Quark spin coupling in baryons - revisited

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Abstract

A direct connection can be made between mixing angles in negative parity baryons and the spin coupling of constituent quarks. The mixing angles do not depend on spectral data. These angles are recalculated for gluon exchange and pion exchange between quarks. For pion exchange the results of Glozman and Riska are corrected. The experimental data on mixing are very similar to those derived from gluon exchange but substantially different from the values obtained for pion exchange.

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I. INTRODUCTION

The spin-spin coupling between two fermions has two terms: a “tensor” term and a “contact” term. In atoms, electron spins interact with the nuclear spin and this explains the hyperfine structure: the tensor term is ordinary magnetic dipole-dipole interaction, and the contact term is part of the same interaction when the dipoles are at the same point. In nuclei, nucleon spins interact through pion exchange and there is a similar coupling with a different weight for the contact term. For constituent quarks in baryons there is a controversy in the literature between “gluon exchange” (OGE) which mimics the magnetic coupling and “pion exchange” (OPE). In the early days of constituent quarks OGE was applied to ground state [1] and excited baryons [2, 3, 4] with some success. Pion exchange was also tried particularly in the context of bag models [5]. More recently it has been argued that the entire spin dependent coupling between constituent quarks is due to Goldstone Boson Exchange [6], a generalization of OPE. This proposal was criticized [7, 8].

We deal here with a single issue: the mixing of states in the lowest mass negative parity nucleons. These negative parity nucleons have internal orbital angular momentum \( L = 1 \), which couples with an overall quark spin of \( S = 1/2 \) or \( 3/2 \) to give the total angular momentum \( J \). The physical states of \( J = 1/2 \) (or \( 3/2 \)) are mixtures of doublet and quartet spin states, and this mixing can be determined from decay data. This issue has been discussed already [7, 8]. However, we find that the discussion has been flawed since [7] used the estimates of [6] and the estimates of [6] are based on fitting the experimental mass spectrum. However, as we show below, these mixing angles are independent of the mass spectrum and depend only on the coupling and wavefunctions. We re-evaluate these angles and find significant changes from those appearing in [6, 7]. The differences between OPE and OGE become larger and the data favors more clearly OGE, the same coupling as between electrons and nuclei (ie., magnetic dipole type hyperfine interactions). All this will be discussed in detail below, as well as some items in the literature.

II. QUARK-QUARK HYPERFINE INTERACTIONS

Our discussion here follows [3], although for the sake of clarity we repeat some of the material.
A. One Gluon Exchange (OGE)

This effective hyperfine interaction between two quarks in a baryon has the form of magnetic dipole-dipole interaction in Electrodynamics, (with dipoles produced by current loops, see [9]):

\[
H_{\text{OGE}} = A\left\{ \left( \frac{8\pi}{3} \right) \vec{S}_1 \cdot \vec{S}_2 \delta^3(\vec{\rho}) + \left( 3S_1 \cdot \hat{\rho} \cdot S_2 \cdot \hat{\rho} - S_1 \cdot \vec{S}_2 \right) \rho^{-3} \right\}.
\]

(1)

Here \( S_1, S_2 \) are the spins of the two quarks, \( \sqrt{2}\vec{\rho} = \vec{r}_1 - \vec{r}_2 \) is a vector joining them and \( \hat{\rho} = \vec{\rho}/|\vec{\rho}| \) is a unit vector and \( A \) is an overall constant which determines the strength of the interaction. We do not need the value of \( A \) in what follows, since we do not engage in fitting spectra (but \( A > 0 \)). Nor does it matter if the value of \( A \) is too large to be interpreted as single gluon exchange. The first term is called the (Fermi) contact term and the second is the “tensor” term, but these names obscure the origin of the second term which is the ordinary dipole-dipole interaction for two separated dipoles of spin one half. Recall that the contact term only contributes when the two dipoles are in an orbital s-wave state (\( l_{12} = 0 \)), while the tensor term only contributes when the two dipoles are in an orbital state with \( l_{12} \) different from zero (\textit{unity} here). It is also important to note that these two terms are parts of the same physical interaction. In Eq. (1) one assumes that the quarks are point-like.

B. One Pion Exchange (OPE)

Here we assume that the two quarks interact by exchanging a massless pseudoscalar, the “pion”, and the coupling takes the form [6]:

\[
H_{\text{OPE}} = B\left\{ \left( -\frac{4\pi}{3} \right) \vec{S}_1 \cdot \vec{S}_2 \delta^3(\vec{\rho}) + \left( 3S_1 \cdot \hat{\rho} \cdot S_2 \cdot \hat{\rho} - S_1 \cdot \vec{S}_2 \right) \rho^{-3} \right\} \vec{\lambda}_{1,2} \cdot \vec{\lambda}_{1,2}^{\dagger},
\]

(2)

where \( B \) is another constant and \( \vec{\lambda}_{1,2} \) are the eight \( 3 \times 3 \) Gell-Mann SU(3) flavor matrices for quarks number 1 and 2. If we consider strictly pion exchange we should replace these \( 3 \times 3 \) matrices by isospin matrices \( \vec{\tau}_{1,2} \). For a pair of nonstrange quarks the difference between the two is small (see below). As noted already in the previous section, if one is interested
only in the mixing angles (and not in fitting mass spectra) the value of the constant B is immaterial, as we shall see. The main difference between Eq. (1) and Eq. (2) is the extra factor of $\vec{\lambda}_1 \cdot \vec{\lambda}_2$ in $H_{OPE}$ and the coefficient of the contact term relative to the tensor term ($8\pi/3$ versus $-4\pi/3$, a factor of minus one half to go from OGE to OPE). It is interesting to note that the coefficient of $(-4\pi/3)$ in the pion exchange case is the same as for the interaction of two electric dipoles [9]. For finite mass pions there are corrections to Eq. (2) which will be discussed elsewhere (J. Chizma, to be published).

C. Negative Parity Eigenstates

The low mass negative parity baryons are assigned to a 70-plet of $SU(6)$ which means that the spatial wavefunctions have mixed permutational symmetry (see [3, 4, 10]). In the notation we use [3], the spatial wavefunction $\psi$ has two components $\psi^\lambda$ and $\psi^\rho$ which transform under permutations of the three quarks as a two dimensional irreducible representation. The notation is explained for example in [10]. The total wavefunction $\Psi$ is a sum of products of spatial $\psi$, spin $\chi$ and flavor $\phi$ wavefunctions. For spin 3/2, the spin wavefunction is totally symmetric under permutations while for spin 1/2 there are again two states of mixed symmetry $\chi^\lambda$ and $\chi^\rho$. The flavor wavefunctions for I=1/2, also have mixed symmetry. Ignoring the color wavefunction (which is antisymmetric) the total wavefunction is totally symmetric under all permutations, and has the following forms:

$$S = 3/2 : \Psi^4P = \frac{1}{\sqrt{2}} \chi^s \{ \psi^\lambda \phi^\lambda + \psi^\rho \phi^\rho \}, \quad (3a)$$

$$S = 1/2 : \Psi^2P = \frac{1}{2} \{ \chi^\lambda \psi^\rho \phi^\rho + \chi^\rho \psi^\lambda \phi^\rho + \chi^\rho \psi^\rho \phi^\lambda - \chi^\lambda \psi^\lambda \phi^\lambda \}. \quad (3b)$$

III. COMPUTATIONS

The spin angular momentum $S=1/2, 3/2$ has to be coupled with the orbital angular momentum $L=1$ to give the total angular momentum $J=L+S$. As a result there are two states each at $J=1/2$ and $J=3/2$, namely spin doublet and spin quartet: $^2P_{1/2}, ^4P_{1/2}$ and $^2P_{3/2}, ^4P_{3/2}$. The physical eigenstates are linear combinations of these two states, and can
be obtained by diagonalizing the Hamiltonian \( H_{\text{OGE}} \) or \( H_{\text{OPE}} \) in this space of states. For example, the \( J^P = 3/2^- \) states are eigenstates of the matrix:

\[
\begin{pmatrix}
\langle 4P_{3/2} \mid H \mid 4P_{3/2} \rangle & \langle 4P_{3/2} \mid H \mid 2P_{3/2} \rangle \\
\langle 2P_{3/2} \mid H \mid 4P_{3/2} \rangle & \langle 2P_{3/2} \mid H \mid 2P_{3/2} \rangle
\end{pmatrix},
\]

where \( H \) is either \( H_{\text{OGE}} \) or \( H_{\text{OPE}} \), which are given in Eq. (1) or Eq. (2) for a single pair of quarks. The total Hamiltonian sums over all three quark pairs, and since the wavefunctions in Eq. (3) are symmetric under all permutations, we can pick a single pair of quarks \( H^{(12)} \) and multiply the result by three. The computation of these matrix elements are simple but a little tedious. We illustrate the computation for the case of \( J^P = 3/2^- \) given above, successively for both gluon and pion exchange. Before we start, we comment that the doublet-doublet matrix elements only receive contributions from the contact terms, while the doublet-quartet matrix elements only come from tensor terms. The quartet-quartet matrix element receives contributions from both tensor and contact terms. Thus the relative size of contact and tensor terms come into play. We find

\[
\langle 4P_{3/2} \mid H_{\text{OPE}} \mid 4P_{3/2} \rangle = \left( \frac{3}{2} \right) \{ \langle \chi^s \psi^\lambda \mid H^{12} \mid \chi^s \psi^\lambda \rangle \langle \phi^\rho \mid \vec{\lambda}_1 \cdot \vec{\lambda}_2 \mid \phi^\rho \rangle + \langle \chi^s \psi^\lambda \mid H^{12} \mid \chi^s \psi^\rho \rangle \langle \phi^\rho \mid \vec{\lambda}_1 \cdot \vec{\lambda}_2 \mid \phi^\rho \rangle \},
\]

where the leading factor of 3/2 comes from the number of pairs and the normalization in Eq. (3). We further take note that

\[
\langle \phi^\lambda \mid \vec{\lambda}_1 \cdot \vec{\lambda}_2 \mid \phi^\lambda \rangle = 4/3 \quad \text{and} \quad \langle \phi^\rho \mid \vec{\lambda}_1 \cdot \vec{\lambda}_2 \mid \phi^\rho \rangle = -8/3.
\]

It is amusing that these flavor matrix elements have coefficients similar to the contact interaction in pion or gluon exchange, but this is a simple numerical coincidence. We also assume harmonic oscillator spatial wavefunctions \( \psi_{1M}^{\rho,\lambda} \) in common with \([3, 6]\). Then there is a further simplification: the contact term which contains a delta function \( \delta^3(\vec{\rho}) \) vanishes in the state \( \psi^\rho \) and only receives a contribution in the state \( \psi^\lambda \). The tensor term only survives in \( \psi^\rho \), which has unit orbital angular momentum \( l_\rho = 1 \). As a result we can write:

\[
H_{\text{OGE}}(4P) = (3/2) \{ \langle \chi^s \psi^\lambda \mid H_{\text{contact}}^{12} \mid \chi^s \psi^\lambda \rangle + \langle \chi^s \psi^\rho \mid H_{\text{tensor}}^{12} \mid \chi^s \psi^\rho \rangle \},
\]
where for one gluon exchange

\[ \langle \chi^\lambda \psi^\lambda | H^{12}_{\text{contact}} | \chi^\lambda \psi^\lambda \rangle = A(8\pi/3)\langle \chi^\lambda | \vec{S}_1 \cdot \vec{S}_2 | \chi^\lambda \rangle \langle \psi^\lambda | \delta^3(\vec{r}) | \psi^\lambda \rangle, \]  
\[ = (2/3)A\alpha^3\pi^{-1/2}. \]  

(8)

Here \( \alpha \) is an oscillator parameter; the corresponding tensor term is

\[ \langle \chi^\rho | H^{12}_{\text{tensor}} | \chi^\rho \rangle = (8/15)A\alpha^3\pi^{-1/2}. \]  

(9)

Inserting Eqs. (8), (9) into Eq. (7) we obtain, (in agreement with [3])

\[ \langle 4P_{3/2} | H_{\text{OGE}} | 4P_{3/2} \rangle = (3/2)\{(2/3) + (8/15)\}, \]  
\[ = (9/5) \text{ (in units } A\alpha^3\pi^{-1/2}). \]  

(10)

Similarly for pion exchange one obtains

\[ \langle 4P_{3/2} | H_{\text{OPE}} | 4P_{3/2} \rangle = (3/2)\{(-1/2)(2/3)(4/3) + (1)(8/15)(-8/3)\}, \]  
\[ = (-14/5) \text{ (in units } B\alpha^3\pi^{-1/2}). \]  

(11)

Where the factor of (-1/2) is the change in contact term from Eq. (1) to Eq. (2). There are only two more matrix elements (for each of pion and gluon exchange), and they are

\[ \langle 2P_{3/2} | H_{\text{OGE}} | 4P_{3/2} \rangle = (10)^{-1/2}A\alpha^3\pi^{-1/2}, \]  
\[ \langle 2P_{3/2} | H_{\text{OGE}} | 2P_{3/2} \rangle = -A\alpha^3\pi^{-1/2}, \]  
\[ \langle 2P_{3/2} | H_{\text{OPE}} | 4P_{3/2} \rangle = (-8/3)(10)^{-1/2}B\alpha^3\pi^{-1/2}, \]  
\[ \langle 2P_{3/2} | H_{\text{OPE}} | 2P_{3/2} \rangle = (-7/3)B\alpha^3\pi^{-1/2}. \]  

(12a, 12b, 12c, 12d)

With these matrix elements we find for OGE, the Hamiltonian for J=3/2 to have the form

\[ \begin{pmatrix} 9/5 & 10^{-1/2} \\ 10^{-1/2} & -1 \end{pmatrix} \begin{pmatrix} 4P_{3/2} \\ 2P_{3/2} \end{pmatrix}, \]  

(13)

where we have omitted the common units \( A\alpha^3\pi^{-1/2} \). We now find the mixing \( \sin \theta_d \simeq \)
The definition we follow has the lowest energy state \( \left| E_{\text{low}} \right\rangle = \sin \theta_d \left| ^1P_{3/2} \right\rangle + \cos \theta_d \left| ^2P_{3/2} \right\rangle \). This means that the lowest eigenstate of the matrix above is: \( |J^P = 3/2^-; \text{OGE}\rangle = -0.110 |^4P_{3/2}\rangle + 0.994 |^2P_{3/2}\rangle \). We emphasize that this mixing is the same for all possible values of the constant \( A \alpha^3 \pi^{-1/2} \), whether they fit the masses or not. Similarly for \( H_{\text{OPE}} \) we have to diagonalize the matrix:

\[
\begin{pmatrix}
-14/5 & -(8/3)10^{-1/2} \\
-(8/3)10^{-1/2} & -7/3
\end{pmatrix}
\begin{pmatrix}
|^4P_{3/2}\rangle \\
|^2P_{3/2}\rangle
\end{pmatrix},
\]

(14)

With this matrix, the mixing angle \( \theta_d \) is found to be \( \theta_d = -52.7^\circ \). This means that the lowest eigenstate of \( H_{\text{OPE}} \) has the composition: \( |J^P = 3/2^-; \text{OPE}\rangle = 0.796 |^4P_{3/2}\rangle + 0.606 |^2P_{3/2}\rangle \). This is very different from the composition of the state with OGE coupling, given above. Whereas with OPE coupling the lowest 3/2\(^-\) state is about 63\% spin-quartet, with OGE it is about 1\% spin quartet. The decay data favors a 1\% contamination \cite{11}. Furthermore, \cite{6, 7} quote a mixing angle \( \theta_d = \pm 8^\circ \) for OPE which differs substantially from \(-53^\circ \). Note that had we used a coupling \( \vec{\tau}_1 \cdot \vec{\tau}_2 \) instead of \( \vec{\lambda}_1 \cdot \vec{\lambda}_2 \) the OPE composition would change slightly to \( 0.78 |^4P_{3/2}\rangle + 0.63 |^2P_{3/2}\rangle \).

We give now briefly the corresponding numbers in the \( J^P = 1/2^- \) sector, referring to the lowest energy states in units of \( A \alpha^3 \pi^{-1/2} \) (for OGE) and \( B \alpha^3 \pi^{-1/2} \) (for OPE):

\[
\begin{align*}
\text{OGE}: \left| E = -1.62 \right\rangle &= 0.526 |^4P_{1/2}\rangle + 0.85 |^2P_{1/2}\rangle; \ \theta_s = -32^\circ, \\
\text{OPE}: \left| E = -3.60 \right\rangle &= -0.43 |^4P_{1/2}\rangle + 0.903 |^2P_{1/2}\rangle; \ \theta_s = +25.5^\circ.
\end{align*}
\]

(15)\hspace{1cm}(16)

References \cite{6} and \cite{7} quote a mixing angle \( \theta_s = \pm 13^\circ \). The data \cite{11} supports a composition close to OGE and a mixing angle of \(-32^\circ \). For reference we quote the OPE Hamiltonian in the 1/2\(^-\) sector in matrix form, (in units \( B \alpha^3 \pi^{-1/2} \))

\[
\begin{pmatrix}
2 & 8/3 \\
8/3 & -7/3
\end{pmatrix}
\begin{pmatrix}
|^4P_{1/2}\rangle \\
|^2P_{1/2}\rangle
\end{pmatrix},
\]

(17)
Note that for OPE coupling the lowest lying state is predominantly (81%) spin doublet, while for OGE coupling the ground state is 72% spin doublet.

IV. SUMMARY AND DISCUSSION

In addition to the spin-spin couplings of Eqn’s (1) or (2) discussed above, spin in the negative parity baryons also couple to the orbital angular momentum. This is the spin-orbit coupling ($\vec{L} \cdot \vec{S}$). It is an empirical observation that this coupling is rather weak in negative parity nucleons, and as a result it has been neglected in some of the literature [3, 4, 6]. There is a great deal of discussion about the physical origin of this effect [7, 8]. If some spin-orbit coupling is included it will contribute to the diagonal matrix elements of the two by two matrices which we diagonalize, and can shift the mixing angles. The inclusion of this effect will however negate a parameter free determination of the mixing angles. That is, one must use the spectroscopic mass data in order to find the relative strengths of the hyperfine interaction (Eqn’s 1,2) and the spin-orbit interaction. This was done in a very preliminary manner, and we find the changes to the mixing angles to be small - less than the experimental error of 10°. The spectroscopic data utilized was the nucleon, delta ($P_{33}$ resonance) and the $D_{13}$ (low), $D_{15}$ mass splitting. With these splittings, we find that the OGE mixing angle changes from $-32°$ to $-36°$, while for OPE the angle changes from $25.5°$ to $27.5°$ (both for the $J^P = 1/2^-$ sector). For more complete results, one should attempt a reasonable fit to all states in the multiplet and this has not yet been done.

We summarize in Table I the results quoted in section III. We emphasize again that these results are independent of spectral fits to the masses of these states. The results depend only on the couplings and the wavefunctions assumed. The strength of the coupling (either A or B here) factorizes from the mixing matrices and the same mixing is obtained regardless of this coupling strength. The wavefunctions were assumed to be harmonic oscillator - appropriate for these states since they are the ground state in the negative parity sector. Moreover this assumption is common to [3, 4] and [6]. We further assumed in the couplings for OPE that the pion mass is zero; results for non-zero pion masses will be given elsewhere. They do not change significantly the numbers given in Table I.

Table I shows a substantial change from the values for OPE in the literature [6, 7]; this
| Coupling | Reference | Mixing Angle | \( \%^4P_j \) |
|----------|-----------|--------------|----------------|
| \( J^P = 3/2^- \) | OPE | 6, 7 | \( \pm 8^\circ \) | 2% |
| | OPE | this ref. | \(-52.7^\circ \) | 63% |
| | OGE | 3, 4 & this ref. | \(+6^\circ \) | 1% |
| | EXP. | 11 | \(+10^\circ \) | 3% |
| \( J^P = 1/2^- \) | OPE | 6, 7 | \( \pm 13^\circ \) | 5% |
| | OPE | this ref. | \(+25.5^\circ \) | 19% |
| | OGE | 3, 4 & this ref. | \(-32^\circ \) | 28% |
| | EXP. | 11 | \(-32^\circ \) | 28% |

The change is relevant since the error of the “experimental” value is of the order of 10° [11], and the preference for the OGE solution is now unambiguous. It has been argued [8] that the addition of vector meson exchange to pseudoscalar exchange will remedy this problem. That may indeed be the case, but one should recall that the primary controversy is whether the quark coupling in baryons is OGE or OPE, and the data answers this question unequivocally. One may just as well argue that atomic hyperfine interactions - which has the same form as Eq. (1) - is really due to the superposition of a pseudoscalar and massive vector field, rather than a massless gauge field. Similar mixings for OPE have also been obtained elsewhere [12], however the emphasis on the independence from spectral data is missing.

Finally, there have been comments on the issue of color versus flavor exchange. In particular, [13] fits the mass spectrum in the \( L=1 \) sector in a rather ingenious way, using only permutation symmetry and SU(6), with a number of free parameters, essentially reduced matrix elements. But by treating the matrix elements corresponding to the contact and tensor terms as independent parameters, one sidesteps the controversy between vector exchange [1, 3, 4] and pseudoscalar exchange [6]. In addition, as noted, if one has well defined Hamiltonians and wavefunctions these mixings are independent of mass fits. Although not in these precise words, similar conclusions are stated by the authors of [13].
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