Study of expansion ratio on dual bell nozzle of LOX-RP1 engine for replacing the existing bell nozzle to dual bell nozzle

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Abstract. Numerical study was conducted on the LOX-RP1 engine bell nozzle to replace the dual bell nozzle for determining the optimum expansion ratio and greater thrust at sea level. Here base nozzle is a conventional Rao’s TIC nozzle, where throat radius, exit radius, inflection angle and exit angles are 138mm, 826mm, 33° and 8° respectively. The total length of dual bell nozzle and existing bell nozzle are kept same. Numerical analyses were carried out in ANSYS FLUENT software on different dual bell nozzle geometry to evaluate the thrust and expansion ratio. Numerical analysis is performed by two dimensional, axi-symmetric, steady state, pressure based solver with SST k-ω turbulence model at different ambient pressures to recreate the patterns of flow in the nozzle at different nozzle pressure ratio (NPR) and expansion ratio. The profile has made by commercial software SOLID WORKSTM. Numerical simulations and flow separation locations are validated with the experimental data published earlier. It is observed that with area ratio 150 and inflection angle 15° the thrust has increased by 5.46% at sea level and around 10.5% in vacuum, where the increase in exit radius and mass is only 96 mm and 6 kg respectively.

1. Introduction
In single stage to orbit (SSTO) mission, present launcher has first stage with high performance liquid rocket engine which has dual bell nozzle configuration that operate from sea level to vacuum. During the twentieth century ending and beginning of twenty first century several researchers are focussed on the nozzle performance and reliability improvement for space launcher and supersonic rocket engine. This research is still continuing and intensified recently. As a result, many launchers like Japan-H2, Ariane 5-V2, American Space Transportation System etc. have significantly improved their performance and payload gain. Hence it is very important in future space programme like unified launch vehicle (ULV), reusable launch vehicle (RLV) and SSTO mission.

First stage engines of unified launcher operate in a varying pressure environment from sea level to nearly vacuum conditions. To avoid flow separation phenomena in the nozzle at sea level, area ratio has to be limited, thus it is limiting the high altitude performance. For this reason the conventional bell nozzle is having a single adaptation altitude cannot yield maximum performance along the whole trajectory. The ideal propulsion device would be an engine with adapted nozzle during its whole operating phase to provide high engine performance.

The dual bell nozzle (DBN) has unique features of altitude adaptation which is achieved only by wall inflection at the inflection point, which is pre designed for a particular altitude. It consists of convergent section, throat and divergent section. The divergent section features two bells with different exit area, which are geometrically connected by the inflection point. The area, at wall
inflection point of base nozzle is the effective cross sectional exit area. This area can be designed in such a way that the effective exit pressure for this section is matched to the low altitude pressure condition. The inflection point acts as a separation point where flow separation happened and the separated flow is contained in the additional axisymmetric area given by the nozzle extension contour. As the ambient pressure is decreasing the separation point will move towards the exit of the nozzle.

This study mainly focused on expansion ratio of DBN, which is the preliminary stage for replacing the conventional existing bell contour to dual bell contour. The base nozzle contour has been designed by conventional Rao’s nozzle, where throat radius, exit radius, inflection angle and exit angle are 138mm, 826mm, 33° and 8° respectively. RP1 and LOX are the fuel and oxidizer, where RP1 is highly refined Kerosene. For the design and configuration of second bell, there are two imposed constraints; first one is the inflection angles which lead to the constant isobaric contour extension. The second constraint is the present dynamic enveloped area, so total length of nozzle is kept constant. Base nozzle CFD analysis has been done in different NPR and obtained the thrust per weight ratio to determine the length of base nozzle and second inflection point. The base nozzle flow simulations are validated with the test data which has published by S.B. Verma, et al. [1]. For keeping the same outer geometrical periphery, there are six DBN models with area ratio of the existing bell nozzle (AR 35.826) and numerical analysis has been done for obtaining the thrust. Similarly there are three DBN model with three different inflection angles with AR 50 and two DBN model with two different inflection angles with AR 150, and numerical analysis has been done for obtaining the thrust. All the DBN profiles has been made using commercial software SOLID WORKSTM and CFD analysis carried out using commercial software ANSYS FLUENT. CFD analysis parameters are taken from various research papers [1-4]. Two dimensional, steady state, axi-symmetric, pressure based solver with SST k-ω turbulence model is used for this analysis. The grid independent study has been carried out before finalising the grid for the CFD analysis.

2. Literature review

The concept of nozzle wall contour inflection is to control the flow separation which was first introduced by Chalom in 1948 as well as Foster et al. [5] in 1949. This was an alternative to conventional nozzle in flow separation study and the first test was conducted at Rocket Division by Horn and Fisher. They had tested four contour combinations to find the extension contour for most favourable flow characteristic at high altitude condition, and compared to the performance of two base line contour. They used the Rao optimum contour as base nozzle for each test nozzle. Horn M et al. [6] found that a dual bell nozzle could provide enough thrust to carry 12.1% more payload than a conventional nozzle of the same area ratio. Cold and hot flow nozzle test were performed by Haidinger FA et al. [7] in co-operation with the Keldysh Research Center. Manski, D et al. [8] and proved that application of dual bell nozzle could reduce the lift off mass by 4.5% in SSTO mission. Design verification of the DBN from nozzle of sustained engine is carried out by analytical verification and validated using experimental results. For this purpose G Hagemann et al. [9] has optimized the equal area ratio and kept the total length of the nozzle almost constant. In the design criterion of DBN the CFD study has done and specially investigated the effect of deflection angle with respect to the nozzle length. Hirotaka Otsu et al. [10]. Frederick L Shope [11] has proposed the technique of design of nozzle contour for supersonic and hypersonic wind tunnels by proposing the solution algorithm by the method of characteristics.

In the last three years research is going on wide directions of application of DBN for performance optimization and evaluation application in launch vehicle. In the sea level DBN transition behaviour proposed by R Stark et al. [2] with respect to length and pressure. Numerical and experimental study has conducted to investigate the transitional behaviour by using different chemical reaction mechanism on DBN done by R stark et al. [12]. The experimental investigation of identifying the location of origin of side loads on TIC nozzle is done by S.B. Verma, et al. [1]. The experimental investigation on subscale level TIC for identifying the flow separation under sea level and high altitude condition carried out by R. Stark and C. Genin[13]. In the year of 2014, Stark R et al. [4] had
studied the various performance characteristics and introduced the extension of DBN to gain the payload of Ariane 5 launch vehicle for around 450kg.

3. Numerical model

The numerical model is employed two-dimensional axisymmetric, steady state, pressure based, RANS equations with SST k-ω turbulence model. Yong SHAN et al. [14] has proved that SST k – ω model has given the close and accurate result with test result during the numerical study. In the boundary condition, fully combusted gas for LOX-RP1 mixture is assumed as the main combustion gas and it has provided at nozzle inlet. Also it is assumed that inside the chamber complete combustion occurs and no reaction is occurring in the nozzle inside section. Specific heat capacity ($C_p$), thermal conductivity (K) and other properties for the combustion gas are calculated by using NASA CEA code (Chemical Equilibrium Application), which is also available in the open source. As the boundary conditions; it is assumed that adiabatic wall along the nozzle surface and inflow gas condition at the nozzle inlet. On the axis of symmetry, the symmetry condition is applied. At the outlet of the computational domain, Zeroth order extrapolation is given. In order to determine the pressure, at the separation point where transition takes place, the exit pressure varied from 1 bar to 2.8 bar. At the inlet combustion gas properties are $P_c = 180$ bar, $T_c = 3800$ K, $k = 1.573 \text{ W/m-K}$, $\mu = 1.209 \times 10^{-4}$ PS, $C_p = 5675 \text{ J/kg-K}$, Molecular weight = 23. Numerical analysis has been done in ANSYS Fluent 2019 R1 software, where pressure based analysis also given stable solution with hybrid initialization.

4. Geometry and grid

4.1. Existing nozzle configuration

In 1958 G V Rao American aerospace engineer has optimised the bell nozzle in research paper [15] “Exhaust Nozzle Contour for Optimum Nozzle” ARS j: 30561, 1960 by GVR Rao, where parabola shape has optimized. The truncated ideal contour (TIC) nozzle also has the parabolic profile which has basic equation and principles are like Rao’s nozzle.

4.2. Calculation for design of existing nozzle contour

Rao’s bell nozzle design has three points, which is obtained by using three curves, first is an initial large circle starting from convergent to throat, second is a small circle at the throat and last is a parabola to extend the approximate bell contour to the exit plane. Rao’s bell nozzle profile is 2D, axisymmetric, where nozzle axis is X direction and radius is Y direction. Origin of X-Y plane is at throat radius. Throat area profile is consisting of two circles, one is $1.5R_t$ towards entry and other is $0.382R_t$ towards exit, which is mentioned in Figure-1. The small radius $0.382R_t$ and the polynomial curve are joined together tangentially at point N. The polynomial constants are to be evaluated. This parabolic curve entry and exit angle are defined as $\theta_n$ and $\theta_e$. Hence generating this profile three parameters, are required ($R_t$, $\theta_n$ and $\theta_e$). Rao’s base nozzle configuration is shown in Figure 1.

![Figure 1. Rao's base nozzle configuration (Source: [3] ARS j:30561,1960 GVR Rao” Exhaust contour for optimum thrust).](image-url)
Nozzle coordinate system is defined as axial (X) axis passing through the line of symmetry and radial axis i.e. (Y) axis is passing through the centre of the throat. The first and second curves are defined as entrance and exit of throat of the nozzle for the circular curve.

\[ x^2 + [y - (R_t + 1.5 R_e)]^2 = (1.5 R_e)^2 \]  
\[ x^2 + [y - (R_t + 0.382 R_e)]^2 = (0.382 R_e)^2 \]

The equation of the parabola curve 3 takes as \( x = ay^2 + by + c \). The coefficients are determined by the derivatives at the point where the circle from the throat meets the beginning of the parabola (\( X_N \)). To determine \( X_N \) the angle \( \theta_N \) needs to be defined, then the derivative of the second curve should be equal to its tangent.

\[ \frac{dy}{dx} = \tan(\theta_N) = \frac{X_N}{\sqrt{(0.382 R_t)}} \]  
\[ X_N = a R_N^2 + b R_N + c \]  
\[ \frac{dy}{dx} = \tan(\theta_N) = \frac{1}{2 a R_N + b} \]  
\[ \frac{dy}{dx} = \tan(\theta_e) = \frac{1}{2 a R_e + b} \]

Length of the nozzle also can determine by \( L_N = \frac{k(\sqrt{e-1})R_t}{\tan(\theta_e)} \) (7)

For solving above these equations, throat radius (\( R_t \)) is 138mm, contour inlet angle (\( \theta_h \)) is 33°, contour exit angle (\( \theta_e \)) is 8°, exit radius (\( R_e \)) is 826 mm, are taken as input. And by solving, the parabolic constants are \( a = 0.0041026, b = 0.3377, c = -106.63 \).

### 4.3 Dual bell nozzle configuration
Rao’s bell nozzle equations are used for designing the second bell also. The base nozzle separation point is used as the inflection point of dual bell, from where the second parabolic curve is starting. Four inputs are required for generating the second parabolic curve, these are \( X_{N2}, \theta_{N2}, R_{B2} \) and \( \theta_{e2} \) which are obtained from base nozzle CFD analysis of flow separation. The second bell starts from the inflection point and ends at the exit radius. The parabolic constants are calculated by solving the matrix. The existing nozzle contour and DBN contour length are kept constant, due to dynamic enveloped area constrain.

### 4.4 Grid generation and 3D model
Parabolic curve has generated in SOLID WORKSTM by equation driven curve and existing base nozzle profile is shown in Figure 2.
Since nozzle geometry is a two-dimensional cylindrical geometry, structured curvilinear body fitted grid and orthogonal curvilinear grid has been chosen for the CFD analysis. In body fitted grid all the domain boundaries coincide with coordinate lines, so that the domain is being incorporated accurately. The domain is two part, one is nozzle part which is X axis 3536 mm, as nozzle length and Y axis is 826 mm as nozzle exit radius. Other part is external domain part which is 1500 mm in Y axis and 18600 mm in X axis. Grid generation has been done in ANSYS Design Moduler. Inflation factor and number of division tools has used to the nozzle wall with biasing factor, for boundary layer analysis. Enlarged view of the Grid near the throat is shown in Figure 3.

4.5 Grid independent study
Grid independent study (GIS) of existing nozzle model and DBN model of expansion ratio 150 with 15° inflection angle has been done and results are mentioned in Table 1 and Table 2 respectively. GIS can give the numerical accuracy in computational result, by eliminating or reducing the influence the number of grid or grid size. All three Grids are produced the identical result with negligible difference, hence Grid-3 has been chosen for CFD numerical analysis.

Table 1. GIS result of existing nozzle.

| Grid     | Element Size (mm) | No of node | Thrust (kN) |
|----------|-------------------|------------|-------------|
|          | Nozzle Domain     |            |             |
| Grid - 1 | 78                | 8          | 401292      | 1173.5 |
| Grid - 2 | 1011              | 212585     | 1173.5 |
| Grid - 3 | 1213              | 151517     | 1173.8 |
5. Methodology

To determine the second wall inflection point and wall separation pressure for a particular area ratio, many CFD analyses has carried out with different NPR. At the inflection point atmospheric pressure is always lower than the nozzle wall pressure and this ratio must be lower than the wall inflection point pressure ratio. Base nozzle has truncated at six area ratios like 50%, 55%, 60%, 65%, 70% and 75% and corresponding DBN model has numerically analyzed to determine the thrust per weight ratio. The profile of six DBN model with AR 35.826 and 15° inflection angle are mentioned in Figure 4. Two profiles of AR 150 with two inflection angle are mentioned in Figure 5. For optimized thrust per weight ratio and wall inflection pressure ratio, 60% area ratio of base nozzle has taken for further CFD analysis of DBN model. During the sea level, the pressure limit has given by the line of separation criterion, and it must be intersecting the wall pressure profile with in the pressure drop at wall inflection point. The wall pressure profile of DBN, within the base nozzle is same as of conventional bell nozzle. There are six DBN models with AR 35.826, three models with different inflection angles and AR 50 and two models with two different inflection angles and AR 150. CFD analysis inlet conditions and other parameters are kept constant and outlet pressure only changed. Analysis method is coupled in pressure velocity and discretization in energy, momentum, pressure, density, gradient, turbulence are in upwind second order with relaxation factor 0.95 to 1.0 with hybrid initialization.

Table 2. GIS result of DBN model of AR 150.

| Grid     | Element size (mm) | No of node | Thrust (kN) |
|----------|-------------------|------------|-------------|
| Grid - 1 | 7                 | 8          | 474501      | 1244.1      |
| Grid - 2 | 10                | 11         | 251431      | 1143.4      |
| Grid - 3 | 12                | 13         | 179848      | 1143.3      |
| Grid - 4 | 15                | 16         | 119376      | 1142.8      |

Figure 4. DBN profiles of AR 35.825.  Figure 5. DBN profiles of AR 150.
6. Results and discussion

CFD analysis of the existing nozzle has repeated by increasing the nozzle exit pressure. During this, the inlet pressure has kept constant and hence the NPR is increasing. When the NPR was increased, the flow was separated from the nozzle wall, and as a result thrust, Mach no and exit area was start reducing. These comparison profile parameters and flow separation parameters of existing nozzle are given in Table 3. Existing nozzle flow separation results comparison is mentioned in Figure 6.

| Percentage of Area Ratio | 75  | 70  | 65  | 60  | 55  | 50  |
|--------------------------|-----|-----|-----|-----|-----|-----|
| $X_{N2}$, mm             | 2229.19 | 2083.40 | 1935.3 | 784.90 | 1637.47 | 1487.57 |
| $\theta_{N2}$, degree    | 9.10 | 9.42 | 9.74 | 10.12 | 10.54 | 11.01 |
| Ma                       | 3.302 | 3.308 | 3.232 | 3.202 | 3.181 | 3.162 |
| Pr$_{in}$, Bar           | 1.6 | 1.8 | 2.2 | 2.3 | 2.4 | 2.8 |
| NPR                      | 112 | 100 | 82 | 78 | 75 | 64 |
| Thrust (kN)              | 1138 | 1085 | 1046 | 1015 | 1004 | 964 |

Table 3. Existing nozzle flow separation parameters.

![Flow Separation Parameters Comparison](image)

**Figure 6** Existing nozzle flow separation result

6.1. Validation

For the validation of the CFD analysis, the test parameters of the research paper “Origin of side load in a subscale truncated ideal contour nozzle” by S B Verma, Abdellah Hadjadj, and Oskar Haidn [1] are taken. In this paper the results of the cold flow test of subscale level truncated ideal contour (TIC) nozzle at NAL Lab Bangalore, India is published. Instrumentation and data accusation system are well calibrated and up to date for measuring all the mechanical properties, which details are mentioned in the paper [1]. Liquid nitrogen is used as fluid with mass flow rate of 4.2 kg/s. During the
test in each NPR the feeding pressure was increase and decrease at 0.1MPa. The average ambient temperature and stagnation temperature during the test was 275K and 260K respectively.

The nozzle configurations are throat diameter 0.02m, area ratio 20.66, inflection angle 16°, exit angle 5.84°, and by using these four parameters, parabolic profile has generated. CFD analysis was carried out with cold flow considering LN2 instead of combustion gas, where boundary conditions and analysis parameters were remain same. Flow separation parameters were plotted in the graph, which is shown in Figure 7 and the graph from the research paper is shown in Figure 8. It is observed that both the graphs are showing the same trend and data are closely matching. The Figure 9 represents the Ma contours for the cold flow (LN2) and combustion gas. And it is observed that the separation point is near the throat for cold flow (LN2). The flow separation details like NPR, \( P_{\text{inc}} R_t \), \( P_{\text{inc}}/P_a \) are given in Table 4. The average \( X/R_t \) value is deviated 0.12 from comparison graph \( X/R_t \) value. This deviation is well below less than 1% and these results are very close to the test result.

**Table 4. CFD analysis results of existing nozzle contour with cold flow test.**

| SL No | NPR | \( X/R_t \) | \( P_{\text{inc}}/P_a \) |
|-------|-----|-------------|----------------------|
| 01    | 60  | 12.24       | 0.365                |
| 02    | 50  | 14.1        | 0.289                |
| 03    | 40  | 11.14       | 0.278                |
| 04    | 30  | 8.71        | 0.298                |
| 05    | 20  | 5.81        | 0.303                |

![Figure 7. CFD result for comparison.](image-url)
Figure 8. Test result for comparison [1].

Figure 9. Mach contour of LN2 and RP1-LOX.
6.2. **CFD analysis of DBN**

CFD analysis has carried out for six DBN models with AR 35.826, where outer geometrical periphery is same and obtained thrust variation from 1181 kN to 1192 kN. Existing nozzle thrust is 1173 kN. Hence, it is clear that an increase of expansion ratio is necessary for obtaining higher thrust. Three DBN model has generated in the location of 60% AR of existing nozzle with three different inflection angles 15°, 16.94°, and 20.39° with AR 50. These profiles are generated as referred from this paper “Design and Performance of the Dual Bell Nozzle” [16], where it is seen that total expansion ratio kept 50 and obtained the maximum thrust. The second bell parabolic profile design is starting from the 0.6 times length of base nozzle, which has taken the reference from various research papers, listed in the bibliography at serial no [17], [2], [3], [18], [4]. These DBN models are analysed and obtained the maximum thrust 1223 kN, with inflection angle 20.39° where radius increased only 80 mm from the existing nozzle exit radius. Though higher inflection angle has given the higher thrust, but side load also will be high. For resisting the high side load mechanical strength has to increase, so more mass penalties will be there. Also in the research papers [4], [2] have shown that higher inflection angle has given less performance. Hence in the further study this higher inflection angle has eliminated. Similarly DBN model has generated in 60% AR location of existing nozzle with inflection angle of 15 degree and 16.94 degree in AR 150. These DBN models are analyzed and obtained the thrust of 1243 kN and 1198 kN respectively at sea level. In the inflection angle 16.94° thrust is reduced due to flow has separated at sea level, which can be observed in ANSYS Mach contour given in Figure 10. But same DBN has given higher performance in vacuum and thrust is 1297 kN with exit pressure of 0.2 bar as on boundary.

The flow patterns of Mach contour are mentioned in Figure 10, where we can observe oblique shock. As we know, in the ascent phase of nozzle, ambient pressure continuously decreases. Also we know that wall pressure and separation pressure profile will intersect each other at a certain ambient pressure. In this paper nozzle extension with negative wall pressure gradient is being used for analysis. At the wall inflection point and low pressure ratio $P_i/P_a$, the flow separation observed is stable and controlled. But in high pressure ratio at the nozzle extension, the flow separation characteristics and features are stable and uncontrolled. In the sea level operation, a flow boundary layer separates from the second bell wall i.e inflection point, due to high pressure yield. In this case, a small expansion develops a back pressure, whose value is slightly lower than the atmospheric pressure, hence generating the oblique shock. Once the atmospheric pressure is low enough to reach the value of reattachment wall pressure, the flow is attached to the second bell and nozzle is switched to high altitude mode.
7. Conclusion
A theoretical simulation is carried out to study the role of expansion ratio on the performance of a dual bell nozzle. Nozzle profiles are generated with various inflection angles and expansion ratios. Each of the generated profiles was subjected to CFD simulations and studied their performance.

It is seen that the dual bell with expansion ratio same as the original single bell the thrust increase is minimal. On the other hand if the expansion ratio of the second bell is increasing there is a considerable increase in the thrust. The present study revealed that for getting optimum thrust the second bell expansion ratio has to be increased. Also it is to be noted that transition behaviour, side loads and cooling methods are to be studied and hot and cold flow tests are to be conducted for the DBN configuration before finalizing the design.

Acknowledgement
Author expresses his sincere gratitude to Dr. Sunil Kumar S, DD PRS/LPSC, Shri Damodaran P DH, PLSD and Miss Sharmistha Choubey Sci/Eng. LPSC/ISRO, for their guidance and support.

Nomenclature:

| Symbol | Description |
|--------|-------------|
| AR     | Area Ratio, $A/A_t$ |
| $R_t$  | Throat radius, mm |
| $\theta_e$ | Nozzle exit angle, degree |
| $R_N$  | Inflection radius, mm |
| $P_a$  | Atmospheric pressure, Bar |
| $T_c$  | Chamber combustion temperature, Kelvin |
| NPR    | Nozzle Pressure Ratio, $P_c/P_a$ |
| $\theta_N$ | Inflection angle, degree |
| $X_N$  | Inflection point and throat distance, mm |
| $M_a$  | Mach number |
| $P_c$  | Chamber combustion pressure, Bar |
| $M_w$  | Molecular weight, Number |
LOX  Liquid oxygen  RP1  Refined petroleum

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