Kinematic analysis of a spatial mechanism for estimating shaking effects

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Abstract. Spatial mechanisms are the most general category of kinematic devices. They offer the greatest capability to accomplish any desired kinematic task. A mechanism exerts forces and moments on its supporting frame, which result in vibration. Besides of its effect on efficiency, reducing vibration has become inevitable in the current industrial environment where stern standards on noise and vibration prevail. Balancing of shaking forces and shaking moments in mechanisms is important in order to improve their dynamic performance and fatigue life by reducing vibration, noise and wear. The analysis and synthesis of spatial mechanisms which involves extensive vector mathematics and linear algebra is to be simplified to be taught to engineers in undergraduate education. In the present paper the kinematic analysis of a spatial four-link RSCR mechanism is done to get the velocities and accelerations of its various links which is necessary for the estimation of inertia forces in a mechanism.

1. Introduction
In a world where the standards of living increase every day, today’s engineers face an incredible challenge to use technology to answer the growing needs of the population and therefore keep searching for better performing machinery, for higher efficiency. Building high-performance machinery requires limiting machine vibration and the associated dynamic problems, such as fatigue, noise and wear. Besides its effect on efficiency, reducing vibration has become inevitable in the current industrial environment where stern standards on noise and vibration prevail. A possible side effect of machine vibrations is for instance the transmission of significant forces to the machine floor, resulting in disturbing effects in the building in which the machine is placed as well as its surroundings. With this in mind current research focuses on reducing the vibrations caused by the mechanisms in high-speed machinery. A high speed mechanism exerts fluctuating forces and moments on its supporting frame. These forces and moments, of which the resultants are denoted as the shaking force and the shaking moment, induce vibration, on the frame\cite{1,2}. One strategy to suppress these vibrations is to add counterweights to the mechanism moving links. Allowing more counterweight mass, for instance, may give better balancing results, but increases cost or compromises the start procedure of the mechanism and its flexible behavior. Also the space for counterweights is not always available and creating such space can increase the cost. Balancing of shaking forces and shaking moments in spatial mechanisms is important in order to improve their dynamic performances and fatigue life by reducing vibration, noise and wear\cite{3,5}. Spatial mechanisms have become the subject of extensive research interest recently. Robotic devices are becoming increasingly important in manufacturing, materials manipulation
in hazardous environment, and in numerous fields of application. Also it is feasible to incorporate complex kinematic requirements in the rational design of such mechanisms. However, prior to wide adaption of spatial mechanisms in high-speed and precision-operating mechanical systems, we must have better knowledge of their dynamic characteristics[6,7]. In the present paper, the kinematic analysis of a spatial four-link RSCR mechanism having two revolute joints, one spherical joint along with one cylindrical joint is done to get the velocities and accelerations of its various links.

2. Spatial mechanisms
Well defined analysis and synthesis techniques of planar mechanisms are readily available and are taught to many mechanical engineers. Additionally, there is a variety of synthesis software available for planar mechanism design. On the other hand, spatial mechanism synthesis and analysis is typically not taught to engineers in undergraduate education. Techniques for analysis and synthesis of spatial mechanisms usually involve extensive vector mathematics and linear algebra techniques. Even for the experienced mechanical designer, it is labor intensive and difficult to design a spatial mechanism. The visualization of these devices can even be difficult. An unbalanced linkage running at high speed transmits shaking forces and shaking moments to its foundation (frame). The shaking force is the resultant inertia force exerted on the frame and is equal to the vector sum of the inertia forces associated with the moving links of the mechanism. The shaking moment about an axis in the frame is the vector sum of the inertia torques and the moments of the inertia forces about the axis. These forces and moments cause vibrations, fluctuations in the input torque and stresses, and therefore impose limitations on the performance of high-speed machinery. There has been a need to develop the optimum balancing of general three-dimensional mechanism. The majority of mechanisms synthesized and found in application are planar devices. These mechanisms have motion such that all elements move in one plane or in parallel planes. While planar mechanisms have been broadly applied, they lack the ability to perform many general motion-control tasks. In fact, planar mechanisms, as well as spherical mechanisms, form a subset of spatial mechanisms. Spatial mechanisms are the most general category of kinematic devices. They offer the greatest capability to accomplish any desired kinematic task. Spatial mechanisms can be composed of any number of links and can include joints with any combination of rotational and translational freedom. It should be noted that robotic manipulators provide controlled actuation of all the joints in the system, which results in a large number of degrees of freedom. This flexibility may be not necessary for the given task, in which case the manipulator could be replaced with a single degree-of-freedom spatial mechanism. If spatial mechanisms could be synthesized quickly and easily, there could be many suitable applications. Some examples include the aerospace industry, exercise equipment, and the rehabilitation medical field. For example, spatial mechanisms are especially useful in satellite design and deployment. Basically, spatial mechanisms could be applied to any task that requires general spatial motion.

3. Kinematic analysis of a spatial four-link RSCR mechanism
The industry is giving top priority to keep up with the current stern norms on noise and vibrations and to limit the workers’ exposure to ‘physical agents like vibrations and noise. If a choice between two ‘equivalent’ machines is possible, the less noisy/vibrating machine will be the natural preference. Less vibrating machines will contribute to consolidating and extending market. The present kinematic analysis is useful in estimating the inertia forces acting on the frame which in turn produce vibrations. The method of constant-distance equation for the kinematic analysis of linkages discussed by Rao [2] is based on fact that a constant-distance equation is formulated, wherever the distance between two pair-centers of rigid link remains
constant throughout its motions. This results in a much simpler kinematic analysis of the linkages. To illustrate the procedure and the feasibility of the method, the case of a single degree of freedom spatial four-link RSCR mechanism [2] with coupler points as shown in Figure 1 is considered. 

\[ R_1 = 0.148 \text{m}, \quad R_2 = 0.105 \text{m}, \quad R_3 = 0.100 \text{m}, \quad R_4 = 0.105 \text{m}, \quad \psi = 50 \text{ degrees}, \quad \gamma = 35 \text{ degrees}. \]

\[ U_p = 0.5, \quad V_p = 1, \quad W_p = 1, \quad X_c = 1, \quad Z_c = 1. \]

The set of equations thus form a displacement mathematical model of this mechanism and describes uniquely the linkage configuration. By differentiating the set of equations with respect to time, a complete solution of linkage-kinematics is obtained.

In an N-bar spatial linkage, the pairs of the linkage, which can be of any type, are designed by \( a, b, i, j, k, m, n \). Considering the constant distances between the pair-centers, a set of constant-distance equations can be written in the form

\[ F_n(X_a, Y_b, Z_c, X_j, Y_j, Z_j, \ldots X_n, Y_n, Z_n) = 0, \quad (1) \]

where \( n = 1, 2, \ldots, g \) and \( g \) is the number of constant distance equations. Using specified geometrical constraints and known quantities (eg., as demonstrated below by equations 7 and 8 for the case of RSCR Mechanism) equation 1 can be reduced to a set of non-linear, second order algebraic equations with \( g \) unknowns in the form

\[ [X_1, Y_2, Z_3, Y_4, \ldots Z_g]^T. \quad (2) \]

Solving above two equations and expanding by Taylor’s method, we have

\[ F_n(X_1, Y_2, Z_3, Z_4) + \delta X_1 \left[ \frac{\partial F_n}{\partial X_1} \right] + \delta Y_2 \left[ \frac{\partial F_n}{\partial Y_2} \right] + \delta Z_g \left[ \frac{\partial F_n}{\partial Z} \right] = 0. \quad (3) \]

Where \( \delta X_1, \delta Y_2, \delta Z_g \) are the correction terms equation 3 is valid only for small increments of the input angle \( \theta \) of the linkage. In a spatial four-bar RSCR linkage with coupler point P, the driver \( R_2 \) rotates about the Z axis. \( \theta \) is the input angle and \( \phi \) is the output angle. \( R_3 \) is the coupler link with the coupler point P, and b is the origin of the moving co-ordinate system, UVW. \( R_4 \) is the output link, rotates about the Y- axis. \( \gamma \) and \( \psi \) are the constant angles determining the orientation of \( R_1 \) and \( R_2 \). The following set of constant-distance equation can be written

\[ [X_c - X_b]^2 + [Y_c - Y_b]^2 + [Z_c - Z_b]^2 = [R_3]^2, \]

\[ [X_d - X_c]^2 + [Z_d]^2 = [R_4]^2, \]

\[ [X_b, Y_b, Z_b] = [(R_2 \cos(\psi) \sin(\theta)), (R_2 \cos(\theta)), (R_2 \sin(\psi))], \]

\[ [X_c, Y_c, Z_c] = [(X_d + R_4 \sin(\phi)), (Y_b), (R_4 \cos(\phi))], \]

\[ [X_d, Y_d, Z_d] = [(R_2 \sin(\gamma)), (R_1 \cos(\gamma)), (0)]. \quad (4) \]
By Rearranging
\[ F_1(X, Z) = [X_c - R_2 \cos(\psi) \sin(\theta)]^2 + [Z_c - R_2 \sin(\psi)]^2 - [R_3]^2 = 0, \]  
\[ F_2(X, Z) = [X_c - R_1 \sin(\gamma)]^2 + [Z_c]^2 - [R_4]^2 = 0. \]  
The output angular displacement \( \theta \) is computed from
\[ \phi = \arctan\left[\frac{X_c - R_1 \sin(\gamma)}{Z_c}\right]. \]  
The coordinates of the coupler point P are given by the transformation matrix \( \lambda \) for this case
\[ \lambda = [(\lambda)_x u(0) \lambda_x w; (0)(1)(0); (\lambda)_z u(0)(\lambda)_z w], \]
\[ (\lambda)_x u = (\lambda)_z w = [X_c - R_2 \cos(\psi) \sin(\theta)]/R_3, \]
\[ (\lambda)_x w = -(\lambda)_z u = [R_2 \sin(\psi) - Z_c]/[R_3], \]
\[ [X_p; Y_p; Z_p] = [X_b; Y_b; Z_b] = [\lambda]([U_p]; [V_p]; [W_p]), \]
we have
\[ X_p = (\lambda)_x u(U_p) + (\lambda)_x w(W_p) + X_b, \]
\[ Y_p = V_p + Y_b, \]
\[ Z_p = (\lambda)_z u(U_p) + (\lambda)_z w(W_p) + Z_b. \]
Applying the constant distance equation 3 for the RSCR mechanism then we get the
\[ F_1 + \delta(X_c)[\partial F_1/\partial X_c] + \delta Z_c[\partial F_1/\partial Z_c] = 0, \]
\[ F_2 + \delta(X_c)[\partial F_2/\partial X_c] + \delta Z_c[\partial F_2/\partial Z_c] = 0. \]
Differentiating equations 4 and 5 with respect to the \( X_c, \) and \( Z_c \)
\[ [\partial F_1/\partial X_c] = 2[X_c - R_2 \cos(\psi) \sin(\theta)], \]
\[ [\partial F_1/\partial Z_c] = 2[Z_c - R_2 \sin(\psi)], \]
\[ [\partial F_2/\partial X_c] = 2[X_c - R_1 \sin(\gamma)], \]
\[ [\partial F_2/\partial Z_c] = 2[Z_c]. \]
Differentiating Equations (6) and (9) and rearranging, we obtain the set of linear equations for velocity and acceleration.

4. Results and discussions
The \( X, Y \) and \( Z \) coordinates of the linear displacement, linear velocity and linear acceleration of coupler point P of RSCR mechanism for different values of input angle \( \theta \) from 0 to 360 is calculated by writing a program in MATLAB and variation is shown in Figures 2 to 4. The angular displacement, angular velocity and angular acceleration of output link \( R_4 \) of Spatial RSCR mechanism for different values of input angle \( \theta \) from 0 to 360 is calculated by writing a program in MATLAB and variation is shown in Figure 5.
5. Conclusion
Because of their complexity, it is generally not practical to perform an analysis of spatial linkages by hand computation or by graphical methods. Spatial linkages have therefore attracted much research interest, following the advent of the high speed digital computers. Due to the inability of practicing engineers to design spatial mechanisms, many spatial kinematic tasks are either done by a robotic manipulator, a machine with multiple planar actuators, or are simply ignored for automation and are performed by human operators. The analysis and synthesis of spatial mechanisms which involves extensive vector mathematics and linear algebra is to be simplified to be taught to engineers. The present kinematic analysis is useful for estimating the complete shaking force and shaking moment acting on a spatial mechanism.
Figure 5. Variation of angular displacement, velocity and acceleration of output link with crank angle.

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