Constraint on Axial-Vector Meson Mixing Angle from Nonrelativistic Constituent Quark Model

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Abstract

In a nonrelativistic constituent quark model we find a constraint on the mixing angle of the strange axial-vector mesons, $35^\circ \lesssim \theta_K \lesssim 55^\circ$, determined solely by two parameters: the mass difference of the $a_1$ and $b_1$ mesons and the ratio of the constituent quark masses.

Key words: quark model, potential model, axial-vector mesons

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1 Introduction

It is known that the decay of the $I = 1/2 \ 1 \ ^3P_1$ and $1 \ ^1P_1$ mesons, $K_1(1270)$ and $K_1(1400)$, with masses $1273 \pm 7$ MeV and $1402 \pm 7$ MeV, respectively [1], satisfies a dynamical selection rule such that

\[ \Gamma (K_1(1270) \rightarrow K\rho) \gg \Gamma (K_1(1270) \rightarrow K^*\pi), \]
\[ \Gamma (K_1(1400) \rightarrow K^*\pi) \gg \Gamma (K_1(1400) \rightarrow K\rho), \]

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which, following the classical example of neutral kaons, suggests a large mixing (with a mixing angle close to $45^\circ$) between the $I = 1/2$ members of two axial-vector and nonets, $K_{1A}$ and $K_{1B}$, respectively, leading to the physical $K_1$ and $K'_1$ states. Carnegie et al. [4] obtained the mixing angle $\theta_K = (41 \pm 4)^\circ$ as the optimum fit to the data as of 1977. In a recent paper by Blundell et al. [3], who have calculated strong OZI-allowed decays in the pseudoscalar emission model and the flux-tube breaking model, the $K_{1A}-K_{1B}$ mixing angle obtained is $\approx 45^\circ$. Theoretically, in the exact $SU(3)$ limit the $K_{1A}$ and $K_{1B}$ states do not mix, similarly to their $I = 1$ counterparts $a_1$ and $b_1$. As for the $s$-quark mass greater than the $u$- and $d$-quark masses, $SU(3)$ is broken and these states do mix to give the physical $K_1$ and $K'_1$. If the $K_{1A}$ and $K_{1B}$ are degenerate before mixing, the mixing angle will always be $\theta_K = 45^\circ$ [3, 4]. As pointed out by Suzuki [7], the data on $K\pi\pi$ production in $\tau$-decay may confirm or refute this simple picture: if $\theta_K = 45^\circ$, production of the $K_1(1270)$ and $K_1(1400)$ would be one-to-one up to the kinematic corrections, since in the $SU(3)$ limit only the linear combination $(K_1(1270) + K_1(1400))/\sqrt{2}$ would have the right quantum numbers to be produced there. After phase-space correction, the $K_1(1270)$ production would be favored over the $K_1(1400)$ one by nearly a factor of 2. However, current experimental data are very uncertain. The measurements made by the TPC/Two-Gamma collaboration give [8]

$$B(\tau \to \nu K_1(1270)) = (0.41 + 0.41 - 0.35) \times 10^{-2}, \quad (1)$$
$$B(\tau \to \nu K_1(1400)) = (0.76 + 0.40 - 0.33) \times 10^{-2}, \quad (2)$$
$$B(\tau \to \nu K_1) = (1.17 + 0.41 - 0.37) \times 10^{-2}. \quad (3)$$

Alemany [9] combines the CLEO and ALEPH data [10] to obtain

$$B(\tau \to \nu K_1) = (0.77 \pm 0.12) \times 10^{-2}, \quad (4)$$

which is smaller, but consistent with, the TPC/Two-Gamma Results. Conversely, the claim from the CLEO collaboration is that the $\tau$ decays preferentially into the $K_1(1270)$. If one assumes, however, that the production of the $K_1(1400)$ is favored over that of $K_1(1270)$ by nearly a factor of 2 (as follows from (1),(2) if the experimental errors are ignored), one would arrive at $\theta_K \approx 33^\circ$ [3]. A very recent analysis by Suzuki of the experimental data on the two-body decays of the $J/\psi$ and $\psi'$ into an axial-vector and a pseudoscalar mesons from the BES collaboration [11] shows that any value of $\theta_K$ between $30^\circ$ and $60^\circ$ can be consistent with the $1^{++}$ modes of both the $J/\psi$ and $\psi'$ that have been so far measured [12].

The purpose of this work is to consider the $K_{1A} - K_{1B}$ mixing angle within the framework of a constituent quark model. In our previous papers [13, 14] this model was successfully applied to $P$- and $D$-wave meson spectroscopy in order to explain the common mass near-degeneracy of two pairs of nonets, $(1^3P_0, 1^3P_2), (1^3D_1, 1^3D_3),$ in the isovector and isodoublet channels, as observed in experiment, and to make predictions regarding the masses of missing and problematic $q\bar{q}$ states. As we shall see, the nonrelativistic constituent quark model provides a very simple constraint on the $K_{1A} - K_{1B}$ mixing angle determined solely by the mass difference of the isovector
counterparts of the corresponding nonets, the $a_1$ and $b_1$ mesons, and the ratio of the constituent quark masses.

## 2 Nonrelativistic constituent quark model

In the constituent quark model, conventional mesons are bound states of a spin 1/2 quark and spin 1/2 antiquark bound by a phenomenological potential which has some basis in QCD [15]. The quark and antiquark spins combine to give a total spin 0 or 1 which is coupled to the orbital angular momentum $L$. This leads to meson parity and charge conjugation given by $P = (-1)^{L+1}$ and $C = (-1)^{L+S}$, respectively. One typically assumes that the $q\bar{q}$ wave function is a solution of a nonrelativistic Schrödinger equation with the generalized Breit-Fermi Hamiltonian\(^1\) $H_{BF}$,

$$H_{BF} \psi_n(r) \equiv (H_{kin} + V(p, r)) \psi_n(r) = E_n \psi_n(r),$$

where $H_{kin} = m_1 + m_2 + p^2/2\mu - (1/m_1^3 + 1/m_2^3) p^4/8$, $\mu = m_1 m_2/(m_1 + m_2)$, $m_1$ and $m_2$ are the constituent quark masses, and to first order in $(v/c)^2 = p^2 c^2/E^2 \simeq p^2/m^2 c^2$, $V(p, r)$ reduces to the standard nonrelativistic result,

$$V(p, r) \simeq V(r) + V_{SS} + V_{LS} + V_T,$$

with $V(r) = V_V(r) + V_S(r)$ being the confining potential which consists of a vector and a scalar contribution, and $V_{SS}, V_{LS}$ and $V_T$ the spin-spin, spin-orbit and tensor terms, respectively, given by [3]

$$V_{SS} = \frac{2}{3m_1 m_2} s_1 \cdot s_2 \triangle V_V(r),$$

$$V_{LS} = \frac{1}{4m_1^2 m_2^2 r} \left\{ [(m_1 + m_2)^2 + 2m_1 m_2] \mathbf{L} \cdot \mathbf{S}_+ + (m_2^2 - m_1^2) \mathbf{L} \cdot \mathbf{S}_- \right\} \frac{dV_V(r)}{dr}$$

$$- [(m_1^2 + m_2^2) \mathbf{L} \cdot \mathbf{S}_+ + (m_2^2 - m_1^2) \mathbf{L} \cdot \mathbf{S}_-] \frac{dV_S(r)}{dr},$$

$$V_T = \frac{1}{12m_1 m_2} \left( \frac{1}{r} \frac{dV_V(r)}{dr} - \frac{d^2V_V(r)}{dr^2} \right) S_{12}. $$

Here $S_+ \equiv s_1 + s_2$, $S_- \equiv s_1 - s_2$, and

$$S_{12} \equiv 3 \left( \frac{(s_1 \cdot r)(s_2 \cdot r)}{r^2} - \frac{1}{3} s_1 \cdot s_2 \right).$$

\(^1\)The most widely used potential models are the relativized model of Godfrey and Isgur [16] for the $q\bar{q}$ mesons, and Capstick and Isgur [17] for the $qqq$ baryons. These models differ from the nonrelativistic quark potential model only in relatively minor ways, such as the use of $H_{kin} = \sqrt{m_1^2 + p_1^2} + \sqrt{m_2^2 + p_2^2}$ in place of that given in (5), the retention of the $m/E$ factors in the matrix elements, and the introduction of coordinate smearing in the singular terms such as $\delta(r)$. 

\[ \]
For constituents with spin $s_1 = s_2 = 1/2$, $S_{12}$ may be rewritten in the form

$$S_{12} = 2 \left( 3 \frac{(S \cdot r)^2}{r^2} - S^2 \right), \quad S = S_+ \equiv s_1 + s_2. \quad (11)$$

Since $(m_1 + m_2)^2 + 2m_1m_2 = 6m_1m_2 + (m_2 - m_1)^2$, $m_1^2 + m_2^2 = 2m_1m_2 + (m_2 - m_1)^2$, the expression for $V_{LS}$, Eq. (8), may be rewritten as

$$V_{LS} = \frac{1}{2m_1m_2} \frac{1}{r} \left[ \left( 3 \frac{dV_V(r)}{dr} - \frac{dV_S(r)}{dr} \right) + \frac{(m_2 - m_1)^2}{2m_1m_2} \left( \frac{dV_V(r)}{dr} - \frac{dV_S(r)}{dr} \right) \right] \mathbf{L} \cdot \mathbf{S}_+ + \frac{m_2^2 - m_1^2}{4m_1^2m_2^2} \frac{1}{r} \left( \frac{dV_V(r)}{dr} - \frac{dV_S(r)}{dr} \right) \mathbf{L} \cdot \mathbf{S}_- \equiv V_{LS}^+ + V_{LS}^- \quad (12)$$

Since two terms corresponding to the derivatives of the potentials with respect to $r$ are of the same order of magnitude, the above expression for $V_{LS}^+$ may be rewritten as

$$V_{LS}^+ = \frac{1}{2m_1m_2} \frac{1}{r} \left( 3 \frac{dV_V(r)}{dr} - \frac{dV_S(r)}{dr} \right) \mathbf{L} \cdot \mathbf{S} \left[ 1 + \frac{(m_2 - m_1)^2}{2m_1m_2} O(1) \right]. \quad (13)$$

### 3 P-wave meson spectroscopy

We now wish to apply the Breit-Fermi Hamiltonian to the $P$-wave mesons. By calculating the expectation values of different terms of the Hamiltonian defined in Eqs. (7),(11),(12), taking into account the corresponding matrix elements $\langle \mathbf{L} \cdot \mathbf{S} \rangle$ and $S_{12}$ [13], one obtains the relations [4, 13]

\[
\begin{align*}
M(3P_0) &= M_0 + \frac{1}{4} \langle V_{SS} \rangle - 2 \langle V_{LS}^+ \rangle - \langle V_T \rangle, \\
M(3P_2) &= M_0 + \frac{1}{4} \langle V_{SS} \rangle + \langle V_{LS}^+ \rangle - \frac{1}{10} \langle V_T \rangle, \\
M(a_1) &= M_0 + \frac{1}{4} \langle V_{SS} \rangle - \langle V_{LS}^+ \rangle + \frac{1}{2} \langle V_T \rangle, \\
M(b_1) &= M_0 - \frac{3}{4} \langle V_{SS} \rangle, \\
\begin{pmatrix} M(K_1) \\ M(K_1') \end{pmatrix} &= \begin{pmatrix} M_0 + \frac{1}{7} \langle V_{SS} \rangle - \langle V_{LS}^+ \rangle + \frac{1}{2} \langle V_T \rangle & \sqrt{2} \langle V_{LS}^- \rangle \\ \sqrt{2} \langle V_{LS}^- \rangle & M_0 - \frac{3}{4} \langle V_{SS} \rangle \end{pmatrix} \begin{pmatrix} K_{1A} \\ K_{1B} \end{pmatrix},
\end{align*}
\]

where $M_0$ stands for the sum of the constituent quark masses and binding energies in either case. The $V_{LS}$ term acts only on the $I = 1/2$ singlet and triplet states giving rise to the spin-orbit term mixing between these states, and is responsible for

\footnote{The spin-orbit $^3P_1 - ^1P_1$ mixing is a property of the model we are considering; the possibility that another mechanism is responsible for this mixing, such as mixing via common decay channels [6] should not be ruled out, but is not included here.}
the physical masses of the \( K_1 \) and \( K'_1 \). The masses of the \( K_{1A} \) and \( K_{1B} \) are determined by relations which are common for all eight \( I = 1, 1/2 \) \( P \)-wave mesons, \( b_1, a_0, a_1, a_2, K_{1B}, K_0^*, K_{1A}, K_2^* \):

\[
M^{(1)P_1} = m_1 + m_2 + E_0 - \frac{3}{4} \frac{a}{m_1 m_2}, \quad (14)
\]

\[
M^{(3)P_0} = m_1 + m_2 + E_0 + \frac{1}{4} \frac{a}{m_1 m_2} - \frac{2b}{m_1 m_2} - \frac{c}{m_1 m_2}, \quad (15)
\]

\[
M^{(3)P_1} = m_1 + m_2 + E_0 + \frac{1}{4} \frac{a}{m_1 m_2} - \frac{b}{m_1 m_2} + \frac{c}{2m_1 m_2}, \quad (16)
\]

\[
M^{(3)P_2} = m_1 + m_2 + E_0 + \frac{1}{4} \frac{a}{m_1 m_2} + \frac{b}{m_1 m_2} - \frac{c}{10m_1 m_2}, \quad (17)
\]

where \( a, b \) and \( c \) are related to the matrix elements of \( V_{SS}, V_{LS} \) and \( V_T \) (see Eqs. (7),(9),(13)), and assumed to be the same for all of the \( P \)-wave states. \( E_0 \) is a nonrelativistic binding energy which may in general be absorbed in the definition of a constituent quark mass \[^{[13,14]}\]. We assume also \( SU(2) \) flavor symmetry: \( m(u) = m(d) \equiv n, m(s) \equiv s \).

The correction to \( V_{LS}^+ \) in the formula (13), due to the difference in the masses of the \( n \) and \( s \) quarks, is ignored. Indeed, these effective masses, as calculated from (14)-(17) in the case where \( E_0 \) is absorbed into their definition, are

\[
n = \frac{3b_1 + a_0 + 3a_1 + 5a_2}{24}, \quad (18)
\]

\[
s = \frac{6K_{1B} + 2K_0^* + 6K_{1A} + 10K_2^* - 3b_1 - a_0 - 3a_1 - 5a_2}{24}. \quad (19)
\]

With the physical values of the meson masses (in GeV), \( a_1 \simeq b_1 \simeq 1.23, a_0 \simeq a_2 \simeq 1.32, K_{1A} \simeq K_{1B} \simeq 1.34, K_0^* \simeq K_2^* \simeq 1.43 \), the above relations give

\[
n \simeq 640 \text{ MeV}, \quad s \simeq 740 \text{ MeV}, \quad (20)
\]

so that the abovementioned correction, according to (13), is \( \sim 100^2/(2 \cdot 640 \cdot 740) \simeq 1\% \), i.e., comparable to isospin breaking on the scale considered here, and so completely negligible.

It follows from (14)-(17) that

\[
\frac{9a}{m_1 m_2} = M^{(3)P_0} + 3M^{(3)P_1} + 5M^{(3)P_2} - 9M^{(1)P_1}, \quad (21)
\]

\[
\frac{12b}{m_1 m_2} = 5M^{(3)P_2} - 3M^{(3)P_1} - 2M^{(3)P_0}, \quad (22)
\]

\[
\frac{18c}{5m_1 m_2} = 3M^{(3)P_1} - 2M^{(3)P_0} - M^{(3)P_2}. \quad (23)
\]

\[^3\text{In the following, } a_0 \text{ stands for the mass of the } a_0, \text{ etc.} \]
By expressing the ratio \( n/s \) in three different ways, viz., dividing the expressions (21)-(23) for the \( I = 1/2 \) and \( I = 1 \) mesons by each other, one obtains the relations

\[
\frac{n}{s} = \frac{K_0^* + 3K_{1A} + 5K_2^* - 9K_{1B}}{a_0 + 3a_1 + 5a_2 - 9b_1} = \frac{5K_0^* - 3K_{1A} - 2K_0^*}{5a_2 - 3a_1 - 2a_0} = \frac{2K_0^* + K_2^* - 3K_{1A}}{2a_0 + a_2 - 3a_1}. \tag{24}
\]

It follows from the last relation of (24) that

\[
(K_2^* - K_0^*)(a_2 - a_1) = (K_2^* - K_{1A})(a_2 - a_0). \tag{25}
\]

This formula explains the common mass degeneracy of the scalar and tensor meson nonets in the isovector and isodoublet channels. Using now (24) and (25), one arrives, by straightforward algebra, at

\[
\frac{n}{s} = \frac{K_{1A} - K_{1B}}{a_1 - b_1}. \tag{26}
\]

This relation is an intrinsic property of the model we are considering; it depends neither on the values of the input parameters, \( n, s, a, b, c \), nor the presence of \( E_0 \) in the relations (14)-(17). We shall now use this relation in order to obtain a constraint on the \( K_{1A} - K_{1B} \) mixing angle.

### 4 Constraint on the \( K_{1A} - K_{1B} \) mixing angle

Since, on general grounds, \( n \leq s \), it follows from (26) that

\[
|K_{1A} - K_{1B}| \leq |a_1 - b_1| \equiv \Delta, \tag{27}
\]

which may be rewritten as

\[
K_{1A}^2 + K_{1B}^2 - 2K_{1A}K_{1B} \leq \Delta^2. \tag{28}
\]

Moreover, independent of the mixing angle,

\[
K_{1A}^2 + K_{1B}^2 = K_1^2 + K_1^{'2}. \tag{29}
\]

It then follows from (28),(29) that

\[
2K_{1A}K_{1B} \geq K_1^2 + K_1^{'2} - \Delta^2. \tag{30}
\]

To obtain a constraint on the \( K_{1A} - K_{1B} \) mixing angle, we now use the formula

\[
\tan^2(2\theta_K) = \left( \frac{K_1^{'2} - K_{1A}^{'2}}{K_{1B}^2 - K_{1A}^2} \right)^2 - 1,
\]

which may be rewritten as

\[
\cos^2(2\theta_K) = \left( \frac{K_{1B}^2 - K_{1A}^2}{K_1^{'2} - K_{1A}^{'2}} \right)^2. \tag{31}
\]
It follows from (29), (30) that
\[
\left(K_{1B}^2 - K_{1A}^2\right)^2 = (K_{1A}^2 + K_{1B}^2)^2 - 4K_{1A}^2K_{1B}^2
\]
\[
\leq \left(K_1^2 + K_1^2\right)^2 - \left(K_1^2 + K_1^2 - \Delta^2\right)^2 \simeq 2\Delta^2 \left(K_1^2 + K_1^2\right), \tag{32}
\]
since \(\Delta \sim 50\) MeV (see below), and therefore \(\Delta^2 \ll K_1^2 + K_1^2\). Thus, Eq. (31) finally reduces to
\[
\cos^2(2\theta_K) \leq \frac{2\Delta^2(K_1^2 + K_1^2)}{(K_1^2 - K_1^2)^2}, \tag{33}
\]
and therefore
\[
\left|\cos(2\theta_K)\right| \leq \frac{\Delta\sqrt{2(K_1^2 + K_1^2)}}{|K_1^2 - K_1^2|}. \tag{34}
\]
The value of \(\Delta\) is determined by current experimental data on the \(a_1\) and \(b_1\) meson masses [I]: \(a_1 = 1230 \pm 40\) MeV, \(b_1 = 1231 \pm 10\) MeV. Therefore, \(\Delta \leq 50\) MeV, and one obtains, from (34),
\[
33.6^o \leq \theta_K \leq 56.4^o, \tag{35}
\]
consistent with the recent result of Suzuki [12], \(30^o \leq \theta_K \leq 60^o\). The above constraint may be tightened further by using the ratio of the constituent quark masses given in (21). Then from (26) we obtain
\[
\left|K_{1A} - K_{1B}\right| = \frac{n}{s} \left|a_1 - b_1\right| \leq \frac{0.64}{0.74} 50\) MeV \(\simeq 43\) MeV \(\equiv \Delta'. \tag{36}\]
With this \(\Delta'\) being used in (34) in place of \(\Delta\), one obtains
\[
35.3^o \leq \theta_K \leq 54.7^o. \tag{37}\]
Both the ranges (35) and (37) are consistent with the value \(\theta_K = (37.3 \pm 3.2)^o\) obtained in our previous work [13].

5 Conclusion remarks

As we have shown, a nonrelativistic constituent quark model provides a simple constraint on the \(K_{1A} - K_{1B}\) mixing angle, in terms of the mass difference of the \(a_1\) and \(b_1\) mesons and the squared masses of the physical states \(K_1\) and \(K_1'\). The numerical value of the allowed interval for the mixing angle, \(33.6^o \leq \theta_K \leq 56.4^o\), is consistent with that provided by the very recent analysis by Suzuki [12]. This interval may be constrained further by using the ratio of the constituent quark masses. In the mass degenerate case \(a_1 = b_1\), the model considered shows a similar mass degeneracy for the corresponding strange mesons, \(K_{1A} = K_{1B}\), independent of the input parameters, and so requiring a precise 45° mixing. We conclude, therefore, that more precise experimental data on the mass of the \(a_1\) meson are required to obtain a better estimate of the \(K_{1A} - K_{1B}\) mixing angle.
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