What is chaos and how is it relevant for philosophy of mind?1

John M. Ostrowick
Philosophy Department, University of Cape Town, South Africa
john@ostrowick.co.za

This paper argues, in agreement with Neuringer, Beggs and others, that if we wish to scientifically characterise human choice, we have to work with a probabilistic or chaos-derived model. This has a number of implications for philosophy of mind. Firstly, it is in theory possible to describe human behaviour with some form of law-like equation; it is just a matter of figuring out what equation best captures the states of our central nervous system. Secondly, it means that our choices are not random but are chaotic: deterministic, but just hard to predict due to internal complexity. According to the current research, our actions are most likely caused by ‘precipitated avalanches’ of neural activity, which are difficult to predict due to chaos.

1. Introduction
This paper explains Chaos, and discusses some implications of Chaos for philosophy, specifically philosophy of mind.

In Section 2, I explain Chaos, as many readers may have the prior assumption that ‘Chaos’ just means ‘randomness.’ Once this has been discussed, I will then go on to Section 3, where I introduce how Chaos affects philosophical considerations, particularly in philosophy of mind. I then examine some evidence from neuroscience (Neuringer, Voss, Gazzaniga, Beggs, et al.) which suggests that our brains (and hence, presumably, our minds), may be, in some suitably qualified sense, chaotic. It will then be argued that evidence that we have, both from neuroscience and from common observation, supports the view that human choices are, in some suitably qualified sense, chaotic.

The contribution of the paper, then, is that it argues that, contrary to generally received opinions in philosophy of mind, the brain is neither quantum-random (Kane 1996, 2002, for example), nor ‘hard’-deterministic (Pereboom 2001, for example),2 but rather chaotic. So, the paper argues, if the brain’s operations are chaotic, and if some form of materialism is true, it follows that the mind is chaotic, in some qualified sense. And if that is true, the paper argues, Davidson’s conjecture, that there are no law-like ways to characterise human actions, would be false, since chaos mathematics provides us with law-like equations.

In Section 4, I finish by contemplating some criticisms of the position, and find that they are not strong enough to dismiss the argument.

It is worth pointing out that this paper assumes the truth of physicalism, that is, that the mind is the activities of the brain. Whether non-reductive materialism, supervenience, anomalous monism, functionalism, epiphenomenalism, or eliminativism are true, will not be discussed.3 I will assume,

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1 An earlier version of this paper was presented at the Annual Congress of the Philosophical Society of Southern Africa in Port Elizabeth, South Africa, 12–14 January 2015.
2 ‘An action is free in the sense required for moral responsibility only if it is not produced by a deterministic process that traces back to causal factors beyond the agent’s control’ (Pereboom 2001: p. 3).
3 One might argue, as Janich (2009: p. 73) does, that ‘it is not brains that communicate, but interacting persons’. However, in brief, the argument can be replied to as follows: brains give rise to mental states, speech acts, bodily acts, and as such, whether we wish to label those behaviours or phenomena as ‘belonging’ to a ‘person’ or ‘self’ is moot. The point is, the mind/mental states has/have to be caused by something—presumably the brain. See Gazzaniga, op. cit. or more. One can then object to reductive materialism, as does Habermas (2005: p. 156) (see also Habermas 2001: pp. 16, 20). However, there are a wide variety of materialisms propounded in philosophy of mind, some of which are reductive, some of which are not reductive, and which are listed above. For example, the well-known critique from Nagel (1974), *What is it like to be a bat?* provides a strong argument against reductive materialism, in that it is apparently not possible for
for the sake of the article, that the brain fully determines mental content, and that the relationship is either simultaneous or unidirectional; that is, that mental states arise at the same time and in the same space as brain states or, that brain states determine mental states. The exact nature of the relationship between the mental and the neurological will not be discussed further, and, importantly, the paper’s findings do not entail that we lack free-will.4

2. What is Chaos?
2.1 What is Chaos, and how does it relate to concepts of randomness?
There are a variety of types of randomness that we need to differentiate before we can proceed with explicating Chaos itself. These forms of randomness are: Entropy, True or Stochastic Randomness (Quantum randomness), Chaos and/or Pseudo-randomness.5

Entropy, covered by the Second Law of Thermodynamics, refers to ‘the amount of additional information needed to specify the exact physical state of a system’ (Giancoli 1990: p. 752 et seq); hence, the more disordered a system, the more information would be required to describe it, the greater the entropy. Entropy is thus a statistical measure of randomness, regardless of how that randomness came about.

True, Quantum or Stochastic randomness is the type of disordered state generated by quantum fluctuations, in which no complete knowledge of the system and its antecedent causes is possible;6 one can know the particles’ momentum or position, but not both. Quantum Mechanics says that the position or momentum of a particle cannot be simultaneously known, but are instead mutually uncertain. Quantum Mechanics implies that everything is inherently random (truly random), at the subatomic level.7

Chaos, however, is a kind of pseudo-randomness; that is, it covers cases which seem to be random but which are in fact deterministic. Chaos is a deterministic result of the long-term evolution of complex behaviour from certain antecedent conditions to unpredictable later circumstances. Chaotic systems are extremely sensitive to small changes, and they can have simple initial conditions that rapidly become complex (Williams 1997: p. 23).8 Due to sensitivity to initial conditions, divergence happens rapidly, and in the longer term, results in unpredictability and irretrievability (as per the Second Law of Thermodynamics). But whilst being locally unpredictable, a Chaos system is also globally stable (Gleick 1988: p. 48). So, for example, we cannot say what weather we will get in a few days’ time, and we cannot predict whether a storm will be especially violent (before it forms), but we can say that it generally rains in the afternoon in my home town, and that the storms are quite violent. Consider, also, snowflakes: each one is unique (unpredictable), but they are all similar (globally stable).

Chaos implies that since many states in nature, such as the weather, are non-linear, we would find that it is impossible to extrapolate from the graph of known data points, to where the data will go next. Consider the Ideal Gas Law, \( PV = nRT \). This law states that given a constant value, \( R \), and a constant amount of substance \( n \), the pressure and volume \( PV \) of a gas are proportional to the gas’s temperature. This is a linear equation; as \( T \) increases, so does \( P \), so \( P \) is proportional to, or a function

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4 It is worth pointing out that the dominant view of philosophers of mind is that reductionism is not freedom-detrimental, i.e. that reductionism would not destroy our free-will or our ability to interact qua persons. For more on this, see Frankfurt (1969, 1982) and Strawson (1962, 1974), op. cit.
5 Giancoli 1990: Physics: Principles with Applications. London: Prentice-Hall: p. 752 et seq.
6 Quantum Mechanics is generally considered to entail the Copenhagen Interpretation, in which states of matter or energy are unavoidable statistically or uncertain and remain so until observed or measured, at which point, the quantum state or wave function undergoes ‘wave function collapse,’ which is to settle into a known value (Wimmel 1992).
7 Lyapunov exponent is calculated by comparing the relative trajectories of points over time; the more they diverge over time, the more chaotic the system. (Williams 1997: p. 53). This variance can also be detected by autocovariance or autocorrelation, or comparing how a function differs from, or correlates with, its own corresponding phases earlier in time (ibid.: pp. 86–88).
of \( T \), and the change is linear, and not sensitive to minor errors. Chaotic equations differ in those two respects; they are neither linear, nor are they tolerant of minor errors.

If one thinks, say, of typical statistical graphs, which have a straight line drawn through the approximate centre of a scatter plot of many nearby points, this illustrates how we try, in science, to find neat linear causal or correlational relationships in nature. But in many real cases, we do not have any such neat direct relationship. What Chaos attempts to do is explain why such points do not sit on a neat line (Williams 1997: p. 19). The reason is, for the most part, that natural systems are ‘dynamical’—they change over time or, in cases of systems that have mathematically continuous change, they are said to ‘flow’. Thus, Chaos is the study of nonlinear dynamical systems (Williams 1997: p. 19).

Examples include pseudo-random numbers generated by a computer algorithm, the structure of organisms, the weather, turbulence and swirls in smoke and fluids generally, the pattern of veins in a leaf, all the unique snowflake structures, the evolution of organisms, population changes, market prices, and the path of a fork of lightning. Indeed, Chaos may be the rule and linear dynamics the exception (Williams 1997: p. 21).

2.2 Pseudo-randomness and the logistic equation

I have said thus far that Chaos is pseudo-random. What is meant by this? What is meant is that Chaos is not stochastic or quantum-random (truly random) but is rather deterministic. As an example, consider present-day computers. Computers do not generate truly random numbers. On a computer, to generate a ‘random’ number, one typically provides a ‘seed number,’ which is then passed through an algorithm to render it unrecognisable, and then one then restricts it to a desired range of numbers using the modulus function, which effectively strips decimal remainders. Once the result is obtained, it is memorised as the seed number for the next iteration or repeat of the program (Metzger 1994: p. 111). So, for example, 2 ; 4 ; 8 ; 16, is recognisibly non-random, because the multiplier (in this case) is very obviously 2. The equation is easy to discern: it is \( x_{n+1} = 2x \), and the first seed number was 1. However, if one uses a more unrecognisable number, like 6.023 as a multiplier, the numbers become the more pseudo-random: 12.046 (seed: 2); 72.553058 (seed: 12.046); 436.987068334 (seed: 72.553...), etc. Thus the equation for this pseudo-random series is \( x_{n+1} = 6.023x \). The more complex the algorithm, the harder it is to see how the results appear, and the more random they appear.

Now, an algorithm is said to be self-referring or recursive if the results of its computations are fed back into the algorithm again each time it runs, which is what we have seen in the above examples. This is what we call a ‘feedback loop’ (Williams 1997: p. 20). Enter Chaos. Chaos equations are recursive. The most commonly-used equation (of many equations) is the Logistic Equation:

\[
x_{n+1} = rx_n(1- x_n)
\]

If we use 1 or 2 for \( r \), the equation’s output generates a regular graph. But the range (or ‘period’) \( r = 3.55 \) to \( r = 4.0 \) generates apparently random decimal numbers:

\[
0.36 ; 0.9216 ; 0.28901376 ; 0.82193922612265 ; 0.5854205387342 ; 0.97081332624944 ; 0.11333924730376
\]

Apart from repeating islands of stability, then, the Logistic Equation is unstable between approximately \( r = 3.55 \) and \( r = 4.0 \). Indeed, after \( r = 4.0 \), it rapidly becomes infinite or exponential. We will see the graphs of this equation later on. Importantly, Chaos can emerge from just one variable (in this case, \( x \)); it is not true to say that many variables have to interact to cause Chaos.

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9 This is mentioned as a technical point in case the reader sees the term ‘flow’. Chaotic systems can have discrete iterative changes (as we will see with the Logistic Equation later), or they can flow (be mathematically continuous). Flow equations use the mathematics of calculus, which deals with continua. However, discrete equations, which treat flows as a series of approximate measurements, are easier to solve, and can often approximate the more complex equations from calculus (Williams 1997: p. 20).
Moreover, there is no ‘error’ factor here in measurement; the data is entirely self-generated, deterministic, and internal to the algorithm (Williams 1997: p. 16). Yet it seems random.\textsuperscript{10}

2.3 Fractals and attractors

Chaotic equations are also marked by self-similarity\textsuperscript{11} or repetition, and as mentioned, non-linearity (Williams 1997: p. 18).\textsuperscript{12} This self-similarity appears, when graphed, as repeating patterns known as ‘fractals’, which never end. The reader can refer to Figures 1 and 2 for examples of fractals, specifically the Mandelbrot Set. We will see more about self-similarity, later, when we discuss how we detect the presence of Chaos in a data set.

Chaotic systems are described mathematically as a function or line plotted on a graph (2D—two-space, or 3D, three-space) called the Phase Space or State Space (Williams 1997: p. 28). Each variable in the function is plotted on an axis. This graphing is often done to show the hidden structure of the apparently random data, which is often not visible on merely inspecting the data, the algorithm, or a 2D plot of the data as a scatter plot (Williams 1997: p. 30). Indeed, this is one way to tell whether random data is stochastic or chaotic.

One of the interesting approaches in Chaos Theory is to take any arbitrary data sequence, and try to detect what algorithm might cause it, so that we can tell whether it is in fact a chaotic system or rather a stochastic system. The purpose of such an exercise, of course, is prediction, which is only possible by detecting whether in fact an apparently random system is really orderly (but chaotic). A simple example of this detection procedure is outlined below (autocorrelation; that is, seeing whether a data set correlates with itself).

Instead of being a straight-line graph, the outcome of a complex or chaotic system tends towards a pattern known as an ‘attractor’, which, despite appearing random, will have a common underlying structure as seen in the fractals above (Gleick 1988: p. 152). A fractal or attractor is thus a graphical representation of a complex system’s algorithm.\textsuperscript{13} An attractor is, mathematically speaking, an area on a plot or graph of a chaotic function that the data tends to appear within; as if the data were ‘attracted’ to that point. And particularly, when the data does not cover the entire phase space or plot area of a graph of a chaotic function, that attractor is called a ‘strange attractor’ (Gershenson 2004: p. 9). As an example of a strange attractor, consider the Lorenz attractor, Figure 3.

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{mandelbrot_set.png} \hspace{1cm} \includegraphics[width=0.4\textwidth]{mandelbrot_set_close_up.png}
\caption{The Mandelbrot Set (source: my image) \hspace{1cm} The Mandelbrot Set, close up (source: my image)}
\end{figure}

\textsuperscript{10} I have spent a bit of time on this detail so that the reader understands literature cited later.

\textsuperscript{11} What Mandlebrot called ‘self-affine’.

\textsuperscript{12} Technically, a linear equation produces a straight line graph, whereas a nonlinear equation does not.

\textsuperscript{13} ‘The term fractal is also used to describe a continuous function which is not differentiable at a dense set of points’ (Prof. A. Knopfmacher, personal communication). Also see Gershenson (2004: p. 2): an attractor is a self-similar fractal representing the changes in a chaotic system, plotted in phase space.
3. How does chaos apply to philosophy?

3.1 Chaos Theory and philosophy

There are at least two points at which Chaos Theory directly impacts philosophy: Firstly, consider our understanding of predictability and explanation in epistemology. One can no longer assume that there are straightforward regular relationships between causes and effects. We will discuss this in a bit more detail below when we look at Van Inwagen.

Secondly, though much attention has been given to quantum mechanics, and particularly, its possible influence on our decision-making abilities (e.g. the work of Kane 1996, 2002), it is unclear whether quantum states are relevant at the neurological level.\footnote{Although there is some evidence for this in some recent experiments; see e.g. Purdy et al. 2012 and Conte 2009.} If quantum states were pertinent above the atomic level, it seems that we might well just flail about randomly without any pattern of behaviour at all. It takes statistically significant tendencies or biases in near-random data to produce a particular shift or tendency in a behaviour; whether we are talking about crowds, flocks, or individuals. These biases or significant tendencies do appear in chaotic data, but do not appear in true stochastic data. Hence, perhaps Chaos offers us some hope of understanding human volition.

3.2 Is the brain chaotic?

The brain is a complex system. Complex systems produce behaviours or results that are ‘greater than the sum of their parts’ (Gazzaniga 2012: p. 71), a phenomenon known as ‘emergence’. Traffic is the easiest example of this. You cannot understand what traffic is by looking at a car’s parts, or a car. Traffic is a function of how the car interacts with other cars, traffic laws, weather conditions,

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{lorenz_attractor.png}
\caption{The Lorenz Attractor; produced by three equations graphed in three dimensions, it is generated by Lorenz’s equations that model the weather. Image courtesy of Wikimedia Commons.}
\end{figure}
road conditions, other drivers, etc., all of which are slightly different every day of the week, and at any different time of day, and subsequently produce approximately unpredictable results. Like the weather, or the structure of a termite colony mound, traffic is not readily predictable, even if it has a general ‘look’ or ‘character’. The same applies to the brain (Gazzaniga 2012: p. 71). Looking at the neurons will not tell you what consciousness is like (cf. Nagel 1974: op. cit.), or what decision would be taken, because it may well be the case that choice and/or consciousness are emergent properties or features of what might turn out to be a chaotic system: the brain.

Have you ever experienced that eerie feeling of a thought popping into your head as if from nowhere, with no clue as to why you had that particular idea at that particular time?... In reality, your brain operates on the edge of Chaos. Though much of the time it runs in an orderly and stable way, every now and again it suddenly and unpredictably lurches into a blizzard of noise...The quintessential example of self-organised criticality is a growing sand pile. As grains build up, the pile grows in a predictable way until, suddenly and without warning, it hits a critical point and collapses. These ‘sand avalanches’ occur spontaneously and are almost impossible to predict, so the system is said to be both critical and self-organising. Earthquakes, avalanches and wildfires are also thought to behave like this, with periods of stability followed by catastrophic periods of instability that rearrange the system into a new, temporarily stable state (Robson 2009: p. 1).15

If this is true of the human mind, we could not exactly predict what a person will do, but we could say that they have a certain stable character.

3.3 Implications for Philosophy of Mind

Now, as we know, Van Inwagen (defending incompatibilism,16 himself a libertarian17), says in his ‘Consequence Argument’ that if determinism is true, where \( P \) is a future action choice, no-one would have a choice that \( P \) if \( P \) is caused by antecedent states \( P_0 \) and the laws of the universe \( L \) (Van Inwagen 1989: p. 405; 2000: p. 2). However, given Chaos Theory, it seems very unlikely that the premises and mode of necessity in Van Inwagen’s original argument could be correct, since Chaos Theory precludes complex systems such as \( P_0 \) and \( L \) from necessitating just one particular subsequent state, such as \( P \). And Davidson argued that ‘[t]here are no strict deterministic laws on the basis of which mental events can be predicted and explained.’18 But if Chaos is the correct explanation for persons’ behaviours, it seems that choices and other mental states might be at least somewhat predictable in terms of some chaotic equation: Davidson was wrong (Neuringer and Voss 1993: pp. 113–119). And so is Van Inwagen. \( L \) and \( P_0 \) might yield any range of chaotic deterministic consequents \( \{P_1, P_2, P_3\}... \).19 Now, according to Gazzaniga (2012), and Neuringer and Voss (1993), and subsequent reviews of Neuringer and Voss’s work; the brain is chaotic in a sense (Neuringer and Voss 1993: p. 119). Let us consider the evidence.

3.4 Neuringer and Voss

Neuringer and Voss ran three experiments with the same human subjects. These subjects were asked to choose random numbers to predict what numbers a computer would choose. The computer was outputting values from the Logistic Difference Equation (a variant on the Logistic Equation).20 The subjects’ choices of numbers fell on a parabolic curve shown at Figure 4a, with few outliers. This shows that the choices were self-similar and algorithmic, even though the experimental subjects believed their choices to be random. Neuringer and Voss (1993: pp. 113–119) concluded that human choice is chaotic, and specifically follows the Logistic Difference Equation.

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15 See also Beggs and Plenz (2003, 2004). It is my view that this ‘avalanche’ model best describes recall or associative memory, or perhaps the trigger moment when an agent chooses to act (Libet’s W).
16 The view that freedom of will is incompatible with determinism.
17 Someone who holds that free-will exists, and that determinism is false.
18 Davidson. http://plato.stanford.edu/entries/anomalous-monism/ -- [Accessed 9 April 2013]; also in Davidson (1970: p. 249).
19 I will discuss this further in a follow-up paper.
20 \( x_n = r x_{n-1}(1-x_{n-1}) \) — this equation differs in that it uses the iteration prior rather than forward \( (x_n) \) instead of \( (x_{n+1}) \).
This demonstration is very interesting because it seems to be consistent with the idea that unpredictable human behavior nevertheless could be rule-governed and even strictly deterministic. (a) Subjects could learn to approximate the output of the logistic function when given the input, (b) subjects’ outputs were about as entropic as the outputs of the logistic, and (c) subjects’ outputs when iterating the learned function (i.e. using their own outputs on trial n-1 as the input for the output on trial n without feedback) diverged rapidly from those of the (computer-run) logistic they had been approximating (with feedback), thus displaying sensitive dependence on initial conditions, a defining feature of Chaos (Ward and West 1994: p. 232).21

### 3.4.1 Autocorrelation in Neuringer and Voss’ experiment

By performing what is known as an autocorrelation, we can find correlations in a signal or data set with itself, and therefore detect if it is stochastic, algorithmic or chaotic. A signal (series of numbers) which is self-similar is likely a chaotic signal. If a signal looks random and does not autocorrelate, it is stochastic. A graph can be produced for any signal that has an ordered structure. In the case of the Logistic Equation, a parabola is produced when performing autocorrelation (Figure 4a).

If we plot the Logistic Equation in two and then three dimensions, comparing various values of $x$, and following Neuringer and Voss (1993), using a constant value of $r=4$, we obtain the graphs shown at Figure 4. The purpose of this exercise is to see how the Logistic Equation changes over time relative to itself, by comparing $x$ to near values ($x_{n+1}$ and $x_{n+2}$) of itself. This is called autocorrelation. ‘Lag-1 Autocorrelation’ is a specific case where one compares the current ($x_n$) value to the previous most recent value (one step behind, or lagged, $x_{n-1}$).

Figure 4b is a 3D plot with the Y-axis using a Lag-2 Autocorrelation and the Z-axis using a Lag-1 autocorrelation. Strictly speaking, of course, these graphs show one ($x_{n+1}$) (Figure 4a) and two ($x_{n+2}$) (Figure 4b) steps ahead rather than behind, but mathematically it demonstrates the same correlation. The plot is of $x_n$ (x-axis) against $x_{n+1}$ (y-axis) and $x_{n+2}$ (z-axis).

To demonstrate how these graphs work, let us take the graph Figure 4a as an example, using point 0.2 on the x-axis ($x_n$) against the y-axis ($x_{n+1}$).

$$x_{n+1} = rx_n(1-x_n)$$

$$= 4 \times 0.2 \times (1 - 0.2) \quad [r \text{ given as 4}]$$

$$= 0.8 \times (0.8)$$

$$= 0.64$$

which we see corresponds to Figure 4a.

Consider, now, the graph of the Logistic Equation of $r$ against $x_n$, called the ‘Bifurcation Diagram’ (Figure 5). It has some predictable portions (up to about $r=3.6$), but during the period $[r=3.6; r=4]$, it becomes chaotic, with only small islands of stability (white) visible at certain key points like $r=3.84$. (This is why Neuringer and Voss used the coefficient $r=4$ in their computer program; to ensure maximum entropy). From these results, we can see that Neuringer and Voss (1993)’s findings seem to show that their experimental subjects produced output that followed the Logistic Equation, and, therefore, that humans are chaotic.

Fractal structure offers the kind of infinitely self-referential quality that seems so central to the mind’s ability to bloom with ideas, decisions, emotions (Gleick 1988: pp. 299).

### 4. Objections

#### 4.1 Davidson

One objection, to my mind, which has some force, is that if a person’s behaviours are chaotic and unpredictable, it makes no sense to reject Davidson’s conjecture that there is no law-like way to capture human behaviour. However, as we have already mentioned, chaos mathematics aims to make predictions from apparently random data. So, if a person’s behaviour seems random, it may

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21 Ward and West (1994) are referring to feeding back the results (n-1) into the current algorithm’s value (n), i.e. a feedback loop.
Figure 4a and Figure 4b: 2D and 3D plots of Logistic Equation, Image courtesy of Wikimedia Commons (modified by this author).

Figure 5: Bifurcation diagram courtesy of Wikimedia Commons from the ‘Logistic Equation’: $x_{n+1} = rx_n(1-x_n)$. If one examines this diagram, it is apparent that after a few initially predictable splits, it rapidly becomes ‘random’. Note that other chaotic equations can also produce bifurcation diagrams.
indicate that it is chaotic (not stochastic), and if this is so, then there are potentially one or more equations which could describe and predict a person’s behaviour in a law-like way. In particular, if Neuringer and Voss (1993) are correct, persons’ behaviour should be graphable along a parabola. The big unanswered question is how to quantify and plot peoples’ choices. If I choose tea instead of coffee, or to write a philosophy paper instead of browsing Facebook, is that 1.1 or 1.2, or 3.5, or 2.7?

The applicability of such models may force separation of the issue of predictability from that of determinism altogether. Moreover, when such models apply, the idea of making precise, detailed quantitative predictions about the state of a system may have to be given up, but not the goal of predictability altogether. In fact, knowing that a chaotic model applies to some human behaviour would enable precise qualitative predictions (relative to the values of the state variables) about large-scale properties (attractors) to be made, particularly concerning under what conditions behaviour would change in form (move between attractors, become chaotic, etc.) and when the precise quantitative predictability of behaviours must give way to probabilistic estimates (Ward and West 1994: p. 235).

Now, it may seem obvious or uncontroversial to say that in some common-sense way, putting neuroscience aside, that what we do is generally in line with our characters, and that our behaviours are regular in some way. So in that sense, it does not seem to be quite so novel to suggest that human behaviour might be regular or law-like in some way. And thus, the argument might go, the claim that this paper makes is uninteresting.

However, this is not all that there is to it. The claim being made is stronger. The claim goes as follows. If Chaos is algorithmic, and if our brains operate chaotically, and if our brains cause our behaviours, it follows that our behaviours would be algorithmic (albeit chaotic). Human behaviour, then, would not be linear. Davidson is right in that sense; we will not derive any linear equation or algorithm that predicts our behaviour. However, if our brains (and hence behaviours) are chaotic, then it suggests that there is indeed an algorithm, or set of algorithms, which might ultimately characterise our behaviour in a reasonably predictable way (albeit, not in a linear and regular way) (Ward and West 1994: p. 235).

These two claims—that we have regular characters, and that our minds chaotically produce behaviours—are not entirely distinct. The first claim, that we have regular behaviours, stands as evidence for the second—that our actions are chaotic, rather than stochastic. For recall: the hallmark of Chaos is not that it is completely random, but that it creates random-looking patterns. So, human behaviour, whilst it seems spontaneous or random, nonetheless follows a general pattern, or, perhaps, a strange attractor. I will discuss this in more detail in (4.4) below.

4.2 Criticisms of Neuringer and Voss
Some researchers have argued that Neuringer and Voss (1993) have failed to show that humans are chaotic:
(a) Neuringer and Voss’s results were derived from a very small pool of experimental subjects.
(b) Two of the subjects, by Neuringer and Voss’s own admission, never seemed to learn to produce chaotic sequences.
(c) Choosing numbers are not really canonical examples of free action; we tend to think of our choices as being morally pertinent, in cases where free-will actually matters. And in such cases, it certainly seems that there would be many factors impinging on a moral agent, such that the single-variable Logistic Equation is hopelessly inadequate to cope with predicting which choice an agent would make. This objection, I think, is the most pertinent one.

Metzger, the first author to respond to Neuringer and Voss in the same journal, argues that their experiment amounted to asking subjects to memorise the coordinates of a parabola’s line. Not only is this relatively easy to do, but the task becomes easier with more coordinates and more practice. In the case of the original chaotic equation, the subjects were merely memorising which number is typically paired with which other number, i.e. which value of \( x \) was paired with which value of \( x_{n-1} \). This, Metzger argues, is tantamount to just memorising an \( x \) and \( y \) coordinate series of a two-dimensional graph. And indeed, the lag-1 autocorrelation graph of the Logistic
Difference Equation is a parabola, which has $x_{n-1}$ (or $x_{n+1}$, in the case of the Logistic Equation) as the $y$-coordinate. So, all that Neuringer and Voss have achieved, Metzger argues, is to demonstrate that subjects can rote-learn the coordinates of a parabola’s graph line. The subjects were merely learning what ‘style’ of number to expect in the sequence, and were merely replicating the style of the sequence (Metzger 1994: op. cit.). Ward and West (1994: p. 233) raise the same objection, after achieving the same results in a replication of Neuringer and Voss (1993)’s experiment (Ward and West 1994: pp. 233–235). So, by analogy, one might argue, if subjects were given the sequence 2, 4, 6, 8, and correctly guessed the next five numbers, we would conclude that humans were governed by a linear deterministic equation, because they produce linear deterministic sequences.

(d) The number of significant digits (decimal points), furthermore, was insufficiently precise in the experiment to generate a truly chaotic sequence.

It seems to me that the problem with Neuringer and Voss is not their idea (or indeed their conclusion, which seems as if it must be correct, given what we know about chaotic systems), but rather, that it is their experimental methodology and setup that was problematic. Perhaps a more plausible demonstration would be to ask subjects to randomly choose numbers themselves without any prompting from a computer, and then see if the numbers chosen follow a chaotic equation’s predictions, or whether the generated series auto-correlates.

Another criticism of Neuringer and Voss might go as follows: their evidence does not seem strong enough to make the claim that there is a deterministic reason for whatever comes to mind, which said reason may be merely too complex to allow prediction. At most, their evidence shows that our behaviours have patterns or are algorithmic, not that they are deterministic.

There are two responses to this criticism. The first response, is that the criticism seems to misunderstand the relationship between determinism, Chaos, and algorithms. An algorithm is deterministic, in the sense that coupled with its structure, its inputs guarantee its outputs. Chaos equations are algorithms; and, likewise, their inputs fully determine their outputs. The necessary and sufficient causes of the outputs are just the inputs and the algorithm’s structure. So, for example, if we consider an ideal situation with zero air friction, and zero coefficient of friction, the movement of a trolley on a smooth surface can be given by a linear equation $v_f = v_0 + v_0t + \frac{1}{2}at^2$. There is nothing else needed to obtain a deterministic output for $v_f$ other than the initial velocity $v_0$, the time travelled $t$, and the acceleration $a$ imparted to the trolley. The same applies to non-linear Chaos equations. The output of the formula $x_{n+1} = rx_n(1-x_n)$ is still completely predetermined by the input. The reason the outputs appear random is that the algorithm ensures that minor changes in the inputs cause large and prima facie unpredictable changes in the outputs.

A second response to the criticism would go as follows: Yes, Neuringer and Voss (1993)’s experiments do not show something, but it is not what the critic thinks. Instead of failing to show determinism, their experiments fail to adequately shown that our output behaviours are specifically chaotic.

### 4.3 Avalanches in the brain

More modern authors such as Beggs and Plenz disagree with the idea that the brain is fully chaotic, and hence, again, Neuringer and Voss may be overstating their case, as mentioned above.

The idea that the brain might be fundamentally disordered in some way first emerged in the late 1980s, when physicists working on Chaos theory...suggested it might help explain how the brain works.

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22 Thanks go to my reviewer for suggesting this argument.
... Researchers built elaborate computational models to test the idea, but unfortunately they did not behave like real brains...

In the 1990s, it emerged that the brain generates random noise, and hence cannot be described by deterministic Chaos. When neuroscientists incorporated this randomness into their models, they found that it created systems on the border between order and disorder—self-organisation of criticality.

More recently, experiments have confirmed that these models accurately describe what real brain tissue does. They build on the observation that when a single neuron fires, it can trigger its neighbours to fire too, causing a cascade or avalanche of activity that can propagate across small networks of brain cells. This results in alternating periods of quiescence and activity—remarkably like the build-up and collapse of a sand pile (Robson 2009: p. 2).

Beggs and Plenz found that firing neurons typically only triggered one other neuron’s firing. This, Beggs argues, is sufficient to cause a cascade, but not complete Chaos. If the brain were actually chaotic, we would see behaviours more like epileptic fits (in Robson 2009: p. 2).

But is this an objection or a qualification? A model of Chaos-induced ‘neural avalanches’ that explain the spurts of activity that we experience, such as suddenly remembering a song, or suddenly making a move (Libet’s W)23, not only explains why we behave as we do, but it remains chaotic.24

The proposal here is that Libet’s measurement of the intention to move, or the inclination to move (his ‘W’), which loosely corresponds to what we might call the ‘will’, is in fact the output of such an ‘avalanche’. This also explains the ‘bubbling up’ or ‘popping up’ that Dennett (1984: pp. 78–80) and Robson (op. cit.) cite.

4.4 Why should we accept the notion that human actions are chaotic rather than stochastic?
The evidence from Neuringer and Voss does not seem strong enough, given the above criticisms, to warrant the claim that our actions are chaotic, rather than stochastic.

Response: It seems that even if Neuringer and Voss’s work is insufficient, it nonetheless makes sense to accept the view that our behaviours are chaotic rather than stochastic-random, for the following reasons:

(i) People show spontaneity, which suggests some sort of randomness, which we could get from stochastic randomness, to be sure, however, people also do show regular, overarching character traits, which bespeaks determinism of some sort. True stochastic randomness would probably yield spasmodic twitches rather than goal-oriented or patterned movements.

(ii) We need to have reasons for actions, if our actions are not to be mere motions (think of a rock rolling down a hill: it has no reason to roll other than that someone may have pushed it; hence, the rock rolling down the hill is a motion, not an action). But if reasons can act as causes, e.g. the reasons we may have to perform X, say, to push a rock down a hill, rather than Y (to refrain), it seems that those reasons influence us in some sort of deterministic way. Since if our reasons for X are in some sense stronger than our reasons for Y, it is highly probable (but not inevitable) that we will choose X rather than Y. If this is true, then in order for something to qualify as an action or a choice, our actions must be determined.

(iii) This line of thought proposes, then, that why we seem spontaneous, but nonetheless why we seem to choose things appropriate for our characters, shows that there is some sort of determinism, e.g. by reasons, or neural events, but that there is nonetheless some randomness. And the only model which allows causal regularities to be coupled with apparent randomness, is Chaos.

23 Libet 1985, q.v. This paper will not address the matter of Libet’s experiments further.
24 Further support for human chaotic mentation comes from Ward and West (1998) where they also managed to replicate chaotic behaviour that approximated a Chaos equation.
5. Conclusion

Now, human behaviour does seem arbitrary and somewhat random; indeed, it is our apparent ability to arbitrarily choose this or that which leads us to believe that our choices are, in a sense ‘free’. The beauty of postulating that humans are chaotic, then, is that we can accommodate both points of view, that is, that humans are both spontaneous or random, and yet deterministic. Given that chaos mathematics aims to predict the behaviours of apparently unpredictable complex systems, it seems as if chaos mathematics is the right sort of explanation and predictive model for our behaviours, and that there is some hope of explaining our choices in terms of a deterministic model. This echoes the nature of Chaos: strange attractors, which look random close-up, are nonetheless algorithmic and deterministic.

This model can also preserve free-will and compatibilism. As Frankfurt argued, as long as we can get what we want, and we get what we want because we wanted it, we are free; our being antecedently determined does not speak against our being able to choose. Furthermore, the model can account for the apparently random manifestation of Libet’s W; W, on this model, is some sort of neural avalanche that eventually triggers a spontaneous action, e.g. moving the hand. Empirical research will explain this for us.

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25 Again, for considerations of space, we will not debate the merits of compatibilism (soft determinism) here.
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