Constraints on the Birth Aggregate of the Solar System

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ABSTRACT

Using the observed properties of our solar system, in particular the isotopic compositions of meteorites and the regularity of the planetary orbits, we constrain the star formation environment of the Sun within the scenario of (external) radioactive enrichment by a massive star. This calculation yields a probability distribution for the number of stars in the solar birth aggregate. The Sun is most likely to have formed within a stellar group containing \( N = \langle N \rangle \approx 2000 \pm 1100 \) members. The \textit{a priori} probability of a star forming in this type of environment is \( P \approx 0.0085 \), i.e., only about 1 out of 120 solar systems are expected to form under similar conditions. We discuss additional implications of this scenario, including possible effects from the radiation fields provided by the putative cluster environment and dynamical disruption of the Kuiper Belt. The constraints of this paper place tight restrictions on the properties of the solar birth aggregate for the scenario of external enrichment by a massive star; alternately, these tight constraints slightly favor a self-enrichment scenario for the short-lived radioactive species.

Subject headings: open clusters and associations: general – stars: formation – solar system: formation

1. INTRODUCTION

A substantial fraction of star formation in our galaxy takes place in groups or clusters, i.e., associations containing many young solar systems. As a consequence, our own solar system may have formed within such a crowded environment. This hypothesis is bolstered by observations of meteorites, which indicate that unexpectedly large quantities of short-lived radioactive nuclei were present at the epoch of planet formation. One traditional explanation for this set of abundance
anomalies is that the solar nebula was enriched in radioactive species by a nearby massive star (see, e.g., the recent review of Goswami and Vanhala 2000; Cameron et al. 1995; Cameron 1978). Enrichment is usually envisioned to occur through a supernova explosion, although Wolf-Rayet winds have also been invoked. In any case, this scenario requires the presence of a nearby massive star, which in turn implies that our solar system formed within a reasonably large stellar group. For completeness, we note that thermally pulsing asymptotic giant branch stars have also been suggested as an enrichment source (see Brusso, Gallino, and Wasserburg 1999), but the probability that such a star is associated with a molecular cloud is relatively low (Kastner and Myers 1994).

We also know that our solar system could not have formed within a cluster containing too many stellar members. In a sufficiently crowded environment, the solar system would be disrupted by gravitational scattering effects from passing stars, binary systems, and other solar systems. The observed orbital elements of the outer planets exhibit low eccentricities and are (almost) confined to the same orbital plane. This relatively well-ordered configuration places modestly tight restrictions on the characteristics of the solar birth aggregate. Some previous work has addressed this issue. The inclinations of the orbits of Uranus and Neptune are sensitive to gravitational perturbations from passing stars in the solar birth cluster; this effect implies a bound (Gaidos 1995; see also Tremaine 1991) on the product of the stellar number density and the residence time in the cluster: \[ \int ndt < 3 \times 10^4 \text{ pc}^{-3} \text{ Myr}. \] In this present work, we constrain the size of the solar birth aggregate from both directions: It must be large enough to provide a sufficiently massive star for enrichment and small enough to allow for well-ordered planetary orbits.

We must keep in mind, however, another possible way to explain the inferred abundances of radioactive species: The solar nebula can be self-enriched through energetic processes in its early formative stages (e.g., Lee et al. 1998). This latter scenario allows for the solar system to form in relative isolation. Recent evidence (McKeegan et al. 2000) provides support for the self-enrichment scenario. This work indicates the presence of $^{10}$Be, a short-lived radioactive species that cannot be produced in supernovae and hence must be produced by nuclear reactions from energetic particles (as would be expected in a picture of self-enrichment). An important issue for solar system formation is to decide between these two alternate enrichment scenarios. As we show here, the external enrichment scenario places strong constraints on the birth environment of the solar system. These tight constraints, in turn, imply a low probability of success for the external enrichment scenario and thus suggest that self-enrichment may be more viable.

In this paper, we explore the compromise necessary to have the solar system formed within a stellar group large enough to produce a nearby massive star and, simultaneously, small enough to allow the planetary orbits to remain relatively unperturbed. We first determine the probability of a stellar cluster containing a sufficiently massive star as a function of the number $N$ of cluster members (§2.1). We then calculate the cross sections for passing stars to disrupt the orbits of the outer planets in the solar system (§2.2), and use the results to estimate the probability of disruption as a function of cluster size $N$ (§2.3). Folding these results together, we find the probability distribution for the size $N$ of the solar system’s birth aggregate (§2.4), the corresponding expectation value $\langle N \rangle$ for the
cluster size, and the *a priori* probability of a star forming in such an environment (§2.5). Next, we use these results to reconstruct the expected ultraviolet radiation field impinging upon the solar nebula during the epoch of planet formation (§2.6). Finally, we calculate the cross sections for the scattering of Kuiper Belt objects through gravitational interactions with passing stars (§2.7) and thereby place further constraints on the birth aggregate. We conclude (§3) with a summary and discussion of our results.

### 2. CONSTRAINTS ON THE SOLAR BIRTH ENVIRONMENT

#### 2.1. Isotopic Abundances and the Probability of a Massive Star

The observed isotopic abundances in solar system bodies provide strong constraints on the material originally contained within the solar nebula. In particular, isotopic studies of meteorites indicate that the solar nebula was enriched in many short-lived nuclides while the solar system was still forming (for a recent review, see Goswami and Vanhala 2000). One leading explanation for this observed enrichment is that a nearby supernova explosion detonated during the formative stages of the solar system and thereby enriched the solar nebula. Possible enriching stellar sources include not only a supernova by a massive star (with mass $M_* > 25M_\odot$; Cameron et al. 1995), but also a non-exploding Wolf-Rayet star (with mass $M_* > 60M_\odot$; Arnould et al. 1997). In either case, however, the Sun would have to be born in close proximity to a massive star. The required distance is $\sim 2\, \text{pc}$ (Goswami and Vanhala 2000), so the Sun would have to be contained with the same cluster as the massive star, but would not require a particularly special location within the cluster. As mentioned earlier, however, this massive star solution is not the only one: Self-enrichment by cosmic rays produced by the early Sun itself remains a possibility (Lee et al. 1998), and the constraints derived in this paper may ultimately help distinguish between these two competing scenarios.

For this work, we want to calculate the probability that the birth aggregate of the Sun contained a sufficiently massive star (to either provide the supernova or Wolf-Rayet effects). And we need to know this probability as a function of the number $N$ of stars in the birth aggregate. The probability $P_{>M}$ of a group of stars having a massive star, say with mass $M_* > 25M_\odot$, is equal to $1 - P_{\text{not}}$, where $P_{\text{not}}$ is the probability of the group *not* having a massive star. We choose this approach because the probability of not having a massive star is straightforward to calculate. Let $p_C$ be the probability that a star is *not* larger than some pre-specified mass scale $M_C$. The probability of a star (a group containing only one star) not having a massive star is thus $P_{\text{not}} = p_C$. The probability for a group of $N$ stars not having a massive star is thus $P_{\text{not}} = p_C^N$. The probability $P_{>M}$ of the group containing a massive star is then given by the expression

$$P_{>M} = 1 - p_C^N.$$  \hspace{1cm} (1)

The probability $p_C$ is determined by the stellar initial mass function (IMF). Since only the
most massive stars enter into this present application, we need to specify the high mass tail of the IMF (with an appropriate normalization). We let $F_{SN}$ denote the fraction of stars that are large enough ($M_* > M_{SN} \approx 8M_\odot$) to explode as supernovae at the end of their nuclear burning lives; for a standard IMF, $F_{SN} \approx 0.003$. The observed solar system enrichment requires larger stars with mass, say, $M_C = 25 - 60M_\odot$. For a given mass requirement $M_C$, the fraction of stars that are heavy enough to provide enrichment is then given by the expression $F_C = F_{SN}(M_{SN}/M_C)^\gamma$, where $\gamma$ is the power-law index of the IMF for high masses [i.e., $df/d\ln m \sim m^{-\gamma}$, where $\gamma = 1.35$ is the traditional Salpeter (1955) value]. Observations of high mass stars in rich clusters (e.g., Brandl et al. 1996) indicate slightly larger indices for the high end of the IMF, $\gamma \sim 1.6$, although the values show some variation from cluster to cluster. Using $\gamma = 1.6$ and the required mass scale $M_C = 25 M_\odot$, we obtain $F_C = 4.85 \times 10^{-4}$, which implies $p_C = 1 - F_C = 0.999515$. We adopt this value for the remainder of the paper. To include other possible parameter choices, we can immediately generalize equation [1] to obtain

$$P_{\geq M} = 1 - \left[1 - F_{SN}(M_{SN}/M)^\gamma\right]^N.$$  

For our standard choice of parameters, we can directly find the number of stars required for a cluster to have a 50-50 chance of containing a sufficiently massive star: $N = -\ln 2/ \ln p_C \approx 1430$.

### 2.2. Cross Sections for Orbital Disruption

We want to ultimately calculate the probability that a solar system will be disrupted as a function of the number $N$ of stars in its birth aggregate. The disruption rate of a solar system is given by

$$\Gamma = n\sigma v,$$

where $\sigma$ is the disruption cross section, $n$ is the mean density of other systems, and $v$ is the relative velocity (typically, $v \sim 1$ km/s).

Using our planet scattering code developed previously (Laughlin and Adams 1998, 2000), we can calculate the cross sections for the disruption of our solar system. In particular, we want to find the effective cross section $\langle \sigma \rangle$ for a specified change in orbital parameters resulting from scattering encounters with other cluster members (which are mostly binaries). Although we should, in principle, consider all possible encounters, no matter how distant, only sufficiently close encounters have an appreciable contribution to the cross section. We thus define our effective cross sections through the relation

$$\langle \sigma \rangle \equiv \int_0^\infty f_D(a)(B\pi a^2)p(a)da,$$

where $a$ is the semi-major axis of the binary orbit and $p(a)$ specifies the probability of encountering a binary system with a given value of $a$. For a given value of $a$, we thus include only those scattering interactions within the predetermined area $B\pi a^2$, where $B$ is a dimensionless factor of order unity.
The function $f_D(a)$ specifies the fraction of encounters which result in a particular outcome (for scattering between the Solar System and a binary of semi-major axis $a$). Because we neglect the contribution to the cross section from scattering interactions outside the area $B\pi a^2$, equation [4] provides a lower limit to the true cross section.

The distribution $p(a)$ is determined by the observed distribution of binary periods and by the normalization condition

$$\int_0^\infty p(a)da = 1.$$  \hspace{1cm} (5)

We model the observed period distribution, and hence obtain $p(a)$, by fitting the results of Kroupa (1995). The observed binary period distribution peaks at $P = 10^5$ days, but the distribution is relatively broad and significant numbers of binaries have periods longer than $10^7$ days. For this set of scattering experiments, however, we only include binaries with $a < 1000$ AU because binaries with larger values of $a$ have little contribution to the cross sections.

The set of possible encounters which can occur between the solar system and a field binary is described by 10 basic input parameters. These variables include the binary semi-major axis $a$, the stellar masses, $m_{*1}$ and $m_{*2}$, of the binary pair, the eccentricity $\epsilon_b$ and the initial phase angle $\ell_b$ of the binary orbit, the asymptotic incoming velocity $v_{\text{inf}}$ of the solar system with respect to the center of mass of the binary, the angles $\theta$, $\psi$, and $\phi$ which describe the impact direction and orientation, and finally the impact parameter $h$ of the collision. Additional (intrinsic) parameters are required to specify the angular momentum vector and initial orbital phases of the planets within the solar system.

To compute the fraction of disruptive encounters $f_D(a)$ and hence the corresponding cross sections, we perform a large number of separate scattering experiments using a Monte Carlo scheme to select the input parameters. Individual encounters are treated as $N$-body problems in which the equations of motion are integrated using a Bulirsch-Stoer scheme (Press et al. 1986). During each encounter, we require that overall energy conservation be maintained to an accuracy of at least one part in $10^8$. For most experiments, both energy and angular momentum are conserved to better than one part in $10^{10}$. This high level of accuracy is needed because we are interested in the resulting planetary orbits, which carry only a small fraction of the total angular momentum and orbital energy of the entire system (for further detail, see Laughlin and Adams 1998, 2000).

For each scattering experiment, the initial conditions are drawn from the appropriate parameter distributions. More specifically, the binary eccentricities are sampled from the observed distribution (Duquennoy and Mayor 1991). The masses of the two binary components are drawn separately from a log-normal initial mass function (IMF) which is consistent with the observed distribution of stellar masses (as advocated by Adams and Fatuzzo 1996). For both the primary and the secondary, we enforce a lower mass limit of $0.075 M_\odot$ and hence our computed scattering results do not include brown dwarfs. Observational surveys indicate that brown dwarf companions are intrinsically rare (Henry 1991); in addition, this cutoff has only a small effect because our assumed IMF peaks in the stellar regime. The impact velocities at infinite separation, $v_{\text{inf}}$, are sampled from a Maxwellian
distribution with dispersion $\sigma_v = 1 \text{ km/sec}$, which is a typical value for stellar clusters (Binney and Tremaine 1987). The initial impact parameters $h$ are chosen randomly within a circle of radius $2a$ centered on the binary center of mass (using a circular target of radius $2a$ implies that $B=4$ in equation [4]).

Using the Monte Carlo technique outlined above, we have performed $N_{\text{exp}} \approx 50,000$ scattering experiments for collisions between binary star systems and the outer solar system. These 7-body interactions involve all four giant planets, the Sun, and the two binary members. From the results of these experiments, we compute the cross sections for orbital disruption of each outer planet (according to equation [4]).

The cross sections for the giant planets to increase their orbital eccentricities are shown in Table 1. For each given value of eccentricity $\epsilon$, the entries give the cross sections [in units of $(\text{AU})^2$] for the eccentricity to increase to any value greater than the given $\epsilon$; these cross sections include events leading to either ejection of the planet or capture by another star. The listings for $\epsilon = 1$ thus give the total cross sections for planetary escape and capture (taken together). In the two additional lines below the main part of the table, we also present the cross sections for planetary escapes and captures separately. For each cross section listed in Table 1, we also provide the one standard deviation error estimate for the Monte Carlo integral; this quantity provides a rough indication of the errors due to the statistical sampling process (Press et al. 1986). In this work, we are mostly interested in the largest possible cross sections for disruption, which ultimately provide the tightest constraints on the environment of the early solar system. Of the four giant planets, Neptune is the most easily sent into an alternative orbits, as expected (see Table 1). For the disruption of Neptune, we find a cross section $\langle \sigma \rangle \approx 143,000 \text{ AU}^2$ to increase its eccentricity to $\epsilon > 0.1$ and $\langle \sigma \rangle \approx 167,000 \text{ AU}^2$ to double its eccentricity. For this work, we use the cross section for Neptune to double its orbital eccentricity [about $(400 \text{ AU})^2$] to represent the effective cross section for solar system disruption (through eccentricity increases).

Another way for the solar system to be disrupted is by changing the planes of the planetary orbits. Table 2 shows the cross sections for the inclination angles of the planetary orbits to increase by varying amounts. The scattering experiments start with all four giant planets in the same plane. After each collision, the angular momentum vectors of the planets will not, in general, be aligned. The quantity $\Delta \theta_i$ in the table is the largest angle (given in radians) between any two of the angular momentum vectors for the four planets. The cross section for any one of the planets to escape or be captured is $\langle \sigma \rangle \approx 17100 \pm 420$ (in units of $\text{AU}^2$). The second column of Table 2 gives the cross sections for all four planets to remain bound to the Sun, but have at least one of the relative angles exceed $\Delta \theta_i$. The final column gives the total disruption cross sections, including planet escapes, planet captures, and the inclination angle increases. Notice that the cross section for planetary escape and/or capture is larger than the cross section for scattering events to increase the inclination angles beyond $\Delta \theta_i \approx 2.4 \approx 3\pi/4$. As is well known (Shu 1980), the inclination angles for the (present-day) planetary orbits in our solar system show a small spread, only about 3.5 degrees or 0.061 radians. The cross section for the inclination angles to increase to
this maximum allowed value is about $\langle \sigma \rangle \approx 158,000 \text{ AU}^2$, which is comparable to the cross section for the maximum allowed eccentricity increase determined earlier.

To summarize, our solar system can be disrupted within its birth aggregate by scattering events which increase both the eccentricities and the inclination angles of the orbits of the outer planets. But these two effects are not independent – scattering events that pump up the orbital eccentricity also increase the inclination angles – so we cannot add the cross sections. As a rough benchmark, the total cross section for significant disruption (large enough to be inconsistent with observations of the present day solar system) is thus $\langle \sigma \rangle \approx 160,000 \text{ AU}^2 = (400 \text{ AU})^2$. We use this cross section as a representative value for the rest of this paper.

### 2.3. Cluster Evolution and the Probability of Solar System Disruption

In order to assess the likelihood of planetary disruption, we must fold into the calculation considerations of the background cluster environment. The effective “optical depth” $\tau$ to disruption can be written in the form

$$\tau = \int \Gamma dt = \langle \sigma \rangle \int_0^{t_{cl}} vndt,$$

(6)

where the cross section $\langle \sigma \rangle$ is now considered as a known quantity. The integral is taken over the total lifetime $t_{cl}$ of the cluster environment. The velocity scale $v$, the mean stellar density $n$, and the lifetime $t_{cl}$ are (on average) increasing functions of the number $N$ of stars in the system. We are implicitly assuming that the Sun stays in the birth cluster for most of the cluster lifetime. This assumption is reasonable because the Sun is relatively heavy and will tend to sink toward the cluster center rather than become immediately evaporated. In addition, the Sun must stay in the cluster long enough for massive stars to evolve and die (in order to have the solar system enriched in short-lived radioactive species). For purposes of this paper, the integral can be approximated as follows:

$$\int_0^{t_{cl}} vndt \approx 50n_0v_0t_{R_0},$$

(7)

where the subscripts indicate that the quantities are evaluated at their initial values and where $t_{R_0}$ is the initial relaxation time. In making the approximation [7], we have used the result that the typical lifetime of a cluster is about 100 times the initial dynamical relaxation time $t_{R_0}$ (see Binney and Tremaine 1987), but the number density of other stellar systems (both singles and binaries) is less than the starting value (averaged over $t_{cl}$). Since $n_0 \approx N/4R^3$ and $t_R \approx (R/v)N/(10 \ln N)$, we find that $n_0v_0t_{R_0} \sim N^\mu$, where the index $\mu \approx 2$. Assuming this power-law scaling for the $N$-dependence, we can “evaluate” the optical depth integral and write it in the form

$$\tau = (N/N_C)^\mu,$$

(8)

where $N_C$ is the number of stars in the system required to make the optical depth unity. Throughout this work, both $N$ and $N_C$ refer to the number of singles and binaries, where the binaries are counted
as one other stellar system. In other words, \( N \) and \( N_C \) represent the number of primaries. For our adopted standard value \( \langle \sigma \rangle = (400 \text{ AU})^2 \approx 4 \times 10^{-6} \text{ pc}^2 \), equation (7) implies that

\[
\frac{N_C}{\sqrt{\ln N_C}} \approx (1.25 \langle \sigma \rangle)^{-1/2} R \approx 447 \left( \frac{R}{1 \text{ pc}} \right).
\]

For the representative value \( R = 2 \text{ pc} \), we find \( N_C \approx 2500 \), which we take as our standard value for the remainder of this paper. For observational comparison, the Trapezium cluster in Orion has approximately 2300 stars in its central region of size \( R \sim 2 \text{ pc} \) (Hillenbrand and Hartmann 1998); if the cluster lives for 100 initial relaxation times \( t_{R0} \) (again, see Binney and Tremaine 1987), where \( t_{R0} \approx 15 \text{ Myr} \), then the effective optical depth for the Trapezium cluster will be about \( \tau \approx 0.9 \), in reasonable agreement with the choices of parameters taken here.

The probability \( P(t) \) of the solar system surviving (i.e., not being disrupted) is given by the solution to the simple differential equation

\[
\frac{dP}{dt} = -\Gamma P \quad \rightarrow \quad P(t) = \exp\left[ -\int t \Gamma dt \right],
\]

where we have used the fact that \( P(t = 0) = 1 \). Putting the above results together, we obtain the probability \( P_{\text{dis}} \) for the solar system surviving disruption as a function of the cluster size \( N \), i.e.,

\[
P_{\text{dis}}(N) = \exp\left[ -(N/N_C)\mu \right],
\]

where we expect \( \mu = 2 \) and \( N_C \approx 2500 \).

### 2.4. Probability Distribution for the Size of the Solar Birth Aggregate

By considering both constraints derived above, i.e., by assuming that the early solar system experienced a supernova and that the outer planets were not severely disrupted, we obtain the probability distribution that the Sun formed in a birth aggregate containing \( N \) members. This probability distribution \( P_\odot \) is given by

\[
P_\odot(N) = P_{>M} P_{\text{dis}} = (1 - p_C^N) \exp\left[ -(N/N_C)^\mu \right].
\]

The resulting joint probability distribution is shown in Figure 1. The peak of the distribution occurs at \( N \approx 1465 \). However, the more relevant quantity is the expectation value, \( \langle N \rangle \equiv \int N P_\odot(N) dN / \int P_\odot dN \), which is somewhat larger, \( \langle N \rangle \approx 1970 \). We also obtain a measure of the range of allowed cluster sizes: The variance of the distribution \( \langle (\Delta N)^2 \rangle^{1/2} = 1090 \), so we obtain \( N \approx 2000 \pm 1100 \). Alternately, the allowed range can be defined by the half-maximum points of the probability distribution, i.e., 425 < \( N < 3000 \).

To further illustrate the conditions within the required birth aggregate, let’s now consider the evolution of a stellar system with \( N = 2000 \) stars. This number of stars should be calculated after
gas removal from the cluster. In the initial stages of cluster formation, the system will contain some fraction of its mass in gaseous form as well as additional stars. The stars obey a distribution of velocities. After gas removal, the high velocity stars leave the system and the low velocity stars remain behind. As a benchmark, if the star formation efficiency is 50%, and the initial velocity distribution of the stars is isotropic, then the cluster must initially contain about 2560 stars (Adams 2000). This value implies that the solar birth aggregate is comparable to (but somewhat larger than) the present day Trapezium cluster in Orion (which has about 2300 stars within its central 2 pc; see Hillenbrand and Hartmann 1998).

After gas removal, our benchmark cluster contains 2000 stars and has a total mass of about 1400 $M_\odot$. With a typical radial size of $R = 1.5$ pc, the velocity scale will be about 2 km/s. The dynamical relaxation time of the system is initially $t_{R_0} \sim 20$ Myr, whereas the crossing time is only $t_{\text{cross}} \sim 0.75$ Myr. The characteristic time scale $t_P$ for nebular disks to retain their gas and form giant gaseous planets is $\sim 10$ Myr (e.g., Lissauer 1993), which is roughly comparable to (but shorter than) the dynamical relaxation time $t_R$, but much longer than the crossing time $t_{\text{cross}}$ of the system. All of these time scales are much less than the total cluster lifetime $t_{\text{cl}}$, which is, in turn, much shorter than the current age of the solar system $t_\odot \approx 4.6$ Gyr. The time scales involved in the problem thus obey the ordering

$$t_{\text{cross}} \ll t_P \ll t_R \ll t_{\text{cl}} \ll t_\odot,$$

which determines the timing of relevant events. The solar system will experience many orbits through the birth cluster while its planets form, but the background gravitational potential and structure of the cluster will not change appreciably. The solar system will thus randomly sample the cluster volume during the planet forming epoch. Since the planet formation time is much shorter than the total cluster lifetime, $t_P \ll t_{\text{cl}}$, the planets are available for disruption for most of the cluster’s life (as implicitly assumed above). Furthermore, because the planets have a relatively small (but significant) chance of being disrupted during the entire life span of the cluster, they have a much smaller chance of being disrupted during their early formative stages (smaller by a factor of $t_P/t_{\text{cl}} \ll 1$). In general, if the cluster is sufficiently diffuse to allow the planetary orbits to remain unperturbed (after planet formation), then the cluster environment can have relatively little effect on the planet formation process (via dynamical scattering processes; radiation can play an important role as discussed in §2.6).

The initial stellar density of the cluster is about 200 pc$^{-3}$ when averaged over the whole system. As the cluster evolves, the half-mass radius of the cluster remains roughly constant, with the central regions shrinking inwards and the outer regions expanding (Binney and Tremaine 1987). The average density over the cluster lifetime (about 100 initial relaxation times or $\sim 1$ Gyr), is thus $n \sim 50 - 100$ pc$^{-3}$. The quantity $\int n dt \sim 10^5$ pc$^{-3}$ Myr, which is comparable to a previous constraint on this quantity (derived in Gaidos 1995). Not surprisingly, the expected optical depth to significant scattering events, $\tau = \langle \sigma \rangle v \int n dt \approx 0.4 \langle v \rangle (1 \text{ km/s})^{-1}$, implies that solar system has a reasonable but not overwhelming probability of disruption. In order for this value of the
optical depth to agree with our approximation of equation [8], we would need an average velocity
dispersion of $\langle v \rangle \approx 1.6 \text{ km/s}$ (again, a reasonable value).

We can generalize this calculation to include other choices of parameters. One quantity that
is not well determined is the number of stars $N_C$ required for a cluster to have optical depth unity
for solar system disruption (see equations [6 – 9]). This parameter $N_C$ ultimately depends on the
degree of cluster concentration and the immediate galactic environment, which show some variation
and are not completely well known. We also might want to consider other choices for the mass
scale $M_C$ of the required massive star (e.g., the Wolf-Rayet scenario requires a larger value) or
different choices of the IMF, both of which lead to different values of $p_C$. Fortunately, however, we
can analytically determine the expectation value $\langle N \rangle$ for the size of the solar birth aggregate (for
the standard choice of $\mu = 2$ in equation [12]). We find

$$\langle N \rangle = \beta N_C \frac{e^{\beta^2 [1 - \text{Erf}(\beta)]}}{1 - e^{\beta^2 [1 - \text{Erf}(\beta)]}},$$

(14)

where Erf$(x)$ is the error function and where we have defined a dimensionless parameter $\beta \equiv 0.5 N_C (\ln p_C)$. Equation [14] thus provides the expectation value for the size of the solar birth
aggregate for any value of $N_C$ and/or $p_C$.

### 2.5. Probability of a Star Forming under Solar Conditions

Next, we can find the overall probability that a star forms in an environmental group containing
$N = \langle N \rangle \approx 2000$ members by using the probability distribution $P_\odot$ given in equation [12]. This
calculation gives us a feeling for how common (or uncommon) our solar system should be, provided
that it forms according to the scenario of external radioactive enrichment. We let $dP_{cl}/dN$ be the
probability density for a star forming within a cluster containing $N$ members. For the high end
of the distribution of cluster sizes $N$, we use the result that only about ten percent of stars form
within “big” clusters (e.g., see Roberts 1957; Elmegreen and Clemens 1985; Battinelli and Capuzzo-
Dolcetta 1991; Adams and Myers 2000; and others). Here, “big” clusters are those sampled by the
observational cluster surveys, which are complete down to some minimum cluster size $N_*$ that is
not precisely specified, but lies in the range $100 < N_* < 500$. Since the putative birth cluster for
the solar system must be much larger, with $N \approx 2000$, it lies safely in the size range for which the
observational surveys are complete. Furthermore, the mass distribution for true (relatively large)
clusters can be represented by a power-law function $df/dN \sim N^{-2}$ (Elmegreen and Efremov 1997).
To obtain the probability density function $dP_{cl}/dN$ for a star forming in a cluster of a given size,
we must multiply by one power of $N$ to obtain

$$\frac{dP_{cl}}{dN} = N \frac{df}{dN} = A_{cl} N^{-1}.$$  

(15)

We specify the normalization constant $A_{cl} \approx (40 \ln 10)^{-1}$ by integrating over the allowed range of
cluster sizes (which we take to be $N_1 \equiv 10^2 \leq N \leq 10^6 \equiv N_2$). The probability $P$ that star forming
environments provide the proper conditions for solar system formation is thus given by the integral
\[ P = \int_{N_1}^{N_2} \frac{dP_{\text{cl}}}{dN} P_{>M_{\text{dis}}} dN \approx 0.0085. \] (16)

In other words, the considerations of this paper imply that 1 out of 120 solar systems in the galaxy form in a dense enough environment to be radioactively enriched by a supernova and, at the same time, a sufficiently diffuse environment to allow outer planetary orbits to remain relatively unperturbed.

The probability \( P \) depends on the parameters of the problem, in particular the size \( N_C \) of a cluster required for the solar system to have optical depth unity to scattering events and the probability \( p_C \) for the cluster to contain a sufficiently massive star. One may wish to consider varying scales \( M_C \) for the required massive star, or variations in the IMF, both of which change the value of \( p_C \). Similarly, one may wish to consider different types of disruption events, which have different cross sections \( \langle \sigma \rangle \), and hence lead to different values of \( N_C \). The probability of a given type of cluster environment, as given by equation [16], can be evaluated analytically as a power series for the standard case of \( \mu = 2 \), i.e.,
\[ P = \frac{A_{\text{cl}}}{2} \sum_{k=0}^{\infty} (-1)^k \alpha^{k+1} \frac{\Gamma[(k+1)/2]}{\Gamma[k+2]}, \] (17)

where \( \Gamma(x) \) is the gamma function and where we have defined \( \alpha \equiv N_C(-\ln p_C) \), which has a "standard" value of \( \alpha \approx 1.2 \). Unfortunately, however, the series converges only for \( \alpha \leq 2 \). Because the dependence of \( P \) on the fundamental physical quantities is somewhat opaque in equation [17], we numerically evaluate the result and plot the resulting probability \( P \) as a function of both \( N_C \) and \( M_C \) in Figure 2. As the mass scale required for enrichment increases to \( M_C = 60 \, M_\odot \), for example, \( P \) decreases to about 0.0025, which corresponds to odds of only 1 part in 400. The solar system is thus much less likely to have been radioactively enriched by being born within an environment containing a 60 \( M_\odot \) star than one containing a 25 \( M_\odot \) star.

2.6. Radiation Environment of the Early Solar System

Given the constraints on the solar birth aggregate derived above, we can place corresponding constraints on the radiation fields that the early solar system experienced. These radiation fields can play an important role in removing gas from the early solar nebula and can thereby strongly influence planet formation (Shu, Johnstone, and Hollenbach 1993; Störzer and Hollenbach 1999). If the solar system did indeed form within a large enough birth aggregate to provide external radioactive enrichment, then the ultraviolet (UV) flux incident upon the outer solar system will be dominated by the background radiation field of the cluster rather than by the intrinsic radiation field of the early Sun. We substantiate this claim below.
To determine the UV radiation field provided by the cluster environment, we first need to estimate the distance from the solar system to the massive stars in the birth aggregate (time averaged over the planet formation epoch). The crossing time $t_{\text{cross}}$ is short compared to both the dynamical relaxation time $t_R$ and the planet formation time $t_P$, but $t_P < t_R$, so we consider the solar system to be on a random orbit in a fixed cluster structure (see eq. [13]). Current observations and theoretical simulations indicate that the most massive stars may form in the cluster centers (e.g., Bonnell and Davies 1998), so we take the massive stars to lie at the origin. We must then calculate the expectation value for the total UV flux, averaged over a typical orbit. For a centrally condensed cluster (like a King model), the speed is nearly constant over the orbit. For each pass through the cluster, the solar system experiences an average flux $\langle F \rangle$ given by

$$\langle F \rangle \approx \frac{1}{2R} \int_{-R}^{R} \frac{L}{4\pi b^2 + s^2} \approx \frac{L}{8Rb},$$

(18)

where $b$ is the impact parameter (the distance of closest approach to the center). In a collapse model of cluster formation, the stellar orbits are nearly radial outside the core and nearly isotropic inside (Adams 2000); we therefore expect the typical impact parameter $b$ to be given by the core radius. In open clusters, the core radii are about ten times smaller than the cluster sizes (Binney and Tremaine 1987), so we take $b \approx R/10$. The quantity $L$ is the total luminosity of the massive stars in the cluster center. Since we are interested in the ionizing ultraviolet flux $F_{\text{uv}}$, we take $L = L_{\text{uv}}$, which is determined by integrating over the stellar initial mass function. As before, we let the IMF take the form $df/d\ln m \sim m^{-\gamma}$ for high mass stars. We also assume that the ultraviolet luminosity can be modeled as a simple power-law over the stellar mass range of interest, i.e., $L_{\text{uv}}(m) = L_C(m/m_1)^q$. We take $m_1 = M_{\text{SN}} = 8 M_\odot$, the minimum stellar mass for a supernova, because smaller stars have little contribution to the ultraviolet radiation background. We also need to impose an upper mass scale, which we take to be $m_2 = 100 M_\odot$, to keep the integrals finite. Fitting the UV fluxes of massive stars, we find $q \approx 3.14$ and $L_C \approx 4.2 \times 10^{46} \text{ sec}^{-1}$ (ionizing photons per second). The mean UV luminosity $\langle L_{\text{uv}} \rangle$ thus becomes

$$\langle L_{\text{uv}} \rangle = \int_{m_1}^{m_2} A_m m^{-(1+\gamma)} \, dm \, L_C (m/m_1)^q = A_m L_C m_1^{-q} \frac{1}{q-\gamma} (m_2^{q-\gamma} - m_1^{q-\gamma}),$$

(19)

where $A_m$ is the normalization constant of the stellar IMF. Normalizing $A_m$ to account for the total number of stars $N$ in the cluster, we find $A_m = F_{\text{SN}} N \gamma m_1^{\gamma}$. Combining equations [18] and [19], we obtain the total ionizing UV flux impinging upon the solar system from the background cluster environment

$$\langle \langle F_{\text{uv}} \rangle \rangle = \frac{F_{\text{SN}} N L_C}{8bR} \frac{\gamma}{q-\gamma} [ (m_2/m_1)^{q-\gamma} - 1 ].$$

(20)

Inserting numerical values, we can evaluate this result to find

$$\langle \langle F_{\text{uv}} \rangle \rangle \approx 1.6 \times 10^{12} \text{ cm}^{-2} \text{ sec}^{-1} \left( \frac{N}{2000} \right) \left( \frac{R}{1 \text{ pc}} \right)^{-2}.$$  

(21)
For comparison, the ionizing UV luminosity of the early Sun cannot be larger than about $L_{\text{uv}} \approx 10^{41}$ (Gahm et al. 1979; see also the discussion given in Shu et al. 1993) and hence the corresponding UV flux is found to be

$$F_{\text{uv}} \odot = 3.5 \times 10^{13} \text{cm}^{-2} \text{sec}^{-1} \left(\frac{\varpi}{1 \text{AU}}\right)^{-2}, \quad (22)$$

where $\varpi$ is the (cylindrical) radial coordinate centered on the Sun. The background flux of the cluster exceeds the local UV flux of the Sun for radial positions $\varpi > 4.7$ AU, which is close to the current value for the semi-major axis of Jupiter’s orbit. As a result, almost the entire region of the solar nebula that participates in giant planet formation is dominated by the ionizing UV flux from the cluster environment, rather than the Sun.

We can also compare the total number of ionizing UV photons intercepted by the solar nebula from both the Sun and the background cluster. The disk is embedded in the UV radiation field and both sides will be exposed; the disk thus receives UV photons from the cluster at a rate $\Phi_{\text{uv}} \approx 2 \times 10^{42}$ sec$^{-1}$, where $R_d \approx 30$ AU is the radial size of the disk. The disk will also intercept a fraction of the $\Phi_{\odot} \approx 10^{41}$ sec$^{-1}$ UV photons generated by the nascent Sun. For the limiting case of a flat disk which is optically thick but spatially thin, the fraction of directly intercepted photons is 25% (Adams and Shu 1986); because the disk can be flared and because of additional scattered (diffuse) photons, the actual fraction is somewhat greater, about 50% (Shu et al. 1993). The total rate of intercepted solar UV photons is thus about $5 \times 10^{40}$ sec$^{-1}$. As a result, the cluster environment provides 40 times more ionizing UV photons to the solar nebula than the Sun itself.

This enhanced UV flux can drive an enhanced rate of photoevaporation from the disk. The mass loss rate for the simplest models (Shu et al. 1993; Hollenbach et al. 1994) scale as $\dot{M} \propto \Phi_{\text{uv}}^{1/2}$, so the mass loss rate is (at least) $\sim 6.3$ times greater if the solar nebula lives within a large birth aggregate. For some regimes of parameter space, the far-ultraviolet (as opposed to ionizing UV) photons dominate the mass loss mechanism and the scalings are different (Störzer and Hollenbach 1999); one could perform an analogous calculation for mass loss due to far-UV photons. In any case, for models of a disk immersed in the radiation field of a cluster, the mass loss rate can be large $\dot{M}_D \approx 10^{-7}$ $M_\odot$ yr$^{-1}$ (again, see Störzer and Hollenbach 1999). For the minimum solar nebula with disk mass $M_D \approx 0.01 M_\odot$, this mass loss rate would destroy the disk in only $10^5$ yr, far shorter than the time scale required for giant planet formation (Lissauer 1993). Although the details depend on the exact orbit of the solar system through its birth environment and other undetermined parameters, this putative cluster of $N = 2000$ stars comes dangerously close to preventing planet formation from taking place.

A related question is to ask what fraction of all stars are born in sufficiently rich clusters so that the UV flux of the background cluster dominates the intrinsic UV radiation field of the star. In general, clusters large enough to be included in observational cluster surveys (systems with a few hundred members or more) are large enough to dominate the UV radiation field (Adams & Myers 2000); as a result, the fraction of stars that are exposed to intense radiation fields is 8–10 percent. About 90 percent of all solar systems thus have their UV radiation fields dominated by their central
stars (and can presumably form planets without interference from the background environment).

### 2.7. Effects on the Kuiper Belt

The large birth aggregate required to provide external radioactive enrichment will also have a substantial impact on Kuiper Belt objects (hereafter KBOs). The population of bodies in the Kuiper Belt is both complex and still under investigation, but enough observations have been made to provide preliminary constraints (e.g., see the reviews of Jewitt and Luu 2000; Malhotra, Duncan, and Levison 2000; Farinella, Davis, and Stern 2000; and references therein). Briefly, the Kuiper Belt contains a population of KBOs in nearly circular orbits with semi-major axes in the range 30 – 50 AU, a second population of KBOs in resonances with Neptune, and a third population of KBOs with high eccentricities and larger ($a > 50$ AU) semi-major axes. Although a great deal of dynamical evolution has taken place between solar system formation and the present-day observations, a (hypothetical) large birth cluster will nonetheless have a dramatic impact on KBOs during the first $\sim 100$ Myr of solar system evolution.

To study the interplay between the Kuiper Belt and the solar birth aggregate, the first step is to calculate the cross sections for the scattering of KBOs by gravitational interactions with passing stars in the birth cluster. The procedure is analogous to that described in §2.2. In this case, we start the scattering experiments with small bodies in circular orbits. This suite of numerical experiments uses orbital radii of $a = 30, 40, 50, 60,$ and $70$ AU. Because KBOs are small (the combined mass of the Kuiper Belt is estimated to be less than an Earth mass), they act like test particles in the scattering simulations. For computational convenience, we take all of the bodies to have a mass of $10^{-6} M_\oplus$; this mass scale is small enough that the bodies are indistinguishable from test particles and large enough to allow the code to conserve energy and angular momentum to good accuracy. By including the KBO at $a = 30$ AU (which would clearly not survive because of Neptune), we obtain a consistency check by comparing the results with those for Neptune scattering (Table 1).

The resulting cross sections for KBO scattering, listed here as a function of the final (post-scattering) eccentricity, are shown in Table 3. Notice that the cross sections for the KBO at $a = 30$ AU are roughly comparable to those for Neptune (compare Tables 1 and 3); this finding implies that even Neptune acts like a test body (for the most part) during scattering interactions. We have not included the giant planets in this set of scattering calculations. These planets themselves can be scattered during the interactions and then can lead to additional disruption of the putative KBOs; this secondary effect is not included in this calculation and hence the cross sections listed in Table 3 represent lower limits to possible disruption of KBO orbits.

The scattering cross sections obtained here can be used in two ways: We can assume that the solar birth aggregate is “known” (from the previous results of this paper) and then predict undiscovered properties of the outer Kuiper Belt. Alternately, we can use the observed KBO populations to place further constraints on the solar birth aggregate.
For example, if we assume that the previous sections specify the properties of the solar birth cluster, then we know that it initially contained \( N \approx 2000 \) members and the effective optical depth \( \tau \) to scattering events is given by

\[
\tau = \frac{\langle \sigma \rangle}{(400 \text{AU})^2} \left( \frac{N}{N_C} \right)^2 = \frac{\langle \sigma \rangle}{(500 \text{AU})^2},
\]

where \( \langle \sigma \rangle \) is the cross section for the scattering event of interest. The corresponding probability of the given event not occurring is thus \( \exp[-\tau] \). Using the cross sections in Table 3, we find that essentially all KBOs beyond \( \sim 50 \text{ AU} \) must attain nonzero eccentricities. For example, KBOs at \( a = 50 \text{ AU} \) will typically attain eccentricities \( \epsilon > 0.2 \) and KBOs at \( a = 70 \text{ AU} \) will attain \( \epsilon > 0.4 \). This type of dynamical excitation (which also includes increased inclination angles of the orbits) spreads out the KBO population and thereby lowers the apparent surface density of objects on the sky; this reduction in surface density can appear as an apparent “edge” to the solar system as recent observations suggest (Allen, Bernstein, and Malhotra 2000). However, we must stress once again that the Kuiper Belt will undergo substantial dynamical evolution of its own after the solar system leaves its birth cluster.

Although the cross sections listed in Table 3 are relatively large, as expected, it is significant that a large portion of the table has cross sections less than the fiducial value \( \langle \sigma \rangle \approx (400 \text{ AU})^2 \) required for solar system disruption (see §2.2–2.3). As a result, some fraction of the KBOs in the range \( 40 \text{ AU} \leq a \leq 70 \text{ AU} \) will survive the birth cluster. The KBOs can be removed (in the short term) either through direct ejection (or capture) or by attaining a large enough eccentricity to cross the orbit of Neptune. Although KBOs with larger radii \( (a) \) have larger cross sections, they need to be scattered to larger eccentricities to encounter Neptune. These two trends nearly compensate for each other and yield a nearly constant cross section for KBOs to be (promptly) removed from the solar system: \( \langle \sigma \rangle \approx (350 \text{ AU})^2 \). This value implies that the probability of KBO survival runs at about \( \exp[-\tau] \approx 0.6 \). In other words, about 40 percent of the KBOs will be removed from the solar system while the Sun remains in its birth cluster (and an additional population will be removed later through longer term dynamical interactions).

As an alternate approach, we can use our results from KBO scattering to place further constraints on the solar birth aggregate. We first note that we could use the survival of the Kuiper Belt as the criterion for solar system to not be disrupted. We then repeat the analysis of §2.3–2.5 using the cross section for KBO removal \( \langle \sigma \rangle \approx (350 \text{ AU})^2 \). Because this cross section is somewhat lower than that used previously (that for disruption of the giant planet orbits), the derived constraints on the solar birth cluster are correspondingly weaker: \( \langle N \rangle = 2250 \pm 1250 \) and \( P = 0.0095 \). Using only the survival of the Kuiper Belt, we thus obtain a (1 \( \sigma \)) upper limit on the size of the birth cluster: \( N \leq 3500 \).

We can also repeat the probability analysis by requiring that the solar system survive with both its giant planet orbits and its Kuiper Belt intact. In this case, the joint probability distribution
\( P_{\odot} \) for solar system survival takes the form
\[
P_{\odot}(N) = P_{>M}P_{\text{dis}} = (1 - p_C^N) \exp[-(N/N_C)^\mu] \exp[-(N/N_{CK})^\mu],
\] (24)
where the second exponential factor represents the survival of the Kuiper Belt (and where we consider the two processes to be independent). The quantity \( N_{CK} \approx 2860 \) is the number of stars in the cluster required to make the scattering optical depth unity for scattering interactions leading to KBO removal. In this case, because we have an added constraint, the resulting bounds are somewhat more restrictive than before: \( \langle N \rangle = 1520 \pm 827 \) and \( P = 0.0067 \) (which corresponds to odds of one part in 150).

Before leaving this section, we note that the Oort cloud of comets may be even easier to disrupt than the Kuiper Belt. For a given model of comet formation, one could thus find the corresponding constraints on the birth aggregate of the solar system. We leave this issue for future work.

3. CONCLUSIONS and DISCUSSION

In this paper, we have explored the consequences of the solar system being formed within a group environment that is large enough to contain a massive star that enriches the early solar system in radioactive species and is also sufficiently diffuse to allow the planetary orbits to remain unperturbed. In particular, we have obtained the following results:

[1] We have calculated the cross sections for the outer planets in our solar system to experience orbital changes due to scattering interactions with binary systems in a cluster environment. We find the cross sections for eccentricity increases (see Table 1) and for increases in the relative inclination angles of the planetary orbits (see Table 2). The cross section for the scattering events to increase either the eccentricities or the inclination angles beyond the currently observed values is \( \langle \sigma \rangle \approx (400 \text{ AU})^2 \). The cross section for planetary ejection and/or capture is somewhat lower, about \( \langle \sigma \rangle \approx (130 \text{ AU})^2 \). However, all of these cross sections are substantially larger than the area subtended by the solar system, \( \text{Area} \approx \pi (30 \text{ AU})^2 \).

[2] We have estimated the probability distribution for the number \( N \) of stars in the birth aggregate for our solar system (see Figure 1). Using the coupled constraints that the group was large enough to contain a massive star (to enrich the solar system in radioactive elements) and small enough so that the outer planetary orbits are not severely disrupted, we find that \( N = \langle N \rangle \approx 2000 \pm 1100 \). The expectation value \( \langle N \rangle \) varies relatively slowly with the parameters of the problem and has the analytic solution given by equation [14].

[3] The \textit{a priori} probability for a star being born in the type of environment required for the external enrichment scenario for our solar system (i.e., subject to the probability distribution depicted in Figure 1) is \( P \approx 0.0085 \). The odds of the solar system forming in this type of environment are thus about 1 in 120. This result can be readily generalized to accommodate other choices of parameters (see eq. [17] and Figure 2).
The time scales for the cluster environment obey a particular ordering (see equation \([13]\)):

The crossing time \(t_{\text{cross}}\) of the cluster is much shorter than the time \(t_P\) required to form giant planets, so the solar system randomly samples the cluster environment over that epoch. The dynamical relaxation time \(t_R\) is somewhat longer than \(t_P\), and the total cluster lifetime \(t_{\text{cl}} \gg t_P\), so the cluster does not change its structure appreciably while the planets form.

Within the scope of this external enrichment scenario, we have reconstructed the radiation field provided by the birth aggregate of the solar system. The early solar nebula receives about 40 times more ionizing UV photons from the background cluster environment than from the early Sun. Only the inner portion of the nebula, at radii \(r < 5\) AU, has its ionizing UV flux dominated by the Sun. This intense flux of UV radiation can severely ablate the early solar nebula and greatly compromise giant planet formation.

Objects forming in the Kuiper Belt will also be scattered due to interactions in the birth cluster. We have calculated the scattering cross sections for KBOs on initially circular orbits with radii in the range \(30\) AU \(\leq a \leq 70\) AU (Table 3). The cross sections for prompt removal are \(\langle \sigma \rangle \approx (350\) AU\(^2\) over the outer part of this radial range. If we include the required survival of the Kuiper Belt in the probability analysis, we obtain slightly tighter constraints on the solar birth aggregate: \(\langle N \rangle = 1520 \pm 827, P = 0.0067\), and \textit{a priori} odds of one part in 150.

Some authors have suggested that the formation of our solar system must be triggered by the same supernova that is postulated to provide the radioactive enrichment (e.g., Boss and Foster 1998). We stress here that external enrichment does not necessarily imply a triggered collapse. The time scale for cluster formation is relatively short (a few Myr; e.g., Elmegreen 2000) and the time scale for the collapse of an individual star forming site is much shorter (about \(10^5\) yr; e.g., Myers and Fuller 1993, Adams and Fatuzzo 1996). Thus, the solar system could be formed within a cluster and yet be formed through a spontaneous (un-triggered) collapse. Another motivation for the collapse being triggered is that the ambipolar diffusion time scale is generally long (about \(10^7\) years), too long for the survival of the necessary radioactive nuclei. However, observational evidence shows that the time scale for molecular cloud cores to shed their magnetic support and begin dynamic collapse is much shorter (about 1 Myr) for all cores (e.g., Jijina et al. 1999; Myers and Lazarian 1998); the ambipolar diffusion time scale is thus not an insurmountable obstacle for star formation in clusters.

The results of this paper have important ramifications for the ongoing debate concerning radioactive enrichment of the early solar system. The meteoritic data strongly indicate that such enrichment took place. However, both the external scenario of enrichment by a massive star and the internal scenario of self-enrichment have some difficulty reproducing all of the short-lived radioactive species (e.g., see Goswami and Vanhala 2000; Lee et al. 1998; and references therein). Although the results are not definitive, this paper tends to favor the self-enrichment scenario for two reasons: (1) A star forming environment that is simultaneously large enough to provide external enrichment and diffuse enough to not disrupt the planetary orbits is \textit{a priori} an unlikely event (at the level of 1 part
in 120). (2) If the solar system formed in the large birth aggregate required for external enrichment, the corresponding radiation fields are likely to have compromised giant planet formation.

Nevertheless, the external enrichment scenario is not conclusively ruled out. Events with long odds (100:1) do indeed sometimes happen. If one adopts the external enrichment scenario, a consistent solution exists and the results of this paper place tight constraints on the birth environment of our solar system: The birth cluster contained \( N \approx 2000 \) other stars and subjected the early solar nebula to an intense UV radiation field. Scattering interactions in the birth cluster would then be responsible (at least in part) for the observed nonzero (but small) inclination angles and eccentricities of the giant planet orbits. Furthermore, objects in the Kuiper Belt would be dynamically excited by these scattering interactions. This highly disruptive environment must be accounted for in a complete description of solar system formation.

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Table 1

Cross Sections for Planet Scattering: Eccentricity Increase

[all cross sections in units of (AU)$^2$]

| $\epsilon$ | Jupiter    | Saturn     | Uranus     | Neptune    |
|------------|------------|------------|------------|------------|
| 0.05       | 54300 ± 707| 67700 ± 775| 126000 ± 1030| 167000 ± 1150|
| 0.10       | 40700 ± 611| 55000 ± 700| 106000 ± 958| 143000 ± 1080|
| 0.20       | 29200 ± 521| 42900 ± 621| 83300 ± 860| 113000 ± 983|
| 0.30       | 22800 ± 458| 36700 ± 576| 69500 ± 794| 95600 ± 915|
| 0.40       | 18600 ± 410| 32400 ± 544| 59300 ± 738| 82900 ± 861|
| 0.50       | 15500 ± 374| 28800 ± 517| 52300 ± 697| 73300 ± 816|
| 0.60       | 13700 ± 351| 25800 ± 491| 46600 ± 661| 64900 ± 774|
| 0.70       | 11900 ± 327| 22700 ± 462| 41500 ± 626| 58700 ± 740|
| 0.80       | 10500 ± 306| 19800 ± 432| 36700 ± 592| 53200 ± 710|
| 0.90       | 9120 ± 286 | 17500 ± 407| 32300 ± 558| 47700 ± 678|
| 0.95       | 8520 ± 276 | 16400 ± 393| 30400 ± 542| 45100 ± 662|
| 1.00       | 7970 ± 267 | 15300 ± 381| 28400 ± 526| 41900 ± 640|
| escape     | 7290 ± 255 | 14000 ± 365| 25000 ± 492| 35400 ± 584|
| capture    | 684 ± 81.4 | 1300 ± 115 | 3320 ± 194 | 6640 ± 280 |
Table 2
Cross Sections for Planet Scattering: Inclination Increase

[all cross sections in units of (AU)$^2$]

| $\Delta \theta_i$ | angle increase | total       |
|------------------|----------------|-------------|
| 0.06             | $149900 \pm 1100$ | $158000 \pm 1130$ |
| 0.10             | $114600 \pm 1010$  | $132000 \pm 1050$  |
| 0.20             | $85000 \pm 880$    | $102000 \pm 943$   |
| 0.30             | $71800 \pm 815$    | $89000 \pm 888$    |
| 0.40             | $63600 \pm 770$    | $80800 \pm 850$    |
| 0.50             | $57600 \pm 736$    | $74800 \pm 822$    |
| 0.60             | $53100 \pm 707$    | $70300 \pm 798$    |
| 0.70             | $49500 \pm 684$    | $66700 \pm 780$    |
| 0.80             | $46400 \pm 663$    | $63600 \pm 763$    |
| 0.90             | $43900 \pm 647$    | $61000 \pm 750$    |
| 1.00             | $41400 \pm 629$    | $58500 \pm 736$    |
| 1.10             | $39500 \pm 616$    | $56600 \pm 726$    |
| 1.20             | $37700 \pm 602$    | $54800 \pm 715$    |
| 1.30             | $35700 \pm 585$    | $52800 \pm 702$    |
| 1.40             | $33700 \pm 570$    | $50900 \pm 690$    |
| 1.50             | $32400 \pm 559$    | $49500 \pm 682$    |
| 1.60             | $30900 \pm 546$    | $48000 \pm 672$    |
| 1.70             | $29400 \pm 534$    | $46500 \pm 663$    |
| 1.80             | $27700 \pm 518$    | $44900 \pm 652$    |
| 2.00             | $24600 \pm 488$    | $41700 \pm 630$    |
| 2.20             | $21400 \pm 459$    | $38500 \pm 609$    |
| 2.40             | $17100 \pm 411$    | $34300 \pm 576$    |
| 2.60             | $12300 \pm 349$    | $29400 \pm 537$    |
| 2.80             | $6870 \pm 263$     | $24000 \pm 490$    |
| 3.00             | $2020 \pm 143$     | $19100 \pm 441$    |
Table 3
Cross Sections for KBO Scattering

[all cross sections in units of (AU)^2]

| $\epsilon$ | $a = 30$ AU | $a = 40$ AU | $a = 50$ AU | $a = 60$ AU | $a = 70$ AU |
|-----------|-------------|-------------|-------------|-------------|-------------|
| 0.05      | 172000 ± 1670 | 206000 ± 1780 | 236000 ± 1860 | 263000 ± 1780 | 286000 ± 1820 |
| 0.10      | 147000 ± 1580 | 179000 ± 1700 | 206000 ± 1780 | 229000 ± 1720 | 251000 ± 1770 |
| 0.20      | 119000 ± 1450 | 145000 ± 1570 | 169000 ± 1670 | 189000 ± 1630 | 209000 ± 1680 |
| 0.30      | 99600 ± 1350  | 123000 ± 1480 | 143000 ± 1570 | 161000 ± 1540 | 179000 ± 1600 |
| 0.40      | 86700 ± 1270  | 106000 ± 1390 | 126000 ± 1500 | 140000 ± 1460 | 156000 ± 1530 |
| 0.50      | 76100 ± 1200  | 92800 ± 1310  | 110000 ± 1420 | 123000 ± 1390 | 138000 ± 1460 |
| 0.60      | 66700 ± 1120  | 82900 ± 1250  | 98600 ± 1350  | 109000 ± 1320 | 123000 ± 1400 |
| 0.70      | 59600 ± 1070  | 74100 ± 1190  | 88300 ± 1300  | 97800 ± 1270  | 111000 ± 1350 |
| 0.80      | 52000 ± 1000  | 66400 ± 1130  | 78800 ± 1240  | 87300 ± 1210  | 99400 ± 1290  |
| 0.90      | 46000 ± 947   | 59100 ± 1080  | 70400 ± 1180  | 77800 ± 1160  | 88900 ± 1230  |
| 1.00      | 39800 ± 889   | 52700 ± 1030  | 61800 ± 1110  | 67700 ± 1090  | 78000 ± 1170  |
REFERENCES

Adams, F. C. 2000. Theoretical models of young open star clusters: Effects of a gaseous component and gas removal. Astrophys. J. 542, 964 – 973.

Adams, F. C., and M. Fatuzzo 1996. A theory of the initial mass function for star formation in molecular clouds. Astrophys. J. 464, 256 – 271.

Adams, F. C., and P. C. Myers 2000. Modes of multiple star formation. Astrophys. J. submitted.

Adams, F. C., and F. H. Shu 1986. Infrared spectra of rotating protostars. Astrophys. J. 308, 836 – 853.

Allen, R. L., G. M. Bernstein, and R. Malhotra 2000. The edge of the solar system. Preprint.

Arnould, M., G. Paulus, and G. Meynet 1997. Short-lived radionuclide production by non-exploding Wolf-Rayet stars. Astron. Astrophys. 321, 452 – 464.

Battinelli, P., and R. Capuzzo-Dolcetta 1991. Formation and evolutionary properties of the galactic open cluster system. Mon. Not. R. Astron. Soc. 249, 76 – 83.

Binney, J., and S. Tremaine 1987. Galactic Dynamics. Princeton Univ. Press, Princeton.

Bonnell, I. A., and M. B. Davies 1998. Mass segregation in young clusters. Mon. Not. R. Astron. Soc. 295, 691.

Boss, A. P., and P. N. Foster 1998. Injection of short-lived isotopes in the presolar cloud. Astrophys. J. 494, L103 – L106.

Brandl, B., B. J. Sams, F. Bertoldi, A. Eckart, R. Genzel, S. Drapatz, R. Hofmann, M. L"owe, and A. Quirrenbach 1996. Adaptive optics near-infrared imaging of R136 in 30 Doradus: The stellar population of a nearby starburst. Astrophys. J. 466, 254 – 273.

Brusso, M., R. Gallino, and G. J. Wasserburg 1999. Nucleosynthesis in asymptotic giant branch stars: Relevance for galactic enrichment and solar system formation, Ann. Rev. Astron. Astrophys. 37, 239 – 309.

Cameron, A.G.W. 1978. Physics of the primitive solar accretion disk, Moon and Planets 18, 5 – 40.

Cameron, A.G.W., P. Hoeftlich, P. C. Myers, and D. D. Clayton 1995. Massive supernovae, Orion gamma rays, and the formation of the solar system. Astrophys. J. 447, L53 – L56.

Duquennoy, A. and M. Mayor 1991. Multiplicity among solar type stars in the solar neighborhood II. Distribution of the orbital elements in an unbiased sample. Astron. Astrophys. 248, 485 – 524.
Elmegreen, B. G. 2000. Star formation in a crossing time, *Astrophys. J.* **530**, 277 – 281.

Elmegreen, B. G., and C. Clemens 1985. On the formation rate of galactic clusters in clouds of various mass. *Astrophys. J.* **294**, 523 – 532.

Elmegreen, B. G., and Y. N. Efremov 1997. A universal formation mechanism for open and globular clusters in turbulent gas. *Astrophys. J.* **480**, 235 – 245.

Farinella, P., D. R. Davis, and S. A. Stern 2000. Formation and collisional evolution of the Edgeworth-Kuiper Belt. In Protostars and Planets IV (V. Mannings, A. P. Boss, and S. S. Russell, eds.), pp. 1255 – 1284. Univ. Arizona Press, Tucson.

Gahm, G. F., K. Fredga, R. Liseau, and D. Dravins 1979. The far-UV spectrum of the T Tauri star RU Lupi. *Astron. Astrophys.* **73**, L4 – L6.

Gaidos, E. J. 1995. Paleodynamics – Solar System formation and the early environment of the Sun. *Icarus* **114**, 258 – 268.

Goswami, J. N., and H.A.T. Vanhala 2000. Extinct radionuclides and the origin of the solar system. In Protostars and Planets IV (V. Mannings, A. P. Boss, and S. S. Russell, eds.), pp. 963 – 994. Univ. Arizona Press, Tucson.

Henry, T. J. 1991. A systematic search for low mass companions orbiting nearby stars and the calibration of the end of the stellar main sequence. PhD Thesis, University of Arizona, Tucson.

Hillenbrand, L. A., and L. W. Hartmann 1998. A preliminary study of the Orion nebula structure and dynamics. *Astrophys. J.* **492**, 540 – 553.

Hollenbach, D., D. Johnstone, S. Lizano, and F. H. Shu 1994. Photoevaporation of disks around massive stars and application to ultracompact HII regions. *Astrophys. J.* **428**, 654 – 669.

Jewitt, D. C., and J. X. Luu 2000. Physical nature of the Kuiper Belt. In Protostars and Planets IV (V. Mannings, A. P. Boss, and S. S. Russell, eds.), pp. 1201 – 1230. Univ. Arizona Press, Tucson.

Jijina, J., P. C. Myers, and F. C. Adams 1999. Dense cores mapped in ammonia: A database. *Astrophys. J. Suppl.* **125**, 161 – 236.

Kastner, J. H., and P. C. Myers 1994. An observational estimate of the possibility of encounters between mass-losing evolved stars and molecular clouds. *Astrophys. J.* **421**, 605 – 614.

Kroupa, P. 1995. The dynamical properties of stellar systems in the galactic disc. *Mon. Not. R. Astron. Soc.* **277**, 1507 – 1521.

Laughlin, G., and F. C. Adams 1998. The modification of planetary orbits in dense open clusters. *Astrophys. J.* **508**, L171 – L174.
Laughlin, G., and F. C. Adams 2000. The frozen Earth: Binary scattering events and the fate of the solar system. *Icarus* **145**, 614 – 627.

Lee, T., F. H. Shu, H. Shang, A. E. Glassgold, and K. E. Rehm 1998. Protostellar cosmic rays and extinct radioactivities in meteorites. *Astrophys. J.* **506**, 898 – 912.

Lissauer, J. J. 1993. Planet formation. *Ann. Rev. Astron. Astrophys.* **31**, 129 – 174.

Malhotra, R., M. J. Duncan, and H. F. Levison 2000. Dynamics of the Kuiper Belt. In Protostars and Planets IV (V. Mannings, A. P. Boss, and S. S. Russell, eds.), pp. 1231 – 1254. Univ. Arizona Press, Tucson.

McKeegan, K. D., M. Chaussidon, and F. Robert 2000. Evidence for the in situ decay of $^{10}$Be in an Allende CAI and implications for short-lived radioactivity in the early solar system. *Lunar Plan. Sci.* XXXI, abs. no. 1999.

Myers, P. C., and G. A. Fuller 1993. Gravitational formation times and stellar mass distributions for stars of mass $0.3 – 30 M_\odot$. *Astrophys. J.* **402**, 635 – 642.

Myers, P. C., and A. Lazarian 1998. Turbulent cooling flows in molecular clouds. *Astrophys. J.* **507**, L157 – L160.

Press, W. H., B. P. Flannery, S. A. Teulkolsky, and W. T. Vetterling 1986. *Numerical Recipes: The art of scientific computing*. Cambridge Univ. Press, Cambridge.

Roberts, M. S. 1957. The numbers of early type stars in the galaxy and their relation to galactic clusters and associations. *Pub. Astron. Soc. Pacific* **69**, 59 – 64.

Salpeter, E. E. 1955. The luminosity function and stellar evolution, *Astrophys. J.* **121**, 161 – 167.

Shu, F. H. 1980. *The Physical Universe*. Univ. Science Books, Mill Valley.

Shu, F. H., D. Johnstone, and D. Hollenbach 1993. Photoevaporation of the solar nebula and the formation of the giant planets. *Icarus* **106**, 92 – 101.

Störzer, H. and D. Hollenbach 1999. Photodissociation region models of photoevaporating circumstellar disks and application to the propyls in Orion. *Astrophys. J.* **515**, 669 – 684.

Tremaine, S. 1991. On the origin of the obliquities of the outer planets. *Icarus* **89**, 85 – 92.
Fig. 1.— Probability distributions for the number of stars in the solar birth aggregate. The dashed curves show the probability $P_{>M}(N)$ of a cluster containing a sufficiently massive star for radioactive enrichment (the increasing function of $N$) and the probability $P_{\text{dis}}(N)$ of the solar system remaining undisrupted (the decreasing function of $N$). The solid curve shows the joint probability $P_{\odot}(N)$ of the solar birth aggregate being simultaneously large enough to contain a massive star (with mass $M_\star > M_C = 25 \, M_\odot$) and small enough to allow the planetary orbits to not be disrupted.

Fig. 2.— Probability of the solar system being born in a cluster environment that is rich enough to contain a massive star (with mass greater than $M_C$) and diffuse enough to not disrupt the orbits of the giant planets. The solid line shows the probability $\mathcal{P}$ as a function of the number $N_C$ of stars required to make the birth aggregate optically thick to scattering events. The dashed curve shows the probability $\mathcal{P}$ as a function of the required mass scale $M_C$ (where the numbers on the horizontal axis must be divided by 100 to express the mass $M_C$ in units of $M_\odot$).
