Nucleon resonances in $\pi N$ scattering up to energies $\sqrt{s} \leq 2.0$ GeV

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Abstract

A meson-exchange model for pion-nucleon scattering was previously constructed using a three-dimensional reduction scheme of the Bethe-Salpeter equation for a model Lagrangian involving $\pi$, $\eta$, $N$, $\Delta$, $\rho$, and $\sigma$ fields. We thereby extend our previous work by including the $\eta N$ channel and all the $\pi N$ resonances with masses $\sim 2$ GeV, up to the $F$ waves. The effects of the $\pi\pi N$ channels are taken into account by introducing an effective width in the resonance propagators. The extended model gives an excellent fit to both $\pi N$ phase shifts and inelasticity parameters in all channels up to the $F$ waves and for energies below 2 GeV. We present a new scheme to extract the properties of overlapping resonances. The predicted values for the resonance masses and widths as well as resonance pole positions and residues are compared to the listing of the Particle Data Group.

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1 Introduction

Pion-nucleon scattering is of interest because of its fundamental nature. In the 1950’s, it was widely regarded as the dynamical problem because of the special role pions and nucleons play in the family of particles [1]. Soon it became one of the main sources of information about the baryon spectrum. The pion-nucleon interaction also plays a fundamental role in the description of nuclear dynamics for which the $\pi N$ off-shell amplitude serves as the basic input to most of the existing nuclear calculations at intermediate energies. Knowledge about the off-shell $\pi N$ amplitude is also essential in interpreting the experiments performed at the intermediate-energy electron accelerators in order to unravel the internal structure of these hadrons. As an example, the importance of the $\pi N$ off-shell $t$-matrix in a dynamical description of pion electromagnetic production has been demonstrated in recent years [2, 3, 4, 5]. For further progress it is now necessary to improve our previous description of the $\pi N$ interaction and to extend it to higher energies.

It is commonly accepted that Quantum Chromodynamics (QCD) is the fundamental theory of the strong interaction. However, due to the confinement problem, it is still practically impossible to derive the $\pi N$ interaction directly from QCD. On the other hand, models based on meson-exchange pictures [6, 7] have been very successful in describing the $NN$ scattering. Over the last decade, similarly successful models have also been constructed for $\pi N$ scattering [3, 8, 9, 10, 11, 12, 13, 14, 15, 16]. Most of the recent attempts in this direction were obtained by applying various three-dimensional reductions of the Bethe-Salpeter equation, except for Ref. [12] in which the four-dimensional Bethe-Salpeter equation was solved. Because the effective Lagrangian used in these models includes only the first few low-lying resonances, in addition to pion and nucleon as well as $\sigma$ and $\rho$ mesons, the energy region is restricted to low and intermediate energies.

In previous works we constructed several meson-exchange $\pi N$ models within the Bethe-Salpeter formulation [3, 10, 14] and investigated their sensitivity with respect to various three-dimensional reduction schemes. The model Lagrangian included only $\pi, N, \Delta, \rho,$ and $\sigma$ fields, and it was found that all the resulting meson-exchange models can yield similarly good descriptions of the $\pi N$ scattering data up to 400 MeV. The model obtained with the Cooper-Jennings reduction scheme [17] was recently extended up to a c.m. energy of 2 GeV in the $S_{11}$ channel by including the $\eta N$ channel and a set of higher $S_{11}$ resonances [18]. An excellent fit to the $t$-matrix in both $\pi N$ and $\eta N$ channels was obtained. In addition, when analyzing the pion photoproduction data, we obtained background contributions to the imaginary part of the $S$-wave multipole which differ considerably from the result based on the $K$-matrix approximation. The resulting resonance contributions required to explain the pion photoproduction data led to a substantial change of the extracted electromagnetic helicity amplitudes. In the present paper we further extend our model to include the higher partial waves up to the $F$ waves. The spin-$\frac{3}{2}$
resonances are treated as Rarita-Schwinger particles while we use simple Breit-Wigner forms for the resonance propagators with spins $\frac{5}{2}$ and $\frac{7}{2}$. Since the importance of $\pi\pi N$ final states grows with energy, also these channels have to be taken care of. Instead of including them like the $\sigma N$, $\rho N$, and $\pi\Delta$ states directly in the coupled-channels calculation as done in Ref. [15], we follow the recipe of Ref. [18] to account for the $\pi\pi N$ channels by introducing a phenomenological term in the resonance propagators. It turns out that this approximation works quite well in most of the considered channels.

The question of whether a resonance be a three-quark state dressed by the meson cloud or be generated dynamically is an issue still under investigation in the literature. At one extreme, there is the conjecture [19] that baryon resonances not belonging to the large-$N_c$ ground states may be generated by coupled-channel dynamics. On the other hand, in Jülich $\pi N$ model [20, 21], it was found that only the Roper resonance $P_{11}(1440)$ can be understood in this way, while other resonances like $S_{11}(1535), S_{11}(1650)$, and $D_{13}(1520)$ had to be included in the model explicitly, in direct contrast to results of [9] where Roper resonance is included explicitly but $S_{11}(1535)$ generated dynamically. Here we take another extreme and assume all the nucleon resonances are fundamentally three-quark states dressed by coupling to meson-nucleon channels. Such a picture has been found to describe well the $\Delta(1232)$ [5, 22, 23] and $S_{11}$ resonances up to $2 GeV$ [18] in $\pi N$ scattering and pion electromagnetic production.

In Sec. II, we summarize the meson-exchange $\pi N$ model constructed in our previous work. We extend the model to include the $\eta N$ channel and the higher resonances in Sec. III. Our results are presented in Sec. IV, and some conclusions are given in Sec. V.

## 2 Meson-exchange $\pi N$ model

Let us first outline the content of our previous meson-exchange model describing the $\pi N$ interaction at low and intermediate energies [14]. The reaction of interest is

$$\pi(q) + N(p) \rightarrow \pi(q') + N(p'),$$

where $q, p, q', p'$ are the four-momenta of the respective particles. We further define the total and relative four-momentum, $P = p + q$ and $k = p\eta_{\pi}(s) - q\eta_N(s)$, respectively, where $s = P^2 = W^2$ is the Mandelstam variable. The dimensionless variables $\eta_{\pi}(s)$ and $\eta_N(s)$ represent the freedom in choosing a three-dimensional reduction, and are constrained by the condition $\eta_N + \eta_{\pi} = 1$. An often used definition for these variables is given by

$$\eta_N(s) = \frac{\varepsilon_N(s)}{\varepsilon_N(s) + \varepsilon_{\pi}(s)}, \quad \eta_{\pi}(s) = \frac{\varepsilon_{\pi}(s)}{\varepsilon_N(s) + \varepsilon_{\pi}(s)},$$

with $\varepsilon_N(s) = (s + m_N^2 - m_{\pi}^2)/2\sqrt{s}$ and $\varepsilon_{\pi}(s) = (s - m_N^2 + m_{\pi}^2)/2\sqrt{s}$. For further details we refer the reader to Ref. [14].
The Bethe-Salpeter (BS) equation for $\pi N$ scattering takes the general form

$$T_{\pi N} = B_{\pi N} + B_{\pi N}G_0T_{\pi N},$$

(3)

where $B_{\pi N}$ is the sum of all irreducible two-particle Feynman amplitudes and $G_0$ the free relativistic pion-nucleon propagator. The BS equation can be cast into the form

$$T_{\pi N} = \hat{B}_{\pi N} + \hat{B}_{\pi N}\hat{G}_0T_{\pi N},$$

(4)

with

$$\hat{B}_{\pi N} = B_{\pi N} + B_{\pi N}(G_0 - \hat{G}_0)\hat{B}_{\pi N},$$

(5)

where a three-dimensional reduction of Eq. (3) is obtained by use of an appropriate propagator $\hat{G}_0(k;P)$. It is also convenient to choose $\hat{G}_0$ such that two-body unitarity is maintained by reproducing the $\pi N$ elastic cut. There is still a wide range of possible propagators which satisfy this requirement. A standard choice of the propagator has the form [17, 24]

$$\hat{G}_0(k;P) = \frac{1}{(2\pi)^3} \int \frac{ds'}{s - s'} f(s, s')\left[\alpha(s, s')P + \hat{k} + m_N\right]$$

$$\times \delta^{(+)}([\eta_N(s')P + \hat{k}]^2 - m_N^2)\delta^{(+)}([\eta_N(s')P - \hat{k}]^2 - m_N^2),$$

(6)

with $P' = \sqrt{\frac{s}{s'}}P$. The superscript $(+)$ associated with $\delta$-functions signifies that only the positive energy part is kept in the propagator. Concerning the Dirac matrices and the Lorentz metrics we use the notation of Bjorken and Drell [25]. Furthermore, the variables $f$ and $\alpha$ are dimensionless variables containing the freedom of reduction, they are constrained by the conditions $f(s, s) = 1$ and $\alpha(s, s) = \eta_N(s)$, which ensure the reproduction of the elastic cut. In the Cooper-Jennings reduction scheme [17] they take the form

$$\alpha(s, s') = \eta_N(s), \quad f(s, s') = \frac{4\sqrt{ss'}\varepsilon_N(s')\varepsilon_\pi(s')}{ss' - (m_N^2 - m_\pi^2)^2}.$$  

(7)

The integral over $s'$ in Eq. (6) can be performed. Expressed in the c.m. frame, the result is

$$\hat{G}_0(k;\bar{k}) = \frac{1}{(2\pi)^3} \int \frac{\delta(k_0 - \varepsilon(k, \bar{k}))}{\sqrt{s} - \sqrt{\bar{s}}} \frac{2\sqrt{s}}{\sqrt{s} + \sqrt{\bar{s}}} f(s, s_k)\frac{\alpha(s, s_k)\gamma_0\sqrt{s} + \hat{k} + m_N}{4E_N(\bar{k})E_\pi(\bar{k})},$$

(8)

where $E_N(\bar{k})$ and $E_\pi(\bar{k})$ are the nucleon and pion energies for the three-momentum $\bar{k}$, $\sqrt{\bar{s}} = E_N(\bar{k}) + E_\pi(\bar{k}) = E$ is the total energy in the c.m. frame, and $\varepsilon(s, k) = E_N(\bar{k}) - \eta_N(s_k)\sqrt{\bar{s}}$. By use of these relations we obtain the following $\pi N$ scattering equation:

$$t(\bar{k};\bar{k}; E) = v(\bar{k}, \bar{k}; E) + \int d\hat{\bar{k}}'v(\bar{k}, \bar{k}'; E)g_0(\bar{k}'; E)t(\bar{k}'; \bar{k}; E).$$

(9)

The explicit relations between the variables of Eqs. (9) and (3) are
\[ t(\vec{k}', \vec{k}; E) = \int dk'_0 dk_0 \delta(k'_0 - \eta') T(k', k; E) \delta(k_0 - \eta), \]

\[ v(\vec{k}', \vec{k}; E) = \int dk'_0 dk_0 \delta(k'_0 - \eta') B(k', k; E) \delta(k_0 - \eta), \]

\[ g_0(\vec{k}; E) = \int dk_0 \hat{G}_0(k; E), \]

(10)

with \( \eta' = \tilde{\eta}(s_{\vec{k}'}, \vec{k}') \) and \( \eta = \tilde{\eta}(s_{\vec{k}}, \vec{k}) \).

Because our previous work concentrated on the \( \pi N \) scattering process at low and intermediate energies, we only considered the degrees of freedom due to the \( \pi, N, \sigma, \rho, \) and \( \Delta(1232) \) fields, and approximated the sum of all irreducible two-particle Feynman amplitudes, \( B(k', k; E) \) in Eq. (10), by the tree approximation of the following interaction Lagrangian:

\[ L_I = \frac{f^{(0)}_{\pi NN}}{m_\pi} \bar{N} \gamma_\mu \vec{\pi} \cdot \partial^\mu \bar{N} \sigma - g_{\sigma NN} m_\pi \sigma(\vec{\pi} \cdot \bar{\pi}) - \frac{g^{(v)}_{\pi NN}}{2m_\pi} \sigma(\partial^\mu \bar{\pi} \cdot \partial_\mu \bar{\pi}) - g_{\sigma NN} \bar{N} \sigma N \]

\[ - g_{\rho NN} \bar{N} \{ \gamma_\mu \bar{\rho}^\mu + \frac{\kappa_\rho}{4m_N} \sigma_{\mu\nu}(\partial^\mu \bar{\rho}^\nu - \partial^\nu \bar{\rho}^\mu) \} \cdot \frac{1}{2} \bar{\pi} N \]

\[ - g_{\rho NN} \bar{N} \{ (\bar{\pi} \times \bar{\sigma}_\mu \bar{\pi}) - \frac{g_{\rho NN}}{4m_\rho^2}(\delta - 1)(\partial^\mu \bar{\rho}^\nu - \partial^\nu \bar{\rho}^\mu) \cdot (\bar{\partial}_\mu \bar{\pi} \times \bar{\partial}_\nu \bar{\pi}) \}

\[ + \frac{g_{\pi N \Delta}}{m_\pi} \Delta_\mu [g^{\mu\nu} - (Z + \frac{1}{2})\gamma_\mu \gamma_\nu] \bar{T}^{\Delta}_\Delta N \cdot \partial_\mu \bar{\pi}, \]

with \( \Delta_\mu \) the Rarita-Schwinger field operator for the \( \Delta \) resonance and \( \bar{T}^{\Delta}_\Delta N \) the isospin transition operator between the nucleon and the \( \Delta \). The resulting driving term consists of the direct and crossed \( N \) and \( \Delta \) diagrams as well as the t-channel \( \sigma \)- and \( \rho \)-exchange contributions.

The procedure of Afnan and collaborators [27] was followed to constrain the \( P_{11} \) phase shift by imposing the nucleon pole condition. This treatment leads to a proper renormalization of both nucleon mass and \( \pi NN \) coupling constant. It also yields the important cancelation between the repulsive nucleon pole contribution and the attractive background, such that a reasonable fit to the \( \pi N \) phase shifts in the \( P_{11} \) channel can be achieved.

To complete the model we further introduced form factors to regularize the driving term \( v(\vec{k}, \vec{k}') \) of Eq. (10). For this purpose covariant form factors of the form

\[ F(p^2) = \left[ \frac{n\Lambda^4}{n\Lambda^4 + (m^2 - p^2)^2} \right]^n, \]

(12)

are associated with each legs of the vertices, where \( p \) is the four-momentum and \( m \) the mass of the respective particle. This parameterization is similar to the prescription of Ref. [8] and in Ref. [14] both \( n = 10 \) and 2 were considered. However, in our previous work we used the value \( n = 10 \) [18].
The parameters that were allowed to vary in fitting the empirical phase shifts are the products \( g_{\sigma NN} \), \( g_{\sigma NN}^{(s)} \), and \( g_{\rho NN} \) as well as \( \delta \) for the t-channel \( \sigma \) and \( \rho \) exchanges, \( m_{\Delta}^{(0)} \), \( g_{\pi NN}^{(0)} \), and \( Z \) for the \( \Delta \) mechanism, and the cut-off parameters \( \Lambda \) of the form factors given by Eq. (12). In the crossed \( N \) diagram, the physical \( \pi NN \) coupling constant is used. For the crossed \( \Delta \) diagram, the situation is not so clear since the determination of the "physical" \( \pi N\Delta \) coupling constant depends on the non-resonant contribution in the \( P_{33} \) channel. In principle, it can be determined by carrying out a renormalization procedure similar to that used for the nucleon. However, this would require a much more difficult numerical task, because the \( \Delta \) pole is complex. In accordance with Refs. [3, 8, 9], we therefore did not carry out such a renormalization for the \( \Delta \) but simply determined the coupling constant in the crossed \( \Delta \) diagram by a fit to the data. The resulting coupling constant was denoted as \( g_{\pi N\Delta} \).

3 Extension to higher energies: inclusion of the \( \eta N \) channel and higher resonances

As the energy increases, two-pion channels like \( \sigma N, \eta N, \pi\Delta, \rho N \) as well as a non-resonant continuum of \( \pi\pi N \) states become increasingly important, and at the same time more and more nucleon resonances appear as intermediate states. The \( \pi N \) model described in Sec. 2 was therefore extended for the \( S_{11} \) partial wave by explicitly coupling the \( \pi, \eta \) and \( \pi\pi \) channels and including the couplings with higher baryon resonances [18]. In particular, in the case of only one contributing resonance \( R \), the Hilbert space was enlarged by the inclusion of a bare \( S_{11} \) resonance \( R \) which acquires a width by its coupling with the \( \pi N \) and \( \eta N \) channels through the Lagrangian

\[
\mathcal{L}_I = i g_{\pi NN}^{(0)} \bar{R} \tau N \cdot \pi + i g_{\eta NN}^{(0)} \bar{R} \eta N + h.c.,
\]

(13)

where \( N, R, \pi, \) and \( \eta \) denote the field operators for the nucleon, bare resonance \( R \), pion, and eta meson, respectively. The full \( t \)-matrix can be written as a system of coupled equations,

\[
t_{ij}(E) = v_{ij}(E) + \sum_k v_{ik}(E) g_k(E) t_{kj}(E),
\]

(14)

with \( i \) and \( j \) denoting the \( \pi \) and \( \eta \) channels and \( E = W \) is the total c.m. energy.

In general, the potential \( v_{ij} \) is the sum of non-resonant \( (v_{ij}^B) \) and bare resonance \( (v_{ij}^R) \) terms,

\[
v_{ij}(E) = v_{ij}^B(E) + v_{ij}^R(E).
\]

(15)

The non-resonant term \( v_{\pi}^B \) for the \( \pi N \) elastic channel is given by the results of Sec. 2 and contains contributions from the \( s \)- and \( u \)-channels, Born terms and \( t \)-channel contributions with \( \omega, \rho, \) and \( \sigma \) exchange. The parameters in \( v_{\pi\pi}^B \) are fixed from the analysis of the pion scattering phase shifts for the \( s \)- and \( p \)-waves at low energies \( (W < 1300 \text{ MeV}) \) [14]. In channels involving the \( \eta \), the potential \( v_{\eta N}^B \) is taken to be zero because the \( \eta NN \) coupling is very small [26].
The bare resonance contribution arises from the excitation and de-excitation of the resonance $R$,

$$v_{ij}^R(E) = \frac{h_{ij}^{(0)}}{E - M_R^{(0)}},$$

(16)

where $h_{ij}^{(0)}$ and $M_R^{(0)}$ denote the bare vertex operator for $R \to \pi/\eta + N$ and the bare mass of the resonance $R$, respectively. The matrix elements of the potential $v_{ij}^R(E)$ can be symbolically expressed in the form

$$v_{ij}^R(q, q'; E) = \frac{f_i(\hat{\Lambda}_i, q; E) g_j^{(0)}(\hat{\Lambda}_j, q'; E)}{E - M_R^{(0)} + \frac{1}{2} \Gamma_{2\pi}^R(E)},$$

(17)

where $q$ and $q'$ are the pion (or eta) momenta in the initial and final states, and $g_j^{(0)}$ is the resonance vertex couplings. As in [14], we associate with each external line of the particle $\alpha$ in a $\pi N R$ vertex a covariant form factor $F_\alpha = \left[ n \Lambda_\alpha^4 / (n \Lambda_\alpha^4 + (p_\alpha^2 - m_\alpha^2)^2) \right]^n$, where $p_\alpha$, $m_\alpha$, and $\Lambda_\alpha$ are the four-momentum, mass, and cut-off parameter of particle $\alpha$, respectively, and $n = 10$. As a result, $f_i$ depends on the product of three cut-off parameters, i.e., $\hat{\Lambda}_\pi \equiv (\Lambda_N, \Lambda_R, \Lambda_\pi)$.

In Eq. (17) we have included a phenomenological term $\Gamma_{2\pi}^R(E)$ in the resonance propagator to account for the $\pi\pi N$ decay channel. Therefore, our "bare" resonance propagator already contains some renormalization or "dressing" effects due to the coupling with the $\pi\pi N$ channel. With this prescription we assume that any further non-resonant coupling mechanism with the $\pi\pi N$ channel is small. Following Refs. [28, 29] we take

$$\Gamma_{2\pi}^R(E) = \Gamma_{2\pi}^{(0)} \frac{q_{2\pi}}{q_0} \left( \frac{X_R^2 + q_0^2}{X_R^2 + q_{2\pi}^2} \right)^{l+2},$$

(18)

where $l$ is the pion orbital momentum, $q_{2\pi} = q_{2\pi}(E)$ the momentum of the compound two-pion system, $q_0 = q_{2\pi}(E = M_R^{(0)})$ and the quantity $\Gamma_{2\pi}^{(0)}$ is the $2\pi$ decay width at resonance. We note that this form accounts for the correct energy behavior of the phase space near the three-body threshold [28]. In our present work, $\Gamma_{2\pi}^{(0)}$ and $X_R$ are considered as a free parameters. As a result, one isolated resonance will in general contain six free parameters, the bare mass $M_R^{(0)}$, the decay width $\Gamma_{2\pi}^{(0)}$, two bare coupling constants $g_i^{(0)}$ and $g_j^{(0)}$, and two cut-off parameters $\Lambda_R$ and $X_R$. The generalization of the coupled channels model to the case of $N$ resonances with the same quantum numbers is then given by

$$v_{ij}^R(q, q'; E) = \sum_{n=1}^N v_{ij}^{R_n}(q, q'; E),$$

(19)

with free parameters for the bare masses, widths, coupling constants, and cut-off parameters for each resonance.
Having solved the coupled channel equations, our next task is the extraction of the physical (or "dressed") masses, partial widths, and branching ratios of the resonances. It is known that this procedure is model dependent because the background and the resonance contributions cannot be separated in a unique way. Of course, the solution to this problem becomes more and more difficult with an increasing number of overlapping resonances in the same channel. In the literature, there are two schemes used to separate the total $t-$matrix into background and resonance contributions. For simplicity, we illustrate these two methods for the uncoupled channel case, i.e., assuming that only the $\pi N$ channel is open. In this case the potential operator describing the excitation of a bare resonance $R$ takes the form

$$v_{\pi N}^R(E) = \frac{h_{\pi R}(0)^\dagger h_{\pi R}(0)}{E - M_R^{(0)}},$$

with $h_{\pi R}(0)$ the bare vertex.

The first scheme was suggested by Afnan and collaborators [30] and recently used in the dynamical model calculation of pion scattering and pion photoproduction [4]. By use of the two-potential formulation the $t-$matrix is written as

$$t_{\pi N}(E) = \tilde{t}^B_{\pi N}(E) + \tilde{t}^R_{\pi N}(E),$$

where $\tilde{t}^B_{\pi N}(E)$ is defined as

$$\tilde{t}^B_{\pi N}(E) = v_{\pi N}^B + v_{\pi N}^B g_0(E) \tilde{t}^B_{\pi N}(E).$$

We will call $\tilde{t}^B_{\pi N}(E)$ the "non-resonant" background because it does not contain any resonance contribution from $v_{\pi N}^R$ of Eq. (20). The resonance term $\tilde{t}^R_{\pi N}(E)$ takes the form

$$\tilde{t}^R_{\pi N}(E) = \bar{h}_{\pi R}(E) \frac{1}{E - M_R^{(0)} - \Sigma_R(E)} h_{\pi R}(E),$$

with the definitions

$$h_{\pi R}(E) = h_{\pi R}(0) + h_{\pi R}(0) g_0(E) \tilde{t}^B_{\pi N}(E),$$

$$\bar{h}_{\pi R}(E) = h_{\pi R}(0)^\dagger + \tilde{t}^B_{\pi N}(E) g_0(E) h_{\pi R}(0)^\dagger,$$

and the self-energy $\Sigma_R(E)$ given by

$$\Sigma_R(E) = h_{\pi R}(0)^\dagger g_0 h_{\pi R}(E) = h_{\pi R}(0)^\dagger h_{\pi R}(0) g_0 + h_{\pi R}(0)^\dagger \tilde{t}^B_{\pi N}(E) g_0 h_{\pi R}(0)^\dagger.$$

Graphical representations of the dressed vertex $h_{\pi R}(E)$ and the self-energy $\Sigma_R(E)$ are depicted in Figs. 1 and 2, respectively, where the solid circle on the l.h.s. of Fig. 1 denotes the dressed vertex $h_{\pi R}$ while the $\pi NR$ vertices, $h_{\pi R}^{(0)}$, on the r.h.s. of Fig. 1 correspond to the excitation
of a bare resonance $R$. The small solid circles in Figs. 1 and 2 represent the "non-resonant" background $\tilde{t}_{\pi N}^R(E)$ as defined in Eq. 22.

The information about the physical mass and the total width of the resonance $R$ are contained in the dressed resonance propagator given in Eq. (23). The complex self-energy $\Sigma(E)$ leads to a shift from the real "bare" mass to a complex and energy-dependent value. However, we characterize the resonances by energy-independent parameters that are obtained by solving the equation

$$E - M_R^{(0)} - \text{Re}\Sigma_R(E) = 0.$$  

(27)

The solution of this equation, $E = M_R$, corresponds to the energy at which the dressed propagator in Eq. (23) becomes purely imaginary and is used to define the "physical" or "dressed" mass,

$$M_R = M_R^{(0)} + \text{Re}\Sigma_R(M_R),$$

(28)

and the width of the resonance

$$\Gamma_R(M_R) = -2\text{Im}\Sigma_R(M_R).$$

(29)

We hasten to add that in some of the literature the resonance mass is defined by the energy at which the phase of the resonance contribution $\tilde{t}_{\pi N}^R(E)$ passes through 90°. Because of the non-resonant background, however, the numerator of $\tilde{t}_{\pi N}^R$ becomes a complex number with the phase $2\delta^B$. Therefore, the position of the resonance and consequently the total width will now differ from our Eqs. (28) and (29). We believe that the physical masses and widths defined
in Eqs. (27)-(29) are more closely related to what one would eventually calculate in lattice QCD.

All of the above equations are based on the two-potential formulation [31]. The extension of this method to the case of several overlapping resonances in the same partial channel $\alpha$ complicates the problem. In particular, we can not express the $t$–matrix as a simple sum of a smooth background and overlapping resonances,

$$t_{\pi N}(E) \neq \tilde{t}^{B}_{\pi N}(E) + \sum_{i=1}^{N} \tilde{t}^{R}_{i}(E).$$

(30)

We therefore prefer to separate the resonance and background contributions in the framework of Refs. [5, 18]. In this approach the full pion-nucleon scattering matrix is decomposed as follows:

$$t_{\pi N}(E) = t^{B}_{\pi N}(E) + t^{R}_{\pi N}(E),$$

(31)

where

$$t^{B}_{\pi N}(E) = v^{B}_{\pi N} + v^{B}_{\pi N} g_{0}(E) t_{\pi N}(E),$$

(32)

$$t^{R}_{\pi N}(E) = v^{R}_{\pi N} + v^{R}_{\pi N} g_{0}(E) t_{\pi N}(E).$$

(33)

Comparing $t^{B}_{\pi N}$ with $\tilde{t}^{B}_{\gamma \pi}$ of Eq. (22), the "background" $t^{B}_{\pi N}$ now includes contributions not only from the background rescattering but also from intermediate resonance excitation. This is compensated by the fact that the resonance contribution $t^{R}_{\pi N}$ now contains only the terms that start with the bare resonance excitation. Expressed in terms of self-energy and vertex functions, we obtain the result

$$t^{R}_{\pi N}(E) = \frac{\bar{h}_{\pi R}(E) h^{(0)}_{\pi R}}{E - M^{(0)}_{R}(E) - \Sigma_{R}(E)},$$

(34)

which differs from Eq. (23) where dressed vertex appears in both the initial and final states. On the other hand, we note that the resonance propagators of the two approaches are identical. Therefore, the physical masses and total widths determined in the two methods will be the same.

The second method can be easily extended to the case of $N$ overlapping resonances by the following decomposition of the full $\pi N$ scattering matrix into background and resonance contributions,

$$t_{\pi N}(E) = t^{B}_{\pi N}(E) + \sum_{i=1}^{N} t^{R}_{i}(E).$$

(35)

The contribution from each resonance $R_{i}$ can be expressed in terms of the bare $h^{(0)}_{\pi R_{i}}$ and dressed $h_{\pi R_{i}}(E)$ vertex operators as well as the resonance self energy derived from one-pion $\Sigma_{R_{i}}^{1\pi}(E)$ and two-pion $\Sigma_{R_{i}}^{2\pi}(E)$ channels, that is

$$t^{R}_{i}(E) = \frac{\bar{h}_{\pi R_{i}}(E) h^{(0)}_{\pi R_{i}}}{E - M^{(0)}_{R_{i}} - \Sigma_{R_{i}}^{1\pi}(E) - \Sigma_{R_{i}}^{2\pi}(E)},$$

(36)
where $M_{R_i}^{(0)}$ is the bare mass of the $i$-th resonance. The contributions from the two-pion channel, $\Sigma^{2\pi}_{R_i}$, is defined phenomenologically as in Eq. (18). The vertices for the resonance excitation are obtained from the following equations,

$$h_{\pi R_i}(E) = h_{\pi R_i}^{(0)} + h_{\pi R_i}^{(0)} g_0(E) t_{\pi N}^{B_i}(E),$$

$$\bar{h}_{\pi R_i}(E) = h_{\pi R_i}^{(0)\dagger} + t_{\pi N}^{B_i}(E) g_0(E) h_{\pi R_i}^{(0)\dagger},$$

where

$$t_{\pi N}^{B_i}(E) = v_i(E) + v_i(E) g_0(E) t_{\pi N}^{B_i}(E),$$

$$v_i(E) = v_{\pi N}^B + \sum_{j \neq i} v_{\pi N}^{R_j}(E),$$

with $v_{\pi N}^{R_j}(E)$ arising from the excitation of the resonance $R_j$ as given in Eq. (17). The one-pion self-energies arising from $t_{\pi N}^{B_i}(E)$ of Eq. (39) are given as

$$\Sigma^{1\pi}_{R_i}(E) = h_{\pi R_i}^{(0)} g_0 \bar{h}_{\pi R_i}(E) = h_{\pi R_i}^{(0)} g_0 h_{\pi R_i}^{(0)\dagger} + h_{\pi R_i}^{(0)} g_0 t_{\pi N}^{B_i} g_0 h_{\pi R_i}^{(0)\dagger},$$

and the one-pion branching ratio at the dressed resonance is

$$\beta^{1\pi}_i = \frac{\Sigma^{1\pi}_{R_i}(M_R)}{\Sigma^{1\pi}_{R_i}(M_R) + \Sigma^{2\pi}_{R_i}(M_R)}.$$

The pole positions in the complex energy plane and the complex residues of the scattering amplitudes at these poles are calculated using the speed plot technique for the pion-nucleon partial waves. For details see Refs. [32, 33].

Finally, it is not difficult to see from Eqs. (37) and (38) that the matrix elements of both $h_{\pi R_i}$ and $t_{\pi N}^{B_i}(E)$ would have the same phase $\phi_{R_i}(E)$, i.e.,

$$< h_{\pi R_i}(E) > = | < h_{\pi R_i}(E) > | \exp(i\phi_{R_i}), \quad t_{\pi N}^{B_i}(E) = | < t_{\pi N}^{B_i}(E) > | \exp(i\phi_{R_i}),$$

where the parenthesis $<>$ is used to denote matrix element of the operator sandwiched between same set of initial and final states. It then follows that also the numerator on the r.h.s. of Eq. (36) carries the phase $\phi_{R_i}(E)$. Information about this phase is very important for the phenomenological Breit-Wigner parametrization of the resonance contributions.

We emphasize that in the formulation of Eqs. (35-41), the nucleon resonances are treated in a completely symmetrical way. In addition, the self-energy and the dressing of any resonance receive contributions from all other resonances.

4 Results and discussion

4.1 $\pi N$ scattering amplitudes

With the extended meson exchange model described in Sec. 3, we have fitted the $\pi N$ phase shifts and inelasticity parameters in all channels up to the $F$-waves and for energies less than 2 GeV.
The results for the real and imaginary parts of the partial wave amplitudes $t_{\pi N}$ are shown in Figs. 3-6. The solid lines are the best fit within our model, while the dashed lines correspond to the non-resonant background $\tilde{t}_{\pi N}^R$ of Eq. (22). The open circles represent the data according to Ref. [35]. These figures show an excellent description for both the real and the imaginary parts of the pion-nucleon scattering amplitudes in all cases except for the $D_{35}$ and $F_{17}$ channels. For the $D_{35}$, our problem lies mostly within the real part as seen in Fig. 5. In the case of the $F_{17}$ in Fig. 6, the inclusion of further resonances does not improve on the non-resonant background shown by the data, neither for the real nor for the imaginary part of the scattering amplitude.

4.2 Resonance parameters

Let us now look at the resonance parameters whose determination was one of the main issues of our investigation. Before going into details by comparing with the PDG values, we point out that our data analysis requires four very broad resonances, $S_{11}$ (1878), $D_{13}$ (2152), $P_{13}$ (2204), and $P_{31}$ (2100), states that are not in the current listing of the PDG [36]. Furthermore, we can not remove the discrepancy between the background contributions and the data in the $F_{17}$ channel by adding the $F_{17}(1990)$-resonance listed by the PDG, which is in line with the results of the SAID analysis [35].

The physical mass $M_R$, total width $\Gamma_R$, single-pion branching ratio $\beta_{1\pi}^R$, and background phase $\phi_R$ defined for each overlapping nucleon resonance $R$ have been determined from Eqs. (28-29) and (42-43). The results are presented in Tables 1 and 2 for the isospin-$\frac{1}{2}$ and isospin-$\frac{3}{2}$ resonances, respectively. Using the speed-plot technique, we have also calculated the resonance pole positions in the complex energy plane and the complex residues at these poles. The results are listed in Tables 3 and 4 for the isospin-$\frac{1}{2}$ and isospin-$\frac{3}{2}$ resonances, respectively. In Tables 1-4 we also compare our results to the listings of the PDG.

A word of caution is necessary when comparing the resonance parameters obtained in our present work with the PDG values. In many investigations the resonance mass is defined as the energy at which the phase of the resonance contribution, $\tilde{t}_{\pi N}^R$ given by Eq. (23), takes the value $\pi/2$. However, in our present work we define the resonance position as the energy at which the phase of the denominator in Eq. (23) runs through 90°, i.e., as the solution of $(E - M_R^{(0)} - \Sigma_R(E)) = 0$ given in Eq. (27). In our opinion this definition has a better physical interpretation.

S-waves

As reported in Ref. [18], we need four $S_{11}$ resonances to fit the $\pi N$ scattering amplitude in this channel, instead of the three resonances listed by the PDG. The additional resonance $S_{11}(1878)$ was found to play an important role in pion photoproduction as well [18], but was not seen in both the $\pi N \rightarrow \eta N$ reaction and recent measurements of $\eta$ photoproduction from the pro-
ton [39]. There also arise some differences in the resonance parameters between our present results and those given by Ref. [18] because of the different definitions for the resonance masses and widths explained in the previous text. It turns out that the choice of these definitions has little effect on the extracted masses of all four $S_{11}$ resonances. However, the extracted widths for the first and third resonances depend very much on the definitions, which leads to an increase of the width with regard to earlier work, from 90 MeV to 130 MeV and from 265 MeV to 508 MeV, respectively. Our results obtained for the pole position via the speed plot technique generally agree with the PDG values for the real parts of the pole positions. However, we obtain much smaller values for the imaginary parts of the pole positions and also for the residues at the pole.

For the isospin-3/2 channel, our extracted masses and widths differ from the PDG values by more than 100 MeV, all except the first resonance $S_{31}$ (1620). The values obtained for the pole positions agree with the PDG values for the lower resonances. However, the imaginary part of the pole position for the $S_{31}$ (1900) and also its residue at the pole comes out very small.

**P-waves**

For P-waves with isospin-1/2, our results are in good agreement with the PDG values regarding both pole positions and residues. However, the extracted widths are much larger than the corresponding PDG values. We also need an extra resonance $P_{13}(2204)$ in order to fit the scattering amplitude in this channel.

For the isospin-3/2 resonance $P_{31}$ (1750) and $P_{33}(1920)$, we extract widths of about 500 MeV and 800 MeV, respectively, both very much above the PDG values. On the other hand, the residue and the imaginary part of the pole position for $P_{31}$ (1750) comes out much below the PDG listings.

**D-waves**

For both isospin-1/2 and 3/2 D-waves our resonance parameters generally agree with the PDG values, except for the fact that we do not find a pole corresponding to the $D_{13}$ (1700).

**F-waves**

Besides the fact that we can not describe the $F_{17}$ channel, our results for the $F$–wave resonance parameters are in good agreement with the PDG listings.

5 **Summary and conclusion**

In earlier work we constructed a meson-exchange model for the $\pi N$ interaction which describes the $\pi N$ elastic scattering data up to pion laboratory energy of 400 MeV [10, 14]. Our approach was based on a three-dimensional reduction scheme of the Bethe-Salpeter equation for a model Lagrangian involving $\pi, N, \Delta, \sigma,$ and $\rho$ fields. This model was later extended to ener-
gies up to 2 GeV in the $S_{11}$ channel by explicitly including the $\eta N$ channel and several higher resonances [18]. The influence of the $2\pi$ channels was accounted for by adding a phenomenological term in the resonance propagator. Good agreement was obtained with the data from the $\pi N \rightarrow \eta N$ reaction and pion photoproduction.

In the present work, the hadron-exchange coupled channels model has been further extended to energies of 2 GeV and partial wave channels including the $F-$waves. We have assumed that all the resonances observed in $\pi N$ scattering are fundamentally three-quark states dressed by the coupling to the meson-nucleon continuum. Based on such a scheme, we are able to achieve a very good description of the $\pi N$ elastic scattering amplitudes in all the partial-waves and over the energy range up to 2 GeV, except for the $F_{17}$ channel. However, the fit to the data requires four additional resonances with very large widths, $S_{11}(1878), D_{13}(2152), P_{13}(2204)$, and $P_{31}(2100)$, which are not listed by the PDG [36].

We have developed a scheme to extract the parameters of overlapping resonances in a completely symmetrical way with respect to the resonances. This scheme allows us to include the dressing of each particular resonance due to all the other resonances in the same channel. We have chosen to define the resonance energy such that the effect of vertex dressing is not included in the self-energy of a resonance, contrary to many previous investigations. Furthermore, the pole positions and the residues of the scattering amplitudes at the pole have been determined by means of the speed-plot technique. The comparison of the extracted resonance parameters with the PDG values yields a qualitative agreement in general but considerable discrepancies in some cases, in particular for the widths and residues of some higher resonances. Further investigations will be necessary to understand these differences in detail.

The $\pi N$ model developed in this work will be used to study the meson cloud effects on the electromagnetic transition form factors of the higher resonances. It will also allow us to extract the helicity amplitudes of all resonances in a more consistent and reliable way.

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Figure 3: The real and imaginary parts of $t_{\pi N}$ in the S-waves as function of the total c.m. energy $W$. The solid (red) lines are the best fits of our meson-exchange model, the dashed (blue) lines correspond to the contributions of non-resonant background $\tilde{t}_B$ of Eq. (22). The open circles are the results of partial waves analysis of Ref. [35].
Figure 4: The real and imaginary parts of $t_{\pi N}$ in the P-waves as function of the total c.m. energy $W$. Notation as in Fig. 3.
Figure 5: The real and imaginary parts of $t_{\pi N}$ in the D-waves as function of the total c.m. energy $W$. Notation as in Fig. 3.
Figure 6: The real and imaginary parts of $t_{\pi N}$ in the F-waves as function of the total c.m. energy $W$. Notation as in Fig. 3.
Table 1: Bare ($M_R(0)$) and physical ($M_R$) resonance masses as well as total widths $\Gamma_R$, all in units of MeV, single pion branching ratios $\beta_1^{1\pi}$, and background phases $\phi_R$ of Eq. (43) for isospin-1/2 resonances. Upper lines: our results, lower lines: results of the PDG [36].

| $N^*$       | $M_R(0)$  | $M_R$     | $\Gamma_R$ | $\beta_1^{1\pi}$ (%) | $\phi_R$(deg) |
|------------|-----------|-----------|------------|----------------------|---------------|
| $P_{11}(1440)$  | 1612      | 1418      | 436        | 32                   |
| $\star\star\star$ | $1445 \pm 25$ | $325 \pm 125$ | $65 \pm 10$ |                     |
| $D_{13}(1520)$  | 1590      | 1520      | 94         | 62                   |
| $\star\star\star$ | $1520 \pm 5$ | $115 \pm 15$ | $60 \pm 15$ |                     |
| $S_{11}(1535)$  | 1559      | 1520      | 130        | 43                   |
| $\star\star\star$ | $1535 \pm 10$ | $150 \pm 25$ | $45 \pm 10$ |                     |
| $S_{11}(1650)$  | 1727      | 1678      | 200        | 73                   |
| $\star\star\star$ | $1655 \pm 10$ | $165 \pm 20$ | $77 \pm 17$ |                     |
| $D_{15}(1675)$  | 1710      | 1670      | 154        | 49                   |
| $\star\star\star$ | $1675 \pm 5$ | $147 \pm 17$ | $40 \pm 5$  |                     |
| $F_{15}(1680)$  | 1748      | 1687      | 156        | 7.9                  |
| $\star\star\star$ | $1685 \pm 5$ | $130 \pm 10$ | $67 \pm 2$  |                     |
| $D_{13}(1700)$  | 1753      | 1747      | 156        | 5                    |
| $\star\star\star$ | $1700 \pm 50$ | $100 \pm 50$ | $10 \pm 5$  |                     |
| $P_{11}(1710)$  | 1798      | 1803      | 508        | 40                   |
| $\star\star\star$ | $1710 \pm 30$ | $180 \pm 100$ | $15 \pm 5$  |                     |
| $P_{13}(1720)$  | 1725      | 1711      | 278        | 18                   |
| $\star\star\star$ | $1725 \pm 25$ | $225 \pm 75$ | $15 \pm 5$  |                     |
| $P_{13}(1900)$  | 1922      | 1861      | 1000       | -3.5                 |
| $\star\star\star$ | $1879 \pm 17$ | $498 \pm 78$ | $26 \pm 6$  |                     |
| $P_{15}(2000)$  | 1928      | 1926      | 58         | 18                   |
| $\star\star\star$ | $1903 \pm 87$ | $490 \pm 310$ | $8 \pm 5$   |                     |
| $D_{13}(2080)$  | 1972      | 1946      | 494        | 5                    |
| $\star\star\star$ | $1804 \pm 55$ | $450 \pm 185$ | $\sim 4$   |                     |
| $S_{11}(xxx)$   | 1803      | 1878      | 508        | 41                   |
| $\star\star\star$ | $2180 \pm 80$ | $350 \pm 100$ | $18 \pm 8$  |                     |
| $P_{11}(2100)$  | 2196      | 2247      | 1020       | 32                   |
| $\star\star\star$ | $2125 \pm 75$ | $260 \pm 100$ | $12 \pm 2$  |                     |
| $D_{13}(xxx)$   | 2162      | 2152      | 292        | 14                   |
| $\star\star\star\star$ | $2125 \pm 75$ | $260 \pm 100$ | $12 \pm 2$  |                     |
| $P_{13}(xxx)$   | 2220      | 2204      | 406        | 15                   |
| $D_{15}(2200)$  | 2300      | 2286      | 532        | 16                   |
| $\star\star\star\star$ | $2180 \pm 80$ | $400 \pm 100$ | $10 \pm 3$  |                     |
Table 2: Bare ($M_R^{(0)}$) and physical ($M_R$) resonance masses as well as total widths $\Gamma_R$, all in units of MeV, single pion branching ratios $\beta_R^{1\pi}$, and phases $\phi_R$ for isospin-3/2 resonances. Notation as in Table 1.

| $N^*$ | $M_R^{(0)}$ | $M_R$  | $\Gamma_R$ | $\beta_R^{1\pi}$ (%) | $\phi_R$(deg) |
|-------|-------------|---------|------------|-----------------------|---------------|
| $P_{33}(1232)$ | 1425 | 1233 | 132 | 100 | 12 |
| **  | 1232 ± 1 | 118 ± 2 | 100 |
| $P_{33}(1600)$ | 1575 | 1562 | 216 | 6 | -9 |
| **  | 1600 ± 100 | 350 ± 100 | 17 ± 7 |
| $S_{31}(1620)$ | 1654 | 1616 | 160 | 32 | -41 |
| **  | 1630 ± 30 | 142 ± 18 | 25 ± 5 |
| $D_{33}(1700)$ | 1690 | 1650 | 260 | 15 | -5 |
| **  | 1710 ± 40 | 300 ± 100 | 15 ± 5 |
| $P_{31}(1750)$ | 1765 | 1746 | 554 | 4 | -24 |
| *   | 1744 ± 36 | 300 ± 120 | 8 ± 3 |
| $S_{31}(1900)$ | 1796 | 1770 | 430 | 8 | -44 |
| **  | 1900 ± 50 | 190 ± 50 | 2 ± 1 |
| $F_{35}(1905)$ | 1891 | 1854 | 534 | 11 | -12 |
| **  | 1890 ± 25 | 335 ± 65 | 12 ± 3 |
| $P_{31}(1910)$ | 1953 | 1937 | 226 | 14 | -21 |
| **  | 1895 ± 25 | 230 ± 40 | 22 ± 7 |
| $P_{33}(1920)$ | 1856 | 1827 | 834 | 12 | 3 |
| **  | 1935 ± 35 | 220 ± 70 | 12 ± 7 |
| $D_{35}(1930)$ | 2100 | 2068 | 426 | 15 | -20 |
| **  | 1960 ± 60 | 360 ± 140 | 10 ± 5 |
| $D_{33}(1940)$ | 2100 | 2092 | 310 | 6 | -10 |
| *   | 2057 ± 110 | 460 ± 320 | 18 ± 12 |
| $F_{37}(1950)$ | 1974 | 1916 | 338 | 47 | 13 |
| **  | 1932 ± 17 | 285 ± 50 | 40 ± 5 |
| $F_{35}(2000)$ | 2277 | 2260 | 356 | 11 | -26 |
| **  | 2200 ± 125 | 400 ± 125 | 16 ± 5 |
| $P_{31}(xxx)$ | 2160 | 2100 | 492 | 35 | -25 |
| $S_{31}(2150)$ | 2118 | 1942 | 416 | 70 | -44 |
| *   | 2150 ± 100 | 200 ± 100 | 8 ± 2 |
Table 3: Pole positions $W_p - \frac{1}{2}i\Gamma_p$ and absolute values of the residues $|r|$ at the pole, all in MeV, as well as the phases $\theta$ of the residues for isospin-1/2 resonances. Notation as in Table 1.

| $N^*$ | $W_p$  | $\Gamma_p$ | $|r|$ | $\theta$(deg) |
|-------|--------|-------------|-------|---------------|
| $P_{11}(1440)$ | 1366  | 179  | 47 | -87 |
| *** | 1365 ± 15 | 190 ± 30 | 46 ± 10 | -100 ± 35 |
| $D_{13}(1520)$ | 1516  | 123  | 40 | -6 |
| *** | 1510 ± 5 | 114 ± 10 | 35 ± 3 | -10 ± 4 |
| $S_{11}(1535)$ | 1449  | 67  | 11 | -46 |
| *** | 1510 ± 20 | 170 ± 80 | 96 ± 63 | 15 ± 45 |
| $S_{11}(1650)$ | 1642  | 97  | 21 | -73 |
| *** | 1655 ± 15 | 165 ± 15 | 55 ± 15 | -75 ± 25 |
| $D_{15}(1675)$ | 1657  | 132  | 24 | -22 |
| *** | 1660 ± 5 | 137 ± 12 | 29 ± 6 | -30 ± 10 |
| $F_{15}(1680)$ | 1663  | 115  | 38 | -28 |
| *** | 1672 ± 8 | 122 ± 12 | 38 ± 6 | -23 ± 7 |
| $D_{13}(1700)$ | not seen | not seen | not seen | not seen |
| *** | 1680 ± 50 | 100 ± 50 | 6 ± 3 | 0 ± 50 |
| $P_{11}(1710)$ | 1721  | 185  | 5 | -163 |
| *** | 1720 ± 50 | 230 ± 150 | 10 ± 4 | -175 ± 35 |
| $P_{13}(1720)$ | 1683  | 239  | 15 | -64 |
| *** | 1675 ± 15 | 195 ± 80 | 13 ± 7 | -139 ± 51 |
| $P_{13}(1900)$ | 1846  | 180  | 7 | -75 |
| ** | not listed | not listed | not listed | not listed |
| $F_{15}(2000)$ | 1931  | 62  | 1.3 | -272 |
| ** | not listed | not listed | not listed | not listed |
| $D_{13}(2080)$ | 1834  | 210  | 13 | -134 |
| ** | 1950 ± 170 | 200 ± 80 | 27 ± 22 | ~ 0 |
| $S_{11}(2090)$ | 2065  | 223  | 16 | -138 |
| * | 2150 ± 70 | 350 ± 100 | 40 ± 20 | 0 ± 90 |
| $P_{11}(2100)$ | 1869  | 238  | 7 | -216 |
| * | 2120 ± 240 | 240 ± 80 | 14 ± 7 | 35 ± 35 |
| $D_{15}(2200)$ | 2188  | 238  | 21 | -27 |
| ** | 2100 ± 60 | 360 ± 80 | 20 ± 10 | -90 ± 50 |
Table 4: Pole positions $W_p - \frac{1}{2} i \Gamma_p$ and absolute values of the residues $|r|$ at the pole, all in MeV, as well as the phases of the residues $\theta$ for isospin-3/2 resonances. Notation as in Table 1.

| $N^*$ | $W_p$   | $\Gamma_p$ | $|r|$  | $\theta$(deg) |
|-------|---------|------------|-------|---------------|
| $P_{33}(1232)$ | 1218 | 89 | 42 | -35 |
| ** ** | 1210 ± 1 | 100 ± 2 | 53 ± 2 | -47 ± 1 |
| $P_{33}(1600)$ | 1509 | 236 | 35 | -197 |
| ** ** | 1600 ± 100 | 300 ± 100 | 17 ± 4 | -150 ± 30 |
| $S_{31}(1620)$ | 1598 | 136 | 22 | -99 |
| ** ** | 1600 ± 10 | 118 ± 3 | 16 ± 3 | -110 ± 20 |
| $D_{33}(1700)$ | 1609 | 133 | 9.5 | -52 |
| ** ** | 1650 ± 30 | 200 ± 40 | 13 ± 3 | -20 ± 25 |
| $P_{31}(1750)$ | 1729 | 70 | 1 | -123 |
| * | 1748 | 524 | 48 | 158 |
| $S_{31}(1900)$ | 1775 | 36 | 1 | -166 |
| ** | 1870 ± 40 | 180 ± 50 | 10 ± 3 | -20 ± 40 |
| $F_{35}(1905)$ | 1771 | 190 | 11 | -47 |
| ** ** | 1830 ± 5 | 280 ± 20 | 25 ± 8 | -50 ± 20 |
| $P_{31}(1910)$ | 1896 | 130 | 6 | -118 |
| ** ** | 1880 ± 30 | 200 ± 40 | 20 ± 4 | -90 ± 30 |
| $P_{33}(1920)$ | 2149 | 400 | 38 | -59 |
| ** ** | 1900 ± 50 | 300 ± 100 | 24 ± 4 | -150 ± 30 |
| $D_{35}(1930)$ | 1992 | 270 | 18 | -75 |
| ** ** | 1900 ± 50 | 265 ± 95 | 18 ± 6 | -20 ± 40 |
| $D_{33}(1940)$ | 2070 | 267 | 7 | -31 |
| * | 1900 ± 100 | 200 ± 60 | 24 ± 4 | 135 ± 45 |
| $F_{37}(1950)$ | 1860 | 201 | 43 | -45 |
| ** ** | 1880 ± 10 | 240 ± 20 | 50 ± 7 | -33 ± 8 |
| $F_{35}(2000)$ | 2218 | 219 | 11 | -36 |
| ** | 2150 ± 100 | 350 ± 100 | 16 ± 5 | 150 ± 90 |
| $S_{31}(2150)$ | 2012 | 148 | 6 | -155 |
| * | 2140 ± 80 | 200 ± 80 | 7 ± 2 | -60 ± 90 |