New Massive Gravity on de Sitter Space and Black Holes at the Special Point

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We study New Massive Gravity on de Sitter background and discuss several interrelated properties: The appearance of an enhanced symmetry point at linearized level where the theory becomes partially massless; its absence at full nonlinear level, and its relation with the existence of static black hole solutions and their hair parameter.

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I. INTRODUCTION

A fully nonlinear covariant theory of massive gravity in 2 + 1 dimensional spacetime, dubbed New Massive Gravity (NMG), has been introduced in Ref. [1]. At the linearized level, the theory is equivalent to the Fierz-Pauli action (FP) for a massive spin-2 field. It also passes highly nontrivial consistency checks at the nonlinear level[2, 3].

In this note, we revisit the formulation of NMG on de Sitter (dS) spacetime. We analyze the theory linearized about dS and study the appearance of an enhanced symmetry at a special point of the parameter space, similarly as it happens in FP gravity on dS [4–7]. This symmetry enhancement has also been observed in Ref. [8] within the canonical approach; here, we go further by relating this symmetry with the existence of dS black holes at the very special point of the parameter space.

II. NEW MASSIVE GRAVITY ON DE SITTER SPACE

The action of NMG is given by [1]

\[ S = \frac{1}{16\pi G} \int d^3x \sqrt{-g} \left[ R - 2\lambda - \frac{1}{m^2}K \right], \quad K = R_{\mu\nu}R^{\mu\nu} - \frac{3}{8}R^2. \]  

(1)

The square-curvature terms \( K \) satisfy the remarkable property \( g^{\mu\nu}\delta K/\delta g^{\mu\nu} = K \), which is important for the unitarity of the theory about flat background.

The equations of motion derived from (1) admit dS spacetime as exact solution provided \( \lambda \leq m^2 \), with the Hubble constant \( H^2 = 2m^2(1 \pm \sqrt{1 - \lambda m^{-2}}) \).

We consider perturbations of the form \( g_{\mu\nu} = \gamma_{\mu\nu} + h_{\mu\nu} \), where \( \gamma_{\mu\nu} \) denotes the metric of dS space. We denote the covariant derivative with respect to the background metric by \( \nabla \) and \( \nabla_\gamma h_{\mu\nu} = 0 \), and the equations of motion read

\[ -\frac{1}{2}\Box h_{\mu\nu} + \lambda h_{\mu\nu} - \frac{1}{2m^2} \left( -\Box^2 h_{\mu\nu} + 2H^2\Box h_{\mu\nu} + \frac{5}{2}H^2\Box h_{\mu\nu} - \frac{11}{2}H^4h_{\mu\nu} \right) = 0, \]

(2)

where we used that \( \nabla^\rho\nabla_\nu h_{\mu\rho} = \nabla_\mu\nabla^\rho h_{\nu\rho} + 3H^2h_{\mu\nu} - H^2\gamma_{\mu\nu}h \). Interestingly, the equations (2) factorize into

\[ (\Box - m^2 - \frac{5}{2}H^2)(\Box - 2H^2)h_{\mu\nu} = 0. \]

(3)

This splits the space of solutions into two: the General Relativity modes, \( h_{\mu\nu} \), that solve equation \((\Box - 2H^2)h_{\mu\nu} = 0\), and the massive mode, \( \tilde{h}_{\mu\nu} \), that solves \((\Box - m^2 - 5H^2/2)\tilde{h}_{\mu\nu} = 0\). In the limit \( m \to \infty \), the latter decouples.

III. CONFORMAL SYMMETRY AT THE SPECIAL POINT

Here, we will be concerned with the special points \( \lambda = m^2 \), that is \( H^2 = 2m^2 \). At this point, the theory exhibits a special property. To see this, consider the variation of action (1) under the transformation

\[ \delta g_{\mu\nu} = \gamma_{\mu\nu} \phi, \]

(4)
for an arbitrary function $\phi$. Up to a total derivative, the variation of the action is

$$\delta S = \int d^3x \sqrt{-g} \left[ \frac{1}{2} \left( 1 - \frac{H^2}{2m^2} \right) \left( \nabla^\sigma \nabla_\sigma + \left( \lambda - \frac{H^4}{4m^2} \right) \phi \right) \right]. \quad (5)$$

We see that, if $\lambda = m^2$ (equivalently, $H^2 = 2m^2$), the variation does vanish for arbitrary $\phi$; therefore, at this point the linearized theory exhibits a symmetry enhancement. This is reminiscent of the special point of FP gravity on dS. In the case of FP theory, the enhanced symmetry is of the form $\delta g_{\mu\nu} = (H^{-2} \nabla_\mu \nabla_\nu + \gamma_{\mu\nu}) \phi$. NMG is generally covariant, so that the term $\nabla_\mu \nabla_\nu \phi$ represents a symmetry by itself; it is the $\gamma_{\mu\nu} \phi$ term that becomes enhanced at $\lambda = m^2$.

**IV. BLACK HOLES AT THE SPECIAL POINT**

It turns out that, at $\lambda = m^2$, NMG exhibits another peculiar property: it admits dS black hole solutions \[9, 10\]. The metrics is

$$ds^2 = -(Hr)^2 + 2bHr - \mu \right) dt^2 + \frac{dr^2}{-(Hr)^2 + 2bHr - \mu} + r^2 d\Omega^2, \quad (6)$$

where $b$ and $\mu$ are two arbitrary parameters. In the range $0 < \mu \leq b^2$, metric \[6\] describes a black hole that asymptotes dS$_3$ spacetime. The black hole exhibits a curvature singularity at the origin, it being covered by an event horizon located at $r_- = (b/H)(1 - \sqrt{1 - \mu/b^2})$. The cosmological horizon of dS$_3$ space is located at $r_+ = (b/H)(1 + \sqrt{1 - \mu/b^2})$.

Considering $b$ as small parameter and expanding to linear order one gets

$$ds^2 \approx g^0_{\mu\nu} dx^\mu dx^\nu - 2bHr dt^2 = \frac{2bHr}{(\mu + (Hr)^2)} dr^2, \quad (7)$$

where $g^0_{\mu\nu}$ corresponds to metric \[7\] with $b = 0$. Now, we observe that a special combination of a conformal symmetry and coordinate transformations of the form

$$\delta g_{\mu\nu} = \nabla_{(\mu} \xi_{\nu)} + g^0_{\mu\nu} \phi, \quad \text{with} \quad \xi^\nu = -\frac{bHr^2}{\mu} \delta^\nu_r, \quad \phi = \frac{2bHr}{\mu}, \quad (8)$$

precisely cancels the terms proportional to $b$ in \[7\]. This means that \[7\] can be generated by a transformation \[4\]. At the nonlinear level, the symmetry \[4\] is absent.

Away from the special point, the linear analysis of NMG reveals the existence of a non-propagating tensor mode of 3D massless General Relativity, as well as the helicity-2, helicity-1, and the helicity-0 modes of a 3D massive graviton (helicity-2 being nondynamical). In contrast, at the special point, the kinetic term of the helicity-0 mode vanishes, and it becomes infinitely strongly coupled since the corresponding conformal symmetry is present at the linear level only. These facts get reflected onto the black hole solution \(6\) as follows: as shown above, the black hole hair is a gauge artifact in the linearized theory, while this hair is not removable in the full nonlinear case\(^1\).

It appears that the consistency with no-hair theorems prevents the existence of the black holes away from the special point since for $\lambda \neq m^2$ the extra longitudinal mode, which would provide the hair, is a propagating field. At $\lambda = m^2$ the black holes are possible only in the regime where the hair is carried by the longitudinal mode that is very (or perhaps infinitely) strongly coupled, at least when the dS horizon is approached. This feature is what seems to be responsible for the evasion of the no-hair theorems. It may be interesting to study similar questions in 3D ghost-free massive gravity \[11, 12\]. The latter theory differs from NMG by the absence in it of the massless 3D GR field. This field is responsible for negative mass, $-\mu$ in \[6\].

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\[^1\] Note, however, that on the full black hole solution \(6\) the helicity-0 model may or may not acquire its kinetic term due to nonzero $\mu$ and/or $b$.  

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