Local Interaction Based Formation Shape Control and Its Application to Lateral Expansion

Yoshinobu INADA\textsuperscript{1)} and Ryosuke TAKAHASHI\textsuperscript{2)}

\textsuperscript{1)Course of Aerospace, Department of Aeronautics and Astronautics, School of Engineering, Tokai University, Kanagawa 259-1292, Japan}
\textsuperscript{2)Course of Aeronautics and Astronautics, School of Engineering, Graduate School of Tokai University, Kanagawa 259-1292, Japan}

Air surveillance using multiple aircraft is an efficient method for information gathering. Aircraft distributed in a wide area, gathering data at different points simultaneously, can provide a regional database with spatial and temporal expanses. The control of flight formation using multiple aircraft was conducted to realize such efficient air surveillance using a control with three kinds of local interaction among the aircraft, i.e. “attraction,” “repulsion,” and “parallel orientation.” The similarity between the shape of the interaction field and the formation shape realized efficient formation shape control by changing the shape of the interaction field of each aircraft without assigning the position of all vehicles. The local interaction parameters and PID control parameters were adjusted to satisfy the desirable performance of formation shape control. Consequently, the lateral expansion of the formation shape was controlled successfully with a convergence to the command value and a certain amount of robustness in formation shape control was confirmed.

Key Words: Formation Shape Control, Aircraft, Local Interaction

Nomenclature

\begin{itemize}
\item $A$: attraction-repulsion vector
\item $A$: attraction-repulsion gain
\item $K$: proportional gain
\item $N$: total number of aircraft in a formation
\item $N_b$: number of interacting neighbors
\item $N_{b,\text{max}}$: maximum number of interacting neighbors
\item $P$: parallel orientation vector
\item $P$: parallel orientation gain
\item $r$: position vector
\item $r_n$: neutral distance
\item $T_D$: derivative time
\item $T_I$: integral time
\item $v$: velocity vector
\item $y$: coordinate of $y$-axis on the formation fixed coordinates
\item $\alpha$: interaction vector
\item $\delta$: expansion ratio of interaction field along $y$-axis
\item $\Phi$: lateral length of formation
\end{itemize}

Subscripts

\begin{itemize}
\item $i$: value of the $i$-th aircraft
\item $ij$: value determined between the $i$-th and the $j$-th aircraft
\item $c$: command value
\end{itemize}

1. Introduction

Aerial information gathering or reconnaissance using multiple aircraft is an effective method because distributed aircraft can survey wide area and gather information at different points simultaneously, thus providing a regional database with spatial and temporal expanses. This kind of survey system is generally called a “sensor network.”\textsuperscript{1,2) Although it is accessible and convenient, the control of flight formation involving multiple aircraft has technical difficulties to realize a safe flight without aircraft colliding and a collaborated flight with many aircraft maintaining the integrity of the formation without splitting into sub-groups or several aircraft dropping out of the formation.

Furthermore, configuring the formation shape, i.e. the positioning of aircraft in the formation, has another difficulty because the number of positions to be controlled increases as the number of aircraft in the formation increases when the formation shape is changed to deal with various missions, such as horizontal distribution for ground monitoring or vertical distribution for monitoring vertical change of temperature or humidity, etc.

In this study, formation shape control was investigated to raise the effectiveness of flight formation so that it can deal with various missions flexibly. The flight formation control in this study was based on three kinds of local interaction among aircraft: “attraction,” “repulsion,” and “parallel orientation.”\textsuperscript{3-5) Attraction and repulsion are used to maintain a desirable distance between aircraft, and parallel orientation is used to adjust the heading of aircraft in the same direction.

In this research, the lateral expansion of formation shape was investigated as an example of formation shape control. This was realized by utilizing the remarkable characteristics of local interaction, i.e. the similarity between the shape of the interaction field and the formation shape, without assigning the position of all aircraft to fit a new formation shape, thus providing a simple and efficient method to control the
formation shape. The details are explained in the following sections.

2. Methods

2.1. Flight formation control model

The necessary functions for flight formation are the adjustment of inter-aircraft distance to avoid collisions while maintaining the integrity of formation without any split or drop-out, and the adjustment of heading to make the aircraft move in the same direction. For these purposes, local interaction control consisting of three kinds of interaction, “attraction,” “repulsion,” and “parallel orientation,” are employed. Attraction and repulsion adjust the inter-aircraft distance by making the aircraft approach its neighbor when it is distant and move away from it when it is too close. Parallel orientation adjusts the aircraft’s heading to orient it in the same direction as its neighbor. The interaction field shown in Fig. 1 illustrates this mechanism. The field is divided into two sub-fields, attraction and repulsion fields, by the border with radius \( r_n \), which is called “neutral distance,” and the interaction field has the radius \( R \) which corresponds to the range of a detection sensor.

Attraction and repulsion controls are switched so that the attraction operates when the inter-aircraft distance is larger than \( r_n \) and the repulsion operates when it is lower than \( r_n \), while the parallel orientation always operates. These interactions can be formulated as follows:

\[
A_{ij} = A(|r_{ij}| - r_n) \frac{r_{ij}}{|r_{ij}|} \quad (1)
\]

\[
P_{ij} = P \frac{v_j}{|v_j|} \quad (2)
\]

\[
\alpha_{ij} = A_{ij} + P_{ij} \quad (3)
\]

where \( A_{ij} \) is the attraction-repulsion vector and \( P_{ij} \) is the parallel orientation vector. Both are determined between the \( i \)-th aircraft, the aircraft at the center of the interaction field, and the \( j \)-th aircraft in the interaction field of the \( i \)-th aircraft as shown in Fig. 1. \( A \) and \( P \) are the attraction-repulsion gain and the parallel orientation gain, respectively. \( r_{ij} \) is the position vector of the \( j \)-th aircraft with respect to the \( i \)-th aircraft, and \( |r_{ij}| \) is the distance between the \( i \)-th and the \( j \)-th aircraft. \( v_j \) is the velocity vector of the \( j \)-th aircraft and \( \alpha_{ij} \) is the interaction vector between the \( i \)-th and the \( j \)-th aircraft. When the \( i \)-th aircraft interacts with multiple neighbors, \( \alpha_{ij} \) for those neighbors is summed and the command value of the interaction vector of the \( i \)-th aircraft \( \alpha_{ic} \) is calculated as follows:

\[
\alpha_{ic} = \frac{N_b}{\sum_j N_b \alpha_{ij}} \left( \sum_j N_b \alpha_{ij} \right) \quad (N_b \leq N_{b,\max}) \quad (4)
\]

where \( N_b \) is the number of neighbors in the interaction field, which has the upper limit \( N_{b,\max} \) because of the assumed limit of sensing ability of mutual interaction. When there are more neighbors than \( N_{b,\max} \), \( N_b \) neighbors are selected in ascending order of deviation from the front because the detection sensors are supposed to face forward to follow or avoid collision with front neighbors. There may be rear neighbors undetected, but they detect front neighbors with their sensors as well, and thus can conduct necessary operations of following or collision avoidance.

The \( i \)-th aircraft is controlled to advance in the direction of vector \( \alpha_{ic} \). In other words, the error vector \( e = \alpha_{ic} - \alpha_i \) is controlled to be zero, where \( \alpha_i \) is the unit vector that orients in the current direction of the \( i \)-th aircraft. To consider the error in the measurement of position and direction of neighbors, a random error which has one degree of standard deviation is added to \( \alpha_{ic} \).

The block diagram for this control is shown in Fig. 2. A PID controller is used to calculate the angles of control surfaces to reduce the error vector \( e \). The “Dynamics” block is composed of the six-degrees-of-freedom Newtonian equation of motion with body parameters of the aircraft as shown in Table 1, obtained from an actual micro-aircraft discussed in literature.6)

2.2. Formation shape control model

Formation shape control in this study does not use the positioning of each aircraft in the formation to form a certain shape, but uses the geometrical similarity between the shape of the interaction field and the formation shape. Figure 3(a) shows the concept of this control. Aircraft existing in the interaction field of a certain aircraft tend to exist around the surface of the sphere with the radius \( r_n \) because of the attrac-

Fig. 1. Interaction field.

Fig. 2. Block diagram of formation control.
interaction and repulsion balance on this surface (hereafter this surface is called “neutral surface”). When the same situation occurs in other aircraft, the spherical shape of the formation is generated. When the interaction field of the aircraft is deformed into another shape, like the laterally expanded spheroid shown in Fig. 3(b), aircraft in the interaction field move to fit the spheroid neutral surface, thus leading to the generation of a spherical formation. Using this similarity between the shape of the interaction field and the formation shape, the immense degree of freedom required for the positioning of all aircraft in the formation can be reduced to a couple of geometrical parameters to determine the shape of the interaction field, e.g. the expansion or reduction ratio of the interaction field along the x, y, z axes.

In the following sections, lateral expansion of the formation shape is considered. In the case of lateral expansion, the expansion ratio $\delta_y$ along the y axis is considered, which means the lateral length of the interaction field is magnified to $\delta_y$ times that of the original lateral length. When there is a difference between the commanded lateral expansion $\Phi_{yc}$ and the observed $\Phi_y$, the error $\varepsilon = \Phi_{yc} - \Phi_y$ is input to the controller, where $\Phi_{yc}$ and $\Phi_y$ are the commanded and observed values of the geometrical lateral length of the formation, respectively. These are defined by the average absolute value of the y-coordinate of aircraft as follows:

$$\Phi_y = \frac{1}{N} \sum_{i=1}^{N} |y_i|$$

where $N$ is the number of aircraft in the formation and $y_i$ is the $y$-coordinate of the $i$-th aircraft on the formation-fixed coordinates, the origin of which is the center of formation and the x-axis orients in the direction of the formation. In a real flight formation, these positions may be taken by the onboard GPS sensor and shared by the aircraft via intercommunication. The controller calculates $\delta_y$ to reduce $\varepsilon$ using the following PID control equation:

$$\delta_y(t) = K\left[\varepsilon(t) + \frac{1}{T_i} \int_{0}^{t} \varepsilon(t) dt + T_D \frac{d\varepsilon(t)}{dt}\right]$$

where $K$, $T_i$, and $T_D$ are the proportional gain, the integral time, and the derivative time, respectively. The block diagram is shown in Fig. 4.

### 2.3 Parameter tuning

There are four parameters in the formation shape control to be tuned for the desired performance: neutral distance $r_0$, attraction-repulsion gain $A$, parallel orientation gain $P$, and...
maximum number of interacting neighbors $N_{b,\text{max}}$. There is another parameter $R$, which is the radius of the interaction field. This parameter corresponds to the range of the detection sensor. One sensor available for detecting the distance of an object is the ultrasonic sensor. A small ultrasonic sensor available for small aircraft generally has a range up to 10 m and this range generally cannot be changed manually. So the value of $R$ is fixed to 30 BL here, where BL means the body length of the aircraft (= 0.33 m), and thus, 30 BL is 9.9 m and then excluded from the subject of parameter tuning.

The tuning of the above four parameters is based on Ziegler-Nichols method. When a command value of lateral expansion $\Phi_y$, is given as a step input, e.g. $\Phi_y = 3.0$ m, the proportional gain $K$ is increased until stable vibration with a constant amplitude and frequency of $\Phi_y$ appears. Here, the integral and the derivative gains, $K/T_I$ and $K/T_D$, are set to zero, namely $T_I = \infty$ and $T_D = 0$.

The value of $K$ at the stable vibration varies as the parameter value changes. Thus, the feasible value of the parameter is determined when $K$ takes the maximum value because the large proportional gain $K$ realizes a quick response and convergence to the command value. The calculation of the $K$ value is executed 10 times for each set of four parameters with different flight conditions, where the different random seeds were used for the error input, and the results from ten calculations are averaged.

During this parameter tuning, a collision number is counted to check the safety of flight formation. The collision number is the number of times when the distance between any aircraft drops below the collision limit. Here, the colli-

| Table 2. Interaction parameters. |
|---------------------------------|
| Parameter                      | Symbol | Nominal | Feasible |
| Neutral distance (BL)          | $r_n$  | 10.0    | 2.0      |
| Attraction-repulsion gain      | $A$    | 0.5     | 0.5      |
| Parallel orientation gain      | $P$    | 10.0    | 4.0      |
| Maximum number of interacting neighbors | $N_{b,\text{max}}$ | 4      | 10      |

Fig. 5. Snapshot of calculation.

Fig. 6. Parameter tuning.
The collision number showed a value of stable vibration. The value of exceeding the allowable limit at several interaction vector of standard deviation was added to the command value of in-each aircraft, even though the random error with one degree of freedom with one degree of standard deviation was added to the command value of interaction vector $\alpha_c$, as mentioned in Section 2.1.

3. Results and Discussion

A snapshot of formation flight calculations with the nominal parameter values listed in Table 2 is shown in Fig. 5. The simulation started with 50 aircraft initially distributed randomly within a certain space and oriented in the same direction. After a short initial fluctuation, the formation kept a stable flight pattern without large variations in the position of each aircraft, even though the random error with one degree of standard deviation was added to the command value of interaction vector $\alpha_c$, as mentioned in Section 2.1.

3.1. Parameter tuning

In the parameter tuning, four parameters were given initially as shown in the nominal value of Table 2. The neutral distance $r_n$ was first varied from 1 BL to 10 BL at 1 BL intervals and the proportional gain $K$ was increased at each $r_n$ until the stable vibration of $\Phi_y$ appeared. Figure 6(a) shows the value of $K$ that gave the stable vibration at each $r_n$. The collision number was also plotted on the graph. When $\Phi_y$ did not show stable vibration, $K$ at such $r_n$ was eliminated from the graph. In Fig. 6(a), $K$ at $r_n = 1.0$ BL was eliminated for this reason, and $K$ at $r_n = 2.0$ BL shows the maximum value with the allowable collision number ($<3.0$). Therefore, $r_n = 2.0$ BL was selected as the feasible value.

Next, the attraction-repulsion gain $A$ was varied from 0.1 to 1.0 at 0.1 intervals while the other values except $r_n$ took the values in Table 2; $r_n$, took the feasible value of the above calculation ($= 2.0$ BL). The value of $K$ at the stable vibration and the collision number were plotted at each $A$ in Fig. 6(b). The value of $K$ at $A = 0.1$ was eliminated because of the lack of stable vibration. The value of $K$ increased as $A$ increased. The collision number showed fluctuation at $A > 0.6$, exceeding the allowable limit at several $A$ values, and thus $A > 0.6$ was eliminated from consideration. Therefore, $A = 0.5$ was selected as the feasible value for attraction-repulsion gain.

For the other parameters, similar procedures were conducted and four feasible parameter values were determined as shown in Table 2. The stable vibration of $\Phi_y$ with those feasible parameter values is shown in Fig. 7.

PID parameters, $K$, $T_I$, and $T_D$, were then determined using Ziegler-Nichols method. The feasibility of this method was confirmed previously in the PID parameter tuning of small aircraft such as a helicopter or a quadrotor UAV and thus this method was used in this study. The base value, the value of $K$ to realize the stable vibration, was already calculated in the last step of parameter tuning process, which was the one when the interaction parameters took the feasible values in Table 2. Then the values of $T_I$ and $T_D$ were given as the recommended values for Ziegler-Nichols method, $K = 0.6K_u$, $T_I = 0.5P_u$, and $T_D = P_u/8.0$, where $K_u$ is $K$ at the stable vibration and $P_u$ is the period of stable vibration. The values of $K$, $T_I$, and $T_D$ were then determined as follows:

$$K = 0.000085788,$$
$$T_I = 5.69375,$$
$$T_D = 1.4234375.$$
shots of formation at the initial stage and converged stage are shown in Figs. 8(a) and 8(b), respectively, where the command value $\Phi_{yc} = 3\text{ m}$ was used as the step input. The time history of $\Phi_y$ is shown in Fig. 9. After the initial fluctuation, $\Phi_y$ converged to the command value $\Phi_{yc}$.

Next, the availability of formation control parameters, PID and interaction parameters, in various command values $\Phi_{yc}$ was analyzed. The initial value of $\Phi_{yc}$ was 3.0 m and it was increased or decreased in increments of 0.5 m. When $\Phi_{yc}$ was increased, $\Phi_y$ converged to the command value $\Phi_{yc}$ until $\Phi_{yc} = 4.0$ m and diverged when $\Phi_{yc} \geq 4.5$ m. Figures 10(a) and 10(b) show the time history of $\Phi_y$ for $\Phi_{yc} = 4.0$ m and 4.5 m, respectively.

When $\Phi_{yc}$ was decreased, $\Phi_y$ converged to the command value $\Phi_{yc}$ until $\Phi_{yc} = 2.0$ m. When $\Phi_{yc} = 1.5$ m, $\Phi_y$ converged to $\Phi_{yc}$, but aircraft collided frequently. The collision number was 6.9 on average. This was not successful because the allowable collision number was 3.0, as mentioned previously. When $\Phi_{yc} = 1.0$ m and $\Phi_{yc} = 0.5$ m, $\Phi_y$ converged to a larger value than the command value $\Phi_{yc}$. Figures 10(c) and 10(d) show the time history of $\Phi_y$ for $\Phi_{yc} = 1.0$ m and 0.5 m, respectively.

The reason for these results could be as follows: As the command value $\Phi_{yc}$ increased, the overshoot of $\Phi_y$ also increased, which was shown in Figs. 9 and 10(a). The overshoot was about 3 m in Fig. 9 when $\Phi_{yc} = 3.0$ m and 5m in Fig. 10(a) when $\Phi_{yc} = 4.0$ m. This might cause the expansion of inter-aircraft distance and eventually cause some aircraft to break away from the interaction field of their neighbors. This would lead to the separation of aircraft from their neighbors and possibly cause the formation to split, resulting in the divergence of $\Phi_y$. In contrast, when the command value $\Phi_{yc}$ decreases, the inter-aircraft distance might decrease and eventually fall below the neutral distance $r_e$. This might cause the frequent occurrence of collisions and repulsions, thus increasing the collision number and eliciting the impediment to converge to the command value.

Fig. 9. Time history of $\Phi_y$.

Fig. 10. Time history of $\Phi_y$ for various command values $\Phi_{yc}$.
The result of this analysis, however, confirmed that the interaction parameters and PID parameters determined with a specified command value like $\Phi_{\text{cmd}} = 3.0 \text{ m}$ could be applied to different command values, such as 2.0 m to 4.0 m, which implies a certain amount of robustness in the control of formation shape.

4. Conclusion

Formation shape control based on the local interaction among aircraft was investigated using “repulsion,” “attraction,” and “parallel orientation” controls as the basis of formation control. Formation shape was controlled by using the similarity between the shape of the interaction field and the formation shape. Lateral expansion of the formation shape was investigated as an example of formation shape control. The interaction parameters were adjusted to realize quick response and convergence to the command value. The PID parameters were determined using Ziegler-Nichols method. Consequently, efficient formation shape control was realized, showing a certain amount of robustness in the lateral expansion of formation shape.

Acknowledgments

This research was supported by JSPS KAKENHI Grant Number 23360382.

References

1) Yick, J., Mukherjee, B., and Ghose, D.: Wireless Sensor Network Survey, Computer Networks, 52, 12 (2008), pp. 2292–2330.
2) Akyildiz, I. F., Su, W., Sankarasubramaniam, Y., and Cayirci, E.: A Survey on Sensor Networks, IEEE Communications Magazine, 40, 8 (2002), pp. 104–112.
3) Inada, Y. and Takanobu, H.: Dependency of Collective Motion Control of Air Vehicles on Interaction Parameters, Theor. Appl. Mech. Jpn., 58 (2010), pp. 185–195.
4) Inada, Y. and Takanobu, H.: Flight-Formation Control of Air Vehicles Based on Collective Motion Control of Organisms, Proceedings of the 18th IFAC Symposium on Automatic Control in Aerospace, 2010.
5) Takahashi, R. and Inada, Y.: Formation Shape Control of Unmanned Air Vehicles, Proceeding of the 28th International Congress of the Aeronautical Sciences, ICAS 2012, 2012.
6) Fujinaga, J., Tokutake, H., and Sunada, S.: Flight Controller Design and Autonomous Flight Tests of 60cm-sized UAV, 7th European Micro Air Vehicle Conference and Flight Competition, Toulouse, France, September, 2007, pp. 17–21.
7) Franklin, G. F., Powell, J. D., and Emami-Naeini, A.: Feedback Control of Dynamical Systems, 2nd ed., Addison-Wesley Publishing Company, Reading, Massachusetts, 1991.
8) Majchrzak, J., Michalski, M., and Wiczynski, G.: Distance Estimation with a Long-Range Ultrasonic Sensor System, IEEE Sensors J., 9, 7 (2009), pp. 767–773.
9) Spinka, O. and Holub, O.: Low-Cost Reconfigurable Control System for Small UAVs, IEEE Trans. Ind. Electron., 58, 3 (2011), pp. 880–889.
10) Dong, W., Gu, G.-Y., Zhu, X., and Ding, H.: Modeling and Control of a Quadrotor UAV with Aerodynamic Concepts, World Academy of Science, Engineering and Technology, 7 (2013), pp. 377–382.

©2015 JSASS 72