ABSTRACT

Similarity search is the task of retrieving data items that are similar to a given query. In this paper, we introduce the time-sensitive notion of similarity search over endless data-streams (SSDS), which takes into account data quality and temporal characteristics in addition to similarity. SSDS is challenging as it needs to process unbounded data, while computation resources are bounded. We propose Stream-LSH, a randomized SSDS algorithm that bounds the index size by retaining items according to their freshness, quality, and dynamic popularity attributes. We analytically show that Stream-LSH increases the probability to find similar items compared to alternative approaches using the same space capacity. We further conduct an empirical study using real world stream datasets, which confirms our theoretical results.

Keywords

Similarity search, Stream-LSH, Retention policy

1. INTRODUCTION

Users today are exposed to massive volumes of information arriving in endless data streams: hundreds of millions of content items are generated daily by billions of users through widespread social media platforms [39, 40, 4]; fresh news headlines from different sources around the world are aggregated and spread by online news services [30, 18]. In this era of information explosion it has become crucial to ‘fish in the stream’, namely, identify stream content that will be of interest to a given user. Indeed, search and recommendation services that find such content are ubiquitously offered by major content providers [18, 19, 30, 15, 17].

A fundamental building block for search and recommendation applications is similarity search, an algorithmic primitive for finding similar content to a queried item [37, 8]. For example, a user reading a news item or a blog post can be offered similar items to enrich his reading experience [42]. In the context of streams, many works have observed that applications ought to take into account temporal metrics in addition to similarity [19, 30, 29, 25, 31, 40, 23, 36, 39, 34]. Nevertheless, the similarity search primitive has not been extended to handle endless data-streams. To this end, we introduce in Section 2 the problem of similarity search over data streams (SSDS).

In order to efficiently retrieve such content at runtime, an SSDS algorithm needs to maintain an index of streamed data. The challenge, however, is that the stream is unbounded, whereas physical space capacity cannot grow without bound; this limitation is particularly acute when the index resides in RAM for fast retrieval [39, 33]. A key aspect of an SSDS algorithm is therefore its retention policy, which continuously determines which data items to retain in the index and which to forget. The goal is to retain items that best satisfy the needs of users of stream-based applications.

In Section 3, we present Stream-LSH, an SSDS algorithm based on Locality Sensitive Hashing (LSH), which is a widely used randomized similarity search technique for massive high dimensional datasets [22]. LSH builds a hash-based index with some redundancy in order to increase recall, and Stream-LSH further takes into account quality, age, and dynamic popularity in determining an item’s level of redundancy.

A straightforward approach for bounding the index size is to focus on the freshest items. Thus, when indexing an endless stream, one can bound the index size by eliminating the oldest items from the index once its size exceeds a certain threshold. We refer to this retention policy as Threshold. Although such an approach has been effectively used for detecting new stories [39] and streaming similarity self-join [34], it is less ideal for search and recommendations, where old items are known to be valuable to users [27, 42, 11].

We suggest instead Smooth – a randomized retention policy that gradually eliminates index entries over time. Since there is redundancy in the index, items do not disappear from it at once. Instead, an item’s representation in the index decreases with its age. Figure 1 illustrates the probability to find a similar item with the two retention policies using the same space capacity. In this example, the index size suffices for Threshold to retain items for 20 days. We see that Threshold is likely to find fresh similar items, but fails to find items older than 20. Using the same space capacity, Smooth finds similar items for a longer time period with a gradually decaying probability; this comes at the cost of a lower probability to find very fresh items. We further show that Smooth exploits capacity resources more efficiently so that the average recall is larger than with Threshold.

Fishing in the Stream:
Similarity Search over Endless Data

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2. SIMILARITY SEARCH OVER DATA-STREAMS

We extend the problem of similarity search and define similarity search over unbounded data-streams. Our stream similarity search is time-sensitive and quality-aware, and so we also define a recall metric that takes these aspects into account.

2.1 Background: Similarity Search

Similarity search is based on a similarity function, which measures the similarity between two vectors, $u, v \in V$, where $V = (\mathbb{R}_d^n)^d$ is some high $d$-dimensional vector space [16].

**Definition 2.1 (Similarity Function).** A similarity function $\text{sim}: V \times V \to [0, 1]$ is a function such that $\forall u, v \in V$, $\text{sim}(u, v) = \text{sim}(v, u)$ and $\text{sim}(v, v) = 1$.

The similarity function returns a similarity value within the range $[0, 1]$, where 1 denotes perfect similarity, and 0 denotes no similarity. We say that $u$ is $s$-similar to $v$ if $\text{sim}(u, v) = s$.

A commonly used similarity function for textual data is angular similarity [39, 36], which is closely related to cosine similarity [13, 16]. The angular similarity between two vectors $u, v \in V$ is defined as:

$$\text{sim}(u, v) = 1 - \frac{\theta(u, v)}{\pi},$$

where $\theta(u, v) = \arccos\left(\frac{u \cdot v}{\|u\| \cdot \|v\|}\right)$ is the angle between $u$ and $v$.

Given a (finite) subset of vectors, $U \subseteq V$, similarity search is the task of finding vectors that are similar to some query vector. More formally, an exact similarity search algorithm accepts as input a query vector $q \in V$ and a similarity radius $R \in [0, 1]$, and returns $\text{Ideal}(q, R)$, a unique ideal result set of all vectors $v \in U$ satisfying $\text{sim}(q, v) > R$.

The time complexity of exact similarity search has been shown to be linear in the number of items searched, for high-dimensional spaces [43]. Approximate similarity search improves search time complexity by trading off efficiency for accuracy [37, 8]. Given a query $q$, it returns $\text{Appx}(q, R)$, an approximate result set of vectors, which is a subset of $q$'s $R$-ideal result set. We refer to approximate similarity search simply as similarity search in this paper.

2.2 SSDS

SSDS considers an unbounded item stream $U \subseteq V$ arriving over an infinite time period, divided into discrete ticks. The (finite) time unit represented by a tick is specified by the application, e.g., 30 minutes or 1 day. On every time tick, 0 or more new items arrive in the stream, and the age of a stream item is the number of time units that elapsed since its arrival. Note that each item in $U$ appears only once at the time it is created. Each item is associated with a query-independent quality score, which is specified by a given weighting function $\text{quality}: V \to [0, 1]$.

**Similarity search over data-streams.** An SSDS algorithm’s input consists of a query vector $q \in V$ and a three-dimensional radius, $(R_{\text{sim}}, R_{\text{age}}, R_{\text{quality}})$, of similarity, age, and quality radii, respectively. An exact SSDS algorithm returns a
unique ideal result set

\[ \text{Ideal}(q,R_{sim},R_{age},R_{quality}) \triangleq \{ v \in U | \text{sim}(q,v) \geq R_{sim} \land \text{age}(v) \leq R_{age} \land \text{quality}(v) \geq R_{quality} \}. \]

An (approximate) SSDS algorithm \( A \) returns a subset \( \text{Appx}(A,q,R_{sim},R_{age},R_{quality}) \) of \( q \)'s ideal result set.

**Recall**

**Definition 2.2** (Recall at radius). The recall at radius of algorithm \( A \) for query \( q \) and radius \((R_{sim}, R_{age}, R_{quality})\) is

\[ \text{Recall}(A,R_{sim},R_{age},R_{quality})(q) \triangleq |\text{Appx}(A,q,R_{sim},R_{age},R_{quality})|/|\text{Ideal}(q,R_{sim},R_{age},R_{quality})|. \]

The recall at radius \( \text{Recall}(A,R_{sim},R_{age},R_{quality}) \) of \( A \) is the mean recall over the query set \( Q \).

**Dynamic popularity.** We consider a second unbounded stream \( I \) which consists of items from the item stream \( U \) and arrives in parallel to \( U \). We call \( I \) the interest stream. The arrival of an item at some time tick in \( I \) signals interest in the item at that point in time. Note that an item may appear multiple times in the interest stream.

We capture an item’s dynamic popularity by a weighted aggregation of the number of times it appears in the interest stream, where weights decay exponentially with time [28]: Let \( t_0, \ldots, t_n \) denote time ticks since the starting time \( t_0 \), and the current time \( t_n \). The indicator \( a_i(x) \) is 1 if item \( x \) appears in the interest stream at time \( t_i \) and is 0 otherwise. A parameter \( 0 < \alpha < 1 \) denotes the interest decay, which controls the weight of the interest history and is common to all items.

**Definition 2.3** (Item popularity). The function \( \text{pop} : U \rightarrow [0,1] \) assigns a popularity score \( \text{pop}(x) \) to an item \( x \in U \):

\[ \text{pop}(x) \triangleq (1-\alpha) \sum_{i=0}^{n} a_i(x)\alpha^{(n-i)}. \]

Given an assignment of popularity scores to items, we are interested in the retrieval of items within a popularity radius \( R_{pop} \in [0,1] \), i.e., with a popularity score that is not lower than \( R_{pop} \). We define recall in a similar manner to the previous definitions.

3. **STREAM-LSH**

STREAM-LSH is an extension of Locality Sensitive Hashing (overviewed in Section 3.1) for unbounded data-streams, augmented with age, quality, and dynamic popularity dimensions. STREAM-LSH consists of a retention policy that defines which items are retained in the index and which are eliminated as new items arrive.

3.1 **Background: Locality Sensitive Hashing**

Locality Sensitive Hashing (LSH) [24, 22] is a widely used approximate similarity search algorithm for high-dimensional spaces, with sub-linear search time complexity. LSH limits the search to vectors that are likely to be similar to the query vector instead of linearly searching over all the vectors. This reduces the search time complexity at the cost of missing similar vectors with some probability.

LSH uses hash functions that map a vector in the high-dimensional input space \( \mathbb{R}^d \) into a representation in a lower dimension \( k << d \), so that the hashes of similar vectors are likely to collide. LSH executes a pre-processing (index building) stage, where it assigns vectors into buckets according to their hash values. Then, given a query vector, the similarity search algorithm computes its hashes and searches vectors in the corresponding buckets. The LSH algorithm is parametrized by \( k \) and \( L \), where \( k \) is the hashed domain’s dimension, and \( L \) is the number of hash functions used, as explained below. Formally [13]: a locality sensitive hashing with similarity function \( \text{sim} \) is a distribution on a family \( \mathcal{H} \) of hash functions on a collection of vectors, \( \mathcal{H} : V \rightarrow \{0,1\} \), such that for two vectors \( u, v \),

\[ \Pr_{h \in \mathcal{H}}[h(u) = h(v)] = \text{sim}(u,v). \] (2)

We use here a hash family \( \mathcal{H} \) for angular similarity [13]. In order to increase the probability that similar vectors are mapped to the same bucket, the algorithm defines a family \( \mathcal{G} \) of hash functions, where each \( g(v) \in \mathcal{G} \) is a concatenation of \( k \) functions chosen randomly and independently from \( \mathcal{H} \). In the case of angular similarity, \( g : V \rightarrow \{0,1\}^k \), i.e., \( g \) hashes \( v \) into a binary sketch vector, which encodes \( v \) in a lower dimension \( k \). For two vectors \( u, v \), \( \Pr_{g \in \mathcal{G}}[g(u) = g(v)] = \text{sim}(u,v)^k \), for any randomly selected \( g \in \mathcal{G} \). The larger \( k \) is, the higher the precision.

In order to mitigate the probability to miss similar items, the LSH algorithm selects \( L \) functions randomly and independently from \( \mathcal{G} \). The item vectors are now replicated in \( L \) hash tables \( H_i \), \( 1 \leq i \leq L \). Upon query, search is performed in \( L \) buckets. This increases the recall at the cost of additional storage and processing.

3.2 **Stream-LSH**

Stream-LSH, presented in Algorithm 1, extends LSH’s indexing procedure to operate on an unbounded data-stream. Every time tick, Stream-LSH accepts a set of newly arriving items in the item stream \( U \) and indexes each item into its LSH buckets. Stream-LSH selects an item’s initial redundancy according to its quality: it indexes the item into each bucket with a probability that equals its quality, independently of other buckets. In addition, in order to bound the index size, in each time tick, Stream-LSH eliminates items from the index according to the retention policy it uses. Note that the two operations – indexing new items and eliminating old ones – are independent, and indexing and elimination work independently in each bucket.

3.3 **Retention Policies**

We first describe the Threshold [39, 34] and Bucket [36] policies, which have been used by prior art in other contexts. Then, we describe our randomized Smooth policy that gradually eliminates item’s copies from the index as a function of its age.

3.3.1 **Threshold and Bucket**
The threshold retention policy [39, 34] presented in Algorithm 2 sets a limit $T_{size}$ on table size, and eliminates the oldest items from all $L$ tables once the size limit is exceeded. Note that with Threshold, the number of copies of an item in the index does not vary with age.

Algorithm 2 Threshold retention policy

1: function H.Eliminate
2: remove |H| - $T_{size}$ oldest items in table
3: end function

The bucket retention policy [36] given in Algorithm 3 sets a limit $B_{size}$ on bucket size (rather than on table size), and eliminates the oldest items in each bucket once its size limit is exceeded. Note that with Bucket, the number of copies of an item in the index varies with age, since each bucket is maintained independently. The probability of an item to be eliminated from a bucket depends on the data distribution, i.e., on the probability that newly arriving items will be mapped to that item’s bucket.

Algorithm 3 Bucket retention policy

1: function H.Eliminate
2: foreach $B_i \in H$ do
3: remove $|B_i| - B_{size}$ oldest items in bucket
4: end function

### 3.3.2 Smooth

In Algorithm 4 we present Smooth, our randomized retention policy that gradually eliminates item copies from the index as a function of their age. Smooth accepts as a parameter a retention factor $p$, $0 < p < 1$. Upon a time tick, Smooth eliminates each existing item copy from its bucket with probability $1 - p$, independently of the elimination of other items. The number of buckets an item is indexed into thus exponentially decays over time. As we show in Section 4.1, Smooth entails an expected bounded index size that is a function of $p$.

Although as described in Algorithm 4 Smooth entails a linear scan of all items in all hash tables at each time unit, it can be implemented efficiently by randomly and uniformly selecting a fraction of $1 - p$ of the items to eliminate from each table.

Algorithm 4 Smooth retention policy

1: function H.Eliminate
2: foreach item in $H$ do
3: with probability $1 - p$, remove item from $H$
4: end function

DynaPop extends Stream-LSH indexing procedure to dynamically re-index items based on signals of user interests, as reflected by the interest stream $I$. Here, an item’s redundancy increases as the interest in it increases. At each time tick, DynaPop re-indexes an item that arrives in $I$ into each of its buckets with probability $quality(x)\mu$ independently of other buckets; the insertion factor, $0 < u < 1$, is a parameter to the algorithm. Note that in this context, an item’s quality may also change dynamically over time. At each time tick, the current quality value is considered.

### 4. Analysis

In this section, we analyze Stream-LSH’s index size and prove that it maintains a bounded index in expectation. We additionally analyze the probability that Stream-LSH finds items within similarity, age, quality, and popularity radii.

#### 4.1 Index Size and Number of Retained Copies

**Index size.** We analyze the index size according to Stream-LSH’s retention policies. For the sake of the analysis, we assume that a constant number of new items $\mu$ arrive at each time unit, and their mean quality is $\phi$. By definition, Threshold and Bucket guarantee a bounded index size. We analyze the index size while using Smooth with a retention factor $p$. Consider one specific hash table, and denote time ticks as $t_0, \ldots, t_n$. At time $t_0$, Smooth stores $\mu \phi$ items in the hash table in expectation. This set of $\mu \phi$ items are scanned independently $n$ times until time $t_n$, and thus the expected number of items that arrive at $t_0$ and survive elimination until $t_n$ is $p^n \mu \phi$. It follows that the expected number of items in the table when processing an infinite stream is $\sum_{x=0}^{\infty} p^n \mu \phi = \frac{\mu \phi}{1 - p}$. The retention process is performed independently in each of the $L$ hash tables, therefore:

**Proposition 1.** Assuming $\mu$ new items with mean quality $\phi$ arrive at each time unit, the expected size of an index with $L$ hash tables when using Smooth with retention factor $p$ is $\frac{\mu \phi}{1 - p} L$.

**Number of retained copies.** Next, we analyze the evolution of an item’s number of copies in the index as a function of its age and quality according to Threshold and Smooth. We omit Bucket from our analysis due to its dependency on the data distribution. We examine Threshold and Smooth when using the same index size: $20 \mu \phi L$ in expectation, and so we set $T_{size} = 20 \mu \phi$ for Threshold, $p = 0.95$ for Smooth (Proposition 1), and $L = 15$ for both.

Let $x$ be some item. Threshold retains $quality(x) L$ copies of $x$ in expectation if $age(x) < 20$, and zero copies otherwise. Smooth retains $quality(x) p^{age(x)} \mu \phi L$ copies of $x$ in expectation. Figure 2 illustrates the number of index copies of an item with quality 1 (solid line), and of an item with quality 0.5 (dashed line), as a function of the item’s age. Due to
its quality-based indexing. Stream-LSH (for both Threshold and Smooth) retains a smaller number of copies of low quality items compared to high quality ones. Additionally, as Smooth's name suggests, it smoothly decays an item's number of copies in contrast with Threshold. This difference between the two policies impacts their effectiveness as we analyze in Section 4.2.

4.2 Success Probability

Success probability quantifies the probability of an algorithm to find some item \( x \) given a query \( q \). For an LSH algorithm \( A \) with parameters \( k \) and \( L \), we denote this probability by \( SP(A(k, L)) \). We sometimes omit \( k, L \) where obvious from the context.

According to LSH theory for angular similarity \cite{Indyk-Motwani98}, the probability of an LSH \((k, L)\) algorithm to find \( x \) that is \( s \)-similar to \( q \) in bucket \( g_i(q) \) equals \( s^k \), under a random selection of \( g_i \in G \). LSH searches in \( L \) buckets \( g_1(q), \ldots, g_L(q) \) independently, thus, LSH \((k, L)\) finds \( x \) with probability \( 1 - (1 - s^k)^L \).

4.2.1 Retention Policies

We analyze Stream-LSH success probability when using Threshold and Smooth. We do not analyze Bucket, as its behavior depends on the data distribution, which we do not model.

Success probability. We denote by \( SP(A(k, L), s, a, z) \) the probability of algorithm \( A(k, L) \) to find an item \( x \) for query \( q \), s.t. \( sim(q, x) = s \), \( age(x) = a \), and \( quality(x) = z \).

Stream-LSH indexes a newly arriving item \( x \) into each bucket \( g_i(x) \) independently with probability \( z \). For a constant arrival rate \( \mu \), a mean quality \( \phi \), and a size limit \( T_{size} \), Threshold eliminates items that reach age \( T_{age} = \frac{T_{size}}{\mu \phi} \). Thus,

\[
SP(\text{Threshold}(k, L), s, a, z) = \begin{cases} 
1 - (1 - s^k)^L, & \text{if } a < T_{age} \\
0, & \text{otherwise}
\end{cases}
\]

(3)

Smooth retains an item \( x \) in the index with probability \( p^{age(x)} \), thus,

\[
SP(\text{Smooth}(k, L), s, a, z) = 1 - (1 - p^a s^k)^L.
\]

(4)

Numerical illustration. We compare the success probabilities of Threshold and Smooth. In order to achieve a fair comparison, we fix \( k, L \), and the index size. Given that our treatment of quality is orthogonal to the retention policies, our example ignores quality, and so we assume \( quality(x) = 1 \) for all items \( x \). For the purpose of the illustration, we select a configuration where \( k = 10 \) and \( L = 15 \); we set \( T_{size} = 20\mu \) and \( p = 0.95 \) yielding a common index size for both policies (Proposition 1).

Figure 3 illustrates as ‘heat maps’ the success probabilities of Threshold and Smooth for this configuration. The \( x \) axis denotes similarity values \( s \), and the \( y \) axis denotes age values \( a \). Figure 3(a) depicts \( SP(\text{Threshold}(10, 15), s, a, 1) \), while figure 3(b) depicts \( SP(\text{Smooth}(10, 15), s, a, 1) \).

As Threshold completely eliminates all item’s copies that reach age 20, the success probability for \( a \geq 20 \) is 0 (colored white). The success probability of newer items behaves according to standard LSH, i.e., the more similar an item is to the query (\( s \) is closer to 1), the higher the success probability (color tends towards red). Fixing an \( s \) value, the success probability remains constant as \( a \) increases, since the number of buckets an item is indexed into remains constant. With Smooth, on the other hand, for a fixed \( s \) value, the success probability gradually decays as \( a \) increases. Smooth retains items for a longer time period than Threshold, and thus the success probability is non-zero for items older than 20.

Cumulative success probability. We next formulate the cumulative success probability (CSP) over similarity, age, and quality radii. Given a query \( q \), a similarity radius \( R_{sim} \), an age radius \( R_{age} \), and a quality radius \( R_{quality} \), CSP quantifies an algorithm’s probability to find an item \( x \) for which \( sim(q, x) \in [R_{sim}, 1], age(x) \in [0, R_{age}], \) and \( quality(x) \in [R_{quality}, 1] \). CSP is the expected SP over all choices of \( s \in [R_{sim}, 1], a \in [0, R_{age}], \) and \( z \in [0, R_{quality}] \) and is given by the following equation:

\[
\text{CSP}(A, R_{sim}, R_{age}, R_{quality}) = \int_{s=R_{sim}}^{1} \int_{a=0}^{R_{age}} \int_{z=0}^{R_{quality}} f(s, a, z) SP(A, s, a, z) ds \, da \, dz,
\]

where \( f(s, a, z) \) denotes the joint probability density function of similarity \( s \), age \( a \), and quality \( z \),
and

$$\psi(R_{sim}, R_{age}, R_{quality}) = \int_{x=R_{quality}}^{1} \int_{a=0}^{R_{age}} \int_{s=R_{sim}}^{1} f(s, a, z) ds da dz,$$

is a normalization factor.

Threshold keeps items up to age $\frac{1}{\mu}$; from (3) and (4):

$$CSP(Threshold(k, L), R_{sim}, R_{age}, R_{quality}) = \int_{x=R_{quality}}^{1} \int_{a=0}^{R_{age}} \int_{s=R_{sim}}^{1} f(s, a, z)(1 - (1 - s^k L)) ds da dz$$

and

$$CSP(Smooth(k, L), R_{sim}, R_{age}, R_{quality}) = \int_{x=R_{quality}}^{1} \int_{a=0}^{R_{age}} \int_{s=R_{sim}}^{1} f(s, a, z)(1 - (1 - p s^k L)) ds da dz$$

Numerical illustration. We compare $CSP(A, R_{sim}, R_{age}, R_{quality})$ of Stream-LSH with Threshold and Smooth. We pose the following assumptions: we focus on the effect of the retention policy and hence we assume a constant quality function $\forall x \text{ quality}(x) = 1$. In general, the items’ similarity distribution is data-dependent, here, we assume a uniform distribution. We consider a discrete age distribution because time is partitioned into discrete time ticks. We assume a constant number of items arriving at each time unit, hence items’ age is distributed uniformly. Last, we assume that similarity and age are independent. Under these assumptions we get:

$$CSP(Threshold(k, L), R_{sim}, R_{age}) = \sum_{a=0}^{R_{age}} \int_{s=R_{sim}}^{1} (1 - (1 - s^k L)) R_{age}(1 - R_{sim}) ds,$$

and

$$CSP(Smooth(k, L), R_{sim}, R_{age}) = \sum_{a=0}^{R_{age}} \int_{s=R_{sim}}^{1} (1 - (1 - p^a s^k L)) R_{age}(1 - R_{sim}) ds.$$

For the purpose of the illustration, we select the same configuration as above: $k = 10$, $L = 15$, $T_{size} = 20 \mu$, and $p = 0.95$. Figures 4(a) and 4(b) depict CSP for fixed $R_{sim}$ values 0.8 and 0.9, respectively, and a varying age radius $R_{age}$. The graphs show a freshness-similarity tradeoff between Threshold and Smooth. Smooth has a better CSP for high age radii ($R_{age} exceeds 20$), for any $R_{sim}$ value. This comes at the cost of a decreased CSP for similarity radius 0.8 when $R_{age} \leq 20$.

4.2.2 Quality-Sensitivity

Some applications, most notably social media ones, commonly handle content of varying quality, and in particular, large amounts of low quality user-generated content [7, 10, 12]. We show that for such applications, quality-sensitive indexing is expected to be attractive, as it can better utilize space for improving the CSP of high quality items. We compare Stream-LSH’s quality-sensitive indexing, which indexes an item with redundancy that is proportional to its quality, to a quality-insensitive Stream-LSH, which indexes $L$ copies of each item regardless of its quality. We use the Smooth retention policy and the same index size for both variants.

Assuming an average quality of 0.5, the expected number of newly indexed items per table is $\mu$ according to quality-insensitive indexing, and $0.5\mu$ according to quality-sensitive indexing. To obtain a common index size of $10\mu L$, we use $p = 0.9$ in the quality-insensitive algorithm and $p = 0.95$ in the quality-sensitive one (Proposition 1). Figure 5 illustrates the two Stream-LSH variants for $R_{sim} = 0.8$ and varying $R_{age}$ radii. In Figure 5(a), we examine items above the mean quality ($R_{quality} = 0.5$), and in Figure 5(b) we examine high-quality items ($R_{quality} = 0.9$). Both figures show that quality-sensitive indexing increases the CSP compared to the quality-insensitive variant. The improvement is even more marked for high-quality items ($R_{quality} = 0.9$).

Figure 4: Cumulative success probability of Threshold and Smooth by age radius for a common index size.

Figure 5: Cumulative success probability comparison of quality-sensitive and quality-insensitive Stream-LSH by age radius for a common index size.

4.2.3 Dynamic Popularity

We next analyze an item’s success probability according to Stream-LSH when using DynaPop and the Smooth retention policy. We first formulate the bucket probability (SB) to find an item in a bucket it is hashed to. We then use SB to formulate an item’s success probability.

For the sake of the analysis, we assume that the interest in an item does not vary over time. At each time $t$, $x$ is included in the interest stream with some probability $p_x$, which we call the item’s interest probability. That is, for
all \( t_i \), \( x \) appears with probability \( \rho_x \). According to Definition 2.3 and due to the linearity of expectation:

\[
E(\text{pop}(x)) = (1 - \alpha) \sum_{i=0}^{n} E(a_i(x))a_i^{(n-i)},
\]

which is a geometrical series that converges to \( E(a_i(x)) \) when \( n \to \infty \). As our stream is infinite and \( E(a_i(x)) = \rho_z \):

\[
E(\text{pop}(x)) = \rho_x.
\] (5)

When clear from the context, we omit \( x \) and denote \( \rho \) for brevity.

**Bucket probability.** We denote by \( SB(p, u, \rho, z, n) \) the probability that \( x \) is stored in its bucket at time \( t_x \), where \( u \) and \( p \) are the insertion and retention factors, respectively, \( \text{quality}(x) = z \), and \( \rho \) is \( x \)'s interest probability. We denote by \( E_i \), \( 0 \leq i \leq n \), the event that \( x \) is inserted to its bucket at time \( t_i \), and survives elimination until time \( t_u \), but is not selected for insertion to its bucket at any subsequent time \( t_j \), \( i < j \leq n \). Then

\[
Pr(E_i) = zu p^{(n-i)}(1 - zu p)^{n-i} = zu p^i (1 - zu p)^{n-i}
\]

Since \( SB(p, u, \rho, z, n) = Pr(\bigcup_{i=0}^{n} E_i) \) and \( \bigcup_{i=0}^{n} E_i \) is a union of pairwise disjoint events, it follows that

\[
SB(p, u, \rho, z, n) = \sum_{i=0}^{n} zu p^i (1 - zu p)^{n-i}.
\]

\( SB(p, u, \rho, z, n) \) is a geometric series that converges to \( \frac{zu p}{1 - (1 - zu p)} \) when \( n \to \infty \). Our interest stream is infinite, thus:

**Proposition 2.** Given an item \( x \), the probability \( SB(p, u, \rho, z) \) to find item \( x \) in its bucket when using Stream-LSH with DynaPop and the Smooth retention policy is \( \frac{zu p}{1 - (1 - zu p)} \), where \( u \) and \( p \) are the algorithm's insertion and retention factors respectively, \( \text{quality}(x) = z \), and \( \rho \) is \( x \)'s interest probability.

**Numerical illustration.** We illustrate SB for an interest probability that follows a Zipf distribution (typical in social phenomena [14]) which implies a small number of very popular items and a long tail of rare ones. We consider a Zipf distribution where the \( r \)-th ranked item \( x \) has an interest probability \( \rho_x = 1/r \). We set the quality \( z \) to 1.

Figure 6 illustrates the SB according to interest probability rank, for different values of \( u \) and \( p \). In Figure 6(a) we fix \( p = 0.95 \) and examine the effect of the insertion probability \( u \). The graphs illustrate that increasing \( u \) increases SB most notably for popular items. In Figure 6(b) we fix \( u = 1 \) and examine the effect of the retention probability \( p \). The graphs illustrate that when \( p \) increases, DynaPop retains additional items of lower popularity.

**Success probability.** We denote by \( SP(DynaPop(k, L), s, w, z) \) the probability of Stream-LSH with DynaPop and the Smooth retention policy to find an item \( x \) for query \( q \), s.t. \( \text{sim}(q, x) = s \), \( E(\text{pop}(x)) = w \), and \( \text{quality}(x) = z \). By applying Proposition 2 and as \( w = \rho \) (Equation 5):

\[
SP(DynaPop(k, L), s, w, z) = 1 - \left(1 - \frac{zu w}{1 - p(1 - zu p)}\right)^{s L}
\] (6)

Figure 6: Effect of parameters on the probability to find an item in its bucket according to DynaPop.

The cumulative success probability is computed similarly to the cumulative success probability analysis in Section 4.2.1, and in particular depends on the distribution of \( w \).

**Numerical illustration.** Figure 7 depicts \( SP(DynaPop(k, L), s, w, z) \) as a function of \( w \)'s rank. We illustrate SP for three \( w \) values: 0.7, 0.8, and 0.9. We fix \( k = 10 \), \( L = 15 \), \( z = 1 \), \( p = 0.95 \) and \( u = 1 \). As the graphs show, an item’s success probability increases as its similarity to the query increases. Additionally, an item’s success probability increases as its expected popularity increases.

5. **EMPIRICAL STUDY**

We conduct an empirical study of Stream-LSH using real world stream datasets and evaluate its effectiveness using the recall metric.

5.1 **Methodology**

**External libraries.** We use Apache Lucene 4.3.0 [1] search library for the indexing and retrieval infrastructure. For retrieval, we override Lucene’s default similarity function by implementing angular similarity according to Equation 1. For the LSH family of functions, we use TarsosLSH [3].

**Datasets.** We use Reuters RCV1 [2] news dataset and Twitter [45, 34] social dataset. In both datasets, each item is associated with a timestamp denoting its arrival time. The Reuters dataset consists of news items from August 1996 to August 1997, and the Twitter dataset consists of Tweets.
collected in June 2009. These datasets do not contain quality information and so we assume $quality(x) = 1$ for all items. In order to evaluate quality-sensitivity, we use a smaller Twitter dataset [5], denoted TwitterNas, consisting of a stream of Nasdaq related Tweets spanning 97 days from March 10th to June 15th 2016. TwitterNas contains number of followers of Tweets authors, which we use for assigning quality scores to Tweets (see Section 5.3). In all datasets, we represent an item as a (sparse) vector whose dimension is the number of unique terms in the entire dataset, and each vector entry corresponds to a unique term, weighted according to Lucene’s TF-IDF formula.

**Train and test.** We partition each dataset into (disjoint) train and test sets. The train set is the prefix of the item stream up to a tick that we consider to be the current time. The test set is the remainder of the dataset, which was not previously seen by the Stream-LSH algorithm. We randomly sample an evaluation set $Q$ of 3,000 items from the test set and compute recall over $Q$ according to the given radii. Table 1 summarizes the train and test statistics.

### 5.2 Retention Policies

We evaluate the performance of the three retention policies for the Reuters and Twitter datasets as a function of age. As the retention aspect of our algorithm is orthogonal to the quality-sensitive indexing aspect, we assume here that $quality(x) = 1$ for all items. In order to achieve a fair comparison, we use $k = 10$ and $L = 15$ for the three retention policies, and configure them to use approximately the same index size: We set $T_{size} = 45,000$ and $B_{size} = 45$ in Reuters; $T_{size} = 180,000$ and $B_{size} = 177$ in Twitter; $p = 0.95$ in both datasets.

Figure 8 depicts our recall results for Reuters in the top row, and Twitter in the bottom row. Our goal is to retrieve items that are similar to the query, hence we focus on $R_{sim}$ values 0.8, and 0.9. As we are also interested in the retrieval of items that are not highly fresh, we evaluate recall over varying age radii values.

When considering $R_{sim} = 0.8$ (leftmost column) there is a tradeoff between Threshold and Smooth: when focusing on the highly fresh items ($R_{age} < 20$), Threshold’s recall is slightly larger than Smooth’s. Indeed, Threshold is effective when only the retrieval of the highly fresh items is desired. However, Smooth outperforms Threshold when the age radius increases to include also less fresh items. For example, in Reuters, Smooth achieves a recall of 0.69 for items that are at least 0.8-similar to the query and are not older than age 50, and Threshold achieves a lower recall of 0.42. Bucket’s recall is higher than Threshold’s for ages that exceed 20, as unlike Threshold, Bucket does not eliminate items at once. Yet, Smooth outperforms Bucket when increasing the age radius due to applying an explicit gradual elimination over all items. When increasing the similarity radius to $R_{sim} = 0.9$ (leftmost column), the advantage of Smooth over Threshold becomes pronounced. For example, in Twitter, Smooth achieves a recall of 0.97 for items that are not older than age 50, whereas Threshold only achieves a recall of 0.7.

### 5.3 Quality-Sensitivity

We move on to evaluating Stream-LSH’s quality-sensitive approach. We experiment with the TwitterNas dataset,
quality-sensitive Smooth is 0.88. When considering $R_{age} = 90$, the recall of quality-insensitive Smooth is 0.45, whereas the recall of quality-sensitive Smooth is 0.76. A similar trend is observed for $R_{quality} = 0.9$ (Figure 9(b)). Note that the graphs of $R_{quality} = 0.9$ and $R_{quality} = 0.5$ differ due to the different distributions of similarity, age, and quality, when considering different radii values in real data. As we noted in our analysis, the advantage of the quality-sensitive approaches is most pronounced when there exists a large amount of low quality items in the dataset. Indeed, in our setting, 73% of the items are assigned a quality value below 0.5. In such cases, using quality-sensitive Stream-LSH is expected to be appealing for similarity-search stream applications.

5.4 Dynamic Popularity

We wrap up by studying Stream-LSH when using DynaPop and the Smooth retention policy. We experiment with $u = 0.95$, $p = 0.95$. As our datasets do not contain temporal interest information, we simulate an interest stream $I$ by considering query results as signals of interests in items [14], as follows: We use the first 75% items in the train set as the item stream $U$. We construct a query set $Q^*$ by randomly sampling each item from the remaining 25% of the train set with probability 0.1. For each query $q \in Q^*$, we retrieve its top 10 similar items in $U$ and include them in the interest stream $I$ at $q$’s timestamp $t_q$, as well as at their original arrival times in $U$. Table 2 summarizes the item and interest stream statistics. We compute popularity scores at the current time according to Definition 2.3 with $\alpha = 0.95$.

Figure 10 depicts recall as a function of $R_{pop}$ for similarity radii 0.8 and 0.9. For both datasets and similarity radii, the recall increases as the popularity radius increases. DynaPop provides high recall when searching for the most popular items in the dataset: For example, in the Reuter’s dataset (Figure 10(a)), for $R_{sim} = 0.8$ (blue curve) and $R_{pop} = 0.05$ (capturing the 3.5% most popular items in the data set), the recall is 0.86. When increasing the similarity radius to $R_{sim} = 0.9$ (green curve), the recall increases and is 0.97. DynaPop’s recall is lower when we also search for less popular items: In the Reuter’s dataset, for $R_{sim} = 0.8$ and $R_{pop} = 0.01$ (capturing the 24% most popular items in the data set), the recall is 0.72. When increasing the similarity radius to $R_{sim} = 0.9$, the recall is 0.97. Overall, DynaPop achieves good recall for popular items that are similar to the query while also retrieving similar items that are less popular albeit with lower recall; the latter is beneficial for applications such as query auto-completion [9] and product recommendation [46].

6. RELATED WORK

Previous work on recommendation over streamed content [20, 18, 30, 26, 29, 31, 12] focused on using temporal information for increasing the relevance of recommended items. Stream recommendation algorithms extend techniques originally designed for static data, such as content-based and collaborative-filtering [6], and apply them to streamed data by taking into account the temporal characteristics of stream generation and consumption within the algorithm internals. In the context of search, many works extend ranking methods to consider temporal aspects of the data, (see [25] for a survey), and quality features such as a social post’s length or the author’s influence [40]. However, these search and recommendation works do not tackle the challenge of bounding the capacity of their underlying indexing data-structures. Rather, they assume an index of the entire stream with temporal information is given. Our work is thus complementary to these efforts in the sense that we offer a retention policy that may be used within their similarity search building block.
| Item stream | Interest stream |
|-------------|-----------------|
| **Num. items** | **Num. ticks** | **Num. items** | **Num. ticks** |
| Reuters      | 540,882         | 252            | 226,890        | 95             |
| Twitter      | 13,124,853      | 2,000          | 4,267,518      | 1,500          |

Table 2: DynaPop item and interest streams statistics.

TI [14] and LSHII [44] improve realtime indexing of stream data using a policy that determines which items to index online and which to defer to a later batch indexing stage. Both assume unbounded storage and are thus complementary to our work. In addition, the TI focuses on highly popular queries, whereas we also address the tail of the popularity distribution. LSHII addresses the tail, however, it assumes exact search while we focus on approximate search, which is the common approach in similarity search [22].

A few previous works have addressed bounding the underlying index size in the context of stream processing [36, 39, 34, 33]. Two papers [36, 39] have focused on first story detection, which detects new stories that were not previously seen. Both use LSH as we do. Petrović et al. [36] maintain buckets of similar stories, which are used in realtime for detecting new stories using similarity search. In order to bound the index, they define a limit on the number of stories kept within a bucket, and eliminate the oldest stories when the limit is reached. We call this retention policy Bucket. Sundaram et al. [39]'s primary goal is to parallelize LSH, in order to support high-throughput data streaming. They bound the index size using a retention policy we call Threshold, by eliminating the oldest items when the entire index exceeds a given space limit. Both papers focus on the first story detection application, while our work focuses on the similarity search primitive. Their retention policies are well-suited for first story detection, where the index is searched in order to determine whether any recent matching result exists (indicating that the story is not the first), and are less adequate for similarity search, where multiple relevant results to an arbitrary query are targeted. We evaluate their retention policies in our Stream-LSH algorithm and find that our Smooth policy provides much better results in our context.

Morales and Gionis propose streaming similarity self-join (SSSJ) [34], a primitive that finds pairs of similar items within an unbounded data stream. Similarly to us, SSSJ needs to bound its underlying search index. Our work differs however in several aspects: First, we study a different search primitive, namely, similarity search, which searches for items similar to an arbitrary input query rather than retrieving pairs of similar items from the stream. Second, SSSJ only retrieves items that are not older than a given age limit. It thus bounds the index using a variant of Threshold. In contrast, we do not assume that an age limit on all queries is known a priori. In this context, we propose Smooth, which better fits our setting as we show in our evaluation. Third, we tackle approximate similarity search whereas SSSJ searches for an exact set of similar pairs.

Magdy et al. [33] propose a search solution over stream data with bounded storage, which increases the recall of tail queries. Their work differs from ours in the retrieval model, more specifically, they assume the ranking function is static and query-independent, e.g., ranking items by their age. Each item’s score is known a priori for all queries, and can be used to decide at indexing time which items to retain in the index. This approach is less suitable to similarity search, where scores are query-dependent and only known at runtime.

In addition, we note that the aforementioned works on bounded-index stream processing [36, 39, 34, 33] do not take into account quality and dynamic popularity as we do.

Several papers have focused on improving the space complexity of LSH via alternative search algorithms [21, 41, 38], via decreasing the number of tables used at the cost of executing more queries [35], or by searching more buckets [32]. Unlike Stream-LSH, these works consider static (finite) data rather than a stream.

7. CONCLUSIONS AND FUTURE WORK

We introduced the problem of similarity search over endless data-streams, which faces the challenge of indexing unbounded data. We proposed Stream-LSH, an SSDS algorithm that uses a retention policy to bound the index size. We showed that our Smooth retention policy increases recall of similar items compared to methods proposed by prior art.

In addition, our Stream-LSH indexing procedure is quality-sensitive, and is extensible to dynamically retain items according to their popularity.

While our work focuses on similarity search, our approach may prove useful in future work, for addressing space constraints in other stream-based search and recommendation primitives.

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