PSR B1133+16: radio emission height and plasma composition

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ABSTRACT

Recent operation of LOFAR by Hassall et al has produced severe constraints on the size and altitude of the 40 MHz emission region in this pulsar. It is shown that these limits, given a limited number of unexceptionable assumptions, demonstrate that an electron-positron plasma cannot be the source of the emission. A physically-acceptable plasma source composed of protons and ions arises naturally in pulsars having positive corotational polar-cap charge density. Acceptance of this would greatly clarify the classification of pulsar types within the whole.

Key words: instabilities - plasma - stars: neutron - pulsars: general

1 INTRODUCTION

PSR B1133+16 is a small-dispersion-measure pulsar whose radio emission exhibits both nulls and sub-pulse drift, but is otherwise unremarkable. Its radio brightness has enabled Hassall et al (2012), using LOFAR observation in the interval 40 - 190 MHz, to give extremely small upper limits for $H_{\alpha}$ emission. The maximum distance for the 40 MHz emission is the size of the emitting region, $r_{\text{max}} = 1.1 \times 10^7$ cm from the centre of the neutron star, equivalent to $\sim 10^{-3} R_{\text{LC}}$, where $R_{\text{LC}}$ is the light-cylinder radius. The section of the open magnetosphere defined by these limits can then be described as a narrow black box, subject to a zero-potential condition on the boundary with the closed region, within which the plasma mode(s) responsible for the emission couples strongly with the radiation field, that is, modes which can propagate in free space.

Unfortunately, and for technical reasons, the authors are unable to quote confidence levels, but as they note, these results are of some significance for the problem of understanding the physics of the emission process. The canonical model for this is a dense plasma of secondary electron-positron pairs formed by the conversion of curvature radiation (CR) photons or by inverse Compton scattering (ICS) from the primary accelerated electrons. An unstable, possibly quasi-longitudinal, plasma O-mode grows in amplitude from the primary accelerated electrons. An unstable plasma O-mode grows in amplitude from the primary accelerated electrons. An unstable plasma O-mode grows in amplitude from the primary accelerated electrons.

2 THE PREDICTED BLACK-BOX SPECTRUM

The assumptions about the pair plasma in B1133+16 to which we refer are to some degree inter-dependent, but are listed as follows.

(i) There is a frame of reference, with Lorentz factor $\gamma_c$ relative to the rotating neutron-star frame, in which the bulk electron-positron plasma has zero momentum.

(ii) The only natural frequencies present in the magnetized plasma within the black box are the rest-frame plasma frequency $\omega_p$, the critical frequency for curvature radiation $\omega_c$, and the cyclotron frequency $\omega_B$. At $r_{\text{max}}$, the transition rate from the electron first Landau excited state to the ground is $4\omega_B^2 e^2 / 3mc^3 \sim 10^{10} \text{ s}^{-1}$ and $\omega_B$ is many orders of magnitude greater than any angular frequency in the radio spectrum. Therefore, the plasma is very near the high-field limit and does not constitute a complex system likely to have unforeseen properties.

(iii) Coherent curvature radiation can be generated within the black box and has both O and E-mode amplitudes. The critical frequency, above which the radiation spectrum cuts off exponentially, is $\omega_c = 3c\gamma_c^2 / 2p$, where $p$ is the flux-line radius of curvature. Whilst propagation of the O-mode is subject to some limitations, the E-mode propa-
gates freely in the high-field limit. The conditions necessary for significant energy transfer to the E-mode were described in the original paper of Ruderman & Sutherland (1975). Suitable \( \gamma_c \) and flux-line radius of curvature may exist in the black box to give values of \( \omega_c \) within the main part of the B1133+16 radio spectrum, but a suitable charge density structure is also needed to produce coherent emission. In the plasma rest-frame, this can only be associated with \( \omega_p^c \). The radiation wavelength in the neutron-star or observer frame is \( 2\pi c/\omega_c \) and there can be no effective coherence if, within this interval, there are a large number of charge density fluctuations of alternating sign. Thus in the limit \( \omega_c \ll \gamma_c \omega_p^c \) there is no significant energy transfer to either O or E-modes.

(iv) The number density of primary accelerated electrons is little different from the Goldreich-Julian density and on average, each primary electron produces \( \lambda \) pairs. This is also not inconsistent with recent studies of the force-free magnetosphere using the techniques of numerical plasma kinetics (see Chen & Beloborodov 2013, Timokhin & Arons 2013) in which it is assumed that the open magnetosphere current density is determined, principally, by the force-free solution for the whole magnetosphere.

(v) Outside the black box, the radiative modes do not interact strongly with the magnetosphere except that there is the possibility of cyclotron resonant absorption in the open magnetosphere region near the light cylinder. Inside the black box, the unstable plasma mode gains amplitude until non-linearity, and possibly turbulence, is reached and then couples strongly with a distribution of radiative-mode angular frequencies.

(vi) The physical assumptions on which the treatment of aberration and retardation by Hassall et al are based are valid. Thus the energy transfer to radiative modes within the black box and in the neutron-star frame of reference is to modes with wave-vector locally parallel with the magnetic flux density \( \mathbf{B} \). Cancellation of an aberration and retardation time difference, otherwise detectable in principle, by the effect of rotation-produced toroidal sweep-back of magnetic flux is not a realistic possibility for B1133+16. The estimate of its size made by Shitov (1983), for a vacuum magnetosphere, is dependent on \( (r/R_{LC})^3 \) and would be negligible at the radii \( r \) found by Hassall et al. Also, these radii lie well inside the spherical inner boundary assumed in current numerical studies of the force-free magnetosphere. Although field geometry in the real magnetosphere is likely to be very different from the vacuum case at \( r \sim 10^{-1} R_{LC} \) or larger, there is no reason to suppose this could be true at the very small radii considered here.

Following these assumptions, the angular frequency of the mode must be close to the local plasma frequency which is, in the plasma rest-frame,

\[
\omega_p^c = \frac{8\pi n_G e^2 \lambda}{m \gamma_c^2} \left( \frac{\rho_0}{n_G} \right)^{1/2} \tag{1}
\]

where \( n_G, \lambda \) is the Goldreich-Julian density number and \( m \) is the electron mass. Its Lorentz transformation to either the neutron-star or observer frame gives a typical radiation frequency,

\[
\nu_{obs} = \gamma_c \omega_p^c / \pi \approx 0.26 \gamma_c^{1/2} \lambda^{1/2} \text{ GHz}, \tag{2}
\]

for B1133+16 when evaluated at \( r_{max} \), for the value of the angle between \( \mathbf{B} \) and the spin \( \mathbf{\Omega} \) cited by Hassall et al. and for a polar surface magnetic flux density of \( 2.13 \times 10^{13} \) G.

The spectrum of B1133+16 is unremarkable (see, for example, Sieber 2002). The ATNF pulsar catalogue (Manchester et al 2005) fluxes at 400 and 1400 MHz give a spectral index \( \alpha = -1.65 \). A low-frequency turnover at \( \nu_{max} \approx 60 \) MHz has been observed by Despande & Radhakrishnan (1992) and above 1400 MHz, the spectrum steepens to a more negative \( \alpha \). The pulsar is observable up to 8.35 GHz (Honmappa et al 2012). Assuming a cut-off at \( \nu < \nu_{max} \) and a constant \( \alpha \) at \( \nu > \nu_{max} \), the fraction of the total energy flux at frequencies above \( \nu \) is \( (\nu/\nu_{max})^{1+\alpha} \). Therefore, the frequency below which a fraction \( f \) of the total radiofrequency spectral energy is contained is,

\[
\nu = \nu_{max} (1 - f)^{1/(1+\alpha)}. \tag{3}
\]

Thus half the energy flux is contained below \( 2.9\nu_{max} \). Referring back to equation (2), typical cited values of the free parameters it contains are \( \gamma_c = 50 \) and \( \lambda = 10^7 \), but it must be emphasized that these are either extremely model-dependent or are merely estimates that are deemed plausible. However, for much smaller values of \( \lambda \), the plasma density becomes too low to allow the relative bulk velocity of the electrons and positrons to adjust to that local charge density which is needed to give only a very small electric field component parallel with \( \mathbf{B} \), and hence maintain only small values of \( \gamma_c \). Therefore, values \( \lambda \gamma_c \approx 1 \) are not a realistic possibility and comparison with equation (2) shows that the bulk of the spectral energy of B1133+16 is at frequencies far too low to be consistent with that expected for an electron-positron plasma. Coherent curvature radiation is not a plausible explanation for the reason stated in (iii) above.

Specifically, there seems to be no obvious mechanism for the large-scale displacement of spectral energy to frequencies well below those given by equation (2). In fact, we anticipate that the black box has the opposite property. Nonlinearity is itself a source of harmonic generation. But more significantly, the transfer of energy to a distribution of degrees of freedom with higher wavenumbers is a general property of developing turbulence (see, for example, Batchelor 1967 for the case of incompressible fluids and Weatherall 1997 for the pulsar plasma). It occurs through the formation of a region of self-similarity in which the energy density as a function of wavenumber \( k \) has a power-law form, \( \propto k^\beta \). It is then natural to associate \( \nu_{max} \) with the frequencies predicted by equation (2), and the power-law high-frequency tail of the radio spectrum with a region of self-similarity, although the closeness of \( \alpha \) for B1133+16 to the Kolmogorov index \( \beta = -1.67 \) for fluid turbulence must be accidental. More specifically to the pulsar magnetosphere, Weatherall (1997) has demonstrated directly that energy transfer to high wavenumbers does occur.

The source of the radio emission proposed here is a plasma of protons and ions (mass \( M \), mass number \( A \), charge \( Z \)) which is relativistic, but with quite low Lorentz factors in the neutron-star frame. This requires the pulsar to have rotational spin such that \( \mathbf{\Omega} \cdot \mathbf{B} < 0 \). The details of how this plasma is formed have been described elsewhere (Jones 2012, 2013). Here, it is sufficient to note that equation (2)
The condition for CR pair formation is $X > X$. The unstable mode eventually becomes too small for observable positron plasma decreases until there is a bifurcation in creation for either sign of acceleration potentials which can certainly support CR pair formation. Secondary pair plasma could be formed only by ICS photons or by higher-multipole enhancement of flux-line curvature. It is always possible that, in a specific pulsar, there could be some curvature enhancement. However, the condition is indicative, not rigorous, because it can be expressed in terms of a parameter $X = \frac{Z}{A}$, where $Z$ is the polar magnetic flux density in units of $10^{12} G$.

$$\nu_{\text{obs}} = 4.2 \left(\frac{\gamma_{\text{A,Z}}}{3}\right)^{1/2} \text{MHz},$$

constructed from the ion rest-frame plasma frequency at $r_{\text{max}}$, in which $\gamma_{\text{A,Z}}$ is the Lorentz factor of the ion in the neutron-star frame. The plasma is essentially cold: there is a substantial difference between the proton and ion Lorentz factors and no doubt that the relativistic Penrose condition (Buschauer & Benford 1977) necessary for amplitude gain is satisfied. The anticipated Lorentz factor is of the order of $\gamma_{\text{A,Z}} \sim 10$ (Jones 2012) which gives a frequency of $\sim 10$ MHz, but the $r_{\text{max}}$ found by Hassall et al is an upper limit and the ion rest-frame plasma frequency is increased by a factor $(r_{\text{max}}/1)^{3/2}$ at smaller $r$, so that $\nu_{\text{max}} = 60$ MHz for B1133+16 is easily attainable. The power-law high-frequency tail at $\nu > \nu_{\text{max}}$ can be identified with the region of self-similarity described in the previous paragraph so that the GHz radiation is easily understood. On the basis that turbulent energy transfer to higher wavenumbers is the source of the high-frequency tail in the spectrum, and given that the unstable mode itself is quasi-longitudinal, the radiation emission is unlikely to be consistent with any simple form of radius-to-frequency mapping.

3 CONCLUSIONS

Rates of self-sustaining pair creation by the magnetic conversion of curvature radiation or of ICS photons have been published by Hibschman & Arons (2001) and Harding & Muslimov (2002, 2011); here we refer to Fig. 1 of Harding & Muslimov (2002). For dipole field geometry, the constraints on rotation period $P$ and polar magnetic flux density $B$ can be expressed in terms of a parameter $X = B12 P^{-1.6}$, where $B12$ is here the polar magnetic flux density in units of $10^{12} G$. The condition for CR pair formation is $X > X_c \approx 6.5$, and is far from being satisfied by B1133+16 for which $X = 1.6$. A secondary pair plasma could be formed only by ICS photons or by higher-multipole enhancement of flux-line curvature. However, the condition is indicative, not rigorous, because it is always possible that, in a specific pulsar, there could be some curvature enhancement.

It is unfortunate that the engagement with electron-positron plasma has been so exclusive in pulsar physics because there is no observational evidence that $X < X_c$ pulsars in general produce pairs. They are so old that there are no observable associated nebulae.

Pulsars with $X > X_c$ generate very large maximum acceleration potentials which can certainly support CR pair creation for either sign of $\mathbf{\Omega} \cdot \mathbf{B}$. Those LAT-detected pulsars found by Abdo et al (2010) conform with this class. The detected GeV $\gamma$-rays are emitted from the light-cylinder region, presumably from a substantial flux of electrons and positrons. Of the 46 pulsars listed in Table 1 of Abdo et al, only 7 have $X < X_c$ and without exception, these are millisecond pulsars.

As $X$ decreases with pulsar age toward $X_c$, the density of pair plasma decreases until there is a bifurcation in evolutionary path. In the $\mathbf{\Omega} \cdot \mathbf{B} > 0$ case, the growth rate of the unstable mode eventually becomes too small for observable radio emission. Those with $\mathbf{\Omega} \cdot \mathbf{B} < 0$ can continue to produce a proton-ion plasma and display phenomena such as nulls and subpulse drift (see Jones 2012, 2013).

The final paragraph of Jones (2013) suggested that the operation of LOFAR might well provide evidence for this latter class of pulsar through observations on the shape of low-frequency spectra. But this is not so. By direct plotting from the ATNF pulsar catalogue, it can be seen that the spectral index $\alpha$ formed from fluxes at 400 and 1400 MHz is almost uncorrelated with $X$. Also, for example, low-frequency emission, down to 10 MHz and with no observed turnover, is strong in the Crab pulsar (see Sieber 2002). In fact, it appears that measurements of $r_{\text{max}}$, where possible, in extensions to the work of Hassall et al will be decisive in determining plasma composition. Whilst the present paper is concerned specifically with B1133+16, the remaining three pulsars investigated by Hassall et al could have been discussed in the same way with broadly similar, though less strong, conclusions.

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