New insights on the spin-up of a neutron star during core-collapse

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ABSTRACT

The spin of a neutron star at birth may be impacted by the asymmetric character of the explosion of its massive progenitor. During the first second after bounce, the spiral mode of the Standing Accretion Shock Instability (SASI) is able to redistribute angular momentum and spin-up a neutron star born from a non-rotating progenitor. Our aim is to assess the robustness of this process. We perform 2D numerical simulations of a simplified setup in cylindrical geometry to investigate the timescale over which the dynamics is dominated by a spiral or a sloshing mode. We observe that the spiral mode prevails only if the ratio of the initial shock radius to the neutron star radius exceeds a critical value. In that regime, both the degree of asymmetry and the average expansion of the shock induced by the spiral mode increase monotonously with this ratio, exceeding the values obtained when a sloshing mode is artificially imposed. With a timescale of 2-3 SASI oscillations, the dynamics of SASI takes place fast enough to affect the spin of the neutron star before the explosion. The spin periods deduced from the simulations are compared favorably to analytical estimates evaluated from the measured saturation amplitude of the SASI wave. Despite the simplicity of our setup, numerical simulations revealed unexpected stochastic variations, including a reversal of the direction of rotation of the shock. Our results show that the spin up of neutron stars by SASI spiral modes is a viable mechanism even though it is not systematic.

Key words: hydrodynamics – instabilities – shock waves – stars: neutron – stars: rotation – supernovae: general

1 INTRODUCTION

Neutron star spin at birth is a key parameter to associate pulsars and their progenitors. It carries information about the massive stars which explode in a core-collapse supernova and give birth to a neutron star. Natal spins can be estimated via an extrapolation of the spin-down of observed pulsars and the determination of their current age. This indicates that a significant fraction of neutron stars are born with modest rotation periods in a broad range from a few tens to a few hundreds milliseconds (Popov & Turolla 2012; Noutsos et al. 2013). Population synthesis studies suggest that the initial pulsar spin distribution may be modeled by a Gaussian distribution centered around 300 ms (Noutsos et al. 2013). Neutron star spin at birth has to be resolved by better stellar evolution models and/or a better description of the role played by core collapse dynamics.

Angular momentum transport processes throughout the stellar evolution play an important role by setting the rotation period of the core prior to collapse. Recent asteroseismic observations of red-giant stars (Beck et al. 2012; Mosser et al. 2012) seem to require a more efficient angular momentum transport than usually assumed (Cantiello et al. 2014; Deheuvels et al. 2014). Considering the transport of angular momentum by magnetic torques driven by a dynamo mechanism, Heger, Woosley & Spruit (2005) estimated pulsar spins of $10^{-15}$ ms – slower than previous estimates but still rather fast compared to observational constraints. Transport of angular momentum by internal gravity waves (see e.g. Lee, Neiner & Mathis (2014); Talon & Charbonnel (2003); Fuller et al. 2013) is also a possibility to explain the distribution of pulsar spins at birth. Considering estimated periods of 20 ms to 400 ms due to an influx of internal gravity waves during the latest burning stages. An-
gular momentum redistribution by hydrodynamic instabilities during the collapse has not been considered in these previous works.

Detailed numerical simulations of core-collapse supernovae show that massive stars do not explode in spherical symmetry (Liebendörfer et al. 2001) except for the low-mass end of supernova progenitors (Kitaura, Janka & Hillebrandt 2006). Neutrino-driven convection (Herant et al. 1994; Janka & Muelled 1996) and the Standing Accretion Shock Instability (SASI) (Blondin, Mezzacappa & DeMarino 2003) are able to generate large-scale asymmetric motions and may play a crucial role in the success of the explosion (see e.g. Foglizzo et al. (2007) for a recent review). Such asymmetric motions have been confirmed as the consequence of a linear instability using perturbative methods (Foglizzo et al. 2007). The resulting asymmetric explosions are supported by a series of observational evidences. The typical pulsar velocities of a few hundreds km/s (Arzoumanian, Chernoff & Cordes 2002) is much higher than its progenitor ones. A pulsar kick imparted on the neutron star by a global deformation [l = 1 has the potential to explain this velocity distribution (Scheck et al. 2004; 2006). Also, the distribution of $^{44}$Ti observed in Cassiopea A suggests a large scale asymmetric explosion (Grefenstette et al. 2014).

Unstable modes of SASI can develop sloshing motions of the shock along a symmetry axis and also spiral motions when the axisymmetric constraint is released. SASI spiral modes can redistribute angular momentum during the collapse and significantly modify the neutron star spin from what could be inferred by angular momentum conservation (Blondin & Mezzacappa 2007). Using a simplified adiabatic model Blondin & Mezzacappa (2007) showed that a spiral mode can dominate the dynamics and is able to spin-up a neutron star to periods as short as 50 ms even if the progenitor does not rotate. In this idealized scenario, some angular momentum is expelled in the explosion whereas the opposite angular momentum is accreted onto the surface of the neutron star.

This mechanism relies on the dominant action of a spiral mode. In the linear regime of SASI a sloshing mode can be decomposed as two counter-rotating spiral modes with similar amplitudes and identical growth rates if the progenitor is non-rotating. A robust spiral mode may dominate the dynamics only if the symmetry between these counter-rotating spiral modes breaks nonlinearly. Spiral modes were obtained by Blondin & Shaw (2007) in 2D, Fernández (2010) in 3D using an approximation of neutrino cooling. This numerical result has been confirmed by an experimental shallow-water analog of SASI (Foglizzo et al. 2013). Spiral modes dominate occasionally the dynamics in 3D numerical simulation of the collapse of a 27 $M_\odot$ progenitor using various approximations of neutrino transport (Hakke et al. 2013; Couch & O’Connor 2014; Abdikamalov et al. 2014). Employing an analytical approach Guilet & Fernández (2014) estimated the amount of angular momentum deposited when a single spiral mode dominates the dynamics in a non-rotating progenitor and showed that SASI has the potential to explain initial pulsar periods of a few tens to a few hundreds milliseconds. The slow end of this range of periods is compatible with the range 100 ms - 8 s obtained by Wongwathanarat, Janka & Müller (2013) in their simulations of 15 $M_\odot$ and 20 $M_\odot$ progenitors. An efficient spin-up mechanism driven by the SASI would require a long-lasting SASI activity up to the point of explosion. Müller, Janka & Hegel (2012) observed such a dynamics in their general relativistic neutrino hydrodynamics axisymmetric simulation of a 27 $M_\odot$ progenitor.

However, the non-linear dynamics of SASI may not always be dominated by a spiral mode. Indeed, no separation of angular momentum was obtained by Iwakami et al. (2008) considering a model with both neutrino heating and cooling. Investigating the flow pattern below the shock wave, Iwakami, Nagakura & Yamada (2014) showed that either sloshing or spiral modes dominate the dynamics depending on the mass accretion rate and neutrino luminosity considered, illustrating the stochasticity of the angular momentum distribution in hydrodynamics simulations. Spin-up by the SASI can be effective only if the symmetry breaking leading to a single spiral mode has occurred before the explosion takes place. Even in this case, the amount of angular momentum accreted is sensitive to the position of the mass cut radius (Rantsiou et al. 2011).

We propose to study the timescale and rotational consequences of the symmetry breaking, which determines the respective roles of sloshing and spiral modes. We perform a set of 2D simulations of a simplified model of an accretion flow restricted to the equatorial plane, using cylindrical geometry. The set of parameters considered shed some light on the non-systematic and non-deterministic features of the symmetry breaking.

The paper is organized as follows. The physical and numerical models are described in section 2. Section 3 focuses on the properties of the symmetry breaking and the non-linear evolution of SASI to evaluate their consequences on the distribution of angular momentum. Our simulations are confronted to the dynamics of less idealized environments in section 4 in order to discuss the potential role of SASI on the initial neutron star spin.

2 METHODS

2.1 Physical model

Our model consists of a standing accretion shock centered around a proto-neutron star in a stationary and non-rotating flow. For the sake of simplicity we focus our study on the equatorial plane of the collapsing core, using cylindrical coordinates in a setup similar to Yamasaki & Foglizzo (2008).

The main advantage of this model is to allow for non-axisymmetric modes of SASI in 2D. The accreting matter is modeled by a perfect gas with adiabatic index $\gamma = 4/3$. Above the shock, the supersonic matter falls inwards radially and reaches the shock radius $r_{sh}$ with an incident Mach number $M_{sh} = 5$. Below the shock, the matter accretes subsonically onto the surface of the proto-neutron star which radius is noted $r_\star$. A cooling function is included to mimic the neutrino emission due to electron capture with the approximation $L_\nu \propto P^{3/2} \rho$ (Blondin & Mezzacappa 2006), where $\rho$ and $P$ are respectively the density and the pressure. Neutrino heating is neglected in order to suppress buoyancy induced convective motions and concentrate on SASI in its simplest form. The gravity is Newtonian and self-gravity is neglected.
The initial solution is computed by solving the time independent continuity, Euler and entropy equations below and above the shock. The two solutions are connected by the Rankine-Hugoniot jump conditions neglecting the dissociation of nuclei at the shock. The resulting dynamics only depends on the ratio of the initial shock to the proto-neutron star radii. Typical values of the radii ratio $R \equiv r_{sh}/r_*$ are $R \approx 2$ (Couch & O’Connor 2014) and $R \approx 4$ (Marek & Janka 2009), depending on the progenitor structure.

When converting to physical units, we choose a proto-neutron star with radius $r_*=50\text{km}$ and mass $M_*=1.3M_\odot$, and a constant mass accretion rate $\dot{M}=0.3M_\odot\text{s}^{-1}$ as typical values for the stalled shock phase of a core-collapse supernova during the first second after bounce.

### 2.2 Numerical model

To run our two-dimensional time-dependent hydrodynamic simulations we use a version of the RAMSES code (Teyssier 2002; Fromang, Hennebelle & Teyssier 2006) adapted to cylindrical coordinates $(r, \phi)$ for which the grid is uniformly spaced. RAMSES is a second-order finite volume code, which uses the MUSCL-Hancock scheme. The simulations are performed using the HLLD Riemann solver (Miyoshi & Kusano 2005) and the monotinized central slope limiter. We set periodic boundary conditions in the azimuthal direction at $\phi=0$ and $\phi=2\pi$. The radial domain covers the interval $[540, 1300]$ depending on the simulation. 1000 cells are imposed reflexive inner boundary conditions and free outer boundary conditions as in Blondin & Mezzacappa (2006); Fernández & Thompson (2009b). Resolution effects are minimized by fixing the number of radial cells below the initial shock to 150. The total number of radial cells is then in the range $[540, 1300]$ depending on the simulation. 1000 cells are used in the azimuthal direction, which is significantly larger than what can be afforded by current 3D simulations (e.g. 176 cells in Hanke et al. 2013; Melson et al. 2013). High resolution is required to properly resolve the steep gradients of the flow dynamics in the vicinity of the proto-neutron star.

An entropy cutoff is applied to the cooling function in order to avoid the divergence of the numerical solution in the vicinity of the proto-neutron star (Fernández & Thompson 2009b; Fernández 2015). The cooling function is written

$$\mathcal{L} \equiv \mathcal{L}_0 \exp \left( \frac{S - S_{\text{min}}}{k S_{\text{min}}} \right)^2 ,$$

where $S \equiv (\gamma - 1) \ln (P/\rho^\gamma)$ defines the entropy, $S_{\text{min}}$ its value at $r = r_*$ and the value of $k$ is chosen to introduce only minimal modifications to the stationary state flow.

Once the numerical solution has relaxed on the grid for a few hundreds numerical timesteps, two density perturbations at pressure equilibrium are introduced ahead of the shock to trigger two counter-rotating spiral modes $m = \pm 1$ or $m = \pm 2$ as described in appendix A. Perturbations are decomposed as Fourier modes in the azimuthal direction according to the general form

$$\delta A(r, t) \equiv \text{Re} \left( \sum_m \tilde{c}_m(r) e^{i(m \omega t - \nu t)} \right)$$

where $\tilde{c}_m(r, t)$ is the complex amplitude. We define the initial $\epsilon$--asymmetry between spiral modes as:

$$\epsilon \equiv \frac{|\tilde{c}_m|^2 - |\tilde{c}_{-m}|^2}{|\tilde{c}_m|^2 + |\tilde{c}_{-m}|^2}$$

with $|\epsilon| \lesssim 1$. Note that $\epsilon = 0$ corresponds to a mirror-symmetric sloshing mode and $\epsilon = \pm 1$ to a single spiral mode.

Our aim is to estimate the timescale for a symmetry breaking, after the phase of linear growth which lasts less than $\sim 3$ SASI oscillations ($\lesssim 100\text{ms}$). Two different methods have been developed for that purpose. The first one is based on the time evolution of the angular momentum flux through the inner boundary. This flux is very close to zero for a sloshing mode and starts to deviate from zero once one of the spiral modes dominates. The second method is based on the angular tracking of the minimum shock radius. This point corresponds to one of the triple points that form in the shock wave. Its rotation rate evolves rather erratically for a sloshing mode but becomes fairly constant for a spiral mode. The two methods are consistent within a SASI period which is sufficient for our study.

Our code has been carefully tested to check that it does not introduce any artificial source of asymmetry. If the initial density perturbation is mirror-symmetric, i.e. $\epsilon = 0$, the mirror-symmetry is conserved and the sloshing mode oscillates along a fixed axis. Moreover, two simulations with opposite initial perturbations, $\epsilon$ and $-\epsilon$, show two dynamical evolutions that remain mirror-symmetric within machine precision. Besides, the robustness of the code has been tested by comparing the growth rates and oscillatory frequencies measured in our simulations to those obtained with a perturbative analysis by Yamazaki & Foglizzo (2008). The discrepancies are less than 8% for the growth rates and less than 2% for the oscillatory frequencies. This is similar to the good agreement obtained by Fernández & Thompson (2009b).

### 3 RESULTS

#### 3.1 A critical ratio for the symmetry breaking

We performed a total of 80 simulations varying the two parameters $R$ and $\epsilon$ such that $R = \{1.67, 2, 2.22, 2.5, 3, 3.5, 4\}$ and $10^{-3} \lesssim |\epsilon| \lesssim 1$. Our simulations show that contrary to what was obtained in some studies (Blondin & Mezzacappa 2007; Fernández 2014), a spiral mode does not always dominate the late evolution. The ratio $R$ was found to determine whether the symmetry breaking occurs or not. When $R \leq 2$, the late evolution is dominated by a robust sloshing mode, even if a single spiral mode (i.e. $|\epsilon|=1$) was used to perturb the stationary flow (Figure 1 left). The azimuthal index of the sloshing mode can either be $m = 1$ or $m = 2$, depending on the value $R$. In this regime, angular momentum is not significantly redistributed.

A totally different behavior is observed when $R > 2$. A spiral mode dominates the late evolution (Figure 1 right)
even for weak $\epsilon$–asymmetry, enabling a redistribution of angular momentum. These results raise the question of the mechanism responsible for this symmetry breaking. The dynamics of SASI observed in our simulations may help to characterize this mechanism as discussed in section 3.7.

### 3.2 Timescale for the symmetry breaking

We apply the methods described in section 2.2 to compute the timescale to reach a symmetry breaking as a function of $R > 2$ and $\epsilon$ (Figure 2). For $R = \{2.5, 3, 4\}$ the symmetry breaking occurs within 2 to 10 SASI oscillations in the non-linear phase, which is fast enough to potentially redistribute angular momentum before the explosion. In the case $R = 2.22$, a spiral mode dominates only after 20 to 30 SASI oscillations. This timescale may be too slow to impact the neutron star spin. The ratio $R = 2.22$ illustrates the continuity between a rapid symmetry breaking and an absence of symmetry breaking.

The influence of the initial asymmetry on the timescale is not straightforward. If the asymmetry is large enough (i.e. $|\epsilon| \gtrsim 0.2$), the timescale decreases with $|\epsilon|$ as would be intuitively expected. The trend seems rather chaotic and the uncertainties on the timescale are as large as the variability of the results when $|\epsilon| \lesssim 0.2$. Furthermore, the direction of rotation of the spiral mode is not always the one determined by the initial asymmetry. Indeed, approximately half of our simulations with $|\epsilon| \lesssim 0.2$ show a symmetry breaking in the other direction (square symbols in Figure 2). The code has been extensively tested to prevent numerical artifacts from inducing asymmetries. This non-deterministic feature could instead be generated by several non-linear processes which we mention as possible paths towards an explanation. The first one is based on the parasitic instabilities, such as Kelvin-Helmholtz and Rayleigh-Taylor that have been proposed to explain the saturation amplitude of the SASI (Guilet, Sato & Foglizzo 2010). The parasites which develop on SASI spiral modes might modify the asymmetry level in a stochastic way before the symmetry breaking (Figure 3). The second non-linear process relies on secondary shocks that arise before the symmetry breaking. Multiple secondary shocks, shown in Figure 4, were witnessed by Fernández & Thompson (2009a). These shocks may interact with the global advective-acoustic cycle. The entropy and the vorticity produced by secondary shocks may produce an acoustic feedback in the azimuthal direction opposite to the initial acoustic wave. This phenomenon might be able to alter the competition between counter-rotating spiral modes and add some stochasticity before a symmetry breaking occurs. An example is shown in Figure 5 where a secondary shock is able to generate opposite angular momentum well inside the outer shock wave.

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**Figure 1:** Left: Entropy snapshot at $t = 960$ ms for $R = 2$ and $\epsilon = 1$. A sloshing motion dominates the non-linear regime despite a spiral perturbation. Right: Entropy snapshot at $t = 187$ ms for $R = 3$ and $\epsilon = 0.1$. The symmetry breaking has already occurred. (Animated versions of these figures are available in the online journal.)

**Figure 2:** Number of SASI oscillations before reaching a symmetry breaking with respect to the initial asymmetry $\epsilon$. From top to bottom are shown ratios $R = \{2.22, 2.5, 3, 4\}$. Square symbols show cases for which the direction of rotation is opposite to the one of the initial asymmetry.
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3.3 Reversal of the direction of rotation

This section and the following one are dedicated to measuring the rotation induced by the spiral mode. We focus on a set of 7 simulations where the radii ratio $R$ is varied and the initial asymmetry is set to $\epsilon = 1$. The angular momentum density profile

$$ l_z(r, t) \equiv r^2 \int \rho v_\phi \, d\phi $$

is averaged in time over ten SASI periods in the non-linear phase as shown in Figure 4. Two counter-rotating regions are observed and the radius separating them is labeled $r_0$. The angular momentum $L_z(t)$ contained between the radius $r_0$ and the shock wave is computed by:

$$ L_z(t) \equiv \int_{r_0}^{r_{sh}} l_z(r, t) \, dr. $$

(5)

If there is no symmetry breaking ($R \leq 2$), the average profile is best defined using the linear phase only. The angular momentum $L_z(t)$ contained between the radius $r_0$ and the shock wave is computed by:

$$ L_z(t) \equiv \int_{r_0}^{r_{sh}} l_z(r, t) \, dr. $$

(5)

Figure 7 shows the time evolution of the enclosed angular momentum for our set of simulations. The cases $R = 4$ (solid red line) and $R = 2.22$ (dashed black line) exhibit a surprising inversion of the direction of rotation of the spiral wave. Both events take place in the fully non-linear regime.

For $R = 2.22$, the change of direction lasts approximately 8 SASI periods during which a sloshing mode dominates. The angular momentum produced by SASI is very...
low during that period. Even though a symmetry breaking has occurred, we observe that the non-linear dynamics of the SASI is able to cancel the angular momentum redistribution for a significant time. In the case $R = 4$, the change of the direction is achieved on a much shorter timescale, less than a SASI period.

The robustness of this behavior was confirmed by repeating these two puzzling simulations varying slightly one parameter such as the numerical resolution or the perturbation amplitude. This intriguing phenomenon calls for a physical interpretation. A possibility might be that the secondary shocks discussed in Sect. 3.2 break the advective-acoustic cycle and establish temporarily a new one between the second shock and the feedback region. This process is illustrated by Figure 9. The conditions for this adverse contribution to be able to reverse the direction of rotation remain to be determined.

### 3.4 Estimate of the pulsar spin

Guilet & Fernández (2014) derived an analytical estimate of the angular momentum redistribution driven by a single spiral mode in spherical geometry. This approach has been adapted to the cylindrical geometry in appendix B.

Birth periods of neutron stars are inferred from our simulations using a moment of inertia of $I = I_{45} \times 10^{37}$ g cm$^2$. Guilet & Fernández (2014) derived an analytical estimate of the angular momentum redistribution driven by a single spiral mode in spherical geometry. This approach has been adapted to the cylindrical geometry in appendix B.

Simulations indicate that in this regime the parasitic instabilities are not as efficient at stopping the growth of SASI as predicted by Guilet, Sato & Foglizzo (2010). This suggests either that a more elaborate description of the parasitic instabilities is necessary or that another process is responsible for the saturation of SASI.

### 3.5 Saturation amplitude of SASI

The saturation amplitude of SASI $\Delta r$ is a key element of the spin-up by spiral modes because the amount of angular momentum redistributed scales as $\Delta r^2$ (Eq. B18). The increase of the saturation amplitude with the ratio $R$ (Figure 8) is consistent with the highest spin obtained for highest values of $R$ (Figure 8).

However, the saturation amplitude obtained by applying the formalism of Guilet, Sato & Foglizzo (2010) decreases with increasing $R$ (Figure 8 left). The higher saturation amplitudes observed at large values of $R$ in the simulations indicate that in this regime the parasitic instabilities are not as efficient at stopping the growth of SASI as predicted by Guilet, Sato & Foglizzo (2010). This suggests either that a more elaborate description of the parasitic instabilities is necessary or that another process is responsible for the saturation of SASI.

The shock expansion due to SASI is increasing with $R$ more steeply than the saturation amplitude: between $R = 2$ and $R = 4$, it increases by a factor $\simeq 4$ while $\Delta r/r_{sh}$ increases by a factor $\simeq 2$ (Figure 9). This is consistent with the shock expansion varying quadratically with the saturation amplitude as might be expected for a non-linear effect. A similar trend was observed by Fernández & Thompson (2000b) with a slighter increase which may be attributed to the geometry difference.
For $R > 2$ the saturation properties of the spiral mode ($\epsilon = 1$) are compared with the ones of a sloshing mode obtained by imposing mirror-symmetric initial perturbations ($\epsilon = 0$) (Figure 10 left). If the ratio $R$ is close to the threshold for symmetry breaking, the average shock radius and the saturation amplitude are almost equal between a sloshing mode and a spiral mode. When the ratio $R$ is large enough for the domination of a spiral mode, the shock radius and the saturation amplitude are increased by up to about 40% compared to the mirror symmetric evolution.

These results confirm the work of Fernández (2013) which showed that a spiral mode in 3D may lower the critical neutrino luminosity compared to a sloshing mode in an axisymmetric case that generates less non-radial kinetic energy. In our simulations with the highest ratios $R$, spiral modes are indeed able to double the total non-radial kinetic energy in the non-linear regime (dashed black line). Right : Variation of the shock radius compared to its initial value $r_{sh0}$ as a function of $R$ for $\epsilon = 1$.

3.6 Differences between spiral and sloshing modes

For $R > 2$ the saturation properties of the spiral mode ($\epsilon = 1$) are compared with the ones of a sloshing mode obtained by imposing mirror-symmetric initial perturbations ($\epsilon = 0$) (Figure 10 left). If the ratio $R$ is close to the threshold for symmetry breaking, the average shock radius and the saturation amplitude are almost equal between a sloshing mode and a spiral mode. When the ratio $R$ is large enough for the domination of a spiral mode, the shock radius and the saturation amplitude are increased by up to about 40% compared to the mirror symmetric evolution.

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3.7 A possible path towards the symmetry breaking mechanism

A description of the physical processes responsible for the symmetry breaking would be helpful to anticipate the efficiency of SASI at spinning-up neutron stars in more realistic models. A first constraint is that no spiral mode dominates the non-linear dynamics if $R \leq 2$. Additional clues may be inferred from the following properties:

- Unlike for $m = 1$ modes, the $m = 2$ sloshing mode in our setup is never transformed non-linearly into a spiral mode.
- In the case $R = 2$, despite a linear domination of the mode $m = 2$, the mode $m = 1$ eventually prevails due to a non-linear coupling between these modes (Figure 11). However, the transition between these modes does not lead to a spiral mode (Figure 11 left).
- Linearly, the mode $m = 2$ dominates the mode $m = 1$ for $R \leq 2$. Interestingly, the critical ratio for the symmetry breaking is close to this linear transition.
- The efficiency of the symmetry breaking seems to be linked to the difference of saturation amplitudes between the spiral and the sloshing modes. A fast symmetry breaking corresponds to a significantly larger amplitude of the spiral mode, while the amplitudes are approximately equal when the symmetry breaking is slow ($R = 2.22$, see Figures 2 and 11 left).

4 DISCUSSION

Several simplifications have been made in our model to study the physics of SASI in its simplest form and less idealized models might modify some aspects of our results. The dimensionality and the geometry are important points to raise. The density and velocity profiles in cylindrical geometry are...
the development of spiral modes in a rotating progenitor, Blondin & Mezzacappa (2007) showed that SASI could surprisingly decelerate a neutron star which accretes SASI induced angular momentum opposed to the initial rotation of the stellar core. They also showed that this mechanism could even lead to the formation of a counter rotating neutron star. Yamazaki & Foglizzo (2008) confirmed that rotation favors prograde spiral modes and showed that SASI growth rates depend linearly on the angular momentum of the progenitor. These results raise the issue of the critical rotation rate of the progenitor above which the neutron star spin at birth is mostly determined by the conservation of initial angular momentum. A crude estimate can be made by evaluating the angular momentum accreted by a spiral mode may increase even if the centrifugal force is negligible. The mutual influence of the initial rotation and the SASI induced dynamics on the birth period of neutron stars will be addressed in a forthcoming paper.

Taking into account neutrino heating would add a source of stochasticity through the development of neutrino-driven convection, and help address the diversity of explosion paths Fernández et al. (2014); Cardall & Budiarja (2013). Pre-collapse convective asymmetries may also add stochasticity to post-bounce dynamics and affect the fate of the massive star (Couch & Ott 2013; Müller & Janka 2015). Iwakami, Nagakura & Yamada (2014) explored the diversity of flow patterns behind the stalled shock and observed that the symmetry breaking is not systematic in their SASI-dominated model A with $\dot{M} = 0.2 \, M_\odot \, s^{-1}$, $L_\nu = 2 \times 10^{52} \, \text{erg} \, s^{-1}$ (see table 1 in Iwakami, Nagakura & Yamada 2014). Changing slightly the numerical resolution or the noise in the initial conditions either lead to a quasi-stationary sloshing or spiral mode. However, their other SASI dominated models exhibit only spiral modes. This opens the question of the existence and the value of a critical ratio $R$ for the symmetry breaking in more complex models which may be more subject to stochasticity. Without neutrino heating in 3D simulations Fernández (2010) observed that the amount of angular momentum redistributed by the dynamics of SASI is greatly reduced for $R > 1.67$ compared to $R = 2$. These results are consistent with the fact that $R = 1.8$ is the transition between $l = 1$ and $l = 2$ linear modes (Foglizzo et al. 2007). A similar transition takes place in cylindrical geometry for $R \approx 2.2$.

5 CONCLUSION

A simplified setup in cylindrical geometry has been used to investigate the flow pattern in the non-linear regime of SASI for a non-rotating progenitor. A symmetry breaking between counter rotating spiral modes occurs only if the ratio of the initial shock to neutron star radii $R > 2$. If this condition is satisfied, the dynamics is dominated by a spiral mode, independently of the initial conditions and SASI has the potential to spin-up a neutron star to initial periods of a few tens to a few hundreds milliseconds. However, if $R \leq 2$, there is no sign of symmetry breaking and a sloshing mode dominates the dynamics. This case leads to very slowly ro-

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Figure 11. Time evolution of the amplitudes of the Fourier modes $m = 1$ (thin blue line) and $m = 2$ (thick green line).

Figure 12. Density profile (left panel) and radial velocity profile (right panel) computed in cylindrical coordinates (solid red line) and in spherical coordinates (dashed blue line).

compared in Figure 12 to those obtained in spherical geometry for the same parameters used in Sect. 2. In the subsonic region of the flow, the density profile is independent of the geometry but the advection time is shorter in spherical geometry. Remembering that SASI frequencies and growth rates scale like the advection rate, these quantities are higher for the same parameters used in Sect. 2. In the subsonic region of the flow, the density profile is independent of the geometry but the advection time is shorter in spherical geometry. A symmetry breaking between SASI spiral modes may therefore occur earlier in a 3D spherical model than in 2D cylindrical geometry. The dimensionality of the model impacts the amount of angular momentum via its dependence on the saturation amplitude. The latter was found to be weakly sensitive to the dimensionality (Fernández 2010; Hanke et al. 2014; Fernández 2015). However, drawing conclusions on this issue may require to clarify the divergence between the predicted and the measured saturation amplitudes observed in our study (Figure 10).

The initial rotation of the progenitor has been neglected for the sake of simplicity, but could dominate the angular momentum budget if it is fast enough. Considering
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APPENDIX A: DENSITY PERTURBATIONS

In the linear regime, the two spiral modes of index ±m are triggered by over-densities injected at the outer boundary of the domain:
\[ \delta \rho_\pm (\theta, t) \equiv A (1 \pm \epsilon) \cos (\omega_r t \mp m \theta) \]  
(A1)
where A and \( \omega_r \) are the amplitude of the perturbations and the oscillatory frequency of the SASI mode, respectively. In this formulation \(-1 \leq \epsilon \leq 1\) and the sign of \( \epsilon \) selects the dominant spiral mode in the linear regime. The overall perturbation are written as:
\[ \delta \rho (\theta, t) = H(t) (\delta \rho_+ (\theta, t) + \delta \rho_- (\theta, t)) \]  
(A2)
where \( H(t) \) is a function used to smoothen the perturbation such that:
\[
H(t) \equiv \begin{cases} 
\exp \left\{ - \left( \frac{t - (t_0 + \tau_{adv}/4)}{\sigma} \right)^2 \right\} & \text{if } t_0 \leq t \leq \tau_{adv}/4 \\
1 & \text{if } \tau_{adv}/4 \leq t \leq 5 \tau_{adv}/4 \\
\exp \left\{ - \left( \frac{t - (t_0 + 5 \tau_{adv}/4)}{\sigma} \right)^2 \right\} & \text{if } 5 \tau_{adv}/4 \leq t \leq 6 \tau_{adv}/4 \\
0 & \text{otherwise}
\end{cases}
\]  
(A3)
where \( t_0 \) is the time when the perturbations start to be advected through the outer boundary, \( \tau_{adv} = 2\pi/\omega_r \) is the advection time and \( \sigma \) is a coefficient used to vary the amplitude of \( H(t) \) from \( 10^{-16} \) to 1 over a timescale \( \tau_{adv}/4 \).

APPENDIX B: ANGULAR MOMENTUM REDISTRIBUTION BY SASI SPINAL MODES IN CYLINDRICAL GEOMETRY

In this appendix, we adapt the formalism developed by Guilet & Fernández (2014) for the angular momentum redistribution by a SASI spiral mode in spherical geometry, to the cylindrical setup considered in this paper. Because the derivation of the equations follows very similar steps, we do not reproduce all of them here but highlight the differences linked to the change of geometry. The end result takes a very similar form to the spherical geometry, differing only in the numerical factor.

As in the rest of the paper, we consider a 2D accretion flow in cylindrical geometry, assumed to be invariant in the vertical direction and described using cylindrical coordinates \( \{r, \phi\} \). The surface integrated angular momentum density is defined in equation 1 (where \( \rho \) is to be understood as a vertically integrated surface density). While in Guilet & Fernández (2014) the surface integration was done...
on spherical shells, it is here performed on cylinders. Angular momentum conservation can then be written as in Guilet & Fernández (2014)

\[ \partial t \mathbf{l}_z + \partial_r \mathbf{F} = 0, \quad (B1) \]

where \( \mathbf{F} \) is the angular momentum flux integrated over a cylindrical surface

\[ \mathbf{F}(r, t) \equiv r^2 \int \rho \mathbf{v} \mathbf{v} \, d\phi. \quad (B2) \]

As in Guilet & Fernández (2014), the flow is described as a stationary background with superimposed small amplitude perturbations

\[ \rho(r, \phi, t) = \rho_0(r) + \delta \rho(r, \phi, t) + \delta^2 \rho(r, \phi, t) + \ldots \quad (B3) \]

\[ v_r = v_0 + \delta v_r + \delta^2 v_r + \ldots \quad (B4) \]

\[ v_\phi = \delta v_\phi + \delta^2 v_\phi + \ldots \quad (B5) \]

where \( \delta \) and \( \delta^2 \) denote first- and second order Eulerian perturbations, respectively, with \( \delta \gg \delta^2 \). With this decomposition, the surface integrated angular momentum density and flux read

\[ \mathbf{l}_z = \frac{\dot{M} r v_0}{2\pi} \int \left[ \frac{\delta \rho \delta v_r}{v_0} + \delta^2 v_r \right] d\phi, \quad (B6) \]

\[ \mathbf{F} = l_z v_0 + T_{Rey} \quad (B7) \]

where \( \dot{M} \equiv -2\pi r \rho_0 v_0 \) is the stationary mass flux, and \( T_{Rey} \) is the surface-integrated Reynolds stress

\[ T_{Rey}(r, t) = -\frac{\dot{M} r v_0}{2\pi} \int \delta v_r \delta v_r \, d\phi. \quad (B8) \]

First order perturbations are then decomposed into a superposition of spiral modes with sinusoidal angular dependence with Fourier index \( m \) (this replaces the spherical harmonics decomposition used in Guilet & Fernández (2014), and the time-dependence of a plane wave with complex frequency \( \omega = \omega_r + i \omega_i \), with \( \omega_r \) and \( \omega_i \) the real and imaginary parts, respectively. The space and time dependence of an arbitrary first-order perturbation \( \delta A \) is therefore

\[ \delta A(r, \phi, t) = \sum_m \text{Re} \left[ \hat{\delta A}_m(r) e^{-i(\omega t - m \phi)} \right] \quad (B9) \]

where \( \delta A_m(r) \) is the complex amplitude. The radial structure and eigenfrequencies of these modes can be computed with a linear analysis as in Yamasaki & Foglizzo (2008).

Using this linear eigenmodes decomposition, the Reynolds stress can be written

\[ T_{Rey} = -\frac{\dot{M} r v_0}{2} \sum_m \text{Re} \left[ \frac{\delta \tilde{v}_r}{v_0} \delta^2 \tilde{v}_r \right] e^{2\omega_i, m t}. \quad (B10) \]

Defining \( T_{Rey, 0, m} \) as the Reynolds stress amplitude of a given mode with Fourier index \( m \) and with the time dependence scaled out,

\[ T_{Rey, 0, m}(r) = \frac{\dot{M} r v_0}{2} \text{Re} \left[ \frac{\delta \tilde{v}_r}{v_0} \delta^2 \tilde{v}_r \right] \quad (B11) \]

we can write equation (B10) as

\[ T_{Rey} = \sum_m T_{Rey, 0, m}(r)e^{2\omega_i, m t}. \quad (B12) \]

The angular momentum density \( l_z \) can therefore be written as the sum of contributions from different Fourier modes, with each term given by (see Guilet & Fernández 2014, for more details on the derivation)

\[ l_z, m = -\frac{T_{Rey, 0, m}}{v_0} + \frac{e^{-2\omega_i, m t}}{2} \int_{r_{sh}}^r \left[ 2\omega_i, m e^{2\omega_i, m t} T_{Rey, 0, m} \right] \, dr, \quad (B13) \]

where \( T_{adv}(r) = \int_{r_{sh}}^r \, dr/v_0 \) is the advection time from the shock radius \( r_{sh} \) to a radius \( r < r_{sh} \).

### B1 Angular momentum density below the shock

The angular momentum density below the shock due to a spiral SASI mode follows from equations (B11) and (B14),

\[ l_{sh, m} = -\frac{T_{Rey, 0, m}}{v_{sh}} \quad (B15) \]

This expression can be evaluated using the boundary conditions of linear eigenmodes at a shock with a constant dissociation energy

\[ l_{sh, m} = -\frac{\omega_r r_{sh}}{2\pi v_{sh}} f(\kappa, M_1) \left( \frac{\Delta r}{r_{sh}} \right)^2, \quad (B16) \]

where \( v_{sh} \) is the radial velocity below the shock, \( \Delta r \) is the amplitude of the shock deformation induced by the SASI spiral mode, \( \kappa \) is the compression ratio of the shock, \( M_1 \) is the upstream Mach number, and \( f(\kappa, M_1) \) is a dimensionless factor

\[ f(\kappa, M_1) \equiv \pi (\kappa - 1)(1 - 1/\kappa)\frac{1 + 1/M_1^2}{(\gamma - (\gamma + 1)/\kappa + 1/\gamma M_1^2)} \quad (B17) \]

### B2 Approximate expression for the angular momentum contained in a spiral wave

Using the same method as Guilet & Fernández (2014) we obtain an approximate expression for the total angular momentum contained in the spiral wave (i.e. between the shock and the radius where the angular momentum changes sign)

\[ L_z \simeq (r_{sh} - r_s)l_{sh} \simeq m f(\kappa, M_1) \frac{\omega_r}{2\pi v_{sh}} (r_{sh} - r_s) \left( \frac{\Delta r}{r_{sh}} \right)^2 = \frac{\Delta r}{r_{sh}} \quad (B18) \]
This is the same equation as Guilet & Fernández (2014), but note that the numerical factor \( f(\kappa, \mathcal{M}_1) \) differs by a factor \( 4\pi \). This is mostly due to the different normalization of the Fourier modes considered here compared to the spherical harmonics used in Guilet & Fernández (2014). Considering the moment of inertia \( I \) of the neutron star, this can be translated into a minimum period of uniform rotation

\[
P \simeq \frac{2\pi I}{mf(\kappa, \mathcal{M}_1) \omega_r (r_{sh} - r_*)} \frac{1}{M \Delta r} \left( \frac{r_{sh}}{\Delta r} \right)^2 .
\]

(B19)

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