A new independent limit on the cosmological constant/dark energy from the relativistic bending of light by Galaxies and clusters of Galaxies

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ABSTRACT
We derive new limits on the value of the cosmological constant, \( \Lambda \), based on the Einstein bending of light by systems where the lens is a distant galaxy or a cluster of galaxies. We use an amended lens equation in which the contribution of \( \Lambda \) to the Einstein deflection angle is taken into account and use observations of Einstein radii around several lens systems. We use in our calculations a Schwarzschild–de Sitter vacuole exactly matched into a Friedmann–Robertson–Walker background and show that a \( \Lambda \)-contribution term appears in the deflection angle within the lens equation. We find that the contribution of the \( \Lambda \)-term to the bending angle is larger than the second-order term for many lens systems. Using these observations of bending angles, we derive new limits on the value of \( \Lambda \). These limits constitute the best observational upper bound on \( \Lambda \) after cosmological constraints and are only two orders of magnitude away from the value determined by those cosmological constraints.

Key words: gravitation – gravitational lensing – cosmology: theory.

1 INTRODUCTION
Cosmic acceleration and the dark energy associated with it constitute one of the most important and challenging current problems in cosmology and all physics, see for example the reviews (Weinberg 1989; Sahni & Starobinsky 2000; Turner 2000; Carroll 2001; Padmanabhan 2003; Peebles & Ratra 2003; Upadhye, Ishak & Steinhardt 2005; Albrecht et al. 2006; Ishak 2007, and references therein). The cosmological constant, \( \Lambda \), is among the favoured candidates responsible for this acceleration. Current constraints on \( \Lambda \) are coming from cosmology (see e.g. Riess et al. 1998; Perlmutter et al. 1999; Bennett et al. 2003; Knop et al. 2003; Page et al. 2003; Spergel et al. 2003; Riess et al. 2004; Tegmark et al. 2004; Seljak et al. 2005; Spergel et al. 2007), and it is important to obtain constraints or limits from other astrophysical observations.

Very recently, the authors of reference (Rindler & Ishak 2007) demonstrated that, contrarily to previous claims (e.g. Islam 1983; Freire, Bezerra & Lima 2001; Kerr, Hauck & Mashhoon 2003; Kagramanova, Kunz & Lammerzahl 2006; Sereno & Jetzer 2006; Finelli, Galaverni & Gruppuso 2007), when the geometry of the Schwarzschild–de Sitter (SdS) space–time is taken into account, the cosmological constant does contribute to the light-bending around a concentrated source and hence to the corresponding Einstein deflection angle. This result was confirmed in Lake (2007), Sereno (2007) and Schucker (2007).

In this paper, we incorporate that result into the broadly used lens equation and then apply it to current observations of Einstein radii around distant galaxies and clusters of galaxies. Using observational data of a selected list of Einstein radii around clusters and galaxies, we show that the contribution of the cosmological constant to the bending angle can be larger than the second-order term of the Einstein bending angle. These new results allow us to put new independent upper bounds on the value of the cosmological constant based on the observations of the bending angle by galaxies and clusters of galaxies. The results provide an improvement of eight orders of magnitude on previous upper bounds on \( \Lambda \) from planetary or stellar systems (see e.g. Kagramanova et al. 2006; Sereno & Jetzer 2006). Interestingly, these limits provide the best observational upper bound on \( \Lambda \) after cosmological constraints and are only two orders of magnitude away from the value determined by those cosmological constraints.

2 THE BENDING ANGLE IN THE PRESENCE OF A COSMOLOGICAL CONSTANT
We outline here the main steps of the calculation of Rindler & Ishak (2007) and extend it using the second-order terms for the solution of the null geodesic equation. We consider the SdS metric (Kottler 1918):

\[
\text{d} s^2 = f(r)\text{d} t^2 - f(r)^{-1}\text{d} r^2 - r^2[\text{d}\vartheta^2 + \sin^2(\vartheta)\text{d}\phi^2],
\]

where

\[
f(r) = 1 - \frac{2m}{r} - \frac{\Lambda r^2}{3},
\]

and where we use relativistic units (\( c = G = 1 \)), \( m \) being the mass of the central object.
As shown in many text books (e.g. Misner, Thorne & Wheeler 1973; Rindler 2006) the null geodesic equation in SdS space–time is given exactly by

\[
d\frac{u^2}{u^2} + u = 3m u^2, \quad (u \equiv 1/R).
\] (3)

In the usual way, the null orbit is obtained as a perturbation of the undeflected line (i.e. the solution of equation 3 without the right-hand side)

\[
r \sin(\phi) = R.
\] (4)

After substitution of (4) into (3), one obtains the following equation for \( u \) (equation 11.64 in Rindler 2006):

\[
1 \frac{1}{r} = u = \frac{\sin(\phi)}{R} + \frac{3m}{2R^2} \left[ 1 + \frac{1}{3} \cos(2\phi) \right],
\] (5)

where \( R \) is a constant related to the physically meaningful area distance \( r_0 \) of closest approach (when \( \phi = \pi/2 \) by

\[
1 = \frac{1}{r_0} + \frac{m}{R^2}.
\] (6)

Many authors (see e.g. Misner et al. 1973; Wald 1984) use the impact parameter \( b \) to discuss the bending of light in Schwarzschild space–time, but SdS space–time is not asymptotically flat and one needs to define another parameter such as \( R \). As shown in Rindler & Ishak (2007), the contribution of \( \Lambda \) to the bending angle comes from the space–time metric itself, independently of the parameterization of the null geodesic equation.

It was shown in Rindler & Ishak (2007) that the angle \( \theta \) of our Fig. 1 (denoted by \( \psi \) in Rindler & Ishak 2007) is given by

\[
\tan(\theta) = \frac{f(r)^{1/2}}{|d\phi/dr|}
\] (7)

with \( f(r) \) as in equation (2) above \( f(r) = \alpha(r) \) in Rindler & Ishak 2007) and

\[
d\frac{r}{d\phi} = \frac{mr^2}{R^2} \sin(2\phi) - \frac{r^2}{R} \cos(\phi)
\] (8)

to lowest order. The total bending angle \( \alpha \) (at coordinate \( \phi = 0 \), just so as to have some standard position at which to measure it) was found in Rindler & Ishak (2007) to be

\[
\alpha \approx \frac{4m}{R} - \frac{\Lambda R^3}{6m}.
\] (9)

to first order in \( m/R \). This result shows that a positive \( \Lambda \) diminishes \( \alpha \), as might well be expected from the repulsive effect of \( \Lambda \). The first term in (9) is simply the classical Einstein bending angle to first order.

Now, since we plan to compare the observations, it is useful to expand the calculation to higher orders including the second-order solution to the null geodesic equation. In the usual way (see e.g. Bodenner & Will 2003), we write

\[
u = u_0 \left[ \sin(\phi) + (mu_0)u_1 + (mu_0^2)u_2 \right],
\] (10)

where \( u \equiv \frac{1}{r} \) and \( u_0 \equiv \frac{1}{R} \). Substituting this into equation (3) and collecting terms of equal powers of \( Mu_0 \) gives the following two equations:

\[
d\frac{\delta u_1}{d\phi^2} + \delta u_1 = 3 \sin^2 \phi,
\] (11)

\[
d\frac{\delta u_2}{d\phi^2} + \delta u_2 = \sin 2\phi.
\] (12)

Solving (11) and (12) for \( \delta u_1 \) and \( \delta u_2 \) and substituting them into (10) gives the solution

\[
\frac{1}{r} = \frac{1}{R} + \frac{3m}{2R^2} \left[ 1 + \frac{1}{3} \cos(2\phi) \right] + \frac{3m^2}{16R^2} \left[ 10\pi \cos \phi \right.
\]

\[
- 20\phi \cos \phi - \sin 3\phi \] (13)

Now we differentiate (13) and multiply by \( r^2 \) to obtain

\[
d\frac{r}{d\phi} = -\frac{r^2}{R} \cos \phi - \frac{mr^2}{R^2} \sin 2\phi + \frac{15m^2r^2}{4R^3} \left[ \cos \phi \right.
\]

\[
+ \frac{3}{20} \cos 3\phi + \left( \frac{\pi}{2} - \phi \right) \sin \phi \].
\] (14)

After some manipulation, it follows from (7) and (14) that the total bending angle (at \( \phi = 0 \)) to the third order is given by

\[
\alpha \approx \frac{4m}{R} + \frac{15\pi m^2}{4R^2} + \frac{305m^3}{12R^3} - \frac{\Lambda R^3}{6m}.
\] (15)

The coefficients for the first- and second-order terms in this expansion are the same as the ones in the expansion in terms of the impact parameter \( b \) (see e.g. Keeton & Petters 2005), which is used for the asymptotically flat Schwarzschild space–time. In the next section, we put our results into an observational context using systems where the lens is a galaxy or a cluster of galaxies.

**Figure 1.** The lens equation geometry. Observer, lens and source are at \( O \), \( L \) and \( S \), respectively. The position of the un lensed source is at an angle \( \beta \), the apparent position is at the angle \( \theta \) and the deflection angle is \( \alpha \). The distance from the observer to the source is \( D_{OS} \), from the observer to the lens is \( D_{OL} \), and from the lens to the source is \( D_{LS} \). The angle \( \phi \) is as shown on the figure. As usual, the lens equation follows from the geometry as \( \theta, D_{OS} = \beta D_{OS} + \alpha D_{LS} \).
3 OBSERVATIONS OF EINSTEIN RADII AND THE CONTRIBUTION OF THE COSMOLOGICAL CONSTANT TO THE DEFLECTION

As one might expect, while the cosmological constant has a very negligible effect on small scales this is not the case at the level of galaxies and clusters of galaxies. In this section, we evaluate the contribution of the cosmological constant to the bending of light using observations of large Einstein radii where the lens is a galaxy or a cluster of galaxies.

Equations (9) and (15) above were derived based on a source and an observer located in a SdS background. We will derive here the corresponding equation in a Friedmann–Lemaître–Robertson–Walker background (FLRW). For that we consider a SdS vacuole exactly embedded into an FLRW space–time using the Israel–Darmois formalism (Darmois 1927; Israel 1966). The relations between radial coordinates $r_b$ at the boundary of the vacuole are simple and well known in the literature (see e.g. Einstein & Strauss 1945; Schucking 1954), and are given by the following two equations:

$$r_b \text{ in } SdS = a(t) \ r_b \text{ in } FLRW \tag{16}$$

and

$$m_{SdS} = \frac{4\pi}{3} r_b^3 \text{ in } SdS \times \rho_{\text{matter in FLRW}} \tag{17}$$

Thus, for a given cluster mass, equation (17) provides a boundary radius where the space–time transitions from a SdS space–time to an FLRW background. We will assume that all the light-bending occurs in the SdS vacuole according to our previous formulae, and that once the light transitions out of the vacuole and into FLRW space–time, all A-bending stops. Unlike the mass-effect, which falls off quickly, the $\Lambda$-effect on the bending of light increases with distance from the source (the ‘$\Lambda$-repulsion’ is proportional to distance); hence, the question of where to cut-off the integration becomes important. The choice of the boundary of the vacuole ($r_b$) in the Einstein–Strauss model seems physically the most appropriate, whereas the choice $\phi = 0$ in reference (Rindler & Ishak 2007) was purely conventional.

Now, for the small angle $\phi_b$ at the boundary, equation (5) gives

$$u_b = \frac{1}{r_b} = \frac{\phi_b}{R} = \frac{2m}{R^2}$$

and equation (8) gives

$$|A| = \frac{r_b^2}{R} \left( 1 - \frac{2\phi_b m}{R} \right).$$

Next, inserting (18) and (19) into equation (7) yields after a few steps

$$\theta \approx \tan \theta \approx \phi_b + \frac{2m}{R} - \frac{\Delta \phi_b r_b^2}{6} + \text{higher order terms.} \tag{20}$$

The bending angle, $\alpha$, is given, to the smallest order in $m/R$ and $\Lambda$, by

$$\alpha \approx \frac{\theta - \phi_b}{2} \approx \frac{2m}{R} - \frac{\Delta \phi_b r_b^2}{6}. \tag{21}$$

Now, equation (18) yields, to the smallest order, $\phi_b = R/r_b$, so we can finally write from equation (21)

$$\alpha \approx \frac{4m}{R} - \frac{\Lambda R r_b}{3}, \tag{22}$$

where $R$ is related to the closest approach by equation (6) and $r_b$ is the boundary radius between SdS and FLRW, and is given by equation (17).

Perhaps a caveat that one need to address is that the Einstein–Strauss model that was used here is known to have some instability to radial perturbations at the boundary as, for example, discussed in Krasinski (1997), and references therein. However, our work hinges on finding a cut-off location where the $\Lambda$-bending of the lens can be regarded as accomplished. In the predecessor paper to this one (Rindler & Ishak 2007), we chose $\phi = 0$ as the only readily available standard cut-off point. This paper is an improvement over the previous one in this respect, in that we now have a cut-off point tailored to each individual lens, namely the edge of the vacuole. The vacuole model as such is not used except for this one purpose, namely to give us a realistic order-of-magnitude estimate of the ‘range of influence’ of the lens. Moreover, as it is widely used in gravitational lensing studies, one could also resort to approximation methods where the inhomogeneity is modelled by a gravitational potential that is embedded in an FLRW background (Mellier 1999; Bartelmann & Schneider 2001; Carroll 2004). Such an alternative treatment of the questions addressed here has been recently carried out in Ishak (2008) and has confirmed the findings of this work.

Also, our result is expressed in terms of the vacuole boundary $r_b$ that is evaluated at some instant in time. The vacuole and its boundary expand as the Universe expands thus when we calculate $r_b$ from equation (17) we must use the density of the universe as it was when light passed by the lens. We are aware of the instability of $r_b$ but since we need it at one instant, the instability should not affect our result.

Next, using equations (13) and (14), we can expand the result to

$$\alpha \approx \frac{4m}{R} + \frac{15\pi m^2}{4 R^2} + \frac{305 m^3}{12 R^3} - \frac{\Lambda R r_b}{3} \tag{23}$$

Finally, following the usual procedure (see e.g. Mellier 1999; Bartelmann & Schneider 2001), we put our results into the lens equation which is given from the geometry (see Fig. 1) and small–angle relations as follows

$$\theta D_{OS} = \beta D_{OS} + \alpha D_{LS} \tag{24}$$

or in the familiar form

$$\theta = \beta + \alpha \frac{D_{LS}}{D_{OS}} \tag{25}$$

where all the quantities are as defined in Fig. 1, and the angular-diameter distance is given by

$$D(z) = \frac{c}{H_0 (1 + z)} \int_0^z \frac{dz'}{\sqrt{\Omega_m (1 + z')^3 + \Omega_\Lambda}}. \tag{26}$$

where, for the spatially flat concordance cosmology, $\Omega_m = 0.27$, $\Omega_\Lambda = 0.73$ and $H_0 = 71 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

Thanks to the advancement of observational techniques, one can find in the literature a number of distant galaxies and clusters of galaxies that are lenses with large Einstein radii, making them very interesting for applying our results. The selected systems are shown in Table 1 along with our evaluation of the deflection first-order term, the second-order term and the $\Lambda$-term, and some of their ratios. Despite the smallness of the cosmological constant, $\Lambda$, we find that the Einstein first-order term in the bending angle due to these systems is only by some $10^3$ bigger than the $\Lambda$-term. Interestingly, we find that for the lens systems in Table 1, the contribution of the cosmological constant term is larger than the second-order term of the Einstein bending angle.
Table 1. Contributions of the cosmological constant to the Einstein bending angle by distant clusters of galaxies. Column 8 shows that the $\Lambda$-term contribution is larger than the second-order term in the Einstein bending angle for these lens systems. The last column shows limits on the cosmological constant based on observations of the bending angle. These limits provide the best upper bound on $\Lambda$ after cosmological constraints and are only two orders of magnitude away from the value determined for $\Lambda$ by those cosmological constraints, i.e. $1.29 \times 10^{-56}$ cm$^{-2}$. Previously, the best upper bound after cosmology was determined from planetary or stellar systems and is $\Lambda \leq 10^{-56}$ cm$^{-2}$ (see e.g. Rindler 1969; Kagramanova et al. 2006, and references therein).

| Cluster or galaxy Name and references | Einstein Radius (Kpc) | Mass in $M_{\odot} h^{-1}$ | First order term (radials) | Second order term (radials) | $\Lambda$-term (radials) | Ratio 1 first/$\Lambda$-term | Ratio 2 $\Lambda$-term/second | Upper limit on $\Lambda$ (cm$^{-2}$) |
|---------------------------------------|----------------------|-----------------------------|---------------------------|---------------------------|-------------------------|---------------------------|-----------------------------|-----------------------------|
| Abell 2744 (Smail et al. 1991; Allen 1998) | 96.4 | $1.97 \times 10^{13}$ | 5.53E−05 | 2.25E−09 | 1.68E−08 | 3.28E+03 | 7.48 | 4.23E−54 |
| Abell 1689 (Allen 1998; Limousin 2007) | 138.2 | $9.36 \times 10^{13}$ | 1.88E−04 | 2.61E−08 | 4.52E−08 | 3.52E+03 | 1.73 | 5.37E−54 |
| SDSS J1004+4112 (Sharon 2006) | 110.0 | $4.26 \times 10^{13}$ | 1.05E−04 | 8.06E−09 | 2.22E−08 | 4.70E+03 | 2.76 | 6.07E−54 |
| 3C 295 (Wold et al. 2002) | 127.7 | $7.1 \times 10^{13}$ | 1.50E−04 | 1.66E−08 | 3.06E−08 | 4.90E+03 | 1.84 | 6.33E−54 |
| Abell 2219L (Sharon 2006; Smail et al. 1995a; Allen 1998) | 86.3 | $3.22 \times 10^{13}$ | 1.01E−04 | 7.47E−09 | 1.85E−08 | 5.44E+03 | 2.48 | 7.01E−54 |
| AC 114 (Smail et al. 1995b; Allen 1998) | 54.6 | $9.23 \times 10^{12}$ | 4.57E−05 | 1.54E−09 | 7.38E−09 | 6.19E+03 | 4.80 | 7.99E−54 |

4 A NEW LIMIT ON THE COSMOLOGICAL CONSTANT FROM LIGHT-BENDING

From cosmology (e.g. using supernova magnitude–redshift relation and the cosmic microwave background radiation), the value of the cosmological constant, $\Lambda$, is found to be about $1.29 \times 10^{-56}$ cm$^{-2}$ (using $H_0 = 71$ km s$^{-1}$/Mpc and $\Omega_\Lambda = 0.73$, see e.g. Rindler 1969; Riess et al. 1998; Perlmutter et al. 1999; Bennett et al. 2003; Knop et al. 2003; Page et al. 2003; Spergel et al. 2004; Riess et al. 2004; Tegmark et al. 2004; Seljak et al. 2005; Spergel et al. 2007). It is very desirable to obtain other limits on $\Lambda$ that come from other astrophysical constraints. As we show, when we consider the uncertainty in the measurements of the bending angle (which is around $\Delta \alpha \sim 5$–10 per cent for several of the systems considered in Table 1), we find that the bending angle due to distant galaxies and clusters can provide interesting limits on the value of the cosmological constant. Indeed, if the contribution of $\Lambda$ cannot exceed the uncertainty in the bending angle for these systems, then it follows that

$$\Lambda \leq \frac{3 \Delta \alpha}{R_{\theta_b}}. \quad (27)$$

For example, with $\Delta \alpha = 10$ per cent, we find from the system Abell 2744 (Smail et al. 1991; Allen 1998) that

$$\Lambda \leq 4.23 \times 10^{-54} \text{ cm}^{-2}. \quad (28)$$

The other limits are in Table 1. Interestingly, these limits are the best observational upper bound on the value of $\Lambda$ after cosmological constraints and are only two orders of magnitude away from the value determined from cosmological constraints. In fact, $\Lambda$ also enters into the expression of the angular-diameter distance but our estimation is that it can affect our limit by a factor of 2 or less. Previously, the best upper bound after cosmology was provided from planetary or stellar systems and is $\Lambda \leq 10^{-56}$ cm$^{-2}$ (see e.g. Kagramanova et al. 2006; Sereno & Jetzer 2006, and references therein).

5 CONCLUSION

In conclusion, we showed that a $\Lambda$-contribution term appears in the deflection angle within the lens equation. This contribution is larger than the second-order term in the Einstein bending angle for many cluster lens systems. These results allow us to put new upper bounds on the cosmological constant, $\Lambda$, based on observations of the bending angle by galaxies and clusters of galaxies. These results provide an improvement of eight orders of magnitude on previous upper bounds on $\Lambda$ that were based on planetary or stellar systems (e.g. Kagramanova et al. 2006; Sereno & Jetzer 2006). The limits provide the best upper bound on $\Lambda$ after cosmological constraints and are only two orders of magnitude away from the value determined for $\Lambda$ from those cosmological constraints.

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