Pion and Kaon form factors in the perturbative QCD approach

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Abstract

We show the complete perturbative QCD calculation for pion electromagnetic form factor at different powers corresponding to the conformal expansion of distribution amplitudes in twist, which is further explored for the kaon form factor. Both the lowest Fock state and high Fock state with quark-gluon-antiquark configuration are taken into account in the calculation, the behaviour of power expansion in twist is confirmed by the numerical result, the chiral enhancement effect at subleading power is examined to exist in the PQCD working and the SU(3) asymmetry in kaon form factor is estimated to no larger than 30%.

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I. INTRODUCTION

Quantum chromodynamics (QCD) has two fundamental properties, the quark confinement in low energy region and the asymptotic freedom in high energy region, which lead to the hadron formatted particles and the perturbative QCD representation, respectively. Form factor is the physical quantum that describes the redistribution of parton momenta inside a hadron when an external interaction happens, i.e., a constituent is hit by an energetic photon with the hadron do not fall apart, so it carries both the information of the hadron structure and the hard scattering amplitude, whose discrepancy is separated by the factorization theory [1–4]. The electromagnetic (e.m.) form factor of pion, being the simplest but simultaneously also the most fundamental QCD observed quantity, attracts much attention theoretically [5–8] and experimentally [9–11].

The statements from different theoretical approaches are not the same for the form factor, i.e., the light-cone sum rules (LCSRs) believes that the soft dynamics gives dominate contribution [6], while in the perturbative QCD (PQCD) the form factor is described by a hard scattering amplitude [7], actually they have different optimum applicabilities exceeding which the approach is not valid well again. The LCSRs approach is more reliable starting from the intermediate region, and the prediction power of PQCD approach for pion form factor holds well in the large momentum transfer squared region, i.e., $Q^2 \geq 10 \text{ GeV}^2$, even with considering the resummation effects, right now the lattice QCD (LQCD) evaluation is still at a few small $Q^2$ points [8] and the direct experiment measurement is reliable also below 3 GeV$^2$ [10, 11]. In this paper we compensate the higher power corrections to pion and kaon form factors up to twist-four of the meson DAs, aiming to check the power expansion behaviour from one side, and from another side to improve the theoretical accuracy in the framework of PQCD approach.

The rest of the paper is organized as follows. In Sec. II, the PQCD calculation of the spacelike pion form factor is performed up to twist-four with considering both the quark-antiquark and quark-gluon-antiquark configurations, the analytical expressions of the hard amplitudes are given. In Sec. III, we present the procedure of PQCD approach to calculate pion form factor, several highlight concerns are proposed. Section IV is the numerics and we conclude in Sec. V.

II. POWER CORRECTIONS

Pion form factor is defined by the nonlocal matrix element

$$\langle \pi^- (p_2) | J_{\mu}^{e.m.} \pi^- (p_1) \rangle \equiv e_q (p_1 + p_2) F_\pi (Q^2),$$  \hspace{1cm} (1)
we are interest in the case that the smallness of relative distance is ensured by the "external reason", says the large momentum transfer between the hadrons\(^1\), where \(\pi \hat{p}_i \sim 1\) and the parameter determined the role of the each given operator is the twist (dimension minus spinor). To separate the amplitude of the matrix element contributed from the short- and long-distance interactions, we replace directly the lines with large virtuality by free propagators in the first approximation, while retain the lines with small virtuality in the Heisenberg operator due to the nonperturbative property. The matrix element is then written in the factorizable formula as,

\[
\langle \pi^-(p_2)|J_{\mu}^{\text{em.}}|\pi^-(p_1)\rangle = \oint dz_1 dz_2 \left\{ \delta_{ij} \langle \pi^-(p_2) \left\{ N_\alpha(0) \exp \left( ig_s \int_{z_2}^{0} d\sigma_{\nu} A_{\nu}(\sigma) \right) u_\beta(z_2) \right\} \right\}_{k_l} \left| 0_\mu \right|, \\
H_{ij\delta\alpha}^{ijkl}(z_1, z_2) \cdot \langle 0 \left\{ \pi_\alpha(z_2) \exp \left( ig_s \int_{z_1}^{z_2} d\sigma_{\nu} A_{\nu}(\sigma) \right) d_\delta(z_1) \right\}_{kl} \left| \pi^-(p_1) \right|_{\mu t},
\]

where \(\gamma, \beta, \alpha, \delta\) and \(i,j,k,l\) are the spinor and color indexes, respectively. The hard kernel for the lowest Fock state is

\[
H_{ij\delta\alpha}^{ijkl}(z_1, z_2) = (-1) [ig_s\gamma_\mu]_{\alpha\beta} T^{ij} \left[ (ie_q\gamma_\mu) S_0(0 - z_1)(ig_s\gamma_\mu) \right]_{\gamma\delta} T^{kl} \left[ -iD_{mn}^{0}(z_1 - z_2) \right],
\]

in which the factor \((-1)\) comes from the anticommunicativity of the quark file operator and the free propagators in the coordinate space are

\[
S_0(z) = \frac{i}{2\pi z}, \quad D_{mn}^{0}(z) = \frac{1}{4\pi} \frac{g_{mn}}{z^2}.
\]

The nonlocal matrix elements describe the amplitudes, i.e., for the meson break-up into a pair of the soft quarks, which is in further expanded in terms of the definition of \(\pi\) meson distribution amplitudes (DAs) with different twists,

\[
\langle 0 \left\{ \pi_\alpha(z_2) \exp \left( ig_s \int_{z_1}^{z_2} d\sigma_{\nu} A_{\nu}(\sigma) \right) d_\delta(z_1) \right\}_{kl} \left| \pi^-(p_1) \right|_{\mu t} = \delta_{il} \frac{1}{3} \left\{ \frac{1}{4} (\gamma_5 \gamma_\mu)^\delta \langle 0 | \Bar{\pi}(z_2) \exp \left( ig_s \int_{z_1}^{z_2} d\sigma_{\nu} A_{\nu}(\sigma) \right) (\gamma_\rho \gamma_5) d(z_1) | \pi^-(p_1) \rangle_{\mu t} \\
+ \frac{1}{4} (i\gamma_5)^\delta \langle 0 | \Bar{\pi}(z_2) \exp \left( ig_s \int_{z_1}^{z_2} d\sigma_{\nu} A_{\nu}(\sigma) \right) (i\gamma_5) d(z_1) | \pi^-(p_1) \rangle_{\mu t} \\
+ \frac{1}{8} (\sigma^{\tau\tau'}\gamma_5)^\delta \langle 0 | \Bar{\pi}(z_2) \exp \left( ig_s \int_{z_1}^{z_2} d\sigma_{\nu} A_{\nu}(\sigma) \right) (i\sigma^{\tau\tau'}\gamma_5) d(z_1) | \pi^-(p_1) \rangle_{\mu t} \\
+ \cdots \right\},
\]

the ellipsis indicates the remaining terms in the Fierz transformation, the truncated scale of the integral over internal momentum \(\mu_t\) is usually known as the factorizable scale\(^2\). The definition of LCDAs with different twists are collected in appendix A.

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\(^1\) Rather than the "internal reason" by the \(W\)-boson mass, the heavy \(b\)-quark mass where the operator product is used at the small distance region \(\pi \ll 1/\mu_t\).

\(^2\) We will drop this index in the following part for the concise.
The form factor at leading twist is obtained straightforwardly from Eq.2.

\[ F_{\pi}^{t2}(Q^2) = \frac{8}{9} \alpha_s \pi f_\pi^2 Q^2 \int \frac{d^2k_{1T}}{(2\pi)^2} \frac{d^2k'_{1T}}{(2\pi)^2} \int_0^1 dx \int_0^1 dy \varphi_\pi(x)\varphi_\pi(y) \frac{\bar{y}}{\Delta_1^2 \Delta_2^2}, \]  

(6)

with the internal propagators carrying momenta \( \Delta_1 = \bar{y}p_2 - p_1 = (-Q/\sqrt{2}, \bar{y}Q/\sqrt{2}, \mathbf{k}) \) and \( \Delta_2 = \bar{x}p_1 - \bar{y}p_2 = (\bar{x}Q/\sqrt{2}, -\bar{y}Q/\sqrt{2}, \mathbf{k} - \mathbf{k}') \) (\( \bar{y} = 1 - y \) and \( \bar{x} = 1 - x \)). The contribution to the form factor from two-parton twist-three DAs is

\[ F_{\pi}^{t3.2p}(Q^2) = \frac{16}{9} \alpha_s \pi f_\pi^2 m_\pi^2 \int \frac{d^2k_{1T}}{(2\pi)^2} \frac{d^2k'_{1T}}{(2\pi)^2} \int_0^1 dx \int_0^1 dy \left( \frac{1}{\Delta_1^2 \Delta_2^2} \cdot \left[ -y \varphi_\pi^P(x)\varphi_\pi^P(y) - \frac{1}{6} \varphi_\pi^P(x)\varphi_\pi^P(y) \left( \frac{yQ^2}{\Delta_1^2} + \frac{(\bar{x} - \bar{y})Q^2}{\Delta_2^2} + 1 + \frac{(2 - x)yQ^2}{\Delta_2^2} \right) \right] \right). \]  

(7)

We also get

\[ F_{\pi}^{t2@4.2p}(Q^2) = \frac{16}{9} \alpha_s \pi f_\pi^2 \int \frac{d^2k_{1T}}{(2\pi)^2} \frac{d^2k'_{1T}}{(2\pi)^2} \int_0^1 dx \int_0^1 dy \left( \bar{y}Q \left[ \frac{1}{\Delta_1^2 \Delta_2^2} + \frac{\bar{y}(2 - x)Q^2}{\Delta_1^2 \Delta_2^2} + \frac{1}{\Delta_1^2 \Delta_2^2} \right] \left[ \varphi_\pi(x)g_{1\pi}(y) - \varphi_\pi(x)g_{2\pi}(y) \right] \right) + \left[ \bar{y}Q^2 \left( \frac{1}{\Delta_1^2 \Delta_2^2} + \frac{\bar{y}Q^2}{\Delta_1^2 \Delta_2^2} + \frac{1}{\Delta_1^2 \Delta_2^2} \right) \varphi_\pi(x)g_{2\pi}(y) \right], \]  

(8)

for the contribution associated with the convoluting of twist-two and twist-four DAs in two-parton-to-two-parton scattering, which piece has been studied in the LCSRs approach [12], and here is the firstly time to been compensated in PQCD approach. To obtain this result, we have defined an auxiliary DA \( g_2^i(x) = \int_0^x dx' g_2(x') \) with the bound condition \( g_2^i(x = 0, 1) = 0 \), and used the following Fourier transformation,

\[ \frac{1}{x^2} \leftrightarrow -i4\pi^2 \frac{1}{p^2}, \quad \frac{x_\alpha}{x^2} \leftrightarrow 8\pi^2 \frac{p_\alpha}{(p^2)^2}, \quad \frac{x_\alpha}{x^2} \leftrightarrow 2\pi^2 \frac{p_\alpha}{p^2}, \]  

\[ x_\alpha x_\beta \leftrightarrow -i8\pi^2 \frac{g_{\alpha\beta} - 4p_\alpha p_\beta}{(p^2)^2}, \quad x_\alpha x_\beta \leftrightarrow -i2\pi^2 \frac{g_{\alpha\beta} - 2p_\alpha p_\beta}{p^2}. \]  

(9)

With dropping the subleading terms proportional to the transversal momenta on the numerator in the large momentum transferred processes, the second term on the right hand side of Eq.8 vanishes, and the contributions associated with twist-three DAs and twist-two \& twist-four DAs reduce to

\[ F_{\pi}^{t3.2p}(Q^2) \to \frac{16}{9} \alpha_s \pi f_\pi^2 m_\pi^2 \int \frac{d^2k_{1T}}{(2\pi)^2} \frac{d^2k'_{1T}}{(2\pi)^2} \int_0^1 dx \int_0^1 dy \left( \frac{1}{\Delta_1^2 \Delta_2^2} \cdot \left[ -y \varphi_\pi^P(x)\varphi_\pi^P(y) + \frac{1}{6y} \varphi_\pi^P(x)\varphi_\pi^P(y) \right] \right), \]  

(10)

\[ F_{\pi}^{t4@4.2p}(Q^2) \to \frac{16}{9} \alpha_s \pi f_\pi^2 \int \frac{d^2k_{1T}}{(2\pi)^2} \frac{d^2k'_{1T}}{(2\pi)^2} \int_0^1 dx \int_0^1 dy \left( \frac{1}{\Delta_1^2 \Delta_2^2} \cdot \left[ g_{2\pi}(x)\varphi_\pi(y) + \left( \frac{y + \bar{y}}{\bar{x}} + \frac{3}{2} \right) \varphi_\pi(x)g_{2\pi}(y) \right] \right). \]  

(11)
The parton transversal momenta in the lowest Fock state brings a gauge dependence, which is cancelled with the gauge dependence associated with the three-parton configuration[13]. The gauge invariance guarantees the factorization formula up to subleading powers, such as the contributions from two-parton high twists DAs and from three-parton DAs of meson. It is also argued that the dominate contribution in the three-parton-to-three-parton scattering comes from the Feynman diagram with a four-gluon vertex [13], this statement is demonstrated by the naive power counting analysis: a valence soft gluon attached to the internal quark propagators introduces another power suppression such as $O(1/(\bar{y}Q^2))$, while the suppression is weaker when the soft gluon attaches to the internal hard gluon, i.e., $O(1/(\bar{x}yQ^2))$, so the leading contribution with three-parton DAs is associated with the assignment that both the soft gluons from initial and final mesons attach to the hard gluon at a same vertex, and one part of this contribution is written in terms of the twist-three DAs $\varphi_{3\pi}(x_i)$.

$$F_{3\pi}^{3,3p}(Q^2) = \frac{16}{3} \alpha_s \pi f_{3\pi}^2 Q^2 \int \frac{D^2 k_T}{(2\pi)^2} \frac{D^2 k'_T}{(2\pi)^2} \int_0^1 D x_i \int_0^1 D y_i \varphi_{3\pi}(x_i) \varphi_{3\pi}(y_i) \frac{1-y_i}{\Delta_1^2 \Delta_2^2 \Delta_3^2}.$$  (12)

with the momenta $\Delta_1 = p_1 - p_2 + k_2$, $\Delta_2 = p_2 - k_2 - (p_1 - k_1)$ and $\Delta_3 = \bar{k}_1 - \bar{k}_2$, the momenta carried by the quark lines are $k_1 = (x_1 p^+_1, 0, k_{1\perp})$ and $k_2 = (0, y_1 p^-_2, k_{2\perp})$ for the initial and final meson, respectively, and the antiquark lines carry momenta $\bar{k}_1 = (x_2 p^+_1, 0, k_{1\perp})$ and $\bar{k}_2 = (0, y_2 p^-_2, k_{2\perp})$. The integral variable $D x_i = dx_1 dx_2 dx_3 \delta(1 - x_1 - x_2 - x_3)$, $D^2 k_T = d^2 k_{1T} d^2 k_{2T}$, and then $\int_0^1 D x_i = \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \int_0^{1-x_1-x_2} dx_3$. It is easy to find that this part is at subleading power ($O(1/Q^2)$) with comparing to the form factor at leading twist DAs, another part contribution in the three-parton-to-three-parton scattering is associated with twist-four DAs,

$$F_{\pi}^{4,4p}(Q^2) = \frac{8}{3} \alpha_s \pi f^2 \int \frac{D^2 k_T}{(2\pi)^2} \frac{D^2 k'_T}{(2\pi)^2} \int_0^1 D x_i \int_0^1 D y_i \frac{1}{\Delta_1^2 \Delta_2^2 \Delta_3^2} \cdot \left\{ \varphi\parallel(x_i) \varphi\perp(y_i) \left[ \frac{2Q^2}{\Delta_2^4} \left( -2(1 - y_1) + \frac{y_2}{2} \right) + \frac{5Q^2 y_2}{\Delta_2^4} + \frac{2Q^4}{\Delta_2^4} \right] (1 - y_1) y_2 x_2 + \frac{4Q^4}{\Delta_2^4} \left( 1 - y_1 \right) y_2 x_2 \right\}$$

$$\left[ + \varphi\parallel(y_i) \varphi\perp(x_i) \left[ 4 + \frac{Q^2}{\Delta_2^2} (1 - y_1) - \frac{Q^2}{\Delta_2^2} (1 - y_1)(1 - x_1) + \frac{Q^2}{\Delta_3^2} (1 - y_1)x_2 \right] + \varphi\parallel(y_i) \varphi\perp(x_i) \left[ - \frac{Q^2}{\Delta_2^2} (1 - y_1) y_1 + \frac{Q^2}{\Delta_3^2} y_1 y_2 \right] + \varphi\perp(y_i) \varphi\perp(x_i) \left[ 5y_1 + \left[ \varphi \rightarrow \bar{\varphi} \right] \right] \right\},$$  (13)

where the auxiliary DAs $\varphi\parallel(x_i) \equiv \int_0^{x_i} dx'_i \varphi\parallel(x'_i, x_2, x_3)$ and $\varphi\perp(y_i) \equiv \int_0^{y_i} dy'_i \varphi\perp(y_1, y'_i, y_3)$ are introduced again, with the bound condition $\varphi\parallel(x_1 = 0/1, x_2, x_3) = 0$ and $\varphi\perp(y_1, y_2 = 0/1, y_3) = 0$, respectively.
III. THE PQCD FORMULAS

Let us discuss the end-point behaviours of the form factors contributed at different powers, at leading power accuracy the form factor \( F_2^{t\pi}(Q^2) \) is end-point safety due to the exchanging symmetry when two valence quarks form a pion in the perturbative limit, the end-point problem starts to emerge at subleading power \( \mathcal{O}(1/Q^2) \), in terms of the logarithm singularity (i.e., the second term of Eq.7 and the first term of Eq.8), linear singularity (i.e., the second term of Eq.8) and their products (i.e., the first term of Eq.7 and Eq.12), the square singularity appears at next-to-next-leading power \( \mathcal{O}(1/Q^4) \), as seen in Eq.13. We recall the transversal momentum for each external quark field to regularize the end-point singularity with the off-shellness \( k_i^2 \), performing resummation of the large logarithm \( \ln(Q^2/k_i^2) \) (appeared in the high orders correction to hard kernel) leads to the Sudakov factor,

\[
S(x_i, y_i, b, b', \mu) = \sum_{i=1,2} \left[ s\left(x_i \frac{Q}{\sqrt{2}} b\right) + s_q(b, \mu) \right] + \sum_{i=1,2} \left[ s\left(y_i \frac{Q}{\sqrt{2}} b'\right) + s_q(b', \mu) \right],
\]

where the factorization scale is set to the maximal virtuality in the hard amplitude \( \mu = \max(1/b_1, 1/b_2, \sqrt{\bar{y}Q}) \). The first type terms collect the double and single logarithms in the vertex correction associated with an energetic light quark [14–16], while the second type terms resum the single logarithms in the quark self-energy correction [4, 17],

\[
s_q(b, \mu) = -\frac{1}{\beta_1} \ln \left( \frac{\ln(\mu/\Lambda^{(5)})}{-\ln(b\Lambda^{(5)})} \right) - \frac{\beta_2}{2\beta_1} \left[ \ln[2 \ln(\mu/\Lambda^{(5)})] + 1 \right] - \frac{\ln[-2 \ln(b\Lambda^{(5)})] + 1}{-\ln(b\Lambda^{(5)})}.
\]

Eq.15 is obtained with using the strong coupling at two-loop accuracy, the \( \beta \) functions are \( \beta_1 = (33 - 2n_f)/12 \) and \( \beta_2 = (153 - 19n_f)/24 \). The number of the active quarks is chosen as

\[
n_f(\mu) = \text{Which} \left[ 0 < \mu < m_c, 3, m_c \leq \mu < m_b, 4, m_b \leq \mu < m_t, 5 \right].
\]

For the hadronic scale we take it from PDG [18] in the \( \overline{\text{MS}} \) scheme, which is obtained by using the four-loop expression of \( \alpha_s \) and doing the three-loop matching at the quark pole masses

\[
\Lambda = \text{Which} \left[ n_f < 3, 0.332, 3 \leq n_f < 4, 0.292, 4 \leq n_f < 5, 0.210 \right],
\]

with the pole masses \( m_c = 1.3 \text{ GeV} \), \( m_b = 4.2 \text{ GeV} \), and \( m_t = 173 \text{ GeV} \), respectively.

The longitudinal momentum fractions in the initial and final states also generate large logarithm (i.e., the double logarithm \( \alpha_s \ln^2 x \)) in the end-point regions, which is resumed, in the convariant gauge \( \partial \cdot A = 0 \), to all order to form a universal jet function [19–21],

\[
J(x) = -\exp\left( \frac{\pi}{4} \alpha_s C_F \right) \int_{-\infty}^{\infty} \frac{dt}{\pi} (1 - x)^{\exp(t)} \sin\left( \frac{\alpha_s C_F t}{2} \right) \exp\left( -\frac{\alpha_s}{4\pi} C_F t^2 \right).
\]
The jet function could be factorized out from the meson wave functions and regarded as a part in the hard kernel, and in practical, for convenience, it is usually parameterized approximately by another Sudakov factor [22, 23],

\[
S_t(x) = \frac{1}{\sqrt{\pi}} \frac{\Gamma(\frac{3}{2} + c)}{\Gamma(1 + c)} [x(1 - x)]^c. \tag{19}
\]

This formula satisfies, without doubt, to the two fundamental properties of the jet function obtained by resolving the running function in Eq.18, one is the vanishment in the end-point region, and the other one is the normalization condition in the perturbative limit \( \alpha_s \to 0 \) (\( c \to 0 \)). We remark here that the threshold resummation happens only for the high twist contributions, and the jet function modifies the shapes of high twist LCDAs, especially in the end-point region, to be proportional to \( x(1 - x) \) (as parameterized in Eq.19), which then eliminates effectively the end-point singularity in the power suppressed contributions.

The mixed logarithm \( \ln(\zeta^2/k_F^2) \ln x \) also brings visible effect in the transversal-momentum-dependent (TMD) pion wave function, and the variable \( \zeta^2 \equiv 4(p \cdot n)^2/n^2 \) (\( p \) is the meson momentum, \( n \) is a vector deviated lightly from the light-cone) introduces the factorization-scheme dependence on a typical choice of Wilson line. The joint resummation with off-shell Wilson line has been proposed to resolve this problem and the improved pion wave function highlights the moderate \( x \) and small \( b \) regions for the momentum distribution [24], as an supplement to the conventional \( k_T \) and threshold resummations. Considering the fact that the complicated expression of joint-resummed improved wave function brings an minor impact on the pion form factor, in this work we would still adopt the conventional TMD pion wave function, with setting \( \zeta^2 = Q^2 \), to estimate the different power contributions.

The formulas in Eqs.(14,18) are derived specially for the lowest Fock state of the mesons, for the high Fock state, such as the quark-gluon-antiquark configuration, these formulas are not available any more since the Sudakov factor associated with a valence gluon must differ from that associated with a valence quark. To evade the Sudakov factor for the valence gluon which is still missing in the factorization theorem, we consider only the effective Sudakov factor associated with the most energetic quarks in the three-parton Fock state, and neglect the Sudakov factors associated with the gluon and the soft quarks [13], the approximation is taken as

\[
S^3(x_1, y_1, b_1, \mu) = s\left((1 - x_1) \frac{Q}{\sqrt{2}}, b_1\right) + s\left(x_2 \frac{Q}{\sqrt{2}}, b_2\right) + s\left((1 - y_1) \frac{Q}{\sqrt{2}}, b'_1\right) + s\left(y_2 \frac{Q}{\sqrt{2}}, b'_2\right), \tag{20}
\]

and the factorization scale is modified to

\[
\mu = \text{Max}[1/b_1, 1/b_2, 1/b'_1, \sqrt{(1 - y_1)Q}] . \tag{21}
\]

3 A nondipolar gauge link for the TMD pion wave function, much simpler than the long-standing dipolar Wilson lines with a complicated soft subtraction [25], is suggested recently [26, 27] to eliminate the pinched singularity in the self-energy correction of the nonlightlike Wilson line. In this work we would not deal with the pinched singularity problem because the NLO pion wave function with the nondipole definition is still missing at subleading twist.

4 In fact, \( b_2 = b'_2 \) due to the Gaussian integral in Eq.23.
For the transversal component of the momentum integration, it is more convenient to process in the coordinate space conjugated to the transversal momenta, the Fourier transformation with two propagators reads

\[
\int db_1^2 db_2^2 \exp\left(-ik_1 \cdot b_1 - ik'_1 \cdot b'_1\right) \int \frac{d^2k_{1T}}{(2\pi)^2} \frac{d^2k'_{1T}}{(2\pi)^2} \frac{1}{\alpha + k_1^2} \frac{1}{\beta + (k'_1 - k_1)^2} 
\]

\[
= \int_0^\infty b_1 db_1' db_2' K_0(\sqrt{\beta}b_1') \left[\Theta(b_1 - b_1')I_0(\sqrt{\alpha}b_1')K_0(\sqrt{\alpha}b_1) - [b_1 \leftrightarrow b_1']\right],
\]

(22)

where \(I_0\) and \(K_0\) is the modified Bessel function of the first and second kind, respectively, \(K_0\) is also called as Basset function. For the contribution with three internal propagators, the transversal integration is transferred to

\[
\int db_1^2 db_2^2 db_3^2 \exp\left(-ik_1 \cdot b_1 - ik'_1 \cdot b'_1 - ik_2 \cdot b_2 - ik'_2 \cdot b'_2\right) \frac{d^2k_{1T}}{(2\pi)^2} \frac{d^2k_{2T}}{(2\pi)^2} \frac{d^2k_{3T}}{(2\pi)^2} \frac{1}{\alpha + k_1^2} \frac{1}{\beta + (k'_1 - k_1)^2} \frac{1}{\gamma + (k'_2 - k_2)^2} 
\]

\[
= \int_0^\infty b_1 db_1' db_2' K_0(\sqrt{\beta}b_1') \left[\Theta(b_1 - b_1')I_0(\sqrt{\alpha}b_1')K_0(\sqrt{\alpha}b_1) - [b_1 \leftrightarrow b_1']\right] \int_0^\infty b_2 db_2' K_0(\sqrt{\gamma}b_2').
\]

(23)

In the past ten years, the PQCD approach has stepped greatly forward with the next-to-leading order (NLO) QCD corrections. Here we give a brief summary of the NLO progresses for light meson form factors, the NLO calculation for pion e.m form factor associated with two-parton twist-two and twist-three DAs are carried out in Ref. [28] and Ref. [29], respectively, following which, the NLO correction to timelike pion form factor is obtained by the analytical continuum technology [30, 31]. Another important correction is for the scalar pion form factor appeared in the factorizable annihilation diagrams [32], which provides the dominate strong phase in PQCD approach to deal with two-body nonleptonic charmless \(B\) decay. Recently the NLO calculation has implemented for the \( \rho \pi\) transition process to determine the strong coupling \(g_{\rho\pi}\) [33], and for the \(\rho\) form factors [34]. All the calculations turn out that the convergency of perturbative expansion is good in the corresponding energy region, which examines the prediction power of PQCD at NLO. We would also include the NLO QCD corrections in the following numerics analysis, for the convenience of the reader, we quote here the correction functions [28, 29] in two-parton-to-two-parton scattering,

\[
F^{(1)}_{12}(x_1, t, Q^2) = \frac{\alpha_s C_F}{4\pi} \left[ \frac{3}{4} \ln \frac{t^2}{Q^2} - \ln^2 x_1 - \ln^2 x_2 + \frac{45}{8} \ln x_1 \ln x_2 
+ \frac{5}{4} \ln x_1 + \frac{77}{16} \ln x_2 + \frac{\ln 2}{2} + \frac{5}{48} \pi^2 + \frac{53}{4} \right],
\]

(24)

\[
F^{(1)}_{13}(x_1, t, Q^2) = \frac{\alpha_s C_F}{4\pi} \left[ \frac{9}{4} \ln \frac{t^2}{Q^2} - \frac{53}{16} \ln(x_1 x_2) - \frac{1}{8} \ln^2 x_2 
- \frac{23}{16} \ln x_1 - \frac{2}{9} \ln x_2 - \frac{137}{96} \pi^2 + \frac{\ln 2}{4} + \frac{337}{64} \right].
\]

(25)
IV. NUMERICS

The pion form factor contributed from quark-antiquark (two-parton-to-two-parton scattering) and quark-gluon-antiquark (three-parton-to-three-parton scattering) configurations are rewritten compactly as follow,

\[
F_{\pi}^{2p}(Q^2) = \frac{8}{9} \alpha_s \pi f_\pi^2 Q^2 \int_0^1 dx \int_0^1 dy \int_0^{1/\Lambda} b_1 db_1 b_1' db_1' e^{-S(x, y, b, b', \mu)} \left\{ \bar{y} \varphi_{\pi}(x) \varphi_{\pi}(y) \left[ 1 + F_{12}^{(1)}(x, y, t, Q^2) \right] \mathcal{H} + \frac{2m_0^2}{Q^2} \left[ -y \varphi''_{\pi}(x) \varphi''_{\pi}(y) \right] \left[ 1 + F_{13}^{(1)}(x, y, t, Q^2) \right] \mathcal{H} + \frac{1}{6} \varphi''_{\pi}(x) \varphi''_{\pi}(y) \left[ -y Q^2 \mathcal{H}_1 - (\bar{x} - \bar{y} - 2\bar{y} + x\bar{y})Q^2 \mathcal{H}_2 - 1 \right] S_t(y) + \frac{2}{Q^2} g_{2\pi}(x) \varphi_{\pi}(y) x\bar{y}Q^2 \mathcal{H}_2 + \varphi_{\pi}(x) g_{2\pi}(y) \left[ y^2Q^2 \mathcal{H}_2 + (\bar{y}^2 - \frac{\bar{y}^2}{2})Q^2 \mathcal{H}_1 + 1 \right] + \left[ \varphi_{\pi}(x) y_1(x) - \varphi_{\pi}(x) g_{2\pi}(y) \right] \left[ 2y^2Q^2(\mathcal{H}_1 + \mathcal{H}_2 + \bar{y}(2 - x)Q^2 \mathcal{H}_3) \right] S_t(y) \right\},
\]

\[
F_{\pi}^{3p}(Q^2) = \frac{16}{3} \alpha_s \pi f_\pi^2 Q^2 \int_0^1 D x_1 \int_0^1 D y_1 \int_0^{1/\Lambda} b_1 db_1 b_1' db_1' b_2 db_2 e^{-S_3(x, y_1, b_1, \mu)} \left\{ \frac{f_\pi^2}{f_\pi^2} (1 - y_1) \varphi_{3\pi}(x_1) \varphi_{3\pi}(y_1) \mathcal{H'} + \frac{1}{2Q^2} \left[ \varphi_{\parallel}(x_1) \varphi_{\parallel}(y_1) \right] \left[ -4(1 - y_1) + (1 - y_1)y_2 \right] Q^2 \mathcal{H}_2' + 5(1 - y_1)y_2Q^2 \mathcal{H}_3' \right\},
\]

in which the hard functions associated with different DAs are written in terms of Bessel functions,

\[
\mathcal{H}(\alpha, \beta, b_1, b_1') = K_0(\sqrt{3}b_1') \left[ \Theta(b_1 - b_1') I_0(\sqrt{3}b_1') K_0(\sqrt{3}b_1) - [b_1 \leftrightarrow b_1'] \right],
\]
\[
\mathcal{H}_1(\alpha, \beta, b_1, b_1') = K_0(\sqrt{3}b_1') \left[ \frac{b_1'}{2\sqrt{3}} \Theta(b_1 - b_1') I_0(\sqrt{3}b_1') K_1(\sqrt{3}b_1) - [b_1 \leftrightarrow b_1'] \right],
\]
\[
\mathcal{H}_2(\alpha, \beta, b_1, b_1') = \frac{b_1'}{2\sqrt{3}} K_1(\sqrt{3}b_1') \left[ \Theta(b_1 - b_1') I_0(\sqrt{3}b_1') K_0(\sqrt{3}b_1) - [b_1 \leftrightarrow b_1'] \right],
\]
\[
\mathcal{H}_3(\alpha, \beta, b_1, b_1') = \frac{b_1'}{2\sqrt{3}} K_0(\sqrt{3}b_1') \left[ \frac{b_1'}{2\sqrt{3}} \Theta(b_1 - b_1') I_0(\sqrt{3}b_1') K_1(\sqrt{3}b_1) - [b_1 \leftrightarrow b_1'] \right],
\]
\[
\mathcal{H'}(\alpha', \beta', \gamma, b_1, b_1', b_2) = K_0(\sqrt{3}b_2) K_0(\sqrt{3}b_1') \left[ \Theta(b_1 - b_1') I_0(\sqrt{3}b_1') K_0(\sqrt{3}b_1) - [b_1 \leftrightarrow b_1'] \right],
\]
\[
\mathcal{H}_1'(\alpha', \beta', \gamma, b_1, b_1', b_2) = K_0(\sqrt{3}b_2) K_0(\sqrt{3}b_1') \left[ \frac{b_1'}{2\sqrt{3}} \Theta(b_1 - b_1') I_0(\sqrt{3}b_1') K_1(\sqrt{3}b_1) - [b_1 \leftrightarrow b_1'] \right],
\]
\[
\mathcal{H}_2'(\alpha', \beta', \gamma, b_1, b_1', b_2) = \frac{b_1'}{2\sqrt{3}} K_0(\sqrt{3}b_2) K_1(\sqrt{3}b_1') \left[ \Theta(b_1 - b_1') I_0(\sqrt{3}b_1') K_0(\sqrt{3}b_1) - [b_1 \leftrightarrow b_1'] \right],
\]
\[
\mathcal{H}_3'(\alpha', \beta', \gamma, b_1, b_1', b_2) = \frac{b_2}{2\sqrt{3}} K_1(\sqrt{3}b_2) K_0(\sqrt{3}b_1') \left[ \Theta(b_1 - b_1') I_0(\sqrt{3}b_1') K_0(\sqrt{3}b_1) - [b_1 \leftrightarrow b_1'] \right].
\]
and the internal virtualities entered in are

\[ \alpha \equiv \bar{y}Q^2, \quad \beta \equiv \bar{x}\bar{y}Q^2, \]
\[ \alpha' \equiv (1 - y_1)Q^2, \quad \beta' \equiv (1 - x_1)(1 - y_1)Q^2, \quad \gamma \equiv x_2y_2Q^2. \]

To obtain the form factor for kaon meson, we just need to do the replacement \( f_\pi \to f_K \), \( m_{\pi}^0 \to m_{K}^0 \) and also for the nonperturbative parameters in meson DAs. The power expansion is shown explicitly in Eqs.(26,27), which reads

\[ \mathcal{O}(1) : \mathcal{O}\left(\frac{m_{\pi}^0}{Q^2}\right) : \mathcal{O}\left(\frac{f_{\pi}^3}{m_{\pi}^0 Q^2}\right) : \mathcal{O}\left(\frac{\delta^2_{\pi}}{m_{\pi}^0 Q^2}\right) \]

corresponding to the contributions associated with leading twist, two-parton twist-three, twist-two convoluting with twist-4, three-parton twist-three and twist-four DAs, respectively.

### TABLE I. Hadronic parameters for \( \pi \) and \( K \) meson DAs in our evaluation.

| \( \pi \) | \( \mu = 1 \text{ GeV} \) | \( K \) | \( \mu = 1 \text{ GeV} \) | Remarks/Refs |
|---|---|---|---|---|
| \( f_\pi \) | 0.130 | \( f_K \) | 0.156 | in unit of GeV, [18] |
| \( m_{\pi}^0 \) | 1.91 | \( m_{K}^0 \) | 1.90 | in unit of GeV, [35, 36] |
| \( a_{1}^{\pi} \) | 0 | \( a_{1}^{K} \) | 0.10 ± 0.04 | [37] |
| \( a_{2}^{\pi} \) | 0.18 ± 0.05 | \( a_{2}^{K} \) | 0.25 ± 0.15 | \( a_{n\geq2} = 0 \), [38] |
| \( f_{3\pi} \) | 0.0045 ± 0.0015 | \( f_{3K} \) | 0.0045 ± 0.0015 | in unit of GeV², [39, 40] |
| \( \omega_{3\pi} \) | −1.5 ± 0.7 | \( \omega_{3K} \) | −1.2 ± 0.7 | [40] |
| \( \lambda_{3\pi} \) | 0 | \( \lambda_{3K} \) | 1.6 ± 0.4 | [40] |
| \( \delta_{\pi}^{2} \) | 0.18 ± 0.06 | \( \delta_{K}^{2} \) | 0.20 ± 0.06 | in unit of GeV², [40–42] |
| \( \omega_{4\pi} \) | 0.2 ± 0.1 | \( \omega_{4K} \) | 0.2 ± 0.1 | [36, 40] |
| \( \kappa_{4\pi} \) | 0 | \( \kappa_{4K} \) | −0.12 ± 0.01 | [36, 40] |

We take the PDG value \( m_s(2 \text{ GeV}) = 96^{+8}_{-4} \text{ MeV} \) corresponding to \( m_s(1 \text{ GeV}) = 125^{+10}_{-5} \text{ MeV} \). The well-known chiral perturbative theory (ChPT) relations [35]

\[ R \equiv \frac{2m_s}{m_u + m_d} = 24.4 ± 1.5, \quad Q^2 \equiv \frac{m_s^2 - (m_u + m_d)^2/4}{m_d^2 - m_u^2} = (22.7 ± 0.8)^2 \]

is used to determine the chiral masses of light mesons

\[ m_{\pi}^0 = \frac{m_s^2 R}{2m_s}, \quad m_{K}^0 = \frac{m_K^2}{m_s \left[ 1 + \frac{1}{R} \left( 1 - \frac{R^2 - 1}{4Q^2} \right) \right]}, \]

(32)
without involving the light quark masses $m_u$ and $m_d$ because we neglect them elsewhere besides in $m_0^\pi$ and $m_0^K$. Parameters for meson DAs chosen for numerical evaluation are displaced in Table I, in which excepting the second gegenbauer moment $a_2^\pi$ is evaluated from LQCD, all others are calculated from QCD sum rules.

Our prediction of pion and kaon form factors is depicted in Fig. 1, where the contributions from different powers are displaced separately, the contributions at leading (Red-Dash curves) and subleading twists (Blue-Dotted curves) include the NLO QCD corrections [28, 29] already. The chiral enhancement at twist-three in the lowest Fock state is shown evidently, and this effect is more apparent in the kaon form factor due to the $SU(3)$ asymmetry, we define the ratio between subleading and leading twist contributions as $R_{\pi}(Q^2) \equiv \frac{F_{T2\pi}(Q^2)}{F_{T3\pi}(Q^2)}$ and take the deviation of their relative magnitude from the unit, $A \equiv 1 - R_{\pi}(Q^2)/R_{K}(Q^2)$, to estimate the $SU(3)$ asymmetry, the result shows that this effect does not exceed 30% in the considered energy region and vanishes in the perturbative limit. Fig.1 also indicates explicitly the power expansion as we claimed following Eqs. 30, where the contributions from three-parton Fock states is at least one power lower than the leading contributions from lowest Fock state in the larger energy regions, i.e., $Q^2 \geq 10\text{GeV}^2$, the contribution from the two-parton Fock states with the DAs convoluted in terms of twist-two and twist-four is a litter bit larger than it from three-parton Fock state, but they are still in the same order.

As displaced in Tab.II, we suggest to compare our result with the LCSRs prediction [6, 12] at the energy point

\[Q^2 \geq 10\text{GeV}^2\]

Recently, the feasibility of calculating the pion DAs from suitably chosen Euclidean correlation functions at large momentum is investigated since this method allow us to study higher-twist DAs from LQCD[43, 44], the result for the twist-four parameter $\delta_2^\pi$ consists with it estimated from QCD sum rules, even though the systematic errors is still not yet under control.
$Q^2 = 10 \text{GeV}^2$ to end this section, where the theoretical uncertainty in our calculation mainly comes from the DAs input, two sources of uncertainty in LCSRs approach are the DAs inputs and the parameters of the approach itself. The choice of the scale for the nonperturative parameters affects weakly in the larger energy regions so we do not consider this uncertainty. We find that the prediction of pion and kaon form factors is comparable in the chosen energy point with considering the uncertainties, and the disparity of the numerical result obtained in these two approaches becomes less and less smaller when increasing $Q^2$ to more larger.

V. CONCLUSION

We study the pion and kaon electromagnetic form factors with considering high power contributions up to twist-four of the meson DAs, the PQCD calculation confirms the convergence behaviour of twist expansion, which shows that the contribution from the three-parton Fock state is at least one order of magnitude smaller than it from the lowest Fock state. The chiral enhancement of the subleading power contribution depends strongly on the corresponding DAs, and this effect is quite obvious in our choice of the conformal expansion of twist-three DAs. The superficial comparison between pion and kaon form factors contributed from two-parton-to-two-parton scattering indicates the $\text{SU}(3)$ asymmetry is no more than 30% in the considered energy regions. Because the current lattice QCD evaluation and experiment measurement of meson form factors are still in small $Q^2$, our calculation can not interplay directly with them now, but we look forward to see more data in the intermediate energy regions at Jefferson Lab with the 12 GeV upgrade, with which the precise PQCD predictions presented in this paper can be forward to extract the nonperturbative parameters in meson DAs, such as the moments in the gegenbauer expansion. In this time we compare our result with the prediction from LCSRs approach in the intermediate energy regions, and show the parallel prediction power of them. The further improvement in this project is to combine the precise measurement of timelike pion and kaon form factors in the resonance energy regions with the PQCD calculation at the large energy regions, in order to determine the meson distribution amplitudes under in the PQCD approach.

| $Q^2$ (GeV^2) | $Q^2 F_{\pi}^{\text{PQCD}} (Q^2)$ | $Q^2 F_{\pi}^{\text{LCSRs}} (Q^2)$ | $Q^2 F_{K}^{\text{PQCD}} (Q^2)$ | $Q^2 F_{K}^{\text{LCSRs}} (Q^2)$ |
|--------------|----------------|----------------|----------------|----------------|
| 10           | 0.75(10)       | 0.51(15)       | 1.08(15)       | 0.76(22)       |
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Appendix A: Definition of the distribution amplitudes

Light-cone distribution amplitudes (LCDAs) for pseudoscalar meson with two-parton configuration is defined by the nonlocal matrix element sandwiched between the meson state and vacuum \([40, 45]\),

\[
\langle 0 | \pi(z_2)(\gamma_\rho \gamma_5) q(z_1) | \mathcal{P}^{-} (p) \rangle = f_P \int_0^1 dx e^{-ixpz_1 - ipz_2} \left\{ i p_\rho \left[ \varphi_p(x) + (z_1 - z_2)^2 g_1(p)(x) \right] + \left[ (z_1 - z_2)_\rho \frac{p_\rho(z_1 - z_2)^2}{p(z_1 - z_2)} \right] g_2(p)(x) \right\}, \tag{A1}
\]

\[
\langle 0 | \pi(z_2)(\sigma_{\tau\tau} \gamma_5) q(z_1) | \mathcal{P}^{-} (p) \rangle = f_P m_0^P \int_0^1 dx e^{-ixpz_1 - ipz_2} \left( 1 - \frac{m_0^2}{(m_0^P)^2} \right) \left[ p^\tau(z_1 - z_2)_{\tau\tau} - p^{\tau'}(z_1 - z_2)_{\tau\tau} \right] \varphi^{\tau'}_P(x), \tag{A2}
\]

\[
\langle 0 | \pi(z_2)(i\gamma_5) q(z_1) | \mathcal{P}^{-} (p) \rangle = f_P m_0^P \int_0^1 dx e^{-ixpz_1 - ipz_2} \varphi_{P^\tau}(x), \tag{A3}
\]

where \(f_P\) is the decay constant, \(m_0^P\) is the chiral mass of pseudoscalar meson, \(\varphi_p\), \(\varphi_P^{\tau,\sigma}\) and \(g_1g_2\) corresponds to the DAs at twist-two, twist-three and twist-four, respectively.

For the quark-gluon-antiquark configuration, the DAs are defined with the matrix element with the gluon field strength tensor operator \(G_{\kappa\kappa'} = g_\kappa g^{\kappa'}_\kappa \lambda^\alpha/2\),

\[
p^+ \langle 0 | \pi(z_2)(\sigma_{\tau\tau} \gamma_5) G_{\kappa\kappa'}(z_0) q(z_1) | \mathcal{P}^{-} (p) \rangle = f_{3P} \int Dz \: e^{-ixpz_1 - ipz_2 - i\lambda z_0} \left[ (p_{\kappa} p_{\tau} g_{\kappa'\tau} - p_{\kappa'} p_{\tau} g_{\kappa\tau'}) - (p_{\kappa} p_{\tau} g_{\kappa'\tau} - p_{\kappa'} p_{\tau} g_{\kappa\tau'}) \right] \varphi_{3P}(x), \tag{A4}
\]

\[
p^+ \langle 0 | \pi(z_2)(\gamma_5) G_{\kappa\kappa'}(z_0) q(z_1) | \mathcal{P}^{-} (p) \rangle = f_P \int Dz \: e^{-ixpz_1 - ipz_2 - i\lambda z_0} \left[ p_\kappa (z_1 - z_2)_{\kappa\kappa'} - p_{\kappa'} (z_1 - z_2)_{\kappa\kappa'} \right] \varphi_{\|}(x) + (g_{\kappa'} p_{\kappa'} - g_{\kappa} p_{\kappa}) \varphi_{\perp}(x), \tag{A5}
\]

\[
p^+ \langle 0 | \pi(z_2)(\gamma_5) \tilde{G}_{\kappa\kappa'}(z_0) q(z_1) | \mathcal{P}^{-} (p) \rangle = f_P \int Dz \: e^{-ixpz_1 - ipz_2 - i\lambda z_0} \left[ p_\kappa (z_1 - z_2)_{\kappa\kappa'} - p_{\kappa'} (z_1 - z_2)_{\kappa\kappa'} \right] \tilde{\varphi}_{\|}(x) + (g_{\kappa'} p_{\kappa'} - g_{\kappa} p_{\kappa}) \tilde{\varphi}_{\perp}(x), \tag{A6}
\]

where \(\tilde{G}_{\kappa\kappa'} = 1/2 \epsilon_{\kappa\kappa'\eta\eta'} G^{\eta\eta'}\), the location of gluon file strength is at \(z_0 = u z_1 + \pi z_2\) with the free variable \(u \in [0, 1]\), \(\varphi_{3P}\) is the twist-three DA, and \(\varphi_{\|, \perp}, \tilde{\varphi}_{\|, \perp}\) are twist-four DAs. When \(q = d, s\), the meson \(\mathcal{P} = \pi, K\), respectively.
Appendix B: Expressions of the distribution amplitudes

LCDAs can be obtained by using the conformal partial expansion, and the most familiar expression is the leading twist DAs written in terms of the Gegenbauer polynomials,

\[ \varphi_p(x, \mu) = 6x\bar{x} \sum_{n=0} \alpha_n(\mu) C_n^{3/2}(2x - 1). \] (B1)

Two-particle twist-three DAs are related to the three-particle DA \( \varphi_{3P}(x_i) \) and also to the leading twist DA \( \varphi_p \) by the QCD equation of motion (EOM), the parameter \( \rho^P = (m_u + m_d)/m_0^P \) is introduced to reflect the quark masses terms in the EOM, in our calculation we only take into account the strange quark mass, with neglecting the \( u, d \) quark masses unless in the chiral masses \( m_0^P \). To next-to-leading order in conformal spin and to the second moments in truncated conformal expansion of \( \varphi_p \), we get

\[ \varphi^P_p(x, \mu) = 1 + 3\rho^P \left( 1 - 3a_1^P + 6a_2^P \right) (1 + \ln x) - \frac{\rho^P}{2} \left( 3 - 27a_1^P + 54a_2^P \right) C_1^{1/2}(2x - 1) \]
\[ + 3 \left( 10\eta_{3P} - \rho^P(a_1^P - 5a_2^P) \right) C_2^{1/2}(2x - 1) + \left( 10\eta_{3P}\lambda_{3P} - \frac{9}{2}\rho^P a_2^P \right) C_3^{1/2}(2x - 1) \]
\[ - 3\eta_{3P}\omega_{3P} C_4^{1/2}(2x - 1), \] (B2)

\[ \varphi^P_p(x, \mu) = 6x(1 - x) \left\{ 1 + \frac{\rho^P}{2} \left( 2 - 15a_1^P + 30a_2^P \right) + \rho^P \left( 3a_1^P - \frac{15}{2}a_2^P \right) C_1^{3/2}(2x - 1) \right. \]
\[ + \frac{1}{2} \left( \eta_{3P}(10 - \omega_{3P}) + 3\rho^P a_2^P \right) C_2^{3/2}(2x - 1) + \eta_{3P}\lambda_{3P} C_3^{3/2}(2x - 1) \]
\[ + 3\rho^P \left( 1 - 3a_1^P + 6a_2^P \right) \ln x \left\} \right. \] (B3)

\[ \varphi_{3P}(x_i) = 360x_1x_2x_3^2 \left\{ 1 + \lambda_{3P}(x_1 - x_2) + \omega_{3P} \frac{1}{2}(7x_3 - 3) \right\} , \] (B4)

where the contributions from the three-particle and from the two-particle by EOM are separated clearly, the three parameters \( f_{3P}, \lambda_{3P}, \omega_{3P} \) can be defined by the matrix element of local twist-three operators, and their evolution have the mixing terms with the quark mass \([40]\).

For the two-particle twist-four DAs, the definition considered in the strictly light-cone expansion in Eq. A1 is more convenient to be used in the QCD calculation, and their relations to the invariant amplitudes \( \psi_{4P}, \phi_{4P} \) defined in the Lorentz structure are,

\[ g_{2P}(x) = -\frac{1}{2} \int_0^x dx' \psi_{4P}(x'), \quad g_{1P}(x) = \frac{1}{16} \phi_{4P}(x) + \int_0^x dx' g_{2P}(x'). \] (B5)

The relations between different operators by EOM indicate that these Lorentz invariant amplitudes are written in terms of the "genuine" twist-four contribution from the three-particle DAs \( \varphi_{\parallel}(x_i) \), \( \varphi_{\perp}(x_i) \) and the Wandzura-Wilczek-type mass corrections from the two-particle lower twist DAs, distinguishing by parameters \( \delta_{P}^P \) and \( m_{P}^2 \), respectively. The
corrected expressions are [36]

\[
\psi_{\alpha P}(x) = \delta_P^2 \left[ \frac{20}{3} C_2^{1/2} (2x - 1) + \frac{49}{2} a_1^P C_3^{1/2} (2x - 1) \right] \\
+ m_P^2 \left\{ 6\rho^P \left( 1 - 3a_1^P + 6a_2^P \right) C_0^{1/2} (2x - 1) \\
- \left[ \frac{18}{5} a_1^P + 3\rho^P \left( 1 - 9a_1^P + 18a_2^P \right) + 12\kappa_{4P} \right] C_1^{1/2} (2x - 1) \\
+ \left( -2 - 6\rho^P \left( a_1^P - 5a_2^P \right) + 60\eta_3^P \right) C_2^{1/2} (2x - 1) \\
+ \left( \frac{18}{5} a_1^P - 9\rho^P a_2^P + \frac{16}{3} \kappa_{4P} + 20\eta_3^P \lambda_3^P \right) C_3^{1/2} (2x - 1) \\
+ \left( \frac{9}{4} a_2^P - 6\eta_3^P \omega_3^P \right) C_4^{1/2} (2x - 1) \right\} \\
+ 6m_q^2 \left( 1 - 3a_1^P + 6a_2^P \right) \ln x, \tag{B6}
\]

\[
\phi_{\alpha P}(x) = \delta_P^2 \left\{ \left( \frac{200}{3} + 196(2x - 1)a_1^P \right) x^2 \bar{x}^2 \\
+ 21 \omega_4^P \left( x \bar{x} (2 + 13x \bar{x}) + [2x^3(6x^2 - 15x + 10) \ln x] + [x \leftrightarrow \bar{x}] \right) \\
- 14a_1^P \left( x \bar{x} (2x - 1)(2 - 3x \bar{x}) - [2x^3(x - 2) \ln x] + [x \leftrightarrow \bar{x}] \right) \right\} \\
+ m_P^2 \left\{ \frac{16}{3} \kappa_{4P} \left( x(2x - \bar{x})(1 - 2x \bar{x}) + [5(x - 2)x^3 \ln x] - [x \leftrightarrow \bar{x}] \right) \\
+ 4\eta_3^P x \bar{x} \left[ 60x + 10\lambda_3^P \left( (2x - 1)(1 - x \bar{x}) - (1 - 5x \bar{x}) \right) \\
- \omega_3^P \left( 3 - 21x \bar{x} + 28x^2 \bar{x}^2 + 3(2x - 1)(1 - 7x \bar{x}) \right) \right] \\
- \frac{36}{5} a_2^P \left[ \frac{1}{4} x \bar{x} (4 - 9x \bar{x} + 110x^2 \bar{x}^2) + [x^3(10 - 15x + 6x^2) \ln x] + [x \leftrightarrow \bar{x}] \right) \\
+ 4x \bar{x} (1 + 3x \bar{x}) \left( 1 + \frac{9}{5} (2x - 1)a_1^P \right) \right\}, \tag{B7}
\]

with $\eta_{3P} = f_{3P} / (f_P m_0^P)$. It is noticed in Eq. B6 that $\psi_{\alpha P}(x)$ has a logarithm end-point singularity for the finite quark mass, while this singularity is not existed in $\phi_{\alpha P}(x)$. The conformal expansion of three-particle twist-four DAs reads:

\[
\psi_\parallel(x_i) = 120x_1x_2x_3 \left\{ \delta_P^2 \left[ \frac{21}{8} (x_1 - x_2) \omega_4^P + \frac{7}{20} a_1^P (1 - 3x_3) \right] \\
+ m_P^2 \left[ - \frac{9}{20} (x_1 - x_2)a_2^P + \frac{1}{5} \kappa_{4P} \right] \right\}, \tag{B8}
\]

\[
\psi_\perp(x_i) = 30x_3^2 \left\{ \delta_P^2 \left[ \frac{1}{3} (x_1 - x_2) + \frac{7}{10} a_1^P (3x_1 - 1 - 3x_3) + 3(x_1 - x_2)^2 \right] \\
+ \frac{21}{4} \omega_4^P (x_1 - x_2)(1 - 2x_3) \right\} \\
+ m_P^2 \left[ (1 - x_3) \left[ \frac{9}{40} (x_1 - x_2) - \frac{1}{3} \kappa_{4P} \right] \right], \tag{B9}
\]

\[
\hat{\psi}_\parallel(x_i) = -120x_1x_2x_3 \delta_P^2 \left\{ \frac{1}{3} + \frac{7}{4} a_1^P (x_1 - x_2) + \frac{21}{8} \omega_4^P (1 - 3x_3) \right\}, \tag{B10}
\]

\[
\hat{\psi}_\perp(x_i) = 30x_3^2 \left\{ \delta_P^2 \left[ \frac{1}{3} (1 - x_3) - \frac{7}{10} a_1^P (x_1 - x_2)(4x_3 - 3) + \frac{21}{4} \omega_4^P (1 - x_3)(1 - 2x_3) \right] \\
+ m_P^2 \left[ \frac{9}{40} a_2^P (x_1^2 - 4x_1 x_2 + x_2^2) - \frac{1}{3} (x_1 - x_2) \kappa_{4P} \right] \right\}, \tag{B11}
\]

in which three nonperturbative parameters $\delta_P^2, \omega_4^P, \kappa_{4P}$ are introduced. We close this section by noticing that all
parameters in the conformal expansion of DAs have the scale dependence and the behaviours of their evolutions can be found in Ref.[40].

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