The Application of Optimal Frequency Hopping Patterns in Telemetry Communication Systems

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Abstract. In telemetry communication, it is crucial to establish correct frame synchronization, a scheme for applying optimal frequency hopping patterns to aerospace telemetry communication systems is proposed, and the architecture of the transceiver system is given. Compared to one-dimensional pseudo-random sequences, optimal frequency hopping patterns have ideal two-dimensional autocorrelation and cross-correlation performance, which can effectively reduce or eliminate the interference caused by delay and Doppler shift and solve the problem of unable to synchronize frames that is caused by low signal-to-noise ratio and strong interference.

1. Introduction

In current telemetry systems, in order to correctly synchronize the signal at the receiving end, one-dimensional pseudo-random sequences is generally inserted into the start position of the signal as frame synchronization flag, which can resist the interference of noise and other factors to a certain extent and improve the anti-interference ability of the signal in the time domain dimension.

Telemetry is long-distance wireless communication, Compared with short-range wireless transmission. The signal will encounter more complicated interference during the propagation process, such as strong interference and low signal to noise ratio, for these problems the signal may to be unable to be frame-synchronized at the receiving end, so that the signal cannot be correctly demodulated. This is a major problem in the field of telemetry communication.

In order to overcome the shortcomings of one-dimensional synchronization sequences in telemetry systems, this paper proposes to apply optimal frequency hopping patterns to telemetry systems. Compared to the structure of one-dimensional pseudo-random sequences based on time domain, but optimal frequency hopping patterns is described in two dimensions(frequency domain and time domain), two-dimensional cyclic shift method can be used to construct frequency hopping patterns with a gap row or gap column based on the algebraic structure of the Welch Costas arrays, or to construct a frequency hopping pattern with a gap row and a gap column based on the algebraic structure of the Golomb Costas arrays, optimal frequency hopping patterns constructed by the two methods both have ideal autocorrelation and cross-correlation performance, which can effectively reduce or eliminate the interference caused by delay and Doppler shift and can work stably and reliably under the scene of low signal to noise ratio and strong interference, so the receiving end performs precise synchronization of the signals.

This paper first introduces the algebraic structure based on Welch Costas arrays and Golomb Costas arrays, and the method for constructing optimal frequency hopping patterns by two-dimensional cyclic
shift method, and combined with specific examples for analysis and explanation; Then it focuses on the application of optimal frequency hopping patterns in telemetry systems, and analyzes the autocorrelation and cross-correlation performance of optimal frequency hopping patterns in combination with the simulation diagram; Finally, the architecture of the transceiver system that applies optimal frequency hopping patterns to telemetry systems is given.

2. Optimal frequency hopping patterns based on Welch Costas arrays
Provided the finite field $GF(p)$, $p$ is a prime number, $\alpha$ and $\beta$ are the primitive and non-zero elements of $GF(p)$, respectively, $C$ is a permutation matrix of $(p-1)$ order, and the placement function of $C$ is:

$$y(k) = \eta \alpha^k \pmod{p}, \ 1 \leq k \leq p-1.$$  

$$y(k) = \alpha^k \pmod{p}, \ 1 \leq k \leq p-1. \quad (2)$$

where (1) is a necessary and sufficient condition for $C$ to be a Welch Costas arrays, and the array obtained by (1) can be regarded as arrays obtained by cyclically shifting the array obtained by (2) in the horizontal direction [1].

Other Welch Costas arrays can be obtained by cyclically shifting the known Welch Costas arrays. In the vertical direction, the optimal frequency hopping pattern $m_3 = [7, 6, 3, 1, 2, 5]$ obtained by $m_2$ loops up 4 distances to get, which has a gap line; In the horizontal direction, $m_2$ is shifted to the left by 3 distances to obtain the Welch Costas array $m_4 = [4, 5, 1, 3, 2, 6]$, where $m_1, m_2, m_3, m_4$ correspond to (a), (b), (c), (d) in Fig. 1 respectively.

![Figure 1 Welch Costas arrays cyclic shift](image)

In telemetry systems, the maximum delay or maximum Doppler shift is often known, the ideal autocorrelation and cross-correlation performance of optimal frequency hopping patterns are obtained by designing the minimum delay distance or minimum Doppler distance between different optimal frequency hopping patterns. As shown in Fig. 1, $m_2$ is cyclically moved up by 4 distances in the vertical direction (i.e. the frequency dimension) to obtain $m_3$, the Doppler distance of $m_2$ and $m_3$ is 3, so there is no interference of Doppler shift between the two arrays when the maximum Doppler shift between $m_2$ and $m_3$ is less than 3 (as shown in Fig. 2 (b)); Similarly, it can also resist delay interference within a limited range (i.e. the maximum relative delay of the array is less than the minimum delay distance of the array) when the optimal frequency hopping pattern is cyclically shifted in the horizontal direction. Therefore, the larger the Doppler (or delay) distance between the two optimal frequency hopping patterns, the larger the maximum Doppler shift (or maximum delay) that can be resisted, but the cyclic shift distance should be used according to the actual situation, can not exceed half of the order of frequency hopping patterns, that is $(p-1)/2$.

The delay interference problem can be solved when the minimum delay distance $R$ of optimal frequency hopping patterns satisfy (3); The Doppler shift interference problem can be solved when the minimum Doppler distance $S$ of frequency hopping patterns satisfy (4), Fig. 2 (b).

$$|t| \leq R - 1.$$  

$$|d| \leq S - 1.$$  

where $t$ represents the maximum delay in (3), and $d$ represents the maximum Doppler shift in (4).
Fig. 2 is the autocorrelation and cross-correlation function of optimal frequency hopping patterns \( m_2 = [3, 2, 6, 4, 5, 1] \) (the frequency hopping pattern has a gap line in the top row of the frequency hopping pattern) and \( m_3 = [7, 6, 3, 1, 2, 5] \), where \( m_3 \) is obtained by cyclically shifting \( m_2 \) by 4 distances in the vertical direction.

Fig. 2 is autocorrelation and cross-correlation function of Welch Costas arrays \( m_1 = [5, 4, 6, 2, 3, 1] \) and \( m_2 = [3, 2, 6, 4, 5, 1] \).

Comparison of Fig. 2 and Fig. 3 can be found, the autocorrelation function of optimal frequency hopping patterns and the Welch Costas arrays both have sharp peaks and side lobes with a maximum value of no more than 1, as shown in Fig. 2(a) and Fig. 3(a); the cross-correlation function is shown in Fig. 2(b) and Fig. 3(b), the cross-correlation performance of the optimal frequency hopping pattern within the maximum Doppler shift range is significantly better than the Welch Costas array, which can effectively improve the signal synchronization performance.

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{fig2a.png}
\includegraphics[width=0.4\textwidth]{fig2b.png}
\caption{Optimal frequency hopping patterns autocorrelation and cross-correlation function}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{fig3a.png}
\includegraphics[width=0.4\textwidth]{fig3b.png}
\caption{Welch Costas autocorrelation and cross-correlation function}
\end{figure}

3. Optimal FREQUENCY Hopping patterns based on Golomb Costas arrays

Provided the finite field \( GF(p) \), \( q = p^m \), \( p \) is a prime number, \( m \in \mathbb{Z}^+ \), \( \alpha \), \( \beta \) are primitive elements of the finite field \( GF(p) \), \( C \) is a permutation matrix of order \( (q - 2) \), the placement function is:

\[ y(k) = \log_\beta (1 - \alpha^k), \quad 1 \leq k \leq q - 2 . \]  
\[ \alpha + \beta^{y(k)} = 1, \quad 1 \leq q - 1 . \]

where (5) is a necessary and sufficient condition for the array \( C \) to be a Golomb Costas array [4], when \( (k, y(k)) \) satisfies (6), "1" is placed at the corresponding coordinate position of the cell, then a Golomb Costas array is available.

Golomb Costas arrays are \( (q - 2) \times (q - 2) \) array, since Golomb Costas arrays have \( (q - 1) \) cycles in both the horizontal and vertical directions, the cyclic shift of Golomb Costas arrays in the horizontal direction contains a gap column, the cyclic shift in the vertical direction contains a gap line, a \( (q - 1) \times (q - 1) \) pattern with a gap column and a gap row can be constructed by the cyclic shift of the Golomb Costas array in the horizontal direction and in the vertical direction respectively, then the optimal frequency hopping pattern is obtained.
If the maximum delay \( R \) (or the maximum Doppler shift \( S \)) between the two optimal frequency hopping patterns satisfy (7) (or (8)): \[
|\tau| \leq R - 1. \tag{7}
\]
\[
|\delta| \leq S - 1. \tag{8}
\]
where \( \tau \) represents the maximum delay in (7), and \( \delta \) represents the maximum Doppler shift in (8). Within the range of maximum delay (or maximum Doppler shift), the cross-correlation function of two optimal frequency hopping patterns have a maximum value of no more than 1, that is the cross-correlation function performance is ideal.

In [4], optimal frequency hopping patterns based on Golomb Costas arrays are constructed by the method of two-dimensional cyclic shift, autocorrelation and cross-correlation performance of optimal frequency hopping patterns are analyzed. Therefore applying optimal frequency hopping patterns based on Golomb Costas arrays to telemetry communication systems can also effectively solve the interference problem caused by delay and Doppler frequency shift.

4. Computer Simulation

Fig. 4 is autocorrelation and cross-correlation simulation diagram of the Welch Costas arrays \( m_1 = [5, 4, 6, 2, 3, 1] \), \( m_2 = [3, 2, 6, 4, 5, 1] \).

Fig. 5 is autocorrelation and cross-correlation simulation diagram of optimal frequency hopping patterns \( m_2 = [3, 2, 6, 4, 5, 1] \) (the frequency hopping pattern has a gap line in the top line of the frequency hopping pattern), \( m_3 = [7, 6, 3, 1, 2, 5] \), where \( m_3 \) is obtained by cyclically shifting \( m_2 \) by 4 distances in the vertical direction, the Doppler distance of \( m_2 \) and \( m_3 \) is 3, there is no interference caused to the frame synchronization when the Doppler shift is less than 3 shift distances.

It can be seen from the simulation results of MATLAB in Fig. 4 and Fig. 5 that Welch Costas arrays and optimal frequency hopping patterns have the same ideal autocorrelation performance, that is both of them have sharp peaks and side lobes values of not more than 1, but in cross-correlation, the optimal frequency hopping pattern shows ideal performance, values of the cross-correlation function does not exceed 1 within the range of the maximum Doppler shift, therefore, optimal frequency hopping patterns can effectively resist the interference of Doppler shift in telemetry communication.

![Correlation function of the Welch Costas arrays](image)

(a) Autocorrelation function of \( m_1 \)  (b) Cross-correlation function of \( m_1 \)

Figure 4 Correlation function of the Welch Costas arrays

![Correlation function of the optimal frequency hopping patterns](image)

(a) Autocorrelation function of \( m_3 \)  (b) Cross-correlation function of \( m_2 \) and \( m_3 \)

Figure 5 Correlation function of the optimal frequency hopping patterns based on the Welch Costas arrays
5. Application of optimal Frequency Hopping patterns in Telemetry Systems

In telemetry systems, the system structure of the transmitting end is mainly composed of sensors, frequency control unit and modulator, the system structure of the receiving end is mainly composed of demodulator, frequency control unit, fuzzy function calculator(the fuzzy function obtaining algorithm mentioned in [2]) and decoders, the structure of the data frame transmitted in telemetry systems is shown in Fig. 6 (the marker represents the frame synchronization sequences generated by frequency control unit).

The structure of the transmitting end is shown in Fig. 7, Sensors receive the original signals and convert them into baseband signals, each baseband signal is encapsulated into data packets in chronological order, and the FCU (Frequency Control Unit) periodically generates a frame synchronization sequences by the generated optimal frequency hopping pattern, and then inserts the frame synchronization sequences into the start position of the data packet as a frame synchronization flag. Finally, the baseband signal u(t) is modulated by the modulator and transmitted.

The structure of the receiving end is shown in Fig. 8, the demodulator demodulates the received signal y(t) into baseband signal u(t), and then sends u(t) to the fuzzy function calculator, the fuzzy function calculator uses the DFT to calculate the fuzzy value of the signal u(t) and the sequences v(t) generated by the FCU (Frequency Control Unit), the correct frame synchronization position can be obtained by the fuzzy value, and the starting position of the data is also obtained, finally the data is restored to the original signal by the decoder.

6. Conclusion

With the development of the aerospace industry, to accurately and effectively establish correct frame synchronization between the aircraft telemetry equipment and the ground telemetry equipment is critical. In recent years, pseudo-random code has been used as a frame synchronization code, there is an exposed problem that the probability of false synchronization and leakage synchronization of telemetry systems becomes large under the condition of large Doppler shift and strong interference, then the demodulator failed to correctly demodulate the signal at the receiving end. This paper proposes a scheme to apply optimal frequency hopping patterns to the telemetry frame synchronization systems, with the ideal autocorrelation and cross-correlation performance, optimal frequency hopping patterns can effectively solve the problem of being unable to synchronize under the condition of large Doppler shift, low signal to noise ratio or strong interference, by which the receiver can accurately demodulate
the signal in the receiving end, and the communication quality is greatly improved. Optimal frequency hopping patterns will vigorously promote the development of telemetry communication field and has broad application prospects.

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