Fractional instantons
in supersymmetric gauge theories

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Abstract

We consider evidence for the existence of gauge configurations with fractional charge in pure $N = 1$ supersymmetric Yang-Mills theory. We argue that these field configurations are singular and have to be treated as distributions. It is shown that the path integral representation of constant Green’s functions can be reduced to a finite dimensional integral. The fractional configurations are essentially the zero size limit of the usual instantons and they have a reduced number of fermionic zero modes. At the end we comment on the status of the $D$-instanton/YM-instanton correspondence within $AdS/CFT$ correspondence.

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1 Introduction

In the last few years a number of remarkable results on the non-perturbative behavior in
gauge theories and string theories have appeared. The exact solutions for supersymmetric
gauge theories are striking in their self-consistency and seem to refute any doubts in their
validity. One of the basic result is the existence of a non-zero gluino condensate in pure
$N = 1$ supersymmetric Yang-Mills theory. There are different consistent approaches to get
non-zero gluino condensate, from the Veneziano-Yankielowicz effective action \[1\] to the in-
stanton calculations \[2\]. Also the existence of a non-zero gluino condensate is consistenly
incorporated in the full picture of supersymmetric gauge and string theories. Nevertheless
within the framework of QFT there is still a confusing question: What kind of field con-
figurations gives rise the gluino condensate itself? The solution of this problem might be
relevant to the understanding of confinement in the supersymmetric gauge theories. There
has been an attempt to answer this question in a finite volume \[3\] by introducing twisted
boundary conditions. To the author’s knowledge the only attempt to answer this question
in infinite volume is by introducing multivalued field configurations \[4\]. In the present letter
we try to give an answer for the problem without any direct references to the ideas of \[3\]
and \[4\].

The main puzzle is that the existence of a non-zero gluino condensate requires gauge con-
figurations with fractional topological charge. This goes beyond our present understanding
of QFT. There are no smooth gauge field configuration with finite action which can have a
reduced number of fermionic zero modes. According to standard wisdom, all gauge config-
urations are classified by integer topological charge and the path integral is just a sum over
different topological sectors. The above problem can be resolved if one is willing to study
non smooth configurations which nevertheless have finite action. We are going to look for
singular configurations with the right properties. We argue that in fact the best candidate
for this role is a configuration with point support which can be regarded as zero-size instan-
ton. We think that it is an important lesson for gauge theories to understand that perhaps
the path integral is not just a sum over different topological sectors of smooth configurations
but also a sum over some singular configurations.

The organization of the paper is as follows. In section 2 we review the basic facts about
pure supersymmetric Yang-Mills theories. We recall the chiral selection rules and show that
non-zero gluino condensate requires a gauge configuration with two fermionic zero modes.
In section 3 we consider a QFT with constant Green’s functions (SYM is example of such
theory). Using these constant Green’s functions we argue that the configurations with point

\[1\] In the sense that one has a unique recipe to define the action for such configurations.
support are important in the path integral and that only the topological part survives in the action (therefore it is just a number). In section 4 we consider the configurations with point support and show that in fact they have the right number of fermionic zero modes. We point out that the zero-size instantons are essentially merons, configurations which were introduced by hand in \[5\] to get the area law for Wilson loop operator. In section 5 we give our conclusions and comment on the status of the $D$-instanton/YM-instanton correspondence within AdS/CFT correspondence.

2 Evidence for the fractional configurations

First of all let us recall the basic relevant facts about the pure supersymmetric gauge theory in four dimensions. The SYM Lagrangian describing the gluodynamics of gluons $A_\mu$ and gluinos $\lambda_\alpha$ with a general compact gauge group has the form

$$L = -\frac{1}{4g_0^2}G^a_{\mu\nu}G^a_{\mu\nu} + \frac{i}{g_0^\alpha}D^{\dot{\alpha}\beta}\lambda_\beta + \frac{i\theta}{32\pi^2}G^a_{\mu\nu}\tilde{G}^a_{\mu\nu},$$

where $G^a_{\mu\nu}$ is the gluon field strength tensor, $\tilde{G}^a_{\mu\nu}$ is the dual tensor and $D^{\dot{\alpha}\beta}$ is the covariant derivative and all quantities are defined with respect to the adjoint representation of the gauge group. This Lagrangian may be written in terms of the gauge superfield $W_\alpha$ with physical components $(\lambda_\alpha, A_\mu)$ as follows

$$L = \frac{1}{8\pi}Im \int d^2\theta \tau_0 W^\alpha W_\alpha,$$

where the bare gauge coupling $\tau_0$ is defined to be $\tau_0 = \frac{4\pi i}{g_0^2} + \frac{\theta_0}{2\pi}$. The model possesses a discrete global $Z_{2C_2}$ symmetry\[1\] a residual non-anomalous subgroup of the anomalous chiral $U(1)$. The discrete chiral symmetry $Z_{2C_2}$ is spontaneously broken by a non-zero gluino condensate $\langle \lambda^\alpha \lambda^\alpha \rangle \equiv \langle \lambda \lambda \rangle$. In term of the strong coupling scale $\Lambda$ the gluino condensate has the form

$$\langle \lambda \lambda \rangle = c \Lambda^3 e^{\frac{2\pi ik}{C_2}}, \quad k \in \mathbb{Z}.$$ 

where the constant $c$ can be absorbed in a redefinition of $\Lambda$. Therefore the system has $C_2$ different values of the gluino condensate and can be in any one of the $C_2$ vacua. In what follows we suppose that the system sits in one of the vacua and that all calculations are valid for this phase. The supersymmetry requires that the Green’s functions of the composite

\[2C_2\] denotes the Dynkin index (the quadratic Casimir) with $C_2 = 1/2$ normalization for the fundamental representations of $SU(N)$, $Sp(N)$ and with $C_2 = 1$ normalization for the vector representation of $SO(N)$. So for adjoint representations we use the following values: $C_2(SU(N)) = N$, $C_2(Sp(2N)) = N+1$, $C_2(SO(N)) = N-2$, $C_2(E_6) = 12$, $C_2(E_7) = 18$, $C_2(E_8) = 30$, $C_2(F_4) = 9$, $C_2(G_2) = 4$. 

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operator $\lambda\lambda$ do not depend on the coordinates. Due to the cluster decomposition property we have the following equalities

$$\langle \lambda\lambda(x_1)\lambda\lambda(x_2)\ldots\lambda\lambda(x_n) \rangle = \Lambda^{3n}, \quad \langle \lambda\lambda(x) \rangle = \Lambda^3. \quad (4)$$

We mainly concentrate our attention on SYM with an $SU(2)$ gauge group.

Now we consider the arguments supporting the existence of fractional topological charge. It is believed that the path integral is a sum over different topological sectors. Let us look at the Green’s function restricted to one topological sector. Consider a chiral rotation of fermions $\lambda \to e^{i\alpha}\lambda$ which gives the following rotation in the path integral representation of the relevant Green’s functions

$$\langle \lambda\lambda(x_1)\lambda\lambda(x_2)\ldots\lambda\lambda(x_n) \rangle = e^{(2in\alpha - if)} \int DA D\lambda \lambda\lambda(x_1)\lambda\lambda(x_2)\ldots\lambda\lambda(x_n)e^{-S(A,\lambda)} \quad (5)$$

where $f = \nu_+ - \nu_-$ and $\nu_+$ ($\nu_-$) is number of fermionic zero modes with positive (negative) chirality. The equality (5) is just a change of variables in path integral. The exponent in front of the path integral comes from an explicit rotation of the fermion fields and the anomalous rotation of the measure. The last contribution can be derived using the Fujikawa’s method \[7\]. Since the expression is independent of $\alpha$ we can take derivative with respect to $\alpha$ and get the following chiral selection rules

$$(2n - f)\langle \lambda\lambda(x_1)\lambda\lambda(x_2)\ldots\lambda\lambda(x_n) \rangle = 0. \quad (6)$$

In fact the number fermionic zero modes is related to the properties of the gauge field. We are interested in the continuous gauge configurations which give a finite action. Therefore all such continuous configurations are characterized by the third homotopy group $\pi_3(G)$. The corresponding topological charge is given by the following expression

$$Q = \frac{1}{32\pi^2} \int d^4x \text{Tr}(e^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma}) \quad (7)$$

which is called the second Chern or first Pontrjagin number, depending on the group. The Atiyah-Singer theorem gives the dependence between the number of zero modes of the Dirac operator in the gauge background and the second Chern number of this background $2C_2Q = f$. After all one can rewrite the expression (6) as follows

$$(n - C_2Q)\langle \lambda\lambda(x_1)\lambda\lambda(x_2)\ldots\lambda\lambda(x_n) \rangle = 0. \quad (8)$$

Therefore one concludes that that the n-point function is saturated by the gauge configurations with topological number $Q = n/C_2$. Since we consider fermions in the adjoint representation $C_2$ is quite “big”. The non-vanishing gluino condensate has $Q = 1/C_2$, but $Q$
is supposed to be integer. In the work on instantons \cite{2}, to avoid contradiction, the following average over degenerate vacua was taken

\[C_2 \sum_{k=1}^{C_2} \langle \lambda \lambda(x_1) \lambda \lambda(x_2) \ldots \lambda \lambda(x_n) \rangle = 0, \quad \frac{n}{C_2} \notin \mathbb{Z} \]  

(9)

where \(k\) labels the different vacua. So these averaged Green’s functions vanish when \(Q\) is fractional. From general principlees of QFT we have no right to average Green’s functions over different degenerate vacua. The worlds with different gluino condensate live completely independent lives (with no correlations between them). Thus as soon as we accept the existence of a non-vanishing gluino condensate and an unbroken supersymmetry we have to accept the existence of all \(n\)-point Green’s functions (4) for the operator \(\lambda \lambda\). We still consider a system which sits in one of the vacua.

To resolve this problem we have to assume that the gauge configurations which give rise the Green’s functions are not continuous (and as a result not smooth). If we accept this then the topological charge cannot classify these configurations by definition. To be able to work with such configurations one has to go to the notion of generalised function (distribution). It is an essential starting point for the axiomatic approach to QFT to think about fields as operator valued distributions. It can be shown that considering fields as ordinary functions (objects defined at each point of space time) leads to contradictions in QFT or triviality of QFT \cite{6}. Further we will argue that the role of fractional instantons may be played by field configurations with point support. We have to understand that such configurations are not instantons in the usual sense, they are singular and any reasonable calculation requires regularization of these configurations. Nevertheless these configurations satisfy the self-duality conditions (in a generalized sense) and we may view them as instantons, at least formally.

### 3 Constant Green’s functions

In this section we are going to use the property that the relevant Green’s functions are constant. This property is a result of unbroken supersymmetry. To simplify the notation we will consider an abstract QFT and identify the composite operator \(\lambda \lambda\) with the fundamental field. All arguments can be straightforwardly specified to a situation with certain field content. Let us consider a QFT with some operator \(\phi(x)\) which has a constant non vanishing \(n\)-point Green’s function \(\langle \phi(x_1) \phi(x_2) \ldots \phi(x_n) \rangle\). Due to the cluster decomposition property we have the following equalities

\[\langle \phi(x_1) \phi(x_2) \ldots \phi(x_n) \rangle = A^n, \quad \langle \phi(x) \rangle = A.\]  

(10)
The generating functional for the Green functions is

\[ Z[J(x)] = \int D\phi \ e^{-S(\phi) + \int d^4 x \phi(x) J(x)}, \quad Z[0] = 1 \] (11)

which one can expand in the standard way

\[ Z[J(x)] = \sum_{n=0}^{\infty} \frac{1}{n!} \int \cdots \int d^4 x_1 d^4 x_2 \cdots d^4 x_n J(x_1) J(x_2) \cdots J(x_n) \langle \phi(x_1) \phi(x_2) \cdots \phi(x_n) \rangle \] (12)

Using the equlities (10) we can rewrite the expression (12) as follows

\[ Z[J(x)] = \sum_{n=0}^{\infty} A^n \frac{1}{n!} \int d^4 x_1 J(x_1) \int d^4 x_2 J(x_2) \cdots \int d^4 x_n J(x_n) = e^{A \int d^4 x J(x)} \] (13)

In general it is very difficult to treat rigirously the path integral representation of the generating functional since this object is non-linear functional of the source. But the expression (13) is somewhat particular. Usually one defines such constructions on the appropriate Banach space. Let us consider the generating functional on the space \( L_1(R^4) \) (the Banach space with norm \( \|J\|_{L_1} = \int d^4 x |J(x)| < \infty \)). The key observation is that the logarithm of the generating functional (13) is a linear continious functional on \( L_1(R^4) \) (\( |\log Z[J(x)]| \leq A \|J\|_{L_1} \)). Therefore we can define the generating functional on some dense subset of the space and afterwards get a unique continuation to the whole space. We will consider sources which have the following special form

\[ J(x) = \begin{cases} J = \text{const}, & x \in \mathcal{B}; \\ 0, & x \notin \mathcal{B}. \end{cases} \] (14)

where \( \mathcal{B} \) is a set of finite volume \( V \) in \( R^4 \) which is homeomorphic to the ball. The set of linear combinations of the functions (14) is a dense subset in \( L_1(R^4) \). Hence for every function \( J(x) \in L_1(R^4) \) one can find sequence \( J_n(x) \) which is a finite linear combination of sources \( J_i \) defined in (14)

\[ J(x) = \lim_n \sum_i J_i(x) \] (15)

where any numbers are included in the definitions of \( J_i(x) \). The limit in (15) is understood with respect to topology given by \( \|\cdot\|_{L_1} \). Thus we can write the following definition of the generating functional on arbitrary function from \( L_1 \)

\[ \log Z[J(x)] = \lim_n \sum_i (\log Z[J_i(x)]) \] (16)

We conclude that it is enough to consider the generating functional restricted on the subset of sources with form (14). Let us concentrate our attention on the one-point function. The integral over the source (14) is \( \int d^4 x J(x) = JV \). We see that the functional \( Z[J,V] = e^{AV} \)
is essentially a function of $J$ and $V$. Thus one has the following definition of the relevant Green’s functions

$$\langle \phi(x) \rangle = \frac{1}{V} \left. \frac{\partial Z}{\partial J} \right|_{J=0} = \frac{1}{V} \left. \frac{\partial Z}{\partial J} \right|_{V=0} = \frac{1}{V} \left. \frac{\partial Z}{\partial V} \right|_{V=0}. \quad (17)$$

These definitions of constant Green’s functions are somewhat surprising since, after taking derivatives, one has to take the zero volume limit. Let us look more carefully at the construction

$$Z[J,V] = \int D\phi e^{-S(\phi)+J \int_\mathcal{B} d^4 x \phi(x)} \quad (18)$$

where the action is an integral over all of space-time $S(\phi) = \int d^4 x \mathcal{L}(\phi)$. Every function $\phi(x)$ can be decomposed as $\phi(x) = \phi_1(x) + \phi_2(x)$ where $\phi_1(x)$ has support inside the ball $\mathcal{B}$ and $\phi_2(x)$ has support outside ball $\mathcal{B}$. In the same way we can decompose the action $S(\phi) = S_1(\phi) + S_2(\phi)$ where the first part is integral over the ball and second is integral over its complement. In this separation of the action the values of $\phi(x)$ on the boundary of the ball are important since non-trivial boundary conditions can give rise important surface contributions. We assume that all possible surface contributions are included in $S_1$. Therefore the generating functional can be written in the following form

$$Z[J,V] = \frac{\int D\phi_2 e^{-S_2(\phi_2)} \int D\phi_1 e^{-S_1(\phi_1)+J \int_\mathcal{B} d^4 x \phi(x)}}{\int D\phi_2 e^{-S_2(\phi_2)} \int D\phi_1 e^{-S_1(\phi_1)}} \quad (19)$$

where in the denominator we have written out the contribution $Z[J(x) = 0]$ which we normalized to one in (11). In (19) we did not write explicitly the integration over collective coordinates. The ball $\mathcal{B}$ has a center $x_0$ and one has to integrate out this dependence in the expression (19). In the supersymmetric models we can put this ball in superspace and thus we have two collective coordinates $x_0$ and its superpartner $\theta_0$. Hence we can write the generating functional as follows

$$Z[J,V] = \int d^4 x_0 \int d^4 \phi e^{-\int_\mathcal{B} d^4 x \mathcal{L} + J \int_\mathcal{B} d^4 x \phi(x)} \quad (20)$$

where in general the integration over appropriate collective coordinates are assumed. Now let us use the definition (17) of the Green’s functions

$$\langle \phi(x) \rangle = \frac{1}{V} \left. \frac{\partial Z}{\partial J} \right|_{V=0} = \int d^4 x_0 \int D\phi \left( \frac{\phi}{V} \right) e^{-\int_\mathcal{B} d^4 x \mathcal{L}} \bigg|_{V \to 0}. \quad (21)$$

where $\phi \equiv \int d^4 x \phi(x)$. We have to take the zero volume limit in the expression (21). In this limit in the action only topological contributions survive which are independent of the
volume $S(\phi) \rightarrow S_{\text{top}}$. In the zero volume limit only the configurations with point support survive

$$\frac{\phi}{V} \rightarrow \phi \delta(x - x_0) \iff \int d^4x \frac{1}{V} \phi(x) \rightarrow \int d^4x \delta(x - x_0) \phi(x), \quad V \rightarrow 0. \quad (22)$$

These configurations give rise to constant Green’s functions. Therefore schematically we can write down the following expression for the Green’s functions

$$\langle \phi(x) \rangle = \sum_{\text{top}} \int d^4x_0 \int d\phi \phi \left( \delta(x - x_0) \right) e^{-S_{\text{top}}} \quad (23)$$

where $\int d\phi$ is just the finite dimensional integral over the relevant moduli. Also the sum over different topological contributions has to be assumed in the most general situation. At this point we have to issue a warning. We cannot regard the expression (23) as a simple recipe for calculating the relevant Green’s functions. The configurations with point support is UV singular and any reliable calculation will need an UV regularization. In the theory of distributions one cannot make sense of products of delta functions except by regularization. Aslo one has to worry about the proper definition the topological contribution $S_{\text{top}}$ in the action for singular configurations. As we will see in the case of Green’s functions of the composite operator $\lambda \lambda$, one can define the topological contribution using the Atiyah-Singer theorem.

In this section we have tried to give arguments in favour of the rather intuitively obvious idea that the constant Green’s functions are saturated by the configurations localized at a point. We have shown that only the configurations with non trivial topology (the action on these configurations is proportional the topological charge) can give rise the constant Green’s functions. The space of such configurations is finite dimensional, therefore the path integral representation of the relevant Green’s functions is reduced to a finite dimensional integral over the corresponding collective coordinates. In the next section we will see that the configurations with point support have the right number of fermionic zero modes.

## 4 Fractional instantons

In the last section we have argued that the field configurations with point support give rise constant Green’s functions. Now let us turn to the discussion of the situation for $SU(2)$ SYM. As we have seen in Section 2, for saturation of the gluino condensate one needs two zero

3In the expression (22) we do not specify the delta function (for example, $\delta^4(x)$ or $\delta(x^2)$). It may depend of the details of the limiting procedure.

4This property is related to the fact that the relevant Green’s functions are a set of disconnected coordinate independent pieces.
fermionic modes and a gauge configuration with \( Q = 1/2 \). Let us look at the fermionic zero modes for self-dual (antiself-dual) gauge configurations. A supersymmetry transformation applied to the bosonic solution \( G^{\alpha}_{\mu
u} \) generates two fermionic zero modes

\[
\lambda^{\alpha}_{\alpha} \sim G^{\alpha}_{\alpha\beta} \epsilon^{\beta}
\]

where \( \epsilon^{\beta} \) is the spinor parameter of transformation. The remaining two zero modes can be obtained by applying a superconformal transformation to the bosonic solution

\[
\lambda^{\alpha}_{\alpha} \sim G^{\alpha}_{\alpha\beta}(x - x_0)^{\beta \gamma} \xi_{\beta}
\]

where \( (x - x_0)^{\beta \gamma} \xi_{\beta} \) is coordinate dependent parameter of transformation. The first two zero modes \( (24) \) correspond to the physical symmetry of the QFT. The existence of the second two modes \( (25) \) correspond to the symmetry of the classical equations of motion. Therefore we can sacrifice the superconformal modes. We can try to solve the problem by brute force, requiring the superconformal modes to vanish

\[
G^{\alpha}_{\alpha\beta}(x - x_0)^{\beta \gamma} \xi_{\beta} = 0.
\]

This means that the superconformal transformation acts trivially on the bosonic solution \( G^{\alpha}_{\alpha\beta} \). Equation \( (26) \) has only one solution in terms of distributions which due to the dimension of the field can be written as follows \( G \sim \delta((x - x_0)^2) \). We have thus found that the field configuration with point support has precisely two fermionic zero modes as we need for non-zero gluino condensate.

The solution found above can be regarded as the zero size limit of the one instanton solution

\[
G^{\alpha}_{\mu\nu} = -4 \eta^{\alpha}_{\mu\nu} \frac{\rho^2}{((x - x_0)^2 + \rho^2)^2} \quad \rho \rightarrow 0 \quad -4 \eta^{\alpha}_{\mu\nu} \delta((x - x_0)^2).
\]

where \( \eta^{\alpha}_{\mu\nu} \) are the 't Hooft symbols. The \(-4 \eta^{\alpha}_{\mu\nu} \delta((x - x_0)^2)\) is the generalized solution of the self-duality equation. Therefore formally one can write the expression for the action as follows \( S = 8\pi^2 Q/g^2 \) where \( Q \) is defined by \( (7) \). In the case of zero size instanton we cannot calculate the action or topological charge directly since there are singularities (in general expression like \( \delta(x)\delta(x) \) do not make sense). The most natural way is to use the Atiyah-Singer theorem as a definition of the topological charge and the corresponding action. It is important to note that zero size instanton is an independent object in the sense that the standard instanton is not the only possible regularization of the zero size instanton. There are infinitely many such regularizations. The consideration in [4] supports the point that such singular configurations exist independently.

\[ ^5 \text{Using the delta-like sequences one can show that } \delta(x)\delta(x) \rightarrow \delta(x^2). \]
Another very interesting point is that if we consider the gauge potential for the instanton in the zero size limit

\[ A_\mu^a = 2\eta^a_{\mu\nu}\frac{(x - x_0)_\nu}{(x - x_0)^2 + \rho^2} \xrightarrow{\rho \to 0} 2\eta^a_{\mu\nu}\frac{(x - x_0)_\nu}{(x - x_0)^2} \] (28)

then one can recognize in this expression the meron configuration introduced in [5]. There are two remarkable facts about this configuration. First in the meron background the Wilson loop has area law and therefore there is confinement. Second the meron is a point like defect since it can be gauge away everywhere except one point \(x_0\).

The supersymmetry transformation applied to the zero size instanton generate two fermionic modes

\[ \lambda_\alpha^a \sim -4\eta^{a\beta}_\alpha \epsilon^\beta \delta((x - x_0)^2). \] (29)

which correspond to two collective coordinate, the center of zero size instanton \(x_0\) and its superpartner \(\theta_0\). The superconformal transformations act trivially and give zero. This is natural since the size of the instanton \(\rho\) and its superpartner are lacking. In a superspace language the relevant superfield configuration has the form

\[ W^\alpha W_\alpha \sim \delta((x - x_0)^4)(\theta - \theta_0)^2. \] (30)

Further one has to integrate over the collective coordinate to get the gluino condensate

\[ \langle W^\alpha W_\alpha \rangle \sim M^3 \int d^4 x_0 d^2 \theta_0 \delta((x - x_0)^4)(\theta - \theta_0)^2 e^{-\frac{4\mu^2}{\sigma^2}} \] (31)

where \(M\) is a parameter with dimension of mass which is needed to keep the dimensions right. After these calculations we get the gluino condensate

\[ \langle \lambda \lambda \rangle \sim cM^3 e^{-\frac{4\mu^2}{\sigma^2}} \sim c\Lambda^3 \] (32)

where \(c\) is numerical constant and \(\Lambda\) is the strong coupling scale. Of course the above calculations are naive since we did not talk about the regularization which we need to consider for the product of two delta functions \(\delta(x)\delta(x)\). Thus we did not worry about numerical factors.

In this context the standard instanton’s calculations [2] can be thought of as a way of introducing a regularization. The size of instanton plays the role of UV-cut-off. The additional fermionic modes have to be introduced to keep supersymmetry unbroken within these calculations.
5 Discussions and conclusions

In the present letter we have tried to answer the question of what kind of field configurations give rise the gluino condensate. We have addressed this problem within the QFT language. The existence of non-zero gluino condensate and unbroken supersymmetry gives us two puzzles. The first is that, from general principles, the non-vanishing matrix element $\langle \lambda \lambda \rangle$ requires the existence of fractional instantons or more precisely; field configurations with a reduced number of fermionic zero modes. Second, a non-vanishing gluino condensate gives rise a tower of constant Green’s functions for the composite operator $\lambda \lambda$. In fact both puzzles have the same answer. We have shown that the constant Green’s functions can be saturated by field configurations with point support. At the same time such configurations can be thought of as the zero size instanton and they have exactly right number of fermionic zero modes.

Also within this context it is interesting to note the correspondence between $D$-instantons of Type IIB superstring on $AdS_5 \times S^5$ and YM instantons in SYM living on the boundary of $AdS_5$. Within this correspondence the size of instanton play the role of UV cut-off (distance of $D$-istanton to the boundary of $AdS_5$) and instanton itself is an object with point support on the boundary. In the large N-limit (’t Hooft limit) the instantons do not survive and only fractional configurations with action $S \sim 1/N$ can survive. Therefore it is natural to have configuration with point support (thus with reduced number of zero modes) on the boundary. Hence one can call this correspondence - $D$-instanton/fractional instanton correspondence. We do not want to speculate about this subject except to point out the possible parallels between the different ideas.

It would be interesting to go further on this subject. For example, it would be nice to gain a better understanding of how to calculate different condensates in supersymmetric gauge theories using configurations with point support without direct use of instantons. Another problem for further research could be the understanding of the twenty years old calculations of Wilson loop in the light of the presented arguments.

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Note added
After completing this work we learned about the work [9] by J.Brodie where similar ideas were discussed from the string theory point of view. Also we learned about some evidence [10] within lattice calculations in favor of the fractional instantons.

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