Solitonic d-5 Brane and Vortex Defects on the World-Sheet

Kazuo Ghoroku

Department of Physics, Fukuoka Institute of Technology, Wajiro, Higashiku, Fukuoka 811-02, Japan

abstract

We examine the behavior of the vortex defects on the world-sheet when a solitonic d-5 brane is existing in the d-dimensional target space, and it is seen that the vortices of any charge should be dissociated to the free-gas phase when they approach to the brane. The meanings of this fact are addressed through the analysis of the renormalization group equations on the world sheet.
1 Introduction

The rapid developments in the recent string theories have made clear the importance of the (D)p-branes in understanding the non-perturbative aspect of string- and field-theories. So it is meaningful to examine the properties of p-branes from various point of views.

There is an approach to this subject from the world-sheet action of the string based on the principle of the conformal invariance. The world sheet action should be corrected when the branes are in the target space. As an example, the recoil effect of the D-brane coming from the scattering of the string and the D-brane has been presented on the world sheet action in terms of the appropriate operators [1, 2]. Another kind of effect of the D-brane on the world sheet has been proposed [3] in terms of the deformed sine-Gordon action. In this proposal, the sine-Gordon term is introduced as the representation of the appearance and disappearance of the virtual D-branes.

Here we examine the problem of the solitonic p-brane in a (noncritical) d-dimensional target-space through the world-sheet action, in which the sine-Gordon term is introduced as the representation of the vortex defects on the sheet. And this term is expected to be a kind of medium which provides the dynamical information of the background including the p-brane. As for the p-brane configuration, we consider the d-5 brane, which is an extended form of the 5-brane solution obtained in the 10d supergravity [7]. And this solution plays an important role in understanding the duality of the strings in ten-dimension.

The sine-Gordon term, which is obtained on the sphere, has been used in [3], but this term would receive a modification from the requirement of the conformal invariance [1, 3] of the world-sheet action in our approach. This modification reflects how the brane configuration in the target space and the sine-Gordon term on the world sheet are affected each other. Our purpose in this paper is to see what kind of the implications are obtained from the modified sine-Gordon term. This analysis would shed some light on the problem of the vacuum configuration of the target space.

Since we are considering in the noncritical target-space dimension, the world-sheet action contains the Liouville field or the conformal mode of two dimensional gravity quantized in the conformal gauge. In the previous works [3, 5], the sine-Gordon term is written by the Liouville field. However, there is no reason to take this assignment in our case. In fact, any bosonic field on the world-sheet is useful to represents the sine-Gordon potential, and it could lead to the the desirable vortex free-energy. Here the sine-Gordon field is not assigned to the Liouville field but
to some other scalar field on the world-sheet. The Liouville field is insteadly used here to represent a physical scale in the 2d curved surface [4] in order to derive the renormalization group equations of the parameters of the world-sheet action. When the sine-Gordon field is assigned to one of the coordinates transverse to the d-5 brane, we can show that the sine-Gordon potential becomes relevant near the brane even if how large the vortex charge is. Namely, the vortices are in the gaseous phase. In other words, any vortex-pair strongly bounded through a large vortex charge dissociates to the free gas due to the d-5 brane.

In the next section, the solitonic d-5 brane is derived from the low-energy effective target-space action. Then, vortices are introduced in §3 in terms of the sine-Gordon action. This is done such as the theory is conformally invariant. In §4, the renormalization group analysis of the parameters are studied, and the summary and discussions are given in the final section.

2 d-5 Solitonic Brane

In d=10, N=1 supergravity, 5-brane solitonic solution is given in [7]. Considering this solution as a prototype, we extend it to the case of the non-critical dimension, $d \neq 10$.

Our starting point is the following low energy effective action [10, 11] of d-dimension,

$$S_t = \frac{1}{4\pi} \int d^dX \sqrt{G} e^{-2\Phi} [R - 4(\nabla\Phi)^2 + (\nabla T)^2 + v(T) + \frac{1}{2 \times 4!} H_3^2 - \kappa],$$

(1)

where $H_3$ is the field strength of the 2-form potential $(B_{MN})$ given in (11) and

$$v(T) = -2T^2 + \frac{1}{6}T^3 + \cdots.$$  

(2)

The parameter $\kappa$ is remained finite since noncritical dimension $d$ is considered. For bosonic case, it is written as, $\kappa = (25 - c)/3$, where $c$ represents the central charge of the theory. Other differences of this action from the supersymmetric one in [7] are (i) the absence of the Yang-Mills field and (ii) the presence of the tachyon field. The first point is not important since we are considering a simple solution, but the tachyon term is necessary here since this part is assigned to the sine-Gordon potential in the world sheet action, which is introduced in the next section.

The solution of (1) is obtained by solving the following equations of motion,

$$\nabla^2 T - 2\nabla\Phi \nabla T = \frac{1}{2} v'(T),$$  

(3)

$$\nabla^2 \Phi - 2(\nabla\Phi)^2 = -\frac{\kappa}{2} + \frac{1}{2} v(T) - \frac{1}{6} H_3^2,$$  

(4)

3
\[ R_{MN} - 2\nabla_M \nabla_N \Phi = -\nabla_M T \nabla_N T - \frac{1}{4} H_{KL}^M H^{NKL}, \]  
(5)  
\[ \partial_M (\sqrt{G} e^{-2\Phi} H^{M NK}) = 0, \]  
(6)  

where the indices \( M, N, \cdots \) run over \( d \)-dimensions, \( 0 \sim d - 1 \). The \( d \)-dimensional coordinates are denoted as \( X^M = (x^\mu, y^m) \), where \( \mu = 0 \sim d - 5 \) and \( m = d - 4 \sim d - 1 \). Here, \( x^\mu \) denotes the coordinates of the world volume of \( d \)-5 brane, and \( y^m \) are the transverse ones to the brane. Further, the ”time” \( x^0 \) is identified with the Liouville field or the conformal mode (\( \rho \)) on the world sheet.

We solve the above equations in the Euclidean metric by assuming \( T = 0 \) and taking the following ansatz,

\[ ds^2 = \delta_{\mu\nu} dx^\mu dx^\nu + e^{2B(y)} \delta_{mn} dy^m dy^n, \]  
(7)  
\[ H^{mnl} = \frac{2 e^{mns} \partial_s \Phi(y)}{\sqrt{\det g_{ij}}}, \]  
(8)  

where \( y = \sqrt{\delta_{mn} y^m y^n} \), the distance from the brane. And \( g_{ij} \) represents the metric of the transverse four dimensional space. Tangential components of \( H_3 \), whose indices include at least one of the tangential directions denoted by \( \mu \), are zero.

Before solving our equations, we remind the supersymmetric 5-brane solution for the case of \( d = 10 \) (\( \kappa = 0 \)). It has been obtained [7] in the following form,

\[ \Phi(y) = B(y) = \Phi_p, \quad \exp(2\Phi_p) = e^{2\phi_0} + \frac{Q_D}{y^2}, \]  
(9)  

under the above ansatzs (7), (8). Here, \( \phi_0 \) is a constant and \( Q_D \) denotes the ”magnetic” charge, which is supported by \( H_3 \), of the 5-brane.

Here we must find a solution for \( d \neq 10 \) and \( \kappa \neq 0 \). For the case of \( \kappa = 0 \), the extended solution to the diverse dimension (\( d \neq 10 \)) has been given in [8], and the solution is written by one harmonic function similar to (9). However, in the case of \( \kappa \neq 0 \), the asymptotic form of the d-5 brane solution should take the linear dilaton vacuum. So we must modify at least the dilaton part in the solutions obtained in [7], [8]. The simplest solution is obtained by changing the configuration of the flat inner space \( x^\mu, \mu = 0 \sim d - 5 \), of the brane to the linear dilaton form. Then our solution is obtained in the following form,

\[ \Phi = \Phi_0 + \Phi_p(y), \quad \Phi_0 = \frac{1}{2} Q \rho, \quad B(y) = \Phi_p(y), \]  
(10)  

where \( \Phi_p(y) \) is the one given in (9), and \( Q = \sqrt{\kappa} \).

Here we give some comments on this solution. (i) It may be possible to consider a more general form for \( \Phi_0 \) in terms of the constants \( c_\mu \) as follows, \( \Phi_0 = \Sigma c_\mu x^\mu / 2 \) with
\( \Sigma \mu^2 = \kappa \). However, we do not take this general form here since we are considering a theory, in which the dilaton should have its asymptotic form of \((\mathbb{1})\). This is naturally derived from the quantization of the string theory at noncritical dimension. (ii) \(B(y)\) is different from \(\Phi\) by \(\Phi_0\). Then this solution would not satisfy the requirement of the supersymmetry, which demands that the solution must be written by one common function. So the d-5 brane given here would not keep a supersymmetry. (iii) Due to the linear dilaton part \(\Phi_0\), it seems impossible to obtain a ”fundamental” string soliton-solution, which can be obtained from the combined action of \(S_t\) (\(\mathbb{1}\)) and the two dimensional source action \((S_2)\) embedded in the d-dimension. For \(\kappa = 0\), this solution can be found even if \(d \neq 10\) \([8, 12]\).

3 Vortices and Monopoles

Next, we introduce the vortices on the world-sheet from the viewpoint of the path-integral formulation of the world-sheet action. The action, which is responsible for the target space action \((\mathbb{1})\), can be written in the form of the following non-linear sigma model,

\[
S = \frac{1}{4\pi} \int d^2z \sqrt{|\hat{g}|} \frac{1}{2}(G_{MN}(X) + B_{MN}(X))(\hat{g}^{\alpha\beta} + \frac{i}{\sqrt{\hat{g}}} \epsilon^{\alpha\beta}) \partial_\alpha X^M \partial_\beta X^N + \hat{R}\Phi(X) + T(X),
\]

where \(X^M\) are assigned as in the previous section, \(X^M = (x^\mu, y^m)\). The theory on the sheet is quantized in the conformal gauge, \(g_{\mu\nu} = e^{2\rho} \hat{g}_{\mu\nu}\), and the conformal mode \((\rho)\), which is aliving through the quantum measure, is assigned as the ”time”, \(x^0 = \rho\).

The vortex configuration on the world-sheet can be introduced as a topological defect \([13]\) through a scalar field, say \(X_v(z)\). The effective action of such vortices can be represented by the sine-Gordon potential, \(\cos(pX_v)\), where \(p\) denotes the vorticity or the vortex charge. Then the gas of vortices, whose total charge is zero, is expressed by the following lagrangian,

\[
\frac{1}{2}(\partial X_v)^2 + \lambda \cos(pX_v),
\]

for the flat surface. Here \(\lambda\) is a parameter for the vortex gas and it will be discussed in the following.

From the viewpoint of the conformal field theory, the same partition function with the one for the vortex gas can be derived \([13]\) in terms of the following potential,

\[
\lambda \cos(q[X(z) - X(\bar{z})]),
\]
where we notice that the usual scalar field has the plus combination, $X(z, \bar{z}) = X(z) + X(\bar{z})$. The corresponding configuration to the minus combination is called as the monopole [9] which is dual to the vortex in the sense that the vortex charge $p$ and the monopole charge $q$ satisfies the Dirac quantization condition, $pq = 2\pi n$, where $n$ is some integer. This monopole defect is also considered in [3] as the representation of the appearance and disappearance of virtual D-branes. We will briefly comment on this monopole below.

First, we consider the case of vortex which can be included in the above action (11) by making an assignment of $X_v$ to the one of $X^M$ in (11) and adding the sine-Gordon potential of (12) which is written by the assigned coordinate field. But it should be modified due to the quantization of the 2d surface, so our first task is to find the modified sine-Gordon potential. Since this newly added potential can be assigned to the tachyon part, $T(X)$ of (11), then its modified form can be obtained by solving the equation (3). This is corresponding to obtaining a conformal invariant action under the condition that the added sine-Gordon term is a perturbation to a conformal invariant vacuum, which is described here by the d-5 brane. According to the procedure of [4], we assume that the modification for the sine-Gordon potential can be expressed by the dressed factor, which is given as follows,

$$T_v = \lambda e^{\gamma \rho} \cos(pX), \quad (13)$$

where the original factor $e^{2\rho}$ is replaced by $e^{\gamma \rho}$. Then from the equation (3), we obtain the value of $\gamma$ by assuming $\lambda$ being small and linearizing the equation with respect to $T = T_v$.

In solving (3), we should notice the following points; (i) The assignments of $X_v$ to the one of $X^M$ are separated into the following two groups, (a) the one of $x^\mu$ ($\mu \neq 0$) (the inner space of the d-5 brane) and (b) the one of $y^m$ (the transverse space). Although we need a modification of the potential in both cases, the second case (b) is more interesting as shown below. (ii) Secondly, the d-5 brane background given above must be considered in writing explicitly (3).

The linearized form of (3) with respect to $T$ is written as

$$(G^{MN} \partial_M \partial_N - Q \partial_0 + 2)T = 0, \quad (14)$$

where we have used (7) and (10), and the term like $\partial B \partial T$ has cancelled out due to the characteristic form of the d-5 brane.

First, we consider the case of (a) where $X$ being assigned to $x^\mu$. Substituting (13) to (14), then we obtain

$$\gamma = \frac{1}{2}(Q - \sqrt{Q^2 + 4p^2} - 8), \quad (15)$$
which is the same one obtained in the linear dilaton vacuum previously [4] for the sine-Gordon model coupled to the 2d gravity. At this order of the approximation, the d-5 brane does not affect on $\gamma$. Then the Kosteritz-Thouless (KT) transition point, $p^2 = 2$ which is given in [4], is not changed.

Next, consider the second case (b), where $X$ is assigned to one of $y^m$. In this case, the coefficient of the differential operator depends on $y$, so we solve the equation by assuming the constancy of $y$. Namely, we consider the problem on a special surface in the target d-dimensional space, where the distance ($y$) from the d-5 brane is fixed. Then we obtain the following solution,

$$
\gamma = \frac{1}{2} (Q - \sqrt{Q^2 + 4\tilde{p}^2 - 8}),
$$

where

$$
\tilde{p}^2 = e^{-2\Phi_p(y)} p^2,
$$

and $\Phi_p(y)$ is given in (9). In this case, the critical vortex-charge varies with $y$, and it is given by

$$
p^2_{cr} = 2e^{2\Phi_p(y)} = 2(1 + \frac{Q_D}{y^2}),
$$

which approaches to infinity when $y$ goes to zero, just on the d-5 brane. This fact implies that all of the vortices of any charge are in the plasma phase near the d-5 brane. Namely, the vortex and anti-vortex pair dissociate to the free gas even if how large the vortex charge is.

Nextly, we consider the case of monopoles. Since the monopole charge ($q$) and the vortex charge ($p$) should satisfy the Dirac condition, $pq = 2\pi n$ where $n$ is the integer, the monopole is expected to be in dipole phase if the vortex is in the gaseous phase. This expectation is true for the flat background. We examine this point in the case of the presence of the d-5 brane in the background. Then, we solve (14) by replacing $T$ by

$$
T_m = X' e^{\gamma \rho} \cos(p[X(z) - X(\bar{z})]).
$$

In this case, we must define the operation of $\partial^2$ on $T_m$ in (13). Since $\partial^2$ is the laplacian in the flat space, it might be replaced by the Virasoro operator, $L_0 + \bar{L}_0$, defined for a free scalar field $X$. According to the usual operator formalism,

$$
X(z) = \frac{x}{2} - ik \ln z + i \sum_{m \neq 0} \frac{1}{m} a_m z^{-m},
$$

$$
X(\bar{z}) = \frac{x}{2} - ik \ln \bar{z} + i \sum_{m \neq 0} \frac{1}{m} a_m \bar{z}^{-m},
$$

$$
L_0 = \bar{L}_0 = \sum_{m=1}^{\infty} a_m a_{-m} ;
$$
where $x$ is the center of mass of $X$, and $k$ can be taken as zero except for the Liouville field. Since we do not assign $X$ to the Liouville field here, $k$ is taken zero. Then we can solve (14), and we obtain $\gamma'$ in the same form with (15) and (16) for the two kinds of assignments respectively. This result implies that the dual phase relation of monopole and the vortex is broken near the brane since both critical charges become infinite near the d-5 brane and both defects would be in the plasma phase near the d-5 brane. Then we can say that the d-5 brane dissociate all the topological defects, which are in a dipole phase, into the free-gas phase.

4 Renormalization group Analysis

In order to understand the above result more deeply, we examine here the behaviors of the parameters, $\lambda$ and $p$, of the vortex through the renormalization group analysis. The renormalization group equations of these parameters are obtained according to the method in [4, 5]. It is as follows. First, the effective action on the world sheet is separated into the conformally exact part and a small perturbation which is characterized by a small parameter (here $\lambda$). To restore the conformal invariance, which is broken by the small perturbation, the first exact solution must be modified order by order. Namely the new exact solution can be expanded in the power series of $\lambda$. After getting the effective action at some order of $\lambda$, the renormalization group equations are obtained by shifting the conformal field, $\rho$, by a constant in the world sheet action and absorbing this shift in the parameters. For the first order approximation, the effective action can be obtained by solving the equations (3) $\sim$ (6).

Strictly speaking, the configuration of d-5 brane given here is not an exact one since the starting action (1) is an approximate target space action where higher derivative terms and the massive modes are abbreviated. As an exact solution, we consider here the linear dilaton vacuum, $G_{MN} = \delta_{MN}, \Phi = \Phi_0$ and others are zero. So the d-5 brane solution is expanded around this vacuum configuration in powers of $Q_D/y^2$, which is assumed small, and further add the sine-Gordon term as a small perturbation. Therefore, there are two small parameters in this case, $\lambda$ and $Q_D/y^2$. For the sake of the brevity, we consider the case of $Q_D/y^2 \sim \lambda^2$ hereafter.

First, we consider the case of the assignment, $X_v = x^1$. Far away from the brane, where the relation $Q_D/y^2 \sim \lambda^2$ is satisfied, we can expand $G_{\mu\nu}, \Phi, H$ and $T$ as follows,

$$
G_{MN} = \delta_{MN} + \lambda^2 h_{MN} + \frac{Q_D}{y^2} \delta^m_M \delta^n_N + \cdots.
$$

\[23\]
\[ \Phi = \Phi^{(0)} + \lambda^2 \Phi^{(2)} + \frac{Q_D}{2y^2} + \cdots, \quad (24) \]

\[ T = \lambda(T^{(0)} + \lambda T^{(1)} + \cdots), \quad (25) \]

\[ H^{mn} = -\epsilon^{mnst} y^s Q_D + \cdots, \quad (26) \]

where \( \cdots \) denotes the higher order terms. Since \( H \) is the order of \( 1/y^3 \), this field can be neglected hereafter in solving the equations. Since \( T^2 \) is the lowest power in the eqs. (4) and (5), it follows that the lowest order corrections to \( G_{\mu \nu} \) and \( \Phi \) are of the order \( O(\lambda^2) \). Then, we make an anzatz,

\[ h_{MN} = \delta^1_M \delta^1_N h(\rho), \quad (27) \]

as in [4]. The reason of this setting is that the lowest order correction should appear as the renormalization of the field \( x^1 \) because of its self-interaction through the sine-Gordon potential. Then, the equations of \( O(\lambda^2) \) are obtained as follows,

\[ (\partial^2 - 2Q \partial_\rho) \Phi^{(2)} + \frac{Q}{4} \partial_\rho h = -T^2_0, \quad (28) \]

\[ 2\partial^2_\rho \Phi^{(2)} - \frac{1}{2} \partial^2_\rho h = (\partial_\rho T^0_0)^2, \quad (29) \]

\[ 2\partial_\rho \partial_\phi \Phi^{(2)} = \partial_\rho T^0_0 \partial_\phi T^0_0, \quad (30) \]

\[ \frac{1}{2} (\partial^2_\rho - Q \partial_\rho) h - 2\partial^2_\phi \Phi^{(2)} = -(\partial_\phi T^0_0)^2, \quad (31) \]

\[ \partial_\rho \partial_\rho \Phi^{(2)} = \partial_\phi \partial_\phi \Phi^{(2)} = \partial_\rho \partial_\phi \Phi^{(2)} = 0, \quad (32) \]

\[ (\partial^2 - Q \partial_\rho + 2)T^1_1 = \frac{1}{4} T^2_0, \quad (33) \]

where the higher order terms like \( 1/y^3 \) are neglected. As a result, these equations are equivalent to the one obtained under the assumption of \( Q_D/y^2 << \lambda^2 \) since the terms depending on \( Q_D \) cancel or are neglected as the higher order terms.

Rescaling \( T \) by factor four, these equations are solved near \( \gamma \sim 0 \), and we obtain

\[ \Phi^{(2)} = \frac{1}{256} \cos(2p\phi) + \frac{1}{32Q} \rho + O(\gamma), \quad (34) \]

\[ h = \frac{p^2}{16Q} \rho + O(\gamma), \quad (35) \]

\[ T^1_1 = \frac{1}{32} (1 - \cos 2p\phi) + O(\gamma). \quad (36) \]

As a result, the effective action is obtained up to \( O(\lambda^2) \) near \( \gamma = 0 \) where \( p = \sqrt{2} \). Then, the renormalization group equations are obtained as mentioned above through a shift \( \rho \rightarrow \rho + 2dl/\alpha \). Here, \( dl \) denotes a small scale of the length and \( \alpha \) represents
the dressed factor for the cosmological constant. And, \( \alpha \) is determined by solving equation (14) with \( T = \mu^2 \exp(\alpha \rho) \). Denoting as \( p = \sqrt{2} + \epsilon \), we obtain
\[
\dot{\lambda} = \frac{4\sqrt{2}}{\alpha Q} \epsilon \lambda, \quad \dot{\epsilon} = \frac{\sqrt{2} p^2}{16 \alpha Q} \lambda^2, \tag{37}
\]
where \( \dot{a} \) means the derivative, \( \dot{a} = -da/d \ln l \). From these results, we can see that the sine-Gordon term is irrelevant for \( \epsilon > 0 \) where the vortex is in dipole phase. On the other hand, it becomes relevant in the gaseous phase where \( \epsilon < 0 \). And \( \lambda \) becomes large in the infrared limit and the coordinate of the target space assigned to \( X_v \) would be fixed at some value of the periodic copies.

For the case of the assignment, \( X_v = y^m \) (\( m = d - 4 \)), the ansatz (27) for the perturbation of the metric is replaced by,
\[
h_{MN} = \delta_M^{d-4} \delta_N^{d-4} h(\rho), \tag{38}
\]
and other fields are expanded in the same way with the former case. The resultant equations of motion and the solutions are equivalent to the above case except for the fact that the role of the suffix 1 is replaced by \( d - 4 \) in this case, and we obtain the same renormalization group equations of \( \lambda \) and \( \epsilon \) with the ones given in (37).

As mentioned in the previous section, the vortices of any charge, which is large enough to bind them to the dipole pairs, are in the gaseous phase (\( \epsilon < 0 \)) near the brane when we assigned as \( X_v = y^m \). As a result, the sine-Gordon term becomes relevant there in this case, and the coordinate \( y^m \) might be fixed at some value. This phenomenon seems to imply that the effective central charge of the noncritical string would be suppressed by the d-5 brane, since one of \( y^m \) does not contribute to the central charge. In this sense, the brane could open a new possibility of the noncritical string theory in the higher dimension (\( d > 2 \)).

Another aspect of this result is a possibility to interpret this phenomenon as a signal of a dynamical compactification since one of \( y^i \) could be restricted to a finite region due to the sine-Gordon potential. If this idea is possible, it is interesting to consider an extended form of the sine-Gordon potential such as
\[
T_v = \lambda e^{\gamma \rho} \sum_{i=d-4}^{d-1} \cos(p_i y^i), \tag{39}
\]
and to see in what circumstances this potential becomes relevant or not. It is easy to perform the same analysis given above with this potential, but we cannot obtain a simple result of the renormalization group equations since new counter terms are necessary even in the first order of the approximation in this case. Then, it seems to be necessary to extend the approximation of the analysis to more higher orders in order to arrive at a meaningful result. This is out of our scope here.
5 Summary and Discussions

The solitonic d-5 brane is considered in the d-dimensional target space as an extension of the 5-brane in the 10d supergravity. From the standpoint of noncritical string, the influence of the d-5 brane on the world-sheet action is examined through the renormalization group analysis on the world-sheet, where the sine-Gordon term is introduced to represent the vortex defects from the path-integral formalism.

Our result implies that the allowed region of one of the coordinate $y^i$ (the transverse direction of the d-5 brane) might be restricted to the region around the minimum of the sine-Gordon potential when $y^i$ is assigned to $X_v$ (the vortex field). This means that at least one scalar field on the world-sheet becomes effectively a constant and one central charge vanishes as a result. Then the d-5 branes might be considered as a key ingredient in obtaining a new noncritical string.

On the other hand, this phenomenon might be related to the dimensional compactification of the direction represented by $y^i$. If this idea is meaningful, it is interesting to extend the form of the sine-Gordon potential such that it includes many coordinates. Analysis in this direction will be given in the near future.

Finally we comment on the possibility of the sine-Gordon term as a realistic, condensed form of the tachyon part of a noncritical string as suggested from a different origin [14], where the condensed tachyon has a different form. In this case, the string fields propagate in the potential which is periodic in some direction. Then the spectrum of the field shows a band structure as we can see in the case of an electron within a crystal. It is interesting if we can observe such a evidence in some field’s spectrum in our macroscopic world.

Acknowledgment: The author thanks to the members of the high-energy group of Kyushu University for useful discussions and comments.
References

[1] W. Fishler, S. Paban and M. Rozali, Phys. Lett. B381(1996)62.

[2] I. I. Kogan, N. E. Mavromatos and J. F. Wheater, Phys. Lett. B387(1996)483.

[3] J. Ellis, N. E. Mavromatos and D. V. Nanopoulos, hep-th/9704169; hep-th/9706125.

[4] J. Ambjorn and K. Ghoroku, Int. J. Mod. Phys. A32(1994)5689.

[5] C. Schmidhuber, Nucl. Phys. B404(1993)1833.

[6] B. Ovrut and S. Thomas, Phys. Lett. B257(1991)292.

[7] C. G. Callan, Jr and J. A. Harvey and A. Strominger, Nucl. Phys. B359(1991)611; B359(1991)611, A. Strominger, Nucl. Phys. B343(1990)167.

[8] M. J. Duff and J. X. Lu, Nucl. Phys. B416(1994)301; M. J. Duff, R. R. Khuri and J. X. Lu, Phys. Rep. 259(1995)213.

[9] B. Ovrut and S. Thomas, Phys. Rev. D43(1991)1314.

[10] A. Cooper, L. Susskind and L. Thorlacius, Nucl. Phys. B363 (1991) 132.

[11] S. R. Das and B. Sathiapalan, Phys. Rev. Lett. 56 (1986) 2664; C. Itoi and Y. Watabiki, Phys. Lett. B198 (1987) 486; A. A. Tseytlin, Phys. Lett. B264 (1991) 311.

[12] G. T. Horowitz and A. A. Tseytlin, Phys. Rev. D50(1994)5204.

[13] J. Zinn-Justin, “Quantum field theory and critical phenomena” Oxford Science Pub. 1989.

[14] K. Ghoroku, hep-th/9612201, FIT-HE-96-82, to be published in Classical and Quantum Gravity.