Mapping $\sigma_p$ into $\sigma_{fus}$ for the $^8B^{+58}Ni$ system

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Abstract. A fusion excitation function for the $^8B^{+58}Ni$ proton-halo system at near Coulomb barrier energies was recently published (Aguilera E. F et al. 2011 Phys. Rev. Lett. 107 092701). The respective fusion cross sections ($\sigma_{fus}$) were determined from the integrated cross sections for the evaporated protons ($\sigma_p$), by using proton multiplicities ($M_p$) calculated with the code PACE. In this code, the transmission coefficients $T_i$ are calculated only for the compound-nucleus values of $A$ and $Z$ and an extrapolation is then made for subsequent decays. In the present work, the reliability of the whole procedure is further investigated both, by studying the effect of variations in the input parameters to PACE and by computing the $M_p$ values with the alternate code LILITA. An explicit calculation of all necessary transmission coefficients is made in the latter code.

1. Introduction

The radioactive nucleus $^8B$ is adjacent to the proton drip line, it is a short-lived nucleus ($T_{1/2} = 0.8$ s) and has a very small proton separation energy of only 0.138 MeV. This radioactive nucleus plays a role in the production of high-energy neutrinos in the sun [1]. In addition, due to the work done in Refs. [2–4], it has attracted much attention in the past decade. The experimental angular and energy distributions of $^7Be$ after $^8B$ breakup on a $^{58}Ni$ target [2, 3], and the large reaction cross sections observed for the same system [4], provide evidence about the proton halo character of $^8B$. In this matter, recently the fusion excitation function for the $^8B^{+58}Ni$ system at near Coulomb barrier energies was reported [5], showing an unusual enhancement most probably related also to proton-halo effects.

Much experimental and theoretical effort has been dedicated to investigate nuclear reactions of exotic neutron-halo nuclei [6–8]. The situation is quite different for the proton-halo nuclei since there is not so much information on them. For neutron-halo systems there is a suppression of the fusion cross section, $\sigma_{fus}$, above the Coulomb barrier. In contrast, the data of Ref. [5] show an enhancement of $\sigma_{fus}$ below and above the barrier, specifically for the $^8B^{+58}Ni$ system.

The respective values of $\sigma_{fus}$ in Ref. [5] were determined from the integrated cross sections for the evaporated protons ($\sigma_p$), by using proton multiplicities ($M_p$) calculated with the code PACE [9]. In the present work, the reliability of the whole procedure used to determine $\sigma_{fus}$, is further investigated both, by studying the effect of variations in the input parameters to PACE [9] and by computing the $M_p$ values with the alternate code LILITA [10, 11].
2. Antecedents

Since the present work relies on the results given in Ref. [5], in this section an insight of the most important details, regarding the experimental arrangement and other considerations related with the determination of \( \sigma_{\text{fus}} \), will be given.

The \(^8\text{B}\) beams were generated using the \(^3\text{He}(^7\text{Li},^8\text{B})\) reaction at the TwinSol facility [12] at the University of Notre Dame during four different stages. Primary \(^4\text{Li}^{3+}\) beams with energies between 31 and 38 MeV produced \(^8\text{B}\) beams with incident (lab) energies between 22.4 and 30.0 MeV. Primary beam currents for the different stages varied from 50 to 220 particle-nanoamperes (pnA), yielding secondary beam rates of 0.5 - 2.7 \times 10^4 \text{s}^{-1}. In order to detect protons evaporated from the fused system \((^8\text{B}+^{58}\text{Ni})\), several \(\Delta E - E\) silicon surface-barrier telescopes were placed at backward angles that ranged from 112.5° up to 157.5°. At forward angles two additional detectors were used to monitor the beam.

The secondary beam from TwinSol is generally contaminated by unwanted ions having the same magnetic rigidity. A bunched beam was used in order to identify the reaction products of interest, and to reject direct protons, by time of flight. Because of their partial time overlap with the \(^8\text{B}\) beam, the most relevant contaminants and typical yields (relative to \(^8\text{B}\)) were \(^3\text{He}^{2+}\) (~45%) and \(^7\text{Li}^{3+}\) (~45%). In order to determine the contributions of the \(^7\text{Li}\) and \(^3\text{He}\) contaminant beams to the proton yields, additional experiments were performed to measure protons produced by these two projectiles on a \(^{58}\text{Ni}\) target. It turned out that the \(^7\text{Li}\) component was not a concern because the \(^7\text{Li}\) beam energies were always well below the Coulomb barrier. This, combined with the fact that proton multiplicities for \(^7\text{Li}+^{58}\text{Ni}\) are about a factor of 2 lower that those for \(^8\text{B}+^{58}\text{Ni}\), leads to a negligible effect (~1%) on the proton yields. On the other hand, the involved \(^3\text{He}\) energies were above the respective Coulomb barrier, with important fusion cross section values [13]. A contribution of up to 10% coming from the \(^3\text{He}\) beam component was corrected for in a few cases. A typical proton spectrum is shown in Fig. 1. It can be seen that the experimental points closely follow the predictions of the evaporation model, computed with the code FACE [9].

As mentioned above, the experiment was performed in four stages. In the first stage a 1.36 mg/cm\(^2\) natural Ni target was used and the bombardment was done at \(E_{\text{lab}} = 27.5\) and 30.0 MeV. It is worth mentioning that in the present paper we follow the notation defined in Ref. [5] for the energies. In other words, \(E_{\text{lab}}\) refers to the lab energy of the projectile before entering the target while \(E_{\text{fus}}\) indicates the effective energy for fusion in the center of mass reference frame. As mentioned below, \(E_{\text{fus}}\) includes corrections for energy-loss in the target, properly weighted by the respective fusion cross section. For the second stage, the target was enriched \(^{58}\text{Ni}\) (0.924 mg/cm\(^2\)) and the measurements were done at \(E_{\text{lab}} = 25.8, 27.6\) and 29.1 MeV. In the third stage two targets were used. The first one was an enriched \(^{58}\text{Ni}\) (0.725 mg/cm\(^2\)) with measurements at \(E_{\text{lab}} = 24.6\) and 26.9 MeV. The second target, used for the measurement at \(E_{\text{lab}} = 22.4\) MeV, was a natural Ni foil (5.6 mg/cm\(^2\)). Finally, a natural Ni target (2.22 mg/cm\(^2\)) was also used in the fourth stage. The measurements were done at \(E_{\text{lab}} = 23.9, 26.5\) and 28.6 MeV. Background determinations were performed during stages 3 and 4 by using pairs of identical target frames. One of them had no target and was used for blank-target measurements right after the respective runs with a target. No modifications of the setup or the beam conditions were made. In addition to providing the proper corrections for the data taken in stages 3 and 4, these blank-target data showed a smooth energy dependence that was used to estimate a background correction for the measurements of stages 1 and 2.

The evaporation protons were measured at the backward angles, so that it was possible to determine the proton cross section for each angle. Since the respective angular distribution is flat, an integration over the whole solid angle can be performed to obtain the total proton cross section, \(\sigma_p\). The procedure is illustrated in Fig. 2, where the differential proton cross sections determined for each detector angle are shown for the bombardment at \(E_{\text{lab}} = 30.0\) MeV. The solid
curve, calculated with PACE, was used to obtain the integrated cross section. Finally, a mapping was done to get the fusion cross section, $\sigma_{\text{fus}}$, by using the proton multiplicities calculated with the code PACE [9]. The fusion cross sections reported are based on the assumption that complete fusion (CF) is the only mechanism responsible for the observed proton yields. One concern was the possibility of having incomplete fusion (ICF), where the residual $^7\text{Be}$ after projectile breakup fuses with the target. For a given energy $E$, the effect of this process on the extracted cross sections depends on the factor $f = \text{CF}/(\text{CF+ICF})$ and on the ratio of the corresponding proton multiplicities $r(E) = M_p(^7\text{Be})/M_p(^8\text{B})$. If $r(E) = 1$ for all $E$, the reported values of $\sigma_{\text{fus}}$ would correspond to total fusion (TF=CF+ICF). Estimations based on PACE [9] calculations indicate that $r(E)$ is actually an increasing function of $E$ such that, for the experimental energy range, $0.72 \leq r(E) \leq 0.85$. As a result, the effect of ICF is more important at lower energies. A value $f = 0.7$ would have a negligible effect on $\sigma_{\text{fus}}$ for all values of $E$ as long as the fusion cross section is associated with TF. Even a factor as small as $f = 0.5$ should leave the values of $\sigma_{\text{fus}}$ within the reported error bars for all energies except the lowest one, where $\sigma_{\text{fus}}$ would increase by 20% (the respective error bar is 14%). Similar results are obtained for $^4\text{He}$ or $^3\text{He}$ capture. The correction factors are somewhat larger in these cases, but it is worth noting that the already large cross sections will always be increased if there is substantial ICF since the proton multiplicities for ICF are always smaller than those for CF. Note also that the measured total reaction cross section places an upper bound on $\sigma_{\text{fus}}$. Therefore, while we cannot rule out the ICF possibility, this will not substantially change the reported values of $\sigma_{\text{fus}}$ as long as they are taken to refer to total fusion.

![Figure 1](image1.png)  
**Figure 1.** Typical proton spectrum ($E_{\text{lab}} = 22.4$ MeV) compared with a PACE [9] calculation. Data taken from Ref. [5].

![Figure 2](image2.png)  
**Figure 2.** Differential proton cross section measured (symbol) and corresponding PACE [9] calculation (line) for $E_{\text{lab}} = 30.0$ MeV.

The experimental fusion excitation function, reported in Ref. [5], is represented by the filled circles in Fig. 3. The reported energies were determined iteratively and include a correction for mean energy loss in the target obtained by using successive Wong-model [14] fits to the data as weighting functions. Appropriate corrections were introduced to account for the respective isotopic composition when natural Ni targets were used. The repeat point at $E_{\text{fus}} = 22.1$ MeV, which was taken first using a $^{58}\text{Ni}$ target and later with a natural Ni target, validates the correction procedure. Two points were averaged to get the $E_{\text{fus}} = 23.3$ MeV point, leaving ten points in the graph. Also, in Fig. 3 the predictions of a barrier penetration model using
the bare potential (solid curve), given by the São Paulo Potential (SPP) [15], are shown. With respect to these predictions, it can be seen that there is a strong enhancement of the fusion cross section below and above the respective Coulomb barrier for this proton-halo system. This behavior, which is different from the usual sub-barrier enhancement observed for many heavy-ion systems [7, 16–19], is possibly associated with possible proton-halo effects [5, 19], but an appropriate theoretical explanation is still due.

Figure 3. Fusion excitation function obtained in Ref. [5]. The dashed curve helps to guide the eye. The solid curve represents a one-dimensional BPM calculation using the barrier parameters obtained from the São Paulo Potential (SPP) [15].

Figure 4. Values of the level density parameter $a$, as obtained from an analysis of 265 nuclei. The straight line corresponds to $a = A/9$ and approximates the average trend. Figure taken from Ref. [20].

Summarizing, in this section the most important facts regarding the determination of $\sigma_{fus}$ were given. While $\sigma_p$ is an experimental quantity which is essentially model-independent, one might wonder about a possible model-dependency introduced by the mapping $\sigma_p \rightarrow \sigma_{fus}$. In the following section the reliability of the respective procedure is investigated both, by studying the effect of variations in the input parameters to PACE [9] and by computing the $M_p$ values with the alternate code LILITA [10, 11].

3. Level density effects and PACE vs LILITA

In general, both PACE and LILITA are Monte Carlo codes. The most important difference between the two codes lies in the way the transmission coefficients ($T_l$) are obtained. In PACE, the transmission coefficients are calculated for the compound-nucleus values of $A$ and $Z$, and an extrapolation is made for subsequent decays [9]. On the other hand, in the present version of LILITA, an explicit calculation of all necessary transmission coefficients is made [11].

In Ref. [5], default values were used for most input parameters in PACE [9]. More specifically, the Yrast line was always determined by the liquid drop rotational energy, the A. J. Sierk fission barrier was assumed throughout the calculations, the level density parameter was $a = A/7.5$ (which is the value suggested by the author [9]), and the regular Wapstra mass table supplied with the code was used for all involved nuclei. In addition, the experimental fusion cross sections were used as an input (in an iterative way), which means that the code internally shifts the respective optical model transmission coefficients to reproduce these values.
The level density is one of the crucial ingredients of any statistical model calculation. This is given to the code through the level density parameter $a$. In Fig. 4, values of this level density parameter obtained from the analysis of 265 nuclei are shown [20]. The rapid decrease of $a$ near shell closures is evident.

![Figure 5](image1.png)  
**Figure 5.** A zoom of the previous figure in the region $A<90$. The straight line corresponds to $a=A/7.5$ and approximates the average trend in the region $70<A<90$.

![Figure 6](image2.png)  
**Figure 6.** The level density parameter $a$ plotted as a function of atomic mass number for $60<A<70$. The straight line with $a=A/8.6$ approximates the average trend in this region, with upper (lower) limit given by $a=A/7.9$ ($a=A/9.5$), respectively.

In Fig. 5, the curve of the level density parameter given by $a=A/7.5$, corresponding to the default value in PACE [9], is plotted. This curve approximates the average trend of nuclei with atomic mass number $A$ between 70 and 90. However, the atomic mass number of the compound nucleus corresponding to the $^8\text{B}+^{58}\text{Ni}$ system is $A=66$ ($^{66}\text{As}$), which is below the group of nuclei described by the curve $a=A/7.5$. Taking this into account, the first step was to investigate the effect on the predictions of $M_p$, given by the code PACE, by changing $a=A/7.5$ to $a=A/8.6$. The curve corresponding to $a=A/8.6$, plotted in Fig. 6, approximates the average trend for nuclei with $A$ between 60 and 70, which is the region where the mass number of the compound nucleus and its residual nuclei after evaporation would lie. The second step was to determine the predictions of $M_p$, given by the code PACE, by varying $a$ within extreme limits. The limits were determined for $60<A<70$, with an upper limit given by $a=A/7.9$ and lower limit given by $a=A/9.5$, as shown by the dashed lines in Fig. 6. As the final step, the predictions of $M_p$ with $a=A/8.6$ were determined with the code LILITA [11]. The input parameters for LILITA were chosen as to have a meaningful comparison with the respective PACE calculations.

4. Results and discussion
The predictions of $M_p$ for each energy, given by the code PACE [9] by replacing $a=A/7.5$ with $A/8.6$, are listed in Table 1. The maximum shift in $M_p$ occurs at the highest energy. Based on these results, an estimated systematic uncertainty of $\sim5\%$ must be assigned to the $\sigma_{\text{fus}}$ values reported in Ref. [5] because of this effect.

In Fig. 7, the fusion cross sections obtained by applying the above correction to the level density parameter $a$, are compared to the previous values. The circles are the experimental
Table 1. Shifts in $M_p$ predicted by PACE [9] when changing the level density parameter $a$ as discussed in the text.

| $E_{c.m.}$ (MeV) | $M_p$ A/7.5 | $M_p$ A/8.6 | $\Delta M_p$ (%) A/7.5 vs A/8.6 |
|------------------|-------------|-------------|---------------------------------|
| 18.9             | 2.41        | 2.30        | 4.2                             |
| 20.1             | 2.42        | 2.32        | 4.1                             |
| 21.1             | 2.43        | 2.33        | 4.0                             |
| 22.1             | 2.42        | 2.32        | 4.1                             |
| 22.1             | 2.42        | 2.32        | 4.1                             |
| 23.2             | 2.44        | 2.33        | 4.4                             |
| 23.4             | 2.44        | 2.33        | 4.4                             |
| 23.7             | 2.43        | 2.32        | 4.5                             |
| 23.8             | 2.44        | 2.32        | 4.6                             |
| 25.0             | 2.44        | 2.32        | 4.8                             |
| 25.6             | 2.43        | 2.31        | 5.0                             |

values of $\sigma_{fus}$ reported in Ref. [5] while the open squares are the respective values obtained for $a = A/8.6$. It can be seen that after considering this correction, $\sigma_{fus}$ would still be well within the uncertainty limits previously reported.

On the other hand, when comparing the predictions of $M_p$ (with code PACE [9]) for $a = A/8.6$ versus the ones obtained for the lower limit ($a = A/9.5$), a maximum systematic error of $\sim 4\%$ results. Similarly, by comparing the $a = A/8.6$ case versus the results obtained for the upper limit ($a = A/7.9$), a maximum systematic error of $\sim 3\%$ is obtained. By taking these and the
previous results into account, properly convoluting the respective uncertainties, we conclude that the sensitivity of $\sigma_{fus}$ to changes in the level density parameter $a$ within extreme limits, can be accounted for by a total systematic uncertainty of $\sim 6\%$.

### Table 2.

Shifts in $M_p$ obtained by using the codes PACE [9] and LILITA [11] (for $a = A/8.6$).

| $E_{c.m.}$ (MeV) | $M_p$ PACE | $M_p$ LILITA | $\Delta M_p(\%)$ |
|------------------|------------|--------------|------------------|
| 18.9             | 2.30       | 2.21         | 4.0              |
| 20.1             | 2.32       | 2.22         | 4.2              |
| 21.1             | 2.33       | 2.25         | 3.3              |
| 22.1             | 2.32       | 2.26         | 2.9              |
| 22.1             | 2.32       | 2.26         | 2.7              |
| 23.2             | 2.33       | 2.35         | 0.6              |
| 23.4             | 2.33       | 2.36         | 1.4              |
| 23.7             | 2.32       | 2.33         | 0.5              |
| 23.8             | 2.32       | 2.31         | 0.5              |
| 25.0             | 2.32       | 2.40         | 4.0              |
| 25.6             | 2.31       | 2.40         | 4.0              |

In Table 2, the predictions of $M_p$ with $a = A/8.6$ by using the codes PACE [9] and LILITA [11], are compared. A maximum shift in $M_p$ of $\sim 4\%$ is observed. In Fig. 8, the diamonds are the respective $\sigma_{fus}$ values obtained with LILITA. Once again, it is clear that after considering the $M_p$ results obtained with the code LILITA, the respective $\sigma_{fus}$ values would not show any important change. The effects of both, the sensitivity to changes in $a$ (within PACE) and using an alternate code (LILITA), can be included in a global systematic uncertainty of $\sim 7\%$.

### 5. Conclusions

The fusion cross sections ($\sigma_{fus}$) reported in Ref. [5] for the $^{8}\text{B}+^{58}\text{Ni}$ system, were reviewed to investigate possible model-dependency in the involved $\sigma_{p} \rightarrow \sigma_{fus}$ mapping. The key quantities in this mapping, the proton multiplicities ($M_p$), were previously calculated with the code PACE [9] by using default values for the respective input parameters. In the present work, the sensitivity of $M_p$ to changes both in the involved level densities and in the relevant transmission coefficients, was investigated.

The maximum shift in $M_p$, obtained with the code PACE by changing the level density parameter within extreme limits, can be accounted for by assigning a systematic uncertainty of $\sim 6\%$ to the $\sigma_{fus}$ values reported in Ref. [5].

On the other hand, a maximum shift of $\sim 4\%$ is observed in $M_p$ when comparing with the results of the code LILITA [11], which, in contrast to PACE, explicitly calculates the necessary transmission coefficients. When applying the mentioned $M_p$ shifts to the individual experimental points, the respective $\sigma_{fus}$ values do not show any important change (Fig. 8).

The effects of both, the sensitivity to changes in $a$ (within PACE) and using the alternate code (LILITA), can be included in a global systematic uncertainty of $\sim 7\%$. This would not change the conclusions discussed in Ref. [5]. The reliability of the whole procedure was thus confirmed.
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