The identification of van Hiele level students on the topic of space analytic geometry

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Abstract. Geometry topics are still considered difficult by most students. Therefore, this study focused on the identification of students related to van Hiele levels. The task used from result of the development of questions related to analytical geometry of space. The results of the work involving 78 students who worked on these questions covered 11.54% (nine students) classified on a visual level; 5.13% (four students) on analysis level; 1.28% (one student) on informal deduction level; 2.56% (two students) on deduction and 2.56% (two students) on rigor level, and 76.93% (sixty students) classified on the pre-visualization level.

1. Introduction

Many have found in the field that students' understanding of geometry especially analytical geometry of space is still low. This can be seen from the results of some previous studies obtained by researchers results of student work is low in the geometry of space [1]. This can also be influenced by one’s ability to think [2]. When it comes to geometry, it means something to do with van Hiele's theory. Van Hiele says that one's ability to work on geometry is influenced by good geometry learning experience [3]. The patented van Hiele test consists of 25 multiple choice questions and every five questions consist of one level of van Hiele thinking. Reminiscent of van Hiele, Piaget says a person is classified by his age from his birth to the end of his life. The older someone is, the more complex knowledge he has [4]. Finally it can be concluded that actual theory of van Hiele and Piaget theory are contradictory.

Based on differences revealed between van Hiele's theory and Piaget's theory above, the researchers tried to focus on van Hiele's theory only to temporarily ignore the theory conveyed by Piaget related to age. Researchers argue that the 25 questions patented by van Hiele should be able to be developed on certain topics, such as lines, angles, fields, and spaces. This means that there is a "local" test package that can be developed based on the descriptors given by van Hiele himself.

Based on van Hiele’s notion, the researchers want to identify the levels of van Hiele related to the topic of space analytic geometry germane to ellipsoid, hyperbolic, paraboloid, and sphere. The five questions given have been validated by a team of experts from the Mathematics Education Study Program, Faculty of Teacher Training and Education, University of Jember, Indonesia. Each question is related to five levels (visualization, analysis, informal deduction, deduction and rigor) as details of number 1 level 0, number 2 level 1, number 3 level 2, number 4 level 3, and number 5 level 4.

To solve the problem of geometry, the most appropriate theory used is van Hiele’s theory. Therefore van Hiele's theory specifically discusses geometry-related topics. Van hiele said, the level of visualization is related to appearance of the object as a whole; the level of analysis related to how students know the
properties of the observed object; the level of informal deduction is related to the relationship between observed objects; the level of deduction related to one's knowledge of axioms, definitions, and theorems [5,6]; and the level of rigor associated with deductive proof [3,7,8]. Van Hiele test results mostly reach the deduction level and very few students reach the rigor level. It because they learn from elementary school to university do not pay attention to descriptors that have been standardized by van Hiele. Like research in elementary school of Jember city, the level of visualization is more dominant than (Analysis until rigor) [9,10]. If viewed from the theory of anticipation, it is possible that learning Geometry will be better.

Anticipation is actually needed if you want to improve a person's ability to understand a problem, such as geometry problems [6,7]. Similarly, which was initiated by van Hiele based on the results of his research, that students who have been on a certain level are certainly able to solve the problems of geometry at the previous level [9,14].

At 0 level (visualization) a person has known geometric shapes, including triangle, cube, sphere, square, circle, but students can’t understand yet the properties of these builds. Although a model has been determined on basis of the characteristics, a person at this level is not yet aware of that characteristic. At 0 level, the person’s thinking is dominated by their perception. At 1 level (Analysis) a person has known the properties of objects’ geometry he observes. Someone is able to mention the regularity contained in geometric objects. for example when one looks at a rectangle, he has known that there are two pairs of opposite sides and the two pairs of sides are parallel to each other. In this stage it has not been able to know the related relationship between an object geometry and other geometry objects. At 2 level (informal deduction) a person has known geometric forms and understands the properties and it has been able to sequence geometric forms with one another interconnected. for example the square is also a rectangle. So at this stage the students have been able to understand the sequencing of geometric forms, even though deductive thinking has not developed or in other words it has just begun. at this stage the students can’t answer the question on why the two diagonals of the rectangle have the same length. At 2 level (deduction) the suitability of deduction as a way of building geometry in axiomatic systems has been understood. Someone has to compile the proof, not only accept the proof [14]. The structure of complete axiom system with the axioms, definitions, theorems, consequences and postulates what is implicitly present at 2 level, now be the explicit object of his thinking. There is more than one possibility of developing proof. The difference between the statement and conversations can be made. At the deduction level it is clear that the square diagonals share each other’s and can realize the necessity of proving through a series of deductive reasons. At 4 level (rigor) person can work in various axiomatic systems. This means that he is able to study non-Euclidean geometry. A person at the rigor level can be said capable of going through 0 level to 3 level and it means he has reversible thinking ability [10-11] and is most likely categorized in anticipation of analysis and exploration, because both of anticipations can help a person achieve the right level of thinking within Solve the problem [1,12].

2. Methods
The test given to 78 students is van Hiele test developed by researcher and team based on van Hiele theory descriptors. The test consists of 5 questions which each question represent one van hiele level, in sequence ie level 0 for question number 1, level 1 for question number 2, level 2 for question number 3, level 3 for question 4, and level 4 for question number 5. The test given to 78 students structurally who is taking analytical geometry course in the event semester of academic year 2016/2017. The seventy-eight students are assigned to work fifth problems developed. Students' work results analyzed and classified according to van Hiele level. The test used can be seen in Table 1.
Table 1. Van Hiele Test Analytical Geometry Space

| No | Test | Level       |
|----|------|-------------|
| 1  | Given equations \((bcx)^2 - (acy)^2 - (abz)^2 - (abo)^2 = 0\). In your opinion, what is the space model meant by the equation? Explain! | Visualization |
| 2  | Given equations \((b^2 + c - 2)\), \((ab)^2\), \(z + (ay)^2\), \(c = 0\). Describe all the elements that you know! | Analysis |
| 3  | Given equations \(\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1\). Based on the equation in question number 3, moving the variable and constant. What happened? Explain! | Informal Deduction |
| 4  | Given the equation of lines \(g_1\) and \(g_2\). The two lines cross each other in space. Prove that \(g_1 \perp g_2\)! | Deduction |
| 5  | Consider the picture beside! If the value is known \(a^2 = 4\), \(b^2 = 9\), and \(c^2 = 16\), then make each equation in the right coordinate system, cylinder, and sphere. | Rigor |

3. Result and Discussion

The five questions used in the research are result of the development of twenty-five questions that have been developed by van Hiele [3,5,18]. Because the authors assume that the twenty-five existing questions have been patented and can be used by elementary school students to adults, therefore researchers delve into developing five questions related to space analytical geometry. The boarding guidance used in the test above is that each question has a maximum score of 20, if each question meets the characteristics of each level of van Hiele. If it is only complete as necessary but it is still related to the parts asked on the question, then it will get a score of 10. As for the score of 5-10 marked at the transition level, and if it is more than 10 then it belongs to that level [7,13,19].

The students solve the questions in Table 1 above in 90 minutes. Based on above scoring tests and guidelines, 78 students taking the test are classified in Table 2.

Table 2. Van Hiele Test Results

| Level          | Total | Percentage |
|----------------|-------|------------|
| Visualization  | 9     | 11.54%     |
| Analysis       | 4     | 5.13%      |
| Informal Deduction | 1    | 1.28%      |
| Deduction      | 2     | 2.56%      |
| Rigor          | 2     | 2.56%      |
| Total          | 18    | 23.07%     |

The results of van Hiele test written in Table 2 above reveal that only 18 out of 78 students can be classified on the five van Hiele levels, while the remaining 60 (76.93%) the students can be classified in pre-visualisation, because on question number 1 it is already unable to solve the problem given [8,19,20]. Nonetheless, on the number 3 problem there are 14 students who are able to solve the problem well. This has caused van Hiele level reduce the result, as it is commonly known as "jumps" and can not be classified at the van Hiele level [7,21,22].
4. Conclusion
The results of the work of 78 students who had worked on these questions evinced the following results: 11.54% (nine students) classified on a visual level; 5.13% (four students) on a analysis level; 1.28% (one student) on a informal deduction level; 2.56% (two students) on a deduction and 2.56% (two students) on a rigor level, and 76.93% (sixty students) classified on the pre-visualization level.

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