ANOMALOUS COUPLINGS OF TYPE 0 D-BRANES

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Abstract
Closed type 0 string theories and their D-branes are introduced. The full Wess-Zumino action of these D-branes is derived. The analogy with type II is emphasized throughout the argument.

Type 0 Strings and D-branes. Type 0 string theory is a non-supersymmetric, modular invariant theory of closed strings. The presence of a tachyonic state in their spectrum makes type 0 strings much harder to analyse than the supersymmetric type II strings. Nevertheless, type 0 and type II strings have many features in common. In this contribution this fact will be exploited to show that type 0 D-branes have anomalous terms in their worldvolume action. The analysis is based on Ref. [1], to which we refer for a more detailed treatment and for a more complete list of references.

In the Neveu-Schwarz-Ramond formulation, type II string theories are obtained by imposing independent GSO projections on the left and right moving part. This amounts to keeping the following (left,right) sectors:

\[ \begin{align*}
\text{IIB} & : (\text{NS}+, \text{NS}+) , \ (\text{R}+, \text{R}+) , \ (\text{R}+, \text{NS}+) , \ (\text{NS}+, \text{R}+) ; \\
\text{IIA} & : (\text{NS}+, \text{NS}+) , \ (\text{R}+, \text{R}−) , \ (\text{R}+, \text{NS}+) , \ (\text{NS}+, \text{R}−) ,
\end{align*} \]

where for instance R± is the Ramond sector projected with \( P_{\text{GSO}} = (1 \pm 1^{F})/2 \), \( F \) being the world-sheet fermion number.
The type 0 string theories contain instead the following sectors:

$$0B : (\text{NS}+, \text{NS}+), (\text{NS}−, \text{NS}−), (\text{R}+, \text{R}+), (\text{R}−, \text{R}−);$$

$$0A : (\text{NS}+, \text{NS}+), (\text{NS}−, \text{NS}−), (\text{R}+, \text{R}−), (\text{R}−, \text{R}+).$$

These theories do not contain bulk spacetime fermions, which would have to come from “mixed” (R,NS) sectors. The inclusion of the NS-NS sectors with odd fermion numbers means that the closed string tachyon is not projected out. The third difference with type II theories is that the R-R spectrum is doubled. For instance, in the 0B case, beside the IIB R-R potentials $C_{p+1}$ contained in bispinors of the (R+,R+) sector, there are the potentials $C'_{p+1}$ from bispinors of the (R−,R−) sector. Note that the bispinors containing the primed and unprimed R-R potentials have opposite chirality. This implies a sign difference in the Poincaré duality relations among the field strengths. Thus, for instance, in type 0B there is an unconstrained five-form field strength, whose self-dual (anti-self-dual) part is the unprimed (primed) field strength.

For our purposes, convenient combinations of $C_{p+1}$ and $C'_{p+1}$ are

$$(C_{p+1})_± = \frac{1}{\sqrt{2}} (C_{p+1} ± C'_{p+1}).$$

(1.1)

For $p = 3$ these are the electric (+) and magnetic (−) potentials [2]. We will adopt this terminology also for other values of $p$. There turn out to be four types of “elementary” D-branes for each $p$: an electric and a magnetic one (i.e., charged under $(C_{p+1})_±$), and the corresponding antibranes [2].

**Anomalous Couplings.** The open strings stretching between two like branes are bosons, just like the bulk fields of type 0. However, a boundary state computation shows that fermions appear from strings between an electric and a magnetic brane [2]. Thus one could wonder whether there are chiral fermions on the intersection of an electric and a magnetic brane. Consider such an orthogonal intersection with no overall transverse directions. If the dimension of the intersection is two or six, the computation reveals that there are precisely enough fermionic degrees of freedom on the intersection to form one chiral fermion.

In type II string theory, the analogous computation shows that chiral fermions are present on two or six dimensional intersections of two orthogonal branes with no overall transverse directions. That observation has had far reaching consequences. Namely, the presence of chiral fermions has been shown to lead to gauge and gravitational anomalies on those intersections of D-branes [3]. In a consistent theory, such
anomalies should be cancelled by anomaly inflow. In the present case, the anomaly inflow is provided by the anomalous D-brane couplings in the Wess-Zumino part of the D-brane action [3]. These anomalous couplings have an anomalous variation localized on the intersections with other branes.

To sketch how this anomaly inflow comes about, let us focus on the case of two type IIB D5-branes (to be denoted by D5 and D5') intersecting on a string. The Wess-Zumino action on D5 contains a term of the form $\int_{D5} C_2 \wedge Y_4$, where $C_2$ is the R-R two-form potential and $Y_4$ a certain four-form involving the field strength of gauge field on D5 and the curvature two-forms of the tangent and normal bundles of D5. To be precise, one should replace this term by $\int_{D5} H_3 \wedge \omega_3$, with $Y_4 = d\omega_3$ and $H_3$ the complete gauge-invariant field strength of $C_2$ (which generically differs from $dC_2$). Since the gauge variation of the “Chern-Simons” form $\omega_3$ is given by $\delta\omega_3 = dI_2$ for some two-form $I_2$, the anomalous term on D5 have a variation localized on the intersection with D5’:

$$\delta \int_{D5} H_3 \wedge \omega_3 = \int_{D5} dH_3 \wedge I_2 = \int_{D5} d*H_7 \wedge I_2 = \int_{D5} \delta_{D5'} \wedge I_2 , \quad (1.2)$$

which can thus cancel the anomaly due to the chiral fermions living on the intersection. A careful analysis of all the anomalies [3] shows that the anomalous part of the Dp-brane action is given, in terms of the formal sum $C$ of the various R-R forms, by

$$S_{WZ} = T_p \frac{\kappa}{\kappa} \int_{p+1} C \wedge e^{2\pi\alpha' F+B} \wedge \sqrt{A(R_T)/A(R_N)} . \quad (1.3)$$

Here $T_p/\kappa$ denotes the Dp-brane tension, $F$ the gauge field on the brane and $B$ the NS-NS two-form. Further, $R_T$ and $R_N$ are the curvatures of the tangent and normal bundles of the D-brane world-volume, and $A$ denotes the A-roof genus.

Let us now return to type 0 string theory. As stated above, here chiral fermions live on intersections of electric and magnetic type 0 D-branes. The associated gauge and gravitational anomalies on such intersections match the ones for type II D-branes. To cancel them, the minimal coupling of a Dp-brane to a $(p+1)$-form R-R potential should be extended to the following Wess-Zumino action [1]:

$$S_{WZ} = T_p \frac{\kappa}{\kappa} \int_{p+1} (C)_\pm \wedge e^{2\pi\alpha' F+B} \wedge \sqrt{A(R_T)/A(R_N)} . \quad (1.4)$$

The $\pm$ in Eq. (1.4) distinguishes between electric and magnetic branes. Note that $T_p/\kappa$ denotes the tension of a type II Dp-brane, which can be computed to be $\sqrt{2}$ times the type 0 Dp-brane tension [2].
The argument that the variation of this action cancels the anomaly on the intersection is a copy of the one described above in the type II case, apart from one slight subtlety. For definiteness, consider the intersection of an electric and a magnetic D5-brane on a string. Varying the electric D5-brane action (exhibiting the \((C_2)_+\) potential, or rather, its field strength \((H_3)_+\)), one finds that the variation is localized on the intersection of the electric D5-brane with branes charged magnetically under the \((H_3)_+\) field strength. Using Eq. (1.1), the different behaviour under Poincaré duality of the primed and unprimed R-R field strengths shows that these are precisely the branes carrying (electric) \((H_7)_-\) charge, i.e. what we called the magnetic D5-branes. Schematically,

\[
\delta \int_{D5_+} (H_3)_+ \wedge \omega_3 = \int_{D5_+} d(H_3)_+ \wedge I_2 = \int_{D5_+} d*(H_7)_- \wedge I_2 = \int_{D5_+} \delta_{D5_+} \wedge I_2 .
\]

(1.5)

A completely analogous discussion goes through for the variation of the magnetic D5-brane action.

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