Bound States of Heavy Mesons by Pion-Exchange Approach
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Abstract
We calculate the two-pion exchange potential of $D\bar{D}$ and $D\bar{D}^*$ by using Gaussian type form factor to regularize loop integrals. Then we discuss the possibility of the molecule states composed of D mesons. Our results show it is very difficult to bind $D\bar{D}$ by pure pions exchange. On the contrary, the possibility of the $D\bar{D}$ molecule state existence cannot be completely ruled out. We also extend our discussion to $B\bar{B}$ and $B\bar{B}^*$.

1 Introduction
The idea of molecular state of mesons was first raised in [1, 2], and it was systematically studied by N.A.Tornqvist in [3]. This idea is often used to study the resonances with "strange " decay property as well as a mass closed to a respective two-meson threshold. One of the newest example is the hadron state of X(3872). A lot of papers suggested it to be a molecular state of $D_0^0D^*_0$.

The X(3872) was first discovered by Belle [4] in $J/\psi\pi^+\pi^-$ channel, and soon confirmed by CDF [5], DO [6], and BaBar [7]. Its mass is $M_X = 3871.2 \pm 0.5 MeV$ and width is $\Gamma < 2.3 MeV$. The study of the angular distributions and correlations of the $J/\psi\pi^+\pi^-$ final state shows that $J^{PC} = 1^{++}$ and $J^{PC} = 2^{-+}$ are two possible quantum number assignments. Since the $2^1P_1$ state in quark model is at least 50 MeV heavier than X(3872) and the decay of a pure charmonium state into $J/\psi\pi\pi$ violates isospin symmetry, it is very difficult to assign X(3872) into charmonium. Many new reasonable explanations were raised, such as explaining X(3872) as a molecule state, a hybrid charmonium, a diquark anti-diquark bound state, a tetraquark state or so. Since X(3872)'s mass is just a half MeV below $D\bar{D}^*$ mass threshold and its branch width in the $D\bar{D}^*$ channel is ten times larger than that in the $J/\psi\pi^+\pi^-$ channel, the molecule picture becomes one of most favorable model though this picture meets a problem in explanation of the decay $X(3872) \rightarrow J/\psi\pi^+\pi^-$. The molecule picture for X(3872) have been discussed in many papers [8, 9, 10, 11, 12]. As the proximity of X(3872) and $D_0^0\bar{D}^*$, threshold, the X(3872) resonance may be just a loosely bound state of $D_0^0\bar{D}^*$. Such loosely bound state is always built in a potential model. In [10] Swanson suggested the potential is mainly supplied by exchanging pion mesons and quarks since the one pion exchange is not good enough to bind $D_0^0\bar{D}^*$ together. In [13] the authors traced back to pion exchange potential of deuteron in nuclear physics, where the single pion exchange cannot bind proton and neutron either. Only by introducing the sigma meson exchange would the deuteron to be stable. So they had introduced the sigma meson exchange in the $D_0^0\bar{D}^*$. However their result showed the situation became worse. A bound state of $D_0^0\bar{D}^*$ could appear only when the parameter of energy cutoff $\Lambda$ become unphysically large. Their results disfavors the X(3872) being a bound state of $D\bar{D}^*$. In [14] C.E.Thomas and F.E.Close calculated the pion exchange of $D$ and $D^*$ in the quark level. And they did find a bound state in their potential. And the bind energy is sensitive to the energy cutoff parameter $\Lambda$ and the coupling constant $g$. Beside the pion exchange, other mesons exchange, such as $\sigma, \rho, \omega$ and $\eta$ exchange between $D$ and $D^*$, has been discussed in [15]. The authors found a loosely bound state of $D\bar{D}^*$ when the energy cutoff $\Lambda = 1150 MeV$. Their results favors the X(3872) being a loosely bound state of $D$ and $D^*$ mesons.

Until now, from both of the theory and the experiment, there are two opposite opinions about the molecule picture of the resonance X(3872). However, more new resonances, such as X,Y,Z series of the charmonium, are
found continuously in experiments. Some of them are also difficult to be explained as the pure $\bar{c}c$ states. Therefore, regardless of whether the $X(3872)$ is a molecule state of $\bar{D}D^*$, the interaction between $D$ mesons in hadron level will be still interesting.

Our work starts from the thinking of the two-pion exchange potential between $D$ and(or) $D^*$ mesons. In nuclear physics, the two-pion exchange potential plays an important role in explaining the inside structure of nucleons. We guess such effects are also important between the $D$ and(or) $D^*$ mesons though it was argued that the chiral perturbative expansion converges more quickly in heavy mesons. At first, we calculate the two-pion exchange potential between $D$ and $\bar{D}$ by using Gaussian type form factor to regularize loop integral and study the bound state of $\bar{D}D$. We find, though there is not the one-pion exchange potential, a stable bound state of $\bar{D}D$ can exist at $\Lambda = 1750\,MeV$. Then we perform the similar calculation for the two-pion exchange potential between $D$ and $\bar{D}^*$. Meanwhile, we also use the same form factor to regularize the one-pion exchange potential between $D$ and $\bar{D}^*$ for comparison. We find the chiral perturbative expansion is quite convergent even when the cutoff $\Lambda \sim 5\,GeV$. This result shows the regularization scheme works quite well. Similarly, we use the obtained potential to study the bound state of $\bar{D}D^*$ and find a stable bound state exists only when $\Lambda > 4.7\,GeV$. This result is better than [13] though it is still negative for the existence of the molecule $X(3872)$. Our results show, even in heavy mesons, the multi-pion exchange is still important.

We organize our paper as the following. In section 2, we give the effective Lagrangian that we use in the calculation. This Lagrangian is derived from the most general one given by Mark.B.Wise in [16]. In section 3, we first calculate the two-pion exchange potential between $D\bar{D}$, then we numerically solve the schrödinger equation to find whether this potential can form a bound state. In section 4, we calculate the potential of $D\bar{D}^*$ and discuss the possibility of the existence of the molecule $X(3872)$. In section 5, we extend our discussion to the case of the $B$ and $\bar{B}^*$ mesons. Finally we give a brief summary and conclusion.

## 2 Effective Chiral Lagrangian

The effective chiral lagrangian for heavy mesons was first given by Mark.B.Wise [16]:

$$\mathcal{L} = -i\text{Tr} \mathcal{P}_{\mu} \partial^\mu H_a + \frac{1}{2} i\text{Tr} \mathcal{P}_a H_b \gamma^\mu (\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger) + \frac{1}{2} \bigl(ig\text{Tr} \mathcal{P}_a H_b \gamma^\mu \gamma^5 (\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger) \bigr) + \cdots, \quad (1)$$

where

$$H_a = \frac{1 + \gamma^5}{2} (P_{a\mu} \gamma^\mu - P_a \gamma_5) \quad (2)$$

$$\mathcal{P}_a = (P_{a\mu}^* \gamma^\mu + P_a^* \gamma_5) \frac{1 + \gamma^5}{2}, \quad (3)$$

and the $P$ and $P^*$ are meson fields and $\xi$ is the nonlinear realization of the SU(3) goldstone fields.

For our purpose, we can expand the lagrangian [11] as

$$\mathcal{L} = \frac{2}{f^2} \bar{P} t \cdot (\pi \times \hat{n}) P - \frac{2g}{f} (P^* \sqrt{2} \bar{t} \cdot \vec{\gamma}_\pi P + P^* \sqrt{2} \bar{t} \cdot \vec{\gamma}_\pi) - \frac{2}{f^2} \bar{P}^* t \cdot (\pi \times \hat{n}) P^*, \quad (4)$$

where $t = \frac{1}{2} \sigma$, $f = 130\,MeV$ is the $\pi$ constant, $g$ is the coupling constant (we use $g = 0.59$ in the calculation below). Same as in [16], the factors of $\sqrt{M_D}$ and $\sqrt{M_{D^*}}$ have been absorbed into the $P$ and $P^*$ fields, and the Lagrangian is independent of the heavy-quark mass.

## 3 $D\bar{D}$ Potential and Bound State

Now we start to calculate the potential of $D$ and $\bar{D}$. Since there is no one-pion exchange between $D$ and $\bar{D}$ mesons, we are directly going to calculate the two-pion exchange potential. There are four two pion-exchange Feynman Diagrams as showed in Fig.1.
Our calculation is very similar to that for the nucleon-nucleon potential\cite{17}\cite{21}, except that the parallel box diagram in our case is a little bit different. In the nucleon case, it should be careful to extract "iterated one-pion exchange" contribution\cite{18}. Therefore, the "old-fashioned" time-ordered perturbation theory is widely used\cite{17}. However, there is not one-pion exchange between $D$ and $\bar{D}$ mesons, we can calculate directly using covariant perturbation theory.

The mass difference $\Delta = M_{D^\ast} - M_D$ plays an important role in our calculation, so we write the propagator of the $D^\ast$ meson like (for example of Fig.1a)

$$T_{D^\ast} = \frac{M_{D^\ast} \gamma_{\mu\nu}}{(p_1 - \frac{\vec{l}}{2} - q_2)^2 - M_{D^\ast}^2 + i\epsilon} = \frac{\gamma_{\mu\nu}}{-l_0 - 2\Delta + i\epsilon},$$  

(5)

where we use the conditions $p_1^2 = M_D^2$ and $q_0 = 0$. Then the rest calculation can be carried out directly like in\cite{17}.

### 3.1 Two-pion Exchange Potential

As usual, we work in center-of-mass(c.m.) system. And denote $p$ as the intial momentum, $p'$ as the final momentum, $q \equiv p - p'$ as the transferred momentum. In the following, we will use the symbols:

$$\omega_1 = \sqrt{(\vec{q} - \vec{l})^2 + 4m^2_\pi}$$  

(6)

$$\omega_2 = \sqrt{(\vec{q} + \vec{l})^2 + 4m^2_\pi}$$  

(7)

$$a = 2\Delta = 2(M_{D^\ast} - M_D) > 0$$  

(8)

After straightforward calculating each figure in Fig.1, we can get the potentials in momentum space: for the diagram a in Fig.1, parallel box digram:

$$V_a = -\frac{1}{16} \left( \frac{g}{f} \right)^4 (3 - 8t_1 \cdot t_2) \int \frac{d^3l}{(2\pi)^3} \frac{(l^2 - q^2)^2}{2a} \left[ \frac{1}{\omega_1(\omega_1 + a)} - \frac{1}{\omega_2(\omega_2 + a)} \right] \frac{1}{\omega_2 - \omega_1},$$  

(9)

the diagram b in Fig.1, crossing box digram:

$$V_b = -\frac{1}{16} \left( \frac{g}{f} \right)^4 (3 + 8t_1 \cdot t_2) \int \frac{d^3l}{(2\pi)^3} \frac{(l^2 - q^2)^2}{2a} \left[ \frac{1}{\omega_1(\omega_1 + a)^2} - \frac{1}{\omega_2(\omega_2 + a)^2} \right] \frac{1}{\omega_2^2 - \omega_1^2},$$  

(10)

the diagram c in Fig.1:

$$V_c = \frac{1}{16} \left( \frac{2g}{f} \right)^2 \frac{2}{f^2} t_1 \cdot t_2 \int \frac{d^3l}{(2\pi)^3} \frac{(l^2 - q^2)^2}{2} \left[ \frac{1}{\omega_1(\omega_1 + a)} - \frac{1}{\omega_2(\omega_2 + a)} \right] \frac{1}{\omega_2^2 - \omega_1^2},$$  

(11)

the diagram d in Fig.1:
\[ V_d = \frac{1}{4} \frac{1}{f^2} t_1 \cdot t_2 \frac{d^{3}l}{(2\pi)^3} \left( \frac{\omega_1}{\omega_1^2 - \omega_2^2} + \frac{\omega_2}{\omega_2^2 - \omega_1^2} \right), \]  

where \( t_1 \) and \( t_2 \) are the isospin matrices in the line 1 and 2 respectively.

To transform the potential from momentum space into coordinate space, the momentum space cutoff is used. This is a conceptually simple way and is mathematically easy too, we just need to bring a cutoff function and choose a suitable cutoff energy. Although it is not theoretically exactly true in physics, but this has been used widely in phenomenology physics, and is believed to be consistent with how the real world is going on. Like in the reference\[19\] [20] [21], we add a Gaussian form factor \( \exp \left( -\frac{q^2}{2\Lambda^2} \right) \) in each vertex (in momentum space) in the diagrams of Fig.1, that can regulate the loop integrals in the potential. Thus, the integrals can easily be carried out numerically. When we do the transformation, we use the formulas and techniques in Ref [21]. Here we only give the simple potential formulas.

After the Fourier transformation, the potential between \( DD \) can be obtained as following (where the function \( F(\lambda, r) \) can be found in Appendix):

\[ V(r) = V_a(r) + V_b(r) + V_c(r) + V_d(r), \]  

where

\[
V_a(r) = -\frac{1}{16} \frac{1}{\pi} \frac{g^4}{f^4} \frac{1}{(a^2 + \lambda^2)^2} \int d\lambda \frac{1}{a^2 + \lambda^2} \left[ \frac{2}{r^2} F'(\lambda, r) F'(\lambda, r) + F''(\lambda, r) F''(\lambda, r) \right] 
\]

\[
V_b(r) = -\frac{1}{16} \frac{1}{\pi} \frac{g^4}{f^4} \frac{1}{(a^2 + \lambda^2)^2} \int d\lambda \frac{a^2 - \lambda^2}{a^2 + \lambda^2} \left[ \frac{2}{r^2} F'(\lambda, r) F'(\lambda, r) + F''(\lambda, r) F''(\lambda, r) \right] 
\]

\[
V_c(r) = \frac{1}{16} \frac{1}{\pi} \frac{2g^4}{f^4} \frac{2}{t_1} \cdot t_2 \int d\lambda \frac{\lambda^2}{a^2 + \lambda^2} \left( F'(\lambda, r) F'(\lambda, r) \right) 
\]

\[
V_d(r) = \frac{2}{4\pi} \frac{1}{f^2} t_1 \cdot t_2 \int d\lambda \lambda^2 F^2(\lambda, r) 
\]

The potential is a function of energy cutoff parameter \( \Lambda \). In Fig.2 we show the potential with different \( \Lambda \). We can see the potential is sensitive to the cutoff \( \Lambda \). The value of \( \Lambda \) is not known right now, however, it is generally believed that \( \Lambda \) is around 1GeV. At most, it should not be larger than the mass of the D meson. We will discuss its value in the following.

It should be also mentioned here that, what we deal with the \( DD \) potential is also available for the \( DD \) potential, i.e., there is no difference between \( DD \) and \( DD \) up to the order of the two-pion exchange. Therefore, in the following what we talk about \( DD \) is also valid for \( DD \).
Fig.2 The potentials of $D\bar{D}$ with various $\Lambda$. The left is in the channel $I=0$ and the right is in the channel $I=1$.

3.2 Numerical Results

We numerically solve the Schrödinger equation to find the bound state of $D\bar{D}$. Our result is certainly dependent on $\Lambda$. For the channel $I=0$, we find that a stable bound state exists only when $\Lambda > 1750$ MeV. For $I=1$, a stable bound appear after $\Lambda$ increases to 2750 MeV. We summarize our results in Table 1. Although the potential in both of the channel $I=0,1$ is attractive, it seems that it is not deep enough to bound a molecule state if we believe $\Lambda$ should not be larger than 1 GeV. However, the experimental data about nucleus support a larger $\Lambda = 1150$ MeV [20]. Considering the mass of the D meson is heavier than nucleon’s, it could not completely exclude the possibility of the existence of the $D\bar{D}$ molecule state.

### Table 1

| $\Lambda$ [MeV] | $E_{\text{bind}}$ [MeV] | $E = M - E_{\text{bind}}$ [MeV] | $\Lambda$ [MeV] | $E_{\text{bind}}$ [MeV] | $E = M - E_{\text{bind}}$ [MeV] |
|-----------------|--------------------------|-----------------------------|-----------------|--------------------------|-----------------------------|
| 1700            | Not found                | -                           | 2700            | Not found                | -                           |
| 1750            | -4.54                    | 3725.12                     | 2750            | -1.53                    | 3728.13                     |
| 1800            | -21.1                    | 3708.56                     | 2800            | -14.71                   | 3714.95                     |
| 1850            | -48.57                   | 3681.09                     | 2850            | -36.19                   | 3693.47                     |
| 1900            | -90.68                   | 3638.98                     | 2900            | -68.68                   | 3660.98                     |
| 1950            | -151.04                  | 3578.62                     | 2950            | -114.53                  | 3615.13                     |
| 2000            | -233.54                  | 3496.12                     | 3000            | -176.06                  | 3553.60                     |

Table 1 The binding energy and the corresponding $\Lambda$ for the $D\bar{D}$ bound state. The left one is in the channel $I=0$, the right one is $I=1$.

4 $D\bar{D}^*$ Potential and Bound State

The calculation of the potential between $D$ and $D^*$ up to the order of the two-pion exchange can be carried out in the same way in the framework of the lagrangian [4]. The only exception is that the one-pion exchange plays an important role in the case of $D\bar{D}^*$. The existence of the one-pion exchange also makes the treatment of the parallel box diagram different. After giving the potential, we will discuss the possibility of the existence of the $D\bar{D}$ molecule state.

4.1 Two-pion Exchange Potential

Since there is the one-pion exchange between $\bar{D}D$, it needs to extract the ”iterated one-pion exchange” part from the parallel box diagram. Normally, such extraction can be done by using the time-ordered perturbation theory. A
more convenient way for us is done in the covariant perturbation theory. As pointed out in [18], in the covariant formulism, the "iterated one-pion exchange" part in the parallel box diagram can be extracted by simply omitting two poles, which is moving towards each other in heavy mass limit, in the \( l_0 \) integration. Then we obtain the two-pion exchange potential of \( D\bar{D}^* \) in the momentum space:

\[
\begin{align*}
V_{aD\cdot D} & = -\frac{1}{16} \frac{g}{f} (3 - 8t_1 \cdot t_2) \int \frac{d^4q}{(2\pi)^3} (i^2 - q^2) (\vec{i} - \vec{q})\alpha (\vec{i} + \vec{q})\beta \frac{1}{2} \left[ \frac{1}{\omega_1(\omega_1 + a)^2} - \frac{1}{\omega_2(\omega_2 + a)^2} \right] \frac{1}{\omega_1^2 - \omega_2^2} \\
V_{bD\cdot D} & = -\frac{1}{16} \frac{g}{f} (3 + 8t_1 \cdot t_2) \int \frac{d^4q}{(2\pi)^3} (i^2 - q^2) (\vec{i} - \vec{q})\alpha (\vec{i} + \vec{q})\beta \frac{1}{2} \left[ \frac{1}{(\omega_1^2 - a^2)(\omega_2^2 - a^2)} \right] \\
V_{c1D\cdot D} & = \frac{1}{4} \frac{g}{f^2} 2t_1 \cdot t_2 \int \frac{d^4l}{(2\pi)^3} (\vec{i} - \vec{q})\alpha (\vec{i} + \vec{q})\beta \frac{1}{2} \left[ \frac{2a}{\omega_2^2 - \omega_1^2 - \omega_2^2 - \omega_1^2} + \frac{1}{\omega_1^2 - \omega_2^2} \right] \\
V_{c2D\cdot D} & = \frac{1}{4} \frac{g}{f^2} 2t_1 \cdot t_2 I_{\alpha\beta} \int \frac{d^4l}{(2\pi)^3} (i^2 - q^2)^2 \frac{1}{2} \left[ \frac{\omega_1}{\omega_2^2 - \omega_1^2 + \omega_2^2 - \omega_1^2} \right] \\
V_{dd\cdot D} & = \frac{1}{4} \frac{g}{f^2} 2t_1 \cdot t_2 \int \frac{d^4l}{(2\pi)^3} I_{\alpha\beta} \left( \frac{\omega_1}{\omega_1^2 - \omega_2^2 + \omega_2^2 - \omega_1^2} \right)
\end{align*}
\]

(18)

The transformation to coordinate space can be done in the same way as we did to \( D\bar{D} \). Then we can get the two-pion exchange potential in coordinate space:

\[
V_{DD\cdot r} = V_{aD\cdot D}(r) + V_{bD\cdot D}(r) + V_{c1D\cdot D}(r) + V_{c2D\cdot D}(r) + V_{dd\cdot D}(r),
\]

(19)

where

\[
\begin{align*}
V_{aD\cdot D}(r) & = \frac{1}{16} \frac{1}{\pi} \frac{g}{f} (3 - 8t_1 \cdot t_2) (I - \frac{s \cdot \vec{r}s \cdot \vec{r}}{r^2})_{\alpha\beta} \int \frac{d^2l^2}{r^2} \left[ \frac{2a}{\omega_1^2 - \omega_2^2} \left( F'(\lambda, r) F'(\lambda, r) + F''(\lambda, r) F''(\lambda, r) \right) \right] \\
V_{bD\cdot D}(r) & = \frac{1}{16} \frac{1}{\pi} \frac{g}{f} (3 + 8t_1 \cdot t_2) (I - \frac{s \cdot \vec{r}s \cdot \vec{r}}{r^2})_{\alpha\beta} \frac{1}{2a} \left\{ \frac{2}{\omega_1^2 - \omega_2^2} \left[ \frac{2a}{\omega_1^2 - \omega_2^2} \right] \right\} \\
V_{c1D\cdot D} & = \frac{1}{4} \frac{g}{f^2} 2t_1 \cdot t_2 (I - \frac{s \cdot \vec{r}s \cdot \vec{r}}{r^2})_{\alpha\beta} \frac{1}{2a} \left[ \frac{2}{\omega_1^2 - \omega_2^2} \right] \left[ \frac{2a}{\omega_1^2 - \omega_2^2} \right] \\
V_{c2D\cdot D}(r) & = \frac{1}{4} \frac{g}{f^2} 2t_1 \cdot t_2 I_{\alpha\beta} \frac{1}{\pi} \int \frac{d^2l^2}{r^2} F'(\lambda, r) F'(\lambda, r) \\
V_{dd\cdot D}(r) & = \frac{1}{4} \frac{g}{f^2} 2t_1 \cdot t_2 I_{\alpha\beta} \frac{1}{\pi} \int d\lambda \frac{\lambda^2}{a^2 + \lambda^2} F'(\lambda, r) F'(\lambda, r)
\end{align*}
\]

(20)

where \( s \) is the spin matrix of the \( D^n \) and \( \alpha\beta \) is the indices of \( s \).

Since \( a = 2\Delta > 2m_\pi \), the term \( \sqrt{4m_\pi^2 - a^2} \) in above formulism is a imaginary number. This makes the potential [19] to be complex. In fact, the one-pion exchange potential of \( D\bar{D}^* \) is also complex. In the following we only choose the real part of \( V_{DD\cdot r} \) in solving the Schrödinger equation as many authors did in the studies of the \( D\bar{D}^* \) molecule state. Besides, we will average the matrix \( (I - \frac{s \cdot \vec{r}s \cdot \vec{r}}{r^2})_{\alpha\beta} \) in the numerically calculation.
4.2 One-pion Exchange Potential

The one-pion exchange potential is:

\[
V_{1\pi}(q) = -\frac{1}{3} \left( \frac{g_f}{f} \right)^2 t_1 \cdot t_2 \frac{q^2}{q^2 + m_\pi^2 - \Delta^2}
\]  

(21)

In the transformation to coordinate space, we also use the Gaussian type form factor for consistency. The corresponding potential is

\[
V_{1\pi}(r) = -\frac{1}{6} \left( \frac{g_f}{f} \right)^2 t_1 \cdot t_2 \Lambda^2 \frac{d}{d\Lambda} I_2(\sqrt{m_\pi^2 - \Delta^2}, r)
\]  

(22)

Since the monopole type form factor \( F(q) = \frac{\Lambda^2 - m_\pi^2}{\Lambda^2 - q^2} \) is used widely to regularize the one-pion exchange potential of \( D\overline{D}^* \), it is deserved to compare these two different type form factors by varying \( \Lambda \). We find \( \Lambda = 1000\,\text{MeV} \) in the Gaussian type is about \( \Lambda = 900\,\text{MeV} \) in the monopole type, therefore there is almost no difference between these two types form factors in the region of \( \Lambda \) where we are interested in. Before proceeding the discussion of the bound state, it is also useful to consider the convergence of the chiral expansion. In Fig.5, we show the dependence of the one-pion and the two-pion exchange potentials on the cutoff \( \Lambda \). In fact, even when \( \Lambda = 5\,\text{GeV} \), the chiral expansion is convergent quite well. So the regularization scheme works well.

4.3 Bound State

| \( \Lambda [\text{MeV}] \) | 4650 | 4700 | 4750 | 4800 | 4850 | 4900 | 4950 | 5000 |
|---|---|---|---|---|---|---|---|---|
| \( E_{\text{bind}} [\text{MeV}] \) | Not Found | -0.99 | -2.58 | -4.61 | -7.14 | -10.23 | -13.91 | -17.90 |

Table.2 The binding energy of the \( D\overline{D}^* \) bound state in the channel I=1 and the corresponding \( \Lambda \).

In Table.2 we summarize the binding energy of the \( D\overline{D}^* \) bound state and the corresponding value of \( \Lambda \) in I=1 state. One can find a stable bound state appears only when \( \Lambda > 4700\,\text{MeV} \). So the pure pion-exchange cannot bind \( D \) and \( \overline{D}^* \) as the X(3872).

5 B and B* Bound State

It is easy to extend above results to the case of \( B \) and \( B^* \). The only difference is that the mass difference between \( B \) and \( B^* \) is smaller than the mass of \( \pi \), so all potentials are real. The values of the \( B \) and \( B^* \) mesons in our calculation are \( M_B = 5279.5\,\text{MeV} \) and \( M_{B^*} = 5325.1\,\text{MeV} \). These masses are much bigger than the masses of \( D \) and \( D^* \), the bound states of \( B\overline{B} \) and \( B^*\overline{B} \) are more possible to exist. We list our results in Table.3 and Table.4.

| \( \Lambda [\text{MeV}] \) | \( E_{\text{bind}} [\text{MeV}] \) | \( E = M - E_{\text{bind}} [\text{MeV}] \) | \( \Lambda [\text{MeV}] \) | \( E_{\text{bind}} [\text{MeV}] \) | \( E = M - E_{\text{bind}} [\text{MeV}] \) |
|---|---|---|---|---|---|
| 1450 | Not Found | \( E \) | 1600 | Not Found | \( E \) |
| 1500 | -1.59 | 10557.41 | 1650 | -1.71 | 10557.29 |
| 1550 | -16.71 | 10542.29 | 1700 | -6.72 | 10552.28 |
| 1600 | -50.42 | 10508.58 | 1750 | -16.06 | 10542.94 |
| 1650 | -107.32 | 10451.68 | 1800 | -31.03 | 10527.97 |
| 1700 | -193.49 | 10365.51 | 1850 | -53.09 | 10505.91 |

Table.3 The binding energy of the \( B\overline{B} \) bound state and the corresponding \( \Lambda \). The left one is the I=0 state and the right one is the I=1 state.
\begin{table}[h]
\begin{tabular}{|c|c|c|}
\hline
$\Lambda$ [MeV] & $E_{\text{bind}}$ [MeV] & $E = M - E_{\text{bind}}$ [MeV] \\
\hline
1700 & Not Found & - \\
1800 & -0.12 & 10604.58 \\
1900 & -1.47 & 10603.13 \\
2000 & -3.98 & 10600.62 \\
2100 & -7.88 & 10596.72 \\
2200 & -13.36 & 10591.24 \\
\hline
\end{tabular}
\caption{The bind energy of the $BB^*$ bound state in the channel $I=1$ and the corresponding $\Lambda$.}
\end{table}

6 Summary and Conclusion

In this work, we calculate the two-pion exchange potential between $D$ mesons and(or) $D^*$ mesons by using Gaussian type form factor to regularize loop integrals. We check the convergence of the chiral perturbative expansion by varying the cutoff $\Lambda$ and make sure the validness of the regularization scheme. By using the obtained potentials we discuss the possibility of the existence of the heavy-meson molecule states. Opposite to the naive expectation, we find the existence of the $DD$ molecule state is more possible than that of the $DD^*$ molecule state. The existence of a $DD$ molecule state requires a large cutoff $\Lambda = 1750$ MeV. Comparing with $\Lambda = 1150$ MeV in the deuteron case\cite{20}, a $DD$ molecule state indeed is not favorable. However, if we consider the fact that the $D$ meson is heavier than nucleons, we cannot rule out completely the possibility of the existence of the $DD$ molecule states. Similar to \cite{13}, our result excludes that the pure-pion exchange can bind $D$ and $D^*$ mesons as a molecule state. We also extend our study to the case of $B$ and $B^*$ mesons. Since the mass of $B$ meson is much heavier than the $D$ meson’s, the existence of the $B$ and(or) $B^*$ molecule state is more possible than that in the $D$ meson case. However, the situation does not become much better than that of the $DD$ case.

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References

[1] J. D. Weinstein and N. Isgur, Phys. Rev. Lett. 48, 659 (1982). J. D. Weinstein and N. Isgur, Phys. Rev. D 27, 588 (1983).
[2] T. Barnes, Phys. Lett. B 165, 434 (1985).
[3] N. A. Tornqvist, Z. Phys. C 61, 525 (1994) \href{http://arxiv.org/abs/hep-ph/9310247}{arXiv:hep-ph/9310247}.
[4] S. K. Choi et al. [Belle Collaboration], Phys. Rev. Lett. 91, 262001 (2003) \href{http://arxiv.org/abs/hep-ex/0309032}{arXiv:hep-ex/0309032}.
[5] D. E. Acosta et al. [CDF II Collaboration], Phys. Rev. Lett. 93, 072001 (2004) \href{http://arxiv.org/abs/hep-ex/0312021}{arXiv:hep-ex/0312021}.
[6] V. M. Abazov et al. [D0 Collaboration], Phys. Rev. Lett. 93, 162002 (2004) \href{http://arxiv.org/abs/hep-ex/0404050}{arXiv:hep-ex/0404050}.
[7] B. Aubert et al. [BABAR Collaboration], Phys. Rev. D 71, 071103 (2005) \href{http://arxiv.org/abs/hep-ex/0406022}{arXiv:hep-ex/0406022}.
[8] F. E. Close and P. R. Page, Phys. Lett. B 578, 119 (2004) \href{http://arxiv.org/abs/hep-ph/0309253}{arXiv:hep-ph/0309253}.
[9] M. B. Voloshin, Phys. Lett. B 579, 316 (2004) \href{http://arxiv.org/abs/hep-ph/0309307}{arXiv:hep-ph/0309307}.
[10] E. S. Swanson, Phys. Lett. B 588, 189 (2004) \href{http://arxiv.org/abs/hep-ph/0311229}{arXiv:hep-ph/0311229}.
[11] C. Y. Wong, Phys. Rev. C 69, 055202 (2004) \href{http://arxiv.org/abs/hep-ph/0311088}{arXiv:hep-ph/0311088}.
Appendix A

Here we list the functions used in the potentials:

\[ F(\lambda, r) = e^{\frac{\lambda^2}{2}} I_2(\sqrt{(2m\pi)^2 + \lambda^2}, r) \]

\[ I_2(m, \lambda) = \frac{m}{4\pi} \phi^0_c(m, r) \]

\[ \phi^0_c(m, r) = e^{\frac{m^2}{4\pi}} \left[ e^{-mr} \text{Erfc}(-\frac{\Lambda r}{2} + \frac{m}{\Lambda}) - e^{-mr} \text{Erfc}(\frac{\Lambda r}{2} + \frac{m}{\Lambda}) \right] \frac{1}{2mr} \]

\[ \phi^0_{so}(m, r) = e^{\frac{m^2}{4\pi}} \left\{ (1 + mr)e^{-mr} \text{Erfc}(-\frac{\Lambda r}{2} + \frac{m}{\Lambda}) - (1 - mr)e^{-mr} \text{Erfc}(\frac{\Lambda r}{2} + \frac{m}{\Lambda}) \right\} - \frac{4}{\sqrt{\pi}} \left( \frac{\Lambda r}{2} \right) e^{-\frac{(\Lambda r)^2}{4}} \frac{1}{2(mr)^3} \]

\[ \phi^1_c(m, r) = \phi^0_c(m, r) - \frac{1}{2\sqrt{\pi}} \left( \frac{\Lambda}{m} \right)^3 e^{-\frac{(\Lambda r)^2}{4}} \]

\[ \frac{d}{dr} \phi^0_c(m, r) = -m^2 r \phi^0_{so}(m, r) \]

\[ \frac{d^2}{dr^2} \phi^0_c(m, r) = m^2 \phi^1_c(m, r) - \frac{2}{r} \frac{d}{dr} \phi^0_c(m, r), \]

where the function \( \text{Erfc}(x) \) is the complementary error function. More details can be found in the Appendix of Ref[21].

Appendix B

Here we show the figures of two-pion and single-pion exchange potential of \( \bar{D}D \).
Fig. 3 In this figure, we show the potential of $D \bar{D}^*$ in the channel $I=1$ when $\Lambda = 1000\text{MeV}$. The real line is the two-pion exchange potential, the dashed line is the one-pion exchange potential, and the dotted line is the total potential.

Fig. 4 In this figure, we show the total potential of $D \bar{D}^*$ in the channel $I=1$ with different $\Lambda$. 
Fig. 5 Here we show the single-pion exchange potential and two-pion exchange potential of $DD^*$ in the channel I=1 with different $\Lambda$. 