Shock-induced ejecta from a layer of spherical particles. Part II: modeling with the non-equilibrium two-phase model of a granular medium

A A Serezhkin\textsuperscript{1}, I S Menshov\textsuperscript{1,2}, M S Egorova\textsuperscript{1}, S A Dyachkov\textsuperscript{1},
A N Parshikov\textsuperscript{1}, V V Zhakhovsky\textsuperscript{1}, D B Rogozkin\textsuperscript{1,3}, S E Kuratov\textsuperscript{1}

\textsuperscript{1} Dukhov Research Institute of Automatics (FSUE VNIIA), Moscow, 127055, Russian Federation
\textsuperscript{2} Keldysh Institute for Applied Mathematics RAS, Moscow 125581, Russian Federation
\textsuperscript{3} Moscow Engineering Physics Institute, National Research Nuclear University, Kashirskoe Shosse 31, 115409 Moscow, Russian Federation

aaserezhkin@gmail.com

Abstract. In this work we consider the problem of high-velocity impact of a solid impactor on the layer of spherical solid particles. The impact initiates a wave-type process of compaction of particles in the layer, and the ejection of small solid particles from the compacted layer surface to the air. The first part of the paper addresses modelling the process of particle compaction and the initial stage of solid ejection with the SPH method. In the second part presented below, we consider the process of compaction and solid mass ejection using a new non-equilibrium two-phase model of the granular medium. This model makes it possible to describe mechanics of two-phase gas-solid medium in a wide range of the phase volume fraction, from regimes of isolated particle flows to regimes of dense packed particles.

Keywords: two-phase heterogeneous medium, numerical simulation, PDV.

1. Introduction

Process of high-velocity impact on a layer of spherical particles can be separated on two stages: the first stage is the compaction of particles and the second one is ejection of particles or fragments of particles from the free surface of the layer. Provided that we use continual approach, the choice of the physical model is of the first importance.

On the one hand, to simulate the compaction of particles, we need a model that should take into account mechanical interaction between the particles, plastic deformation, non-equilibrium in stress in the particles and in the gas filling the volume between the particles. The most appropriate model for this case is the 7-equation Baer-Nunziato model \cite{1}. This model is based on the continuum theory of mixtures. Either the solid phase or the gas phase is considered as a continuum with proper physical and mechanical properties. This model works well for the regimes of flow when the volume fraction
of the solid phase is greater than a critical value $\varphi_s^*$ that corresponds to the volume fraction of the close-bed structure of the medium.

On the other hand, the simulation of the ejection stage is best fitted by the 5-equations model of a cloud of particles [2]. The solid volume fraction is assumed to be less then a critical value of $\varphi_s^*$. This model works well for the regimes of flow when solid particles are dispersive and do not interact with each other.

These two models form two classes of models of two-phase heterogeneous mixture mechanics. Models of one class differ by the closing relations and the model control parameters. If we want to simulate the both processes (compaction and ejection) we need to combine these two models in one numerical code, or develop a new model that could be capable to simulate the both regimes of flow in a unique way.

In the current work we use the second approach. We construct a new model for two-phase continuum dynamics that smoothly switches between the 5-equations model and the 7-equations model when the solid volume fraction crosses over the critical value $\varphi_s^*$.

One purpose of this work is interpretation of PDV measurements of disperse phase velocity in experiments [3], [4], [5]. The experimental data shows that the interaction of the compaction wave with the layer surface initiates ejection of a cloud of particles to the surrounding air. A simple analysis of velocity slowing-down given in [3] has shown that a typical diameter of particles in the cloud do not correspond to the diameter of particles in the layer.

In the first part of the present paper, the impact problem on a layer of particles is simulated with the SPH method. The results of SPH simulations show that the interaction of the compaction wave with the layer surface initiates ejection in the form of cumulative jets that are broken down in process. Jet fragments form a kind of fraction of solid particles that move with a velocity greater than the velocity of particles in the layer. Such a the velocity of the fragments is measured in experiments.

Simulating in framework of the continual model, one can not describe the formation of cumulative jets because this process occurs at the mesoscale level. Therefore, for simulation of the interaction of the compaction wave with the layer surface we use the results of SPH simulations as initial data.

Results of the SPH simulation show that the mass ejecting from the layer surface depends on the distribution of particles in the layer. Therefore, these results do not provide reliable data for the volume fraction in the jet domain because the initial arrangement of particles in the layer is chaotic. Nevertheless, the ejecting mass is small enough, so that we can use the drag coefficient for a single spherical particle for the simulation of slowing-down of fragments.

Along with simulating experiments with gold particles [3] we validate the proposed model in a similar problem of the impact with a smaller impact velocity on nickel particles [5]. Under these conditions, no cumulative jets appear, and therefore this experiment can be simulated by using the continuum model.

2. A wide-range model for two-phase gas-solid flows

The derivation of the system of governing equation is carried out with the continual approach proposed in [2]. We consider a small Euler volume that consists of two parts. The first part corresponds the solid phase (particles), the second one is the volume filled in gas. The ratio of the phase volume the whole one represents volume fractions ($\varphi_S^$, $\varphi_G^$). Here and after, indexes S and G indicate solid and gas phases respectively. Each phase is described by distributions of density ($\rho_S$, $\rho_G$), velocity ($u_S$, $u_G$), internal energy ($e_S$, $e_G$) and pressure ($P_S$, $P_G$). The evolution in time of conservative variables as mass, momentum and total energy ($E_S = 0.5u_S^2 + e_S$, $E_G = 0.5u_G^2 + e_G$) in the whole volume is defined by corresponding conservation laws written for corresponding sub-domains with taking into account change the volume fractions.

The system of governing equations that describes the evolution of conservative variables can be written in following form:
\[
\begin{align*}
\frac{\partial}{\partial t} \varphi_s \rho_s + \nabla (\varphi_s \rho_s u_s) &= 0 \\
\frac{\partial}{\partial t} \varphi_s \rho_s u_s + \nabla (\varphi_s \rho_s u_s) + \nabla (\varphi_s P_s) &= P_I \nabla \varphi_s + K (u_s - u_G) \\
\frac{\partial}{\partial t} \varphi_s \rho_s E_s + \nabla (\varphi_s \rho_s E_s u_s) + \nabla (\varphi_s P_s u_s) &= P_I u_s \nabla \varphi_s + A_{\text{def}} + K u_I (u_s - u_G) \\
\frac{\partial}{\partial t} \varphi_G \rho_G + \nabla (\varphi_G \rho_G u_G) &= 0 \\
\frac{\partial}{\partial t} \varphi_G \rho_G u_G + \nabla (\varphi_G \rho_G u_G) + \nabla (\varphi_G P_G) &= -P_I \nabla \varphi_s - K (u_s - u_G) \\
\frac{\partial}{\partial t} \varphi_G \rho_G E_G + \nabla (\varphi_G \rho_G E_G u_G) + \nabla (\varphi_G P_G u_G) &= -P_I u_s \nabla \varphi_s - A_{\text{def}} - K u_I (u_s - u_G)
\end{align*}
\] (1)

where \(P_I\) and \(u_I\) are the average pressure and the average velocity on the interphase boundary, \(K\) is the drag coefficient \([2]\), \(A_{\text{def}}\) is the work done for change the volume fraction of phases in the small Euler volume. The latter we determine in the following form:

\[
A_{\text{def}} = -P_I \frac{d \varphi_s}{d t} + \lambda_S \nabla u_S + \lambda_G \nabla u_G
\]

where \(\lambda_S\) and \(\lambda_G\) are parameters of model to be defined. The system of equations (1) is not closed. One need to define yet the following values: \(\varphi_s, \varphi_G, P_s, P_G, P_I, u_I, \lambda_S\) and \(\lambda_G\).

First of all to do this, we use equations of state for each phase \(e_s = e_s(\rho_s, P_s)\) and \(e_G = e_G(\rho_G, P_G)\), and a condition \(\varphi_s + \varphi_G = 1\). Then we employ the total entropy inequality \([6]\):

\[
\varphi_s \rho_s \frac{d S_s}{d t} + \varphi_G \rho_G \frac{d S_G}{d t} \geq 0
\]

that must be valid for all regimes of flow. Using first the law of thermodynamics, equations of state, and the entropy definition we can the equation for the entropy evolution:

\[
\begin{align*}
\varphi_s \rho_s \frac{d S_s}{d t} + \varphi_G \rho_G \frac{d S_G}{d t} &= \varphi_s \rho_s \frac{d}{d t} \left( \varphi_s \rho_s \frac{d S_s}{d t} + (\varphi_s \rho_s c_s^2 + \lambda_s A) \nabla u_s \left( \frac{P_s - P_I}{T_s} - \frac{P_G - P_I}{T_G} \right) \right) \\
&+ \varphi_G \rho_G \frac{d}{d t} \left( \varphi_G \rho_G \frac{d S_G}{d t} + (\varphi_G \rho_G c_G^2 + \lambda_G A) \nabla u_G \left( \frac{P_G - P_I}{T_G} \right) \right) \\
&- \frac{\varphi_s \rho_s c_s^2}{Z} \left( P_s - P_I \right) - \frac{\varphi_G \rho_G c_G^2}{Z} \left( \frac{P_G - P_I}{T_G} \right) \\
&+ K (u_s - u_G) \left( \frac{\varphi_s \Gamma_s (u_I - u_s) + \varphi_G \Gamma_G (u_I - u_G)}{Z} \right) \left( \frac{P_s - P_I}{T_s} - \frac{P_G - P_I}{T_G} \right) + \left( \frac{u_I - u_s}{T_s} - \frac{u_I - u_G}{T_G} \right)
\end{align*}
\] (2)
where

\[ A = \left( T_G - T_S \right) \left( \varphi_G \rho_S C_S^2 + \varphi_S \rho_G C_G^2 \right) + \left( \varphi_G \Gamma_S T_S + \varphi_S \Gamma_G T_G \right) \left( P_S - P_G \right) / \varphi_S \varphi_G \left( \left( P_S - P_I \right) T_G - \left( P_G - P_I \right) T_S \right) \]

\( T_S \) and \( T_G \) are the temperatures of phases,

\[ Z = \varphi_G \rho_S C_S^2 + \varphi_S \rho_G C_G^2, \quad C_S^2 = \left( \frac{P_S - \bar{\varepsilon}_S}{\rho_S} \right) \frac{\bar{\varepsilon}_S}{\partial P_S}^{-1}, \quad C_G^2 = \left( \frac{P_I - \bar{\varepsilon}_G}{\rho_G} \right) \frac{\bar{\varepsilon}_G}{\partial P_G}^{-1} \]

Note, that in above formulas \( C_S \) and \( C_G \) are the standard thermodynamic sounds velocities of the solid and gas phase. \( C_{SI} \) and \( C_{GI} \) are effective sound velocities at the interphase as proposed in [7]. In general \( C_{SI}^2 \) or \( C_{GI}^2 \) might be negative. In this case vacuum vapors arise nearby boundary. This situation is out of scope of the present study. \( \Gamma_S \) and \( \Gamma_G \) are Gruneisen coefficients of the phases.

To guarantee the entropy inequality to be valid, we suggest the following relations considered as additional closure equations:

\[ \frac{d_P}{dt} C^2 + \left( \rho_G C_G^2 + \lambda_G A \right) \bar{\varepsilon}_G = - \frac{P_G - P_I}{\tau T_G} \]  \hspace{1cm} (3)

\[ \frac{d_P}{dt} C^2 + \left( \rho_S C_S^2 - \lambda_S A \right) \bar{\varepsilon}_S = - \frac{P_S - P_I}{\tau T_S} \]  \hspace{1cm} (4)

\[ u_I = \frac{\varphi_G \Gamma_S u_S + \varphi_S \Gamma_G u_G}{\varphi_G \Gamma_S + \varphi_S \Gamma_G} \]  \hspace{1cm} (5)

where \( \tau \) - is the pressure time relaxation in computations it could be takes equal the time of shock wave passing the distance of several particle size, \( \tau = nd_p C_s^{-1} \). Providing (3)-(5), equation (2) can be rewritten as:

\[ \varphi_S \rho_S \frac{d_S}{dt} + \varphi_G \rho_G \frac{d_G}{dt} = \frac{\left( P_S - P_I \right) T_S - \left( P_G - P_I \right) T_G}{\tau Z} \]  \hspace{1cm} (6)

Due Gruneisen coefficient and temperature are positive, first two terms in the right-hand side of the equation (6) are positive. Therefore to ensure the entropy inequality for every thermodynamic process possible in the considered two-phase mixture, the last term must be vanish. This provides a quadratic equation for \( P_I \):
\[ P_i^2(\varphi_G T_s G + \varphi_S T_s G) - P_i \left( P_s + \rho_G^2 \frac{\partial \varphi_G}{\partial \rho_G} \right) \varphi_3 T_s G = P_s + \rho_S^2 \frac{\partial \varphi_S}{\partial \rho_S} \varphi_0 T_s G + \left( P_s + \rho_S^2 \frac{\partial \varphi_S}{\partial \rho_S} \right) \varphi_0 T_s G \]  

(7) 

One can prove that this equation always has one solution on an interval \([\min(P_s, P_G), \max(P_s, P_G)]\). If \( P_i = P_G \), the left hand side of equation (7) equals to \( \varphi_S \rho_G C_G^2 \left( \frac{P_G - P_s}{T_s} \right) \), and has the same sign as \( (P_s - P_G) \). If \( P_i = P_s \), the left hand side of equation (7) equals to \( \varphi_G \rho_s C_s^2 \left( \frac{P_G - P_s}{T_s} \right) \), and has the same sign as \( (P_G - P_s) \). This proves existing and uniqueness of the solution.

Using (3) - (5) and (7), we can get the equation for the volume fraction of the solid phase:

\[ \frac{d_s \varphi_s}{dt} = -\frac{\rho_s}{\tau_s} \left( \varphi_s \rho_s C_s^2 + \varphi_s \rho_s C_s^2 \right) + \frac{\varphi_s \rho_s (P_s - P_i)}{\tau_s} + \frac{\varphi_s \rho_s (P_G - P_s)}{\tau_s} \left( \frac{P_G - P_s}{T_s} \right) \]

(8)

Equation (8) is not a closing relation but it allows us to write (1) as a closed system of governing equations:

\[ \frac{\partial Q}{\partial t} + \frac{\partial F}{\partial x} + \nabla \cdot \overline{B} Q = H \]

(9)

where \( \overline{Q} \) is a vector of conservative variables \( \varphi_s \rho_s, \varphi_s \rho_s u_s, \varphi_s \rho_s E_s, \rho_s, \varphi_s \rho_s, \varphi_g \rho_g, \varphi_g \rho_g u_g, \varphi_g \rho_g E_g \); \( F \) is the vector of conservative fluxes; the matrix \( \overline{B} \) is the matrix of the non-conservative part; \( H \) is non-differential right-hand side.

Closing relations (3) - (5) and (7) are not enough to close the system (1) because there are undefined parameters \( \lambda_s \) and \( \lambda_G \). To estimate their values we study mathematical properties of equation (9). To correctly describe evolutionary process, equation (9) must be hyperbolic. Let's recast equation (9) in a non-conservative form as follows:

\[
\begin{pmatrix}
\rho_s & u_s \\
0 & u_s \\
0 & \rho_s C_s^2 - \lambda_s A \\
0 & -\rho_s \varphi_s W \lambda_s \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{pmatrix}
\begin{pmatrix}
\rho_s \\
\rho_s \\
\rho_s \\
\rho_s \\
\rho_s \\
\rho_s \\
\rho_s \\
\rho_s
\end{pmatrix}
= 
\begin{pmatrix}
-\rho_s B_s (u_G - u_s) \\
0 \\
\rho_s B_s (u_G - u_s) \\
0 \\
0 \\
0 \\
0 \\
0
\end{pmatrix}
\]

(10)
where $h$ is the non-differential right-hand side:

$$h = \begin{pmatrix}
0 \\
K(u_s - u_G)/(\varphi_S \rho_S) \\
-(P_s - P_G)/(\tau T_s) \\
\varphi_S \rho_G ((P_s - P_1)/(\tau T_s) - (P_G - P_1)/(\tau T_G))/Z \\
0 \\
-((u_s - u_G)/(\varphi_G \rho_G)) \\
-(P_G - P_1)/(\tau T_G)
\end{pmatrix}$$

$$W = Z(\varphi_G \Gamma_s + \varphi_S \Gamma_G - \varphi_S \varphi_G A), \quad B_S = Z\varphi_G \rho_S C^2_S, \quad B_G = Z\varphi_S \rho_G C^2_G.$$  

The matrix of this system has 7 eigenvalues that depend on $\lambda_S$ and $\lambda_G$. If $\lambda_S = \lambda_G = 0$, then the eigenvalues take the form:

$$\lambda_S = u_s; \quad \lambda_2 = B_su_s + B_G u_G; \quad \lambda_{3,4} = u_s \pm C_s; \quad \lambda_5 = u_G; \quad \lambda_{6,7} = u_G \pm C_G.$$  

In this case all eigenvalues are real and the governing system is hyperbolic. Also values of $\lambda_1, ..., \lambda_7$ show that small perturbations in the medium propagate with velocities of phases (or its linear combinations) and sound velocities. Therefore it can be assumed that this case corresponds to regimes of flow when solid directly interact each other. Particles forms the dense packed structure and the volume fraction of the solid phase is larger then the critical value, $\varphi_S \geq \varphi^*_S$.

If $\lambda_G = 0, \lambda_S A = \rho_S C^2_S$, then the eigenvalues are:

$$\lambda_{1,2} = u_s; \quad \lambda_{3,4} = \frac{1}{2}(u_s + B_s u_s + B_G u_G) \pm \frac{1}{2}\left((B_s(u_G - u_s))^2 + 4\varphi_G C^2_S (P_s - P_1) W/A \rho_S \right)^{1/2}$$  

$$\lambda_5 = u_G; \quad \lambda_{6,7} = u_G \pm C_G.$$  

In this case $\lambda_{1,2}$ might be complex, and hyperbolicity of the system violated. In order to overcome this difficulty we suggest to transfer to pressure equilibrium model by setting the pressure time relaxation parameter in (3) and (4), $\tau \to 0$. In this case $P_s - P_G \to 0$, and $\lim_{P_s - P_G \to 0} (W/A) = Z\rho_s \varphi_G$, the system (10) is hyperbolical. Small perturbations propagate in this case with mass velocities of phases (or its linear combinations) and with sound velocity of the gas phase. Therefore we can assume that this case corresponds to a structure with separated particles which forms a cloud of particles in gas. We suggest apply this case when the solid volume fraction satisfies an inequality of $\varphi_S < \varphi^*_S$.

3. Numerical method

For numerical simulations we use in-house code named TIS. The basic idea of the numerical method used in this code is splitting the governing equations in accordance with main physical processes [8]. By using this approach, solving the problem can be reduced to more simple sub-problems without loss of fidelity. The dynamical process in the two-phase granular medium can be thought of as the combination of two sub-processes. One is hydrodynamic where the medium behaves like a two-phase fluid without the relaxation or drag process between the phases. Another sub-process is considered as Lagrangian one. It goes on the interface between phases and includes processes of drag and pressure relaxation. It means that calculation of one time step can be split in two stages. This splitting is mathematically described by 2 systems of equations: for the hydrodynamic stage.
\[ \frac{\partial Q}{\partial t} + \frac{\partial F}{\partial x} + B \frac{\partial Q}{\partial x} = 0 \]  
\[ (11) \]

and for the Lagrangian stage:

\[ \frac{\partial Q}{\partial t} = H \]
\[ (12) \]

First, we solve non-conservative system of equations (11). For this we use a Godunov-type scheme with an approximate of Riemann solver HLLEM proposed in [9]. The method HLLEM is the extension of the HLL method [10] to non-conservative systems of the form (11). The solution is found on a moving eulerian grid. At the second stage system of ODEs (12) is solved with the Gear method for stiff systems.

4. Numerical results

4.1. Impact on a layer of nickel particles.

We consider a 0.35mm layer of nickel spherical particles with a diameter of 40 μm. The layer is posed on a 3 mm thickness copper plate. The process is initiated by an iron impactor of 10 mm thickness that runs over the copper plate with a velocity of 450m/s. The numerical domain is represented by 3 blocks that correspond to the impactor, the copper plate and the layer of particles. Each block is divided on 100 uniformly distributed numerical cells. Motion of the grid is defined by the velocity of block interfaces. Block boundaries correspond to positions of plate surfaces and boundary of the domain of solid particles. Maximal velocity of particles (km/s) it is shown in figure 1 vs time along with the experimental data [5].

![Figure 1](image)

Figure 1. Comparison numerical and experimental data for the impact problem over the layer of nickel particles.

The agreement between numerical and experimental results confirms correctness of the numerical model in description process of compaction and ejection. Small difference in the results in an initial time period of 2 mks can be explained by mesoscale effects of the compaction wave at the particle layer surface.
4.2. Impact on a layer of gold particles

We simulate the experiment of ejecta from surface of spherical particles layer [3]. We consider a 58.3 μm layer of gold spherical particles with a diameter of 9 μm. The layer is posed on a denal plate. Impact is initiated by plane-wave generator. Part I of this work addresses the simulation of this problem by using SPH method. It is shown that the interaction of the compaction wave with the layer surface initiates ejection of the solid phase in the form of mesoscale cumulative jets that breaks down into separate fragments. The velocity of fragments is measured in experiments [3]. Simulation in the continual approach do not give adequate results because do not take into account mesoscale effects. However we can use results of SPH simulations as initial data (figure 2 and 3) and calculate slowing-down of expansion of cumulative jet fragments in air. Results of SPH simulation do not gives exact value of volume fraction of solid phase in area of cumulative jets, because in SPH simulations periodical position of particles in layer is proposed, and volume fraction depends on structure of particles position. Nevertheless we can use these results as a first approximation.

![Figure 2. Results of SPH simulation: the solid phase distribution in the cumulative jets.](image1)

![Figure 3. Results of SPH simulation: distribution of solid phase velocity in the cumulative jet.](image2)

We approximate data of the distribution of volume fraction from results of SPH simulation using following equation:

$$\phi_S = 0.4e^{-5.4x\cdot10^{-2}x} - 25x + 0.0065$$  \hspace{1cm} (13)

The velocity distribution is nearly linear (figure 3). We assume that jet fragments spherically shaped. Moreover we need to define typical size of the fragments. We assume in the calculations two type of particles (fragments): of 1 μm diameter small particles - fragments of jets tips located at a distance of [50, 240] μm, and large 3 μm particles - fragments of the jet base, located at [0, 50] μm. The calculation domain consists of three blocks: [0, 50] μm that corresponds position of the particles cloud from the jet base, [50, 240] μm that corresponds the cloud of particles from the jets tips, and [240, 1000] μm that corresponds air. Each block is divided into 100 uniform cells. The grid motion is defined by velocities of particles at blocks boundaries.

In figure 4 we present the comparison of experiment and numerical data for the maximal velocity of particles. Black line corresponds experiments, red line corresponds numerical results, and blue lines velocities at block interfaces. In the beginning small particles move faster than large particles. Due drag effects in air slowing-down of small particles is more than that of large particles, so that at a time moment of 2.7 mks the velocities became equal.
Figure 4. Maximal velocity of particles (km/s) vs time (mks).
Comparison of experimental data and results of numerical simulations.

We can define the distribution of typical jet fragments more exactly by carrying out the following simulations. We assign any typical size of jet fragments and simulate expansion of only part of fragments cloud from the base of jets to any point in the middle of total cloud. In results we get curve of maximal velocity of particles. We are varying right boundary position of part of fragments cloud till tangent of the curves of maximal velocity of particles from experiments and from simulations. We can receive distribution of jets fragments typical size vs coordinate \( x \) or vs velocity of solid phase in jet. Results of particles size distributions definition are presented on figure 5 and 6.

Figure 5. Estimation of the fragments size.

Figure 6. Fragments size (μm) vs the ratio of particles velocity to velocity of impactor surface.
5. Conclusion

In this work we have considered the problem of high-velocity impact of a solid impactor on the layer of spherical solid particles using the continual two-phase non-equilibrium approach. We have designed a new model that makes it possible to describe in a unique way mechanics of a two-phase gas-solid medium in a wide range of the phase volume fraction, from the regimes of isolated particle flows to the regimes of dense packed particles.

A continual approach model is not capable to simulate of mesoscale effects of cumulative jet ejection in the case of very high-velocity impact. Therefore, for the problem of interpretation of experimental PDV measurements of velocity, we have used the results of SPH simulation published in the part I of the work as initial data.

References

[1] Baer M R and Nunziato J W 1986 A two-phase mixture theory for the deflagration-to-detonation transition (DDT) in reactive granular materials Int. J. Multiphase flow vol 12 6 pp 861-889

[2] Khomenko Y P, Ischenko A N and Kasimov V Z 1999 Mathematical modelling of interior ballistic processes in barrel systems (Novosibirsk: Publishing House of SB RAS) p 256

[3] Prudhomme G, Mercier P and Berthe L 2014 PDV experiments on shock-loaded particles J. of Physics: Conf. Series. vol 500 14 142027

[4] Bandurkin K V, Kamenev V G, et al 2015 Experimental (Laser-Heterodyne Method PDV) and Numerical Investigation of the Movement of the Dispersed Phase Physical-Chemical Kinetics in Gas Dynamics vol 16 4

[5] Fedorov A V, Mikhailov A L, et al 2014 Study of lead bheavior feachures at shock loading and further unloading. Zababakhin Scientific Talks Int. Conf.

[6] Nigmatulin R I 1991 Dynamics of multiphase media (Hemisphere Publ. Corp.) vol I

[7] Saurel R and Abgrall R 1999 A Multiphase Godunov Method for Compressible Multifluid and Multiphase Flows J. of Computational Physics 150 pp 425–467

[8] Menshov I and Serezhkin A 2013 Modelling non-equilibrium two-phase flow in elastic-plastic porous solids 11th World Congress on Computational Mechanics (WCCM XI) 5th European Conf. on Computational Mechanics (ECCM V) 6th European Conf. on Computational Fluid Dynamics (ECFD VI)

[9] Dumbser M and Balsara D 2016 A new efficient formulation of the HLLEM Riemann solver for general conservative and non-conservative hyperbolic systems J. of Computational Physics vol 304 pp 275 - 319

[10] Harten A, Lax P D and van Leer B 1983 On upstream differencing and Godunov-type schemes for hyper-bolic conservation laws SIAM Rev. 25(1) pp 35–61