Quantum coherence and non-Markovianity of an atom in a dissipative cavity under weak measurement

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Quantum coherence and non-Markovianity of an atom in a dissipative cavity under weak measurement are investigated in this work. We find that: the quantum coherence obviously depends on the initial atomic state, the strength of the weak measurement and its reversal, the atom–cavity coupling constant and the non-Markovian effect. It is obvious that the weak measurement effect protects the coherence better. The quantum coherence is preserved more efficiently for larger atom–cavity coupling. The stronger the non-Markovian effect is, the more slowly the coherence reduces. The quantum coherence can be effectively protected by means of controlling these physical parameters.

Keywords: quantum coherence, non-Markovianity, dissipative cavity

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1. Introduction

Quantum coherence marks the departure of quantum mechanics from classical physics. It is increasingly recognized as a vital physical resource in both quantum theory and quantum information. There are several measurement methods of quantum coherence, such as the relative entropy of coherence $C_{REL}(\rho) = S(\rho_{Diag}) - S(\hat{\rho})$, where $S$ is the von Neumann entropy and $\rho_{Diag}$ denotes the state obtained from $\rho$ by deleting all off-diagonal elements, and the intuitive $l_1$ norm of coherence $C_{l_1}(\rho) = \sum_{i,j(i \neq j)} |p_{ij}|$. A growing number of applications can be certified to rely on various incarnations of quantum coherence as a primary ingredient. This has been the main motivation for recent researches to quantify and develop a number of measures of quantum coherence. Quantum coherence has been linked with other quantum resources, such as non-locality,[2,3] non-Markovianity,[4,5] entanglement,[6,7] and quantum discord.[8,9]

Recently, quantum coherence has been established as an important notion, which has been gaining a great deal of attention as a signature of quantum nature and a resource for quantum technologies. For example, Chiranjib et al.[10] explored quantum coherence in superposition with non-distinguishable pointers, it is plausible that the results will be useful in quantum multiple-slit experiments and interferometeric set-ups with feedback loops. The authors in Ref. [5] studied the non-Markovianity and coherence of a moving qubit inside a leaky cavity. Zhang et al.[11] used the relative entropy of coherence as the quantification of coherence in multimode Fock spaces. Peng et al.[12] found that only maximally coherent states should achieve the maximal value for a coherence measure. Recently, the applications of weak measurement[13,14] have been proposed as a practical method to protect the quantum coherence.[15] The weak measurement and its reversal can be utilized to improve the fidelity of a system. In this paper, we study the quantum coherence of an atom in a dissipative cavity under weak measurement. We find that the quantum coherence can be effectively protected by adjusting the strength of the weak measurement and its reversal, the atom–cavity coupling and the spectral width of the reservoir. We also analyze the effect of the qubit on the degree of non-Markovianity.

The paper is organized as follows. In Section 2, we give the Hamiltonian of an atom in the dissipative cavity and determine a state evolution of the atom referring to this Hamiltonian under weak measurement. In Section 3, we discuss the coherence of the atom and the influence of the non-Markovianity. Finally, a brief summary is presented in Section 4.

2. Physical model

2.1. Atom in a dissipative cavity

Consider an atom interacts with a cavity, where the cavity is coupled to a bosonic environment.[16] The Hamiltonian reads

$$H = \frac{1}{2} \omega_0 \sigma_z + \omega_\alpha a^\dagger a + \Omega (a \sigma_+ + a^\dagger \sigma_-) + \sum_k \omega_k c_k^\dagger c_k + (a^\dagger + a) \sum_k g_k (c_k^\dagger + c_k),$$

(1)

where $a$ and $a^\dagger$ are the creation and annihilation operators of the cavity, $\sigma_\pm$ and $\omega_\alpha$ are the transition operators and Bohr fre-
quency of the atom.\cite{17} $\Omega$ is the coupling constant between the atom and its cavity, and $c_k$ and $c_{\bar{k}}$ are the creation and annihilation operators of the reservoir. The coupling constant between the cavity and its environment is denoted by $g_k$.

By assuming one initial excitation and the reservoir at zero temperature, the non-Markovian master equation for the density operator $R(t)$ in the dressed-state basis $\{|E_{1+}\rangle, |E_{1-}\rangle, |E_0\rangle\}$ is

$$
\dot{R}(t) = -i[H_{\text{JC}}, R(t)] + \gamma(\omega_0 + \Omega, t) \left( \frac{1}{2} |E_0\rangle\langle E_{1+}| R(t) |E_{1+}\rangle\langle E_0| - \frac{1}{4} |E_{1+}\rangle\langle E_{1+}| R(t) |E_{1+}\rangle\langle E_0| \right) $$

$$
+ \gamma(\omega_0 - \Omega, t) \left( \frac{1}{2} |E_0\rangle\langle E_{1-}| R(t) |E_{1-}\rangle\langle E_0| - \frac{1}{4} |E_{1-}\rangle\langle E_{1-}| R(t) |E_{1-}\rangle\langle E_0| \right),
$$

(2)

where $|E_{1\pm}\rangle = (|1\rangle \pm |0\rangle) / \sqrt{2}$ are the eigenstates of $H_{\text{JC}}$ with one total excitation, with energy $\omega_0/2 \pm \Omega$, and $|E_0\rangle = |0\rangle$ is the ground state, with energy $-\omega_0/2$. The time-dependent decay rates for $|E_{1-}\rangle$ and $|E_{1+}\rangle$ are $\gamma(\omega_0 - \Omega, t)$ and $\gamma(\omega_0 + \Omega, t)$, respectively.

If the reservoir at zero temperature is modeled with a Lorentzian spectral density\cite{18}

$$
J(\omega) = \frac{\lambda_0 \lambda^2}{2\pi (\omega - \omega_0)^2 + \lambda^2},
$$

(3)

where $\lambda$ defines the spectral width of the coupling, which is connected to the reservoir correlation time and $\lambda_0$ is the system–environment coupling strength. For a weak regime, we mean the case $\lambda > 2\lambda_0$, in this regime, the behavior of dynamical evolution of the system is a Markovian exponential decay controlled by $\lambda_0$ essentially. In the strong coupling regime, for $\lambda < 2\lambda_0$, non-Markovian effects become relevant.\cite{19–21} We consider the spectrum is peaked on the frequency of the state $|E_{1-}\rangle$, i.e., $\omega_0 = \omega_0 - \Omega$, the decay rates for the two dressed states $|E_{1\pm}\rangle$ are respectively expressed as\cite{22}

$$
\gamma(\omega_0 - \Omega, t) = \gamma_0 (1 - e^{-\lambda t})
$$

and

$$
\gamma(\omega_0 + \Omega, t) = \frac{\lambda_0 \lambda^2}{4\Omega^2 + \lambda^2} \left\{ 1 + \frac{2\Omega}{\lambda} \sin(2\Omega t) - \cos(2\Omega t) \right\} e^{-\lambda t}.
$$

In the dressed-state basis, if $R_{ij}(0)$ $(i, j = 1, 2, 3)$ describe the matrix elements of the initial atom–cavity state, we can acquire the matrix elements at all times from Eq. (2) as

$$
R_{11}(t) = A_{11}^{22} R_{11}(0), \quad R_{12}(t) = A_{12}^{22} R_{12}(0),
$$

$$
R_{13}(t) = A_{13}^{22} R_{13}(0),
$$

$$
R_{22}(t) = A_{22}^{11} R_{22}(0), \quad R_{23}(t) = A_{23}^{11} R_{23}(0),
$$

$$
R_{33}(t) = A_{33}^{11} R_{33}(0) + A_{33}^{22} R_{22}(0) + A_{33}^{33} R_{33}(0).
$$

(4)

Here,

$$
A_{11}^{11} = e^{-\frac{1}{2}t^2}, \quad A_{12}^{12} = e^{-2\Omega t^2} e^{-\frac{1}{2}(t^2 + \frac{t}{t^2})},
$$

$$
A_{13}^{13} = e^{-i(\omega_0 + \Omega) t} e^{-\frac{1}{2}t^2},
$$

$$
A_{22}^{22} = e^{-\frac{1}{2}t^2}, \quad A_{23}^{23} = e^{-i(\omega_0 - \Omega) t} e^{-\frac{1}{2}t^2},
$$

$$
A_{33}^{33} = 1 - A_{11}^{11}, \quad A_{33}^{33} = 1 - A_{22}^{22}, \quad A_{33}^{33} = 1.
$$

(5)

and

$$
I_+ = \lambda_0 t^2 + \lambda_0^2 (e^{\lambda t} - 1),
$$

$$
I_- = \lambda_0 t^2 + \lambda_0^2 \left[ \frac{4\Omega t e^{-\lambda t} \sin(2\Omega t)}{4\Omega^2 + \lambda^2} + \frac{2\Omega^2 - \lambda^2}{\lambda (4\Omega^2 + \lambda^2)} \right].
$$

(6)

We consider the coherence of the single atom in the dissipative cavity. The atom density operator in the standard state is

$$
\rho = \left( \begin{array}{cc} \rho_{11}(t) & \rho_{12}(t) \\ \rho_{21}(t) & \rho_{22}(t) \end{array} \right),
$$

(7)

where

$$
\rho_{11}(t) = \frac{1}{2} (\rho_{11}(t) - \rho_{11}(t) - \rho_{21}(t) + \rho_{22}(t)),
$$

$$
\rho_{12}(t) = \frac{1}{2} (\sqrt{2}\rho_{13}(t) - \sqrt{2}\rho_{23}(t)),
$$

$$
\rho_{21}(t) = \frac{1}{2} (\sqrt{2}\rho_{31}(t) - \sqrt{2}\rho_{32}(t)),
$$

$$
\rho_{22}(t) = \frac{1}{2} (\rho_{11}(t) - \rho_{11}(t) + \rho_{21}(t) + \rho_{22}(t) + 2\rho_{33}(t)).
$$

(8)

2.2. Weak measurement

In the dissipative cavity, the weak measurement and its reversal\cite{22} can be applied in two places: one before and the other after the atom interacting with the dissipative cavity. The operator of the weak measurement can be described as

$$
b_1 = \left( \begin{array}{cc} 1 & 0 \\ 0 & \sqrt{1 - p_1} \end{array} \right),
$$

(9)

where $p_1$ is the strength of the weak measurement. The larger the $p_1$ value is, the stronger the weak measurement is, the weak measurement becomes the strong measurement when $p_1 = 1$. Let the initial state be

$$
\varphi_0 = \cos \frac{\theta}{2} |\epsilon\rangle + \sin \frac{\theta}{2} |\delta\rangle,
$$

(10)

the atomic state after the weak measurement can be described as
Then, the atom interacts with the dissipative cavity, so the atom state at time \( t \) can be described
\[
p'(t) = \begin{pmatrix} \rho'_{11}(t) & \rho'_{12}(t) \\ \rho'_{21}(t) & \rho'_{22}(t) \end{pmatrix}.
\] (12)

From Eqs. (4), (8), and (11), we can obtain the matrix elements as
\[
\rho'_{11}(t) = \frac{1}{4}(A_{11}^0 + A_{12} + A_{21}^0 + A_{22}^0)p_{11}(0),
\]
\[
\rho'_{12}(t) = \frac{1}{2}(A_{13}^0 + A_{23}^0)p_{12}(0),
\]
\[
\rho'_{21}(t) = \frac{1}{2}(A_{31}^0 + A_{32}^0)p_{21}(0),
\]
\[
\rho'_{22}(t) = \frac{1}{2} \left( \frac{1}{2}A_{11}^0 - \frac{1}{2}A_{12}^0 + \frac{1}{2}A_{22}^0 - \frac{1}{2}A_{21}^0 \right)
\times \rho_{11}(0) + 2\rho_{12}'(0).
\] (13)

The operator of the reversal is given by
\[
b_2 = \begin{pmatrix} \sqrt{1-p_2} & 0 \\ 0 & 1 \end{pmatrix},
\] (14)
where \( p_2 \) is the strength of the reversal.

The atom state can be described as
\[
p^w(t) = b_2p'(t)b_2^\dagger = \begin{pmatrix} \rho^w_{11}(t) & \rho^w_{12}(t) \\ \rho^w_{21}(t) & \rho^w_{22}(t) \end{pmatrix}.
\] (15)

In the following, we calculate the quantum coherence of the atom by using the intuitive \( \ell \) norm of coherence, it is
\[
C_p = \sum_{i,j} |\rho^w_{ij}| = \rho^w_{12}(t) + \rho^w_{21}(t),
\] (16)
where \( \rho^w_{12}(t) \) and \( \rho^w_{21}(t) \) are the off-diagonal elements of the system density matrix, respectively.

3. Discussions and results

3.1. Quantum coherence

To investigate the quantum coherence evolution of the atom in the dissipative cavity, we numerically calculate the quantum coherence and analyze the behavior of the coherence in different conditions. Figure 1 shows the behavior of \( C_p \) as a function of the initial state \( |\phi_0\rangle \). In Fig. 1, we choose \( \Omega = \lambda_0, \lambda = 5\lambda_0, \lambda_{dt} = 10, p_1 = 0.5, \) and \( p_2 = 0.5 \). The result clearly shows that the coherence changes cyclically. In addition, the coherence reaches the maximum value at \( \theta = \pi/2 \) or \( 3\pi/2 \), i.e., when the initial state is \( |\phi_0\rangle = (|e\rangle + |g\rangle)/\sqrt{2} \), there is the maximal coherence.

We depict the coherence as a function of parameters \( p_1 \) and \( p_2 \) in Fig. 2. One can clearly see that the behavior of the coherence \( C_p \) depends on the parameters \( p_1 \) and \( p_2 \). The numerical results indicate that the coherence decreases by increasing the strength of the weak measurement or its reversal. This means that the better coherence is maintained when weak measurement is more obvious. In the optimal initial state, \( p_1 = 0 \) and \( p_2 = 0 \), the atom has the maximal coherence.

Figures 3 and 4 are the images of quantum coherence as functions of \( \Omega/\lambda_0 \). If \( \lambda = 3\lambda_0 \), the reservoir is in the Markovian regime. When the cavity is dissipative, the coherence will decrease with time. In Fig. 3, we find that the coherence almost reduces to zero when the coupling constant between the atom and its cavity is small. The decay of coherence is slower when \( \Omega \) increases. In particular, if the coupling constant is large enough, the coherence tends to a finite value after a long time. Figure 4 is a more detailed description of the quantum coherence as the functions of \( \Omega \). It can clearly display the influence of coupling constant \( \Omega \) on the decay behavior of the coherence.

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is that, the interaction between the atom and its cavity is large enough, so that the impact of the reservoir on the atom is relatively weak. The results show that the quantum information will be imprisoned in the atom–cavity and the quantum coherence has been effectively protected.

![Fig. 3](image_URL)

Fig. 3. (color online) The coherence as a function of the coupling constant $\Omega$ and time $\lambda t$. Here, $p_1 = 0$, $p_2 = 0$, $\lambda = 3\lambda_0$. The initial state is $|\psi_0\rangle = (|e\rangle + |g\rangle)/\sqrt{2}$.

![Fig. 4](image_URL)

Fig. 4. (color online) The dotted red line represents, $\Omega = \lambda_0$; the solid blue line, $\Omega = 10\lambda_0$; the dot-dashed green line, $\Omega = 40\lambda_0$.

![Fig. 5](image_URL)

Fig. 5. (color online) The coherence for $\lambda/\lambda_0$ given in Eq. (12). Here, $p_1 = 0$, $p_2 = 0$, $\Omega = \lambda_0$. The initial state is $|\psi_0\rangle = (|e\rangle + |g\rangle)/\sqrt{2}$.

We depict the coherence $C_\rho$ as a function of $\lambda/\lambda_0$ and $\lambda_0 t$ in the case of $\Omega = \lambda_0$ in Figs. 5 and 6. Figure 5 clearly shows that the coherence $C_\rho$ of the atom in the dissipative cavity will quickly decay to zero when $\lambda > 2\lambda_0$ (in the Markovian regime). For $\lambda < 2\lambda_0$, the non-Markovian effects become relevant. When $\lambda/\lambda_0$ approaches to 0, the coherence is well maintained and reaches a fixed value. For a further understanding of the coherence $C_\rho$ under different coupling strengths, figure 6 demonstrates the coherence $C_\rho$ as a function of $\lambda_0 t$ in different $\lambda$. We can see that the coherence is greatly influenced by the reservoir when $\Omega = \lambda_0$. That is to say, the coherence in the non-Markovian regime reveals distinct features: although the coherence will decay to zero in a long time when $\lambda = \lambda_0$, the decay rate of coherence in this case is obviously smaller than in Markovian regime ($\lambda = 3\lambda_0$). Comparing $\lambda = 0.01\lambda_0$ and $\lambda = 0.1\lambda_0$, it is seen that, the smaller the value of $\lambda$ is, the stronger the non-Markovian effect is, the slower the entanglements reduce. The coherence will tend to 0.36 when the non-Markovian effect is strong enough.

![Fig. 6](image_URL)

Fig. 6. (color online) The solid blue line represents, $\lambda = 0.01\lambda_0$; the dotted yellow line, $\lambda = 0.1\lambda_0$; the dot-dashed green line, $\lambda = \lambda_0$; the dashed red line, $\lambda = 3\lambda_0$.

3.2. Non-Markovianity

The measure $N(\Phi)$ for the non-Markovianity of the quantum process $\Phi(t)$ has been defined in Ref. [24]. By considering a quantum process $\Phi(t)$, this quantity depends on time $t$ and on the initial states $\rho_{1,2}(0)$ with corresponding time evolutions $\rho_{1,2}(t) = \Phi(t,0)\rho_{1,2}(0)$. The non-Markovianity $N(\Phi)$ is defined as

$$N(\Phi) = \max_{\rho_{1,2}(0)} \int_{\sigma > 0} dt \sigma[t, \rho_{1,2}],$$

(17)

where $\sigma[t, \rho_{1,2}(0)]$ is the rate of change of the trace distance

$$\sigma[t, \rho_{1,2}(0)] = \frac{d}{dt} D[\rho_1(t), \rho_2(t)].$$

The trace distance $D$ describing the distinguish ability between the two states is defined as [25]

$$D(\rho_1, \rho_2) = \frac{1}{2} \|\rho_1 - \rho_2\|,$$

(18)

where $\|M\| = \sqrt{M^\dagger M}$ and satisfying $0 \leq D \leq 1$. $\sigma[t, \rho_{1,2}(0)] \leq 0$ corresponds to all dynamical semigroups and all time-dependent Markovian processes; a process is non-Markovian if there exists a pair of initial states at a certain time $t$ such that $\sigma[t, \rho_{1,2}(0)] > 0$. We take the maximum over all initial states $\rho_{1,2}(0)$ to calculate the degree of non-Markovianity. By drawing a sufficiently large sample of random pairs of initial states, the optimal state pair is attained for the initial states are $\rho_1(0) = |g\rangle\langle g|$ and $\rho_2(0) = |e\rangle\langle e|$.

Figure 7 shows the degree of non-Markovianity as a function of the parameter $\lambda/\lambda_0$. From Fig. 7, we know that $N(\Phi)$ is obviously dependent on $\lambda/\lambda_0$. The bigger the value of $\lambda/\lambda_0$ is, the larger the degree of non-Markovianity is. Especially, when $\lambda/\lambda_0$ increases from 0.01 to 0.1, the damping of $N(\Phi)$ is vary fast but $N(\Phi)$ is still
great and is larger than 5. We again observe the solid blue line ($\lambda/\lambda_0 = 0.01$) and the dotted yellow line ($\lambda/\lambda_0 = 0.1$) in Fig. 6 and find that their coherence slowly decays to their stable value. In other words, the non-Markovianity could effectively control the coherence decay. With $\lambda/\lambda_0$ increasing, $N(\Phi)$ is reduced, the coherence will disappear after a long time, shown in Fig. 5. When the degree of non-Markovianity is not equal to zero, there is a flow of information from the environment back to the open system, this is the key feature of non-Markovian dynamics. Therefore, the information is more from reservoir back to the atom–cavity system and the protection of coherence is the best when $N(\Phi)$ is larger.

![Fig. 7. (color online) The non-Markovianity $N(\Phi)$ for this model as a function of the $\lambda/\lambda_0$. The optimal state pair attained for the initial states is $\rho_1(0) = |g\rangle\langle g|$ and $\rho_2(0) = |e\rangle\langle e|$. We choose $p_1 = 0$, $p_2 = 0$, and $\Omega = \lambda_0$.](image)

4. Conclusion

In this paper, we mainly study the quantum coherence and non-Markovianity of an atom in a dissipative cavity under weak measurement. The results show that the quantum coherence obviously depends on the initial atomic state, the strength of the weak measurement and its reversal, the atom–cavity coupling constant and the spectral width of the reservoir. The atom has the best coherence when the atomic initial state is $|\phi_0\rangle = (|e\rangle + |g\rangle)/\sqrt{2}$. The weak measurement and its reversal can protect coherence. When $p_1 \rightarrow 0$ and $p_2 \rightarrow 0$, the weak measurement effect is more obvious, therefore, the better the protection of coherence is.

As the coupling constant $\Omega$ is large enough, the coherence will not be affected by the reservoir, i.e., does not decay to 0 or even reach a fixed value. The smaller the value of $\lambda$ is, the stronger the non-Markovian effect is, and the more slowly the coherence reduces. The coherence will tend to 0.36 after a long time when the non-Markovian effect is strong enough. On the other hand, the result shows that the degree of non-Markovianity can act as a protector of qubit coherence. The greater degree of non-Markovianity is when $\lambda/\lambda_0 \rightarrow 0$. By this time, there is a greater amount of information about the reservoir back flow, so there is better protection of the quantum coherence of the atom.

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