A novel three axis Semi Elliptic Manipulator Configuration for simultaneous Multi Point Operations

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Abstract. A new three axes robotic configuration named as “Semi Elliptic Configuration” has been explored here. The configuration’s topology, working principle, forward kinematics, inverse kinematics and application have been described in this article. The inverse and forward kinematic equations have been derived using more fundamental mathematical tools like plane coordinate geometry, trigonometry and properties of triangles instead of using DH parameters, Transformation and Jacobian matrices. The configuration’s name has been derived from the unconventional geometry of the work volume within which it operates. Unlike the cubically shaped work volume of a typical XYZ Cartesian robotic configuration, its shape has been warped towards the edge containing the vertical Z-axis. It was found that the edges enclosing the face opposite to the Z-axis is part of an Ellipse. Two mutually perpendicular co-planar arms originating from the Z-axis enclose the semi elliptic boundary. Two manipulators of such configuration could be accommodated and be made to work independently within the cubical volume occupied by the perpendicular arms. It was found that the configuration’s work volume leaves a void region within the volume of occupancy of the configuration’s structure, which proved to be a disadvantage in certain applications and an advantage in others. An enhancement has been proposed to work around this restriction. The modification allows the configuration to expand its work volume into the void regions. The superior version of the Semi elliptic configuration was named “Enhanced Semi Elliptic Configuration”.

1. Introduction
There are four fundamental robotic configurations, namely: Cartesian, Cylindrical, Polar, Jointed Arm Configurations [1]. The Cartesian configuration is the most widely used in industrial machinery such as Gantry cranes, CNC milling machines, drilling machines, routers, laser cutters, engravers, Pick-n-Place machines, 3D printers, et cetera as it is intuitively three dimensional along the Cartesian coordinates, simple in principle and construction [2]. It has three mutually perpendicular prismatic actuation axes, each for the three Cartesian X, Y and Z coordinate axes. The configuration’s governing mathematics is straight forward as it involves linear forward and inverse kinematics. The motion dynamics is easy to model and the fabricated structure could be made very stable with little design calculation effort. Tweaking around with the arrangement of few links and joints on the kinematic chain form or the skeletal form of the Cartesian configuration shown in figure 1, a new configuration was obtained as shown in figure 2, later named as a “Semi elliptic” configuration. The reasoning
behind the moniker “Semi elliptic” coined for and used in reference to the new configuration all along this article would be justified in the later sections.

The work volumes were traced for both the configurations as depicted in the two figures. The immediately striking observation is that for a given actuation length limit of the prismatic joints in both configurations, the work volume of the Semi elliptic configuration is less than half the cubical volume of the Cartesian configuration. Its work volume has a concaved side, which in later sections of this article, has been showed to be of an Elliptic curve profile. The new configuration was studied further using 3D modelling CAD software such as SolidWorks [3] and mathematical tools.

2. Configuration topology and features
The topology of the proposed configuration is shown in figure 3 and is simple to comprehend. The topology shows the way in which the links and the joints have been arranged and also the fashion in which they are connected to one another, but eliminates details like actuators, gears, belts, mechanical components et cetera to make it easy for the reader to appreciate the configuration rather than understand how it is constructed for an end user application. It is evident that the height positioning of the end-effector along the vertical Z axis is easy and straight forward. It could be achieved by simply actuating the prismatic joint $P_1$ along the vertical Z axis, for example, by the means of a linear actuator or a motor driven belt system or even a lead screw mechanism. However, due to the particular arrangement of links in this configuration, the end-effector positioning in the two dimensional horizontal plane along the $X$ axis and $Y$ axis could be a little tricky as it would need some mathematical computation of condition bound kinematic equations. For example, an interdependence of motion can be immediately observed between prismatic joints $P_2$ and $P_3$. If $P_2$ moves towards the positive $X$ axis, then $P_3$ would need to move towards the negative $Y$ axis by a certain distance in relation to the displacement of $P_2$ as the two prismatic joints are connected by a link and mechanically constrained to their axis of movement. Thus, $P_2$ and $P_3$ cannot be independently actuated as fancied, for they are mechanically bound to move as per the other. Only either one of them may be driven by an actuator such as a Servo motor, lest it should damage the actuators or the mechanical components such as the joint bearings or the links.
2.1. Topology of links and joints in the horizontal plane

The configuration consists of two perpendicular arms in the shape of an ‘L’ in the horizontal plane. The arms contain two prismatic joint moving along the X and Y axis. The two prismatic joints \( P_2 \) and \( P_3 \) on their respective X and Y axis arms are joined by a link of fixed length called the \textit{Oblique} axis by respective revolute joints \( R_1 \) and \( R_2 \). The prismatic joint \( P_4 \) moves the end-effector along the \textit{Oblique} axis by Stepper, DC or AC Servo motor driven linear actuation mechanisms based on Timer Belts or Lead Screws (again, not explicitly shown in the figure to draw attention to the skeletal configuration).

The length of the \textit{Oblique} axis is fixed and is rigid which ensures the movement of joints \( P_2 \) and \( P_3 \) are mutually related by a simple mechanical constraint. Due to this constraint, either one of the prismatic joints, \( P_1 \) or \( P_3 \), could be actuated by the means of a motor driven linear actuation system to control the orientation of the \textit{Oblique} axis. This means that the angle subtended by the \textit{Oblique} axis on the horizontal X axis (about the revolute joint \( R_1 \)) is dependent on the actuation length of the prismatic joint \( P_2 \) (along the X axis) or \( P_3 \) (along the Y axis). This feature is unique to the Semi elliptic configuration as, in typical Cartesian X, Y, Z configuration shown in figure 1; all the links and prismatic joints are independent of each other in terms of orientation. This implies that the orientation of the end-effector changes depending on the actuation length of \( P_2 \) (or \( P_3 \)) as this rotates the \textit{Oblique} axis (containing the end-effector). The end-effector can only revolve in the horizontal plane about its vertical axis, but not independently due to the inherent mechanical relation of links. This makes the configuration only suitable for applications where the end-effector tool orientation in the horizontal plane is not a concern as it cannot be independently controlled. Such applications include CNC routing, laser engraving and 3D printing to name a few. To work around this constraint and orient the tool as desired, a revolute joint (about the vertical axis of the end-effector) acting as a wrist joint could be added. However we would not be discussing that path of design any further as it is an application specific accessory and the article focuses on the new configuration as it is. The joints discussed so far move the end-effector within the work area existing in the two dimensional horizontal plane.

2.2. Three dimensional movement

The prismatic joint \( P_1 \) at the intersection of the X and Y axis arms traverses the whole horizontal plane assembly along the Z axis, thus positioning the end-effector at any given height from the base plane. With the addition of a motor driven linear actuation mechanism attached to \( P_1 \), the end-effector can move within a three dimensional work volume. The three motor driven prismatic joint actuations \( P_1 \),

![Figure 3. Configuration topology.](image1)

![Figure 4. Work and Void volume.](image2)
$P_2$ and $P_4$ (or $P_1$, $P_3$ and $P_5$) are sufficient to completely define the position and orientation of the end-effector \cite{6}, thus the configuration has three DOF (Degrees Of Freedom). As shown in figure 4 the work area in the horizontal plane is extruded vertically upward along the $Z$ axis to demarcate the *work volume*. It is observed that there is a *void region* within the *enclosing volume* encapsulated by the arm links of the configuration within which the end-effector would not be able to traverse. As mentioned earlier, the work volume of the Semi elliptic configuration is less than the diagonally halved cubical volume of the Cartesian configuration due to this *void region*. This too is a unique feature of the Semi elliptic configuration which could in some cases, restrict its application and in others, make it more preferred. To eliminate the *void region*, an alternative configuration derived from the Semi elliptic configuration has been discussed later.

**3. Governing mathematics**

In this section the mathematical equations governing and modelling the behaviour of the proposed configuration is discussed. Instead of employing transformation matrices and Denavit–Hartenberg parameters, a simpler approach using basic mathematical tools like analytical geometry, properties of triangles and trigonometric equations has been adopted to perform forward and inverse kinematics \cite{4}. It is also interesting and important to note that the kinematic complexity of the configuration is in the planar actuation assembly containing the *Oblique* axis and the two arms along the $X$ and $Y$ axis in the horizontal plane.

**3.1. The semi elliptic trajectory**

To quantitatively analyse the Semi elliptic configuration, consider the illustration shown in figure 5 is an Ellipse inscribed within a Square, such that its diagonals are collinear with the major and minor axes of the Ellipse. The Ellipse $KLMN$ is inscribed within the Square $OPQR$ with points of tangency at points $K$, $L$, $M$ and $N$ with sides $RO$, $OP$, $PQ$ and $QR$. Each enclosing contour of the figure formed by the point of tangency on the ellipse with the square’s corners contains a part of the inscribed Ellipse. The closed sectors $KOL$, $LPM$, $MQN$ and $NRK$ form “semi elliptic wedges” as they contain a part of the Ellipse. If among the four wedges, the small wedges $LPM$ and $NRK$ are removed and the remaining two larger wedges $KOL$ and $MQN$ are isolated, then the resultant topology is identical to the planar motion assembly of the proposed configuration. The semi elliptic work area $KOL$ and $MQN$ of the planar motion assembly is formed with their respective curved boundaries $KL$ and $MN$ being part of an Ellipse as shown in figure 6. It is this curved boundary that forms the concaved face of the work volume associated with the Semi elliptic configuration shown earlier in figure 2 and figure 4. Thus, the design of the planar motion assembly about the $X$ and $Y$ axis shown in figure 3 is termed “Semi elliptic” as the locus of the points covered by the end-effector at the radially farthest points from the $Z$ axis within the work area is part of an Ellipse, or, in other words traverses an Elliptical trajectory.

![Figure 5. Ellipse inscribed in a Square.](image1)

![Figure 6. Semi-elliptic areas isolated.](image2)
3.2. General planar motion assembly annotations and constrains
The top view of the Semi elliptic configuration focussing on the planar actuation of the end-effector in the horizontal plane is shown in figure 7. In this plane of actuation, two degrees of freedom are sufficient to position the end-effector about the two dimensional coordinate system \([5]\). The lengths \(L_1\) and \(L_2\) of the perpendicular arms are fixed.

![Figure 7. Two dimensional motion plane annotation.](image)

The point \(A\) and \(B\) in figure 7 correspond to the ends of the link along the Oblique axis joining the respective revolute joints \(R_1\) and \(R_2\) shown in figure 3. Point \(O (0,0)\) is the projection of the configuration’s origin at the base plane along the Z axis on the planar motion assembly. For simplicity of analysis and derivation of kinematic equations, the joint distances between the axis of joint pair \(P_2, R_1\) and pair \(P_3, R_2\) is considered negligible and ignored. The joint distance simply adds an invariant offset to the position of the end-effector and plays no role in determining its path. The Pythagorean triangle \(\Delta AOB\) has hypotenuse line \(AB\) along the Oblique axis having a fixed length \(L_3\). The end-effector represented by point \(P\) having an arbitrary \((x, y, z)\) coordinate is positioned as desired in the three dimensional space by simply actuating prismatic joint \(P_1\) by a length \(l_a\) along Z axis as per the height specified by the z coordinate. The end-effector could be positioned to a maximum height of \(Z_{max}\) which is the length of the vertical pole holding the joint \(P_1\). The planar positioning of the end-effector is done by resolving the \(x, y\) coordinates into the actuation stroke \(OA\) (of length \(l_a\)) on prismatic joint \(P_2\) (at point \(A\)) along the horizontal X axis and stroke \(AP\) (of length \(l_b\)) on prismatic joint \(P_4\) (at point \(E\)) along the Oblique axis. If the fixed length \(L_3 \leq \min(L_1, L_2)\) then the Oblique axis can completely coincide with the perpendicular arms on the \(X\) or the \(Y\) axes when \(P_2\) or \(P_3\) joint actuators are at maximum. For uniformity and symmetry, the arm lengths are kept equal such that \(L_3 \leq L_1 = L_2\). The joint actuation lengths are limited by the mechanical constrains of the configuration. These limits in general can be expressed as follows:

\[
L_3 \leq L_1 = L_2 \\
0 \leq l_a \leq L_3 \\
0 \leq l_b \leq L_3 \\
0 \leq l_c \leq Z_{max}
\]
As mentioned earlier, the end-effector can revolve in the horizontal plane about its vertical axis, but cannot be controlled independently. Thus the kinematic equations described in the following two sections only deal with the translating component of motion without considering the orientation.

3.3. Forward kinematics

The forward kinematic equations determine the unknown end-effector position \( P(x, y, z) \) from the known actuation lengths \( l_a, l_b \) and \( l_c \) which satisfy their constrains in equation (1.2), (1.3) and (1.4) respectively. The equations can be derived by considering the corresponding sides of Pythagorean triangles \( \Delta BDP, \Delta PCA \) and \( \Delta BOA \) from the simple fact that all the three are similar.

\[
\begin{align*}
  x &= l_b (L_3 - l_b) (L_3)^{-1} \\
  y &= l_b |(L_3 y^2 - l_a y^2)|^{1/2} (L_3)^{-1} \\
  z &= l_c
\end{align*}
\]

Thus these forward kinematic equations are useful to determine the position \((x, y, z)\) of the end-effector from known actuation lengths \((l_a, l_b, l_c)\).

3.4. Inverse kinematics

The inverse kinematic equations determine the unknown actuation lengths \( l_a, l_b \) and \( l_c \) from the known or the desired end-effector position \( P(x, y, z) \). Unlike the forward kinematic equations, the inverse kinematic equations are expected to render a set of solutions for one input value of \( P(x, y, z) \). The most suitable solution could be extracted from the set of solutions by subjecting it to certain mathematical conditions. Now, the equation of line \( AB \) in slope-intercept form \([7]\) is:

\[
\frac{x - (OA)}{|(OB)|} + \frac{y - (OB)}{|(OA)|} = 1
\]

By squaring both sides of the equation and rearranging the terms, we get a Quartic equation which can render a set of four possible solutions for \( l_a \) in terms of \( x \) and \( y \) as follows:

\[
l_a^{4} - l_a^{3} (2 x) + l_a^{2} (y^{2} + x^{2} - L_3^{2}) + l_a (2 x L_3 y^{2}) - (L_3 x)^{2} = 0
\]

\[
{x, y, L_3} \in R \\
0 \leq l_a \leq L_3
\]

Here, the variable and constants in the constants in the Quartic polynomial are real since practically all of them represent physical quantities. Similarly, to derive an equation to find \( l_b \) in terms of \( x \) and \( y \), consider the following trigonometric equations \([8]\) and properties:

\[
\sin(\theta) = y (l_b)^{-1} \\
\cos(\theta) = x (L_3 - l_b)^{-1} \\
\sin^{2}(\theta) + \cos^{2}(\theta) = 1
\]

On solving these three equations for \( l_b \) and rearranging the terms, we now get a Quartic equation which can render a set of four possible solutions for \( l_b \) in terms of \( x \) and \( y \) as follows:

\[
l_b^{4} - l_b^{3} (2 L_3) - l_b^{2} (y^{2} + x^{2} - L_3^{2}) + l_b (2 L_3 y^{2}) - (L_3 y)^{2} = 0
\]

\[
{x, y, L_3} \in R \\
0 \leq l_b \leq L_3
\]
For the vertical actuation length \( l_c \) the relation is straightforward as follows:

\[
l_c = z
\]

Thus equations (5), (9) and (10) are the inverse kinematic equations that determine the unknown actuation lengths of its actuators to achieve the desired end-effector position in the three dimensional space. To extract the most suitable solution from the set of four solutions of the Quartic equations, the following mechanical constrains and mathematical conditions \(^9\) are applicable:

- The solutions of \( l_a, l_b \) and \( l_c \) must satisfy the mechanical constrain equations (1.2), (1.3) and (1.4) to be applicable on the physical system.
- The solution numerically closer to the previous actuation lengths actuated by the actuators during any continuous motion is considered if multiple real solutions satisfy the mechanical constrain equations.
- If \( \Delta_D \) is the discriminant of the Quartic equation and \( \Delta_D > 0 \); then either all the four solutions are real or are imaginary. Unless all solutions are imaginary, the real solutions must be further subjected to the mechanical constrains to arrive at the applicable solution.
- If \( \Delta_D < 0 \); then there are two real and distinct solutions with two conjugate complex ones. In this case the complex solutions must be ignored and the real solutions must be further subjected to the mechanical constrains to arrive at the applicable solution.
- If \( \Delta_D = 0 \); then multiple possibilities of solutions are possible which need to be subjected to the mechanical constrains.

3.5. Yet another proof of Semi elliptic configuration

At this point, it is convenient to draw the reader’s attention to yet another justification on why this unique configuration was named Semi elliptic configuration. Consider the equations (6), (7) and (8) mentioned in the previous subsection. By rearranging the terms in equation (8) after substituting the equivalent value of \( \sin(\theta) \) from equation (6) and the value of \( \cos(\theta) \) from equation (7), we get the following equation:

\[
[x^2 (L_3-l_b)^2] + [y^2 (l_b)^2] = 1
\]

(11)

If the term \( l_b \) is a constant and also that \( l_b \leq L_3 \) at all times for a given length \( l_b \) of actuation on the Oblique axis, then the equation (11) is a typical equation of an Ellipse \(^7\). Hence, we conclude that the planar trajectory of the end-effector is that of a portion of an Ellipse, thus the name Semi elliptic!

4. Some applications of Semi elliptic configuration

As mentioned earlier, the configuration is only suitable for applications where the end-effector tool orientation in the horizontal plane is not a concern as it cannot be independently controlled. Some applications enjoying the advantage of having a void region and those others at a disadvantage have been explored in this section. At the end of this section, another configuration derived from the Semi elliptic configuration would be explored which overcomes its inherent limitations.

4.1. 3D printers using Semi elliptic configuration

This configuration can be introduced in applications where simultaneous 3D printing for two designs is to be executed as depicted in figure 8. Existing Cartesian 3D printing bots are capable of sequential printing where multiple models are developed layer by layer sequentially one after the other \(^{10}\). This reduces the pace of printing a single model. The primary limitation is that it consumes more printing time, though occupying the same workspace as that of the new Semi elliptic configuration. With the Semi elliptic configuration, two independently operational Semi elliptic robots can be accommodated at the two opposite corners of the square workspace as shown in figure 9.
The 3D printing extruder mounted on the two identical manipulators deposit molten plastic layer by layer to build two separate models simultaneously instead of sequentially building the two models one after the other. Another form of 3D printer based on the Semi elliptic configuration is shown in figure 10. The two models are being built on a turn table which assists in moving the completed models out of the printing area for collection by a conveyor or exchange the work pieces’ position as per requirement of operation where different materials may have to be layered over the model being built. Different nozzles can be used at a time on the two individual manipulators for printing. The flexibility in the number of nozzles operational at a time is more as compared to a multi extruder 3D printer.

**4.2. Pick-n-place applications**

As shown in figure. 11, the gripper 1 picks the objects from conveyor 1 and places it in the containers placed on the conveyor belt 2. Similarly the gripper 2 also performs the same operation. This increases the productivity of the pick and place operations by reducing the time required and increasing throughput. The sequence of pick and place can be coordinated among the two manipulators. For example both the grippers pick the alternative objects and place them in the containers etc. The activity and mode of operation can be decided by the control computer depending on the workload and pace of the overall assembly line’s conveyors feeding the system. The void region in between the two work spaces can be used to store the unnecessary objects.

**Figure 8.** Two models simultaneously 3D printed.

**Figure 9.** Two independent work spaces.

**Figure 10.** Turn table assisted dual Semi elliptic 3D printer.
volumes could be used to mount object detection cameras and infrared light blasters. The easy integration of such a manipulator into existing assembly lines in the industries is another advantage.

![Figure 11. Pick-n-place system.](image)

Though suitable for applications like these, the presence of a void region within the work space turns out to be a disadvantage, despite being used in a productive way by installing supporting hardware such as cameras and sensors. The void region does not allow the manipulators to perform tasks within them. This calls for some modifications in the proposed Semi elliptic configuration as described in the following section to eliminate the void region and utilize the space as part of the work volume.

5. The Enhanced Semi elliptic configuration

With reference to figure 3 previously shown in this article, the reason why the void region exists within the volume of occupancy of the configuration is that the link along the Oblique axis joining the prismatic joints $P_2$ and $P_3$ on the perpendicular arms is of fixed length. With reference to figure 7, the link segment $AB$ is of fixed length, thus tracing the Semi elliptic boundary of the work area. To work around this drawback, a mechanism must be introduced to allow the fixed length of the Oblique axis link to be variable as needed, in order to enable the end-effector reach within the void regions. Such a variable length link has been employed along the Oblique axis in the derived Semi elliptic configuration named as “Enhanced Semi elliptic configuration” shown in figure 12. The design is such that the variable length link connecting the prismatic joints on the two perpendicular arms carries the end effector which can slide along its whole length as shown in figure 13. The length of the variable link can be varied by independently actuating the prismatic joints $P_2$ and $P_3$ with separate actuators. This was not possible in the original Semi elliptic configuration as the movements of $P_2$ and $P_3$ were locked into inter dependence by the mechanical constrain of the fixed length.
This scheme allows the length of the Oblique axis link to be adjusted in real time as required during operations. The length is adjusted such that the end effector can reach the previously void regions. This feature would alter the kinematics of the system as the variable link length introduces a new variable in determining the motion and path of traversal of the end-effector. With the new variable introduced, the system would become a 4 DOF system as the link length of the variable link must be defined along with the existing three actuation lengths to fully define the position of the end-effector. Thus, the in depth exploration of the kinematics of the Enhanced Semi elliptic configuration is off the
The purpose of this section was to just introduce the solution to the known drawbacks of the Semi elliptic configuration.

6. Conclusion
The exploration of the Semi elliptic configuration to a certain extent (within the scope of this paper) has been described. The topology and features of the proposed configuration has been established and the kinematic equations have been derived. Some real applications of the Semi elliptic configuration were studied and ways to improve the application was discovered. The properties and limitations of the configuration were explored which ultimately led to the conception of an improved configuration derived from the original Semi elliptic configuration called the “Enhanced Semi elliptic configuration”. The working principle and advantages of the Enhanced Semi elliptic configuration has been described in brief. The whole concept, constraints and applications of Semi elliptic configuration has been described with fair amount detail to the reader.

7. References
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