Comparison of Constitutive Relationships of Tubes Established Using Uniaxial Tensile Tests and Tube Hydroforming Experiments

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Abstract. The aim of the present paper is to evaluate existing constitutive models and to fitting hardening laws of SS304 tubes for the accurate prediction of the deformation behaviors of the tubes in hydroforming. Uniaxial tensile test (UTT) and free hydro-bugling (FHB) experiments were conducted on SS304 tubes, and a hi-speed three-dimensional (3D) digital image correlation (DIC) system was applied to obtain the deformation data of the samples. Eight constitutive relationships of the tubes were then established by fitting the equivalent stress and strain data with the four existing constitutive models of Hollomon, Ghosh, Voce and Ghosh/Voce, and the fitting accuracy of the obtained constitutive relationships were analyzed and compared. The results show that Ghosh/Voce model holds the highest accuracy in describing the deformation behaviors of the tubes in UTT and FHB, followed by the Ghosh model and then the Hollomon model. The Voce model holds the lowest accuracy. A distinct discrepancy between the constitutive relationships obtained using UTT and FHB experiments are observed in present research conditions.

Keywords. Tube; Constitutive relationship; Uniaxial tensile test; Free hydro-bugling

1. Introduction

Tube hydroforming (THF) is an advanced near net-shape technology of hollow parts with complex cross-sections. It has been widely used in automotive and aircraft industries owing to its improvement of structural stiffness and weight reduction in products [1]. An accurate constitutive relationship is essential for the analyses of the forming process of THF. Constitutive relationship is an equation to describe the relationship between the equivalent stresses and strains of metallic materials under plastic deformation. Uniaxial tensile test (UTT) is widely used to establish the constitutive relationship of metals in plastic deformation. However, free hydro-bugling (FHB) experiments have increasingly been adopted to establish the constitutive relationships of metallic tubes for THF, due to their consistent biaxial stress state. For instance, N H Wang et al. [2] fitting the polynomial hardening law for tubes based on the plastic incremental theory by conducting UTT and THF. T Hakoyama et al. [3] fitting the hardening law of multiaxial tubes using the Ghosh model by conducting FHB experiments. The aim of this paper is to fitting hardening laws of SS304 tubes using UTT and FHB experiments, respectively. In the experiments, a hi-speed three-dimensional (3D) digital image correlation (DIC) system was applied to obtain the deformation data of the samples. Eight constitutive expressions were then
obtained by fitting the equivalent stress and strain data with four existing constitutive models of Hollomon, Ghosh, Voce and Ghosh/Voce. The obtained constitutive relationships were analyzed and compared.

2. Constitutive models
Diversified constitutive models exist at present. Among them, Hollomon and Ghosh model are both power functions. Hollomon model, a special form of Ghosh model, was put forward by Hollomon [4] in 1944. Hollomon model is commonly used to describe the stress-strain relation of steels and nonferrous metals in plastic deformation. Voce model is an exponential function that was put forward by Voce [5] in 1948. The Ghosh/Voce model, a combination model of the power function and the exponential function, have been used to describe the stress-strain relationship in the deformation process of metal materials [6, 7].

In the present research, four typical models including Hollomon, Ghosh, Voce, Ghosh/Voce, as shown in Tab. 1, are selected to investigate based on the features of THF.

Table 1. The constitutive models and the values of the material parameters obtained by fitting the data of UTT and FHB

| Constitutive models | Expressions of the models | Material Parameters | Values based on UTT (Fitting precision) | Values based on FHB (Fitting precision) |
|---------------------|---------------------------|--------------------|----------------------------------------|----------------------------------------|
| Hollomon            | $\sigma = k \epsilon^n$ | $k, n$             | $k=1594$, $n=0.4491$ ($R^2=0.9987$)     | $k=1540$, $n=0.4517$ ($R^2=0.9838$)     |
| Ghosh               | $\sigma = a (\epsilon_a + \epsilon)^{1-c}$ | $a, b, c, \sigma_y$ | $(R^2=0.999, \text{RMSE}=6.638)$         | $(R^2=0.9845)$                         |
|                     | $\epsilon_a = \sqrt{\sigma_y / a}$ |                      |                                        |                                        |
| Voce                | $\sigma = a - b e^{-c}$ | $a, b, c$          | $a=1767, b=0.3871, c=203.1$ ($R^2=0.999$) | $a=1733, b=0.51, c=105.6$ ($R^2=0.984$) |
|                     |                           |                    | $\sigma_y=360$MPa                     | $\sigma_y=360$MPa                     |
| Ghosh/Voce          | $\sigma = k (\text{Ghosh}) + (1-k) (\text{Voce})$ | $k$               | $k=1.017 (R^2=0.9999, \text{RMSE}=6.512)$ | $k=0.9955 (R^2=0.985)$                |

(Tips: $R^2$, the determination coefficient, indicate the fitting accuracy, and a smaller value of $|R^2-1|$ indicates a higher fitting precision. RMSE, the root-mean-square error, judge the deviation between the fitted values and the experimental values, and a bigger value of $|\text{RMSE} - 1|$ indicates a higher fitting precision.)

3. Descriptions of experiments

3.1. Uniaxial Tensile Test (UTT)

**Figure 1.** Dimensions of the specimen for UTT

SS304 tubes with the out diameter of 32 mm and wall thickness of 0.6 mm were adopted in present experiments. The specimens for the UTT were shown in Fig. 1. The UTT were performed on an electronic universal testing machine, a model of WDW-100E, made by Time Group Inc. The testing machine holds the following specifications: maximum capacity of 100 KN, moving speed range of 0.005-500 mm/min,
etc. The UTT were carried out with the tension speed 6 mm/min at room temperature. A hi-speed 3D DIC system, as shown in Fig. 2, was applied to obtain the deformation data of the specimens. The images of the deformation zone of the specimens were synchronously captured by the CCD cameras. The deformation data of 3D displacement, strain and other parameters were calculated by the analysis system based on the images. The effective stresses and strains were then obtained by dealing with the deformation data using the method described in reference [2]. Finally, four constitutive expressions, as shown in Tab. 1, were achieved by fitting the effective stresses and strains in Matlab.

3.2. Free hydro-bugling (FHB) experiments
Tubular specimens, with the initial length \( L = 110 \) mm and the bulging length \( l = 50 \) mm, for FHB were made from the SS304 tubes. The FHB experiments were conducted on the THF system, as shown in Fig. 3. It consisted of a hydraulic power generation system, a hi-speed 3D DIC system and a FHB device.

![Figure 3. Configuration of self-developed THF system](image)

In the pressure supply system, required hydraulic fluid with a certain pressure for the hydro-bugging was created. The hydraulic fluid was then led into the tubular specimen to bulge it to crack. During the entire process of bulging, the hydraulic pressure was recorded by a sensor, and the deformation data on the bulging zone were synchronously captured by the hi-speed 3D DIC system. The effective stress and strain were determined by the methods as described in reference [2]. After the equivalent stress and strain in FHB process were obtained, four constitutive models, as shown in Tab. 1, were achieved by fitting the effective stresses and strains in the software Matlab.

4. Results and discussion

![Figure 4. Constitutive curves of tubes](image)

The eight constitutive curves plotted according to the constitutive expressions in Tab. 1 are shown in Fig. 4. In view of the degree of the approximation of the fitting curves to the experimental data, it is obvious to observe that Ghosh/Voce model holds the best approximation, followed by the Ghosh model and the Hollomon model, and the Voce model holds the worst approximation on either UTT or FHB cases.
Moreover, with respect to fitting accuracy, compared with the constitutive curves based on FHB, the constitutive curves based on UTT are closer to their own experimental data. The constitutive curves based on the three constitutive models and the experimental data from the UTT and FHB are shown in Fig. 5. In this section, a relative deviation $\delta=(\sigma_{\text{UTT}} - \sigma_{\text{FHB}})/\sigma_{\text{FHB}}$ is introduced to indicate the maximum difference between the stress $\sigma_{\text{UTT}}$ obtained from UTT and the stress $\sigma_{\text{FHB}}$ obtained from FHB. In the case of the Hollomon model, as illustrated in Fig. 5(a), a relative deviation $\delta=3.81\%$ happens when the strain arrives 0.33, and $\delta=3.68\%$ when the strain comes to 0.5, the end of the deformation. In the case of the Ghosh model, as illustrated in Fig. 5(b), a relative deviation $4.69\%$ appears when the strain comes to 0.15, despite a relative deviation of 38.33\% at the start of the deformation. In the case of the Ghosh/Voce model, as illustrated in Fig. 5(c), a relative deviation $4.90\%$ appears when the strain comes to 0.15, despite a relative deviation of 39.43\% at the start of the deformation.

![Figure 5. Comparison of the constitutive curves based on the experimental data between the UTT and FHB and the constitutive models of: (a) Hollomon, (b) Ghosh, and (c) Ghosh/Voce](image)

5. Conclusions

Based on the analyses mentioned above, the following conclusions can be drawn from the study:

1. Among Hollomon, Ghosh, Voce, Ghosh/Voce constitutive models, the deformation behaviors of SS304 tubes in UTT and FHB can best be described by the Ghosh/Voce model, followed by the Ghosh model and the Hollomon model, but worst by Voce model.

2. With respect to fitting accuracy, compared with the constitutive curves based on FHB, the constitutive curves based on UTT are closer to their own experimental data.

3. There are distinct difference between the constitutive curves based on UTT and FHB. The maximum differences of the two constitutive curves based on UTT and FHB are 3.81\%, 4.69\% and 4.90\% as for using the Hollomon, Ghosh and Ghosh/Voce models, respectively.

6. References

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Acknowledgements

The authors gratefully acknowledge the support of the National Natural Science Foundation of China (grant number 51271062), Guangxi Natural Science Foundation (grant number 2013GXNFAA019305).