Dynamical Behavior of dilaton in de Sitter space

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Abstract

We study the dynamical behavior of the dilaton in the background of three-dimensional Kerr-de Sitter space which is inspired from the low-energy string effective action. The perturbation analysis around the cosmological horizon of Kerr-de Sitter space reveals a mixing between the dilaton and other fields. Introducing a gauge (dilaton gauge), we can disentangle this mixing completely and obtain one decoupled dilaton equation. However it turns out that this belongs to the tachyon. The stability of de Sitter solution with $J = 0$ is discussed. Finally we compute the dilaton absorption cross section to extract information on the cosmological horizon of de Sitter space.
I. INTRODUCTION

Recently an accelerating universe has proposed to be a way to interpret the astronomical data of supernova [1–3]. The inflation is employed to solve the cosmological flatness and horizon puzzles arisen in the standard cosmology. Combining this observation with the need of inflation leads to that our universe approaches de Sitter geometries in both the infinite past and the infinite future [4–6]. Hence it is very important to study the nature of de Sitter (dS) space and the dS/CFT correspondence [7]. However, there exist difficulties in studying de Sitter space. First there is no spatial infinity and global timelike Killing vector. Thus it is not easy to define the conserved quantities including mass, charge and angular momentum appeared in asymptotically de Sitter space. Second the dS solution is absent from string theories and thus we do not have a definite example to test the dS/CFT correspondence. Finally it is hard to define the $S$-matrix because of the presence of the cosmological horizon.

Accordingly most of works on de Sitter space were concentrated on the massive scalar propagation and its quantization [8–11]. Also the bulk-boundary relation for the scalar was introduced to study the dS/CFT correspondence [12]. Hence it is important to find a model which can accommodate the de Sitter space solution. In this work we introduce an interesting model which is motivated from the low-energy string action in (2+1)-dimensions [13]. This model includes a nontrivial scalar so-called the dilaton $\phi$. Actually we will use the dilaton to investigate the nature of the cosmological horizon in de Sitter space.

It is known that the cosmological horizon is very similar to the event horizon in the sense that one can define its thermodynamic quantities using the same way as is done for the black hole. Two important quantities to understand the black hole are the Bekenstein-Hawking entropy and the absorption cross section (greybody factor). The former specifies intrinsic property of the black hole itself, while the latter relates to the effect of spacetime curvature surrounding the black hole. We emphasized here the greybody factor for the black hole arises as a consequence of scattering off the gravitational potential surrounding the event horizon [14]. For example, the low-energy $s$-wave greybody factor for a massless scalar has a universality such that it is equal to the area of the horizon for all spherically symmetric black holes [17]. This can be obtained by solving the wave equation explicitly. The entropy for the cosmological horizon was first discussed in literature [18]. However, there exist a few of the wave equation approaches to find the greybody factor for the cosmological horizon [11]. A similar work for the four-dimensional Schwarzschild-de Sitter black hole appeared in [19] but it focused mainly on obtaining the temperature of the eternal black hole. Also the absorption rate for the four-dimensional Kerr-de Sitter black hole was discussed in [20].

In this paper we compute the absorption cross section of the dilaton in the background of three-dimensional de Sitter (dS$_3$) space with the cosmological horizon. For this purpose we first confine the wave equation only to the southern diamond where the time evolution of waves is properly defined even if this area does not include spatial infinity. And then we solve the wave equation to find the greybody factor in the low-energy and low-temperature limits.

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$^1$Previously we are interested in anti de Sitter black hole like the BTZ black hole [14]. In the BTZ black hole and the three-dimensional black string, the role of the dilaton was discussed in ref. [15].
The organization of this paper is as follows. In section II we briefly review our model inspired from the low-energy string action and its Kerr-de Sitter solution. We introduce the perturbation to study the cosmological horizon in the background of Kerr-de Sitter space in section III. In section IV we perform the potential analysis to check whether the de Sitter solution is or not stable. In section V we calculate the absorption cross section for $j = 1, 2$-angular modes of the dilaton explicitly. Finally we discuss our results in section VI.

II. KERR-DE SITTER SOLUTION

We start with the low-energy string action in string frame \[ \text{(1)} \]

$$S_{l-s} = \int d^3 x \sqrt{-g} e^\Phi \left\{ R + (\nabla \Phi)^2 + \frac{8}{k} \Phi - \frac{1}{12} H^2 \right\},$$

where $\Phi$ is the dilaton, $H_{\mu\nu\rho} = 3 \partial_{[\mu} B_{\nu\rho]}$ is the Kalb-Ramond field, and $k$ the cosmological constant. This action was widely used for studying the BTZ black hole and the black string \[ \text{[13]} \]. Although $k$ was originally proposed to be positive, here we assume to extend it to be negative for our purpose. The equations of motion lead to

$$R_{\mu\nu} - \nabla_\mu \nabla_\nu \Phi - \frac{1}{4} H_{\mu\rho\sigma} H^{\rho\sigma}_{\nu} = 0, \quad (2)$$

$$\nabla^2 \Phi + (\nabla \Phi)^2 - \frac{8}{k} - \frac{1}{6} H^2 = 0, \quad (3)$$

$$\nabla_\mu H^{\mu\nu\rho} + (\nabla_\nu \Phi) H^{\mu\rho}_\mu = 0. \quad (4)$$

The Kerr-de Sitter solution to Eqs. (2)-(4) is found to be \[ \text{[9]} \]

$$\bar{g}_{xx} = -\frac{2r}{\ell}, \quad \bar{\Phi} = 0, \quad k = -2\ell^2,$$

$$\bar{g}_{\mu\nu} = \begin{pmatrix} - (\bar{M} - r^2 / \ell^2) & -J/2 & 0 \\ -J/2 & r^2 & 0 \\ 0 & 0 & f^{-2} \end{pmatrix} \quad (5)$$

with the metric function $f^2 = \bar{M} - r^2 / \ell^2 + J^2 / 4r^2$. The above Kerr-de Sitter solution is obtained from the BTZ black hole \[ \text{[14]} \] by replacing both $\bar{M}$ and $\ell^2$ by $-\bar{M}$ and $-\ell^2$. The metric $\bar{g}_{\mu\nu}$ is singular at $r = r_\pm$,

$$r^2_\pm = \frac{M\ell^2}{2} \left\{ 1 \pm \left[ 1 + \left( \frac{J}{M\ell} \right)^2 \right]^{1/2} \right\} \quad (6)$$

with $M = (r^2_+ + r^2_-) / \ell^2 = (r^2_+ - r^2_-)$ and $J = 2r_+ r_- / \ell$. For convenience we introduce $r^2_- \equiv -r^2_+ > 0$ due to $r^2_- < 0$ in Kerr-de Sitter space. We note that in (2+1)-dimensions, there is no black hole horizon for Kerr-de Sitter space because the black hole degenerates to a conical singularity at the origin $r = 0$. This singularity gives rise to some difficulties to analyze the wave equation in the southern diamond of Kerr-de Sitter space. In this work we consider the cosmological horizon $r_c = r_+$ with interest. For convenience, we list the
Hawking temperature $T_c$, the area of the cosmological horizon $A_c$, and the angular velocity at the horizon $\Omega_c$ as

$$T_c = \frac{r_+^2 + r_-^2}{2\pi \ell^2 r_+}, \quad A_c = 2\pi r_+, \quad \Omega_c = J/2r_+^2. \quad (7)$$

For $J = 0$ case, one finds de Sitter solution which gives us with $r_+ = \ell$ and $r_- = 0$

$$M = 1, \quad J = 0, \quad T_c = \frac{1}{2\pi \ell}, \quad A_c = 2\pi \ell, \quad \Omega_c = 0. \quad (8)$$

### III. PERTURBATION AROUND KERR-DE SITTER SOLUTION

To study the propagation of all fields in Kerr-de Sitter space specifically, we introduce the small perturbation fields $[15]$

$$H_{t\phi r} = \bar{H}_{t\phi r} + \delta H_{t\phi r}, \quad \Phi = \bar{\Phi} + \phi, \quad g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \quad (9)$$

around the background solution of Eq.(5). For convenience, we introduce the notation

$$\hat{h}_{\mu\nu} = h_{\mu\nu} - \bar{g}_{\mu\nu}h/2$$

with $h = h^\rho_\rho$. And then one needs to linearize Eqs.(2)-(4) to obtain

$$\delta R_{\mu\nu}(h) - \bar{\nabla}_\mu \bar{\nabla}_\nu \phi - \frac{1}{2} \bar{H}_{\mu\rho\sigma} \bar{H}^{\rho\sigma}_\nu + \frac{1}{2} \bar{H}_{\mu\rho\sigma} \bar{H}^{\rho\sigma}_\nu h^\rho = 0, \quad (10)$$

$$\bar{\nabla}^2 \phi - \frac{1}{6} \{2 \bar{H}_{\mu\rho\sigma} \bar{H}^{\rho\sigma}_\nu - 3 \bar{H}_{\mu\rho\sigma} \bar{H}^{\rho\sigma}_\nu h^\mu \} = 0, \quad (11)$$

$$\nabla_\mu \bar{H}^{\mu\nu\rho} - (\nabla_\mu h_\nu^\rho) \bar{H}^{\mu\beta\rho} + (\nabla_\mu h_\beta^\rho) \bar{H}^{\mu\beta\nu} - (\nabla_\mu \bar{h}_\nu^\rho) \bar{H}^{\alpha\nu\rho} + (\partial_\mu \phi) \bar{H}^{\mu\nu\rho} = 0, \quad (12)$$

where the Lichnerowicz operator $\delta R_{\mu\nu}(h)$ is given by

$$\delta R_{\mu\nu} = -\frac{1}{2} \bar{\nabla}^2 h_{\mu\nu} + \bar{R}_{\sigma(\nu} h_{\mu)\sigma} - \bar{R}_{\sigma\mu\rho\nu} h^{\rho\sigma} + \bar{\nabla}_{(\nu} \bar{\nabla}_{|\mu|} h_{\rho)$. \quad (13)$$

These are the bare perturbation equations. It is desirable to examine whether we make a choice of perturbation and gauge which can simplify Eqs.(10)-(12) significantly. For this purpose we wish to count the physical degrees of freedom. A symmetric traceless tensor has $D(D+1)/2-1$ in $D$-dimensions. $D$ of them are eliminated by the gauge condition. Also $D-1$ are eliminated from our freedom to take further residual gauge transformations. Thus gravitational degrees of freedom are $D(D+1)/2-1-D-(D-1)=D(D-3)/2$ in three dimensions we have no propagating degrees of freedom for $h_{\mu\nu}$. Also two-form $B_{\mu\nu}$ has no physical degrees of freedom for $D=3$. Hence the physical degree of freedom in the Kerr-de Sitter solution is just the dilaton $\phi$.

Considering the $t$ and $\phi$-symmetries of the background spacetime Eq.(5), we can decompose $h_{\mu\nu}$ into frequency ($\omega$) and angular $(j = 0, 1, 2, \cdots)$ modes in these variables

$$h_{\mu\nu}(t, \phi, r) = e^{-i\omega t} e^{ij\phi} H_{\mu\nu}(r). \quad (14)$$

For simplicity, one chooses the same perturbation as in Eq.(14) for Kalb-Ramond field and dilaton as
\[ \mathcal{H}_{tφr}(t, φ, r) = \tilde{\mathcal{H}}_{tφr} \mathcal{H}(t, φ, r) = \tilde{\mathcal{H}}_{tφr} e^{-iωt} e^{ijφ} \tilde{\mathcal{H}}(r), \]  
\[ \varphi(t, φ, r) = e^{-iωt} e^{ijφ} \tilde{\varphi}(r). \]  

Since the dilaton is only a propagating mode, we are interested in the dilaton equation (11). Note that Eq. (10) is irrelevant to our analysis, because it belongs to the redundant relation. Eq. (11) can be rewritten as

\[ \vec{∇}^2 \varphi + \frac{4}{l^2} (h - 2H) = 0. \]

If we start with full six degrees of freedom of Eq. (14), we should choose a gauge. Conventionally, we choose the harmonic gauge \((\vec{∇}_\mu \hat{h}^{\mu} = 0)\) to describe the propagation of gravitons in \(D > 3\) dimensions [21]. It turns out that a mixing between the dilaton and other fields of \(h, H\) is not disentangled completely with the harmonic gauge condition. Here we focus on the propagation of the dilaton \(\varphi\). Fortunately if we introduce the dilaton gauge \((\vec{∇}_\mu \hat{h}^{\mu} = h_{\muν} \Gamma^\mu_{νρ})\), this difficulty may be resolved. Actually this gauge was designed for the dilaton propagation [22]. We attempt to disentangle the last term in Eq. (17) by using both the dilaton gauge and Kalb-Ramond equation (12). Each component \((ρ = t, φ, r)\) of the dilaton gauge condition gives rise to

\[ t : (\partial_r + \frac{1}{r}) h^{tr} - iω h^{tt} + ij h^{tφ} + \frac{1}{2} iω h \hat{g}^{tφ} - \frac{1}{2} ij h \hat{g}^{tφ} = 0, \]
\[ \phi : (\partial_r + \frac{1}{r}) h^{φr} - iω h^{φt} + ij h^{φφ} + \frac{1}{2} iω h \hat{g}^{φφ} - \frac{1}{2} ij h \hat{g}^{φφ} = 0, \]
\[ r : (\partial_r + \frac{1}{r}) h^{rr} - iω h^{rt} + ij h^{rφ} - \frac{1}{2} (\partial_r h) \hat{g}^{rr} = 0. \]

And each component \((ν, ρ)\) of the Kalb-Ramond equation (12) leads to

\[ tφ : -\partial_r (\varphi + \mathcal{H} - h^t_t - h^φ_φ) + \frac{1}{rf^2} \left( -M + \frac{3r^2}{l^2} + \frac{J^2}{4r^2} \right) h^{r}_{r} + iω h^{t}_{r} - ij h^{φ}_{r} = 0, \]
\[ tr : -ij (\varphi + \mathcal{H} - h^t_t - h^r_r) - \frac{1}{r} h^{r}_{φ} + 2f^2 h^{φ}_{r} - \partial_t h^{φ}_{φ} + iω h^{t}_{φ} = 0, \]
\[ φr : -iω (\varphi + \mathcal{H} - h^{φ}_φ - h^r_r) + \frac{1}{r} h^{r}_{t} - \frac{2rf^2}{l^2} h^{t}_{r} + \partial_r h^{φ}_{t} + ij h^{φ}_{t} = 0. \]

Solving six equations (18)-(23), one finds an important constraint

\[ \partial_μ(2φ + 2\mathcal{H} - h) = 0, \quad μ = t, φ, r \]

which leads to \(h - 2\mathcal{H} = 2φ\). This means that \(h\) and \(\mathcal{H}\) belong to the redundant field if one chooses the perturbations along Eqs. (14)-(16). It confirms that our counting for degrees of freedom is correct. We note that the harmonic gauge with the Kalb-Ramond equation (12) leads to the same constraint as in Eq. (24). As a result, Eq. (17) becomes a decoupled dilaton equation

\[ \nabla^2 \varphi + \frac{8}{l^2} \varphi = 0 \]
which can be rewritten explicitly as

\[
\begin{aligned}
& f^2 \partial_\tau^2 f + \left\{ \frac{1}{r}(\partial_r rf^2) \right\} \partial_r - \frac{J \omega}{r^2 f^2} + \frac{\omega^2}{f^2} + \left( \frac{-M + r^2/\ell^2}{r^2 f^2} j^2 \right) \tilde{\phi} + \frac{8}{\ell^2} \tilde{\phi} = 0,
\end{aligned}
\]

(26)

It is noted that if the last term is absent, Eq. (26) corresponds to the wave equation of a free scalar in Kerr-de Sitter space. Comparing this dilaton equation with the massive scalar equation

\[
\bar{\nabla}^2 \phi_m - m^2 \phi_m = 0,
\]

(27)

it seems that the dilaton propagates on Kerr-de Sitter space with the tachyonic mass \( m^2 \phi = -8/\ell^2 \). However, this observation is not the whole of story in de Sitter space.

**IV. STABILITY ANALYSIS FOR DE SITTER SPACE**

It is not easy to solve the wave equation Eq. (26) of the dilaton on the southern diamond including the cosmological horizon \( r = r_c \) and the origin \( r = 0 \). The main difficulty comes from the fact that the black hole horizon degenerates to give a conical singularity at \( r = 0 \). In other words, the Kerr-de Sitter solution represents a spinning point mass \( M \) in de Sitter space. This makes it hard to express the solution to the wave equation on the southern diamond in terms of the hypergeometric function. Then we cannot calculate the dilaton absorption cross section to test the cosmological horizon. Hence, hereafter, we confine ourselves to the pure de Sitter solution with \( J = 0 \). Then the origin is just that of the coordinate and thus there is nothing to worry about singularity on the southern diamond. Even though we give up the Kerr-de Sitter background, we can study the nature of de Sitter space using the dilaton.

Eq. (26) reduces to the differential equation for \( r \)

\[
(1 - r^2) \tilde{\phi}''(r) + \left( \frac{1}{r} - 3r \right) \tilde{\phi}'(r) + \left( \frac{\omega^2}{1 - r^2} - \frac{j^2}{r^2} - m^2 \phi \right) \tilde{\phi}(r) = 0, \quad m^2 \phi = -8
\]

(28)

where the prime (') denotes the differentiation with respect to its argument and for simplicity we take \( \ell \) to be 1. The original equation from (26) with \( J = 0, \ell \neq 1 \) takes the same form as in Eq. (28) if \( r/\ell \rightarrow \tilde{r}, \omega \ell \rightarrow \omega, m_\varphi \ell \rightarrow m_\varphi \). This information will be used for obtaining the absorption cross section in section V. From Eq. (28) it is obscure to know how the dilaton wave propagates in the southern diamond. In order to show it clearly, we must transform the wave equation into the Schrödinger-like equation by introducing a tortoise coordinate \( r^* \). Then we can get information through the potential analysis. We introduce \( r^* = g(r) \) with \( g'(r) = 1/r(1 - r^2) \) to transform Eq. (28) into the Schrödinger-like equation with the energy \( E = \omega^2 \)

\[
- \frac{d^2}{dr^*^2} \tilde{\varphi} + V_\varphi(r) \tilde{\varphi} = E \tilde{\varphi}
\]

(29)

with the potential
\[ V_\varphi(r) = \omega^2 + r^2(1 - r^2)\left[m_\varphi^2 + \frac{j^2}{r^2} - \frac{\omega^2}{1 - r^2}\right]. \]  

(30)

Considering \( r^* = g(r) = \int g'(r)dr \), one finds

\[ r^* = \ln r - \frac{1}{2} \ln(1 - r^2), \quad e^{2r^*} = \frac{r^2}{1 - r^2}, \quad r^2 = \frac{e^{2r^*}}{1 + e^{2r^*}}. \]

(31)

Here we confirm that \( r^* \) is a tortoise coordinate such that \( r^* \to -\infty (r \to 0) \), whereas \( r^* \to \infty (r \to 1) \). Let us express the potential as a function of \( r^* \)

\[ V_\varphi(r^*) = \omega^2 + \frac{e^{2r^*}}{(1 + e^{2r^*})^2} \left[m_\varphi^2 + \frac{1 + e^{2r^*}}{e^{2r^*}} j^2 - (1 + e^{2r^*}) \omega^2\right]. \]

(32)

First of all we mention that this potential is the energy-dependent potential. Let us consider the low-energy limit of \( \omega^2 \ll 1 \). For \( m_\varphi^2 = -8, j = 0, \omega = 0.1 \), the shape of this takes a potential well near \( r^* = 0 \). Due to the potential well, this potential induces an exponentially large dilaton which is obviously contradicted to the genuine small value of the perturbation. Hence the \( j = 0(s)\)-mode of the dilaton seems to be unstable. Naively speaking, this means that the cosmological horizon in our model does not truly exist. However we have some ambiguity to define this \( s\)-mode in de Sitter space. Hence it is not clear from \( s\)-mode analysis that the cosmological horizon is unstable.

For \( j \neq 0 \)-modes with the low-energy of \( \omega^2 \ll 1 \), one finds that \( V_\varphi(r^* = 0) = -8 + j^2/2 + \omega^2/2 \). Hence for \( j = 1, 2, 3 \)-modes one finds the potential wells, which confirm that the cosmological horizon is unstable. For \( j \geq 4 \)-modes the potential well disappears. In addition one finds the potential step with its height \( \omega^2 + j^2 \) on the left-hand side. All potentials decrease exponentially to zero as \( r^* \) increases on the right-hand side. This means that we always develop a well-defined wave near the cosmological horizon of \( r^* = \infty \). But near \( r^* = -\infty (r = 0) \) it is not easy to develop the genuine waves. It is expected that the scattering to give a finite absorption cross section will occur if \( E = \omega^2 \sim V_\varphi(r^*) \). This case is possible if \( \omega^2 \gg j^2 \). This means that the relevant scattering in de Sitter space may be arisen if the frequency of the external field is larger than its angular momentum quantum number. This corresponds to the low-temperature limit of \( \omega > T_c \). In this case the scattering can be defined even for \( j = 1, 2, 3 \) cases.

V. ABSORPTION CROSS SECTION

The absorption coefficient by the cosmological horizon is defined by the ratio of the outgoing flux at \( r = 0 \) to the outgoing flux at \( r = r_c \) as

\[ \mathcal{A} = \frac{\mathcal{F}_{\text{out}}(r = r_c)}{\mathcal{F}_{\text{out}}(r = 0)}. \]

(33)

Up to now we do not insert the curvature radius \( \ell \) of dS3 space. The correct absorption coefficient can be recovered by replacing \( \omega(m_\varphi) \) with \( \omega\ell(m_\varphi\ell) \). Here we do not repeat the procedure of the flux computation but refer ref. [11]. Then the dilaton absorption cross section in three dimensions is defined by
\[
\sigma_{\text{abs}} = \frac{A}{\omega} = \left[ \frac{A_r^2 + B_i^2}{A_r B_i} \right] \ell |\alpha_{-\omega \ell, j}|^2
\]  

(34)

where

\[
|\alpha_{-\omega \ell, j}|^2 = \frac{|\Gamma(1 + j)|^2 |\Gamma(i \omega \ell)|^2}{|\Gamma[2 + j/2 + i \omega \ell/2]|^2 |\Gamma[j/2 + i \omega \ell/2]|^2}.
\]  

(35)

We observe that \(s(j = 0)\)-mode cross section is ill-defined because \(\mathcal{F}_{\text{out}}(r = 0) = 0\) for \(j = 0\). Furthermore the normalization factor of \(\left[ \frac{A_r^2 + B_i^2}{A_r B_i} \right]\) is not fixed by the theory. In order to obtain the explicit form, let us calculate \(|\alpha_{-\omega \ell, j}|^2\) according to values of the angular momentum quantum number \(j\). One finds

\[
\sigma_{\text{dil}, \omega \ell < 1}^{\text{abs}, j = 1} = \left[ \frac{A_r^2 + B_i^2}{A_r B_i} \right] \frac{8 \mathcal{A}_ch}{\pi^2 \omega \ell \sinh[\pi \omega \ell]} \frac{\omega \ell}{(9 + (\omega \ell)^2)(1 + (\omega \ell)^2)} \times \frac{\cosh[\pi \omega \ell/2]^2}{(16 + (\omega \ell)^2)(4 + (\omega \ell)^2)(\pi \omega \ell/2)^2}.
\]  

(36)

For \(j = 1\), this takes the form

\[
\sigma_{\text{dil}, \omega \ell > 1}^{\text{abs}, j = 1} = \left[ \frac{A_r^2 + B_i^2}{A_r B_i} \right] \frac{8 \mathcal{A}_ch}{\pi^2 (\omega \ell)^2} \times \frac{\omega \ell}{(9 + (\omega \ell)^2)(1 + (\omega \ell)^2)}.
\]  

(37)

In the low-energy limit of \(\omega \ell < 1\), this reduces to

\[
\sigma_{\text{dil}, \omega \ell > 1}^{\text{abs}, j = 1} = \left[ \frac{A_r^2 + B_i^2}{A_r B_i} \right] \frac{8 \mathcal{A}_ch}{9 \pi^3 (\omega \ell)^2}.
\]  

(38)

On the other hand, its low-temperature limit is given by

\[
\sigma_{\text{dil}, \omega \ell > 1}^{\text{abs}, j = 1} = \left[ \frac{A_r^2 + B_i^2}{A_r B_i} \right] \frac{8 \mathcal{A}_ch}{\pi^2 (\omega \ell)^5}.
\]  

(39)

For \(j = 2\), this leads to

\[
\sigma_{\text{dil}, \omega \ell < 1}^{\text{abs}, j = 2} = \left[ \frac{A_r^2 + B_i^2}{A_r B_i} \right] \frac{16 \mathcal{A}_ch \omega \ell \sinh[\pi \omega \ell]}{\sinh[\pi \omega \ell/2]^2} \times \frac{(\pi \omega \ell/2)^2}{(16 + (\omega \ell)^2)(4 + (\omega \ell)^2)(\pi \omega \ell/2)^2}.
\]  

(40)

In the low-energy limit of \(\omega \ell < 1\), this reduces to
\[
\sigma_{dil, \omega \ell < 1}^{\text{abs}, j=2} = \left[ \frac{A_r^2 + B_i^2}{A_r B_i} \right] \frac{1}{16\pi} \frac{\mathcal{A}_{ch}}{(\omega \ell)^2}.
\]

But its low-temperature limit is given by
\[
\sigma_{dil, \omega \ell > 1}^{\text{abs}, j=2} = \left[ \frac{A_r^2 + B_i^2}{A_r B_i} \right] \frac{1}{16\pi} \frac{64 \mathcal{A}_{ch}}{\pi^2} \frac{1}{(\omega \ell)^7}.
\]

VI. DISCUSSION

We calculate the absorption cross section of the dilaton which propagates on the southern diamond of three-dimensional de Sitter space. One of the striking results is that the low-energy \( s(j = 0) \)-wave absorption of the dilaton is not defined properly. This mainly rests on being unable to calculate its finite flux at \( r = 0 \). This contrasts sharply to the cases found in the symmetric black holes whose \( s \)-wave cross sections are well-defined and proportional to the area of the event horizon [17].

On the other hand, the \( j \neq 0 \)-angular modes of the dilaton can be used for exploring the dynamical aspects of the cosmological horizon. We expect from the black hole analysis that the low-energy limit (\( \omega R \to 0 \)) of the \( l \neq 0 \)-angular mode absorption cross section is proportional roughly to \( (\omega R)^4l \) for the D=7 black hole which is induced from D3-branes [23]. For D=5 black hole, it is proportional to \( (\omega r_o)^2l \) [24]. However, one finds from Eqs. (38) and (41) that those for \( j \neq 0 \) in the low-energy limit of \( \omega \ell < 1 \) are given by \( (\omega \ell)^{-2} \) which implies that the absorption cross section is greater than the area of the cosmological horizon. This is not the case what we want to get. From the potential analysis in section IV, it conjectures that for \( j = 1, 2, 3 \) cases, the low-energy absorption cross section with \( E = (\omega \ell)^2 \ll 1 \) is meaningless because these become the unstable case. This implies that to obtain the finite absorption cross section, \( \omega \ell \) should be large such as \( \omega \ell > j \). This corresponds to the low-temperature limit of \( \omega > T_c \). The low-temperature limit is meaningful in de Sitter space since its cross section appears less than \( \mathcal{A}_{ch} \). For example, we find from Eqs. (39) and (12) that the absorption cross sections takes \( \sigma_{dil, \omega \ell > 1}^{\text{abs}, j=1} \sim (\omega \ell)^{-2} \) roughly. This is consistent with the potential analysis. According to the this, the potential height is proportional to \( \omega^2 + j^2 \) which implies that the absorption cross section decreases as \( j \) increases. On the other hand, the free scalar absorption cross section takes the same form \( (\omega \ell)^{-2} \) as in the dilaton in the low-energy limit, while it is given by \( \sigma_{\text{free}, \omega \ell > 1}^{\text{abs}, j>1} \sim (\omega \ell)^{-(2j+1)} \) in the low-temperature limit [11]. As a result, we confirm that the low-temperature limit (not the low-energy limit) of \( j \neq 0 \)-angular mode absorption cross section will be used to test the cosmological horizon in de Sitter space. This contrasts sharply to the fact that the low-energy \( s \)-mode plays an important role to test the black hole event horizon.

In conclusion, to get information about the cosmological horizon, we have to inject the test field with high frequency into the de Sitter background.

Finally we mention that the AdS bulk absorption cross section can be also calculated from the two-point function of CFT defined on the boundary if one assumes the AdS/CFT correspondence using the boundary-bulk Green function [25]. Hence we propose that our results for the dS bulk space can be recovered from the Euclidean CFT by making use of the dS/CFT correspondence [8,10] and the corresponding boundary-bulk Green function [12].
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