Magnetic permeability of near-critical 3d abelian Higgs model and duality

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ABSTRACT: The three-dimensional abelian Higgs model has been argued to be dual to a scalar field theory with a global $U(1)$ symmetry. We show that this duality, together with the scaling and universality hypotheses, implies a scaling law for the magnetic permeability $\chi_m$ near the line of second order phase transition: $\chi_m \sim t^\nu$, where $t$ is the deviation from the critical line and $\nu \approx 0.67$ is a critical exponent of the $O(2)$ universality class. We also show that exactly on the critical lines, the dependence of magnetic induction on external magnetic field is quadratic, with a proportionality coefficient depending only on the gauge coupling. These predictions provide a way for testing the duality conjecture on the lattice in the Coulomb phase and at the phase transition.

KEYWORDS: Field Theories in Lower Dimensions, Thermal Field Theory.

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1. Introduction

Understanding phase transitions in gauge theories is important for the physics of the early Universe \([1]\) and heavy ion collisions \([2]\). In some cases, e.g., electroweak theory with a small ratio of the Higgs mass to the \(W\) mass, the phase transition can be treated reliably using perturbative techniques \([3]\). In many other cases (e.g., in the electroweak theory with \(m_H/m_W \sim 1\) or in QCD) perturbative calculations are unreliable and one has to resort to numerical simulations and other non-perturbative methods to learn about the nature of the phase transitions.

It is thus instructive to investigate simpler models where the phase transitions can be studied in detail. The abelian Higgs model (AHM, also called the Ginzburg-Landau model), which describes the metal-superconductor transition, is an example of a simple theory with a rather nontrivial phase diagram. This model has two distinct phases: the Higgs phase, where the gauge boson (photon) is massive, and the Coulomb phase with a massless photon. The two phases must be separated by a phase transition. The temperature-induced phase transition is first order deep in the type-I regime \((m_H \ll m_W)\), as shown by perturbative calculations \([3]\), but becomes second order as one goes to the type-II regime \((m_H \gtrsim m_W)\). The latter has been demonstrated by direct numerical simulations \([3, 4, 5]\).
That the phase transition in the AHM can be second order is somewhat surprising, given that it is always first order in $4 - \epsilon$ dimensions with small $\epsilon$ [7]. This fact, as has been argued, might have connection with a duality picture, according to which the three-dimensional AHM allows a dual description as a theory of a complex scalar field. The role of the elementary scalar in the dual theory is played by the Abrikosov-Nielsen-Olesen (ANO) vortex of the AHM [8, 9, 10, 11]. Although the exact form of the dual lagrangian is not known, quantitative predictions of the duality picture are possible near the second order phase transition, where only the symmetries of the dual lagrangian are important. If the duality is valid, certain quantities in the AHM must behave singularly near the phase transition with the critical exponents of the $O(2)$ universality class (i.e., of the $XY$ model). In this way one can test the duality picture on the lattice. Numerical tests of this sort have been carried out in the Higgs phase, where, according to duality, the tension of an ANO vortex is equal to the mass of the dual scalar, and hence approaches zero as $t^\nu$, where $t$ is the distance to the critical line and $\nu \approx 0.67$ is a critical exponent of the $XY$ model.¹ Lattice results are still inconclusive, some appear to be inconsistent with this prediction [5, 13].

In this paper, we suggest some other tests of the duality hypothesis. In addition to measuring of the vortex tension, we propose to consider the AHM in its Coulomb phase and at the phase transition. What one should measure in the Coulomb phase is the magnetic permeability $\chi_m$, which goes to zero as one approaches the critical line where the Meissner effect (i.e., the Higgs mechanism) starts taking place and the system is perfectly diamagnetic. We shall show that $\chi_m$ is proportional to the square of the decay constant of the Goldstone boson in the dual theory. This mapping is precise (provided duality is valid) and involves only quantities which are not renormalized. Using scaling and universality hypotheses, we then show that the critical behavior for $\chi_m$ is $\chi_m \sim t^\nu$. Exactly on the critical line, the magnetic permeability vanishes and the dependence of magnetic induction $B$ on external magnetic field $H$ is nonlinear. We shall demonstrate, by using simple scaling arguments, that this dependence is quadratic: $B \sim H^2$, with a proportionality coefficient depending only on the gauge coupling $e$.

The paper is organized as follows. In section 2 we review the duality picture. Section 3 is devoted to the study of the magnetic permeability of the Coulomb phase. The main line of logic in this section consists of three steps. In the first step (section 3.1) one relates the magnetic permeability of the AHM with the susceptibility of the dual vacuum to the $U(1)$ chemical potential. The second step (section 3.2) establishes, in the dual theory, the connection between the susceptibility with the decay constant of the Goldstone boson. The third step (section 3.3) determines the critical behavior of

¹The critical behavior of the photon mass is discussed in ref. [12].
the decay constant. Each step involves fairly well-known arguments, but we believe their synthesis is new. In section 3 we discuss the response of the AHM to an external magnetic field exactly on the critical line. Section 5 contains concluding remarks.

2. Review of the duality picture

Although we are mostly interested in the phase transition driven by temperature, thanks to dimensional reduction we can describe the static long-distance physics by an Euclidean three-dimensional AHM theory. Changing the temperature in the (3+1)d theory corresponds varying the parameter of the 3d dimensionally reduced theory. We shall thus start directly from the AHM in three spatial dimensions. It is sometimes useful, especially in our discussion of duality, to turn one spatial dimension into a temporal dimension; we then have a (2+1)d AHM, where the ANO vortex is a particle. This particle is the elementary scalar in the dual theory. The duality was first argued by using a formal representation of the partition function of the AHM in terms of loops [8, 9, 10]. Subsequently it has been given an operator form in 2+1 dimensions [11].

| abelian Higgs model                                      | complex scalar theory                                      |
|----------------------------------------------------------|-----------------------------------------------------------|
| magnetic induction (total magnetic field)                | $U(1)$ charge density                                       |
| external magnetic field                                  | chemical potential                                         |
| Coulomb phase                                            | broken $U(1)$ (superfluid phase)                           |
| photon                                                   | Goldstone boson                                            |
| Higgs particle                                           | global vortex                                             |
| magnetic permeability $\chi_m$                          | square of decay constant $f^2$                            |
| Higgs phase                                              | unbroken $U(1)$ (Mott insulator)                           |
| Abrikosov-Nielsen-Olesen vortex                          | scalar particle                                           |
| vortex tension                                           | scalar mass                                                |
| critical magnetic field                                  | scalar mass                                                |

**Table 1:** The duality maps between the abelian Higgs model and the dual theory of a complex scalar. Some of the mappings are explained further in the paper.

We summarize the correspondence between the AHM and the dual complex scalar theory in table 1. The Higgs phase is dual to the phase with unbroken $U(1)$ symmetry (the Mott insulator phase), and the Coulomb phase is dual to the phase where the $U(1)$ global symmetry is broken (the superfluid phase) by the condensation of the dual scalar field (“vortex condensation”). The dual of the massless photon in the AHM Coulomb phase is the superfluid Goldstone boson. This is possible because photon has only one
polarization in 2+1 dimensions. The vortices in the AHM and the scalar theory, when exist, correspond to a particle in the other theory.

For the purpose of this paper, the most important equation of the duality picture comes from the identification of the magnetic field in the AHM with the $U(1)$ current in the dual theory,

$$\frac{e}{2\pi} \epsilon^{\mu\nu\lambda} F_{\nu\lambda} = j^\mu. \quad (2.1)$$

The conservation of the $U(1)$ current is an automatic consequence of eq. (2.1). The correspondence (2.1) implies that the magnetic flux is proportional to the $U(1)$ charge:

$$eF_{12}/2\pi = j_0,$$

where the proportionality coefficient $e/2\pi$ is such that the quantum of the magnetic flux corresponds to an unit charge. This suggests that the vortex of the AHM is the elementary scalar particle in the dual theory.

If one works in 2+1 dimensions, it is also possible to construct in the AHM a local operator creating unit magnetic flux, which plays the role of the order parameter in the dual theory [1]. The precise form of this local order parameter is not as important to us as the fact that it exists. We denote the order parameter as $V$; the Mott insulator phase has $\langle V \rangle = 0$ and the superfluid phase has $\langle V \rangle \neq 0$. The exact form of the lagrangian for $V$, $\mathcal{L}(V)$, is not known and also not important for further discussion. On the other hand, the expression for the $U(1)$ charge density operator is known exactly,

$$j_0 = -i(\pi V - \pi^* V^*), \quad (2.2)$$

where $\pi$ is the operator canonically conjugate to $V$.

### 3. Magnetic permeability of the Coulomb phase

#### 3.1 Magnetic permeability and susceptibility of the dual vacuum

We now turn on an external magnetic field $H$ in the AHM. Assuming that the external field is aligned along the $z$ axis, the lagrangian becomes

$$\mathcal{L} = \mathcal{L}_0 - HF_{12}. \quad (3.1)$$

From the point of view of the physics in (2+1)d, the hamiltonian is changed by

$$\mathcal{H} = \mathcal{H}_0 - HF_{12}. \quad (3.2)$$

Once $H$ is turned on, $B \equiv F_{12}$ obtains an expectation value. Following established tradition, we shall call the total magnetic field $\langle B \rangle$ the magnetic induction, and $H$ the external magnetic field. The magnetic permeability is defined as $\chi_m = \partial \langle B \rangle / \partial H |_{H=0}$. 
In the (2+1)d dual theory, $B$ is proportional to the charge density. Thus, the external magnetic field in the AHM corresponds to the chemical potential coupled to the $U(1)$ charge in the dual theory \[14\],
\[
\mathcal{H}(\pi, V) = \mathcal{H}_0(\pi, V) + i\mu(\pi V - \pi^* V^*),
\]
where $\mathcal{H}_0(\pi, V)$ is the hamiltonian in the absence of the chemical potential. The relation between the chemical potential in the dual theory and the external magnetic field in AHM, according to eq. (2.1), is
\[
\mu = \frac{2\pi}{e} H.
\]
(3.4)

The magnetic permeability of the AHM is thus proportional to the susceptibility of the vacuum of the dual theory with respect to the $U(1)$ chemical potential,
\[
\chi_m \equiv \frac{\partial \langle B \rangle}{\partial H} = \frac{4\pi^2}{e^2} \frac{\partial \langle j_0 \rangle}{\partial \mu} \equiv 4\pi^2 \frac{e^2}{\chi} \chi.
\]
(3.5)

Therefore, in order to find the critical behavior of $\chi_m$, one needs to find that of $\chi$ in the dual theory. Notice that $e^2\chi_m$ receives no renormalization in the AHM, and $\chi$ is not renormalized in the dual theory.

Before turning to the Coulomb phase, which is the main subject of this paper, let us make a comment about the response of the Higgs phase to an external magnetic field. According to the duality hypothesis, the Higgs phase of the AHM corresponds to the Mott insulator phase of the dual theory where the $U(1)$ symmetry is not spontaneously broken. This phase has a mass gap $m$ equal to the mass of the elementary scalar. At zero temperature, a chemical potential less than $m$ has no effect on the vacuum, since creating an excitation costs positive energy ($m - \mu$ for a particle and $m + \mu$ for an antiparticle). In particular, the charge density remains zero if $|\mu| < m$. The counterpart of this is the Meissner effect in the AHM: the total magnetic field inside a superconductor remains zero if one turns on a small external magnetic field. If the chemical potential in the dual theory exceeds $m$, then it becomes energetically favorable to create particles. This corresponds, in the AHM, to the existence of a critical magnetic field (more precisely, the lower critical magnetic field $H_{c1}$), above which the magnetic field begins to penetrate the type-II superconductor.

### 3.2 Susceptibility and decay constant of the Goldstone boson

Let us now return to the Coulomb phase, or the superfluid phase from the dual point of view. Due to the breaking of the $U(1)$ symmetry, the low-energy dynamics of the theory contains only one Goldstone mode, which is the phase $\varphi$ of the order parameter $V = |V|e^{i\varphi}$. Let us first assume no external magnetic field (chemical potential in the
dual theory). In contrast to the lagrangian for $V$ which is not known, the form of the low-energy effective lagrangian for the Goldstone mode is completely fixed,

$$L_{\text{eff}} (\varphi) = \frac{f^2}{2} (\partial_\mu \varphi)^2 ,$$  \hspace{1cm} (3.6)$$

where $f$ is some parameter to be called the “decay constant” of the Goldstone boson, since it is the analog of the pion decay constant $f_\pi$ in the chiral lagrangian of QCD.\footnote{In condensed matter literature $f^2$ is called the stiffness.} Notice, however, that in $d$ dimensions $f^2$ has dimension $d-1$, i.e, in 3d $f^2$ has the dimension of energy. The effective theory (3.6) is valid below the typical energy of non-Goldstone excitations, which we denote as $m_\sigma$.

If the chemical potential $\mu$ is small compared to $m_\sigma$, its effect can be captured within the framework of the effective theory. Moreover, the way $\mu$ enters the effective lagrangian can also be fully determined \cite{15}. First we notice that, by taking the Legendre transform of eq. (3.3), the lagrangian for $V$ in the presence of a chemical potential can be obtained from the lagrangian with no chemical potential $L_0 (V)$ by making the replacement

$$\partial_0 V \to \partial_0 V - i \mu V .$$  \hspace{1cm} (3.7)$$

In other words, $\mu$ enters the lagrangian as the zeroth component of a fictitious gauge field arising from gauging the global $U(1)$ symmetry.

A useful trick is to consider this fictitious gauge field as a fully dynamical field $A_\mu$ and make the substitution $A_0 = \mu$, $A_i = 0$ at the very end \cite{14}. The lagrangian for $V$ is invariant under gauge transformations $V \to e^{i\alpha} V$, $A_\mu \to A_\mu + \partial_\mu \alpha$. If $A_\mu$ is small and slowly varying, this gauge invariance must also be a property of the effective lagrangian for $\varphi$ as well. Noticing that $\varphi$ transforms as $\varphi \to \varphi + \alpha$, one sees that the gauge invariance can be preserved if in the effective lagrangian (3.6) $\partial_\mu \varphi$ is replaced by $\partial_\mu \varphi - A_\mu$. Substituting in the final answer $A_\mu = (\mu, 0)$, one discovers the effective lagrangian for the Goldstone mode in the presence of the chemical potential:

$$L_{\text{eff}} (\varphi) = \frac{f^2}{2} [(\partial_0 \varphi - \mu)^2 - (\partial_i \varphi)^2] .$$  \hspace{1cm} (3.8)$$

The ground state free energy (i.e., pressure) is obtained by putting $\varphi = \text{const}$ into eq. (3.8), and is equal to $f^2 \mu^2 / 2$. The susceptibility is, by definition, the second derivative of the pressure with respect to the chemical potential $\mu$, i.e.,

$$\chi = f^2 .$$  \hspace{1cm} (3.9)$$

Together with eq. (3.5), the magnetic permeability of the AHM is now related to the decay constant of the Goldstone boson.
3.3 Critical behavior of the decay constant

Now let us find out the critical behavior of $f^2$. This can be done by using various arguments. The simplest one is purely dimensional in nature: one notices that $\varphi$ is a dimensionless phase variable, defined mod $2\pi$, so it is not renormalized. Therefore $f^2$ has the canonical dimension $d - 2 = 1$. According to the scaling hypothesis, the mass scale of non-Goldstone excitation $m_\sigma$ is the only dimensionful scale near the critical point, so $f^2 \sim m_\sigma^{d-2}$, i.e., $f^2 \sim t^{(d-2)\nu} = t^\nu$ in 3d.

Another argument is similar to the one originally used by Josephson [10] for the stiffness parameter of superfluid helium, and in ref. [17] for the pion decay constant near the chiral phase transition. Let us decompose the complex order parameter $V$ into the real and imaginary parts,

$$V = V_1 + iV_2,$$

and choose the ground state so that $\langle V_1 \rangle = \langle V \rangle$ and $\langle V_2 \rangle = 0$. At distances large compared to $m_\sigma^{-1}$, the amplitude of the order parameter is effectively frozen, and $V_2$ is proportional to the Goldstone field, $V_2 = \langle V \rangle \varphi$. Therefore, the correlator of $V_2$ can be found from the effective lagrangian for $\varphi$, eq. (3.6),

$$\int d^3x e^{-iq \cdot x} \langle V_2(x)V_2(0) \rangle = \frac{\langle V \rangle^2}{f^2q^2}, \quad q \ll m_\sigma. \quad (3.11)$$

On the other hand, at distances small compared to $m_\sigma^{-1}$, the system is effectively at the critical line and is $O(2)$ symmetric. In this regime the correlator of $V_2$ have the same form as that of the order parameter at the Wilson-Fisher fixed point, i.e.,

$$\int d^3x e^{-iq \cdot x} \langle V_2(x)V_2(0) \rangle = \int d^3x e^{-iq \cdot x} \langle V_1(x)V_1(0) \rangle = \frac{c}{q^{2-\eta}}, \quad q \gg m_\sigma, \quad (3.12)$$

where $c$ is a constant independent of the distance $t$ to the critical line. The two formulas (3.11) and (3.12) are valid in two opposite regimes, but must smoothly match to each other at $q \sim m_\sigma$. From this condition one finds the critical behavior of $f^2$,

$$f^2 \sim \langle V \rangle^2 m_\sigma^{-\eta} \sim t^{2\beta-\eta \nu}. \quad (3.13)$$

By using a well-known (hyperscaling) relation between the critical exponents,

$$2\beta = \nu(d - 2 + \eta), \quad (3.14)$$

one finally obtains $f^2 \sim t^{(d-2)\nu} = t^\nu$, which agrees with the previous dimensional argument. Notice that $f \sim t^{\nu/2}$ scales differently from the order parameter, $\langle V \rangle \sim t^\beta$, 

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in contrast to what one may naively expect, but the difference is numerically small because of the smallness of \( \eta \) in eq. (3.14). Since, as we have shown in eqs. (3.5) and (3.9), the magnetic permeability of the AHM is proportional to \( f^2 \), one concludes that \( \chi_m \) approaches 0 as

\[
\chi_m \sim t^{\nu},
\]

which is one of the main results of this paper.

4. Magnetic response at the critical line

Let us now turn our attention to the response of the critical AHM to an external magnetic field. On the critical line \( \chi_m = 0 \), so the dependence of the magnetic induction \( B \) on the external field \( H \) must be nonlinear. The dependence of \( B \) on \( H \) can be found in the most intuitive way by going to the dual theory, where it governs the dependence of the charge density \( j_0 \) on the chemical potential \( \mu \) at the critical line. Since at the critical point the dual theory is a conformal field theory with no intrinsic scale, the only dimensionful parameter is \( \mu \). Due to charge conservation, \( j_0 \) has the canonical dimension, i.e., \( d - 1 \). Therefore, \( j_0 \sim \mu^{d-1} \), which in three dimensions reads

\[
j_0 = C \mu^2,
\]

From eqs. (2.1), (3.4) and (4.1), we find that the magnetic induction is quadratic over the external magnetic field:

\[
B = \left( \frac{2\pi}{e} \right)^3 C H^2.
\]

The dependence (4.1) can be found by another (related) argument, which elucidates the magnetic response of the near-critical AHM. Let us slightly go away from the critical line toward the Coulomb phase. The dual theory is in the superfluid phase, characterized by a small decay constant \( f \) and a small mass scale of non-Goldstone excitations \( m_\sigma \). If the chemical potential \( \mu \) is small compared to \( m_\sigma \), then \( j_0 \) is linear of \( \mu \), with the proportionality coefficient equal to the susceptibility \( \chi = f^2 \),

\[
j_0 = f^2 \mu, \quad \mu \ll m_\sigma.
\]

In the opposite regime \( \mu \gg m_\sigma \), the slight deviation from the critical line is unimportant, and the dependence must be of the form \( j_0 = C \mu^n \), where \( C \) is independent of \( t \) and \( n \) needs to be found. Recalling that \( f^2 \sim m_\sigma^{d-2} \), this power-law behavior can match with eq. (4.3) at \( \mu \sim m_\sigma \) if and only if \( n = d - 1 = 2 \), i.e., it must be of the form (4.1).

As the infrared fixed point of the dual theory is unique, one should expect the constant \( C \) in eq. (4.1) to be universal. In this case the proportionality coefficient
between $B$ and $H^2$ in eq. (4.2) depends only on the gauge coupling $e$, but not on the Higgs self-coupling (provided the phase transition is second order).

5. Conclusion

One should note that there exists a rather trivial way to test duality in the Coulomb phase (in fact, also in the Higgs phase), which is based on the measurement of the specific heat. However, this method is potentially difficult from the numerical point of view due to the smallness of the critical exponent $\alpha$ in the $O(2)$ universality class. Therefore, one should look for a quantity with a stronger critical behavior. We propose here to use the magnetic permeability, which goes to zero as $\chi_m \sim t^{\nu}$, for this purpose. We have demonstrated that the scaling law for $\chi_m$ can be found by invoking fairly standard arguments of duality, scaling, and universality. In addition, we have shown that on the line of second order phase transition the dependence of magnetic induction $B$ on external magnetic field $H$ is quadratic, $B \sim e^{-3} H^2$, and the proportionality coefficient does not depend on the Higgs self-coupling.

These predictions, in principle verifiable on the lattice, are rather nontrivial tests of duality. These tests, relevant for the Coulomb phase and the critical line, should be considered as supplements to the measurement of the vortex tension in the Higgs phase. Looking from a broader perspective, we hope that the investigation of the phase transition in the AHM will further elucidate the physics at the phase transitions of more realistic theories, e.g., the standard model or QCD.

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