On the Temperatures of Planetary Magnetosheaths at the Subsolar Points

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Abstract This research explores the relationship between the temperatures of the solar corona and planetary magnetosheaths. Based on the second law of thermodynamics, the maximum temperature of the planetary magnetosheaths cannot exceed that of the solar corona. A theoretical investigation is presented into the expansion of the solar corona, the propagation of solar wind, and the compression of planetary magnetosheaths by bow shocks. The method used is general and fits the dynamics of multiple components, thermal anisotropy, and non-Maxwellian plasmas in the steady state, and approximate formulas are obtained. The results indicate that, for the steady state, planetary magnetosheaths at the subsolar points in the solar system have approach peak mean temperatures. Second, a systematic statistical survey of the average temperature of the planetary magnetosheaths is presented and shows that the average plasma temperature of the subsolar point magnetosheaths of Earth and Saturn are 206 eV (2.39 MK) and 171 eV (1.98 MK), respectively, which are close to that of the corona. The statistical results are consistent with the theoretical estimations. These results are of significant use for estimating the thermal properties of the planetary magnetospheres.

Plain Language Summary This research focuses on the relationship between the temperatures of the planetary magnetosheaths and that of the solar corona. It is thus an interdisciplinary problem in solar-terrestrial physics. A theoretical investigation is presented on the expansion of the solar corona, the propagation of the solar wind, and the compression of the planetary magnetosheath by bow shocks. An approximate formula for the relationship between the temperatures of the solar corona and that of planetary magnetosheaths is obtained. The quantitative results indicate that the peak temperature of the planetary magnetosheaths are comparable and approach that of the solar corona. A statistical investigation is made of the average temperatures of the magnetosheaths of several planets, and it shows that, although the proton temperatures are several times the electron temperatures, the average plasma temperatures of the magnetosheaths of Earth and Saturn are almost the same as that of the solar corona. This work advances our understanding of the thermal properties of planetary magnetosheaths and also advances research into the formation of plasma sheets.

1. Introduction

Magnetosheaths are important sources of plasmas in the planetary magnetospheres; they partially control the thermal state of magnetospheric plasma and play a critical role in the dynamical evolution of the planetary magnetospheres (Axford & Hines, 1961; Chapman & Ferraro, 1930; Dungey, 1961; Fujimoto et al., 1998; Phan et al., 2000; Song et al., 1990, 1992, 1994, 1999a,b; Southwood & Kivelson, 1995; Taylor et al., 2008; Wang et al., 2012). The solar wind, which originates from the solar corona, interacts with the intrinsic magnetic field of planets and shapes the magnetosheaths. Upon impacting the magnetosheaths, the supersonic solar wind forms bow shocks, which surround the magnetosheaths with conicoid surface shapes (Chao et al., 2002; Dmitriev et al., 2003; Shen et al., 2007, 2020; Shue, 1998). The upstream solar wind is compressed by the bow shocks to form the downstream magnetosheath plasmas between the bow shocks and the magnetopause (Chao & Zhang, 1995; Génot, 2009). The density and temperature of the magnetosheath plasmas are maximal at the stagnation point and gradually decrease downstream (Song et al., 1990, 1999a; Spreite & Alksne, 1969). Therefore, the thermal properties of the magnetosheath plasmas should be closely related to the features of the solar corona.

The corona is extremely hot, with a temperature of millions of kelvin, which is much hotter than the photosphere of the Sun (≈5,770 K), and the heating mechanism for the corona remains unclear and is still under investigation.
The extremely hot corona expands outward and generates outflowing solar wind into interplanetary space (Barnes, 1992; Cranmer et al., 2017; Marsch, 1999; Parker, 1958). Near the Sun, the solar wind accelerates rapidly (McComas et al., 2007) and, with increasing distance from the Sun, the velocity of the solar wind increases whereas its density and temperature gradually decrease (Barnes et al., 1992; Cranmer et al., 2017; Koet et al., 1999; Marsch, 1999; McComas et al., 2007, 2008; Parker, 1958). As per Parker’s model (Parker, 1958), the velocity $V$ of the solar wind far from the Sun varies with the heliocentric distance $r$ and approximately follows the formula $V = V_r \sqrt{\ln(r/r_0)}$, where $r_0$ is the Parker critical heliocentric distance and $V_r$ is the critical velocity, and the density $n$ of the solar wind is $n \propto (V/r)^2$ due to conservation of matter. The temperature of the solar wind decreases with distance from the Sun as $T \propto r^{-2}$, where the factor $\alpha$ ranges from 2/7 to 4/3 (Barnes, 1992; Cranmer et al., 2009; McComas et al., 2008; Scudder, 2015). Weber and Davis (1967) established a steady-solar-wind model that considers the azimuthal component of the solar wind and indicates that the magnetic field in the solar wind may apply a torque to the Sun and lead to the loss of the angular momentum of the Sun. As the solar wind reaches the Earth at a heliocentric distance of 1 AU, measurements (Burlaga and Szabo, 1999) reveal that its mean velocity $\langle V \rangle \approx 400 \text{ km/s}$, its mean density $\langle n \rangle \approx 5 \text{ cm}^{-3}$, its average electron temperature $\langle T_e \rangle \approx 0.1-0.2 \text{ MK}$ ($12-25 \text{ eV}$), its average proton temperature $\langle T_p \rangle \approx 0.01-0.40 \text{ MK}$ ($1.2-50 \text{ eV}$), and its average plasma temperature is $\langle T \rangle_{sw} \approx \frac{1}{3} \langle T_e + T_p \rangle \approx 0.13 \text{ MK}$ ($15 \text{ eV}$), if helium ions are omitted (Burlaga and Szabo, 1999; Cranmer et al., 2017).

On the other hand, the solar wind expands with the magnetic field frozen in. According to the Parker spiral field model, the radial component of the interplanetary magnetic field (IMF) $B_r \propto 1/r^2$ and its azimuthal component $B_\phi \propto (r - r_0)/r^2$ (Parker, 1958). Near Earth at 1 AU, the magnetic strength of the IMF is $\approx 6 \text{ nT}$ with the IMF spiral angle being $\approx 45^\circ$, and the ratio of thermal to magnetic energy in the solar wind is $\beta \approx 0.1-6.0$ (Burlaga and Szabo, 1999).

If the expansion of corona and solar wind can be regarded as a heat engine process, its thermal efficiency at 1 AU is $\zeta = \langle (T)_{sw} - (T)_{sw} \rangle / \langle (T)_{sw} \rangle \approx (30 - 1.3) \times 10^2 / (3 \times 10^2) = 96\%$. Thus, the solar corona heat engine rather effectively transfers heat into kinetic energy. When the solar wind passes by the planets, such as Mercury, Earth, Jupiter, or Saturn, it impacts their magnetic field to produce planetary magnetospheres and bow shocks. The upstream solar wind traverses the bow shocks and is compressed to form the denser and hotter plasmas of the magnetosheaths (Chapman et al., 2004; Masters et al., 2011; Petricec & Russell, 1997). The physical parameters of the upstream solar wind and downstream magnetosheath plasmas approximately obey the Rankine–Hugoniot jump conditions (Hudson, 1970; Liu et al., 2007). The observations by the Mercury Surface, Space Environment, Geochemistry, and Ranging (MESSENGER) craft show that, the proton temperature in Mercury’s magnetosheath at the subsolar point is $T_p \approx 1.2-8.4 \text{ MK}$ ($150-980 \text{ eV}$) with the most probable proton temperature being $T_{p,\text{mp}} \approx 3 \text{ MK}$ ($350 \text{ eV}$) (Gershman et al., 2013). Under extreme solar-wind conditions, the proton temperature in Mercury’s magnetosheath can reach up to 6.0 MK ($700 \text{ eV}$) (Slavin et al., 2014). According to observations by the Double Star Project from 2004 to 2005 (Liu et al., 2005; Shen & Liu, 2005) the time-averaged electron temperature of Earth's magnetosheath on the dayside is $T_e \approx 50 \text{ eV}$, while the time-averaged ion temperature is $T_i \approx 200 \text{ eV}$ (Shen et al., 2008). Therefore, the average plasma temperature of Earth's magnetosheath on the dayside is about $\langle T \rangle_{\text{sh}} \approx \frac{1}{2} \langle T_e + T_i \rangle \approx 125 \text{ eV}$. A statistical analysis of the THEMIS observations (Wang et al., 2012) indicates that, at the subsolar point of Earth's dayside magnetosheath, the mean electron and ion temperatures are $T_e \approx 40 \text{ eV}$ and $T_i \approx 210 \text{ eV}$, respectively; thus, the average plasma temperature at the subsolar point of Earth's magnetosheath is $\langle T \rangle_{\text{sh}} \approx \frac{1}{2} \langle T_e + T_i \rangle \approx 125 \text{ eV}$. Given a fast solar wind, the mean electron and ion temperatures at the subsolar point of Earth's magnetosheath are $T_e \approx 53 \text{ eV}$ and $T_i \approx 400 \text{ eV}$, respectively, with the average plasma temperature being $\langle T \rangle_{\text{sh}} \approx \frac{1}{2} \langle T_e + T_i \rangle \approx 227 \text{ eV}$. Based on the measurements by Voyager 1 and 2 of Jupiter and Saturn, Richardson (1987, 2002) revealed that protons in their...
magnetosheaths have a double-Maxwellian distribution and are composed of both cold and hot components with temperatures $T^p_C \approx 100\text{ eV}$ and $T^p_H \approx 600\text{ eV}$, respectively. These two proton components have comparable densities, so the average proton temperature in the magnetosheaths of Jupiter and Saturn is estimated as $T^p = (T^p_C + T^p_H)/2 \approx 350\text{ eV}$. The explorations of Saturn by Cassini find that the average ion temperature of Saturn’s magnetosheath is $T^i \approx 210–370\text{ eV}$ (Sergis et al., 2013). Thomsen et al. (2018) surveyed the features of Saturn’s magnetosheath in detail based on Cassini measurements and showed that the mean temperatures of the electrons and protons at the subsolar point of Saturn’s magnetosheath are $T^e \approx 34\text{ eV}$ and $T^i \approx 340\text{ eV}$, respectively, with the average temperature of Saturn’s magnetosheath being $(T^i)_{av} = \frac{1}{2} (T^e + T^i) \approx 187\text{eV}$. The temperature of both electrons and protons from Saturn’s magnetosheath gradually decreases as we move away from the noon (Thomsen et al., 2018). Therefore, the observations indicate that the plasma temperatures of the planetary magnetosheaths are comparable to each other to within several MK or several hundred eV and are very close to the temperature of the solar corona.

Satellite observations of velocity distribution functions (VDFs) from solar wind and the magnetosheaths of planets (such as Earth, Mercury, Saturn, and Uranus) frequently exhibit non-Maxwellian features, which feature suprathermal tails at high energies and quasi-Maxwellian behavior at low energies (Christon et al., 1998; Maksimovic et al., 1997; Pierrard et al., 2004). Non-Maxwellian VDFs have also been found in the solar wind and around Earth’s bow shock with two different temperatures and densities: a dense “core” thermal population superimposed on a hot “halo” superthermal population (Feldman et al., 1975, 1983a; 1983b; Gaelzer et al., 2008; Lin, 1998; Marsch, 2006). Distributions of superthermal tails are well modeled by the kappa distribution since it fits both the Maxwellian (thermal) and non-Maxwellian (superthermal) high-energy part of the distribution (Pierrard & Lemaire, 1996; Schippers et al., 2008). Electron VDFs in Earth’s magnetosheath and magnetosphere have also been observed with flat tops instead of a quasi-Maxwellian low-energy part that can be fit by neither Maxwellian nor kappa distribution functions. Such flat-top VDFs often have one or two distinct components and are well fit by a generalized $(r,q)$ distribution function (Qureshi et al., 2004, 2019). The magnetosheaths are also rather turbulent and filled with various waves, such as fast and slow magnetosonic waves, Alfvén waves, mirror modes, whistler waves, and even solitary waves (Sckopke et al., 1990; Song et al., 1990, 1992, 1994; Southwood & Kivelson, 1995).

As for the simplified cases when the magnetic field, solar gravity, and coronal heat conduction may be omitted, the expansion of the solar corona may be regarded as an adiabatic process, with the plasma entropy conserved. The coronal plasmas with an extremely high temperature expand outward from a stationary state, accelerate into interplanetary space, and decelerate at bow shocks in the vicinity of planets to form the hot magnetosheath plasmas with exceedingly small bulk velocities. According to the second law of thermodynamics, the plasma temperatures of the planetary magnetosheaths cannot exceed the maximum temperature of the source region plasmas—the solar corona (i.e., $T_{sh} \leq T_{cor}$). However, in actual situations, some of the thermal energy of the corona is spent to overcome the pull of solar gravity, and the magnetic field in the corona may also accelerate the solar wind. Furthermore, the electrons and ions of the solar wind or the magnetosheaths have different temperatures of the electrons and protons at the subsolar point of Saturn’s magnetosheath are $A_A e \approx 34\text{ eV}$ and $A_A p \approx 340\text{ eV}$, respectively. These two proton components have comparable densities, so the average proton temperature in the magnetosheaths of Jupiter and Saturn is estimated as $A_A p_C \approx 100\text{ eV}$ and $A_A p_H \approx 600\text{ eV}$, respectively. These two proton components have comparable densities, so the average proton temperature in the magnetosheaths of Jupiter and Saturn is estimated as $T^p = (T^p_C + T^p_H)/2 \approx 350\text{ eV}$. Thomsen et al. (2018) surveyed the features of Saturn’s magnetosheath in detail based on Cassini measurements and showed that the mean temperatures of the electrons and protons at the subsolar point of Saturn’s magnetosheath are $T^e \approx 34\text{ eV}$ and $T^i \approx 340\text{ eV}$, respectively, with the average temperature of Saturn’s magnetosheath being $(T^i)_{av} = \frac{1}{2} (T^e + T^i) \approx 187\text{eV}$. The temperature of both electrons and protons from Saturn’s magnetosheath gradually decreases as we move away from the noon (Thomsen et al., 2018). Therefore, the observations indicate that the plasma temperatures of the planetary magnetosheaths are comparable to each other to within several MK or several hundred eV and are very close to the temperature of the solar corona.

This research analyzes theoretically in Section 2 the motion of the solar wind and presents approximations relating the mean temperatures of the solar corona to that of the planetary magnetosheaths for the steady-state situations. Section 3 pursues statistical investigations into features of the plasma temperatures in planetary magnetosheaths, and Section 4 presents the discussion and conclusions.

2. Theoretical Analysis of Physical Processes

We investigate the propagation of the solar wind from the corona to the planetary magnetosheaths, considering its multiple components, thermal anisotropy, and non-Maxwellian features. Multicomponent magnetohydrodynamics (MHD) describes approximately the coronal expansion and the propagation of the solar wind (Eichm et al., 2011; Parker, 1958).
In the solar system, the planets orbit the Sun in the ecliptic plane, so we investigate the steady propagation of the solar wind in this plane, as illustrated in Figure 1. To make the physics explicit and facilitate the analysis, we first present a short derivation of Bernoulli’s equation that is applicable to the solar wind with its multiple components, thermal anisotropy, and non-Maxwellian features.

The steady coronal solar wind and magnetosheath plasmas obey the continuity equation:

$$\frac{d\rho}{dt} = -\nabla \cdot (\rho \mathbf{V}) = 0,$$

where \( \mathbf{V} \) and \( \rho \) are the bulk velocity and mass density of the plasmas, respectively. For simplicity, we assume that the plasma electrons and ions have the same bulk velocities. Generally, the notation \( \mathbf{V} = \mathbf{V}_n \) is used, where \( \mathbf{n} \) is the unit radial vector with respect to the heliocenter. The mass density of the plasmas can be expressed as \( \rho = \sum a n_a m_a \), where \( m_a \) and \( n_a \) are the mass and number densities of species \( a \), respectively.

The steady expansion of the corona, the propagation of the solar wind, and the compression of the magnetosheath plasmas also obey the following equation of energy (Rossi & Olbert, 1970; Echim et al., 2011):

$$\frac{\partial}{\partial t} \left( \frac{1}{2} \rho \mathbf{V}^2 + \varepsilon_T + \varepsilon_{em} + \rho \phi \right) = -\nabla \cdot \left( \mathbf{P} \cdot \mathbf{V} + \frac{1}{2} \rho \mathbf{V}^2 + \varepsilon_T \right) \mathbf{V} + \mathbf{S}_{em} + (\rho \phi) \mathbf{V} + \mathbf{q} = 0,$$

where the total energy of the solar wind is composed of kinetic, thermal, electromagnetic, and gravitational potential energy. The thermal energy density of the solar wind is

$$\varepsilon_T = \sum_a \varepsilon_T^a = \sum_a n_a \left( \frac{1}{2} k T^a_T + k T^a_{\perp} \right),$$

where \( \varepsilon_T^a \) is the thermal energy density of species \( a \), \( T^a_T \) and \( T^a_{\perp} \) are the temperature of the species \( a \) parallel and perpendicular to the magnetic field, respectively, \( k \) is Boltzmann’s constant, and \( \varepsilon_{em} \) is the electromagnetic energy density. The gravitational potential \( \phi = -GM_s/r \), where \( G \) is the gravitational constant, \( M_s \) is the mass of the Sun, and \( r \) is the heliocentric distance. In contrast with the solar gravity, the gravity due to planets is rather weak so any effects on the solar wind are omitted. The thermal pressure tensor of species \( a \) is defined as \( P^a_T = \int v f_a(x, p) d^3p \) in the frame of reference of the plasma bulk velocity \( \mathbf{V} \), where \( f_a(x, p) \) is the phase-space density of species \( a \) at position \( x \) and momentum \( p \) (Rossi & Olbert, 1970). The phase-space density of the solar wind and of the planetary magnetosheath plasmas can be non-Maxwellian (Maksimovic et al., 1997; Richardson, 2002; Qureshi et al., 2014; Qureshi et al., 2019; Vasyliunas et al., 1968). The components of the total thermal pressure tensor \( \mathbf{P} \) of the magnetized plasmas in the coordinates of the magnetic field are

$$P^a_T = \begin{pmatrix} P^a_{\perp} & 0 & 0 \\ 0 & P^a_T & 0 \\ 0 & 0 & P^a_{\perp} \end{pmatrix},$$

where the pressure of species \( a \) parallel and perpendicular to the magnetic field is \( P^a_T = n_a k T^a_T \) and \( P^a_{\perp} = n_a k T^a_{\perp} \), respectively. The flux density of electromagnetic energy is

$$\mathbf{S}_{em} = \frac{\mathbf{E} \times \mathbf{B}}{\mu_0}.$$

Generally, the magnetic flux is frozen in the plasmas and \( \mathbf{E} = -\nabla \times \mathbf{B} \), so that \( \mathbf{S}_{em} = \frac{1}{\rho_0} (\mathbf{B} \times \mathbf{V}) \times \mathbf{B} = \frac{1}{\rho_0} [\mathbf{B}^2 \mathbf{V} - (\mathbf{B} \cdot \mathbf{V}) \mathbf{B}] \). In Equation 2, \( \mathbf{q} \) is the total heat flux in the solar wind. Presently, we still do not understand the heating mechanism of the solar corona and it is still under investigation (Cranmer et al., 2017; Klimchuk, 2015; McComas et al., 2007; Parnell & De Moortel, 2012). Given that the heating and radiative losses are at equilibrium in the core region of the solar corona, the temperature of the coronal plasmas...
is extremely high. This investigation assumes that the coronal plasma expands under the thermodynamic force or pressure and thereby produces the outward-flowing solar wind. Therefore, any heating and radiative losses in the coronal region are not considered here.

Applying the equation of conservation of energy (2) in the steady state to a tube with cross-sectional area \( A \) enclosing the solar wind (see Figure 1) yields

\[
\mathbf{A} \cdot \left[ \mathbf{P} \cdot \mathbf{V} + \left( \frac{1}{2} \rho \mathbf{V}^2 + \varepsilon_T \right) \mathbf{V} + \mathbf{S}_m + (\rho \phi) \mathbf{V} + \mathbf{q} \right] = \text{constant},
\]

or

\[
\left[ P_{\text{mm}} V + \left( \frac{1}{2} \rho V^2 + \varepsilon_T \right) V + \frac{1}{\mu_0} B_0^2 V - \rho \frac{G M_s}{r} V + q_0 \right] A = \text{constant},
\]

where \( P_{\text{mm}} = \mathbf{n} \cdot \mathbf{P} \cdot \mathbf{n} \) and \( S \cdot \mathbf{n} = \frac{1}{\mu_0} (\mathbf{B} \times \mathbf{V}) \cdot \mathbf{n} = \frac{1}{\rho_0} (\mathbf{B} \times \mathbf{\hat{n}}) \cdot (\mathbf{B} \times \mathbf{V}) = \frac{B_0^2}{\rho_0} V + \frac{1}{\rho_0} B_0^2 V \). Here, \( B_i \) is the component of the magnetic field perpendicular to the radial direction.

On the other hand, the continuity Equation 1 reduces to

\[
\rho \mathbf{V} \cdot \mathbf{A} = \text{constant.}
\]

Comparing the Equations 6 and 7 yields

\[
\left( \frac{P_{\text{mm}}}{\rho} + \frac{1}{2} \rho V^2 + \frac{\varepsilon_T}{\rho} + \frac{B_0^2}{\mu_0 \rho} - \frac{G M_s}{r} + \frac{q_0}{\rho V} \right) = \text{constant.}
\]

Equation 8 is Bernoulli’s equation for solar wind plasmas with multiple components, thermal anisotropy, and non-Maxwellian features. Note that only the laws of conservation of matter and energy (Equation 1 and 2) have been used in this derivation, whereas conservation of momentum was not applied. Thus, Equation 8 is not a complete description of the dynamical evolution of the corona, solar wind, and magnetosheaths. Equation 8 relates the thermal properties of the corona, solar wind, and planetary magnetosheaths but does not describe definite features of their motions. Their velocities are given by direct observations.

Denote the angle between \( \mathbf{V} \) and \( \mathbf{B} \) as \( \theta \). In the steady state, \( \theta \approx 0^\circ \) in the outer corona, whereas \( \theta \approx 45^\circ \) at 1 AU. In Cartesian coordinates with respect to the magnetic field, as illustrated in Figure 2,

\[
\mathbf{n} = \cos \theta \hat{\mathbf{e}}_x + \sin \theta \hat{\mathbf{e}}_z, \quad \mathbf{P} = P_x \hat{\mathbf{e}}_x + P_y \hat{\mathbf{e}}_y + P_z (\hat{\mathbf{e}}_x + \hat{\mathbf{e}}_y + \hat{\mathbf{e}}_z),
\]

so

\[
P_{\text{mm}} = \mathbf{n} \cdot \mathbf{P} \cdot \mathbf{n} = \cos^2 \theta P_{\parallel} + \sin^2 \theta P_{\perp} = \sum_a \left( \cos^2 \theta n_a kT_a^\parallel + \sin^2 \theta n_a kT_a^\perp \right).
\]

Here the transverse plasma beta is defined as \( \beta_t = \varepsilon_T / B_0^2 / 2 \mu_0 \), that is, the ratio of the total thermal energy \( \varepsilon_T \) to the transverse magnetic field energy \( B_0^2 / 2 \mu_0 \). Thus, Bernoulli’s Equation 8 becomes

\[
\frac{1}{2} V^2 + \frac{1}{\rho} \left( P_{\text{mm}} + \varepsilon_T + \frac{2}{\beta_t} \varepsilon_T \right) = \frac{G M_s}{r} + \frac{q_0}{\rho V} = \text{constant},
\]

where the term \( \frac{G M_s}{r} = \frac{R_s}{r} \frac{G M_s}{R_s} \equiv \frac{V_e^2}{V_e^2} \frac{G M_s}{R_s} \equiv V_e^2 \), and the escape velocity \( V_e \approx 438 \text{ km/s} \). Here, the gravitational constant \( G \approx 6.67 \times 10^{-11} \text{ m}^3 / \text{kg} \cdot \text{s}^2 \), the radius of the Sun is \( R_s \approx 6.69 \times 10^8 \text{ m} \), and the solar mass \( M_s \approx 1.99 \times 10^{30} \text{ kg} \). Then Bernoulli’s Equation 10 may be written as

\[
\frac{1}{2} V^2 + \frac{1}{\rho} \left[ P_{\text{mm}} + (1 + 2 \beta_t^{-1}) \varepsilon_T \right] = \frac{R_s}{r} \frac{V_e^2}{V_e^2} + \frac{q_0}{\rho V} = \text{constant}.
\]
Commonly, the ion ratio $\text{He}^{2+}/\text{H}^+$ in the solar wind is less than 6% and averages 4.5% (Cranmer et al., 2017; Song, Russell, Zhang, et al., 1999; McComas et al., 2008). In the plasma of the solar wind, heavy elements are exceedingly rare, so we just consider electrons, protons and $^4\text{He}^{2+}$ (i.e., $\alpha$ particles) and neglect other ions, which is a reasonable approximation. We further assume the ion ratio $\text{He}^{2+}/\text{H}^+$ in the solar wind is $\eta < 0.06$ and that the number densities of protons, $^4\text{He}^{2+}$ and electrons are $n_p$, $n_{\alpha} = \eta n_p$, and $n_e = n_p + 2n_{\alpha} = (1 + 2\eta)n_p$, respectively. The total number density in the plasmas is then $n = n_e + n_p + n_{\alpha} = (1 + 2\eta)n_p + n_p + \eta n_p = (2 + 3\eta)n_p$, and the mass density of the solar wind is

$$
\rho = n_p m_p + n_{\alpha} m_{\alpha} + (1 + 2\eta)n_p m_e = n\mu m_p,
$$

where $m_e$, $m_p$, and $m_\alpha$ are the electron, proton, and $^4\text{He}^{2+}$ masses. Given $m_e \ll m_p$ and $m_\alpha \approx 4m_p$, the average atomic weight in the solar plasma is $\mu \approx (1 + 4\eta)/(2 + 3\eta)$. Therefore, Equation 9 yields

$$
\frac{P_{\text{in}}}{\rho} = \frac{k}{(2 + 3\eta)\mu m_p} \left\{ \cos^2 \theta \left[ (1 + 2\eta)T^e_\parallel + T_\parallel^p + \eta T^\alpha_\parallel \right] + \sin^2 \theta \left[ (1 + 2\eta)T^e_\perp + T_\perp^p + \eta T^\alpha_\perp \right] \right\}
$$

or

$$
\frac{P_{\text{in}}}{\rho} = \frac{k}{(2 + 3\eta)\mu m_p} \left[ (1 + 2\eta)T^e_\parallel + T_\parallel^p + \eta T^\alpha_\parallel \right].
$$

where the radial temperature of species $a$ ($e, p, \alpha$) is defined as

$$
T^a_\parallel = \cos^2 \theta T^e_\parallel + \sin^2 \theta T^\alpha_\parallel.
$$

The total thermal energy density is

$$
\varepsilon_T = \sum_a \varepsilon^a_T = \sum_a n_a \left( \frac{1}{2} k T^a_\parallel + kT^a_\perp \right).
$$

The omnidirectional temperature of a species can be defined as

$$
\frac{3}{2} kT^a_\parallel \equiv \frac{1}{2} kT^e_\parallel + kT^\alpha_\parallel,
$$

in which case the total thermal energy density in Equation 16 becomes

$$
\varepsilon_T = \frac{3}{2} n k \left[ (1 + 2\eta)T^e_\parallel + T_\parallel^p + \eta T^\alpha_\parallel \right]/(2 + 3\eta).
$$

The average temperature of the plasmas is then

$$
\langle T \rangle \equiv \left[ (1 + 2\eta)T^e_\parallel + T_\parallel^p + \eta T^\alpha_\parallel \right]/(2 + 3\eta).
$$

When the helium abundance is very small, the contribution of $^4\text{He}^{2+}$ ions can be neglected, and the average temperature of the plasmas takes the form $\langle T \rangle = \left( T^e_\parallel + T_\parallel^p \right)/2$

Therefore, Bernoulli’s Equation 11 reduces to

$$
\frac{1}{2} V^2 + \frac{k}{\mu m_p} \left[ \frac{(1 + 2\eta)T^e_\parallel + T_\parallel^p + \eta T^\alpha_\parallel}{2 + 3\eta} + \frac{3}{2} (1 + 2\beta^{-1}) \langle T \rangle \right] - \frac{R_s}{\rho} V^2 + \frac{q_n}{\rho V} = \text{constant}.
$$

Relating the corona to the solar wind and planetary magnetosheaths yields

$$
\frac{1}{2} V^2_{\text{cor}} + \frac{k}{\mu m_p} \left[ \frac{(1 + 2\eta)T^e_\parallel + T_\parallel^p + \eta T^\alpha_\parallel}{2 + 3\eta} + \frac{3}{2} (1 + 2\beta^{-1}) \langle T \rangle \right]_{\text{cor}} - \frac{R_s}{\rho} V^2_{\text{cor}} + \frac{q_n}{\rho V_{\text{cor}}} = \text{constant}.
$$
where the subscript “cor”, “sw,” and “sh” denote the corona, solar wind and magnetosheaths, respectively. Here, we neglect the heat fluxes of the solar wind and the planetary magnetosheath collisionless plasmas. The influence of the gravity on both the solar wind and magnetosheaths are neglectable. At the subsolar point of the planetary magnetosheaths, the temperatures reach maximal values, whereas the bulk velocities of the downstream plasmas are small. Obviously, Equation 21 contains the matter continuity and energy conservation parts of the shock jump conditions constrain the upstream and downstream plasmas in the vicinity of the planetary bow shocks.

As shown in the Introduction, the temperature of the solar wind has decreased considerably, and \((T)_{sw} \approx 15 \text{eV}\), and \((\langle T \rangle_{cor} - \langle T \rangle_{sw}) / (\langle T \rangle_{cor} \approx 96\%). So that the part of the thermal energy for the solar wind in Equation 21 is rather small.

For the steady corona, the magnetic field \(B\) is almost in the radial direction due to the drawing of the outflowing streams, so \(B \approx 0\) and the transverse plasma beta \(\beta_t = \varepsilon_T / \varepsilon_{2n} \gg 1\). Usually, all the species in the solar corona are thermally isotropic due to the rapid Coulomb collisions (Cranmer et al., 2017). Therefore, the perpendicular temperature and the parallel temperature for the species \(a\) in the corona are the same, that is, \(T_a^\parallel \approx T_a^\perp \approx T_a\) (Equation 17). So that we get \(T_a^\parallel \approx T_a^\perp \approx T_a\) from Equation 15, indicating that the transverse temperature and the average temperature are approximately equal. According to Equation 19, we have in the corona \(((1 + 2\eta)T_a^\parallel + T_a^\perp + \eta T_a^\parallel) / (2 + 3\eta) \approx (1 + 2\eta)T_a^\parallel + T_a^\perp + \eta T_a^\parallel / (2 + 3\eta) = \langle T \rangle\). Furthermore, in the coronal region with the highest temperature \(T_a^{\text{cor}}\), the temperature gradient is almost zero (i.e., \(\nabla T_{\text{cor}} \approx 0\)), so the heat flux is neglected in these zones. Therefore, Equation 21 becomes

\[
\frac{\mu m_p}{k} \left( \frac{1}{2} V_{\text{cor}}^2 - \frac{R_e}{r_{\text{cor}}} V_E^2 \right) + \frac{5}{2} \langle T \rangle_{\text{cor}} \approx \frac{1}{2} \frac{\mu m_p}{k} V_{\text{sw}}^2 + \frac{1 + 2\eta}{2 + 3\eta} \left( T_a^\parallel + T_a^\perp + \eta T_a^\parallel \right) \langle T \rangle_{\text{sw}} + \frac{3}{2} \left( 1 + 2\beta_t^{-1} \right) \langle T \rangle_{\text{sh}}.
\]

The above formula shows the relationship between the temperatures of the solar corona, solar wind and the planetary magnetosheaths at the subsolar points in the steady state. The maximum temperature of the planetary magnetosheaths at the subsolar points is determined by the highest temperature of the solar corona. The transverse plasma beta \(\beta_t\) of the magnetosheaths on the right-hand side of Equation 22 is still not fixed because the equation of momentum was not included in this investigation. The transverse plasma beta \(\beta_t\) of the magnetosheaths can be determined observationally.

We now consider the statistically averaged situations. Although the temperatures of protons and electrons in the planetary plasmasheds have anisotropy generally, there exist a trend of isotropization caused by various instabilities stimulated, thus the velocity distributions of the particles do not deviate from isotropy too much, especially for high beta plasmas. Here we make the approximation that the perpendicular temperature and the parallel temperature for the species \(a\) in the magnetosheaths are equal statistically, that is, \(T_a^\parallel \approx T_a^\perp \approx T_a\). Then, the transverse temperature of electrons, protons, and He\(^{2+}\) (a) ions in the planetary magnetosheaths statistically equals their average temperature, that is, \(T_a^\parallel \approx T_a^\perp \approx T_a\). So that in the planetary magnetosheaths, \(((1 + 2\eta)T_a^\parallel + T_a^\perp + \eta T_a^\parallel) / (2 + 3\eta) \approx \left( (1 + 2\eta)T_a^\parallel + T_a^\perp + \eta T_a^\parallel / (2 + 3\eta) = \langle T \rangle \right)\). In the solar wind, the thermal energy density is actually too small compared with the kinetic energy density, so that the term related to the temperature of the solar wind in Equation 22 can be neglected. Thus, given Equation 19, Equation 22 takes the form

\[
\frac{\mu m_p}{k} \left( \frac{1}{2} V_{\text{cor}}^2 - \frac{R_e}{r_{\text{cor}}} V_E^2 \right) + \frac{5}{2} \langle T \rangle_{\text{cor}} \approx \frac{1}{2} \frac{\mu m_p}{k} V_{\text{sw}}^2 + \frac{1 + 2\eta}{2 + 3\eta} \left( T_a^\parallel + T_a^\perp + \eta T_a^\parallel \right) \langle T \rangle_{\text{sw}} + \frac{3}{2} \left( 1 + 2\beta_t^{-1} \right) \langle T \rangle_{\text{sh}}.
\]

Thomsen et al. (2018) have statistically obtained an empirical formula for the relationship between the speed of the solar wind and the velocity and temperature of the Saturn’s magnetosheath, that is,
\[ \frac{1}{2} m_p V_{sw}^2 = 1.687 \times \left( \frac{1}{2} m_p V_{sh}^2 + kT_p + \frac{v^2}{\mu_n} \right), \]

where \( n \) is the number density of protons, which has certain similarity to the above equation as omitting the contribution of the thermal energy of the solar wind.

According to the observations of Wang et al. (2012) of Earth’s magnetosheath and on the observations of Thomsen et al. (2018) of Saturn’s magnetosheath, \( V_{sh} \leq 100 \text{ km/s}, \frac{V_{sh}}{V_{sw}} \leq 1/4 \), and \( V_{sh}^2/V_{sw}^2 \leq 0.060 \) in general. Upon approaching the nose (the stagnant point) of the magnetopause along the Sun-planetary line, the velocity of the magnetosheath plasma decreases to about zero, so the kinetic energy is much less than the total energy in the planetary magnetosheaths at the subsolar points and thus can be neglected. Generally, in the planetary magnetosheaths, \( \beta \approx 0.1–20 \) (Gershman et al., 2013; Richardson, 2002; Sergis et al., 2013; Thomsen et al., 2018). As shown in Figure S1-S3 in Supporting Information S1, the average plasma beta of the magnetosheaths of Mercury, Earth and Saturn are 2.4, 7.3 and 9.6, respectively, that is, \( 2.4 \leq \langle \beta \rangle_{sh} \leq 9.6 \). Then the corresponding average transverse plasma beta obey \( 4.8 \leq \langle \beta \rangle_{sh} \approx 2\langle \beta \rangle_{sw} \leq 19.2 \). So that we can make an approximation that \( \langle \beta \rangle_{sh} \gg 1 \), that is, the magnetic field is assumed to have a neglect role on the evolution of the solar wind. Then Equation 23 reduces to

\[
\frac{2\mu n_p}{5k} \left( \frac{1}{2} V_{sw}^2 - \frac{R_S}{r_{cor}} V_p^2 \right) + \langle T \rangle_{cor} \approx \frac{1}{5} \mu m_p \frac{V_{sh}^2 + \langle T \rangle_{sh}}{k}.
\]

Firstly, as indicated by Equation 24, if the velocity of the plasmas at the subsolar magnetosheaths are very small, the average temperatures of all the planetary magnetosheaths at the subsolar point have approximately the same value, which is controlled by the corona.

Secondly, Equation 24 indicates that the mean temperatures of planetary magnetosheaths are closely related to the mean temperature of the solar corona. Here we introduce the effective temperature of the corona as

\[
\langle T \rangle_{cor, eff} \equiv \frac{2\mu n_p}{5k} \left( \frac{1}{2} V_{sw}^2 - \frac{R_S}{r_{cor}} V_p^2 \right) + \langle T \rangle_{cor},
\]

which is the expected temperature of the corona with no heat source and gravitation that can expand to the same average temperature. Here we introduce the effective temperature of the corona as

\[
\langle T \rangle_{cor, eff} \approx \langle T \rangle_{sh},
\]

where the contribution of the kinetic energy of magnetosheath plasmas is omitted on considering \( V_{sh}^2/V_{sw}^2 \leq 0.060 \). This implies the average temperatures of the planetary magnetosheaths are about equal to the effective temperature of the solar corona. The observational investigations in the following Section 3 shows that the average plasma temperatures of the magnetosheaths of Earth and Saturn at the subsolar points are 184 eV (2.13 MK) and 171 eV (1.98 MK), respectively. On the other hand, there is still no sufficient observational information about the corona temperature. As shown in the next Section, based on Kohl et al. (2006), the average temperature in the polar coronal holes at solar minimum are about \( \langle T \rangle_{cor} \approx 146 eV \approx 1.7 MK \). This implies that the average temperatures of the planetary magnetosheaths are possibly very close to that of the solar corona. Therefore, it is most likely that the first term in the right side hand of Equation 25 are actually in balance and cancel each other, and thus \( \langle T \rangle_{cor} \approx \langle T \rangle_{sh} \).

This result is consistent with the second law of thermodynamics if we neglect the plasma bulk velocity in the planetary magnetosheath at the subsolar point and assume the magnetic field have an omissible role in the propagation of the solar wind with very high plasma beta.

Therefore, in the present work, it is expected that, in the steady state, the temperatures of all planetary magnetosheaths in the solar system at their subsolar points are comparable and possibly close to the temperature of the solar corona.

The above assumptions are correct for the steady state and can be valid for general situations but for transient processes such as coronal mass ejections. Nevertheless, the theoretical results obtained above are supported by the statistical survey of planetary magnetosheaths, as discussed in the next section.
3. Statistical Investigations

We now present a statistical investigation of the plasma temperature in the magnetosheaths of Mercury, Earth, Jupiter, and Saturn and compare the results with the theoretical results from the previous section.

The Supporting Information (jgra55009-sup-0001-2021JA029108-si) gives the plots (Figures S1–S3 in Supporting Information S1) of the distributions of plasma betas in the magnetosheaths of Mercury, Earth and Saturn as well as the plots (Figures S4–S9 in Supporting Information S1) of the distributions of ion and electron temperatures in the magnetosheaths of Mercury, Earth, Jupiter and Saturn.

Figure 3 shows the distributions of ion temperature in the magnetosheaths of Mercury, Earth, Jupiter, and Saturn, respectively.

For Mercury, we use the data from the Messenger satellite from 2012 to 2013 (Slavin et al., 2007). It is found that the Messenger satellite was in Mercury’s magnetosheath region for 94 days during 2012 and 2013. Based on the proton measurements during these periods, we obtain the distribution of proton temperatures in Mercury’s magnetosheath. The proton temperature is drawn from the NTP data in FIPS-DDR with a time resolution of 1 min (Andrews et al., 2007). We are still in lack of the data for the temperatures of other ions, for example, $\text{He}^+$, $\text{He}^{2+}$ and $\text{O}^+$. Considering that the protons make up most of the ions in the magnetosheath, we would use the proton temperature to approximate the ion temperature in Mercury’s magnetosheath. Figure 3 and also Figure S4 in Supporting Information S1 (jgra55009-sup-0001-2021JA029108-si) show that the maximum ion temperature in Mercury’s magnetosheath can reach as high as 1,450 eV, with the most probable ion temperature being $T_{i\text{,max}} \approx 275$ eV, and the average ion temperature being $T_i \approx 414$ eV with its standard deviation $S_i \approx 232eV$.

As for Earth, we analyze the data from the MMS1 satellite (Burch et al., 2016; Pollock et al., 2016; Torbert et al., 2015) acquired from 2015 to 2021. We obtain 169 days when the detector was in the subsolar region (with the zenith angles from the X axis being <30°) of Earth’s magnetosheath. Using the data within this interval, we obtain the distribution of ion and electron temperatures in Earth’s magnetosheath. The ion temperature is derived from the ion vertical temperature $T_{i\parallel}$ and the ion parallel temperature $T_{i\|}$ in FPI_FAST_L2_DIS-MOMS of MMS1 (Pollock et al., 2016), and the electron temperature is obtained from the electron vertical temperature $T_{e\parallel}$ and the electron parallel temperature $T_{e\|}$ in FPI_FAST_L2DES-MOMS of MMS1 (Pollock et al., 2016). The time resolution of the data is 4.5 s. The calculation formulas for the total temperatures of ions and electrons are $T_i = \frac{1}{3}T_{i\parallel} + \frac{2}{3}T_{i\perp}$ and $T_e = \frac{1}{3}T_{e\parallel} + \frac{2}{3}T_{e\perp}$, respectively, as shown in Equation 17. Figure 3 as well as Figure S5 in Supporting Information S1 show the distribution of ion temperature in Earth’s magnetosheath. These results
show that the maximum ion temperature in Earth's magnetosheath can reach up to 1,200 eV, the average ion temperature is $T_i \approx 362\text{eV}$, and the most probable ion temperature is $T_{i,\text{prob}} \approx 185\text{eV}$. The standard deviation of the ion temperature is $\sigma_i \approx 221\text{eV}$. The average ion temperature of the subsolar magnetosheath of Earth obtained here based on MMS measurements is consistent with that of Wang et al. (2012) based on THEMIS observations within the range of error. Figure 4 as well as Figure S6 in Supporting Information S1 present the distribution of electron temperature in Earth's subsolar magnetosheath. These results show that the maximum electron temperature can reach 140 eV, the average electron temperature is $T_e \approx 49\text{eV}$, and the most probable electron temperature is $T_{e,\text{prob}} \approx 35\text{eV}$, with a standard deviation $\sigma_e \approx 18\text{eV}$. The mean electron temperature of the subsolar magnetosheath of Earth based on MMS measurements is in agreement with the statistical result of Wang et al. (2012) from THEMIS observations within the range of error.

For Jupiter, we have used the data of Voyager 2 satellite (Kohlhase & Penzo, 1977) from July 2 to 5 July 1979. Ion temperatures were derived from the ION-L-MODE data from the Plasma Subsystem (PLS) which has a time resolution of 96 s (Bridge et al., 1977). Figure 3 and also Figure S7 in Supporting Information S1 show the distribution of ion temperature in Jupiter's magnetosheath. The maximum ion temperature in Jupiter's magnetosheath can reach 560 eV, the average ion temperature is $T_i \approx 309\text{eV}$, and the most probable ion temperature is $T_{i,\text{prob}} \approx 330\text{eV}$, with a standard deviation $\sigma_i \approx 75\text{eV}$.

For Saturn, we apply the data of the Cassini satellite from 2007 to 2008 (Matson et al., 2002). Counting the data from 2007 to 2008, we find 66 days during which the detector was located in the subsolar region of Saturn's magnetosheath (with the zenith angles from the X axis being <30°). Using the data during these 66 days, we obtain the distribution of plasma temperature in Saturn's subsolar magnetosheath. The proton temperature is derived from the DDR-ION-MOMENTS data from the Cassini Plasma Spectrometer (CAPS) with a time resolution of approximately 7 min (Young et al., 2004), and the electron temperature is generated from the DDR-ELE-MOMENTS data from the CAPS with a time resolution of approximately 32s (Young et al., 2004). Similar to the situation of Mercury previously, we use the proton temperature to approximate the ion temperature in the magnetosheath. Figure 3 and also Figure S8 in Supporting Information S1 show the distribution of ion temperature in Saturn's magnetosheath. The ion temperature can reach 800 eV, the average ion temperature is $T_i \approx 304\text{eV}$, and the most likely ion temperature is $T_{i,\text{prob}} \approx 270\text{eV}$, with a standard deviation $\sigma_i \approx 104\text{eV}$. Figure 4 and also Figure S9 in Supporting Information S1 show the distribution of electron temperature in the magnetosheath of Saturn. These results show that the electron temperature can reach 100 eV, the average electron temperature is $T_e \approx 37\text{eV}$, and the most probable electron temperature is $T_{e,\text{prob}} \approx 33\text{eV}$, with a standard deviation $\sigma_e \approx 12\text{eV}$. These results are
in well agreements with those of Thomsen et al. (2018) for the subsolar point magnetosheaths of Saturn within the range of error.

Table 1 lists the relevant parameters for the ion and electron temperature in the magnetosheaths of Mercury, Earth, Jupiter, and Saturn. Unfortunately, no electron data are available for the magnetosheaths of Mercury and Jupiter.

The average plasma temperature of Earth's magnetosheath plasmas is $\langle T \rangle \approx 206 \text{ eV} \approx 2.39 \text{ MK}$ (as defined in Equation 19), which is very close to the average temperature of the solar corona. Its standard deviation is $S_i \approx \frac{1}{2} (S_e + S_i) \approx \frac{1}{2} (221 + 18) \text{ eV} \approx 120 \text{ eV}$. The average plasma temperature of Saturn's magnetosheath is $\langle T \rangle \approx 171 \text{ eV} \approx 1.98 \text{ MK}$, which is again approximately the average temperature of the solar corona. Its standard deviation is $S_i \approx \frac{1}{2} (S_e + S_i) \approx \frac{1}{2} (104 + 12) \text{ eV} \approx 58 \text{ eV}$. And also, based on the proton and electron measurements from the Jovian Auroral Distributions Experiment (JADE) (McComas, Alexander, et al., 2017) on board Juno, recently Ranquist et al. (2019) have statistically found that the average temperature of Jupiter's magnetosheath at the region with the zenith angle between 15° and 35° (a little dawn-side) is 197 eV, which is in consistence with the results here. As shown in Table 1 and Figures 3 and 4, S1–S6 in Supporting Information S1, the deviations from the average temperatures generally become smaller and the distributions of the temperatures grow more Gaussian with the distances of the planets away from the Sun.

Kohl et al. (2006) presented that the electron and proton temperatures in the polar coronal holes at solar minimum are about $\langle T^e \rangle \approx 1.5 \text{ MK}$ and $\langle T^p \rangle \approx 2.0 \text{ MK}$, respectively, with the average temperature being approximately $\langle T \rangle \approx (\langle T^e \rangle + \langle T^p \rangle) / 2 \approx 1.7 \text{ MK} \approx 146 \text{ eV}$. (Considering the high temperature of Alpha particles, it can be enhanced even further.) Cramer et al. (2017) have given a slightly smaller value. It is expected that the average temperature of the plasmas in equatorial corona is larger than that in the polar coronal holes (Kohl et al., 2006). Kohl et al. (2006) have also shown that the average proton temperature of the corona streamers is 2.0–3.2 MK, or 170–280 eV. So that the average temperatures of the planetary magnetosheaths are very close to that of the solar corona within the statistical deviations. These statistical results support the theoretical results presented in the previous section.

In addition, for both Earth's and Saturn's magnetosheaths, the electron temperatures are much less than the proton temperatures. The ratio of ion-to-electron temperature in the magnetosheaths of Earth and Saturn are 7.4 and 8.2, respectively, which indicates that, as the solar wind streams out from the solar corona, the electrons cool significantly. It is possibly caused by the ambipolar diffusion process (Lemaire & Pierrard, 2001; Parks, 2018).

### 4. Discussion and Conclusions

The magnetosheaths supply matter and energy to planetary magnetospheres and plays a critical role in the evolution of the magnetospheres (Axford & Hines, 1961; Dungey, 1961; Phan et al., 2000; Fujimoto et al., 2008; Wang et al., 2012). The upstream solar wind plasmas, which originate from the solar corona, are compressed by the bow shocks and form the downstream magnetosheath plasmas. The thermal properties of the planetary magnetosheaths are closely related to the features of the solar corona. Numerous observational investigations indicate that the temperature of planetary magnetosheaths at subsolar points is on the order of several hundred eV or several MK (Gershman et al., 2013; Richardson, 1987, 2002; Slavin et al., 2014; Shen & Liu, 2005; Sergis et al., 2013; Wang et al., 2012).
et al., 2012), which is very close to the mean temperature of the solar corona (Delaboudinière et al., 1995; Laming et al., 1995; Schrijver et al., 1999; Schmelz & Winebarger, 2015; Tu et al., 1999). This research seeks to find a quantitative relationship between the temperature of the solar corona and that of the planetary magnetosheaths.

The thermal energy of the solar corona that is converted into kinetic energy to accelerate the solar wind is almost entirely converted back to thermal energy when the plasma crosses a planetary bow shock. As viewed from the second law of thermodynamics, the maximum temperature of a planetary magnetosheath generally cannot exceed that of the solar corona if we omit the role of magnetic field for high plasma beta situations. This investigation includes a detailed theoretical analysis of the steady expansion of the solar corona, the propagation of the solar wind, and the compression of planetary magnetosheaths by the bow shocks with the general Bernoulli's equation. The approach is universal and considers the dynamics of multiple components, thermal anisotropy, and non-Maxwellian plasmas in the steady state. In the core region of the solar corona, the heating input and the radiative loss reach thermal equilibrium, thereby maintaining the extremely high temperature of the corona plasmas. At present, no clear understanding exists of the real heating mechanism of the solar corona (Cranmer et al., 2017; Klimchuk, 2015; McComas et al., 2007; Parnell & De Moortel, 2012). In this research, we only study the outward expansion of the outer corona under the thermodynamic driving and evade possible heating and radiation losses. This approximation is reasonable and does not seriously affect the results obtained herein. It is somewhat odd that the Rankine–Hugoniot shock jump conditions have not been directly applied in this investigation related to planetary bow shocks. Actually, Bernoulli's equation and the matter conservation and energy conservation parts of Rankine–Hugoniot shock jump conditions are equivalent for describing the bow shocks. In this study, we have not omitted the Rankine–Hugoniot shock jump conditions, but applied Bernoulli's equation instead to describe the variations of the quantities at the two sides of the bow shock.

We propose an approximate formula relating the temperature of the solar corona to that of the planetary magnetosheaths. The quantitative results indicate that the average temperatures of all planetary magnetosheaths at the subsolar points are comparable. In general, the peak temperatures of the planetary magnetosheaths at the subsolar regions are very close to that of the solar corona. These theoretical results are consistent with measurements of planetary magnetosheaths (Gershman et al., 2013; Richardson, 1987, 2002; Slavin et al., 2014; Shen et al., 2008; Wang et al., 2012; Sergis et al., 2013).

We also provide a systematic statistical investigation into the average temperatures of the magnetosheaths of Mercury, Earth, Jupiter, and Saturn. The results indicate that the average proton/ion temperatures of the magnetosheaths of Mercury, Earth, Jupiter, and Saturn are 414, 362, 309, and 304 eV, respectively, whereas the average electron temperatures of the magnetosheaths of Earth and Saturn are 49 and 37 eV, respectively (no electron data are available for Mercury and Jupiter at present). The average plasma temperatures of the magnetosheaths of Earth and Saturn are 206 and 171 eV (or 2.39 and 1.98 MK), respectively, which are very close to the average temperature of the solar corona. The statistical results are consistent with the theoretical results. However, the electrons cool considerably as they travel away from the Sun in the solar wind.

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The quantitative relationship obtained herein regarding the temperature of planetary magnetosheaths can be applied to the steady solar wind propagation and planetary magnetosheaths. These results can also be meaningful for investigating the magnetosheaths of Venus and Mars without involving the intrinsic magnetic field (Øieroset et al., 2004). Observations indicate that the heliosheath temperature is approximately 2 MK (Liu et al., 2007), which can also be explained by the theoretical results of Section 2. The planetary magnetosheaths, interplanetary coronal mass ejection sheaths, and heliosheath are similar in terms of hot protons, which should also be the case for the steady-state fast solar wind originating from the coronal holes. However, the results obtained in this investigation do not hold for the explosive processes of the coronal mass ejections, during which the coronal magnetic energy contributes to the outward acceleration of the solar wind.

The relationship obtained herein between the temperature of the solar corona and that of planetary magnetosheaths is useful for evaluating the thermal features of the planetary magnetospheres based on the conditions of the solar corona. The plasmas in the tail plasma sheet originate mainly from the magnetosheath. The ratio of proton to electron temperature is approximately seven in Earth's plasma sheet, which is about the same as that in the magnetosheath. The higher the temperature of the magnetosheath, the higher the temperature of the plasma sheet.
Data Availability Statement

The authors are thankful to the Energetic Particle and Plasma Spectrometer (EPFP) team and Magnetometer (MAG) team for providing Messenger data (https://pds-ppi.igpp.ucla.edu/search/?t=Mercury%26sc=Messenger%26facet=SPACECRAFT_NAME%26depth=1), the Plasma Subsystem (PLS) team and MAG PI team for providing Voyager 2 data (https://pds-ppi.igpp.ucla.edu/search/?t=Jupiter%26sc=Voyager_2%26facet=SPACECRAFT_NAME%26depth=1), the Cassini Plasma Spectrometer (CAPS) team for providing Cassini data (https://pds-ppi.igpp.ucla.edu/search/?t=Saturn%26sc=Cassini%26facet=SPACECRAFT_NAME%26depth=1), and the MMS team for providing the MMS data (https://cdaweb.gsfc.nasa.gov/pub/data/mms/).

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