A calibration of the Belle II hadronic tag-side reconstruction algorithm with $B \to X\ell\nu$ decays

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Abstract

Tag-side reconstruction is an important method for reconstructing $B$ meson decays with missing energy. The Belle II tag-side reconstruction algorithm, Full Event Interpretation, relies on a hierarchical reconstruction of $B$ meson decays with multivariate classification employed at each stage of reconstruction. Given the large numbers of classifiers employed and decay chains reconstructed, the performance of the algorithm on data and simulation differs significantly. Here, calibration factors are derived for hadronic tag-side $B$ decays by measuring a signal side decay, $B \to X\ell\nu$, in 34.6 fb$^{-1}$ of Belle II data. For a very loose selection on the tag-side $B$ multivariate classifier, the calibration factors are $0.65 \pm 0.02$ and $0.83 \pm 0.03$ for tag-side $B^+$ and $B^0$ mesons, respectively.
1. INTRODUCTION

The Belle II experiment \[1\] is an $e^+e^-$ collider experiment in Japan, which began its main physics runs in early 2019 and has collected 74 fb$^{-1}$ of data at a centre-of-mass (CM) energy, $\sqrt{s}$, corresponding to the mass of the $\Upsilon(4S)$ resonance. The clean environment of $e^+e^-$ collisions together with the unique event topology of Belle II, in which an $\Upsilon(4S)$ meson is produced and subsequently decays in a pair of $B$ mesons, allows a wide range of physics measurements to be performed that are difficult or impossible at hadron colliders. In particular, measurements in which there is missing energy, which includes semileptonic decays with missing neutrinos, can benefit substantially from the additional constraints provided by the collision environment of Belle II. This includes the measurement of the ratio of branching fractions, $R(D^*) = \mathcal{B}(B \rightarrow D^{(*)}\tau\nu)/\mathcal{B}(B \rightarrow D^{(*)}\ell\nu)$, inclusive determinations of the CKM matrix elements $|V_{ub}|$ and $|V_{cb}|$ from $B \rightarrow X_{u/c}\ell\nu$ decays and searches for the rare decay $B \rightarrow K^*\nu\bar{\nu}$.

Full Event Interpretation \[2\] is an algorithm for tag-side $B$ meson reconstruction at Belle II. The algorithm utilises a hierarchical reconstruction of exclusive decay chains of $B$ mesons, with multivariate classifiers utilised to identify each unique sub-decay channel. Given the large number of decay chains reconstructed and multivariate classifiers employed, there can be significant differences between the tag-side reconstruction efficiency in simulation and data. In order to correct for this, a calibration can be performed by measuring a decay with a well known branching fraction and sufficient available statistics after selection. A suitable choice, given the current Belle II dataset, is inclusive $B \rightarrow X_{\ell}\nu$ decays due to their substantial branching fraction of $\sim20\%$.

2. DETECTOR AND SIMULATION

The Belle II detector \[1, 3\] operates at the SuperKEKB asymmetric-energy electron-positron collider \[4\], located at the KEK laboratory in Tsukuba, Japan. The detector consists of several nested detector subsystems arranged around the beam pipe in a cylindrical geometry.

The innermost subsystem is the vertex detector, which includes two layers of silicon pixel detectors and four outer layers of silicon strip detectors. Currently, the second pixel layer is installed in only a small part of the solid angle, while the remaining vertex detector layers are fully installed. Most of the tracking volume consists of a helium- and ethane-based small-cell drift chamber.

Outside the drift chamber, a Cherenkov-light imaging and time-of-propagation detector provides charged-particle identification in the barrel region. In the forward endcap, this function is provided by a proximity-focusing, ring-imaging Cherenkov detector with an aerogel radiator. Further out is an electromagnetic calorimeter, consisting of a barrel and two endcap sections made of CsI(Tl) crystals. A uniform 1.5 T magnetic field is provided by a superconducting solenoid situated outside the calorimeter. Multiple layers of scintillators and resistive plate chambers, located between the magnetic flux-return iron plates, constitute the $K_L$ and muon identification system.

The data used in this analysis were collected at a CM energy, $\sqrt{s}$, of 10.58 GeV, cor-
responding to the mass of the $\Upsilon(4S)$ resonance. The energies of the electron and positron beams are 7 GeV and 4 GeV, respectively, resulting in a boost of $\beta\gamma = 0.28$ of the CM frame relative to the lab frame. The integrated luminosity of the data is 34.6 fb$^{-1}$. In addition, a smaller sample of 3.23 fb$^{-1}$ off-resonance data was collected at a CM energy of 10.52 GeV.

The analysis utilises several samples of simulated events. These include a sample of $e^+e^- \rightarrow (\Upsilon(4S) \rightarrow BB)$ with generic $B$-meson decays, generated with EvtGen [5], and corresponding to an integrated luminosity of 100 fb$^{-1}$. A 100 fb$^{-1}$ sample of continuum $e^+e^- \rightarrow q\bar{q}$ ($q = u, d, s, c$) is simulated with KKMC [6] interfaced with PYTHIA [7]. All data samples were analyzed (and, for Monte Carlo (MC) events, generated and simulated) in the basf2 [8] framework.

3. THE ALGORITHM

The Full Event Interpretation employs a hierarchical reconstruction of exclusive $B$ meson decay chains, in which each unique decay channel of a particle has its own designated multivariate classifier. The algorithm utilises several stages of reconstruction, which are shown in Fig. [1]. The algorithm starts by selecting candidates for stable particles, which include muons, electrons, pions, kaons, protons and photons, from tracks and EM clusters in the event. Subsequently, the algorithm carries out several stages of reconstruction of intermediate particles such as $\pi^0$, $K^0_S$, $J/\psi$, $D$ and $D^*$ mesons and, in addition, $\Sigma$, $\Lambda$ and $\Lambda_c$ baryons. The addition of baryonic modes is a recent extension of the algorithm. Intermediate particles are reconstructed in specific decay modes from a combination of stable and other intermediate particle candidates. The final stage of the algorithm reconstructs the $B^+$ and $B^0$ mesons in 36 (8) and 31 (8) hadronic (semileptonic) modes.

![FIG. 1. The stages of reconstruction employed by Full Event Interpretation.](image)

Each stage consists of pre-reconstruction and post-reconstruction steps. In the pre-reconstruction step, candidates for particles are reconstructed, an initial pre-selection is ap-
FIG. 2. (a) Comparison of the distribution of log $P_{\text{tag}}$ in early Belle II data to the shape expectation from simulation. Here, log $P_{\text{tag}}$ is the logarithm of the tag-side $B^+$ meson classifier output, $P_{\text{tag}}$. Reference selection criteria of $P_{\text{tag}} > 0.1$ and $P_{\text{tag}} > 0.5$ are illustrated. (b) Fits to the beam-constrained-mass, $M_{bc}$, distribution of reconstructed $B^+$ (top) and $B^0$ (bottom) tag-side $B$ mesons in data. A looser selection criteria of $P_{\text{tag}} > 0.1$ (left) and a tighter selection criteria of $P_{\text{tag}} > 0.5$ (right) are applied on the $B$ meson classifier $P_{\text{tag}}$ to select samples with different levels of purity.

plied and a best candidate selection is made on a discriminating variable. Subsequently, in the post-reconstruction step, vertex fits are performed where applicable, pre-trained classifiers are applied and a best-candidate selection is made on the classifier output. Classifiers for stable particles utilise kinematic and particle identification information as features; meanwhile, intermediate and $B$ classifiers utilise the kinematic information from all daughters, daughter classifier outputs and information from vertex fits as features.

The algorithm requires a training procedure, in which all of the particle classifiers are trained. For the calibration studies performed here, the training was performed on simulated $\Upsilon(4S) \to BB$ events corresponding to an integrated luminosity of 100 fb$^{-1}$. The training of the algorithm utilises an equivalent reconstruction procedure to produce training datasets for each particle decay channel classifier.

Subsequently, the tag-side $B$ classifier, $P_{\text{tag}}$, can be used to select a pure sample of correctly reconstructed tag-side $B$ mesons. This is demonstrated in Fig. 2 which shows fits to the beam constrained mass distribution, $M_{bc} = \sqrt{E_{\text{beam}}^2 - (p_{\text{tag}}^\text{CM})^2}$, for reconstructed tag-side $B^0$ and $B^+$ mesons, for selections requiring $P_{\text{tag}}$ to be greater than 0.1 and 0.5. The contribution from correctly reconstructed tag-side $B$ mesons is parametrised by a Crystal Ball function [9]; backgrounds from $e^+e^- \to q\bar{q}$ and incorrectly reconstructed $B$ mesons are modelled with an Argus function [10]. By applying a tighter selection on the classifier output, a higher purity sample of tag-side $B$ mesons can be selected with the sacrifice of a lower tag-side efficiency, which is proportional to the yield of correctly reconstructed tag-side $B$ mesons.
4. SELECTION

The selection process begins by requiring that there is at most one tag-side $B$ meson candidate in each event. This is achieved by selecting the tag-side candidate with the highest tag-side $B$ classifier output, $P_{\text{tag}}$. For correctly reconstructed tags, the beam energy difference, $\Delta E$, should peak around 0 with some mode-dependent resolution, which is asymmetric with a skew towards lower values for modes containing $\pi^0 \rightarrow \gamma\gamma$ decays. Therefore, an asymmetric requirement of $-0.15 < \Delta E < 0.1$ GeV is placed on the beam energy difference. To reduce background from $e^+ e^- \rightarrow q\bar{q}$ events, a requirement on the event-level-normalised second Fox-Wolfram moment to be less than 0.3 is made. Fig. 3 shows a breakdown of the $M_{bc}$ distribution in data into several categories of tag-side decay mode after the above selection and the loose purity requirement that $P_{\text{tag}} > 0.01$. The dominant tag-side decay mode categories are $D\pi$, $D^*\pi$, $Dn\pi$ and $D^*n\pi$. The recently added baryonic modes result in a small increase in the tag-side efficiency, boosting the number of correctly reconstructed tag-side $B$ mesons by roughly 3% (2%) for tag-side $B^+$ ($B^0$) mesons. The final selection applied to the tag-side candidate, is a requirement that $M_{bc}$ is greater than 5.27 GeV/$c^2$, which selects the region containing correctly reconstructed tag-side $B$ mesons as can be seen in Fig. 2.

![Belle II preliminary](image)

**FIG. 3.** Contribution of different tag-side decay modes to the $M_{bc}$ distribution in data for $B^+$ (left) and $B^0$ (right) parents for $P_{\text{tag}} > 0.01$. Contributions from the newly added baryonic modes can also be seen.

After the tag-side selection, the signal-side selection is applied. In particular, a lepton is selected with $p^*_\ell > 1$ GeV/$c$, where $p^*_\ell$ refers to the momentum of the lepton in the rest frame of the signal-side $B$ meson, which can be determined using the four-momentum of the recoiling tag-side $B$ meson. The distance of closest approach between each track and the interaction point is required to be less than 2 cm along the $z$ direction (parallel to the beams) and less than 0.5 cm in the transverse $r-\phi$ plane. Particle identification information from several sub-detectors, including Cherenkov time of propagation (TOP), Aerogel ring
imaging Cherenkov and dedicated muon detectors, is combined into a likelihood for each of
electron and muon hypotheses in order to select each lepton species. The selection on $p_\ell^*$ to be greater than 1 GeV/c was motivated by the fact that lepton identification performance is found to degrade significantly below 1 GeV/c.

5. CALIBRATION PROCEDURE

The calibration factor is defined as $\epsilon = N_{Xe\nu}^{\text{Data}} / N_{Xe\nu}^{\text{MC}}$, where the yield of $B \rightarrow Xe\nu$ decays in data, $N_{Xe\nu}^{\text{Data}}$, is determined by fitting the $p_\ell^*$ distribution and the expected yield, $N_{Xe\nu}^{\text{MC}}$, is determined using MC simulation.

The fitting procedure maximises a binned likelihood, $\mathcal{L}$, defined by the following equation,

$$-2 \log \mathcal{L} = -2 \log \prod_i P(\nu_i^{\text{obs}} | \nu_i^{\text{exp}}) + \theta^T \Sigma^{-1} \theta + (k - k_{\text{constraints}})^T \Sigma_{\text{constraints}}^{-1} (k - k_{\text{constraints}}),$$

where the probability to observe $\nu_i^{\text{obs}}$ events in bin $i$ of $p_\ell^*$ given that $\nu_i^{\text{exp}}$ events were expected is $P(\nu_i^{\text{obs}} | \nu_i^{\text{exp}})$ and is governed by a Poisson distribution. Here, $\nu_i^{\text{exp}}$, is given by

$$\nu_i^{\text{exp}} = \sum_j \nu_j p_i^j (1 + \theta_i^j) \sum_k p_k^j (1 + \theta_k^j),$$

where $p_i^j$ defines the probability for an event of process type $j$ to have a reconstructed value of $p_\ell^*$ in bin $i$. The nuisance parameters, $\theta_i^j$, account for both MC template statistics and additional systematic effects. The associated bin-to-bin correlations arising from systematic uncertainties are accounted for in the covariance matrix, $\Sigma_y$.

The fit has three yields associated with three probability density functions (pdfs), which describe the $B \rightarrow Xe\nu$ signal decays, background from $e^+e^- \rightarrow q\bar{q}$ events, and background in which the lepton is fake or secondary. “Secondary” here refers to the situation in which the lepton is not produced directly in the decay of the $B$ meson but rather through a secondary cascade decay of a charmed meson. Meanwhile, “Fake” refers to the case in which a hadron is mis-reconstructed as a lepton. The $B \rightarrow Xe\nu$ signal pdf has four sub-components, which include $B \rightarrow D^*e\nu$, $B \rightarrow Dl\nu$, $B \rightarrow X_u e\nu$ and any remaining $B \rightarrow X_c e\nu$ decays ($B \rightarrow D^{*+}e\nu$ and $B \rightarrow D^{(*)}n\pi l\nu$). The relative contributions of these four components are parametrised by three fractions $(f_D, f_{D^*}, \text{ and } f_{X_u})$.

The last term, $(k - k_{\text{constraints}})^T \Sigma_{\text{constraints}}^{-1} (k - k_{\text{constraints}})$, in Equation 1 allows for constraints on parameters in the fit. The parameter vector $k = (N(e^+e^- \rightarrow q\bar{q}), f_D, f_{D^*}, f_{X_u})$ contains the subset of fit parameters, which are subject to constraints. The vector $k_{\text{constraints}}$ contains the corresponding nominal values to which these parameters are constrained. The continuum yield, $N(e^+e^- \rightarrow q\bar{q})$, is constrained to its expectation based on counting off-resonance events and scaling up to account for luminosity. The constraints on the three fractions are obtained from MC expectation after all branching fraction corrections are made.

Fit results for the channels $B^+e^-, B^+\mu^-, B^0e^-$ and $B^0\mu^-$ with a selection of $P > 0.001$ are shown in Fig. 4. A good agreement between data and the fitted models is observed
FIG. 4. Fits to $p^*_\ell$ in data for charged (top) and neutral (bottom) tag-side $B$ mesons combined either with electron (left) or muon (right) signal-side $B \to X\ell\nu$ decays.

across all channels. Fig. 5 shows the $B^+\ell^-$ fit channels in the region where $p^*_\ell > 2$ GeV/$c$. In this region, the contribution from $B \to X_u\ell\nu$ decays becomes evident due to the lower kinematic endpoint of $B \to X_c\ell\nu$ decays. This allows one to better constrain the albeit small contribution from $B \to X_u\ell\nu$ decays.

6. SOURCES OF SYSTEMATIC UNCERTAINTY

The calibration procedure is affected by a number of sources of systematic uncertainty. These can influence the determination of the MC expected yield (normalisation uncertainties) or the shapes of pdfs in the fitting procedure (shape uncertainties).

We first discuss the estimation of systematic uncertainties for the MC expected yield,
The first source of systematic uncertainty considered is that arising from the knowledge of the $B \to X\ell\nu$ branching fractions. Several branching fractions of the $B \to X\ell\nu$ decay modes, including $B \to D\ell\nu$, $B \to D^*\ell\nu$ and $B \to X_u\ell\nu$, were first corrected to their latest PDG values. After having applied these corrections, the overall charged and neutral $B \to X\ell\nu$ branching fractions were scaled to match those in the PDG: $B(B^+ \to X\ell\nu) = 10.99 \pm 0.28$ and $B(B^0 \to X\ell\nu) = 10.33 \pm 0.28$. The corresponding uncertainties are treated as a source of systematic uncertainty. In addition to correcting several branching fractions, the form factors of $D\ell\nu$ and $D^*\ell\nu$ decays are updated to the BGL parametrisations of Ref. [11, 12], with the central parameter values in Ref. [13]. The associated uncertainties on the form factor parameters of these parameterisations are propagated in the analysis using one-sigma variations in an uncorrelated eigenbasis of form factor parameters of the corresponding BGL parametrisations. The form factor uncertainties can influence $N_{X\ell\nu}^{MC}$ due to the selection of $p_t^* > 1$ GeV/c.

The next sources of uncertainty relate to tracking and particle identification. Due to mismatches in the reconstruction of tracks between simulation and data, a systematic error of 0.91% is assigned for the single signal-side track. The performance of lepton identification also differs between data and MC. Consequently, the lepton identification rates and $\pi \to \ell$ and $K \to \ell$ fake rates are corrected in bins of lepton momentum and polar angle using corrections derived from data samples of $J/\psi \to \ell^+\ell^-$, $D^{*+} \to (D^0 \to K^-\pi^+)\pi^+$ and $K_S^{0} \to \pi^+\pi^-$ decays. The systematic uncertainty associated with these corrections is determined by generating gaussian variations on these weights according to their systematic and statistical uncertainties, while assuming that the systematic uncertainties across bins are 100% correlated. The final considered source of systematic uncertainty on $N_{X\ell\nu}^{MC}$ is the statistical size of the MC sample used to estimate $N_{X\ell\nu}^{MC}$.

A number of systematic effects can impact the expected $p_t^*$ distribution from simulation. These include the Monte Carlo statistics, the $B \to D^{(*)}\ell\nu$ form factors, lepton identification and the composition of $B \to X\ell\nu$ decays. The uncertainty associated with the composition...
of $B \to X\ell\nu$ is propagated into the fit through the freedom of the $B \to X\ell\nu$ pdf to change according to aforementioned sub-pdf fractions. A multivariate Gaussian constraint on these fractions is estimated, which accounts for the PDG uncertainty on several branching fraction updates and Monte Carlo statistics. Given that the contribution from $B \to D^{**}\ell\nu$ and $B \to D^{(*)}\nu\nu\ell\nu$ is not very well known, the overall branching fraction of these transitions is assigned a 20% uncertainty.

The shape impact for the remaining systematic sources of uncertainty are accounted for by using the nuisance parameters associated with each bin of a sub-pdf. For each systematic source of uncertainty, $s$, a $N_{\text{dim}} \times N_{\text{dim}}$ covariance matrix, $\Sigma_s$, is estimated, where $N_{\text{dim}} = N_{\text{bins}} \times N_{\text{pdfs}}$. For lepton identification, $\Sigma_{\text{LID}}$, is estimated by filling histograms with each independent weight variation. Meanwhile, for the $D^{(*)}$ form factors, $\Sigma_{D^{(*)}\text{FF}}$ is estimated by combining covariance matrices associated with one-sigma eigen-variations of BGL form factor parameters. Lastly, for MC statistics, $\Sigma_{\text{MC}}$ is determined using Poisson statistics and is purely diagonal. The total covariance matrix $\Sigma_\theta = \sum_s \Sigma_s$ is used in the nuisance parameter constraint term of Equation 1.

7. RESULTS

Final results for the calibration factors as determined from the fitted yields are shown in Fig. 6. The corresponding numerical results are itemised in Appendix A along with the simulated and fitted yields of $B \to X\ell\nu$ decays. Calibration factors for tag-side $B^0$ and $B^+$ mesons are found to agree well for both lepton channels with the $B^+$ and $B^0$ calibration factors ranging from 0.60-0.63 and 0.70-0.83, respectively. For tag-side $B^0$ mesons, the calibration factors with a looser selection on the tag-side $B$ classifier output, $P_{B^0\text{tag}}$, are generally observed to be higher. This appears to be due to the fact that a looser cut increases the contribution of certain modes in the lower purity region. The sources uncertainties for the calibration factors are shown in Table I for the threshold of $P > 0.001$. The dominant systematic uncertainty is associated with the shape freedom in the fit, which ranges from 2 to 4%, depending on the channel. The next largest sources of uncertainty are those associated with $B(B^{+/0} \to X\ell\nu)$ (2.1%) and tracking (0.91%).

The calibration factors are subsequently averaged across lepton modes as displayed in Table II and in Fig. 6. The averaging procedure uses a weighted average, that accounts for the relative uncertainties and correlations of the measurements. In particular, the uncertainties from tracking, $B(B^{+/0} \to X\ell\nu)$, and the $D^{(*)}\ell\nu$ form factors are deemed to be 100% correlated.

| Channel | MC Stat. | $B(B^{+/0} \to X\ell\nu)$ | Tracking | $D\ell\nu$ | FF | Lepton ID | $D^{(*)}\ell\nu$ | FF | Fit Stat. | Fit Model |
|---------|----------|-----------------|----------|------------|----|-----------|-----------------|----|----------|-----------|
| $B^+\mu^{-}$ | 0.37 | 2.1 | 0.91 | 0.06 | 2.13 | 0.38 | 0.86 | 2.93 |
| $B^+\mu^{-}$ | 0.62 | 2.1 | 0.91 | 0.07 | 0.73 | 0.43 | 1.22 | 3.72 |
| $B^0\mu^{-}$ | 0.6 | 2.09 | 0.91 | 0.06 | 2.13 | 0.41 | 1.19 | 3.17 |

TABLE I. Itemisation of the percentage contribution from the sources of uncertainty on the calibration factors for the selection $P_{\text{tag}} > 0.001$.  

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FIG. 6. (a) Calibration factors for each of the different channels and different signal probability, $P_{\text{tag}}$, selection choices. Good agreement is seen between the muon and electron channels for the signal-side $B \to X\ell\nu$ decay. (b) $\epsilon_{\text{MC}}^{\text{tag}} \times \epsilon_{\text{cal}}$ against purity for $P_{\text{tag}} > 0.001$, 0.01 and 0.1 for $B^0$ and $B^+$ mesons.

| $B^+$     | $P_{\text{tag}}$ | $\epsilon$ | uncertainty [%] |
|-----------|------------------|-------------|-----------------|
|           | 0.001            | 0.65 ± 0.02 | 3.0             |
|           | 0.01             | 0.61 ± 0.02 | 3.1             |
|           | 0.1              | 0.64 ± 0.02 | 3.3             |

| $B^0$     | $P_{\text{tag}}$ | $\epsilon$ | uncertainty [%] |
|-----------|------------------|-------------|-----------------|
|           | 0.001            | 0.83 ± 0.03 | 3.4             |
|           | 0.01             | 0.78 ± 0.03 | 3.5             |
|           | 0.1              | 0.72 ± 0.03 | 3.9             |

TABLE II. Final calibration factors averaged over lepton type. A weighted average taking into account the uncertainties and correlated systematics is used.

The final calibration factors, $\epsilon_{\text{cal}}$, in Table II can be applied in order to correct the tag-side efficiency in simulation, $\epsilon_{\text{MC}}^{\text{tag}}$. In Fig. 6 the corrected tag-side efficiency from simulation, $\epsilon_{\text{MC}}^{\text{tag}} \times \epsilon_{\text{cal}}$, is shown against purity, for the $P_{\text{tag}}$ thresholds of 0.001, 0.01 and 0.1. Here, the tag-side efficiency, $\epsilon_{\text{MC}}^{\text{tag}}$, refers to ratio of the number of events containing a correctly reconstructed tag-side $B$ meson in the region $M_{bc} > 5.27$ GeV$/c^2$ to the total number of simulated $\Upsilon(4S) \to BB$ events. Meanwhile the purity is the ratio of the number of events containing a correctly reconstructed tag-side $B$ meson in this region to the number of events containing a reconstructed tag-side $B$ meson.
8. CONCLUSIONS

At Belle II, hadronic tag-side reconstruction will be a critical part of the physics program, allowing a number of challenging final states with missing energy to be measured. This includes measurements of $R(D^{(*)})$ with $B \to D^{(*)}\tau\nu$ decays, measurements of the CKM matrix elements $|V_{ub}|$ and $|V_{cb}|$ using inclusive $B \to X_{c/u}\ell\nu$ transitions and searches for the rare decay $B \to K^*\nu\bar{\nu}$.

The Belle II experiment’s tag-side reconstruction algorithm, Full Event Interpretation, relies on a hierarchical reconstruction of around 10000 $B$ meson decays with over 200 multivariate classifiers. In order to employ the algorithm in a physics analysis, it is necessary to account for differences in the performance of the algorithm between data and simulation. Here, first calibration factors were derived in order to correct for these effects by measuring a well-known signal side of $B \to X\ell\nu$ decays. Calibration factors are determined for both $B^0$ and $B^+$ mesons for a range of selections on the tag-side $B$ multivariate classifier. For a very loose selection, the calibration factors are $0.653 \pm 0.020$ and $0.830 \pm 0.029$ for tag-side $B^+$ and $B^0$ mesons, respectively.

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Appendix A

A summary of all fitted yields, \( N_{X\ell\nu}^{Data} \), MC expected yields, \( N_{X\ell\nu}^{MC} \), and the corresponding calibration factors are provided in Table III.

| Channel | \( N_{X\ell\nu}^{MC} \) | \( N_{X\ell\nu}^{Data} \) | \( \epsilon \) |
|---------|----------------|----------------|----------|
| \( B^+ e^- \) | \((4.46 \pm 0.11) \times 10^4 \) | \((2.94 \pm 0.08) \times 10^4 \) | 0.66 \pm 0.02 |
| \( B^+ \mu^- \) | \((4.78 \pm 0.11) \times 10^4 \) | \((3.10 \pm 0.10) \times 10^4 \) | 0.65 \pm 0.03 |
| \( B^0 e^- \) | \((1.75 \pm 0.04) \times 10^4 \) | \((1.46 \pm 0.07) \times 10^4 \) | 0.83 \pm 0.04 |
| \( B^0 \mu^- \) | \((1.85 \pm 0.06) \times 10^4 \) | \((1.54 \pm 0.05) \times 10^4 \) | 0.83 \pm 0.04 |

| Channel | \( N_{X\ell\nu}^{MC} \) | \( N_{X\ell\nu}^{Data} \) | \( \epsilon \) |
|---------|----------------|----------------|----------|
| \( B^+ e^- \) | \((2.65 \pm 0.07) \times 10^4 \) | \((1.63 \pm 0.05) \times 10^4 \) | 0.62 \pm 0.02 |
| \( B^+ \mu^- \) | \((2.88 \pm 0.09) \times 10^4 \) | \((1.71 \pm 0.05) \times 10^4 \) | 0.59 \pm 0.03 |
| \( B^0 e^- \) | \((1.11 \pm 0.03) \times 10^4 \) | \((0.84 \pm 0.04) \times 10^4 \) | 0.76 \pm 0.04 |
| \( B^0 \mu^- \) | \((1.18 \pm 0.04) \times 10^4 \) | \((0.94 \pm 0.03) \times 10^4 \) | 0.80 \pm 0.04 |

| Channel | \( N_{X\ell\nu}^{MC} \) | \( N_{X\ell\nu}^{Data} \) | \( \epsilon \) |
|---------|----------------|----------------|----------|
| \( B^+ e^- \) | \((1.10 \pm 0.03) \times 10^4 \) | \((0.71 \pm 0.03) \times 10^4 \) | 0.65 \pm 0.03 |
| \( B^+ \mu^- \) | \((1.21 \pm 0.04) \times 10^4 \) | \((0.78 \pm 0.04) \times 10^4 \) | 0.64 \pm 0.03 |
| \( B^0 e^- \) | \((0.60 \pm 0.02) \times 10^4 \) | \((0.43 \pm 0.02) \times 10^4 \) | 0.72 \pm 0.04 |
| \( B^0 \mu^- \) | \((0.64 \pm 0.02) \times 10^4 \) | \((0.46 \pm 0.02) \times 10^4 \) | 0.72 \pm 0.04 |

TABLE III. Results for \( N_{X\ell\nu} \) as determined from the fits to data and simulation together with total uncertainties. The corresponding calibration factors computed from the ratio of these yields are also shown for each channel.