Research on several constitutive models to predict the flow behaviour of GCr15 continuous casting bloom with heavy reduction

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Abstract

In this work, the modified Johnson-Cook (J-C) model was developed. The predictive abilities of the JC model, new modified JC model, modified Zerilli-Armstrong (ZA) model, and strain-compensated Arrhenius model of the flow behaviour of GCr15 continuous casting bloom with heavy reduction (HR) were compared. The deformation mechanism was explored. Experimental stress-strain data from single-pass thermo-simulation compression experiments over a wide range of temperatures (1023–1573 K), strains (0.05–0.7) and strain rates (0.001–0.1 s−1) were applied to deduce the material parameters of these constitutive models. The applicability of the models was analysed by calculating the correlation coefficient; the average relative error (AARE) of the model; and the number of model parameters, which represents the computation time of each model. The experimental results showed that the new modified JC model was more suitable for describing the flow behaviour of GCr15 continuous casting bloom with HR because of its convenient calculation and better accuracy.

1. Introduction

Metals typically could not be used without hot working, which changes their shape and mechanical properties to meet application requirements. It is necessary to understand the hot deformation behaviour of metals through constitutive modelling [1]. The main types of constitutive models are a phenomenological constitutive model, a physical-based constitutive model and an artificial neural network [2].

The phenomenological constitutive model has been widely used because of its simple form and because its material parameters can be solved for by fitting limited experimental data [3]. The Johnson-Cook [4] (JC) and Arrhenius [5] models are two common phenomenological models for hot working. The JC model includes only 5 parameters, and its form is simple. However, the JC model assumes that the strain, strain rate and temperature are independent, although these features actually affect one another [6, 7], thereby reducing the prediction accuracy of the model. In order to improve the accuracy, some researchers have modified the JC model by studying, for example, how the coefficient of strain-rate enhancement varies with the strain and strain rate [8–10], the effect of temperature on the parameters [11, 12], and the coupled effects of the strain, strain rate, and temperature on the flow stress [3]. Lin et al. [13] and Li et al. [14] not only considered the coupled effects of the strain, strain rate and temperature on the flow stress but also used a polynomial to describe a nonlinear relationship in which the yield stress and strain enhancement terms varied with the strain.

The Arrhenius model was first proposed by Sellars and Tegart [5], and the Z parameter proposed by Zener and Hoolomon was introduced to describe the coupled effects of the strain rate and temperature on the flow stress [15]. The original Arrhenius-type constitutive equation could not consider true strain effects on the flow stress; therefore, some researchers have devoted more attention to increasing the accuracy of this model and reducing its calculation complexity. Pu et al. [16] first proposed material parameters for the constitutive equation as a function of strain to predict the flow stress of a TiAl alloy at high temperatures. Li et al. [17] first proposed the compensation of the strain and strain rate, and their methods were suitable for GCr15 [18], 9Cr1Mo [19], and austenitic stainless steel [20], among others. Cabrera et al. [21] introduced the Young’s modulus and a self-
diffusion coefficient to modify the Arrhenius model based on physical meaning, which achieved high prediction accuracy for 17-4PH stainless steel [22], 9Cr1Mo steel [1], and medium-carbon microalloy steel [23]. Mirzadeh introduced characteristic stresses, such as peak stress, which represents stress curves to calculate the material constants in the model to simplify the calculation [24, 25].

Many researchers had noted that the microstructure of metals substantially affects their flow behaviour. Zerilli and Armstrong proposed constitutive equations should precisely distinguish the crystal lattice types of materials with considering the lattice type of metal crystals and the effects of the temperature, strain, and strain rate on the flow stress [26]. Many materials changed their lattice type with different temperature conditions. For example, GCr15 is austenitic after treatment at 1173 K, but the crystal structure contains cementite and austenite features when it is treated between 1023 and 1173 K [27, 28], which dramatically increases the difficulty of constructing constitutive equations. The Zerilli-Armstrong (ZA) model does not consider the coupled influences of the temperature, strain and strain rate on the flow stress [2]. Scholars carried out numerous studies to increase the accuracy of the ZA model [29, 30]. Samantaray et al [6] first proposed a modified ZA model that considered the effects of thermal softening, strain rate hardening and isotropic hardening, as well as the coupled effects of temperature and strain and of the strain rate and temperature, on the flow stress. The modified ZA model was suitable for predicting the flow behaviour of austenitic steels [6], ferritic steels [7], 28CrMnMoV steel [31], and 7050 aluminium alloy [32].

Researchers had also compared the applicability of different models to different metals, such as 9Cr1Mo steel [7], 28CrMnMoV steel [31], 20CrMo alloy steel [33], and aluminium alloy under hot-working conditions [32]. By using tensile test data and creep test data from Wray [34] and Suzuki et al [35], Kozlowski et al [36] systematically analysed and compared the suitability, accuracy, and calculation complexity of four different constitutive models used in the field of regular continuous casting. Koric and Thomas [37] compared two different elastic-visco-plastic constitutive laws for plain-carbon steel.

Notably, few studies have been performed on the flow behaviour of metal in continuous casting blooms during Heavy Reduction (HR), which could improve the centre segregation, porosity and shrinkage of the slab [38, 39]. In this process, the strain rate is approximately $10^{-3}$–$10^{-1}$ $s^{-1}$, which is significantly different from that in the regular continuous casting process ($10^{-6}$–$10^{-3}$ $s^{-1}$) and that in the hot-working process, in which the strain rate is greater than $10^{-1}$ $s^{-1}$ [40]. Therefore, the metal flow behaviour of continuous casting during HR warrants further attention. The present authors [40] have previously discussed the flow behaviour of GCr15 continuous casting blooms with HR in depth and have compared the predicted accuracy of model I [36] in four traditional constitutive models, a conventional Arrhenius-type constitutive model and a physically based Arrhenius-type constitutive model.

In the present work, due to the wide range of the experimental temperatures (1023–1573 K), the temperature is divided into two regions (1023–1123 K and 1173–1573 K) based on whether GCr15 is fully transformed into the austenite structure, avoiding the influence of large-scale temperature variations on the accuracy of the constructed constitutive equations. The new modified JC model was developed. The ability of the JC model, the new modified JC model, the modified ZA model, and the strain-compensated Arrhenius model to predict the flow behaviour of a GCr15 continuous casting bloom with HR at 1023–1573 K is compared. The applicability of each model is evaluated by using the correlation coefficient (R); average relative error (AARE); and number of model parameters, which represents the computation time of each model. A good model that has not only high prediction accuracy but also a relatively simple solution for its parameters is selected.

2. Materials and experimental methods

The MMS-200 thermal simulation machine developed by Northeastern University was used to conduct single-pass compression experiments on bearing steels. The experimental temperature ranged from 1023 K to 1573 K, the strain rates of single-pass compression were 0.001, 0.01, and 0.1 $s^{-1}$, and the strains ranged from 0 to 0.7 with an interval of 0.05. The hot compress specimens had a gage high of 12 mm and a diameter of 8 mm. Figure 1 shows the sample before/after isothermal compressive experiment. The compressed sample was cut perpendicular to the direction of compression, polished, and then etched with saturated picric acid. Details of the sampling methods, experimental methods, and experimental results are available elsewhere [40].

3. Results and discussion

3.1. The flow stress curve and microstructure of compressed sample under various conditions

The flow stress-strain curve of the compressed specimen was displayed in figure 2. The stress increase with increasing strain rate and decreasing temperature.
The degree of softening is discussed through the difference of peak stress $\sigma_P$ and steady stress $\sigma_s$, as showed in figure 3. It is obvious that the variation of $(\sigma_P - \sigma_s)$ significantly decreases with temperature while modestly changes with strain rate, which reflects the enhanced DRX process due to increasing temperature. At higher temperature, the smaller critical strain for triggering dynamic recrystallization is caused due to the more accumulated energy and the increased diffusion kinetics resulting in easier dislocation motion. Some researchers have done a lot of research on dynamic recrystallization [41, 42].

The work hardening rate ($\theta = d\sigma / d\varepsilon$) plot could reflect material variation in microstructure [43]. The variation of work hardening rate with true strain was explored, as shown in figure 4. The work hardening rate in both different temperatures and different strain rates obviously varied until the true strains of 0.1 and 0.2, respectively, were reached. The original work hardening rate at 1073 K was much higher than that of other temperatures, irrespective of strain rate. Additionally, it was also evident that the dependency of work hardening rate on temperature was much higher at 1023–1173 K than that of other temperatures, which reflected the
weaken multiplication and pinning of dislocations at higher temperature due to the occurrence of dynamic soft mechanism. The true strain corresponding to zero work hardening rate value decreased with increasing temperature and decreasing strain rate, which demonstrated the enhanced dynamic soft mechanism.

The microstructures of compressed samples were showed in figure 5. The grain size was much smaller with the increasing strain rate at a fixed temperature due to the increase in density of dislocation, which increased the
resistance of the grain growth. At the same time, it had more time for grains to grow at lower strain rate. The fraction of area of grain boundary under high strain rate was higher than that of lower strain as a result of the difference of hardening capacity between inner grain and grain boundary, and the hardening capacity of grain boundary was higher [44]. It was very well explained the phenomenon that the flow stress increased with increasing strain rate. The fraction of dynamic recrystallization increased with increasing temperature and decreasing strain rate.

3.2. Research on constitutive relationship of GCr15 under HR

3.2.1. The JC constitutive model of GCr15 under HR

The JC model was widely used because of simple form. The JC model consisted of a strain reinforcement term, a strain rate reinforcement term and a thermal softening term, and three parts were considered to be independent. The JC model can be expressed as

\[ \sigma = [A + B\varepsilon^n][1 + C \ln \dot{\varepsilon}^*] \left\{ 1 - \left[ \frac{T^*}{(T_m - T^*)^m} \right]^n \right\} \]  

(1)

first, second and third terms in parenthesis in (1) refer to strain hardening, strain rate hardening and thermal softening. Where \( \sigma \) is the stress, \( A \) is the yield stress at the reference temperature and reference strain rate, which can be read directly from the stress-strain curve. The curve shows no obvious yield platform; thus, the flow stress corresponding to 0.2% strain is the yield stress. \( B \) is the coefficient of strain hardening, \( n \) is the strain hardening exponent, \( C \) is the coefficient of strain-rate enhancement, and \( m \) is the exponent of thermal softening. The units of \( A \) and \( B \) are MPa, \( n \), \( C \), and \( m \) are dimensionless coefficients. \( T^* = (T - T_T) \), where \( T \) and \( T_T \) are the current and reference temperature, and \( T_m \) is the melting temperature of GCr15 steel, which is 1603 K. \( \dot{\varepsilon} \) is strain, \( \dot{\varepsilon}^* = \dot{\varepsilon} / \dot{\varepsilon}_T \), \( \dot{\varepsilon} \) is the strain rate, and \( \dot{\varepsilon}_T \) is the reference strain rate. The reference temperature is 1023 K at 1023 K–1123 K and 1173 K at 1173 K–1573 K. \( T \) is the absolute temperature, and the reference strain rate is 0.1 s\(^{-1}\).

The method of obtaining parameters of JC model is similar to that of previous methods [3, 6, 7]. The material parameters of the JC mode are presented in table 1.

![Figure 5. Microstructures of GCr15 steel after deformation at different conditions (a) 0.01 s\(^{-1}\), 1473 K (b) 0.1 s\(^{-1}\), 1473 K (c) 0.1 s\(^{-1}\), 1523 K.](image-url)
The predicted and experimental values are compared, as shown in figure 6. The JC model is not suitable to describe the flow behavior of GCr15 within entire deformation condition, and predicted values show a monotonous downward trend at various strain rates. The deviation between predicted values and experimental values is large.

| Parameters | A (MPa) | B (MPa) | n     | C     | m     |
|------------|---------|---------|-------|-------|-------|
| Temperature range 1023–1123 K | 130 | 90.945 49 | −0.1964 | 0.110 95 | 0.965 32 |
| Temperature range 1173–1573 K | 80 | 16.3572 | −0.381 | 0.091 99 | 0.644 14 |

Figure 6. Comparison between the experimental and predicted flow stress dates by JC model at (a) strain rate of 0.001 s$^{-1}$, (b) strain rate of 0.01 s$^{-1}$, (c) strain rate of 0.1 s$^{-1}$.
3.2.2. The modified ZA constitutive model of GCr15 under HR

A modified ZA model [6] based on the original ZA model proposed by Zerilli and Armstrong in 1987 [13] is proposed; the reference temperature is introduced due to the unavailable stress at 0 K, and the original ZA model for FCC material did not consider the absolute effect of strain rate on stress at 0 K, thus the modified model is expressed as

$$\sigma = [C_1 + C_2 \varepsilon^e] \exp \left[ -\left( C_3 + C_4 \varepsilon \right) \cdot T^* + (C_5 + C_6 T^*) \cdot \ln \varepsilon^e \right]$$

(2)

where $\sigma$, $\varepsilon$, $T^*$, $T$, $\varepsilon^e$ and $T$ have the same meanings as those part 3.2.1; and the units of $C_1$ and $C_2$ are MPa. Parameters $C_3$, $C_4$, $C_5$, $C_6$ and $n$ are material parameters as well as dimensionless. $C_1$ is the yield stress at reference temperature and reference strain rate, $C_2 \varepsilon^e$ represents the effect of strain hardening. $C_3$ incorporates the effect of thermal softening, while $C_4$ indicates the coupled effect of temperature and strain on flow stress. The parameter $C_5$ defines the effect of strain rate on flow stress at reference temperature and the parameter $C_6$ represents the coupled effect of temperature and strain rate on flow stress. The modified ZA model not only considers the effect of strain hardening, thermal softening, strain rate hardening on flow stress, but also the coupled effect of temperature and strain and of strain rate and temperature.

The method of obtaining parameters of the modified ZA model is similar to that of previous methods [6, 7, 31–33]. Parameters of the modified ZA model is shown in table 2.

Comparison of predicted values of the modified ZA model and experimental values is shown in figure 7. The modified ZA model are not suitable to describe the flow behavior of GCr15 at 1023–1273 K, showing a monotonous downward trend at various strain rates. The deviation between the predicted values and the experimental values is large. While the modified ZA model could well capture the flow behaviour of GCr15 at 1323–1573 K.

3.2.3. The new modified JC constitutive model of GCr15 under HR

The original JC model assumed that the strain, strain rate and temperature are independent. However, these parameters interact with one another [6, 7, 45, 46]. Lin et al [13] used a second-order polynomial to fit a nonlinear relationship in which the yield stress and strain enhancement terms vary with the strain and considered the coupled effects of the temperature and strain rate on the flow stress. The modified JC model is expressed as

$$\sigma = [A_1 + B_1 \varepsilon + B_2 \varepsilon^2] \left( 1 + C_1 \cdot \ln \varepsilon^e \right) \exp \left[ -\left( \lambda_1 + \lambda_2 \ln \varepsilon^e \right) \cdot (T - T_r) \right]$$

(3)

where $A_1$, $B_1$, $B_2$, $B_3$, $C_1$, $\lambda_1$, and $\lambda_2$ are material coefficients; the units of $A_1$, $B_1$, $B_2$, $B_3$, $B_4$ are MPa;$C_1$, $\lambda_1$, and $\lambda_2$ are dimensionless. represents the effect of strain hardening on flow stress. $C_1$ defines the effect of strain rate on flow stress at reference temperature. $\lambda_1$ represents the effect of temperature on flow stress, and $\lambda_2$ references to the coupled effect of strain rate and temperature on flow stress.

In this work, a fourth-order polynomial was used to find better fitting of the nonlinear relationship in which the yield stress and strain enhancement terms vary with the strain, and the equation (3) is expressed as:

$$\sigma = [A_1 + B_1 \varepsilon + B_2 \varepsilon^2 + B_3 \varepsilon^3 + B_4 \varepsilon^4] \left( 1 + C_1 \cdot \ln \varepsilon^e \right) \exp \left[ -\left( \lambda_1 + \lambda_2 \ln \varepsilon^e \right) \cdot (T - T_r) \right]$$

(4)

The method of obtaining parameters of the modified JC model is similar to that of previous methods [13]. All of parameters of equation (4) have been found, as shown in table 3.

The comparison of predicted values and experimental values is made, as shown in figure 8. The predicted values agree well with the experimental values except for the deviation occurring at peak stress under 0.001–0.01 s⁻¹. As a result, the model formula has to be modified. Traditional $C_1$ parameter cannot fully reflect the mechanical characteristics of the GCr15 under HR.

The $C_1$ is considered as constant with various strain. However, $C_1$ varies drastically with true strain, as shown in figure 9. The polynomial is used to increase the predicted accuracy. After many attempts, the 5th degree polynomial is suitable for describing the relationship between $C_1$ and strain:

| Table 2. Parameters of the modified Zerilli-Armstrong (ZA) model. |
|---------------------------------------------------------------|
| Parameters | Temperature range | Temperature range |
|------------|------------------|------------------|
| $C_1$ (MPa) | 130              | 80               |
| $C_2$ (MPa) | 90.587 003 84    | 14.035 421 24    |
| $C_3$      | 0.005 040 326    | 0.004 068 028    |
| $C_4$      | -0.001 375 136   | 0.000 285 058    |
| $C_5$      | 0.149 814 069    | 0.085 469 958    |
| $C_6$      | 0.000 016 142    | 0.000 341 295    |
| $m$        | -0.204 463 946   | -0.410 341 985   |

Q Zhou et al
The nonlinear fitting is used to solve parameters of polynomial.

\[ C_4 = x_0 + x_1 \varepsilon + x_2 \varepsilon^2 + x_3 \varepsilon^3 + x_4 \varepsilon^4 + x_5 \varepsilon^5 \] (5)
Figure 8. Comparison between the experimental and predicted flow stress data by modified JC at (a) strain rate of 0.001 s⁻¹, (b) strain rate of 0.01 s⁻¹, (c) strain rate of 0.1 s⁻¹.

Figure 9. Variation of true strain with $C_1$. (a) $1023–1123$ K, (b) $1173–1573$ K.
The expression of the new modified JC model is:

\[
\sigma = [A_1 + B_1 \varepsilon + B_2 \varepsilon^2 + B_3 \varepsilon^3 + B_4 \varepsilon^4](1 + C_1 \cdot \ln \dot{\varepsilon}^* \exp \left[-(\lambda_1 + \lambda_2 \ln \dot{\varepsilon}^*) \cdot (T - T_1)\right] \cdot \exp[-(\lambda_1 + \lambda_2 \ln \dot{\varepsilon}^*) \cdot (T - T_1)]
\]

where \(A_1, B_1, B_2, B_3, B_4, \lambda_1,\) and \(\lambda_2\) are material coefficients; the units of \(A_1, B_1, B_2, B_3, B_4\) are MPa; \(\lambda_1\) and \(\lambda_2\) are dimensionless; \(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7\) are material coefficients as well as dimensionless.

\[A_1 + B_1 \varepsilon + B_2 \varepsilon^2 + B_3 \varepsilon^3 + B_4 \varepsilon^4\] represents the effect of strain hardening on flow stress. \(\lambda_1\) represents the effect of temperature on flow stress, while \(\lambda_2\) references to the coupled effect of strain rate and temperature. \(C_1\) not only defines the effect of strain rate on flow stress at reference temperature, but also indicates the coupled effect of strain and strain rate. Compared with the modified JC model, the new modified JC model not only takes the coupled effect of strain rate and temperature on flow stress into consideration, but defines the coupled effect of strain rate and strain on flow stress.

Through the above calculation, the parameters of the new modified JC model are shown in tables 4 and 5. The comparison of predicted values and experimental values is made, as shown in figure 10. Predicted values agree well with the experimental values. Especially, the deviation occurring at peak stress under 0.001–0.01 s\(^{-1}\) almost disappears.

### 3.2.4. The Arrhenius constitutive model of GCr15 under HR

Arrhenius model directly shows the effect of the temperature and strain rate on the flow stress, which is expressed as

\[
\dot{\varepsilon} = A \cdot \exp(\frac{Q}{RT}) \cdot F(\sigma)
\]

where \(F(\sigma)\) is the stress function. According to the different stress states, the stress function can be expressed as

\[
F(\sigma) = \begin{cases} \sigma^n & \alpha \sigma < 0.8 \\ \exp(\beta \sigma) & \alpha \sigma > 1.2 \\ [\sinh(\alpha \sigma)]^n & \text{for all } \sigma \end{cases}
\]

The parameter \(Z\) [15] is introduced to describe the effect of the temperature and strain rate on the flow stress and is expressed as

\[
Z = \dot{\varepsilon} \cdot \exp\left(\frac{Q}{RT}\right)
\]

where \(Q\) is the deformation activation energy in J \(\cdot\) mol\(^{-1}\), \(R\) is the ideal gas constant (8.314 J \(\cdot\) mol\(^{-1}\) \(\cdot\) K\(^{-1}\)), \(n\) is the material stress index, and \(A, \alpha, \beta, \) and \(n_1\) are material constants, where \(\alpha = \beta/n_1\).
At all stress levels \( F(\sigma) = \sinh(\alpha \sigma^n) \), the relationship between the \( Z \) parameters and the flow stress is obtained through equations (7) and (9) and is expressed as

\[
\sigma = \frac{1}{\alpha} \cdot \ln \left\{ \left( \frac{Z}{A} \right)^{1/n} + \left[ \frac{Z}{A} \right]^{1/2} + 1 \right\}
\]

(10)

The material parameters vary with the strain according to the calculated values. A sixth-degree polynomial is used to fit the relationship between material parameters and strain.

\[
\begin{align*}
Q &= B_0 + B_1\varepsilon + B_2\varepsilon^2 + B_3\varepsilon^3 + B_4\varepsilon^4 + B_5\varepsilon^5 + B_6\varepsilon^6 \\
\ln A &= C_0 + C_1\varepsilon + C_2\varepsilon^2 + C_3\varepsilon^3 + C_4\varepsilon^4 + C_5\varepsilon^5 + C_6\varepsilon^6 \\
n &= D_0 + D_1\varepsilon + D_2\varepsilon^2 + D_3\varepsilon^3 + D_4\varepsilon^4 + D_5\varepsilon^5 + D_6\varepsilon^6 \\
\alpha &= E_0 + E_1\varepsilon + E_2\varepsilon^2 + E_3\varepsilon^3 + E_4\varepsilon^4 + E_5\varepsilon^5 + E_6\varepsilon^6
\end{align*}
\]

(11)

A detailed parameter solution can be found in the present authors work [40]. The coefficients of the sixth-degree polynomial are shown in tables 6 and 7 [40].
According to previous discussion [40], the Arrhenius model also has good predictive performance for GCr15 under HR. Thus, the comparison of predicted values between the new modified JC and strain-compensated Arrhenius model was made.

Comparison of predicted performance between the strain-compensated Arrhenius model and new modified JC model is shown in figure 11. It is observed that both constitutive model exhibit fine predictive performance over entire deformation condition. However, when the temperature is 1173 K, there is a remarkable deviation between experimental values and predicted values of the Arrhenius model at 0.01 s$^{-1}$ and 0.1 s$^{-1}$, while the new modified JC model exhibits better predictive performance at 0.01 s$^{-1}$ and 0.1 s$^{-1}$. There is almost no deviation between experimental values and predicted values of the new modified JC model.

4. Discussion

According to the above calculation results, predicted values and experimental values are compared in figures 6, 7, 10, 11. The JC and modified ZA model could not accurately capture the flow behavior of GCr15 under HR.

To further quantify the prediction accuracy of models, the correlation coefficient (R) and the average relative error (AARE) are used. R is used to represent the strength of the linear relationship between experimental and predicted values. If the model is biased toward higher or lower values, then higher values of R may not express a better correlation [47]. The AARE is computed through a term-by-term comparison of the relative error and is an unbiased statistical parameter [48]:

$$R = \frac{\sum_{i=1}^{N}(E_i - \bar{E}) \cdot (P_i - \bar{P})}{\sqrt{\sum_{i=1}^{N}(E_i - \bar{E})^2 \cdot \sum_{i=1}^{N}(P_i - \bar{P})^2}}$$

(12)

$$AARE(\%) = \frac{1}{N} \sum_{i=1}^{N} \left| \frac{E_i - P_i}{E_i} \right| \times 100$$

(13)

where $E_i$ is the experimental value, $P_i$ is the predicted value of the constitutive model, $\bar{E}$ and $\bar{P}$ are the mean values of experimental values and predicted values respectively.

The R of four models at 1023–1573 K are shown in table 8, the new modified JC model is enough for providing better correlation.

At the same time, the AARE and number of parameters of four models are discussed, as shown in table 9. The original JC with 5 parameters model could not describe the deformation behaviour of GCr15 well, the AARE is highest. The original JC model has good predictions only at the reference temperature mainly because the original JC model considers the effects of the temperature, strain, and strain rate on the flow stress to be

Table 6. Polynomial fitting coefficients of the material parameters at 1023–1123 K ($X$ represents $B$, $C$, $D$, and $E$ Polynomial) [40].

| Coefficients | $Q$ (J mol$^{-1}$) | lnA  | n   | $\alpha$ |
|--------------|------------------|------|-----|----------|
| $X_0$        | 589 038.7        | 58.69| 12.57| 0.007 62 |
| $X_1$        | $-3256\ 290.0$  | $-324.81$| $-124.97$| $-0.023 \ 76$ |
| $X_2$        | 11 257 500.0    | 1168.57| 773.17| 0.112 74 |
| $X_3$        | $-8243\ 070.0$  | $-777.75$| $-2464.05$| $-0.158 \ 75$ |
| $X_4$        | $-36\ 199\ 000.0$| $-4053.59$| 4305.89| $-0.018 \ 34$ |
| $X_5$        | 79 767 700.0   | 8723.79| $-3933.27$| 0.236 61 |
| $X_6$        | $-46\ 456\ 700.0$| $-5059.00$| 1469.84| $-0.158 \ 02$ |

Table 7. Polynomial fitting coefficients of the material parameters at 1173–1573 K ($X$ represents $B$, $C$, $D$, and $E$ Polynomial) [40].

| Coefficients | $Q$ (J mol$^{-1}$) | lnA  | n   | $\alpha$ |
|--------------|------------------|------|-----|----------|
| $X_0$        | 824 297.9        | 65.62| 8.10 | 0.029 49 |
| $X_1$        | $-7297\ 500.0$  | $-628.84$| $-76.04$| $-0.048 \ 41$ |
| $X_2$        | 51 118 100.0    | 4465.25| 523.61| 0.266 40 |
| $X_3$        | $-169\ 929\ 000.0$| $-15\ 039.42$| $-1787.61$| $-0.060 \ 39$ |
| $X_4$        | 300 289 000.0   | 26 917.37| 3288.95| $-1.347 \ 99$ |
| $X_5$        | $-274\ 727\ 000.0$| $-24\ 911.21$| $-3119.92$| 2.368 38 |
| $X_6$        | 103 311 000.0  | 9452.88| 1204.46| $-1.237 \ 16$ |
independent [2, 13], even though they actually influence one another. The AARE of JC model increases with increasing temperature, as shown in figure 12.

From the figure 13(a), the relationship of the \(\ln (\sigma - A)\) and \(\ln \varepsilon\) is nonlinear due to occurring of work hardening and dynamic softening. It can be deduced that two slopes with different signs are required to represent the data. As a result, JC model is segmented according to whether the strain is greater than the peak strain or not. Parameters \(n\) and \(B\) are calculated, as shown in figure 13(b). Corresponding parameters are shown in table 10. The AARE of the segmented JC model is 15.35% at 1023–1123 K and 18.15% at 1173–1573 K, the average relative error is still large. Therefore, the main problem is that the JC model does not incorporate the couple effects of temperature, strain and strain rate.

The modified ZA model with 7 parameters has better predictive ability for GCr15 than that of JC model. Especially at 1173–1573 K, the modified ZA model has higher prediction accuracy. However, predicted values of modified ZA model cannot agree well with the experimental values at 1023–1273 K. As shown in figure 14, the least squares method is used for deducing parameters \(C_3\) and \(C_4\), and the relationship between parameter \(S_1\) and strain is nonlinear, which may be contribute to the low predicted accuracy of modified ZA model. Consequently, the modified ZA model is not suitable for describing the deformation behavior of GCr15 under HR.

Figure 11. Comparison between strain-compensated Arrhenius model [40] and the new modified JC of predicted flow stress at (a) strain rate of 0.001 s\(^{-1}\), (b) strain rate of 0.01 s\(^{-1}\), (c) strain rate of 0.1 s\(^{-1}\).
After using the 5th degree polynomial to describing the relationship between $C_1$ and true strain, the new modified JC model with 13 parameters is feasible to describe flow behavior of GCr15. It may attributes to the coupled effect of strain and strain rate on the flow stress.

When a sixth-order polynomial was used to fit the relationship between strain and material parameters, predicted values of the Arrhenius model showed good agreement with the experimental values except for

Table 8. The correlation coefficient (R) of four models.

| Model          | 1023–1123 K | 1173–1573 K |
|----------------|-------------|-------------|
| JC             |             |             |
| Modified ZA    |             |             |
| New modified JC|             |             |
| Arrhenius [40] |             |             |

14
deviation between experimental values and predicted values of the Arrhenius model at 0.01 s$^{-1}$ and 0.1 s$^{-1}$, and the strain-compensated Arrhenius model contained 16 parameters.

As shown in figure 15 (AARE$_0$ corresponding to original values, $\varepsilon_0$, $\dot{\varepsilon}_0$ corresponding to original strain, $\dot{\varepsilon}_0$ corresponding to original strain rate), the JC model only needs three steps to acquire parameters of model. The modified ZA model also simply need two steps to obtain parameters of model, $C_5$ and $C_6$ corresponding to minimum AARE are desired values, which increases the calculated quantities. The new modified JC model need three steps to obtain parameters of model, and it requires repeated calculation under different strain rate to obtain values of $\lambda_1$ and $\lambda_2$. The parameters of Arrhenius model under a fixed strain could be obtained through

| Model            | Temperature range | AARE | R   | Temperature range | AARE | R   | Number of parameters |
|------------------|-------------------|------|-----|-------------------|------|-----|----------------------|
| JC               | 1023–1123 K       | 16.15% | 0.91 | 1173–1573 K       | 18.50% | 0.97 | 5                    |
| New modified JC  |                   | 3.29%  | 0.99 |                   | 6.22%  | 0.99 | 13                   |
| Modified ZA      |                   | 9.39%  | 0.94 |                   | 7.24%  | 0.98 | 7                    |
| Arrhenius        |                   | 3.74%  | 0.99 |                   | 5.76%  | 0.99 | 16                   |

Figure 12. AARE versus the temperature for the different models. (a) 1023–1123 K. (b) 1173–1573 K.

Figure 13. Parameters $n$ and $B$ of JC model at 1173–1573 K (a) Least squares method of solving for parameters $n$ and $B$ (b) Least squares method of solving for parameters $n$ and $B$ of JC model based on whether the strain is greater than the peak strain.
three steps, and it requires repeated calculation to obtain parameters under different strain, then polynomial fit is used to deduce coefficient of polynomial which is suitable for describing the relationship between parameters of Arrhenius model and strain. Compared with other models, the calculated quantities for parameters are high, and the calculation is complicated. Although the JC model and modified ZA model have simple calculation process of parameters, their predicted accuracy is not desired. The strain-compensated Arrhenius model has the highest accuracy at 1173–1573 K but experiences complicated calculation. Although the predicted accuracy of the new modified JC model is slightly lower than that of the strain-compensated Arrhenius model at 1173–1573 K, parameters of the new modified JC model are relatively easy acquired. Moreover, the new modified JC model has the highest predicted accuracy at 1023–1123 K.

Figure 16 summarizes the ideal predictability of the flow stress by some phenomenal constitutive model for some metals and alloy [3, 6, 7, 14, 19, 20, 23, 29, 31–33, 49]. From the statistical parameters (R and AARE) in

| Parameters | Temperature range 1023–1123 K | Temperature range 1173–1573 K |
|------------|-------------------------------|-------------------------------|
| ε (> 0.2)  | ε ≤ 0.2                       | ε > 0.2                       |
| A (MPa)    | 130                           | 80                            |
| B (MPa)    | 407.8172                      | 57.3526                       |
| n          | 0.4716                        | −0.7683                       |
| C          | 0.110 95                      | 0.110 95                      |
| m          | 0.965 52                      | 0.965 52                      |

Figure 14. Detailed least squares fitting process for parameters $C_3$ and $C_4$ of modified ZA model at 1173 K to 1573 K.

Figure 15. Calculation process of parameters for different constitutive models.
Figure 16, which AARE varies from 4.34% to 10.14% and R varies from 0.980–0.995, it could be inferred that the new modified JC model with acceptable accuracy and fewer parameters is the desired model for predicting the flow stress of GCr15 steel under HR.

5. Conclusion

The experiments in this work were designed to study the applicability and prediction accuracy of four models of GCr15 steel under HR conditions over a wide range of temperatures (1023–1573 K), strains (0.05–0.7) and strain rates (0.001–0.1 s\(^{-1}\)). The new modified JC model was developed based on the modified JC model proposed by Lin et al\[13\]. The conclusions are summarized as follows:

1. The flow stress increased with decreasing temperature and increasing strain rate due to the competing occurrence of dynamic softening and working hardening mechanisms. The degree of DRX was discussed through the difference of peak stress and steady stress as well as the evolution of microstructure, additional, the work hardening rate was explored. The process of DRX was enhanced with higher temperature and lower strain rate.

2. The original JC and modified ZA model were inappropriate to make a good description for predicting the flow behaviour of GCr15 steel under HR. The values predicted by the JC and modified ZA model deviated substantially from the experimental values, and the prediction accuracy of both models were not desired. Considering the linearly fitting the parameters \(n\) and \(B\) of the JC model, JC model is segmented according to whether the strain is greater than the peak strain or not, but the segmented JC model did not have desired prediction accuracy. The main problem is that the JC model did not incorporate the couple effects of temperature, strain and strain rate on flow stress.

3. The new modified JC model was proposed to describe the flow behaviour of alloy. Compared with the modified JC model, the new modified JC model retarded considerable discrepancy between predicted values and experimental values before the peak stress at 0.001 and 0.01 s\(^{-1}\) by considering couple effect of strain and strain rate. The new modified JC model not only had highest prediction accuracy at 1023–1123 K but also accurately described the deformation behaviour of GCr15.

4. The strain-compensated Arrhenius model had highest prediction accuracy at 1173–1573 K. However, compared with the new modified JC model, the Arrhenius model included many parameters, the calculation was complex. However, when the temperature is 1173 K, there is a remarkable deviation between experimental values and predicted values of the Arrhenius model at 0.01 s\(^{-1}\) and 0.1 s\(^{-1}\), while the new modified JC model exhibits better predictive performance at 0.01 s\(^{-1}\) and 0.1 s\(^{-1}\). The accuracy of the new modified JC model was slightly lower than that of the Arrhenius model; Comprehensively, the new modified JC model could be suitable for predicting the flow behaviour of GCr15.
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