Quantum secret sharing without entanglement

Guo-Ping Guo∗, Guang-Can Guo†

Key Laboratory of Quantum Information, University of Science and Technology of China, Chinese Academy of Sciences, Hefei, Anhui, P.R.China, 230026

After analysing the main quantum secret sharing protocol based on the entanglement states, we propose an idea to directly encode the qubit of quantum key distributions, and then present a quantum secret sharing scheme where only product states are employed. As entanglement, especially the inaccessible multi-entangled state, is not necessary in the present quantum secret sharing protocol, it may be more applicable when the number of the parties of secret sharing is large. Its theoretic efficiency is also doubled to approach 100%.

PACS number(s): 03.67.Hk, 89.70.+c

I. INTRODUCTION

Suppose the president of a bank, Alice, wants to give access to a vault to two vice presidents, Bob and Charlie, who are not entirely trusted. Instead of giving the combination to any one individual, it may be desirable to distribute information in such a way that no vice president alone has any knowledge of the combination, but both of them can jointly determine the combination. Classical cryptography provides an answer which is known as secret sharing [1]. Alice creates two coded messages and one of them is sent to Bob and the other to Charlie. Each of the encrypted message contains no information about her original message, but together they contain the complete message.

However, either a fourth party or the dishonest member of the Bob-Charlie pair gains access to both of Alice’s transmissions can learn the contents of her message in this classical procedure. Quantum secret sharing protocols have been proposed to accomplish this work securely [2,3,4,5] where multi-photon entanglement is employed. Recently, many kinds quantum secret sharing with entanglement have been proposed [6,7,8,9]. As the quantum key distribution and classical sharing protocol can be used straightforward to accomplish secret sharing safely, the aim of the quantum secret sharing protocols is to allow one to determine whether an eavesdropper has been active during the secret sharing and reduce the resources necessary to implement such multi-party secret sharing tasks [2]. But the efficiency of those quantum secret sharing protocols using entanglement can only approach 50% in principle, the lack of the multi-parties entanglement also holds their experimental implementation. The efficiency of preparing even tripartite or four-partite entangled states is very low [10,11].

There is another thing that should be noted in this quantum secret sharing scheme with GHZ states. It uses the correlation of the measurement value of the three qubits of the GHZ state [2]:

$$\langle \sigma_x^a \otimes \sigma_y^b \otimes \sigma_x^c \rangle = -\langle \sigma_y^a \otimes \sigma_x^b \otimes \sigma_y^c \rangle = -\langle \sigma_y^a \otimes \sigma_y^b \otimes \sigma_x^c \rangle = 1,$$

where $\sigma_i^j$ means the measurements value $\pm 1$ of qubit $i$ along basis $j$ with $i = a, b, c$ and $j = x, y, z$.

∗Electronic address: harryguo@mail.ustc.edu.cn

†Electronic address: gcguo@ustc.edu.cn
\( j = x, y, z \). So only the cases that even of the three qubits are measured along the \( y \) direction are kept to share secret in this scheme and its theory efficiency is only 50%.

In the security analysis of the case that Bob eavesdrops both qubits \( b \) and \( c \), the authors assume that he measures them jointly in the \((|00\rangle \pm |11\rangle)/\sqrt{2} \) or \((|00\rangle \pm i|11\rangle)/\sqrt{2} \) basis. But we find if Bob measures qubits \( b \) and \( c \) separately (measures qubits \( b \) and \( c \) in product states), he can cheat and get Alice’s secret without being detected. For example, suppose Bob measures the qubits \( b \) and \( c \) in the product basis \( \sigma^b_y \otimes \sigma^c_y \), he can thus infer Alice’s value if she measures in the basis \( \sigma^a_x \). Then Bob can send a qubit in arbitrary state to Charlie.

In the procedure of announcing basis, Bob tries to announce his direction after Charlie and cheats his announcement to let Alice keep the choose that she has measured his particle in the basis \( \sigma^a_x \). Different from the eavesdropping analysis in the paper [2], this eavesdropping will not introduce a higher than usual failure rate. For example, if Charlie announces that he has chosen his measurement in the basis \( \sigma^c_x \), Bob can announce that his measurement is also in the basis \( \sigma^b_x \). Then if Alice also makes her measurement in the basis \( \sigma^a_x \), she will believe that there is correlation between those three man’s measurement value and keep this measurement value as secret. But Bob has successfully eavesdropped this secret value.

Although Bob has changed the state of qubit sending to Charlie, he can cheat again in the check procedure if Charlie announces his check bit before Bob and then escapes detection. For example, when he had measured the qubits \( b \) and \( c \) in the product basis such as \( \sigma^b_y \otimes \sigma^c_y \), Bob can infer that if Alice measures her particle in the basis \( \sigma^a_x \) will gets the value +1. When Charlie that his measurement is \(-1\), Bob can announces that his measurement is also \(-1\). Then no error will occur in this check procedure. Thus Bob successfully eavesdrops the secret without being detected by Alice and Charlie. It means that if one party (say Bob) can reveal his directions and values after the other party (say Charlie), he is able to discover Alice’s secret bit value, without any assistance from Charlie, and escape being detected. The feature of this eavesdropping is that Bob directly measures qubits \( b \) and \( c \) in product basis to learn Alice’s value. He cheats in the basis announcing procedure and check procedure again. Bob needn’t know Alice measurement direction, but he can let Alice only keep the measurements he desired by announcing his direction according to Charlie’s measurement direction. Then this eavesdropping is more powerful than those having been analyzed in the paper [2].

In order to detect this kind of eavesdropping, Alice should randomly require one party to reveal his directions before the other one, sometimes Bob before Charlie, sometimes Charlie before Bob. And in the check procedure, she also randomly requires one party to reveal his value before the other. This random can ensure the security of this quantum secret sharing protocol based on GHZ state correlation. It has also been pointed out in the paper [3], Alice and Bob can prevent this eavesdropping by releasing the outcomes for the check bits before the announcement of their measurement directions. We can see that the paper [1] introduce the EPR state correlations to split the secret. The efficiency of those quantum secret sharing protocols with entanglement can only reach 50% in principle.

In this paper, we propose the idea to directly encode the qubits of quantum key distributions classically and present a quantum secret sharing scheme employing product states to achieve the aim mentioned above. Its security is based on the quantum no-cloning theory just as the BB84 quantum key distribution. Comparing with the efficiency 50% limiting for the existing quantum secret sharing protocols with quantum entanglement, the present scheme can be 100% efficient in principle. It has been pointed out in the paper [2] that the quantum key distribution can accomplish the task of secret sharing, the resources necessary can be much smaller with our present protocol although it is very simple and alike the BB84 quantum key distribution scheme. Our emphasis is that the classically encoding the qubits of quantum key distributions simply may be more economical than the idea of the paper [3] to use EPR states correlations to split secret. This may suggests that the introduce of entanglement into the quantum secret sharing may be unworthiness.
II. EFFICIENCY QUANTUM SECRET SHARING WITHOUT ENTANGLEMENT

Now we present a quantum secret sharing protocol where no entanglement is employed. Its essence is to encode the sources of two BB84 key distribution schemes. The particular process of this quantum secret sharing is as follows:

1. Alice creates two random $n$-bit string $L$ and $A$. For each bit of $L$ and $A$, she creates a two-qubit product state $|bc\rangle$ in the basis $\oplus$ (if the corresponding bit of $L$ is 0) or the basis $\otimes$ (if the bit of $L$ is 1) where the XOR result of the two qubits $b$ and $c$ equals to the bit value of the string $A$. The two-qubit vectors of the basis $\oplus$ and $\otimes$ are $S1(|\alpha\beta\rangle_{bc}) = \{|00\rangle_{bc}, |10\rangle_{bc}, |01\rangle_{bc}, |11\rangle_{bc}\}$ and $S2(|\alpha'\beta'\rangle_{bc}) = \{|++\rangle_{bc}, |--\rangle_{bc}, |+-\rangle_{bc}, |--\rangle_{bc}\}$ respectively where $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $|--\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$. Table 1 summarizes the coding two-qubit states in the corresponding basis. For example, if the bits of $L$ and $A$ are 1 and 0 respectively, then the two-qubit state Alice prepared can be $|++\rangle_{bc}$ or $|--\rangle_{bc}$ each with 50% probability. But she knows exactly which pair he prepares.

2. Alice sends the resulting two strings $B$ and $C$ to Bob and Charlie separately. It should be emphasized that every bit of the two strings $B$ and $C$ are corresponding and their XOR result equals to the corresponding bit of the string $A$.

3. When both Bob and Charlie have announced the receiving of their strings $B$ and $C$, Alice announces the string $L$.

4. Bob and Charlie then measure each qubit of their string in the basis $\oplus$ or $\otimes$ according to the corresponding bit value of string $L$.

5. In the check procedure, Bob and Charlie are required to announce the values of their check bits. If Alice finds too few of these values agree, they abort this round of operation and restart from the first step.

6. The rest unchecked bits of strings $A$, $B$ and $C$ can be used as raw keys for secret sharing. Alice’s secret encrypted by her keys can only be decrypted by Bob and Charlie when they cooperate with each other.

Obviously, the qubits $b$ and $c$ need not be sent out simultaneously in this quantum secret sharing scheme. Alice can send them separately to Bob and Charlie. They can hold their strings until Alice wants them to extract out her information and announces her the string $L$. It is virtually a combination of two modified BB84 quantum key distribution protocols, Alice to Bob and Alice to Charlie, where the states Alice sent out have been classically encoded so that her keys are shared in the strings $B$ and $C$. Either of them can learn nothing about Alice’s keys without the cooperation of the other one. Furthermore, this secret sharing protocol is almost 100% efficient as all the keys can be used in the ideal case of no eavesdropping, while the quantum secret sharing protocols with entanglement states $\{\}$ can be at most 50% efficient in principle. However, in this procedure of making measurements after the announcement of bases, quantum memory is required to store the qubits which has been shown available in the present experimental technique $[12]$. Adjusting the measurements order in the similar way the BB84 quantum key distribution protocol can also double its efficiency of to nearly 100% $[13]$. When no quantum memory is employed, Bob and Charlie measure their qubits before Alice’s announcement of basis in our protocol, the efficiency of the present protocol falls to 50%.

As the present scheme is more efficient than employing two BB84 procedures, one between Alice and Bob and the other between Alice and Charlie, which has been briefly discussed in the references $[2][3]$, it allows the possible eavesdropper, who obtains both of the particles that Alice sent, to gain more information than the two-BB-84 scheme, when the qubits $b$ and $c$ are sent simultaneously. Suppose an eavesdropper, Eve, obtains both of the particles that Alice sent, and measures them in the Bell basis. Then when the two qubits $b$ and $c$ are measured in the state $|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$, Eve can infers that the secret is 1. And if they in the state $|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$, he knows that the secret is 0. But we
can prevent this kind of information leakage causing by the jointed measurements, either by sending the two qubit un-synchronously or by encoding the two qubit as Table 2. Then in the same security degree, the present protocol is double efficient as the two-BB-84 scheme.

### III. EAVESDROPPING AND GENERALIZATION TO THE MULTI-PARTY SECRET SHARING

In this quantum secret sharing scheme, only product states are employed and one cannot get any information about the value of one qubit from measurement on the other qubit. In fact it is exactly two independent BB84 quantum key distribution protocols, which has been proved security generally [4,5]. The only difference is that the qubits Alice sent out have been classical encoded. Obviously, any eavesdropping on individual quantum channel for example Alice to Charlie will get nothing useful and will unavoidably introduce errors.

Now suppose an adversary (who could also be either Bob or Charlie) eavesdrops the qubits $b$ and $c$ jointly. His object is to extract out Alice’s information without any assistance from Charlie in a way that cannot be detected. But whatever he does, if Bob eavesdrops qubit $c$, Alice will find some error in the check procedure although Bob could cheat again in this check procedure. Then the security for the present quantum secret sharing is guaranteed.

The generalization of this quantum secret sharing scheme to multi-parties is straightforward. For each bit of $L$ and $A$, Alice can creates an $N$-qubit product state $|b_1b_2...b_N⟩$ in the basis $⊕$ (if the corresponding bit of $L$ is 0) or the basis $⊗$ (if the bit of $L$ is 1) where the XOR result of the $N$ qubits $b_1,b_2...b_N$ equals to the bit value of the string $A$. The two-qubit vectors of the basis $⊕$ and $⊗$ are $S_1(|α_1α_2...α_N⟩_{⊕12...N}) = \{ |00⟩_{12...N}, |01⟩_{12...N}, ..., |11⟩_{12...N} \}$ and $S_2(|α_1′α_2′...α_N⟩_{⊗12...N}) = \{ |++⟩_{12...N}, |+-⟩_{12...N}, ..., |--⟩_{12...N} \}$. Obviously, if there are odd numbers $|1⟩$ in $|α_1α_2...α_N⟩_{⊕12...N}$ (|−⟩ in $|α_1′α_2′...α_N⟩_{⊗12...N}$), the bit value of $A$ is 1. The case of even numbers $|1⟩$ stands for 0. The following steps are the same as the above two-party sharing case. As no entanglement, especially inaccessible multi-parties entangled states, are required, it may be more experimentally realizable than the original one [3].

Finally, let us discuss the resources necessary to implement this quantum secret sharing protocol. In the most obvious way of sharing secret, Alice first must establish mutual keys among different pairs of parties, and then use quantum cryptographic protocols to send each of the bit strings which result from the classical procedure. While in our secret sharing protocol of direct encoding the qubits of the two BB84 quantum key distribution, once the key has been established, Alice needs to send only one string of classical bits to either Bob or Charlie. Then in order to share a classical bit(cbit) information between two parties, Bob and Charlie, quantum cryptography as BB84 protocol ideally needs 2 qubits, 2 cbit and no ebit on average. The quantum secret sharing with EPR states needs 4 qubits, 1cbit and 2ebits. While our protocol with product states needs only 2 qubits, 1cbit and no ebit. In general, the more parts into which the secret is split, the greater the difference between the number of classical bits which be sent in those two ways. We see the direct encoding to the source qubits is able to act as a substitute for transmitted random bits. Furthermore, the efficiency of the existing entangled-state scheme can only reach 50% in principle while the present secret sharing protocol can be 100% efficient.

### IV. CONCLUSION

In this paper, we analyze the main quantum secret sharing protocol based on the GHZ-state and propose a quantum secret sharing scheme where only product states are employed.
As entanglement, especially the inaccessible multi-party entangled state, is not necessary in the present quantum secret sharing protocol, it may be more applicable when the number of the parties of secret sharing is large. Its theoretic efficiency is also doubled to approach 100%.

Although those quantum secret sharing with entanglement can be a demonstrator of concepts of various entanglement applications, the inherent efficiency limiting of these protocols and the difference for the preparation of multi-party entanglement may suggest that the employing of the quantum entanglement in quantum secret sharing be unworthiness in some cases. After all the practicality is an important pursuit of the quantum information theory.

This work was funded by National Fundamental Research Program(2001CB309300), National Natural Science Foundation of China, the Innovation funds from Chinese Academy of Sciences, and also by the outstanding Ph. D thesis award and the CAS’s talented scientist award entitled to Luming Duan.

[1] B. Schneier, Applied Cryptography (Wiley, New York, 1996) p. 70; see also J. Gruska, Foundations of Computing (Thomson Computer Press, London, 1997) p. 504.

[2] M. Hillery, V. Buzek, and A. Berthiaume, Phys. Rev. A 59, 1829 (1999).

[3] W. Tittel, H. Zbinden, and N. Gisin, Phys. Rev. A 63, 042301 (2001).

[4] D. Gottesman, Phys. Rev. A 61, 042311 (2000).

[5] A. C. A. Nascimento, J. M. Quade, and H. Imai, Phys. Rev. A 64, 042311 (2002).

[6] A. Karlsson, M. Koashi, and N. Imoto, Phys. Rev. A 59, 162 (1999).

[7] R. Cleve, D. Gottesman, and H. K. Lo, Phys. Rev. Lett. 83, 648 (1999).

[8] V. Karimipour, A. Bahraminasab, and S. Bagherinezhad, Phys. Rev. A 65, 042320 (2002).

[9] S. Bagherinezhad, and V. Karimipour, quant-ph/0204124

[10] D. Bouwmeester et al., Phys. Rev. Lett. 82, 1345 (1999).

[11] J. W. Pan et al., Phys. Rev. Lett. 86, 4435 (2001).

[12] G. C. Guo, and G. G. Guo, quant-ph/0206041

[13] C. H. Bennett, and G. Brassard, Advances in Cryptology: Proceedings of Crypto84, August 1984,
[14] P. W. Shor, and J. Preskill, Phys. Rev. Lett. 85, 441 (2000).

[15] N. J. Cerf, M. Bourennane, A. Karlsson, and N. Gisin, Phys. Rev. Lett. 88, 127902 (2002).
|    | L   | A | B | C |
|----|-----|---|---|---|
|    | 0   | 1 | 0 | 1 |
| A  | 0   | 1 | 0 | 1 |
| B C| 0 0 | 0 1 | ++ | -+ |
|    | 1 1 | 1 0 | - - | + - |

Table 1: The corresponding bits values of String L, A, B and C.
| L | 0 | 1 |
|---|---|---|
| A | 0 | 1 | 0 | 1 |
| BC | 0+ | 0- | +0 | -0 |
|    | 1- | 1+ | 1- | +1 |

Table 2: The corresponding bits values of String L, A, B and C.