Flexible Design for $\alpha$-Duplex Communications in Multi-Tier Cellular Networks

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Abstract

Full-Duplex (FD) transceivers are expensive to implement at the users’ equipment (UEs) side due to the increased complexity, power consumption, and price. Therefore, backward compatibility between FD base stations (BSs) with half-duplex (HD) UEs, without sacrificing the foreseen FD communication gain, is required. In this context, 3-node topology (3NT) is recently proposed to harvest FD gains with FD BSs with HD UEs. In this paper, we present a flexible tractable modeling framework for FD multi-tier cellular networks, which is based on stochastic geometry and accounts for different network topologies as well as the intrinsic vulnerability of uplink transmissions. The model is location aware and enables fine-grained design for the overlap between uplink and downlink channels to maximize the overall rate. To this end, the results confirm that FD UEs are not necessary for harvesting the FD gains, and hence, offloads the burden to implement the FD transceivers in the UEs. The results show that implementing FD transceivers at the UEs offers a maximum of 8% rate gain over the 3NT, which is a marginal gain compared to the burden to implement the FD transceivers at the UEs’ side. To this end, we shed light on practical scenarios in which 3NT operation with FD BSs and HD UEs outperforms the 2NT operation where both FD BSs and FD UEs. Finally, we propose a location dependent mixed 3NT/2NT strategy to maximize the overall rate when UEs FD receivers with different efficiencies.

Index Terms

Full duplex, half duplex, stochastic geometry, network interference, ergodic rate, network topology.

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Part of this work is submitted to 2016 IEEE International Conference on Communications (ICC) [1].
I. INTRODUCTION

Time division duplexing (TDD) and frequency division duplexing (FDD) are the commonly used techniques to protect receivers from their overwhelming self-interference (SI). This implies that the resources (i.e., time or frequency) are divided between forward and reverse links. TDD and/or FDD, denoted as HD communications, create a performance tradeoff between forward and reverse links because each has limited access to a common pool of resources. This tradeoff can be eliminated using SI cancelation techniques, which emerged from recent advances in analog and digital circuit design [2]. SI cancelation enables in-band FD communication, which gives the forward and reverse links the opportunity to simultaneously utilize the complete set resources. In this case, transceivers are capable of sufficiently attenuating (up to -110 dB [3]) their own interference (i.e., SI) and simultaneously transmit and receive on the same channel, which offers higher bandwidth (BW) for FDD systems and longer transmission time for TDD systems. Consequently, FD communication improves the performance of both the forward and reverse links, in which the improvement depends on the efficiency of SI cancelation.

In addition to the SI cancellation, receivers in FD large-scale setup experience more severe mutual interference when compared to the HD case. This is because each FD link contains two active transmitters while each HD link contains an active transmitter and a passive receiver. Therefore, rigorous studies that capture the effect of the networks interference on FD communication is required to draw legitimate conclusions about its operation in large-scale networks. In this context, stochastic geometry is an elegant mathematical tool that can be used to model FD operation in large scale networks and understand its behavior [4]. Stochastic geometry succeeded to provide a systematic mathematical framework for modeling both ad-hoc and cellular networks [4]–[6].

Despite the higher interference injected into the network, recent studies have shown that FD communications outperform HD communications if sufficient SI cancellation is achieved. For instance, the asymptotic study in [7] shows a maximum improvement of 80% rate gain, which monotonically decreases in the link distance, for FD communication over the HD case. A more realistic ad-hoc network setup in [8] shows that FD offers an average of 33% rate gain when compared to the HD operation. In the case of cellular networks, [9] shows around 30% improvement in the total rate for FD when compared to the HD case. However, [3] reveals
that the FD gains are mainly confined to the DL due to the high disparity between UL and DL transmission powers. Furthermore, the authors in \[10\]–\[12\] show that when a constrained power control is employed in the UL, the FD communication gains in the DL can come at the expense of high degradation in the UL. Therefore, \[10\] advocates using pulse shaping along with partial overlap between UL and DL spectrum to neutralize DL to UL interference and avoid deteriorating UL rate. With pulse shaping and partial UL/DL overlap, \[10\] shows a simultaneous improvement of 33% and 28% in the UL and DL, respectively.

To harvest the aforementioned gains, FD transceivers are required on both sides of each link. However, in the context of cellular networks, operators can only upgrade their BSs and do not have direct access to upgrade UEs. Furthermore, FD transceiver may be expensive in terms of complexity, power consumption, and price that impedes their penetration to the UEs’ domain. Therefore, techniques to achieve FD gains in cellular networks with FD BSs and HD UEs are required. For instance, 3NT is proposed in \[13\]–\[16\] to harvest FD gains by serving two HD UEs within each FD BS. In 3NT, the BSs have SI capabilities and can simultaneously serve UL and DL users on the same channels. That is, each BS can merge each UL/DL channel pair into a larger channel and reuse that channel to serve an UL and a DL users simultaneously. The studies in \[13\]–\[15\] show the potential of 3NT to harvest HD gains. However, the results in \[13\] are based on simulations, and the results in \[14\]–\[16\] are based on a simplistic system model.

In this paper, we present a unified mathematical framework, based on stochastic geometry, to model 3NT (i.e., FD BSs and HD users) and 2NT (i.e., FD BSs and UEs) FD communications in multi-tier cellular networks. Different from \[14\]–\[16\], the presented system model accounts for the explicit performance of UL and DL for CCU and CEU in a multi-tier cellular network. It also captures more realistic system parameters than \[14\]–\[16\] by accounting for pulse-shaping, matched filtering, UL power control, and maximum power constraint for UEs, and different configurations for each BSs’ tier. The proposed mathematical framework is then used to conduct a rigorous comparison between 3NT and 2NT. In contrast to \[14\]–\[16\], we exploit a fine-grained duplexing strategy that allows partial overlap between the UL and DL channels, which we denote as $\alpha$-duplex ($\alpha$D) scheme \[10\]. The parameter $\alpha \in [0, 1]$ controls the amount of overlap between UL and DL channels and captures the HD (at $\alpha = 0$) and FD (at $\alpha = 1$) as special cases. Hence, the parameter $\alpha$ is used to visualize the gradual effect of the interference induced via FD communication on the system performance, and to optimize the amount of the overlap between
UL and DL channels.

The results show that 3NT can achieve close performance (within 8%) when compared to a 2NT with UEs that have efficient SI cancelation. On the other hand, if the UEs in the 2NT have poor SI cancelation, the 3NT achieves a better performance. In both cases, it is evident that network operators do not need to carry the burden of implementing SI cancelation in the UEs to harvest FD gains. To this end, we show that cellular networks with FD UEs can still use the 3NT for CEUs in addition to the 2NT for cell center users, denoted as location dependent mixed network topology (MNT), to maximize the overall rate.

The rest of the paper is organized as follows: in Section II, we present the system model and methodology of the analysis. In Section III, we analyze the performance of the $\alpha$-duplex system. Numerical and simulation results with discussion are presented in Section IV before presenting the conclusion in Section V.

**Notations:** $\mathbb{E}[]$ denotes the expectation over all the random variables (RVs) inside $[]$, $\mathbb{E}_x[.]$ denotes the expectation with respect to (w.r.t.) the RV $x$, $\mathbb{1}_{\{\cdot\}}$ denotes the indicator function which takes the value 1 if the statement $\{\cdot\}$ is true and 0 otherwise, $\ast$ denotes the convolution operator and $S^\ast$ denotes the complex conjugate of $S$, $L_x(.)$ denotes the Laplace transform (LT) of the RV $x$ and *Italic* letters are used to distinguish the variables from constants.

## II. System Model

### A. Network Model

A multi-tier cellular network is considered, where the BSs in each tier is modeled via an independent homogeneous 2-D Poisson point processes (PPPs) $\Phi_d(i)$, where $i \in \{1, 2, ..., K\}$ and $K$ is the total number of tiers, with intensity $\lambda_i$, where the location of the $j^{th}$ BS in the $i^{th}$ tier is denoted by $x_{i,j} \in \mathbb{R}^2$. Beside simplifying the analysis, the PPP assumption for abstracting cellular BSs is verified by several experimental studies [5], [6]. UEs are also distributed according to a PPP $\Phi_u$, which is independent from the BSs locations, with intensity $\lambda_u$, where $\lambda_u \gg \sum_{i=1}^{K} \lambda_i$.

It is assumed that all BSs in each tier transmit with a constant power $P_d(i)$. In contrast, UEs employ a truncated channel inversion power control with maximum transmit power constraint of $P_u$ [17]. That is; each UE compensates for its path-loss to maintain a tier-specific target average power level of $\rho(i)$ at the serving BS. UEs that cannot maintain the threshold $\rho(i)$ transmit with their maximum power $P_u$. UEs who can keep the threshold $\rho(i)$, are denoted by cell center users
Fig. 1: Channel allocation, types of interference, and path-loss exponents for a) 2NT and b) 3NT.

(CCUs), while UEs who transmit with their maximum power are denoted by cell edge users (CEUs).

The power of all transmitted signals experiences a power law path loss attenuation with exponent $\eta > 2$. We denote the path loss exponent for the paths between two BSs (DL to UL), two UEs (UL to DL), and a BS and a UE (UL to UL) by $\eta_{du}$, $\eta_{ud}$, and $\eta_{uu}$, respectively, as shown in Fig. 1. Note that the path loss exponent for interfering signals from the DL on DL, denoted by $\eta_{dd}$, is identical to $\eta_{uu}$, so both symbols will be used interchangeably in order to provide unified expressions. Also, Rayleigh fading channels are assumed such that the channel power gains are independent and identically distributed (i.i.d) exponential RVs with unit means.

B. Operation Modes and Spectrum Allocation

Without loss of generality, we consider the $\alpha$D scheme proposed in [10], which captures the FD and HD as special cases. We denote the BWs used in the HD case in the UL and DL, respectively, as $B_{\text{HD}}^u$ and $B_{\text{HD}}^d$, in which $B_{\text{HD}}^u$ and $B_{\text{HD}}^d$ are not necessarily equal. To avoid adjacent channel interference, the BSs utilize a guard band of $\epsilon B$ between each UL-DL pair of bands, where $B = \min(B_{\text{HD}}^u, B_{\text{HD}}^d)$. As shown in Fig. 2, we denote the BW used in the DL by $B_d(\alpha) = B_{\text{HD}}^d + \alpha(\epsilon + 1)B$ and in the UL by $B_u(\alpha) = B_{\text{HD}}^u + \alpha(\epsilon + 1)B$. Note that the parameter $\alpha$ controls the partial overlap between the UL and DL frequency bands. Also, the HD

\footnote{The scheme proposed in [10] is captured by setting $\epsilon$ to zero, since no guard bands are assumed there.}
Fig. 2: Frequency bands allocation.

and FD modes are captured as special cases by setting $\alpha$ to 0 and 1, respectively. Each BSs tier has its own duplexing parameter $\alpha_i$, where $i$ is the tier index.

For simplicity, we assume that $f_{u1}^{HD} < f_{d1}^{HD} \ll f_{u2}^{HD} < f_{d2}^{HD}$ to avoid adjacent channel interference between different UL-DL pairs. It is worth noting that the idealized rectangular frequency domain pulse shapes shown in Fig. 2 are for illustration only. However, as will be discussed later, we use time-limited pulse shapes that impose adjacent channel interference due to the out of band ripples in the frequency domain. Without loss of generality, we assume that each BS has only two pairs UL-DL channel that are universally reused across the network.

In a 2NT, users have FD transceivers and can use the UL and DL belonging to the same UL-DL pair for their $\alpha$-duplex operation. In contrast, 3NT UEs have HD transceivers and cannot transmit and receive on overlapping channels. Hence, each HD user is assigned his UL and DL channels from two different UL-DL pairs as shown in Fig. 1 and Fig. 2. Consequently, 3NT UEs can benefit from the larger BW channels without SI. Note that in both cases, the BS would experience SI shown in Fig. 1. Particularly, in the 2NT (3NT), the UL and DL channels of the same (different) UEs interferes together and at the BS. Note that the DL for 2NT UEs also experiences SI that requires SI cancellation techniques. In contrast, 3NT experience intra-cell interference that can be mitigated by scheduling and multi-user diversity techniques.

For the BSs and 2NT UEs, we denote the SI attenuation power as $\beta_u h_s$ and $\beta_d h_s$, respectively, where $\beta_u$, $\beta_d$ are positive constants that represent the mean attenuation power values in the UL and DL, respectively, and $h_s$ follows a general unit mean distribution with probability density
function (PDF) given by $f_{H_s}(x)$. Two special cases of interest for $f_{H_s}(x)$ are considered and compared in this paper, namely, constant attenuation where $f_{H_s}(x)$ is a degenerate distribution as in [10], [19] and random attenuation where $f_{H_s}(x)$ is an exponential distribution as in [15].

C. UEs-BSs Associations

We consider a biased and coupled BS-UE association scheme. To neutralize the effect of the high power disparity between the different BSs, biasing factors are used to encourage UEs to connect to lower power Bs to balance the load served by each BSs across the network [21]. We define a distance dependent biasing factor $\tau$ and assume that all BSs within the same tier have the same biasing factor. Hence, a UE will connect to $j^{th}$ tier if $\{\tau_j r_j < \tau_i r_i \forall i \in \{1, ..., K\}, i \neq j\}$.

Using this general scheme, we can capture different association schemes as special cases. For example, if $\tau$ is set to the same value for all tiers, then closest BS association is considered, if $\tau_j = \left(\frac{P_d^{(j)}}{\eta d^2}\right)^{1/\alpha}$ then the UE will connect to the BSs based on the received signal strength (RSS). The main difference between different schemes is in the BSs association areas as shown in Fig. 3 where three tiers network is shown, green squares represent macro BSs with transmit power equals to 10W and the diamonds represent micro BSs with transmit power equals to 5W, and the circles represent pico BSs with transmit power 1W. In Fig. 3a, nearest BS association is considered so the associations areas construct Voronoi tessellation [23]. In Fig. 3b, UE will connect to the BSs based on RSS, in this case the association areas construct multiplicative weighted Voronoi tessellation or circular tessellation [23] since BSs with larger transmit power will have larger association areas.

D. Pulse Shaping

We employ time-limited pulse shapes with unit energy, denoted as $s(t, BW, b_v) \leftrightarrow_{FT} S(f, BW, b_v)$, where BW is the pulse null-to-null BW and $b_d^{(i)}, b_u^{(i)}$ indicate the pulse types used by the $i^{th}$ tier in the DL and UL, respectively. We assume a flexible pulse shaping scheme, where each tier has its own pulse shapes in the DL and UL, however, BSs within the same tier use the same pulse

$^2$Decoupled association was analyzed before using stochastic geometry in [20] for traditional HD multi-tier network, extending this analysis decoupled association is postponed to future work.

$^3$The values of the transmit powers are based on [22].
shapes. To have a unified effective BW for all values of $\alpha_i$ in the $\alpha$D mode, the null-to-null BW of the pulse-shapes is kept equal to the channel BW. Hence, the pulse shapes are functions of the parameter $\alpha_i$ also.

\section*{E. Base-band Signal Representation}

For the sake of simple presentation, we use $\alpha_i$, $b^{(i)}_d$ and $b^{(i)}_u$ to denote the duplexing factor, and the used pulse shapes in the DL and UL, respectively in the $i^{th}$ tier. Also, we use $v, \bar{v}, w$ to indicate the desired transmission, where $v, \bar{v}, w \in \{d, u\}, v \neq \bar{v}$, for DL and UL, respectively, and $i, k$ as BSs’ tier index, where $i, k \in \{1, ..., K\}$. Exploiting this notion, the received baseband signal at the input of the matched filter of a test transceiver in the $i^{th}$ tier (BS or UE) can be expressed as

$$y_v^{(i)}(t) = \sqrt{P_v^{(i)}} r_v^{m_{vv}} h_v s(t, B_v(\alpha_i), b_v^{(i)}) + \sum_{k=1}^{K} \sum_{j \in \tilde{\Psi}_d^{(k)}} J_d^{(k) \rightarrow v^{(i)}}(t) + \sum_{k=1}^{K} \sum_{j \in \tilde{\Psi}_u^{(k)}} J_u^{(k) \rightarrow v^{(i)}}(t) + n(t).$$

(1)

where $A$ is the intended symbol which is drawn from a bi-dimensional, symmetric unit energy constellation, $h_v$ is the intended channel power gain, $P_v^{(i)}$ is the transmitted power by the desired
transmitter, \( r_o \) is the serving distance between the UE and its serving BS, \( \Psi_d^{(k)} \subseteq \Psi_d^{(k)} \) is the set of interfering BSs in the \( k \)th tier, \( \gamma_j^{(k)\rightarrow v(i)}(t) \) is the DL interference from the \( j \)th BS in the \( k \)th tier, \( \tilde{\Psi}_u \subset \Psi_u \) is the set of interfering UEs associated with the \( k \)th tier, \( \gamma_j^{u(k)\rightarrow v(i)}(t) \) is the UL interference from \( j \)th UE connected to the \( k \)th tier. \( \gamma_s^{(i)}(t) \) is the SI term affecting the \( v \) direction, and \( n(t) \) is a white complex Gaussian noise with zero mean and two-sided power spectral density \( N_o/2 \). The symbols transmitted by interfering network elements are abstracted via Gaussian codebooks as in [24]. It is shown in [25]–[27] that such abstraction has negligible effect on the interference and signal-to-interference-plus-noise-ratio (SINR) distributions. In this case, the interference in (1) can be expressed as

\[
\begin{align*}
\gamma_j^{d(k)\rightarrow v(i)}(t) &= \Gamma_{d_j}^{(k)} s(t, B_d(\alpha_i), b_d^{(k)}) \sqrt{P_d^{(k)} h_d^{(k)}} \left( r^{(k)}_{d_j} \right)^{-\eta_d} \exp \left( j2\pi \left( f_d^{(k)} - f_s^{(i)} \right) t \right), \\
\gamma_j^{u(k)\rightarrow v(i)}(t) &= \Gamma_{u_j}^{(k)} s(t, B_u(\alpha_i), b_u^{(k)}) \sqrt{P_u^{(k)} h_u^{(k)}} \left( r^{(k)}_{u_j} \right)^{-\eta_u} \exp \left( j2\pi \left( f_u^{(k)} - f_s^{(i)} \right) t \right).
\end{align*}
\]

where \( \Gamma_{d_j}^{(k)} \) and \( \Gamma_{u_j}^{(k)} \) are independent zero-mean unit-variance complex Gaussian random variables representing the interfering symbol from, respectively, the \( j \)th interfering BS, the \( j \)th interfering UE in the \( k \)th tier. \( h_d^{(k)} \) and \( h_u^{(k)} \) are the channel power gains between the tagged receiver and the \( j \)th interfering BS and UE in the \( k \)th tier, \( r^{(k)}_{d_j} \) and \( r^{(k)}_{u_j} \) are the distances between the tagged receiver and the \( j \)th interfering BS and UE in the \( k \)th tier. \( P_d^{(k)} \) is the transmitted power of the \( j \)th interfering UE associated with the \( k \)th tier, and \( P_u^{(k)} \) is the transmitted power of an interfering BS in the \( k \)th tier, note that the interfering BS index is removed since we assumed that all BSs in the same tier transmit with the same power. Last but not least, \( f_d^{(k)} \) and \( f_u^{(k)} \) represent the center frequencies of the DL and UL frequency bands used by the \( k \)th tier which depends on the duplexing factor \( \alpha_i \) (see Fig. 1). The SI term in (1) is given by

\[
\begin{align*}
\gamma_s^{(i)}(t) &= \Gamma_s \sqrt{\beta_s^{(i)} h_s} s(t, B_d(\alpha_i), b_d^{(i)}) \exp \left( j2\pi f_s^{(i)} t \right), \\
\gamma_s^{(i)}(t) &= \begin{cases} 
\Gamma_s \sqrt{\beta_d h_s} s(t, B_u(\alpha_i), b_u^{(i)}) \exp \left( -j2\pi f_s^{(i)} t \right), & 2NT \\
0, & 3NT
\end{cases}
\end{align*}
\]

where, \( \beta_d \) represents the average attenuation power of the SI in the DL, \( \beta_u^{(i)} \) is the average attenuation power of the SI affecting a BS in the \( i \)th tier in the UL, hence, each tier can have a different SI cancellation capabilities depending on their sizes and complexity. \( P_{u_o} \) is the transmit power of the tagged UE and
\[ \Delta f^{(i)} = f^{(i)}_u - f^{(i)}_d, \]  

which represents the difference between the UL and DL center frequencies in the \( i^{th} \) tier. Note that this difference also depends on the chosen tier, since each tier can have a different duplexing factor \( \alpha_i \) which leads to a different BWs and frequencies.

**F. Methodology of Analysis**

The analysis is conducted on a test transceiver on a test channel pair, which is a BS for the UL and a UE for the DL, located at the origin. According to Slivnyak’s theorem [5], there is no loss of generality in this assumption. Also, there is no loss of generality to focus on a test channel pair as interference on different bands are statistically equivalent. The impact of FD communication can be assessed via the ergodic rate, which is defined as

\[ \mathcal{R} = \mathbb{E} [\text{BW} \log_2 (1 + \text{SINR})]. \]  

In (7), the degraded SINR inside the \( \log_2(.) \) function is compensated by the increased linear BW term. Hence, (7) can be used to fairly assess the performance of FD operation. The outage probability represents an alternative way to judge the effect of FD communication on the network performance. The outage probability is defined as the probability that the current link capacity is less than the desired rate \( (R_d) \), which is expressed as

\[ O(R_d) = P \{ \text{BW} \log_2 (1 + \text{SINR}) < R_d \}. \]  

In the analysis, we start by modeling the effect of the matched and low-pass filtering on the baseband signal. Then, based on the base-band signal format after filtering, the expressions for the SINR in different cases (i.e., UL, DL, 3NT, and 2NT) are obtained. The performance metrics in (7) and (8) are then expressed in terms of the LT of the PDF of the interference, which is obtained later to evaluate (7) and (8).
III. Performance Analysis

The received signal is first convolved with the conjugated time-reversed pulse shape template, passed through a low-pass filter, and sampled at \( t = t_o \). The baseband signal after filtering and sampling at the input of the decoder is given by:

\[
y_v^{(i)}(t) = y_v^{(i)}(t) \ast h_v^{(i)}(t - t_o)_{|t = t_o}
\]

\[
= A \sqrt{P_v^{(i)} r_o^{\eta_v}} h_o \mathcal{I}_v(\alpha_i, \alpha_i) + \sum_{k=1}^{K} \sum_{j \in \Psi_v^{(i)}} \Gamma_v^{(k)} \sqrt{P_v^{(k)} h_v^{(k)} \left( f_v^{(k)} \right)^{-\eta_v}} \mathcal{I}_v(\alpha_i, \alpha_k) + \sum_{k=1}^{K} \sum_{j \in \Psi_v^{(i)}} \Gamma_v^{(k)} \sqrt{P_v^{(k)} h_v^{(k)} \left( f_v^{(k)} \right)^{-\eta_v}} C_v(\alpha_i, \alpha_k) + \sqrt{N_o} \mathcal{I}_v(\alpha_i, \alpha_i) \leq f \leq \frac{B_v(\alpha_i)}{2}.
\]

(9)

where \( h_v^{(i)}(t) \) is the combined matched and low-pass filter impulse response for a transceiver in the \( i^{th} \) tier, with the following frequency domain representation

\[
H_v^{(i)}(f) = \begin{cases} 
S^*(f, B_v(\alpha_i), b_v^{(i)}) & \quad -\frac{B_v(\alpha_i)}{2} \leq f \leq \frac{B_v(\alpha_i)}{2} \\
0 & \quad \text{elsewhere.}
\end{cases}
\]

(10)

where \( S(f, B_v(\alpha_i), b_v^{(i)}) \) represents the used pulse shape as discussed in section II-D.

The factors \( \mathcal{I}(\ldots) \) and \( C(\ldots) \) in (9) represent the intra-mode (i.e., from UL-UL or DL-DL) and cross-mode (i.e., from UL-DL or vice versa) effective received energy factor, respectively. From (2), (3), (10), and expressing the convolution in the frequency domain, the pulse shaping and filtering factors are obtained as,

\[
\mathcal{I}_v(\alpha_i, \alpha_k) = \int_{-B_v(\alpha_i)/2}^{B_v(\alpha_i)/2} S^*(f, B_v(\alpha_i), b_v^{(i)}) S(f - f_v^{(k)}) B_v(\alpha_k, b_v^{(k)}) df,
\]

(11)

\[
C_v(\alpha_i, \alpha_k) = \int_{-B_v(\alpha_i)/2}^{B_v(\alpha_i)/2} S^*(f, B_v(\alpha_i), b_v^{(i)}) S(f - f_v^{(k)}) B_v(\alpha_k, b_v^{(k)}) df,
\]

(12)

It should be noted that although same mode users use similar pulse shapes in the same tier, the effective energy received from intra-mode transmitters is not unity as shown in (11). This is because (10) includes the combined impulse response of the matched and low-pass filters, which extracts the desired frequency range from the received signal. Consequently, the energy outside
the desired BW is lost and the energy contained within the pulse shape is no longer unity. Also, the cross mode interference factor in (12) is strictly less than unity due to low-pass filtering, the possibly different pulse shapes, and the partial overlap between cross-mode channels.

Let $\Xi = \{ r_o, r_v^{(i)}, h_o, h_v^{(i)}, P_v^o, P_v^\prime, h_s; \forall i = \{1, ..., K\}, v \in \{u, d\} \}$, then conditioning on $\Xi$ the SINR is given by

$$\text{SINR}^{(i)}_v(\Xi) = \frac{P_v^o r_o^{-\eta_o} |\mathcal{I}_v(\alpha_i, \alpha_i)|^2}{\sum_{k=1}^{K} \sum_{j \in \Psi_k} P_v^{(k)} h_v^{(k)} (r_v^{(k)})^{-\eta_v} |\mathcal{I}_v(\alpha_i, \alpha_k)|^2 + \sum_{k=1}^{K} \sum_{j \in \Psi_k} P_v^{(k)} h_v^{(k)} (r_v^{(k)})^{-\eta_v} |\mathcal{C}_v(\alpha_i, \alpha_k)|^2 + \sigma^2_v(\alpha_i) + N_o |\mathcal{I}_v(\alpha_i, \alpha_i)|^2},$$

where,

$$|\mathcal{I}_d(\alpha_i, \alpha_k)|^2 = \frac{|\mathcal{I}_d(\alpha_i, \alpha_k)|^2}{|\mathcal{I}_d(\alpha_i, \alpha_i)|^2}, \quad (14)$$

$$|\mathcal{C}_d(\alpha_i, \alpha_k)|^2 = \frac{|\mathcal{C}_d(\alpha_i, \alpha_k)|^2}{|\mathcal{I}_d(\alpha_i, \alpha_i)|^2}, \quad (15)$$

and $\sigma^2_v(\alpha_i)$ represents the residual SI power and $\bar{\sigma}^2_v(\alpha_i)$ is the residual SI power normalized by $|\mathcal{I}_d(\alpha_i, \alpha_i)|^2$. From (41) and (42), $\bar{\sigma}^2_v$ can be expressed for the UL and DL as

$$\bar{\sigma}^2_{s_u}(\alpha_i) = \beta_u^{(i)} h_s P_d^{(i)} |\mathcal{I}_d(\alpha_i, \alpha_i)|^2. \quad (16)$$

$$\bar{\sigma}^2_{s_d}(\alpha_i) = \begin{cases} \beta_d h_s P_u \mathcal{C}_d(\alpha_i, \alpha_i)|^2, & 2\text{NT} \\ 0, & 3\text{NT} \end{cases} \quad (17)$$

The SINR in (13) is used in the next section to evaluate the outage provability and rate as discussed in Section II-F.

A. Performance Metrics

From (8), the outage probability is given by,

$$\mathcal{O}(R_d) = P\{\text{BW log}_2 (1 + \text{SINR}) < R_d\},$$

$$= P\{\text{SINR} < 2^\frac{R_d}{\text{BW}} - 1\}. \quad (18)$$
From equation (13), the outage probability in the link \( v \in \{u, d\} \) in the \( i^{th} \) tier can be written as,

\[
\mathcal{O}_v^{(i)}(R_d) = P \left\{ \frac{P^{(i)}_{v_o} r_{o,v}^{-\eta_{o,v}} h_o}{\sum_{k=1}^{K} \lambda^v_{v}(\alpha_{i}, \alpha_k)^2 + \sum_{k=1}^{K} \lambda^v_{v}(\alpha_{i}, \alpha_k) + N_o} < \bar{R}_d \right\}.
\]  

(19)

where in general, \( \lambda^v_{v}(\alpha_{i}, \alpha_k) = \sum_{j \in \Psi^v_{v}} P^{(k)}_{v_j} h_{v_j} \left( r^{(k)}_{v_j} \right)^{-\eta_{o,v}} \) and \( \bar{R}_d = 2R^{d}_{W} - 1 \). By exploiting the exponential distribution of \( h_o \), it can be written as

\[
\mathcal{O}_v^{(i)}(R_d) = 1 - \mathbb{E} \left\{ e^{-\bar{R}_d h_o} \prod_{k=1}^{K} \frac{-\sigma^2 \lambda^v_{v}(\alpha_{i}, \alpha_k)^2}{P^{(i)}_{v_o}} \right\}.
\]  

(20)

where the expectation is over \( \{r_o, P^{(i)}_{v_o}, \sigma^2\} \). Since the \( \{r_o, P^{(i)}_{v_o}\} \) depends on the UEs type (CCU or CCE) as discussed in section II-A we will first study the characteristics of each type starting by the serving distance which is given by the following Lemma.

**Lemma 1.** The serving distance distribution for a randomly selected CCU or CCE given that it is connected to the \( i^{th} \) tier denoted by \( f^{(i)}_{R^{(i)}_e}(r) \) and \( f^{(i)}_{R^{(i)}_c}(r) \), respectively, are given by the following equations,

\[
f^{(i)}_{R^{(i)}_e}(r) = \frac{2\pi \bar{\lambda}_i r \exp \left( -\pi \bar{\lambda}_i r^2 \right)}{1 - \exp \left( -\pi \bar{\lambda}_i \frac{P_u}{\rho^{(i)}_{u}} \right) \frac{r}{\eta_{sd}}} \mathbb{I} \left\{ 0 \leq r \leq \left( \frac{\rho^{(i)}_{u}}{P_u} \right)^{\frac{1}{\eta_{sd}}} \right\}(r),
\]  

(21)

\[
f^{(i)}_{R^{(i)}_c}(r) = 2\pi \bar{\lambda}_i r \exp \left( -\pi \bar{\lambda}_i r^2 + \pi \bar{\lambda}_i \frac{P_u}{\rho^{(i)}_{c}} \right) \frac{r}{\eta_{sd}} \mathbb{I} \left\{ \left( \frac{\rho^{(i)}_{c}}{P_u} \right)^{\frac{1}{\eta_{sd}}} < r < \infty \right\}(r).
\]  

(22)

where \( \bar{\lambda}_i = \sum_{k=1}^{K} \frac{\eta_{sd}}{\eta_{sd}} \lambda_k \).

Proof: Refer to Appendix A.

From Lemma 1, it is straightforward to find the probability that a randomly selected UE from the \( i^{th} \) tier is a CCU or a CCE, these probabilities are given by,

\[
P \{\text{CCU} \} = 1 - \exp \left( -\pi \bar{\lambda}_i \frac{P_u}{\rho^{(i)}_{u}} \right)^{\frac{1}{\eta_{sd}}}.
\]  

(23)

\[
P \{\text{CEU} \} = \exp \left( -\pi \bar{\lambda}_i \frac{P_u}{\rho^{(i)}_{c}} \right)^{\frac{1}{\eta_{sd}}}.
\]  

(24)
By using the previous Lemmas, the outage probability for the CCUs and the CEUs will be found separately, denoted by $O^{(i)}_{v_c}$ and $O^{(i)}_{v_e}$, respectively, and then the average outage probability is found by the following equation,

$$
\bar{O}^{(i)}_{v}(R_d) = O^{(i)}_{v_c}(R_d)\mathbb{P}\{\text{CCU}\} + O^{(i)}_{v_e}(R_d)\mathbb{P}\{\text{CEU}\}.
$$  \hfill (25)

Next, we will find the LTs of the aggregated interference. In order to evaluate (20), the LT of the aggregate interference $I^{(k,i)}_{w\to v}$ from each tier is required, which depends on the spatial distribution of the set of interfering BSs and UEs in the tier, $\tilde{\Psi}^{(k)}_d$ and $\tilde{\Psi}^{(k)}_u$, respectively. The set of interfering BSs $\tilde{\Psi}^{(k)}_d$ in the $k^{th}$ tier is the same as the original set of BSs $\Psi^{(k)}_d$ excluding the transmitting BS itself in the UL and the serving BS in the UL. Hence, $\tilde{\Psi}^{(k)}_d$ is a PPP with intensity $\lambda_k$. Since each BS assigns a unique channel to each of the associated users and that $\lambda_u \gg \sum_{k=1}^{K} \lambda_k$, the intensity of the interfering UEs $\tilde{\Psi}^{(k)}_u$ on a certain channel in the $k^{th}$ tier is also $\lambda_k$. However, $\tilde{\Psi}^{(k)}_u$ is not a PPP because only one UE is using the channel in each Voronoi-cell, which imposes correlations among the positions of the interfering UEs on each channel and violates the PPP assumption. Furthermore, the employed association makes the set of interfering UEs $\tilde{\Psi}^{(k)}_u$ and the set of interfering BSs $\tilde{\Psi}^{(k)}_d$ correlated. The inter-correlations between the interfering UEs and the cross-correlations between the UEs and BSs impede the model tractability. Hence, to maintain the tractability, we ignore these correlations. The used assumptions to keep the model tractability are formally stated below.

**Assumption 1.** The set of interfering UEs $\tilde{\Psi}^{(k)}_u$ in the $k^{th}$ tier is a PPP with intensity $\lambda_k$.

**Assumption 2.** The point process $\tilde{\Psi}^{(k)}_d$ for the interfering BSs and the point process $\tilde{\Psi}^{(k)}_u$ for the interfering UEs both in the $k^{th}$ tier are independent.

**Assumption 3.** The point processes $\tilde{\Psi}^{(k)}_u$'s which represent the interfering UEs connected to different tiers are independent from each other.

**Remark 1.** The previous assumptions are necessary to maintain the model tractability. Assumption 1 has been used and validated in [9], [10], [12], [17], [18], Assumption 2 in [10], [12], and Assumption 3 in [17]. It is important to mention that the assumptions ignore the mutual correlations between the interfering sources, however, the correlation between the interfering sources and the test receiver are captured through the proper calculation for the interference exclusion region enforced by association and/or UL power control. The accuracy of the developed
model under these assumptions is validated via independent Monte Carlo simulation in Section IV.

Based on the previous assumptions, the LT of the aggregated interference is always generated from a PPP $\Phi$, but with different interference exclusion regions as will be shown later. The following Lemma presents the LT of the aggregated interference generated from a PPP with a general interference exclusion region, intensity, and transmit powers.

**Lemma 2.** Let $L_I(s)$ be the LT of the aggregate interference generated from a PPP network with intensity $\lambda$, i.i.d transmit powers $P_j$, unit means i.i.d exponentially distributed channel power gains, and interference protection region of $B(o, a)$, where $B(o, a)$ is a ball centered at the origin \(o\) and has a radius $a$. Then, $L_I(s)$ is given by,

$$L_I(s) = \exp \left( -\frac{2\pi \lambda}{\eta} \frac{E_P}{2} \left[ a^{2-\eta} s P \right. \left. 2F_1 \left[ 1, 1 - \frac{2}{\eta}; 2 - \frac{2}{\eta}; -a^{-\eta} P s \right] \right] \right), \tag{26}$$

where $2F_1$ is the hyper-geometric function [28], $E_P[.]$ is the expectation over the transmitted power of the sources, and $\eta > 2$ is a general path loss exponent. For the special case of $a = 0$, equation (26) reduces to,

$$L_I(s) = \exp \left( -\frac{2\pi \lambda}{\eta} E_P \left[ (sP)^{\frac{2}{\eta}} \right] \csc \left( \frac{2}{\eta} \right) \right). \tag{27}$$

*Proof: Refer to Appendix B.*

Based on the previous Lemma, in order to find the LT of the aggregated interference generated from a PPP network with $a \neq 0$, we need to know its intensity, the PDF of the transmit power of its nodes, and the interference protection region. If $a$ is independent of the transmit powers, then the expression given by (26) can be lower-bounded by the simplified expression given in the following Lemma.

**Lemma 3.** Let $L_I(s)$ be the LT of the aggregate interference generated from a PPP network with intensity $\lambda$, i.i.d transmit powers $P_j$, unit means i.i.d exponentially distributed channel power gains, and interference protection region of $B(o, a)$, where $B(o, a)$ is a ball centered at the origin \(o\) and has a radius $a$. Assuming that $a$ is independent from $P_j$, $\forall j$, then $L_I(s)$ can be lower-bounded by,
\[ \mathcal{L}_3(s) \geq \exp \left( \frac{-2\pi \lambda_k (\rho^{(k)})^{1-\frac{2}{\eta_{uu}}}}{\eta_{uu} - 2} \mathbb{E} \left[ P_u^{(k)} \right] \right) \left[ 1, 1 - \frac{2}{\eta_{uu}}, 2 - \frac{2}{\eta_{uu}}, -\rho^{(k)} \right] \right) \right), \] (28)

**Proof:** Refer to Appendix C.

Lemma 4. Let \( \mathcal{L}^{(e)}_{I_{u\rightarrow u}^{(i,k)}}(s) \) and \( \mathcal{L}^{(e)}_{I_{d\rightarrow u}^{(i,k)}}(s) \) represent the LTs of the UL to UL, DL to UL, DL to DL, and UL to DL aggregated interference generated from the \( k \)th tier affecting a CCU (CCE) and its serving BS given that both of them are in the \( i \)th tier, then these LTs are given by

\[
\mathcal{L}^{(e)}_{I_{u\rightarrow u}^{(i,k)}}(s) = \exp \left( -\frac{2\pi \lambda_k (\rho^{(k)})^{1-\frac{2}{\eta_{uu}}}}{\eta_{uu} - 2} \mathbb{E} \left[ P_u^{(k)} \right] \right) \left[ 1, 1 - \frac{2}{\eta_{uu}}, 2 - \frac{2}{\eta_{uu}}, -\rho^{(k)} \right] \right), \]

\[
\mathcal{L}^{(e)}_{I_{d\rightarrow u}^{(i,k)}}(s) = \mathcal{L}^{(e)}_{I_{d\rightarrow u}^{(i,k)}}(s) = \exp \left( -\frac{2\pi \lambda_k (\rho^{(k)})^{1-\frac{2}{\eta_{dd}}}}{\eta_{dd} - 2} \mathbb{E} \left[ P_d^{(k)} \right] \right) \left[ 1, 1 - \frac{2}{\eta_{dd}}, 2 - \frac{2}{\eta_{dd}}, -\rho^{(k)} \right] \right), \]

\[
\mathcal{L}^{(e)}_{I_{d\rightarrow d}^{(i,k)}}(s) = \exp \left( -\frac{2\pi \lambda_k (\rho^{(k)})^{1-\frac{2}{\eta_{dd}}}}{\eta_{dd} - 2} \mathbb{E} \left[ P_d^{(k)} \right] \right) \left[ 1, 1 - \frac{2}{\eta_{dd}}, 2 - \frac{2}{\eta_{dd}}, -\rho^{(k)} \right] \right), \]

\[
\mathcal{L}^{(e)}_{I_{u\rightarrow d}^{(i,k)}}(s) = \exp \left( -\frac{2\pi \lambda_k (\rho^{(k)})^{1-\frac{2}{\eta_{ud}}}}{\eta_{ud} - 2} \mathbb{E} \left[ P_u^{(k)} \right] \right) \left[ 1, 1 - \frac{2}{\eta_{ud}}, 2 - \frac{2}{\eta_{ud}}, -\rho^{(k)} \right] \right), \]

where,

\[
\mathbb{E} \left[ \left( P_u^{(k)} \right) \right] = \frac{\zeta^{(k)} \gamma \left( \frac{\zeta^{(k)}}{\rho^{(k)}} + 1, \pi \lambda_k \left( \frac{P_u}{\rho^{(k)}} \right) \right)}{\left( \pi \lambda_k \right)^{\zeta^{(k)}}} + \left( P_u \right)^{\zeta} \exp \left( \pi \lambda_k \left( \frac{P_u}{\rho^{(k)}} \right) \right), \]

(35)
and $U^{(i,k)}_{1}(r_o,s)$ is the intra-cell interference in the 3NT case, which is expressed as

$$U^{(i,k)}_{1}(r_o,s) =$$

$$\begin{cases} 
\left( \frac{P}{\rho^{(i)}} \right)^{\kappa_{ud}} \int_{0}^{\pi} \frac{P\{CCU\}f_{R_u^{(i)}}(r)}{\pi + \pi \rho^{(i)} (1 + (\cos (\theta))^2 - 2 \rho \cos (\theta))} d\theta dr + \int_{0}^{\pi} \frac{P\{CEU\}f_{R_u^{(i)}}(r)}{\pi + \pi \rho^{(i)} (r^2 + r^2 - 2r_o r \cos (\theta))} d\theta dr, & 3NT \\
1, & \text{O.W.}
\end{cases} \quad (36)$$

and $\gamma(\cdot, \cdot)$ is the lower incomplete gamma function \[28\].

**Proof:** Refer to Appendix D.

Note that $U^{(i,k)}_{1}(r_o,s)$ that appears in equations \(33\) and \(34\) represents the intra-cell interference, so it only had an effect in 3NT and in the LT of the interference from the tier that tagged transceiver belongs to it.

By Exploiting the previous Lemmas and \(20\), the outage probability is given by the following theorem.

**Theorem 1.** The outage probabilities in the UL and in the DL for a BS and a UE in the $i$th tier assuming a CCU and a CEU are given by,

$$O^{(i)}_{u}(R_d) = 1 - e^{-\frac{N_o R_u}{\rho^{(i)}}} U_{S_1 u}^{(i)} \left( \frac{\rho^{(i)}}{R_d} \right) \prod_{k=1}^{K} \mathcal{L}^{(c)}_{\alpha_{i,k}, \alpha_k} \left( -\frac{\tilde{R}_d \tilde{I}_u(\alpha_{i,k})^2}{\rho^{(i)}} \right) \mathcal{L}^{(c)}_{\beta_{i,k}, \beta_k} \left( -\frac{\tilde{R}_d \tilde{I}_u(\alpha_{i,k})^2}{\rho^{(i)}} \right), \quad (37)$$

$$O^{(i)}_{d}(R_d) = 1 - \int_{0}^{\pi} e^{-\frac{N_o R_d}{\rho^{(i)}}} U_{S_1 d}^{(i)} \left( \frac{P_d}{R_d} \right) f_{R_u^{(i)}}(r_o) \times \prod_{k=1}^{K} \mathcal{L}^{(c)}_{\alpha_{i,k}, \alpha_k} \left( -\frac{\tilde{R}_d \tilde{I}_d(\alpha_{i,k})^2}{P_u} \right) \mathcal{L}^{(c)}_{\beta_{i,k}, \beta_k} \left( -\frac{\tilde{R}_d \tilde{I}_d(\alpha_{i,k})^2}{P_u} \right) dr_o, \quad (38)$$

$$O^{(i)}_{d}(R_d) = 1 - \int_{0}^{\pi} e^{-\frac{N_o R_d}{\rho^{(i)}}} U_{S_1 d}^{(i)} \left( r_{d,\tilde{R}_d} \right) f_{R_u^{(i)}}(r_o) \times \prod_{k=1}^{K} \mathcal{L}^{(c)}_{\alpha_{i,k}, \alpha_k} \left( -\frac{\tilde{R}_d \tilde{I}_d(\alpha_{i,k})^2}{P_d^{(i)}} \right) \mathcal{L}^{(c)}_{\beta_{i,k}, \beta_k} \left( -\frac{\tilde{R}_d \tilde{I}_d(\alpha_{i,k})^2}{P_d^{(i)}} \right) dr_o, \quad (39)$$
\[ \mathcal{O}^{(i)}_d (R_d) = 1 - \int_{0}^{\infty} e^{-\frac{B_u(\alpha_i)}{(g+1) \ln(2)}} U_{SL_u}^{(i)} \left( \frac{\rho^{(i)}}{g} \right) \prod_{k=1}^{K} \mathcal{L}^{(c)}_{\alpha_i, \alpha_k} \left( \frac{-g|\hat{\mathcal{C}}_u(\alpha_i, \alpha_k)|^2}{\rho^{(i)}} \right) \mathcal{L}^{(c)}_{\alpha_i, \alpha_k} \left( \frac{-r^{\eta_{uu}} g|\hat{\mathcal{C}}_u(\alpha_i, \alpha_k)|^2}{P_u} \right) \, dr_o. \quad (40) \]

where,

\[ U_{SL_u}^{(i)} (x) = \int_{0}^{\infty} \exp \left( -\frac{\beta_u^{(i)} h P_d^{(i)} |\hat{\mathcal{C}}_u(\alpha_i, \alpha_k)|^2}{x} \right) f_{H_u}(h) \, dh, \quad (41) \]

\[ U_{SL_a}^{(i)} (x) = \begin{cases} \int_{0}^{\infty} \exp \left( -\frac{\beta_d^{(i)} h P_d^{(i)} |\hat{\mathcal{C}}_u(\alpha_i, \alpha_k)|^2}{x} \right) f_{H_u}(h) \, dh, & \text{2NT} \\ 1, & \text{3NT} \end{cases} \quad (42) \]

and \( \tilde{R}_d = 2^{\frac{R_d}{\eta_{uu}(\alpha_i) - 1}} \), \( f_{H_u}(.) \) is the distribution of the SI cancellation power. \( f_{R_e^{(i)}(.)} \) and \( f_{R_i^{(i)}(.)} \) are given in equations (21) and (22), respectively, and the LTs are given in Lemma 4.

**Proof:** Refer to Appendix E.

As mentioned earlier, to find the total average outage probability including both UEs’ types, Theorem 1 with equation (25) are used. Similarly, the total averaged ergodic rate can be found by,

\[ \mathcal{R}_{v^{(i)}}^{(i)} = \mathcal{R}_{v^{(i)}}^{(i)} \mathbb{P} \{ \text{CCU} \} + \mathcal{R}_{v^{(i)}}^{(i)} \mathbb{P} \{ \text{CEU} \}, \quad (43) \]

where, \( \mathcal{R}_{v^{(i)}}^{(i)} \) and \( \mathcal{R}_{v^{(i)}}^{(i)} \) are the ergodic rates for CCUs and CEUs which are given by the following theorem.

**Theorem 2.** The ergodic rates in the UL and in the DL for a BS and a UE in the \( i^{th} \) tier assuming a CCU and a CEU are given by,

\[ \mathcal{R}_{u^{(i)}}^{(i)} = \int_{0}^{\infty} \frac{B_u(\alpha_i)}{(g+1) \ln(2)} U_{SL_u}^{(i)} \left( \frac{\rho^{(i)}}{g} \right) \prod_{k=1}^{K} \mathcal{L}^{(c)}_{\alpha_i, \alpha_k} \left( \frac{-g|\hat{\mathcal{C}}_u(\alpha_i, \alpha_k)|^2}{\rho^{(i)}} \right) \mathcal{L}^{(c)}_{\alpha_i, \alpha_k} \left( \frac{-r^{\eta_{uu}} g|\hat{\mathcal{C}}_u(\alpha_i, \alpha_k)|^2}{P_u} \right) \, dg. \quad (44) \]

\[ \mathcal{R}_{a^{(i)}}^{(i)} = \int_{0}^{\frac{\eta_{uu}}{\rho}} \int_{0}^{\infty} f_{R_e^{(i)}(r_o)} B_u(\alpha_i) e^{-\frac{-r^{\eta_{uu}} g}{\rho}} U_{SL_u}^{(i)} \left( \frac{P_u r^{\eta_{uu}}}{g} \right) \prod_{k=1}^{K} \mathcal{L}^{(c)}_{\alpha_i, \alpha_k} \left( \frac{-r^{\eta_{uu}} g|\hat{\mathcal{C}}_u(\alpha_i, \alpha_k)|^2}{P_u} \right) \mathcal{L}^{(c)}_{\alpha_i, \alpha_k} \left( \frac{-r^{\eta_{uu}} g|\hat{\mathcal{C}}_u(\alpha_i, \alpha_k)|^2}{P_u} \right) \, dg \, dr_o. \quad (45) \]
Theorem 1 in equation of the outage probability given in equation where, are given in equations
where $f$ is given by,

$$\text{UEs to operate in 2NT if both of them are CCUs, otherwise, both the UEs will operate in 3NT,}
\text{MNT only affects the DL performance (see UL expressions in Theorem 1)}$$

These expressions are used in the next section to compare the performance and draw conclusions about 2NT and 3NT operation. Another interesting topology that we will present denoted by mixed network topology (MNT). In MNT, all the UEs have SI cancellation capabilities, but the BSs allow its UEs to operate in 2NT if both of them are CCUs, otherwise, both the UEs will not exploit their SI cancellation capabilities and will operate in 3NT. Similar to the 2NT and 3NT, MNT only affects the DL performance (see UL expressions in Theorem 1 & 2). The following theorem gives the DL ergodic rate using the MNT.

**Theorem 3.** The DL ergodic rate for a UE in $i^{th}$ tier under the MNT, where the BSs allow its UEs to operate in 2NT if both of them are CCUs, otherwise, both the UEs will operate in 3NT, is given by,

$$R^{(i)}_{d} = R^{(i)}_{d(cc)} P \{ \text{CCU} \}^2 + P \{ \text{CCU} \} P \{ \text{CEU} \} \left( R^{(i)}_{d(cc)} + R^{(i)}_{d(cc)} \right) + R^{(i)}_{d(cc)} P \{ \text{CEU} \}^2 ,$$

where $f_{H_s}(\cdot)$ is the distribution of the SI cancellation power: $f_{R^{(i)}_d}(\cdot)$, $f_{R^{(i)}_d}(\cdot)$, $U^{(i)}_{SL_u}(\cdot)$ and $U^{(i)}_{SL_d}(\cdot)$ are given in equations (21), (22), (41) and (42) respectively, and the LTs are given in Lemma 4.

**Proof:** The ergodic rate given by equation (7) can be expressed as,

$$R = E [BW \log_2 (1 + \text{SINR})] = \int_0^\infty \left\{ BW \log_2 (1 + \text{SINR}) > t \right\} dt = \int_0^\infty [1 - \mathcal{O}(t)] dt,$$

where, (i) follows from the fact that the SINR is always positive and (ii) from the definition of the outage probability given in equation (8). Then by substituting the expressions given in Theorem 1 in equation (41), expressions in Theorem 2 are obtained.

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Theorem 1 and Theorem 2 provide unified outage and rate expressions for 2NT and 3NT cases. These expressions are used in the next section to compare the performance and draw conclusions about 2NT and 3NT operation. Another interesting topology that we will present denoted by mixed network topology (MNT). In MNT, all the UEs have SI cancellation capabilities, but the BSs allow its UEs to operate in 2NT if both of them are CCUs, otherwise, both the UEs will not exploit their SI cancellation capabilities and will operate in 3NT. Similar to the 2NT and 3NT, MNT only affects the DL performance (see UL expressions in Theorem 1 & 2). The following theorem gives the DL ergodic rate using the MNT.
where,

\[ R_{dcc}^{(i)} = \left( \frac{\rho_{d}}{\rho_{d}^{(r)}} \right) \frac{1}{\eta_{ad}} \int_{0}^{\infty} \int_{0}^{(g+1)\ln(2)} f_{R_{d}^{(i)}(r_{o})} B_{d}(\alpha_{i}) e^{-\frac{N_{d} r_{d}^{(a)g}}{r_{d}} s} \left( \frac{\beta_{4} h P_{u} \bar{C}_{d}(\alpha_{i}, \alpha_{k})^{2}}{P_{d}^{(i)}} \right) f_{H_{d}(h)} dH_{d} \]  

\[ \times \prod_{k=1}^{K} \mathcal{L}_{c}^{(c)} \left( \frac{-r_{d}^{(a)g} \bar{I}_{d}(\alpha_{i}, \alpha_{k})^{2}}{P_{d}^{(i)}} \right) \mathcal{L}_{c}^{(c)} \left( \frac{-r_{d}^{(a)g} \bar{C}_{d}(\alpha_{i}, \alpha_{k})^{2}}{P_{d}^{(i)}} \right) \]  

\[ R_{dce}^{(i)} = \left( \frac{\rho_{d}}{\rho_{d}^{(r)}} \right) \frac{1}{\eta_{ad}} \int_{0}^{\infty} \int_{0}^{(g+1)\ln(2)} f_{R_{d}^{(i)}(r_{o})} B_{d}(\alpha_{i}) e^{-\frac{N_{d} r_{d}^{(a)g}}{r_{d}} s} \left( \frac{\beta_{4} h P_{u} \bar{C}_{d}(\alpha_{i}, \alpha_{k})^{2}}{P_{d}^{(i)}} \right) \right) f_{R_{e}^{(i)}(r)} d\theta dr \]  

\[ \times \prod_{k=1}^{K} \mathcal{L}_{c}^{(c)} \left( \frac{-r_{d}^{(a)g} \bar{I}_{d}(\alpha_{i}, \alpha_{k})^{2}}{P_{d}^{(i)}} \right) \mathcal{L}_{c}^{(c)} \left( \frac{-r_{d}^{(a)g} \bar{C}_{d}(\alpha_{i}, \alpha_{k})^{2}}{P_{d}^{(i)}} \right) \]  

\[ R_{dce}^{(i)} = \left( \frac{\rho_{d}}{\rho_{d}^{(r)}} \right) \frac{1}{\eta_{ad}} \int_{0}^{\infty} \int_{0}^{(g+1)\ln(2)} f_{R_{d}^{(i)}(r_{o})} B_{d}(\alpha_{i}) e^{-\frac{N_{d} r_{d}^{(a)g}}{r_{d}} s} \left( \frac{\beta_{4} h P_{u} \bar{C}_{d}(\alpha_{i}, \alpha_{k})^{2}}{P_{d}^{(i)}} \right) \right) f_{R_{e}^{(i)}(r)} d\theta dr \]  

\[ \times \prod_{k=1}^{K} \mathcal{L}_{c}^{(c)} \left( \frac{-r_{d}^{(a)g} \bar{I}_{d}(\alpha_{i}, \alpha_{k})^{2}}{P_{d}^{(i)}} \right) \mathcal{L}_{c}^{(c)} \left( \frac{-r_{d}^{(a)g} \bar{C}_{d}(\alpha_{i}, \alpha_{k})^{2}}{P_{d}^{(i)}} \right) \]  

\[ L_{f_{u,i}^{(cc)}}^{(c)}(s) = \exp \left( -\frac{2\pi \lambda_{k}(P_{u})^{\frac{1}{\eta_{ad}}} s}{\eta_{ad}} \right) \]  

\[ \exp \left( -\rho_{k}^{(a)g} \frac{2}{\eta_{ad}} s \right) \csc \left( \frac{2\pi}{\eta_{ad}} \right) \]  

and \( L_{f_{u,i}^{(c)}}^{(c)}(s), L_{d}^{(c)}(s), f_{R_{d}^{(i)}(r)}, \) and \( f_{R_{e}^{(i)}(r)} \) are given in equations (21), (22), (33), and (34), respectively.
### Table I: Parameters Values

| Parameter | Value     | Parameter | Value     |
|-----------|-----------|-----------|-----------|
| $P_u$     | 2 W       | $P_d$     | 5 W       |
| $\lambda$| 1 BSs/Km$^2$ | $N_0$     | -90 dBm   |
| $P_{d}^{HD}$ | 1 MHz    | $P_{d}^{BD}$ | 1 MHz     |
| $\beta_d$ | -75 dB    | $\beta_u$ | -110 dB   |
| $\rho$    | -70 dBm   | $\epsilon$ | 0.03134   |
| $b_d$     | R         | $b_u$     | T         |
| $\eta_{au},\eta_{dd}$ | 4     | $\eta_{du},\eta_{ud}$ | 4     |

**Proof:** Similar to the proof of Theorem 2, except that for the LTs in Lemma 3, $U_1^{(i,k)}(s)$ is set to 1 in the case of two CCUs, for the other cases similar derivation of $U_1^{(i,k)}(s)$ in Appendix D is followed. Then by multiplying by the probability of each case, the expressions in Theorems 3 is found.

IV. Results and Discussion

Throughout this section, we verify the developed mathematical paradigm via independent system level simulations, where the BSs are realized via a PPP over an area of 400 Km$^2$. Then, the UEs are distributed uniformly over the area such that each BS has at least two UEs within its association area. Each BS randomly selects two UEs to serve. The SINR is calculated by summing the interference powers from all the UEs and the BSs after multiplying them by the effective interference factors. In the uplink, the transmit powers of the UEs are set according to the power control discussed in Section II. The results are taken for UEs and BSs within a square of 4Km$^2$ around the center to avoid the edges effect. Unless otherwise stated, the parameters values in Table 1 are used. Note that for the average SI cancellation power, the maximum reported value according to [3] is -110 dB, so we set $\beta_u$ to -110 and consider $\beta_d$ to be less or equal to $\beta_u$, since the BSs are more likely to have more powerful SI cancellation capabilities.

For the pulse shaping, we consider two basic pulse shapes, namely, triangular and rectangular pulse shapes, which have the following FTs are given by

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4Employing and designing more sophisticated pulse shapes for specific purposes is left for future work.
(a) 2NT DL ergodic rate.

(b) 3NT DL ergodic rate.

(c) MNT DL ergodic rate.

(d) UL ergodic rate.

Fig. 4: Ergodic Rates vs $\alpha$ under different network topology, where the solid and dashed line represent the results obtained analytically by Theorems 1, 2, and 3, and the diamonds represent the results obtained by simulations.

$$S(f, BW, b) = \begin{cases} \frac{\text{SINC}(\frac{f}{BW})}{\sqrt{\int_{-\infty}^{\infty} \text{SINC}^2(\frac{2f}{BW}) df}} & b = R, \\ \frac{\text{SINC}^2(\frac{f}{BW})}{\sqrt{\int_{-\infty}^{\infty} \text{SINC}^4(\frac{2f}{BW}) df}} & b = T. \end{cases}$$  \hspace{1cm} (56)

where $b = R$ when the rectangular pulse is considered and $b = T$ when triangular pulse is considered. For the SI cancellation power distribution $f_{H_s}(.)$, it is assumed to be exponentially distributed with unity mean unless otherwise stated.

Fig. 4 shows the ergodic rate behavior for UL and DL versus $\alpha$ under different network topologies. The close match between the analysis and simulation results validates the developed
mathematical model and verifies the accuracy of the mentioned assumptions in Section III-A and the bound presented in Lemma 2. As the figure shows, in general, the CCUs have better performance compared to the CEUs in all cases as expected; since CEUs’ service distances are larger than the CCUs’ service distances, the average received power from their serving BSs suffers a more severe path loss that deteriorates the DL performance. The UL performance of CEUs, on the other hand, is lower than the CCUs due to the constrained transmit power of the UEs since CEUs can’t achieve the desired threshold $\rho$ at their serving BSs. The figure also shows that choosing the suitable network topology highly depends on $\beta_d$ and $\alpha$, hence, we plot Fig. 5 to visualize the rate behavior with $\beta_d$. The figure compares the performance of 2NT, 3NT, and MNT for different values of $\alpha$; $\alpha=1$ (FD, where the DL rate is maximized), $\alpha=0.28859$ (where the UL rate is maximized), and $\alpha=0$ (the conventional HD case). As the figure shows, the effect of $\beta$ is more severe on the CEUs rate, this is because CEUs transmit with their maximum power, which makes the residual SI power more dominant compared to CCUs. Also, operating in FD mode is not suitable for CEUs and setting $\alpha$ to 0.28856 can achieve better performance for them, this is also because CEUs are more susceptible to the increase in the SI and the aggregated interference with $\alpha$.

Fig. 5 also shows that there is a turning point at which the 2NT outperforms 3NT for values of $\beta_d$ less than it, this point occurs when the SI experienced by DL UEs in the 2NT becomes more significant than the intra-cell interference experienced by DL-UEs in 3NT. Interestingly, even when the SI is negligible, the 2NT does not offer significant gains when compared to the 3NT that experience intra-cell interference. Hence, network operators can harvest FD gains by HD UEs almost similar to the gains harvested by FD UEs with efficient SI cancellation capabilities. In the case that FD UEs have poor SI cancellation capabilities, the 3NT can offer significant gains when compared to the 2NT case. In all cases, network operators do not need to carry the burden of implementing SI cancelation in the UEs to harvest FD gains.

If UEs have already SI cancellation capabilities, then we present MNT, where the UEs operate in 2NT or 3NT depending on their serving distances so that it can be considered as location-dependent topology. As the figure shows, MNT offers higher gain than both 2NT and 3NT at

\footnote{The UL performance is maximized at this particular value of $\alpha$ due to the orthogonality between the used pulse shapes, for more details refer to \cite{10}.}
Fig. 5: DL ergodic Rates vs $\beta_d$ under different network topology, where FD denotes $\alpha = 1$ and $\alpha D$ denotes $\alpha \approx 0.2886$.

A particular region of $\beta_d$. For the average rate, when $\beta_d \in [-112, -97]$ the MNT achieves the best performance, note that this region contains the maximum reported value of $\beta_d$ as reported in [3]. Also, for low (high) values of $\alpha$, the performance of MNT is close to the 2NT (3NT) especially for low values of $\alpha$, e.g. $\alpha = 0.28856$. Hence, if the UEs have different $\beta_d$, then by using the MNT we can exploit their SI cancellation capabilities without the need of going to the extremes; 2NT which will harm the UEs with low $\beta_d$ and 3NT which will waste the capabilities of UEs who have a sufficient (low) values of $\beta_d$.

Last but not least, Fig. 6 shows the rate behavior with $\alpha$ for exponential and degenerate distributions for SI power cancellation. The figure shows that the performance in both cases is almost the same. This implies that the models and insights obtained for any of the two cases in
Fig. 6: Average DL and UL ergodic rates, assuming exponentially distributed and constant SI cancellation power, where the solid and the dashed lines represent the results obtained analytically while the diamonds the results obtained by simulation both for $\beta_d = \beta_u = -75$ dBm.

the literature hold for the other.

V. CONCLUSION

This paper presents a mathematical paradigm for multi-tier cellular networks with FD BSs and HD/FD users. The presented model captures detailed system parameters including pulse shaping, filtering, imperfect self-interference cancellation (SIC), partial uplink/downlink overlap, uplink power control, limited users’ transmit powers, and UE-BS association. To this end, unified rate expressions for 2 node topology (2NT) with FD users and 3 node topology (3NT) with HD users are presented and used to compare their performance. The results show that there exist a turning point, that depends on the efficiency of self-interference cancellation, at which the performance of 3NT outperforms the 2NT. The results also show that even when SI is efficiently canceled, the 2NT does not offer significant gains when compared to the 3NT operation. This implies that network operators can harvest FD gains by implementing FD transceivers at their BSs regardless of the state of the users (i.e., FD or HD). We also proposed a location dependent topology, denoted as mixed network topology (MNT), that has a particular importance if the UEs have already different SIC capabilities, since it achieves a close performance to the 2NT for UEs with powerful SIC, a close to the 3NT for UEs with poor SIC, and it outperforms both topologies for the currently achievable range of SIC.
APPENDIX A
PROOF OF LEMMA 1

Let $R_k$ denotes the distance between the origin and the nearest BS in the $k^{th}$ tier, then the PDF of $R_k$, using the null probability of the PPP, is given by,

$$f_{R_k}(r) = 2\pi \lambda_k r \exp \left( -\pi \lambda_k r^2 \right) \quad r \geq 0. \quad (57)$$

Since the PPPs representing the tiers locations are independent, these distance ($R_k$’s) are also independent. Given that the UE is connected to the $i^{th}$, the cumulative distribution function (CDF) of its serving distance is given by,

$$F_{R_i}(r) = \Pr \{ R_i \leq r \} = \Pr \{ R_i \leq r | R_i \leq \frac{\tau_i}{r_i} \forall j \mathrel{\in} \{1, ..., K\}, j \neq i \} = \frac{\Pr \{ R_i \leq r \cap R_i \leq \frac{\tau_i}{r_i} \forall j \mathrel{\in} \{1, ..., K\}, j \neq i \}}{\Pr \{ R_i \leq \frac{\tau_i}{r_i} \forall j \neq i \}} \quad (58)$$

The denominator is given by,

$$\Pr \{ R_i \leq \frac{\tau_i}{r_i} \forall j \neq i \} = \int_0^\infty f_{R_i}(r_i) \left[ \prod_{j=1, j \neq i}^M \int_{\left( \frac{\tau_i}{r_i} \right)}^\infty f_{R_j}(r_j) dr_j \right] dr_i = \frac{\lambda_i}{\sum_{j=1}^M \frac{\tau_i^2}{r_j^2} \lambda_j} \quad (59)$$

and the nominator is given by,

$$\Pr \{ r_i \leq r \cap r_i \leq \frac{\tau_j}{r_i} \forall j \neq i \} = \int_0^\infty f_{R_i}(r_i) \left[ \prod_{j=1, j \neq i}^M \int_{\left( \frac{\tau_i}{r_i} \right)}^\infty f_{R_j}(r_j) dr_j \right] dr_i = \frac{\lambda_i \left( 1 - \exp \left( -\pi r^2 \sum_{j=1}^M \frac{\tau_i^2}{r_j^2} \lambda_j \right) \right)}{\sum_{j=1}^M \frac{\tau_i^2}{r_j^2} \lambda_j} \quad (60)$$

By substituting equations (59) and (60) in (58) it results in,

$$F_{R(i)}(r) = 1 - \exp \left( -\pi r^2 \sum_{j=1}^M \frac{\tau_i^2}{r_j^2} \lambda_j \right) = 1 - \exp \left( -\pi \bar{\lambda}_i r^2 \right) \quad r \geq 0. \quad (61)$$

where $\bar{\lambda}_i = \sum_{j=1}^M \frac{\tau_i^2}{r_j^2} \lambda_j$ and the PDF is given by,

$$f_{R(i)}(r) = 2\pi \bar{\lambda}_i r \exp \left( -\pi \bar{\lambda}_i r^2 \right) \quad r \geq 0. \quad (62)$$

Given that the UE is a CCU, we need to truncate the PDF in (62) by the possible distances for the CCU which is based on the used channel inversion power control. Let $R_c(i)$ denote the
serving distance for a chosen CCU connected to the $i^{th}$ tier, then its PDF is given by,

$$f_{R,(i)}(r) = \frac{2\pi \bar{\lambda}_i r \exp(-\pi \bar{\lambda}_i r^2)}{\int_0^r 2\pi \bar{\lambda}_i r \exp(-\pi \bar{\lambda}_i r^2) dr} \mathbb{I}\left\{0 \leq r \leq \left(\frac{P_{\rho \lambda}(i)}{\rho \lambda(i)}\right)^{\frac{1}{\eta}}\right\}(r). \tag{63}$$

Similarly, for the CEUs,

$$f_{R,(i)}(r) = \frac{2\pi \bar{\lambda}_i r \exp(-\pi \bar{\lambda}_i r^2)}{\int_0^{\infty} 2\pi \bar{\lambda}_i r \exp(-\pi \bar{\lambda}_i r^2) dr} \mathbb{I}\left\{0 \leq r \leq \left(\frac{P_{\rho \lambda}(i)}{\rho \lambda(i)}\right)^{\frac{1}{\eta}} \right\}(r). \tag{64}$$

**APPENDIX B**

**PROOF OF LEMMA 2**

The proof is as follows,

$$L_I(s) = \mathbb{E} \left[ \exp \left( \sum_{j \in \Phi} -s P_j h_j r_j^{-\eta} \mathbb{I}\{r_j > a_j\} \right) \right],$$

$$(i) \equiv \mathbb{E}_\Phi \left[ \prod_{r_j \in \Phi} \mathbb{E}_{h_j, r_j} \left[ \exp \left( -s P_j h_j r_j^{-\eta} \mathbb{I}\{r_j > a_j\} \right) \right] \right],$$

$$(ii) \equiv \exp \left( -2\pi \bar{\lambda} \mathbb{E}_P \left[ \int_a^{\infty} \mathbb{E}_h \left[ (1 - \exp \left( -s Ph r^{-\eta} \right) \right] r dr \right] \right),$$

$$(iii) \equiv \exp \left( - \frac{2\pi \bar{\lambda}}{\eta - 2} \mathbb{E}_P \left[ a^{2-\eta} s P \text{$_2F_1$} \left[ 1, 1 - \frac{2}{\eta}, 2 - \frac{2}{\eta}, -a^{-\eta} s P \right] \right] \right), \tag{65}$$

where, $(i)$ follows from the independence between $\tilde{\Psi}$ and $h_j$, $(ii)$ by using the probability generation functional (PGFL) of PPP and $(iii)$ by using the LT of $h$ and by evaluating the integral.

**APPENDIX C**

**PROOF OF LEMMA 3**

The proof is based on Jensen’s inequality [29, Section 3.1.8], hence, we need to prove that the exponent in equation (28) is convex w.r.t $P$. One way to do it, is by proving that the second derivative of the exponent is positive in everywhere w.r.t $P$ [29, Section 3.1.4], or equivalently, if we define $y = a^{-\eta} s P$, which is positive, and we proved that the second derivative of $G(y)$ is positive everywhere, where $G(y)$ is defined as

$$G(y) = -y \text{$_2F_1$} \left[ 1, 1 - \frac{2}{\eta}, 2 - \frac{2}{\eta}, -y \right] \tag{66}$$
The second derivative of \( G(y) \) is given by

\[
\frac{d^2 G(y)}{dy^2} \overset{(i)}{=} - (1 - \frac{2}{\eta}) \left( \frac{1 - \frac{2}{\eta}}{\eta + 1} - \frac{2 F_1 \left(1,1 - \frac{2}{\eta}; 2 - \frac{2}{\eta}; -y\right)}{y} \right) - \frac{1}{(y + 1)^2}
\]

\[
\overset{(ii)}{=} \left( \frac{2 F_1 \left(1,1 - \frac{2}{\eta}; 2 - \frac{2}{\eta}; -y\right)}{y} \right) - \frac{1}{\eta} + \frac{1}{(y + 1)^2} \left(1 - \frac{2}{\eta}\right)
\]

where, \( (i) \) is found by using \([28]\) Equations (15.2.2, 15.2.10-15.2.27) and \( (ii) \) by simple mathematical simplifications. Since \( \frac{1}{(1+y)^2} \left(1 - \frac{2}{\eta}\right), \frac{2}{\eta}, \text{ and } y \) are positive for \( \eta > 2 \), if we prove that \( G_2(y) \) is also positive, where \( G_2(y) = \left(2 F_1 \left(1,1 - \frac{2}{\eta}; 2 - \frac{2}{\eta}; -y\right) - \frac{1}{\eta + 1}\right) \), then the second derivative is positive everywhere. The first step is to use the integral definition of the hypergeometric function given by \([28]\) Equation (15.3.1)], by projecting it on our case it results in,

\[
2 F_1 \left(1,1 - \frac{2}{\eta}; 2 - \frac{2}{\eta}; -y\right) = \frac{\Gamma \left(2 - \frac{2}{\eta}\right)}{\Gamma \left(1 - \frac{2}{\eta}\right)} \int_0^1 \frac{t^{\frac{-2}{\eta}}}{1 + ty} dt
\]

\[
\overset{(i)}{=} \left(1 - \frac{2}{\eta}\right) \int_0^1 \frac{t^{\frac{-2}{\eta}}}{1 + ty} dt \overset{(ii)}{>} \int_0^1 \frac{t^{\frac{-2}{\eta}}}{1 + ty} dt
\]

where \( (i) \) follows by equation \([28], 6.1.15\] and \( (ii) \) follows from the assumption of \( \eta > 2 \). The first derivative of the integrand w.r.t \( t \) in equation \((68)\) is given by,

\[
\left(\frac{t^{-2/\eta}}{ty + 1}\right)' = \frac{t^{-2/\eta} \left(- (\eta + 2) ty - 2\right)}{\eta (ty + 1)^2}
\]

since \( \eta, y \) and \( t \) are positive, the first derivative is negative everywhere, hence, the integrand is a decreasing function in \( t \) and its minimum occurs at the boundary \( t = 1 \). Since the minimum is at \( t = 1 \), we can lower-bound the integral by evaluating the integrand at \( t = 1 \) multiplied by the region width which is one,

\[
\int_0^1 \frac{t^{\frac{-2}{\eta}}}{1 + ty} dt > \frac{1}{1 + y}
\]

By substituting the inequality in \( G_2(y) \) we get,

\[
G_2(y) > \frac{1}{1 + y} - \frac{1}{1 + y} = 0.
\]

hence, the second derivative is positive everywhere is terms of \( y \), or equivalently the exponent in \((28)\) is convex w.r.t \( P \) and Jensen’s inequality can be applied which results in,

\[
-\frac{2\pi\lambda}{\eta - 2} a^{2 - \eta} s E[P] ~ 2 F_1 \left[1,1 - \frac{2}{\eta}; 2 - \frac{2}{\eta}; -a^{-\eta} E[P] s\right] \leq \frac{2\pi\lambda}{\eta - 2} E_P \left[a^{2 - \eta} s P ~ 2 F_1 \left[1,1 - \frac{2}{\eta}; 2 - \frac{2}{\eta}; -a^{-\eta} Ps\right]\right]
\]

\[(72)\]
APPENDIX D
PROOF OF LEMMA 4

Starting by the 2NT, based on Lemma 2 and 3, we only need to determine the interference exclusion region (IER) for each case \((a_j)\). Starting by the DL to DL interference, denoted by \(L_{i,k}^{(i,k)}(s)\) based on Section II-C, \(r_o \tau_i \leq r_j \tau_k\) is always satisfied, so the IER is defined by \(r_j < r_o \tau_i \tau_k\), by substituting \(a\) in Lemma 2 by \(r_o \tau_i \tau_k\) the expression for \(L_{i,k}^{(i,k)}(s)\) is found for both the CCU and CEUs. For the UL to UL, based on the power inversion for CCUs and following [17], \(a = \left(\frac{P_{u_1}}{\rho^o}\right)^{1/2}\). For CEU, \(a\) is approximated by \(a = r_o [18]\), since \(a\) is independent from the transmit powers, Lemma 3 is used for this case. Moving to the UL to DL, CCU are assumed to be allocated by their serving BSs (since the distance between them is limited) as in [10], so \(a = \left(\frac{P_{u_1}}{\rho^o}\right)^{1/2}\) as in the UL to UL case. For the CEUs, their is no IER, so \(a = 0\). For the DL to UL their is no IER in both cases (CCUs and CCUs), so \(a = 0\).

For the 3NT, all the previous regions are valid except for the case of UL to DL interference, since UEs will suffer from intra-cell interference if \(i = k\), which violates the defined IER, so we have to obtain it separately. Let \(P_{u_i}, h_{1-o}, r_{1-o},\) and \(r_1\) denote the transmitted power of the interfering user, the channel gain between the two users, the distance between them and the distance between the interfering UE and the serving BS, respectively, then the LT of the interfering power can be expressed as which is denoted by \(U_{1}^{(i,k)}(s)\),

\[
U_{1}^{(i,i)}(s) = \mathbb{E}\left[ e^{-sP_{u_i} h_{1-o} r_{1-o}^{-\eta}} \right],
\]

\[
= \mathbb{E}\left[ e^{-sP_{u_i} h_{1-o} (r_o^2 + r_1^2 - 2r_or_1 \cos(\theta))^{-\eta/2}} \right],
\]

where \((i)\) follows by expressing \(r_{1-o}^2\) as \(r_o^2 + r_1^2 - 2r_or_1 \cos(\theta)\) by using the cosine rule, where \(\theta\) is the uniformly distributed between 0 and \(\pi\). When the other UE is a CCU, which has a probability \(\mathbb{P}\{\text{CCU}\}\), then \(P_{u_1} = \rho^{(j)} r_1^\eta\), and when it is CEU, which has a probability \(\mathbb{P}\{\text{CEU}\}\), then \(P_{u_1} = P_u\) by substituting these values and by exploiting the exponential distribution of \(h_{1-o}\), the expression for \(U_{1}^{(i,k)}(s)\) is found.

APPENDIX E
PROOF OF THEOREM 1

Starting by the UL outage probability, for CCUs the transmitted power is equal to \(\rho r_o^\eta\), and for the CEUs the transmitted power is set to the maximum \(P_u\), by substituting these values in (20) we get equations (37) and (38) except \(U_{1}^{(i)}\) which is found by substituting \(\tilde{\sigma}_s^2\) by its value.
given in (16) and then by averaging over $h_s$ while conditioning on $r_o$. Similar steps are followed to find the outage in the DL direction.

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