Study on Prediction and Application of Initial Chord Elastic Modulus with Resonance Frequency Test of ASTM C 215

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Abstract: For accurate design, construction, and maintenance, it is important to identify the elastic modulus of concrete. This is usually achieved using a destructive test based on American Society for Testing and Materials (ASTM) C469. However, obtaining an appropriate static elastic modulus ($Ec$) requires many specimens, and the testing is difficult and time-consuming. Thus, a dynamic elastic modulus ($Ed$) is often obtained through a natural frequency for a specific size (e.g., the longitudinal (LT) or transverse (TR) mode) based on a resonance frequency test. However, this method uses a gradient at a very low-stress part of the stress–strain curve and assumes a completely elastic body. In fact, the initial chord elastic modulus ($Ei$) of the stress–strain curve in a concrete fracture test differs from the Ed, owing to the non-homogeneity and inelasticity of the concrete. The $Ei$ of the experimental value may be more accurate. In this study, the $Ei$ was predicted using machine learning methods for natural frequencies. The prediction accuracy for $Ei$ was analyzed based on $f1$–$f4$, as calculated through the LT and TR modes. The predicted $Ei$ had higher correlations with the actual $Ec$ and compressive strength ($fc$) than $Ed$. Thus, more accurate prediction of concrete mechanical properties is possible.

Keywords: initial chord elastic modulus; resonance frequency test; static elastic modulus; dynamic elastic modulus; compressive strength; non-destructive testing; concrete; machine learning

1. Introduction

The elastic modulus of concrete is an important factor for the trustworthy design, construction, and maintenance of structures, and can predict the deformation in an actual stress state [1,2]. The static elastic modulus ($Ec$) of concrete uses the secant elastic modulus of the stress–strain curve as obtained by ASTM C469. However, a large number of specimens is required to obtain a reliable representative value, and it is difficult to provide a sufficient assessment, owing to constraints on time and the difficulty of testing [3,4]. One alternative method comprises obtaining a dynamic elastic modulus ($Ed$), while resonance frequency test and ultrasonic pulse velocity method can be applied according to ASTM C215 and ASTM C597, respectively [5,6]. Among ultrasonic test methods, a pressure wave (P-wave) measurement provides easy and convenient testing, but the distribution of the data is large, owing to uneven distributions of aggregates, moisture, and voids in the test piece. In addition, although a large correlation between $Ec$ and compressive strength ($fc$) has been reported as a result of using a shear wave (S-wave), which has a larger energy than a P-wave, it is difficult to measure and obtain consistent data (especially at low ages), owing to the influences of voids and moisture in the concrete [7–10]. The resonance frequency test is known for consistently predicting the Ed with less fluctuation in
the collected data, as it is simple to measure and grasps the overall dynamic characteristics of the specimen. The resonance frequency test assumes that the concrete has a homogeneous elastic modulus, density, Poisson’s ratio, etc., and uses a method to calculate a theoretical elastic modulus for an initial stress range [6,11]. In the ASTM, the relationship between the dynamic modulus for the longitudinal (LT) and transverse (TR) modes was proposed using physical parameters such as the dimensions, mass, and first resonance frequency \( (f_f) \). Subramaniam et al. applied \( f_1 \) and the second resonance frequency \( (f_2) \) of the LT mode in the Rayleigh Ritz theory, proposing a general equation for the ED and dynamic Poisson’s ratio [11]. The \( E_{ds} \) measured by these non-destructive examination (NDE) methods were larger than the value of \( E_c \), and several empirical equations for predicting \( E_c \) through a certain reduction in the \( Ed \) ratio have been proposed [12–14]. However, the \( Ed \) measured according to the NDE method has a large deviation and degree [10]. In addition, the \( Ed \) measured with the resonance frequency test has been assumed to be the initial chord elastic modulus \( (E_i) \), owing to the deformation of an initial very small stress in the stress-strain curve. Nevertheless, concrete is a nonhomogeneous, inelastic material, and thus the value of \( Ed \) as measured with the theoretical equation such as ASTM’s is different from the actual \( E_i \) value [10]. In previous studies, the \( E_i \) was defined as a slope in the range of 10–50 \( \mu \) in the stress-strain curve, and was compared to \( Ed \) and \( E_c \). The modulus of elasticity was \( Ed > E_i \) (10 \( \mu \)-50 \( \mu \) > \( E_c \), and it was determined that there was a difference of 11.62% between the \( E_c \) and \( E_i \) [15]. However, if the range of \( E_i \) is set to (10–50 \( \mu \)) as in the previous studies, the correlation with \( E_c \) may be small, as the measured value in the initial part of the stress-strain curve is unstable, and has high variation [9,16]. Therefore, if the correct \( E_i \) value is extracted, the correlation of \( E_i \)–\( E_c \) is expected to be greater than that of \( E_d \)–\( E_c \). To overcome the limitations of the fracture test and predict \( E_c \) more accurately, a non-destructive method for accurately predicting \( E_i \) is required.

In recent years, studies have been conducted to overcome the nonlinearity and improve the prediction accuracy of the mechanical properties (such as the \( E_c \) and \( f_c \)) of concrete by considering various variables such as the water/binder ratio, type of binders/aggregates and corresponding ratios by machine learning (ML), rather than by general regression analysis. These studies focused on predicting the concrete strength and integrity more accurately with general ML algorithms, such as the support vector machine (SVM), ensemble and artificial neural network (ANN), and provided predictions with relatively improved accuracy [17–20]. For example, Erdal et al. used ensemble and ANN methods with cement, blast-furnace slag, water, and aggregate, and compared the accuracies of \( f_c \) prediction for high-performance concrete; they found that the ensemble method was slightly better [21]. Park et al. predicted the \( E_c \) and \( f_c \) using four \( E_{ds} \) with SVM, ensemble, ANN, and multiple linear regression (MLR) methods, and announced that the ensemble and ANN were suitable [8]. Yan and Shi predicted the \( E_c \) by the \( f_c \) using an ANN, SVM, linear regression, and four theoretical equations, and reported that the ML methods were more suitable than linear regression and theoretical equations [22]. Young et al. used an ANN, SVM, and linear regression to predict a 28-day intensity from data with varying mixing ratios and reported that ML methods were more accurate than linear regression [23]. Cihan used ANN, ensemble, and SVM methods to estimate the \( f_c \) and slump values of concrete and reported that the ANN and ensemble approaches were superior to the SVM [24]. Among the ML methods, the ANN and ensemble methods overcome the nonlinear behavior of concrete and provide suitable contributions to quality prediction. However, although some studies have been conducted on the prediction of \( E_c \) and \( f_c \) with \( Ed \), few studies have been conducted with \( E_i \). Therefore, it is necessary to analyze the relationships among \( Ed \), \( E_c \), and \( f_c \) for the application of \( E_i \), and to predict the correct \( E_i \).

In this study, the difference between the dynamic elastic modulus (\( Ed \)) and initial chord elastic modulus (\( E_i \)) values obtained with the ASTM theoretical equation was analyzed based on the results of the resonance frequency test, and the relationship between the \( E_c \) and \( f_c \) was analyzed through accurate prediction of the \( E_i \). Three \( Ed \) values were obtained using the theoretical equations suggested by ASTM C215-14 and Subramaniam, as the frequencies of the LT and TR modes of the resonance frequency test. In addition, the exact \( E_i \) value at the origin was extracted through curve fitting to the stress–strain curve, and was compared with the \( E_{ds} \). The \( E_i \) was predicted using ML (ensemble, ANN) for the
2. Materials and Methods

2.1. Materials and Preparation of Specimens

The concrete consisted of Type I Portland cement, river sand, supplementary cementitious materials (SCMs), and crushed granite (up to 25 mm in size). The concrete in this study replaced approximately 50% of the cement with granulated blast furnace slag (GBFS) and fly ash. It possessed high strength in the long term, but the initial strength development could be slow. Two concrete mixtures (Mix 1, Mix 2) were prepared with W/B ratios of 0.45 and 0.35, respectively, and were expected to develop fcs of 20 and 40 MPa at 28 days, respectively. The ratios of concrete mixture are summarized in Table 1. 300 cylinders were cast to a size of 150 × 300 mm according to ASTM C31/C31M-12 [25]. The Ec, fc, and Ed values were tested at different ages, i.e., 4, 7, 14, and 28 days. The 28-day fcs for the Mix1/Mix2 cylinders were 19.26 MPa /43.99 MPa, similar to the expected fcs.

| ID              | Cement Type | W/B   | S/A | W  | C  | S  | G  | Unit Quantity (kg/m³) | Chemical Admixture |
|-----------------|-------------|-------|-----|----|----|----|----|------------------------|--------------------|
| Mix1 (20 MPa)   | Type I      | 0.45  | 0.46| 259| 121| 777| 934| 58                     | FA 0.9 GBFS 89      |
| Mix2 (40 MPa)   |             | 0.35  | 0.47| 308| 166| 761| 886| 81                     | AE SP 1            |

Table 1. Proportions of the concrete mixture groups

2.2. Destructive Tests for Elastic Modulus and Compressive Strength

For the measurement of the static elastic modulus (Ec) and compressive strength (fc), each cylinder was positioned vertically, with both ends polished, and the protrusions on the specimen surface removed. As shown in Figure 1, the Ec and fc for each concrete cylinder were measured with a universal testing machine (UTM, Instruments, Instron, MA, USA) with a capacity of 1000 kN, according to ASTM C39/C39M-14a and ASTM C469/C469M-14 [3,26]. The UTM was operated at a speed of approximately 0.28 MPa/s.

![Figure 1](image)

Figure 1. Experimental setup for static destructive test.

2.3. Dynamic Elastic Modulus Measurements with Resonance Frequency Tests

As shown in Figure 2, the three to four longitudinal and transverse resonance frequencies of the concrete cylinders were measured based on ASTM C215-14 [6]. A steel ball hammer with a diameter...
the most dominant frequency was considered as the basic resonance frequency of the longitudinal modes, with four resonance frequencies in the LT mode and three resonance frequencies in the TR mode. The first of these frequency values was used to calculate the Ed of the LT and TR modes, using Equations (1) and (2) from ASTM C215-14.

\[ ASTM.LT = a_{LT} m f_{LT}^2 (Pa) \]  
\[ ASTM.TR = a_{TR} m f_{TR}^2 (Pa) \]  

where \( a_{LT} \) and \( a_{TR} \) are the constant values calculated with the procedure outlined in ASTM C215-14, \( m \) is the mass of the cylinders in kg, \( f_{LT} \) and \( f_{TR} \) are the first and second resonance frequencies of transverse mode.

An oscilloscope (NI-PXIe 6366) was used to measure the dynamic response of the concrete cylinders. The time signals measured with the accelerometer were stabilized by a signal conditioner (PCB 482C16, PCB, Depew, NY, USA) and were digitized with 1 MHz sampling through an oscilloscope (NI-PXIe 6366). The time signals were transformed into the frequency domain by a fast Fourier transform algorithm.

Figure 3 shows typical spectral amplitudes of the concrete cylinders at 4, 7, 14, and 28 days in the Mix 2 cylinder. The resonance frequency appears as a large amplitude in the amplitude spectrum. The most dominant frequency was considered as the basic resonance frequency of the longitudinal \( f_{LT} \) and transverse \( f_{TR} \) modes, with four resonance frequencies in the LT mode and three resonance frequencies in the TR mode. The first of these frequency values was used to calculate the Ed of the LT and TR modes, using Equations (1) and (2) from ASTM C215-14.

In the above, \( ASTM.LT \) and \( ASTM.TR \) are the Ed values calculated with \( f_{LT} \) and \( f_{TR} \), respectively, as the first resonance frequencies. \( a_{LT} \) has a constant value with regard to the dimensions of the specimen \((=5.093 \times (L/d^2))\), where \( d \) is the diameter and \( L \) is the length. \( a_{TR} \) has a constant value with
regard to Poisson’s ratio and the size of the cylinders (=1.6067 × (L^3T/d^4)). T is a correction factor corresponding to the Poisson’s ratio, and m is the mass of the cylinders in kg [6].

In addition, as in the method of predicting the dynamic elastic modulus (Ed) with two frequencies as proposed by Subramaniam et al., the dynamic Poisson’s ratio was obtained through Equation (3), and the Ed was predicted through Equation (4).

\[
\mu = A_1 \left( \frac{f_2}{f_1} \right)^2 + B_1 \left( \frac{f_2}{f_1} \right) + C_1
\]

(3)

\[
f_1, f_2LT = 2(1 + \mu) \rho \left( \frac{2\pi f_1 R_0}{f_{n1}^3} \right)^2
\]

(4)

Here, \( f_1, f_2, LT \) is the Ed value calculated with \( f_1 \) and \( f_2 \), \( f_1 \) is the first frequency, \( f_2 \) is the second frequency, \( R_0 \) is the diameter of the cylinder, \( A_1, B_1, \) and \( C_1 \) are correction factors, \( \rho \) is the density, and \( f_{n1}^2 \) is a correction factor based on the dynamic Poisson’s ratio [11].

2.4. Initial Chord Elastic Modulus Measurement

To calculate the initial chord elastic modulus (Ei) from the measured stress and strain data, a predictive equation for the stress-strain curve of the existing concrete (such as Equations (5) and (6)) can be used [27,28]. However, the existing theoretical equations are suitable for hardened concrete; when measuring the slope of the 10 µ and 50 µ strain ranges applied in previous studies, the initial data value was unstable, and the variability was large. Thus, the error could be large. Therefore, as shown in Figure 4, instead of extracting the Ei with an existing theoretical equation, the curve was predicted through best curve fitting from the origin to the max \( f_c' \), and the slope of Ei was calculated from the origin.

\[
f_c = \frac{2f_c'(\frac{x}{\ce})}{1 + \left( \frac{x}{\ce} \right)^2}
\]

(5)

\[
f_c = f_c' \left[ 2 \left( \frac{x}{\ce} \right) - \left( \frac{x}{\ce} \right)^2 \right]
\]

(6)

![Figure 4](image_url). Extraction of initial chord elastic modulus (Ei) from stress-strain curve.
3. Machine Learning Methods

3.1. Ensemble Method

The ensemble method combines several weak learners to create learners with strong predictive power. This study used a regression tree ensemble method and boosting method to combine weak learners. The boosting method performs slightly better than a bagging method and shows a higher predictive power. In addition, the boosting method can compensate for weaknesses in learning the next regression tree, by assigning weights to reduce the error between the predicted and output values of the previously learned regression tree. Here, a least squares method was used to calculate the error. As shown in Figure 5, least squares boosting (LSBoost) is a sequential ensemble method that sequentially builds a decision tree. It works in a way that compensates for errors in the previous tree and is defined as shown in Equation (7) [29,30].

\[ F_T(x) = \sum_{i=1}^{T} f_i(p) \]  

Here, \( x, f_i, \) and \( F_T(p) \) represent the input variable, weak learner, and strong learner, respectively. Each weak learner produces an output which is a hypothesis for each sample of the training set. One weak learner is selected in each repetitive step \( t \). Then, a coefficient is allocated to minimize the sum of the training errors at the final \( t \)-step acceleration classifier, as follows (Equation (8)):

\[ E_t = \sum_i E[F_{t-1}(p_i) + \alpha_i h(p_i)] \]  

In the above equation, \( F_{t-1}(p) \) and \( E(F) \) are the accelerated learners and error functions generated up to the previous training stage, respectively, and \( f_i(p) = \alpha_i h(p) \) is weak learner considered for the final learner. In each iteration step of the training process, a weight of a value equal to the current error \( E(F_{t-1}(p_i)) \) for the training data set is reflected in the next data set, thereby compensating for the error.

![Figure 5. Procedure of ensemble from an input sample.](image)

3.2. Artificial Neural Network (ANN)

An ANN is a statistical learning algorithm based on a biological neural network, i.e., its structure and function. ANNs make use of connected artificial neurons and perform nonlinear modeling through neurons explaining their unique behaviors by learning input parameters. The multilayer perceptron (MLP) is the most commonly employed ANN architecture; it is comprised of input, hidden, and output layers, as shown in Figure 6 [31]. As shown in Equation (9), all the neurons connected in every layer of
the ANN include weights \((w)\) and biases \((b)\). In addition, the neuron values of the previous layers are modified by various weights and compensated through biases.

\[
y_i = f(\text{net}) = f\left(\sum_{i=1}^{n} a_i p_i + b_j\right)
\]

In the above, \(p\) is the input value \((i = 1, \ldots, n)\), \(b\) denotes the bias of the neuron, \(w\) denotes the weight vector between neurons, \(f\) is the activation function, and \(y\) represents the output value. An MLP is ordinarily trained using an inverse propagation algorithm, where interconnected weights in the network are repeatedly adjusted to minimize errors (defined as root mean square errors) (RMSEs) \([32]\). Previous studies have reported that the Levenberg–Marquardt algorithm is suitable as it produces coherent results for the majority of ANNs \([33]\).

![Figure 6. Procedure of artificial neural network from an input sample.](image)

4. Results and Discussion

4.1. Experimental Consistency of Static and Dynamic Tests

The static elastic modulus \((Ec)\), initial chord elastic modulus \((Ei)\), and compressive strength \((fc)\) values were collected through static testing of the concrete according to ASTM C469/C469M-14 and ASTM C39/C39M-14a \([3,26]\), and the \(Eds\) \((ASTM.LT, f1,f2.LT,\) and \(ASTM.TR)\) values calculated as the first and second resonance frequencies were calculated using Equations \((9)\) for frequencies obtained by dynamic testing according to ASTM C215-14. The ranges of physical and mechanical properties of the specimens in the Mix1 and Mix2 were summarized in Table 2 and the coefficient of variation \((\text{COV})\) which means the standard deviation \((\omega)\) divided by the mean value \((\mu)\) of the same group was used to evaluate the consistency of the test results in Table 3. Outliers were detected using the Z-score method, and some data were removed from the statistical analysis \([34]\). The COV of the \(Ec\) by static testing ranged from 4.60% to 14.31%, and the COV of the \(fc\) ranged from 3.03% to 5.47%. The 28-day \(fc\) values of Mix1 and Mix2 were 19.26 MPa and 43.99 MPa, respectively, i.e., similar to the target \(fc\). However, the COV of the \(Ec\), which was slightly higher in Mix 1 (low curing age), was slightly distorted on the opposite side of the test piece; thus, the deformation during compression was not uniform. This appears to make the \(Ec\) more sensitive than the \(fc\) in static testing. The COV of the \(Ei\) ranged from 4.31% to 7.89%, and by using the method shown in Figure 4, it was possible to overcome the instability in the initial part of the stress–strain curve to collect consistent values. The COV ranges of \(ASTM.LT\) and \(ASTM.TR\) as measured by the longitudinal and transverse resonance frequency tests...
were 2.14%–4.36% and 2.71%–5.16%, respectively. The COV of $f_1/f_2.LT$, in which the first and second frequencies in the longitudinal mode of the resonance frequency test were used, ranged from 2.35% to 4.71%. For ASMT.LT, $f_1,f_2.LT$, and ASTM.TR, the COV remained reasonably consistent.

Table 2. The range of physical and mechanical properties in the Mix 1 and 2.

| w/c | The Number of Specimens | Weight (kg) | Dimension | Density (kg/m³) | Age (Day) | $f_c$ (MPa) | $E_c$ (MPa) |
|-----|-------------------------|-------------|-----------|-----------------|-----------|-------------|-------------|
|     |                         |             | Diameter (mm) | Length (mm) |           |             |             |
| 0.45 (Mix1) | 91 | 10.66 | 150 | 290.95 | 2051.58 | 4 | 7.33–8.56 (7.91) | 9202–15,929 (10,368) |
|     | | −12.12 (11.19) | −150 | −298.10 (293.97) | −2340.38 (2153.14) | 7 | 9.31–10.85 (10.07) | 10,930–13,256 (11,784) |
|     | | | | | | 14 | 12.67–14.63 (13.87) | 12,723–16,883 (14,735) |
|     | | 11.10 | 150 | 293.30 | 2125.88 | 28 | 20.96–26.19 (24.12) | 14,566–23,879 (16,796) |
| 0.35 (Mix2) | 194 | −12.08 (11.63) | −150 | −299.70 (297.50) | −2290.24 (2212.76) | 7 | 27.21–32.56 (29.62) | 15,431–19,807 (18,079) |
|     | | | | | | 14 | 32.82–42.38 (38.17) | 15,523–24,009 (20,904) |
|     | | 11.10 | 150 | 293.30 | 2125.88 | 28 | 40.21–47.34 (43.99) | 18,686–26,530 (22,860) |

Table 3. Summary of statistical analysis.

| Days/Variable | ASTMLT [MPa] | $f_1/f_2.LT$ [MPa] | ASTM.TR [MPa] | $E_i$ [MPa] | $E_c$ [MPa] | $f_c$ [MPa] |
|---------------|--------------|--------------------|--------------|-----------|-------------|-------------|
| | Mix 1 Mix 2 Mix 1 Mix 2 Mix 1 Mix 2 Mix 1 Mix 2 Mix 1 Mix 2 Mix 1 Mix 2 |
| Day 4 | N | 24 49 | 24 49 | 24 49 | 24 49 | 24 49 | 24 49 | 24 49 | 24 49 | 24 49 | 24 49 |
| | $\mu$ | 15,378 24,348 | 15,643 24,813 | 14,850 23,914 | 10,745 18,650 | 10,368 16,796 | 7.91 24.12 |
| | COV | 4.36% 3.81% | 4.71% 4.00% | 4.56% 4.01% | 7.89% 7.36% | 14.31% 7.84% | 4.29% 5.10% |
| Day 7 | N | 19 49 | 19 49 | 19 49 | 19 49 | 19 49 | 19 49 | 19 49 | 19 49 | 19 49 | 19 49 |
| | $\mu$ | 17,643 26,166 | 18,003 26,561 | 17,151 25,924 | 13,301 20,467 | 11,784 18,079 | 10.07 29.62 |
| | COV | 4.30% 2.39% | 4.36% 2.57% | 5.16% 2.71% | 4.36% 4.31% | 4.90% 4.60% | 4.57% 3.03% |
| Day 14 | N | 25 50 | 25 50 | 25 50 | 25 50 | 25 50 | 25 50 | 25 50 | 25 50 | 25 50 | 25 50 |
| | $\mu$ | 20,696 28,543 | 21,057 28,973 | 20,074 27,530 | 16,804 24,003 | 14,735 20,904 | 13.87 38.17 |
| | COV | 3.72% 2.47% | 3.80% 2.64% | 3.88% 3.01% | 5.35% 5.81% | 6.10% 6.96% | 3.35% 5.47% |
| Day 28 | N | 23 46 | 23 46 | 23 46 | 23 46 | 23 46 | 23 46 | 23 46 | 23 46 | 23 46 | 23 46 |
| | $\mu$ | 23,814 30,348 | 24,259 30,805 | 22,959 29,652 | 19,648 25,853 | 17,041 22,860 | 19.26 43.99 |
| | COV | 3.94% 2.14% | 4.35% 2.35% | 3.92% 2.73% | 6.25% 4.90% | 6.88% 6.21% | 4.09% 4.32% |

4.2. Relationship among Static and Initial Chord and Dynamic Elastic Modulus

The static elastic modulus ($E_c$) of concrete can be determined using a dynamic test method. Various empirical equations have been proposed for the prediction of $E_c$ in several studies, using measured $E_d$ values. Popovics considered the density of concrete in the relationship between $E_d$ and $E_c$ and proposed Equation (10) for lightweight concrete and general concrete.

$$E_c = \frac{446.09E_d^{1.4}}{\omega_c} \text{(MPa)} \quad (10)$$

As another example, the British standard BS8110 Part 2 proposed Equation (11). This equation does not apply to concrete or lightweight aggregate concrete containing more than 500 kg of cement per 1 m³ of concrete.

$$E_c = 1.25E_d - 19000 \text{(MPa)} \quad (11)$$

Lydon and Balendran proposed an empirical relationship (Equation (12)) between the $E_d$ and $E_c$.

$$E_c = 0.83E_d \text{(MPa)} \quad (12)$$
The values of dynamic elastic modulus \((Ed)\) as measured by the three methods in Figure 7 were almost similar, and the values followed \(Eds\) \((f1, f2.LT, ASTM.LT,\) and \(ASTM.TR\)) calculated as the first and second resonance frequencies. Compared to the added empirical equation, the predicted static elastic modulus \((Ec)\) and measured value of do not match, and in fact, the \(Ed\) value may vary greatly depending on the test methods and size and type of cylinder. Therefore, it is difficult to select the correct empirical equation for producing the minimum error for the various dynamic tests and cylinders. As expected, the initial chord elastic \((Ei)\) had a large margin of error with the \(Ed\), and was in the middle of the \(Ed\) and \(Ec\). In addition, the \(Ei\) is less variable and more consistent than the \(Ed\) in relation to the \(Ec\). Table 4 summarizes the correlations between \(Ed\), \(Ei\), and \(Ec\). The correlation between the three \(Ed\) values and \(Ec\) was 0.93, and \(Ei\) and \(Ec\) were analyzed to have a greater correlation, at 0.96. Therefore, it is necessary to accurately predict the \(Ei\) as having a greater correlation with \(Ec\).

![Figure 7](image_url) - Comparison of static elastic modulus \((Ec)\), \(Ei\), and dynamic elastic modulus \((Ed)\).

**Table 4.** Correlation between static elastic modulus \((Ec)\) and E-modulus.

| Correlation          | ASTM.LT-Ec | f1,f2.LT-Ec | ASTM.TR-Ec | Initial Chord Elastic Modulus (Ei)-Ec |
|----------------------|------------|-------------|------------|---------------------------------------|
| Value                | 0.9376     | 0.9362      | 0.9364     | 0.9645                                |

Next, the E-moduli are compared, based on the most commonly used ASTM.LT value among the resonance frequency test for predicting the \(Ed\). By comparing ASTM.LT values, other \(Eds\) \((ASTM.LT, f1,f2.LT,\) and \(ASTM.TR\)) show linear trend, but the \(Eds\), initial chord elastic modulus \((Ei)\), and static elastic modulus \((Ec)\) have nonlinear relationship. \(Ei\) and \(Ec\) show similar trends and data distributions; thus, it is more appropriate to use the \(Ei\) rather than \(Eds\), according to the theoretical equation for \(Ec\) prediction.

In Table 5, the errors and correlations of \(Ed\) and \(Ei\) are analyzed. MSE, RMSE, and MAPE were used as methods for confirming errors. The MSE and MAPE are defined in Equations (13) and (14), respectively, where \(n\) is the number of data, \(A_i\) is the \(Ei\) value, and \(P_i\) is the \(Ed\) value.

\[
MSE = \frac{1}{n} \sum_{i=1}^{n} (A_i - P_i)^2
\]  
(13)
As the three $Eds$ have almost similar values and trends for the same cylinder, the correlation with $Ei$ was close to 0.95. ASTM.TR had the smallest MAPE with the $Ei$.

### Table 5. Comparison of errors and correlations between $Ei$ and dynamic elastic moduli ($Eds$).

| Type of Errors                          | ASTM.LT | $f_1,f_2.LT$ | ASTM.TR |
|-----------------------------------------|---------|--------------|---------|
| Mean square error (MSE)                 | $2.53 \times 10^7$ | $2.95 \times 10^7$ | $1.88 \times 10^7$ |
| Root MSE (RMSE)                         | 5031    | 5427         | 4338    |
| Mean absolute percentage error (MAPE)   | 26.32\% | 28.39\%     | 22.59\% |
| R                                       | 0.9572  | 0.9556       | 0.9551  |

Figure 8 shows the relationships between the ASTM.LT and initial chord elastic modulus ($Ei$) according to mixes and ages. The MAPEs of the dynamic elastic modulus ($Ed$) and $Ei$ were large at low age and decreased as age increased. In addition, the difference between the $Ed$ and $Ei$ in Mix 2, a higher strength mixture, was smaller. This trend is considered to decrease the difference between the $Ed$ value and $Ei$ in the theoretical equation, as the moisture content inside the concrete is small (because the w/c ratio is small).

The difference between the dynamic elastic modulus ($Ed$) and initial chord elastic modulus ($Ei$) values from the three theoretical equations was confirmed using the method shown in Figure 9, and an analysis was conducted to confirm how accurately the $Ei$ could be predicted by correcting for the $Eds$. The correction factors and MAPE according to the mixes and ages are summarized in Table 6. Although the MAPE with the $Ei$ could be lowered to 4.69\%, 2.81\%, 3.12\%, and 3.43\% (for days 4, 8, 14, 28, respectively) with corrections based on mixes and ages for the $Eds$ values, this process was quite complicated and difficult. Therefore, the entire data were applied for the $ASTM.LT$, $f_1,f_2.LT$, and ASTM.TR, average correction factors of 0.80, 0.79, and 0.82. Compared to the actual $Ei$ value, the MAPE values of $ASTM.LT$, $f_1,f_2.LT$, and ASTM.TR according to the correction factors showed errors of 6.40\%, 6.50\%, and 6.43\%, respectively. However, the $Ei$ is not suitable for predicting the entire section, as it is nonlinear with the theoretical equations.
Next, as a method for predicting the $E_i$, an analysis was conducted with the fundamentally collected frequencies in the resonance frequency test. Figure 9 shows the relationship between the first resonance frequency in the longitudinal mode and the E-modulus. The first to fourth frequencies are defined as $f_1$ to $f_4$. For $f_1$, the ASTM equation has an exponential function form. The $E_i$ and $E_c$ have the same form. Based on these results, the $E_i$ and $E_c$ could be predicted using the ASTM method (correction coefficient, etc.), and the $E_i$ and $E_c$ could be predicted using the resonance frequencies (such as $f_1$ and $f_2$) collected in the resonance frequency test. Therefore, an analysis was conducted with the $f_1$ to $f_4$ resonance frequencies collected through the resonance frequency test.

Figure 10a shows the relationship between the four frequencies in the longitudinal (LT) mode and initial chord elastic modulus ($E_i$), and Figure 10b shows the relationship between the three frequencies in transverse (TR) mode and $E_i$. The number of used data and frequency range for the frequency combination of LT and TR modes are summarized in Tables 7 and 8. The data with unclear peaks or inconsistent with others in the frequency domain were excluded from the analysis. As both the LT and TR modes increased from $f_1$ to $f_4$, the variance of the data increased, and the consistency of data decreased. $f_1$ and $f_2$ of the LT and TR modes have relatively small coefficient of variation (COV) values and are statistically stable, whereas the trend can be identified from $f_3$, but the statistical stability is
poor. The ratio of $f_1, f_2, f_3,$ and $f_4$ at the bottom and top of the LT mode was similar to 1.9, 2.43, and 2.82, but in the TR mode, $f_1-f_2$ was 1.85, and $f_1-f_3$ showed a large error.

![Graphs showing frequency and $E_i$ relationships](image)

**Figure 10.** Relationship between the frequencies and $E_i$ of the resonance frequency test. (a) LT frequencies-$E_i$, (b) TR frequencies-$E_i$.

### Table 7. Summary of used data in the LT mode for machine learning analysis.

| Type of Mode | The Number of Specimens | Variable | $f_1$ (Hz) | $f_2$ (Hz) | $f_3$ (Hz) | $f_4$ (Hz) | Weight (kg) | Diameter (mm) | Length (m) | Density (kg/m$^3$) |
|--------------|-------------------------|----------|------------|------------|------------|------------|-------------|---------------|-------------|-----------------|
| LT.$f_1$     | 285                     | Range    | 4450-6400  | 8500-12,000| 10,550-15,500| 12,400-20,250| 10.66-12.12 | 11.49-15.00  | 2051.58-2340.38|               |
|              |                         | Average  | 5641       | 10,757     | 13,677     | 15,838     | 290.95-299.70| 296.39-299.40| 2193.72-2194.86|               |
| LT.$f_2$     | 283                     | Range    | 4450-6400  | 8500-12,000| 10,550-15,500| 12,400-20,250| 10.66-12.12 | 11.49-15.00  | 2051.58-2340.38|               |
|              |                         | Average  | 5640       | 10,757     | 13,677     | 15,838     | 290.95-299.70| 296.39-299.40| 2193.72-2194.86|               |
| LT.$f_3$     | 275                     | Range    | 4450-6400  | 8500-12,000| 10,550-15,500| 12,400-20,250| 10.66-12.12 | 11.49-15.00  | 2051.58-2340.38|               |
|              |                         | Average  | 5640       | 10,757     | 13,677     | 15,838     | 290.95-299.70| 296.39-299.40| 2193.72-2194.86|               |
| LT.$f_4$     | 230                     | Range    | 4450-6400  | 8500-12,000| 10,550-15,500| 12,400-20,250| 10.66-12.12 | 11.49-15.00  | 2051.58-2340.38|               |
|              |                         | Average  | 5640       | 10,757     | 13,677     | 15,838     | 290.95-299.70| 296.39-299.40| 2193.72-2194.86|               |
| LT.$f_1,f_2$ | 283                     | Range    | 4450-6400  | 8500-12,000| 10,550-15,500| 12,400-20,250| 10.66-12.12 | 11.49-15.00  | 2051.58-2340.38|               |
|              |                         | Average  | 5640       | 10,757     | 13,677     | 15,838     | 290.95-299.70| 296.39-299.40| 2193.72-2194.86|               |

### Table 8. Summary of used data in the LT mode for machine learning analysis.

| Type of Mode | The Number of Specimens | Variable | $f_1$ (Hz) | $f_2$ (Hz) | $f_3$ (Hz) | Weight (kg) | Diameter (mm) | Length (m) | Density (kg/m$^3$) |
|--------------|-------------------------|----------|------------|------------|------------|-------------|---------------|-------------|-----------------|
| TR.$f_1$     | 285                     | Range    | 2750-3900  | 7750-12,250| 10.71-12.12| 10.66-12.12 | 11.49-15.00  | 2051.58-2340.38|               |
|              |                         | Average  | 3441       | 6276       | 9919       | 290.95-299.70| 296.39-299.40| 2193.72-2194.86|               |
| TR.$f_2$     | 105                     | Range    | 2750-3850  | 5150-7700  | 10.71-12.12| 10.66-12.12 | 11.42-15.00  | 2051.58-2340.38|               |
|              |                         | Average  | 3351       | 6276       | 9919       | 290.95-299.70| 296.39-299.40| 2193.72-2194.86|               |
| TR.$f_3$     | 105                     | Range    | 2750-3850  | 5150-7700  | 10.71-12.12| 10.71-12.12 | 11.42-15.00  | 2051.58-2340.38|               |
|              |                         | Average  | 3351       | 6276       | 9919       | 290.95-299.70| 296.39-299.40| 2193.72-2194.86|               |
| TR.$f_1,f_2$ | 105                     | Range    | 2750-3850  | 5150-7700  | 10.71-12.12| 10.71-12.12 | 11.42-15.00  | 2051.58-2340.38|               |
|              |                         | Average  | 3351       | 6276       | 9919       | 290.95-299.70| 296.39-299.40| 2193.72-2194.86|               |
| TR.$f_1,f_2,f_3$ | 105                | Range    | 2750-3850  | 5150-7700  | 10.71-12.12| 10.71-12.12 | 11.42-15.00  | 2051.58-2340.38|               |
|              |                         | Average  | 3351       | 6276       | 9919       | 290.95-299.70| 296.39-299.40| 2193.72-2194.86|               |
Next, the correlation between the frequencies of the LT and TR modes and initial chord elastic modulus \((E_i)\) was analyzed, as shown in Table 9. In the LT mode, the correlation between the frequency and \(E_i\) was \(f_1 > f_2 > f_3 > f_4\). The correlation values of \(f_1\) and \(f_2\) were almost identical and had a high correlation with \(E_i\). In the TR mode, \(f_1-E_i\) had the largest correlation at 0.94, but the correlation greatly decreased from \(f_2\). It was confirmed that the \(E_i\) can be predicted with \(f_1\) and \(f_2\) with high correlation, through analysis of the variability of the resonance frequency for each mode and correlation with the \(E_i\).

\[
\begin{array}{cccccccc}
\text{Correlation} & \text{LT.}\ f_1-E_i & \text{LT.}\ f_2-E_i & \text{LT.}\ f_3-E_i & \text{LT.}\ f_4-E_i & \text{TR.}\ f_1-E_i & \text{TR.}\ f_2-E_i & \text{TR.}\ f_3-E_i \\
\text{Values} & 0.9420 & 0.9397 & 0.8629 & 0.7071 & 0.9413 & 0.8992 & 0.7915 \\
\end{array}
\]

4.3. Prediction of Initial Chord Elastic Modulus with Multiple Linear Regression

Subramaniam et al. applied \(f_1\) and \(f_2\) to Equation (3) to obtain a dynamic Poisson’s ratio, and proposed an equation for predicting the \(E_d\) through Equation (4) \[11\]. The multiple linear regression (MLR) results using the LT mode’s \(f_1\) and \(f_2\) values with high correlations with the \(E_i\) and small data variability were compared with those from the theoretical equation of Subramaniam. Equations (15) and (16) for \(f_1\) and \(f_2\) in LT: \(E_d\) measured by the first and second frequency in LT modes, \(E_i\) values were extracted using the curve fitting toolbox of MATLAB R2019b and were compared in three dimensions, as shown in Figure 11.

\[
E_i = -3.088 f_1^4 + 4.104 \times f_1 + 2.575 \times f_2 
\]

\[
f_1, f_2.LT = -2.853 f_1^4 + 11.9 \times f_1 - 1.247 \times f_2 
\]

Generally, in the LT mode, \(f_2\) has a value of 1.8–2 times \(f_1\). The MLR plane has a similar specific gravity to \(f_1\) and \(f_2\) and has a positive correlation. In the Subramaniam equation, \(f_1\) has a high specific gravity and positive correlation, and \(f_2\) has a small specific gravity and negative correlation. The MAPE of the \(E_i\) as predicted by the MLR with \(f_1\) and \(f_2\) and actual \(E_i\) could be lowered by 5.04%, i.e., higher accuracy than the predicted result through general correction to the theoretical equation.

**Figure 11.** Comparison of prediction results with two frequencies. (a) Predicted \(E_i\) with multiple linear regression (MLR), (b) Predicted \(E_d\) with Equation (4).

4.4. Prediction of Initial Chord Elastic Modulus with Ensemble Method

Recently, studies have been conducted to overcome the nonlinearity of data by applying machine learning (ML) methods, and to improve prediction accuracy and reliability. In this study, ML methods...
are applied to overcome the nonlinearity of data that is difficult to predict using basic regression analysis and correction factors. The errors of the initial chord elastic modulus \(E_i\) and actual \(E_i\) values as predicted by various combinations of frequencies by the ensemble method are summarized in Tables 10 and 11. As a result of the analysis, when a single frequency was used, MAPE values of 3.90% and 4.31% were found for the longitudinal (LT) and transverse (TR) modes, respectively. In the ensemble method, using only \(f_1\) can reduce the error by 1–2% more than in the correction coefficient and MLR method. Thus, it is possible to predict a sufficiently accurate \(E_i\) value using only \(f_1\). As the number of frequencies being used increased, the accuracy tended to increase. In the LT/TR modes, maximum MAPE values of 3.51%/3.40% were shown. Through the correlation analysis shown in Table 10 and the predicted accuracy values in Tables 9 and 10, approximate contributions to \(f_1\) to \(f_4\) can be inferred. However, as they are not exact values, an additional contribution analysis was performed.

Table 10. \(E_i\) predicted by LT mode frequencies with ensemble.

| LT Mode | \(f_1\) | \(f_2\) | \(f_3\) | \(f_4\) | \(f_1\)\(^{-2}\) | \(f_1\)\(^{-3}\) | \(f_1\)\(^{-4}\) |
|---------|--------|--------|--------|--------|-------------|-------------|-------------|
| MSE     | 1.18 \times 10^6 | 1.21 \times 10^6 | 2.75 \times 10^6 | 4.73 \times 10^6 | 1.08 \times 10^6 | 9.74 \times 10^5 | 9.06 \times 10^5 |
| RMSE    | 1080   | 1100   | 1660   | 2170   | 1040        | 987         | 952         |
| MAPE    | 3.90%  | 3.95%  | 5.69%  | 8.00%  | 3.67%       | 3.52%       | 3.51%       |
| R       | 0.9716 | 0.9705 | 0.9332 | 0.8900 | 0.9738      | 0.9768      | 0.9798      |

Table 11. \(E_i\) predicted by TR mode frequencies with ensemble.

| TR Mode | \(f_1\) | \(f_2\) | \(f_3\) | \(f_1\)\(^{-2}\) | \(f_1\)\(^{-3}\) |
|---------|--------|--------|--------|-------------|-------------|
| MSE     | 1.36 \times 10^6 | 1.64 \times 10^6 | 1.83 \times 10^6 | 1.13 \times 10^6 | 7.74 \times 10^5 |
| RMSE    | 1160   | 1280   | 1350   | 1060        | 879         |
| MAPE    | 4.31%  | 4.44%  | 5.79%  | 3.87%       | 3.40%       |
| R       | 0.9668 | 0.9620 | 0.9577 | 0.9740      | 0.9822      |

The relative importance (RI) was calculated using the Statistics and Machine Learning Toolbox of MATLAB R2019b to identify the detailed contribution of each variable from a combination of four frequency variables. As presented in Figures 12 and 13, the results of the contribution analysis showed that \(f_1\) was more than 75% in all combinations and had a dominant effect on the \(E_i\) prediction. However, even if the contribution of other factors is low, they can contribute to some prediction accuracy. Also, the difference in the contribution of each variable for each case is due to the data difference, and especially when consistent data from \(f_1\) to \(f_4\) were used, the contribution of \(f_1\) tends to be large.

Figure 12. Relative importance according to combination of variables in LT mode.
Next, an analysis was conducted by considering various variables for the concrete cylinder. The $E_i$ was predicted by the ML method by adding the mass ($M$), length/diameter ($L/d$), area ($A$), volume ($V$), and density ($\rho$) as dimensional variables, with the frequency obtained by the resonance frequency test. It was found that $M$, $L/d$, $A$, $V$, and $\rho$ do not contribute to $E_i$ prediction, as they have constant values for the same specimen. Additionally, the $E_i$ was predicted with variables such as the curing age ($day$), $W/B$, frequencies of longitudinal (LT) and transverse (TR) mode, and RIs for each variable, as summarized in Figures 12 and 13. In the LT and TR modes, $f_1$ contributed more than 87%, and $f_2$ contributed more than 6%, accounting for most of the ratio. The $W/B$ ratio was expected to have an impact but did not contribute to the actual $E_i$ prediction. Thus, it is considered that this factor is contained in the main resonance frequencies such as $f_1$ and $f_2$, such that the elastic modulus and strength can be accurately predicted using only the main resonance frequency, i.e., without $W/B$ information.

Although the day variable was very small, it contributed partly to the accuracy of initial chord elastic modulus ($E_i$) prediction, and the prediction results are shown in Figure 14. When the number of frequencies (including the day) is increased from $f_1$ to $f_4$ for the LT mode, the errors are 3.79%, 3.69%, 3.51%, and 3.47% in the ensemble method, respectively, and 3.66%, 3.57%, 3.42%, and 3.31 in ANN method, respectively. Even in the TR mode, the predicted results including the day improved from 0.1% to 0.2%. This means that the frequency obtained by the resonance frequency test includes the day information, and it is sufficient to use only the frequency to predict the $E_i$. Moreover, although the TR mode frequency predicted the $E_i$ with higher accuracy, it is more appropriate to use the LT mode, which appears accurately up to the fourth mode. This is because $f_3$ of the TR mode has a large dispersion and low consistency, and some frequencies are unclear.
Additionally, the initial chord elastic modulus \( (E_i) \) values predicted with the ensemble, artificial neural network (ANN), multiple linear regression (MLR), and correction factor methods are compared with the actual \( E_i \) for the LT mode frequency (i.e., with a greater correlation with the \( E_i \) than the TR mode). As a result of comparing the predicted \( E_i \) and actual \( E_i \) values, the MALR of the MLR method was 5.04%, and the correction factor method had an error of 6.33%. The MLR method using two frequencies has a higher predictive performance than the normal correction. In addition, the nonlinearity of the low age mix was partially compensated for by using two frequencies. For the ANN and ensemble methods, the MAPEs were 3.31% and 3.51%, respectively, and the use of ML reduced the errors as compared to normal calibration and MLR analysis. This is because the ANN and ensemble methods can compensate for nonlinearity of data by providing weight correction for additional frequencies. Therefore, it is possible to more accurately predict the \( E_i \) through the ML method.

### 4.5. Relationship between Initial Chord and Static Elastic Modulus

Table 12 shows the correction factors and MAPE for the \( E_i \) and \( E_c \) values extracted from Figure 4. For Mix 1 and Mix 2, by applying the correction factors of 0.91–0.95 and 0.89–0.90 for each age, respectively, the MAPE could be minimized. As a result of applying the average correction factor for each mix, the \( E_c \) could be predicted with an error of approximately 5%. In addition, it was found that the correction factor of 0.89 had the lowest MAPE for the entire data set (Mix 1 + Mix 2).

**Table 12. Comparison of correction factors and MAPE of \( E_i \) and \( E_c \) according to mix and age.**

| Type   | Day 4   | Day 7   | Day 14  | Day 28  | Average |
|--------|---------|---------|---------|---------|---------|
| Mix 1  | 0.95    | 0.89    | 0.87    | 0.91    | 0.91    |
|        | (5.51%) | (2.97%) | (3.94%) | (6.31%) | (5.27%) |
| Mix 2  | 0.90    | 0.88    | 0.88    | 0.89    | 0.89    |
|        | (4.15%) | (2.97%) | (4.78%) | (4.88%) | (4.14%) |

The initial chord elastic modulus \( (E_i) \) values predicted by the four methods had errors of 3–6% from the actual \( E_i \), so the correction factors with the static elastic modulus \( (E_c) \) were calculated to be equal to 0.89. The \( E_c \) values were predicted by applying a correction factor (0.89) to the predicted \( E_i \), which is compared with the actual \( E_c \) in Figure 15. The LT mode has a higher correlation with the \( E_i \) than the TR mode, and the frequency of the LT mode is used, because the resonance frequency is clearly seen up to the higher-order mode.

![Figure 15. Comparison of \( E_c \) predicted with \( E_i \) and actual \( E_c \).](image-url)
The predicted static elastic modulus \( (E_c) \) through the correction of \( ASTM.LT \) (\( E_d \) measured by the first frequency in LT modes) and measured mean absolute percentage error (MAPE) of \( E_c \) was 6.57% and had the largest error. It is difficult to overcome the nonlinearity of concrete, as the theoretical ASTM equation based on the resonance frequency test has a constant density and Poisson’s ratio within a general range. The MAPEs between the \( E_c \) values predicted by the ensemble, ANN, and MLR methods and the actual \( E_c \) were determined to be 4.63%, 4.69%, and 5.53%, respectively. Two or more frequencies are used to compensate for the nonlinearity of concrete, and have a linear relationship with the actual \( E_c \).

4.6. Relationship between Initial Chord and Compressive Strength

In the concrete cylinder in this study, materials similar to crushed cobblestone and granulated blast furnace slag (GBFS) were used, and Equation (17) (proposed by Noguchi) was used for comparison with the theoretical strength curve; \( k_1 \) is the aggregate, and \( k_2 \) is the correction coefficient of the SCMs [35]. According to Table 13, 0.95 for both \( k_1 \) and \( k_2 \) was used. Such concrete has the advantage of slow strength development at an early age but demonstrates significant progress for long-term strength.

\[
E_c = k_1 k_2 33500 \left( \frac{f_c}{60} \right)^{1/3} \left( \frac{\omega_c}{2400} \right)^2 \text{ (MPa)}
\]  

(17)

| Lithological Type of Coarse Aggregate | \( k_1 \) | Type of Addition                                      | \( k_2 \) |
|--------------------------------------|--------|------------------------------------------------------|--------|
| Crushed limestone, calcined bauxite  | 1.20   | Silica fume, ground-granulated blast-furnace slag, fly ash fume | 0.95   |
| Crushed quartzitic aggregate, crushed andesite, crushed basalt, crushed clay slate, crushed cobblestone | 0.95   | Fly ash                                              | 1.10   |
| Coarse aggregate, other than above   | 1.00   | Addition other than above                             | 1.00   |

Figure 16 shows the relationship between the static elastic modulus \( (E_c) \) \( (E_c \) predicted from \( E_i \), actual \( E_c \) \) and compressive strength \( (f_c) \). The theoretical strength curve of the Noguchi equation and trend of the measured \( E_c \) and \( f_c \) data are suitable, and close to the line. Therefore, the \( E_c \) and \( f_c \) are well-measured, and the values of \( E_c \) and \( f_c \) are within a reasonable range. The results were the same for the \( E_c \) values predicted using the initial chord elastic modulus \( (E_i) \). Thus, it was confirmed that there is a high correlation (with intensity) when predicting the correct \( E_c \) using the \( E_i \).
In this study, to confirm the accurate prediction and applicability of actual initial chord elastic modulus ($E_i$) values, the $E_i$ was compared with the dynamic elastic modulus ($E_d$) of three theoretical equations, using a resonance frequency test. In addition, the relationship among the predicted $E_i$ and static elastic modulus ($E_c$) values and the compressive strength ($f_c$) was analyzed. It was found that the $E_i$ is more correlated with the $E_c$ than the $E_d$, confirming the need for accurate $E_i$ prediction. In addition, it was confirmed that the correlation between the frequencies collected by the resonance frequency test according to the ASTM method (but not the fracture test) and the $E_i$ and $E_c$ was high, and that the $E_i$ and $E_c$ could be predicted using the resonance frequency. As a result of predicting the $E_i$ using ML (ensemble, ANN), multiple linear regression (MLR), and correction factor methods from the collected frequency, it was possible to overcome the nonlinear behavior of concrete in the range of the initial $E_c$. Thus, the prediction error was smaller than when using the MLR or correction factor methods. The predicted $E_i$ showed a higher correlation with the $E_c$ and $f_c$ than the $E_d$ and was able to identify the applicability of the $E_i$.

5. Conclusions

Predicting the static elastic modulus ($E_c$) is important for concrete structures. The ASTM resonance frequency test is commonly used in predicting a consistent dynamic elastic modulus ($E_d$) using a nondestructive method. The resonance frequency test calculates the $E_d$ for a very small stress in the stress–strain curve and assumes that the $E_d$ is equal to the initial chord elastic modulus ($E_i$). However, the calculated $E_d$ derives a large error for utilizing the $E_c$ and $f_c$, owing to the nonlinearity of the concrete. Therefore, the correct $E_i$ as extracted through the curve fitting of the stress-strain curve showed a better correlation between the $E_c$ and $f_c$ than the $E_d$. These results indicate that it is desirable to measure and utilize accurate $E_i$ values. In this study, it was possible to predict the actual $E_i$ quite accurately by applying the ensemble, ANN, MLR, and correction factor methods to the $f_1$ to $f_4$ frequencies of longitudinal (LT) and transverse (TR) modes collected by the ASTM resonance frequency test. If several frequency variables were used, the contributions of the variables were extracted, and the relationships with the $E_c$ and $f_c$ were analyzed using the predicted $E_i$. The following conclusions were drawn.

- The $E_d$ values calculated by three theoretical equations (ASTM, Rayleigh Ritz) of the resonance frequency test were in the order of $f_1$, $f_2$.LT > ASTM.LT > ASTM.TR, and had nearly the same values. The size of the elastic modulus as measured by static and dynamic tests was $E_d$ > $E_i$ > $E_c$. In addition, it is determined that it is desirable to utilize the $E_i$, as the correlation with $E_c$ is analyzed as $E_i$ > $E_d$.
- The Popovis equation for the relationship between $E_c$ and $E_d$ gives results similar to the $E_d$s of the ASTM, and the Lydon and Balendran equations are similar to $E_i$ values. BS8110 Part 2 is not suitable, as because it has a large error from the $E_d$ and $E_i$ in the resonance frequency test.
- As a result of comparing an E-modulus based on $ASTM.LT$, $f_1$, $f_2$.LT, and $ASTM.TR$ had a clear linear relationship, and they were close in the line of equality. They were identified as having a nonlinear relationship with the $E_i$ and $E_c$. As the theoretical equations assumed the concrete as a perfectly elastic body for microscopic stress, it was difficult to overcome the nonlinear behavior of the actual $E_i$ and $E_c$, owing to challenges in considering inhomogeneity and inelasticity of concrete. Thus, it is more appropriate to accurately predict and utilize the $E_i$, which has a similar nonlinear behavior with the $E_c$.
- As a result of applying $ASTM.LT$, $f_1$, $f_2$.LT, and $ASTM.TR$ to the correction factors, the MAPE in the $E_i$ could be lowered to 6.40%, 6.50%, and 6.43%, respectively. In addition, the $E_d$ in the three equations and $E_i$ of the MAPE decreased in order in days 4, 7, 14, and 28. The theoretical equation is suitable for concrete after 28 days but is considered difficult to use to accurately predict lower ages.
• In the relationship between $E_i$ and frequency, the correlation of $E_i$-$f_1$ was the largest, the nonlinearity increased as the mode appeared later, and the density and consistency of the data gradually decreased. In addition, the $E_i$ and $E_c$ values and first frequency of the resonance frequency test tended to be similar to an exponential function, indicating that prediction of the $E_i$ and $E_c$ based on frequency was possible.

• As a result of predicting the $E_i$ using only frequencies through the ensemble and ANN methods, the MAPE decreased by 3.90% in the case of using only $f_1$, and by 3.51% in the case of using $f_1$-$f_4$. Accordingly, the nonlinear behavior could be overcome by using ML.

• As a result of analyzing the contributions of variables in predicting the $E_i$, $f_1$ and $f_2$ were dominant, the RI of the size factor was 0, and 0.3% of the day variables contributed to the $E_i$ prediction. Therefore, it is possible to predict a sufficiently accurate $E_i$ using only the frequencies, i.e., without other variables.

• As a result of predicting the $E_c$ by applying a correction factor of 0.89 to the predicted $E_i$ in four ways, the MAPE ranged from 4.6% to 6.57%, and the correlation between the predicted $E_c$ and $f_c$ was high. Therefore, far more accurate $E_i$ values can be predicted by the ASTM method in the future, and more accurate design, construction, and maintenance will be possible if this approach is used for calculating the $E_c$ and $f_c$.

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Abbreviations

ANN artificial neural network
ASTM.LT dynamic elastic modulus measured by the first frequency in longitudinal modes
ASTM.TR dynamic elastic modulus measured by the first frequency in transverse modes
COV coefficient of variation
$f_1, f_2$,LT dynamic elastic modulus measured by the first and second frequency in longitudinal modes
GBFS granulated blast furnace slag
LSBoost least squares boosting
LT longitudinal
MAPE mean absolute percentage error
ML machine learning
MLP multilayer perceptron
MLR Multiple linear regression
MSE mean squared error
RI relative importance
RMSE root mean square error
SCMs supplementary cementitious materials
SVM support vector machine
TR transverse
$E_c$ static elastic modulus
$E_d$ dynamic elastic modulus
$E_i$ initial chord elastic modulus
$f_c$ compressive strength
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