The effect of sensor noise on structural modal parameter identification

Shuai Li¹*, Ting Wang¹, Yang Zhao¹, Congying Deng¹, Xinghong Li¹

¹School of Advanced and Manufacturing Engineering, Chongqing University of Posts and Telecommunications, Chongqing, 400065, China

*Email: S182131015@stu.cqupt.edu.cn

Abstract. When errors occur in the sensor networks, there are noise in the acquired data. In this paper, based on the response correlation function between two points of the structure, the least squares complex exponential method is used to identify the influence of noise on the structural modal parameters. Through the construction of the steel beam platform experiment, the experiment verifies the influence of the noise in the collected data on the recognition of modal parameters. The results show that when there is noise in the collected response data, the noise component will cause errors in the identification of structural modal parameters, and the lower the modal order, the more sensitive the noise level.

1. Introduction

Structural health monitoring [1] (SHM) aims to evaluate the structural health status based on the response data of the structure with ambient excitation, and monitor the real-time physical status of the structure through the change of the structural system state relative to the ambient excitation. The recognition of modal parameters under ambient excitation has received extensive attention in the engineering community, especially in the aspect of operational modal analysis, which only needs to extract modal characteristic information from structural response data. However, in the health monitoring of large structures (such as dams [2], bridges [3], high-rise buildings [4], etc.), the sensor collection nodes are generally distributed in various parts of the main structure, and the collection points have a certain degree of discreteness, thus, the acquired data exists certain noise. This may have a certain impact on the estimation of structural modal parameters, resulting in misjudgment of the structural state.

Huang [5] et al. have developed and proposed a machine vibration monitoring system based on WSN, and designed a wireless sensor node with dual processor architecture to improve the accuracy of synchronous acquisition. This solution is based on cross-layer hardware design, which brings cost, power consumption and performance problems to a certain extent. Later, Shamim N et al. [6] designed and developed an acceleration sensor node for structural health monitoring to meet the requirements of structural vibration monitoring and modal recognition. These collection nodes exhibit powerful and scalable performance even when the number of hops is required for communication. In the paper [7], the author studies the influence of time delay on the reconstruction of mode shapes in the wireless sensor acquisition network. The WISAN method is proposed to realize the synchronous acquisition and control the wireless synchronous acquisition error within the range of less than \( \pm 10\mu s \).

On the basis of the above literature research, based on the similar expressions of the cross-correlation function and impulse response function of the response between two points of the structure under the ambient excitation, after the cross-correlation function of the response between two
2. Identification of structural modal parameters in ambient excitation

Under the condition of general viscous damping, physical-modal coordinate transformation is adopted to decouple the original coupled differential equations of motion and transform them into a set of modal equations which can be solved independently, and each independent equation can be regarded as a modal model unit system [9].

The mathematical theory of modal analysis of vibration system is constructed from the structural dynamics theory. For a general viscous damped structural system with 2\(n\) degrees of freedom, its dynamic differential vibration equation can be expressed as:

\[ M \ddot{x}(t) + C \dot{x}(t) + Kx(t) = f(t) \]  

Where, \([M],[C]\) and \([K]\) respectively represent the input vector of the system, and \([x(t)]\) represents the state vector. Based on complex modal theory and mode shape superposition method, the relationship between dynamic response and modal parameters is established, and the mathematical model of output response of the structure under a certain excitation can be obtained as follows [8]:

\[
x_{ik}(t) = \sum_{r=1}^{2n} \frac{\{\phi_i^r\} \{\phi_j^r\}^T}{a_r} \int_{-\infty}^{\infty} \{f(\tau)\} \exp[\lambda_r(t-\tau)]d\tau
\]

When \(f(t)\) is impulse excitation, there are:

\[
x_{ik}(t) = \sum_{r=1}^{2n} \frac{\phi_i^r \phi_j^r}{m^r \omega_n^r} \exp(-\xi^r \omega_n^r t) \sin(\omega_n^r t)
\]

Where, \(\xi^r\) is the \(r\)-order modal damping factor, \(\omega_n^r\) is the \(r\)-order modal frequency, \(\omega_n^r\) is the \(r\)-order modal damping frequency. Given the response of \(i\) and \(j\) points, its correlation function can be expressed as:

\[
R_j^j(T) = E[x_{ik}(t+T)x_{jk}(t)]
\]

When there is an error in the collected data, substitute in Equation (3), so the correlation function of noise component in the response of \(i\) point and \(j\) point is as follows:

\[
R_j^j(T) = E \left\{ \sum_{r=1}^{2n} \sum_{s=1}^{2n} \phi_i^r \phi_j^s \phi_k^r \phi_j^s \int_{-\infty}^{\infty} \exp\left[-\xi^r \omega_n^r (t+T) - \xi^s \omega_n^s t\right] \sin(\omega_n^r (t+T)) \sin(\omega_n^s t) dt \right\}
\]

\[
= \sum_{r=1}^{2n} \sum_{s=1}^{2n} \phi_i^r \phi_j^s \phi_k^r \phi_j^s \int_{0}^{\infty} \exp\left[-\xi^r \omega_n^r (t+T) - \xi^s \omega_n^s t\right] \sin(\omega_n^r (t+T)) \sin(\omega_n^s t) dt
\]

\[
= \sum_{r=1}^{2n} \frac{\phi_i^r A_j^r}{m^r \omega_n^r} \exp(-\xi^r \omega_n^r T) \sin(\omega_n^r T + \theta_r)
\]

That is, the correlation function of noise component in the response of \(i\) point and \(j\) point is as follows:

\[
R_j^j(T) = \sum_{r=1}^{2n} \frac{\phi_i^r A_j^r}{m^r \omega_n^r} \exp(-\xi^r \omega_n^r T) \sin(\omega_n^r T + \theta_r)
\]

Obviously, its correlation function has a similar expression to the impulse response function in Equation (3), so the correlation function can be used to replace the impulse response function to obtain the modal parameters of the system.

The linear equations of each order impulse response amplitude at each measurement point are constructed from the impulse response data sequence. The linear equations of auto regressive
coefficient are obtained through sampling at different starting points. Since the impulse response function obtained by measurement is a discrete time series, then \( t_k = k\Delta t \):

\[
h'_k = h(k\Delta t) = \sum_{r=1}^{2n} A_r e^{r \cdot k\Delta t} = \sum_{r=1}^{2n} A_r V_r^k, \quad k = 0, 1, 2, 3, \ldots, L
\] (7)

Among them \( V_r = e^{r \cdot \Delta t} \), since \( A_r \) is a real number, the equations in the above equation can be seen as the roots of the \( 2n \)-order polynomial equation. It can be written as:

\[
\sum_{r=1}^{2n} t_k V_r^k = \prod_{r=1}^{2n} (V - V_r^k)
\] (8)

The least squares solution of the equations is obtained by the pseudo-inverse method, that is:

\[
\{\tau\} = (|h|^T |h|^{-1}) (|h|^T \{h\})
\] (9)

Substituting Equation (9) into Equation (8), as shown in the following:

\[
\sum_{r=1}^{2n} t_k V_r^k = \tau_0 + \tau_1 V + \tau_2 V^2 + \ldots + V^{2n}
\] (10)

The roots of polynomial \( V_r \) with coefficients \( t_k \) are found, from which the modal frequency \( \omega \), and damping ratio \( \xi \), can be obtained.

\[
\begin{align*}
R_r &= \ln V_r = s_r \Delta t \\
\omega_r &= |R_r| / \Delta t \\
\xi_r &= \sqrt{\frac{1}{1 + (\text{Im}(R_r) / \text{Re}(R_r))}}
\end{align*}
\] (11)

3. Experimental analysis

In order to verify the influence of collected noise on modal parameter identification, we built a steel beam experimental platform, added "artificial noise" to the collected data, and calculated its modal parameters. For verify the validity of the experimental data and more accurate modal parameters of the beam structure, the finite element modeling (FEM) and simulation analysis of the steel beam structure was carried out before the experiment.

The two ends of the steel beam are fixed by the vibration experimental platform, and six triaxial acceleration sensors (as shown in Figure 1) are arranged on the steel beam at equal intervals, which are connected to the DHDNS data acquisition instrument through cable, and the data collected by the acquisition instrument is connected to the computer through Gigabit network cable for processing and analysis. The random excitation steel beam structure is simulated by the vibration exciter, and the relevant response data are collected.

\[
\text{Figure 1. Modal test of beam structure}
\]

In the response data collected by the above experimental platform, noise is added to the data collected by node 4 artificially, and the structural modal parameters are identified by the algorithm. Figure 2 (a) shows the modal shape of the steel beam reconstructed by the identified modal parameters without adding noise. Figure 2 (b) shows the modal shapes reconstructed by adding artificial noise to the node response data. Through the comparison of the two figures, it can be seen that when the
collected data contains noise, the modal shape of structural reconstruction will be affected, and it can be seen that the noise effect is particularly obvious at the node 4 where the noise is added.

![Figure 2 (a). Vibration mode of noiseless structure](image1)
![Figure 2 (b). Vibration mode of noise structure](image2)

The collected experimental data, the artificial noise data and the finite element simulation data are drawn into a table. According to Table 1, when there is no noise in the experimental data, the error between the identified modal frequency and the finite element simulation data is less than 5%. However, when there is noise in the collected data, there is a certain error between the identified modal natural frequency and the finite element simulation data, the error of the first and second order modal frequency is as high as about 10% (considering the relevant error in theory, it is reasonable to be about 5%). It can be seen that when the collected data contain noise components, there will be some errors in the structural modal parameters.

| Modal order | FEM Modal frequency/Hz | Actual data Modal frequency/Hz | Add noise Modal frequency/Hz | Relative error (%) |
|-------------|------------------------|-------------------------------|-----------------------------|-------------------|
| 1           | 33.122                 | 31.51                         | 28.953                      | 12.58             |
| 2           | 91.282                 | 89.42                         | 78.841                      | 13.63             |
| 3           | 178.950                | 170.30                        | 173.240                     | 3.19              |
| 4           | 295.840                | 298.61                        | 285.920                     | 3.35              |

Figure 3 shows the first four modal frequencies of the steel beam structure. The blue dots represent the finite element simulation results, the yellow ones represent the modal frequencies obtained from the actual experimental data (no noise group), and the red ones represent the modal frequencies identified after adding artificial noise to the collected response data. It can be seen from the comparison that the lower order modes are more sensitive to the noise in the sensor data.

The experimental results of steel beam show that when errors occur in the acquisition network, there is noise in the collected data, which will lead to "false modal parameters" in the later identification results. Moreover, the lower the modal order is, the more sensitive it is to the noise in the sensor data. In the traditional structural modal parameter analysis, due to the existence of modal error caused by the collected data noise, it needs an additional method to eliminate the "false mode" in the structure. How to eliminate this influence in modal analysis is the next work arrangement, which needs further discussion.
4. Conclusions
In this paper, the cross-correlation function of two-point response signals under ambient excitation is approximate to impulse response function, and the influence of noise in response data on structural modal parameter identification is analyzed. An experimental steel beam platform is built to verify the theoretical analysis. The experimental results show that the noise in the sensor data will cause the modal natural frequency to jitter. This discovery estimates the influence of the acquisition error on the accuracy of modal parameters. The conclusions are as follows:

(1) When the jitter of the acquisition system results in the noise of the collected data, the modal parameters of the structure will produce additional "false modal parameters".

(2) When there is noise in the collected data, the lower order modes are more sensitive to the noise level.

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