LAST HOPE for an astrophysical solution to the solar neutrino problem.

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Abstract

We discuss what appears the last hope for an astrophysical solution to the solar neutrino problem: a correlated variation of the astrophysical factors for the helium burning cross sections ($S_{33}$ and $S_{34}$) and either $S_{17}$ or the central temperature $T_c$. In this context, we recognize the important role played by the CNO neutrinos. In fact, we can obtain a fair fit to the experimental data only if three conditions are met simultaneously: the astrophysical factor $S_{33}$ is about 200 times what is presently estimated, the astrophysical factor $S_{17}$ is about 3 times larger and the $^{13}$N and $^{15}$O neutrino fluxes are negligible compared to the ones predicted by standard solar models. These conditions are not supported by the present data and their correlated combination is improbable.

A. Introduction

The essence of the solar neutrino problem (SNP) is that all four solar-neutrino experiments [1–4] detect signals considerably smaller than the ones predicted by the standard solar models (SSM). This deficit is illustrated by Table I.

There exist non standard solar models (NSSM) that predict a low boron-neutrino flux $\Phi(B)$ which is in agreement with Kamiokande (see Table). This agreement is achieved due to a combination of the following factors: the use of the data [5] which indicate that the astrophysical factor $S_{17}$ could be smaller than the standard value $S_{17} = 22.4$ eV barn; a few percent decrease of the solar central temperature caused either by collective plasma effects [6] or by slightly lower heavy element abundances [7]; a small increase of the astrophysical factor $S_{33}$; and a coincidence of the $S_{33}$-solar-heating effect with the CNO neutrino energy.
due to a hypothetical low-energy resonance in the reaction $^3\text{He} + ^3\text{He}$, and a small decrease of the $S_{34}$ within the experimental uncertainties of the $^3\text{He} + ^4\text{He}$ cross section. The most important factor in reducing the boron flux, and consequently in bringing NSSM in agreement with Kamiokande, is the first one, i.e. reducing $S_{17}$.

However, the construction of such NSSM with low boron flux has not solved the SNP, it has only shifted the emphasis from boron neutrinos to beryllium neutrinos $^7\text{Be}$. Now it is the deficit of $^7\text{Be}$ neutrinos $[8,14]$, or a too low ratio of beryllium to boron neutrino flux $\Phi(\text{Be})/\Phi(\text{B})$ $[14]$, that constitutes the present SNP.

Several model independent analyses which use different combinations of experimental data show clearly that the problem is real:

1. The combination of the Homestake and Kamiokande data implies that, if neutrinos are standard (as in the SM of electroweak interactions), the beryllium flux has an unphysical negative value at the 92% confidence level (C.L.) $[15–17]$. This result is very robust. It does not depend either on uncertainties in nuclear reactions or in details of the SSM. We only need the very reliable assumption that the $\nu_e + ^{37}\text{Cl} \rightarrow ^{37}\text{Ar} + e^+$ cross section is not overestimated $[4,17]$. Therefore, the combination of these two experiments strongly disfavors an astrophysical solution (with uncertainties in nuclear cross-sections included).

2. The gallium experiments by themselves imply a deficit of the $^7\text{Be}$ neutrinos when combined with the solar luminosity sum rule $[18]$: we find that now $\Phi(\text{Be})/\Phi^{SSM}(\text{Be}) < 1/2$ at the 90% C.L.

3. If we arbitrary exclude one of the four experiments from the analysis, e.g. we can exclude either Homestake or Kamiokande, the discrepancy between the remaining experiments still exists. Updating the analyses of Refs. $[9,11]$ leads to the limit $\Phi(\text{Be}) < 1.5 \cdot 10^9 \text{ cm}^{-2} \text{ s}^{-1}$ at the 97% C.L., which is equivalent to

$$ r_{\text{Be}} \equiv \frac{\Phi(\text{Be})}{\Phi^{SSM}(\text{Be})} < \frac{2}{7} , $$

where $\Phi^{SSM}(\text{Be})$ is any of the SSM fluxes from Table $[14]$.

4. Finally, if we use the information from all four experiments, we find that $\Phi(\text{Be}) < 0.7 \cdot 10^9 \text{ cm}^{-2} \text{ s}^{-1}$ at the 97% C.L. By using any of the SSM in Table $[14]$, this model independent limit can be given as

$$ r_{\text{Be}} \equiv \frac{\Phi(\text{Be})}{\Phi^{SSM}(\text{Be})} < \frac{1}{7} . $$

We shall use this limit in our analysis.

We have estimated the above limits and confidence levels by means of $\chi^2$ analyses similar to those of Refs. $[14,14]$ with the addition of Monte Carlo simulations $[19]$ and using the new experimental data from Table $[14]$

It is worth observing that the restrictions discussed above are in fact bounds on the sum of the $^7\text{Be}$ and CNO ($^{13}\text{N} + ^{15}\text{O}$) neutrinos. In particular, $\Phi(\text{Be}) + \Phi(\text{CNO}) < 0.7 \cdot 10^9 \text{ cm}^{-2} \text{ s}^{-1}$ at the 97% C.L. The reason is that $^7\text{Be}$ and CNO neutrinos have similar energies and, therefore, similar interaction cross-sections in the detectors. More precisely, the energy averaged cross sections for the CNO neutrinos are slightly larger than the cross sections for $^7\text{Be}$ neutrinos. We can replace the CNO cross sections with that for $^7\text{Be}$ neutrinos with the only consequence of slightly underestimating the contribution of the CNO neutrinos to the signals. Therefore, all the above bounds apply to the sum of the $^7\text{Be}$, $^{13}\text{N}$ and $^{15}\text{O}$ fluxes,
since underestimating the contribution of the CNO neutrinos only makes those inequalities stronger.

In other words, the so-called $^7$Be neutrino problem is actually the problem of the intermediate energy solar neutrinos.

All the above-mentioned arguments strongly suggest that non-standard neutrinos (beyond the standard model of electroweak interactions) are needed to solve the SNP. In particular, we remind that both the MSW mechanism [20,21] and vacuum oscillations [22–24] are able to explain simultaneously all four solar-neutrino experiments.

However, as we have already briefly discussed in Ref. [25], there is one last hope for finding an astrophysical solution to the SNP. It consists of the following two steps:

(1) We strongly increase the astrophysical factor $S_{33}$, motivated by a hypothetical resonance in the cross section $^3$He + $^3$He $\rightarrow$ $^4$He + 2 p, until the $\Phi$(Be) is suppressed below the “observed” upper limit $\Phi_{\text{SSM}}$(Be)/7.

(2) Since the first step has the undesired side effect of strongly suppressing also the boron flux $\Phi$(B), we boost $\Phi$(B) back to the experimental value by increasing either the astrophysical factor $S_{17}$ (the boron flux is directly proportional to this factor) or the central temperature $T_c$ (since $\Phi$(B)/$\Phi$(Be) $\sim$ $T_c^{10}$, $\Phi$(B) grows faster than $\Phi$(Be)).

This game could obviously be played also with the somewhat less stringent bounds obtained by excluding one of the experiments. Apart from Ref. [25], similar ideas were privately discussed also by M. Altman, I. Barabanov and S. Gershtein.

The purpose of this paper is to make a quantitative analysis of this last hope for an astrophysical solution to the SNP. We shall study whether it is possible to reconcile present experimental data by simultaneously increasing $S_{33}$ and either $S_{17}$ or $T_c$.

We shall demonstrate the important role played by CNO neutrinos, especially for the temperature solution.

**B. Analytical approach**

For the following semiquantitative analysis, we use two scaling laws [26]:

$$\Phi(\text{Be}) \sim T_c^9 S_{34} S_{33}^{-1/2}$$

(3)

and

$$\Phi(\text{B}) \sim T_c^{22} S_{17} S_{34} S_{33}^{-1/2}$$

(4)

Had we used instead the not too different scaling laws of Ref. [27], results would have been similar. Moreover, for simplicity, we only discuss the roles of the boron and beryllium fluxes; however, we shall eventually comment on the relevance of the other fluxes, especially those of the CNO cycle.

The fact that the beryllium and boron fluxes depend on $S_{33}$ and $S_{34}$ only through the ratio [17]

$$X \equiv S_{33}/S_{34}^2$$

(5)
implies that our analysis automatically includes not only an increase of $S_{33}$ but also a
decrease of $S_{34}$. In the following we shall use the notation
\[
x \equiv \frac{X}{X^{SSM}} = \frac{S_{33}}{S_{SSM}^{33}} \left(\frac{S_{34}}{S_{SSM}^{34}}\right)^2
\]
\[s_{17} \equiv S_{17}/S_{SSM}^{17}
\]
\[t_c \equiv T_c/T_c^{SSM}.
\]

First, let us analyze the solution that involves only adjusting the nuclear cross sections
and assumes $T_c = T_c^{SSM}$, i.e. $t_c = 1$. Using the scaling law of Eq. (3) one obtains from the
bound on beryllium flux, Eq. (2):
\[
r_{Be} \equiv \frac{\Phi(Be)}{\Phi^{SSM}(Be)} = \frac{1}{\sqrt{x}} \leq 1/7,
\]
which results in
\[x \geq 50.
\]
From the scaling law of Eq. (4), using the one-sigma lower limit for the $^8$B flux from the
Kamiokande experiment, $\Phi(B) \geq 2.3 \times 10^6$ cm$^{-2}$ s$^{-1}$, and the SSM value $\Phi^{SSM}(B) = 6.62 \times 10^6$ cm$^{-2}$ s$^{-1}$ we find:
\[
r_{B} \equiv \frac{\Phi(B)}{\Phi^{SSM}(B)} = \frac{s_{17}}{\sqrt{x}} \leq 0.35
\]
Combined with the bound on $x$, given by Eq. (10), it yields
\[s_{17} > 2.4.
\]
If we use Bahcall scaling relations [27], instead of Eqs (3) and (4), we obtain the stronger
limits
\[x \geq 130
\]
\[s_{17} > 2.4.
\]

Now, let us examine the “temperature solution”, assuming $S_{17} = S_{SSM}^{17}$ and increasing
$T_c$. Since in this case it is not possible to derive a strict inequality, we use the central value
for the Kamiokande result $\Phi(B) = 2.73 \times 10^6$ cm$^{-2}$ s$^{-1}$, and the upper limit for the beryllium
flux (a more complete numerical analysis that takes properly into account uncertainties, other fluxes, etc. can be found in the next section). With these fluxes we obtain
\[
r_{Be} = \frac{t_c^0}{\sqrt{x}} \approx 1/7
\]
and
\[
r_{B} = \frac{t_c^{22}}{\sqrt{x}} \approx 0.41,
\]
and thus

\[ t_c \approx 1.08 \]  \hspace{1cm} (17)

\[ x \approx 210. \] \hspace{1cm} (18)

We can conclude as follows:

1. In both cases, an extremely high value of \( x \), and therefore of \( S_{33} \), is necessary. It clearly implies a so far undetected, and theoretically disfavored, very-low-energy (say \( E_r < 20 \text{ KeV} \)) resonance in the \(^3\text{He} + ^3\text{He} \) channel. Nonetheless, this possibility cannot be completely ruled out and it is presently being investigated in an experiment at the LNGS underground laboratory \[28\].

2. The first case (\( x \) and \( s_{17} \) increase), in addition to a large value of \( S_{33} \), requires also a value of \( S_{17} \) almost three times larger than the one used in the SSM: \( S_{17}^{\text{SSM}} = 22.4 \text{ eV barn} \). Should we accept the recently proposed smaller value \[5\], the situation would be hopeless.

3. In the second case (\( x \) and \( t_c \) increase), in addition to very large value of \( S_{33} \), an increase of the central temperature by 8\% is also needed. This value is difficult to reconcile with the helioseismological data. In addition, and more important, if one tries to enhance the solar temperature, the CNO cycle gains efficiency, and the production of \(^{13}\text{N} \) and \(^{15}\text{O} \) neutrinos grows as fast as the one of \(^8\text{B} \) neutrinos. This effect will be analyzed numerically in the next section.

4. Moreover, in the SSM, the CNO neutrinos alone already saturate the bound \( \Phi(\text{Be}) + \Phi(\text{CNO}) < 0.7 \cdot 10^9 \text{ cm}^{-2} \text{ s}^{-1} \). Even if we were able to suppress \(^7\text{Be} \) neutrinos almost to zero and, at the same time, we could make the boron flux compatible with experiments, there will be still a conflict with experiments due to the CNO neutrinos. This is the crucial point for understanding why both attempts outlined above fail even more miserably when we perform the numerical calculations taking into account the CNO neutrinos.

\[ \text{C. Numerical analysis} \]

In this section we verify numerically the conclusions reached by the semiquantitative analytical analysis in the previous section. Here we include in the calculations the CNO neutrinos, treat more accurately the dependence on \( S_{33} \), \( S_{17} \) and \( T_c \) and take into account uncertainties in the input parameters. Regarding the dependence of the boron and beryllium flux on \( T_c \), it is worth observing that a low energy \(^3\text{He} + ^3\text{He} \) resonance suppresses beryllium neutrinos more than boron neutrinos: \( r_{\text{Be}} = 0.76 \cdot r_{\text{B}} \) \[17\]. The reason is that this low energy resonance is more effective at lower temperatures, and thus it is more efficient in the outer (cooler) region where \(^7\text{Be} \) neutrinos are produced. This effect is taken into account in the numerical calculations, which, for simplicity, we still present in terms of an effective \( s_{33} = S_{33}/S_{33}^{\text{SSM}} \).

Without loss of generality and for the sake of two-dimension graphical presentation we shall use \( S_{34} = \text{const} \). If one is interested in the explicit dependence on \( S_{34} \), he can replace everywhere \( s_{33} \) with \( x \) given by Eq. \[8\] as discussed in the previous section.
1. The correlated variations of $S_{33}$ and $S_{17}$

The numerical results confirm the analytical estimates: extremely large values of $S_{33}$ and $S_{17}$ are needed. They are larger than in the analytical estimate mainly due to the already mentioned contribution of the CNO neutrinos. As it is clearly seen in Fig. 1 even for values of $s_{33} \approx 200$ and $s_{17} = 3.4$, the $\chi^2$ is still above 15 (at the 99% C.L. the $\chi^2$ for 4 degrees of freedom should be less than 13.28).

This result can be understood from Fig. 2. The solid line going from the diamond labeled SSM to the point 1 shows the effect of increasing $s_{33}$ by a factor 170: the final value of $\Phi(\text{Be}) + \Phi(\text{CNO})$ is still twice the value allowed at the 95% C.L. by the experiments and it is therefore useless to try to adjust the boron flux by increasing $S_{17}$ (solid line between points 1 and 2).

The same figure shows that the main problem is due to neutrinos from the CNO cycle (consider the dashed lines): if we arbitrary switch off the $^{13}\text{N}$ and $^{15}\text{O}$ fluxes (point labeled NO CNO) our game of taking $s_{33} = 170$ (point 4) and then $s_{17} = 3$ produces the point 5, which shows that it is possible to reach the region allowed at the 95% C.L. (solid ellipse). However, even in this unrealistic case the values of $S_{33}$ and/or $S_{34}$ are too high.

2. The correlated variations of $S_{33}$ and $T_c$

The numerical results here are even more discouraging than in the previous case (see Fig. 3). The minimum $\chi^2$ is this time larger than 20. Moreover, it is clear that increasing the temperature does not help, and the “best” results are actually obtained for reduced values of the temperature.

Figure 2 can again help us to understand the reason of such behavior. As before taking $s_{33} = 170$ (point 1) does not reduce sufficiently the sum of the beryllium and CNO fluxes; in addition, if we increase $T_c$ the CNO fluxes increase as fast as the boron flux with the result that the point 3 is far away from the allowed region (solid ellipse).

As in the previous case, we can stress the importance of the CNO neutrinos by considering the same solar model with their contribution reduced to zero. Now increasing the temperature we are able to barely reach the 95% C.L. allowed region (see dashed line from point 4 to point 6).

3. Eliminate one experiment?

Given the well-known “incompatibility” of the experimental results, one might think that disregarding one of the experimental result could be the solution to our problems. We find that the situation does not change drastically. In particular, the most favorable case, which corresponds to neglecting the Kamiokande result and to a variation of $s_{33}$ and $s_{17}$, still gives us a $\chi^2$ greater than 8 (at the 95% C.L. the $\chi^2$ for 3 degrees of freedom should be less than 8) for $s_{33} = 200$ and $s_{17} = 2.5$. The basic reason is that even if we have eliminated the Kamiokande constraint on the boron flux, this flux cannot be much smaller than before, as it can be seen comparing the two 95% C.L. regions in Fig. 2: the one obtained using all four experiments (solid ellipse) and the other obtained using only the chlorine and gallium data.
(dotted ellipse). In fact the chlorine result implies that the contribution from beryllium and CNO neutrinos must increase if the boron flux is too low, but the gallium result forbids a too high beryllium flux. Therefore, we are still only able to get close to the allowed region, if the CNO neutrino fluxes are not much smaller than the ones predicted in SSM (see solid line from point 1 to point 2).

D. Conclusions

We have discussed what appeared to be the last hope for an astrophysical solution to the SNP, i.e. a correlated variation of $S_{33}/S_{34}$ and either $S_{17}$ or the central temperature $T_c$. The important role played by the CNO neutrinos has been properly emphasized in our discussion.

We have concluded that:

1. If the calculated fluxes of the CNO neutrinos ($^{13}\text{N}$ and $^{15}\text{O}$) are not greatly overestimated, there is absolutely no chance of solving the SNP by adjusting $S_{33}$ and/or $S_{34}$, and either $S_{17}$ or the central temperature $T_c$.

2. Even if the CNO fluxes were negligible and a hypothetical low energy resonance allowed us to increase $S_{33}$ at our convenience, we would still need an astrophysical factor $S_{17}$ about 3 times larger than the SSM value. There is no experimental indication of such an enhancement; on the contrary, it is claimed [5] that the actual value is even smaller.

3. The situation is even worse if one tries to increase the solar temperature. Sooner or later the CNO cycle becomes efficient and one is again producing too many intermediate energy neutrinos (see Fig. 2).

4. If one arbitrarily disregards any single experiment, we still need a strong reduction of the CNO fluxes and a large increase (close to 200) of $S_{33}$.

Thus, the last hope turned out to be a no-hope case.
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TABLE I. Comparison of the most recent experimental data (Experiment), and a selected sample of theoretical predictions including some from low-boron-neutrino-flux models (SS93 and DS94). We also report predictions for the main fluxes (pp, $^7$Be, $^{13}$N, $^{15}$O and $^8$B) and for the central temperature $T_c$, and the model input values for the astrophysical factors $S_{33}$, $S_{34}$ and $S_{17}$. Only the sum $^{13}$N + $^{15}$O is reported for SS93. For the experimental data we give separately statistical and systematic errors, while for the theoretical predictions errors are 1σ “effective” errors.

|                  | Standard models | Low-flux models | Experiment |
|------------------|-----------------|-----------------|------------|
|                  | TCL93           | CDF94          | BP95       | SS93       | DS94       |       |
| pp               | 60.2            | 60.0           | 59.1       | 61         | 60.4       |       |
| $[10^9 \text{ cm}^{-2} \text{ s}^{-1}]$ |                |                |            |            |            |       |
| $^7$Be           | 4.33            | 4.79           | 5.15       | 3.9        | 4.30       |       |
| $[10^9 \text{ cm}^{-2} \text{ s}^{-1}]$ |                |                |            |            |            |       |
| $^{13}$N         | 0.382           | 0.47           | 0.618      | 0.3        | 0.075      |       |
| $[10^9 \text{ cm}^{-2} \text{ s}^{-1}]$ |                |                |            |            |            |       |
| $^{15}$O         | 0.318           | 0.40           | 0.545      | 0.22       |           |       |
| $[10^9 \text{ cm}^{-2} \text{ s}^{-1}]$ |                |                |            |            |            |       |
| $^8$B/Kamiokande | $4.4 \pm 1$     | 5.6            | $6.6^{+0.9}_{-1.1}$ | 3.0       | 2.77       | $2.75^{+0.20}_{-0.18} \pm 0.41$ f |
| $[10^6 \text{ cm}^{-2} \text{ s}^{-1}]$ |                |                |            |            |            |       |
| GALLEX           | 122 ± 7         | 130 ± 7        | $137^{+8}_{-7}$ | 114      | 109        | $77.1 \pm 8.5^{+4.4}_{-5.4}$ g |
| [SNU]            |                |                |            |            |            |       |
| SAGE             | 122 ± 7         | 130 ± 7        | $137^{+8}_{-7}$ | 114      | 109        | $69 \pm 11 \pm 6$ h |
| [SNU]            |                |                |            |            |            |       |
| Chlorine         | 6.36 ± 1.3      | 7.8 ± 1.4      | $9.3^{+1.2}_{-1.4}$ | 4.5     | 4.2        | $2.55 \pm 0.17 \pm 0.18$ i |
| [SNU]            |                |                |            |            |            |       |
| $S_{33}$         | 5.24            | 5.00           | 4.99       | 5.6       | 5.6        |       |
| [MeV barn]       |                |                |            |            |            |       |
| $S_{34}$         | 0.533           | 0.524          | 0.45       |           |           |       |
| [KeV barn]       |                |                |            |            |            |       |
| $S_{17}$         | 22.4            | 22.4           | 22.4       | 20.2      | 17         |       |
| [eV barn]        |                |                |            |            |            |       |
| $T_c$            | 1.543           | 1.564          | 1.584      | 1.545     | 1.571      |       |
| [10^7 K]         |                |                |            |            |            |       |

aRef. [29]  
bRef. [26]  
cRef. [30]  
dRef. [7]  
eRef. [31]  
fRef. [2]  
gRef. [3]  
hRef. [4]  
iRef. [1]
FIGURES

**FIG. 1.** Contours of equal $\chi^2$ for the neutrino fluxes in nonstandard solar models parameterized by the $S_{33}$ and $S_{17}$ astrophysical factors, which have been normalized to their SSM values (5.00 [MeV barn] and 22.4 [eV barn], respectively). Solid contours correspond to $\chi^2$ equal to 40, 35, 30, 25 and 20; broken contours correspond to values in between. Note that values of $\chi^2 > 13.28$ have less than 1% probability for the four data (chlorine, GALLEX, SAGE and Kamiokande).

**FIG. 2.** Beryllium plus CNO fluxes vs. boron flux. The solid ellipse confines the region allowed at the 95% C.L. by the four current experiments (chlorine, GALLEX, SAGE and Kamiokande) with the only constraint due to the luminosity sum rule. The dotted ellipse confines the region allowed by only the chlorine and gallium experiments. The diamond shows the SSM prediction. When increasing $S_{33}$ (or more generally $x$), the theoretical point moves along the solid line and reaches point 1 at $x = 170$. If, starting from this point 1, we increase $t_c \equiv T_c/T_{SSM}$, the point moves away from the allowed region towards point 3 and reaches it at $t_c = 1.07$. This unsuccessful game with the ”temperature solution” is caused by the increase of the CNO flux with the temperature. If instead, starting again from point 1, we increase $s_{17}$, the theoretical point moves towards point 2 and reaches it at $s_{17} = 3$. This point is still outside the allowed region. The same SSM, but with the $^{13}$N and $^{15}$O fluxes reduced to zero, is labeled NO CNO. Points 4, 5, 6 are the analogues of points 1, 2, 3, respectively. The “NO CNO” track clearly illustrates the role of CNO neutrinos. However, even for this track (absence of CNO neutrinos), the theoretical point gets into allowed region at too large values of $S_{33}$ and $S_{17}$.

**FIG. 3.** Contours of equal $\chi^2$ for the neutrino fluxes in nonstandard solar models parameterized by the $S_{33}$ astrophysical factors and the central temperature $T_c$, both of which have been normalized to their SSM values (5.00 [MeV barn] and $1.564 \times 10^7$ [K], respectively). Solid contours correspond to $\chi^2$ equal to 40, 35, 30 and 25; broken contours correspond to values in between. Note that values of $\chi^2 > 13.28$ have less than 1% probability for the four data (chlorine, GALLEX, SAGE and Kamiokande).
Fig. 2

$[10^{6} \text{ cm}^{-2} \text{s}^{-1}] \Phi$
