Remarks on monopoles in Abelian projected continuum Yang-Mills theories

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A possible mechanism accounting for monopole configurations in continuum Yang-Mills theories is discussed. The presence of the gauge fixing term is taken into account.

1. Introduction

The understanding of confinement in non-abelian gauge theories is one of the major challenges in theoretical physics. The idea that confinement could be explained as a dual Meissner effect for type II superconductors is largely accepted, with confirmations from lattice simulations.

A key ingredient for the picture of dual superconductivity is the mechanism of Abelian projection introduced by ‘t Hooft\textsuperscript{1}, which consists of reducing the gauge group $SU(N)$ to an Abelian subgroup, identified with the Cartan subgroup $U(1)^{N-1}$, by means of a partial gauge fixing. This is achieved by choosing any local composite operator $X(x)$ which transforms in the adjoint representation, $X'(x) = UX(x)U^\dagger$. The gauge is partially fixed by requiring that $X$ becomes diagonal, $X'(x) = \text{diag}(\lambda_1(x), \ldots, \lambda_N(x))$, where $\lambda_i(x)$ denote the gauge invariant eigenvalues. As shown in\textsuperscript{2}, monopoles configurations appear at the points $x_0$ of the space-time where two eigenvalues coincide, \textit{i.e.} $\lambda_{i+1}(x_0) = \lambda_i(x_0)$. Further, the gauge field is decomposed into its diagonal and off-diagonal parts. The diagonal components correspond to the generators of the Cartan subgroup and behave as photons. The off-diagonal components are charged with respect to the Abelian residual subgroup and may become massive\textsuperscript{2,3}, being not protected by gauge invariance. This mass should set the confinement scale, allowing for the decoupling of the off-diagonal fields at low energy. The final Abelian projected theory turns out thus to be described by an effective low-energy theory in which the relevant degrees of freedom are identified with the diagonal components of the gauge fields and with a certain amount of monopoles, whose condensation should account for the confinement of all chromoelectric charges. Lattice simulations\textsuperscript{4,5} have provided evidences for the Abelian dominance hypothesis, according to which QCD in the low-energy regime is described by an effective Abelian theory. This supports the realization of confinement through a dual Meissner effect, although the infrared Abelian dominance in lattice calculations seems not to be a general feature of any Abelian gauge\textsuperscript{3}. Furthermore, many conceptual points remain to be clarified in order to achieve a satisfactory understanding of confinement in the continuum. Certainly, the problem of the derivation of the Abelian dominance from the QCD Lagrangian is a crucial one. Also, the characterization of the effective low-energy Abelian projected theory and of its monopoles content is of great relevance. There, one usually starts by imposing the so called Maximal Abelian Gauge (MAG)\textsuperscript{6}, which allows for a manifest residual subgroup $U(1)^{N-1}$. The presence of monopoles in the MAG follows then from $\Pi_2(SU(N)/U(1)^{N-1}) = Z^{N-1}$. However, being the MAG a gauge-fixing condition, it is manifestly noncovariant. Therefore,
monopoles here do not seem to be directly related to the singularities occurring for coinciding eigenvalues in the process of diagonalization of a local covariant operator \( X(x) \). Rather, they are associated to singular configurations of the fields \( A \).

The purpose of this contribution is to discuss a possible mechanism accounting for the presence of monopoles in the MAG, for continuum gauge theories. The argument turns out to be generalized to any renormalizable gauge, the main idea being that of showing that \'t Hooft Abelian projection can be suitably carried out in the presence of gauge fixing terms.

2. Monopoles in quantized Yang-Mills theories

In what follows we present a simple way in order to account for monopoles in continuum quantized Yang-Mills theories. In particular, we point out that it is possible to introduce in the path integral a covariant local quantity whose diagonalization is compatible with the gauge fixing, reproducing at the end the usual form of the Yang-Mills partition function in the presence of monopoles \( \Omega \).

Let us start by considering the partition function for the quantized \( SU(N) \) Yang-Mills theory

\[
Z = N \int [D\Phi][DA]\exp \left( -\int d^4x \frac{1}{4} F_{\mu\nu} F_{\mu\nu} - S_{\text{GF}}(A, b, c, \bar{c}) \right)
\]

where \( S_{\text{GF}} \) denotes the gauge-fixing action including the Faddeev-Popov ghosts. We do not specify further the term \( S_{\text{GF}} \), which can be any renormalizable gauge fixing action as, for instance, the MAG condition, the Landau gauge, etc. The measure \( [D\Phi] \) denotes integration over the Lagrange multiplier \( b \) and the ghost fields \( c, \bar{c} \).

We proceed by rewriting the term \( \text{Tr} F_{\mu\nu} F_{\mu\nu} \) in a first order formalism by introducing an antisymmetric two-form field \( B_{\mu\nu} \)

\[
\text{Tr} \left( \frac{1}{4} F_{\mu\nu} F_{\mu\nu} \right) \rightarrow \text{Tr} \left( \frac{i}{2} F_{\mu\nu} B_{\mu\nu} \right) + \text{Tr} \left( \frac{1}{4} B_{\mu\nu} B_{\mu\nu} \right)
\]

Therefore, for the partition function we get

\[
Z = N \int [D\Phi][DA][DB]\exp \left( - \frac{1}{4} \text{Tr} \left( \frac{i}{2} F_{\mu\nu} B_{\mu\nu} \right) + \frac{1}{4} \text{Tr} \left( B_{\mu\nu} B_{\mu\nu} \right) \right) \]

Notice that the field \( B_{\mu\nu} \) transforms covariantly under a gauge transformation of \( SU(N) \)

\[
B_{\mu\nu} \rightarrow B_{\mu\nu}^U = U B_{\mu\nu} U^\dagger,
\]

from which it follows that the quadratic term \( \text{Tr} B_{\mu\nu} B_{\mu\nu} \) is left invariant

\[
\text{Tr} B_{\mu\nu} B_{\mu\nu} = \text{Tr} B_{\mu\nu}^U B_{\mu\nu}^U.
\]

Also, it is worth remarking that the field \( B_{\mu\nu} \) does not appear in the gauge fixing term \( S_{\text{GF}}(A, b, c, \bar{c}) \). According to \'t Hooft procedure, we can now pick up any component of \( B_{\mu\nu} \), say \( B_{12} \), and, due to its hermiticity, diagonalize it by a suitable transformation \( \Omega \) of \( SU(N) \), namely

\[
B_{12} \rightarrow B_{12}^\text{diag} = \Omega B_{12} \Omega^\dagger.
\]

Due to the invariance of \( \text{Tr} B_{\mu\nu} B_{\mu\nu} \), we have

\[
\text{Tr} B_{\mu\nu} B_{\mu\nu} = \text{Tr} \left( 2B_{12} B_{12}^\dagger + B_{jk} B_{jk} \right) = \text{Tr} \left( 2\Omega B_{12} \Omega^\dagger \Omega B_{12}^\dagger \Omega^\dagger \right)
\]

\[
= \text{Tr} \left( 2B_{12}^\text{diag} B_{12}^\text{diag} \right)
\]

\[
+ \text{Tr} \left( \Omega B_{jk} \Omega^\dagger \Omega B_{jk} \Omega^\dagger \right),
\]

where the sum over the indices \((j, k)\) does not include the component \( B_{12} \). The partition function \( Z \) becomes

\[
Z = N \int [D\Phi][DA][DB][D\Omega]\exp \left( - \frac{1}{4} \text{Tr} \left( \frac{i}{2} F_{\mu\nu} B_{\mu\nu} \right) + \frac{1}{4} \text{Tr} \left( 2\Omega B_{12} \Omega^\dagger \Omega B_{12}^\dagger \Omega^\dagger \right) \right)
\]

where we have inserted the integration measure \([D\Omega]\) over the gauge transformations which diagonalize \( B_{12} \). This is always possible, thanks to eq. (4). Performing now the change of variables

\[
B_{\mu\nu} \rightarrow \Omega^\dagger B_{\mu\nu} \Omega, \quad \Omega \rightarrow \Omega,
\]
we obtain

\[
Z = N \int [D\Phi][DA][DB][D\Omega] \exp \left( \int d^4x \text{Tr} \left[ -\frac{i}{2} \Omega F_{\mu\nu} \Omega^\dagger B_{\mu\nu} - \frac{1}{4} B_{\mu\nu} B_{\mu\nu} \right] - S_{GF} \right)
\]

The change of variables (5) has the effect of moving the $\Omega$’s from the quadratic term $BB$ to the first term $FB$. Recalling then that the $\Omega$’s are precisely those transformations which diagonalize $B_{12}$, it follows that

\[
\Omega F_{\mu\nu} \Omega^\dagger = \Omega \left( \partial_\mu A_\nu - \partial_\nu A_\mu - [A_\mu, A_\nu] \right) + (\partial_\mu, \partial_\nu) \Omega^\dagger \Omega^\dagger, \quad F_{\mu\nu} = [D_\mu, D_\nu].
\]

Finally, we can path integrate the field $B$ obtaining the expression

\[
Z = N \int [D\Phi][DA][D\Omega] \exp \int d^4x \left( \text{Tr} \left[ -\frac{1}{4} (F_{\mu\nu}^{reg} + F_{\mu\nu}^{sing})^2 \right] - S_{GF} \right)
\]

with

\[
F_{\mu\nu}^{sing} = (\partial_\mu, \partial_\nu) \Omega^\dagger \Omega.
\]

Notice that $F_{\mu\nu}^{sing}$ is nonvanishing when the transformation $\Omega$ is singular. This occurs for coinciding eigenvalues of the $B_{12}$. Needless to say, these singularities correspond to monopoles. The sum over the singular transformations may be represented as an integration over the surfaces corresponding to the closed loop currents of the monopoles.

We end up thus with the usual form of the Yang-Mills partition function in which monopole configurations appear explicitly. Of course, the introduction of $B_{\mu\nu}$ has to be regarded as a trick for inserting in the path integral a covariant field which can be diagonalized, according to ’t Hooft procedure. It is apparent that the field $B_{\mu\nu}$ represents indeed the field strength $F_{\mu\nu}$. The meaning of this procedure is that we should be able to introduce monopole configurations in the expression for the partition function, regardless of the particular gauge fixing adopted.

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