Theoretical research of the distortion of quantum circuit in Grover's algorithm

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Abstract. Grover's algorithm is a quantum search algorithm among unsorted elements that can do several operations at the same time due to their wave like properties. In addition, it could solve problems of global optimization and graph coloring. This work presents the results of the simulation of Grover's algorithm and research of its resistance to the effects of perturbations of quantum logic circuit elements. These dependencies can be useful for creating optical circuit.

Introduction
Currently, the field of quantum information and quantum computing is a perspective direction of research. Quantum computers use quantum properties of matter, which gives to algorithms, working on the rules of quantum information, the ability to provide a significant increase in calculation speed compared to classical. For example, the problem of searching in unsorted list in classical algorithm actually reduces to sequential scan, and uses O (N) steps. Grover's algorithm is the fastest algorithm for solving such problems, giving quadratic speedup computations [1]. Using this algorithm to search, you can significantly speed up the well-known classical algorithms that require solving the problem of the search in an unsorted space [2].

Grover’s algorithm applies for searching a specified element in unsorted list. The essence of the algorithm is to increase the amplitude of the state that satisfies the search condition, due to the decrease of the amplitudes of all other states. Therefore, amplitude of marked state increase, and increase the probability of detection correct answer.

Application of Grover's algorithm, to adapt it to accelerate the works of well-known classical algorithms, is promising. In [3, 4] proved the possibility of using Grover's algorithm for solving the problem of global optimization and shows the different ways of implementing the oracle.

In papers [5, 6] presented a quantum Dijkstra's algorithm, which improves the work of the classical algorithm. Classical Dijkstra's algorithm solves the problem of graph coloring and its quantum implementation can be used to finding shortest path in network routing.

Grover's algorithm is a perspective direction of research and has a number of applications that can be solved with its help. In this regard, there is a problems of creating an optical circuit that implements the algorithm and the calculation of the effect of the distortion circuit elements on the work of Grover's algorithm. In this research, we simulated Grover's algorithm and tested its resistance to the effects of perturbations of quantum logic circuit elements using the unitary distortion of quantum operators.
Simulation of perturbations
Scheme of the Grover’s algorithm is on figure 1-2. At the initial time, the system is in the zero state. Hadamard gate transforming the system into a state of superposition. Then follow iterative process consists from two functions - controlled phase rotation and diffusion function.

Figure 1. Flowchart of the Grover’s algorithm.

Phase rotation controlled by a search condition, which is enclosed in oracle. Oracle is a quantum operator that can recognize the decision and mark it by phase rotation.

Diffusion function makes a calculation of the amplitudes relatively its average. Therefore, amplitude of marked state increase, and increase the probability of its detection.

Histograms on figure 3 presented results of the simulation algorithm on 6 qubits. Such system has 64 different unsorted states, and if it is necessary to find 1 element from them, it should be better to use Grover’s algorithm, which can find it for 6 iterations, instead of classically scan at least half of all elements.

Figure 3. Results of the Grover’s algorithm simulation on 6 qubits for superposition state, 3 and 6 iterations. Marked state equal 27.
Obviously, that at the beginning probability of detection states is the same. After 3-d iteration probability of detection marked state increase by 59.1% and probability of detection other states equal 0.65%. After 6-th iteration marked state detects in 99.7% cases. Through formula (1) was calculated that six iterations is enough to reach highest probability.

\[ I = \frac{\pi}{4} \sqrt{2^n} \]  

(1)

To simulate the Grover’s algorithm and research distortion system on 3 qubits was used. If \( n = 3 \), then we have \( 2^n = 8 \) different states of qubits. In classical physics, each state has its own probability and sum of them equals one. In quantum physics, each state has its own amplitude, and the magnitude of the sum of all amplitudes squares is equal to one. All perturbations of this research is caused by unitary operators ensures the sum of probabilities of all possible outcomes of any event is always 1.

First step of the algorithm is the Hadamard operator that transform initial state of the system to the superposition state. In such system amplitudes of any states is the same, so probabilities of detection any states is the same too. For system on 3 qubits and after correct Hadamard transformation, amplitudes at the superposition state equal \( \approx 0.35 \).

In this research have been explored different ways of setting the value of the perturbation. They could divided into 2 groups: light deviations and strong deviations (figure 4). For example if amplitude of marked state distorted for 10%, other states should distorted for 10% summary. In this case amplitude of other states distorted just a little, and amplitude of the marked state distorted more. This is light deviations (figure 4a). For strong deviations, amplitudes of other states distorted as well as amplitude of the marked state (figure 4b). Moreover, amplitude of marked state can increase on 10% or decrease on 10%. Therefore, we have 4 possible cases:

- **Light deviations**
  - amplitude of marked state increase (LI);
  - amplitude of marked state decrease (LD);

- **Strong deviations**
  - amplitude of marked state increase (SI);
  - amplitude of marked state decrease (SD).

![Figure 4](image)

**Figure 4.** Initial amplitudes of the states after 10% perturbations for light and strong deviations. Amplitude of marked state underlined and equal \( |011\rangle \). Left values from a) and b) – amplitude of marked state increase, right values – decrease.

In the following histograms presented changes in the probability of detection states at different perturbations value. Zero percent means that algorithm working without any perturbations. It is ideal Grover’s algorithm.
Figure 5. Dependencies of detection probability and value of perturbations for light deviations and case LI (when initial amplitude of marked state increase). ProbabilityM - probability of detection marked state, ProbabilityO – probability of detection other states, InitialM – initial probability of detection marked state, InitialO - initial probability of detection other states.

Histogram for light deviations and increases initial amplitude of marked state at figure 5 shows that probability of detection marked state decrease while perturbation value grows. Moreover, probability of detection other states increase, causing the growth of the frequency of registering wrong answer.

Figure 6. Dependencies of detection probability and value of perturbations for strong deviations and case SI (when initial amplitude of marked state increase). ProbabilityM - probability of detection marked state, ProbabilityOI – probability of detection other states, which initial amplitude increases, ProbabilityOD – probability of detection other states, which initial amplitude decreases.

Results for strong deviations and case SI is on figure 6. Simulation shows strong decrease of detection probability of marked state. Probability of detection other states increase strongly too.
Figure 7. Dependencies of detection probability and value of perturbations for light deviations and case LD (when initial amplitude of marked state decrease). ProbabilityM - probability of detection marked state, ProbabilityO – probability of detection other states, InitialM – initial probability of detection marked state, InitialO - initial probability of detection other states.

Figure 8. Dependencies of detection probability and value of perturbations for strong deviations and case SD (when initial amplitude of marked state decrease). ProbabilityM - probability of detection marked state, ProbabilityOI – probability of detection other states, which initial amplitude increases, ProbabilityOD – probability of detection other states, which initial amplitude decreases.

Light deviations and decreases initial amplitude shows effect of a little increase of detection probability of marked state (figure 7), but it's gone when deviation of states amplitude becomes much stronger (figure 8). On the figure 8 also shown, that probability of detection other states increase.
Results
In this research, we simulated Grover's algorithm and verified its resistance to the effects of perturbations of quantum logic circuit elements. The results is shown in table 1. As it shown, probability of detection marked state decreases. For highest perturbations values that researched in this work, probability decreases on 34%. Moreover, probability of detection other states increases, causing the growth of the frequency of registering wrong answer in 10 times for highest perturbation. On the other hand, probability of detection marked state is still higher others, so Grover’s algorithm has good resistance to the effects of perturbations of quantum logic circuit elements.

| Perturbation value, % | Marked state, % | Other states, *100% |
|-----------------------|-----------------|---------------------|
| 5                     | -0.06           | 1.21                |
| 20                    | -1              | 1.96                |
| 35                    | -3.1            | 2.9                 |
| 50                    | -6.6            | 4.07                |
| 65                    | -11.9           | 5.52                |
| 80                    | -19.8           | 7.4                 |
| 95                    | -34.1           | 10.3                |

Table 1. Results of Grover’s algorithm simulation. Dependencies detection probability of states from perturbation value.

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