NEUTRINO-DRIVEN JETS AND RAPID-PROCESS NUCLEOSYNTHESIS

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ABSTRACT
We have studied whether the jet in a collapse-driven supernova can be a key process for the rapid-process ($r$-process) nucleosynthesis. We have examined the features of a steady, subsonic, and rigidly rotating jet in which the centrifugal force is balanced by the magnetic force. As for the models in which the magnetic field is weak and angular velocity is small, we found that the $r$-process does not occur because the final temperature is kept too high and the dynamical timescale becomes too long when the neutrino luminosities are set to be low. Even if the luminosities of the neutrinos are set to be low, which results in the low final temperature, we found that the models do not give a required condition to produce the $r$-process matter. Furthermore, the amount of the mass outflow seems to be too little to explain the solar system abundance ratio in such low-luminosity models. As for the models in which the magnetic field is strong and angular velocity is large, we found that the entropy per baryon becomes too small and the dynamical timescale becomes too long. This tendency is, of course, a bad one for the production of the $r$-process nuclei. As a conclusion, we have to say that it is difficult to cause a successful $r$-process nucleosynthesis in the jet models in this study.

Subject headings: nuclear reactions, nucleosynthesis, abundances — stars: magnetic fields — stars: rotation — supernovae: general

1. INTRODUCTION
It is one of the most important astrophysical problems that the sites where the rapid-process ($r$-process) nucleosynthesis occurs are not still known exactly. There are at least three reasons that make the study of $r$-process nucleosynthesis important. One of them is a very pure scientific interest. The mass numbers of the products of $r$-process nucleosynthesis are very high ($A = 80-250$), which means that the most massive nuclei in the universe are synthesized through the $r$-process. You can guess easily that the situation in which the $r$-process nuclei are synthesized is a very peculiar one in the universe. We want to know where, when, and how the $r$-process nuclei are formed. The second reason is that some $r$-process nuclei can be used as chronometers. For example, the half-lives of $^{232}$Th and $^{238}$U are $1.405 \times 10^{10}$ and $4.468 \times 10^{10}$ yr, respectively. Thus, if we can predict the mass spectrum of the products of $r$-process nucleosynthesis precisely, we can estimate the ages of metal-poor objects that contain the $r$-process nuclei by observing its abundance ratio. The third reason is that some $r$-process nuclei can be used as tools of the study on the chemical evolution in our Galaxy (e.g., Ishimaru & Waniyo 1999), which has a potential to reveal the history of the evolution of our Galaxy itself. Because of the reasons mentioned above, the study on the $r$-process nucleosynthesis is very important.

The conditions in which the $r$-process nucleosynthesis occurs successfully are (e.g., Hoffman, Woosley, & Qian 1997) (1) neutron-rich ($n_n \geq 10^{20}$ cm$^{-3}$), (2) high entropy per baryon, (3) small dynamical timescale, and (4) small $Y_e$. This is because $r$-process nuclei are synthesized through the nonequilibrium process of the rapid neutron capture on the seed nuclei that is synthesized through the $s$-rich freezeout (e.g., Hoffman et al. 1997). In other words, an explosive and neutron-rich site with high entropy will be a candidate for the location where the $r$-process nucleosynthesis occurs.

The most probable candidates for the sites are collapse-driven supernovae (e.g., Woosley et al. 1994, hereafter W94) and/or neutron star mergers (e.g., Freiburghaus, Rosswog, & Thielemann 1999). This is because these candidates are thought to have a potential to meet the requirements mentioned above. However, we think that the collapse-driven supernovae are thought to be more probable sites than the neutron star mergers because metal poor stars already contain the $r$-process nuclei (e.g., Freiburghaus et al. 1999). In fact, McWilliam et al. (1995) reported that the abundance of Eu can be estimated in 11 out of 33 metal-poor stars. These observations prove that $r$-process nuclei are produced from the early stage of the star formation in our Galaxy. Taking the event rate of collapse-driven supernovae ($10^{-2}$ yr$^{-1}$ gal$^{-1}$; van den Bergh & Tammann 1991) and neutron star merger ($10^{-5}$ yr$^{-1}$ gal$^{-1}$; van den Heuvel & Lorimer 1996; Bethe & Brown 1998) into consideration, collapse-driven supernovae are favored since they can supply the $r$-process nuclei from the early stage of the star formation in our Galaxy. Also, Cowan et al. (1999) reported that the abundance ratio of $r$-process nuclei in metal-poor stars is very similar to that in the solar system. This proves that $r$-process nuclei are synthesized through similar conditions. This will be translated that at least most of the $r$-process nuclei are from one candidate. Thus, we assume in this paper that most of the $r$-process nuclei are synthesized in the collapse-driven supernovae.

There are many excellent and precise analytic and/or numerical calculations on the $r$-process nucleosynthesis in the collapse-driven supernovae. However, it would be able to be said that there is no report that the $r$-process nuclei can be reproduced completely. For example, Takahashi, Witti, & Janaka (1994) performed numerical simulations assuming Newtonian gravity and reported that entropy per baryon in the hot bubble is about 5 times smaller than the required value. Qian & Woosley (1996, hereafter QW96) also reported analytic treatments of the neutrino-driven winds from the surface of the proto–neutron star. At the same time, their analytical treatments are tested and confirmed by numerical methods. However, the derived
entropy by their wind solutions is shown to fall short, by a factor of 2–3, of the value required to produce a strong r-process (Hoffman et al. 1997). In order to solve this difficulty, QW96 included a first post-Newtonian correction to the gravitational force equation. As a result, they reported that the entropy is increased and the dynamical timescale is reduced by a factor of ≈2. Cardall & Fuller (1997) developed this argument by considering a fully general relativistic treatment. They showed that a more compact neutron star leads to higher entropy and a shorter dynamical timescale in the neutrino-driven wind. In order to confirm their conclusion quantitatively, Otsuki et al. (2000) have surveyed the effects of general relativity parametrically. They reported that r-process can occur in the strong neutrino-driven winds \( L_v \sim 10^{52} \text{ erg s}^{-1} \) as long as a massive \( \sim 2.0 M_\odot \) and compact \( \sim 10 \text{ km} \) proto–neutron star is formed. It is very interesting because such a solution cannot be found in the framework of Newtonian gravity (QW96). Such a solution is confirmed by the excellent numerical calculations (Sumiyoshi et al. 2000). However, the equation of state (EOS) of the nuclear matter has to be very soft to achieve such conditions. Although a few non-standard models of EOS can meet them (Wiringa, Fiks, & Fabrocini 1988) as long as the matter is cold enough, it seems to be very difficult to achieve them in the phase of the proto–neutron star. In fact, the r-process nuclei cannot be produced in the numerical simulations with a normal EOS (Sumiyoshi et al. 2000). Thus, it seems that the difficulty cannot be solved by the effects of general relativity alone.

There is only one report that r-process nucleosynthesis occurred successfully. That is the work done by W94. In their numerical simulation, the entropy per baryon becomes higher and higher as time goes on. Finally, at a very late phase of neutrino-driven wind \( \sim 10 \text{ s after the core collapse} \), successful r-process occurs. However, there are some questions concerning their results. One is that the reason why the entropy per baryon at the late phase becomes as high as their results is unclear. In fact, when we adopt the analytic formulation of QW96, it is concluded that such a high entropy should not be obtained. It is true that general relativistic effects are included in W94, but such a high entropy could not be obtained in the work done by Otsuki et al. (2000). Hence, the discrepancy between W94 and QW96 cannot be explained by general relativistic effects alone. In addition, W94 has a problem that the nuclei whose mass numbers are \( \sim 90 \) are overproduced in the early stage of the neutrino-driven winds. So if we try to reproduce the mass spectrum of the solar system abundances, we have to abandon the matter that is made at the early stage of the neutrino-driven winds. Even worse, the successful mass spectrum at the late phase of neutrino-driven winds is destroyed when the reactions of neutrino–current neutrino spallations of nucleons from \(^4\text{He} \) are taken into consideration (Meyer 1995). In their paper, it is reported that a 30%–50% increase in the entropy is needed in order to restore the \( A = 195 \) peak, which is an extremely large modification to the model. Thus, although the work done by W94 is very remarkable and interesting, it would not be concluded that the problem of r-process nucleosynthesis has been solved completely.

Because of the reason mentioned above, it will be natural to think that there may be an effect(s) that will help the r-process nucleosynthesis. There are some papers that mention the effects of a jet (Symbalisty 1984; Shimizu, Yamada, & Sato 1994; Nagataki 2000) that is generated around the polar region of the highly rotating (the period \( T \sim 1 \text{ ms} \) and/or magnetized \( \sim 10^{15} \text{ G} \) proto–neutron star (LeBlanc & Wilson 1970; Symbalisty 1984; Shimizu et al. 1994; Yamada & Sato 1994; Fryer & Heger 2000). Since the physical conditions in the jet are different from those in a spherical explosion, the products of the nucleosynthesis are expected to be different. In fact, it is reported that the products of the explosive nucleosynthesis in a jetlike explosion in Si- and O-rich layers are much different from those in a spherical explosion (Nagataki et al. 1997; Nagataki 2000). Hence, it is natural to think that the products of explosive nucleosynthesis in the Fe core (that is, in the hot bubble) will also be changed as a result of the effects of the jet. This is the reason why we should examine whether the effects of the jet can help the synthesis of r-process nuclei.

Since there is no numerical simulation on the r-process nucleosynthesis in the jet during the neutrino-driven wind phase, our final goal is to perform such realistic numerical simulations. However, such a numerical simulation will be a difficult task. We also think that the results of such numerical simulations will not be understood completely without analytical studies. Thus, before performing such numerical simulations, we examine the physical conditions of the jet using a simple model. In this paper we study a steady, subsonic, and rigidly rotating flow of the jet that is driven by neutrinos. We study the effects of the jet on the physical conditions such as the entropy per baryon and the dynamical timescale. Finally, we discuss whether the effects of the jet can help the synthesis of r-process nuclei. In § 2 we explain the formulation for the jet in the hot bubble. In § 3 we show the results. Summary and discussions are presented in § 4.

2. FORMULATION OF JET

2.1. Basic Equations

In Gaussian units, the Euler equation acted on by electromagnetic forces can be written as (Shapiro & Teukolsky 1983)

\[
\frac{dv}{dt} = -\frac{1}{\rho} \nabla P - \nabla \Phi - \frac{1}{8\pi\rho} \nabla B^2 + \frac{1}{4\pi\rho} (B \cdot \nabla)B .
\]

(1)

Here

\[
\frac{d}{dt} = \frac{\partial}{\partial t} + v \cdot \nabla
\]

(2)

is the Lagrangian time derivative following a fluid element.

In this paper we study a steady jet that has \( \phi \)-symmetry around the polar region of the proto–neutron star. Thus, we use the cylindrical coordinate \((r, \phi, z)\) for convenience. The origin \( z = 0 \) is set at the center of the proto–neutron star. In the cylindrical coordinate, the Euler equation can be written as

\[
\begin{align*}
\frac{v_r}{c^2} + \frac{v_\phi}{r} + \frac{v_z}{c} - \frac{v_\phi^2}{r} &= -\frac{1}{\rho} \left( \frac{\partial P}{\partial r} \right) - \frac{GM}{z^2} \\
\frac{v_r}{c^2} + \frac{v_\phi}{r} + \frac{v_z}{c} &= -\frac{1}{\rho} \left( \frac{\partial P}{\partial z} \right) - \frac{GM}{z^2} \\
\frac{v_r}{c^2} + \frac{v_z}{c} &= -\frac{1}{\rho} \left( \frac{\partial P}{\partial z} \right) - \frac{GM}{z^2}
\end{align*}
\]
where free nucleons) as on free nucleons, neutrino scattering processes on the elec-
tions (QW96). In this paper we consider three neu-
ejecta and the radius of the jet, respectively.

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The equation for the evolution of material energy, \( \epsilon \), is

\[
\rho \dot{q} = \nabla \cdot (\rho \epsilon \mathbf{v}) + P \mathbf{v} \cdot \nabla \mathbf{v} + \frac{1}{r} \frac{\partial}{\partial r} (r \frac{\partial}{\partial r} (B_\phi)) \]

where \( \dot{M} \) and \( D \) are the constant mass outflow rate in the

\[
\dot{q} = \dot{q}_N + \dot{q}_v e - \dot{q}_e N ,
\]

and

\[
\epsilon = \frac{11\pi^2}{60} \frac{c^4 T^4}{\hbar^3 c^3} \rho (\text{ergs g}^{-1}) ,
\]

where \( k \) and \( h \) are Boltzmann and Planck constants, respec-
tively. These are the basic equations in this paper. Precisely,

\[
B = [0, B_\phi(r, z), 0] .
\]

Next, we seek a solution for the jet rotating rigidly. That is, we seek a solution \( \mathbf{v} = [0, \Omega r, v_z(r, z)] \) at \( z \geq R \), where \( \Omega \)
is the angular velocity of the proto–neutron star. From equation

\[
\dot{q}_v e = 3.48 \times 10^{-6} N_A T_{\text{MeV}}^6 \rho_b
\]

\[
\times \frac{1-x}{R_o^2} \text{ (ergs s}^{-1} \text{ g}^{-1} ) ,
\]

and

\[
\dot{q}_e N = 3.63 \times 10^{-6} N_A T_{\text{MeV}}^6 \text{ (ergs s}^{-1} \text{ g}^{-1} ) .
\]

Here \( R_o \) is the neutrino sphere radius in units of \( 10^6 \) cm,

\[
x = (1 - R^2/z^2)^{1/2} , \quad N_A \text{ is Avogadro’s number, } L_{\nu, 51} \text{ is the}
\]

\[
\text{individual neutrino luminosity in } 10^{34} \text{ ergs s}^{-1} , \text{ and } \epsilon_{\nu, \text{MeV}} \text{ is an appropriate neutrino energy } \epsilon_{\nu} \text{ in MeV (QW96). In this paper we set } \dot{q} = 0 \text{ at } T \leq 0.5 \text{ MeV because free}
\]
nucleons are bound into \( \alpha \)-particles and heavier nuclei and

\[
\text{electron-positron pairs annihilate into photons. The pressure } P \text{ and internal energy } \epsilon \text{ are determined}
\]

\[
P = \frac{11\pi^2}{180} \frac{k^4 T^4}{\hbar^3 c^3} \text{ (dyn cm}^{-2} )
\]

The pressure \( P \) and internal energy \( \epsilon \) are determined approx-
натмy by the relativistic electrons and positrons and photon radiation as long as \( T \geq 0.5 \) MeV. Thus, the pressure and internal energy can be written as

\[
\frac{d\rho}{dz} = - \frac{P \dot{q} / v_z + \rho G M / z^2 - 2 P v_z^2}{(P / \rho)(\rho + P / \rho - v_z^2) / 4}
\]

It is clearly understood from equation (16) that the solution
holds valid as long as \( \Omega D \) is small enough. In this case, equation (4) can be rewritten as

\[
\dot{M} = \pi D^2 \rho v_z .
\]

Here we also note that the derivatives of \( \rho \) and \( T \) by \( z \) for the spherical explosion model can be written as

\[
\frac{d\rho}{dz} = - \frac{P \dot{q} / v_z + \rho G M / z^2}{(P / \rho)(\rho + P / \rho - v_z^2) / 4}
\]

and

\[
\frac{dT}{dz} = \frac{\dot{q} / v_z + (\rho / \rho + P / \rho^2)(dp/dz)}{4 \epsilon / T} .
\]
by these parameters as (QW96) the explosions in conditions between the neutrino-driven jets and spherical equations are used to discuss the difference of the physical respectively (see also eqs. [1], [2], and [3] in QW96). These solutions to the case in which inapplicable to the case of strong magnetic fields, we extend stronger. So, although the truth is that these solutions are can happen as the magnetic fields are getting stronger and in this paper. On the other hand, we also want to see what equation (15) that small gradient is negative. We are also able to see clearly from these solutions to the case of strong magnetic fields qualitatively. We call such By so doing, we will be able to find at least what happens in the case of strong magnetic fields qualitatively. We call such solutions “strong-field solutions” in this paper. In § 3 we show the results for the weak- and strong-field solutions.

### 2.3. Boundary Conditions

In this section the surface of the proto–neutron star is considered as the inner boundary. The inner boundary conditions are composed of density, luminosities of neutrinos, mass and radius of the proto–neutron star, velocity of the outflow, and $\Omega D$. Initial temperature and electron fraction at the time when $x$-rich freezeout takes place are determined by these parameters as (QW96)

$$T_i = 1.19 \times 10^{19} \left[1 + \frac{L_{\nu_e}}{L_{\nu_e}} \left(\frac{\epsilon_{\nu_e, \text{MeV}}}{\epsilon_{\nu_e, \text{MeV}}}\right)^2\right]^{1/6}$$

and

$$Y_e = \left(1 + \frac{L_{\nu_e} \epsilon_{\nu_e, \text{MeV}} - 2\Delta + 1.2\Delta^2/\epsilon_{\nu_e, \text{MeV}}}{L_{\nu_e} \epsilon_{\nu_e, \text{MeV}} + 2\Delta + 1.2\Delta^2/\epsilon_{\nu_e, \text{MeV}}}\right)^{-1}, \quad (22)$$

where $L_{\nu_e, 51}$ is the individual neutrino luminosity in $10^{51}$ ergs s$^{-1}$, $R_o$ is the neutron star radius in $10^6$ cm, $\Delta = 1.293$ MeV is the neutron-proton mass difference, and $\epsilon_{\nu_e, \text{MeV}}$ is the neutrino energy in MeV. We assume that the neutron star radius is equal to the neutrino sphere radius.

In this paper the luminosities of neutrinos are assumed to be common (QW96; Otsuki et al. 2000). The energy of neutrinos is assumed to be 12, 22, and 34 MeV for $\nu_e$, $\bar{\nu}_e$, and other neutrinos, respectively (W94; QW96; Otsuki et al. 2000). Surface density is assumed to be $10^{10}$ g cm$^{-3}$ (Otsuki et al. 2000). In previous works, initial velocity of the outflow is chosen so that $\dot{M}$ becomes less than $M_{\text{crit}}$, where $M_{\text{crit}}$ is the critical value for supersonic solution (QW96; Otsuki et al. 2000). In this paper we also adopt this assumption so that the flow becomes subsonic and contains no critical point. In particular, we take $\dot{M}$ as $\dot{M} \sim M_{\text{crit}}$ in this study. This means that the initial velocity (velocity at the surface of the proto–neutron star) is set to be maximal because surface density is set to be constant ($10^{10}$ g cm$^{-3}$). In case we try to survey the flow that contains a transition point like a shock front, we cannot use equations (6), (14), (16), and (17). This is because these differential equations diverge and break down. In order to treat such a flow that contains a discontinuity, we have to use the Rankine-Hugoniot relation instead of these equations. We will examine such flows in a forthcoming paper. D is assumed to be $10^3$ cm, which is about 1/10 of the proto–neutron star radius. We note that an appropriate environment can exist around the jet, that is, there is a solution that meets equation (14) for $r \geq D$ in which $B_o$ and $\nu_e$ become zero at $r \to \infty$. Thus, the rigid rotating flow at $r \leq D$ is not an unrealistic solution at all. In this paper we consider only the region $r \leq D$ in which the matter rotates rigidly for simplicity. Other parameters are changed parametrically. The parameters employed as well as the model names are given in Table 1. We take $z = 10^9$ cm for the radius of the outer boundary (Otsuki et al. 2000). Since the radius of the Fe core is about $10^8$ cm, we think that the radius of the outer boundary is large enough to investigate the $r$-process nucleosynthesis that occurs in the hot bubble.

### 3. RESULTS

#### 3.1. Weak-Field Solutions

Output parameters are shown in Table 2. Entropy per baryon ($S$), dynamical timescale ($\tau_{\text{dyn}}$), analytically estimated dynamical timescale ($\tau_{\text{dyn, ana}}$), electron fraction ($Y_e$), and temperature ($T$) at the outer boundary ($z = 10^9$ cm) are shown in the table. Entropy per baryon in radiation-dominated gas can be written as

$$\frac{S}{k} = \frac{11\pi^2}{45} \frac{k^3}{h^3 c^3} \frac{T^3}{\rho/m_N} \quad (23)$$

as long as $T \geq 0.5$ MeV; $m_N$ is the nucleon rest mass. Even if the annihilation of the electron-positron pairs occurs, the entropy per baryon is conserved. The definition of the dynamical timescale is the time for the temperature to decrease from 0.5 to 0.2 MeV. This is because $r$-process

### Table 1

| Model       | Mass ($M_\odot$) | Radius (km) | $L_{\nu_e}$ ($10^{51}$ ergs s$^{-1}$) | $\dot{M}$ ($M_\odot$ s$^{-1}$) | $\Omega D$ (cm s$^{-1}$) |
|-------------|------------------|-------------|----------------------------------------|-------------------------------|--------------------------|
| 10AW        | 1.4              | 10          | 3.00                                   | 8.4 ($-8$)                    | 0                        |
| 10BW        | 1.4              | 10          | 1.00                                   | 1.0 ($-8$)                    | 0                        |
| 10CW        | 1.4              | 10          | 0.60                                   | 5.8 ($-9$)                    | 0                        |
| 10DW        | 2.0              | 10          | 3.00                                   | 5.1 ($-9$)                    | 0                        |
| 10EW        | 2.0              | 10          | 1.00                                   | 8.1 ($-9$)                    | 0                        |
| 10FW        | 2.0              | 10          | 0.60                                   | 3.5 ($-9$)                    | 0                        |
| 10GW        | 1.4              | 10          | 0.10                                   | 3.1 ($-10$)                   | 0                        |
| 10GS1       | 1.4              | 10          | 0.10                                   | 3.1 ($-10$)                   | 1.0 ($+8$)                |
| 10GS2       | 1.4              | 10          | 0.10                                   | 3.3 ($-10$)                   | 1.0 ($+9$)                |
| 10GS3       | 1.4              | 10          | 0.10                                   | 5.1 ($-10$)                   | 1.0 ($+10$)               |
| 10HW        | 1.4              | 10          | 0.05                                   | 9.8 ($-11$)                   | 0                        |
| 10IW        | 1.4              | 10          | 0.01                                   | 6.6 ($-12$)                   | 0                        |
| 10JW        | 2.0              | 10          | 0.10                                   | 1.8 ($-10$)                   | 0                        |
| 10KW        | 2.0              | 10          | 0.05                                   | 5.7 ($-11$)                   | 0                        |
| 10LW        | 2.0              | 10          | 0.01                                   | 3.9 ($-12$)                   | 0                        |
| 30AW        | 1.4              | 30          | 3.00                                   | 13.0 ($-6$)                   | 0                        |
| 30BW        | 1.4              | 30          | 10.0                                   | 2.2 ($-7$)                    | 0                        |
| 30CW        | 1.4              | 30          | 6.00                                   | 1.1 ($-7$)                    | 0                        |
| 30DW        | 1.4              | 30          | 1.00                                   | 6.0 ($-9$)                    | 0                        |
| 30GW        | 1.4              | 30          | 0.50                                   | 1.9 ($-9$)                    | 0                        |
| 30FW        | 1.4              | 30          | 0.10                                   | 1.4 ($-10$)                   | 0                        |
| 30GW        | 1.4              | 30          | 0.05                                   | 4.4 ($-11$)                   | 0                        |
| 30HW        | 1.4              | 30          | 0.01                                   | 3.0 ($-12$)                   | 0                        |

Note.—Mass and radius of the neutron star, total luminosity of neutrinos, mass outflow rate, and $\Omega D$ are shown.
nucleosynthesis occurs in this temperature range (W94; Takahashi et al. 1994; QW96). The reason why the dynamical timescales are not written in some models is that temperature does not decrease to 0.2 MeV within $z = 10^9$ cm.

There is a tendency that $S$ and $T_\text{e}$ (temperature at the outer boundary) in the models studied here are higher than those in the models of QW96. For example, we show in Figure 1 the outflow velocity, temperature, and density as a function of $z$ for models 10CW and 10C in QW96. The inner boundary conditions (density, luminosities of neutrinos, mass and radius of the proto–neutron star) in model 10C in QW96 are the same as those in model 10CW except for the outflow velocity. Outflow velocity is taken so that $M$ becomes $M \sim M_{\text{crit}}$ in model 10C.

Furthermore, the definition of the analytically estimated dynamical timescale is described later (see eq. [33]).

In Figure 1 the outflow velocity, temperature, and density as a function of $z$ for models 10CW and 10C in QW96. The inner boundary conditions (density, luminosities of neutrinos, mass and radius of the proto–neutron star) in model 10C in QW96 are the same as those in model 10CW except for the outflow velocity. Outflow velocity is taken so that $M$ becomes $M \sim M_{\text{crit}}$ in model 10C.

Also show those for model 10CW in QW96 in Figure 3. Lines (a)–(g) correspond to $4e/T$, $(P/\rho)(\epsilon + P/\rho)$, $v_\text{o}^2$, $\rho GM/z^2$, $Pq/v_\text{o}^2\epsilon$, $(\epsilon/\rho + P/\rho)(dp/dz)$, and $d\rho/dz$, as a function of $z$, respectively. In Figure 3, line (i), which represents $2Pq^2/r$, is also added (see eq. [19]). As can be seen from Figures 2 and 3, $dp/dz$ in both models can be approximated by

$$\frac{dp}{dz} \sim -\frac{\rho GM/z^2}{(P/\rho)(\epsilon + P/\rho)}.$$  

Furthermore, $dT/dz$ can be approximated by

$$\frac{dT}{dz} \sim -\frac{(\epsilon/\rho + P/\rho^2)(dp/dz)}{4e/T} \sim -\frac{m_N GM}{S}.$$  

From equation (25) we can see that the temperature gradient $dT/dz$ is steeper when the entropy per baryon is smaller. Thus, we can expect that the entropy per baryon is larger in model 10CW than in model 10C in QW96, which is verified in Figure 4. Hence, the problem of why $T_\text{e}$ becomes high is translated into the problem of why the entropy per baryon becomes high in model 10CW. The answer is shown in Figure 2. The velocity at small radii in model 10CW is smaller than that in model 10C in QW96. The outflow velocity for model 10CW is almost supersonic because $M$ is set to be $M_{\text{crit}}$, as is explained in § 2.3. Inner boundary conditions (density, luminosities of neutrinos, mass and radius of the proto–neutron star) in model 10C in QW96 are set to be the same as those in model 10CW except for the outflow velocity. Outflow velocity is taken so that $M$ becomes $M \sim M_{\text{crit}}$ in model 10C.

For comparison, we show the absolute values (in cgs units) of the components in equations (16) and (17) for model 10CW in Figure 2. From this figure we can see what determines the gradients of temperature and density. For comparison, we show the absolute values (in cgs units) of the components in equations (16) and (17) for model 10CW in Figure 2. From this figure we can see what determines the gradients of temperature and density. For comparison, we also show those for model 10C in QW96 in Figure 3. Lines (a)–(g) correspond to $4e/T$, $(P/\rho)(\epsilon + P/\rho)$, $v_\text{o}^2$, $\rho GM/z^2$, $Pq/v_\text{o}^2\epsilon$, $(\epsilon/\rho + P/\rho)(dp/dz)$, and $d\rho/dz$, as a function of $z$, respectively. In Figure 3, line (i), which represents $2Pq^2/r$, is also added (see eq. [19]). As can be seen from Figures 2 and 3, $dp/dz$ in both models can be approximated by

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**TABLE 2**

| Model      | $S$ ($\Delta t$) | $\tau_{\text{dyn}}$ (s) | $\tau_{\text{dyn,ana}}$ (s) | $Y_\text{e}$ | $T_\text{e}$ (MeV) |
|------------|-----------------|---------------------|------------------------|-------------|---------------------|
| 10AW       | 92              | 0.43                | 0.43                   | 68          |                     |
| 10BW       | 102             | 0.43                | 0.43                   | 37          |                     |
| 10CW       | 110             | 0.43                | 0.43                   | 36          |                     |
| 10DW       | 129             | 0.43                | 0.43                   | 62          |                     |
| 10E         | 146             | 0.43                | 0.43                   | 38          |                     |
| 10F         | 156             | 0.43                | 0.43                   | 32          |                     |
| 10G         | 136             | 0.43                | 0.43                   | 14          |                     |
| 10G1        | 137             | 0.43                | 0.43                   | 15          |                     |
| 10G2        | 132             | 0.43                | 0.43                   | 15          |                     |
| 10G3        | 4.7             | Undefined           | 0.43                   | 50          |                     |
| 10G4        | 150             | 0.43                | 0.43                   | 11          |                     |
| 10I         | 196             | 0.43                | 0.43                   | 92          |                     |
| 10J         | 196             | 0.43                | 0.43                   | 12          |                     |
| 10K         | 216             | 0.43                | 0.43                   | 11          |                     |
| 10L         | 274             | 0.43                | 0.43                   | 61          |                     |
| 30A         | 37              | 0.43                | 1.21                   |             |                     |
| 30B         | 42              | 0.43                | 0.92                   |             |                     |
| 30C         | 42              | 0.43                | 0.60                   |             |                     |
| 30D         | 50              | 0.43                | 0.31                   |             |                     |
| 30E         | 55              | 0.43                | 0.28                   |             |                     |
| 30F         | 64              | 0.43                | 0.11                   |             |                     |
| 30G         | 72              | 0.43                | 0.11                   |             |                     |
| 30H         | 90              | 0.43                | 0.11                   |             |                     |

Note.—Entropy per baryon, dynamical timescale ($\tau_{\text{dyn}}$), analytically estimated dynamical timescale ($\tau_{\text{dyn,ana}}$), electron fraction, and temperature at the outer boundary are shown. The reason why dynamical timescales ($\tau_{\text{dyn}}$ and $\tau_{\text{dyn,ana}}$) are not written in some models is that temperature does not decrease to 0.2 MeV within $z = 10^9$ cm.
heating process is effective only at small radii, which is proved in Figure 4. So, if the velocity at small radii is small, there is a long time for the outflow matter to get heat from neutrinos. As a result, higher entropy per baryon can be achieved and becomes higher. This is the case with the jet models studied in this paper. As for the reason why the initial velocity has to be smaller in model 10CW than in model 10C in QW96, it is relatively difficult to discern. At least we can say that \( \frac{dv_z}{dz} \) can be written from equation (18) as
\[
\frac{dv_z}{dz} = -\frac{v_z \frac{dp}{d\rho}}{\rho \frac{dz}{r}} \text{ in model 10CW .} \tag{26}
\]
On the other hand, it can be written as
\[
\frac{dv_z}{dz} = -\frac{v_z \frac{dp}{d\rho}}{\rho \frac{dz}{r}} \text{ in model 10C} \tag{27}
\]
because \( 4\pi z^2 \rho v_z \) is constant in model 10C in QW96. Taking these equations and equations (16) and (19) into consideration, the velocity gradient is steeper in model 10CW as long as \( v_z(z) \), \( \rho(z) \), and \( T(z) \) are the same in both models. Hence, we can guess that the initial \( v_z \) has to be smaller in model 10CW so as not to achieve the adiabatic sound speed, \( v_s = (4P/3\rho)^{1/2} \). This will be the reason why the initial velocity in model 10CW has to be smaller. Obviously, this result reflects the difference in the physical structure between the cylindrical jets and spherical explosion because the above discussion is based on equations (16), (19), (26), and (27).

Next, we consider the dynamical timescale. As mentioned above, the dynamical timescale becomes infinity when \( T_b \) is above 0.2 MeV. Since \( T_b \) in the jet models tends to be higher, we cannot determine the dynamical timescale in many jet models. This is a bad tendency to produce r-process nuclei. From Table 2, the dynamical timescale can be determined as long as \( L_{\nu_e,51} \) is small. We will discuss later whether r-process can occur or not in such models. Before we continue with further discussion, we explain the way to derive \( \tau_{\text{dyn,ana}} \). The definition of \( \tau_{\text{dyn,ana}} \) is the analytically estimated dynamical timescale for the case in which the tem-
perature at the outer boundary becomes lower than $\sim 0.2$ MeV. Otherwise, this estimate does not hold. Thus, $\tau_{\text{dyn,ana}}$ is defined only for the case in which the temperature at the outer boundary becomes lower than $\sim 0.2$ MeV (see also Table 2).

Like model 10C in QW96, from equations (6) and (14) we can derive the following equation as long as $\Omega D$ is small:

$$\dot{q} = v_z \frac{d}{dz} \left( \frac{v_z^2}{2} + \frac{TS}{m_N} - \frac{GM}{z} \right).$$  \hspace{1cm} (28)

Thus, when the heating/cooling process becomes non-effective, there is a conserved quantum

$$\epsilon_{\text{flow,f}} = \left( \frac{v_z^2}{2} + \frac{TS}{m_N} - \frac{GM}{z} \right).$$  \hspace{1cm} (29)

In the subcritical case, $v_z \leq v_s$ everywhere, we can approximate equation (29) as

$$\frac{TS}{m_N} - \frac{GM}{z} \sim T_b S.$$  \hspace{1cm} (30)

When $T_b \ll 0.5$ MeV, we can take

$$\frac{TS}{m_N} \sim \frac{GM}{z} \text{ at } 0.5 \text{ MeV}$$  \hspace{1cm} (31)

(see QW96 for details). In this case, $T \propto z^{-1}$ near $T \sim 0.5$ MeV because $S$ is also a conserved quantum as long as the heating/cooling process does not work. From equation (23), $\rho \propto z^{-3}$. From equation (18), $v_z \propto z^3$. Thus, $\tau_{\text{dyn,ana}}$ can be approximated as

$$\tau_{\text{dyn,ana}} = \int_{T=0.2 \text{ MeV}}^{T=0.5 \text{ MeV}} \frac{dt}{dz} \frac{dt}{dT} dT = \left( \frac{z}{v T^2} \right)_{T=0.5 \text{ MeV}}^{T=0.2 \text{ MeV}} T dT$$

$$\sim 0.42 \left( \frac{z}{v} \right)_{T=0.5 \text{ MeV}} \text{ (s).}$$  \hspace{1cm} (32)

For further discussion, we try to express the dynamical timescale by input parameters (see QW96 for details). From simple calculations, we can derive the entropy per baryon and the mass outflow rate in model 10CW as

$$\frac{S}{k} \sim 235 L_{v_s,51}^{-1/6} \epsilon_{v_s,\text{MeV}}^{-1/3} R_6^{-2/3} \left( \frac{M}{1.4 M_\odot} \right)$$  \hspace{1cm} (33)

and

$$\dot{M} \sim \frac{D^2}{4 R^2} \times 1.14 \times 10^{-10} L_{v_s,51}^{5/3} \epsilon_{v_s,\text{MeV}}^{10/3} R_6^{5/3}$$

$$\times \left( \frac{1.4 M_\odot}{M} \right)^2 \left( M_\odot \text{ s}^{-1} \right),$$  \hspace{1cm} (34)

respectively. At last, we can derive the expression for $\tau_{\text{dyn,ana}}$ from equations (18), (23), (31), (33), (34), and (35):

$$\tau_{\text{dyn,ana}} \sim 10.5 L_{v_s,51}^{-4/3} \epsilon_{v_s,\text{MeV}}^{-8/3} R_6^{5/3} \left( \frac{M}{1.4 M_\odot} \right)$$  \hspace{1cm} (35)

This value for each model is written in Table 2. We can find that $\tau_{\text{dyn,ana}}$ agrees with $\tau_{\text{dyn}}$ within a factor of 2–3 in all models in which the dynamical timescale can be defined, although $\tau_{\text{dyn,ana}}$ tends to be smaller. This precision of $\tau_{\text{dyn,ana}}$ is about the same as that in the analysis in QW96.

We compare $\tau_{\text{dyn,ana}}$ derived here with that in QW96 in order to see the effects of a jet. It is written as

$$\frac{\tau_{\text{dyn,ana}}}{\tau_{\text{dyn,QW}}} = 0.15 \frac{R_6^{3/3}}{L_{v_s,51}^{1/3} \epsilon_{v_s,\text{MeV}}^{2/3}}.$$  \hspace{1cm} (36)

For example, when $L_{v_s,51} = 0.1$, $R_6 = 1$, and $\epsilon_{v_s,\text{MeV}} = 22$, $\tau_{\text{dyn,ana}}/\tau_{\text{dyn,QW}} = 0.04$. Even if we take the uncertainty (a factor of 2–3) of $\tau_{\text{dyn,ana}}$ into consideration, it seems that a short dynamical timescale can be realized in jet models. So, as long as $T_b$ can be small (that is, $L_{v_s,51}$ is small enough), we think that a short dynamical timescale can be realized in jet models.

Now we discuss whether r-process nucleosynthesis can occur in jet models. We can use Hoffman's criterion (Hoffman et al. 1997) for the judgment. Hoffman's criterion can be written as

$$S \geq 2 \times 10^3 Y_e \left( \frac{\tau_{\text{dyn}}}{1 \text{ s}} \right)^{1/3}$$  \hspace{1cm} (37)

for $Y_e \geq 0.38$. This is the criterion for production of r-process nuclei with mass number $A \sim 200$ (Hoffman et al. 1997). Substituting equations (34) and (36) into equation (38), we can translate the criterion into

$$R_6 \leq 0.74 \left( \frac{M}{1.4 M_\odot} \right)^{6/11}$$  \hspace{1cm} (38)

where $\epsilon_{v_s,\text{MeV}}$ is assumed to be 22. $C$ is the factor of 2–3 that represents the tendency that $\tau_{\text{dyn,ana}}$ tends to be smaller than $\tau_{\text{dyn}}$. In the case of $M = 1.4 M_\odot$ and $C = 2$, we can see from equation (39) that $R_6$ can be larger than 1 as long as $L_{v_s,51}$ is larger than 8.6. This means that it seems to be possible to produce r-process nuclei without an unrealistically soft EOS as long as $L_{v_s,51}$ is larger than 8.6. However, we found from Table 2 that $T_b$ cannot be lower than 0.2 MeV when the neutrino luminosity is so high. Thus, we have to conclude that r-process cannot occur in these models.

However, we can find that the requirement to produce r-process nuclei seems to be relaxed in these jet models. In fact, for the models of QW96, the criterion like equation (39) can be written as

$$R_{6,\text{QW}} \leq 0.19 L_{v_s,51}^{1/6} \left( \frac{M}{1.4 M_\odot} \right)^{2/3}.$$  \hspace{1cm} (39)

Hence, $R_{6,\text{QW}}$ can be larger than 1 when $L_{v_s,51}$ is larger than $2.3 \times 10^4$, which cannot be realized in numerical simulations of supernovae (W94). We can compare the required radius in jet models ($C = 2$) with that in models of QW96 as

$$R_6/R_{6,\text{QW}} = 3.2 L_{v_s,51}^{2/3} \left( \frac{M}{1.4 M_\odot} \right)^{-4/3}.$$  \hspace{1cm} (40)

For example, in the case of the model of $M = 1.4 M_\odot$ and $L_{v_s,51} = 1$, $R_6/R_{6,\text{QW}} = 2.8$. This suggests that the EOS of the nuclear matter does not necessarily have to be too soft, as Otsuki et al. (2000) requires, when the jet models are adopted. Thus,
the requirement to achieve $r$-process nucleosynthesis can be relaxed for the jet models as long as the temperature at the outer boundary becomes lower than $\sim 0.2$ MeV, although $r$-process nucleosynthesis is still not able to occur in the jet models. When we include the effects of a jet and general relativity at the same time, a successful solution may be obtained. We will discuss this topic in a forthcoming paper.

Finally, we consider the mass outflow rate. Taking the event rate of collapse-driven supernovae ($10^{-2}$ yr$^{-1}$ gal$^{-1}$; van den Bergh & Tammann 1991), one needs $10^{-6}$ to $10^{-4}$ $M_\odot$ of ejected $r$-process material per collapse-driven supernova event to explain the observed solar abundance ratio (Käppeler, Beer, & Wisshak 1989; W94). We give a rough estimate of whether each model can produce enough $r$-process matter to explain the solar system abundance. Taking the gravitational binding energy of a neutron star $\sim 3 \times 10^{43}$ ergs into consideration, the duration time, $t$, can roughly be estimated as

$$t \leq \frac{300}{6L_{r,51}} \left( \text{s} \right),$$

where the factor 6 represents the number of flavor of neutrinos. Hence, the required $\dot{M}_{\text{req}}$ to explain the solar abundance ratio is

$$\dot{M}_{\text{req}} \geq (10^{-4} \text{ to } 10^{-6}) \frac{L_{r,51}}{50} \left( M_\odot \text{ s}^{-1} \right).$$

Thus, $\dot{M}$ in Table 1 has to meet the relation

$$2 \dot{M} \sim \dot{M}_{\text{req}},$$

where the factor 2 represents that the matter is ejected from both (north and south) sides of the polar region. We can find from equations (43), (44), and Table 1 that low-luminosity models like 10GW–10LW and 30FW–30HW cannot meet the relation mentioned above, although the high-luminosity models like 10AW–10FW and 30AW–30EW may be able to meet the relation.

At last we can derive the conclusion. In the framework of the steady neutrino-driven jet with weak magnetic fields, high-luminosity models cannot work because $T_\beta$ becomes high and dynamical timescale becomes too long. Furthermore, low-luminosity models do not present a required condition to produce the $r$-process nuclei in the framework of Newtonian gravity. Although a successful solution may be obtained when we include the effects of a jet and general relativity at the same time, the amount of mass outflow seems to be too little to explain the solar system abundance ratio in such low-luminosity models. As a conclusion, we have to say that it is difficult to cause a successful $r$-process nucleosynthesis in the weak-field solutions.

### 3.2. Strong-Field Solutions

In this section we examine the models in which $\Omega D \neq 0$. As stated in § 2.2, the truth is that the solutions studied in this paper are applicable to the case of strong magnetic fields. However, we want to see what can happen as the magnetic fields are getting stronger and stronger. At the same time, we should investigate the range of application of weak-field solutions.

Here we consider three models (models 10GS1–10GS3) for strong-field solutions. These are listed in Table 1. The strength of the magnetic fields corresponds to $2.5 \times 10^{13}$, $2.5 \times 10^{14}$, and $2.5 \times 10^{15}$ G, respectively (see eq. [15]).

The results are shown in Table 2. We can find that the values of output parameters change drastically in model 10GS3. This result suggests that the weak-field solutions can be applied at least in the range of order $B \leq 10^{14}$ G. This means that the weak-field solutions will be valid for the typical proto–neutron stars. In case we consider the nucleosynthesis in a magnetar (Woods et al. 1999), we will have to investigate using strong-field solutions.

In order to understand what happens in model 10GS3, we show the outflow velocity, temperature, density, and entropy per baryon as a function of $z$ of model 10GS3 in Figure 5. We also show the absolute values (in cgs units) of the components of equations (16) and (17) for model 10GS3 in Figure 6. Lines (a)–(h) correspond to $4e/T$, $(P/\epsilon)(\epsilon + P/\rho)$, $v_\alpha^2$, $\rho GM/z^2$, $P q / v_\alpha \epsilon$, $(\epsilon/\rho + P/\rho^2)(d \rho / dz)$, $q / v_\alpha$, and $\Omega^2 D^2 / 4$ as a function of $z$, respectively. As can be seen from Figure 6, $d \rho / dz$ at $r = D$ can be approximated by

$$\left. \frac{d \rho}{dz} \right|_{r=D} \sim \frac{\rho GM/z^2}{\Omega^2 D^2 / 4}.$$

In addition, $dT/dz$ can be approximated by

$$\frac{dT}{dz} \sim \frac{T(\epsilon/\rho + P/\rho^2)(d \rho / dz)}{4 \epsilon}$$

for $z \leq 8 \times 10^1$ and $z \geq 2 \times 10^3$

$$\sim \frac{T q}{4 \epsilon v_\alpha}$$

for $8 \times 10^1 \leq z \leq 2 \times 10^3$.

![Fig. 5.—Outflow velocity, temperature, density, and entropy per baryon as a function of height ($z$) from the center of the proto–neutron star in model 10GS3. These quanta are written in units of $10^8$ cm$^{-1}$, 1 MeV, $10^9$ g cm$^{-3}$, and the Boltzmann constant, respectively. The outflow velocity for model 10GS3 is almost supersonic because $M$ is set to be $M_{\text{crit}}$, as is explained in § 2.3.](image)
We note that the denominator of equation (45) is $\Omega^2 D^2/4$, which is constant throughout. Since the denominator is large, the value of the density gradient becomes small. Thus, the density gradient in 10GS3 becomes nearly zero at smaller $z$ (compare Figs. 1 and 5). As a result, the entropy per baryon is getting lower in the range $8 \times 10^8 \leq z \leq 2 \times 10^9$ cm. This is the reason why the entropy per baryon becomes small in model 10GS3. We also note that the temperature gradient becomes too small and the dynamical timescale is long in the strong-field solution 10GS3. This tendency is a bad one for the production of $r$-process nuclei. The problem whether the flow contains $r$-process nuclei or not depends sensitively on the initial transition points or not depends sensitively on the initial. Hence, we cannot say that we have proved that a successful $r$-process nucleosynthesis does not occur in a neutrino-driven jet in a collapse-driven supernova. For example, we assumed that the flow is subsonic and there is no critical point, which is the common assumption in the previous studies of $r$-process nucleosynthesis (WQ96; Otsuki et al. 2000). However, we think that we do not need to restrict the solutions in such a way, that is, there may be a transition point at which equations (6), (14), (16), and (17) break down. In most cases, the transition point will be a shock front. This means that the flow will gain entropy at the transition point, which will be a good sense to produce the $r$-process nuclei. The problem whether the flow contains transition points or not depends sensitively on the initial velocity on the surface of the proto–neutron star. Thus, our final goal is to determine physically the velocity at the surface of the proto–neutron star. This means that $M$ should be not given as a input parameter; it should be an output parameter. We have to investigate the mechanism for determining the outflow velocity at the surface of the proto–neutron star for further discussions. We also assumed that the flow is steady. As seen in § 3.2, the density gradient becomes nearly zero and acceleration does not occur. Thus, the situation may be different, and acceleration occurs in the case of an unsteady flow. We will investigate the features of such an unsteady flow by numerical tests in the near future.

4. SUMMARY AND DISCUSSION

We have studied whether the jet in a collapse-driven supernova can be a key process for the $r$-process nucleosynthesis. We have studied the effects of a jet using a simple model because the results of a realistic numerical simulation concerned with such a theme will not be understood completely without analytical study. Although our final goal is to perform such realistic numerical simulations, this work is a necessary process to understand the effects of a jet on the $r$-process nucleosynthesis. We have studied two cases, that is, the cases in which the magnetic fields are weak/strong. In both cases, we assumed that the flow is steady and subsonic. As for the weak-field solutions, we have concluded that high-luminosity models cannot work because $T_b$ becomes higher than 0.2 MeV and $t_{\text{dyn}}$ becomes infinite in this case. In addition, low-luminosity models do not present a required condition to produce the $r$-process nuclei in the framework of Newtonian gravity. Although a successful solution may be obtained when we include the effects of a jet and general relativity at the same time, the total mass of the outflow seems to be too little to explain the solar system abundance ratio in such low-luminosity models. On the other hand, we found that the entropy per baryon is small and the dynamical timescale is long in the strong-field solution. We can tell that this tendency is a bad one for the production of $r$-process nuclei. The truth is that the solutions studied in this paper are inapplicable to the case of strong magnetic fields. However, similar tendency may also be found in the exact solutions. If so, we will be able to conclude that strong-field solutions are unsuitable for $r$-process nucleosynthesis. As a conclusion, we have to say that it seems to be difficult to cause a successful $r$-process nucleosynthesis in the jet models in this study.

We have to emphasize that there are some assumptions in this study. Hence, we cannot say that we have proved that a successful $r$-process nucleosynthesis does not occur in a neutrino-driven jet in a collapse-driven supernova. For example, we assumed that the flow is subsonic and that there is no critical point, which is the common assumption in the previous studies of $r$-process nucleosynthesis (WQ96; Otsuki et al. 2000). However, we think that we do not need to restrict the solutions in such a way, that is, there may be a transition point at which equations (6), (14), (16), and (17) break down. In most cases, the transition point will be a shock front. This means that the flow will gain entropy at the transition point, which will be a good sense to produce the $r$-process nuclei. The problem whether the flow contains transition points or not depends sensitively on the initial velocity on the surface of the proto–neutron star. Thus, our final goal is to determine physically the velocity at the surface of the proto–neutron star. This means that $M$ should be not given as a input parameter; it should be an output parameter. We have to investigate the mechanism for determining the outflow velocity at the surface of the proto–neutron star for further discussions. We also assumed that the flow is steady. As seen in § 3.2, the density gradient becomes nearly zero and acceleration due to the strong magnetic fields does not occur in the framework of the steady flow. Thus, the situation may be different, and acceleration occurs in the case of the unsteady flow. We

![Figure 6](image_url)
should investigate the features of the unsteady jet flow. It will be investigated by numerical tests assuming a simple environment. We will perform such numerical tests in the near future. The form of the jet is also assumed in this study. Since the conclusion in this study that the temperature does not decrease and the dynamical timescale becomes long for the case of the neutrino-driven jets results from the fact that the initial outflow velocity cannot be as fast as that in the spherical explosion because of its cylindrical structure, its tendency will not change even if the form of the jet is slightly changed. That is, the tendency that the initial outflow velocity cannot be as fast will be valid as long as the form of the explosion is jetlike. However, it will be worthwhile to survey physical conditions using a variety of forms of the jet because we will be able to understand more precisely and correctly the results of the realistic numerical simulations. Of course, our final goal is to perform realistic numerical simulations for the jet-induced explosion in a collapse-driven supernova and seek a final answer to the question of whether the jet in a collapse-driven supernova is a key process for the $r$-process nucleosynthesis.

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