Description of composite systems in the spectral integration technique: the gauge invariance and analyticity constraints for the radiative decay amplitudes

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Abstract

The constraints followed from gauge invariance and analyticity are considered for the amplitudes of radiative transitions of composite systems when composite systems are treated in terms of spectral integrals. We discuss gauge-invariant amplitudes for the transitions $S \rightarrow \gamma S$ and $V \rightarrow \gamma S$ with scalar $S$ and vector $V$ mesons being two-particle composite systems of scalar (or pseudoscalar) constituents, and we demonstrate the mechanism of cancellation of false kinematical singularities. Furthermore, we explain how to generalize the performed consideration for quark-antiquark systems, in particular, for the reaction $\phi(1020) \rightarrow \gamma f_0(980)$. Here we also consider in more detail the quark-model non-relativistic approach for this reaction.

1 Introduction

Presently there exists rich information for radiative decays of mesons with masses at 1000–1800 MeV, and one may expect the appearance of more data in future, see e.g. [1, 2, 3, 4] and references therein. The data on partial widths of radiative decays provide us with an important knowledge about the quark-gluon structure of hadrons. However, to suffice this expectation and avoid misleading conclusions one needs to work with an adequate technique for the description of radiative processes involving composite systems.

To describe the low-lying hadronic states, namely, $S$-wave mesons of the 36-plet and baryons of the 56-plet in terms of the SU(6)-symmetry, the non-relativistic quark-model approach is an appropriate technique. The investigation of radiative decays carried out decades ago played crucial role in the establishing of quark model, by operating with constituent quark as a universal object for mesons and baryons [5]. However, for higher states, e.g. for the $P$-wave quark-antiquark states, relativistic effects become important. Nevertheless, till now the use of non-relativistic formulae for radiative decays of mesons with masses about 1000–1500 MeV is rather common practice.
So there is a necessity to use relativistic technique for the description of radiative decays. The scheme of relativistic description of the composite system interacting with electromagnetic field was suggested in [6, 7, 8, 9]. Within this approach, form factors of composite systems are represented as double spectral integrals over the masses of composite systems. The spectral integration technique is a direct generalization of non-relativistic quantum-mechanical approximation, and the processes considered within this technique are time-ordered, like in quantum mechanics. The energy in the intermediate state is not conserved but the particles are mass-on-shell. In this method, vertex functions of the transitions composite system \( \rightarrow \) constituents are defined by the scattering amplitude of constituents.

The scheme applied to the composite-system form factors in [6] is as follows: partial scattering amplitude \( A^{(J)}(s) \) (\( s \) is total energy squared of the scattered constituents and \( J \) is the total angular momentum) is considered within dispersion \( N/D \) representation [10]. In this technique the amplitude is represented as a sum of dispersion \( N/D \) loop diagrams shown in Fig. 1a. In case when the \( D \)-function does not have CDD-poles [11], partial amplitude of the \( J \)-state reads:

\[
A^{(J)} = \frac{N_j(s)}{1 - B_j(s)} Q^{(j)}(p^0) ,
\]

where \( Q^{(j)}(p^0) \) is covariant angular momentum-J operator which depends on relative momentum of particles 1 and 2: \( p_{\perp \mu} = g^{\perp \mu \nu} p_\nu \); here \( p = p_1 - p_2 \), and \( g^{\perp \mu \nu} \) is the metric tensor which works in the space orthogonal to \( P = p_1 + p_2 \): \( g^{\perp \mu \nu} = g_{\mu \nu} - P_\mu P_\nu / P^2 \) (for equal particle masses 1 and 2, one has \( p_{\perp} = p \)). If the scattered particles are spinless and \( J = L \), where \( L \) is the orbital momentum of the system, we replace \( Q^{(j)}(p^0) \rightarrow X^{(L)}(p_{\perp}) \), where for the lowest waves, according to [12], the operators \( X^{(L)}(p_{\perp}) \) are determined as follows:

\[
X^{(0)}(p_{\perp}) = 1 , \quad X^{(1)}(p_{\perp}) = p_{\perp \mu} , \quad X^{(2)}(p_{\perp}) = \frac{3}{2} \left( p_{\perp \mu_1} p_{\perp \mu_2} - \frac{1}{3} p^2_{\perp} g^{\perp \mu_1 \mu_2} \right) .
\]

When the particles 1 and 2 are fermions, like quarks or nucleons, then the operators \( Q^{(j)}_{\mu_1...\mu_j} \) are constructed by using \( \gamma \)-matrices, see [12] for details.

If there is a bound state with mass \( M \) in the partial wave, the sum of diagrams shown in Fig. 1a creates a pole at \( s = M^2 \), see Fig. 1b, and the vertex of the pole diagram determines the wave function of the bound state.

The form factor of bound state can be defined by the process of Fig. 2a when the photon is emitted by interacting constituents. The amplitudes of the initial state (interaction block before the photon emission) and final one (that after photon emission) contain the poles \( s = M^2 \) and \( s' = M^2 \), see Fig. 2b, so the two-pole amplitude defines form factor of the composite system: the form factor is the residue in these poles, it is shown separately in Fig. 2c.

To be more understandable in explaining the form factor calculus within gauge invariance and analyticity constraints, we use a simplified variant of the \( N \)-function, with separable forces. The hypothesis of separability of the interaction blocks can be successfully applied to the realistic description of composite systems such as the deuteron is [6]. Here we use this hypothesis to simplify rather cumbersome presentation, and this simplification does not influence the
principal statements. Correspondingly, we use separable interaction with \( N_J(s) \to g_J^2(s) \). Then the amplitude of Fig. 2a for the emission of photon by the two-particle system with total angular momenta \( J \) and \( J' \) in initial and final states, respectively, reads as follows:

\[
A^\alpha_{\mu_1...\mu_J}(p) = Q_{\mu_1...\mu_J}(p) \frac{g_J(s)}{1 - B_J(s)} \Gamma^\mu_{\nu_1...\nu_J}(P, P', q) \frac{g_{J'}(s')}{1 - B_{J'}(s')} Q_{\nu_1...\nu_J}(p') ,
\]

where \( \Gamma^\mu_{\nu_1...\nu_J}(P, P', q) \) is the three-point interaction amplitude at \( P^2 = s \) and \( P'^2 = s' \). The amplitude \( A^\alpha_{\mu_1...\mu_J}(p) \) is represented by Fig. 3a as a chain of loop diagrams; the three-point interaction amplitude is depicted in Fig. 3a in the middle of the chain of loop diagrams.

The loop diagram \( B_f(s) \), under the ansatz of separable interaction, is equal to:

\[
B_J(s) = \int_0^\infty \frac{d\bar{s}}{\pi} \frac{g^2_J(\bar{s})}{\bar{s} - s - i0} \rho_J(\bar{s}) ,
\]

where \( m_1 \) and \( m_2 \) are the masses of scattered particles and \( \rho_J(s) \) is the phase space in the state with total angular momentum \( J \). For scalar constituent particles and total angular momentum \( J = 0 \), the phase volume is defined as follows:

\[
\rho(s) = \int d\Phi_2(P; p_1, p_2) ,
\]

\[
d\Phi_2(P; p_1, p_2) = \frac{d^3p_1}{2p_10(2\pi)^3} \frac{d^3p_2}{2p_20(2\pi)^3} (2\pi)^{4} \delta(4)(P - p_1 - p_2) ,
\]

where we have redenoted \( \rho_0(s) \to \rho(s) \). For \( J \neq 0 \), the convolution of the operators \( Q_{\mu_1...\mu_J}(p)Q_{\mu_1...\mu_J}(p') \) should be inserted in the right-hand side of \([5]\) for the calculation of \( \rho_J(s) \).

The D-function zeros at \( s = M^2 \) and \( s' = M'^2 \) correspond to the existence of bound states:

\[
1 - B_J(M^2) = 0 , \quad 1 - B_{J'}(M'^2) = 0 .
\]

In this way, the D-functions near the pole read:

\[
1 - B_J(s) \simeq \frac{dB_J(M^2)}{ds}(M^2 - s) , \quad 1 - B_{J'}(s') \simeq \frac{dB_{J'}(M'^2)}{ds'}(M'^2 - s') .
\]

Therefore, the amplitude \([3]\) near the poles takes the form:

\[
A^\alpha_{\mu_1...\mu_J}(p) \simeq Q_{\mu_1...\mu_J}(p) \frac{G_J(s)}{M^2 - s} \frac{\Gamma^\mu_{\nu_1...\nu_J}(P, P', q)}{\sqrt{\frac{dB_J(M^2)}{ds}}} \frac{G_{J'}(s')}{M'^2 - s'} Q_{\nu_1...\nu_J}(p') ,
\]

where \( G_J(s) = g_J(s)/\sqrt{dB_J(M^2)/ds} \).

One can introduce the wave functions as follows:

\[
\psi_J(s) = \frac{G_J(s)}{M^2 - s} , \quad \psi_{J'}(s') = \frac{G_{J'}(s')}{M'^2 - s'} .
\]
The radiative transition amplitude \( \text{meson-J} \to \text{meson-J'} \) is determined by residues in the poles \( s = M^2 \) and \( s' = M'^2 \):

\[
\Gamma^{\text{meson-J} \to \text{meson-J'}}_{\mu_1 \ldots \mu_J \alpha_1 \ldots \nu_{J'}}(P, P'; q) = \left[ \frac{\Gamma^{(J \to J')}_{\mu_1 \ldots \mu_J \alpha_1 \ldots \nu_{J'}}(P, P'; q)}{\sqrt{dB_J(M^2)/ds \ dB_{J'}(M'^2)/ds'}} \right]_{s=M^2, s'=M'^2}.
\]

(10)

In this way, the magnitudes \( M^2 \) and \( M'^2 \) are fixed in [6]. This means that we should discriminate between analytical properties of the amplitude of photon emission by unbound particles and those of radiative transition amplitude \( \text{meson-J} \to \text{meson-J'} \). Analytical properties of the amplitude as function of \( s \) and \( s' \) for the emission of photon by unbound particles are determines by all diagrams shown in Fig. 3.

By studying analytical one should take into account that moment operators, which give the spin dependence of the form factor \( \Gamma^{(J \to J')}_{\mu_1 \ldots \mu_J \alpha_1 \ldots \nu_{J'}}(P, P'; q) \), may have false kinematical singularities. In the whole amplitude of Fig. 3 just these singularities should cancel each other.

This paper is devoted to the problem of cancellation of false singularities: we use as an example the processes when \( J = J' = 0 \) (Section 2) and \( J = 1, J' = 0 \) (Section 3). We deal with scalar (or pseudoscalar) constituent particles with equal masses, \( m_1 = m_2 = m \): this does not affect generality but simplify cumbersome calculations. Furthermore, in Section 4, we present the generalization for quark constituents.

In Section 2, the transition \( (J = 0) \to (J' = 0) \) is considered; we denote this transition as \( S \to \gamma S \). The three-point amplitude for this transition \( \Gamma^{(0 \to 0)}_{\alpha}(P, P'; q) \) can be expanded in respect to two spin operators: the transverse one, \( (P_\perp + P'_\perp)_\alpha = 2(P_\alpha - q_\alpha(Pq)/q^2) \), and longitudinal one, \( q_\alpha \). Transverse spin operator contains kinematical singularity \( 1/q^2 \), which should be cancelled in the whole amplitude.

The necessity to use transverse and longitudinal spin operators, which are orthogonal to each other \( (P_\perp + P'_\perp)_\alpha q_\alpha = 0 \), is dictated by the specifics of the spectral-representation method for form factors of the \( S \to \gamma S \) transition. Vertex function of the photon emission is expanded in respect to independent and orthogonal operators \( (P_\perp + P'_\perp)_\alpha \) and \( q_\alpha \) as follows:

\[
\Gamma^{(0 \to 0)}_{\alpha}(P, P'; q) = (P_\perp + P'_\perp)_\alpha F^{(0 \to 0)}_T(s, s', q^2) + q_\alpha F^{(0 \to 0)}_L(s, s', q^2).
\]

(11)

Then, \( F^{(0 \to 0)}_T(s, s', q^2) \) and \( F^{(0 \to 0)}_L(s, s', q^2) \) are defined by dispersion integrals over \( s \) and \( s' \), which are in due course determined by transverse and longitudinal components of the triangle diagram only.

The mechanism of singularity cancellation in \( \Gamma^{(0 \to 0)}_{\alpha}(P, P'; q) \) was considered in [6] for the variant of \( F^{(0 \to 0)}_T(s, s', q^2) \) being double spectral integral without subtraction terms. However, as was shown in [6], one cannot deal without subtraction terms at all: subtraction terms in \( F^{(0 \to 0)}_L(s, s', q^2) \) are important for cancelling false singularities in [11]. In Section 2, we consider more general case of \( \Gamma^{(0 \to 0)}_{\alpha}(P, P'; q) \) with subtraction term in \( F^{(0 \to 0)}_T(s, s', q^2) \): such a variant serves us as a guide to consider transition form factors with angular momenta \( J = 1, J' = 0 \).
The vertex function for the transition \((J = 1) \rightarrow (J' = 0)\), or \(V \rightarrow \gamma S\), is considered in Section 3. The spin structure of such a vertex, \(\Gamma^{(1\rightarrow0)}(P, P'; q)\), is determined by two independent tensors, one of them can be chosen as metric tensor operating in the two-dimensional space and being orthogonal to \(P\) and \(q\):

\[
g^{\perp\perp}_{\mu\alpha} = g_{\mu\alpha} + \frac{q^2}{(Pq)^2 - P^2 q^2} P_{\mu} P_{\alpha} + \frac{P^2}{(Pq)^2 - P^2 q^2} q_{\mu} q_{\alpha} - \frac{(Pq)}{(Pq)^2 - P^2 q^2} (P_{\mu} q_{\alpha} + q_{\mu} P_{\alpha}) , \tag{12}
\]

while the second tensor is defined as

\[
4 L_{\mu\alpha} = \frac{q^2}{(Pq)^2 - P^2 q^2} P_{\mu} P_{\alpha} + \frac{P^2}{(Pq)^2 - P^2 q^2} q_{\mu} q_{\alpha} - \frac{(Pq)}{(Pq)^2 - P^2 q^2} q_{\mu} P_{\alpha} . \tag{13}
\]

These two tensors satisfy gauge invariance requirements:

\[
P_{\mu} g^{\perp\perp}_{\mu\alpha} = 0 , \quad g^{\perp\perp}_{\mu\alpha} q_{\alpha} = 0 , \quad P_{\mu} L_{\mu\alpha} = 0 , \quad L_{\mu\alpha} q_{\alpha} = 0 , \tag{14}
\]

and \(L_{\mu\alpha}\) is constructed to be orthogonal to \(g^{\perp\perp}_{\mu\alpha}\):

\[
L_{\mu\alpha} g^{\perp\perp}_{\mu\alpha} = 0 . \tag{15}
\]

Vertex function for \(V \rightarrow \gamma S\) is defined by two form factors which correspond to operators in the form \(\text{(12)}\) and \(\text{(13)}\):

\[
\Gamma^{(1\rightarrow0)}(P, P'; q) = g^{\perp\perp}_{\mu\alpha} F_{T}^{(1\rightarrow0)}(s, s', q^2) + L_{\mu\alpha} F_{L}^{(1\rightarrow0)}(s, s', q^2) . \tag{16}
\]

The operators written in \(\text{(12)}\) and \(\text{(13)}\) are singular: they contain poles at \((Pq)^2 - P^2 q^2 = 0\) and \((Pq) = 0\). In \(\text{(16)}\) these singularities must be compensated by zeros of the form factors \(F_{T}^{(1\rightarrow0)}(s, s', q^2)\) and \(F_{L}^{(1\rightarrow0)}(s, s', q^2)\); corresponding mechanism is considered in detail in Section 3. We demonstrate that to compensate false singularities in the reactions \(V \rightarrow \gamma S\) the subtraction terms play an important role; from this point of view the compensation mechanism in the amplitudes \(S \rightarrow \gamma S\) and \(V \rightarrow \gamma S\) is similar.

At \(q^2 = 0\), the only form factor \(F_{T}^{(1\rightarrow0)}(s, s', q^2)\) determines the transition amplitude \(V \rightarrow \gamma S\). In Section 3, we discuss in detail the problem of unambiguous determination of the amplitude in terms of the spectral integration technique. The matter is that at \(q^2 \rightarrow 0\) one of independent operators of \(\text{(12)}\) and \(\text{(13)}\) become nilpotent one. Indeed,

\[
g^{\perp\perp}_{\mu\alpha}(0) = g_{\mu\alpha} + \frac{4 s}{(s - s')^2} q_{\mu} q_{\alpha} - \frac{2}{s - s'} (P_{\mu} q_{\alpha} + q_{\mu} P_{\alpha}) , \tag{17}
\]

\[
L_{\mu\alpha}(0) = \frac{s}{(s - s')^2} q_{\mu} q_{\alpha} - \frac{1}{2(s - s')} P_{\mu} q_{\alpha} ,
\]

and the second operator has a zero norm:

\[
L_{\mu\alpha}(0) L_{\mu\alpha}(0) = 0 . \tag{18}
\]
Due to Eqs. (15) and (18) any combination of $g_{\mu\alpha}^{\perp\perp}(0)$ and $L_{\mu\alpha}(0)$,

$$g_{\mu\alpha}^{\perp\perp}(0) + C(s, s') L_{\mu\alpha}(0),$$

(19)
can be equally used to define the transverse form factor $F_T(s, s', 0)$. In Section 3, this property is demonstrated directly by considering two sets of operators, the first one given by (17) and the second set appeared after the substitution as follows:

$$g_{\mu\alpha}^{\perp\perp}(0) \rightarrow g_{\mu\alpha}^{\perp\perp}(0) + 4 L_{\mu\alpha}(0) = g_{\mu\alpha} - \frac{2}{s - s'} q_{\mu} P_{\alpha}.$$  

(20)

Such a choice of operators for the illustration of (19) is motivated by the existing statements (for example, see [13]) that only the form of (20) provide us with a correct operator expansion of transition amplitude for $V \rightarrow \gamma(q^2 = 0) S$. However, our consideration performed in Section 3 proves directly that it is not so.

Concerning analytical properties of amplitudes in the transitions $V \rightarrow \gamma S$, with emission of photon ($q^2 = 0$), they are as follows.

The whole amplitude of the transition of unbound particles is a sum of three contributions shown in Fig. 3a,b,c:

$$\frac{g_1(s)}{1 - B_1(s)} \left[ g_{\mu\alpha}^{\perp\perp}(0) T(s, s', 0) + L_{\mu\alpha}(0) L(s, s', 0) \right] \frac{g_0(s')}{1 - B_0(s')}.$$  

(21)

The analyticity of this amplitude requires for transverse component $T(s, s', 0)$ to have zero of the first order at $s = s'$, namely, $T(s, s, 0) = 0$, while the combination $L(s, s', 0) + 4T(s, s', 0)$ should have zero of the second order.

Transverse amplitude for the transition of bound vector state to bound scalar state, $F_{T}^{(1\rightarrow 0)}(s = M^2, s' = M'^2, 0)$, which is defined by the diagram of Fig. 2c, is not indebted to be zero at $\omega = M - M' = 0$.

In Section 4, we show the way of generalization of our results for constituent quarks. We present the transition form factor for $\phi(1020) \rightarrow \gamma f_0(980)$ both in the form of relativistic spectral integral [7] and non-relativistic quark model approach.

Direct form factor calculations performed for the transition $\phi(1020) \rightarrow \gamma f_0(980)$ both in relativistic and non-relativistic approximations demonstrate the absence of zero at $\omega \rightarrow 0$. This is in accordance with general result obtained in the analysis of the whole amplitude (21).

In Conclusion we summarise briefly the results focusing our attention to advantages of the spectral integration technique and the problems which this method faces on.

2 Interaction of scalar two-particle composite system in the state $J^P = 0^+$ with electromagnetic field: $S \rightarrow \gamma S$

Diagrammatical representation of the interaction amplitude in terms of the dispersion-relation graphs is shown in Fig. 3. These diagrams are obtained by coupling the photon to all the
constituents (both internal and external) in the diagrams related to the constituent scattering amplitude. Besides, there is an uncoupled diagram of Fig. 3d corresponding to non-interacting constituents.

We assume that two constituents may form the bound state, and the form factor of composite system is determined by residues in the amplitude poles, see Fig. 2b.

For the sake of simplicity, the constituent masses are put to be equal to each other, \( m_1 = m_2 = m \), though constituents are not identical; in this way we do not symmetrize the states of constituents.

### 2.1 Diagrams of the Fig. 3a type

Consider the sum of the diagrams shown in Fig. 3a, when constituents interact in the S-wave state both before and after the emission of photon. The sum of diagrams of such a type is written as:

\[
A^{(0 \rightarrow 0)}_{\alpha}(P, P'; q) = \frac{g_0(s)}{1 - B_0(s)} \Gamma^{(0 \rightarrow 0)}_{\alpha}(P, P'; q) \frac{g_0(s')}{1 - B_0(s')},
\]

where \( \Gamma^{(0 \rightarrow 0)}_{\alpha} \) is the three-point function shown separately in Fig. 4a; its representation through form factors \( F_T^{(0 \rightarrow 0)}(s, s', q^2) \) and \( F_L^{(0 \rightarrow 0)}(s, s', q^2) \) is given in [6, 7, 8, 9].

The dispersion representation of the triangle graph can be found in [6, 7, 8, 9]. Here we briefly repeat the scheme keeping in mind to apply it not only to three-point diagrams but also to the two-point ones describing the photon emission by constituents in the initial and final states too, see Fig. 4b,c.

Let us start with the Feynman expression for the triangle diagram \( \Gamma^{(0 \rightarrow 0)}_{\alpha}(P, P'; q) \):

\[
\int \frac{d^4k_1}{i(2\pi)^4} \frac{g_0(\bar{P}^2; k_1, k_2^2)(k_1 + k_1')(\bar{P}'^2; k_1', k_2^2)}{(m^2 - k_1^2)(m^2 - k_2^2)(m^2 - k_1'^2)}.
\]

The following steps are necessary to write down the dispersion integral starting from this amplitude:

1. We should calculate the double discontinuity of the Feynman diagram [23], with fixed energy squared of initial and final states, \( \bar{E}^2 = \bar{s} \) and \( \bar{P}'^2 = \bar{s}' \). This implies the substitution of operators in the intermediate states as follows:

\[
(m^2 - k_1^2)^{-1}(m^2 - k_2^2)^{-1} \rightarrow \theta(k_{10})\delta(k_1^2 - m^2)\theta(k_{20})\delta(k_2^2 - m^2)
\]

\[
(m^2 - k_1'^2)^{-1} \rightarrow \theta(k_{10}')\delta(k_1'^2 - m^2)
\]

as well as integration over three-particle phase space in both channels at fixed \( \bar{s} \) and \( \bar{s}' \):

\[
\frac{d^4k_1}{(2\pi)^4} \delta(m^2 - k_1^2) \delta(m^2 - k_2^2) \delta(m^2 - k_2^2) \rightarrow d\Phi_{tr}(\bar{P}, \bar{P}'; k_1, k_1', k_2),
\]

\[
d\Phi_{tr}(\bar{P}, \bar{P}'; k_1, k_1', k_2) = d\Phi_2(\bar{P}; k_1, k_2)d\Phi_2(\bar{P}'; k_1', k_2')2k_20(2\pi)^3\delta^3(k_2 - k_2'),
\]
The constituents in the intermediate state are mass-on-shell. The double discontinuity is calculated at \( \tilde{q} = \tilde{P} - \tilde{P}' \neq q \) but \( \tilde{q}^2 = q^2 \).

(ii) The vertex functions are to be replaced as

\[
g_0(\tilde{P}^2; k_1^2, k_2^2) \rightarrow g_0(\tilde{P}'^2; k_1'^2, k_2'^2) \longrightarrow g_0(\tilde{s}) g_0(\tilde{s}') \tag{26}
\]

that actualize the treatment of composite system as a true two-particle state.

(iii) The invariant part of the triangle diagram should be singled out by expanding the spin factor \((k_1 + k_1')_\alpha\) in the vectors \((\tilde{P} + \tilde{P}')_\perp\) and \(\tilde{q} = \tilde{P} - \tilde{P}'\):

\[
(k_1 + k_1')_\alpha = \alpha(\tilde{s}, \tilde{s}', q^2) \left( \tilde{P}_\alpha + \tilde{P}'_\alpha - \frac{\tilde{s} - \tilde{s}'}{q^2} \tilde{q}_\alpha \right) + \beta(\tilde{s}, \tilde{s}', q^2) \tilde{q}_\alpha \tag{27}
\]

The coefficients \(\alpha(\tilde{s}, \tilde{s}', q^2)\) and \(\beta(\tilde{s}, \tilde{s}', q^2)\) are given below, in Eq. (30). As a result, we have the following expressions for double discontinuities (double spectral densities) for the triangle diagram:

\[
disc_2 disc_2 F_T^{(0-0)}(\tilde{s}, \tilde{s}', q^2) = \alpha(\tilde{s}, \tilde{s}', q^2) g_0(\tilde{s}) g_0(\tilde{s}') d\Phi_{tr}(\tilde{P}, \tilde{P}'; k_1, k_2, k_1'),
\]

\[
disc_2 disc_2 F_L^{(0-0)}(\tilde{s}, \tilde{s}', q^2) = \beta(\tilde{s}, \tilde{s}', q^2) g_0(\tilde{s}) g_0(\tilde{s}') d\Phi_{tr}(\tilde{P}, \tilde{P}'; k_1, k_2, k_1') \tag{28}
\]

The form factor \(F_i^{(0-0)}(s, s', q^2)\) is determined by its spectral density as follows:

\[
F_i^{(0-0)}(s, s', q^2) = f_i^{(0-0)}(s, s', q^2) + \int_{4m^2}^{\infty} \frac{d\tilde{s} d\tilde{s}'}{\pi} disc_2 disc_2 F_i^{(0-0)}(\tilde{s}, \tilde{s}', q^2) \tag{29}
\]

where \(f_i^{(0-0)}(s, s', q^2)\) are the subtraction terms with zero double spectral density. Within the approach, where partial amplitude is described by a set of dispersion diagrams of Fig. 1a, the subtraction term \(f_i^{(0-0)}(s, s', q^2)\) is an arbitrary function determined by diagrams where photon interacts with other particles, not constituents, for example, with mesons which determine the forces between constituents.

The expansion coefficients \(\{27\}\) are calculated under the orthogonality requirements. Indeed, by projecting \(\{27\}\) onto \(\tilde{P}_\alpha + \tilde{P}'_\alpha - \tilde{q}_\alpha (\tilde{s} - \tilde{s}')/q^2\) and \(\tilde{q}_\alpha\), we obtain the equations for \(\alpha(\tilde{s}, \tilde{s}', q^2)\) and \(\beta(\tilde{s}, \tilde{s}', q^2)\). We have:

\[
\alpha(\tilde{s}, \tilde{s}', q^2) = - \frac{q^2(\tilde{s} + \tilde{s}' - q^2)}{\lambda(\tilde{s}, \tilde{s}', q^2)},
\]

\[
\beta(\tilde{s}, \tilde{s}', q^2) = 0,
\]

\[
\lambda(\tilde{s}, \tilde{s}', q^2) = -2q^2(\tilde{s} + \tilde{s}') + q^4 + (\tilde{s} - \tilde{s}')^2. \tag{30}
\]

Here we took into account that \(k_i^2 = m^2\) and \((k_1 - k_1')^2 = q^2\). Therefore, \(F_L^{(0-0)}(s, s', q^2)\) has zero double spectral density, and it is defined by the subtraction term only:

\[
F_L^{(0-0)}(s, s', q^2) = f_L^{(0-0)}(s, s', q^2). \tag{31}
\]
For $F_T^{(0 \rightarrow 0)}(s, s', q^2)$, after integrating in (25) over the momenta $k_1, k'_1$ and $k_2$ at fixed $s$ and $s'$, we obtain the following equation:

$$
F_T^{(0 \rightarrow 0)}(s, s', q^2) = F_T^{(0 \rightarrow 0)}(s, s', q^2) + \int_{4m^2}^{\infty} \frac{d\tilde{s} d\tilde{s}'}{\pi^2} \cdot \frac{g_0(\tilde{s}) g_0(\tilde{s}')}{\tilde{s} - s \tilde{s}' - s'} \cdot \frac{\Theta(-\tilde{s} \tilde{s}' q^2 - m^2 \lambda(\tilde{s}, \tilde{s}', q^2))}{16 \sqrt{\lambda(\tilde{s}, \tilde{s}', q^2)}} \cdot \alpha(\tilde{s}, \tilde{s}', q^2) .
$$

(32)

Here the $\Theta$-function is defined as follows: $\Theta(X) = 1$ at $X \geq 0$ and $\Theta(X) = 0$ at $X < 0$.

Now let us come back to the requirements the amplitude should obey. First, as was said above, it must be analytical function, that is, kinematical singularities should be absent. Concerning the $q^2 = 0$ singularity of the considered amplitude, the term we should care for is:

$$
-\frac{s - s'}{q^2} F_T^{(0 \rightarrow 0)}(s, s', q^2) q_\alpha .
$$

(33)

Let us calculate the form factor in the limit $q^2 \rightarrow 0$. To this aim let us introduce new variables in (32):

$$
\sigma = \frac{1}{2}(\tilde{s} + \tilde{s}' ) ; \quad \Delta = \tilde{s} - \tilde{s}' , \quad Q^2 = -q^2 ,
$$

(34)

and then consider the case of interest, $Q^2 \rightarrow 0$. The form factor formula reads:

$$
F_T^{(0 \rightarrow 0)}(s, s', q^2) = F_T^{(0 \rightarrow 0)}(s, s', q^2) + \int_{4m^2}^{\infty} \frac{d\sigma}{\pi} \cdot \frac{\sigma}{s, q^2} \cdot \frac{g_0^2(\sigma)}{(\sigma - s)(\sigma - s')} \cdot \int_{-b}^{b} d\Delta \cdot \frac{\alpha(\sigma, \Delta, Q^2)}{16 \pi \sqrt{\Delta^2 + 4 \sigma Q^2}} ,
$$

(35)

where

$$
b = \frac{Q}{m} \sqrt{\sigma(\sigma - 4m^2)} , \quad \alpha(\sigma, \Delta, Q^2) = \frac{2\sigma Q^2}{\Delta^2 + 4 \sigma Q^2} .
$$

(36)

As a result we have:

$$
F_T^{(0 \rightarrow 0)}(s, s', 0) = F_T^{(0 \rightarrow 0)}(s, s', 0) + \frac{B_0(s) - B_0(s')}{s - s'} ,
$$

(37)

where $B_0(s)$ is the loop diagram:

$$
B_0(s) = \int_{4m^2}^{\infty} \frac{d\tilde{s}}{\pi} \cdot \frac{g_0^2(\tilde{s})}{\tilde{s} - s} \cdot \rho(\tilde{s}) , \quad \rho(s) = \frac{1}{16 \pi} \sqrt{\frac{s - 4m^2}{s}} .
$$

(38)

In (38) the index $J$ for the phase volume with $J = 0$ is omitted.

In the limit $q^2 \rightarrow 0$ the amplitude takes the form:

$$
\Gamma_\alpha^{(0 \rightarrow 0)}(P, P'; q^2 \rightarrow 0) = \left[ P_\alpha + P'_\alpha - \frac{s - s'}{q^2} \cdot q_\alpha \right] \left[ F_T^{(0 \rightarrow 0)}(s, s', q^2 \rightarrow 0) + \frac{B_0(s) - B_0(s')}{s - s'} \right]
+ q_\alpha F_L^{(0 \rightarrow 0)}(s, s', q^2 \rightarrow 0) .
$$

(39)
The amplitude (39) should not have pole singularity $1/q^2$: the presence in the right-hand side of (39) of singular factor $q_\alpha (s-s')/q^2$ is an artifact of our expansion of the amplitude $\Gamma^{(0\to0)}_\alpha$ in the transverse and longitudinal components. Therefore, the subtraction term in $f_L^{(0\to0)}(s, s', q^2 \to 0)$ must contain expressions which cancel the singularity $1/q^2$. The cancellation of singular terms lead to the requirement:

\[
f_L^{(0\to0)}(s, s', q^2 \to 0) = \frac{1}{q^2} \left( (s-s')f_T^{(0\to0)}(s, s', 0) + B_0(s) - B_0(s') \right) . \tag{40}
\]

After the fulfilment of (40), we have for $\Gamma^{(0\to0)}_\alpha(s, s'; 0)$:

\[
\Gamma^{(0\to0)}_\alpha(s, s'; 0) = (P_\alpha + P'_\alpha) \left( f_T^{(0\to0)}(s, s', 0) + \frac{B_0(s) - B_0(s')}{s - s'} \right) . \tag{41}
\]

This formula has been obtained in (4) for the case of $f_T^{(0\to0)} \equiv 0$. In this approximation we come to a well-known Ward identity for the triangle diagram: $q_\alpha \Gamma^{(0\to0)}_\alpha(s, s', 0) = B_0(s) - B_0(s')$. With non-zero subtraction term, the Ward identity looks as follows:

\[
q_\alpha \Gamma^{(0\to0)}_\alpha(s, s'; 0) = (s-s')f_T^{(0\to0)}(s, s', 0) + B_0(s) - B_0(s') . \tag{42}
\]

### 2.2 Diagrams of Fig. 3b,c type

Consider the amplitude for the diagram of Fig. 3b; it reads as follows:

\[
A^{(-\to0)}_\alpha(P, P'; q) = \Gamma^{(-\to0)}_\alpha(P, P'; q) \frac{g_0(s')}{1 - B_0(s')} . \tag{43}
\]

Here $\Gamma^{(-\to0)}_\alpha$ stands for the vertex representing the emission of photon by the incoming constituent, it is shown in Fig. 4b.

By singling out the $S$-wave state from the initial state of the amplitude of Fig. 4b, $\Gamma^{(-\to0)}_\alpha \to \Gamma^{(S\to0)}_\alpha$, we can represent $\Gamma^{(S\to0)}_\alpha$ as the spectral integral. In this way the amplitude is written as follows:

\[
\Gamma^{(S\to0)}_\alpha(P, P'; q) = \left( P_\alpha + P'_\alpha - \frac{s-s'}{q^2} q_\alpha \right) F_T^{(S\to0)}(s, s', q^2) + q_\alpha F_L^{(S\to0)}(s, s', q^2) . \tag{44}
\]

The spectral integrals for $F_T^{(S\to0)}$ and $F_L^{(S\to0)}$ are obtained in the same way as before. Namely, we project the Feynman of Fig. 4b on the $S$-wave state by averaging over the phase space of initial particles, $k_1$ and $k_2$:

\[
\int \frac{d\Phi_2(P; k_1, k_2)}{\rho(s)} (k_1 + k'_1) \alpha \frac{g_0(\tilde{P}; k_1^0; k_2^0)}{m^2 - k_1^2} . \tag{45}
\]

The discontinuity of the amplitude (44) is calculated for the mass-on-shell constituent:

\[
(m^2 - k_1^2)^{-1} \to \theta(k_{10}^2) \delta(k_1^2 - m^2) . \tag{46}
\]
with the phase space integration in the channel \( s' \):

\[
(2\pi)^3 2k_{20}g^{(3)}(k_2 - k'_2) \, d\Phi_2(P', k'_1, k'_2) .
\]

Besides, it is necessary to expand the spin factor \((k_1 + k'_1)\alpha\) in the vectors \(P_\alpha, \tilde{P}_\alpha\) and \(q\) with the constraint \(\tilde{q}^2 = q^2\). Invariant expansion coefficients, \(\alpha(s, \tilde{s}', q^2)\) and \(\beta(s, \tilde{s}', q^2)\), are given by Eqs. (27) and (30).

After the substitution \(g_0(\tilde{P}^2; k_1^2, k_2^2) \rightarrow g_0(\tilde{s}')\) and spectral integration, we have the following representation for \(F^{(S \rightarrow 0)}_{\alpha}(s, s', q^2)\):

\[
F^{(S \rightarrow 0)}_T(s, s', q^2) = f^{(S \rightarrow 0)}_T(s, s', q^2) + \int_{4m^2}^{\infty} \frac{d\tilde{s}'}{\pi} \frac{g_0(\tilde{s}')}{\tilde{s}' - s'} \, d\Phi_2(P, \tilde{P}', k_1, k_2) ,
\]

\[
F^{(S \rightarrow 0)}_L(s, s', q^2) = f^{(S \rightarrow 0)}_L(s, s', q^2) .
\]

Here we took into account that \(\beta(s, \tilde{s}', q^2) = 0\), see (30); the equation for \(F^{(S \rightarrow 0)}_L(s, s', q^2)\) has zero double spectral density, and it is completely determined by its subtraction term.

For \(F^{(S \rightarrow 0)}_T(s, s', q^2)\), after integrating over \(k_1, k'_1\) and \(k_2\) at fixed \(s'\), we have the following expression:

\[
F^{(S \rightarrow 0)}_T(s, s', q^2) = f^{(S \rightarrow 0)}_T(s, s', q^2) + \int_{4m^2}^{\infty} \frac{d\tilde{s}'}{\pi} \frac{g_0(\tilde{s}')}{(s - \tilde{s}') q^2 - 16m^2\lambda(s, \tilde{s}', q^2)} \Theta(-s, \tilde{s}' q^2 - m^2\lambda(s, \tilde{s}', q^2)) \alpha(s, \tilde{s}', q^2) .
\]

For the factor \(F^{(S \rightarrow 0)}_T(s, s', q^2)\) in the limit \(q^2 \rightarrow 0\), after the same calculations as for previous diagrams, we obtain:

\[
F^{(S \rightarrow 0)}_T(s, s', 0) = f^{(S \rightarrow 0)}_T(s, s', 0) + \frac{g_0(s)}{s - s'} .
\]

The amplitude (44) in the limit \(q^2 \rightarrow 0\) has the form:

\[
\Gamma^{(S \rightarrow 0)}_\alpha(P, P'; q^2 \rightarrow 0) =
\]

\[
= \left( P_\alpha + P'_\alpha - \frac{s - s'}{q^2} q_\alpha \right) \left[ f^{(S \rightarrow 0)}_T(s, s', q^2 \rightarrow 0) + \frac{g_0(s)}{s - s'} \right] +
\]

\[
+ q_\alpha f^{(S \rightarrow 0)}_L(s, s', q^2 \rightarrow 0) ,
\]

and the requirement of absence of singularity \(1/q^2\) in the amplitude \(\Gamma^{(S \rightarrow 0)}_\alpha(P, P'; q^2 \rightarrow 0)\) leads to the formula:

\[
f^{(S \rightarrow 0)}_L(s, s', q^2 \rightarrow 0) = \frac{1}{q^2} \left( (s - s') f^{(S \rightarrow 0)}_T(s, s', 0) + g_0(s) \right) .
\]

Taking this condition into account for the vertex \(\Gamma^{(S \rightarrow 0)}_\alpha(P, P'; 0)\), one obtains:

\[
\Gamma^{(S \rightarrow 0)}_\alpha(P, P'; 0) = (P_\alpha + P'_\alpha) \left( f^{(S \rightarrow 0)}_T(s, s', 0) + \frac{g_0(s)}{s - s'} \right) .
\]
The amplitude for diagrams of Fig. 3c-type is treated similarly. After the $S$-wave is extracted for outgoing constituents, we have:

$$A^{(0\to S)}_\alpha(P, P'; q) = \frac{g_0(s)}{1 - B_0(s)} \Gamma^{(0\to S)}_\alpha(P, P'; q),$$

(54)

where

$$\Gamma^{(0\to S)}_\alpha(P, P'; q) = \left(P_\alpha + P'_\alpha - \frac{s - s'}{q^2} q_\alpha\right) F_T^{(0\to S)}(s, s', q^2) + q_\alpha F_L^{(0\to S)}(s, s', q^2),$$

(55)

and

$$F_L^{(0\to S)}(s, s', q^2) = f_L^{(0\to S)}(s, s', q^2),$$

(56)

$$F_T^{(0\to S)}(s, s', q^2) = f_T^{(0\to S)}(s, s', q^2) + \int_{4m^2}^\infty \frac{d\tilde{s}}{\pi} \frac{g_0(\tilde{s})}{s - \tilde{s}} \theta(-\tilde{s} s' q^2 - m^2 \lambda(\tilde{s}, s', q^2)) \frac{\alpha(\tilde{s}, s', q^2)}{16 \sqrt{\lambda(\tilde{s}, s', q^2)}}.$$

In the limit $q^2 \to 0$, the amplitude takes the form:

$$\Gamma^{(0\to S)}_\alpha(P, P'; q^2 \to 0) =$$

$$= \left(P_\alpha + P'_\alpha - \frac{s - s'}{q^2} q_\alpha\right) \left[f_T^{(0\to S)}(s, s', q^2 \to 0) - \frac{g_0(s')}{s - s'}\right] +$$

$$+ q_\alpha f_L^{(0\to S)}(s, s', q^2 \to 0),$$

(57)

and the requirement of cancellation of the singularity $1/q^2$ works out the following formula:

$$f_L^{(0\to S)}(s, s', q^2 \to 0) = \frac{1}{q^2} \left((s - s') f_T^{(0\to S)}(s, s', 0) - g_0(s')\right).$$

(58)

That results in

$$\Gamma^{(0\to S)}_\alpha(P, P'; 0) = (P_\alpha + P'_\alpha) \left(f_T^{(0\to S)}(s, s', 0) - \frac{g_0(s')}{s - s'}\right).$$

(59)

### 2.3 Connected diagrams in the limit $q^2 \to 0$

First, consider the case when there are no subtraction terms in the transverse form factor, $f_T^{(S\to 0)}(s, s', 0) = f_T^{(0\to S)}(s, s', 0) = 0$: such a variant has been considered in [6]. It is easy to see that the sum of all connected diagrams of Fig. 3a, 3b and 3c, is equal to zero in this limit. Indeed, according to [11], [53] and [59], it is equal to

$$(P_\alpha + P'_\alpha) \left[\frac{g_0(s)}{s - s'} \frac{g_0(s')}{1 - B_0(s')} + \frac{g_0(s)}{1 - B_0(s)} \frac{B_0(s) - B_0(s')}{s - s'} - \frac{g_0(s')}{1 - B_0(s')} \frac{B_0(s) - B_0(s')}{s - s'}\right] = 0.$$  

(60)
Generally, when subtraction terms differ from zero, the amplitude $A^{(conected)}_{\alpha}(P, P', q^2)$ should also turn into zero at $q^2 \to 0$:

$$A^{(conected)}_{\alpha}(P, P', q^2 \to 0) = A^{(S\to 0)}_{\alpha}(P, P', q^2 \to 0) + A^{(0\to S)}_{\alpha}(P, P', q^2 \to 0) = 0 ,$$

that is equivalent to the requirement (see formulae (41), (53), (59)):

$$f_T^{(0\to 0)}(s, s', 0) = \frac{B_0(s) - 1}{g_0(s)} f_T^{(S\to 0)}(s, s', 0) + f_T^{(0\to S)}(s, s', 0) \frac{B_0(s') - 1}{g_0(s')} .$$

(62)

Let us bring our attention to the following. The existence of bound state requires $B_0(M^2) = 1$; therefore, we have on the basis of (62):

$$f_T^{(0\to 0)}(M^2, M^2, 0) = 0 .$$

(63)

This means that charge form factor of composite system $F(q^2) = F_T^{(0\to 0)}(M^2, M^2, q^2)$ is determined at $q^2 = 0$ by the triangle graph only, without subtraction terms. Such a property is not surprising: relying on $B_0(M^2) = 1$, we assumed actually that composite system is true two-particle one, and the condition for charge form factor of the composite system,

$$F(0) = 1 ,$$

(64)

is the normalization condition for the wave function of this system.

Accounting for (63), one can impose more general constraint

$$f_T^{(0\to 0)}(M^2, M^2, q^2) = 0 ,$$

(65)

that is equivalent to the suggestion that charge form factor of composite system is defined by double spectral integral only. Within this approximation the form factor of deuteron as two-nucleon system was calculated in [6] as well as form factor of the pion treated as $q\bar{q}$-system [8].

### 3 Transition $V \to \gamma S$

Consider now the transition of vector state to scalar one, $V \to \gamma S$: this is the transition of the $P$-wave two-constituent state to the $S$-wave one. Such a reaction, as in the previous case, is represented by a set of diagrams shown in Fig. 3a,b,c. The vertex function for $V \to \gamma S$, $\Gamma_{\mu\alpha}^{(1\to 0)}$, depends on the two spin indices: $\mu$ stands for vector state of constituents and $\alpha$, as before, is related to photon.

In the diagrams shown in Fig. 4a,b,c the following factor carries spin indices:

$$(k_1 - k_2)_\mu (k_1 + k'_1)_\alpha ,$$

(66)

13
where \((k_1 - k_2)_\mu\) provides the \(P\)-wave of the initial state and \((k_1 + k'_1)_\alpha\) determines gauge-invariant vertex photon–constituent. Let us expand the factor \((k_1 - k_2)_\mu (k_1 + k'_1)_\alpha\) in the spin operators. As was said in Introduction, there exists a freedom in the choice of expansion operators. To reveal the consequences of this freedom for spectral amplitudes, consider in parallel two sets of operators. In the first case the operators are as follows:

Expansion I: \(g^{\bot\bot \alpha}_{\mu \alpha}, L_{\mu \alpha}\), \(g^{\bot\bot \alpha}_{\mu \alpha}, L_{\mu \alpha}\) \(\text{as follows:} \)

and in the second one:

Expansion II: \(\bar{g}^{\bot\bot \alpha}_{\mu \alpha} = g_{\mu \alpha} - \frac{q_{\mu} P_\alpha}{(Pq)} = g^{\bot\bot \alpha}_{\mu \alpha} - 4 L_{\mu \alpha}, \ L_{\mu \alpha}\) \(\text{as follows:} \)

Recall that the operators \(g^{\bot\bot \alpha}_{\mu \alpha}\) and \(L_{\mu \alpha}\) were introduced in \(12\) and \(13\).

The convolutions of operators \(g^{\bot\bot \alpha}_{\mu \alpha}, \bar{g}^{\bot\bot \alpha}_{\mu \alpha}\) and \(L_{\mu \alpha}\) are equal to:

\[
\begin{align*}
g^{\bot\bot \alpha}_{\mu \alpha} g^{\bot\bot \alpha}_{\mu \alpha} &= 2, \quad L_{\mu \alpha} L_{\mu \alpha} = \frac{q^2 P^2}{16 (Pq)^2}, \quad L_{\mu \alpha} g^{\bot\bot \alpha}_{\mu \alpha} = 0, \\
\bar{g}^{\bot\bot \alpha}_{\mu \alpha} \bar{g}^{\bot\bot \alpha}_{\mu \alpha} &= 2 + \frac{q^2 P^2}{(Pq)^2}, \quad L_{\mu \alpha} \bar{g}^{\bot\bot \alpha}_{\mu \alpha} = \frac{q^2 P^2}{4 (Pq)^2}.
\end{align*}
\]

We see that the operators from the second set are not orthogonal to one another but the orthogonality is restored at \(q^2 \to 0\). At \(q^2 = 0\) one has:

\[
\begin{align*}
g^{\bot\bot \alpha}_{\mu \alpha}(0) g^{\bot\bot \alpha}_{\mu \alpha}(0) &= 2, \quad L_{\mu \alpha}(0) L_{\mu \alpha}(0) = 0, \quad L_{\mu \alpha}(0) g^{\bot\bot \alpha}_{\mu \alpha}(0) = 0, \\
\bar{g}^{\bot\bot \alpha}_{\mu \alpha}(0) \bar{g}^{\bot\bot \alpha}_{\mu \alpha}(0) &= 2, \quad L_{\mu \alpha}(0) \bar{g}^{\bot\bot \alpha}_{\mu \alpha}(0) = 0,
\end{align*}
\]

that means the equivalence of both sets of operators.

In terms of the considered operators the spin factor,

\[
S_{\mu \alpha} = (k_1 - k_2)_\mu (k_1 + k'_1)_\alpha ,
\]

is represented as:

Expansion I: \(S_{\mu \alpha} = \xi_T(s, s', q^2) g^{\bot\bot \alpha}_{\mu \alpha} + \xi_L(s, s', q^2) L_{\mu \alpha}\)

\[
\begin{align*}
\xi_T(s, s', q^2) &= 2 \left( m^2 + \frac{ss'q^2}{\lambda(s, s', q^2)} \right), \\
\xi_L(s, s', q^2) &= \frac{2(s + s' - q^2)(s - s' - q^2)(s - s' + q^2)}{\lambda(s, s', q^2)}.
\end{align*}
\]

Expansion II: \(S_{\mu \alpha} = \bar{\xi}_T(s, s', q^2) \bar{g}^{\bot\bot \alpha}_{\mu \alpha} + \bar{\xi}_L(s, s', q^2) L_{\mu \alpha}\)

\[
\begin{align*}
\bar{\xi}_T(s, s', q^2) &= 2 \left( m^2 + \frac{ss'q^2}{\lambda(s, s', q^2)} \right), \\
\bar{\xi}_L(s, s', q^2) &= 2 \left( 4m^2 + \frac{4ss'q^2 + (s + s' - q^2)(s - s' - q^2)(s - s' + q^2)}{\lambda(s, s', q^2)} \right)
\end{align*}
\]
with function $\lambda(s, s', q^2)$ given in (30); the calculation of coefficients is carried out in Appendix A. As one can see,

$$\xi_T(s, s', q^2) = \tilde{\xi}_T(s, s', q^2), \quad (74)$$

and this is important for the discussion.

### 3.1 The amplitude of Fig. 3a.

The whole amplitude is a sum of amplitudes of three types represented by Fig. 3a,b,c. Let us start with the diagram of Fig. 3a, which contains double pole term. Corresponding amplitude is written as follows:

$$A^{(1\rightarrow 0)}_\alpha(s, s', q^2) = (p_1 - p_2)_\mu \frac{g_1(s)}{1 - B_1(s)} \frac{\Gamma^{(1\rightarrow 0)}(P, P'; q) g_0(s')} {1 - B_0(s')}, \quad (75)$$

The functions $g_1(s)$ and $g_0(s')$ are vertices of vector and scalar states; $\Gamma_{\mu\alpha}$ is the three-point amplitude of Fig. 4a, for which the expressions for different choices of operators read:

- Expansion I: $\Gamma^{(1\rightarrow 0)}_{\mu\alpha}(P, P'; q) = g_{\mu\alpha}^{1 \rightarrow -1} F^{(1\rightarrow 0)}_T(s, s', q^2) + L_{\mu\alpha} F^{(1\rightarrow 0)}_L(s, s', q^2)$,

- Expansion II: $\Gamma^{(1\rightarrow 0)}_{\mu\alpha}(P, P'; q) = g_{\mu\alpha}^{1 \rightarrow -1} \tilde{F}^{(1\rightarrow 0)}_T(s, s', q^2) + L_{\mu\alpha} \tilde{F}^{(1\rightarrow 0)}_L(s, s', q^2). \quad (76)$

Here $F^{(1\rightarrow 0)}_T$, $\tilde{F}^{(1\rightarrow 0)}_T$, $\tilde{F}^{(1\rightarrow 0)}_L$ and $\tilde{F}^{(1\rightarrow 0)}_L$ are the form factors, for which the dispersion relations can be written in similarly to what was described in the previous Section.

The form factor $F^{(1\rightarrow 0)}_i(s, s', q^2)$ in Expansion I reads:

$$F^{(1\rightarrow 0)}_i(s, s', q^2) = f^{(1\rightarrow 0)}_i(s, s', q^2) + \int_{4m^2}^{\infty} \frac{d\bar{s} d\bar{s}'}{\pi \pi} disc_\bar{s} disc_\bar{s}' F^0_i(s, s', q^2), \quad i = T, L. \quad (77)$$

Here $f^{(1\rightarrow 0)}_i(s, s', q^2)$ is the subtraction term, and the double spectral density is:

$$disc_\bar{s} disc_\bar{s}' F^0_i(s, s', q^2) = \xi_i(\bar{s}, \bar{s}', q^2) g_1(\bar{s}) g_0(\bar{s}') d\Phi_{tr} \left( \bar{P}, \bar{P}'; k_1, k_1', k_2 \right). \quad (78)$$

The form factor $\tilde{F}^{(1\rightarrow 0)}_i(s, s', q^2)$ in Expansion II is written similarly to (77) but with differently defined double spectral density given by (78): one should substitute $\xi_i(\bar{s}, \bar{s}', q^2) \rightarrow \tilde{\xi}_i(\bar{s}, \bar{s}', q^2)$.

Using dispersion relations for the form factors, one can investigate these two variants for any $q^2$. But the subject of our interest is the case of $q^2 \rightarrow 0$, so concentrate our attention just here. At $q^2 \rightarrow 0$ the amplitude in Expansion I takes the form:

$$\Gamma^{(1\rightarrow 0)}_{\mu\alpha}(s, s'; q^2 \rightarrow 0) =$$

$$= \left[ g_{\mu\alpha} + \frac{4s}{(s-s')^2} q_\mu q_\alpha - \frac{2}{s-s'} (P_\mu q_\alpha + q_\mu P_\alpha) \right] F^{(1\rightarrow 0)}_T(s, s', 0) +$$

$$+ \left[ \frac{s}{(s-s')^2} q_\mu q_\alpha - \frac{1}{2(s-s')} P_\mu q_\alpha \right] F^{(1\rightarrow 0)}_L(s, s', 0) =$$
\[
= \left[ g_{\mu\alpha} - \frac{2}{s - s'} q_\mu P_\alpha \right] F_T^{(1\rightarrow 0)}(s, s', 0) + \\
+ \left[ \frac{s}{(s - s')^2} q_\mu g_\alpha - \frac{1}{2(s - s')^2} P_\mu q_\alpha \right] \left( F_L^{(1\rightarrow 0)}(s, s', 0) + 4 F_T^{(1\rightarrow 0)}(s, s', 0) \right) .
\]

(79)

Hence one has:
\[
\tilde{F}_T^{(1\rightarrow 0)}(s, s', 0) = F_T^{(1\rightarrow 0)}(s, s', 0) ,
\]
\[
\tilde{F}_L^{(1\rightarrow 0)}(s, s', 0) = F_L^{(1\rightarrow 0)}(s, s', 0) + 4 F_T^{(1\rightarrow 0)}(s, s', 0) .
\]

(80)

The calculation of three-point form factors in the limit \( q^2 \rightarrow 0 \) is given in Appendix B for both expansion types, it is similar to the calculation of the three-point amplitude performed in previous Section for the transition \( S \rightarrow \gamma S \). Here we present the case of Expansion I only, for the amplitudes of Expansion II are defined by (80). One gets:

Expansion I:
\[
F_i^{(1\rightarrow 0)}(s, s', 0) = f_i^{(1\rightarrow 0)}(s, s', 0) + \frac{B_i^{(1\rightarrow 0)}(s) - B_i^{(1\rightarrow 0)}(s')}{s - s'} , i = T, L .
\]

(81)

where the loop diagram \( B_i^{(1\rightarrow 0)}(s) \) is equal to:
\[
B_i^{(1\rightarrow 0)}(s) = \int_4^{\infty} \frac{d\bar{s}}{\pi} \frac{g_1(\bar{s})g_0(\bar{s})}{\bar{s} - s} \rho(\bar{s}) \zeta_i(\bar{s}) ,
\]
\[
\zeta_T(s) = 2m^2 \sqrt{\frac{s}{s - 4m^2}} \ln \frac{1 + \sqrt{(s - 4m^2)/s}}{1 - \sqrt{(s - 4m^2)/s}} - s ,
\]
\[
\zeta_L(s) = 4s \left[ \sqrt{\frac{s}{s - 4m^2}} \ln \frac{1 + \sqrt{(s - 4m^2)/s}}{1 - \sqrt{(s - 4m^2)/s}} - 2 \right] .
\]

(82)

Here we come to a clue result of our study: the form factor related to the transverse component (80) does not depend on the choice of expansion operators. The choice of the expansion results in the definition of \( F_L^{(1\rightarrow 0)}(s, s', 0) \), but the amplitude \( A_{L \mu\alpha}^{(1\rightarrow 0)} \), in its turn, does not contribute to cross sections of physical processes with real photons because \( A_{L \mu\alpha}^{(1\rightarrow 0)} \epsilon_{\alpha}(\gamma) = 0 \).

Summing up, we conclude that in the limit \( q^2 \rightarrow 0 \) the amplitude of Fig. 3a-type diagrams is determined unambiguously:
\[
A_{\alpha}^{(1\rightarrow 0)}(s, s'; 0) = (p_1 - p_2)_\mu \frac{g_1(s)}{1 - B_1(s)} \Gamma_{\mu\alpha}^{(1\rightarrow 0)}(s, s'; 0) \frac{g_0(s')}{1 - B_0(s')} ,
\]

(83)

where \( \Gamma_{\mu\alpha}^{(1\rightarrow 0)}(s, s'; 0) \) is given by (79).

### 3.2 The amplitudes for the processes of Fig. 3b,c.

Furthermore, consider the diagram of Fig. 3b, when the photon interacts with constituents in the initial state. Corresponding amplitude for the diagrams of such a type is given by (13):
recall that $\Gamma^{(s\rightarrow 0)}_{\alpha}(P, P' ; q)$ is a function represented diagrammatically by Fig. 4b. By studying the transitions $V \rightarrow \gamma S$, one needs to single out the $P$-wave component in the initial state of the pole amplitude of Fig. 4b. In Appendix C the expansion of pole diagram in partial waves is presented in more detail. After singling out the $P$-wave, the amplitude $\Gamma^{(s\rightarrow 0)}_{\alpha}(P, P' ; q)$ turns into $\Gamma^{(P\rightarrow 0)}_{\mu\alpha}(P, P' ; q)$:

$$
\Gamma^{(P\rightarrow 0)}_{\mu\alpha}(P, P' ; q) = \frac{3}{p^2} \int \frac{d\Phi_2(P ; p_1, p_2)}{\rho(s)} (p_1 - p_2)_{\mu} \Gamma^{(s\rightarrow 0)}_{\alpha}(P, P' ; q), \quad (84)
$$

where $p^2 = (p_1 - p_2)^2 = 4m^2 - s$.

Therefore, the amplitude for diagrams with the $P$-wave initial state takes the form:

$$
A^{(P\rightarrow 0)}_{\alpha}(P, P' ; q) = (p_1 - p_2)_{\mu} \Gamma^{(P\rightarrow 0)}_{\mu\alpha}(P, P' ; q) \frac{g_0(s')}{1 - B_0(s')}, \quad (85)
$$

Now we can perform the expansion similar to what has been done in Section 3.1, namely:

Expansion I : $\Gamma^{(P\rightarrow 0)}_{\mu\alpha}(P, P' ; q) = g_{\mu\alpha} F_T^{(P\rightarrow 0)}(s, s', q^2) + L_{\mu\alpha} F_L^{(P\rightarrow 0)}(s, s', q^2)$,

Expansion II : $\Gamma^{(P\rightarrow 0)}_{\mu\alpha}(P, P' ; q) = -g_{\mu\alpha} F_T^{(P\rightarrow 0)}(s, s', q^2) + L_{\mu\alpha} F_L^{(P\rightarrow 0)}(s, s', q^2). \quad (86)$

The form factors $F_T^{(P\rightarrow 0)}$, $F_L^{(P\rightarrow 0)}$ and $F_T^{(P\rightarrow 0)}$, $F_L^{(P\rightarrow 0)}$ entering this expression may be found in the same way as for $S \rightarrow \gamma S$, see also Appendix C. For Expansion I we have:

$$
F_i^{(P\rightarrow 0)}(s, s', q^2) = f_i^{(P\rightarrow 0)}(s, s', q^2) + \frac{3}{p^2} \int_{4m^2}^{\infty} \frac{d\tilde{s}'}{\pi} \frac{g_0(\tilde{s}')}{\tilde{s}' - s'} \xi_i(s, \tilde{s}', q^2)d\Phi_{tr}(P, \tilde{P}' ; k_1, k_1', k_2), \quad i = T, L, \quad (87)
$$

with factors $\xi_i(s, \tilde{s}', q^2)$ determined by the formula (72). The formulae for $f_i^{(P\rightarrow 0)}(s, s', q^2)$ are given by an equation similar to (87), with the substitution $\xi_i(s, \tilde{s}', q^2) \rightarrow \tilde{\xi}_i(s, \tilde{s}', q^2)$.

In the limit $q^2 \rightarrow 0$, the amplitude (86) reads:

$$
\Gamma^{(P\rightarrow 0)}_{\mu\alpha}(s, s', q^2 \rightarrow 0) = 
\frac{4s}{(s - s')^2} q_\mu q_\alpha - \frac{2}{s - s'} \left( P_\mu q_\alpha + q_\mu P_\alpha \right) F_T^{(P\rightarrow 0)}(s, s', 0) + 
\frac{s}{(s - s')^2} q_\mu q_\alpha - \frac{1}{2(s - s')} P_\mu q_\alpha \right) F_L^{(P\rightarrow 0)}(s, s', 0) = 
\frac{4s}{(s - s')^2} q_\mu q_\alpha - \frac{2}{s - s'} P_\mu q_\alpha \right) \left( F_L^{(P\rightarrow 0)}(s, s', 0) + 4 F_T^{(P\rightarrow 0)}(s, s', 0) \right) \cdot (88)
$$

Hence the form factors at Expansions I and II are related to each other as:

$$
\tilde{F}_T^{(P\rightarrow 0)}(s, s', 0) = F_T^{(P\rightarrow 0)}(s, s', 0),
\tilde{F}_L^{(P\rightarrow 0)}(s, s', 0) = F_L^{(P\rightarrow 0)}(s, s', 0) + 4 F_T^{(P\rightarrow 0)}(s, s', 0), \quad (89)
$$
that is similar to (80). The calculation of form factors in the limit $q^2 \to 0$ is performed in Appendix B for Expansion I. We have:

\[
\text{Expansion I : } \quad F_{i}^{(P \to 0)}(s, s', 0) = \int_{s}^{s'} \frac{g_{0}(s)}{s - s'} \frac{3 \zeta_{i}(s)}{4m^2 - s}, \quad i = T, L, \quad (90)
\]

where $\zeta_{T}(s)$ and $\zeta_{L}(s)$ are given in (82). Let us emphasize that the factor $\xi_{i}(s)/(4m^2 - s)$ in (90) is analytical at $s = 4m^2$, since $\xi_{i}(4m^2) = 0$.

The amplitude for diagrams of Fig. 3c-type, with a separated $S$-wave in the final state, reads as follows:

\[
A_{\alpha}^{(1 \to S)}(P, P'; q) = (p_1 - p_2)_{\mu} \frac{g_1(s)}{1 - B_1(s)} \Gamma_{\mu \alpha}^{(1 \to S)}(s, s'; q^2), \quad (91)
\]

where $\Gamma_{\mu \alpha}^{(1 \to S)}$ is the function represented by Fig. 4c, where the separation of the $S$-wave has been carried out. For Expansion I, this function is written as follows:

\[
\text{Expansion I : } \quad \Gamma_{\mu \alpha}^{(1 \to S)}(P, P'; q) = g_{\mu \alpha} \frac{f_{T}^{(1 \to S)}(s, s', q^2) + L_{\mu \alpha} f_{L}^{(1 \to S)}(s, s', q^2)}{4m^2}, \quad (92)
\]

The calculation of $F_{i}^{(1 \to S)}$ and $\tilde{F}_{i}^{(1 \to S)}$ can be done quite similarly to a former case. As a result, we have the following expressions for the form factors:

\[
\text{Expansion I : } \quad F_{T, L}^{(1 \to S)}(s, s', q^2) = f_{T, L}^{(1 \to S)}(s, s', q^2) + \int_{4m^2}^{\infty} d\tilde{s} \frac{g_{1}(\tilde{s})}{\tilde{s} - s} \xi_{T, L}(\tilde{s}, s', q^2) d\Phi_{2}(\tilde{P}; k_1, k_2), \quad (93)
\]

In the limit $q^2 \to 0$, which is just a subject of our investigation, we have:

\[
\text{Expansion I : } \quad F_{T, L}^{(1 \to S)}(s, s', 0) = f_{T, L}^{(1 \to S)}(s, s', 0) + \frac{g_{1}(s')}{s' - s} \zeta_{i}(s'). \quad (94)
\]

For Expansion I, the final amplitude at $q^2 \to 0$ reads:

\[
A_{\alpha}^{(1 \to S)}(s, s'; 0) = (p_1 - p_2)_{\mu} \frac{g_1(s)}{1 - B_1(s)} \times \quad (95)
\]

\[
\times \left[ \left( g_{\mu \alpha} + \frac{4s}{(s - s')^2} q_{\mu} q_{\alpha} - \frac{2}{s - s'} (P_{\mu} q_{\alpha} + q_{\mu} P_{\alpha}) \right) + \frac{s}{(s - s')^2} q_{\mu} q_{\alpha} - \frac{1}{2(s - s')} P_{\mu} q_{\alpha} \right] \left( f_{T}^{(1 \to S)}(s, s', 0) + \frac{g_{1}(s')}{s' - s} \zeta_{T}(s') \right) \times \quad (95)
\]

\[
\times \left( f_{T}^{(1 \to S)}(s, s', 0) + \frac{g_{1}(s')}{s' - s} \zeta_{T}(s') \right) + \frac{1}{2(s - s')} \right] f_{L}^{(1 \to S)}(s, s', 0) + \frac{g_{1}(s')}{s' - s} \zeta_{L}(s'). \quad (95)
\]

It can be easily rewritten in the form of Expansion II:

\[
A_{\alpha}^{(1 \to S)}(s, s'; 0) = (p_1 - p_2)_{\mu} \frac{g_1(s)}{1 - B_1(s)} \times \quad (96)
\]
where

\[ 3.3 \text{ Analytical properties of the amplitude } V \to \gamma S \]

Now let us turn to the whole amplitude, which is the sum of processes shown in Fig. 3a,b,c, and investigate its analytical properties in the limit \( q^2 \to 0 \).

For the two representations of the amplitude corresponding to Expansions I and II, one has:

\[
A_{\mu \alpha}^{(\text{connected})}(s, s'; 0) = \frac{g_1(s)}{1 - B_1(s)} \times
\]

\[
\times \left[ \left( g_{\mu \alpha} + \frac{4s}{s - s'} q_{\mu q_\alpha} - \frac{2}{s - s'} P_{\mu q_\alpha} \right) T(s, s', 0) + \frac{g_0(s')}{1 - B_0(s')} = \right.
\]

\[
\times \left( g_{\mu \alpha} - \frac{2}{s - s'} q_{\mu P_\alpha} \right) T(s, s', 0)
\]

\[
+ \frac{g_1(s)}{1 - B_1(s)} \left[ \left( g_{\mu \alpha} - \frac{2}{s - s'} q_{\mu P_\alpha} \right) T(s, s', 0)
\right.
\]

\[
+ \frac{s}{(s - s')^2} q_{\mu q_\alpha} - \frac{1}{2(s - s')} P_{\mu q_\alpha} \right] \left( L(s, s', 0) + 4 T(s, s', 0) \right) \frac{g_0(s')}{1 - B_0(s')},
\]

where

\[
T(s, s', 0) = \frac{1 - B_1(s)}{g_1(s)} f_T^{(p \to 0)}(s, s', 0) + \frac{1 - B_1(s)}{g_1(s)} \frac{g_0(s) 3 \zeta_T(s)}{s - s' 4m^2 - s} + (98)
\]

\[
+ f_T^{(1 \to 0)}(s, s', 0) + \frac{B_T^{(1 \to 0)}(s)}{s - s'} +
\]

\[
+ f_T^{(1 \to S)}(s, s', 0) \frac{1 - B_0(s')}{g_0(s')} + \frac{g_1(s')}{s' - s} \zeta_T(s') \frac{1 - B_0(s')}{g_0(s')}
\]

and

\[
L(s, s', 0) = \frac{1 - B_1(s)}{g_1(s)} f_L^{(p \to 0)}(s, s', 0) + \frac{1 - B_1(s)}{g_1(s)} \frac{g_0(s) 3 \zeta_L(s)}{s - s' 4m^2 - s} + (99)
\]

\[
+ f_L^{(1 \to 0)}(s, s', 0) + \frac{B_L^{(1 \to 0)}(s)}{s - s'} +
\]

\[
+ f_L^{(1 \to S)}(s, s', 0) \frac{1 - B_0(s')}{g_0(s')} + \frac{g_1(s')}{s' - s} \zeta_L(s') \frac{1 - B_0(s')}{g_0(s')}.
\]
Looking on the last equation in (37), which corresponds to Expansion II, we conclude that the analyticity of the amplitude \( A_{\mu \alpha}^{(\text{connected})}(s, s', 0) \) requires the following ultimate expressions be fulfilled at \( s \to s' \):

\[
\left[ T(s, s', 0) \right]_{s \to s'} = 0 ,
\]

and

\[
\left[ L(s, s', 0) + 4 T(s, s', 0) \right]_{s \to s'} = 0 ,
\]

\[
\left[ L(s, s', 0) + 4 T(s, s', 0) \right]_{s \to s'} = 0 .
\]

After satisfying the requirements given by (100) and (101), the point \( s = s' \) is not singular for \( A_{\mu \alpha}^{(\text{connected})}(s, s', 0) \).

First, consider the condition (100) for the transverse amplitude \( T(s, s', 0) \). This amplitude is defined in (38), it contains pole singularities \( 1/(s - s') \), which are due to both \( A_{\mu \alpha}^{(P \to 0)}(s, s', 0) \) and \( A_{\mu \alpha}^{(1 \to S)}(s, s', 0) \), see (31). These singularities should be cancelled by similar singular points, correspondingly, in \( f_{T}^{(P \to 0)}(s, s', 0) \) and \( f_{T}^{(1 \to S)}(s, s', 0) \). Namely, at \( s \to s' \) we should deal with finite limits for:

\[
f_{T}^{(P \to 0)}(s, s', 0) + \frac{g_{0}(s)}{s - s'} \frac{3 \xi_{T}(s)}{4 m^{2} - s} \equiv l_{T}(s, s', 0) ,
\]

\[
f_{T}^{(1 \to S)}(s, s', 0) - \frac{g_{1}(s)}{s - s'} \xi_{T}(s) \equiv r_{T}(s, s', 0) .
\]

Therefore, \( l_{T}(s, s, 0) \) and \( r_{T}(s, s, 0) \) should be analytical functions of \( s \). In terms of \( l_{T}(s, s, 0) \) and \( r_{T}(s, s, 0) \), the equation (100) reads:

\[
-f_{T}^{(1 \to 0)}(s, s, 0) = \frac{1 - B_{1}(s)}{g_{1}(s)} l_{T}(s, s, 0) + F_{T}^{(1 \to 0)}(s, s, 0) + r_{T}(s, s, 0) \frac{1 - B_{0}(s)}{g_{0}(s)} .
\]

Here we use the equality

\[
F_{T}^{(1 \to 0)}(s, s', 0) = \frac{B_{T}^{(1 \to 0)}(s) - B_{T}^{(1 \to 0)}(s')}{s - s'} ,
\]

that results in \( F_{T}^{(1 \to 0)}(s, s, 0) = dB_{T}^{(1 \to 0)}(s)/ds \). The freedom in choosing subtraction terms makes (100) being fulfilled. Assuming that transition form factor of composite systems is defined by double spectral integral only, the following requirement should be imposed:

\[
f_{T}^{(1 \to 0)}(M_{1}^{2}, M_{0}^{2}, q^{2}) = 0 ,
\]

in particular, at \( q^{2} = 0 \): \( f_{T}^{(1 \to 0)}(M_{1}^{2}, M_{0}^{2}, 0) = 0 \). We see that the requirement (103) does not contradict the amplitude analyticity constraint given by (102).

Likewise, using the freedom for the choice of subtraction functions,

\[
f_{L}^{(P \to 0)}(s, s', 0) , \quad f_{L}^{(1 \to 0)}(s, s', 0) , \quad f_{L}^{(1 \to S)}(s, s', 0) ,
\]

one can satisfy the analyticity constraints given by (101); still, we would not present explicitly these constraints, for they are rather cumbersome and do not teach us of a something new.
4 Quark model

To rewrite formulae of Sections 2 and 3 for composite fermion–antifermion systems does not provide problems. For such case one needs to introduce spin variables and substitute vertices of scalar (pseudoscalar) constituents by fermion ones:

\[
\begin{align*}
    g_0 & \rightarrow (\bar{u} u) g_0, \\
    p_\perp g_1 & \rightarrow (\bar{u} \gamma_\mu u) g_1,
\end{align*}
\]

(108)

where \( u \) is the four-spinor. Then the consideration of fermion–antifermion composite system \( f \bar{f} \) remains in principle the same as for scalar (pseudoscalar) constituents. Namely, one should consider the \( (f \bar{f}) \) scattering amplitude of Fig. 1, and the pole of the amplitude \( f \bar{f} \rightarrow f \bar{f} \) determines the bound state of the \( f \bar{f} \)-system. Its form factor is defined by triangle graph of Fig. 2c, which is a residue in the amplitude poles of the transition \( (f \bar{f})_{in} \rightarrow \gamma + (f \bar{f})_{out} \), Fig. 2b. Triangle diagram shown separately in Fig. 4a determines form factor of composite particle.

Still, quarks do not leave the confinement trap as free particles do, so one cannot use for quarks the above-described scheme in a full scale.

The logic of the quark model tells us that we may treat constituent quarks inside the confinement region as free particles, and the region, where they are "allowed" to be free, is determined by quark wave function. This means that, within quark model, one can calculate the three-point form factor amplitudes, which refer to the interaction of photon in the intermediate state: these are the diagrams of Fig. 4a-type. The diagrams with photon interacting with incoming/outgoing particle (Fig. 4b,c-type) should be treated using hadronic language. So we come to a combined approach, where incoming/outgoing particles in the processes of Fig. 3 are mesons, while constituents of the triangle diagram are quarks. It is obvious that in such a combined approach to the amplitude the relations [100] and [101] imposed by analyticity are kept; they may be satisfied, without any problem, by the constraints similar to [104] and [106]. Triangle diagrams determined as residues in amplitude poles, see Fig. 2b, stand for quark form factors, which are obtained in accordance with gauge invariance and analyticity constraints.

Now, to illustrate the above statements, let us repeat the main items of the method of singling out the quark form factor of the transition \( \phi(1020) \rightarrow \gamma f_0(980) \) from hadronic processes. We cannot use directly the process \( (q\bar{q})_{incoming} \rightarrow \gamma(q\bar{q})_{outgoing} \) for the definition of quark form factor: being rigorous we are not allowed to treat quarks as free particles at large distances. In this way, the reaction of the type \( K \bar{K} \rightarrow \gamma \pi \pi \) is to be considered; then the amplitude \( \phi(1020) \rightarrow \gamma f_0(980) \) should be extracted as a double residue in the amplitude poles of \( K \bar{K} \rightarrow \gamma \pi \pi \). The amplitude itself, \( \phi(1020) \rightarrow \gamma f_0(980) \), may be considered in terms of constituent quarks, namely, as triangle diagram with constituent quarks.

The form factor amplitude for radiative transition of vector meson to scalar one, of the type of \( \phi(1020) \rightarrow \gamma f_0(980) \), was calculated in terms of constituent quarks [7, 9]. The spectral integral for \( F_T^{(1\rightarrow 0)}(M_V^2, M_S^2, 0) \) was given by Eq. (32) of [9], it was denoted there as \( A_{V \rightarrow \gamma S}(0) \).
Up to the charge factor $Z_{V\to\gamma S}$, which we omit here, it reads:

$$F_T^{(1\to0)}(M_V^2, M_S^2, 0) = \int_{4m^2}^{\infty} \frac{ds}{\pi} \Psi_V(s)\Psi_S(s) \left[ \frac{m}{4\pi} \sqrt{s(s - 4m^2)} - \frac{m^3}{2\pi} \ln \frac{\sqrt{s + \sqrt{s - 4m^2}}}{\sqrt{s - \sqrt{s - 4m^2}}} \right].$$  \hspace{1cm} (109)

The quark wave functions for vector and scalar states, $\Psi_V(s)$ and $\Psi_S(s)$, are normalized as follows:

$$\int_{4m^2}^{\infty} \frac{ds}{\pi} \Psi_V^2(s) \frac{s + 2m^2}{12\pi} \sqrt{s - 4m^2} = 1, \quad \int_{4m^2}^{\infty} \frac{ds}{\pi} \Psi_S^2(s) \frac{s - 4m^2}{8\pi} \sqrt{s - 4m^2} = 1.$$ \hspace{1cm} (110)

Masses of light constituent quarks $u$ and $d$ are of the order of 350 MeV, and the strange-quark mass $\sim 500$ MeV.

The function in square brackets of the integrand (109) is positive at $s > 4m^2$, so the transition form factor $F_T^{(1\to0)}(M_V^2, M_S^2, 0) \neq 0$ at arbitrary $M_V$ and $M_S$, $M_V = M_S$ included, if the wave functions $\Psi_V(s)$ and $\Psi_S(s)$ do not change sign, and just this feature (absence of zeros in radial wave function) is a signature of basic states with the radial quantum number $n = 1$.

One point needs special discussion, that is, the possibility to apply spectral-integration technique, with mass-on-shell intermediate states, to the calculation of quark diagrams of Fig. 4a. The fact that quarks–constituents do not fly off at large distances does not restrain directly the calculation technique: calculations may be performed with mass-on-shell particles in the intermediate states as well as mass-off-shell ones, like Feynman integrals. Recall that in non-relativistic quantum mechanics, that is, in non-relativistic quark model, the particles in the intermediate states are mass-on-shell, that does not prevent the use of confinement models. The flying-off of quarks at large distances corresponds to threshold singularities of the amplitude: in the triangle diagram of Fig. 4a they are at $s = 4m^2$ and $s' = 4m^2$. The suppression of contribution from large distances means the suppression in the momentum space from the regions $s \sim 4m^2$ and $s' \sim 4m^2$. Such a suppression is implemented by the properties of vertices, or wave functions, of composite systems. As concern the threshold singularities, they are present in other techniques too, like Feynman or light-cone ones. Therefore, in all representations one should suppress the contributions from the regions $s \sim 4m^2$ and $s' \sim 4m^2$. The spectral-integration technique provides us a possibility to control the near-threshold contributions.

**Transition $^3S_1q\bar{q} \to ^3P_0q\bar{q}$ in the non-relativistic approach**

In the non-relativistic limit, formula (109) turns into standard expression of the quark model transition $^3S_1 \to ^3P_0$, and non-relativistic amplitude is obtained by expanding the expression in square brackets in a series in respect to relative quark momentum squared $k^2$, where $k^2 = s/4 - m^2$. Here we present the form factor $F_T^{(1\to0)}(M_V^2, M_S^2, 0)$ in the non-relativistic approach: this very transition is responsible for the decays $\phi(1020) \to \gamma f_0(980)$ and $\phi(1020) \to \gamma a_0(980)$, if $f_0(980)$ and $a_0(980)$ are the $q\bar{q}$ states.
In the non-relativistic approximation, after re-definition $\Psi_V(s) \rightarrow \psi_V(k^2)$ and $\Psi_S(s) \rightarrow \psi_S(k^2)$, Eq. (109) turns into:

$$F_T^{(1 \rightarrow 0)}(M_V^2, M_S^2, 0) \rightarrow \left[ F_T^{(1 \rightarrow 0)}(M_V^2, M_S^2, 0) \right]_{\text{non-relativistic}} = \int_0^\infty \frac{d k^2}{\pi} \psi_V(k^2) \psi_S(k^2) \frac{8}{3 \pi} k^3 , \quad (111)$$

Normalization conditions for wave functions $\psi_V(k^2)$ and $\psi_S(k^2)$ should be also re-written in the non-relativistic limit, they read:

$$1 = \int_0^\infty \frac{d k^2}{\pi} \psi_S^2(k^2) \frac{2 k^3}{\pi m} , \quad 1 = \int_0^\infty \frac{d k^2}{\pi} \psi_V^2(k^2) \frac{m k}{2 \pi} . \quad (112)$$

For exponential parametrization of wave functions,

$$\psi_S(k^2) = N_S e^{-b_S k^2} , \quad \psi_V(k^2) = N_V e^{-b_V k^2} , \quad (113)$$

one can easily calculate integrals (111) and (112). Normalization constants are determined as

$$N_S^2 = \frac{8 \sqrt{2}}{3} \pi^{3/2} m b_S^{5/2} , \quad N_V^2 = 2 \sqrt{2} \pi^{3/2} \frac{1}{m} b_S^{3/2} , \quad (114)$$

and transition form factor is equal to:

$$\left[ F_T^{(1 \rightarrow 0)}(M_V^2, M_S^2, 0) \right]_{\text{non-relativistic}} = 8 \sqrt{\frac{2}{3}} \frac{b_V^{3/4} b_S^{5/4}}{(b_V + b_S)^{5/2}} . \quad (115)$$

At $b_V \simeq b_S = b$, one has

$$\left[ F_T^{(1 \rightarrow 0)}(M_V^2, M_S^2, 0) \right]_{\text{non-relativistic}} \simeq \frac{2}{\sqrt{3b}} . \quad (116)$$

For loosely bound systems $1/b \sim \sqrt{m \epsilon}$, where $\epsilon$ is the binding energy, so the right-hand side of Eq. (116) contains the suppression factor inherent in the E1-transition.

In Fig. 5 we demonstrate the calculated form factors $F_T^{(1 \rightarrow 0)}$ for the transition $\phi(1020) \rightarrow \gamma f_0(980)$ both for non-relativistic approximation, Eq. (111), and relativistic spectral integrals, Eq. (109), with the $n\bar{n}$ and $s\bar{s}$ components. We use $b_V = 10 \text{ GeV}^{-2}$ that corresponds to the $\phi$-meson radius of the order of pion radius, and for the $f_0$-meson we change the wave function slope within the limits $2 \text{ GeV}^{-2} < b_S < 12 \text{ GeV}^{-2}$, that means the change of the radius squared in the interval $0.5 R_p^2 < R_f^2 < 2 R_p^2$, for more detail see [7, 9].

It is seen that form factors calculated in both relativistic and non-relativistic approaches do not differ significantly one from another, that makes puzzling the statement about impossibility of the quark-model description of the reactions $\phi(1020) \rightarrow \gamma f_0(980)$ and $\phi(1020) \rightarrow \gamma a_0(980)$ under the assumption about $f_0(980)$ and $a_0(980)$ being $q\bar{q}$ states: recall that the data can be indeed described [7, 9] by using spectral-integral formula (109).
5 Conclusion

The use of dispersion technique for the calculation of form factors of composite system has certain advantages, of which we would underline two.

(i) In the dispersion technique, or in the spectral integration technique, the content of bound state is controlled. The interaction of constituents with each other due to meson exchanges does not lead to the appearance of new components in the bound state related to these mesons.

(ii) The dispersion technique, as well as spectral integration technique, works with the energy-off-shell amplitudes, and the particles in the intermediate states are mass-on-shell. This provides us an easy possibility to construct spin operators, that in its turn allows one to consider without problems the amplitudes for composite particles with a large spin (see [12] for more detail).

However, as it happens often, the advantages make it necessary to take special care about other aspects of the approach: in the spectral integration technique, when radiative processes are considered, one should impose "by hand" the constraints related to gauge invariance and analyticity. In [6], this problem was considered in connection with charge form factors, for example, for the transitions of (S → γS)-type, where S is scalar meson. However, more complicated radiative processes were investigated later on [3], such as V → γS (V is vector meson) and P → γγ, S → γγ, T → γγ (P and T are pseudoscalar and tensor mesons). In these cases the spin structure of amplitude is more complicated, hence more complicated constraints for amplitudes are needed, that is related to gauge-invariant operators which perform the moment-operator expansion. Although in principle the reconstruction of analyticity and transition form factors is analogous to that used in [6], the statements of Ref. [13] concerning transition form factors need certain comment.

In the present paper, we have carried out the study of constraints owing to gauge invariance and analyticity using as an example the process V → γS. As the first step, we accept the mesons V and S to be two-particle composite systems of scalar or pseudoscalar particles–constituents (Section 3). Such an approach makes calculations simple, less cumbersome, without affecting basic points. This approach can be generalized for fermion-antifermion system without principal complications, and the necessary changes are related only to the phase space (spin factors should be included) and the form of vertices. In this way, one can generalize the present results for quark-antiquark systems (Section 4), i.e. one can apply the method to the consideration of radiative decays of q̅q-mesons too.

Concerning the transition V → γS, we demonstrate that two independent operators are responsible for the spin structure of this reaction, $g^{±}_μα$ and $L_μα$, given by (12) and (13). The operators $g^{±}_μα$ and $L_μα$ determine transverse and longitudinal amplitudes. In the spectral integration technique, the essential point is the use of operators, which are responsible for total spin space, that is, two orthogonal operators $g^{±}_μα$ and $L_μα$, with $g^{±}_μα L_μα = 0$. At $q^2 = 0$ the longitudinal operator turns into a nilpotent one, $L_μα(0)L_μα(0) = 0$. 

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Also, we demonstrate that meson-decay form factors are determined by residues in the poles of scattering blocks. For example, the form factor for the decay $\phi(1020) \to \gamma f_0(980)$ is given by the residue of the amplitude of the reaction $\phi(1020) \to \gamma \pi \pi$, so analytical properties of these two amplitudes are different. In particular, the amplitude of the transition $\phi(1020) \to \gamma \pi \pi$ should be zero at $(M_\phi - M_{\pi \pi}) \to 0$, while for the transition $\phi(1020) \to \gamma f_0(980)$ an analogous requirement is absent for $M_\phi = M_{f_0}$. Moreover, if $\phi(1020)$ and $f_0(980)$ are members of the basic $q \bar{q}$ nonets, $1^3S_1$ and $1^3P_0$, the transition amplitude cannot be zero at any meson masses, including $M_\phi = M_{f_0}$. An opposite statement is declared in [13].

Present investigation does not confirm the use of a unique form of the spin operator in $S \to \gamma V$ declared in [13]: generally, at $q^2 \neq 0$, two independent spin operators exist related to the transverse and longitudinal amplitudes, $g_{\perp \perp}^\mu$ and $L_\mu$. At $q^2 \to 0$ one operator, $L_\mu(0)$, turns into nilpotent one, thus giving us a freedom in writing spin operator for the transition with real photon.

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Appendix A: Coefficients $\xi_T(s, s', q^2)$ and $\xi_L(s, s', q^2)$

Let us calculate coefficients in the expansion (72) for spin factor $S_\mu$. To this aim we need the convolutions as follows:

\begin{align*}
g_{\mu}(k_1 - k_2)_{\mu}(k_1 + k'_1)_{\alpha} &= 4m^2 - \frac{1}{2}(s + s' + q^2), \\
P_{\mu\alpha}(k_1 - k_2)_{\mu}(k_1 + k'_1)_{\alpha} &= 0, \\
q_{\mu\alpha}(k_1 - k_2)_{\mu}(k_1 + k'_1)_{\alpha} &= 0, \\
P_{\mu\alpha}(k_1 - k_2)_{\mu}(k_1 + k'_1)_{\alpha} &= 0, \\
q_{\mu\alpha}(k_1 - k_2)_{\mu}(k_1 + k'_1)_{\alpha} &= \frac{1}{4}(s + s' - q^2)(-s + s' + q^2).
\end{align*}

(117)

Projecting $S_\mu$ on the operators $g_{\mu\alpha}^{\perp \perp}$ and $L_{\mu\alpha}$, one obtains:

\begin{align*}
\xi_T(s, s', q^2) \left(g_{\mu\alpha}^{\perp \perp}\right)^2 &= 4 \left(m^2 + \frac{ss'q^2}{\lambda(s, s, q^2)}\right), \\
\xi_L(s, s', q^2) \left(L_{\mu\alpha}\right)^2 &= -\frac{q^2s}{2\lambda(s, s', q^2)}\frac{(s + s' - q^2)(-s + s' + q^2)}{(s - s' + q^2)}. \\
\end{align*}

(118)

As a result, we have the coefficients given in (72). Taking into account that $g_{\mu\alpha}^{\perp \perp} = \tilde{g}_{\mu\alpha}^{\perp \perp} + 4L_{\mu\alpha}$, we obtain coefficients for Expansion II, $\tilde{\xi}_T(s, s', q^2)$ and $\tilde{\xi}_L(s, s', q^2)$.
Appendix B: Form factors in the limit \( q^2 \to 0 \)

Form factors of the three-point diagram of Fig. 4a

Let us calculate form factors in the limit \( q^2 \to 0 \) for the diagram of Fig. 4a for two variants of the expansion given by (76).

**Expansion I:** To get the formula (77) for form factors in the limit \( q^2 \to 0 \) let us insert new variables: \( \sigma, \Delta \) and \( Q^2 \), see (34), and integrate over phase space, that leads at small \( Q^2 \) to

\[
F_i^{(1\to 0)}(s, s', q^2 \to 0) = f_i^{(1\to 0)}(s, s', q^2 \to 0) + \int_{4m^2}^{\infty} \frac{d\sigma}{\pi} \frac{g_1(\sigma)g_0(\sigma)}{(\sigma-s)(\sigma-s')} \int_b^\infty d\Delta \frac{\xi_i(\sigma, \Delta, Q^2)}{16\pi \sqrt{\Delta^2 + 4\sigma Q^2}},
\]

where \( b \) is determined in (36) and

\[
\xi_T(\sigma, \Delta, Q^2) = 2 \left( m^2 - \frac{Q^2\sigma^2}{\Delta^2 + 4\sigma Q^2} \right),
\]

\[
\xi_L(\sigma, \Delta, Q^2) = 4\sigma \left( \frac{\Delta^2}{\Delta^2 + 4\sigma Q^2} \right).
\]

Integrating over \( \Delta \), we obtain final formula for form factors in Expansion I, see (81) and (82).

**Expansion II:** Likewise, the same procedure is carried out for Expansion II, though with other coefficients defining double discontinuity of the form factor:

\[
\tilde{\xi}_T(\sigma, \Delta, Q^2) = \xi_T(\sigma, \Delta, Q^2),
\]

\[
\tilde{\xi}_L(\sigma, \Delta, Q^2) = 4 \left( 2m^2 + \frac{\sigma(\Delta^2 - 2\sigma Q^2)}{\Delta^2 + 4\sigma Q^2} \right).
\]

As a result, we obtain: \( \tilde{F}_T^{(1\to 0)}(s, s', q^2) = F_T^{(1\to 0)}(s, s', 0) \) and

\[
\tilde{F}_L^{(1\to 0)}(s, s', q^2) = \tilde{f}_L^{(1\to 0)}(s, s', 0) + \frac{\tilde{B}_L^{(1\to 0)}(s) - \tilde{B}_L^{(1\to 0)}(s')}{s-s'},
\]

\[
\tilde{B}_L^{(1\to 0)}(s) = \int_{4m^2}^{\infty} \frac{d\tilde{s}}{\pi} \frac{g_1(\tilde{s})g_0(\tilde{s})}{\tilde{s}-s} \rho(\tilde{s}) \tilde{\zeta}_L(\tilde{s}),
\]

\[
\tilde{\zeta}_L(s) = 4 \left( 2m^2 + s \right) \sqrt{\frac{s}{4m^2-s}} \ln \frac{1 + \sqrt{(s-4m^2)/s}}{1 - \sqrt{(s-4m^2)/s}} - 3s.
\]
Two-point diagram of Fig. 4b

For the diagram of Fig. 4b, the form factor calculations in the limit \( q^2 \to 0 \), see (87), is carried out as before, by using the variables \( \sigma, \Delta \) and \( Q^2 \) determined in (34):

\[
F_i^{(P \to 0)}(s, s', q^2 \to 0) = f_i^{(P \to 0)}(s, s', q^2 \to 0) + \frac{3}{p^2} \int \frac{d\sigma}{4m^2} \frac{g_0(\sigma)}{\pi} \frac{\sigma - s'}{\sigma - s} \delta(\sigma - s) \int d\Delta \frac{\xi_i(\sigma, \Delta, Q^2 \to 0)}{16\pi \sqrt{\Delta^2 + 4\sigma Q^2}}.
\]

(123)

where \( \xi_i(s, s', q^2) \) are given by (120). Performing calculations analogous to the above ones, we obtain (90).

Appendix C: Separation of the \( P \)-wave in the initial state of the process of Fig. 4b

The Feynman diagram shown in Fig. 4b reads:

\[
\Gamma^{(- \to 0)} = (p_1 + k_1')_\alpha \frac{g_0(P^{\prime 2}; k_1'^2, k_2^2)}{m^2 - k_1'^2}.
\]

(124)

The amplitude \( \Gamma^{(- \to 0)} \) can be expanded in a series with respect to orbital momenta of the initial states as follows:

\[
\Gamma^{(- \to 0)} = \Gamma^{(S \to 0)} + (p_1 - p_2)_\mu \Gamma^{(P \to 0)}_{\mu \alpha} + X^{(2)}_{\mu_1 \mu_2}(p) \Gamma^{(D \to 0)}_{\mu_1 \mu_2 \alpha} + \cdots
\]

(125)

There is summation over \( \mu \) and \( \mu_1 \mu_2 \). To find out \( \Gamma^{(P \to 0)}_{\mu \alpha} \), one should multiply \( \Gamma^{(- \to 0)}_\alpha \) by

\[
(p_1 - p_2)_\mu \equiv p_\mu
\]

(126)

and integrate over \( d\Omega/(4\pi) \) or \( \int d\Phi_2(P; p_1, p_2)/\rho(s) \). The right-hand side of (126) gives us:

\[
\int \frac{d\Phi_2(P; p_1, p_2)}{\rho(s)} (p_1 - p_2)_\mu (p_1 - p_2)_\mu \Gamma^{(P \to 0)}_{\mu \alpha} = \frac{p^2}{3} \Gamma^{(P \to 0)}_{\mu \alpha},
\]

(127)

for one can replace in the integrand of (127):

\[
(p_1 - p_2)_\mu (p_1 - p_2)_\mu' \to q_{\mu \mu'} \frac{p^2}{3}, \quad p^2 = 4m^2 - s.
\]

(128)

The left-hand side of (125) with the account for (124) gives us:

\[
\int \frac{d\Phi_2(P; k_1, k_2)}{\rho(P^2)} (k_1 - k_2)_\mu (k_1 + k_1')_\alpha \frac{g_0(P^{\prime 2}; k_1'^2, k_2^2)}{m^2 - k_1'^2}.
\]

(129)
In the integrand we re-denoted the momenta $p_1$ and $p_2$ as $k_1$ and $k_2$. To get the spectral integral, let us calculate the discontinuity; to this aim we consider intermediate state as mass-on-shell:

$$(m^2 - k_1'^2)^{-1} \longrightarrow \theta(k_{10}')\delta(m^2 - k_1'^2) ,$$

(130)

and substitute the vertex function as follows:

$$g_0(\tilde{P}^2; k_1'^2, k_2^2) \longrightarrow g_0(\tilde{s}') .$$

(131)

Then we expand the spin factor of the amplitude:

$$(k_1 - k_2)_{\mu}(k_1 + k_1')_{\alpha} = g^{\perp \perp}_{\mu\alpha} \xi_T(s, \tilde{s}', q^2) + L_{\mu\alpha} \xi_L(s, \tilde{s}', q^2) ,$$

(132)

with the coefficients $\xi_T$ and $\xi_L$ given in (72). As a result, the discontinuity in the $s'$-channel reads:

$$\text{disc}_{s'} F_{T,L}^{(P \rightarrow 0)}(s, s', q^2) = \frac{1}{\rho(s)} \xi_{T,L}(s, \tilde{s}', q^2) g_0(\tilde{s}') d\Phi_{tr}(P', P; k_1, k_1', k_2) .$$

(133)

So we have the following dispersion representation for the pole diagram of Fig. 4b:

$$F_{T,L}^{(P \rightarrow 0)}(s, s', q^2) = f_{T,L}^{(P \rightarrow 0)}(s, s', q^2) +$$

$$+ \frac{3}{p^2} \frac{1}{\rho(s)} \int_{4m^2}^{\infty} \frac{d\tilde{s}'}{\pi} \frac{g_0(\tilde{s}')}{\tilde{s}' - s'} \xi_{T,L}(s, \tilde{s}', q^2) d\Phi_{tr}(P, P'; k_1, k_1', k_2) ,$$

(134)

that gives [87].

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Figure 1:  a) Representation of partial scattering amplitude as a set of the dispersion loop diagrams. b) Partial amplitude at the energies close to the mass of a bound state: the main contribution is given by the pole at $s = M^2$. 
Figure 2: Diagrams determining form factor of the composite system: a) the photon emission by interacting constituents; b) the diagram of Fig. 2a near pole corresponding to bound states; c) form factor of bound state determined as a residue of the amplitude of Fig. 2b in the poles at \( s = M^2 \) and \( s' = M'^2 \).
Figure 3: a) The process shown diagrammatically in Fig. 2a in terms of the dispersion loop diagrams; b,c) diagrams with the photon interaction in the initial and final states; d) emission of photon by non-interacting constituents.
Figure 4: a) Three-point form factor amplitude. The blocks for the emission of photon in the initial (b) and final (c) states.
Figure 5: Form factor $F_{T}^{1-0}(M_{V}^2, M_{S}^2)$ for the decay $\phi(1020) \rightarrow \gamma f_{0}(980)$ calculated in relativistic (for $n\bar{n}$ and $s\bar{s}$ components) and non-relativistic approaches: solid and dashed lines.