Efficient Beamforming for MIMO Relaying Broadcast Channel with Imperfect Channel Estimation

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Abstract—We consider a multiple-input multiple-output (MIMO) relaying broadcast channel in downlink cellular networks, where the base station and the relay stations are both equipped with multiple antennas, and each user terminal has only a single antenna. In practical scenarios, channel estimation is imperfect at the receivers. Aiming at maximizing the SINR at each user, we develop two robust linear beamforming schemes respectively for the single relay case and the multi-relay case. The two proposed schemes are based on singular value decomposition (SVD), minimum mean square error (MMSE) and regularized zero-forcing (RZF). Simulation results show that the proposed scheme outperforms the conventional schemes with imperfect channel estimation.

Index Terms—MIMO relaying broadcast, MMSE receiver, RZF precoding, SINR, Singular value decomposition.

I. INTRODUCTION

In recent years, MIMO relay networks have drawn considerable interest due to the advantages to increase the data rate and extend coverage in the cellular edge. The MIMO relay network with perfect channel state information (CSI) have been studied in [1], [2]. In [1], the authors investigate the linear processing at relay for MIMO relay networks with fairness requirement. In [2], the authors investigate the regularized zero-forcing (RZF) precoder at relays, which is observed to have an advantage to the zero-forcing (ZF) and the matched filter (MF) precoders. But the RZF precoder is not optimized and constantly chooses one as the regularizing factor. The MIMO relaying broadcast network has been considered in [3], where the singular value decomposition (SVD) and ZF precoder are respectively used to the backward channels (BC) and the forward channels (FC) to optimize the joint precoding. The authors use an iterative method to show that the optimal precoding matrices always diagonalize the compound channel.

All the above works consider perfect CSIs. However, perfect CSI is usually difficult to be obtained for a practical system. In [4], MMSE based precoding has been considered in multiple antenna broadcast channel with imperfect CSI at the source. In [5], the authors optimized a QR based beamformings with imperfect R-D CSI due to large delay. Works for limited feedback in MIMO relay networks are studied in [6], [7], and in MIMO relaying broadcast channel are studied in [8]–[10]. In [8], the authors further study the impact of feedback bits of BC and FC on the achievable rates for the linear processing scheme in [3]. In [9], based on MMSE criteria, robust ZF precoding are considered at the relay using the limited feedback of CSI to the relay. But only imperfect forward channel (FC) is considered. In [10], the authors propose an MMSE based beamforming design in a MIMO relay broadcast channel with finite rate feedback.

In this paper, we study MIMO relaying downlink broadcast channel in a wireless cellular network. Focusing on linear beamformings, we propose a robust beamforming scheme considering both imperfect channel estimation at relay and user terminals. The proposed scheme is based on SVD-RZF for the single relay case and MMSE-RZF for the multi-relay case. By maximizing the derived signal-to-interference noise ratio (SINR), we optimize the MMSE receiver and RZF precoder. Simulation results show that the proposed robust SVD-RZF and MMSE-RZF outperform other conventional beamformers.

In this paper, boldface lowercase letter and boldface uppercase letter represent vectors and matrices, respectively. Notation $\mathbb{C}^N$ denotes an $N \times 1$ complex vector. The $\text{tr}(\mathbf{A})$ and $\mathbf{A}^H$ denote the trace and the conjugate transpose of a matrix $\mathbf{A}$, respectively. $(\mathbf{a})_k$ and $(\mathbf{A})_{j,k}$ represent the $k$-th entry of vector $\mathbf{a}$ and the $(j,k)$-th entry of matrix $\mathbf{A}$ respectively. $\mathbf{I}_N$ denotes the $N \times N$ identity matrix. Finally, we denote the expectation operation by $\mathbb{E}\{\cdot\}$.

II. SYSTEM MODEL

We consider a MIMO relaying broadcast network which consists a base station, $R$ fixed relays, and $K$ user terminals as depicted in Fig. 1. The base station is equipped with $M$ antennas, each relay is equipped with $N$ antennas and each user terminal only has a single antenna. It is supposed that $M, N \geq K$ so that the network can support $K$ independent data streams. A broadcast transmission is composed of two phases. During the first phase, the base station broadcasts $M$
precoded data streams to the relays after applying a linear precoder to the original data vector \( s \in \mathbb{C}^K \), where \( E\{ss^H\} = I_K \). We denote the precoding matrix at the base station as \( F \) and suppose that the base station transmit power is \( P_r \). Because we have \( \rho \sim \mathcal{CN}(0,1) \), we denote the precoding matrix at the base station as \( F \), \( F \) distributed with zero mean and unit variance, and \( \rho \) and \( \rho_s \) are respectively independent of \( \Omega_{1,r} \) and \( \Omega_{2,r} \).

\[
\rho_s = \sqrt{\frac{P_r}{\text{tr}(FF^H)}}.
\]

Denoting the received signal at the \( k \)-th user terminal as \( y_k \), the received vector at user terminals can thus be written as

\[
y = [y_1, y_2, \ldots, y_K]
\]

\[
y = \sum_{r=1}^{R} \rho_r G_r W_r (\rho_s H_r F s + n_r) + n_D,
\]

where \( n_D \in \mathbb{C}^K \) denotes the noise vector at the user terminals, in which, all entries are i.i.d Gaussian distributed with zero mean and \( \sigma_r^2 \) variance, \( G_r \) is the is the Rayleigh FC matrix of the \( r \)-th relay.\n
Considering imperfect channel estimation at both the relay and user terminals, we model the channel state information (CSI) as

\[
H_r = \hat{H}_r + e_1 \Omega_{1,r},
\]

where \( \hat{H}_r \) is the estimated CSI of the \( r \)-th relay to the \( k \)-th user channel. The entries of \( \Omega_{1,r} \) and \( \Omega_{2,r} \) are i.i.d complex Gaussian distributed with zero mean and unit variance, \( H_r \) and \( \hat{H}_r \) are the estimated CSIs and they are respectively independent of \( \Omega_{1,r} \) and \( \Omega_{2,r} \). \( e_1^2 \) and \( e_2^2 \) denotes the channel estimation error powers. We suppose that each user has the same channel estimation error power for simplicity.

III. SINR AT USER TERMINALS

Considering channel estimation errors, (3) becomes

\[
y = \sum_{r=1}^{R} \rho_r \hat{G}_r W_r \hat{H}_r F s + \rho_s \rho_r G_r W_r \Omega_{1,r} F + e_2 \Omega_{2,r} W_r \hat{H}_r F s
\]

\[
+ \rho_s \rho_r (\hat{G}_r + e_2 \Omega_{2,r}) W_r n_r + n_D,
\]

where we omitted the term involving \( e_1 e_2 \) because we assume \( e_1, e_2 \ll 1 \). We can write (9) as

\[
y = H_{\text{eff}} s + n,
\]

where \( H_{\text{eff}} \) is the first term and \( n \) is the rest terms in the right-hand-side of (6). Then the SINR at the \( k \)-th user terminal can be calculated by

\[
\text{SINR}_k = \frac{|(H_{\text{eff}})_{k,k}|^2}{\sum_{j=1,j\neq k}^{K} |(H_{\text{eff}})_{k,j}|^2 + E\{n_k n_k^*\}},
\]

where

\[
E\{n_k n_k^*\} = \sum_{r=1}^{R} \left( \frac{\epsilon_1^2 \rho^2 \rho_r^2}{K} \text{tr}(H_r H_r^H) \text{tr}(\hat{G}_r W_r W_r^H \hat{G}_r^H)
\]

\[
+ \epsilon_2^2 \rho^2 \rho_r^2 \text{tr}(\hat{H}_r F_s \hat{H}_r^H W_r W_r^H \hat{G}_r^H)
\]

\[
+ \frac{\rho^2 \sigma_r^2}{K} \text{tr}(\hat{G}_r W_r W_r^H \hat{G}_r^H)
\]

\[
+ \rho^2 \epsilon_2^2 \rho^2 \text{tr}(W_r \hat{H}_r F_s \hat{H}_r^H W_r W_r^H \hat{G}_r^H)
\]

\[
+ \sigma_r^2.
\]
IV. ROBUST BEAMFORMING DESIGN

A. SVD-RZF based design for the single relay case

If there is only one relay, for the first phase, the transmission is similar to a point-to-point MIMO system. Therefore, we propose an SVD-based beamforming for the backward channel [11]. Using singular value decomposition (SVD), the imperfect BC matrix can be decomposed as

\[ \hat{H} = \hat{H}_1 = U \Sigma V^H, \]

where \( U \in \mathbb{C}^{N \times N} \) and \( V \in \mathbb{C}^{M \times M} \) are both unitary matrices, and \( \Sigma = [\Theta(0)], \) with \( \Theta = \text{diag}(\sqrt{\theta_1}, \ldots, \sqrt{\theta_M}) \) and \( 0 \) being an \( N \times (N - M) \) zero matrix. Then, we propose the precoding matrix \( F \) at the base station as the first \( K \) columns of \( V \) and the receiving matrix \( W = W_1 \) at the relay as \( U^H \). Thus we have

\[ \rho_s = \sqrt{\frac{P_s}{1 + (F^H F)_{11}}} = \sqrt{\frac{P_s}{K}}. \]

For the second phase, the transmission is a broadcast channel. Instead of zero-forcing (ZF) or matched-filter (MF) in tradition, we design a robust regularized zero-forcing (RZF) precoder for the forward channel (FC). Given the imperfect BC matrix, the RZF at relay is \( \hat{G}^H (\hat{G}^H + \alpha I_k)^{-1} \). We aim at optimizing \( \alpha \) in the RZF precoder in terms of SNRs of BC and FC and the powers of channel estimation errors \( e_1^2 \) and \( e_2^2 \). Since the power penalty problem of ZF mostly exists in the case \( N = K \) [12], we assume \( N = K \). Generally, a non-zero \( \alpha \) will bring interference, but can reduce the power penalty. To optimize \( \alpha \), we need to derive the SINR in terms of \( \alpha \) at each user. In the following we will see that \( \alpha \) can be optimized based on the SINR expressed by the eigenvalues of the instantaneous CSI at each user terminal, and for large \( K \) case, the \( \alpha \) is independent of the instantaneous CSI. For SVD-RZF, we have

\[ F = V, \quad W = W_1 = \hat{G}^H (\hat{G}^H + \alpha I_k)^{-1} U^H. \]

In the following derivation, we use the decomposition

\[ \hat{G}^H G = Q \text{diag}(\lambda_1, \ldots, \lambda_K) Q^H. \]

Substituting (13) and (14) into (2), we have the power control factor respectively as

\[ \rho_s = \left( \frac{P_s}{K} \right)^\frac{1}{2}, \]

and (15) which is written at the top of the next page.

Substituting (12) and (13) into (5), through some manipulations, we have the power of effective noise

\[ N(\theta, \lambda) = \left( \frac{\rho_s^2 \rho_r e_1^2 + \rho_r^2 \sigma_1^2}{\rho_s^2 \rho_r e_2^2 + \rho_r^2 \sigma_2^2} \right) \sum_{\lambda} \frac{\lambda^2}{(\lambda + \alpha)^2} + \left( \frac{1}{K} \rho_r^2 \rho_r e_2^2 \sum_{\lambda} \frac{\lambda}{(\lambda + \alpha)^2} + \sigma_2^2 \right). \]

where in the derivation, we have taken expectation over unitary matrix \( Q \). The received data signal vector at the user terminals can be calculated as

\[ \rho_s \rho_r \hat{G}_r \hat{H}_r \mathbf{F} = \rho_s \rho_r \hat{G}_r \hat{H}_r (\hat{G}_r \hat{H}_r + \alpha I_k)^{-1} \mathbf{F}. \]

From the above expression, we see that the effective channel matrix is not diagonal when \( \alpha \) is not zero. So the received signal by a user terminal consists of the desired signal and the interference from other users’ signal. To divides the interference from the desired signal, we introduce the following two lemmas.

**Lemma 1:** If \( A = QAQ^H \), then

\[ E \left\{ (A)_{k,k}^2 \right\} = \frac{\beta}{K(K+1)} (\sum_{\lambda} \lambda^2)^2 \mu(\lambda). \]

The proof of Lemma 1 can be directly obtained in [12].

**Lemma 2:** If \( A = QAQ^H \), then

\[ E \left\{ (A)_{k,j}^2 \right\} = \frac{1}{(K-1)(K+1)} (\sum_{\lambda} \lambda^2)^2 \nu(\lambda), \quad \text{for } k \neq j. \]

**Proof:** Because \( A^H A \) is a conjugate symmetric matrix, we have

\[ E \left\{ (A)_{k,j}^2 \right\} = \frac{1}{K} \sum_{\lambda} \lambda^2. \]

Since \( E \left\{ (A)_{k,j}^2 \right\} \) are all equal for \( j \neq k \), we have

\[ E \left\{ (A)_{k,j}^2 \right\} = \frac{1}{(K-1)(K+1)} \sum_{\lambda} \lambda^2 = \frac{1}{(K-1)K(K+1)} (\sum_{\lambda} \lambda^2)^2. \]

Therefore, for user-\( k \), if we denote \( A = \hat{G}^H (\hat{G}^H + \alpha I_k)^{-1} \), we can calculate the power of desired signal as

\[ E \left\{ \| A_{k,k} \theta_k(s) \|_2^2 \right\} = \rho_s^2 \rho_r^2 \mu \left( \frac{\lambda}{\lambda + \alpha} \right). \]

The power of interference is

\[ E \left\{ \sum_{j=1, j \neq k}^{K} \| A_{k,j} \theta_j(s) \|_2^2 \right\} = \rho_s^2 \rho_r^2 \left( \sum_{j=1, j \neq k}^{K} \theta_j \right) \nu \left( \frac{\lambda}{\lambda + \alpha} \right). \]

Finally, the SINR at user-\( k \) is

\[ \text{SINR}_k = \frac{\rho_s^2 \rho_r^2 \mu \left( \frac{\lambda}{\lambda + \alpha} \right)}{\rho_s^2 \rho_r^2 \left( \sum_{j=1, j \neq k}^{K} \theta_j \right) \nu \left( \frac{\lambda}{\lambda + \alpha} \right) + N(\theta, \lambda)}. \]

Note that in the above expression, the SINR is based on the eigenvalue of instantaneous imperfect CSIs. To maximize the SINR expression, we introduce the following lemma which is a conclusion of the Appendix B in [12].
\[
\rho_r = \left( \frac{\text{tr} \left( \hat{G} \hat{G}^H (\hat{G} \hat{G}^H + \alpha I_k)^{-2} \left( \rho_k^2 \Theta^2 + \rho_r^2 e_1^2 \Omega_1 \Omega_1^H + \sigma_r^2 I_k \right) \right)}{P_r \left( \rho_k^2 \sum \theta + \rho_r^2 P_s + \sigma_r^2 \right) \frac{\lambda}{(\lambda+\alpha)^2}} \right)^{\frac{1}{2}}.
\]  

(18)

\[
\text{SINR}_{r,N} \rightarrow \frac{P_r \left( R e_1^2 e_3^\lambda \right)^2}{\left( \frac{P_r}{M} \sum \theta + \rho_r^2 P_s + \sigma_r^2 \right) \frac{\lambda}{(\lambda+\alpha)^2} + \sigma_r^2 \rho_r^2 - 2}.
\]  

(19)

**Lemma 3:**

\[
\text{SINR}(\alpha) = \frac{A \left( \sum \frac{\lambda}{\lambda+\alpha} \right)^2 + B \sum \frac{\lambda^2}{(\lambda+\alpha)^2}}{C \sum \frac{\lambda}{(\lambda+\alpha)^2} + D \sum \frac{\lambda^2}{(\lambda+\alpha)^2} + E \left( \sum \frac{1}{\lambda+\alpha} \right)^2}.
\]  

(25)

for large \( K \), is maximized by \( \alpha = C/D \).

Using Lemma 3, we finally get the optimized

\[
\alpha_{SVD-RZF,\text{opt}} = \frac{\frac{\lambda}{K} \sum \frac{\lambda}{\lambda+\alpha} + \frac{\lambda^2}{(K-1)(K+1)} + e_1^2 + \frac{\lambda^2}{P_r}}{\frac{\sum \frac{\lambda}{\lambda+\alpha}}{K-1} + \frac{\lambda^2}{P_r}}.
\]  

(26)

For large \( K \), we have

\[
\alpha_{SVD-RZF,\text{opt}} \approx \frac{\frac{\lambda}{K} \sum \frac{\lambda}{\lambda+\alpha} + \frac{\lambda^2}{K} + \frac{\lambda^2}{P_r}}{\frac{\sum \frac{\lambda}{\lambda+\alpha}}{K-1} + \frac{\lambda^2}{P_r}}
\]

\[
\approx K \left( 1 + \frac{e_1^2 + \frac{\lambda^2}{P_r}}{1 + \frac{\lambda^2}{P_r}} \right).
\]  

(27)

**B. MMSE-RZF based design for multi-relay case**

Although SVD is advantageous, it can only be implemented in the single relay case. For the multi-relay case, the relays have to work in a cooperative mode to diagonalize the channel as SVD or the base station needs the CSI of all the backward channels which will lead to considerable delay. Therefore, for the multi-relay case, we propose another beamforming scheme which is based on MMSE-RZF instead of SVD-RZF.

It is known that MMSE receiver is widely used in point-to-point MIMO systems. The MMSE receiver can be viewed as a duality of the RZF precoder, where the difference is that the RZF precoder is frequently used in multiantenna multiuser communication. Our main idea is to obtain the optimal regularizing factor in MMSE receiver to reduce the effect of channel estimation error of the backward channels.

The MMSE receiver at the \( r \)-th relay is \( \hat{H}_r \hat{H}_r \left( \hat{H}_r \hat{H}_r + \alpha \text{MMSE} I_k \right)^{-1} \). For the same reason as RZF, MMSE receiver is most superior to other linear receivers (e.g. ZF) when \( M = N \). So we consider \( M = N = K \) for the multi-relay case. Because the aim of MMSE receiver is to reduce the effect of channel estimation error of BC, we optimize \( \alpha_{\text{MMSE}} \) by idealizing the forward channels as Gaussian channels, i.e., the forward channel is considered as \( \hat{G}_r = G_r = I_N \).

In the following analysis, we use the decompositions,

\[
\hat{H}_r^H \hat{H}_r = P_r \text{diag} \{ \theta_1, \ldots, \theta_N \} P_r^H,
\]

\[
\hat{G}_r \hat{G}_r^H = Q_r \text{diag} \{ \lambda_1, \ldots, \lambda_N \} Q_r^H,
\]

where \( P_r \) and \( Q_r \) are unitary matrices. For the \( r \)-th relay, the signal vector processed by an MMSE receiver is

\[
\nu_r = \rho_r \left( \hat{H}_r^H \hat{H}_r + \alpha_{\text{MMSE}} \text{I}_M \right)^{-1} \hat{H}_r^H \nu_r = \rho_r \left( \hat{H}_r^H \hat{H}_r + \alpha_{\text{MMSE}} \text{I}_M \right)^{-1} \hat{H}_r^H \nu_r
\]

(28)

(29)

At the destination, the received vector is from all the \( R \) relays. So the desired signal is scaled by \( R^2 \) and the interference and the noise inherited from the relays are scaled by \( R \). Therefore, by idealizing the forward channels, we have the SINR of the \( k \)-th stream as

\[
\text{SINR}_{r,k}^D \approx \frac{P_r \rho_r^2 \left( \frac{\theta}{\theta+\alpha_{\text{MMSE}}} \right)}{R P_r \left( M-1 \right) \mu \left( \frac{\theta}{\theta+\alpha_{\text{MMSE}}} \right) + R e_1^2 \left( P_r + \sigma_r^2 \right) \sum \frac{\theta}{(\theta+\alpha_{\text{MMSE}})^2} + \sigma_r^2 \rho_r^2}.
\]  

(32)

where the power control factor \( \rho_r \) at relay normalizes the noise at the destination. We use the same \( \rho_r \) for all the relays for the simplicity of analysis by taking expectation to the denominator in \( \theta \). Using Lemma 3, we obtain

\[
\alpha_{\text{MMSE, opt}} = \frac{P_r e_1^2 + M \rho_r^2 \sigma_r^2}{P_r e_1^2 + \rho_r^2 \sigma_r^2} + \frac{P_r e_1^2 + M \rho_r^2 \sigma_r^2}{M} = \left( e_1^2 + \frac{\sigma_r^2}{P_s} \right) \frac{M + \rho_r^2}{1 + \frac{P_r}{M}}.
\]  

(33)

To obtain the optimal \( \alpha_{\text{RZF}} \), we need to derive the asymptotic SINR of the system. Again, we separate the desired
signals from the interference and the noise and finally derive the SNR at the k-th user terminal as
\[
\text{SINR}_k^D = \frac{P}{M \sum_{j=1,j\neq k}^K |(H_{\text{SD}})_{k,k}|^2} N (G_r, H_r),
\] (34)
where
\[
H_{\text{SD}} = \sum_{r=1}^R \rho_r \hat{G}_r W_r \hat{H}_r,
\] (35)
and
\[
N (G_r, H_r) = (e_r^2 P_s + \sigma_r^2) + \frac{P e_r^2}{M} \sum_{r=1}^R \rho_r^2 \text{tr} (W_r \hat{H}_r \hat{H}_r^H W_r^H)
\]
+ \frac{P e_r^2}{M} \sum_{r=1}^R \rho_r^2 \text{tr} (W_r \hat{H}_r \hat{H}_r^H W_r^H).
\] (36)
For the case of large R, using Law of Large Number, we have
\[
\frac{1}{(H_{\text{SD}})_{i,i}} \approx w.p. \rho_r \left( \mathbb{E} \left( \hat{G}_r W_r \hat{H}_r \right)_{i,i} \right)
\]
\[
= R \rho_r \mathbb{E} \left( Q_r \frac{\lambda_r}{\lambda_r + \alpha \text{RZF}_M} \Theta_r P_r \Theta_r \text{MMSE}_m \text{I}_r N \right)_{i,i}
\]
\[
= R \rho_r \mathbb{E} \left( Q_r \frac{\lambda_r}{\lambda_r + \alpha \text{RZF}_M} \Theta_r P_r \Theta_r \text{MMSE}_m \text{I}_r N \right)_{n,n}
\]
\[
= R \rho_r \mathbb{E} \left( \frac{\lambda_r}{\lambda_r + \alpha \text{RZF}} \right) \mathbb{E} \left( \frac{\lambda_r}{\lambda_r + \alpha \text{RZF}} \right),
\] (37)
and
\[
| (H_{\text{SD}})_{i,j} |^2 \approx \sum_{r=1}^R \left( Q_r \frac{\lambda_r}{\lambda_r + \alpha \text{RZF}} \right)_{i,k} \left( Q_r \frac{\lambda_r}{\lambda_r + \alpha \text{RZF}} \right)_{i,k}^* (P_r)_{l,m} (P_r)_{l,m} \]
\[
\approx \sum_{k,m,n,r} \frac{1}{M^2} \left( \frac{\lambda_r}{\lambda_r + \alpha \text{RZF}} \right)_{i,k} \left( \frac{\lambda_r}{\lambda_r + \alpha \text{RZF}} \right)_{i,k}^* \mathbb{E} \left( \frac{\theta^2}{\theta + \alpha \text{RZF}} \right) \mathbb{E} \left( \frac{\lambda^2}{\lambda + \alpha \text{RZF}} \right),
\] (38)
where in (a) we approximate \( \mathbb{E} \left\{ \left| (Q_r)_{i,k} \right|^2 \right\} \approx \frac{1}{M^2} \).
In fact, this expectation is \( \frac{1}{m(m+1)} \) if \( i = l \) or \( \frac{1}{m^2} \) if \( i \neq l \) [12]. Here we denote \( \lambda \) and \( \theta \) without subscript \( r \) for simplicity, because all the channels for different relays are i.i.d. Let us define the expectations as
\[
\mathcal{E}_1^\theta = \mathbb{E} \left\{ \frac{\theta^2}{\theta + \alpha \text{MMSE}} \right\}, \quad \mathcal{E}_2^\theta = \mathbb{E} \left\{ \frac{\theta^2}{\theta + \alpha \text{MMSE}} \right\}, \quad \mathcal{E}_3^\theta = \mathbb{E} \left\{ \frac{\theta^2}{\theta + \alpha \text{RZF}} \right\},
\]
\[
\mathcal{E}_1^\lambda = \mathbb{E} \left\{ \frac{\lambda^2}{\lambda + \alpha \text{MMSE}} \right\}, \quad \mathcal{E}_2^\lambda = \mathbb{E} \left\{ \frac{\lambda^2}{\lambda + \alpha \text{RZF}} \right\}.
\]
Substituting (38) into (34), we obtain the asymptotic SINR at each user terminal as [19], at the top of the last page, where
\[
\rho_r^2 = \frac{1}{P_r} \mathbb{E} \left\{ \left( F_k (H_k^H \hat{H}_k + \sigma_r^2 M \hat{H}_k^H) + \sigma_s^2 \right) \mathbb{E} \left\{ \left( F_k \hat{H}_k^H \right) \right\} \right\}
\]
\[
= \frac{P}{P_r} \alpha_\text{RZF,opt} \approx (P_r RZF + \sigma_s^2) \mathcal{E}_1^\theta + \left( \frac{\sigma_s^2}{\alpha_\text{RZF}^2 \mathcal{E}_3^\theta} \right) \mathcal{E}_1^\theta.
\] (39)

The calculation of (35) can follow the same line as (37). Generally, the expectations in the asymptotic SINR are difficult. Fortunately, if we approximate the expectations by the arithmetic mean, for large \( R \), then the asymptotic SINR can be maximized by using Lemma 3. Finally, we obtain
\[
\alpha_\text{RZF,opt} \approx (P_r RZF + \sigma_s^2) \mathcal{E}_1^\theta + \left( \frac{\sigma_s^2}{\alpha_\text{RZF}^2 \mathcal{E}_3^\theta} \right) \mathcal{E}_1^\theta.
\] (40)

Note that although we maximize the SINR for large \( K \), and large \( R \) for multi-relay case, we will see from the numerical simulation that the obtained beamforming is robust enough for small \( K \) and \( R \) when channel estimation error occurs.

V. SIMULATION RESULTS

In this section, numerical simulations have been carried out. For the single relay case, we compare the SINR at each user terminal of the robust SVD-RZF beamforming with SVD-ZF and SVD-MF in [3], MMSE-RZF in [9], and two other relative beamforming schemes such as ZF-ZF and SVD-RZF for references. For MMSE-RZF, \( \alpha_\text{MMSE} = K \sigma_s^2 / P_s \), and \( \alpha_\text{RZF} = K \sigma_s^2 / P_r \). We also consider the robust MMSE-RZF proposed for multi-relay case for \( R = 1 \). For the multi-relay case, we compare with the conventional MMSE-RZF and ZF-ZF. All the results are averaged over 10000 different channel realizations.

A. SINR performances for the single relay case

Fig. 2 shows the SINRs of different beamforming schemes versus the SNR of BC. We observe that the proposed robust SVD-RZF beamforming has consistently advantage to others. Robust MMSE-RZF underperforms robust SVD-RZF and SVD-MF, which shows the superior of SVD. Fig. 3 shows the SINR performances versus the SNR of FC. The SINR of SVD-RZF even falls and converges to SVD-ZF when the SNR of FC increases, because the \( \alpha \) converges to zero, which should remain nonzero if estimation error is considered. Fig. 4 shows the SINR performances versus the number of users \( K \). We see that the robust SVD-RZF also outperforms others when \( K \) is small. The advantage of robust SVD-RZF comes from the fact that the SVD beamforming outperforms robust MMSE receiver although the former ignores the estimation error. For
the broadcast phase, the robust RZF compensates well the estimation error compared to ZF and RZF.

**B. SINR performances for the multi-relay case**

For multi-relay case where SVD can not be implemented, we only compare the proposed robust MMSE-RZF with MMSE-RZF and ZF-ZF. Fig. 5 shows the average SINR performances versus the power of channel estimation error ($e_1^2 = e_2^2$). This is because that the $\alpha_{\text{MMSE}}$ and $\alpha_{\text{RZF}}$ increase with $e_1$ and $e_2$ to decrease the effect of estimation error. This can be directly seen from Fig. 6. Fig. 7 shows the sum rate performances versus the number of relays ($R$) with perfect and imperfect channel estimation. We see that all sum rates grow logarithmically with $R$ and the superior of robust MMSE-RZF increases when channel estimation is imperfect or the number of relays grows. This is because that comparing with conventional MMSE-RZF, the robust one considers both imperfect channel estimation and multiple relays.

**VI. CONCLUSION**

In this paper we propose the robust SVD-RZF and robust MMSE-RZF beamformers which consider imperfect channel estimation for a multiuser downlink MIMO relaying network. For the single relay case, the SINR expression at user terminals based on the eigenvalue of BC and FC matrix is derived to obtain the optimized RZF. For the multi-relay case, the asymptotic SINR is derived to obtain the optimized MMSE and RZF. Simulation results show that the proposed robust SVD-RZF and MMSE-RZF outperform the conventional schemes for various conditions of SNR of channels, power of estimation errors, the number of antennas, users and the relays.
Fig. 6. $\alpha_{\text{MMSE}}$ and $\alpha_{\text{RZF}}$ for different beamforming schemes versus the power of channel estimation error in the multi-relay case. $P_s/\sigma^2_1 = 10\text{dB}$, $P_r/\sigma^2_2 = 20\text{dB}$, $e_1 = e_2$, $R = 10$.

Fig. 7. The sum rates for different beamforming schemes versus the number of relays $(R)$ in the multi-relay case with perfect channel estimation and channel estimation errors. $P_s/\sigma^2_1 = P_r/\sigma^2_2 = 20\text{dB}$, $e_1 = e_2 = 0$ or $e_1^2 = e_2^2 = 0.2$. The sum rate are averaged by $0.5\log_2(1 + \text{SINR}_k)$. The factor 0.5 is due to the two time slots transmission.

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