Gravitational wave and collider signals in complex two-Higgs doublet model with dynamical CP-violation at finite temperature

Xiao Wang,1,2 Fa Peng Huang,3 and Xinmin Zhang1,2

1Theoretical Physics Division, Institute of High Energy Physics, Chinese Academy of Sciences, 19B Yuquan Road, Shijingshan District, Beijing 100049, China
2University of Chinese Academy of Sciences, Beijing, China
3Department of Physics and McDonnell Center for the Space Sciences, Washington University, St. Louis, MO 63130, USA

(Dated: September 9, 2019)

Abstract

Dynamical CP-violating source for electroweak baryogenesis can appear only at finite temperature in the complex two-Higgs doublet model, which might help to alleviate the strong constraints from the electric dipole moment experiments. In this scenario, we study the detailed phase transition dynamics and the corresponding gravitational wave signals in synergy with the collider signals at future lepton colliders. For some parameter spaces, various phase transition patterns can occur, such as the multi-step phase transition and supercooling. Gravitational wave in complementary to collider signals can help to pin down the underlying phase transition dynamics or different patterns.
I. INTRODUCTION

After the observation of the gravitational wave (GW) by the Advanced Laser Interferometer Gravitational Wave Observatory [1], a new era of GW astronomy has been initiated and the GW detector provides a new technique to study the fundamental physics. Especially, electroweak (EW) baryogenesis [2-4], which is aimed to explain the baryon asymmetry of the Universe, becomes a promising and testable mechanism after the discovery of GW and Higgs boson. To generate the observed baryon asymmetry of the Universe, all three Sakharov conditions need to be satisfied [5]. These conditions are baryon number violation, C and CP violation, and the departure from the thermal equilibrium or CPT violation. An essential ingredient for a successful EW baryogenesis is the process of a strong first-order phase transition (FOPT) which can achieve the departure from thermal equilibrium. As a by product, the phase transition GW signal induced by a strong FOPT can potentially be detected by the future space-based GW interferometers.

In the standard model (SM), the discovery of Higgs boson by ATLAS [6] and CMS [7] shows that a strong FOPT can not be generated for a 125 GeV Higgs boson based on lattice simulation. It is just a smooth crossover for 125 GeV Higgs boson in the SM. The CP violation is also too weak in the SM. Thus, the extension of the SM are needed to give a strong FOPT and a large enough CP violation for successfully EW baryogenesis. One of the simplest extension of the SM, which is the so-called 2-Higgs Doublet Model (2HDM), is the SM with an additional $SU(2)_L$ scalar doublet, where the sphaleron process was studied in Ref. [8]. However, current electric dipole moments (EDM) experiments [9] have put strong constraints on the CP-violating source at zero temperature for most of the new physics models. In this work, we focus on the complex 2HDM (C2HDM) or the spontaneous CP-violating model. Recent study [10] has shown that there are viable parameter spaces in the C2HDM which can produce a strong FOPT with spontaneous CP violation based on the criterion $v_c/T_c > 1$. They also discuss the collider phenomenology including the Higgs trilinear coupling modification and Higgs boson pair production at hadron collider. Further, Ref. [11] has revisited the constraints from colliders and EDM, and predictions in details. Based on these two comprehensive studies [10, 11], we investigate the phase transition dynamics with different phase transition patterns. Besides the dynamical CP-violating behavior, we also find the multi-step phase transition patterns and supercooling
patterns. The dynamical process might help to provide the CP-violating source for successful EW baryogenesis. We discuss other possible approaches to explore this scenario in C2HDM. On one hand, during a strong FOPT, detectable GWs can be produced by three mechanisms: bubble collisions, sound waves, and magnetohydrodynamic turbulence. Based on the viable parameters from Refs. [10, 11], we discuss the possibility to detect the GW signals by the future space-based experiments, such as the approved Laser Interferometer Space Antenna (LISA) [12] (launch in 2034 or even earlier), Deci-hertz Interferometer Gravitational wave Observatory (DECIGO) [13, 14], Ultimate-DECIGO (U-DECIGO) [15], Big Bang Observer (BBO) [16], Taiji [17, 18], and TianQin [19, 20]. The dynamical CP-violation behavior can escape the strong constrains from electric dipole moment (EDM) measurements [21–24]. On the other hand, the strong FOPT could obviously modify the Higgs trilinear coupling and thus can be tested by the future lepton collider, such as Circular Electron-Positron Collider (CEPC) [25], International Linear Collider (ILC) [26] as well as Future Circular Collider (FCC-ee) [27]. Combined with the GW signals, they can make a complementary test on this scenario and the underlying phase transition patterns.

This paper is organised as follows. In Section II, we describe the C2HDM and the basic idea of dynamical CP-violation at finite temperature. In section III, the one-loop effective potential at finite temperature and the renormalization prescription are presented. In section IV, we investigate the phase transition dynamics including the corresponding GW signals and its correlation with the collider signatures. We discuss the consistent check of the dynamical CP-violation and supercooling case in section V. Section VI contains our conclusions.

II. MODEL WITH DYNAMICAL CP-VIOLATION

The tree-level potential of the C2HDM can be written as

\[ V_{\text{tree}} = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left( m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right) + \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \left( \frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \text{h.c.} \right), \]

(1)

1 In appendix A, we present the thermal correction of the mass for the C2HDM in the Landau gauge. In appendix B, we derive the field dependent mass matrix elements for the gauge bosons, the scalar bosons and the top quark for C2HDM in the Landau gauge.
where $m_{12}^2$ and $\lambda_5$ are complex numbers, and $\text{arg}(\lambda_5) \neq 2\text{arg}(m_{12}^2)$. It is obvious that the C2HDM has a softly broken $Z_2$ symmetry ($\Phi_1 \rightarrow \Phi_1, \Phi_2 \rightarrow -\Phi_2$). At zero temperature, we have

$$\Phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} \rho_1 + i\eta_1 \\ v_1 + \zeta_1 + i\psi_1 \end{pmatrix}, \quad \Phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \rho_2 + i\eta_2 \\ v_2 + \zeta_2 + i\psi_2 \end{pmatrix}. \quad (2)$$

This model has been extensively studied including the EDM constraints and collider phenomenology, such as the recent works [10, 11] and references therein. However, at finite temperature, there would be dynamical CP-violating behavior [10] as

$$\Phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} \rho_1 + i\eta_1 \\ \tilde{v}_1 + \zeta_1 + i\psi_1 \end{pmatrix}, \quad \Phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \tilde{v}_{CB} + \rho_2 + i\eta_2 \\ \tilde{v}_2 + i\tilde{v}_{CP} + \zeta_2 + i\psi_2 \end{pmatrix}. \quad (3)$$

The $\tilde{\phi}$ with tilde represents the VEV at finite temperature. This is the starting point of this work. It means there exists extra CP-violation at high temperature, which might provide the CP-violating source for successful EW baryogenesis. At zero temperature, this extra CP-violating source disappears to escape the severe EDM constraints. To consider more general situation, we also assume there is charge-breaking at high temperature. In Section V, we show the numerical results on the evolution of these CP-violating source with the decreasing of the temperature which confirms the starting point is consistent. Various phase transition patterns can also be trigged based on Eq. (3), which are discussed carefully in next section.

For more compact form, the vacuum expectation value (VEV)s at zero temperature are denoted as

$$\tilde{v}_1(T = 0) = v_1, \quad \tilde{v}_2(T = 0) = v_2, \quad \tilde{v}_{CP}(T = 0) = v_{CP} = 0, \quad \tilde{v}_{CB}(T = 0) = v_{CB} = 0, \quad (4)$$

with this convention,

$$v = \sqrt{v_1^2 + v_2^2 + v_{CP}^2 + v_{CB}^2} = \sqrt{v_1^2 + v_2^2}, \quad (5)$$

where $v \approx 246 \text{ GeV}$ is the SM VEV, and the stationary conditions are

$$\frac{\partial V_{\text{tree}}}{\partial \Phi_i} \bigg|_{\Phi_i = \langle \Phi_i \rangle} = 0, \quad \frac{\partial V_{\text{tree}}}{\partial \Phi_i^\dagger} \bigg|_{\Phi_i = \langle \Phi_i \rangle} = 0, \quad i = 1, 2, \quad (6)$$

and give the following relations

$$m_{11}^2 = \text{Re}(m_{12}^2)\frac{v_2}{v_1} - \frac{v_1^2}{2}\lambda_1 - \frac{v_2^2}{2}\lambda_{345}, \quad (7)$$
\[ m_{22}^2 = \text{Re}(m_{12}^2) \frac{v_1}{v_2} - \frac{v_2^2}{2} \lambda_2 - \frac{v_1^2}{2} \lambda_{345}, \quad (8) \]

\[ \frac{v_1 v_2 \text{Im} (\lambda_5)}{2} = \text{Im} (m_{12}^2), \quad (9) \]

where

\[ \lambda_3 + \lambda_4 + \text{Re} (\lambda_5) \equiv \lambda_{345}. \quad (10) \]

We introduce a mixing angle \( \beta \), which is defined as

\[ \tan \beta = \frac{v_2}{v_1}, \quad (11) \]

then transform the fields into a new basis

\[ \zeta_3 = -\sin \beta \psi_1 + \cos \beta \psi_2, \quad A = \cos \beta \psi_1 + \sin \beta \psi_2. \quad (12) \]

In the C2HDM, the neutral components \( \zeta_1, \zeta_2 \) and \( \zeta_3 \) mix into the neutral mass eigenstates \( H_i (i = 1, 2, 3) \) through the mixing matrix

\[
\begin{pmatrix}
H_1 \\
H_2 \\
H_3
\end{pmatrix}
= R
\begin{pmatrix}
\zeta_1 \\
\zeta_2 \\
\zeta_3
\end{pmatrix}.
\quad (13)
\]

The mixing matrix \( R \) can diagonalize the neutral mass matrix

\[ M_{ij} = \frac{\partial^2 V}{\partial \zeta_i \partial \zeta_j}, \quad (14) \]

and derive

\[ R M R^T = \text{diag}(m_1^2, m_2^2, m_3^2), \quad (15) \]

where \( m_1 \leq m_2 \leq m_3 \) are the masses of the neutral Higgs bosons. We can parameterise the matrix \( R \) as the following [28]

\[
R = \begin{pmatrix}
c_1 c_2 & s_1 c_2 & s_2 \\
-c_1 s_2 s_3 - s_1 c_3 & c_1 c_3 - s_1 s_2 s_3 & c_2 s_3 \\
-c_1 s_2 c_3 + s_1 s_3 & -c_1 s_3 - s_1 s_2 c_3 & c_2 c_3
\end{pmatrix},
\quad (16)
\]

where \( s_i = \sin \alpha_i, c_i = \cos \alpha_i (i = 1, 2, 3) \), and \(-\frac{\pi}{2} \leq \alpha_i < \frac{\pi}{2}\) [10, 11]. Note the above mixing matrix is valid at zero temperature. When we consider the finite-temperature situation in the next section, this result should be modified. The Higgs potential in Eq.(1) has 9
independent parameters. We follow Ref. [29] and choose 9 input parameters \( v, \tan \beta, m_{H^{\pm}}, \alpha_1, \alpha_2, \alpha_3, m_1, m_2, \) and \( \text{Re}(m_{12}^2) \). For these input parameters, \( m_3 \) can be expressed as

\[
m_3^2 = \frac{m_{13}^2 R_{13}(R_{12} \tan \beta - R_{11}) + m_2^2 R_{23}(R_{22} \tan \beta - R_{21})}{R_{33}(R_{31} - R_{32} \tan \beta)}. \tag{17}
\]

The analytic relations between the above parameter set and the coupling parameters in the original lagrangian can be written as [30]

\[
\begin{align*}
\lambda_1 &= \frac{1}{v^2 \cos^2 \beta} \left[ m_1^2 c_1^2 c_2^2 + m_2^2 (c_3 s_1 + c_1 s_2 s_3)^2 + m_3^2 (c_1 c_3 s_2 - s_1 s_3)^2 - \mu^2 \sin^2 \beta \right], \\
\lambda_2 &= \frac{1}{v^2 \sin^2 \beta} \left[ m_1^2 s_1^2 c_2^2 + m_2^2 (c_3 c_1 - s_1 s_2 s_3)^2 + m_3^2 (s_3 s_1 s_2 + c_1 s_3)^2 - \mu^2 \cos^2 \beta \right], \\
\lambda_3 &= \frac{1}{v^2 \sin \beta \cos \beta} \left[ (m_1^2 c_2^2 + m_2^2 (s_2 s_3 - c_3^2) + m_3^2 (s_2 c_3^2 - s_3^2)) c_1 s_1 \\
&\quad + (m_3^2 - m_2^2) (c_1^2 - s_1^2) s_2 c_3 s_3 \right] - \frac{\mu^2 - 2 m_{H^{\pm}}^2}{v^2}, \\
\lambda_4 &= \frac{m_1^2 s_2^2 + (m_2^2 s_3^2 + m_3^2 c_3^2) c_2^2 + \mu^2 - 2 m_{H^{\pm}}^2}{v^2}, \\
\text{Re}(\lambda_5) &= -\frac{m_1^2 s_2^2 - (m_2^2 s_3^2 + m_3^2 c_3^2) c_2^2 + \mu^2}{v^2}, \\
\text{Im}(\lambda_5) &= \frac{2 c_2}{v^2 \sin \beta} \left[ (-m_1^2 + m_2^2 s_3^2 + m_3^2 c_3^2) c_1 s_2 + (m_2^2 - m_3^2) s_1 s_3 c_3 \right], \tag{18}
\end{align*}
\]

where

\[
\mu^2 = \frac{v^2}{v_1 v_2} \text{Re}(m_{12}^2). \tag{19}
\]

In general, 2HDM can be classified into type I, type II, lepton-specific and flipped, according to the interactions of the fermions to the Higgs doublets. In this work we only study type I case, and only consider the top quark’s contribution to the EW phase transition among all the fermions.

### III. PHASE TRANSITION DYNAMICS AND CP-VIOLATION AT FINITE TEMPERATURE

To study the phase transition dynamics in the C2HDM, we use the finite-temperature effective field theory [31,33]. The full one-loop finite-temperature effective potential reads

\[
V_{\text{eff}}(\tilde{v}, T) \equiv V_{\text{tree}}(\tilde{v}) + V_{\text{CW}}(\tilde{v}) + V_{\text{CT}}(\tilde{v}) + V_{T}(\tilde{v}, T), \tag{20}
\]
where $V_{\text{tree}}$, which is obtained by replacing the doublets with their classical background fields $(\tilde{v}_1, \tilde{v}_2, \tilde{v}_{CP}, \tilde{v}_{CB})$ from Eq. (3), is the tree-level potential at zero temperature as shown in the following

$$V_{\text{tree}}(\tilde{v}) = \frac{1}{2}m_{11}^2\tilde{v}_1^2 + \frac{1}{2}m_{22}^2(\tilde{v}_2^2 + \tilde{v}_{CP}^2 + \tilde{v}_{CB}^2) - Re(m_{12}^2\tilde{v}_1\tilde{v}_2 + Im(m_{12}^2\tilde{v}_1\tilde{v}_{CP} + \frac{1}{8}\lambda_1\tilde{v}_1^4
+ \frac{1}{8}\lambda_2(\tilde{v}_2^2 + \tilde{v}_{CP}^2 + \tilde{v}_{CB}^2)^2 + \frac{1}{4}\lambda_3\tilde{v}_1^2(\tilde{v}_2^2 + \tilde{v}_{CP}^2 + \tilde{v}_{CB}^2) + \frac{1}{4}\lambda_4\tilde{v}_1^2(\tilde{v}_2^2 + \tilde{v}_{CP}^2)
+ \frac{1}{4}Re(\lambda_5)\tilde{v}_1^2(\tilde{v}_2^2 - \tilde{v}_{CP}^2) - \frac{1}{2}Im(\lambda_5)\tilde{v}_1^2\tilde{v}_2\tilde{v}_{CP}.
$$

(21)

$V_{\text{CW}}$ is the Coleman-Weinberg potential (CW) at zero temperature. In the $\overline{\text{MS}}$ scheme, the CW potential can be written as

$$V_{\text{CW}}(\tilde{v}) = \frac{1}{64\pi^2}\sum_s n_s m_s^4(\tilde{v}) \left[ \log \frac{m_s^2(\tilde{v})}{\mu^2} - C_s \right],$$

(22)

where $\tilde{v} \equiv \{\tilde{v}_1, \tilde{v}_2, \tilde{v}_{CP}, \tilde{v}_{CB}\}$, and $m_s^2(\tilde{v})$ is the eigenvalue for the particle $s$ in the mass matrix in terms of the background fields $\tilde{v}$, details see Appendix B. $n_s$ denotes the numbers of the degree of freedom. Because of the charge-breaking VEV, photon becomes massive. And we have to take into account different masses and numbers of degree of freedom for the charge conjugated particles. For each particle $s$, the numbers of degree of freedom are $\{n_{H_1}, n_A, n_{H^+}, n_{H^-}, n_{G^+}, n_{G^-}, n_{W^+}, n_{W^-}, n_Z, n_\gamma, n_t, n_{\bar{t}}\} = \{1, 1, 1, 1, 1, 3, 3, 3, 3, 3, -6, -6\}$ and the constants $C_s$ are

$$C_s = \begin{cases} 
\frac{5}{6}, & s = W^\pm, Z, \gamma \\
\frac{3}{2}, & \text{others}
\end{cases}.$$

(23)

The masses and the mixing angles with one-loop corrections are different from those extracted from the tree-level potential. To enforce the one-loop corrected masses and the mixing angles to be equal to the tree-level values, we use the on-shell renormalisation prescription as in Refs. [10, 34, 35]. Then, a counterterm potential $V_{\text{CT}}$ is added to the one-loop effective potential. The general formula of the counterterm contribution $V_{\text{CT}}$ reads

$$V_{\text{CT}} = \sum_{i=1}^n \partial V_{\text{tree}} / \partial p_i \delta p_i + \sum_{k=1}^m \delta T_k(\phi_k + \tilde{v}_k),$$

(24)

where $\delta p_i$ and $n$ are the counterterms and the number of parameters of the tree-level potential, respectively. $\delta T_k$ denotes the counterterms of the tadpole $T_k$, and $m$ is the number
of background field or the number of field that is allowed for the development of a non-zero VEV. In the C2HDM, the counterterm potential can be written as

\[ V_{CT} = \delta m_{11}^2 \Phi_1^4 + \delta m_{22}^2 \Phi_2^4 - [(\delta Re(m_{12}^2) + i \delta Im(m_{12}^2))\Phi_1^4 \Phi_2 + \text{h.c.}] + \frac{1}{2} \delta \lambda_1 (\Phi_1^4)^2 + \frac{1}{2} \delta \lambda_2 (\Phi_2^4)^2 + \delta \lambda_3 (\Phi_1^4)(\Phi_2^4) + \delta \lambda_4 (\Phi_1^4)(\Phi_2^4) + \frac{1}{2} [(\delta Re(\lambda_5) + i \delta Im(\lambda_5))(\Phi_1^4)^2 + \text{h.c.}] + \delta T_1 (\zeta_1 + \tilde{\nu}_1) + \delta T_2 (\zeta_2 + \tilde{\nu}_2) + \delta T_{CP} (\psi_2 + \tilde{\nu}_{CP}) + \delta T_{CB} (\rho_2 + \tilde{\nu}_{CB}). \]  

(25)

The on-shell renormalisation conditions at zero temperature are

\[ \partial_{\phi_i} V_{CW}(\phi) \bigg|_{\phi = \langle \phi^c \rangle_{T=0}} + \partial_{\phi_i} V_{CT}(\phi) \bigg|_{\phi = \langle \phi^c \rangle_{T=0}} = 0, \]

\[ \partial_{\phi_i} \partial_{\phi_j} V_{CW}(\phi) \bigg|_{\phi = \langle \phi^c \rangle_{T=0}} + \partial_{\phi_i} \partial_{\phi_j} V_{CT}(\phi) \bigg|_{\phi = \langle \phi^c \rangle_{T=0}} = 0, \]

(26)

where

\[ \phi_i \equiv \{ \rho_1, \eta_1, \rho_2, \eta_2, \zeta_1, \psi_1, \zeta_2, \psi_2 \}, \]  

(27)

\[ \langle \phi^c \rangle_{T=0} = \{ 0, 0, 0, 0, v_1, 0, v_2, 0 \}. \]  

(28)

The second derivatives of the CW potential lead to the well-known problem of infrared (IR) divergences for the Goldstone bosons \[36\, \[39\] in the Landau gauge. In practice, we can introduce an IR regulator for the Goldstones and then discard the terms proportional to the IR divergence. Previous study \[36\] has dealt with this problem and derived analytic formulae for the first and second derivatives of the CW potential in the physical basis

\[ \partial_{\phi_i} V_{CW}(\phi) \bigg|_{\phi = \langle \phi^c \rangle_{T=0}} = O^H_{ij} \sum_s \frac{(-1)^{x_s}(1 + x_s)}{32\pi^2} m_{(s)a}^2 \lambda_{(s)ab} \left( \log \frac{m_{(s)a}^2}{\mu^2} - C_s + \frac{1}{2} \right), \]  

(29)

\[ \partial_{\phi_i} \partial_{\phi_j} V_{CW}(\phi) \bigg|_{\phi = \langle \phi^c \rangle_{T=0}} = o^H_{ij} \sum_s \frac{(-1)^{x_s}(1 + x_s)}{32\pi^2} S_{ij} \left[ \lambda_{(s)abj} \lambda_{(s)ba} \left( f^{(1)}_{(s)ab} - C_s + \frac{1}{2} \right) \right] \]

\[ + \lambda_{(s)ab} m_{(s)a}^2 \left( \log \frac{m_{(s)a}^2}{\mu^2} - C_s + \frac{1}{2} \right), \]

(30)

with

\[ f^{(1)}_{(s)ab} = \sum_{x=1}^2 \frac{m_{(s)a_x}^2 \log \frac{m_{(s)a_x}^2}{\mu^2}}{\prod_{y \neq x} \left( m_{(s)a_x}^2 - m_{(s)a_y}^2 \right)}, \]  

(31)
where $\chi_s$ is the spin of different particles, $m^2_{(s)a}$ is the physical mass of particle $s$ at zero temperature, $O^H_{ij}$ is the rotation matrix that transform scalar fields from Landau gauge basis to mass eigenstate basis, $S_{ij}$ denotes symmetrisation with respect to the two indices, $\lambda_{(s)abi}$ and $\lambda_{(s)abij}$ are the cubic and quartic couplings for particle $s$ in mass eigenstate basis. Note that we need to deal with degenerate mass limit carefully in Eq. (31), for more detail, see Ref. [36]. Then the counterterms can be expressed in terms of the derivatives of the CW potential. For the analytic formulae of the counterterms, see Refs. [10, 35]. $V_T$ is the one-loop thermal corrections including daisy resummation [40, 41] at finite temperature. The thermal correction reads

$$V_T = \sum F \frac{T^4}{2\pi^2} n_F J_F \left( \frac{m^2_F}{T^2} \right) + \sum B \frac{T^4}{2\pi^2} n_B J_B \left( \frac{m^2_B}{T^2} \right),$$

with the thermal functions

$$J_{B/F} = \int_0^\infty dx x^2 \log \left[ 1 \pm e^{-\sqrt{x^2+m^2_i/T^2}} \right],$$

where the plus sign is for fermions and the minus sign is for bosons. In order to include the contribution of daisy resummation, we make the following replacement for the scalar boson mass and the longitudinal components of the gauge boson mass

$$m^2_B \rightarrow \overline{m}^2_B = m^2_B + \Pi_B,$$

where $\Pi_B$ is the thermal correction of the scalar boson and the longitudinal components of gauge boson at finite temperature, which can be found in Appendix A. The Debye corrected masses are applied in the all terms of $J_B$ and also used in the CW potential [41]. It is worth noticing that Parwani scheme is used in this work, while Arnold-Espinosa scheme is used in Ref. [10].

With the full effective potential in Eq. (20), we can use the method that is introduced below to numerically calculate the phase transition dynamics. From the comprehensive studies of the C2HDM [10], we know there are viable parameter space to induce a strong FOPT. According to the Ref. [10], we scan the viable parameter space within the allowed parameter spaces by the current collider and EDM constraints from Ref. [11]. Here, for simplicity, we only show 12 benchmark points which can induce various representative phase transition patterns, and we choose $H_1$ to be the SM Higgs boson. Multi-step FOPTs, supercooling and second-order phase transition (SOPT) can occur.
In Tab. I, we show 8 benchmark sets. Each parameter set can give a one-step strong FOPT, and the FOPT take place as \((0, 0, 0, 0) \xrightarrow{\text{FOPT}} (\tilde{v}_1, \tilde{v}_2, \tilde{v}_{CP}, \tilde{v}_{CB}) \xrightarrow{T \to 0} (v_1, v_2, 0, 0)\) with the temperature decreasing from high value to zero. Only one strong FOPT happens for these benchmark sets.

In Tab. II, two parameter sets are shown. Each benchmark set can induce two FOPTs and they evolve as \((0, 0, 0, 0) \xrightarrow{\text{FOPT}} (\tilde{v}_1, \tilde{v}_2, \tilde{v}_{CP}, \tilde{v}_{CB}) \xrightarrow{\text{FOPT}} (\tilde{v}_1, \tilde{v}_2, \tilde{v}_{CP}, \tilde{v}_{CB}) \xrightarrow{T \to 0} (v_1, v_2, 0, 0)\) with the temperature decreasing from high value to zero.

Three-step phase transition can be produced for the two benchmark sets in Tab. III. And they evolve like \((0, 0, 0, 0) \xrightarrow{\text{SOPT}} (\tilde{v}_1, \tilde{v}_2, \tilde{v}_{CP}, \tilde{v}_{CB}) \xrightarrow{\text{FOPT}} (\tilde{v}_1, \tilde{v}_2, \tilde{v}_{CP}, \tilde{v}_{CB}) \xrightarrow{\text{FOPT}} (\tilde{v}_1, \tilde{v}_2, \tilde{v}_{CP}, \tilde{v}_{CB}) \xrightarrow{T \to 0} (v_1, v_2, 0, 0)\) with the temperature decreasing from high value to zero. For these two benchmark sets, two FOPTs and one SOPT happen.

| \(v\) [GeV] | \(m_1\) [GeV] | \(m_2\) [GeV] | \(m_{H^\pm}\) [GeV] | \(\text{Re}(m_{12}^2)\) [GeV^2] | \(\alpha_1\) | \(\alpha_2\) | \(\alpha_3\) | \(\tan \beta\) |
|-------------|-------------|-------------|----------------|-----------------|-----------|-----------|-----------|-------|
| BP_1        | 246         | 125.09      | 356.779        | 581.460         | 29939     | 1.470     | 0.0223    | -0.097 | 4.17   |
| BP_2        | 246         | 125.09      | 603.699        | 629.564         | 73628     | 0.817     | 3.687 \times 10^{-3} | -1.557 | 1.216  |
| BP_3        | 246         | 125.09      | 455.834        | 685.479         | 85376     | 0.880     | -0.0156   | 1.568  | 1.399  |
| BP_4        | 246         | 125.09      | 458.834        | 683.632         | 85376     | 0.880     | -0.0156   | 1.568  | 1.399  |
| BP_5        | 246         | 125.09      | 490.698        | 525.220         | 20392     | 0.932     | 0.0101    | -0.514 | 1.608  |
| BP_6        | 246         | 125.09      | 485.698        | 530.220         | 20392     | 0.932     | 0.0101    | -0.514 | 1.608  |
| BP_7        | 246         | 125.09      | 495.698        | 525.220         | 20192     | 0.932     | 0.0101    | -0.514 | 1.608  |
| BP_8        | 246         | 125.09      | 481.698        | 533.220         | 20192     | 0.932     | 0.0101    | -0.514 | 1.608  |

**TABLE I:** One-step phase transition benchmark points.

| \(v\) [GeV] | \(m_1\) [GeV] | \(m_2\) [GeV] | \(m_{H^\pm}\) [GeV] | \(\text{Re}(m_{12}^2)\) [GeV^2] | \(\alpha_1\) | \(\alpha_2\) | \(\alpha_3\) | \(\tan \beta\) |
|-------------|-------------|-------------|----------------|-----------------|-----------|-----------|-----------|-------|
| BP_9        | 246         | 125.09      | 430.698        | 500.220         | 20192     | 0.832     | 0.0101    | -0.514 | 1.458  |
| BP_10       | 246         | 125.09      | 440.698        | 500.220         | 20092     | 0.832     | 0.0101    | -0.514 | 1.458  |

**TABLE II:** Two-step phase transition benchmark points with two FOPTs.

To obtain the parameter sets in the above Tables, we need to know the bubble dynamics during the phase transition process. The essential quantity of bubble dynamics is the bubble nucleation rate per unit time per unit volume

\[
\Gamma = \Gamma_0 e^{-S_f},
\]
where $S_E(T) = S_3/T$ is the Euclidean action of a critical bubble and $\Gamma_0 \propto T^4$. $S_3$ is the three-dimensional Euclidean action, which can be denoted as

$$S_3 = 4\pi \int drr^2 \left[ \frac{1}{2} \left( \frac{d\tilde{v}_i}{dr} \right)^2 + V_{\text{eff}}(\tilde{v}_i, T) \right],$$

(36)

where $\tilde{v}_i = \{\tilde{v}_1, \tilde{v}_2, \tilde{v}_{CB}, \tilde{v}_{CP}\}$. To calculate the nucleation rate, we need to obtain the bubble profiles of the four scalar fields by solving the following bounce equations

$$\frac{d^2\tilde{v}_i}{dr^2} + \frac{2}{r} \frac{d\tilde{v}_i}{dr} = \frac{\partial V_{\text{eff}}}{\partial \tilde{v}_i}, \quad i = 1, 2, CP, CB,$$

(37)

with the boundary conditions

$$\lim_{r \to \infty} \tilde{v}_i = \tilde{v}_f, \quad \frac{d\tilde{v}_i}{dr} \bigg|_{r=0} = 0,$$

(38)

where $\tilde{v}_f$ is the false VEVs. Conventionally, we use the so-called overshooting (undershooting) method \[42, 43\] to solve the single-field bounce equation. However, the multi-field case becomes much more complicated. We use the path deformation method, which is introduced by Ref. \[44\], to find a proper path that connects the initial and final vacuum state. In our analysis, we make use of the public available package \textit{cosmoTransitons} to solve the four differential bounce equations. Then the nucleation temperature $T_n$ is defined as the temperature at time $t_n$ at which $\Gamma$ becomes large enough to nucleate a bubble per horizon volume with the probability is $O(1)$,

$$\int_0^{t_n} dt \frac{\Gamma}{H^3} \simeq 1,$$

(39)

where $H$ is the Hubble parameter. In other words, this condition can be simplified as

$$\frac{S_3(T_n)}{T_n} = 4\ln \left( \frac{T_n}{100\text{GeV}} \right) + 137.$$  

(40)

The properties of the bubbles are illustrated by two key parameters $\alpha$ and $\beta$. Note $\alpha$ is the ratio of the latent heat $\epsilon(T_n)$ to the energy density of the radiation bath $\rho_{\text{rad}}$. It is defined as

$$\alpha = \frac{\epsilon(T_n)}{\rho_{\text{rad}}(T_n)},$$

(41)
where \( \rho_{\text{rad}}(T) = g_\star \pi^2 T^4 / 30 \), and \( g_\star \) is the number of the relativistic degree of freedom in the thermal plasma at \( T \). And \( \epsilon(T_n) \) can be written as

\[
\epsilon(T_n) = \left[ -V_{\text{eff}}(\phi, T) + T \frac{\partial V_{\text{eff}}(\phi, T)}{\partial T} \right] \bigg|_{T=T_n} .
\] (42)

Moreover, the parameter \( \beta \) is defined as

\[
\beta \equiv -\frac{dS_E}{dt} \bigg|_{t=t_n} \simeq \frac{1}{\Gamma} \frac{d\Gamma}{dt} \bigg|_{t=t_n} .
\] (43)

However, in the actual calculations, the renormalised parameter \( \tilde{\beta} \) is more convenient:

\[
\tilde{\beta} = T_n \frac{d}{dT} \left( \frac{S_3(T)}{T} \right) \bigg|_{T=T_n} .
\] (44)

The parameter \( \alpha \) describe the strength of the phase transition, namely, the larger value of \( \alpha \) corresponds to a stronger phase transition. In addition, the inverse of the parameter \( \beta \) is related to the time scale of phase transition. Based on the above approaches, we can numerically know the phase transition dynamics and calculate the phase transition parameters of all of the benchmark point sets.

**IV. COLLIDER AND GRAVITATIONAL WAVE SIGNATURES**

After the three parameters \( \alpha, \tilde{\beta} \) and \( T_n \) are extracted from the finite-temperature effective potential, we can predict the phase transition GW signals which are produced by three mechanisms: bubbles collisions, sound waves, and magnetohydrodynamic turbulence in the plasma after collisions. Based on the envelope approximation \([45-48]\), the numerical simulation gives the formula of the GW spectrum from bubble collisions \([49-51]\):

\[
h^2 \Omega_{\text{co}}(f) \simeq 1.67 \times 10^{-5} \tilde{\beta}^{-2} \left( \frac{\kappa \alpha}{1+\alpha} \right)^2 \left( \frac{100 g_\star}{g_\star} \right)^{\frac{1}{3}} \left( \frac{0.11v_b^3}{0.42 + v_b^2} \right)^2 \frac{3.8(f/f_{\text{co}})^{2.8}}{1 + 2.8(f/f_{\text{co}})^{3.8}},
\] (45)

where \( g_\star \) is the total number of degrees of freedom at \( T_n \). The coefficient \( \kappa \), which denotes the fraction of the latent heat transformed into the fluid kinetic energy, is function of \( \alpha \) \([48]\). The bubble wall velocity \( v_b \) can be calculated using the following formula \([48]\):

\[
v_b = \frac{1/\sqrt{3} + (\alpha^2 + 2\alpha/3)^{1/2}}{1 + \alpha} .
\] (46)
The peak frequency is

\[ f_{co} \simeq 1.65 \times 10^{-5} \text{Hz} \left( \frac{0.62}{1.8 - 0.1v_b + v_b^2} \right) \tilde{\beta} \left( \frac{T_n}{100 \text{GeV}} \right) \left( \frac{g_*}{100} \right)^{1/6} . \]  

(47)

The second source is generated by the sound waves of the bulk motion, and numerical simulation gives \([52, 53]\)

\[ h^2 \Omega_{sw}(f) \simeq 2.65 \times 10^{-6} \tilde{\beta}^{-1} \left( \frac{\kappa_v \alpha}{1 + \alpha} \right)^2 \left( \frac{100}{g_*} \right)^{1/3} v_b \left( \frac{f/f_{sw}}{} \right)^{3} \left( \frac{7}{4 + 3(f/f_{sw})^2} \right)^{7/2} , \]  

(48)

with the peak frequency

\[ f_{sw} \simeq 1.9 \times 10^{-5} \text{Hz} \left( \frac{T_n}{100 \text{GeV}} \right) \left( \frac{g_*}{100} \right)^{1/6} . \]  

(49)

The turbulence contribution to the GW spectrum is \([54, 55]\)

\[ h^2 \Omega_{turb}(f) \simeq 3.35 \times 10^{-4} \tilde{\beta}^{-1} \left( \frac{\kappa_{turb} \alpha}{1 + \alpha} \right)^{3/2} \left( \frac{100}{g_*} \right)^{1/3} v_b \left( \frac{f/f_{turb}}{} \right)^{3} \left( 1 + f/f_{turb} \right)^{11/3} \left( 1 + 8\pi f/h_* \right) , \]  

(50)

with the peak frequency

\[ f_{turb} \simeq 2.7 \times 10^{-5} \text{Hz} \left( \frac{T_n}{100 \text{GeV}} \right) \left( \frac{g_*}{100} \right)^{1/6} , \]  

(51)

and

\[ h_* = 1.65 \times 10^{-5} \text{Hz} \left( \frac{T_n}{100 \text{GeV}} \right) \left( \frac{g_*}{100} \right)^{1/6} . \]  

(52)

Note that, for relativistic bubbles

\[ \kappa_v \simeq \frac{\alpha}{0.73 + 0.083\sqrt{\alpha + \alpha}} , \]  

(53)

and \( \kappa_{turb} \simeq 0.1\kappa_v \).

Combined the three contributions, we show the numerical results of the total GW spectra in the C2HDM for the above benchmark points. Strong GW signal favors supersonic bubble wall velocity. However, the EW baryogenesis prefers subsonic bubble wall velocity. Actually, the bubble wall velocity obtained from Eq. \([46]\) is not accurate enough here since these formula is obtained in the simplest scalar model. It is still possible that the real bubble wall velocity in this model is smaller than the velocity of sound wave for non-supercooling case. To tell the difference between two velocities, we show the GW spectrum of the same benchmark sets with a bubble wall velocity calculated by Eq. \([46]\) and a fixed input subsonic velocity \( v_b = 0.5 \).
Since the GW given by the supercooled phase transition is still controversial, we need a more detailed study. It is worthy noticing that the above formulae of the GW spectrum for the three sources, which is given by numerical simulation, are based on a rapid phase transition process and $\alpha < 1$. Since a supercooling FOPT may induce a longer and stronger transitions [56, 57], it is not clear whether these formulae are applicable to this situation. Therefore, we just give the GW spectra of the benchmark points without supercooling.

In Fig. 1 for the supersonic bubble wall velocity case, the GW spectrum of benchmark sets $BP_1$, $BP_2$ can be certainly detected by DECIGO, U-DECIGO, and BBO. The GW spectrum of benchmark point $BP_2$ can also be certainly detected by LISA and Taiji. For the subsonic bubble wall velocity case, the GW spectrum of $BP_1$ and $BP_2$ can also certainly be detected by DECIGO, U-DECIGO, and BBO. However, only $BP_2$ can be marginally detected by LISA for subsonic case.

In Fig. 2 for the supersonic case, we can see both of the GW signals generated by the two benchmark sets $BP_5$, $BP_6$ can be certainly detected by LISA, DECIGO, U-DECIGO, and BBO. And the spectrum of $BP_5$ can only be marginally observed by Taiji. For the subsonic case, the signals become slightly weaker, which makes that $BP_5$ becomes difficult to be detected by LISA and Taiji.

Fig. 3 presents the GW spectra for two-step FOPTs. Even though the signals are not strong enough compared to the current GW detection proposals, they are still intriguing phase transition patterns. They can produce two copies of GW signals with different peak frequencies [58–60]. Their signals are different from the one-step FOPT as shown in Fig. 1 and 2 where exists only one copy of GW signals for given benchmark sets.

Fig. 4 shows the GW spectra of the benchmark points with three-step phase transition. For the supersonic and the subsonic case, both of these parameter set indicate that the first FOPT of the three-step phase transition is much more weaker than the second one, and the GW signals induced by the first FOPT can not reach the sensitivity of space based GW experiments. For the supersonic case, the GW signals, which are produced by the second FOPT, of $BP_{11}$ and $BP_{12}$ can be certainly detected by BBO and U-DECIGO. DECIGO can only marginally detect the GW signals from $BP_{11}$ and $BP_{12}$. From the GW spectra, we find the bubble wall velocity have small effects on the strength of the GW signals. Since the sound wave give the dominant contribution to the GW spectrum, Eq. (48) shows that it linearly depends on the bubble wall velocity. For the supersonic and the subsonic case, it
gives small modifications to the GW signals.

FIG. 1: The GW spectra of the one-step phase transition for $BP_1$ and $BP_2$. The color shaded regions correspond to the expected sensitivity of the GW interferometers LISA, DECIGO, U-DECIGO, BBO, Taiji, and TianQin, respectively. Left: The black and the blue line denote the GW spectrum of $BP_1$ and $BP_2$ with the bubble wall velocity determined by Eq. (46). Right: The black and the blue line denote the GW spectrum of $BP_1$ and $BP_2$ with the bubble wall velocity given by hand as $v_b = 0.5$.

FIG. 2: The GW spectra of the one-step phase transition for $BP_5$ and $BP_6$. Left: The black line and the blue line denote the GW spectrum of $BP_5$ and $BP_6$ with bubble velocity calculated by Eq. (46), respectively. Right: The black line and the blue line denote the GW spectrum of $BP_5$ and $BP_6$ with bubble velocity given by hand as $v_b = 0.5$.

Besides the detectable GW signals, the strong FOPT could also induce obvious deviation
FIG. 3: The GW spectra of the two-step phase transition for $BP_9$ and $BP_{10}$. Top Left: The black line and the blue line denote the GW spectrum of the first and the second FOPT of $BP_9$ with bubble wall velocity calculated by Eq. (46). Top Right: The black line and the blue line denote the GW spectrum of the first and the second FOPT of $BP_9$ with a bubble wall velocity $v_b = 0.5$. Bottom Left: The black line and the blue line represent the GW spectrum of the first and the second FOPT of $BP_{10}$ with bubble wall velocity that is determined by Eq. (46). Bottom Right: The black line and the blue line show the GW spectrum of the first and the second FOPT of $BP_{10}$ with bubble wall velocity $v_b = 0.5$.

of Higgs trilinear coupling compared to the SM as the following

$$\mathcal{L}_{hhh} = -\frac{1}{3!}(1 + \delta_h)A_h h^3.$$  

(54)

In Tab. IV Tab. V and Tab. VI we show the deviation of the Higgs trilinear coupling for each benchmark set. The deviation of Higgs trilinear coupling $\delta_h$ from SM roughly varies from 1.049 to 1.863 at one-loop level for these benchmark points by BSMPT [10, 35].
FIG. 4: The GW spectra of the three-step phase transition for $BP_{11}$ and $BP_{12}$. Top Left: The black line and blue line depict the GW spectrum of the first FOPT and the second FOPT of $BP_{11}$ with a bubble wall velocity calculated by Eq. (46). Top Right: The black and blue line denote the GW spectrum of the first FOPT and the second FOPT of $BP_{11}$ with a bubble wall velocity $v_b = 0.5$. Down Left: The black and blue line denote the GW spectrum of the first FOPT and the second FOPT of $BP_{12}$ with a bubble wall velocity calculated by Eq. (46). Down Right: The black and blue line denote the GW spectrum of the first FOPT and the second FOPT of $BP_{12}$ with a bubble wall velocity $v_b = 0.5$.

LHC, it may be not easy to pin down this deviation. However, the significant modification of Higgs trilinear coupling can be measured by the Higgs pair production at future hadron collider. This obvious deviation can also modify the cross section of $e^+e^- \rightarrow ZH$ process at one-loop level. Therefore, it can be indirectly tested by the precise measurements of the cross section for the Z boson and Higgs boson associated production at the future lepton collider, such as CEPC or ILC and FCC-ee [61–65]. The deviation of the ZH cross section
can be defined as
\[ \delta(ZH) = \frac{\sigma_{2HDM}^{ZH}}{\sigma_{SM}^{ZH}} - 1 \]  
(55)

At 240 GeV CEPC with 5.6 $ab^{-1}$ integrated luminosity, the estimated precision of $\sigma_{HZ}$ is about 0.5%, which means all the benchmark sets are within the sensitivity of CEPC \[25\].

The sensitivity for FCC-ee is about 0.4%. The corresponding numerical results for each benchmark set are shown in Tab. IV, Tab. V and Tab. VI. In the Tables, each benchmark set corresponds to $\alpha, \beta, T_n$ (they determine the GW signal) and $\delta_h$ (it determines the collider signal), which means the GW signal and collider signal are correlated by the EW phase transition physics. Therefore, the future lepton colliders in complementary to GW experiments \[61-65\] can help to unravel different phase transition dynamics. Namely, these two complementary experiments can help us to understand whether the phase transition process is one-step FOPT, or two-step FOPTs or even three-step phase transitions in the early universe.

| pattern | $T_n$ [GeV] | $\epsilon(T_n)$[GeV$^4$] | $v_b$ | $\alpha$ | $\beta$ | $\delta_h$@one-loop | $\delta(ZH)$ |
|---------|-------------|----------------|-------|----------|--------|-----------------|-------------|
| $BP_1$ 1-step | 59.653 | $6.892 \times 10^7$ | 0.825 | 0.192 | 648.048 | 1.135 | 1.816% |
| $BP_2$ 1-step | 45.291 | $4.493 \times 10^7$ | 0.875 | 0.376 | 630.773 | 1.338 | 2.141% |
| $BP_3$ 1-step | 25.964 | $2.771 \times 10^7$ | 0.964 | 2.149 | 471.699 | 1.677 | 2.684% |
| $BP_4$ 1-step | 23.644 | $2.714 \times 10^7$ | 0.974 | 3.060 | 414.956 | 1.711 | 2.737% |
| $BP_5$ 1-step | 40.912 | $2.954 \times 10^7$ | 0.874 | 0.372 | 915.233 | 1.652 | 2.643% |
| $BP_6$ 1-step | 36.639 | $2.61 \times 10^7$ | 0.895 | 0.510 | 313.287 | 1.672 | 2.674% |
| $BP_7$ 1-step | 26.529 | $2.121 \times 10^7$ | 0.952 | 1.509 | 100.331 | 1.720 | 2.752% |
| $BP_8$ 1-step | 27.621 | $2.188 \times 10^7$ | 0.947 | 1.325 | 81.825 | 1.680 | 2.687% |

TABLE IV: Correlation between the GW parameters ($\alpha, \beta, T_n$) and the collider parameter (the modification of Higgs trilinear coupling at one loop $\delta_h$) for the one-step phase transition pattern. $\delta(ZH)$ represents the corresponding loop-induced modification of ZH cross section at 240 GeV CEPC.
TABLE V: Correlation between the GW parameters \((\alpha, \beta, T_n)\) and the collider parameter (the modification of Higgs trilinear coupling at one loop \(\delta_h\)) for the two-step phase transition pattern (two consecutive FOPTs at different temperature). \(\delta(ZH)\) represents the corresponding loop-induced modification of ZH cross section at 240 GeV CEPC.

| pattern | \(T_n\) [GeV] | \(\epsilon(T_n)\) [GeV\(^4\)] | \(v_b\) | \(\alpha\) | \(\tilde{\beta}\) | \(\delta_h\) @ one-loop | \(\delta(ZH)\) |
|---------|----------------|------------------|--------|--------|-------|------------------|---------|
| \(BP_9\) 2-step | 96.995 | \(1.532 \times 10^7\) | 0.638 | 0.00610 | 107292.81 | 1.049 | 1.678% |
|         | 93.997 | \(4.077 \times 10^7\) | 0.677 | 0.018 | 1279659.55 |             |         |
| \(BP_{10}\) 2-step | 93.462 | \(2.56 \times 10^7\) | 0.659 | 0.0118 | 20542.25 | 1.104 | 1.766% |
|         | 91.920 | \(4.892 \times 10^7\) | 0.690 | 0.0241 | 479401.89 |             |         |

TABLE VI: Correlation between the GW parameters \((\alpha, \beta, T_n)\) and the collider parameter (the modification of Higgs trilinear coupling at one loop \(\delta_h\)) for the three-step phase transition pattern (two FOPTs and one SOPT). \(\delta(ZH)\) represents the corresponding loop-induced modification of ZH cross section at 240 GeV CEPC.

| pattern | \(T_n\) [GeV] | \(\epsilon(T_n)\) [GeV\(^4\)] | \(v_b\) | \(\alpha\) | \(\tilde{\beta}\) | \(\delta_h\) @ one-loop | \(\delta(ZH)\) |
|---------|----------------|------------------|--------|--------|-------|------------------|---------|
| \(BP_{11}\) 3-step | 68.046 | \(2.15 \times 10^6\) | 0.624 | 0.00353 | 1457261.58 | 1.863 | 2.980% |
|         | 51.316 | \(2.966 \times 10^7\) | 0.807 | 0.151 | 2235.16 |             |         |
| \(BP_{12}\) 3-step | 69.380 | \(2.864 \times 10^6\) | 0.629 | 0.00436 | 1225417.53 | 1.854 | 2.966% |
|         | 55.586 | \(3.354 \times 10^7\) | 0.792 | 0.124 | 3142.96 |             |         |

V. DISCUSSIONS

A. Consistent check on our assumptions: The evolution of the dynamical CP-violation

As mentioned above, we assume CP-violating VEV \(\tilde{v}_{CP}\) can get non-zero value at finite temperature and disappear at zero temperature. To verify our assumption, we do the numerical consistent check for the different phase transition patterns, and show the evolution of CP-violating VEV with temperature in Fig. 5, Fig. 6 and Fig. 7, respectively. For example, Fig. 5 depicts the one-step FOPT pattern, and it shows that the CP-violating VEV \(\tilde{v}_{CP}\) increases with temperature, when it is below critical temperature. And when the temperature decreases, the CP-violating VEV \(\tilde{v}_{CP}\) gradually evolves to zero. This is con-
FIG. 5: Evolution of the CP-violating VEV $\tilde{v}_{CP}$ for one-step phase transition.

sistent with our assumption. For other phase transition patterns, they are also consistent. As for the charge-breaking VEV, it also numerically shows the similar behavior except the VEV value is much smaller compared to CP-violating case. The extra CP-violating source at finite temperature may provide enough CP violation for successful EW baryogenesis. And this extra CP-violating source evolves to zero at zero temperature to avoid the strong constraints from EDM data.

B. Supercooling case

The benchmark sets $BP_3$, $BP_4$, $BP_7$ and $BP_8$ can produce supercooling pattern of FOPT, where $\alpha > 1$. In this case, according to the definition of $\alpha$, the PT latent heat density dominates the plasma energy density. In the previous figures, we show the GW signals for the $\alpha \ll 1$ since both the bubble dynamics and GW spectra are well studied for several decades. However, for $\alpha > 1$ case ($BP_3$, $BP_4$, $BP_7$ and $BP_8$), it is still unclear and is still under investigating, such as Ref. [66]. For a strong supercooling, bubbles become very thin and relativistic. In this case, the bubble wall velocity $v_b$ quickly approaches the speed of light. This is the so-called runaway bubbles in vacuum [49], which means phase transitions occur in a vacuum-dominated epoch. In principle, these benchmark sets can
FIG. 6: Evolution of the CP-violating VEV $\tilde{v}_{CP}$ for two-step phase transition with two FOPTs.

FIG. 7: Evolution of the CP-violating VEV $\tilde{v}_{CP}$ for three-step phase transition with two FOPTs and one SOPT.

trigger even stronger GW signals. However, for some scenarios, recent study [67] shows that it results in weaker overall GW signals as compared to previous conclusion in literatures. Therefore, we leave the precise study of the GW spectrum for the supercooling case in our future work. As for the implication from this C2HDM, from numerical calculations, we find
that supercooling favors relatively large coupling constants. And in some narrow parameter spaces, the nucleation temperature decreases as the mass hierarchy of the two neutral Higgs bosons decreases. More reliable results rely on further lattice simulations.

VI. CONCLUSION

We have studied the detailed phase transition dynamics with the existence of dynamical CP-violation at finite-temperature in the complex two-Higgs doublet Model. Various phase transition patterns have been investigated, including multi-step phase transition and supercooling case in this scenario. The dynamical CP-violation can not only provide a possible cosmological origin of CP-violation source, but also make the phase transition dynamics more abundant. The corresponding GW signals in synergy with collider signals have also been discussed, which can be used to make complementary test on this scenario and further unravel the underlying phase transition dynamics or different patterns in the early universe. The detailed study on EW baryogenesis and gravitational waves from supercooling are left for our future work.

Acknowledgments

XW would like to thank Phillipp Basler for correspondence regarding the BSMPT, as well as Carroll L. Wainwright for useful discussions of the cosmoTransitions package. FPH deeply appreciates Eibun Senaha’s various discussion. XW and XMZ are supported in part by the Ministry of Science and Technology of China (2016YFE0104700), the National Natural Science Foundation of China (Grant NO. 11653001), the CAS pilot B project (XDB23020000). FPH is supported in part by the McDonnell Center for the Space Sciences.

A. TEMPERATURE CORRECTION OF MASS IN C2HDM

There are contributions to the ring diagrams from the gauge boson and Higgs boson. We need to calculate the self-energy of the gauge boson and Higgs boson in the IR limit. First, we consider the self-energy of the Higgs boson. The Higgs self-energy can be derived from the propagator of the Higgs boson with Higgs boson, gauge boson, and top quark loops.
We work in the original basis, where the relevant fields are \( \phi_i \equiv \{ \rho_1, \eta_1, \rho_2, \eta_2, \zeta_1, \psi_1, \zeta_2, \psi_2 \} \), then the contributions to the Higgs self-energy from the Higgs boson are

\[
\Pi^S_{\phi_i \phi_i} = \frac{T^2}{24} (6 \lambda_1 + 4 \lambda_3 + 2 \lambda_4) \quad \phi_i = \{ \rho_1, \eta_1, \zeta_1, \psi_1 \},
\]

(56)

\[
\Pi^S_{\phi_i \phi_i} = \frac{T^2}{24} (6 \lambda_2 + 4 \lambda_3 + 2 \lambda_4) \quad \phi_i = \{ \rho_2, \eta_2, \zeta_2, \psi_2 \}.
\]

(57)

The contributions come from the gauge bosons are

\[
\Pi^{GB}_{\phi_i \phi_i} = \frac{T^2}{16} (g^2 + g'^2).
\]

(58)

The contribution from top-quark loop is

\[
\Pi^F_{\phi_i \phi_i} = \frac{T^2}{4} y_t^2 \quad \phi_i = \{ \rho_2, \eta_2, \zeta_2, \psi_2 \}.
\]

(59)

Thus, the total contributions to the Higgs boson self-energy in the C2HDM are

\[
\Pi^1_{\phi_i \phi_i} = \frac{T^2}{48} (12 \lambda_1 + 8 \lambda_3 + 4 \lambda_4 + 3(3g^2 + g'^2)) \quad \phi_i = \{ \rho_1, \eta_1, \zeta_1, \psi_1 \},
\]

(60)

\[
\Pi^2_{\phi_i \phi_i} = \frac{T^2}{48} (12 \lambda_2 + 8 \lambda_3 + 4 \lambda_4 + 3(3g^2 + g'^2) + 12y_t^2) \quad \phi_i = \{ \rho_2, \eta_2, \zeta_2, \psi_2 \}.
\]

(61)

Next, we calculate the self-energy of gauge boson. There are two relevant fields in original basis \( W^a, B^\mu \). Then the contributions to the gauge boson self-energy come from the gauge bosons, Higgs boson, and top quark, respectively. Hence, the total self-energy for the gauge boson in the C2HDM are

\[
\Pi_{W^a W^a} = 2g^2 T^2,
\]

\[
\Pi_{BB} = 2g'^2 T^2.
\]

(62)

B. FIELD DEPENDENT MASS MATRIX ELEMENTS OF C2HDM

Since we introduce a charge-breaking VEV, the mass matrix of gauge boson and Higgs boson in the original basis are fully mixed. We can not give the analytic form of the field-dependent mass for each physical particle. Instead, we derive the mass matrix in the original basis, and then numerically calculate the eigenvalues which are the physical masses of the particles. The field-dependent mass matrix elements of the gauge bosons in the original basis can be written as

\[
m_{11}^G = m_{22}^G = m_{33}^G = \frac{1}{4} g^2 (\tilde{v}_1^2 + \tilde{v}_2^2 + \tilde{v}_{CP}^2 + \tilde{v}_{CB}^2),
\]

23
\[
m^{G}_{44} = \frac{1}{4} g^2 (\bar{v}_1^2 + \bar{v}_2^2 + \bar{v}_{CP}^2 + \bar{v}_{CB}^2),
\]
\[
m^{G}_{14} = \frac{1}{2} gg' \bar{v}_2 \bar{v}_{CB},
\]
\[
m^{G}_{24} = \frac{1}{2} gg' \bar{v}_{CP} \bar{v}_{CB},
\]
\[
m^{G}_{34} = -\frac{1}{4} gg' (\bar{v}_1^2 + \bar{v}_2^2 + \bar{v}_{CP}^2 - \bar{v}_{CB}^2).
\]

The mass matrix is
\[
M_{GB} = \begin{pmatrix}
m^{G}_{11} & 0 & 0 & m^{G}_{14} \\
0 & m^{G}_{22} & 0 & m^{G}_{24} \\
0 & 0 & m^{G}_{33} & m^{G}_{34} \\
m^{G}_{14} & m^{G}_{24} & m^{G}_{34} & m^{G}_{44}
\end{pmatrix}.
\]

For the longitudinal components of the gauge bosons, we need to consider the Debye corrected masses, which are the eigenvalues of
\[
\overline{M}_{GB} = M_{GB} + \text{diag}(\Pi_{W^*W^*}, \Pi_{W^*W^*}, \Pi_{W^*W^*}, \Pi_{BB}).
\]

The mass matrix elements of Higgs boson in the original basis can be expressed as
\[
M_{11} = m_{11}^2 + \frac{1}{2} \lambda_1 \bar{v}_1^2 + \frac{1}{2} \lambda_3 (\bar{v}_1^2 + \bar{v}_{CP}^2) + \frac{1}{2} \lambda_{345} \bar{v}_{CB}^2,
\]
\[
M_{22} = m_{22}^2 + \frac{1}{2} \lambda_1 \bar{v}_1^2 + \frac{1}{2} \lambda_3 (\bar{v}_1^2 + \bar{v}_{CP}^2) + \frac{1}{2} \lambda_{345} \bar{v}_{CB}^2,
\]
\[
M_{33} = m_{33}^2 + \frac{1}{2} \lambda_2 (\bar{v}_2^2 + \bar{v}_{CP}^2 + 3 \bar{v}_{CB}^2) + \frac{1}{2} \lambda_3 \bar{v}_1^2,
\]
\[
M_{44} = m_{44}^2 + \frac{1}{2} \lambda_2 (\bar{v}_2^2 + \bar{v}_{CP}^2 + \bar{v}_{CB}^2) + \frac{1}{2} \lambda_3 \bar{v}_1^2,
\]
\[
M_{55} = m_{55}^2 + \frac{3}{2} \lambda_3 \bar{v}_1^2 + \frac{1}{2} \lambda_{345} \bar{v}_2^2 + \frac{1}{2} \lambda_{345} \bar{v}_{CP}^2 + \frac{1}{2} \lambda_{345} \bar{v}_{CB}^2 - \text{Im}(\lambda_5) \bar{v}_2 \bar{v}_{CP},
\]
\[
M_{66} = m_{66}^2 + \frac{1}{2} \lambda_3 \bar{v}_1^2 + \frac{1}{2} \lambda_{345} \bar{v}_{CP}^2 + \frac{1}{2} \lambda_{345} \bar{v}_2^2 + \frac{1}{2} \lambda_3 \bar{v}_{CB}^2 + \text{Im}(\lambda_5) \bar{v}_2 \bar{v}_{CB},
\]
\[
M_{77} = m_{77}^2 + \frac{1}{2} \lambda_2 (3 \bar{v}_2^2 + \bar{v}_{CP}^2 + \bar{v}_{CB}^2) + \frac{1}{2} \lambda_{345} \bar{v}_1^2,
\]
\[
M_{88} = m_{88}^2 + \frac{1}{2} \lambda_2 (\bar{v}_2^2 + 3 \bar{v}_{CP}^2 + \bar{v}_{CB}^2) + \frac{1}{2} \lambda_{345} \bar{v}_1^2,
\]
\[
M_{12} = \frac{1}{2} \text{Im}(\lambda_5) \bar{v}_{CB},
\]
\[
M_{13} = -\text{Re}(m_{12}^2) - \frac{1}{2} \text{Im}(\lambda_5) \bar{v}_1 \bar{v}_{CP} + \frac{1}{2} (\text{Re}(\lambda_5) + \lambda_4) \bar{v}_1 \bar{v}_{CP},
\]
\[
M_{14} = \frac{1}{2} gg' \bar{v}_2 \bar{v}_{CB},
\]
\[
M_{24} = \frac{1}{2} gg' \bar{v}_{CP} \bar{v}_{CB},
\]
\[
M_{34} = -\frac{1}{4} gg' (\bar{v}_1^2 + \bar{v}_2^2 + \bar{v}_{CP}^2 - \bar{v}_{CB}^2).
\]
\[ M_{14} = \text{Im}(m_{12}^2) - \frac{1}{2} \text{Im}(\lambda_5) \bar{v}_1 \bar{v}_2 + \frac{1}{2} (\lambda_4 - \text{Re}(\lambda_5)) \bar{v}_1 \bar{v}_{CP} , \]

\[ M_{15} = \frac{1}{2} [(\lambda_4 + \text{Re}(\lambda_5)) \bar{v}_2 \bar{v}_{CB} - \text{Im}(\lambda_5) \bar{v}_{CP} \bar{v}_{CB}] , \]

\[ M_{16} = \frac{1}{2} [\text{Im}(\lambda_5) \bar{v}_2 \bar{v}_{CB} + (\lambda_4 + \text{Re}(\lambda_5)) \bar{v}_{CP} \bar{v}_{CB}] , \]

\[ M_{17} = \frac{1}{2} (\lambda_4 + \text{Re}(\lambda_5)) \bar{v}_1 \bar{v}_{CB} , \]

\[ M_{18} = -\frac{1}{2} \text{Im}(\lambda_5) \bar{v}_1 \bar{v}_{CB} , \]

\[ M_{23} = -\text{Im}(m_{12}^2) + \frac{1}{2} \text{Im}(\lambda_5) \bar{v}_1 \bar{v}_2 + \frac{1}{2} (\text{Re}(\lambda_5) - \lambda_4) \bar{v}_1 \bar{v}_{CP} , \]

\[ M_{24} = -\text{Re}(m_{12}^2) + \frac{1}{2} (\lambda_4 + \text{Re}(\lambda_5)) \bar{v}_1 \bar{v}_2 - \frac{1}{2} \text{Im}(\lambda_5) \bar{v}_1 \bar{v}_{CP} , \]

\[ M_{25} = \frac{1}{2} [\text{Im}(\lambda_5) \bar{v}_2 \bar{v}_{CB} - (\lambda_4 - \text{Re}(\lambda_5)) \bar{v}_{CP} \bar{v}_{CB}] , \]

\[ M_{26} = \frac{1}{2} [(\lambda_4 - \text{Re}(\lambda_5)) \bar{v}_2 \bar{v}_{CB} + \text{Im}(\lambda_5) \bar{v}_{CP} \bar{v}_{CB}] , \]

\[ M_{27} = \frac{1}{2} \text{Im}(\lambda_5) \bar{v}_1 \bar{v}_{CB} , \]

\[ M_{28} = -\frac{1}{2} (\lambda_4 - \text{Re}(\lambda_5)) \bar{v}_1 \bar{v}_{CB} , \]

\[ M_{35} = \lambda_3 \bar{v}_1 \bar{v}_{CB} , \]

\[ M_{37} = \lambda_2 \bar{v}_2 \bar{v}_{CB} , \]

\[ M_{38} = \lambda_2 \bar{v}_{CP} \bar{v}_{CB} , \]

\[ M_{56} = \text{Re}(\lambda_5) \bar{v}_2 \bar{v}_{CP} + \frac{1}{2} \text{Im}(\lambda_5)(\bar{v}_2^2 - \bar{v}_{CP}^2) , \]

\[ M_{57} = -\text{Re}(m_{12}^2) + \lambda_{345} \bar{v}_1 \bar{v}_2 - \text{Im}(\lambda_5) \bar{v}_1 \bar{v}_{CP} , \]

\[ M_{58} = \text{Im}(m_{12}^2) - \text{Im}(\lambda_5) \bar{v}_1 \bar{v}_2 + \bar{\lambda}_{345} \bar{v}_1 \bar{v}_{CP} , \]

\[ M_{67} = -\text{Im}(m_{12}^2) + \text{Re}(\lambda_5) \bar{v}_1 \bar{v}_{CP} + \text{Im}(\lambda_5) \bar{v}_1 \bar{v}_2 , \]

\[ M_{68} = -\text{Re}(m_{12}^2) + \text{Re}(\lambda_5) \bar{v}_1 \bar{v}_2 - \text{Im}(\lambda_5) \bar{v}_1 \bar{v}_{CP} , \]

\[ M_{78} = -\frac{1}{2} \text{Im}(\lambda_5) \bar{v}_1^2 + \lambda_2 \bar{v}_2 \bar{v}_{CP} , \]

(66)

where

\[ \lambda_{345} = \lambda_3 + \lambda_4 + \text{Re}(\lambda_5) , \]

\[ \bar{\lambda}_{345} = \lambda_3 + \lambda_4 - \text{Re}(\lambda_5) . \]

(67)
The mass matrix is

\[
M_S = \begin{pmatrix}
M_{11} & M_{12} & M_{13} & M_{14} & M_{15} & M_{16} & M_{17} & M_{18} \\
M_{12} & M_{22} & M_{23} & M_{24} & M_{25} & M_{26} & M_{27} & M_{28} \\
M_{13} & M_{23} & M_{33} & 0 & M_{35} & 0 & M_{37} & M_{38} \\
M_{14} & M_{24} & 0 & M_{44} & 0 & 0 & 0 & 0 \\
M_{15} & M_{25} & M_{35} & 0 & M_{55} & M_{56} & M_{57} & M_{58} \\
M_{16} & M_{26} & 0 & 0 & M_{66} & M_{67} & M_{68} & 0 \\
M_{17} & M_{27} & M_{37} & 0 & M_{77} & M_{78} & 0 & 0 \\
M_{18} & M_{28} & M_{38} & 0 & M_{88} & M_{78} & M_{68} & M_{58}
\end{pmatrix}.
\]

The Debye corrected mass of the scalar bosons are given as the eigenvalues of

\[
\overline{M}_S = M_S + \text{diag}(\Pi^1_{\phi_1,\phi_1}, \Pi^1_{\phi_2,\phi_2}, \Pi^2_{\phi_1,\phi_1}, \Pi^2_{\phi_1,\phi_1}, \Pi^1_{\phi_1,\phi_1}, \Pi^2_{\phi_1,\phi_1}, \Pi^2_{\phi_1,\phi_1}) .
\]

Since we just consider the top quark in our work, the field dependent mass of top quark can be easily derived as

\[
m_t^2 = \frac{1}{2} y_t^2 (\tilde{v}_2^2 + v_{CP}^2).
\]
[7] S. Chatrchyan et al. [CMS Collaboration], “Observation of a New Boson at a Mass of 125 GeV with the CMS Experiment at the LHC,” Phys. Lett. B 716, 30 (2012) [arXiv:1207.7235 [hep-ex]].

[8] B. M. Kastening, R. D. Peccei and X. Zhang, “Sphalerons in the two doublet Higgs model,” Phys. Lett. B 266, 413 (1991).

[9] V. Andreev et al. [ACME Collaboration], “Improved limit on the electric dipole moment of the electron,” Nature 562, no. 7727, 355 (2018).

[10] P. Basler, M. Mühlleitner and J. Wittbrodt, “The CP-Violating 2HDM in Light of a Strong First Order Electroweak Phase Transition and Implications for Higgs Pair Production,” JHEP 1803, 061 (2018) [arXiv:1711.04097 [hep-ph]].

[11] D. Fontes, M. Mühlleitner, J. C. Romão, R. Santos, J. P. Silva and J. Wittbrodt, “The C2HDM revisited,” JHEP 1802, 073 (2018) [arXiv:1711.09419 [hep-ph]].

[12] K. Danzmann et al. [LISA Collaboration], ”Laser Interferometer Space Antenna,” arXiv:1702.00786

[13] N. Seto, S. Kawamura and T. Nakamura, “Possibility of direct measurement of the acceleration of the universe using 0.1-Hz band laser interferometer gravitational wave antenna in space,” Phys. Rev. Lett. 87, 221103 (2001) [astro-ph/0108011].

[14] S. Kawamura et al., “The Japanese space gravitational wave antenna: DECIGO,” Class. Quant. Grav. 28, 094011 (2011).

[15] H. Kudoh, A. Taruya, T. Hiramatsu and Y. Himemoto, “Detecting a gravitational-wave background with next-generation space interferometers,” Phys. Rev. D 73, 064006 (2006) [gr-qc/0511145].

[16] V. Corbin and N. J. Cornish, “Detecting the cosmic gravitational wave background with the big bang observer,” Class. Quant. Grav. 23, 2435 (2006) [gr-qc/0512039].

[17] W. R. Hu and Y. L. Wu, “The Taiji Program in Space for gravitational wave physics and the nature of gravity,” Natl. Sci. Rev. 4, no. 5, 685 (2017).

[18] W. H. Ruan, Z. K. Guo, R. G. Cai and Y. Z. Zhang, “Taiji Program: Gravitational-Wave Sources,” [arXiv:1807.09495] [gr-qc].

[19] J. Luo et al. [TianQin Collaboration], “TianQin: a space-borne gravitational wave detector,” Class. Quant. Grav. 33, no. 3, 035010 (2016) [arXiv:1512.02076 [astro-ph.IM]].

[20] X. C. Hu et al., “Fundamentals of the orbit and response for TianQin,” Class. Quant. Grav.
35, no. 9, 095008 (2018) [arXiv:1803.03368 [gr-qc]].

[21] I. Baldes, T. Konstandin and G. Servant, “A first-order electroweak phase transition from varying Yukawas,” Phys. Lett. B 786, 373 (2018) [arXiv:1604.04526 [hep-ph]].

[22] I. Baldes, T. Konstandin and G. Servant, “Flavor Cosmology: Dynamical Yukawas in the Froggatt-Nielsen Mechanism,” JHEP 1612, 073 (2016) [arXiv:1608.03254 [hep-ph]].

[23] S. Bruggisser, T. Konstandin and G. Servant, “CP-violation for Electroweak Baryogenesis from Dynamical CKM Matrix,” JCAP 1711, no. 11, 034 (2017) [arXiv:1706.08534 [hep-ph]].

[24] S. Bruggisser, B. Von Harling, O. Matsedonskyi and G. Servant, “Baryon Asymmetry from a Composite Higgs Boson,” Phys. Rev. Lett. 121, no. 13, 131801 (2018) [arXiv:1803.08546 [hep-ph]].

[25] J. B. Guimaraes da Costa et al. [CEPC Study Group], “CEPC Conceptual Design Report: Volume 2 - Physics & Detector,” arXiv:1811.10545 [hep-ex].

[26] K. Fujii et al., “Physics Case for the 250 GeV Stage of the International Linear Collider,” arXiv:1710.07621 [hep-ex].

[27] A. Abada et al. [FCC Collaboration], “FCC-ee: The Lepton Collider : Future Circular Collider Conceptual Design Report Volume 2,” Eur. Phys. J. ST 228, no. 2, 261 (2019).

[28] A. W. El Kaffas, P. Osland and O. M. Ogreid, “CP violation, stability and unitarity of the two Higgs doublet model,” Nonlin. Phenom. Complex Syst. 10, 347 (2007) [hep-ph/0702097 [HEP-PH]].

[29] A. W. El Kaffas, W. Khater, O. M. Ogreid and P. Osland, “Consistency of the two Higgs doublet model and CP violation in top production at the LHC,” Nucl. Phys. B 775, 45 (2007) [hep-ph/0605142].

[30] D. Fontes, J. C. Romão and J. P. Silva, “$h \to Z\gamma$ in the complex two Higgs doublet model,” JHEP 1412, 043 (2014) [arXiv:1408.2534 [hep-ph]].

[31] S. R. Coleman and E. J. Weinberg, “Radiative Corrections as the Origin of Spontaneous Symmetry Breaking,” Phys. Rev. D 7, 1888 (1973).

[32] L. Dolan and R. Jackiw, “Symmetry Behavior at Finite Temperature,” Phys. Rev. D 9, 3320 (1974).

[33] M. Quiros, “Finite temperature field theory and phase transitions,” hep-ph/9901312.

[34] P. Basler, M. Krause, M. Mühlleitner, J. Wittbrodt and A. Wlotzka, “Strong First Order Electroweak Phase Transition in the CP-Conserving 2HDM Revisited,” JHEP 1702, 121
P. Basler and M. Mühlleitner, “BSMPT (Beyond the Standard Model Phase Transitions): A tool for the electroweak phase transition in extended Higgs sectors,” Comput. Phys. Commun. 237, 62 (2019) [arXiv:1803.02846 [hep-ph]].

J. E. Camargo-Molina, A. P. Morais, R. Pasechnik, M. O. P. Sampaio and J. Wessén, “All one-loop scalar vertices in the effective potential approach,” JHEP 1608, 073 (2016) [arXiv:1606.07069 [hep-ph]].

S. P. Martin, “Taming the Goldstone contributions to the effective potential,” Phys. Rev. D 90, no. 1, 016013 (2014) [arXiv:1406.2355 [hep-ph]].

J. Elias-Miro, J. R. Espinosa and T. Konstandin, “Taming Infrared Divergences in the Effective Potential,” JHEP 1408, 034 (2014) [arXiv:1406.2652 [hep-ph]].

J. A. Casas, J. R. Espinosa, M. Quiros and A. Riotto, “The Lightest Higgs boson mass in the minimal supersymmetric standard model,” Nucl. Phys. B 436, 3 (1995) Erratum: [Nucl. Phys. B 439, 466 (1995)] [hep-ph/9407389].

P. B. Arnold and O. Espinosa, “The Effective potential and first order phase transitions: Beyond leading-order,” Phys. Rev. D 47, 3546 (1993) Erratum: [Phys. Rev. D 50, 6662 (1994)] [hep-ph/9212235].

R. R. Parwani, “Resummation in a hot scalar field theory,” Phys. Rev. D 45, 4695 (1992) Erratum: [Phys. Rev. D 48, 5965 (1993)] [hep-ph/9204216].

S. R. Coleman, “The Fate of the False Vacuum. 1. Semiclassical Theory,” Phys. Rev. D 15, 2929 (1977) Erratum: [Phys. Rev. D 16, 1248 (1977)].

C. G. Callan, Jr. and S. R. Coleman, “The Fate of the False Vacuum. 2. First Quantum Corrections,” Phys. Rev. D 16, 1762 (1977).

C. L. Wainwright, “CosmoTransitions: Computing Cosmological Phase Transition Temperatures and Bubble Profiles with Multiple Fields,” Comput. Phys. Commun. 183, 2006 (2012) [arXiv:1109.4189 [hep-ph]].

A. Kosowsky, M. S. Turner and R. Watkins, “Gravitational radiation from colliding vacuum bubbles,” Phys. Rev. D 45, 4514 (1992).

A. Kosowsky, M. S. Turner and R. Watkins, “Gravitational waves from first order cosmological phase transitions,” Phys. Rev. Lett. 69, 2026 (1992).

A. Kosowsky and M. S. Turner, “Gravitational radiation from colliding vacuum bubbles:
envelope approximation to many bubble collisions,” Phys. Rev. D 47, 4372 (1993) [astro-ph/9211004].

[48] M. Kamionkowski, A. Kosowsky and M. S. Turner, “Gravitational radiation from first order phase transitions,” Phys. Rev. D 49, 2837 (1994) [astro-ph/9310044].

[49] C. Caprini et al., “Science with the space-based interferometer eLISA. II: Gravitational waves from cosmological phase transitions,” JCAP 1604, no. 04, 001 (2016) [arXiv:1512.06239 [astro-ph.CO]].

[50] S. J. Huber and T. Konstandin, “Gravitational Wave Production by Collisions: More Bubbles,” JCAP 0809, 022 (2008) [arXiv:0806.1828 [hep-ph]].

[51] R. Jinno and M. Takimoto, Phys. Rev. D 95, no. 2, 024009 (2017) doi:10.1103/PhysRevD.95.024009 [arXiv:1605.01403 [astro-ph.CO]].

[52] M. Hindmarsh, S. J. Huber, K. Rummukainen and D. J. Weir, “Gravitational waves from the sound of a first order phase transition,” Phys. Rev. Lett. 112, 041301 (2014) [arXiv:1304.2433 [hep-ph]].

[53] M. Hindmarsh, S. J. Huber, K. Rummukainen and D. J. Weir, “Numerical simulations of acoustically generated gravitational waves at a first order phase transition,” Phys. Rev. D 92, no. 12, 123009 (2015) [arXiv:1504.03291 [astro-ph.CO]].

[54] C. Caprini, R. Durrer and G. Servant, “The stochastic gravitational wave background from turbulence and magnetic fields generated by a first-order phase transition,” JCAP 0912, 024 (2009) [arXiv:0909.0622 [astro-ph.CO]].

[55] P. Binetruy, A. Bohe, C. Caprini and J. F. Dufaux, “Cosmological Backgrounds of Gravitational Waves and eLISA/NGO: Phase Transitions, Cosmic Strings and Other Sources,” JCAP 1206, 027 (2012) [arXiv:1201.0983 [gr-qc]].

[56] S. Iso, P. D. Serpico and K. Shimada, “QCD-Electroweak First-Order Phase Transition in a Supercooled Universe,” Phys. Rev. Lett. 119, no. 14, 141301 (2017) [arXiv:1704.04955 [hep-ph]].

[57] A. Kobakhidze, C. Lagger, A. Manning and J. Yue, “Gravitational waves from a supercooled electroweak phase transition and their detection with pulsar timing arrays,” Eur. Phys. J. C 77, no. 8, 570 (2017) [arXiv:1703.06552 [hep-ph]].

[58] B. Imtiaz, Y. F. Cai and Y. Wan, “Two-field cosmological phase transitions and gravitational waves in the singlet Majoron model,” Eur. Phys. J. C 79, no. 1, 25 (2019) [arXiv:1804.05835].

30
[hep-ph]].

[59] A. Addazi, Y. F. Cai and A. Marciano, “Testing Dark Matter Models with Radio Telescopes in light of Gravitational Wave Astronomy,” Phys. Lett. B 782, 732 (2018) [arXiv:1712.03798 [hep-ph]].

[60] F. P. Huang and X. Zhang, “Probing the gauge symmetry breaking of the early universe in 3-3-1 models and beyond by gravitational waves,” Phys. Lett. B 788, 288 (2019) [arXiv:1701.04338 [hep-ph]].

[61] F. P. Huang, P. H. Gu, P. F. Yin, Z. H. Yu and X. Zhang, “Testing the electroweak phase transition and electroweak baryogenesis at the LHC and a circular electron-positron collider,” Phys. Rev. D 93, no. 10, 103515 (2016) [arXiv:1511.03969 [hep-ph]].

[62] F. P. Huang, Y. Wan, D. G. Wang, Y. F. Cai and X. Zhang, “Hearing the echoes of electroweak baryogenesis with gravitational wave detectors,” Phys. Rev. D 94, no. 4, 041702 (2016) [arXiv:1601.01640 [hep-ph]].

[63] F. P. Huang and J. H. Yu, “Exploring inert dark matter blind spots with gravitational wave signatures,” Phys. Rev. D 98, no. 9, 095022 (2018) [arXiv:1704.04201 [hep-ph]].

[64] Q. H. Cao, F. P. Huang, K. P. Xie and X. Zhang, “Testing the electroweak phase transition in scalar extension models at lepton colliders,” Chin. Phys. C 42, no. 2, 023103 (2018) [arXiv:1708.04737 [hep-ph]].

[65] F. P. Huang, Z. Qian and M. Zhang, “Exploring dynamical CP violation induced baryogenesis by gravitational waves and colliders,” Phys. Rev. D 98, no. 1, 015014 (2018) [arXiv:1804.06813 [hep-ph]].

[66] R. Jinno, H. Seong, M. Takimoto and C. M. Um, “Gravitational waves from first-order phase transitions: Ultra-supercooled transitions and the fate of relativistic shocks,” [arXiv:1905.00899 [astro-ph.CO]].

[67] J. Ellis, M. Lewicki and J. M. No, “On the Maximal Strength of a First-Order Electroweak Phase Transition and its Gravitational Wave Signal,” JCAP 1904, 003 (2019) [arXiv:1809.08242 [hep-ph]].