Infrared modified QCD couplings and Bjorken sum rule

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Abstract. We test the recently proposed “Massive” Perturbation Theory (MPT) for the description of the $\Gamma_{1}^{p-n}$ data at low momentum transfers. The MPT constructed on the two grounds: the first is pQCD with only one parameter added, an effective “glueball mass” $m_{\rho} \leq M_{gl} \leq 1$ GeV; serving as an infrared “regulator”; the second stems out of the ghost-free Analytic Perturbation Theory comprising non-power perturbative expansion that makes it compatible with linear integral transformations. It is regular in the low-energy region and could serve as a practical means for the analysis of data below 1 GeV up to the IR-limit. We study the non-perturbative Bjorken sum rule higher twists correction by using the MPT, the integral representation for infinite sum of higher twists coefficients and the QCD-inspired model for the $Q^{2}$-dependence of the generalized Gerasimov-Drell-Hearn sum rule.

1. Introduction

The perturbative QCD (pQCD) is a firmly established part of the particle interaction theory. Starting with gauge-non-invariant quantization, it correlates several dozen of experiments at quite different scales from a few up to hundreds of GeV. At the same time, pQCD meets troubles in the low-energy domain, below a few GeV, at the scales marked by the QCD parameter $\Lambda_{QCD}$.

To avoid the unwanted singularity of the QCD running coupling in the low energy region, several modifications (for example, [1–3]) of the pQCD have been devised. Recently, one of them, the Analytic Perturbation Theory ([4] and a latter review paper [5]) (APT), has proved to be good [6] in describing the polarized $\Gamma_{1}^{p-n}(Q^{2})$ moment of the Bjorken Sum Rule (BSR) down to a few hundred MeV.

To approach the global fitting of data, one needs a modified perturbation theory (MPT) with two essential properties: correspondence with common pQCD in ultra-violet limit (that is above a few GeV) and regularity and finiteness of the modified effective coupling $\alpha^{MPT}(Q^{2})$ and matrix elements in the low-energy domain. As a primary launch pad for this construction, the above-mentioned APT seems good. It satisfies the first condition and, partially, the second one. To exempt the APT-like scheme from its last drawback – the singularity (infinite derivatives) in the infra-red limit, one has to disentangle it from the ultra-violet logs. To this goal the infra-red regulator has been introduced just by the shift of the $Q^{2}$ scale [7], $\ln \left(\frac{Q^{2}}{\Lambda_{QCD}^{2}}\right) \rightarrow \ln \left(\frac{(Q^{2} + M_{gl}^{2})}{\Lambda_{QCD}^{2}}\right)$, with the only fitting parameter added, an effective glueball mass, $M_{gl}$. 
2. Description of the methods
Let us briefly discuss methods of the QCD analysis of the $\Gamma_1^{p-n}$ data base on the MPT.

Away from the large $Q^2$ limit, the BSR is given by a double series in powers of $\alpha_s$ and in powers of $1/Q^2$ and can be written as

$$\Gamma_1^{p-n}(Q^2) = \left| g_{AV} \right| \left[ 1 - \Delta_B(Q^2) \right] + \sum_{i=2}^{\infty} \frac{\mu_{2i}}{Q^{2i-2}},$$

where $|g_{AV}| = 1.2723$ [8] is the nucleon axial charge, $\mu_{2i}$ are the higher twist (HT) coefficients, $\Delta_B(Q^2)$ is the perturbative correction, which at the four-loop (N$^3$LO) level in the massless case reads [9]

$$\Delta_B(Q^2) = 0.318 \alpha_s + 0.363 \alpha_s^2 + 0.652 \alpha_s^3 + 1.804 \alpha_s^4.$$  \hspace{1cm} (2)

In the framework of MPT perturbative $\alpha_s$-series replaced by expansions over MPT functions $A_k$:

$$\sum_k c_k \alpha_s^k \to \sum_k c_k A_k,$$

where $A_1(Q^2) = \alpha^{MPT}(Q^2)$ is MPT running coupling, which is the two-loop massive renormalization group solution in the denominator representation (for details, see [10]) has a following form

$$A_1(Q^2) = \frac{\alpha_0}{1 + \alpha_0 \beta_0 L^* + \alpha_0 \beta_1 \ln \left( 1 + \alpha_0 \beta_0 L^* \right) / \beta_0}, \quad L^* = \ln \left( \frac{Q^2 + M_{gl}^2}{\Lambda_{QCD}^2} \right).$$

The MPT preserves an essential APT feature, namely, the non-polynomiality of perturbative expansion over a set of higher functions $A_k$ ($k > 1$). These functions are connected by the differential recurrent relations (at NLO)

$$\beta_0 A_{k+1}(Q^2) = -\frac{Q^2 + M_{gl}^2}{k} \frac{d}{dQ^2} A_k(Q^2) - \beta_1 A_{k+2}(Q^2).$$

In the following analysis, we use the natural condition

$$\Gamma_1^{p-n}(Q^2 = 0) = 0,$$

which is motivated by finiteness of the the spin-dependent cross-sections in real photon limit.

In the spirit of [11, 12], where Gerasimov–Drell–Hearn and Burkhardt–Cottingham sum rules considered for the purpose of a smooth continuation of $\Gamma_1^{p,n}(Q^2)$ to the non-perturbative region $0 \leq Q^2 \lesssim \Lambda_{QCD}^2$, we obtain slope of $\Gamma_1^{p-n}$ moment at the IR-limit $Q = 0$:

$$d \frac{d}{dQ^2} \Gamma_1^{p-n}(Q^2 = 0) = -\frac{(\mu_p - 1)^2 + \mu_n^2}{8M^2},$$

where $\mu_p = 2.79$ and $\mu_p = -1.91$ are proton and neutron magnetic moments [8], respectively, and $M = 0.938$ GeV is a nucleon mass.

We use the non-perturbative series summation procedure proposed in [13]. This procedure allows to represent a non-perturbative operator product expansion series introducing a single free parameter $m_{ht}$

$$\sum_{n=1}^{\infty} \frac{\mu_{2n+2}^{p-n}}{Q^2} \left( \frac{m_{ht}^2}{Q^2} \right)^n \to \frac{\mu_4^{p-n} m_{ht}^2}{Q^2 + m_{ht}^2}. $$
We use the conditions (5) and (6) to connect the free parameters \( M_{gl}, m_{ht} \) and \( \mu_{4}^{p-n} \). As a result we obtain an approach with only one free parameter – effective “glueball mass” \( M_{gl} \):

\[
\Gamma_{1}^{p-n}(Q^2, M_{gl}^2) = \frac{|g_{A/V}|}{6} \left[ 1 - 0.318 A_1(Q^2, M_{gl}^2) - 0.363 A_2(Q^2, M_{gl}^2) - 0.652 A_3(Q^2, M_{gl}^2) - 1.804 A_4(Q^2, M_{gl}^2) + \ldots \right] + \frac{\mu_{4}^{p-n} m_{ht}^2}{Q^2 + m_{ht}^2}. \tag{8}
\]

\[
\Gamma_{1}^{p-n}(Q^2, M_{gl}^2) = \frac{\mu_{4}^{MPT} m_{ht}^2}{Q^2 + m_{ht}^2}. \tag{9}
\]

3. Results of fit and discussion
Using expression (1) fitted to the low \( Q^2 \) data [14–16], we can extract the HT coefficients \( \mu_{4}^{p-n} \) and parameters of “effective glueball mass” \( M_{gl} \) and parameter \( m_{ht}^2 \) in the HT sum.
In the Table 1 we present our result for the coefficient \( \mu_{4} \) obtained in different PT orders.

| Order | \( M_{gl}^2 \), GeV\(^2\) | \( m_{ht}^2 \), GeV\(^2\) | \( \mu_{4} \), GeV | \( \chi^2_{d.o.f} \) |
|-------|-----------------|-----------------|-------------|----------------|
| LO    | 0.782           | 0.736           | -0.170      | 6.24           |
| NLO   | 0.954           | 0.724           | -0.146      | 6.71           |
| N^2LO | 0.648           | 0.039           | 0.025       | 5.11           |
| N^3LO | 0.546           | 0.143           | -0.055      | 0.73           |

One can see that the value \( \mu_{4}^{MPT} = -0.055 \) is compatible with previously extracted \( \mu_{4}^{APT} = -0.050(2) \) [6].

Figure 1 shows the fits curves in various orders of MPT and for comparison four-loop APT curve from [6].

![Figure 1](image-url)
In conclusion we stress, that MPT approach combined the IR-frozen and the non-polynomiality of perturbative expansions is a next step for understanding low-energy QCD. The MPT together with a duly modified HT sum allows one to fit the data on $\Gamma_{T}^{p-n}$ down to the IR limit $Q^2 = 0$.

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References
[1] Shirkov D V, Mikhailov S.V. 1994 Z. Phys. C 63 463
[2] Simonov Yu A 1995 Phys. Atom. Nucl. 58 107
[3] Nesterenko A V, Papavassiliou J 2006 J. Phys. 32 1025
[4] Solovtsov I L, Shirkov D V 1999 Theor. Math. Phys. 120 1220
[5] Shirkov D V, Solovtsov I L 2007 Theor. Math. Phys. 150 132
[6] Khandramai V L, Pasechnik R S, Shirkov D V, Solovtsova O P, Teryaev O V 2012 Phys. Lett. B 706 340
[7] Shirkov D V 2013 Phys. Part. Nucl. Lett. 10 186
[8] Olive K A et al. 2014 Chin. Phys. C 38 090001
[9] Baikov P A, Chetyrkin K G, Kühn J H 2010 Phys. Rev. Lett. 104 132004
[10] Shirkov D V Mass 1982 Theor. Math. Phys. 49 1039
[11] Soffer J, Teryaev O V 1993 Phys. Rev. Lett. 70 3373
[12] Soffer J, Teryaev O V 2004 Phys. Rev. 70 116004
[13] Teryaev O V 2013 Nucl. Phys. Proc. Suppl. 245 195
[14] Deur A et al. 2004 Phys. Rev. Lett. 93 212001
  Deur A et al. 2007 Phys. Lett. B 650 244
  Deur A et al. 2008 Phys. Rev. D 78 032001
  Deur A et al. 2008 Phys. Lett. B 665 349
  Deur A et al. 2014 Phys. Rev. D 90 012009
[15] Alekseev M G et al. 2010 Phys. Lett. B 690 466
[16] Abe K et al. 1997 Phys. Rev. Lett. 79 26
  Anthony P I et al. 2000 Phys. Lett. B 493 19