An Integrated Optimization Approach of $p$-Hub Median Problem and Service Facility Configuration

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Abstract: Traditional network design approach neglected the impact of network-wide location of service facility configuration on hub network designing scheme. In order to improve the mutual adaptability between hub network scheme and service facility configuration, the selections including the location of hubs, the itinerary of OD pairs and the allocation of both types and number of service facilities on each link were considered as the decision variables. The limitations of the maximum expected number of service facilities deploying on each link and the total service time of each facility type were treated as constraints. Then an integrated mathematical model of $p$-hub median problem and service facility configuration was constructed. A numeric example with the scale of 9 cities, 4 types of aircraft and 72 OD pairs in domestic airline industry in 2016 is used to verify the feasibility. The results indicate that the approach can decrease the hub network designing cost by 1.34% and the adaptability of hub network scheme to service facility configuration is significantly raised as compared to the traditional approach.

1. INTRODUCTION
The objective of $p$-hub median problem is to determine the locations of $p$ hubs, the itineraries of each origin-destination flows, and the link relations between hubs as well as hub and non-hubs. In the scope of future planning, it is necessary to determine the address of the $p$-hub node, the connection relationship between the hub and the non-hub, and the transportation path of each OD (Origin-Destination) stream. It's the core of hub network design and widely used in aviation, logistics, postal and telecommunications industries. In response to this problem, the early research results mainly focused on the discussion of modeling skills of this type of problem (See O'Kell[1], Skorin-Kapov [2], Campbell [3] and Ernst [4], etc. for details). Subsequent research focused more on the economies of scale of the hub network, and used a discount factor on the edge between hub nodes to influence the hub network model (see O'Kelly [5], Ebery [6], Wojahn [7], Elhedhli [8] and Camargo [9] etc.). In view of the uncertainty of the hub network design environment, some scholars regard the hub network design parameters (such as unit flow cost, OD flow demand) as random variables, and study the robust optimization design of the hub network from the perspective of risk control (such as interval robust optimization [10], random optimization [11]) problems (see Conde
When using the above method to research the design of hub network, the network design parameters such as the link cost of the future network and the traffic of each OD should be given in advance. But in the actual hub network design, the link cost on the network depends on the type of service equipment configured on the link and its quantity. For example, the service cost between two places in the logistics, aviation, and postal industries depends on the type and frequency of the transportation vehicles configured, and the cost of data transmission between the two places in the telecommunication industry depends on its laying cable type and quantity. Faced with this situation, the above research process only used discount factors to try to reflect the differences in the types of service equipments and the number of service devices configured on each link of the network, which may lead to a large deviation from the actual network design results and generate a sub-optimal hub network solution. Therefore, recent research has focused on the hub network’s economies of scale, and explored hub network design methods by introducing fixed costs of service equipment (see Qin [17], Campbell [18], O’Kelly [19], and Ghodratnama [20], etc.). However, just considering the fixed cost of service equipment as a known network design parameter is difficult to reflect the relationship between the link cost of the network and the type, quantity of service equipment configured on that link. In fact, the transport path of each OD pair in the hub network design process determines the type of OD pairs and their traffic carried on each link of the network, and thus determines the type and quantity of service equipment required on each side to achieve the minimum network design cost. This decision result in turn determines the actual link cost in the network, which in turn will affect the design result of the hub network. Therefore, hub network design and service equipment configuration are two types of decision problems that affect each other.

In view of this, this paper first gives the mathematical model of the $\text{p}$-hub median problem with the hub setting cost, and analyzes the relationship between the service equipment configuration decision and the $\text{p}$-hub median problem; then, choosing the location of the hub, the path of each OD pair, and the type and number of service equipment configured on each link as the decision variables, combined with factors such as the maximum number of service equipment expected on each link and the total service time limit of various types of equipment, an integrated optimization mathematical model of $\text{p}$-hub median problem and service equipment configuration is constructed; finally, the city pair data of domestic air transport are used to verify the effectiveness of the method.

2. PROBLEM DESCRIPTION AND MODEL STRUCTURE

2.1. $p$-hub Median Problem and Model Structure

In a no-capacity hub network with nodes, it is known that the flow $W_{ij}$ of each OD pair, the cost $H_k$ of setting hub and unit cost $c_{ikmj}$ of each OD pair $(i, j)$ flowing through the hub $k$ in the network ($\alpha, \beta, \gamma$ is the discount factor for non-hub and hub, inter-hub, hub and non-hub, respectively, and $0 \leq \beta \leq \alpha, \gamma \leq 1$ holds). It is required to find the setting address of $p$ hub nodes, and the proportion $x_{ikmj}$ of each OD pair $(i, j)$ that chooses to transit through the hub node $k,m$ to reach the destination $j$, so as to minimize the cost of designing the hub network after transporting all OD pairs of traffic.

In a hub network, the unit flow cost between any two nodes is usually proportional to the distance, and satisfies the triangle inequality relationship [21]. Therefore, direct transportation must be adopted between hub nodes, and the number of non-hub and hub node transfers will not exceed two times. Using the CAMPBELL four subscript model [3] expressions, the problem can be expressed as shown in equations (1)-(8).

$$\min \left[ \sum_{j=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{p} \sum_{m=1}^{p} c_{ikmj} W_{ij} x_{ikmj} + \sum_{k=1}^{p} H_k y_k \right]$$

(1)
\[ s.t. \quad \sum_{i=1}^{n} y_i = p, \quad (2) \]

\[ \sum_{m=1}^{n} x_{iakj} + \sum_{m=1}^{n} x_{makj} \leq y_k, \quad \forall i, k, j, \quad (3) \]

\[ \sum_{m=1}^{n} \sum_{l=1}^{n} x_{iakj} = 1, \quad \forall i, j, \quad (4) \]

\[ \sum_{m=1}^{n} \sum_{l=1}^{n} x_{iakj} \leq 1 - y_i, \quad \forall i, j, \quad (5) \]

\[ \sum_{m=1}^{n} \sum_{l=1}^{n} x_{iakj} \leq 1 - y_j, \quad \forall i, j, \quad (6) \]

\[ x_{iakj} \geq 0, \quad \forall i, k, m, j, \quad (7) \]

\[ y_i \in \{0,1\}, \quad \forall i. \quad (8) \]

Among them: the objective function (1) minimizes the design cost of the hub network (transportation cost and hub installation cost). Constraints (2) specify the number of hub nodes. Constraints (3) specify that any OD pair must finish transferring traffic at the hub node. Constraints (4) stipulate that any OD traffic task must be completed. Constraints (5) and (6) stipulate that any OD pair has a start point and an end point and there is only one hub. Only once transfer is allowed if needed. When the start and end points are both hubs, they can only take direct transportation. Equations (7)-(8) are the variable types and value ranges.

Constraints (5) and (6) not only define the OD transfer method between different types of nodes, but also define the path expression of this transfer method: when nodes \( i \) and \( j \) are both hubs, the variables in the left-hand terms of constraints (5) and (6) must all be zero, resulting in the path form of the OD pair being limited to \( i-i-j-j \). When one of nodes \( i \) and \( j \) is a hub and one is a non-hub, there must be a variable in the left-hand terms of constraints (5) and (6) that are all zero, resulting in the path form of the OD pair being limited to \( i-i-m-j \) or \( i-k-j-j \).

In a non-hub network without the functions of remittance, transshipment, and distribution [21], the unit flow cost of each side is determined by the type, number of devices serving that side and the flow only serving the corresponding OD pair on the side. However, in the hub network, other OD pairs flow through the side due to remittance, transshipment and distribution, and lead to increased traffic on the side, which can use equipment services with greater carrying capacity ("economies of scale"). As such, formulas (1)-(8) use discount factors to express this economies of scale. Although this method is feasible, it ignores the cost of unit flow on the edge of the network is neglected by the total cost of transportation caused by all types of OD pairs flowing through the edge and their flow rate (as shown by the denominator in the right-hand term of equation (9)), and the type, number of service devices configured (as shown by the numerator in the right-hand term of equation (9)). Therefore, traditional methods often produce sub-optimal, even unfeasible hub network design schemes in hub design.

\[
\sum_{t=1}^{q} c_{tj} F_{tj} z_{tj} = \frac{\sum_{t=1}^{q} \sum_{m=1}^{F_{tj}} \sum_{n=1}^{m} w_{mnt} x_{mnt} + \sum_{t=1}^{q} \sum_{n=1}^{F_{tj}} \sum_{m=1}^{n} w_{mnt} x_{mnt} + \sum_{t=1}^{q} \sum_{n=1}^{F_{tj}} w_{mnt} x_{mnt}}{q} \quad (9)
\]

In formula (9), \( c_{tj} \) represents the single service cost of the service equipment \( t \in \{1,2,\ldots,q\} \) on the edge \( (i,j) \). \( q \) is the number of service equipment types. \( F_{tj} \) is the maximum number of configurable service equipment on the edge \( (i,j) \) of the network, and it is used to comprehensively reflect the upper limit of the flow service capacity of any side \( (i,j) \) of the network and the connected nodes \( i \) and \( j \) within a certain period. \( z_{tj} \) represents the proportion of the number of various types of service
equipment configured on the edge \((i,j)\) of the network. so \(F_{ij} z_{ij}'\) reflects the actual configuration number of each service equipment type \(t\) on the side \((i,j)\), and \(c_{ij}' F_{ij} z_{ij}'\) is the service cost of the service equipment type \(t\) on the side \((i,j)\). In the formula (9), the denominator represents the total flow of all OD pairs flowing through that side when any side \((i,j)\) may perform the functions of remittance, transshipment and distribution.

2.2. Integrated Optimization Problem and Model Structure

We believe that the fundamental solution to the above-mentioned shortcomings lies in the integrated optimization of the p-hub median problem and the service equipment configuration. The problem can be described as: based on the flow \(W_{ij}\) of OD pair \((i,j)\), the cost \(H_k\) of hub node set-up, the maximum number of service devices \(F_{ij}\) expected to configure of each side \((i,j)\) and the single service cost \(c_{ij}'\) of service devices configured \(t\) on this side, single service time \(h_{ij}\), maximum load capacity \(Cap_t\) and its average load rate \(L_{ij}\) in a given network, to minimize the cost of designing a hub network that transports all OD traffic, it is required to find the setting address \(y_j\) of the hub node, the proportion of each OD pair \((i,j)\) that chooses to transfer through the hub node \(k,m\) to the destination \(j\), and the proportion \(z_{ij}'\) of the number of service devices \(t\) configured on each side of the network.

Convert equation (9) and form a new integrated planning objective function. Equation (9) can be transformed into equation (10) after removing the denominator.

\[
\sum_{t=1}^{s} c_{ij}' \left( \sum_{k=1}^{n} W_{ik} x_{ik} + \sum_{k=1}^{n} W_{in} x_{kn} + \sum_{k=1}^{n} W_{km} x_{km} \right) = \sum_{t=1}^{s} c_{ij}' F_{ij} z_{ij}' .
\]

The left-hand side of equation (10) is the flow service cost of any side \((i,j)\) in the network, which is equal to the unit flow cost on that edge multiplied by the sum of all the OD pairs flowing through the edge; The right-hand side is another way of expressing the service cost of the side, which is the service cost generated by the type of service equipment and the number of services used on the side. Then, for the total cost of services in the network, the service costs on all sides can be accumulated and obtained on the basis of equation (10), as shown in equation (11).

\[
\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{s} c_{ij}' \left( \sum_{t=1}^{s} W_{ik} x_{ik} + \sum_{t=1}^{s} W_{in} x_{kn} + \sum_{t=1}^{s} W_{km} x_{km} \right) \Rightarrow \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{s} c_{ij}' F_{ij} z_{ij}' .
\]

The cumulative sum of the three polynomials in the brackets of the left-hand term of equation (11) has nothing to do with the cost of the unit stream on that side, so equation (11) can be changed to equation (12).

\[
\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{s} (c_{ij} W_{ini} x_{ini} + c_{ij} W_{jin} x_{jin} + c_{ij} W_{jmk} x_{jmk}) - \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{s} c_{ij}' F_{ij} z_{ij}' .
\]

Because of \(i,k,m,j \in \{1,2,\ldots,n\}\), the expression form of the subscript symbols in the left-hand items in equation (12) can be changed. For example, \(c_{ij} W_{ini} x_{ini}\) can be rewritten as \(c_{ij} W_{ini} x_{ini}\), and equation (13) holds.

\[
\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{s} (c_{ij} W_{ini} x_{ini} + c_{ij} W_{jin} x_{jin} + c_{ij} W_{jmk} x_{jmk}) = \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{s} c_{ij}' F_{ij} z_{ij}' .
\]

Because of \(c_{ini} = c_{ij} + c_{in} + c_{nj}\), after merging similar terms on the left-hand side of equation (13), equation (14) holds.
\[
\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{m} C_{ikj} x_{ij} = \sum_{j=1}^{n} \sum_{k=1}^{m} y_{jk} F_{i}^{j} \_x. \tag{14}
\]

Therefore, the objective function formula (1) of the integrated optimization model can be rewritten as formula (15).

\[
\min \left[ \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{m} c_{ij}^{k} x_{ij} + \sum_{k=1}^{m} y_{jk} F_{i}^{j} \_x \right]. \tag{15}
\]

The constraints of the integrated optimization model consist of equations (2)-(8) and equations (16)-(19).

\[
\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{m} W_{i}^{k} X_{ikj}^{k} + \sum_{i=1}^{n} \sum_{i=1}^{n} \sum_{k=1}^{m} W_{i}^{k} X_{ikj}^{k} + \sum_{i=1}^{n} \sum_{i=1}^{n} \sum_{k=1}^{m} W_{i}^{k} X_{ikj}^{k} \leq \sum_{i=1}^{n} L_{i}^{k} Cap_{i} F_{i}^{k} \_x, \forall i \neq j, \tag{16}
\]

\[
\sum_{i=1}^{n} \sum_{j=1}^{n} h_{i} F_{i}^{j} \_x \leq \Gamma_{i}, \forall t, \tag{17}
\]

\[
\sum_{i=1}^{n} z_{i} \leq 1, \forall i, j, \tag{18}
\]

\[
z_{i} \geq 0, \forall i, j, t. \tag{19}
\]

Among them: constraint (16) is the constraint of supply and demand, which stipulates that the sum of the traffic of all OD pairs flowing through any side \((i, j)\) in the network within a certain period of time does not exceed the amount that it can actually carry. Constraint (17), on the basis of the specific device type serving the side \((i, j)\) single service time \(h_{i} F_{i}^{j} \_x\), it is required that the total service time of the device in the network should not exceed its specified maximum service capacity \(\Gamma_{i}\), which is used to reflect the upper limit of service time due to factors such as equipment maintenance and human resource limitations of service equipment within a certain period of time. Constraint (18) stipulates that the sum of the number of various devices configured on any side \((i, j)\) does not exceed the maximum number that can be configured on that side, which is used to reflect the upper limit of the flow service capacity of any side of the network and the nodes connected to it in a certain period of time. Equation (19) is the variable type and value range. It should be noted that, since constraints (5) and (6) define the path expression of OD pairs corresponding to different node types, the flow of different OD pairs flowing on any side corresponding to the left-hand term of equation (16) don’t calculate repeatedly, which can ensure the correctness of constraints.

3. **NUMERICAL EXAMPLES**

The 9 cities with the highest passenger throughput in the air transportation industry (sequence numbers 1 ~ 9 in turn) are selected for verification and analysis. The unit transportation costs of the aircraft types (150, 200, 250, and 300 seat classes) and the maximum service capabilities of each aircraft type fleet in 9 cities are from the production and operation data of a domestic airline in 2016. The data related to OD passenger flow and expected flight time between 9 city pairs comes from the 2016 International Air Transport Association MIDT (Market Information Data Types).The frequency of flights between cities (the data mainly comes from the SRS (Schedules Reference Service) flight plan). We use the frequency of flight operations as the number of service devices expected to be deployed on each side in this article.

The construction cost of hub airports in nine cities is summarized by the "13th Five-Year Plan of Civil Aviation of China" and the 2016 Statistical Bulletin of Civil Aviation Industry Development, and it is obtained after a certain proportion of corrections [21,22] according to the proportion of the annual passenger throughput of each city airport.

These data all above are ignored due to the space limitation.

Assume that the expected passenger load factor on each side is 85% and the number of required...
hubs is $p = 3$. The entire algorithm program is implemented using the YALMIP tool based on MATLAB, using a CPU 2.4Hz, 8G memory machine for calculation.

3.1. Analysis and Comparison of Results

(1) Integrated decision-making results. The results are shown in Table 1. The hub address is $\{1, 2, 4\}$, and the network capacity allocation scheme is shown in Table 2, with a total cost of 76.936 billion YUAN.

(2) Decision results of traditional methods. First, use the route capacity configuration method to obtain the model and frequency of each side, thereby obtaining the unit flow cost. Secondly, use it as a network design parameter, and then use equations (1)-(8) to design the hub network (Each discount factor is shown in the first column of Table 1), and then make the optimal decision of route capacity allocation, the calculation results are shown in Table 1. When $\alpha = 1.0$, $\beta = 0.8$, $\gamma = 1.0$, the hub network scheme is $\{1, 2, 8\}$, which is different from the integrated decision-making hub network scheme. Based on this network scheme, the capacity is optimized and the total cost is estimated to be 77.986 billion Yuan, which is 1.050 billion YUAN higher than the total cost of the integrated decision model. The transportation capacity allocation plan for the entire route is shown in Table 3. When $\alpha = 0.8$, $\beta = 0.4$, $\gamma = 0.8$, the hub network plan is $\{1, 2, 8\}$. However, when optimizing route capacity based on the latter two network solutions, due to the maximum flight frequency on the route and the maximum production capacity of each type of aircraft, there is no feasible capacity configuration solution, that is, it can’t meet all OD passenger traffic.

(3) Comparison of calculation results. Comparing the above data, it can be found that the traditional method of hub network design followed by route capacity allocation will result in inadequate model and route adaptability, or even no feasible capacity allocation plan. The main reason is that the hub network model of formulas (1)-(8) only considers the average unit flow cost and reflects the economies of scale through discount factors when performing network planning, which is quite different from the actual optimal route capacity allocation scheme and cost. Therefore, the integrated decision-making method established in this paper is superior to the traditional method.

| Table 1 Comparison of the results of the two methods |
|---------------------------------------------|
| model            | Hub address | Total cost / 100 million yuan |
| integrated decision method | $\{1,2,4\}$ | 769.36 |
| $\alpha=1.0, \beta=0.8, \gamma=1.0$ | $\{1,2,8\}$ | 779.86 |
| Traditional method | $\alpha=0.8, \beta=0.4, \gamma=0.8$ | -- |

Note: When $\alpha = 0.8$, $\beta = 0.4$, $\gamma = 0.8$, the total cost can’t be calculated.

4. CONCLUSIONS

This paper proposes an integrated optimization mathematical model of p-hub median problem and service equipment configuration. This model essentially considers service equipment configuration decisions during the hub network design stage, and takes the minimum total cost of traffic for all OD services as the objective function, considers the constraints in the hub network design stage and the resource constraints in the service equipment configuration decision stage. The results of numerical examples show that this model can make the mutual adaptability between the hub network and the service equipment configuration higher, and make the decision results more in line with the actual requirements. Follow-up can further study the integrated decision-making problem of hub network
design and capacity allocation based on the change of each index value.

### Table 2 integrated Decision Network Solution

| number | 1   | 2       | 3   | 4       | 5   | 6   | 7   | 8   | 9   |
|--------|-----|---------|-----|---------|-----|-----|-----|-----|-----|
| 1      | --  | 0.05⁵  | 0.04³ | 0.15¹  | 0.09³ | 0.12⁴ | 0.10¹ | 0.05¹ | 0.24¹ |
| 2      | 0.03¹,0.02⁴ | 0.23² | 0.14² | 0.11¹  | 0.16² | 0.25³ | 0.38¹ | 0.13⁴ |
| 3      | 0.01¹,0.03² | 0.31¹ | --   | 0.73³  | --   | --   | --   | --   | --   |
| 4      | 0.15³  | 0.14²  | 0.92² | 0.25²,0.75¹ | 1.00⁴ | 0.49³ | 0.31² | 0.47³ |
| 5      | 0.09³  | 0.11³  | --   | 0.25²,0.75¹ | 1.00⁴ | 0.49³ | 0.31² | 0.47³ |
| 6      | 0.18³  | 0.21¹  | --   | 1.00⁴  | --   | --   | --   | --   | --   |
| 7      | 0.10³  | 0.20²  | --   | 0.49³  | --   | --   | --   | --   | --   |
| 8      | 0.05³  | 0.38³  | --   | 0.31³  | --   | --   | --   | --   | --   |
| 9      | 0.14³  | 0.20³  | --   | 0.59³  | --   | --   | --   | --   | --   |

Note: $b$ in $a^b$ represents the model number; $a$ represents the proportion of flight frequency of the corresponding aircraft type.

### Table 3 Network solutions of traditional methods ($\alpha = 1.0$, $\beta = 0.8$, $\gamma = 1.0$)

| number | 1   | 2       | 3   | 4       | 5   | 6   | 7   | 8   | 9   |
|--------|-----|---------|-----|---------|-----|-----|-----|-----|-----|
| 1      | --  | 0.09¹  | 0.05¹ | 0.19² | 0.11² | 0.12³ | 0.10¹ | 0.05¹ | 0.16¹ |
| 2      | 0.09¹ | --   | 0.15²  | 0.11³ | 0.11¹  | 0.10³  | 0.11³ | 0.01³  | 0.08⁴ |
| 3      | 0.04²  | 0.15²  | --   | --   | --   | --   | --   | 0.79³  | --   |
| 4      | 0.11²  | 0.11³  | --   | --   | --   | --   | --   | 0.74⁴  | --   |
| 5      | 0.11²  | 0.11³  | --   | --   | --   | --   | --   | 0.62²  | --   |
| 6      | 0.19¹  | 0.10³  | --   | --   | --   | --   | --   | 0.19⁴  | --   |
| 7      | 0.10³  | 0.08³,0.02⁴ | --   | --   | --   | --   | --   | 0.38³  | --   |
| 8      | 0.05¹  | 0.19³  | 0.79³  | 0.74³  | 0.62²  | 0.19³  | 0.34³  | --   | 0.89¹ |
| 9      | 0.16¹  | 0.18¹  | --   | --   | --   | --   | --   | 0.78¹  | --   |

Note: $b$ in $a^b$ represents the model number; $a$ represents the proportion of flight frequency of the corresponding aircraft type.

**ACKNOWLEDGMENT**

The National Natural Science Foundation of China(U1733127), Scientific Research Project of Education Department of Sichuan(18ZB0682). The Project of Civil Aviation Transportation Planning Intelligent decision-making Research Institute of CAFUC (JG2019-32).

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