Multifractal Properties of Temporal Rainfall Series in Peninsular Malaysia

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Abstract. Studies on the investigation of scale-invariance of rainfall are still in the early development stage in the humid tropical regions. In order to further enhance understanding of the rainfall variability across a range of temporal scales in these regions, this study aims to investigate and characterize rainfall in a multifractal framework from areas in Peninsular Malaysia. It is done by studying the scaling of the statistical properties of rainfall. Data set consisting of 15 min rainfall observations from 56 rain gauge stations were adopted and the stations were grouped into four regions to identify the possibility that the multifractal properties of rainfall could depend on the geographical locations and local climatology. The results show that the temporal structure of rainfall in Peninsular Malaysia displays a multifractal behaviour for scales ranging from 15 min to about 3 days. The multifractal parameters that described the scaling properties of rainfall in terms of intermittency and multifractality revealed to be different for each region in the peninsula, particularly between the west and east coasts. The rainfall process for east coast was characterized by less intermittent rain with the rainfall fluctuations displaying a strong degree of multifractality. While for the west coast which consists of northwest, west and southwest regions, they exhibited a similar distribution of rainfall occurrences which is very intermittent and unsmoothed. Rainfall fluctuations, however, were spikier in northwest regions than in the west and southwest regions.

1. Introduction
Rainfall is a complex atmospheric process that displays enormous variability over a wide range of scales in space and time. This variability is the consequence of several characteristics. Among them are intermittency (the occurrence of rain and no rain), rain intensity, rain and drought extremes, and multiple scaling regimes [1]. Due to the complexity process, rainfall is hard to be analysed and modelled. Undoubtedly, understanding and quantifying the temporal (and spatial) nonlinear variability of rainfall, especially at small scales, are crucial tasks as its knowledge can be utilized for the purposes in water-related sectors such as water resource management, water infrastructure design, hydrology, and agriculture.

One of the ways to explore all features of rainfall variability is using scale-invariance properties and multifractality that are manifested in the structure of rainfall process over a range of scales [2]. Rain process is said to exhibit scale-invariance if the structures at different scales, on large and small scales, are related by the same scaling law without showing any preferred mode. The invariance of
properties can be described by multifractal theories that are based on the concept of multiplicative cascade process. The existing of this process in natural phenomena such as rain, cloud, temperature, and wind have long been believed to provide a primary mechanism of injection of energy flux from larger to smaller scales. The attractive feature of the multifractal approach is that it can provide a parsimonious description of statistical properties of complex geophysical data at all orders and over a wide range of scales in terms of a few parameters [3,4].

Study on characterizing scale properties of rainfall process is continuously being explored by many researchers in the field to get a better understanding of rainfall variability at different scales and in different climate and topographic conditions [5–9]. Nonetheless, it seems that practical applications of multifractals are still in an early development stage for some areas. For instance, only a few studies are concentrated on the humid tropical regions [10–13]. Because of this, more research on the multifractal behaviour of rainfall series is still needed and should be continued, in particular rainfall process from areas in Peninsular Malaysia. Within the present study areas, the climate differs greatly with geographical location, especially between the west coast and east coast of the peninsula and slightly less so between the north and south [14]. The variation of annual rainfall observed across the country and together with the marked seasonal rainfall regime, has given a strong impact on the environment and society such as floods, droughts, soil erosion or spread of pollutants. In view of this, the present study aimed to explore and characterize the scale-invariant properties that best describes the dynamics of rainfall regime in Peninsular Malaysia. By characterizing the rainfall behaviour, not only would this improve the understanding of the hydrological regime in the peninsula [4,15,16] but also be useful for the data generation purposes [17–20] as well as for regionalization that can solve the problem of scarce temporal rainfall data [21,22].

With this purpose in mind, a rainfall dataset consisting of 15 min rainfall observations for the periods ranging from 18 to 42 years is analyzed. As differences of local rainfall regime are observed within the country, the selected 56 stations will be divided into four topographical areas of the peninsula. The multifractal behaviour for each region will then be investigated and characterized by studying the scaling of the statistical moments of rainfall in a multifractal framework. A model proposed by Schertzer and Lovejoy [23] called Universal Multifractal model is adopted to describe the scaling statistical properties of rainfall in terms of rainfall intermittency and its multifractality. The present study is also attempted to connect the multifractal temporal structure of rainfall with the physical phenomena that influenced the distribution of Peninsular Malaysia rainfall.

2. Materials and Methods

2.1. Data and Study Area

The rainy season in Peninsular Malaysia is influenced by two main monsoon seasons, namely the southwest monsoon and northeast monsoon which occur in May to August and November to February, respectively. During the southwest monsoon, the whole country experiences a drier period, while the northeast monsoon is a wet season with the more severely affected areas are the east coast of the peninsula. However, during the two inter-monsoon seasons, that is in April and October, the west coast receives more rain than the east coast. The rain has usually occurred in the form of convective rain.

The collection of 15 min rainfall data from 56 rain gauge stations over Peninsular Malaysia were grouped into four regions, namely northwest, west, southwest, and east. The datasets have been obtained from Malaysian Meteorological Services and Drainage and Irrigation Department for the duration of 18 to 42 years with most of the records were greater than 22 years. The study regions along with the locations of 56 selected rainfall stations are mapped onto figure 1.
2.2. Multifractal Analysis

Most of the geophysical processes including rainfall are non-conservative or non-stationary. Multiplicative cascade models are, however, built to generate conservative fields or series. In the present study, an adequate approximation to fractional differentiation is adopted to produce a stationary field by taking small-scale fluctuations of the original field [24–26]. This approach consists of taking $\varepsilon_\lambda$ as the absolute value of rainfall fluctuations of at fine resolution $R_\lambda$ and normalizing it to have $(\varepsilon_\lambda) = 1$, equation (1),

$$\varepsilon_\lambda = \frac{|R_\lambda(i + 1) - R_\lambda(i)|}{E(|R_\lambda(i + 1) - R_\lambda(i)|)}$$

Then $\varepsilon_\lambda$ is obtained by consecutive averaging of pairs of contiguous values. Here $\lambda$ is defined as the scale ratio $L/l$, a ratio between the scale $L$ chosen for studying the phenomenon to any time interval $l$ (commonly, the time interval $l$ increased gradually from fine resolution to scale $L$ by a factor of 2). The presence of multifractality in rainfall time series can be investigated by studying its statistical moments. Moments at different time scales (i.e., on large and small-time scales) are related by a scale ratio $\lambda$ [4]. The statistical moments $M_\lambda(q)$ of (arbitrary) order $q$ were estimated for each time scale of resolution $\lambda$ as $E(\varepsilon^q_\lambda)$. The rainfall fluctuations are said to be multifractal if the following relation holds for a range of scales without a characteristic scale as shown in equation (2):

$$M_\lambda(q) = E(\varepsilon^q_\lambda) \approx \lambda^{K(q)}$$

where $K(q)$ is the convex-shaped moment scaling function of $q$ that signify the multifractality of the rainfall process, $K(q)$ can be obtained by estimating the slope $M_\lambda(q)$ against $\lambda$ in a logarithmic domain for various moment orders $q$.

The moment scaling function $K(q)$ can be theoretically expressed with the help of Universal Multifractal model (UM model) [23]. This model has a multiplicative cascade structure and is based
on log-Levy stable random variables [4], where only two parameters are needed to describe the \( K(q) \) function and statistical properties of a conservative multifractal field. The moment scaling function can be modelled as shown in equation (3):

\[
K(q) = \begin{cases} 
C_1(q^\alpha - q)/(\alpha - 1), & \alpha \neq 1 \\
C_1 q \log q, & \alpha = 1
\end{cases}
\]  

Both parameters of \( K(q) \) function have a physical significance. The parameter \( C_1 \in [0,D] \) \((D = 1\) for rainfall time series) is the co-dimension parameter that describes the degree of intermittency or sparseness of the mean of the process. The value of \( C_1 \) is a good first-order measure of the overall intermittency of the time series [27]. The value of \( C_1 \) increases as the values in the rainfall fields deviate from the mean value of the process in which majority of the values are higher and lower than the mean value. The parameter \( \alpha \in [0,2] \) is the multifractality index that relates to the spikiness or presence of extreme fluctuations in the rainfall series. This index indicates the degree of deviation of the rainfall series from monofractality wherein the rain process is said to exhibit a monofractal behaviour when \( \alpha = 0 \). A large value of \( \alpha \) implies that the rainfall series are dominated by high-valued and localized spikes; on the contrary, a smaller value of \( \alpha \) means that less frequency of extreme rainfall values and less domination of the high values since they are spread throughout the field. For case \( \alpha \neq 1 \) (equation (3)), since the parameter \( \alpha \) is in the numerator and \( C_1 \) is in the denominator, the parameters act in opposite directions. Small values of \( \alpha \) and/or large values of \( C_1 \) are representative of tendencies to large fluctuations in the data; large values of \( \alpha \) and/or small values of \( C_1 \) are representative of greater temporal homogeneity [28]. The parameters \( C_1 \) and \( \alpha \) can be estimated using the Double Trace Moment (DTM) technique [29].

3. Results and Discussions

3.1. The Scaling Regime

The analysis was carried out for a range of scales from 15 min (\( \lambda = 2048 \)) to approximately 21 days (30720 min, \( \lambda = 1 \)) since the aim of this study is to investigate the presence of multifractality from the observation scale up to a few days only.

Figure 2 shows the log-log plots of curves \( M_{\lambda}(q) \) against \( \lambda \) using values of \( q \) between 0 and 3 with four stations coded as N10, W09, S06, and E05 selected to represent each region in Peninsular Malaysia; northwest, west, southwest, and east, respectively. As indicated by the well fitted straight lines in Figure 2, the moments manifested scale invariance of properties for time scale spanning from 15 min (\( \lambda = 2048 \)) to about 3 days (\( \lambda = 8 \)). Similar plots were also observed for other stations in each region. However, \( M_{\lambda}(q) \) curves for station E05 exhibited a well-defined straight-line behaviour over scaling regime from 15 min to about 3 days only for moments \( q < 2.5 \). The departure of the straight lines over the entire scaling regime for higher order moments is due to the effect of large-scale storms with heavy and long duration rainfall that are prevalent in the east coast rainfall. The high order empirical moments have influenced by large observations and are, therefore highly variable (Kumar et al. 1994).

3.2. The Existence of Multifractal Behaviour

The moments observed for scaling regime between 15 min to ~3 days were then fitted with power laws in equation (2) by using linear regression to obtain the corresponding scaling exponents \( K(q) \). The slightly convex appearance of the empirical functions of \( K(q) \), as can be seen in figure 3, shows that a multifractal behaviour is presented in the rainfall fluctuations of Peninsular Malaysia. Stations N10, W09, and S06 showed similar shapes of the empirical scaling exponent functions \( K(q) \) but, the curve of \( K(q) \) for station E05 has deviated from the rest of the stations, particularly for higher order moments.
Figure 2. An example of scaling regimes obtained through moment scaling analysis for stations (a) N10, (b) W09, (c) S06, and (d) E05.

Figure 3. An example of empirical and theoretical $K(q)$ of rainfall fluctuations for stations with code N10, W09, S06, and E05. The points denoted the empirical $K(q)$ and the solid lines denoted theoretical $K(q)$. 
As previously observed in figure 2, we can notice the differences in slopes of empirical moments between station E05 and the other three stations. Station E05 exhibited a lower value of $K(3)$, for instance, as compared to stations N10, W09, and S06 (Figure 3). A low value of $K(3)$ indicates the smooth signal of rainfall intensity with small fluctuations across the scaling regime while the high-valued $K(3)$ obtained when the intensities are more variable and intermittent, with the presence of uneven peaks [9,16]. The heavy and smooth rainfall observed in the east region, which mainly contributed by rainfall during the northeast monsoon, has produced lower values of the empirical $K(q)$ at higher order moments. As a result, the steepness of the slope of the $K(q)$ function for station E05 is lesser than the slope of the other three regions in the peninsula, rain of which is dominated with weak rainfall and high intermittent. These findings are in agreement with results of Over and Gupta [30] who observed that the changes of the slope of $K(q)$ are dependent on the large-scale forcing, defined as average rain rate, where a larger rain rate will result in a less steeply slope of $K(q)$.

3.3. The Multifractal Properties
The parameters of UM model, $\alpha$ and $C_1$ that describe the behaviour of $K(q)$ are estimated using the Double Trace Moment method. The values of intermittency index $C_1$ for rainfall series in Peninsular Malaysia are in the range from 0.47 to 0.67 and the degree of multifractality, as expressed by $\alpha$, ranged from 0.24 to 0.59. The high-valued $C_1$ indicates a non-smooth rainfall field in the peninsula for the scales from 15 min to ~3 days. This could be reasoned by the high frequency of short rainfall events with dry periods within the day. The average duration of rain events for peninsula is 4.5 hours, of which is the characteristic of convective rainfall. Past studies observed that the values of $C_1$ tend to be higher for rainfall with predominant convective than frontal rainfall [6,31].

3.4. The Spatial Distribution of Multifractal Properties
As the next step to further understand the influence of topographic features of each region in Peninsular Malaysia to the multifractal temporal structure of rainfall process, the present study investigated the presence of spatial patterns of multifractal parameters for the scaling regime from 15 min to ~3 days. The spatial distributions of multifractal properties were obtained by applying the kriging technique on the peninsula by using information from the selected 56 stations. Figure 4 displays the spatial maps of parameters $\alpha$ and $C_1$, respectively. As expected, the multifractal properties vary strongly with location, in particular between the west and east coast of the peninsula.

An inspection of the maps in figure 4 (left) reveals that eastern region is characterized by high and moderate values of $\alpha$ ranged between 0.46 and 0.54 and 0.38 and 0.46, respectively. The highest $\alpha (>0.54)$ is observed in the eastern region of latitudes between 4.2º and 5º, while the lowest values of $\alpha$ are found in the west and southwest regions of latitudes between 1.3º and 3.7º and longitudes between 101º and 103.5º. Most part in the northwest region exhibited $\alpha$-values between 0.33 and 0.38 and a small area in this region between latitude 5.6º and 6º are characterized by moderate degree of multifractality. On the contrary, the intermittency index $C_1$ is high in west and southwest regions with values ranged between 0.62 and 0.67, while the east region exhibited a low value of $C_1$ (<0.57). Areas between latitudes 1.4º and 2.3º in the southwest region is found to have the highest value of $C_1$ (>0.67). Also note that the multifractal temporal structure for central part of Peninsular Malaysia is characterized by moderate values of parameter $\alpha$ and $C_1$.

From figure 4, we can say that a part of northwest region practically shared a similar multifractal behaviour with regions in the west and southwest where the rainfall occurrences are sparsely distributed in time and the fluctuation of rain is less spiky, while the other part of it is characterized by a lower intermittency and a stronger multifractality which is similar to rainfall behaviour in central region and upper part of east region. Rain process in east region with predominant persistent and smooth rainfall events is characterized by lower parameters of $C_1$ and higher values of parameter $\alpha$. This implies that rain in this region is less intermittent than rain in other regions and with a steadily varying background field punctuated by occasional large spikes.
4. Conclusions

The characteristics of the multiscale variability of rainfall time series in Peninsular Malaysia was investigated through the analysis of fine resolution, 15 min rainfall data recorded by 56 rain gauge stations located in four topographical regions, namely northwest, west, southwest, and east for length of data ranged between 18 and 42 years. The rainfall variability was characterised in a multifractal framework by describing the observed rainfall series as the outcome of multiplicative cascade processes. A statistical moment analysis was employed to analyse the multifractal behaviour in rainfall series and the behaviour was then modelled by Universal Multifractal (UM) model. The two parameters of UM model described the multifractal behaviour of rainfall processes associated with rainfall intermittency (i.e. the variability of rainfall support), $C_1$ and the degree of multifractality, $\alpha$.

From the moment scaling analysis, the scale invariance of rainfall intensity fluctuations in Peninsular Malaysia is found to hold from 15 min to about 3 days. The empirical $K(q)$ exhibits a nonlinear behaviour implies that rainfall fluctuations in the peninsula were well represented by a multifractal behaviour. The fitting parameters of UM model showed that the rainfall process in the peninsula is characterized by moderate- to high-valued intermittency index $C_1$ and low- to moderate-valued multifractality index $\alpha$. A high value of $C_1$ indicates the peninsula rainfall fields are sparsely scattered for scales between 15 min and ~3 days. The dominance of convective storms in rainfall events across peninsular, in particular regions along the west coast of peninsular, lead to high intermittency.

Comparing each region in Peninsular Malaysia, it is obvious that the characteristics of rainfall strongly differ associated with the geographical and topography as well as the local climatology and monsoon winds. The occurrences of rain and no rain in west and southwest regions are sparser and less continuous for scales from 15 min to about 3 days. On the other hand, the rainfall fields in the east region are characterized by smaller intermittency and sparseness and the multifractality of rain is stronger as they experienced more frequent of rainfall extremes and with longer durations. Rainfall series in the northwest region were shown to be much intermittent similar to data series in west and southwest regions but the series were more fluctuated. The finding could be due to the exposure of this region to the southwest monsoon wind influence while west and southwest regions are shielded by the mountain ranges in Sumatra during this monsoon season. On another note, the existence of the Titiwangsa mountain range, which blocks the north easterly wind from arriving northwest, west and southwest regions could explain the large difference between the east coast and west coast of peninsula in the context of intermittency and multifractality of rainfall.
The dynamics of rainfall characterized by the multifractal approach does enhance our understanding of the hydrological regime in Peninsular Malaysia. The gained information will help us to build practical tools to improve the rainfall data input, particularly at fine resolution, for hydrological models and consequently, the accuracy and reliability of the output may significantly increase.

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