Joint timing-offset and channel estimation for physical layer network coding in frequency selective environments

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Abstract
This paper considers the problem of joint timing-offset and channel estimation for physical-layer network coding systems operating in frequency-selective environments. Three different algorithms are investigated for the joint estimation of the channel coefficients and the fractional timing offset. The first algorithm is based on the maximum-likelihood (ML) criterion assuming baud-rate (BR) sampling. The second algorithm also assumes BR sampling and is based on the special properties of Zadoff-Chu training sequences. In the third algorithm, oversampling at double the baud-rate (DBR) is used and the least-squares (LS) estimation criterion applied. While the above algorithms assume that the integer timing offset is known, three generalized-likelihood-ratio tests (GLRTs) are also considered for integer offset error correction that integrate very well with the proposed estimation algorithms. Our simulation studies show that the DBR-LS estimator provides the highest estimation accuracy, significantly outperforming both BR estimators and performing very close to the corresponding Cramer–Rao bound. A gain of 4 dB is observed in symbol-error-rate performance using the DBR-LS algorithm. The DBR-GLRT also provides substantially higher probability of error correction.

1 INTRODUCTION

Physical-layer network coding (PLNC) [1] is an important communication paradigm that continues to attract the interest of many researchers. The appeal of PLNC lies in exploiting the superposition of electromagnetic waves and the broadcast nature of wireless channels during its multiple access phase and broadcast phase, to achieve superior spectral efficiency. PLNC is capable of achieving 100% improvement in throughput over traditional scheduling that is based on point-to-point transmissions, and 33% improvement over conventional network coding schemes [2]. The PLNC framework provides an attractive solution to meet the demands of various applications envisaged for 5th generation (5G) wireless networks. In particular, PLNC can play an important role in facilitating efficient device-to-device (D2D) communication [3, 4], where the numerous inactive users can be exploited as relays to extend the range of D2D communication. Other promising applications of PLNC include multi-way relaying [2], cognitive relay networks [5] and visible light communication (VLC) [6].

Asynchronous PLNC, where the transmissions of the two sources are not perfectly synchronized, has also attracted the attention of researchers [7–10]. Perfect timing synchronization may be difficult to achieve in practice, and the presence of a timing offset can result in appreciable performance penalties in terms of bit-error rate (BER) [7]. The timing offset may be difficult to avoid in some applications, especially, Internet-of-Things (IoT) applications involving internet-based processing entities or networks constructed with basic software-defined radio technology [8]. A general framework for decoding in asynchronous PLNC was proposed in [7] based on belief propagation. The decoding of convolutional codes for asynchronous PLNC was studied in [11]. Asynchronous multiuser relay communication was studied in [9] with appropriate relay selection strategies. While the aforementioned works considered the impact of the presence of timing offsets, they assumed that these timing offsets are perfectly known at the relay.

Timing-offset estimation in PLNC systems was investigated in [12, 13], where pilot-based estimation algorithms were developed using constant amplitude zero-autocorrelation (CAZAC
sequences [14] with both baud-rate (BR) and double-baud-rate (DBR) estimation algorithms. These works focused only on estimating the timing offset, assuming that the channel coefficients were perfectly known.

Joint channel and timing-offset estimation for PLNC was studied in [15, 16]. In [15], only integer offset estimation was considered, while fractional timing offset was ignored. Least-squares (LS) estimation was used to obtain channel estimates for each candidate timing offset. An exhaustive search was then employed to obtain the LS integer offset estimate. A discrete Fourier transform (DFT)-based channel and delay estimation method for asynchronous PLNC was proposed in [16]. The DFT properties were exploited to design pilot sequences that maintain their orthogonality in the frequency domain. The channel coefficients were estimated in the frequency domain, and then inverse DFT was applied and the relative delay was extracted from the estimated time-domain channel information. Joint timing-offset and channel estimation was also investigated for amplify-and-forward (AF) two-way relay networks (TWRNs) in [17] using both pilot-based and semi-blind estimation techniques. Unlike PLNC, however, in AF TWRNs the estimation takes place at the terminals, which changes the nature of the estimation problem due to the presence of self-interference.

The aforementioned estimation algorithms in [12, 13, 15, 16] all assume flat-fading channels. When the channels are frequency selective, the joint estimation problem becomes significantly more challenging, with intersymbol interference (ISI) due to both the timing misalignment and the time-dispersive nature of the channel. OFDM-based PLNC Receiver architectures were proposed in [18] to handle frequency selective conditions, assuming that the channel coefficients are known and that the relative timing offset of the two sources is shorter than the cyclic prefix (CP). Moreover, channel estimation and carrier frequency offset (CFO) compensation schemes for OFDM-PLNC systems were proposed in [19]. While OFDM can be employed to combat timing-offset errors, in OFDM the symbol offset in the time domain is transformed to a phase shift as long as it is within the cyclic prefix. However, as pointed out in [15], OFDM-PLNC is very sensitive to CFOs, unlike point-to-point systems, CFOs in OFDM-PLNC systems cannot be fully compensated at relay even when the CFOs of both terminals are perfectly known [19], which can lead to performance degradation.

This work addresses the problem of joint timing-offset and channel estimation for PLNC systems operating in frequency selective environments. We investigate three different algorithms for estimating the channel coefficients and the fractional timing offset, offering different tradeoffs between estimation accuracy and computational complexity. The first two algorithms assume BR sampling, while the third uses DBR sampling. The first algorithm is the pilot-based maximum-likelihood (ML) joint timing-offset and channel estimator. The second algorithm assumes that the pilot sequences are Zadoff-Chu (ZC) sequences and exploits their special properties to estimate the channel and timing-offset parameters. Although this algorithm offers lower computational complexity than the ML estimator, it imposes some constraints on the required number of pilots and pilot structure, and it is prone to error floors when these constraints are not satisfied. In the third algorithm, we use oversampling in the form of DBR sampling to extract more information from the continuous-time signal. LS estimation is then applied to estimate the channel and timing-offset parameters.

The above three estimation algorithms assume that the integer timing offset is known and focus on jointly estimating the channel coefficients and the fractional timing offset. The integer timing offset can be estimated using the methods developed in [12]. As noted in [12], however, it is common for integer timing-offset estimation to encounter an error of ±1. To address this issue, we investigate three generalized-likelihood-ratio tests (GLRTs) that can be employed to correct this error, and that integrate very well with the three proposed joint estimation algorithms.

We use simulations to investigate the performance of the proposed algorithms. Our simulations show that the BR-based ML estimator slightly outperforms the BR-based ZC estimator in terms of estimation accuracy. The DBR-based estimator, however, provides substantially better estimation accuracy than both BR estimators, with up to 6 dB improvement in the mean-squared-error (MSE) for the estimation of the channel coefficients. In terms of integer offset error correction, the DBR GLRT also provides significantly better correction performance. The symbol-error-rate (SER) performance of the three estimators is also investigated assuming DBR sampling for data detection, and a minimum-mean-squared-error (MMSE) linear receiver. The DBR estimator yields a gain of approximately 4 dB over the two BR estimators.

The rest of the paper is organized as follows. In Section 2, we present our system model for both BR and DBR sampling. The BR-based joint estimation algorithms are developed in Section 3. The DBR-based LS estimation algorithm is presented in Section 4. The computational complexities of the three algorithms are discussed in Section 5. Methods for integer offset error correction are investigated in Section 6. Data detection is discussed in Section 7. Our simulation results are presented in Section 8. Finally, the conclusions are in Section 9.

**Notation**

For a matrix $\mathbf{G}$, the transpose and Hermitian are denoted by $\mathbf{G}^T$, $\mathbf{G}^\dagger$, respectively. For a vector $\mathbf{v}$, $\|\mathbf{v}\|$ denotes the two-norm, while $\ell[\mathbf{v}]$ denotes the $\ell$th element. For a complex number $\mathbf{c}$, $\mathbb{R}[\mathbf{c}]$, $\mathbb{I}[\mathbf{c}]$, $\Re\{\mathbf{c}\}$, $\Im\{\mathbf{c}\}$, $\Re$, $\Im$ and $|\mathbf{c}|$ represent the real part, the imaginary part, the conjugate, the phase and the magnitude of $\mathbf{c}$, respectively. $\mathbf{I}_N$ is the $N \times N$ identity matrix. The notation $\mathcal{C}\mathcal{C}\mathcal{N}(0, \sigma^2 \mathbf{I}_N)$ describes a circularly complex Gaussian random vector with mean zero and covariance matrix $\sigma^2 \mathbf{I}_N$. We use $\lfloor \cdot \rfloor$ to denote the floor of a real number. The symbol $*$ is used to represent continuous-time convolution.

2 | **SYSTEM MODEL**

We consider a PLNC system consisting of three single antenna half-duplex nodes: two terminals $\mathcal{T}_1$, $\mathcal{T}_2$ and a single relay $\mathcal{R}$,
shown in Figure 1. The terminals \(T_1\) and \(T_2\) exchange information bidirectionally through the help of \(R\), a process that is completed in two phases. In the first phase, both terminals transmit their information symbols to the relay. The relay processes the received signal, decodes the information symbols of both terminals and then re-encodes them and broadcasts the resulting signal to both terminals in the second phase.

The symbol period is denoted by \(T_s\) and \(\rho'(t)\) denotes the pulse-shaping function for the baseband signal. We assume that a rectangular pulse is employed, that is,

\[
\rho'(t) = \begin{cases} 
1, & 0 \leq t \leq T_s \\
0, & \text{otherwise}. 
\end{cases}
\]  

(1)

Because there is no perfect synchronization, the symbols transmitted by the two terminals do not arrive at the relay simultaneously. We assume that the signal transmitted by \(T_1\) arrives at \(R\) ahead of that transmitted by \(T_2\) by a timing offset \(\tau > 0\). The offset \(\tau\) can be expressed as

\[
\tau = n_\tau T_s + \lambda, 
\]

(2)

where \(n_\tau = \left\lfloor \frac{\tau}{T_s} \right\rfloor\) is the integer timing offset, and \(\lambda\) is the fractional timing offset, which satisfies \(0 < \lambda < T_s\).

As in [15] and [16], we focus on estimating the relative delay between the two terminals, assuming optimal sampling time for the first received signal\(^1\). We adopt the symbol-spaced tap-delay-line model to characterize the frequency selective channels \(T_1 \rightarrow R\) and \(T_2 \rightarrow R\). We also assume a slow-fading channel, such that the channel gains for different paths remain unchanged during the pilot and data transmission. For simplicity, and without loss of generality, we assume that the two channels have the same length. We denote by \(h \triangleq [h[0], \ldots, h[L]]^T\) and \(g \triangleq [g[0], \ldots, g[L]]^T\) the complex coefficients representing the channels \(T_1 \rightarrow R\) and \(T_2 \rightarrow R\), respectively.

Before data transmission, each terminal broadcasts a sequence of \(N\) pilot symbols, which are used to estimate the channel coefficients and the timing offset. The received signal at \(R\) is processed through a matched filter. The pilot-carrying continuous-time received signal at the relay after matched-filtering is given by

\[
\mathbf{z}(t) = \sum_{k=0}^{L} \sum_{m=1}^{N} h[k] s_1[m] \rho(t - (m + k - 1)T_s) + \sum_{k=0}^{L} \sum_{m=1}^{N} g[k] s_2[m] \rho(t - (m + k - 1)T_s - \tau) + \omega(t),
\]

(3)

where \(s_1[m]\) and \(s_2[m]\) represent the transmitted pilot symbols of terminals \(T_1\) and \(T_2\), respectively, and are non-zero for \(1 \leq m \leq N\), and zero otherwise. Moreover, \(\rho(t) \triangleq \rho'(t) * \rho'(-t)\) is the effective pulse-shaping filter after matched-filtering at the relay, and \(\omega(t) \triangleq \omega'(t) * \rho'(-t)\) is the noise process at the output of the matched-filter, whereas \(\omega'(t)\) is the continuous-time additive white Gaussian noise (AWGN) process at the input of the matched filter.

Since the rectangular pulse is not bandlimited, the BR sampled signal does not capture all the information embedded in the continuous-time signal. Thus, oversampling can be used to extract more information from the continuous-time signal and to enhance the performance of channel estimation and data detection. Hence, we will consider both BR sampling and DBR sampling in our work.

2.1 BR Sampling

Assuming BR sampling, the samples are acquired at \(t = nT_s\) where \(n \in \mathbb{Z}\). It can be shown that the resulting information-carrying discrete-time samples are given by

\[
\mathbf{z}[n] = \sum_{k=0}^{L} h[k] s_1[n + 1 - k] + \sum_{k=0}^{L} \left( g[k] \rho(-\lambda) s_2[n + n_\tau - k + 1] + p(T_s - \lambda) s_2[n - n_\tau - k] \right) + \omega[n],
\]

(4)

for \(n = 0, \ldots, N + n_\tau + L\), where \(p(-\lambda) = \frac{T_s - \lambda}{T_s}\), \(p(T_s - \lambda) = \frac{\lambda}{T_s}\) and \(\omega[n]\) is AWGN with mean zero and variance \(\sigma^2\). The assumption that the noise \(\omega[n]\) at the output of the BR sampler is white is commonly used in the PLNC literature. We note, however, that in practice BR-sampling is sensitive to spectral nulls, which may compromise the whiteness of the noise. We let \(N_t \triangleq N + n_\tau + L + 1\) be the total number of information-carrying discrete-time samples.

Letting \(\mu \triangleq \frac{\lambda}{T_s}\), and defining the received signal vector \(\mathbf{z} \triangleq [z[0], \ldots, z[N_t - 1]]^T\), Equation (4) can be rewritten in matrix notation as

\[
\mathbf{z} = \mathbf{S}_1 \mathbf{h} + (1 - \mu) \mathbf{S}_2 \mathbf{g} + \mu \mathbf{S}_2 \mathbf{g} + \mathbf{w},
\]

(5)

\(^1\) In practical systems, a sampling offset may exist with respect to the optimal sampling time as discussed in [18]. A discussion of the impact of the sampling offset can be found in [18].
where $S_1$ is the $N_t \times (L + 1)$ matrix whose $k$th column is $s_{1,k} \triangleq [0_{1\times(k-1)} , \tau_k , 0_{1\times N-1-k} ]^T$, $S_2$ is the $N_t \times (L + 1)$ matrix whose $k$th column is $s_{2,k} \triangleq [0_{1\times (n+k-1)} , \tau_k , 0_{1\times (N-N-k-n_t+1)} ]^T$ and $S_3$ is the $N_t \times (L + 1)$ matrix whose $k$th column is $s_{3,k} \triangleq [0_{1\times (n+k-1)} , \tau_k , 0_{1\times (N-N-k-n_t)} ]^T$. Finally, $\omega$ is the $N_t \times 1$ AWGN vector with mean $0$ and covariance matrix $\sigma^2 I_{N_t}$.

### 2.2 | DBR sampling

In DBR sampling, the samples are acquired from $z(t)$ at $t = nT_s / 2$, where $n \in \mathbb{Z}$. The resulting discrete-time signal is given by

$$ z^{(d)}[n] = z(nT_s / 2) = \sum_{k=-\infty}^{\infty} \sum_{m=1}^{N} \left[ b[k] r_1[m] \rho\left( nT_s / 2 - (m + k - 1) T_s \right) + g[k] s_2[m] \rho\left( nT_s / 2 - (m + k) T_s - \tau \right) \right] + \omega(nT_s / 2). $$

We can separate the above samples into two groups that correspond to even values of $n$ ($n = 2\ell$, $\ell \in \mathbb{Z}$) and odd values of $n$ ($n = 2\ell + 1$, $\ell \in \mathbb{Z}$). The samples for even values of $n$ coincide with the BR samples of Equations (4) and (5), since $z^{(d)}[2\ell] = z[\ell]$.

On the other hand, for odd $n$, we have

$$ z^{(d)}[2\ell + 1] = \sum_{k=-\infty}^{\infty} \sum_{m=1}^{N} \left[ b[k] r_1[m] \rho\left( \ell - (m + k - 1) T_s + T_s / 2 \right) + g[k] s_2[m] \rho\left( \ell - m + k - 1 + n_t T_s + T_s / 2 - \lambda \right) + \omega(\ell T_s + T_s / 2) \right]. $$

In order to simplify Equation (7), we need to consider when the effective matched-filter components affecting the symbols $s_1[m]$ and $s_2[m]$ are non-zero. Given that $\rho(\ell)$ is triangular for the duration $-T_s \leq \ell \leq T_s$ and zero otherwise, the only possible non-zero samples of the filter affecting $s_1[m]$ (for odd $n$) are $\rho(-\frac{T_s}{2}) = \frac{1}{2}$ and $\rho\left( \frac{T_s}{2} \right) = \frac{1}{2}$, which correspond to $\ell = m + k - 1$ and $\ell = m + k - 2$, respectively.

Now consider the matched-filter components affecting $s_2[m]$. We let $\kappa = \ell - m + k - 1 + n_t$, such that the filter component under consideration is $\rho(\kappa T_s + T_s / 2 - \lambda)$.

The values of $\kappa$ for which the matched-filter can be non-zero are $\kappa \in \{-1, 0, 1\}$. In particular, we have $\rho\left( \frac{T_s}{2} - \lambda \right)$ for $\kappa = 0$, $\rho\left( \frac{T_s}{2} - \lambda \right)$ for $\kappa = -1$ and $\rho\left( \frac{3T_s}{2} - \lambda \right)$ for $\kappa = 1$.

Based on the above, we have that two distinct cases for the discrete-time received signal model for odd $n$, depending on the value of $\lambda$. In particular, if $0 < \lambda < T_s / 2$ then

$$ \rho\left( \frac{T_s}{2} - \lambda \right) = \frac{1}{2} + \mu, \quad \rho\left( \frac{3T_s}{2} - \lambda \right) = 0, $$

In this case, the number of information-carrying samples for odd $n$ will be $N_t$, that is, the same as the case for even $n$.

On the other hand, when $T_s / 2 < \lambda < T_s$, we have

$$ \rho\left( \frac{T_s}{2} - \lambda \right) = \frac{3}{2} - \mu, \quad \rho\left( \frac{3T_s}{2} - \lambda \right) = 1 - \mu, $$

Hence, if $0 < \lambda < T_s / 2$ then $y^{(d)} = y$, and if $T_s / 2 < \lambda < T_s$, then $y^{(d)} = y + \omega$. In this case, the number of information-carrying samples for odd $n$ will be $N_t + 1$.

To distinguish between the signal models in both cases, we let $y^{(d)}[\ell] = \rho(\ell) \triangleq \rho^{(d)}[\ell] + 1$, and define the vectors $y \triangleq [y[0], y[1], y[N_t - 1]]^T$ and $\bar{y} \triangleq [y[0], y[1], y[N_t - 1]]^T$. We also define the signals $y^{(1)}[\ell] = y^{(d)}[\ell] - 1$ for $0 < \lambda < T_s / 2$, and $y^{(2)}[\ell] = y^{(d)}[\ell] - 1$ for $T_s / 2 < \lambda < T_s$. We further define the vectors $y^{(1)} \triangleq [y^{(1)}[0], y^{(1)}[1], y^{(1)}[N_t - 1]]^T$ and $y^{(2)} \triangleq [y^{(2)}[0], y^{(2)}[1], y^{(2)}[N_t - 1]]^T$. Hence, if $0 < \lambda < T_s / 2$ then $y^{(1)} = y$, and if $T_s / 2 < \lambda < T_s$, then $y^{(2)} = y + \omega$. In order to simplify $y^{(d)}$, we need to consider when the effective matched-filter components affecting the symbols $s_1[m]$ and $s_2[m]$ are non-zero. Given that $\rho(\ell)$ is triangular for the duration $-T_s \leq \ell \leq T_s$ and zero otherwise, the only possible non-zero samples of the filter affecting $s_1[m]$ (for odd $n$) are $\rho(-\frac{T_s}{2}) = \frac{1}{2}$ and $\rho\left( \frac{T_s}{2} \right) = \frac{1}{2}$, which correspond to $\ell = m + k - 1$ and $\ell = m + k - 2$, respectively.

Now consider the matched-filter components affecting $s_2[m]$. We let $\kappa = \ell - m + k - 1 + n_t$, such that the filter component under consideration is $\rho(\kappa T_s + T_s / 2 - \lambda)$. The values of $\kappa$ for which the matched-filter can be non-zero are $\kappa \in \{-1, 0, 1\}$. In particular, we have $\rho\left( \frac{T_s}{2} - \lambda \right)$ for $\kappa = 0$, $\rho\left( \frac{T_s}{2} - \lambda \right)$ for $\kappa = -1$ and $\rho\left( \frac{3T_s}{2} - \lambda \right)$ for $\kappa = 1$.

Based on the above, we have that two distinct cases for the discrete-time received signal model for odd $n$, depending on the value of $\lambda$. In particular, if $0 < \lambda < T_s / 2$ then

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In this case, the number of information-carrying samples for odd $n$ will be $N_t$, that is, the same as the case for even $n$.

On the other hand, when $T_s / 2 < \lambda < T_s$, we have

$$ \rho\left( \frac{T_s}{2} - \lambda \right) = \frac{3}{2} - \mu, \quad \rho\left( \frac{3T_s}{2} - \lambda \right) = 1 - \mu, $$

Hence, if $0 < \lambda < T_s / 2$ then $y^{(d)} = y$, and if $T_s / 2 < \lambda < T_s$, then $y^{(d)} = y + \omega$. In this case, the number of information-carrying samples for odd $n$ will be $N_t + 1$.

To distinguish between the signal models in both cases, we let $y^{(d)}[\ell] = y^{(d)}[\ell] - 1$, and define the vectors $y \triangleq [y[0], y[1], y[N_t - 1]]^T$ and $\bar{y} \triangleq [y[0], y[1], y[N_t - 1]]^T$. We also define the signals $y^{(1)}[\ell] = \rho^{(d)}[\ell] - 1$ for $0 < \lambda < T_s / 2$, and $y^{(2)}[\ell] = \rho^{(d)}[\ell] - 1$ for $T_s / 2 < \lambda < T_s$. We further define the vectors $y^{(1)} \triangleq [y^{(1)}[0], y^{(1)}[1], y^{(1)}[N_t - 1]]^T$ and $y^{(2)} \triangleq [y^{(2)}[0], y^{(2)}[1], y^{(2)}[N_t - 1]]^T$. Hence, if $0 < \lambda < T_s / 2$ then $y^{(1)} = y$, and if $T_s / 2 < \lambda < T_s$, then $y^{(2)} = y$.
the \((N_t + 1) \times (L + 1)\) matrix whose \(k\)th column is \(s^{(\xi)}_{2,k} = [0_{1 \times (N_t + k + 1)}]^{\top} \). Moreover, \(\omega^{(2)}\) represents the AWGN vector with mean zero and covariance matrix \(\Sigma_{N_t + 1}^{(2)}\).

Although the noise samples within each of the vectors \(\omega^{(1)}\) and \(\omega^{(2)}\) are uncorrelated, there is a correlation of \(3/2\) between consecutive even and odd noise samples, that is, between the noise samples affecting \(\xi^{(1)}[2f]\) and \(\xi^{(1)}[2f + 1]\), due to the oversampling.

### 3 Joint Timing Offset and Channel Estimation Using BR Sampling

In this section, we derive two joint timing-offset and channel estimators for the system model under consideration. The first estimator is based on BR sampling and utilizes the ML estimation criterion, without making any specific assumptions regarding the structure of the pilot sequences \(s_1\) and \(s_2\). The second estimator is also based on BR sampling, but it additionally assumes that both \(s_1\) and \(s_2\) are designed using ZC sequences, whose zero-correlation properties are exploited in the estimation process.

#### 3.1 Pilot-based ML estimation (BR-ML)

We begin by rewriting the received signal \(\xi\) in Equation (5) as

\[
\xi = S_1 h + S_2 g + \mu V g + \omega,
\]

where \(V = S_2 - S_3\). Because the noise vector \(\omega\) is AWGN, the ML criterion coincides with LS criterion in this case\(^3\). In particular, the ML estimates of \(h, g\) and \(\mu\) are given by

\[
\hat{h}_{ML}, \hat{g}_{ML}, \hat{\mu}_{ML} = \arg \min_{h, g, \mu} \|\xi - S_1 h - S_2 g - \mu V g\|^2. \tag{13}
\]

Taking the derivative of our objective function with respect to \(h^\top\) and setting it to zero, we obtain the following solution for \(h\) in terms of the other parameters

\[
\hat{h}_{ML}(g, \mu) = \left( \begin{bmatrix} S_1^{\top} \end{bmatrix} \right)^{-1} \left( S_1 \xi - S_1^{\top} S_2 g - \mu S_1^{\top} V g \right). \tag{14}
\]

To simplify the derivation, we let \(B = \left( S_1^{\top} S_1 \right)^{-1} S_1^{\top} \xi\) and \(C = - \left( S_1^{\top} S_1 \right)^{-1} S_1^{\top} S_2\) and \(D = - \left( S_1^{\top} S_1 \right)^{-1} S_1^{\top} V\). Hence, we may rewrite Equation (14) as

\[
\hat{h}_{ML}(g, \mu) = B + C g + \mu D g. \tag{15}
\]

Substituting \(\hat{h}_{ML}(g, \mu)\) back into the objective function, we obtain the updated minimization in terms of \(g\) and \(\mu\), as follows

\[
\begin{aligned}
\{ \hat{g}_{ML}, \hat{\mu}_{ML} \} &= \arg \min_{g, \mu} \|\xi - S_1 B - (S_1 C + S_2) g - \mu (S_1 D + V) g\|^2. \tag{16}
\end{aligned}
\]

We further simplify our notation by letting \(\hat{\xi} = \xi - S_1 B, U_1 \equiv (S_1 C + S_2)\) and \(U_2 \equiv (S_1 D + V)\). Hence, Equation (16) becomes

\[
\begin{aligned}
\{ \hat{g}_{ML}, \hat{\mu}_{ML} \} &= \arg \min_{g, \mu} \|\hat{\xi} - U_1 g - \mu U_2 g\|^2. \tag{17}
\end{aligned}
\]

Taking the derivative of the updated objective function with respect to \(g^\top\) and setting it to zero, we obtain

\[
\hat{g}_{ML}(\mu) = \left( U_1^{\top} U_1 + \mu^2 U_2^{\top} U_2 + \mu (U_1^{\top} U_2 + U_2^{\top} U_1) \right)^{-1} \times \left( U_1^{\top} \hat{\xi} + \mu U_2^{\top} \hat{\xi} \right).
\]

After substituting \(\hat{g}_{ML}(\mu)\) into the objective function in Equation (16), the resulting function depends only on the real parameter \(\mu\). Hence, the estimate \(\hat{\mu}_{ML}\) can be found using one-dimensional line search. Once \(\hat{\mu}_{ML}\) has been obtained, it can be substituted into Equation (18) to obtain \(\hat{g}_{ML}\), and then both \(\hat{h}_{ML}\) and \(\hat{g}_{ML}\) are substituted into Equation (14) to obtain \(\hat{h}_{ML}\).

#### 3.2 Pilot-based estimation using ZC sequences (BR-ZC)

We now consider an alternative pilot-based estimator that exploits the properties of ZC sequences\(^4\). The ZC sequence is a CAZAC sequence, that is, the out-of-phase autocorrelation of a ZC sequence is zero.

The training sequence of each terminal consists of a ZC sequence of length \(N_{ZC}\), as well as a cyclic prefix and cyclic suffix, both of length \(N_t\). Furthermore, the ZC sequence of terminal \(T_2\) is obtained from that of \(T_1\) through a circular shift by \([N_{ZC}/2]\) positions. This will be exploited to eliminate interference during the estimation process.

To be consistent with the notation in earlier sections, the pilot sequences of \(T_1\) and \(T_2\) are still referred to as \(s_1\) and \(s_2\), respectively. The total length of each pilot sequence is still \(N_t\), where \(N = N_{ZC} + 2 N_t\). Hence, the ZC sequence of \(T_i\) occurs at the symbols \(s_i[N_t + 1], \ldots, s_i[N_t + N_{ZC}]\) for \(i = 1, 2\). We let \(\tilde{s}_i \equiv [s_i[N_t + 1], \ldots, s_i[N_t + N_{ZC}]]^T\).

The estimation will happen in two stages. In the first stage, we will estimate the channel vector \(h\) while in the second stage we will estimate the channel vector \(g\) and the timing offset \(\mu\). The frequency-selective nature of the transmission means that the received vector \(\xi\) involves several shifted copies of both ZC sequences \(s_1\) and \(s_2\). Hence, it is important to make sure that

\(^3\) We note that the derived estimator can still be used if the noise is coloured. However, in this case, it will correspond to the LS criterion, not the ML criterion, and it will not be optimum \(^5\).

\(^4\) We refer the reader to a paper by another author for a detailed discussion of the CAZAC sequence and its properties.

\(^5\) We note that the derived estimator can still be used if the noise is coloured. However, in this case, it will correspond to the LS criterion, not the ML criterion, and it will not be optimum.
the estimation does not encounter error floors. It can be verified that error floors are avoided if the following conditions are met:

1. \( N_g \geq n_r + L + 1 \).
2. \( N_{ZC} \geq 2|n_r + L + 2| \).

### 3.2.1 Stage 1

To estimate the vector \( \hat{h} \), the relay cross-correlates the received signal with the ZC sequence \( \hat{x} \). The cross-correlation yields the samples \( \varphi_i \), given by

\[
\varphi_i = \sum_{n=1}^{N_{ZC}} n_1 \left[ n + N_g \right]^\star \zeta[i + N_g - 1 + n], \quad i = 0, \ldots, L.
\]

Hence, we have a total of \( L + 1 \) samples. Each of these samples will result in an estimate of the corresponding channel coefficient. Defining the vector \( \hat{h}_{ZC} \) as the BR-ZC estimate of \( h \), we have

\[
\hat{h}_{ZC}[i] = \frac{1}{N_{ZC}} \varphi_i,
\]

for \( i = 0, \ldots, L \).

### 3.2.2 Stage 2

Now we consider the samples that will be used in the estimation of \( g \) and \( \mu \). These samples are the following:

\[
\varphi_i = \sum_{n=1}^{N_{ZC}} n_2 \left[ n + N_g \right]^\star \zeta[i + n_r + N_g - 1 + n], \quad i = 0, \ldots, L + 1.
\]

We collect the above correlation samples into the vector \( \varphi \). If conditions 1, 2 are satisfied, it can be verified that \( \varphi \) has the following form

\[
\varphi = (1 - \mu)Xg + \mu Yg + \hat{n},
\]

where

\[
X \triangleq \begin{bmatrix} I_{L+1} \\ 0_{1 \times (L+1)} \end{bmatrix},
\]

and

\[
Y \triangleq \begin{bmatrix} 0_{1 \times (L+1)} \\ I_{L+1} \end{bmatrix}.
\]

While the vector \( \hat{n} \) consists of coloured Gaussian noise, it is well-approximated as white noise [12], with mean \( 0 \) and covariance matrix \( N_{ZC} \sigma^2 I_{L+2} \). We can thus apply the ML criterion to find the estimates of \( g \) and \( \mu \). This yields the following estimate of \( g \) in terms of \( \mu \)

\[
\tilde{g}_{ZC}(\mu) = (X^H X + \mu X^H Q + \mu Q^H X + \mu^2 Q^H Q)^{-1} \\
\times (X^H \varphi + \mu Q^H \varphi),
\]

where \( Q \triangleq Y - X \). It remains to find the parameter \( \mu \). This is done by substituting the expression of \( \tilde{g}_{ZC}(\mu) \) into the corresponding log-likelihood function (LLF) and performing a one-dimensional line search with respect to \( \mu \) to obtain \( \mu_{ZC} \). Once \( \mu_{ZC} \) is available, we substitute it into Equation (25), obtaining the estimate \( \tilde{g}_{ZC} \).

The main advantage of this estimator is its low complexity, since the dimension of the correlated signal is significantly lower that of the original signal. It is prone to error floors, however, when the aforementioned conditions 1, 2 are not satisfied.

### 4 Joint Estimation with DBR Sampling

In this section, we consider joint timing-offset and channel estimation under DBR sampling. As we discussed in Section 2, the received signal structure under DBR sampling depends on whether \( 0 < \lambda < T_s / 2 \) or \( T_s / 2 < \lambda < T_s \). As the receiver has no prior knowledge of the value of \( \lambda \), both possibilities have to be considered in the estimation process. Hence, we define the following two hypotheses:

\[
H_1 : \quad 0 < \lambda < \frac{T_s}{2},
\]

\[
H_2 : \quad \frac{T_s}{2} < \lambda < T_s.
\]

For even \( n \), the range of \( \lambda \) does not affect the overall structure of the signal \( \varphi^{[n]} \). Hence, under both \( H_1 \) and \( H_2 \) the received signal samples for even \( n \) are collected into the vector \( \varphi \) given in Equation (5). For odd \( n \), however, the structure of the received signal vector will depend on whether \( H_1 \) or \( H_2 \) holds. In particular, if \( H_1 \) holds, then the received vector will be given by \( y^{(1)} \) (see Equation (10)), whereas if \( H_2 \) holds the received vector will be given by \( y^{(2)} \) (see Equation (11)).

Since the receiver does not know beforehand which hypothesis holds, it has to obtain two sets of estimates of the desired parameters corresponding to both hypotheses, and then determine which of the two sets is more likely. For the DBR-based joint estimator, we propose to use the LS criterion, since the ML criterion would be difficult to apply in the presence of coloured noise. Hence, we will refer to this estimator as the DBR-LS estimator.

Under hypothesis \( H_1 \), we will obtain the following estimates:

\[
\left\{ \hat{h}^{(1)}, g^{(1)}, \mu^{(1)} \right\} = \arg \min_{h, g, \mu} \| \varphi - S_h - S_g - \mu V g \|^2 \\
+ \| y - \frac{1}{2} \left( S_1^{(1)} + S_2^{(1)} \right) \hat{h} - \left( \frac{1}{2} - \mu \right) S_2^{(1)} g - \left( \frac{1}{2} + \mu \right) S_2^{(1)} g \|^2.
\]

(27)
Under hypothesis $H_2$, we obtain the following estimates:

$$\begin{align*}
\{ \hat{h}^{(2)}, \hat{g}^{(2)}, \hat{\mu}^{(2)} \} &= \arg\min_{h, g, \mu} \| z - S_1 h - S_2 g - \mu V g \|^2 \\
+ \| \hat{y} - \frac{1}{2} \left( \hat{S}_1^{(2)} + \hat{S}_2^{(2)} \right) h - \left( \frac{3}{2} - \mu \right) \hat{S}_2^{(2)} g - \left( \frac{1}{2} - \mu \right) \hat{S}_2^{(2)} g \|^2.
\end{align*}$$

(28)

We now illustrate the estimation under hypothesis $H_1$. The estimation under Hypothesis $H_2$ follows similar steps. Letting $D_1 \triangleq \frac{1}{2} (\hat{S}_1^{(1)} + \hat{S}_1^{(1)})$, $D_2 \triangleq \frac{1}{2} (\hat{S}_2^{(1)} + \hat{S}_2^{(1)})$ and $D_3 \triangleq (\hat{S}_2^{(1)} - \hat{S}_2^{(1)})$, we rewrite Equation (27) as

$$\begin{align*}
\{ \hat{h}^{(1)}, \hat{g}^{(1)}, \hat{\mu}^{(1)} \} &= \arg\min_{h, g, \mu} \| z - S_1 h - S_2 g - \mu V g \|^2 \\
+ \| y - D_1 h - D_2 g - \mu D_3 g \|^2.
\end{align*}$$

(29)

Hence, it can be shown that the optimal solution for $h$ in terms of $g$ and $\mu$ is given by

$$\hat{h}^{(1)}(g, \mu) = \left( S_1^{(1)} S_1 + D_1^{(1)} D_1 \right)^{-1} \times \left( S_1^{(1)} z - S_1^{(1)} S_2 g - \mu S_1^{(1)} V g + D_1^{(1)} y - D_1^{(1)} D_2 g - \mu D_1^{(1)} D_3 g \right).$$

(30)

We rewrite $\hat{h}^{(1)}(g, \mu)$ as

$$\hat{h}^{(1)}(g, \mu) = B^{(d)} + C^{(d)} g + \mu D^{(d)} g,$$

(31)

where

$$B^{(d)} \triangleq \left( S_1^{(1)} S_1 + D_1^{(1)} D_1 \right)^{-1} \left( S_1^{(1)} z + D_1^{(1)} y \right),$$

(32)

$$C^{(d)} \triangleq \left( S_1^{(1)} S_1 + D_1^{(1)} D_1 \right)^{-1} \left( -S_1^{(1)} S_2 - D_1^{(1)} D_2 \right),$$

(33)

and

$$D^{(d)} \triangleq \left( S_1^{(1)} S_1 + D_1^{(1)} D_1 \right)^{-1} \left( -S_1^{(1)} V - D_1^{(1)} D_3 \right).$$

(34)

Substituting Equation (36) into $g$ in Equation (35) yields an objective function that depends only on $\mu$ and that can be solved using a one-dimensional line search.

The solution under hypothesis $H_2$ follows a very similar procedure and is not repeated to avoid redundancy. Once the two sets of estimates have been obtained, we need to compare their likelihoods and select the one with the higher likelihood. The respective likelihoods are chosen as follows:

$$\mathcal{L}(H_1 | y, z) = -\frac{1}{\sigma^2} \| z - S_1 \hat{h}^{(1)} - S_2 \hat{g}^{(1)} - \hat{\mu}^{(1)} V \hat{g}^{(1)} \|^2$$

$$- \frac{1}{\sigma^2} \| y - \frac{1}{2} (\hat{S}_1^{(1)} + \hat{S}_2^{(1)}) \hat{h}^{(1)} - \left( \frac{1}{2} - \hat{\mu}^{(1)} \right) \hat{S}_2^{(1)} g \|^2,$$

(37)

and

$$\mathcal{L}(H_2 | y, z) = -\frac{1}{\sigma^2} \| z - S_1 \hat{h}^{(2)} - S_2 \hat{g}^{(2)} - \hat{\mu}^{(2)} V \hat{g}^{(2)} \|^2$$

$$- \frac{1}{\sigma^2} \| y - \frac{1}{2} (\hat{S}_1^{(2)} + \hat{S}_2^{(2)}) \hat{h}^{(2)} - \left( \frac{3}{2} - \hat{\mu}^{(2)} \right) \hat{S}_2^{(2)} g \|^2,$$

(38)

We note that the above likelihoods do not take into account the noise correlation. It is possible to take it into account but that would increase the complexity of the testing.

### 5 COMPUTATIONAL COMPLEXITY

The computational complexities of the BR-ML, BR-ZC and DBR-LS estimators are shown in Table I in terms of the number

| Algorithm      | Computational complexity |
|----------------|-------------------------|
| BR-ML Estimator | $(L + 1)(4KN + 4N_f + 2KL + 3K + 4L + 3)$ + $KN_f + K + N_f$ |
| BR-ZC Estimator | $(L + 1)(2KN + 2KN_f - 2KN_f + 6KL + 11K + 4L + 7) + 2KN + 2KN_f + 4KL + 8K$ |
| DBR-LS Estimator | $(L + 1)(12KN_f + 8N_f + 2KL + 4K + 7L + 4) + 9KN_f + 4K + 3N_f + 2$ |
of complex mathematical operations. The variable $K$ denotes the number of points considered in the line search for $\mu$. We note that we only show the number of run-time operations for each algorithm. Mathematical operations that do not depend on the received signal (including all matrix inversions) can be done off-line, and are thus excluded from the calculation of the computational complexity.

Among the three algorithms, the BR-ZC estimator has the lowest computational complexity since $N_{ZC}$ is typically much smaller than $N$. The DBR-LS estimator has the highest complexity, which is expected since DBR sampling increases the number of available samples and hence the dimension of the problem. Moreover, the DBR-LS estimator considers two hypotheses ($H_1$ and $H_2$) and involves a hypothesis testing step, unlike the BR-ML estimator. Nonetheless, the computational complexity of the DBR-LS estimator is not significantly higher than that of the BR-ML estimator. In fact, both of them are $O(KLN + KL^2)$. Moreover, as we shall see in Section 8, the moderately increased complexity of the DBR-LS estimator is more than justified by the resulting substantial improvements in estimation accuracy.

## 6 INTEGER OFFSET ESTIMATION ERROR CORRECTION

The three joint estimation algorithms proposed in Sections 3 and 4 assume that the integer timing offset $n_t$ is perfectly known. In practice, as observed in [12], it is common to encounter an error of $\pm 1$ in the estimation of $n_t$, especially at low SNR or if the fractional offset is too large or too small. If this error goes uncorrected, it will severely affect the quality of the estimates for the channels $h$ and $g$ and the fractional timing offset $\lambda$. In this section, we address this issue by proposing several GLRT-inspired schemes for correcting existing errors in the estimation of $n_t$. These schemes integrate very well with the proposed joint estimation algorithms.

We let $\hat{n}_t$ be the available estimate of $n_t$. We have that $\hat{n}_t = n_t + \delta$ where $\delta$ is the error in the estimation of $n_t$. Without loss of generality, we assume that $\delta \in \{-1, 0, 1\}$. We thus focus on the following three hypotheses:

- $H_0 : n_t = \hat{n}_t - 1$,
- $H_1 : n_t = \hat{n}_t$,
- $H_2 : n_t = \hat{n}_t + 1$.

Our goal is to select the most likely hypothesis among $H_0$, $H_1$, and $H_2$ based on the available samples. To do this, we first need to perform the joint timing-offset and channel estimation to acquire the corresponding estimates under each hypothesis. We begin by presenting the BR-based GLRT criteria that use the vector $\varphi$ and the parameter estimates acquired using the BR-ML and BR-ZC estimators. We then propose a DBR-based GLRT criterion that uses the estimates acquired using the DBR-LS estimator.

### 6.1 BR-GLRT for integer offset error correction

We let $\hat{n}_t^{(0)} = \hat{n}_t - 1, \hat{n}_t^{(1)} = \hat{n}_t$ and $\hat{n}_t^{(2)} = \hat{n}_t + 1$. Since the error is assumed to be in the set $\{-1, 0, 1\}$, one of the values $\hat{n}_t^{(i)}, i = 0, 1, 2$ is the correct value of $n_t$. We also let $L_{BR}^{(i)} \equiv N + \hat{n}_t^{(i)} + L + 1$ be the number of information-carrying samples under each hypothesis.

We also denote by $\hat{\xi}$ the vector obtained by acquiring $N + \hat{n}_t + L + 2$ BR samples from the received signal $\xi(t)$ in Equation (3), starting with the sample at $i = 0$. The vector $\hat{\varphi} \equiv [\hat{\varphi}_1, ..., \hat{\varphi}_N, \hat{n}_t^{(i)} + L + 1]^T$ corresponds to the information-carrying samples under hypothesis $H_{i}$, for $i = 0, 1, 2$. We further let $\hat{\theta}^{(i)} \equiv [\hat{h}^{(i)}, \hat{g}^{(i)}]^T$ be the BR-ML estimates of the parameters of interest $\theta \equiv [h^T, g^T, \mu]^T$ under hypothesis $H_{i}, i = 0, 1, 2$.

The corresponding BR likelihood for each hypothesis will be given by

$$
L(H|\hat{\varphi}; \hat{\theta}^{(i)}) = -\frac{1}{\sigma^2} \| \varphi - S_1^{(i)} \hat{h}^{(i)} - S_2^{(i)} \hat{g}^{(i)} - V \hat{\theta}^{(i)} \|^2, \quad (39)
$$

where $S_1^{(i)}$ is the $N_{BR}^{(i)} \times (L + 1)$ matrix where $j$th column is $s_1^{(j)} = [0_{1 \times (N-1)}, s_1^T, 0_{1 \times (N_0-N_j+j+1)}]^T$, $S_2^{(i)}$ is the $N_{BR}^{(i)} \times (L + 1)$ matrix where its $j$th column is $s_2^{(j)} = [0_{1 \times (N_0-N_j+N_0-N_j+j+1)}]^T$ and $S_2^{(i)}$ is the $N_{BR}^{(i)} \times (L + 1)$ matrix where its $j$th column is $s_2^{(j)} = [0_{1 \times (N_0-N_j+j+1)}]^T$ and $N_{BR}^{(i)} \equiv N + \hat{n}_t^{(i)} + L + 1$, and $V \equiv \hat{\mu}^{(i)} - S_2^{(i)}$.

After computing the likelihood for each hypothesis, we select the one with the highest likelihood, and the corresponding set of estimates for the channel coefficients and the timing offset. We refer to the scheme thus described as BR-ML-GLRT. Alternatively, we can follow the same approach, but utilizing the estimates $\hat{h}_{ZC}, \hat{g}_{ZC}$ and $\hat{\mu}_{ZC}$. We refer to this as BR-ZC-GLRT.

### 6.2 DBR-GLRT for integer offset error correction

We now consider integer offset error correction using DBR sampling. Hence, we use both integer and fractional samples to determine the most likely hypothesis among $H_0$, $H_1$ and $H_2$. We denote as $\hat{\varphi}$ the vector obtained by acquiring $N + \hat{n}_t + L + 3$ fractional samples from the received signal $\xi(t)$, starting with the discrete-time sample $\xi(-T_s/2)$. We also define the vectors $\hat{\varphi}_i \equiv [\hat{\varphi}_1, ..., \hat{\varphi}_N, \hat{n}_t^{(i)} + L + 1]^T$ for $i = 1, 2, 3$. 

DATA DETECTION

In this section, we discuss symbol detection for the system under consideration. We assume that after pilot transmission, a sequence of $K$ data symbols is transmitted by each source terminal. We also assume that there is a guard interval of sufficient length between pilot and data transmission to avoid interference between the two transmissions. We will first show that BR sampling during data transmission is highly restrictive in terms of the size of the data vector. We will then proceed to explore data detection using DBR sampling.

We let $t_1$ and $t_2$ be the $K \times 1$ vectors of data symbols transmitted by $T_1$ and $T_2$, respectively. Assuming BR sampling, we let $q$ be the corresponding $N_D \times 1$ received signal vector, where $N_D = K + L + n_t + 1$ is the total number of information-carrying samples. The vector $q$ can be expressed as

$$q = T_1 h + (1 - \mu) T_2 g + \mu T_2 g + n,$$  \hspace{1cm} (42)$$

where $T_1$ is the $N_D \times (L + 1)$ matrix whose $k$th column is $t_{1,k} = \begin{bmatrix} 0_{1 \times (k-1)} & t_{1,k} \end{bmatrix}^T$, $T_2$ is the $N_D \times (L + 1)$ matrix whose $k$th column is $t_{2,k} = \begin{bmatrix} 0_{1 \times (K-k-n_t)} & t_{2,k} \end{bmatrix}^T$, and $T_2$ is the $N_D \times (L + 1)$ matrix whose $k$th column is $i_{2,k} = \begin{bmatrix} 0_{1 \times (K-k-n_t)} & t_{2,k} \end{bmatrix}^T$. Furthermore, $n$ is the AWGN noise vector with mean $0$ and covariance matrix $\sigma^2 I_{N_D}$.

Before we proceed, it is useful to define the $(K + L) \times K$ matrices $H$ and $G$, as follows

$$H \triangleq \begin{bmatrix} h[0] & 0 & \ldots & \ldots & 0 \\ h[1] & h[0] & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ h[L] & \ldots & h[0] & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \ldots & \ldots & 0 & h[L] \end{bmatrix},$$ \hspace{1cm} (43)$$

$$G \triangleq \begin{bmatrix} g[0] & 0 & \ldots & \ldots & 0 \\ g[1] & g[0] & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ g[L] & \ldots & g[0] & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \ldots & \ldots & 0 & g[L] \end{bmatrix}.$$ \hspace{1cm} (44)$$

We can now rewrite Equation (42) as

$$q = H t_1 + (1 - \mu) G t_2 + \mu G t_2 + n,$$ \hspace{1cm} (45)$$
where
\[
\tilde{H} \equiv \begin{bmatrix} H \\ 0_{(\nu+1)\times K} \end{bmatrix}, \quad \tilde{G} \equiv \begin{bmatrix} 0_{\nu\times K} \\ G \end{bmatrix}, \quad \bar{G} \equiv \begin{bmatrix} 0_{(\nu+1)\times K} \\ 0_{1\times K} \end{bmatrix}.
\]  

(46)

Observing Equation (45), it is clear that BR sampling is very restrictive in the context of data detection. In particular, we are trying to detect 2\(K\) unknowns \((t_1, t_2)\) using \(N_D = K + L + \nu + 1\) samples. Hence, we would need to satisfy the constraint \(K \leq L + \nu + 1\). Using DBR sampling during data detection effectively resolves this problem and is also expected to improve data detection accuracy, as reported in [13].

When DBR sampling is used during data transmission, the received samples can be grouped into two vectors, one consisting of the integer samples (at even multiples of \(T_s/2\)) and the other consisting of the fractional samples (at odd multiples of \(T_s/2\)). The vector corresponding to integer samples is the same as \(q\). Similar to the pilot transmission case, the form of the vector of fractional samples depends on the value of \(\lambda\). In particular, if \(0 < \lambda < T_s/2\), then the vector of fractional samples is given by
\[
q^{(1)} = \frac{1}{2} \left( \tilde{H}^{(1)} + \tilde{H}^{(1)} \right) t_1 + \left( \frac{1}{2} - \mu \right) \bar{G}^{(1)} t_2
\]
\[
+ \left( \frac{1}{2} + \mu \right) \tilde{G}^{(1)} t_2 + n^{(1)}
\]  

(47)

where \(\tilde{H}^{(1)} \equiv \tilde{H}, \tilde{G}^{(1)} \equiv \tilde{G}, \bar{G}^{(1)} \equiv \bar{G}, \)
\[
\tilde{H}^{(1)} \equiv \begin{bmatrix} H \\ 0_{\nu\times K} \end{bmatrix},
\]  

(48)

and \(n^{(1)}\) is \(N_D \times 1\) AWGN vector with mean 0 and covariance matrix \(\sigma^2 I_{N_D}\).

On the other hand, if \(\lambda > T_s/2\), then the fractional samples have the form
\[
q^{(2)} = \frac{1}{2} \left( \tilde{H}^{(2)} + \tilde{H}^{(2)} \right) t_1 + \left( \frac{3}{2} - \mu \right) \bar{G}^{(2)} t_2
\]
\[
+ \left( \frac{1}{2} - \mu \right) \tilde{G}^{(2)} t_2 + n^{(2)}
\]  

(49)

where
\[
\tilde{H}^{(2)} \equiv \begin{bmatrix} H \\ 0_{(\nu+2)\times K} \end{bmatrix}, \quad \tilde{H}^{(2)} \equiv \begin{bmatrix} 0_{1\times K} \\ H \end{bmatrix},
\]  

(50)

\[
\tilde{G}^{(2)} \equiv \begin{bmatrix} 0_{(\nu+1)\times K} \\ G \end{bmatrix}, \quad \bar{G}^{(2)} \equiv \begin{bmatrix} 0_{(\nu+1)\times K} \\ 0_{1\times K} \end{bmatrix},
\]  

(51)

and \(n^{(2)}\) is the AWGN vector with mean 0 and covariance matrix \(\sigma^2 I_{N_D+1}\).

In the DBR case, the number of available samples is either \(2N_D\) or \(2N_D + 1\), while the number of unknown data symbols is \(2K\). Since \(N_D = K + L + \nu + 1\), we always satisfy \(2N_D > 2K\), and thus there is no limiting constraint on the value of \(K\), unlike the BR case.

While ML detection is the optimal detection criterion, it is not considered here due to its extremely high complexity. Instead we will consider linear detection criteria. To apply detection in the DBR case, we need to take into account both received vectors \(q\) and \(q^{(i)}\), \(i \in \{1, 2\}\). We thus define the augmented vector
\[
\hat{q}^{(i)} \equiv \begin{bmatrix} q \\ q^{(i)} \end{bmatrix},
\]  

(52)

where \(i = 1, 2\), depending on the value of \(\lambda\). We can rewrite \(\hat{q}^{(i)}\) as
\[
\hat{q}^{(i)} = \Lambda^{(i)} t + \tilde{n}^{(i)}.
\]  

(53)

where
\[
\Lambda^{(1)} \equiv \begin{bmatrix} \tilde{H} \\ \frac{1}{2} (\tilde{H}^{(1)} + \tilde{H}^{(1)}) \end{bmatrix} \begin{bmatrix} (1 - \mu) \hat{G} + \mu \bar{G} \\ \left( \frac{1}{2} - \mu \right) \tilde{G}^{(1)} + \left( \frac{1}{2} + \mu \right) \bar{G}^{(1)} \end{bmatrix}^T
\]  

(54)

\[
\Lambda^{(2)} \equiv \begin{bmatrix} \tilde{H} \\ \frac{1}{2} (\tilde{H}^{(2)} + \tilde{H}^{(2)}) \end{bmatrix} \begin{bmatrix} (1 - \mu) \hat{G} + \mu \bar{G} \\ \left( \frac{3}{2} - \mu \right) \tilde{G}^{(2)} + \left( \frac{1}{2} - \mu \right) \bar{G}^{(2)} \end{bmatrix}^T
\]  

(55)

\[t \equiv [t_1^T, t_2^T]^T, \quad \tilde{n}^{(i)} \equiv [\tilde{n}^T, \tilde{n}^{(i)}]^T.
\]

Zero-forcing (ZF) detection obtains the vector
\[
q_{ZF}^{(i)} = \Lambda^{(i)\dagger} \hat{q}^{(i)},
\]  

(56)

where \(\Lambda^{(i)\dagger}\) is the Moore–Penrose pseudo-inverse of \(\Lambda^{(i)}\). However, ZF detection is known to cause noise enhancement, which is expected to result in performance degradation, especially that the noise is already coloured due to DBR sampling. Hence, MMSE detection is a more promising approach in this scenario. The MMSE detector utilizes the covariance matrix \(C^{(i)}\) of \(\hat{q}^{(i)}\). For \(i = 1\), we have
\[
C^{(1)} \equiv \sigma^2 \begin{bmatrix} I_{N_D} & R^{(1)} \end{bmatrix},
\]  

(57)

where
\[
R^{(1)} \equiv \begin{bmatrix} 1 & 1/2 & 0 & \ldots & 0 \\ 1 & 1/2 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \ldots & \ldots & \ldots & 0 \\ 0 & \ldots & \ldots & \ldots & 1/2 \\ \end{bmatrix}.
\]  

(58)
For \( i = 2 \), we have
\[
C^{(2)}(2) \triangleq \sigma^2 \begin{bmatrix} I_{N_D} & R^{(2)} \\ R^{(2)T} & I_{N_D+1} \end{bmatrix},
\] (59)
where
\[
R^{(2)}(2) \triangleq \begin{bmatrix} 1 & 2 & 0 & \cdots & 0 \\ 2 & 1 & 0 & \cdots & 0 \\ 0 & 1 & 2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 & 2 \end{bmatrix}.
\] (60)

The MMSE linear detector is given by
\[
q_{\text{MMSE}}^{(i)} \triangleq \Lambda^{(i)H} \left( \Lambda^{(i)} \Lambda^{(i)H} + C^{(i)} \right)^{-1} q^{(i)}.
\] (61)

In practice, since the channel coefficients are not perfectly known, the estimated coefficients will be utilized to form the MMSE detector. In Section 8, we will show the SER performance of this detector when the estimates \( \hat{\Lambda}^{(i)}_{\text{ML}}, \hat{\Lambda}^{(i)}_{\text{ZC}} \) and \( \hat{\Lambda}^{(i)}_{\text{DBR}} \) are used in the detection process, where each of these three matrices is formed using the corresponding channel and timing-offset estimates.

8 SIMULATION RESULTS

In this section, we use MATLAB simulations to compare the performance of the proposed estimators. In all our simulations, the terminals use ZC sequences in their pilot transmission. Each terminal transmits a ZC sequence of length \( N_{ZC} = 20 \), preceded by a cyclic prefix of size \( N_g = 10 \) and followed by a cyclic suffix of the same size. The ZC sequence of terminal \( T_2 \) is obtained from that of terminal \( T_1 \) by circular shift of \( N_{ZC}/2 \). The channel length parameter \( L \) is set as \( L = 5 \). The channel coefficients are all generated as independent circularly complex Gaussian with mean zero and variance 1 (\( CCN(0, 1) \)). The timing offset \( \tau \) is selected uniformly in the interval \( (0, (N_g - L - 1)/T) \). All the plots are obtained by averaging over 100 independent realizations of the channel coefficients \( h \) and \( g \) and the timing offset \( \tau \). For the SER plots, the transmitted symbols are selected using QPSK modulation.

In Figure 2, we show the sum MSE performances of the BR-ML, BR-ZC and DBR-LS estimators for the estimation of the channel vectors \( h \) and \( g \). With regards to BR estimators, it is clear that the BR-ML estimator outperforms the BR-ZC estimator. Importantly, it is obvious that the DBR-LS estimator significantly outperforms both BR estimators, and performs very close to the DBR-CRB for the whole SNR range, showing the optimality of this estimator. In fact, there is a gap of approximately 6 dB between the BR-CRB and the DBR-CRB, which shows that the proposed DBR estimation provides substantial improvements in terms of estimation accuracy and power consumption.

For the same settings, Figure 3 shows the MSE of all the estimators for the estimation of the fractional timing offset \( \lambda \). The same trends are observed, whereby the BR-ML outperforms the BR-ZC, while the DBR-LS gives the best performance with substantial improvements in accuracy. However, the improvement in accuracy of the DBR is much more substantial for \( \lambda \) than the channel parameters. In particular, there is an improvement of about 15 dB. This demonstrates the huge advantages of over-sampling in the scenario under consideration.

Figure 4 shows the SER performance for all the estimators versus SNR for \( L = 5 \). As a benchmark, we also show the ideal
SER performance when all the parameters are perfectly known (Genie). The SER for all the estimators is obtained assuming DBR sampling during data transmission. MMSE detection is applied at the receiver, and the MMSE detector for each estimator is formed using the corresponding channel and timing-offset estimates obtained in the estimation stage. It is clear from Figure 4 that the DBR-LS estimator provides the best SER performance, with an improvement of approximately 4 dB over the BR-ML estimator at SER of $10^{-2}$. In fact, the SER of the DBR-LS estimator almost overlaps with the ideal SER performance for the whole SNR range, confirming the advantage of this proposed estimator and oversampling scheme. The BR-ML estimator is approximately 1.5 dB better than the BR-ZC estimator.

Finally, we consider the case where there is an error in the estimation of $n_\tau$ and investigate the performances of the three GLRT schemes for the integer offset error correction proposed in Section 6. In particular, we assume that the available estimate of $n_\tau$ is given by $\hat{n}_\tau = n_\tau + \delta$ where $\delta \in \{-1, 0, 1\}$, and that the three possible values of $\delta$ occur with equal probability. The BR-ML-GLRT, BR-ZC-GLRT and DBR-GLRT schemes are used to correct the error in $n_\tau$, and the resulting probability of correct detection is shown in Figure 5. It is clear that the DBR-GLRT offers the best probability of correct detection. In fact, it offers a gain of approximately 10 dB over the BR-ML-GLRT and the BR-ZC-GLRT at 90% correct detection probability. Importantly, the DBR-GLRT performs very well even at low SNR. The probability of correct detection at 0 dB is approximately 88% for the DBR-GLRT, whereas it is approximately 71% for the other two schemes. No noticeable difference is seen between the two BR GLRT schemes.

9  |  CONCLUSIONS

This paper considered the problem of joint timing-offset and channel estimation for PLNC in frequency selective channel environments. Three algorithms were developed to address this problem, providing various tradeoffs between accuracy and complexity. The first two algorithms were based on BR sampling, using the ML estimation criterion and the special properties of ZC sequences, respectively. The third algorithm was based on DBR sampling and the LS estimation criterion. While the first two algorithms have lower complexity, our simulations showed that the DBR-LS estimator offers substantially higher estimation accuracy, and up to 4 dB improvement in SER. We also proposed three GLRT-based schemes to correct errors in integer offset estimation, which integrate very well with the proposed estimation algorithms. The DBR-GLRT offered the best error-correction performance, with up to 10 dB improvement over the BR-GLRT schemes. Based on these results, the DBR-LS approach provides a convenient solution for the problem of joint timing-offset and channel estimation in frequency selective environments, with substantial gains in accuracy and power efficiency. These gains are achieved with a moderate increase in run-time computational complexity. While we considered the rectangular pulse in this work, in future work we can consider joint timing-offset and channel estimation for Nyquist pulses, which may be approximated as time-limited as done in [17].

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