Temporally heterogeneous dynamics in granular flows

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Granular simulations are used to probe the particle scale dynamics at short, intermediate, and long time scales for gravity driven, dense granular flows down an inclined plane. On approach to the angle of repose, where motion ceases, the dynamics become intermittent over intermediate times, with strong temporal correlations between particle motions – temporally heterogeneous dynamics. This intermittency is characterised through large scale structural events whereby the contact network periodically spans the system. A characteristic time scale associated with these processes increases as the stopped state is approached. These features are discussed in the context of the dynamics of supercooled liquids near the glass transition.

Granular simulations of chute flows have captured features of the rheology with remarkable accuracy [10,11]. Three principal flow regimes are observed for chutes with rough bases (see Fig. 1). If $\theta$ is the inclination angle of the chute: i) no flow occurs below the angle of repose $\theta_r$, ii) steady state flow exists in the region $\theta_r < \theta < \theta_u$, and iii) unstable flow occurs for $\theta > \theta_u$. For all $\theta < \theta_u$, the density is constant throughout the bulk, away from the free top surface and lower boundary, with smoothly varying depth profiles of the flow velocity and the principle and shear stresses [10]. The location of $\theta_r$, also depends on the height of the flowing pile $h$ [12,13]. Pouliquen introduced the quantity $h_{stop}(\theta)$ that encodes all the (undetermined) information about the roughness and effective friction of the base to relate the dependency of $\theta_r$ on $h$. In the language of glasses and colloids, $h_{stop}(\theta)$ represents the glass transition or yield stress line, respectively.

Gravity driven flows continue to be studied extensively due to their ubiquity throughout nature – avalanches, debris flows, and sand dunes – and also because of their importance in materials handling throughout industry. More recently, there has been a focus of attention towards understanding the properties of chute flows because of the ability of obtaining well-defined and reproducible steady state flow regimes. Despite the fact that recent experiments have been able to determine, in detail, macroscopic information of granular flows [12,13], probing the internal dynamics remains problematic due to the opacity of the particles. Still, such flows offer a relatively clean system in which to develop theoretical models for constitutive relations as well as describing the flow at the level of the grain size [14]. This is where the role of simulation has proved extremely useful.

Granular materials remain at the forefront of contemporary research due to the extreme richness in their dynamics, either under shear [1], flowing out of a hopper [2], or driven by gravity [3]. Because thermodynamic temperature plays little role in determining these features, granular materials are widely recognized as a macroscopic analogue of athermal systems far from equilibrium [4]. The notion of jamming [4] provides a unifying framework to describe the behaviour of a wide range of systems, from the molecular level (supercooled liquids and glasses), microscale (colloidal glasses), through to the macroscale (granular matter), near their jamming transition. The motivation for this work comes from the view that granular flows can be used as a macroscopically accessible analogue through which we can gain a better understanding of the critical slowing down of the dynamics in systems near their point of jamming. One of the features associated with the approach of the glass transition is the concept of spatially heterogeneous dynamics [6]. This work presents features of a driven granular material flowing down an inclined plane – chute flows – that exhibit large scale, cooperative events on approach to the angle of repose where flow stops – its jamming point.

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particle motions, through the relaxation and reformation of mechanically stable clusters that periodically span the system. A characteristic time scale \( \tau_c \), associated with these correlations is found to increase as \( \theta - \theta_c \rightarrow 0 \).

The simulations were performed using the discrete element technique with interactions appropriate for granular materials \(^{10}\). Monodisperse (soft) spheres of diameter \( d \) and mass \( m \) flow down a roughened, inclined plane due to gravity \( g \). Initially, \( \theta \) is set to \( \theta_0 \) to remove any preparation history, then incrementally reduced in steps of \( 0.01^\circ \leq \Delta \theta \leq 0.5^\circ \). Particles interact only on contact (cohesionless), along directions normal and tangential (via static friction) to the vector joining their centres. Contacts are defined as two overlapping neighbours. All runs were performed with a particle friction coefficient \( \mu = 0.5 \) and coefficient of restitution \( \epsilon = 0.88 \). Most of the results presented are for \( N = 8000 \) particles flowing down a chute that is \( L_x = 20d \) long, \( L_y = 10d \) wide (the \( xy \)-plane employs periodic boundary conditions), and an average flow height \( h \approx 40d \) (with a free top surface). See Fig. 1. Massively, large-scale, parallel simulations were used for \( N = 160000 \), with \(( L_x, L_y, h ) = (100, 40, 40)d \), to demonstrate that the observed features are not a system size artifact. Length scales are non-dimensionalised by the particle diameter \( d \) and time in units of \( \tau = \sqrt{d/g} \).

The intermittent regime was first identified through measurements of the kinetic energy as shown in Fig. 1. In the continuous-flow, steady state regime the energy is approximately constant with small fluctuations about the mean value. As the inclination angle is incrementally decreased towards \( \theta_c \), changes in the time profile of the energy first become apparent for \( \theta_i \approx 20.2^\circ \).

![FIG. 2: Average kinetic energy per particle over an intermediate time window for \( N = 8000 \) and different inclination angles as indicated. The symbols are for \( N = 160000 \) at \( \theta = 19.5^\circ \).](image)

Because of the convective nature of the flow, the mean-squared displacement (MSD), shown in Fig. 3, is measured normal to the inclined plane in the \( z \) direction: \( \Delta z^2 \equiv \langle (z(t) - z(0))^2 \rangle \). During continuous flow, there is a clear crossover between the ballistic regime, for \( t/\tau < 10^{-1} \), and diffusive regime, \( t/\tau > 10 \). Closer to the jamming point, the MSD oscillates over times intermediate between them. The appearance of novel dynamics in the vicinity of \( \theta_c \) is a consequence of strong correlations emerging in time-time quantities such as the velocity autocorrelation function, \( C(t) \equiv \langle v_z(t)v_z(0) \rangle \), where \( v_z \) is the velocity component in the direction normal to the plane. See Fig. 4(a). These fluctuating quantities are a signature of temporally heterogeneous dynamics. To investigate this further, the average coordination number \( z \), Fig. 4(a), is shown over the intermediate time window, demonstrating that the structural processes associated with these correlations involve large-scale cooperative events. During the flowing phase of the intermittent regime, \( z \) remains lower than the mechanically stable limit for frictional spheres \( z_c = 4 \) \(^{15}\). In the static phase, the systems attains a value \( z \approx z_c \), thus almost coming to rest. (There is always a residual creeping motion as flow does not completely stop until \( \theta_r \).

To facilitate such processes, the contact force network undergoes temporal transitions between static and flowing phases. This evolution is captured in the (distinct) particle-particle contact force-force time correlation function \( F(t) \), as shown in Fig. 4(c). The physical picture of the intermittent regime is thus: over some characteristic time scale the system oscillates between almost total mechanical stability and fluid-like flow. This involves time-dependent, system-spanning, structural relaxation events through the break-up and reformation of particle contacts. This process is best depicted in the simulation snapshots of Fig. 5.

The intermittent regime can therefore be considered as temporally biphasic. During one phase, \( z \approx z_c \), so the
The intermittent regime is peculiar due to the complex interplay between several different characteristic length and time scales. Identifying the structural entities of characteristic size $\ell$, remains a matter of debate \cite{9, 16}. However, the general picture is as follows: one expects $\ell$ to increase with decreasing $\theta$, or correspondingly, the shear rate $\dot{\gamma}$. Then, $\theta_r$ is the angle where $\ell \approx h$, resulting in the transition from a dynamic state to a static one. The development of an intermittent regime suggests another competing time scale, whereby $\ell \approx h$ can occur, but other mechanisms drive the system away from stability before the system permanently jams. This seems to coincide with the regime where the scaling of the flow velocity differs from that of the continuous flow regime \cite{11}. Presently, it is not clear how to approach this problem without a more sophisticated particle scale analysis.

Progress can still be made by identifying the characteristic time scales of the intermittent regime. One is the periodicity in the kinetic energy, correlation functions, and $z$. This time scale is rather insensitive to $\theta$ in the intermittent phase. However, $C(t)$ has an envelope of decay that corresponds to the extent of the intermittency. A time scale $\tau_c$, that characterises the initial decay in $C(t)$ is shown in Fig. 7. Interestingly, a power-law divergence: $\tau_c \sim (\phi - \phi_r)^{-\gamma}$, with $\gamma \approx 0.65$, captures the behaviour quite well. As a comparison, a modified Vogel-Fulcher form is also shown in Fig. 7 which is satisfactory close to the jamming point, but deviates further away. The available range for analysis is somewhat restricted through this particular definition of $\tau_c$, so it remains unclear how

depending on the time window of observation.

![FIG. 4: Intermediate time window associated with the oscillations in the MSD, for 19.35° (dashed line), 19.5° (thick), and 24° (dotted): (a) coordination number $z$, (b) velocity auto-correlation function $C(t)$, and (c) force-force time correlation function $F(t)$.](image)

![FIG. 5: The contact force network at $\theta = 20^\circ$ for three successive times. (a) Snapshot corresponds to a peak in the coordination number, (b) a trough, and (c) the next peak. The shade and thickness of the lines represent the magnitudes of the normal forces between two particles in contact. The inclination has been removed in the figures and the frame is a guide to the eye. The simulation axes are indicated.](image)

system is close to mechanical stability, while the flowing phase has a considerably lower coordination. A signature of this is seen in the probability distribution function of the normal contact forces $P(f)$. The partial distributions shown in Fig. 6 are for the high-$z$ phase (defined as configurations with $z > 3.5$) which resembles that of a static packing – exponential decay at high forces – and separately the low-$z$ phase ($z < 2$), with a high-$f$ tail that decays more slowly. This suggests that it may be possible to distinguish between different types of flows depending on the time window of observation.

![FIG. 6: Probability distribution function $P(f)$ of the normal contact forces (normalised so $\bar{f} = 1$) for $\theta = 20^\circ$; total (solid line), the high-$z$ phase (dashed), and the low-$z$ phase (dotted). In the high-$z$ phase, the contact forces are binned into a histogram if $z > 3.5$ for that configuration, and similarly if $z < 2.0$ in the low-$z$ curve. The full $P(f)$ for $\theta = 22^\circ$ (symbols) is shown for comparison.](image)
this time scale might correspond to the more familiar definitions of relaxation times in supercooled fluids [17]. A more appropriate definition of $\tau_c$ is being investigated.

![Graph](image)

FIG. 7: Characteristic time scale $\tau_c$, with error bars, associated with the intermittent structural events at intermediate times as a function of distance from the jamming point ($\theta - \theta_i$). The solid line is a power law fit: $\tau \sim (\theta - \theta_i)^{-\gamma}$, with $\gamma \approx 0.65$; whereas the dashed line is of the modified Vogel-Fulcher form: $\tau_c \sim \exp\{A(\theta - \theta_i)^{-\gamma}\}$, with $A \approx 0.66$ and $p \approx 0.39$.

In conclusion, gravity-driven, dense, granular flows down an inclined plane exhibit intermittent dynamics over intermediate time scales in the vicinity of the angle of repose. The intermittent regime signals the onset of temporal heterogeneous dynamics, whereby the short time motion is ballistic, at long times diffusive, but at intermediate times a combination of creeping flow and trap-like dynamics exists. The system becomes trapped in temporally metastable states [18] through large scale cooperative events. Correlations between these events decay, but do so more slowly as the jamming point is approached. The behaviour of the characteristic time scale $\tau_c$, shares similarities with the properties of supercooled liquids in the vicinity of the glass transition. Such dynamical similarities were recently probed experimentally in vibrated bead packs [20]. Therefore, in a typically heuristic fashion, an analogy with glassy dynamics theory can be made. $\theta_r$ marks the point where all dynamics cease and in this sense plays the role of the glass transition temperature $T_g$. Indeed, earlier work [12] showed that the flowing-static transition resembles the way a model liquid approaches the glassy phase [13]. However, the way the dynamics change at $\theta_r$, is similar is spirit to the way the dynamics in a supercooled fluid changes at the mode coupling temperature $T_c$. [13]. With this view in mind, one can associate $\theta_i$ with $T_c$. These results therefore hint towards a unifying picture of dynamical heterogeneities in dense, amorphous systems. Because the time scales associated with the onset of dynamical heterogeneity and cooperativity are experimentally accessible, granular flows offer a convenient system to address questions often associated with glasses at the molecular and colloidal level. This emerging picture thus begs the question, can existing theories that are currently being applied to traditional glasses and colloids be extended to granular systems?

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