Redshift-space effects in voids and their impact on cosmological tests
Part I: the void size function

Carlos M. Correa,1,2* Dante J. Paz,1,2 Ariel G. Sánchez,3 Andrés N. Ruiz,1,2
Nelson D. Padilla4,5 and Raúl E. Angulo6,7

1 Instituto de Astronomía Teórica y Experimental, UNC-CONICET, Laprida 854, X5000BGR Córdoba, Argentina
2 Observatorio Astronómico de Córdoba, Universidad Nacional de Córdoba, Laprida 854, X5000BGR Córdoba, Argentina
3 Max-Planck-Institut für Extraterrestrische Physik, Postfach 1312, Giessenbachstr, D-85741 Garching, Germany
4 Instituto de Astrofísica, Pontificia Universidad Católica de Chile, Av. Vicuña Mackenna 4860, Santiago, Chile
5 Centro de Astroingeniería, Pontificia Universidad Católica de Chile, Av. Vicuña Mackenna 4860, Santiago, Chile
6 Donostia International Physics Centre (DIPC), Paseo Manuel de Lardizabal 4, E-20018 Donostia-San Sebastian, Spain
7 IKERBASQUE, Basque Foundation for Science, E-48013 Bilbao, Spain

Accepted XXX. Received YYY; in original form ZZZ

ABSTRACT
Cosmic voids are promising cosmological probes. Nevertheless, every cosmological test based on voids must necessary employ methods to identify them in redshift-space. Therefore, systematic effects, such as redshift-space distortions (RSD) and the Alcock-Paczynski effect (AP), are expected to have an impact on the void identification process itself, generating additional distortion patterns in observations. We developed a statistical and theoretical framework that describes physically the void finding mechanism. We used a spherical void finder and made a statistical comparison between real and redshift-space voids. We found that redshift-space voids above the shot noise level have a unique real-space counterpart spanning the same region of space, are systematically bigger and their centres preferentially shifted along the line-of-sight. The expansion effect is a by-product of the RSD induced by tracer dynamics at scales around the void radius, whereas the off-centring effect constitutes a different class of RSD induced at larger scales, which resembles a void dynamics. The volume of voids is also altered by the fiducial cosmology assumed to measure distances, this is the AP change of volume. These three systematics have an impact on cosmological statistics. In this work, we focus on the void size function. We developed a theoretical framework to model these effects and tested it with a numerical simulation, recovering the statistical properties of the abundance of voids in real-space. This description depends strongly on cosmology. Hence, we lay the foundations for improvements in current models for the abundance of voids in order to obtain unbiased cosmological constraints from redshift surveys.

Key words: large-scale structure of Universe – galaxies: distances and redshifts – methods: data analysis, statistical – cosmological parameters

1 INTRODUCTION

Cosmic voids are vast underdense regions. Since they are the largest observable structures, voids are proving to be powerful cosmological laboratories, as they encode information about the expansion history and geometry of the Universe. This is very valuable, since one of the major goals of modern cosmology is to understand the nature of the accelerated expansion of space, which is believed to be caused by a dark energy component.

Voids offer two special advantages over the high density regime, which make them attractive to theoretical modelling and observational test designing. On the one hand, void dynamics can be treated linearly, assuming spherical symmetry for the velocity and density fields. In this way, it is easier to model systematics such as redshift-space distortions: Paz et al. (2013), Hamaus et al. (2015), Cai et al. (2016), Hamaus et al. (2016), Achitouv (2017), Achitouv et al. (2020).
Voids are identified measuring densities from tracer positions. Despite the intrinsic differences between the available void finding methods, they all share this basic property. In fact, this is inherent to void definition. Therefore, it is expected that the RSD and the AP effects also affect void properties. This was specially noted in Nadathur et al. (2019a) and Nadathur et al. (2019b), where they show that there were four hypotheses commonly assumed when modelling RSD, which are only valid for voids identified in real-space, and are violated for voids identified in redshift-space. Specifically, these hypotheses are: (1) conservation of void number, (2) isotropy of the density field, (3) isotropy of the velocity field, and (4) invariance of void centre positions. In Nadathur et al. (2019b), they show that this problem can be tackled in two different ways: (i) using a reconstruction technique (Eisenstein et al. 2007), or (ii) analysing physically the void finding mechanism. They focused on the first approach. Reconstruction is an algorithm to recover the real-space positions of galaxies from redshift-space based on the Zel’dovich approximation. This technique has been used for BAOs analyses as well. The idea is to apply the reconstruction before performing the void finding step. As this is a cosmology dependent procedure, it can be used to measure the growth rate factor if the reconstruction plus the void finding step are applied iteratively.

In this paper, instead, we focus on the second approach, namely, we analyse the void finding method in order to understand physically the redshift-space systematics affecting voids. To do this, we use a spherical void finder and make a statistical comparison between the resulting real and redshift-space voids. We find two relevant results. First, there is a one-to-one relationship between voids. This means that each redshift-space void has a unique real-space counterpart and vice versa, spanning the same region of space. In this sense, condition (1) of void number conservation is not violated. This is a consequence of our void definition: voids are large spherical regions with very low integrated density, and hence, mostly expanding. Second, redshift-space voids are systematically bigger than their real-space counterparts, and their centre positions have shifted preferentially along the LOS. These phenomena can be attributed to two physical effects: expansion and off-centring, which in turn, can be theoretically described based on both tracer and void dynamics. Moreover, this description depends strongly on cosmology.

This paper is the first of two publications concerning the impact of redshift-space effects in voids on cosmological statistics. Here, we study the void size function. We leave the implications of these effects on the void-galaxy correlation function for the second part. Up to our knowledge, this is the first time that redshift-space systematics on the VSF are treated. The community has concentrated their efforts on modelling the true underlying real-space VSF with the excursion set formalism. The intention of this work is to lay the foundations for a full modelling, leaving for future investigation to link both developments.

This paper is organised as follows. In Section 2, we describe the data set, that is, the numerical N-body simulation and the void catalogues. In Section 3, we explain the bijective mapping between real and redshift-space voids. In Section 4, we explain theoretically the expansion and off-centring effects, along with the additional AP change of volume. Then, in Section 5, we provide a statistical analysis confirming them. After that, in Section 6, we analyse the implications of these effects on the VSF as a cosmological test. Finally, we summarize and discuss our results in Section 7.
2 DATA SET

2.1 Simulation setup

We used the Millennium XXL N-body simulation (Angulo et al. 2012, hereafter MXXL) which extends the previous Millennium and Millennium-II simulations (Springel et al. 2005; Boylan-Kolchin et al. 2009) and follows the evolution of 6720\(^3\) dark matter particles inside a cubic box of length 3000 \(h^{-1}\)Mpc. The particle mass is 8.46\(\times\)10\(^3\) \(h^{-1}\)M⊙ in a flat ΛCDM cosmology with the same cosmological parameters as the previous runs: \(Ω_m = 0.25\), \(Ω_Λ = 0.75\), \(Ω_b = 0.045\), \(Ω_r = 0.0\), \(h = 0.73\)^\(^1\), \(n_s = 1.0\) and \(σ_8 = 0.9\). We used the snapshot belonging to redshift \(z_{\text{box}} = 0.99\), obtained from the spherical collapse model (Gunn & Gott 1972; Lilje & Lahav 1991) by fixing a final spherical perturbation of \(Δ_{\text{id}} = -0.9\) for \(z = 0\).

We applied the spherical void finder described in Ruiz et al. (2009), which is a modified version of the algorithm of Padilla et al. (2005). The main characteristics of the void catalogues and selected samples used in this work.

2.2 Void catalogues

We applied the spherical void finder described in Ruiz et al. (2015), which is a modified version of the algorithm of Padilla et al. (2005). Since the goal of this paper is to understand the implications of redshift-space systematics on void identification, we detail below the steps of this procedure:

(i) A Voronoi tessellation is performed to obtain an estimation of the density field: each halo has an associated cell with volume \(V_\text{cell}\), and a density given by the inverse of that volume: \(ρ_\text{cell} = 1/V_\text{cell}\). We used a parallel version of the public library voro++ (Rycroft 2009).

(ii) A first selection of underdense regions is done by selecting all Voronoi cells which satisfy the criterion \(δ_\text{cell} := ρ_\text{cell}/\bar{ρ} - 1 < -0.7\), where \(\bar{ρ}\) is the mean number density of haloes. Each underdense cell is considered the centre of a potential void.

(iii) Centred on each candidate, the integrated density contrast \(Δ(\rho) := δ(< r)\) is computed in spheres of increasing radius \(r\) until the overall density contrast satisfies a redshift dependent threshold of \(Δ_{\text{id}} = -0.853\) for \(z_{\text{box}} = 0.99\), obtained from the spherical collapse model (Gunn & Gott 1972; Lilje & Lahav 1991) by fixing a final spherical perturbation of \(Δ_{\text{id}} = -0.9\) for \(z = 0\).

(iv) Once these first void candidates are identified, step (iii) is repeated several times, but starting in a randomly displaced centre proportional to 0.25 times the radius of the candidate. Then, the void centre is updated to a new position if the new radius is larger than the previous one. This procedure mimics a random walk around the original centre in order to obtain the largest possible sphere in that local minimum of the density field.

(v) Finally, the list of void candidates is cleaned so that each resulting sphere does not overlap with any other. This cleaning is done by ordering the list of candidates by size and starting from the largest one. The final result is a catalogue of non-overlapping spherical voids with radii \(R_i\) and overall density contrast \(Δ(R_i) = Δ_{\text{id}}\).

We applied the void finder in two ways, resulting in two types of catalogues. In the first case, we adopted the same cosmology of the MXXL simulation in order to compute distances and densities, needed in void definition. In turn, we performed the identification both in real and redshift-space (hereafter r-space and z-space respectively), in order to study the impact of RSD. We will refer to these catalogues as the true cosmology (TC) void catalogues.

In the second case, we modified the simulation coordinate system according to two different cosmologies, in order to study the impact of the combined RSD and AP effects. Specifically, we fixed all the MXXL global parameters with the exception of \(Ω_m\), for which a lower and an upper fiducial values were chosen: \(Ω_m^l = 0.20\) and \(Ω_m^u = 0.30\). In this case, the identification was only performed in z-space (see Section 6 for more details). We will refer to these catalogues as the fiducial cosmology (FC) void catalogues. Table 1 shows the main characteristics of the void catalogues and selected samples used in this work.

3 BIJECTIVE MAPPING

We begin the analysis with a visual inspection of r-space and z-space voids. Fig. 1 shows two slices of the MXXL simulation box using the TC void catalogues. The aim of this section is to study the impact of RSD alone, postponing the analysis of the combined RSD and AP effects until Section 6, where we will use the FC void catalogues. The left panel is a slice in the range 500 ≤ \(x_1/h^{-1}\)Mpc ≤ 1000, 500 ≤ \(x_2/h^{-1}\)Mpc ≤ 1000 and 95 ≤ \(x_3/h^{-1}\)Mpc ≤ 105. Hence, it is a representation of the POS distribution of haloes and voids. Here, r-space void centres are represented with blue dots, whereas z-space centres, with red squares. The right panel, on the other hand, is a slice in the range 500 ≤ \(x_1/h^{-1}\)Mpc ≤ 1000, 95 ≤ \(x_2/h^{-1}\)Mpc ≤ 105 and
the radius increases with a functional shape similar to those line: (i) on the left, small voids dominated by shot noise, and (ii) on the right, large voids decreasing their number as the radius increases with a functional shape similar to those

Table 1. Main characteristics of the void samples used in this work. From left to right: sample name, catalogue type, cosmology used in void definition, $\Omega_m$ choice, space where the identification was performed, completeness regarding the bijective filtering, number of voids, and type of systematics taken into account.

| Sample         | Catalogue | Cosmology | $\Omega_m$ | Space     | Completeness | Number of voids | Systematics |
|----------------|-----------|-----------|------------|-----------|--------------|-----------------|-------------|
| TC-rs-f        | TC        | MXXL      | 0.25       | r-space   | full         | 463,690         | none        |
| TC-rs-b        | TC        | MXXL      | 0.25       | r-space   | bijective    | 318,784         | none        |
| TC-zs-f        | TC        | MXXL      | 0.25       | z-space   | full         | 455,482         | RSD         |
| TC-zs-b        | TC        | MXXL      | 0.25       | z-space   | bijective    | 318,784         | RSD         |
| FC-I           | FC        | Fiducial  | 0.20       | z-space   | full         | 375,560         | AP + RSD    |
| FC-u           | FC        | Fiducial  | 0.30       | z-space   | full         | 526,552         | AP + RSD    |

$500 \leq \delta_3/\hbar^{-1}\text{Mpc} \leq 1000$, i.e. it shows the LOS distribution of haloes and voids. From the figure, it is clear that z-space and r-space voids approximately span the same regions of space.

In order to link r-space and z-space voids, we looked for their correspondence by cross-correlating both distributions. Specifically, for each z-space centre, we picked the nearest r-space centre with the condition that it must lay inside 1 $R^z$ (or $R^r$ and $R^z$ will denote r-space and z-space radius respectively). Then, we filtered those voids if no partner could be found. Note that this z-space $\rightarrow$ r-space mapping is a well defined function, since the condition of the nearest r-space neighbour assigns only one object to each z-space void. Furthermore, this mapping is also injective, since the non-overlapping condition implies that each r-space void can only be reached by a single z-space one. Even further, the filtering condition guarantees then a one-to-one relationship between z-space and r-space voids. For this reason, these voids constitute what we call the bijective samples (TC-rs-b and TC-zs-b in Table 1). Note that, by construction, these samples have the same number of elements. Moreover, it is ensured in this way that a void and its associated counterpart span the same region of space. In order to distinguish the bijective samples from the entire catalogues, we will refer to the last ones as the full samples (TC-rs-f and TC-zs-f in Table 1). Going back to Fig. 1, bijective voids are represented with circles around their centres, which correspond to the intersections of the spherical voids with the mid-plane of the slice. The rest are voids of the full samples without a partner in the other space.

In order to enquire deeper into the relation between r-space and z-space voids, we computed from the radius distribution of each sample, expressing the void counts as comoving differential number densities, $dn_v$, and normalising them with the logarithmic sizes of the radius bins, $d\ln R_v$. The solid lines represent the abundances of the full samples, both in r-space (blue) and z-space (red), whereas the dashed lines, the abundances of the bijective samples. The vertical dashed line represents the median of the z-space full sample, and will serve as a reference line throughout the work. The qualitative behaviour of the VSFs is consistent with previous studies, where we can distinguish two main behaviours separated by the vertical line: (i) on the left, small voids dominated by shot noise, and (ii) on the right, large voids decreasing their number as the radius increases with a functional shape similar to those predicted by theory (see Sheth & van de Weygaert 2004 and Jennings et al. 2013). Small voids, in this sense, are not reliable for any cosmological analysis, hence, we will mainly focus on large voids throughout this work.

Note that the full and bijective abundances (solid and dashed lines) tend to the same values as the radius increases. This means that the loss of voids in the z-space $\rightarrow$ r-space mapping is only significant in the region dominated by shot noise, whereas all the large and relevant voids are conserved. This is better shown in the right-upper panel, where we present the corresponding fractional differences of void counts between the full and bijective samples. For all radii of interest, the loss of voids decreases as the radius increases, being less than 25% in the worst case. We arrive here at the first and one of the most important conclusions of this work: large voids identified in an observational catalogue are true voids, i.e., they have a real-space counterpart. Therefore, it is valid to treat the full and bijective samples indistinctly in their properties.

Comparing now the r-space and z-space abundances (blue and red lines), it is clear that z-space voids are systematically bigger than their r-space counterparts. This is better shown with the fractional differences in the right-lower panel. In this case, the differences increase as the radius increases, and they can be high for the largest sizes.

4. THEORETICAL FRAMEWORK

The goal of this section is to study theoretically the possible physical mechanisms responsible of the transformation of r-space voids into their associated z-space counterparts. We will do this in the context of the four non-valid hypotheses commonly assumed in any r-space $\rightarrow$ z-space mapping noticed by Nadathur et al. (2019b):

(1) void number conservation,
(2) isotropy of the density field,
(3) isotropy of the velocity field,
(4) invariance of centre positions.

In next section, we will provide statistical evidence of the framework developed here.

4.1 Void number conservation

Strictly speaking, condition (1) of void number conservation is violated. This is evident for the r-space and z-space full samples, since they have different number of voids (see Table 1) and different void abundances (Fig. 2). Nevertheless,
Figure 1. Slices of the Millennium XXL simulation box showing the distribution of haloes and voids from the TC samples of Table 1. Real-space void centres are represented with blue dots, whereas redshift-space centres, with red squares. Bijective voids, in turn, are represented with circles, the intersections of the spherical voids with the mid-plane of the slice. Left panel. Slice in the range $500 \leq x_1/\hMpc \leq 1000$, $500 \leq x_2/\hMpc \leq 1000$ and $95 \leq x_3/\hMpc \leq 105$, a representation of the plane of the sky distribution. Right panel. Slice in the range $500 \leq x_1/\hMpc \leq 1000$, $95 \leq x_2/\hMpc \leq 105$ and $500 \leq x_3/\hMpc \leq 1000$, a representation of the line of sight distribution. Bijective voids span the same regions in both spaces. Voids expand and their centres shift when they are mapped from r-space into z-space.

Figure 2. Left panel. Void size functions of the TC void samples. The solid lines represent the VSFs of the full samples, both in r-space (blue) and z-space (red). The dashed lines, the VSFs of the bijective samples. The vertical dashed line is the median of the z-space full sample (TC-zs-f), which separates the small voids dominated by shot noise (at the left) from the large ones relevant for cosmological analyses (at the right). Right-upper panel. Fractional differences of void counts between the bijective and full samples. It quantifies the void loss in the z-space $\rightarrow$ r-space mapping. Note that large voids are almost bijective. Right-lower panel. Fractional differences of void counts between the z-space and r-space samples.
condition (1) is not violated in the context of the bijective mapping that we defined. This is supported by two reasons. First, bijective voids are, by definition, the same entities spanning the same regions of space. This is why the bijective samples have the same number of voids. Second, large and relevant voids are conserved after this mapping.

### 4.2 Expansion effect

In Section 3 we showed that z-space voids are systematically bigger than their r-space counterparts. This suggests that voids expand when they are mapped from r-space into z-space. Left panel of Fig. 3 depicts schematically this expansion effect. An r-space void (blue circle with some galaxies) of radius $R^r_s$, appears elongated along the LOS in z-space (orange ellipse) due to the RSD induced by the LOS components of the peculiar velocities of the tracers surrounding it. The r-space sphere has been transformed into a z-space ellipsoid with semi-axes $(s_\parallel, s_\perp)$, where $s_\parallel$ is the POS semi-axis (equal for both $x_1$ and $x_2$ directions), and $s_\perp$, the LOS semi-axis.

We derive new analytical expressions for the semi-axes. We will assume as a first approximation that RSD do not affect the void dimensions across the POS. Hence, we can consider that $s_\parallel = R^r_{\parallel}$. An expression for $s_\perp$, on the other hand, can be obtained by means of Eq. (1) adapted for the case of void-centric comoving distances:

$$
\tilde{r}_\parallel = r_\parallel + \frac{v_\parallel}{H(z_\text{box})}(1 + z_\text{box}),
$$

where $r_\parallel$, $\tilde{r}_\parallel$ and $v_\parallel$ are the void-centric analogues of $x_3$, $v_3$ and $v_3$ in that equation. In this case, $r_\parallel$ and $\tilde{r}_\parallel$ must be replaced by $s_\parallel$ and $R^r_{\parallel}$ respectively. The next ingredient is an expression for $s_\perp$, which can be obtained from the void-centric velocity profile characterising the peculiar velocity field around voids. This can be derived following linear theory via mass conservation up to linear order in density and assuming spherical symmetry (Peebles 1976; Gazet al. 2013; Hamaus et al. 2015, 2016; Correa et al. 2019):

$$
v(r) = \frac{1}{3} \frac{H(z)}{1+z} \beta(z)r \Delta(r),
$$

where $\Delta(r)$ is the integrated density contrast profile characterising the density field around voids, and $\beta(z) = f(z)/b$, the ratio between the logarithmic growth rate of density perturbations, $f(z)$, and the linear tracer-mass bias parameter, $b$. In this profile, $v_\parallel = v(r_\parallel) = v(R^r_{\parallel})$, for which $\Delta(r)$ must be evaluated at $r = R^r_{\parallel}$, which in turn is equal to the threshold of void identification: $\Delta(R^r_{\parallel}) = \Delta_0$ (see Section 2.2, where we introduced the quantity $\Delta_0$). In this way, combining Eqs. (3) and (4) with the mentioned replacements, we get an expression for $s_\parallel$:

$$
s_\parallel = R^r_{\parallel} \left(1 - \frac{1}{3} \beta(z_\text{box})\Delta_0\right).
$$

Note that here, we assumed the validity of hypotheses (2) and (3) of isotropy of the density and velocity fields in r-space in order to explain a z-space phenomenon, even if this isotropy is no longer valid for z-space voids.

Given that we are using a spherical void finder, it is more appropriate to think in the equivalent sphere with the same volume of the ellipsoid. This is also depicted in the left panel of Fig. 3 with a red circle. Calling $R^z_s$ the radius of this new sphere, equating both volumes, and using Eq. (5), it is straightforward to get an expression for $R^z_s$:

$$
R^z_s = q_{\text{RSD}} R^r_{\parallel}, \quad q_{\text{RSD}} = \left(1 - \frac{1}{3} \beta(z_\text{box})\Delta_0\right).
$$

Note that the factor $q_{\text{RSD}}$ is independent of the scale, it is only a constant of proportionality. Moreover, it depends on $\beta$, hence it is cosmology dependent. To get an explicit value for $q_{\text{RSD}}$, we need the corresponding values of $\Delta_0$ and $\beta(z_\text{box})$. As specified in Section 2.2, $\Delta_0 = -0.853$ for $z_\text{box} = 0.99$. The value of $\beta$ corresponding to the MXXL simulation was obtained following the procedure of Correa et al. (2019) by fitting Eq. (4) to a measured r-space velocity profile with a Levenberg-Marquardt algorithm, getting in this way a value of $\beta < 0.5$. Finally, these two quantities imply that $q_{\text{RSD}} = 1.058$.

In reality, the factor $q_{\text{RSD}}$ may not be correct, since the density may also change across the POS due to RSD itself, and hence, the approximation $s_\parallel = R^r_{\parallel}$ might not be valid. Therefore, instead of considering a sphere with the same volume of the ellipsoid, we considered an alternative approximation where the equivalent sphere has a radius equal to the mean value between $R^r_{\parallel}$ and $s_\parallel$. In this way, we get an expression slightly different from Eq. (6):

$$
R^z_s = q'_{\text{RSD}} R^r_{\parallel}, \quad q'_{\text{RSD}} = 1 - \frac{1}{6} \beta(z_\text{box})\Delta_0.
$$

where we find that $q'_{\text{RSD}} = 1.092$.

### 4.3 Alcock-Paczyński change of volume

Up to here, a true distance scale was implicitly assumed. Note however that the only information available from observational catalogues are angular positions and redshifts of astrophysical objects like galaxies. These observables must be transformed into a Mpc-scale, which involves the use of a fiducial cosmology. A deviation between the true and fiducial cosmologies will lead to additional distortions in the spatial distribution of galaxies. This is a manifestation of the AP effect, and will also affect the volume of voids. To understand this effect, let us consider the distribution of galaxies in r-space (free of RSD) for the following analysis.

The size of a spherical void can be quantified by a POS angular radius $\Delta_\theta$, and a LOS redshift radius $\Delta z$. These observables are related to physical dimensions $(R_{\parallel}, R_{\perp})$ by the following transformation equations:

$$
R_{\parallel} = D_M(z_\text{box})\Delta_\theta, \quad R_{\parallel} = \frac{c}{H(z_\text{box})} \frac{dM}{dz}(z_\text{box}) \Delta z,
$$

where $D_M$ is the comoving angular diameter distance. Hence, the pair of Eqs. (8) depend on cosmology through the Hubble parameter (Eq. 2). Note that if one knew the true cosmology, then it would not be necessary to distinguish between the POS and LOS dimensions. Both would be equal to the r-space void radius: $R^r_s = R_{\parallel} = R_{\perp}$. However, assuming a fiducial cosmology leads to discrepancies between both quantities, and a spherical void will appear as an ellipsoid in the underlying coordinate system. Nevertheless, unlike the RSD-ellipsoids from the expansion effect, the AP-ellipsoids are distorted in both the POS and LOS directions. Even more, the net result is not necessary an expansion, it can also be
Here, $\varepsilon$ sphere with the same volume of the ellipsoid, and calling underlying cosmology. Finally, considering the equivalent the index “true” to refer to quantities based on the true $q$ get a value of in Section 2.2. For the FC-l catalogue, with $q$ mic structures. $q$ information about the expansion history and geometry of $\beta$ the RSD factors depend only on $q$ only on the background cosmological parameters, whereas $q$ partial approach to that used for the expansion effect. Consid-

ering that the AP-ellipsoid has semi-axes $(R_{\|}^\text{fid}, R_{\perp}^\text{fid})$, given by Eqs. (8) with fiducial values $H_{\text{fid}}$ and $D_M^{\text{fid}}$, then a direct comparison with $(R_{\|}^\text{true}, R_{\perp}^\text{true})$ leads to the following relations:

$$s_{\perp} = q_{AP}^\perp s_{\perp}^\text{true}, \quad s_{\|} = q_{AP}^\parallel s_{\|}^\text{true},$$  \hspace{1cm} (9)

where

$$q_{AP}^\perp = \frac{H_{\text{fid}}(\Omega_{\text{box}})}{H^\text{fid}} \quad q_{AP}^\parallel = \frac{H_{\text{true}}(\Omega_{\text{box}})}{H_{\text{fid}}(\Omega_{\text{box}})}.$$  \hspace{1cm} (10)

Here, $e(z)$ is the square root term of Eq. (2), and we adopted the index “true” to refer to quantities based on the true underlying cosmology. Finally, considering the equivalent sphere with the same volume of the ellipsoid, and calling $R_V^\text{fid}$ this new radius, we get an expression similar to Eq. (6):

$$R_V^{z\text{fid}} = q_{AP} R_V^{z\text{true}}.$$  \hspace{1cm} (11)

Like the RSD factors, $q_{AP}$ is also a constant of proportionality and cosmology dependent. However, there is an interesting difference between them. The AP factor depends on the background cosmological parameters, whereas the RSD factors depend only on $\beta$. Therefore, $q_{AP}$ encodes information about the expansion history and geometry of the Universe, whereas $q_{RSD}$, about the growth rate of cosmic structures.

In the development of Section 6 we will need explicit values of the $q_{AP}$ factors for the FC void catalogues defined in Section 2.2. For the FC-l catalogue, with $q_{m}^{z} = 0.30$, a value of $q_{AP}^{z} > 0.960$. Note that $q_{AP}^{z} > 1$ for the former, hence according to Eq. (11) it is expected an expansion of the z-space voids. Conversely, $q_{AP}^{z} < 1$ for the latter, hence it is expected a contraction.

4.4 Combined AP and RSD contributions

The volume of a void will be affected by the combined contributions of the AP and RSD effects, which are indistinguishable in observations. A priori, it is not trivial to ensure that both effects can be treated independently as we did. However, in Section 6 we will provide evidence of this. From a theoretical point of view, the fact that the $q_{RSD}$ and $q_{AP}$ factors encode different cosmological information is a good sign of this assumption.

Assuming this independence, we can relate the z-space and r-space void radii making a two-step correction: first, we apply Eq. (11) to correct for the AP effect, and then, we apply Eq. (6) (or Eq. 7) to correct for the RSD expansion effect:

$$R_V^{z\text{fid}} = q_{AP} q_{RSD} R_V^{z\text{true}}.$$  \hspace{1cm} (12)

4.5 Off-centring effect

A simple visual inspection of Fig. 1 shows that z-space void centres are shifted with respect to their r-space counterparts. This off-centring is a direct consequence of the failure of hypothesis (4) concerning the invariance of centre positions when voids are mapped from r-space into z-space. Nadathur et al. (2019b) remarks that this hypothesis is equivalent to assuming that void positions do not suffer RSD themselves. On the other hand, Lambas et al. (2016), Ceccarelli et al. (2016) and Lares et al. (2017) demonstrate that voids move as whole entities with a net velocity $V_v$. Inspired by these results, then the off-centring effect can be simply understood

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{Schematic illustration of the three z-space effects in voids. In all panels, a hypothetical r-space void is represented with a blue circle of radius $R_v^R$ with some galaxies. The LOS direction is vertical. \textit{Left panel. Expansion effect.} The r-space void appears elongated along the LOS in z-space (orange ellipse) with LOS semi-axis $s_{\|}$. The void finder identifies an equivalent sphere of radius $R_v^S$ (red circle). This effect is a by-product of tracer dynamics at scales around the void radius (t-RSD). \textit{Middle panel. AP change of volume.} The r-space void is distorted into an ellipse with LOS semi-axis $s_{\|}^\text{fid}$ and POS semi-axis $s_{\perp}^\text{fid}$ (orange ellipse). The void finder identifies an equivalent sphere of radius $R_v^{z\text{fid}}$ (red circle). This is a geometrical effect caused by the selection of a fiducial cosmology in order to transform the observable dimensions of the void, an angular radius $\Delta \theta$ and a redshift radius $\Delta z$, into a Mpc-scale (no t-RSD are included here for a better comprehension). \textit{Right panel. Off-centring effect.} The r-space void centre appears shifted along the LOS in z-space. This effect is a by-product of tracer dynamics at larger scales, which resembles a void dynamics (v-RSD). The three effects can be treated independently.}
\end{figure}
as a new kind of RSD induced by void dynamics, and therefore it is expected that void centres appear shifted along the LOS when they are identified in z-space, in the same way as tracers do. The right panel of Fig. 3 depicts schematically this effect. Going back to Fig. 1, the off-centring effect is expected to be most pronounced in the right panel, where the LOS distribution of haloes and voids is shown. Although this is difficult to see by eye because of the residual displacements in the POS directions, we will provide solid evidence of this effect in Section 5.2.

We can make an analytical prediction of this effect considering it as a dynamical phenomenon. The void finder used in this work provides the positions \(X_v = (X_{v1}, X_{v2}, X_{v3})\) [\(h^{-1}\text{Mpc}\)] and peculiar velocities \(V_v = (V_{v1}, V_{v2}, V_{v3})\) [\(\text{km/s}\)] of void centres (see Section 5 for more details about how the velocities were calculated). Therefore, in order to account for the LOS shifting of centres, it is only necessary to write an expression equivalent to Eq. (1) for voids:

\[
\tilde{X}_{v3} = X_{v3} + V_{v3} \left(1 + \frac{z_{\text{cen}}}{H(z)}\right), \tag{13}
\]

where \(\tilde{X}_{v3}\) denotes the shifted \(X_{v3}\)-coordinate. As was the case of Section 4.4, it is not trivial if the expansion and off-centring effects are independent from each other. In Section 5.3 we will provide evidence of this. In this way, Eq. (3) is still valid, provided that \(r_2 = |X_{v3} - X_1|\) and \(v_1 = |V_{v3} - v_1|\).

Note that the expansion effect is a by-product of RSD induced by tracer dynamics at scales around the void radius. At these scales, the velocity field of tracers responds to a divergence originated in the local density minimum located at the void. On the other hand, the off-centering effect is a result of RSD induced at larger scales. Its source is the bulk motion of galaxy tracers in the whole region containing the void following the large scale dynamics of the gravitational field (Lares et al. 2017). This last aspect resembles a void dynamics, that presents itself as a different kind of RSD. Therefore, it is expected that both effects leave a footprint on cosmological statistics such as the void size function and the void-galaxy correlation function. In some occasions, we will refer to them as the t-RSD expansion effect and the v-RSD off-centring effect, hinting at their different nature: t for tracer dynamics, and v for void dynamics.

5 STATISTICAL ANALYSIS

The statistical analysis of this section aims to provide evidence about the t-RSD expansion and v-RSD off-centring effects introduced in last section. In next section, we will complete the analysis incorporating the additional AP change of volume. For this reason, we will continue using the TC catalogues, specifically the bivariate samples (TC-rs-b and TC-zs-b in Table 1).

This analysis is based on correlations between three statistics that characterise the volume alteration and movement of a void: (i) z-space to r-space radius ratio \(R_{v3}^z/R_{v3}^r\), (ii) z-space displacement of the centre \(d_v = (d_{v1}, d_{v2}, d_{v3})\), and (iii) r-space net velocity \(V_v = (V_{v1}, V_{v2}, V_{v3})\). Specifically, \(d_v\) was calculated as the displacement of the void centre in going from r-space into z-space normalised to the r-space radius:

\[
d_v = \frac{\tilde{X}_{v3} - X_v}{R_v^r}, \tag{14}
\]

while \(V_v\), on the other hand, was computed summing all the individual velocities of haloes inside a spherical shell with dimensions 0.8 ≤ \(r/R_v^r\) ≤ 1.2. This velocity is an unbiased and fair estimation of the bulk flow velocity of the void, as was demonstrated in Lambas et al. (2016) (see their Fig. 1).

5.1 Correlations between r-space and z-space void radii

The left panel of Fig. 4 shows the 2D distribution \((R_v^r, R_v^z)\) as a heat map. From blue to red, colours span from low to high void counts \(N_v\). There is a linear trend between both radii, whose slope can be correctly described by the theoretical RSD factors (dashed line for \(q_{\text{RSD}}\) and solid line for \(q'_{\text{RSD}}\)). Note that both factors work well at small scales, but \(q_{\text{RSD}}\) is more suitable than \(q'_{\text{RSD}}\) at large scales. This is seen more clearly in the right panel, where the 1D distribution of \(R_v^r, R_v^z\) is shown. The upper panel represents the overall distribution. Note that \(q_{\text{RSD}}\) fits very well the median. While it is true that \(q'_{\text{RSD}}\) is biased to the right, after dividing the void population in two as shown in the lower panel, then \(q_{\text{RSD}}\) fits the median for small voids (red dashed line) and \(q'_{\text{RSD}}\) for the large ones (blue solid line). We shall recall that we are interested in large voids for cosmological studies.

From this analysis, we arrive at the second important conclusion of this work: voids expand when they are mapped from r-space into z-space, and this expansion can be statistically quantified as an increment in void radius by a factor \(q_{\text{RSD}}\) (or \(q'_{\text{RSD}}\) for large voids). These results give support to the t-RSD expansion effect postulated in Section 4.2.

5.2 Correlations between displacement of centres and net velocity

Fig. 5 shows the 2D distribution \((|V_v|, |d_v|)\), where the moduli of these vectors were taken. This distribution contains information about the dynamics of voids as whole entities. The cross in the figure indicates the mode of the 2D distribution, which shows that voids tend to move with a speed of 290 km/s, and their centres tend to displace an amount of 0.17 \(R_v^r\). It is clear then, that voids can not be considered at rest.

Concerning the velocities, Fig. 6 shows the distribution of the components of \(V_v\) along the three directions of the simulation box. They show Gaussian shapes, centred at 0 km/s, with a dispersion of 231 km/s. This was expected, since there is not any privileged direction of motion for voids in simulations.

Concerning the displacements, the left panel of Fig. 7 shows the distribution of the components of \(d_v\) along the three directions of the simulation box. They all show Gaussian shapes centred at 0. However, unlike velocities, displacements are different depending on the direction. On the one hand, the POS distributions (green dotted and blue dashed lines) are almost identical, as expected, with a dispersion of 0.25. On the other hand, the LOS distribution (red solid line) has a dispersion of 0.3. Nevertheless, after correcting the LOS displacements with Eq. (13), the POS distribution.
Redshift-space effects in voids I

Figure 4. Left panel. 2D distribution \((R_v^r, R_v^s)\) shown as a heat map. From blue to red, the colours indicate low to high number of void counts. There is a linear trend, whose slope is correctly described by the theoretical RSD factors \(q_{\text{RSD}}\) (dashed line) and \(q'_{\text{RSD}}\) (solid line). Right-upper panel. Overall distribution of \(R_v^s/R_v^r\). The \(q_{\text{RSD}}\) factor fits the median of the distribution. Right-lower panel. Distribution of \(R_v^s/R_v^r\) after dividing the void population in two. The \(q_{\text{RSD}}\) factor fits the median for the smaller subsample (red dashed line), whereas \(q'_{\text{RSD}}\) for the larger one (blue solid line). This is a statistical demonstration of the t-RSD expansion effect.

Figure 5. 2D distribution \((|V_v|, |d_v|)\). The cross indicates the 2D mode, which shows that voids tend to move with a speed of 290 km/s, and their centres tend to shift an amount of 0.17 \(R_v^s\). The phenomenon described in the last paragraph is more evident in the left panel of Fig. 8, where the 2D distribution \((V_v^3, d_v^3)\) is shown. There is a linear trend between both quantities, which is correctly described by Eq. (13) rep-

is recovered, as shown in the right panel. Note that these distributions are the residual and isotropic displacements detected in Section 3, when we were inspecting Fig. 1 visually, and can be attributed to Poisson noise when the void finder tries to localise the optimum centre (step iv in Section 2.2).
respected by the dashed line. Specifically, the slope of this line is given by the term \((1 + z_{\text{box}})/H_{\text{box}}\). After correcting the LOS displacements with this equation, the correlation disappears, leading to a 2D distribution that is almost identical to the corresponding ones of the POS components, \((v_1, d_1)\) and \((v_2, d_2)\). This is shown in the right panel.

From this analysis, we arrive at the third important conclusion of this work: void centres shift preferentially along the LOS when they are mapped from r-space into z-space, and this displacement can be statistically quantified by means of Eq. (13). These results give support to the v-RSD off-centring effect postulated in Section 4.5.

5.3 Cross correlations

It only remains to test if the expansion and off-centring effects are statistically independent by looking for cross correlations between the statistics that characterise the volume and movement of voids. Fig. 9 shows the 2D distributions \((d_1, R_{\text{v1}}/R_{\text{d1}})\) (left panel) and \((v_2, R_{\text{v1}}/R_{\text{d2}})\) (right panel). The horizontal lines are the theoretical estimations \(q_{\text{RSD}}\) (dashed) and \(q_{\text{RSD}}\) (solid). No correlations can be seen. It is worth mentioning that the POS distributions \((d_1, R_{\text{v1}}/R_{\text{d1}})\), \((d_2, R_{\text{v2}}/R_{\text{d2}})\), \((v_1, R_{\text{v2}}/R_{\text{d1}})\) and \((v_2, R_{\text{v1}}/R_{\text{d2}})\) show a similar behaviour. Therefore, these results provide a good evidence of this independence. This is the fourth important conclusion of this work.

6 IMPACT ON THE VOID SIZE FUNCTION

This section has a double intention. On the one hand, we finish the analysis of the last section incorporating the additional AP change of volume not treated yet. On the other hand, we study the impact of all the z-space effects studied in this work on the void size function. For this reason, we turn now to the FC void catalogues, fully affected by z-space systematics (see Table 1).

6.1 Generalities of the VSF modelling

We start with some generalities concerning the VSF modelling. The real-space VSF can be modelled using the excursion set formalism combined with the spherical expansion of matter underdensities derived from perturbation theory (Sheth & van de Weygaert 2004). This model is analogous to that used to describe the abundance of dark matter haloes, for which spherical collapse is used instead of expansion. There is another difference between the abundance of haloes and voids. The first is a one (overdense) barrier problem, whereas the second, a two barrier problem: one underdense and one over dense in order to take into account both the void-in-void and void-in-cloud evolution modes. The former corresponds to voids embedded in underdense regions and expanding, whereas the latter, to voids embedded in an overdense shell shrinking due to gravitational collapse. One barrier is the overdense threshold from which collapse occurs, whereas the other is the underdense threshold from which shell-crossing occurs.

There are two main approaches for this modelling: the Sheth & van de Weygaert (2004, SvdW) model, and the Jennings et al. (2013, Vdn) model. The key assumption of the former is isolated expansion, namely, it assumes that the comoving number density is conserved during expansion. However, this leads to a cumulative volume fraction of voids that exceeds unity. In order to fix this problem, the latter
assumes that, instead of the number density, the comoving volume fraction is conserved during expansion.

It is important to highlight that both models are only applicable to dark matter voids. Halo or galaxy voids are substantially different in their statistical properties. Nevertheless, many authors claim that both types of voids can still be related to each other with a linear bias approach (Furlanetto & Piran 2006; Pollina et al. 2017, 2018; Chan et al. 2019; Contarini et al. 2019; Fang et al. 2019; Pollina et al. 2019; Schuster et al. 2019; Chan et al. 2020), and hence, the SvdW and Vdn models are still valid. Such a model should fit the r-space abundances of Fig. 2 (blue lines).

In practice, z-space galaxy voids are used in observations. Therefore, the z-space systematics proposed in this work are expected to have a strong impact on the VSF. We will tackle this problematic using the theoretical machinery.
developed in Section 4. As we explained there, this framework depends strongly on cosmology, hence it must be combined with the excursion set formalism in order to obtain unbiased cosmological constraints from redshift surveys. In this way, we lay the foundations for a complete treatment of the VSF modelling, leaving for a future investigation a full analysis combining both developments.

6.2 Alcock-Paczynski correction

The left panel of Fig. 10 shows the void abundances (upper panel) and fractional differences of void counts (lower panel) of the two FC void catalogues, which are fully affected by z-space systematics, and hence, mimic two possible observational measurements. The VSF of the FC-l sample, which assumes a lower fiducial value with respect to that of the simulation, is represented with a green dot-dashed line, whereas the VSF of the FC-u sample, which assumes an upper fiducial value (Ω_m = 0.30), with a purple dashed line. The goal of this section is to verify if the theoretical framework developed in Section 4 allows to correct these abundances curves in order to recover the true underlying r-space one, unaffected by any of the z-space systematics (blue solid line in the plot, corresponding to the TC-rs-f sample).

To do this, it is sufficient to correct each void radius just applying Eq. (12), using the values of the AP and RSD factors that we have derived (q_{RSD}^l = 1.092, q_{AP}^l = 1.046 and q_{AP}^u = 0.960).

Instead of performing this correction directly, we will split it in a two-step procedure in order to discuss the different physical mechanisms involved. In this subsection, we discuss the first step, correcting for the AP change of volume with the AP factors. In the next subsection, we discuss the second step, correcting for the t-RSD expansion effect with the RSD factor. Therefore, the goal of this subsection is to recover the z-space VSF which is affected by RSD but unaffected by the AP effect (red solid line in the plot, corresponding to the TC-zs-f sample). This correction is shown in the right panel of Fig. 10.

The first aspect clearly seen in the left panel of Fig. 10 when comparing the abundances of the FC-l and FC-u samples with respect to the z-space VSF of reference, is that a higher VSF is obtained when a lower value of Ω_m is assumed, whereas the opposite behaviour occurs when a higher value of Ω_m is assumed. In the context of the bijective mapping, this means that the FC-l voids are systematically bigger, whereas the FC-u ones are smaller. This is in agreement with our discussion of Section 4.3, where we expected an AP-expansion for the FC-l voids since q_{AP}^l > 1, and an AP-contraction for the FC-u ones since q_{AP}^u < 1. Note that after correcting for the AP change of volume (right panel), both curves coincide with the z-space VSF of reference remarkably well for all radii of interest. Furthermore, this is also a clear signature that this effect is independent of the other z-space systematics.

We arrive here at the fifth important conclusion of this work: the volume of voids is also affected by the cosmological metric assumed to measure distances, which manifests as an overall expansion or contraction, depending on the chosen fiducial parameters. Moreover, this effect is independent of any other z-space systematics, and can be statistically quantified as a change of radius by a factor q_{AP}. These results give support to the AP change of volume postulated in Section 4.3.

6.3 Expansion effect correction

In this subsection, we discuss the second step of the correction: the t-RSD expansion effect. The goal now is to recover the r-space VSF of reference. This is shown in Fig. 11. The left panel is the same as the right panel of Fig. 10, except for the fact that the fractional differences are referred now to the r-space sample. The right panel shows the correction per se. This is satisfactory for all radii of interest, although there are some appreciable deviations for the smallest scales. Note that the large differences between z-space and r-space voids, already noted in Fig. 2, which can be ∆N_r/N_i > 4 for the largest scales, have been reduced to ∆N_r/N_i < 0.8 in the worst case.

For the analysis up to here, we have used the full r-space and z-space samples as references (TC-rs-f and TC-zs-f respectively). This was motivated by the fact that, in the spirit of the bijective mapping analysis, the full and bijective samples can be treated indistinctly. Moreover, we have used Eq. (7) (with q_{RSD}^l) to correct for the expansion effect instead of Eq. (6) (with q_{RSD}^u). In order to test the impact of the impurity of the reference samples regarding the bijective filtering, and the performance of both RSD factors, we repeated the analysis of the last paragraph using now the bijective r-space and z-space samples as references (TC-rs-b and TC-zs-b respectively). Moreover, as the AP correction works well at all scales, we have put aside the FC void samples, and only focused on correcting the bijective z-space VSF towards the r-space one. This is shown in Fig. 12. Note that the z-space VSF (red dashed line) and the r-space VSF (blue dashed line) are the same as in Fig. 2. The brown dot-dashed line represents the correction made with q_{RSD}^l, whereas the orange dot-dashed one, the correction with q_{RSD}^u. Two conclusions can be made. First, q_{RSD}^l performs better than q_{RSD}^u, specially at large scales. This confirms our suggestion that q_{RSD}^l is more suitable than q_{RSD}^u for large voids, the ones of interest for cosmological analyses. Second, unlike Fig. 11, there are not appreciable deviations at small scales. Therefore, these deviations can be attributed to the contamination of non-bijective voids in the full samples at these scales.

6.4 Free of off-centring effect

The achievements of the correction process proves another important fact: the VSF is unaffected by the r-RSD off-centring effect. This was implicitly assumed in the two-step correction, and constitutes the sixth important conclusion of this work.

In summary, the only two necessary ingredients to correct an observational VSF are the q_{AP} and q_{RSD} factors, which relate void radii in r-space and z-space. As we discussed previously, these factors are only two constants of proportionality, independent of the scale, and strongly cosmology dependent: q_{AP} depends only on the background cosmological parameters, whereas q_{RSD} depends only on β, encoding different cosmological information in a decoupled way. Therefore, the framework developed in this work must be combined with the excursion set used to model void abundances in order to obtain unbiased cosmological constraints.
Figure 10. Alcock-Paczynski correction of void abundances. \textit{Left panel.} VSFs of the FC void samples. The VSF of the FC-l sample, which assumes a fiducial value of $\Omega_m = 0.2$, is represented with the green dot-dashed line, whereas the VSF of the FC-u sample, which assumes a fiducial value of $\Omega_m = 0.3$, with the purple dashed line. By way of comparison, the VSF of the TC full r-space and z-space samples (blue and red solid lines respectively) are also shown as references. The corresponding fractional differences of void counts between the FC samples and the TC z-space one are shown at the bottom. \textit{Right panel.} The same as the left panel, but after correcting the FC abundance curves for the AP change of volume.

Figure 11. t-RSD expansion effect correction of void abundances. \textit{Left panel.} The same as the right panel of Fig. 10, except for the fact that the fractional differences were taken with respect to the r-space sample. \textit{Right panel.} The same as the left panel, but after correcting the abundance curves for the t-RSD expansion effect.

from redshift surveys. This is the seventh and last important conclusion of this work.

7 CONCLUSIONS

Cosmic voids are promising cosmological probes provided that the z-space systematics that affect them are properly treated. One approach is to use a reconstruction technique
to recover the r-space position of tracers before applying the void finding step. While this method has proved to be accurate in recovering the r-space void statistics, such as the void size function and the void-galaxy correlation function, and in extracting cosmological information from them, it looses the physical information about the structure and dynamics of voids that manifest when they are identified in z-space.

In this work, we explored an alternative approach: we analysed the void finding method in order to understand physically the z-space systematics affecting them. We used a spherical void finder and made a statistical comparison between the resulting real and redshift-space voids, in the context of the four (non-valid) hypotheses commonly assumed in any r-space → z-space mapping: (1) void number conservation, (2) isotropy of the density field, (3) isotropy of the velocity field, and (4) invariance of centre positions.

The main conclusions of this work can be summarised in the following statements.

1. There is a bijective mapping between z-space and r-space voids for scales not dominated by shot noise. This means that each z-space void has a unique r-space counterpart spanning the same region of space and vice-versa. In this context, condition (1) of void number conservation is not violated.

2. Voids in z-space are systematically bigger than their r-space counterparts. This can be understood as an expansion effect and statistically quantified as an increment in void radius by a constant factor \( q_{\text{RSD}} \) (Eq. 6). Actually, the slightly modified factor \( q'_{\text{RSD}} \) (Eq. 7) has proved to be more suitable for large voids, the ones of interest for cosmological studies. For this analysis, we assumed the validity of hypotheses (2) and (3) concerning the isotropy of the density and velocity fields in r-space in order to explain a z-space phenomenon, even if this isotropy is no longer valid for z-space voids. This expansion effect is a by-product of the RSD induced by tracer dynamics (t-RSD) at scales around the void radius.

3. Void centres are systematically shifted along the LOS when they are identified in z-space. It is a direct consequence of the violation of hypothesis (4) concerning the invariance of centre positions. This off-centring effect can be statistically quantified by means of Eq. (13). Hence, it constitutes a different class of RSD induced by large scale flows in the matter distribution. Interpreting voids as whole entities moving in space with a net velocity, this effect can be thought as a by-product of the RSD induced by void dynamics (v-RSD).

4. The expansion and off-centring effects are statistically independent, since they manifest in observations as two uncoupled effects.

5. The volume of voids is also altered by the fiducial cosmology assumed to transform angular positions and redshifts into distances, which manifests itself as an overall expansion or contraction, depending on the chosen fiducial parameters. This is the AP change of volume. Moreover, this effect is independent of the other two z-space systematics, and can be statistically quantified as a change of radius by a constant factor \( q_{\text{AP}} \) (Eq. 11). Therefore, all z-space systematics of this paper can be treated separately.

6. The void size function is affected by the t-RSD expansion effect and the AP change of volume, but it is free of the v-RSD off-centring effect. Therefore, an observational VSF can be corrected in order to recover the true underlying r-space VSF by a simple two-step correction given by Eq. (12).

7 The AP and RSD constant factors are strongly cosmology dependent: \( q_{\text{AP}} \) depends only on the background cosmological parameters, whereas \( q'_{\text{RSD}} \) depends only on \( \beta \), encoding in this way different cosmological information in a decoupled way. The former encodes information about the expansion history and geometry of the Universe, whereas the latter, about the growth rate of cosmic structures. Therefore, the framework developed in this work must be combined with the excursion set theory used to model void abundances in order to obtain unbiased cosmological constraints from redshift surveys.

Although the VSF is unaffected by the v-RSD off-centring effect, this is not the case for the void-galaxy correlation function. In a follow-up paper (Correa et al. in prep.), we will show that this effect plays a significant role, inducing new distortion patterns in observations. In the literature, only t-RSD are taken into account when modelling this statistic.
ACKNOWLEDGEMENTS
This work was partially supported by the Consejo de Investigaciones Científicas y Técnicas de la República Argentina (CONICET) and the Secretaría de Ciencia y Técnica de la Universidad Nacional de Córdoba (SeCyT). This project has received financial support from the European Union’s Horizon 2020 Research and Innovation programme under the Marie Skłodowska-Curie grant agreement number 734374 - project acronym: LACEGAL. This research was also partially supported by the Munich Institute for Astro- and Particle Physics (MIAPP) which is funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany’s Excellence Strategy àÅ EXC-2094 àÅ 390783311. CMC acknowledges the hospitality of the Max Planck Institute for Extraterrestrial Physics (MPE), where part of this work has been done. ANR acknowledges the financial support of the National Agency of Investigation Científica y Técnica of Argentina (PICT 2016-2017). NP was supported by Fondecyt Regular 1191813, and ANID project Basal AFB-170002, CATA. REA would like to specially thank Daniela Taborda for helping with the design of Fig. 3 and, fundamentally, for her unconditional support.

DATA AVAILABILITY
The data underlying this article will be shared on reasonable request to the corresponding author.

REFERENCES
Achitouv I., 2016, Phys. Rev. D, 94, 103524
Achitouv I., 2017, Phys. Rev. D, 96, 083006
Achitouv I., 2019, Phys. Rev. D, 100, 123513
Achitouv I., Neyrinck M., Paranjape A., 2015, MNRAS, 451, 3964
Achitouv I., Blake C., Carter P., Koda J., Beutler F., 2017, Phys. Rev. D, 100, 123513
Achitouv I., 2019, Phys. Rev. D, 99, 123524
Achitouv I., 2017, Phys. Rev. D, 96, 083006
Alcock C., Paczynski B., 1979, Nature, 281, 358
Angulo R. E., Springel V., White S. D. M., Jenkins A., Baugh C. M., Frenk C. S., 2012, MNRAS, 426, 2046
Barreira A., Cautun M., Li B., Baugh C. M., Pascoli S., 2015, J. Cosmology Astropart. Phys., 8, 028
Boylan-Kolchin M., Springel V., White S. D. M., Jenkins A., Benson G., 2009, MNRAS, 398, 1150
Cai Y.-C., Padilla N., Li B., 2015, MNRAS, 451, 1036
Cai Y.-C., Taylor A., Peacock J. A., Padilla N., 2016, MNRAS, 462, 2465
Cautun M., Paillas E., Cai Y.-C., Bose S., Armijo J., Li B., Padilla N., 2018, MNRAS, 476, 3195
Ceccarelli L., Ruiz A. N., Lares M., Perez A., Maldonado V. E., Luparello H. E., Garcia Lambas D., 2016, MNRAS, 461, 4013
Chan K. C., Hamaus N., Biagetti M., 2019, Phys. Rev. D, 99, 121004
Chan K. C., Li Y., Biagetti M., Hamaus N., 2020, ApJ, 889, 89
Chuang C.-H., Kitaura F.-S., Liang Y., Font-Ribera A., Zhao C., McDonald P., Tao C., 2017, Phys. Rev. D, 95, 063528
Clampitt J., Cai Y.-C., Li B., 2013, MNRAS, 431, 749
Clifton T., Ferreira P. G., Padilla A., Skordis C., 2012, Phys. Rep., 513, 1
Contarini S., Ronconi T., Marulli F., Moscardini L., Veropalumbo A., Baldi M., 2019, MNRAS, 488, 3526
Correa C. M., Paz D. J., Padilla N. D., Ruiz A. N., Angulo R. E., Sánchez A. G., 2019, MNRAS, 485, 5761
Davies C. T., Cautun M., Li B., 2019, MNRAS, 490, 4907
Eisenstein D. J., Seo H.-J., Spergel D. N., 2007, ApJ, 664, 675
Falck B., Koyama K., Zhao G.-B., Cautun M., 2018, MNRAS, 475, 3262
Fang Y., et al., 2019, MNRAS, 490, 3573
Furlanetto S. R., Piran T., 2006, MNRAS, 366, 467
Gunn J. E., Gott III J. R., 1972, ApJ, 176, 1
Hamaus N., Sutter P. M., Wandelt B. D., 2014, Physical Review Letters, 112, 251302
Hamaus N., Sutter P. M., Lavaux G., Wandelt B. D., 2015, J. Cosmology Astropart. Phys., 11, 036
Hamaus N., Pisani A., Sutter P. M., Lavaux G., Escoffier S., Wandelt B. D., Weller J., 2016, Physical Review Letters, 117, 091302
Hamaus N., Cousinou M.-C., Pisani A., Aubert M., Escoffier S., Weller J., 2017, J. Cosmology Astropart. Phys., 7, 014
Hawken A. J., et al., 2017, A&A, 607, A54
Hawken A. J., Aubert M., Pisani A., Cousinou M.-C., Escoffier S., Nadathur S., Rossi G., Schneider D. P., 2020, J. Cosmology Astropart. Phys., 2020, 012
Jenkins E., Li Y., Hu W., 2013, MNRAS, 434, 2167
Joyce A., Lombriser L., Schmidt F., 2016, Annual Review of Nuclear and Particle Science, 66, 95
Kaiser N., 1987, MNRAS, 227, 1
Koyama K., 2016, Reports on Progress in Physics, 79, 046902
Lam T. Y., Clampitt J., Cai Y.-C., Li B., 2015, MNRAS, 450, 3319
Lambas D. G., Lares M., Ceccarelli L., Ruiz A. N., Paz D. J., Maldonado V. E., Luparello H. E., 2016, MNRAS, 455, L99
Lares M., Ruiz A. N., Luparello H. E., Ceccarelli L., Garcia Lambas D., Paz D. J., 2017, MNRAS, 468, 4822
Li B., Zhao G.-B., Koyama K., 2012, MNRAS, 421, 3481
Lilje P. B., Lahav O., 1991, ApJ, 374, 29
Nadathur S., Percival W. J., 2019, MNRAS, 483, 3472
Nadathur S., Hotchkiss S., Diego J. M., Iliev I. T., Gottlöber S., Watson W. A., Yepes G., 2016, in van de Weygaert R., Shandarin S., Saar E., Einaasto J., eds, IAU Symposium Vol. 308, The Zeldovich Universe: Genesis and Growth of the Cosmic Web, pp 542–545 (arXiv:1412.8372), doi:10.1017/S1743921316010541
Nadathur S., Carter P. M., Percival W. J., Winther H. A., Bautista J. E., 2019a, Phys. Rev. D, 100, 023504
Nadathur S., Carter P., Percival W. J., 2019b, MNRAS, 482, 2459
Padilla N. D., Ceccarelli L., Lambas D. G., 2005, MNRAS, 363, 977
Paillas E., Cautun M., Li B., Cai Y.-C., Padilla N., Armijo J., Bose S., 2018, preprint, (arXiv:1810.02864)
Paz D., Lares M., Ceccarelli L., Padilla N., Lambas D. G., 2013, MNRAS, 436, 3480
Peebles P. J. E., 1976, ApJ, 204, 318
Pisani A., Sutter P. M., Madau M., Alizadeh E., Biswas R., Wandelt B. D., Hirata C. M., 2015, Phys. Rev. D, 92, 083531
Pollina G., et al., 2019, MNRAS, 487, 497
Pollina G., et al., 2019, arXiv e-prints, (arXiv:1911.09130)
Pollina G., et al., 2019, MNRAS, 487, 497
R Core Team 2013, R: A Language and Environment for Statistical Computing. R Foundation for Statistical Computing, Vienna, Austria, http://www.R-project.org/
Ronconi T., Marulli F., 2017, A&A, 607, A24
Ronconi T., Contarini S., Marulli F., Baldi M., Moscardini L., 2019, MNRAS, 488, 5075
Ruiz A. N., Paz D. J., Lares M., Luparello H. E., Ceccarelli L., Lambas D. G., 2015, MNRAS, 448, 1471
Rycroft C. H., 2009, Chaos, 19, 041111
Sahlén M., Silk J., 2018, Phys. Rev. D, 97, 103504
Schuster N., Hamaus N., Pisani A., Carbone C., Kreisch C. D., Pollina G., Weller J., 2019, J. Cosmology Astropart. Phys., 2019, 055
Sheth R. K., van de Weygaert R., 2004, MNRAS, 350, 517
Springel V., et al., 2005, Nature, 435, 629
Verza G., Pisani A., Carbone C., Hamaus N., Guzzo L., 2019, J. Cosmology Astropart. Phys., 2019, 040
Wickham H., 2016, ggplot2: Elegant Graphics for Data Analysis. Springer-Verlag New York, https://ggplot2.tidyverse.org
Zivick P., Sutter P. M., Wandelt B. D., Li B., Lam T. Y., 2015, MNRAS, 451, 4215

This paper has been typeset from a \TeX/\LaTeX file prepared by the author.