Multi-Response Optimization of Mechanical Properties of Hybrid (Fiberglass / Abaca Woven) in Polyester Matrix using Desirability Function based on DOE

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Abstract. The study of the application and mechanical behavior of new composite and hybrid materials within the bodywork industry in its different parts (external and internal) has led to perform different characterizations and analysis of results which determined the material with the best mechanical properties in many cases depending on the subjectivity of the researcher. In the present study the optimization of the mechanical properties tensile and flexural hybrid composite made of natural fiber and synthetic fiber was made. To produce the hybrid fabric, abaca (AbF) and cotton (CF) over glass (FG), combined with a polymer matrix polyester resin was used. The configuration of the fibers and the type of drying (ambient and oven), were taken as input factors. The influence of these factors on the overall responses were analyzed using factorial design. A function of desirability was used for the optimization of experimental responses. The results indicated that the optimal factors to take advantage of the experimental responses or mechanical properties, were the configuration of 20% FV + 7.75% FAb (45°) + 2.25% FAI (135°) and the type of oven drying, with a value of global desirability of 0.8208.

1. Introduction

The study, application and the use of new composite materials of polymeric matrix have had a great development within the world industry, from the obtaining of prototypes, parts and pieces to the manufacture of large constructions, where the insertion of metallic reinforcements, particles and fibers (synthetic and natural) have tried to improve physical, chemical and mechanical properties. This has caused the increase in the range of materials and even increasing the market competitiveness of this type of elements [1, 2]. At the same time, the continuous interest in the preservation of environment and increase in use of green composites materials has led to an inclination to research composite materials and hybrids with natural fibers [3]. Research efforts in this field are aimed at finding possible plant fibers that match or exceed the characteristics of a synthetic fiber. In addition to the above, these fibers have been combined to improve the final properties at the lowest possible cost. [4, 5].

The application of these engineering materials in the secondary industry has grown in recent times, when metal parts began to be replaced by compounds with similar properties but obtaining a greater advantage in the reduction of structural weight and manufacturing costs. Synthetic fibers (carbon, glass, kevlar, hony comb, etc.) have been used mainly in composite materials within the automotive, body, naval and aeronautical industries [6-9]. The advantages of natural fibers in the manufacture of parts and pieces for several years have been investigated and applied, reaching an excellent behavior, physical, chemical and mechanical. These fibers can act alone in a polymeric matrix or, sometimes, combining with synthetic fibers forming the so-called hybrid materials, which have great results in
mechanical resistance and economic. This is the case of the Mercedes Benz E-Class cars of the company Daimler AG, where it was possible to improve the original material of the panels of doors to the replacement by epoxy resin embedded in sisal fiber, obtaining a notable weight reduction of 20% [10].

In Ecuador there are several projects related to composite materials and the use of vegetable fiber, such as the developed in the project PIC-08-493 "Development of new materials for structural and industrial applications" [1] in which aspects such as the type of matrix, the morphology of fibers, surface treatments and mechanical properties. This has favored the use of greenmaterials such as abaca fiber, jute, coconut, cabuya, among others, in production activities of the industrial sector, thus benefiting the creation of new sources of employment with the possibility of promoting sustainable economic development in areas rural [11]. Ecuador is the second largest producer of abaca after the Philippines. The areas of Quinindé, Santo Domingo de los Tsáchilas, Quevedo and La Maná have a combined production of 10,000 tons per year. The abaca plant is an herbaceous structure belonging to the musaceae family (musa textiles), it is native to the Philippines and it grows in humid tropics. The fibers come from the sheath of the leaves that make up the pseudostem of the plant, their cells are characterized by being long and thin [12]. These fibers have a length ranging from 1 to 2 m or more with a diameter of 0.1 - 0.28 and are chemically formed by cellulose, hemicellulose, pectin, lignin. There are 5 grades of abaca fibers that are increasing according to their grade of quality, being grade 1 those of better quality and properties [5]. Nowadays, the experimental designs assisted with statistical tools allow an appropriate analysis of the mechanical behavior of the materials, as well as to predict said behavior for a given application [13, 14]. The use of DOE in the industry is for the purpose of solving a problem or checking an idea through statistical tests. This corresponds to effectively identifying which tests should be performed and how to do them, obtaining data that, when analyzed statistically, provide objective evidence to answer the questions posed [15]. Studies such as the analysis of the flexural strength of mild steel A36 by design of experiments, determine in the field of metalworking the best welding process under specific conditions [16]. Applied to thermodynamics DOE allows to optimize the accuracy of a certain temperature reading, as mentioned by F. Barari et al. [17]. In the development of composite materials E.H.Agung et al. [18] chemically modifies a high impact polystyrene composite reinforced with abaca fiber and uses a Box-Behnken design with desirable function to choose the acceptable combination of weight percentages of its components. Suardi et al. [19], evaluates the statistical properties of the diameter of abaca fibers and identifies a statistical method (weakest link, Weibull, t-test) to suppress the dispersion of tensile strength.

The present work used an approach based on the design of DOE experiments which seeks to analyze statistically and with the help of the desirability function [18][20][21] the experimental results of the proposed combinations (configuration and type of drying) in order to establish the optimal combination of the input factors for the tensile and flexural mechanical properties of the hybrid composite material made of abaca (AbF) and cotton (CF) plus glass (FG), combined with a matrix polyester resin polymer.

2. Methodology

2.1. Materials and Equipment

2.1.1. Materials. The woven fibers of abaca, cotton and fiberglass are adhered to the polymeric matrix to form a hybrid composite material. Unsaturated polyester resin was used as the thermostable polymer matrix. For this study, quality abaca fiber 1 with a diameter of approximately 0.252 mm was used and obtained from the localities of Quevedo and Santo Domingo (Ecuador). The following table summarizes the materials used in the preparation of the composite:
### Table 1. Materials.

| Material                      | Brand     | Characteristic                  | Country of origin |
|-------------------------------|-----------|---------------------------------|-------------------|
| Fiberglass                    | Jushi     | Synthetic fiber of 375 gr/m²     | China             |
| Abaca fiber                   | -         | Natural fiber                   | Quevedo - Ecuador |
| Polyester resin               | Andercol 836 | Unsaturated resin               | Medellin          |
| Cobalt octoate                | Andercol  | 12% w/w                         | Colombia          |
| Peroxide metal - ethyl ketone | -         | Polymerization initiator        | Ecuador           |
| Desmoldable wax               | Simoniz   | Paraffin paste wax              | Colombia          |

#### 2.1.2. Equipment.

For the laboratory tests of tensile and flexural, was used the Universal testing machine Metro Test 50 kN for polymeric, ceramic and composite materials. ASTM D3039 regulations, ASTM D7264 were used to prepare specimens for each test. An infrared curing furnace available at the Faculty of Mechanical Engineering, Technical University of Ambato was used to dry the specimens.

#### 2.2. Experimental process

By means of the compression stratification process, the test pieces were obtained with the following volumetric fractions: 70% matrix (polyester resin) and 30% reinforcement (natural fiber plus glass fiber). An A 36 steel mold was fabricated to make the specimens based on the minimum number of specimens in accordance with ASTM D standards. Different configurations were made for the hybrid compound. Cotton fiber plus abaca (CF + AbF) with orientations of 45 °, 90 ° and 135 ° plus fiberglass (FG) with random orientation. Seven cases of studies that are shown in Table 2 were analyzed.

### Table 2. Cases of hybrid material.

| Cases | Configuration                                      |
|-------|---------------------------------------------------|
| Case 1 | 20% FG + 7.75% AbF (0 °) + 2.25% CF (90 °)         |
| Case 2 | 10% FG + 17.75% AbF (0 °) + 2.25% CF (90 °)        |
| Case 3 | 20% FG + 7.75% AbF (45 °) + 2.25% CF (135 °)       |
| Case 4 | 10% FG + 17.75% AbF (45 °) + 2.25% CF (135 °)      |
| Case 5 | 23.25% AbF (0 °, 45 °, 135 °) + 6.75% CF (90 °, 135 °, 225 °) |
| Case 6 | 10% FG + 20% Long Abaca Fiber                      |
| Case 7 | 10% FG + 20% Short Abaca Fiber                     |

Two types of curing were carried out: ambient and oven. The specimens were subjected to the curing process at an ambient temperature of 18 °C to 21 °C and cured in an infrared drying oven, at a temperature of 80 °C. Specimens of the dimensions of 250x25x3 mm were obtained for the tensile tests and 160x13x3 for the flexural tests.

After the laboratory tests a database was obtained that is composed of the following information: Tensile tests: There are five observations and three properties analyzed, maximum tensile strength, tensile modulus of elasticity and strain, giving a total of 70 processed data by property. Flexural tests: There are five observations and four analyzed properties, deflection, maximum flexural strength, flexural secant modulus of elasticity and flexural strain, giving a total of 70 processed data by property. These data are necessary for the development of the design of experimental.
2.3. Experimental Design

The design of experiments aims to answer the question posed of comparing the combinations tested (configuration and type drying) in order to choose the best. For this study it is required measuring the global quality of the adjustment of the regression models of three different experimental designs by means of the determination coefficients ($R^2$ and $R^2_a$), to choose the model that best fits the data. It was proposed to use a 7x2 factorial design, understanding that the effect of the factors, such as configuration and type of drying, is significant; as well as the effect of interaction of these factors on the answers. The factors and levels are described in table 3.

| Table 3. Experiment factors and levels. |
|-----------------------------------------|
| Levels | Configuration | Drying Type |
| Level 1 | Case 1 | Ambient |
| Level 2 | Case 2 | Oven |
| Level 3 | Case 3 | |
| Level 4 | Case 4 | |
| Level 5 | Case 5 | |
| Level 6 | Case 6 | |
| Level 7 | Case 7 | |

2.4. Desirability function analysis

The desirability function analysis used in this study is based on transforming the responses predicted by the model $\hat{y}$ of each mechanical properties of the tested hybrid for different combinations in dimensionless values within the interval [0,1] denominated as individual desirabilities $d_i$.

Individual desirability when you want to maximize the output response.

$$d_i = \begin{cases} 0, & \hat{y} \leq y_{min} \\ \left(\frac{\hat{y} - y_{min}}{y_{max} - y_{min}}\right)^r, & y_{min} \leq \hat{y} \leq y_{max}, r \geq 0 \\ 1, & \hat{y} \geq y_{max} \end{cases}$$  \hspace{1cm} (1)

Individual desirability when you want to minimize the output response.

$$d_i = \begin{cases} 1, & \hat{y} \leq y_{min} \\ \left(\frac{\hat{y} - y_{max}}{y_{min} - y_{max}}\right)^r, & y_{min} \leq \hat{y} \leq y_{max}, r \geq 0 \\ 0, & \hat{y} \geq y_{max} \end{cases}$$  \hspace{1cm} (2)

Where, $y_{max}$ is the highest specification of response, $y_{min}$ is the lowest specification response and $r$ is a constant that defines the shape of the desirability function for each response.

After calculating the individual desirability, we proceed to establish a value that measures the average convenience in each combination, which converts the problem of optimizing multiple responses into a problem of a single, objectively analyzable response. Equation 3 is used to calculate global convenience.

$$DG = (d_1^{w_1} \ast d_2^{w_2} \ast \ldots \ast d_l^{w_l})^{1/w}$$  \hspace{1cm} (3)

Where, $w_i$ is the relative importance of each response and $w$ is the sum of the relative importance.
3. Results and discussions

3.1. Analysis of variance of the mechanical properties

It is determined that the regression model that best fits the data is the model obtained from the complete factorial design. This model presents values of the coefficient of determination closer to 100% compared to other designs of experiments such as the One-way ANOVA (configuration) and the Two-factor ANOVA (configuration, drying). The coefficients of determination are described in Table 4.

The analysis of variance identifies the importance of each of the factors as well as their interrelation with the response variables. The p-value of the model, of each factor and of the interrelationships for all the mechanical properties analyzed have a value less than 0.05. Therefore, all of them are defined as statistically significant, there being representative differences between the combinations. The applied experiment combines all input factors, obtaining a total of 14 combinations as dictated by the complete 7x2 factorial design. Five replicas were made for each analyzed property.

Table 4. Coefficients of determination for each model analyzed

| Mechanical properties | One-way ANOVA | Two-factor ANOVA | Full factorial |
|-----------------------|--------------|----------------|---------------|
|                       | R<sup>2</sup> | R<sub>aj</sub><sup>2</sup> | R<sup>2</sup> | R<sub>aj</sub><sup>2</sup> | R<sup>2</sup> | R<sub>aj</sub><sup>2</sup> |
| Maximum tensile strength | 69.08% | 66.14% | 75.23% | 72.43% | 92.31% | 90.52% |
| Tensile modulus of elasticity | 34.67% | 28.45% | 67.98% | 64.36% | 80.69% | 76.21% |
| Strain | 47.23% | 42.20% | 69.39% | 65.93% | 81.62% | 77.35% |
| Deflection | 57.67% | 53.64% | 76.68% | 74.05% | 86.37% | 83.20% |
| Maximum flexural strength | 58.85% | 54.93% | 74.18% | 71.26% | 88.37% | 85.71% |
| Flexural secant modulus of elasticity | 62.79% | 59.24% | 83.33% | 81.45% | 96.12% | 95.22% |
| Flexural strain | 61.19% | 57.49% | 74.59% | 71.72% | 84.33% | 80.69% |

3.2. Desirability Function Analysis

In the individual desirability, the criterion of maximizing the output response is used by equation 1 for the properties: Maximum Traction Effort, Elasticity Module, Elongation Percentage, Maximum Flexion Effort and Bending Sequence Elasticity Module. On the other hand, the criterion of minimizing the output response was used using Equation 2 for: Deflection and Maximum Deformation. For all cases, the value of 1 was assigned to the constant that defines the form of the function, while for the values of the specifications the values described in Table 5 were used.

Table 5. Response specifications.

| Mechanical properties | y<sub>min</sub> | y<sub>max</sub> | Objective |
|-----------------------|----------------|----------------|-----------|
| Maximum tensile strength [MPa] | 19.380 | 77.150 | Maximize |
| Tensile modulus of elasticity [MPa] | 458.480 | 2459.880 | Maximize |
| Strain [%] | 1.890 | 4.940 | Maximize |
| Deflection [mm] | 4.752 | 17.952 | Minimize |
| Maximum flexural strength [MPa] | 11.250 | 174.928 | Maximize |
| Flexural secant modulus of elasticity [MPa] | 495.370 | 17547.240 | Maximize |
| Flexural strain [%] | 0.934 | 3.180 | Minimize |
The relative importance was established, these constants being equal to 1. Applying this data to equation 3, a point value was generated that is comparative for each combination. The values of individual desirability and global desirability are detailed in Table 6.

Table 6. individual desirability and overall desirability for answers

| Combinations | Individual desirability | DG   | Rank |
|--------------|-------------------------|------|------|
|              | $d_1^a$ | $d_2^b$ | $d_3^c$ | $d_4^d$ | $d_5^e$ | $d_6^f$ | $d_7^g$ |     |
| Case 1 Ambient | 1.0000 | 0.7361 | 0.6911 | 0.5190 | 0.5553 | 0.2706 | 0.5227 | 0.5748 | 5   |
| Case 2 Ambient | 0.6865 | 0.7000 | 0.4380 | 0.2077 | 0.4011 | 0.1417 | 0.2342 | 0.3450 | 10  |
| Case 3 Ambient | 0.6160 | 0.6704 | 0.4007 | 0.6845 | 0.5118 | 0.2788 | 0.6073 | 0.5166 | 6   |
| Case 4 Ambient | 0.4364 | 0.6118 | 0.2643 | 0.1997 | 0.3673 | 0.1632 | 0.1941 | 0.2879 | 12  |
| Case 5 Ambient | 0.6510 | 0.6182 | 0.4852 | 0.1849 | 0.1934 | 0.0569 | 0.1149 | 0.2399 | 13  |
| Case 6 Ambient | 0.9013 | 1.0000 | 0.3489 | 0.5098 | 0.4586 | 0.2140 | 0.4746 | 0.4968 | 7   |
| Case 7 Ambient | 0.2785 | 0.5201 | 0.1587 | 1.0000 | 0.1028 | 0.1458 | 1.0000 | 0.3201 | 11  |
| Case 1 Oven   | 0.8366 | 0.6024 | 0.7154 | 0.9623 | 0.7065 | 0.8584 | 0.9457 | 0.7940 | 2   |
| Case 2 Oven   | 0.7889 | 0.4281 | 1.0000 | 0.4633 | 0.7200 | 0.4157 | 0.4630 | 0.5785 | 4   |
| Case 3 Oven   | 0.8234 | 0.6606 | 0.6197 | 0.8639 | 1.0000 | 1.0000 | 0.8620 | 0.8208 | 1   |
| Case 4 Oven   | 0.1939 | 0.3340 | 0.3036 | 0.6035 | 0.6426 | 0.4730 | 0.6589 | 0.4218 | 9   |
| Case 5 Oven   | 0.0478 | 0.0510 | 0.6839 | 0.4810 | 0.0272 | 0.0124 | 0.1923 | 0.0910 | 14  |
| Case 6 Oven   | 0.6195 | 0.5173 | 0.5902 | 0.8618 | 0.7180 | 0.6506 | 0.7952 | 0.6699 | 3   |
| Case 7 Oven   | 0.2110 | 0.2625 | 0.4334 | 0.8573 | 0.4808 | 0.4790 | 0.8468 | 0.4546 | 8   |

- $a$ Desirability of maximum tensile strength
- $b$ Desirability of tensile modulus of elasticity
- $c$ Desirability of strain
- $d$ Desirability of deflection
- $e$ Desirability of maximum flexural strength
- $f$ Desirability of flexural secant modulus of elasticity
- $g$ Desirability of flexural strain

Figure 1 shows a visual interpretation of the overall desirability for each combination. Consequently, the combination 10 that corresponds to Case 3 (Baked Drying), with an overall desirability of 0.8208, is established as optimum.

It was determined that the regression model of the complete factorial design explains from 80.69% to 96.12% the variability of the analyzed mechanical properties, which is why the model is considered as the best fit to the analyzed data. These values are comparable with similar investigations; as for example, E. H. Agung applied an experimental design with response surface methodology in which values of the coefficient of determination were obtained between 96.72 and 98.17%. [18]
4. Conclusions
This experimental study uses desirability function based on a full factorial design to establish the optimal combination of input factors for the tensile and flexural mechanical properties of the hybrid composite made of abaca (AbF) and cotton (CF) plus glass (FG), combined with a polymeric polyester resin matrix. From the results of the research you can take of the following conclusions.

1. The configuration and type of drying factors, together with the interaction between these factors explain a high percentage of variability observed in the mechanical properties of the hybrid material analyzed, this percentage is 96.12%.

2. Desirability function with the full factorial design convert a subjective analysis of the mechanical properties, in an objective analysis using a comparable value in each combination.

3. The configuration of hybrid composite is the most significant factor for all response variables or mechanical properties.

4. The best configuration of hybrid material is when use: 20% glass fiber, 7.75% abaca fiber arranged at 45 ° woven with 2.25% of cotton fiber arranged at 135 °, in a polymer matrix polyester and drying oven.

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