SUPERMASSIVE BLACK HOLE BINARY EVOLUTION IN AXISYMMETRIC GALAXIES: THE FINAL PARSEC PROBLEM IS NOT A PROBLEM

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ABSTRACT

During a galaxy merger, the supermassive black hole (SMBH) in each galaxy is thought to sink to the center of the potential and form an SMBH binary; this binary can eject stars via three-body scattering, bringing the SMBHs ever closer. In a static spherical galaxy model, the binary stalls at a separation of about a parsec after ejecting all the stars in its loss cone—this is the well-known final parsec problem. Earlier work has shown that the centrifugal orbits in triaxial galaxy models are key in refilling the loss cone at a high enough rate to prevent the black holes from stalling. However, the evolution of binary SMBHs has never been explored in axisymmetric galaxies, so it is not clear if the final parsec problem persists in these systems. Here we use a suite of direct N-body simulations to follow SMBH binary evolution in galaxy models with a range of ellipticity. For the first time, we show that mere axisymmetry can solve the final parsec problem; we find the SMBH evolution is independent of N for an axis ratio of $c/a = 0.8$, and that the SMBH binary separation reaches the gravitational radiation regime for $c/a = 0.75$.

Key words: black hole physics – Galaxy: center – galaxies: kinematics and dynamics

1. INTRODUCTION

Supermassive black holes (SMBHs), with masses from $10^8$ to $10^{10} M_\odot$ are thought to dwell at the centers of most galaxies (e.g., Kormendy & Richstone 1995; Ferrarese & Ford 2005), and since we know that galaxy mergers are commonplace, binary SMBHs will almost inevitably form deep within the galaxy at some point during galaxy assembly (Begelman et al. 1980). There is ample observational evidence for two widely separated SMBHs within the same host (e.g., Comerford et al. 2009), as well as growing evidence for true SMBH binaries (Komossa 2006; Bon et al. 2012). Upon coalescence, SMBH binaries emit copious gravitational radiation; they are expected to be the loudest sources of gravitational waves, and future space-based gravitational wave detectors, such as the New Gravitational wave Observatory (Hughes 2003; Barack & Cutler 2004; Danzmann et al. 2011), will be able to detect them to extreme cosmological distances, even breaching the electromagnetic barrier presented by the cosmological dark ages.

The paradigm for the evolution of SMBH binaries can be broken into three distinct phases (Begelman et al. 1980), where the physics that governs the binary is quite different. First, dynamical friction drives the two SMBHs to sink toward the galaxy center, and there they eventually form a bound pair with semimajor axis (of the relative motion of the binary) $a \sim r_h$. Here, $r_h$ is the radius of influence of binary, typically is defined as the distance within the stellar distribution that encloses twice the binary mass. Dynamical friction continues to bring the SMBHs ever closer until the system becomes a hard binary (Quinlan 1996; Yu 2002); here the separation is $a \sim a_h$:

$$a_h := \frac{G\mu_r}{4\sigma^2}, \quad (1)$$

where $\mu_r$ is the reduced mass of the binary and $\sigma$ is the one-dimensional velocity dispersion. In the second phase, three-body encounters between stars and the SMBH binary provide a slingshot mechanism to eject stars which extracts energy and angular momentum from the SMBH binary. If three-body scattering can shrink the binary orbit by a factor of order a hundred, then the SMBHs will be close enough to emit strong gravitational radiation. This gravitational wave emission in the third and final phase will drain orbital energy from the binary and will drive the final coalescence in roughly 100 Myr. The transition from dynamical friction to the three-body scattering can be quite prompt, provided that the galaxy merger is a major one (with a mass ratio greater than 0.1; Callegari et al. 2011). In contrast, the transition from three-body scattering to the gravitational radiation regime could be a bottleneck that effectively prevents binary SMBHs from ever coalescing within a Hubble time. The root cause of this hang up is simply a lack of low-angular momentum stars capable of interacting with the binary via three-body scattering. This theoretical bottleneck has become well-known as the final parsec problem.

Theorists commonly invoke gas to move these SMBHs through the final parsec. This is because the SMBHs lose energy and angular momentum against the gas very efficiently, as long as the gas is not forming stars (e.g., Mayer et al. 2007; Escala et al. 2005). Unfortunately, relying on gas to solve the final parsec problem will leave out the most massive SMBHs, since their host galaxies are massive ellipticals and are therefore gas-poor (Kannappan 2004).

There has been significant work to describe the final parsec problem and to solve it with a purely stellar dynamical approach. For example, Yu (2002) first pointed out that flattening and non-axisymmetry could circumvent this binary coalescence bottleneck, and using the structural parameters from a sample of
nearby ellipticals and spiral bulges (Faber et al. 1997), calculated SMBH merger times of less than a billion years for equal mass black holes in triaxial models. Later, several studies built self-consistent cuspy, triaxial models with a single central SMBH and showed that when they were populated with a large fraction of centrifophilic orbits, the loss cone refilling rate was rapid; in principle, this could speed up the binary hardening rate—if a binary were present (e.g., Merritt & Poon 2004; Holley-Bockelmann et al. 2002; Holley-Bockelmann & Sigurdsson 2006). Direct N-body simulations of SMBH binary evolution in isolated rotating galaxy models also showed that the final parsec problem could be solved by purely stellar dynamical models that develop strong triaxiality through a bar instability (Berczik et al. 2006; Berentzen et al. 2009). Finally, the most recent studies show that the hardening rates of SMBH binaries in equal-mass galaxy mergers are substantially higher than those found in spherical models (Khan et al. 2011, 2012b; Preto et al. 2011), presumably because these remnants are highly triaxial (e.g., Barnes 1992; Naab et al. 2006).

Though these experiments all point to a solution to the final parsec problem that involves more realistic non-spherical galaxy shapes, the relative importance of the shape versus the bulk dynamics in solving the final parsec problem is not well understood. The consensus has so far been that the orbit structure inherent in non-axisymmetric systems is key to bringing the SMBHs together. Triaxial galaxy models are made by a large population of stars on regular centrifophilic orbits deep within the galaxy potential, such as pyramid or box orbits with formally zero angular momentum (Schwarzschild 1979, 1982; de Zeeuw 1985; Gerhard & Binney 1985; Levinson & Richstone 1987; Statler 1987; Miralda-Escude & Schwarzschild 1989; Valluri & Merritt 1998; Holley-Bockelmann et al. 2001; Merritt & Vasiliev 2011). These orbits are present even when the galaxy is only mildly triaxial (Hoffman et al. 2010; Valluri et al. 2010; van den Bosch & de Zeeuw 2010; Bryan et al. 2012), since by breaking the symmetry of the system, the number of conserving integrals is reduced. Outside the influence radius of the SMBH, many of the centrifophilic orbits are chaotic in a triaxial potential, though some regular resonant boxlet orbits remain (Poon & Merritt 2001; Holley-Bockelmann et al. 2002). However, because the numerical studies thus far have included rotation and highly evolving potentials, it is difficult to disentangle the effects of shape from these other mechanisms. This paper is the first in a series of papers dedicated to isolating the key causes behind the solution of the final parsec problem.

We begin by exploring the problem in equilibrium, non-rotating axisymmetric models. In contrast to the wealth of work on solving the final parsec problem in triaxial galaxies, comparatively little attention has been paid to exploring binary coalescence in axisymmetric systems. Recent work on the capture of stars from the loss cone in axisymmetric nuclei looks promising for the final parsec problem, in that the stellar capture rate is several times higher in flattened systems (Magorrian & Tremaine 1999; Vasiliev & Merritt 2013). In an axisymmetric galaxy, the centrifophilic orbit family contains saucer or cone orbits (Sridhar & Touma 1999), and these can, in principle increase three-body scattering rates. However, no detailed N-body simulations have been done to test this; it is not even clear that these centrifophilic orbits can remain stable in a model with a binary black hole. Therefore, the degree to which axisymmetry can solve the final parsec problem is unknown. In this paper, we explore SMBH coalescence in gas-free axisymmetric galaxies using high resolution N-body simulations of equilibrium axisymmetric galaxy models with a range of intrinsic flattening.

2. INITIAL CONDITIONS AND NUMERICAL TECHNIQUE

2.1. The Equilibrium Host Galaxies and Their SMBHs

We generated our equilibrium axisymmetric systems from initially spherical SMBH-embedded η models (Tremaine et al. 1994) by applying a slow, steady velocity drag on the particles in the z direction (see Holley-Bockelmann et al. 2001, for details). This adiabatic squeezing technique slowly changes the phase space distribution function of the system and alters the orbit content as well, but the spherically-averaged density profile remains unchanged as the galaxy shape evolves (Holley-Bockelmann et al. 2002). The suite we present here has an inner density slope, γ, of 1.0. Our model has a mass of 1.0 in system units and the half mass radius is 2.41. The central SMBH has a mass of 0.005 which is in the range of observed SMBH masses when compared to the mass of bulge hosting them (Merritt & Ferrarese 2001; H"aring & Rix 2004; Scott et al. 2013). A second equal mass SMBH is introduced into the outskirts of the equilibrium model at a distance of 0.5 in model units with 70% of the circular velocity.

2.2. Hardware and Numerical Methods

Our suite of N-body simulations were conducted with a self-developed ϕGRAPE+GPU code, a unique parallel, direct summation N-body code that takes advantage of Graphic Processing Units (GPU) cards to accelerate the pairwise gravitational force calculations between each particle. The first version of the code was written from scratch in C and originally was designed to use the GRAPE6a clusters for the N-body task integration (Harfst et al. 2007).

The particles are advanced in the ϕGRAPE+GPU code by a fourth-order Hermite integrator with hierarchical individual block time-steps. This requires calculating acceleration a and its first time derivative ̇a for each particle; we use GPU cards in parallel to speed up this calculation. We use the SAPPORO (Gaburov et al. 2009) library to determine our forces, which emulates standard GRAPE-6 library calls on the GPU. Multi-GPU support is achieved through effective MPI parallelization. Each individual MPI process uses only a single GPU, but we instantiate two or more MPI processes per node to effectively use both the multi core CPU and the multi GPU cluster architecture.

In more detail, the MPI parallelization is done within the ϕGRAPE-GPU code in a “j” particle parallelization scheme. All the particles are divided equally between the working nodes (using the MPI_Bcast() command) and in each node we calculate only the fractional forces for the particles in the current time step—these are the “active” or “i” particles. We obtain the full forces from all the particles acting on the active particles by applying a global MPI_Allreduce() communication routine.

The ϕGRAPE+GPU code has been extensively tested and has been successfully used in several earlier SMBH publications (Berczik et al. 2005, 2006; Berentzen et al. 2009; Preto et al. 2011; Khan et al. 2012b). The original public version of the ϕGRAPE+GPU code is now available here: ftp://ftp.mao.kiev.ua/pub/berczik/phi-GRAPE.

The ϕGRAPE+GPU code does not include the regularization of close encounters or binaries (Mikkola & Aarseth 1998), so we apply a small softening for star–star encounters to avoid the
Table 1

| Run   | N  | γc/a | q  |
|-------|----|-----|----|
| Spha  | 1000k | 1.0  | 1.0 |
| Flat9a| 1000k | 1.0  | 0.9 |
| Flat9b| 800k  | 1.0  | 0.9 |
| Flat9c| 500k  | 1.0  | 0.9 |
| Flat9d| 250k  | 1.0  | 0.9 |
| Flat9e| 125k  | 1.0  | 0.9 |
| Flat9f| 1000k | 1.0  | 0.8 |
| Flat9g| 800k  | 1.0  | 0.8 |
| Flat9h| 500k  | 1.0  | 0.8 |
| Flat9i| 250k  | 1.0  | 0.8 |
| Flat9j| 125k  | 1.0  | 0.8 |
| Flat9k| 1000k | 1.0  | 0.75|
| Flat9l| 800k  | 1.0  | 0.75|
| Flat9m| 500k  | 1.0  | 0.75|
| Flat9n| 250k  | 1.0  | 0.75|
| Flat9o| 125k  | 1.0  | 0.75|

Notes. Column 1: galaxy model. Column 2: number of particles. Column 3: central density slope γ. Column 4: axes ratio. Column 5: SMBH mass ratio.

formation of tight binaries during our simulations. We use a typical Plummer type of softening between the star particles.

The softening length for star–star encounters $\epsilon_s$ is deliberately selected to be smaller than the minimum separation reached by SMBH binaries in our study ($\epsilon_s = 10^{-4}$ in our model units). We set the softening for the interaction between the SMBH particles $\epsilon_b$ exactly equal to zero for the entire suite of runs. For interactions between black holes and star particles, we apply a “mixed” gravitational softening: $\epsilon_{bs} = \frac{1}{2}(\epsilon_b^2 + \epsilon_s^2)$, where $\epsilon_{bs}$ is the softening for black hole–star interactions (and vice versa).

The suite of $N$-body experiments were carried out on a high-performance GPU computing cluster ACCRE employing 192 GPUs at Vanderbilt University, Nashville, TN.

3. SMBH BINARY EVOLUTION IN FLATTENED GALAXY MODELS

We created models with axis ratios $c/a = [1.0, 0.9, 0.8, 0.75]$ and particle number $N$ (see Table 1) in order to study the SMBH binary evolution as a function of flatness and particle number.

Figure 1 describes the comparison between the evolution of the SMBH binary in spherical and flattened galaxy models ($c/a = 0.9$). In the first phase, which can be seen before $T = 10$, the separation between the two SMBHs shrink due to dynamical friction. The evolution of SMBHs in this phase is very similar for both the spherical and flattened galaxy models. However the subsequent evolution, governed by three-body encounters, happens at much faster rate in flat galaxy models, as can be seen from the evolution of both the separation and the semimajor axis of the binary SMBH (the top and middle panels). The SMBH binary forms with an eccentricity, $e$, much less than 0.1 and it remains small during the binary evolution. Although the semimajor axis of SMBH binary evolves faster than that of the spherical galaxy model, it still shows clear dependence on $N$ (see middle panel)—a sure sign that the binary evolution in this simulation is plagued by the numerical effects of two-body relaxation. Note, though, that the SMBH binary pair experiences Brownian motion as well, as each interaction with a star imparts a small recoil kick to the binary (e.g., Merritt 2001; Hensendorf et al. 2002; Chatterjee et al. 2003; Merritt et al. 2007); this Brownian motion also decreases with larger $N$ because the simulated potential is smoother.

The hardening rates $s = (d/dt)(1/a)$ of the SMBH binary are calculated by straight lines fit to $a^{-1}(t)$ in the hard binary phase from $T = 40–70$. Figure 2 shows the hardening rates for all our models as function of $N$. For models with $c/a = 0.9$, the value of $s$ decreases with increasing $N$, and again this suggests that
numerical two-body relaxation is important for SMBH binary evolution in both of these models. Note, though, that $s$ is roughly four times higher for the SMBH binary evolution in flattened systems ($c/a = 0.9$) compared to a spherical galaxy model. In flattened galaxy models, then, there appears to be an additional supply of stars that interact with the massive binary.

The SMBH binary evolution for non-rotating models with $c/a = 0.8$ are shown in Figure 3. On the face of it, the evolution of the SMBH binary in flattened models with $c/a = 0.8$ is very similar to $c/a = 0.9$. However, there is a striking difference for runs with $N$ greater than 500k in the sense that SMBH binary evolution becomes independent of $N$. This implies that the system is no longer dominated by artificial two-body relaxation, and we are uncovering the accurate SMBH binary evolution.

Again, we calculated SMBH binary hardening rates in interval between $T = 40$ and $T = 70$. Note that for $N > 500k$, $s = 22$ is eight times higher than the spherical galaxy model at the highest particle resolution. Also for these models hardening rates are two times larger than $c/a = 0.9$ models (see Figure 2).

Figure 4 features the SMBH binary evolution for the flattest model we explored so far, $c/a = 0.75$, and we confirm the both $N$-independence and the rapid hardening rate seen in the $c/a = 0.8$ runs. As is perhaps expected, the hardening rate is slightly enhanced compared to the $c/a = 0.8$ model and the SMBHs reach an even smaller separation.

3.1. Estimates for the Relativistic Regime

At the time when we stop our simulations, the SMBH binary separation is still far larger than what is required for gravitational wave emission to dominate and guide the binaries on their way to coalescence. To estimate the coalescence time for these binaries, we adopt the approach of Berentzen et al. (2009) and Khan et al. (2012b), which assumes a constant hardening rate in the three-body scattering regime and Peters (1964) formula to estimate the hardening in the gravitational wave regime. Such estimates agree well with coalescence time obtained from simulations that follow the binary evolution using post-Newtonian ($\mathcal{P}N$) terms in the equation of motion of the SMBH binary:

$$\frac{da}{dt} = \left( \frac{da}{dt} \right)_{\text{NB}} + \left( \frac{da}{dt} \right)_{\text{GW}} = -sa^2(t) + \left( \frac{da}{dt} \right)_{\text{GW}}. \quad (2)$$

The Peters (1964) formalism finds orbit-averaged expressions for the rates of change of the semimajor axis and eccentricity of a binary due to gravitational wave emission, and include the lowest $\mathcal{P}N$ dissipative order at 2.5 as follows:

$$\left( \frac{da}{dt} \right)_{\text{GW}} = -\frac{64}{5} \frac{G^3 M_1 M_2 (M_1 + M_2)}{a^5 c^5 (1 - e^2)^{3/2}} \times \left( 1 + \frac{73}{24} c^2 + \frac{37}{96} c^4 \right), \quad (3a)$$
Figure 4. Evolution of the binary SMBH separation (top), semimajor axis (middle) and eccentricity (bottom) of the SMBH binary in N-body integrations for spherical and flattened models with \(c/a = 0.75\) (see Table 1). Time and \(R\) are measured in model units.

\[
\langle \frac{de}{dt} \rangle_{GW} = -\frac{304}{15} e^3 \frac{G^3 M_1 M_2 (M_1 + M_2)}{a^5 (1 - e^2)^{3/2}} \times \left(1 + \frac{121}{304} e^2\right). \tag{3b}
\]

Estimation of the evolution of the SMBH binary in the gravitational radiation regime requires some physical length and mass scales. We compare the mass and influence radius of the SMBH in our model to the observed SMBH mass and influence radius for the galaxy NGC 4486A. One unit of length and mass in our model units is equivalent to 0.3 kpc and \(2.6 \times 10^9\) solar masses, respectively. The value of speed of light is \(c = 1550\) and one time unit is equal to 1.4 Myr.

We next present an estimate of the coalescence time for the run Flat75a, which is a typical representation of the \(c/a = 0.75\) runs. At \(T = 70\) in model units, the semimajor axis \(a\) is \(5 \times 10^{-4}\) and the eccentricity \(e\) is 0.1, which makes the estimated coalescence time 2.4 Gyr. This coalescence is a few times longer than the coalescence times for nearly radial galaxy mergers in Khan et al. (2012b), but it is still well within a Hubble time. We speculate that the main reason behind this difference is that the eccentricity here is quite low. Indeed, we are currently exploring the eccentricity evolution in axisymmetric galaxy models to determine if low eccentricity is a generic feature of flattened equilibrium models.

4. SUMMARY AND CONCLUSIONS

We present the first results of SMBH binary evolution in a fully self-consistent equilibrium axisymmetric galaxy model. We find, in general, that SMBH binaries evolve faster and reach smaller separations for even mildly flattened galaxy models. For \(c/a = 0.8\), we begin to see hints of \(N\)-independence for \(N > 500k\), and here the hardening rate is eight times faster than in a spherical model. For our flattest model \(c/a = 0.75\), the system evolves to a gravitational radiation regime regardless of the particle resolution; for sufficiently flattened systems, we show that the final parsec problem is not a problem. These models may best pertain to pseudobulges and S0s; a recent example is the lenticular galaxy NGC 1277, which seems to host a new class of ultramassive black hole (van den Bosch et al. 2012). Note, though, that the flattening in our models is very modest compared to these systems, where the axis ratios are often \(c/a = 0.6\) or smaller (Kormendy & Kennicutt 2004), and we have also yet to include rotation. Both of these effects should act to increase the hardening rate.

Contrary to previous work that consistently predict highly eccentric SMBH binaries as they pass to the gravitational radiation regime (Khan et al. 2011, 2012a, 2012b), we find that the SMBH binaries here all have quite low eccentricity, with the most eccentric orbit only having \(e = 0.1\). It is unclear whether this is due to triaxiality or to the fact that previous models were products of radial mergers, and therefore the system itself—both the SMBH and the particles in the model—contained a large fraction of highly radial orbits. More work is needed, both to examine the orbital content of our equilibrium models, and to explore the effect of the SMBH initial eccentricity, to unravel the eccentricity evolution of these SMBH binaries.

In this pilot study, all of our flattened galaxy models had the same density profile, central black hole mass, black hole mass ratio, and initial black hole orbital eccentricity. Since the SMBH binary evolution will likely depend on the internal structure of the host galaxy and on details of the black hole orbit, we are launching a systematic study of SMBH binary evolution in axisymmetric galaxy models of various cusp slopes, mass ratios, flattening and orbit type.

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