Origin of supermassive black holes

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Abstract

The origin of supermassive black holes in the galactic nuclei is quite uncertain in spite of extensive set of observational data. We review the known scenarios of galactic and cosmological formation of supermassive black holes. The common drawback of galactic scenarios is a lack of time and shortage of matter supply for building the supermassive black holes in all galaxies by means of accretion and merging. The cosmological scenarios are only fragmentarily developed but propose and pretend to an universal formation mechanism for supermassive black holes.

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1 Introduction

The physics of Black Holes (BHs) is the most developed branch of general relativity. There were elaborated numerous mechanisms for the formation of BH of different mass-scales and proposed a list of convincing observational signatures in favor of BH existence. A possible range of BH masses in the Universe is thought to be extremely wide: from the Plank mass, \( M_{\text{Pl}} \sim 10^{-5} \, \text{g} \), up to the huge mass of the most luminous quasars, \( M_{\text{BH}} \sim 10^{10} \, M_\odot \). Conventionally we will distinguish the stellar-mass BHs with mass in the range \( \sim 3-100 \, M_\odot \), the Intermediate Mass Black Holes (IMBHs) with mass in the range \( \sim 10^3-10^5 \, M_\odot \) and the Supermassive Black Holes (SBHs) with mass \( \geq 10^6 \, M_\odot \). The stellar-mass BHs are rather definitely originated from the collapse of exhausted massive stars during supernova explosions. At the same time the origin of other types of BHs nowadays is quite uncertain.

Modern astrophysics provides strong indications of the existence of BHs as among the remnants of evolved stars and in the centers of spiral and elliptic galaxies. Meanwhile to declare the real discovery of BHs we need the proof that the suspicious candidates have a distinctive BH feature — an event horizon.

In contrast to the evident origin of stellar mass BHs the origin of SBHs with a mass exceeding \( 10^6 \, M_\odot \) in the galactic nuclei is quite unclear. With a present state of art of observations and theory we have nowadays only the list of more or less elaborated scenarios leading to the SBH formation. We will review here these known ideas and models with discussions of possible crucial signatures to discriminate between them and find out the universal mechanism (or several
mechanisms) of SBH formation. The recent advance and current observational status of SBH searches is extensively reviewed e.g. in [4, 5].

The SBHs is not those things that can be “touched by hands” or be investigated thoroughly in the laboratory (it must be noted that formation of short-lived microscopic BHs is forecasted in particles collisions at LHC in accordance with some extra-dimensional theories). Nevertheless, a supposition of their existence permits to explain a lot of observations. The Seyfert and radio galaxies, blazars, quasars and huge jets emitted from the galactic centers could be explained as various aspects of the same unique phenomenon: an activity of SBH in the galactic center. It is an interaction of SBHs and matter in the host galaxies that produces and explains different forms of Active Galactic Nuclei (AGN) phenomenon [1, 2, 3].

A recent discovery of very distant quasars reveals the deficiencies in the understanding of SBH formation. The SBHs are inevitably the main engine of quasars and the same fact of observation of so distant SBHs is a source of many questions. Indeed, how could SBHs as massive as $10^9 - 10^{10} M_\odot$ be formed so early, at $z = 6 - 7$, i.e. at 800 million years after the Big Bang? After nucleation, a BH duplicates its mass during the time $t_{acc} \approx 4 \cdot 10^7$ yrs, if an accretion near the Eddington limit takes place. In this case the smallest period of time necessary to increase an initial BH mass $M_0$ up to some value $M$ is

$$ t \sim \frac{4 \cdot 10^7}{\ln 2} \ln \frac{M}{M_0} \text{ yrs.} $$

Hence about billion years is needed to increase the BH mass from e.g. the Solar mass to the observable values. There is no room for all stages preceding nucleation of the Solar mass BHs — from a star formation to its explosion.

Nevertheless, the multi-scale simulations indicate that luminous quasars at $z \sim 6.5$ could be formed within the framework of standard $\Lambda$CDM paradigm. It was shown [7] that the most massive halos after several mergers could reproduce in general the observed properties of SDSS J1148+5251, the most distant quasar detected at $z = 6.42$, [8].

2 Theoretical fundamentals

The physics of BH is based on the Einstein-Hilbert equations of general relativity. Specific details and features of BH are investigated since 1916, when Karl Schwarzschild found the famous static spherically symmetric solution for a general relativistic body [9]. In spite of complexity of the subject and permanent disputes on the BH existence, scientific community has came to agreement on the general BH features. There are a few important formulas of BH physics which are useful in interpretation of observational data and astrophysical applications.
A size of BH is characterized by its gravitational radius

\[ r_g = \frac{2GM}{c^2} \simeq 3 \times 10^5 \frac{M}{M_\odot} \text{ cm}, \]

where \( G \) is the gravitational constant and \( M \) is a BH mass. Another important scale is the minimal radius \( r_{st} \) of stationary circular orbit of a massive body moving around the BH:

\[ r_{st} = 3r_g \]

in the case of nonrotating BH.

The SBHs are the most bright objects in the Universe due to radiation of accreting matter. In a dense environment of galactic nucleus the mass of SBH is increased in several orders of magnitude during the lifetime of the galaxy. In the stationary spherically symmetric case (Bondi accretion) the growth of BH is defined mostly by the density of surrounded gas. The corresponding rate for BH mass growth is

\[ \frac{dM_B}{dt} = 4\pi \alpha \frac{G^2 M^2 \rho}{c_s^3}, \]

where a dimensionless parameter \( \alpha \) depends on the equation of state of an accreting gas and is of the order of unity, \( c_s \) is a sound speed of the gas and \( \rho \) is its density at infinity.

When a particle is falling onto BH, some part of its energy is converted into radiation and the rest part increases the mass of BH. Hence the rate of accretion and the rate of BH mass growth is proportional to its luminosity. At the same time the accretion luminosity is limited by the Eddington condition — equality of the pressure of radiated photons and gravitational forces experienced by accreted plasma. This condition leads to the maximal value of luminosity for the stationary spherically symmetric accretion (the Eddington luminosity limit):

\[ L_E = 4\pi GM m_p c \sigma_T \simeq 10^{38} \frac{M}{M_\odot} \text{ erg s}^{-1}, \]

where \( m_p \) is the proton mass and \( \sigma_T \) is the Thomson cross-section.

Equation for BH mass growth in the case of stationary accretion of baryonic gas is

\[ \frac{dM}{dt} = \frac{M}{t_{Sal}}, \]

were the Salpeter time \( t_{Sal} \) is

\[ t_{Sal} = \frac{\varepsilon}{1 - \varepsilon} \frac{M c^2}{L} = 4.1 \times 10^8 \frac{\varepsilon}{1 - \varepsilon} \frac{L_E}{L} \text{ yr}. \]

Here \( \varepsilon \) is an efficiency of accretion and \( L \) is an observable luminosity of BH. It is also reasonable to suppose that \( L \propto M \) as in the Eddington luminosity limit, see (4). In this case the solution of equation (5) is rather simple:

\[ M(t) = M_{in} \exp\left[\frac{t - t_{in}}{t_{Sal}}\right]. \]
where index “in” corresponds to the initial values.

An accretion efficiency $\varepsilon$ is an emitted fraction of the total energy of accreted particle. To estimate this value, let us consider a particle with mass $m$ orbiting around a nonrotating BH at some radius $r$. If the orbit is stationary, a total energy of particle is

$$E = mc^2 \frac{1 - 2GM/c^2r}{(1 - 3GM/c^2r)^{1/2}}.$$  \(8\)

A difference between the energy at last stationary orbit \((r \to \infty)\) and those at infinity is $\Delta E = 0.0572mc^2$. Respectively, for the extremely rotating Kerr BH this value is greater, $\Delta E = 0.42mc^2$. A real efficiency differs from an “absolute” value $\varepsilon = 0.0572$ for the Schwarzschild and $\varepsilon = 0.42$ for the extreme Kerr case because some part of radiation is lost within the BH. In addition, some energy will be radiated during the particle motion between the horizon and the last stable orbit. For estimations it is usually used $\varepsilon \simeq 0.1$ for an accretion efficiency of nonrotating BH.

A gravitational redshift $z$ is the productive instrument in modern astrophysics. Let us denote a difference between the Newtonian gravitational potentials of two points as $\Delta \Phi_N$. The observers located in these points will measure the different frequencies $\nu_1$ and $\nu_2$ of the same electromagnetic wave. These values define the redshift:

$$1 + z \equiv \frac{\nu_1}{\nu_2} \simeq 1 + \frac{1}{c^2} \Delta \Phi_N.$$  \(9\)

An another group of formulae is related to the BH binaries and with the processes of their collisions and merging accompanied by an emission of gravitational waves. Investigation of gravitational wave generation in the Universe is the topical subject in view of coming completion of huge interferometric laser gravitational waves detectors. It is known that a collision of two BHs proceeds through a stage of quasi-periodic motion in close binary after their mutual gravitational capture. An orbital period for two BHs with mass $M_1$ and $M_2$ is

$$T_{\text{orb}} = 2\pi \sqrt{\frac{a^3}{G(M_1 + M_2)}},$$  \(10\)

where $a$ is a semi-major axis of the Keplerian orbit. Obviously, the gravitational waves emitted by binary have the same period. Corresponding duration of this Keplerian stage of gravitational waves emission by binary is \[10\]

$$t_{\text{GW}} = \frac{5}{256F(e)} \frac{a^4\varepsilon^5}{G^3(M_1 + M_2)^2\mu};$$  \(11\)

$$F(e) = (1 - e^2)^{7/2} \left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right),$$  \(12\)

where $e$ is an orbit eccentricity and $\mu = M_1M_2/(M_1 + M_2)$. 

5
3 Observational signatures of SBHs

Strictly speaking we have so far only numerous indirect indications in favor of BHs existence in the Universe. For this reason the claimed BHs of stellar mass in binaries and SBHs in galactic nuclei are only the candidates, not finally proven BHs. There is still some room for alternatives to BHs among suspected candidates. Decisive future experiment would be a demonstration or discovery of the BH event horizon, which is the most striking manifestation of strong gravitational field and space-time curvature. At the present time, the crucial point of BH astrophysics is a revealing of observational signatures of event horizon in suspected BH candidates. The discovery of a BH event horizon would be the most crucial verification of the General Relativity as well.

3.1 Active galactic nuclei and quasars

Observations of host galaxies of quasars, i.e. the Quasi Stellar Objects (QSOs), are rather difficult because of the huge luminosity of central objects in comparison with stellar components and because of the large distances. Nevertheless, the connection between QSOs and their host galaxies has been very definitely established during the last decade though at least one exception is known - the bright quasar at the edge of a huge gas cloud without any visible stellar host galaxy [11]. One of the explanation consists of a possibility that the SBH powering the QSO was ejected from the galaxy due to gravitational slingshot of three or more SBHs in the merger process of smaller galaxies [12]. The close similarity of the X-ray spectra of low- and highest-z QSOs means that SBHs with accretion disks were already formed \( z \geq 6 \). By using of some type of a “continuity equation” with “source” in redshift space it is possible to provide the link between SBHs in distant QSOs and SBHs in the centers of local galaxies [13, 14, 15, 16]. This analysis is suitable for a reconstruction of the accretion history of SBHs [17, 18]. Different aspects of the accretion physics in QSOs are reviewed in [19].

3.2 Dynamics of gas and stars near SBH

Specific details of motion of gas and stars near the suspected SBH supply us with valuable information on the gravitational field in the vicinity of SBH. The best fit of numerous observations gives for a value of SBH mass at the Galactic Center \( M_{\text{BH}} \approx 3.6 \cdot 10^6 M_\odot \).

Infra-Red (IR) imaging technic allowed to track the Keplerian orbits of several stars around the Galactic Center. Velocities of the stars approach 9000 km s\(^{-1}\) (compare with the average velocity of stars 200 – 300 km s\(^{-1}\)). Pericenter distances from the SBH are as small as 90 AU that is well inside a radius of gravitational influence of the SBH [20].

If a star passes close enough to SBH it could be ripped apart by the tidal gravitational forces. The conditions for such effect are favorable in the dense stellar nuclei of galaxies. The flare of UV- and X-radiation should be emitted
by the stream of stellar debris that plunges into the BH. These star destructions are really visible phenomena. Gezari et al. [21] observed the flare and its tail produced by stellar destruction near SBH in the galaxy at $z = 0.37$. Multi-wavelength data collected for 2 years beginning from the time of the event are in excellent agreement with theoretical predictions for the tidal disruption of a star. Gas streams from ripped stars serve the fuel for QSO activity.

On the other hand, a vicinity of SBH is an appropriate area for the process of young star formation. Well known fact is the gravitational instability of massive accretion discs [22]. It could lead to formation of new stars if a cooling process in the disk is effective. Many young stars where discovered in the nearest vicinity of our Galactic Center, some of them about 0.03 pc from the center [23]. The processes of star formation and gas accretion could interfere meaningfully [24].

In general, a final stage of matter accretion is poorly investigated yet because of inevitable observational difficulties in resolving small angular scales. An inner parsec of AGN contains dense molecular gas of $H_2O$, $SiO$ and $CH_3OH$, which is constantly heated by the accretion generated radiation. A detection of distribution of water maser emission line at 1.35 cm wavelength is a powerful investigation method of the near vicinity of SBH. A maser activity was first discovered in 1965 [25]. A water maser outside of our Galaxy was discovered in the spiral arms of the galaxy M33 by [26]. Up to now, in total about 60 events are found [27].

Observations of the broad fluorescence $K_\alpha$ iron line with energy 6.4 keV is also explained in the framework of accreting SBH in the distant galactic center [28]. Due to the combined Doppler effect in the fast rotating accretion disk and the gravitational field of the central BH the observed profile of emission line has an asymmetric double-peaked shape [29, 30, 31, 32]. The fitting of line profile corresponds to the emission of iron atoms at a distance of only a few gravitational radius of SBH. From the detailed spectroscopy of accretion disks it is possible also to evaluate the Kerr BH spin [33, 34, 35, 36].

New observational evidences continuously confirm the SBH paradigm. For example, it was reported recently [37] the activity of region close to $SgrA^*$ at a wavelength 3.5 mm. This region surrounds the central BH in the Galaxy having the size about 1 AU. On the other hand, the mass of the central object within this scale is well known, $\sim 3 \cdot 10^6 M_\odot$. It permits to calculate an average density in the Galactic center, which appears to be $\sim 6.5 \cdot 10^{21} M_\odot / pc^3$ — too much to consists of gas in any state and/or stars.

### 3.3 Galaxy — SBH correlations

There are some kinds of observational data which must be explained by any model of SBH formation. On the other hand, these data give the “guiding thread” to choose the successful models.

Early observations of normal galaxies, quasars and AGN recovered a simple relation $M_{SBH} \propto L$ between a mass $M_{SBH}$ of the central SBHs in galactic nuclei and a total luminosity of host galaxies or their central stellar bulges [38, 39, 40]. However an intrinsic data spread in the above linear relation was very high,
even larger than the observational errors. More tight correlations were found recently \cite{11} between a mass $M_{\text{SBH}}$ and velocity dispersion $\sigma_e$ at the bulge half-optical-radius:

$$M_{\text{SBH}} \propto \sigma^\alpha,$$

where $\alpha = 3.75 \pm 0.3$. An other analysis \cite{42} based on poorer set of observational data revealed the correlation with a somewhat different power index $\alpha = 4.80 \pm 0.54$. In \cite{13} it was shown that difference of slopes $\alpha$ arises mostly due to systematic difference in the velocity dispersions used by different authors for the same galaxies. More detailed analysis \cite{43} of the set of 31 galaxies yields $\alpha = 4.02 \pm 0.32$. Recently, a correlation between $M_{\text{SBH}}$ and host galaxy velocity dispersion for QSOs in the Data Release 3 of the Sloan Digital Sky Survey was discovered \cite{44}. For small redshifts $z < 0.5$ these results agree with ones for the nearby galaxies. For a range $0.5 < z < 1.2$ there is indication on the evolution with $z$: the bulges are too small for their central accreting SBHs. However, this evolution can be attributed to the observational biases. It have to be mentioned presumable correlation between the SBH mass and the age of the galactic stellar population \cite{45}. This last type of correlation could be inconsistent with the scenarios of primordial SBHs origin.

The observed correlations imply one-to-one relation between the processes of the central SBH and bulge formation and evolution in the all types of galaxies. Theoretical interpretation of these correlation is not very easy due to extremely different length-scales: the galactic bulge scale is a few kpc, whereas the scale of SBH gravitational influence is millions times smaller. Specific mechanism is needed for mass transfer from the bulge to its innermost part. One of the possibilities is that SBH formation depends on the dynamical properties of bulge through the rate of gas supply. In this case a velocity dispersion $\sigma$ measures the depth of the gravitational potential in which a SBH forms and grows. Another discussed possibility is the energy feedback between an accreting SBH and bulge which results in a self-regulated accretion and a corresponding growth of the central SBH \cite{46}. This model predicts the following relation, $M_{\text{BH}} \propto \sigma^5$. Third model \cite{47} supposes that bulges and SBHs were formed during the multiple galactic merging. The resulting correlation between $M_{\text{BH}}$ and $\sigma_e$ in the galactic merging model is in a good agreement with observations. A forth model proceeds from the hypothesis that observed correlations are stochastic in origin \cite{48}. The basic assumption of stochastic model is an existence of a pre-galactic or primordial SBH population. A similar hypothesis of an existence of pre-galactic SBH population was used \cite{49, 208} for interpretation of the QSO evolution.

### 3.4 Black holes of intermediate mass

Some mechanisms of SBH formation imply the Intermediate Mass BHs (IMBHs). The latter are believed to be an intermediate stage of SBHs growth in astrophysical evolution scenarios with an accretion and merging. Fast variability of compact X-ray source in the starburst galaxy M82 indicates that this source is an accreting BH \cite{50}. With the observed luminosity of $10^{41}$ erg s$^{-1}$, a mass of
the supposed BH is of the order of $\sim 10^3 M_\odot$ provided the emission is isotropic and corresponds to the Eddington limit. The IMBH in M82 exists in starburst environment. This is a hint on a possible formation of the IMBH in the M82 by a runaway merging of stellar mass BHs [51]. An other suspected IMBH in the star cluster G1 of the Andromeda galaxy has mass $2 \cdot 10^4 M_\odot$ [52], but there is still a room for a tight cluster of massive stars in this case.

Some of nearby Globular Clusters may contain IMBHs according to the measurements of stellar kinematics. On the basis of detailed observations of Globular Clusters it was stated that IMBHs may inhabit a half of Globular Clusters in the Galaxy [53]. These Globular Clusters are very old in general. In particular, the mass of IMBH containing in M15 is about $4000 M_\odot$ [54]. It is interesting that masses of suspected IMBHs in Globular Clusters are almost those as expected from the extrapolated $M_{SBH}\cdot \sigma$ correlation for galaxies, in spite of poor statistics of the central BHs with $M_{SBH} < 10^7 M_\odot$ in galactic nuclei.

It was shown [55] that the primordial IMBHs could be connected with such dense Dark Matter (DM) objects like neutralino stars through their common origin from primordial density perturbations. The neutralino stars with mass $\sim (0.01 \div 1) M_\odot$ were proposed to account for observed microlensing events in the Galactic halo [56]. These neutralino stars consist of weakly interacting DM particles, and were formed in the expanding Universe from adiabatic density perturbations. The existence of neutralino stars should give rise to a large number of primordial IMBHs with a mass $\sim 10^5 M_\odot$. These IMBHs may constitute a large fraction of the Galactic halo mass.

### 3.5 Prospects for gravitational waves detection

Coalescence of BHs in the galaxies and clusters are inevitably accompanied by strong bursts of gravitational radiation. The planned ESA/NASA space observatory LISA will be capable to detect these coalescence events in very distant galaxies. The combined theories of galaxies and SBH merges are developed to predict the principally observed gravitational waves signals [57, 58, 59]. Future observations of binary merging with LISA will allow to test General Relativity in details, place bounds on alternative theories of gravity and study the merger history of massive BHs [60].

### 3.6 Astrophysical constraints on SBHs

There are several purely astrophysical constraints on the mass and number of SBHs. If SBHs provide the dominant part of DM in the Galaxy, then they must tidally interact and disrupt some part of Globular Clusters. The SBH mass was constrained in this case, $M_{BH} \leq 10^4 M_\odot$ [61]. At $\Omega_{BH} \sim 1$, primordial BHs are capable of distorting the CMB spectrum if they are formed about 1 s after the annihilation of $e^+e^-$-pairs [62]. Mass accretion at the pre-galactic and present epochs contributes to the background radiation in different wavelength ranges. However, calculations are strongly model dependent and yield $\Omega_{BH} \leq 10^{-3}$.
10^{-1}$ for $M_{BH} \sim 10^5 M_{\odot}$. In [63], the constraint on the fraction of intergalactic BHs, $\Omega_{BH} < 0.1$, was obtained from a condition of absence of the reliable gamma-ray-burst lensing events for $10^5 M_{\odot} < M_{BH} < 10^9 M_{\odot}$. A more stringent lensing constraint, $\Omega_{BH} < 0.01$ for the mass range $10^6 M_{\odot} < M_{BH} < 10^8 M_{\odot}$, was obtained from VLBI observations of compact radio sources [64].

3.7 Dark matter spike in the Galactic center

Annihilation rate of weakly interacting cold DM particles at the Galactic Center could be greatly enhanced by the growth of density spike around the central SBH. The resulting annihilation signal may be observed in the form of high-energy gamma-rays, positrons, antiprotons and radio emission [65]. It could facilitate the study of DM content. At the same time this annihilation spike may be the observational signature of the SBH in the Galactic center. The form and amplitude of this spike is rather uncertain due to influence of different dynamical processes such as sinking of baryonic gas and hierarchical merging of sub-halos [66] [67]. Searching for annihilation signals could set an upper bound to the DM cusp, DM particle annihilation cross-section and the merger history of the Galaxy. The DM spikes could also arise around primordial IMBHs [48], and the DM annihilation in these spikes is in observable range of future detectors [68]. A model of adiabatic growth of a seed BH in collisionless DM halo through the DM accretion is proposed in [69]. The supposed combined evolution of SBH and DM halo results in the observed $M_{SBH} - \sigma$ relation.

4 Astrophysical scenarios of SBH formation

4.1 Dynamical evolution of galactic nuclei

Physically the most simple and natural possibility for SBH formation is a dissipative dynamical evolution of a dense stellar cluster in the galactic nucleus ending by its total gravitational collapse. This idea was first proposed at the dawn of quasar discovery and then was elaborated in some details in numerous works.

An evolved galactic nucleus contains dense central stellar cluster and probably subsystem of compact stellar remnants, such as neutron stars and stellar mass BHs. This subsystem come from the dynamical evolution of star cluster in the galactic nucleus through merging of stars, thereby forming the short-living massive stars. The clusters of compact remnants form almost inevitably.

To perform estimations, let us consider a central stellar cluster in the galactic nucleus of mass $M$ and radius $R$ (the radius can be effectively defined through the sphere which contain half of the cluster mass), consisting of $N \gg 1$ of identical stars with mass $m = M/N$ and radius $r_\star$. The virial theorem gives the velocity dispersion in the cluster $v \simeq (GM/2R)^{1/2}$.

A characteristic measure of the stellar system compactness is a depth of its gravitational potential $|\phi| \simeq v^2$. That is why the value of star velocity
dispersion $v$ is crucial parameter in evolution history of the stellar system. In the case $v \ll c$ the Newtonian approximation is applicable to a star motion in cluster. The Newtonian approximation is failed when a stellar system becomes relativistically compact, with $|\phi| \simeq c^2$. At this stage of evolution the radius of the systems is close to its gravitational radius. The cluster as a whole or its major part becomes dynamically unstable and collapses into the SBH.

Dynamical evolution of the cluster at nonrelativistic stage was widely discussed (see e.g. [70, 71, 89, 72] and references therein) and almost exhaustively studied in the Fokker-Plank approximation. It turns out that very important factors of the evolution are the tidal interactions of stars, the binary stars formation and the star mass segregation. Due to the mass segregation, the binaries and most massive stars are concentrated in a central part of the cluster.

Dynamical evolution of stellar systems those as galactic nuclei which are dense enough, proceeds through the stellar collision and coalescence stage [73, 74, 76, 72, 70]. This process is enhanced by the formation and hardening of star binaries formed by tidal two-body interactions [77, 78, 79]. The newly formed hard binaries result in heating of the cluster. The resulting evolution of star cluster is determined by the competition of tidal dissipation processes and heating by newly formes binaries. In the clusters with a high enough velocity dispersion $v$ dissipative processes dominate and accelerate the contraction and growth of $v$. Later, due to the growth of dispersion $v$ the stellar collisions become catastrophic and destroy the colliding stars. As a result the compact stellar cluster transforms into the Supermassive Star (SMS) with an appreciable fraction of neutron stars and and BHs of stellar mass [80, 81, 82, 83, 84, 85].

There are two distinct ways of IMBH or SBH formation in the evolving star cluster:

(i) Central core of the cluster reaches relativistic state and collapses as a whole.

(ii) Stellar mass BHs and neutron stars merge (aggregate), progressively increasing their mean mass.

In the following we suppose for simplicity that stellar cluster consists of identical stars of mass $m$. The main virial parameters of a self-gravitating stellar system are a total mass $M = Nm$, a mean radius $R$ within which $N/2$ stars are contained, a star velocity dispersion is $v$. These parameters obey a virial theorem [80]:

$$v^2 = \frac{GmN}{2R}.$$  \hspace{1cm} (14)

A total energy of the self-gravitating system is

$$E = -\frac{1}{2}mNv^2.$$  \hspace{1cm} (15)

These virial relations are established in the dynamical time $t_{\text{dyn}} = R/v$. The corresponding dynamical evolution equation on the time-scales exceeding $t_{\text{dyn}}$ is derived by the differentiation of viral relations (14) and (15):

$$\frac{\dot{R}}{R} = -\frac{\dot{E}}{E} + 2\frac{\dot{M}}{M}.$$  \hspace{1cm} (16)
A central stellar cluster in a galactic nucleus must be compact enough to evolve during the life-time of the galaxy. Due to the Coulomb character of the gravitational interactions of stars the evolution of star orbits in the cluster proceed diffusively with a characteristic two-body relaxation time \[ t_r = \left( \frac{2}{3} \right)^{1/2} \frac{v^3}{3\pi G^2 m^2 n \Lambda}, \] \[ (17) \]

where \( m \) is a constituent stellar mass, \( n \) is a local star number density, \( \Lambda \simeq \log(0.4 N) \) is a gravitational Coulomb logarithm. For many-body stellar systems with number of stars \( N \gg 1 \), the relaxation time \( t_r \) is much greater than a dynamical time \( t_{\text{dyn}} \):

\[ \frac{t_r}{t_{\text{dyn}}} \sim \frac{N}{\log(N)} \gg 1 \text{ at } N \gg 1. \] \[ (18) \]

This means that stars in the self-gravitating dynamically stationary systems have regular orbits with integrals of motion (energy and angular momentum) slowly changing at time-scales \( t \ll t_r \).

The smallness of the ratio of relaxation time \( t_r \) and the age of the galaxy \( T_g \sim H^{-1} \) is a measure of stellar cluster compactness needed for a substantial evolution. Stellar clusters with \( t_r/T_g \sim H^{-1} \ll 1 \) are called (dynamically) evolved: they have enough time for substantial change of initial orbital energies and angular momenta of stars in the cluster due to two-body interactions. The examples of dynamically non-evolved stellar clusters with \( t_r/T_g \sim H^{-1} \gg 1 \): stellar disks of spiral galaxies, Globular Clusters, galactic DM halos. The conditions for substantial evolution are realized only in the central most compact parts of these objects. For the same reason the central parts of the Globular Clusters and galactic nucleus are suspected for the final gravitational collapse.

**4.1.1 Non-dissipative contraction of star cluster**

A non-dissipative dynamical evolution is governed by the evaporation of fast stars from the system. The rate of fast star evaporation due to two-body interactions is

\[ \dot{N} = -\alpha_{\text{ev}} \frac{N}{t_r}, \] \[ (19) \]

where numerical coefficient \( \alpha_{\text{ev}} \simeq 10^{-2} \) in the Fokker-Plank approximation. Due to diffusive nature of star orbital energy changing, an evaporating star escapes from the stellar system with a nearly zero total energy, i.e. with a nearly parabolic velocity \( v_{\text{esc}} = 2v \). Hence the total energy of the stellar system is nearly conserved during star evaporation, \( E \simeq \text{const} \). This defines the scaling of the virial parameters: the evolutionary growing of a star velocity dispersion with a diminishing of the number of stars, \( v \propto N^{-1/2} \), and a corresponding diminishing of the stellar system radius, \( R \propto N^{-1/2} \). Integration of evolution equation (16) at the condition \( E = \text{const} \) by using (19) and by neglecting
variation of logarithmic factor $\Lambda$ gives a simple law for non-dissipative dynamical evolution

$$R(t) = R(0) \left(1 - \frac{t}{t_{\text{ev}}}\right)^{4/7},$$  \hspace{1cm} (20)

where the lifetime of the system $t_{\text{ev}} = (2/7\alpha_{\text{ev}})^{1/2}$ and $t_{r}(0)$ is a relaxation time at the initial moment $t = 0$. According to this formula, a contracting stellar system reaches the relativistic stage, $v \rightarrow c$. At the relativistic stage, not the evaporation of stars but the dissipative processes such as gravitational radiation and star collisions become the crucial dynamical evolution factor. The dissipation processes hasten the contraction of cluster towards the onset of dynamical instability and final gravitational collapse. This scenario of evolution is valid in clusters consisting of degenerate stars like neutron stars and/or BHs of stellar masses. In the case of contracting cluster of normal stars, the dissipative tidal interactions and the star collisions become more important than star evaporations long before the relativistic stage of evolution.

### 4.1.2 Dissipative contraction of star cluster

The influence of dissipative tidal interactions and star collisions on the dynamical evolution is crucial in the star clusters with a large value of stellar velocity dispersion $v$, namely when $v \sim v_p$, where $v_p = (2Gm/r_*)^{1/2}$ is a surface parabolic velocity of constituent stars. For example, in the case of the Sun $v_p \approx 620 \text{ km s}^{-1} \ll c$.

In close encounters the non-radial oscillations are excited by tidal interactions of stars. Some part of kinetic orbital energy transferred into these oscillations. Finally the energy of non-radial oscillations is thermalized inside stars and reradiated. This is also the mechanism for dissipative tidal friction of stars and the tidal capture mechanism of close binary star formation [77]. The orbital energy deposited into oscillations during one close encounter equals [79]

$$\Delta E = 2^{7/3} \frac{Gm^2}{r_*} \left(\frac{r_*}{R_{\text{min}}}\right)^{10}$$  \hspace{1cm} (21)

for a specific case of stars with $n = 3$ polytropic structure. Here $R_{\text{min}}$ is a periastron distance. The resulting tidal “cooling” rate of the stellar system in the case of the Maxwellian distribution of stars over velocities is [90, 91, 92]

$$\dot{E}_{\text{dis}} = -\Gamma(0.9)g^{-1} \frac{N}{M_r}, \quad g = 2^{37/30} \left(\frac{Gm^2\beta}{r_*}\right)^{-9/10}.$$  \hspace{1cm} (22)

In this equation $\Gamma(0.9) \approx 1.069$ is the Gamma-function and $\beta^{-1} = m\nu^2/3$ is a mean kinetic energy of stars in a cluster (or cluster’s “temperature”). The tidal dissipative friction of stars in the cluster accelerates its contraction. Under the influence of both evaporation of stars and tidal friction the evolution equation (16) according to (19) and (22) takes the form

$$\frac{\dot{R}}{R} = -\left[\left(\frac{v}{v_{\text{dis}}}\right)^{9/5} + 2\right] \frac{\alpha_{\text{ev}}}{t_{r}},$$  \hspace{1cm} (23)
where

\[ v_{\text{dis}} = \left( \frac{3}{2} \right)^{1/2} 2^{-67/54} \left[ \frac{3(\pi/2)^{1/2} \alpha_{\text{ev}} \Lambda}{\Gamma(0.9)} \right]^{5/9} v_p. \]  

(24)

For a case of cluster consisting of the Sun type stars, \( v_{\text{dis}} \simeq 190 \text{ km s}^{-1} \). According to equation (23) dynamical evolution follows non-dissipative evaporative scenario (20) until \( v \ll v_{\text{dis}} \). It also concerns the central parts of the nowadays Globular clusters with \( v \sim 10 \text{ km s}^{-1} \). Only a very small fraction of stars in Globular Clusters survives in the final steps of dynamical evolution. Additionally a lot of hard binaries (with binding energies exceeding \( \beta^{-1} \)) are formed in Globular Clusters by dissipative two-body encounters [77]. As a result the heating of cluster by these hard binaries [78] prevents the formation of a massive BHs in the Globular Clusters (see for details [91, 92]).

On the contrary, in stellar systems such as the galactic nuclei with \( v \geq v_{\text{dis}} \), the tidal friction dominates over star evaporation and heating by hard binaries. As a result the cluster contracts formally to a zero radius having a finite number of stars in the remnant [90, 92]. Physically this dissipative stage of evolution is finished by the direct collisions, merging and destruction of stars. Collisions of stars with relative velocity \( u < v_p \) result in merging of stars or their explosion of a supernova type. A head-on collisions of stars with \( u > v_p \) are more dangerous and inevitably end with a total destruction of these stars [73]. In this limit the rate of disruptive collisions of stars in the system is equal to

\[ \dot{N} = \left( \frac{v}{v_p} \right)^4 \frac{N}{\Lambda t_r}. \]  

(25)

A contracting star system with \( v > v_p \) consists of a decreasing number of stars and an increasing number of moving gas clouds — remnants of destructive star collisions. These gas clouds participate in the virial balance as the peer entities until they are small compared with the size of the remaining system. In a simple approximation, suppose that the two clouds with a nearly equal masses are formed as a result of the two stars catastrophic collision, and so \( N m = M = \text{const} \). Integration of equations (16) and (25) in this approximation gives

\[ R(t) = R(0) \left( 1 - \frac{t}{t_{\text{coll}}} \right)^{2/7}, \]  

(26)

where the collision evolution time \( t_{\text{coll}} = [v_p/v(0)]^4 \Lambda t_r(0) \). The system contracts according to (26) until its radius reaches \( R \sim N^{1/2} \alpha \), at which point \( t_{\text{coll}} \sim t_{\text{dyn}} \). At this point the system ceases to exist as a collection of separate stars and transforms into the compact self-gravitating massive cloud of gas — the Supermassive Star (SMS) [93, 94, 95, 98]. The possible evolution tracks of stellar clusters are shown in the Fig. (1)

### 4.2 Collapse of supermassive star

A Supermassive Star (SMS) was first proposed long ago as a possible source of quasar activity and nowadays is considered as a short intermediate stage of
Figure 1: Evolution tracks of stellar systems in the plane “number of stars $N$, velocity dispersion $v$” (see [92] for details). $A$ — globular clusters, $B$ — normal galaxies with low $v$, $C$ — galactic nuclei with high $v$. In region I, the tidal dissipative effects and heating of stellar system by newly produced binaries do not influence the evolution of the system which follows to the non-dissipative evaporative scenario (see Section 4.1.1). The boundary between regions I and II (and between III and IV) corresponds to condition $\dot{E} = 0$, when dissipative effects in the system are compensated by the heating by binaries. In the region II, the heating of the system by newly formed binaries dominates over the tidal dissipation, $\dot{E} > 0$. The boundary between regions II and III is determined by the condition $\dot{v} = 0$, or equivalently $v = v_{\text{crit}}^{(+)}$. In the region III, where $v > v_{\text{crit}}^{(-)} \sim 190 \, \text{km s}^{-1}$, the heating by binaries prevails over the tidal dissipation and $v$ decreases with the decrease of $N$. In the region IV, the evolution of stellar system is mainly affected by tidal dissipative energy losses which lead to a rapid contraction of the system toward to the final collapse into SBH. Above the dotted line the rate of star collisions becomes greater than the rate of star evaporation. The dashed-dotted line corresponds to $t_r = 10^{10} \, \text{yr}$.
galactic nucleus evolution toward the formation of SBH. To prevent a fragmentation before a SMS formation, a primordial gas must be hot and magnetized. The natural way for SMS production is stellar collisions stage of evolution of compact galactic nuclei. Fast evolving SMSs include an unstable hydrodynamic relativistic radial mode and eventually collapse to form one or several close SBHs. Recently numerical simulations provides the possibility to analyze the final state of the SMS gravitational collapse.

The SMSs may be naturally formed in the galactic nuclei from gas produced in the destructive collisions of stars in the evolved stellar clusters with a velocity dispersion $v \geq v_p$. Characteristic time-scale for the dynamical evolution of stellar cluster is the (two-body) relaxation time, $t_r \approx 4.6 \times 10^8 N_8^2 \left( \frac{v}{v_p} \right)^{-3}$ yr, where $N = 10^8 N_8$ is the number of stars in the cluster. The corresponding virial radius of the stellar cluster is $R = GNm/2v^2$. At $v > v_p$, where $v_p$ is an escape velocity from the surface of constituent star, the time-scale for self-destruction of normal stars in mutual collisions is $t_{coll} = (v_p/v)^4 t_r$. Numerical modelling of catastrophic stellar collisions has been performed by e.g. We choose $v \approx v_p$ as a characteristic threshold value for a complete destruction of two stars and final production of unbound gas cloud. As a result, the stellar cluster in the evolved galactic nucleus with $v \geq v_p$ transforms into the SMS due to catastrophic stellar collisions. At $v \approx v_p$ the time-scale for the formation of SMS due to destructive collisions of stars is of the same order as the relaxation time, $t_{coll}(v_p) \sim t_r(v_p)$. A total mass of the gas produced by destruction of normal stars composes the major part of a progenitor central stellar cluster in the galactic nucleus. The natural range of masses for the formed SMS is $M_{SMS} = 10^7 - 10^8 M_\odot$.

A newly formed SMS with mass $M_{SMS}$ and radius $R_{SMS}$ gradually contracts due to radiation with the Eddington luminosity, $L = L_E$. A nonrotating SMS is a short-lived object that collapses due to post-Newtonian instability. Rotation provides the stabilization of SMS if the rotation energy is an appreciable part of its total energy $E_{SMS} \simeq (GM_{SMS}^2/2R_{SMS})$. In general an evolution time of SMS is the Kelvin-Helmholtz time-scale $t_{SMS} = E_{SMS}/L_E \propto R_{SMS}^{-1}$. Stabilization of SMS by rotation (and additionally by internal magnetic field) ensures in principle its gradual contraction up to the gravitational radius $R_{GR}$. A resulting maximum evolution time of SMS is of the order of the Eddington time $t_E = 0.1 M_{SMS} c^2/L_E \simeq 4.5 \times 10^7$ yr. The distribution of gas in a SMS can be approximated by the polytropic model with an adiabatic index $\gamma = 4/3$. For this value of adiabatic index the central gas density in SMS is $\rho_c = k_c v_{SMS} m_p$, and central sound velocity $c_{s,c} = k_s v_{SMS}$, where $n_{SMS}$ is a SMS mean number density, $v_{SMS} = (GM_{SMS}/2R_{SMS})^{1/2}$ is a SMS virial velocity and numerical constants $k_c \simeq 54.2$ and $k_s \simeq 1.51$ respectively.

A SMS, formed in the galactic nucleus, may contain compact remnants of
exhausted stars in the form of neutron stars and stellar mass BHs. Due to dynamical friction these compact objects are settled down to the central part of the SMS. They form a very compact and fast evolving self-gravitating subsystem which collapses into the massive BH earlier than the host SMS. An observational signature of these nearly collapsing SMS in the distant galactic nuclei may be a long-wavelength gravitational radiation \[113\] and a power flux of high-energy neutrino \[84, 85\].

4.3 Collapse of neutron star cluster

Neutron star cluster evolves much faster than a host SMS and collapses finally into the massive BH. The dense clusters of neutron stars and stellar-mass BHs can undergo the catastrophic events of gravitational collapse due to general relativistic instability. The collapse goes through the avalanche-type contraction described firstly in \[114\] and confirmed by the detailed numerical calculations of \[81, 82, 83, 115, 116, 117, 118\]. They found that even in the case of extremely centrally condensed cluster configuration with an extensive Newtonian halo, a significant fraction (several percents) of total mass collapses into a central BH in a few dynamical times. The collapse begins when the central redshift approaches \( z \simeq 0.5 \) and a corresponding virial radius of the cluster diminishes up to \( R \sim 3R_g \). The ‘avalanche’ arises because the falling of stars onto the center leads to an increase of the gravitational field. The latter acts on the other particles/stars, whose orbits contract in turn, and so on. The non-circularity of star orbits are of principle significance because they connect the different layers of the cluster. Only stars at highly elongated orbits are involved in the collapse and accreted into the growing BH. After the collapse the cluster settle down to a new, dynamically stable equilibrium state: central BH embedded into quasi-Newtonian cluster of stars and compact stellar remnants. This evolutionary track is time-consuming because it requires the prior evolution of normal stars and subsequent settle down of their remnants to a highly concentrated system.

The further growth of a seed massive BH with a mass \( M_{\text{BH}} \geq 10^4M_\odot \) is governed by an accretion of ambient stars, gas and DM particles (see e.g. \[1, 120, 121\]). During a lifetime of the normal galaxy a seed massive BH should grow significantly, up to \( M_{\text{BH}} \sim 10^6 - 10^8M_\odot \).

4.4 Merging of stellar-mass black holes

The alternative model of IMBH formation in a star cluster is the multiple merging of stellar mass BHs when. The evolution of mass distribution usually studied by using the Smoluchowski equation \[51, 122, 123\]. Under the conditions that may exist in some star clusters the characteristic timescale for runaway merging is \( \sim 10^7 \) yr, which is short enough for the formation of BH with mass \( \sim 10^3M_\odot \). More massive BHs sink into the cluster center under the influence of dynamical friction and increase the merger probability in the core. Such a ‘mass segregation’ is very important process for the evolution of the cluster core. After merging of hosted protogalaxies, the IMBHs may sink to the galactic centers.
and merge into SBHs. It should be mentioned that this evolutionary track is rather time-consuming and probably can not explain the observed early QSO activity. Additional prolific accretion of baryons or DM is required [124]. It is considered also a possibility of the super-Eddington accretion regime, [125, 126] which strongly accelerates a BH growth.

A peculiar example is an accretion onto BHs of the Dark Energy (DE) with an equation of state $P(\rho) < 0$. In the extreme case of phantom energy (with $P + \rho < 0$), the accretion is accompanied with a gradual decrease (rather than increase, as in a case of the usual matter) of the BH mass [127, 128]. Respectively, masses of all BHs tend to zero while the phantom energy Universe approaching to the Big Rip.

4.5 Merging trees of galaxies and SBHs

Galaxies are formed by hierarchical merging of smaller protogalaxies, and even nowadays a significant fraction of low redshift galaxies still experience merging. When galaxies merge, their central SBHs get a chance to merge too. The dynamical friction in the field of stars and DM brings the SBHs into the central part of coalesced galaxies, where they finally merged into a bigger SBH. The timescale for a $10^8 M_\odot$ SBH to spiral down into the nucleus of typical giant galaxy from an initial radius of 10 kpc is $\sim 10^{10}$ yr. A subsequent closing of two SBHs to one another proceeds even faster $\sim 10^7$ yr due to a high number density of stars in the galactic nucleus, which produces a strong dynamical friction [129, 131]. On the final stage of merging the gravitational radiation dominates. However, there are calculations which predict the long living binary SBHs, longer than the Hubble time [132].

Merging of small protogalaxies containing the IMBHs is a promising mechanism of SBH formation. The seed IMBH could be formed initially in the minihaloes collapsing at $z \sim 20$ from high-$\sigma$ density fluctuations [133, 134, 135]. They increase their masses up to modern values $10^6 - 10^9 M_\odot$ due to multiple merging and accretion [136, 137]. Direct evidence of SBHs merging was founded by VLA telescope from observation of jets flipping (X-shape of radio lobes) in NGC 326 galaxy [138]. It is interesting that a total energy transferred from the binary SBH to the field stars is comparable with a binding energy of galactic core [129]. A combined upper limit on the mutual evolution of SBHs and their host galaxies was obtained in [130].

Recent X-ray observations clearly demonstrate the existence of binary AGN in the galactic system Arp 299 consisting of two galaxies in an advanced merging state, NGC 3690 and IC 694 to the east, plus a small compact galaxy [138]. The other cases of two AGN in a merging system is NGC 6240 [139] and NGC 6104 [140]. A close encounter and coalescence of spiral galaxies triggers an inflow of nuclear gas, which fuels both a powerful starburst and strong accretion onto the central SBH [141, 142].

Coalescence of SBHs in galaxies must be accompanied by the strong bursts of gravitational radiation. The planned ESA/NASA space observatory LISA will be capable to detect these coalescences and verify model predictions for
SBHs merging rate\textsuperscript{143} \textsuperscript{144}. A binary SBH leaves an imprint on a galactic nucleus in the form of a “mass deficit”, a decrease in the mass of the nucleus due to ejection of stars by the binary\textsuperscript{145}. Comparison with observed mass deficits implies between 1 and 3 mergers for most galaxies, in accordance with the hierarchical galaxy formation models.

A mean number of galactic mergers is rather low to ensure the growth of stellar mass BHs to SBHs in most galaxies with the only exception of giant CD galaxies in the centers of rich clusters. Another difficulty is the “final parsec problem”: quite large distance between SBHs in binaries after the merging of host galaxies to provide the strong enough gravitational wave radiation losses for merging of SBHs in binaries\textsuperscript{146}. Nevertheless, it seems that SBHs populate near all spiral and elliptical galaxies. This may be a hint in favor of some universal mechanism of SBH formation long before the formation of galaxies in the early Universe. The merging rate of galaxies and their central SBHs is insufficient to ensure the needed mass growth at least at low redshifts, \(z < 0.36\)\textsuperscript{147}.

5 Cosmological scenarios of SBH formation

The present state of art in understanding of SBH origin provides only a list of different scenarios, more or less elaborated. Even Moreover, we even non confident in the choosing between two major alternatives: the galactic or the cosmological origin of these objects. In view of these uncertainties we will itemize in this review the most popular known scenarios with a brief elucidation of underlying physics. It is a challenge for future theoretical investigations and detailed astrophysical observations to recover the real mechanism(s) of SBH formation.

The problem of SBH origin becomes more acute with a discovery of very distant quasars. Most distant of them are:

\[
\begin{align*}
J114816.64 + 525150.3, & \quad z = 6.43, M \simeq 3 \times 10^9 M_\odot; \\
J103027.10 + 052455.0, & \quad z = 6.28, M \simeq 3.6 \times 10^9 M_\odot; \quad (28) \\
J130608.26 + 035626.3, & \quad z = 5.99, M \simeq 2.4 \times 10^9 M_\odot.
\end{align*}
\]

Observations of high redshift quasars lead to an unambiguous conclusion that host galaxies of SBHs are formed very early. The existence of quasar with redshift \(z = 6.43\) indicates that SBHs with mass around \(10^9 M_\odot\) already existed when the Universe was less than 1 Gyr old. In order of magnitude, it is just a lifetime of an ordinary massive star. Such stars produce BHs with masses of order \(1 M_\odot\) after explosion. Using formula (7) one can obtain the time needed a BH to grow up in \(10^9\) times. In case of the Eddington accretion, a BH increases its mass in \(10^9\) times during 800 million years. There is no room for first ordinary star formation.

Nowadays we have a list of competing models to solve this puzzle. This list consists of two parts: cosmological (pre-galactic) astrophysical (galactic)
models. The latter were discussed in previous Section. Before the discussion of cosmological models let us consider a couple of “intermediate” models where SBHs and galaxies are formed at the same time approximately.

5.1 Population III stars or supermassive stars

BHs with masses about 100М\(_{\odot}\) are thought to be produced as remnants of first stars (Population III stars) which are supposed as massive as 200М\(_{\odot}\)\cite{148,149,150,151}. A lifetime of these stars is quite short — after several millions years they collapse into BHs.

According to a hierarchical model of structure formation, less massive DM objects are formed earlier. Formation of gravitationally bound objects with \(\sim 10^5 - 10^7\text{M}_{\odot}\) is expected to be before \(z \sim 10\). The short-lived massive stars with mass \(\sim 100\text{M}_{\odot}\) can appear from the metal-free gas clouds in the first DM sub-haloes, with a several massive stars per a DM sub-halo.

Formation of population III stars requires an effective cooling mechanisms at early epoch of fragmentation of primeval gas clouds into the stars. Usually believed that the line emission of molecular hydrogen is the basic cooler in these primeval gas clouds. If the gas cloud evolves without fragmentation to stars, it can reach the state of a single Super-Massive Star (SMS) with mass in the range \(\sim 10^3 - 10^5\text{M}_{\odot}\). A SMS is a very short-lived object and inevitably collapses into an IMBH.

5.2 Collapse of massive primordial clouds

SBHs could be formed as a result of dissipation and collapse of primordial gas during the early stages of galaxy formation\cite{6,13,152,153,154,155,156,157,158}. Initial masses of such SBHs are of order 10^5\text{M}_{\odot}. Another scenario of SBH formation in the low angular momentum haloes was proposed and recently considered in details by\cite{152}. In this scenario the baryons cool and sink to the center of the halo and finally settle down into a rotating disk. Due to viscosity, an angular momentum is transferred outward from an inner region of the disk. A model of the formation of lightweight BHs (a few solar mass) by the direct collapse of gaseous clouds (without a preceding formation of stars) was elaborated in\cite{153}. It is supposed that these BHs are growing up to the supermassive ones by accretion of massive envelops with the super-Eddington rate.

The haloes are distributed over their angular momenta. This distribution depends only on the spectrum of primordial density perturbations and was obtained from analytical calculations and numerical simulations\cite{154,155,156,157,158}. Only those haloes from low angular momentum tail of the distribution are capable to collapse. According to\cite{152}, haloes more massive than a critical threshold about \(\sim 7 \cdot 10^7\text{M}_{\odot}\) at red shifts \(z \sim 15\) contain the gravitationally unstable discs with an efficient viscosity. The authors of\cite{152} also showed that under reasonable assumptions, this model leads to the observed correlation between BH masses and spheroid properties.
Combined dynamics of baryons and DM in the halo of forming galaxy was analyzed in [159]. The cooling and contraction of baryonic component both lead to the formation of SBH which grows rapidly due to accretion as baryons [160] and the DM [161, 162]. Absorption of DM in [161] and [162] was examined by taking into account the scattering of DM particles on stars near the galactic center. It was shown that significant flux of DM on a seed BH in the galactic center of galaxy dominates. Respectively, a fraction of DM in the total mass of the SBH in this scenario is significant.

5.3 Pre-galactic SBH

As it was mentioned above there were proposed some mechanisms of massive BH production before the formation of first stars. Below we shortly list the proposed mechanisms of primordial BH formation (see also the detailed review [167]) and discuss some of the listed models in more details in the next sections.

Adiabatic fluctuations at radiation-domination stage. According to [62, 96, 97, 168], density fluctuations at the radiation-dominated evolutionary stage of the Universe give rise to primordial BHs. The formation of BHs is quite efficient in the case of “blue” spectra of power-law density perturbations \( n \geq 1.1 - 1.2 \).

Isocurvature (or isothermal) fluctuations after inflationary stage. Variety of models with the flat or bumped spectra of isocurvature fluctuations were proposed [169, 170, 171, 172, 173, 174, 175, 176, 177, 178]. A very promising mechanism of SMB formation from the large amplitude isothermal fluctuations in baryonic charge density was proposed by Dolgov and Silk [178]. In this mechanism the bumped spectrum of isothermal fluctuations is a byproduct of vacuum bubble collapses during phase transition at the inflationary stage.

Artificial form of the inflaton potential. Models of such sort represent a modification of the previous model. It is known that the inflationary stage is the reason of density fluctuations. Spectrum of the fluctuations depends on the form of the inflaton potential, see i. g. [179, 180]. As it was shown in [181], it opens a possibility to produce high density fluctuations that collapse into SBHs in the following.

SBHs from phase transitions. BHs with masses \( \sim 1 \text{M}_\odot \) possibly formed at quark-hadron phase transition at the cosmological moment \( 10^{-6} \) s, [182, 183]. Such BHs would be a component of DM today.

Soft equation of state. Suppose that some massive non-relativistic particles dominates at an early period of the Universe evolution. In that case the pressure is negligible and could not prevent gravitational collapse of high density regions [168].

Bubble collision. False vacuum decay is usually accompanied by formation of spherical walls with a true vacuum inside them [184, 185, 186, 187, 188, 189, 190, 191]. The walls quickly expand and collide what could lead to collapse of islands of false vacuum into BH. Most massive BHs of order \( 1 \text{M}_\odot \) are produced during the period of quark-hadron phase transition.
**Closed domain walls.** A mechanism of BH formation from the closed domain walls was proposed in \([192, 193, 194]\). These domain walls could be originated due to evolution of a scalar field during inflation. An initial non-equilibrium distribution of scalar field imposed by the background de-Sitter fluctuations gives rise to the spectrum of BHs, which covers a wide range of mass — from the subsolar up to supermassive ones. The primordial BHs of smaller masses are concentrated around the most massive ones within a fractal-like cluster. It was revealed that this mechanism is a rather common for many inflationary models and it worth to discuss its essential details at the end of the review.

5.4 Adiabatic fluctuations at radiation-domination stage

Gravitational collapse and the formation of a primordial BH occurs if a relative radiation density fluctuation \(\delta_H = (\rho - \rho_c)/\rho_c\) satisfies some conditions \([62, 96, 97]\) at the moment \(t\) of its separation from the cosmological expansion. Namely, the BH is nucleated if

\[
\delta_c \leq \delta_H \leq 1,
\]

where \(\delta_c = 1/3\) \([62]\). The left-hand inequality implies that the radius of the perturbed region at the time \(t\) exceeds the Jeans radius \(ct/\sqrt{3}\). The right-hand inequality corresponds to the formation of a primordial BH rather than an isolated Universe. The primordial BHs are produced with a near horizon size, and thus their masses are connected with the formation time as

\[
t = \frac{GM}{c^3} = 0.5 \left( \frac{M}{10^5 \text{M}_\odot} \right) \text{ s}.
\]

In recent years, numerical experiments have revealed a near-critical gravitational collapse of primordial perturbations with less than the horizon scales \([195, 196, 197, 198]\). In this process a mass of forming primordial BH is

\[
M_{\text{BH}} = AM_H(\delta_H - \delta_c)\gamma,
\]

where \(A \sim 3, \gamma \simeq 0.36, \text{ and } \delta_c \simeq (0.65 \pm 0.7)\). The mass \([91]\) may be much smaller than the horizon mass \(M_H\). However, as shown in \([199]\), the primordial BH mass distribution for critical gravitational collapse is concentrated near \(M_{\text{BH}} \sim M_H\), and the contribution of low masses to the cosmological primordial BH density is modest.

If the power spectrum of primordial cosmological perturbations is a power law with an index \(n > 1\), then the primordial BHs are formed in a wide range of masses. However a power law spectrum with \(n > 1\) is not favored by recent CMB observations, and a corresponding number density of resulting primordial BHs is extremely (exponentially!) small. This is because the primordial BHs form at the tail of the Gaussian distribution with probability

\[
\beta = \int_{\delta_c}^{1} \frac{d\delta_H}{\sqrt{2\pi}\Delta_H} \exp\left(-\frac{\delta_H^2}{2\Delta_H^2}\right) \simeq \frac{\Delta_H}{\delta_c\sqrt{2\pi}} \exp\left(-\frac{\delta_c^2}{2\Delta_H^2}\right).
\]
The value of r.m.s. fluctuation at the horizon scale $\Delta H$ can’t reach value $\simeq 0.05$ which is necessary to achieve $\beta \sim 10^{-8}$ and correspondingly the cosmological mass fraction of primordial BHs $\sim 10^{-3}$ today.

If, however, the spectrum has a peak at some scale, then primordial BHs are formed mostly in a narrow range of masses, near the mass that corresponds to this peak. The spectrum with a sharp maximum at some scale arises in some inflation models if the inflationary potential $V(\phi)$ has a flat segment (see e. g. [179, 180]). As it was shown in [181], the strong density fluctuations are generated in this models and collapse into SBHs. At the same time the spectrum beyond the maximum may be of the standard Harrison-Zeldovich form and reproduce the usual scenario of large-scale structure formation in the galactic distribution. The primordial BHs production from the maximum may be connected to the formation of noncompact DM objects at the matter-dominated epoch [200].

For the early formed primordial BHs, the process of mass increasing by the secondary accretion of DM is possible. Indeed, in the uniform Universe a primordial BH with mass $M_h$ inside the sphere containing total mass $M$ produces the gravitational fluctuation $\delta_{eq} = M_h/M$ at the epoch of matter-radiation equality $t_{eq}$. Subsequent growth of this fluctuation obeys the well-known law for DM fluctuations. As a result the primordial BHs would be “dressed” by the DM halo with a steep density profile, $\rho \propto r^{-9/4}$. We call this combined spherical volume of the “primordial BH +halo” by the “induced halo”. It was shown [48] that for primordial BH with a mass $\sim 10^5 M_\odot$ the resultant mass of induced halo is of the order of mass of a typical dwarf galaxy, $\sim 10^7 M_\odot$. During a hierarchical clustering of DM the induced halos enter into the haloes of ordinary galaxies. Many of these induced halos are capable to sink down to the galactic center during the Hubble time under the influence of dynamical friction.

5.5 Clusters of BHs from closed domain walls

5.5.1 General idea

Some inflationary models suppose a creation of our Universe either near a maximum of potential of inflaton field or near its saddle point(s) to realize a desired slow rolling providing a sufficient number of e-foldings (see details, e. g. in [201, 202]). As it will be shown below these models include the possibility of the formation of macroscopically large closed walls from a scalar field. After the end of inflation these closed walls collapse to BHs if these walls are large and heavy enough [192, 193]. This mechanism is realized in well known models like the Hybrid Inflation [203] and the Natural Inflation [204]. A scalar field could be the inflaton itself or some additional field.

First of all we consider a general mechanism of closed wall formation based on the quantum fluctuations near unstable point(s) like a saddle point or a maximum of potential of scalar field. An evolving scalar field may be split into a classical part, governed by the classical equation of motion, and the quantum
fluctuations. To facilitate the analysis, let us approximate a potential near its maximum as

$$V = V_0 - \frac{m^2}{2} \phi^2,$$  
(33)

where the maximum is assumed at $\phi = 0$ without the loss of generality. Then, a probability density to find a certain field value $\phi$ has a form (adapted to the considered case):

$$dP(\phi, T; \phi_{in}, 0) = d\phi \sqrt{\frac{a}{\pi(e^{2\mu T} - 1)}} \exp \left[ -a \frac{(\phi - \phi_{in} e^{\mu T})^2}{e^{2\mu T} - 1} \right].$$  
(34)

Here $a = \mu/\sigma^2$, $\mu \equiv m^2/3H$ and $\sigma = H^{3/2}/2\pi$, where the Hubble parameter $H \simeq \sqrt{8\pi V_0/(3M_{Pl})}$.

Let us choose a positive value for the initial field, $\phi_{in} > 0$, as illustrated in the Fig. Then an average field value will increase with time, ultimately reaching a minimum of the potential at some value $\phi_{+} > 0$. It means that a greater part of space will be finally filled with the field value $\phi = \phi_{+}$. Meanwhile, the field in some (small) space domain could jump over the maximum due to the quantum fluctuations. In the following, an average value of the field representing this

Figure 2: Quantum creation of walls during inflation. Right black point relates to the initial field value, $\phi_{in}$.
fluctuation tends to another minimum of the potential, \( \phi_- < 0 \). As the result, the space at the final stage will be filled by vacuum \( \phi_+ \) while some space domain is characterized by the field value \( \phi = \phi_- < 0 \). If one starts to move from inside of the domain to the outside, the path would start from a space point with \( \phi_- \) and finish at a space point with \( \phi_+ \). Hence, the path must contain a point with the maximum value of potential. It means that a wall is formed inevitably between such space domains and the “outer” space with \( \phi = \phi_+ \).

The “dangerous” values of fluctuations are those with \( \phi \leq 0 \). Such space domains will be surrounded by closed walls and if their number is too large it would strongly influence the dynamics of the early Universe. It is useful to calculate the probability of nucleation of these fluctuations. The latter depends on the initial field value \( \phi_{in} \) at the moment of creation of our Universe. The corresponding probability

\[
P_0(\phi_{in}, T) = \int_{\phi = -\infty}^{\phi = 0} dP(\phi, T; \phi_{in}, 0)
\]

(35)

to find the field value \( \phi \leq 0 \) at some space point for reasonable values of parameters is represented in Fig. 3. This probability determines a ratio of spatial volumes with different signs of the field. This probability is highly sensitive to the initial field value \( \phi_{in} \): a closer to the potential maximum it is nucleated, a greater part of the Universe will be covered with walls at the final stage.

If a fraction of space surrounded by the walls is not very large, the resulting massive BHs, which are formed from the walls, could explain the early formation of quasars [208]. In opposite case a contribution of BHs into the DM density is unacceptably large [207]. A further increasing of wall content leads to the wall-dominated Universe [209].

A mass and space distribution of resulting BHs strongly depends on a specific model and parameters of involved Lagrangian. It is instructive to demonstrate the mechanism of massive primordial BHs production in the framework of the hybrid inflation model [203] following to the results of [207].

5.5.2 SBH formation in the Hybrid Inflation

A potential of the hybrid inflation model according to [203] has the form

\[
V(\chi, \sigma) = \kappa^2 \left( M^2 - \frac{\chi^2}{4} \right)^2 + \frac{\lambda^2}{4} \chi^2 \sigma^2 + \frac{1}{2} m^2 \sigma^2.
\]

(36)

Inflationary expansion of the Universe takes place during a slow rolling along the valley \( \chi = 0, \sigma > \sigma_c \). When a field \( \sigma \) passes through the critical point\n
\[
\sigma_c = \sqrt{2M} \chi
\]

a motion along the line \( \chi = 0, \sigma < \sigma_c \) becomes unstable. As a result, a field \( \chi \) quickly moves to one of the minima, \( \chi_\pm = \pm 2M, \sigma = 0 \), see Fig. 4.
Figure 3: Part of space occupied by an another vacuum state, depending on the time measured by e-folds. The initial field value is \( \phi_{\text{in}} = \phi_{\text{max}} + 3H \), where \( \phi_{\text{max}} \) is a field value at a maximum of the potential. The parameter \( m \) is chosen to be \( m = 0.3H \), where \( H \) is the Hubble parameter at the top of potential.

where potential \( \phi_{\text{in}} \) for the hybrid inflation model is represented. Inflation is finished with the intensive field fluctuations around one of an accidentally chosen minimum.

It is clear now that this well elaborated picture suffers a serious problem nevertheless. During an inflationary stage, when the fields \( \sigma \) and \( \chi \) move classically along the line \( \chi = 0 \), the space is divided into many causally disconnected regions. The values of scalar fields within these regions are slightly different due to quantum fluctuations. An each e-fold produces approximately \( e^3 \approx 20 \) space domains. Hence there are about \( e^{180} \approx 10^{78} \) space domains right before the end of inflation. The values of field are chaotically distributed around the point \( \chi = 0, \sigma = \sigma_c \). The domains with a field value \( \chi < 0 \) tend to the left minimum \( \chi_- = -2M, \sigma = 0 \). Another part fall to the right minimum \( \chi_+ = +2M, \sigma = 0 \). A lot of walls between these domains appear and we come to a well known problem of the wall-dominated Universe [205].

The only way for our Universe to evolve into the modern state is to be created with an initial field value \( \chi_{\text{in}} \neq 0 \) at the beginning of inflation. During inflation, the field \( \chi \) must slowly approach to the critical line \( \chi = 0 \). If, in the middle of inflation, an average field value approaches to the critical line \( \chi = 0 \), the field in some part of space domains could cross this line due to fluctuations. In the future, these domains will be filled by vacuum, say, \( \chi_- \) surrounded by a sea of another vacuum \( \chi_+ \). These two vacua are separated by a closed wall as it was discussed above. A number of these walls strictly depends on the
Figure 4: A form of potential for the hybrid inflation model. Arrows indicate directions of classical motion of the fields. A black dot indicates the critical point.
initial conditions at the creation moment of our Universe.

Let us estimate the energy and size of closed walls in the framework of the hybrid inflation. To this end, let us suppose that a field in some volume crossed the critical line at e-folds number $N$ before the end of inflation. Its size is about the Hubble radius, $H^{-1}$, and it will be increased in $e^N$ times during inflation. A surface energy density of the domain wall after inflation defined by the potential (36) is

$$\epsilon = \frac{8\sqrt{2}}{3} \kappa M^3.$$  \hspace{1cm} (37)

Thus an energy $E_{\text{wall}}$ contained in the wall after inflation is at least

$$E_{\text{wall}} \simeq 4\pi \epsilon \left(H^{-1}e^N\right)^2 = 4\sqrt{2} \frac{M^2_{\text{Pl}}}{\kappa M} e^{2N}.$$  \hspace{1cm} (38)

A numerical value $N$ varies in the interval $(0 < N < N_U \simeq 60)$. These walls will collapse into the BHs with mass $M_{\text{BH}} \simeq E_{\text{wall}}$ (for details see [193]).

Let us estimate masses of these BHs for a representative values of parameters $\kappa = 10^{-2}$ and $M = 10^{16}$ GeV. If $N = 40$, we obtain for the mass of formed BH

$$M_{\text{BH}} \simeq 3 \times 10^{59} \text{ GeV} \sim 100M_{\odot}.$$  

A similar estimation for a smallest BHs, which are created at the e-fold number $N = 1$ before the end of inflation, gives

$$M_{\text{BH,small}} \simeq 10^6 M_{\text{Pl}}.$$  

Therefore, the hybrid inflation leads to the formation of BHs with mass in a rather wide range, $M_{\text{BH}} > 10^{25}$ GeV with an upper limit of $\sim 10^2 M_{\odot}$. An amount of massive BHs depends on how close an average field value approaches to the critical line $\chi = 0$. The last, in turn, depends on the initial conditions and specific values of parameters of this model.

5.5.3 Evolution of massive cluster of primordial BHs

The main lesson of previous discussion is that the massive primordial BHs are formed after inflation as a rule rather than as an exception. As was shown in [194] the mechanism described above leads to the nucleation of groups (clusters) of primordial BHs rather than to a single primordial BH. Here we discuss the idea that these clusters of primordial BHs could be the seeds for early galaxy formation [208]. Our consideration is based on the results published in [192, 193, 206], where the “Mexican hat” potential for the scalar field was considered. The calculations with the “Mexican hat” potential give the plausible parameters of clusters of primordial BHs.

In numerical calculations, the cluster of primordial BHs is modelled by a spherically symmetric system with a radius $r < ct$, consisting of primordial BHs with a total mass $M_h$ inside the radius $r$. To fix the cosmological evolution after inflation it is also supposed that the density of radiation is $\rho_r$, the density
of ordinary DM is $\rho_{DM}$ and the density of $\Lambda$-term is $\rho_\Lambda$. The main, most massive BH in a chosen cluster is in the center of the system. The density of radiation (and obviously the density of $\Lambda$-term) is homogenous. Therefore, the fluctuations induced by the primordial BHs are classified as entropy fluctuations. The scale under consideration is smaller than the horizon scale, and we use the Newtonian gravity with the prescription of treating the gravitation of homogenous relativistic components, $\rho \rightarrow \rho + 3p/c^2$.

The evolution of any fictitious spherical shell of considered cluster with an initial radius $r_i < r$ obeys the equation

$$\frac{d^2 r}{dt^2} = -\frac{G(M_h + M_{DM})}{r^2} - \frac{8\pi G \rho_r r}{3} + \frac{8\pi G \rho_\Lambda r}{3}, \quad (39)$$

with an approximate initial conditions at the moment $t_i$: $\dot{r} = H r$, $r(t_i) = r_i$. In obtaining equation (39), it was taken into account that $\varepsilon_r + 3p_r = 2\varepsilon_r$, $\varepsilon_\Lambda + 3p_\Lambda = -2\varepsilon_\Lambda$. Numerical calculations on the basis of equation (39) with an initial conditions indicate that the radius of the shell increases initially, and gradually its expansion decelerates. At some instant, a speed of expansion diminishes to zero. The shell is separated from the cosmological expansion and starts to shrink. All components of nonrelativistic matter — the DM, primordial BHs and baryons follow the dynamics of the shell surrounding them. As a result, the solutions of (39) for shells with different initial radii provide us the density distribution of the DM and primordial BHs in the evolving cluster.

It is suitable to rewrite the main equation by using a new variable $b(t)$.

$$r(t) = \xi a(t)b(t) \quad (40)$$

In this parametrization, $\xi$ is a comoving length, $a(t)$ is a scale factor of the Universe normalized to the present moment $t_0$ as $a(t_0) = 1$ and the function $b(t)$ characterizes the deflection of the cosmological expansion from the Hubble law. The variable $\xi$ is connected to the mass of DM inside the considered spherical volume (excluding total BHs mass) by the relation $M_{DM} = (4\pi/3)\rho_{DM}(t_0)\xi^3$, where $\rho_{DM}(t_0)$ is the modern DM density. The function $a(t)$ obeys the Friedman equation, which can be rewritten as $\ddot{a}/a = H_0 E(z)$, where $z = a^{-1} - 1$ is the redshift, $H_0$ is the current value of the Hubble constant and function $E(z)$ has a form

$$E(z) = [\Omega_{r,0}(1 + z)^4 + \Omega_{m,0}(1 + z)^3 + \Omega_{\Lambda,0}]^{1/2}, \quad (41)$$

where $\Omega_{r,0}$ is a density parameter of radiation, $\Omega_{m,0} \simeq 0.3$, $\Omega_{\Lambda,0} \simeq 0.7$, and $h = 0.7$. Using the second Friedman equation (for $\ddot{a}$), one can rewrite (39) as follows

$$\frac{d^2 b}{dz^2} + \frac{db}{dz}S(z) + \left(1 + \frac{\delta_h}{b^2} - b\right)\frac{\Omega_{m,0}(1 + z)}{2E^2(z)} = 0, \quad (42)$$

where function

$$S(z) = \frac{1}{E(z)}\frac{dE(z)}{dz} - \frac{1}{1 + z} \quad (43)$$

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and a value of fluctuation \( \delta_h = M_h / M_{DM} \). Equation (42) is equivalent to those obtained in [211] in the case \( \Omega_\Lambda = 0 \). We start tracing the evolution of cluster at a high redshift \( z_i \) when the considered shell crosses the horizon \( r \sim ct \).

Initial mass profile \( M_h(r_i) \) of the cluster of primordial BHs was calculated following to formalism developed in [194] and has the form presented in the Fig. 5 (upper panel). This numerical result is a starting point for the following analysis. The mass \( M_{DM}(r_i) \) of DM inside the same spherical shells is also shown for a comparison. The radius \( r_i \) is the physical size of sphere at the moment \( t_i \) and the temperature \( T_i \) corresponds to the moment when the sphere is crossing the cosmological horizon. Note that the radiiuses of shells in Fig. 5 are taken at different instants \( t_i \). Therefore, the shown mass of uniformly distributed DM not follows to the law \( M_{DM} \propto r^3 \) as it must be for a fixed moment, common for all spheres. Physical size at the temperature \( T_i \) is smaller than that in the modern epoch in \( T_0 / T_i \) times, where \( T_0 = 2.7 \text{ K} \). A density in the local center is so high that a lot of primordial BHs appear to be inside their common gravitational radius \( r_g = 2GM/c^2 \). According to numerical calculations with specific values of parameters, a total mass of primordial BHs appears inside the horizon from the beginning is \( 2.7 \times 10^4 \text{M}_\odot \). Hence, the massive BH with this mass is formed at first.

A widespread prejudice is that a mass of formed primordial BH could not exceed a total mass under the horizon at the same time, \( M \sim (t/t_{Pl})M_{Pl} \) [212]. As a result, the mass of primordial BH seems must be not larger than \( 10^3 \text{M}_\odot \). It is true indeed if one considers the formation of primordial BHs density fluctuations. On the contrary, in our case the mass of the formed primordial BHs is determined by the area of the closed walls, formed and stretched during inflation. Size of wall could be greater its horizon size in orders of magnitude. BH is nucleated when the wall collapse under its horizon.

The following dynamics of the cluster of primordial BHs together with DM component could finally lead to the formation of galaxies provided that the numerous collisions of galaxies are taken into account. Results of calculations made below are applicable both for an inner part of the cluster, composed mainly from primordial BHs and collapsing at the radiation dominated stage, and for an outer regions of the cluster, where DM is a main dynamical component. The outer regions are detached from the cosmological expansion at the matter dominated epoch.

The most early epoch in our calculations corresponds to the formation of central BH with a mass \( 2.7 \times 10^4 \text{M}_\odot \) described above. The temperature of the Universe at that time is \( \simeq 16 \text{ MeV} \). We suppose that DM has been already decoupled from radiation at this temperature. For example, neutralino DM particles with mass 100 GeV and slepton with mass 1 TeV decouples at the temperature \( \simeq 150 \text{ MeV} \) [213], well before the considered epoch. Therefore, the neutralino DM can be treated as moving freely under the influence of gravitational force only. As a result the clustering of two-component medium (BHs+DM) is described by the equation (42) from the very beginning. The same situation is realized for DM composed of the super-heavy particles with mass \( m_\chi \sim 10^{13} - 10^{14} \text{ GeV} \), which probably never been in the kinetic equi-
Figure 5: In the top panel the initial mass profile $M_h(r_i)$ of the cluster of primordial BHs and the mass profile $M_{DM}(r_i)$ of DM are shown. In the bottom panel, the r.m.s. density perturbation at the moment $t_{eq}$ of matter-radiation equality is shown. Two cases are compared: the total density fluctuations produced in the presence of the cluster of primordial BHs and those produced in a standard way as remnants of the inflationary stage.
librium with radiation. In the opposite case (for some another DM particles candidates), the growth of fluctuations in DM medium is suppressed by the friction on radiation until the kinetic decoupling, while the BHs are clustering. The super-heavy particles are preferable for our model in comparison with the neutralino because their annihilation cross-section is very small, $\propto m^{-2}_\chi$. In this case the problems with a huge annihilation signal from the considered protogalaxies are absent.

All scenarios imply a further merging of BHs during the multiple coalescences of host minihalos [214, 215, 216, 217] accompanied also by an additional gas accretion. It is worth to estimate the probability to find a nowadays galaxy without the SBH in the framework of the scenario discussed above. Induced galaxies containing massive BHs and ordinary small protogalaxies without BHs have masses around $M_{DM} = 10^8 M_\odot$, while nowadays galaxies are as massive as $M_{DM} = 10^{12} M_\odot$. Every collision of an induced galaxy with an ordinary protogalaxy produces the next generation of protogalaxy with a massive BH in its center. So about $10^4$ collisions have happened up to now. Suppose that the amount of induced galaxies is about 0.1% in comparison with the ordinary ones. Upper limit of probability to find a nowadays galaxy without a SBH is $0.999^{10^4} \simeq 4.5 \times 10^{-5}$. Hence, even a very small amount of induced galaxies is able to explain the observable abundance of e.g. the AGN.

6 Conclusions and Discussions

The existence of SBHs at the centers of galaxies looks like an unavoidable fact. The SBHs play an important role in the formation and global evolution of galaxies and the intergalactic medium. Nevertheless a valuable theory of SBHs formation is still absent. It appears not so easy to explain the whole set of observations which are biased and sometimes contradictive. The SBHs mass correlate with the mass of the host galaxies, though exceptions are known. For example, a suspected SBH was found recently in the dwarf galaxy VCC128 [218]. The maximum accretion rate onto the SBHs, at redshift $z \sim 0.7$ almost coincides with a peak density of luminous infrared galaxies at $z \sim 0.8$. One could expect a quick growth of already existing SBHs. At the same time, the most massive BHs are found much earlier, at $z > 6.0$.

At the recent epoch we are witnesses of impressive competition of two main directions in BH investigations. Several decades ago an idea was accepted that all BHs, if exist, result from the supernova explosions. Formation of galaxies seemed definitely precede a formation of BHs. But gradually this picture becomes more and more tangled. New observational data during the last 20 years indicate that almost all massive galaxies contain SBHs with a mass in the range $10^6 - 10^8 M_\odot$. Moreover, the discovery of quasars at high redshifts means a combined evolution of the galactic nuclei and SBHs. The ubiquity of SBHs in galaxies may be the indication of some special and universal mechanism for SBH formation: simultaneously with the galaxies or even before the formation of galaxies.
The nowadays observations reveal a very early formation of SBHs: they already formed at the epoch of 800 million years after the Big Bang. Considerable efforts are needed to explain so early existence of SBHs originated from the remnants of stars. It seems that an idea of Population III stars with mass exceeding $10^2 M_\odot$ is able to solve the problem. Nevertheless, it is not excluded that astrophysical scenario for SBH formation will not succeed.

Meantime new direction of primordial BHs research becomes more and more popular. The idea of primordial BHs is not very new — the first paper were published in 1966. [96, 97]. It was supposed that primordial BHs were produced from density fluctuations long before the star formation. These BHs are very light, $M_{BH} \ll 10^{15}$ g, and should be quickly evaporated due to the Hawking radiation. These BHs could not merge effectively into the rather massive ones. Nevertheless, an idea of primordial BHs was survived. Several absolutely different mechanisms of massive and even supermassive primordial BHs formation has been revealed and widely discussed [167, 175, 180, 182, 185, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194].

Competition between astrophysical and cosmological schemes of SBHs origin is going on. New observational data are analyzed from the both points of view. Both astrophysical and cosmological scenarios posses as advantages and some defects. The astrophysical scenarios of SBHs formation has rather firm basis but suffer from the lack of time and shortage of matter supply for building the SBHs in galaxies by means of accretion and merging. Meanwhile, the cosmological scenarios are based only on the supposed mechanisms but provide the way of early SBHs formation. It is not excluded that both scenarios of SBHs formation were realized in the Universe. More elaborate and predictive models and new detailed observations are requested to resolve the SBH enigma. The upcoming gravitational wave laser interferometers provide a very promising new channel for the exploration of SBH problem.

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References

[1] Begelman, M.C., Blandford, R.D., and Rees, M.C., 1984, Rev. Mod. Phys., 56, 255.
[2] Rees, M.J. 1984, Ann. Rev. Astron. Astrophys., 22, 471.
[3] Antonucci, R.R.J. 1993, Ann. Rev. Astron. Astrophys., 31, 473.
[4] Ferrarese, L., and Ford, H. 2004, arXiv:astro-ph/0411247
[5] Falcke, H., and Hehl, F.W. (Eds.) 2003, The Galactic black holes, Series in high energy physics, cosmology and gravitation. IOP Publishing Ltd, London.

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[6] Gnedin, O.Y. 2001, Class. Quant. Grav., 18, 3983.

[7] Li, Y., et al. 2006, arXiv:astro-ph/0608190.

[8] Fan, X., et al. 2003, Astron. J., 125, 1649.

[9] K. Schwarzschild, 1916, Sitzungsber. Preuss. Akad. Wiss., Berlin (Math. Phys.) 189; (English translation in arXiv:physics/9905030).

[10] Peters, P.S., and Mathews, J. 1963, Phys. Rev. D, 131, 435.

[11] Magain, P. et al. 2005, Nature 437, 381.

[12] Haehnelt, M.G., Davies, M.B., and Rees, M.J. 2006, Mon. Not. Roy. Astron. Soc., 366, L22.

[13] Cavaliere, A., and Vittorini, V. 2001, arXiv:astro-ph/0110644.

[14] Haehnelt, M.G., Natarajan, P., and Rees, M.J. 1998, Mon. Not. Roy. Astron. Soc., 300, 817.

[15] Kauffmann, G., and Haehnelt, M.G. 2000, Mon. Not. Roy. Astron. Soc., 311, 576.

[16] Choi, Y., Yang, J., and Yi, I. 2000, arXiv:astro-ph/0005590.

[17] Hopkins, P.F. et al. 2005, Astrophys. J., 630, 705.

[18] Mack, K.J., Ostriker, J.P., and Ricotti, M. 2006, arXiv:astro-ph/0608642.

[19] Marziani, P., Dultzin-Hacyan, D., Jack, W., and Sulentic, J.W. 2006, in New Development in Black Hole Research, Kreitler P.V. (Editor), New York: Nova Science Publishers, pp. 123-183.

[20] Tal, A. 2006, Phys. Rep., 419, 65.

[21] Gezari, et al. 2006, Astrophys. J., 630, L25.

[22] Rice, W.K.M., Lodato, G., and Armitage, P.J. 2005, Mon. Not. Roy. Astron. Soc., 364, L56.

[23] Paumard, T. et al. 2006, Astrophys. J., 643, 1011.

[24] Nayakshin, S. 2007, arXiv:astro-ph/0701150.

[25] Weaver, H., et al. 1965, Nature, 208, 29.

[26] Churchwell, E., et al. 1977, Astron. Astrophys., 54, 969.

[27] Kondratko, P.T., Greenhill, L.J., and Moran 2006, Astrophys. J., 652, 136.

[28] Nandra, K., et al. 1997, Astrophys. J., 477, 602.
[29] Chen, K., and Halpern, J.P. 1998, Astrophys. J., 344, 115.
[30] Fabian, A.C., Rees, M., Stella, L., and White, N.E. 1989, Mon. Not. Roy. Astron. Soc., 238, 729.
[31] Matt, G., Perola, G.C., and Stella, L. 1993, Astron. Astrophys., 267, 643.
[32] Zakharov, A.F. 1991, Soviet Astron., 35, 30.
[33] Fabian, A.C., Iwazawa, K., Reynolds, C.S., and Young, A.J. 2000, Publ. Astron. Soc. Pac., 112, 1145.
[34] Zakharov, A.F. et al. 2005, arXiv:gr-qc/0507118.
[35] Pariev, V.I., and Bromley, B.C. 1998, Astrophys. J., 508, 590.
[36] Pariev, V.I., Bromley, B.C., and Miller, W.A. 2001, Astrophys. J., 547, 649.
[37] Shen, Z.-Q., Lo, K.Y., Liang, M.-C., Ho, P.T.P., and Zhao, J.H. 2005, Nature, 438, 62.
[38] Kormendy, J., and Richstone, D.O. 1995, Ann. Rev. Astron. Astrophys., 33, 581.
[39] Magorrian, J. et al. 1998, Astron. J, 115, 2285.
[40] Kazantzidis, S. et al. 2005, Astrophys. J., 623, L67.
[41] Gebhardt, K. et al. 2000, Astrophys. J., 539, L642.
[42] Ferrarese, L., and Merritt, D. 2000, Astrophys. J., 539, L9.
[43] Tremaine, S. et al. 2002, Astrophys. J., 574, 740.
[44] Salviander, S., Shields, G.A., Gebhardt, K., and Bonning, E.W. 2007, Astrophys. J., 662, 131.
[45] Merrifield, M.R., Forbes, D.A., and Terlevich, A.I. 2000, Mon. Not. Roy. Astron. Soc., 313, L29.
[46] Silk, J., and Rees, M.J. 1998, Astron. Astrophys., 331, L4.
[47] Haehnelt, M.G., and Kauffmann, G. 2000, Mon. Not. Roy. Astron. Soc., 318, L35.
[48] Dokuchaev, V.I., and Eroshenko, Yu.N. 2003, Astron. Astrophys. Trans., 22, 727.
[49] Fukugita, M., and Turner, E.L. 1996, Astrophys. J., 460, L81.
[50] Kaaret, P. et al. 2001, Mon. Not. Roy. Astron. Soc., 321, L29.
[51] Mouri, H., and Taniguchi, Y. 2002, Astrophys. J., 566, L17.
[52] Gebhardt, K., Rich R.M., and Ho L.C. 2002, Astrophys. J., 578, L41.
[53] Trenti, M. 2006. arXiv:astro-ph/0612040
[54] van der Marel, R.P. et al. 2002. arXiv:astro-ph/0209314
[55] Dokuchaev, V.I., and Eroshenko, Yu.N. 2002, Zh. Eksp. Teor Fiz, 121, 5 (JETP, 2002 94, 1).
[56] Gurevich, A.V., Zybin, K. P., and Sirotc, V.A. 1997, Uspekhi Fiz, Nauk, 40, 913 (Physics — Uspekhi, 1997, 40, 869).
[57] Haehnelt, M.G. 1994, Mon. Not. Roy. Astron. Soc., 269, 199; ibid. 2003, Class. Quant. Grav., 20, S31.
[58] Menou, K., Haiman, Z., and Narayanan, V.K. 2001, Astrophys. J., 558, 535.
[59] Wyithe, J.S.B., and Loeb, A. 2003, Astrophys. J., 590, 691.
[60] Berti, E., Buonanno, A., and Will, C.M. 2005, Class. Quant. Grav., 22, S943.
[61] Moore, B. 1993, Astrophys. J. Lett., 413, 93.
[62] Carr, B.J. 1975, Astrophys. J., 201, 1.
[63] Nemiroff, R.J., Marani, G.F., Norris, J.P., and Bonnel, J.T. 2001, Phys. Rev. Lett., 86, 580.
[64] Wilkinson, P.N. et al. 2001, Phys. Rev. Lett., 86, 584.
[65] Gondolo, P., and Silk, J. 1999, Phys. Rev. Lett., 83, 1719.
[66] Ulio, P., Zhao, H., and Kamionkowski, M. 2001, Phys. Rev. D, 64, 043504.
[67] Merritt, D., Milosavljevic, M, Verde, L., and Jimenez, R. 2002, Phys. Rev. Lett., 88, 191301.
[68] Zhao, H., and Silk, J. 2005, Phys. Rev. Lett., 95, 011301.
[69] MacMillan, J.D., and Henriksen, R.N. 2002, Astrophys. J., 569, 83.
[70] Spitzer, L. Jr. 1987, Dynamical evolution of Globular Clusters, Princeton Univ. Press, Princeton, New Jersey.
[71] Saslaw, W. 1987, Gravitational physics of stellar and galactic systems, Cambridge Univ. Press, Cambridge.
[72] Dokuchaev, V.I. 1991. Soviet Phys. Usp., 34, 447 (1991, Usp. Fiz. Nauk, 161, 1).
[73] Spitzer, L. Jr., and Saslaw W. 1966, Astrophys. J., 143, 400.
[74] Sanders, R.H. 1970, Astrophys. J., 162, 791.
[75] Spitzer, L., and Hart, M.H. 1971, Astrophys. J., 164, 399.
[76] Merritt, D. 2006, Rep. Prog. Phys., D69, 2513.
[77] Fabian, A.C., Pringle, J.E., and Rees, M.J. 1975, Mon. Not. Roy. Astron. Soc., 172, 15P.
[78] Heggie, D.C. 1975, Mon. Not. Roy. Astron. Soc., 173, 729.
[79] Press, W.H., and Teukolsky, S.A. 1977, Astrophys. J., 213, 183.
[80] Begelman, M.C., and Rees, M.J. 1978, Mon. Not. Roy. Astron. Soc., 185, 847.
[81] Quinlan, G.D., and Shapiro, S.L. 1987, Astrophys. J., 321, 199.
[82] Quinlan, G.D., and Shapiro, S.L. 1989, Astrophys. J., 343, 725.
[83] Quinlan, G.D., and Shapiro, S.L. 1990, Astrophys. J., 356, 483.
[84] Berezinsky, V.S., and Dokuchaev, V.I. 2001, Astropart. Phys., 15, 87.
[85] Berezinsky, V.S., and Dokuchaev, V.I. 2006, Astron. Ap., 454, 401.
[86] Landau, L.D., and Lifshitz, E.M. 1960, Mechanics, Eddison-Wesley, Reading Mass.
[87] Chandrasekhar, S. 1943, Rev. Mod. Phys., 15, 21.
[88] Spitzer, L., and Harm, R. 1958, Astrophys. J., 127, 544.
[89] Lightman, A.P., and Shapiro S.L. 1978, Rev. Mod. Phys., 50, 437.
[90] Milgrom, M., and Shapiro S.L. 1978, Astrophys. J., 223, 991.
[91] Ozernoy, L.M., and Dokuchaev, V.I. 1982, Astron. Ap., 111, 1.
[92] Dokuchaev, V.I., and Ozernoy, L.M. 1982, Astron. Ap., 111, 16.
[93] Hoyle, F., and Fowler, W. 1963, Nature, 197, 533.
[94] Fowler, W. 1966, Astrophys. J., 144, 180.
[95] Fowler, W. 1966, Rev. Mod. Phys., 36, 545 (erratum 36, 1104).
[96] Zel’dovich, Ya.B., and Novikov, I.D. 1966, Astron. Zh., 43, 758 (Sov. Astron. — A. J., 1976, 10, 602).
[97] Hawking, S. W. 1971, Mon. Not. Roy. Astron. Soc., 152, 75.
[98] Zel’dovich, Ya.B., and Novikov, I.D. 1971, Relativistic Astrophysics, Vol. 1: Stars and Relativity, Univ. of Chicago Press, Chicago.

[99] Haehnelt, M.G., and Rees, M.J. 1993, Mon. Not. Roy. Astron. Soc., 263, 168.

[100] Eisenstein, D.J., and Loeb, A. 1995, Astrophys. J., 443, 11.

[101] Iben, I. 1963, Astrophys. J., 138, 1090.

[102] Chandrasekhar, S. 1964, Phys. Rev. Lett., 12, 114 (erratum 12, 437).

[103] Chandrasekhar, S. 1964, Astrophys. J., 140, 417.

[104] Chandrasekhar, S. 1965, Astrophys. J., 142, 1488.

[105] Lee, H.M. 1987, Astrophys. J., 319, 851.

[106] Saijo, M.T., Baumgarte, W., Shapiro, S.L., and Shibata, M. 2002, Astrophys. J., 569, 349.

[107] Shibata, M., and Shapiro, S.L. 2002, Astrophys. J., 572, L39.

[108] Shibata, M. 2003, Astrophys. J., 595, 992.

[109] Benz, W., and Hills, J.G. 1992, Astrophys. J., 389, 546.

[110] Lai, D., Rasio, F.A., and Shapiro, S.L. 1993, Astrophys. J., 412, 593.

[111] Shapiro, S.L., and Teukolsky, S.A. 1983, Black Holes, White Dwarfs and Neutron Stars, Willey, New-York.

[112] New, K.C., and Shapiro, S.L. 2001, Astrophys. J., 548, 439.

[113] Kimberly, C.B., and Shapiro, S.L. 2001, Class. Quant. Grav., 18, 3965.

[114] Zel’dovich, Ya.B., and Podurets, M.A. 1965, Astron. Zh., 42, 963 (Sov. Astron. — A. J., 1966, 9, 742).

[115] Shapiro, S.L., and Teukolsky, S.A. 1985, Astrophys. J., 292, L41.

[116] Shapiro, S.L., and Teukolsky, S.A. 1985, Astrophys. J., 298, 34.

[117] Shapiro, S.L., and Teukolsky, S.A. 1985, Astrophys. J., 298, 58.

[118] Shapiro, S.L., and Teukolsky, S.A. 1986, Astrophys. J., 307, 575.

[119] Dokuchaev, V.I., and Ozernoy, L.M. 1977, Pisma Astron. Zh., 3, 391 (Sov. Astron. Lett., 1977, 3, 209).

[120] Dokuchaev, V.I. 1991., Mon. Not. Roy. Astron. Soc., 251, 564.

[121] Ilyin, A.S., Zybin, K.P., and Gurevich, A.V. 2004, Zh. Eksp. Teor. Fiz., 98, 5 (JETP, 2004, 98, 1).
[122] Benson, A.J., Kamionkowski, M., and Hassani, S.H. 2005, Mon. Not. Roy. Astron. Soc., 357, 847.

[123] Erickcek, A.L., Kamionkowski, M., and Benson, A.J. 2006, Mon. Not. Roy. Astron. Soc., 371, 1992.

[124] Tyler, C., Janus, B., and Santos-Noble, D. 2003, \texttt{arXiv:astro-ph/0309008}.

[125] Rees, M.J. 1992, in Physics of Active Galactic Nuclei, eds. W.J. Duschl, S.J. Wagner, Springer-Verlag, Berlin, 662.

[126] Kawaguchi, T., Aoki, K., Ohta, K. and Collin, S. 2004, Astron. Astrophys., 420, L23.

[127] Babichev, E., Dokuchaev, V., and Eroshenko, Yu. 2004, Phys. Rev. Lett., 93, 021102.

[128] Babichev, E., Dokuchaev, V., and Eroshenko, Yu. Zh. Eksp. Teor. Fiz. 127, 597 (JETP, 2005 100, 528

[129] Fukushima, T., Ebisuzaki, T., and Makino, J. 1992, Astrophys. J., 396, L61.

[130] Hopkins, P.F., Robertson, B., Krause, E., Hernquist, L., and Cox, T.J. 2006, Astrophys. J., 652, 107.

[131] Haehnelt, M.G., and Kauffmann, G. 2002, Mon. Not. Roy. Astron. Soc., 336, L61.

[132] Valtaoja, L., and Valtonen, M.J. 1989, Astrophys. J., 343, 47.

[133] Volonteri, M., Haardt, F., and Madau, P. 2003, Astrophys. J., 582, 559.

[134] Bromm, V., and Loeb, A. 2003, Astrophys. J., 596, 34.

[135] Islam, R.R., Taylor, J.E., and Silk, J. 2004, Mon. Not. Roy. Astron. Soc., 354, 427.

[136] Cattaneo, A., Blaizot, J., Devriendt, J., and Guiderdoni, B. 2005, Mon. Not. Roy. Astron. Soc., 364, 407.

[137] Volonteri, M., Lodato, G., and Natarajan P. 2007, \texttt{arXiv:0709.0529 [astro-ph]}.

[138] Ballo, L. et al. 2004, Astrophys. J., 600, 634.

[139] Komossa, S., et al. 2003, Astrophys. J., 582, L15.

[140] Smirnova, A.A., Moiseev, A.V., and Afanasiev, V.L. 2006, Astron. Lett., 32, 520.
[141] Springel, V., Di Matteo, T., and Hernquist, L. 2005, Astrophys. J., 620, L79.
[142] Escala, A., Larson, R.B., Coppi, P.S., and Mardones, D. 2005, Astrophys. J., 630, 152.
[143] Merritt, D., and Ekers, R.D. 2002, Science, 297, 1310.
[144] Campanelli, M. 2005, Class. Quant. Grav., 22, S387.
[145] Merritt, D. 2006, Astrophys. J., 648, 976.
[146] Berczik, P., Merritt, D., Spurzem, R., and Bischof, H.-P. 2006, Astrophys. J., 642, L21.
[147] Masjedi, M. et al. 2006, Astrophys. J., 644, 54.
[148] Madau, P., Rees, M.J. 2001, Astrophys. J., 551, L27.
[149] Fryer, C. L., Woosley, S. E., and Heger, A. 2001, Astrophys. J., 550, 372.
[150] Larson, R.B. 2000, in Star Formation from the Small to the Large Scale, eds. F. Favata, A. Kaas and A. Wilson, ESA SP-445, ESA, Noordwijk, p. 13.
[151] Schneider, R., Ferrara, A., Ciardi, B., Ferrari, V., and Matarrese, S. 2000, Mon. Not. Roy. Astron. Soc., 317, 385.
[152] Koushiappas, S.M., Bullock, J.S., and Dekel, A. 2004, Mon. Not. Roy. Astron. Soc., 354, 292.
[153] Begelman, M.C. 2007, To appear in First Stars III, proc. of the conf. in Santa Fe, NM, July 16-20, 2007, eds. B. O’Shea, A. Heger and T. Abel. AIP Conf. Proc.; [arXiv:0709.0545 [astro-ph]].
[154] Susa, H., Sasaki, M., and Tanaka T. 1994, Prog. Theor. Phys., 92, 961.
[155] Loeb, A. 1993, Astrophys. J., 403, 542.
[156] Loeb, A., and Rasio, F.A. 1994, Astrophys. J., 432, 52.
[157] Eisenstein, D., and Loeb, A., 1995, Astrophys. J., 443, 11.
[158] Eisenstein, D.J., and Loeb, A. 1995, Astrophys. J., 439, 520.
[159] Gurevich, A.V., and Zybin, K.P. 1990, Zh. Eksp. Teot. Fiz., 97, 20.
[160] Sirota, V.A., Ilyin, A.S., Zybin, K.P., and Gurevich, A.V. 2004, [arXiv:astro-ph/0403023]
[161] Ilyin, A.S., Zybin, K.P., and Gurevich, A.V. 2004, JETP, 98, 1.
[162] Zelnikov, M.I., and Vasiliev, E.A. 2005, JETP Lett., 81, 85.
[163] Haehnelt, M.G., and Rees, M.J. 1993, Mon. Not. Roy. Astron. Soc., 263, 168.
[164] Loeb, A., and Rasio, F.A. 1994, Astrophys. J., 432, 52.
[165] Eisenstein, D.J., and Loeb, A. 1995, Astrophys. J., 443, 11.
[166] Bromm, V., and Loeb, A. 2003, Astrophys. J., 596, 34.
[167] Carr, B. 2005. arXiv:astro-ph/0511743
[168] Khlopov, M.Yu., and Polnarev, A.G. 1980, Phys. Lett. B 97, 383.
[169] Barrow, J.D., and Turner, M.S. 1981, Nature, 291, 469.
[170] Bond, J.R., Kolb, E.W., and Silk, L. 1982, Astrophys. J., 255, 341.
[171] Fukujita, M., and Rubakov, V.F. 1986, Phys. Rev. Lett., 56, 988.
[172] Dolgov, A.D., Illarionov, A.F., Kardashev, N.S., and Novikov, I.D. 1987, Zh. Eksp. Teor. Fiz., 94, 1 (Sov. Phys. JETP, 1988, 67, 213].
[173] Kofman, L.A., and Linde, A.D. 1987, Nucl. Phys., B282, 555.
[174] Kofman, L.A., and Pogosyan, D.Yu. 1988, Phys. Lett. B 214, 508.
[175] Dolgov A.D., and Kirillova, D.P. 1991, J. Moscow Phys. Soc, 1, 217.
[176] Chizhov, M.V., and Dolgov, A.D. 1992, Nucl. Phys. B327, 527.
[177] Yokoyama J., Suto, Y. 1991, Astrophys. J., 379, 427.
[178] Dolgov A., and Silk, L. 1993, Phys. Rev. D 47, 4244.
[179] Starobinsky, A.A. 1992, Pisma Zh. Eksp. Teor. Fiz., 55, 477 (Sov. JETP Lett., 1992, 55, 489).
[180] Ivanov, P., Naselsky, P., and Novikov, I. 1994, Phys. Rev. D., 50, 7173.
[181] Yokoyama, J. 1998, Phys. Rev. D, 58, 083510.
[182] Jedamzik, K. 1997, Phys. Rev. D, 55, 5871.
[183] Jedamzik, K. 1998, Phys. Rep., 307, 155.
[184] Crawford, M., and Schramm, D.N. 1982, Nature, 298, 538.
[185] Hawking, S.W., Moss, I., and Stewart, J. 1982, Phys. Rev. D, 26, 2681.
[186] Kodama, H., Sato, K., and Sasaki, M. 1982, Prog. Theor. Phys., 68, 1979.
[187] La, D., and Steinhardt, P.J. 1989, Phys. Lett. B., 220, 375.
[188] Moss, I.G. 1994, Phys. Rev. D, 50, 676.
[189] Khlopov, M.Yu., Konoplich, R.V., Rubin, S.G., and Sakharov, A.S. 1999, Grav. Cosm., 2, 1.
[190] Konoplich, R.V., Rubin, S.G., Sakharov, A.S., and Khlopov, M.Yu. 1999, Phys. Atom. Nuc., 62, 1593.
[191] Dynnikova, I., Kozel, L., Khlopov, M. and Rubin, S. 2000 Grav.& Cosm., vol. 6, pp. 311–318.
[192] Rubin, S.G., Khlopov, M.Y., and Sakharov, A.S. 2000, Grav.& Cosm., 56, 51.
[193] Rubin, S.G., Khlopov. M.Y., and Sakharov, A.S. 2001, JETP, 92, 921
[194] Khlopov. M.Y., Rubin, S.G., and Sakharov, A.S., 2005, Astrop. Phys., 23, 265.
[195] Niemeyer, J.C., and Jedamzik, K. 1998, Phys. Rev. Lett., 80, 5481.
[196] Niemeyer, J.C., and Jedamzik, K. 1999, Phys. Rev. D 59, 124013.
[197] Niemeyer, J.C., and Jedamzik, K. 1999, Phys. Rev. D 59, 124014.
[198] Choptuik, M.W. 1993, Phys. Rev. Lett., 70, 9.
[199] Yokoyama, J. 1998, Phys. Rev. D, 58 107502.
[200] Dokuchaev, V.I., and Eroshenko, Yu.N. 2002, Zh. Eksp. Teot. Fiz, 121, 5 (JETP, 2002, 94, 1).
[201] Dvali, G., and Kachru, S. 2003, arXiv:hep-th/0309095
[202] Blanco-Pillado, J.J. et al. 2004, JHEP, 0411, 063.
[203] Linde, A. 1991, Phys. Lett. B 259, 38.
[204] Dolgov, A., Freese, K., Rangarajan, R., and Srednicki, M. 1997, Phys. Rev. D, 56, 6155.
[205] Starobinsky, A. 1986, in Field Theory, Quantum Gravity and Strings, ed. H.J. de Vega and N. Sanchez, 107.
[206] Khlopov, M.Yu., and Rubin, S.G. 2004, Cosmological Pattern of Microphysics in the Inflationary Universe, Kluwer Academic, Dordrecht, Netherlands.
[207] Rubin, S.G. 2003, in Proc. conf. I. Ya. Pomeranchuk and physics at the turn of the century, Moscow, p. 413, arXiv:astro-ph/0511181
[208] Dokuchaev, V., Eroshenko, Y., and Rubin, S. 2005, Grav. Cosm. 11, 41.
[209] Zeldovich, Ya.B., Kobzarev, I.Iu., and Okun, L.B. 1974, Zh. Eksper. Teor. Fiz., 67, 3 (Sov. Phys. — JETP, 1975, 40, 1).
[210] Tolman, R.C. 1930, Phys. Rev., 35, 875.
[211] Kolb, E.W., and Tkachev, I.I. 1994, Phys. Rev. D, 50, 769.
[212] Carr, B.J., Gilbert, J.H., and Lidsey J.E. 1994, Phys. Rev. D, 50, 4853.
[213] Schwarz, D.J. 2003, Annalen Phys., 12, 220.
[214] Koushiappas, S.M., and Zentner, A.R. 2006, Astrophys. J., 639, 7.
[215] Koushiappas, S.M., Bullock, J.S., and Dekel, A. 2004, Mon. Not. Roy. Astron. Soc., 354, 292.
[216] Volonteri, M., Haardt, F., and Madau, P. 2003, Astrophys. J., 582, 559.
[217] Bromm, V., and Loeb, A. 2003, Astrophys. J., 596, 34.
[218] Debattista, V.P., et al. 2006, Astrophys. J., 651, L97.