Octupole response and stability of spherical shape in heavy nuclei.

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Abstract

The isoscalar octupole response of a heavy spherical nucleus is analyzed in a semiclassical model based on the linearized Vlasov equation. The octupole strength function is evaluated with different degrees of approximation. The zero-order fixed-surface response displays a remarkable concentration of strength in the $1\hbar\omega$ and $3\hbar\omega$ regions, in excellent agreement with the quantum single-particle response. The collective fixed-surface response reproduces both the high- and low-energy octupole resonances, but not the low-lying $3^-$ collective states, while the moving-surface response function gives a good qualitative description of all the main features of the octupole response in heavy nuclei. The role of triangular nucleon orbits, that have been related to a possible instability of the spherical shape with respect to octupole-type deformations, is discussed within this model. It is found that, rather than creating instability, the triangular trajectories are the only classical orbits contributing to the damping of low-energy octupole excitations.

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I. INTRODUCTION

It is well known that there is an intimate connection between the shell structure in quantum systems like nuclei and metallic clusters and the properties of classical trajectories within these systems (see e.g. [1], p. 579). In particular, for nuclei it has been argued that closed orbits of triangular shape might lead to an instability of the spherical shape against octupole-type deformations in the region beyond $^{208}\text{Pb}$ [1], p.560. Here we would like to investigate in detail this possibility by using a semiclassical theory of nuclear response based on the linearized Vlasov equation [2,3]. Instabilities are expected to show up as some kind of pathological behaviour in response functions (vanishing eigenfrequencies and diverging response) and the semiclassical theory of [2] and [3], that has already been shown to give good qualitative results for lower multipolarities [4–6], is an ideal tool to study the role played by classical trajectories in determining the response of large quantum systems. Of course quantum corrections are expected to modify the results of this theory [7,8], especially
at very low energy, however having a clear picture of what should be expected already at
the classical level might help in making progress. We study the isoscalar octupole response
of a sample “nucleus” of $A = 208$ nucleons contained in a square-well potential of radius
$R = r_0 A^{1/3}$, with $r_0 = 1.2$ fm. This is done with different degrees of approximation: in a first
approximation, discussed in Sect. II, the interaction between nucleons is neglected, while
in Sect. III this interaction is taken into account in a separable approximation. Finally in
Sect. IV the effect of surface vibrations on the octupole response function is included.

Because of an interesting scaling property of the zero-order semiclassical response, the
$A$-dependence can be factorized and becomes trivial. Thus, although we perform explicit
calculations only for $A = 208$, our results on stability apply also to the region beyond $^{208}$Pb.

II. SINGLE-PARTICLE RESPONSE (FIXED SURFACE)

Following Ref. [1], the connection between shell structure and classical trajectories is
most easily illustrated by considering the quantum energy levels of a particle in a spherical
potential (no spin-orbit interaction). In this case the energy levels depend on the radial and
angular momentum quantum numbers $n_r$ and $l$ only. For sufficiently large values of these
quantum numbers, the energy difference between two neighbouring levels can be approxi-
imated as

$$
\epsilon(n'_r, l') - \epsilon(n_r, l) \approx (n'_r - n_r) \frac{\partial \epsilon}{\partial n_r} + (l' - l) \frac{\partial \epsilon}{\partial l}.
$$

(2.1)

Now the important point is that the derivatives $\partial \epsilon/\partial n_r$ and $\partial \epsilon/\partial l$ are essentially the
derivatives of the classical Hamiltonian with respect to the corresponding action variable,
hence they can be recognized as the radial ($\omega_r$) and angular ($\omega_\phi$) frequencies of the classical
orbits respectively. Thus

$$
\epsilon(n'_r, l') - \epsilon(n_r, l) \approx (n'_r - n_r)\omega_r + (l' - l)\omega_\phi,
$$

(2.2)

(we use units such that $\hbar = c = 1$). This combination of frequencies is exactly that appear-
ing in the denominator of the zero-order response function obtained in [2] from the solution
of the linearized Vlasov equation with fixed-surface boundary conditions. Moreover, from a
comparison of the zero-order Vlasov response function with the analogous quantum propa-
gator [8], it can be seen that in the Vlasov propagator the quantum matrix elements of the
excitation operator are replaced by appropriate Fourier coefficients that can be evaluated
as integrals along the classical trajectories. Since here we are interested in the octupole
response, we report the propagator describing the response of a nucleus to an external field
of the type

$$
Q(r) = r^3 Y_{3M}(\hat{r})
$$

(2.3)
in the single-particle approximation. This propagator is given by [5]

$$
R^0_{L=3}(s) = \frac{9A R^6}{8\pi \epsilon_F} \sum_{n=-\infty}^{\infty} \sum_{N=\pm 1, \pm 3} C_{3N}^2 \int_0^1 dx x^2 s_{nN}(x) \frac{(Q^{(3)}_{nN}(x)/R^3)^2}{s + i\varepsilon - s_{nN}(x)}.
$$

(2.4)
Instead of the frequency $\omega$, as independent variable we have used the dimensionless quantity $s = \omega/(v_F/R)$, as a consequence the $A$-dependence of this propagator is factorized as $AR^6 \propto A^3$. The eigenfrequencies (2.2) are accordingly replaced by the functions (for a square-well potential)

$$s_{nN}(x) = \frac{n\pi + N\alpha(x)}{x}. \quad (2.5)$$

The variable $x$ is related to the classical angular momentum $\lambda$ of a nucleon. The relation is $x = \sin \alpha$, where $\alpha$ is the angle spanned by the radial vector when the particle moves from the inner to the outer turning point. Clearly, for a square-well potential one has $\cos \alpha = \lambda/\bar{\lambda}$, where $\bar{\lambda}$ is the maximum particle angular momentum $\bar{\lambda} = p_F R$, $p_F$ is the Fermi momentum, while $v_F$ and $\epsilon_F$ are the corresponding velocity and energy. The sum over discrete angular momentum values of the quantum propagator is replaced here by the integration over $x$, thus the particle angular momentum is treated as a continuous variable.

The quantities $C_{3N}$ in Eq. (2.4) are classical limits of the Clebsh-Gordan coefficients coming from the angular integration of the quantum matrix elements [8]. Their explicit value is $C_{3\pm 1} = \frac{\sqrt{3}}{4}$ and $C_{3\pm 3} = \frac{\sqrt{5}}{4}$. In principle the integer $N$ takes values between $-L$ and $L$, however only the coefficients $C_{L,N}$ where $N$ has the same parity as $L$ are nonvanishing. The coefficients $Q_{nN}^{(3)}(x)$ appearing in the numerator of Eq. (2.4) have been defined in Ref. [2], they are essentially the classical limit of the radial matrix elements of the octupole operator $r^3$ and can be evaluated explicitly as

$$Q_{nN}^{(3)}(x) = (-)^n R^3 \frac{3}{s_{nN}^2(x)} \left(1 + \frac{4}{3} N \sqrt{1 - \frac{x^2}{s_{nN}^2(x)}} - \frac{2}{s_{nN}^2(x)} + 4(|N| - 1) \frac{1 - x^2}{s_{nN}^2(x)}\right). \quad (2.6)$$

For $N = \pm 1$ this expression coincides with that appearing in the (compression) dipole response (cf. Eq.(A.5) of Ref. [5]), however for the octupole response we also need terms with $N = \pm 3$. These new modes have an interesting property since the associated eigenfrequencies $s_{nN}(x)$ can vanish in the interval $0 < \alpha(x) < \frac{\pi}{2}$ [the equation $(n\pi + N\alpha) = 0$ has a solution for $\alpha = \frac{\pi}{2}$, corresponding to closed triangular orbits]. In Ref. [1] it has been pointed out that the vanishing of this eigenfrequency might give rise to a possible instability against octupole-type deformations in nuclei heavier than $^{208}\text{Pb}$. Although at first sight it might seem that the coefficients (2.6) would diverge when $s_{nN}(x) \to 0$, it is actually possible to check that

$$\lim_{x \to \frac{\pi}{2}} Q_{\pm 1 \pm 3}^{(3)}(x) = -\frac{1}{4} R^3. \quad (2.7)$$

The very fact that this limit is finite is important for our discussion about the role of triangular nucleon orbits. The linear response function in the single-particle approximation is well-behaved because

$$\lim_{s \to 0} \left(\text{Im } R_3^0(s)\right) = 0, \quad (2.8)$$

and

$$\lim_{s \to 0} \left(\text{Re } R_3^0(s)\right) = -\frac{t_0^6}{8\pi \epsilon_F} A^3, \quad (2.9)$$
a result that we shall need later. Since the zero-order octupole strength function is proportional to the imaginary part of the response function (2.4), we can see that the contribution of triangular trajectories to the octupole strength function does not give any pathology, at least at the single-particle level. Of course instabilities could still arise in the collective response, so we study also this response by using the same model, however, before doing this we look more in detail to the single-particle octupole strength given by the Vlasov theory.

The zero-order octupole strength function 

\[ S_{L=3}(E) = -\frac{1}{\pi} \text{Im} \mathcal{R}_3^0(E), \]  

is shown in Fig.1. It can be seen that the single-particle octupole strength is concentrated in two regions around 8 and 24 MeV. As pointed out already in [2], in this respect our semiclassical response is strikingly similar to the quantum response, which is concentrated in the \( 1\hbar\omega \) and \( 3\hbar\omega \) regions. The modes that contribute most are those with \( (n, N) = (0, 1), (1, -1), (-1, 3), (2, -3) \), for the \( 1\hbar\omega \) region and with \( (n, N) = (0, 3), (1, 1), (2, -1), (3, -3) \), for the \( 3\hbar\omega \) region. This concentration of strength is quite remarkable because our static distribution, which is taken to be of the Thomas-Fermi type

\[ f_0(r, p) \propto \theta(\epsilon_F - h_0(r, p)), \]

does not include any shell effect, however, because of the close connection between shell structure and classical trajectories expressed by Eqs. (2.2) and (2.6), we still obtain a strength distribution that is very similar to the one usually attributed to shell effects. Clearly the integers \( n \) and \( N \) correspond to the difference of radial and angular momentum quantum numbers in Eq. (2.2).

### III. COLLECTIVE RESPONSE (FIXED SURFACE)

The zero-order response function (2.4) gives only a first approximation to the nuclear response. When the residual interaction between nucleons is taken into account, a collective response function can be obtained by solving the Vlasov equation with appropriate boundary conditions [2,3]. If the interaction is assumed to be of the octupole-octupole type,

\[ V(r_1, r_2) = \kappa_3 r_1^3 r_2^3 \sum_M Y^*_M(\hat{r}_1) Y_M(\hat{r}_2) \]  

the collective fixed-surface octupole response function is given by [9]

\[ \mathcal{R}_3(s) = \frac{\mathcal{R}_3^0(s)}{1 - \kappa_3 \mathcal{R}_3^0(s)}. \]  

The parameter \( \kappa_3 \) specifies the strength of the residual interaction, its value can be estimated in a self-consistent way, giving ([1], p.557),

\[ \kappa_{BM} = -\frac{4\pi m\omega_0^2}{3AR^4} \approx -1.10^{-5}\text{MeV/fm}^6 \]  

for the octupole case. The parameter \( \omega_0 \) is given by \( \omega_0 \approx 41A^{-\frac{1}{3}}\text{MeV} \). Since this estimate is based on a harmonic oscillator mean field and we are assuming a square-well potential instead, we expect some differences. Hence we shall determine the parameter \( \kappa_3 \).
phenomenologically, by requiring that the peak of the high-energy octupole resonance agrees with experiment. This requirement implies $\kappa_3 \approx 2\kappa_{BM}$, which is in agreement with the prescription obtained in the quadrupole case [6]. In Fig.2 we report the collective octupole strength function given by Eq. (3.2), with this value of $\kappa_3$. We can clearly see the effects of collectivity that result in a shift and concentration of the strength into two sharp peaks around 20 Mev and 6-7 Mev. The experimentally observed [10] concentration of isoscalar octupole strength in the two regions usually denoted by HEOR (high energy octupole resonance) and LEOR (low energy octupole resonance) is qualitatively reproduced, however the considerable strength experimentally observed at lower energy (low-lying collective states) is absent from our fixed-surface response function. Like for the quadrupole response [6], we need to consider a different solution of the linearized Vlasov equation (in which the nuclear surface is allowed to vibrate [3]) in order to account for this feature of the response.

The poles of the collective response function are determined by the vanishing of the denominator in Eq. (3.2). A solution of this equation at zero or purely imaginary frequency could be interpreted as an instability of the spherical shape against an octupole-type deformation of the ground state, however we can see that this could happen only for interactions stronger than

$$\kappa_{3cr} = \frac{1}{\mathcal{R}_3^0(0)} \approx -3.14 \times 10^{-5} \text{MeV/fm}^6.$$

(3.4)

Our value of $\kappa_3$ is smaller (in absolute value) than this, so in the present model the spherical shape is stable. This is valid for any value of $A$, as long as we take $\kappa_3 = 2\kappa_{BM}$, since the product $\kappa_{BM}\mathcal{R}_3^0(0)$ is $A$-independent.

The triangular orbits do not enter directly into the discussion of this collective response, but the fact that they do not generate pathologies at the level of the single-particle response $\mathcal{R}_3^0(s)$ simplifies the discussion of the stability in the collective response also.

### IV. COLLECTIVE RESPONSE (MOVING SURFACE)

The collective fixed-surface response function (3.2) does reproduce two important features of the octupole response: the HEOR and the LEOR, however it misses the experimentally observed low-lying collective $3^-$ states. In order to account for these low-lying states, a different solution of the linearized Vlasov equation has been proposed in Ref. [3]. In that approach the nuclear surface is allowed to vibrate according to the usual liquid-drop model expression

$$R(\theta, \varphi, t) = R + \sum_{LM} \delta R_{LM}(t) Y_{LM}(\theta, \varphi).$$

(4.1)

A self-consistency condition involving the surface tension is then used to determine the time-dependence of the additional collective variables $\delta R_{LM}(t)$. Then, always for a separable residual interaction of the kind (3.1), the collective fixed-surface response function (3.2) is replaced by the following moving-surface response function [6]

$$\tilde{\mathcal{R}}_3(s) = \mathcal{R}_3(s) + \mathcal{S}_3(s),$$

(4.2)
with $R_3(s)$ still given by Eq. (3.2), while $S_3(s)$ gives the moving-surface contribution. For the separable interaction (3.1) the function $S_3(s)$ can be evaluated explicitly as [6]

$$S_3(s) = -\frac{R^6}{1 - \kappa_3 R_3^0(s)} \frac{[\chi^0_3(s) + \kappa_3 \varrho_0 R^3 R_3^0(s)]^2}{C_3 - \chi_3(s)} \left[1 - \kappa_3 R_3^0(s) + \kappa_3 R^6 [\chi^0_3(s) + \varrho_0 R^3]^2\right],$$

(4.3)

with $C_3 = 10\sigma R^2 + (C_3)_{\text{coul}}$ ($\sigma \approx 1\text{MeV fm}^{-2}$ is the surface tension parameter obtained from the mass formula, $(C_3)_{\text{coul}}$ gives the Coulomb contribution to the restoring force) and $\varrho_0 = A/\frac{4}{3}R^3$ the equilibrium density.

The functions $\chi^0_3(s)$ and $\chi_3(s)$ are defined as

$$\chi^0_3(s) = \frac{9A}{4\pi} \sum nN C^2_{3N} \int_0^1 dx x^2 s_{nN}(x) \frac{(-)^n(Q^{(3)}_{nN}(x)/R^3)}{s + i\varepsilon - s_{nN}(x)},$$

(4.4)

and

$$\chi_3(s) = -\frac{9A}{2\pi} \epsilon_3 f_3 (s + i\varepsilon) \sum nN C^2_{3N} \int_0^1 dx x^2 \frac{1}{s + i\varepsilon - s_{nN}(x)},$$

(4.5)

their structure is similar to that of the zero-order propagator (2.4).

In Fig.3 we report the octupole strength function given by the moving-surface response function (4.2) and compare it to the collective fixed-surface response given by Eq. (3.2). The most relevant change induced by the moving surface is the large double hump appearing at low energy. This feature is in qualitative agreement both with experiment [10] and with the result of RPA-type calculations [11,12], moreover it is rather similar to that found in Ref. [13] within the same model, but with different excitation operator and residual interaction. We interpret this low-energy double hump as a superposition of surface vibrations and LEOR. In order to explain why we do not obtain one or more sharp $3^-$ states at low energy, we have to analyze our moving-surface response function in some detail. Since the explicit expression (4.3) looks rather involved, we consider the limit of non-interacting nucleons. If we let $\kappa_3 \to 0$, the function (4.3) becomes

$$S^0_3(s) = -R^6 \frac{[\chi^0_3(s)]^2}{C_3 - \chi_3(s)},$$

(4.6)

and the full moving-surface response function (4.2)

$$\tilde{R}^0_3(s) = R^0_3(s) + S^0_3(s).$$

(4.7)

In Fig.4 we show the octupole response function evaluated both for $\kappa_3 = 2\kappa_{BM}$ (solid curve) and for $\kappa_3 = 0$ (dashed curve). We can see from this figure that, while the residual interaction changes drastically the response in the giant resonance region, at low energy the octupole response is affected only very slightly by the interaction (3.1). Thus the low-energy octupole response can be analyzed by using the simpler formula (4.6), rather than Eq. (4.3).

The eigenfrequencies of low-energy collective modes are approximately determined by the vanishing of the denominator in Eq. (4.6), moreover at low frequency the function $\chi_3(\omega)$ can be expanded as [4]
\[ \chi_3(\omega) = i\omega\gamma_3 + D_3\omega^2 + \ldots, \]  
\hspace{1cm} (4.8)

implying that the parameters \( \delta R_{3M}(t) \), that describe octupole surface vibrations in Eq. (4.1), approximately satisfy an equation of motion of the damped oscillator kind:

\[ D_3\ddot{\delta}R_{3M}(t) + \gamma_3\dot{\delta}R_{3M}(t) + C_3\delta R_{3M}(t) = 0. \]  
\hspace{1cm} (4.9)

It has already been pointed out in [4] that for \( A = 208 \) the numerical values of parameters in this equation are such that this oscillator is actually overdamped, moreover, always in [4], the coefficients \( \gamma_L \) have been evaluated analytically in the low-frequency limit, giving (for a generic \( L \))

\[ \gamma_L = \gamma_{wf} \frac{(4\pi)^2}{2L + 1} \sum_{N=1}^{L} \frac{1}{N} |Y_{LN}(\pi/2, \pi/2)|^2 \sum_{n=1}^{+\infty} \cos \alpha_{nN} \sin^3 \alpha_{nN} \Theta(\pi/2 - \alpha_{nN}), \]  
\hspace{1cm} (4.10)

with \( \gamma_{wf} = \frac{3}{4} \theta_0 p_F R^4 \) and \( \alpha_{nN} = \frac{nN}{N} \pi \). The angles \( \alpha_{nN} \) are related to the nucleon trajectories as discussed in Sect. II. In our case the coefficient \( \gamma_{L=3} \) gets a contribution only from the term with \( n = 1 \) and \( N = 3 \), thus we see that only nucleons moving along closed triangular trajectories can contribute to the damping of octupole surface vibrations. In order to check that indeed the closed triangular orbits are the main source of Landau damping in the low-energy octupole response, we study also the response that is obtained when the contribution of these orbits is excluded. This exclusion can be made simply by avoiding a small interval of length \( 2\Delta \) about the value \( x = \sqrt{\frac{3}{2}} \) in the \( x \)-integration in Eqs. (2.4), (4.4) and (4.5). In Fig. 5 we show the response obtained with a value of \( \Delta = 0.02 \). The shape of the low-energy hump changes dramatically because of the lack of damping due to the missing triangular trajectories. A very sharp peak is now developed at low energy. The position of this peak is influenced by the value of the mass parameter \( D_3 \), omitting the triangular orbits changes also the numerical value of this parameter. Once again we see that the closed triangular orbits, rather than generating a shape instability in the octupole channel, are the main source of Landau damping in this response function. Of course quantum effects can play an essential role at this level since in quantum mechanics the angular momentum is quantized and there could be no value of angular momentum corresponding to triangular trajectories.

We would also like to point out that our moving-surface response function can display a shape instability in the octupole channel, which can arise if the restoring force parameter \( C_3 \) vanishes. This could happen if the repulsive Coulomb term \( (C_3)_{coul} \) exactly balances the attractive surface-tension part of the restoring force. In this case our moving-surface response function develops a pole at the origin, as shown in Fig. 6. For odd multipolarities the instability condition in our model is the same as for the liquid-drop model [4].

Before concluding, in Fig. 7 we compare our moving surface response function with the response function of the overdamped oscillator (4.9). This figure supports our interpretation of the lower-energy hump as due to overdamped surface vibrations.

V. CONCLUSIONS

We have analyzed the octupole response function of a hypothetical heavy nucleus containing \( A = 208 \) nucleons in a square-well potential by employing a semiclassical model
that relates the response to features of the classical trajectories of nucleons. Triangular trajectories are particularly relevant for the octupole response because, at the single-particle level, a simple combination of their characteristic frequencies, appearing in this response function, can vanish. This fact had already been noticed long ago and some speculations had been made on the possible connection between these classical orbits and the onset of a shape instability in heavy spherical nuclei against octupole-type deformations [1]. Our detailed calculations performed in a semiclassical approach show that the vanishing of the eigenfrequencies associated with triangular trajectories has no consequences on the stability of the spherical shape in heavy nuclei, rather, triangular orbits are essential in providing a damping of low-energy octupole excitations.
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FIGURES

Fig.1 Semiclassical octupole strength function analogous to quantum single-particle strength function. Note the strength concentration in the $1\hbar\omega$ and $3\hbar\omega$ regions. Calculations are for $A = 208$ nucleons in a square well potential of radius $R = 1.2\sqrt[3]{A}$ fm.

Fig.2 Collective strength function (solid curve) evaluated for fixed-surface boundary conditions. The residual interaction parameter is $\kappa_3 = -2.10^{-5}$MeV/fm$^6$. The dashed curve is the strength function of Fig.1.

Fig.3 The solid curve shows the octupole strength function evaluated by using moving-surface boundary conditions in the Vlasov equation, the dashed curve instead, corresponds to fixed-surface boundary conditions.

Fig.4 Moving-surface strength function for $\kappa_3 = -2.10^{-5}$MeV/fm$^6$ (solid) and for $\kappa_3 = 0$ (dashed).

Fig.5 Moving-surface strength function with (solid) and without (dashed) contribution of closed triangular orbits.

Fig.6 Collective moving-surface strength function (solid) displaying a divergence at vanishing excitation energy when $C_3 \to 0$ (dashed). This divergence is interpreted as a shape instability.

Fig.7 Collective moving-surface strength function (solid) compared to overdamped oscillator strength function (dashed) with appropriate parameters. The two functions practically coincide for $E < 2$ MeV.
Fig. 1

$S_3(E) \times 10^4 \text{ fm}^6 / \text{MeV}$

$A=208$
Fig. 2

$S_3(E) \left( 10^{-4} \text{ fm}^6 / \text{MeV} \right)$

$E (\text{MeV})$

$A = 208$
$S_3(E) \left( 10^4 \text{ fm}^6 / \text{MeV} \right)$

$E \text{ (MeV)}$

A=208

Fig.3
\[ S_3(E) \left( 10^4 \text{ fm}^6 / \text{MeV} \right) \]

Figure 4

A = 208
Fig. 5

$A = 208$

$S_3(E) \left( 10^4 \text{ fm}^6 / \text{MeV} \right)$

$E \,(\text{MeV})$

$S_3(E)$ vs $E$ for $A = 208$.
\begin{align*}
S_3(E) &= 10^4 \text{ fm}^6 / \text{MeV} \\
E &= \text{MeV}
\end{align*}

Fig. 6
Fig. 7

A = 208

$S(E)_3 \left(10^4 \text{ fm}^6 / \text{MeV}\right)$ vs $E$ (MeV)