High spin states in the heavy nucleus $^{208}$Pb and the coupling of one-particle one-hole states to platonic shapes

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Abstract. Structure, spin, and parity of states in $^{208}$Pb at $9 < E_x < 17$ MeV are explained by the weak coupling of the $3^-$, $4^+$, $6^+$ yrast, and the $12^+$ yrare states to one-particle one-hole yrast and yrare states. The spins of the particle and the hole are coupled in the stretched or nearly stretched mode. The $n$-isomerism of three states is shown to derive from the exchange of an intruder hole and the $p_{1/2}$ hole.

1. Introduction
In $^{208}$Pb about 500 neutron bound ($S(n) = 7368$ keV) and proton bound ($S(p) = 8004$ keV) states are known [1]. The mean distance between two states is 3.0 keV. The excitation energies are known with an uncertainty of 10 eV for the lowest states and a few 100 eV for higher lying states. Most states are described in the shell model as one-particle one-hole (1p1h) configurations. Thirty-six states have different structures [2, 3]. Eighteen states are described by the coupling of 1p1h states to the $3^-$ yrast state [4], four states as pairing vibrations [5], ten states are suggested as tetrahedral configurations [6], and five collective states with spins $3^-$, $5^-$, $5^+$, $6^+$, and $12^+$ are not described by any existing theory, the platonic shape of dodecahedrons or icosahedrons is suggested [3].

Deep inelastic scattering on $^{208}$Pb revealed a $\gamma$-cascade starting with the 16375 keV state and feeding 28 states down to the 9061 keV state; $\gamma$-transitions proceed further down to the 4895 keV $10^+$ isomer and finally to the ground state [7]. The 4895 keV $10^+$ state is an isomer with the extraordinary life time of 0.5 $\mu$s [1]. It mainly feeds the $7^+$ yrast state which bears only 70% of the configuration $g_{9/2}f_{5/2}$ [8]; the remaining strength is of a type differing from 1p1h configurations.

2. Discussion
The spin of the 6101 keV state was first determined by the Heisenberg group with $(e,e')$ [9] and verified with $(p,p')$ [10]. The new data from Broda et al. [7] confirmed spin and parity beyond any doubt by the feeding from the $13^-$ yrast state and the feeding of the $11^+$ yrast and yrare states [11] (Fig. 1). The 6435 keV, 6449 keV, 6744 keV states were identified by $(e,e')$ as the $12^-$, $13^-$, $14^-$ members of the $j_{13/2}i_{13/2}$ multiplet [1, 12, 13].

In the weak coupling model (WCM) [14] the coupling of the $3^-$ yrast state to the 6435 keV $12^-$, 6449 keV $13^-$, 6744 keV $14^-$ states in a stretched mode predicts three states with spins $15^+$,
Figure 1. High spin states in $^{208}$Pb at $6 < E_x < 10$ MeV. At left an extract from the level scheme determined by Broda et al. (Fig. 4 in [7]) is shown.

$16^+, 17^+$ at $E_x^{WCM} = 9050, 9064, 9359$ keV. The 9061, 9103, 9394 keV states are interpreted to correspond to the predicted $15^+, 16^+, 17^+$ states. The similarity of the $\gamma$-transitions for the $12^-, 13^-, 14^- j_{15/2,13/2}$ states and for the three states generated by the coupling to the $3^-$ yrast state is striking (Fig. 1).

$\gamma$-transitions from the highest observed state in $^{208}$Pb at $E_x = 16375$ keV feed the 9394 keV suggested with spin $17^+$. Among them five transition multipolarities were measured [7].

An extended WCM explains the excitation energies, spin, and parity as well as the structure of seven states in the $\gamma$-cascade from 13675 keV to 9394 keV (Fig. 2) by the coupling of the tetrahedral state $3^-_1$ [6] and $6^+_1, 12^+_2$ suggested as dodecahedral or icosahedral states [3] to 1p1h yrast states $5^-_1, 6^-_2, 9^+_{1,2}, 14^+_{1,2}$. In the WCM the hamiltonian $H_{WCM}$ has four constituents

$$H_{WCM} = H_{Th} + H_{6^+_1} + H_{12^+_2} + H_{1p1h} + H_{residual},$$

where $H_{residual} \approx 0$ defines the weak coupling.

In Eq. (1) $H_{Th}$ describes a tetrahedral state, $3^+_1$ or $4^+_1$ are relevant for explaining seventeen states excited in the $\gamma$-cascade from 13632 keV to 9061 keV. $H_{6^+_1}$ and $H_{12^+_2}$ are suggested as icosahedral configurations; the two collective states do not appear in shell model calculation [2, 3]. $H_{1p1h}$ are 1p1h states with spin $I$ where particle $LJ^{+1}$ and hole $lj^{-1}$ are coupled in a (nearly) stretched mode,

$$J^{+1} - j^{-1} - x \leq I \leq J^{+1} + j^{-1},\quad 0 \leq x \leq 2, \quad 1 \leq M \leq 4.$$
Figure 2. High spin states in $^{208}$Pb at $9 < E_x < 14$ MeV. At left the level scheme shown by Fig. 4 in [7] is reproduced.

The sum of the excitation energies of the constituents defines the energy

$$E_{WCM}^x = E_{Th}^x + E_{6^-}^{I_{6^-}} + E_{12^2}^{I_{12^2}} + E_{1p1h}^{1p1h}.$$  

The coupling of the spins of the constituents in the stretched mode describes the total spin

$$I_{WCM}^x = I_{Th}^{I_{Th}} + I_{6^-}^{I_{6^-}} + I_{12^2}^{I_{12^2}} + I_{1p1h}^{1p1h}.$$ 

The product of the parities yields the parity

$$\pi_{WCM}^x = \pi_{Th}^{\pi_{Th}} \times \pi_{6^-}^{\pi_{6^-}} \times \pi_{12^2}^{\pi_{12^2}} \times \pi_{1p1h}^{\pi_{1p1h}}.$$  

From the analysis of 1p1h states at $E_x < 6.2$ MeV the mean deviation of the excitation energy from shell model calculations is determined to be about 30 keV [2]. Allowing a factor four for the additional residual interaction with the three other constituents the WCM should predict the observed state within a range

$$-100 \lesssim E_{WCM}^x - E_{exp}^x \lesssim +100 \text{ keV.}$$  

Starting from a state with known spin and parity $I_i^{\pi_i}$, the next higher lying state connected by the measured transition multipolarity $E \lambda$ may have spins from $I_i - (2\lambda - 1)$ to $I_i + 2\lambda - 1$. Broda et al. [7] measured four transition multiplicities connecting the states 9394 keV 17$^+$ $\rightarrow$ 10342 keV 18$^-$ $\rightarrow$ 11361 keV 21$^+$ $\rightarrow$ 12949 keV 21$^+$ $\rightarrow$ 13675 keV 24$^-$. Eqs. (1)- (4) restrict the choice for the sequence in a unique manner (Fig. 2). Similarly the spins of the intermediate
Table 1. States in $^{208}\text{Pb}$ excited in the $\gamma$-cascade from $E_x = 13675$ keV to $E_x = 9061$ keV. Underlined values of excitation energies indicate ns-isomers [7]. Constituting states in the WCM with excitation energies below 7 MeV are shown at bottom.

| $E_x^{\exp}$ | $I^\pi$ | $E_x^{WCM}$ | $I_x^{WCM}$ | $\gamma$-transition | configuration |
|-------------|--------|-------------|-------------|---------------------|---------------|
| [keV]       |        | [keV]       | [keV]       |                     |               |
| 1 13675 24$^-$  | yrare  | 13726       | -51         | E3 to 3             | $\nu\ g_{9/2}\ i_{13/2}\ 9_1^+$ yrast |
|             |        |             |             | E2 to 2             |               |
| 2 13536 22$^+$  | yrare  | 13488       | +48         | ? to 3              | $\nu\ i_{11/2}\ p_{1/2}\ 6_2^-$ yrare stretched |
| 3 12949 21$^+$  | yrast  | 12922       | +27         | E2 to 5             | $\nu\ i_{11/2}\ p_{1/2}\ 6_2^-$ yrare stretched |
|             |        |             |             | M1 to 4             |               |
| 4 11958 20$^+$  | yrare  | 11914       | +44         | ? to 5              | $\nu\ g_{9/2}\ p_{1/2}\ 5_1^-$ yrast stretched |
| 5 11361 21$^+$  | yrare  | 11263       | +98         | E3 to 6             | $\pi\ h_{9/2}\ h_{11/2}\ 9_2^+$ yrare |
| 6 10342 18$^+$  | yrare  | 10307       | +35         | E1 to 7             | $\nu\ i_{11/2}\ p_{1/2}\ 6_2^-$ yrare stretched |
| 7 9394 17$^+$  | yrast  | 9359        | +35         | M1+E2 to 8          | $\nu\ j_{15/2}\ i_{13/2}\ 14_1^-$ yrast stretched |
| 8 9103 16$^+$  | yrast  | 9064        | +39         | ? to 9              | $\nu\ j_{15/2}\ i_{13/2}\ 13_1^-$ yrast |
| 9 9061 15$^+$  | yrast  | 9050        | +11         | E3 to 10            | $\nu\ j_{15/2}\ i_{13/2}\ 12_1^-$ yrast |
| 10 6744 14$^+$  | yrast | M1+E2 to 11 |             | ν $j_{15/2}\ i_{13/2}\ 14_1^-$ yrast stretched |
| 11 6449 13$^-$  | yrast  |             |             | ν $j_{15/2}\ i_{13/2}\ 13_1^-$ yrast |
| 6435 12$^-$  | yrast  |             |             | ν $j_{15/2}\ i_{13/2}\ 12_1^-$ yrast |
| 6101 12$^+$  | yrare  |             |             | collective          |
| 4424 6$^+$  | yrast  |             |             | collective          |
| 2615 3$^-$  | yrast  |             |             | collective          |

11958 keV and 13536 keV states are found. By assuming transition multipolarities $\lambda \leq 3$, spins of more states are determined (Fig. 3).

3. Summary

Fig. 3 shows the $\gamma$-cascade observed by deep inelastic scattering on $^{208}\text{Pb}$ [7] at $9 < E_x < 17$ MeV. Table 2 shows results. The mean deviation between the excitation energies calculated by the weak coupling model (WCM) from the observed excitation energies is

$$\langle E_x^{WCM} - E_x^{\exp} \rangle = +30 \text{ keV.} \quad (5)$$

The size and the sign equals the mean deviation found for 1p1h states at $E_x < 6.2$ MeV [2].

Many states at $E_x < 9$ MeV have the 3$^-$ yrast state (suggested as a tetrahedral configuration) and the 12$^+$ yrare state (suggested as an icosaheiral configuration) as one constituent.

The highest observed state ($E_x = 16362$ keV) with spin 26$^+$ has the 3$^-$ yrast, the 6$^+$ yrast, the 12$^+$ yrare, and the lowest 1p1h state (3192 keV 5$^-$) as constituents.

$\gamma$-transitions from ns-isomers involve the exchange of an intruder hole ($h_{11/2}$ or $i_{13/2}$) with a $p_{1/2}$ hole.

The success of the WCM may show the description of the collective 3$^-$, 4$^+$, 6$^+$ yrast states and the 12$^+$ yrare state by platonic shapes differing from 1p1h and 2p2h configurations to be a good assumption.
Figure 3. High spin states at $9 < E_x < 17$ MeV. Eight states complement the $\gamma$-cascade displayed in Fig. 2. All shown states have 1p1h states with either an intruder hole or the $p_{1/2}$ hole except for 10552 keV states with a $p_{3/2}$ hole and the 14883 keV state without a 1p1h state.

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