SCALING PROPERTIES OF HADRON PRODUCTION IN \( \pi^- - p \) AND \( \pi^- - A \) COLLISIONS AT HIGH-\( P_T \)

G.P. Škoro 1,* M.V. Tokarev 2,* Yu.A. Panebratsev 2,♮ and I.Zborovský 3,♣

1 Institute of Nuclear Sciences "Vinča", Faculty of Physics, University of Belgrade, Belgrade, Yugoslavia
2 Laboratory of High Energies, Joint Institute for Nuclear Research, 141980, Dubna, Moscow region, Russia
3 Nuclear Physics Institute, Academy of Sciences of the Czech Republic, Újez, Czech Republic
* E-mail: goran@rudjer.ff.bg.ac.yu
♮ E-mail: tokarev@sunhe.jinr.ru
♣ E-mail: zborovsky@ujf.cas.cz

Abstract

Scaling features of particles produced in \( \pi^- - p \) and \( \pi^- - A \) collisions over a high-\( P_T \) range at high energies are studied. The general concept of \( z \)-scaling is applied for the analysis of \( \pi^- - p \) and \( \pi^- - A \) experimental data on the \( E d^3\sigma/dq^3 \) inclusive cross section. The scaling function \( \psi(z) \) and scaling variable \( z \) are constructed and the anomalous dimension \( \delta_\pi \) is found. The \( A \)-dependence of particle production in pion-nucleus collisions is studied. The predictions of the inclusive cross section of the \( \pi^0 \)-mesons produced in \( \pi^- - A \) collisions in the central rapidity range at high energies have been made.

Key-words: pion-proton and pion-nucleus collisions, high energy, scaling

1 Introduction

Particle production with a high transverse momentum is traditionally connected with fundamental phenomena of elementary constituent interactions. The hypothesis of parton-hadron duality [1] states, in particular, that the features of high-\( P_T \) hadron spectra reflect the features of hard parton-parton interactions. It means that partons retain information about the collision during particle formation. Therefore, the features of single inclusive particle spectra in hadron-hadron and hadron-nucleus collisions of particle having a different flavour content are of interest to search for unusual properties of particle itself and its formation. Such features could be very useful to search for complementary signatures of unusual phenomena such as the phase transition of nuclear matter, new type of particle interactions and quark compositeness.

One of the methods to study the properties of nuclear matter is to search for the violation of known scaling laws established in elementary collisions such as the Bjorken and Feynman scaling laws.

1The work is partially supported by the grant No. 020475 of the Czech Ministry of Education, Youth and Physical Training.
In this paper we study scaling features of hadron production in \( \pi^- - p \) and \( \pi^- - A \) collisions over a high \( p_T \) range. Experimental data on cross section \([2, 3, 4, 5] \) and \([6, 7] \) are used for the analysis.

The \( z \)-scaling was proposed in \([8] \) to describe the features of charged hadron production in \( p - p \) and \( \bar{p} - p \) collisions. New presentation (\( z \)-presentation) of experimental data can be obtained using the experimental observables, the inclusive cross section \( Ed^3\sigma/dq^3 \), the multiplicity density of charged particles \( dN/d\eta|_{\eta=0} = \rho(s) \) and kinematical quantities of the reaction (collision energy, momenta and masses of initial and produced particles). As shown in \([8, 10, 14, 11] \) the data \( z \)-presentation reveals the symmetry properties, the independence on center-of-mass energy \( \sqrt{s} \) and the angle of produced particle \( \theta \) over a wide kinematical range. The energy dependence of the \( \pi^- - p \) experimental data is used to find the anomalous dimension \( \delta_\pi \). The scaling function \( \psi(z) \) describes the probability density to form a particle with formation length \( z \). The scaling variable \( z \) reveals the property of fractal measure \( z = z_0 \epsilon^{-\delta} \), where \( \epsilon \) is the scale resolution, and has a relevance to the geometry of space-time \([12, 13] \). It was shown \([11] \) that the \( A \)-dependence of high-\( p_T \) hadron production in the framework of \( z \)-presentation is described by the function \( \alpha(A) \) depending on the single parameter, the atomic weight \( A \).

The existence of the scaling and its properties is assumed to reflect the fundamental features of particle structure, constituent interaction and particle production such as self-similarity, locality, fractality and scale-relativity.

2 Z-scaling

The idea of \( z \)-scaling is based on the assumption \([15] \) that the gross features of the inclusive particle distributions for reaction \((1) \) at high energies

\[
P_1 + P_2 \rightarrow q + X.
\]

(1)
can be described in terms of the corresponding kinematic characteristics of the exclusive subprocess written in the symbolic form

\[
(x_1 M_1) + (x_2 M_2) \rightarrow m_1 + (x_1 M_1 + x_2 M_2 + m_2).
\]

(2)
The scale-invariant fractions \( x_1 \) and \( x_2 \) of the incoming four-momenta of colliding objects are expressed via momenta and masses of incident and produced particles \( (P_1, P_2, q \) and \( M_1, M_2, m_1) \) and determine a minimum energy, which is necessary for the production of the secondary particle with mass \( m_1 \) and four-momentum \( q \). The parameter \( m_2 \) is introduced to satisfy the internal conservation laws (for isospin, baryon number, and strangeness).

2.1 Fractions \( x_1 \) and \( x_2 \)

The elementary parton-parton collision is considered as a binary sub-process which satisfies the condition

\[
(x_1 P_1 + x_2 P_2 - q)^2 = (x_1 M_1 + x_2 M_2 + m_2)^2.
\]

(3)
The equation reflects minimum recoil mass hypothesis in the elementary sub-process. To connect kinematic and structural characteristics of the interaction, the coefficient \( \Omega \) is introduced. It is chosen in the form

\[
\Omega(x_1, x_2) = m(1 - x_1)^{\delta_1}(1 - x_2)^{\delta_2},
\]

(4)
where $m$ is a mass constant and $\delta_1$ and $\delta_2$ are factors relating to the fractal structure of the colliding objects. The fractions $x_1$ and $x_2$ are determined to maximize the value of $\Omega(x_1, x_2)$, simultaneously fulfilling condition $d\Omega(x_1, x_2)/dx_1|_{x_2=x_2(x_1)} = 0$. Expressions for $x_1$ and $x_2$ as a function of the momenta and masses of the colliding and produced particles are given in [10]. The variables $x_{1,2}$ are equal to unity along the phase space limit and cover the full phase space accessible at any energy.

### 2.2 Scaling function $\psi(z)$ and variable $z$

The scaling function $\psi$ expressed via the invariant differential cross section for the production of the inclusive particle $m_1$ is introduced as follows (see [10])

$$
\psi(z) = -\frac{\pi s_A}{\rho_A(s, \eta) \sigma_{inel}} J^{-1} E \frac{d\sigma}{dq^3}.
$$

(5)

Here, $s_A \simeq s \cdot A$ and $s$ are the center-of-mass energy squared of the corresponding $h-A$ and $h-N$ systems and $A$ is the atomic weight. The factor $J$ is a known function of kinematic variables [10]. The expression (5) relates the inclusive differential cross section and the average multiplicity density $\rho_A(s, \eta)$ to the scaling function $\psi(z)$.

The scaling function is normalized as

$$
\int_{z_{min}}^{\infty} \psi(z) dz = 1.
$$

(6)

The equation allow us to give the physical meaning of the scaling function $\psi$ as a probability density to form a particle with a corresponding value of the variable $z$.

The variable $z$ as argued in [10] can be interpreted as a particle formation length. It is chosen in the form

$$
z = \frac{\sqrt{s_{\perp}}}{\Omega \cdot \rho_A(s)},
$$

(7)

where $s_{\perp}^{1/2}$ is the transverse kinetic energy of subprocess (2); $\Omega$ is the measure given by (4) and $\rho_A(s) = \rho_A(s, \eta = 0)$. We would like to note that the form of $z$ determines its variation range. The boundaries of the range are 0 and $\infty$, as defined by (6) and (4). These values are scale independent and kinematically accessible at any energy.

### 2.3 Fractality and scale-relativity

Fractality in particle and nuclear physics concerns the internal structure of particles and their interactions. It is manifested by their self-similarity on any scale. This general principle is described by power law dependencies of the corresponding quantities [4, 10].

The equation (3) written in the form $x_1 x_2 - x_1 \lambda_2 - x_2 \lambda_1 = \lambda_0$, does not change under the scale transformation $\lambda_{1,2} \rightarrow \rho_{1,2} \cdot \lambda_{1,2}$, $x_{1,2} \rightarrow \rho_{1,2} \cdot x_{1,2}$, $\lambda_0 \rightarrow \rho_{1} \cdot \rho_{2} \cdot \lambda_0$. The transformation with the scale parameters $\rho_{1,2}$ allows us to consider the collisions of the complex objects in terms of suitable sub-processes of the interacting elementary constituents. The coefficient $\Omega$, given by (4), connects the kinematic and fractal characteristics of the interaction. The factors $\delta_1$ and $\delta_2$ are anomalous fractal dimensions of the colliding objects. The fractal structure itself is defined by the structure of the interacting constituents, which is not an

---

2The anomalous dimensions are found to be $\delta_{1,2} = \delta_h$ and $\delta_1 = \delta_h$, $\delta_2 = \delta_A = \delta_N \cdot A$ for $h-h$ and $h-A$ collisions, respectively.
elementary one either. In this scheme, high energy hadron-hadron, hadron-nucleus and nucleus-nucleus interactions are considered as interactions of fractals.

The variable $z$ written in the form $z = z_0 \cdot \epsilon^{-\delta}$ (where $z_0 = \sqrt{s_{\perp}/\rho(s)}$ and $\epsilon^{-1} = [m(1-x_1)^A(1-x_2)^A]^{-1}$) reveals the properties of a fractal measure and $\delta$ is the anomalous fractal dimension describing the intrinsic structure of the interaction constituents revealed at high energies. The nontrivial features of mechanism of particle formation is that the formation length $z$ increases with resolution $\epsilon^{-1}$.

3. $\pi^- p$, $\pi^- p$-nucleus collisions and $z$-scaling

Experimental data sets of cross sections for $\pi^{\pm,0}, K^{\pm}, \bar{p}$ hadrons produced in $\pi^- p$ and $\pi^- A$ collisions at high transverse momentum $p_T$ are presented in [2, 3, 4, 5] and [6, 7], respectively. The measurements were made at pion momentum $p_{lab} = 40, 200, 300$ GeV/c over the range $0.8 < p_T < 10$. GeV/c. The nuclear targets Be, Cu and W were used.

We would like to note that all the cross section data demonstrate the strong energy dependence of the cross section on transverse momentum, the tendency that difference between hadron yields increases with transverse momentum and energy $\sqrt{s}$ and the non-exponential behavior of the spectra at $p_T > 1$ GeV/c.

3.1 $\pi^- p$ collisions

In this section we study the properties of $z$-scaling for hadrons produced in $\pi^- p$ collisions. The charged particle density $\rho(s, \eta)$ in $\pi^- p$ collisions was simulated by PYTHIA [8] in the energy range $\sqrt{s} = 10 - 200$ GeV. The results of simulations show that the energy dependence of the density $\rho(s)$ for processes $\pi^- p$ and $p - p$ is practically the same one (taking into account the errors) and can be parameterized in the form $\rho = as^b$. The values of the parameters were found to be $a = 0.74 \pm 0.12$, $b = 0.105 \pm 0.011$ and $a = 0.59 \pm 0.08$, $b = 0.126 \pm 0.017$ for $p - p$ and $\pi^- p$, respectively. The PYTHIA results give the relation $\sigma_{\pi p} = 0.67\sigma_{pp}$ expected from quark counting rule too. We do not have enough experimental data for $\rho(s)$ of $\pi^- p$ at high $\sqrt{s}$ and $p_T$ and the available experimental data [8] are not in disagreement with MC results. Therefore we use in our analysis of $z$-scaling in $\pi^- p$ collisions the experimentally measured dependence of the average charged particle multiplicity density for $p - p$ collisions. As we will show later the replacement does not destroy the general properties of $z$-scaling in $\pi^- p$ particle production.

We verify the hypothesis of energy scaling for data $z$-presentation for hadron production in $\pi^- p$ collisions using the available experimental data.

Figures 1(a)-2(a) show the dependence of the cross section $Ed^3\sigma/dq^3$ of $\pi^+$ and $K^-$-mesons produced in $\pi^- p$ on transverse momentum $p_T$ at $p_{lab} = 40, 200, 300$ GeV and the produced angle $\theta_{cm}$ near 90°. Note that the data cover the wide transverse momentum range, $p_T = 1 - 6$ GeV/c.

Figures 1(b)-2(b) show $z$-presentation of the same data sets. Taking into account the experimental errors we can conclude that the scaling function $\psi(z)$ demonstrates an energy independence over a wide energy and transverse momentum range at $\theta_{cm} \approx 90^\circ$. The energy dependence of the $\pi^- p$ experimental data is used to find the value of the anomalous dimension $\delta_n$. It is equal to 0.1.

To analyze the angular dependence of the scaling function $\psi(z)$ of charged hadrons
\[ \pi^\pm, K^\pm, \bar{p} \] produced in \( \pi^- - p \) collisions we use the data set obtained at Protvino. The data set includes the results of measurements of the invariant cross section \( Ed^2\sigma/dq^3 \) at the pion incident momentum \( p_{lab} = 40 \text{ GeV} \) over the momentum and angular ranges of \( p_T = 1.05 - 3.75 \text{ GeV/c} \) and \( \theta_{cm}^{\pi^N} = 49^0 - 93^0 \). The obtained results show that experimental errors are large enough and more high accuracy data on the cross section are necessary to verify carefully the angular independence of the scaling function \( \psi \) of hadrons produced in \( \pi^- - p \) collisions as a function of energy \( \sqrt{s} \), transverse momentum \( p_T \) and and \( \theta_{cm}^{\pi^N} \).

### 3.2 \( \pi^- - A \) collisions

In this section, we study the properties of \( z \)-scaling for hadron production in \( \pi^- - \)nucleus collisions. The experimental data sets \([2, 3, 4, 5]\) and \([6, 7]\) are used in the analysis.

According to the procedure of \( z \)-analysis of the \( p - A \) experimental data, the function \( \psi \) is calculated for every nucleus using the normalization factor \( \sigma_{inel}^{pp}/\sigma_{inel}^{A} \) in the expression for the inclusive cross section \([4]\). The factor \( \sigma_{inel}^{A} \) is the total inelastic cross section for \( pA \) interactions. The \( A \)-dependence of the ratio \( \sigma_{inel}^{pA}/\sigma_{inel}^{pp} \) is taken from \([16]\). The relevant multiplicity densities of charged particles obtained by the Monte Carlo simulation generator HIJING \([17]\) for different nuclei \( (A = 7 - 197) \) are taken in the form \( \rho_A(s) \simeq 0.67 \cdot A^{0.18} \cdot s^{0.105} \) \([14]\). In the present analysis we use for the multiplicity density \( \rho_A(s) \) and the \( A \)-dependence of the ratio \( \sigma_{inel}^{pA}/\sigma_{inel}^{pp} \) of \( \pi^- - A \) the results obtained for \( p - A \) collisions. The possibility of such replacement will be argued by obtained results for data \( z \)-presentation.

The symmetry transformations \( z \rightarrow \alpha(A) \cdot z, \psi \rightarrow \alpha^{-1}(A) \cdot \psi \) of the function \( \psi(z) \) and the argument \( z \) are used to compare the functions \( \psi \) for different nuclei.

Figures 3(a),4(a) show the dependence of the inclusive cross section for \( \pi^+, K^- \) produced in \( \pi^- - Be, Cu, \) and \( W \) collisions on the transverse momentum \( p_T \) at \( p_{lab} = 200 \) and 300 \text{ GeV/c} \). The incisive cross section data for \( W \) nucleus demonstrate the energy dependence, which enhances as the transfers momentum of produced particle increases.

Figure 5(a) presents cross sections for \( \pi^0 \)-mesons produced on \( Be \) and \( Cu \) nuclear targets at \( \sqrt{s} \simeq 31 \text{ GeV} \). The dependencies of the scaling function \( \psi \) on \( z \) of the same experimental data are shown in Figures 3(b)-5(b). As seen from Figure 5(b) the asymptotic regime (the power law for the scaling function, \( \psi(z) \sim z^{-\beta} \)) is achieved over a high-\( p_T \) range for \( \pi^0 \)-meson production on nuclei \( Be \) and \( Cu \) at \( \sqrt{s} \simeq 31 \text{ GeV} \). The value of the slope parameter \( \beta \) is found to be \( \simeq 9.37 \) over a wide range of high transverse momentum \( (3 < q_T < 7.5 \text{ GeV/c}) \).

We use the properties of \( z \)-scaling to calculate the cross section of \( \pi^0 \)-meson production in \( \pi^-Be \) and \( \pi^-Cu \) collisions at high energies. The results are shown in Figures 6(a,b).

### 4 Conclusions

The scaling features of \( \pi^+, K^- \) hadrons produced in \( \pi^- - p \) and \( \pi^- - A \) collisions at high energies in terms of \( z \)-scaling are studied. The experimental data sets \([2, 3, 4, 5]\) and \([6, 7]\) on the inclusive cross sections are used in the analysis. The momentum of incident pion beam \( p_{lab} \) changes from 40 to 515 \text{ GeV/c} over the high transverse momentum range \( (p_T = 0.2 - 10 \text{ GeV/c}) \).

The \( z \)-presentation of experimental data is constructed and the anomalous fractal dimension \( \delta_\pi \) is found to be 0.1. The value is allowed us to reproduce the general properties

\[ \text{\footnotesize{3The other normalization factor, } \sigma_{inel}, \text{ was used in [16]}}.\]
of $z$-scaling established in $p - p$, $\bar{p} - p$ and $p - A$ collisions.

The $A$-dependence of data $z$-presentation is studied and it is shown that the dependence is described by the function $\alpha = \alpha(A)$. The fractal dimension of nuclei $\delta_A$ for $\pi^- - A$ is found to be the same $\delta_A = A \cdot \delta_N$ as for the hadrons produced in $p - A$ collisions.

The asymptotic regime of the scaling function, $\psi(z) \sim z^-\beta$, is observed and the asymptotic value of the slope parameter $\beta$ is determined to be $\beta \approx 9.37$ at $\sqrt{s} \approx 31\ GeV$ and over the range $3 < p_T < 7.5\ GeV/c$.

Using the properties of $z$-scaling, the dependence of the cross sections of $\pi^0$-mesons produced in $\pi^- - Be$, $\pi^- - Cu$ and $\pi^- - Au$ collisions on transverse momentum over the central rapidity range at high energy $\sqrt{s} = 60, 200$ and $500\ GeV$ is predicted.

Thus, the obtained results show that data $z$-presentation of hadrons produced in $\pi^- - p$ and $\pi^- - A$ collisions demonstrates general properties of the particle formation mechanism such as self-similarity, locality, scale relativity and fractality. As one can assume the properties reflect through the anomalous dimension $\delta_r$ the features of elementary constituent substructure too.

References

[1] A. Capella, U. Sukhatme, C.I. Tan, and J. Tran Thanh Van, Phys. Rep. 236, 225 (1994).

[2] N. D. Giokaris et al., Phys. Rev. Lett. 47, 1690 (1981); H. J. Frisch et al., Phys. Rev. D27, 1001 (1983).

[3] G. J. Donaldson et al., Phys. Rev. Lett. 36, 1110 (1976); Phys. Rev. Lett. 40, 917 (1978); Phys. Lett. B73, 375 (1978).

[4] C. DeMarzo et al., Phys. Rev. D36, 16 (1987).

[5] L. K. Turchanovich et al., Yad.Fiz. 56(10), 116 (1993).

[6] G. Alverson et al., Phys. Rev. D48, 5 (1993).

[7] L. Apanasevich et al. Phys. Rev. Lett. 81, 2642 (1998).

[8] I.Zborovský, Yu.A.Panebratsev, M.V.Tokarev, G.P.Skoro, Phys. Rev. D54 (1996) 5548.

[9] L.Nottale, Fractal Space-Time and Microphysics. World Scientific Publishing Co.Pte. Ltd. 1993.

[10] I.Zborovský, M.V.Tokarev, Yu.A.Panebratsev, and G.P.Škoro, Phys. Rev. C59, 2227 (1999).

[11] M.Tokarev, I.Zborovský, Yu.Panebratsev, G.Skoro, Int. J. Mod. Phys. A16 (2001) 1281.

[12] I.Zborovský, hep-ph/0101018

[13] I.Zborovský, M.Tokarev, Yu.Panebratsev, G.Skoro, JINR Preprint E2-2001-41, Dubna, 2001.

[14] I.Zborovský, M.Tokarev, Yu.Panebratsev, G.Skoro, JINR Preprint E2-97-24, Dubna, 1997.

[15] V.S. Stavinsky, Physics of Elementary Particles and Atomic Nuclei 10, 949 (1979).

[16] J. Carroll et al., Phys. Lett. B 80, 319 (1979).

[17] X.N. Wang and M. Gyulassy, Phys. Rev. D 44, 3501 (1991); Phys. Rev. D 45, 844 (1992).

[18] T. Sjostrand, Computer Physics Communications, 82, 74 (1994).

[19] M. Adamus et al., IHEP preprint 88-121, Serpukhov, 1988.
Figure 1. (a) Dependence of the inclusive cross section of $\pi^+$-meson production on transverse momentum $q_T$ at $p_{lab} = 40, 200$ and $300 \text{ GeV/c}$ and $\theta_{\pi p} \simeq 90^0$ in $\pi^- - p$ collisions. Experimental data are taken from [2, 5]. (b) The corresponding scaling function $\psi(z)$.

Figure 2. (a) Dependence of the inclusive cross section of $K^-$-meson production on transverse momentum $q_T$ at $p_{lab} = 40, 200$ and $300 \text{ GeV/c}$ and $\theta_{\pi p} \simeq 90^0$ in $\pi^- - p$ collisions. Experimental data are taken from [2, 5]. (b) The corresponding scaling function $\psi(z)$. 
Figure 3. (a) Dependence of the inclusive cross section of $\pi^+$-meson production on transverse momentum $q_T$ in $\pi^- - A$ collisions at $p_{lab} = 200, 300$ GeV/c. Experimental data are taken from [2]. (b) The corresponding scaling function $\psi(z)$.

Figure 4. (a) Dependence of the inclusive cross section of $K^-$-meson production on transverse momentum $q_T$ in $\pi^- - A$ collisions at $p_{lab} = 200, 300$ GeV/c. Experimental data are taken from [2]. (b) The corresponding scaling function $\psi(z)$. 
Figure 5. (a) Dependence of the inclusive cross section of $\pi^0$-meson production on transverse momentum $q_T$ in $\pi^- - A$ collisions at $\sqrt{s} = 31$ GeV. Experimental data are taken from [6, 7]. (b) The corresponding scaling function $\psi(z)$.

Figure 6. Dependence of the inclusive cross section of $\pi^0$-meson production on transverse momentum $q_T$ at $\theta_{\pi^0N} \approx 90^0$ in $\pi^- - Be$ (a), $\pi^- - Cu$ (b) and $\pi^- - Au$ (c) collisions. The calculated results are shown by points and solid lines ($\diamond - 60$ GeV, $\circ - 200$ GeV, $+$ - 500 GeV). Experimental data ($\triangle$) are taken from [6, 7].