Comparison of coherently coupled multi-cavity and quantum dot embedded single cavity systems

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Abstract: Temporal group delays originating from the optical analogue to electromagnetically induced transparency (EIT) are compared in two systems. Similar transmission characteristics are observed between a coherently coupled high-Q multi-cavity array and a single quantum dot (QD) embedded cavity in the weak coupling regime. However, theoretically generated group delay values for the multi-cavity case are around two times higher. Both configurations allow direct scalability for chip-scale optical pulse trapping and coupled-cavity quantum electrodynamics (QED).

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References and links
1. S. Kocaman, X. Yang, J. F. McMillan, M. B. Yu, D. L. Kwong, and C. W. Wong, “Observations of temporal group delays in slow-light multiple coupled photonic crystal cavities,” Appl. Phys. Lett. 96(22), 221111 (2010).
2. M. D. Lukin and A. Imamoglu, “Controlling photons using electromagnetically induced transparency,” Nature 413(6853), 273–276 (2001).
3. L. V. Hau, S. E. Harris, Z. Dutton, and C. H. Behroozi, “Light speed reduction to 17 metres per second in an ultracold atomic gas,” Nature 397(6720), 594 (1999).
4. J. J. Longdell, E. Fraval, M. J. Sellars, and N. B. Manson, “Stopped light with storage times greater than one second using electromagnetically induced transparency in a solid,” Phys. Rev. Lett. 95(6), 063601 (2005).
5. M. F. Yanik, W. Suh, Z. Wang, and S. Fan, “Stopping light in a waveguide with an all-optical analog of electromagnetically induced transparency,” Phys. Rev. Lett. 93(23), 233903 (2004).
6. D. D. Smith, H. Chang, K. A. Fuller, A. T. Rosenberger, and R. W. Boyd, “Coupled-resonator-induced transparency,” Phys. Rev. A 69(6), 063804 (2004).
7. G. Lenz, B. J. Eggleton, C. K. Madsen, and R. E. Slusher, “Optical delay lines based on optical filters,” IEEE J. Quantum Electron. 37(4), 525–532 (2001).
8. T. Gu, S. Kocaman, X. Yang, J. F. McMillan, M. Yu, G. Q. Lo, D. L. Kwong, and C. W. Wong, “Deterministic integrated tuning of multi-cavity resonances and phase for slow-light in coupled photonic crystal cavities,” arXiv preprint arXiv:1012.5805 (2010).
9. L. Maleki, A. B. Matsko, A. A. Savchenkov, and V. S. Ilchenko, “Tunable delay line with interacting whispering-gallery-mode resonators,” Opt. Lett. 29(6), 626–628 (2004).
10. B. Peng, S. K. Ödemen, W. Chen, F. Nori, and L. Yang, “What is and what is not electromagnetically induced transparency in whispering-gallery microcavities,” Nat. Commun. 5, 5082 (2014).
11. N. Liu, L. Langguth, T. Weiss, J. Kästel, M. Fleischhauer, T. Pfau, and H. Giessen, “Plasmonic analogue of electromagnetically induced transparency at the Drude damping limit,” Nat. Mater. 8(9), 758–762 (2009).
12. Z. Duan, B. Fan, T. M. Stace, G. J. Milburn, and C. A. Holmes, “Induced transparency in optomechanically coupled resonators,” Phys. Rev. A 93(2), 023802 (2016).
13. P. Tassin, L. Zhang, R. Zhao, A. Jain, T. Koschny, and C. M. Soukoulis, “Electromagnetically induced transparency and absorption in metamaterials: the radiating two-oscillator model and its experimental confirmation,” Phys. Rev. Lett. 109(18), 187401 (2012).
14. H. Yan, T. Low, F. Guinea, F. Xia, and P. Avouris, “Tunable phonon-induced transparency in bilayer graphene nanoribbons,” Nano Lett. 14(8), 4581–4586 (2014).
15. J. Gu, R. Singh, X. Liu, X. Zhang, Y. Ma, S. Zhang, S. A. Maier, Z. Tian, A. K. Azad, H. T. Chen, A. J. Taylor, J. Han, and W. Zhang, “Active control of electromagnetically induced transparency analogue in terahertz metamaterials,” Nat. Commun. 3, 1151 (2012).
16. T. Baba, “Slow light in photonic crystals,” Nat. Photonics 2(8), 465–473 (2008).
17. T. F. Krauss, “Why do we need slow light?” Nat. Photonics 2(8), 448–450 (2008).
18. Q. Xu, P. Dong, and M. Lipson, “Breaking the delay-bandwidth limit in a photonic structure,” Nat. Phys. 3(6), 406–410 (2007).
Several groups made use of various structures such as ring resonators, microspheres and photonic crystal cavities to demonstrate interesting dispersive effects [16–20]. Strong light-matter interactions with low number of photons in the optical cavities could also play a critical role in many applications. For example, in quantum information processing, cavity quantum electrodynamics (CQED) in photonic crystal cavities has been used to realize quantum gates and quantum memories [21–24].

1. Introduction

Quantum and classical optics might be considered to have separate descriptions for light in their own ways. However, recent demonstrations have showed that some effects, which are known as quantum only, also present in classical optics with a potential for variety of applications in many fields. One such field is the large-scale communications as photons will keep being the dominant transport medium in the optical fiber network infrastructures and parallel computing systems. An important subset of the recent work in this area is about the dynamic storage of light has been shown to be possible as a result of the quantum coherence in atoms leading to electromagnetically induced transparency (EIT) [2–4]. Then, further studies revealed that EIT-like spectra and light trapping can also be observed with integrated cavity systems in all-optical classical analog systems [5–10] as well as by utilizing metamaterials, graphene based systems and optomechanically coupled systems [11–15]. Several groups made use of various structures such as ring resonators, microspheres and photonic crystal cavities to demonstrate interesting dispersive effects [16–20]. Strong light-matter interactions with low number of photons in the optical cavities could also play a critical role in many applications. For example, in quantum information processing, cavity quantum electrodynamics (CQED) in photonic crystal cavities has been used to realize quantum gates and quantum memories [21–24].
critical role in power management considering the huge amount of energy consumption at the optical interconnects where electrical and optical signals are converted to each other [21–23]. With the continuously increasing data rates, the optimization of power requirements is quite critical and cavity quantum electrodynamics provides an interesting potential [24,25]. Therefore, as the proposals such as N-photon quanta are realized [26], physical systems spanning quantum computers and quantum networks will play crucial roles in improving particular tasks [27–30]. Thus, developing a solid theoretical understanding covering the limits is quite essential.

Previous work on EIT-like behavior in optical systems generally have focused on the transmission spectra and comparative study between various possible systems on the related characteristics such as the delay have not been discussed. Here, the optical analogue to electromagnetically induced transparency is modeled for two separate systems with the same formalism and the spectral characteristics together with the generated group delays are compared. First system is a coherently coupled high-Q two-cavity array which represents the classical EIT and is limited by the finite broadening of the cavity. Second one is a single embedded quantum dot (QD) cavity system in the weak coupling regime, a cavity-QD EIT (can also be called dipole induced transparency [31,32]), that depends on both QD broadening and cavity properties [33]. For the case of similar spectral characters, former generated theoretically two times higher group delays.

2. Theoretical model

When discussing the interactions between the quantum dots and the cavities, the interaction could be in strong or weak excitation regime depending on the coupling strength. Strong coupling is an excited topic and due to the strong interaction between the cavity and the dipole emitter, there are a number of important applications such as the phase shifts on single atoms where the dipole can convert the cavity from high transmission to high reflection leading to a quantum phase gate [34]. However, it has been shown that a perfect switch between transmission and reflection is possible even if the dipole field coupling rate, usually denoted as $g$, is much smaller than the cavity decay rate [35].

Obtaining transmission alteration in the weak coupling regime is quite interesting as the switch does not happen due to normal mode splitting but there is a destructive interference of cavity field, which is the analogous of the EIT [2]. This difference also leads to functional behavior variation between the regimes [35]. This point is particularly important as EIT-like
behavior in weak coupling regime means quantum information processing could be feasible in the solid state systems where phonon dephasing and fabrication may prevent strong coupling [36]. In addition, on-chip semiconductor implementations of CQED requires high cavity-waveguide coupling rate in order not to lose field out of plane extensively which makes the strong coupling even more difficult. Therefore, the systems that have a potential to be utilized in chip scale implementation and are discussed here are in the weak coupling regime where the quantum dot is predominantly in the ground state and the cavities have considerably higher extrinsic cavity decay rates compared the bare cavity decay rate.

First, cavity quantum dot subsystem shown in Fig. 1(a), where there is only one cavity interacting with a quantum dot, is investigated. If $\kappa_i$ is the external cavity decay rate, input and output relations can be written as the following:

$$\hat{a}_{\text{out}} = \hat{b}_{\text{in}} - \sqrt{\kappa_i} \hat{c}$$  \hspace{1cm} (1)

$$\hat{b}_{\text{out}} = \hat{a}_{\text{in}} - \sqrt{\kappa_i} \hat{c}$$  \hspace{1cm} (2)

The Heisenberg equations of motion are shown below [30–32]:

$$\frac{d\hat{c}}{dt} = -i[\hat{c}, H] - \Gamma \hat{c} + \sqrt{\kappa_i} (\hat{a}_{\text{in}} + \hat{b}_{\text{in}})$$  \hspace{1cm} (3)

$$\frac{d\hat{\sigma}^-}{dt} = -i[\hat{\sigma}^-, H] - \gamma \hat{\sigma}^- + \sqrt{\gamma} \hat{\sigma}^+$$  \hspace{1cm} (4)

Here, $2\Gamma$ defines the total cavity decay rate ($\Gamma = (\kappa_i + 2\kappa_i)/2$; $\kappa_i$ is the intrinsic cavity decay rate), $\hat{\sigma}^-$ represents the descending operator of the interacting two level quantum dot (QD) with $\gamma$ total decay rate and $\hat{\sigma}^+$ is the vacuum noise operator related to the decay rate. $H$ is the subsystem Hamiltonian where $g$ is the coupling strength between the cavity mode and the dipolar transition between the excited and ground states of the QD, $\omega$ is the cavity mode resonant frequency and $\omega_r$ is the transition frequency of the QD [30].

$$H = \omega_c \hat{c}^+ \hat{c} + \omega g \sigma^-, \sigma^- + [g \sigma^+ \hat{c} + h.c.]$$  \hspace{1cm} (5)

If the motions equations are solved in the weak excitation limit (please see the Appendix for a detailed derivation), the input output relations become as follows:

$$\hat{b}_{\text{in}} (\omega) = \left( \frac{\kappa_i}{-\alpha + \Gamma - \kappa_i} \right) \hat{a}_{\text{in}} (\omega) + \left( \frac{-\alpha + \Gamma}{-\alpha + \Gamma - \kappa_i} \right) \hat{a}_{\text{out}} (\omega)$$  \hspace{1cm} (6)

$$\hat{b}_{\text{out}} (\omega) = \left( \frac{\alpha - \Gamma + 2\kappa_i}{\alpha - \Gamma + \kappa_i} \right) \hat{a}_{\text{in}} (\omega) + \left( \frac{\kappa_i}{\alpha - \Gamma - \kappa_i} \right) \hat{a}_{\text{out}} (\omega)$$  \hspace{1cm} (7)

where $\alpha = i(\omega - \omega_r) + \frac{|g|^2}{i(\omega - \omega_r) - \gamma}$

Thus, the transport relation is

$$\begin{pmatrix}
\hat{b}_{\text{in}} (\omega) \\
\hat{b}_{\text{out}} (\omega)
\end{pmatrix} = \frac{1}{\alpha - \Gamma + \kappa_i} \begin{pmatrix}
-\kappa_i & -\alpha - \Gamma \\
\alpha - \Gamma + 2\kappa_i & \kappa_i
\end{pmatrix} \begin{pmatrix}
\hat{a}_{\text{in}} (\omega) \\
\hat{a}_{\text{out}} (\omega)
\end{pmatrix}$$  \hspace{1cm} (8)

This relation defines the single cavity-QD EIT system. In order to get the classical EIT configuration [Fig. 1(b)], the coupling coefficient, $g$, needs to go to zero and the transport...
relation for the waveguide (just a phase component of $\theta = 2\pi$) is required to be added and these transportation relations can easily be generalized [1,30].

\[
\begin{pmatrix}
\hat{b}_{in}(\omega) \\
\hat{b}_{out}(\omega)
\end{pmatrix} = \frac{1}{\beta_1 - \Gamma - \kappa_1} \begin{pmatrix}
-\kappa_2 & \beta_1 - \Gamma \\
\beta_2 - \Gamma - 2\kappa_2 & \kappa_1
\end{pmatrix} \begin{pmatrix}
0 & e^{i2\pi} \\
e^{i2\pi} & 0
\end{pmatrix} \begin{pmatrix}
-\kappa_1 & \beta_1 - \Gamma \\
\beta_1 - \Gamma - 2\kappa_1 & \kappa_1
\end{pmatrix} \begin{pmatrix}
\hat{a}_{in}(\omega) \\
\hat{a}_{out}(\omega)
\end{pmatrix}
\]

where $\beta_j = i(\omega - \omega_j)$.

Assuming initial field begins in mode $\hat{a}_{in}$, reflection and transmission coefficients can be calculated from $r = \hat{a}_{out}/\hat{a}_{in}$ and $t = \hat{b}_{out}/\hat{a}_{in}$. In addition, phase of the transmitted field is given by $\phi = \angle(\hat{b}_{out}/\hat{a}_{in})$.

Figure 2 shows the comparison between calculated transmission spectra, extracted phase values imposed on the transmitted field around the transparency window and the corresponding calculated group delay values, $d\phi/d\omega$, for both configurations. As can be seen in the figure, the transmission characteristics [Figs. 2(a) and 2(b)] and the corresponding phase/group delay values [Figs. 2(c) and 2(d)] look similar for both the cavity-cavity coherent interaction configuration (classical EIT) and the single embedded QD cavity configuration (cavity-QD EIT). The methods used for tuning the characteristics are as follows:

In the classical EIT case [Fig. 2(e)], as mentioned in the Theoretical model section, the coupling strengths is assumed to be zero. As the coherent interaction can only occur if the phase accumulation in the bus waveguide is an integer multiple of $2\pi$ in this case [1], $2\pi$ is used as the $\theta$. The two cavities are identical except for the resonant frequency and the detunings in this configuration are achieved by modifying the resonant frequency of the second cavity, $\omega_{c2}$ ($\Delta\omega$ is 0.5, 1.33, and 5$\Gamma$ for the solid (red), dashed (grey) and dotted (black) lines respectively).

In the cavity-QD EIT case [Fig. 2(f)], the intrinsic and the extrinsic cavity decay rates are assumed to be same with the classical EIT case. Besides, quantum dot is resonant with the cavity (transition frequency of the QD, $\omega_r$, is equal to the resonant frequency of the cavity, $\omega_c$). The splitting of the transmitted field, which has been also termed as the dipole induced transparency, is governed by the coupling strength, $g$, between the cavity mode and the QD transition. If $g$ is equal to $\eta\Gamma$, the transmission deeps in the spectrum occur $\omega_c \pm \eta\Gamma$ leading to a transparency peak at the resonant frequency with $\sim 2\eta\Gamma$ detuning between the deeps [30–35].

Here, the extrinsic cavity decay rate, $\kappa_i$, is adopted as 50 times the intrinsic cavity decay rate for all the cavities. In the rest of the manuscript, in order to simplify the discussion without loss of generality, extrinsic cavity decay rates are taken equal to each other in each single calculation and that’s why $\kappa$ is used instead of $\kappa_i$ in the formulism. It is also noted that the detunings here are normalized, so there is a detuning variation in absolute terms with increasing total cavity decay rate.
3. Spectral character and the group delay comparison

In order to compare the configurations explained above, the transmission and the group delay characteristics need to be analyzed. Initially Eq. (8) is solved for $t(\omega)$:

$$\text{Fig. 2. Spectral character and the generated group delay for the subsystems in Fig. 1. a (b), The transmission spectrum for classical EIT (cavity-QD EIT). c(d), Calculated group delay values for the classical EIT (cavity-QD) case in a (b). Insets: Calculated phase imposed on the transmitted field for a zoom-in range for normalized detunings. e (f), Change in the transmission spectrum for various detunings for the case in a (b).}$$
Then, the transparency peak value can be calculated from the transmission coefficient at $\omega = 0$,

$$t(\omega) = \frac{\omega^2 - \frac{\gamma \kappa_0}{2} - g^2 + i\omega \left(\gamma + \frac{\kappa_0}{2}\right)}{\omega^2 - \frac{\gamma \kappa_0}{2} - g^2 + i\omega \left(\kappa + \gamma + \frac{\kappa_0}{2}\right)}$$

(10)

When the phase angle of transmitted field is extracted from Eq. (10) and its derivative is calculated at $\omega = 0$, the propagation delay is found. As a result, calculated delay value can be normalized with the total cavity rate, $1/2 \Gamma$ leading to the following expression:

$$\frac{\tau_{qd}}{\tau_{00}} = \frac{\kappa \left(-\gamma^2 + g^2\right) \left(2\kappa + \kappa_0\right)}{\kappa^2 \gamma^2 + \kappa_0^2 \gamma^2 + g^2 \left(\kappa_0^2 + \frac{g^2}{2}\right)}$$

(12)

Next, a similar derivation is performed to get the transmission coefficient for the classical EIT case using the Eq. (9). Here, the cavity resonances, $\omega_c$, are selected as $\pm \Delta \omega/2$ so that the detuning between the cavities is equal to $\Delta \omega$. Then, the peak values for the transmission coefficient and the normalized group delay at $\omega = 0$ are given as follows:

$$t_c(0) = \frac{\kappa_0^2 + \Delta \omega^2}{4} \left(\kappa_0^2 + \Delta \omega^2\right)$$

(13)

$$\frac{\tau_{c}}{\tau_{00}} = \frac{\kappa \left(-\kappa_0^2 + \frac{\Delta \omega^2}{2}\right) \left(2\kappa + \kappa_0\right)}{\kappa \kappa_0^2 + \kappa_0^2 + \Delta \omega^2 \left(\kappa_0^2 + \frac{\Delta \omega^2}{2}\right)}$$

(14)

The quantum dot decay rate, $\gamma$, in Eq. (11) can be set to $\kappa_0$ [30]. As the coupling strength is smaller than the total cavity decay rate in the weak coupling regime, this indeed helps satisfying the large Purcell factor necessary in order to make the system transparent [35]. The selection here is also realistic in terms of the experimentally measured cavity and quantum dot decay rates [37,38]. In addition, $\kappa \gg \kappa_0$ condition, which is desired for chip-scale devices, needs to be applied. Consequently, the dominant terms in Eq. (11) and Eq. (13) are given by the expressions below:

$$t_{qd}(0) = \frac{g^2}{\kappa \kappa_0^2 + g^2}$$

(15)

$$t_c(0) = \frac{\Delta \omega^2 / 4}{\kappa \kappa_0^2 + \Delta \omega^2 / 4}$$

(16)

Comparison of Eq. (15) and Eq. (16) indicates that the approximate transmission is converging to the same value for both cases if the coupling strength in cavity-QD EIT is chosen equal to the half of the resonant detuning in classical EIT ($g = \Delta \omega/2$). This is indeed consistent with the transmission deeps that occur at $\pm g$ in cavity-QD EIT case. Therefore, the spectral features and transmission characteristics consists of quite similar components for both cases. This is an interesting and important result as it means that same functionality can
be achieved with the two separate dynamics that can be utilized in the on-chip information processing applications.

Furthermore, if the same procedure is applied to Eq. (12) and Eq. (14), the group delay characteristics can be found [Eq. (17) and Eq. (18)] in the high extrinsic decay rate condition.

$$\frac{\tau_{\text{sl}}}{\tau_{\text{sl}}} = \frac{\kappa (2\kappa + \kappa_0)}{(\kappa \kappa_0 + g^2)}$$  (17)

$$\frac{\tau_{\scriptscriptstyle g}}{\tau_{\scriptscriptstyle g}} = \frac{2\kappa (2\kappa + \kappa_0)}{(\kappa \kappa_0 + \Delta \omega^2 / 4)}$$  (18)

Results derived from the Eq. (17) and Eq. (18) for propagation delay are even more interesting. This time, there is a factor of 2 difference in favor of the classical EIT case and this might be an important decision parameter in system design problems involving integrated switches and modulators for wavelength division multiplexing (WDM) applications. As it seems the difference is coming from the extrinsic cavity decay rate coefficients, this suggests that the higher group delay in classical EIT case originates from higher number of waveguide coupling incidents.

Next, calculations with various parameters for both cases is discussed for further analysis. To begin with, the resonant frequency of the second cavity, $\omega_{c2}$, in the classical EIT case is tuned from 0.2$\Gamma$ to 1.5$\Gamma$ and the coupling strength, $g$, is tuned from 0.1$\Gamma$ to 0.75$\Gamma$ for the cavity-QD case while the extrinsic cavity decay rate ($\kappa$) is kept 50 $\kappa_0$. Then, the peak values for the transparency windows are extracted from the transmission and the generated group delay spectra. Results are summarized in Fig. 3 below.

As it is shown in Fig. 3(a), the transmission characteristics are almost identical in all the calculated points. This shows that the simplification done between Eq. (11) [Eq. (13)] and Eq. (15) [Eq. (16)] is valid and the two configurations will result in almost identical transmission characteristics. Then, if the generated group delays are compared for the same parameters, it also verifies the interesting observation above that the group delay in the classical EIT case is more than twice as the group delay in the cavity-QD case [Fig. 3(b)].

Another important observation here is that the group delay for the given extrinsic cavity decay seems to be monotonically decreasing and eventually vanishes when the detuning gets closer to 1.5$\Gamma$ for both systems. As a reminder, to be able to have a meaningful comparison, in the cavity-QD case, the parameter driving the normalized detuning is defined as $2g$.

Additional analysis is performed with various extrinsic cavity decay rate, $\kappa$, values as it forms the key parameter that directly affects the transmission spectrum and the generated delay. Variation from 5$\kappa_0$ to 250$\kappa_0$ with equal steps are performed and the transmission spectra and the corresponding group delay values are calculated for both configurations. The coupling strength, $g$, is set to 0.25$\Gamma$ and the resonance frequency of the second cavity, $\omega_{c2}$, is equal to 0.5$\Gamma$ in order to get a comparable systems. The results are summarized in Figs. 3(c) and 3(d). Similar to the detuning variation in Figs. 3(a) and 3(b), the transmission values are almost identical and there is the expected alteration in the group delay values. The classical EIT case again provides around two times longer delays for almost all the calculated points.
Fig. 3. Comparison of the classical EIT and cavity-QD EIT cases in terms of the transmission spectra and the generated group delay values. 

a, The change of the transparency peak transmission with respect to various detunings in the second cavity resonance (coupling strength) in the classical EIT (cavity-QD) case for $\kappa = 50$. 

b, The calculated group delay values for the cases in a. 

c, The change of the transparency peak transmission with respect to various external cavity decay rate with normalized detuning of 0.5 $\Gamma$. 

d, The calculated group delay values for the cases in c.

Moreover, the calculations in Figs. 3(a) and 3(b) are repeated for a smaller extrinsic cavity decay rate ($\kappa = 10$) and the results are shown in Figs. 4(a) and 4(b). Regarding the transmission characteristics, the match between the two configurations especially in the lower detuning values is slightly broken as expected. However, curves are still quite close to each other. On the other hand, delay profile is changed more significantly. This time, there is a maximum normalized delay point, which could be utilized for the on-chip application optimizations. This is due to the non-negligible dependency of the Eq. (12) on the coupling strength for the cavity-QD EIT case and the similar behavior in the Eq. (14) on the resonance frequency difference for the classical EIT case. This dependency is getting much less significant when the extrinsic cavity decay rate is increased so that it becomes more dominant. This is indeed why maximum normalized delay behavior is not captured in the simplified delay equations [Eq. (17) and Eq. (18)].

Then, Figs. 3(c) and 3(d) are also repeated with a smaller detuning value and the calculations are summarized in Figs. 4(c) and 4(d). For the smaller values of the extrinsic decay rate, the ratio between the generated group delays in two configurations are more than twice higher. For instance, for $\kappa$ values of 11 and 21, classical EIT provides 7.86 and 2.56 times more group delay respectively. Then, the ratio converges to the approximated value 2 as $\kappa$ starts being significantly dominant.
Fig. 4. Comparison of the classical EIT and cavity-QD EIT cases in terms of the transmission spectra and the generated group delay values. a, The change of the transparency peak transmission with respect to various detunings in the second cavity resonance (coupling strength) in the classical EIT (cavity-QD) case for $\kappa = 10$. b, The calculated group delay values for the cases in a. c, The change of the transparency peak transmission with respect to various external cavity decay rate with normalized detuning of 0.2$\Gamma$. d, The calculated group delay values for the cases in c.

The natural next step in the analysis above is to look at the bandwidth of the transparency windows for both cases. Cavities and the systems discussed here are in the class of optical filters and the dispersive nature of the optical filters introduces limitations on the broadband optical delays. In particular, as the resonance gets sharper in the spectrum, optical pulses which has a duration that is shorter than the inverse of the resonance’s bandwidth is distorted due to the dispersion. Therefore, this trade-off leads to smaller bandwidth for a higher peak value in the generated group delay [39]. However, it is important note that if there is a dynamical tuning possible, the ultrafast tuning of photonic structures can help break the delay bandwidth limit by utilizing special techniques including EIT-like behavior [5,18].

When the transmission spectra from the calculations for Figs. 3(c) and 3(d) are studied in order to get the full width half-maximum (FWHM) of the transparency peaks, the delay bandwidth limitation is observed as presented in Fig. 5. The cavity-QD [Fig. 5(b)] configuration has a wider bandwidth than the classical EIT configuration [Fig. 5(a)]. This leads to a lower transparency peak quality factor for cavity-QD EIT case since the quality factor is essentially inversely proportional with the linewidth of the transmission peak. Therefore, the results observed in Fig. 5 are expected in comparison with the Fig. 3. It is noted that the stepwise behavior in Fig. 5 is due to numerical discretization.
Fig. 5. The bandwidth of the transparency peaks and the corresponding quality factor Figs. 3c-d. a, Classical EIT case. b, Cavity-QD EIT case. The cavity-QD configuration has a wider bandwidth than the classical EIT configuration leading to a lower transparency peak quality factor.

4. Conclusion

In conclusion, the temporal group delays in the optical analogue to EIT are compared using two systems. For almost identical transmission levels between coherently coupled high-Q multi-cavity array and single QD embedded cavity in the weak coupling regime, generated delay is theoretically two times higher for multi-cavity case. Both configurations allow direct scalability for chip-scale optical pulse trapping and coupled-cavity QED which may be used in many applications such as the filters, switches and modulators in WDM systems.

5. Appendix

If $\kappa_1$ is the external cavity decay rate, input and output relations are taken as the following:

$$\hat{a}_{\text{in}} = \hat{b}_{\text{out}} - \sqrt{\kappa_1} \hat{c}$$

$$\hat{b}_{\text{in}} = \hat{a}_{\text{out}} - \sqrt{\kappa_1} \hat{c}$$

The motion equations [30–32]:

$$\frac{d\hat{c}}{dt} = -i[\hat{c}, H] - \Gamma \hat{c} + \sqrt{\kappa_1} \left( \hat{a}_{\text{in}} + \hat{b}_{\text{in}} \right)$$

$$\frac{d\hat{\sigma}}{dt} = -i[\hat{\sigma}, H] - \gamma \hat{\sigma} + \sqrt{\gamma} \hat{\sigma}^+$$

Here, $2\Gamma$ defines the total cavity decay rate ($\Gamma = (\kappa_1 + 2\kappa_0)/2$; $\kappa_0$ is the intrinsic cavity decay rate), $\hat{\sigma}_{\text{-}(+)}$ represents the descending (ascending) operator of the interacting two level quantum dot (QD) with $\gamma$ total decay rate and $\hat{\sigma}^+$ is the vacuum noise operator related to the decay rate. $H$ is the subsystem Hamiltonian where $g$ is the coupling strength between the cavity mode and the dipolar transition between the excited ($|e\rangle$) and ground ($|g\rangle$) states of the QD, $\omega$ is the cavity mode resonant frequency and $\omega_0$ is the transition frequency of the QD. The subsystem Hamiltonian is as follows [30]:

$$H = \omega \hat{c}^+ \hat{c} + \omega_0 \hat{\sigma}_- \hat{\sigma}_+ + [g \hat{\sigma}_- \hat{c}^+ + \text{h.c.}]$$

Here, $\hat{\sigma}_- |e\rangle = |g\rangle$ and $\hat{\sigma}_+ |g\rangle = |e\rangle$, so $\hat{\sigma}_- = |g\rangle \langle e|$ and $\hat{\sigma}_+ = |e\rangle \langle g|$ resulting in $\hat{\sigma}_+ = \hat{\sigma}_-^\dagger$. Therefore, the Hamiltonian becomes as:
\[ H = \omega \hat{c}^\dagger \hat{c} + \omega \hat{\sigma} \hat{\sigma} + g \hat{\sigma} \hat{c} + g^* \hat{c}^\dagger \hat{\sigma} \]  

(24)

If this is inserted into the commutation relations:

\[
[\hat{c}, H] = \omega \left[ \hat{c}, \hat{c}^\dagger \right] + \omega \left[ \hat{\sigma}, \hat{\sigma} \right] + g \left[ \hat{c}, \hat{\sigma} \right] + g^* \left[ \hat{c}^\dagger, \hat{\sigma} \right]
\]

(25)

\[
[\hat{\sigma}, \hat{c}] = \omega \left[ \hat{\sigma} \hat{c} \hat{c}^\dagger \hat{\sigma} \right] + \omega \left[ \hat{\sigma} \hat{\sigma} \hat{\sigma} \hat{\sigma} \right] + g \left[ \hat{\sigma} \hat{\sigma} \hat{\sigma} \hat{\sigma} \right] + g^* \left[ \hat{\sigma} \hat{\sigma} \hat{\sigma} \hat{\sigma} \right]
\]

(26)

As the annihilation operator and ascending (descending) operators commute and for a standard normalization for a single mode [32],

\[
[\hat{c}, \hat{\sigma}] = [\hat{c}, \hat{\sigma}^\dagger] = 0
\]

(27)

\[
[\hat{c}, \hat{c}^\dagger] = 1
\]

(28)

Using the rules for evaluating the commutators [40], commutations can be simplified as follows:

\[
[\hat{c}, H] = \omega \hat{c} + g^* \hat{\sigma}
\]

(29)

\[
[\hat{\sigma}, \hat{c}] = \omega \left[ \hat{\sigma} \hat{c} \hat{c}^\dagger \hat{\sigma} \right] + \omega \left[ \hat{\sigma} \hat{\sigma} \hat{\sigma} \hat{\sigma} \right] + g \left[ \hat{\sigma} \hat{\sigma} \hat{\sigma} \hat{\sigma} \right] + g^* \left[ \hat{\sigma} \hat{\sigma} \hat{\sigma} \hat{\sigma} \right]
\]

(30)

In the weak excitation limit, \([\hat{\sigma}, \hat{\sigma}]\) which is equal to \((1 - 2\hat{\sigma} \hat{\sigma})\) can be further simplified [31]. Therefore,

\[
[\hat{\sigma}, H] = \omega \hat{\sigma} + g \hat{c}
\]

(31)

If these are inserted into the motion equations and ignoring the noise term:

\[
\frac{d\hat{\sigma}}{dt} = -i \omega \hat{\sigma} - ig \hat{c} - \gamma \hat{\sigma}
\]

(32)

\[
\frac{d\hat{c}}{dt} = -i \left( \omega \hat{c} + g^* \hat{\sigma} \right) - \Gamma \hat{c} + \sqrt{\kappa_i} \left( \hat{a}_m + \hat{b}_m \right)
\]

(33)

Taking Fourier Transform of both sides,

\[
-\omega \hat{\sigma}(\omega) = -i \omega \hat{\sigma}(\omega) - ig \hat{c}(\omega) - \gamma \hat{\sigma}(\omega)
\]

(34)

\[
-\omega \hat{c}(\omega) = -i \omega \hat{c}(\omega) - ig^* \hat{\sigma}(\omega) - \Gamma \hat{c}(\omega) + \sqrt{\kappa_i} \left( \hat{a}_m(\omega) + \hat{b}_m(\omega) \right)
\]

(35)

Solving for \(\hat{c}(\omega)\)

\[
\hat{c}(\omega) = \frac{\sqrt{\kappa_i} \left( \hat{a}_m(\omega) + \hat{b}_m(\omega) \right)}{-i(\omega - \omega_r) + \Gamma - \frac{|g|^2}{i(\omega - \omega_r) - \gamma}}
\]

(36)

Thus, the input-output relations are modified as follows:

\[
\hat{\omega m}(\omega) = \hat{b}_m(\omega) - \frac{\kappa_i \left( \hat{a}_m(\omega) + \hat{b}_m(\omega) \right)}{-i(\omega - \omega_r) + \Gamma - \frac{|g|^2}{i(\omega - \omega_r) - \gamma}}
\]

(37)
\[ \hat{b}_{\text{out}}(\omega) = \hat{a}_{\text{in}}(\omega) - \frac{\kappa_i (\hat{a}_{\text{in}}(\omega) + \hat{b}_{\text{in}}(\omega))}{-i(\omega-\omega_i) + \Gamma - \frac{|g|^2}{i(\omega-\omega_i) - \gamma}} \quad (38) \]

If simplified,

\[ \hat{b}_{\text{in}}(\omega) = \left( \begin{array}{c} \kappa_i \\ -\alpha + \Gamma - \kappa_i \end{array} \right) \hat{a}_{\text{in}}(\omega) + \left( \begin{array}{c} -\alpha + \Gamma \\ -\alpha + \Gamma - \kappa_i \end{array} \right) \hat{a}_{\text{out}}(\omega) \quad (39) \]

\[ \hat{b}_{\text{out}}(\omega) = \left( \begin{array}{c} \alpha - \Gamma + 2\kappa_i \\ \alpha - \Gamma + \kappa_i \end{array} \right) \hat{a}_{\text{in}}(\omega) + \left( \begin{array}{c} \kappa_i \\ \alpha - \Gamma + \kappa_i \end{array} \right) \hat{a}_{\text{out}}(\omega) \quad (40) \]

where \( \alpha = i(\omega-\omega_i) + \frac{|g|^2}{i(\omega-\omega_i) - \gamma} \)

Writing in the matrix form,

\[ \begin{pmatrix} \hat{b}_{\text{in}}(\omega) \\ \hat{b}_{\text{out}}(\omega) \end{pmatrix} = \frac{1}{\alpha - \Gamma + \kappa_i} \begin{pmatrix} \kappa_i & \alpha - \Gamma \\ \alpha - \Gamma + 2\kappa_i & \kappa_i \end{pmatrix} \begin{pmatrix} \hat{a}_{\text{in}}(\omega) \\ \hat{a}_{\text{out}}(\omega) \end{pmatrix} \quad (41) \]

where \( \alpha = i(\omega-\omega_i) + \frac{|g|^2}{i(\omega-\omega_i) - \gamma} \)

In order to get the classical EIT configuration, \( g = 0 \) & the second cavity and a bus waveguide with phase component of \( \theta \) is inserted,

\[ \begin{pmatrix} \hat{b}_{\text{in}}(\omega) \\ \hat{b}_{\text{out}}(\omega) \end{pmatrix} = \frac{1}{\beta_i - \Gamma + \kappa_i} \begin{pmatrix} -\kappa_i & \beta_i - \Gamma \\ \beta_i - \Gamma + 2\kappa_i & \kappa_i \end{pmatrix} \begin{pmatrix} 0 & e^{i\theta} \\ e^{i\theta} & 0 \end{pmatrix} \begin{pmatrix} \hat{a}_{\text{in}}(\omega) \\ \hat{a}_{\text{out}}(\omega) \end{pmatrix} \quad (42) \]

where \( \beta_j = i(\omega-\omega_j) \)

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