Theoretical Distributions of Short-lived Radionuclides for Star Formation in Molecular Clouds

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Abstract

Short-lived radioactive nuclei (half-life $\tau_{1/2} \sim 1$ Myr) influence the formation of stars and planetary systems by providing sources of heating and ionization. Whereas many previous studies have focused on the possible nuclear enrichment of our own solar system, the goal of this paper is to estimate the distributions of short-lived radionuclides (SLRs) for the entire population of stars forming within a molecular cloud. Here we focus on the nuclear species $^{60}$Fe and $^{26}$Al, which have the largest impact due to their relatively high abundances. We construct molecular-cloud models and include nuclear contributions from both supernovae and stellar winds. The resulting distributions of SLRs are time dependent with widths of $\sim$3 orders of magnitude and mass fractions $\rho_{\text{SLR}}/\rho_*$ $\sim 10^{-11} - 10^{-8}$. Over the range of scenarios explored herein, the SLR distributions show only modest variations with the choice of cloud structure (fractal dimension), star formation history, and cluster distribution. The most important variation arises from the diffusion length scale for the transport of SLRs within the cloud. The expected SLR distributions are wide enough to include values inferred for the abundances in our solar system, although most of the stars are predicted to have smaller enrichment levels. In addition, the ratio of $^{60}$Fe/$^{26}$Al is predicted to be greater than unity, on average, in contrast to solar system results. One explanation for this finding is the presence of an additional source for the $^{26}$Al isotope.

Unified Astronomy Thesaurus concepts: Star forming regions (1565); Star formation (1569); Interstellar medium (847); Giant molecular clouds (653)

1. Introduction

Star formation environments can influence their constituent stars and planetary systems through a variety of channels, including dynamical interactions and background radiation fields, as well as exposure to high-energy particles and radioactive nuclei (see, e.g., Adams 2010; Pfalzner et al. 2015; Parker 2020). Most stars form within embedded clusters (Lada & Lada 2003; Porras et al. 2003), and the immediate cluster properties largely determine the degree of dynamical disruption (Adams et al. 2006; Malmberg et al. 2007; Portegies Zwart 2009; Pfalzner 2013) and the intensity of the background radiation (Fatuzzo & Adams 2008; Lee & Hopkins 2020; Parker et al. 2021). The particle contribution includes both high-energy cosmic radiation and short-lived radioactive nuclei, which are produced via supernovae (SNe) and stellar winds (Vasileiadis et al. 2013; Adams et al. 2014; Lichtenberg et al. 2016; Kuffmeier et al. 2016; Nicholson & Parker 2017; Kaur & Sahijpal 2019). The accelerated particles can propagate beyond the immediate cluster environment and thereby influence additional star-forming regions within the molecular cloud. This paper focuses on the large-scale distribution of the short-lived radionuclides (SLRs), which affect star and planet formation through several mechanisms (as outlined below). The goal is to determine the distribution of enrichment levels for SLRs over the entire population of stars and planetary systems forming within the molecular cloud.

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others), spallation in the interstellar medium (Desch et al. 2010), local protostellar spallation (Shu et al. 1997; Lee et al. 1998), and distributed enrichment from many different SNe within the molecular cloud that formed the Sun (Gounelle & Meynet 2012; Gounelle 2015).

Much of the aforementioned work has primarily focused on finding an explanation for the degree of radioactive enrichment of our solar system. Because many different nuclei are observed via meteoritic analysis (including $^{10}\text{Be}$, $^{26}\text{Al}$, $^{36}\text{Cl}$, $^{41}\text{Ca}$, $^{60}\text{Fe}$, $^{53}\text{Mn}$, $^{107}\text{Pd}$, and $^{129}\text{I}$), the required ensemble of abundance SLR ratios places tight constraints on enrichment scenarios. For completeness, we note that the same SNe that provide nuclear enrichment could also trigger the star formation event that led to the solar system (Boss 2017).

On the other hand, it remains possible for the solar system SLR abundances to be closer to those expected for star formation in clouds (see the discussion of Jura et al. 2013, and Young 2014, 2016, 2020, and references therein). If the galactic supply of $^{26}\text{Al}$ is distributed only within molecular clouds, rather than throughout all of the gas in the interstellar medium, then the galaxy-wide abundance ratio becomes $^{26}\text{Al}/^{27}\text{Al} \sim 2-3 \times 10^{-5}$, roughly comparable to solar system values. In addition, white dwarf stars that are polluted by rocky bodies show evidence that the impactors are differentiated, which implies heating at levels comparable to those expected from (inferred) solar system abundances of $^{26}\text{Al}$. As a result, the observed solar system SLRs may not be anomalous.

Independent of the SLR inventory of our solar system, this paper takes a wider view and seeks to estimate the distribution of radioactive-enrichment levels for the entire population of forming stars and planets within a molecular cloud. For purposes of heating and ionization, the particular nuclei $^{60}\text{Fe}$ and $^{26}\text{Al}$ provide the dominant contribution and are the focus of this work (although we also briefly discuss $^{36}\text{Cl}$ and $^{41}\text{Ca}$). Both of these principal SLRs are synthesized in SNe, as considered here. In addition, stellar winds from evolved stars provide an additional source of $^{26}\text{Al}$, which is also included. We ignore spallation$^4$ and other local sources (although their contributions can be considered separately).

This work is the analog of a previous paper (Adams et al. 2014), where we estimated the distributions of radioactive abundances for the scenario where circumstellar disks are enriched by SNe exploding within their birth clusters (see related work by Lichtenberg et al. 2016 and Nicholson & Parker 2017). In this paper, we estimate the corresponding distributions for the radioactive-enrichment scenario where the SLRs propagate through the entire star-forming region. Instead of focusing on specific cases that account for the full range of isotopes inferred for our solar system (e.g., Gounelle & Meynet 2012; Gounelle 2015), the goal of this work is to construct the distributions of $^{26}\text{Al}$ and $^{60}\text{Fe}$ for all stars forming in the cloud. These distributions can then be used to assess the degree of radioactive heating, ionization, and other processes that are present during planet formation across the entire stellar population. Previous work along these lines has been carried out using detailed numerical simulations of molecular clouds (Vasileiadis et al. 2013; Kuffmeier et al. 2016). These numerical studies provide distributions of the SLRs as a function of time for the specific scenario under consideration. This present work—which is complementary to these earlier studies—considers a parametric model, which allows for the exploration of a wider range of input parameters, albeit within the restricted geometries and star formation prescriptions of the model. We thus determine how the distributions of SLR abundances vary with the time and spatial dependence of star formation, the fractal dimension of the molecular cloud, cluster properties, propagation efficiencies, and other inputs. Although this paper focuses on nuclear enrichment on the scales of molecular clouds, we note that additional studies have also considered enrichment on galactic scales (e.g., Fujimoto et al. 2018; Côte et al. 2019; Kaur & Sahijpal 2019). For example, this latter study estimates the time evolution of the galactic-scale abundances of SLRs and also constructs a working scenario to explain the observed isotopic abundances in our solar system.

Although the goal of this paper is to determine the expected distributions of SLRs for the general population of stars forming in molecular clouds, the observed abundances inferred for our solar system provide a useful reference point. For the isotope $^{26}\text{Al}$, meteorite studies imply a mass fraction $X_{26} \sim 4 \times 10^{-9}$ (starting with Cameron & Truran 1977). For the case of $^{60}\text{Fe}$, however, different studies find a wide range of abundances. The largest estimates are $X_{60} \sim 10^{-9}$ with many measurements indicating much smaller values (compare Tachibana et al. 2006, Moynier et al. 2011, Telus et al. 2012, Mishra & Goswami 2014, and Trappitsch et al. 2018). In this paper, when we compare with solar system values for the SLRs, we use the aforementioned $^{26}\text{Al}$ mass fraction and the high end of the inferred $^{60}\text{Fe}$ mass fraction ($10^{-9}$) for reference values. One should keep in mind that the latter mass fraction is highly uncertain.

This paper is organized as follows. Section 2 formulates our model for the nuclear enrichment of molecular clouds. We must specify the cloud structure, distribution of star clusters, star formation history, stellar initial mass function (IMF), nuclear yields, and the mechanism for the propagation of radioactive material through the cloud. The results for our benchmark model are then presented in terms of the distributions of enrichment levels (mass fractions of the SLRs) as a function of time. Section 3 explores how the resulting distributions vary with the chosen input parameters, including cloud structure, star formation history, and the propagation of radioactive nuclei. Although the distributions are wide, with nuclear abundances ranging over several orders of magnitude, the distributions themselves are relatively robust. The paper concludes in Section 4 with a summary of our results and a discussion of their implications.

### 2. Baseline Model

#### 2.1. Stellar Distribution in the Molecular-cloud Environment

Giant molecular clouds (GMCs) are complex and varied structures (e.g., Dobbs & Pringle 2013) with evolutionary histories that remain under investigation. Given the current uncertainties concerning the physical characteristics of these environments, we first develop a baseline model that uses standard assumptions about the GMC stellar population and the SLR transport mechanisms. We then consider several variations to our baseline model that explore other viable scenarios in order to gauge the sensitivity of our results to the physical input parameters.

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$^4$ Note that spallation can produce $^{26}\text{Al}$ but not $^{60}\text{Fe}$. 
In all cases, we embed a GMC within a well-defined cuboid structure with a 50 pc body diagonal (e.g., Heyer & Dame 2015). The cloud is assumed to contain $10^6 M_\odot$ of gas out of which $N^*_{\text{new}} = 50,000$ stars form. The three physical dimensions of the cuboid are generated by randomly drawing numbers $w_i$ between $e^{0.5}$ and $e^2$ (where $i = \{x, y, z\}$), and assigning $l_i = \ln w_i$ as relative lengths that are then normalized in accordance to the body diagonal. The resulting GMC volumes for this scheme, which range from 6550 pc$^3$ to 24,100 pc$^3$, have an expectation value $\langle V_{\text{GMC}} \rangle \approx 20,000$ pc$^3$. We note that a sphere with the same volume would have a radius of $r \approx$ 17 pc.

In our baseline model, the formation of stars occurs within clusters whose stellar memberships $N$ span from $10^2$ to $10^4$ in accordance to the distribution function (e.g., Lada & Lada 2003):

$$P_c(N) \propto N^{-2}. \quad (1)$$

Clusters are drawn from this distribution function until their collective stellar content reaches $N^*_{\text{tot}}$, with the last drawn cluster’s membership reduced as needed to achieve that outcome. Each cluster is assigned a radius through the empirical formula (Adams 2010)

$$R_e = 1 \text{ pc} \left(\frac{N}{300}\right)^{1/3}. \quad (2)$$

For our set assumptions, the mean cluster membership is $\langle N \rangle = 465$, so that a GMC is expected to have $\sim 10^3$ clusters with radii $R_e = 0.7$–3.2 pc. We note, however, that the cumulative probability for finding stars in clusters with membership size $N$ or smaller is given by the expression

$$P_s(N) = \frac{\ln[N/N_{\text{min}}]}{\ln[N_{\text{max}}/N_{\text{min}}]}, \quad (3)$$

indicating that stars populate clusters in a logarithmic sense. In other words, stars can be found with roughly equal probability in each decade of cluster membership size $N$. For our assumed cluster distribution, half of the GMC stars are expected to belong to clusters with membership $N \leq 10^3$.

Following the fractal generating scheme of Goodwin & Whitworth (2004), clusters are randomly placed within a cuboid GMC region with spacial fractal dimension $d$ (with $d = 2$ assumed for our baseline model). The main GMC structure is evenly divided into eight substructures, the centers of which are seeded with a “child” that has a probability $P = 2^{d-3}$ of maturing. If a child does not mature, the substructure it was born in becomes inactive, and no clusters are placed within it. If the child matures, the substructure is further divided evenly into eight substructures, and the process is repeated until enough mature children exist to populate the required number of clusters. One complication in the adopted scheme is that the volume of the largest clusters exceeds that of the substructures associated with the final generation of mature children. As such, we stray from the process outlined in Goodwin & Whitworth (2004) by keeping all mature children (as opposed to keeping only the last generation). Larger clusters are placed randomly first at locations with mature children born into substructures that are minimally larger than the cluster volume, and all higher-generation “active” substructures within are removed from the pool of possible cluster sites. The process is repeated until all clusters are placed with no overlapping volume.

In our baseline model, star formation within a given cluster begins at a time chosen randomly between $t = 0$–10 Myr, and proceeds with equal probability over a time interval of $\Delta t = 2$ Myr. The corresponding stellar-birth distribution function for the entire GMC is therefore given by the piecewise function

$$b(t) = \begin{cases} 
  t/(20 \text{ Myr}^2) & 0 \leq t \leq 2 \text{ Myr} \\
  1/(10 \text{ Myr}) & 2 \text{ Myr} \leq t \leq 10 \text{ Myr} \\
  (12 \text{ Myr} - t)/(20 \text{ Myr}^2) & 10 \text{ Myr} \leq t \leq 12 \text{ Myr} \\
  0 & t > 12 \text{ Myr}.
\end{cases} \quad (4)$$

The 12 Myr duration of the star formation epoch is comparable to the crossing time of the molecular cloud (for transport speed $\sim1 \text{ km s}^{-1}$ and size scale $\sim$10 pc), which is characteristic of molecular-cloud operations (e.g., Elmegreen 2000; Dobbs & Pringle 2013). Since SLRs are produced exclusively by massive stars ($8 M_\odot \leq M_* \leq 120 M_\odot$), we adopt an IMF

$$f(m) = 0.173 m^{-2.5}, \quad (5)$$

(where $m = M_\odot/M_\odot$) that mimics the high-mass portion of the observed IMF (e.g., Scalo 1998; Kroupa 2001). The normalization is chosen so that the distribution has a given probability that a star has a mass $M_* \geq 8 M_\odot$, i.e.,

$$\int_8^{120} f(m)dm = 0.005. \quad (6)$$

For completeness, we note that stars near the high end of the allowed mass range can collapse directly to black holes (e.g., see the discussion of Woosley 2017). If black hole formation is efficient, then the SLRs produced during stellar evolution will be incorporated into the black holes instead of being distributed into the molecular cloud. Although the probability of black hole formation is not known, this process would reduce the radioactive yields estimated in this paper, as illustrated below (see Figure 8).\(^5\)

Massive stars are placed at the center of their respective cluster and evolve into SN events upon reaching an age given by

$$\log_{10}\left(\frac{\tau_{\text{SN}}}{\text{Myr}}\right) = \frac{1.4}{(\log_{10} m)^{1.5}}, \quad (7)$$

where this form represents an empirical fit (Williams & Gaidos 2007) to results from an ensemble of stellar-evolution simulations (Schaller et al. 1992).

The locations of the remaining “field” stars are then specified within each cluster using a formalism similar to that used to place clusters in the molecular cloud (with fractal dimension $d = 2$ for our baseline model). However, as stars are effectively point masses, they can all be placed at the active sites of the final generation of mature children, which is done randomly. In addition, the lengths of the cube in which stars are placed for a

\(^5\) For example, if all stars with $m > 60$ collapse to black holes without releasing SLRs, then the total radioactive-enrichment levels of this paper would be reduced by $\sim$14%.
given cluster is scaled as \( r^* c = aR_c \) (with \( a = 2 \) for the baseline model), but active sites that are farther than \( r^* c \) from the cluster center are eliminated as possible locations (thereby turning a cubic structure into a spherical one). We note that this process for placing field stars does not accurately reflect the dynamics of cluster evolution, but, nevertheless, produces structures that are visually similar to those observed in GMC environments.

The structure of a baseline GMC generated using our scheme is shown in the left panel of Figure 1, where blue spheres represent clusters (with radii \( R_c \)), black dots show the location of massive stars, and gray spheres denote the location of each field star. For illustrative purposes, we also show the corresponding column-density map in the right panel of Figure 1, where a Hernquist (Hernquist 1990) density profile is used to calculate the density at each point of the region, and \( M_\infty \) is set to 20 \( M_\odot \) (ignoring any stellar mass contribution). Noting that the mass contained within a radius \( r = c \) for a Henrquist profile is \( M_\infty / 4 \), we define a characteristic density for star-forming regions as

\[
\rho_\star \equiv \frac{M_\infty / 4}{4\pi c^3/3} = 150 \, M_\odot \, \text{pc}^{-3}.
\]

For ease of comparison to observed values of mass fractions in our solar system, the mass densities of \( ^{26}\text{Al} \) and \( ^{60}\text{Fe} \) obtained in our work will be presented in terms of \( \rho_\star \).

### 2.2. Distribution of Short-lived Radionuclides

Massive stars \((M_\star \geq 8 \, M_\odot)\) born in GMCs, while quite rare, live only a few million years before exiting in cataclysmic fashion. The resulting SN explosions that follow the death of those massive stars spread the byproducts of the nuclear fusion that powers stellar life, as well as nuclei created through the rapid capture of neutrons during the SN event, into the surrounding GMC environment. The most massive of these stars \((M_\star \geq 25 \, M_\odot)\) also inject nucleosynthesis products into the surrounding medium via strong winds during a short-lived Wolf–Rayet phase prior to their cataclysmic ends. Our main interest lies in the small fraction of those elements that are unstable and will eventually undergo radioactive decay on timescales \( \sim 1 \) Myr. Specifically, \( ^{26}\text{Al} \) and \( ^{60}\text{Fe} \) are produced with relatively large abundances and have half-lives \( t_{1/2} \) of 0.72 and 2.6 million years, respectively—long enough for them to reach many of the stellar systems forming within the molecular cloud, but short enough that their ensuing radioactive decay can efficiently heat and ionize the disks within those systems. For completeness, we also consider the production of \( ^{26}\text{Al} \) via stellar winds during the Wolf–Rayet phase, as well as the
production of the lower-yield elements $^{36}\text{Cl}$ ($t_{1/2} = 0.30$ Myr) and $^{41}\text{Ca}$ ($t_{1/2} = 0.10$ Myr) in SN events.

The integrated abundance of $^{26}\text{Al}$ produced during the Wolf–Rayet phase of massive stars is presented in Gounelle & Meynet (2012) for several different initial masses (see their Figure 3). While the time evolution of this process is complicated, the most important feature for our study is the plateau reached just prior to the ensuing SN event. As a result, we adopt a highly idealized empirical model based on simple fits to the results of Gounelle & Meynet (2012) wherein the mass-dependent integrated abundance of $^{26}\text{Al}$ has the form

$$M_{\text{Al,\w}}(m) = 10^{-4.7+1.66\log_{10}[m]-1.48} M_\odot,$$

and remains constant over a time

$$\tau_w = \left[0.4 + \frac{1.6(m-30)}{90}\right] \text{Myr},$$

prior to the SN event for stellar masses $M_\w \geq 30 M_\odot$. The abundance then diminishes due to radioactive decay following the SN event. Lower-mass stars produce substantially lower yields, and are therefore disregarded. Our highly idealized model is presented in Figure 2, and can be compared directly to Figure 3 of Gounelle & Meynet (2012).

The SN yields of the SLRs used in this work are shown in Figure 3 over the range of relevant stellar masses. These yields are taken from the stellar-nucleosynthesis calculations of Chiefi & Limongi (2013); see also Sukhbold et al. (2016) and Limongi & Chiefi (2018). These simulations provide updated yields for the SLRs and include the effects of rotation, compared with the earlier stellar-evolution models of Woosley & Weaver (1995) and Limongi & Chiefi (2006); see also Timmes et al. (1995a, 1995b), Woosley et al. (2002), and Rauscher et al. (2002). The stellar models are computed for discrete mass values, with the yields shown by the solid circles, and with intermediate values determined by interpolation (carried out in log-log space). Since results from the stellar models are listed with a minimum mass of $m = 13$, yields for smaller progenitor masses are extrapolated assuming constant yields over the range $8 < m < 13$. The resulting yields are shown in Figure 3 for the SLRs of interest: $^{26}\text{Al}$ (blue curve), $^{60}\text{Fe}$ (red curve), $^{41}\text{Ca}$ (green curve), and $^{36}\text{Cl}$ (magenta curve). For completeness, Figure 3 also includes the plateau $^{26}\text{Al}$ wind-generated abundances shown in Figure 2, along with the corresponding fit from Equation (10).

For a given realization of a GMC environment, the total mass enrichment of a given element as a function of time resulting from all SN events, each occurring at time $t_i$, is given by

$$M_A(t) = \sum_{i=1}^{N_{\text{SN}}} \left[H[t - t_i] Y_A \int_0^{t_i} b(t) H[t - t_{\text{SN}}(m)] \right.\left. e^{-\lambda_A(t-t_i)} dt_e \right] dm,$$

where $A$ denotes the species, $\lambda_A$ is the corresponding decay constant, and $H[x]$ is the well-known step function. Each realization, in turn, generates a value of $M_A(t)$ that is a member of the parent population which statistically describes mass enrichment for our assumed GMC scenario. The expectation value of the corresponding parent distribution can be calculated by integrating over both the IMF and stellar-birth functions

$$\langle M_A(t) \rangle = N_{\text{mc}} \int f(m) \int_0^{t_{\text{SN}}(m)} b(t_e) H[t - t_{\text{SN}}(m)] \right.\left. e^{-\lambda_A(t-t_i)} dt_e \right] dm,$$

where $t_e$ represents the stellar-birth time, $t_{\text{SN}} = t_{\text{SN}} + t_e$, and $b(t_e)$ is given by Equation (4) for our baseline model.
variance can also be calculated via the expression

$$\sigma^2(t) = \frac{\langle M_\alpha(t)^2 \rangle - \langle M_\alpha(t) \rangle^2}{\langle N_{SN}(t) \rangle - 1},$$

(14)

where

$$\langle M_\alpha(t) \rangle = \frac{\langle N_{SN}(t) \rangle \int_8^{120} f(m) \int_0^t b(t_s) H[t - t_{SN}(m)]}{\int_8^{120} f(m) \int_0^t b(t_s) H[t - t_{SN}(m)]} \times [Y_\alpha(m) e^{-\lambda_\alpha(t-t_{SN}(m))}]^2 \ dt_s \ dm,$$

(15)

and

$$\langle N_{SN}(t) \rangle = \int_8^{120} f(m) \int_0^t b(t_s) H[t - t_{SN}(m)] \ dt_s \ dm.$$

(16)

Similar expressions can be obtained for the distribution of $^{26}$Al yields produced by winds, for which a given realization of a GMC environment generates a total mass enrichment of

$$M_{Al,w}(t) = \sum_{i=1}^{\infty} H[t - t_i - \tau_t] \times M_{Al,w,i} e^{-\lambda_\alpha(t-t_i)},$$

(17)

The expectation values calculated using the above integral expressions are in complete agreement with mean value obtained from $10^5$ randomly generated realizations of GMCs for our baseline model (with values from each realization calculated using Equation (12)). Likewise, the variances calculated for the parent populations are in good agreement with those obtained from our randomly generated sample, though the latter are slightly wider owing to undersampling of the IMF at high masses. The mean values of the total mass enrichment obtained from our numerical simulations of our baseline GMC model are presented by the solid curves in Figure 4, with the corresponding shading depicting the $\pm 1\sigma$ band. For reference, the mean values of the total mass enrichment at 5 Myr intervals of GMC age, along with the corresponding characteristic relative abundance for star-forming regions,

$$\eta_A \equiv \frac{\langle M_\alpha \rangle}{\langle V_{GMC} \rangle},$$

(18)

are compiled in Table 1, where the subscripts Al and Al + refer to SN yields only and SN plus wind yields, respectively.

Further insight is provided by the skewness and kurtosis of the generated $^{26}$Al and $^{60}$Fe distributions, which are plotted along with the normalized first-central moments $\langle M_\alpha \rangle / \sigma$ as a function of GMC age in Figure 5. Consistent with the results shown in Figure 4, the first-central moments (solid curves) increase with age, indicating that the distributions narrow with age as the death of more stars increases the sample size of SN events which contribute to the yields. The skewness and kurtosis of the distributions decrease with age from fairly large initial values to values of around 0.5 and 3.1, respectively, both of which are slightly larger than their normal counterparts of 0 and 3. These results indicate that the distributions evolve from highly non-Gaussian forms to nearly normal distributions with a median value given by $\sim \langle M_\alpha \rangle - \sigma / 6$. Moreover, the tails are slightly heavier and longer at higher mass-enrichment values than normal distributions. To illustrate this point, we show in Figure 6 the histogram of the $^{26}$Al mass-enrichment values generated at a GMC age of 30 Myr that correspond to the results shown in Figures 4 and 5, along with a Gaussian fit to the data. Finally, the probability functions for $\log(M_\alpha)$ (with and without wind contributions) and $\log(M_{Fe})$ resulting from our analysis are shown at 5 Myr intervals of GMC age in

![Figure 3](https://example.com/figure3.png)

**Figure 3.** Yields $Y_\alpha$ of short-lived radioactive nuclei $^{26}$Al (blue), $^{60}$Fe (red), $^{41}$Ca (green), and $^{36}$Cl (magenta) as a function of progenitor mass. Yields are taken from the stellar-evolution models of Chiefi & Limongi (2013). The curves show the interpolations and extrapolations used to determine yields for any given mass. For completeness, the plateau $^{26}$Al wind-generated abundances from Figure 2 are presented by black diamonds along with the corresponding fit from Equation (10).
Figure 4. The solid curves depict the mean values of the distributions of total mass enrichment resulting from all SN events in a GMC obtained from 10^5 realizations of our baseline model. The shading around the solid curves represents ±σ boundaries of the corresponding distributions. The color scheme is the same as in Figure 3: 60Fe (red), 26Al from SN events only (blue), 41Ca (green), 36Cl (magenta), 26Al from Wolf–Rayet winds (black).

Table 1

Mean Values of the Total Mass Enrichment from 10^5 Realizations of Our Baseline GMC Model at Six Different GMC Ages, Along with the Corresponding Characteristic Relative Abundance η for Star-forming Regions (as Defined in Equation (18))

|       | 5 Myr | 10 Myr | 15 Myr | 20 Myr | 25 Myr | 30 Myr |
|-------|-------|--------|--------|--------|--------|--------|
| M_{Al} | 1.9 x 10^{-4} | 9.4 x 10^{-4} | 1.0 x 10^{-3} | 3.9 x 10^{-4} | 1.7 x 10^{-4} | 1.3 x 10^{-4} |
| M_{Al+} | 3.7 x 10^{-4} | 1.3 x 10^{-3} | 1.2 x 10^{-3} | 3.9 x 10^{-4} | 1.7 x 10^{-4} | 1.3 x 10^{-4} |
| M_{Fe} | 1.7 x 10^{-4} | 1.9 x 10^{-3} | 2.9 x 10^{-3} | 1.7 x 10^{-3} | 8.3 x 10^{-4} | 4.0 x 10^{-4} |
| M_{Cl} | 1.1 x 10^{-5} | 5.6 x 10^{-5} | 6.5 x 10^{-5} | 2.6 x 10^{-5} | 1.2 x 10^{-5} | 9.7 x 10^{-6} |
| M_{Ca} | 5.6 x 10^{-6} | 3.0 x 10^{-5} | 3.2 x 10^{-5} | 1.2 x 10^{-5} | 6.5 x 10^{-6} | 5.2 x 10^{-6} |
| η_{Al} | 6.4 x 10^{-11} | 3.1 x 10^{-10} | 3.5 x 10^{-10} | 1.3 x 10^{-10} | 5.8 x 10^{-11} | 4.2 x 10^{-11} |
| η_{Al+} | 1.2 x 10^{-10} | 4.2 x 10^{-10} | 4.0 x 10^{-10} | 1.3 x 10^{-10} | 5.8 x 10^{-11} | 4.2 x 10^{-11} |
| η_{Fe} | 5.8 x 10^{-11} | 6.4 x 10^{-10} | 9.5 x 10^{-10} | 5.7 x 10^{-10} | 2.8 x 10^{-10} | 1.3 x 10^{-10} |
| η_{Ca} | 3.7 x 10^{-12} | 1.9 x 10^{-11} | 2.2 x 10^{-11} | 8.8 x 10^{-12} | 4.0 x 10^{-12} | 3.2 x 10^{-12} |
| η_{Cl} | 1.9 x 10^{-12} | 1.0 x 10^{-11} | 1.1 x 10^{-11} | 3.9 x 10^{-12} | 2.2 x 10^{-12} | 1.7 x 10^{-12} |

Note. All masses are given in units of M_⊙.

Figure 7. While the means of the 26Al and 60Fe distributions reach their highest values at t ≈ 15 Myr when only SN events are considered, the contribution from winds maximizes the mass enrichment of 26Al at t ≈ 10 Myr.

The discussion thus far assumes that stellar progenitors of all masses contribute to the production of SLRs. In some cases, however, SN explosions can stall, so that the stars directly collapse to form black holes. In such cases, the SLRs that are produced via stellar nucleosynthesis are not available for distribution throughout the cloud. Unfortunately, the masses for which black hole production occurs is not well determined. Simulations of SN detonation show complicated behavior, with some progenitor masses producing black holes and comparable masses having successful explosions. Moreover, the trends are not monotonic and different sets of simulations are not in complete agreement (e.g., Sukhbold et al. 2016; Couch et al. 2020; Boccioli et al. 2021). Nonetheless, the likelihood of black hole formation generally increases with progenitor mass. In addition, gravity-wave experiments (Abbott et al. 2016, 2021) are finding black holes in the mass range 20–120 M_⊙. After correcting for observational biases (e.g., Croker et al. 2021), the underlying mass distribution of black holes displays a broad peak at M_{bh} ≈ 60 M_⊙. The corresponding distribution of progenitor masses (for stars that produce black holes) is expected to peak at somewhat higher mass.

In order to assess the loss of SLRs due to black hole formation, we calculate the total supernova remnant mass enrichment for 26Al and 60Fe in our baseline model, but with the contributions from stars with mass M ≥ 100, 80, and 60 M_⊙ removed. The results are shown in Figure 8, along with the curves corresponding to the full mass range and corresponding ±σ boundaries shown in Figure 4. We note that the formation of black holes could have a noticeable effect on our results for GMC ages less than 15 Myr due primarily to the delay in the...
onset of SLR injection (we note that a 60 $M_{\odot}$ star remains on the main sequence for $\sim 1$ Myr longer than a 120 $M_{\odot}$ star), but also because the total contribution of SLRs from stars of mass $\geq 60 M_{\odot}$ is about 14% of the total. Nevertheless, as can be seen by comparing the curves in Figure 8 to the $\pm \sigma$ boundaries of the full mass range contribution, the effect is comparable to (or less than) the statistical fluctuations that arise naturally in the system. However, the cutoff at progenitor mass 60 $M_{\odot}$ could be considered conservative (see the aforementioned references). If an even larger fraction of the stellar population produces (large)

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{Normalized central moments as a function of GMC age of the $^{60}$Fe (red) and $^{26}$Al (blue) distributions of total mass enrichment as a function of GMC age obtained from $10^5$ realizations of our baseline model. Solid curves depict the first normalized central moment $\langle M_1 \rangle/\sigma$, the dashed curves depict the skewness, and the dotted curves depict the kurtosis.}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure6.png}
\caption{Histogram of the values of total mass enrichment of $^{26}$Al in a GMC of age 30 Myr obtained from $10^5$ realizations of our baseline model, along with the corresponding normal distribution with the same mean and standard deviation as the resulting sample.}
\end{figure}
Figure 7. Probability functions for the values of the total GMC mass enrichment (in units of $M_\odot$) for $^{26}$Al (top panel) and $^{60}$Fe (bottom panel). These results were obtained from $10^5$ realizations of our baseline model at time $t = 5$ (black), 10 (blue), 15 (light blue), 20 (green), 25 (magenta) and 30 (red) Myr. Solid curves denote enrichment from SN events only, whereas the dashed curves in the top panel include $^{26}$Al produced by the winds of massive stars during their Wolf--Rayet phase. The vertical lines denote the mean values of the corresponding mass-enrichment distributions (e.g., $\log(M_\odot)$). Note that no SN occurred in many realizations of our model at 5 Myr, and these results, while not shown in the figure, are included in the calculation of the mean value for the corresponding distributions.

Figure 8. The solid curves depict the mean values of the distributions of total mass enrichment resulting from all SN events in a GMC obtained from $10^5$ realizations of our baseline model, as shown in Figure 4. The shading around the solid curves represents $\pm \sigma$ boundaries of the corresponding distributions ($^{60}$Fe, red; $^{26}$Al from SN events only, blue). The remaining curves depict the same values, but with contributions from stars with masses $\geq 100 M_\odot$ (long dashed), 80 $M_\odot$ (short dashed), and 60 $M_\odot$ (dotted-dashed) excluded.
black holes, then the reduction in SLRs would be correspondingly greater.

2.3. Relative Abundances of Short-lived Radionuclides in Star Formation Sites

Determining the distributions of the relative abundances of SLRs in the dense regions where stars form requires an understanding of how radioactive nuclei propagate through the GMC environments. Detailed numerical simulations of SN remnants indicate that the blast wave and ejecta propagate freely for distances of order $1–2$ pc and then interact strongly with the background molecular cloud (e.g., Pan et al. 2012). The ejecta are efficiently mixed into the cloud within a relatively thin layer, with thickness $\sim0.5$ pc, and within a time frame of $\sim30,000$ yr. Subsequent movement of the radioactive nuclei must take place diffusively. The nuclei themselves are tied to the magnetic fields of the cloud, but the gyroradius is small compared to the size scales of interest. As a result, the subsequent propagation of the SLRs occurs primarily through turbulent diffusion.

Since the mixing size and timescales are smaller than those of interest for the propagation of SLRs throughout the cloud, we assume here that each SN immediately delivers its SLRs to a thin shell with radius $R = 2$ pc, and calculate the ensuing density profiles (including decay) for both $^{26}$Al and $^{60}$Fe relative to the shell center by assuming point diffusion from each part of the shell. Doing so yields an expression

$$
\rho(r, t) = \frac{2M_A Dt e^{-(r^2+R^2)/(4Dt)}}{rR[4\piDt]^{3/2}} e^{-\lambda_A t} \sinh\left[\frac{2rR}{4Dt}\right],
$$

for the density of the diffusing particles at radius $r$ from the SN event at time $t$ measured relative to that event. The diffusion constant (see Klessen & Lin 2003) is estimated through the expression

$$
D = 2v_T \ell,
$$

where $v_T$ is the turbulent transport speed and $\ell$ is the mean free path. The speed $v_T$ is identified with the observed nonthermal linewidths ($\Delta v$) of the molecular clouds, where ($\Delta v$) scales with the size $L$ of the observed region according to

$$
v_R \sim (\Delta v) \sim 1\text{km s}^{-1} \left(\frac{L}{1\text{ pc}}\right)^b,
$$

where the index $b = 0.4–0.5$ (Larson 1981; Jijina et al. 1999). Equation (21) holds over a range of size scales $L = 1–30$ pc, with a corresponding range of velocity scales $\sim1–5$ km s$^{-1}$. These values imply that the effective diffusion constant lies in the range $D \sim 1–150$ (km s$^{-1}$)pc. Given the uncertainty within this possible range, we adopt $D = 10$ (km s$^{-1}$)pc for our baseline model (roughly, the geometric mean of the range), but will also explore how our results vary for other values.

Figure 9. Baseline model probability functions for the logarithmic values of SLR densities at stellar sites for $^{26}$Al (top panel, dashed curves include wind) and $^{60}$Fe (lower panel) at 5 (black), 10 (blue), 15 (light blue), 20 (green), 25 (magenta), and 30 (red) Myr. The lower vertical lines denote the mean value of the corresponding distribution. The upper solid vertical lines show the corresponding values of $\log([M_\odot]/V_{\text{obs}})$. All densities are in units of the mean stellar site density $\rho_\odot = 150 M_\odot$ pc$^{-3}$. Note that no SN occurred in many realizations of our model at 5 Myr, and these results, while not shown in the figure, are included in the calculation of the mean value for the corresponding distributions.

---

$^6$ Note that in the limit where the diffusion constant becomes large, the propagation of the SLRs becomes effectively free streaming. As a result, the diffusive model includes the free-streaming limit.
In contrast to the SN ejecta, the $^{26}$Al produced during the Wolf–Rayet phase of a massive star is carried by stellar winds out to the boundary where shocks stall the winds. Given that wind speeds are typically $\sim 100$ km s$^{-1}$, we assume that this process instantaneously distributes the produced $^{26}$Al uniformly within a sphere of radius $R_w = 5$ pc centered on the massive star (Gounelle & Meunet 2012; Deharveng et al. 2010).

For a given realization of our baseline model for a GMC, the densities $\rho_{\alpha}$ and $\rho_{\beta}$ at every field star location are determined at a given GMC age by summing over the contributions of all SN events. The process is then repeated for a total of $10^4$ GMC realizations (thereby building up a total sample of $\approx 5 \times 10^8$ field point locations). Corresponding values for the relative abundances of SLRs are obtained by dividing the calculated densities by the characteristic density of a star-forming region $\rho_\star = 150$ M$_\odot$ pc$^{-3}$ (see Equation (9)). The resulting probability functions for $\log[\rho_{\alpha}/\rho_{\beta}]$ (with and without wind contributions) and $\log[\rho_{\beta}/\rho_{\alpha}]$ at 5 Myr intervals of GMC age are shown in Figure 9. The mean values for each distribution are indicated by the corresponding long vertical lines. For comparison, the characteristic relative abundances presented in Table 1 are shown by the short vertical lines at the top of the figure.

The distributions depicted in Figure 9 show a number of trends. Early on, SLRs injected into the GMC have not had much time to propagate, resulting in large spatial density fluctuations characterized by broad distributions with mean values larger than the expected characteristic values. Over time, diffusion smooths out the fluctuations, and the relative abundance distributions become somewhat narrower. Taken as a whole, the distributions of SLRs span several orders of magnitude, with the bulk of the results falling in the range $10^{-11}$–$10^{-8}$ for both radioactive species of interest (where this range is consistent with previous estimates; see Vasileiadis et al. 2013; Kuffmeier et al. 2016). Significantly, the high end of the range includes solar system benchmark values ($X_{\alpha} \sim 4 \times 10^{-9}$ and $X_{\beta} \sim 10^{-8}$), although the typical mass fractions are somewhat lower.

In addition, we note that diffusion competes with radioactive decay to remove SLRs from the GMC environment with a characteristic timescale that depends on the diffusion constant,$$
\tau_D = \frac{R_{\text{GMC}}^2}{4D} = 7 \text{ Myr} \left( \frac{R_{\text{GMC}}}{17 \text{ pc}} \right)^2 \left( \frac{D}{10 \text{ (km s}^{-1}) \text{pc}} \right)^{-1},
$$
and which is comparable to the GMC age for our baseline model. Since $\tau_D$ is larger than the half-lives of our SLRs, diffusion is not expected to have a significant effect on lowering the SLR content for the GMC for our standard scenario (notice, however, that we consider more-efficient SLR propagation out of the cloud in the following section). We also note that the difference between the distribution means and expected characteristic values become more pronounced at later ages for the longer-living $^{60}$Fe.

In addition to the absolute values of the radioactive abundances, it is also interesting to consider the ratio of different SLRs. We complete our analysis by plotting the probability distributions for the abundance ratios $\log[\rho_{\beta}/\rho_{\alpha}]$ (with and without winds) calculated in the star-forming regions at our six representative ages in Figure 10. The corresponding ratios for $\log[\rho_{\beta}/\rho_{\alpha}]$ and $\log[\rho_{\alpha}/\rho_{\beta}]$ are shown in Figure 11.

The distributions of SLR abundance ratios for $^{60}$Fe/$^{26}$Al shown in Figure 10 display a peak at $\sim 2.5$ and extend down to $\sim 0.3$. This lower value is roughly comparable to the benchmark value $X_{\beta}/X_{\alpha} \approx 0.25$ that we would expect for the early solar system, if $^{60}$Fe enrichment occurs at levels corresponding to the upper end of the measured range. For comparison, the abundance ratio inferred from gamma-ray observations (Diehl et al. 2013; Kuffmeier et al. 2016).
et al. 2006; Diehl 2013) is ∼0.6, which corresponds to a galactic scale and steady-state average. Although this value falls within the expected range, it is below both the peak and the mean of the distribution. We also note that these distributions are generally above the upper limits used in many considerations of the nascent solar system Gounelle (2015). One possible interpretation of this finding is that another mechanism (e.g., spallation) could have contributed to the production of $^{26}$Al when our solar system formed, thereby lowering the ratio.

3. Analysis of Different Enrichment Scenarios

In this section we perform a sensitivity analysis of our nuclear-enrichment model by considering several variations of the input parameters. Specifically, we calculate the $^{26}$Al and $^{60}$Fe abundance distributions at star formation sites for different star formation scenarios, varying physical parameters that determine molecular-cloud structure, and different properties for the transport of radioactive material through the cloud. In all cases, the number of stars is held fixed at $N^* = 50,000$, and the GMC dimensions are sampled as in our baseline model. The resulting distributions of SLR abundances can then be compared to the those shown in Figure 9.

3.1. Instantaneous Star Formation

One limiting case for the star formation history of a GMC is to have all of the stars form at once. In this scenario, all stars form at $t = 0$, so that the stellar-birth distribution function is given by $b(t) = \delta(t)$. All other model parameters are the same as for our baseline model. As can be seen from Figure 12, the immediate onset of star formation injects SLRs into the GMC at earlier times, leading to the yields of both $^{60}$Fe and $^{26}$Al peaking at around 4–5 Myr (compare with Figures 2 and 3 from Voss et al. 2009).

The resulting probability functions for $\log(\rho_{Al}/\rho_*)$ (with and without wind contributions) and $\log(\rho_{Fe}/\rho_*)$ at 5 Myr intervals of GMC age are shown in Figure 13. The mean values for each distribution are indicated by the corresponding lower vertical lines, and can be compared directly to their baseline model counterparts, which are shown by the upper vertical lines. Not surprisingly, instantaneous star formation produces higher SLR densities for the first 10 Myr, and lower SLR densities at later times.

3.2. Sequential Star Formation

In this scenario, we consider the possibility that star formation sweeps across the GMC (e.g., Elmegreen & Lada 1977). For the sake of definitiveness, we implement this scenario by correlating the time that star formation begins within a given cluster to the relative center-of-mass position of that cluster along the longest axis of the GMC, and set the sweeping time to 10 Myr. All other model parameters are the same as for our baseline model. Once star formation begins within a cluster, it proceeds with equal probability over a time interval $\Delta t = 2$ Myr, resulting in the same stellar-birth distribution function for the entire GMC as our baseline model. As such, the total SLR mass enrichment as a function of time is identical to that shown in Figure 4. However, as shown in Figure 14, the correlation between stellar position and stellar birth generally leads to slightly lower values for the probability functions of $\log(\rho_{Al}/\rho_*)$ and $\log(\rho_{Fe}/\rho_*)$ at star-forming sites (the exception being at 5 Myr).
Figure 12. The solid curves depict the mean values of the distributions of total mass enrichment resulting from all SN events in a GMC obtained from $10^5$ realizations of our instantaneous star formation model. The shading around the solid curves represent $\pm \sigma$ boundaries of the corresponding distributions. The color scheme is the same as in Figure 3: $^{56}$Fe (red), $^{26}$Al from SN events only (blue), $^{41}$Ca (green), $^{36}$Cl (magenta), $^{26}$Al from Wolf-Rayet winds (black).

Figure 13. Instantaneous star formation model probability functions for the logarithmic values of SLR densities at stellar sites for Al (top panel, dashed curves include wind) and Fe (lower panel) at 5 (black), 10 (blue), 15 (light blue), 20 (green), 25 (magenta), and 30 (red) Myr. The lower vertical lines denote the mean value of the corresponding distribution. The upper solid vertical lines denote the mean value of the corresponding distribution for our baseline model (as shown by the lower vertical lines in Figure 9). All densities are in units of the mean stellar site density $\rho_\ast = 150 \, M_\odot \, pc^{-3}$. Note that no SN occurred in many realizations of our model at 5 Myr, and these results, while not shown in the figure, are included in the calculation of the mean value for the corresponding distributions.
3.3. Cloud Structure Variations

In the standard model, stars are placed within clusters, which in turn are placed within the GMC using a scheme that creates a structure with fractal dimension \( d = 2 \). We now consider what effect cloud structure has on our results by considering opposite extremes. Specifically, we first consider a model where the same scheme is used to place clusters within the GMC and stars within clusters as for the baseline model, but with a fractal dimension of \( d = 1.6 \) for placing both the clusters in the cloud and the stars within the cluster. This scenario leads to an increase in the substructure within the GMC. At the other extreme, we consider a model where all stars are placed randomly within the GMC in absence of any clusters.

For the lower-fractal-dimension scenario, star formation evolves in the same manner as our baseline model. Specifically, star formation within a given cluster begins at a time chosen randomly between \( t = 0–10 \) Myr, and proceeds with equal probability over a time interval of \( \Delta t = 2 \) Myr. For consistency, stellar birth in the random placement model is assigned through the same stellar-birth distribution function that is used to describe the scenario with lower fractal dimension (which in turn is the same as for our baseline model). All other model parameters are the same as for our baseline model.

As illustrated in Figure 15, lowering the fractal dimension from \( d = 2 \) to \( d = 1.6 \) has a minimal effect on how SLRs are distributed throughout the GMC. The resulting probability distributions are nearly the same. In contrast, the scenario with randomly placed stars (shown in Figure 16) leads to lower SLR abundances at stellar sites. In this random model, the stellar locations are spread out more evenly over the entire GMC and the SNe themselves are less clustered. As a result, the random distribution of stellar locations leads to less concentration of the SLRs and a corresponding deficit of high abundances.

3.4. Smaller Cluster Membership

While the cluster distribution function chosen for our baseline model is based on our current understanding of star formation, local clusters (within 1–2 kpc) have been observed to have memberships that range from \( N = 30 \) to 2000 (Lada & Lada 2003; Porras et al. 2003). Motivated by this local sample, we also consider the scenario in which the cluster membership \( N \) spans a smaller range from 10^2 to 10^3. All other model parameters are the same as for our baseline model.

Within the smaller range of stellar membership size \( N \), clusters are drawn from (the usual) distribution function given by Equation (1). As before, we sample the distribution until the collective stellar content reaches \( N_{\text{mc}} \), with the last drawn cluster’s membership reduced as necessary to achieve that outcome. For this set of assumptions, the mean cluster membership size is only \( \langle N \rangle = 256 \), so that a GMC is expected to have \( \sim 200 \) clusters with radii \( R_c = 0.7–1.5 \) pc. We note that for our assumed cluster distribution, half of the GMC stars are expected to belong to clusters with membership \( N \leq 316 \).

As shown in Figure 17, the resulting distributions are shifted toward lower values compared to the standard case. The smaller clusters, which are more populous, act to distribute the...
stars more evenly across the cloud (but not as evenly as in the case of the random-distribution model of Section 3.3). As a result, fewer locations are near multiple SNe, and the SNe themselves are more spread out, so that fewer locations have high abundances of SLRs.

3.5. Propagation Parameters

As noted in Section 2.3, particle transport in GMC environments is poorly constrained both observationally and theoretically. We can address this issue by considering varying values for the diffusion constant, which determines the manner in which the SLRs are transported across the molecular cloud. Given the timescales of interest \( t \sim 10 \text{ Myr} \) and the standard value of the diffusion constant \( D = 10 \text{ (km s}^{-1})\text{pc} \), the typical transport distances are comparable to the typical sizes of molecular clouds \( r \sim \sqrt{D} t \sim 10 \text{ pc} \). As a result, larger values of the diffusion constant allow for a substantial fraction of the SLRs to be swept out of the cloud. In contrast, smaller values lead to much less transport and allow for enhanced localized enrichment.

Here, we first consider the case where the diffusion constant has the smaller value \( D = 1 \text{ (km s}^{-1})\text{pc} \), as shown in Figure 18. The resulting distributions of SLRs show enhanced abundances due to the smaller diffusion constant. In this scenario, the SLRs stay relatively close to their birth clusters and are effective at enriching stars forming in the vicinity. This enhancement effect is larger for \(^{56}\text{Fe}\) compared to \(^{26}\text{Al}\) due to its longer half-life.

For larger values of the diffusion constant, \( D = 100 \text{ (km s}^{-1})\text{pc} \), Figure 19 shows that the SLR abundances are significantly lower than in the standard baseline model. This result is expected, as the larger diffusion constant allows for a substantial fraction of the SLRs to be swept out of the cloud.

Note that of all the parameters varied in this section, the value of the diffusion coefficient is the least constrained. With its possible range of two orders of magnitude, this quantity has the largest effect on the resulting distributions of SLRs abundances.

3.6. Anisotropic SLR Transport

The use of a single diffusion coefficient to describe particle transport in a GMC is highly idealized. Indeed, recent insights about the evolution of molecular clouds indicate that massive-star feedback erodes clouds within a few Myr, and that wind-blowed bubbles are important sites where radioisotopes spread. Far from homogeneous, GMCs are highly nonuniform, and exhibit thin filamentary structures extending over \( \sim 100 \text{ pc} \). The spherical injection and propagation models adopted in our work are therefore too simplistic to fully describe the transport of matter in such a hot and stirred massive-star environment.

While creating a particle-transport model that takes into account the structural complexities of GMCs is beyond the scope of our work, we model the effects on nonuniformity in the GMC environment by selecting the diffusion coefficient used to determine the density at a given stellar site (via Equation (19)) from a distribution function \( F(D) \). In order to
generate this distribution function, we first calculate the column density along a 4 pc line of sight extending in six directions from the position of each field star for one realization of our GMC. Doing so yields \( \approx 300,000 \) values of column densities whose distribution characterizes the nonuniform structure of our GMC environments. Since the diffusion constant is expected to be inversely proportional to the mean free path, we then generate a corresponding distribution of diffusion constants through the relation 

\[
D_i = 10 \frac{\langle N \rangle}{N_i} \text{pc},
\]

where \( \langle N \rangle \) is the mean of the column-density distribution, and \( N_i \) is the \( i \)th member of the column-density distribution. The resulting distribution of diffusion constants \( F(D) \) is shown in Figure 20.

The spread in \( F(D) \) indicates that particle diffusion will not proceed uniformly in all directions, but rather will vary in accordance with the diffusion-constant distribution. Note that the width of the distribution is approximately a factor of 2 on either side of the mean. With this result, we perform a calculation using our baseline-model parameters, but with the diffusion constant \( D \) used to calculate the SLR densities at each star formation site (via Equation (19)) randomly selected from our generated distribution function \( F(D) \). The results of this procedure are shown in Figure 21 (solid curves) along with the corresponding distributions from the baseline model (dotted curves), for which \( D = 10 \) km s\(^{-1}\) pc. The inclusion of nonuniform propagations slightly broadens the density-distribution functions (as expected) with slightly lower mean values. However, as can be seen by comparing the solid and dotted curves, the overall effect for our model is quite modest. We also note that filamentary clouds allow for more complicated transport behavior, wherein gas streaming away from one filament can reach other filaments. As a result, future work should explore this issue further, including a dynamical cloud medium shaped by feedback of winds and explosions.

4. Conclusions

Given the possible importance of SLRs during the processes of star and planet formation, this paper considers the expected distributions of SLRs on the size and mass scales of molecular clouds. SNe (for both \(^{60}\)Fe and \(^{26}\)Al) and stellar winds (for \(^{26}\)Al only) provide SLRs for the star-forming population produced within the cloud. The main result of this work is the distribution of possible radioactive-enrichment levels for forming stars and planets. The resulting values for the mass fractions of \(^{26}\)Al are listed in Table 2 for a collection of cloud ages and for the different scenarios considered in this work. Table 3 shows the corresponding mass fractions for \(^{60}\)Fe.

4.1. Summary of Results

Our main conclusions can be summarized as follows:

1. Distributed enrichment of SLRs from SN sources produces a wide distribution of abundances of radioactive nuclei across molecular clouds. For the isotope \(^{60}\)Fe, we find abundance levels in the approximate range \( \rho_{\text{SLR}} / \rho_\star \sim 10^{-11} - 10^{-8} \), where such enrichment is
expected during the first $\sim 10$ Myr of star formation. The corresponding enrichment levels for $^{26}$Al are somewhat smaller but roughly comparable (see Figure 9). The distributions of SLR abundances are time dependent, and decrease steadily after the epoch of star formation has finished (here, for times later than 12 Myr).

2. The ratio of $^{60}$Fe/$^{26}$Al also displays a wide distribution (Figure 10), with typical values somewhat greater than unity ($\sim 1$–$2$). Significantly, SNe lead to greater enrichment levels for $^{26}$Fe, compared to $^{26}$Al, which is opposite to the trend observed/inferred for our solar system (and for the Galaxy as a whole). This trend arises because the two isotopes are produced with roughly comparable abundances in SNe, whereas $^{60}$Fe has a significantly longer half-life (2.6 Myr, compared to 0.72 Myr for $^{26}$Al). These results correspond to the typical values sampled from the entire distributions found by this paper. Any particular realization still could have more $^{26}$Al than $^{60}$Fe. For example, the scenario with efficient SLR propagation (large diffusion constant) and evaluated at late evolution times allows for larger values of $^{26}$Al/$^{60}$Fe (see Figure 19).

3. The distributions for the absolute enrichment levels indicate that this GMC-wide enrichment scenario could explain the absolute values of the abundances of SLRs inferred for the early solar system. The probability of attaining the observed solar system mass fractions depends on time (as determined by the star formation history of the cloud). For our standard scenario, forming stars have a $\sim 10\%$ chance of reaching $^{26}$Al enrichment levels for cloud times $t = 10$–$15$ Myr, with considerably lower probabilities outside that span of time. On the other hand, the distributions of abundance ratios $^{60}$Fe/$^{26}$Al are generally not consistent with current estimates.

4. In addition to our baseline-enrichment scenario (Figure 9), we have varied the input parameters of the model. Specifically, this paper provides SLR distributions for instantaneous star formation (Figure 13), sequential star formation (Figure 14), different fractal dimensions for cloud structure (Figure 15), uniform–random distribution of star formation sites (Figure 16), smaller clusters (Figure 17), varying SLR propagation efficiencies (Figures 18 and 19), and for the effects of nonspherical propagation (Figures 20 and 21). Over most of this parameter space, the distributions of SLRs display only modest variations. The most important variable—in terms of changing the distributions of SLRs abundances—is the diffusion coefficient for the propagation of SLRs through the molecular cloud. If the diffusion constant becomes sufficiently large ($D \gtrsim 30$ pc$^2$ Myr$^{-1}$), then the enrichment levels decrease substantially. The key issue required for maintaining significant levels of nuclear enrichment is the retention of the SLRs within the molecular cloud.

4.2. Discussion

In terms of the absolute SLR abundances that affect forming stars and planets, we find intermediate results. The expected nuclear enrichment is large enough to provide significant
sources of heating and ionization for circumstellar disks and planets, but is not large enough to completely dominate the picture (see the discussion of Lichtenberg et al. 2019, Reiter 2020, and references therein). Moreover, the distributions of SLRs abundances are wide, roughly comparable to their mean values. As a result, enrichment levels must be described in terms of probability distributions, i.e., we cannot assign a single, typical value to the expected degree of radioactive enrichment.

Although the focus of this paper is to provide the distributions of SLRs for the entire population of stars forming within a cloud, it is useful to put our solar system in context. Here, our results present a somewhat complicated picture for nuclear-enrichment scenarios. On one hand, the probability of reaching the abundance levels for $^{60}\text{Fe}$ and $^{26}\text{Al}$ inferred for our solar system are high enough that the observed values are not problematic. In this sense, our solar system is not out of the ordinary (consistent with previous work, from Jura et al. 2013 to Young 2020). On the other hand, the abundance ratio of iron to aluminum, $^{60}\text{Fe}/^{26}\text{Al}$, is generally larger than unity, rather than smaller than unity as observed for the solar system (where $^{60}\text{Fe}/^{26}\text{Al} \lesssim 0.25$). This ratio is also smaller than unity on galactic scales, $^{60}\text{Fe}/^{26}\text{Al} \sim 0.4$–0.9, as measured by gamma-ray lines (Wang et al. 2020).

This study considers the enrichment provided by the transport of SLRs across molecular clouds, where the sources include SNe and stellar winds. Additional sources of radioactive material could also play a role, including spallation in the interstellar medium (Desch et al. 2010) and spallation from local sources of cosmic rays (Shu et al. 1997). Significantly, spallation can provide an additional source for $^{26}\text{Al}$, whereas only stellar nucleosynthesis is known to produce $^{60}\text{Fe}$. As a result, these additional sources of radioactive nuclei act to decrease the abundance ratio $^{60}\text{Fe}/^{26}\text{Al}$.

Taken together, the results of this paper suggest that the expected abundances of both $^{60}\text{Fe}$ and $^{26}\text{Al}$ in molecular clouds are large enough to affect star and planet formation. Specifically, enrichment levels comparable to those inferred for the early solar nebula can be attained with reasonable probability, although such values fall toward the high end of the distribution. In any case, the expected enrichment levels provide significant contributions to ionization and heating in forming stars and planets. On the other hand, enrichment through SNe and stellar winds alone does not provide a full explanation for all the solar system SLRs, including their exact abundance ratios. In particular, the mass ratios $^{60}\text{Fe}/^{26}\text{Al}$ predicted from this study are generally larger than estimates for both the early solar system and the Galaxy as a whole (although the tail of the distribution includes smaller values). Additional work is thus necessary to obtain a full understanding of the SLR abundances for the early solar system, the Galaxy, and the current population of forming stars and planets.

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![Figure 18.](image-url)
Figure 19. $D = 100\text{ km s}^{-1}\text{pc}$ model probability functions for the logarithmic values of SLR densities at stellar sites for Al (top panel, dashed curves include wind) and Fe (lower panel) at 5 (black), 10 (blue), 15 (light blue), 20 (green), 25 (magenta), and 30 (red) Myr. The lower vertical lines denote the mean value of the corresponding distribution. The upper solid vertical lines denote the mean value of the corresponding distribution for our baseline model (as shown by the lower vertical lines in Figure 9). All densities are in units of the mean stellar site density $\rho_s = 150 M_\odot\text{ pc}^{-3}$. Note that no SN occurred in many realizations of our model at 5 Myr, and these results, while not shown in the figure, are included in the calculation of the mean value for the corresponding distributions.

Figure 20. The distribution function of diffusion constants built through the relation $D_i = 10 \langle N_i \rangle / N_i \text{pc}$, where $N_i$ is the value of the i-th member of the distribution of column densities calculated along a 4 pc line of sight extending in six directions from the position of each field star for one realization of our GMC, and $\langle N \rangle$ is the mean of the column-density distribution.
Figure 21. Nonuniform model probability functions (solid curves) for the logarithmic values of SLR densities at stellar sites for Al (top panel) and Fe (lower panel) at 5 (black), 10 (blue), 15 (light blue), 20 (green), 25 (magenta), and 30 (red) Myr. Dotted curves show the corresponding distribution for our baseline model (as shown in Figure 9). All densities are in units of the mean stellar site density $\rho_*$ = 150 $M_\odot$ pc$^{-3}$. Note that no SN occurred in many realizations of our model at 5 Myr, and these results, while not shown in the figure, are included in the calculation of the mean value for the corresponding distributions.

### Table 2
Mean Values of the Density of $^{26}$Al Produced by SN Events at the Star Formation Sites at Six Different GMC Ages for the Different Models Explored in this Work

| Model      | Figure | 5 Myr     | 10 Myr    | 15 Myr    | 20 Myr    | 25 Myr    | 30 Myr    |
|------------|--------|-----------|-----------|-----------|-----------|-----------|-----------|
| Standard   | 9      | $8.0 \times 10^{-11}$ | $1.8 \times 10^{-10}$ | $3.1 \times 10^{-10}$ | $1.1 \times 10^{-10}$ | $4.9 \times 10^{-11}$ | $3.3 \times 10^{-11}$ |
| Inst. SF   | 13     | $6.8 \times 10^{-10}$ | $2.3 \times 10^{-10}$ | $8.0 \times 10^{-11}$ | $4.0 \times 10^{-11}$ | $3.1 \times 10^{-11}$ | $2.5 \times 10^{-11}$ |
| Seq. SF    | 14     | $9.1 \times 10^{-11}$ | $1.6 \times 10^{-10}$ | $2.8 \times 10^{-10}$ | $9.9 \times 10^{-11}$ | $4.6 \times 10^{-11}$ | $3.2 \times 10^{-11}$ |
| $d = 1.6$  | 15     | $8.4 \times 10^{-11}$ | $1.7 \times 10^{-10}$ | $3.1 \times 10^{-10}$ | $1.2 \times 10^{-10}$ | $5.0 \times 10^{-11}$ | $3.3 \times 10^{-11}$ |
| Random     | 16     | $3.3 \times 10^{-11}$ | $7.6 \times 10^{-11}$ | $1.2 \times 10^{-10}$ | $4.7 \times 10^{-11}$ | $2.0 \times 10^{-11}$ | $1.3 \times 10^{-11}$ |
| Small cluster | 17   | $6.3 \times 10^{-11}$ | $1.5 \times 10^{-10}$ | $2.4 \times 10^{-10}$ | $9.2 \times 10^{-11}$ | $3.9 \times 10^{-11}$ | $2.5 \times 10^{-11}$ |
| $D = 1$ km s$^{-1}$ pc$^{-1}$ | 18   | $3.2 \times 10^{-10}$ | $1.7 \times 10^{-10}$ | $3.4 \times 10^{-10}$ | $1.5 \times 10^{-10}$ | $6.1 \times 10^{-11}$ | $3.9 \times 10^{-11}$ |
| $D = 10^2$ km s$^{-1}$ pc$^{-1}$ | 19   | $3.4 \times 10^{-11}$ | $8.6 \times 10^{-11}$ | $1.1 \times 10^{-10}$ | $3.8 \times 10^{-11}$ | $1.7 \times 10^{-11}$ | $1.2 \times 10^{-11}$ |
| Nonuniform | 21     | $7.0 \times 10^{-11}$ | $1.7 \times 10^{-10}$ | $2.8 \times 10^{-10}$ | $1.0 \times 10^{-10}$ | $4.5 \times 10^{-11}$ | $3.0 \times 10^{-11}$ |

**Note.** All densities are in units of the mean stellar site density $\rho_*$ = 150 $M_\odot$ pc$^{-3}$. Wind-produced $^{26}$Al is excluded.

### Table 3
Mean Values of the Density of $^{60}$Fe Produced by SN Events at the Star Formation Sites at Six Different GMC Ages for the Different Models Explored in this Work

| Model      | Figure | 5 Myr     | 10 Myr    | 15 Myr    | 20 Myr    | 25 Myr    | 30 Myr    |
|------------|--------|-----------|-----------|-----------|-----------|-----------|-----------|
| Standard   | 9      | $8.2 \times 10^{-11}$ | $4.2 \times 10^{-10}$ | $7.3 \times 10^{-10}$ | $3.4 \times 10^{-10}$ | $1.4 \times 10^{-10}$ | $6.9 \times 10^{-11}$ |
| Inst. SF   | 13     | $1.3 \times 10^{-9}$ | $5.5 \times 10^{-10}$ | $2.9 \times 10^{-10}$ | $9.5 \times 10^{-11}$ | $6.0 \times 10^{-11}$ | $4.6 \times 10^{-11}$ |
| Seq. SF    | 14     | $9.3 \times 10^{-11}$ | $2.4 \times 10^{-10}$ | $6.9 \times 10^{-10}$ | $3.1 \times 10^{-10}$ | $1.2 \times 10^{-10}$ | $6.6 \times 10^{-11}$ |
| $d = 1.6$  | 15     | $8.7 \times 10^{-11}$ | $4.0 \times 10^{-10}$ | $7.4 \times 10^{-10}$ | $3.4 \times 10^{-10}$ | $1.4 \times 10^{-10}$ | $6.9 \times 10^{-11}$ |
| Random     | 16     | $3.5 \times 10^{-11}$ | $2.2 \times 10^{-10}$ | $3.8 \times 10^{-10}$ | $2.0 \times 10^{-10}$ | $8.7 \times 10^{-11}$ | $4.0 \times 10^{-11}$ |
| Small cluster | 17   | $6.5 \times 10^{-11}$ | $3.8 \times 10^{-10}$ | $6.2 \times 10^{-10}$ | $3.0 \times 10^{-10}$ | $1.3 \times 10^{-10}$ | $5.9 \times 10^{-11}$ |
| $D = 1$ km s$^{-1}$ pc$^{-1}$ | 18   | $3.4 \times 10^{-10}$ | $4.6 \times 10^{-10}$ | $1.4 \times 10^{-9}$ | $9.4 \times 10^{-10}$ | $4.3 \times 10^{-10}$ | $2.0 \times 10^{-10}$ |
| $D = 10^2$ km s$^{-1}$ pc$^{-1}$ | 19   | $3.4 \times 10^{-11}$ | $1.2 \times 10^{-10}$ | $1.6 \times 10^{-10}$ | $6.2 \times 10^{-11}$ | $2.3 \times 10^{-11}$ | $1.3 \times 10^{-11}$ |
| Nonuniform | 21     | $7.3 \times 10^{-11}$ | $3.8 \times 10^{-10}$ | $6.4 \times 10^{-10}$ | $3.0 \times 10^{-10}$ | $1.2 \times 10^{-10}$ | $5.9 \times 10^{-11}$ |

**Note.** All densities are in units of the mean stellar site density $\rho_*$ = 150 $M_\odot$ pc$^{-3}$. 
