Determination Price Volatility of Bitcoin with Autoregressive Conditional Heteroscedasticity Models

Bitcoin’s Fiyat Oynaklığının Otoregressif Koşullu Değişen Varyans Modellemeleri ile Belirlenmesi

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Abstract

**Purpose:** The purpose of this research is to analyze the price movements of bitcoin, which has become a new phenomenon in financial markets since 2009, the first year of its release, and can be defined as virtual money or crypto money, to be seen as a financial investment tool.

**Design/Methodology:** In the study, volatility, return behavior and reliability as a financial investment tool are examined with autoregressive Conditional Variable Variance modeling. In this context, symmetrical and asymmetrical ARCH models were used.

**Findings:** As a result of the analysis, it has been found that it has an asymmetric effect in the first period for the bitcoin return series examined with symmetric and asymmetric ARCH models. In addition, it has been determined that shocks occurring in the bitcoin return series according to the half-life criteria are exposed to the volatility effect for more than 30 days in each period. It has been determined that bitcoin, which is examined by periods, has higher volatility in its first years.

**Limitations:** The volatility of bitcoin, which has become a new phenomenon in financial markets today, can be defined as virtual money or crypto money, has been analyzed.

**Originality/Value:** In fact, there are many virtual currencies or cryptocurrencies traded in the market. However, among many virtual currencies, bitcoin is the most known and the most market volume. Analysing the price movements of bitcoin, which has started to be seen as a financial investment tool, is of great importance in the framework of reliability. The examination made in this respect constitutes the original value of the research.

**Keywords:** Bitcoin, Cryptocurrency, Volatility, Symmetric and Asymmetric ARCH Models

**Öz**

**Amaç:** Bu araştırmanın amacı, ilk çıkan yıldır olan 2009’dan günümüze finsal piyasalarda yeni bir fenomen haline gelen, sanal para veya kripto para olarak tanımlanabilen bitcoinnin, finsal bir yatırım aracı olarak görülmesi yönündeki fiyat hareketlerinin analiz edilmesidir.

**Yöntem:** Çalışmadada, Otoregresif Koşullu Değişen Varyans modellemeleri ile oynaklık, getiri davranış ve finsal bir yatırım aracı olarak güvenilirliği incelenmektedir. Bu kapsamda simetrilık ve asimetrik etkili ARCH modellemeleri kullanılmıştır.

**Bulgular:** Analiz sonucunda, simetrilık ve asimetrik etkili ARCH modellemeleri ile incelenen bitcoin getiri serisi için ilk dönemde asimetrik etkiye sahip olduğu bulunmustur. Ayrıca half-life ölçütüne göre bitcoin getiri serisinde meydana gelen şoklar her dönemde 30 günden daha fazla oynaklılık kalıcılığı ektikine maruz kıldığı tespit edilmiştir. Dönemler itibarında incelenen bitcoinin ilk yıllarında daha yüksek oynaklığı sahip olduğu tespit edilmiştir.

**Sonuçlar:** Günümüzde finsal piyasalarda yeni bir fenomen haline gelen, sanal para veya kripto para olarak tanımlanabilen bitcoinin oynaklığının analiz edilmesi

**Anahtar Kelimeler:** Bitcoin, Kriptopara, Oynaklık, Simetrik ve Asimetrik ARCH Modelleri

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1. INTRODUCTION

The rapid development of the computer age and the increase in the volume of online shopping has given rise to virtual currencies as a new concept. Virtual currencies maintain their characteristic to be a controversial concept since their emergence in the market due to the fact that they are not connected with a central bank, contrary to national currencies, and not guaranteed by any institution or organization.

Bitcoin is the first currency which emerged as a virtual currency. The simple definition of bitcoin is given as follows: “The digital version of the most known and accepted currencies such as dollar and euro” (De Martino, 2018).

Bitcoin was announced by its founder or founders in 2008 with an article and defined as given: “It is an end-to-end electronic cash flow transaction that allows digital payments to be transferred directly from one side to another without going through a financial institution” (Nakamoto, 2008).

Because of the fact that bitcoin transfer is not limited for countries which are included in the monitoring list or imposed an embargo, its speed and flexibility are higher in comparison to other currencies regulated by banks (Dyhrberg, 2016).

Bitcoin is based on the idea of a peer-to-peer exchange without a third party involvement and accordingly, it has brought about many discussions since the day it was released in the market. The fluctuations that occur in the price of bitcoin has given rise to differences of opinion among experts, investors and regulatory bodies regarding the definition of bitcoin as an independent currency. Furthermore, the characteristic of bitcoin as a financial asset has started to be stressed as a research topic recently. Glasser et al. (2014) revealed that bitcoin is demanded as a speculative asset rather than being traded as a currency.

The main purpose of the present study was to determine price volatility and price movements of bitcoin with the modelling family of Autoregressive Conditional Heteroscedasticity (ARCH) which is the main method of time series econometrics. The study aimed to determine the price dynamics more accurately by analyzing the volatility modelling and price movements in the form of three different data sets. The data sets were created by receiving the daily closing price of bitcoin (Unit/Dollar) from blockchain. The first data set covered the period that starts from the year 2010 to the mid-2013. This data set represented the period which bitcoin had followed a smooth course before reaching to three digit numbers. The second data set covered the period that bitcoin had reached to three digit numbers and had more tendencies to increase from mid-2013 to the first quarter of 2017. Lastly, the third data set covered the period from the first quarter of 2017 to April 2018, in other words, the period that the unit price of bitcoin expressed with thousands of dollars and had the highest relative volatility.

As a result, the study aimed to determine the risk level of cryptocurrency for investors by determining price dynamics and volatility. Within this framework, the points that investors who want to add bitcoin in their investment portfolio should take into consideration were emphasized and the results obtained in the light of the analysis were explained and suggestions were provided.

The first section of the study defined the concepts of virtual currency and bitcoin; the second section addressed bitcoin as a currency and an investment tool and reviewed the literature, the third section presented the method to be employed in the study, the fourth section evaluated the implementation and results, and finally, the fifth section provided the results and the discussion. Also, contribution and originality of this paper that bitcoin price volatility examined by different period would give point of view researchers and investors.

2. BITCOIN AS A CURRENCY OR AN INVESTMENT TOOL AND LITERATURE

In order to analyze the price movements of bitcoin from the moment that it entered the market, the price graph should be examined as the first step.
It is evident that bitcoin has been on increase since the day it started to be mining. A unit of bitcoin did not even have the value of a half dollar in 2009, yet today it has a value of thousands of dollars. In Figure 1, 2011 is the date that the price movements of bitcoin have started to be examined, as it is the date that the price of bitcoin was higher than one dollar. However, it is not sufficient to address this increase separately. The price movements of bitcoin should be predictable and decrease in mean deviation to be accepted as a currency or as an investment tool.

The euro-dollar parity graph presented in Figure 2 has a more balanced structure in comparison to the bitcoin price graph. As the visual analysis demonstrates, the price movements of bitcoin have a very different structure than the most-known currencies. In the contemporary global market, observing the consistent price movements associated with other investment tools of virtual currencies that have a history shorter than 10 years.

Figure 3 illustrates the financial bubble chart of Rodrigue. The price movements of bitcoin show considerable similarities with the financial bubble graph. The experts who considered bitcoin as a financial bubble associate the price movements of bitcoin with the financial bubble chart of Rodrigue (2011). Due to the fact that the concept of crypto money is a relatively new concept, it is difficult to
compare the price movements of bitcoin with stocks or currencies such as euro or dollar. Therefore, given the fact that price movements of bitcoin include statistically significant information, analyzing them with a univariate time series method will be a more appropriate approach.

**Figure 3: Phases of Financial Bubble**

Volatility plays major role in risk modelling and price analysing of complex financial derivative products. Conditional variance analysis of financial time series data has received a growing interest in literature. Therefore, investigation volatility effects in bitcoin price as a financial time series organised primary issue of this study.

### 2.1. Literature

Bitcoin took its place in the world financial news as a fixture in late 2013 and early 2014. The "virtual currency", which was started to be used by computer enthusiasts, rapidly increased exchange rates and the value of a bitcoin increased many times against currencies. Although Bitcoin is considered as a medium of exchange, like money, its use has displayed high volatility levels by showing different prices in different exchanges. When the discussions in the literature are examined, there are various results on the use of bitcoin. Firstly, Nakamoto (2008) stated that it is an end-to-end electronic cash flow process that allows digital payments to be transferred directly from one side to another without going through a financial institution. According to Yermack (2013) in the future studies; Pricing with bitcoin in consumer goods is very difficult and requires a very zero decimal pricing. Again, bitcoin faces daily hacking and theft risks. It is not used to refer to consumer loan or loan agreements as futures. Bitcoin looks more like an investment transaction that behaves more speculatively than a trading currency. It is not used to refer to consumer loan or loan agreements as futures. Bitcoin looks more like an investment transaction that behaves more speculatively than a trading currency. Also "Is bitcoin a real currency?" with his work named has been at the center of these discussions.

Christopher (2013) analyzed the operation and functioning dimensions of bitcoin within the framework of anti-money laundering laws of the USA and emphasized the role of bitcoin as a money laundry tool and also potential crimes and difficulties that can arise in the practice of law due to the use of bitcoin. Atik et al. (2015) investigated the impact of bitcoin on foreign exchange rates and determined that there is a one-way causality from Japanese Yen to Euro, and these two currencies are affected from each other.

Szetela, Mentel and Gedek (2016) in their work, according to the results of the ARMA process, Bitcoin acts independently from other currencies in the analysis. The conditional variance of Bitcoin modeled with the GARCH process is affected by the Euro, Dollar and Yuan returns.

Dyrhberg (2016) determined that bitcoin has a place in the market between gold and American Dollar as a medium of exchange. The given study indicated that according to Asymmetric
Garch model results, including bitcoin in the portfolio could be useful in terms of risk management, and it is ideal for investors who avoid risks to protect themselves from the negative shocks that can occur in the market. Similar to the present study, Bouri et al. (2016) conducted a study on the change in price volatility of bitcoin based on the instant breaks which happened in bitcoin price in 2013. As a result of the research study, they found out that positive shocks that occur in bitcoin return before the breaks in 2013 affected volatility more than negative shocks. Kocoglu et al. (2016) evaluated the development of bitcoin as a virtual currency and its use as an investment tool. As a result, they stressed that bitcoin is a risky investment tool and could not prove its significance as the currencies of developed countries.

In Katsiampa (2017), Volatility forecast for Bitcoin: Making a comparison of GARCH models and exploration of GARCH-type models to determine bitcoin price volatility. As a result of the analysis in terms of compliance with the data, it was concluded that the suitable model is AR-CGARCH.

Baur et al. (2018), Bitcoin: Is it a medium of exchange or a speculative asset? According to the study, not affiliated with any government or authority and the independent commodity trading in the virtual environment is the currency. Analysis of bitcoin data traded in their accounts shows that bitcoin is not actually an alternative currency and exchange tool, it has shown that it is mostly used as a speculative investment tool.

Çütçü and Kılıç (2018) investigated the medium and long-term relationship between the dollar rate and bitcoin prices. In this context, Maki cointegration test, one of the new generations econometric analysis, was used and found a cointegration relationship between the dollar rate and bitcoin prices in the medium and long term. In addition, according to the Hacker-Hatemi-J Bootstrap causality test, they found a one-way causality relationship from dollar rate to bitcoin prices. According to the analysis results of this study, the change in dollar exchange rates significantly affects the prices of Bitcoin in the medium and long term.

Baur et al. (2018) stated that Bitcoin follows a different volatility process compared to other assets and it has been found to have a specific risk-return characteristic. In addition, analysis results showed that no relationship could be determined between bitcoin and gold and dollar. Chaim and Laurini (2018), In their studies, they explored the development of bitcoin daily returns and volatility. Not captured properly by traditional conditional volatility models, changing average volatility and created models of discontinuous yield increases.

Katsiampa et al. (2019) they determined the bidirectional volatility spreading effects between three pairs of cryptocurrencies through econometric analysis in their studies. According to the study, a cryptocurrency's own past shocks and its volatility again, it significantly affects its current conditional variance. But the important thing here is as evidence of the bidirectional shock realization effects, between Bitcoin and Ether and between Bitcoin and Litecoin one way shock from Ether to Litecoin has been found. Finally, it has been shown that positive correlations are predominant, with conditional correlations that change over time.

Gemici and Polat (2019) in this study, the relationship between bitcoin price and bitcoin volume was investigated by symmetric and asymmetric causality tests. According to the standard causality test results, there is a causality relationship between the price and volume. An asymmetric causality relationship was found between the positive and negative shocks of the variables in the analyzes. Also, it has been determined that the relationship between bitcoin price and bitcoin volume is cointegrated.

3. ECONOMETRIC METHODOLOGY

In this section, the econometric time series models are presented that lay the foundation for the price movements of bitcoin which will be analyzed. ARMA models which examine the linear dimension of time series and GARCH-family processes that model the condition of time series to be linear and non-linear on average.
3.1. Autoregressive Moving Average Models (ARMA)

The model which emerges as the combination of AR(p) and MA(q) models is called ‘ARMA(p,q) model’. This model is expressed as a linear combination of the lagged values of the y series itself and current and lagged values of the error term.

\[ y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} + \theta_1 u_{t-1} + \theta_2 u_{t-2} + \ldots + \theta_q u_{t-q} + u_t \]  

(1)

The equation is presented in 1. In this equation, the degree of the AR model can be expressed as “p” and the degree of the MA model can be expressed as “q”.

3.2. Autoregressive Conditional Heteroscedasticity Models

Granger and Andersen (1978) proposed the model of the conditional heteroscedasticity that can be explained by the past values of its dependent variable in the following equation.

\[ y_t = \epsilon_t y_{t-1} \]  

(2)

In the equation (2) conditional heteroscedasticity can be expressed as \( \sigma^2 y^2_{t-1} \). Engle (1982) eliminated this model which was developed by Granger due to the fact that conditional heteroscedasticity is equal to zero or infinite. The model proposed by Engle (1982) can be expressed as follows.

\[ y_t = \epsilon_t h_{t}^{1/2} \]  

(3)

\[ h_t = \alpha_0 + \alpha_1 y^2_{t-1} \]  

(4)

Engle defined the model in which the variance of the error term is \( V(\epsilon_t) = 1 \) as ‘Autoregressive Conditional Heteroscedasticity Model’ (ARCH). Although the defined model is not purely a bilinear model, it is considerably approximate to one. When the normal distribution assumption is added to the model with \( \psi_{t-1} \), the model can be expressed as follows.

\[ y_t | \psi_{t-1} \sim N(0, h_t) \]  

(5)

\[ h_t = \alpha_0 + \alpha_1 y^2_{t-1} \]  

(6)

The variance function of the Autoregressive Conditional Heteroscedasticity Model can be demonstrated with its general lines as follows.

\[ h_t = h(y_{t-1}, y_{t-2}, \ldots, y_{t-p}, \alpha) \]  

(7)

The given “p” in the equation (7) implies the level of the ARCH process and “\( \alpha \)” implies the unknown parameter vector.

ARCH regression model states the mean of \( y_t \) as linear combination of \( \chi, \beta \)’s internal and external lags. The model includes \( \psi_{t-1} \) information set and \( \beta \) unknown vector parameters (Engle, 1982).

3.3. GARCH Model

The fact that the ARCH model is simple it often requires several parameters in practice. For this reason, alternatives models were required (Tsay, 2010). The initial empiric implementations of the ARCH model investigated the relationship between inflation volatility and inflation level. Engle (1982, 1983) needed to calculate the high-level lag q parameter to predict appropriate condition-variance function in the ARCH model. The high lag parameter might violate the parameter limitations of the ARCH model. Bollerslev (1986) and Taylor (1986) by adding the past values of the conditional
variance to the obtained conditional variance function, the Generalized Autoregressive Conditional
Variable Variance (GARCH) model is given in equation (7) (Bera & Higgins, 1993).

Bollerslev (1986) defines the GARCH model as: Let \( \varepsilon_t \) denote a real-valued discrete-time
stochastic process, and \( \psi_t \) the information set (a-field) of all information through time \( t \). The GARCH
\((p, q)\) process (Generalized Autoregressive Conditional Heteroscedasticity) is then given by:

\[
\varepsilon_t | \psi_{t-1} \sim N(0, h_t)
\]

\[
h_t = a_0 + \sum_{i=1}^{q} a_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i}
= a_0 + A(L)\varepsilon_{t-i}^2 + B(L)h_t
\]

In the GARCH model, it is assumed that \( p \geq 0, q > 0, a_0 > 0, a_j \geq 0, j = 1, ..., q, \gamma_i \geq 0 \). For
\( p = 0 \), the model is reduced to the ARCH model without any doubt. For \( p = q = 0 \), the process is
expressed as a white noise process. In the ARCH\( (q) \) process, the conditional variance is defined as a
linear function of past sample variances in the model only. On the other hand, the GARCH \((p, q)\)
process allows lagged conditional variances as well. When the error terms are left alone in the
GARCH \((p, q)\) model, the model is as follows.

\[
\varepsilon_t = y_t - \chi'_t b
\]

In the equation (10)’da \( y_t \) is the dependent variable, \( \chi_t \) is the vector of explanatory variables
and \( b \) is the vector of unknown parameters. In the case that all roots of \( A(1) + B(1) < 1 \) condition
is fulfilled (Bollerslev, 1986).

4. IMPLEMENTATION

The study covers the period between the years 2011 and 2018. The daily closing prices of
bitcoin (Dollar/BTC) were used in the analysis. The analysis process was separated into three periods
as 1st Period.

10.02.2011 - 02.10.2013, 2nd Period 03.10.2013 - 31.12.2016 and 3rd Period 01.01.2017 -
27.04.2018. In total 2633 observations were performed. The logarithms of the series were calculated to
determine the return series. Returns were calculated in the following formulas.

\[
R_t = \ln \left( \frac{P_t}{P_{t-1}} \right)
\]

The descriptive statistics that were subjected to an inter-period analysis showed that the 2nd
Period had the highest variation coefficient. The variation coefficient that was found later on as 14.04
belongs to the 2nd Period. In the context of variation coefficient, the lowest volatility coefficient -
relatively- belongs to the 3rd period with 11.04. Given the fact that bitcoin returns gained a significant acceleration in the second period, it is possible that it has a more volatile structure in this period. The third period which has the lowest variation coefficient proves that 3rd-period bitcoin returns became more consistent. As a characteristic feature of the daily return series, the normality assumption could not be fulfilled in all periods. The series had high-level extreme kurtosis and the return series of the 2nd and 3rd period have negative skewness.

Table 2: Descriptive Statistics

| Variable         | 1. Period (10.02.2011-03.10.2013) | 2. Period (03.10.2013-31.12.2016) | 3. Period (01.01.2017-27.04.2018) |
|------------------|----------------------------------|----------------------------------|----------------------------------|
| Mean             | 0.004719                         | 0.001863                         | 0.004669                         |
| Median           | -0.000194                        | 0.001194                         | 0.008075                         |
| Maximum          | 0.515324                         | 0.230735                         | 0.246615                         |
| Minimum          | -0.478305                        | -0.268621                        | -0.225712                        |
| Standard Deviation | 0.066296                       | 0.038771                         | 0.051551                         |
| Variation Coefficient | 14.04887                  | 20.811057                        | 11.041122                        |
| Skewness         | 0.779086                         | -0.173239                        | -0.298518                        |
| Kurtosis         | 17.50661                         | 11.41256                         | 5.739517                         |
| Jarque-Bera      | 8559.136                         | 3503.205                         | 157.5556                         |
| Probability      | 0.0000                           | 0.0000                           | 0.0000                           |
| Number of Observations | 965                  | 1186                            | 481                             |

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Figure 4 examined the graph of the return series. The visual analysis of the return series highlighted that the 2nd period has a more volatile structure as it can be also seen in the descriptive statistics. The general observations of the return series revealed that a positive volatility is followed by a positive volatility, and a negative volatility was followed by a negative volatility. In this context, it is revealed that the series have a volatility clustering. Unit root tests examined for the return series and results showed at the following.

Figure 4: Time Path Charts of Return Series
As can be seen from the unit root results return series are I(0) for all periods. ARMA model trials have been made for return series and the most suitable model has been reported according to the selected information criteria.

Table 4: Selected ARIMA Models

| Models            | AIC*   | BIC    | LO     | HQ     |
|-------------------|--------|--------|--------|--------|
| 1. PERIOD         |        |        |        |        |
| ARIMA (3,1,3)     | -2.6129| -2.5725| 1270.053| -2.5975|
| ARIMA (2,1,4)     | 2.6111 | -2.5708| 1269.193| -2.5958|
| ARIMA (2,1,2)     | 2.6058 | -2.5755| 1264.633| -2.5943|
| ARIMA (1,1,2)     | -2.6003| -2.5751| 1260.978| -2.5907|
| ARIMA (1,1,1)     | -2.5986| -2.5784| 1259.135| -2.5909|
| 2. PERIOD         |        |        |        |        |
| ARIMA (3,1,3)     | -3.6902| -3.6559| 2196.299| -3.6637|
| ARIMA (1,1,3)     | -3.6749| -3.6492| 2185.247| -3.6652|
| ARIMA (1,1,2)     | -3.6617| -3.6403| 2176.428| -3.6536|
| ARIMA (0,1,0)     | -3.6597| -3.6512| 2172.247| -3.6565|
| ARIMA (1,1,1)     | -3.6586| -3.6414| 2173.550| -3.6521|
Table 4 (Continued): Selected ARIMA Models

| Period | Models       | AIC*   | BIC    | LO     | HQ     |
|--------|--------------|--------|--------|--------|--------|
| 3      | ARIMA (6,1,6)| -3.1024| -2.9808| 760.1306| -3.0546|
|        | ARIMA (5,1,5)| -3.0874| -2.9832| 754.5312| -3.0465|
|        | ARIMA (1,1,0)| -3.0821| -3.0561| 744.2621| -3.0719|
|        | ARIMA (1,1,2)| -3.0739| -3.0305| 744.2809| -3.05687|

* Information criterion which was accepted as a reference in model selection

To select the most appropriate ARIMA models, Akaike, Bayes, Hannan-Quinn information criteria were provided in Table 3. The Akaike Information Criterion was accepted as the reference criterion that directs the model selection process. The ones which are specified with bold fonts are the selected models. For the 1st and 2nd periods the ARIMA (3,1,3) model, and for the 3rd period, the ARIMA (6,1,6) were preferred. Suitability of the selected models depends on some assumptions. Rediulas should not be autocorrelated and heteroskedastic. Autocorrelation and heteroskedasticity tests showed at the following.

Table 5: ARMA Models’ Residuals Autocorrelation Test Results

| Period  | ARMA Models | Lag1 | Lag5 | Lag10 | Lag20 | Lag30 |
|---------|-------------|------|------|-------|-------|-------|
|         | ARMA (3,3)  |      |      |       |       |       |
|         | AC          | 0.372| 0.127| 0.092 | 0.045 | 0.004 |
|         | PAC         | 0.372| 0.075| 0.048 | 0.029 | -0.036|
|         | Q.Stat.     | 133.76| 283.08| 315.77| 353.08| 388.03|
|         | Prob.       | 0.000| 0.000| 0.000 | 0.000 | 0.000 |
|         | ARMA (6,6)  |      |      |       |       |       |
|         | AC          | 0.113| 0.110| 0.020 | 0.048 | -0.013|
|         | PAC         | 0.113| 0.103| 0.006 | 0.034 | -0.033|
|         | Q.Stat.     | 6.227| 13.229| 17.867| 24.408| 27.832|
|         | Prob.       | 0.013| 0.021| 0.057 | 0.225 | 0.579 |

Autocorrelation tests results show that return series’ residuals autocorrelated even in thirtieth lag. Only last period’s autocorrelation have disappeared at the end of twentieth lag. To sum up all the ARMA models’ residuals autocorrelated. Heteroskedasticity tests indicate that ARMA model residuals are heteroskedastic even at 36th lag. ARMA models’ residuals suffered both autocorrelation and heteroskedasticity so these models inadequate. Heteroskedasticity should be modelled for the further analysis. Therefore, variance component of ARMA models can be nonlinear. Nonlinearity of return series should be examined.

Table 6: ARCH-LM Heteroskedasticity Test Performed on the Error Terms of the ARIMA Models

| Period | Number of Lags | Test Statistic | Probability Value |
|--------|----------------|----------------|------------------|
| 1      | 1              | 133.8566       | 0.000            |
|        | 2              | 166.1298       | 0.000            |
|        | 4              | 166.3109       | 0.000            |
|        | 12             | 176.4546       | 0.000            |
|        | 24             | 209.4356       | 0.000            |
|        | 36             | 222.8497       | 0.000            |
| 2      | 1              | 145.5468       | 0.000            |
|        | 2              | 157.1399       | 0.000            |
|        | 4              | 159.2385       | 0.000            |
|        | 12             | 192.9329       | 0.000            |
|        | 24             | 216.9208       | 0.000            |
|        | 36             | 236.0558       | 0.000            |
Table 6 (Continued): ARCH-LM Heteroskedasticity Test Performed on the Error Terms of the ARIMA Models

| Number of Lags | Test Statistic | Probability Value |
|---------------|----------------|-------------------|
| 1             | 6.2724         | 0.0123            |
| 2             | 6.3169         | 0.0425            |
| 4             | 7.1608         | 0.1276            |
| 12            | 13.9313        | 0.3051            |
| 24            | 19.5778        | 0.7205            |
| 36            | 23.9165        | 0.9386            |

BDS nonlinearity tests indicates that return series has nonlinear components.

Table 7: BDS Test Results

| Dimension | BDS-Stat. | Std-Error | z-stat. | Prob. |
|-----------|-----------|-----------|---------|-------|
| 1. Period |           |           |         |       |
| 2         | 0.0426    | 0.0040    | 10.5668 | 0.0000|
| 3         | 0.0800    | 0.0064    | 12.4297 | 0.0000|
| 4         | 0.1056    | 0.0077    | 13.7218 | 0.0000|
| 5         | 0.1204    | 0.0080    | 14.9438 | 0.0000|
| 6         | 0.1269    | 0.0078    | 16.2619 | 0.0000|

| Dimension | BDS-Stat. | Std-Error | z-stat. | Prob. |
|-----------|-----------|-----------|---------|-------|
| 2. Period |           |           |         |       |
| 2         | 0.0398    | 0.0034    | 11.6682 | 0.0000|
| 3         | 0.0764    | 0.0054    | 14.0487 | 0.0000|
| 4         | 0.0990    | 0.0065    | 15.2293 | 0.0000|
| 5         | 0.1118    | 0.0068    | 16.4482 | 0.0000|
| 6         | 0.1181    | 0.0065    | 17.9509 | 0.0000|

| Dimension | BDS-Stat. | Std-Error | z-stat. | Prob. |
|-----------|-----------|-----------|---------|-------|
| 3. Period |           |           |         |       |
| 2         | 0.0147    | 0.0043    | 3.3862  | 0.0007|
| 3         | 0.0268    | 0.0069    | 3.8680  | 0.0001|
| 4         | 0.0389    | 0.0083    | 4.6850  | 0.0000|
| 5         | 0.0480    | 0.0086    | 5.5342  | 0.0000|
| 6         | 0.0548    | 0.0084    | 6.5266  | 0.0000|

There are doubts that the series has a volatility clustering and permanence. In this context, it was thought that the predicted ARIMA processes might have heteroskedasticity and autocorrelation. Also BDS test results show that nonlinearity of residuals should be modelled.

Table 8: Volatility Model Results

| Periods  | Volatility Model | Coefficient | \( a_0 \) | \( a_1 \) | \( a_2 \) | \( a_3 \) | \( \beta_1 \) | \( \beta_2 \) | \( \beta_3 \) | \( \gamma_3 \) |
|----------|------------------|-------------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1.Period | ARCH (1)         |             | 0.0011***|         |         |         |         |         |         |         |
|          |                  |             | 1.3023***|         |         |         |         |         |         |         |
|          | ARCH (2)         |             | 0.0010***|         |         |         |         |         |         |         |
|          |                  |             | 0.8514***|         | 0.2928***|         |         |         |         |         |
|          | ARCH (3)         |             | 0.0007***|         |         |         |         |         |         |         |
|          |                  |             | 0.2526***|         | 0.4438***|         |         |         |         | 0.4483***|
| 2.Period | GARCH (1,1)      |             | 0.0005***|         |         |         |         |         |         |         |
|          |                  |             | 0.9279***|         |         |         |         |         |         |         |
|          | ARCH (2)         |             | 0.0004   |         |         |         |         |         |         |         |
|          |                  |             | 0.6145   |         |         |         |         |         |         | 0.3188  |
|          | ARCH (3)         |             | 0.0004   |         |         |         |         |         |         |         |
|          |                  |             | 0.5725   |         |         |         |         |         |         | 0.2495  |
| 3.Period | GARCH (1,2)      |             | 0.0021   |         |         |         |         |         |         |         |
|          |                  |             | 0.1549   |         |         |         |         |         |         |         |
|          | ARCH (2)         |             | 0.0017   |         |         |         |         |         |         |         |
|          |                  |             | 0.1682   |         |         |         |         |         |         | 0.1850  |
|          | ARCH (3)         |             | 0.0011   |         |         |         |         |         |         |         |
|          |                  |             | 0.2117   |         |         |         |         |         |         | 0.2454  |

\[
h_t = a_0 + \sum_{j=1}^{q} a_j \mu_{t-j}^2. \tag{12}\]

| Periods  | Volatility Model | Coefficient | \( a_0 \) | \( a_1 \) | \( a_2 \) | \( a_3 \) | \( \beta_1 \) | \( \beta_2 \) | \( \beta_3 \) | \( \gamma_3 \) |
|----------|------------------|-------------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1.Period | GARCH (1,1)      |             | 0.000065***|         |         |         |         |         |         |         |
|          |                  |             | 0.1916***|         |         |         |         |         |         |         |
|          | GARCH (1,2)      |             | 0.000070***|         |         |         |         |         |         |         |
|          |                  |             | 0.2093***|         |         |         |         |         |         | 0.7086***|
| 2.Period | GARCH (1,1)      |             | 0.0000373***|         |         |         |         |         |         |         |
|          |                  |             | 0.2184***|         |         |         |         |         |         | 0.7861***|
|          | GARCH (1,2)      |             | 0.0000440***|         |         |         |         |         |         |         |
|          |                  |             | 0.27207***|         |         |         |         |         |         | 0.4560***|
| 3.Period | GARCH (1,1)      |             | 0.0000861***|         |         |         |         |         |         |         |
|          |                  |             | 0.1992***|         |         |         |         |         |         | 0.7906***|
|          | GARCH (1,2)      |             | 0.00011***|         |         |         |         |         |         |         |
|          |                  |             | 0.2623***|         |         |         |         |         |         | 0.30454*|

\[
h_t = a_0 + \sum_{j=1}^{p} a_j \mu_{t-j}^2 + \sum_{i=1}^{q} \beta_i h_{t-i}. \tag{13}\]
Table 8 (Continued): Volatility Model Results

| Periods | Volatility Model | Coefficient | $a_0$ | $a_1$ | $a_2$ | $a_3$ | $\beta_1$ | $\beta_2$ | $\beta_3$ | $\gamma_3$ |
|---------|------------------|-------------|-------|-------|-------|-------|-----------|-----------|-----------|-----------|
| 1. Period | TGARCH (1,1) | 0.0000867*** | 0.1864*** | 0.8147*** | 0.0214 |
|          | TGARCH (1,2) | 0.0000865*** | 0.1871*** | 0.8104*** | 0.0036 | 0.0214 |
| 2. Period | TGARCH (1,1) | 0.0000374*** | 0.2061*** | 0.7853*** | 0.0261 |
|          | TGARCH (1,2) | 0.0000441*** | 0.2589*** | 0.4366*** | 0.2964*** | 0.0287 |
| 3. Period | TGARCH (1,1) | 0.0000722*** | 0.1933*** | 0.7735*** | 0.0671 |
|          | TGARCH (1,2) | 0.000120*** | 0.2015*** | 0.2467* | 0.4525*** | 0.1698*** |

*: % 10, **: % 5, ***: % 1 represent the statistically significant coefficients according to the significance level.

\[ h_t = a_0 + \sum_{i=1}^{q} a_i \mu_{t-i}^2 + \gamma_i \mu_{t-i}^2 \alpha_{t-i} \sum_{i=1}^{p} \beta_i h_{t-i} \]  \hspace{1cm} (14)

| Periods | Volatility Model | Coefficient | $a_0$ | $a_1$ | $a_2$ | $a_3$ | $\beta_1$ | $\beta_2$ | $\beta_3$ | $\gamma_3$ |
|---------|------------------|-------------|-------|-------|-------|-------|-----------|-----------|-----------|-----------|
| 1. Period | EGARCH (1,1) | -0.4388*** | 0.3284*** | 0.9634*** | 0.0055 |
|          | EGARCH (1,2) | -0.4737*** | 0.3687*** | 0.7621*** | 0.1999 | 0.0089 |
|          | EGARCH (2,1) | -0.4551*** | 0.4230*** | -0.0856 | 0.9614*** | -0.0174 |
|          | EGARCH (2,2) | -0.8104*** | 0.3791*** | 0.2470*** | 0.0490 | 0.8859*** | 0.0305 |
|          | EGARCH (2,3) | -0.9178*** | 0.2809*** | 0.4371*** | -0.0534 | 0.3039*** | 0.6787*** | 0.0672*** |
|          | EGARCH (3,1) | -0.8595*** | 0.4165*** | 0.2982*** | -0.0621*** | -0.0303*** | 0.9631*** | -0.0123*** |
|          | EGARCH (3,2) | -1.5519*** | 0.2821*** | 0.6011*** | 0.3271*** | -0.3728*** | 0.3322*** | 0.9190*** | 0.0656*** |
|          | EGARCH (3,3) | -0.8595*** | 0.2821*** | 0.6011*** | 0.3271*** | -0.3728*** | 0.3322*** | 0.9190*** | 0.0656*** |
| 2. Period | EGARCH (1,1) | 0.0000374*** | 0.2061*** | 0.7853*** | 0.0261 |
|          | EGARCH (1,2) | 0.0000443*** | 0.2589*** | 0.4366*** | 0.2964*** | 0.0287 |
|          | EGARCH (2,1) | -0.5519*** | 0.2821*** | 0.6011*** | 0.3271*** | -0.3728*** | 0.3322*** | 0.9190*** | 0.0656*** |
|          | EGARCH (2,2) | -0.8104*** | 0.3791*** | 0.2470*** | 0.0490 | 0.8859*** | 0.0305 |
|          | EGARCH (2,3) | -0.9178*** | 0.2809*** | 0.4371*** | -0.0534 | 0.3039*** | 0.6787*** | 0.0672*** |
|          | EGARCH (3,1) | -0.8595*** | 0.4165*** | 0.2982*** | -0.0621*** | -0.0303*** | 0.9631*** | -0.0123*** |
|          | EGARCH (3,2) | -1.5519*** | 0.2821*** | 0.6011*** | 0.3271*** | -0.3728*** | 0.3322*** | 0.9190*** | 0.0656*** |
|          | EGARCH (3,3) | -0.8595*** | 0.2821*** | 0.6011*** | 0.3271*** | -0.3728*** | 0.3322*** | 0.9190*** | 0.0656*** |

| Periods | Volatility Model | Coefficient | $a_0$ | $a_1$ | $a_2$ | $a_3$ | $\beta_1$ | $\beta_2$ | $\beta_3$ | $\gamma_3$ |
|---------|------------------|-------------|-------|-------|-------|-------|-----------|-----------|-----------|-----------|
| 1. Period | PARCH (1,1) | 0.0078*** | 0.1568*** | 0.8538*** | 0.4595*** |
|          | PARCH (1,2) | 0.0088*** | 0.1769*** | 0.7112*** | 0.1221 | 0.4608*** |
| 2. Period | PARCH (1,1) | 0.000012 | 0.2111*** | 0.7760*** | 2.3112*** |
|          | PARCH (1,2) | 0.000025 | 0.2687*** | 0.4336*** | 0.2918*** | 2.21405*** |
| 3. Period | PARCH (1,1) | 0.0045 | 0.1652*** | 0.8352*** | 0.5360** |
|          | PARCH (1,2) | 0.0077 | 0.1802*** | 0.5573*** | 0.2667 | 0.5360** |

\[ \log(h_t) = a_0 + \sum_{i=1}^{q} \beta_i \log(h_{t-j}) + \sum_{i=1}^{p} a_i \frac{\mu_{t-i}}{\sqrt{h_{t-i}}} + \sum_{k=1}^{r} \gamma_k \frac{\mu_{t-k}}{\sqrt{h_{t-k}}} \]  \hspace{1cm} (15)

ARCH, GARCH, TGARCH, EGARCH, PARCH models examined for all periods in Table 8. Some models have insignificant parameters and they are eliminated from heteroskedasticity test. If the problem of heteroskedasticity in models can not be solved. It can be said that ARCH/GARCH models are insufficient in volatility modeling.

\[ \sigma_t^d = a_0 + \sum_{i=1}^{q} a_i (|e_{t-i}^d + \gamma_i e_{t-i}^d + \sum_{i=1}^{p} \beta_i \sigma_t^d \]  \hspace{1cm} (16)
| Periods | ARCH (1) | Lag.1 | Lag.5 | Lag.10 | Lag.20 | Lag.30 |
|---------|----------|-------|-------|--------|--------|--------|
|         | F stat.  | 3.1191| 3.3289| 2.6471 | 2.4706 | 2.0876 |
|         | F stat. Prob. | 0.0777| 0.0055| 0.0035 | 0.0004 | 0.0006 |
|         | Obs. $R^2$ | 3.1155| 16.4618| 26.0482| 47.9669| 60.5732|
|         | $R^2$ Prob. | 0.0775| 0.0056| 0.0037 | 0.0004 | 0.0008 |

**ARCH (2)**

|         | F stat.  | 2.2235| 1.7464| 1.4923 | 1.2895 | 1.8521 |
|         | F stat. Prob. | 0.1363| 0.1214| 0.1369 | 0.1765 | 0.0338 |
|         | Obs. $R^2$ | 2.2229| 8.7071| 14.8619| 25.6608| 54.1369|
|         | $R^2$ Prob. | 0.1360| 0.1213| 0.1372 | 0.1773 | 0.0044 |

**ARCH (3)**

|         | F stat.  | 0.2577| 0.4728| 0.4769 | 0.0880 | 0.5875 |
|         | F stat. Prob. | 0.6121| 0.4728| 0.4769 | 0.0880 | 0.5875 |
|         | Obs. $R^2$ | 0.2577| 4.5635| 9.6154 | 28.9113| 27.7734|
|         | $R^2$ Prob. | 0.6116| 0.4714| 0.4749 | 0.0895 | 0.5824 |

**ARCH (1)**

|         | F stat.  | 1.1852| 2.9972| 3.7614 | 2.3192 | 2.1892 |
|         | F stat. Prob. | 0.2107| 0.0108| 0.0001 | 0.0009 | 0.0002 |
|         | Obs. $R^2$ | 1.5691| 14.8731| 36.7820| 45.3972| 63.7633|
|         | $R^2$ Prob. | 0.2103| 0.0109| 0.0001 | 0.0010 | 0.0003 |

**ARCH (2)**

|         | F stat.  | 0.7169| 0.9113| 2.4628 | 1.7113 | 1.7529 |
|         | F stat. Prob. | 0.3973| 0.4734| 0.0065 | 0.0262 | 0.0076 |
|         | Obs. $R^2$ | 0.7176| 4.5573| 24.3463| 33.8434| 51.6250|
|         | $R^2$ Prob. | 0.3969| 0.4723| 0.0067 | 0.0272 | 0.0084 |

**ARCH (3)**

|         | F stat.  | 0.2164| 0.6549| 2.7679 | 1.8332 | 1.7188 |
|         | F stat. Prob. | 0.6418| 0.6577| 0.0022 | 0.0139 | 0.0096 |
|         | Obs. $R^2$ | 0.2168| 3.2825| 27.2919| 36.1790| 50.6654|
|         | $R^2$ Prob. | 0.6415| 0.6565| 0.0023 | 0.0147 | 0.0106 |

**ARCH (1)**

|         | F stat.  | 0.0751| 2.0989| 1.6347 | 1.4143 | 1.0316 |
|         | F stat. Prob. | 0.7841| 0.0644| 0.0940 | 0.1101 | 0.4230 |
|         | Obs. $R^2$ | 0.0754| 10.3966| 16.1644| 27.8471| 30.9529|
|         | $R^2$ Prob. | 0.7836| 0.0647| 0.0950 | 0.1131 | 0.4177 |

**ARCH (2)**

|         | F stat.  | 0.0187| 2.3201| 1.9538 | 1.5042 | 1.1700 |
|         | F stat. Prob. | 0.8912| 0.0424| 0.0366 | 0.0730 | 0.2492 |
|         | Obs. $R^2$ | 0.0188| 11.4663| 19.1916| 29.5054| 34.7830|
|         | $R^2$ Prob. | 0.8909| 0.0429| 0.0379 | 0.0782 | 0.2505 |

**ARCH (3)**

|         | F stat.  | 0.0495| 1.7048| 1.7209 | 1.1423 | 0.9720 |
|         | F stat. Prob. | 0.8240| 0.1319| 0.0734 | 0.3024 | 0.5112 |
|         | Obs. $R^2$ | 0.0497| 8.4792| 16.9863| 22.7563| 29.2808|
|         | $R^2$ Prob. | 0.8236| 0.1317| 0.0747 | 0.3009 | 0.5029 |

**GARCH (1.1)**

|         | F stat.  | 0.3809| 0.5316| 0.7339 | 0.6108 | 0.6148 |
|         | F stat. Prob. | 0.5374| 0.7524| 0.6926 | 0.9056 | 0.9471 |
|         | Obs. $R^2$ | 0.3822| 2.6768| 7.3967 | 12.4528| 18.9735|
|         | $R^2$ Prob. | 0.5364| 0.7496| 0.6875 | 0.8996 | 0.9406 |

**GARCH (1.2)**

|         | F stat.  | 0.0334| 0.1087| 0.1386 | 0.2853 | 0.4314 |
|         | F stat. Prob. | 0.8549| 0.9904| 0.9992 | 0.9992 | 0.9969 |
|         | Obs. $R^2$ | 0.0335| 0.5470| 1.4008 | 5.8004 | 13.1986|
|         | $R^2$ Prob. | 0.8547| 0.9903| 0.9992 | 0.9991 | 0.9966 |
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Table 9 (Continued): Heteroskedasticity Test Results for Volatility Models

| 2. Period | 3. Period |
|-----------|-----------|
| **GARCH (1,1)** | **TGARCH (1,1)** |
| **F stat.** | **F stat.** |
| 3.8539 | 0.9867 |
| **F stat. Prob.** | **F stat. Prob.** |
| 0.0499 | 0.8667 |
| **Obs. R^2** | **Obs. R^2** |
| 3.8479 | 0.8049 |
| **R^2 Prob.** | **R^2 Prob.** |
| 0.0498 | 0.4792 |

| **GARCH (1,2)** | **TGARCH (1,2)** |
| **F stat.** | **F stat.** |
| 1.5674 | 0.9576 |
| **F stat. Prob.** | **F stat. Prob.** |
| 0.2108 | 0.0028 |
| **Obs. R^2** | **Obs. R^2** |
| 1.5680 | 0.9575 |
| **R^2 Prob.** | **R^2 Prob.** |
| 0.2105 | 0.9575 |

| **GARCH (1,1)** | **TGARCH (1,1)** |
| **F stat.** | **F stat.** |
| 0.4990 | 0.0282 |
| **F stat. Prob.** | **F stat. Prob.** |
| 0.4803 | 0.8665 |
| **Obs. R^2** | **Obs. R^2** |
| 0.5005 | 0.0282 |
| **R^2 Prob.** | **R^2 Prob.** |
| 0.4792 | 0.9576 |

| **GARCH (1,2)** | **TGARCH (1,2)** |
| **F stat.** | **F stat.** |
| 0.9576 | 0.0282 |
| **F stat. Prob.** | **F stat. Prob.** |
| 0.9645 | 0.4221 |
| **Obs. R^2** | **Obs. R^2** |
| 0.9575 | 0.6465 |
| **R^2 Prob.** | **R^2 Prob.** |
| 0.9575 | 0.6465 |

| **GARCH (1,1)** | **TGARCH (1,1)** |
| **F stat.** | **F stat.** |
| 3.2830 | 3.2794 |
| **F stat. Prob.** | **F stat. Prob.** |
| 0.0703 | 0.0702 |
| **Obs. R^2** | **Obs. R^2** |
| 3.2794 | 0.6465 |
| **R^2 Prob.** | **R^2 Prob.** |
| 0.2987 | 0.4214 |

| **GARCH (1,2)** | **TGARCH (1,2)** |
| **F stat.** | **F stat.** |
| 0.2563 | 0.1294 |
| **F stat. Prob.** | **F stat. Prob.** |
| 0.2560 | 0.2563 |
| **Obs. R^2** | **Obs. R^2** |
| 0.2560 | 0.2563 |
| **R^2 Prob.** | **R^2 Prob.** |
| 0.2560 | 0.2560 |

| **GARCH (1,1)** | **TGARCH (1,1)** |
| **F stat.** | **F stat.** |
| 1.1552 | 1.1573 |
| **F stat. Prob.** | **F stat. Prob.** |
| 0.2830 | 0.2820 |
| **Obs. R^2** | **Obs. R^2** |
| 1.1573 | 0.1972 |
| **R^2 Prob.** | **R^2 Prob.** |
| 0.2830 | 0.6572 |

| **GARCH (1,2)** | **TGARCH (1,2)** |
| **F stat.** | **F stat.** |
| 0.0383 | 0.0383 |
| **F stat. Prob.** | **F stat. Prob.** |
| 0.8449 | 0.8447 |
| **Obs. R^2** | **Obs. R^2** |
| 0.0383 | 0.0383 |
| **R^2 Prob.** | **R^2 Prob.** |
| 0.8447 | 0.8447 |

| **EGARCH (3,3)** | **EGARCH (3,3)** |
| **F stat.** | **F stat.** |
| 0.3443 | 0.3449 |
| **F stat. Prob.** | **F stat. Prob.** |
| 0.5575 | 0.5575 |
| **Obs. R^2** | **Obs. R^2** |
| 0.3449 | 0.3443 |
| **R^2 Prob.** | **R^2 Prob.** |
| 0.5570 | 0.5570 |

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### Table 9 (Continued): Heteroskedasticity Test Results for Volatility Models

|                | Lag 1 | Lag 5 | Lag 10 | Lag 20 | Lag 30 |
|----------------|-------|-------|--------|--------|--------|
| **EGARCH (1,1)** |       |       |        |        |        |
| F stat.        | 3.1585| 1.3868| 1.4234 | 0.8746 | 0.8136 |
| F stat. Prob.  | 0.0758| 0.2265| 0.1642 | 0.6206 | 0.7514 |
| Obs. R²        | 3.1554| 6.9285| 14.1957| 17.5448| 24.5503|
| R² Prob.       | 0.0757| 0.2260| 0.1643 | 0.6174 | 0.7466 |
| **EGARCH (1,2)** |       |       |        |        |        |
| F stat.        | 0.7443| 1.0427| 1.1454 | 0.7251 | 0.6954 |
| F stat. Prob.  | 0.3884| 0.3909| 0.3244 | 0.8030 | 0.8901 |
| Obs. R²        | 0.7451| 5.2171| 11.4505| 14.5833| 21.0490|
| R² Prob.       | 0.3880| 0.3900| 0.3235 | 0.7997 | 0.8863 |
| **PARCH (1,1)** |       |       |        |        |        |
| F stat.        | 1.5279| 1.0063| 0.8148 | 0.6543 | 0.5816 |
| F stat. Prob.  | 0.2170| 0.4133| 0.6144 | 0.8705 | 0.9638 |
| Obs. R²        | 1.5294| 5.0418| 8.1983 | 13.3143| 17.9883|
| R² Prob.       | 0.2162| 0.4108| 0.6095 | 0.8635 | 0.9587 |
| **PARCH (1,2)** |       |       |        |        |        |
| F stat.        | 0.6240| 0.7595| 0.7491 | 0.9258 | 0.9871 |
| F stat. Prob.  | 0.2172| 2.6301| 6.7991 | 11.8646| 15.7597|
| Obs. R²        | 0.6412| 0.7568| 0.7443 | 0.9207 | 0.9847 |

**2. Period**

|                | Lag 1 | Lag 5 | Lag 10 | Lag 20 | Lag 30 |
|----------------|-------|-------|--------|--------|--------|
| **EGARCH (1,1)** |       |       |        |        |        |
| F stat.        | 1.5279| 1.0063| 0.8148 | 0.6543 | 0.5816 |
| F stat. Prob.  | 0.2170| 0.4133| 0.6144 | 0.8705 | 0.9638 |
| Obs. R²        | 1.5294| 5.0418| 8.1983 | 13.3143| 17.9883|
| R² Prob.       | 0.2162| 0.4108| 0.6095 | 0.8635 | 0.9587 |
| **PARCH (1,1)** |       |       |        |        |        |
| F stat.        | 0.0220| 0.2410| 0.2040 | 0.4147 | 0.4608 |
| F stat. Prob.  | 0.8821| 0.9443| 0.9960 | 0.9894 | 0.9945 |
| Obs. R²        | 0.0220| 1.2113| 2.0502 | 8.4076 | 14.0861|
| R² Prob.       | 0.8819| 0.9438| 0.9959 | 0.9888 | 0.9940 |
| **PARCH (1,2)** |       |       |        |        |        |
| F stat.        | 0.0118| 0.2124| 0.1669 | 0.3653 | 0.4366 |
| F stat. Prob.  | 0.9132| 0.9573| 0.9983 | 0.9954 | 0.9966 |
| Obs. R²        | 0.0119| 1.0679| 1.6860 | 7.4139 | 13.3561|
| R² Prob.       | 0.9131| 0.9569| 0.9982 | 0.9951 | 0.9962 |
| **PARCH (1,1)** |       |       |        |        |        |
| F stat.        | 2.3379| 0.9533| 1.0186 | 0.7070 | 0.6833 |
| F stat. Prob.  | 0.1265| 0.4455| 0.4251 | 0.8220 | 0.9009 |
| Obs. R²        | 2.3372| 4.7716| 10.1935| 14.2253| 20.6858|
| R² Prob.       | 0.1263| 0.4444| 0.4237 | 0.8189 | 0.8974 |
| **PARCH (1,2)** |       |       |        |        |        |
| F stat.        | 0.1248| 0.8029| 1.1257 | 0.7318 | 0.7056 |
| F stat. Prob.  | 0.2641| 0.5475| 0.3390 | 0.7956 | 0.8804 |
| Obs. R²        | 0.1249| 4.0215| 11.2548| 14.7172| 21.3518|
| R² Prob.       | 0.2637| 0.5463| 0.3380 | 0.7924 | 0.8766 |
| **PARCH (1,1)** |       |       |        |        |        |
| F stat.        | 1.9858| 0.9482| 0.7431 | 0.6026 | 0.6033 |
| F stat. Prob.  | 0.1594| 0.4494| 0.6837 | 0.9115 | 0.9534 |
| Obs. R²        | 1.9859| 4.7335| 7.4885 | 12.2900| 18.6309|
| R² Prob.       | 0.1588| 0.4467| 0.6787 | 0.9057 | 0.9474 |
| **PARCH (1,2)** |       |       |        |        |        |
| F stat.        | 1.2647| 0.6952| 0.7459 | 0.5755 | 0.6777 |
| F stat. Prob.  | 0.2613| 0.6273| 0.6810 | 0.9293 | 0.9024 |
| Obs. R²        | 1.2664| 3.4946| 7.5162 | 11.7539| 20.8292|
| R² Prob.       | 0.2604| 0.6242| 0.676 | 0.9243 | 0.8912 |

**3. Period**

|                | Lag 1 | Lag 5 | Lag 10 | Lag 20 | Lag 30 |
|----------------|-------|-------|--------|--------|--------|
| **EGARCH (3,2)** |       |       |        |        |        |
| AC             | -0.006 | 0.029 | 0.008 | -0.018 | -0.022 |
| PAC            | -0.006 | 0.029 | 0.007 | -0.018 | -0.015 |
| Q.Stat.        | 0.0293 | 1.2686| 2.3106 | 10.996 | 17.146 |
| Prob.          | 0.864  | 0.938 | 0.993 | 0.946  | 0.971  |
| **EGARCH (3,3)** |       |       |        |        |        |
| AC             | 0.014  | -0.015| -0.018 | 0.009  | -0.016 |
| PAC            | 0.014  | -0.014| -0.017 | 0.009  | -0.014 |
| Q.Stat.        | 0.1830 | 0.6809| 1.1647 | 3.7260 | 7.8501 |
| Prob.          | 0.669  | 0.984 | 1.000 | 1.000  | 1.000  |
According to Table 9, for ARCH (1) model we can see that null hypothesis can be rejected at %10 significance level which is residuals are heteroskedastic at first lag and first period. Residuals are also heteroskedastic in fifth, tenth, twentieth even in thirtieth lag at the first period. ARCH (2) model also suffer from heteroskedasticity. At the second and third periods have same problem according to ARCH (1) and ARCH (2) models. ARCH (3) model’s residuals are also heteroskedastic in the second period at tenth, twentieth even in thirtieth lag. Heteroskedasticity test have been done for all models in table 9 but this occupy lots of place. Therefore, autocorrelation results have reported for only selected models.

### Table 10: Autocorrelation Test Results for Selected Models

|          | Lag 1 | Lag 5 | Lag 10 | Lag 20 | Lag 30 |
|----------|-------|-------|--------|--------|--------|
| **1. Period** |
| EGARCH (3,2) | -0.006 | 0.029 | 0.008 | -0.018 | -0.022 |
| AC       | -0.006 | 0.029 | 0.007 | -0.018 | -0.015 |
| PAC      | 0.0293 | 1.2686 | 2.3106 | 10.996 | 17.146 |
| Q.Stat.  | 0.864 | 0.938 | 0.993 | 0.946 | 0.971 |
| Prob.    | 0.669 | 0.984 | 1.000 | 1.000 | 1.000 |

| **2. Period** |
| EGARCH (1,1) | 0.052 | -0.013 | 0.021 | -0.005 | 0.015 |
| AC       | 0.052 | -0.007 | 0.010 | -0.008 | 0.016 |
| PAC      | 3.165 | 7.2365 | 15.372 | 18.490 | 25.306 |
| Q.Stat.  | 0.214 | 0.355 | 0.445 | 0.555 | 0.710 |
| Prob.    | 0.387 | 0.390 | 0.306 | 0.801 | 0.897 |

| **3. Period** |
| EGARCH (1,2) | 0.022 | 0.041 | 0.025 | 0.053 | -0.010 |
| AC       | 0.022 | 0.038 | 0.020 | 0.049 | -0.010 |
| PAC      | 0.218 | 2.9679 | 7.7352 | 14.171 | 17.373 |
| Q.Stat.  | 0.640 | 0.705 | 0.655 | 0.822 | 0.968 |

|          | Lag 1 | Lag 5 | Lag 10 | Lag 20 | Lag 30 |
|----------|-------|-------|--------|--------|--------|
| **1. Period** |
| EGARCH (3,3) | 0.014 | -0.015 | -0.018 | 0.009 | -0.016 |
| AC       | 0.014 | -0.014 | -0.017 | 0.009 | -0.014 |
| PAC      | 0.1830 | 0.6809 | 1.1647 | 3.7260 | 7.8501 |
| Q.Stat.  | 0.669 | 0.984 | 1.000 | 1.000 | 1.000 |
| Prob.    | 0.010 | 0.010 | 0.006 | 0.010 | 0.006 |

| **2. Period** |
| EGARCH (1,1) | 0.052 | -0.007 | 0.008 | -0.004 | 0.001 |
| AC       | 0.052 | -0.003 | 0.001 | -0.007 | 0.003 |
| PAC      | 0.7474 | 5.2142 | 11.692 | 14.562 | 20.702 |
| Q.Stat.  | 0.387 | 0.390 | 0.306 | 0.801 | 0.897 |
| Prob.    | 0.010 | 0.010 | 0.006 | 0.010 | 0.006 |

| **3. Period** |
| EGARCH (1,2) | 0.022 | 0.041 | 0.025 | 0.053 | -0.010 |
| AC       | 0.022 | 0.038 | 0.020 | 0.049 | -0.010 |
| PAC      | 0.218 | 2.9679 | 7.7352 | 14.171 | 17.373 |
| Q.Stat.  | 0.640 | 0.705 | 0.655 | 0.822 | 0.968 |
All tests had been done for volatility models in Table 9 and Table 10. Only ARCH (1) and ARCH (2) models have autocorrelation and heteroskedasticity problems for according to selected lags. Other tried models' residuals have not heteroskedasticity and autocorrelation problems. According to determine the most suitable model for volatility modeling, Theil Inequality Coefficient (TIC), Mean Absolute Error (MAE) and Root Mean Square Error (RMSE) values calculated. The best models for return series have been selected both parameter significance and prediction fitness.

When the calculated TIC coefficients are evaluated in Table 11, smallest theil value is the main criteria for periods. Also small RMSE and MAE criteria is important for some situations. The EGARCH (3,2) model has been determined as the most suitable model for the first period. EGARCH (1,2) is the most suitable model for the second period. GARCH (1,1) is the most suitable model for the last period.

### Table 11: Results of Volatility Models’ Comparison

| Periods       | Models     | TIC   | RMSE | MAE  |
|---------------|------------|-------|------|------|
| 1. Period     | ARCH (1)   | 0.8824| 0.4828| 0.3101|
|               | ARCH (2)   | 0.9962| 18.0496| 8.5107|
|               | ARCH (3)   | 0.9651| 0.0662| 0.0371|
|               | GARCH (1,1)| 0.9509| 0.0662| 0.0371|
|               | GARCH (1,2)| 0.9553| 0.0662| 0.0371|
|               | TGARCH (1,1)| 0.9567| 0.0662| 0.0371|
|               | TGARCH (1,2)| 0.9353| 0.0663| 0.0371|
|               | EGARCH (1,1)| 0.9765| 0.0664| 0.0369|
|               | EGARCH (1,2)| 0.9788| 0.0664| 0.0369|
|               | EGARCH (2,1)| 0.7994| 0.0716| 0.0462|
|               | EGARCH (2,2)| 0.9785| 0.0664| 0.0369|
|               | EGARCH (2,3)| 0.9662| 0.0662| 0.0370|
|               | EGARCH (3,2)| 0.9626| 0.0663| 0.0372|
|               | EGARCH (3,3)| 0.9626| 0.0663| 0.0372|
|               | PARCh (1,1)| 0.9698| 0.0665| 0.0369|
|               | PARCh (1,2)| 0.9758| 0.0663| 0.0370|
|               | PARCh (1,3)| 0.9331| 0.0386| 0.0233|
|               | PARCh (2,1)| 0.9360| 0.0387| 0.0234|
|               | PARCh (2,2)| 0.9385| 0.0387| 0.0233|
|               | PARCh (2,3)| 0.9149| 0.0386| 0.0232|
|               | PARCh (1,1)| 0.9149| 0.0386| 0.0232|
|               | PARCh (1,2)| 0.9163| 0.0386| 0.0232|
|               | TGARCH (1,1)| 0.9161| 0.0386| 0.0232|
|               | TGARCH (1,2)| 0.9161| 0.0386| 0.0232|
|               | EGARCH (1,1)| 0.8698| 0.0509| 0.0359|
|               | EGARCH (1,2)| 0.8670| 0.0509| 0.0357|
|               | PARCh (1,1)| 0.8914| 0.0510| 0.0362|
|               | PARCh (1,2)| 0.8914| 0.0510| 0.0362|
| 2. Period     | ARCH (1)   | 0.8525| 0.0509| 0.0359|
|               | ARCH (2)   | 0.8253| 0.0509| 0.0357|
|               | ARCH (3)   | 0.8336| 0.0510| 0.0362|
|               | GARCH (1,1)| 0.8158| 0.0506| 0.0360|
|               | GARCH (1,2)| 0.8345| 0.0509| 0.0357|
|               | TGARCH (1,1)| 0.8398| 0.0508| 0.0357|
|               | TGARCH (1,2)| 0.8444| 0.0508| 0.0357|
|               | EGARCH (1,1)| 0.8462| 0.0508| 0.0357|
|               | EGARCH (1,2)| 0.8470| 0.0510| 0.0358|
|               | PARCh (1,1)| 0.8695| 0.0510| 0.0361|
|               | PARCh (1,2)| 0.8914| 0.0514| 0.0361|
| 3. Period     | ARCH (1)   | 0.8158| 0.0506| 0.0360|
|               | ARCH (2)   | 0.8345| 0.0509| 0.0357|
|               | ARCH (3)   | 0.8336| 0.0510| 0.0362|
|               | GARCH (1,1)| 0.8158| 0.0506| 0.0360|
|               | GARCH (1,2)| 0.8345| 0.0509| 0.0357|
|               | TGARCH (1,1)| 0.8398| 0.0508| 0.0357|
|               | TGARCH (1,2)| 0.8444| 0.0508| 0.0357|
|               | EGARCH (1,1)| 0.8462| 0.0508| 0.0357|
|               | EGARCH (1,2)| 0.8470| 0.0510| 0.0358|
|               | PARCh (1,1)| 0.8695| 0.0510| 0.0361|
|               | PARCh (1,2)| 0.8914| 0.0514| 0.0361|

The coefficients for the EGARCH (3, 2) model in the first period were calculated as follows. **Model 1:** Calculated Results in the First Period
Leverage parameter is statistically insignificant for EGARCH (2, 1) model in the first period. There is an asymmetric effect in the first period. So negative shock has more effect than positive shock in this period. We can say that first period of bitcoin volatility dependent of market news.

The coefficients for the EGARCH (1, 2) model in the second period were calculated as follows.

**Model 2: Calculated Results in the Second Period**

| $\alpha_1$ | $\beta_1$ | $\beta_2$ | $\gamma_1$ |
|------------|-----------|-----------|-------------|
| 0.2589     | 0.4366    | 0.2964    | 0.0287      |

Leverage parameter is statistically insignificant for EGARCH (1, 2) model in the second period. There is no asymmetric effect in the second period. So positive and negative shocks have same effect in this period. The effect of the shock on the bitcoin return series last for approximately 85.22 days in the second period.

**Model 3: Calculated Results in the Third Period**

| $\alpha_0$ | $\alpha_1$ | $\beta_1$ |
|------------|------------|-----------|
| 0.00000661 | 0.1992     | 0.7906    |

Last period suits GARCH (1,1) model. The sum of coefficients of GARCH (1,1) parameters is 0.9898. Volatility shocks have temporary effect on bitcoin return series in last period. Affecting of bitcoin return series shocks has %19.92 of past shocks while %79.06 caused a previous period shocks. $(\alpha_1 + \beta_1)$ measurement is an indicator of volatility persistence.

HL (halflife) indicator can be calculated with $\text{HL} = \frac{\ln(0.5)}{\ln(\alpha_1 + \beta_1)}$ formula. HL measurement shows period of shock and also shows recovery from shock (Topaloğlu, 2020: 33). The effect of the shock on the bitcoin return series last for approximately 67.61 days in the last period.

In the first period there is an asymmetric effect bitcoin return series sensitive to market news. In the second period bitcoin return series have high volatility because a shock has been continued approximately 3 months and there is not any asymmetric affect. In the last period volatility more temporary than second period so the effect of the shock on the bitcoin return series last for approximately 67.61 days.

**5. RESULTS AND DISCUSSION**

Bitcoin returns have an asymmetric effect in the first period so it is sensitive to market news. Bitcoin returns depend on market news at the first period. As a result of the speculative flows occurred between the dates, 10.02.2011- 02.10.2013, which Bitcoin has started to draw attention for the first time and it is relatively calm than other periods. Second period have not any asymmetric affect, also have the highest volatility persistence. Thus, half-life indicator shows that effect of the shock on the bitcoin return series last for approximately 85.22 days in the second period. In this framework, the observed return series reached very high volatility levels and cause negative impacts on the consistency of the model in the second period. Last period is relatively calm than second period because the effect of the shock on the bitcoin return series last for approximately 67.61 days. There is not any asymmetric affect have been found in the last period. The return behaviours of bitcoin in the given periods caused to the permanence of volatility which was identified in the model. The return volatility of bitcoin was high and close to be non-stationary in ARCH modelling. The given situation may stem from the fact that virtual currency rates are not accepted by many countries and do not have an advanced market. Bitcoin, which is not accepted in many countries and traded in a relatively narrow market in comparison to the acknowledged currencies of developed countries, has become successful in the recent period in terms of having a -relatively- predictable volatility but still a risky asset for investors.
As a result, investors who enjoy risk may follow the recent development in virtual currencies and include bitcoin in their basket with predictable volatilities. The inclusion of bitcoin by investors in their portfolio will reduce the portfolio risk of bitcoin which is not associated with other investment tools.

Ethics Statement: In this study, no method requiring the permission of the “Ethics Committee” was used

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