BREAKING ELECTROWEAK SYMMETRY STRONGLY* †

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ABSTRACT

The problem of electroweak symmetry breaking is reviewed with discussion of future relevant experimentation at LHC and $e^+e^-$ linear colliders. The possibility of strong electroweak symmetry breaking is examined in more detail, using the BESS (Breaking Electroweak Symmetry Strongly) model as a basis for the discussion. The formal constructions are briefly presented and the possible expectations at future colliders are summarized.

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1 Introduction

It is a honour to dedicate this contribution to the memory of Robert E. Marshak, a man who gave so much to our profession. Besides important scientific work, such as his contribution to $V - A$ theory together with George Sudarshan, Robert Marshak will be recalled for having during all his life effectively operated for the progress of high energy physics, especially through promotion of international collaboration. The Rochester Conferences were a substantial but not isolated aspect of the sense of service to the community that Robert Marshak professed, as he was always dedicated to social progress, to peace, and to cooperation.

The problem of electroweak symmetry breaking, studied under certain aspects and from some points of view, will be the main subject of this contribution. The standard model describes the elementary world in terms of quarks and leptons and their gauge interactions. The gauge structure consists of color, responsible for strong forces, with group structure $SU(3)_c$, and electroweak charges from $SU(2)_L \otimes U(1)_Y$. Quarks and leptons form three generations, $(u, d, e, \nu_e)$, $(c, s, \mu, \nu_\mu)$, $(t, b, \tau, \nu_\tau)$. The photon, the $W$ and $Z$, and the gluon are the gauge bosons.

Within such a beautiful synthesis there is one aspect which is usually regarded as perhaps uncomplete and unsatisfactory. It is the mechanism for symmetry breaking, which is responsible for the masses of $W$ and $Z$.

The gauge couplings are uniquely fixed from the gauge principle. On the other hand the gauge principle alone does not directly provide for an understanding of the symmetry breaking and of the masses. At this point the conventional approach is to introduce an elementary scalar field, and an ad-hoc scale, expressed by its expectation value on the vacuum. The gauge theory, to be stressed again, does not say something unique on the scalar sector; in a way, one should look at such a sector as a kind of grafting operated on the gauge theory, though not in contrast to the gauge principle itself.

The simplest but somewhat artificial way to operate the grafting is to introduce one or more scalar fields. The predictive power of the gauge theory is greatly reduced; and one would expect in the simplest and standard formulation a massive neutral scalar particle, the so-called “physical Higgs”, at some yet unknown mass. For a slightly more complicated structure, one would expect two additional neutral and one charged (of both charges) scalars. So, even within this simplest realization of the symmetry breaking mechanism, in terms of elementary scalars, one at least expects a new, yet unseen, particle, whose discovery would of course be crucial.

The more satisfactory realizations, from a theory point of view, are however more complicated. As such, they lead to the expectation that higher energy accelerators, than the ones we can presently use, might reveal a new realm of particles and interactions. A general, common, theoretical idea, is that our parametrization in terms of scalar couplings (for instance scalar mass term, scalar self-coupling, all the list of Yukawa couplings) is fundamentally the effective low energy manifestation of a more complicated dynamics, with additional particles and interactions. The mass scale related to the Higgs vacuum value provides then for a valuable information for the scale where these new particles may be found.

The new dynamics may, for instances, have the form of a new strong interaction \[2\]. A simplest idea is that of reusing our technical knowledge from QCD, supplementing it with additional inputs (for instance, extended technicolor \[3\]).
The idea of a composite model would be more radical. In these models quarks and leptons are composites.

Another intensively discussed possibility is supersymmetry [], according to a number of different formulations. New particles, such as superpartners, are then introduced, and the Higgs picture emerges in a stabilized form (non-renormalization result).

This stabilization is thought to be useful to avoid the difficult theoretical problem of naturalness, which afflicts the simple breaking scheme based on elementary scalar fields alone. In this sense the discovery of a Higgs of light mass would still leave the simple-minded Higgs picture in face of its naturalness problem. It would rather suggests supersymmetry as the next theoretical alternative, suggesting additional Higgs, superpartners, etc.

Confirmation of the supersymmetry concept would essentially come from the discovery of the massive superpartners. Also, all supersymmetry schemes have at least two doublets of Higgs. Thus charged Higgs and more neutral ones would be expected in these schemes.

In the standard model with a single scalar doublet one has a theoretical lower mass bound for the Higgs mass. The limit arises from the requirement that the Higgs potential (as calculated at one loop) be bound from below. In other words it is a stability limit for the theory itself. For a top mass of 150 GeV such a stability requires a Higgs mass larger than 92 GeV. For a top of 200 GeV one would have instead a limit of 175 GeV.

Other limits on the Higgs mass, again in the simplest standard model, have to do with perturbative unitarity. One requires that the amplitudes calculated in perturbation theory, in practice at one-loop order or for leading logarithms, satisfy unitarity. This is interpreted in terms of bounds for the Higgs mass.

Within the standard model, it is known that the interaction in the scalar-longitudinal sector becomes stronger when the Higgs mass parameter, $m_H$, of the scalar potential becomes very large []. Specifically, for large $m_H$, partial wave amplitudes for scattering among longitudinal $W$, $Z$ and Higgs violate unitarity, when calculated at their lowest order.

For a very massive Higgs one obtains that at energy higher than 1.5 TeV unitarity (to be called strictly, perturbative unitarity) would be violated. Then either the Higgs is not very massive, or the true amplitude is not the one calculated in perturbation theory. This relatively low value of 1.5 TeV has motivated much interest in higher energy colliders (SSC, LHC).

A related theoretical speculation is to find out beyond which value of the Higgs mass the high energy limit of $WW$ scattering violates perturbative unitarity. In the standard model with one scalar doublet, this happens for Higgs masses larger than 1.2 TeV. On the other hand at those masses the Higgs has a width already as large as its mass. This can already be taken as an indication of failure of perturbation theory.

In the approximation known as large $N$ limit one can construct an effective lagrangian which explicitly exhibits multiple vertices between longitudinally-polarized $W$'s and $Z$'s, together with modified propagators, with unitarity visibly restored [].

The effective lagrangian constructed in such large $N$-model exhibits the scalar Higgs resonance, whose properties coincide with those of the usual physical Higgs for small
$m_H$. This resonance becomes broader for increasing $m_H$, and its mass saturates at an upper bound of about 0.8 TeV \[7\].

A comprehensive discussion of the phenomenology to be expected when the limit of failure of perturbative unitarity is approached has been given by Chanowitz and Gaillard \[8\], \[9\]. A result, which relates the amplitudes among longitudinally polarized gauge bosons to the corresponding Goldstone amplitudes \[10\] \[8\], is useful for these developments.

It is not yet known whether a large $m_H$ indeed generates a strong interacting sector. The problem is theoretically difficult to solve also for reasons connected with the possible triviality of $\varphi^4$. The considerations which lead to the BESS (Breaking Electroweak Symmetry Strongly) model \[11\] did in fact assume the existence of a strong interacting longitudinal-scalar sector, but they were not necessarily bound to the hypothetical mechanism of large $m_H$. The model was rather constructed as a way of parametrizing the most relevant phenomenological effects of a possible strong interacting sector.

Complex poles, of various spin-parities, might be present in a complete treatment of a strong interacting longitudinal-scalar sector, of whatever origin. Particularly interesting would be vector or axial poles. In fact, because of their quantum numbers such poles could mix with the $W$ and $Z$ and thus originate visible deviations in the accessible phenomenology.

In ref. \[11\] the discussion of $J = 1$ poles was made using an approach which goes back to the work by Callan, Coleman, Wess and Zumino \[12\] on non-linear realizations of symmetry. The approach used the notion of hidden local symmetry of non-linear $\sigma$-models, recently applied to describe vector mesons in strong interactions \[13\].

The non-linear $\sigma$-model appears in this context when one takes the formal limit of infinite $m_H$. By the classical limit the isoscalar degree of freedom gets thus frozen. Beyond this limit quantum fluctuations come in and the Higgs mass plays the role of a cut off within the non-renormalizable non-linear $\sigma$-model \[14\].

Explicit gauge bosons correspond to hidden local symmetries and classically they appear as auxiliary fields. The physical hypothesis here is that higher order effects provide for their kinetic terms, as it happens in known two-dimensional examples \[13\]. This is a hypothesis, and it simply adds to the many uncertainties in the underlying dynamics (role of possible $\varphi^4$ triviality \[16\] \[17\], irrelevance of the $\sigma$-model limit at higher loop orders \[18\], various conjectures on fixed point mechanisms to prevent triviality in Higgs sector \[19\], possible independence from the strength of the quartic coupling \[20\]).

In view of the essentially unknown dynamics one may be lead to consider the model developed in ref. \[11\] as an alternative to the standard model in its realization of symmetry breaking.

One can see, in fact, quite easily, that in general it is impossible to linearly realize a spontaneous symmetry breaking mechanism in such a way that all scalar degrees of freedom be eaten up via the Higgs mechanism.

In fact if the scalars transform linearly under the gauge group (supposed connected, compact, and semisimple) and if they all have to be absorbed, the invariant potential could only be an overall constant, being constant over a connected compact subspace of same dimensionality as the representation.
When however the symmetry breaking is realized through the mechanism of the non-linear condition, this conclusion can be avoided. Note however that only geometric arguments are used here and the possible plague of non-renormalizability is not taken into account.

In the standard model the scalar degrees of freedom would be three, corresponding to the coordinates of the quotient space $SU(2)_L \otimes SU(2)_R / SU(2)_{\text{diagonal}}$, the right number to give masses to $W$ and $Z$. In the model of ref. [11] the hidden $SU(2)_V$ is supposed to have related gauge bosons $(V^\pm, V^0)$ and the quotient coordinates, which are now six, are again all absorbed to give masses to $W, Z$ and $V$.

2 Experimental prospects at future colliders

Assuming the validity of the one-doublet standard model and leaving aside its probably unsatisfactory theoretical background, the phenomenological discussion, both for the past SSC project and for LHC [21], has focused on the possibility of detecting a heavy Higgs. These studies provide at least a well defined ground for exercises, particularly to formulate detector requirements. The subject has been widely discussed in conferences and workshops. We shall here present a rough overall picture, certainly not the final one.

Such a heavy Higgs ($m_H > 0.5$ TeV and up to 1 TeV) would essentially decay into $WW$ and twice less frequently into $ZZ$, with a total width approximately given by $\Gamma_H (\text{TeV}) = 1/2 (m_H/1\text{TeV})^3$, rather independent of $m_t$. It would be produced mostly by gluon fusion and $WW$ or $ZZ$ fusion, with gluon fusion dominant at the lower masses, and (especially for small $m_t$) $WW$ and $ZZ$ fusion becoming dominant in the higher mass range.

In the channel $ZZ \to 4$ leptons, peaks would be present in the invariant $m_{ZZ}$ mass. These peaks would be attributed to a heavy Higgs disintegrating into $ZZ$, with expected branching ratio of the order of $4 \times 10^{-3}$. One will have to cut on the low $Z$ transvers momenta and on the high $Z$ rapidities, and introduce some jet separation cut for the two jet process $qq \to qqZZ$.

A factor $\approx 6$ in the branching ratio could be gained by observing the channel $ZZ \to l^+l^-\nu\bar{\nu}$, but in that case a missing momentum cut excluding momenta lower than, say, 100 GeV, will have to be introduced. However, the less neat situation introduces a fake background, due to $Z$ produced in association with jets simulating a missing momentum.

To avoid the fake background one must demand that the hadron calorimeter should cover a full range of hadronic rapidities up to four or five units. For such channel, $ZZ \to l^+l^-\nu\bar{\nu}$, one should look for peaks in transverse $Z$ mass or transverse $Z$ momentum for the reconstructed $Z$.

On such exercise-type heavy Higgs, the lower mass limit for the Higgs could probably be pushed up to 0.6 TeV at LHC (we shall here refer to the original project) with $10^4 \text{ pb}^{-1}$ or 0.8 TeV with $10^5 \text{ pb}^{-1}$ within the $4l$ mode. Provided all detector conditions be met, the lower luminosity using the $ll\nu\nu$ mode might allow to go higher than 0.6 TeV. All this is thinkable only provided adequate detectors for the high luminosity can be built.

The heavy Higgs case study we have just discussed should be regarded as the typical study in order to define on a simple example the potentialities of colliders. As we had
discussed previously, the theoretical frame, for a standard model with heavy Higgs and nothing else, contains unconvincing features. One may than ask whether other similar exercises should not be pursued allowing for different scenarios.

The simplest candidate for an alternative exercise is the BESS-model. The model, in its simplest form, predicts a new triplet of (strong-interacting) vector bosons $V^\pm$ and $V^0$ of almost degenerate mass in the $TeV$ range. They would show up as resonances in $pp \rightarrow W^\pm Z + X$, where $W^\pm$ could be seen in $W \rightarrow l\nu$, and $Z$ in $Z \rightarrow ll$. The $pp \rightarrow ZZ + X$ is non-resonant in BESS, and $pp \rightarrow W^+W^- + X$ is dominated by $pp \rightarrow t\bar{t} + X$ with both $t \rightarrow bW$.

The resonant signal would come from $q\bar{q} \rightarrow V \rightarrow W^+_L Z_L$ (the suffix $L$ referring to longitudinal polarization) and from $W_L Z_L \rightarrow V \rightarrow W_L Z_L$, over a number of non-resonant contributions such as continuum $WZ$ production, $\gamma W \rightarrow W Z$, etc. Analysis of the experimental possibilities, leads to a discovery limit for $V$ of $\approx 2 TeV$ in one year LHC (always the original project) running at $10^{34} cm^{-2} s^{-2}$.

It is important in the whole context to also discuss the possibilities for intermediate Higgs search, that is $m_H$ beyond the LEP2 limit up to the $2Z$ threshold. Here gluon fusion is the dominant $pp$ production process, and decay is mostly into $b\bar{b}$, with small branching ratios into other modes. A most effective possibility seems to be $pp \rightarrow ZZ^* X$ ($Z^*$ stands for virtual $Z$) with $ZZ^* \rightarrow 4l$ which would work in the range $130 - 160 GeV$.

Other experiments would be $pp \rightarrow WHX$ with $W \rightarrow l\nu$ and $H \rightarrow \gamma\gamma$, to possibly cover a lower mass range, and, also $pp \rightarrow \gamma\gamma X$, based on the small $H \rightarrow \gamma\gamma$ branching ratio, requiring very hard detector conditions.

In the intermediate range, $e^+e^-$ machines at adequate energies and luminosities ($\geq 10^{32} cm^{-2} s^{-1}$) would have excellent prospects. The intermediate $H$ would be looked at in its dominant mode $H \rightarrow b\bar{b}$.

For a high energy $e^+e^-$, such as CLIC, one would look at the heavy Higgs in $H \rightarrow WW, H \rightarrow ZZ$, and one expects for $30 fb^{-1}$ to reach limits $\approx 0.5 TeV$.

A few words on supersymmetry. As we have said in the previous section, supersymmetry remains a valid theoretical alternative. Supersymmetry can be formulated according to a variety of models. Within each model many parameters are not fixed; calculations can be made for a given set of parameters. This possibility of changing the model and its parameters does not unfortunately allow for universal predictions from supersymmetry.

The so-called minimal supersymmetric standard model is in general used for a first insight into supersymmetry, but it does not seem to possess fundamental theoretical reasons to be preferred to other models.

Changing the model may lead to brutal changes in the expected experimental signatures, as, for instance, if one goes to models where R-invariance does not hold (R-invariance is assumed to hold in the minimal supersymmetric standard model). One will have, therefore, to qualify the meaning to be attributed to statements about “reasonable” lower limits for the masses of supersymmetric partners that may be reached at different future accelerators.

Supersymmetry predictions are at present object of intense studies. At LHC in the main $pp$ option the search for squarks and gluinos appears as the most promising among the superpartners searches, with expected “reasonable” mass limits of the rough order
of magnitude of 1 TeV for both \( \tilde{q} \) and \( \tilde{g} \) for a total \( 10^4 \text{ pb}^{-1} \), and possibly 1.5 TeV for \( 10^5 \text{ pb}^{-1} \).

SSC would have allowed for still higher limits. Tevatron, for comparison, could push the limit for \( \tilde{q} \) and \( \tilde{g} \) to 0.2 TeV. LHC in the \( ep \) option would not compete with LHC-\( pp \) on such a limit. A 2 TeV \( e^+e^- \) with 500 \( fb^{-1} \) would not go beyond 0.8 – 0.9 TeV for this limit, but it would on the other hand allow to push up the slepton mass lower limit, also to a similar value of 0.8 – 0.9 TeV.

In this section we have given a rapid regard to some of the possibilities that future accelerators can offer to look for standard Higgs, even up to rather heavy mass values, and we have given a quick assessment of their possibilities with regard to supersymmetry, which is always considered to be an interesting valid alternative to the standard model with some definite theoretical advantages. In the present contribution we shall essentially concentrate on BESS (breaking electroweak symmetry strongly). We shall first discuss the theoretical frame and then come back to the expectations at future colliders.

3 BESS

For a breaking of a group \( G \) into a subgroup \( H \) the Goldstones can be taken as the coordinates of \( G/H \). We know that \( H \) must contain \( U(1)_{\text{e.m.}} \). We need three Goldstones to give masses to \( W \) and \( Z \). In addition we can guarantee for the standard parameter \( \rho \) the value \( \rho = 1 \) apart from weak corrections, if we have a "custodial" \( SU(2) \) [22].

The minimal \( H \) in this case would have to be \( SU(2) \). In the SM the breaking is realized linearly with scalars originally transforming as the \((\frac{1}{2}, \frac{1}{2})\) of \( SU(2)_L \otimes SU(2)_R \). Such a direct product breaks into \( SU(2)_{\text{diagonal}} \), with corresponding breaking of \((\frac{1}{2}, \frac{1}{2})\) into \( 1 \oplus 3 \), describing the physical Higgs and the 3 absorbed Goldstones.

The non-linear realization can be seen classically as corresponding to the limit of infinite \( m_H \). The scalars can indeed be represented as proportional to a unitary matrix \( U \). In the formal limit \( m_H \rightarrow \infty \) one is just freezing the proportionality factor to the vacuum expectation value (called \( f \) to emphasize the formal similarity with \( f_\pi \), the pion-decay constant of strong chiral theory) and the scalar lagrangian is simply

\[
\mathcal{L} = \frac{f^2}{4} Tr[(\partial_{\mu} U)(\partial^{\mu} U^\dagger)]
\]

Such a lagrangian is obviously invariant under \( SU(2)_L \otimes SU(2)_R \), namely under \( U \rightarrow g_L g^\dagger_R \) where \( g_L, g_R \) belong to \( SU(2)_L, SU(2)_R \) respectively. The breaking into the diagonal \( SU(2) \) is demanded by the (non-linear) unitarity condition \( U^\dagger U = 1 \).

Before coming back to the our physical problem we want to summarize here some general formal considerations. They have been known in different ways in literature [12], [13], [14], [15]. We shall give here a systematic presentation of the main relevant points.

Such considerations will be applied in the following not only to the construction of the simplest original BESS, but also to derive different extensions of BESS, that we shall also discuss and examine, in view of possible phenomenological consequences.
Let us start by considering a local map $g(x)$ of the Minkowski space into a compact connected Lie group $G$. One can from $g(x)$ construct the Maurer-Cartan form $\omega$. The form $\omega$ is globally invariant under left group multiplication.

Suppose $H$ is a connected subgroup of $G$, which we shall identify as the “unbroken subgroup”. One has also for $H$ a local map that we call $h(x)$.

We recall that $F_{\mu\nu}(\omega)$ vanishes by construction. One can decompose $g(x)$ as a product

$$g(x) = e^{iq(x)}h(x) \quad (3.1)$$

In eq. $3.1$ $q(x)$ has components along the generators $X_i$ of $G$ not belonging to the Lie algebra of $H$: $q_i(x) = Tr(q(x)X_i)$. The $q_i(x)$ are the coordinates of the non-linear realization.

Let us assume right-multiplication invariance under the local $H$. The Maurer-Cartan form $\omega$ can be decomposed as

$$\omega = \omega_\parallel + \omega_\perp \quad (3.2)$$

Parallel and orthogonal in $3.2$ is intended in relation to the unbroken subgroup $H$.

Under the local right-multiplication one has

$$\omega_\parallel \to h^\dagger \omega_\parallel h + h^\dagger \partial h \quad (3.3)$$

On the other hand

$$\omega_\perp \to h^\dagger \omega_\perp h \quad (3.4)$$

Under the restriction of the global $G$ into the global $H$ the coordinates $q_i(x)$ will in general transform non-linearly.

This construction can now be specialized to a symmetric space, in which case one adopts a standard basis for $\text{Lie}[G]$. In such a basis one has $[T_\mu, T_\nu] = i f_{\mu\nu\lambda} T_\lambda$ for $T_\mu \in \text{Lie}[H]$, and $[T_\mu, X_i] = ig_{\muij} X_j$, $[X_i, X_j] = ig_{ij\mu} T_\mu$. Also we recall that there exists in such a case a parity operation acting as an automorphism of the algebra.

The standard non-linear realization of Callan, Coleman, Wess, and Zumino is obtained by gauge-transforming $Tr(\omega_\perp^2)$ with $h^{-1}(x)$. One obtains

$$\omega_\perp = \left[ e^{-iq(x)} \partial e^{iq(x)} \right]_\perp \quad (3.5)$$

One can use the formula of Baker-Campbell-Hausdorf to write $\omega_\perp$ as

$$\omega_\perp = i[\text{adj}(iq(x))]^{-1} \sinh[\text{adj}(iq(x))] \partial q(x) \quad (3.6)$$

where the $\text{adj}$ of an operator is the adjoint defined in $\text{Lie}[G]$.

The standard results (current algebra, PCAC, etc.) are recovered by expanding

$$f^2 Tr[\omega_\perp^2] = -f^2 Tr \left[ (\partial q(x))^2 - \frac{1}{3} \partial q(x)[q(x), [q(x), \partial q(x)]] + \ldots \right] \quad (3.7)$$

The additional step is to introduce the gauge field $\eta$ of the local unbroken subgroup $H$

$$\eta \to h^\dagger \eta h + h^\dagger \partial \eta \quad (3.8)$$
One defines a covariant derivative (remark the arrow to the left)

\[ Dg(x) = g(x)(\partial + \eta) \]  

(3.9)

For the field \( \zeta \) defined as

\[ \zeta = \omega_{\parallel} - \eta \]  

(3.10)

one has

\[ \zeta \to h^\dagger \zeta h \]  

(3.11)

Eq. 3.11 allows for a new symmetric term

\[ f'^2 Tr[\omega^2] \]  

(3.12)

with a new constant \( f' \) which appears in addition to \( f \) (\( f \) is for instance \( f_\pi \) in QCD).

Now, \( \eta \) remains as an auxiliary field as long as it does not develop a kinetic term. This would give back the standard theory of non-linear realizations. The physical assumption we shall make is that such kinetic term arises in the renormalized theory.

In fact nothing prevents its appearance within the overall symmetry frame, and special examples may lead to suggest that it will indeed appear \[15\]. We shall in the following assume this to be the case. Or, at least, construct our models as realizing such a possibility.

Let us now come back to the physical problem.

For a description of the scalar sector evidencing the hidden local symmetry of the model one introduces local group elements \( L(x) \), \( R(x) \) belonging to \( SU(2)_L \) and \( SU(2)_R \) respectively. Under global transformations of \( SU(2)_L \otimes SU(2)_R \) these group elements are multiplied to their left by the corresponding global group transformation. In addition one can ask invariance under right-multiplication by a local group element of the unbroken \( SU(2)_V \).

The Maurer-Cartan form \( \omega^\mu dx_\mu \), where \( \omega^\mu = (\omega^\mu_L, \omega^\mu_R) = (L^\dagger \partial^\mu L, R^\dagger \partial^\mu R) \), is decomposed into the component \( \omega^\mu_{\parallel} \), parallel to the subgroup \( SU(2)_V \), and the component \( \omega^\mu_{\perp} \), orthogonal to \( SU(2)_V \): \( \omega^\mu = (\omega^\mu_{\parallel})_a T^a_V + (\omega^\mu_{\perp})_a T^a_A \), where \( T^a_V \) and \( T^a_A \) are the vector and axial vector generators of \( SU(2)_L \otimes SU(2)_R \).

One has

\[ (\omega^\mu_{\parallel})_a = Tr \left[ \frac{1}{2} \tau^a (L^\dagger \partial^\mu L + R^\dagger \partial^\mu R) \right], \quad (\omega^\mu_{\perp})_a = Tr \left[ \frac{1}{2} \tau^a (L^\dagger \partial^\mu L - R^\dagger \partial^\mu R) \right] \]  

(3.13)

The parallel Maurer-Cartan component transform under the local \( SU(2)_V \) invariance according to such local invariance, whereas the orthogonal component transforms as if such symmetry were only global (that is it develops no inhomogeneous term under the gauge transformation). In addition one introduces the \( SU(2)_V \) gauge field: \( \eta^\mu = (\eta^\mu)_a T^a_V \).

One considers terms which are invariant under the whole set of global \( SU(2)_L \otimes SU(2)_R \) transformations and local \( SU(2)_V \) transformations, as defined. For constructing an effective lagrangian one limits to terms containing at most two derivatives and which are linearly independent. The simplest term is \( -f^2 Tr[\omega^2_{\perp}] \). If one writes \( U = LR^\dagger \) one can rewrite such term as

\[ -f^2 Tr[\omega^2_{\perp}] = -\frac{1}{4} f^2 Tr \left[ (L^\dagger \partial^\mu L - R^\dagger \partial^\mu R)^2 \right] = \frac{1}{4} f^2 Tr \left[ (\partial^\mu U)^\dagger (\partial^\mu U) \right] \]  

(3.14)
showing that one has only reexpressed the $\sigma$-model in terms of alternative degrees of freedom.

However the availability of the field $\eta^\mu$ allows for the new term $-f'^2 Tr[(\omega^\parallel - \eta)^2]$, with a new independent vacuum value $f'$. On the other hand in absence of a kinetic term for $\eta^\mu$ the field equations derived by adding the two terms would simply lead back to the original non linear $\sigma$-model. The main physical assumption is that $\eta^\mu$ becomes a dynamical field.

The gauging of $SU(2)_L \otimes U(1)$ only implies the substitution of the ordinary derivatives with covariant left and right derivatives, acting on the left or right group elements respectively:

\[
D^{(L)}_\mu L = \partial_\mu L + W^{(0)}_\mu L - L \eta_\mu , \quad D^{(R)}_\mu R = \partial_\mu R + Y_\mu R - R \eta_\mu
\]  

(3.15)

where $Y_\mu = Y^\mu_\tau / 2$, $W^{(0)}_\mu = \tilde{W}^{(0)}_\mu \cdot \tau / 2$, $\eta_\mu = \tilde{\eta}_\mu \cdot \tau / 2$, and we have added a superscript zero to $W$ to allow us later for use of the simpler symbols for the physical fields that will emerge after mixing. Notice the different positioning of for instance $W^{(0)}_\mu$ and $\eta^\mu$ in the covariant derivatives.

The final lagrangian will contain the term built up from the transverse Maurer-Cartan component, the term corresponding to the field $(\omega^\parallel - \eta)$ and the kinetic energies of the gauge bosons $W^{(0)}_\mu$, $Y_\mu$, and also of $\eta_\mu$:

\[
\mathcal{L} = -\frac{1}{4} f'^2 Tr[(L^\dagger D^{(L)}_\mu L - R^\dagger D^{(R)}_\mu R)^2] - \frac{1}{4} f'^2 Tr[(L^\dagger D^{(L)}_\mu L + R^\dagger D^{(R)}_\mu R - \eta_\mu)^2] + \text{kinetic terms for the gauge fields}
\]  

(3.16)

The Higgs mechanism gives masses to all gauge bosons, except for the photon. All scalar degrees of freedom are absorbed. Formally one finds that one has to perform the following gauge transformation ($\Omega = RL^\dagger$) :

\[
\tilde{W}^{(0)} = \Omega^\dagger \tilde{W} \Omega + \Omega^\dagger \partial \Omega , \quad \tilde{\eta} = R^\dagger \tilde{\eta} R + R^\dagger \partial R
\]  

(3.17)

Finally one performs a rescaling of the fields according to $\tilde{W} \rightarrow g \tilde{W}$, $Y \rightarrow g' Y$, $2 \tilde{V} \rightarrow g'' \tilde{V}$ and after separate diagonalization of the $2 \times 2$ charged and $3 \times 3$ neutral sectors one derives the physical vector boson states $W^\pm$, $V^\pm$ and $A$, $Z^0$, $V^0$ with masses and mixing angles.

As we have said, in absence of kinetic term for the gauge field of the hidden local symmetry, one can only recover the original gauged non linear $\sigma$-model. The rescaling we have performed for the field $\tilde{V}_\mu$ allows however for a different way of looking at such a limit. When $g'' \rightarrow \infty$ the limit is again reobtained. Therefore $g'' \rightarrow \infty$ must lead back to the standard electroweak theory. This indeed happens quite evidently from the expressions for the masses and mixings [11].

The fermions, quarks and leptons, are those of the standard families, left-handed fermions $\psi_L$ and right-handed fermions $\psi_R$. Under the local $SU(2)_V$ they are assumed to be singlets. Their couplings are then uniquely determined

\[
\bar{\psi}_L i \gamma^\mu \left( \partial_\mu + \tilde{W}^{(0)}_\mu \frac{\tau}{2} + \frac{1}{2} (B - L) Y_\mu \right) \psi_L + \bar{\psi}_R i \gamma^\mu \left( \partial_\mu + \left( \frac{\tau_3}{2} + \frac{1}{2} (B - L) Y_\mu \right) \right) \psi_R
\]  

(3.18)
from which one obtains the coupling constants.

An alternative procedure to discuss the ”hidden gauge symmetries” for a quite general model, that is perhaps more related to usually employed notions, is only to enlarge the initial symmetry and correspondingly enlarge the scalar sector and the number of the needed non-linear conditions.

To obtain the model of ref. [11] one adds to $SU(2)_L \otimes SU(2)_R$ a group $SU(2)_V$ and realizes non-linearly the breaking $SU(2)_L \otimes SU(2)_R \otimes SU(2)_V \rightarrow SU(2)_{\text{diagonal}}$. The Goldstones are six (coordinates of the quotient). They are all absorbed, giving masses to $W, Z$ and $V$.

The Goldstones are described by two unitary matrices $L$ and $R$, which transform as $L \rightarrow g_L L h$, $R \rightarrow g_R R h$ where $g_L, g_R, h$ belong to $SU(2)_L, SU(2)_R, SU(2)_V$ respectively. Forgetting the unitarity conditions one would have $L, R$ transforming as $(\frac{1}{2}, 0, \frac{1}{2})$ and $(0, \frac{1}{2}, \frac{1}{2})$ under $SU(2)_L \otimes SU(2)_R \otimes SU(2)_V$. The unitarity conditions $L L^\dagger = 1, R R^\dagger = 1$ lead to the wanted breaking.

The procedure then consists in writing down the most general Lagrangian with at most two derivatives invariant under $SU(2)_L \otimes SU(2)_R \otimes SU(2)_V$ for the unitary local matrices $L$ and $R$ and satisfying the symmetry $L \leftrightarrow R$. One then introduces gauge fields for the subalgebra $SU(2)_L \otimes U(1)_Y \otimes SU(2)_V$ and adds kinetic terms for them. The related gauge couplings are $g, g', g''$ as usual, and $g''$ for $SU(2)_V$.

In the formal strong coupling limit, $g'' \rightarrow \infty$, the kinetic term of the $SU(2)_V$ gauge fields vanishes, and the fields become auxiliary. Their elimination brings back to the non-linear formulation of the SM.

The model developed in ref. [11] contains the massive dynamical gauge bosons corresponding to the ”hidden” $SU(2)_V$ gauge symmetry. As we have said, the importance of such vector bosons, in comparison to other composite degrees of freedom, is that they can mix with $W$ and $Z$, and therefore play an important role in phenomenology. A model with one extra triplet of vector bosons, based on $SU(2)_L \otimes SU(2)_V \otimes U(1)_Y$ and constrained by $\rho = 1$ at tree level, has also been considered [23]; BESS is obtained by specialization of the parameter space of the model.

The same remarks however would also apply to axial-vector bosons which also could mix. In the absence of a complete dynamical treatment, which would enlighten us on the relative role of vector and of axial bosons, one can only develop a general scheme which contains both, and then discuss and compare the various phenomenological predictions. From the point of view of our way of treating ”hidden symmetries”, that is by adding the ”hidden symmetries” at the start and then increasing the number of scalar fields and of non-linear conditions, the inclusion of the axial degrees of freedom appears indeed as a natural and in principle very simple extension [24].

One has to start from an initial symmetry $G$ consisting of a global $SU(2)_L \otimes SU(2)_R$ times a local $SU(2)_L \otimes SU(2)_R$ and introduce besides $L$ and $R$, transforming as $L \sim (\frac{1}{2}, 0, \frac{1}{2}, 0)$ and $R \sim (0, \frac{1}{2}, \frac{1}{2}, 0)$ under the above sequence of groups, an additional $M$ transforming as $M \sim (0, 0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$. The matrix $U = L M^\dagger R^\dagger$ will then transform only globally and be unaltered by the local (hidden) group. By imposing the non-linear, unitarity conditions $L L^\dagger = R R^\dagger = M M^\dagger = 1$ we realize again the breaking from $G = [SU(2)_L \otimes SU(2)_R]_{\text{global}} \otimes [SU(2)_L \otimes SU(2)_R]_{\text{local}}$ down to the diagonal subgroup $H = SU(2)_{\text{diagonal}}$. 
Altogether we have 9 Goldstone modes. At this point we can write down the most general two-derivative lagrangian, invariant under the group $G \otimes P$, where $P$ parity transformation ($L \leftrightarrow R$ and $M \leftrightarrow M^\dagger$), required to make contact with the non-linear $\sigma$-model limit of the standard model.

We first build up covariant derivatives with respect to the local group:

$$D_\mu L = \partial_\mu L - L L_\mu$$
$$D_\mu R = \partial_\mu R - R R_\mu$$
$$D_\mu M = \partial_\mu M - M L_\mu + R_\mu M$$

(3.19)

where $L_\mu$ and $R_\mu$ are the Lie algebra valued gauge fields of $(SU(2)_L)_{local}$ and $(SU(2)_R)_{local}$ respectively.

One now constructs the invariants of our original group extended by the parity operation. One finds

$$I_1 = \text{Tr}(L^\dagger D_\mu L - M^\dagger D_\mu M - M^\dagger R^\dagger (D_\mu R) M)^2$$

(3.20)

$$I_2 = \text{Tr}(L^\dagger D_\mu L + M^\dagger R^\dagger (D_\mu R) M)^2$$

(3.21)

$$I_3 = \text{Tr}(L^\dagger D_\mu L - M^\dagger R^\dagger (D_\mu R) M)^2$$

(3.22)

$$I_4 = \text{Tr}(M^\dagger D_\mu M)^2$$

(3.23)

Using these invariants is it now possible to write down the most general Lagrangian with at most two derivatives in the form:

$$\mathcal{L} = -\frac{v^2}{16}(a I_1 + b I_2 + c I_3 + d I_4) + \text{kinetic terms for the gauge fields}$$

(3.24)

where $a$, $b$, $c$, $d$ are free parameters and furthermore the gauge coupling constant for the fields $L_\mu$ and $R_\mu$ is the same.

It is not difficult to see that this Lagrangian is the same one would obtain from the hidden gauge symmetry approach [13]. The requirement of getting back the non-linear $\sigma$-model in the limit in which the gauge fields $L_\mu$ and $R_\mu$ are decoupled is satisfied by imposing the following relation among the parameters $a, b, c, d$

$$a + \frac{cd}{c + d} = 1$$

(3.25)

The gauging of the previous effective Lagrangian with respect to the standard gauge group $SU(2)_L \otimes U(1)_Y$ is obtained by the following substitutions:

$$D_\mu L \rightarrow D_\mu L = \partial_\mu L - L(V_\mu - A_\mu) + W_\mu L$$

(3.26)

$$D_\mu R \rightarrow D_\mu R = \partial_\mu R - R(V_\mu + A_\mu) + Y_\mu R$$

(3.27)

$$D_\mu M \rightarrow D_\mu M = \partial_\mu M - M(V_\mu - A_\mu) + (V_\mu + A_\mu) M$$

(3.28)

where $V_\mu = (R_\mu + L_\mu)/2$ and $A_\mu = (R_\mu - L_\mu)/2$ are the fields describing the new vector and axial-vector resonances.

Is it possible to fix the gauge so that $L = R = M = 1$, obtaining the following Lagrangian:

$$\mathcal{L} = -\frac{v^2}{4} \left[ a \text{ tr}(W - Y)^2 + b \text{ tr}(W + Y - 2V)^2 + c \text{ tr}(W - B + 2A)^2 + d \text{ tr}(2A)^2 \right] + \text{kinetic terms for } V_\mu, A_\mu, W_\mu, Y_\mu$$

(3.29)
We still want to mention another, more intuitive, approach to obtain the Lagrangian \(3.29\). One starts from the transformation properties of the gauge fields \(W_\mu^3\), \(Y_\mu\), \(V_\mu^3\) and \(A_\mu^3\) under the electromagnetic \(U(1)_{em}\) gauge transformations:

\[
\begin{align*}
\delta W_\mu^3(x) &= -\frac{1}{g} \partial_\mu \lambda(x) \\
\delta B_\mu(x) &= -\frac{1}{g'} \partial_\mu \lambda(x) \\
\delta V_\mu^3(x) &= -\frac{2}{g''} \partial_\mu \lambda(x) \\
\delta A_\mu^3(x) &= 0
\end{align*}
\] (3.30)

In the limit \(g' = 0\), an \(SU(2)\) global symmetry is defined, under which \(W\), \(V\) and \(A\) transform as triplets.

The lagrangian \(\mathcal{L}\) is just that of a massive Yang-Mills theory invariant under the \(U(1)_{em}\) gauge transformations given in eq. 3.30 and the "custodial" \(SU(2)\). The constraint given in eq. 3.25 is obtained by asking that in the limit \(g'' \rightarrow \infty\) the lagrangian \(\mathcal{L}\) reproduces the SM terms:

\[
\mathcal{L}_{SM} = -\frac{v^2}{4} \text{tr}(W - Y)^2 + \mathcal{L}_{kin}(W,Y)
\] (3.31)

To summarize: the model describes the interactions of the vector and axial-vector gauge bosons \(A\) and \(V\) of \([SU(2)_L \otimes SU(2)_R]_{local}\) with the gauge bosons \(W\), \(Z\) and \(\gamma\) of \(SU(2)_L \otimes U(1)_Y\). The original 9 Goldstone bosons are eaten up by \(V\), \(A\), \(W\) and \(Z\).

The parameters are: \(f\), \(g\), \(g'\); three independent coefficients in front of the lagrangian invariants and the additional gauge coupling constants \(g_A\) and \(g_V\) of the "hidden symmetry" group.

An important distinction from other schemes appears at this stage. Our framework does not allow for vector resonant \(SU(2)\) singlets, such as the state which in the hadronic language would correspond to the \(\omega\) meson, as it instead naturally happens in schemes which try to mimic the QCD behavior. The reason for this is that our starting \(SU(2)_L \otimes SU(2)_R\) global symmetry can be enlarged only by additional \(SU(2)\) factors which, once gauged, give rise to triplets of massive gauge bosons. On the other hand, if one tries to gauge only a particular subgroup \(U(1)\) of an extra \(SU(2)\), unwanted Goldstone bosons appear, with embarrassing phenomenological consequences. It would be different, of course, if the starting global symmetry were \(SU(3)_L \otimes SU(3)_R\). One thus expects, on merely symmetry grounds, that technicolor, for instance, has general distinctive features with respect to the scheme we have just described.

Couplings to fermions can be introduced following, for instance, ref \([11]\). In a minimal choice, fermions couple to the new vector bosons \(V\) and \(A\) only through the mixing of \(V\), \(A\) with \(W\) and \(Z\).

The couplings among fermions and gauge bosons as well as the low-energy (\(\sqrt{s} \ll m_W\)) charged and neutral currents lagrangian can be straightforwardly derived. One observes that \(G_F = (\sqrt{2} f^2)^{-1}\), as in the SM, and, more remarkably, \(\rho = 1\) at tree level. Therefore low-energy charged currents are unaffected, whereas the neutral ones are only modified in the expression for the Weinberg angle and by the presence of an extra \(j_{e.m.}^2\) term. Of course a sizeable modification is given by the shift of the ordinary gauge boson masses.
The minimal chiral structure $SU(2)_L \otimes SU(2)_R$ of the original BESS can be easily extended to a larger $SU(N)_L \otimes SU(N)_R$.

The most apparent feature, in such a case, is the appearance of spin-zero pseudogoldstones, due to the spontaneous breaking of the global $SU(N)_L \otimes SU(N)_R$ to the diagonal $SU(N)_V$. They are therefore $N^2 - 1$; three of them give mass to the $W$ and $Z$. The others in general will not remain massless, due to the interactions explicitly breaking the global symmetry group. For instance the standard model gauge interactions contribute to the pseudo-Goldstone mass spectrum [25].

It is however clear that other interactions explicitly breaking the global symmetry group must also be present and taken into account. We are here referring in particular to the mechanism which is responsible for the generation of the masses of the ordinary fermions.

If for instance we think to an extended technicolor scheme, the gauge interactions associated to the generators connecting ordinary fermions to technifermions will in general break the chiral symmetry $G$, which in this model is related to the technifermion sector. Since the interactions considered are those responsible for the generation of the fermion masses, it is natural to expect that the induced pseudo-Goldstone masses are somehow related to the fermionic mass spectrum. A quantitative analysis is presented in [26].

Extended BESS contains explicit vector and axial-vector resonances. The phenomenology of ordinary technicolor, in its low energy limit, would correspond to a specialization of extended BESS.

The simplest construction for extended BESS uses a local copy of the global chiral symmetry and goes through classification of the relevant invariants, as shown before for $G = SU(2) \otimes SU(2)$. The same results follow from the hidden gauge symmetry approach. The standard electroweak $SU(2) \otimes U(1)$ and $SU(3)$ are gauged and a definite mixing scheme emerges for the gauge bosons and the vector and axial-vector resonances. The physical photon and the physical gluon remain automatically massless and coupled to their conserved currents.

The quantitative estimates have been restricted to the "historical" case $N = 8$, although a number of results are more general. Through their mixing with the gauge bosons of $SU(2)_L \otimes U(1) \otimes SU(3)_c$, some of the vector and axial vector resonances acquire a coupling to quarks and leptons, and are thus expected to be produced at proton-proton and electron-positron colliders of sufficient high energy. In $SU(8)$ these spin-1 bosons are an $SU(2)$ vector triplet and axial triplet, an overall singlet, and a vector color octet, the last one susceptible to be produced through the stronger color interaction.

The effective charged current-current interaction of extended BESS reproduces the SM interaction, after identification of the relevant scale parameter with the square root of the inverse Fermi coupling. Also, for any chiral $SU(N)_L \otimes SU(N)_R$, it can be seen that the neutral current-current interaction strength corresponds to a $\rho$-parameter of 1, because of the diagonal $SU(N)$ which is supposed to remain unbroken. All these results are of course corrected by radiative effects.

If one tries to compare $SU(8)$-BESS with the original $SU(2)$-BESS one sees that one main difference, concerning low energy effective interaction, lies in the role of the additional singlet vector-resonance, mentioned above. In addition the extension has new features, notably the appearance of pseudogoldstones.
Testing the symmetry breaking mechanism of the electroweak interactions will be one of the main tasks of future colliders. We have already discussed the strategies for the Higgs search, and now we address the topic of possible signatures of a strong symmetry braking sector, taking BESS as a simple parametrization containing all the relevant elements.

In its minimal version, the BESS model has three independent parameters; the mass $M_V$ of the new triplet of vector bosons, their gauge coupling $g''$, assumed to be much larger than $g$ and $g'$, and a parameter $b$ giving the direct coupling of the $V$'s resonances to fermions. The parameter $b$, even if it is free in an effective lagrangian approach, can be thought of as generated by radiative corrections and therefore is expected to be small [27]. The Standard Model is obtained in the limit $g'' \to \infty$ and $b = 0$.

Ordinary gauge bosons ($W$ and $Z$) mix to the new vector bosons $V$ with a mixing angle of the order $g/g''$, at least in the approximation $M_V >> M_W$. Due to this mixing the $V$'s are coupled to fermions even in absence of a direct coupling ($b = 0$).

It is important to notice that the mixing angle does not disappear in the limit $M_V \to \infty$, but it has an asymptotic value $g/g''$: therefore there is no decoupling, and for this reason the observables far from the resonance turn out to be quite insensitive to the value of $M_V$ (at least in the limits of validity of the model).

The new resonances from the strong symmetry breaking influence masses and couplings of ordinary gauge bosons and couple to fermions. Therefore one expects small deviations with respect to the Standard Model predictions already at collider energies far below the production threshold $M_V$.

These small virtual effects can be seen in $e^+e^-$ colliders, where high-precision measurements are possible. In absence of such deviations one can put bounds on the parameter space of the model.

Of course if the mass $M_V$ of the new resonances is below the maximal c.m. energy of the collider there will be a peak in the $e^+e^-$ annihilation cross section; tuning the collider at an energy $\sqrt{s} \approx M_V$ would provide for a $V$'s factory allowing to measure properties and couplings of the new particles. But one may expect to see dominant peaks below the maximum c.o.m. energy even without tuning the beam energies, due to beamstrahlung.

If the new vector bosons are too heavy to be produced as resonances, one has to look for deviations from the Standard Model values of the observables. At LEP1, at the $Z$ resonance, the relevant couplings are those among the $Z$ and the fermions, which enter in the process $e^+e^- \to f \bar{f}$. In BESS they differ from the SM ones up to terms proportional to $g/g''$ or $b$. Furthermore, due to the mixing, also the values of the masses of $W$ and $Z$ bosons get shifted.

Putting together the data from LEP1 and CDF/UA2 on the masses of $W$ and $Z$, the widths of $Z$ into leptons and hadrons, and asymmetries, one gets severe restrictions on the parameter space of the model. This space is essentially the plane $(b, g/g'')$ because the observables are almost $M_V$ independent. For instance at $b = 0$ one finds $g/g'' < 0.06$.

Future $e^+e^-$ linear colliders with different c.o.m. energies and luminosities have been proposed; a collider with energy up to $500 GeV$ has concentrated most of the studies [28], but at the same time possibilities of c.o.m. energies of 1 or 2 $TeV$ have been discussed.
In order to test the hypothesis of a strongly interacting symmetry breaking sector the channel $e^+e^- \rightarrow W^+W^-$ is particularly interesting, and large deviations from the Standard Model predictions may be obtained. This is due to the strong coupling between the longitudinal $W$ bosons and the new neutral resonance $V^0$; furthermore in BESS the Standard Model cancellation among the $\gamma-Z$ exchange diagrams and the neutrino contribution is destroyed. Therefore the differential cross section grows with the energy.

However, explicit calculations show that the leading term in $s$ is suppressed by a factor $(g/g'')^4$ and, at the energies considered here, it is the constant term of the order $(g/g'')^2$ that matters.

Final $W$ polarization reconstruction can be done considering one $W$ decaying leptonically and the other hadronically [29], and it is relevant to constrain the model, even if already at the level of unpolarized cross section one gets important restrictions. Assuming an integrated luminosity of $20\ fb^{-1}$, $\sqrt{s} = 500\ GeV$ and $b = 0$, it is possible to improve the LEP1 limit on $g/g''$ over the whole $M_V$ range if polarization is measured, up to $M_V \approx 1\ TeV$ for unpolarized $W$.

$W^+W^-$ pairs can be produced also through a mechanism of fusion of a pair of ordinary gauge bosons, each being initially emitted from an electron or a positron. This potentially interesting process allows, for a given c.m. energy, to study a wide range of mass spectrum for the $V$ resonance, but it becomes important for energies bigger than $2\ TeV$.

Measurements of the various observables (cross sections and asymmetries) of the fermionic channel $e^+e^- \rightarrow f\bar{f}$ will not give a real improvement with respect to the existing bounds from LEP1. The most sensitive observables are the left-right asymmetries, which need polarized $e^+e^-$ beams; but also in this case the bounds improve only for $M_V$ close to the value of the collider energy.

In conclusion, concerning $e^+e^-$ colliders, we can say that they could give the possibility to study the neutral sector of symmetry breaking; $V^0-Z$ mixing, $V^0f\bar{f}$ and $V^0W^+W^-$ couplings. As we will see below there is complementarity with respect to $pp$ colliders (LHC), allowing to explore $V^\pm$ resonances through the decay channel $W^\pm Z$. At proton colliders, as mentioned before, the channel $V^0 \rightarrow W^+W^-$ is difficult to study due to background problems, and $V^0 \rightarrow l^+l^-$ has a very low rate.

Proton-proton colliders, as LHC, have a great potentiality for discovering new strong interacting gauge bosons, but of course they are not as clean as $e^+e^-$ colliders; the high hadronic jets background makes the signals difficult to analyze, and if new particles are found their properties could not be investigated in detail.

At proton colliders there are two possible mechanisms to produce $V$ resonances; $q\bar{q}$ annihilation and $WW(WZ, ZZ)$ fusion. In the first mechanism a quark-antiquark pair annihilates into a $V$, which decays mostly into a pair of ordinary gauge bosons because the couplings $V^0W^+_LW^-_L$ and $V^\pm W^\mp Z_L$ are strong (of the order $g''$). We stress that this process of annihilation always takes place in BESS, even if $b = 0$, due to the mixing between ordinary and new gauge bosons. We notice that in BESS there is no coupling $V^0ZZ$.

The second mechanism goes through fusion of a pair of ordinary gauge bosons, both of them initially emitted from a quark or antiquark leg, to give a $V$ resonance decaying into a pair $W^\pm Z$ or $W^+W^-$. The cross section is obtained by a double convolution of the fusion cross section with the luminosities of the initial $W/Z$’s inside the quarks and
the structure function of the quarks inside the protons. In the $q\bar{q}$ annihilation only the convolution with the structure functions of the quarks is needed. The amplitude of the elementary fusion process is strong in BESS; in fact the scattering of two longitudinally polarized $W/Z$’s proceeds via the exchange of a $V$ vector boson with large couplings (of the order $g''$ at each vertex).

As we pointed out before, the interesting channel at proton colliders is $pp \rightarrow W^\pm Z + X$, because the $W^+W^-$ channel has a strong background from $pp \rightarrow t\bar{t} + X$ and the $ZZ$ one is not resonant in BESS. Monte Carlo simulations have been performed on the channel $WZ$ [30], considering only the leptonic decays of $W$ and $Z$. To isolate the signal from the background, coming from Standard Model $WZ$ and $t\bar{t}$ production, one requires three isolated leptons, two of them reconstructing a $Z$ with high transverse momentum. At LHC, with c.o.m. energy of 16 $TeV$, it turns out that to reach a 2 $TeV$ mass for the $V$ an integrated luminosity greater than $10^5 pb^{-1}$ is needed.

So far we have discussed the effects of the triplet of vector resonances $V$. The real situation could be more complex: axial-vector resonances might modify in a relevant way the predictions of the minimal model with only vectors [24]. As a general feature, virtual effects and deviations from the Standard Model coming from the vector and axial-vector sector tend to cancel each other, and the final physical effects depend on the relative weight of the two contributions. In some region of the parameter space of the model there could be complete cancellations and no deviations from the Standard Model would be observed, at least at energies below the new resonances. The discovery of a strong electroweak sector only through virtual effects and precision measurements could therefore be difficult and ambiguous. The direct discovery of new resonances at the $TeV$ scale would be in such a case determinant.

In the extended BESS model a richer phenomenology appears. There are $N^2 - 1$ vector and $N^2 - 1$ axial-vector new resonances, associated to the local copy of the global $SU(N)_L \otimes SU(N)_R$. These resonances mix with the ordinary gauge bosons: in the case $N = 8$ the neutral gauge sector involves the mixing of the fields $W^3, Y, V^3, A^3, V_D$. $V_D$ is a chiral singlet, and its mixing makes the colorless gauge sector of $SU(8)$-BESS different from the model based on $SU(2)_L \otimes SU(2)_R$. The $W^\pm, V^\pm$ and $A^\pm$ sector is like in $SU(2)$-BESS. Concerning the colored sector, the $SU(3)_C$ gluons mix with a color octet of vector resonances $V_8^\alpha$

Another new feature is of course the presence of pseudo-Goldstone bosons. We will indicate with $P^\pm (P^0)$ the lightest charged (neutral) ones, discussing in the following possible signatures at future accelerators [31].

Linear $e^+e^-$ colliders give the possibility to study the production of pairs of charged pseudo-Goldstone bosons. They can be produced at the $V$ resonance through the process $e^+e^- \rightarrow V \rightarrow P^+P^-$. The main decay mode of a charged $P$ is $P^+ \rightarrow t\bar{b}$, if the pseudo-Goldstone is heavy enough. We have therefore to analyze the final state $P^+P^- \rightarrow t\bar{b}b$, and compare it with the background. There are three background sources: $e^+e^- \rightarrow W^+W^-$, $e^+e^- \rightarrow ZZ$, $e^+e^- \rightarrow t\bar{t}$ and they have been already studied in the process of charged Higgs boson production [32]. Tagging one $b$ in the final state easily reduce the background $e^+e^- \rightarrow W^+W^-$, while the others two sources are smaller than the signal, at least in a reasonable range of the model parameter space.

At LHC pseudo-Goldstone bosons can be produced from a decay of a $V$ resonance, previously produced from quark-antiquark annihilation or from a fusion process. The
charged channel, $pp \to V^\pm \to P^\pm P^0 + X$, gives the signal $t\bar{b}b\bar{b}$ or $t\bar{b}gg$, because $P^0$, the lightest pseudo-Goldstone, decays mainly in $b\bar{b}$ and $gg$. For the neutral channel $pp \to V^0 \to P^+P^- + X$ one looks for the signal $t\bar{b}b$. The backgrounds are expected to be large, and a careful study is needed. In a study done for the case of charged Higgs boson pair at LHC [33] it has been shown that a good $b$ tagging is necessary to identify the signal.

5 Conclusion

The problem of electroweak symmetry breaking has acted in these last years as a dominant stimulus for imagining new physics beyond the standard model.

In this contribution we have first reviewed the theoretical situation and the different perspectives on the problem. We have then shortly summarized prospects at existing and future colliders, relevant to the question of electroweak symmetry breaking.

In the main part of this work we have concentrated on BESS (Breaking Electroweak Symmetry Strongly) as a simple scheme to describe an alternative breaking scheme avoiding elementary scalars. We have discussed the mathematical frame, according to two possible general constructions, the possible directions for extensions, specialization to the technicolor phenomenology, and general characteristic features.

Finally we have tried to summarize the work done to put limits on the BESS parameters from presently available precision data, and the exploratory work on BESS predictions for future colliders such as LHC and $e^+e^-$ linear colliders at very high energy.

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