Weak Mixing Angles as Dynamical Degrees of Freedom

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abstract

In an analogy to the case of axion, which converts the $\Theta$-angle into a dynamical degree of freedom, we are trying to imagine a situation where the quark mixing angles turn out to be dynamical degrees of freedom (pseudo-Goldstone bosons), and their vacuum expectation values are obtained from the minimization of the vacuum energy. We present an explicit supersymmetric model with horizontal symmetry, where such a mechanism can be realized. It implies one relation between the quark masses and the CKM mixing angles:

$$s_{13}s_{23}/s_{12} = (m_s/m_b)^2,$$

which is fulfilled within present experimental accuracy. We believe, however, that the idea might be more general than this concrete model, and it can be implemented in more profound frameworks.
1 Introduction

The problem of CP violation in strong interaction, so-called Θ-problem, can be most naturally resolved by the introduction of the axion field which converts Θ parameter into a dynamical degree of freedom [1, 2, 3].

The pseudo-Goldstone boson, axion, is related to the chiral, flavour nonchanging, transformations of quarks: global $U(1)_{PQ}$ symmetry by Peccei and Quinn [1]. This symmetry group can be extended to the rotations including a change between different generations. Such a generalization of the Peccei-Quinn symmetry would lead to the appearance of a set of Goldstone bosons – familons [4].

Axion, being massless at the classical level, acquires small mass due to quantum corrections – more precisely, due to axial anomaly – and thus becomes a pseudo-Goldstone boson. The axion field acquires the vacuum expectation value (v.e.v.) which fixes the effective value of the Θ parameter. Namely, the minimum of the vacuum energy corresponds to $Θ = 0$ resulting in the absence of strong CP violation.

The v.e.v.’s of the familons are unfixed as long as the familons remain true Goldstone bosons. However, though they cannot acquire masses by the same mechanism as the axion, they can nevertheless have small masses due to the explicit breaking of the corresponding symmetry, possibly through the radiative corrections. If this happens the v.e.v.’s of the familon fields would fix the mixing angles of the quarks, i.e. the Cabibbo–Kobayashi–Maskawa (CKM) matrix, in the same way as the v.e.v. of the axion field fixes the Θ parameter.

In other words, we are trying to imagine a situation where the quark mixing angles turn out to be dynamical degrees of freedom (pseudo-Goldstone bosons) and their vacuum expectation values are obtained from the minimum of the vacuum energy. We shall present an explicit example how such a mechanism can be realized. We believe, however, that the idea might be more general than the concrete model described in this paper.

The complex, generally non-hermitian mass matrices of the up and down quarks can always be represented in the form:

$$M_u = U_R^\dagger M_{\text{diag}}^u U_L, \quad M_d = V_R^\dagger M_{\text{diag}}^d V_L,$$

where

$$M_{\text{diag}}^u = \text{diag}(m_u, m_c, m_t), \quad M_{\text{diag}}^d = \text{diag}(m_d, m_s, m_b),$$

and $U_R, U_L, V_R, V_L$ are the unitary matrices which connect the quark mass eigenstates with the symmetry states ("current quarks") as the latter appear in the Lagrangian. Evidently, the products $M_u^\dagger M_u$ and $M_d^\dagger M_d$ transform under the left-handed rotations of quarks:

$$M_u^\dagger M_u = U_L^\dagger (M_{\text{diag}}^u)^2 U_L, \quad M_d^\dagger M_d = V_L^\dagger (M_{\text{diag}}^d)^2 V_L,$$

while $M_u M_u^\dagger$ and $M_d M_d^\dagger$ are related to the right-handed rotations:

$$M_u M_u^\dagger = U_R^\dagger (M_{\text{diag}}^u)^2 U_R, \quad M_d M_d^\dagger = V_R^\dagger (M_{\text{diag}}^d)^2 V_R.$$
The mixing of left-handed quarks in weak interaction is given by the CKM matrix \( K_L = U_L V^\dagger_L \). The analogous matrix for the right-handed quarks, \( K_R = U_R V^\dagger_R \), has not much physical sense in the absence of the right-handed weak interactions.

Assume now that the fermion masses are actually the v.e.v.'s of certain fields. It can be a very natural situation that the minimum of the relevant Higgs potential, at least in the tree approximation, would fix only the eigenvalues of \( M_u \) and \( M_d \) (i.e. the quark masses in \( M^u_{\text{diag}} \) and \( M^d_{\text{diag}} \)) while the matrices \( U_R, U_L, V_R, V_L \) would remain undefined. To have this property it is sufficient that the Higgs potential would contain only the terms depending on the quantities \( \text{Tr}(M^u_{\text{diag}} M^u_{\text{diag}}) \) and \( \text{Tr}(M^d_{\text{diag}} M^d_{\text{diag}}) \) but not on \( M_u \) or \( M_d \) themselves. (Of course, we actually have in mind the appropriate Higgs fields whose v.e.v.'s represent \( M_u \) and \( M_d \)).

Furthermore, the potential in tree approximation may depend, or not depend, on the structures

\[
\text{Tr} \left[ M^u_{\text{diag}} M^u_{\text{diag}} M^d_{\text{diag}} M^d_{\text{diag}} \right] = \text{Tr} \left[ (M^u_{\text{diag}})^2 K_L (M^d_{\text{diag}})^2 K^\dagger_L \right],
\]

or the structures with a reverse order of \( M_{u,d} \):

\[
\text{Tr} \left[ M^u_{\text{diag}} M^u_{\text{diag}} M^d_{\text{diag}} M^d_{\text{diag}} \right] = \text{Tr} \left[ (M^u_{\text{diag}})^2 K_R (M^d_{\text{diag}})^2 K^\dagger_R \right].
\]

If it does not, the dependence on the CKM matrix \( K_L \) will anyway appear when the usual weak interaction is taken into account. Indeed, when the quark masses are fixed, radiative corrections from weak interaction will lead to the contribution to the effective potential exactly of the form (5) through the loop diagram shown in Fig. 1. Indeed, it is straightforward to see that this diagram yields the contribution to the effective potential which depends on \( K_L \):

\[
V_{\text{eff}} \sim \sum_{i,j=1}^3 |K_{Li}||^2 m^2_{ui} m^2_{dj}.
\]

where \( m_{ui} = (M^u_{\text{diag}})_{ii} \), \( m_{dj} = (M^d_{\text{diag}})_{jj} \) are the masses of the up and down quarks. Due to Eq. \( (3) \), this expression exactly coincides with (5).

One can ask whether it is reasonable to assume the absence of the contribution (5) to the effective potential in the tree approximation if it anyway appears through the diagrams of Fig. 1? Clearly, the tree potential should include counterterm of the same structure. The situation is analogous to one of the pioneering work by Coleman and Weinberg \( (3) \). For a fixed, not too large value

\[\text{1 In the absence of weak interactions, the total chiral symmetry of massless quarks would be } U(3)_{u_R} \times U(3)_{u_L} \times U(3)_{d_R} \times U(3)_{d_L}, \text{ so the scalars inducing the quark mass matrices } M_u \text{ and } M_d \text{ are respectively in representations } (3,3,1,1) \text{ and } (1,1,3,3). \text{ Clearly, no mixed structures like } (3) \text{ or } (5) \text{ are allowed by this } U(3)^4 \text{ global symmetry. However, weak interactions are not invariant against independent rotations of } u_L \text{ and } d_L \text{ states and thus the term } (3) \text{ becomes possible.} \]
of a cutoff the contribution of the loop diagrams is smaller than the value of the tree potential. The smallness of the loop contribution should be attributed then to the additional powers of the dimensionless coupling constant.

In the standard model $SU(2) \times U(1)$ the left-handed quarks $q_{Li} = (u_L, d_L)_i$ transform as the doublets of the electroweak symmetry while the right-handed quarks $u_{Ri}, d_{Ri}$ are the weak singlets ($i = 1, 2, 3$ is a family index). The quark masses emerge via the Yukawa couplings

$$\mathcal{L}_{\text{Yuk}} = G^{ij}_u \bar{u}_{Ri} q_{Lj} \tilde{H} + G^{ij}_d \bar{d}_{Ri} q_{Lj} H \tag{8}$$

where $H = (H^+, H^0)$ is the standard Higgs doublet with the v.e.v. $\langle H^0 \rangle = v$ ($v = (2\sqrt{2} G_F)^{-1/2} = 174$ GeV), and the Yukawa coupling constants $G^{ij}_u$ and $G^{ij}_d$ are $3 \times 3$ complex matrices. The quark mass matrices are:

$$M_u = G_u v, \quad M_d = G_d v \tag{9}$$

Therefore, actually these are Yukawa coupling constants which we treat as dynamical degrees of freedom, assuming that they are given by v.e.v.’s of certain fields. In particular, we assume that the eigenvalues of the matrices $G_u$ and $G_d$, i.e. the values of quark masses, are frozen by the requirement of the minimum of the tree-level potential of these fields. In what follows, they will be treated as fixed constants. At the same time the CKM matrix is related to a set of dynamical degrees of freedom, the angles which enter the CKM matrix are the v.e.v.’s of the pseudo-Goldstone bosons similar to axion, and their values should be determined by the minimum of effective potential at the radiative level. In other words, we minimize the energy of the ground state with respect to the form of the unitary matrices in (1).

In Figs. 2,3 we show the loop diagrams induced due to the Yukawa couplings (8). 3-loop diagrams of Fig. 2 in fact contribute to the vacuum energy, and they all have a structure $\sim \Lambda^4 \text{Tr}[G_u^i G_u G_d^i G_d]$, where $\Lambda$ is a cutoff scale (for the moment we omit the loop factors). The quadratically divergent 2-loop diagrams like the one of Fig. 3, where we deliberately inserted v.e.v.’s of Higgs $H$, give a structure $\sim v^2 \Lambda^2 \text{Tr}[G_u^i G_u G_d^i G_d]$. For the fixed Yukawa matrices this diagram represents a contribution to the Higgs doublet mass (among the other quadratically divergent contributions). However, for the given value of $H$ it can be treated as an effective potential term for the Yukawa degrees of freedom.

Clearly, the quadratic divergency in the diagram of Fig. 3 will be removed as soon as one considers the supersymmetric theory [3]. In the case of unbroken supersymmetry the radiative corrections of Figs. 2,3 are exactly vanishing. Once supersymmetry is broken at the scale $m_S$ which can be from few hundred GeV to few TeV (i.e. roughly $m_S \sim v$), the contribution of Fig. 3 becomes $\sim m_S^2 v^2$. On the other hand, in the supersymmetric theory the vacuum energy is in general quadratically divergent. Hence, contribution of diagrams in Fig. 2 should be $\sim m_S^2 \Lambda^2$. For the special choice of structures of the supersymmetry breaking soft
terms the quadratic divergency can be removed also in the vacuum energy \[7\], in which case the contribution of Fig. 2 would become \(\sim m_4^4\). However, in the rest of the paper we will not consider this specific case.

Thus, in the context of our discussion the diagrams of Fig. 1 and Figs. 2,3, in spite of different degree of their divergency, are very similar: in fact, they all reproduce the structure \(8\). The insertion of the Higgs v.e.v. in the diagram of Fig. 3 ensures that the quarks become massive, which fact was implicitly assumed in the diagram of Fig. 1. One can say that the diagram of Fig. 1 has been calculated after the spontaneous symmetry breaking had already taken place while the diagram of Fig. 3 is used before the symmetry breaking occurs. The \(W\) exchange in Fig. 1 is altered to the exchange of the charged Higgs boson in Fig. 3, which stays now instead of the longitudinal \(W\) boson. Indeed, if the gauge coupling constant goes to zero, then the contribution of the diagram of Fig. 1 does not vanish, as it may seem at first glance, if one would substitute \(M_W \sim gv\). This concurs with the non-vanishing contribution of the diagram of Fig. 3.

To summarize, we assume that the eigenvalues of the mass matrices \(M_u\) and \(M_d\), i.e. quark masses \(2\), are frozen by the requirement of the minimum of the tree-level potential. In what follows, they will be treated as fixed constants. At the same time the CKM matrix is related to a set of dynamical degrees of freedom, the angles which enter the CKM matrix are the v.e.v.’s of the pseudo-Goldstone bosons similar to axion. We have argued that if the term fixing the relative orientation of \(M_u^\dagger M_u\) and \(M_d^\dagger M_d\) is absent in a tree-level potential, then it is induced radiatively. The effective potential, which must fix the v.e.v.’s of these fields, i.e. weak mixing angles, should at least contain the term \(3\) since this structure is dictated by usual weak interactions.

On the other hand, if the structure \(3\) is absent in the tree-level potential, then it will not emerge after the radiative corrections as long as the right-handed fermions do not have \(SU(2)_R\) gauge interactions and the related Yukawa couplings. That means that the relative rotation angles of the right-handed quarks correspond to the true Goldstone degrees of freedom. They will not be considered in this paper as well as any other true Goldstones – familons.

According to our scenario the v.e.v.’s of the pseudo-Goldstone bosons (so to say, ”pseudo-familons”) fix the mixing angles, just like the v.e.v. of axion field fixes the \(\Theta\)-angle \(4\). The masses of these bosons are related to the absolute value of the loop contribution of the type of \(4\) to the effective potential. It is very

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2 Let us remark that besides the axion, there exists one more example when a rotation angle is actually a dynamical degree of freedom. This is an usual pion field corresponding to chiral rotations of quarks in the isotopic space. In the chiral limit when the current masses of the light quarks vanish, \(m_u = m_d = 0\), pions are true Goldstone bosons and their v.e.v.’s are undefined. For \(m_u, m_d \neq 0\) pions become pseudo-Goldstones and their v.e.v.’s turn out to be zero, \(\langle \pi^a \rangle = 0\) (in a ”reference frame” in O(4) chiral space determined by the condition \(\langle \bar{q}q\rangle \neq 0, \langle \bar{q}\gamma^a\gamma_5q\rangle = 0\)).
difficult to estimate the value of $\nu_{\text{eff}}$. However, we shall argue below that if
the cutoff is actually given by the TeV scale supersymmetry, the pseudo-familon
masses can be in the MeV range.

Unfortunately, the potential which contains only a term (4) does not lead to
nontrivial mixing angles: the angles all vanish if it enters $\nu_{\text{eff}}$ with a negative
coefficient, or all equal to $\pi/2$ if this coefficient is positive. Therefore, we are
obliged to add some different structures. This problem will be discussed in the
next sections.

The paper is organized as follows. In Section 2 we continue to discuss a
possible symmetry structure of the effective potential which could provide a non-
trivial solution for the CKM angles. The effective potential is presented in a
special parametrization. In Section 3 a concrete model based on a chiral horizontal
$SU(3)_{H}$ symmetry of generations is considered, in which naturally emerges
the general structure of the effective potential assumed in Section 2. In Section 4
we discuss a toy model with two generations of fermions. This example allows to
explain the underlying physical mechanism, and also serves as a technical tool in
considering the more complicated realistic case of three generations. The latter
case is considered in Section 5. The mixing angles are found and one physical
relation between the angles is established (see Eq. (65)), which is satisfied within
the present experimental accuracy. In Section 6 we estimate a range of possible
masses of pseudo-Goldstone bosons related to the CKM mixing angles. Some
concluding remarks are given at the end of the paper.

2 Effective potential for the CKM matrix

In the standard model the gauge interactions of fermions obey the global symme-
try related to the inter-family transformations of the different types of fermions:
$U(3)_{uR} \times U(3)_{dR} \times U(3)_{qL}$. As it was explained above, one can imply by the
fermion "masses" the appropriate Higgs v.e.v.'s, so the Higgses responsible for
$M_{u}$ and $M_{d}$ transform respectively as $(3, 1, 3)$ and $(1, 3, 3)$ representations of this
$U(3)^{3}$ group.

The simplest assumption is that the effective potential in the tree approx-
imation depends only on the traces of the powers of $M_{u}^{\dagger}M_{u}$ and $M_{d}^{\dagger}M_{d}$. This
would leave all the rotation matrices in Eq. (11) to be the Goldstone degrees of
freedom. Then radiative corrections induce the structure (4) in the effective po-
tential which lifts the vacuum degeneracy with respect to CKM angles, and thus
fixes their values. However, as we have already mentioned, if only the structure
(4) is present in the effective potential, the mixing angles are trivial.

Next in simplicity would be an assumption that the third generation of the
fermions is somewhat different from the first two. Qualitatively we can express
this by considering the terms $\text{Tr}[M_{u}^{\dagger}M_{u}\lambda_{8}]$ and $\text{Tr}[M_{d}^{\dagger}M_{d}\lambda_{8}]$. We assume that
these terms indeed appear together with the term (4) in the effective potential,
which now acquires a form:

$$\mathcal{V}_{\text{eff}} = A \mathrm{Tr} \left[ M_u^8 M_u \lambda_8 \right] + B \mathrm{Tr} \left[ M_d^8 M_d \lambda_8 \right] + C \mathrm{Tr} \left[ M_u^8 M_u M_d^8 M_d \right], \quad (10)$$

where $A, B$ and $C$ are some unknown constants.

In the next section we shall present an explicit model which has exactly these properties. In this model all the structures in (10) emerge at the radiative level due to the spontaneous symmetry breaking rather than in explicit manner.

Substituting Eqs. (3) in (10), one obtains:

$$\mathcal{V}_{\text{eff}} = A \mathrm{Tr} \left[ (M_{\mathrm{diag}}^u)^2 U \lambda_8 U \dagger \right] + B \mathrm{Tr} \left[ (M_{\mathrm{diag}}^d)^2 V \lambda_8 V \dagger \right]$$

$$+ C \mathrm{Tr} \left[ (M_{\mathrm{diag}}^u)^2 K (M_{\mathrm{diag}}^d)^2 K \dagger \right] \quad (11)$$

where $U \equiv U_L, V \equiv V_L$, and $K = UV \dagger$. This basic expression can be reorganized in the following way. First we notice that the matrix $\lambda_8 \sim \text{diag}(1, 1 - 2)$ can be changed to $\lambda_0 \sim \text{diag}(0, 0, 1)$, since the terms (11) with the unit matrix $V$ instead of $\lambda_8$ does not depend on $U$ and $V$. Then, without lose of generality, one can also subtract from $(M_{\mathrm{diag}}^u)^2$ and $(M_{\mathrm{diag}}^d)^2$ respectively the unit matrices $m_u^2 I$ and $m_d^2 I$. Therefore, the expression (11) can be presented in the form:

$$\mathcal{V}_{\text{eff}} = A \sum_{i=2,3} |U_{i3}|^2 \bar{m}_u^2 + B \sum_{i=2,3} |V_{i3}|^2 \bar{m}_d^2 + C \sum_{i,j=2,3} |K_{ij}|^2 \bar{m}_u^2 \bar{m}_d^2, \quad (12)$$

where

$$\bar{m}_c^2 = m_c^2 - m_u^2 \simeq m_c^2, \quad \bar{m}_t^2 = m_t^2 - m_u^2 \simeq m_t^2,$$

$$\bar{m}_s^2 = m_s^2 - m_d^2 \simeq m_s^2, \quad \bar{m}_b^2 = m_b^2 - m_d^2 \simeq m_b^2. \quad (13)$$

Of course, $A, B, C$ in Eq. (12) are not the same as in Eq. (11). In the following for $\bar{m}^2_{c,s,t,b}$ we use their approximate values (13):

We shall parametrize the $3 \times 3$ unitary matrices $U$ and $V$ by three consecutive unitary transformations acting between the (1, 2), (2, 3) and (1, 2) generations:

$$U = U_{12} U_{23} U_{12} \dagger, \quad V = V_{12} V_{23} V_{12} \dagger \quad (14)$$

$$K = UV \dagger = U_{12} U_{23} S_{12} V_{23} V_{12} \dagger, \quad S_{12} = U_{12} \dagger V_{12} \dagger.$$

The advantages of this parametrization are obvious. First, since the matrices $U_{12}$ and $V_{12}$ commute with $\lambda_8$, they drop out in two first terms for the potential in the expression (11). Second, only their product, $S_{12}$, remains in the third term. To introduce the necessary 6 independent phases in $U$ and $V$ we include three phases in each matrix $U_{12}, U_{23}$:

$$U_{12} = \begin{pmatrix} e^{i \alpha_{12}} \cos \Theta_{12} & e^{i \beta_{12}} \sin \Theta_{12} & 0 \\ -e^{i \gamma_{12}} \sin \Theta_{12} & e^{i \delta_{12}} \cos \Theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \alpha_{12} - \beta_{12} = \gamma_{12} - \delta_{12},$$
where $s_{12}$ depends only on three combinations of the phases, namely:

$$\alpha_{23} - \beta_{23} = \gamma_{23} - \delta_{23},$$

and analogously for $V_{12}$ and $V_{23}$, but with the change of notations: $\Theta_{12} \rightarrow \bar{\Theta}_{12}, \alpha_{12} \rightarrow \bar{\alpha}_{12}, \ldots, \Theta_{23} \rightarrow \bar{\Theta}_{23}, \alpha_{23} \rightarrow \bar{\alpha}_{23}, \ldots, \text{etc.}$

The matrices $U_{12}'$ and $V_{12}'$ actually can be chosen orthogonal. Only their product, $S_{12} = U_{12}' V_{12}'$, enters the expression (11) for $\mathcal{V}_{\text{eff}}$. We parametrize:

$$S_{12} = \begin{pmatrix} \cos \omega & \sin \omega & 0 \\ -\sin \omega & \cos \omega & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The straightforward calculation of the effective potential (12) shows that it depends only on three combinations of the phases, namely:

$$\begin{align*}
\Phi_1 &= \alpha_{23} - \beta_{23} - \bar{\alpha}_{23} + \bar{\beta}_{23}, \\
\Phi_2 &= \alpha_{23} - \beta_{23} - \bar{\alpha}_{12} + \bar{\beta}_{12} + \bar{\beta}_{23}, \\
\Phi_3 &= \alpha_{12} - \beta_{12} - \beta_{23} - \bar{\alpha}_{23} + \bar{\beta}_{23}.
\end{align*}$$

Indeed, substituting (14) in Eq. (12) we obtain:

$$\mathcal{V}_{\text{eff}} = A m_1^2 \cos^2 \Theta_{23} + A m_2^2 \cos^2 \Theta_{12} \sin^2 \Theta_{23} + B m_3^2 \cos^2 \bar{\Theta}_{23} + C m_1^2 m_2^2 \cos \Theta_{23} \cos \bar{\Theta}_{23} + \sin \Theta_{23} \sin \bar{\Theta}_{23} \cos \omega e^{i\Phi_1} \left| \Phi_1 \right|^2$$

$$+ C m_1^2 m_3^2 \cos \bar{\Theta}_{12} \left( \cos \Theta_{23} \sin \bar{\Theta}_{23} - \sin \Theta_{23} \cos \bar{\Theta}_{23} \cos \omega e^{i\Phi_1} \right)$$

$$- \sin \Theta_{23} \sin \bar{\Theta}_{12} \sin \omega e^{i\Phi_2} \left| \Phi_2 \right|^2 + C m_2^2 m_3^2 \cos \Theta_{12} \left( \sin \Theta_{23} \cos \bar{\Theta}_{23} \right)$$

$$- \cos \Theta_{23} \sin \bar{\Theta}_{23} \cos \omega e^{i\Phi_1} + \sin \bar{\Theta}_{23} \sin \Theta_{12} \sin \omega e^{i\Phi_3} \left| \Phi_3 \right|^2$$

$$+ C m_2^2 m_3^2 \cos \Theta_{12} \sin \bar{\Theta}_{12} \left( \sin \Theta_{23} \sin \bar{\Theta}_{23} + \cos \Theta_{23} \cos \bar{\Theta}_{23} \cos \omega e^{i\Phi_1} \right)$$

$$+ \cos \Theta_{12} \cos \Theta_{23} \sin \bar{\Theta}_{12} \sin \omega e^{i\Phi_2} - \sin \Theta_{12} \cos \Theta_{12} \cos \bar{\Theta}_{23} \sin \omega e^{i\Phi_3}$$

$$+ \sin \Theta_{12} \sin \bar{\Theta}_{12} \cos \omega e^{-i\Phi_1 + i\Phi_2 + i\Phi_3} \right|^2.$$ (18)

In the following, for parametrization of the CKM matrix we adopt the "standard" choice advocated by the Particle Data Group [8]:

$$K = \begin{pmatrix}
c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\
-s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & s_{13} c_{13} \\
s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta} & s_{23} e^{-i\delta}
\end{pmatrix}$$

(19)

where $s_{12} = \sin \vartheta_{12}$, $c_{ij} = \cos \vartheta_{12}$, etc., and $\delta$ is the CP violating phase.
3 The model

To carry out explicitly the program which was outlined in the previous sections we use the model with the chiral horizontal \( SU(3)_H \) symmetry between the generations \[9\]. In this model left-handed quarks \( q^a_{Li} = (u, d)_{Li} \) transform as triplets of \( SU(3)_H \), whereas the right-handed ones \( u^i_R \) and \( d^i_R \) are anti-triplets \( i = 1, 2, 3 \) is an index of generations). In this paper we will not consider leptons, though clearly they can be included in a straightforward way.

Using only left-handed fields and, consequently, \( u^c_L = C \bar{u}^R \) and \( d^c_L = C \bar{d}^R \) instead of \( u^R \) and \( d^R \), we can write the simplest Yukawa couplings which can lead to the appearance of the quark masses in the following form (we skip subscript \( L \) since we use only the left-handed fermions):

\[
 u^c_i q^a_j \epsilon_{a\beta} H^{ij,\beta}_2 + d^c_i q^a_j \epsilon_{a\beta} H^{ij,\beta}_1
\]  

(20)

where \( H^{ij,\beta}_2 \) and \( H^{ij,\beta}_1 \) represent a set of the Higgs doublets of \( SU(2) \times U(1) \) (index \( \alpha, \beta = 1, 2 \)) which simultaneously transform as \( \bar{6} \) or \( 3 \) under \( SU(3)_H \). The problem with the couplings (20) is that they lead to the flavour changing neutral currents (FCNC), as always happens when more than one Higgs doublet gives masses to the quarks with the same charge of different generations \[10\]. It is not easy to suppress naturally these currents \[9\].

One way to overcome this difficulty is to change the fields \( H^{ij,\alpha}_2 \) and \( H^{ij,\alpha}_1 \) in Eq. (20) by the products of the Higgs fields which are transformed trivially by each of the groups \( SU(3)_H \) and \( SU(2) \times U(1) \) \[11\]. Namely, let us put \[12\]:

\[
 H^{ij,\beta}_2 = \frac{\chi^{ij} \cdot H^3_2}{M}, \quad H^{ij,\beta}_1 = \frac{\xi^{ij} \cdot H^1_1}{M},
\]  

(21)

where \( \chi^{ij} \) and \( \xi^{ij} \) are transformed as \( 3 \) or \( \bar{6} \) of \( SU(3)_H \) and are singlets of \( SU(2) \times U(1) \), while \( H_{1,2} \) are doublets of \( SU(2) \times U(1) \) and the \( SU(3)_H \) singlets. \( M \) is the mass parameter which is introduced to preserve the right dimension of the fields. In other words, we consider non-renormalizable interactions

\[
 \frac{\chi^{ij}}{M} \cdot u^c_i q^a_j \epsilon_{a\beta} H^{ij,\beta}_2 + \frac{\xi^{ij}}{M} \cdot d^c_i q^a_j \epsilon_{a\beta} H^{ij,\beta}_1
\]  

(22)

One sees that for large enough \( M \) the interaction of the quarks with the scalars can be made as weak as necessary whereas the usual values of the masses of the fermions can be set up by the appropriate choice of the v.e.v.’s of \( \chi^{ij} \) and \( \xi^{ij} \). In fact the ratios \( \langle \chi^{ij} \rangle/M \) and \( \langle \xi^{ij} \rangle/M \) are nothing but the matrices \( G_u \) and \( G_d \) of the Yukawa coupling constants in the standard model.

\[3\] Notice, that actual global chiral symmetry of the terms (22) is \( U(3)_H = SU(3)_H \times U(1)_H \), where \( U(1)_H \) is related to phase transformation \( q, u^c, d^c \rightarrow e^{i\varphi} q, u^c, d^c \), \( \chi, \xi \rightarrow e^{-2i\varphi} \chi, \xi \). Thus the fermion mass hierarchy in fact is a reflection of the v.e.v.’s hierarchy in the chiral \( U(3)_H \) symmetry breaking \( U(3)_H \rightarrow U(2)_H \rightarrow U(1)_H \rightarrow nothing \). In fact, \( U(1)_H \) can serve as the Peccei-Quinn symmetry unless it is explicitly broken in the potential of \( \chi \) and \( \xi \) \[12\].

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As a matter of fact, what we actually have in mind in considering this model, is a supersymmetric theory. In other words, \( q \), \( u^c \) and \( d^c \) are chiral superfields of quarks, \( H_{1,2} \) are the MSSM Higgs doublets and \( \chi \) and \( \xi \) are ‘horizontal’ Higgs superfields breaking the \( SU(3)_H \) symmetry. For the completeness of the theory, in principle one has to introduce also the Higgs superfields \( \bar{\chi} \) and \( \bar{\xi} \) in representations conjugated to \( \chi \) and \( \xi \), but these do not play a relevant role in our further considerations. Eq. (22) actually are the superpotential terms responsible for quark masses.

There are different ways to justify the appearance of the non-renormalizable interactions (22). Maybe the most natural and simplest way is to introduce the additional vector-like set of heavy fermions \[11\], namely, the weak isosinglets transforming as triplet representation of \( SU(3)_H \) \[12\]. In other words, per each generation we introduce the left chiral (\( SU(2) \)-singlet) partners \( U_i, U^c_i \) and \( D_i, D^c_i \) \((i = 1, 2, 3)\), with the same electric and colour charges as \( u, u^c, d, d^c \) but with the following transformation properties under \( SU(3)_H \):

\[
U_i, D_i \sim 3, \quad U^c_i, D^c_i \sim \bar{3}.
\]

The assignment (23) allows the large mass terms ("survival hypothesis") for the states \( U, U^c \) and \( D, D^c \):

\[
M(U^c_i U_i), \quad M(D^c_i D_i),
\]

as well as their couplings

\[
(U^c_i U_j)\Sigma^j_i, \quad (D^c_i D_j)\Sigma^j_i,
\]

with the scalar \( \Sigma \) in an adjoint (octet) representation of \( SU(3)_H \): \( \Sigma \sim 8 \). It is natural to assume that due to a tree-level potential, \( \Sigma \) develops the v.e.v. proportional to \( \lambda_8 \): \( \langle \Sigma \rangle \sim \text{diag}(1, 1, -2) \). Of course, the mass parameter \( M \) can be different in \( U \) and \( D \) mass terms as well as the coupling constants for the two structures of (23). However, this is irrelevant for our discussion.

The Yukawa couplings which lead now to the masses of the light quarks are:

\[
(u^c_i U_j)\chi^{ij}, \quad (d^c_i D_j)\xi^{ij}
\]

and

\[
(U^c_i q^\alpha_i)\epsilon_{\alpha\beta}H_2^\beta, \quad (D^c_i q^\alpha_i)\epsilon_{\alpha\beta}H_1^\beta,
\]

where we absorb the coupling constants into the Higgs fields.

All Yukawa couplings of the light and heavy fermions, in the basis of \( (u, d, U, D) \) and \( (u^c, d^c, U^c, D^c) \) states, can be presented in the form of a field-dependent mass

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\[\text{In the following, as it is usually adopted, we distinguish the fermion and Higgs superfields by their matter parity, negative for fermions and positive for Higgses.}\]
matrix:
\[
\mathcal{M} = \begin{pmatrix}
0 & 0 & H_0^2 & H_0^1 \\
0 & 0 & H_2^+ & H_1^0 \\
\chi & 0 & M + \Sigma & 0 \\
0 & \xi & 0 & M + \Sigma
\end{pmatrix}.
\] (28)

The constant mass matrix emerges when the scalars are changed by their v.e.v.’s.

The effective non-renormalizable Lagrangian (22) emerges through the diagrams of the type of Fig. 4 in the limit \( M \gg \chi, \xi, \Sigma \). Hence, mass matrices of the up- and down-quarks are connected to the v.e.v.’s of \( \chi \) and \( \xi \):

\[
M_u = G_u \langle H_2^0 \rangle, \quad G_{ij}^u = \frac{\langle \chi_{ij} \rangle}{M} ; \quad M_d = G_d \langle H_1^0 \rangle, \quad G_{ij}^d = \frac{\langle \xi_{ij} \rangle}{M} .
\] (29)

Each of these v.e.v.’s can be brought to diagonal form: \( \langle \chi \rangle \sim \text{diag}(\chi_1, \chi_2, \chi_3) \) and \( \langle \xi \rangle \sim \text{diag}(\xi_1, \xi_2, \xi_3) \), so that in the ‘seesaw’ limit \( \chi_i, \xi_i \ll M \) the quark masses in (2) are essentially the ratios \( \chi_i/M \) and \( \xi_i/M \). Clearly, the large value of the top mass requires \( \chi_3 \sim M \), whereas other v.e.v.’s should be much smaller than \( M \). Actually, for the top mass one has to use more precise formula (see e.g. in ref. [13]) rather than the one given in seesaw limit, Eq. (29). However, this is not of principal importance for our consideration. In addition, since our model has a rather illustrative character, we do not take into account the renormalization running of masses from the scale of the horizontal symmetry down to the electroweak scale.

In spirit of our proposal, we assume that a tree-level superpotential of \( \chi, \xi \) and \( \Sigma \) contains only the self-interaction terms of these fields like \( \text{Tr} (\bar{\chi} \chi), \text{Tr} (\bar{\chi} \chi \bar{\chi}), \text{Tr} (\Sigma^2), \text{Tr} (\Sigma^3) \), etc., but does not contain crossing terms like \( \text{Tr} (\bar{\chi} \xi), \text{Tr} (\bar{\chi} \chi \Sigma) \), etc. At this level, potential can fix a shape of v.e.v.’s of each of these fields, but the relative orientation of the v.e.v.’s of \( \chi, \xi \) and \( \Sigma \) remains unfix. In other words, superpotential has a global symmetry \( SU(3)_\chi \times SU(3)_\xi \times SU(3)_\Sigma \) related to independent unitary transformations of \( \chi, \xi \) and \( \Sigma \).

The Yukawa terms do not respect the \( SU(3)^3 \) global symmetry, and hence radiative corrections should violate it also in the Higgs potential. Nevertheless, if supersymmetry is unbroken, no additional structures will emerge in radiative corrections and thus the CKM angles would remain the true Goldstone modes. However, once supersymmetry is broken, radiative corrections will become effective. They remove the vacuum degeneracy and give rise to certain terms in the effective potential which link these scalars to each other. The soft supersymmetry breaking can be accounted by the spurion superfield \( z = m_S \theta^2 (\bar{z} = m_S \bar{\theta}^2) \), where \( m_S \sim v \) is the soft mass scale. Then the desired structures (10) could emerge from D-terms

\[
\left[ \text{Tr} (\chi^+ \chi \Sigma) zz \right]_D , \quad \left[ \text{Tr} (\xi^+ \xi \Sigma) zz \right]_D , \quad \left[ \text{Tr} (\chi^+ \xi^+ \xi) zz \right]_D .
\] (30)

The first two terms in (30) indeed emerge from the one-loop supergraphs shown in Fig. 5, after inserting the spurion fields into the internal lines or vertices.
By taking into account that $\langle \Sigma \rangle \sim \lambda_8$, these terms would immediately translate into the first two terms of the effective potential (10). Clearly, from the similar diagrams (with insertion of $\Sigma$ instead of mass entry $M$), also the terms like $\text{Tr} (\xi^+ \Sigma^+ \Sigma)$ will be induced. However, these in fact do not create new structures in (10), since $\lambda_8^2$ is a combination of the unit matrix and $\lambda_8$ itself.

The third term in (30) emerges from the 3-loop graph of Fig. 6, where under the non-renormalizable vertices we actually imply the effective operators induced by the heavy (with mass $M$) fermion exchanges as in Figs. 4. For the momenta smaller than $M$, when our theory effectively reduces to the non-renormalizable operators (22), this graphs effectively reduce to the ones given in Fig. 2, which (in supersymmetric case) are quadratically divergent. Therefore, $M$ actually acts as a cutoff scale and the contribution of this diagram is $\sim m_3^2 M^2 \text{Tr}(G_u^+ G_u G_d^+ G_d)$.

Thus, after the supersymmetry breaking the following terms emerge in the effective potential of the scalars $\chi, \xi$ and $\Sigma$ (the loop factors are omitted):

\[
\frac{m_3^2}{M} \text{Tr} (\chi^+ \chi \Sigma), \quad \frac{m_3^2}{M} \text{Tr} (\xi^+ \xi \Sigma), \quad \frac{m_3^2}{M^2} \text{Tr}(\chi^+ \chi \xi^+ \xi). 
\]

which after substituting the basic tree-level v.e.v.’s of these fields reduce to the $\mathcal{V}_{\text{eff}}$ of the structure given by Eq. (11). The same order of magnitude of all three terms can be achieved by properly chosen values of $\langle \Sigma \rangle \ll M$ and of the coupling constants in Eq. (25).

The following comment is in order. For the vacuum expectation value of $\Sigma$ we have assumed that actually only one component of the octet does not vanish: $\langle \Sigma \rangle \sim \lambda_8$. Such a solution indeed emerge in an unique way from the superpotential of $\Sigma$, $W(\Sigma) = m \Sigma^2 + \Sigma^3$. One can expect the similar properties for sextets and triplets. Certainly, there is no reason for $\chi$ or $\xi$ to have several non-vanishing components but rather one non-vanishing eigenvalue for each matrices $\xi^{ij}$ and $\chi^{ij}$. In other words, their v.e.v.’s can be (independently) rotated to the form $\sim (0, 0, 1)$. As a result, only one up- and one down-quark would acquire the non-zero masses. However, the other non-zero eigenvalues in $\xi$ and $\chi$ can be induced by their interactions to the other set of superfields $\xi'$ and $\chi'$ in some representations of $SU(3)_H$ which themselves do not couple to fermions. It this way all quarks can get masses. Furthermore our assumption is that the Higgs superpotential is organized in such a way that it is invariant under the separate $SU(3)$ rotations of all fields composing $\xi, \chi, \Sigma$. In other words, we assume that it has a form

\[
W = W(\xi, \xi') + W(\chi, \chi') + W(\Sigma) 
\]

respecting the accidental global symmetry $SU(3)_\chi \times SU(3)_\xi \times SU(3)_\Sigma$ related to the independent unitary transformations of $\chi(\chi'), \xi(\xi')$ and $\Sigma$. Then the relative

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5 Alternatively, one could introduce several Higgs fields in the place of $\chi$ or $\xi$, say a set of sextets and triplets for each, with v.e.v.'s on different components. What we actually mean then by $\chi^{ij}$ and $\xi^{ij}$ are in fact the relevant combinations of these fields in which they couple to fermions.
SU(3) orientation of $\xi$ and $\chi$ as well as their relative orientation to $\Sigma$ will be fixed by the loop contributions to $V_{\text{eff}}$ leading to the expression (10).

Concluding, in the case of the exact supersymmetry the structures (31), once they are absent in tree-level potential, would not appear in radiative corrections. Broken supersymmetry allows to generate such terms, however suppresses their values so that they are proportional to $m_S^2$. They appear in effective potential with values $\sim m_S^2 M^2$, much smaller than typical size ($\sim M^4$) of the tree-level terms like $(\chi^+ \chi)^2$ etc. (certainly, there is also an additional suppression due to the loop factors). Therefore, pseudo-familons are indeed light, with masses $< m_S^6$. Below we shall try to estimate the magnitude of the loop diagrams of Figs. 5,6, and hence the values of the pseudo-familon masses. At the moment we confine ourselves by the observation that the model indeed leads to the $V_{\text{eff}}$ of the structure given by Eq. (11).

4 A toy model for two generations

In this section we shall consider the non-realistic model of two generations of fermions: $u_i = (c, t)$, $d_i = (s, b)$. The purpose of this exercise is twofold: first, this simplified version very well illustrates the physics related to our approach. Second, below we shall use the results of this section in treating the realistic case of three generations. In the expression (11) for the effective potential we change $\lambda_8 \to \text{diag}(0, 1)$.

Let us parametrize the $2 \times 2$ unitary matrices $U$ and $V$ as

\[
U = \begin{pmatrix} e^{i\alpha} \cos \Theta & e^{i\beta} \sin \Theta \\ -e^{i\gamma} \sin \Theta & e^{i\delta} \cos \Theta \end{pmatrix}, \quad \alpha - \beta = \gamma - \delta,
\]

\[
V = \begin{pmatrix} e^{i\tilde{\alpha}} \cos \tilde{\Theta} & e^{i\tilde{\beta}} \sin \tilde{\Theta} \\ -e^{i\tilde{\gamma}} \sin \tilde{\Theta} & e^{i\tilde{\delta}} \cos \tilde{\Theta} \end{pmatrix}, \quad \tilde{\alpha} - \tilde{\beta} = \tilde{\gamma} - \tilde{\delta}.
\]

(33)

Then the same arguments which were used to obtain Eq. (13) lead to the result:

\[
V_{\text{eff}} = Am_i^2 |U_{33}|^2 + Bm_b^2 |V_{33}|^2 + Cm_t^2 m_b^2 \left| (UV^\dagger)_{33} \right|^2 = \\
= Am_i^2 \cos^2 \Theta + Bm_b^2 \cos^2 \tilde{\Theta} + Cm_t^2 m_b^2 \left| \cos \Theta \cos \tilde{\Theta} + e^{i\Phi} \sin \Theta \sin \tilde{\Theta} \right|^2, \\
\Phi = \delta - \tilde{\delta} - \gamma + \tilde{\gamma}.
\]

(34)

The extremum of this potential corresponds to $\sin \Phi = 0$, i.e. $\Phi = 0, \pi$. By allowing both signs for $\Theta, \tilde{\Theta}$, we can choose $\Phi = 0$ and simplify Eq. (34) in the

\footnote{Let us remark that there are other interesting examples when the flat directions of the supersymmetric theory give rise to light states in the particle spectrum. One popular example is, e.g. when the MSSM Higgs doublets appear as the pseudo-Goldstone bosons of the accidental global symmetry [14].}
following way, omitting an unessential additive constant:

\[ V_{\text{eff}} = a \cos \chi_1 + b \cos \chi_2 + c \cos \chi_3 \]

(35)

where \( \chi_1 = 2\Theta, \chi_2 = 2\tilde{\Theta}, \chi_3 = 2(\Theta - \tilde{\Theta}) \) and

\[ a = \frac{1}{2} A m_i^2, \quad b = \frac{1}{2} B m_b^2, \quad c = \frac{1}{2} C m_t^2 m_b^2 \]

(36)

To gain some physical intuition it is useful to interpret the expression (35) as a potential energy of the system of three interacting two-dimensional unit vectors: \( \vec{n}_0(1,0), \vec{n}_1(\cos \chi_1, \sin \chi_1), \vec{n}_2(\cos \chi_2, \sin \chi_2) \). In terms of these vectors

\[ V_{\text{eff}} = a (\vec{n}_0 \cdot \vec{n}_1) + b (\vec{n}_0 \cdot \vec{n}_2) + c (\vec{n}_1 \cdot \vec{n}_2). \]

(37)

Each positive coefficient (say \( a > 0 \)) describes the "repulsion" of the corresponding pair of vectors. The minimum of the corresponding term, i.e. \( a(\vec{n}_0 \cdot \vec{n}_1) \), is reached for \( (\vec{n}_0 \cdot \vec{n}_1) = -1 \), i.e. \( \chi_1 = \pi \). Any negative coefficient (e.g. \( b < 0 \)) can be understood as an attraction, and the minimum corresponds now to \( (\vec{n}_0 \cdot \vec{n}_2) = 1 \), i.e. \( \chi_2 = 0 \).

Clearly, if all three couplings are attractive: \( a, b, c < 0 \), then the absolute minimum of \( V_{\text{eff}} \) is obtained when all three vectors \( \vec{n}_{0,1,2} \) are parallel: \( \chi_1 = \chi_2 = \chi_3 = 0 \). If there are two repulsions and one attraction (say, \( a > 0, b > 0 \) and \( c < 0 \)), then \( \chi_1 = \chi_2 = \pi, \chi_3 = 0 \): two vectors \( \vec{n}_1 \) and \( \vec{n}_2 \) are stuck to each other but oriented in the opposite direction to the third one \( \vec{n}_0 \).

There are two cases when one can expect the nontrivial configuration of the vectors and therefore nontrivial mixing angles (see Fig. 7) First case corresponds to three repulsions, \( a, b, c > 0 \). Second is realized for two attractions and one repulsion (for example, \( a, b < 0 \) and \( c > 0 \)). In this latter case vectors \( \vec{n}_1 \) and \( \vec{n}_2 \) are attracted to \( \vec{n}_0 \), but their mutual repulsion does not allow them to stick to each other and to \( \vec{n}_0 \).

The equation \( dV_{\text{eff}}/d\chi_{1,2} = 0 \), besides the trivial solution \( \sin \chi_1 = \sin \chi_2 = \sin \chi_3 = 0 \), has a non-trivial one:

\[ \cos \chi_1 = \frac{1}{2} \left( \frac{bc}{a^2} - \frac{b - c}{b} \right), \quad \cos \chi_2 = \frac{1}{2} \left( \frac{ac}{b^2} - \frac{a - c}{a} \right), \quad \cos \chi_3 = \frac{1}{2} \left( \frac{ab}{c^2} - \frac{a - b}{a} \right). \]

(38)

To analyze these solutions, it is useful to write down the expressions for \( \sin^2 \Theta_i \) (here we put \( c = 1 \), i.e. rescale \( a/c, b/c \to a, b \):

\[ \sin^2 \Theta = \frac{1}{2} (1 - \cos \chi_1) = \frac{1}{4a^2b} (ab + a + b)(ab + a - b) , \]

\[ \sin^2 \tilde{\Theta} = \frac{1}{2} (1 - \cos \chi_2) = \frac{1}{4ab^2} (ab + a + b)(ab - a + b) , \]

\[ \sin^2(\Theta - \tilde{\Theta}) = \frac{1}{2} (1 - \cos \chi_3) = \frac{1}{4ab} (ab + a + b)(a + b - ab) . \]

(39)
The requirement that all \( \sin^2 \Theta_i \) should be positive leads to the region of the allowed values of \( a \) and \( b \) shown in Fig. 8 (for \( c = 1 \) regions I and III, for \( c = -1 \) regions II and IV). It is also easy to prove that in this region \( \sin^2 \Theta_i < 1 \).

One can see from Fig. 8 that, in order to have nontrivial mixing angles for \( c = 1 \), one should either have \( a \) and \( b \) both positive or both of them negative. An additional feature is that for positive \( a, b \), i.e. for the case of all three repulsions, for enough large values of \( a \) and \( b \), when \( ab - a - b > 0 \), there is no non-trivial mixing. Consider, for example, the simple case when \( a = b \). Then for \( a > 2 \) the repulsion of \( \vec{n}_1 \) and \( \vec{n}_2 \) from \( \vec{n}_0 \) is so strong that they stick to each other in the opposite direction to \( \vec{n}_0 \), in spite of repulsion between themselves. The value \( a = b = 2 \) is sort of a "threshold": for the smaller values of \( a \) a small angle between \( \vec{n}_1 \) and \( \vec{n}_2 \) appears. This angle grows as \( a \) decreases and reaches 120° for \( a = b = 1 \). In this latter symmetric case (we remind that \( c = 1 \)) \( \vec{n}_0, \vec{n}_1 \) and \( \vec{n}_2 \) compose a configuration with all the angles equal 120°.

The similar phenomenon has place in the case of \( a, b < 0 \). If the attraction of \( \vec{n}_1 \) and \( \vec{n}_2 \) to \( \vec{n}_0 \) is very strong, so that \( ab + a + b > 0 \) (e.g.\( a < -2 \) for \( a = b \)), all three vectors are stuck to each other. Only for smaller \(|a|, |b|\) the mixing angle appears.

On the other hand, it is easy to show that when \( 0 \leq \sin^2 \Theta_i \leq 1 \), i.e. inside the regions of Fig. 8, the non-trivial solution (38) (or (39)) leads to lower energy than the trivial ones. For example, for the case of three repulsions, \( a, b, c > 0 \), the magnitude of the effective potential (35) for the cos \( \chi_i \) from (38) is the following:

\[
V_{\text{eff}}^{(\text{ext})} = -a - b + c - \frac{(a + bc - ab)^2}{2abc},
\]

which is always smaller as compared to the magnitude \( V_{\text{eff}}^{(0)} = -a - b + c \) at the trivial extremum \( \chi_{1,2} = \pi, \chi_3 = 0 \).

For the case \( a, b < 0 \) but \( c > 0 \) we have:

\[
V_{\text{eff}}^{(\text{ext})} = a + b + c - \frac{(ab + bc + ac)^2}{2abc},
\]

which is again less than the magnitude \( V_{\text{eff}}^{(0)} = a + b + c \) at the trivial extremum \( \chi_{1,2,3} = 0 \). In fact, the mixing angles outside the regions of Fig. 8 become trivial (zero or \( \pi \)) not because the energy of the trivial solution becomes lower than the energy corresponding formally to (38), but solely because that there are no physical solutions fulfilling the condition \( 0 \leq \sin^2 \Theta_i \leq 1 \).

### 5 Three generations: fixing the CKM angles

We now pass to the realistic case of three generations. The basic expression for \( V_{\text{eff}} \), which we use in what follows, is given by Eq. (38).
The solution with $\sin \Phi_i = 0$, $i = 1, 2, 3$, is certainly an exact extremum of $\mathcal{V}_{\text{eff}}$. One can argue that it is unlikely to have different extremums corresponding to non-trivial phases.

Let us first focus on the dependence of $\mathcal{V}_{\text{eff}}$ on $\Phi_1$. The leading term is proportional to $m_i^2m_b^2$:

$$\mathcal{V}_{\text{eff}} \sim 2Cm_i^2m_b^2 \cos \Phi_1 \cos \Theta_{23} \cos \tilde{\Theta}_{23} \sin \Theta_{23} \sin \tilde{\Theta}_{23} \cos \omega .$$

(42)

Though it is difficult to have a rigorous proof concerning the next terms proportional to $m_i^2m_b^2$, $m_i^2m_s^2$ and $m_e^2m_j^2$ they seem to be negligible as compared to (33). For example, the term $\sim m_i^2m_b^2 \cdot \cos(\Phi_1 - \Phi_2)$ contains even more sinuses then the leading contribution (42) and it is very difficult to imagine what could compensate the smallness of $m_i^2/m_b^2$.

Leaving only the contribution (42), we see that the non-trivial solution of the equation $d\mathcal{V}_{\text{eff}}/d\Phi_1 = 0$ (i.e. when $\Theta_{23}, \tilde{\Theta}_{23} \neq 0, \pi/2$, $\Theta'_{12} \neq \pi/2$) implies that $\sin \Phi_1 = 0$. Thus, $\Phi_1 = 0$ or $\Phi_1 = \pi$. We shall choose $\Phi_1 = 0$ and see that this value corresponds to a minimum of $\mathcal{V}_{\text{eff}}$. Indeed, from (42) one has:

$$\left(\frac{d^2\mathcal{V}}{d\Phi_1^2}\right)_{\Phi_1=0} = -2Cm_i^2m_b^2 \cos \Theta_{23} \cos \tilde{\Theta}_{23} \sin \Theta_{23} \sin \tilde{\Theta}_{23} \cos \omega .$$

(43)

For the solution described below: $C < 0$, $\Theta_{23}$ and $\tilde{\Theta}_{23}$ are small and have the same sign, and $\omega = 0$. Therefore $(d^2\mathcal{V}/d\Phi_1^2)_{\Phi_1=0} > 0$.

Similar arguments are applicable to $\Phi_2$ and $\Phi_3$, after we insert $\Phi_1 = 0$. We assume therefore that $\Phi_2 = \Phi_3 = 0$. For the solution given below ($\omega = 0$) we obtain analogously to (42):

$$\mathcal{V}_{\text{eff}}(\Phi_2, \Phi_3) \sim 2Cm_i^2m_b^2 \cos(\Phi_2 + \Phi_3) \cos \Theta_{12} \cos \tilde{\Theta}_{12} \times \cos(\Theta_{23} - \tilde{\Theta}_{23}) \sin \Theta_{12} \sin \tilde{\Theta}_{12} .$$

(44)

Again $\Theta_{12}$ and $\tilde{\Theta}_{12}$ are small and have the same sign. Therefore, $\mathcal{V}_{\text{eff}}$ has a minimum at $\Phi_2 = \Phi_3 = 0$.

Next step is to find the minimum in $\omega$. Contrary to the case of the phases $\Phi_i$, $\omega = 0$ is not an exact solution of the equation $d\mathcal{V}_{\text{eff}}/d\omega = 0$. However, if one neglects non-leading contributions and leaves only the term $\sim m_i^2m_b^2$, then $\omega = 0$ is indeed an extremum. We adopt this approximation and write down the simplified expression for $\mathcal{V}_{\text{eff}}$ for $\Phi_1 = \Phi_2 = \Phi_3 = \omega = 0$:

$$\begin{align*}
\mathcal{V}_{\text{eff}} &= Am_i^2 \cos^2 \Theta_{23} + Am_e^2 \cos^2 \Theta_{12} \sin^2 \Theta_{23} + Bm_b^2 \cos^2 \tilde{\Theta}_{23} + Bm_b^2 \cos^2(\Theta_{23} - \tilde{\Theta}_{23}) \\
&\quad + Cm_i^2m_b^2 \cos^2 \Theta_{12} \sin^2 \tilde{\Theta}_{23} + Cm_i^2m_b^2 \cos^2(\Theta_{12} + \tilde{\Theta}_{12}) + Cm_i^2m_b^2 \cos^2(\Theta_{23} - \tilde{\Theta}_{23}) \\
&\quad + Cm_i^2m_b^2 \left[ \sin \Theta_{12} \sin \tilde{\Theta}_{12} + \cos \Theta_{12} \cos \tilde{\Theta}_{12} \cos(\Theta_{23} - \tilde{\Theta}_{23}) \right]^2.
\end{align*}$$

(45)
To find explicitly the mixing angles we shall use the mass hierarchy and the smallness of the mixing angles. With the accuracy of order of $10^{-4} - 10^{-5}$ the leading terms in Eq. (15) are:

$$V_{\text{eff}} = A m_t^2 \cos^2 \Theta_{23} + B m_b^2 \cos^2 \bar{\Theta}_{23} + C m_t^2 m_b^2 \cos^2 (\Theta_{23} - \bar{\Theta}_{23})$$  \hspace{1cm} (46)

Thus the problem of finding $\Theta_{23}$ and $\bar{\Theta}_{23}$ reduces to the two-generation case considered in Section 4. As it was explained in this section, the only way to get the non-trivial mixing angles is to have two negative and one positive coefficients in the expression for $V_{\text{eff}}$. If one rewrites (46) in the form of Eq. (37) and identifies the cosines of the double angles as the scalar products of the unit vectors:

$$\cos 2\Theta_{3} = \vec{n}_1 \cdot \vec{n}_0, \cos 2\bar{\Theta}_{23} = \vec{n}_2 \cdot \vec{n}_0, \cos 2(\Theta_{23} - \bar{\Theta}_{23}) = \vec{n}_1 \cdot \vec{n}_2,$$

this case would correspond to an attraction of a two pairs of the vectors and one repulsion. We shall choose the situation shown in Fig. 7B corresponding to $A < 0$, $C < 0$, $B > 0$ when $\vec{n}_0$ and $\vec{n}_2$ are both attracted to $\vec{n}_1$ but repulse from each other.

Clearly, in this case the "gluey" vector $\vec{n}_1$ should be placed between $\vec{n}_0$ and $\vec{n}_2$.

To use the results of Section 4 we rewrite the expression (46), omitting an unessential additive constant, in the form

$$\frac{V_{\text{eff}}}{2|C|m_t^2 m_b^2} = -a \cos 2\Theta_{23} + b \cos 2\bar{\Theta}_{23} - \cos 2(\Theta_{23} - \bar{\Theta}_{23})$$  \hspace{1cm} (47)

$$a = \left| \frac{A}{C m_t^2} \right|, \quad b = \left| \frac{B}{C m_t^2} \right|.$$

In Eq. (39) $a$ and $b$ mean actually $a/c$ and $b/c$ with $c = 1$. Since the expression (47) differs from (35) by the change $a \rightarrow -a$ and $c \rightarrow -c = -1$, we can directly use the expression (39) changing the ratios $a/c \rightarrow a/c$, $b/c \rightarrow -b/c$, i.e. $a \rightarrow a$, $b \rightarrow -b$. Thus we get:

$$\sin^2 \Theta_{23} = \frac{1}{4a^2b} (ab - a + b)(a + b - ab),$$

$$\sin^2 \bar{\Theta}_{23} = \frac{1}{4ab^2} (ab - a + b)(a + b + ab),$$

$$\sin^2 (\Theta_{23} - \bar{\Theta}_{23}) = \frac{1}{4ab} (ab - a + b)(ab + a - b).$$  \hspace{1cm} (48)

It is now clear that at the end we shall be able to get just one relation for the three physical mixing angles. Indeed, Eqs. (18) and similar relations for the angles $\Theta_{12}$ and $\bar{\Theta}_{12}$ (see below, Eq. (24)) express all mixing angles (3 of them physical) through two unknown parameters $|A/C|$ and $|B/C|$. However, it is necessary first to connect the physical angles entering the CKM matrix with $\Theta_{23}$, $\bar{\Theta}_{23}$, $\Theta_{12}$, $\bar{\Theta}_{12}$.

One can easily get the relations between the "standard" angles $\vartheta_{12}$, $\vartheta_{23}$, $\vartheta_{13}$ in (19) and $\Theta_{23}, \bar{\Theta}_{23}, \Theta_{12}, \bar{\Theta}_{12}$. Using our definition of $K$, Eq. (17), and taking
into account that $S_{12} = I$ (i.e. $\omega = 0$) according to our solution of the equation $dV_{\text{eff}}/d\omega = 0$, one obtains:

\[
\begin{align*}
S_{12} &= \frac{\sin \Theta_{12} \cos \tilde{\Theta}_{12} \cos(\Theta_{23} - \tilde{\Theta}_{23}) - \cos \Theta_{12} \sin \tilde{\Theta}_{12}}{\sqrt{1 - \sin^2 \Theta_{12} \sin^2(\Theta_{23} - \tilde{\Theta}_{23})}} \approx \sin(\Theta_{12} - \tilde{\Theta}_{12}) \\
S_{23} &= \frac{\cos \Theta_{12} \sin(\Theta_{23} - \tilde{\Theta}_{23})}{\sqrt{1 - \sin^2 \Theta_{12} \sin^2(\Theta_{23} - \tilde{\Theta}_{23})}} \approx \cos \Theta_{12} \sin(\Theta_{23} - \tilde{\Theta}_{23}) \\
S_{13} &= \sin \Theta_{12} \sin(\Theta_{23} - \tilde{\Theta}_{23})
\end{align*}
\]

where we have approximated $\cos(\Theta_{23} - \tilde{\Theta}_{23}) \approx 1$ and neglected $\sin^2 \Theta_{12} \sin^2(\Theta_{23} - \tilde{\Theta}_{23}) \approx 0$. We have kept $\cos \Theta_{12}$ since the angle $\Theta_{12}$ is slightly bigger than $\Theta_{23} - \tilde{\Theta}_{23}$:

\[
\Theta_{12} \approx \tan \Theta_{12} = \frac{\sin \Theta_{12}}{\cos \Theta_{12}} \sim 0.1, \quad \Theta_{23} - \tilde{\Theta}_{23} = s_{23} \sim 0.04.
\]

We can now resolve the last of the equations (48) using the smallness of $\sin^2(\Theta_{23} - \tilde{\Theta}_{23}) = s_{23}^2/c_{12}^2$. We assume that this smallness is ensured by the relation $ab - a + b \approx 0$, which also leads to the smallness of $\Theta_{23}$ and $\tilde{\Theta}_{23}$ separately. In the linear approximation in $s_{23}^2$ one has:

\[
ab - a + b = 2 \left( \frac{s_{23}^2}{c_{12}^2} \right) \quad \Rightarrow \quad b = \frac{1}{a + 1} \left[ a + 2 \left( \frac{s_{23}^2}{c_{12}^2} \right) \right] \approx \frac{a}{a + 1}.
\]

We shall see below that $a \sim 50$ while $2s_{23}^2/c_{12}^2 \sim 3 \cdot 10^{-3}$. Thus, the second term in the square brackets in (50) is completely negligible as compared to $a$.

Eq. (50) is the only physical consequence of the minimum of the potential (47). According to Eq. (47) the "trivial" (vanishing) mixing angles, $\Theta_{23} = \tilde{\Theta}_{23} = 0$, correspond to

\[
\left[ \frac{V_{\text{eff}}}{2|C|m_2^2m_b^2} \right]_{\text{trivial}} = -a + b - 1.
\]

whereas for the solutions (48) one gets:

\[
\left[ \frac{V_{\text{eff}}}{2|C|m_2^2m_b^2} \right]_{\text{non-trivial}} = -a + b + 1 - \frac{1}{2ab}(ab - a + b)^2.
\]

Thus the non-trivial minimum is deeper than the trivial one.

In order to find the angles $\Theta_{12}$ and $\tilde{\Theta}_{12}$, we consider the next terms in (48):

\[
V_{\text{eff}}(\Theta_{12}, \tilde{\Theta}_{12}) = \left[ Am_c^2 \sin^2 \Theta_{23} + Cm_{e}^2m_{b}^2 \sin^2(\Theta_{23} - \tilde{\Theta}_{23}) \right] \cos^2 \Theta_{12}
+ \left[ Bm_s^2 \sin^2 \tilde{\Theta}_{23} + Cm_{e}^2m_{s}^2 \sin^2(\Theta_{23} - \tilde{\Theta}_{23}) \right] \cos^2 \tilde{\Theta}_{12}
+ Cm_{e}^2m_{s}^2 \cos^2(\Theta_{12} - \tilde{\Theta}_{12}),
\]

(53)
where we have approximated \( \cos(\Theta_{23} - \tilde{\Theta}_{23}) = 1 \) in the last term of (15). Substituting here \( \Theta_{23} \) and \( \tilde{\Theta}_{23} \) from (18), one obtains

\[
\frac{V_{\text{eff}}(\Theta_{12}, \tilde{\Theta}_{12})}{2|C|m_{e}^{2}m_{s}^{2}} = -a' \cos 2\Theta_{12} + b' \cos 2\tilde{\Theta}_{12} - \cos 2(\Theta_{12} - \tilde{\Theta}_{12}) ,
\]

with

\[a' = \frac{m_{b}^{2}}{m_{s}^{2}} \frac{1}{2b} (ab - a + b) = \frac{\lambda_{b}}{b} , \quad b' = \frac{m_{t}^{2}}{m_{s}^{2}} \frac{1}{2a} (ab - a + b) = \frac{\lambda_{t}}{a} , \quad (54)\]

where from Eq. (50) we have:

\[
\lambda_{b} = \left( \frac{m_{b}^{2}}{m_{s}^{2}} \right) \left( \frac{s_{23}^{2}}{c_{12}^{2}} \right) , \quad \lambda_{t} = \left( \frac{m_{t}^{2}}{m_{s}^{2}} \right) \left( \frac{s_{23}^{2}}{c_{12}^{2}} \right) . \quad (55)\]

These equations show that, as anticipated, the two unknown parameters \( a' \) and \( b' \) are expressed through one unknown number \( a \), or \( b \) (\( a \) and \( b \) are not independent since are connected by Eq. (50)) and physical mixing angles.

The angles \( \Theta_{12} \) and \( \tilde{\Theta}_{12} \) can be now found through \( a' \) and \( b' \) exactly in the same form as \( \Theta_{23}, \tilde{\Theta}_{23} \) through \( a \) and \( b \) (Eq. (48)):

\[
\sin^{2}\Theta_{12} = \frac{1}{4a'b'} (a'b' - a' + b')(a' + b' - a'b') ,
\]

\[
\sin^{2}\tilde{\Theta}_{12} = \frac{1}{4a'b'^{2}} (a'b' - a' + b')(a' + b' + a'b') ,
\]

\[
\sin^{2}(\Theta_{12} - \tilde{\Theta}_{12}) = \frac{1}{4a'b'} (a'b' - a' + b')(a'b' + a' - b') . \quad (56)\]

We again can use the smallness of \( \Theta_{12}, \tilde{\Theta}_{12} \) to assume that \( a'b' - a' + b' \approx 0 \). Substituting here \( a' \) and \( b' \) from (54) and using the relation (50), we get the equation for \( b \). For the quantity \( 1 - b \) this equation has the form:

\[
\lambda_{t}(1 - b)^{2} - (\lambda_{t}\lambda_{b} + \lambda_{t} + \lambda_{b})(1 - b) + \lambda_{b} = 0 . \quad (57)\]

It is easy to see that \( b \) is close to unity. Indeed, \( \lambda_{t} \gg \lambda_{b} \), namely \( \lambda_{t} \sim 35 \) and \( \lambda_{b} \sim 2.5 \). Choosing the proper sign for the root of (57) we get

\[
1 - b = \frac{1}{2\lambda_{t}} \left[ \lambda_{t}\lambda_{b} + \lambda_{t} + \lambda_{b} - \sqrt{(\lambda_{t}\lambda_{b} + \lambda_{t} + \lambda_{b})^{2} - 4\lambda_{t}\lambda_{b}} \right] \approx \frac{\lambda_{b}}{\lambda_{t}(1 + \lambda_{b})} . \quad (58)\]

Thus \( b \) differs from 1 only by a small correction:

\[
b = 1 - \Delta, \quad \Delta = \frac{m_{b}^{2}m_{e}^{2}}{m_{s}^{2}m_{t}^{2}} \cdot \left( 1 + \frac{m_{b}^{2}s_{23}^{2}}{m_{s}^{2}c_{12}^{2}} \right)^{-1} \approx 0.02 . \quad (59)\]
The parameter $a$ is indeed large, $a = b/1 - b \simeq 50$, while the parameters $a'$ and $b'$ are

$$a' = \frac{m_b^2}{m_s^2} s_{23}^2 (1 + \Delta) \simeq 2.6 \quad b' = \frac{m_t^2}{m_c^2} s_{23}^2 \Delta \simeq 0.7.$$  \hfill (60)

It is straightforward now to derive the relation between the physical mixing angles. From (49) and (56) we get

$$\frac{\sin^2(\Theta_{12} - \tilde{\Theta}_{12})}{\sin^2 \Theta_{12}} = \frac{s_{12}^2 (s_{23}^2 + s_{13}^2)}{s_{13}^2} = a' \frac{a' + a'b' - b'}{a' - a'b' + b'}.$$  \hfill (61)

This is already the sought relation because $a'$ and $b'$ are already expressed through the mixing angles, Eqs. (59) and (60). To present this relation in a more transparent way we use the constraint $a'b' - a' + b' \approx 0$. Slightly more accurately it reads:

$$b' = 1 + \frac{1}{a' + 1} (a' + 2s_{12}^2).$$  \hfill (62)

(Note that the accuracy of $a'b' - a' + b' = 0$ was quite adequate for the previous estimates. Its change to (62) leads only to a small change of $\Delta = 1 - b$ in Eq. (58): $\lambda_b \rightarrow \lambda_b + 2s_{12}^2$. Substituting (62) into (63) we obtain:

$$\frac{s_{12}^2 (s_{23}^2 + s_{13}^2)}{s_{13}^2} = a'^2 \left[ 1 + s_{12}^2 \left( 1 - \frac{1}{a'^2} \right) \right].$$  \hfill (63)

Or, with a linear accuracy in $s_{12}^2$, $(s_{13}/s_{23})^2$ and $\Delta$:

$$\frac{s_{13}s_{23}}{s_{12}^2} (1 + \Delta) \left[ 1 + \frac{1}{2} s_{12}^2 \left( 1 - \frac{1}{a'^2} \right) \right] \left( 1 - \frac{1}{2} s_{13}^2 s_{23}^2 \right) = \frac{m_s^2}{m_b^2}.$$  \hfill (64)

Here $\Delta$ and $a'$ are given by (59) and (60). Thus, neglecting the corrections which are of about $(3 - 4)\%$, the final result comes out:

$$\frac{s_{13}s_{23}}{s_{12}} = \frac{m_s^2}{m_b^2}.$$  \hfill (65)

Neither the left-hand nor the right-hand side of this equation is well known. However (65) is satisfied within the present experimental accuracy. Indeed, according to [8], we have $s_{12} = 0.22$, $s_{23} = 0.040 \pm 0.005$ and $s_{13}/s_{23} = 0.08 \pm 0.02$. Substituting these in the left side of Eq. (64), we obtain that $m_s/m_b = (2.4 \pm 0.6) \cdot 10^{-2}$, in agreement with the present understanding of the quark mass spectrum.

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7We have learned recently that the same relation was obtained in ref. [13] in the completely different approach, namely by considering the mass matrix ansatze with universal strength of the Yukawa couplings. We thank G. Branco for bringing his paper to our attention.
6 Pseudo-familon masses

The equation (65) is the only relation between physical quantities following from the symmetry structure of the effective potential. On the other hand, the value of the effective potential, or more precisely, the value of its second derivative, determines the masses of the "pseudo-familons" — the pseudo-Goldstone bosons corresponding to mixing angles. Though no reliable estimate of these masses seems possible we shall try to suggest a guess of what they could have been.

The typical pseudo-familon mass can be presented as

\[ m_{PF}^2 = \frac{d^2 V_{eff}}{d\Phi_F^2} = \frac{1}{F^2} \frac{d^2 V_{eff}}{d\Theta^2} \sim \frac{1}{F^2} V_{eff}. \]  

(66)

Here \( \Phi_F \) is the pseudo-familon field, \( \Theta \) and \( F \) are the corresponding angle and the scale, \( \Phi_F = F\Theta \). The last equality in (66) expresses the obvious fact that the differentiation in angles do not change dramatically the value of the effective potential.

For the estimate of the value of \( V_{eff} \) we can use one of the diagrams of Figs. 5,6. We prefer to consider a three-loop diagram shown in Fig. 6 since it does not contain an unknown parameter \( \langle \Sigma \rangle \). Approximately

\[ V_{eff} \sim \frac{m_S^2}{(8\pi^2)^3 M^2} \text{Tr}(\xi^+ \chi^+ \chi) = \frac{m_S^2 M^2}{(8\pi^2)^3} \text{Tr}(G_u^+ G_u G_d^+ G_d), \]  

(67)

where \( m_S \sim v \) is the supersymmetry breaking scale, and \( M \) is a mass of heavy fermions which in fact is a cutoff scale for quadratic divergency of this diagram. \( G_u \) and \( G_d \) are the MSSM Yukawa coupling matrices which are related to v.e.v.'s of \( \chi \) and \( \xi \) by Eqs. (29). We also introduce a factor \( 1/(8\pi^2) \) for each Feynman loop.

Let us consider for example pseudo-familons related to 23 (or 13) mixing angles. Then relevant terms in (67) are those involve the Yukawa constants of the third family: \( G_t = \chi_3/M \) and \( G_b = \xi_3/M \), and the relevant scale \( F \) is essentially the smallest scale amongst \( \chi_3 \) and \( \xi_3 \), i.e. presumably \( \xi_3 \) if \( G_b \ll G_t \) (The larger scale \( \chi_3 \) the corresponds to the "true" familon if the \( SU(3)_H \) symmetry is global; it is absorbed by the horizontal gauge boson if \( SU(3)_H \) is local). Then from Eqs. (66) and (67) we obtain:

\[ m_{PF}(23) \sim \frac{G_t G_b m_S M}{(8\pi^2)^{3/2} \xi_3} \sim \frac{G_t m_S}{16\sqrt{2\pi^3}} \]  

(68)

which say for \( m_S = 700 \text{ GeV} \) is of the order of GeV. As for the familons related to the 12 mixing angles, one can obtain the similar estimate by taking the charm and strange quark constants \( G_c \) and \( G_s \) instead of \( G_t, G_b \), and scale \( \xi_2 \sim \langle G_s/G_b \rangle \xi_3 \). Therefore, these are lighter, with the mass of several MeV:

\[ m_{PF}(12) \sim (m_c/m_t)m_{PF}(23). \]
Such massive pseudo-familons can decay into light quarks. For example, for 12 pseudo-familons with mass say 10 MeV we estimate the decay width into light quarks as

$$\Gamma \sim \frac{1}{8\pi^2} \left(\frac{m_{u,d}}{F}\right)^2 m_{PF}(12), \quad \tau \sim \left(\frac{F}{10^{11} \text{GeV}}\right)^2 \cdot 10^4 \text{s}$$

(69)

since the ratio $m_f/F$, where $m_f$ is a fermion mass, determines the strength of the familon coupling to fermion. Rather arbitrarily we have chosen the value $F = \xi_2 \simeq 10^{11} \text{ GeV}$, keeping in mind the typical value discussed usually for the breaking of the Peccei-Quinn symmetry. Note also that $F$ cannot be much smaller than $10^{10} \text{ GeV}$, due to the experimental bounds on the FCNC with the emission of the familon, like $K^+ \rightarrow \pi^+ + \text{familon}$ [4]. It is easy to see that the lifetime of 23 pseudo-familons approximately scales with respect to that of 12 ones as inverse ratio of their masses, i.e. is smaller by about 2 orders of magnitude.

If pseudo-familons were in equilibrium in the early universe, then such long lifetimes ($\tau > 1 \text{ s}$) can be somewhat problematic for nucleosynthesis. However, if the inflationary reheating temperature is considerably below the scale $F > 10^{10} \text{ GeV}$, which indeed seems to be the case e.g. due to constraint from the gravitino production, then familons would not be produced after the inflation.

7 Discussion

Summarizing the content of this work we would like to separate the general idea which has been put forward from its concrete implementation. The idea is that the weak mixing angles might be actually the dynamical degrees of freedom — the pseudo-Goldstone bosons similar to the axion. The vacuum expectation values of these fields fix the Cabibbo–Kobayashi–Maskawa matrix. We believe that this general assumption may survive even if the concrete scenario turns out to be quite different from the one suggested in this paper. In that respect what has been done may be considered as an existence proof. A clear lack of this model is that it cannot naturally explain the smallness of the angles $s_{12}$ and $s_{23}$ in the CKM matrix (e.g. in terms of the mass ratios), but rather implies certain fine tunings in adjusting their values to the experiment. At the same time we cannot help feeling a pleasant surprise that a model which we have chosen has led us to the relation (55) which is in a reasonably good agreement with the experiment.

One interesting feature of our model is that $CP$-violating phase is vanishing in the CKM matrix. In other words, weak interactions cannot be responsible for the $CP$ violation in our model. However, $CP$ violation in the $K^0 - \bar{K}^0$ system could emerge from the supersymmetric contributions to both $\epsilon_K$ and $\epsilon'_K$ parameters [16], due to the flavour non-diagonal quark-quark-gluino couplings. Interestingly,

\footnote{In this case typically one would obtain very small and maybe even negative $\epsilon'_K$, so that $CP$ violation in the $K^0 - \bar{K}^0$ system could mimic the superweak mechanism.}
the horizontal symmetry itself controls that there can be no big flavour changing fermion-sfermion couplings to neutral gauginos. In particular, if the horizontal $SU(3)_H$ symmetry is global, then the considered model satisfies the criteria given in ref. \[7\] and thus no flavour-changing effects would emerge at all beyond the usual MSSM ones. (The latter could not induce CP-violation in $K^0 - \bar{K}^0$ system once the CKM matrix is real.) Nevertheless, if the $SU(3)_H$ symmetry is local, the flavour changing and CP-violating effects could be induced by the D-terms contributions \[8\]. Alternatively, one could introduce some additional fermion states heavier than $U$ and $D$. Then some flavour changing effects could emerge at their decoupling \[9\].

Coming back once again to our initial point, if only the structure (5) emerges in the effective potential, then the CKM mixing angles are trivial. In order to deviate them from zeroes, some other terms should be introduced. In particular, we have included additional terms in the form of Eq. (10). However, the additional terms in principle could have completely different structures. For example, in the context of the left-right symmetric models one can imagine the situation when the structure (5) emerges in effective potential as well, while $K_L$ and $K_R$ are related through certain symmetry relations (in other words, left- and right rotation angles are not independent but do not coincide). In this case one could obtain a natural solution. Another possibility can be related to the grand unification theories, which introduce leptons into the consideration and thus could create alternative structures. The renormalization group effects could be also important for obtaining the non-trivial mixing angles.

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Figure Captions

Fig. 1. The diagram due to the $W$ boson exchange which generates the term (7).

Fig. 2. Diagrams contributing the vacuum energy due to the Yukawa interactions, and leading to a structure (3).

Fig. 3. An example of diagram with the charged Higgs exchange.

Fig. 4. Diagrams generating the fermion masses via exchange of the heavy fermions $U$ and $D$.

Fig. 5. Supergraphs generating the first two terms in (31) after supersymmetry breaking. Insertions of spurions $z, \bar{z}$ are not shown.

Fig. 6. Supergraph generating third term in (31). Under the non-renormalizable vertices the tree-level graphs of Fig. 4 are understood.

Fig. 7. Stable configurations of vectors with interactions given by Lagrangian (37): (A) the case of three repulsions, $a, b, c > 0$; (B) the case of two attractions and one repulsion, $a, b < 0, c > 0$.

Fig. 8. Contours restricting the parameter regions with the non-trivial minimum. Regions I and III correspond to the case $c = 1$, and regions II and IV – to the case $c = -1$. 

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