Dynamic analysis and application in medical digital image watermarking of a new multi-scroll neural network with quartic nonlinear memristor

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Abstract Memristor is widely used in various neural bionic models because of its excellent characteristics in biological neural activity simulation. In this paper, a piecewise nonlinear function is used to transform the quartic memristor, which is introduced into the ternary Hopfield neural network (HNN) with self-feedback, and a piecewise quartic memristive chaotic neural network model with multi-scroll is constructed. Through simulation analysis, the number of scroll layers changes with memristor parameters and has significant coexistence of multi-scroll attractors and high initial value sensitivity has been found. Using its excellent unpredictability, a digital watermarking algorithm based on wavelet transform is improved and used in the protection of personal medical data. The results show that it not only improves the confidentiality and convenience, but also ensures its robustness and has good encryption effect.

1 Introduction

Neural network is a mathematical model that imitates the structure and function of biological network. It is composed of multiple artificial neurons and can show complex dynamic characteristics through the interaction between these neurons. Therefore, neural network is a highly nonlinear dynamic system. As a powerful nonlinear system, neural network has a wide range of applications in information processing, expert system, robot control, pattern recognition and other fields [1–6].

Chaos has been widely studied by researchers since last century because of its extreme sensitivity to initial values, ergodicity, simple implementation and fast encryption speed [7–16]. Studies have shown that chaotic dynamic behavior may exist in biological neurons in human brain, and it plays an important role in associative memory. By analyzing the chaotic phenomenon of the brain, a neural network model with chaotic dynamics is established, which is called chaotic neural network model. In recent years, chaotic neural network has been widely used in random number generator [17–19], system optimization [20–22], network synchronization [23–27] and electronic circuit [28–31].

Memristor is an electronic component that characterizes the internal relationship between charge and magnetic flux. Since the HP memristor model was discovered in HP Laboratory in 2008 [32], various mathematical and physical memristor models and memristive chaotic systems have been proposed successively [33–40]. What is important is the memory characteristics of the memristor, which can effectively simulate the synapses of neurons. Moreover, the low power consumption and nanometer size enable the memristor to achieve high-density distribution of the human brain, which can be used to construct neural networks based on the memristor [41,42]. Memristive neural network (MNN) not only inherits the advantages of low power consumption and nano-size of memristor, but also contributes to the improvement of neural network performance [43,44]. In [45], a memristor model with multistability was constructed. Then multistable memristor was used to simulate synaptic connections in the Hopfield neural network (HNN), and the MNN successfully generated an infinite number of coexisting chaotic attractors. Based on HNN, Ref. [46] simulated neural burst by modeling two kinds of neural network models, and the simple neural network model proposed can produce rich bursting dynamics.

Compared with general chaotic systems, multi-scroll or multi-wing chaotic systems have more complex dynamic characteristics, so they are more suitable for random number generator, image encryption and secure communication [47–50]. Multi-scroll MNNs have become a research hotspot in artificial neural networks in recent years because they can show more complex chaotic behaviors and better simulate the interactions between neurons in biological neural networks. Zhang et al. [51] constructed a novel memristive HNN model with multi-double-scroll attractors by introducing a non-ideal flux-controlled memristor model into HNN. The parity of the number of double scrolls can be controlled flexibly by internal parameters of memristor. In [52], a no-equilibrium Hindmarsh–Rose (HR) neuron model with memristive electromagnetic radiation effect was proposed, which can generate multi-scroll hidden attractors with sophisticated topological structures.

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At present, with the development of the Internet, the popularity of content creation and the emergence of personal information disclosure, copyright protection has become a highly valued aspect. Gaps and deficiencies in the security of medical data and patient privacy, particularly during the COVID-19 pandemic, have left them vulnerable. Therefore, in the research field of secure communication, digital watermarking research hotspot arises at the historic moment. Digital watermarking can embed specific information into the original carrier and has good concealment. Changes in the carrier cannot be recognized by the human visual system, and effectively protect individual rights and interests. In 1993, Van Schyndel et al. [53] first proposed the concept of digital watermarking. With the advancement of research, the traditional digital watermarking has changed from basic time-domain system, and effectively protect individual rights and interests. In 1993, Van Schyndel et al. [53] first proposed the concept of communication, digital watermarking research hotspot arises at the historic moment. Digital watermarking can embed specific information into the original image to achieve better robustness. Swanson et al. [56] realized embedding watermark in video and had good effect. Ng et al. [57] used Laplace operator to replace the traditional Gaussian model to realize adaptive strength selection. However, the above research still cannot meet the security requirements in many use scenarios, such as personal medical data. Because of the particularity of these information, its security is the primary premise of use. Therefore, the use of chaotic encryption to further strengthen the confidentiality of digital watermarking has become a research hotspot.

In this paper, a magnetron piecewise nonlinear quartic memristor model is proposed based on the step piecewise function, and its hysteresis characteristics are verified. Then, the memristor model is introduced into an improved three neuron HNN with self-feedback, resulting in a MNN model with multi-scroll attractors. Through the simulation analysis of the attractor phase diagram, Lyapunov exponential spectrum and bifurcation diagram of the proposed MNN system, it is found that the system has controllable multi-scroll attractors, and the number of scrolls can change with the odd-even property of the memristor. It is further found that the system has complex coexisting chaotic attractors. Then, the time series file generated by the system has a high initial value sensitivity, and it is applied to the traditional wavelet transform digital watermarking algorithm, which effectively improves the security of the algorithm, while taking into account the invisibility and robustness, and realizes the protection of personal medical image data.

2 Mathematical model

2.1 Memristor model

The appearance of memristors makes the research of traditional chaotic neural networks burst out new vigor. Its memory, low power consumption and large capacity are very consistent in simulating synapses and building neurons. It can effectively improve the performance of neural network and simulate the dynamic characteristics of neural network. According to the memristor theory, a generalized voltage controlled or magnetic controlled memristor can be expressed as:

\[
\begin{aligned}
  i &= W(u) v \\
  \frac{du}{dt} &= f(u, v)
\end{aligned}
\]  

where \( W(u) \) is a continuous function as a variable, \( i, v, u \) represents the current, voltage and internal state variables through the memristor, respectively.

In 2018, Xia et al. [58] improved the two magnetron smooth multi-segmented quadratic nonlinear memristor and introduced them into the three-dimensional jerk system to establish a memristive multi-scroll jerk circuit. On this basis, a new nonlinear magnetron piecewise quartic memristor model is proposed and introduced into HNN to obtain a multi-scroll MNN chaotic system. The memristor can be expressed as follows:

\[
\begin{aligned}
  i &= W(u) v = (m + n f^4(u)) v \\
  \frac{du}{dt} &= f(u, v) = hv - lf(u)
\end{aligned}
\]  

where \( m, n, h, l \) is the positive parameter and \( f(u) \) is the internal state function of memristor, as shown below:

\[
f(u) = \begin{cases} f_1(u), & \text{for odd - scroll} \\ f_2(u), & \text{for even - scroll} \end{cases}
\]  

where

\[
\begin{aligned}
  f_1(u) &= \begin{cases} u, & i = 0 \\ u - \sum_{i=1}^{I} \text{sgn}(u + (2i - 1)) \\ -\sum_{i=1}^{I} \text{sgn}(u - (2i - 1)), & I = 1, 2, 3 \ldots \end{cases} \\
  f_2(u) &= \begin{cases} u - \text{sgn}(u), & j = 0 \\ u - \text{sgn}(u) - \sum_{j=1}^{J} \text{sgn}(u + 2j) \\ -\sum_{j=1}^{J} \text{sgn}(u - 2j), & J = 1, 2, 3 \ldots \end{cases}
\end{aligned}
\]
When the selected memristor internal functions are different, there will be different numbers of attractors with odd and even numbers, and the number of chaotic scroll layers can be controlled by changing the parameters. The number increases with the increase of $I$ and $J$. The step function $\text{sgn}(u)$ is a special continuous time function, a process from 0 to 1, which belongs to a singular function.

In order to verify our proposed memristor, we set parameters $m = 0.11$, $n = 0.03$, $I = 1$. The sinusoidal voltage source is $v = A_m \sin (Ft)$. When fixed $A_m = 1$, the variation of hysteresis curve with frequency $F$ is shown in Fig. 1a. When fixed $F = 0.6$, the change with amplitude $A_m$ is shown in Fig. 1b. It can be clearly seen that the hysteresis loop of the simulator shrinks at the origin and presents a lobe shape, and the curve lobe area changes with the change of amplitude and frequency. It is verified that the simulator has memristor characteristics and excellent properties.

2.2 Multi-scroll MNN model

A theoretically ideal neural network, HNN was proposed by Hopfield in 1984. Because of its excellent nonlinear characteristics and efficient mathematical model, it is widely used in the dynamic analysis of various biological nervous system activities. In recent years, HNN is also widely used in the field of nonlinear chaos. HNN is represented by a set of nonlinear ordinary differential equations corresponding to neurons. Its mathematical model can be expressed as follows:

$$\frac{C_i \, dx_i}{dt} = -\frac{x_i}{R_i} + \sum_{j=1}^{n} w_{ij} v_j + I_{i\text{ext}} \quad (6)$$

among them, $C_i$, $R_i$, and $x_i$ are the membrane capacitance, membrane resistance, and membrane potential of neurons $i$, respectively. $w_{ij}$ represents the connection weight between neurons $i$ and $j$, $v_j = \tanh(x_j)$ represents the activation function of neurons $I_{i\text{ext}}$ represents the external external stimulation.

According to Ref. [59], $C_i = 1$, $R_i = 1$, $I_{i\text{ext}} = 0$. In this paper, a new topological connection of ternary MNN is obtained by replacing a synaptic weight in Eq. (7) with a piecewise nonlinear memristor. The three neurons are connected to each other, as shown in Fig. 2. The synaptic weight matrix can be described as follows:

$$w = \begin{pmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{pmatrix} = \begin{pmatrix} -1.4 & 1.1 & kW(u) \\ 1.2 & 0 & -2 \\ -6 & 2.8 & 4 \end{pmatrix} \quad (7)$$

By replacing the synaptic weight $W(u)$ with the memristor model $W_{13}$ and adding a new control parameter, namely neuron coupling strength $K$, a new HNN model can be obtained as follows:

$$\begin{cases} x = -x - 1.4 \tanh(x) + 1.2 \tanh(y) - 6 \tanh(z) \\ y = -y + 1.1 \tanh(x) + 2.8 \tanh(z) \\ z = -z + kW(u) \tanh(x) - 2 \tanh(y) + 4 \tanh(z) \\ u = h \tanh(y) - lf(u) \end{cases} \quad (8)$$

where $x$, $y$, $z$, $u$ is the system variable, $m$, $n$, $h$, $l$, and $k$ are positive parameters, $k$ is the connection strength of memristor resistance coupling. In this system, the second neuron is selected as the input of memristor resistance simulator and affects the connection between the first neuron and the third neuron. The hyperbolic tangent function $\tanh$ is a neuron activation function, and its coefficient represents the connection strength between adjacent neurons.
In order to study its equilibrium point, we select \( f_2(u), J = 1, m = 0.53, n = 0.03, h = 3, l = 1.3 \) and set \( P = (\xi_1, \xi_2, \xi_3, \xi_4) \) as an equilibrium point of the equation, then the equilibrium point can be obtained from Eq. (9):

\[
\begin{align*}
0 &= -\xi_1 - 1.4 \tanh(\xi_1) + 1.2 \tanh(\xi_2) - 6\tanh(\xi_3) \\
0 &= -\xi_2 + 1.1 \tanh(\xi_1) + 2.8\tanh(\xi_3) \\
0 &= -\xi_3 + kW(\xi_4) \tanh(\xi_1) - 2 \tanh(\xi_2) + 4\tanh(\xi_3) \\
0 &= 3\tanh(\xi_2) - 1.3 f(\xi_4)
\end{align*}
\]  

(9)

where \( W(\xi_4) = 0.53 + 0.03 f^4(\xi_4) \) and \( f(\xi_4) = \xi_4 - \text{sgn}(\xi_4) - \text{sgn}(\xi_4 + 2) - \text{sgn}(\xi_4 - 2) \)

The above equations can be obtained by simplifying the second and fourth equations:

\[
\begin{align*}
\xi_1 &= \text{arctanh} \left( (\xi_2 - 2.8 \tanh(\xi_3)) / 1.1 \right) \\
\xi_2 &= \text{arctanh} \left( 1.3(\xi_4 - \text{sgn}(\xi_4) - \text{sgn}(\xi_4 + 2) - \text{sgn}(\xi_4 - 2)) / 3 \right)
\end{align*}
\]  

(10)

Substituting (10) into (9) can obtain the trajectories of Eqs. (1) and (3) in (9), which are marked with red and blue curves, respectively, as shown in Fig. 3. The equilibrium point is the green intersection of the red and blue curves in Fig. 3. It can be seen from Fig. 3 that in this case, the above equation has 17 focal points, that is, 17 equilibrium points, which are symmetrically distributed at the center. The symmetrical equilibrium points corresponding to Fig. 3 are calculated as shown in Table 1. The value of the symbolic function will affect the number of scrolls and equilibrium points, indicating that the memristor can control the number of vortices and produce controllable multistable chaos.
Table 1   Equilibrium points corresponding to Fig. 3

| Equilibrium points | a1(-3.5021, -0.3766) | a2(-3, 0) | a3(-2.4979, 0.3766) |
|--------------------|------------------------|-----------|----------------------|
|                    | -                      | b1(-2,0)  | -                    |
|                    | c1(-1.5021, -0.3766)   | c2(-1, 0) | c3(-0.4979, 0.3766)  |
|                    | d1(-3.5021, -0.3200)   | d2(0,0)   | d3(0.3200)           |
|                    | e1(0.4979, -0.3766)    | e2(1,0)   | e3(1.5021,0.3766)    |
|                    | -                      | f1(2,0)   | -                    |
|                    | g1(2.4979,0.3766)      | g2(3,0)   | g3(3.5021,0.3766)    |

Fig. 4   Phase portrait of plane $z$-u and time-domain waveform with: a & (1) I = 0; b & (2) I = 1; c & (3) I = 2

The Jacobian matrix near the equilibrium point can be expressed as:

$$
J = \begin{bmatrix}
-1 - 1.4t_1 & 1.2t_2 & -6t_3 & 0 \\
1.1t_1 & -1 - 2.8t_3 & 0 \\
kW (\xi_4) t_1 & -2t_2 & 4t_3 & kW' (\xi_4) \tanh(\xi_1) \\
0 & 3t_3 & 0 & -1.3f' (\xi_4)
\end{bmatrix}
$$

(11)

where $t_1 = sech^2 (\xi_1)$, $t_2 = sech^2 (\xi_2)$, $t_3 = sech^2 (\xi_3)$. Select $f_1 (u)$, $f_2 (u)$, and the corresponding $I$, $J$ take 1, 2 and 3, and the initial value set as $(0.1, 0, 0, 0)$. The system phase portrait can be obtained, as shown in Figs. 4 and 5. It can be seen that the number of scrolls has different parity with the selected symbolic function. As the value of $I$ and $J$ increases gradually, the number of scrolls also increases, and eventually there will be hyperchaos of infinite scrolls, which means that the symbolic function in the memristor model has a strong scroll control effect.
3 Dynamical analysis and numerical simulations

3.1 Dynamics of coupling strength \( K \) with memristor

In order to further study and analyze the system model, this section uses the R2018a MATLAB software platform and uses the fourth-fifth order Runge–Kutta algorithm, that is, the ode45 algorithm of MATLAB platform to solve the equations. The phase diagram, bifurcation diagram, Lyapunov exponent and other data are used for dynamic research and analysis.

The coupling strength \( k \) is used as the control parameter, \( f_2(u) \) and \( J = 1 \) are selected, the initial values are \((0.1, 0, 0, 0)\) and \((-0.1, 0, 0, 0)\). The corresponding bifurcation diagram is calculated, as shown in Fig. 6. The red and blue curves are the results when the initial value is \( x(0) = \pm 1 \). It can be seen that when \( k \) is in the range of \([0, 2]\), the dynamic behavior of system formula (8) changes from periodic limit cycle to chaotic state, and finally evolves from inverse period doubling bifurcation path to periodic limit cycle near \( k = 1.55 \). It has extremely complex bifurcation behavior, such as stable point, period, chaos, reverse period doubling bifurcation scenario. When the initial value is slightly adjusted, it can be seen that the bifurcation diagram becomes axisymmetric with \( x = 0 \), and coexistence hyperchaos will occur in the range of \([1.1, 1.55]\), which means that the MNN is sensitive to the initial value.

In order to reflect the sensitivity of the system to initial values, Lyapunov exponential spectrum is used to describe the dynamic characteristics of the system. Lyapunov exponent reflects the overall behavior of the trajectory in the phase space of the dynamic system, that is, as long as there is a positive Lyapunov exponent in the system, any two adjacent orbits in the system will inevitably separate at an exponential rate and enter a chaotic state. Therefore, whether the system is a chaotic system can be judged by...
calculating the exponent, the Lyapunov exponential spectrum with the initial value of \((0.1, 0, 0, 0)\) is calculated as Fig. 7. Within the range of \([1, 2]\), the system has a positive exponent, which is consistent with the phenomenon of bifurcation diagram, and the system has hyperchaotic characteristics.

3.2 Coexistence attractor

According to the specific combination of parameters, different initial values will lead to different trajectories of the system. Some trajectories may eventually converge to one attractor and others to another, and these attractors are called coexistence attractors. If the convergent attractor has the same morphology and dynamic behavior, it is called a homogenous coexistence attractor. If they have different dynamic behavior characteristics and different attractor forms, they are called heterogeneous coexistence attractors.

According to the analysis of the above bifurcation diagram and Lyapunov exponent diagram, the coexistence of chaotic attractors is obvious when the coupling strength \(K\) is in the range \([1.3, 1.55]\), so we choose the second memristor \(f_2(u) J = 1 k = 1.55\) to obtain the coexistence attractor, as shown in Fig. 8, in which the initial value of the orange track is \((0.1 0 0 0)\), the initial value of the blue track is \((-0.1, 0, 0, 0)\), and Fig. 8a shows the coexistence chaos. It can be seen that although the initial trends are different, their trajectories are intertwined, and finally tend to the same chaotic attractor form, belonging to homogeneous coexistence attractors. Figure 8b shows the coexistence periodic attractor. Finally, its trajectory tends to be the same in shape, but in the opposite direction.

The attraction basin is an important dynamic index in the coexistence research and analysis, which can clearly describe the relevant properties of the system when the initial values of different states change on the two-dimensional plane. Under the conditions of the above graphical coexistence attractor, the simulated attraction basin is shown in Fig. 9. The red region represents the divergent state, the red region represents the periodic state, and the Yellow region represents the chaotic state. The distribution and boundary of each region in the diagram can be clearly seen, which is highly consistent with the phase diagram, which verifies the initial value sensitivity of the system.
4 Digital watermarking application

4.1 Principle of digital watermarking

Digital watermarking technology directly embeds and stores some content with specific identification or representing some information into the carrier content through a specific algorithm without changing the actual use and original value of the carrier content. It can not only resist the loss in actual use through the algorithm, but also well hide the identification information from human perception system. So it has a high degree of robustness on the basis of ensuring information security. With the advancement of research, the traditional digital watermarking has changed from the basic DCT time domain processing to the frequency domain transformation processing with better performance. However, it still cannot meet the needs of use, so there is a research hotspot of using chaotic encryption to further strengthen the confidentiality of digital watermarking.

In image processing, the frequency domain has more operation space than the time domain. For human visual system, the frequency domain processing also has better performance than the time-domain processing. The chaotic digital watermarking method based on wavelet transform is a research hotspot. However, due to the influence of capacity, the chaotic sequence used in most algorithms is not completely aperiodic. The corresponding algorithm can successfully remove or destroy most of the hidden watermark information.

In this paper, a chaotic digital watermarking method based on the above chaotic system is proposed. Based on the existing chaotic digital watermarking, this method uses a more complex chaotic system to generate a specific sequence file, convert it into a two-dimensional code and embed it into the carrier image, which can improve the confidentiality and anti-interference of the digital watermarking algorithm.

Discrete wavelet transform is developed on the basis of Fourier transform. It expands the image according to different levels and can get images with different frequencies, including low-frequency areas where the main information of the image is stored, and medium and high-frequency areas where the details of the image are stored. The human eye does not perceive the change of smooth low-frequency region obviously, so this part is generally selected as the embedded region.

Wavelet transform can be described as:

\[ \psi_j (t) = a_0^{-j} \psi \left( a_0^{-j} t - k b_0 \right) \]  (12)

Wavelet coefficients:

\[ C_{j,k} = \int_{-\infty}^{\infty} f(t) \psi_{j,k}(t) dt \]  (13)
Fig. 10  Digital watermarking algorithm flow

Fig. 11  Carrier original and watermark original

DWT inverse transform: Wavelet coefficients:

\[ f(t) = C \sum_{j=1}^{\infty} \sum_{k=1}^{\infty} C_{J,k} \psi_{J,k}(t) \]  \hspace{1cm} (14)

PSNR value and NC value are two commonly used analysis indexes to evaluate the hiding and robustness of digital watermarking. PSNR is the full name of peak signal-to-noise ratio, which can evaluate the similarity between the reconstructed image and the original image. The greater its value, the more similar it is. It can be described as:

\[ PSNR = 10 \log_{10} \frac{M \ast N \ast 255}{\sum_{m} \sum_{n} (\omega(m,n) - \omega'(m,n))^2} \]  \hspace{1cm} (15)

The full name of NC value is normalized correlation coefficient. The closer its value is to 1, the better the watermark effect. It can be described as:

\[ NC = \frac{\sum_{m} \sum_{n} \omega(m,n) \ast \omega'(m,n)}{\sum_{m} \sum_{n} (\omega(m,n))^2} \]  \hspace{1cm} (16)

The digital watermarking algorithm used in this paper has four processes in total, as shown in Fig. 10. The first is the embedded watermark generation process in the green area. Using the system proposed above, under the condition of selecting the second memristor \( f_2(u), J = 2 \), the chaotic time series is generated by using the initial value \((0.1, 0, 0, 0)\), and the QR code watermark is generated by using this sequence. On this basis, it is scrambled, encrypted and wavelet transform to generate subband data as the preparation for the next step. The watermark effect generated in the experiment is shown as Fig. 11.

The second step is shown in the blue area. Firstly, the carrier image is preprocessed, and at the same time, the wavelet transform is used to convert the carrier image into frequency domain information and block it. Select the appropriate area, embed the frequency
domain watermark information obtained in the first step into the carrier, and then reconstruct the carrier image containing watermark through inverse wavelet transform.
The third step, as shown in the red area, simulates the possible impact and malicious attacks that the carrier image may encounter during use, such as changing the image size, adding Gaussian noise, being cut, filtering attack, rotation attack, random noise attack, etc.

The fourth step, as shown in the yellow area, extracts the watermark from the carrier image after the above attack, restores and decrypts it, and evaluates its performance. The first is the numerical evaluation. NC and PSNR are used to evaluate the watermark effect, and the visual observation is used to judge the actual effect. The combination of the two methods can obtain more accurate and comprehensive test results.

4.2 Analysis of watermark attack

Through experiments, the quality of watermarks under various attacks is evaluated. The values of NC and PSNR that are not attacked are used as the control group. The closer the values of the experimental group after being attacked, the better the effect is. The experimental results show that the effect is better when the NC value is greater than 0.92 and the PSNR value is more than 15, which will not affect the use. The specific data are shown in the Figs. 12, 13, 14, 15, 16, 17 and 18. It can be seen that the carrier image after adding the watermark has no change that can be recognized by the naked eye, and the watermark directly extracted from the image that has not been attacked has not been affected. However, the watermark extracted after different attacks can still...
directly extract the information stored in the watermark and has good restoration. Among them, scaling attack and filtering attack have little impact on the watermark, and there is basically no damage; Gaussian noise and random noise have a slight impact on the watermark quality, but do not affect the watermark effect; Shear attack and rotation attack have a slightly larger impression on the watermark, but still do not affect the watermark reading; The experimental results show that the watermark algorithm has improved the confidentiality and capacity of the watermark, does not reduce the hiding and robustness of the watermark, and better balances the performance of all aspects.

5 Conclusion

In this paper, the nonlinear magnetron piecewise quartic memristor simulator is used to replace a connecting synapse in the three neuron self feedback HNN, and a multi-scroll MNN model with controllable parity is constructed. Through theoretical analysis and experimental simulation, there is obvious that the model has the characteristics of controllable multi-scroll number varying with memristor parameters. Complex symmetric coexisting attractors can be generated with the change of coupling parameters. At the same time, the system has high initial value sensitivity can be found from the attraction basin image. Based on the excellent
unpredictability of this system, the wavelet transform digital image watermarking algorithm is improved and applied to the protection of medical information data. It not only improves the protection, but also balances the robustness and invisibility of the watermark, which has considerable practical application value.

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**Data Availability Statement** This manuscript has associated data in a data repository. [Authors’ comment: All data used to support the findings of this study are available from the corresponding author upon request.]

**Declarations**

**Conflict of interest** The authors declare that they have no conflict of interest. Data sets generated and/or analyzed during the current study may be obtained from the corresponding authors upon reasonable request.

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