Neutralino–Nucleus Elastic Scattering
in the MSSM with Explicit CP Violation

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Abstract

We re–investigate the neutralino–nucleus elastic scattering as a promising dark matter detection mechanism including contributions from the scalar–pseudoscalar mixing of neutral Higgs states and the induced phase between two Higgs doublets due to the CP–violating phases of the scalar top and bottom sectors in the minimal supersymmetric standard model. The spin–dependent part of the cross section turns out to be hardly affected by the CP–violating induced phase due to a mutually destructive unavoidable suppression mechanism of various relevant supersymmetric parameters. On the other hand, although the phase $\Phi_{\mu}$ of the higgsino mass parameter is set to be zero, the spin–independent part, which can dominate over the spin–dependent part for heavy nuclei, can be strongly dependent on the CP–violating scalar–pseudoscalar mixing and the induced phase, in particular, for a large $|\mu|$, a small charged Higgs boson mass, and a large trilinear term $|A|$ compared to the SUSY breaking scale. For a small $\tan \beta$ and a large $|\mu|$, the spin–independent cross section is enhanced by an order of magnitude as the phase $\Phi_A$ of the trilinear term increases up to $\pi$, while for a large $\tan \beta$ and a large $|\mu|$ the spin–independent cross section is significantly suppressed for non–zero values of the phase $\Phi_A$.

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I. INTRODUCTION

Supersymmetry (SUSY) is one of the most promising theoretical frameworks for a successful unification of gravity with all other fundamental forces and is the most appealing perturbative solution to the gauge hierarchy problem of the standard model (SM). However, SUSY is not an exact symmetry of nature. The minimal supersymmetric SM (MSSM) \cite{1}, the minimal SUSY realization, must break SUSY softly in order to accomplish agreement with experimental observations and its breaking scale should not be much larger than a few TeV in order to retain naturalness. In general, the SUSY breakdown introduces a large number of unknown parameters, many of which can be complex \cite{2}. CP–violating phases associated with sfermions of the first and, to a lesser extent, second generations are severely constrained by bounds on the electric dipole moments of the electron, neutron and muon. The present experimental upper bounds on the neutron EDM $d_n$ and electron EDM $d_e$ are very tight \cite{3}: $|d_n| < 1.12 \times 10^{-25}$ e cm and $|d_e| < 0.5 \times 10^{-26}$ e cm at the 2–σ level. As a result of the CP crises, some fine–tuning mechanisms are necessary in generic supersymmetric theories to avoid these problems. There have been several phenomenologically attractive solutions \cite{4,5} to evade these constraints without suppressing the CP–violating phases. One option is to make the first two generations of scalar fermions rather heavy so that one–loop EDM constraints are automatically evaded. As a matter of fact one can consider so–called effective SUSY models \cite{5} which seem to combine all healthy features of both the MSSM and technicolor theories. The main virtue of the effective SUSY model is that any non–SM source of CP violation and FCNC involving the first two generations is suppressed by allowing their respective soft–SUSY–breaking masses to be as high as 20 TeV, whereas third generation scalar quarks and leptons may naturally be light well below the TeV scale. Another possibility is to arrange for partial cancellations among various contributions to the electron and neutron EDM’s \cite{6}.

Following the suggestions that the CP–violating phases do not have to be always suppressed, many important works on the effects due to the CP–violating phases in the MSSM have been already reported; the effects are very significant in extracting the parameters in the SUSY Lagrangian from experimental data \cite{7}, estimating dark matter densities and scattering cross sections and Higgs boson mass limits \cite{8,9}. CP violation in the $B$ and $K$ systems \cite{10}, and so on. In particular, it has been found \cite{10} that the Higgs–sector CP violation induced via loop corrections of soft CP–violating Yukawa interactions may drastically modify the couplings of the light neutral Higgs boson to the gauge bosons. As a result, the production cross sections as well as the decay branching ratios \cite{11} of neutral Higgs bosons changes so significantly. One of the crucial modifications is that the current experimental lower bound on the lightest Higgs boson mass may be dramatically relaxed up to a 60–GeV level in the presence of large CP violation in the Higgs sector of the MSSM. Moreover, the explicit CP violation in the MSSM Higgs sector radiatively induces a finite unremovable misalignment \cite{12} between two Higgs doublets. This additional phase can be as large as the original CP phases in certain portions of the MSSM parameter space and affect the chargino and neutralino systems. Therefore, it would be very important to re–visit all the relevant phenomena by including the CP–violating induced phase while avoiding the severe constraints from the neutron and electron EDMs. In particular, one of the most phenomenologically interesting subjects is to investigate the possibility of detecting lightest neutralinos
when a large portion of the dark matter in the Universe is composed of the lightest and (almost) stable lightest supersymmetric particle (LSP).

The observations of the dynamics of galaxies and clusters of galaxies [14], and the constraints on the baryon density from big bang nucleosynthesis [14] requires the existence of a considerable amount of non-baryonic dark matter. Therefore, it is almost universally accepted that most of the mass in the Universe and most of the mass in the Galactic halo is dark and the dark matter consists of some new, as yet undiscovered, weakly–interacting massive particle (WIMP). Of the many WIMP candidates, one of the best motivated and the most theoretically developed is the lightest neutralino, the lightest supersymmetric particle (LSP) in most SUSY theories. In this light, there have been an intensive investigation for its detection and identification [16]. In the present work, we re–investigate the LSP–nucleus elastic scattering process in the MSSM framework with R-parity and with CP–violating complex parameters. There have been already several works [9] on the effects of the phase $\Phi_\mu$ of the higgsino mass parameter $\mu$ on the neutralino–nucleus elastic scattering as well as the neutralino relic density [17]. So, referring to those works for the effects from the phase $\Phi_\mu$, we will mainly concentrate on the impact of the scalar–pseudoscalar mixing and the induced phase between two Higgs doublets in the MSSM Higgs sector stemming from the radiative corrections due to the CP–violating scalar top and bottom sectors in the MSSM on the neutralino–nucleus elastic scattering. For a more concrete, quantitative investigation, our analysis through the paper is based on a specific scenario with the following assumptions:

- The first and second generation sfermions are very heavy so that they are decoupled from the theory. In this case, there are no constraints on the CP–violating phases from the neutron and electron EDMs. On the other hand, the annihilation of the neutralinos into tau pairs through the exchange for relatively light scalar tau leptons guarantees that the cosmological constraints on the dark matter densities be satisfied.

- The explicit CP violation in the Higgs sector through the CP–violating radiative corrections from the scalar top and bottom sectors is included.

- Simultaneously, the effects of the induced CP–violating phase between two Higgs doublets on the chargino and neutralino systems are explicitly included.

- It is necessary to avoid the possible constraints from the so–called Barr-Zee–type diagrams [18] to the electron and neutron EDMs as well as from the null results of the Higgs boson searches at LEP [19]. We take two values 3 and 30 for $\tan \beta$, the ratio of the vacuum expectation values of two neutral Higgs fields in the present analysis.

Certainly, the major issue concerning the supersymmetric dark matter is its detection and identification. Indeed, there are a multitude of ongoing experiments involved in the direct and indirect detection of dark matter, many with a specific emphasis on searching for supersymmetric dark matter [10]. The event rates for either direct or indirect detection depend crucially on the LSP-nucleon, or LSP-nucleus, cross-section. Because the neutralinos have Majorana mass terms, their interactions with matter are generally spin dependent, coming from an effective interaction term of the form $(\bar{\chi}_i \gamma^\mu \gamma_5 \chi_j)(\bar{f}_j \gamma_\mu \gamma_5 f_i)$. In the regions of the MSSM parameter space where the LSP is a mixture of both gaugino and Higgsino
components, there is also an important contribution to the scattering cross-section due to a term in the interaction Lagrangian of the form $(\bar{\chi}\chi)(\bar{f}f)$ which is spin independent. These terms are particularly important for scattering off of large nuclei, where coherent nucleon scattering effects can quickly come to dominate all others. The scalar–pseudoscalar mixing and the modified couplings of the neutral Higgs bosons to fermions and neutralinos affect the spin–independent part while the CP–violating induced phase affects both the spin–dependent and the spin–independent parts through its modifications of the structure of the neutralino mass matrix. In this light it is worthwhile to make a systematic investigation of the effects of the CP–violating phases on the neutralino–nucleus scattering cross section, which is the goal of the present work.

The organization of the present paper is as follows. In Section II, a brief review on the explicit CP violation in the Higgs sector is given following the work by Pilaftsis and Wager. Section III is devoted to a detailed analysis of various effects of the CP–violating induced phase between two Higgs doublets on the chargino and neutralino sectors. Then we give the fully analytic expressions for the spin–dependent and spin–independent neutralino–nucleus scattering cross sections in Section IV and give a detailed numerical analysis of the dependence of the cross sections on the CP–violating phases as well as real SUSY parameters such as $\tan \beta$, the size of the higgsino mass parameter $|\mu|$ and the trilinear terms $|A_{t,b}|$. Finally, we summarize our findings and conclude in Section V.

II. CP VIOLATION IN THE MSSM HIGGS SECTOR

A. CP–violating radiative corrections

The MSSM introduces several new parameters in the theory that are absent from the SM and could, in principle, possess many CP–violating phases. Specifically, the new CP phases may come from the following parameters: (i) the higgsino mass parameter $\mu$, which involves the bilinear mixing of the two Higgs chiral superfields in the superpotential; (ii) the soft SUSY–breaking gaugino masses $M_a$ ($a = 1, 2, 3$), where the index $a$ stands for the gauge groups $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$, respectively; (iii) the soft bilinear Higgs mixing masses $m_{12}^2$, which is sometimes denoted as $B\mu$ in the literature; (iv) the soft trilinear Yukawa couplings $A_f$ of the Higgs particles to scalar fermions; and (v) the flavor mixing elements of the sfermions mass matrices. If the universality condition is imposed on all gaugino masses at the unification scale $M_X$, the gaugino masses $M_a$ have a common phase, and if the diagonal boundary conditions are added to the universality condition for the sfermion mass matrices at the GUT scale, the flavor mixing elements of the sfermions mass matrices vanish and the different trilinear couplings $A_f$ are all equal, i.e. $A_f = A$.

The conformal–invariant part of the MSSM Lagrangian has two global $U(1)$ symmetries; the $U(1)_Q$ Peccei–Quinn symmetry and the $U(1)_R$ symmetry acting on the Grassmann–valued coordinates. As a consequence, not all CP–violating phases of the four complex parameters $\{\mu, m_{12}^2, M_a, A\}$ turn out to be physical, i.e. two phases may be removed by redefining the fields accordingly. Employing the two global symmetries, one of the Higgs doublets and the gaugino fields can be rephased such that $M_a$ and $m_{12}^2$ become real. In this case, $\arg(\mu)$ and $\arg(A)$ are the only physical CP–violating phases in the low–energy
MSSM supplemented by universal boundary conditions at the GUT scale. Denoting the scalar components of the Higgs doublets \( H_1 \) and \( H_2 \) by \( H_1 = -i\tau_2 \Phi_1^+ \) (\( \tau_2 \) is the usual Pauli matrix) and \( H_2 = \Phi_2 \), the most general CP–violating Higgs potential of the MSSM can be conveniently described by the effective Lagrangian

\[
\mathcal{L}_V = \mu_2^2(\Phi_1^+\Phi_1) + \mu_1^2(\Phi_2^+\Phi_2) + m_2^2(\Phi_1^+\Phi_1) + m_1^2(\Phi_2^+\Phi_2) + m_{12}^2(\Phi_1^+\Phi_2) + m_{12}^2(\Phi_2^+\Phi_1) \\
+ \lambda_1(\Phi_1^+\Phi_1)^2 + \lambda_2(\Phi_2^+\Phi_2)^2 + \lambda_3(\Phi_1^+\Phi_1)(\Phi_2^+\Phi_2) + \lambda_4(\Phi_1^+\Phi_2)(\Phi_2^+\Phi_1) \\
+ \lambda_5(\Phi_1^+\Phi_2)^2 + \lambda_5^*(\Phi_2^+\Phi_1)^2 + \lambda_6(\Phi_1^+\Phi_1)(\Phi_2^+\Phi_2) + \lambda_6^*(\Phi_1^+\Phi_1)(\Phi_2^+\Phi_1) \\
+ \lambda_7(\Phi_2^+\Phi_2)(\Phi_1^+\Phi_2) + \lambda_7^*(\Phi_2^+\Phi_2)(\Phi_1^+\Phi_1). \tag{1}
\]

In the Born approximation, the quartic couplings \( \lambda_{1,2,3,4} \) are solely determined by the gauge couplings and \( \lambda_{5,6,7} \) are zero. However, beyond the Born approximation, the quartic couplings \( \lambda_{5,6,7} \) receive significant radiative corrections from trilinear Yukawa couplings of the Higgs fields to scalar–top and scalar–bottom quarks. These parameters are in general complex and so lead to CP violation in the Higgs sector through radiative corrections. The explicit form of the couplings with radiative corrections can be found in Refs. [10][22].

It is necessary to determine the ground state of the Higgs potential to obtain physical Higgs states and their self–interactions. To this end we introduce the linear decompositions of the Higgs fields

\[
\Phi_1 = \left( \frac{1}{\sqrt{2}}(v_1 + \phi_1 + ia_1) \right), \quad \Phi_2 = e^{i\xi} \left( \frac{1}{\sqrt{2}}(v_2 + \phi_2 + ia_2) \right), \tag{2}
\]

with \( v_1 \) and \( v_2 \) the moduli of the vacuum expectation values (VEVs) of the Higgs doublets and \( \xi \) their CP–violating induced relative phase. These VEVs and the relative phase can be determined by the minimization conditions on \( \mathcal{L}_V \), which can be efficiently performed by the so–called tadpole renormalization techniques [10][23]. It is always guaranteed that one combination of the CP–odd Higgs fields \( a_1 \) and \( a_2 \) (\( G^0 = \cos \beta a_1 - \sin \beta a_2 \)) defines a flat direction in the Higgs potential and so it is absorbed as the longitudinal component of the \( Z \) boson. [Here, \( \sin \beta = v_2/\sqrt{v_1^2 + v_2^2} \) and \( \cos \beta = v_1/\sqrt{v_1^2 + v_2^2} \).] As a result, there exist one charged Higgs state and three neutral Higgs states that are mixed in the presence of CP violation in the Higgs sector. Denoting the remaining CP–odd state \( a = \sin \beta a_1 + \cos \beta a_2 \), the \( 3 \times 3 \) neutral Higgs–boson mass matrix describing the mixing between CP–even and CP–odd fields can be decomposed into four parts in the weak basis \( (a, \phi_1, \phi_2) \) :

\[
\mathcal{M}_H^2 = \begin{pmatrix} \mathcal{M}_P^2 & \mathcal{M}_{PS}^2 \\ \mathcal{M}_{SP}^2 & \mathcal{M}_S^2 \end{pmatrix}, \tag{3}
\]

where \( \mathcal{M}_P^2 \) and \( \mathcal{M}_S^2 \) describe the CP–preserving transitions \( a \rightarrow a \) and \( (\phi_1, \phi_2) \rightarrow (\phi_1, \phi_2) \), respectively, and \( \mathcal{M}_{PS}^2 = (\mathcal{M}_{SP}^2)^T \) contains the CP–violating mixings \( a \leftrightarrow (\phi_1, \phi_2) \). The analytic form of the sub–matrices can be found in Ref. [10].

On the other hand, the charged Higgs-boson mass \( m_{H^\pm} \) is related to the pseudoscalar mass term \( m_a \) as

\[
m_a^2 = m_{H^\pm}^2 - \frac{1}{2}\lambda_4 v^2 + R(\lambda_5 e^{2i\xi})v^2. \tag{4}
\]
Taking this very last relation between $m_{H^\pm}$ and $m_a$ into account, we can express the neutral Higgs–boson masses as functions of $m_{H^\pm}$, $\mu$, $A$, a common SUSY scale $M_{\text{SUSY}}$, $\tan\beta$, and the physical phase $\xi$, which cannot be rotated away in the presence of the chargino and neutralino contributions. Clearly, the CP–even and CP–odd states mix unless all of the imaginary parts of the parameters $\lambda_{5,6,7}$ vanish. Since the Higgs–boson mass matrix $M^2_H$ describing the scalar–pseudoscalar mixing is symmetric, we can diagonalize it by means of an orthogonal rotation $O$: $O^T M^2_H O = \text{diag}(m^2_{H_3}, m^2_{H_2}, m^2_{H_1})$ with the ordering of masses $m_{H_1} \leq m_{H_2} \leq m_{H_3}$. The neutral Higgs–boson mixing affects the couplings of the Higgs fields to fermions, gauge bosons, and Higgs fields themselves as shown in the following.

The CP–violating effects due to radiative corrections to the Higgs potential is characterized by a dimensionless parameter $\eta_{CP}$

$$\eta_{CP} = \frac{m^4_{\ell b}}{v^4} \left( \frac{|\mu||A|}{32\pi^2 M^2_{\text{SUSY}}} \right) \sin\Phi_{CP},$$  

(5)

where $\Phi_{CP} = \text{arg}(A\mu) + \xi$, i.e. the sum of three CP–violating phases. So, for $|\mu|$ and/or $|A|$ values larger than the SUSY–breaking scale $M_{\text{SUSY}}$, the CP–violating effects can be significant.

B. An induced CP–violating phase

As shown in the previous section, the first derivatives of the Higgs potential with respect to the neutral fields $\{\phi_1, \phi_2, a\}$ do not vanish any longer; hence, one has to redefine the Higgs doublet fields with a relative phase $\xi$. This CP–violating induced phase can be obtained analytically by combining the two relations for the minimization. First of all, we make use of the fact that a U(1)$_{PQ}$ rotation allows us to take $m^2_{12}$ to be real and for a notational convenience define $\tilde{\lambda}_6$, $\delta$, and $\delta'$:

$$\tilde{\lambda}_6 = \lambda_6 c^2_\beta + \lambda_7 s^2_\beta,$$

$$\delta = \left( \frac{m^2_{H^\pm}}{v^2} - \frac{\lambda_4}{2} + \lambda_5 \right) \sin 2\beta, \quad \delta' = \left( \frac{m^2_{H^\pm}}{v^2} - \frac{\lambda_4}{2} - \lambda_5 \right) \sin 2\beta. \quad (6)$$

Then, the CP–violating induced phase $\xi$ is determined by the relations;

$$\sin \xi = -\frac{1}{|\delta|^2} \left\{ R(\delta) \mathcal{I}(\tilde{\lambda}_6) - \mathcal{I}(\delta) \sqrt{|\delta|^2 - \mathcal{I}^2(\tilde{\lambda}_6)} \right\},$$

$$\cos \xi = +\frac{1}{|\delta|^2} \left\{ \mathcal{I}(\delta) \mathcal{I}(\tilde{\lambda}_6) + R(\delta) \sqrt{|\delta|^2 - \mathcal{I}^2(\tilde{\lambda}_6)} \right\}, \quad (7)$$

and the soft–breaking positive bilinear mass squared $m^2_{12}$ is given by

$$m^2_{12} = \frac{v^2}{2|\delta|^2} \left\{ \mathcal{I}(\delta \delta') \mathcal{I}(\tilde{\lambda}_6) + R(\delta \delta') \sqrt{|\delta|^2 - \mathcal{I}^2(\tilde{\lambda}_6)} - |\delta|^2 R(\tilde{\lambda}_6) \right\}. \quad (8)$$

We note that the induced phase vanishes if $\text{arg}(A\mu)$, the sum of the phases $\Phi_\mu$ and $\Phi_A$, vanishes, even if each of the CP–violating phases might not have to vanish. On the other hand,
the size of $\delta$ or $\delta'$ is proportional to the pseudoscalar mass $m_a$ to a very good approximation so that if it becomes large, i.e., decoupled, the induced phase $\xi$ is diminished. Since the size of the induced phase is also inversely proportional to $\sin 2\beta$, the phase grows with increasing $\tan \beta$. In general, this induced phase will remain as a non–trivial physical phase and lead to a modification in the chargino and neutralino mass matrices.

Analytically, the induced phase $\xi$ plays a role of rotating the vacuum expectation value $v_2$ into $v_2 e^{i\xi}$. So, the chargino mass matrix is given in the $(\tilde{W}^-, \tilde{H}^-)$ basis by

$$M_C = \begin{pmatrix} M_2 & -\sqrt{2} m_W c_\beta |\mu| e^{i\Phi_\mu} \\ \sqrt{2} m_W s_\beta e^{i\xi} & |\mu| e^{i\Phi_\mu} \end{pmatrix},$$

which is built up by the fundamental SUSY parameters; the SU(2) gaugino mass $M_2$, the higgsino mass parameter $|\mu|$, its phase $\Phi_\mu$ and the ratio $\tan \beta$ of the vacuum expectation values of the two neutral Higgs doublet fields. The gaugino mass $M_2$ was made real already by appropriate field redefinitions. Since the chargino mass matrix $M_C$ is not symmetric, two different unitary matrices acting on the left– and right–chiral $(\tilde{W}^-, \tilde{H}^-)$ states are needed to diagonalize the matrix:

$$U_{L,R} \begin{pmatrix} \tilde{W}^- \\ \tilde{H}^- \end{pmatrix} = \begin{pmatrix} \tilde{\chi}_1^- \\ \tilde{\chi}_2^- \end{pmatrix},$$

and the mass eigenvalues are given by

$$m_{\tilde{\chi}_{1,2}^\pm}^2 = \frac{1}{2} \left[ M_2^2 + |\mu|^2 + 2 m_W^2 \mp \Delta \right],$$

with $\Delta$ involving the phases $\Phi_\mu$ and the induced phase $\xi$:

$$\Delta = \left\{ (M_2^2 - |\mu|^2)^2 + 4 m_W^4 \cos^2 2\beta + 4 m_W^2 (M_2^2 + |\mu|^2 - M_2 |\mu| \sin 2\beta \cos(\Phi_\mu - \xi)) \right\}^{1/2}. \quad (12)$$

As a matter of fact, an additional field redefinition enables one to find that every CP–violating phenomenon due to the chargino mixing is dependent on the difference $\Phi_\mu - \xi$ of two phases $\Phi_\mu$ and $\xi$. Keeping in mind this point, one can infer the following aspects:

- Even if $\Phi_\mu$ vanishes there is still a source of CP violation due to the presence of the induced phase $\xi$ stemming from the CP–violating phases in the scalar top and bottom sectors.

- If both $\Phi_\mu$ and $\Phi_A$ vanish as in the CP–invariant theory, then the induced phase $\xi$ vanishes, leaving no source of CP violation.

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1We note that the vacuum expectation value of the Higgs doublet $H_1$ has an opposite sign to the conventional one in the literature. This is the reason why there appears a negative sign in the (12) component of the chargino mass matrix.
These effects may be investigated through the chargino pair production as soon as the collider energies become high enough to go over their production thresholds. However, this investigation is beyond the regime of the present work and so, it will be not touched upon here.

Similarly, the neutralino mass matrix describing neutralino mixing is modified by the introduction of the CP–violating induced phase. Following the same prescription for the chargino mass matrix yields the neutralino mass matrix in the \((\tilde{B}, \tilde{W}^3, \tilde{H}_0^1, \tilde{H}_0^2)\) basis:

\[
M_N = \begin{pmatrix}
|M_1| e^{i\Phi_1} & 0 & m_Z s_W c_\beta & m_Z s_W s_\beta e^{i\xi} \\
0 & M_2 & -m_Z c_W c_\beta & -m_Z c_W s_\beta e^{i\xi} \\
m_Z s_W c_\beta & -m_Z c_W c_\beta & 0 & -|\mu| e^{i\Phi_\mu} \\
m_Z s_W s_\beta e^{i\xi} & -m_Z s_W s_\beta e^{i\xi} & -|\mu| e^{i\Phi_\mu} & 0
\end{pmatrix}
\]

The neutralino mass matrix \(M_N\) is a complex but symmetric matrix so that it can be diagonalized by just one unitary matrix \(N\) such that \(N^* M_N N^\dagger = \text{diag}(m_{\tilde{\chi}^0_1}, m_{\tilde{\chi}^0_2}, m_{\tilde{\chi}^0_3}, m_{\tilde{\chi}^0_4})\) with the increasing ordering in masses. A simple orthogonality transformation of a phase matrix enables one to confirm that any physical observable related with the neutralino mixing depends only on the phase \(\Phi_1\) and the combination \(\Phi_\mu - \xi\) of the phases \(\Phi_\mu\) and \(\xi\) as in the chargino system. So, it is clear that except for the phase \(\Phi_1\) the neutralino system exhibits a similar dependence on the phase \(\Phi_\mu\) and the induced phase \(\xi\). In the present work, we take the assumption of gaugino mass unification for which the phase \(\Phi_1\) should be zero at least up to one–loop level against the renormalization group running from the unification scale to the electroweak scale. In this scenario, there exists only one CP–violating rephasing–invariant phase \(\Phi_\mu - \xi\) in the chargino and neutralino mass sectors.

### III. NEUTRALINO–NUCLEUS ELASTIC SCATTERING

#### A. Four–Fermi effective Lagrangian

In this section we investigate the importance of the CP-violating phases on the elastic scattering cross-sections of neutralinos on nuclei. To this end, we calculate the four-Fermi effective \(\chi\)-quark interaction Lagrangian with the inclusion of the CP violating phases \(\Phi_\mu\), \(\Phi_1\), and \(\xi\) for the standard spin–dependent and spin–independent neutralino–nucleon interactions. There are two types of diagrams contributing to the elastic scattering; \(Z\)-exchange diagram and neutral Higgs boson exchange diagrams as shown in Fig. 1. In order to derive the analytic expression for the scattering cross sections, it is necessary to determine the interactions of the \(Z\) and Higgs bosons to neutralinos and fermions.

Firstly, the interactions of the neutral Higgs fields with SM fermions are described by the Lagrangian

\[
\mathcal{L}_{Hff} = -H_{4-\alpha} \left\{ -\frac{g_{m_1}}{2m_W s_\beta} \bar{d}[O_{2\alpha} - i s_\beta O_{1\alpha} g_5] d + \frac{g_{m_1}}{2m_W c_\beta} \bar{u}[O_{3\alpha} - i c_\beta O_{1\alpha} g_5] u \right\}.
\]

(14)

Here, \(u\) denotes one of up–type fermions and \(d\) one of down–type fermions. Obviously, the Higgs–fermion–fermion couplings are significant for the third–generation fermions, \(t, b\) and
because of their relatively large Yukawa couplings. On the contrary, because any ordinary nucleus is mainly composed of the first (and second) generation fermions, the couplings are very small. Nevertheless, these contributions become important for a nucleus with a large atomic/mass number, because they can contribute to the spin-independent cross section in a coherent manner. We note in passing that the effect of CP-violating Higgs mixing is to induce a simultaneous coupling of $H_i$ ($i = 1, 2, 3$) to CP-even and CP-odd fermion bilinears $\bar{f}f$ and $\bar{f}\gamma_5 f$. This can lead to a sizable phenomenon of CP violation in the Higgs decays into polarized top-quark or tau-lepton pairs [23,24].

Secondly, the interactions of the Higgs bosons to neutralinos involve the neutralino mixing and the Higgs boson mixing simultaneously. As a result, the expression can become lengthy. So, for a notational convenience, we introduce the expression $G_\alpha$ defined in terms of the induced phase $\xi$, the neutralino diagonalization matrix $N$, and the Higgs diagonalization matrix $O$ as follows:

$$G_\alpha = (N_{12} - t_W N_{11}) \left[ i (N_{13} s_\beta + N_{14} c_\beta e^{i\xi}) O_{1a} + N_{13} O_{2a} + N_{14} e^{i\xi} O_{3a} \right].$$  (15)

Then the interaction Lagrangian for the couplings of the neutral Higgs bosons to a lightest neutralino–pair is cast into a very simple form

$$\mathcal{L}_{H\chi\chi} = \frac{g}{2} \sum_{\alpha=1}^{3} \bar{\chi} \left[ R(G_\alpha) + i I(G_\alpha) \gamma_5 \right] \chi H_{4-\alpha}. \quad (16)$$

Here and from now on, we use a simplified notation $\chi$ instead of the conventional notation $\tilde{\chi}_1^0$ to denote the lightest neutralino state. We note that a sizable coupling is expected when the LSP has the significant compositions of both the gaugino states and the higgsino states.

Thirdly, since both $\tilde{W}^0$ and $\tilde{B}$ have $T_3 = Q = 0$, these neutral gaugino states do not couple to the $Z$ boson. Therefore, the neutral current coupling of the $Z$ boson to neutralinos occurs only from the higgsino component of neutralinos. The interaction Lagrangian is given by

$$\mathcal{L}_{Z\chi\chi} = \frac{g}{4c_W} \left[ |N_{13}|^2 - |N_{14}|^2 \right] \bar{\chi} \gamma^\mu \gamma_5 \chi Z_\mu, \quad (17)$$

Note that the $Z$ boson couples to only an axial–vector current, reflecting the Majorana property of neutralinos. One the other hand, the coupling of the $Z$ boson to fermions are not changed even in the presence of new CP–violating phases.

Since the recoil momenta of the nucleus in the elastic scattering of the neutralinos with fixed nuclei are very small compared to the masses of the exchanged $Z$ and neutral Higgs bosons, it is appropriate to have an effective four–Fermi Lagrangian by taking the momentum transfer to be zero. The general form of the four-Fermi effective Lagrangian can be written as

$$\mathcal{L} = \alpha_{1f} (\bar{\chi} \gamma^\mu \gamma_5 \chi) \left( \bar{f} \gamma_\mu f \right) + \alpha_{2f} (\bar{\chi} \gamma^\mu \gamma_5 \chi) \left( \bar{f} \gamma_\mu \gamma_5 f \right) + \alpha_{3f} (\bar{\chi} \chi) \left( \bar{f} f \right) + \alpha_{4f} (\bar{\chi} \gamma_5 \chi) \left( \bar{f} f \right) + \alpha_{5f} (\bar{\chi} \chi) \left( \bar{f} \gamma_5 f \right) + \alpha_{6f} (\bar{\chi} \gamma_5 \chi) \left( \bar{f} f \right). \quad (18)$$

The effective Lagrangian should be summed over fermions and the coefficients $\alpha_{if}$ ($i = 1$ to 6) based on the effective Lagrangian can be obtained by evaluating two Feynman diagrams in Fig. 1 as
\[ \alpha_{1f} = \frac{G_F}{\sqrt{2}} \left[ |N_{13}|^2 - |N_{14}|^2 \right] (T^f_3 - 2Q_f s_W^2), \]
\[ \alpha_{2f} = -\frac{G_F}{\sqrt{2}} \left[ |N_{13}|^2 - |N_{14}|^2 \right] T^f_3, \]
\[ \alpha_{3f} = -\frac{g Y_f}{2\sqrt{2}} \sum_{a=1}^{3} \frac{\mathcal{R}(G_a)}{m^2_{H^4-a}} \left\{ O_{3\alpha} \text{ for } u \right. \}
\[ \left. \left\{ O_{2\alpha} \text{ for } d \right. \right\}. \]
\[ \alpha_{4f} = -\frac{g Y_f}{2\sqrt{2}} \sum_{a=1}^{3} \frac{\mathcal{T}(G_a)}{m^2_{H^4-a}} O_{1\alpha} \left\{ c_\beta \text{ for } u \right. \}
\[ \left. \left\{ s_\beta \text{ for } d \right. \right\}. \]
\[ \alpha_{5f} = +\frac{ig Y_f}{2\sqrt{2}} \sum_{a=1}^{3} \frac{\mathcal{R}(G_a)}{m^2_{H^4-a}} O_{1\alpha} \left\{ c_\beta \text{ for } u \right. \}
\[ \left. \left\{ s_\beta \text{ for } d \right. \right\}. \]
\[ \alpha_{6f} = -\frac{ig Y_f}{2\sqrt{2}} \sum_{a=1}^{3} \frac{\mathcal{T}(G_a)}{m^2_{H^4-a}} \left\{ O_{3\alpha} \text{ for } u \right. \}
\[ \left. \left\{ O_{2\alpha} \text{ for } d \right. \right. \] \( ) \]

In these expressions, \( G_F \) is the Fermi constant, \( Y_f \) the Yukawa coupling of the fermion \( f \), which is \( g m_u / \sqrt{2} m_W s_\beta \) for \( u \)-type fermions and \( g m_d / \sqrt{2} m_W c_\beta \) for \( d \)-type fermions, and \( T^f_3 \) the third component of the isospin of the fermion \( f \). In the limit of vanishing CP-violating phases, these expressions agree with those in [16] and [26].

Among the six independent terms in the effective Lagrangian, only the terms with the coefficients \( \alpha_{2f} \) and \( \alpha_{3f} \) survive in the vanishing momentum-transfer limit. The coefficient \( \alpha_{2f} \) contains contributions from the \( Z \) boson exchange while the coefficient \( \alpha_{3f} \) has contributions from the neutral Higgs–boson exchanges. The spin–dependent contribution from the \( \alpha_{2f} \) terms contains are not suppressed by the fermion mass and can be large over much of the parameter space. In contrast, the spin–independent contribution from the \( \alpha_{3f} \) terms holds a small spin–dependent cross section. However, the spin–independent cross-section can be enhanced by the effects of coherent scattering in a nucleus and can dominate over the spin–dependent cross-section for heavy nuclei. In the following subject, we present the analytic form of both the spin–dependent and spin–independent elastic cross sections and investigate the physical parameters determining them.

**B. Elastic cross sections: spin–dependent versus spin–independent**

The elastic scattering cross sections based on \( \alpha_{2,3f} \) have been conveniently expressed in [16]. The spin–dependent cross-section can be written as
\[ \sigma_s = \frac{32}{\pi} G_F^2 m_r^2 \Lambda^2 \mathcal{J} (J + 1), \]
where \( m_r = m_f m_N / (m_f + m_N) \) is the reduced neutralino-nucleus mass, \( J \) is the spin of the nucleus, the value of which is 4.5 for \(^{73}\text{Ge}\) and 0.5 for \(^{19}\text{F}\), respectively, and the quantity \( \Lambda \) is given by
\[ \Lambda = \frac{1}{f} (a_p \langle S_p \rangle + a_n \langle S_n \rangle), \]  

(21)

with the coefficients \( a_p \) and \( a_n \):

\[ a_p = \sum_{f=u,d,s} \frac{\alpha^2 f}{\sqrt{2} G_F} \Delta_f^{(p)} , \quad a_n = \sum_{f=u,d,s} \frac{\alpha^2 f}{\sqrt{2} G_F} \Delta_f^{(n)}. \]  

(22)

The factors \( \Delta_f^{(p,n)} \) depend on the spin content of the nucleus and the values of the factors are taken to be \[ \Delta_u^{(p)} = +0.77, \quad \Delta_d^{(p)} = -0.38, \quad \Delta_s^{(p)} = -0.09, \]

\[ \Delta_u^{(n)} = -0.38, \quad \Delta_d^{(n)} = +0.77, \quad \Delta_s^{(n)} = -0.09, \]  

(23)

in our analysis. The expectation values \( \langle S_{p,n} \rangle \) are the averaged values of the spin content in the nucleus and therefore are dependent on each target nucleus. We will display results for scattering off of a \(^{73}\text{Ge}\) target and a \(^{19}\text{F}\) for which in the shell model

\[ \langle S_p \rangle_{\text{Ge}} = 0.011, \quad \langle S_n \rangle_{\text{Ge}} = 0.491, \]

\[ \langle S_p \rangle_{\text{F}} = 0.415, \quad \langle S_n \rangle_{\text{F}} = -0.047. \]  

(24)

For a more detailed information on the these quantities, we refer to the review paper by Jungman, Kamionkowski and Griest \[ [16], \]

On the other hand, the spin–independent cross section is written as

\[ \sigma_i = \frac{4m_r^2}{\pi} [Zf_p + (A - Z)f_n]^2, \]  

(25)

where \( Z \) and \( A \) are the atomic number and the mass number of the nucleus, respectively, and the coefficients \( f_p \) are given by

\[ \frac{f_p}{m_p} = \sum_{q=u,d,s} f^{(p)}_{Tq} \frac{\alpha q}{m_q} + \frac{2}{27} f^{(p)}_{TG} \sum_{q=c,b,t} \frac{\alpha q}{m_q}, \]  

(26)

and \( f_n \) is given by an expression similar to that for \( f_n \). The parameters \( f^{(p)}_{Tq} \) are defined by

\[ \langle p | m_q \bar{q} q | p \rangle = m_p f^{(p)}_{Tq}, \]  

while \( f_{TG} = 1 - (f_{Tu} + f_{Td} + f_{Ts}) \) \[ [28]. \] For our numerical analysis, we adopt \[ [29], \]

\[ f^{(p)}_{Tu} = 0.019, \quad f^{(p)}_{Td} = 0.041, \quad f^{(p)}_{Ts} = 0.140, \]

\[ f^{(n)}_{Tu} = 0.023, \quad f^{(n)}_{Td} = 0.034, \quad f^{(n)}_{Ts} = 0.140. \]  

(27)

There exist additional contributions due to one–loop Higgs couplings to gluons and so-called twist-2 operators; however the change from a more careful treatment of loop effects for heavy quarks and the inclusion of twist-2 operators is expected to be numerically small \[ [30,31]. \]
IV. LSP–NUCLEUS ELASTIC SCATTERING CROSS SECTIONS

A. Independent SUSY parameters

The elastic scattering cross sections depend on a large number of SUSY parameters; more than ten parameters. So, in order to make a realistic analysis, it will be necessary to make a few assumptions which are reasonable for physics point of view.

Firstly, we note that the neutral Higgs mass spectrum depends on the chargino mass $m_{H^\pm}$, a SUSY breaking scale $M_{\text{SUSY}}$, two trilinear terms $|A_t|, |A_b|$ and their phases $\Phi_{A_t}, \Phi_{A_b}$ as well as $\tan \beta$, the higgsino mass parameter $|\mu|$ and its phase $\Phi_\mu$. Let us assume a universal trilinear parameter in our analysis:

$$|A_t| = |A_b| \equiv |A|, \quad \Phi_{A_t} = \Phi_{A_b} \equiv \Phi_A,$$

(28)

This assumption will not forbid us from finding out the general trend of the Higgs masses and their couplings to fermions and neutralinos. The size of the Higgs boson mixing is determined by the dimensionless parameter $\eta_{\text{CP}}$. We take in our analysis $M_{\text{SUSY}} = 0.5 \text{TeV}$, $|A| = 1.5 \text{TeV}$. (29)

The charged Higgs mass plays a crucial role in determining the contribution of the spin–independent cross section so that it will be treated as a free parameter along with the parameters $\{\tan \beta, |\mu|, \Phi_\mu\}$.

Secondly, the neutralino masses and mixing are determined by the SU(2) gaugino mass parameter $M_2$, the U(1) gaugino mass $|M_1|$ and its phase $\Phi_1$ as well as the parameters $\{\tan \beta, |\mu|, \Phi_\mu\}$. It will be reasonable to take the gaugino mass unification condition between two gaugino masses so that at least up to the one–loop level one have

$$|M_1| = \frac{5}{3} t_W^2 M_2 \sim 0.5 M_2, \quad \Phi_1 = 0,$$

(30)

where $t_W = \tan \theta_W$. In this case, the lightest neutralino will be Bino–like for $|\mu| \gg M_2$ and it will be higgsino–like for $|\mu| \ll M_1$. Note that the higgsino parameter $|\mu|$ and its phase $\Phi_\mu$ affect both the neutralino mixing and neutral Higgs mixing. For a large Higgs mixing, a large $|\mu|$ is preferred. On the contrary, a large neutralino mixing requires a relatively small value of $|\mu|$. In this light, $|\mu|$ is a crucial SUSY parameter in determining the relative importance of the spin–dependent and spin–independent contributions. We will take $|\mu|$ and $\Phi_\mu$ as free parameters.

Thirdly, the spin–independent cross section strongly depends on fermion masses. For the fermion masses, we will use their maximum values coded by the Particle Data Group [32]:

$$m_u = 5.0 \text{MeV}, \quad m_d = 9.0 \text{MeV}, \quad m_s = 170 \text{MeV},$$

$$m_c = 1.4 \text{GeV}, \quad m_b = 4.4 \text{GeV}, \quad m_t = 174 \text{GeV}.$$  

(31)

It is certain that there still exist large uncertainties in the values of the fermion masses. Nevertheless, our numerical analysis will be qualitatively reasonable and even quantitatively meaningful with some improved determinations of fermion masses.
Consequently, the independent SUSY parameters which we manipulate in our numerical estimates of the LSP–nucleus scattering cross sections are

$$\tan \beta, \ M_2, \ |\mu|, \ m_{H^\pm}, \ \Phi_\mu, \ \Phi_A. \quad (32)$$

We set $\Phi_\mu = 0$ and take two values 3 and 30 for $\tan \beta$ which will satisfy the constraints from the Higgs search experiments at LEP \[19\] and the 2–loop Barr–Zee–type electron and neutron EDMs.

**B. Numerical results**

We are now ready to show the importance of the CP–violating phases in the LSP detection through the LSP–nucleus elastic scattering. For a systematic analysis, it is useful to understand the dependence of the CP–violating induced phase on the parameters $|\mu|, \Phi_\mu$ and $\Phi_A$. Incidentally, the induced phase depends only on one combination of two phases $\Phi_\mu + \Phi_A$. So, for this analysis, we simply introduce $\Phi = \Phi_\mu + \Phi_A$ and set the phase to be $\pi/2$, which will give (almost) the maximal absolute value of $\sin \xi$. In this case, the sine of the induced phase is determined by the higgsino mass parameter $|\mu|$ and the charged Higgs mass for which we take into account two values; 250 GeV and 500 GeV. Also, $\sin \xi$ in this limit is is given by a simple analytical expression

$$\sin \xi = -\frac{|\tilde{\lambda}_6|}{m_a^2 \sin 2\beta}. \quad (33)$$

Therefore, $\sin \xi$ is always negative and it is expected that the existence of $\sin 2\beta$ in the denominator forces its absolute value to increase with increasing $\tan \beta$.

Figure 1 shows $|\sin \xi|$ as a function of the higgsino mass parameter $|\mu|$ by taking four different sets of $\{\tan \beta, m_{H^\pm}\}$: $\{3, 250 \text{ GeV}\}$ (solid line), $\{30, 250 \text{ GeV}\}$ (dashed line), $\{3, 500 \text{ GeV}\}$ (dot–dashed line), and $\{30, 500 \text{ GeV}\}$ (dotted line). The values of the other SUSY parameters are given in the previous section. It is clear that $|\sin \xi|$ increases with increasing $\tan \beta$ but decreases with increasing the charged Higgs mass $m_{H^\pm}$. It depends very strongly on the higgsino mass parameter $|\mu|$; for a small $\tan \beta$ the absolute value of $\sin \xi$ can be at most as large as 0.1 for a large value of $|\mu|$ and for a relatively small value of $m_{H^\pm}$, but it can be as large as unity for a large value of $|\mu|$ and a small value of $m_{H^\pm}$. This is due to the fact that for a small value of $\tan \beta$ the scalar top quarks contribute (almost) exclusively, but for a large value of $\tan \beta$ the scalar bottom quarks as well as the scalar top quarks contribute to the CP–violating induced phase. In this light, one may expect that the CP–violating induced mass $\xi$ can affect the chargino and neutralino sectors significantly for a certain regime of the SUSY parameter space. However, we note that the relative size of the induced phase contribution to the chargino or neutralino mass spectrum is at most as large as $(m_W/|\mu|)^2 \sin 2\beta$ for $|\mu|$ much larger than $M_2$ so that the enhancement effect in the induced phase by a large $|\mu|$ and a large $\tan \beta$ is washed out through the strong suppression by the same parameters. Therefore, in most cases there are no significant modifications in the chargino and neutralino mass spectrums.

To begin with, we estimate the spin–dependent cross section. For this case we consider the scattering of neutralinos on fluorine $^{19}F$, for which the spin–dependent contribution
typically dominates by a factor of 20 \cite{16}. As shown in Section 2B, the crucial parameters determining the spin–dependent cross section are the coefficients $\alpha_{2f}$ and on the whole by the coefficients $a_p$ and $a_n$. Because there is simply one $Z$–exchange contribution to the spin–independent process, all the $\alpha_{2f}$ coefficients are proportional to the coupling of the $Z$ boson to neutralinos except for their relative factors; $|N_{13}|^2 - |N_{14}|^2$. We find by a comprehensive numerical scan on the relevant SUSY parameters that the CP–violating induced phase hardly changes the values of $a_p$ and $a_n$. This property is in a sharp contrast with the aspect that the coefficients and the spin–dependent cross section are very sensitive to $\Phi_\mu$, the phase of the higgsino. For the details of the significant dependence on the phase $\Phi_\mu$, we refer to the work by Falk, Ferstl and Olive \cite{9}.

On the other hand, the spin–independent cross section is dominant in much of the SUSY parameter space for scattering off heavy nuclei so that we consider the scattering of neutralinos on $^{73}$Ge. The cross section is determined by the coefficients $\alpha_{3f}$ which involve the contributions from the neutral Higgs exchanges and the scalar part of their couplings to neutralinos and fermions. Because the explicit CP violation through radiative corrections to the Higgs sector can lead to a large mixing among scalar Higgs bosons and pseudoscalar Higgs boson, it is naturally expected to exhibit a rather strong dependence of the cross section on the phases such as the phase $\Phi_A$ and the induced phase $\xi$, even if the phase $\Phi_\mu$ is taken to be zero in favor of the naturalness conditions at the GUT scale. Nevertheless, the structure of the coupling $G_\alpha$ requires the lightest neutralino state to be composed of both gauginos and higgsinos with significant compositions. This means that a large cross section can be obtained when the gaugino mass $M_2$ and the higgsino mass parameter $|\mu|$ are comparable in size.

Keeping in mind these aspects, we take for our numerical analysis two values of $\tan \beta$ (3 and 30), set $m_{H^\pm} = 250$ GeV and use the same values for the other SUSY parameters as those given in the previous analyses. Figure 3 shows $f_p$ for five values of the higgsino mass parameter $|\mu|$; 200 GeV (solid line), 400 GeV (long dashed line), 600 GeV (dot–dashed line), 800 GeV (dotted line) and 1000 GeV (dashed line). The value of $\tan \beta$ is taken to be 3 in the left frame and 30 in the right frame, respectively. We note several interesting aspects from two figures:

- The magnitude of $f_p$ (as well as $f_n$) is very sensitive to the value of $|\mu|$. In most cases except for the region of large values of $\Phi$, it decreases with increasing $|\mu|$. The main reason for the suppression is that the LSP is gaugino–dominated for a large value $|\mu|$, while an intermediate state of the LSP is required to have a sizable $f_p$.

- Comparing the small and large $\tan \beta$ cases, one can find that the magnitude of $f_p$ increases with increasing $\tan \beta$.

- The relative sensitivity of $f_p$ to the phase $\Phi$ is very much enhanced for a large value of $|\mu|$, in particular for a small value of $\tan \beta$. In this case, $f_p$ increases by one order

\footnote{The dependence of $f_n$ on the phase $\Phi$ and the parameter $|\mu|$ is very similar to that of $|f_p|$ and furthermore we find that two quantities have a very similar value for the whole scanned space of the SUSY parameters.}
of magnitude as $\Phi$ changes from zero to 180 degrees. As a result, one can expect to have a large LSP detection rate even for a large $|\mu|$.

- On the contrary, for a large $\tan \beta$ and a large $|\mu|$, the value of $f_p$ tends to be suppressed for non–trivial values around $\Phi = 90^\circ$.

On the whole, it is clear that $f_p$ is very sensitive to $|\mu|$ and $\tan \beta$, and it becomes sensitive to the phase $\Phi$, equivalently, $\Phi_A$ in the present work, for a large $|\mu|$. However, the behavior of $f_p$ with respect to the phase $\Phi$ strongly depends on the value of $\tan \beta$.

From the discussion in the previous paragraph, it is expected that the spin–independent cross section also will show the same behavior with respect to the SUSY parameters and CP–violating phases. With the fact that $f_p$ and $f_n$ are (almost) similar to each other in size, one can see that the spin–independent cross section is proportional to the square of $f_p$ so that the dependence of the cross section is enhanced. Figure 4 shows the dependence of the spin–independent cross section $\sigma_I(\chi^{73}\text{Ge} \rightarrow \chi^{73}\text{Ge})$ on the phase $\Phi$ with the other SUSY parameters as those in Figure 3. As expected, the behavior of the cross section is similar to that of $f_p$. For $\tan \beta = 3$ and $|\mu| = 1000 \text{ GeV}$, the cross section changes by one order of magnitude, depending on the phase, while the cross section tends to be suppressed for non–trivial phases in the case of a large $\tan \beta$.

V. SUMMARY AND CONCLUSIONS

In this paper, we have re–visited the neutralino–nucleus elastic scattering process portion of dark matter in the Universe under a specific SUSY scenario (which has been recently suggested to avoid the severe EDM constraints); in the scenario, the first and second generation sfermions are so heavy that they are decoupled from the low–energy supersymmetric theories, while the third generation sfermions are required to be relatively light not to spoil naturalness. In this case, the complex parameters, in particular, the phase of the trilinear terms and the phase of the higgsino mass parameter, of the third–generation stop and sbottom sectors can lead to a significant mixing between scalar neutral Higgs bosons and a pseudoscalar neutral Higgs boson. As a result, there appears a CP–violating induced phase due to the misalignment between two Higgs doublet fields. We have focussed on the impact of the scalar–pseudoscalar mixing and the induced phase on the LSP–nucleus elastic scattering process.

For the sake of our numerical analysis, we have assumed a universal trilinear parameters $|A|$ and $\Phi_A$ and set the phase of the higgsino mass parameter $\Phi_\mu$ to be zero, and then we have varied the charged Higgs boson mass, $\tan \beta$, $|\mu|$ and the phase $\Phi_A$ while keeping $|A| = 2 \text{ TeV}$ and $M_{\text{SUSY}} = 0.5 \text{ TeV}$, the SUSY breaking scale. To summarize, we have several interesting aspects related with the induced phase and the detection of the neutralino dark matter:

- The induced phase $\xi$ itself can be large if $\tan \beta$ is large, and $|\mu|$ is comparable to $M_{\text{SUSY}}$ or larger. However, we have found that in this case its contribution to chargino and neutralino masses is strongly suppressed because the contribution is determined by the combination $(m_W/|\mu|)^2 \sin 2\beta$. 

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• The coupling of the $Z$ boson to neutralinos is affected only through the CP–violating induced phase from the third generation stop and sbottom sectors. We have found that because of the mutually destructive properties mentioned before the CP–violating induced phase hardly changes the spin–dependent cross section.

• The spin–independent part, to which the main contribution comes from the neutral Higgs boson exchanges can undergo a big change with respect to the phase $\Phi_A$ and the induced phase $\xi$ as well. The Higgs mass spectrum and the couplings of the neutral Higgs bosons to neutralinos and fermions vary very significantly with respect to the phases as well as the other SUSY parameters. The dimensionful parameter $f_p$ as well as $f_n$ dictating the size of the spin–independent cross section increases with decreasing $|\mu|$ and increasing $\tan \beta$, while the sensitivity of the parameter to the phase $\Phi$. For a small value of $\tan \beta = 3$, the sensitivity of $f_p$ and the spin–independent cross section to the phase $\Phi$ can be huge for a large value of $|\mu|$. On the contrary, the spin–independent cross section is suppressed for non–trivial values of the phase $\Phi$ for a large value of $\tan \beta$.

Even though we have not presented here, we have confirmed the result by Falk, Ferstl and Olive [9] that the spin–dependent and spin–independent cross sections are strongly dependent on the phase $\Phi_\mu$. In some cases, for a broad range of non–zero $\Phi_\mu$, there are cancellations in the cross sections which reduce both the spin–dependent and spin–independent cross sections by more than an order of magnitude. In other cases, there may be enhancements as one varies $\Phi_\mu$.

In general there are several CP–violating phases which can affect many important physics phenomena. In addition to the supersymmetric CP–violating phase $\Phi_\mu$, we have found that even in the scenario of the gaugino mass unification and the decoupling of the first and second sfermion states, the CP–violating scalar–pseudoscalar neutral Higgs boson mixing and the CP–violating induced phase between two Higgs doublets can affect the spin–independent part of the neutralino–nucleus elastic scattering cross section significantly, depending on the values of $\tan \beta$, the charged Higgs boson mass, the SUSY breaking scale, and so on. Therefore, we can conclude that in the present situation of no SUSY signatures it is important to clearly understand the impact of all the CP–violating phases, (which are not constrained by low–energy measurements such as the electron and neutron EDMs), on the neutralino–nucleus elastic scattering, one of the most promising dark matter detection mechanism.

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Note added

While finalizing our paper, we became aware of one very recent work [33] by Gondolo and Freese which treats some of the features we have been studying here. In the paper, they have mainly concentrated on the scalar–pseudoscalar mixing and have provided a global parameter scan in estimating the elastic scattering cross section. Instead, our work have studied the effect of the CP–violating induced phase between two Higgs doublets as well as of the scalar–pseudoscalar mixing.
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FIG. 1. Two types of Feynman diagrams contributing to the neutralino–nucleus elastic scattering process; (a) the spin–dependent Z-exchange diagram and (b) the spin–independent Higgs–boson–exchange diagram.
FIG. 2. The absolute value $|\sin \xi|$ of the CP-violating induced phase $\xi$ for $\Phi = \pi/2$ as a function of the higgsino mass parameter $|\mu|$. We take four different sets of $\{\tan \beta, m_{H^\pm}\}$; $\{3, 250 \text{ GeV}\}$ (solid line), $\{30, 250 \text{ GeV}\}$ (dashed line), $\{3, 500 \text{ GeV}\}$ (dot–dashed line) and $\{30, 500 \text{ GeV}\}$ (dotted line). The values of the other SUSY parameters are given in the text.
FIG. 3. The dimensionful coefficient $f_p$ as a function of the phase $\Phi$ with $\Phi_\mu = 0$ for the spin–independent neutralino–nucleus elastic scattering. The solid line is for $|\mu| = 200$ GeV, the long dashed line for $|\mu| = 400$ GeV, the dot–dashed line for $|\mu| = 600$ GeV, the dotted line for $|\mu| = 800$ GeV and the dashed line for $|\mu| = 1000$ GeV. The value of $\tan \beta$ is taken to be 3 in the left frame and 30 in the right frame. In both cases, the charged Higgs boson mass is 250 GeV. The values of the other remaining SUSY parameters are given in the text.

FIG. 4. The spin–independent elastic scattering cross section $\sigma_I(\chi^{73}\text{Ge} \rightarrow \chi^{73}\text{Ge})$ as a function of the phase $\Phi$ with $\Phi_\mu = 0$ for the spin–independent neutralino–nucleus elastic scattering. The legend for the lines is the same as that in Fig. 3.