Influence of statistical sequential decay on isoscaling and symmetry energy coefficient in the GEMINI simulation

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Extensive calculations on isoscaling behavior with the sequential-decay model GEMINI are performed for the mediate-heavy nuclei in the mass range $A = 60$–120 at excitation energies up to 3 MeV per nucleon. The comparison between the products after the first-step decay and the ones after entire-steps decay demonstrates that there exists strong sequential decay effect on the final isoscaling parameters and the apparent temperature. Results show that the apparent symmetry energy coefficient $\gamma_{app}$ does not reflect the initial symmetry energy coefficient $C_{sym}$ embedded in the mass calculation in the present GEMINI model.

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One of main goals of the isospin physics is to determine the isospin dependence of the in-medium nuclear effective interactions and the equation of state (EOS) of isospin asymmetric nuclear matter or finite nuclei, particularly its isospin-dependent term, i.e., the density dependence of the nuclear symmetry energy. Knowledge of nuclear symmetry energy is essential for understanding not only many problems in nuclear physics, such as the dynamics of heavy-ion collisions induced by radioactive beams and the structure of exotic nuclei, but also a number of important issues in astrophysics, such as the supernova simulation and neutron star models, which require inputs of the nuclear equation of state at extreme values of density and asymmetry [1, 2]. Recently impressive progress has been made both experimentally and theoretically, a number of earlier reviews on isospin physics with heavy-ion reactions can be found in several references [3–5].

Symmetry energy could be extracted from heavy-ion collision using the isoscaling approach [6–15]. Isoscaling law means that the ratio of isotope yields $R_{21}(N, Z) = Y_2(N, Z)/Y_1(N, Z)$, from two similar reactions, denoted as reaction 1 and 2, which are different only in their isospin asymmetry, is found to exhibit an exponential relationship as a function of the neutron number $N$ and proton number $Z$ [6], i. e.,

$$R_{21}(N, Z) = \frac{Y_2(N, Z)}{Y_1(N, Z)} = C \exp(\alpha N + \beta Z), \quad (1)$$

where $Y_2(N, Z)$ and $Y_1(N, Z)$ are the fragment yields from the neutron-rich and the neutron-deficient reaction, respectively. $C$ is an overall normalization factor, and $\alpha$ and $\beta$ are fitted parameters. The isoscaling parameter $\alpha$ is related to the symmetry energy coefficient $C_{sym}$ of EOS in microcanonical and canonical frames by following relation [6],

$$\alpha = \frac{4C_{sym}}{T} \left( \frac{Z}{A} \right)^2 - \left( \frac{Z}{A} \right)_{s1}^2 \equiv \frac{4C_{sym}}{T} \Delta \left( \frac{Z}{A} \right)_{s}, \quad (2)$$

and

$$\beta = \frac{4C_{sym}}{T} \left( \frac{N}{A} \right)^2 - \left( \frac{N}{A} \right)_{s1}^2 \equiv \frac{4C_{sym}}{T} \Delta \left( \frac{N}{A} \right)_{s}, \quad (3)$$

where $Z_{s1}$, $Z_{s2}$, $N_{s1}$, $N_{s2}$, $A_{s1}$, $A_{s2}$ are the charge number, neutron number and mass numbers of the sources from the two systems, $T$ is their temperature and $C_{sym}$ is the symmetry energy coefficient. This relation has also been evidenced in other model frameworks [7–15]. A great deal of effort has been devoted to investigate the nuclear symmetry energy and its density/temperature dependences [3, 5, 16–19].

Ideally, primary fragments should be detected right after emission in order to extract information about the collisions, and Eq. (2) is derived based on the primary reaction products bypassing secondary decays. However, the detected experimental data are for cold products after the secondary decays from hot products. Isoscaling has also been reasonably reproduced in the sequential decay codes [15, 20]. However there are still arguments on the sequential decay effect on isoscaling, some models show that the effect from sequential decays on isoscaling is negligible, but some efforts show that sequential decay affects on the isoscaling parameters, and then distort the extraction of symmetry energy coefficient $C_{sym}$ [21, 22]. There are some issues still keep unsolved or unclear, such as the sequential decay effect on isoscaling parameters, derived apparent temperature $T_f$ from the experimental measurement and the isospin evolution of the decaying sources, these factors affect the extraction of symmetry energy coefficient $C_{sym}$.

The statistical GEMINI model [23] calculates the decay of compound nuclei by modes of sequential binary decays. The model employs a Monte Carlo technique

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to follow the decay chains of individual compound nuclei through sequential binary decays until the resulting products are unable to undergo further decay. GEMINI has been widely used to simulate the hot equilibrium source de-excitation, or as an "afterburner" code to analyze the hot fragments decay after dynamical simulation [24–26]. Isoscaling has been investigated by statistical sequential secondary decay code GEMINI [23], in which only the first-step sequential decay was simulated and Eq. (2) was confirmed for the fragments which are decayed directly from the initial sources [15]. In the present work, we investigate the entirely decayed fragments from excited sources, comparing with the only first-decay fragments from the same source. The influences of sequential decays on isoscaling parameters $\alpha$, $\beta$, and the apparent temperature $T_r$ are discussed, and the apparent symmetry energy coefficient $\gamma_{app}$ is extracted.

The detailed description of GEMINI code can be found in [15, 23], the same configuration and parameters of the GEMINI code were adopted as in Ref. [15]. Several pairs of equilibrated sources are considered at various initial excitation energy $E_{ex}=1.0$, 1.4, 2.0, 2.4, and 3.0 MeV/nucleon. We selected source pairs with the same proton number $Z_s$ but different mass number $A_s$ to systematically study the isoscaling behavior. In this case, possible effects of different magnitudes of Coulomb interaction on isotopic distributions are avoided. The equilibrated source pairs are chosen in different mass region and system isospin asymmetry $N/Z$. Two groups of the source pairs have been used: (1) $Z_s=50$ with $A_s=100$, 105, 110, and 115, respectively; (2) $Z_s=30$ with $A_s=60$, 63, 66, and 69, respectively. Following literature the index "2" denotes more neutron-rich system as widely used in convention, and index "1" denotes the more neutron-deficient system. In our previous work [15], the statistical decay stops after one particle emitted from the source, that was called the "first-step" decay in this paper, which is a simple picture and the decay procedure can be expressed definitely and clearly, isoscaling has been confirmed for the first-step decay products in detail, and the reasonability of extracting symmetry energy coefficient $C_{sym}$ from the simulation results via experimental analysis technique. But the first-step decay was not the real case, experiments measure the final products after multi-step decays until no fragments produced or gamma rays emitted, which was called "entire-steps" decay in this paper.

Isoscaling is analyzed from the emitted light fragments in above both cases, namely first-step decay only and entire-steps decay chains for all the simulated systems. As an example shown in Fig. 1 the comparison of the isoscaling parameters $\alpha$ and $\beta$ is plotted as a function of emitted light fragment proton number $Z$ and neutron number $N$, between the first-step decay and entire-steps decay fragments for the source pairs of $Z_s = 50$. As we can see, isoscaling parameters $\alpha$ and $\beta$ are essentially independent on $Z$ or $N$ of fragments, for both first-step and entire-steps decay fragments, especially for $Z \leq 4$ or $N \leq 5$. It has been evidenced that in the first-step decay case, the probability of producing a cluster with a given $Z$ and $A$ at $T$ depends exponentially on the free energy of that cluster, $F(Z, A, T)$, in GEMINI simulation, the cluster free energies depend on the strength of the symmetry term in the liquid-drop energy through the Eq. (2) with $C_{sym} \approx 24$ MeV[15]. If the entire-steps sequential secondary decay is included, the source isospin $N/Z$ and temperature $T$ as well as isoscaling parameters vary after each step of decay. Similarly, in previous study, time dependence of isoscaling parameters have been discussed in molecular dynamics model [14], indicating that the final values of these parameters could be related to the last part of the reaction where the fragments finish cooling by particle evaporation. In present GEMINI model, fragment yields are strongly expected degraded after entire-steps decay, so do the isotopic yield ratios. It is not surprising that in Fig. 1 that parameters $\alpha$ and $\beta$ extracted from isotopic yield ratios of the final emitted light fragments show discrepancy, especially for the intermediate mass fragments like $Z \geq 5$ in some cases, since those heavier fragments experience strong multi-step decay and feeding-down effects. Finally, $\alpha$ and $\beta$ change a lot comparing with the only first-step decay case, and the fluctuation increases in the entire-steps decay case. Average $\alpha$ and $\beta$ values over fragments $Z$ and $N$ are used in following discussions to observe the overall property.

In Fig. 2 we show the comparison of isoscaling parameters $\alpha$ and $\beta$ as a function of excitation energy of sources from different source pairs. If the entire-steps decay chains are included, which are depicted by the open symbols in Fig. 2, $\alpha$ and $|\beta|$ values show significant decrease. This reduction is about 20% in average, consistent with the result in Ref. [20, 21] for the $C_{sym} \approx 25$ MeV case, but the excitation energy dependent trend of $\alpha$ and $\beta$ does not change.
As already discussed, the isoscaling parameter $\alpha$ is related to the symmetry energy coefficient $C_{sym}$ of the nuclear binding energy through Eq. (2) and (3), this relation provides a direct link between the measurable quantities and the nuclear symmetry energy coefficient. It should be noticed that the parameters $\alpha$ and $\beta$ refer to the hot primary fragments, which have to undergo sequential decays into the cold fragments. It was assumed that the secondary decay on the yield of a specific isotope is similar for the two reactions, thus the effect of the sequential decays on $R_{21}(N, Z)$ is small and that $R_{21}(N, Z)$ reflects the properties of the primary source. In our present study, the first-step statistical sequential decay process stems from a fixed initial source with definite excitation energy (temperature) and isospin asymmetry, and it has been verified by Eq. (2) and Eq. (3), to reflect the link between the measurable quantity $R_{21}(N, Z)$ and the symmetry energy coefficient $C_{sym}$.

It has been found in Fig. 1, the isoscaling behavior still presents after considering the entire-steps decay chains. But in this case, the isoscaling parameters $\alpha$ and $\beta$ decrease comparing with the only first-step decay case as seen in Fig. 2. In the statistical sequential decay, the source temperature $T$ and isospin asymmetry $N/Z$ also change after each step of the decays, thus the parameters $T$ and $\Delta(Z/A)^2$ (or $\Delta(N/A)^2$) varies during the sequential decay process, where many intermediate sources are different from the initial source.

To explore the validity of Eq. (2) and (3) in the entire-steps statistical sequential decay, we plot $\alpha T$ as a function of $\Delta(Z/A)^2$ and $\beta T$ as a function of $\Delta(N/A)^2$ in Fig. 3. For the first-step only decay, data points depicted by the solid symbols, with using the initial source temperature $T_i$ which was calculated by input excitation energy as shown in Table I and isoscaling parameters after the first step decay. The linear fit (the solid line) gives a symmetry energy coefficient $C_{sym} = 24.2 \pm 0.3$ MeV.

To investigate the decay effect on symmetry energy coefficient, data points from the entire-steps decays are also plotted in Fig. 3 as shown by the open symbols. As we have mentioned for the entire-steps decay chains, there are many intermediate-state sources with different temperature $T$ and isospin asymmetry $Z/A$ or $N/A$. Nevertheless, the final isoscaling parameters $\alpha$ and $\beta$ still show similar rules as the first-step decay only case, i.e. $\alpha T$ and $\Delta(Z/A)^2$ (or $\Delta(N/A)^2$) can be still fitted by another linear function, namely

$$\alpha = \frac{4\gamma_{app}}{T} \left[ \left( \frac{Z}{A} \right)_{s1}^2 - \left( \frac{Z}{A} \right)_{s2}^2 \right] = \frac{4\gamma_{app}}{T} \Delta \left( \frac{Z}{A} \right)_s^2,$$  

and

$$\beta = \frac{4\gamma_{app}}{T} \left[ \left( \frac{N}{A} \right)_{s1}^2 - \left( \frac{N}{A} \right)_{s2}^2 \right] = \frac{4\gamma_{app}}{T} \Delta \left( \frac{N}{A} \right)_s^2.$$  

which gives the apparent symmetry energy coefficient $\gamma_{app} = 19.65 \pm 0.25$ MeV if the same $T_i$ are used.

In fact, for the source decays with entire steps, the intermediate-state sources have different isospin asymmetry $N/Z$ ranging from initial isospin asymmetry to the...
stable line or the evaporation attract line [27] and different temperature \( T \) ranging from initial temperature to zero. In this case, principally both \( T \) and \( \Delta(Z/A)^2 \) need to be corrected to reflect the intermediate sources. In the simulation, the initial source temperature \( T \) can be calculated, and the intermediate source tracing the sequential decay chains can also be performed. From an experimental point of view, temperature and isospin asymmetry of the intermediate source can be extracted from evaporation products which reflects the whole decay chains.

Traditionally, temperature can be extracted from the measurements of spectral slopes or double isotopic ratios at lower energies [28, 29]. In the present work, initial temperature \( T_i \) are calculated directly in the GEMINI code by the input excitation energy [15], and the final-state temperature \( T_f \) can be obtained by the neutron and proton spectra fitting when the entire-steps decay chains are included in the GEMINI calculation. The results are displayed in TABLE. I.

When the temperature \( T_f \) is used in Eq. (4) and (5) to fit the linear slope parameter \( \gamma_{app} \), it leads to the production of the parameter \( \gamma_{app} \) as shown in Fig. 3 (short dashed line). Its slope gives an apparent symmetry energy coefficient \( \gamma_{app} = 15.84 \pm 0.18 \) MeV, which is one-third reduction comparing with the only first-step decay case. In this context, we should be careful to use the apparent symmetry energy derived directly from the final fragments which could be distorted due to the multi-step sequential decays. Of course, the present results are specific for the use of GEMINI to describe the secondary decay, i.e. they may depend on the details of the sequential decay code.

In summary, we performed the isoscaling analysis for both light fragments from only the first-step decay and the entire-steps decay chains with GEMINI code, it is found that isoscaling can still be observed and the Eq. (4) and (5) which are used to extract the symmetry energy coefficient also work after the entire-steps decay is taken into account. However, the statistical sequential decay leads to the decreasing of isoscaling parameters \( \alpha \) and \( \beta \) as well as temperature. Therefore, the reduced (apparent) source temperature together with the reduced isoscaling parameters leads to a smaller symmetry energy parameter \( \gamma_{app} \) in comparison with the initial symmetry energy coefficient \( C_{sym} \), which is constrained from the only first-step statistical decay calculation. From the present GEMINI model calculations, we shall carefully consider the multi-step sequential decay effect on the extraction of the symmetry energy coefficient via the final cold products.

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| \( E_{ex} \) | \( A \_s \) | \( T_i \) | \( T_f \) | \( T_s \) | \( T_r \) (MeV) |
|---|---|---|---|---|---|
| 60/63 | 2.8/2.9 | 2.3/2.4 | 2.9/2.9 | 2.4/2.4 |
| 60/63 | 3.4/3.5 | 2.8/2.9 | 3.5/3.5 | 2.9/3.0 |
| 66/69 | 4.1/4.2 | 3.5/3.6 | 4.2/4.2 | 3.6/3.6 |
| 66/69 | 4.5/4.6 | 3.8/3.9 | 4.6/4.7 | 3.9/4.0 |
| 4.5/4.6 | 4.3/4.4 | 5.2/5.3 | 4.4/4.5 |

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