Intrinsic Hall response of the CuO$_2$ planes in a chain-plane-composite system of YBa$_2$Cu$_3$O$_y$

Kouji Segawa and Yoichi Ando

Central Research Institute of Electric Power Industry, Komae, Tokyo 201-8511, Japan
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The Hall coefficient is measured in YBa$_2$Cu$_3$O$_y$ untwinned single crystals for a wide range of doping. We show that the Hall conductivity and the Hall angle of the CuO$_2$ planes in YBa$_2$Cu$_3$O$_y$ can be extracted from measurable transport properties regardless of the conduction of the Cu-O chains nor the in-plane anisotropy of the CuO$_2$ planes. The present analysis allows us to discuss the genuine Hall effect in the CuO$_2$ planes alone in YBa$_2$Cu$_3$O$_y$ without any complications due to the Cu-O chains.

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I. INTRODUCTION

The origin of the peculiar normal-state properties of the high-$T_c$ superconductors is not fully understood. The strongly temperature-dependent Hall coefficient ($R_H$) is one of the best-known peculiar features in the normal state of high-$T_c$ cuprates. While $R_H$ shows a complicated temperature dependence, the cotangent of the Hall angle ($\cot \Theta_H = \rho_{xy}/\rho_{xx} = \rho_{xx}/\rho_{yy}$) was reported to show a simple $T^2$-dependence in YBa$_2$Cu$_3$O$_y$ (YBCO) [Ref. 2]; it is widely believed that the peculiar behavior of $R_H$ comes from an anomalous coexistence of the $T^2$-law in $\cot \Theta_H$ and the pronounced $T$-linear resistivity. This fact suggests that two kinds of scattering times dominate charge transport in high-$T_c$ cuprates.

However, the YBCO system has the Cu-O chains which are likely to show one-dimensional (1D) electronic conduction, which is believed to cause the in-plane resistivity anisotropy in the highly-doped region. On the other hand, in the underdoped region, the 1D-conduction of the Cu-O chains is expected to disappear quickly with reducing oxygen content, because oxygens are removed from the Cu-O chains, creating vacancies in the chains, and 1D systems are known to be very sensitive to disorder. However, we have demonstrated that the in-plane anisotropy does not disappear in the underdoped region, which indicates that the CuO$_2$ plane itself is anisotropic. In such a material with in-plane anisotropy as well as the chains, resistivity and $\cot \Theta_H$ measured in twinned crystals are mixtures of the intrinsic transport properties. The original report of the $T^2$-law in $\cot \Theta_H$ [Ref. 2] was based on the result in twinned single crystals, and therefore the conductive Cu-O chains may have affected the temperature dependences of $\cot \Theta_H$ and resistivity. Detwining the crystals allows us to observe the in-plane anisotropy in resistivity; however, even the results in untwinned single crystals are not free from contribution of the Cu-O chains, because the Cu-O chains can indirectly modify the Hall coefficient, as described later. Therefore, measurement on untwinned crystals and estimating the contribution from the Cu-O chains are necessary to obtain intrinsic transport properties in YBCO in the whole doping region.

In this paper, we report the way to deduce the genuine Hall response of the CuO$_2$ planes in YBa$_2$Cu$_3$O$_y$ untwinned single crystals. We show that the Hall conductivity ($\sigma_{xy}$) and the cotangent of the Hall angle ($\cot \Theta_H$) of the CuO$_2$ planes can be obtained from measurable transport properties such as the $a$($b$)-axis in-plane resistivity $\rho_{ab}$ and the Hall coefficient $R_H$. The result shows that the electronic state of the CuO$_2$ planes is responsible for the observed ‘60-K anomalies’ in YBCO, such as a decrease in the Hall coefficient just above $T_c$ and the enhancement of the Hall conductivity and the Hall mobility in samples with $y \sim 6.65$.

II. EXPERIMENTS

The YBCO single crystals are grown in Y$_2$O$_3$ crucibles by a conventional flux method. A wide range of oxygen contents are achieved by annealing under various conditions as shown in Table I and quenching from the set temperatures of the annealing. Slightly overdoped ($y \approx 7.00$) and optimally doped ($y \approx 6.95$) crystals are obtained by annealing in oxygen atmosphere for a long time. Samples with $y = 6.70$–6.85 are obtained by annealing in air for 12–48 hours at temperatures which increase with decreasing oxygen contents. For more underdoped samples, annealing in air is not applicable because it is difficult to quench samples quickly enough for preventing absorption of oxygen onto the surface of the crystals at very high temperatures. Instead, in order to obtain oxygen contents of $y \leq 6.65$ we seal crystals in a quartz tube together with polycrystalline powders with controlled oxygen contents. The amount of the buffer powder should be as large as possible for obtaining homogeneous oxygen distribution; however, if the amount of powder is too large (e.g., more than 100 mg in ~7 c.c. sealed quartz tube) we observe superconducting transition of minor phases at around 60 K. Therefore, we use ~50 mg of powder for each annealing, keeping the amount of single crystals less than 1% of powder. Only samples with $y = 6.30$ are annealed by using special furnace in which we can control the oxygen partial pressure. We keep sam-
samples at a room temperature for at least a week after any heat treatments, because the room-temperature annealing effect is observed in the time scale of a few days. All crystals are detwinned at $\leq 210 ^{\circ} C$ in flowing nitrogen after the annealing, because annealing at high temperatures above $\sim 600 ^{\circ} C$ destroys an untwinned state. We confirmed that the experimental results are not affected by the order of annealing and detwinning for samples with $y = 6.80$ and 6.85 [Ref. 20], which can be detwinned either before or after the annealing. We decrease the detwinning temperature with decreasing oxygen contents; for example, the detwinning temperature for $y = 7.00$ samples is $\sim 210 ^{\circ} C$, and that for $y = 6.30$ samples is $\sim 120 ^{\circ} C$ under an uniaxial pressure of $\sim 0.1$ GPa. The oxygen content $y$ is determined by iodometric titration on powders which are annealed with the crystals, because the volume of the crystals themselves is too small for the titration. The error in $y$ is $\pm 0.02$ [Ref. 18].

The Hall coefficient data are taken by sweeping the magnetic field (along the $c$-axis) to both plus and minus polarities up to 10–14 T at fixed temperatures. For each composition, the $R_H$ data are presented only for those temperatures at which the Hall voltage is perfectly proportional to the magnetic field. Figure 1 shows the field dependences of the Hall resistivity for an optimally doped sample at various temperatures. The origin for each temperature is shifted along $y$-axis by 0.5 $\mu \Omega$ cm for 93–100 K and 0.25 $\mu \Omega$ cm for 100–300 K for clarity. We cannot extract $R_H$ from the data for 93 and 95 K, because the field dependences deviate from the $H$-linear behavior by superconducting fluctuation.

### TABLE I: Annealing conditions for our YBCO crystals.

| $y$ | $T_{\text{anneal}}$ | keeping time | atmosphere | $T_{\text{quench}}$ (°C) | $T_c$ (K) |
|-----|---------------------|--------------|------------|--------------------------|-----------|
| 6.70| 400 $\geq 10$ days  | $\sim 2$ atm $O_2$ | air        | 69                        |
| 6.85| 535 48 hours        |              | air        | 62                        |
| 6.75| 600 12 hours        | sealed       | air        | 60                        |
| 6.50| 600 12 hours        | sealed       | air        | 55                        |
| 6.45| 600 12 hours        | sealed       | air        | 20                        |
| 6.35| 700 6 hours         | sealed       | $3.5 \times 10^{-4}$ atm $O_2$ | –          |
| 6.30| 550 36 hours        |              |            | –                         |

According to the Onsager’s reciprocal relation, $[\rho_{yx}(H) = \rho_{yx}(-H)]$, when the magnetic field is applied along the $c$-axis, $R_H$ measured in a sample with the current along the $a$-axis ($\rho_{a}$-sample) is the same as that with the current along the $b$-axis ($\rho_b$-sample) unless time-reversal symmetry is broken. However, YBCO is a complicated composite system containing two-dimensional (2D) CuO$_2$ planes and one-dimensional (1D) Cu-O chains and thus it is not self-evident whether the Onsager relation is satisfied in YBCO. Harris et al. mentioned that the Onsager relation was accurately satisfied, but no actual data were shown in their paper. Figure 2 shows the temperature dependences of $R^{b}_{H}$ and $R^{b}_{H}$ in an optimally doped sample, which were measured with the current along the $a$-axis and the $b$-axis; it should be noted that these data were measured for the same sample with the same electrodes by re-detwinning the crystal without changing the oxygen content. Therefore, we have eliminated the uncertainty in the estimation of the sizes of the crystal and electrodes from comparison. The two curves shown in Fig. 2 do not coincide but are very close to each other. $R^{b}_{H}$ is slightly (at most $\sim 4 \%$) smaller than $R^{b}_{H}$, but this difference is most likely due to an extrinsic effect of the Hall measurement on a finite-sized sample combined with the resistivity anisotropy. Therefore,
we can say that the Onsager relation indeed holds within an error of \( \sim 5\% \).

**B. Measured Hall coefficient**

Figure 3 shows the temperature dependences of the measured Hall coefficient \( R_{H}^{\text{meas}} \) for samples with various oxygen contents in a semi-log plot. In general, the magnitude of \( R_{H}^{\text{meas}} \) increases with decreasing oxygen content. At high temperatures for most samples \( R_{H}^{\text{meas}} \) increases with decreasing temperature, while at low temperatures various behaviors are observed. For slightly underdoped samples with \( y=6.60–6.85 \), \( R_{H}^{\text{meas}} \) shows a decrease at low temperatures below \( \sim 120 \) K unlike highly doped samples with \( y=6.95–7.00 \). The observed decrease appears to be the strongest in samples with \( y=6.75–6.80 \). These results are roughly consistent with the previously published data for twinned crystals with \( y \geq 6 \) [Ref. 26]. For less doped samples the low-temperature decrease becomes weaker and eventually almost constant \( R_{H}^{\text{meas}} \) is observed at low temperatures for samples with \( y=6.45–6.55 \). In the lightly-doped nonsuperconducting samples with \( y = 6.30 \) and \( 6.35 \), \( R_{H}^{\text{meas}} \) increases with decreasing temperature at low temperatures, which is due to the charge localization observed in the in-plane resistivity [15].

In the lightly doped region, nearly constant \( R_{H}^{\text{meas}} \) may allow us to discuss carrier concentrations extracted from the Hall coefficients. Figure 4 shows the temperature dependence of the apparent carrier concentration which is calculated by \( \frac{V}{N_e R_{H}^{\text{meas}}} \), where \( V \) is the volume of a unit cell and \( N \) is the number of Cu atoms per unit cell [28]. The carrier concentration at the superconductor-insulator boundary in YBCO appears to be a reasonable value of \( \sim 4.5\% \) and thus the Hall coefficient in this region is apparently not affected by the conduction of the Cu-O chains. However, for more doped samples \( R_{H} \) shows the complicated temperature dependence, which is possibly modified by a contribution of the Cu-O chains if the Cu-O chains are conductive. Next we try to extract the Hall coefficient in the CuO planes for samples with possibly conductive Cu-O chains.
IV. ANALYSIS AND DISCUSSION

In this section, we extract the physical properties of the CuO$_2$ planes by evaluating the effect of 1D conduction of the Cu-O chains. Our analysis is based on the parallel resistor model (PRM) along the $b$-axis, by which a $T^2$-law in the resistivity of the Cu-O chains was successfully extracted in optimally doped YBCO [Ref. 13] and YBa$_2$Cu$_4$O$_8$ (Y124) [Ref. 29]. Excellent $T^2$-law is also observed in the resistivity of the Cu-O chains ($\rho^{\text{ch}}_y$) in our slightly overdoped samples with $y=7.00$ as shown in Fig. 5(a). In addition, the value of the residual resistivity for $\rho^{\text{ch}}$ (35.7 $\mu\Omega$cm) is roughly a third of the previously reported value$^{12}$, which evinces the cleanliness of our crystals. In the PRM we assume that (1) the Cu-O chains show electronic conduction along their direction, (2) inter-chain conduction is negligible, and (3) conduction bands of the Cu-O chain layer and the CuO$_2$ planes do not mix. Since the Cu-O chains run along the $b$-axis, the total conductivity along the $b$-axis is

$$\sigma^\text{tot}_b = \sigma^{\text{pl}}_b + \sigma^{\text{ch}}_b,$$

where $\sigma^{\text{pl}}$ and $\sigma^{\text{ch}}$ are the conductivity of the CuO$_2$ planes and the Cu-O chains, respectively. On the other hand, the total conductivity along the $a$-axis is

$$\sigma^\text{tot}_a = \sigma^{\text{pl}}_a,$$

since the conduction perpendicular to the Cu-O chains is neglected. The in-plane anisotropy of the electrical conduction of the CuO$_2$ planes is neglected for the moment (we will discuss the case of anisotropic CuO$_2$ planes later).

A. Hall coefficient

First we extract the Hall coefficient of the CuO$_2$ planes in the $\rho_y$-sample because the process is simpler than that in the $\rho_x$-sample. In the $\rho_y$-sample the current flows separately through the CuO$_2$ planes and the Cu-O chains so that the current flowing through the CuO$_2$ planes ($j^{\text{pl}}_x$) is given by

$$j^{\text{pl}}_x = j^\text{tot}_x \left( \frac{\sigma^{\text{pl}}_x}{\sigma^{\text{pl}}_x + \sigma^{\text{ch}}_x} \right) = j^\text{tot}_x \left( \frac{\sigma^\text{tot}_x}{\sigma^\text{tot}_b} \right) = j^\text{tot}_x \left( \frac{\rho_a}{\rho_b} \right),$$

where $j^\text{tot}_x$ is the total current. In the PRM it is expected that the Hall voltage is produced only by this current $j^{\text{pl}}_x$ and the Cu-O chains do not affect the Hall electric field. (Although some finite Hall coefficient is observed in quasi-1D systems$^{30,31}$, as we discuss in Section IV.E contribution from the Cu-O chains to $R_H$ can be neglected in the case of YBCO. Therefore, the Hall coefficient of the CuO$_2$ planes which are assumed to be isotropic, $R^\text{pl(iso)}_H$, is calculated as

$$R^\text{pl(iso)}_H = R^\text{meas}_H \left( \frac{j^{\text{pl}}_x}{j^\text{tot}_x} \right) = R^\text{meas}_H \left( \frac{\rho_a}{\rho_b} \right).$$

The same result is obtained also in the $\rho_x$-sample. The longitudinal current $j^{\text{tot}}_y$ flows only through the CuO$_2$ planes in the $\rho_x$-sample, and thus $j^{\text{pl}}_y = j^{\text{tot}}_y$ and $j^{\text{ch}}_y = 0$ are satisfied along the $a$-axis. Naively, the measured Hall coefficient itself may seem to signify the Hall coefficient of the CuO$_2$ planes; however, the observed Hall voltage is reduced by the short-circuiting effect of the Cu-O chains in the $\rho_x$-sample, because the transverse electric field $E_y$ takes effect not only in the CuO$_2$ planes but also in the Cu-O chains. Since $E_y$ is parallel to the Cu-O chains, a current $j^{\text{ch}}_y$ inevitably flows through the Cu-O chains. Therefore, in the Cu-O chains

$$E_y = \rho^{\text{ch}} y j^{\text{ch}}_y$$

is satisfied. On the other hand, a counter current of the same magnitude of $j^{\text{ch}}_y$ must flow through the CuO$_2$ planes because of the condition $j^{\text{tot}}_y = 0$, where $j^{\text{tot}}_y$ is the transverse current of the total system along the $y$-axis. Therefore, $E_y$ in the CuO$_2$ planes is calculated as

$$E_y = E_H - \rho^{\text{pl}}_y j^{\text{pl}}_y,$$

where $E_H$ is the Hall electric field which is generated to compensate the Lorentz force and $E_H$ is given by

$$E_H = \frac{\rho^{\text{pl(iso)}}_y}{j^{\text{pl}}_y}.$$

where $\rho^{\text{pl(iso)}}_y$ is the Hall resistivity of the isotropic CuO$_2$ planes. Since $E_y$ is identical in the CuO$_2$ planes and in the Cu-O chains,

$$E_H - \rho^{\text{pl}}_y j^{\text{ch}}_y = \rho^{\text{ch}} y j^{\text{ch}}_y$$

is satisfied. We can express $j^{\text{ch}}_y$ by using measurable properties as

$$j^{\text{ch}}_y = \frac{E_y}{\rho^{\text{ch}}} = E_y \left( \frac{1}{\rho_b} - \frac{1}{\rho_a} \right).$$

Therefore, we obtain

$$E_H = E_y + \rho_a j^{\text{ch}}_y = E_y \left( \frac{\rho_a}{\rho_b} \right),$$

and by dividing both sides of this equation by $j^{\text{tot}}_x (= j^{\text{pl}}_x)$ Eq. 1 is obtained (using $\rho^{\text{pl(iso)}}_y = E_H/j^{\text{pl}}_x$ and $\rho^{\text{meas}}_y = E_y/j^{\text{tot}}_x$).

In the above analysis, the anisotropy of the resistivity in the CuO$_2$ planes is ignored. In other words, the global in-plane anisotropy is assumed to be caused only by the Cu-O chains. In recent measurements, however, we observed considerable anisotropy in the in-plane resistivity in oxygen deficient YBCO [Ref. 15]. It is unlikely that oxygen deficient Cu-O chains show good 1D conduction, and thus the CuO$_2$ planes themselves must be anisotropic in the underdoped YBCO. If we re-calculate Eq. 1 under the assumption of the anisotropic CuO$_2$ planes, we obtain

$$R^\text{pl}_H = R^\text{meas}_H \left( \frac{\rho_a}{\rho_b} \right) \left( \frac{\rho^{\text{pl}}_y}{\rho^{\text{pl}}_a} \right).$$

(2)
where $R_{H}^{\text{pl}}$ is the Hall coefficient of the CuO$_2$ planes which can be anisotropic and $\rho_{a(b)}^{\text{pl}}$ is the resistivity of the CuO$_2$ planes along the $a(b)$-axis. Unfortunately, $\rho_{b}^{\text{pl}}$ is not measurable and thus $R_{H}^{\text{pl}}$ cannot be obtained if the CuO$_2$ planes are anisotropic and the Cu-O chains are conductive.

Figure 5(b) shows the temperature dependence of $R_{H}^{\text{pl(iso)}}$ and $R_{H}^{\text{meas}}$ with $y = 7.00$. Figure 6(a) shows the temperature dependence of $R_{H}^{\text{pl(iso)}}$ for samples with $y \geq 6.60$. The significant decrease at low temperatures in samples with $y \sim 6.75$ we noted in Fig. 3 does not diminish in $R_{H}^{\text{pl(iso)}}$. Figures 6(b) and 6(c) show the $y$-dependences of $R_{H}^{\text{meas}}$ and $R_{H}^{\text{pl(iso)}}$ at 150 K and 300 K, respectively. One should note that the obtained $R_{H}^{\text{pl(iso)}}$ just gives the maximum possible value of $R_{H}^{\text{pl}}$ in the limit where the Cu-O chains are conductive and the CuO$_2$ planes are isotropic. On the other hand, if the Cu-O chains are insulating, $R_{H}^{\text{pl}}$ becomes equal to $R_{H}^{\text{meas}}$. Therefore, the above analysis of the Hall coefficient provides a range of the $R_{H}^{\text{pl}}$ values. In the heavily underdoped samples where chains are broken, the chain conduction is expected to be negligible and the observed peculiar anisotropy is most likely due to the planes, in this case, one can see from Eq. (2) that $R_{H}^{\text{meas}}$ is a true measure of $R_{H}$ of the planes.

### B. Hall conductivity

In contrast to the Hall resistivity (Hall coefficient), we can unambiguously obtain the Hall conductivity of the CuO$_2$ planes regardless of anisotropy of the CuO$_2$ planes. Since the Hall conductivity is calculated by $\sigma_{xy} = \rho_{yx}/(\rho_{xx}\rho_{yy} - \rho_{yx}\rho_{xy}) \approx \rho_{yx} / (\rho_{xx}\rho_{yy})$, in YBCO we calculate it as

$$\sigma_{xy}^{\text{pl}} = \frac{\rho_{yx}^{\text{pl}}}{\rho_{a}\rho_{b}}.$$  

Note that this result tells us that the Hall conductivity in the CuO$_2$ planes is the same as that of the total system regardless of the anisotropy of the CuO$_2$ planes, which means that $\sigma_{xy}^{\text{pl}} = \sigma_{xy}$ always holds in YBCO. Thus, in the following we do not discriminate $\sigma_{xy}^{\text{pl}}$ from $\sigma_{xy}$.

Figure 7(a) shows the temperature dependences of $\sigma_{xy}$ for samples with 5 representative compositions in a semilog plot. We omitted other samples only for clarity. For the samples with $y \sim 6.75$ the low-temperature decrease remains in $\sigma_{xy}$ and thus the observed decrease in $R_{H}$ at low temperatures is not an artifact of some feature of the Cu-O chains but is an intrinsic feature of the CuO$_2$ planes. The $y$ dependence of $\sigma_{xy}$ at fixed temperatures...
is shown in Fig. 7(b). \( \sigma_{xy} \) at low temperatures appears to be suppressed at the oxygen content of \( \sim 6.75 \) [Ref. 20]; this anomaly also has nothing to do with the Cu-O chains. As we discussed in Ref. 20, this anomaly must be due to a peculiar doping dependence of the mobility of the carriers.

C. Hall angle

Next, let us analyze the cotangent of the Hall angle (\( \cot \Theta_H \)). As mentioned above, the Hall coefficient \( R_H \) appears to show no in-plane anisotropy as far as magnetic fields are applied along the \( c \)-axis, which is expected from the Onsager relation. On the other hand, the in-plane resistivity is observed to be anisotropic\(^{18}\) in a whole doping range in YBCO. The parameter \( \cot \Theta_H \) is calculated by \( \rho_{xx}/\rho_{yx} \), and thus in general \( \cot \Theta_H \) is expected to be anisotropic. With careful considerations, it turns out that one can obtain the Hall angle of the CuO\(_2\) planes from measurable properties only in the \( \rho_0 \)-sample. We can express \( \cot \Theta_H = (E_x/E_y) = \rho_{xx}/\rho_{yx} \) of the CuO\(_2\) planes along the \( b \)-axis as

\[
\cot \Theta_H^{pl} = \frac{\rho_b^{pl}}{\rho_{yx}^{pl}},
\]

where \( \Theta_H^{pl} \) is the Hall angle of the CuO\(_2\) planes with the current along the \( b \)-axis, and from Eq. 2 we obtain

\[
\cot \Theta_H^{pl} = \frac{\rho_0}{\rho_{yx}^{meas}},
\]

where the non-measurable property \( \rho_0^{pl} \) is canceled out.

Fig. 8(a) shows a \( \cot \Theta_H^{pl} \) vs. \( T^2 \) plot for samples with \( y = 6.30-7.00 \), where the origin for each sample is shifted for clarity. In highly doped samples with \( y = 6.95 \) and 7.00 \( \cot \Theta_H \) appears to be fitted well by a function of \( AT^2 \) except for high temperatures (dashed lines in the figure). Note that only one fitting parameter is used for the fitting and the residual component is zero. This result is consistent with the reported result of \( \rho_0/\rho_{yx} \) for a 90-K sample\(^{17}\). The data for \( y = 6.85 \) and 6.80 can also be fitted by the function of \( AT^2 \) (dashed lines in the figure); however, at low temperatures some upward deviation is observed. The low-temperature deviation seen in samples with \( y = 6.60-6.85 \) is clearly corresponding to the decrease in \( R_H \) at low temperatures and thus this anomaly seems to be one of the 60-K anomalies rather than an effect of the pseudogap\(^{22,32}\). The \( T^2 \)-dependences of \( \cot \Theta_H \) for samples with \( y \leq 6.75 \) can be fit by a function of \( AT^{\alpha} + C \) with \( \alpha > 2 \) (solid lines in the figure) except for the upturn at low temperatures. Figures 8(b-d) show \( y \) dependences of the fitting parameters. The power \( \alpha \) is 2 for \( y \geq 6.80 \) and increases with decreasing oxygen content from 6.75. In the slightly-doped region \( \alpha(y) \) shows a peak at \( y \sim 6.45 \) and decreases with decreasing \( y \); \( \alpha \) is shown to be 2 in the low doping limit\(^{25}\).

The ‘residual’ component \( C \) becomes finite only for \( y \) below \( \sim 6.60 \), and this is reminiscent of the result of the normal-state orbital magnetoresistance\(^{25}\), in which the residual component \( b \) in its temperature dependence \( (aT^2 + b)^{-2} \) becomes finite for \( y \leq 6.60 \).

Of course we can also calculate \( \cot \Theta_H \) with the current along the \( a \)-axis (\( \cot \Theta_{H(a)} \)), which may contain con-
the temperature dependences of the resistivities and/or mobilities of YBCO, because Y124 has doubled Cu-O chains which are conductive. Figure 11(a) shows a cot ΘH(a) vs. T^2 plot for samples with y=6.30–7.00 (the origin for each sample is shifted for clarity). In highly doped samples with y ≥ 6.85, cot ΘH appears to be fitted well by a function of AT^α + C except for high temperatures (dashed lines in the figure); however, unlike cot Θ_H^pl, the residual component C becomes negative. The solid lines are fits to the data by a function AT^α + C with α > 2. For y = 6.70 – 6.80 the data can be fitted only below ~200 K, whereas for more underdoped samples with y ≤ 6.65 the data are fitted well. The y dependences of the fitting parameters are shown in Figs. 11(b–d). Observed tendency is qualitatively similar to that in cot Θ_H^pl except for the residual component for highly doped samples with y ≥ 6.85.

From the Hall angle we can extract the Hall mobility µ_H; Figs. 10(a) and 10(b) show the y dependences of µ_H with the current along the b-axis and the a-axis, respectively, at various temperatures. Of course, the meaning of µ_H with the current along the b-axis is more transparent, but both data of µ_H are qualitatively consistent. At low temperatures the Hall mobility increases with decreasing y from 6.80 to 6.65. This enhancement of the mobility compensates the effect of decreasing carrier concentration and thus an overlapping of ρ_a(T) is observed for samples with y = 6.65–6.80 [Ref. 20].

D. Comparison with YBa_2Cu_3O_8

YBa_2Cu_3O_8 (Y124) can be a good reference system of YBCO, because Y124 has doubled Cu-O chains which are believed to show good conduction. Figures 11(a–d) shows the temperature dependences of ρ_a, ρ_b, R_H^meas, R_H^pl(iso), σ_xy and cot Θ_H^pl for our YBCO samples (y = 7.00 and/or 6.75) together with those for Y124, which are extracted from the data in published papers. Fig. 11(a) shows the temperature dependences of ρ_a and ρ_b. While these data are not very different between YBCO_{7.00} and Y124, the anisotropy of the resistivity in Y124 is larger than that in YBCO_{7.00}; this strong anisotropy is considered to come from the doubled Cu-O chains. The temperature dependence of the measured Hall coefficient is very different [Fig. 11(b)]; that of Y124 is much weaker than YBCO. However, when we calculate R_H^pl(iso) from the data, it shows qualitatively similar behavior in both YBCO_{7.00} and Y124. This fact suggests that the very conductive Cu-O chains mask a strong temperature-dependence of the Hall coefficient in Y124. Figure 11(c) shows σ_xy(T) in Y124 and YBCO samples with y = 7.00 and 6.75. The Hall conductivity of Y124 appears to be very close to that of YBCO_{6.75} rather than YBCO_{7.00}. This is understandable because Y124 is naturally underdoped. The cotangent of the Hall angle along the b-axis is shown in Fig. 11(d). Data for Y124 can be fitted well by a function of AT^2 below ~ 230 K (solid lines). Thus, the transport properties of the CuO_2 planes in Y124, which are extracted by the analysis presented in this paper, appear to be similar to those in YBCO, and this fact gives confidence in the validity of our analysis.

E. Finite Hall coefficient in quasi-1D systems

Our analysis assumes that the Cu-O chains do not contribute to the Hall electric-field, whereas in actual quasi-1D systems the Hall resistivity has been observed to be not negligible. This is mostly due to the large resistivity in the direction perpendicular to the chains, ρ_{yy}.
One can easily understand this situation by noticing that the Hall resistivity $\rho_{yx}$ is expressed as

$$\rho_{yx} = \sigma_{xy} \rho_{xx} \rho_{yy},$$

where $\sigma_{xy}$ is the Hall conductivity of the quasi-1D system and $\rho_{xx}$ is the resistivity along the chain direction. (In quasi-1D metals, $\rho_{yy} \gg \rho_{xx}$.) This formula tells us that a sizable $\rho_{yx}$ can be observed for a very small $\sigma_{xy}$, if $\rho_{yy}$ is very large. In fact, if we calculate $\sigma_{xy}$ for the case of PrBa$_2$Cu$_4$O$_{y}$ (Pr124) using published data, it turns out that $\sigma_{xy}$ of Pr124 is only $10^{-4}$ of that of YBCO. This estimate indicates that the contribution of $\sigma_{xy}$ of the Cu-O chains to the total Hall conductivity is negligible in YBCO. More generally speaking, in a chain-plane composite system, the Hall conductivity of the total system, $\sigma_{xy}^{tot}$, is given by

$$\sigma_{xy}^{tot} = \sigma_{xy}^{pl} + \sigma_{xy}^{ch},$$

where $\sigma_{xy}^{pl}$ and $\sigma_{xy}^{ch}$ are the individual Hall conductivity of the planes and the chains. In the chain layer, $\sigma_{xy}^{ch}$ is always very small even when $\rho_{yx}$ is sizable, as we discussed above. Therefore, one can safely neglect the contribution of $\sigma_{xy}^{ch}$ in the composite system and consider that the behavior of $\sigma_{xy}^{tot}$ is governed almost solely by $\sigma_{xy}^{pl}$.

F. Anomalous decrease in $R_H$ at low temperatures

Finally, let us discuss the pronounced decrease in $R_H$ observed only for samples near $y \simeq 6.75$ at low temperatures. We emphasize that the present data are measured only at $T > T_c$ by sweeping the magnetic fields up to at least 10 T, where the field dependence of the Hall resistivity is $H$-linear in the whole region. Therefore, the decrease in $R_H$ reflects the normal-state property and is irrelevant to the negative Hall anomaly which is observed only at low fields and below $T_c$. Our observation is somewhat reminiscent of the decrease in $R_H$ observed in La$_{2-x-y}$Nd$_y$Sr$_x$CuO$_4$ [Ref. 36], and thus the present result may actually be related to charged stripes. In fact, the carrier concentration of YBCO$_{6.75}$ is consistent with the hole concentration $p \sim 1/8$ (per Cu atom in the CuO$_2$ planes) [Refs. 33, 37, 38], where so-called ‘1/8 anomaly’ is observed in cuprate superconductors; therefore, it is likely that the observed anomaly in $R_H$ is related to the ‘1/8 problem’. Note, however, that the in-plane resistivity anisotropy is observed to become small in samples with $y \sim 6.75$ at low temperatures. Such a result may seem to imply that one-dimensionality of charge dynamics in the CuO$_2$ planes is minimal, which is inconsistent with the picture of static stripes at $p \sim 1/8$. However, the possibility of fluctuating charge stripes is not excluded, because the stripe liquid can be in an ‘isotropic’ phase: in this case, the decrease in $R_H$ can be caused by a realization of the particle-hole symmetry when the hole concentration becomes $\sim 1/8$ and the stripes become $1/4$ filled.

VI. ACKNOWLEDGMENTS

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