We consider the class of metrics that can be obtained from those of nonextreme black holes by limiting transitions to the extreme state such that the near-horizon geometry expands into a whole manifold. These metrics include, in particular, the Rindler and Bertotti - Robinson spacetimes. The general formula for the entropy of massless radiation valid either for black-hole or for acceleration horizons is derived. It is argued that, as a black hole horizon in the limit under consideration turns into an acceleration one, the thermodynamic entropy $S_q$ of quantum radiation is due to the Unruh effect entirely and $S_q = 0$ exactly. The contribution to the quasilocal energy from a given curved spacetime is equal to zero and the only nonvanishing term stems from a reference metric. In the variation procedure necessary for the derivation of the general first law, the metric on a horizon surface changes along with the boundary one, and the account for gravitational and matter stresses is an essential ingredient of the first law. This law confirms the property $S_q = 0$. The quantum-corrected geometry of the Bertotti - Robinson spacetime is found and it is argued that backreaction of quantum fields mimics the effect of the cosmological constant $\Lambda_{eff}$ and can drastically change the character of spacetime depending on the sign of $\Lambda_{eff}$ — for instance, turn $AdS_2 \times S_2$ into $dS_2 \times S_2$ or $Rindler_2 \times S_2$. Two latter solutions can be thought of as the quantum versions of the cold and ultracold limits of the
I. INTRODUCTION

In recent years, distinction between extreme and nonextreme black holes became an object of intensive studies. A nontrivial peculiarity of the extreme state consists in that its properties depend crucially on the way the limiting transition is performed. Consider, for instance, the Reissner-Nordström black hole as the simplest example. If its mass \( m \) is put equal to its charge \( e \), the proper distance between the horizon and any other point outside it diverges, the topology corresponding to the annulus of an infinite size. As a result, the vicinity of the horizon responsible for the entropy does not contribute to the Euclidean action. This property gave grounds to reason that classical extreme black holes possess unusual thermodynamic properties in the sense that their temperature is arbitrary and is not connected with the Hawking one (which is zero in the limit in question), while the entropy \( S = 0 \) [1]. On the other hand, as was shown in [2], [3], there exists a limiting transition \( m \to e \) in the topological sector of nonextreme black holes such that the proper distance between the horizon and points outside it remains finite. Although either the surface gravity or the Hawking temperature in this case are equal to zero, the physical temperature defining properties of a thermodynamic ensemble is finite, and the entropy \( S \) has the Bekenstein-Hawking value \( A/4 \), where \( A \) the a horizon area.

Thus, we have two types of limiting states of black holes with essentially different properties. For the sake of shortness we will further refer to the first type as 1 and to the second type as 2. Types 1 and 2 possess different Euler characteristics [4]. A sharp distinction between them manifests itself not only in thermodynamics and Euclidean approach but also in Lorentzian quantum geometrodynamics [5]. In fact, the states of type 2 represent the direct product of two-dimensional spaces — for example, \( AdS_2xS_2 \) (see below in a more detail). In this sense, they are not real black holes. However, as they arise as a result of the limiting
transition from the true ones whose horizons certainly possess thermodynamic properties, it is necessary to elucidate what happens to these properties in the limiting states.

Up to now our treatment concerned classical black holes. Meanwhile, studying quantum effects in the background of extreme black holes is of special interest. If a black hole is in state 1 there are good grounds to believe that the principal conclusion $S = 0$ made in [1] for classical black holes loses its validity when effects of backreaction from quantum fields surrounding a hole are taken into account. These effects force the temperature to take the Hawking value $T_H = 0$ since otherwise the stress-energy tensor of quantum fields diverges on a horizon [3], so the possibility of thermodynamic description of extreme black holes becomes questionable. In this respect, we are faced with the curious situation when even weak switching on dynamical interaction between a hole and its quantum environment changes drastically thermal properties of the system.

Correspondingly, the question arises about the role of quantum effects for state 2. In the paper [4] for the particular case of the Bertotti - Robinson (BR) metric [8], which is the limiting form of the Reissner-Nordström black hole [3], the quantum correction to thermodynamic entropy $S_q$ was found. It turned out that this correction does not change the thermodynamic properties of a system dressed by a quantum field as compared with a bare one and, moreover, $S_q = 0$. In the present paper we generalize this result and show that it retains its validity for a wide set of the limiting state belonging to class 2 independently of the particular form of the metric. Thus, the role of quantum effects in thermal properties proves to be very different for both types of states.

We consider also the influence of quantum effects on geometrical properties of the type 2 states. First of all, even in the absence of quantum effects the nature of a horizon in state 2 is different from that of a black hole whose limiting form the state 2 represents. Thus, for the BR metric, the existence and properties of a horizon is an observer-dependent effect due to geodesic incompleteness in some accelerated frames [3], so the horizon of a Reissner-Nordström black hole turns into an acceleration one. The geometry of quantum-corrected acceleration horizons possesses two main features. It retains the general form of the
direct product of two two-dimensional spacetimes typical of the classical counterpart and, in this sense, our approach confirms the recent observation made for the particular case of the geometry which represents the direct product of anti-deSitter space with a sphere. However, we argue that the concrete type of geometry may be changed by quantum backreaction in an essential way. For example, it can lead to the appearance of a second, cosmological-like horizon which is absent for a classical counterpart of the metric.

In turn, the general structure of a metric and, in particular, the difference between an acceleration horizon as compared to a black hole one, affects the thermodynamics in what concerns the formulation of the general first law. In the black hole case, the usual picture implies that the horizon radius is a free parameter allowed to vary, whereas the metric on a boundary of a system may be kept fixed. However, for a typical metric with an acceleration horizon the radius of a sphere entering an angular part of a metric is constant, so if it is kept fixed the first law turns into an empty identity. Therefore, we are faced here with a somewhat unusual situation when the first law acquires a nontrivial meaning only under condition that a boundary metric itself is varied. Another peculiarity consists in that the quasilocal energy density of such a system coming from a gravitational action is identically zero and nonzero contribution stems entirely from a reference metric (usually chosen as a flat one).

The paper is organized as follows. In Sec. II we derive the general expression for the entropy of quantum massless radiation valid in spacetimes with either black hole horizons or acceleration ones and show that in the latter case, \( S_q = 0 \). This derivation relies on the definition of the stress-energy tensor, its conservation law, the scale properties of massless radiation, and does not use the first law.

In Sec. III we give a qualitative explanation to the found property \( S_q = 0 \) as connected with the Unruh effect in, generally speaking, curved spacetimes. We discuss relationship between different pairs of spacetimes which represent Minkowski - Rindler analogues in curved manifolds and show how in some limit this analogy becomes literal coincidence.

In Sec. IV, we show that if in the formulation of the first law one properly accounts
for spatial gravitational stresses and the pressure of quantum fields, this law confirms the property $S_q = 0$.

In Sec. V, we derive the explicit form of the quantum-corrected geometry and find that there are three qualitatively different types of solutions depending on whether the curvature of submanifold in time-radial directions is negative, positive or zero.

In Sec. VI, we summarize briefly the results obtained and mark some possible problems for future research.

## II. ENTROPY OF HAWKING RADIATION IN TERMS OF STRESS-ENERGY TENSOR

Consider the Euclidean metric of the form

$$ds^2 = d\tau^2 b^2 + \alpha^2 dy^2 + r^2(y)d\omega^2. \quad (1)$$

If $r$ is not constant, it can be chosen as a new radial variable and we arrive at a generic spherically symmetrical spacetime. The special class of metric arises when $r = const \equiv r_+$. It just corresponds to the limiting transition to the extreme state of nonextreme black holes since in that limit all points of a manifold take the same value of $r$ [3]. In the particular case $b = r_+ \sinh y$, $\alpha = r_+$ we obtain the BR spacetime. While deriving in this section the formula for quantum entropy, we will not specify further the coefficients $b$ and $\alpha$.

The total entropy of the system is equal to

$$S = S_0 + S_q. \quad (2)$$

Here $S_0$ is the Bekenstein-Hawking entropy $S_0 = A/4$ where $A$ is the area of the event horizon and $S_q$ is the entropy of Hawking radiation. While $S_0$ appears as the result of the zero-loop approximation in the path-integral approach [11], $S_q$ is due to quantum fields. The quantity $S_0$ is determined solely by one characteristics of the event horizon and in this sense manifests the universality of laws of black hole physics. On the contrary, the quantity $S_q$
depends strongly on the kind of fields and concrete details of matter distribution. If $r$ is not constant, the expression for the entropy of Hawking radiation in terms of the renormalized stress-energy tensor is obtained for a wide class of metric

$$ds^2 = d\tau^2 f\left(\frac{r_+}{r}\right) + dr^2 f^{-1} + r^2 d\omega^2, \quad d\omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$$

(3) in [12]. Here $r_+$ is a horizon radius. The Schwarzschild metric belongs to this class with $f(x) = 1 - x$. In the state of thermal equilibrium the entropy of massless quantum field in this background inside a cavity of a radius $r_B$ is equal to [12]

$$S_q = 16\pi^2 r_+ |f'(1)| \int_+^{r_B} drr^2 (T^r_r - T^0_0 - T^\mu_\mu \ln \frac{r_B}{r})$$

(4)

where $T^\nu_\mu$ are the components of the renormalized stress-energy tensor in the Hartle-Hawking state. As such a tensor is finite on a horizon in an orthonormal frame, all components entering (4) are finite and the entropy converges. It is worth stressing that this is just thermodynamic entropy which reveals itself in physical experiments but not a statistical-mechanical part of it which for black holes has no direct physical meaning and even diverges [13]. This expression contains, apart from two first terms typical of a classical thermal gas, also a purely quantum anomaly part which is necessary for the general first law to hold [12].

Meanwhile, we are willing to obtain the entropy of quantum field valid for the BR spacetime

$$ds^2 = r_+^2 (d\tau^2 \sinh^2 x + dx^2 + d\omega^2)$$

(5)

and its generalization [11]. As the coefficient at $d\omega^2$ may now be constant, we cannot use the result [11] of [12] directly. Below we suggest derivation suitable for both cases, when $r$ can be either variable or constant.

It is convenient to normalize the radial coordinate in such a way that $y = 0$ at a horizon and $y = 1$ on the boundary, the period of the Euclidean time $\beta_0$ is chosen to be $2\pi$. It is implied that a system is situated in a cavity of a finite size. Let the surface area of a horizon be equal to $\pi r_+^2$. We choose $y = l/l_B$, $l$ is a proper distance from a horizon ($l = l_B$ for a boundary) and assume that metric coefficients take the form
\[ r = r_+q(l/r_+) \equiv r_+q(zy), \quad b = r_+d(l/r_+) \equiv r_+d(zy), \quad \alpha = l_B, \quad z = l_B/r_+ \quad (6) \]

which embraces both cases (3) and (8). Now we will show how the expression for the entropy can be recovered from the components of the stress-energy tensor.

Let us consider the variation of the metric which preserves the form (6), so only parameters of the metric are allowed to change. This metric has two such parameters - for instance, \( z \) and \( r_+ \). Then the variation of the Euclidean action \( I \) of quantum field inside a cavity splits to two parts: \( \delta I = \delta_1 I + \delta_2 I \). Here the first term has the standard form:

\[
\delta_1 I = \frac{1}{2} \int d^4 x \sqrt{g} T_{\mu\nu} \delta g^{\mu\nu},
\]

\( x^0 = \tau, \ x^1 = y, \ x^2 = \theta, \ x^3 = \phi \). It is worth bearing in mind, however, that this term does not exhausts the total variation since the formula (7) implies that the metric on a boundary is fixed. Meanwhile, we consider generic variation which affects the boundary surface. Below we will see that the term \( \delta_2 I \sim \delta r_B \) can be recovered from the scale properties of the action.

The terms with the variation of the metric can be written as

\[
T_{\mu\nu} \delta g^{\mu\nu} = -2T_0^0 \frac{\delta b}{b} - (T_2^2 + T_3^3) \frac{\delta r}{r} - T_1^1 \frac{\delta \alpha}{\alpha}.
\]

It follows from (8) that \( \delta b/b = \delta r_+/r_+ + \delta zy d'/d, \ \delta r/r = \delta r_+/r_+ + \delta zy q'/q, \ \delta \alpha/\alpha = \delta z/z + \delta r_+/r_+ \), where the prime denotes the derivative with respect to argument. Performing integration over angle variables and Euclidean time we obtain for the first part of variation:

\[
\delta_1 I/8\pi^2 = -\frac{\delta r_+}{r_+} \int_0^1 dy \sqrt{g} T_0^0 \delta z + \int_0^1 dy y \sqrt{g} [T_0^0 \frac{\delta d}{d} + T_1^1 T_0^0 + (T_2^2 + T_3^3) \frac{\delta z}{z}].
\]

\( \sqrt{g} = b \alpha r^2 \)

where \( r_+^4 z q^2 d \) takes into account the fact that the factor \( 8\pi^2 \) due to integration over angles and Euclidean time is already singled out. Now let us make use of the conservation law \( T_{\nu1}^\nu = \frac{(T_0^0 \sqrt{g})}{\sqrt{g}} - \frac{1}{2} \frac{\partial g_{\alpha\beta}}{\partial x_1} T_{\alpha\beta} = 0 \). Taking into account the explicit form of the metric (11) and scale properties of it we have: \( \frac{(q^2 dT_0^0)}{q^2 d} - \frac{[\frac{d}{d} T_0^0 + (T_2^2 + T_3^3) \frac{\delta z}{z}]}{q} = 0 \). By substituting this expression into \( \delta I \) we obtain after integration by parts:

\[
\frac{\delta_1 I}{8\pi^2} = -\frac{\delta r_+}{r_+} \int_0^1 dy \sqrt{g} T_0^0 - \delta z \frac{\sqrt{g}}{z} T_1^1(z).
\]

7
Let us write down also the term $\delta I/8\pi^2 \equiv \gamma \delta r_B$. It follows from (6) that $\delta r_B = \delta r_+ q + q' r_+ \delta z = \delta r_+ \frac{q' r_+}{r_+} + q' r_+ \delta z$, whence

$$\frac{\delta I}{8\pi^2} = \delta r_+ (\gamma q - \frac{1}{r_+} \int_0^1 dy \sqrt{\tilde{g} T^\mu_\mu}) + \delta z [\gamma q' r_+ - \frac{\sqrt{\tilde{g}}}{z} T^1_1(z)].$$  \hspace{1cm} (10)

This general formula describes the response of the action to the change of two variables $r_+$ and $z$.

Now let us take into account scale properties of the action of the massless quantum field. It is a dimensionless quantity which must depend on dimensionless arguments. In general, one can compose two such combinations from the parameters of the problem: $z$ and $l_0/r_+$, where $l_0$ is the Planck length. Correspondingly, one can write down $I = I(z, l_0/r_+)$. However, in the semiclassical approximation, when $l_0 \ll r_+$ and all effects of high-order loops are neglected, only the first parameter is relevant: $I = I(z, 0) \equiv I(z)$ plus negligible corrections. (Let me recall that in the Schwarzschild case when either the entropy or the action of quantum fields are calculated explicitly, they depend on one variable $r_B/r_+$ only, where $r_B$ is a radius of a system [14]; meanwhile, now we use $z = l_B/r_+$ instead of $r_B/r_+$). Thus, the semiclassical action of massless fields depends only on one variable $z$: $I = I(z)$. I stress that this fact, as we will see below, is consistent with the presence of quantum anomaly terms in the action.

This means that the coefficient at $\delta r_+$ should be equal to zero, whence

$$\gamma = (r_+ q)^{-1} \int_0^1 dy \sqrt{\tilde{g} T^\mu_\mu}. \hspace{1cm} (11)$$

By substitution into the part proportional to $\delta z$ we obtain

$$\frac{\delta I}{8\pi^2} = \delta z \left( q' \int_0^1 dy \sqrt{\tilde{g} T^\mu_\mu} - q \int_0^1 dy \sqrt{\tilde{g} T^1_1} \right). \hspace{1cm} (12)$$

From dimension grounds it also follows that the stress-energy tensor has the general form

$$T^\nu_\mu = r_+^{-4} t^\nu_\mu(r/r_+) \equiv r_+^{-4} f^\nu_\mu,$$  \hspace{1cm} (13)

where $t^\nu_\mu(q(zy)) \equiv f^\nu_\mu(zy)$. In eq. (12) the value of $T^1_1$ is to be taken at the boundary where $y = 1$. After simple transformations we have
\[ \frac{1}{8\pi^2} \frac{dI}{dz} = \frac{q'}{q} \int_0^z dx q^2(x) f^\mu_\nu(x) - f^1_1(z) d(z) q^2(z). \] (14)

Now we may find \( I \) by direct integration. The constant of integration is determined by the demand that \( I = 0 \) when \( z = 0 = l_B \) (no room for radiation). Changing the order of integration in the first term we may get rid of a double integral. Using the thermodynamic formula \( I = -\int d^4x \sqrt{g} T^0_0 - S_q \) let us write down the result at once for the entropy:

\[ S_q = \int_0^z dx q^2(x) d(x) [f^1_1 - f^0_0 - f^\mu_\mu \ln \frac{q(z)}{q(x)}]. \] (15)

Returning to dimensional variables we may rewrite this formula as

\[ S_q = \int dV_4 (T^1_1 - T^0_0 - T^\mu_\mu \ln \frac{r_B}{r}) \] (16)

Here \( dV_4 = \beta_0 d^3x \sqrt{g} \) is the element of Euclidean four-volume which in our particular coordinate system has the form (after integration over angles and time variable) \( dV_4 = 8\pi^2 dx r^3 q^2(x) d(x) \) where \( x = yz \). Let me stress that the contribution of conformal anomalies, as seen from (16), is taken into account.

The formula (16) holds for any spacetime whose metric has the form (3). There are two typical classes of it. In the first case, the metric can be rewritten as

\[ ds^2 = d\tau^2 U(r) + dr^2 V^{-1}(r) + r^2 d\omega^2. \] (17)

where \( U \) and \( V \) depend on \( r_+ \) via combination \( r_+ / r \). In particular, the Schwarzschild metric belongs to this class, in which case \( \beta_0 = 4\pi r_+ \), \( dV_4 = 16\pi^2 r_+ r^2 dr \) and we return to the result (4) describing a nonextreme black hole. The second class can be obtained from (17) by a well-defined limiting transition to the extreme state such that a local Tolman temperature remains finite nonzero quantity at any point outside a horizon. In so doing, \( r \to r_+ \) for all points of manifolds. As a result, the metric takes the form (3) with \( r(y) = r_+ = const \) and \( b \) typically representing a combination of hyperbolic functions (see [3] for details). In so doing, the components \( T^\nu_\mu \) of the stress-energy tensors in an orthonormal frame in any spacetime with a regular horizon pick up their values from a horizon: \( \lim_{r \to r_+} T^\nu_\mu(r) = T^\nu_\mu(r_+) \). Then
the regularity condition on a horizon $T_0^0 - T_1^1 = 0$ holds for a whole manifold. With the condition $r = r_+$, this means that both terms in entropy (15), (16) (either "normal" or "anomalous" ones) are equal to zero, so $S_q = 0$. In particular, the previous result [7] for the Bertotti-Robinson spacetime (for which $q = \sinh zy$ and $T_\mu^\nu \sim \delta_\mu^\nu r_+^{-4}$ [15], [16]) is reproduced.

It is also worth paying attention to the degenerate case corresponding to the "ultraextreme" case with $U''(r_+) = V''(r_+) = 0$. Then the limiting procedure elaborated in [3] leads to the metric which is a direct product of two-dimensional Rindler space and a sphere:

$$ds^2 = d\tau^2 l^2 + dl^2 + r_+^2 d\omega^2. \tag{18}$$

In spite of $T_\mu^\nu \neq 0$ due to effects of curvature of spacetime, the entropy of radiation $S_q = 0$ according to the general properties $T_1^1 - T_0^0 = 0$ and $r = r_+$.

One reservation is in order here. The possibility to replace components $T_\mu^\nu$ by their horizon values due to taking the limit under discussion implies that a horizon itself is regular in the original metric in the extreme state. This is not the case for dilatonic black holes and, as a result, the coefficient at the angular part does not turn into a constant in this limit but retains some dependence on $l$ [17]. Correspondingly, some residual dependence on $l$ may survive for $T_\mu^\nu$ and there is no reason to expect $S_q = 0$ in dilatonic backgrounds. We will not, however, discuss this case here further.

Thus, for a generic metric obtained as a finite-temperature extreme limit of nonextreme black holes with a regular extreme state the entropy of quantum massless Hawking radiation $S_q = 0$. Thus, the total entropy of such systems dressed by Hawking radiation (which turn into spacetimes with acceleration horizons in the limit under consideration) is equal to those of bare ones.

### III. UNRUH EFFECT AND LIMITS OF SPACETIMES

What physical explanation can be suggested for the property $S_q = 0$? Metrics discussed above share the following feature: they are obtained by a special kind of the limiting transition $r \to r_+$. As a result, spacetime picks up the sharp strip of near horizon geometry
which expands into a whole manifold with a finite Euclidean four-volume, so this is not approximation but the example of taking the "spacetime limit" procedure which maps an original manifold onto a new one [18]. In so doing, a new spacetime inherits properties of a vicinity of a horizon where a metric looks like that perceived by an accelerated observer, so as a matter of fact we deal with the Unruh effect [19]. This effect, however, has a pure kinematic nature and does not need the existence of a true black horizon; in a sense, it is too weak to gain nonzero entropy of Hawking radiation as there are no "true" quanta of it. (To avoid possible confusion in terminology, let me stress that we distinguish here the Unruh and Hawking effects as connected with the presence of acceleration and true black hole horizons, correspondingly. Thus, we use these terms in a way different from the book [20], where the thermal properties of the Hartle-Hawking state are prescribed, by definition, to the Unruh effect in a curved spacetime independently of the nature of a horizon — in particular, in the Schwarzschild background, i.e. in a true black hole metric. On the other hand, the term "Hawking effect" is used therein to describe a dynamical process of particles creation.)

Such a role of an acceleration horizon could serve as one more manifestation of the kinematical nature of Hawking radiation which may or may not be connected with an entropy associated with a horizon [21]. However, it is worth noting that in our case, in contrast to what was discussed in [21], the zero-loop entropy connected with the information loss does exist and kinematical properties of a spacetime reveal themselves only in cancellation of the one-loop part of entropy.

In general, metric obtained after the limiting transition in question, is curved; if quantum backreaction is neglected, it is the BR spacetime which is nothing else than a direct product of anti-de Sitter space and a sphere \((\text{AdS}_2 \times S_2)\) since the curvature of \((\tau, r)\) submanifold is a negative constant. Such a spacetime has three independent Killing vectors [9] and an observer moving along a Killing orbit may feel horizons with nonzero or zero Hawking temperature or see the absence of a horizon at all that justifies purely kinematic nature of the effect in question.
In this sense there is some analogy in relationship between different sections of BR spacetime, on one hand, and relationship between the Minkowski and Rindler spaces, on the other one. Now we show that in the properly adjusted ”large mass limit” \( r_+ \to \infty \) this analogy turns into literal correspondence. Let us write down the Lorentzian form of the BR metric relevant for description of the extreme limit of nonextreme black holes (BR1):

\[
ds^2 = r_+^2(-dt^2 \sinh^2 x + dx^2 + d\theta^2 + d\phi^2 \sin^2 \theta). \tag{19}
\]

This metric possesses a horizon at \( x = 0 \) which, however, is not of black hole type but rather is an acceleration horizon. One can perform the transformation into another frame in which an observer moving along orbits of another Killing vector will see no horizons at all \([9]\). Namely, after the transformation

\[
cosh t \sinh x = \sinh \chi, \quad \cosh x = \cos \tilde{t} \cosh \chi \tag{20}
\]

we arrive at the metric BR2

\[
ds^2 = r_+^2(-d\tilde{t}^2 \cosh^2 \chi + d\chi^2 + d\theta^2 + d\phi^2 \sin^2 \theta). \tag{21}
\]

Let us perform the transformation \( \theta = \vartheta + \pi/2, \quad x = l/r_+, \quad \tilde{t} = T/r_+, \quad \chi = Z/r_+, \quad \vartheta = X/r_+, \quad \phi = Y/r_+ \). Then after the limit \( r_+ \to \infty \) is taken, the metric (19) turns into the Rindler one

\[
ds^2 = -dt^2l^2 + dl^2 + dX^2 + dY^2, \tag{22}
\]

while (21) becomes the Minkowski metric

\[
ds^2 = -dT^2 + dZ^2 + dX^2 + dY^2. \tag{23}
\]

Expanding eq. (20) in powers of \( r_+^{-1} \) and retaining main non-vanishing terms, we obtain the formulae

\[
Z = l \cosh t, \quad Z^2 - T^2 = l^2 \tag{24}
\]

which describe just the connection between the Minkowski and Rindler metrics. As far as the entropy of thermal gas is concerned, it remains zero in the process of the limiting
transition for both sections of the BR spacetime \((19), (21)\). Thus, the analogy between the Rindler/Minkowski and BR1/BR2 metrics in what concerns pure kinematic nature of Hawking radiation and the property \(S_q = 0\) becomes literal in the limit at hand.

It is worthwhile to note that, strictly speaking, the result \(S_q = 0\) for the Rindler spacetime \((22)\) does not follow from the general formula \((16)\) directly since the metric \((22)\) does not belong (in contrast to \((18)\)) to the limiting class of metrics \((1)\) for which \((16)\) was derived. However, the above limiting relation makes this property transparent since \(S_q(\text{Rindler}) = \lim S_q(\text{BR}) = 0\). Nevertheless, it is also instructive to trace another kind of limiting transition — directly from the Schwarzschild metric since, as we will see, it exhibits features similar to those for the limiting transition which brings a nonextreme Reissner-Nordström black hole to the extreme limit. Consider the Schwarzschild metric

\[
\text{ds}^2 = -dt^2(1 - \frac{r_+}{r}) + dr^2(1 - \frac{r_+}{r})^{-1} + r^2(d\theta^2 + \sin^2\theta d\phi^2),
\]

where \(-\pi \leq \phi \leq \pi\), \(0 \leq \theta \leq \pi\). Here \(r_+ = 2M\) is a horizon radius of a black hole with mass \(M\). We will assume that a black hole is situated in a cavity whose boundary ensures thermal equilibrium between a black hole itself and its Hawking radiation (the Hartle-Hawking state). Then, as was shown in \([12]\), the entropy of massless Hawking radiation is described by eq. \((4)\) with \(|f'(1)| = 1\). Due to scale properties of massless quantum radiation \([13]\) this equation can be rewritten as

\[
S_q = 16\pi^2 \int_w^1 \frac{du}{u}u^{-2}[f'_r - f'_0 - f'_\mu \ln(u/w)],
\]

where \(w = r_+/r\). The formulas \((3), (24)\) hold for any kind of massless radiation including either bosons with generic type of coupling to gravity or fermions \([14]\).

In the canonical ensemble the event horizon radius \(r_+\) is not arbitrary but is a function of either \(r_B\) or a local Tolman temperature \(\beta^{-1}\) on a boundary according to the equation \([22]\)

\[
\beta = 4\pi r_+ \sqrt{1 - \frac{r_+}{r_B}}.
\]

Accounting for a finite size of a system has the crucial consequences for thermodynamics: it allows one to define the canonical ensemble for black holes in a self-consistent way, leads
to the appearance of the stable branch of solutions, etc. [22]. Now we will consider the large mass limit which takes into account properly that the system has a finite size, so $r_+ \leq r \leq r_B$. We will also assume that the limiting transition preserves the value of $\beta$. These assumptions mean that the process of limiting transition is performed in such a way that $r_+ \rightarrow \infty$, $r_B \rightarrow \infty$, while a square root in eq. (27) tends to zero and $r_+/r \rightarrow 1$ for all points of the manifold. Thus, the coordinate $r$ becomes degenerate and is to be properly rescaled. In a similar way, as the inverse Hawking temperature $T_H^{-1} = 4\pi r_+ \rightarrow \infty$, the time coordinate needs to be rescaled too. Before taking such a limit, let us perform the change of variable $\theta = \vartheta + \pi/2$, so $-\pi/2 \leq \vartheta \leq \pi/2$ and introduce new coordinates according to

$$
\vartheta = X/r_+, \phi = Y/r_+, t = \tau/2\pi T_H = 2\tau r_+, \ r = r_+ + l^2/4r_+.
$$

Then after the limiting transition at hand the original metric (25) turns into the Rindler metric (22). It follows from (13) that $T^{\nu}_{\mu} \rightarrow 0$ in accordance with the fact that in the state of thermal equilibrium the stress-energy tensor in the Hartle - Hawking state cancels for the Rindler metric [23]. It is worth noting that in the Hartle - Hawking state, the components $T^{\nu}_{\mu}$ of the stress-energy tensor in an orthonormal frame are finite on the event horizon of the Schwarzschild black hole, so the quantities $f^{\nu}_{\mu}$ are also finite at $u \rightarrow 1$. Therefore, it is seen from (26) that $S_q \rightarrow 0$ in the limit at hand: the thermodynamical entropy of Hawking radiation cancels for a Rindler wedge. In a sense, thermal gas of Rindler quanta is a rather peculiar object from the thermodynamic point of view: in spite of its temperature being nonzero, either its energy defined as $- \int d^3 x \sqrt{g} T^0_0$ or the entropy are equal to zero. I stress that the statement $S_q = 0$ is related just to the thermodynamic entropy (i.e. the quantity just having direct physical meaning) and should not be confused with the properties of statistical-mechanical one [24], [25] (in a similar way, the property $S_q = 0$ for a thermal gas in the BR background [4] should not be confused with the behavior of quantum correction to the entropy of a black hole itself [26]).

It is worth stressing that the limiting procedure for any spacetime is not unique and depends, for example, on a particular choice of a coordinate system in which parameters of
a system tend to their limiting values \[18\]. In our case the limit \(r_+ \to \infty\) is distinguished by the demand that a local temperature on a boundary is fixed, so it has clear thermodynamic meaning. In so doing, however, the boundary itself drastically changes: a sphere turns into a plane. As in the limit at hand \(r_+ \to \infty\) in such a way that \(r_+/r_B \to 1\), the zero-loop entropy of a black hole, equal to its Bekenstein-Hawking value, behaves like \(S_0 = \pi r_+^2 \simeq \pi r_B^2 \simeq A/4\) where \(A = \int dXdY\) is the surface area of a plane \(l = \text{const}\), so \(S_0\) diverges but the entropy per unit area is finite \[27\]. It is worthwhile to note that the derivation of the formula for the entropy \(S_0\) in \[27\] relies at once on the metric of the Rindler wedge and the term \(A/4\) comes from a boundary, so the connection between this term and the horizon remained not quite clear (any surface \(l = \text{const}\) has the infinite area \(A\) for the Rindler metric). Meanwhile, the limiting transition performed above clearly shows that the Rindler wedge inherits the formula \(S_0 = A/4\) from the Schwarzschild spacetime where it originates from an event horizon.

It is interesting that, although \(A \to \infty\), the proper distance \(L\) between a horizon and any other fixed point outside, including a boundary, is finite. Indeed, in the Schwarzschild metric we have \(L = r_+[\sqrt{x(x-1)} + \ln(\sqrt{x} + \sqrt{x-1})]\) where \(x = r/r_+\). In the limit \(r_+ \to \infty\) performed in coordinates \(28\), \(L \to l\). In other words, we have a plane situated at the proper distance \(l_B\) from the origin of coordinates, where \(l_B\) is the value of the \(l\)-coordinate of the boundary in the original Schwarzschild metric, this plane having the same temperature as a boundary sphere in the Schwarzschild metric.

In general, the total entropy of a system possessing a horizons comes, according to \(2\), from either the horizon (the term \(S_0\)) itself or quantum fields (the term \(S_q\)). The fact that in our case the entropy is determined by the area of the horizon only, this horizon having kinematical nature (the acceleration horizon instead of the black hole one), manifests the kinematic character of the Unruh effect in the given context.

Recently, an interesting interpretation of the Hawking effect as the Unruh one in some embedding auxiliary flat space of higher dimensionality has been suggested \[28\]. On the contrary, in our approach we trace the passage from one spacetime to another remaining
within a physical four-dimensional curved manifold. And what we want to stress is that the equivalence between two effects traced in [28] for temperatures and zero-loop entropies, breaks down for the entropies of quantum radiation.

IV. GENERAL FIRST LAW FOR ACCELERATION HORIZONS

The Euclidean canonical action for a spherically-symmetrical bounded self-gravitating charged system obeying the Hamiltonian constraint and Gauss law reads [27]

\[ I = \beta E - S - \beta \phi e, \]  

(29)

where \( \beta \) is an inverse Tolman temperature on a boundary, \( \phi \) is a blueshifted potential difference between the horizon and boundary, and \( e \) is charge. For a given set of boundary data \( (\beta, r_B, \phi) \) a small variation in a horizon radius gives, according to the action principle \( \delta I = 0 \), the form of the first law under conditions that all the field equations are satisfied, so terms arising due to equations of motions cancel [30]. The energy \( E = 4\pi r_B^2 \varepsilon \), where quasilocal energy density \( \varepsilon \) entering a thermodynamic energy \( E \) is equal to \( (k - k_0)/8\pi \) [31], [30], \( k \) is an extrinsic curvature of two-dimensional boundary embedded into a three-dimensional space, \( k_0 \) is that for the same boundary metric embedded into a reference flat space to have \( E = 0 = I \) for a flat metric. In a spherically-symmetrical spacetime of the form (17) \( E = r_B[1 - \sqrt{V(r_B)}] \), where the first contribution corresponds to the flat space term \( k_0 \).

In attempting to apply the first law to metrics under discussion which have \( r = cons = r_+ \) one immediately faces the following oddities. The term with \( k \) in energy is equal to zero identically. It follows either from definition of \( k \) or from the above formula for \( E \). Indeed, the metric in question is obtained by the limit \( r \to r_+ \) for all points of manifold including a boundary. As a result, the coefficient \( V \) in the formula for \( E \) picks up its value from a horizon where \( V = 0 \). Apart from this, the quantity \( r_+ \) cannot be any longer considered as a free parameter independent of boundary data. Now the two-dimensional metric induced on
a horizon coincides with that of a boundary. These two circumstances do not mean, however, that the first law loses its sense. Rather, it leads us to the necessity to consider its extended form including from the very beginning the contribution from the changes of a boundary metrics. According to [30], [32], such a contribution can be represented as $-\frac{1}{2} \int \delta \sigma_{ab} s^{ab} \sqrt{\sigma} \beta$ where indices $a,b$ are related to a two-dimensional boundary with the metric $\sigma_{ab}$, $s^{ab}$ are components of spatial stresses. For a spherically symmetrical spacetime with $\beta = \text{const}$ on a boundary this reduces to $\lambda \delta r_B$ where $\lambda = -4 \pi r_B \beta s^a_a$. It follows from eq. (6.9) of [30] that, in our notations, $\lambda = 2 \pi [b - \frac{\partial (b r)}{\partial l}]_B$. Apart from gravitational contribution, we must take into account the change of the action of quantum fields $8 \pi \gamma \delta r_B$ with $\gamma$ from eq. (11).

Comparing with (29), we have

$$\beta \delta E - \beta \phi \delta e - \delta S = 2 \pi \{[b - \frac{\partial (b r)}{\partial l}]_B + 4 \pi r_B^{-1} \int_0^1 dy \sqrt{\tilde{g}} T^\mu_\mu \} \delta r_B.$$  \hspace{1cm} (30)

In such a form the first law must hold for any spacetime of the form (1) under the presence of an electromagnetic field. Now we apply it to the spacetimes which are the $r \rightarrow r_+$ limits of (17) in the sense discussed above.

First, consider the classical BR spacetime for which in (1) $b = r_+ \sinh l/r_+$, $T^\nu_\mu$ is neglected. The energy $E = r_B = r_+$. Integrating the Maxwell equation $F_{ij}^{0 \mu} = (F_{01} \sqrt{g}) / \sqrt{g} = 0$ it is easy to find the value of $\phi \equiv b_B^{-1} [A_0(1) - A_0(0)]$, where $A_0(1) - A_0(0)$ is the difference of electrostatic potentials $A_0$ between a horizon and boundary: $\phi = \tanh l/2 r_+$ where we have taken into account that for a BR spacetime the charge $e = r_+$ (see for details below).

Substituting $\beta = 2 \pi b_B$ and $S = \pi r_+^2$ into (30) we see that the first law is satisfied.

Let now quantum backreaction be taken into account, the total stress-energy tensor $T^\nu_\mu(\text{tot}) = T^\nu_\mu(\text{em}) + T^\nu_\mu$ representing the sum of contributions from an electromagnetic field and quantum one. For a metric (1) with $r = r_+ = \text{const}$ the nonvanishing components of the Einstein tensor are $G^0_0 = -1/r_+^2 = G^1_1$, $G^2_2 = G^3_3 = b^{-1} \frac{\partial b}{\partial r}$. If the Gauss law $\frac{\partial A_0}{\partial l} = eb/r_+^2$ is taken into account, the electromagnetic part of the energy-momentum tensor has the standard form $T^\nu_\mu(\text{em}) = e^2/8 \pi r_+^4 \text{diag}(-1,-1,1,1)$. For a massless radiation the stress-energy in the BR background is
\[ T_\mu^\nu = \frac{B}{8\pi} \delta_\mu^\nu r_+^{-4} \]  

where \( B = \text{const} \) and the factor \((8\pi)^{-1}\) is introduced for convenience. Einstein equations give us \( e^2 = r_+^2 + B, \) \( b = \rho \sinh \rho^{-1} l \) where \( \rho^{-2} = r_+^2(1 + 2B/r_+^2) \). In the main approximation with respect to \( B \) we have \( \gamma = \frac{B}{2\pi r_+}(\cosh l_B/r_+ - 1) \). From the Gauss law it follows that \( \phi \equiv b^{-1}[A_0 - A_0(0)] = \frac{e}{r_+ \rho \tan \frac{\rho l}{2}} \). Substituting these expressions into (30), we may check directly that the general first law is satisfied provided \( \delta S = 2\pi r_+ \delta r_+ \). Integrating this equality we obtain that \( S = \pi r_+^2 + c \) where \( c \) is some constant. Here the first term represent Bekenstein - Hawking entropy whereas the second one is responsible for Hawking radiation. From the demand that the second contribution vanishes when a boundary approaches the surface of a horizon (no room for radiation) we obtain that \( c = 0 \). Thus, we arrive at the same conclusion as was made above: entropy of Hawking radiation in the BR background is equal to zero exactly.

V. QUANTUM-CORRECTED GEOMETRY OF SPACETIMES WITH ACCELERATION HORIZONS

Limiting geometries found in [3] relied on the general assumptions of the limiting transition from nonextreme black hole metrics to the extreme state with a finite local temperature in any point between a horizon and physical boundary. No field equations with or without backreaction of quantum fields on a metric were used in [3]. Meanwhile, account for such equations restricts strongly the possible type of limiting metrics. As follows from the formulae of the previous section, the quantum-corrected BR spacetime has the form

\[
\begin{aligned}
    ds^2 &= d\tau^2 \rho^2 \sinh^2 \frac{l}{\rho} + dl^2 + r_+^2 d\omega^2, \\
    \rho &= \frac{e}{r_+ \rho \tan \frac{\rho l}{2}}, \\
    \phi &= \frac{e}{r_+ \rho \tan \frac{\rho l}{2}}, \\
    \rho^{-2} &= r_+^{-2}(1 + 2B/r_+^2), \\
    e^2 &= r_+^2 + B.
\end{aligned}
\]  

There is also another solution

\[
\begin{aligned}
    ds^2 &= d\tau^2 \rho^2 e^{2\rho l} + dl^2 + r_+^2 d\omega^2
\end{aligned}
\]  

18
with the same \( \rho \).

Recently it was argued by Solodukhin that the product spacetime \( AdS_2 \times S_2 \) is an exact solutions of semiclassical field equations with quantum backreaction taken into account \([10]\).

In fact, as in \((r, \tau)\) submanifold the curvature \( R_2 = -2\rho^{-2} \) is constant and \( R_2 < 0 \), our formula \((32)\) is in conformity with this statement. This metric is based on the perturbative expression for \( T_\mu^\nu \) valid in the region \(|B| \ll r_+^2\). It is instructive, however, to extend (not quite rigorously) a semiclassical approach to pure quantum domain for which \(|B| \sim r_+^2\). Then for \( B = -|B| \) the \((32)\) equation gives us a qualitatively new type of solutions if \(|B| \geq r_+^2/2\):

\[
    ds^2 = d\tau^2 \sigma^2 \sin^2 \frac{l}{\sigma} + dl^2 + r_+^2 d\omega^2, \quad \phi = \frac{e\sigma}{r_+^2} \tan \frac{l}{2\sigma}, \quad \text{(34)}
\]

\[
    \sigma^{-2} = r_+^{-2} \left( \frac{2|B|}{r_+^2} - 1 \right), \quad e^2 = r_+^2 - |B|. \quad \text{(35)}
\]

Formally, the metric \((34)\) is obtained from \((32)\) by the substitution \( \rho = i\sigma \). It is seen from \((34)\) that the solution of the form \((34)\) may exist only under the condition

\[
    r_+^2/2 \leq |B| \leq r_+^2 \quad \text{(35)}
\]

, i.e. for strong backreaction, and in this sense is pure quantum, the curvature of \((r, \tau)\) submanifold \( R_2 = 2\sigma^{-2} = \text{const} \geq 0 \). In these respects the \((r, \tau)\) part of \((34)\) resembles the one found in \([33]\) for 2D dilaton gravity. Thus, in addition to the \( AdS_2 \times S_2 \) found in \([10]\), in our problem there exists also spacetime which is a direct product of de Sitter space and a sphere \((dS_2 \times S_2)\). As for this solution \( T_{\mu}^{\nu}(\text{em}) = 0 \) and \( B < 0 \), the energy density is positive everywhere including a horizon. In this respect it can be considered as a BR-like counterpart of black holes which may possess the extreme state due to positive energy density on a horizon whose existence was qualitatively conjectured in \([34]\).

For the solution under discussion the ratio of squared radii of two pieces of spacetime \( 0 \leq r_+^2/\sigma^2 \leq 1 \). The minimum value of this ratio is achieved at \( R_2 = 2\sigma^{-2} = 0 \), \( |B| = r_+^2/2 \) when we obtain the spacetime \( Rindler_2 \times S_2 \) \([18]\) with the electrostatic potential \( \phi = 2^{-3/2}l/r_+ \). In contrast to \((32)\) - \((34)\), the solution at hand has no 2D counterpart: in the latter case \([33]\) a metric can be flat only under condition that either an electromagnetic
field or backreaction cancel whereas now either each contribution separately or their sum differs from zero: $T_0^{0(\text{tot})} = T_1^{1(\text{tot})} = -1/8\pi r_+^2$. Such a spacetime can be regarded as the example of physical realization of the ultraextreme limit of nonextreme black holes [3] that shows how a Rindler metric may appear as a nontrivial result of special tuning between electromagnetic forces and quantum backreaction: $e^2 = |B|^2$ and tangential stresses vanish, $T_2^{2(\text{tot})} = T_3^{3(\text{tot})} = 0$.

The maximum value of $r_+^2/\sigma^2$ corresponds to $e = 0$, $B = -r_+^2$, $\sigma = r_+$. The possibility $e = 0$ due to quantum effects was suggested by Solodukhin [10] for the $AdS_2 \times S_2$ solution. Our formulae, however, do not admit such a possibility for the $AdS_2 \times S_2$ case since it is inconsistent with the property $R_2 < 0$ according to (32). This difference in properties of solutions under discussion can be explained by the fact that we are dealing with a conformally invariant scalar field, whereas in [10] this field has minimal coupling. Meanwhile, for $dS_2 \times S_2$ solutions $e = 0$ is indeed possible, in which case radii of both two-dimensional subspaces coincide.

It is instructive to suggest qualitative explanation of rather unusual consequences in the structure of spacetime caused by quantum backreaction. The key moment consists in the structure of the energy-momentum tensor (31). It is seen from eq. (31) that the stress-energy tensor in the BR metric mimics the effect of the cosmological constant: $T^\nu_\mu = \Lambda_{\text{eff}} \delta^\nu_\mu$, where the effective cosmological constant $\Lambda_{\text{eff}} = -Br_+^{-4}$. This constant is absent on the classical level and is caused in our case by quantum effects entirely. If $B < 0$, $\Lambda_{\text{eff}} > 0$. If a system possesses either an electric charge or the positive cosmological constant, we have, in general, the Reissner-Nordstrom-de Sitter solution (RNdS) with three horizons - the inner one $r_i$, the outer black hole horizon $r_o$ and the cosmological one $r_c$. In the particular case, when the radii $r_o$ and $r_c$ merge, one obtains the charged version of the Nariai solution [35]. It can be obtained as the so-called cold limit of the RNdS metric [36]. In this limit, the volume in the region $r_o < r < r_c$ remains finite despite $r_o \to r_c$, and the new metric, arising as a result of the limiting transition, looks just like (34). The surface gravity of each horizon in question tends to zero (that motivates the name "cold") but it is essential that the physical Tolman
temperature in every point outside the horizon remains finite nonzero quantity (in Ref. [37] such a kind of limiting transitions is considered in a general setting without specifying the concrete type of the metric). Substituting $|B| = \Lambda_{\text{eff}} r_+^4$ into eq. (35), we obtain inequalities inherent to the charged Nariai solution, $e^2 < \Lambda_{\text{eff}} r_+^4$ and $\Lambda_{\text{eff}} r_+^2 < 1$ (the modern discussion of the Nariai solution and its properties as well as a number of references can be found in [38]). The case of the equalities corresponds to the ultracold one (see below). The physical interest in spacetimes under discussion is dictated, in particular, by their role in pair creation of black holes in cosmological backgrounds (see, for instance, Refs. [39], [40] and references therein).

In the case when all three horizons merge ($r_i \to r_o \to r_c$) the so-called ultracold limit of the RNdS spacetimes arises [36]. The corresponding Euclidean metric reads (18) [39] and, thus, coincides with the quantum-corrected geometry obtained by us above.

Thus, the structure of the resulting spacetime depends on the sign of $\Lambda_{\text{eff}}$ and can be thought of as the result of different types of limiting transitions to the extreme state. If $\Lambda_{\text{eff}} < 0$, we have $AdS_2 \times S_2$ which can be considered as a result of the extreme limit for nonextreme black holes [2], [3]. If $\Lambda_{\text{eff}} > 0$, the metric has the form $dS^2 × S_2$ (34) which appears after the limiting transition to the state with $r_o = r_c \neq r_i$, or Rindler$^2 × S_2$ when radii of all three horizons merge.

Let me stress that the qualitatively new point as compared with Refs. [36], [38], [39], [40] and references therein consists in that the $\Lambda$-term in our problem is absent classically, so the appearance of analogs of the limiting forms of the RNdS solutions is a pure quantum effect. It is also worth noting that we did not consider the RNdS with the consequent limiting transition but started at once with the BR-like spacetimes and showed how account for backreaction of quantum fields may change the properties of a spacetime.

The constant $B$, responsible for quantum effects, can be written as $al_0^2$, where $l_0$ is the Planck length, $a$ is numerical coefficient. According to (32) - (34), the solution with $e = 0$ is possible only for $a = - |a| < 0$. In particular, a massless conformally invariant field for which $a = (2880\pi^2)^{-1} > 0$ [16] seems to be not a suitable candidate for such type of solutions. If
$e = 0$, the radius $r_+$ which measures the curvature of a sphere acquires planckian scale in agreement with Solodukhin’s observation: $r_+ = |a| l_0$ (In fact, even $r_+ \ll l_0$, but one can attain $r_+ \sim l_0$ for a sufficiently large number of field species).

One reservation is in order. From the formal viewpoint, the metric (32) was based on a semiclassical expression for the stress-energy tensor $T_{\mu}^{\nu} = (8\pi)^{-1}Br_+^{-4}\delta_{\mu}^{\nu}$ calculated on a given BR background, so the extension to the domain $|B| \sim r_+^2$ is nothing else than extrapolation. Nevertheless, the striking similarity to the 2D case where a semiclassical stress-energy tensor is known exactly, strongly supports the validity of found solutions. More rigorous justification of possibilities indicated in [10] and in the present paper needs a fully self-consistent quantum treatment. One cannot exclude in advance the existence of pure quantum solutions with $e = 0$ for any kind of quantum field.

Anyway, the indicated class of solutions (34) indebted entirely to quantum effects hints that strong backreaction can modify the structure of spacetime qualitatively and, in particular, change the sign of curvature. Drastic changes may also happen not only to solutions (32)-(34) themselves but also with black holes whose near-horizon geometry may be approximated by these constant curvature solutions. One such possibility consists in the existence of an extreme quantum Schwarzschild-like black hole with a zero surface gravity [34], [10]. Here we would like to draw attention to another possibility: the appearance, according to (34), of the solution with a cosmological horizon.

Thus, on one hand, quantum effects preserve the general form of the metric as a direct product of two two-dimensional submanifolds in accordance with [10]. On the other hand, the concrete set of possible kinds of such a structure includes not only spacetimes with $R_2 < 0$ indicated in [10] but also those with both other possible types $R_2 > 0$ and $R_2 = 0$.

VI. CONCLUSION

In general, one may distinguish three areas of application of thermodynamic approach to systems with horizons: nonextreme black holes, extreme black holes and acceleration
horizons. The present paper is devoted to the third case and, in this sense, fills a gap in the relationship between thermodynamics and horizon mechanics. A typical representative of the spacetimes with an acceleration horizon is the BR metric. The interest to different aspects of BR-like spacetimes has increased in recent years [41]. Here, we considered quantum-corrected BR-like spacetimes and showed that their acceleration horizons exhibit the following universal property: the entropy of Hawking massless radiation $S_q = 0$. The procedure of taking spacetimes limits, with the help of which the metrics with an acceleration horizons are obtained from black hole ones, showed that the result $S_q = 0$ is intimately connected with the Unruh effect rather than with the Hawking one. The general first law does have sense for the metrics in question in spite of some restrictions on the variation procedure which now implies that the boundary radius is to be varied together with that of the horizon. If formulated properly, this law confirms that the entropy of radiation does not contribute to thermodynamics of a system. The result $S_q = 0$ can serve as a test for checking various renormalization schemes in calculations of quantum entropy of black holes which possess the extreme state.

While quantum effects do not reveal themselves directly in thermodynamics of acceleration horizons, they can have crucial consequences for a structure of spacetime. In particular, the strong backreaction seems to lead to the possible change of the sign of two-dimensional $R_2$ curvature of $(r, \tau)$ submanifold and the appearance of the quantum version of the Nariai solution with cosmological horizon but without the cosmological constant or to the possibility to have a flat $(r, \tau)$ submanifold as an exact solution of semiclassical field equations.

In the present paper we restricted ourselves by the case of massless fields since their scale properties simplify the problem at once in two points: (i) the action depends on one variable and (ii) the thermal stress-energy tensor has the general structure (13) (see Sec.II above). Meanwhile, it is of interest to obtain the formula for the entropy of quantum massive fields in terms of the stress-energy tensor which would replace eq. (16) derived for massless ones, and check the validity of the property $S_q = 0$. The case of massive fields is especially important in connection with the issue of quantum renormalization of the black hole entropy.
It was shown in [12] without using the brick-wall model that Pauli-Villars regularization correctly reproduces the Bekenstein-Hawking entropy, if the gravitational constant is properly renormalized (this approach was elaborated further for many-dimensional cases in [13]). Therefore, the desired formula for the thermodynamic entropy, being combined with the results of [12], would enable us, in particular, to trace in detail the behavior of different parts of the black entropy near the extreme state. We hope to address this issue in a subsequent research.

Apart from extending the approach to another kinds of fields, it is also worthwhile to consider more general geometries - in particular, spacetimes which may be obtained by the spacetime limits taken in the background of distorted black holes [37]. Of special interest is the problem of finding either quantum geometries or the stress-energy tensor in a background with acceleration horizons in a fully self-consistent manner.

In this paper, we restricted ourselves to the limiting states of type 2 which are obtained by certain limiting transitions to the extreme state within the topological sector of nonextreme black holes. The separate issue deserving special treatment is the influence of quantum backreaction on states of type 1 which represent topologically true extreme black holes.

VII. ACKNOWLEDGMENTS

I am grateful to Ted Jacobson for comment on my paper [3] with the interpretation of limiting geometries found therein as describing the Unruh effect in the AdS$_2$ background. I am also grateful to Sergey Solodukhin for helpful correspondence. This work is supported by the International Science Education Program, grant # QSU082068.

[1] Hawking S W, Horowitz G T and Ross S F 1995 Phys. Rev. D 51 4302

[2] Zaslavskii O B 1996 Phys. Rev. Let. 76 2211
[3] Zaslavskii O B 1997 Phys. Rev. D 56 2188

[4] Wang B and Su R K 1998 Phys. Lett. B 432 69

[5] Kiefer C and Louko J 1999 Annalen Phys. 8 67

[6] Lorantz D J, Hiscock W A and Anderson P R 1995 Phys. Rev. D 52 4554

[7] Zaslavskii O B 1998 Phys. Rev. D 57 6265

[8] Robinson I 1959 Bull. Acad. Pol. Sci. 7 351

Bertotti B 1959 Phys. Rev. 116 1331

[9] Carter B 1973 General theory of stationary black hole states Black Holes, edited by De Witt C and De Witt B S (New York: Gordon and Breach)

Lapedes A S 1978 Phys. Rev. D 17 2556

[10] Solodukhin 1999 S N Phys. Lett. B 448 209

[11] Gibbons G W and Hawking S W 1977 Phys. Rev. D 15 2752

[12] Zaslavskii O B 1993 Phys. Lett. A 181 105

1996 Class. Quant. Grav. 13 L23

[13] Frolov V P 1995 Phys. Rev. Lett. 74 3319

[14] Hochberg D, Kephart T W and York J W Jr 1993 Phys. Rev. D 48 479

P R Anderson, Hiscock W A, Whitesell J and York J W Jr 1994 Phys. Rev. D 50 6427

Hochberg D and Sushkov S V 1996 Phys. Rev. D 53 7094

[15] Kofman L A and Sahni V 1983 Phys. Lett. 127 B 197

[16] Anderson P R, Hiscock W A and Loranz D J 1995 Phys. Rev. Lett. 74 4365

[17] Zaslavskii O B 1997 Phys. Rev. D 56 6695

[18] Geroch R 1969 Commun. Math. Phys. 13 180
[19] Birrell N D and Davies P C W 1982 *Quantum Fields In Curved Space* (Cambridge: Cambridge University Press)

[20] Wald R 1995 Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics (Chicago and London: The University of Chicago Press)

[21] Visser M 1998 *Phys. Rev. Lett.* **80** 3436
Matyjasek J 1998 *Phys. Rev. D* **57** 7615

[22] York J W Jr 1986 *Phys. Rev. D* **33** 2092

[23] Ginzburg V L and Frolov V P 1989 Vacuum un a homogeneous gravitational field and exitation of an uniformly accelerated detector *Quantum Theory and Gravitation, Proceedings of the Lebedev Institute* edited by Markov M A **197** 8 - 62 (Moscow: Nauka)

[24] Solodukhin S N 1995 *Phys. Rev. D* **51** 609

[25] Zerbini S, Cognola G and Vanzo L 1996 *Phys. Rev. D* **54** 2699

[26] Mann R B and Solodukhin 1998 *Nucl. Phys. B* **253** 293

[27] Laflamme R 1987 *Phys. Lett. B* **196** 449

[28] Deser S and Levin O 1997 *Class. Quant. Grav.* **14** L163

1998 "Mapping Hawking into Unruh thermal properties", [hep-th/9809159](https://arxiv.org/abs/hep-th/9809159)

[29] Braden H W, Brown J D, Whiting B F and York J W Jr 1990 *Phys. Rev. D* **42** 3376

[30] Brown J D and York J W Jr 1993 *Phys. Rev. D* **47** 1407

[31] Zaslavskii O B 1991 *Phys. Lett. A* **152** 463

[32] Brown J D and York J W Jr 1993 *Phys. Rev. D* **47** 1420

[33] Zaslavskii O B 1998 *Phys. Lett. B* **242** 271

[34] Balbinot R and Barletta A 1988 *Class. Quant. Grav.* **5** L11
[35] Nariai H 1951 *Science Reports of the Tohoku Univ.* **35** 62

[36] Romans L J 1992 *Nucl. Phys.* **B** 332 395

[37] Zaslavskii O B 1998 *Class. Quant. Grav.* **15** 3251

[38] Bousso R *Phys. Rev.* D **55** 3614

[39] Mann R B and Ross S F 1995 *Phys. Rev.* D **52** 2254

[40] Booth I S and Mann R B 1999 *Nucl. Phys.* B **539** 267

[41] Maldacena J 1998 *Adv.Theor.Math.Phys.* 2 231

Gubser S S, Klebanov I R and Polyakov A M 1998 *Phys.Lett.* B **428** 105

Witten E 1998 *Adv.Theor.Math.Phys.* 2 253

[42] Demers J-G, Lafrance R and Myers R C 1995 *Phys. Rev.* D **52** 2245

[43] Kim S P, Kim S K, Soh K-S, Yee J H 1997 *Phys. Rev.* D **55** 2159