Interplay of CMB Temperature, Space Curvature, and Expansion Rate Parameters

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The cosmic microwave background (CMB) temperature, \( T \), surely the most precisely measured cosmological parameter, has been inferred from local measurements of the blackbody spectrum to an exquisite precision of 1 part in \( \sim 4700 \). On the other hand, current precision allows inference of other basic cosmological parameters at the \( \sim 1\% \) level from CMB power spectra, galaxy correlation and lensing, luminosity distance measurements of supernovae, as well as other cosmological probes. A basic consistency check of the standard cosmological model is an independent inference of \( T \) at recombination. In this work we first use the recent Planck data, supplemented by either the first year data release of the dark energy survey (DES), baryon acoustic oscillations (BAO) data, and the Pantheon SNIa catalog, to extract \( T \) at the \( \sim 1\% \) precision level. We then explore correlations between \( T \), the Hubble parameter, \( H_0 \), and the global spatial curvature parameter, \( \Omega_k \). Our parameter estimation indicates that imposing the local constraint from the SH0ES experiment on \( H_0 \) results in significant statistical preference for departure at recombination from the locally inferred \( T \). However, only moderate evidence is found in this analysis for tension between local and cosmological estimates of \( T \), if the local constraint on \( H_0 \) is relaxed. All other dataset combinations that include the CMB with either BAO, SNIa, or both, disfavor the addition of a new free temperature parameter even in the presence of the local constraint on \( H_0 \). Analysis limited to the Planck dataset suggests the temperature at recombination was higher than expected at recombination at the \( \gtrsim 95\% \) confidence level if space is globally flat. An intriguing interpretation of our results is that fixing the temperature to its locally inferred value would result in a preference for spatially closed universe, if \( T(z) \) is assumed to evolve adiabatically and the analysis is based only on the Planck dataset.

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\section{I. INTRODUCTION}

The cosmic microwave background (CMB) radiation was measured by the COBE/FIRAS experiment to have a black body spectral distribution with the exquisitely precise temperature \( T_0 = 2.72548 \pm 0.00057 \) at the present epoch [1]. Independent inferences of the temperature at redshifts of a few from measurements of molecular absorption lines in galaxies at redshifts of a few, and from spectral measurements of the Sunyaev-Zeldovich (SZ) effect towards nearby galaxy clusters, though much less precise, are still consistent with an adiabatic evolution ‘history’ \( T(z) = T_0(1+z) \), e.g. [2-4] and most recently [5]. The scarcity of directly detectable objects at high redshifts, and substantial uncertainties in quantifying local astrophysical conditions, render these tests rather limited; consequently, it is assumed that adiabatic \( z \)-dependence applies (essentially) at all times.

In analyses of datasets that include CMB measurements the value of \( T_0 \) is commonly assumed for the temperature [1], but because this physical parameter enters in the scalings of key quantities (such as radiation and matter energy densities), and essentially determines the timing of the recombination epoch, it is quite important to gauge the impact of relaxing this assumption. Clearly, this is of particular interest in assessing the precision and possible bias when inferring global parameter values. Even though no efficient thermalization mechanism of the CMB past \( z \sim 10^6 \) is known (or expected), taking the temperature to be a free parameter allows for independent determination of its value based largely on CMB features imprinted at the recombination era. Assuming that the standard model (SM) reliably describes our universe, systematics-free CMB measurements are expected to independently yield a result consistent with the locally-inferred value, albeit with a considerably larger statistical uncertainty. This test of the SM is well motivated in its own right, and also for quantifying any possible deviations from the standard scaling.

For example, it has been shown recently that the (much debated) tension between the locally measured value of \( H_0 \), e.g. [6-9], and that inferred from the small degree of anisotropy and minute polarization levels of the CMB [10], could be explained by deviation from the locally measured value of \( T_0 \), if the adiabatic scaling of the temperature with redshift still holds [11, 12]. In sharp contrast to distance ladder methods that probe the local expansion rate,

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Example figure caption.}
\end{figure}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
\textbf{Parameter} & \textbf{Value} & \textbf{Uncertainty} & \textbf{Reference} \\
\hline
\text{\( T_0 \)} & 2.72548 \pm 0.00057 & 1 part in 4700 & [1] \\
\text{\( H_0 \)} & 73.4 \pm 2.4 \text{ km s}^{-1} \text{ Mpc}^{-1} & 1 \text{ km s}^{-1} \text{ Mpc}^{-1} & [6-9] \\
\text{\( \Omega_k \)} & 0.018 \pm 0.004 & 95\% confidence level & [10] \\
\hline
\end{tabular}
\caption{Example table caption.}
\end{table}
the CMB provides an inference of $H_0$ at recombination, $z_* \approx 1100$. Earlier studies of the potential impact of CMB temperature uncertainties on CMB observables, or on parameter estimation in particular, are reported in [13-16].

In this work we explore possible implications that a deduced value of $T_*$, the value of $T$ at recombination, could have for the ‘Hubble tension’ and the expected relation to spatial curvature. This is based on the realization that a lower than expected $T_*$ implies earlier decoupling between baryonic matter and radiation, which in turn results in a smaller sound horizon at last scattering, i.e. smaller angular scale on the sky unless incoming light rays are focused in positively curved space. Thus, a strong $T_*-\Omega_k$ correlation would be expected, as has indeed been shown in the analysis of [12].

On the other hand, assuming flat space, i.e. $\Omega_k = 0$, it is clear that a lower $T_*$ implies earlier decoupling and consequently higher inferred $H_0$. Naturally then, the parameter trio $T - H_0 - \Omega_k$ is the main focus of the present work. This relatively simple and flexible picture becomes much more constrained when CMB data are supplemented by other large-scale structure (LSS) and SN data, although the latter do not explicitly depend on $T$. Doing so removes certain parameter degeneracies and results in more stringent constraints on key cosmological parameters that correlate with $T$, thereby severely limiting the freedom for the latter parameter to stray away from the value expected based on the standard adiabatic scaling as is shown below.

While the idea that $T$ and $H_0$ are correlated has been considered in [11] & [12] in the context of the Hubble tension and both analyses employed the Planck & BAO data (the latter was included in the analysis in order to break certain parameter degeneracies), these analyses did not use the currently available DES 1 yr data & Pantheon SNIa catalog. In addition, model comparison was carried out [11] in a somewhat simplistic frequentist approach. No model comparison has been carried out in [12]. In the present work we fill in these gaps by including the DES 1 yr & SNIa datasets, and we do carry out a detailed model comparison based on the popular Deviance Information Criterion (DIC) – a compromise between the Bayesian and frequentist approaches – for comparison between the standard and extended cosmological models [17].

The paper is structured as follows. In section II we briefly summarize the status of certain known tensions between values of key cosmological parameters. The extended cosmological model that we consider and assess its statistical likelihood is described in section III, followed by an outline in section IV of the datasets used in this work and the model comparison criteria we adopt. Our main results are described in section V followed by summary in section VI.

II. PARAMETER ‘TENSION’

Currently available high-quality cosmological datasets have allowed a sub-percent inference of several key cosmological parameters, but the diverse sets of measurements have also revealed significant inconsistencies between the deduced values of some of the global $\Lambda$CDM parameters. A much-discussed example is the $\gtrsim 5\sigma$ ‘Hubble tension’ between the results from cosmologically-based and local distance-ladder-based methods. These two fundamentally different approaches for measuring the current expansion rate of the universe have been in tension for quite some time now, with the Planck data (referred to as P18) alone favoring a value $H_0 = 67.36 \pm 0.54$ km/s/Mpc [10], while a range of local measurements seem to indicate systematically higher values, e.g. $H_0 = 74.03 \pm 1.42$ km/s/Mpc deduced by the SH0ES collaboration by calibrating the luminosity of nearby SNIa (using the period-luminosity relation of Cepheid variable stars [18]). Alternative calibration methods infer, e.g. $H_0 = 69.6 \pm 1.9$ km/s/Mpc, with calibration based on the tip of the red giant branch (TRGB) [19], or $H_0 = 73.3 \pm 3.9$ km/s/Mpc by using variable red giant stars for calibration [20]. Other local measurements, based on either strong lensing time-delay, e.g. [8, 9] or other methods, result in higher-than-Planck values, e.g. [21, 22]. Compared to local distance-ladder based inferences, CMB anisotropy and polarization results are generally considered more reliable due to the fact that the relevant physical processes and conditions are well-founded and can be (essentially) fully quantified in the linear regime. In addition, some doubts have been raised recently on the validity of certain elements of the analysis reported by both the SH0ES collaboration, e.g. [23], and the standard treatment using the TRGB approach [24]. Nevertheless, the convergence of a range of other approaches based on local measurements towards $H_0 \gtrsim 70$ km/s/Mpc is a rather compelling argument in favor of a claimed ‘Hubble tension’, albeit not necessarily as high as is usually claimed.

In addition, it has been claimed that other parameter values are in tension when inferred from different combinations of datasets, most notably the dimensionless spatial curvature parameter $\Omega_k$, e.g. [10, 25], and the matter perturbations at a scale of 8 Mpc, $\sigma_8$, rescaled to $S_8 \equiv \sigma_8/\sqrt{\Omega_m}$ in terms of $\Omega_m$, the current energy density of non-relativistic matter, e.g. [26]. Actually, all these are different aspects of the same fundamental tension because $H_0$, $\Omega_k$ and $S_8$ are all strongly correlated.

Another anomalous measurement (which is not quantitatively explored in the present work) is that of the lensing amplitude of the CMB anisotropy and polarization. It seems that there is simply not enough matter in a flat background universe to explain the observed lensing of CMB anisotropy and polarization by the intervening LSS. This mismatch is quantified by a dimensionless parameter $A_{tens}$ which should be consistent with unity in the SM.
Observationally, $A_{\text{lens}}$ is larger than unity at the $\geq 2\sigma$ confidence level. It has already been pointed out [25] that $\Omega_k$ and $A_{\text{lens}}$ are strongly correlated, which is indeed expected given the fact that incoming light rays are more focused at the background level as $\Omega_k$ decreases.

It should be noted that while these tensions are intriguing as they may imply the need for new physics, it would be statistically more appropriate to quantify the tension between datasets (rather than individual parameters) given a multivariate likelihood function in terms of, e.g. the ‘Mahalanobis metric’, the ‘index of inconsistency’ [27], or the ‘update difference in mean parameters’ [28]. In this spirit, and rather than attempting to quantify tension between datasets, we adopt the SM comparison approach and look for the best-fit model assuming that the available datasets are systematics-free. Nevertheless, the possible resolution or alleviation of discordance between certain parameters is a tantalizing possibility that we do consider in the present work.

III. EXTENDED COSMOLOGICAL MODEL

We explore the implications of treating the CMB temperature, whose ‘local’ value was precisely measured, as a free parameter. We introduce a new dimensionless parameter $A_{T_0}$ – the temperature in units of 2.72548 K, such that $T_0$ is replaced by $A_{T_0}T_0$ everywhere in the Boltzmann code CAMB within CosmoMC, still maintaining the adiabatic evolution law $T(z) \propto 1 + z$. The reasoning is clear: $A_{T_0} < 1$ means that recombination began earlier (than with $A_{T_0} = 1$), implying a smaller acoustic scale at recombination, and a higher inferred $H_0$. The latter effect can be masked by a spatially closed universe ($\Omega_k < 0$), where incoming light rays are focused and distant objects look larger. In other words, $A_{T_0}$ is expected to correlate with $\Omega_k$, as noted above. These parameters are degenerate with other parameters, which could be better constrained in joint analyses of CMB data with other cosmological probes.

In a vacuum-dominated universe (presumably, the present SM epoch) $H_0$ is strongly-correlated with $\Omega_\Lambda$ and thus similarly anti-correlated with $\Omega_m$ (because their sum is fixed at unity in a flat spacetime with negligible radiation), i.e. a lower temperature implies lower values of $\Omega_m$ and higher $\Omega_\Lambda$. In this context we mention that the relative heights of the odd and even acoustic peaks in CMB anisotropy, which reflect maximum compression and rarefaction of the acoustic plasma waves prior to and at last scattering, are regulated by the ratio of $\Omega_m/\Omega_\Lambda$. Thus, at least in flat space, any mechanism that boosts the inferred $H_0$ value (thereby increasing $\Omega_\Lambda$ in a DE-dominated, i.e. asymptotically de-Sitter, universe) would necessarily lower $\Omega_m$, which would in turn require a lower $\Omega_b$. Consistency of the efficiency of photon-baryon interaction prior to decoupling would then require a lower $A_{T_0}$, which is again consistent with a higher $H_0$. However, whereas the physics of the acoustic waves is sensitive to the ratio of energy densities $\Omega_{\text{rad}}/\Omega_b$, i.e. to $A_{T_0}^2/\Omega_b$, BBN yields sensitively depend on the ratio of abundance numbers $\eta_b = \Omega_b/A_{T_0}^2$. Obviously, it is impossible to keep both ratios fixed exactly to their standard values if $A_{T_0}$ extrapolated to the present is allowed to depart from 1. However, in practice these values are still free to vary within measurement precision.

Moreover, this relatively simple picture becomes more complicated once $\Omega_k$ is also treated as a free parameter, as we will see in section V. Since the BBN constraints in CosmoMC were calculated assuming the locally measured $T_0$, we disable the BBN constraints in our simulations. However, we do include the helium abundance $Y_{He}$ as a free parameter in our statistical analysis, a parameter which could be affected by $A_{T_0}$ departure from unity. Ideally, one would couple BBN and Boltzmann solver codes for consistency and vary all parameters simultaneously, including the baryon density and $A_{T_0}$, but this is beyond the scope of the present work.

IV. DATASETS AND CRITERIA FOR COMPARATIVE ANALYSIS

Our baseline model is described by the parameter vector $\theta = (\Omega_b h^2, \Omega_c h^2, \theta_{MC}, \tau, Y_{He}, A_s, n_s, A_{T_0})$, as well as 21 likelihood parameters in case of Planck 2018, and 20 additional likelihood parameters in case of DES 1 yr and a few more parameters in case of Pantheon datasets, when applicable. The cosmological parameters have their standard meanings; $\Omega_b$, $\Omega_c$, are the energy density of baryons and cold dark matter in critical density units, respectively; $\theta_{MC}$ is the ratio of the acoustic scale at recombination and the horizon scale, $\tau$ is the optical depth at reionization, $Y_{He}$ is the helium abundance, $A_s$ & $n_s$ are the amplitude and tilt of the primordial power spectrum of scalar perturbations, respectively. In our analysis $H_0$, and the modified mass fluctuation $S_8 = \sigma_8\sqrt{\Omega_m}/0.3$ (a mass fluctuation measure which is optimal for inference by LSS probes such as DES), are derived parameters. In our extended model we allow for a non-vanishing spatial curvature parameter, $\Omega_k$.

The datasets included here are Planck 2018 temperature anisotropy and polarization as well as lensing extraction data, the DES 1yr (cosmic shear, galaxy auto-and cross-correlations), BAO (data compilation from BOSS DR12, MGS, and 6DF), Pantheon data (catalog of 1048 SNIa in the redshift range $0.01 \lesssim z < 2.26$), and (when applicable) a gaussian prior on $H_0$ based on local inference by the SH0ES collaboration, all are included in the 2019 version of CosmoMC. The Planck likelihood functions employed in our analysis include the likelihood function $\text{plikHM}_{\text{TTTEEE}}$. 
(the TT, TE and EE correlations over the multipole range 30 < \ell < 2500), \texttt{lowl} \& \texttt{lowE} (TT and EE autocorrelations over 2 \leq \ell \leq 29), and the CMB lensing likelihood function constructed from the 4-point correlation function of the CMB. In our analysis we consider five different baseline dataset combinations: P18, P18+DES, P18+BAO, P18+Pantheon and P18+BAO+Pantheon. We do so with and without the SH0ES prior, as well as with and without curvature per each baseline dataset combination. We adopt default CosmoMC flat priors for key cosmological parameters which are shown in Table I (along with fiducial values used as initial guess) except for $A_{T_0}$ whose values are drawn from the range $[0.5, 1.5]$.

| Parameter | Fiducial | prior       |
|-----------|----------|-------------|
| $\Omega_b h^2$ | 0.0221 | $[0.005, 0.1]$ |
| $\Omega_c h^2$ | 0.12 | $[0.001, 0.99]$ |
| 100$\theta_{MC}$ | 1.0411 | $[0.5, 10]$ |
| $\tau$ | 0.06 | $[0.01, 0.8]$ |
| $\Omega_k$ | 0 | $[-0.3, 0.3]$ |
| $Y_{He}$ | 0.245 | $[0.1, 0.5]$ |
| $\ln(10^{10} A_s)$ | 3.1 | $[1.61, 3.91]$ |
| $n_s$ | 0.96 | $[0.8, 1.2]$ |
| $A_{T_0}$ | 1 | $[0.5, 1.5]$ |

TABLE I: The basic cosmological parameters, their fiducial values, and flat priors are specified. The derived value of $H_0$ was constrained to the interval $[40, 100]$ km/(s Mpc).

Sampling from posterior distributions is done using the fast-slow dragging algorithm with a Gelman-Rubin [29] convergence criterion $R - 1 < 0.02$ (where $R$ is the scale reduction factor). Our analysis of curved models (with a free $A_{T_0}$ parameter) using only the Planck dataset (with or without the SH0ES prior) did not satisfy this convergence criterion; results of this analysis are not considered in this work. This is clearly due to the strong correlation of $A_{T_0}$ with other parameters, especially with $\Omega_k$, which is not strongly constrained by the Planck data alone.

Model comparison criteria `penalize' complicated models for additional parameters which are not well constrained by the data. The $AIC \equiv \chi^2_{\text{min}} + 2p$ (Akaike information criterion) and the $BIC \equiv \chi^2_{\text{min}} + p \ln N$, where $p$ and $N$ are the free model parameters and number of data points used, respectively, represent inherently different statistical approaches: Whereas the $AIC$ follows a frequentist approach the $BIC$ represents a Bayesian-based decision criterion. In this work we adopt the DIC [17]

$$DIC \equiv 2\chi^2(\theta) - \chi^2(\overline{\theta}),$$

(1)

where $\theta$ is the vector of free model parameters and bars denote averages over the posterior distribution $P(\theta)$. By `Jeffreys scale' convention, a model characterized by $\Delta \chi^2$ (with respect to some reference model) $<1$, $1.0-2.5$, $2.5-5.0$, and $>5.0$ lower than the SM (reference model) would be considered as inconclusively, weakly/moderately, moderately/strongly, or decisively favored [30], respectively. While both $AIC$ & $BIC$ depend only on the peak of the likelihood function, the DIC depends on the mean parameter likelihood and on the effective number of parameters (i.e. those parameters which are well-constrained by the data). Another useful diagnostic is the number of parameters actually constrained by the data [17]

$$p_D \equiv \chi^2(\overline{\theta}) - \chi^2(\bar{\theta}).$$

(2)

This suggests that an extension of a model by $N$ additional parameters is deemed to be reasonable if as a result $p_D$ increases by $\sim N$.

V. RESULTS

The possibility that the $H_0$ tension may reflect a more basic $T_0$ tension has been considered very recently [11, 12]. It was found that for this to be the case, there has to be a noticeable departure of the temperature, extracted from a snapshot of the universe at $z \approx 1100$, such that $A_{T_0} > 1$. As discussed in the next section (and noted in [12]), this result can be explained by the strong $\Omega_k - T_0$ correlation present in the CMB data that dominates over $H_0 - T_0$ anti-correlation expected when (it is assumed that) $\Omega_k = 0$. Whereas these analyses were limited only to P18 or P18+BAO data with the SH0ES prior, we extend the analysis by including also the DES 1 yr and SNIa data. Since
The ‘degradation’ in the precision inference introduced by the addition of $A_T0$ is shown in the right-most column; it is clear that parameters which are significantly biased also suffer from significant degradation commensurate with the interpretation that they are most affected by allowing $T0$ to depart from its FIRAS-deduced value. The value of $A_T0$ quoted in the second column is adopted from [1].

$A_T0$ is strongly correlated with other cosmological parameters, the inclusion of DES and SNIa datasets jointly with the P18 and or P18+BAO data severely limits the range over which $A_T0$ is effectively free to vary. This is akin to the fact that the Planck dataset alone implies that $\Omega_k < 0$ in the SM, but the addition of other probes, most notably BAO, ‘restores’ $\Omega_k = 0$.

In this work we compare the fit of four different cosmological models to various combinations of the datasets. In addition to the flat SM, we refer to the SM with $\Omega_k \neq 0$ as ‘SM+K’; the SM with $A_T0$ as ‘SM+T’, and to the extension of SM+T model to allow for curved space as ‘SM+K+T’. In Table II we show results obtained from the joint analyses of the Planck dataset with the SH0ES prior in the SM and SM+T models. We focus on this particular dataset of SM+T model to allow for curved space as ‘SM+K+T’. In Table II we show results obtained from the joint analyses of the Planck dataset applied to the standard cosmological model (second column) and its extension with a free $A_T0$ parameter (third column) assuming $\Omega_k = 0$. The fourth column shows the relative bias (in 1σ units) between the standard and extended model. For most of the parameters the absolute value of the bias is non-negligible $\sim 3$. Since $H_0$ and $T$ are strongly anti-correlated, a lower $A_T0$ implies a larger $H_0$, the latter ($H_0 = 72.5 \pm 1.3$) is now well within the $1 - \sigma$ uncertainty of the locally inferred value from the SH0ES: $H_0 = 74.03 \pm 1.43$. The ‘degradation’ in the precision inference introduced by the addition of $A_T0$, is shown in the right-most column; it is clear that parameters which are significantly biased also suffer from significant degradation commensurate with the interpretation that they are most affected by allowing $T0$ to depart from its FIRAS-deduced value. The value of $A_T0$ quoted in the second column is adopted from [1].

In Table II we show results obtained from the joint analyses of the Planck dataset with the SH0ES prior in the SM and SM+T models. We focus on this particular dataset combination because it results in the most significant differences between results for the SM and SM+T models. Most of the key parameters are biased by $\sim 3$ standard deviations in the SM as compared with the SM+T model. We define a dimensionless bias for a given parameter $\alpha$ between two models ‘a’ and ‘b’ as $\delta_{\alpha,ab} \equiv (\alpha_b - \alpha_a)/\sqrt{\sigma^2_{\alpha,a} + \sigma^2_{\alpha,b}}$, where $\alpha_i$ and $\sigma_{\alpha,i}$ are the values of $\alpha$ and its uncertainty, respectively, inferred in the $i$th (a or b) model. In case of asymmetric posterior distributions we use the largest error of the two sides of the distribution in the denominator of $\delta_{\alpha,ab}$. In addition, results for the SM seem to underestimate the uncertainties in cosmological parameters when ignoring their correlations with $A_T0$. It is clear from the values shown in the rightmost column of the table that the SM underestimates the uncertainties in many of the parameters by a factor $\sim 3 - 7$, and up to $\sim 46$ in the extreme case of $\theta_{MC}$. While thawing fixed constants always results in increased errors, correlations, and bias, we note that if indeed the temperature evolves non-adiabatically with redshift in a fashion that is theoretically unknown, then these levels of bias and precision degradation may constitute the actual (rather than nominal) precision available at present.

Results for the four models with various Planck, DES, BAO, SNIa dataset combinations, with and without the SH0ES prior, are listed in Table III. The first five lines in Table III correspond to datasets that include the SH0ES prior. From the DIC values it is clear that both P18+SH0ES and P18+DES+SH0ES decisively favor the SM+T over the SM, and ‘benefit’ mainly from adding $A_T0$ as a free parameter (relatively) less so from allowing for a non-flat geometry. The lower DIC obtained for the SM+T model with the P18+DES+SH0ES data combination results when $A_T0 < 1$, which is due to the larger $H_0$ by virtue of the anti-correlation of these two parameters. Specifically, with the P18+SH0ES data we obtain $A_T0 = 0.934 \pm 0.020$ & $0.908 \pm 0.047$ at the 68% C.L., assuming the SM+T & SM+K+T, respectively. These correspond to departure from the canonical value $A_T0 = 1$ at the $\gtrsim 99.9\%$ & $\lesssim 95\%$ C.L, respectively. For the P18+DES+SH0ES our analysis yields $A_T0 = 0.950 \pm 0.013$ & $0.956 \pm 0.025$ at the 68% C.L.

### Table II: Marginalized average values of key cosmological parameters along with their 68% confidence regions for the joint P18+SH0ES analysis applied to the standard cosmological model (second column) and its extension with a free $A_T0$ parameter (third column) assuming $\Omega_k = 0$. The fourth column shows the relative bias (in 1σ units) between the standard and extended model.

| Parameter | Standard Model | Standard Model + $T$ | Bias | Degradation |
|-----------|----------------|----------------------|------|-------------|
| $\Omega_b h^2$ | 0.02259 ± 0.00019 | 0.0182 ± 0.0012 | -3.6 | 6.3 |
| $\Omega_c h^2$ | 0.1182 ± 0.0011 | 0.0980 ± 0.0057 | -3.5 | 5.2 |
| $1000 h_{MC}$ | 1.0435 ± 0.00052 | 1.115 ± 0.021 | 3.1 | 46.2 |
| $\tau$ | 0.0604 ± 0.0070 | 0.0562 ± 0.0078 | -0.4 | 1 |
| $Y_{He}$ | 0.251 ± 0.012 | 0.239 ± 0.013 | -0.7 | 1.1 |
| $\ln(10^{10} A_s)$ | 3.055 ± 0.014 | 3.047 ± 0.016 | -0.3 | 1 |
| $n_s$ | 0.9720 ± 0.0067 | 0.9635 ± 0.0072 | -0.9 | 1.1 |
| $H_0$ | 68.34 ± 0.57 | 72.5 ± 1.3 | 2.9 | 2.3 |
| $\Omega_L$ | 0.6972 ± 0.0072 | 0.779 ± 0.022 | 3.5 | 3.1 |
| $\Omega_m$ | 0.3028 ± 0.0069 | 0.223 ± 0.022 | -3.5 | 3.2 |
| $\Omega_{m0} h^2$ | 0.1414 ± 0.0010 | 0.1169 ± 0.0069 | -3.5 | 6.9 |
| $\sigma_8$ | 0.8107 ± 0.0070 | 0.880 ± 0.023 | 2.9 | 3.3 |
| $S_8$ | 0.814 ± 0.012 | 0.758 ± 0.020 | -2.4 | 1.7 |
| $A_T0$ | 1.0 ± 0.000209 | 0.934 ± 0.020 | -3.3 | 95.6 |
in the SM+T & SM+K+T cases, respectively, with similar C.L for departure from the canonical value as deduced for the P18+SH0ES combination.

Comparison with LSS probes is especially important as it provides an important cross-check. As is well-known, DES yr 1 analysis [26] resulted in $S_8 = \sigma_8(\Omega_m/0.3)^{0.5} = 0.773^{+0.026}_{-0.026}$ and $\Omega_m = 0.267^{+0.030}_{-0.032}$, with $S_8$ systematically lower than inferred for the SM with the P18 data. In the SM, $A_T^P$ is fixed to unity and so $H_0$ cannot increase appreciably, even with the inclusion of the SH0ES prior (due to its strong anti-correlation with $A_T^P$) and consequently $S_8$ cannot correspondingly decrease. We emphasize that compatibility of Planck with LSS probes, such as DES, in the $H_0 - S_8$ plane with the SH0ES prior is not guaranteed, and is thus a non-trivial test for any extension of the SM purported to address the ‘Hubble tension’. For example, it has been noted recently [31] that ‘early dark energy’ models, e.g. [32], while seem to relieve the Hubble tension, bring the CMB- and LSS-derived mass clustering parameter $S_8$ to a more significant tension level than already exists in the SM. To get a sense of the compatibility of the Planck+DES-based inference of $S_8$ in the models explored in this work we mention that when the P18+SH0ES combination is considered $S_8 = 0.814 \pm 0.012, 0.797 \pm 0.013 \& 0.758 \pm 0.020$ in case of the SM, SM+K & SM+T, respectively. For comparison, when P18+DES+SH0ES is considered we obtain $S_8 = 0.8013 \pm 0.0099, 0.790 \pm 0.010, 0.776 \pm 0.012 \& 0.776 \pm 0.013$ in case of the SM, SM+K, SM+T & SM+K+T, respectively. The lower values for $S_8$ obtained in the SM+T & SM+K+T models for P18+SH0ES (i.e. even when the DES is not included in the analysis) bring this dataset combination to a better agreement with the DES collaboration results reported in [26] by virtue of the strong $A_T^P - S_8$ correlation: A higher $H_0$ implies lower $A_T^P$, and $S_8$ as is clearly demonstrated in Figures 1 & 2. In Figure 1 we show the 1- and 2-$\sigma$ confidence contours of $\Omega_b h^2$, $A_T^P$, $H_0$ & $S_8$, along with their posterior distributions for the SM+T model (with and without the SH0ES prior and for all five data set combinations considered in this work). Similarly, shown in Figure 2 are the results for the SM+K+T model, and in Figure 3 for both the SM and SM+T models based on the Planck dataset with the SH0ES prior.

Another statistical diagnostic that supports the SM+T (and to a smaller extent SM+K or SM+K+T) extension for data sets involving Planck & DES is the value of $p_D$ (Eq. 4.2), the number of parameters actually constrained by the data set in question. For all data set combinations considered in this work, $p_D$ increases by $\lesssim 0.2 - 0.3$, except

| Datasets             | $D_{IC_{SM}}$ | $D_{IC_{SM+T}}$ | $D_{IC_{SM+K}}$ | $D_{IC_{SM+K+T}}$ |
|----------------------|---------------|-----------------|-----------------|-------------------|
| P18+SH0ES            | 2828.80       | 2818.25         | 2821.87         | -                 |
|                      | -10.55        | -6.93           | -11.79          | -                 |
| P18+DES+SH0ES        | 3364.84       | 3352.08         | 3355.07         | 3353.05           |
|                      | -12.76        | -9.77           | -11.79          | -                 |
| P18+BAO+SH0ES        | 2833.91       | 2834.05         | 2832.06         | 2831.28           |
|                      | 0.14          | -1.85           | -2.63           | -                 |
| P18+SN+SH0ES         | 3863.59       | 3858.92         | 3857.52         | 3858.49           |
|                      | -4.67         | -6.07           | -5.1            | -                 |
| P18+BAO+SN+SH0ES     | 3869.17       | 3869.04         | 3866.86         | 3866.94           |
|                      | -0.13         | -2.31           | -2.23           | -                 |
| P18                  | 2809.23       | 2806.65         | 2807.72         | -                 |
|                      | -2.58         | -1.51           | -               | -                 |
| P18+DES              | 3349.71       | 3347.56         | 3349.66         | 3344.72           |
|                      | -2.15         | -0.05           | -4.99           | -                 |
| P18+BAO              | 2816.21       | 2816.38         | 2816.87         | 2817.17           |
|                      | 0.17          | 0.66            | 0.96            | -                 |
| P18+SN               | 3844.87       | 3845.63         | 3845.57         | 3845.07           |
|                      | 0.76          | 0.70            | 0.20            | -                 |
| P18+BAO+SN           | 3850.74       | 3851.23         | 3851.82         | 3851.71           |
|                      | 0.49          | 1.08            | 0.97            | -                 |

TABLE III: Model comparison between the SM, SM+T, SM+K, and SM+K+T. For each data combination we calculate the DIC for each of the four models, and three values of $\Delta D_{IC}$ when compared with the SM. We emphasize that while nested models always have lower $\chi^2$ values, their DIC does not have to be lower than that of the extended model due to the ‘penalty’ for additional poorly-constrained parameters. The latter case is best illustrated by the $\Delta D_{IC} > 0$ values of either P18+BAO, P18+SN or P18+BAO+SN for which none of the extended models considered in our work is justified.
for the cases P18+DES & P18+DES+SH0ES. In the former case \( p_D \) increases by 0.23, 1.74 & 1.04 when the SM is extended to SM+K, SM+T, and SM+K+T, respectively, while in the case of P18+DES+SH0ES it increases by 0.63, 0.96, and 1.07. From this perspective the addition of the temperature as a free parameter is warranted in both cases, especially when the SH0ES prior is excluded from the data. In comparison, extension of the SM by adding a free spatial curvature parameter only moderately increases \( p_D \).

The last five lines of Table III correspond to dataset combinations excluding the SH0ES prior. It is clear that in this case only the P18+DES combination benefits (equally well) from allowing for a free \( A_T \), and non-flat geometry. The other three dataset combinations (P18+BAO, P18+SN, P18+BAO+SN) do not favor any of these SM extensions; the improved fit to the data is outweighed by the penalty (albeit small) incurred by adding either \( \Omega_k \), or \( A_T \), or both.

Our interpretation of these results is as follows: P18+SH0ES favor a high value of \( H_0 \) which is anti-correlated with \( A_T \), thus improving the fit when this parameter is added. With the DES dataset a systematically lower \( S_8 \) is obtained which is compatible with the other LSS datasets (e.g. [26]). This parameter is anti-correlated with \( H_0 \), which in turn anti-correlates with \( A_T \). Therefore, the fit with either P18+DES+SH0ES and (to a lower extent) P18+DES is more statistically acceptable when \( A_T \neq 1 \). It is also clear from comparing \( DIC_{SM+T} \) and \( DIC_{SM+K} \) (Table III) when the SH0ES prior is imposed, that in those cases where the fit does not ‘benefit’ from adding \( A_T \), the fit improves somewhat by allowing for curved space, consistent with our expectation for a partial degeneracy between these two parameters. This is a consequence of the fact that a lower \( T_0 \) implies earlier recombination which could be partially mimicked by an open geometry (\( \Omega_k > 0 \)), where light rays are ‘defocused’ and length scales look smaller than they really are. Thus, \( A_T \) is expected to be correlated with \( \Omega_k \); lowering \( \Omega_k \) enhances focusing of the radiation, thereby compensating for the lower temperature effect. This strong correlation qualitatively explains the result of [11] that while the P18+SH0ES combination leads to a relatively low temperature, adding BAO data restores agreement with the local \( T_0 \) measurement. This is the case simply because BAO data result in a strong preference for flat space; fixing the temperature to its locally measured value results also in \( H_0 \) attaining the CMB-inferred value. Thus, it is clear that the parameter trio \( A_T - H_0 - \Omega_k \) is highly correlated, and that when the BAO dataset is added, all these parameters approach their ‘concordance’ values.

It is important to determine whether there is an appreciable statistical preference (based on current datasets) for one of the extended models over the other. Our analysis shows that for virtually all data combinations (without the SH0ES constraint on \( H_0 \)) adding the parameter \( A_T \) to the SM results in a lower DIC when than \( \Omega_k \) is added as a free parameter. This lends additional support to the idea that the small \( \Omega_k \)-tension between Planck and, e.g. BAO data, is no more than a tension between local inference of \( T_0 \) versus the value inferred from CMB features imprinted at recombination, i.e. that the interpretation of closed spatial geometry advocated in [25] could be replaced by the statement that, assuming that space is flat, \( T \) at recombination is higher than is thought based on adiabatic evolution history. Nevertheless, our analysis shows that except for the P18+DES, data combinations that exclude the SH0ES constraint clearly indicate no preference for both SM extensions as they result in even slightly higher DIC values. More interesting in that respect are dataset combinations that do involve the SH0ES constraint. In this case, either P18+SH0ES or P18+DES+SH0ES results in a better fit with SM+T, whereas P18+BAO+SH0ES, P18+SN+SH0ES, or P18+BAO+SN+SH0ES yield better fits to the SM+K+T model. Thus, when the SH0ES constraint is included there is no clear preference for either model extension.

Figure 1 is a triangle plot that shows the 1-\& 2-\( \sigma \) contours and posterior distributions of several parameters for the SM+T model. Shown are results for the P18, P18+DES, P18+BAO, P18+SNIa and P18+BAO+SNIa dataset combinations with (upper panel) and without (lower panel) SH0ES prior. Corresponding results (except for the case of Planck alone) for the SM+K+T model are shown in Figure 2. Allowing for a free \( A_T \) results in a factor of a few larger uncertainties on several parameters, e.g. \( H_0 \). In addition, certain parameters correlated with \( A_T \) are in a \( \sim 2-3 \sigma \) tension with those obtained from the SM-based analysis with the locally determined \( T_0 \).

As we have shown, \( H_0 \) is strongly correlated with \( A_T \); according to our analysis using the recent P18+SH0ES data \( H_0 \) increases to \( \gtrsim 72.5 \pm 1.3 \) \( \text{km/sec/Mpc} \), as compared to the value deduced for the ‘vanilla’ SM (\( \Omega_k = 0 \)), 68.3\( \pm 0.57 \) \( \text{km/sec/Mpc} \). Allowing for non-vanishing spatial curvature only slightly changes this value from 71.3\( \pm 1.2 \) to 72.3\( \pm 1.4 \) \( \text{km/sec/Mpc} \). Therefore, while allowing for a non-vanishing curvature significantly alleviates the Hubble tension, this cannot be equally said for the CMB temperature. Whereas the correlation with \( A_T \) brings the cosmologically-inferred \( H_0 \) closer to its locally-inferred value, perhaps more important is the fact that the uncertainty in this parameter increases by a factor \( \sim 3 \) compared to the value deduced in the standard analysis. These two changes weaken the Hubble tension from \( \gtrsim 5 \sigma \) to only \( \sim 1 - 2 \sigma \) with the local inference of \( H_0 \) from strong lensing and SH0ES, and to \( \lesssim 1 \sigma \) with other local probes. The degeneracy of \( A_T \) with \( S_8 \) and \( H_0 \) brings them closer to locally measured values, thereby alleviating long-standing tensions of cosmologically versus locally inferred values.

The impact of adding either \( \Omega_k \) or \( A_T \) to the best-fit SM can be clearly illustrated by considering \( \chi^2_{\text{SH0ES}} \), which is calculated by CosmoMC when the SH0ES prior is imposed. In the case of P18+SH0ES data combination we obtain \( \chi^2_{\text{SH0ES}} = 16.2 \pm 3.2, 4.3 \pm 3.5 \) and \( 2.1 \pm 2.3 \) for the SM, SM+K and SM+T models, respectively. A qualitatively similar impact is seen for the P18+DES+SH0ES combination, for which the corresponding values are 13.5 \( \pm 2.6 \),
2.8 ± 2.3, 4.1 ± 2.3, and 3.9 ± 3.0, respectively. Assuming a-priori the fixed values \( \Omega_k = 0 \) and \( A_{T_0} = 1 \) yields a low \( H_0 \), and consistency with the SH0ES prior results in a \( \chi^2 \) value that is higher by \( O(10) \). When these parameters are allowed to vary, a significantly lower \( \chi^2 \) and (even more relevant) overall DIC values are obtained.

VI. SUMMARY

Despite the remarkable success of the SM, over the last decade significant evidence has mounted that various degrees of tension exist between locally inferred cosmological parameters and inference from the earlier universe. While it is clearly of interest to explore the possibility that these tensions could be at least partially resolved, our other key objective is the inference of \( T(z) \) at \( z \approx 1100 \) as part our ongoing interest in direct determination of \( T(z) \) over the widest possible redshift range. The analysis reported here has resulted in an improved determination of the CMB temperature at recombination in both flat or curved space, with and without the local SH0ES prior on \( H_0 \).

Whereas the physics of recombination is very sensitive to the CMB temperature correlations of \( A_{T_0} \) with most of the other key cosmological parameters limits the precision of \( A_{T_0} \) extraction from the P18 dataset to the percent level. In principle, the spectrum of the thermal component of the SZ effect could be an ideal probe of the evolution of the temperature with redshift due to its exponential dependence on the temperature and lack of correlation with other cosmological parameters. However, high-quality measurements of the effect towards clusters at sufficiently high redshifts are rare and certainly not competitive with the \( z_* \approx 1100 \) leverage provided by CMB anisotropy. In addition, marginalization over the cluster comptonization parameter and gas temperature (the latter is required for marginalization over relativistic corrections to the non-relativistic effect), as well as over the cluster bulk velocity, significantly degrade the nominal achievable precision based on the spectral shape of the thermal effect. Nevertheless, near future surveys with e.g. the Simons Observatory are expected to detect thousands of clusters; this opens up the possibility of complementing CMB anisotropy with an independent probe that continuously samples \( T(z) \) up to \( z \approx 2 \). Of course, the efficiency of this probe is expected to depend on the specific non-standard \( T(z) \) models.

Independent inference of \( T \) at recombination is an important consistency test of the SM, and of the assumption of adiabatic evolution history in particular. Unlike the case of local measurements that are based on the steep dependence of the Planck spectrum on \( T \), the inference from temperature anisotropy and polarization at \( z \approx 1100 \) carried out in this work is based on athermal angular power spectra aided by luminosity distance measurements, as well as other observables that do not explicitly depend on the CMB temperature. In addition, the temperature perturbation measurements, from which we extract \( T(z_*) \), are \( O(10^{-5}) \) times smaller than \( T_0 \). Nevertheless, the 68% uncertainty level on the inferred temperature is only \( O(10^2) \) larger than the uncertainty obtained from local measurements due to the strong dependence of the CMB power spectra on \( T(z_*) \). This \( \sim 1\% \) precision elevates precision determination of \( T(z_*) \) to the current typical precision level of other cosmological parameters.

Insight gained on the Hubble tension is that treating the CMB temperature as a free parameter considerably lowers the previous \( \sim 5\sigma \) tension at a ‘cost’ of a moderately lower temperature, at the \( 2\sigma \) level, than would be expected if the locally measured \( T \), extrapolated to \( z \approx 1100 \), holds at the FIRAS-deduced precision level. Similarly reduced level of Hubble tension can also be achieved by a slightly positive value of \( \Omega_k \), i.e., in a model with geometrically open spacetime. Without the local Hubble prior all dataset combinations considered in this work favor either a lower \( T \) at last scattering, or the FIRAS value, and either flat or closed universe. Notable departures from the ‘vanilla’ \( T(z) \) values are obtained for SM+T model when contrasted with the P18 \& P18+DES datasets; in this case \( A_{T_0} = 1.189 \pm 0.080 \) \& \( A_{T_0} = 0.968 \pm 0.015 \), respectively, where the former nicely illustrates the \( \Omega_k - T \) correlation. For the SM+K model with only Planck data are included, \( A_{T_0} = 1 \), but only if space is flat. The \( \gtrsim 2\sigma \) higher CMB temperature at recombination that we find here will increase to \( \sim 3\sigma \), if \( \Omega_k \) increases in a SM+K+T model (as found also in [12]). Much like in the SM+K model, we obtain in the SM+T model the anomalously small value \( H_0 = 53 \pm 6 \text{ km/sec/Mpc} \).

Allowing for non-vanishing curvature in the SM+K+T model we obtain \( \Omega_k = -0.0081^{+0.0046}_{-0.0050} \) \& \( A_{T_0} = 0.940 \pm 0.022 \) when the P18+DES dataset combination. When the same model is contrasted with the P18+SN dataset combination we deduce \( \Omega_k = -0.0089 \pm 0.0063 \) \& \( A_{T_0} = 0.977 \pm 0.020 \). In these cases \( A_{T_0} \) is \( \sim 1\sigma \) lower than in the corresponding dataset combinations in the SM+T model, simply due to the \( \Omega_k - T \) correlation; imposing flat space artificially increases the temperature at last scattering. In the same vein, these same dataset combinations are consistent with flat space if the SM+K model is considered, again due to the \( \Omega_k - T \) correlation; letting the temperature to be different than the FIRAS value allows both \( \Omega_k \) \& \( T \) to drop to their data-favored values.

The above result of a possibly higher temperature at recombination than extrapolated from local measurements (under the assumption of adiabatic evolution) has an intriguing implication: The observed locations of the acoustic peaks in the anisotropy power spectrum can only be consistent with a higher temperature if (global) spatial curvature is negative, i.e. if the the radiation is de-focused. If so, then by not allowing the temperature to be a free parameter would result in a larger curvature radius, infinite (flat space), or even a closed universe. The preference seen in the Planck data for a spatially closed geometry is based on this exact premise – that the comoving temperature at
recombination is fixed to its locally inferred value. Therefore, it is possible that the deduced ‘closed’ space at the ~ 2σ confidence level actually implies that space is flat, but that T at recombination is ~ 2σ (i.e. a few percent) higher than inferred locally. Since other cosmological probes are sensitive to the spatial geometry, but are insensitive to $A_T$, they all favor flat space (in clear contrast with the indication from the Planck data). We conclude that it is a viable possibility that – rather than suggesting a closed spatial geometry – the Planck result could be interpreted as an indication for a non-standard evolution of the CMB temperature, i.e. that the universe has been cooling down somewhat faster than expected in the SM.

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FIG. 1: Confidence contours and posterior distributions for selected parameters assuming the SM+T model. SH0ES prior included (top panel) or excluded (bottom panel).
FIG. 3: Confidence contours and posterior distributions for selected parameters assuming the SM+K+T model: SH0ES prior.
FIG. 3: Confidence contours and posterior distributions for selected parameters obtained for the SM+T and SM models extracted from the P18+SH0ES data.