PWIA calculation [14] was employed to generate $A_{T'}$ as a function of $G_M^n$ in the same 30 MeV-wide $\omega$ region. In doing so, spectrometer acceptance effects were taken into account. By comparing the measured asymmetries with the PWIA predictions, $G_M^n$ values could be extracted.

Results for $G_M^n$ were obtained in two ways: (a) by taking the weighted average of $A_{T'}$ from three neighboring 10 MeV bins around the quasi-elastic peak (30 MeV total for $\omega$) and then extracting $G_M^n$ from this average asymmetry, and (b) by first extracting $G_M^n$ from each these 10 MeV bins separately and then taking the weighted average of the resulting $G_M^n$ values. Both methods yield essentially the same results (within 0.1%).

The systematic uncertainty in $G_M^n$ is almost entirely due to the systematic error from the determination of the beam and target polarizations, which is 1.7% in $A_{T'}$ and 0.85% in $\delta G_M^n/G_M^n$. Such a high precision in the determination of beam and target polarizations can be achieved by using elastic polarimetry [3]. An additional systematic error occurs in the extraction of $G_M^n$ due to the experimental uncertainty in the determination of the energy transfer $\omega$. The uncertainty due to this source is 1.4% at $Q^2 = 0.3$ and becomes negligible (< 0.5%) at the higher $Q^2$ points.

The model uncertainty inherent in the extraction procedure depends on the various ingredients of the calculation, such as the NN potential, the proton nucleon form factors, relativity, and the reaction mechanism, including FSI and MEC. The main processes neglected in PWIA are FSI and MEC; therefore, these two contributions are expected to dominate the overall model uncertainty. As mentioned, we used results from the non-relativistic Faddeev calculation carried out up to a $Q^2$ value of 0.4 (GeV/c)$^2$ to estimate the uncertainties resulting from the
Based on the Faddeev calculation [17], we find that MEC decrease exponentially as $Q^2$ increases. Similar conclusions have been drawn from studies of the quasi-elastic $d(\vec{e},e')$ process [26]. We estimate the uncertainty due to the neglect of the MEC effect in PWIA for $A_{T'}$ on top of the quasi-elastic peak to be 3.6%, 2.4%, 1.0%, and 1.0% for $Q^2$ of 0.3, 0.4, 0.5, and 0.6 (GeV/c)$^2$, respectively.

The effect of various different off-shell prescriptions [27] was studied in the framework of the PWIA calculation, and the contribution to the uncertainty of extracting $G_M^n$ from $A_{T'}$ was found to be negligible. Difference in $G_M^n$ arising from different choices of NN potential and other nucleon form factor parametrizations was found to be about 1%.

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Results for $G_M^n$ extracted at $Q^2 = 0.3$ to 0.6 (GeV/c)$^2$ using the PWIA calculation are presented in Table I along with statistical, systematic, and model uncertainties. The model uncertainties are obtained based on studies described previously, which may represent the lower limits only. The results are plotted in Fig. 2 along with the previously reported $G_M^n$ results [12] at $Q^2 = 0.1$ and 0.2 (GeV/c)$^2$, which were extracted using the Faddeev calculation. All other published results since 1990 are also shown. The error bars shown on our data are the quadrature sum of the statistic and systematic uncertainties reported in Table I, which do not include the estimated model uncertainty.

While limitations exist in our approach, we note that our results are in very good agreement with the recent deuteron ratio measurements from Mainz [10], and in disagreement with results by Bruins et al. [11].

In conclusion, we have measured the spin-dependent asymmetry $A_{T'}$ in the quasi-elastic $^3$He$(\vec{e},e')$ process with high precision at $Q^2$-values from 0.1 to 0.6 (GeV/c)$^2$. In this Rapid Communication, we report the extraction of $G_M^n$ at $Q^2$ values of 0.3 to 0.6 (GeV/c)$^2$ based on PWIA calculations, which are expected to be reasonably reliable in our range of $Q^2$. We estimate the total uncertainty of our results to be about 4-6%. A more precise extraction of $G_M^n$ at these $Q^2$ values requires a fully relativistic three-body calculation, which is unavailable at present. Efforts are underway to extend the theory into this regime [3].

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