Abstract—Distributed fuzzy neural networks (DFNNs) have attracted increasing attention recently due to their learning abilities in handling data uncertainties in distributed scenarios. However, it is challenging for DFNNs to handle cases in which the local data are non-independent and identically distributed (non-IID). In this article, we propose a federated fuzzy neural network (FedFNN) with evolutionary rule learning (ERL) to cope with non-IID issues as well as data uncertainties. The FedFNN maintains a global set of rules in a server and a personalized subset of these rules for each local client. ERL is inspired by the theory of biological evolution; it encourages rule variations while activating superior rules and deactivating inferior rules for local clients with non-IID data. Specifically, ERL consists of two stages in an iterative procedure: a rule cooperation stage that updates global rules by aggregating local rules based on their activation statuses and a rule evolution stage that evolves the global rules and updates the activation statuses of the local rules. This procedure improves both the generalization and personalization of the FedFNN for dealing with non-IID issues and data uncertainties. Extensive experiments conducted on a range of datasets demonstrate the superiority of the FedFNN over state-of-the-art methods.

Index Terms—Data uncertainty, evolutionary rule learning (ERL), federated fuzzy neural network, federated learning (FL), non-IID dataset.

I. INTRODUCTION

Combining fuzzy neural networks with fuzzy logic [1], [2], fuzzy neural networks (FNNs) [3], [4], [5], [6], [7] have been proposed with powerful learning capabilities and uncertainty handling abilities in centralized scenarios. However, due to increasing data privacy concerns, the samples collected from distributed parties must be processed locally. Distributed FNNs (DFNNs) [8], [9], [10], [11], [12] address this issue by learning a global FNN via the integration of local models. However, the existing DFNNs are fragile to non-independent and identically distributed (non-IID) data. In addition, as DFNNs tend to learn a shared group of global rules for all clients, their personalization ability is limited and their learned global rules are less adaptive. Furthermore, they regard the local model integration process as a convex optimization problem and solve it with the alternating direction method of multipliers [13], which overlooks the powerful learning ability of feedforward FNNs.

Fortunately, federated learning (FL) models [14], [15], [16] offer decentralized learning architectures that allow local models to optimize their parameters using gradient descent. The FL approach learns a shared model by aggregating the updates obtained from local clients without accessing their data. Notably, data distribution heterogeneity is also one of the key challenges for FL. Yet, many efforts [17], [18], [19], [20], [21] have been made to handle this issue in FL, in which many approaches consider solutions that allow clients to have personalized models. However, few of the existing FL methods are able to simultaneously cope with data uncertainties and non-IID issues in distributed learning scenarios. Although several studies have adopted Bayesian treatments [22] and Gaussian processes [20], [21] to enable FL methods to handle data uncertainties, their performance heavily relies on the learning of good posterior inferences and kernel functions, which is very time-consuming.

To solve the aforementioned issues, in this article, we propose a federated fuzzy neural network (FedFNN) with evolutionary rule learning (ERL) to handle data uncertainties and non-IID issues. As shown in Fig. 1(a), the theory of biological evolution [23] states that variants of the same species can evolve to adapt to their different living environments by selectively activating and expressing their genes. Inspired by this, we use FNNs as our local models and consider them as compositions of fuzzy rules, which capture valuable local data information from multiple views, such as distributions. Similar to the genes of a species, each rule of the FedFNN is a basic functional component that can be activated or deactivated for clients according to their performance on local data. Thus, our FedFNN aims to obtain a group of global fuzzy rules that can be selectively activated for local clients to enable them to outperform competing approaches on non-IID data. It is worth noting that our ERL is a novel algorithm different from the FedFNN over state-of-the-art methods.
Fig. 1. (a) A brief demonstration of how a species evolve variants to survive in diverse living environments based on genes selective activation and expressions. (b) A brief demonstration of how FedFNN selectively activate a personalized subset of contributive rules for clients to effectively deal with their local non-IID data.

The contributions of this article are as follows,

1) We are the first to propose FedFNN that integrates fuzzy neural networks into a FL framework to handle data non-IID issues as well as data uncertainties in distributed scenarios. FedFNN is able to learn personalized fuzzy if–then rules for local clients according to their non-IID data.

2) Inspired by the theory of biological evolution, we design an ERL algorithm for the learning of FedFNN. ERL encourages the global rules to evolve while selectively activating superior rules and eliminating inferior ones during the training procedure. This procedure ensures the ability of generalization and personalization of FedFNN.

II. RELATED WORK

A. Distributed Fuzzy Neural Networks

DFNNs [8], [9], [10], [11], [12] have been proposed to handle the uncertainties encountered in distributed applications. Fierrmonte et al. [8] proposed a DFNN model that randomly sets the parameters in antecedents and only updates the parameters in consequent layers. Later, they extended this work to an online DFNN model [9]. Their models assume that all clients share the information in antecedent layers, making this technically not a seriously distributed method. To avoid this problem, a fully DFNN [10] model was proposed by adopting consensus learning in both the antecedent and consequent layers. As its subsequence variant, a semisupervised DFNN model [12] was presented to enable the DFNN to leverage unlabeled samples by using the fuzzy C-means method and distributed interpolation-based consistency regularization. However, the existing DFNNs cannot handle situations in which the data distribution varies across clients. The authors in [11] proposed a DFNN with hierarchical structures to process the heterogeneity existing in the variables of training samples. However, instead of processing the data heterogeneity across distributed clients, they focused on variable composition heterogeneity, which meant that data variables were collected from different sources. Generally, by employing the well-known Takagi–Sugeno (T–S) fuzzy if–then rules [2], the existing DFNN models build the antecedent layers of their local models in traditional ways (e.g., K-means) and calculate the corresponding consequent layers with closed-form solutions. Then, the original DFNNs are transformed into convex optimization problems. While efficient and effective, they are not able to learn local models with personalized rule sets. Worse, they fail to utilize the strong learning abilities of neural networks that enable local FNNs to investigate more adaptive rules.
B. Federated Learning

FL [14] is an emerging distributed paradigm in which multiple clients cooperatively train a neural network without revealing their local data. Recently, many solutions [14], [25], [26] have been presented to solve FL problems, among which the most known and basic solution is federated averaging (FedAvg) [14], which aggregates local models by calculating the weighted average of their updated parameters.

However, FL has encountered various challenges [27], among which the non-IID issue is the core problem that makes the local model aggregation process harder and leads to performance degradation. Numerous FL algorithms have been presented to solve the non-IID problem, e.g., stochastic controlled averaging for FL (SCAFFOLD) [17]; FedProx [18]; model-contrasted FL (MOON) [28], which attempts to increase the effect of local training on heterogeneous data by minimizing the dissimilarity between the global model and local models; FedMA [22] and FedNova [29], which improve the aggregation stage by utilizing Bayesian nonparametric methods and local update normalization, respectively; CCVR [30], which calibrates the constitutive classifiers using virtual representations to eliminate the global model bias caused by local distribution variance; and FedEM [31], which introduces expectation maximization to make the learned model robust to data heterogeneity. Though these methods have been proposed based on FedAvg by trying to learn a more robust global model, they focus on learning a shared global model, which degrades their performance when the data distributions heavily vary across clients.

Recently, personalized FL (PFL) [19], [20], [32], [33] has been proposed; this approach aims to process heterogeneous local data with personalized models. Many of the existing PFL methods were proposed to solve the distributed meta-learning problem [33], [34], [35], [36]. Among the methods that target normal FL tasks, multitask learning [37] is applied to learn personalized local models by treating each client as a learning task; model mixing [38], [39] achieves the same goal by allowing clients to learn a mixture of the global model and local models. Hanzely and Richtárík [40] presented a new local model structure that comprises a global feature encoder and a personalized output layer. By contrast, LG-FedAvg provides clients with a local feature encoder and a global output layer.

However, very few of the mentioned FL methods are able to handle data uncertainties, except for that of [22] and [29], which adopts Bayesian treatment, and that of [20], which adopts a Gaussian process. In addition, building Bayesian posteriors and Gaussian kernels is time-consuming. In contrast, our study uses FNNs as local models, which are viewed as assemblies of fuzzy rules. Thus, taking rules as basic functional units, we break down the task of learning a global model into learning global fuzzy rules, each of which can independently investigate its local sample space and contribute to the local training process.

III. FEDERATED FUZZY NEURAL NETWORK

In this section, we describe the general structure of our FedFNN. As depicted in Fig. 2, the FedFNN includes one server and several local clients. The server is responsible for communication with local clients and maintaining a group of global rules by aggregating the uploaded local rules. Local clients download the global rules as local rules for constructing their FNNs, which are then updated via training on their own data. Due to the concerns of data privacy, each local agent learns from its own data without accessing the data of other agents and communicates, while the server communicates with all local agents and aggregates the locally learned rules according to their activation status. An overview of a local FNN is shown in Fig. 3.

To mimic the selective activation of genes, the rules in the local clients are activated selectively to make the FedFNN personalized and properly adapted to local non-IID data. Thus, we introduce an activation vector containing the rule status $s_{q,k}^q$ of each client. Accordingly, the global server can help local clients activate useful rules and deactivate useless or harmful rules based on their own local data. For example, if $s_{q,k}^q = 0$, the server will deactivate the $k$th rule for the FNN on the $q$th client;
otherwise, the corresponding rule will be activated and involved in the operations of the $q$th client.

In the FedFNN, we adopt the fuzzy logic presented in the first-order T–S fuzzy system [2]. Suppose that our FedFNN holds $Q$ clients; then, the dataset owned by the $q$th client can be denoted as $D^q := \{x^q_i, y^q_i\}_{i=1}^N$ where $x^q_i = [x^q_{i1}, x^q_{i2}, \ldots, x^q_{iD}]^T$ and $y^q_i \in \mathbb{R}^C$ are the $i$th sample and its one-hot vector label, respectively, and $C$ and $N^q$ denote the category number and the local dataset size. Then, the $k$th fuzzy rule of the local FNN on the $q$th client can be described as

\[ h^q_k(x^q_i; \tau^q_k) = \begin{cases} 1; & \text{if } \phi^q_k(x^q_i; \theta^q_k) > 0 \\ 0; & \text{otherwise} \end{cases} \]

where $\phi^q_k(x^q_i; \theta^q_k)$ denotes the consequent parameters. Many types of membership functions can be employed to describe the fuzzy set of rule $k$ with the consequent output of the $k$th rule $h^q_k(x^q_i; \tau^q_k)$ and can be calculated by

\[ g^q_k(x^q_i; \theta^q_k) = [1; \theta^q_k]^T \theta^q_k \]

where $\theta^q_k \in \mathbb{R}^{(D+1) \times C}$ denotes the consequent parameters. Thus, by considering rule statuses, the FedFNN is able to eliminate the interruption of deactivated rules for local clients. The antecedent layer $h^q(x^q_i; m^q, \sigma^q)$ and the consequent layer $g^q(x^q_i; \theta^q)$ of the $q$th client in our model are

\[ h^q_k(x^q_i; m^q, \sigma^q) = (h^q_1(x^q_i; m^q_1, \sigma^q_1), \ldots, h^q_K(x^q_i; m^q_K, \sigma^q_K)) \quad (4) \]

and

\[ g^q_k(x^q_i; \theta^q_k) = (g^q_1(x^q_i; \theta^q_1), \ldots, g^q_K(x^q_i; \theta^q_k)) \quad (5) \]

respectively. A classification head is further added to the tail to generate the final predictions by considering the outputs of all $K$ rules. The classification head connects all rules, which guarantees that the gradient of the loss function can successfully propagate backward to every component of the local FNNs. Thus, the predictions of the $q$th local FNN $f^q(x^q_i; m^q, \sigma^q, \theta^q, s^q)$ can be represented as

\[ f^q(x^q_i; m^q, \sigma^q, \theta^q, s^q) = \text{softmax} (\tau) \quad (6) \]

where $\tau = \sum_{k=1}^K h^q_k(x^q_i; m^q_k, \sigma^q_k, s^q_k) g^q_k(x^q_i; \theta^q_k)$. Suppose that $w^q = (m^q, \sigma^q, \theta^q)$ collect the parameters of all local rules on client $q$; the loss of the $q$th local FNN can then be defined as

\[ \ell^q(x^q_i; w^q, s^q) = -\sum_{c=1}^C y_{ic} \log(\hat{y}_{ic}) \quad (7) \]
where $\hat{y}_{ic}$ is the cth output of $f^q(x^q_i; w^q, s^q)$, and its goal is to optimize

$$\min_{w^q, s^q} \mathbb{E}_{(x, y) \sim D^q}[\ell^q((x, y); w^q, s^q)].$$

(8)

Thus, the training objective of our proposed FedFNN can be given by

$$\arg\min_{\Theta} \frac{1}{Q} \sum_{q=1}^{Q} \ell^q(x^q_i; w^q, s^q)$$

$$= \arg\min_{\Theta} \frac{1}{Q} \sum_{q=1}^{Q} \frac{1}{N_q} \sum_{i=1}^{N_q} \ell^q(x^q_i; w^q, s^q)$$

(9)

where $\Theta$ denotes the set of personal parameters $\{w^q, s^q\}_{q=1}^{Q}$.

IV. EVOLUTIONARY RULE LEARNING

In this section, we present an in-depth introduction of the ERL method, which enables the FedFNN to be personalized and achieve superior performance on non-IID data. As shown in Fig. 4, ERL includes a rule cooperation stage that improves the generalization of global rules and a rule evolution stage that enhances the personalization of the FedFNN. The above two stages are presented in Section IV.A and IV.B, respectively.

A. Rule Cooperation Stage

In this stage, we focus on the learning of more general global rules. Generally, the global rules activated by multiple clients are able to capture informative representations across these clients. Thus, updating the global rules requires cooperation among the local clients.

Technically, the rule cooperation stage highly relies on the rule activation status vectors of the local clients. These vectors are randomly initialized at the beginning and updated in the rule evolution stage. During each communication round, the local clients download the global rules from the server as their local rules. Then, the global server selects the activated rules according to the corresponding rule activation status vectors to build the local FNNs. As described in the Local Training of the algorithm 1, the constructed personalized local FNNs are then trained on their associated non-IID local data to update the activated local rules.

Afterward, each global rule is updated by calculating the weighted average of the activated local rules. Thus, the parameters of the kth global rule $w_k$ are updated by activation-status driven weight averaging

$$w_k = \sum_{q=1}^{Q} \frac{N_q^k s^q_k}{\gamma_k} w_q^k$$

(10)
Algorithm 2: Adaptive Evolutionary Stage of the ERL Method.

Input: Number of clients $Q$, number of global rules $K$, global rule parameter set $w(t)$, hyperparameter $\beta$.

Output: The global rule set parameter $w(t+1)$, the rule status $s(t+1)$, and the rule number $K$ for the next round.

Server executes:
for $q = 1, 2, \ldots, Q$ in parallel, do
  for $k = 1, 2, \ldots, K$ in parallel, do
    update $s_k^q(t+1)$ via (12).
  end for
if (13) and (14) both hold or $\sum s^q(t+1) = 0$ then
  generate a new rule for client $P^q$ and randomly initialize its parameter $w_{K+1}$.
  expand global parameter set $w(t+1)$ via $w(t+1) \leftarrow [w(t), w_{K+1}]$.
  expand rule status $s(t+1)$ via $s(t+1) \leftarrow [s(t), 0]$, $s_{K+1}^q(t) = 1$.
  update $K$ via $K = K + 1$
end if
end for
for $k = 1, 2, \ldots, K$ do
  if $\sum s^q_k = 0$ then
    remove rule $k$ from global rule list and $w_k$ from global parameter set $w$.
    update $K$ via $K = K - 1$
  end if
end for

where $\gamma_k = \sum_{q=1}^Q N^q s_k^q$ is the factor that measures the contribution of a global rule. This stage is conducted for $L$ rounds before stepping to the rule evolution stage to ensure that the global rules are effectively learned under cooperation among the local clients.

It is worth noting that each global rule is updated by aggregating its corresponding activated local rules instead of aggregating all of them. This is different from existing FL methods that aggregate local models without checking whether their updates are helpful or not. Our ERL approach only shares the useful pieces of information that contribute to each other while avoiding misleading information. This procedure enables the FedFNN to be more general.

B. Rule Evolution Stage

After $L$ rounds of coevolutionary learning, the FedFNN optimization process falls into a bottleneck since the capability of the current model structure has been fully explored. In this stage, we allow the server to inspect the performance of the learned model on local samples and then implement the FedFNN to increase its performance and personalization for handling non-IID data. In general, as described in algorithm 2, the rule evolution stage includes three major mutations: evolving new global rules, activating superior rules, and deactivating local inferior rules.

We adopt a contribution factor $\pi_k^q$ to measure the importance of the $k$th rule on the $q$th client, which can be calculated as

$$
\pi_k^q = \sum_{i=1}^{\lvert D^q \rvert} h_k^q(x_i^q : n_k^q, \sigma_k^q, s_k^q)
$$

where $\lvert D^q \rvert$ is the size of $D^q$. Here, we introduce a threshold $\bar{\pi}$ to trigger the rule activation procedure. $\bar{\pi}$ is calculated based on the average contribution of the activated rules as $\bar{\pi} = \beta \sum_{k=1}^K \pi_k^q / K^q$, where $\beta$ is a hyperparameter and $K^q$ is the number of activated rules in the $q$th client. A rule is meaningless to client $k$ when its contribution level is smaller than $\bar{\pi}$. Consequently, the server deactivates this rule for the corresponding clients and eliminates the involvement of those clients in the rule aggregation process. To assure that this step is completed, the rule status $s_k^q(t)$ in the $t$th round is updated by a status adjusting operation

$$
s_k^q(t) = \begin{cases} 
1, & \pi_k^q > \bar{\pi}_k^q \\
0, & \text{otherwise}
\end{cases}
$$

It is worth noting that each client should inspect its activated rules to check if the combination of these rules is sufficient for evaluating its local dataset. If not, it means that some of the unique features are not captured by the current activated rule settings. To solve this problem, we introduce two conditions to evaluate the capabilities of local models from different views

$$
\sum_{l=1}^L \ell^q(t-l+1) - \ell^q(t-l) > 0
$$

$$
\left( \ell^q(t) - \sum_{q=1}^Q \ell^q(t) / Q \right) > 0
$$

where $t$ is the current training round. The first condition (13) serves as a self-evaluation of the learning performance for each client by monitoring the loss value trends. If (13) holds, then the current local rules cannot be further improved. The second condition (14) serves as a peer evaluation by comparing the loss of the $q$th client with the average loss across all clients. If (14) holds, then the current local rules cannot handle non-IID data well. If both of these conditions hold, then the current architecture in the $q$th client is unable to attain high performance. In this case, a new rule should be generated to improve the local
To avoid the extreme situation when certain agents have no rules to use, the server will also create new rules for agents once their activated rule number is detected as zero.

This stage imitates the selective activation process of genes by updating the activation status of each rule based on its contribution to the local clients. Consequently, ERL selects useful rules for the local clients from the well-learned global rules, increasing the personalization of the FedFNN. In addition, the updated rule activation schemes benefit the optimization process in the next round of the rule cooperation stage in turn because of the better rule selection effect.

V. EXPERIMENTS

In this section, we conduct extensive experiments on seven datasets of different types in various settings to evaluate the effectiveness of the proposed FedFNN. Collected from multiple scenes, the selected datasets are representative and widely used. The descriptive statistics of each dataset are listed in Table I. In addition, to verify the superior performance of our model, state-of-the-art DFNNs and FL methods for constructing deep models are adopted as comparison methods. The details of all algorithms adopted in this section and their settings are introduced as follows.

1) DFNN: This is the fully DFNN algorithm proposed in [10], which adopts consensus learning in both the parameter learning and structure learning procedures and achieves state-of-the-art performance among distributed fuzzy models. As mentioned before, this model learns a consensus FNN for all clients, which limits its applications in non-IID scenarios. We use DFNN+ and DFNN* to denote homogeneous and heterogeneous federated setups, respectively.

2) FedAvg [14]: This is the most basic and popular algorithm for developing FL models. Similarly, we use FedAvg+ and FedAvg* to refer to the FedAvg models that cope with homogeneous and heterogeneous scenarios, respectively. Considering that the preferable deep model structures for different datasets vary from each other, to make the comparison convincing, we design more than 20 types of deep models for the comparison and report the best one for each dataset in the following tables and figures.

3) MOON [28]: This is the state-of-the-art FL approach for deep models. MOON adapts individual clients based on their dissimilarity with the server, which bestows the outperforming learning ability on the network model in solving heterogeneous distributed scenarios. Here, we utilize MOON for heterogeneous situations, namely, MOON*, for the comparison. Similar to FedAvg, we choose 20 deep models with different structures and list their best performance, as shown as follows.

4) FedFNN: This is the model proposed in this article, which adopts a rule evolution strategy to make full use of each fuzzy rule and acquire better estimation results for each individual dataset. We set the global number of rules as $K = 15$, the number of ERL iterations as $L = 10$, and the number of coevolutionary rounds as $N = 10$. For the hyperparameter setting, $\beta$ is set as 0.7.

In our experiment, all the distributed models are assigned with five local clients, each of which can only access their own dataset. To simulate heterogeneous local datasets for these clients, we generate five non-IID local data partitions using the Dirichlet distribution $\text{Dir}(\alpha)$, where $\alpha \in (0, 100)$ is the concentration parameter and can be seen as the indicator of the non-IID level of the data. Technically, the sample proportions of all categories for all clients are sampled from $\text{Dir}(\alpha)$. The local datasets are thus generated based on random sampling from the original dataset based on the obtained category proportions.

It is worth noting that for the fairness of the experiments, the features in all datasets are first normalized between $-1$ and $1$. 

| Algorithm | Dataset | GSAD | SDD | SC | MGT | WFRN | FM | WIL |
|-----------|---------|------|-----|---|-----|------|----|-----|
| DFNN+     |         | 36.64/1.08 | 16.26/0.67 | 63.02/4.66 | 83.22/0.49 | 53.04/1.59 | 60.56/6.02 | 83.40/3.48 |
| DFNN*     |         | 34.91/3.38 | 16.74/1.96 | 51.08/7.12 | 79.02/3.66 | 51.50/4.12 | 57.22/5.41 | 82.20/2.20 |
| FedFNN*   |         | 90.13/0.89 | 75.98/3.49 | 93.12/4.34 | 86.58/0.56 | 91.99/3.56 | 93.33/2.36 | 96.35/0.86 |
| MOON*     |         | 68.56/0.50 | 73.68/3.58 | 87.52/4.38 | 75.30/3.45 | 78.05/1.30 | 73.00/2.59 | 91.34/1.19 |
| FedAvg*   |         | 65.49/0.62 | 63.38/4.02 | 79.08/0.35 | 64.25/0.56 | 71.94/3.16 | 63.33/7.07 | 69.17/5.45 |
| FedAvg+   |         | 70.80/0.72 | 89.43/0.71 | 98.35/0.20 | 82.14/0.69 | 82.69/1.82 | 75.67/9.25 | 94.63/1.41 |

| Algorithm | Dataset | GSAD | SDD | SC | MGT | WFRN | FM | WIL |
|-----------|---------|------|-----|---|-----|------|----|-----|
| FedDNN    |         | 2,003,462 | 1,591,051 | 1,973,255 | 32,070 | 539,396 | 549,124 | 1,578,756 |
| FedFNN    |         | 15,450 | 9,525 | 1,320 | 630 | 2,220 | 3,930 | 690 |
using the well-known mapminmax normalization method. Then, a certain proportion of the features is randomly polluted by noise generated from a normal Gaussian distribution to simulate uncertainty in the different datasets. This perturbed sample proportion is considered the uncertainty level and is used to verify the uncertainty processing abilities of the comparison algorithms. In this section, all experiments are conducted with fivefold cross-validation, and each experiment is repeated 10 times. The final reported results are the mean average precision (mAP) values obtained on the test data during these runs.

A. Performance of the FedFNN on Non-IID Datasets

We conduct extensive experiments with the aforementioned algorithms on all seven datasets with a non-IID level of 0.5 and an uncertainty level of 10% to verify the effectiveness of our FedFNN. The obtained results are summarized in Table II, in which the green-colored values are those for which our FedFNN is superior to MOON* on different datasets. According to the table, our model outperforms the existing state-of-the-art FL method MOON* in terms of test accuracy by an average of 11.43%, which proves the extraordinary heterogeneity handling ability of our FedFNN.

To verify the robustness of our FedFNN when dealing with different levels of non-IID data, we choose the concentration parameter $\alpha$ from the set of $\{0, 0.1, 0.5, 5, 20, 50, 100\}$ and compare our FedFNN with FedAvg and MOON on the GSAD dataset. The results are shown in Fig. 5, in which the test accuracy curve of the FedFNN for different non-IID levels is flatter and higher than those of the compared FL algorithms. Thus, we can conclude that our FedFNN achieves superior performance when dealing with a wide range of non-IID data. In addition, when the given data are uncertain, our FedFNN* that processes non-IID local data can even beat the FedAvg+ version that processes local IID data on most datasets, which further proves the robustness of our FedFNN.
It is worth noting that our model is far more lightweight than existing FL algorithms. As listed in Table III, our model has an average of approximately 100 times fewer parameters than existing FL algorithms and achieves a better heterogeneity processing capability. Besides, we compared the computation overhead of our FedFNN and the DFNN on the Quadro P5000 GPU device with running storage of 26 384 MiB. Our results are listed in Table IV. Though the ERL consumes extra time compared with DFNN, our FedFNN increases more than 40% on average test accuracy than DFNN.

In addition, the personalization abilities of the algorithms are discussed in this section. To explicitly demonstrate the high level of the heterogeneous federated scenario, we plot the number of samples in each category for each local client of the GSAD dataset and the performance achieved by each local model when $\alpha = 0.5$ in Fig. 8. According to Fig. 8, our local models outperform MOON* and FedAvg* on all clients and achieve much higher test accuracy on the fifth client. MOON* and FedAvg* fail to learn the features of all categories because of the severe data heterogeneity.

Clearly, the global models of MOON and FedAvg tend to deal with the samples in the first five categories and overlook the information learned for the sixth category after several rounds of aggregation; this is because the samples in the sixth category are mostly allocated on client 5, whose learned information is easily disturbed by other clients. Instead, our personalized local FNN for the fifth client achieves relatively high performance on all clients by automatically generating new rules to estimate the unique samples in the sixth category. The newly generated rules are unique to this client and cannot be disturbed by other clients; thus, our proposed personalized local FNN exhibits superiority in dealing with heterogeneous local clients based on its flexible structure.

B. Performance of the FedFNN on Datasets With Uncertainty

To investigate the effectiveness of the FedFNN when dealing with data uncertainty, we conduct experiments under multiple uncertainty level settings $\{0\%, 10\%, 20\%, 30\%\}$. The performances of all algorithms on all datasets, except the FM dataset, under these uncertainty levels are depicted in Fig. 9. Intuitively, our methods achieve state-of-the-art performance for all uncertainty levels and all datasets. In addition, we list the performance attained by all algorithms when processing datasets with 10% uncertainty in Table II. From the table, our model outperforms the DFNN* in terms of test accuracy by an average of 36.40%. Thus, the extraordinary ability of our FedFNN to deal with data uncertainties can be confirmed.

C. Convergence Analysis of the FedFNN With ERL

To verify the convergence ability of our proposed FedFNN, the test accuracies it achieves throughout the optimization
process are depicted in Fig. 10. As the results in this figure show, the FedFNN gradually converges with the increase in the number of communication rounds on all datasets. In addition, the functions and contributions of the rule cooperation stage and the rule evolution stage are clearly proven in Fig. 10. During the \( L \)th round of the rule cooperation stage in each ERL iteration, the local clients cooperate with each other to learn more general global rules, and consequently, as shown in Fig. 10, the network performance stably improves on all datasets. However, the performance of the local FNNs gradually falls into a bottleneck along with the execution of the rule cooperation stage, as shown in Fig. 10, where the test accuracy hardly increases at each ERL iteration. The subsequent rule evolution stage solves this issue by updating the local model architecture. As shown in Fig. 10, the performance of the local clients dramatically increases because of the mutation of local rules. Eventually, these rapid performance improvements induced by the latter learning stage vanish after each local client learns all their required activated local rules.

**D. Analysis of Key Parameter Robustness**

We conducted extensive experiments to verify the robustness of our key parameters, including \( \alpha \), \( \beta \) and the initialized global rule number \( K \). Their corresponding results are depicted in Figs. 5, 6, and 7, respectively. As shown in Figs. 5 and 7, our FedFNN can still achieve as good performance when setting wide range of parameters. In addition, we can achieve high performance when setting \( \beta \geq 0.7 \) from the results drawn in Fig. 6. Intuitively, our FedFNN is proved to be robust to different parameter settings.

**VI. CONCLUSION**

This article proposes a FedFNN with ERL to handle non-IID issues and data uncertainties in distributed scenarios. The proposed FedFNN integrates fuzzy if–then rules into an FL framework. By considering these fuzzy rules as the basic optimization units, our FedFNN is able to learn a group of general global rules and selectively activate an effective subset of these rules for each local client. This flexible composition approach for fuzzy rules increases the personalization of the local models with respect to handling non-IID data. Inspired by the theory of biological evolution, the proposed ERL method not only encourages the cooperation of local clients at the rule level to improve the generalization of the global rules, but also updates the rule activation statuses for all clients to make their local models more personalized. Unlike most existing FL methods that implement aggregation among all local models, the FedFNN with ERL only aggregates the activated rules for their corresponding local clients. This enables the server to selectively aggregate only beneficial local updates, preventing the disturbances brought by harmful updates. Consequently, the FedFNN with ERL provides an effective learning framework for dealing with non-IID issues as well as data uncertainties. Comprehensive experiments verify the superiority and effectiveness of the proposed FedFNN over state-of-the-art methods. In the future, we will improve the ERL to increase the robustness and communication efficiency.
of FedFNN in scenarios, where the agent number is large. In addition, we will investigate the applications in interpretable AI using our FedFNN.

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