Design of Dual-Rate PIM-Disturbance Regulators for a DC-DC Switching Converter

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Abstract. Three types of dual-rate digital controllers are designed to suppress the output voltage fluctuations caused by load-current changes for a buck DC-DC switching converter running at 300kHz PWM. The slower-rate outer loop is designed using the IA-PIM method, which stabilizes the outer loop at a slow rate of 25kHz. Since the time lag in sensing the disturbance is critical, a faster loop of 300kHz is added based on three different approaches. The resulting dual-rate control laws are implemented on a single 16-bit fixed-point microcomputer with the clock frequency of 140MHz, via 12-bit A/D and D/A converters. Experiments on the fluctuation of the output voltage caused by load-current transients show that, while the analog regulator suppresses the maximum output fluctuation to be 4.7% of the output voltage, one of the three digital control methods achieves 4%, without adding extra analog components, such as capacitors as in our previous paper.

1. Introduction

While analog controllers have played a major role in the development of high-performance power converters [1-3], they are being taken over by digital controllers [4-8] due to increased computational power at low costs. As the clock-speed of the processor increases, the frequencies of switching, PWM, and the converter itself follow suits, resulting in never ending quest for algorithms with high accuracy at low or modest sampling rates. Areas of application of such controllers are expanding quickly to higher frequency domains, and demands for control laws that require relatively low sampling rates still exist. A slower sampling rate usually leads to a lower computational load and a reduced power consumption, but often to poorer or less stable performances. This should be overcome in view of looming expectations that digital control methods are practical tools to realize high performances for the next generation of power supplies [9].

A pulse-frequency-modulation (PFM)-based digital controller is designed in [10]-[11] for an LLC resonant DC-DC converter. Since the sudden change in the output voltage can cause shut-down and even break-down of the devices connected to the power supply, the regulator must respond quickly to such changes. As the plant input is a change in the switching frequency, a variable sampling-period is required to achieve soft-switching, for which an adequate digital control theory still lacks in dealing with variable rate technique. In [12], a pulse-width-modulation (PWM)-based digital controller is considered, which can be implemented using a commonly used scheme of a uniform sampling period. The analog controller is modified, taking into account the possible performance degradation in the disturbance rejection properties due to discretization. This is achieved by increasing the output...
capacitance, and the analog controller is redesigned. While this redesigned analog controller is successfully discretized into an Integral-Action Plant-Input-Mapping (IA-PIM) digital controller, an increase in the number of analog components is not desirable due to higher costs, larger size, and lower reliability. In the present paper, therefore, an internal fast-rate loop is added such that the duty ratio of the PWM is adjusted to counteract the disturbance, thus without adding analog components.

In Section 2, the IA-PIM design method is reviewed very briefly. In Section 3, this method is applied to the buck DC-DC converter, to obtain the digital controller running at the slow sampling rate. Since the microcomputer has spare computational capacity for additional calculations, three methods are proposed and implemented. The duty-ratio is adjusted to suppress the output-voltage fluctuations against load-current changes. Experimental results are also compared. Conclusions are drawn in Section 4.

2. Low-rate stabilization

2.1. System Models

Figure 1 is a schematic of the switching DC-DC buck converter considered in the study, where the control input is the duty ratio $D$ of the PWM signal and the output is the voltage $V_o$. To design a model-based control system, a plant model is needed, such as a state-space plant model based on a state averaging method [3], from which the dynamic model can be extracted and the transfer functions can be derived [13]. In the end, the output voltage is determined by $\Delta D$, the change in the duty ratio of the PWM, $\Delta I_o$, the change in the output current, and $\Delta V_i$, the change in the input voltage [14], as

$$\Delta V_O(s) = G_1(s)\Delta D(s) + G_2(s)\Delta I_o(s) + G_3(s)\Delta V_i(s),$$

where

$$G_1(s) = \frac{\Delta V_O(s)}{\Delta D(s)} = \frac{C r_c s + 1}{P(s)} V_i,$$

$$G_2(s) = \frac{\Delta V_o(s)}{-\Delta I_o(s)} = \frac{L C r_c s^2 (L + C r_c s + r_i)}{P(s)},$$

$$G_3(s) = \frac{\Delta V_i(s)}{\Delta V_i(s)} = \frac{C r_c s + 1}{P(s)} D,$$

and

$$P(s) = L C s^2 + C (r_r + r_c) s + 1.$$

The device used in the study is TPS40170EVM-758 by Texas Instruments, whose input voltage is 12V, the output 5V, $L=8.2\mu H$, $C=64\mu F$, and the PWM frequency 300kHz. Transfer function $G_i(s)$ is identified experimentally [14] as
\[ G_s(s) = \frac{5.305 \times 10^{-7} s + 1}{2.585 \times 10^{-20} s^3 + 1.961 \times 10^{-6} s + 1}. \] (6)

In the rest of the paper, the deviation \( \Delta \) terms will be denoted by the lower-case letters.

2.2. Analog Control

The control objective is to maintain the output voltage constant under changes in the load-current. As shown in Figure 2, the analog controller senses the output voltage and compare it with the reference voltage. Their difference is used to adjust the duty ratio to the analog PWM, which in turn activates the gate-driver to switch the transistor S1 and S2 so that the supply voltage of 12V is regulated down to 3V. The analog controller used is TPS40170, for which the frequency response was measured and the transfer function derived [12] as

\[ C(s) = \frac{6728 \times 8.331 \times 10^{-10} s^2 + 5.793 \times 10^{-5} s + 1}{s(6.608 \times 10^{-13} s^2 + 1.626 \times 10^{-6} s + 1)}. \] (7)

The gain margin is 15dB, the phase margin 54 degrees, and the bandwidth 43.5kHz. Experiments have been conducted when the load-current is changed from 2A to 4A. A typical result is shown in Figure 3, where the maximum amplitude of the deviation is about 140mV from the nominal value of 3V (4.7%).

![Figure 2. The buck converter under analog control.](image)

![Figure 3. Output-voltage regulation under analog control for load-current change from 2A to 4A](image)

2.3. Tustin Discretization

The most popular Tustin’s method, where analog controller blocks are individually discretized, by substituting

\[ s = \frac{\varepsilon}{T} \frac{z - 1}{2 + \varepsilon}, \quad \varepsilon = \frac{z - 1}{T} \] (8)

with \( z \) the usual \( z \) operator and \( T \) the sampling period [15]. These discrete-time blocks can be implemented as in Figure 4. It is known that its performance approaches that of the analog original as the sampling rate increases. However, unless the sampling rate is sufficiently high, its performance is usually poor and can even be unstable. The device used for digital controllers converts 12V supply to 5V output, while the analog version is from 12V to 3V. Figure 5 shows the result of Tustin control running at 300kHz. The output voltage is oscillatory and is not regulated.
Figure 4. The buck converter under digital control.

Figure 5. Output-voltage regulation under Tustin control at 300kHz for load-current change from 2A to 4A.

2.4. IA-PIM Discretization

The IA-PIM method [16] is a digital redesign method, where a type-one analog controller is discretized into an equivalent type-one digital controller with guaranteed stability for non-pathological sampling intervals. Thus, the IA-PIM method is used to convert an analog controller in a stable manner at a low sampling frequency. The backbone of this method is the PIM, which is briefly reviewed in the following:

For a plant given by

\[ G(s) = \frac{n(s)}{d(s)}, \]

with \( u(t) \) the input and \( y(t) \) the output, an analog controller (Figure 6) is designed such that the transfer function from the reference input \( r(t) \) to the plant input \( u(t) \) can be written as

\[ M(s) = \frac{u(s)}{r(s)} = \frac{A(s)C(s)}{1+G(s)C(s)B(s)}. \]

This is called the plan-input transfer-function [17]. In essence, the plant-input-mapping (PIM) method uses the step-invariant-model of the plant, which is given by

\[ G(\varepsilon) = \text{SIM}\{G(s)\} = \frac{n(\varepsilon)}{d(\varepsilon)}, \]

and the matched-pole-zero model of the plant-input transfer-function, given by

\[ M(\varepsilon) = \text{MPZ}\{M(s)\} = \frac{\mu(\varepsilon)d(\varepsilon)}{d_m(\varepsilon)}. \]

The PIM digital controller is realized as in Figure 7 [16], where each control block is determined by

\[ A(\varepsilon) = \frac{\mu(\varepsilon)}{\lambda(\varepsilon)}, B(\varepsilon) = \frac{\beta(\varepsilon)}{\lambda(\varepsilon)}, C(\varepsilon) = \frac{\lambda(\varepsilon)}{\alpha(\varepsilon)}. \]

\[ d_m(\varepsilon) = \beta(\varepsilon)m(\varepsilon) + \alpha(\varepsilon)d(\varepsilon). \]

These choices achieve eq. (10). The numerator \( \mu \) of block A is the part of the numerator in Eq. (7) that is not the denominator of the SIM of the plant Eq. (3). This is called the controller zero principle [18]. Eq. (14) is the design equation to be solved for polynomials \( \alpha \) and \( \beta \). \( \lambda \) is an arbitrary stable...
observer polynomial of appropriate degree [16]. A general PIM method may change the system types before and after discretization. The IA-PIM method [16] solves Eq. (7) such that the resulting block $C$ contains the pole at the origin, making the digital controller type-one.

The IA-PIM controller is designed for the plant, eq. (2), and the controller, eq. (7), at the sampling frequency of 25kHz as

$$B(\varepsilon) = \frac{-1.064 \times 10^{-6} \varepsilon^4 + 4.520 \times 10^{-5} \varepsilon + 1}{6.400 \times 10^{-10} \varepsilon^2 + 1.600 \times 10^{-7} \varepsilon + 1}$$

$$C(\varepsilon) = \frac{6.400 \times 10^{-6} \varepsilon^2 + 1.600 \times 10^{-4} \varepsilon + 1}{\varepsilon(1.461 \times 10^{-7} \varepsilon^2 + 8.548 \times 10^{-5} \varepsilon + 1)}$$

and implemented as shown in Figure 8 at the. As shown in Figure 9, although the resulting output voltage is stabilized at this low sampling frequency, there is a large transient of about 800mV (16%).

The performance of the IA-PIM might improve if the processor were capable of running the control algorithm at a faster rate. However, the CPU used for experiments can handle only up to about 100kHz, at which the result is better but the maximum deflection is still large with the 680mV amplitude (13.6%), as seen in Figure 10.
3. High-rate disturbance alleviation

Since the processor can handle extra computations using the IA-PIM method with 25kHz, a simple algorithm can be added. As it is suspected that a faster sampling rate is required to counteract loading disturbances, three approaches are attempted at a higher sampling rate of 300kHz; two by mimicking current feedback and one by previewing the disturbance using an inverse plant-model. In all cases, the disturbance characteristics of the plant is to be improved and other properties, such as stability and steady-state performances, are not intended to be changed, so that the original controller block $C(s)$ remains to meet requirements other than those for the disturbance. However, $C(s)$ can still be redesigned using the modified plant if desired or necessary. This is not carried out in the present study so that comparisons can be made more easily on the same bases.

3.1. Addition of Current Feedback

Methods 1 and 2 are based on feeding back the inductor current in a faster inner-loop, which modifies the plant. By adding the inductor-current feedback as shown in Figure 11, the characteristic polynomial of the plant, eq. (5), is modified as

$$\hat{P}(s) = LCs^2 + C(r_L + r_T + KV)s + 1,$$

which could increase the damping ratio for a positive gain $K$. When designing IA-PIM controller using the same $C(s)$ for this modified plant model, eq. (17) is used in both Methods 1 and 2 to be explained next.

3.2. Method 1: Current Observer

Since no extra sensor is available to measure directly the inductor current, it is estimated by using the state-observer or obtained through a filter. In Method 1, the averaged state-space model mentioned in
Section 2.1 is used to design a full-order (second-order) state-observer [15] for estimating and feeding back the inductor current, ignoring the loading-current fluctuation in these equations. It is then discretized as the step-invariant model in the shift-form at 300kHz sampling frequency, as

\[
\begin{bmatrix}
i_L(k) \\
v_c(k)
\end{bmatrix} = \begin{bmatrix}
0.8280 & -18.75 \\
0.03657 & -0.8280
\end{bmatrix}
\begin{bmatrix}
i_L(k) \\
v_c(k)
\end{bmatrix} + \begin{bmatrix}
18.34 \\
0.1264
\end{bmatrix} y(k) + \begin{bmatrix}
4.838 \\
0.1264
\end{bmatrix} u(k),
\]

(18)

where \( u \) is the change in the duty ratio and \( y \) the output voltage. This is then implemented as shown in Figure. 12. This modifies the plant model and eq. (17) is used for both Methods 1 and 2 to redesign the IA-PIM controller at 25kHz, and gives

\[
B(\varepsilon) = \frac{3.628 \times 10^{-5} \varepsilon^2 + 1.306 \times 10^{-4} \varepsilon + 1}{2.56 \times 10^{-3} \varepsilon^2 + 3.2 \times 10^{-4} \varepsilon + 1}
\]

(19)

\[
C(\varepsilon) = 1281 \frac{2.56 \times 10^{-3} \varepsilon^2 + 3.2 \times 10^{-4} \varepsilon + 1}{\varepsilon (8.151 \times 10^{-3} \varepsilon^2 + 5.832 \times 10^{-5} \varepsilon + 1)}
\]

(20)

A typical experimental result is shown in Figure. 13 with \( K=0.12 \). It shows that the maximum amplitude of deviation is reduced to almost one-third (6.4%) during the transient stage of the load-current disturbance compared with one in Figure. 10. However, the output fluctuation continues into the steady-state phases.

It is suspected that a computational delay associated with the second-order observer affect such fluctuation and a lower order approach is attempted next.

![IA-PIM 25kHz + Current Observer 300kHz](image)

**Figure 13.** Output-voltage regulation under IA-PIM control at 25kHz with current observer at 300kHz.

3.3. **Method 2: Current Filter**

Rather than using the second-order state-observer, the first-order state-equation can be extracted [14] and Laplace-transformed to obtain the current filter (Figure. 14) as

\[
i_L(s) = \frac{C_s}{Cr_c s + 1} v_c.
\]

(21)

Taking noises into account, the low-pass filter with the corner frequency twice the system bandwidth is added, resulting in

\[
KH(s) = 0.12 \frac{6.4 \times 10^{-3} s}{3.183 \times 10^{-5} s + 1}.
\]

(22)
This inner-loop current filter is discretized using the Tustin’s method at 300kHz and implemented. As for the outer-loop, the same controller \( C(s) \) is used with the modified plant of eq. (15) for IA-PIM at 25kHz, resulting in the same block as given by eq. (19) and (20).

A typical experimental result is shown in Figure 15. It is observed that the maximum deviation during the transient is 200mV (4%) and the output voltage regulation is satisfactory. A better performance may be achieved by re-designing \( C(s) \), which was not attempted this time for comparison with other two methods and left for further studies.

![Figure 15. Output-voltage regulation under IA-PIM control at 25kHz with current filter at 300kHz.](image)

3.4. Method 3: Inverse Filter

To improve the disturbance rejection property, the inverse model of the plant is used in the third method, rather than using current-based feedback. An estimate \( \hat{d} \) of the actual disturbance \( d \) may be calculated from the measured output \( v_o \) and the available plant input \( u \), as

\[
\hat{d}(s) = v_o - \hat{G}_i^{-1} u ,
\]

where \( \hat{G}_i^{-1} \) is the inverse of the plant model. For most plant, this inverse is non-proper and needs to be filtered to make this to be bi-proper, as explained shortly. Now, let \( f \) be the fictitious portion of the plant input that is capable of causing the plant output equivalent to the disturbance \( d \). Subtracting \( f \) from the plant input a priori, the effect of the actual disturbance \( d \) on the plant output can be expected to be reduced. Figure 16 shows such a control system, which includes an algebraic loop. However, when implemented in a discrete-time form, the computation of \( f \) introduces one sample delay, which makes the loop dynamic.

Since the inverse filter \( G_i^{-1}(s) \) is improper and cannot be implemented on-line, a low-pass filter \( F(s) \) given by

\[
F(s) = \frac{1}{3.183 \times 10^{-4} s + 1}
\]

is multiplied to make it bi-proper and then discretized at 300kHz using the Matched-Pole-Zero method [17], so that

\[
G_i^{-1}(e) = 0.0833 \frac{5.273 \times 10^{-10} e^2 + 4.263 \times 10^{-6} e + 1}{1.715 \times 10^{-11} e^2 + 8.475 \times 10^{-6} e + 1}.
\]

(For some reason, Tustin’s method does not work well.) The IA-PIM controllers for the plant \( G_i \) at the sampling frequency of 25kHz are the same as those given by eqs. (15) and (16). A typical
experimental result is shown in Figure. 17, which illustrates that after the load current changes, the output voltage fluctuates and the transient dip has the magnitude of 230 mV (4.6%).

**Figure 17.** Output-voltage regulation under IA-PIM control at 25 kHz with inverse filter at 300 kHz.

4. Conclusion

Through dual-rate digital control, improvements on disturbance rejection properties have been attempted for a switching DC-DC buck converter. As in the previous study, a slower outer-loop controller was designed by discretizing an existing analog controller, which satisfies all the design requirements, using the so-called IA-PIM method, which guarantees the stability for any non-pathological sampling interval. In the present study, to suppress the amplitude of a sudden dip in the output voltage for a change in the load-current, a faster inner-loop controller was added. For this purpose, three types of digital controllers have been designed and tested experimentally; those using the current-observer, the current-filter, and the inverse-filter. The results obtained using the current-filter was superior to the current-observer and the inverse-filter, at this stage. However, further tuning of parameters in the digital controllers should be attempted. Besides, since the analog performance is achieved by fine-tuning various component values, a better digital performance may be possible by re-tuning the parameters and taking some measures to counter such issues as computational and conversion delays and quantization errors. Larger transients with the digital controllers are more eminent in high-frequency applications as in the present study. Nevertheless, since the conventional Tustin method running at 300 kHz did not provide stable results, the stable performance of the proposed methods at 25 kHz is a good start. In terms of percentage maximum fluctuation from the output voltage, better performances of the digital regulator (4% and 4.6%) than those of the analog regulator (4.7%) have been experimentally confirmed. However, the digital performance has small ripples compared with the analog performance. The approaches presented in the present study are based on pole-relocations using feedback. Those based on zero-relocations using feedforward are under way, to further improve the digital performance.

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