SU(2) Family Symmetry and the Supersymmetric Spectrum

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Abstract

We present a supersymmetric extension of the Standard Model with a gauged SU(2) family symmetry for the leptons. It is shown that this family symmetry can be consistently broken at the TeV scale along with supersymmetry. If supersymmetry breaking is driven by anomaly mediation, this model can provide positive squared masses for the sleptons and thus cure the tachyon problem. We analyze the constraints and consequences of this scenario. A characteristic feature of this model is the non–degeneracy of the first two family sleptons. The model predicts large value of \( \tan \beta \) and observable \( \tau \to e\gamma \) and \( B \to \mu^+\mu^- \) decay rates.

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1 Introduction

Family symmetries based on non–Abelian gauge groups may be useful in addressing the mass hierarchy and flavor mixings among quarks and leptons. In a supersymmetric context, an $SU(2)_F$ family symmetry, realized either as a global symmetry \([1, 2]\) or as a local symmetry \([3]\) can also provide a solution to the SUSY flavor problem. A concrete realization of this idea would have the first two family fermions transforming as doublets of $SU(2)_F$, while the third family fermions are singlets of the group. In the exact $SU(2)_F$ symmetric limit, the soft SUSY breaking mass parameters for the first two families would be degenerate, alleviating the SUSY flavor problem significantly. The same setup can also explain the observed mass and mixing hierarchies of quarks and leptons, including the neutrinos \([3]\).

In this paper we propose a SUSY extension of the Standard Model (SM) with a gauged family $SU(2)_F$ symmetry for the leptons. When the model is embedded into the anomaly mediated SUSY breaking (AMSB) scenario \([4, 5]\), we show that the model can cure the tachyonic slepton problem. This is our main motivation.\(^4\) In order to achieve this, we maintain asymptotic freedom of the $SU(2)_F$ interactions and insist that the symmetry breaks at the TeV scale along with SUSY. (If the symmetry were broken at a higher scale, due to the ultraviolet insensitivity of AMSB scenario, there would be no new contributions to the slepton masses.) One of our major tasks is to show that phenomenologically consistent $SU(2)_F$ gauge models broken at the TeV scale can be designed without violating flavor changing neutral current (FCNC) constraints. The main concern would be FCNC mediated by the $SU(2)_F$ gauge bosons (we denote them as $V^a_{\mu}$). In our construction we show that owing to approximate symmetries present in the model, excessive FCNC processes do not occur. We then show the consistency of the symmetry breaking and study the salient features of the SUSY spectrum. Although the first two family sleptons are degenerate in mass in the $SU(2)_F$ symmetric limit, symmetry breaking effects lift this degeneracy by factors of order one. While excessive flavor violation is absent in the model, there are residual effects, which we study. It turns out that consistency of the model requires $\tan \beta$ to be large, $\tan \beta \geq 40$. As a result,

\(^4\)Higher family symmetries, such as $SU(3)$ have found application in solving the tachyon problem of AMSB \([6]\). The realization presented here is somewhat simpler.
the model predicts branching ratio for the process \( B_s \rightarrow \mu^+\mu^- \) very close to the current experimental limit. While rare decays involving the muon are suppressed in the model, decays such as \( \tau \rightarrow 3e \) and \( \tau \rightarrow e\gamma \) are allowed. The branching ratio for the latter is found to be within reach of future experiments.

A number of attempts to resolve the tachyonic slepton problem in AMBS have appeared. For example, a non-decoupling universal bulk contribution to all the scalar masses has been considered \([7,8]\). In this case the ultraviolet (UV) sensitivity of the theory is no longer guaranteed. Our TeV scale family symmetry resembles somewhat the phenomenology of the universal bulk contributions, but differs from it in many crucial aspects. One common feature is that the neutral Wino is still the lightest supersymmetric particle, which is nearly mass degenerate with the lightest chargino. The possibilities to detect such a quasi-degenerate pair at the Tevarton Run 2 as well as at the LHC was considered in Refs. \([9]\). The possibility that the neutral Wino is the cold dark matter candidate has also been studied \([10]\). Other approaches to solving the problem include new TeV scale particle \([11]\), or interactions \([12]\), or \(D\)-term contributions from a new \(U(1)\) broken either at a high scale \([13,14]\) or at the weak scale \([15]\). Non-decoupling effects of heavy fields at higher orders have been studied \([16]\) in attempts to solve the tachyon problem. In the model presented here the UV insensitivity of AMSB scenario is maintained. There are a variety of ways in which our model can be tested in forthcoming experiments.

The plan of this paper is as follows. In section 2 we present our model based on \(SU(2)\) family symmetry. In section 3 we analyze the Higgs potential of the model. The SUSY spectrum is presented in section 4. We discuss our numerical results in section 5 for a specific choice of input parameters. Here we show explicitly the positivity of all slepton squared masses. In section 6 we discuss the experimental implication of the model of immediate interest. We give our conclusions in section 7.

### 2 \(SU(2)_F\) family symmetry

Our model, which is supersymmetric, is based on the gauge symmetry \(G \equiv SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(2)_F\). The matter fields of the model and their transformation properties under the gauge symmetries are listed in Table 1. Here \(SU(2)_F\) is a horizontal symmetry that acts on the first two families of lepton fields. The third family leptons
transform as singlets of $SU(2)_F$. This symmetry is broken by the vacuum expectation values of a pair of $SU(2)_F$ doublet Higgs fields $\{\phi_u, \phi_d\}$ which are singlets of the SM gauge symmetry. In the exact $SU(2)_F$ symmetric limit, the electron and the muon would be degenerate. In order to lift this degeneracy, we introduce a pair of $SU(2)_F$ singlet vector–like leptons $\{E, E^c\}$. The $\Psi_N$ field is introduced for the cancelation of $SU(2)_F$ Witten anomaly. Note that the quark fields are all singlets of $SU(2)_F$. This feature helps maintain asymptotic freedom of the $SU(2)_F$ interactions, a key ingredient for generating positive squared masses for the sleptons when the model is embedded in AMSB. Although we do not explicitly use them, we note that if a pair of $SU(2)_F$ singlet colored fields $(3, 1, -1/3) + (3^*, 1, 1/3)$ are added to the model, along with the $\{E, E^c\}$ fields they would form complete $5 + \overline{5}$ of $SU(5)$ and would lead to unification of the three gauge couplings associated with the SM gauge symmetries, just as in the MSSM.

It should be noted that the particle content of any SUSY $SU(2)_F$ model that is asymptotically free is highly constrained. The spectrum of Table 1 is one of the few possibilities. With the spectrum of Table 1, the $SU(2)_F$ gauge beta function turns out to be negative, $\beta_{g_F} = -3g_F^3/(16\pi^2)$.

We assume an unbroken $R$–parity as in the MSSM. We take the $R$–parity of the $\Psi_N$ field to be odd, similar to the SM fermions. The superpotential of the model consistent with the gauge symmetries reads as:

$$\begin{align*}
W &= (Y_u)_{ij} Q_i H_u u^c_j + (Y_d)_{ij} Q_i H_d d^c_j + f_\mu \psi^T i\tau_2 \psi^c H_d + f_\tau L_\tau \tau^c H_d \\
&+ f_{\tau E} L_\tau E^c H_d + f_{eE} E \psi^c T i\tau_2 \phi_d + \mu H_u H_d + \mu' \phi_u i\tau_2 \phi_d + M_{EE} E^c.
\end{align*}$$

Here we have suppressed the $SU(2)_L$ and $SU(3)_C$ indices, but have shown the $SU(2)_F$ contraction explicitly.

The following additional terms are allowed in the superpotential by the gauge symmetry:

$$W' = f_{eE} E \psi^c T i\tau_2 \phi_u + f_N \psi^T i\tau_2 \Psi_N H_u.$$  

5Adding $SU(2)_F$ triplet Higgs fields that are $SU(2)_L$ doublets is another option, but the $SU(2)_F$ beta function will then be positive, which we wish to avoid.

6It is conceivable that there is a separate $SU(2)_F$ acting on the quark fields, which would make the model compatible with quark–lepton symmetry. We assume that such a family symmetry, if present, is broken at a very high scale.
Table 1: Particle content and charge assignment of the model. The indices $i$ take values $i = 1 - 3$.

It turns out that the terms in $W'$ are not desirable. The first term in $W'$ would lead to unsuppressed flavor changing neutral currents ($\mu \to 3e$, $\mu \to e\gamma$ etc) via the exchange of $SU(2)_F$ gauge boson, if $SU(2)_F$ is broken at the TeV scale. The second term would generate Dirac neutrino masses for the SM neutrinos. $\Psi_N$ cannot have a large Majorana mass, as it is needed in the low energy theory for consistency. We observe that the terms in $W'$ can be eliminated consistently by making use of a $Z_4$ symmetry present in $W$ and not in $W'$. Under this $Z_4$ the fields with nontrivial transformation are:

$$\phi_u \to -i\phi_u, \; \phi_d \to i\phi_d, \; E \to -iE, \; E^c \to iE^c, \; L_\tau \to -iL_\tau, \; \tau^c \to i\tau^c, \; \Psi_N \to -\Psi_N.$$

Although smallness of the couplings in Eq. (2) is adequate for consistent phenomenology, we shall set $W'$ to zero by invoking this $Z_4$ symmetry.
The neutrinos in the model get masses from the following non-renormalizable operators (which we assume do not respect the $Z_4$ symmetry)$^7$

\[
\frac{L_L L_H H_u}{M^2}, \quad (\psi^T i\tau_2 \phi_{d,u}) L_L H_u \frac{H_u H_u}{M'^2}, \quad (\psi^T i\tau_2 \phi_{d,u})(\psi^T i\tau_2 \phi_{d,u}) \frac{H_u H_u}{M''^3}.
\] (4)

Consistent neutrino phenomenology can be realized if the mass parameters in Eq. (4) are of order $\{M, M', M''\} \sim \{10^{14}, 10^9, 10^7\}$ GeV. The fermionic components of $\Psi$ fields will acquire masses through effective operators of the form $(\Psi^T i\tau_2 \phi_{u,d})^2/M$, which are expected to be of order sub-eV.

3 Symmetry breaking

The symmetry breaking is assumed to follow the following hierarchical pattern:

\[
SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(2)_F \quad \xrightarrow{\langle \phi_u \rangle, \langle \phi_d \rangle} \quad SU(3)_C \times SU(2)_L \times U(1)_Y \quad \xrightarrow{\langle H_u \rangle, \langle H_d \rangle} \quad SU(3)_C \times U(1)_{EM}.
\]

The tree level Higgs potential can be written as

\[
V(H_u, H_d, \phi_u, \phi_d) = (m^2_{H_u} + \mu^2)|H_u|^2 + (m^2_{H_d} + \mu^2)|H_d|^2 + B \mu(H_u H_d + c.c.)
+ \frac{(g_2^2 + g_1^2)}{8}(|H_u|^2 - |H_d|^2)^2 + \frac{g_2^2}{2}|H_u H_d|^2 + \frac{g_F^2}{8}(|\phi_u|^2 - |\phi_d|^2)^2
+ \frac{g_F^2}{2}|\phi_u^T i\tau_2 \phi_d|^2 + (m^2_{\phi_u} + \mu^2)|\phi_u|^2 + (m^2_{\phi_d} + \mu^2)|\phi_d|^2
+ B' \mu'(\phi_u^T i\tau_2 \phi_d + c.c.).
\] (5)

The soft mass parameters $m^2_{H_u}$ and $m^2_{H_d}, m^2_{\phi_u}$ and $m^2_{\phi_d}$ are determined in terms of a single mass parameter $M_{aux}$ and the Yukawa and gauge couplings within AMSB. We present these masses in the Appendix, see Eqs. (63)-(66). The $B$ and $B'$ parameters are assumed to be a priori free in the model.$^8$ The Higgs fields acquire vacuum expectation values of the form

\[
\langle H_u \rangle = \begin{pmatrix} 0 \\ v_u \end{pmatrix}, \quad \langle H_d \rangle = \begin{pmatrix} v_d \\ 0 \end{pmatrix}, \quad \langle \phi_u \rangle = \begin{pmatrix} 0 \\ u_u \end{pmatrix}, \quad \langle \phi_d \rangle = \begin{pmatrix} u_d \\ 0 \end{pmatrix}.
\] (6)

$^7$If the $Z_4$ is broken in the $\nu_R$ sector, the breaking effects will show up only very weakly in the effective low energy theory.

$^8$In some special class of models, the $B$ parameters are determined in terms of $M_{aux}$ and the gamma functions of the Higgs fields. We do not assume these special values for $B$ and $B'$ here.
The desired symmetry breaking patterns can be achieved if the hierarchy $u_u, u_d \gg v_u, v_d$ can be realized.

Minimization of the Higgs potential $V$ leads to the following conditions:

$$
\sin 2\beta = \frac{-2B\mu}{2\mu^2 + m_{H_u}^2 + m_{H_d}^2}, \quad \mu^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{M_Z^2}{2}, \quad (7)
$$

$$
\sin 2\beta' = \frac{-2B'\mu'}{2\mu'^2 + m_{\phi_u}^2 + m_{\phi_d}^2}, \quad \mu'^2 = \frac{m_{\phi_d}^2 - m_{\phi_u}^2 \tan^2 \beta'}{\tan^2 \beta' - 1} - \frac{M_Z^2}{2}. \quad (8)
$$

Here we have introduced the notation $\tan \beta = v_u/v_d$, $\tan \beta' = u_u/u_d$, and $u^2 = u_u^2 + u_d^2$. $M_Z^2 = \frac{g^2}{2}(u_u^2 + u_d^2)$ is the common mass of the three gauge boson associated with $SU(2)_F$.

To find the physical Higgs boson masses, we parameterize the Higgs fields (in the unitary gauge) as

$$
H_u = \begin{pmatrix} H^+ \sin \beta \\ u_u + \frac{1}{\sqrt{2}}(\phi_2 + i \cos \beta \phi_3) \end{pmatrix}, \quad (H_d) = \begin{pmatrix} v_d + \frac{1}{\sqrt{2}}(\phi_1 + i \sin \beta \phi_3) \\ H^- \cos \beta \end{pmatrix},
\quad (9)
$$

$$
\phi_u = \begin{pmatrix} \phi' \sin \beta' \\ u_u + \frac{1}{\sqrt{2}}(\phi_4 + i \cos \beta' \phi_5) \end{pmatrix}, \quad \phi_d = \begin{pmatrix} u_d + \frac{1}{\sqrt{2}}(\phi_6 + i \sin \beta' \phi_5) \\ \phi'^* \cos \beta' \end{pmatrix}.
$$

The masses of the CP–odd Higgs bosons $\{\phi_3, \phi_5\}$ are

$$
m_A^2 = \frac{-2B\mu}{\sin 2\beta}, \quad m_{A'}^2 = \frac{-2B'\mu'}{\sin 2\beta'}. \quad (10)
$$

The mass matrices for the CP–even neutral Higgs bosons $\{\phi_1, \phi_2\}$ and $\{\phi_4, \phi_6\}$ are decoupled. They are given by

$$
(M^2)_{cp-even} = \begin{pmatrix} m_A^2 \cos^2 \beta + M_Z^2 \sin^2 \beta & -\{m_A^2 + M_Z^2\} \frac{\sin 2\beta}{2} \\ -\{m_A^2 + M_Z^2\} \frac{\sin 2\beta}{2} & m_A^2 \sin^2 \beta + M_Z^2 \sin^2 \beta \end{pmatrix}, \quad (11)
$$

$$
(M^2)_{cp-even} = \begin{pmatrix} m_A^2 \cos^2 \beta' + M_V^2 \sin^2 \beta' & -\{m_A^2 + M_V^2\} \frac{\sin 2\beta'}{2} \\ -\{m_A^2 + M_V^2\} \frac{\sin 2\beta'}{2} & m_A^2 \sin^2 \beta' + M_V^2 \sin^2 \beta' \end{pmatrix}. \quad (12)
$$

Finally, the masses of the charged Higgs boson $H^\pm$ and that of $\phi'$ are given by

$$
m_{H^\pm}^2 = m_A^2 + M_W^2, \quad m_{\phi'}^2 = m_{A'}^2 + M_V^2. \quad (13)
$$

The (complex) $\phi'$ fields are electrically neutral, but they are “charged” under $SU(2)_F$.
The Majorana mass matrix of the neutralinos \( \{ \tilde{B}, \tilde{W}_3, \tilde{H}_d^0, \tilde{H}_u^0, \tilde{B}_H, \tilde{\phi}_d^0, \tilde{\phi}_u^0 \} \) is found to be

\[
\mathcal{M}^{(0)} = \begin{pmatrix}
M_1 & 0 & -\frac{\nu}{\sqrt{2}} g_1 & \frac{\nu}{\sqrt{2}} g_1 & 0 & 0 & 0 \\
0 & M_2 & \frac{\nu}{\sqrt{2}} g_2 & -\frac{\nu}{\sqrt{2}} g_2 & 0 & 0 & 0 \\
-\frac{\nu}{\sqrt{2}} g_1 & \frac{\nu}{\sqrt{2}} g_2 & 0 & -\mu & 0 & 0 & 0 \\
\frac{\nu}{\sqrt{2}} g_1 & -\frac{\nu}{\sqrt{2}} g_2 & -\mu & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & M_2 & \frac{\nu}{\sqrt{2}} g_F & -\frac{\nu}{\sqrt{2}} g_F & 0 \\
0 & 0 & 0 & 0 & \frac{\nu}{\sqrt{2}} g_F & 0 & -\mu' \\
0 & 0 & 0 & 0 & -\frac{\nu}{\sqrt{2}} g_F & -\mu' & 0
\end{pmatrix},
\]

(14)

where \( M_1, M_2 \) and \( \tilde{M}_2 \) are the gaugino masses for \( U(1)_Y, SU(2)_L \) and \( SU(2)_F \) which are listed for the case of AMSB in Eq. (62) of Appendix A.4. The physical neutralino masses \( m_{\tilde{\chi}_i^0} \) \((i=1–7)\) are obtained as the eigenvalues of this mass matrix Eq. (14).

In the basis \( \{ \tilde{W}^+, \tilde{H}_u^+ \}, \{ \tilde{W}^-, \tilde{H}_d^- \} \), the chargino (Dirac) mass matrix is

\[
\mathcal{M}^{(c)} = \begin{pmatrix}
M_2 & g_2 v_d \\
2 & g_2 v_u \\
\mu & \mu
\end{pmatrix}.
\]

(15)

Similarly, for the \( SU(2)_F \) sector, we have

\[
\tilde{\mathcal{M}}^{(c)} = \begin{pmatrix}
\tilde{M}_2 & g_F u_d \\
g_F u_u & \mu'
\end{pmatrix}.
\]

(16)

The three \( SU(2)_F \) gauge boson masses are given by

\[
\mathcal{M}_V^2 = \frac{g_F^2}{2} (u_u^2 + u_d^2).
\]

(17)

### 3.1 Lepton masses

Now we describe how realistic lepton masses with \( m_e \neq m_\mu \) are generated in the model. We have introduced \( E \) and \( E^c \) fields in the superpotential Eq. (1) for the purpose of breaking \( e - \mu \) degeneracy. These new fields mix with the usual leptons leading to the mass matrix

\[
\mathcal{L}_{mass} = \begin{pmatrix}
e & \mu & \tau & E
\end{pmatrix} \begin{pmatrix}
f_\mu v_d & 0 & 0 & 0 \\
0 & f_\mu v_d & 0 & 0 \\
0 & 0 & f_\tau v_d & f_\tau E v_d \\
f_{eE} u_d & 0 & 0 & M_E
\end{pmatrix} \begin{pmatrix}
e^c \\
\mu^c \\
\tau^c \\
E^c
\end{pmatrix}.
\]

(18)
The muon field decouples from the rest of the leptons. This enables us to define an approximate muon number\(^9\) which guarantees that there is no excessive FCNC processes involving the muon (for which the experimental constraints are the most stringent). We are left with a \(3 \times 3\) mass matrix for the \(e, \tau\) and \(E\) fields. The eigenvalue equation can be solved using the hierarchy \(m_e \ll m_\tau \ll m_E\) and for \(\frac{\nu_d}{M_E} \ll 1\) (corresponding to large \(\tan \beta\)) with the result

\[
\begin{align*}
m_{\mu} &= f_\mu v_d \\
\frac{m_e}{m_\mu} &\approx \left[1 + \frac{f^2 e \nu^2}{M^2 E} \left(1 + \frac{f^2 \nu^2}{f^2 \tau}\right)\right]^{-\frac{1}{2}} \\
m_\tau &\approx f_\tau v_d \left[1 + \frac{f^2 \nu^2}{f^2 \tau} \frac{u^2}{M^2 E} + \frac{u^2}{M^2 E + f^2 e \nu^2} \right]^{\frac{1}{2}} [1 + \Delta_\tau] \equiv y_\tau v_d (1 + \Delta_\tau), \\
m_E &\approx \left[M^2 E + f^2 e \nu^2\right]^{\frac{1}{2}}. \tag{19}
\end{align*}
\]

Note that \(m_e \neq m_\mu\), showing consistency of the model. Although there is no flavor violation in the muon sector, violation of \(e\) and \(\tau\) lepton numbers do arise in the model. Owing to the violation of GIM mechanism in the left–handed lepton sector, there is a \(\bar{e} \tau Z\) coupling in the model. However, this coupling is of order \((m_\mu m_\tau / m^2_E) \sim 10^{-10}\), which is too small to be observed. In Section 5 we discuss lepton number violating \(\tau\) decays arising from \(e - \tau - E\) mixing and mediated by the \(SU(2)_F\) gauge bosons.

Since \(\tan \beta\) will turn out to be rather large in the model, finite SUSY loop correction arising through chargino and neutralino exchange are important for the \(\tau\) lepton mass \([17]\). These corrections are indicated in Eq. (19) as \(\Delta_\tau\). The tau mass corrections are dominated by diagrams involving exchange of the Bino/slepton and Higgsino/slepton. We use the following approximate expressions for \(\Delta_\tau\) in our numerical analysis \([18]\):

\[
\Delta_\tau \simeq \mu \tan \beta \left[\frac{\alpha^2}{8 \pi} M_2 (I(\mu^2, m^2_\tau, M^2_2) + 2I(\mu^2, m^2_\tau, M^2_1)) + \frac{3\alpha_1}{20 \pi} M_1 I(m^2_\tau, m^2_\tau, M^2_1) - \frac{3\alpha_1}{40 \pi} M_1 (I(\mu^2, M^2_1) - 2I(\mu^2, m^2_\tau, M^2_2))\right]. \tag{20}
\]

Here the function \(I\) is defined as

\[
I(m^2_1, m^2_2, m^2_3, \ldots) = \frac{1}{m^2_3} \left[\frac{x \ln x}{1 - x} - \frac{y \ln y}{1 - y}\right] \frac{1}{x - y}, \tag{21}
\]

\(^9\)since the \(Z_4\) symmetry is broken in the \(\nu_R\) sector, muon number is only an approximate symmetry in the model.
with \( x = m_1^2/m_3^2 \) and \( y = m_2^2/m_3^2 \).

The corrected running bottom quark mass is given by [17]

\[
m_b = y_b v_d (1 + \Delta_b).
\]

Here \( \Delta_b \) is the finite SUSY loop correction arising from the exchange of gluino and charginos and is given by

\[
\Delta_b \simeq \mu \tan \beta \left[ \frac{2 \alpha_3}{3 \pi} M_3 I(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, M_3^2) + \frac{y_t^2}{16 \pi^2} A_4 I(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2, \mu^2) \right].
\]

## 4 The SUSY spectrum

In this section we present the results for the masses of the SUSY scalars. We will show that the tachyonic slepton problem of AMSB is cured by virtue of the positive contribution from the \( SU(2)_F \) gauge sector to the masses for the first two families. For the \( \tau \) family sleptons, contributions arising from the Yukawa coupling, which are significant for large \( \tan \beta \), render the squared masses positive.

### 4.1 Slepton masses

The slepton masses are obtained from a \( 2 \times 2 \) mass matrix for the \( (\tilde{\mu}, \tilde{\mu}^c) \) sector (since the muon family decouples from the rest of the sleptons) and from a \( 6 \times 6 \) mass matrix for the \( \{e, \tau, E, E^c, \tau^c, E^c\} \) fields. For the scalar muons the mass matrix is

\[
M_{\mu}^2 = \begin{pmatrix}
m_{\tilde{\mu}}^2 & m_{\mu} (A_{f_{\mu}} - \mu \tan \beta) \\
m_{\mu} (A_{f_{\mu}} - \mu \tan \beta) & m_{\tilde{\mu}}^2 \\
\end{pmatrix},
\]

where the diagonal entries in the AMSB scheme are

\[
m_{\tilde{\mu}}^2 = \frac{M_{\mu}^2}{16\pi^2} \left[ 2 f_{\mu} \beta(g_{\mu}) - \left( \frac{3}{2} g_2 \beta(g_2) + \frac{3}{10} g_1 \beta(g_1) + \frac{3}{2} g_F \beta(g_F) \right) \right] + m_{\mu}^2 + \frac{g_F^2}{4} (u_u^2 - u_d^2),
\]

\[
m_{\tilde{\mu}}^c = \frac{M_{\mu}^2}{16\pi^2} \left[ 2 f_{\mu} \beta(g_{\mu}) - \left( \frac{6}{5} g_1 \beta(g_1) + \frac{3}{2} g_F \beta(g_F) \right) \right] + m_{\mu}^2 + \frac{g_F^2}{4} (u_d^2 - u_u^2).
\]

Note the positive contributions arising from the \( SU(2)_F \) gauge sector given by the term \( -\frac{3}{2} g_F \beta(g_F) \), with gauge beta function \( \beta(g_F) = -\frac{3}{16\pi^2} g_F^3 \). For \( g_F \geq 0.9 \), we find the squared masses of all sleptons can be positive. It is important to point out that the \( SU(2)_F \) D–term contribution to the diagonal entries of the mass matrix Eq. (24) is positive for one
slepton and negative for the other. Consistency demands that this contribution be rather small compared to the other terms. Thus \( \tan^2 \beta \) is required in the model.

The mass matrix for the sleptons other than the scalar muons has the form

\[
\mathcal{M}^2 = \begin{pmatrix}
    m_\tilde{e}^2 & 0 & f_{e\mu} f_{\tilde{e} E} v_d u_d & A & 0 & 0 \\
    0 & m_\tilde{\tau}^2 & M_{E \tilde{\tau} E} v_d & 0 & B & C \\
    f_{e\mu} f_{\tilde{e} E} v_d u_d & M_{E \tilde{\tau} E} v_d & \tau^2 & D & 0 & M_{E \tilde{E} E} \\
    A & 0 & D & m_\tilde{\tau}^2 & 0 & M_{E \tilde{E} E} \\
    0 & B & 0 & 0 & m_\tilde{\tau}^2 & f_{\tau \tilde{E} E} v_d^2 \\
    0 & C & M_{E \tilde{E} E} & M_{E \tilde{E} E} & f_{\tau \tilde{E} E} v_d^2 & m_\tilde{E}^2 \\
\end{pmatrix}, \quad (26)
\]

where we have defined

\[
m_\tilde{e}^2 = \frac{M_{aux}^2}{(16\pi^2)} \left[ 2 f_{e\mu} \beta (f_{e\mu}) - \left( \frac{3}{2} g_2 \beta (g_2) + \frac{3}{10} g_1 \beta (g_1) + \frac{3}{2} g_F \beta (g_F) \right) \right] + f_{e\mu}^2 v_d^2 + \frac{g_F^2}{4} (u_d^2 - u_u^2),
\]

\[
m_\tilde{\tau}^2 = \frac{M_{aux}^2}{(16\pi^2)} \left[ 2 f_{e\mu} \beta (f_{e\mu}) - \left( \frac{3}{2} g_2 \beta (g_2) + \frac{3}{10} g_1 \beta (g_1) + \frac{3}{2} g_F \beta (g_F) \right) \right] + \frac{f_{e\mu}^2 v_d^2 + f_{\tilde{e} E}^2 u_u^2 + \frac{g_F^2}{4} (u_u^2 - u_d^2)}{4}.
\]

\[
m_\tilde{\tau}^2 = \frac{M_{aux}^2}{(16\pi^2)} \left[ 2 f_{e\mu} \beta (f_{e\mu}) - \left( \frac{3}{2} g_2 \beta (g_2) + \frac{3}{10} g_1 \beta (g_1) + \frac{3}{2} g_F \beta (g_F) \right) \right] + \frac{f_{e\mu}^2 v_d^2 + f_{\tilde{e} E}^2 u_u^2 + \frac{g_F^2}{4} (u_u^2 - u_d^2)}{4}.
\]

\[
m_\tilde{E}^2 = \frac{M_{aux}^2}{(16\pi^2)} \left[ 2 f_{e\mu} \beta (f_{e\mu}) - \left( \frac{3}{2} g_2 \beta (g_2) + \frac{3}{10} g_1 \beta (g_1) + \frac{3}{2} g_F \beta (g_F) \right) \right] + \frac{f_{e\mu}^2 v_d^2 + f_{\tilde{E} E}^2 u_u^2 + \frac{g_F^2}{4} (u_u^2 - u_d^2)}{4}.
\]

\[
A = f_{e\mu} (A_{e\mu} v_d + \mu v_u)
\]

\[
B = f_{\tau} (A_{\tau} v_d + \mu v_u)
\]

\[
C = f_{\tau} (A_{\tau} v_d + \mu v_u)
\]

\[
D = f_{\tau} (A_{\tau} v_d + \mu v_u)
\].

The requirement that the slepton masses are positive puts constraints on the couplings \( f_{\tau}, f_{eE}, f_{\tau e} \). We find \( f_{\tau}, f_{eE}, f_{\tau E} \geq 0.5 \) are needed.

The \( \Psi_N \) scalar masses are give by

\[
m_{\tilde{N}_1} = \frac{M_{aux}^2}{(16\pi^2)} \left[ \frac{3}{2} g_F \beta (g_F) \right] + \frac{g_F^2}{4} (u_d^2 - u_u^2)
\]
\[ m_{N_2} = \frac{M_{aux}^2}{(16\pi^2)} \left[ -\frac{3}{2} g_F (g_F) + \frac{g_F^2}{4} (u_v^2 - u_u^2) \right]. \]  

(28)

4.2 Squark masses

The mixing matrix for the squark sector is identical to the usual MSSM with no contributions from the $SU(2)_c$ sector. The diagonal entries of the up and the down squark mass matrices are given by

\[
m_{U_i}^2 = (m_{soft}^2)_{U_i} + m^2_{U_i} + \frac{1}{6} (4M_W^2 - M_Z^2) \cos 2\beta, \\
m_{U_i}^2 = (m_{soft}^2)_{U_i} + m^2_{U_i} - \frac{2}{3} (M_W^2 - M_Z^2) \cos 2\beta, \\
m_{U_i}^2 = (m_{soft}^2)_{D_i} + m^2_{D_i} + \frac{1}{6} (2M_W^2 + M_Z^2) \cos 2\beta, \\
m_{D_i}^2 = (m_{soft}^2)_{D_i} + m^2_{D_i} + \frac{1}{3} (M_W^2 - M_Z^2) \cos 2\beta,
\]  

(29)

were $m_{U_i}$ and $m_{D_i}$ are the quark masses of the different generations with $i = 1, 2, 3$. The soft masses are obtained in AMSB from the RGE as

\[
(m_{soft}^2)_{U_i} = \frac{M_{aux}^2}{16\pi^2} \left( Y_{u_i} \beta(Y_{u_i}) + Y_{d_i} \beta(Y_{d_i}) - \frac{1}{30} g_1 \beta(g_1) - \frac{3}{2} g_2 \beta(g_2) - \frac{8}{3} g_3 \beta(g_3) \right), \]  

(30)

\[
(m_{soft}^2)_{U_i} = \frac{M_{aux}^2}{16\pi^2} \left( 2Y_{u_i} \beta(Y_{u_i}) - \frac{8}{15} g_1 \beta(g_1) - \frac{8}{3} g_3 \beta(g_3) \right), \]  

(31)

\[
(m_{soft}^2)_{D_i} = \frac{M_{aux}^2}{16\pi^2} \left( 2Y_{d_i} \beta(Y_{d_i}) - \frac{2}{15} g_1 \beta(g_1) - \frac{8}{3} g_3 \beta(g_3) \right). \]  

(32)

The RGE for quark sector Yukawa couplings are listed in Appendix A.2.

5 Numerical results

Here we present our numerical results for the SUSY spectrum. Our analysis follows the procedure of Ref. [6]. The input values of the SM gauge couplings [19] used are:

\[
\alpha_{EM}^{-1}(M_Z) = 128.91 \pm 0.02, \\
\sin^2 \theta_W(M_Z) = 0.23120 \pm 0.00015, \\
\alpha_3(M_Z) = 0.1182 \pm 0.0027.
\]  

(33)

We extrapolate these couplings to $M_{SUSY} \approx 1$ TeV using the SM renormalization group equations. We use the central value of the top mass taken to be $M_t = 174.3$ GeV.
The scale of SUSY breaking, \( M_{\text{aux}} \), should be in the range \( 40 - 100 \) TeV in order for the sparticle masses to be in the range \( 0.2 - 2 \) TeV. Since the positivity of the mass–squared of the slepton of the third family depends on the Yukawa couplings, we find that the couplings should obey \( f_\tau, f_\tau E, f_\epsilon E \geq 0.5 \). This will lead to a large value of \( \tan \beta \geq 40 \).

For the positivity of the first two family slepton masses, the \( SU(2)_F \) gauge coupling should obey \( g_F \geq 0.9 \).

The parameters of the model are highly constrained. Besides the positivity of the slepton squared masses, one should ensure that the lightest Higgs boson mass is above the current experimental limit, \( m_H \geq 114 \) GeV. Furthermore, symmetry breaking should be consistently achieved with the hierarchy \( u_u \sim u_d \gg v_u \gg v_d \). This hierarchy is needed to guarantee that the \( SU(2)_F \) gauge bosons are heavier than the \( W \) and the \( Z \) bosons.

We have not performed a systematic parameter search within the model. By performing a “spot search” we were able to find consistent solutions. If we “move around” a solution, we found that the solution quickly disappears. This feature indicates that the model is highly predictive.

The first two family fermion masses do not play any significant role in our fit, but the third family fermion masses do. We choose \( m_\tau(m_\tau) = 1.777 \) GeV as an input. The running mass \( m_\tau(M_{\text{SUSY}}) = 1.769 \) GeV. The input value of \( b \)-quark mass is taken to be \( m_b(m_b) = 4.8 \) GeV, corresponding to \( m_b(M_{\text{SUSY}}) = 2.8 \) GeV. In computing \( \tau \) and \( b \) masses we include the finite SUSY loop corrections as noted in Eqs. (20) and (23).

We present a specific fit in Table 2. The parameters used for the fit are indicated in the Table caption. As shown in the Table, all slepton squared masses are positive. An interesting feature of the model is that the \( \tilde{e} \) and \( \tilde{\mu} \) are not nearly degenerate. For example, \( m_{\tilde{e}} = 538 \) GeV, while \( m_{\tilde{\mu}} = 834 \) GeV. Non-degenerate sleptons is a characteristic feature of our model.

The lightest neutral Higgs boson mass is \( m_H = 120 \) GeV in our fit. This is obtained after including the leading one–loop and two–loop radiative corrections for \( m_H \). We follow the procedure outlined in Ref. [20]. Since the entire SUSY spectrum is relatively heavy, including the pseudoscalar Higgs boson, we decouple all SUSY particles at \( M_{\text{SUSY}} = 1 \) TeV, and use the SM RGE to compute the evolution of the Higgs quartic coupling between 1 TeV and \( m_t \). There is a light scalar \( m'_H = 86 \) GeV, but this is mostly from the \{\phi_u, \phi_d\} sector and has very weak couplings to the SM fermions and gauge bosons.
The lightest supersymmetric particle is found to be the neutral Wino which is nearly mass degenerate with one of the charginos. (The mass splitting between the two is about 230 MeV.) The three $SU(2)_F$ gauge bosons have a common mass is found to be $\sim 1.91$ TeV with this set of input parameters. The heavy Higgs bosons, Higgsinos and squarks masses are in the range $0.6 - 2.0$ TeV.

| Particles                        | Symbol                  | Mass (TeV)          |
|----------------------------------|-------------------------|---------------------|
| Neutralinos                      | $\{m_{\tilde{\chi}^0_1}, m_{\tilde{\chi}^0_2}, m_{\tilde{\chi}^0_3}, m_{\tilde{\chi}^0_4}\}$ | $\{0.149, 0.235, 0.614, 0.912\}$ |
| Neutralinos                      | $\{m_{\tilde{\chi}^0_1}, m_{\tilde{\chi}^0_2}, m_{\tilde{\chi}^0_3}\}$ | $\{0.917, 1.593, 2.452\}$ |
| Charginos                        | $\{m_{\tilde{\chi}^\pm_1}, m_{\tilde{\chi}^\pm_2}\}$ | $\{0.149, 0.915\}$ |
| Charginos ($SU(2)_F$)            | $\{m_{\tilde{\chi}^\pm_1}, m_{\tilde{\chi}^\pm_2}\}$ | $\{1.585, 2.457\}$ |
| Gluino                           | $M_3$                   | 1.319               |
| Neutral Higgs bosons             | $\{m_h, m_H, m_A\}$     | $\{0.120, 0.910, 0.910\}$ |
| Neutral Higgs bosons             | $\{m_{H'}, m_{H''}, m_{A'}\}$ | $\{0.086, 1.318, 2.319\}$ |
| Charged Higgs bosons             | $m_{H^\pm}$             | 0.913               |
| Charged Higgs bosons $SU(2)_F$   | $m_{H^\pm}$             | 2.321               |
| R.H smuon                        | $\{m_{\tilde{\mu}_1}\}$ | $\{0.834\}$         |
| L.H smuon                        | $\{m_{\tilde{\mu}_2}\}$ | $\{0.628\}$         |
| R.H sleptons                     | $\{m_{\tilde{\tau}_R}, m_{\tilde{\tau}_L}, m_{\tilde{E}_R}\}$ | $\{0.538, 0.207, 2.522\}$ |
| L.H sleptons                     | $\{m_{\tilde{\tau}_L}, m_{\tilde{\tau}_R}, m_{\tilde{E}_L}\}$ | $\{0.636, 0.401, 1.018\}$ |
| Snuetrinos                       | $\{m_{\tilde{\nu}_e}, m_{\tilde{\nu}_\mu}, m_{\tilde{\nu}_\tau}\}$ | $\{0.636, 0.834, 0.414\}$ |
| Scalar $\Psi_N$                 | $\{m_{N_1}, m_{N_2}\}$ | $\{0.673, 0.863\}$ |
| R.H down squarks                 | $\{m_{\tilde{d}_R}, m_{\tilde{d}_L}, m_{b_L}\}$ | $\{1.241, 1.241, 1.171\}$ |
| L.H down squarks                 | $\{m_{\tilde{u}_L}, m_{\tilde{u}_R}, m_{b_L}\}$ | $\{1.231, 1.231, 1.016\}$ |
| R.H up squarks                   | $\{m_{\tilde{t}_R}, m_{\tilde{e}_R}, m_{\tilde{b}_L}\}$ | $\{1.233, 1.233, 0.946\}$ |
| L.H up squarks                   | $\{m_{\tilde{t}_L}, m_{\tilde{e}_L}, m_{\tilde{b}_L}\}$ | $\{1.228, 1.228, 1.133\}$ |
| $SU(2)_F$ gauge boson            | $M'_Z$                  | 1.910               |

Table 2: Partiacle masses in Model 1 for the choice $M_{aux} = 57.605$ TeV, $y_b = 0.95$, $f_\tau = 0.55$, $f_e = 1.2$, $f_{\tau e} = 0.53$, $g_F = 1.0$, $M_E = 0.0149$ TeV and $M_L = 0.1743$ TeV, $u = 2.702$ TeV, $\tan \beta = 58.2$, $\tan \beta' = 1.08$, $\mu = -0.908$ TeV, $\mu' = 0.236$ TeV, $B = 0.016$ TeV, $B' = -3.676$ TeV, $B_E = 0.007$ TeV.
6 Experimental implications

In this section we list the salient experimental signatures of the model.

(i) Non-degeneracy of the first two family sleptons is a characteristic feature of our model. This is unlike most models of supersymmetry breaking. The origin of this splitting can be traced back to the Yukawa couplings and the $SU(2)_F$ $D$-terms.

(ii) The model predicts large value of $\tan \beta \geq 40$. There are observable experimental consequences, which will be discussed below.

(iii) Three degenerate vector gauge bosons with masses of order TeV are predicted by the model. These gauge bosons do not mix with the $Z$ boson, nor do they couple to quarks. Experimental discovery of these bosons will be hard at a hadron collider, but should be easy at a lepton collider. Electroweak precision observables are left intact by the new gauge sector, since there is no $Z-V$ mixing. As noted earlier, the presence of an approximate $Z_4$ symmetry in the model prevents $\mu \to 3e$ and $\mu \to e\gamma$ decays that could have been mediated by the $V$ gauge bosons. For the fit given in Table 2, the $SU(2)_F$ gauge bosons are degenerate with a mass $M_V = 1.910$ TeV. The most stringent constraint on $M_V$ arises from the process $e^+e^- \to \mu^+\mu^-$. LEP II has set severe constraints on lepton compositeness [21, 19] from this process. For $\Lambda (ee\mu\mu) > 9.5$ TeV [21, 19], we obtain the limit $M_V > 1.6$ TeV (for $g_F = 1.0$). This limit is satisfied in our model.

(iv) Because $\tau$ and $e$ lepton numbers are not conserved in the model, one would expect decays such as $\tau \to 3e$ and $\tau \to e\mu^+\mu^-$. These are mediated by the $V$ gauge bosons. Note that the leptonic mass matrix of Eq. (18) has both $e-E$ and $\tau-E$ mixings. We denote by $\theta_{e\tau}^{L,R}$ the $(e, \tau)$ entry of the matrix $O_{L,R}^T \text{diag}[1, -1, 0, 0]. O_{L,R}$, where $O_{L}^T M_{l} O_{R} = M_{l}^{\text{diag}}$. For our fit, these mixing angles are found to be $\theta_{e\tau}^{L} = -2.1 \times 10^{-4}$ and $\theta_{e\tau}^{R} = -2.3 \times 10^{-3}$. Since $\theta_{e\tau}^{L}$ is an order of magnitude larger than $\theta_{e\tau}^{R}$, we ignore the latter and obtain the following approximate expressions for the decay rates $\Gamma(\tau \to 3e)$ and $\Gamma(\tau \to \mu^+\mu^-)$:

\[ \Gamma(\tau \to 3e) \approx \frac{3}{8(192\pi^3)} \frac{g_F^4 m_{\tau}^3}{16 M_V^4} |\theta_{e\tau}^{L}|^2, \]
\[ \Gamma(\tau \to \mu^+\mu^-) \approx \frac{1}{(192\pi^3)} \frac{g_F^4 m_{\tau}^3}{16 M_V^4} |\theta_{e\tau}^{L}|^2. \]  

We find the Branching ratios $Br(\tau \to 3e) \approx 6.9 \times 10^{-14}$ and $Br(\tau \to \mu^+\mu^-) \approx 1.84 \times 10^{-13}$. These are clearly well below the current experimental sensitivity.

(v) The decay $\tau \to e\gamma$ is mediated by SUSY scalar exchange. The dominant contri-
bution to this decay amplitude arises from the exchange of Bino and sleptons. The rate for the decay is given by
\[
\Gamma(\tau \rightarrow e\gamma) \simeq \frac{\alpha_1}{4} m_\tau^3 \left| \frac{3\alpha_1}{20\pi m_\tau^2} m_B F(m_B^2/m_\tau^2)(\delta_{LR})^2 \right|^2 \tag{36}
\]
where (in the standard notation) \(\delta_{LR}^L = \delta_{LR}^R \times \delta_{RR}^L\) which is found to be 0.044 (0.719 \times 0.061) in our model and
\[
F(x) = \frac{(1 + 4x - 5x^2 + 4x \ln(x) + 2x^2 \ln(x))}{2(1-x)^4} \tag{37}
\]
We find the branching ratio of \(Br(\tau \rightarrow e\gamma) = 3.74 \times 10^{-8}\) which is very close to the current experimental limit and within reach of future experiments.

(vi) Since \(\tan \beta\) is large, the Higgs boson mediated decay \(B_s \rightarrow \mu^+\mu^-\) has a large rate. To estimate this we follow the analysis of Ref. \cite{23}.
\[
BR(B^0 \rightarrow \mu^+\mu^-) \simeq \frac{\eta_{QCD}^2 m_B^3}{64\pi} f_B^2 g_y^2 y_t^2 |V_{t(d,s)}^* V_{\mu t}|^2 \chi_{FC}^2, \tag{38}
\]
where \(\chi_{FC}\) is given by
\[
\chi_{FC} = \frac{-\epsilon_u y_t^2 \tan \beta}{(1 + \epsilon_g \tan \beta)[1 + (\epsilon_g + \epsilon_u y_t^2) \tan \beta]}, \tag{39}
\]
\(\epsilon_g\) and \(\epsilon_u\) are given by
\[
\epsilon_u = \mu \tan \beta \left[ \frac{2\alpha_3}{3\pi} M_A I(m_{b_1}^2, m_{b_2}^2, M_3^2) \right],
\]
\[
\epsilon_g = \mu \tan \beta \left[ \frac{y_t^2}{16\pi^2} A_t I(m_{t_1}^2, m_{t_2}^2, \mu^2) \right]. \tag{40}
\]
For the fit of Table 2, we find \(\epsilon_g = -8.9 \times 10^{-3}\) and \(\epsilon_u = -2.1 \times 10^{-3}\) which leads to \(\chi_{FC} = 0.427\). Consequently, the branching ratio \(BR(B^0 \rightarrow \mu^+\mu^-) \simeq 7.1 \times 10^{-8}\) (with \(\eta_{QCD} = 1.5\)), and using analogous expressions, \(BR(B^0_d \rightarrow \mu^+\mu^-) \simeq 2.8 \times 10^{-9}\). These decays are within reach of ongoing experiments at the Tevatron and/or the LHC.

(vii) The lightest R–odd SUSY particle in the model is the neutral Wino (\(\tilde{\chi}_0^1\)) which is nearly mass degenerate with the chargino. \(\tilde{\chi}_1^0\) is stable and can be a candidate for cold dark matter \cite{10}.

7 Conclusion

In this paper we have presented a realistic supersymmetric model based on a gauged \(SU(2)\) family symmetry for the leptons. The \(SU(2)\) symmetry is broken at the TeV
scale along with supersymmetry. We have shown how such a scenario can be made phenomenologically consistent.

In the context of anomaly mediated SUSY breaking, the model presented provides a simple solution to the tachyonic slepton problem. Just as the color interactions make the squared masses of squarks positive, the $SU(2)_F$ interactions make the slepton squared masses positive. A large value of $\tan \beta \geq 40$ is predicted by the model, as needed for the positivity of the third family slepton masses.

An intriguing feature of the model is that, although the first two family sleptons are degenerate in mass in the $SU(2)_F$ symmetric limit, symmetry breaking effects render them non–degenerate. This is one of the few models where a non–degeneracy of first two family sleptons is observed. The SUSY spectrum is relatively heavy with masses spanning the range 500 GeV - 2 TeV for most particles. The lightest $R$–odd particle is the neutral Wino, which is a candidate for cold dark matter.

Other salient features of the model include observable rates for $\tau \rightarrow e\gamma$ and $B_s \rightarrow \mu^+\mu^-$ decays.

A Appendix

In this Appendix we give the one-loop anomalous dimensions for the matter fields, beta-function for the gauge and Yukawa couplings and for the soft SUSY breaking masses in the $SU(2)_F$ symmetric model.

A.1 Anomalous dimensions

The one–loop anomalous dimensions for the various matters fields in our model are:

$$16\pi^2\gamma_\psi = f_{e\mu}^2 - \left( \frac{3}{10} g_1^2 + \frac{3}{2} g_2^2 + \frac{3}{2} g_F^2 \right),$$

$$16\pi^2\gamma_{\psi^c} = 2f_{e\mu}^2 + f_{eE}^2 - \left( \frac{6}{5} g_1^2 + \frac{3}{2} g_F^2 \right),$$

$$16\pi^2\gamma_{L_\tau} = f_{\tau}^2 + f_{\tau E}^2 - \left( \frac{3}{10} g_1^2 + \frac{3}{2} g_2^2 \right),$$

$$16\pi^2\gamma_{\tau^c} = 2f_{\tau}^2 - \frac{6}{5} g_1^2,$$

$$16\pi^2\gamma_{Q_{ij}} = (Y_d Y_d^\dagger)_{ji} + (Y_u Y_u^\dagger)_{ji} - \delta_{ij} \left( \frac{1}{30} g_1^2 + \frac{3}{2} g_2^2 + \frac{8}{3} g_3^2 \right),$$
\begin{align*}
16\pi^2 \gamma_{U_{ij}} &= 2(Y_u^i Y_u^j) - \delta_i^j \left( \frac{8}{15} g_1^2 + \frac{8}{3} g_2^2 \right), \\
16\pi^2 \gamma_{D_{ij}} &= 2(Y_d^i Y_d^j) - \delta_i^j \left( \frac{2}{15} g_1^2 + \frac{8}{3} g_2^2 \right), \\
16\pi^2 \gamma_{H_d} &= 3Y_b^2 + 4 f_{\nu\mu}^2 + f_{\tau E}^2 + f_{\tau}^2 - \frac{3}{10} g_1^2 - \frac{3}{2} g_2^2, \\
16\pi^2 \gamma_{H_u} &= 3Y_t^2 - \frac{3}{10} g_1^2 - \frac{3}{2} g_2^2, \\
16\pi^2 \gamma_{\phi_{d}} &= f_{\nu E}^2 - \frac{3}{2} g_F^2, \\
16\pi^2 \gamma_{\phi_{u}} &= -\frac{3}{2} g_F^2, \\
16\pi^2 \gamma_{E} &= 2f_{\nu E}^2 - \frac{6}{5} g_1^2, \\
16\pi^2 \gamma_{E^c} &= 2f_{\tau E}^2 - \frac{6}{5} g_1^2.
\end{align*}

\section*{A.2 Beta functions}

The beta functions for the Yukawa couplings appearing in the superpotential, Eq. (4), are:

\begin{align*}
\beta(Y_b) &= \frac{Y_b}{16\pi^2} \left( 6Y_b^2 + Y_t^2 + f_{\tau}^2 + f_{\tau E}^2 + 4f_{\nu\mu}^2 - \frac{13}{15} g_1^2 - 3g_2^2 - \frac{16}{3} g_3^2 \right), \\
\beta(Y_t) &= \frac{Y_t}{16\pi^2} \left( 6Y_t^2 + Y_b^2 - \frac{13}{15} g_1^2 - 3g_2^2 - \frac{16}{3} g_3^2 \right), \\
\beta(Y_{\tau}) &= \frac{Y_{\tau}}{16\pi^2} \left( 4Y_{\tau}^2 + 3Y_b^2 + 2f_{\tau E}^2 + 2f_{\nu\mu}^2 - \frac{9}{5} g_1^2 - 3g_2^2 \right), \\
\beta(f_{\nu E}) &= \frac{f_{\nu E}}{16\pi^2} \left( 4f_{\nu E}^2 + 2f_{\nu\mu}^2 - \frac{12}{5} g_1^2 - 3g_2^2 \right), \\
\beta(f_{\tau E}) &= \frac{f_{\tau E}}{16\pi^2} \left( 4f_{\tau E}^2 + 2f_{\tau}^2 + 4f_{\nu\mu}^2 + 3Y_b^2 - \frac{9}{5} g_1^2 - 3g_2^2 \right), \\
\beta(f_{\nu\mu}) &= \frac{f_{\nu\mu}}{16\pi^2} \left( 7f_{\nu\mu}^2 + 2f_{\tau E}^2 + 2f_{\tau}^2 + 2f_{\nu E}^2 + 3Y_b^2 - \frac{9}{5} g_1^2 - 3g_2^2 - 3g_F^2 \right).
\end{align*}

The gauge beta function of the model are

\[
\beta(g_i) = b_i \frac{g_i^3}{16\pi^2},
\]

where \(b_i = \left( \frac{39}{5}, 1, -3, -3 \right)\) for \(i = 1 - 4\) with \(g_F\) being the gauge coupling associated with the \(SU(2)_F\) gauge group.
A.3 $A$ terms

The trilinear soft SUSY breaking terms are given by

$$A_Y = -\frac{\beta(Y)}{Y} M_{\text{aux}}, \quad (61)$$

where $Y = (Y_u, Y_d, Y_e, f_{\tau E}, f_{\tau})$.

A.4 Gaugino masses

The soft masses of the gauginos are given by:

$$M_i = \frac{\beta(g_i)}{g_i} M_{\text{aux}}, \quad (62)$$

where $i = 1, 2, 3, 4$, corresponding to the gauge groups $U(1)_Y, SU(2)_L, SU(3)_C, SU(2)_F$ with $\beta(g_i)$ given as in Eq. (60).

A.5 Soft SUSY masses

The soft masses of the squarks and the sleptons are given in the text. For the $H_u, H_d, \nu^c, S_+, S_-$ fields they are:

$$\langle \tilde{m}_{\text{soft}}^2 \rangle_{H_u} = \frac{M_{\text{aux}}^2}{16\pi^2} \left( 3Y_i \beta(Y_i) - \frac{3}{10} g_1 \beta(g_1) - \frac{3}{2} g_2 \beta(g_2) - 2 \left( \frac{x}{2} \right)^2 g_F \beta(g_F) \right), \quad (63)$$

$$\langle \tilde{m}_{\text{soft}}^2 \rangle_{H_d} = \frac{M_{\text{aux}}^2}{16\pi^2} \left( 3Y_h \beta(Y_h) + Y_\tau \beta(Y_\tau) + Y_{\tau E} \beta(Y_{\tau E}) - \frac{3}{10} g_1 \beta(g_1) - \frac{3}{2} g_2 \beta(g_2) - 2 \left( -\frac{x}{2} \right)^2 g_F \beta(g_F) \right), \quad (64)$$

$$\langle \tilde{m}_{\text{soft}}^2 \rangle_{\phi_u} = \frac{M_{\text{aux}}^2}{16\pi^2} \left( -\frac{3}{2} g_F \beta(g_F) \right), \quad (65)$$

$$\langle \tilde{m}_{\text{soft}}^2 \rangle_{\phi_d} = \frac{M_{\text{aux}}^2}{16\pi^2} \left( f_{\tau E} \beta(f_{\tau E}) - \frac{3}{2} g_F \beta(g_F) \right). \quad (66)$$

The soft mass parameters of the sleptons and the squarks are given in the text.

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