Grand Unification of the Sterile Neutrino

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The simplest way to simultaneously understand all existing indications of neutrino oscillations from solar and atmospheric neutrino deficits and the LSND experiment, seems to be to postulate a sterile neutrino. We present a realistic grand unified model based on the gauge group \( SO(10) \times SO(10)' \) that leads to the desired masses and mixings for the sterile and the known neutrinos needed to understand the above observations while fitting those of the known charged fermions. The model is a grand unified realization of the recently proposed idea that the sterile neutrino is the lightest neutrino of a mirror sector of the universe which has identical matter and gauge content as the standard model. The two \( SO(10)'s \) operate on the two sectors in a mirror symmetric way and are connected by a mixed Higgs representations whose net effect is to connect the superheavy right handed neutrinos of the two sectors.

UMD-PP-98-118

I. INTRODUCTION

A turning point in the search for new physics beyond the standard model may be at hand in the arena of neutrino physics. The standard model predicts zero neutrino masses but there are now strong indications for neutrino oscillations (and hence nonzero neutrino masses) from the five solar neutrino experiments (Kamiokande, Homestake, Gallex, Sage and Super-Kamiokande [1,2,11]), four atmospheric neutrino observations [2–4] and the direct laboratory observation in the LSND experiment [12]. Furthermore, to explain all three experiments, three different scales for mass differences (\( \Delta m^2 \)) are needed. Since with the three known neutrinos one can get only two independent \( \Delta m^2 \)'s, it has been suggested [8] that a fourth sterile neutrino be invoked. In the presence of this extra neutrino species (\( \nu_s \)), one can construct several scenarios for solving the three neutrino puzzles [8]. In this letter, we will be interested in the scheme [8], where the solar neutrino puzzle is solved via the oscillation of the \( \nu_e \) to \( \nu_s \) using the MSW mechanism [8,10] and the atmospheric neutrino puzzle is solved via the \( \nu_\mu - \nu_\tau \) oscillation. The solar neutrino puzzle fixes the \( \Delta m^2_{\nu_e - \nu_s} \approx (0.3 - 1.0) \times 10^{-5} \text{ eV}^2 \), whereas the atmospheric neutrino puzzle implies that \( \Delta m^2_{\nu_\mu - \nu_\tau} \approx 10^{-2} - 10^{-3.5} \text{ eV}^2 \) and the LSND experiment keeps the 0.3 eV\(^2 \leq \Delta m^2_{\nu_e - \nu_\mu} \leq 10 \text{ eV}^2 \). In this picture the \( \nu_\mu \) and \( \nu_\tau \) are nearly degenerate and near maximally mixed. If the existence of the sterile neutrino becomes confirmed say, indirectly by KARMEN [11] observing \( \nu_\mu - \nu_\tau \) oscillation or directly by SNO neutral current data to come in the early part of the next century, a key theoretical challenge will be to construct an underlying theory that embeds the sterile neutrino along with the others with appropriate mixing pattern, while naturally explaining its ultralightness. In this letter, we propose a grand unified model which not only explains the smallness of neutrino masses via the seesaw mechanism but it also incorporates the sterile neutrino, whose ultralight mass is naturally explained. The model at the same time predicts mass degeneracy between maximally mixed \( \nu_\mu \) and \( \nu_\tau \).

The underlying framework for the present work will be the suggestion that there is a parallel standard model [2,8,11,12] which is an exact copy of the known standard model (i.e. all matter and all gauge forces identical). The two “universes” communicate only via gravity or other forces that are equally weak. At an overall level, such a picture emerges quite naturally in superstring theories which lead to “universes” communicating only via gravity or other forces that are equally weak. At an overall level, such a picture
a hierarchical pattern for all fermion masses including neutrinos. However the LSND results in conjunction with the atmospheric neutrino data imply that $m_{\nu_\tau} \approx m_{\nu_\tau}$. The challenge is therefore to construct an SO(10) model which will lead to the above non-generic prediction.

We impose a mirror symmetry [16] between the two SO(10) sectors so that the field contents as well as the Yukawa couplings in the two sectors are identical to each other and all differences between them arise from the process of spontaneous symmetry breaking. In order to constrain the model further, we impose the requirement that it conserve R-parity automatically without using any extra global symmetries. This ensures that there is a natural cold dark matter candidate in the model. We also impose an additional global permutation symmetry $S_3$ which plays a key role in ensuring the mass degeneracy between the tau and muon neutrinos. This leads to a three neutrino texture which is similar in form to the one discussed in Ref. [17]. The connection between the visible and the mirror sector occurs via the mixing of the heavy right-handed neutrinos [18].

The nontrivial nature of the problem arises from the fact that in a GUT framework the neutrino couplings are intimately linked to the charged fermion couplings and it is by no means obvious that with a simple set of Higgs fields one can make the observed hierarchical pattern of the charged fermion masses and mixings compatible with the apparent non-hierarchical mass and mixing pattern for the neutrinos.

II. THE MODEL: EACH SECTOR

The fermions of each generation are assigned to the $(16,1) \oplus (1,16')$ representation of the gauge group. We denote them by $\Psi_{e,\mu,\tau}$ in the visible sector and by corresponding symbols with a prime in the mirror sector (as we do for all fields). The SO(10) symmetry is broken down to the left-right symmetric model by the combination of $45 \oplus 54$ representations in each sector. The SU(2)$_R \times U(1)_{B-L}$ gauge symmetry in turn is broken by the $126 \oplus \overline{126}$ representations and we take three such representations (and denote them by $\Delta_{0,1,2} \oplus \overline{\Delta}_{0,1,2}$). The role of the these fields is two-fold: First, they guarantee automatic R-parity conservation and second, they lead to the see-saw suppression for the neutrino masses [19].

The standard model symmetry is then broken by the $10$-dim. Higgs fields of which we take three $H_{0,1,2}$. As is well-known, the $126$-dim. representation contains in it left-handed triplets with $B - L = 2$. Due to the presence of the $54$-Higgs field $S$ in the model, couplings of type $\Delta \Delta S$ and $HHS$ are allowed and they lead to induced $B - L$ breaking vev's $v_L$ which give a direct Majorana mass to the neutrinos leading to the so called type II see saw formula [20] written symbolically as

$$m_{\nu} \approx f v_L - \frac{m^2_{\nu_R}}{f v_R}$$  

(1)

where $v_R$ is the generic vev of the $\nu^c \nu^c$ component of $126$. As was shown in [20], the detailed minimization of the potential in such theories leads to the conclusion that $v_L$ is also suppressed by a see saw like formula (i.e. $v_L \approx v_{L}^2 / \lambda v_R$ where $\lambda$ is an unknown parameter in the superpotential). If we choose $v_R \approx 10^{14} - 10^{15}$ GeV (so that it is not far from the GUT scale) and $\lambda \approx 0.1 - 0.01$, then we get $v_L$ in the eV range. Note that while the second term in Eq. (1) arising from the conventional see saw formula leads to a hierarchical mass pattern for the neutrinos, the first term has no such obligation. Thus, if we require some neutrinos to be nearly degenerate, the first term has to be given the dominant role as we do here.

A second point is that in the effective MSSM derived from the model, the low energy Higgs doublets will be assumed to be linear combinations of the doublets present in all $10$ as well as $126$ dimensional multiplets. In principle this situation can be realized by appropriate arrangement of parameters.

Next we assume the invariance of the action under a discrete permutation symmetry $S_3$ under which the $(\Psi_{e}, \Psi_{\mu}, \Delta_{1,\Delta_2})$ and $(H_1, H_2)$ transform as doublets whereas $\Psi_{\tau}, H_0, \Delta_0$ and the rest of the fields transform as singlets. The same discrete operates in the mirror sector (i.e. no mirror version of $S_3$). This then restricts the form of the Yukawa part of the superpotential to the following form:

$$W_Y = h_1 \Psi_{e}\Psi_{\mu} H_0 + h_2 (\Psi_{\mu}\Psi_{\mu} + \Psi_{\tau}\Psi_{\tau}) \ H_0 + h_3 \Psi_{e}(\Psi_{\mu} H_1 + \Psi_{\tau} H_2) + h_4 \ [ (\Psi_{\mu} \Psi_{\mu} - \Psi_{\tau} \Psi_{\tau}) \ H_1 + 2 \Psi_{\mu} \Psi_{\tau} H_2] + f_1 \Psi_{e} \Delta_0 + f_2 (\Psi_{\mu}\Psi_{\mu} + \Psi_{\tau}\Psi_{\tau}) \ \overline{\Delta}_0 + f_3 \ [ (\Psi_{\mu} \Psi_{\mu} - \Psi_{\tau} \Psi_{\tau}) \overline{\Delta}_1 + 2 \Psi_{\mu} \Psi_{\tau} \overline{\Delta}_2]$$

$$+ f_4 \Psi_{e} (\Psi_{\mu} \overline{\Delta}_1 + \Psi_{\tau} \overline{\Delta}_2)$$  

(2)

Using Eq. 2, we can write down the quark and lepton mass matrices for the visible sector as follows.
of both sectors get connected. This in conjunction with the Dirac masses of both sectors introduces a mixing matrix \( \nu \) for neutrino masses and mixings, we need to know the structure of the mass matrices in the mirror sector and the connection between the visible and the mirror sector.

For neutrino masses and quark mixings and vanishing charged lepton mixings. The exact mirror symmetry between the visible and the mirror sector implies that at the level of the superpotential, all couplings in the mirror sector are identical to those in the visible sector. We will assume that the spontaneous invariance of the theory which yields eight relations among them. They are \( \frac{c_a}{c_a} = \frac{c_b}{c_b} = \frac{c_c}{c_c} = \frac{c_d}{c_d} \). We now proceed to determine the remaining parameters in such a way they give rise to observed fermion masses and quark mixings and vanishing charged lepton mixings.

Even though apriori, it may appear from Eq. 3 and 4 that there are 24 parameters in the visible sector mass matrices, the actual number is 16 due to the \( S_3 \) invariance of the MSSM. From Eq. 1 is of order \( 10^{-2} \) eV in the 33 element and much smaller in other places. As far as the first term goes, its form is dictated by Eq. 2 and we choose it as follows:

\[
M_{\nu
u} = \begin{pmatrix} 0 & A_l & A'_l \\ A_l & B_l & D_l \\ A'_l & D_l & -B_l \end{pmatrix} \text{ in eV}
\]

Where \( A_l = B_l \). This mass matrix looks very similar to the one analyzed in [17]. In order to obtain the predictions for neutrino masses and mixings, we need to know the structure of the mass matrices in the mirror sector and the connection between the visible and the mirror sector.

The exact mirror symmetry between the visible and the mirror sector implies that at the level of the superpotential, all couplings in the mirror sector are identical to those in the visible sector. We will assume that the spontaneous symmetry breaking breaks the mirror symmetry so that actual mass matrices will exhibit differences. For simplicity, we will assume that all doublet vev’s in the mirror sector differ by a common ratio from those in the visible sector (i.e. \( v'_w/v_w = \xi \)). Since this asymmetry will effect the fermion masses in the two MSSM’s, we will expect the \( B - L \)-breaking scales and the GUT scales to be different. This in turn will imply that the induced triplet vev’s will also be different in the two sectors. Our strategy will therefore be to scale the Dirac mass matrix for the mirror sector by a common factor but introduce arbitrary triplet vev’s in the mirror sector.

\[
M'_{\nu
u} = \begin{pmatrix} \alpha & A_m & A'_m \\ A_m & B_m & D_m \\ A'_m & D_m & -B_m \end{pmatrix} \text{ in eV}
\]

Where, \( A_m = q_1 A_l \), \( A'_m = q_1 A_l \), \( D_m = q_2 D_l \) and \( B_m = q_2 B_l \). Let us now try to connect the two sectors which we do by postulating the Higgs fields (16,16') \( \oplus (\bar{16},16') \) (denoted by \( \chi \oplus \bar{\chi} \)). There can now be a connecting term between the two sectors given by

\[
W' = g_c \Psi_\chi \Psi'_{\bar{\chi}} + g'_c (\Psi_\mu \Psi'_\mu + \Psi_\tau \Psi'_{\bar{\tau}}) \chi
\]

We now give a vacuum expectation value to the \( \nu' \nu' \) element of \( \chi \oplus \bar{\chi} \) fields. Then only the right handed neutrinos of both sectors get connected. This in conjunction with the Dirac masses of both sectors introduces a mixing matrix between the two sectors of the following form (assuming for simplicity \( g_c = g'_c \)):


\[ M_{\nu\nu'} = g_c M_{\nu\nu'} M_{\nu\nu'}^{-2} M_{\nu\nu} < \chi > \]

Where,

\[ M_{\nu\nu'} = \begin{pmatrix} 0 & A_r & A'_r \\ A_r & B_r & D_r \\ A'_r & D_r & -B_r \end{pmatrix} \text{ in GeV} \]

(10)

Where, \( A_r = l_1 A_l, A'_r = l_1 A'_l, B_r = l_2 B_l \) and \( D_r = l_2 D_l \). We diagonalize the complete neutrino mass matrix to obtain the following absolute values of the mass eigenvalues (in eV's):

\[ m_{\nu_e} = 90.56, m_{\nu_\mu} = -90.56, m_{\nu_\tau} = 0.0034 \]

\[ m_{\nu_e} = 1.51, m_{\nu_\mu} = -1.509, m_{\nu_\tau} = 0.001. \]

The squared mass differences (in eV²) are \( \Delta m^2_{\alpha\beta} = 9.9 \times 10^{-6}, \Delta m^2_{\mu\tau} = 0.003 \) and \( \Delta m^2_{\nu_{\mu\tau}} = 2.27 \), where the numbers are given in eV². The fitted values of the parameters are, \( \alpha = 0.005, A_l = 0.0253, A'_l = 0.050, B_l = 0.675, D_l = 1.35 \) given in eV units, \( q_1 = 10, q_2 = 60, l_1 = 1.2 \times 10^{15}, l_2 = 0.5 \times 10^{15} \) and \( g_c < \chi > = 6.9 \times 10^{12} \) given in GeV units. The mixing matrix \( O^\nu \) of the six neutrinos in the basis \( (\nu_e, \nu_\mu, \nu_\tau, \nu'_e, \nu'_\mu, \nu'_\tau) \) is approximately given as,

\[ O^\nu = \begin{pmatrix} -0.99 & 0.037 & 0 & 0.039 & -0.00025 & 0 \\
-0.031 & -0.85 & -0.52 & -0.00072 & 0 & 0 \\
0.019 & 0.525 & -0.85 & -0.00043 & 0 & 0 \\
-0.042 & 0.0014 & 0.00071 & -0.999 & 0.0062 & 0 \\
0 & 0 & 0 & -0.0053 & -0.850 & -0.525 \\
0 & 0 & 0 & 0.0032 & 0.52 & -0.85 \end{pmatrix} \]

(11)

Combining this with the mixing angle for the charged leptons, we obtain the final mixing matrix among the four neutrinos which looks identical to the corresponding top-left 4 × 4 submatrix of \( O_\nu \) with only the \( \nu_e - \nu_\tau \) entry reduced by a factor of 2 because of the presence of a small 13 element in the charged leptonic mass matrix. We have varied the parameters of the model to see the allowed range for the \( m_{\nu_e} \) relevant for the LSND experiment and find consistent solutions for the range \( 0.5 \leq m_{\nu_e}/eV \leq 1.5 \). Therefore, the \( \nu_\mu \) and \( \nu_\tau \) together could play the role of the hot dark matter for the upper allowed range of the masses. Also note that the zeros in the neutrino mixing matrix simply means that those entries are less than \( 10^{-4} \).

We thus see that in this model not only are all three positive indications of neutrino oscillations are explained but the mixing between the heavier sterile neutrinos \( \nu'_\mu \) and \( \nu'_\tau \) and the active neutrinos are consistent with all known oscillation data such as for example the one from the CHOOZ \[21\]. What we find very interesting is that with only six parameters describing the entire 6 × 6 neutrino mass matrix (three active and three sterile) and every other parameter fixed by the charged fermion masses, four neutrino masses and 12 mixing parameters that link the active to sterile neutrinos which could have observable consequences are all completely consistent with known data.

Turning now to the consistency of our model with big bang nucleosynthesis (BBN), we recall that present observations of Helium and deuterium abundance can allow for as many as 4.53 neutrino species \[22\] if the baryon to photon ratio is small. There are also a new interesting possibility for generation of lepton asymmetry in the presence of sterile neutrinos which will effect the upper limit on the neutrino number \[23\]. The relevant parameter that determines if extra neutrinos contributes via oscillation of the known ones is \( \Delta m^2 / 2 \equiv \delta_{BBN} \). It has been argued that in the absence of neutrino asymmetries, \( \delta_{BBN} \leq 10^{-7} \) eV². For large masses of the two mirror neutrinos, this bound is not satisfied. We therefore invoke the new mechanism proposed in Ref. \[23\] to evade these bounds thus restoring consistency with the BBN constraints.

The second feature of mirror universe models is the existence of the mirror photon, which could have experimental manifestations. One arena would be the BBN; but as was discussed in Ref. \[2\], this problem can be ameliorated by the assumption of asymmetric inflation between the two sectors \[24\]. A second constraint comes from the fact that \( \gamma - \gamma' \) kinetic mixing \[20\] is highly constrained by the BBN considerations. In our model, since it arises from the mixed Higgs field \( \chi \) and thus depends on the splittings among the various sub-multiplets in the mixed field \( \chi \), at the phenomenological level, this will serve to fix the intra-multiplet splitting. It was noted by Carlson and Glashow \[20\] that positronium-mirror-positronium oscillation could also constrain the \( \gamma - \gamma' \) mixing. But in our model, we assume the mirror weak scale to be about ten times large so that \( e^+ e^- \) bound state will have mass of about 10 MeV, preventing the possibility of this oscillation.

In conclusion, we have presented a supersymmetric grand unified model for the sterile neutrino which can explain the solar, atmospheric and LSND data. The new features of the model are (a) the use of the type II seesaw mechanism to explain the smallness of the neutrino masses while at the same time accomodating maximal \( \nu_\mu - \nu_\tau \) oscillation with degenerate neutrinos and (b) the prediction of the neutrino mixings among six light neutrinos (three active and three mirror) with very few parameters, which are consistent with all known constraints.

This work is supported by the National Science Foundation under grant no. PHY-9421385. We wish to thank Markus Luty for some discussions.
TABLE I. The fitted values of $a_i$, $b_i$, $c_i$ and $d_i$. 

| $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ |
|-------|-------|-------|-------|-------|-------|
| 0.140 | 0.230 | 0     | 50.54 | -55.30| 0     |

| $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ | $c_6$ |
|-------|-------|-------|-------|-------|-------|
| 0.0016| 0.0023| -0.00012| 0.579 | -0.55 | 0.0288 |

| $b_1$ | $b_2$ | $b_3$ | $b_4$ | $b_5$ | $b_6$ |
|-------|-------|-------|-------|-------|-------|
| -0.230| 0     | -0.508| 0     | -0.139| 5.64  |

| $d_1$ | $d_2$ | $d_3$ | $d_4$ | $d_5$ | $d_6$ |
|-------|-------|-------|-------|-------|-------|
| 0.00076| 0.0043| 0.0016| 0.0096| 0.0037| -0.0152|