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Evaporating the Milky Way halo and its satellites with inelastic self-interacting dark matter

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ABSTRACT
Self-interacting dark matter provides a promising alternative for the cold dark matter paradigm to solve potential small-scale galaxy formation problems. Nearly all self-interacting dark matter simulations so far have considered only elastic collisions. Here we present simulations of a galactic halo within a generic inelastic model using a novel numerical implementation in the AREPO code to study arbitrary multi-state inelastic dark matter scenarios. For this model we find that inelastic self-interactions can: (i) create larger subhalo density cores compared to elastic models for the same cross section normalisation; (ii) lower the abundance of satellites without the need for a power spectrum cutoff; (iii) reduce the total halo mass by about 10%; (iv) inject the energy equivalent of $\mathcal{O}(100)$ million Type II supernovae in galactic haloes through level de-excitation; (v) avoid the gravothermal catastrophe due to removal of particles from halo centres. We conclude that a $\sim 5$ times larger elastic cross section is required to achieve the same central density reduction as the inelastic model. This implies that well-established constraints on self-interacting cross sections have to be revised if inelastic collisions are the dominant mode. In this case significantly smaller cross sections can achieve the same core density reduction thereby increasing the parameter space of allowed models considerably.

Key words: cosmology: dark matter – galaxies: haloes – methods: numerical

1 INTRODUCTION
The Cold Dark Matter (CDM) paradigm has been proven to be very successful in describing the large-scale distribution of galaxies (e.g. Springel et al. 2005) and serves as the cornerstone of our current understanding of galaxy formation and evolution (e.g. Vogelsberger et al. 2014a; Schaye et al. 2015; Khandai et al. 2015; Dubois et al. 2016; Springel et al. 2017). At sub-galactic scales however, the CDM paradigm remains to be verified with various outstanding challenges that have appeared over the last decades (for a recent review see Bullock & Boylan-Kolchin 2017). Among the most relevant challenges are: the under-abundance of dwarf galaxies in the Milky Way (the missing satellites problem, Klypin et al. 1999; Moore et al. 1999) and in the field (Zavala et al. 2009; Papastergis et al. 2011; Klypin et al. 2015), the unexpected inner dark matter density profile in low surface brightness galaxies and dwarf galaxies (the cusp-core problem, e.g. de Blok & McGaugh 1999; Walker & Peñarrubia 2011), the deficit of dark matter in the inner regions of massive dwarf galaxies (the too-big-to-fail problem, e.g. Boylan-Kolchin et al. 2011; Papastergis et al. 2015), and the unexpected diversity in the shape of dwarf rotation curves (Oman et al. 2015, 2016).

Most of these problems have been identified by contrasting dark-matter-only simulations with observations, which is clearly an oversimplified comparison that does not take into account the complex baryonic physics at play. It is therefore possible that some or even all of these challenges to CDM can be solved through the proper modelling of baryonic physics and by carefully considering observational biases. Plausible solutions have been presented separately for each problem. For instance, the existence of dark matter cores could be explained by the gravitational transfer of energy from supernovae into the orbits of dark matter particles (e.g. Navarro et al. 1996; Governato et al. 2012; Oñorbe et al. 2015; Chan et al. 2015; Read et al. 2016). The resulting inner dark matter densities, in combination with strong environmental effects like tidal stripping and heating, have been invoked to alleviate the too-big-to-fail problem in the Milky Way (e.g. Zolotov et al. 2012; Brook & Di Cintio 2015; Sawala et al. 2016; Wetzel et al. 2016). Furthermore, accounting for observational biases influences the in-
terpretation of the severity of the dwarf abundance problem (in the Milky Way satellites e.g. Koposov et al. 2008; Kim et al. 2017 and in the field e.g. Brooks et al. 2017), as well as the too-big-to-fail problem in the field (e.g. Dutton et al. 2016; Verbek et al. 2017), and more recently the diversity problem of dwarf rotation curves (Santos-Santos et al. 2018).

Nevertheless, to this date, a comprehensive solution to all these CDM challenges remains elusive since there is yet no consensus on a definitive implementation of baryonic physics galaxy formation models. In particular, it is uncertain how to couple the sub-resolution physics of the supernova explosion with the effective energy injection into the interstellar medium, and ultimately into the dark matter distribution. This coupling between supernovae and dark matter, which is seemingly crucial in solving the CDM challenges, depends on the stellar mass content of the galaxy relative to the depth of the potential well (e.g. Peñarrubia et al. 2012; Di Cintio et al. 2014), which sets the energy requirements for the cusp-core transformation: the smaller the galaxy, the less likely it is that this transformation is viable. Another key ingredient is how efficiently the available energy is deposited in the interstellar medium, and on which timescales it is injected relative to the dynamical time scales of the inner regions of the halo. Large modifications to the dark matter distribution are associated with “bursty” star formation histories, which have been shown to prevail in galaxies with stellar masses in the range $10^8 - 10^{10} M_\odot$ (Kauffmann 2014), however, at lower masses, where the CDM challenges are more severe, the time resolution needed to settle this issue remains unachievable (Weisz et al. 2014).

Given these outstanding CDM challenges, it is sensible to consider the possibility that these problems actually have a root in the CDM assumptions about the underlying nature of dark matter. The fact that two of the most well-studied CDM candidates, the axion and the thermal WIMP (Weakly Interacting Massive Particle), have not been detected yet, gives further motivation to study CDM alternatives. In the context of structure formation theory, the two key assumptions of CDM where dark matter is assumed to be cold and collisionless, can be relaxed in the following two ways: (i) introduction of a galactic-scale cutoﬀ in the linear power spectrum in the early Universe, either through free streaming of dark matter particles, known as Warm Dark Matter, (WDM, e.g. Colin et al. 2000; Bode et al. 2001), or through interactions between dark matter particles and relativistic particles (known as interacting dark matter, e.g. Brehm et al. 2002; Buckley et al. 2014); (ii) allowing for strong dark matter self-interactions in the late Universe, known as Self-Interacting Dark Matter (SIDM, e.g. Spiegel & Steinhardt 2000).

We note that these two modifications can actually be present within the same particle model (e.g. van den Aarssen et al. 2012), and can be studied generically within the framework of a generalised theory of structure formation (see the ETHOS framework, Cyr-Racine et al. 2016; Vogelsberger et al. 2016; Lovell et al. 2018).

The CDM challenges can be alleviated to different degrees by these two modifications. For instance, a cutoff in the power spectrum reduces both the abundance of dark matter structures, and their inner densities, which helps in solving the missing dwarfs challenge (e.g. Zavala et al. 2009) and the too-big-to-fail problem (e.g. Lovell et al. 2012). In the context of thermal WDM, the relevance of this possibility as a solution is small due to constraints on thermal WDM particles from Ly-α forest observations (e.g. Viel et al. 2013; Irsic et al. 2017), which require $\sigma_{\rm WDM} \gtrsim 3.5 \text{keV}^{-2}$, which is too large to have a meaningful impact on the CDM challenges (e.g. $\sigma_{\rm WDM} \sim 2 \text{keV}^{-2}$ Schneider et al. 2014), although see Garzilli et al. (2017). In the context of interacting dark matter, dark-matter-only simulations have shown that this scenario substantially alleviates the missing satellite problem (Boehm et al. 2014), the too-big-to-fail problem (Vogelsberger et al. 2016), and the diversity of rotation curves problem for the smallest dwarfs (Vogelsberger et al. 2016, Moseley et al. 2018 in prep.). Since this model includes a complex cutoff in the power spectrum with dark acoustic oscillations, a detailed analysis with current Ly-α constraints remains to be done in order to assess how relevant these models remain for the CDM challenges (Bose et al. 2018 in prep.).

Without SN feedback, neither WDM nor interacting DM can result in the creation of dark matter cores. The inner density profiles have a lower normalisation compared to CDM, but the profile remains cuspy since there is no relevant mechanism to form a core (primordial thermal motions can set a maximum to the phase space density, hence a core, but the size of allowed cores is too small to be astrophysically significant, e.g. Macciò et al. 2012; Shao et al. 2013). SIDM is thus needed as an alternative mechanism to create dark matter cores, by transferring energy from the outside in, thermallyising the inner dark matter distribution (e.g. Colin et al. 2002). By now, this is a well understood process that has been shown to create $\mathcal{O}(\text{kpc})$ size cores in the centre of haloes with allowed transfer cross sections per unit mass $\sigma v m_\chi \sim 1 \text{ cm}^2 \text{g}^{-1}$. This is enough to alleviate the core-cusp problem and the too-big-to-fail problem (Vogelsberger et al. 2012; Rocha et al. 2013; Zavala et al. 2013), and it seemingly enhances the diversity of rotation curve shapes relative to CDM for a fixed baryonic physics implementation (Kamada et al. 2017; Creasey et al. 2017). Current constraints on the cross sections are however too strong for SIDM to have a relevant impact on the dwarf abundance problem (cross sections an order of magnitude larger than current limits are needed to evaporate substructure during halo-subhalo particle interactions). By now, the SIDM model has been studied extensively both within particle theories and within structure formation using numerical simulations, becoming an appealing alternative to the CDM model (for a recent SIDM review see Tulin & Yu 2017).

With a few exceptions (Todorki & Medvedev 2017 a,b), structure formation within the SIDM scenario has been restricted to elastic scattering only, i.e., during a collision between dark matter particles, kinetic energy is conserved. This does not have to be the case however, since well-motivated particle models exist that include “excited” dark matter states (e.g. Arkani-Hamed et al. 2009; Loeb & Weiner 2011; Schutz & Slatyer 2015). In the simplest two-state case, there is a ground ($\chi^0$) and an excited state ($\chi^+$); a transition from the lower to the upper level, up-scattering, absorbs energy (endothermic reaction), while down-scattering releases energy (exo thermic reaction). The latter case is especially interesting for structure formation since the preferential velocity kick imparted to the dark matter particles upon down-scattering from the centre of haloes diminishes the inner dark matter content, augmenting the efficiency of core creation relative to purely elastic SIDM. In addition, and contrary to the purely elastic case, interactions between halo and subhalo particles along the orbit of a subhalo, might lead to velocity kicks that are large enough to evaporate the subhalo, which might be a viable mechanism to reduce the abundance of dwarf galaxies. Similar velocity kicks have been studied in the case of late decays from an excited-state population, e.g. Wang et al. (2014), but unlike the decay rate, the scattering rate is velocity-dependent and enhanced in regions of high density. The magnitude of the velocity kick in the exothermic case depends on the exact energy splitting, $\delta$, between the ground state, with mass $m_{\chi^0}$ and the excited state, with mass $m_{\chi^+} \pm \delta v_{\text{kick}} = \sqrt{\delta m_{\chi}}$. The velocity
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2 INELASTIC SIDM MODEL

We aim for a numerical exploration of the effects of inelastic dark matter self-interaction on the evolution of structure of a galactic dark matter halo. This requires a simple dark matter particle physics model, which has a tractable amount of free parameters such that numerical studies can be used to sample the relevant parameter space. Here, we use the two-state dark matter model of self-scattering including a nearly-degenerate excited state that has been presented in Schutz & Slatyer (2015). This model is based on accurate analytic approximations for the corresponding elastic and inelastic s-wave cross sections, which are valid outside the perturbative regime provided the particle velocity is sufficiently low.

The underlying particle physics model is characterised by four parameters: the mass splitting $\delta$ between ground ($\chi^1$) and excited ($\chi^2$) state, the coupling constant $\alpha$ between the dark matter particle ($\chi^1$-$\chi^2$) and the mediator ($\phi$), the mass of the dark matter particle $m_{\chi^1,2}$, and the mass of the force mediator $m_\phi$. We note that although this is clearly not the only well-motivated dark matter model with inelastic self-scattering, it is the only such model where the description of scattering at the velocity scales of dwarf galaxies has proven analytically tractable thus far. Furthermore, the involved Yukawa potential has proven ubiquitous in different areas in particle physics, rendering this model a useful representative example for studies of inelastic dark matter self-scattering more generally.

Small mass splittings between states, similar to those we consider, could also be present in the context of atomic dark matter (e.g. Kaplan et al. 2010; Fan et al. 2013; Foot & Vagnozzi 2015; Choquette & Cline 2015; Ghalasai & McQuinn 2018), non-Abelian dark sectors where the dark matter is part of a nearly-degenerate multiplet of states (e.g. Chen et al. 2009; Cirelli & Cline 2010), or in scenarios where the DM forms stable bound states (e.g. Wise & Zhang 2014). However, it should be noted that the scenario we consider requires that transitions from the heavier to the lighter state occur primarily through collisional de-excitation; it is necessary but not sufficient to have small mass splittings. In models that include a very light force carrier, such as classic dark atom models, the excited states are typically depleted by radiative decay rather than collisions, leading to different effects on structure formation (e.g. Wang et al. 2014). Our calculation has other qualitative features that are not universal to dark-sector models with small mass splittings: for example, that there are only two states that significantly participate in the phenomenology, and that de-excitation requires two excited-state particles to simultaneously de-excite. Thus while this model serves as a valuable illustrative example of the effects of inelastic self-scattering, detailed quantitative results for other dark-sector models would generally require dedicated analyses.

Schutz & Slatyer (2015) found that at the velocity scale of $\sim 10 \text{ km s}^{-1}$, relevant for the dynamics of dwarf galaxies, the range of particle physics parameters that result in interesting cross sections are: $m_{\chi^1} \in [0.1, 300] \text{ GeV c}^{-2}$, $m_\phi \in [10^{-4}, 1] \text{ GeV c}^{-2}$, $\alpha \in [10^{-3}, 10^{-1}]$, $\delta \in [0.1, 10] \text{ keV}$. We select one particular model from this range with $\delta = 10 \text{ keV}$, $m_{\chi^1} = 10 \text{ GeV c}^{-2}$, $m_\phi = 30 \text{ MeV}$ and $\alpha = 0.1$ (see top panel of Fig. 1 for a schematic representation). This choice results in

![Figure 1. Inelastic self-interacting dark matter model. Top panel: Schematic overview of the two-state inelastic SIDM model. The ground state ($\chi^1$) and excited state ($\chi^2$) are split by $\delta = 10 \text{ keV}$. This two-state model allows for exo- and endothermic reactions. Bottom panel: Self-scattering cross section per unit mass for the different reactions of the two-state inelastic dark matter model. Up-scattering ($\chi^1 + \chi^1 \rightarrow \chi^2 + \chi^2$; purple) is suppressed in the Milky Way environment since it can only occur for large relative velocities, $v > 2 \sqrt{2\delta/m_{\chi^1}} \approx 848 \text{ km s}^{-1}$. The elastic cross sections ($\chi^1 + \chi^1 \rightarrow \chi^1 + \chi^1$, red) and ($\chi^2 + \chi^2 \rightarrow \chi^2 + \chi^2$, blue) are nearly flat over the whole velocity range relevant for the Milky Way environment. Down-scattering ($\chi^2 + \chi^2 \rightarrow \chi^1 + \chi^1$, green) mainly occurs for rather low velocities ($v < 10 \text{ km s}^{-1}$). This reaction is exothermic corresponding to a velocity kick of $\sqrt{2\delta/m_{\chi^1}} \approx 424 \text{ km s}^{-1}$. The largest cross sections occur for mixed-state elastic scattering ($\chi^1 + \chi^2 \rightarrow \chi^2 + \chi^1$, black), with a normalisation that exceeds $\sim 10 \text{ cm}^2 \text{ g}^{-1}$ for low velocities.](image)
an elastic cross section per unit mass\(^1\) of a few \(\text{cm}^2 \text{g}^{-1}\) at the scale of dwarf galaxies, while the velocity kick is of the order of \(v_{\text{kick}} \sim 424 \text{ km s}^{-1}\). We remark that this model has not been fine-tuned, and only represent a benchmark point in the relevant parameter space. The model has five different reactions and corresponding cross sections, which are presented in the bottom panel of Fig. 1: elastic scattering in the excited state \((\chi^4 \rightarrow \chi^1 + \chi^1)\), elastic scattering in the excited state \((\chi^1 \rightarrow \chi^1 \rightarrow \chi^1 + \chi^1)\), endothermic up-scattering \((\chi^1 \rightarrow \chi^1 
rightarrow \chi^3 + \chi^3)\), exothermic down-scattering \((\chi^3 \rightarrow \chi^3 
rightarrow \chi^1 + \chi^1)\), and elastic Yukawa scattering \((\chi^1 + \chi^3 \rightarrow \chi^1 + \chi^1)\). We note that our benchmark point practically forbids up-scattering for the typical velocities of dark matter particles in the Milky Way environment, since the required energy splitting is too large for this reaction to occur frequently. This can be seen in the bottom panel of Fig. 1, where the cross section for up-scattering is zero for relative velocities \(v < 2 \sqrt{2} \gamma / m_{\chi} \approx 848 \text{ km s}^{-1}\).

Besides specifying the model parameters, we also have to specify the initial conditions for the abundance of dark matter species; i.e., what fraction of dark matter is in which state initially – ground state \((\chi^1)\) or excited state \((\chi^3)\). Obviously, putting all particles initially into the excited state will maximise the effect of energy release during structure formation. On the other hand, putting all particles initially in the ground state will do the opposite, and behave like a purely elastic SIDM model with multiple different cross sections. To explore the relevant range for inelastic SIDM, we consider in the following two initial configurations: all particles initially at \(z = 127\) as described below, in the excited state \((\chi^\text{init} = 100\%)\), or half the particles initially in the excited state \((\chi^\text{init} = 50\%)\).

We note that in the generic framework of the model we are considering here, the presence of a potential mediated by light vector exchange automatically implies that both the ground and excited states can annihilate into the light vectors. For the large dark-sector coupling \((\alpha = 0.1)\) and relatively light DM mass \((10 \text{ GeV} c^{-2})\) considered in this work, the natural cross section for this process is \(\langle \sigma v \rangle = \pi \alpha^2 / m_{\chi}^2 \sim 3 \times 10^{-21} \text{ cm}^3 \text{s}^{-1}\), which is much larger than the thermal relic cross section, implying a depletion of the dark matter states in the early Universe. To justify a large primordial abundance of the excited state, a non-thermal origin for these species could be invoked, which could occur through the decay of thermally produced heavier species, preferentially into the excited state (as discussed in Loeb & Weiner 2011). In order to have \(\sim 100\%\) of all particles in the excited state by the starting redshift of the simulations \(z = 127\), it is necessary to suppress down-scattering (or delay the production of the excited state) until around the epoch of recombination \((z = 1100)\).

If instead, down-scattering is allowed already by the time of matter-radiation equality \((z_{\text{eq}} \sim 3400)\), then we can make a simple estimate of the ratio of excited to ground states by the time the simulation starts at \(z_{\text{sim}} = 127\).

\[
\chi = 1 - \int_{z_{\text{eq}}}^{z_{\text{sim}}} \frac{\Gamma(z)}{H(z)(1+z)} dz
\]

where \(H(z)\) is the Hubble expansion rate and \(\Gamma(z)\) is the scattering rate for de-excitation:

\[
\Gamma(z) = \rho_{\text{dm}}(z) v_{\text{typ}}(z) \sigma_{\chi^3}(v_{\text{typ}}(z))/m_{\chi}
\]

where \(v_{\text{typ}}(z)\) is the characteristic velocity of dark matter particles, which at redshifts prior to \(z = 127\) is roughly equal to the velocity dispersion of unclustered dark matter particles \(v_{\text{typ}} \sim T(z)/(m_{\chi} T_{\text{kd}})\)\(^{1/2}\) where \(T(z)\) is the radiation temperature and \(T_{\text{kd}}\) is the dark matter kinetic decoupling temperature. After decoupling, the dark matter temperature and radiation temperature scale as \(T_{\chi} = T^2 / T_{\text{kd}}\). We fix \(T_{\text{kd}} = 10 \text{ MeV}\), but note that the dependence of \(\chi\) on \(T_{\text{kd}}\) is only mild. The cross section for de-excitation \(\sigma_{\chi^3}(v)/m_{\chi}\) is extrapolated down to the very small typical velocities of dark matter particles at early redshifts, from the behaviour in the bottom panel of Fig. 1. This calculation results in \(\chi \sim 0.76\).

If the excited state is populated and down-scattering is unsuppressed at times prior to matter-radiation equality, but after the dark matter temperature drops below the mass splitting, then down-scattering could deplete the excited-state population by a larger factor (Loeb & Weiner 2011).

Plausible models with late-time decays would need to avoid a series of constraints in the early Universe, for instance in their gravitational effects on the cosmic microwave background radiation (e.g., Poulin et al. 2016) and large scale structure. Such constraints may point toward models where the metastable decaying species is only slightly heavier than the dark matter. Since our focus in this paper is on the implications for structure formation of a large metastable population of excited states in the early Universe, we leave a detailed study of possible models for later work. Models with large late-time populations of a metastable excited state have been previously discussed in the context of indirect-detection and direct-detection signals (e.g. Finkbeiner et al. 2009; Cline et al. 2011).

3 NUMERICAL IMPLEMENTATION IN AREPO

Although we study in this paper a specific two-state inelastic SIDM model, we have implemented a more general multi-state dark matter framework into the AREPO code (Springel 2010). This framework is able to handle an arbitrary number of states with an arbitrary number of reactions and corresponding cross sections, and with arbitrary, also non-degenerate, energy level splittings. This code represents a generalisation and complete rewrite of the algorithm presented in Vogelsberger et al. (2012) and Vogelsberger et al. (2016), which has been employed in multiple previous SIDM studies (Vogelsberger & Zavala 2013; Vogelsberger et al. 2014b; Vegetti & Vogelsberger 2014; Dooley et al. 2016; Brinckmann et al. 2018; Sameie et al. 2018). Here we briefly describe this new numerical implementation.

In the following we assume that each dark matter simulation particle \(i\) is in a specific state \(\alpha\), i.e., every simulation particle represents a single state and not a mixture of different states. The simulation volume is then filled with dark matter simulation particles in different states \((\alpha, \beta, \gamma, \delta)\) with a variety of possible two-body scatterings:

\[
\chi_i^{\alpha} + \chi_j^{\beta} \rightarrow \chi_i^{\gamma} + \chi_j^{\delta},
\]

where two particles, \(i\) and \(j\), with initial states \(\alpha\) (particle \(i\)) and \(\beta\) (particle \(j\)), are scattered from state \(\alpha\) to state \(\gamma\) of the other particle.

\(^1\) We remark that for the model considered, the relevant cross section, which is the transfer cross section is the same as the total cross section given the lack of angular dependence of the differential cross section, see Section B3.3 of Schutz & Slatyer (2015). Hereafter we use the terms transfer cross section and cross section interchangeably.

\(^2\) In Eq. 25 of Feng et al. (2010), this choice of \(T_{\text{kd}}\) would correspond to a kinetic mixing parameter \(\epsilon \sim 3 \times 10^{-7}\).
(particle $j$) scatter into two new states $\gamma$ (particle $i$) and $\delta$ (particle $j$). The particle states have masses $m_i^\alpha$, $m_j^\beta$ before the scattering and $m_i^\gamma$, $m_j^\delta$ after the scattering. We note that these are simulation particle masses, whereas the actual dark matter particle masses are $m_\chi$, with $\epsilon = (\alpha, \beta, \gamma, \delta)$, depending on the state. The velocity-dependent transfer cross section for reaction $\alpha\beta \rightarrow \gamma\delta$ is given by

$$\sigma_T^{\alpha\beta\rightarrow\gamma\delta}(v_{\alpha\beta}) = \sigma^{\alpha\beta\rightarrow\gamma\delta}(v^{\alpha\beta}),$$

(4)

where we take the thermal average of the product of the cross section times the relative velocities between particles. Here $\rho^{\alpha\beta}$ measures the local density of dark matter particles in state $\beta$. We convert this mass density to a number density by dividing it by $m_\chi$, the mass of the dark matter particle in state $\beta$. During a scattering reaction an energy $\Delta E^{\alpha\beta\rightarrow\gamma\delta}$ is released (exothermic) or absorbed (endothermic):

$$\Delta E^{\alpha\beta\rightarrow\gamma\delta} \begin{cases} \geq 0, & \text{elastic}, \\ < 0, & \text{inelastic : endothermic}, \\ > 0, & \text{inelastic : exothermic}. \end{cases}$$

(5)

Once particle $i$ is in state $\alpha$ and $j$ in state $\beta$ have been selected to scatter into states $\gamma$ and $\delta$, we perform the scattering in the centre of mass frame and assign new velocities for the particles after the scattering:

$$v_i = m_i^\alpha + m_j^\beta - m_i^\gamma - m_j^\delta = m_i^\gamma + m_j^\delta - m_i^\alpha - m_j^\beta$$

$$v_j = m_i^\alpha + m_j^\beta - m_i^\gamma - m_j^\delta = m_i^\gamma + m_j^\delta - m_i^\alpha - m_j^\beta,$$

(6)

where $v_{cm}$ is the centre of mass velocity of the two particles, $v_{ij}$ their relative velocity, $\hat{e}$ is a random vector on the unit sphere, and $\sigma^{\alpha\beta\rightarrow\gamma\delta}$ is a dimensionless velocity scale factor that depends on the energy splitting related to the reaction the two particles are undergoing:

$$\sigma^{\alpha\beta\rightarrow\gamma\delta} = \frac{\rho^{\alpha\beta}}{\rho^{\gamma\delta}} \left( 1 + \frac{2\Delta E^{\alpha\beta\rightarrow\gamma\delta}}{\mu^{\alpha\beta} \mu^{\gamma\delta}} \right),$$

(7)

where $\rho^{\alpha\beta}$ and $\rho^{\gamma\delta}$ are the reduced masses of the two particles before and after the scattering event; i.e., we take into account the change in particle mass during the scattering. We note however, that this change is usually tiny in the cases of interest where the mass splitting is small. For example, for our nearly-degenerate model, it is negligible ($10^{-6}$) such that the two reduced masses are essentially given by $1/2 m_\alpha$, assuming a constant dark matter simulation particle mass $m_\alpha$. Based on the three cases for $\Delta E^{\alpha\beta\rightarrow\gamma\delta}$ we find accordingly for the dimensionless velocity scale factor:

$$0 \leq \sigma^{\alpha\beta\rightarrow\gamma\delta} \begin{cases} = 1, & \text{elastic}, \\ < 1, & \text{inelastic : endothermic}, \\ > 1, & \text{inelastic : exothermic}. \end{cases}$$

(8)

In the inelastic case this energy is either given or taken from the two scattering particles in equal parts. The endothermic regime is limited by the fully inelastic collision ($\sigma^{\alpha\beta\rightarrow\gamma\delta} = 0$) after which both particles move with the centre of mass velocity. The exothermic case is not limited and can in principle inject an arbitrary amount of energy into the system depending on the energy level splitting.

To decide whether a certain scattering reaction occurs between two simulation particles, we have to evaluate the corresponding scattering probabilities. The pairwise scattering probability for a given reaction $\alpha\beta \rightarrow \gamma\delta$, and the total probability for scattering of a given particle with any of its neighbours are given respectively by:

$$P^{\alpha\beta\rightarrow\gamma\delta}_{ij} = \delta^{\alpha\beta} \rho^{\gamma\delta} W_{ij} \left( \frac{\sigma^{\alpha\beta\rightarrow\gamma\delta}}{\mu^{\alpha\beta} \mu^{\gamma\delta}} \right) v_{ij} \Delta t_i,$$

(9)

$$P^\alpha_{i} = \sum_{j=0}^{N_{ngh}} P^{\alpha\beta\rightarrow\gamma\delta}_{ij},$$

(10)

where $N_{ngh}$ is the number of neighbor particles, $\Delta t_i$ the individual...
time-step of particle \(i\), and \(W_{ij} = w(r_{ij}/h_i, h_i)\) is the cubic spline Kernel function with a 3D normalisation:

\[
w(q, h) = \frac{8}{\pi h^3} \left\{ \begin{array}{ll}
1 - 6q^2 + 6q^3, & 0 \leq q < \frac{1}{3}, \\
2(1 - q)^3, & \frac{1}{3} \leq q < 1, \\
0, & q \geq 1.
\end{array} \right.
\]

(11)

Here \(h_i\) is the smoothing length including the \(N_{\text{ngb}}\) neighbor particles \(j\) around particle \(i\) with spatial distance \(r_{ij}\). We note that \(N_{\text{ngb}}\) does not distinguish the particle states; i.e., it is possible that particles of a certain state are not enclosed in the smoothing length. In this case scattering between the particle in question with this state is not possible. The factor \(1/2\) in the calculation of \(P_{\alpha\beta}^{\gamma\delta} = \delta_{\gamma\delta}\) accounts for the fact that a scatter event always involves two particles, and we therefore need to divide by two to reproduce the correct scattering rate. A given neighbour only contributes to the sum if the initial states of the reaction match the actual particle states. This is guaranteed by the Kronecker delta function \(\delta_{\gamma\delta}\), which is equal to 1 if particle \(i\) is in initial state \(\alpha\) and particle \(j\) is in initial state \(\beta\). Otherwise the function evaluates to zero.

To decide whether a particle is going to scatter and with which reaction, we first arbitrarily order the reactions according to \(\Delta t_i < \min_{\alpha, \beta, \gamma, \delta}(\rho_i^\alpha \sigma_{T,\gamma\delta}^{\alpha\beta} \sigma_i^\alpha / m_{\chi,\alpha} \sigma_i^\alpha)^{-1}\),

\[
\Delta t_i < k \min_{\alpha, \beta, \gamma, \delta}(\rho_i^\alpha \sigma_{T,\gamma\delta}^{\alpha\beta} \sigma_i^\alpha / m_{\chi,\alpha} \sigma_i^\alpha)^{-1},
\]

(14)

where \(\rho_i^\alpha\) is the density of particles in state \(\alpha\) at the location of particle \(i\), and \(\sigma_i^\alpha\) is the corresponding velocity dispersion. \(k\) is a dimensionless parameter that can be adjusted to control the size of the minimal time-step. For our simulations we find that \(k = 0.0025\) is sufficient to avoid multiple scattering and usually sets the time-step to be smaller than the local dynamical time scale in the simulations.

We test our inelastic SIDM implementation by setting up an isolated Hernquist (1990) halo in equilibrium with \(M_{200} = 10^{10}\, M_\odot\) and a concentration\(^3\) of \(c = 8\) for the benchmark dark matter model presented above. The halo is sampled with \(10^7\) particles and gravitational forces are softened with a Plummer-equivalent softening length of \(\epsilon = 10\, \text{pc}\). We simulate this halo in three different dark matter models: CDM, elastic SIDM and inelastic SIDM. The CDM case just follows the evolution of collisionless CDM, whereas the other two cases employ the two-state model with the cross sections discussed above (see bottom panel of Fig. 1) using the numerical implementation described in the previous section. The elastic simulation artificially suppresses the energy release during (de-)excitations, but is otherwise identical to the inelastic SIDM model. This simulation can then be compared to the inelastic case, where velocity kicks play a relevant role. To be more specific, the elastic case is realised by simply setting \(\gamma_{\chi\chi'\gamma\delta} = 1\) during any scattering process, i.e., \(\Delta E_{\chi\chi'}\gamma\delta = 0\). As mentioned above, we explore two different initial configurations for the non-CDM models. In the first configuration all particles are initially in the excited state (\(\chi_{\chi\chi'} = 100\%\)), while in the second only half of the particles are initially in the excited state (\(\chi_{\chi\chi'} = 50\%\)).

In Fig. 2 we present the time evolution of the population split in the halo for the two initial conditions for the inelastic SIDM case, in the left and right panels, respectively. The right axis in each panel indicates the total injected energy into the system (solid white lines) in units of the canonical SNII energy (10\(^{51}\, \text{erg}\)). This measures the total energy released due to de-excitation that is transformed into kinetic energy by the cumulative effect of velocity kicks in each scattering event. As expected, in both cases the ground state population increases over time while the excited state gets de-populated. As mentioned above, this is a consequence of the employed cross sections, where up-scattering is strongly suppressed (see bottom panel of Fig. 1), and does not occur given the typical relative velocities of particles in a 10\(^{10}\, M_\odot\) halo. After a few Gyr, the exothermic reaction has already injected the equivalent of more than one million SNII for the \(\chi_{\chi\chi'} = 100\%\) configuration. After 10 Gyr, the cumulative energy injection of down-scatterings has reached \(\sim 2 \times 10^{57}\, \text{erg}\) of energy into the system for the \(\chi_{\chi\chi'} = 100\%\) configuration, and \(\sim 7 \times 10^{56}\, \text{erg}\) for the \(\chi_{\chi\chi'} = 50\%\) configuration. The ground state population increases from 6% to 10% for the \(\chi_{\chi\chi'} = 100\%\) initial configuration over the simulation time span of 10 Gyr. Similarly also the ground state population for the \(\chi_{\chi\chi'} = 50\%\) initial configuration increases as a function of time, although the total increase is lower in this case. As a reference, we note that the minimum energy required to transform a cusp into a 1 kpc core for a Navarro-Frenk-White (NFW) halo with a mass of 10\(^{10}\, M_\odot\) calculated from the virial theorem assuming initial and final equilibrium configuration is \(\sim 10^{55}\, \text{erg}\) (Peñarrubia et al. 2012). Depending on the stellar mass content, efficiency of energy injection, and star formation history of a given galaxy living in a halo of this size, this energetic requirement might or might not be satisfied by the SNe-driven core formation scenario (e.g. Peñarrubia et al. 2012; Amorisco et al. 2014; Maxwell et al. 2015). In the inelastic SIDM model explored here the cumulative energy available easily exceeds this minimum energy requirement.

\(^3\) Defined as the ratio of \(r_{200}\), the radius where the mean density of the halo is equal to 200 times the critical density, and \(r_{-2}\), the radius where the logarithmic slope of the density profile is equal to \(-2\).
Table 1. Basic properties of the Milky Way-sized halo simulated in the different dark matter models. We list the virial mass ($M_{200}$), virial radius ($r_{200}$), maximum circular velocity ($V_{\text{max}}$), radius where the maximum circular velocity is reached ($R_{\text{max}}$), the number of resolved subhaloes within 300 kpc ($N_{\text{sub}}$), the ground state population of all dark matter particles within 300 kpc, and the injected energy (in units of $10^{51}$ erg), i.e., the canonical energy of a SNIa measured based on the population split between the two dark matter states at all high resolution particles at $z = 0$.

| Model         | $M_{200}$ [10^{12} M_\odot] | $r_{200}$ [kpc] | $V_{\text{max}}$ [km s^{-1}] | $R_{\text{max}}$ [kpc] | $N_{\text{sub}}$ | ground state population [%] (r < 300 kpc) | injected energy [SNIas ($10^{51}$ erg)] |
|---------------|-----------------------------|----------------|-----------------------------|-----------------------|----------------|------------------------------------------|-----------------------------------------|
| CDM           | 1.600                       | 243.85         | 174.87                      | 66.14                 | 16,264        | --                                       | --                                      |
| elastic SIDM ($\chi^2_{\text{init}} = 100\%$) | 1.600                      | 243.39         | 178.12                      | 62.22                 | 14,486        | 7.51                                     | --                                      |
| elastic SIDM ($\chi^2_{\text{init}} = 50\%$) | 1.600                      | 243.38         | 177.51                      | 63.39                 | 14,675        | 52.23                                    | --                                      |
| inelastic SIDM ($\chi^2_{\text{init}} = 100\%$) | 1.478                      | 237.05         | 164.14                      | 77.21                 | 13,171        | 0.65                                     | 755 $\times$ 10^6                      |
| inelastic SIDM ($\chi^2_{\text{init}} = 50\%$) | 1.569                      | 241.81         | 172.44                      | 64.76                 | 14,409        | 51.12                                    | 268 $\times$ 10^6                      |

The mass splitting, which corresponds to a velocity kick of 424 km s^{-1} is sufficient to unbind particles, i.e., to efficiently remove dark matter particles from the 10^{13} M_\odot halo. This is demonstrated in Fig. 3, where we show the dimensionless energy distribution of all gravitationally unbound particles after 10 Gyr. The CDM case (black lines) has a very small fraction of gravitationally unbound particles $O(0.01\%)$, caused by numerical noise over the equilibrium configuration. This population serves as a comparison baseline with the other models. On the other hand, the inelastic model (red lines) has a much larger fraction of gravitationally unbound particles ($\sim 5\% - 10\%$), with a significant number of them populating a nearly log-normally distributed peak at high energies. These are the particles that were predominantly expelled from the halo centre during the velocity kicks imparted in downscatterings. Notice that this population is absent in the elastic case (blue lines), which shows a very similar distribution as the CDM case, but shifted towards higher energies and with a higher fraction of gravitationally unbound particles, of $O(1\%)$. This distinct population of gravitationally unbound particles in the inelastic case is key to understand the further reduction of the core density that happens in this case compared to pure elastic SIDM.

We can roughly estimate the energy shifts in the elastic and inelastic SIDM cases. For the elastic case we assume that the shift seen between the blue and black lines is related to the energy gained by the particles that were barely bound in the inner halo, and that become gravitationally unbound after elastic scattering with particles with more kinetic energy. Since scattering is more frequent deep within the potential, this energy gain is approximately bounded by the the r.m.s. velocity at the maximum of the velocity dispersion profile, which occurs at $r_{\sigma_{\text{max}}} \sim a/5$, where $a$ is the scale radius of the Hernquist profile. In Vogelsberger et al. (2012) we found that the r.m.s. velocity at the core is $\sigma_{\text{rms}}(r_{\sigma_{\text{max}}}) \sim 3\sqrt{\gamma}(r_{\sigma_{\max}}) \sim 0.96 \times GM_{200}/a$ (see Fig. 2 of Vogelsberger et al. 2012). This means that those particles that were barely bound become gravitationally unbound gaining an additional kinetic energy $0.48 \times GM_{200}/a \sim 3.42 \times GM_{200}/r_{200}$, which is roughly 0.5 dex to the right, relative to the CDM distribution where this effect is absent. We note that our analytic estimate is an oversimplification given the radially dependent redistribution of energy. For the inelastic case the elastic collisions still cause a similar shift as in the elastic case since the reaction ($\chi^4 + \chi^2 \rightarrow \chi^4 + \chi^2$) is very frequent (it has the highest cross
section). In addition, during down-scatterings, the resulting ground state particles receive a velocity kick $\sqrt{2E/m}$, which results in a gain in kinetic energy of $\sim 74GM/200$, which is shown in Fig. 3 as a vertical dashed line. This estimate describes the numerical result well, demonstrating that the numerical implementation behaves correctly.

### 4 IMPACT OF INELASTIC SIDM ON A MILKY WAY-LIKE HALO

Next we explore the impact of inelastic SIDM on a galactic halo in a cosmological context. We resimulated the galactic halo ($M_{200} = 1.6 \times 10^{12} M_\odot$) presented in Vogelsberger et al. (2016) with inelastic SIDM for the benchmark model presented above with two different initialisations, $\chi_{\text{init}} = 100\%$ and $\chi_{\text{init}} = 50\%$. The simulations employ the following cosmological parameters: $\Omega_m = 0.302$, $\Omega_\Lambda = 0.698$, $h = 0.69$, $\sigma_8 = 0.839$ and $n_s = 0.967$, which are consistent with recent Planck data (Planck Collaboration et al. 2014; Spergel et al. 2015). The initial conditions are generated at $z = 127$. The gravitational softening length is fixed in comoving coordinates until $z = 9$, and is then fixed in physical units until $z = 0$. The dark matter particle mass resolution is $2.756 \times 10^4 M_\odot$ with a Plummer-equivalent softening length of 72 pc at $z = 0$. A convergence study is presented in Appendix A.

The main properties of this halo are presented in Table 1 for the four different SIDM models, elastic and inelastic with the two different initial state populations explored in this paper. We can see that the virial mass, $M_{200}$, is reduced by nearly $10\%$ for the inelastic model with initially all particles in the excited state ($\chi_{\text{init}} = 100\%$). This mass loss is a consequence of unbinding ground state particles after down-scattering. We also find a rather significant reduction in $V_{\text{max}}$ of about $\sim 8.5\%$. The total number of resolved subhaloes within $300 \text{ kpc}$ is also reduced by $\sim 23\%$ for the inelastic model with $\chi_{\text{init}} = 100\%$. Here we count all subhaloes that are found by our structure finder (Springel et al. 2001) with more than 20 particles corresponding to a lower mass limit of $5.512 \times 10^5 M_\odot$. We note that there is a significant reduction in the abundance of subhaloes in both the elastic and inelastic SIDM model. The fact that this also happens for the elastic model is caused by the rather large elastic cross sections that we employ in our benchmark model (see bottom panel of Fig. 1). In particular, the cross section for the $(\chi^2 + \chi_2 \rightarrow \chi_1 + \chi^2)$ reaction reaches values larger than $10 \text{ cm}^2 \text{ g}^{-1}$ for low relative velocities, and it is around $5 \text{ cm}^2 \text{ g}^{-1}$ at relative velocities $\sim 200 \text{ km s}^{-1}$, which are the relevant ones for subhalo evaporation. For instance, in Vogelsberger et al. (2012) it was demonstrated that an elastic cross section of $\sim 10 \text{ cm}^2 \text{ g}^{-1}$ on galactic scales leads to a significant suppression of substructure. As anticipated, this reduction in the abundance of subhaloes is clearly enhanced if inelastic scattering is included.

As is clear from Table 1, qualitatively similar trends are observed for the $\chi_{\text{init}} = 50\%$ configuration. The second-to-last column of Table 1 shows the present-day population of ground state particles within a radius of 300 kpc from the galactic centre. We find that for the inelastic model the ground state population increases to $\sim 0.7\%$ for the $\chi_{\text{init}} = 100\%$ initial configuration and to $\sim 51.1\%$ for the $\chi_{\text{init}} = 50\%$ initial configuration. On the other hand, for the elastic case we find larger fractions of particles in the ground state at $z = 0$ than in the inelastic case, $\sim 7.5\%$ and $\sim 52.2\%$ for the $\chi_{\text{init}} = 100\%$ and $\chi_{\text{init}} = 50\%$ configurations, respectively. This is because the elastic configuration has been set on purpose to have the same reactions as the inelastic benchmark case, but without the energy release (see Section 3), i.e., there is no velocity kick associated to down-scattering, which means that the associated ground state particles remain bound to the halo. For the inelastic case, we provide in the last column of the table the cumulative injected energy due to de-exitations. For the fully excited initial configuration we find that a total of $\sim 8 \times 10^9 \times 10^{41} \text{ erg}$ are injected. This value is a factor of a few lower ($\sim 3 \times 10^9 \times 10^{41} \text{ erg}$) for the configuration where only $50\%$ of the particles are initially in the upper state. The redshift dependence of the injected energy is presented in Fig. 4. The magnitude of these energies suggest that inelastic SIDM can have a substantial impact on the galactic halo, both in terms of abundance of substructure, and their density structure.

The projected dark matter density distribution of the simulated halo for a CDM universe is shown in the left panel of Fig. 5, while the inelastic SIDM model with $\chi_{\text{init}} = 100\%$ is shown in the right panel. These plots clearly show that in this specific inelastic SIDM model, both the central (sub)halo densities and subhalo abundance are significantly reduced. This latter property is quite distinct from typical elastic SIDM models where a relevant difference in the subhalo abundance relative to CDM is only possible for rather large elastic cross sections on galactic scales ($\sim 10 \text{ cm}^2 \text{ g}^{-1}$).

The inelastic SIDM model with $\chi_{\text{init}} = 100\%$ represents the most extreme scenario of all our simulations. To show the larger diversity of the non-CDM cases we explored, we present the corresponding maps of all of them in the four top panels of Fig. 6. These four panels show the total dark matter density, i.e., taking into account the ground and excited states. Overall, we see a consistent trend on the abundance of substructure being more suppressed in the inelastic cases compared to the elastic ones. The effect is however, considerably stronger in the case where all particles are initially in the excited state (the two leftmost panels are visually distinct, while the rightmost panels are more alike). In the middle and bottom panels of Fig. 6 we show the density maps but only considering particles in the ground state, and excited state, respectively.

The inelastic model with $\chi_{\text{init}} = 100\%$ has a very low ground state density. De-excited particles escape from the halo centres in that model due to the strong velocity kicks. For the associated elas-
Evaporating the Milky Way with inelastic SIDM

Figure 5. Projected dark matter density for the Milky Way-sized halo in CDM and for inelastic SIDM. The left panel shows the CDM case, whereas in the right panel we present the inelastic SIDM case where 100% of the particles are initially in the excited state ($\chi_{\text{init}}^2 = 100\%$). The spatial extent of the maps is 500 kpc across with a depth of 300 kpc, with a colour scale related to the projected value of $\rho^2$. It is apparent that the inelastic SIDM model leads to reduced core densities and an overall reduction in the abundance of subhaloes due to down-scattering. The virial mass for the halo in the inelastic case is reduced by about 10% compared to the CDM case due to the removal of dark matter particles in the ground state following de-excitation. The efficient removal of ground state particles leads to a very small fraction, $\lesssim 1\%$, of such particles within 300 kpc.

tic case, these particles are not removed and therefore stay near the halo centres and are visible in the maps. A visual comparison between the two leftmost panels of Fig. 6 shows very clearly the striking difference between elastic and inelastic SIDM models. For the $\chi_{\text{init}}^2 = 50\%$ configuration, the differences between the ground and excited state populations, and between the elastic and inelastic cases are barely visible in the maps since the increase of ground state particles due to de-excitations is only at the per cent level.

To quantify in detail the distribution of the two-state population in the SIDM models, we show in the top panels of Fig. 7 the radial profiles of the ground state population centered in the Milky Way-sized halo. The differences between the central distributions of the ground and excited state populations are striking, particularly for the case where all particles are initially in the excited state ($\chi_{\text{init}}^2 = 100\%$, left panel). As mentioned above this is due to the large energy imparted to the ground state particles during down-scattering, which is large enough to remove these particles from the halo. These removed particles are actually visible in Fig. 7 far away from the halo centre in the differential ground state population distribution. They appear as peaks in this distribution around and beyond 1 Mpc from the halo centre. Since the velocity kick in our benchmark model is $424 \text{ km s}^{-1}$, the particle would travel $\sim 1 \text{ Mpc}$ already within 3 Gyr. For the elastic case on the other hand, the ground state population rises towards the centre since the down-scattering rate is higher towards the denser and colder halo centre (see bottom panel of Fig. 1), unimpeded due to the lack of energy release. Here we find that the ground state population reaches more than 20% towards the centre of the halo. Even at a distance of $\sim 100 \text{ kpc}$ we still find a cumulative fraction of $\sim 10\%$ of all particles in the ground state for the elastic mode, whereas for the inelastic case, this fraction is much smaller and less than a per cent even out to $\sim 1 \text{ Mpc}$. The right panel of Fig. 7 shows the profiles for the case where 50% of all particles are initially in the excited state ($\chi_{\text{init}}^2 = 50\%$). We find here a similar trend when comparing the elastic and inelastic cases, i.e., the ground state is more densely populated for the elastic case in the inner halo. For the elastic case about 55% of all particles in the centre are in the ground state. Lastly, we note that the cumulative fraction of ground state particles within $\sim 10 \text{ Mpc}$ agrees between the elastic and inelastic cases for both initial state configurations. This population is at the per cent level above the initial ground state population. This radial distance is large enough to include even those particles that were ejected due to de-excitation in the inelastic SIDM model, and thus, the ground state level populations converge to similar values at large radii. Any remaining deviations are due to differences in the detailed scatter reactions that occurred during the assembly history of the halo.

The lower panels of Fig. 7 show the mass-weighted average number of scattering events per particle in each reaction channel. For the $\chi_{\text{init}}^2 = 50\%$ configuration (right panel) we find that the average number of scattering events follows closely the overall ranking of the cross sections presented in the bottom panel of Fig. 1. For example, the mixed elastic reaction ($\chi^+ + \chi^2 \rightarrow \chi^+ + \chi^2$) has the largest cross section of all channels and, correspondingly, it also contributes to the largest number of average scattering events at all radii. The exothermic reaction ($\chi^2 + \chi^2 \rightarrow \chi^4 + \chi^2$) on the other hand, has the smallest cross section, except for very small relative velocities, and therefore leads to the smallest number of scattering events. The inelastic model shows a lower number of scattering events along that channel (long-dashed red lines) compared to the elastic case (long-dashed blue lines), because of the removal of particles once they de-excite into the ground state. The situation
Figure 6. Projected dark matter density for the total dark matter density, the lower level, and the upper level densities for the SIDM models. The total extent of the maps is 500 kpc with a depth of 300 kpc, and we project $\rho^2$. **Top row panels:** Total dark matter density fields for the halo in different SIDM models as indicated. The largest difference compared to the CDM case occurs for the inelastic model with the $\chi^2_{\text{init}} = 100\%$ initial configuration. The reduction of core densities and subhalo abundance is smaller for the $\chi^2_{\text{init}} = 50\%$ initial configuration compared to the $\chi^2_{\text{init}} = 100\%$ case. **Middle row panels:** Dark matter densities of the ground state ($\chi^1$) for the different models. For the inelastic case nearly all excited particles have been removed from the halo centre due to the energy injection during de-excitation of the state. This is not the case for the elastic model, where the ground state particles stay close to the halo centre and are only redistributed due to scattering events. **Bottom row panels:** Dark matter densities of the excited state ($\chi^2$) for the different models. This is quite different for the $\chi^2_{\text{init}} = 100\%$ configuration (left panel) since some channels are initially suppressed because no ground state scattering partners can be found (e.g. $\chi^1 + \chi^2 \rightarrow \chi^1 + \chi^2$). Also, given that all particles are initially in the excited state, the number of scattering events for the $\chi^2 + \chi^2 \rightarrow \chi^2 + \chi^2$ channel is the highest despite having a smaller cross section than the mixed channel. The thick solid lines show the total number of scatters summed over all channels. These are nearly the same for the elastic and inelastic cases in the $\chi^2_{\text{init}} = 50\%$ configuration, whereas there is a clear difference in the $\chi^2_{\text{init}} = 100\%$ case. Again, this is caused by the removal of ground state particles following de-excitation. We note that for all cases, on average, only a few scatters per particle occur in a Hubble time within the inner halo. This number is nearly constant within the central $\sim 20$ kpc of the halo. Beyond $\sim 100$ kpc, the number of scattering events drops below one rapidly.

The net impact of energy release due to de-excitations is seen in Fig. 8 where we show the spherically averaged total radial density profiles (thick lines) for the CDM (black), elastic (blue) and inelastic (red) SIDM models. Velocity dispersion profiles are presented in Appendix B. As in previous figures, on the left (right) we show the $\chi^2_{\text{init}} = 100\%$ ($\chi^2_{\text{init}} = 50\%$) case. One can immediately see that the inelastic case leads to a stronger depletion of the central density (a larger density core) than the elastic case. We stress that the elastic and inelastic cases have exactly the same reaction channels and cross sections, with the only difference being the energy release during down-scattering in the inelastic case. An interesting implication of this result is that it is possible in inelastic models to create a core of the same size and density as in the elastic case but with a smaller scattering cross section. This is an important point since it indicates that in the inelastic case, a wider range of cross section normalisations could significantly modify the inner dark matter density while remaining consistent with other constraints, compared with the range preferred by elastic SIDM simulations.
The thin lines in Fig. 8 show the density profiles of the individual states, ground state ($\chi^1$) and excited state ($\chi^2$) with solid and dashed lines, respectively. For clarity, we have shifted those profiles down by one dex, relative to the total profiles. As anticipated, the density profile for the ground state in the inelastic $\chi^2_{\text{init}} = 100\%$ case (left panel, dashed red line) is strongly suppressed. The average density of the ground state is more than two orders of magnitude lower than in the elastic case, where particles are not kicked out from the halo. This is not the case for the $\chi^2_{\text{init}} = 50\%$ configuration, where the profiles for the different state populations are rather similar and nearly the same between the elastic and inelastic cases.

The lower panels in Fig. 8 present the actual density reduction compared to the CDM case for the four different SIDM scenarios. Elastic collisions alone reduce the central density at 1 kpc already by an order of magnitude compared to the CDM case. As stated, the strongest reduction is for the inelastic $\chi^2_{\text{init}} = 100\%$ model where by $z = 0$, the density is lower by nearly a factor of $\sim 25$ at 1 kpc. Comparing the elastic-CDM and inelastic-CDM ratio profiles in more detail, it becomes also clear that, contrary to the elastic case, the inelastic case does not show an enhancement over the CDM density at intermediate radii (for the $\chi^2_{\text{init}} = 100\%$ case). This is because in inelastic models the particles are not only redistributed within the halo due to the effective inside-out heat transport caused by elastic scattering, but they can, at least for our benchmark model, also be removed, and not contribute to the halo density profiles anymore. This is a distinctive signature between elastic and inelastic SIDM models since all elastic models that produce a core also lead to such a density enhancement at intermediate radii. We finally note that the impact of inelastic down-scattering in the halo density profiles strongly depends on the initial level population of the excited state. For the 50% case (right panel of Fig. 8), we see only a rather small effect compared to the elastic case.

We expect that inelastic SIDM models have a stronger impact on the abundance of subhaloes compared to purely elastic models. In fact, for elastic models a quite large cross section on galactic scales ($\sim 10 \text{ cm}^2 \text{ g}^{-1}$) is needed to create a relevant difference relative to CDM. We present the subhalo velocity ($V_{\text{max}}$) function for our MW-size simulations in the top panels of Fig. 9. Given the relatively large cross sections of our benchmark model, we already see...
The inelastic SIDM models lead to larger cores with lower central densities compared to the elastic SIDM models. This is caused by the removal of ground state particles after exothermic reactions, which is largest for the $\chi^2_{\text{init}} = 100\%$ case, where more energy is available for release. For the inelastic case the total density almost never exceeds the CDM density at intermediate radii. This is distinct from the typical behaviour of elastic SIDM models, where a small density enhancement over the CDM density is always observed due to the redistribution of particles from the inner to the intermediate regions. The ground state density profile for the $\chi^2_{\text{init}} = 100\%$ configuration is very different for the elastic and inelastic cases due to the removal of de-excited particles from the halo centre. For the $\chi^2_{\text{init}} = 50\%$ configuration, on the other hand, the profiles are very similar since the de-excited particles represent only a small fraction of the total ground state population. The bottom panels show the ratio of the total density profiles relative to CDM.

![Figure 8. Radial density profiles for the Milky Way-sized halo split in the two-state populations.](image)

Figure 8. Radial density profiles for the Milky Way-sized halo split in the two-state populations. The inelastic SIDM models lead to larger cores with lower central densities compared to the elastic SIDM models. This is caused by the removal of ground state particles after exothermic reactions, which is largest for the $\chi^2_{\text{init}} = 100\%$ case, where more energy is available for release. For the inelastic case the total density almost never exceeds the CDM density at intermediate radii. This is distinct from the typical behaviour of elastic SIDM models, where a small density enhancement over the CDM density is always observed due to the redistribution of particles from the inner to the intermediate regions. The ground state density profile for the $\chi^2_{\text{init}} = 100\%$ configuration is very different for the elastic and inelastic cases due to the removal of de-excited particles from the halo centre. For the $\chi^2_{\text{init}} = 50\%$ configuration, on the other hand, the profiles are very similar since the de-excited particles represent only a small fraction of the total ground state population. The bottom panels show the ratio of the total density profiles relative to CDM.

In the bottom panel of Fig. 9 we show the median (solid lines) and 1σ region (shaded areas) of the distribution of density profiles of the ten most massive subhaloes at $z = 0$. Similar to the main halo density profile, we find that inelastic SIDM models lead to subhaloes with larger cores and smaller densities compared to the elastic case. We note however, that the effect of inelastic collisions is larger for these smaller systems with lower velocity dispersion than the main MW halo since the cross section for down-scattering strongly increases towards lower relative velocities as shown in the bottom panel of Fig. 1, where the cross section exceeds the main state elastic cross sections for relative velocities below $\sim 10 \text{ km s}^{-1}$.

For the $\chi^2_{\text{init}} = 100\%$ ($\chi^2_{\text{init}} = 50\%$) configuration we find that the core density is reduced by $\sim 4$ (2) compared to the elastic case. Therefore, the inelastic SIDM cross sections can be smaller by a factor of a few compared to the elastic case while creating a core of similar size and density. We quantify this reduction in Fig. 10, where we compare the elastic and inelastic model for the $\chi^2_{\text{init}} = 100\%$ initial configuration with an elastic model, where we have increased the cross section normalisation by factors of 2.5 and 5. These simulations were performed at level-3 resolution, which is sufficiently converged for this test (see Appendix A). The figure shows that the central density reduction for the main halo density profile is comparable for the inelastic model and the elastic model with a five times larger cross section (left panel). This implies that previous conclusions obtained from elastic SIDM simulations on the requirements for core sizes and densities given a certain cross section normalisation, are strongly altered in the presence of inelastic scattering. We also note that inelastic SIDM models will not suffer from the gravothermal catastrophe, as long as down-scattering and the resultant energy release is not suppressed. This is demonstrated in the right panel, where at $z = 0$ the scaled elastic models already started to core-collapse, which does not occur for the inelastic model. This absence of core collapse is distinctly different from elastic SIDM models, where the runaway collapse of the core is unavoidable on a time scale that depends on the normalisation of the cross section (Koda & Shapiro 2011). As for the main halo density profile, we find that the inelastic model does not lead to a density enhancement relative to the CDM case at intermediate radii. As we have argued, this is due to the expulsion of de-excited ground state particles from the halo.

Finally, we remark that the formation of a core in the inelastic case is a combination of the redistribution of energy following elastic scatterings and loss of DM particles during inelastic down-scattering. The degree to which effect is more important depends on the primordial fraction of particles in the excited state, and the hierarchy of the velocity-dependent cross sections of the different scattering channels. A substantial mass loss in the halo centre is a distinct feature of inelastic models relative to elastic SIDM, and it might be of particular relevance for constraining the (inelastic) energy deposition. In elastic SIDM for instance, the mass redistribution in the halo due to elastic scattering can be reversed if the
Conclusions

We have performed high resolution simulations of a galactic halo within an inelastic self-interacting dark matter model using a newly developed self-interaction dark matter implementation in the AREPO code to study multi-state dark matter models. We simulate a generic inelastic model of a nearly degenerate two-state dark matter particle with an energy level splitting of $10^{-6}$ with respect to its ground state, and an interaction cross section of a few cm$^2$g$^{-1}$ on the scale of dwarf galaxies.

We demonstrate that the physics of inelastic self-interacting dark matter leads to interesting new effects, which are distinctly different or absent in elastic self-interacting dark matter models. Most importantly, dark matter particles can be removed from potential wells, which causes large core formation with lower densities compared to elastic models for the same interaction cross section. Furthermore, inelastic collisions also lead to subhalo evaporation reducing efficiently the abundance of subhaloes. Our main findings are:

- Inelastic SIDM creates larger and lower density cores compared to elastic models for the same cross section. Specifically, a ~5 times larger elastic cross section would be required to achieve the same density reduction and core formation as the inelastic model. The exact factor depends on the underlying inelastic dark matter model. This implies that currently existing constraints on inelastic eV-sized dark matter are not satisfied by SIDM. Additionally, this effect provides a new handle to study the presence of inelastic dark matter that may be evaporation reducing efficiently the abundance of subhaloes. Our main findings are:

Figure 9. Abundance and inner structure of subhaloes in CDM and (in)elastic SIDM models. Top panels: Cumulative subhalo velocity ($V_{\text{max}}$) function for the different models and observed satellites of the Milky Way including a sky coverage correction (Polisensky & Ricotti 2011). Despite the relatively large cross sections (see bottom panel of Fig. 1), the elastic model leads only to a minor reduction in subhalo abundance relative to CDM. A substantially larger effect is visible for the inelastic case. For $V_{\text{max}} \lesssim 30$ km s$^{-1}$ ($V_{\text{max}} \lesssim 20$ km s$^{-1}$) the number of subhaloes is reduced by $\sim 5$ ($\sim 4$) for the inelastic model with $\chi^2_{\text{init}} = 100\%$. Bottom panels: Subhalo density profiles. The solid lines show the median profile of the ten most massive subhaloes at $z = 0$, while the shaded region indicates the 1σ scatter of the distribution. Inelastic SIDM leads to significantly larger and lower density cores compared to the elastic case. For the $\chi^2_{\text{init}} = 100\%$ configuration the core density is reduced by $\sim 4$ compared to the elastic case. This implies that previous estimates on cross sections within elastic SIDM simulations based on certain requirements for core densities, are altered in the presence of inelastic SIDM reactions. We also note that such an inelastic SIDM model will not suffer from the gravothermal catastrophe if the elastic scattering cross sections are sufficiently small. 

**5 CONCLUSIONS**

Evaporating the Milky Way with inelastic SIDM

Elbert et al. 2018) or if the gravothermal collapse phase has been triggered. This reversal has been invoked to argue that in elastic SIDM, not all haloes are expected to be cored, and in fact the natural diversity of the inner dark matter densities reflected in the rotation curves of dwarf galaxies (Oman et al. 2015) is to be expected for elastic SIDM with $\sigma_T/m \sim 1$ cm$^2$g$^{-1}$ (Kamada et al. 2017). This reversal of cored profiles into dense profiles would have only a limited extent in the case of inelastic SIDM since the central mass loss is irreversible and the gravothermal collapse cannot be triggered. Thus, it will be quite relevant to explore the interplay of inelastic SIDM and baryonic physics to check under which conditions they can develop cuspy profiles, which is a promising avenue to constrain inelastic SIDM.
self-interaction cross sections have to be revised if inelastic reactions are taken into account.

• Similar to models with a power spectrum cutoff, we find that inelastic SIDM can reduce the amount of substructure for rather small interaction cross sections. The reduction of the subhalo abundance is due to gravitational particle unbinding such that for $V_{\text{max}} \gtrsim 30 \text{ km s}^{-1}$ ($V_{\text{max}} \gtrsim 20 \text{ km s}^{-1}$) the number of subhaloes is reduced by $\sim 3$ ($\sim 4$) compared to the cold dark matter case.

• The de-excitation of the upper level to the ground state can inject the energy equivalent of $O(100)$ million Type II supernovae in Milky Way-like haloes.

• The gravothermal catastrophe that can occur in elastic SIDM models, can be avoided in inelastic SIDM models. This is possible because of the unbinding of particles that de-excite during the scattering process. If this process dominates over elastic scattering, a central density increase is not possible anymore since the dark matter mass is efficiently removed.

• The virial mass of a Milky Way-like halo is reduced by about 10% compared to the cold dark matter case for inelastic self-interacting dark matter.

• Density profiles for elastic and inelastic SIDM models are distinctly different. Elastic models lead to an increased dark matter density at intermediate radii caused by particles being transferred from the core region to outer parts. This effect is not present for inelastic models that can unbind particles. For such models the density profile follows the CDM prediction outside of the core region.

We conclude that inelastic SIDM models have some properties, like the removal of substructure and enhanced core formation, that are absent in typically studied elastic SIDM models. Those features can have a significant impact on properties of Milky Way-like dark matter haloes and can address many of the small-scale challenges of the cold dark matter paradigm. Furthermore, inelastic SIDM models are a rather generic feature of many dark matter particle physics models beyond the canonical WIMPs, which have still not been detected. It is therefore high time to consider also alternative dark matter particles, like inelastic SIDM, and understand how structure formation progresses in those models. In the future it will be interesting to study these inelastic models with baryonic galaxy formation models (e.g. Vogelsberger et al. 2013, 2014c) to understand their impact on galaxy formation in more detail. Furthermore, we also expect changes in the detailed dark matter phase-space structure (e.g. Vogelsberger et al. 2009; Vogelsberger & Zavala 2013) due to the energy injection and particle removal in inelastic self-interacting models.

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We test the numerical convergence of the inelastic SIDM implementation in Figure A1. We present the density profiles for total density and state density split in ground and excited state for three different resolution levels of the $\chi^2_{\text{init}} = 100\%$ initial configuration. The three resolution levels differ each by factors of 8 in mass resolution, and a factor of 2 in softening length. Level-1 corresponds to the highest resolution that has been employed for the paper. We find good convergence of the inelastic SIDM implementation.

APPENDIX B: VELOCITY DISPERSION PROFILE

The total velocity dispersion profile for the $\chi^2_{\text{init}} = 100\%$ initial configuration is presented in Figure B1. An isothermal core of very similar characteristics is formed in both the elastic and inelastic SIDM models, the key differences are that the inelastic case has a colder core (to compensate for the lower central density) and a population of ground state particles that is essentially unbound, moving with large velocities due to the energy injection during downscattering.

Figure A1. Numerical convergence of inelastic SIDM simulations. Density profiles for total density and state density split by ground and excited state for three different resolution levels of the $\chi^2_{\text{init}} = 100\%$ initial configuration.

Figure B1. Total velocity dispersion profiles for the Milky Way-sized halo split in the two-state populations. Both elastic and inelastic SIDM lead to isothermal profiles in the centre. The ground state velocity dispersion is significantly higher for the inelastic model due to the injected kinetic energy gained during the level decay.