Pre-service teachers’ algebraic reasoning and thinking barriers in solving algebraic problem

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Abstract. Algebraic reasoning is a process in which student generalize mathematical ideas from a set of particular instances, establish those generalizations through the discourse of argumentation, and express them in increasingly formal and age-appropriate ways. This is descriptive qualitative research. The primary interest was to explore pre-service teacher’ thinking that might be foundational to algebraic reasoning and to describe their thinking barriers in solving an algebraic problem. There are twelve pre-service teachers who have participated in this study. The task was designed and given to all participants. Documents were the subjects' test results and unstructured interview transcripts. The results showed that the subjects had difficulties in generalizing and connecting mathematical patterns. Based on the subject's difficulties, we concluded that the subjects have barriers that include confirmation bias, mental set, functional fixedness, and unnecessary constraints. The intensity of each barrier will be discussed further in this article.

1. Introduction
Problem solving is an important focus in the school mathematics curriculum, ranging from elementary to high school levels [1]. Problem solving is important as a way of doing, learning and teaching mathematics [2]. The importance of problem solving is reflected in the 2013 Indonesian curriculum where mastery of each competency standard is always equipped with problem solving competencies as stated in Basic Competence 4, known as KD 4 in Indonesian Curriculum 2013. Problem solving is a skill that is related to reasoning skills [3], and one type of reasoning in mathematics learning according to [1] algebraic reasoning.

Blanton & Kaput [4] said that algebraic reasoning is a process in which students generalize mathematical ideas from a set of particular instances, establish those generalizations through the discourse of argumentation, and express them in increasingly formal and age-appropriate ways. The framework of teacher actions to facilitate algebraic reasoning integrates four separated and interlinked components that the study identifies as key to the development of early algebraic reasoning [5]. These include: (1) Teacher awareness of and a purposeful focus on algebraic concepts, (2) Teacher actions to develop and modify tasks and enact them in ways that facilitate algebraic reasoning, (3) Teacher actions to develop classroom practices that provide opportunities for engagement in algebraic reasoning, (4) Teacher actions to develop mathematical practices that support the development of algebraic reasoning. Algebraic reasoning refers to the psychological processes involved in solving problems that can easily express mathematics using algebraic notation [6].
In fact, there are a lot of students who often experience difficulties in algebraic problems [7]. In addition, most pre-service teachers still have difficulties in solving algebraic problems in linear algebraic courses [8]. In the Third International Mathematics and Science Study – Repeat (TIMSS-R), the Philippines was ranked among the lowest three, and the lowest performance of Philippines’ pre-service teacher was in Algebra [9]. Khatimah et al. [10] assumed that the difficulties arose both in learning algebra and solving algebraic problems. It indicates that there are barriers in thinking process. People are often trapped when they try to solve problems caused by barriers [11].

The difficulties in learning algebra can be minimized by identifying the student’s algebraic reasoning so that teachers can design learning activities [12]. Understanding what individuals do in the problem solving process is one of the most important aspects of learning problem solving [13]. Based on a previous exposure, the purpose of this paper is to describe algebraic reasoning and thinking barriers of pre-service teachers in solving a mathematics problem. The research questions are as follows: (1) how is algebraic reasoning of the pre-service teachers in solving mathematics problem? (2) How are thinking barriers of the pre-service teachers in solving mathematics problem?

2. Theoretical Framework

2.1. Algebraic Reasoning

Algebraic reasoning is a process in which subjects generalize mathematical ideas from a set of particular instances, establish those generalizations through the discourse of argumentation, and express them in increasingly formal and age-appropriate ways [4]. Another opinion was expressed by Lins [14] that algebraic thinking arises when people are detecting and expressing structure, whether in the context of problem solving concerning numbers or some modelled situation, whether in the context of resolving a class of problems, or in the context of studying structure more generally. Algebraic reasoning is characterized by its generality and by the role that symbolic expressions play in stating general relationships, comparing and manipulating them, and facilitating many numerical evaluations [15]. Algebraic reasoning is activities in which a person engages in discovering patterns from a mathematical or specific context, making relations between quantities and compiling generalizations through formal symbolic representation and manipulation. Thus, algebraic reasoning is very close to its abilities and makes patterns of generalization.

2.2. Thinking Barriers

2.2.1. Mental Set. One factor that can hinder problem solving is mental set — a frame of mind involving an existing model for representing a problem, a problem context, or a procedure for problem solving. When problem solvers have an entrenched mental set, they fixate on a problem [16]. Mental set refers to the tendency to transfer solutions (techniques) from solving a problem in the past to a new situation [17]. Once a way of reasoning developed that has been effective in the past solving certain problems, the method tends to be maintained to face the new problem [11].

2.2.2. Confirmation Bias. Confirmation bias is defined as the tendency to collect, interpret, and remember evidence in a way that confirms rather than challenges the beliefs that have been held by someone [18,19]. Sometimes confirmation bias is associated with an individual's tendency to gather information related to only one hypothesis and even only considers the possibility that the hypothesis is true or only the possibility that the hypothesis is wrong or in other words not considering both possibilities at one time [20]. Someone who experiences confirmation bias feels that he already knows everything about a problem, resulting in an unwillingness to do research, even though doing research only by examining things that confirm the suitability of the solution you want to use or have believed from the start.
2.2.3. **Functional Fixedness**. Functional fixedness is one of the barriers that prevent insight into problem solving [21]. The tendency to think of an object in terms of its most common use is called functional fixedness [11]. Robert et al. [16] said that becoming free of functional fixedness is what first allowed people to use a reshaped coat hanger to get into a locked car. It is also what first allowed thieves to pick simple spring door locks with a credit card. Functional fixedness is the inability to realize that something known to have a particular use may also be used for performing other functions [22].

2.2.4. **Unnecessary Constraints**. Effective problem solving requires determining everything constraints that solve problems without assuming any constraints that are not needed [23]. People make some assumptions or predictions that impose unnecessary constraints on solving a problem. Unnecessary constraints happen when trying to solve a problem using the previous experience of what has worked in a situation and force it to work in the current situation, rather than looking for a new solution. This barrier can be removed by insight that occurs when people suddenly discover the correct solution of a problem after struggling with it for a while [23].

2.3. **Problem Solving**

Problem solving is a subject in the mathematics curriculum that must be taught [24]. Butterworth & Thwaites [25] said that problems are about using logic to solve problems in decision making. Solving problem activity means the problem solver gets involved in a task which solution method is not known first [24]. Further explained that problem solving activities provide a rationale for teaching arithmetic skills. That is, problem solving is an activity that will involve subjects in their way of thinking. Basic problem solving skills are: selecting relevant data, finding the right procedure to solve problems and comparing data in various forms [25]. By learning problem solving in mathematics subjects will be trained in terms of ways of thinking [1].

Bransford and Stein classify problem solving skills into five stages for problem solvers [26] namely: identifying the problem, determining and representing the problem, thinking of possible strategies, carrying out a strategy, looking back and evaluating the impact of the work. There are four stages of problem solving: (1) understanding the problem, (2) making a plan, (3) carrying out the plan, and looking back [27]. In the first stage subjects must see clearly what is needed and see how various items in the problem are connected to each other. In the second and third stages, subjects must then know how the unknown in the problem has an attachment to the known data to get an idea of the solution so can make a settlement plan and then implement it. While in the final stage, the subject must look back at the solution that has been completed and review it whether it is correct or not. In this study, mathematical problem solving is the process in solving mathematical problems which its steps consist of understanding the problem, planning a solution, implementing the plan and checking the answers.

3. **Method**

This is descriptive qualitative research. The primary interest was to explore the subject's thinking that might be foundational to algebraic reasoning and to describe pre-service teachers’ thinking barriers in solving an algebraic problem. Purposeful sampling was employed to select the twelve subjects (samples) for this study. They were students from a public university in Indonesia enrolled in a 4th year Bachelor of Education program in mathematics. Algebraic reasoning can be stimulated by tasks specifically designed to train subjects to reason algebraically. The task involving algebraic reasoning was designed and given to all the participants. Figure 1 below is the example of instrument that was used in this study.
4. Data collection and analysis procedures
Materials collected for the analysis consisted of transcript of the interview, participant's notes and answers, and researchers' notes during the interview. The process of data analysis consisted of four stages. In stage one, all participants' answers were grouped based on the similarity of procedures in solving problems. Panhuizen in his research classified students’ answers into three categories: correct answers, reasonable responses (this response is not assessed as “correct close reading” or “carrying out tasks precisely,” but it is still considered to be and valued as) and wrong answers [28]. Then in the second stage, student's answers were classified into three categories: correct answers, reasonable responses, and the wrong answer. In the third stage, three participants were selected as representatives of each answer and then interviewed. In the fourth stage, transcript of interview was analysed to describe algebraic reasoning and participants' thinking barriers in solving algebraic problem.

5. Findings And Discussion
We explained and analysed the subjects’ answers of algebraic reasoning based on problem solving steps by Polya. Then, we interviewed the subjects to get information about the barriers more clearly.

5.1. Subject A (Correct Answer)
Subject A had described important information from the problem that determined the first term and the difference of both sequences, as a form of subject understanding of the problem. However, the subject immediately wrote $U_n = a + (n - 1) b$ (arithmetic sequence formula) as her strategy to find a general pattern of the triangle and square arrangement sequences. We asked subject A, "Why do you immediately write the formula of arithmetic sequences?" Subject A responded that she believed with the formula because she had identified both sequences that were suitable with arithmetic sequence problem. Therefore, subject A did not identify and generalize from each term of both sequences when she had the formula that has learned in the previous lesson. Then, the subject found the general pattern of the triangle and square arrangement.
On the other hand, the subject used the algebraic equation to find the sequence term of triangles and squares that have the same value, but the subject did not look back the answer although the answer was correct. This means subject A did not follow few steps of problem solving. The subject stated that she trusted that both sequences had the same values in 6th term because the procedure had been correct.

Actually, the answer of subject A could be understandable but a few steps of problem solving were not sequential (putting strategy at the beginning of a solution) and the last step of problem solving (looking back) was not presented. Subject A tried to write the answer based on her idea without considering whether problem solving was clear or not. Therefore, we indicated subject A got confirmation bias because she just tried to confirm the preconceived solution. Based on the data, subject A also used the formula of arithmetic sequences to solve the problem since the formula had learned in the previous lesson. Subject A felt mental set when relying too heavily on heuristics of problem solving when the subject believed that this way could find a general pattern of both sequences.

5.2. Subject B (Reasonable Response)

Subject B had tried to identify the problem by determining the first term and the difference from both sequences. Although without planning strategy, the subject immediately wrote the general form of a triangle and square arrangement pattern. We asked "Why do you directly write the form of the general pattern of the sequences?" Then, he said, "Because, I understand the form of the sequences in that problem as an arithmetic sequence and as we know, the sequences are some number arranged according to a certain pattern". It indicated that the subject directly related the information and formula that would be used to solve the problems.
Figure 3. Subject B’s Answer

Then, subject B also had an error because the subject directly claimed that both sequences will be equal in 6th term, without going through any procedures. We asked the subject to explain that answer and the subject said that he knew how to get the solution but the procedure did not write in the paper. Finally, the subject just confirmed that their answers were correct.

Based on the data, subject B surely had barriers in confirmation bias. He did not do some procedure of problem solving especially planning and conducting the strategy. The subject directly wrote the general pattern of both sequences that made the answer not clear. The subject also didn't use the method correctly when he didn't finish the general pattern of both sequences and wrote the last solution without justification that made the answer bias. The subject was also had mental set in the problem. He heavily trusted with the formula of arithmetic sequences and didn't use a generalization of particular cases.

5.3. Subject C (Wrong Answer)

Subject C just wrote the general pattern of both sequences but he did not write the procedure. We asked, "What important information did you get from this problem?". Subject C said "Hmm. Maybe the problem is how the two sequences of patterns, determine which term that both sequences will have the same value. Honestly, I don't really understand arithmetic sequences like this". Actually, the subject understood the problem but he confused how to solve it. Then, we asked again "Why do you write immediately the general pattern for triangles is $3n - 1$ and square is $2n + 6$?". He responded, "Because I had found for it (with arithmetic sequence formula) using another paper and I immediately wrote the answer about the pattern in the answer paper".
The subject also tended to strain a method in which he had learned to find the same values in both sequences. We asked “What strategy did you use? Explain your reason”. Subject C said, “The strategy that I used is how abstract concepts in mathematics are related to mathematical problems so that results can be obtained through mathematical steps as well”. Subject assumed that he did not understand how to find the same value of both sequences. He also used the method of LCM (Least Common Multiplication) that was not related to find the solution. He assumed that the LCM showed multiples of the smallest of the same numbers appearing simultaneously. He said, "As for looking for it, I assume how the two sequences of the row pattern are diverted, maybe we can get the answer from there, but to be honest, I haven't tried it yet and that is my assumption”.

We indicated subject C got more barriers because he had many difficulties in problem solving. Confirmation bias surely happened in problem solving of subject C since he did not follow the steps of problem solving and just wrote the wrong answer. When subject C used the arithmetic formula to solve the problem, he also had a mental set. He relied on previous experiences to directly solve the problem. Subject C also got functional fixedness since he did not use the algebraic equation to find the same value of both sequences. He was not aware that the pattern that he found was algebraic form and not connected them. Unfortunately, subject C constrained the method of LCM to solve the problem but it did not relate to solving this problem.

6. Conclusion and Suggestion
The subjects got barriers to solving algebraic reasoning problems with different types and intensities. In conclusion, students with good problem solving skills allow them to have fewer barriers with less intensity than students with poor problem solving abilities. Students with barriers such as confirmation bias and mental set with small intensity are still able to solve problems properly. Contrary, students with the same barriers with greater intensity have not been able to solve the problem properly but still be able to answer rationally. Students are unable to solve the problem if they have been exposed to the functional fixedness and unnecessary constraint because it is certain that the student's answer will be wrong.

Teachers should firstly identify student barriers to determine the treatment. Some of the treatments suggested for the teachers are:

- Familiarizing students to solve problems according to coherent steps so that students' mathematical communication can be structured properly (Confirmation Bias).
- Providing understanding and developing concepts for students rather than providing a standard formula (Mental Set)
- Familiarizing students with critical and creative thinking through problems and tasks (Functional Fixedness)
• Do not orient students’ answers to the result that makes students impose wrong methods
(\textit{Unnecessary Constraints})

References

[1] NCTM 2000 \textit{Principles and Standards for School Mathematics} (Virginia: NCTM)
[2] Chapman O 2005 \textit{Proc. of the 29th Conf. of the Int. Group for the Psychology of Mathematics Education} vol 2 (Melbourne: Eric) p 225
[3] Yurt E and Sünbül A M 2014 A structural equation model explaining 8th grade students’
mathematics achievements \textit{Educ. Sci. Theory Pract.} \textbf{14} 1642–52
[4] Blanton M L and Kaput J J 2011 Functional thinking as a route into algebra in the elementary
grades early algebraization \textit{Advances in Math. Educ.} \textbf{37} 34–42
[5] Hunter J 2015 Teacher actions to facilitate early algebraic reasoning \textit{Annual Meeting of MERGA}
pp 58-67
[6] Carragher D W and Schliemann A D 2007 \textit{Early Algebra and Algebraic Reasoning: Second
Handbook of Research on Mathematics Teaching and Learning} (Charlotte, NC: NCTM)
[7] Femiano R B 2003 Algebraic problem solving in the primary grades \textit{Teach. Child. Math.} \textbf{9} 444-9
[8] Firdaus M, Darma Y and Haryadi R 2003 \textit{J. Edukasi Matematika dan Sains.} \textbf{2} 22-33
[9] Castro B de 2004 Pre-service teachers’ mathematical reasoning as an imperative for codified
conceptual pedagogy in algebra: a case study in teacher education \textit{Asia Pacific Educ. Rev.} \textbf{5}
157–66
[10] Khatimah K, Sa’dijah M and Susanto H 2017 Pemberian scaffolding untuk mengatasi hambatan
berpikir siswa dalam memecahkan masalah aljabar \textit{J. Kaji. Pembelajaran Mat.} \textbf{1} 36–45
[11] Train B, Ahmed R, Bandewe C, Cockcroft K, Crafford A, Greenop K, Stacey M, Tomlinson M,
Tommy J and Dale-jones B 2007 \textit{Introduction to Psychology} (South Africa: Prentice Hall)
[12] Indraswari N F, Budayasa I K and Ekawati R 2018 Algebraic reasoning in solving mathematical
problem based on learning style \textit{J. Phys. Conf. Ser.} \textbf{947} 1–6
[13] Aljaberi N M 2015 University students’ learning styles and their ability to solve mathematical
problems \textit{Int. J. Bus. Soc. Sci.} \textbf{6} 152–66
[14] Lins R L 1990 \textit{Proc. 14th Ann. Conf. Int. Group for the Psychology of Mathematics Education
with the North American Chapter 12th PME-NA Conf.} vol 2 (Mexico) pp 93–101
[15] Smith J and Thompson P W 2007 Quantitative reasoning and the Development of Algebraic
Reasoning \textit{Algebra in the Early Grades} ed J J Kaput, D W Carraher and M L Blanton (New
York: Erlbaum) pp 95-132
[16] Sternberg R J and Sternberg K 2012 \textit{Cognitive Psychology, Sixth Edition} (California: Wadsworth)
[17] Kay N M 2000 \textit{Pattern In Corporate Evolution} (New York: Oxford University Press Inc)
[18] Charness G and Dhave C 2017 Confirmation bias with motivated beliefs \textit{Games Econ. Behav.} \textbf{104} 1–23
[19] Nickerson R S 1998 Confirmation bias: a ubiquitous phenomenon in many guises \textit{Rev. Gen.
Psychol.} \textbf{2} 175–220
[20] Tweney R D and Doherty M E 1983 Rationality and the psychology of inference. \textit{Synthese} \textbf{57}
139–61
[21] Coon D and Mitterer J O 2008 *Introduction to Psychology: Gateways to Mind and Behavior* (Toronto, ON: Wadsworth)

[22] German T P and Barrett H C 2005 Functional fixedness in a technologically sparse culture *Psychol. Sci.* 16 1-5

[23] Weiten W 2007 *Psychology: Themes and Variation, Seventh Edition* (California: Wadsworth)

[24] Posamentier A S and Krulik S 2009 *Problem-Solving in Mathematics Grades 3-6: Powerful Strategies to Deepen Understanding* (Thousand Oaks: Corwin A Sage Company)

[25] Butterworth J and Thwaites G 2013 *Thinking Skill, Critical Thinking and Problem solving* (Italy: L.E.G.O. S.p.A.)

[26] Brookhart S M 2010 *How to Assess Higher-Order Thinking Skills in Your Classroom* (Alexandria: ASCD)

[27] Polya G 1973 *How to Solve It* (New Jersey: Princeton University Press)

[28] Romberg T 2004 *Standards-Based Mathematics Assessment in Middle School: Rethinking Classroom Practice* (United States: Teachers College Press)