Two Higgs Bi-doublet Left-Right Model
With Spontaneous P and CP Violation

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Abstract

A left-right symmetric model with two Higgs bi-doublet is shown to be a consistent model for both spontaneous P and CP violation. The flavor changing neutral currents can be suppressed by the mechanism of approximate global $U(1)$ family symmetry. We calculate the constraints from neural $K$ meson mass difference $\Delta m_K$ and demonstrate that a right-handed gauge boson $W_2$ contribution in box-diagrams with mass well below 1 TeV is allowed due to a cancellation caused by a light charged Higgs boson with a mass range $150 \sim 300$ GeV. The $W_2$ contribution to $\epsilon_K$ can be suppressed from appropriate choice of additional CP phases appearing in the right-handed Cabbibo-Kobayashi-Maskawa matrix. The model is also found to be fully consistent with $B^0$ mass difference $\Delta m_B$, and the mixing-induced CP violation quantity $\sin 2\beta_{J/\psi}$, which is usually difficult for the model with only one Higgs bi-doublet. The new physics beyond the standard model can be directly searched at the colliders LHC and ILC.

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I. INTRODUCTION

Since the discovery of parity (P) violation fifty years ago\cite{1, 2}, it has been realized that both symmetry and asymmetry can play important roles in particle physics. Later on, charge-conjugation-parity (CP) violation was also discovered in kaon decays\cite{3}. The electroweak standard model was established based on the left-handed symmetry $SU(2)_L$ and has well been described by the gauge symmetry $SU(2)_L \times U(1)_Y$\cite{4, 5, 6}. Since then, one of the important issues in particle physics concerns origin of P and CP violations as well as the smallness of flavor changing neutral currents (FCNC). Its solution requires physics beyond the standard model.

The investigation of explicit CP violation in the standard model led to the prediction for the existence of three generation quarks, so that a single Kobayashi-Maskawa CP phase\cite{7} can be introduced to characterize the CP-violating mechanism in the standard model. Such a simple CP-violating mechanism has been found to be remarkable for explaining not only the indirect CP violation in kaon decays, but also the direct CP violation in kaon decays\cite{8, 9} observed by two experimental groups at CERN\cite{10} and Fermilab\cite{11}, and the direct CP violations in B meson decays\cite{12, 13, 14} reported by two B-factories\cite{15, 16}. Nevertheless, the CP violation in the standard model is assumed to be caused from the explicit complex Yukawa couplings put in by hand, thus its origin remains unknown. To understand the origin of CP violation, a spontaneous CP violation mechanism was suggested by Lee in 1973\cite{17, 18} in which scalar fields are responsible to CP violation. Soon after, an interesting spontaneous CP-violating three Higgs doublet model was proposed by imposing discrete symmetries\cite{19} in order to avoid the FCNC, while such a model has been strongly constrained from the low energy phenomena of K and B systems. By abandoning the natural flavor conservation hypothesis\cite{20, 21, 22}, a general two Higgs doublet model (2HDM) motivated from spontaneous CP violation has been investigated in detail\cite{21, 22}, where the FCNC is assumed to be naturally suppressed by the mechanism of approximate global U(1) family symmetry\cite{21, 22}. Of particular, it has been shown in refs.\cite{21, 22} that after spontaneous symmetry breaking the single relative CP phase of two vacuum expectation values can induce rich CP-violating sources, which not only explain the KM CP-violating mechanism in the standard model, but also lead to new type of CP-violating sources in the charged Higgs interactions. Such a model can result in new physics phenomena\cite{23, 24, 25, 26, 27} and
remains consistent with the current experiments.

With the hypothesis that parity is a good symmetry at high energy, a left-right symmetric model was proposed based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ \cite{28, 29, 30}. In such a model, parity violation can naturally be understood via spontaneous symmetry breaking. Also CP asymmetry can be realized as a consequence of spontaneous symmetry breaking \cite{31, 32, 33, 34}. Nevertheless, the spontaneous P and CP-violating left-right model with only one Higgs bi-doublet is strongly constrained \cite{32, 33, 34} from low energy phenomenology:

(i) The neutral kaon mass difference $\Delta m_K$ requires that the right-handed gauge bosons must be very heavy above 2 TeV to suppress the extra box-diagram as the gauge coupling is left-right symmetric. Since the Yukawa couplings for neutral and charged Higgs bosons are fixed to quarks masses and Cabbibo-Kobayashi-Maskawa (CKM) matrices. There is no cancellation occurring among different contributions. For the same reason, the lightest neutral Higgs boson must be above 10 TeV \cite{34, 35, 36} to suppress FCNC. Such a neutral Higgs mass is too heavy in the Higgs bi-doublet sector to make the model natural as the bi-doublet Higgs bosons are expected to be at the electroweak scale which is much lower than the right-handed gauge boson mass; (ii) In the one Higgs bi-doublet model, all the CP violating phases are calculable quantities in terms of quarks masses and ratios of VEVs, which can be directly tested by the experimental data on CP violating observables. It has been shown \cite{34} that the combing constraints from $K$ system and $B$ system actually excluded the so-called minimal one Higgs bi-doublet left-right model with spontaneous CP violation in the decoupling limit, as the model fails to reproduce the precisely measured weak phase angle $\sin 2\beta$ from B factories; (iii) Furthermore, the condition for the spontaneous CP violation requires an unnatural fine tuning of the Higgs potential in the one Higgs bi-doublet LR model \cite{36, 37}. For those reasons, it was motivated to consider the one Higgs bi-doublet LR model with general CP violation \cite{38, 39, 40} instead of spontaneous CP violation. An alternative consideration for spontaneous P and CP violation is to introduce the concept of mirror particles \cite{41, 42}; recently, a maximally symmetric model \cite{43, 44} was constructed along this line by considering mirror quarks and leptons.

In this note, motivated by the general 2HDM as a model for spontaneous CP violation, we shall simply extend the one Higgs bi-doublet left-right model to a two Higgs bi-doublet left-right model with spontaneous P and CP violation, and demonstrate that the above
mentioned stringent phenomenological constraints from neutral meson mixings can be significantly relaxed. It will be shown that the right-handed gauge boson mass can be as low as 600 GeV with the charged Higgs mass around 200 GeV. The FCNC will not impose severe constraints on the neutral Higgs mass, provided small off-diagonal Yukawa couplings via the mechanism of approximate global $U(1)$ family symmetry\[20, 21, 22\].

The paper is organized as follows: in section II, we present a general description for two Higgs bi-doublet left-right model. In section III, we analyze the neutral $K$ system, which includes the mass difference $\Delta m_K$ and indirect CP violation $\epsilon_K$. We observe that the right-handed gauge boson contributions to the mass difference $\Delta m_K$ can be opposite to that from the charged Higgs boson in this extended model and a cancellation between the two contributions is possible in a large parameter space. The suppression of right-handed gauge boson contributions to the indirect CP violation $\epsilon_K$ is found to occur naturally. As a consequence, a light right-handed gauge boson around the current experimental low bound is allowed. In section IV, we discuss in detail the neutral B meson system, the mass difference $\Delta m_B$ and the time dependent CP asymmetry in $B^0 \rightarrow J/\Psi K_S$ decay are found to be consistently characterized in the two Higgs bi-doublet model with spontaneous P and CP violation, which is unlike the one Higgs bi-doublet model. Conclusions and remarks are presented in the last section.

II. PROPERTIES OF A TWO HIGGS BI-DOUBLET LEFT-RIGHT MODEL

The left-handed and right-handed quarks and leptons in the $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ model are all given by the doublets

$$Q_{iL} = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L : (2,1,1/3), \quad Q_{iR} = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_R : (1,2,1/3),$$

$$L_{iL} = \begin{pmatrix} \nu_i \\ l_i \end{pmatrix}_L : (2,1,-1), \quad L_{iR} = \begin{pmatrix} \nu_i \\ l_i \end{pmatrix}_R : (1,2,-1),$$

(1)

where $i = 1, 2, 3$ runs over number of generations. The quantum numbers $(X_L, X_R, Y)$ in parenthesis denote the $SU(2)_L$, $SU(2)_R$ and $U(1)_{B-L}$ representation. $X_{L,R}$ represent dimensions of the $SU(2)_L$ and $SU(2)_R$ representations, and $Y$ is the hypercharge $Y = B-L$. 
As a gauge invariant model, three gauge fields for the symmetry group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ are introduced as $W^\mu_L$, $W^\mu_R$ and $B^\mu$ respectively. The gauge invariant fermion-gauge interactions are constructed as follows

$$\mathcal{L}_f = \sum_{\Psi = (Q), (L)} \bar{\Psi}_L \gamma^\mu \left( i \partial_\mu + g_L \frac{\tau^i}{2} W^i_{L\mu} + g'_{Y} \frac{Y}{2} B^\mu \right) \Psi_L + (L \rightarrow R).$$

(2)

To generate masses of fermions and gauge bosons, we shall introduce scalar fields and apply the Higgs mechanism to break symmetry spontaneously. In order to generate fermion mass matrices, one only needs to introduce one Higgs bi-doublet [29, 30]. However, in view of the above mentioned phenomenological difficulties, here we shall consider a left-right symmetric model with two Higgs bi-doublets

$$\phi = \begin{pmatrix} \phi_0^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi_0^0 & \chi_1^+ \\ \chi_2^- & \chi_2^0 \end{pmatrix} : (2, 2, 0).$$

(3)

The most general Yukawa interaction for quarks is given by

$$\mathcal{L}_Y = - \sum_{i,j} \bar{Q}_{iL} \left( (y_q)_{ij} \phi + (\tilde{y}_q)_{ij} \tilde{\phi} + (h_q)_{ij} \chi + (\tilde{h}_q)_{ij} \tilde{\chi} \right) Q_{jR},$$

(4)

where $\tilde{\phi}(\tilde{\chi}) = \tau_2 \phi^*(\chi^*) \tau_2$ also belong to the representation $(2, 2, 0)$. Parity $P$ symmetry requires $g_L = g_R \equiv g$ and

$$y_q = y_q^i, \quad \tilde{y}_q = \tilde{y}_q^i, \quad h_q = h_q^i, \quad \tilde{h}_q = \tilde{h}_q^i.$$

(5)

When both $P$ and CP are required to be broken down spontaneously, all the Yukawa coupling matrices are real symmetric.

Note that allowing the two Higgs bi-doublet coupling to the same quark field may generate large FCNC at tree level. To suppress FCNC, we shall follow the similar treatment in the general two-Higgs-doublet model [21, 22] by considering the mechanism of approximate global $U(1)$ family symmetry [20, 21, 22]

$$(u_i, d_i) \rightarrow e^{-i\theta_i} (u_i, d_i),$$

(6)

which is motivated by the approximate unity of the CKM matrix. As an consequence, $y$, $\tilde{y}$, $h$ and $\tilde{h}$ are nearly diagonal matrices.
To break $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ to the $U(1)_{em}$, it requires to introduce other Higgs multiplets in addition to $\phi$ and $\chi$. The most popular choice for generating small neutrino masses is to introduce Higgs triplets ($\Delta_L \sim (3, 1, 2), \Delta_R \sim (1, 3, 2)$) \cite{29, 30}

$$\Delta_{L,R} = \begin{pmatrix} \delta_{L,R}^+/\sqrt{2} & \delta_{L,R}^{++} \\ \delta_{L,R}^0 & -\delta_{L,R}^+/\sqrt{2} e \end{pmatrix}.$$ (7)

Introducing the SU(2) triplets breaks the custodial symmetry and leads to corrections to the parameter $\rho = m_W^2/(m_Z^2 \cos^2 \theta_W)$ from unity at tree level, which may subject to strong constraints from the LEP data. However, the corrections from the model involve a number of free parameters such as the ratios among the VEVs of Higgs bidoublets and triplets. The constraint from a single $\rho$ parameter is not severe. Furthermore, the low energy process such as $\mu$ decays and neutral current interactions $\nu N$, $\nu e$ and $eN$ are all affected by the model parameters, which modifies the SM relations among the electroweak precision observables. Useful constraints can be obtained from a global fit to all the relevant data. It has been shown that the combined analysis on both the high and low energy electroweak data within the minimal left-right model only leads to a mild lower bound of $M_2 > 700 \sim 800$ GeV for right-hand gauge boson $W_2$ when the correction to $\rho$ parameter is below 1%. Since the two Higgs bidoublet model considered here contains one more bidoublet which does not violate the custodia symmetry and contain more parameters, the constraints should be even weaker.

The most general form of Higgs potential in this model is rather complicated, which involves the quadratic and quartic terms for the extra bi-doublet field $\chi$ and its mixing with $\phi$ and the two triplets $\Delta_{L,R}$. It has been shown that by simply adding a singlet Higgs field to the one Higgs bi-doublet LR model, the spontaneous CP violation can occur naturally \cite{45}. The two Higgs bi-doublet model has definitely more flexibility in Higgs potential. It is expected that in the most general case the spontaneous CP violation is allowed, which will not be discussed in detail in the present note.

After the spontaneous symmetry breaking, the two Higgs bi-doublet fields can have the following vacuum expectation values(VEVs)

$$\langle \phi \rangle = \begin{pmatrix} v_1 e^{i\delta_1} & 0 \\ 0 & v_2 e^{i\delta_2} \end{pmatrix} \quad \text{and} \quad \langle \chi \rangle = \begin{pmatrix} w_1 e^{i\phi_1} & 0 \\ 0 & w_2 e^{i\phi_2} \end{pmatrix},$$ (8)
which leads to the following mass matrix for quarks

\[
M_u = y_q v_1 e^{i\delta_1} + \bar{y}_q v_2 e^{-i\delta_2} + h_q w_1 e^{i\phi_1} + \bar{h}_q w_2 e^{-i\phi_2},
\]
\[
M_d = y_q v_1 e^{i\delta_2} + \bar{y}_q v_2 e^{-i\delta_1} + h_q w_1 e^{i\phi_2} + \bar{h}_q w_2 e^{-i\phi_1}.
\] (9)

As the mass matrices are symmetric, they can be diagonalized by

\[
M_u = U M_u^D U^T \quad \text{and} \quad M_d = V M_d^D V^T,
\] (10)

with \(M_{u(d)}^D\) being diagonal mass matrices for up(down)-type quarks. It follows that the resultant quark mixing matrices for left-handed and right-handed quarks are complex conjugate to each other

\[
K^L = U^\dagger V \quad \text{and} \quad K^R = U^T V^* = K^*_L.
\] (11)

Note that rotating the left-handed quark mixing matrix to the standard CKM form \(V^L\) with a single CP phase is non-trivial due to the existence of right-handed quark-gauge interactions and charged Higgs Yukawa interactions. In this case, there are in general five additional CP phases \(\alpha_i, (i = 1, 2, 3)\) and \(\beta_i, (i = 1, 2)\) in the right-handed quark mixing matrix which is parametrized as follows

\[
V^R = \eta^u \begin{pmatrix}
(V_{ud}^L)^* e^{2i\alpha_1} & (V_{us}^L)^* e^{i(\alpha_1 + \alpha_2 + \beta_1)} & (V_{ub}^L)^* e^{i(\alpha_1 + \alpha_3 + \beta_1 + \beta_2)} \\
(V_{cd}^L)^* e^{i(\alpha_2 + \beta_1 - \beta_2)} & (V_{cs}^L)^* e^{2i\alpha_2} & (V_{cb}^L)^* e^{i(\alpha_2 + \alpha_3 + \beta_2)} \\
(V_{td}^L)^* e^{i(\alpha_3 - \beta_1 - \beta_2)} & (V_{ts}^L)^* e^{i(\alpha_2 + \alpha_3 - \beta_2)} & (V_{tb}^L)^* e^{2i\alpha_3}
\end{pmatrix} \eta^d.
\] (12)

The sign matrices \(\eta^u, \eta^d\) corresponds to the 32 different sign arrangements of quark masses. In the following considerations, we should focus only on the case with positive quark masses, i.e. \(\eta^u = \eta^d = \mathbb{1}\).

Within the Wolfenstein parametrization, we define \(\beta_L \equiv \arg(V_{td}^L V_{tb}^L)\) which is, to a high precision, one of the angles of the unitarity triangle. As \(V_{tb}^L\) is real by convention, one has \(\beta_L = \arg(V_{td}^{L*})\). Under this parametrization, one can define a similar quantity \(\beta_R \equiv \arg(V_{td}^{R*} V_{tb}^R)\). They satisfy the following relation

\[
\beta_L + \beta_R = -(\alpha_1 - \alpha_3 - \beta_1 - \beta_2),
\] (13)

which is useful for discussing \(B\) meson system. Similarly, one can define phase parameters relevant to the \(K\) meson system, i.e., \(\beta'_L \equiv \arg(V_{td}^L V_{ts}^L)\) and \(\beta'_R \equiv \arg(V_{td}^{R*} V_{ts}^R)\). They are
related by

$$\beta'_{L} + \beta'_{R} = -(\alpha_1 - \alpha_2 - \beta_1).$$

(14)

In the Wolfenstein parametrization, it is known that \(|\arg(V_{L}^{L})/\arg(V_{L}^{L})| \ll 1\), we have in a good approximation

$$\beta'_{L} \simeq \beta_{L} \quad \text{and} \quad \beta'_{R} = \beta_{R} + \alpha_2 - \alpha_3 - \beta_2.$$  

(15)

With enlarged Higgs sector, in this model there are two doubly charged Higgs $H^{++}_i$, ($i = 1, 2$), four singly charged Higgs particles $H^+_i$, ($i = 1, \ldots, 4$), six neutral scalars $h^0_i$, ($i = 1, \ldots, 6$) and four neural pseudo-scalars $A^0_i$, ($i = 1, \ldots, 4$). The doubly charged $H^{++}_i$ contribute only to the leptonic sector such as lepton flavor violation processes\[46\], whereas the singly charged and neutral scalars may have significant effects on mixings and CP violation in quark sector.

For the sake of simplicity, we should work in a simplest scenario that only one charged Higgs (labeled as $H^+$) is light enough to actively contribute to the box-diagrams. Of particular, when the VEVs satisfy the conditions $v_2 \ll v_1$ and $w_2 \ll w_1$ or more precisely $v_2/v_1, w_2/w_1 < m_b/m_t$, which are also needed for obtaining the experimentally allowed small mixing between left-handed and right-handed gauge bosons, then many features of this model are similar to the general 2HDM with spontaneous CP violation\[21, 22\]. Therefore, we consider here the 2HDM-like charged Higgs to be the lightest one, the corresponding Yukawa interaction is parametrized as follows

$$L_C = -(2\sqrt{2}G_F)^{1/2} \bar{u}^{L} \left( \sqrt{m_{u}^2 m_{k}^2} \xi_{u k}^{L} V_{k j} P_{L} - V_{i k}^{L} \sqrt{m_{d}^2 m_{j}^2} \xi_{i j}^{d} P_{R} \right) d^{j} H^{+} + H.c.$$  

(16)

Here we have used the Cheng-Sher parametrization\[47\] in the general 2HDM with $\xi^{u(d)}_{ij}$ the effective Yukawa coupling matrices in the physics basis after spontaneous symmetry breaking. The small off-diagonal terms characterized in $\sqrt{m_{i}^2 m_{j}^2} \xi^{q}_{ij}$ describe the breaking of the global $U(1)$ family symmetry. For future convenience, we denote the diagonal elements as $\xi_{c} \equiv \xi_{22}^{u}$ and $\xi_{t} \equiv \xi_{33}^{u}$ etc.

For the flavor changing neutral Higgs boson interactions, in the same case that $v_2/v_1, w_2/w_1 < m_b/m_t$, it becomes similar to the general 2HDM with spontaneous CP violation. When considering the 2HDM-like neutral Higgs boson $h^0$ to be the lightest one, the dominant interactions can approximately be expressed in the following form

$$L_N = -(\sqrt{2}G_F)^{1/2} \bar{q}_i d_j \eta^{q}_{ij} \ q^0_R h^0 + H.c.$$  

(17)
where $\eta_{ij}^q$ are given by $\xi_{ij}^q$ up to the factors caused by the mixing matrix elements $O_{ij}$ among the neutral Higgs bosons, i.e., $\eta_{ij}^q \sim (O_{1k} \pm iO_{1l})\xi_{ij}^q$. Note that a remarkable difference from the one Higgs bi-doublet LR model with spontaneous CP violation is that the effective Yukawa couplings $\xi_{ij}^q$ or $\eta_{ij}^q$ in the physics basis are in general all complex and no longer symmetric due to spontaneous P and CP violation, they contain more free parameters due to the extra source of CP violation in the VEVs and more Yukawa couplings associated with the extra Higgs bi-doublet. As a consequence, the effective Yukawa couplings $\eta_{ij}^q$ or $\xi_{ij}^q$ in the physics basis are in general all complex and no longer symmetric due to spontaneous P and CP violation, they contain more free parameters due to the extra source of CP violation in the VEVs and more Yukawa couplings associated with the extra Higgs bi-doublet. Similar to the two Higgs doublet model with spontaneous CP violation\cite{21,22}, the effective Yukawa couplings $\eta_{ij}^q$ or $\xi_{ij}^q$ in the two Higgs bi-doublet model are also free parameters which can cause significantly different effects in low energy phenomenology.

The neutral meson mixing can arise from the neutral scalar exchange at tree level, which could be significant. The contributions to the mixing matrix between the neutral meson $P^0$ and $\bar{P}^0$ can easily be obtained. Denoting $P^0$ the bound state of two quarks with quantum number $P^0 \equiv (\bar{q}_i \gamma_5 q_j)$, we have, in the factorization approximation, the following general form

$$M_{12} = \frac{1}{2m_P} \langle P^0 | H_{eff} | \bar{P}^0 \rangle \simeq G_F \sqrt{2f_P^2 m_P B_P^S} \left[ \frac{1}{6} + \frac{m_i^q m_j^q}{(m_i^q + m_j^q)^2} \right] (\eta_{ij}^q - \eta_{ji}^q)^2. \quad (18)$$

**III. NEUTRAL K MESON MIXING**

We proceed to discuss the low energy phenomenological constraints of this model. Since $W_L W_R$ mixing angle is very small from $\mu$ decays, the mass eigenstate $W_{1(2)}$ is almost left(right)-handed. In the left-right model, the strongest constraint comes from K meson system. The $K^0$ meson receives additional contributions from both $W_1 W_2$ loop and charged Higgs loop in box-diagrams. As the internal $(c,c)$ quark loop dominates the whole contribution, any CP violating phases associated with it have to be strongly suppressed in order to accommodate the tiny CP violating parameter $\epsilon_K$. In $K^0$ mixing, the $W_1 W_2$ box-diagrams are proportional to the following CKM factor combinations

$$\lambda_{q}^{LR} \lambda_{q'}^{RL} = V_{qs}^{LR} V_{qd}^{RL} (q,q' = u,c,t). \quad (19)$$

The condition for $(c,c)$ loop to be CP conserving leads to

$$\alpha_1 - \alpha_2 - \beta_1 \simeq 0 \quad \text{or} \quad \beta'_{R} \simeq -\beta'_{L} \simeq -\beta_{L}. \quad (20)$$
As all the CP phases $\alpha_i$ and $\beta_i$ are expected to be small quantities, we neglect the possibility of $\alpha_1 - \alpha_2 - \beta_1 \simeq \pi$. For the charged Higgs contribution, the situation is similar to the general 2HDM: although the Yukawa couplings are less constrained and in general complex, the dominant contribution is only proportional to the left-handed CKM matrix $V^L$ in the same manner as in the SM. Thus the charged Higgs contribution to $(c,c)$ loop remains real up to $\mathcal{O}(\lambda^3)$, with $\lambda$ the Wolfenstein parameter.

Since both of the two contributions are nearly real, their interference is either constructive or destructive. It will be shown below that due to the different chiralities, the contribution from charged Higgs loop interferes always destructively with the $W_1W_2$ loop in the CP conserving case of Eq. (20). This provides a possibility of a nearly complete cancellation, which may greatly reduce the mass lower bounds for both $W_2$ and charged Higgs $H^+$.

The SM $W_1W_1$ loop diagram contribution to $K^0 - \bar{K}^0$ mixing is described by the following effective Hamiltonian

$$H_{eff}^{W_1W_1} = \frac{G_F m_W}{16\pi^2} \left[ (\lambda_c^{LL})^2 \eta_{cc} S_0(x_c) + (\lambda_t^{LL})^2 \eta_{tt} S_0(x_t) + 2\lambda_c^{LL} \lambda_t^{LL} \eta_{ct} S_0(x_c, x_t) \right]$$

$$\bar{d}\gamma^\mu (1 - \gamma_5)s \otimes \bar{d}\gamma_\mu (1 - \gamma_5)s + H.c,$$

where $m_W$ is the mass of the left-handed gauge boson and

$$S_0(x) = \frac{x}{(1 - x)^2} \left[ 1 - \frac{11x}{4} + \frac{x^2}{4} - \frac{3x^2 \ln x}{2(1 - x)} \right],$$

$$S_0(x_c, x_t) = x_c x_t \left[ -\frac{3}{4(1 - x_c)(1 - x_t)} + \frac{\ln x_c}{(x_c - x_t)(1 - x_c)^2} \left( 1 - 2x_c + \frac{x_c^2}{4} \right) + (x_c \leftrightarrow x_t) \right].$$

the Inami-Lim functions. The CKM factors are defined as $\lambda_i^{LL} = V_{qs}^i V_{qd}^{L*}$. The matrix element is given by

$$\langle K^0 | \bar{d}\gamma^\mu (1 - \gamma_5)s \otimes \bar{d}\gamma_\mu (1 - \gamma_5)s | \bar{K}^0 \rangle = \frac{8}{3} f_K^2 m_K^2 B_K,$$

with the normalization $f_K = 159.8$ MeV. The values for QCD corrections are $\eta_{cc} \simeq 1.46$, $\eta_{ct} = 0.47 \pm 0.04$, and $\eta_{tt} = 0.5765 \pm 0.0065$, and the bag parameter is $B_K = 0.86 \pm 0.06 \pm 0.14$.

The effective Hamiltonian for $W_1W_2$ and $S$(Goldstone)$W_2$ has been extensively investigated and reads

$$H_{eff}^{W_1W_2+S} = \frac{G_F^2 m_W^2}{8\pi^2} \sum_{i,j} \lambda_i^{LR} \lambda_j^{RL} \sqrt{x_i x_j} \left[ (4 + x_i x_j \beta) \eta_1^{LR} I_1(x_i, x_j, \beta) - (1 + \beta) \eta_2^{LR} I_2(x_i, x_j, \beta) \right] \bar{d}(1 - \gamma_5)s \otimes \bar{d}(1 + \gamma_5)s + H.c,$$
where \( x_i = \frac{m_i^2}{m_W^2} \) and \( \beta = \frac{m_W^2}{M_W^2} \). The two QCD correction coefficients are \( \eta_1^{LR} = 1.4 \) and \( \eta_2^{LR} = 1.17 \) for \( \Lambda_{QCD} = 0.2 \text{GeV} \). The phases in CKM matrix elements are defined as \( \lambda_q^{LR} = V_{qs}^{L} (V_{qd}^{R})^* \) and \( \lambda_q^{RL} = V_{qs}^{R} (V_{qd}^{L})^* \) with the loop functions:

\[
I_1(x_i, x_j, \beta) = \frac{x_i \ln x_i}{(1-x_i)(1-x_i \beta)(x_i - x_j)} + (i \to j) - \frac{\beta \ln \beta}{(1-\beta)(1-x_i \beta)(1-x_j \beta)}
\]

\[
I_2(x_i, x_j, \beta) = \frac{x_i^2 \ln x_i}{(1-x_i)(1-x_i \beta)(x_i - x_j)} + (i \to j) - \frac{\ln \beta}{(1-\beta)(1-x_i \beta)(1-x_j \beta)}.
\] (25)

In the limit of \( x_i = x_j \) and \( \beta \ll 1 \), they reduce to:

\[
I_1(x, \beta) \simeq \frac{1}{(1-x)} + \frac{\ln x}{(1-x)^2} - \beta \ln \beta,
\] (26)

\[
I_2(x, \beta) \simeq \frac{x}{1-x} + \frac{(2-x) x \ln x}{(1-x)^2} - \ln \beta.
\] (27)

Compared with the loop functions \( S_0 \) for \( W_1W_1 \) loop, the functions \( I_1(x, \beta) \) and \( I_2(x, \beta) \) have different mass dependencies: (i) For very small \( x_c \ll 1 \), the loop functions can be further simplified as \( I_1(x_c, \beta) \simeq \ln x_c + 1 \), and \( I_2(x_c, \beta) \simeq - \ln \beta \). It is easy to see that the combination \( 4 \eta_1^{LR} I_1(x_c, \beta) - \eta_2^{LR} I_2(x_c, \beta) \) is always negative. The sign of the amplitude is of crucial importance when there are multiple sources of contributions. The negative contribution may lead to cancellations with other amplitudes such as Higgs loops. (ii) The functions grow slowly with the internal quark masses. The typical values (for \( M_2 = 1 \text{ TeV} \)) are \( \beta x_c I_1(2)(x_c, \beta) = -1.3 \times 10^{-5}(9.2 \times 10^{-6}) \) and \( \beta x_t I_1(2)(x_t, \beta) = -4.3 \times 10^{-3}(0.077) \). Comparing with \( S_0(x_c(x_i)) = 2.8 \times 10^{-4}(2.6) \), one sees that for \( t \)-quark loop, the loop functions \( I_1(x_t, \beta) \) and \( I_2(x_t, \beta) \) are much smaller than \( S_0(x_t) \), which significantly suppresses the \( W_1W_2 \) loop contribution to \( B^0 \) mixing. While for \( c \)-quark loop, the \( W_1W_2 \) loop correction can be significant. Thus the main constraint for this model comes from neutral \( K \) meson system.

The matrix element for the scalar operator is given by

\[
\langle K^0| \bar{d}(1-\gamma_5)s \otimes \bar{d}(1+\gamma_5)s|\bar{K}^0 \rangle = \left[ \frac{1}{3} + \frac{2m_K^2}{(m_s+m_d)^2} \right] f_K m_K^2 B_K^S
\] (28)

with \( B_K^S \) the bag factor for scalar operator.

Since the \( c \)-quark mass dominates over the \( s,d \) quarks, only the first term in eq.(18) is considered which involves left-handed CKM matrix \( V^L \). The dominant contributions are:

\[
H_{eff}^{H^\pm W_1^\pm H^\pm} = \frac{G_F^2}{16\pi^2} m_W x_c y_c (\lambda_c^{LL})^2 \left[ 2\eta_c^{HH} |\xi_c|^2 B_V^{HH}(y_c, y_W) + \frac{1}{4} \eta_c^{HH} |\xi_c|^4 B_V^{HH}(y_c, y_W) \right] \]

\[
\cdot \bar{d}\gamma^\mu(1-\gamma_5)s \otimes \bar{d}_c(1-\gamma_5)s + \text{H.c.,}
\] (29)
with \( y_c = m_c^2/m_{H^\pm}^2 \) and \( y_W = m_W^2/m_{H^\pm}^2 \). The loop functions are given by

\[
B_{VW}^{HH}(y, y_W) = \frac{y_W - \frac{1}{4}}{(1 - y)(y - y_W)} + \frac{y_W - \frac{1}{4}y}{(1 - y)^2(1 - y_W)} \ln y + \frac{3}{4} \frac{y_W^2 \ln(y_W/y)}{(y_W - y)^2(1 - y_W)},
\]

\[
B_{HH}(y, y_W) = \frac{1 + y}{(1 - y)^2} + \frac{2y}{(1 - y)^3} \ln y. \tag{30}
\]

Note that the \( H^\pm H^\pm \) loop is proportional to \( |\xi_c^4| \), which significantly enhances the charged Higgs contribution at large \( \xi_c \).

The contributions to the effective Hamiltonian can also arise from the flavor changing neutral current interactions via neutral Higgs exchange at tree level, which is denoted as \( H_{\text{eff}}^{h^0} \). Summing up all the individual contributions, the total effective Hamiltonian is

\[
H_{\text{eff}} = H_{\text{eff}}^{W_1 W_2} + H_{\text{eff}}^{W_1 W_2 + S W_2} + H_{\text{eff}}^{H^\pm W_1 + H^\pm H^\pm} + H_{\text{eff}}^{h^0}. \tag{31}
\]

### A. \( K^0 - \bar{K}^0 \) mass difference

The mass difference for \( K \) meson is simply given by

\[
\Delta m_K \simeq 2 \text{Re}(M_{12}), \tag{32}
\]

which is dominated by internal \( c-\) quark loop for all loop diagrams, and can be calculated numerically. In Fig.1 the individual contribution from \( W_1 W_2 \) diagram is shown, the figure indicates a large negative \( M_{12} \) relative to the observed \( K^0 \) mass difference \( \Delta m_K \) at low \( M_2 < 1 \text{TeV} \), which reflects the difficulties to have a light \( W_2 \) bellow \( \text{TeV} \) in the one Higgs bi-doublet LR model. The situation is different when the charged Higgs contribution is taken into account. For the same set of CKM matrix element, the charged Higgs gives a positive contribution to \( M_{12} \), which is comparable to the \( W_2 \) term for light charged Higgs mass with sufficiently large couplings. Taking only the dominant \( H^\pm H^\pm \) loop contribution, we find that a cancellation between \( W_1 W_2 \) and \( H^\pm H^\pm \) loop requires

\[
\eta_{c c}^{H^\pm} x_c |\xi_c| |\xi_c| \frac{M_2^2}{m_H^2} \simeq - 24 \left[ \eta_1^{LR}(\ln x_c + 1) + \frac{\eta_2^{LR}}{4} \ln \left( \frac{m_W^2}{M_2^2} \right) \right] \frac{m_k^2}{(m_s + m_d)^2} B_{K}^S. \tag{33}
\]

A numerical calculation including all the contributions is shown in Fig.2. Numerically, for \( m_{H^+} \sim 150 \text{GeV} \) and Yukawa coupling \( \xi \sim 25 \), the charged Higgs can compensate a opposite contribution from a light \( W_R \) at \( M_2 \sim 600 \text{ GeV} \). The large Higgs contribution relies on the fact that the \( H^\pm W_1 \) loop is proportional to \( |\xi_c|^2 \) and \( H^\pm H^\pm \) loop proportional to \( |\xi_c|^4 \), which grow rapidly with \( |\xi_c| \) increasing.
In Fig. 3 we give total loop contributions from $W_1 W_2$ loop, $W_1 H$ loop, $HH$ loop together with the $W_1 W_1$ loop in the SM. The mass of $W_2$ is set to 600GeV. One sees that in the range of $150 \text{GeV} < m_H^+ < 250 \text{GeV}$ and $25 < \xi_c < 30$, the whole contribution can coincide with the experimental data of $\Delta m_K = (3.483 \pm 0.006) \times 10^{-15} \text{GeV}$. Generically, the needed charged Higgs mass $m_H^+$ grows with the mass of $W_2$. For a heavier $W_2$ at 1TeV and the same Yukawa coupling $\xi_c$ the allowed value of $m_H^+$ is around 250 $\sim$ 350 GeV. The numerical result for this case is shown in Fig. 4.

We now check the contributions from the flavor changing neutral current interactions via neutral Higgs exchanges at tree level. From eq. (18), we have

$$M_{h^0}^{12} \simeq 6.0 \times 10^{-16} \left( \frac{200 \text{GeV}}{m_h^0} \right)^2 \frac{(\eta_{12}^d - \eta_{21}^d)^2}{(0.1)^2},$$

(34)

where we have taken $m_d = 9\text{MeV}$ and $m_s = 180\text{MeV}$. By requiring that the $h^0$ contribution can not exceed the experimental data of $\Delta m_K$, one arrives at a upper bound of

$$\frac{\sqrt{\text{Re}[(\eta_{12}^d - \eta_{21}^d)^2]}}{m_{h^0}} \leq 8.3 \times 10^{-4} \text{GeV}^{-1}.$$

(35)

For $m_{h^0}$ around 200 GeV, $\sqrt{\text{Re}[(\eta_{12}^d - \eta_{21}^d)^2]} \leq 0.16 \approx \mathcal{O}(0.1)$, which is in agreement with the approximate global U(1) family symmetry of $|\eta_{12}^d| \ll 1$. It will be shown below that a more stringent constraint can arise from the indirect CP violation $\epsilon_K$. From that constraint, the contributions to the mass different from the flavor changing neutral Higgs interactions can be neglected.

B. Indirect CP violation $\epsilon_K$

The indirect CP violation parameter $\epsilon_K$ arises from imaginary part of the mixing amplitude, dominated by internal $(c, t)$ quarks and $(t, t)$ quarks. With different CP phases, the interference among $W_1 W_1$, $W_1 W_2$ and $H^\pm H^\pm$ are rather complicated and the allowed parameter space is large.

The expression for indirect CP violation is given by

$$|\epsilon_K| \simeq \frac{1}{2\sqrt{2}} \left( \frac{\text{Im}M_{12}}{\text{Re}M_{12}} + 2\xi_0 \right) \simeq \frac{\text{Im}M_{12}}{\sqrt{2}\Delta m_K},$$

(36)

where $\xi_0$ is the weak phase of $K \rightarrow \pi\pi$ decay amplitude with isospin zero.
Let us first examine the simplest case in which $|\xi_t|$ is tiny so that $H^±H^±$ loop is negligible. In this case the only extra contribution is from the $W_1W_2$ loop. It is straightforward to see that once the $(c, c)$ loop is set real as in Eq.(20), the $(c, t)$ and $(t, t)$ loop contributions become real as well because

$$\text{Im}[\lambda_c L_R^c \lambda_t R_L^t + (L \rightarrow R)] = -2|V_{cs}^L V_{cd}^L V_{ts}^L V_{td}^L| \cos(\alpha_2 + \alpha_3 - \beta_2 + \phi) \times \sin(\alpha_1 - \alpha_2 - \beta_1),$$

(37)

and

$$\text{Im}(\lambda_t L_R^c \lambda_t R_L^t) = -|V_{ts}^L V_{td}^L|^2 \sin(\alpha_1 - \alpha_2 - \beta_1),$$

(38)

where $\phi$ stands for $\arg(V_{cs}^L V_{cd}^L V_{ts}^L V_{td}^L)$ and $\phi \simeq \beta'_L \simeq \beta_L$ in the Wolfenstein parametrization. Thus, there is no contribution to $\epsilon_K$ from $W_1W_2$ loop.

In the case of non-negligible $\xi_t$, the charged Higgs contributes which involve the term proportional to $|\xi_t|^4$ and $|\xi_t \xi_c|^2$. The constraints are very similar to the general 2HDM case which has been analyzed in details in Ref.[25]. For a light charged Higgs mass $m_{H^+} = 150 \sim 300 \text{ GeV}$, the typically allowed values are

$$|\xi_c| \sim \mathcal{O}(10) \quad \text{and} \quad |\xi_t| \sim \mathcal{O}(10^{-1}),$$

(39)
FIG. 2: Contribution to $2\text{Re}(M_{12})$ from the lightest charge Higgs $H^+$. Three curves corresponds to $\xi_c = 30$ (solid), 25 (dashed) and 15 (dotted).

FIG. 3: Sum of all loop contributions, including the SM contribution to the $2M_{12}$ normalized to $\Delta m_K$ with $M_2 = 600$ GeV. Three curves corresponds to $|\xi_c| = 30$ (solid), 25 (dashed) and 15 (dotted) respectively.
FIG. 4: The same as in Fig. 3 with $M_2 = 1$ TeV.

which is consistent with the previous discussion. As $\xi_t$ is a free parameter, the constraints on $\xi_c$ is not severe. Nevertheless, it indicates that a small $|\xi_t|$ is needed to meeting the experiments.

The $\text{Im} M_{12}^{h_0}$ contribution to the indirect CP violation $\epsilon_K$ from the flavor changing neutral current via neutral Higgs change at tree level can be significant

$$\epsilon_K^{h_0} \simeq 4.25 \times 10^{-4} \left( \frac{200 \text{GeV}}{m_{h_0}} \right)^2 \frac{\text{Im}[(\eta_{12}^d - \eta_{21}^{d*})^2]}{(0.01)^2}.$$  \hspace{1cm} (40)

From the requirement $|\epsilon_K^{h_0}| < \epsilon_K^{\text{exp}}$, we arrive at the constraint

$$\frac{|\text{Im} (\eta_{12}^d - \eta_{21}^{d*})^2|^{1/2}}{m_{h_0}} < 6.9 \times 10^{-5} \text{GeV}^{-1},$$  \hspace{1cm} (41)

which is a more stringent constraint on the imaginary part of the Yukawa couplings. For $m_{h_0} = 200 \text{GeV}$, one has $|\text{Im} (\eta_{12}^d - \eta_{21}^{d*})^2|^{1/2} < 0.014$. It requires that either the off-diagonal coupling $\eta_{12}^d$ should be very small or CP-violating phase must be tuned to be very small, or the neutral scalars must be very heavy, above 1 TeV. Here we shall consider the small off diagonal coupling via the mechanism of approximate U(1) family symmetry, which allows to have a light Higgs boson at the electroweak scale. In other words, the flavor changing neutral Higgs interactions in the two Higgs bi-doublet model can really be suppressed via such a mechanism, namely

$$|\eta_{12}^d| \leq 0.01, \quad \text{for} \quad m_{h_0} \sim 200 \text{ GeV},$$  \hspace{1cm} (42)
which is significantly different from the case in the one Higgs bi-doublet model 35, 36, 56, 57, 58, in which the off-diagonal coupling is fixed to $(V_{ij})_1 m_i^q (V_{ji})_1 v_1^*/(|v_1|^2 - |v_2|^2)$.

IV. NEUTRAL $B$ MESON SYSTEM

In the previous section, we have illustrated that a light right-handed gauge boson can coincide with the $K$ mixing data. In this section, we shall show that it is consistent with the $B$ mixing measurements as well. Unlike the case in the Kaon system, the $B$ meson mixing is dominated by internal $t$–quark loop. Due to the weaker dependence of loop functions $I_{1,2}(x, \beta)$ on quark masses. $I_1(x_t, \beta)$ and $I_2(x_t, \beta)$ from the $W_1W_2$ loop are only a few percent of $S_0(x_t)$, which greatly suppresses it’s phenomenological significance in $B$ mixing and decays. The charged Higgs contribution is dominated by an extra parameter, the Yukawa coupling $\xi_t$, and is suppressed if $|\xi_t|$ is small.

In the first step let us take a close look at the $W_1W_2$ contribution. The effective Hamiltonian for $\Delta B = 2$ process with $W_1W_2$ loop is similar to the $\Delta S = 2$ case

$$H_{eff} \simeq \frac{G_F^2 m_W^2}{8\pi^2} \lambda_t^L \lambda_t^R \beta x_t \left[ (4 + x_t^2 \beta) I_1(x_t, \beta) - (1 + \beta) I_2(x_t, \beta) \right]$$

$$\bar{d}(1 - \gamma_5) b \otimes \bar{d}(1 + \gamma_5) b + \text{H.c.} \quad (43)$$

The QCD corrections at scale $m_b$ are $\eta_1 \simeq 1.8$ and $\eta_2 \simeq 1.7$ 57. The matrix element is given by

$$M_{12}^{W_1W_2} = \frac{G_F^2 m_W^2}{8\pi^2} |V_{td}^L|^2 \beta x_t \left[ 4\eta_1 I_1(x_t, \beta) - \eta_2 I_2(x_t, \beta) \right] \left( \frac{m_B^2}{m_b^2} + \frac{1}{6} \right) f_B^2 m_B B_B^S$$

$$= \frac{G_F^2 m_W^2}{8\pi^2} |V_{td}^L|^2 e^{-i(\beta_L + \beta_R)} \left[ \frac{m_t^2}{M_2^2} \left( 4\eta_1 I_1(x_t, \frac{m_W^2}{M_2^2}) - \eta_2 I_2(x_t, \frac{m_W^2}{M_2^2}) \right) \right]$$

$$\times \left( \frac{m_B^2}{m_b^2} + \frac{1}{6} \right) f_B^2 m_B B_B^S \quad (44)$$

(45)

In the limit that all $\alpha$'s are vanishing, one has $\beta_R = -\beta_L$. The bag parameters from QCD sum rule gives $B_B^S(m_b)/B_B(m_b) = 1.2 \pm 0.2$ 34, and $f_B\sqrt{B_B} = 0.228 \pm 0.030 \pm 0.010 \text{ GeV}$. The total contribution is given by

$$M_{12} = M_{12}^{SM} + M_{12}^{W_1W_2} = \frac{G_F^2 m_W^2}{6\pi^2} |V_{td}^L|^2 e^{-2i\beta_L} f_B^2 m_B B_B \{ \eta_B S_0(x_t)$$

$$+ \frac{3}{4} e^{-i(\beta_L - \beta_R)} \frac{m_t^2}{M_2^2} \left[ 4\eta_1 I_1(x_t, \frac{m_W^2}{M_2^2}) - \eta_2 I_2(x_t, \frac{m_W^2}{M_2^2}) \right] \left( \frac{m_B^2}{m_b^2} + \frac{1}{6} \right) \frac{B_B^S}{B_B} \} \quad (46)$$

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with QCD correction factor $\eta_B = 0.551 \pm 0.007$. The neutral $B$ meson mass difference is given by
\[
\Delta m_B \simeq 2 |M_{12}|.
\] (47)

The latest data give $\Delta m_B = (3.337 \pm 0.003) \times 10^{-13}$ GeV. The $B^0$ and $B^0_s$ mixing are the most important for determining the CKM matrix elements $V_{td}$ and $V_{ts}$ in the SM. The SM global fit gives $|V_{td}^L| = (7.4 \pm 0.8) \times 10^{-3}$ [60]. In the presence of new physics, the connection between $\Delta m_B$ and the CKM matrix elements is in general complicated (see, e.g., [60]). The right-handed gauge boson will contribute to both mixing amplitude and phases.

With the pollution from $W_2$, the time-dependent decay $B \to J/\psi K_S$ only measures an effective phase angle which may differ from $\beta_L$. The expression for $\beta_{eff}$ is
\[
2\beta_{eff} = \text{Im} \left( \frac{q \bar{A}}{p A} \right) = \text{Im} \sqrt{\frac{M_{12}^*}{M_{12}}} = \arg(M_{12}^*)
\] (48)

Using the measured experimental value of $\Delta m_B$ and $\beta_{eff}$ one can obtain the value of $\beta_L$ as a function of $\beta_R$ only. In the limit $m_W^2 \ll M_2$, $\beta_L$ is close to $\beta_{eff}$ we have in a good approximation
\[
\tan 2\beta_L \simeq \tan 2\beta_{eff} \left[ 1 - r \frac{\sin(\beta_R - \beta_{eff})}{2 \sin 4\beta_{eff}} \right],
\] (49)
where $r$ is the ratio between $W_1W_1$ and $W_1W_2$ box diagrams
\[
r = \frac{3m_t^2(4\eta_1 I_1(x_t, \beta) - \eta_2 I_2(x_t, \beta))}{4M_2^2 S(x_t)\eta_B} \left( \frac{m_B^2}{m_b^2} + \frac{1}{6} \right) \frac{B_B^S}{B_B}.
\] (50)

The above expression also lead to a bound on $\beta_L$ expressions
\[
\tan 2\beta_{eff} \left[ 1 - \frac{r}{2 \sin 4\beta_{eff}} \right] \leq \tan 2\beta_L \leq \tan 2\beta_{eff} \left[ 1 + \frac{r}{2 \sin 4\beta_{eff}} \right].
\] (51)

In Fig.5, we plot the $\beta_L$ as a function of $\beta_R$ with different right-handed gauge boson mass $M_2$. As mentioned before, the $W_2$ contribution to $B$ mixing is rather limited. One sees that for a light $W_2$ around 600 GeV, and $\beta_R$ varying from $-180^\circ$ to $180^\circ$, the modification to $\beta_L$ is less than $2^\circ$.

Once the $\beta_L$ is obtained, one can evaluate the matrix element $|V_{td}^L|$ which given in Fig.6 as a function of $\beta_R$. One sees again that the changes in $|V_{td}^L|$ is small for the whole range of $\beta_R$. Comparing with the global SM fit value of $|V_{td}^L|$ the modifications is within the 1$\sigma$ error range. Thus the model can easily accommodate both the data of $\Delta m_B$ and $\sin 2\beta_{J/\psi}$. For the left-right model with only one Higgs bi-doublet, since both $\beta_L$ and $\beta_R$ are calculable
quantities which depend only on the quark masses and ratios of VEVs, there is little room to meet the CP violation in both $K$ and $B$ system. Due to the suppression of CP phase form $\epsilon_K$, the predicted $\sin 2\beta_{J/\psi}$ has to be small and can not exceed 0.1.

The constraints from $B$ system to charged Higgs couplings is quite similar to the general 2HDM. For $\xi \ll 1$, the box-diagram contribution can be safely neglected. The constraint
on the neutral Higgs FCNC couplings can be easily obtained from Eq.(18).

\[ M_{12}^{h_0} \simeq 1.0 \times 10^{-12} \left( \frac{200\text{GeV}}{m_{h_0}} \right)^2 (\eta_{13}^d - \eta_{31}^{d*})^2. \]  

(52)

Using the experimental data \( \Delta m_B = 3.337 \times 10^{-3} \), one get a upper bound of

\[ \frac{|\eta_{13}^d - \eta_{31}^{d*}|}{m_{h_0}} \leq 2.4 \times 10^{-3}\text{GeV}^{-1}. \]  

(53)

For typical \( m_{h_0} = 200\text{GeV} \), \( |\eta_{13}^d - \eta_{31}^{d*}| < 0.41 \).

Similarly, from the recently measured \( \Delta m_{B_s} = 17.77 \pm 0.01 \pm 0.07\text{ps}^{-1} [61] \), one can infer a bound for the couplings \( \eta_{23}^d \) and \( \eta_{32}^d \)

\[ \frac{|\eta_{23}^d - \eta_{32}^{d*}|}{m_{h_0}} \leq 2.7 \times 10^{-3}\text{GeV}^{-1}. \]  

(54)

For typical \( m_{h_0} = 200\text{GeV} \), we get \( |\eta_{13}^d - \eta_{31}^{d*}| < 0.54 \). Both bounds satisfies the condition of \( |\eta_{ij}^q| \ll 1 \) from the approximate global \( U(1) \) family symmetry.

V. CONCLUSIONS

In summary, motivated by natural spontaneous P and CP violation and the latest low energy experimental results, we have investigated a general left-right symmetric model with two Higgs bi-doublets. This simple extension evades the stringent constraints from \( K \) meson mixing, and lowers the allowed mass of right-handed gauge boson closing to the current direct experimental search bound \( \sim 600\text{GeV} \). Through a negative interference with charged Higgs loop, which automatically occur when the charged Higgs is also light around electroweak scale with large Yukawa couplings. The FCNC can be suppressed by the mechanism of approximate global \( U(1) \) family symmetry. We have illustrated that the off diagonal Yukawa couplings of \( \mathcal{O}(10^{-2}) \sim \mathcal{O}(10^{-1}) \) are consistent with all the constraints. This model has rich sources of CP violation which may show up in lower energy processes such as rare \( B \) decays and the new physics particles can be directly searched in upcoming LHC and future ILC experiments.

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