Modelling of micro vibration energy harvester considering size effect

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Abstract. Considering increase of stiffness caused by size effect, equivalent Young's modulus was introduced for further analysis. Experimental platform was established to test vibration characteristics. Dynamic equation for micro piezoelectric cantilever beam considering size effect was studied with finite element analysis and experiment. Results shows it is accurate. Based on that, dynamic model for micro vibration energy harvester was improved, a T-type micro vibration energy harvester was designed and fabricated. Resonant frequency, tip displacement and output voltage of the harvester were obtained. Comparing with macroscopic model for vibration harvester, improved one reduces errors by 13%, 35% and 22%.

1. Introduction
"Internet of Things"[1-2] includes a huge amount of sensors, traditional power supply can not meet its demands as it is nondurable, storage limited, environmental polluting[3]. Research of new power supply technology is important. Mechanical energy can be transformed to electrical one by piezoelectric energy harvester which is efficient, simple, durable, environmental friendly, and compatible with micro-nano manufacture[4]. Due to size effect, microstructure performance is inconsistent with macroscopic model[5-7]. Micro piezoelectric cantilever beam is a basic structure of micro vibration energy harvester[3], Dynamic model for micro-beam was verified, and a model for harvester was refined. Improved model for harvester reduces error dramatically.

2. Improved dynamic model for micro piezoelectric cantilever beam and verification
Not considering size effect, dynamic model of micro-beam can be expressed as:

\[ EIw^{(4)} + \rho A \ddot{w} = 0 \]  

(1)

In [8], intrinsic length \( l \) that reflects size effect was introduced. Dynamic model for micro-beam was improved by couple stresses theory:

\[ \left( EI + \mu Al^2 \right) w^{(4)} + \rho A \ddot{w} = 0 \quad (h/l = 3.5) \]  

(2)

Using mode superposition method, resonant frequency equation is obtained:

\[ \omega_r = \left( \beta_r \right)^{\frac{1}{2}} \left( \frac{EI + \mu Al^2}{\rho A l^4} \right) \quad (r = 1, 2, 3, ... \)  

(3)

2.1. Finite element analysis of micro piezoelectric cantilever beam
Considering formula (2), $\mu Al^2$ is main reason for increasing of stiffness under size effect\textsuperscript{[9-12]}. Equivalent Young’s modulus $E_i$ was introduced:

$$E_i = E + \frac{\mu Al^2}{I} = E \left[1 + \frac{6}{1+v} \left(\frac{L}{h}\right)^2\right]$$  \hspace{1cm} (4)

Micro-cantilever can be simplified to two-layer whose size are shown in table 1.

**Table 1.** Size of silicon beam and PZT-5H  

|            | Length(μm) | Width(μm) | Thickness(μm) |
|------------|------------|-----------|---------------|
| Si-Beam    | 3150       | 1000      | 13            |
| PZT-5H     | 3150       | 1000      | 1.6           |

Reference to parameters in \textsuperscript{[13]} and formula (4), equivalent Young's modulus is shown in Table 2.

**Table 2.** Equivalent Young's modulus of materials  

|            | Si         | PZT-5H    |
|------------|------------|-----------|
| Equivalent | 178.98     | 18.1733   |
| Young's    |            | 10.0504   |
| Modulus    |            | 15.8328   |
| (GPa)      |            | 4.1303    |
|            |            | 3.5796    |

Infinite element analysis for micro-beam was done. Figure 1 and figure 2 show that first-order vibration modes of different models are 1536Hz and 1803Hz, the difference is 267Hz.

**Figure 1.** Results with macroscopic model  

**Figure 2.** Results with improved model

2.2 Fabrication and experiment test of piezoelectric cantilever beams

Photolithography, sputtering, sol gel, dry etching process was used in a 4-inch SOI substrate to complete fabrication and packaging, the SEM is shown in figure 3 and figure 4.
Figure 3. Top view of SEM image

Use signal generator (YE1311), power amplifier (E8571A), vibration table (E-JZK-5), laser vibration meter (OFV505/5000), data acquisition card (INTEST Main Unit INJ9008U-I), acceleration sensor (CA-YD-1182) and other devices to build a vibration experimental platform for cantilever beam, as shown in Figure 5.

Figure 4. Thickness enlargement of SEM image

Figure 5. Experimental platform

Figure 6. Power spectrum-frequency curve

Set acceleration excitation to 6g (1g=9.8m/s²), frequency of acceleration was sequentially changed in a range of 1600Hz~2000Hz, tip displacement was recorded. Using power spectrum to deal with it, power spectrum and vibration frequency curve were generated, as shown in Figure 6. At 1818Hz, the power spectrum value reaches the maximum, that is the resonant frequency.

Table 3. Frequency comparison between experiment and theoretical analysis

| Resonant Frequency (Hz) | Error |
|-------------------------|-------|
| Experimental Test       | 1818  | 0%    |
| Improved Model          | 1803  | 0.8%  |
| Macroscopic Model       | 1536  | 15.5% |

3. Experimental analysis of micro vibration energy acquisition

Micro-beam resonant frequency is generally above 1000Hz, many scholars[14,15,16] try to reduce resonant frequency by adding a Si mass or a Ni one to the free end of cantilever beam, so that it can perform well in low-frequency vibration environment. In this paper, a Si mass was etched onto the T-type cantilever beam, and a "T-type" harvester was made. The "T-type" vibration energy harvester consists of five layers, which can be simplified to three layers that consists of PZT-5H, Si-beam and mass blocks. The structure is shown in Figure 7.
3.1. Micro vibration energy harvester dynamic analysis

Considering size effect and improved model of micro-beam, the dynamic model of the harvester for free vibration can be expressed as:

\[
EI \frac{\partial^4 w(x,t)}{\partial x^4} + \rho A \frac{\partial^2 w(x,t)}{\partial t^2} = 0
\]  

(5)

When \( \mu Al^2 \) was removed, formula (5) turns to be a macroscopic model. Charge \( Q \) produced by the harvester deformation gathers on the surfaces of PZT-5H to form a capacitor, output voltage of the micro harvester was obtained:

\[
V_r = \frac{Q}{C_s} = \frac{2k \pi w_0 d_31 E_p}{\varepsilon_r S_s} t_{pZT-5H} W_r(L, t)
\]  

(6)

3.2. Finite element analysis of harvester

Finite element analysis can reflect the actual motion state of harvester in a form of cloud map. According to the equivalent elastic modulus in Table 1 and relevant parameters in [13], finite element model of harvester was established. DOF of fixed end of harvester was set to zero and the acceleration was 1g. Output voltage and tip displacement reach maximum at the resonant frequency 300Hz, which were 0.17V and 10.7μm.

3.3. Micro vibration energy harvester performance test

Acceleration excitation was set to 1g, vibration table was in the range of 50Hz~500Hz by specific frequency. Figure 8 and figure 9 show the tip displacement frequency, the output voltage frequency of micro harvester which is obtained from macroscopic model, improved model and experiment. Main information and error compared with experiment are shown in table 4.
Comparing macroscopic model to experiment test, error of resonant frequency was large. The macroscopic model failed to describe dynamic performance of micro energy harvester. When improved model was compared to experiment test, error turned out to be relatively small.

| Table 4. Main information of resonance point and the contrast error |
|---|---|---|
| Resonant frequency | Macroscopic model | Improved model | Experiment test |
| Value (Hz) | 250 | 300 | 310 |
| error | 19.4% | 3.2% | 0% |
| Tip displacement | Value (μm) | 14.26 | 10.72 | 10.13 |
| error | 40.8% | 5.8% | 0% |
| Output voltage | Value (V) | 0.215 | 0.178 | 0.168 |
| error | 28.0% | 6.0% | 0% |

4. Conclusion
Mechanical properties of microstructure considering size effect was studied using micro piezoelectric cantilever beam. Intrinsic length of material was introduced, and dynamic model was improved by couple stresses theory. Mechanical properties of micro-beam were studied, the improved model could describe mechanical properties of microstructures more accurately. Based on that research, dynamic model of micro vibration energy harvester was refined. Output performance was analyzed with finite element and vibration test. Results show that the improved model of harvester can provide theoretical support for design of microstructure.

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