Relative Position Estimation for Formation Control with the Fusion of Predicted Future Information and Measurement Data

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Abstract: This paper addresses a relative position estimation problem for formation control of multiple robots. In the authors’ previous paper, a relative position estimation method has been proposed, which fuses information from distance sensors and wireless communication. In this method, it is assumed that the robots communicate with others by wireless devices at every control sampling time. Therefore, depending on the performance of the wireless devices, the control sampling time should be set to a large value, which can degrade control performance. In this paper, we propose a new relative position estimation method, which is effective even if the communication sampling time is longer than the control sampling time. The idea in this method is to use predicted information on the time-series of the control input from detected robots. We develop a method to generate the time-series of the predicted control input for successful estimation. Finally, we verify the effectiveness of the proposed method by simulations and an experiment.

Key Words: formation control, multi-robot systems, position estimation.

1. Introduction

Recently, formation control of multi-robot systems has been vigorously investigated [1]–[3]. Application examples include a formation movement by unmanned aerial vehicles (UAVs) [4], automatically operated trucks [5], satellite formation [6], a search at disaster sites [7], and so on. We can expect to improve work efficiency and to expand the range of work by employing multiple robots.

For formation control, robots need the information on the relative positions of other robots. In the existing studies [8]–[10], cameras or laser range finders are used to achieve such information. However, it is sometimes difficult to equip small robots with these large and heavy sensors requiring high performance to robots. To overcome this issue, the authors proposed a relative position estimation method [11],[12], which can be carried out via only small distance sensors and wireless communication devices. The papers [11],[12] showed that the relative positions can be estimated even if not all distance sensors are available, by fusing information from distance sensors and wireless communication. However, this method has a drawback: the robots are required to communicate with other robots by wireless devices at every control sampling. Therefore, depending on the performance of the wireless devices, the control sampling time should be set to a large value, which can degrade control performance. In this case, we might not be able to achieve a desired formation.

In this paper, we propose a new relative position estimation method which is effective even if the communication sampling time is longer than the control sampling time. Our idea for this is to utilize predicted information on the time-series of the control input of detected robots through wireless communication. The required information is the predicted future control input before the next communication sampling. Therefore, not all the motions of the robots have to be predetermined. This information is used for the accurate estimation of the relative positions of the detected robots, by fusing it with the information from distance sensors over the extended Kalman filter (EKF). The proposed method enables us to set the control sampling time to a smaller value than the communication sampling time to improve control performance. The problem to be solved here is how the robots generate the predicted future information for successful estimation. In this study, we propose a method to generate it by using the state equations of the relative position and the relative attitude angle of the detected robots. Finally, the effectiveness of the proposed method is demonstrated by not only simulations but also an experiment.

This paper is based on the conference paper of the authors [13]. The update points are as follows: (i) details of the proposed methods are provided, and (ii) the experimental result is added.

![Fig. 1 Line formation.](image_url)
2. Problem Setting

2.1 Target System

Consider \( n \) robots on a 2-dimensional workspace. The set of the numbers of the robots is described as \( \mathcal{V} = \{1, 2, \ldots, N\} \). Robot 1 is the leader, and robots 2 to \( N \) are followers. Assume that the shapes of the robots are given by circles with radius \( r > 0 \). Here, the leader leads the group, and the followers form a line formation as Fig. 1. Assume that the leader is not affected by the others.

As Fig. 2, we have a global coordinate system, common among the robots, and relative coordinate systems, different from each other. The position of robot \( i \in \mathcal{V} \) in the global coordinate system is determined by the center of the circle of the robot, defined as \( \mathbf{p}_i[k] = [x_i[k], y_i[k]]^\top \in \mathbb{R}^2 \), where \( k \) is the discrete time. Let \( \theta_i[k] \in [0, 2\pi) \) be the angle of robot \( i \) from \( x \)-axis of the global coordinate system.

Robot \( i \in \mathcal{V}\{1\} \) follows robot \( i - 1 \). Then, as shown in Fig. 3, the relative position and the relative attitude angle of robot \( i - 1 \) in the relative coordinate system of robot \( i \) are described as \( \mathbf{p}^{(i)}_{\text{rel}}[k] = [p_{\text{rel}}^{(i)}[k], y_{\text{rel}}^{(i)}[k]]^\top \in \mathbb{R}^2 \) and \( \theta_{\text{rel}}^{(i)}[k] \in [0, 2\pi) \), respectively, where the superscript \[\text{[i]}\] shows that the variables are defined in the relative coordinate system of robot \( i \). Then, the relationships among the relative and global variables, \( \mathbf{p}^{(i)}_{\text{rel}}[k], \theta_{\text{rel}}^{(i)}[k], \mathbf{p}_{\text{global}}[k], \mathbf{p}_i[k] \) and \( \theta_i[k] \), are represented as

\[
\mathbf{p}^{(i)}_{\text{rel}}[k] = \mathbf{R}(\theta_i[k]) \mathbf{p}_i[k],
\]

\[
\theta_{\text{rel}}^{(i)}[k] = \theta_i[k] - \theta_{\text{global}}[k],
\]

where the rotation matrix \( \mathbf{R}(\theta) \in \mathbb{R}^{2 \times 2} \) is given as

\[
\mathbf{R}(\theta) = \begin{bmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{bmatrix}.
\]

We assume that the robots are omnidirectionally movable and their velocities and angular velocities are controllable. Then, the dynamics of robot \( i \) is given as

\[
\mathbf{p}_i[k+1] = \mathbf{p}_i[k] + \mathbf{R}(\theta_i[k]) \mathbf{u}_{\text{rel}}^{(i)}[k] \Delta t,
\]

\[
\theta_i[k+1] = \theta_i[k] + \omega_{\text{rel}}[k] \Delta t,
\]

with the control input \( \mathbf{u}_{\text{rel}}^{(i)}[k] = [u_{\text{rel}}^{(i)}[k]]^\top \in \mathbb{R}^2 \) is the velocity input of robot \( i \), and \( \omega_{\text{rel}}[k] \in \mathbb{R} \) is the angular velocity input of robot \( i \), and \( \Delta t \) is the control sampling time.

2.2 Position Estimation and Equipped Devices

In order to achieve a formation, the following controller has been proposed in the authors’ previous paper [14]:

\[
\mathbf{u}_{\text{rel}}^{(i)}[k] = k_{p}\left(\mathbf{p}_{i}[k] - \mathbf{q}_{t} e(\phi_{i}^{(i)})\right),
\]

\[
u_{\text{rel}}[k] = k_{q}q_{t} e(\phi_{i}^{(i)}) + \frac{\pi}{2},
\]

where \( k_{p}, k_{q} > 0 \) are gains, \( e(\theta) = [\cos \theta \ \sin \theta]^\top \in \mathbb{R}^2 \), and \( q_{t} > 0 \) and \( \phi_{i}^{(i)} \in [0, 2\pi) \) are the desired distance and angle, respectively, between robots \( i \) and \( i - 1 \). Then, the formation shape determined by \( q_{t} \) and \( \phi_{i}^{(i)} \in [0, 2\pi) \) is achieved.

To implement the controllers (5) and (6), the value of the relative position \( \mathbf{p}^{(i)}_{\text{rel}}[k] \) of robot \( i-1 \) is required, which has to be estimated with some obtainable information. For this purpose, we assume that distance sensors and wireless communication devices are available.

Each robot is equipped with two distance sensors, which are numbered as \( m = 1, 2 \). A distance sensor \( m \) is placed to robot \( i \) at the position \( \mathbf{s}^m_{i} = [\mathbf{s}_{\text{sim}}^m, \mathbf{s}_{\text{sim}}^m]^\top \in \mathbb{R}^2 \) toward the angle \( \phi_{m}^i \in [0, 2\pi) \) from \( x \)-axis on the relative coordinate system of robot \( i \). If the light of distance sensor \( m \) hits robot \( i - 1 \), robot \( i \) can obtain the information on distance \( \ell_{m}(t) > 0 \).

As for wireless communication, let \( \Delta t \) be the communication sampling time of robots with a positive integer \( L \), which means that the communication sampling time is \( L \) times longer than the control sampling time \( \Delta t \). At every \( \Delta t \), robot \( i \) receives some information from robot \( i - 1 \) through wireless communication. Here, we propose to adopt the time-series of the predicted input \( \hat{\mathbf{u}}_{\text{rel}}^{(i)}[k-\ell][k-\ell+1], \ell = 0, 1, \ldots, L - 1 \) of robot \( i - 1 \) as such information. Here, the predicted and estimated values are described with hats, and \( [k:\ell] \) represents information predicted at time \( k \).

An estimation method has been discussed in the authors’ previous papers [11],[12], where \( L = 1 \) is assumed. In contrast, \( L \) can be arbitrarily chosen in the current paper, which enables us to set the control sampling time to a smaller value than the communication sampling time to improve control performance.

2.3 Problem Setting

At time \( k \), the information obtainable to robot \( i \) through wireless communication from robot \( i - 1 \) is the time-series of the predicted input \( \hat{\mathbf{u}}_{\text{rel}}^{(i)}[k-\ell][k-\ell+1], \ell = 0, 1, \ldots, L - 1 \). The information obtained through the distance sensors is the measurement values, \( d_{ij}[k] \) and/or \( d_{2ij}[k] \). In this paper, we have two objectives. (i) First, we propose a method for robot \( i \) to estimate the relative position \( \mathbf{p}^{(i)}_{\text{rel}}[k+j] \) of robot \( i - 1 \) at time \( k + j (j = 0, 1, \ldots, L - 1) \) by using the obtainable information. (ii) Second, we propose a method for robot \( i \) to generate the time-series of the predicted input \( \hat{\mathbf{u}}_{\text{rel}}^{(i)}[k+j][k+j], \ell = 0, 1, \ldots, L - 1 \) to send to robot \( i + 1 \) for successful estimation. Combining the
methods (i) and (ii), each robot can estimate the relative position of its following robot.

3. Existing Method for Relative Position Estimation

In this section, we explain a relative position estimation method using distance sensors and wireless communication, based on the authors’ previous papers [11,12].

From Eqs. (1), (2), (3), and (4), the state equation is derived as follows:

\[ x_{i-1}^{(i)}[k+1] = f(x_{i-1}^{(i)}[k], u_{i-1}^{(i)}[k], u_i^{(i)}[k]) + v[k], \quad (7) \]

where \( x_{i-1}^{(i)}[k] \) is \( \Phi (x_{i-1}^{(i)}[k])^{\top} \theta_i^{(i)}[k]^{\top} \in \mathbb{R}^2 \times [0, 2\pi] \) is the state, \( v[k] \in \mathbb{R}^2 \times [0, 2\pi] \) is the system noise, and \( f(\cdot) \in \mathbb{R}^2 \times [0, 2\pi] \) is the nonlinear function given by

\[
\begin{align*}
    f(x_{i-1}^{(i)}[k], u_{i-1}^{(i)}[k], u_i^{(i)}[k]) &= [F_i[k]p_i^{(i)}[k] + (R(\theta_i^{(i)}[k])u_{i-1}^{(i)}[k] - u_i^{(i)}[k]) \Delta_t] \\
    &\quad - \delta^{(i)}_1, [k] + (\omega_{i-1}[k] - \omega_i[k]) \Delta_t], \\
\end{align*}
\]

(8)

where

\[
F_i[k] = \begin{bmatrix}
    1 & u_i[k] \Delta_t \\
    -u_i[k] \Delta_t & 1
\end{bmatrix} \in \mathbb{R}^{2 \times 2}.
\]

Let \( y[k] \) be the values obtained from the distance sensors, which are defined as follows:

- If both of the two sensors hit the following robot, \( y[k] = [d_1[k], d_2[k]]^{\top} + w[k] \in \mathbb{R}^2 \).
- If only one sensor \( m \) hits the following robot, \( y[k] = d_{im}[k] + w[k] \in \mathbb{R}^1 \).
- If no sensors hit the following robot, \( y[k] \) is unavailable.

Here, \( w[k] \in \mathbb{R}^2 \) or \( \mathbb{R}^1 \) is the observation noise. Then, the observation equation is given as

\[ y[k] = h(x_{i-1}^{(i)}[k]) + w[k] \quad (9) \]

with the nonlinear function \( h(\cdot) \in \mathbb{R}^2 \) or \( \mathbb{R}^1 \):

\[
h(x_{i-1}^{(i)}[k]) = \begin{bmatrix}
    \begin{cases}
        a_{11}(p_i^{(i)}) - a_{12}(p_i^{(i)}) \sqrt{b_1(\phi_i^{(i)}) - b_2(\phi_i^{(i)})} \\
        a_{21}(p_i^{(i)}) - a_{22}(p_i^{(i)}) \sqrt{b_1(\phi_i^{(i)}) - b_2(\phi_i^{(i)})}
    \end{cases} \\
    \begin{cases}
        a_{11}(p_i^{(i)}) - a_{12}(p_i^{(i)}) \sqrt{b_1(\phi_i^{(i)}) - b_2(\phi_i^{(i)})} \\
        a_{21}(p_i^{(i)}) - a_{22}(p_i^{(i)}) \sqrt{b_1(\phi_i^{(i)}) - b_2(\phi_i^{(i)})}
    \end{cases}
\end{bmatrix},
\]

(10)

where

\[
D_{im} = b_{im}^2(p_i^{(i)}) - a_{im}c_{im}(p_i^{(i)}),
\]

with

\[
a_{im} = 1 + \tan^2 \phi_i^{(i)}.
\]

\[ b_{im}(p_i^{(i)}) = x_{im}^{(i)} + x_{im}^{(i)} \tan^2 \phi_i^{(i)} - (x_{im}^{(i)} - x_{im}^{(i)}) \tan \phi_i^{(i)}.
\]

\[ c_{im}(p_i^{(i)}) = (y_{im}^{(i)} - x_{im}^{(i)} + x_{im}^{(i)} \tan \phi_i^{(i)})^2 + x_{im}^{(i)} - r_i^{(i)}.
\]

Equation (10) is derived from the geometric relations between robots and sensors.

From the state and observation Eqs. (7) and (9), robot \( i \) can estimate the relative position \( p_i^{(i)}[k] \) of robot \( i - 1 \) by the EKF if the communication sampling time is the same as the control sampling time, namely \( L = 1 \). See [11,12] for details.

4. Proposed Method

Now, we consider the situation that the communication sampling time is \( L \) times longer than the control sampling time. We propose the following strategy to estimate \( p_i^{(i)}[k] \) under this situation.

(i) First, we develop a method for robot \( i \in V\setminus\{1\} \) to estimate the relative position \( p_i^{(i)}[k + j] \) of robot \( i - 1 \) at time \( k + j \) \((= 0, 1, \ldots, L - 1) \). Robot \( i \) obtains the information on the time-series of the control input \( \hat{u}_i^{(i)}[k - 1][k - 1 + \ell], \ell = 0, 1, \ldots, L - 1 \) at time \( k \) through wireless communication. This information can be used in Eq. (8) by replacing the control input \( u_i^{(i)}[k + j] \) of robot \( i - 1 \) with \( \hat{u}_i^{(i)}[k - 1][k + j] \). From this, robot \( i \) can estimate the relative position \( p_i^{(i)}[k + j] \) by fusing the information on the predicted input \( \hat{u}_i^{(i)}[k - 1][k + j - 1] \) and the measurement values \( (d_1[k + j] \text{ and/or } d_2[k + j]) \) of the distance sensors by the EKF. The details are omitted here because it directly follows the method in the existing studies [11,12].

(ii) Next, we develop a method for robot \( i \) to generate the time-series of the predicted input \( \hat{u}_i^{(i)}[k + \ell], \ell = 0, 1, \ldots, L - 1 \) at time \( k \). Here, the information on the time-series of the predicted input \( \hat{u}_i^{(i)}[k - 1][k - 1 + \ell], \ell = 0, 1, \ldots, L - 1 \) of robot \( i - 1 \) at the previous time \( k - 1 \) is available, which is sent through wireless communication. Now, we propose the following procedure.

(ii.a) Set the initial values \( \hat{x}_{i-1}^{(i)}[k][k + \ell], \ell = 1, 2, \ldots, L - 1 \) iteratively using

\[
\hat{x}_{i-1}^{(i)}[k][k + \ell] = f(x_{i-1}^{(i)}[k + \ell - 1], \hat{u}_{i-1}^{(i)}[k + \ell - 1]),
\]

(11)

\[
\hat{u}_i^{(i)}[k + \ell] = \begin{bmatrix}
    \hat{u}_i^{(i)}[k + \ell] \\
    \hat{u}_i^{(i)}[k + \ell - 1]
\end{bmatrix}^{\top} \hat{u}_i^{(i)}[k + \ell]^{\top},
\]

(12)

where

\[
\hat{u}_i^{(i)}[k + \ell] = k_p \hat{p}_i^{(i)}[k + \ell] - q_i \phi_i^{(i)}[k + \ell],
\]

\[
\hat{u}_i^{(i)}[k + \ell] = k_p \hat{p}_i^{(i)}[k + \ell] - q_i \phi_i^{(i)}[k + \ell] + \pi / 2.
\]

Equation (11) predicts the state \( \hat{x}_{i-1}^{(i)}[k + \ell], \ell \), obtained from Eq. (8) by replacing \( x_{i-1}^{(i)}[k], u_i^{(i)}[k-1][k], \) and \( u_i^{(i)}[k] \) with \( \hat{x}_{i-1}^{(i)}[k + \ell - 1], \hat{u}_i^{(i)}[k - 1][k + \ell - 1], \) and \( \hat{u}_i^{(i)}[k + \ell - 1], \) respectively. Equation (12) derives the predicted control input \( \hat{u}_i^{(i)}[k + \ell], \ell \), obtained from Eqs. (5) and (6). Then, robot \( i \) can generate the time-series of the predicted input \( \hat{u}_i^{(i)}[k + \ell], \ell = 0, 1, \ldots, L - 1 \), which will be sent to robot \( i + 1 \).

From (i) and (ii), we develop the following algorithm for relative position estimation by robot \( i \).
Algorithm for relative position estimation

[A.1] At time $k$, set $j = 0$, and receive $\hat{u}_{i-1}^{[k-1]}[k-1][k-1 + \ell], \ell = 0, 1, \ldots, L - 1$ from robot $i-1$ through wireless communication.

[A.2] At time $k + j$, update $\hat{x}_{i}^{[k+j]}$ based on the method (i), and decide $u_{i}^{[k+j]}$ from $\hat{p}_{i}^{[k+j]}$ based on Eqs. (5) and (6).

[A.3] If $j = 0$, go to [A.4]. If $j < L - 1$, add one to $j$, and go back to [A.2]. Otherwise, go back to [A.1].

[A.4] Set $x_{i}^{[k]}[k], \hat{u}_{i}^{[k]}[k]$ according to (ii.a), generate $\hat{u}_{i}^{[k]}[k + \ell], \ell = 0, 1, \ldots, L - 1$ according to (ii.b), and send this information to robot $i + 1$ through wireless communication. Add one to $j$, and go back to [A.2].

5. Simulation

We compare the proposed method and the existing method through numerical examples. The number of robots is five ($= N$), where the leader is robot 1, and the followers are robots 2 to 5. The radii of the robots are all $r = 100$ mm. Noise is added to the measurement values of the distance sensors. The communication sampling time of the wireless devices is $0.2$ s. For the proposed method, the control sampling time is set to $\Delta t = 0.02$ s, and $L = 10$ is obtained because $L\Delta t = 0.2$ s is the communication sampling time. The desired distance and angle are given as $q_{i}^{*} = 350$ mm and $\phi_{i}^{*} = 0$ rad, respectively, for all $i$. The controllers are designed as Eqs. (5) and (6) with the gains $k_{p1} = 8$ and $k_{d0} = 0.00004$. The simulation conditions are summarized in Table 1. The parameters for estimation are provided in Table 2. The two values of $R$ in Table 2 correspond to the situations that one or two distance sensors are available. In simulations, the variance of $w[k]$ is set to diag(72, 72) or 72 mm$^2$/s$^2$ as in Table 1, which is different from the supposed variance.

Table 1 Simulation conditions.

| Parameter                  | Symbol | Value       |
|----------------------------|--------|-------------|
| Number of robots           | $N$    | 5           |
| Radius of robot            | $r$    | 100 mm      |
| Position of distance sensor| $s_{i1}^{[0]},s_{i2}^{[0]}$ | $r\pi/18, r(-\pi/18)$ |
| Angle of distance sensor   | $\xi_{i1}^{[0]},\xi_{i2}^{[0]}$ | 0 rad       |
| Observation noise          | $\mathcal{N}(0, 72 \text{ mm}^2)$ |
| Velocity input noise       | $\mathcal{N}(0, 8 \times 10^{-4} \text{ rad}^2/\text{s}^2)$ |
| Angular velocity input noise| $\mathcal{N}(0, 8 \times 10^{-4} \text{ rad}^2/\text{s}^2)$ |
| Initial position           | $p_{1}(0)$ | [300 mm 650 mm]$^\top$ |
|                           | $p_{2}(0)$ | [220 mm 350 mm]$^\top$ |
|                           | $p_{3}(0)$ | [250 mm 50 mm]$^\top$ |
|                           | $p_{4}(0)$ | [230 mm –250 mm]$^\top$ |
|                           | $p_{5}(0)$ | [280 mm –550 mm]$^\top$ |
| Initial attitude angle     | $\theta_{1}(0)$ | 0.5 rad       |
|                           | $\theta_{2}(0)$ | 0.43 rad       |
|                           | $\theta_{3}(0)$ | 0.51 rad       |
|                           | $\theta_{4}(0)$ | 0.43 rad       |
|                           | $\theta_{5}(0)$ | 0.49 rad       |
Table 2 Estimation parameters for simulation.

| Parameter                               | Symbol | Value                                      |
|-----------------------------------------|--------|--------------------------------------------|
| Initial states                          | \(\hat{p}^i_0(0)\) | Measurement value obtained by the existing study [14] |
|                                        | \(\hat{\theta}^i_0(0)\) | 0 rad                                       |
|                                        | \(P(0)\) | diag(100, 100, 8 \times 10^{-3})           |
| Covariance matrix of system noise       | \(\mathcal{Q}\) | diag(0.1, 0.1, 2 \times 10^{-3})          |
| Covariance matrix of observation noise  | \(\mathcal{R}\) | diag(8, 8) or 8                             |
| Control sampling time                   | \(\Delta t\) | 0.02 s                                      |
| Communication sampling time             | \(\Delta t\) | 0.2 s \((L = 10)\)                        |

Fig. 5 Trajectories of the robots with the proposed method.

Fig. 6 \(x\)-coordinate of robot 2 estimated by robot 3.

\(R = \text{diag}(8, 8)\) or 8 in Table 2.

Figure 4 shows the snapshots of the robot positions in the simulation with the proposed method. Each of the sub-figures depicts the robot positions at time \(t = 0\) s, 2 s, 4 s, 6 s, and 8 s, where \(t = k\Delta t\). We can observe that the formation is maintained even while the leader moves. In Fig. 5, the trajectory of the leader is depicted with the solid line, and those of the followers are drawn with the dashed lines. It is shown that although the leader’s trajectory is non-smooth, the followers successfully maintain the formation. The estimation results are shown in Figs. 6–8, which represent the estimation values of \(x\)-coordinate \(\hat{x}_2^3(t)\) and \(y\)-coordinate \(\hat{y}_2^3(t)\) of the relative position, and the relative attitude angle \(\hat{\theta}_2^3(t)\) of robot 2 in the relative coordinate system of robot 3. From Figs. 6–8, we can observe that the estimation values match the actual ones well.

Fig. 7 \(y\)-coordinate of robot 2 estimated by robot 3.

Fig. 8 Angle \(\theta\) from \(x\)-axis of robot 2 estimated by robot 3.

Fig. 9 Estimation error in \(y\)-coordinate.

Fig. 10 Trajectories of the robots with the existing method.

Hence, robot 3 successfully estimates \(x_2^3(t), y_2^3(t), \theta_2^3(t)\) by the proposed method. The number of the sensors detecting robot 2 decreases from 2 to 1 during time \(t = 2.54\) s to 2.64 s,
3.76 s to 4.7 s, and 5.96 s to 6.2 s, which are represented by the shaded areas in Figs. 6–8. The estimation error in $y_{[3]}(t)$ with the proposed method is shown in Fig. 9. We can observe that the error does not become so large even when the number of the detecting sensors decreases.

A simulation is conducted with the existing method [11],[12] for comparison with the proposed one. The simulation setting is the same as the ones for the proposed method, except that the control sampling time is set to 0.2 s. This is because this method requires that the communication and control sampling times are the same. The simulation result with the existing method is shown in Fig. 10, which shows that the formation cannot be maintained anymore after $t = 2$ s. Figure 11 shows the estimation error, which rapidly increases when the number of the detecting sensors decreases. Compared with this method, the proposed method successfully estimates the variables and maintains the formation, which indicates the effectiveness of the proposed method.

6. Experiment

An experiment with five robots is conducted to verify the effectiveness of the proposed method in practical situations. We use the omnidirectional robots shown in Fig. 12, equipped with board computers (Arduino Mega, Arduino), distance sensors (P2Y0A21YK0F, SHARP), and wireless communication devices (XBee ZB module (S2C), Digi International). From the computational performance of the computers, the control sampling time is set to $\Delta t = 0.02$ s. Due to the performance limitation of the wireless communication devices, the communication sampling time is set to $L \Delta t = 0.2$ s, and thus $L = 10$. The radii of the robots are all $r = 100$ mm. The positions and angles of the distance sensors are $s_{1i} = re(\pi/18)$, $s_{2i} = re(-\pi/18)$, and $s_{11}, s_{12} = 0$ rad, respectively. The desired distance and angle are $q^*_i = 430$ mm and $\phi^*_i = 0$ rad, respectively, for all $i$. Controllers are designed as Eqs. (5) and (6) with the gains $k_{pi} = 0.64$ and $k_{thi} = 0.000025$.

The parameters in the estimation are shown in Table 3. The experimental result is shown in Fig. 13, which represents the states of the robots at $t = 0$ s to 20 s. We can see that the
robots successfully maintain the formation while the leader moves. The robots have a function that an equipped LED shines when the light of the corresponding distance sensor does not hit. From Figs. 13(c), 13(d), at $t = 8$ s and 12 s, one of the distance sensors of robots 2 and 3 does not hit. Nevertheless, the formation is maintained due to the accurate estimation by the proposed method. Figure 14 shows the trajectories of the robots, which shows that the robots smoothly move. Note that by using the existing method [11],[12], experiments could not be carried out in this situation because the formation collapsed as soon as experiments started. In contrast, we can maintain the formation by the proposed method, which shows the effectiveness of this method.

### 7. Conclusion

In this paper, we proposed a relative position estimation method, which is effective even if the communication sampling time is longer than the control sampling time. First, we considered estimating the relative positions of the detected robots by using the time-series of the predicted input of the detected robots. Second, we developed a method to generate the time-series of the predicted input to send to the following robots for successful estimation. Both results of simulations and an experiment demonstrated the effectiveness of the proposed method. Future work includes considering more complex formations.

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