Crystallization of anisotropic colloids with a Yukawa potential

Fu-Jun Lin¹,², Jing-jing Liao², and Bao-quan Ai¹∗

¹ Guangdong Provincial Key Laboratory of Quantum Engineering and Quantum Materials, Guangdong-Hong Kong Joint Laboratory of Quantum Matter, School of Physics and Telecommunication Engineering, South China Normal University, Guangzhou 510006, China.
² School of Science, Jiangxi University of Science and Technology, Ganzhou 341000, China.

Abstract

Crystallization in a dense suspension of anisotropic spherical colloidal particles with a Yukawa potential is numerically investigated in a two-dimensional plane. It is found that a strong anisotropy can hinder the particles from crystallizing, while a weak anisotropy but super-strong coupling facilitates colloids to freeze into a hexagonal crystal. Different criterions are employed to describe the phase transition, one can find that a competition between anisotropic degree and coupling strength shall widened the transition region in the phase diagram, where the heterogeneous structures coexist, which render as a quasi-platform stretched across the probability distribution curve of the local order parameter. Our study maybe helpful for the experiments relating to the crystallizing behavior in statistical physics, materials science and biophysical systems.

* Email: aibq@scnu.edu.cn
I. INTRODUCTION

Phase transition is a universal phenomenon in nature as well as in industry, and plays an essential role in statistical physics, materials science, chemistry and biophysics. Crystallization represents the prime example of a phase transition and hence gets a lot of attention. Understanding the crystallization kinetics and microscale processes are not only important for fundamental research, but also for the applications associated with crystalline materials [1] and biotechnology [2,3].

Quantitative crystallization examinations are mostly based on the models of repulsive hard spheres (discs) [4-8], charged spheres [9-11], or attractive spheres [12,13]. As far as we know, it is challenging experimentally to investigate the particles dynamics on an atomic/molecular scale, due to the small size. Therefore, the colloidal particles, are usually viewed as the large atoms with tailorable size, shape and interactions [14], have been extensively studied to explore the physics of crystallization [15-20], because their thermal motion can be visualized by optical microscopy [21] and tracked by image processing [22]. The systematic studies can significantly enhance understanding and help controlling of the crystallization processes. A flowing crystalline structure, named “rheocrystal”, spontaneously forms by removing the particles in experimental work, supported by numerical simulations [23]. By doing colloidal glasses with small amounts of self-propelled particles, the crystallization dynamics can be speeded up [6]. Additionally, the molecular dynamics simulation of equal-sized hard-spheres systems, showing conventional nucleation and growth of crystals at concentrations near melting and crossing over to a spinodal-like regime, and a Monte Carlo ‘constrained aging’ method slows crystallization [24]. It is worth to mentioning that an excellent work on crystallization of monodisperse self-propelled colloidal particles at sufficiently high densities has been done by Bialk et.al. [15], which provides us the inspiration for doing the current work. Furthermore, the crystallization in system of binary hard spheres with different sizes have been reported in experiments [25,26] and theory [27,28], the results indicate that the small spheres can fit between large spheres to stabilize binary crystals. Bommineni et.al. [29] firstly perform the spontaneous formation of AlB$_2$, NaZn$_{13}$ and Laves phases with swap simulations, they point out that the binary hard sphere crystals grow robustly and reproducibly in standard event-driven-molecular dynamics, and the particles are integrated faster into the crystal due to the enhanced diffusion.

Most of the works on particle crystallization mentioned above have focused on spherical or isotropic particles. However, few natural particles have the perfect symmetry. Anisotropy is an intrinsic property, including the shape (e.g., ellipsoids, rods, or cubes), interaction (e.g., janus) or diffusion (e.g., polar particles) anisotropy. Thus relevant researches are necessary and valuable. The needlelike ellipsoidal particles under the action of an external potential exhibit complex motion and destroy the directed transport [30,31]. Triblock Janus colloids, are viewed as potential
building blocks for functional materials, their inherent anisotropy could induce self-assembly at a certain condition, have been used to measure the crystal growth kinetics [32]. A variety of crystalline structures have been discovered by varying the shape anisotropy of spheroids [33]. Predictably, simulations [34] of a generalized Vicsek model, report a mutual exclusion of the polar and the structural order. We are motivated by these works to understand how the ability of particles to form crystalline structures is affected by the anisotropy. In the paper, we consider a spherical colloidal particle with different mobilities in orthogonality direction, and the interaction between particles is determined by the repulsive Yukawa potential.

II. MODEL AND METHODS

In this section, we mainly review the derivation of the equations for Brownian motion of an anisotropic spherical particle in two-dimension. Due to the particle anisotropy, the rotational and translational motion are always coupling in the lab-frame coordinates, which makes the analysis of relevant issues very complicated. For simplicity, we shall initially describe the motion of particle in body-fixed coordinates where the rotational and translational motion are decoupled. Then, the particle’s position vector \( \mathbf{S}(t) \) of its center of mass at a given time \( t \) can be decomposed in the body frame as \((\delta \tilde{x}, \delta \tilde{y})\), corresponding to the coordinates \((\delta x, \delta y)\) in the lab frame. \( \theta(t) \) is the angle between the \( x \) axis of the lab frame and the \( \tilde{x} \) axis of the body frame. Thus, it can be easily obtained from FIG.1(b) that the Langiven equations of overdamped motion of the anisotropic particle in the body frame are [35,36]

\[
\frac{1}{\mu_i} \frac{\partial \delta \tilde{x}}{\partial t} = F_x \cos \theta(t) + F_y \sin \theta(t) + \delta \dot{\eta}_1(t),
\]

\[
\frac{1}{\mu_i} \frac{\partial \delta \tilde{y}}{\partial t} = -F_x \sin \theta(t) + F_y \cos \theta(t) + \delta \dot{\eta}_2(t),
\]
\[
\frac{1}{\mu_2} \frac{\partial \tilde{y}}{\partial t} = F_x \cos \theta(t) - F_y \sin \theta(t) + \tilde{\eta}_x(t), \tag{2}
\]

\[
\frac{1}{\mu_3} \frac{\partial \theta(t)}{\partial t} = \tilde{\eta}_3(t). \tag{3}
\]

Where \(\mu_1, \mu_2\) are the mobilities along \(\tilde{x}\) and \(\tilde{y}\) axis, respectively. \(\mu_3\) is rotational mobility. The noise \(\tilde{\eta}_i\) in the body frame has mean zero and correlations

\[
\langle \tilde{\eta}_i(t) \tilde{\eta}_j(t') \rangle = \frac{2k_BT}{\mu_i} \delta_{ij} \delta(t-t'), \quad i, j = 1, 2, 3, \tag{4}
\]

with \(T\) is temperature and \(k_B\) is the Boltzmann constant. \(F_x\) and \(F_y\) are the components of the force along the \(x\) and \(y\) axis of lab frame. For convenience, the energy, length and time are normalized by \(k_B T\), \(\rho^{-1/2}\) and \((\rho D)^{-1}\), respectively. Here \(\rho\) is the number density and \(D_0\) is the bare diffusion coefficient. For a many-particle system, particles are considered to interact pairwise through the repulsive Yukawa potential: \(u(r) = \frac{\Gamma e^{-\lambda r}}{r}\), with screening length \(\lambda^{-1}\), and the dimensionless coupling strength \(\Gamma = V_0 \sqrt{\rho / k_B T}\), with the bare potential strength \(V_0\). Thus, the force acts on the \(i\)th particle is \(F_i = -\nabla \Sigma_{i,j} u(r_i - r_j)\). We now translate the Eqs.(1)-(3) into that in lab-frame coordinates by means of a straightforward rotation of coordinates, the displacement relationship between the two frames is represented as following:

\[
\delta x = \delta \tilde{x} \cos \theta - \delta \tilde{y} \sin \theta, \tag{5}
\]

\[
\delta y = \delta \tilde{x} \sin \theta + \delta \tilde{y} \cos \theta. \tag{6}
\]

By a series of manipulations, the Langiven equations in the lab frame of overdamped motion of the anisotropic particle can be obtained as following:

\[
\frac{\partial x(t)}{\partial t} = F_x \left( \bar{\mu} + \Delta \bar{\mu} \cos 2\theta(t) \right) + \Delta \bar{\mu} F_y \sin 2\theta(t) + \eta_x(t), \tag{7}
\]

\[
\frac{\partial y(t)}{\partial t} = F_y \left( \bar{\mu} - \Delta \bar{\mu} \cos 2\theta(t) \right) + \Delta \bar{\mu} F_y \sin 2\theta(t) + \eta_y(t), \tag{8}
\]

\[
\frac{1}{\mu_3} \frac{\partial \theta(t)}{\partial t} = \eta_3(t). \tag{9}
\]

Where \(\bar{\mu} = \frac{1}{2}(\mu_1 + \mu_2)\) is the average of the mobility, and \(\Delta \bar{\mu} = \frac{1}{2}(\mu_1 - \mu_2)\) determines the anisotropy of the particle, \(\Delta \bar{\mu} = 0\) indicates that the particle is isotropic, and while \(\Delta \bar{\mu} \to \bar{\mu}\), the particle is prone to diffuse along \(\tilde{x}\) axis. The noise \(\eta_i(t)\) in the lab frame has mean zero and satisfies [35,36]:
\[
\langle \eta_i(t) \eta_j(t') \rangle = 2 D_\theta \delta(t-t'),
\]
\[
\langle \tilde{\eta}_i(t) \tilde{\eta}_j(t') \rangle^{\eta_j} = 2 k_B T \mu_{ij} \delta(t-t'), 
\]
with
\[
\mu_{ij} = \mu_0 \delta_{ij} + \Delta \mu \begin{pmatrix}
\cos 2\theta & \sin 2\theta \\
\sin 2\theta & -\cos 2\theta
\end{pmatrix},
\]
where \( D_\theta = k_B T \mu_0 \) is the rotational diffusion coefficient, describing the angular fluctuation. The superscripts of statistical averages in Eq.(11) mean over which the noises are averaged, and the subscript quantity is kept fixed.

In this paper, we focus on the influence of the particles’ anisotropic degree on the crystallization behavior. Defining \( \Delta \mu = \Delta \mu / \mu_0 \in [0,1] \) to describe the anisotropy, The diffusion coefficient of the particle are \( D_1 = k_B T \mu_1 \) and \( D_2 = k_B T \mu_2 = \left[ (1-\Delta \mu) / (1+\Delta \mu) \right] D_1 \) respectively. We model a system with \( N \) spherical particles of radius \( r_0 \) moving in a 2D box of size \( L_x \times L_y \) with periodic boundary conditions, and numerically investigate the dynamic behavior of the crystallization by integration of the Langevin equations (7)-(9) using the second-order stochastic Runge-Kutta algorithm.

### III. NUMERICAL RESULTS AND DISCUSSION

In our simulations, we choose the same parameters as in Ref.[15]: the total particle number \( N=1936 \), the particle radius \( r_0 = 0.5 \) the rotational diffusion coefficient \( D_\theta = 3.0 \) and the translational diffusion coefficient \( D_t = 1 \), the inverse screening length \( \lambda = 3.5 \). \( L_x / L_y = 2 / \sqrt{3} \) is chosen such that the particles can crystallize into the perfect hexagonal crystal. The initial position and orientation of particles are random. The integration time step \( dt \) is chosen to be smaller than \( 10^{-4} \) and the total integration time as great than \( 10^3 \), which ensure that the system reaches a steady state and the simulation results will not depend on the integration time and time step.

The structural transitions between the different phases can be characterized by the global bond-orientational order parameter \[ \psi_6 \equiv \left( \frac{1}{N} \sum_{i=1}^{N} q_6(i) \right)^2, \]
\[ q_6 = \frac{1}{6} \sum_{j \in \Pi(i)} e^{i \theta_{ij}}, \]
where \( \Pi(i) \) is the set of the six nearest neighbors of the \( i \)th particle and \( \theta_{ij} \) is angle between the vector from particle \( i \) to \( j \) and an predefined direction. For a perfect crystal \( \psi_6 = 1 \) whereas in a disordered phase \( \psi_6 \to 0 \) Following Schweigert et al. [38], the liquid-to-solid transition can be identified by a jump of the order parameter.
above a value of \( \psi_6 = 0.45 \) (see also Ref. [15]). Meanwhile, a dynamical criterion for crystalline process of the anisotropic particles is determined by the abrupt drop of the long-time diffusion coefficient

\[
D = \lim_{t \to \infty} \frac{1}{4} \langle |\Delta r(t)|^2 \rangle,
\]

(14)

with \( \Delta r(t) = r(t) - r(0) \). The value of 0.086 for \( D \) has been proved by Löwen (1996) to be ‘universal’, which does not matter whether freezing occurs continuously via a hexagonal phase or is a conventional first-order transition[39], thus we employ this criterion to describe the dynamical phase transition.

We firstly examine the effect of anisotropic degree \( \Delta \mu \) of particles on the phase transition by monitoring the global bond-orientational order parameter \( \psi_6 \) and the long-time diffusion coefficient \( D \) for different \( \Gamma \) in FIG.2. At the beginning of the

FIG. 2. Cooling curves for (a) the bond-orientational parameter \( \psi_6 \) and (b) the long-time diffusion coefficient \( D \) versus particles’ anisotropic degree \( \Delta \mu \) for different \( \Gamma \). The crossings with the dashed horizontal lines define the position of the structural transition \( \Delta \mu^*_s \) (\( \psi_6 \approx 0.45 \)) and the dynamical freezing \( \Delta \mu^*_D \) (\( D=0.086 \)), respectively.

FIG. 3. Cooling curves for (a) the bond-orientational parameter \( \psi_6 \) and (b) the long-time diffusion coefficient \( D \) versus coupling strength \( \Gamma \) at \( \Delta \mu = 0.5 \).
simulation, a particle configuration with random particle positions and orientations is employed. From FIG.2(a) we find that for a given $\Gamma$, the particles can freeze into an ordered crystalline phase in the small $\Delta \mu$ region, especially for isotropic particles ($\Delta \mu = 0$), $\psi_6$ has its maximum value. With increasing $\Delta \mu$, the Brownian diffusion along a specific axis of a particle becomes increasingly dominant, the random motion hinders system from assembling an ordered structure. Therefore, the order parameter drops to below $\psi_6 = 0.45$ at $\Delta \mu = \Delta \mu^*_D$, indicating a loss of the long-range orientational order in the system, and the system transiting into the liquid phase[40]. Figure 2(b) exhibits dynamical phase transition for different $\Gamma$, the diffusion coefficient increases abruptly near the phase transition point $\Delta \mu^*_D$ which gives the lower bound to liquid region, and even exceeds that of a free passive Brownian particle. Obviously, the phase transition point is significantly shifted to the large $\Delta \mu$ with $\Gamma$ increasing, which is determined by the competition between the particles’

FIG. 4. Snapshots of particle configurations for $\Delta \mu = 0$ (left column), 0.5 (middle column) and 0.9 (right column), the rows correspond to constant $\Gamma$, from top to bottom $\Gamma = 200(0.2886, 0.2806, 0.2014), 700(0.7436, 0.6160, 0.2636)$ and $1400(0.8627, 0.8128, 0.2982)$. Particles are colored according to their $q_6$. 

anisotropic degree and the coupling strength. As shown in FIG.3, with $\Gamma$ increasing, the corresponding temperature of the system is decreased, which leads to the dense colloids gradually freeze into an ordered configuration and the diffusion coefficient drops toward zero.

To effectively distinguish liquidlike from ordered regions, we utilize an order parameter $\bar{q}_b = \text{Re} \sum_{j \in \Pi(i)} \frac{1}{6} \sum_{2} q_b(i) q_b^*(j)$ per particle to describe the local behavior of its neighbours [41]. The snapshots of particle configurations for $\Delta \mu = 0$, 0.5 and 0.9 with different $\Gamma$ are illustrated in FIG.4. Clearly, for either small coupling strength (e.g. $\Gamma = 200$) or large anisotropic degree (e.g. $\Delta \mu = 0.9$), the system always stay liquid with small order parameter. Whereas for the isotropic particles ($\Delta \mu = 0$), the system crystallize into hexagonal structures with $\psi_6 > 0.8$ under strong coupling.

![FIG. 5. (a) Time dependence of the Lindemann parameter for $\Gamma = 1000$. (b) Phase diagram in the $\Gamma - \Delta \mu$ plane.](image)

To assess in more detail the phase transition, we consider a process of melting starting from a perfect hexagonal crystal, introduce the Lindemann-like parameter[42]

$$\gamma_L(t) = \frac{\left| \Delta \mathbf{r}_i(t) - \Delta \mathbf{r}_j(t) \right|^2}{2 \ell^2},$$

(15)
as a criterion to decide the upper bound to solid region. Where the subscripts $i$ and $j$ donate two particles that are initially neighbors, and the lattice spacing of the hexagonal crystal $\ell \equiv 2^{1/2} 3^{-1/4} \approx 1.075$. This criterion states that the melting commences once the vibrational displacement of a particle exceeds a certain fraction of the lattice spacing [15]. In our case, both the small coupling strength and large particles anisotropic degree can induce particles to vibrate sharply, the former has been explicated in Ref.[15]. We now mainly focus on the time dependence of the Lindemann-like parameter for different $\Delta \mu$ at a given $\Gamma$, e.g., $\Gamma = 1000$ as shown in FIG.5(a). It is found that, in the liquid region, the curves of Lindemann-like parameter are abruptly divergent over time. Especially for $\Delta \mu > 0.65$, due to the
existence of intensive diffusion in a special direction, the particles easily escape from their lattice position, and the crystal structure is destroyed. When $\Delta \mu = 0.6$, one can find a quasi-plateau with Lindemann-like parameter $\gamma_L$, we defined this value $\Delta \mu^*_L$ as melting point. Based on these above, the Phase diagram is mapped in FIG.5(b). Clearly, the curves of $\Delta \mu^*_S$ and $\Delta \mu^*_D$ mostly coincide in all parameter space, which indicates that the two criterions we employed are compatible with our model. Note that in the region with large $\Gamma$ and small $\Delta \mu$, the particles crystalized into hexagonal structure, whereas in the small $\Gamma$ or large $\Delta \mu$ region, the bond-orientational order is destroyed due to long-time diffusion. Moreover, there is a transition regime between liquid and crystal, which is characterized by a high structural order and low but nonvanishing diffusion [15]. It is clear that the transition regime widens in the region of moderate parameter space, which we regard as the result of the competition between the particles’ anisotropic degree and coupling strength, as what shows in FIG.2.

We begin by briefly reviewing our results to characterize the solid, liquid and transition regions have been shown in FIG.5(b). Figure 6(a) illustrates the probability distribution for $\bar{q}_6$ with different $\Gamma$ at $\Delta \mu = 0.5$. One can find that the system is less structure in liquid region($\Gamma = 200$), while in the solid region ($\Gamma = 1400$), the particles are well-crystallized with a few “bubbles” due to the long-range bond-orientational order. Notably, For the transition region ($\Gamma = 700$), there is a broad peak probability distribution for $\bar{q}_6$, corresponding to a coexistence phase as the second snapshot of middle column shown in FIG.4. Figure 6(b) displays the probability distributions of $\bar{q}_6$ at $\Delta \mu = 0, 0.5$ and 0.9 for four different global $\psi_6$ value. As what has mentioned that when $\Delta \mu = 0.9$, the system is always disordered ($\psi_6 < 0.3$), thus there is only one curve. One can easily find that the probability distributions of $\bar{q}_6$ are less dependent on $\Delta \mu$. It should be emphasized, a quasi-platform stretched across the distribution curve of $\psi_6 = 0.45$, which means that the transition regime is occupied by a variety of
heterogeneous structures.

IV. CONCLUDING REMARKS

To summarize, we have numerically investigated the crystallization in a dense suspension of spherical colloids with different mobilities in an orthogonal direction. Starting from a random particle configuration, we integrate the Langivin equations using the second-order stochastic Runge-Kutta algorithm in a two-dimensional box with periodic boundary condition. By employing different criterions, the phase diagram in the $\Gamma - \Delta \mu$ plane is obtained. It is found that the anisotropy hinders particles from freezing into a crystalline structure. Especially for a large $\Delta \mu$ or small $\Gamma$ regime, the system always keeps liquid phase. However, the competition between $\Delta \mu$ and $\Gamma$ widened the transition region in the moderate parameter space, in which the suspension is overall ordered but with a lot of heterogeneous structures. Furthermore, our results indicate that the anisotropic colloidal spheres in a dense suspension can be frozen into perfect hexagonal crystals in the case of weak anisotropy and strong coupling.

ACKNOWLEDGMENTS

This work was supported in part by the National Natural Science Foundation of China (Grant No.12075090, 11905086, 11575064), the GDUPS (2016), and the Natural Science Foundation of Guangdong Province (Grant No. 2017A030313029), and the Major Basic Research Project of Guangdong Province (Grant No. 2017KZDXM024), and the Natural Science Foundation of Jiangxi Province (Grant No. 20192BAB212006 and No. GJJ191598), and High-level Scientific Research Foundation for the introduction of talent of Jiangxi University of Science and Technology.

[1] D. Li, H. Zhou and I. Honma, Nat. Mater. 3, 65 (2004).
[2] H. Liu, S. K. Kumar, J. F. Douglas, Phys. Rev. Lett. 103, 018101 (2009).
[3] H. X. Lin, S. Lee, L. Sun, M. Spellings, M. Engel, S. C. Glotzer and C. A. Mirkin, Science 355, 931 (2017).
[4] E. Zaccarelli, C. Valerians, E. Sanz, W. C. K. Poon, M. E. Cates, P.N. Pusey, Phys. Rev. Lett. 103, 135704 (2009).
[5] E. Sanz, C. Valeriana, E. Zaccarelli, W. C. K. Poon, P.N. Pusey, M.E. Cates, Phys. Rev. Lett. 106, 215701 (2011).
[6] R. Ni, M. A. Cohen Stuart, M. Dijkstra and P. G. Bolhuis. Soft Matt. 10, 6609 (2014).
[7] G. Briand, O. Dauchot, Phys. Rev. Lett. 117, 098004 (2016).
[8] A. Mori, Crystals, 7, 102 (2017).
[9] S. Auer, D. Frenkel, J. Phys-Condens. Matter 14, 7667, (2002).
[10] D. Reinke, H. Stark, H. H. von Gr¨unberg, A. B. Schofield, G. Maret and U. Gasser, Phys. Rev. Lett. 98 038301, (2007).
[11] F. Smallenburg, N. Boon, M. Kater, M. Dijkstra, and R. van Roij, J. Chem. Phys. 134, 074505 (2011).
[12] V. J. Anderson, H. N. W. Lekkerkerker, Nature, 416, 811 (2002).
[13] W. C. K. Poon, J. Phys. Condens. Matter 14, R859 (2002).
[14] D. Frenkel, Science, 296, 65 (2002).
[15] Z. Wang, A. M. Alsayed, A. G. Yodh and Y. Han, J. Chem. Phys. 132, 154501 (2010).
[16] J. Bialk´e, T. Speck and H. L¨owen, Phys. Rev. Lett. 108, 168301 (2012).
[17] T. Palberg, J. Phys. Condens. Matter, 26, 333101 (2014).
[18] E. Allahyarov, K. Sandomirski, S. U. Egelhaaf, H. L¨owen, Nat. Commun. 6, 7110 (2015).
[19] B. Li, D. Zhou, Y. Han, Nat. Rev. Mater. 1, 15011 (2016).
[20] D. P. Singh, U. Choudhury, P. Fischer and A. G. Mark, Adv. Mater. 29, 1701328 (2017).
[21] A. Kose, M. Ozaki, K. Takano, Y. Kobayashi, S. J. Hachisu, Colloid Interface Sci. 44, 330 (1973).
[22] J. C. Crocker, D. G. Grier, J. Colloid Interface Sci. 179, 298 (1996).
[23] G. Briand, M. Schindler and O. Dauchot, Phys. Rev. Lett. 120, 208001 (2018).
[24] C. Valeriani, E. Sanz, E. Zaccarelli, W. C. K. Poon, M. E. Cates, P. N. Pusey, J. Phys. Condens. Matter 23, 194117 (2011).
[25] A. B. Schofield, P. N. Pusey, and P. Radcliffe, Phys. Rev. E 72, 031407 (2005).
[26] N. Schaertl, D. Botin, T. Palberg, and E. Bartsch, Soft Matter 14, 5130 (2018).
[27] A. P. Hynninen, L. Filion, and M. Dijkstra, J. Chem. Phys. 131, 064902 (2009).
[28] M. Dijkstra, Adv. Chem. Phys. 156, 35 (2014).
[29] P. K. Bommineni, M. Klement, and M. Engel, Phys. Rev. Lett. 124, 218003 (2020).
[30] B. Q. Ai and J. C. Wu, J. Chem. Phys. 140, 094103 (2014).
[31] J. J. Liao, W. J. Zhu, B. Q. Ai, Phys. Rev. E 97, 062151 (2018).
[32] W. F. Reinhart, A. Z. Panagiotopoulos, J. Chem. Phys. 148, 124506 (2018).
[33] W. Jin, H. K. Chan, Z. Zhong, Phys. Rev. Lett. 124, 248002 (2020).
[34] C. A. Weber, C. Bock, and E. Frey, Phys. Rev. Lett. 112, 168301 (2014).
[35] Y. Han, A. M. Alsayed, M. Nobili, J. Zhang, T. C. Lubensky, and A. G. Yodh, Science, 314, 626 (2006); Y. Han, A. Alsayed, M. Nobili, and A. G. Yodh, Phys. Rev. E 80, 011403 (2009).
[36] R. Grima and S. N. Yaliraki, J. Chem. Phys. 127, 084511 (2007); R. Grima, S. N.
Yaliraki, and M. Barahona, J. Phys. Chem. B 114, 5380 (2010).
[37] B. I. Halperin and D. R. Nelson, Phys. Rev. Lett. 41, 121(1978).
[38] I. V. Schweigert, V. A. Schweigert, and F. M. Peeters, Phys. Rev. Lett. 82, 5293 (1999).
[39] H. Löwen, Phys. Rev. E 53, R29 (1996).
[40] P. Hartmann, G. J. Kalman, Z. Donko and K. Kutasi, Phys. Rev. E 72, 026409 (2005).
[41] W. Lechner and C. Dellago, J. Chem. Phys. 129, 114707 (2008).
[42] K. Zahn and G. Maret, Phys. Rev. Lett. 85, 3656 (2000).