Theoretical and experimental study on magnetic-fluid-based flow sensors

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The dependence of flow volume on the pressure difference between the ports of a U-tube was determined for both laminar and turbulent flows of a magnetic fluid. The results showed that the dependence was linear in the case of laminar flow but was non-linear in the case of turbulent flow. In addition, the inductance and the voltage difference across two coils around the arms of the U-tube were calculated. The voltage difference was proportional to the flow volume and inversely proportional to the square of the coil length. These theoretical and experimental results demonstrate that the design of magnetic-fluid-based flow sensors is feasible.

magnetic fluid, flow, sensor, inductance

The development of magnetic-fluid-based sensor technology is a novel research topic [1]. In principle, a magnetic fluid based flow sensor is a device that determines the flow parameters of a medium in a pipe by making direct measurements of the magnetic field changed by the movement of magnetic fluid.

Typically, a magnetic fluid is a stable colloidal dispersion of nanoscale ferromagnetic particles in a carrier liquid [2]. For the purposes of our study, we use a liquid composed of particles roughly 10 nm in diameter. They are solid, magnetic, and single-domain. The particles are coated with a molecular layer of dispersants and suspended in a carrier liquid [3]. Figure 1 shows the composition of the magnetic fluid. The magnetic fluid has a high magnetization saturation intensity, stability at working temperature, no precipitation in gravitational or electromagnetic fields, perfect fluidity, and is paramagnetic.

Due to the unique properties of magnetic fluids, devices based on these fluids have applications in mechanical systems, electronics, shipping, spaceflight, distance measurement, manufacturing and medical treatments. Typically, magnetic fluids would be used in the fields of sensors or other metering devices [4–9].

International researchers have already demonstrated many applications of magnetic fluids. Acceleration sensors, angle sensors, and pressure difference sensors based on magnetic fluids have been demonstrated [10]. The effect of device size and fluid parameters on sensitivity has also been well explored [11–14]. The interplay between magnetic and electric fields has already been leveraged to create a magnetic-fluid-based electric-field sensor [6]. In addition, an inertial sensor...
An operational schematic of the flow sensor is shown in Figure 2. A medium, such as water, is transported in the pipe. From the Darcy-Weisbach formula [21], the pressure difference at the ports is

\[ \Delta p = \frac{\lambda \rho v^2}{2d}, \]  

where, \( l \) is the length of the pipe, \( \rho \) is the density of the liquid, \( v \) is the velocity of the liquid, \( d \) is the inner diameter of the pipe, and \( \lambda \) is the Darcy-Weisbach friction factor.

1.1 Laminar flow

For Poiseuille flow, the Reynolds number is

\[ Re = \frac{\rho dv}{\mu}, \]

then

\[ \lambda_1 = \frac{64}{Re} = \frac{64\mu}{\rho dv}, \]

where \( \mu \) is the viscosity of the liquid. Then

\[ \Delta p = \frac{\lambda_1 \rho v^2}{2d} = \frac{32\mu v}{d^2}. \]

The velocity of the liquid is

\[ v = \frac{\Delta p \cdot d^2}{32\mu l}. \]  

The flow volume is

\[ q_1 = \frac{\Delta p \cdot \pi d^4}{128\mu l}. \]  

If \( K_1 = \frac{\pi d^4}{128\mu l} \), then

\[ q_1 = K_1 \Delta p. \]  

For our investigation of laminar flow, the distance between the two measurement points was 1 m. The diameter of the pipe was 40 mm. The transport liquid was water. Figure 3 shows the relationship between the flow volume and the pressure difference between the two points. In the allowed error range, the flow volume is linearly proportional to the pressure difference of the two measured points.

1.2 Turbulent flow

For turbulent flow, the Reynolds number is

\[ Re = \frac{\rho dv}{\mu}, \]

where \( v \) is the average velocity of the water in the pipe.

If \( f \) is the relative degree of roughness of the pipe, the friction factor [21], \( \lambda_2 \), is

\[ \lambda_2 = 0.25\left[ \log\left( \frac{f}{3.7} + \left( \frac{5.74}{Re^{0.8}} \right) \right) \right]^2. \]

The flow volume is

\[ q_2 = \frac{\pi d^2}{4} \sqrt{\frac{2d \cdot \Delta p}{\lambda_2 \rho l}}. \]  

If \( K_2 = \frac{\pi d^2}{4} \sqrt{\frac{2d}{\lambda_2 \rho l}} \), then

\[ q_2 = K_2 \Delta p. \]  

Figure 3  Flow volume versus pressure difference for laminar flow.
then

$$q_i = K_i \sqrt{\Delta p}.$$  \hspace{1cm} (11)

From eqs. (7) and (11), flow volume is related to the inner diameter of the pipe, $d$, and the pressure difference, $\Delta p$.

For our investigation of turbulent flow, the distance between the two measurement points was 1 m. The diameter of the pipe was 40 mm. The transport liquid was water. Figure 4 shows the relationship between the flow volume and the pressure difference between the two points.

From this simulation, we demonstrate that for laminar flow the ratio of the flow volume to the pressure difference is linear.

## 2 Output of magnetic-fluid-based sensor inductance

We begin with no height difference in the magnetic core in the U tube. Therefore the pressure difference, $\Delta p$, is zero and the inductances of the two coils are equal. When the pressure difference $\Delta p \neq 0$, the $\Delta h \neq 0$, the flux [22,23] of the coil is:

$$\phi(x) = \frac{2N^2 SL}{I^2} \left[ \mu_0 (l-x) + \mu_m x \right].$$  \hspace{1cm} (12)

where $\mu_0$ and $\mu_m$ are the magnetic permeability of the air and the magnetic fluid, respectively, $x$ is the distance between the magnetic fluid surface and the bottom of the coil, $N$ is the turn number of the coil, $S$ is the area of the magnetic fluid core, $l$ is the length of the coil, and $I$ is the current in the coil and can be written as $I_0 \sin wt$.

The inductance of the coil is

$$L(x) = \frac{2N^2 SL}{I^2} \left[ \mu_0 (l-x) + \mu_m x \right].$$  \hspace{1cm} (13)

If $x=x_0=l/2$, the inductance of the two coils is

$$L_1 = L_2 = \frac{NS}{I} (\mu_m + \mu_0).$$  \hspace{1cm} (14)

If the height of the magnetic fluid in the left arm of the U tube decreases by $\Delta x$, and the height in the right are increases by $\Delta x$, the inductance of the coil will be

$$L(x) = \frac{2N^2 S L w \cos wt}{I^2} \left[ \mu_0 (l-(x-\Delta x)) + \mu_m (x+\Delta x) \right].$$  \hspace{1cm} (15)

And then the inductance difference will be

$$\Delta L = \frac{2N^2 S L w \cos wt}{I^2} \left( \mu_0 + \mu_m \right) \Delta h,$$  \hspace{1cm} (16)

where $\Delta h = 2 \Delta x$.

Then the voltage difference of the coil is

$$\Delta U = \frac{2N^2 S L w \cos wt}{I^2} \left( \mu_0 + \mu_m \right) \Delta h,$$  \hspace{1cm} (17)

where $I_0$ and $w$ are the amplitude and the frequency of the excited current, respectively.

The height difference of the magnetic fluid in the U tube is

$$\Delta h = \frac{\Delta p}{(\rho_m - \rho_0) g},$$  \hspace{1cm} (18)

where $\rho_m$ and $\rho_0$ are the density of the magnetic fluid and air, respectively, and $g$ is the acceleration due to gravity.

Then the voltage difference of the coil is:

$$\Delta U = \frac{2N^2 S L w \cos wt (\mu_0 + \mu_m)}{I^2} \Delta p,$$  \hspace{1cm} (19)

$$\Delta p = \frac{\Delta U (\rho_m - \rho_0) g l^2}{2N^2 S L w \cos wt (\mu_0 + \mu_m)}.$$  \hspace{1cm} (20)

For laminar flow, combining eqs. (7) and (19) yields

$$\Delta U = \frac{2N^2 S L w \cos wt (\mu_0 + \mu_m) g}{K (\rho_m - \rho_0) g l^2}.$$  \hspace{1cm} (21)

From eq. (21), it can be seen that the output voltage difference is proportional to the input flow volume when the parameters of the coil and magnetic fluid are constant.

## 3 Experiment

The experimental results are shown in Figure 5. It can be seen that the output voltage difference is proportional to the input flow volume of the liquid.

## 4 Conclusions

For laminar flow, the flow volume is proportional to the pressure difference measured from the two arms of the U tube. For turbulent flow, the flow volume is proportional to the square root of the pressure difference. For laminar flow, the output voltage difference is proportional to the flow volume.
volume of the liquid. In addition, the greater the magnetic permeability of the magnetic fluid, the greater the ratio of the output voltage difference to the input flow volume will be. The voltage difference is linearly proportional to the pressure difference input in the two ports of the U tube, for a reasonable error allowance. From our calculation, we show that for certain parameters the output is proportional to the input of the flow sensor. Our theoretical and experimental results demonstrated that magnetic-fluid-based flow sensors are developmentally feasible and can be used in real-world applications.

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**Figure 5** Experimental data showing the relationship between the voltage difference and the flow volume.