Chebyshev wavelets approach for the squeeze film lubrication of long porous journal bearings with couple stress fluids

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Abstract
In the present investigation, Chebyshev wavelet-based technique is considered for the study of the effect of couple stress fluid on the squeeze film lubrication in long porous journal bearings. In order to study the pressure distribution, the non-dimensional Reynolds equation is solved by the aid of projected technique. The response of obtained solution with respect to pressure and circumferential co-ordinate are captured. Further, the physical behaviour of pressure with the help of eccentricity ratio, permeability parameter, and couple stress parameter is analysed and presented in terms of plots. According to results obtained, the couple stress fluid effect significantly increases the pressure as compare to the Newtonian case. Also, when the permeability parameter is decreased and the corresponding pressure decreases as compared to the solid case.

Keywords
Chebyshev wavelets, lubrication, porous journal bearing, squeeze film, couple stress fluid.

AMS Subject Classification
76D08, 65T60, 35A24, 65L10.

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1. Introduction

Recently, the study and analysis of hydrodynamic lubrication (HL) magnetize the attention of researchers due to its exact productions for some essential and simulating behaviour describe the various phenomena related to the daily life of living beings. More specifically, the self-lubricating porous bearings have the benefits while decreasing the certain requirements of lubricating tools like pump, breaks, gears, and oil pipes, and also simplifying other complexities associated with the lubrication processes. As a concerned of lubricant for the purpose of modern industries, non-Newtonian fluids are widely used as compared to the Newtonian fluids. In order to reduce the change in viscosity with temperature, the lubricant with stable suspensions of fine particles of insoluble solids with non-Newtonian behaviours is used as a viscosity-index improver. The interactions between these suspended particles of the fluid can be considered as the coupled stress. To exemplify the behavior of non-Newtonian polymeric fluids, recently many authors developed the theories of micro-continuum [1, 2]. In connection with this, Stokes proposed and natured the theory of fluids, and later he applied to understand the effect of couple stress in fluids, usually known as the couple stress theory [25]. With the help of Stokes’s theory, the effect of couple stresses in a squeeze film of synovial joints has been exemplified by Bujurke and Jayaraman [4]. Many authors have been studied the effect of couple stress in order to understand the complexity of the diverse phenomena [5, 12, 19]. In connection with this Morgan and Cameron investigated the nature of the HL of porous bearings and its corresponding consequences [14]. In 1973, Prakash and Vij
investigated the nature of HL for a plane porous slider bearing [18]. Patel and Gupta in [17] consider the slip velocity for an inclined porous slider bearing at the fluid porous interface and later they conclude that in order to raise the capability of the porous slider bearings, we should minimize the slip velocity operator. In order to find the solution for the above-cited mechanism, and due to the revolution in the computer software and mathematical methods, many mathematicians and physicists find the analytical as well as the numerical solution for the HL of porous bearings. For instance, Murti and Naduvinamani investigated the hydrodynamic lubrication of porous bearings by employing the finite difference method, Bujurke and Vaidya in [20] find the numerical solution for the operating conditions of a hydrodynamic porous journal bearing by employing the finite difference method, Bujurke and Naduvinnamani investigated the hydrodynamic lubrication for the rolling bearing case in [5], and the analysis of HL of short porous journal bearings has been studied in [13] by applying Brinlonan-extended Darcy model. Recently, tremen-
dous consideration has been devoted to the study of the theory and methods of wavelets, due to the numerous properties and applicability in diverse branches of science and engineering fields [10, 21, 28]. The main proficiency of wavelet-based techniques is the ability to convert the given differential and integral equations into the system of linear and nonlinear algebraic equations. Due to this, many authors have found the numerical solution for the diverse class of linear and nonlinear differential and integral equations describing various physical and biological phenomena arisen in daily life using different wavelets based methods. Particularly, Haar wavelet [6, 11], Legendre wavelet [27], Laguerre wavelet [24], Hermite wavelet [23] and many others. Among distinct families of wavelets, recently many mathematicians and physicists considered Chebyshev wavelets in order to analyze various model, and proved that the projected wavelets simulates and exemplifies very interesting properties of nonlinear problems in accurate and more efficient manner [22, 26].

2. Wavelets and Chebyshev wavelets

Wavelets constitute a family of functions composed from dilation and translation of a single function called the mother wavelet $ψ(x)$ [9]. We have the following family of continuous wavelets when the dilation ($a$) and translation ($b$) parameters vary continuously and which is presented as [3]:

$$ψ_{a,b}(x) = |a|^{-1/2} ψ\left(\frac{x-b}{a}\right), a, b \in \mathbb{R}, a \neq 0. \quad (2.1)$$

If we consider the dilation($a^{-k}$) and translation($nba^{-k}$) parameters with $a > 1$, $b > 0$, $n$, and $k$ are positive integers, then we admit the following family of discrete wavelets

$$ψ_{n,m}(x) = |a|^\frac{k}{2} ψ\left( ax - nb \right). \quad (2.2)$$

Chebyshev wavelet is a function of four arguments $ψ_{n,m} = ψ(k, n, m, x)$, $n = 1, 2, \ldots, 2^k$, where $k$ can assume any positive integer, $m$ is the degree of Chebyshev polynomials of the first kind and $x$ denotes the time.

$$ψ_{n,m}(x) = \begin{cases} \frac{G_{m}x^{(2k-1)/2}}{\sqrt{k}} T_{m}(2kx - 2n + 1), & x < 0, \\ 0, & \text{otherwise}. \end{cases}$$

Here $G_m = \begin{cases} \sqrt{2}, & m = 0, \\ 2, & m = 1, 2, \ldots, M - 1, \end{cases}$, where $T_m(t)$ represents the $m^{th}$ order Chebyshev polynomials with maximum order $M$, which are orthogonal with respect to weight function $w(t) = \frac{1}{\sqrt{1-t^2}}$ and satisfying the following recursive formulas:

$$T_0(t) = 1,$$

$$T_1(t) = t,$$

$$T_{m+1}(t) = 2tT_m(t) - T_{m-1}(t), \quad m = 1, 2, 3, \ldots$$

For the weight function $w_n(t) = w(2^k + 1 - 2n)$, Chebyshev wavelets are orthogonal.

3. Formulation of the Problem

The lubricant in the film region and also in the porous region is assumed to be a Stokes couple stress fluid [25]. The film thickness($h$) equation as a function of circumferential coordinate ($\theta$) with radial clearance ($C$) and eccentricity of the journal centre ($e$) is defined in [16] as $h = C - e \cos(\theta)$. Under the general assumptions of fluid film lubrication relevant to thin films [7], when the body couples and body forces are absent the equation of motion of an incompressible couple stress fluid within the film region are given by

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 U}{\partial y^2} - \eta \frac{\partial^4 U}{\partial y^4}, \quad (3.1)$$

$$\frac{\partial p}{\partial y} = 0, \quad (3.2)$$

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0. \quad (3.3)$$

The modified Darcy law is governed by the flow of couple stress fluid in a porous matrix. Which accounts for the polar effects and defined as $\bar{Q} = \frac{k}{\mu (1-\alpha)} \nabla p^*$, where $\bar{Q} = (U^*, V^*)$ and $\alpha = (\eta/\mu)/k$. The ratio $(\eta/\mu)^{1/2}$ is of dimensional length and hence characterizes the chain length of the polymer additives. The ratio of the microstructure size to the pore size is represented by $\alpha$. Also, the pressure in the porous region is taken as $p^*$. The governing continuity equation satisfying the Laplace equation presented as

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 0. \quad (3.4)$$

The corresponding boundary constraints of the velocity components are,
1. For boundary surface \( y = h \):
\[
w = 0, \frac{\partial^2 U}{\partial y^2} = 0, V = -V_\theta. \tag{3.5}
\]

2. For porous journal surface \( y = 0 \):
\[
U = 0, \frac{\partial^2 U}{\partial y^2} = 0, V = -V^*. \tag{3.6}
\]

The solution of Eq. (3.1) subject to the boundary conditions (3.5) and (3.6) is
\[
U(x, y) = \frac{1}{2\pi} \frac{\partial}{\partial x} \left\{ y(y - h) + 2l^2 \left[ 1 - \frac{\cosh((2y - h)/2l)}{\cosh(h/2l)} \right] \right\},
\]
where \( l = (\eta/\mu)^{1/2} \) is the couple stress parameter, \( \eta \) represents the material constant responsible for couple stress property and \( \mu \) represents lubricant viscosity. Over the porous layer thickness \( H_0 \), integration Eq. (3.4) with respect to \( y \) and using the equation \( \frac{\partial^2 p^*}{\partial y^2} = 0 \) at \( y = -H_0 \), one can get
\[
\frac{\partial p^*}{\partial y} \bigg|_{y=0} = -H_0 \frac{\partial^2 p}{\partial x^2}.
\]

Assuming that \( H_0 \) is very small and considering the pressure continuity condition \( (p = p^*) \) at the interface \( (y = 0) \) of fluid film and porous matrix, Eq. (3.8) simplifies to
\[
\frac{\partial p^*}{\partial y} \bigg|_{y=0} = -H_0 \frac{\partial^2 p}{\partial x^2}.
\]

At \( y = 0 \), the vertical component of the modified Darcy velocity \( V^* \) is presented as
\[
V^* \bigg|_{y=0} = \frac{kH_0}{(1 - \alpha)} \left( \frac{\partial^2 p}{\partial x^2} \right).
\]

By using the boundary conditions defined in Eqs. (3.5) and (3.6), and relations cited in Eqs. (3.7) and (3.10), and integrating Eq. (3.3) across the fluid film, the modified Reynolds type equation reduces to
\[
\frac{\partial}{\partial x} \left\{ f(h, l) + \frac{12kH_0}{(1 - \alpha)} \frac{\partial P}{\partial x} \right\} = -12\mu V_\theta,
\]
where \( f(h, l) = h^3 - 12l^2h + 24l^3\tan(h/2l) \) and \( V_\theta = -\frac{\partial h}{\partial t} = C \frac{du}{d\theta} \cos(\theta) \). By incorporating the non-dimensional parameters: \( P = \frac{pc^2}{\mu R^2(du/dt)} \), \( \theta = \frac{x}{R} \), \( \bar{H} = \frac{h}{c} \), \( 1 - \cos(\theta) \), \( L = \frac{L}{c} \) and \( S = \frac{kH_0}{\mu c} \). Here, \( R \) denotes the radius of the journal then, the Eq. (3.11) simplifies to
\[
\frac{\partial}{\partial \theta} \left\{ f(\bar{H}, L) + \frac{12S}{(1 - \alpha)} \frac{\partial P}{\partial \theta} \right\} = -12\cos(\theta).
\]

Here \( f(\bar{H}, L) = \bar{h}^3 - 12L^2\bar{h} + 24L^3\tan(\bar{h}/2L) \).

### Squeeze film pressure

The boundary conditions for the fluid film pressure at 180° partial journal bearing are
\[
\bar{P} = 0 \quad \text{at} \quad \theta = \pm \frac{\pi}{2}, \tag{3.13}
\]
\[
\frac{\partial \bar{P}}{\partial \theta} = 0 \quad \text{at} \quad \theta = 0. \tag{3.14}
\]

### 4. Chebyshev wavelets method of solution

Suppose we consider the general second order modified Reynolds type equation is of the form:
\[
\frac{d}{d\theta} \left( B(\theta) \frac{dP}{d\theta} \right) = f(\theta), \tag{4.1}
\]
where \( B(\theta) \) and \( f(\theta) \) are continuous real valued functions with physical conditions for some real constants \( A, B \) and \( D \):
\[
P(\theta_i) = A, P(\theta_n) = B, \frac{dP}{d\theta}(\theta_i) = D.
\]
We assume the Chebyshev wavelets approximate solution of Eq. (4.1) with the unknown coefficients as,
\[
P(\theta) = \sum_{n=1}^{\infty} \sum_{m=0}^{M} a_{n,m} \psi_{n,m}(\theta)
\]

Truncated the Eqn. (4.2), that is,
\[
P(\theta) \approx \sum_{n=1}^{2^{k-1}-1} \sum_{m=0}^{M-1} a_{n,m} \psi_{n,m}(\theta), \tag{4.3}
\]

Substituting the given boundary conditions in the Eq. (4.3), we get,
\[
P(\theta_0) = \sum_{n=1}^{\infty} \sum_{m=0}^{M-1} a_{n,m} \psi_{n,m}(\theta_0) = A, \tag{4.4}
\]
\[
P(\theta_n) = \sum_{n=1}^{\infty} \sum_{m=0}^{M-1} a_{n,m} \psi_{n,m}(\theta_n) = B, \tag{4.5}
\]
\[
\frac{d}{d\theta}(P(\theta_i)) = \frac{d}{d\theta} \sum_{n=1}^{2^{k-1}-1} \sum_{m=0}^{M-1} a_{n,m} \psi_{n,m}(\theta_i) = D. \tag{4.6}
\]

Here, we required \( 2^{k-1}M - 3 \) equations to recover the unknown coefficients \( a_{n,m} \). These equations can be obtained by substituting Eq. (4.3) in Eq. (4.1) with the collocation points, \( \theta_i = \frac{2^{k-1}-i}{2^{k-1}} \), \( i = 1, 2, ..., M - 3 \), we get,
\[
\frac{d}{d\theta} \left[ B(\theta) \frac{d}{d\theta} \sum_{n=1}^{2^{k-1}-1} \sum_{m=0}^{M-1} a_{n,m} \psi_{n,m}(\theta_i) \right] = f(\theta_i). \tag{4.7}
\]

Now, solving the \( 2^{k-1}M \) system of equations from Eqs. (4.4)-(4.6), we obtain the Chebyshev wavelet coefficients \( a_{n,m} \). And then substituting these coefficients in Eq. (4.3), which yields the Chebyshev wavelet based numerical solution of the proposed equation (4.1) with the boundary conditions.
5. Chebyshev wavelet based numerical solution

Now, consider the modified Reynolds type Eq. (3.12) as,

\[
\begin{align*}
\frac{d^2 P}{d\theta^2} &= \left[ f(\bar{\theta}, L) + \frac{12S}{(1 - \alpha)} \right] \frac{d^2 P}{d\theta^2} \\
&\quad + \left( \frac{d}{d\theta} \left[ f(\bar{\theta}, L) + \frac{12S}{(1 - \alpha)} \right] \right) \frac{dP}{d\theta} \\
&= -12\cos(\theta), \quad (5.1)
\end{align*}
\]

where,

\[
\begin{align*}
f(\bar{\theta}, L) &= (1 - \cos(\theta))^3 - 12L^2 (1 - \cos(\theta)) \\
&\quad + 24L^3 \tan \left( \frac{(1 - \cos(\theta))}{2L} \right).
\end{align*}
\]

We assume the Chebyshev wavelet approximation of Eq. (5.1) is given in Eq. (5.2), for \( k = 1 \) and \( M = 5 \).

\[
P(\theta) = \begin{cases} 
(\psi_{1,0}(1,0)(\theta) + \psi_{1,1}(1,1)(\theta) + \psi_{1,2}(1,2)(\theta) \\
+ \psi_{1,3}(1,3)(\theta) + \psi_{1,4}(1,4)(\theta)) & \text{for } \theta \in (0, \pi/2) 
\end{cases}
\]

where

\[
\begin{align*}
\psi_{1,0}(1,0)(\theta) &= \sqrt{2} \frac{1}{\sqrt{\pi}}, \\
\psi_{1,1}(1,1)(\theta) &= \sqrt{2} \frac{2}{\sqrt{\pi}} (2\theta - 1), \\
\psi_{1,2}(1,2)(\theta) &= \sqrt{2} \frac{2}{\sqrt{\pi}} (2(2\theta - 1)^2 - 1), \\
\psi_{1,3}(1,3)(\theta) &= \sqrt{2} \frac{2}{\sqrt{\pi}} (4(2\theta - 1)^3 - 3(2\theta - 1)), \\
\psi_{1,4}(1,4)(\theta) &= \sqrt{2} \frac{2}{\sqrt{\pi}} (8(2\theta - 1)^4 - 8(2\theta - 1)^2 + 1).
\end{align*}
\]

Chebyshev wavelet approximation (5.2) should satisfy the given boundary conditions (3.13 and 3.14) as follows,

\[
P\left( -\frac{\pi}{2} \right) \approx a_{1,0}\psi_{1,0}\left( -\frac{\pi}{2} \right) + a_{1,1}\psi_{1,1}\left( -\frac{\pi}{2} \right) \\
+ a_{1,2}\psi_{1,2}\left( -\frac{\pi}{2} \right) + a_{1,3}\psi_{1,3}\left( -\frac{\pi}{2} \right) \\
+ a_{1,4}\psi_{1,4}\left( -\frac{\pi}{2} \right) \\
= 0 \quad (5.4)
\]

\[
P\left( \frac{\pi}{2} \right) \approx a_{1,0}\psi_{1,0}\left( \frac{\pi}{2} \right) + a_{1,1}\psi_{1,1}\left( \frac{\pi}{2} \right) \\
+ a_{1,2}\psi_{1,2}\left( \frac{\pi}{2} \right) + a_{1,3}\psi_{1,3}\left( \frac{\pi}{2} \right) \\
+ a_{1,4}\psi_{1,4}\left( \frac{\pi}{2} \right) \\
= 0 \quad (5.5)
\]

\[
P'(0) \approx a_{1,0}\psi'_{1,0}(0) + a_{1,1}\psi'_{1,1}(0) \\
+ a_{1,2}\psi'_{1,2}(0) + a_{1,3}\psi'_{1,3}(0) \\
+ a_{1,4}\psi'_{1,4}(0) \\
= 0 \quad (5.6)
\]

Now, substituting Eq. (5.2) in Eq. (5.1) with the collocation points \( \theta = \frac{\pi}{4} \) and \( \theta = \frac{3\pi}{4} \), we get the followings equations,

\[
\begin{align*}
&\left( \left( 1 - \cos \left( \frac{3\pi}{4} \right) \right)^3 - 12L^2 \left( 1 - \cos \left( \frac{3\pi}{4} \right) \right) \\
&+ 24L^3 \tan \left( \frac{1 - \cos \left( \frac{3\pi}{4} \right)}{2L} \right) \right) \left( \psi_{1,0}(1,0) + \psi_{1,1}(1,1) + \psi_{1,2}(1,2) + \psi_{1,3}(1,3) + \psi_{1,4}(1,4) \right) \\
&= -12\cos \left( \frac{3\pi}{4} \right) \quad (5.7)
\end{align*}
\]

Solving the Eqs. (5.4 - 5.8), by the suitable solver with the couple stress parameter \( L = 0.3 \), eccentricity ratio \( e = 0.1 \) and permeability parameter \( S = 0.01 \), fluid parameter \( \alpha = 0.1 \), we get the Chebyshev wavelet coefficients are:

\[
\begin{align*}
a_{1,0} &= 10.78837781780183; a_{1,1} = -12.527721538776992; \\
a_{1,2} &= -1.870312046587394; a_{1,3} = 0.687406881337612; \\
a_{1,4} &= 0.085925860167202.
\end{align*}
\]

Substituting these coefficients in Eq. (5.2), we get the required pressure solution of the Eq. (5.1) as,

\[
P(\theta) = (10.78837781780183) \left( \frac{\sqrt{2}}{\sqrt{\pi}} \right)
\]
Further, in the present investigation, behaviour of the non-dimensional pressure distribution of long porous journal bearings with couple stress fluid is examined by using the proposed method presented in section 3. In order to understand the distribution of pressure in the proposed problem with respect to circumferential co-ordinate ($\theta$), we capture the behaviour of the obtained pressure solution are presented in the Figures 1-3. Here, the parameter $L$ afford the mechanism of interaction of fluid with bearing geometry, since $L$ is the ratio of microstructure size to the redial clearance. In Figure 1 the non-dimensional squeeze film pressure for different values of $L$ with $S = 0.01$ and $e = 0.1$, $\alpha = 0.1$. 

Figure 1. Variation of non-dimensional pressure for different values of $L$ with $S = 0.01$ and $e = 0.1$, $\alpha = 0.1$.

In this paper, we study the squeeze film lubrication problem by using Chebyshev wavelets approach for the long porous journal bearings. The non-dimensional Reynolds equation of the proposed problem is solved by the Chebyshev wavelet based numerical method. From the obtained solution we observed that the couple stress parameter, permeability parameter and the eccentricity ratio are increased leads to increasing in the pressure distribution. Finally we can conclude that, the considered method is highly efficient and methodical to find the solution for diverse class of mathematical models describing various problems arises in fluid dynamics.

6. Conclusion

In this paper, we study the squeeze film lubrication problem by using Chebyshev wavelets approach for the long porous journal bearings. The non-dimensional Reynolds equation of the proposed problem is solved by the Chebyshev wavelet based numerical method. From the obtained solution we observed that the couple stress parameter, permeability parameter and the eccentricity ratio are increased leads to increasing in the pressure distribution. Finally we can conclude that, the considered method is highly efficient and methodical to find the solution for diverse class of mathematical models describing various problems arises in fluid dynamics.
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