Broad angle negative refraction in lossless all dielectric or semiconductor based asymmetric anisotropic metamaterial

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Abstract

In this article, it has been theoretically shown that broad angle negative refraction is possible with asymmetric anisotropic metamaterials (AAMs) constructed by only dielectrics or lossless semiconductors at the telecommunication and relative wavelength range. Though natural uniaxial materials can exhibit negative refraction, the maximum angle of negative refraction and critical incident angle lie in a very narrow range. This problem can be overcome by our proposed structure. In our structures, negative refraction originates from the highly asymmetric elliptical iso-frequency. This is artificially created by the rotated multilayer sub-wavelength dielectric or semiconductor stack, which acts as an effective AAM. This negative refraction is achieved without using any negative permittivity materials such as metals. As we are using simple dielectrics, fabrication of such structures would be less complex than that of the metal based metamaterials. By considering the time harmonic field incidence, negative refraction has been demonstrated for two dimensional bi-dielectric structures for TM polarization with realistic parameters. Our proposed ideas have been validated by the full wave simulations considering both the effective medium approach and realistic structure model. This device might find some important applications in photonics and optoelectronics.

Keywords: metamaterial, all dielectric metamaterial, negative refraction, semiconductor material, iso frequency

(Some figures may appear in colour only in the online journal)

1. Introduction

Negative refraction has been demonstrated from double negative metamaterial [1] to metal–dielectric multilayers [2, 3], nanowire metamaterials [4, 5], polar dielectrics [6], photonic crystals [7], antiferromagnets [8], hyper crystals [9, 10] and so on. Metal dielectric multilayers have been intensively used in constructing so called indefinite or hyperbolic metamaterials (HMMs) [2, 3, 11, 12]. HMMs possess the ability of negative refraction due to its hyperbolic iso-frequency [2–6, 13]. However, as they are made from metals, loss in individual metal layer greatly reduces its performance and practical applications. Very recently, dielectric–dielectric/semiconductor sub-wavelength multilayers have been proposed as anisotropic elliptical metamaterial [13, 14] in their lossless frequency region. It has been already predicted that using such anisotropic metamaterial, wave-guiding performance would be better [14] in comparison with previous metamaterial based fibers [15]. Natural birefringence or uniaxial materials can also show negative refraction [16–21] but in a very narrow incident angle and exhibit very narrow angle of refraction [16–21], which hinders their practical applications.
In this article, to the best of our knowledge, probably for the first time it is shown that broad angle negative refraction is possible by using an asymmetric anisotropic metamaterial (AAM) made from lossless dielectric or semiconductor materials at the telecommunication wavelength band. The maximum angle of negative refraction and the critical incident angle (CIA) (the maximum possible incident angle for negative refraction) with the proposed lossless metamaterials can exceed the properties observed with the natural uniaxial materials [16–21]. It can also be made tunable by changing the fill fraction and the rotation angle of the metamaterial. This negative refraction does not require any negative permittivity like others [4, 5, 9, 10, 22] and there is no resonance like phenomena. So there is no possibility of loss as long as the dielectrics or semiconductors used are lossless in the respected frequency range of interest. Our ideas have been verified by full wave simulations considering exactly the realistic structure. Moreover, no ambiguity has arisen to verify our ideas based on effective medium theory (EMT) [23]. As we are using dielectrics or lossless semiconductors, the fabrication of our proposed structure should not be as complicated as metal based metamaterials [11]. We believe this lossless device with broad angle negative refraction may find some fundamental novel applications in photonics, for example in lensing, routing, beam splitting etc.

2. Theory of negative refraction in all dielectric multilayer AAMs

Before describing the theory of negative refraction in our proposed structure, at first we give some simple and clear differences with other negative refraction mechanisms already reported in literature. Veselago first predicted that if a material can possess negative values for both the permittivity and permeability (i.e. \( \varepsilon < 0 \) and \( \mu < 0 \)), then the index of refraction would be negative, \( n < 0 \) [24]. This was later experimentally proved by Pendry [1] by using artificially structured metamaterials which can achieve the condition \( \varepsilon < 0 \) and \( \mu < 0 \) over a broad frequency range. From Snell’s law, we know that a material having \( n < 0 \), negative refraction will occur. These artificial materials possessing such characteristics are generally known as left handed materials. Negative refraction has been already observed in such left handed materials in the microwave region [25–27]. In case of HMMs [2, 3, 11, 12] or hyper crystals [9, 10] negative refraction originates from the hyperbolic iso-frequency. In our proposed multilayer structure, the whole structure behaves effectively as a material with elliptical iso-frequency or as a birefringent material. It is already known that when such material’s optic axis is rotated with interface normal, for certain incident angle negative refraction will occur [16–21]. Here we have artificially created such material with elliptical iso-frequency with multilayer dielectric stack. So the basic difference between the mechanism of negative refraction of our proposed structure with left handed materials is that for left handed materials both the permittivity and permeability has be to simultaneously negative. This will create a negative refractive index which will cause negative refraction following the Snell’s law. Whereas in our proposed structure, all the materials are non-magnetic and have positive permittivity. But negative refraction occurs due to the artificially created rotated elliptical iso-frequency which is discussed in detail later on.

Figure 1 shows the schematic of the proposed AAM. In the \((x’, y’, z’)\) coordinate system the relative permittivity tensor of a general uniaxial anisotropic medium can be given by [13, 14]

\[
\varepsilon' = \begin{bmatrix}
\varepsilon_\parallel & 0 & 0 \\
0 & \varepsilon_\perp & 0 \\
0 & 0 & \varepsilon_\perp
\end{bmatrix}.
\]

Through rotation transformation around the \(Y\)-axis at an angle \( \varphi \) with respect to the \(X\)-axis, the permittivity tensor in the new coordinate system can be given by

\[
[\varepsilon_{\text{new}}] = R_y[\varepsilon']R_y^T,
\]

where, \( R_y \) is the rotation matrix around the \(Y\)-axis and \( R_y^T \) is the transposed matrix:

\[
R_y = \begin{bmatrix}
\cos \varphi & 0 & -\sin \varphi \\
0 & 1 & 0 \\
\sin \varphi & 0 & \cos \varphi
\end{bmatrix}.
\]
In figure 1 two Cartesian systems have been used, \((x', y', z')\) before rotation and \((x, y, z)\) after rotation. A TM polarized electromagnetic wave (magnetic field polarized along y axis) is incident from the ambient medium (air) to the structure, propagating in \(z-x\) plane forming an angle \(\theta\) with the \(z\) axis. We have taken the space–time dependence of fields as \(e^{-i(\omega t-k_z z-k_x x-k_y y)}\).

In the new \((x, y, z)\) coordinate system after rotation, for nonmagnetic materials, the normal component of the wave vector is derived by rigorous calculation as (see supplementary information for a detailed analysis considering both permittivity and permeability tensor)

\[
k_z = \frac{\varepsilon_{zz} k_x \pm \sqrt{(\varepsilon_{xx} - \varepsilon_{zz}) k_x^2 - k_0^2 \varepsilon_{zz} (\varepsilon_{xx} \varepsilon_{zz} - \varepsilon_{zz} \varepsilon_{xx})}}{\varepsilon_{zz}},
\]

(5)

where \(k_x\) is the tangential component of wave vector or momentum, which is defined by \(k_x \sin \theta\), where \(k_0 = \frac{2 \pi}{\lambda}\) is the wave-number in free space and \(\lambda\) is the wavelength of the incident EM wave. For symmetric case \(\varepsilon_{xx} = \varepsilon_{zz}\). Thus equation (5) reduces to

\[
k_z = \frac{\varepsilon_{zz} k_x \pm \sqrt{(\varepsilon_{xx} - \varepsilon_{zz}) k_x^2 - k_0^2 \varepsilon_{zz} (\varepsilon_{xx} \varepsilon_{zz} - \varepsilon_{zz} \varepsilon_{xx})}}{\varepsilon_{zz}}.
\]

(6)

In equation (6) the \((+)\) and the \((-)\) signs are for the normal wave vector components propagating in the downward and upward directions respectively. For TM polarization, \(H\) field is confined along the \(y\) direction. The time-average poynting vector can be calculated as \(S = \frac{i}{2} \text{Re}[E \times H^*]\). The \(x\) and \(z\) component of the time-average poynting vector have been derived as

\[
\langle S_x \rangle = \frac{i}{2} \frac{|H|^2}{\omega \varepsilon_0} \text{Re} \left( \frac{\varepsilon_{xx} k_x - \varepsilon_{zx} k_z}{\varepsilon_{zz} \varepsilon_{xx} - \varepsilon_{xx} \varepsilon_{zz}} \right),
\]

(7)

\[
\langle S_z \rangle = \frac{i}{2} \frac{|H|^2}{\omega \varepsilon_0} \text{Re} \left( \frac{k_z \varepsilon_{zz} - k_x \varepsilon_{zx}}{\varepsilon_{zz} \varepsilon_{xx} - \varepsilon_{xx} \varepsilon_{zz}} \right).
\]

(8)

The angle of refraction for the wave vector can be obtained as

\[
\tan \theta_{r,k} = \frac{\text{Re} (k_x)}{\text{Re} (k_z)}.
\]

(9)

And the angle of refraction for the poynting vector can be obtained as,

\[
\tan \theta_{t,\varepsilon} = \frac{\text{Re} (\varepsilon_{xx} k_x - \varepsilon_{zx} k_z)}{\text{Re} (k_z \varepsilon_{zz} - k_x \varepsilon_{zx})}.
\]

(10)

(see supplementary information for the details of theoretical derivations). For \(\varphi = 0\) \((x = x', y = y', z = z')\), \(\varepsilon_{xx} = \varepsilon_{zz} = 0\) and equation (10) reduces to the most familiar form [4, 5] for uniaxial metamaterials

\[
\tan \theta_{t,\varepsilon} = \frac{\varepsilon_{xx} k_x}{k_z \varepsilon_{zz} - k_x \varepsilon_{zx}}.
\]

(11)

For HMMs: \(\varepsilon_{xx} \varepsilon_{zz} < 0\). As a result, negative refraction can be achieved [4, 5]. To construct a HMM and to achieve the condition, \(\varepsilon_{xx} \varepsilon_{zz} < 0\), one has to use metal in the structure [2–5, 11].

Now when constructing an anisotropic metamaterial only with sub-wavelength dielectrics, all the tensor components in equation (1) are always positive. Therefore for non-rotated case, \(\varphi = 0\) and \(\varepsilon_{xx} = \varepsilon_{yy}, \varepsilon_{zz} = \varepsilon_{ll}, \varepsilon_{xx} = \varepsilon_{zx} = 0\). The right hand side of equation (11) is always positive (same is true for \(\varphi = \pm 90^\circ\)) and negative refraction cannot be achieved. However, interesting phenomenon can happen when the structure is rotated. From equation (10), depending on the values of \(\varepsilon_{xx}, \varepsilon_{xx} = \varepsilon_{zx}, \varepsilon_{xx} = \varepsilon_{zx}\), the right hand side of equation (10) can be negative. As a result, negative refraction can be achieved. This can be easily visualized by the iso-frequency contour in wave vector space of the proposed metamaterial.

Without using arbitrary values of \(\varepsilon_{xx}, \varepsilon_{xx}\) and \(\varepsilon_{zz}\), we are interested in practical realization of a deep sub-wavelength all dielectric multilayer structure. This multilayer structure is made by alternatively repeating two different dielectric materials with permittivities \(\varepsilon_{ll}\) and \(\varepsilon_{zz}\) and thicknesses \(t_1\) and \(t_2\) as shown in figure1.

The bloch equation of such 1D multilayer structure for non-rotated case can be derived as

\[
\cos (k_z t) = \cos (k_{zz} t_2) \cos (k_{zz} t_1) - \frac{1}{2} \left( \frac{\varepsilon_{zz} k_{zi}}{k_{zz} \varepsilon_{ll} + k_{zz} \varepsilon_{zz}} \right) \sin (k_{zz} t_2) \sin (k_{zz} t_1),
\]

(12)

where, \(k_{zz}^2 = \varepsilon_{ll} k_{zz}^2 - k_{zz}^2\); \(i = 1, 2\) and \(t = t_1 + t_2\) is the thickness of the unit cell. In deeply sub-wavelength limit (\(k_0 t \ll 1\)), the values of permittivity tensor in equation (1) can be calculated from equation (12) as

\[
\varepsilon_{ll} = f \varepsilon_{ll} + (1 - f) \varepsilon_{zz},
\]

(13a)
Figure 2. (a) Effective permittivity (tangential $\varepsilon_{\parallel}$ and perpendicular $\varepsilon_{\perp}$) of air–silicon and SiO$_2$–silicon multilayer metamaterial as a function of fill factor. (b) Effective permittivity $\varepsilon_{xx}$, $\varepsilon_{zz}$ and $\varepsilon_{xy}$ of the asymmetric anisotropic metamaterial as a function of rotation angle around the $y$ axis. Solid lines and symbols are for air–silicon and SiO$_2$–silicon based metamaterial respectively.

\[
\varepsilon_{\parallel} = \left( \frac{f}{\varepsilon_{\parallel}} + \frac{1 - f}{\varepsilon_{\perp}} \right)^{-1},
\]  

(13b)

where, $f$ is fill fraction given by, $f = \frac{h}{h + t}$. Equations (13a) and (13b) are well-known from EMT and widely used in literature [13, 14, 28–30]. The effective permittivity values for the rotated case can be calculated by equation (4) using the values from equations (13a) and (13b).

3. Numerical simulation and discussion

Figure 2(a) shows the effective permittivity ($\varepsilon_{\parallel}$ and $\varepsilon_{\perp}$) of air–silicon (solid lines) and SiO$_2$–silicon (symbols) multilayer metamaterial as a function of fill factor. Here at $\lambda = 1.55$ $\mu$m, permittivity values of air, silicon and SiO$_2$ are 1, 12.099 and 2.08 respectively. Unlike the case metal dielectric multilayers [2, 3, 11], both $\varepsilon_{\parallel}$ and $\varepsilon_{\perp}$ are confined to values between $\varepsilon_{\parallel1}$ and $\varepsilon_{\parallel2}$. Also for any value of fill factor, the values of $\varepsilon_{\parallel}$ and $\varepsilon_{\perp}$ cannot achieve negative values like in [2, 3, 11]. The effect of rotation on the optical properties (permittivity values) the structure is shown in figure 2(b). Here for example, we have considered air–silicon and SiO$_2$–silicon multilayer metamaterial. For any rotation angle $\varphi$ without 0 or $\pm \pi/2$, it can be observed that non-diagonal components of permittivity tensor $\varepsilon_{xy}$($\varepsilon_{xz}$) become non-zero. This non-zero off diagonal component is required to obtain negative refraction of the proposed structure. Also from the figure 2(a), it can be observed that higher difference between the permittivities of layer 1 and 2 results in a higher difference between the effective permittivities $\varepsilon_{\parallel}$ and $\varepsilon_{\perp}$. Higher difference between the permittivities of layer 1 and 2 also causes a higher non-diagonal component $\varepsilon_{xy}$($\varepsilon_{xz}$) of the permittivity tensor. The difference between $\varepsilon_{\parallel}$ and $\varepsilon_{\perp}$ plays a critical role in obtaining maximum angle of negative refraction as described later.

Figure 3 shows the isofrequency contour in $k$-space of the proposed AAM with rotation angle around $y$ axis (a) $\varphi = 0^\circ$ (b) $\varphi = 45^\circ$ (c) $\varphi = -45^\circ$ (d) $\varphi = 90^\circ$. (In this case, we are using air–silicon metamaterials). The blue circle represents the isofrequency contour of air. The green and red arrows indicate the power flow or group velocity direction. Blue lines are drawn to indicate specific tangential ($k_t$) wave vector for specific group velocity direction.
rotated case. Refraction of a wave incident from one medium to another can be easily visualized by the isofrequency contour in the wave vector space. As we know, the group velocity can be expressed as, $v_g = \frac{d\omega}{dk}$ which coincides with the direction of the time averaged Poynting vector or the direction of energy flow. So the direction of energy flow at any point $k$ of the isofrequency can be determined by drawing a normal line to the tangent at that point (indicated by the green and red arrows in figures 3(a)–(d)). Of course, the tangential momentum has to be conserved at the interface. The blue circle represents the isofrequency contour of air and the blue lines are drawn to specify different tangential momentum. Form figure 3(a) (rotation angle, $\varphi = 0^\circ$) it can be visualized that negative refraction would not be possible for the non-rotated case and also for the rotation angle, $\varphi = \pm 90^\circ$ (figure 3(d)), which coincides with the mathematical explanation above. But for any other rotation angle, negative refraction will be possible as indicated by figures 3(b) and (c).

The directions of wave and energy propagation for the extraordinary mode ($p$ polarization) in birefringent uniaxial medium do not coincide [20]. For non-rotated case ($\varphi = 0^\circ$, $\pm 90^\circ$), the group velocity or power flow direction always remains on different side of the surface normal indicating positive refraction. However for the case with rotation and due to elliptical isofrequency, the refracted wave can occur on the same side of the surface normal [18] indicating negative refraction (figures 3(b), (c)). In case of HMMs, negative refraction occurs due to the hyperbolic iso-frequency for which one must use metallic materials. In case of elliptical metamaterials, which is the case of our current discussion, negative refraction occurs due to the elliptical iso-frequency and only for the case when optical axis is rotated at certain angle to the interface normal. The physics behind such negative refraction is similar to the negative refraction occurring in natural birefringent matrual. In this case, birefringence is created artificially and this birefringence can be controlled by choosing suitable materials with higher permittivity difference and also the fill factor of the structure. Though all angle negative refraction will not be possible, broad angle negative refraction can be achieved easily. As lossless materials are used to construct the metamaterial, there will be no absorption and high transmission will be feasible. In addition, as the materials are almost non-dispersive in the considered frequency range, broadband negative refraction will be possible especially in the telecommunication wavelength bands.

Figure 4(a) shows the angle of refraction for the pointing vector as a function of incident angle for different combination of materials (air–SiO$_2$, air–GaAs, air–silicon, SiO$_2$–GaAs, SiO$_2$–silicon where material parameter are $\varepsilon_{air} = 1$, $\varepsilon_{SiO2} = 2.085$, $\varepsilon_{silicon} = 12.099$, $\varepsilon_{GaAs} = 11.38$) with fill factor 0.5 at $\lambda = 1.55 \mu m$ (b) 2D map of refraction angle for the pointing vector as a function of the incident angle and the rotation angle for SiO$_2$–silicon based AAM. The solid black curve corresponds to the angle of refraction equal to zero.
Table 1. Maximum angle of negative refraction and critical angle for different combination of materials along with natural ones.

| Combination (meta-material or natural) | Maximum angle of negative refraction (°) | Maximum critical incident angle (°) |
|--------------------------------------|----------------------------------------|-----------------------------------|
| Air–SiO₂                               | 3.78                                   | 4.54                              |
| Air–GaAs                               | 32.82                                  | 90                                |
| Air–silicon                            | 34.06                                  | 90                                |
| SiO₂–GaAs                              | 18.23                                  | 45.10                             |
| SiO₂–silicon                           | 19.39                                  | 49.99                             |
| SiO₂–germanium                         | 27.54                                  | 90                                |
| CaCO₃ (natural)                        | 5.77                                   | 9.03                              |
| YVO₄ (natural)                         | 5.88                                   | 12.20                             |
| a-BBO (natural)                        | 3.8                                    | 6.10                              |
| Nematic liquid crystal                 | 7.7                                    | 12.5                              |
| E7 [20]                                |                                        |                                    |
| Nematic liquid crystal [21]            | 10.4                                   | 14.5                              |

Optical parameters of materials used (at \( \lambda = 1.55 \text{ µm} \)):
\( \varepsilon_{\text{air}} = 1 \), \( \varepsilon_{\text{SiO₂}} = 2.085 \), \( \varepsilon_{\text{GaAs}} = 12.09 \), \( \varepsilon_{\text{Air}} = 11.38 \),
\( \varepsilon_{\text{germanium}} = 18.28 + 0.05i \) [32].

For CaCO₃ ordinary refractive index, \( n_a = 1.634 \), extraordinary refractive index, \( n_e = 1.477 \).

For a-BBO ordinary refractive index, \( n_{a} = 1.6466 \), extraordinary refractive index, \( n_{e} = 1.5416 \).

For YVO₄ ordinary refractive index, \( n_{a} = 1.9447 \), extraordinary refractive index, \( n_{e} = 2.1555 \).

For nematic liquid crystals data for maximum angle of negative refraction and critical incident angle has been collected from [20, 21].

\[ \theta_{\text{CIA}} = \sin^{-1} \left( \frac{\sin 2\varphi (\varepsilon_2 - \varepsilon_1) \sqrt{\varepsilon_2 \cos^2 \varphi + \varepsilon_1 \sin^2 \varphi}}{\sqrt{4\varepsilon_2 \varepsilon_1 + \sin^2 2\varphi (\varepsilon_2 - \varepsilon_1)^2}} \right) \] (15)

The angle of negative refraction reaches maximum value for rotation angle, \( \varphi_{\text{max}} = \tan^{-1} \sqrt{\varepsilon_1 / \varepsilon_2} \). From equation (14) it can be observed that maximum angle of negative refraction and the rotation angle required for maximum angle of negative refraction depend on both \( \varepsilon_{1} \) and \( \varepsilon_{2} \). From equations (13a) and (13b), it can be observed that \( \varepsilon_{1} \) and \( \varepsilon_{2} \) also fill factor \( f \). It is found that maximum angle of negative refraction is achieved for fill factor of 0.5. This indicates that thickness of layer 1 and 2 has to be same in the unit cell in order to achieve maximum angle of negative refraction. In order to increase maximum angle of negative refraction and also the CIA, materials with high permittivity contrast must be used to construct the multilayer metamaterial. For example, air–silicon AAM provides higher maximum angle of negative refraction than SiO₂–silicon AAM (see figure 4(a)).

In order to compare the negative refraction performance with natural birefringent materials such as YVO₄, CaCO₃, a-BBO, we have calculated maximum angle of negative refraction and CIA for both the natural materials and the metamaterial proposed in this article. Table 1 shows the maximum refraction and CIA for different combinations of materials used for constructing the AAM along with the natural uniaxial anisotropic materials such as YVO₄, CaCO₃, a-BBO [31, 32] and also liquid crystals [20, 21]. Optical properties (permittivity values) of all the materials used are provided in the table. Observing the data of table 1, it can be observed that proposed metamaterial can easily outperform the natural birefringent materials and liquid crystals for both maximum angle of negative refraction and CIA. Highest value of maximum angle of negative refraction and CIA is achieved for the AAM constructed by air and silicon (considering only lossless dielectrics at \( \lambda = 1.55 \text{ µm} \)).

As the materials in the considered frequency (telecommunication wavelength, \( \lambda = 1.55 \text{ µm} \)) range is lossless, probability of absorption loss is extremely low. We have calculated the transmission coefficient of our proposed AAM using \( 4 \times 4 \) transfer matrix [33–35] which takes the permittivity tensor values of equation (4) as input parameter. The method also considers the thickness of the structure; incident and exit medium’s refractive index. Figures 5(a) and (b) show the 2D map of transmission coefficient of a SiO₂–silicon and air–silicon AAM respectively where the thickness of the structure has been considered as \( d = \lambda / 2 \). It can be visualized that very high transmission can be achieved for a broad range of incident and rotation angles.

To verify our theoretical proposal, full wave simulations have been performed. Negative refraction can be observed from both the time average power flow or electric or magnetic field distributions [22, 36], figure 6 demonstrates the map of the real part of transverse magnetic field component \( H_{x} \) when a TM polarized Gaussian beam (\( \lambda = 1.55 \text{ µm} \)) is incident from air to air–silicon multilayer metamaterial. The structure is rotated at an angle \( \varphi \approx 59^\circ \). In figures 6(a) and (b), wave is incident at an angle \( \theta = 0^\circ \) and in figures 6(c) and (d) wave is incident at an angle \( \theta = 25^\circ \). For both \( \theta = 0^\circ \) and \( \theta = 25^\circ \) negative refraction is clearly visible. In figures 6(a) and (c), real multilayer structure has been considered where each layer thickness is 100 nm. But in figures 6(b) and (d), effective medium description (permittivity tensor values calculated from equation (4) for \( \varphi \approx 59^\circ \)) has been used to characterize the metamaterial in full wave simulation. The simulation results considering the real structure and the effective medium approximation are in quite good agreement. At both incident angles, negative refraction of light is evident, which coincides with our theoretical predictions. Two more important simulation results with different materials and different rotation angle are available in the supplementary.

In summary, the possibility of broad angle negative refraction by AAM using only nonmetallic and lossless materials has been theoretically demonstrated in the telecommunication wavelength band. This lossless metamaterial can have superior performance over natural uniaxial materials and also over metal based highly absorbing metamaterials. As dielectrics are less dispersive in the considered frequency range, the proposed metamaterial is expected to be broadband. Moreover, because of the absence of metal in the proposed multilayer metamaterial, its fabrication is expected to be much simpler than that of the metal based multilayer
Figure 5. 2D map of transmission co-efficient of p-polarized wave (wavelength, $\lambda = 1.55 \mu m$) as a function of incident angle and rotation angle calculated by $4 \times 4$ transfer matrix method for (a) SiO$_2$–silicon (b) air–silicon asymmetric anisotropic metamaterial. Thickness of the metamaterial structure considered in calculation, $d = \lambda/2$. Optical parameters of air, SiO$_2$ and silicon are given in table 1.

Figure 6. Full wave numerical simulations (map of the real part of the transverse magnetic field component, $H_y$) demonstrate negative refraction of a monochromatic ($\lambda = 1.55 \mu m$) TM-polarized Gaussian beam. The beam is incident from air to air–silicon multilayer metamaterial (thickness $d = 5 \mu m$) with incident angle (a), (b) $\theta = 0^\circ$ (c), (d) $\theta = 25^\circ$. The structure is rotated at an angle $\varphi \approx 59^\circ$ and each layer thickness is 100 nm. Figures (a) and (c) represent the field components for the real rotated multilayer metamaterial structure. Figures (b) and (d) represent the field components with its effective medium description.
metamaterials. Our ideas have been validated by full wave simulations considering realistic structures. We believe this work may find important applications in photonics, for example in lensing, beam splitting etc and also in new research involving ballistic electrons [17, 37] in semiconductors.

Supplementary article
Supplementary article contains derivations and also two additional full wave simulation results (figures S1 and S2).

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