Abstract—Grid-forming (GFM) control has been considered as a promising solution for accommodating large-scale power electronics converters into modern power grids thanks to its grid-friendly dynamics, in particular, voltage source behavior on the AC side. The voltage source behavior of GFM converters can provide voltage support for the power grid, and therefore enhance the power grid (voltage) strength. However, grid-following (GFL) converters can also perform constant AC voltage magnitude control by properly regulating its reactive current, which may also behave like a voltage source. Currently, it still remains unclear what are the essential difference between the voltage source behaviors of GFL and GFM converters, and which type of voltage source behavior can enhance the power grid strength. In this paper, we will demonstrate that only GFM converters can provide effective voltage source behavior and enhance the power grid strength in terms of small signal dynamics. Based on our analysis, we further study the problem of how to configure GFM converters in the grid and how many GFM converters we will need. We investigate how the capacity ratio between GFM and GFL converters affects the equivalent power grid strength and thus the small signal stability of the system. We give guidelines on how to choose this ratio to achieve a desired stability margin. We validate our analysis using high-fidelity simulations.

Index Terms—Grid strength, grid-forming converters, small signal stability, short-circuit ratio, voltage source behaviors.

I. INTRODUCTION

To achieve net zero, the large-scale integration of power electronics converters into power systems is inevitable, as they act as grid interfaces of renewable energy sources [1], [2]. Currently, most converters apply phase-locked loops (PLLs) in practice, which passively follow the grid frequency, also known as grid-following (GFL) control [3]. However, it has been widely recognized that GFL control cannot support the large-scale integration of converters, because i) the power grid needs some sources to establish the voltage and frequency, and ii) GFL converters may induce instability in weak grids, i.e., power grids with low short circuit ratios (SCRs) [4]–[7].

By comparison, grid-forming (GFM) converters behave as coupled oscillators in a power network, which can establish their own frequencies and spontaneously synchronize with each other [8]–[13]. Moreover, they can adapt to very weak power grids. In this paper, we consider virtual synchronous machine (VSM) as a prototypical GFM control, similar to [9].

We conjecture that the combination of GFM and GFL converters can constitute a resilient power grid, where GFL converters follow the frequency/voltage established by GFM converters.

Another major advantage of GFM converters is their voltage source behavior, namely, they generally behave as voltage sources rather than current sources thanks to the AC voltage control loop (cascaded with current control loop) [12], [14], [15]. Some GFM controls directly give control commands of voltage magnitude and angle to the modulation block without a cascaded voltage and current control loop, which also have voltage source behaviors [13], [16], [17]. The voltage source behavior of GFM converters enables fast voltage support for power grids and enhances the power grid strength in terms of small signal dynamics [9]. Note that under large disturbances, GFM converters will become current sources due to the current limitation, and their ability of enhancing power grid strength under such circumstances still remains to be investigated thoroughly [15]. This paper particularly focuses on how the voltage source behavior of GFM converters enhance the power grid (voltage) strength with regard to small signal dynamics.

Intuitively, since GFM converters behave like voltage sources, installing a GFM converter near a GFL converter should improve the local power grid strength of the GFL converter and thus improve its small signal stability margin (as GFL converters may become unstable in weak grids). This intuition was confirmed in our previous work [9], where we investigated the impact of GFM converters on the small signal stability of power systems integrated with GFL converters. We demonstrated that installing GFM converters is equivalent to enhancing the power grid strength (characterized by the so-called generalized short-circuit ratio (gSCR)). However, it still remains unclear how to configure newly installed GFM converters in the grid and how to decide their capacities (or equivalently, how many GFM converters we will need) to ensure a desired stability margin for the power grid.

Moreover, one important question is: since GFL converters can perform constant AC voltage magnitude control, do they also have effective voltage source behaviors to enhance the power grid strength? To be specific, one can introduce the terminal voltage magnitude as a feedback signal to generate the reactive current reference and regulate the voltage magnitude to a reference value [4], [5]. In this case, though the terminal voltage magnitude is well regulated, it remains unclear if the GFL converters can be considered as effective voltage sources to enhance the power grid (voltage) strength. We believe that it is essential to answer the above question before studying how many GFM converters we will need to enhance the power grid strength, as one may simply resort to modifying GFL...
converters to enable voltage source behaviors if they can be used to enhance the power grid strength.

This paper aims at answering the above questions. Firstly, we compare the dynamical admittance/impedance models of GFL and GFM converters. Note that a large admittance (or equivalently, a small impedance) indicates that the converter’s behavior is closer to a voltage source. To make a fair comparison, we consider the scenario where both GFL control and GFM control aim at regulating the AC voltage and active power. We show that only GFM control can provide effective voltage source behaviors, which justifies the necessity of installing GFM converters. On this basis, we investigate the problem of how many GFM converters are needed to enhance power grid strength. We review the relationship between power grid strength and the small signal stability of a multi-converter system. Then, we give recommendations for the capacity ratio of GFM converters to the stability of a GFM-GFL hybrid system.

The rest of this paper is organized as follows: Section II compares the voltage source behaviors of GFL and GFM converters. Section III reviews the connection between power grid strength and small signal stability. Section IV investigates how GFM converters enhance power grid strength and how many GFM converters are needed to satisfy a prescribed stability margin. Our analysis sheds some light on the perspective of power grid strength and small signal stability.

II. VOLTAGE SOURCE BEHAVIORS OF GFL AND GFM CONVERTERS

In this section, we demonstrate how the voltage source behavior of a converter can be characterized by its impedance model. Then, we will compare the voltage source behaviors of GFL and GFM converters by studying their impedance models.

A. Control structures of GFL and GFM converters

Consider a three-phase power converter that is connected to an AC grid (an infinite bus), as shown in Fig. 1. The converter can be operated either in GFL mode (Mode 1 in Fig. 1) or in GFM mode (Mode 2 in Fig. 1).

In GFL mode, the converter applies a conventional synchronous reference frame (SRF) PLL to realize grid synchronization, which uses the q-axis voltage \( V_q \) to generate the internal frequency (or frequency deviation \( \Delta \omega \)) \([5], [18]\). A current control loop is used for fast current control. The \( d \)-axis current reference \( I_{d,\text{ref}} \) comes from an active power control loop, which regulates the active power \( P_E \) to its reference value \( P_{E,\text{ref}} \). The \( q \)-axis current reference \( I_{q,\text{ref}} \) comes from an AC voltage magnitude control loop, which regulates the \( q \)-axis voltage \( V_q \) (\( V_d \) is the voltage magnitude at steady state) to the reference \( V_{q,\text{ref}} \). Note that \( I_{q,\text{ref}} \) can also come from a reactive power control loop, whilst in this paper, we aim at investigating if GFL converters can behave like a voltage source when it performs constant AC voltage control. The GFL control law is then summarized as

\[
\begin{bmatrix}
\Delta \omega \\
I_{d,\text{ref}} \\
I_{q,\text{ref}} \\
\end{bmatrix} =
\begin{bmatrix}
\text{PI}_{\text{PLL}}(s) & 0 & 0 \\
0 & \text{PI}_{\text{PC}}(s) & 0 \\
0 & 0 & \text{PI}_{\text{VC}}(s) \\
\end{bmatrix}
\begin{bmatrix}
V_q \\
P_{E,\text{ref}} - P_E \\
V_d - V_{q,\text{ref}} \\
\end{bmatrix}
\]

where \( \text{PI}_{\text{PLL}}(s) \), \( \text{PI}_{\text{PC}}(s) \), and \( \text{PI}_{\text{VC}}(s) \) are respectively the transfer functions of the PI regulators in the PLL, active power control loop, and voltage magnitude control loop.

In GFM mode, the internal frequency may come from droop control or an emulated swing equation, which uses the active power signal to generate the frequency \([19]\). Note that one can also consider DC voltage control as a special type of (indirect) active power control to achieve similar functionalities \([20], [21]\). To enable a voltage source behavior, an outer AC voltage control loop is cascaded with the inner current control loop to regulate the \( d \)-axis and \( q \)-axis voltages \([12], [15]\). To be specific, the \( d \)-axis and \( q \)-axis current references respectively come from the \( d \)-axis and \( q \)-axis voltage control loops. Moreover, one may introduce the grid-side current as feedforward...
signals to improve the control performance, as shown in Fig. 1. The GFM control law (ignoring current feedforward) can be summarized as

\[
\begin{bmatrix}
\Delta \omega \\
I_d^\text{ref} \\
I_q^\text{ref}
\end{bmatrix} = \begin{bmatrix}
0 & f_I(s) & 0 \\
0 & 0 & -\pi \text{VC}(s) \\
-\pi \text{VC}(s) & 0 & 0
\end{bmatrix} \begin{bmatrix}
V_q \\
P^\text{ref} - P_E \\
V_d - V^\text{ref}
\end{bmatrix},
\]

(2)

where \( f_I(s) \) is the synchronization control law, and a typical choice is \( f_I(s) = \frac{1}{J_d + s} \) as in VSMs (\( J \) and \( D \) are respectively the virtual inertia and virtual damping coefficients).

By comparing (1) and (2) (or observing the control loop connections in Fig. 1), one can deduce that in our setting, the difference between GFL control and GFM control is characterized by the structural difference between control matrices \( \mathbf{K}_{\text{GFL}}(s) \) and \( \mathbf{K}_{\text{GFM}}(s) \). That is, one can view GFL control and GFM control as connecting control loops in different ways; this observation is consistent with (22). For instance, in GFL control, the \( q \)-axis voltage control loop (i.e., PLL) is used to generate the frequency, while in GFM control, the \( q \)-axis voltage control loop is used to generate the \( q \)-axis current reference (i.e., connected to the \( q \)-axis current control loop). Under our setting, the GFL control and GFM control share exactly the same objectives, i.e., regulating the \( q \)-axis voltage to 0, regulating the \( d \)-axis voltage to \( V^\text{ref} \), and regulating the active power \( P_E \) to its reference \( P^\text{ref} \). Hence, they achieve the same equilibrium at steady state (assume that the grid frequency remains constant), but the dynamic behaviors during the transient are different. Note that under our setting, the GFL control also aims at regulating the \( q \)-axis voltage (via PLL) and the \( d \)-axis voltage (via reactive current control), just like GFM control. However, it is not clear if such an implementation of GFL control can equip the converter with effective voltage source behaviors. We investigate this issue in what follows.

B. Admittance model and voltage source behaviors

Impedance/admittance based modeling and analysis has been popular in studying power converter’s dynamics thanks to its compact representation of the interaction among different control loop, filters, and the power grid (23–26). In this paper, we use impedance/admittance modeling to study the converter’s dynamics and analyze its voltage source behavior. Fig. 2 shows the \( dq \) impedance model (in the global \( dq \) coordinate under the nominal frequency) of the grid-connected converter in Fig. 1, where \( Z_g(s) \) is the impedance model of the grid-side inductor \( L_g \), and \( Z_{\text{GFL}}(s) \) (\( Z_{\text{GFM}}(s) \)) is the impedance model of the converter when it applies GFL control (GFM control); see the detailed derivations in Appendix A and B. Note that \( Z_g(s), Z_{\text{GFL}}(s), \) and \( Z_{\text{GFM}}(s) \) are all \( 2 \times 2 \) transfer function matrices. The impedance \( Z_{\text{GFL}}(s) \) (or \( Z_{\text{GFM}}(s) \) if the converter applies GFM control) represents the “distance” between the point of voltage control (i.e., where the capacitor voltage \( V_{dC} \) is considered) and an ideal voltage source behind the impedance (whose voltage deviation in the global coordinate is \( \Delta U_s = 0 \)), as shown in Fig. 2 i.e.,

\[
\begin{bmatrix}
\Delta V_d' \\
\Delta V_q'
\end{bmatrix} = -Z_{\text{GFL}}(s) \begin{bmatrix}
\Delta I_d' \\
\Delta I_q'
\end{bmatrix},
\]

(3)

where \( \Delta V_d' \) and \( \Delta V_q' \) are respectively the \( d \)-axis and \( q \)-axis capacitor voltage deviations in the global coordinate; \( \Delta I_d' \) and \( \Delta I_q' \) are respectively the \( d \)-axis and \( q \)-axis grid-side current deviations in the global coordinate. If \( Z_{\text{GFL}}(s) = 0 \), then we have \( \Delta V_d' = \Delta V_q' = 0 \). In this case, the converter can be viewed as a perfect voltage source at the point of voltage control, since the voltage deviation is always zero no matter how the current changes. Note that (3) can also be rewritten as (A.20) (given in Appendix A), where \( Y_{\text{GFL}}(s) = Z_{\text{GFL}}^{-1}(s) \) is the corresponding admittance matrix. If the converter applies GFM control, then one can replace \( Z_{\text{GFL}}(s) \) by \( Z_{\text{GFM}}(s) \) in (3) to represent the converter’s dynamics; the corresponding admittance model is \( Y_{\text{GFM}}(s) = Z_{\text{GFM}}^{-1}(s) \) (see also (B.9) in Appendix B). The difference between \( Y_{\text{GFL}}(s) \) and \( Y_{\text{GFM}}(s) \) results from the difference of GFL control and GFM control (see (1) and (2)). These admittance/impedance models fully capture the dynamics of all the control loops and the filters (capacitor and converter-side inductor).

If \( Z_{\text{GFL}}(s) \) (or \( Z_{\text{GFM}}(s) \) if applying GFM control) is “large”, then the converter’s behavior is far away from a voltage source at the point of voltage control. However, it is nontrivial to tell if \( Z_{\text{GFL}}(s) \) is “large” since it is a \( 2 \times 2 \) transfer function matrix rather than a scalar. We deal with this issue in the following subsection.

C. “Dimension” of voltage source behaviors

When the converter applies GFL control, its admittance model in the global \( dq \) coordinate is

\[
Y_{\text{GFL}}(s) = Y_{\text{CL}}(s) + \begin{bmatrix}
Y_{\text{PC}}(s) & 0 \\
Y_{\text{VC}}(s) & Y_{\text{Sync}}(s)
\end{bmatrix},
\]

(4)

where \( Y_{\text{CL}}(s) = \begin{bmatrix}
s C_F & -\omega C_F \\
\omega C_F & s C_F
\end{bmatrix} \) is the admittance matrix of the capacitor, and

\[
Y_{\text{PC}}(s) = \frac{G_I(s) P_{\text{PC}}(s) I_{Cd0} + Y_{V_F}(s)}{1 + G_I(s) P_{\text{PC}}(s) V_{d0}},
\]

\[
Y_{\text{VC}}(s) = -G_I(s) P_{\text{VC}}(s),
\]

\[
Y_{\text{Sync}}(s) = \frac{Y_{V_F}(s) - P_{\text{PLL}}(s) I_{Cd0}}{s + P_{\text{PLL}}(s) V_{d0}}.
\]

In the above, \( I_{Cd0} \) and \( V_{d0} \) are respectively the steady-state values of the \( d \)-axis current and \( d \)-axis voltage; \( G_I(s) \) represents tracking dynamics of the current control loop and \( Y_{V_F}(s) \) represents the dynamics of voltage feedforward, see (A.5). We
have \( G_I(s) \approx 1 \) and \( Y_{VF}(s) \approx 0 \) if the current control loop has sufficiently high bandwidth.

When the converter applies GFM control, its admittance model in the global \( dq \) coordinate is

\[
Y_{GFM}(s) = Y_{CL}(s) + \begin{bmatrix}
Y_0(s) & 0 \\
Y_{Swing}(s) & Y_0(s)
\end{bmatrix},
\]

where

\[
Y_0(s) = \frac{Y_{VF}(s) + G_I(s)P_{VC}(s) + G_I(s)sC_F}{1 - G_I(s)},
\]

\[
Y_{Swing}(s) = \frac{I^2_{d0} - Y^2_0(s)V^2_{d0}}{Js^2 + D_s}.
\]

We provide detailed derivations of \( Y_{GFL}(s) \) and \( Y_{GFM}(s) \) in Appendix A and B respectively, where to simplify the expressions, we assume that 1) the global \( dq \) coordinate is aligned with the controller’s \( dq \) coordinate at steady state, and 2) the steady-state value of the reactive current is zero.

In \( Y_{GFL}(s) \), the diagonal elements \( Y_0(s) \) represents the equivalent admittance caused by the \( d \)-axis and \( q \)-axis voltage control, and we have \( Y_0(s) \to \infty \) when the voltage control is sufficiently fast (i.e., the PI parameters in \( P_{VC}(s) \) are sufficiently large). The non-zero off-diagonal element \( Y_{Swing}(s) \) describes the dynamics of the swing equation combined with fast voltage control. The limit of \( Y_{Swing}(s) \) depends on the virtual inertia and virtual damping (i.e., \( J \) and \( D \)). For instance, if the converter can provide sufficient amount of virtual inertia, one may obtain \( Y_{Swing}(s) \to 0 \); if very little virtual inertia and damping can be provided, one may achieve \( Y_{Swing}(s) \to \infty \). Hence, we do not assign a limit to \( Y_{Swing}(s) \), while we notice that if one can achieve perfect voltage control (with a sufficiently high control bandwidth), \( Y_{GFM}(s) \) approaches

\[
Y_{GFM}(s) \to Y_{CL}(s) + \begin{bmatrix}
\infty & 0 \\
0 & \infty
\end{bmatrix}.
\]

The elements in \( Y_{GFL}(s) \) also have clear interpretation: 1) \( Y_{VC}(s) \) describes the tracking dynamics of active power, and \( Y_{PC}(s) \to I_{C0}/V_{d0} \) when the active power control loop is sufficiently fast (i.e., the PI parameters in \( P_{VC}(s) \) are sufficiently large); 2) \( Y_{VC}(s) \) describes the \( d \)-axis voltage tracking, and we have \( Y_{VC}(s) \to \infty \) when the \( d \)-axis voltage control loop is sufficiently fast (i.e., the PI parameters in \( P_{VC}(s) \) are sufficiently large); and 3) \( Y_{Sync}(s) \) describes the PLL tracking (synchronization) dynamics, and \( Y_{Sync}(s) \to -I_{C0}/V_{d0} \) when the PLL (i.e., \( q \)-axis voltage control loop) is sufficiently fast (i.e., the PI parameters in \( P_{PLL}(s) \) are sufficiently large). Hence, in an ideal case that achieves perfect active power tracking, perfect \( d \)-axis and \( q \)-axis voltage tracking, the admittance of a GFL converter approaches

\[
Y_{GFL}(s) \to Y_{CL}(s) + \begin{bmatrix}
I_{C0}/V_{d0} & 0 \\
0 & -I_{C0}/V_{d0}
\end{bmatrix}.
\]

This ideal case is not implementable since the control bandwidth is limited in practice, while it reflects how the control objectives affect the admittance matrix through GFL control.

We can observe that there are two infinity elements in \( Y_{GFL}(s) \) while there is only one infinity element in \( Y_{GFM}(s) \). This difference results from the structural difference between GFM control and GFL control, even though they share the same control objectives (e.g., \( d \)-axis and \( q \)-axis voltage control); see Section II-A. According to Section II-B, we require the impedance matrix being sufficiently “small”, or equivalently, the admittance matrix being sufficiently “large” such that the converter’s behavior is close to a voltage source. Since the two infinity elements appear on the diagonal of \( Y_{GFM}(s) \) in (7), we refer to it as “two-dimensional” voltage source (2D-VS) behavior; since there is only one infinity element in \( Y_{GFL}(s) \) in (8), we refer to it as “one-dimensional” voltage source (1D-VS) behavior. The 2D-VS indicates that the voltage source behavior is well maintained regardless of the direction of the current vector; by comparison, the 1D-VS indicates that the voltage source behavior can only be maintained with regard to one direction of current vector, namely, there exists one direction of current perturbation that renders the behavior of the converter far away from a voltage source. Hence, only 2D-VS behavior is an effective voltage source behavior in terms of enhancing power grid strength.

These properties can be mathematically validated by performing singular value decomposition to the admittance matrix or the impedance matrix. To be specific, we can focus on the smallest singular value of the admittance matrix, or equivalently, the largest singular value of the impedance matrix, which corresponds to the weakest direction of the voltage source behavior. Let \( Z(s) \) denote the converter’s impedance matrix; we have \( Z(s) = Z_{GFL}(s) = Y_{GFL}^{-1}(s) \) in GFL mode and \( Z(s) = Z_{GFM}(s) = Y_{GFM}^{-1}(s) \) in GFM mode. The largest singular value of \( Z(s) \), denoted by \( \sigma(Z(s)) \), satisfies

\[
\forall w: \sigma(Z(j\omega)) = \max_{\Delta V_{dq}(j\omega) \neq 0} \frac{\|Z(j\omega)\Delta V_{dq}(j\omega)\|_2}{\|\Delta V_{dq}(j\omega)\|_2},
\]

where \( \Delta V_{dq}(j\omega) = \begin{bmatrix} \Delta V_{dj}(j\omega) \\ \Delta V_{dq}(j\omega) \end{bmatrix} \), \( \Delta V_{dq}(j\omega) = \begin{bmatrix} \Delta V_{dj}(j\omega) \\ \Delta V_{dq}(j\omega) \end{bmatrix} \), and \( \| \|_2 \) denotes the \( \ell^2 \) norm.

**Remark 1.** A large \( \sigma(Z(s)) \) indicates that \( Z(s) \) is “large” in a certain direction where the converter’s behavior is far away from a voltage source. In other words, the largest singular value of the impedance matrix can be used to tell how far the converter’s behavior is away from a voltage source; the smaller \( \sigma(Z(s)) \), the closer the converter is to a voltage source. Moreover, \( \sigma(Z_{GFL}(s)) \) is generally larger than \( \sigma(Z_{GFM}(s)) \) due to the 1D-VS behavior of GFL converters and 2D-VS behavior of GFM converters, especially within the bandwidth of voltage control. Hence, only GFM converters can provide effective voltage source behaviors to enhance the power grid strength.

The above analysis is consistent with our previous work [9] based on matrix perturbation theory, which shows that \( Z_{GFM}(s) \) is “small” in the direction of the eigenvector of \( Y_{GFL}(s) \) (pertinent to the PLL instability problem). In fact, our analysis in the above is more general, as it shows that \( Z_{GFM}(s) \) is “small” in any direction. Fig. 5 plots \( \sigma(Z_{GFL}(s)) \) and \( \sigma(Z_{GFM}(s)) \) under typical control parameters (given at
III. POWER GRID STRENGTH AND STABILITY

To link the voltage source behavior to stability problems, we firstly revisit the relationship between power grid strength and stability. It has been widely recognized in power system studies that the system stability is strongly related to the power grid strength, especially when large-scale GFL converters are integrated into the grid [1, 2, 8, 24]. In a single-device-infinite-bus system, the power grid strength can be effectively characterized by SCR, which reflects the distance between the device and the infinite bus (an ideal voltage source). The characterization of power grid strength becomes nontrivial in a multi-device system. In our previous work [27], we rigorously showed that in terms of the small signal stability of a multi-device system, the power grid strength can be characterized by the so-called generalized short-circuit ratio (gSCR). We briefly review this concept in what follows.

Though our approach is general, to illustrate the point we consider the integration of n wind farms (Nodes 1 ~ n) into an inductive power network (with a high X/R ratio), as shown in Fig. 4. The power network has m (≥1) interior nodes (n + 1 ~ n + m) and k (≥1) infinite buses (n + m + 1 ~ n + m + k). The infinite buses can be used to represent some large-capacity synchronous generators or other areas. Let $B_{ij}$ be the susceptance between Node $i$ and Node $j$ in per-unit values ($B_{ij} = 0$, where the per-unit calculation is based on a global capacity $S_{global}$. The extended susceptance matrix of the network is $B \in \mathbb{R}^{(n+m) \times (n+m)}$, where $B_{ij} = -B_{ji}$ (i ≠ j) and $B_{ii} = \sum_{j=1}^{n+m+k} B_{ij}$. The interior nodes can be eliminated by Kron reduction [28], and the Kron-reduced susceptance matrix is $B_s = B_1 - B_2B_4^{-1}B_3$, where $B = \begin{bmatrix} B_1 & B_2 \\ B_3 & B_4 \end{bmatrix}$.

Definition III.1 (gSCR). The gSCR of the system in Fig. 4 is defined as the smallest eigenvalue of $S_B^{-1}B_r$, where $S_B \in \mathbb{R}^{n \times n}$ is a diagonal matrix whose $i$-th diagonal element is the ratio between the $i$-th wind farm’s capacity $S_i$ and the base capacity of per-unit calculation $S_{global}$, i.e., $S_{Bii} = S_i / S_{global}$.

Proposition III.2 (gSCR and stability [27]). When all the wind farms in Fig. 4 adopt GFL control and have homogeneous dynamics, the multi-wind-farm system is (small-signal) stable if and only if gSCR > CgSCR. Here CgSCR denotes the critical gSCR, defined as the value of SCR that renders a wind farm critically stable in a single-wind-farm-infinite-bus system. A larger gSCR indicates a larger stability margin.

We refer the interested readers to [27] for the rigorous proof of Proposition III.2. The gSCR mathematically reflects the weighted connectivity of the power network (i.e., power grid strength). Moreover, it dramatically simplifies the small signal stability of large-scale power systems, as one can focus only on the network part instead of directly calculating the eigenvalues of a large-scale dynamical system. The intuition behind is that the power network should be strong enough, or equivalently, the sources/generators should be close enough to the ideal voltage sources (infinite buses) such that the GFL control can effectively follow the established frequency and voltage. From the power network perspective, one can increase the gSCR (and thus improve the stability) by connecting more voltage sources (infinite buses) to the network, especially to the nodes that are far away from the existing infinite buses. In next section, we will show that the integration of GFM converters has similar effect to installing ideal voltage sources in the network, and we will investigate how large the capacity should be to meet certain stability margins.

IV. GFM CONVERTERS AND POWER GRID STRENGTH

Although GFM control has many superiorities over GFL control (e.g., voltage source behaviors, natural inertia emulations), it also has shortcomings. For instance, due to the current
limitation of power converters, GFM converters have much more complicated transient behaviors than GFL converters under large disturbances \([15], [29], [30]\), and it may be challenging to ensure the transient stability of the grid when it has a great amount of GFM converters. Actually, one open question is: do we need to operate all the converters in a power grid as GFM converters? In our opinion, operating some of them as GFM converters is enough to ensure the small signal stability of the system. We justify this thought below.

Consider a wind farm (with GFL control) that is equipped with an (aggregated) GFM converter, as shown in Fig. 5. Since we focus on the power grid strength on the transmission level, the interaction among different wind turbines inside the wind farm is ignored. Hence, we use an aggregated wind turbine to represent its dynamics, which is connected to the high-voltage grid via two series step-up transformers. The GFM converter is often an energy storage system, which on the one hand, can be used to compensate the fluctuation of wind power, and on the other hand, enhances the power grid strength. This setting has been widely accepted in industry and aligned with many on-going real-world GFM demonstration projects. The GFM converter is also connected to the high-voltage grid via two series step-up transformers. Let \(Y_{\text{local}}\) \((Z_{\text{local}})\) be the per-unit susceptance (reactance) between the internal voltage of the GFM converter and Node \(i\) (see Fig. 3), which includes the converter’s equivalent internal inductor (to approximate \(Z(s)\) at a certain frequency) and the equivalent inductor of transformers. Here the per-unit calculation of \(Y_{\text{local}}\) \((Z_{\text{local}})\) is based on the GFM converter’s capacity so that \(Y_{\text{local}}\) \((Z_{\text{local}})\) remains the same in different wind farms.

As detailed in Section II, GFM converters have effective voltage source behaviors in terms of small signal dynamics, which justifies the assumption below.

**Assumption 1.** In terms of small signal dynamics, a GFM converter can be approximated as an ideal voltage source (i.e., infinite bus) in series with an equivalent internal inductor at the point where AC voltage control is performed.

Fig. 2 shows that a GFM converter can be accurately modeled as an ideal voltage source (i.e., infinite bus) in series with a dynamical impedance \(Z_{\text{GFM}}(s)\). Since \(Z_{\text{GFM}}(s)\) is usually small due to the 2D-VS behavior of GFM converters, we approximate \(Z_{\text{GFM}}(s)\) by an inductor to simplify the following analysis. The choice of the reactance can refer to Fig. 3. For instance, if the frequency of the dominant poles (induced by the PLL dynamics in weak grids) is 10 Hz, then we can choose the reactance to be 0.01 pu since \(\sigma(Z_{\text{GFM}}(s)) = -40 \text{ dB} = 0.01\) at 10 Hz (see Fig. 3). It can be seen that this reactance is very small and thus has negligible impact on the system, which, again, justifies Assumption 1.

According to Assumption 1, the integration of GFM converters should have similar effect to connecting infinite buses to the power network, thereby increasing the power grid strength. This intuition was theoretically confirmed in [9]. It was proved that changing the converters’ control scheme from GFL to GFM is equivalent to increasing the power grid strength thanks to the voltage source behaviors. However, it remains unclear how many GFM converters we will need. To this end, we present the connection between the capacity of GFM converters and the grid strength (i.e., gSCR).

**Proposition IV.1 (gSCR and capacity of GFM converters).** Consider the power network with multiple wind farms in Fig. 4, where each wind farm (with GFL control) is equipped with a GFM converter, and the identical capacity ratio between the GFM converter and the GFL wind farm is \(\gamma\). If Assumption 1 holds, then the gSCR of the system is

\[
gSCR = gSCR_0 + \gamma Y_{\text{local}},
\]

where \(gSCR_0\) is the gSCR value without GFM converters.

**Proof.** The per-unit susceptance between the GFM converter and Node \(i\) becomes \(S_{Bi}Y_{\text{local}}\) when we use the global capacity \(S_{\text{global}}\) for per-unit calculations. With Assumption 1, the integration of a GFM converter in the \(i\)-th wind farm is equivalent to adding a branch between Node \(i\) and an infinite bus with susceptance \(S_{Bi}Y_{\text{local}}\). Then, we have

\[
gSCR = \lambda_1(S_{B}^{-1}(B_y + \gamma Y_{\text{local}}S_B)) = \lambda_1(S_{B}^{-1}B_y + \gamma Y_{\text{local}}I) = \lambda_1(S_{B}^{-1}B_y) + \gamma Y_{\text{local}} = gSCR_0 + \gamma Y_{\text{local}},
\]

where \(\lambda_1(\cdot)\) denotes the smallest eigenvalue of a matrix, and \(I\) is the identity matrix. This completes the proof.

Proposition IV.1 indicates that the installations of GFM converters in the wind farms increase the gSCR and thus the power grid strength. Moreover, the gSCR is a linear function of the capacity ratio \(\gamma\), with the slope being \(Y_{\text{local}}\). In practice, a typical value of \(Y_{\text{local}}\) is 0.16 pu if two step-up transformers are used, as shown in Fig. 5. With these typical values, we can easily calculate the desired capacity ratio \(\gamma\) using (10). Note that once \(\gamma\) is obtained, we can decide the number of GFM converters to satisfy this capacity ratio, based on the typical capacity of a GFM converter provided by the manufacturers.

**Example 1.** Consider the setting in Fig. 5 where the GFM converter and the GFL converter are connected to one common 110 kV bus. In practice, a lot of wind farms are integrated in weak grids, where the wind turbines often have to face a low SCR, e.g., SCR = 1 at its terminal (690 V bus) that can only guarantee the power transmission, or equivalently, gSCR = 1.2 at the transmission level (110 kV). However, currently many manufacturers design their (GFL) wind turbines to operate stably with SCR larger than 1.5 at its terminal (690 V bus), i.e., gSCR \(\geq 2\) at 110 kV. This gap (between 1.2 and 2) can be filled using GFM converter without further investment in enhancing the power network. According to (10), it requires the capacity ratio \(\gamma \geq 12.8\%\) if \(Z_{\text{local}} = 0.16\) pu.

![Fig. 5. Each wind farm is equipped with a GFM converter. The GFM converter and the GFL converter are connected to one common 110 kV bus.](image)
In practice, it is also possible that the wind farm (GFL converter) and the GFM converter are connected to one common 35 kV bus, as shown in Fig. 6. In this case, the electrical distance between the GFL converter and the GFM converter becomes smaller, and one may need less GFM converters to enhance the equivalent power grid strength, as illustrated in the next example.

**Example 2.** Consider the setting in Fig. 6 where the GFM converter and the GFL converter are connected to one common 35 kV bus. Under this setting, when the wind turbines face SCR = 1 at its terminal (690 V bus), we can equivalently calculate the new gSCR as follows:

\[
gSCR = gSCR_i = \frac{1}{1 - \gamma_i} + \frac{\gamma_i Z_{local}}{1 - \gamma_i}.
\]

As an alternative, one can also change some of the wind turbines in the wind farm from GFL mode to GFM mode, without installing a new energy storage system. This can be done by, for instance, using the control method in [31]. Notice that in practice, usually each wind turbine is equipped with this type of converter, and the system has satisfactory performance with a larger γ, i.e., γ = 1.7 at 35 kV. This gap (between 1.1 and 1.7) can be filled using GFM converter without further investment in enhancing the power network. According to (10), it requires the capacity ratio γ ≥ 4.8% since Z_{local} = 0.08 pu.

As an alternative, one can also change some of the wind turbines in the wind farm from GFL mode to GFM mode, without installing a new energy storage system. This can be done by, for instance, using the control method in [31]. Notice that in practice, usually each wind turbine is equipped with this type of converter, and the system has satisfactory performance with a larger γ, i.e., γ = 1.7 at 35 kV. This gap (between 1.1 and 1.7) can be filled using GFM converter without further investment in enhancing the power network. According to (10), it requires the capacity ratio γ ≥ 4.8% since Z_{local} = 0.08 pu.

**Example 3.** Consider the topology depicted in Fig. 5 where the GFM converter (capacity: γS_i) and the GFL converter (capacity: (1−γ)S_i) are connected to one common 35 kV bus. Similar to Example 2, we aim to raise the gSCR at 35 kV from 1.1 to at least 1.7, which can be done by changing some wind turbines from GFL mode to GFM mode. According to (11), it requires the capacity ratio γ ≥ 4.2% as Z_{local} = 0.08 pu.

**V. SIMULATION RESULTS**

Consider four wind farms that are integrated into a two-area power network (with a similar setting as in [9]), as shown in Fig. 7, where each wind farm adopts the setting in Fig. 5, i.e., the GFM converter and the GFL converter are connected to one common 35 kV bus. We consider direct-drive wind turbines that rely on GFL converters for grid connection. We consider the scenario where the system is unstable with gSCR = gSCR_0 = 1.2 (i.e., γ = 0) at the 110 kV bus. Rather than changing the power network, we use GFM converters to improve the power grid strength and stabilize the system according to Proposition IV.1. Fig. 8 shows the responses of the system with different capacity ratios γ. There is a voltage disturbance from the infinite bus at t = 0.2 s (a voltage sag of 5% that lasts 10 ms). It can be seen that the damping ratio of the system is improved when a larger γ is adopted (i.e., with more grid-forming converters), and the system has satisfactory performance with γ = 12.8% (aligned with Example 1).

We next consider the setting of wind farms depicted in Fig. 6, i.e., the GFM converter and the GFL converter are connected to one common 35 kV bus. We consider the scenario where the system is unstable with gSCR = gSCR_0 = 1.1 (i.e., γ = 0) at the 35 kV bus. The other settings for the power grid in Fig. 7 are the same as those described above. Fig. 9 shows the responses of the system with different capacity ratios γ. It can be seen that the damping ratio of the system is improved when a larger γ is adopted (i.e., with more grid-forming...
converters in GFM mode to ensure a desired stability margin. When the GFM converters are connected to medium voltage buses (e.g., 35 kV), we usually need less than 20% of the capacity ratio between the aggregated GFM converter and the power grid strength, rather than modifying GFL converters (e.g., with constant AC voltage magnitude control). Then, we explicitly derived the relationship between the capacity ratio of GFM converters and the power grid strength (characterized by the so-called gSCR), and proved that the installation of GFM converters in a multi-converter system improves the power grid strength and thus the overall small signal stability. Our analysis suggests that in terms of improving small signal stability, it is not necessary to operate all the converters in a power grid in GFM mode. For instance, our Example 1 and simulation results showed that a capacity ratio around 12.8% can already increase the stability margin significantly. Future work can include how to configure GFM converters in the power grid considering frequency stability, transient stability, and small signal stability simultaneously.

VI. CONCLUSIONS

This paper focused on how to determine the capacity/number of GFM converters in a power grid. As a first step, we study the voltage source behaviors of GFM and GFL converters, and demonstrate that only GFM converters can provide effective (i.e., 2D-VS) voltage source behaviors. Our analysis confirms that it is necessary to install GFM converters to provide effective (small signal) voltage support and enhance the power grid strength, as discussed in Section II. By comparison, GFM converters have 2D-VS behaviors due to its control structure and constant AC voltage control. The reason behind is that even with constant AC voltage control, GFL converters can only exhibit 1D-VS behaviors due to its control structure and thus cannot enhance the power grid strength, as discussed in Section II. By comparison, GFM converters have 2D-VS behaviors and can effectively enhance the power grid strength.

As discussed in the previous section, it is also possible to change some of the wind turbines from GFL mode to GFM mode without installing new energy storage systems. Fig. 11 shows the simulation results when different γ is applied (γ is the capacity ratio between the aggregated GFM converter and the whole wind farm). It can be seen that the damping ratio is increased with a larger γ, and the system has satisfactory performance with γ = 4.2% (aligned with Example 3). In short, the above simulation results are fully consistent with our analysis in the previous sections. We note that one can further increase γ to obtain a better damping ratio and performance. Of course, this requires more effort to increase the proportion of GFM converters in the system. Fig. 12 shows the capacity ratios (γ) required to achieve different prescribed values of equivalent gSCR at the terminal 690 V bus under different scenarios.

APPENDIX A

ADMITTANCE MODEL OF GFL CONVERTERS

In what follows, we derive the admittance matrix of the GFL (PLL-based) converter in Fig.1. When modeled in the controller’s rotating dq-frame (whose angular frequency is determined by the PLL), the filters’ equations are

\[ \bar{\vec{U}} - \bar{\vec{V}} = (sL_F + j\omega L_F) \bar{\vec{I}}_C, \]

\[ \bar{\vec{I}}_C - \bar{\vec{I}} = (sC_F + j\omega C_F) \bar{\vec{V}} = : Y_{CL}(s) \bar{\vec{V}}, \]

where \( \bar{\vec{U}} = U_d^* + jU_q^*, \bar{\vec{V}} = V_d + jV_q, \bar{\vec{I}}_C = I_{Cd}^* + jI_{Cq}, \bar{\vec{I}} = I_d + jI_q \) are the corresponding vectors of \( \bar{\vec{U}}_{abc}, \bar{\vec{V}}_{abc}, \bar{\vec{I}}_{abc} \) in the controller’s dq-frame, respectively, \( L_F \) is the converter-side inductance, and \( C_F \) is the LCL capacitance.

The control law of the current control loop is

\[ \bar{\vec{I}}^* = PI_{CC}(s) \left( \bar{\vec{I}}^{ref} - \bar{\vec{I}}_C \right) + j\omega L_F \bar{\vec{I}}_C + f_{VF}(s) \bar{\vec{V}}, \]

where \( PI_{CC}(s) = K_{CCP} + K_{CCl}/s \) is the transfer function of the PI regulator, \( f_{VF}(s) = K_{VF}/(TV_{VS} + 1) \) is a first-order filter that mitigates the high-frequency components of the voltage feed-forward signals, and \( \bar{\vec{I}}^{ref} = I_d^{ref*} + jI_q^{ref} \) is the current reference vector that comes from the outer loops.
By combining (A.1) and (A.3) we obtain
\[ G_I(s)I_{\text{ref}} - Y_{VF}(s)\vec{V} = \vec{I}_C, \tag{A.4} \]
where
\[ G_I(s) = \frac{\text{PI}_{\text{CC}}(s)}{sL_F + \text{PI}_{\text{CC}}(s)}, \quad Y_{VF}(s) = \frac{1 - f_{VF}(s)}{sL_F + \text{PI}_{\text{CC}}(s)} \tag{A.5} \]

We note that the above equations are obtained based on space vectors and complex transfer functions, and they can be conveniently transformed to matrix form considering the following equivalent transformation \[ y_d + jy_q = [G_d(s) + jG_q(s)](x_d + jx_q) \]
\[ \Leftrightarrow \begin{bmatrix} y_d \\ y_q \end{bmatrix} = \begin{bmatrix} G_d(s) & -G_q(s) \\ G_q(s) & G_d(s) \end{bmatrix} \begin{bmatrix} x_d \\ x_q \end{bmatrix}. \tag{A.6} \]

The small signal dynamics of the SRF-PLL that determines the dynamics of the controller’s rotating dq-frame is
\[ \Delta \theta = \frac{\Delta \omega}{s} = \frac{\text{PI}_{\text{PLL}}(s)}{s} \Delta V_q, \tag{A.7} \]
where \( \theta \) (rad) is the phase of the controller’s rotating dq-frame, \( \omega \) (rad/s) is the angular frequency, \( \text{PI}_{\text{PLL}}(s) = K_{\text{PLL}} + K_{\text{PLLI}}/s \) is the transfer function of the PI regulator in PLL.

The converter applies active power control and constant AC voltage control as
\[ I_{d}\text{ref} = \text{PI}_{\text{PC}}(s)(P_{\text{ref}} - P_E), \quad I_{q}\text{ref} = \text{PI}_{\text{VC}}(s)(V_d - V_{\text{ref}}), \tag{A.8} \]
where \( \text{PI}_{\text{PC}}(s) = K_{\text{PCP}} + K_{\text{PC}}/s \) and \( \text{PI}_{\text{VC}}(s) = K_{\text{VC}}, K_{\text{VC}}/s \) are the transfer functions of the PI regulators, \( P_{\text{ref}} \) and \( V_{\text{ref}} \) are the reference values, \( P_E \) is the active power output of the converter, which can be calculated by
\[ P_E = V_dI_{C,d} + V_qI_{C,q}. \tag{A.9} \]

Linearize (A.9) around the equilibrium \( (I_{C,d}, I_{C,q}, V_{d0}, V_{q0}) \) where \( V_{q0} = 0 \) and then combine it with (A.4) and (A.8) to obtain the converter-side equivalent admittance
\[ -\begin{bmatrix} \Delta I_{C,d} \\ \Delta I_{C,q} \end{bmatrix} = \begin{bmatrix} \Delta V_{d} \\ \Delta V_{q} \end{bmatrix} \begin{bmatrix} Y_{11}(s) & Y_{12}(s) \\ Y_{21}(s) & Y_{22}(s) \end{bmatrix}, \tag{A.10} \]
where
\[ Y_{11}(s) = \frac{G_I(s)\text{PI}_{\text{PC}}(s)I_{C,d0} + Y_{VF}(s)}{1 + G_I(s)\text{PI}_{\text{PC}}(s)V_{d0}}, \]
\[ Y_{12}(s) = \frac{G_I(s)\text{PI}_{\text{PC}}(s)I_{C,q0} + Y_{VF}(s)}{1 + G_I(s)\text{PI}_{\text{PC}}(s)V_{d0}}, \]
\[ Y_{21}(s) = -G_I(s)\text{PI}_{\text{VC}}(s), \]
\[ Y_{22}(s) = Y_{VF}(s). \tag{A.11} \]

The equivalent admittance in (A.10) represent the converter-side dynamics in the controller’s dq-frame. We next transform this local admittance to a global coordinate whose angular frequency is a constant (i.e., \( \omega = 100\pi \) rad/s in this paper).

Consider the following coordinate transformation
\[ \vec{V}e^{j\theta} = \vec{V}'e^{j\theta_G}, \tag{A.12} \]
\[ \vec{I}_{C}e^{j\theta} = \vec{I}_{C}'e^{j\theta_G}, \tag{A.13} \]
where \( \vec{V}' = V_d'e^{j\delta} + jV_q'e^{j\phi} \) and \( \vec{I}_{C}' = I_{C,d}' + jI_{C,q}' \) are the corresponding voltage and current vectors in the global coordinate, \( \theta_G \) is the phase of the global coordinate which meets \( s\theta_G = \omega_0 \).

We rewrite (A.12) as \( V_d' = V_d e^{j\delta} \), where \( \delta = \theta - \theta_G \), and then linearize it around \( \delta_0 = \theta_0 - \theta_G \) and \( V_0 = V_{d0} + jV_{q0} \) as
\[ \Delta \vec{V}' = \Delta \vec{V} e^{j\delta_0} + jV_0 e^{j\delta_0} \Delta \delta. \tag{A.14} \]

The matrix form of (A.14) is
\[ \begin{bmatrix} \Delta V_d' \\ \Delta V_q' \end{bmatrix} = e^{j\delta_0} \begin{bmatrix} \Delta V_d \\ \Delta V_q \end{bmatrix} \begin{bmatrix} -V_{d0} \\ V_{q0} \end{bmatrix}, \tag{A.15} \]
where \( e^{j\delta_0} \) is the matrix form of \( e^{j\delta_0} \). Analogously, for the current signals, we have
\[ \begin{bmatrix} \Delta I_{C,d}' \\ \Delta I_{C,q}' \end{bmatrix} = e^{j\delta_0} \begin{bmatrix} \Delta I_{C,d} \\ \Delta I_{C,q} \end{bmatrix} \begin{bmatrix} -I_{C,d0} \\ I_{C,q0} \end{bmatrix}. \tag{A.16} \]

Note that \( \Delta \delta = \Delta \theta \) in small signal dynamics. Hence, we have
\[ \Delta \delta = \frac{\text{PI}_{\text{PLL}}(s)}{s} \Delta V_q, \tag{A.17} \]
By combining (A.10), (A.15), (A.17), we obtain
\[ -\begin{bmatrix} \Delta I_{C,d}' \\ \Delta I_{C,q}' \end{bmatrix} = Y(s) \begin{bmatrix} \Delta V_d' \\ \Delta V_q' \end{bmatrix}, \tag{A.18} \]
where
\[ Y(s) = e^{j\delta_0} \begin{bmatrix} Y_{11}(s) & Y_{12}(s) + \text{PI}_{\text{PLL}}(s)I_{C,q0} \\ Y_{21}(s) & Y_{22}(s) + \text{PI}_{\text{PLL}}(s)I_{C,d0} \end{bmatrix} e^{-j\delta_0}. \tag{A.19} \]

Then, we obtain the admittance model of a PLL-based converter in the global coordinate seen from the grid side
\[ -\begin{bmatrix} \Delta I_{C,d}' \\ \Delta I_{C,q}' \end{bmatrix} = Y_{\text{GFL}}(s) \begin{bmatrix} \Delta V_d' \\ \Delta V_q' \end{bmatrix}, \tag{A.20} \]
\[ Y_{\text{GFL}}(s) = Y_{\text{CL}}(s) + Y(s), \tag{A.21} \]
which includes the admittance of the capacitor of LCL, with \( Y_{\text{CL}}(s) \) being the matrix form of \( \vec{Y}_{\text{CL}}(s) \). To simplify the analysis in this paper, we assume that the d-axis of the global coordinate is aligned with the d-axis of the controller’s coordinate at steady state (i.e., \( \delta_0 = 0 \)) and that the reactive current \( I_{C,q0} \approx 0 \). Under these assumptions, we obtain (4). The corresponding impedance model is \( Z_{\text{GFL}}(s) = Y_{\text{GFL}}(s)^{-1} \).

**APPENDIX B**

**Admittance Model of GFM Converters**

Consider the control law of an AC voltage control loop
\[ \vec{I}_{\text{ref}} = \text{PI}_{\text{VC}}(s)(V_{\text{ref}} - \vec{V}) + j\omega C_F \vec{V} + \vec{I}, \tag{B.1} \]
where the voltage reference \( V_{\text{ref}} \) is a real value, i.e., the voltage reference vector is aligned with the d-axis.

By combining (B.1), (A.2) and (A.4) we obtain
\[ -\Delta \vec{I} = Y'_0(s)\Delta \vec{V}, \tag{B.2} \]
where

\[ Y_0'(s) = \frac{Y_{VF}(s) + G_I(s)P_{IVC}(s) + sC_F}{1 - G_I(s)} + j\omega C_F. \]  

(B.3)

If we exclude the admittance of the capacitor of LCL, then the admittance model (in the controller’s dq-frame) becomes

\[ -\Delta \vec{I}_C = Y_0(s)\Delta \vec{V}, \]

\[ Y_0(s) = \frac{Y_{VF}(s) + G_I(s)P_{IVC}(s) + G_I(s)sC_F}{1 - G_I(s)}. \]  

(B.4)

(B.5)

Note that \( sC_F \) appears in the converter’s admittance \( Y_0(s) \) because the control law \( \text{(B.1)} \) introduces the grid-side current \( \vec{I} \) as a feedforward term. The matrix form of \( \text{(B.4)} \) is

\[ -\begin{bmatrix} \Delta I_{Cd} \\ \Delta I_{Cq} \end{bmatrix} = \begin{bmatrix} Y_0(s) & 0 \\ 0 & Y_0(s) \end{bmatrix} \begin{bmatrix} \Delta V_d \\ \Delta V_q \end{bmatrix}. \]  

(B.6)

The small-signal model of the swing equation is

\[ \Delta \delta = \frac{1}{J_s^2 + D_s} \Delta \phi. \]  

(B.7)

We assume \( \delta_0 = 0 \) and \( I_{C,q0} \approx 0 \), and combine \( \text{(A.15), (A.16), (B.6), (B.7)}, \) leading to

\[ \begin{bmatrix} \Delta I_{Cd} \\ \Delta I_{Cq} \end{bmatrix} = \begin{bmatrix} \frac{Y_0(s)}{J_s + sD_s} & 0 \\ 0 & Y_0(s) \end{bmatrix} \begin{bmatrix} \Delta V_d \\ \Delta V_q \end{bmatrix}. \]  

\[ \begin{bmatrix} \Delta I_d \\ \Delta I_q \end{bmatrix} = Y_{GFM}(s) \begin{bmatrix} \Delta V_d \\ \Delta V_q \end{bmatrix}, \]  

\[ \text{and } Y_{GFM}(s) \text{ is given in (5).} \]

The corresponding impedance model is \( Z_{GFM}(s) = Y_{GFM}(s). \)

The main parameters of the GFM converter are: \( L_F = 0.05 \text{ pu}, \) \( C_F = 0.06 \text{ pu}, \) \( P_{\text{ref}} = 1, \) \( V_{\text{ref}} = 1, \) \( K_{\text{CCP}} = 0.3, \)

\( K_{\text{CCI}} = 10, \) \( K_{VF} = 1, \) \( T_{VF} = 0.02, \) \( K_{VCP} = 2, \) \( K_{VCI} = 10, \) \( J = 2, \) \( D = 50. \)

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