Neutrino masses and cosmic radiation density: Combined analysis

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Abstract. We determine the range of neutrino masses and cosmic radiation content allowed by the most recent CMB and large-scale structure data. In contrast to other recent works, we vary these parameters simultaneously and provide likelihood contours in the two-dimensional parameter space of $N_{\text{eff}}$, the usual effective number of neutrino species measuring the radiation density, and $\sum m_\nu$. The allowed range of $\sum m_\nu$ and $N_{\text{eff}}$ has shrunk significantly compared to previous studies. The previous degeneracy between these parameters has disappeared, largely thanks to the baryon acoustic oscillation data. The likelihood contours differ significantly if $\sum m_\nu$ resides in a single species instead of the standard case of being equally distributed among all flavors. For $\sum m_\nu = 0$ we find $2.7 < N_{\text{eff}} < 4.6$ at 95\% CL while $\sum m_\nu < 0.62$ eV at 95\% CL for the standard radiation content.
1. Introduction

The recent release of the 3-year WMAP data has stimulated several renewed analyses of cosmological neutrino mass limits. Neutrinos are known to have mass from oscillation experiments so that the unknown overall mass scale is unavoidable as a fit parameter of the standard cosmological model. The resulting mass limits range from $\sum m_\nu < 2.0$ eV (95% CL) using the WMAP-3 data alone [1] to $\sum m_\nu < 0.17-0.4$ eV (95% CL) when data from the Lyman-\(\alpha\) forest is included [2, 3].

Translating cosmological limits on the hot dark matter fraction into neutrino mass limits depends on the cosmic neutrino density that is fixed by standard physics and thus not an ordinary cosmic fit parameter. On the other hand, the most direct evidence for the presence of the cosmic neutrino sea derives from big-bang nucleosynthesis and from cosmological parameter fitting so that it is a natural consistency test to study if the cosmic radiation density implied by the cosmological precision parameters reproduces the standard radiation content.

However, since neutrinos are known to have mass, one cannot simply assume that $\sum m_\nu = 0$ when extracting an allowed range for $N_{\text{eff}}$, the effective number of neutrino species that is the usual measure of the radiation content. Nevertheless this has been the standard procedure in most of the recent parameter analyses based on WMAP-3 [2, 4]. The caveat is especially relevant because we found a degeneracy between $\sum m_\nu = 0$ and $N_{\text{eff}}$, based on the cosmological data available in 2003 [5–8]. One result of our present $\sum m_\nu$-$N_{\text{eff}}$-analysis will be that this degeneracy is no longer present in the much smaller allowed range based on the 2006 data (Fig. 2). We will thus conclude that the current cosmological data provide essentially independent limits on $\sum m_\nu$ and $N_{\text{eff}}$.

Another important issue is what one actually means with $N_{\text{eff}}$, as there are several different plausible cases that should be considered. One possibility is that the cosmic number density of the standard neutrinos is different from what is usually assumed, i.e. $\sum m_\nu$ is equally distributed among all species that comprise $N_{\text{eff}}$ (our Case 1, see Table 1). Our second case is that three standard massive neutrinos have equal masses, i.e. the number of equally massive species is $N_m = 3$ so that $N_{\text{eff}} - N_m$ signifies additional radiation in some completely new form unrelated to ordinary neutrinos. Finally, we consider $N_m = 1$ that could represent a situation where the standard neutrinos are nearly massless, i.e. they have hierarchical masses with a largest mass eigenvalue given by the atmospheric scale of about 50 meV, while there is an additional massive species,

| Case | Model | $\sum m_\nu$ (95% CL) | $N_{\text{eff}}$ (95% CL) |
|------|-------|----------------------|-------------------|
| 1    | $N_m = N_{\text{eff}}$ | < 0.62 eV            | $2.7 < N_{\text{eff}} < 4.6$ |
| 2    | $N_m = 3$  | < 0.57 eV            | $3.0 \leq N_{\text{eff}} < 4.6$ |
| 3    | $N_m = 1$  | < 0.41 eV            | $2.7 < N_{\text{eff}} < 4.6$ |

Table 1. Cases of neutrino mass distribution among the $N_{\text{eff}}$ species. For the allowed $\sum m_\nu$ range we have marginalized over $N_{\text{eff}}$ and vice versa.
perhaps a sterile neutrino. This case is largely motivated to demonstrate that the current cosmological data are sensitive to the $\sum m_\nu$ distribution among the flavors.

We begin in Sec. 2 with a brief description of the cosmological data used in our study. In Sec. 3 we derive the range of $\sum m_\nu$ and $N_{\text{eff}}$ allowed by these data and conclude in Sec. 4 with a summary of our findings.

2. Cosmological data and likelihood analysis

In order to study bounds on $\sum m_\nu$ and $N_{\text{eff}}$ we use the same data as in Ref. [3]. We use distant type Ia supernovae measured by the SuperNova Legacy Survey (SNLS) [9] and large-scale structure data from the 2dF [10] and SDSS [11, 12] surveys. From the SDSS we also include the recent measurement of the baryon acoustic oscillation feature in the 2-point correlation function [13]. Finally, we include the precision measurements of the cosmic microwave background anisotropy from the WMAP experiment [4, 14, 15], as well as the smaller-scale measurement by the BOOMERANG experiment [16–18].

We do not include data from the Lyman-\(\alpha\) forest in our analysis. These data were used previously and very strong separate bounds on $\sum m_\nu$ and $N_{\text{eff}}$ were obtained [2]. However, the strength of these bounds is mainly related to the fact that the Lyman-\(\alpha\) analysis used in Ref. [2] leads to a much higher normalisation of the small-scale power spectrum than the WMAP data. Other analyses of the same SDSS Lyman-\(\alpha\) data find a lower normalisation, in better agreement with the WMAP result [19–21]. In this case the Lyman-\(\alpha\) data add little to the strength of the neutrino mass bound [3]. The discrepancy between different analyses of the same data probably points to unresolved systematic issues so that we prefer to exclude the Lyman-\(\alpha\) data entirely.

We then perform a likelihood analysis based on a flat, dark-energy dominated model characterised by the matter density $\Omega_m$, the baryon density $\Omega_b$, the dark energy equation of state $w$, the Hubble parameter $H_0$, the spectral index of the primordial

| parameter | prior |
|-----------|-------|
| $\Omega = \Omega_m + \Omega_{\text{DE}} + \Omega_\nu$ | 1 | Fixed |
| $\Omega_m$ | 0 – 1 | Top hat |
| $h$ | 0.5 – 1.0 | Top hat |
| $\Omega_bh^2$ | 0.014 – 0.040 | Top hat |
| $w_{\text{DE}}$ | $-2.5$ – $-0.5$ | Top hat |
| $n_s$ | 0.6 – 1.4 | Top hat |
| $\alpha_s$ | $-0.5$ – 0.5 | Top Hat |
| $\tau$ | 0 – 1 | Top hat |
| $Q$ | — | Free |
| $b$ | — | Free |

Table 2. Priors on the parameters used in our likelihood analysis.
power spectrum $n_s$, the running of the primordial spectral index $\alpha_s$, and the optical depth to reionization $\tau$. Finally, the normalization of the CMB data $Q$ and the bias parameter $b$ are used as free parameters. The dark-energy density is given by the flatness condition $\Omega_{\text{DE}} = 1 - \Omega_m - \Omega_{\nu}$. Including the neutrino mass $\sum m_\nu$, parameterised in terms of the contribution to the present energy density $\Omega_\nu h^2 = \sum m_\nu / 92.8 \text{ eV}$, and the effective number of neutrino species $N_{\text{eff}}$, our benchmark model has 11 free parameters.

Our priors on these parameters are shown in Table 2. The treatment of data is exactly the same as in Ref. [3]. When calculating constraints, the likelihood function is found by minimizing $\chi^2$ over all parameters not appearing in the fit, i.e. over all parameters other than $\sum m_\nu$ and $N_{\text{eff}}$.

![Figure 1. The 68%, 95%, and 99% confidence level contours for our three cases.](image)
3. Bounds on neutrino properties

Following these procedures we find the 68%, 95%, and 99% likelihood contours for $\sum m_\nu$ and $N_{\text{eff}}$ shown in Fig. 1 for the three cases discussed in the introduction and shown in Table 1. The top panel of Fig. 1 corresponds to a nonstandard number density of the standard neutrinos, assuming a standard velocity dispersion as in all other cases as well. In Fig. 2 we overlay these contours with the analogous ones that we found on the basis of the data available in 2003 [6]. The allowed range of both parameters has shrunk dramatically as expected. Moreover, the pronounced degeneracy between $\sum m_\nu$ and $N_{\text{eff}}$ that was present at that time has now completely disappeared, largely thanks to the baryon acoustic oscillation (BAO) measurements. We conclude that at the level of precision that has now been reached, the cosmological data constrain $\sum m_\nu$ and $N_{\text{eff}}$ almost independently of each other.

Perhaps the physically best motivated case is No. 2 where we have the ordinary neutrinos with mass ($N_\text{m} = 3$) and additional radiation in some new form. In this case we have a hard lower limit $N_{\text{eff}} \geq N_\text{m} = 3$. Otherwise the contours of the middle panel of Fig. 1 are very similar to Case 1 (top panel).

The largest modification appears in Case 3 where we assume that all hot-dark matter mass resides in a single neutrino species. The mass limits are significantly more restrictive in this case. The reason is that for a single massive species the total neutrino energy density is larger in the semi-relativistic regime than if the mass is shared between all flavours [22]. Since the mass bound is such that neutrinos become nonrelativistic very close to the epoch of matter-radiation equality, this equality occurs later in the model with only one massive neutrino. As a consequence, small-scale structure is more suppressed, but the effect can be offset by a slight increase in the matter density. We...
indeed observe that the best-fit value of $\Omega_m$ is higher for $N_m = 1$. However, both the SN Ia and BAO data prefer a low value of $\Omega_m$ and consequently the model with $N_m = 1$ becomes a poor fit to this data at $\sum m_\nu$ around 0.4–0.5 eV. Table 3 shows exactly this effect. Here, $\Delta \chi^2$ for $\sum m_\nu = 0.45$ eV has been broken down into individual contributions from the different data sets. As expected, the main effect comes from SN Ia and BAO data.

| Data set | $N_m = N_{\text{eff}}$ | $N_m = 1$ |
|----------|------------------------|----------|
| CMB      | -1.2                   | -0.9     |
| LSS      | 0.5                    | 0.5      |
| SN Ia    | 1.0                    | 2.3      |
| BAO      | 1.3                    | 2.9      |

Table 3. $\Delta \chi^2$ for $\Omega_\nu h^2 = 0.005$ compared with the best fit model, broken down into individual contributions.

We finally note that for the case with one sterile massive state and three active, almost massless neutrinos (the LSND 3+1 case) the mass bound is 0.45 eV at 95% CL (0.93 eV at 99.99% C.L.), somewhat lower than the 0.62 eV bound in the standard case. That the bound on the 3+1 model is stronger than for the standard case is contrary to what was found in previous studies (see [5–7]). The reason is the low value of $\Omega_m$ preferred by the BAO and SNI-a data.

4. Discussion

We have derived likelihood contours in the two-dimensional parameter space spanned by $\sum m_\nu$ and $N_{\text{eff}}$, based on the latest cosmological precision data, however excluding Lyman-α. We consider two physically motivated cases for $N_{\text{eff}}$ where either the effective number of massive neutrinos differs from the standard scenario, or where there is a new form of radiation besides $N_m = 3$ standard massive neutrinos. The results for these cases differ very little, except that in Case 2 there is hard lower limit $N_{\text{eff}} \geq N_m = 3$.

For the sake of principle we have also considered a third case where all the neutrino mass resides in a single species. Here, the mass limit is more restrictive, reflecting that near the limiting mass of around 0.5 eV neutrinos become nonrelativistic very close to the epoch of matter-radiation equality.

For all cases we provide in Table 1 limits on $\sum m_\nu$ after marginalizing over $N_{\text{eff}}$ and limits on $N_{\text{eff}}$ after marginalizing over $\sum m_\nu$. We stress that the neutrino mass scale can not be avoided as a standard cosmic fit parameter so that one should not derive limits on $N_{\text{eff}}$ while enforcing the neutrino masses to vanish. In practice, because $\sum m_\nu$ and $N_{\text{eff}}$ are no longer degenerate, the allowed range for $N_{\text{eff}}$ is not very different if one assumes $\sum m_\nu = 0$.

The limits on $N_{\text{eff}}$ differ little between our cases. Independently of the exact distribution of masses among the neutrino species we find $2.7 < N_{\text{eff}} < 4.6$ (95% CL),
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except in Case 2 where the lower limit is by definition $3 \leq N_{\text{eff}}$. The bound is significantly stronger than the $2.5 < N_{\text{eff}} < 5.6$ found with WMAP-1 data and without inclusion of the BAO data [23]. The standard case of three massive neutrinos without modified number density and without additional radiation is well within the 95% CL range of $N_{\text{eff}}$, although it is just slightly outside the 68% CL range in Cases 1 and 2. Either way, the cosmological model with a nonstandard density of massive neutrinos or radiation is not significantly favored over the standard case.

Acknowledgments

This work was supported, in part, by the Deutsche Forschungsgemeinschaft under grant No. SFB 375 and by the European Union under the ILIAS project, contract No. RII3-CT-2004-506222. S.H. acknowledges support from the Alexander von Humboldt Foundation through a Friedrich Wilhelm Bessel Award. Use of the CMBFAST code is acknowledged [24].

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