Selection of Plant Location using Consistent FAHP and Goal Programming Technique

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Analytic Hierarchy Process (AHP) is one of the effectively used tools for solving Multi Criteria decision making problem. In AHP the decision maker's ambiguity and uncertainty cannot be handled. To solve this, AHP is extended in Fuzzy environment. In this paper a new method of Fuzzy AHP based on multiplicative consistent fuzzy preference relation is introduced. In this method, an iterative algorithm is given to construct comparison matrices from n (n-1)/2 preferences. From these comparison matrices the weight vectors are evaluated by goal programming technique. The validity of this method is checked by applying this it to a solved problem in the literature and it is found that this method yields the same result. Location of a manufacturing plant has a significant impact on the performance of the company as it minimizes the cost and maximizes the use of resource. A manufacturing industry wants to select a location for its new plant. The method proposed in this paper is applied to select the best location out of several alternatives with the real time data.

Keywords: Analytic Hierarchy process, Consistency, Fuzzy preference relation, Location selection, Multi criteria Decision making problem

I. INTRODUCTION

While constructing a new plant the foremost question to be answered is about the location. The location should be selected in such a way that the cost, time should be minimized. The use of the resources and profit should be maximized. Selection of a plant involves many criteria. So these become Multi Criteria Decision Making (MCDM) problem.

Many approaches have been introduced to solve MCDM problem such as PROMTHEE, ELECTREE, TOPSIS and AHP etc....AHP developed by T.L.Saaty [1] is a widely used approach for decision making problem, in which any decision making problem can be structured into multi-level hierarchical structure. Though AHP has been widely used method, uncertainty, vagueness and ambiguity of human thinking cannot be expressed using AHP. Fuzzy theory introduced by Zadeh [2] is an effective tool to deal with uncertainty and vagueness. So AHP is extended in fuzzy environment, representing the elements of the comparison matrix by fuzzy numbers.

FAHP was first studied by VranLaarhoven and Pedrycz [3]. In their paper they introduced logarithmic least square method for deriving priorities. Later C.G.E.Boender et al [4] made a correction on normalization procedure of formers. L.Mikhailov [5] introduced an alpha-cut method to derive priorities from fuzzy pairwise comparison matrices whose elements are triangular fuzzy numbers. Chang [6] used extent analysis to get weight vectors from comparison matrices. This method has been applied widely by many authors in different fields. But Wang Luo and Huo [7] proved by showing some examples that extent analysis method cannot estimate the true weights and has led to quiet a number of misapplications in the literature. J.J.Buckely [8] used fuzzy ratios in place of exact ratios in hierarchical analysis and used Geometric mean method to derive priorities. The consistence of the comparison matrices are not established in all the above methods. This is the main disadvantage of all the methods. Wang and Chen [9] introduced linear goal programming method to derive priorities and applied to new product screening. Even though this method ensures the consistency, the calculations used in this method are difficult.

Location selection problem has been solved by many authors using several methods like Fuzzy MCDM, Fuzzy TOPSIS, AHP, ANP, Fuzzy AHP, Delphi-AHP-VIKOR methodology etc. Apple [10], Moore [11] gave the list of factors to be considered while selecting a plant location. Yong [12] used Fuzzy TOPSIS method based on linguistic terms for location selection problem. Farahani and Asgari [13] used TOPSIS method to find the supportive centres in military logistic system. Onut and Soner [14] a Fuzzy TOPSIS methodology integrated with AHP to select a suitable site for transshipment of solid waste. D.B Mahalik [15] used AHP with GRA methodology to select a suitable site. B.Vahdani et al [16] used three step Methodology consist of Delphi-AHP-VIKOR to select best location.

In this paper to select a location for a new plant of a manufacturing company, a new method of FAHP is introduced. The novelty of this method is construction of complete consistent comparison matrices using only n-1 values. Goal programming technique is used to derive priorities from the comparison matrices. To do so, this paper is structured as follows.

In the initial section the problem is introduced and the literature survey is given. Section -1 explains the methodology used in this paper. In the next section, to check the validity of the proposed method, it is applied to a solved problem in the literature. Section-3 gives the case analysis. In Final section conclusion is given.
II. METHODOLOGY

2.1 Fuzzy Analytic Hierarchy Process:
In AHP the consistency of the comparison matrix is an important issue. Inconsistency leads to unreliable solution. To focus on consistency the preference relations are introduced.

Among these Multiplicative preference relation by Saaty [1], Fuzzy Preference Relation (FPR) by Orvolosky [18] have received much attention. The definition of FPR is given as follows:

**Def: 1**

Let \( X = \{X_1, X_2, \ldots, X_n\} \) be a fixed set, then \( R = (r_{ij})_{n \times n} \) is called fuzzy Preference relation with the condition \( r_{ij} \geq 0 \), \( r_{ij} + r_{ji} = 1 \), \( i, j = 1, 2, \ldots, n \) where \( r_{ij} \) denotes the degree that the alternative \( X_i \) is prior to the alternative \( X_j \).

To establish a comparison matrix with \( n \) criteria it requires \( \frac{n(n-1)}{2} \) comparisons. As the number of criteria increases, the matrices become inconsistent. To avoid this, Herrera-Videma [19] introduced an algorithm based on additive transitivity property to construct a consistent comparison matrix as follows.

In this paper, consistent matrix is constructed based on multiplicative transitivity property. The definition of Multiplicative Transitivity Tanino [20] and algorithm to construct a consistent comparison matrix is as follows:

**Def: 2**

FPR \( R = (r_{ij})_{n \times n} \) is called a Multiplicative Consistent preference relation, if it satisfies the multiplicative transitivity property:

\[
r_{ij}r_{jk}r_{ki} = r_{ij}r_{kj}r_{ik} \quad i, j, k = 1, 2, \ldots, n
\]

where \( r_j \geq 0 \).

By simple calculations it can be shown that

\[
r_y = \frac{r_{ij}r_{ji}}{r_{ij} + (1 - r_{ij})(1 - r_{ji})} \quad (1)
\]

H. Xia and Z. Xu [21] proved that the above equation is equivalent to

\[
r_y = \frac{\prod_{i=1}^{j-1} r_{ij} \prod_{j=i+1}^{n} r_{ij}}{\prod_{i=1}^{j-1} r_{ij} + \prod_{j=i+1}^{n} (1 - r_{ij})(1 - r_{ji})} \quad (2)
\]

**Algorithm: 2.1**

The method to construct a consistent FPR for a set \( X = \{x_1, x_2, \ldots, x_n\} \) as follows:

Step: 1 Get n-1 preference values \( \{r_{12}, r_{13}, \ldots, r_{(n-1)}\} \) from decision maker and \( r_y = 0.5 \).

Step: 2 Find \( r_j \) for \( j = i + 2 \) by using the formula

Step: 3 Find \( r_j \) for \( j > i + 2 \) by using the formula

Step: 4 find for all \( r_y = 1 - r_y \) for all \( j \geq i \)

Example:
Consider the following matrix on four criteria.

\[
\begin{bmatrix}
0.5 & 0.1 & x & x \\
0.5 & 0.8 & x & x \\
x & 0.5 & 0.9 & x \\
x & x & x & 0.5
\end{bmatrix}
\]

The values \( \{r_{12}, r_{23}, r_{34}\} \) are evaluated by decision maker.

\( r_{12} = 0.308, r_{23} = 0.974 \) are got by equation (1). \( r_{34} = 0.6611 \) are got by equation (2). Remaining elements are calculated by \( r_y = 1 - r_y \).

So we get \( r_{13} = 0.9, r_{13} = 0.698, r_{32} = 0.2 \).

2.2 Goal programming Technique:

Goal programming is a branch of multi objective programming which is started from the work of Charnes and Cooper [22] and further developed by many others. It is used as a technique to find satisfying solution to MCDM problem.

Tsechulin and Jacques [23] formulated a goal programming to derive priority vectors from comparison matrices in AHP. Fuzzy AHP combined with goal programming technique are used by Shaw et al [24], Srivikaya et al [25] and many others.

A simple goal programming model for deriving priority weights from fuzzy comparison matrices as follows:

\[
\min \sum (d_y^+ + d_y^-) \quad \text{for } i, j = 1, 2, \ldots, n
\]

subject to

\[
w_i (a_{ij}^+ - 1) + w_i a_{ij}^+ + d_y^+ - d_y^- = 0;
\]

\[
\sum_{i=1}^{n} w_i = 1;
\]

\[
w_i \geq 0; \quad d_y^+ \geq 0; \quad d_y^- \geq 0;
\]

III. VALIDATION

To check the reliability, of the new method of FAHP and goal programming technique proposed in the paper is applied to a problem with the same data authored by Kong, Liu [26]. In that paper the author evaluated the success factors of e-commerce.

The hierarchical structure of the problem is given below:
Fig 1 Hierarchical structure of the problem

For the comparison matrices the authors used Fuzzy numbers in the interval \([0, 1]\). The criteria comparison matrix is given in Table 1.

Table 1 - Criteria comparison matrix

| Criteria | C1 | C2 | C3 | C4 | C5 |
|----------|----|----|----|----|----|
| C1       | 0.5 | 0.8 | 0.7 | 0.5 | 0.9 |
| C2       | 0.2 | 0.5 | 0.4 | 0.1 | 0.4 |
| C3       | 0.3 | 0.6 | 0.5 | 0.3 | 0.7 |
| C4       | 0.5 | 0.9 | 0.7 | 0.5 | 0.7 |
| C5       | 0.1 | 0.6 | 0.3 | 0.3 | 0.5 |

Taking \(r_{12}, r_{34}, r_{45}\) values from the table and evaluating other values using our procedure we get the matrix.

Table 2 - Criteria comparison matrix by our method

| Criteria | C1 | C2 | C3 | C4 | C5 |
|----------|----|----|----|----|----|
| C1       | 0.5 | 0.8 | 0.7272 | 0.64 | 0.76 |
| C2       | 0.2 | 0.5 | 0.4 | 0.22 | 0.4 |
| C3       | 0.2727 | 0.6 | 0.5 | 0.3 | 0.7 |
| C4       | 0.3642 | 0.778 | 0.7 | 0.5 | 0.7 |
| C5       | 0.246 | 0.6 | 0.5 | 0.3 | 0.5 |

By using goal programming technique the weight vectors derived from the above matrix are \((0.348, 0.08, 0.131, 0.31, \text{ and } 0.13)\). Likewise all other matrices are constructed and weight vectors are evaluated. The results from this method and the method in Kong, Liu are compared and given in following table.

Table 3 - Comparison results

| Criteria | Priority of criteria By the method in Kong, Liu | Priority of criteria By the method in Kong, Liu | Final priority of sub criteria |
|----------|-----------------------------------------------|-----------------------------------------------|-------------------------------|
| C1       | 0.37                                          | 0.06                                          | Final priority of sub criteria |
| C2       | 0.06                                          | 0.08                                          | Final priority of sub criteria |
| C3       | 0.15                                          | 0.06                                          | Final priority of sub criteria |
| C4       | 0.34                                          | 0.31                                          | Final priority of sub criteria |
| C5       | 0.06                                          | 0.13                                          | Final priority of sub criteria |
From this it can be concluded that our method yields the same ranking as that of the paper. Hence our procedure is the reliable. Because of the given automatic algorithm the Decision Maker’s work is reduced. This is the main advantage of this method.

IV. CASE ANALYSIS

In this study, a manufacturing industry X based on Chennai wants to establish its new plant at the options A₁, A₂, A₃. Discussing with top level managers of the company who are the decision maker’s for this problem the criteria are selected. Criteria and Sub criteria are given in the following table.

Table 4: Criteria and sub criteria table

| Criteria          | Sub Criteria                  |
|-------------------|--------------------------------|
| Cost (C)          | Initial Investment Cost (C₁)   |
|                   | Maintenance cost (C₂)          |
|                   | Transportation cost (C₃)       |
|                   | Labor cost (C₄)                |
| Transport facility (T) | Proximity to urban areas(T₁)  |
|                   | Proximity to public transport (T₂) |
|                   | Proximity to warehouse (T₃)    |
|                   | Customs (T₄)                   |
| Environmental Issue (E) | Drainage System (E₁)         |
|                   | Regulations (E₂)               |
|                   | Proximity to energy resources like water, fuel and Natural gas (E₃) |
|                   | Expansion Possibility (E₄)    |
| Work force (W)    | Availability of Labor force (W₁) |
|                   | Availability of Medical facility for work force (W₂) |
|                   | Skilled Labor (W₃)             |
|                   | Availability of Transportation facility to the workforce (W₄) |
| Political Situation (P) | Support from Government (P₁) |
|                   | Subsidiary (P₂)                |

To construct the comparison matrices, the preference values for the consecutive elements should be filled by decision makers. Criteria comparison matrix is given as example.

Table 5: Judgement scale for FAHP

| Verbal term                  | Scale values |
|------------------------------|--------------|
| Extremely not preferred      | 0.1          |
| Very strongly not preferred  | 0.2          |
| Strongly nor preferred       | 0.3          |
| Moderately not preferred     | 0.4          |
| Equally preferred            | 0.5          |
| Moderately preferred         | 0.6          |
| Strongly preferred           | 0.7          |
| Very strongly preferred      | 0.8          |
| Extremely preferred          | 0.9          |

Table 6: Comparison matrix

|   | C | T | E | W | P |
|---|---|---|---|---|---|
| C | 0.5 X |   |   |   |   |
| T | 0.5 x |   |   |   |   |
| E | 0.5 x |   |   |   |   |
| W | 0.5 X |   |   |   |   |
| P | 0.5  |   |   |   |   |

Here x marked cells are filled by Decision maker. Remaining values are evaluated by procedure explained in this paper. The constructed comparison matrix is given as follows.

Table 7: Consistent comparison matrix

|   | C   | T   | E   | W   | P   |
|---|-----|-----|-----|-----|-----|
| C | 0.5 | 0.55| 0.64| 0.92| 0.93|
| T | 0.45| 0.5 | 0.6 | 0.89| 0.91|
| E | 0.36| 0.4 | 0.5 | 0.85| 0.87|
| W | 0.08| 0.11| 0.15| 0.5 | 0.55|
| P | 0.07| 0.09| 0.13| 0.45| 0.5 |

From this matrix weight vectors are derived by Goal Programming technique.

Solution of these equations are:

\[ W₁ = 0.3935749 \]
\[ W₂ = 0.3220159 \]
\[ W₃ = 0.2146772 \]
\[ W₄ = 0.0379 \]
\[ W₅ = 0.03 \]

Likewise weight vectors for all criteria and sub criteria are calculated are listed in the following table.
Table-8 Alternative weights table

| Criteria       | Criteria weights | Sub criteria | Weights of sub criteria | Local weights | Global weights |
|----------------|------------------|--------------|-------------------------|---------------|---------------|
|                |                  |              | A₁ | A₂ | A₃ | A₁ | A₂ | A₃ |
| C              | 0.395            | C₁           | 0.64 | 0.74 | 0.16 | 0.1 | 0.03 | 0.007 | 0.0044 |
|                |                  | C₂           | 0.18 | 0.7 | 0.17 | 0.13 | 0.08 | 0.002 | 0.0016 |
|                |                  | C₃           | 0.11 | 0.32 | 0.42 | 0.26 | 0 | 0.003 | 0.002 |
|                |                  | C₄           | 0.07 | 0.48 | 0.22 | 0.3 | 0 | 0.001 | 0.0015 |
| Overall weights of Cost |              |              | 2.24 | 0.97 | 0.79 | 0.04 | 0.014 | 0.0096 |
| T              | 0.322            | T₁           | 0.49 | 0.31 | 0.41 | 0.28 | 0.03 | 0.052 | 0.036 |
|                |                  | T₂           | 0.21 | 0.33 | 0.38 | 0.29 | 0.01 | 0.02 | 0.0158 |
|                |                  | T₃           | 0.13 | 0.63 | 0.2 | 0.17 | 0.02 | 0.006 | 0.0057 |
|                |                  | T₄           | 0.07 | 0.59 | 0.23 | 0.18 | 0.01 | 0.004 | 0.0033 |
| Overall weights of Transport facility |              |              | 2.82 | 1.91 | 1.27 | 0.1 | 0.092 | 0.0648 |
| E              | 0.215            | E₁           | 0.19 | 0.52 | 0.15 | 0.33 | 0.01 | 0.005 | 0.0113 |
|                |                  | E₂           | 0.15 | 0.16 | 0.33 | 0.51 | 0.11 | 0.008 | 0.0137 |
|                |                  | E₃           | 0.26 | 0.54 | 0.22 | 0.24 | 0.02 | 0.01 | 0.011 |
|                |                  | E₄           | 0.19 | 0.15 | 0.25 | 0.6 | 0 | 0.008 | 0.0205 |
| Overall weights of Environmental facility |              |              | 1.93 | 2.06 | 2.01 | 0.07 | 0.074 | 0.0681 |
| W              | 0.038            | W₁           | 0.5 | 0.32 | 0.52 | 0.16 | 0.06 | 0.104 | 0.032 |
|                |                  | W₂           | 0.21 | 0.38 | 0.32 | 0.3 | 0.03 | 0.026 | 0.0252 |
|                |                  | W₃           | 0.12 | 0.33 | 0.5 | 0.17 | 0.01 | 0.024 | 0.0081 |
|                |                  | W₄           | 0.12 | 0.29 | 0.61 | 0.1 | 0.01 | 0.029 | 0.0048 |
| Overall weights of workforce |          |              | 1.66 | 2.41 | 0.93 | 0.13 | 0.193 | 0.0741 |
| P              | 0.032            | P₁           | 0.12 | 0.37 | 0.33 | 0.3 | 0 | 0.001 | 0.0018 |
|                |                  | P₂           | 0.12 | 0.6 | 0.22 | 0.18 | 0 | 0.001 | 0.001 |
| Overall weights of Political situation |              |              | 2.79 | 1.27 | 0.94 | 0.028 | 0.0136 |
| Overall priority |              |              | 0.39 | 0.41 | 0.23 |     |     |     |
| Alternatives ranking |         |              | 2 | 1 | 3 |     |     |     |

The final weight vectors for the alternatives are A₁=0.395; A₂=0.409; A₃=0.229; from this it can be concluded that second alternative is the best one.

V. CONCLUSION

For a manufacturing industry, location selection is an important issue which depends on several criteria and priority of Decision maker. Since it involves several criteria it can be handled effectively by AHP. Since Goal programming method is used to find the weight vectors, the resultant weight vectors are accurate. Because of the given automatic algorithm to construct the comparison matrix the work of Decision maker is reduced. The validity of this method is also checked. It is found that this method is one of the best and simple method. This method can also be used for other decision making problems.

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