Is the ‘Soft Pomeron’ Valid for the Description of the Data from HERA?

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Abstract

It is established in the paper that experimental data on deeply virtual exclusive electroproduction of $\rho^0$- and $\phi$-mesons may be described fairly well in the framework of a generalized Regge approach without additional, $Q^2$-dependent, singularities in the $J$-plane.
Introduction

Experiments at $ep$-collider HERA have revealed that the cross section for exclusive production of light mesons by the virtual photon grows with increasing energy faster than the cross section for production of the same mesons by the real photon \[1\]. Besides, the cross section for production of heavy mesons ($J/\Psi$ etc) by the real photon also grows with energy faster than the cross-section for production of light mesons ($\rho, \omega, \phi$).

Therefore, with the presence of the second energy scale (besides the collision energy) which is virtuality of the photon $Q^2$ or the mass of the produced vector meson, the dependence on energy gets stronger.

There are several opinions on this interesting effect, and the most popular and wide spread (it seems to be accepted by experimentalists) is that besides the usual, considered the leading (i.e. the most right-hand) singularity in the $J$-plane and called "pomeron \[1\] there exists another singularity, which is located to the right of the former and thus the "pomeron" is not the leading singularity any longer. The peculiarity of the latter singularity (which is called "hard pomeron" sometimes) is that it exists only with the presence of "hardness" in the process (i.e. either high virtuality or heavy meson mass, like $J/\Psi$). The "hard pomeron" is considered to play the main role in the effect. The hypothesis seems to be rather convincing when we look at fig.1 \[1\] and fig.2 \[9\], where the experimental data and the dependence on energy of the cross-section in the framework of the "hard pomeron" dominance are represented.

Nevertheless, it leads us to some serious questions about consistency with general principles of the theory, like, for instance, unitarity or others less common but quite proved ideas like "maximal analyticity of the 2nd type" \[2\].

Let’s consider one of the processes being explored at HERA

\[ \gamma^* + p \rightarrow \rho_0 + p, \]

and suppose that the energy dependence of the cross section

\[ \sigma_{\gamma^*p \rightarrow \rho_0p} \sim \int d(\text{phase space}) \, |T_{\gamma^*p \rightarrow \rho_0p}|^2, \]

is determined by the "hard pomeron"

\[ T_{\gamma^*p \rightarrow \rho_0p} \sim s^{1+\lambda}, \quad \text{where } \lambda \simeq 0.2 \text{ (phenomenological value).} \quad (1) \]

The following inequality takes place

\[ |\text{Im}T_{\gamma^*p \rightarrow \rho_0p}|^2 \leq |\text{Im}T_{\gamma^*p \rightarrow \gamma^*p}||\text{Im}T_{\rho_0p \rightarrow \rho_0p}| \]

According to (1) the left part grows as $s^{2(1+\lambda)}$ the right as $s^{2+\lambda+\Delta}$, where $\Delta = \alpha(0) - 1$ is connected with the "soft" pomeron which controls pure hadronic process $\rho_0 + p \rightarrow \rho_0 + p$. So we obtain

\[ \lambda \leq \Delta, \]

with full contradiction with the hypothesis of the "hard" pomeron dominance.

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1 Historically, pomeron is a trajectory of the pole with even signature and quantum numbers of the vacuum, and $\alpha(0) = 1$. At present it is believed that $\alpha(0) > 1$. 
One could think, that "unitarization" of the powerlike (Born) behaviour would help us to avoid this conclusion, but it seems not to take place and the "paradox" remains [3]. So, it looks not to be with no purpose to search for new explanation of the effect. For example, in [4] an alternative idea was proposed for description of the total cross-section. The idea is that the onset of "preliminary asymptotics", which is reached in "soft" processes, only starts in "hard" processes. In other words, the threshold of "preliminary asymptotics" is increased due to "hardness". Unfortunately, in the paper [4] authors gave just a phenomenological description, which drops out of the pure Regge framework. The description is abound in peculiar parameterizations of residues, and tricky functions, and a great number of parameters. It makes the argument not quite convincing. Preasymptotical character of the energy dependence of structure functions in the energy region of HERA was also disputed in [5].

In this paper we suggest a concrete method based on using Regge pole conception and the scenario of delaying asymptotics is realized in its framework. We formulate the problem in a different way: If it’s possible to describe the data in the Regge-eikonal framework, and if ‘yes’, then what is the $Q^2$-dependence of residues etc due to the data?

The positive answer to the first question raises the problem of the "hard pomeron" status. The result of extraction of the $Q^2$-dependence of the residues puts in fact the problem of the theoretical investigation of transition form-factors (reggeon-vector meson).

2"Preliminary" in the sense that in the region of extra-high energy significant modifications are possible.
Regge-eikonal model for processes with virtual particles.

The amplitude of the process

\[ \gamma^* + N \rightarrow V + N, \]

\[ T_{\gamma^* N \rightarrow VN}(b; s, Q^2), \]

has the following form in the impact parameter representation (* always means "off-mass-shell")

\[
T_{\gamma^* N \rightarrow VN} = \sum_{V'} c_{V'}(Q^2) \frac{\delta_{VV'}}{\delta_{V'V}} \cdot T_{V'N \rightarrow VN} + \sum_{V' \neq V} c_{V'}(Q^2) (\delta_{VV'} - \frac{\delta_{V'V'}}{\delta_{V'V}} \delta_{VV'}) e^{i\delta_{VV'}}
\]

where \( c_V(q^2) = \frac{\mu^2/f_V}{q^2 + Q^2}, \)

\( f_V \) the coupling constant of the meson \( V \) to the electromagnetic current, \( \mu \) the mass of the vector meson, \( \delta_{AB}(b; s, Q^2) \) generalization of the Born amplitude for nondiagonal (and off-mass-shell) process

\[ A + N \rightarrow B + N \]

In the Regge-eikonal framework \( \delta_{AB}(b; s, Q^2) \) has form

\[
\delta_{AB}(b; s, Q^2) = \int \frac{dt}{16\pi s} J_0(b\sqrt{-t}) \sum_n g_{AB}^n(Q^2, t) g_{NN}^n(t) \xi_n(t) \left( \frac{s}{\mu^2 + Q^2} \right)^{\alpha_n(t)}
\]

where sum goes in all relevant reggeons (with trajectories \( \alpha_n(t) \) and signature factors \( \xi_n(t) \)).

These formulas are effective scalar, because we do not take into account different polarizations of vector mesons. In this sense the method used here is valid for processes like \( W^*(Z^*) + N \rightarrow \pi + N \).
For further analysis we retain in (2) only contributions of the pomeron and the secondary reggeon $\alpha_R(t)$ in the linear approximation in $t$ with parameters:

$$\Delta \equiv \alpha_P - 1 = 0.075$$

$$\alpha'_P(0) = 0.25$$

$$\eta \equiv 1 - \alpha_R = 0.46$$

$$\alpha'_R(0) = 1.00$$

partially taken from papers [7], and tested for the total and differential cross-section description.

Formula (2) is adjusted with unitarity, which, generally, in the case of virtual particles does not lead to the Martin-Froissart bound [8] for asymptotic behaviour of the cross-section, and even allows for such a powerlike growth as $s^\Delta$ [6]. Taking it into consideration we will investigate the first Born term only with following estimation of "unitarity corrections".

$$T_{\gamma^* N \rightarrow V N} = \delta_{\gamma^* V}[1 + i\delta_{VV}(s, b)] + i \sum_{V' \neq V} c_{V'}(Q^2)\delta_{V'VV}\delta_{VV},$$

where $\delta_{\gamma^* V} = \sum_{V'} c_{V'}(Q^2)\delta_{V'VV}(s, b, Q^2)$. The analysis shows that the first correction contribution is less than 10% (the second 0.1%), so with present experimental accuracy we decided to consider Born term only.

Then one has

$$\delta_{\gamma^* V}(s, t, Q^2) = c_P(Q^2)e^{\frac{4}{\pi}R^2_{\gamma^*P}t}\xi_P(t) \left(\frac{s}{Q^2 + \mu^2}\right)^{\alpha_P(t)} + c_R(Q^2)e^{\frac{4}{\pi}R^2_{\gamma^*R}t}\xi_R(t) \left(\frac{s}{Q^2 + \mu^2}\right)^{\alpha_R(t)}$$

Let’s transform the scattering amplitude into the impact parameter representation

$$T_{\gamma^* N \rightarrow V N} \simeq \delta_{\gamma^* V}(s, b, Q^2) = \frac{c_P(Q^2)\xi_P(0)}{\pi(Q^2 + \mu^2)R^2_{\gamma^*P}} \left(\frac{s}{Q^2 + \mu^2}\right)^{\Delta} e^{-\frac{k^2}{R^2_{\gamma^*P}}} +$$

$$+ \frac{c_R(Q^2)\xi_R(0)}{\pi(Q^2 + \mu^2)R^2_{\gamma^*R}} \left(\frac{Q^2 + \mu^2}{s}\right)^{\eta} e^{-\frac{k^2}{R^2_{\gamma^*R}}}$$

where $\xi_{P,R}(0)$ stands for signature factors,

$$\bar{R}^2_{\gamma^*P} = 4\alpha'_P(0) \ln \frac{s}{Q^2 + \mu^2} + R^2_{\gamma^*P}(Q^2),$$

$$\bar{R}^2_{\gamma^*R} = 4\alpha'_R(0) \ln \frac{s}{Q^2 + \mu^2} + R^2_{\gamma^*R}(Q^2)$$

are the pomeron and the reggeon "radii". In the framework of Regge approach the $Q^2$-dependence of radii and residues is neither fixed nor limited. It gives us a possibility to describe the data in "pure Regge spirit" (i.e. without any trajectory with the $Q^2$-dependence, or in other words, without "hard pomerons")

Then

$$\sigma_{\gamma^* p \rightarrow V p}(s, Q^2) = 4\pi \int_0^\infty db^2 |T_{\gamma^* p \rightarrow V p}|^2$$
Figures 3 and 4 show descriptions of the cross-sections for the processes $\gamma^* p \rightarrow \rho_0 p$ and $\gamma^* p \rightarrow \phi p$ and the $Q^2$-dependence of residues and radii needed for that.

One can obviously conclude:

1. The data can be described in terms of a generalized Regge-eikonal mechanism and it doesn’t demand introducing new ($Q^2$-dependent) singularities besides the Regge poles.

2. The $Q^2$-dependence of residues is rather arbitrary and does not reveal any contradiction with (quite poor) theory.

Now we shall briefly discuss the $Q^2$-dependence of the cross-section. Let’s analyze the pomeron contribution into the Born term

$$\delta^P_{\gamma^* V} = C^P_{\gamma^* V}(Q^2)e^{\frac{1}{4}R^2_0(Q^2)+\frac{1}{4}R^2+\alpha'(0)log\frac{s}{Q^2+\mu^2})t\left(\frac{s}{Q^2+\mu^2}\right)^{1+\Delta}$$

where all powerlike behaviour on $Q^2$ is dictated by absence of actual (powerlike) scaling violation (i.e. in the leading term) in structure functions of deep inelastic scattering. The $Q^2$-dependence in $C^P_{\gamma^* V}(Q^2)$ is then supposed to be weak.

Then we have

$$\sigma_{\gamma^* V}(s, Q^2) = \left(\frac{1}{Q^2+\mu^2}\right)^{2+2\Delta} \frac{2s^{2\Delta}|C^P_{\gamma^* V}(Q^2)|^2}{R^2_0+R^2+4\alpha'(0)log[s/(Q^2+\mu^2)]} + \ldots .$$

to the extent the Born approximation is valid, the strong $Q^2$-dependence is given by the factor

$$\left(\frac{1}{Q^2+\mu^2}\right)^{2+2\Delta} .$$

The experimental data on the $Q^2$-dependence of cross-sections for exclusive electroproduction of $\rho^0$-mesons with fixed $s$ can be parametrized by a strong dependence like (3) with the exponent $2.05 \pm 0.09[1]$. It is curious to compare it with $2 + 2\Delta \simeq 2.15$.

Good accordance of our prediction with the data gives us additional optimism.

**Conclusion**

Thus, in the paper we have managed to establish that a Regge-eikonal approach generalized for the case of virtual particles is valid for the description of the experimental data on exclusive electroproduction of vector mesons from HERA. The conclusion disproves the opinion that new data from HERA demand existence of other, besides Regge poles, singularities in the $J$-plane. For the sake of justice it is worth noticing that our work does not reject the very possibility for existence of the non-Regge singularities. Nevertheless, we point out that general principles of the theory (unitarity in particular) play the essential role in answering the question on determination of relative weight of Regge poles and possible non-Regge singularities in the complex $J$-plane.

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References

[1] ZEUS Collab.:M.Derrick et al., Phys. Lett. B 356 (1995) 601-616.

[2] G. Chew: "The analytic S matrix" (W. A. Benjamin, Inc. New-York–Amsterdam, 1966).

[3] V.A. Petrov: Nucl. Phys. (Proc. Suppl.) B 54A (1997) 160-162.

[4] E. Martynov, in: "Frontiers in Strong Interactions" (ed. P. Chiapetta, M. Haguenauer and J. Trân Thanh Vân), Editions Frontières 1996, 203.

[5] S.M. Troshin, N.E. Tyurin: preprint hep-ph/9701201.

[6] V.A. Petrov, in: "Frontiers in Strong Interactions" (ed. P. Chiapetta, M. Haguenauer and J. Trân Thanh Vân), Editions Frontières 1996, 139.

[7] Compas Group IHEP. Phys. Rev. D 54 (1996) 192.

P.V. Landshoff and A. Donnachie, Phys. Lett., B 296 (1992) 227.

[8] M. Froissart, Phys. Rev. 123 (1961) 1053.

A. Martin, Phys. Rev. 129 (1963) 1423.

[9] ZEUS Collab.: M. Derrick et al., Phys. Lett. B 80 (1996) 220-234.
Figure 3: The cross-section $\sigma_{\gamma p \to \rho p} (s = W^2)$ and the $Q^2$-dependence of parameters.
Figure 4: The cross-section $\sigma_{\gamma p \to \phi p} (s = W^2)$ and the $Q^2$-dependence of parameters.