Investigation of the stability of a cantilevered pipeline with a fluid flow compressed at the free end by the tracking force

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Abstract. The paper presents a numerical and analytical investigation of the stability of a cantilevered pipeline transporting an incompressible fluid flow at a constant speed compressed by tracking force at the free end. To solve the problem, a dynamic approach based on the analysis of the behavior of the system's own characteristics is used. In contrast to the classical research approaches, which are based on the representation of the solution of the problem using a finite number of basic functions, an algorithm that does not require such a restriction is proposed.

1. Introduction
Consideration of problems related to the interaction of elements of mechanical structures and their streamlined fluid flow is relevant for the modern development of the technical industry, including mechanical engineering. Quite often, structural elements are represented by a system of pipes for which the liquid is the internal medium. The main feature of the interaction of a structure with a liquid can be considered a significant change in the dynamic properties of the structure and the occurrence of vibrations for its individual components, which can lead to its destruction or decommissioning [1, 2]. Quite often, the pipeline is modeled as hollow rods, streamlined by the flow of liquid. Under this assumption, the hydrodynamic component of the system can be eliminated by adding hydrodynamic forces acting from the fluid on the structural elements to the mathematical model of the system. In this case, everything is reduced to solving problems related to the study of dynamics and stability of distributed mechanical systems using the theory of rods.

2. Mathematical model
Let us consider small flat low-frequency bending vibrations of a cantilevered pipeline loaded at the free end by a tracking force, which transports an incompressible fluid flow at a constant speed. In [2, 3], the pipeline oscillation equation is derived. In [4-5], the results of stability studies in the absence of compressive load are presented.

The equation describing the movement of the pipeline taking into account internal friction in the material according to the Kelvin-Voigt hypothesis and the corresponding boundary conditions have the form:

\[
\mu EI \frac{\partial^5 y}{\partial x^5 \partial t} + EI \frac{\partial^4 y}{\partial x^4} + (M \nu^2 + P) \frac{\partial^2 y}{\partial x^2} + 2Mv \frac{\partial^2 y}{\partial x \partial t} + (m + M) \frac{\partial^2 y}{\partial t^2} + \xi \frac{\partial y}{\partial t} = 0,
\]  

(1)
2.

\( y|_{x=0} = \frac{\partial y}{\partial x}|_{x=0} = EI \frac{\partial^2 y}{\partial x^2}|_{x=l} = EI \frac{\partial^3 y}{\partial x^3}|_{x=l} = 0, \) \hspace{1cm} (2)

where \( y(x,t) \) – transverse displacement of the pipeline, \( m \) and \( M \) – per unit length weight of pipeline and liquid, respectively, \( EI \) – is the exural rigidity of the pipeline, \( P \) – is the longitudinal compressive load, \( l \) – is the length of the pipeline, \( \mu \) and \( \xi \) – are the coefficients of internal and external friction, respectively, \( v \) – the speed of fluid flow.

To write the equation of motion (1) and boundary conditions (2) in dimensionless parameters, we replace the variables:

\[ x = l \varphi \quad (0 \leq \varphi \leq 1), \quad t = \left( \frac{m + M}{EI} \right)^{1/2} \ell^2 \tau. \]

As a result, we get:

\[ \gamma \frac{\partial^5 y}{\partial \varphi^5 \partial \tau} + \frac{\partial^4 y}{\partial \varphi^4} + (b + c) \frac{\partial^3 y}{\partial \varphi^3} + 2a \frac{\partial^2 y}{\partial \varphi^2} + \frac{\partial^2 y}{\partial \varphi \partial \tau} + \delta \frac{\partial y}{\partial \tau} = 0, \] \hspace{1cm} (3)

\[ y|_{\varphi=0} = \frac{\partial y}{\partial \varphi}|_{\varphi=0} = \frac{\partial^2 y}{\partial \varphi^2}|_{\varphi=0} = \frac{\partial^3 y}{\partial \varphi^3}|_{\varphi=0} = 0, \] \hspace{1cm} (4)

where:

\[ a = \frac{Mvl}{EI} \left( \frac{EI}{m + M} \right)^{1/2}, \quad b = \frac{Mv^2 \ell^2}{EI} = a^2 \frac{m + M}{M}, \quad c = \frac{P \ell^2}{EI}, \]

\[ \delta = \frac{\xi \ell^2}{EI} \left( \frac{EI}{m + M} \right)^{1/2}, \quad \gamma = \frac{\mu}{\ell^2} \left( \frac{EI}{m + M} \right)^{1/2}. \]

3. Stability study of a compressed cantilevered pipeline with a liquid flow

We will search for the solution of problem (3, 4) using the variable separation method:

\[ y(\varphi, \tau) = W(\varphi) e^{\lambda \tau}. \] \hspace{1cm} (5)

After substituting the expression (5) into equation (3) and boundary conditions (4), a boundary value problem for the eigenvalue problem is obtained:

\[ (1 + \gamma \lambda) \frac{d^4 W(\varphi)}{d \varphi^4} + (b + c) \frac{d^3 W(\varphi)}{d \varphi^3} + 2a \lambda \frac{d^2 W(\varphi)}{d \varphi^2} + (\delta \lambda + \lambda^2) W(\varphi) = 0 \] \hspace{1cm} (6)

\[ W(\varphi)|_{\varphi=0} = \frac{d W(\varphi)}{d \varphi}|_{\varphi=0} = \frac{d^2 W(\varphi)}{d \varphi^2}|_{\varphi=0} = \frac{d^3 W(\varphi)}{d \varphi^3}|_{\varphi=0} = 0 \] \hspace{1cm} (7)

The eigenvalue problem consists of an ordinary fourth-order differential equation (6) and homogeneous boundary conditions (7).

The general solution of the problem (6) has the form:

\[ W(\varphi) = \sum_{i=1}^{4} A_i e^{W_i \varphi}, \]

where \( W_i = W_i(\lambda, a, b, c, \gamma, \delta), \ i = \frac{1}{4}, \) – roots of the characteristic equation

\[ F(\lambda, W) = (1 + \gamma \lambda) W^4 + (b + c) W^2 + 2a \lambda W + (\delta \lambda + \lambda^2) = 0 \] \hspace{1cm} (8)

The coefficients \( A_i (i = \frac{1}{4}) \) are determined from the boundary conditions of problem (7) as a result of solving a system of linear homogeneous algebraic equations.
For the existence of a non-trivial solution of system (9), it is necessary and sufficient that its determinant $D(W_1, W_2, W_3, W_4)$ vanishes:

$$D(W_1, W_2, W_3, W_4) = 0$$  \hspace{1cm} (10)

From equations (8) and (10), we can write a system of five equations with five unknowns considered over a field of complex numbers:

$$\begin{cases}
F(\lambda, W_i) = 0, & i = 1, 4, \\
D(W_1, W_2, W_3, W_4) = 0.
\end{cases}$$  \hspace{1cm} (11)

Starting from zero values of the dimensionless parameter of the flow velocity of the fluid (parameter $a$) and gradually increasing it, and by varying a dimensionless parameter of compressive load (option $c$) from zero to some value when the current value of the dimensionless speed parameter, it is possible to determine critical values of parameters $a$ and $c$, which will cause the loss of stability in the system (figure 1). This happens when the real part of one of the eigenvalues of $\lambda$ in the system (11) becomes positive. All parameters describing the mechanical system, with the exception of the fluid flow rate and compressive load, are fixed. The system (11) is solved by the iterative Newton method, and the initial values are taken as the values in the absence of compressive load and friction losses at zero fluid flow velocity [6], and later, when the parameters $a$ and $c$ are changed, the solutions of the system obtained at the previous step of changing these parameters are taken.

4. The results of the research

The conducted studies allow us to divide the plane of parameters of the dimensionless fluid flow rate and dimensionless load ($a; c$) into stable and unstable regions of the system (figure 1). Four cases are considered in this paper: no friction losses ($\gamma = 0, \delta = 0$, curve 1), the presence of only internal friction ($\gamma = 0.0015, \delta = 0$, curve 2), the presence of only external friction ($\gamma = 0, \delta = 2.52$, curve 3) and the presence of both types of friction ($\gamma = 0.0015, \delta = 2.52$, curve 4). The results are presented for a system corresponding to the flow of water in a steel pipe. The pipe has a length of $l = 1$ m, its external and internal radiuses are $R = 5 \cdot 10^{-3} m$ and $r = 4 \cdot 10^{-3} m$, respectively.

![Figure 1. Stability region of the system.](image_url)
Studies have shown that the loss of stability is of the flutter type and occurs in the second form of vibrations. Figure 2-5 provides a qualitative representation of the hodographs of the first three eigenvalues of the system on the complex plane as the fluid flow rate increases from zero at a constant dimensionless load $c$ for four different cases presented earlier. In all cases, the eigenvalues start moving to the left. Then the eigenvalue corresponding to the second oscillation mode changes its direction and passes into the positive real half-plane (there is a loss of stability). If there is no friction, the eigenvalues begin their movement from the imaginary axis, and if there is friction from the negative real half-plane. If there is only external friction, the movement begins with an axis parallel to the imaginary one, and if there is only internal friction, the real parts of the eigenvalues in the absence of a fluid flow have different values. It can be noted that internal friction has a stronger effect on the older modes of vibrations compared to the younger ones. In the presence of both types of friction, the critical velocity of the fluid flow at which the loss of stability occurs is of the greatest importance compared to the other cases.

**Figure 2.** No friction in the system.

**Figure 3.** The presence of only external friction.

**Figure 4.** The presence of only internal friction.

**Figure 5.** Presence of both types of friction.
Table 1 shows the numerical values of the critical dimensionless load for some values of the dimensionless fluid flow velocity, depending on the parameters of internal and external friction.

**Table 1.** Dependence of the dimensionless load on the dimensionless fluid flow rate.

| a  | \( \gamma = 0 \) | \( \gamma = 0.0015 \) | \( \gamma = 0 \) | \( \gamma = 0.0015 \) |
|----|------------------|------------------|------------------|------------------|
| 0.10 | 17.578 | 20.078 | 20.469 | 20.625 |
| 0.50 | 16.797 | 18.047 | 19.688 | 20.078 |
| 1.00 | 14.453 | 15.391 | 17.422 | 17.891 |
| 1.50 | 10.469 | 14.141 | 13.750 | 14.141 |
| 2.00 | 4.844 | 8.672 | 8.281 | 5.391 |
| 2.30 | 0.703 | 4.609 | 4.219 | 1.250 |

Figure 6-8 shows, respectively, graphs of approximations of the dependence of the real parts of the first three eigenvalues on the dimensionless parameters of the fluid flow velocity \( a \) and the compressive load \( c \). According to the corresponding graphs of the level lines, it can be seen that the loss of stability also occurs in the second form of system oscillations.

**Figure 6.** Dependence of the real part of the first eigenvalue on the system parameters.

**Figure 7.** Dependence of the real part of the second eigenvalue on the system parameters.
Figure 8. Dependence of the real part of the third eigenvalue on the system parameters.

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