Spacecraft Rendezvous Utilizing Invariant Manifolds for a Halo Orbit*

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For spacecraft to rendezvous in a halo orbit around Earth Moon Lagrange Point 2 with an amplitude of several ten-thousand kilometers, we propose selecting a chaser’s rendezvous trajectory from two different types, depending on the phase difference to the target. In the first trajectory, the chaser approaches the target from behind along the orbit, similarly to a rendezvous in a low Earth orbit. The second trajectory utilizes the homoclinic intersection of invariant manifolds of the halo orbit extended toward the Moon, where the chaser’s trajectory is controlled so that it first departs from the halo orbit along an unstable manifold, is connected to a stable manifold through the intersection, and then returns to the halo orbit. We showed that this detour can adjust the time of arrival to the halo orbit with low fuel usage and the total delta-v for the rendezvous can be significantly reduced in comparison to the first trajectory if the initial phase difference is large. The strategy employed can significantly increase the flexibility of the flight plan, increase the launch window of the visiting vehicle to the target and enhance the tolerance against failure compared to the application of a traditional phasing method.

Key Words: Rendezvous, Halo Orbit, Lunar Mission, Invariant Manifold, Homoclinic Intersection

Nomenclature

\( x \): state vector
\( \Delta v \): impulsive velocity change vector
\( t, \tau \): time
\( F(x, t) \): forward integration of the equations of motion for \( t \) given \( x \) as an initial state
\( \mu \): mass of the Moon
\( r_1 \): distance to spacecraft from Earth
\( r_2 \): distance to spacecraft from Moon
\( \lambda \): eigenvalue

1. Introduction

Two potential scenarios toward the goal of human exploration of Mars in the next 25 years, “Asteroid Next” and “Moon Next,” have been integrated into a single reference mission scenario. This entails a set of missions in the lunar vicinity and on the lunar surface in the late 2010s and the 2020s as the first targets in preparation for a manned Mars mission after 2030.\(^1\) The main reasons for this change were capability constraints of the Space Launch System (SLS) being developed by NASA and the early difficulties in the establishment of radiation-proof technology for deep space.\(^2\) Earth Moon Lagrange Point 2 (EML2) has received much attention in this context. The Block 1 SLS can insert Orion’s Multi-Purpose Crew Vehicle (MPCV) into the lunar vicinity.\(^3\) The halo orbit around EML2, located behind the Moon, is one such orbit and became a candidate location for the development of a space base for lunar missions. A space base in a halo orbit can be a great test bed for exploration technologies for future manned Mars missions. It can obviously contribute to lunar missions themselves, provide observations of the lunar far side and poles, enable remote control of rovers on the lunar surface, and become a link for materials supply and communication between the Earth and Moon.

To realize these missions, rendezvous technology for a visiting space vehicle to a space base in a halo orbit is necessary. Because of its dynamical properties, a rendezvous in a halo orbit might be significantly different from one in a low-Earth orbit, where visiting vehicles, like the Japanese H-II Transfer Vehicle\(^4\) (HTV), have successfully rendezvoused with the International Space Station. In comparison to a low-Earth orbit, a halo orbit is not centered by a heavy celestial body but by an equilibrium point, so the gravitational field is quite shallow and its periodic motion takes much longer time than in a low-Earth orbit. Additionally, because of the non-dominant gravitational field, the position relation of the Sun or other planets with respect to the Earth and Moon might affect the relative motion of the vehicle visiting the space base.

A recent study of rendezvous in a halo orbit includes the application of a rendezvous control law for a small halo orbit around EML2\(^5,6\) and a design for rendezvous trajectories between different halo orbits with large Z-amplitude around the L2 point of the Sun-Earth system for a spacecraft using a low-thrust engine, like a variable specific impulse engine.\(^7,8\) The volume of research on optimal guidance of spacecraft to the Lagrange points or their halo orbits from outside is more significant than that on halo orbit rendezvous; however, these two are closely related in that they can both utilize invariant manifolds. It is well known that the use of invariant manifolds in three-body dynamics can significantly save fuel in many ways. A dynamical structure of an unstable manifold and a stable manifold of two halo orbits of different Lagrange points, called the heteroclinic intersection, can provide various kinds of low-fuel transfer trajectories between the orbits.\(^9,10\) A dynamical structure of manifolds of different halo

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orbits belonging to the same Lagrange points or those manifolds between quasi-periodic orbits around a halo orbit, called the homoclinic intersection, can also provide such trajectories. The homoclinic intersection of an individual halo orbit in the planar case can be used to change the phase of the spacecraft in the halo orbit using almost zero fuel. Given a target spacecraft at the end point of the target orbit, these methods can be extended to solve rendezvous problems.

In this work, we investigated a strategy for rendezvous in a halo orbit around EML2 with an amplitude of several ten-thousand kilometers. We assumed that the two spacecraft initially fly in the same halo orbit with a distant phase and finally rendezvous in the halo orbit by allocating a set of maneuvers only to the chaser. An important issue is that the required $\Delta t$ for the rendezvous rapidly increases with the amplitude of the halo orbits and the initial phase difference increases, as earlier work indicates. We addressed this issue by utilizing the homoclinic intersection of the halo orbit. If the initial phase difference is large, we propose to select such a trajectory so that the chaser first departs from the halo orbit along the unstable manifold, connects to the stable manifold through the intersection, and then returns to the halo orbit. The time of arrival in this detour can be adjusted with low fuel consumption, and consequently the total required $\Delta t$ is reduced.

In Section 2, we introduce the dynamical model of the Earth-Moon system and the considered halo orbit where the rendezvous is to occur. In Section 3, we present a rendezvous method in which the chaser (visiting vehicle) approaches the target (space base) from behind along the orbit, like a rendezvous in a low-Earth orbit. In Section 4, we propose a rendezvous method utilizing the homoclinic intersection and demonstrate that it is more fuel efficient than the first method if the initial phase difference of the two spacecraft is large.

2. Dynamics of the Earth-Moon System

2.1. Model

The dynamical model of the Earth-Moon system used in this work is the Circular Restricted Three-Body Problem (CRTBP), where the Earth-Moon motions are modeled as circular orbits around their barycenter, and any effects on them from the spacecraft motion are ignored. Although eccentricity and forces due to other celestial bodies might affect the relative motion of the two spacecraft, since they do not change the characteristics of the halo orbit significantly, we ignored them and assumed this approximation to be sufficient for this fundamental investigation.

The model is normalized by selecting the Earth-Moon distance as the unit of length, the orbital period of their motion as the unit of time, and the sum of their masses as the unit of mass. A rotating frame with its origin at the barycenter is the coordinate frame, where the $xy$ plane is the orbital plane of the Earth and Moon, the $x$ axis lies along the connecting line from the Earth to the Moon, and the $y$ axis is perpendicular to it. In CRTBP, the normalized equations of motion of the spacecraft in the rotating frame are described as follows:

$$\dot{x} = f(x) = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} 2\dot{y} - \frac{\partial U}{\partial x} - 2\dot{z} - \frac{\partial U}{\partial y} - \frac{\partial U}{\partial z} \end{bmatrix}$$

(1)

where $x = [x \ y \ z \ \dot{x} \ \dot{y} \ \dot{z}]^T$ is the state vector consisting of the position and velocity of the spacecraft with respect to the origin, and the potential $U$ is defined as follows:

$$U = -\frac{1}{2}(x^2 + y^2) - \frac{1 - \mu}{r_1} - \frac{\mu}{r_2} - \frac{1}{2}(1 - \mu)\mu$$

(2)

There exist five equilibrium points called Lagrange points, L1 to L5, and the family of periodic orbits around these points is called a halo orbit. In this work, we focus on the L2 point behind the Moon and its halo orbit. Figure 1 shows the coordinate frame of CRTBP and the location of the Earth, the Moon, and the Lagrange points. [n/d] indicates the non-dimensional unit of position.

2.2. Halo orbit

For sufficient visibility from the Earth, a halo orbit with an amplitude of several ten-thousand kilometers can be the orbit where the target space base is placed. Since such a halo orbit is much larger than halo orbits that can be approximated using a linear analytical model, we numerically generated a halo orbit for this work. One of the simplest methods for the calculation of a halo orbit with large amplitude is presented here. A halo orbit in a rotating frame with its origin at the Lagrange point is approximated as follows:

$$\begin{cases} x = -A_x \cos(\lambda t + \alpha) \\ y = k A_x \sin(\lambda t + \alpha) \\ z = A_z \sin(\lambda t + \beta) \end{cases}$$

(3)

where $k$ and $\lambda$ are given as

$$k = \frac{2l}{L_1^2 + 1 - c_2}$$

(4)

$$\lambda = \sqrt{-4 + 2c_2 + 2\sqrt{9c_2^2 - 8c_2}}\frac{2}{2}$$

(5)

$$c_2 = 1 - \frac{\mu}{R_1^3} + \frac{\mu}{R_2^3}$$

(6)

Fig. 1. Coordinate frame of CRTBP and its Lagrange points.
$R_1$ and $R_2$ are the distances to the Lagrange point from the primary body (i.e., Earth) and from the secondary body (i.e., Moon), respectively. This approximation was used as an initial guess in order to numerically generate the halo orbit by differential correction.

Due to the symmetry of the equation of motion with respect to the $xz$ plane for forward and backward time and for continuity, the velocity components in the $x$ and $z$ directions are zero when the orbit intersects with the $xz$ plane and the $z$ coordinate at the two intersections is maximum and minimum, respectively. Considering this property and setting $\alpha = 0, \beta = \pi/2$, the initial estimate of the state $x_0$ at the $xz$ plane with maximum $z$ is given as

$$x_0 = [-A_x, 0, A_z, 0, \lambda k A_x, 0]$$

The initial estimate of the period $T$ is given as

$$T_0 = \frac{2\pi}{\lambda}$$

Based on the symmetry, the $x$ position, $y$ velocity, and half-period time are iteratively corrected in the following way.

$$Z_0 = [-A_x, \lambda k A_x, T_0/2]$$

$$x_i = [Z_i(1), 0, A_z, 0, Z_i(2), 0]$$

$$Y = F(x_i, Z_i(3))$$

$$Z_{i+1} = Z_i - \begin{bmatrix}
\Phi_{2,1} & \Phi_{2,5} & f_2(Y) \\
\Phi_{3,1} & \Phi_{4,5} & f_3(Y) \\
\Phi_{6,1} & \Phi_{6,5} & f_6(Y)
\end{bmatrix}^{-1} \begin{bmatrix}
Y(2) \\
Y(4) \\
Y(6)
\end{bmatrix}$$

$\Phi$ in Eq. (9) is a state transition matrix $\Phi(x_i, Z_i(3))$ obtained by integrating Eq. (10) where $I_6$ is an identity matrix of size six.

$$\Phi(x_i, 0) = I_6$$

$$\begin{bmatrix}
\dot{x} = f(x) \\
\Phi = \nabla_x f(x) \cdot \Phi
\end{bmatrix}$$

In this work, given the Earth to Moon distance as $r_{12} = 3.844 \times 10^8$ km, we used $A_x = 12000/r_{12}$ and $A_z = 10000/r_{12}$, and then obtained a halo orbit around EML2 with amplitudes of 12,320, 35,518, and 12,069 km on each axis shown by the trajectory with a thicker line in Fig. 2. This halo orbit was used for investigation in the following sections.

A specific energy constraint can be added to the equation to precisely adjust the amplitude. Given a specified Jacobi constant $CJ_{val}$, the simultaneous equation is modified as Eq. (11) and the $z$ position becomes a variable to be solved. The differential correction in Eq. (9) is also modified accordingly.

$$Y = F(x, T/2)$$

$$\begin{bmatrix}
Y(2) = 0 \\
Y(4) = 0 \\
Y(6) = 0 \\
CJ(x) - CJ_{val} = 0
\end{bmatrix}$$

Starting from the halo orbit, we computed a family of halo orbits with different Jacobi constants using numerical continuation. The family of halo orbits is shown by trajectories with thinner lines in Fig. 2. Figure 3 shows the orbital period and the maximum value of the $z$ position of the halo orbits with different Jacobi constants. The smaller the Jacobi constant becomes, the amplitudes of the halo orbit, especially the $z$ component, grow larger, and then the orbital period becomes slightly shorter. On the other hand, the larger the Jacobi constant becomes, the smaller the amplitudes of the halo orbit are, and the $z$ component is reduced to zero. Then the orbit becomes a horizontal Lyapunov orbit, at which time the orbital period is the longest. If we give a negative $A_z$ in Eq. (3), a symmetric family of halo orbits with respect to $xz$ plane can be generated.

2.3. Phase angle in halo orbit

Figure 4 shows the definition of the phase angle in the halo orbit used in this work. The intersection point at the $xz$ plane with maximum $z$ was used as reference point A. The phase angle $\theta$ of point B, which takes time $T$ from point A, is $\theta = \omega T$, where $\omega$ is the mean angular velocity. We define $x_{halo}(\theta)$ as the state vector in the halo orbit, corresponding to the phase angle $\theta$, and $T_{halo} = 2\pi/\omega$ as the orbital period of

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the halo orbit. $T_{\text{halo}}$ of the halo orbit used in this work is approximately 14.8064 d, 3.4044 in normalized time.

3. Rendezvous by Phasing along the Orbit

Hereafter, the visiting vehicle is referred to as the chaser and the space base as the target. We assume that the chaser is already inserted in the same halo orbit with the target but their phase angles are different. In this section, we investigate a rendezvous method for a halo orbit that is similar to those used in low-earth orbits. In a low-Earth orbit, a chaser spacecraft is usually first inserted to an orbit lower than that of the target, having a negative phase difference to the target. Because the difference of the orbital period due to the height difference is significant, the chaser can reduce the phase by time, and then finally approach the target by carefully increasing its height with a set of maneuvers. As shown in Fig. 3, the halo orbit also has the dependency of orbital period on its size. The same strategy might be employed in the case of a halo orbit and is worth investigating.

3.1. Formulation of the problem

If the initial distance of the chaser to the target is up to several thousand kilometers, the rendezvous trajectory can be designed in a semi-analytical way by using a state transition matrix. However, we formulated the trajectory design problem using nonlinear optimization so that it can be applied to cases with a large initial distance (such as several ten-thousand kilometers). Given the initial states $x_0$ of the chaser and $x_0'$ of the target, the whole trajectory from the beginning to the rendezvous was divided into $N$ segments, each of which was connected to the adjacent segment by an impulsive velocity change (i.e., an impulsive maneuver). The state variable $Z$ to be optimized includes the state vector of the chaser at each connecting point except the initial point (N vectors), the velocity change vector at each point ($N + 1$ vectors), and the amount of time assigned to each segment ($N$ scalar variables in total).

$$Z = \left[ \Delta v_1, t_1, x_2, \Delta v_2, t_2, x_3, \ldots, \Delta v_N, t_N, x_{N+1}, \Delta v_{N+1} \right]$$

where

$$\Delta v_i = [\Delta v_{i1}, \Delta v_{i2}, \Delta v_{i3}]$$,

$$x_i = [x_{i1}, y_{i1}, z_{i1}, v_{i1}, v_{i2}, v_{i3}]$$.

The chaser conducts the impulsive maneuvers $\Delta v_i$ at each connecting point, whose relative time $t_{i-1}$ with respect to the beginning is $t_{i-1} = \sum_{j=1}^{i-1} t_j$ ($i = 1 \ldots N + 1$). The time of rendezvous is $t_N$.

The objective function $J$ is given as the sum of the Euclidean norm of the velocity change vector.

$$J = \sum_{i=1}^{N+1} (\Delta v_{i1}^2 + \Delta v_{i2}^2 + \Delta v_{i3}^2)$$

Although an objective function expressed in l1-norm is the most suitable for actual fuel consumption, it is possible that one of the l1-norms of $\Delta v_i$ becomes much larger than that of other maneuvers. By choosing the objective function to be expressed in Euclidean norm, the l1-norm of each maneuver can be smoothed. In other words, the maximum $\Delta v$ among all individual maneuvers is suppressed. This is especially preferable for using a low-thrust engine, like an electric propulsion system.

The equality constraints are given in Eq. (15). The third equation indicates that the relative position and velocity between the target and the chaser become zero at the end, which is the rendezvous.

$$x_2 - F([x_0^1, 0, 0, \Delta v_1], t_1) = 0$$

$$x_{i+1} - F([x_i + [0, 0, \Delta v_i], t_i) = 0 \ (i = 2 \ldots N)$$

$$[x_{N+1} + [0, 0, 0, \Delta v_{N+1}]] - F(x_0', \sum_{i=1}^{N} t_i) = 0$$

We considered the use of a low-thrust propulsion system. This is included in the following inequality constraints.

$$\sqrt{\Delta v_{i1}^2 + \Delta v_{i2}^2 + \Delta v_{i3}^2} \leq 0.005 \ (i = 1 \ldots N + 1)$$

$$0.08 \leq t_i \ \text{for} \ i = 1 \ldots N$$

The first constraint limits the amount of velocity change in each maneuver. The second constraint gives sufficient span between each maneuver so that the resulting discrete control input can be converted to a continuous control trajectory, which is attainable for a low-thrust engine like an electric propulsion system. A normalized velocity value of 0.005 corresponds to about 5 m/s and a normalized time value of 0.08 to approximately 0.35 d. We gave an additional inequality constraint to limit the total time to less than 2.5 times of the period of the halo orbit.

$$\sum_{i=1}^{N} t_i \leq 3.4044 \times 2.5$$

3.2. Computation of the rendezvous trajectories

Since the amount of velocity change in each maneuver is limited by Eq (16), segment $N$ should be given a larger number if the sum of the required velocity changes for the whole trajectory gets larger. On the other hand, if large $N$ is given in the computation of a trajectory whose sum of the required velocity change is small, it may unnecessarily elongate the total time because the amount of time assigned to each segment is lower bounded by Eq. (17). Additionally, the $N$ required to have optimization converged depends on the nonlinearity of the dynamic with respect to states and time. In consideration of the above situations, in computation of each rendezvous trajectory, we incremented $N$ until the optimal solution was found from small $N$, resulting in different $N$ for different solutions. $N$ is larger when the total
velocity change is larger and/or the total time is longer.

We computed rendezvous trajectories for various initial phases $\theta$ of the chaser and various phase differences $\psi$ of the target with respect to the chaser. In a nominal flight plan, the chaser might be inserted to the position about 1,000 km behind the target at most, as it is usually in a low-Earth orbit. This distance corresponds to the phase difference of just 1 to 2 deg. Larger phase differences we consider in this work may not be assumed in a nominal flight plan. But the availability of rendezvous from these initial differences is desirable for the flexibility of flight plans. For each set of $\theta$ and $\psi$, an initial guess of the state variable $Z$ was given by equally dividing the halo orbit by time starting from $x_0 = x_{\text{hub}}(\theta)$ and ending at $x_f = x_{\text{hub}}(\theta + \psi)$.

In order to understand the characteristics of the trajectory, we first examined two cases of positive and negative phase difference $\psi$. Figure 5 shows the trajectories of the chaser and target when $\theta = 0$ and $\psi = 60$ deg, and $\theta = 180$ and $\psi = -60$ deg, respectively. The family of halo orbits in Fig. 2 is also shown to express how the trajectory of the chaser utilizes an orbit along the halo orbit of the target. In both cases, the chaser does not exactly take an orbit from the family. But it takes a trajectory having a larger amplitude of $z$ position when $\psi$ is positive, and takes a lower amplitude when $\psi$ is negative. These results are consistent with the relationship between the maximum value of $z$ position and the orbital period shown in Fig. 3. When the target is ahead of the chaser, the chaser takes a trajectory with larger $z$ position to increase the orbital period. On the other hand, if the target is behind the chaser, the chaser takes a trajectory with smaller $z$ position to reduce the orbital period.

Figure 6 shows the time history of the impulse maneuvers in the two cases. In both cases, most of the Euclidean norm of velocity change in each maneuver is almost the upper bound given by Eq. (16). The second case takes a longer time for rendezvous. This is because the trajectory is designed so that the chaser approaches the target from behind even when the chaser is initially inserted ahead of the target.

Figure 7 summarizes the total velocity change and total time required to rendezvous for cases of various $\theta$ and $\psi$, where $\psi$ was varied from 0 deg to 350 deg at intervals of 10 deg. For each $\psi$, 10 cases of the initial phase $\theta$ were investigated by equally dividing 360 deg. As $\psi$ gets larger, larger velocity change and time are required. The total velocity change monotonically increases as $\psi$ increases, becoming the maximum at $\psi = 180$ deg, and then proportionally decreasing as $\psi$ reaches 350 deg. The total time reaches its upper bound given in Eq. (18) around $\psi = 180$ deg. Both the total velocity change and total time are almost independent of the initial phase $\theta$ of the chaser. Table 1 shows the total velocity changes, total time, segment $N$ number, and the rendezvous trajectory of the cases $\psi = 20, 60, 120, 180$ deg with $\theta = 0$. As $\psi$ gets larger, the chaser’s trajectory gets more angled with respect to the $xz$ plane and it occupies the inner side of the halo orbit of the target.

4. Rendezvous by Detouring Invariant Manifolds

In the previous section, we investigated the rendezvous by phasing along the orbit in which the difference of orbital period due to the size of the halo orbit was utilized. Although the computed trajectories showed the effectiveness of utilizing this structure to rendezvous, the required total velocity change proportionally increases as the initial phase difference $\psi$ increases, and the maximum exceeds 400 m/s when $\psi = 180$ deg (i.e., the case chaser is inserted entirely opposite of the target). In order to reduce the required total velocity change in such cases, we investigate another structure of the halo orbit in this section.
demonstrated in several missions.\textsuperscript{15} These manifolds in energy-saving trajectory design\textsuperscript{13,14}) are known as unstable and stable manifold, respectively. The use of trajectories winding into the halo orbit. These families are both unstable and stable eigenvalues, which indicate a family of trajectories winding away from the halo orbit and a family of trajectories winding toward the Moon side. The monodromy matrix of a halo orbit around EML2 has a backward time of one orbital period is called a monodromy matrix.

4.1. Invariant manifolds and their connection

The unstable manifold and stable manifold were computed for a rendezvous on a halo orbit with a large initial phase difference between two spacecraft, we propose using a homoclinic intersection of the unstable and stable manifolds. Figure 8(a) shows the unstable (red) and stable (green) manifolds of the halo orbit extended toward the Moon side. The unstable manifold and stable manifold were computed by integrating perturbed states on the halo orbit until they intersect with the $xz$ plane behind the Moon forward time and backward time, respectively. The perturbation was $10^{-6}$ multiplied by unstable and stable unit eigenvectors, respectively. Figure 8(b) and (c) show how these two manifolds are intersected in the $xz$ plane. From Fig. 8(b), these two manifolds entirely overlap in terms of position. In contrast, Fig. 8(c) shows that the velocities of each manifold only partially overlap, indicating that these two manifolds are not continuously connected. These are not connected naturally, but can be connected by applying small adjustments. Implementing this property in the design of the chaser’s trajectory can address the issue of the total increasing $\Delta v$ with the increasing initial phase difference, as in the previous rendezvous method. The objective is a detour via the homoclinic intersection to adjust the time of the chaser’s arrival back to the halo orbit and reduce the total $\Delta v$ to achieve a lower cost.

| Initial phase difference: 20 deg | Initial phase difference: 60 deg | Initial phase difference: 120 deg | Initial phase difference: 180 deg |
|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| Total $\Delta v$: 61.2 m/s      | Total $\Delta v$: 180.0 m/s     | Total $\Delta v$: 331.2 m/s     | Total $\Delta v$: 397.0 m/s     |
| Total time: 14.5 d              | Total time: 19.7 d              | Total time: 30.2 d              | Total time: 33.9 d              |
| $N = 12$                        | $N = 36$                        | $N = 72$                        | $N = 90$                        |

Fig. 8. (a) Unstable and stable manifolds of the halo orbit. (b) (c) State variables at their intersections in the $xz$ plane.

| Initial phase difference: 20 deg | Initial phase difference: 60 deg | Initial phase difference: 120 deg | Initial phase difference: 180 deg |
|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| Total $\Delta v$: 61.2 m/s      | Total $\Delta v$: 180.0 m/s     | Total $\Delta v$: 331.2 m/s     | Total $\Delta v$: 397.0 m/s     |
| Total time: 14.5 d              | Total time: 19.7 d              | Total time: 30.2 d              | Total time: 33.9 d              |
| $N = 12$                        | $N = 36$                        | $N = 72$                        | $N = 90$                        |

Table 1. Comparison of the rendezvous trajectory in four cases.
The principle of this effect is based on the method of generating invariant manifolds. A set of unstable and stable invariant manifolds taking different times from the halo orbit to the intersection plane can be generated by adjusting the amount of perturbation applied at a point on a halo orbit to the direction of the corresponding eigenvectors. If the perturbation is small, the time (forward for unstable manifold and backward for stable manifold) required to reach the intersection plane becomes longer, and if the perturbation is large, it becomes shorter. By selecting the perturbations and connecting the two manifolds appropriately, various detouring trajectories can be generated with the desired time. We searched for such a trajectory using nonlinear optimization.

4.2. Computation of the rendezvous trajectories

Given the initial phase angle $\theta$ of the chaser and the phase difference $\psi$ of the target, we generated the rendezvous trajectory by nonlinear optimization with the same formulation as the previous section except for an additional inequality constraint shown in Eq. (19).

$$\sqrt{(x_i - (1-\mu))^2 + y_i^2 + z_i^2} \geq 0.03 \quad (i = 2 \ldots N + 1) \quad (19)$$

By giving a lower bound of the distance to the Moon, this constraint keeps the trajectory from getting too close to the Moon. The distance value of 0.03 corresponds to about 10 km. If the trajectory passes near the Moon, it may significantly reduce the required velocity change in the maneuvers due to the gravity effect of the Moon; however, it is sensitive to control error and less robust. We didn’t compute such a trajectory in this work.

For each set of $\theta$ and $\psi$, an initial guess of $Z$ was given by equally dividing such trajectory that the first half is a part of the unstable manifold starting from $x_{\text{halo}}(\theta)$ and the last half is a part of the stable manifold connected with the first half on $xz$ plane in terms of position.

Figures 9 and 10 show an example of the trajectory with $\theta = 0$ and $\psi = 180$ deg. The number of $N$ segments is 60. The chaser first departs from the halo orbit along the unstable manifold, and subsequently returns to the halo orbit along the stable manifold from behind the Moon, and finally rendezvous with the target. Figure 11 shows the trajectory of the two spacecraft in the inertial frame, which presents a more intuitive view of the trajectories. Figure 12 shows the time history of the impulse maneuvers. The total $\Delta v$ is 122.4 m/s and the total time is approximately 2.45 $T_{\text{halo}}$, 36.3 d. Although the total $\Delta v$ is still large, it is much smaller than the rendezvous in the previous section with the same initial phase condition and in the equivalent time for rendezvous. Furthermore, the maximum $\Delta v$ among 61 maneuvers is 4 m/s and the minimum time assigned to each segment is 0.35 d, resulting in a requirement of $1.3 \times 10^{-4}$ m/s² acceleration capability. Given that the chaser mass is 10 tons, similar to the HTV, a propulsion system with about 1.3 N is required. This could be attainable by a set of low-thrust electric engines of several hundred mN class.

4.3. Comparison of the total $\Delta v$ for various phases

We generated the rendezvous trajectories for various $\psi$ from 10 deg to 320 deg at intervals of 10 deg. For each $\psi$,
The total Δv and total time for rendezvous by detouring the invariant manifolds. 10 cases of the initial phase θ were investigated by equally dividing 360 deg. The total Δv and total time for one set of solutions are shown in Fig. 13.

The total Δv is almost independent of the initial phase θ and becomes the minimum at θ = 160 deg. The total time shows some dependence on both θ and ψ. In both figures, the results of cases where a feasible solution was not found are not shown. Except for cases when θ = 0, 300, 330 deg, a feasible rendezvous solution was not found for large ψ. For example, from θ = 210 deg, the rendezvous trajectory gets closer to the Moon to reduce the amount of Δv at maneuvers due to the gravity effect, resulting in shorter time as ψ increases up to 270 deg. But if ψ is more than 270 deg, the trajectory needs to take a closer path to the Moon, lower bounded by Eq. (19), to achieve the rendezvous within other constraints in Eqs. (16), (17), and (18).

The dependence of the result on θ is due to the time that it takes to reach the xz plane from the halo orbit along the unstable manifold even when the same amount of perturbation is different for different θ.

We compared the total Δv of the two rendezvous methods investigated in this work. Given the independence of the initial phase θ, the total Δv for a different initial phase difference ψ is shown in Fig. 14. This comparison is appropriate because the total time is almost the same in both trajectories, as shown in Figs. 7 and 13. The total Δv for each ψ is averaged for θ. The total Δv in the first trajectory (phasing along the orbit) is smaller than the second trajectory (detouring by manifolds) for an initial phase difference smaller than 70 deg, while the second method becomes fuel efficient for larger values up to 280 deg, resulting in the conclusion that, with both methods, the total Δv can be smaller than 220 m/s for any initial phase difference.

5. Conclusion

For a set of missions in the lunar vicinity and on the lunar surface in the late 2010s and in the 2020s, a strategy has been proposed to rendezvous at a target space base located on a halo orbit around EML2 with an amplitude of several thousand kilometers. Considering a target and chaser spacecraft in the same halo orbit with an initial phase difference, the rendezvous trajectory of the chaser can be selected from two different types of trajectories depending on the initial phase difference. In the first trajectory, the chaser approaches the target along the orbit from behind, like a rendezvous in a low-Earth orbit. The second trajectory uses a homoclinic intersection of unstable and stable manifolds of the halo orbit, where the chaser’s trajectory is controlled so that it first departs from the halo orbit along the unstable manifold, connects to the stable manifold, and then returns to the halo orbit along the manifold. We found that this detour can adjust the time of arrival to the rendezvous with low fuel consumption so that the total Δv can be significantly reduced if the initial phase difference is large. Numerical simulations in CRTBP showed that the total Δv in the first method is smaller than that in the second method for an initial phase difference smaller than 70 deg and greater than 280 deg, while the second method is fuel efficient if it is between, resulting in the conclusion that the total Δv can be smaller than 220 m/s for any initial phase difference with both methods. In other words, this work demonstrated that there are at least two local optimal rendezvous trajectories for a given initial phase angle of the target and chaser. It is worth noting that the order of fuel efficiency of these two trajectories changes, and both trajectories have their extremas at the phase difference of around 180 deg. Further investigation of this complex solution space from the viewpoint of nonlinear dynamics will constitute future work.

The implementation of this strategy can significantly increase the flexibility of the flight plan, enlarging the launch window of the visiting vehicle and enhancing the tolerance against failures in comparison with the application of a traditional phasing method. For practice, investigation of the robustness of these two trajectories against disturbance will constitute future work.

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