Mathematical Analysis of Soil-Structure Interaction Including Kinematic and Inertial Interaction Effects

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Abstract

In this research, both kinematic interaction (KI) and inertial interaction (II) effects of soil-structure interaction (SSI) on inelastic seismic demands of structures are investigated. Site effect is also considered only by applying ground motions recorded at site classes D and E (as defined in NEHRP[1] and FEMA-440 [2]) that on them SSI effect is considerable. Carrying out a parametric study, the structure and underlying soil are modeled as a Single Degree Of Freedom (SDOF) structure with elasto-plastic behavior and a mathematical simplified 3DOF system, based on the concept of Cone Models, respectively. Also the foundation is considered as a rigid cylinder embedded in the soil with different embedment ratios. Then the whole soil-structure systems are analyzed under 30 ground motion recorded at site classes D and E and a comprehensive parametric study is performed for a wide range of non-dimensional parameters defining SSI problem. Results indicated that ignoring SSI causes considerable and in some cases un-conservative differences in seismic demands of structures. In the case of embedded foundation, it is observed that rocking input motion due to KI plays the main role and increase the structural demands especially in deep foundation embedment and slender buildings located on soft soils.

Consequently, comparing the results with and without inclusion of SSI effects reveals that both II and KI effects of SSI play an important role in analyses or design procedures and ignoring them may cause un-conservative results in cases of deep embedded foundation and slender structures.

Keywords: soil-structure interaction; cone model; foundation embedment; kinematic interaction (KI); Inertial interaction (II); Strength reduction factor; ductility demand; Elastic and inelastic seismic demands
1. Introduction

The flexibility of structure’s underlying soil affects the response of the structure due to SSI. This phenomenon has two main effects as follows:

1- The difference between stiffness of the foundation and the surrounding soil induces the difference between the motion experienced by the essentially rigid foundation (the foundation input motion (FIM)) and the free-field motion (FFM). This effect is called the KI effect and happens even if the foundation has no mass. In other words, the FIM is the result of geometric averaging of the seismic input motion in the free field [26].

2- The flexibility of soil affects the response of the structure subjected to FIM. In fact, the soil-structure system behaves as a new system with different dynamic properties (longer natural period and usually higher damping). This effect is called II effect.

Numerous researches have been done on the effects of SSI over the past few decades more specifically in 1970s. Veletsos and Meek [3] and Veletsos and Nair [4] recognized that the effects of inertial interaction on elastic structures could be approximated by modifying the fundamental period and the damping ratio of the fixed base replacement oscillator. In addition, the variations of the equivalent natural period and damping ratio have been studied by Wolf [5]; Aviles and Perez-Rocha [6]. But, the inelastic behavior of structures has recently been given more attention by some researchers. Bielak [7] first studied this matter by investigating the harmonic response of a bilinear structure supported on a visco-elastic half-space and found that the resonant structural deformation could be significantly larger than the deformation obtained from the fixed-base structure. Aviles and Perez-Rocha [8] considered a SDOF elasto-plastic structure supported on a rigid foundation embedded in a visco-elastic stratum of constant thickness over a uniform visco-elastic half space. They concluded that the effects of the foundation flexibility and the yielding of structures are beneficial for slender structures with a natural period somewhat larger than the site period, but detrimental if the structural period is shorter than the site period. Aviles and Perez-Rocha [9] employed this replacement oscillator formulation in NEHRP provisions. But most of aforementioned documents were prepared for surface foundations and the KI effect was ignored. In some researches effect of foundation embedment was introduced as simplified factors to modify the soil dynamic stiffness. The dynamic stiffness of embedded foundations was studied by Beredugo and Novak [10] and Elsabee etal. [11]. Morray [12] studied the KI problem of embedded circular foundations parametrically for a varied range of parameters typically found in nuclear reactor design. Luco etal. [13] pointed to the influence of rocking input motion due to KI effect on the response of structures with embedded foundation. The general effect of the foundation embedment on the structural response through simplified methods was also studied by Bielak [14], Kausel etal. [15] and more recently by Aviles and Perez-Rocha [16] and Takewaki et al. [17].

Almost all mentioned researches were conducted on linear soil-structure systems. Examples of early works on the response of nonlinear structures are made by Veletsos and Verbic [18], Bielak [19] and Muller and Keintzel [20]. However, most of the researches are focused on surface foundations, which ignore KI effect and believed that ignoring KI effect is conservative for the structure [21]. NEHRP [1] also ignore the KI effect and FEMA-440 [2] supports this idea by considering the reducing effect of KI due to base slab averaging and foundation embedment. However, it seems that despite the reducing effect of KI on the translational component of the FIM, the resulting rocking component may increase the structural demands especially for soil-structure systems with deep embedded foundations. In this research, soil-structure systems are analyzed parametrically for a set of non-dimensional parameters, which define the soil-structure problem, using 30 ground motions recorded.
at soil sites D and E. Then kinematic and inertial effects of SSI and site effect are investigated on elastic and inelastic seismic demands of structures.

2. Soil-Structure model

The soil-structure system considered in this study is shown in Figure 1. (a) that is based on the following assumptions:

1- The super-structure is modeled as an equivalent elasto-plastic SDOF system with height $h$, mass $m$ and mass moment of inertia $I$, model with soil material damping, which may be considered to be the effective values for the first mode of vibration of a real multi degree of freedom system.

2- The foundation is considered to be rigid with embedment depth $e$ and mass and mass moment of inertia $m_f$ and $I_f$, respectively.

The soil beneath the structure is considered as a homogeneous half-space and replaced by a mathematical discrete model based on the concept of cone models for embedded foundations [22]. Two degrees of freedom (DOFs) are introduced in this model for the foundation which are sway ($u$) and rocking ($\varphi$). An additional internal DOF ($\varphi_1$) is introduced for the soil model to consider frequency dependency of soil stiffness. Representative springs of soil behave elastically but effect of soil nonlinearity is approximately introduced using a degraded shear wave velocity for the soil medium, consistent with the estimated strain level in soil [23]. In NEHRP [1] and FEMA-440 [2], the strain level in soil is related to the peak ground acceleration.

Consequently, a 4-DOF model is formed for the whole soil-structure system as shown in Figure 1. (b).

![Figure 1. (a) The soil-structure system; (b) Mathematical model of soil-structure](image_url)

The structure, foundation and soil related parameters, introduced in Figure 1. (b), are defined as follows:
\[ k_{oh} = \frac{8\rho V_s^2 r}{2-\nu} (1 + \frac{e}{r}) \]  
\[ c_{oh} = \frac{r}{V_s} \gamma_{oh} k_{oh} \]  
\[ k_r = \frac{8\rho V_s^2 e^3}{3(1-\nu)} \left(1 + 2.3 \frac{e}{r} + 0.58 \left(\frac{e}{r}\right)^3\right) \quad k_{0r} = k_r - k_{oh} f_k^2 \]  
\[ c_{0r} = \frac{r}{V_s} \gamma_{0r} k_r \]  
\[ c_{1r} = \frac{r}{V_s} \gamma_{1r} k_r \]  
\[ I_{1r} = (\frac{r}{V_s})^2 \mu_{1r} k_r \]

Where, \( \rho \), \( \nu \), \( V_s \) and \( r \) are the specific mass, Poisson’s ratio, shear wave velocity of the soil and the radius of the cylindrical foundation, respectively. Besides, \( \gamma_{0h}, \gamma_{0r}, \gamma_{1r} \) and \( \mu_{1r} \) are non-dimensional coefficients of the discrete model in terms of \( e/r \) and are calculated using the following formulae:

\[ \gamma_{0h} = 0.68 + 0.57 \frac{e}{r} \]  
\[ \gamma_{0r} = 0.15631 \frac{e}{r} - 0.08906 \left(\frac{e}{r}\right)^2 - 0.00874 \left(\frac{e}{r}\right)^3 \]  
\[ \gamma_{1r} = 0.4 + 0.03 \left(\frac{e}{r}\right)^2 \]  
\[ \mu_{1r} = 0.33 + 0.1 \left(\frac{e}{r}\right)^2 \]

Sway springs and dashpots are connected to the super-structure model with the following eccentricities in order to account for the coupling terms between the sway and rocking degree of DOFs in the stiffness matrix of the embedded foundation:

\[ f_k = 0.25 e \]  
\[ f_r = 0.32 e + 0.03 e \left(\frac{e}{r}\right)^2 \]

The whole soil-structure model is subjected to sway and rocking components of FIM (\( u_s \) and \( \varphi_s \)) as shown in figure 1. (b). More details about components of FIM are described in section 4.

3. Problem parameters

The response of soil-structure system depends basically on the size of the structure, its dynamic properties, and the soil profile as well as the applied excitation. The effect of these factors can be best described by the following non-dimensional parameters [24].

- A non-dimensional frequency as an index for the structure-to-soil stiffness ratio:

\[ a_0 = \frac{\omega_{fs} h}{V_s} \]
Where $\omega_n$ is the circular frequency of the fixed-base structure. This index can have values of up to 3 for conventional building-type structures resting on very loose soils; while infinitesimal values close to zero are representative of fixed-base structures [25].

- Aspect ratio of the building $h/r$, an index for its slenderness ratio.
- Embedment ratio of the foundation defined as $e/r$.
- Ductility demand of the structure defined as:

$$\mu = \frac{u_m}{u_y}$$  \hspace{1cm} (14)

Where, $u_m$ and $u_y$ are the maximum displacement caused by a specific base excitation and the yield displacement of the structural stiffness, respectively.

- Strength reduction factor (SRF) of the structure defined as:

$$R = \frac{F_0}{F_y}$$  \hspace{1cm} (15)

Where, $F_0$ and $F_y$ are the strength required to maintain the structure in elastic range and inelastic strength demand of the structure, respectively.

- Structure-to-soil mass ratio index defined as:

$$\bar{m} = \frac{m}{\rho r^2 h}$$  \hspace{1cm} (16)

This parameter varies between 0.4 and 0.6 for ordinary building-type structures [25] and is set equal to 0.5 in this study.

- Foundation-to-structure mass ratio $m_f / m$ that is assigned 0.1.
- Poisson’s ratio of soil $\nu$ that is considered to be 0.25 for alluvium and soil in this study.
- Material damping ratios of the structure $\zeta_{str}$ that is set to 5% of the critical damping at the effective period of the soil-structure system.

The first three factors not only participate within higher exponents in the equations of motion but also have a vaster range of variations. Thus, they are commonly selected as the key parameters of the system [24]. But the other parameters are those with less importance and were set to typical values for ordinary buildings.

4. Kinematic interaction effect

As introduced in soil-structure model of Figure 1, two different FIM components are produced as a result of KI: Horizontal FIM ($g_u$) and rocking FIM ($g_\phi$).

Horizontal FIM component generally decreases in comparison with FFM especially for more embedment depths. But rocking FIM amplitude has an increase as the depth of embedment increases. To evaluate FIM components, the mathematical (Meek and Wolf [26]) method is used based on the concept of double-cone models. Double cones are used to represent a disk embedded in a full space. An embedded foundation is then replaced by a stack of N disks commencing from the lowermost point of the foundation, $e$, and continuing to the ground surface. In order to provide stress-free conditions on the ground surface, another stack of N disks, which are the mirror images of the former disks, are considered on the other side of the ground surface as demonstrated in Figure 2. These mirror image disks are excited by the same excitations as the original disks; therefore, stress-
free conditions on the ground surface will be guaranteed. By using the green functions at the level of each disk and its mirror image, the N×N flexibility matrix of the free field is evaluated. The inverse of this flexibility matrix is the dynamic stiffness matrix of the free field \( (S_f^T) \). Then by extracting the excavated part of the soil from the model and inserting the rigid foundation, the dynamic stiffness of the embedded foundation can be evaluated. Because the rigid foundation is inserted, the dimension of the stiffness matrix is reduced from N to 2 for introduced sway-rocking foundation model.

This can be done by using an N×2 kinematic conditions matrix \( (A) \), which is calculated based on the foundation geometry. Thus, the dynamic stiffness matrix of the rigid foundation \( (S_g^T) \) is calculated using the mass matrix of the excavated part of the soil \( (M) \) as follows:

\[
S_g = A^T S_f A + \omega^2 M
\]  
(17)

Subsequently, the FIM vector is evaluated using the following equation:

\[
u_g = S_g^{-1} A^T S_f \nu_f
\]  
(18)

Where, \( \nu_f \) is the N×1 vector of the FFM evaluated at the level of the disks and \( \nu_g \) is the 2×1 vector of the FIM comprising the two components of the sway and rocking motions:

\[
u_g = \begin{bmatrix} u_g \\ \phi_g \end{bmatrix}
\]  
(19)

In deriving Equation (18), the following relationship between the dynamic stiffness and motion of the free-field state as well as those of the foundation is used:

\[
A^T S_f \nu_f = S_g \nu_g
\]  
(20)

5. Method of analysis

The soil-structure model, introduced in previous section, has the capability to be used directly in a time domain analysis to assess inelastic response of soil-structure systems. This mathematical model has been analyzed by direct step-by-step integration, using \( \beta \)-Newmark method, subjected to a total of 30 strong motions recorded at soil types D and E (as defined in NEHRP[1] and classified in FEMA-440, Appendix C[2]) on which soil-structure interaction effect is considerable. Details of selected ground motions are listed in Tables 1 and 2.

It is known that for any specific base excitation, inelastic response of fixed-base structures is mainly a function of the natural period of the structure, \( T_{fix} \), and the level of inelastic deformation (the target ductility ratio, \( \mu \) in design procedure or strength reduction factor, \( R \) in analysis procedure. The material damping and the type of hysteretic behavior of structure have been found to be less important. In soil-structure systems the three non-dimensional key parameters \( a_0, h/r \) and \( e/r \) also play an important role. Thus, a parametric study has been conducted using the five above-mentioned parameters \( (T_{fix}, \mu, R, a_0, h/r \) and \( e/r) \). For each earthquake record, a set of 2,160 soil-structure systems consisting of 60 SDOF structures with fixed-base periods ranging from 0.05 to 3 s with three different values of aspect ratio \( (h/r =1, 3, 5) \), four values of embedment ratio \( (e/r=0,0.5,1,2) \) and three values of non-dimensional frequency \( (a_0=0, 1, 3) \) are investigated. Cases with \( a_0 = 0 \) are indeed related to fixed-base state. The response of each system is investigated both with and without inclusion of KI effect. For any given case, the inelastic strength demand of structure \( (F_\mu \) or \( \bar{F}_e \) ) was calculated by iteration in order to reach the target ductility \( (\mu=2, 4) \) in the structure, in addition to the elastic case \( (\mu=1) \), within 1% of accuracy. Consequently, at least a total of 12,960 independent non-linear analyses have been carried out for different soil-structure systems subjected to 30 strong
motions. Using MATLAB mathematical software, a comprehensive code is conducted to support mentioned purposes as following steps:

5.1. Analysis procedure

For each soil-structure model, the following procedure is used to investigate the SSI effect on inelastic demands of the structure:

1- The yield strength demand of the structure in the fixed-base state (ignoring the soil effect) is calculated by iteration to reach a specific ductility level within 1% of accuracy when subjected to a specified free-field ground motion.

2- The ductility demand of the structure, as a part of the soil-structure system, is calculated for soil-structure systems with different values of $a_0$, $h/r$ and $e/r$, providing the same yield strength for the structure as calculated in the fixed-base state.

3- As the input motion to the soil-structure system (in Sec. 2.), both the FFM and the resulted FIM are used in order to investigate the SSI effect with and without KI.

In each case, the difference between the ductility demand of the fixed-base model and that of the structure as a part of the soil-structure system reflects the problem that does exists in conventional design methodology, i.e. the difference between our expectation of structural behavior as a fixed-base model and the way that structures behave in reality when located on flexible soil.

![Figure 2](image)

Figure 2. Model of the embedded foundation comprising stack of N disks and their mirror image

6. Effect of SSI on seismic demands of structures

The effects of SSI on elastic and inelastic demands of structure are investigated as following sections. Comprehensive analyses are performed through statistical study. The results are calculated using a set of non-dimensional parameters introduced in Sec. 3. Results include both KI and II effects and are provided for models with three embedment ratio $e/r = 0, 1$ and $2$, three different aspect ratios, $h/r = 1, 3$ and $5$ as the representatives of squat, medium and slender buildings and for three different values of non-dimensional frequency $a_0=1, 2$ and $3$ in comparison to the fixed-base structure ($a_0 = 0$). The value of $a_0 = 3$ is representative of systems with severe SSI effect for conventional building type structures [25]. The effect of SSI on inelastic seismic demand are also investigated on structures
undergoing three different ductility levels ($\mu = 2, 4$). The results for site classes D and E are indeed the average values for soil-structure systems subjected to 15 different strong motions listed in Table 1 and 2 respectively.

### Table 1: Selected ground motions recorded at site class D

| No. | Date   | Earthquake Name       | Station Name                        | Station No. | Dir. | PGA (cm/s²) | PGD (cm) | Distance (km) |
|-----|--------|-----------------------|-------------------------------------|-------------|------|-------------|----------|---------------|
| D1  | 10/15/79 | Imperial Valley       | Calexico, Fire Station              | 5053        | 225  | 269.6       | 8.981    | 10.45         |
| D2  | 10/15/79 | Imperial Valley       | El Centro #13, Strobel Residence    | 5059        | 230  | 136.2       | 5.771    | 21.98         |
| D3  | 10/17/89 | Loma Prieta           | Gilroy 2, Hwy 101 Bolsa Road Motel  | 47380       | 0    | 394.2       | 7.152    | 11.07         |
| D4  | 10/17/89 | Loma Prieta           | Gilroy 3, Sewage Treatment Plant    | 47381       | 0    | 531.7       | 8.257    | 12.82         |
| D5  | 10/17/89 | Loma Prieta           | Agnews, Agnews State Hospital      | 57066       | 0    | 163.1       | 12.63    | 24.57         |
| D6  | 10/17/89 | Loma Prieta           | Hayward, John Muir School           | 58393       | 0    | 166.5       | 3.893    | 52.68         |
| D7  | 04/24/84 | Morgan Hill           | Gilroy #2, Keystone Rd.             | 47380       | 90   | 207.9       | 2.094    | 13.69         |
| D8  | 04/24/84 | Morgan Hill           | Gilroy #3 Sewage Treatment Plant    | 47381       | 90   | 189.8       | 3.457    | 13.02         |
| D9  | 04/24/84 | Morgan Hill           | Gilroy #4, 2905 Anderson Rd        | 57382       | 360  | 341.4       | 3.114    | 11.54         |
| D10 | 04/24/84 | Morgan Hill           | Gilroy #7, Mantnill Ranch, Jamison Rd | 57425       | 0    | 183.0       | 2.051    | 12.07         |
| D11 | 01/17/94 | Northridge            | Los Angeles, N. Westmoreland       | 90021       | 0    | 393.3       | 2.288    | 26.73         |
| D12 | 02/09/71 | San Fernando          | Los Angeles, Hollywood Storage Bldg. | 135         | 90   | 207.0       | 12.43    | 21.2          |
| D13 | 02/09/71 | San Fernando          | Vernon, Cmd Terminal Building 4814 L.Vista | 288 | 277  | 104.6      | 6.57     | 45.3          |
| D14 | 10/01/87 | WhittierNarrows       | Los Angeles, 116th St School       | 14403       | 270  | 288.4       | 1.973    | 23.29         |
| D15 | 10/01/87 | WhittierNarrows       | Downey, County Maintenance Bldg.    | 14368       | 180  | 193.2       | 3.952    | 20.82         |

### Table 2: Selected ground motions recorded at site class E

| No. | Date   | Earthquake Name       | Station Name                        | Station No. | Dir. | PGA (cm/s²) | PGD (cm) | Distance (km) |
|-----|--------|-----------------------|-------------------------------------|-------------|------|-------------|----------|---------------|
| E1  | 10/17/89 | Loma Prieta           | San Francisco, International Airport | 58223       | 0    | 231.5       | 4.192    | 58.65         |
| E2  | 10/17/89 | Loma Prieta           | San Francisco, International Airport | 58223       | 90   | 322.7       | 6.023    | 58.65         |
| E3  | 10/17/89 | Loma Prieta           | Oakland, Title & Trust Bldg. (2-story) | 58224       | 180  | 191.3       | 3.526    | 72.20         |
| E4  | 10/17/89 | Loma Prieta           | Oakland, Title & Trust Bldg. (2-story) | 58224       | 270  | 239.4       | 7.238    | 72.20         |
| E5  | 10/17/89 | Loma Prieta           | Larkspur Ferry Terminal             | 1590        | 270  | 134.7       | 4.876    | 94.6          |
| E6  | 10/17/89 | Loma Prieta           | Larkspur Ferry Terminal             | 1590        | 360  | 94.6        | 3.267    | 94.6          |
| E7  | 10/17/89 | Loma Prieta           | Emeryville, 6363 Christie Ave.      | 1662        | 260  | 254.7       | 8.398    | 76.9          |
| E8  | 10/17/89 | Loma Prieta           | Emeryville, 6363 Christie Ave.      | 1662        | 350  | 210.3       | 3.790    | 76.9          |
| E9  | 10/17/89 | Loma Prieta           | Foster City (APEEL 1;Redwood Shores) | 58375       | 90   | 277.6       | 6.285    | 43.8          |
| E10 | 10/17/89 | Loma Prieta           | Foster City (APEEL 1; Redwood Shores) | 58375       | 360  | 63.0        | 15.038   | 43.8          |
| E11 | 10/17/89 | Loma Prieta           | Redwood City (APEEL Array Stn. 2)   | 1002        | 43   | 270.0       | 12.610   | 43.23         |
| E12 | 10/17/89 | Loma Prieta           | Redwood City (APEEL Array Stn. 2)   | 1002        | 133  | 222.0       | 6.839    | 43.23         |
| E13 | 10/17/89 | Loma Prieta           | Treasure Island (Naval Base Fire Station) | 58117       | 0    | 112.0       | 4.411    | 77.42         |
| E14 | 10/17/89 | Loma Prieta           | Treasure Island (Naval Base Fire Station) | 58117       | 90   | 97.9        | 11.488   | 77.42         |
| E15 | 10/15/79 | ImperialValley       | El Centro Array 3, Pine Union School | 5057        | 230  | 216.8       | 20.98    | 12.85         |

### 6.1. Effect of SSI on inelastic strength demand spectra

This effect is depicted in Figures 3 and 4 for structures (with $\mu = 2$ and 4) located on site class E. Figure 3 shows the SSI effect on inelastic strength demand of structure with ($\mu = 2$). The effect of SSI on inelastic strength demand of structures to reach a ductility level of ($\mu = 4$) is also shown in Figure 4. All the results have been normalized by the product of mass of structure and peak ground
acceleration (PGA). The results indicate a general trend of lower strength demands for soil-structure systems in comparison to the fixed-base structures. The exceptions are short period buildings with aspect ratio $h/r = 1$. This trend is clearer for the case of $a_0 = 3$ where SSI effect is predominant. Also, it is seen that the SSI effect becomes less important as the structure undergoes more inelastic deformations ($\mu = 4$). It means that SSI affects the elastic and inelastic strength demands of structure in different ways. This trend is identical in different embedment ratio.

**6.2. Effect of SSI on Strength reduction factor (SRF) spectra**

In this section, SRF of soil-structure systems are computed using mentioned non-dimensional parameters. The graphs of SRF for structures located on site class E, with $\mu=2$ and 4 are presented in Figures 5 and 6 as a function of structural period ($T_{fix}$). The results are presented as the average of the results due to different strong motions in each soil category.

All graphs show a common trend of apparent lower SRF for larger values of $a_0$. It means that SSI reduces SRF values considerably and the more SSI effect, the more reduction in SRF. The effect of SSI on SRF is higher for slender buildings ($h/r = 3$ and 5) and for larger target ductility ($\mu=4$). In ATC3-06 provisions [27], it is believed that the SRFs proposed for fixed-base models can be used to approximate the inelastic strength demands of soil-structure systems as well. However, given the results of Figure 5 and 6, it can be concluded that using this idea leads to underestimation of inelastic strength demands of soil-structure systems. Consequently, the structure would experience higher ductility ratios than expected.

**6.3. Effect of SSI on ductility demand of structure**

In this part, ductility demand of the structure, as a part of the soil-structure system, is calculated for soil-structure systems with different values of $a_0$, $h/r$ and $e/r$, providing the same yield strength for the structure as calculated in the fixed-base state. As seen in Figures 7, 8, 9 and 10, for structures with surface foundation ($e/r=0$), there is a threshold period before which the flexible-base ductility is greater than that of the fixed-base one; afterwards, this trend is reversed. The more the aspect ratio, the greater is the difference between ductility demands of the flexible-base and the fixed-base models. As shown in the same figure, though the embedment of structure generally reduces ductility demands of squat buildings with $h/r=1$, it results in higher demands for slender buildings with $h/r=3$ and 5. The effect is intensified by increasing the embedment ratio. Thus, it is observed that the SSI increases the ductility demand of slender structures with deep embedment almost in the whole range of demonstrated period.

All trends discussed above are intensified by increasing the non-dimensional frequency $a_0$. Hence, it can be concluded that foundation embedment is, in general, beneficial for squat structures while it may increase ductility demands for the case of slender structures. Even for slender structures, the increase in ductility demands is not significant for embedment ratios up to $e/r=1$. However, for deeply embedded structures, the ductility demand can be much higher than expected. As seen, the ductility demand for the case of $h/r=3$, as the representative of slender structures, increases with the embedment ratio and reaches a value of 9 in embedment ratio of 2; 50% more than the target ductility. However, for squat structures with $h/r=1$, ductility demand is always less than the presumed fixed-base target ductility.

**6.4. Effect of KI on ductility demand of structure**

As mentioned before, SSI has two main effects. First, it transforms the FFM into FIM through KI effect and second, it affects the response of structures subjected to the resulted FIM due to flexibility of the soil under the structure through II effect. The role of KI effect is investigated in this section. Figures 11 and 12 demonstrate the ductility demand curves evaluated both with and without inclusion of KI effect for soil-structure systems located on site classes D and E, with fixed-base target ductility of 4.
As seen, for squat structures \((h/r=1)\), inclusion of KI effect generally reduces the flexible-base ductility. In fact, the ductility demand of soil-structure systems without KI effect is very close to fixed-base target ductility, for almost the whole range of period. This means that KI plays the main role in reducing ductility demands. This trend is observed more clearly in case of \(a_0=3\). For slender structures with \(h/r=3\) and 5, however, the importance of KI effect depends on the embedment ratio. For shallow foundations \((e/r=0.5)\), the effect of KI is negligible. But, by increasing the embedment ratio, KI affects the ductility demand more considerably leading to a significant effect for \(e/r=2\). In other words, the FIM is considered as a more severe input motion than the original FFM in such cases.

7. Conclusion

Using mathematical equations to model underlying soil, the response of soil-structure systems was studied parametrically to assess the effect of SSI on the elastic and inelastic demands of structures with embedded foundation to consider site effects and both II and KI effects of SSI. Three non-dimensional parameters, the non-dimensional frequency \(a_0\), the aspect ratio of the structure \((h/r)\) and embedment ratio of the foundation \((e/r)\), are selected as the key parameters of the problem. The considered soil-structure model consists of an elasto-plastic SDOF super-structure embedded in a homogeneous elastic half-space. Based on this model and the ground motions recorded at different soil types (site classes D and E), a comprehensive statistical study is carried out and the following conclusions are made:

1- SSI reduces the elastic and inelastic strength demand of structures. But when the structure undergoes more inelastic deformations \((\mu = 4)\), this effect becomes less important.

2- The SSI effect on SRF of structures located on soft soils (site-classes D and E) is particularly important. In this case, SSI reduces SRF, which in turn may result in larger design forces. This conclusion has an important implication in practical design of structures when SSI effect is predominant.

3- For structures with surface foundation \((e/r=0)\), SSI increases the ductility demand of structure, as a part of soil-structure system, before a threshold period which is closely related to the predominant period of the ground motion. It means that structures having periods less than this threshold period may experience larger deformation than predicted by using conventional fixed-base models. In particular, the effect deserves special attention for the case of larger values of non-dimensional frequency \(a_0\) and where the predominant period of the record is long enough to cover the practical range of conventional buildings. It is also observed that increasing the aspect ratio of the structure increases the SSI effect before the threshold period. But for embedded structures, the SSI effect is different. Though the embedment of structure generally reduces ductility demands of squat buildings with \(h/r=1\), it results in higher demands for slender structures with \(h/r=3\) and 5. The effect is intensified by increasing the embedment ratio, \(e/r\) and non-dimensional frequency, \(a_0\). So, it may be concluded that foundation embedment is beneficial for squat structures while it may increase ductility demands in slender buildings.

4- Comparing the results with and without inclusion of KI effect reveals that the rocking input motion due to KI plays the main role in this phenomenon, because the rocking component produces larger acceleration input on the mass of super-structure and leads to more severe structural response. Consequently, the rocking component of FIM, which is not allowed by common commercial softwares and usually ignored in conventional SSI analysis of buildings, may play an important role, especially for tall and slender structures.
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Figure 3. Normalized inelastic strength demand spectra for site-class D. (µ=2)

Figure 4. Normalized inelastic strength demand spectra for site-class D. (µ=4)
Figure 5. Strength reduction factor spectra for site-class E. ($\mu=2$)

Figure 6. Strength reduction factor spectra for site-class E. ($\mu=4$)
Figure 7. Averaged ductility demand of soil-structure systems located on site-class D. ($\mu_{\text{fixed-base}}=2$)

Figure 8. Averaged ductility demand of soil-structure systems located on site-class D. ($\mu_{\text{fixed-base}}=4$)
**Figure 9.** Effect of embedment ratio on ductility demand of soil-structure systems located on site-class E. ($\mu_{\text{fixed-base}}=2$)

**Figure 10.** Effect of embedment ratio on ductility demand of soil-structure systems located on site-class E. ($\mu_{\text{fixed-base}}=4$)
Figure 11. Averaged ductility demand of different soil-structure systems located on site-class D with and without KI effect. (µfixed-base=4)

Figure 12. Averaged ductility demand of different soil-structure systems located on site-class E with and without KI effect. (µfixed-base=4)