Scalar perturbation of the viscosity dark fluid cosmological model

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Abstract

A general equation of state is used to model unified dark matter and dark energy (dark fluid), and it has been proved that this model is equivalent to a single fluid with time-dependent bulk viscosity. In this paper, we investigate scalar perturbation of this viscosity dark fluid model. For particular parameter selection, we find that perturbation quantity can be obtained exactly in the future universe. We numerically solve the perturbation evolution equations, and compare the results with those of \(^{-}\)CDM model. Gravitational potential and the density perturbation of the model studied here have the similar behavior with the standard model, though there exists significant value differences in the late universe.

PACS numbers: 98.80.-k,98.80.Cq
Keywords: dark energy theory, cosmological perturbation theory
I. INTRODUCTION

Astrophysics and cosmology observations in recent years delineate the cosmological picture on its constituents more and more accurately, i.e., the precision cosmology era comes. Except for the long standing puzzling dark matter component, an unknown cosmic matter-energy constituent referred to the so called dark energy may also exist that accelerates our universe expansion now, which contradicts with our traditional attitude on the behavior of conventional matter but it is eventually confirmed by recent observations like SNe Ia [1] [2] and CMB observations [3]. The cosmological dark sector, often divided as the mysterious DM and DE sectors respectively, takes around 95% of total energy budget of our universe. The concord ΛCDM model could be consistent with most of global astrophysics observational results. But the introduction of the cosmological constant simultaneously results in the yet to answer problems related directly to how to understand the fundamental physics theory, like the fine-tuning and the coincidence problems respectively. At the same time, “most” of course does not equal to “all”, some astrophysics problems still need be clarified and solved in the framework of the ΛCDM model [4], such as the the core singularities of the cold matter halo profiles. With the aim to understand the cosmic acceleration or dark energy phenomena, many theoretical models have been proposed, like the scalar field models and the modified Einstein gravity models [5] [6] [7].

Due to the limited scope of our experimental and observational tools, we have not yet been able to understand the “dark” nature and to detect the origin of DM and DE. Purely gravitational probes can not provide enough information to differentiate these two kinds of mysterious constitutions either. Therefore, from the phenomenological and practical point of view, a single (unified DM with DE) fluid description may be more plausible at least in the cosmic evolution description, which utilizes a single equation of state to model the dark matter and dark energy contributions together [8] [9] [10] [11] [12] [13]. Generally, such models has a non-constant equation of state (EoS), which reflects both its dynamical and thermodynamics characters. The density dependent equation of state is widely investigated, such as the famous Chaplygin gas and generalized Chaplygin gas models, which assume an EoS form like the \( p = -A/\rho^\alpha \) where the \( \alpha \) is a model parameter. Another practical method to modify the EoS is by the introduction of cosmic viscosity media contribution which replaces the simplest perfect fluid EoS on a more physical and realistic basis. In the homogeneous and isotropic Friedmann-Robertson-Walker frame, only a bulk viscosity term which behaves as an additive pressure contribution can mimic both the two dark components and their coupling effects by playing a main role to influence the cosmic evolution. Different forms of the viscosity coefficients have been proposed like the density \( \rho \) dependent [14] or the redshift \( z \) dependent [15].

In this letter, we will proceed our effort on the investigation of the viscosity dark fluid model to study the scalar perturbation of this model, for perturbation analysis can provide us a powerful tool to differentiate and constrain cosmology models finely as the calculation of perturbation quantities links the theoretical models with more plentiful and precise observations, like the cosmic microwave background (CMB) and large scale structure observation. There have been some researches on the perturbation evolution of the viscosity models [16] [17] [18], by which we know that after corresponding model parameters chosen properly, the Chaplygin gas formulation can be viewed as a special case of the density dependent viscosity model. In the non-perturbative (zero order) level, the Chaplygin gas model can be exactly solved and fit the observational data well. But it has been found that in the perturbation level, there exist some unacceptable behaviors, like the blow up of density perturbation evolution and other peculiar behaviors [18] [19]. One motivation to build other kinds of the viscosity models is to overcome these difficulties the Chaplygin gas models possess. Here, we will consider a time-dependent viscosity coefficient model, which is equivalent to the introduction of a general Equation of state(EoS) [20]. The general EoS is

\[
p = (\gamma - 1)\rho + p_0 + w_1 H + w_2 H^2 + w_3 H^3 + w_4 H^4.
\] (1)

In the background, this model can fit the current astrophysics observational datasets consistently. We derive
its perturbation equations that govern the evolution of gravitational potential and density perturbation below. We numerically solve the perturbation equation, and compare it with that of conventional ΛCDM model and the Chaplygin gas model finding that the dark fluid model behaves well in different scales. Though there exists some value difference between the ΛCDM model and the dark fluid model in the late time evolution, their gravitational potential and density contrast shape and evolution behavior are similar by plotting respectively.

This paper is organized as follows: In Sec. II, we summarize the calculations of scalar perturbation, and give the general evolution equation of the gravitational potential. In Sec. III, we briefly review the background evolution of the dark fluid model. In Sec. IV, we discuss the perturbation evolution of the dark fluid model. In Sec. V, we numerically solve the perturbation equation and compare it with other models. Finally, we present the conclusions in the last section.

II. CALCULATIONS OF SCALAR PERTURBATION

In this paper, we choose Newtonian gauge to calculate the scalar perturbation

\[ ds^2 = -(1 + 2\phi)dt^2 + a(t)^2\delta_{ij}(1 - 2\psi)dx^i dx^j. \]  

(2)

If making the assumption here that there is no contribution from anisotropy inertia, it concludes that \( \phi = \psi \).

Generally, Einstein field equation with perturbed metric takes the form (for simplicity, we set \( \kappa = 1 \) hereafter.) \[21\] \[22\]

\[ \frac{\dot{a}}{a} \phi + \frac{\dot{a}^2}{a^2} \phi - \frac{1}{3} \frac{\nabla^2}{a^2} \phi = -\frac{1}{6} \delta \rho, \]  

(3)

\[ \dot{\phi} + \frac{\dot{a}}{a} \phi = -\frac{1}{2} (\rho + p) \delta u, \]  

(4)

\[ \ddot{\phi} + 3 \left( \frac{\dot{a}}{a} \right) \dot{\phi} + \left( \frac{\ddot{a}}{a} + \frac{1}{3} \frac{\nabla^2}{a^2} \right) \phi = \frac{1}{6} (\delta \rho + 3 \delta p), \]  

(5)

where \( \delta \rho \) and \( \delta p \) are first order perturbation to zero-order cosmic density \( \rho \) and pressure \( p \) respectively. Perturbation to velocity of cosmic fluid \( \delta u \) is decomposed as \( \delta u_i = \nabla_i \delta u + \delta u'_i \). \( \delta u \) is the scalar velocity potential. \( \delta u'_i \) is a divergenceless vector, which we also assume here contributes no effect. \( \nabla \) denotes gradient with respect to comoving coordinate. From Eq. (3) and (4), we obtain a constrain on first-order perturbation quantity \( \phi, \delta \rho \) and \( \delta u \)

\[ 2 \frac{\nabla^2}{a^2} \phi - \delta \rho + 3 \frac{\dot{a}}{a} (\rho + p) \delta u = 0. \]  

(6)

Also momentum and energy conservation equation to first order in perturbation could be derived

\[ \delta p + \partial_i [(\rho + p) \delta u] + \frac{\dot{a}}{a} (\rho + p) \delta u + (\rho + p) \phi = 0, \]  

(7)

\[ \dot{\delta \rho} + \frac{3\dot{a}}{a} (\delta \rho + \delta p) + \nabla^2 \left[ \frac{1}{a^2} (\rho + p) \delta u \right] - 3(\rho + p) \dot{\phi} = 0. \]  

(8)

A generalized parameterized equation of state may have the form as

\[ p = (\gamma - 1)\rho + f(\rho; \dot{\rho}; \alpha_i) + ... \]  

(9)

or

\[ p = (\gamma - 1)\rho + g(H; \dot{H}; \beta_i) + ..., \]  

(10)
where $\alpha_i$ and $\beta_i$ are model parameters. When the models are discussed in flat universe $k = 0$, two parameterized function $f(\rho)$ and $g(H)$ are categorized in one class. The equation of state reduces to the perfect fluid case with EoS as $p = (\gamma - 1)\rho$ when the model parameters $\alpha_i$ vanish.

For a barotropic equation of state, adiabatic sound speed is defined as $c_a^2 = \frac{d\rho}{d\rho}$, hence the ratio of pressure and density perturbation is $c_a^2$. Therefore we could eliminate $\delta\rho$ from Eq. (3) and (5), then obtain the equation governs the gravitational potential

$$\ddot{\phi} + (4 + 3c_a^2)\dot{\phi} + [2\frac{\ddot{a}}{a} + (1 + 3c_a^2)(\frac{\dot{a}}{a})^2 - c_a^2\nabla^2\phi] = 0 \hspace{1cm} (11)$$

If write metric perturbation as Fourier integral

$$\phi(x,t) = \int d^3q e^{iq\cdot x}\phi_q(t), \hspace{1cm} (12)$$

we can make the substitution $\nabla^2 \rightarrow q^2$, where $q$ is the wave number.

III. BACKGROUND EVOLUTION

In this section, we briefly review the background evolution behavior of the viscosity dark fluid model. A general form of EoS investigated in [20] is

$$p = (\gamma - 1)\rho + p_0 + w_H H + w_{H2} H^2 + w_{dH} \dot{H} \hspace{1cm} (13)$$

One can prove that this generally parameterized EoS can be effectively equivalent to a single fluid with a time-dependent bulk viscosity:

$$\zeta = \zeta_0 + \zeta_1 \frac{\dot{a}}{a} + \zeta_2 \frac{\ddot{a}}{a} \hspace{1cm} \text{(14)}$$

and three parameters in the viscosity coefficient correspond to EoS parameters as

$$w_H = -3\zeta_0 \hspace{1cm} \text{(15a)}$$
$$w_{H2} = -3(\zeta_1 + \zeta_2) \hspace{1cm} \text{(15b)}$$
$$w_{dH} = -3\zeta_2 \hspace{1cm} \text{(15c)}$$

Flat Friedmann-Robertson-Walker metric reads

$$ds^2 = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j \hspace{1cm} (16)$$

The energy-momentum tensor with modified EoS could be written as

$$T_{\mu\nu} = \rho U_\mu U_\nu + p H_{\mu\nu} \hspace{1cm} (17)$$

where $\bar{p}$ represents modified pressure, in model concerned it is Eq.(11), and in comoving coordinate $U^\mu = (1, 0, 0, 0)$ . Due to the correspondence between this modified EoS model and viscosity model, pressure could also be $\bar{p} = p - \zeta \theta$, where $\theta = U_{\mu i} U_i = 3\dot{a}/a$.

Using the EoS above and Friedmann equation, the equation of scale factor $a(t)$ evolution could be obtained

$$\frac{\ddot{a}}{a} = \frac{-(3\gamma - 2)/2 - (\kappa^2/2)w_{H2} + (\kappa^2)w_{dH}}{1 + (\kappa^2)w_{dH}} \left( \frac{\dot{a}}{a} \right)^2 + \frac{-(\kappa^2)w_H}{1 + (\kappa^2/2)w_{dH}} \frac{\dot{a}}{a} + \frac{-(\kappa^2/2)p_0}{1 + (\kappa^2/2)w_{dH}} \hspace{1cm} (18)$$

After redefining model parameters, there will be a compact form of evolution equation, at the same time, this form is comparable to perfect fluid case

$$\frac{\ddot{a}}{a} = -\frac{3\gamma - 2}{2} \left( \frac{\dot{a}}{a} \right)^2 + \frac{1}{T_1} \frac{\dot{a}}{a} + \frac{1}{T_2} \hspace{1cm} (19)$$

4
where

\[ \tilde{\gamma} = \frac{\gamma + (\kappa^2/3)w_{H2}}{1 + (\kappa^2/2)w_{dH}}, \]  

(20)

\[ \frac{1}{T_1} = \frac{- (\kappa^2/2)w_H}{1 + (\kappa^2/2)w_{dH}}, \]  

(21)

\[ \frac{1}{T_2} = \frac{- (\kappa^2/2)p_0}{1 + (\kappa^2/2)w_{dH}}, \]  

(22)

\[ \frac{1}{T^2} = \frac{1}{T_1^2} + \frac{6 \tilde{\gamma}}{T_2^2}. \]  

(23)

The solution of scale factor is

\[ a(t) = a_0 \left\{ \frac{1}{2} \left( 1 + \tilde{\gamma} \theta_0 T - \frac{T}{T_1} \right) \exp \left[ \frac{t - t_0}{2} \left( \frac{1}{T} - \frac{1}{T_1} \right) \right] + \right. \]

\[ \left. \frac{1}{2} \left( 1 - \tilde{\gamma} \theta_0 T + \frac{T}{T_1} \right) \exp \left[ - \frac{t - t_0}{2} \left( \frac{1}{T} - \frac{1}{T_1} \right) \right] \right\} ^{2/3}. \]  

(24)

From Friedmann equation, cosmic density evolution reads

\[ \rho(t) = \frac{1}{3 \kappa^2 \tilde{\gamma}^2} \left[ \left( 1 + \tilde{\gamma} \theta_0 T - \frac{T}{T_1} \right) \left( \frac{T}{T_1} + \frac{T}{T_2} \right) \exp \left( \frac{t - t_0}{T_1} \right) - \left( 1 - \tilde{\gamma} \theta_0 T + \frac{T}{T_1} \right) \left( \frac{T}{T_2} \right) \right]^2. \]  

(25)

The case above is for \( \tilde{\gamma} \neq 0 \), when take the limit of \( \tilde{\gamma} \), solution could be obtained

\[ a(t) = a_0 \exp \left[ \left( \frac{1}{3} \theta_0 T_1 + \frac{T_1}{T_2} \right) \left( \exp \left( \frac{t - t_0}{T_1} \right) - 1 \right) - \frac{T_1(t - t_0)}{T_2} \right], \]  

(26)

and cosmic density evolution

\[ \rho(t) = \frac{3}{\kappa^2} \left[ \frac{1}{3} \theta_0 \exp \left( \frac{t - t_0}{T_1} \right) + \frac{T_1}{T_2} \left( \exp \left( \frac{t - t_0}{T_1} \right) - 1 \right) \right]. \]  

(27)

Using Friedmann equation, Eq. (1) could be converted into a form, r.h.s of which is only the function of density \( \rho \)

\[ p = (\tilde{\gamma} - 1)\rho - \frac{2}{T_1} \sqrt{\frac{\rho}{3}} - \frac{2}{T_2} \rho. \]  

(28)

where parameters are defined the same as (20)-(23).

IV. EVOLUTION OF SCALAR PERTURBATION

A. Evolution equation

In adiabatic perturbation case, ratio of adiabatic density and pressure perturbation equals to the adiabatic sound speed

\[ \frac{\delta p_a}{\delta \rho_a} = c_a^2. \]  

(29)
In this paper we pay our attention on single fluid model and investigate perturbation in adiabatic region, that is, pressure perturbation is proportional to the density perturbation. For a barotropic EoS model, the proportional efficient between pressure and density perturbation is merely a function of density \( \rho \). On the other hand, in the condition of single (dark) fluid, we assume that dark sector interacts with baryon matter and the least dominated radiation negligibly.

With the barotropic EoS \( (28) \), adiabatic sound speed is \( c_s^2 = \gamma - 1 - \frac{1}{\sqrt{3T_1}} \frac{1}{\sqrt{p}} \). From Eq. \( (11) \), we obtain the correspondent equation of single(dark) fluid model

\[
\ddot{\phi}_q + f \dot{\phi}_q + g \phi_q = 0, \tag{30}
\]

where two coefficients are defined by

\[
f(H) = (3\tilde{\gamma} + 1)H - \frac{1}{T_1} \tag{31}
\]

and

\[
g(H; q) = 2\dot{H} + 3\tilde{\gamma}H^2 - \frac{1}{T_1}H - (\tilde{\gamma} - 1 - \frac{1}{3T_1H}) \frac{q^2}{a^2}. \tag{32}
\]

Conveniently, one can decompose metric perturbation as \( \phi_q(t) = v(t)p(t) \), therefore obtains a differential equation about \( v(t) \) and \( p(t) \)

\[
\ddot{v} + (2\frac{\dot{p}}{p} + f)\dot{v} + (\frac{\ddot{p} + \dot{f} + gp}{p})v = 0. \tag{33}
\]

If we choose the function \( p(t) \) properly as

\[
p(t) = \exp\left(-\frac{1}{2} \int_0^t dt' f(t') \right), \tag{34}
\]

then the damping term can be eliminated, so Eq. \( (33) \) reduces to

\[
\ddot{v} - (\frac{1}{2}f + \frac{1}{4}f^2 - g)v = 0. \tag{35}
\]

which takes a harmonic oscillator form. For the short wave limit, we have \( \ddot{v} + g(q; H)v = 0 \) and \( g \approx -(\tilde{\gamma} - 1 - \frac{1}{3T_1H}) \frac{q^2}{a^2} \). If WKB condition is fulfilled, we have the approximate solution

\[
v \approx \frac{1}{g(q; H)^{1/4}} \left\{ c_+ \exp\left( i \int dt \sqrt{g(q; H)} \right) + c_- \exp\left( -i \int dt \sqrt{g(q; H)} \right) \right\}. \tag{36}
\]

**B. The future solution of the gravitational potential**

Eq. \( (30) \) is too complicated to solve exactly, so here we will consider a simpler asymptotic case and try to extract the solution in this limit. If we strict that parameter \( T_1 \) should be negative to confirm that sound speed is real, then we could see from Eq. \( (19) \) in \[20\] that cosmic density approaches a constant value

\[
\rho = \frac{3T_1}{T_2^2} \tag{37}
\]

as \( t \to \infty \), and negative \( T_1 \) also confirms the positive of energy. It concludes that the universe will enter a de Sitter period then: \( H \to H_\Lambda \), where \( H_\Lambda \) is a constant. Also the scale factor evolves exponentially:

\[
a(t) = e^{H_\Lambda t} \quad H_\Lambda = \sqrt{-\frac{T_1}{T_2}}, \tag{38}
\]
Eq. \( \text{(35)} \) becomes
\[
\ddot{v} - \left[ \frac{1}{4} f_\Lambda^2 - g_\Lambda(q) \right] v = 0,
\]  
where \( f_\Lambda = (3\tilde{\gamma} + 1)H_\Lambda - \frac{1}{T_1} \) is the late time asymptotic constant of function \( f(H) \) and \( g_\Lambda(q) = 3\tilde{\gamma}H_\Lambda^2 - \frac{1}{T_1}H_\Lambda - (\tilde{\gamma} - 1 - \frac{1}{3T_1 H_\Lambda}) q^2 \) is the function of \( g(H; q) \) when Hubble parameter approaches constant. Then we can solve the differential equation \( \text{(38)} \), and get
\[
v(t) = C_1 J_m \left( \sqrt{\frac{ma^{-1}(t)}{H_\Lambda}} \right) \left[ 1 + i \frac{m}{H_\Lambda} \right] + C_2 J_{-m} \left( \sqrt{\frac{ma^{-1}(t)}{H_\Lambda}} \right) \left[ 1 - i \frac{m}{H_\Lambda} \right],
\]  
where two parameter \( m \) and \( n \) are defined for simplicity by
\[
m = \frac{1}{4} f_\Lambda^2 - 3\tilde{\gamma}H_\Lambda^2 + \frac{1}{T_1}H_\Lambda = \frac{1}{2} \left[ (3\tilde{\gamma} - 1)^2 H_\Lambda^2 - \frac{1}{T_1} \right]^2,
\]  
\[
n = -\frac{1}{3} (\tilde{\gamma} - 1 - \frac{1}{3T_1 H_\Lambda}) \frac{q^2}{1}
\]  
When the physical wavelength is much longer than the Hubble radius
\[
\lambda_{\text{phys}} \gg H^{-1} \Rightarrow \frac{q}{a(t)} \ll H.
\]  
The solution in this long wave limit could be obtained directly from Eq. \( \text{(30)} \). Assuming the solution takes the form as
\[
\phi_q(t) = e^{bt}.
\]  
After inserting it into Eq. \( \text{(30)} \), we get
\[
b^2 + [(3\tilde{\gamma} + 1)H_\Lambda - \frac{1}{T_1}] b + 3\tilde{\gamma}H_\Lambda^2 - \frac{1}{T_1} H_\Lambda = 0.
\]  
This quadratic equation has two dependent solution, therefore the solution of Eq.\( \text{(30)} \) reads
\[
\phi_q = c_1 \exp \left[ (-H_\Lambda + \frac{1 - H_\Lambda}{T_1}) t \right] + c_2 \exp \left[ (-6\tilde{\gamma}H_\Lambda + \frac{1 + H_\Lambda}{T_1}) t \right]
\]  
This unstable solution exponentially increases or decreases in the infinite future, which depends on the parameters.

For the opposite limit, we consider large wave number solution with \( q \gg aH \), and fix at some conformal wave number \( k = q/a \). Then we have the simple solution in this limit
\[
v \propto e^{\sqrt{lt}},
\]  
where \( l = \tilde{\gamma} - 1 - \frac{1}{3T_1 H_\Lambda} \). Together with the definition \( \text{(34)} \), we see this is an exponential decay solution for the gravitational potential, which is also unstable.

V. NUMERICAL RESULTS

A. Comparison models

In the numerical results presented below, we also calculate perturbation in \( \Lambda \)CDM model and the Chaplygin gas model numerically as a comparison, so we present a short review here.

(a) \( \Lambda \)CDM model: The cosmological constant does not contribute perturbation in the total energy density
\[
\delta \rho_\Lambda = 0.
\]
The density perturbation comes from matter density, \( \delta \rho = \delta \rho_m \). The cosmological constant plays a role in influencing the background evolution of the universe, especially by Hubble parameter, so it will be imprinted into the evolution of matter perturbation. We could obtain the scale-independent gravity perturbation equation from Eqs. (3) and (5):

\[
\ddot{\phi}_{mq} + 4H \dot{\phi}_{mq} + (2 \dot{H} + 3H^2) \phi_{mq} = 0.
\] (49)

In the matter dominated era

\[
H^2 = \frac{1}{3} \rho_m = \Omega_m a^{-3} = \frac{2}{3} t^{-2}.
\] (50)

Also the solution \( \phi_{mq} = \phi_{mq0} + \frac{3 \rho_m}{3} t^{-5/3} \) is well-known.

(b) Chaplygin gas model: It has the EoS form

\[
p = -\frac{A}{\rho}
\] (51)

If we use this EoS to model single fluid universe, then get Hubble parameter

\[
H(z) = [\tilde{\Omega}(1 + z)^6 + (1 - \tilde{\Omega})]^{1/4}
\] (52)

and the adiabatic sound speed

\[
c_a^2 = \frac{A}{\rho^2}
\] (53)

Hence \( H \) and \( c_a^2 \) in Eq. (50) are specified.

B. Effective dark fluid

1. Background

For simplicity, we set parameter \( T_2 \to \infty \), which we call this case effective dark fluid. This means if \( \rho \to 0 \), pressure \( \rho \) vanishes too, and there is not a cosmological constant like pressure contribution, which could be seen from EoS (28). Hence Eq. (19) becomes

\[
\frac{\ddot{a}}{a} = -\frac{3\bar{\gamma}}{2} \left( \frac{\dot{a}}{a} \right)^2 + \frac{1}{T_1} \left( \frac{\dot{a}}{a} \right)
\] (54)

This differential equation about scale factor can be seen as a special case given by a general EoS [24] [25] [26]

\[
p = -\rho - Ap^\alpha - BH^{2\beta},
\] (55)

which can give

\[
\frac{\ddot{a}}{a} = -\frac{3\bar{\gamma}}{2} \left( \frac{\dot{a}}{a} \right) + \lambda \left( \frac{\dot{a}}{a} \right)^m + \mu \left( \frac{\dot{a}}{a} \right)^n + \nu.
\] (56)

Eq. (54) can be converted as a differential equation of \( H(a) \)

\[
a \frac{dH}{da} = -\frac{3\bar{\gamma}}{2} H + \frac{1}{T_1}.
\] (57)

Its solution is

\[
H(a)/H_0 = \Omega a^{-3\bar{\gamma}/2} + (1 - \Omega),
\] (58)
where $\Omega = 1 - \frac{2}{3\tilde{\gamma}_T H_0}$, and we have already set $a_0 = 1$. We use this Hubble parameter with $\tilde{\gamma} = 0.9$ and $\tilde{\gamma} = 1.2$ to calculate the distance

$$D_L(z) = H_0(1 + z) \int_0^z \frac{1}{H(z')} dz'$$

and the distance modulus. We compare it with the supernova data, which is plotted in FIG. 1. We see that model with parameter $\tilde{\gamma} = 0.9$ (blue) and $\tilde{\gamma} = 1.2$ nearly can not be discriminated in the late time (small redshift, more data in this region has been obtained). There are some differences for larger redshift. But in the whole, both two parameters consist with the data well.

![FIG. 1: The relation between distance modulus and redshift. Black and Blue lines correspond to the theoretical calculation curves with $\tilde{\gamma} = 1.2$ and 0.9 respectively. Both of them can fit data in an acceptable level.](image)

2. The gravitational potential and the density perturbation

After inserting the Hubble parameter into Eq. (30), we numerically solve this scale-dependent model. Results are illustrated in FIG. 2. As a comparison, $\Lambda$CDM and Chaplygin gas model are solved and plotted (green and red line respective) too. In the early time, the difference between $\Lambda$CDM and dark fluid model is tiny. Both of them behaves nearly as a constant. In the late time, two models give the same shape of the gravitational potential. Though these two models predicts the decay of potential, there exists a value contrast around $5\% - 10\%$. The quantity of contrast dependents on the value of model parameter $\tilde{\gamma}$. Here we point that the dark fluid model predicts more similar potential as $\Lambda$CDM than the Chaplygin gas model. On the other hand, it can be qualitatively seen the level of scale-dependent of the dark fluid model. We plot different results with $q = 0.005$, 0.5 and 1.5.

For the small scale, the perturbation evolution of the dark fluid model with $\tilde{\gamma} = 0.9$ is significantly different. The gravitational potential decays much earlier, which contradicts with $\Lambda$CDM. Numerical results for $\tilde{\gamma} < 1$ indicate that this condition strongly influences the early evolution of perturbation quantity (we only plot $\tilde{\gamma} = 0.9$ case here.), which puts strict constrain on parameter region. Also, if $\tilde{\gamma}$ is bigger than 1, the positive of adiabatic sound speed could be easily fulfilled.

From Eq. (3), density perturbation could be expressed as

$$\delta \rho = -6H^2 \dot{\phi} - 6H^2 \phi + 2\frac{q^2}{a^2} \phi.$$  

It tells us that once we have the solution of the gravitational potential we could get the information of the density contrast. During the early time ($\dot{\phi} \simeq 0$) and in the very large scale ($q \ll aH$), the density contrast
and the gravitational potential is linked by the Hubble parameter $\delta \rho \simeq -6H^2\phi$. Always define

$$
\delta = \frac{\delta \rho}{\rho} = -2\frac{\dot{\phi}}{\dot{H}} - 2\phi + \frac{2}{3}\frac{q^2}{a^2H^2}\phi,
$$

where Friedmann equation is used in the last step. Numerical results of the density perturbation $\delta$ for different $q$ is plotted in FIG. 3. The numerical curves have the similar shape, but for modified models, the density perturbation is suppressed in the late time. For the large scale, the density perturbation evolution in the dark fluid model increases linearly in the late universe. $\delta$ deviates from that of $\Lambda$CDM in the late time, and suppressed today for different scale. The values enhanced or depressed dependent on parameter $\tilde{\gamma}$ and scale.

VI. CONCLUSION AND DISCUSSION

In this paper, we investigate extensively the dark fluid model proposed in [20], equivalently this model can be viewed as a single fluid with time-dependent bulk viscosity. Scale factor and density evolution can be exactly solved in this model. In the background, the dark fluid model can fit the supernova data acceptable. Our main task in this paper is to analysis the behavior of this model in the perturbation level. We derive equations govern the perturbation quantities. For the condition that $T_1$ is smaller than 0, the universe will enter de Sitter phase in the $t \to \infty$ future. We solve exactly the gravitational equation in this condition and obtain the solution for both long and short wave case. Generally, the perturbation evolution equations are solved numerically. When compare the results with those of $\Lambda$CDM model, we find that
• In the early time and the large scale, both the gravitational potentials of two models behave as a constant.

• Though the gravitational potentials of two models have similar behavior and shape, as can be seen from FIG. 3, there exists about 5% – 10% significant value difference in the late time.

Perturbation analysis also provides constraint on model parameter. For different selection of parameter $\tilde{\gamma}$, both $\tilde{\gamma} < 0$ and $\tilde{\gamma} > 0$ can give consistent prediction curve of distance modulus, but numerical results indicate in $\tilde{\gamma} < 0$ case, the gravitational potential deviates from $\Lambda$CDM significantly from the early time. This result strongly constrain the selection region of $\tilde{\gamma}$. We suggest $\tilde{\gamma} > 0$, which can also produce positive sound speed naturally.

Acknowledgements

This work is partly supported by NSF of China under Grant No.10675062 and by the project of knowledge Innovation Program (PKIP) of Chinese Academy of Sciences under Grant No.KJCX2.YW.W10 through the KITPC.

[1] A. G. Riess et al., Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant, Astron. J. 116 (1998) 1009, arXiv:astro-ph/9805201.
[2] S. Perlmutter et al., Measurements of Omega and Lambda from 42 High-Redshift Supernovae, Astrophys. J. 517 (1999) 565, arXiv:astro-ph/9812133v1.
[3] E. Komatsu et al., Five-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation, Astrophys. J. Suppl. 180 (2009) 330, arXiv:0803.0547.
[4] L. Perivolaropoulos, Six Puzzles for LCDM Cosmology, arXiv:0811.4684.
[5] J. Frieman, M. Turner and D. Huterer, Dark Energy and the Accelerating Universe, Ann. Rev. Astron. Astrophys. 46 (2008) 385, arXiv:0803.0982.
[6] R. Caldwell and M. Kamionkowski, The Physics of Cosmic Acceleration, Ann. Rev. Nucl. Part. Sci. 59 (2009) 397, arXiv:0903.0866.
[7] A. Silvestri and M. Trodden, Approaches to Understanding Cosmic Acceleration, Rept. Prog. Phys. 72 (2009) 096901, arXiv:0904.0024.
[8] J. Ren and X.H. Meng, Dark viscous fluid described by a unified equation of state in cosmology, Int. J. Mod. Phys. D 16 (2007) 1341, arXiv:astro-ph/0605010.
[9] V. Folomeev and V. Gurovich, Viscous dark fluid, Phys. Lett. B 661 (2008) 75, arXiv:0710.0210.
[10] A. Balbi, M. Bruni and C. Quercellini, $\Lambda$CDM: Observational constraints on unified dark matter with constant speed of sound, Phys. Rev. D 76 (2007) 103519, arXiv:astro-ph/0702423.
[11] C. Quercellini, M. Bruni and A. Balbi, Affine equation of state from quintessence and k-essence fields, Class. Quant. Grav. 24 (2007) 5413, arXiv:0708.3687.
[12] D. Pietrobon, A. Balbi, M. Bruni and C. Quercellini, Affine parameterization of the dark sector: constraints from WMAP5 and SDSS, Phys. Rev. D 78 (2008) 083510, arXiv:0807.5077.
[13] M. Kunz, A.R. Liddle, D. Parkinson and C. Gao, Constraining the dark fluid, Phys. Rev. D 80 (2009) 083533, arXiv:0908.3197.
[14] J. Barrow, The Deflationary Universe: An Instability Of The De Sitter Universe, Phys. Lett. B 180 (1986) 335.
[15] X.H. Meng and X. Dou, Friedmann Cosmology with Bulk Viscosity: A Concrete Model for Dark Energy, Commun. Theor. Phys. 52 (2009) 377, arXiv:0812.4904.
[16] J.C. Fabris, S.V.B. Goncalves and R. de Sa Ribeiro, Bulk viscosity driving the acceleration of the Universe, Gen. Rel. Grav. 38 (2006) 495, arXiv:astro-ph/0503362.
[17] B. Li and J. D. Barrow, Does Bulk Viscosity Create a Viable Unified Dark Matter Model?, Phys. Rev. D 79 (2009) 103521, [arXiv:0902.3163].

[18] W.S. Hipolito-Ricaldi, H.E.S. Velten and W. Zimdahl, Non-adiabatic dark fluid cosmology, JCAP 0906 (2009) 016, [arXiv:0902.4710].

[19] H. B. Sandvik, M. Tegmark, M. Zaldarriaga and I.Waga, The end of unified dark matter?, Phys. Rev. D 69 (2004) 123524.

[20] J. Ren and X.H. Meng, Cosmological model with viscosity media (dark fluid) described by an effective equation of state, Phys. Lett. B 633 (2006) 1, [arXiv:astro-ph/0511163].

[21] V. Mukhanov, Physical Foundations of Cosmology (Cambridge University Press, 2005).

[22] S. Weinberg, Cosmology (Oxford University Press, 2008).

[23] M. Hicken et al., Improved Dark Energy Constraints from 100 New CfA Supernova Type Ia Light Curves, Astrophys. J. 700 (2009) 1097, [arXiv:0901.4804].

[24] S. Nojiri and S.D. Odintsov, Inhomogeneous Equation of State of the Universe: Phantom Era, Future Singularity and Crossing the Phantom Barrier, Phys. Rev. D 72 (2005) 023003, [arXiv:hep-th/0505215].

[25] S. Capozziello, V. F. Cardone, E. Elizalde, S. Nojiri, and S.D. Odintsov, Observational constraints on dark energy with generalized equations of state, Phys. Rev. D 73 (2006) 043512, [arXiv: astro-ph/0508350].

[26] S. Nojiri, S.D. Odintsov and T. Tsujikawa, Properties of singularities in (phantom) dark energy universe, Phys. Rev. D 71 (2005) 063004, [arXiv:hep-th/0501025].