Active Disturbance Rejection Control With Linear Quadratic Regulator for Power Output of Hydraulic Wind Turbines

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ABSTRACT In this research, a new method based on Active Disturbance Rejection Control (ADRC) for controlling the power output of hydraulic wind turbines (HWT) is proposed. There are three major problems in the power control system of HWT. The first problem is the existence of multiplicative nonlinearity in the system, which is an essential nonlinearity. The second problem is that the controlled system is a non-minimum phase (NMP) system. The third problem is the wind speed with strong randomness and large fluctuation range. These three problems make it difficult for HWT to achieve power control. Therefore, in this paper, the power transmission mechanism of HWT is analyzed. Then the ADRC controller combined with the linear quadratic regulator (LQR) method is designed and the stability is proved. In the process of controller design, the multiplicative nonlinear problem is regarded as the total disturbance in ADRC. Then the extended system model of ADRC and HWT is deduced, the range of the tuning parameter \(b_0\) is calculated, and the control parameters are tuned through the LQR method. Finally, the effectiveness of the proposed method is verified through simulation experiments.

INDEX TERMS Multiplicative nonlinearity, power smoothing, hydraulic systems, wind power generation, ADRC.

I. INTRODUCTION

The main types of conventional wind turbine transmissions include doubly-fed wind turbines and direct drive wind turbines and the related technologies are relatively mature. Both models are rigid transmissions, as described in [1]–[3]. However, the cost and failure rate of these traditional unit drive systems are relatively high. Therefore, the flexible hydraulic transmission form is introduced into wind turbines [4]–[6]. Compared with conventional wind turbines, the advantage of HWT is that the hydrostatic transmission has flexibility, which makes it more reliable in terms of shock loading. And the transmission ratio is adjustable, so the system does not need additional frequency conversion equipment, which solves the problem of the high failure rate of the double-fed wind generator gearbox.

Wind speed fluctuations and time-varying system parameters will affect the stability of the power grid. The instability of wind power output has become a bottleneck restricting the development of wind power [7]. For the problem of control system uncertainty and disturbance, an output feedback control method is proposed and the effectiveness of the method is verified in [8], [9]. This provides a new idea for solving the disturbance and uncertainty problems of wind turbines.

In recent years, many scholars have studied the power smoothing control of wind turbines. An output power smoothing method based on a fuzzy controller was proposed in [10], which can adjust the average time and produce effective smooth output power. However, this method has the disadvantages of many manual interventions, slow inference speed, and low accuracy. A fuzzy-based discrete Kalman filter approach was proposed in [11], which can smooth the output power fluctuations of the wind and PV generation systems using a battery energy storage system. But the service life and...
safety of battery energy storage systems are a big problem. A novel pitch angle control strategy had developed in [12], which can effectively smooth the generator output power. But this method adjusts the power output by abandoning the wind, so it loses a lot of energy. And the above methods are aimed at the output power control of traditional wind turbines. It is difficult to find the research on the suppression of the output power fluctuation of the HWT. A variable gain PID control strategy was proposed in [13], which realizes the power control of hydraulic wind turbine. But there is no power smoothing control under fluctuating wind speed. A method based on the coupled control of wind turbine rotor energy storage and pitch control was proposed in [14], but due to the multiplicative nonlinearity of the system, the control effects are limited.

Given the literature on HWT power smoothing control, a better power controller to solve fluctuating wind power and multiplying nonlinear problems is still needed. Since the hydraulic transmission system contains multiplicative nonlinearity and wind power fluctuation problems, HTW needs to have a certain comprehensive anti-interference ability. A large number of studies have shown that LADRC still has a good control effect on complex nonlinear systems with uncertainties [15]. Therefore, the power transmission mechanism of HWT is analyzed. And aiming at the problems in the mechanism analysis, the ADRC controller combined with LQR is established, and the stability verification and simulation experiments are carried out.

The rest of this paper is constructed as follows. In section 2, the mathematical model of the HWT is established. In section 3, the power control system model analysis is carried out. In section 4, ADRC with LQR strategy is designed. Then the parameters are adjusted and the stability is analyzed. In section 5, the effectiveness of the control strategy is verified by simulation analysis. In section 6, this work is concluded.

II. MATHEMATICAL MODEL OF HYDRAULIC WIND TURBINE

A. WORKING PRINCIPLE OF HYDRAULIC WIND TURBINE

The HWT working principle [16] is shown in Fig. 1. The main hydraulic transmission system consists of a hydraulic fixed displacement pump–hydraulic variable displacement motor. When the wind speed changes, the output power can be controlled by changing the variable motor displacement.

B. WIND SPEED MATHEMATICAL MODEL

The combined wind speed model description is similar to the natural wind speed, which consists of basic wind, gust, gradual wind, and random wind.

\[ v_w = v_{wB} + v_{wG} + v_{wR} + v_{wN} \]  

where \( v_w \) is the wind speed. \( v_{wB} \) is the basic wind speed. \( v_{wG} \) is the gust wind speed. \( v_{wR} \) is the gradual wind speed. \( v_{wN} \) is the random wind speed.

C. ROTOR AERODYNAMIC MODEL

The rotor aerodynamic model relies on the wind turbine’s inherent parameters. This paper adopts the internationally simplified mathematical model. The power and torque of the turbine rotor can be expressed as

\[ P_r = C_p(\lambda, \beta) \frac{\rho A}{2} v^3 = \frac{1}{2} \rho \pi R^2 v^3 C_p(\lambda, \beta) \]  

\[ T_r = \frac{P_r}{\omega_r} \]  

where \( P_r \) is the wind power absorbed by the rotor. \( C_p \) is the wind energy utilization coefficient of the rotor. \( \rho \) is air density. \( \omega_r \) is the speed of the rotor. \( T_r \) is the rotor aerodynamic torque.

D. MAIN TRANSMISSION HYDRAULIC SYSTEM MATHEMATICAL MODEL

To analyze the mathematical model of the pump-motor closed speed control system, the following assumptions are as follows:

1) The oil volume elastic modulus in the pipeline is a constant, that is, the flow compression is only related to the pressure;
2) The low-pressure pipeline pressure is zero;
3) When the rotor is working, the oil density, leakage coefficient, viscous damping, and transmission efficiency are considered to be constant;
4) The fixed displacement pump is rigidly connected to the rotor, so the two speeds are considered to be equal, \( \omega_r = \omega_p \);
5) After the generator is connected to the grid, the generator speed is constant at 1500rpm, regardless of the influence of the grid frequency fluctuation on the generator speed.

1) THE MODEL OF THE FIXED DISPLACEMENT PUMP

The fixed displacement pump flow continuity equation is

\[ Q_p = D_p \omega_p - C_{p} p_h \]  

The fixed displacement pump torque balance equation is

\[ T_r - T_p = J_p \frac{d\omega_p}{dt} + B_p \omega_p + G_p \theta_p \]
The fixed displacement pump torque equation is

$$T_p = \frac{D_p p_h}{n_{mp}}$$

where $Q_p$ is the pump flow, $D_p$ is the pump displacement, $\omega_p$ is the pump speed, $C_p$ is the pump leakage coefficient, $p_h$ is the pump inlet and outlet pressure differential. $J_p$ is the moment of inertia of the rotor and pump. $B_p$ is the pump damping coefficient. $G_p$ is the pump load spring stiffness. $\theta_p$ is the pump angle.

2) THE MODEL OF THE VARIABLE HYDRAULIC MOTOR
The variable hydraulic motor flow continuity equation is

$$Q_m = K_m \gamma t_\omega m + C_{tm} p_h$$

The variable hydraulic motor displacement equation is

$$D_m = K_m \gamma t_\omega m$$

The variable hydraulic motor output torque equation is

$$T_m = D_m p_h \eta_{mm}$$

The variable hydraulic motor torque balance equation is

$$T_m - T_e = J_m \frac{d\omega_m}{dt} + B_m \omega_m + G_m \omega_m$$

where $Q_m$ is the motor flow, $D_m$ is the motor displacement, $\omega_m$ is the motor speed, $C_{tm}$ is the motor leakage coefficient. $K_m$ is the motor displacement. $\gamma$ is non-dimensional displacement fraction, ranging from 0 to 1. $T_m$ is the motor output torque. $\eta_{mm}$ is the motor mechanical efficiency. $T_e$ is electromagnetic torque acting on the motor. $J_m$ is the moment of inertia of the motor. $B_m$ is the motor damping coefficient. $G_m$ is the motor load spring stiffness. $\theta_m$ is the motor angle.

3) HYDRAULIC PIPING MODEL
The flow equation between the fixed displacement pump and the variable hydraulic motor is

$$Q_c = (V/\beta_c)(dp_h/d_t)$$

The differential value of the high pressure can be obtained from the formula (12)

$$\hat{p}_h = \frac{\beta_c}{V} Q_c$$

where $V$ is the pressure-affected oil volume. $\beta_c$ is the effective oil bulk modulus including a correction for hose expansion. $Q_c$ is the flow rate caused by the oil compression.

4) HYDRAULIC SYSTEM OVERALL MODEL
After the generator is incorporated into the grid, the proportional flow control valve is fully open and the variable hydraulic motor is dragged by the infinite grid. The state space model of the main drive system can be expressed as

$$\begin{align*}
\dot{\omega}_p &= \frac{B_p}{J_p} \omega_p - \frac{D_p p_h}{\eta_{mech,p} J_p} + \frac{1}{J_p} T_w(\omega_p, v) \\
\dot{p}_h &= \frac{D_p p_h}{V} \omega_p - \frac{C_p}{\beta_p} \frac{p_h}{p_h} - \frac{K_p \beta_p \omega_m}{V} \gamma_1
\end{align*}$$

III. POWER CONTROL SYSTEM MODEL ANALYSIS
Through the Laplace transform of the mathematical model of the hydraulic main transmission system, the transfer function of the hydraulic system speed model is established

$$\frac{\omega_p}{\gamma} = \frac{K_m \omega_{ma}}{D_p \gamma} s + 1$$

where $\omega_{hp} = \sqrt[\beta_p D_p V_h C_p p_h}{V_h} \xi_{hp}$

The output power model of the hydraulic system is established. Due to the multiplicative nonlinearity of the system [13], the system is linearly expanded at the operating point $(p_{h0}, \gamma_0)$, taking increments and ignoring high-order infinitesimal quantities

$$P_h = K_m \omega_{ma} p_h \gamma + \frac{\omega_p D_p K_m \omega_{ma} \gamma_0}{C_i + \frac{V_p}{p_c} s} - \frac{K_m^2 \omega_{ma}^2 \gamma_0}{C_i + \frac{V_p}{p_c} s} - \frac{K_m \omega_{ma} \gamma_0}{C_i + \frac{V_p}{p_c} s}$$

The first term on the right side of equation (15) is the power change value corresponding to the variable motor flow rate change caused by the motor swing angle change. The second term is the power change value corresponding to the system pressure change caused by the fixed displacement pump speed change. The third term is the power change value corresponding to the system pressure change caused by the motor swing angle change. The fourth term is the power change value corresponding to the system pressure change caused by the motor speed change. After the generator is connected to the grid, the motor speed is a fixed value, and the corresponding incremental value is zero. Therefore, the simplified formula (15) is

$$P_h = K_m \omega_{ma} p_h \gamma + \frac{\omega_p D_p K_m \omega_{ma} \gamma_0}{C_i + \frac{V_p}{p_c} s} - \frac{K_m^2 \omega_{ma}^2 \gamma_0}{C_i + \frac{V_p}{p_c} s}$$

Using power as output, the system can be transformed into a third order system

$$\ddot{\gamma} + a_2 \dot{\gamma} + a_1 \gamma + a_0 \gamma = b_3 \ddot{u} + b_2 \dot{u} + b_1 u + b u$$

where $a_0 = \frac{C_i \beta_p \omega_{ma} \gamma_0}{V_0}$, $a_1 = \left(1 + \frac{2C_i \beta_p \xi_{hp}}{V_{0hp}}\right) \omega_{hp}$, $a_2 = \frac{C_i \beta_p}{V_0}$, $2 \xi_{hp} \omega_{hp}$

$$b = \left[\frac{(C_i K_m \omega_{ma} \gamma_0 - K_m^2 \omega_{ma}^2 \gamma_0) \beta_c}{V_0} + (K_m \omega_{ma})^2 \gamma_0\right] \omega_{hp}^2$$

$$b_1 = 2 \left(\frac{(C_i K_m \omega_{ma} \gamma_0 - K_m^2 \omega_{ma}^2 \gamma_0) \beta_c \xi_{hp}}{V_{0hp}} + K_m \omega_{ma} \gamma_0\right) \omega_{hp}^2$$

$$b_2 = \left[\frac{(C_i K_m \omega_{ma} \gamma_0 - K_m^2 \omega_{ma}^2 \gamma_0) \beta_c \xi_{hp}}{V_{0hp}} + 2K_m \omega_{ma} \gamma_0\right]$$

Due to the multiplicative nonlinearity of the system and the external wind speed disturbance, it is difficult to control the
system output power. Therefore, the ADRC control method is adopted to integrate all disturbances such as nonlinearity and external disturbances of the system into total disturbances. The total disturbance is calculated by designing the expanded state observer (ESO) and eliminated by designing the controller to achieve the purpose of power control.

But since \( b \) is negative at the operating point, this power control system is an NMP system. When using the conventional ADRC controller parameter configuration method for power control, the system is difficult to stabilize [17]. Therefore, it is considered to set the controller parameters through the LQR method [18], which improves the system stability and achieves the purpose of system output power control.

IV. CONTROLLER DESIGN

A. ADRC DESIGN FOR POWER CONTROL SYSTEM

In equation (18), the total disturbance is defined as

\[
f = -a_2\dot{y} - a_1\dot{y} - a_0y + b_3\ddot{u} + b_2\dot{u} + b_1u
\]  

(18)

Defining states as \( x_1 = y, x_2 = \dot{y}, x_3 = \ddot{y}, x_4 = f \). The state space representation of (17) is

\[
\begin{align*}
\dot{x} &= Ax + Bu + Ef \\
y &= Cx
\end{align*}
\]

(19)

where

\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0
\end{bmatrix},
B = \begin{bmatrix}
0 \\
0 \\
1 \\
0
\end{bmatrix},
C = \begin{bmatrix}
1 & 0 & 0 & 0
\end{bmatrix},
E = \begin{bmatrix}
0 \\
0 \\
0 \\
1
\end{bmatrix}
\]

A fourth order ESO is constructed correspondingly for the system (19)

\[
\dot{\hat{x}} = A\hat{x} - b_0Bu + L(y - \hat{x})
\]

(20)

where \( \hat{x} = [\hat{x}_1 \hat{x}_2 \hat{x}_3 \hat{x}_4]^T \) is estimation of state variables. \( x(t), L = [l_1 l_2 l_3 l_4]^T \) is the observer gain vector, and \( b_0 \) is a tuning parameter, which should be adjusted according to observer gain.

The input \( u(t) \) is designed as follows

\[
u(t) = \frac{(u_0 - \hat{x}_3)}{-b_0} = \frac{K(r - \hat{x})}{-b_0}
\]

(21)

where \( K = [k_1 k_2 k_3 \ 1] \) is the controller gain vector.

The observer gain from the polar arrangement method. When the pole position of the observer is \( -\omega_o \), the observer gain is \( l_1 = 4\omega_o, l_2 = 6\omega_o, l_3 = 4\omega_o, l_4 = \omega_o \).

The block diagram of ADRC is shown in Fig. 2.

B. PARAMETER SETTING

1) RANGE OF TUNING PARAMETER \( B_0 \)

The ADRC controller designed in Section A can be equivalent to a two-degree-of-freedom transfer function form shown in Fig. 3.

where \( H(s) \) is the pre-filter transfer function and \( C(s) \) is the controller transfer function. The equivalent transfer functions for a fourth order ADRC with state feedback are given in (22) and (23), as shown at the bottom of the next page.

Combined with the linearized power control system (24), the characteristic equation of the closed-loop system can be obtained (25)

\[
\frac{Y}{U} = \frac{b_3s^3 + b_2s^2 + b_1s + b}{s^3 + a_2s^2 + a_1s + a_0}
\]

(24)

\[
\lambda(s) = A_7s^7 + A_6s^6 + A_5s^5 + A_4s^4 + A_3s^3 + A_2s^2 + A_1s + A_0
\]

(25)

where \( A_0 = -bk_1l_4 \)

\[
A_1 = a_0b_0k_1 + a_0b_0l_3 - bk_1l_3 - bk_2l_4 - b_1k_1l_4 + a_0b_0k_3l_2
\]

\[
A_2 = a_0b_0k_2 + a_1b_0k_1 + a_0b_0l_2 + a_1b_0l_3 - bk_1l_2 - bk_2l_3 - b_1k_1l_3
\]

\[
A_3 = -bk_2l_4 - bk_1l_1 + a_1b_0k_3l_1 + a_1b_0l_2 + a_1b_0l_1 + a_2b_0l_3
\]

\[
A_4 = a_0b_0 + a_0b_0k_1 + b_0l_3 - b_1l_4 + a_1b_0k_3 + a_2b_0k_2 + a_2b_0k_1
\]

\[
+ a_2b_0l_2 + a_2b_0l_1 + a_2b_0l_3
\]

\[
+ a_2b_0l_2 + a_2b_0l_1 + a_2b_0l_3 + a_2b_0k_2 + a_2b_0k_1 + a_2b_0k_3l_2 - bk_2l_3
\]

\[
- b_3k_1l_4 - b_3k_3l_4 - b_3k_2l_4 + a_0b_0k_3l_1 + a_0b_0l_2 + a_0b_0l_1 + a_2b_0l_3
\]

\[
+ a_2b_0l_1 + a_2b_0l_3 + a_2b_0k_2 + a_2b_0k_1 + a_2b_0k_3l_2 - bk_2l_3
\]

\[
- b_3k_1l_4 - b_3k_3l_4 - b_3k_2l_4 + a_0b_0k_3l_1 + a_0b_0l_2 + a_0b_0l_1 + a_2b_0l_3
\]

\[
+ a_2b_0l_1 + a_2b_0l_3 + a_2b_0k_2 + a_2b_0k_1 + a_2b_0k_3l_2 - bk_2l_3
\]
A5 = a1b0 + b0k2 + b0l2 − b2l4 + a2b0k3 + a2b0l1 + b0k3l1
− b2k1l1 − b2k2l2 − b3k1l2 − b2k3l3 − b3k2l3 − b3k3l4
A6 = a2b0 + b0k3 + b0l1 − b3l4 − b3k1l1 − b3k2l2 − b3k3l3
A7 = b0

The parameter b0 must be adjusted so that all roots of equation (25) are on the left half of the s-plane. However, due to the time-varying state of the system and frequent adjustment of control parameters k1, k2, and k3. Therefore, during the operation of the system, the stability conditions of the closed-loop system are constantly changing, and the lower limit of b0 will dynamically change following the changes in operating conditions and controller parameters.

Because the order of the system is too high and the solution result is too complicated, the analytical formula of b0 for the above variables cannot be listed. Therefore, taking k1 = 320, k2 = 65, k3 = 0.1, p0 = 2 × 107, γ0 = 0.5. The range of b0 should satisfy the formula (26).

$$H(s) = \frac{k_1(s^4 + l_1s^3 + l_2s^2 + l_3s + l_4)}{(k_1l_1 + k_2l_2 + k_3l_3 + l_4)s^3 + (k_1l_2 + k_2l_3 + k_3l_4)s^2 + (k_1l_3 + k_2l_4)s + k_4l_4}$$

$$C(s) = \frac{-(k_1l_1 + k_2l_2 + k_3l_3 + l_4)s^3 + (k_1l_2 + k_2l_3 + k_3l_4)s^2 + (k_1l_3 + k_2l_4)s + k_4l_4}{b_0[s^4 + (k_3 + l_1)s^3 + (k_2 + l_2 + k_l_1)s^2 + (k_1 + l_3 + k_2l_1 + k_3l_2)s]}$$

$$\begin{align*}
H(s) &= \frac{1.52b_0 + 1.1 \times 10^{10}}{1.01b_0 + 5.89 \times 10^9} > 0 \\
&= \frac{7.89b_0 + 4.17 \times 10^9}{1.94b_0 + 1.8 \times 10^7} > 0 \\
&= \frac{4.37b_0 + 7.45 \times 10^5}{b_0} > 0 \\
&= \frac{3.36b_0 - 1.37 \times 10^9}{1.21b_0^2 \times 10^4 - 5.95b_0 \times 10^{12} - 1.02 \times 10^{18}} > 0 \\
&= \frac{1.51b_0 - 6.16 \times 10^8}{-2.34b_0^3 \times 10^8 + 1.17b_0^2 \times 10^{15} - 1.17b_0 \times 10^{22} + 4.12 \times 10^{29}} > 0 \\
&= \frac{8.74b_0^3 + 4.29b_0 \times 10^9 + 7.34 \times 10^{14}}{-1.83b_0^4 \times 10^5 + 1.98b_0^3 \times 10^6 + 3.51b_0^2 \times 10^2 - 4.55b_0 \times 10^9 + 1.69 \times 10^7} > 0 \\
&= \frac{-6.76b_0^3 + 3.38b_0^2 \times 10^9 - 3.37b_0 \times 10^{16} + 1.19 \times 10^{24}}{A(s2) > 0} \\
&= \frac{1.06b_0^2 \times 10^3 + 1.06b_0 \times 10^{12} - 1.41b_0 \times 10^{39}}{5.04 \times 10^{46}} > 0.0001
\end{align*}$$

Since the Rolls criterion is used for the linearized system, the calculation result can only roughly determine the range of b0. Of course, the values of the five parameters can be changed under the conditions consistent with the working conditions, only to roughly determine the magnitude of b0. During the debugging process, simple adjustments to b0 can achieve the control purpose, (26), as shown at the bottom of the page, where, A(s2) and A(s1), as shown at the bottom of the page.

2) CONTROL PARAMETER SETTING

The conventional ADRC controller parameter tuning method is difficult to stabilize the non-minimum phase system, so the LQR parameter tuning method is used to set the controller parameters. (20) is rewritten by

$$(sI - A)\dot{x} = B\dot{U} + L(Y - C\dot{x})$$

$$\dot{x} = (sI - A + LC)^{-1}(B\dot{U} + LY)$$

(27)
By using (21), (24) and (27), the extended system state space model of ADRC and controlled system is established.

\[
Y = \frac{n_3s^3 + n_2s^2 + n_1s + n_0}{m_3s^3 + m_2s^2 + m_1s + m_0} V
\]

(28)

where

\[
n_7 = -3, n_6 = -3l_l - 2, n_5 = -3l_l - 2l_l - 1, n_4 = -3l_l - 2l_l - 1 - b, n_3 = -3l_l - 4 - 3l_l - 2 - b - l_l - 1 - b, n_2 = -3l_l - 6l_l - 3, n_1 = -3l_l - 4 - 3l_l - 3, n_0 = -3l_l - 2
\]

\[
m_7 = b_0, m_6 = b_0l_l + b_0a_{l2} - b_3l_l
\]

\[
m_5 = b_0a_{l1} + b_0l_l + b_0a_{l1} - b_2l_l - b_4
\]

\[
m_4 = b_0l_l + b_0a_{l2} + b_0l_l + b_0a_{l1} - b_2l_l + b_0a_l - b_4
\]

\[
m_3 = b_0a_{l3} + b_0a_{l2} + b_0a_{l1} - b_4 - b_4
\]

\[
m_2 = b_0a_{l3} + b_0a_{l2} + b_0a_l - b_4
\]

\[
m_1 = b_0a_{l4} - b_0a_l - b_4 = 0
\]

Approximately simplify the extended system [19]. There are some differences in the high-frequency part of the system before and after simplification. Therefore, the value of the control parameter can be roughly positioned by this method, and then it needs to be adjusted within a small range to achieve the control effect.

\[
Y \approx \frac{n_3s^3 + n_2s^2 + n_1s + n_0}{m_3s^3 + m_2s^2 + m_1s + m_0} V
\]

(29)

Convert equation (29) into a controllable form of the state space equation

\[
\begin{bmatrix}
    x_1 \\
    \dot{x}_2 \\
    \dot{x}_3
\end{bmatrix}
= \begin{bmatrix}
    0 & 1 & 0 \\
    0 & 0 & 1 \\
    -\frac{m_1}{m_2} & -\frac{m_2}{m_3}
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    x_2 \\
    x_3
\end{bmatrix}
+ \begin{bmatrix}
    h_1 \\
    h_2 \\
    h_3
\end{bmatrix}
\nu
\]

(30)

\[
y = \begin{bmatrix}
    1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    x_2 \\
    x_3
\end{bmatrix}
+ h_0 \nu
\]

where

\[
h_0 = \frac{n_3}{m_3}, h_1 = \frac{n_2}{m_3} - \frac{m_2}{m_3} h_0, h_2 = \frac{n_1}{m_3} - \frac{m_1}{m_3} h_0 - \frac{m_2}{m_3} h_1, h_3 = \frac{n_0}{m_3} - \frac{m_0}{m_3} h_0 - \frac{m_1}{m_3} h_1 - \frac{m_2}{m_3} h_2
\]

At this time, the control block diagram of the closed-loop system can be represented as Fig. 4.

When the optimum gain parameter obtained by the LQR method is defined as \( K \), the manipulated variable \( u_0(t) \) is

\[
u_0(t) = Kx(t)
\]

(31)

Let the quadratic cost function \( J \) be

\[
J = \frac{1}{2} \int_0^\infty (x(t)^T Q x(t) + R u_0(t)^2) dt
\]

(32)

where \( Q(\geq 0) \) is a positive definite symmetric matrix, \( R \) is a positive integer. We focus on the change of \( x_1 \), so the value in the first column of \( Q \) is relatively large, and other variables are relatively less concerned, so the smaller value is used in \( Q \) and \( R \). But it must be ensured that equation (34) has a solution. In this study

\[
R = 1, Q = \begin{bmatrix}
    80000 & 0 & 0 \\
    0 & 100 & 0 \\
    0 & 0 & 100
\end{bmatrix}
\]

(33)

The LQR feedback gain \( K \) that minimizes the evaluation value \( J \) of (32) is

\[
K = R^{-1} P Q^T
\]

(34)

The symmetric matrix \( P > 0 \) is the solution of the Riccati equation [20].

C. STABILITY ANALYSIS

The system is expressed as the following nonlinear time-varying system form

\[
y^{(3)}(t) = f(y^{(2)}(t), y^{(1)}(t), y(t), w(t)) + bu(t)
\]

(35)

where \( w(t) \) is external disturbance

The control law of ADRC is

\[
u = [k_1(r_1 - \hat{x}_1) + k_2(r_2 - \hat{x}_2) + k_3(r_3 - \hat{x}_3) - \hat{x}_4]/b
\]

(36)

Write the system as an expanded state

\[
\begin{bmatrix}
    \hat{x}_1 \\
    \hat{x}_2 \\
    \hat{x}_3 \\
    \hat{x}_4
\end{bmatrix}
= \begin{bmatrix}
    1 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 \\
    0 & 0 & 1 & 0 \\
    0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    x_2 \\
    x_3 \\
    x_4
\end{bmatrix}
+ \begin{bmatrix}
    k_1 \\
    k_2 \\
    k_3 \\
    -b
\end{bmatrix}
\]

(37)

And \([l_1, l_2, l_3, l_4] = [4\omega_0, 6\omega_0^2, 4\omega_0^3, \omega_0^4]\)

Since the structure of \( f \) is unknown, ESO is designed as

\[
\begin{bmatrix}
    \dot{\hat{x}}_1 \\
    \dot{\hat{x}}_2 \\
    \dot{\hat{x}}_3 \\
    \dot{\hat{x}}_4
\end{bmatrix}
= \begin{bmatrix}
    \hat{x}_1 \\
    \hat{x}_2 \\
    \hat{x}_3 \\
    \hat{x}_4
\end{bmatrix}
+ \begin{bmatrix}
    l_1 & l_2 & l_3 & l_4
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    x_2 \\
    x_3 \\
    x_4
\end{bmatrix}
+ \begin{bmatrix}
    k_1 \\
    k_2 \\
    k_3 \\
    -b
\end{bmatrix}
\]

(38)

Let \( \tilde{x}_i = x_i - \hat{x}_i, i = 1, 2, \ldots n \). Then the observed error of the controlled system can be expressed as

\[
\begin{bmatrix}
    \dot{\hat{x}}_1 \\
    \dot{\hat{x}}_2 \\
    \dot{\hat{x}}_3 \\
    \dot{\hat{x}}_4
\end{bmatrix}
= \begin{bmatrix}
    \hat{x}_2 - 4\omega_0\tilde{x}_1 \\
    \hat{x}_3 - 6\omega_0^2\tilde{x}_1 \\
    \hat{x}_4 - 4\omega_0^3\tilde{x}_1 \\
    h(x, w) - \omega_0^4\tilde{x}_1
\end{bmatrix}
\]

(39)
Let $\varepsilon_i = \frac{\hat{x}_i}{\omega_0}$, $i = 1, 2, \cdots n + 1$. Then equation (39) is transformed into

$$\dot{\varepsilon} = \omega_0 A_{\varepsilon} \varepsilon + B_{\varepsilon} \frac{h(x, w)}{\omega_0^3}$$

(40)

where $A_{\varepsilon} = \begin{bmatrix} -4 & 1 & 0 & 0 \\ -6 & 0 & 1 & 0 \\ -4 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \end{bmatrix}$, $B_{\varepsilon} = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T$

According to the literature [21], [22], since the system disturbance is caused by the change of natural wind speed, the change rate $h$ of the total disturbance is bounded. Therefore, the observation error is bounded, and the upper limit of the bound is monotonically decreasing with the observer bandwidth $\omega_0$. However, the observer bandwidth cannot be set too high to eliminate observation errors, otherwise, it will be affected by high-frequency noise. At the same time, there will be $k_1 > 0$, $k_2 > 0$ can make the tracking error of ADRC bounded. Thus for a bounded input $r$, the output of the closed-loop system is bounded. That is, the closed-loop system is BIBO stable.

V. SIMULATION ANALYSIS

Establish a mathematical simulation model in MATLAB/Simulink. The simulation model includes a wind speed model, the rotor model, hydraulic main transmission system model, ADRC control model. The ADRC control model consists of ESO and controller. The simulation parameters are shown in Table 1., and the power smooth output control block diagram is shown in Fig. 5.

To evaluate the degree of system output power smoothness, the following power smoothing coefficient is defined as

$$S = \int_0^t \left| \frac{dP}{dt} \right| \frac{dr}{\overline{P}}$$

(41)

where $P$ is the system output power, $\overline{P}$ is the average value of the system output power in the selected time.
A. POWER CONTROL UNDER CONSTANT WIND SPEED

1) REDUCE THE GIVEN POWER

Under the condition of constant wind speed of 8 m/s, different power given values are input to the controller. By adjusting the controller parameters, the system power response and dynamic characteristics can be observed.

The input of the controller is stepped from 5225 W to 4500 W. Adjust the controller parameters, and the simulation results are shown in Fig. 7.

Fig. 7 a) is the system power response curve. It can be seen that the given power is reduced to 4500 W at 10 s. When different combinations of $k_1$, $k_2$, and $k_3$ are selected, the system power response characteristics are different. When increasing $k_1$, the tracking accuracy of the system will be improved, but the number of overshoots increases. When increasing $k_2$, the accuracy of the system is almost unchanged, and the system response adjustment time and overshoot are significantly reduced. But $k_3$ cannot be too large, otherwise, it will cause system instability.

Fig. 7 b) is the power response curve in 500 s after 10 s in Fig. 7 a). As time increases, the output power gradually stabilizes to a given value. The reason for this phenomenon is that the energy storage element is the turbine rotor, which has the characteristics of large inertia and slow response. At the same time, its speed change will directly affect the size of the input system power. Therefore, the system needs a time to reach the dynamic equilibrium state, and the deviation value and adjustment time that needs to be adjusted will decrease with the decrease of the given power change.

It can be seen from Fig. 7 c) that as the output power decreases, excess energy is stored in the rotor of the wind turbine, and the rotor speed increases. However, the speed of the wind turbine rotor cannot exceed the mechanical system’s bearing range.

Fig. 7 d) is the system high-pressure pressure response curve. It can be seen that the system pressure changes within
2) INCREASE THE GIVEN POWER
The input of the controller is stepped from 5225W to 5600W. Adjust the controller parameters, and the simulation results are shown in Fig.8.

Fig.8 a) is the system power response curve. It can be seen that the given power is increased to 5600W at 10s. When different combinations of $k_1$, $k_2$, and $k_3$ are selected, the system power response characteristics are different.

Fig.8 b) is the power response curve within 500s after 10s in Fig.8 a). The difference with reducing a given power is that as time increases, the power control system will gradually become unstable. The reason for this phenomenon is that as the rotor speed decreases, the power absorbed by the system will gradually decrease. At this time, the system falls into a vicious circle. When the rotor speed decreases to a certain limit, the sum of the rotor absorbed power and the rotor kinetic energy release cannot meet the given power demand, the system will not be stable. Therefore, when the given power is higher than the absorbed power of the rotor, the rotor kinetic energy can be released in a short time to supplement the output power, but the system cannot be in this state for a long time.

It can be seen from Fig.8 c) that as the output power increases, the rotor kinetic energy is converted into hydraulic energy, and the rotor speed decreases. However, the rotor speed cannot be lower than a certain limit, otherwise the system will not be stable.

Fig.8 d) is the system high-pressure pressure response curve. It can be seen that the system pressure changes within a reasonable working range and the system is in a normal working state.

3) GIVEN RATED POWER
Under the condition that the wind speed is stepped from 8m/s to 8.5m/s, rated power given values is input to the controller. By adjusting the controller parameters, the system power response and dynamic characteristics can be observed.

The input of the controller is stepped from 5225W to 6365W. Adjust the controller parameters, and the simulation results are shown in Fig.10.

Figure 10 a) is the system power response curve. It can be seen that as the wind speed becomes 8.5m/s, the given power increases to 6365W at 10s. And 6365W is the rated power under current working conditions. When the given power is the current rated power, the power output can quickly stabilize to the given value without a period of dynamic adjustment such as Fig.7 a) and Fig.8 a).

Fig.10 b) is the power response curve within 500s after 10s in Fig.10 a). It can be seen that the system can be stable in the current state for a long time.
It can be seen from Fig.10 c) that the rotor kinetic energy has hardly changed.

It can be seen from Fig.10 d) that the system pressure changes within a reasonable working range and the system is in a normal working state.

**B. POWER CONTROL UNDER FLUCTUATING WIND SPEED**

Under the condition of fluctuating wind speed of an average of 8m/s, different power given values is input to the controller. By adjusting the controller parameters, the system power response and dynamic characteristics can be observed.

When the control is not added, the system power response is shown in Fig.12. And take the power response value of 200s to 400s to calculate the power smoothing coefficient $S$.

It can be seen from Fig.12 that when power control is not added, the system output power fluctuation range and power smoothing coefficient are larger.

Power control is added at 100s, and the controller input is 5500W. Then the controller input step is 5000W at 400s. Adjust the controller parameters $k_1 = 320$, $k_2 = 65$, $k_3 = 0.1$, and take the power response value of 200s to 400s to calculate the power smoothing coefficient $S$. The simulation results are shown in Fig.13.

Fig.13 a) is the power response curve under fluctuating wind speed. It can be seen that after adding power control, the macro fluctuation of output power has been significantly improved, and the fluctuation range is significantly reduced. The output power fluctuates in a small range around the given power, which proves that the active power control is realized and the output power is smoothed. Note that when the control is added at 100s, the output power gradually transitions from the original larger fluctuation to a smaller fluctuation. And through local magnification, it can be seen that the instantaneous power is relatively smooth in the local range without instantaneous fluctuations.

It can be seen from the Fig.13 b) that the output power smoothing effect is significant at this time, and the power smoothing coefficient is greatly reduced.

It can be seen from the Fig.13 c) that with the change of the given power, the rotor speed decreases first and then increases. The reason for this phenomenon is that when the given power is 5500W in 100s, the system output power is greater than the average power absorbed by the rotor.
Therefore the rotor releases energy. When the given power is 5000W in 400s, the output power of the system is less than the average power absorbed by the rotor. Therefore, excess energy is stored by the rotor. At the same time, the output power fluctuation caused by the wind speed fluctuation is smoothed by the small range fluctuation of the rotor speed.

Fig.13 d) is the system high-pressure pressure response curve. It can be seen that the system pressure changes within a reasonable working range and the system is in a normal working state.

**FIGURE 11.** Fluctuating wind speed.

**FIGURE 12.** System response without power control.

**FIGURE 13.** $k_1 = 320$, $k_2 = 65$, $k_3 = 0.1$ system response.
Change the controller parameters $k_1 = 450$, $k_2 = 240$, $k_3 = 0.1$. The simulation results are shown in Fig. 14.

It can be seen from Fig. 14 a) that after changing the control parameters, the macro fluctuation range of the output power is smaller than that without changing the control parameters. Note that when the control is added at 100s, the output power will firstly fluctuate in a large range and then converge quickly. And through local amplification, we can see that there is a certain instantaneous fluctuation in the output power after convergence.

As can be seen from Fig. 14 b), the output power smoothing effect is significant at this time, and the power smoothing coefficient is greatly reduced.

It can be seen from Fig. 14 c) and Fig. 14 d) that the rotor speed and system pressure are changing within a reasonable working range, and the system is in a normal working state.

And compare the power smoothing coefficients in the above three cases, as shown in Fig. 15.

![FIGURE 14. k_1 = 450, k_2 = 240, k_3 = 0.1 system response.](image1)

![FIGURE 15. Comparison of smoothing coefficients.](image2)

It can be seen that when the controller parameters are $k_1 = 320$, $k_2 = 65$, $k_3 = 0.1$, the power smoothing coefficient is the smallest. Comparing Fig. 13 a) and Fig. 14 a), the macro fluctuation of output power in Fig. 14 a) is smaller than that in Fig. 13 a). However, there is almost no fluctuation in the instantaneous power in Fig. 13 a) in a local range, but there is a certain instantaneous fluctuation in Fig. 14 a). Because of this phenomenon, the power smoothing control in Fig. 13 a) has a better effect.

**C. COMPARISON BETWEEN DIFFERENT CONTROL STRATEGIES**

Under the conditions of the same wind speed as shown in Fig. 11 and the same HWT parameters, the power smoothing effect of the proposed control strategy, and the decoupling control method [14] are compared.

It can be seen from Fig. 16 that the ADRC control method combined with LQR designed in this paper has a significant
effect on power smoothing control. And this method greatly optimizes the power control effect on HWT.

VI. CONCLUSION

In this paper, the ADRC control method combined with LQR is used to solve the control problem of the multiplicative nonlinear system, and the power control of the HWT with rotor energy storage is realized. By establishing the ADRC control model and observing and eliminating the total disturbance caused by multiplicative nonlinearity and external wind speed disturbances, power control with essential nonlinearity and strong disturbances is achieved. However, because the conventional ADRC algorithm is difficult to control the non-minimum phase system, the combination of the LQR parameter tuning method can greatly improve the system stability and achieve the purpose of effective control of the system output power. Moreover, a parameter tuning method under high-order systems is proposed. Through simulation analysis, the system output power is effectively controlled. At the same time, under the fluctuating wind speed, the smoothness of the output power is greatly improved, and the power smoothing control effect is remarkable.

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