A New Hypothesis on the Origin of the Three Generations

S. D. Bass \([1,2,4]\) and A. W. Thomas \([3,4]\)

1 Cavendish Laboratory, University of Cambridge, Cambridge CB3 0HE, U.K.

2 Institut für Kernphysik, KFA-Jülich, D-52425 Jülich, Germany

3 Department of Physics and Mathematical Physics, University of Adelaide, Adelaide 5005, Australia

4 Institute for Theoretical Physics, University of Adelaide, Adelaide 5005, Australia

ABSTRACT

We suggest that the Standard Model may undergo a supercritical transition near the Landau scale, where the U(1) gauge boson couples to the left and right handed states of any given fermion with different charges. This scenario naturally gives rise to three generations of fermion, corresponding to the three critical scales for the right-right, right-left and left-left fermion interactions going supercritical, as well as CP violation in the quark sector.
1 Introduction

The Standard Model has proven very successful in every area of particle physics, including recent high-energy collider experiments – e.g. see Ref.[1]. However, it has three features which are not well understood: the origin of mass, the three fermion generations and the phenomenon of CP violation. The question of mass is usually framed in terms of (fundamental) Higgs fields [2] and why the corresponding Yukawa couplings take particular values – see however Ref.[3]. Instead, we believe that one should ask whether a formulation of the Standard Model with massless fermions makes sense. For example, it is well known that QED with massless electrons is not well defined at the quantum level [4, 5].

In this paper we consider the pure Standard Model with gauge symmetry $SU(3) \otimes SU(2)_L \otimes U(1)$ and no additional interaction; that is, assuming no additional unification. We examine the physical theory that corresponds to the bare Standard Model Lagrangian with no elementary Higgs and just one generation of fermions and gauge bosons which all have zero bare mass. At asymptotic scales, where the U(1) coupling is significantly greater than the asymptotically free SU(3) and SU(2)$_L$ couplings, the left and right handed states of any given charged fermion couple to the U(1) gauge boson with different charges. At the Landau scale there will be three separate phase transitions corresponding to each of the right-right, right-left and left-left interactions becoming supercritical. These transitions correspond to three generations of fermions. As one passes through each transition from a higher scale (shorter distance) the corresponding scalar condensate “melts”, releasing a dynamical fermion into the Dirac phase studied in the laboratory. In this picture the three generations emerge as quasi-particle states built on a “fundamental fermion” interacting self-consistently with the condensates.

Clearly this proposal differs in a fundamental manner from the conventional approaches to the Standard Model. While the conceptual framework is extremely simple and elegant, the techniques for dealing with non-perturbative physics at the Landau scale are not well developed. In particular, at the present stage we are not able to present a rigorous, quantitative derivation of all of the features of the Standard Model. Nevertheless, we believe that the potential for understanding so many phenomena, including mass, CP-violation and the generations, is so compelling that the ideas should be presented at this stage.

The structure of the paper is the following. In order to introduce the ideas we first review the change in the vacuum of pure QED near a point-like nucleus with charge greater than 137, a problem that has received enormous effort [4, 6, 7]. We then consider the analogous case
of QED at the Landau scale, from which we conclude that in pure QED the electron would self-consistently generate its own mass. Having thus introduced the basic notions we turn to the full Standard Model, considering in turn the generations of charged fermions and the origin of CP violation, the neutrinos and the mass of the vector bosons.

2 The Supercritical Phase of Non-Asymptotically Free Gauge Theory

Several decades of work have revealed that non-asymptotically free (NAF) U(1) gauge theories with zero bare fermion mass are capable of generating their own renormalised mass. The prime example of such a theory is, of course, QED with zero bare mass, where both analytic \cite{9, 10, 11} and numerical (lattice) calculations \cite{12} have shown that one finds a finite renormalised mass and a non-trivial, ultraviolet (UV), stable fixed point. Without this the theory is trivial; that is, the charge is completely screened by the interactions so that the theory is equivalent to a free field theory.

The possibility that QED may be trivial was originally suggested by Landau and co-workers \cite{13, 14} and Fradkin \cite{15}. Consider the running coupling in perturbative (weakly coupled) QED. The one loop vacuum polarisation implies

$$\alpha(m^2) = \frac{\alpha(\lambda^2)}{1 + \frac{\alpha(\lambda^2)}{\lambda^2} \ln \frac{\lambda^2}{m^2}}.$$  \hspace{1cm} (1)

If we take $\lambda^2 \to \infty$ (the continuum limit of QED with a finite cut-off) then $\alpha(m^2)$ vanishes for all $m^2$. The same result applies in the limit of zero mass gap (i.e. $m^2 \to 0$), namely the coupling $\alpha(m^2)$ vanishes again. Recent work by Kocić et al. \cite{16} has shown that this “zero charge problem” persists when the magnetic interaction of the electron is also included, despite the fact that it tends to screen the vacuum polarisation.

Another indication of the problem of massless QED is the fact that one cannot renormalise it (perturbatively) on mass shell (which is a necessary condition for the electron to be a physical particle). The only alternative, which was once again suggested by Landau et al. \cite{13} (see also Dirac \cite{17}), is that non-perturbative effects near the Landau scale mean that QED has a non-trivial, UV, stable fixed point. A number of groups \cite{9, 10, 11, 18} have shown that QED, in quenched, ladder approximation, has a non-trivial, UV, stable fixed point at $\alpha_c = \frac{\pi^2}{3}$, which separates the weakly and strongly interacting phases. The theory is trivial for bare coupling
\[ \alpha < \alpha_c, \text{ whereas for } \alpha > \alpha_c \text{ the chiral symmetry of the massless bare theory is spontaneously} \]

\[ \text{broken by the interactions leading to the formation of tightly bound states } - \text{ much like the} \]

\[ Z > 137 \text{ point nucleus problem in QED. Kogut et al. [12] have found that this UV, stable} \]

\[ \text{fixed point survives in unquenched lattice QED. Estimates of the value of } \alpha_c \text{ (the critical bare} \]

\[ \text{coupling) from Schwinger-Dyson and lattice calculations range between 0.8 and 2 [18-21].} \]

Given that a NAF U(1) gauge theory has a non-trivial, UV, stable fixed point, \( \alpha_c \), it follows

\[ \text{that the theory has a two phase structure. We let } \lambda_c \text{ denote the scale at which } \alpha \text{ reaches the} \]

\[ \text{fixed point } \alpha_c \text{ and call the phases at scales above and below the critical scale } \lambda_c \text{ the Landau} \]

\[ \text{and Dirac phases respectively. Perturbative QED (and the Standard Model) is formulated} \]

\[ \text{entirely in the Dirac phase of theory } (\mu < \lambda_c). \text{ The theory seems to behave as a gauged} \]

\[ \text{Nambu-Jona-Lasinio model [22] in the Landau phase [12, 23, 24].} \]

\[ \text{The connection with supercritical phenomena (in particular, the large-}Z, \text{ point-nucleus} \]

\[ \text{problem) suggests a simple physical interpretation of this theory. Since massless, perturbative} \]

\[ \text{QED is not a consistent theory because of the “zero charge problem”, we consider perturbative} \]

\[ \text{QED with a finite renormalised mass and sketch how this mass could be recovered} \]

\[ \text{self-consistently in a complete formulation of QED.} \]

\[ \text{The coupling } \alpha \text{ increases until we reach the critical scale } \lambda_c \text{ where the interaction of the} \]

\[ \text{fermions with the gauge field becomes supercritical. To understand what happens at this} \]

\[ \text{transition it is helpful to consider the analogous problem of a static, large-}Z, \text{ point nucleus in} \]

\[ \text{QED [6, 7, 8]. There the 1s bound state level for the electron falls into the negative energy} \]

\[ \text{continuum at } Z = 137. \text{ If we attempt to increase } Z \text{ beyond 137 the point nucleus becomes a} \]

\[ \text{resonance: an electron moves from the Dirac vacuum to screen the supercritical charge which} \]

\[ \text{then decays to } Z - 1 \text{ with the emission of a positron.} \]

\[ \text{If the electron itself were to acquire a supercritical charge at very large scales, } O(\lambda_c), \text{ it} \]

\[ \text{would not be able to decay into a positive energy bound state together with another electron} \]

\[ \text{because of energy momentum conservation. In this case, the Dirac vacuum itself would decay} \]

\[ \text{to a new supercritical vacuum state. Since the vacuum is a scalar, this transition necessarily} \]

\[ \text{involves the formation of a scalar condensate which spontaneously breaks the (near perfect)} \]

\[ \text{chiral symmetry of perturbative QED at large momenta. The Dirac vacuum is a highly excited} \]

\[ \text{state at scales } \mu \geq \lambda_c \text{ and one must re-quantise the fields with respect to the new ground state} \]

\[ \text{vacuum in the Landau phase of the theory. The Dirac electron of perturbative QED would} \]

\[ \text{freeze out of the theory as a dynamical degree of freedom and the running coupling would freeze} \]

\[ \text{at } \alpha(\lambda_c). \text{ Perturbative QED, which requires a finite electron mass, is formulated entirely in the} \]
Dirac phase of the theory. The normal ordering mismatch between the zero point energies of the scalar vacua in the Dirac and Landau phases of QED means that the electron in the Dirac phase always feels a uniform, local, scalar potential. This potential must be included in the Hamiltonian for perturbative QED. The minimal gauge invariant, local, scalar operator that we can construct is the scalar mass term $m_e|\vec{e}|$. A self-consistent treatment of QED appears to generate its own mass.

3 Generations in the Standard Model

We now discuss how the considerations of the previous section carry over to the Standard Model. The Standard Model differs from QED at very large momentum in that the U(1) gauge boson coupling to a fermion depends on its chirality. The right-right, right-left and left-left fermion interactions have different strengths for the Dirac leptons ($e$, $\mu$ and $\tau$) and the quarks. As we now explain, this important difference means that a non-perturbative solution of the Standard Model requires three generations of fermions. The fermion gauge boson interaction in the electroweak sector is described by the Standard Model Lagrangian with symmetry $SU(3) \otimes SU(2)_L \otimes U(1)$:

$$\bar{\Psi}_L \left( \hat{\partial} - ig_1 \hat{B} - ig_2 \hat{W}_1 \frac{1}{2\tau} \right) \Psi_L + \bar{\Psi}_R \left( \hat{\partial} - ig_1 \hat{B} \right) \Psi_R.$$  (2)

Here $\Psi_L$ and $\Psi_R$ include the left and right handed fermions according to the Standard Model. We use $\alpha_1 = \frac{2\pi}{4\tau}$, $\alpha_2 = \frac{2\pi}{4\tau}$ and $\alpha_s$ to denote the U(1), SU(2) and colour SU(3) couplings respectively.

Consider the Standard Model evolved to some very large scale, much greater than the "unification scales" where the U(1) coupling $\alpha_1 = \alpha_2$ and $\alpha_1 = \alpha_s$. As we approach the Landau scale the $Z^0$ evolves to become the U(1) gauge boson as $\sin^2 \theta_W \rightarrow 1$. Since the SU(2) and SU(3) sectors of the Standard Model are asymptotically free [25] the $W^\pm$, the photon and the gluon have effectively disappeared at these scales. The $Z^0$ mass increases logarithmically with increasing $\mu^2$ and can be treated as negligible at the Landau scale so that the theory behaves as a U(1) gauge field coupling to left and right handed fermions with different charges. The $Z^0$ coupling to the fermions is

$$-ig_1 \gamma_\mu \left( c_L \frac{1 - \gamma_5}{2} + c_R \frac{1 + \gamma_5}{2} \right)$$  (3)

where the left and right handed charges $c_L g_1$ and $c_R g_1$ are given in Table 1. (Here $l$ denotes the charged leptons and $\nu_l$ the corresponding neutrinos. We use $q^*$ and $q_*$ to denote the upper
Table 1: The fermion couplings to the $Z^0$.

| $c_L$       | $c_R$       |
|-------------|-------------|
| $l$         | $-\frac{1}{2} + \sin^2 \theta_W$ | $\sin^2 \theta_W$ |
| $\nu_l$     | $+ \frac{1}{2}$ | $0$ |
| $q^*$       | $+ \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W$ | $- \frac{2}{3} \sin^2 \theta_W$ |
| $q^*$       | $- \frac{1}{2} + \frac{1}{3} \sin^2 \theta_W$ | $+ \frac{1}{3} \sin^2 \theta_W$ |

and lower components of the electroweak quark doublet.)

The idea that we wish to develop is the following. Consider a “fundamental fermion”,
which is defined in the pure Landau phase of the Standard Model. Since the left and right
handed charges have different values, it follows that the left-left, left-right and right-right
fermion interactions will, in general, become sub-critical at different scales as we evolve the
theory through the supercritical transitions to lower $\mu^2$. The first interaction to become sub-
critical as we decrease $\mu^2$ is the left-left interaction, followed by the left-right and then the
right-right interactions. Each transition is associated with the melting of a scalar condensate
which releases a dynamical fermion into the Dirac phase of the Standard Model. These Dirac
fermions interact self-consistently with the condensates in the Landau phase of the theory. In
this picture the three fermion generations emerge as three quasi-particle states in the Dirac
phase which correspond to the “fundamental fermion” in the Landau phase and which couple
to the gauge field with identical charge.

Let us now outline how this structure should be manifest from the opposite direction, as we
evolve the Standard Model upwards from the “low scale” of the laboratory towards the Landau
scale. For simplicity, we first consider the charged leptons. In the absence of any other physics
the Standard Model should undergo a rich series of phase transitions near the Landau scale as each of the right-right, right-left and left-left fermion interactions become supercritical with increasing $\mu^2$. These transitions can be classified into one of two types: “static” transitions and “vacuum” transitions. “Static” transitions involve the decay of the left or right handed component of a “heavy” fermion $\Psi_h$ into a “light” fermion $\Psi_l$ together with the formation of a $(\Psi_h\overline{\Psi}_l)$ bound state (like the decay of a large-$Z$ point nucleus). The supercritical component of $\Psi_h$ becomes a resonance between the critical scales for the static transition and the vacuum transition at which the $\Psi_h$ freezes into the Landau phase. “Vacuum” transitions involve the decay of the fermionic vacuum from the Dirac into the Landau phase and the formation of a scalar condensate. Static transitions do not affect the symmetry or generation structure (which is given by the vacuum transitions). They do affect the scale at which the vacuum transition involving $\Psi_h$ takes place. The charge of the “resonance fermion” increases more slowly with increasing $\mu^2$ than we would predict using perturbative arguments alone so that vacuum transitions which involve the resonance fermion are pushed to a higher scale.

At very large scales where $\sin^2 \theta_W \to 1$, the charges of the left and right handed charged leptons become $c_L \to \frac{1}{2}$ and $c_R \to 1$ respectively. The interaction between two right handed fermion fields (eg. $e_R^-$, $e^+_L$) is the first to go supercritical. One finds the static decays of the right handed muon $\mu_R^-$ and tau $\tau_R^-$, viz. $\tau_R^- \to (\tau_R^- e^+_L) e^-_R$, and also the vacuum transition involving the right handed electron at a critical scale $\lambda^{RR}_e$. Since the vacuum is a scalar, this vacuum transition must be associated with the formation of a scalar condensate. It is important to consider what has happened to the left handed electron at this point. The Dirac vacuum for the left-handed electrons collapses at $\lambda^{RR}_e$ because of the axial anomaly \cite{26}, whereby the chirality of a charged lepton in the Dirac phase is not conserved in the presence of a background gauge field. The anomaly has a simple interpretation in a two phase NAF gauge theory \cite{27}. Consider the gauge-invariant axial-vector current in perturbative QED with an explicit UV cut-off, which we shall take to be equal to $\lambda^{RR}_c$. The anomaly appears as a flux of chirality (or spin) over the cut-off – and into the Landau phase of the theory. If one turns off the anomaly, the Dirac vacuum for the left handed electrons is highly excited with respect to the Landau vacuum for the right handed electrons at $\mu \geq \lambda^{RR}_c$. Via the axial anomaly, the left-handed electrons condense with the right-handed electrons to form the Landau vacuum which is created at $\lambda^{RR}_c$ and the electron completely freezes out of the theory.

At scales $\mu \geq \lambda^{RR}_c$ the remaining charged lepton degrees of freedom are the $\mu$ and the $\tau$. Here the right handed muons and taus are supercritical resonances while the left handed
muons and taus are still perturbative fermions. The left handed charge evolves significantly faster than the right handed charge with increasing $\mu^2$ and the left handed fermions drive the dynamics. The muon freezes into the Landau phase at the critical scale $\lambda^{L R}$ for the left-right vacuum transition and the tau freezes out at the left-left vacuum transition. The latter is catalysed by the axial anomaly in the same way as the right-right vacuum transition. The three self-supercritical transitions (right-right, left-right and left-left) yield three condensates in the Landau phase of the Standard Model.

The same arguments hold in the quark sector but there is one important new point to note. The upper and lower components of the electroweak quark doublet $q^*$ and $q_*$ become self-supercritical at different scales because of the different coupling of the U(1) gauge boson to each of the $q^*$ and $q_*$ quarks. This means that the eigenstates of the $W^\pm$-quark interaction in the Standard Model (which define the components of the quark doublet) and the quark mass eigenstates are not identical. The three generations of quarks mix according to a unitary (Kobayashi Maskawa) matrix which, in general, gives CP violation in the quark sector. To see that we have a CP violating interaction at large scales, consider the vector, vector, axial-vector triangle diagram. This is anomaly free in the pure Dirac phase of the Standard Model when we sum over $l$, $\nu_l$, $q^*$ and $q_*$ propagating in the triangle loop. At intermediate momentum scales, where one component of the quark doublet has frozen into the Landau phase and the other component remains in the Dirac phase, there is a nett three-gauge-boson contact interaction in the Dirac phase of theory which carries the CP-odd quantum numbers of the axial anomaly. This corresponds to regularising the UV behaviour of the triangle amplitude with a slightly different cut-off for each component of the electroweak doublet in perturbation theory. Since this cut-off is so much greater than any mass scales that are currently amenable to experiment this contact interaction does not harm either anomaly cancellation or the renormalisability of the Standard Model.

As the right-handed (Dirac) neutrino is non-interacting in the Standard Model we cannot see how to form a scalar, neutrino condensate at the supercritical transitions. Of course, each type of neutrino will sense the corresponding charged lepton transition (through the coupling $\nu_l \rightarrow W l \rightarrow \nu_l$). While it may be that this coupling gives rise to a mass, $m_l$, of the order of $G_F m_l$ in the Dirac phase, it seems most likely that the neutrinos are massless. (The Standard Model with massive gauge bosons and massless neutrinos can be renormalised on mass shell $[28]$.) If this is the case it is trivial that there should be no Kobayashi-Maskawa matrix in the lepton sector: one can simultaneously diagonalise the eigenstates of mass and the $W^\pm$-lepton
interaction.

The resonance structure offers a possible reason why the top quark is so much heavier than the bottom quark. The relative separation of the left-right and left-left transitions is greater for the $q^*$ (top quark) than the $q_*$ (bottom quark). This means that the top quark has further to evolve than the bottom quark to get from $\lambda_c^{RL}$ to $\lambda_c^{LL}$ with a slowly increasing left handed charge; the top quark freezes out at a much higher scale than the bottom quark and has a much higher mass. Similarly, the charm quark has a lot further to go than the strange quark between the right-right and left-right transitions.

The dynamical chiral symmetry breaking which gives us the fermion masses also gives mass to the gauge bosons. The gauge fixing in the “fundamental” bare Lagrangian (with zero mass) does not involve the $0^{-+}$ Goldstone bosons, which are generated with the dynamical chiral symmetry breaking when we turn on the vacuum polarisation. The propagators for the gauge bosons are transverse in covariant (eg. Landau) gauge:

$$\Pi_{\mu\nu} = f^2 \left( g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right)$$

When we evaluate the gauge boson self-energies using the Schwinger-Dyson equations (eg. in leading logarithm approximation [29]) we find a non-transverse mass term in $\Pi_{\mu\nu}$ which is proportional to $g_{\mu\nu}$. The transversity of $\Pi_{\mu\nu}$ is restored by the mixing of the gauge bosons with the $0^{-+}$ Goldstone bosons. The fermion, gauge-current vertex becomes:

$$\left( \gamma_\mu \frac{1}{2} (1 - \gamma_5) \frac{\tau^a}{2} \right)_{\text{bare}} \rightarrow \left( \gamma_\mu \frac{1}{2} (1 - \gamma_5) \frac{\tau^a}{2} - f_{ab} g_b \frac{p_\mu}{p^2} \right)_{\text{(Standard Model)}}$$

where $f_{ab}$ is the current-Goldstone transition amplitude and $g_b$ denotes the Goldstone-fermion coupling. The Higgs mass and the Goldstone parameters $f_{ab}$ and $g_b$ are determined by the mass of the top quark $m_t$ and the running QCD coupling $\alpha_s(m_t^2)$. As Gribov has emphasised [24], the Schwinger-Dyson equations for the Higgs and Goldstone self-energies involve all the fermions on an equal footing. The top quark becomes important only because of its large mass; it has no special interaction.

## 4 Conclusions

We have argued, on quite general grounds, that in the absence of elementary Higgs (or other, additional, physics) the Standard Model may generate its own mass and three generations of fermions as a result of super-critical phenomena at the Landau scale. We have not considered
gravity and one may worry that, at least in a perturbative treatment, the Landau scale is larger than the Planck mass. However, we believe that the scenario presented in this paper is compelling and certainly merits further investigation. One could speculate that in a non-perturbative treatment the physics of the Planck scale and the Landau scale may in fact be coupled.

It is clearly important to explore the physics of the Landau scale in the laboratory. This is difficult in the U(1) sector because of the large momentum scales involved. On the other hand, in QCD the Landau scale is in the infra-red and it might be that one can learn a little about the mechanism proposed here through the study of phenomena such as quark confinement and hadronisation [30-33].

Acknowledgements

We would like to thank T. Goldman, V. N. Gribov, P. A. M. Guichon, C. A. Hurst, R. G. Roberts, D. Schütte, R. Volkas and A. G. Williams for helpful discussions. We would also like to thank J. Speth for his hospitality at the KFA Jülich where this work began. This work was supported in part by the Australian Research Council and the Alexander von Humboldt Foundation.

References

[1] O.Nachtmann, *Elementary particle physics*, Springer-Verlag (1990);
E. Leader and E. Predazzi, *An introduction to Gauge Theories and the New Physics*, Cambridge UP (1982);
S. Pokorski, *Gauge field theories*, Cambridge UP (1987).

[2] P. W. Higgs, Phys Lett. 12 (1964) 132;
Phys. Rev. Lett. 13 (1964) 132;
Phys. Rev. 145 (1966) 1156

[3] H. Georgi, “Why I would be very sad if a Higgs boson were discovered”, in *Perspectives on Higgs physics*, ed. by G.L. Kane (World Scientific, Singapore 1993).
[4] T. Muta, *Foundations of Quantum Chromodynamics*, World Scientific (1987).

[5] V. N. Gribov, Nucl. Phys. B206 (1982) 103.

[6] I. Pomeranchuk and Ya. Smorodinsky, J. Fiz. USSR 9 (1945) 97;
W. Pieper and W. Greiner, Z Physik 218 (1969) 327;
Ya. B. Zeldovich and V. S. Popov, Uspekhi Fiz. Nauk. 105 (1971) 4.

[7] W. Greiner, B. Müller and J. Rafelski, *Quantum electrodynamics of strong fields*, Springer-Verlag (1985);
E.S. Fradkin, D.M. Gitman and S.M. Shvartsman, *Quantum electrodynamics with unstable vacuum*, Springer-Verlag (1991).

[8] V. N. Gribov, Orsay lectures LPTHE Orsay 92/60 (June 1993) and LPTHE Orsay 94/20 (Feb. 1994).

[9] T. Maskawa and H. Nakajima, Prog. Theor. Phys. 52 (1974) 1326; ibid. 54 (1975) 860.

[10] R. Fukuda and T. Kugo, Nucl. Phys. B117 (1976) 250.

[11] P. I. Fomin, V. P. Gusynin and V. A. Miransky, Phys. Lett. B78 (1978) 136;
P. I. Fomin, V. P. Gusynin, V. A. Miransky and Yu. A. Sitenko, Nucl. Phys. B110 (1976) 445.

[12] J. B. Kogut, E. Dagotto and A. Kocić, Phys. Rev. Lett. 60 (1988) 772.

[13] L. D. Landau and I. Pomeranchuk, Dokl. Akad. Nauk. USSR 102 (1955) 489;
L. D. Landau, A. A. Abrikosov and I. M. Khalatnikov, Nouvo Cim. Suppl. X3 (1956) 80.

[14] V. B. Berestetskii, E. M. Lifshitz and L. P. Pitaevskii, *Quantum Electrodynamics*, Section 133 (Landau and Lifshitz Course of Theoretical Physics, Volume 4, Pergamon Press 1980).

[15] E.S. Fradkin, Sov. Phys. JETP 1 (1955) 604.

[16] A. Kocić, E. Dagotto and J. Kogut, Phys. Lett. B213 (1988) 772.

[17] P. A. M. Dirac, 1941 Bakerian lecture, Proc. Roy. Soc. London 180A (1942) 1.

[18] P. I. Fomin, V. P. Gusynin, V. A. Miransky and Yu. A. Sitenko, “Dynamical symmetry breaking and particle mass generation in gauge field theories”, Riv. Nuovo Cimento 6 (1983) 1.
[19] V. P. Gusynin, Mod. Phys. Lett. A5 (1990) 133.

[20] D. C. Curtis and M. R. Pennington, Phys. Rev. D48 (1993) 4933.

[21] J. Bartholomew et al., Nucl. Phys. B230 (1984) 222.

[22] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122 (1961) 345.

[23] W. A. Bardeen, C. N. Leung and S. T. Love, Phys. Rev. Lett. 56 (1986) 1230;
     C. N. Leung, S. T. Love and W. A. Bardeen, Nucl. Phys. B273 (1986) 649;
     W. A. Bardeen, C. N. Leung and S. T. Love, Nucl. Phys. B323 (1989) 493.

[24] V. A. Miransky and V. P. Gusynin, Prog. Th. Phys. 81 (1989) 426.

[25] H. Georgi, H. R. Quinn and S. Weinberg, Phys. Rev. Lett. 33 (1974) 451;
     T. P. Cheng and L. F. Li, ”Gauge Theory of Elementary Particle Physics” (Clarendon
     Press, Oxford, 1984).

[26] J. S. Bell and R. Jackiw, Nuovo Cimento A51 (1969) 47;
     S. L. Adler, Phys. Rev. 177 (1969) 2426;
     J. Schwinger, Phys. Rev. 82 (1951) 664.

[27] V. N. Gribov, Budapest preprint KFKI-1981-66 (1981);
     V. N. Gribov, Phys. Lett. B194 (1987) 119.

[28] K. Aoki et al., Suppl. Prog. Th. Phys. 73 (1982) 1

[29] V. N. Gribov, Phys. Lett. B336 (1994) 243.

[30] V. N. Gribov, Physica Scripta T15 (1987) 164;
     Lund preprint LU TP 91/7 (May 1991), unpublished.

[31] J. E. Mandula, Phys. Lett. B67 (1977) 175;
     A. J. G. Hey, D. Horn and J. E. Mandula, Phys. Lett. B80 (1978) 90.

[32] T. Goldman and R. W. Haymaker, Phys. Rev. D24 (1981) 724.

[33] Various contributions in “Confinement Physics”, Proc. 1st ELFE Summer School, eds. S.
     D. Bass and P. A. M. Guichon (Editions Frontières,1996).