Gauge-invariant TMD factorization for Drell-Yan hadronic tensor at small $x$

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ABSTRACT: The Drell-Yan hadronic tensor for electromagnetic (EM) current is calculated in the Sudakov region $s \gg Q^2 \gg q^2_\perp$ with $\frac{1}{Q^2}$ accuracy, first at the tree level and then with the double-log accuracy. It is demonstrated that in the leading order in $N_c$ the higher-twist quark-quark-gluon TMDs reduce to leading-twist TMDs due to QCD equation of motion. The resulting tensor for unpolarized hadrons is EM gauge-invariant and depends on two leading-twist TMDs: $f_1$ responsible for total DY cross section, and Boer-Mulders function $h_1^\perp$. The order-of-magnitude estimates of angular distributions for DY process seem to agree with LHC results at corresponding kinematics.

KEYWORDS: Perturbative QCD, Resummation

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1 Introduction

The Drell-Yan (DY) process of production of lepton pairs with large invariant mass in hadronic collisions [1] is one of the most important tools to study QCD. From experimental viewpoint, it is a unique source of information about partonic structure of hadrons [2]. On the theoretical side, it serves as a testing ground for factorization approaches in various kinematics regions, like the classical collinear factorization [3–8], TMD factorization [9–13], and SCET [14–17].

The differential cross section of DY process is determined by the product of leptonic tensor and hadronic tensor. The hadronic tensor $W_{\mu\nu}$ is defined as

$$W_{\mu\nu}(p_A,p_B,q) \overset{\text{def}}{=} \frac{1}{(2\pi)^4} \sum_X \int d^4 x \ e^{-i q x} \langle p_A, p_B | J_\mu(x) | X \rangle \langle X | J_\nu(0) | p_A, p_B \rangle$$

$$= \frac{1}{(2\pi)^4} \int d^4 x \ e^{-i q x} \langle p_A, p_B | J_\mu(x) J_\nu(0) | p_A, p_B \rangle.$$  (1.1)
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where $p_A, p_B$ are hadron momenta, $q$ is the momentum of DY pair, $\sum_X$ denotes the sum over full set of “out” states and $J_\mu$ is either electromagnetic or $Z$-boson current. In this paper I consider only the case of electromagnetic current, the $Z$-boson case will be studied in a separate publication. For unpolarized hadrons, the hadronic tensor $W_{\mu\nu}$ is parametrized by 4 functions, for example in Collins-Soper frame [18]

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) (W_T + W_{\Delta\Delta}) - 2X_\mu X_\nu W_{\Delta\Delta} + Z_\mu Z_\nu (W_L - W_T - W_{\Delta\Delta}) - (X_\mu Z_\nu + X_\nu Z_\mu) W_{\Delta\Delta}$$

(1.2)

where $X$, $Z$ are unit vectors orthogonal to $q$ and to each other (their explicit form is presented in section 8.2).

Conventionally, the analysis of hadronic tensor (1.1) in the Sudakov region $q^2 \equiv Q^2 \gg q^2_\perp$ is performed by using TMD factorization. For example, functions $W_T$ and $W_{\Delta\Delta}$ can be represented in a standard TMD-factorized way [9, 19]

$$W_i = \sum_{\text{flavors}} e_i^2 \int d^2 k_\perp D_{f/A}(x_A, k_\perp) D_{f/B}(x_B, q_\perp - k_\perp) C_i(q, k_\perp) + \text{power corrections} + \text{Y-terms}$$

(1.3)

where $D_{f/A}(x_A, k_\perp)$ is the TMD density of a parton $f$ in hadron $A$ with fraction of momentum $x_A$ and transverse momentum $k_\perp$, $D_{f/B}(x_B, q_\perp - k_\perp)$ is a similar quantity for hadron $B$, and coefficient functions $C_i(q, k)$ are determined by the cross section $\sigma(f f \rightarrow \mu^+ \mu^-)$ of production of DY pair of invariant mass $q^2$ in the scattering of two partons.

There is, however, a problem with eq. (1.3) for the functions $W_L$ and $W_{\Delta}$. The reason is that while $W_T$ and $W_{\Delta\Delta}$ are determined by leading-twist quark TMDs, $W_L$ and $W_{\Delta}$ start from terms $q_\perp^2$ and $\sim q_\perp^2 Q^2$ determined by quark-quark-gluon TMDs. The power corrections $\sim q_\perp^2$ were found in ref. [20] more than two decades ago but there was no calculation of power corrections $\sim q_\perp^2 Q^2$ until recently. Also, the leading-twist contribution is not gauge invariant.¹ It is well known from DVCS studies that check of EM gauge invariance sometimes involves cancellation of contributions of different twists (see e.g. [21–27]) so the fact that we need power corrections to check $q^\mu W_{\mu\nu} = 0$ should not come as a surprise. Still, the absence of gauge invariance may cause discomfort in practical applications of TMD factorization.

In a recent paper [28] A. Tarasov and the author calculated power corrections $\sim q_\perp^2 Q^2$ to total DY cross section production which are determined by quark-quark-gluon operators. In this paper I present the result of calculation of symmetric part of $W_{\mu\nu}(q)$ for unpolarized hadrons at large $s \gg Q^2 \gg q_\perp^2$ relevant for DY experiments at LHC. The method of calculation is based on the rapidity factorization approach developed in refs. [28, 29]. The calculations will be performed in the leading order in perturbation theory, first at the tree level and then in the double-logarithmic approximation for coefficient functions $C_i(q, k)$. In this paper I consider only the production of leptons by virtual photon and leave the case of Z-boson production for future publication.

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¹Hereafter gauge invariance of hadronic tensor means electromagnetic (EM) gauge invariance, namely that $q^\mu W_{\mu\nu} = 0$. 

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To find all functions in eq. (1.2) we need to have gauge-invariant expression for $W_{\mu\nu}$ in terms of TMDs. As noted above, only $W_T$ and $W_{\Delta \Delta}$ come from leading-twist quark-antiquark TMD while two other structures come from higher-twist quark-antiquark-gluon TMDs. Fortunately, in the leading order in $N_c$ the latter are related to the former by QCD equations of motion ([28], see also ref. [20]). Moreover, in the small-$x$ region $x_A, x_B \ll 1$ all structures can be expressed by just two leading-twist TMDs - $f_1(x, k_\perp)$ (responsible for the total cross section) and $h_1^\perp(x, k_\perp)$ (the Boer-Mulders function [30]). The results for four functions in eq. (1.2), presented in next section, are of the type of eq. (1.3) with TMDs $f_1(x, k_\perp)$ and/or $h_1^\perp(x, k_\perp)$ and tree-level coefficient functions constructed of $q$ and $k_\perp$.

The paper is organized as follows. In section 2 I present the resulting gauge-invariant expression for $W_{\mu\nu}$ up to $1/Q^2$ terms which is calculated in the rest of the paper. In section 3 the TMD factorization is derived from the rapidity factorization of the double functional integral for a cross section of particle production. In section 4 I explain the method of calculation of power corrections based on approximate solution of classical Yang-Mills equations. Using this method, DY hadronic tensor for small $x$ is calculated in sections 5, 6, and 7. Section 8 contains results of calculations and order-of-magnitude estimate of angular coefficients of DY cross section. The matching of obtained TMDs and coefficient functions $C_i$ in the double-log approximation is discussed in section 9 and the last section 10 is devoted to conclusions and outlook. The necessary technical details can be found in appendices.

## 2 Gauge-invariant hadronic tensor

To set up the stage, in this section I present the final result for tree-level DY hadronic tensor. It is determined by two leading-twist TMDs: the function $f_1^f(x, k_\perp)$ responsible for the total DY cross section and Boer-Mulders time-odd function $h_1^\perp(x, k_\perp)$ (the explicit definition of these functions is presented in the appendix B). The result reads

$$W_{\mu\nu}(q) = \frac{1}{N_c} \sum_f e_f^2 \int d^2k_\perp \left[ F^f(q, k_\perp) W_{\mu\nu}(q, k_\perp) + H^f(q, k_\perp) W_{\mu\nu}(q, k_\perp) \right]$$

(2.1)

where $e_f$ are electric charges of quarks, $q = x_A p_A + x_B p_B + q_\perp$ and

$$F^f(q, k_\perp) = f_1^f(x_A, k_\perp) f_1^f(x_B, (q - k)_\perp) + f_1^f (\leftrightarrow f_1^f)$$

$$H^f(q, k_\perp) = h_1^\perp(x_A, k_\perp) h_1^\perp(x_B, (q - k)_\perp) + h_1^\perp (\leftrightarrow h_1^\perp)$$

(2.2)

The gauge-invariant structures $W_{\mu\nu}^F$ and $W_{\mu\nu}^H$ are given by

$$W_{\mu\nu}^F(q, k_\perp) = -g_{\mu\nu} + \frac{2(k, q - k)_\perp}{Q^2} g_{\mu\nu} + \frac{2}{Q^2} \left[ x_A p_A \kappa_{\nu}^\perp + x_B p_B \mu \kappa_{\nu}^\perp + \mu \leftrightarrow \nu \right]$$

$$+ \frac{4x_A^2 p_A p_{A\nu} k_{\perp}^2}{Q^4} + \frac{4x_B^2 p_B p_{B\nu} (q - k)_\perp}{Q^4} (q - k)_\perp^2$$

$$m^2 W_{\mu\nu}^H(q, k_\perp) = - \left[ \kappa_{\mu}^\perp (q - k)_\nu + \kappa_{\nu}^\perp (q - k)_{\mu} + g_{\mu\nu} (k, q - k)_\perp \right] - \frac{2g_{\mu\nu}}{Q^2} (q - k)_\perp (q - k)_\perp^2$$

$$- 2x_A \left[ p_{A\mu} (q - k)_\nu + \mu \leftrightarrow \nu \right] \frac{k_{\perp}^2}{Q^2} - 2x_B \left[ p_{B\nu} k_{\nu}^\perp + \mu \leftrightarrow \nu \right] (q - k)_\perp^2$$

$$- \frac{4x_A^2 p_A p_{A\nu} k_{\perp}^2}{Q^4} (k, q - k)_\perp - \frac{4x_B^2 p_B p_{B\nu}}{Q^4} (q - k)_\perp^2 (k, q - k)_\perp$$

(2.3)
where $g_{\mu\nu}^\perp$ and $g_{\mu\nu}^\parallel$ are transverse and longitudinal parts of metric tensor (the explicit form of our notations is specified in the next section, see the paragraph including eq. (3.2)). It is easy to check that $q^\mu W^F_{\mu\nu} = 0$ and $q^\mu W^H_{\mu\nu} = 0$. As we will see below, in some of the structures the corrections to eq. (2.1) are of order $O(x_A)$ and $O(x_B)$ while in others on the top of that there are corrections $\sim O\left(\frac{1}{\sqrt{x}}\right)$ times some other higher-twist TMDs discussed in ref. [28]. It should be also noted that $W^F$ part coincides with the result obtained in refs. [31, 32] using parton Reggeization approach to DY process [33].

In the rest of the paper I will derive the above equations and discuss their accuracy. Let me mention upfront that since the approximations made in eq. (2.1) are close to $x$ approximation and small-$x$ approximation at LHC with $Q^2 \sim 100$ GeV or less. Last but not least, the derivation of the above equations is lengthy so the readers interested in final formulas for structures process at LHC with $Q^2 \sim 100$ GeV or less.

3 TMD factorization from rapidity factorization

As was mentioned in the Introduction, to find the TMD formulas of eq. (1.3) type I use the rapidity factorization approach developed in refs. [28, 29]. Let me quickly summarize basic ideas of this approach. The sum over full set of “out” states in eq. (1.1) can be represented by a double functional integral

$$(2\pi)^4 W_{\mu\nu}(p_A, p_B, q) = \sum_X \int d^4x \ e^{-iqx} \langle p_A, p_B | J_\mu(x) | X \rangle \langle X | J_\nu(0) | p_A, p_B \rangle$$

(3.1)

$$= \lim_{t_1 \to \infty} \lim_{t_1 \to -\infty} \int d^4x \ e^{-iqx} \int A(t) \ D\bar{A}\ D\mu \ D\mu \tilde{\psi}(t) \tilde{\psi}(t) \times P_{PB}(\tilde{A}(t_1), \tilde{\psi}(t_1)) \ e^{-iS_{QCD}(A, \bar{A})} \ e^{-iS_{QCD}(A, \bar{A})} \tilde{J}_\mu(0) | p_A, \tilde{A}(t_1), \tilde{\psi}(t_1) \rangle \langle X | J_\nu(0) | p_A, \tilde{A}(t_1), \tilde{\psi}(t_1) \rangle \psi_{PA}(\tilde{A}(t_1), \tilde{\psi}(t_1))).$$

where $J_\mu = \sum_{flavors} e f \bar{\psi}(t) \gamma_\mu \psi_f$ is the electromagnetic current. In this double functional integral the amplitude $\langle X | J_\mu(0) | p_A, p_B \rangle$ is given by the integral over $\tilde{\psi}, \tilde{A}$ fields whereas the complex conjugate amplitude $\langle p_A, p_B | J_\mu(x) | X \rangle$ is represented by the integral over $\tilde{\psi}, \tilde{A}$ fields. Also, $\Psi_{PA}(\tilde{A}(t), \tilde{\psi}(t))$ denotes the proton wave function at the initial time $t$ and the boundary conditions $\tilde{A}(t) = A(t)$ and $\tilde{\psi}(t) = \psi(t)$ reflect the sum over all states $X$, cf. refs. [35–37].

We use Sudakov variables $p = \alpha p_1 + \beta p_2 + p_\perp$, where $p_1$ and $p_2$ are light-like vectors close to $p_A$ and $p_B$ so that $p_A = p_1 + \frac{m^2}{s} p_2$ and $p_A = p_1 + \frac{m^2}{s} p_2$ with $m$ being the proton mass. Also, we use the notations $x_\parallel \equiv x_\mu p_\parallel^\mu$ and $x_\perp \equiv x_\mu p_\perp^\mu$ for the dimensionless light-cone coordinates ($x_\perp = \sqrt{2} x_+$ and $x_\perp = \sqrt{2} x_-$). Our metric is $g^{\mu\nu} = (1, -1, -1, -1)$ which we
will frequently rewrite as a sum of longitudinal part and transverse part:

\[ g_{\mu\nu} = g_{\parallel}^{\mu\nu} + g_{\perp}^{\mu\nu} = \frac{2}{s} (p_1^{\mu} p_2^{\nu} + p_2^{\mu} p_1^{\nu}) + g_{\perp}^{\mu\nu} \]  

\[ (3.2) \]

Consequently, \( p \cdot q = (\alpha_p \beta_q + \alpha_q \beta_p) \frac{s}{2} - (p, q)_\perp \) where \((p, q)_\perp \equiv -p_3 q_3\). Throughout the paper, the sum over the Latin indices \( i, j, ... \) runs over two transverse components while the sum over Greek indices \( \mu, \nu, ... \) runs over four components as usual.

Following ref. [29] we separate quark and gluon fields in the functional integral (3.1) into three sectors (see figure 1): “projectile” fields \( A_\mu, \psi_A \) with \( |\beta| < \sigma_p \), “target” fields \( B_\mu, \psi_B \) with \( |\alpha| < \sigma_t \) and “central rapidity” fields \( C_\mu, \psi_C \) with \( |\alpha| > \sigma_t \) and \( |\beta| > \sigma_p \), see figure 1.\(^3\) Our goal is to integrate over central fields and get the amplitude in the factorized form, i.e. as a product of functional integrals over \( A \) fields representing projectile matrix elements (TMDs of the projectile) and functional integrals over \( B \) fields representing target matrix elements (TMDs of the target). In the spirit of background-field method, we “freeze” projectile and target fields and get a sum of diagrams in these external fields. Since \( |\beta| < \sigma_p \) in the projectile fields and \( |\alpha| < \sigma_t \) in the target fields, at the tree level one can set with power accuracy \( \beta = 0 \) for the projectile fields and \( \alpha = 0 \) for the target fields — the corrections will be \( O(\frac{m^2}{\sigma_{p,t}}) \) and \( O(\frac{m^2}{\sigma_{p,t}}) \). Beyond the tree level, the integration over \( C \) fields produces logarithms of the cutoffs \( \sigma_p \) and \( \sigma_t \) which match the corresponding logs in TMDs of the projectile and the target, see the discussion in section 9.

From integrals over projectile and target fields in the above equation we see that the functional integral over \( C \) fields should be done in the background of \( A \) and \( B \) fields satisfying

\[ \tilde{A}(t_f) = A(t_f), \quad \tilde{\psi}_A(t_f) = \psi_A(t_f) \quad \text{and} \quad \tilde{B}(t_f) = B(t_f), \quad \tilde{\psi}_B(t_f) = \psi_B(t_f). \]  

\[ (3.3) \]

Combining this with our approximation that at the tree level \( \beta = 0 \) for \( A, \tilde{A} \) fields and \( \alpha = 0 \) for \( B, \tilde{B} \) fields, which corresponds to \( A = A(x, x_\perp), \tilde{A} = \tilde{A}(x, x_\perp) \) and \( B = \tilde{B}(x, x_\perp) \) and \( \tilde{B} = A(x, x_\perp) \) and \( B = \tilde{A}(x, x_\perp) \),

\[ \text{Figure 1. Rapidity factorization for DY particle production} \]
\( B(x_s, x_{\perp}) = \tilde{B}(x_s, x_{\perp}) \), we see that for the purpose of calculation of the functional integral over central fields we can set

\[
A(x_s, x_{\perp}) = \hat{A}(x_s, x_{\perp}), \quad \psi_A(x_s, x_{\perp}) = \hat{\psi}_A(x_s, x_{\perp})
\]

and

\[
B(x_s, x_{\perp}) = \hat{B}(x_s, x_{\perp}), \quad \psi_B(x_s, x_{\perp}) = \hat{\psi}_B(x_s, x_{\perp}). \tag{3.4}
\]

In other words, since \( A, \psi \) and \( \hat{A}, \hat{\psi} \) do not depend on \( x_s \), if they coincide at \( x_s = \infty \) they coincide everywhere. Similarly, since \( B, \psi_B \) and \( \hat{B}, \hat{\psi}_B \) do not depend on \( x_s \), if they coincide at \( x_s = \infty \) they should be equal.

Summarizing, we see that at the tree level in our approximation

\[
\int D\mu \int [C(t)] D\bar{C} \int D\psi_C D\psi_C \int \hat{\psi}_C \hat{\psi}_C \tilde{\psi}_C \tilde{\psi}_C \tilde{\psi}_C \tilde{\psi}_C \psi_A(x_s, x_{\perp}) \psi_B(x_s, x_{\perp}) = O(q, x; A, \psi_A; B, \psi_B), \tag{3.5}
\]

where now \( S_C = S_{QCD}(C + A + B, \psi_C + \psi_A + \psi_B) - S_{QCD}(A, \psi_A) - S_{QCD}(B, \psi_B) \) and \( \tilde{S}_C = S_{QCD}(\tilde{C} + A + B, \tilde{\psi}_C + \psi_A + \psi_B) - S_{QCD}(A, \psi_A) - S_{QCD}(B, \psi_B) \). It is well known that in the tree approximation the double functional integral (3.5) is given by a set of retarded Green functions in the background fields [38–40] (see also appendix A of ref. [29] for the proof). Since the double functional integral (3.5) is given by a set of retarded Green functions in the background fields \( A \) and \( B \), the calculation of the tree-level contribution to \( \bar{\psi}\gamma_\mu\psi \) in the r.h.s. of eq. (3.5) is equivalent to solving YM equation for \( \psi(x) \) and \( A_\mu(x) \) with initial condition that the solution has the same asymptotics at \( t \rightarrow -\infty \) as the superposition of incoming projectile and target background fields.

The hadronic tensor (1.1) can now be represented as

\[
W_{\mu\nu}(p_A, p_B, q) = \frac{1}{(2\pi)^4} \int d^4x \ e^{-iqx} \langle p_A|p_B|O_{\mu\nu}(q, x; A, \hat{\psi}_A; B, \hat{\psi}_B)|p_A\rangle \langle p_B\rangle, \tag{3.6}
\]

where \( O_{\mu\nu}(q, x; A, \hat{\psi}_A; B, \hat{\psi}_B) \) should be expanded in a series in \( \hat{A}, \hat{\psi}_A, \hat{B}, \hat{\psi}_B \) operators and evaluated between the corresponding (projectile or target) states: if

\[
O_{\mu\nu}(q, x; A, \hat{\psi}_A; B, \hat{\psi}_B) = \sum_{m,n} \int dz_m dz'_n c_{m,n}(q, x) \hat{\Phi}_A(z_m) \hat{\Phi}_B(z'_n). \tag{3.7}
\]

where \( c_{m,n} \) are coefficients and \( \Phi \) can be any of \( A_\mu, \psi \) or \( \bar{\psi} \) with appropriate Lorentz indices. We get then

\[
W_{\mu\nu} = \frac{1}{(2\pi)^4} \int d^4x e^{-iqx} \sum_{m,n} \int dz_m c_{m,n}(q, x) \langle p_A|\hat{\Phi}_A(z_m)|p_A\rangle \int dz'_n \langle p_B|\hat{\Phi}_B(z'_n)|p_B\rangle. \tag{3.8}
\]

As we will demonstrate below, the relevant operators \( \hat{\Phi}_A \) and \( \hat{\Phi}_B \) are quark and gluon fields with Wilson-line type gauge links collinear to either \( p_2 \) for \( A \) fields or \( p_1 \) for \( B \) fields.
4 Power corrections and solution of classical YM equations

4.1 Power counting for background fields

As we discussed in previous section, to get the hadronic tensor in the form (3.6) we need to calculate the functional integral (3.5) in the background of the fields (3.4). Since we integrate over fields (3.4) afterwards, we may assume that they satisfy Yang-Mills equations\footnote{As was mentioned above, for the purpose of calculation of integral over $C$ fields the projectile and target fields are “frozen”.}

\[
iD_A^\nu A_{\mu} = 0, \quad D_A^\nu A_{\mu}^a = g^2 \sum_f \bar{\psi}_A^f \gamma_\mu t^a \psi_A^f, \\
iD_B^\nu B_{\mu} = 0, \quad D_B^\nu B_{\mu}^a = g^2 \sum_f \bar{\psi}_B^f \gamma_\mu t^a \psi_B^f, 
\]

(4.1)

where $A_{\mu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$, $D_A^\mu \equiv (\partial^\mu - i[A^\mu, \cdot])$ and similarly for $B$ fields.\footnote{Since we are dealing with tree approximation and quark equations of motion, it is convenient to include coupling constant $g$ in the definition of gluon fields.}

It is convenient to choose a gauge where $A_\perp = 0$ for projectile fields and $B_\perp = 0$ for target fields. (The existence of such gauge was proved in appendix B of ref. [29] by explicit construction.) The relative strength of Lorentz components of projectile and target fields in this gauge was found in ref. [29]

\[
\begin{align*}
\bar{p}_i A_i(x_\parallel, x_\perp) &\sim m_\perp^{5/2}, & \gamma_i \psi_A(x_\parallel, x_\perp) &\sim m_\perp^{3/2}, & \bar{p}_2 \psi_A(x_\parallel, x_\perp) &\sim s\sqrt{m_\perp}, \\
\bar{p}_1 \psi_B(x_\parallel, x_\perp) &\sim s\sqrt{m_\perp}, & \gamma_i \psi_B(x_\parallel, x_\perp) &\sim m_\perp^{3/2}, & \bar{p}_2 \psi_B(x_\parallel, x_\perp) &\sim m_\perp^{5/2}, \\
A_\parallel(x_\parallel, x_\perp) &\sim B_\parallel(x_\parallel, x_\perp) \sim m_\perp^2, & A_i(x_\parallel, x_\perp) &\sim B_i(x_\parallel, x_\perp) \sim m_\perp.
\end{align*}
\]

(4.2)

Here $m_\perp$ is a scale of order of $m$ or $q_\perp$. As discussed in refs. [28, 29], our rapidity factorization (3.8) is applicable in the region where $s, Q^2 \gg q_\perp^2, m^2$, while the relation between $q_\perp^2$ and $m^2$ and between $Q^2$ and $s$ may be arbitrary. Correspondingly, for the purpose of counting of powers of $s$, we do not distinguish between $s$ and $Q^2$ so our power counting will be correct at any Bjorken $x$. The distinction will come at a later time when we specify to small $x$ and disregard $1/x$ in comparison to $1/q^2$ in final expressions for TMDs and/or coefficient functions. Similarly, for the purpose of power counting we will not distinguish between $m$ and $q_\perp$ so we introduce $m_\perp$ which may be of order of $m$ or $q_\perp$ depending on matrix element.

Note also that in our gauge

\[
A_i(x_\parallel, x_\perp) = \frac{2}{s} \int_{-\infty}^{x_\parallel} dx'_\parallel A_i(x'_\parallel, x_\perp), \quad B_i(x_\parallel, x_\perp) = \frac{2}{s} \int_{-\infty}^{x_\parallel} dx'_\parallel B_i(x'_\parallel, x_\perp)
\]

(4.3)

where $A_i \equiv F_i^{(A)}$ and $B_i \equiv F_i^{(B)}$ are field strengths for $A$ and $B$ fields respectively.

Thus, to find TMD factorization formula with power corrections at the tree level we need to calculate the functional integral (3.1) in the background fields of the strength given by eqs. (4.2).
4.2 Approximate solution of classical equations at $q_\perp^2 \ll Q^2$

As we discussed in section 3, the calculation of the functional integral (3.5) over $C$-fields in the tree approximation reduces to finding fields $C_\mu$ and $\psi_C$ as solutions of Yang-Mills equations for the action $S_C = S_{\text{QCD}}(C + A + B, \psi_C + \psi_A + \psi_B) - S_{\text{QCD}}(A, \psi_A) - S_{\text{QCD}}(B, \psi_B)$

$$\left(i\partial + gA + gB + gC\right)(\psi_A^f + \psi_B^f + \psi_C^f) = 0,$$

$$D^\mu F_{\mu\nu}^a(A + B + C) = g^2 \sum_f (\bar{\psi}_A^f + \bar{\psi}_B^f + \bar{\psi}_C^f) \gamma_\mu t^a (\psi_A^f + \psi_B^f + \psi_C^f).$$

The solution of eq. (4.4) which we need corresponds to the sum of set of diagrams in background field $A + B$ with retarded Green functions, see figure 2. The sum of tree diagrams with retarded Green functions gives fields $C_\mu$ and $\psi_C$ that vanish at $t \to -\infty$. Thus, we are solving the usual classical YM equations

$$D^\nu F_{\mu\nu}^a = g^2 \sum_f \bar{\Psi}_f t^a \gamma_\mu \Psi_f,$$  \hspace{1cm} \Psi_f = 0,$$

where

$$\partial_\mu = C_\mu + A_\mu + B_\mu,$$

$$\mathbb{P}_\mu \equiv i\partial_\mu + C_\mu + A_\mu + B_\mu,$$

$$\Psi_f = \psi_C^f + \psi_A^f + \psi_B^f,$$

$$\mathbb{P}_{\mu\nu} = \partial_\mu \partial_\nu - \mu \leftrightarrow \nu - i[A_\mu, A_\nu],$$

with boundary conditions

$$A_\mu(x) \underset{x, t \to -\infty}{\to} A_\mu(x, x_\perp), \quad \psi(x) \underset{x, t \to -\infty}{\to} \psi_A(x, x_\perp),$$

$$B_\mu(x) \underset{x, t \to -\infty}{\to} B_\mu(x, x_\perp), \quad \psi(x) \underset{x, t \to -\infty}{\to} \psi_B(x, x_\perp)$$

following from $C_\mu, \psi_C \underset{t \to -\infty}{\to} 0$. These boundary conditions reflect the fact that at $t \to -\infty$ we have only incoming hadrons with $A$ and $B$ fields.

---

We take into account only $u,d,s,c$ quarks and consider them massless. In principle, one can include “massless” $b$-quark for $q_\perp^2 \gg m_b^2$. 
As discussed in ref. [29], for our case of particle production with $q_\perp \ll 1$ it is possible to find the approximate solution of (4.5) as a series in this small parameter. One solves eqs. (4.5) iteratively, order by order in perturbation theory, starting from the zero-order approximation in the form of the sum of projectile and target fields

$$A^{[0]}_\mu(x) = A_\mu(x_\cdot, x_\perp) + B_\mu(x_\cdot, x_\perp),$$

$$\Psi^{[0]}(x) = \psi_A(x_\cdot, x_\perp) + \psi_B(x_\cdot, x_\perp)$$

and improving it by calculation of Feynman diagrams with retarded propagators in the background fields (4.8).

Let me now explain how the parameter $m_\perp^2/s$ comes up in the rapidity-factorization approach (for details, see ref. [29]). When we expand quark and gluon propagators in powers of background fields, we get a set of diagrams shown in figure 2. The typical bare gluon propagator in figure 2 is

$$\frac{1}{p^2 + i\epsilon p_0} = \frac{1}{\alpha \beta s - p_\perp^2 + i\epsilon(\alpha + \beta)}. \quad (4.9)$$

In the tree approximation, the transverse momenta in tree diagrams are determined by further integration over projectile (“A”) and target (“B”) fields in eq. (3.1) which converge on either $q_\perp$ or $m_N$. On the other hand, the integrals over $\alpha$ converge on either $\alpha q$ or $\alpha \sim 1$ and similarly the characteristic $\beta$’s are either $\beta q$ or $\beta \sim 1$. Since $\alpha q \beta q s = Q_\parallel^2 \gg q_\perp^2$, one can expand gluon and quark propagators in powers of $p_\perp^2/\alpha \beta s$.

$$\frac{1}{p^2 + i\epsilon p_0} = \frac{1}{s(\alpha + i\epsilon)(\beta + i\epsilon)} \left(1 + \frac{p_\perp^2/s}{(\alpha + i\epsilon)(\beta + i\epsilon)} + \ldots\right),$$

$$\frac{\hat{p}}{p^2 + i\epsilon p_0} = \frac{1}{s} \left(\frac{\hat{p}_1}{\beta + i\epsilon} + \frac{\hat{p}_2}{\alpha + i\epsilon} + \frac{\hat{p}_\perp}{(\alpha + i\epsilon)(\beta + i\epsilon)} \left(1 + \frac{p_\perp^2/s}{(\alpha + i\epsilon)(\beta + i\epsilon)} + \ldots\right)\right).$$

After the expansion (4.10), the dynamics in the transverse space effectively becomes trivial: all background fields stand either at $x$ or at 0. Note that in this statement is solely a consequence of $Q_\parallel^2 \gg q_\perp^2$ and does not rely on small-$x$ approximation.

### 4.3 Power expansion of classical quark fields

Now we expand the classical quark fields in powers of $p_\perp^2/p_\parallel^2 \sim m_\perp^2/s$ (the corresponding expansion of classical gluon fields is presented in ref. [29], but we do not need it here). As demonstrated in ref. [28], expanding it in powers of $p_\perp^2/p_\parallel^2$ we obtain

$$\Psi(x) = \Psi_1(x) + \Psi_2(x) + \ldots,$$

$$\quad$$
\[ \Psi_1 = \psi_A + \Xi_1, \quad \Xi_1 = -\frac{\gamma^i B_i}{s} \frac{1}{\alpha + i\epsilon} \psi_A = \frac{i}{s} \sigma_i B^i \frac{1}{\alpha + i\epsilon} \psi_A, \]
\[ \tilde{\Psi}_1 = \tilde{\psi}_A + \Xi_1, \quad \Xi_1 = -\left( \tilde{\psi}_A \frac{1}{\alpha - i\epsilon} \right) \gamma^i B_i \frac{1}{s} = \frac{i}{s} \left( \tilde{\psi}_A \frac{1}{\alpha - i\epsilon} \right) B^i \sigma_i, \]
\[ \Psi_2 = \psi_B + \Xi_2, \quad \Xi_2 = -\frac{\gamma^i A_i}{s} \frac{1}{\beta + i\epsilon} \psi_B = \frac{i}{s} \sigma_i A^i \frac{1}{\beta + i\epsilon} \psi_B, \]
\[ \tilde{\Psi}_2 = \tilde{\psi}_B + \Xi_2, \quad \Xi_2 = -\left( \tilde{\psi}_B \frac{1}{\beta - i\epsilon} \right) \gamma^i A_i \frac{1}{s} = \frac{i}{s} \left( \tilde{\psi}_B \frac{1}{\beta - i\epsilon} \right) A_i \sigma_i, \]
(4.12)

and dots stand for terms subleading in \( \frac{Q^2}{s^2} \) and/or \( \alpha_q, \beta_q \) parameters (hereafter we assume the small-\( x \) approximation \( \alpha_q, \beta_q \ll 1 \) in all calculations). In this formula
\[
\frac{1}{\alpha + i\epsilon} \psi_A(x_\bullet, x_\perp) \equiv -i \int_{-\infty}^{x_\bullet} dx'_\bullet \psi_A(x'_\bullet, x_\perp),
\]
\[
\left( \psi_A \frac{1}{\alpha - i\epsilon} \right)(x_\bullet, x_\perp) \equiv i \int_{-\infty}^{x_\bullet} dx'_\bullet \psi_A(x'_\bullet, x_\perp)
\]
and similarly for \( \frac{1}{\beta + i\epsilon} \). For brevity, in what follows we denote \( \left( \tilde{\psi}_A \frac{1}{\alpha} \right)(x) \equiv \left( \tilde{\psi}_A \frac{1}{\alpha - i\epsilon} \right)(x) \) and \( \left( \tilde{\psi}_B \frac{1}{\beta} \right)(x) \equiv \left( \tilde{\psi}_B \frac{1}{\beta - i\epsilon} \right)(x) \). Let us estimate the relative size of corrections \( \Xi \) in eq. (4.12) at small \( x \). As we will see, \( \frac{1}{\alpha} \) and \( \frac{1}{\beta} \) transform to \( \frac{1}{\alpha_q} \) and \( \frac{1}{\beta_q} \) in our TMDs so
\[
\Xi_1 \sim \psi_A \frac{m_\perp}{\alpha q \sqrt{s}} \sim \psi_A \frac{q_\perp}{Q}, \quad \Xi_2 \sim \psi_B \frac{m_\perp}{\beta q \sqrt{s}} \sim \psi_B \frac{q_\perp}{Q}
\]
(4.14)
if \( \alpha_q \sim \beta_q \sim \frac{Q}{\sqrt{s}} \) (recall that we assume that the DY pair is emitted in the central region of rapidity). For example, the correction \( \sim [\tilde{\psi}_A \gamma_\mu \Xi_2][\tilde{\psi}_B \gamma_\nu \Xi_1] \) will be of order of \( \frac{Q^2}{Q^2} \) in comparison to leading-twist contribution \( [\tilde{\psi}_A \gamma_\mu \psi_B] [\tilde{\psi}_B \gamma_\nu \psi_A] \).

5 Hadronic tensor at \( s \gg Q^2 \gg q^2_\perp \)

In general, our method is applicable for calculation of power corrections at any \( s, Q^2 \gg q^2_\perp, m_N^2 \). However, the expressions are greatly simplified in the physically interesting case \( s \gg Q^2 \gg q^2_\perp \) which is considered in this paper.

As we noted above, we take into account only hadronic tensor due to electromagnetic currents of \( u, d, s, c \) quarks and consider these quarks to be massless. It is convenient to define coordinate-space hadronic tensor multiplied by \( N_c \frac{2}{3} \) (and denoted by extra “check” mark) as follows
\[
W_{\mu\nu}(p_A, p_B, x) \equiv N_c \frac{2}{3} \langle p_A, p_B | J_\mu(x) J_\nu(0) | p_A, p_B \rangle \]
(5.1)
\[
W_{\mu\nu}(p_A, p_B, q) = \frac{s/2}{(2\pi)^4 N_c} \int d^4 x \ e^{-iqx} \tilde{W}_{\mu\nu}(p_A, p_B, x).
\]

\[ \text{The reader may wonder why there are no corrections } \sim \frac{q^2}{Q^2} \text{ coming from next terms in the expansion (4.11) like } [\tilde{\psi}_A(x) \gamma_\mu \psi_B(x)] [\tilde{\psi}_B(0) \gamma_\nu \tilde{\psi}_A(0)] \text{. The reason is that } \frac{1}{\alpha} \text{ between } \tilde{\psi}_B(0) \text{ and } B(0) \text{ does not transform to } \frac{1}{\beta} \text{ and remains } \sim O(1), \text{ see the discussion in the appendix 8.3.4 of ref. [28].} \]

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where the square brackets mean trace over Lorentz and color indices.

As to terms \( \frac{q^2}{Q^2} \) from \( J_{AB}^\mu(x) J_{BA}^\nu(0) \), we can substitute \( Q_\perp^2 \) with \( Q^2 \). In the limit \( s \gg Q^2 \gg q^2 \), this change of variables can only lead to errors of the order of subleading power terms.\(^8\)

As to terms \( \sim \bar{\Psi}(x) \gamma^\mu \Psi(2) \bar{\Psi}(0) \gamma^\nu \Psi(1) \), they can be decomposed using eq. (4.12) as follows:

\[
\begin{align*}
\left[ (\bar{\psi}_A + \Xi_1)(x) \gamma^\mu (\psi_B + \Xi_2)(x) \right] & \left[ (\bar{\psi}_B + \Xi_2)(0) \gamma^\nu (\psi_A + \Xi_1)(0) \right] + x \leftrightarrow 0 \\
= \left[ \bar{\psi}_A(x) \gamma^\mu \psi_B(x) \right] & \left[ \bar{\psi}_B(0) \gamma^\nu \psi_A(0) \right] \\
& + \left[ \bar{\psi}_A(x) \gamma^\mu \psi_B(x) \right] \left[ \bar{\psi}_B(0) \gamma^\nu \psi_A(0) \right] + \left[ \bar{\psi}_A(x) \gamma^\mu \psi_B(x) \right] \left[ \bar{\psi}_B(0) \gamma^\nu \psi_A(0) \right] \\
& + \left[ \bar{\psi}_A(x) \gamma^\mu \psi_B(x) \right] \left[ \bar{\psi}_B(0) \gamma^\nu \psi_A(0) \right] + \left[ \bar{\psi}_A(x) \gamma^\mu \psi_B(x) \right] \left[ \bar{\psi}_B(0) \gamma^\nu \psi_A(0) \right] \\
& + \left[ \bar{\psi}_A(x) \gamma^\mu \psi_B(x) \right] \left[ \bar{\psi}_B(0) \gamma^\nu \psi_A(0) \right] + \left[ \bar{\psi}_A(x) \gamma^\mu \psi_B(x) \right] \left[ \bar{\psi}_B(0) \gamma^\nu \psi_A(0) \right] + x \leftrightarrow 0.
\end{align*}
\]

where the square brackets mean trace over Lorentz and color indices.

\(^8\)Except for the leading-twist term where the difference between \( Q_\perp^2 \) and \( Q^2 \) matters.
First, let us consider the leading-twist term coming from the first term in the r.h.s. of this equation.

5.2 Leading-twist contribution

As we mentioned, the leading-twist term comes from $J_{AB}^\mu(x)J_{BA}^\nu(0)$ and $J_{BA}^\mu(x)J_{AB}^\nu(0)$. Using Fierz transformation (A.1) one obtains

$$\frac{N_c}{s} \left( \langle \bar{\psi}_A(x, x_\perp) \gamma_\mu \psi_B(x_\perp) \right] \langle \bar{\psi}_B(0) \gamma_\nu \psi_A(0) \rangle + \mu \leftrightarrow \nu \rangle + x \rightarrow 0$$

$$= \frac{g_{\mu\nu}}{2s} \left[ - (\bar{\psi}_A \gamma_\mu \psi_A)(\bar{\psi}_B \gamma_\nu \psi_B) + (\bar{\psi}_A \gamma_\mu \gamma_5 \psi_A)(\bar{\psi}_B \gamma_5 \gamma_\nu \psi_B) + (\bar{\psi}_A \gamma_\alpha \gamma_5 \psi_A)(\bar{\psi}_B \gamma_5 \gamma_\alpha \psi_B) 
+ \frac{1}{2} (\bar{\psi}_A \gamma_\alpha \gamma_5 \psi_A)(\bar{\psi}_B \gamma_5 \gamma_\alpha \psi_B) - \frac{1}{2} (\bar{\psi}_A \gamma_\alpha \gamma_5 \psi_A)(\bar{\psi}_B \sigma_{\alpha\beta} \psi_B) 
- \frac{1}{2s} (\bar{\psi}_A \gamma_\mu \gamma_5 \psi_A)(\bar{\psi}_B \gamma_\nu \gamma_5 \psi_B) + \mu \leftrightarrow \nu \right] 
+ \frac{1}{2s} (\bar{\psi}_A \gamma_\nu \psi_A)(\bar{\psi}_B \gamma_\mu \psi_B) + (\bar{\psi}_A \gamma_\sigma \gamma_5 \psi_A)(\bar{\psi}_B \gamma_\sigma \gamma_5 \psi_B) + x \leftrightarrow 0 \tag{5.7}$$

where all parentheses in the r.h.s. are color singlet. As usual, after integration over background fields $A$ and $B$ we promote $A, \psi_A$ and $B, \psi_B$ to operators $\hat{A}, \hat{\psi}$. A subtle point is that our operators are not under T-product ordering so one should be careful while changing the order of operators in formulas like Fierz transformation. Fortunately, all operators in the r.h.s. of eq. (5.7) are separated either by space-like intervals or light-like intervals so they commute with each other.

From parametrization of two-quark operators in section B, it is clear that the leading-twist contribution to $W_{\mu\nu}(q)$ comes from

$$W_{\mu\nu}^{lt} = \frac{1}{2s} \left( g_{\mu\nu} g^{\alpha\beta} - \delta_\mu^\alpha \delta_\nu^\beta - \delta_\mu^\beta \delta_\nu^\alpha \right) \langle \hat{\psi}(x, x_\perp) \gamma_\alpha \gamma_5 \hat{\psi}(0) \rangle A \langle \hat{\psi}(0) \gamma_\beta \gamma_5 \hat{\psi}(x_\perp) \rangle B + \frac{1}{2s} \left( \delta_\mu^\alpha \delta_\nu^\beta - \delta_\mu^\beta \delta_\nu^\alpha \right) \langle \hat{\psi}(x, x_\perp) \sigma_{\alpha\beta} \hat{\psi}(0) \rangle A \langle \hat{\psi}(0) \sigma_{\alpha\beta} \hat{\psi}(x_\perp) \rangle B + x \leftrightarrow 0 \tag{5.8}$$

Hereafter we use notations $\langle O \rangle_A \equiv \langle p_A | O | p_A \rangle$ and $\langle O \rangle_B \equiv \langle p_B | O | p_B \rangle$ for brevity. The corresponding leading-twist contribution to $W_{\mu\nu}(q)$ has the form [41]

$$W_{\mu\nu}^{lt}(\alpha_q, \beta_q, q_\perp) = \frac{1}{16\pi^2 N_c} \int d^2 \frac{1}{x_\perp} \left( e^{-i\alpha_q x_\perp - i\beta_q x_\perp + i(q \cdot x_\perp)} \right) W_{\mu\nu}^{lt}(x)$$

$$= \sum_f e_f^2 \left[ \left( g_{\mu\nu} \left( f^f_\alpha (\alpha_q, k_\perp) f^f_\beta (\beta_q, q_\perp - k_\perp) + f^f_\beta (\alpha_q, k_\perp) f^f_\alpha (\beta_q, q_\perp - k_\perp) \right) 
+ \left( k_\mu^\alpha (q - k) + k_\perp^\alpha (q - k) \right) g_{\mu\nu} (k, q - k_\perp) \right] 
\times \left[ h^f_{\mu f}(\alpha_q, k_\perp) h^f_{\nu f}(\beta_q, q_\perp - k_\perp) + h^f_{\nu f}(\alpha_q, k_\perp) h^f_{\mu f}(\beta_q, q_\perp - k_\perp) \right] \right] \tag{5.10}$$

\footnote{In a general gauge for projectile and target fields these matrix elements read

$$\langle p_A | \hat{\psi}_f(x) \gamma_\mu \hat{\psi}_f(0) | p_A \rangle = \langle p_A | \hat{\psi}_f(x, x_\perp) \gamma_{\mu \nu} \gamma_5 \gamma_{\nu \alpha} \gamma_5 | x_\perp, 0_\perp \rangle \langle x_\perp, 0_\perp | \hat{\psi}_f(0) | p_A \rangle,$$

$$\langle p_B | \hat{\psi}_f(x) \gamma_\mu \hat{\psi}_f(0) | p_B \rangle = \langle p_B | \hat{\psi}_f(x, x_\perp) \gamma_{\mu \nu} \gamma_5 \gamma_{\nu \alpha} \gamma_5 | x_\perp, 0_\perp \rangle \langle x_\perp, 0_\perp | \hat{\psi}_f(0) | p_B \rangle \tag{5.9}$$

and similarly for other operators.}
Let us discuss other terms proportional to different TMDs in parametrizations in section B. To this end, we write down terms from eq. (2.3) that we are looking for in Sudakov variables:

\[ g_\perp(q) \left[ 1 - \frac{q^2}{\alpha_s \beta_0 q s} \right], \quad g_\perp(q) \left[ 1 + \frac{q^2}{\alpha_s \beta_0 q s} \right], \quad g_\parallel(q) \left[ 0 - \frac{q^2}{\alpha_s \beta_0 q s} \right], \quad \frac{2}{\alpha_s q s} (p^\mu q^\nu + \mu \leftrightarrow \nu), \]

(5.11)

Here zero in the third term means that the contribution of order one is actually absent. As discussed in section B, all TMDs considered here can have only logarithmic dependence on Bjorken \( x \) (\( \equiv \alpha_q \) or \( \beta_q \)) but not the power dependence \( \frac{1}{x} \). It is easy to see that other quark-antiquark TMDs give contributions to \( W_{\mu}(q) \) which look like terms in eq. (5.11) but without extra \( \frac{1}{\alpha_q} \) and/or \( \frac{1}{\beta_q} \), so they are power suppressed in low-\( x \) regime \( s \gg Q^2 \).

Let us also specify the terms which we do not calculate. Roughly speaking, they correspond to terms in eq. (5.11) multiplied by \( \frac{m^2}{Q^2} \) or by either \( \alpha_q \) or \( \beta_q \). Our strategy in the next sections is to compare a certain term in \( \tilde{W}_{\mu \nu} \) to terms in eq. (5.11), and, if it is smaller, neglect, if it is of the same size, calculate.

6 Terms coming from \( J_{AB}^{\mu}(x)J_{BA}^{\nu}(0) \)

We separate terms in eq. (5.6) according to number of gluon fields (contained in \( \Xi \)'s )

\[ \tilde{W}_{\mu \nu} \text{ sym} = \tilde{W}_{\mu \nu}^{(1)} + \tilde{W}_{\mu \nu}^{(2a)} + \tilde{W}_{\mu \nu}^{(2b)} + \tilde{W}_{\mu \nu}^{(2c)} \]

(6.1)

where leading-twist terms without gluons (quark-antiquark TMDs) were considered in previous section, and

\[ \tilde{W}_{\mu \nu}^{(1)}(x) = \frac{N_c}{s} \langle A, B \rangle [\bar{\psi}_A(x) \gamma_\mu \psi_B(x)] [\bar{\psi}_B(0) \gamma_\nu \Xi_1(0)] + [\bar{\Xi}_1(x) \gamma_\mu \psi_B(x)] [\bar{\psi}_B(0) \gamma_\nu \psi_A(0)] + [\bar{\psi}_A(x) \gamma_\mu \Xi_2(x)] [\bar{\psi}_B(0) \gamma_\nu \psi_A(0)] \]

(6.2)

\[ \tilde{W}_{\mu \nu}^{(2a)}(x) = \frac{N_c}{s} \langle A, B \rangle [\bar{\psi}_A(x) \gamma_\mu \Xi_2(x)] [\bar{\psi}_B(0) \gamma_\nu \Xi_1(0)] + [\bar{\Xi}_1(x) \gamma_\mu \psi_B(x)] [\bar{\Xi}_2(0) \gamma_\nu \Xi_1(0)] \]

(6.3)

\[ \tilde{W}_{\mu \nu}^{(2b)}(x) = \frac{N_c}{s} \langle A, B \rangle [\bar{\psi}_A(x) \gamma_\mu \Xi_2(x)] [\bar{\Xi}_2(0) \gamma_\nu \psi_A(0)] + [\bar{\Xi}_1(x) \gamma_\mu \psi_B(x)] [\bar{\psi}_B(0) \gamma_\nu \Xi_1(0)] + [\bar{\psi}_A(x) \gamma_\mu \psi_B(x)] [\bar{\Xi}_2(0) \gamma_\nu \Xi_1(0)] + [\bar{\Xi}_1(x) \gamma_\mu \psi_B(x)] [\bar{\Xi}_2(0) \gamma_\nu \psi_A(0)] \]

(6.4)

and

\[ \tilde{W}_{\mu \nu}^{(2c)}(x) = \frac{N_c}{s} \langle A, B \rangle [\bar{\Xi}_1(x) \gamma_\mu \Xi_2(x)] [\bar{\Xi}_2(0) \gamma_\nu \psi_A(0)] + [\bar{\psi}_A(x) \gamma_\mu \psi_B(x)] [\bar{\Xi}_2(0) \gamma_\nu \Xi_1(0)] + [\bar{\Xi}_1(x) \gamma_\mu \psi_B(x)] [\bar{\Xi}_2(0) \gamma_\nu \psi_A(0)] + [\bar{\psi}_A(x) \gamma_\mu \psi_B(x)] [\bar{\psi}_B(0) \gamma_\nu \Xi_1(0)] \]

(6.5)

The corresponding contributions to \( W_{\mu \nu}(q) \) will be denoted \( W_{\mu \nu}^{(1)} \), \( W_{\mu \nu}^{(2a)} \), \( W_{\mu \nu}^{(2b)} \), and \( W_{\mu \nu}^{(2c)} \), respectively. We will consider these contributions in turn.
6.1 Terms with one quark-quark-gluon operator

In this section we consider terms in eq. (6.2) which will lead to $\frac{2}{\alpha_s} p_1^\mu q_1^- + \mu \leftrightarrow \nu$ and $\frac{2}{\alpha_s} p_2^\mu q_2^- + \mu \leftrightarrow \nu$ contributions to $W_{\mu\nu}(q)$.

6.1.1 Term with $\Xi_1$

Let us start with the last term in eq. (6.2). The Fierz transformation (A.1) yields

$$\frac{1}{2} \left[ \bar{\psi}_A(x) \gamma_\mu \psi_B(x) \right] \left[ \bar{\psi}_B(0) \gamma_\nu \Xi_1(0) \right] + \mu \leftrightarrow \nu$$

$$= \frac{g_{\mu\nu}}{4} \left\{ \left[ \bar{\psi}_A(x) \frac{\not{p}^2}{s} \gamma^1 \gamma_\alpha \psi_A(0) \right] \left[ \bar{\psi}_B B_{\mu}^{\alpha} (0) \psi_B(x) \right] \right\} + \frac{1}{4} (\gamma^\alpha \gamma^\beta + \gamma^\beta \gamma^\alpha - g_{\mu\nu} g^{\alpha\beta})$$

$$\times \left\{ \left[ \bar{\psi}_A(x) \gamma_\alpha \frac{\not{p}^2}{s} \gamma^1 \gamma_\alpha \psi_A(0) \right] \left[ \bar{\psi}_B B_{\mu}^{\alpha} (0) \psi_B(x) \right] \right\} - \frac{1}{4} (\gamma^\alpha \gamma^\beta + \gamma^\beta \gamma^\alpha - g_{\mu\nu} g^{\alpha\beta})$$

$$\left[ \bar{\psi}_A(x) \gamma_\alpha \frac{\not{p}^2}{s} \gamma^1 \gamma_\alpha \psi_A(0) \right] \left[ \bar{\psi}_B B_{\mu}^{\alpha} (0) \psi_B(x) \right]$$

$$= \frac{g_{\mu\nu}}{4} \left\{ \left[ \bar{\psi}_A(x) \gamma_\alpha \frac{\not{p}^2}{s} \gamma^1 \gamma_\alpha \psi_A(0) \right] \left[ \bar{\psi}_B B_{\mu}^{\alpha} (0) \psi_B(x) \right] \right\} + \frac{1}{4} (\gamma^\alpha \gamma^\beta + \gamma^\beta \gamma^\alpha - g_{\mu\nu} g^{\alpha\beta})$$

$$\left[ \bar{\psi}_A(x) \gamma_\alpha \frac{\not{p}^2}{s} \gamma^1 \gamma_\alpha \psi_A(0) \right] \left[ \bar{\psi}_B B_{\mu}^{\alpha} (0) \psi_B(x) \right]$$

where we used eq. (4.12) $\Xi_1(0) = -\frac{g_{\mu\nu}}{4} \gamma^1 \tilde{B}^i_\alpha \psi_A(0)$. Note that all colors are in the fundamental representation so e.g. $B^{mn}(x) \equiv (t_m)^m B^n(x)$.

Promoting $A$ and $B$ fields to operators and sorting out the color-singlet contributions we get\(^\text{10}\)

$$\bar{W}_{1\mu\nu}^{(1)}(x) = \frac{N_c}{s} \left\{ A, B \right\} \hat{\psi}_A(x) \gamma_\mu \psi_B(x) \left[ \bar{\psi}_B(0) \gamma_\nu \Xi_1(0) \right] + \mu \leftrightarrow \nu |A, B \rangle + x \leftrightarrow 0$$

$$= \frac{g_{\mu\nu}}{2s^2} \left\{ \left[ \hat{\psi}(x) \gamma_\alpha \frac{\not{p}^2}{s} \gamma^1 \gamma_\alpha \psi(x) \right] A \left[ \bar{\psi}_B(0) \gamma_\nu \psi(x) \right] \right\} B$$

$$+ \frac{1}{2s^2} (\gamma^\alpha \gamma^\beta + \gamma^\beta \gamma^\alpha - g_{\mu\nu} g^{\alpha\beta}) \left\{ \left[ \hat{\psi}(x) \gamma_\alpha \frac{\not{p}^2}{s} \gamma^1 \gamma_\alpha \psi(x) \right] A \left[ \bar{\psi}_B(0) \gamma_\nu \psi(x) \right] \right\} B$$

$$+ x \leftrightarrow 0$$

It is convenient to treat terms $\sim g_{\mu\nu}$ separately so we define $\bar{W}_{1\mu\nu}^{(1)}(x) = \bar{W}_{\mu\nu}^{(1a)}(x) + \bar{W}_{\mu\nu}^{(1b)}(x)$ where

$$\bar{W}_{\mu\nu}^{(1a)}(x) = \frac{g_{\mu\nu}}{2s^2} \left\{ \left[ \hat{\psi}(x) \gamma_\alpha \frac{\not{p}^2}{s} \gamma^1 \gamma_\alpha \psi(x) \right] A \left[ \bar{\psi}_B(0) \gamma_\nu \psi(x) \right] \right\} B$$

$$- \left\{ \left[ \hat{\psi}(x) \gamma_\alpha \frac{\not{p}^2}{s} \gamma^1 \gamma_\alpha \psi(x) \right] A \left[ \bar{\psi}_B(0) \gamma_\nu \psi(x) \right] \right\} B$$

$$+ \frac{1}{2} \left[ \hat{\psi}(x) \gamma_\alpha \frac{\not{p}^2}{s} \gamma^1 \gamma_\alpha \psi(x) \right] A \left[ \bar{\psi}_B(0) \gamma_\nu \psi(x) \right] B + x \leftrightarrow 0$$

Hereafter we omit “hat” notation from from operators: $\langle \mathcal{O} \rangle_{A,B} \equiv \langle \hat{\mathcal{O}} \rangle_{A,B}$ for brevity.

\(^\text{10}\)We will keep different notations $A_i$ and $B_i$ for the projectile and target gluon fields because of the relations (A.10) and (A.14).
Let us now estimate this contribution to $\hat{W}_{\mu\nu}$. First, recall that $B_i$ is of order of $m_\perp$ (more accurately, it will be $\sim q_i$ after the Fourier transformation, see e.g. eq. (C.13) or eq. (D.1)). Next, as demonstrated in section C (see eqs. (C.1), (C.2)), $\frac{1}{\alpha}$ in the target matrix element turns to $\pm \frac{1}{\alpha q}$ after Fourier transformation. Due to this fact we will replace $\frac{1}{\alpha}$ by $\frac{1}{\beta q}$ in our estimates, even in the coordinate space. Similarly, for the estimate of the target matrix elements we will replaces operator $\frac{1}{\beta}$ by $\frac{1}{\beta q}$ whenever appropriate.

Now we will demonstrate that three terms in the r.h.s. of eq. (6.8) are small in comparison to terms listed in eq. (5.11). The projectile matrix element in the first term in the r.h.s. of eq. (6.8) brings factor $s$ (see eq. (B.6)) but the target matrix element can produce only factor $x_i$ so the first term is $\sim \frac{g_{\mu\nu} m_\perp^2}{\alpha q^2}$ which is smaller than $\frac{g_{\mu\nu} q^2}{Q^2}$ that we have in eq. (5.11) (and will calculate in the next section). As to the second term in the r.h.s. of eq. (6.8), it can be rewritten as

$$\begin{eqnarray*}
& & - \left\langle \psi(x) \gamma^i \frac{1}{\alpha} \psi(0) \right\rangle_A \langle \hat{W}^i(0) \gamma^j \psi(x) \rangle_B + \frac{2}{s} \left\langle \psi(x) \frac{1}{\alpha} \psi(0) \right\rangle_A \langle \hat{W}^i(0) \gamma^j \psi(x) \rangle_B \\
& & + \langle \psi(0) \gamma^j \psi(x) \rangle_B \langle \bar{\psi} \gamma^i \frac{1}{\alpha} \psi(0) \rangle_A \langle \bar{\psi} \gamma^j \gamma^i \psi(x) \rangle_B \\
& & \times g_{\mu\nu} \frac{2s}{2s^2} B \\
& & (6.9)
\end{eqnarray*}$$

The projectile matrix element in the first term in the r.h.s. of this equation brings factor $s$ but, as we discussed above, the target matrix element cannot produce factor $s$ so this term is again $\sim \frac{g_{\mu\nu} m_\perp^2}{\alpha q^2} \ll \frac{g_{\mu\nu} m_\perp^2}{Q^2}$. As to the second term, converting three $\gamma$-matrices in the projectile matrix element to a combination of $\gamma$’s and $\gamma \gamma_5$’s and looking at the parametrization of section B, we see that $\frac{1}{\beta} \langle \psi(x) \bar{\psi} \gamma_5 \psi(0) \rangle_A$ is not proportional to $s$. In addition, as discuss in section B, the target matrix element $\langle \bar{\psi} \gamma^i \gamma^j \gamma^i \psi(x) \rangle_B$ knows about $p_1$ only via the direction of Wilson lines so it can be proportional only to $\frac{p_{\mu \perp}}{p_{1 \perp}}$ that does not change at rescaling of $p_1$. Thus, $\langle \bar{\psi} \gamma^i \gamma^j \gamma^i \psi(x) \rangle_B \sim O(1)$ and therefore the second term in eq. (6.9) is even smaller than the first one. Finally, let us discuss the third term in the r.h.s. of eq. (6.8). If both $\alpha$ and $\beta$ are transverse

$$\begin{eqnarray*}
& & \frac{g_{\mu\nu}}{4s^2} \left\langle \psi(x) \gamma^i \frac{1}{\alpha} \psi(0) \right\rangle_A \langle \hat{W}^i(0) \gamma^j \psi(x) \rangle_B \sim \frac{g_{\mu\nu} m_\perp^2}{\alpha q^2} \frac{1}{s^2} \frac{(6.10)}{s^2} \end{eqnarray*}$$

similarly to the first term in eq. (6.9). If both indices are longitudinal, we get

$$\begin{eqnarray*}
& & \frac{g_{\mu\nu}}{s^4} \left\langle \psi(x) \gamma^i \frac{1}{\alpha} \psi(0) \right\rangle_A \langle \hat{W}^i(0) \gamma^j \psi(x) \rangle_B \\
& & = \frac{g_{\mu\nu}}{s^4} \left\langle \psi(x) \gamma^i \frac{1}{\alpha} \psi(0) \right\rangle_A \langle \hat{W}^i(0) \gamma^j \psi(x) \rangle_B \frac{s}{2s^2} \frac{(6.11)}{s^2} \end{eqnarray*}$$

The projectile matrix element brings a factor $s$, but the target one is $\sim O(1)$ due to the reason discussed above, so this contribution is negligible. Finally, let us consider the case when index $\alpha$ is longitudinal and $\beta$ is transverse

$$\frac{g_{\mu\nu}}{2s^2} \left\langle \psi(x) \gamma^i \frac{1}{\alpha} \psi(0) \right\rangle_A \langle \hat{W}^i(0) \gamma^j \psi(x) \rangle_B \frac{s}{2s^2} \frac{(6.12)}{s^2}$$
Again, the target matrix element is $\sim O(1)$ while the projectile one can bring one factor of $s$ as can be seen from parametrization (B.6) by reducing the number of $\gamma$-matrices to two. Thus, the contribution (6.12) is negligible and so is the total contribution (6.8).

We get
\[
\tilde{W}_{\mu\nu}^{(1)}(x) \simeq \tilde{W}_{\mu\nu}^{(1b)}(x) = \frac{1}{2s^2} \left\{ \frac{1}{\alpha} \right\}
\]
\[
\langle \bar{\psi}(x) \gamma_{\mu} p_2 \gamma_{\nu} \psi(0) \rangle_A \langle \bar{\psi} B'(0) \gamma_{\nu} \psi(x) \rangle_B + \langle \psi(0) \otimes \psi(x) \leftrightarrow \gamma_5 \psi(0) \otimes \gamma_5 \psi(x) \rangle
\]
\[
- \frac{1}{2s^2} \left\{ \frac{1}{\alpha} \right\}
\]
\[
\langle \bar{\psi}(x) \gamma_{\mu} p_2 \gamma_{\nu} \psi(0) \rangle_A \langle \bar{\psi} B'(0) \gamma_{\nu} \psi(x) \rangle_B + \langle \psi(0) \otimes \psi(x) \leftrightarrow \gamma_5 \psi(0) \otimes \gamma_5 \psi(x) \rangle
\]
\[
(6.13)
\]
Let us start with the case when both of the indices $\mu$ and $\nu$ are transverse. It is easy to see that in all terms the projectile matrix element is proportional to the corresponding term in eq. (5.11). If one index is first power of $s$ we get
\[
\cancel{\tilde{W}_{\mu\nu}^{(1)}(x)} \simeq \tilde{W}_{\mu\nu}^{(1b)}(x) = \left\{ \frac{1}{\alpha} \right\}
\]
\[
\langle \bar{\psi}(x) \gamma_{\mu} p_2 \gamma_{\nu} \psi(0) \rangle_A \langle \bar{\psi} B'(0) \gamma_{\nu} \psi(x) \rangle_B + \langle \psi(0) \otimes \psi(x) \leftrightarrow \gamma_5 \psi(0) \otimes \gamma_5 \psi(x) \rangle
\]
\[
- \frac{1}{2s^2} \left\{ \frac{1}{\alpha} \right\}
\]
\[
\langle \bar{\psi}(x) \gamma_{\mu} p_2 \gamma_{\nu} \psi(0) \rangle_A \langle \bar{\psi} B'(0) \gamma_{\nu} \psi(x) \rangle_B + \langle \psi(0) \otimes \psi(x) \leftrightarrow \gamma_5 \psi(0) \otimes \gamma_5 \psi(x) \rangle
\]
\[
(6.14)
\]
It is easy to see that both projectile and target matrix elements are proportional to the first power of $s$ so the resulting estimate is $\frac{p_2 \mu \nu}{s} m_2^2 \gamma$ which is $\sim O(\frac{m_2^2}{s})$ in comparison to the corresponding term in eq. (5.11). If one index is $p_1$ and the other $p_2$ we get
\[
\frac{g_{\mu\nu}}{s^2} \left\{ \frac{1}{\alpha} \right\}
\]
\[
\langle \bar{\psi}(x) \gamma_{\mu} p_2 \gamma_{\nu} \psi(0) \rangle_A \langle \bar{\psi} B'(0) \gamma_{\nu} \psi(x) \rangle_B + \langle \psi(0) \otimes \psi(x) \leftrightarrow \gamma_5 \psi(0) \otimes \gamma_5 \psi(x) \rangle
\]
\[
- \frac{1}{2s^2} \left\{ \frac{1}{\alpha} \right\}
\]
\[
\langle \bar{\psi}(x) \gamma_{\mu} p_2 \gamma_{\nu} \psi(0) \rangle_A \langle \bar{\psi} B'(0) \gamma_{\nu} \psi(x) \rangle_B + \langle \psi(0) \otimes \psi(x) \leftrightarrow \gamma_5 \psi(0) \otimes \gamma_5 \psi(x) \rangle
\]
\[
(6.15)
\]
It is easy to see that in all terms the projectile matrix element is $\sim s$ but the target one is $\sim O(1)$ so the corresponding contribution $\sim \frac{q_{\mu} m_2^2}{s}$ is negligible.

Finally, let us consider the case when one of the indices $\mu$ or $\nu$ longitudinal and one transverse. For example, let $\mu$ be longitudinal and $\nu$ transverse, the opposite case will differ by replacement $\mu \leftrightarrow \nu$. Using the decomposition of $g_{\mu\nu}$ in longitudinal and transverse part (3.2) we get
\[
\left( \frac{2p_1 \mu p_2^{\nu'}}{s} + p_1 \leftrightarrow p_2 \right) \tilde{W}_{\mu\nu}^{(1b)}(x) = \left( \frac{2p_1 \mu p_2^{\nu'}}{s^2} + p_1 \leftrightarrow p_2 \right)
\]
\[
\times \left\{ \langle \bar{\psi}(x) \gamma_{\mu} p_2 \gamma_{\nu} \psi(0) \rangle_A \langle \bar{\psi} B'(0) \gamma_{\nu} \psi(x) \rangle_B + \langle \psi(0) \otimes \psi(x) \leftrightarrow \gamma_5 \psi(0) \otimes \gamma_5 \psi(x) \rangle
\]
\[
- \frac{1}{s} \left\{ \langle \bar{\psi}(x) \gamma_{\mu} p_2 \gamma_{\nu} \psi(0) \rangle_A \langle \bar{\psi} B'(0) \gamma_{\nu} \psi(x) \rangle_B + \langle \psi(0) \otimes \psi(x) \leftrightarrow \gamma_5 \psi(0) \otimes \gamma_5 \psi(x) \rangle \right\}
\]
\[
+ x \leftrightarrow 0
\]
\[
(6.16)
\]
The term proportional to \( p_{2\mu} \) in the r.h.s. can be expressed using eq. (A.13) as follows

\[
\frac{p_{2\mu}}{s^3} \left\{ \left[ \left< \bar{\psi}(x)\gamma_{\nu\perp} p_2 \gamma_i \frac{1}{\alpha} \psi(0) \right>_A \left< \bar{\psi}B^i(0) p_1 \psi(x) \right>_B \right] + \left< \bar{\psi}(x) p_1 p_2 \gamma_i \frac{1}{\alpha} \psi(0) \right>_A \left< \bar{\psi}B^i(0) \gamma_{\nu\perp} \psi(x) \right>_B + (\psi(0) \otimes \psi(x) \leftrightarrow \gamma_5 \psi(0) \otimes \gamma_5 \psi(x)) \right\}
\]

(6.17)

\[
= -\frac{p_{2\mu}}{s^3} \left[ \left< \bar{\psi}(x) p_2 \frac{1}{\alpha} \psi(0) \right>_A \left< \bar{\psi}(0) p_1 B_{\nu\perp}(0) \psi(x) \right>_B \right] + \left< \bar{\psi}(x) p_2 \frac{1}{\alpha} \psi(0) \right>_A \left< \bar{\psi}(0) B(0) p_1 \gamma_{\nu\perp} \psi(x) \right>_B
\]

(6.18)

As we shall see below, due to QCD equations of motion \( B \) in the r.h.s. of this equation can be replaced by transverse momentum of the target TMD \( k_{\perp} \). Also, \( \frac{1}{\alpha} \) will be replaced by \( \frac{1}{\alpha_0} \) so from the parametrizations \( (B.1) \) and \( (B.4) \) we see that

\[
\frac{p_{2\mu}}{s^3} \left< \bar{\psi}(x) p_2 \frac{1}{\alpha} \psi(0) \right>_A \left< \bar{\psi}(0) B(0) p_1 \gamma_{\nu\perp} \psi(x) \right>_B \sim \frac{p_{2\mu}}{\alpha_0 s^3} k_{\mu} f \tilde{f}
\]

(6.19)

which is of order of fourth term in eq. (5.11). The second relevant term is

\[
\frac{ip_{2\mu}}{s^3} \left< \bar{\psi}(x) \sigma_{\nu\perp j} \frac{1}{\alpha} \psi(0) \right>_A \left< \bar{\psi}B^j(0) \psi(x) \right>_B - \frac{p_{2\mu}}{s^3} \left< \bar{\psi}(x) \sigma_{\nu\perp i} \frac{1}{\alpha} \psi(0) \right>_A \left< \bar{\psi}B^i(0) \sigma_{\nu\perp} \psi(x) \right>_B
\]

\[
= \frac{p_{2\mu}}{s^3} \left< \bar{\psi}(x) \sigma_{\mu j} \frac{1}{\alpha} \psi(0) \right>_A \left< \bar{\psi}(0) [B_{\nu}(0) \sigma_{\mu j} - \nu \leftrightarrow j] \psi(x) \right>_B - \frac{p_{2\mu}}{s^3} \left< \bar{\psi}(x) \sigma_{\nu\perp} \frac{1}{\alpha} \psi(0) \right>_A \left< \bar{\psi}(0) B(0) p_1 \psi(x) \right>_B
\]

(6.20)

where we used formula \( (A.4) \) and the fact that for unpolarized protons

\[
\left< p | \bar{\psi}(0) | A_i(0) \sigma_{\mu j} - i \leftrightarrow j | \psi(x) | p \right> = 0
\]

(6.21)
from parity conservation.\footnote{A rigorous argument goes like that: the matrix element (6.21) can be rewritten as $\epsilon_{\nu_\perp j} \langle \hat{\psi}(0) | A_{k}(0) | \psi(x) \rangle = \epsilon_{\nu_\perp j} \langle \psi(0) | A(0) | \bar{p}_1 \gamma_5 \psi(x) \rangle$. As demonstrated in section C, $A$ in this formula can be replaced by $k_\perp$ so the contribution is proportional to matrix element $k_\perp | \bar{p}_1 \gamma_5 \psi(x) \rangle = k_\perp \epsilon_{ij} \langle \bar{p}_i \gamma_5 | \psi(x) \rangle$ which vanishes as seen from the parametrization (B.6).} Again, $\frac{1}{\alpha}$ will turn to $\frac{1}{\alpha_q}$ and $B$ can be replaced by $k_\perp$ for the target, so (6.11) is of order of

$$\frac{ip_\perp}{s^2} \left( \bar{\psi}(x) \sigma_{\nu_\perp} \frac{1}{\alpha} \psi(0) \right)_A \langle \bar{\psi}(0) B(0) \bar{p}_1 \psi(x) \rangle_B \sim \frac{p_\perp}{\alpha_q s^2} k_\perp h \tag{6.22}$$

Let us demonstrate that the remaining terms in the r.h.s. of eq. (6.17) are negligible. First, term coming from replacement $\psi(0) \otimes \psi(x) \leftrightarrow \gamma_5 \psi(0) \otimes \gamma_5 \psi(x)$ in eq. (6.18) vanishes since $\langle \bar{\psi}(x) \bar{p}_2 \gamma_5 \psi(0) \rangle_A = 0$ for unpolarized hadrons, see eq. (B.5). Next, term $\frac{p_\perp}{2s} \langle \bar{\psi}(x) \gamma_5 \bar{p}_1 \psi(x) \rangle_B \sim \frac{p_\perp}{\alpha_q s} k_\perp h \sim \frac{p_\perp}{\alpha_q s} k_\perp h \sim 0$ is small because neither projectile no target matrix elements can bring factor $s$. Last, using eq. (A.4) we get

$$x^2 \frac{ip_\perp}{s^2} \left( \bar{\psi}(x) \sigma_{ij} \sigma_{\nu_\perp} \frac{1}{\alpha} \psi(0) \right)_A \langle \bar{\psi} B^j(0) \sigma_{\nu_\perp} \psi(x) \rangle_B \sim \frac{p_\perp}{\alpha_q s^2} k_\perp h \tag{6.23}$$

It is easy to see that neither the projectile nor the target matrix element in the r.h.s. of this equation gives $s$ so these terms can be neglected in comparison to eqs. (6.19) and (6.22).

Thus, the two non-negligible terms in eq. (6.17) give

$$\tilde{W}^{(1)}_{1\mu}(x) = \frac{N_c}{s} \langle A, B | \bar{\psi} A(x) \gamma_{\mu} B(x) \left[ \bar{\psi} B(0) \gamma_\nu \Xi_1(0) \right] + \mu \leftrightarrow \nu | A, B \rangle + x \leftrightarrow 0$$

$$= \frac{p_\perp}{s^2} \left( \bar{\psi}(x_s, x_\perp) \frac{1}{\alpha} \psi(0) \right)_A \langle \bar{\psi} B(0) \bar{p}_1 \psi(x_s, x_\perp) \rangle_B$$

$$+ i \left( \bar{\psi}(x_s, x_\perp) \sigma_{\nu_\perp} \psi(0) \right)_A \langle \bar{\psi} B(0) \bar{p}_1 \psi(x_s, x_\perp) \rangle_B + \mu \leftrightarrow \nu + x \leftrightarrow 0 \tag{6.24}$$

Using formulas (C.1), (C.2), (C.4), (C.7), (C.12), and (C.14) for quark-antiquark-gluon...
operators and parametrizations from section B we get the contribution to $W_{\mu \nu}$ in the form

$$W^{(1)}_{1\mu\nu}(q) = \frac{1}{16\pi^2 s^4} \int dx_1 dx_2 d^2 x_\perp e^{-ix\cdot x - i\beta_\perp + i(q\cdot x)_\perp}$$

$$\times \langle A, B | [\bar{\psi}(x) \gamma_\mu \psi_B(x)] \bar{\psi}_B(0) \gamma_\nu \Xi_1(0) \rangle + x \leftrightarrow 0 \langle A, B \rangle + \mu \leftrightarrow \nu$$

$$= \frac{1}{64\pi^6 N_c^2 s^3} \int d^2 k_\perp \int dx_1 dx_2 d^2 x_\perp e^{-ix\cdot x - i\beta_\perp + i(q\cdot x)_\perp}$$

$$\times \left\{ \langle \bar{\psi}(x) \gamma_\mu \sigma_\nu \bar{\psi}(x') \rangle \bar{\psi}_B(0) \gamma_\nu \Xi_1(0) \right\} + x \leftrightarrow 0 \langle A, B \rangle + \mu \leftrightarrow \nu$$

$$= \frac{p_{1\mu}}{\alpha_q s N_c^2} \int d^2 k_\perp \left\{ (q - k)_\nu \left[ f_1^f(\alpha_q, k_\perp) f_1^f(\beta_q, (q - k)_\perp) + f_1^f(\alpha_q, k_\perp) f_1^f(\beta_q, (q - k)_\perp) \right]$$

$$- k_\perp (q - k)_\perp \left[ h_1^f(\alpha_q, k_\perp) h_1^f(\beta_q, (q - k)_\perp) + h_1^f(\alpha_q, k_\perp) h_1^f(\beta_q, (q - k)_\perp) \right] \right\} + x \leftrightarrow 0 \langle A, B \rangle$$

where terms with replacement $f_1^f \leftrightarrow f_1^f$ and $h_1^f \leftrightarrow h_1^f$ come from $x \leftrightarrow 0$ contribution.

Next we consider the remaining $\sim p_{1\mu}$ term in eq. (6.13) which can be rewritten as

$$\frac{2 p_{1\mu}}{s^2} N_c \langle A, B | [\bar{\psi}_A(x) \gamma_\mu \psi_B(x)] [\bar{\psi}_B(0) \gamma_\nu \Xi_1(0)] \rangle$$

$$= \frac{p_{1\mu}}{s^3} \left\{ \langle \bar{\psi}(x) \gamma_\mu \sigma_\nu \gamma_\nu \frac{1}{\alpha} \gamma_\lambda \bar{\psi}(x') \rangle \bar{\psi}_B(0) \gamma_\nu \Xi_1(0) \right\}$$

$$+ (\psi(0) \otimes \psi(x) \leftrightarrow \gamma_5 \psi(0) \otimes \gamma_5 \psi(x)) \right\}$$

$$+ \frac{p_{1\mu}}{s^3} \left\{ \langle \bar{\psi}(x) \gamma_\mu \sigma_\nu \gamma_\nu \frac{1}{\alpha} \gamma_\lambda \bar{\psi}(x') \rangle \bar{\psi}_B(0) \gamma_\nu \Xi_1(0) \right\}$$

$$+ \frac{i p_{1\mu}}{s^3} \left\{ \langle \bar{\psi}(x) \gamma_\mu \sigma_\nu \gamma_\nu \frac{1}{\alpha} \gamma_\lambda \bar{\psi}(x') \rangle \bar{\psi}_B(0) \gamma_\nu \Xi_1(0) \right\}$$

$$= \frac{p_{1\mu}}{s^3} \left\{ \langle \bar{\psi}(x) \gamma_\mu \gamma_\nu \frac{1}{\alpha} \gamma_\lambda \bar{\psi}(x') \rangle \bar{\psi}_B(0) \gamma_\nu \Xi_1(0) \right\}$$

$$+ (\psi(0) \otimes \psi(x) \leftrightarrow \gamma_5 \psi(0) \otimes \gamma_5 \psi(x)) \right\}$$

$$+ \frac{p_{1\mu}}{s^3} \left\{ \langle \bar{\psi}(x) \gamma_\mu \gamma_\nu \frac{1}{\alpha} \gamma_\lambda \bar{\psi}(x') \rangle \bar{\psi}_B(0) \gamma_\nu \Xi_1(0) \right\}$$

$$+ \frac{p_{1\mu}}{s^3} \left\{ \langle \bar{\psi}(x) \gamma_\mu \gamma_\nu \frac{1}{\alpha} \gamma_\lambda \bar{\psi}(x') \rangle \bar{\psi}_B(0) \gamma_\nu \Xi_1(0) \right\}$$

where again we used formula (A.4). Note that while the matrix elements between projectile states give contributions $\sim \frac{s}{\alpha_q k_\perp}$, the target matrix elements cannot give $s$. Indeed, these target matrix elements know about $p_1$ only through direction of Wilson lines so they should not change under rescaling $p_1 \rightarrow \lambda p_1$, see the discussion in section B. Thus, the r.h.s. of eq. (6.26) is $\sim \frac{p_{1\mu}}{\alpha_q s^2} k_\perp$ which means that the $p_{1\mu}$ term in eq. (6.16) is

$$\frac{2 p_{1\mu}}{s^2} N_c \langle A, B | [\bar{\psi}_A(x) \gamma_\mu \psi_B(x)] [\bar{\psi}_B(0) \gamma_\nu \Xi_1(0)] \rangle \sim \frac{p_{1\mu} q_\perp}{\alpha_q s^2}$$

(6.27)
so the corresponding contribution to $W_{\mu\nu}$ is $\sim \frac{p_1 q_\perp^2}{\alpha q^2 x}$ which is $O(1)$ in comparison to that of eq. (6.28).

Thus, the contribution of the first term in eq. (6.2) to $W_{\mu\nu}$ is

$$W^{(1)}_{\mu\nu}(q) = \frac{1}{16\pi^2} \frac{1}{s} \int dx_1 dx_2 d^2 x_\perp e^{-i \alpha x_1 - i \beta x_2 + i (q, x_\perp)_{\perp}}$$

$$\times \langle A, B | [\bar{\psi}_A(0) \gamma_\mu \psi_B(x)] [\bar{\psi}_B(0) \gamma_\nu \psi_A(0)] + (x \leftrightarrow 0) | A, B \rangle + \mu \leftrightarrow \nu$$

$$= \frac{p_2 \mu}{s N_c} \int d^2 k \left[ (q - k)^\perp F^I(q, k_\perp) - k_\nu \frac{(q - k)^2}{m^2} H^I(q, k_\perp) \right] + \mu \leftrightarrow \nu$$

where $F^I(q, k_\perp)$ and $H^I(q, k_\perp)$ are given by expressions (2.2) with $x_A \equiv \alpha_q$ and $x_B \equiv \beta_q$

$$F^I(q, k_\perp) = f^I_1(\alpha_q, k_\perp) \tilde{f}^I_2(\beta_q, (q - k)_\perp) + f^I_2(\beta_q, (q - k)_\perp) \tilde{f}^I_1(\alpha_q, k_\perp) + \tilde{h}^I_{1f} \leftrightarrow h^I_{1f}$$

(6.29)

Let us consider now the second term in eq. (6.2). The calculation repeats that of the first term so we will indicate here main steps and pay attention to non-negligible terms only. If one of the indices (say, $\mu$) is longitudinal and the other transverse, we get

$$\left( \frac{2p_1 p_2^\prime}{s} + p_1 \leftrightarrow p_2 \right) \frac{N_c}{s} \langle A, B | [\bar{\Xi}_1(x) \gamma_\mu \psi_B(x)] [\bar{\psi}_B(0) \gamma_\nu \psi_A(0)] + \mu' \leftrightarrow \nu | A, B \rangle$$

$$= \langle A, B | p_2 \mu \left[ [\bar{\Xi}_1(x) \gamma_\mu \psi_B(x)] [\bar{\psi}_B(0) \gamma_\nu \psi_A(0)] + [\bar{\Xi}_1(x) \gamma_\nu \psi_B(x)] [\bar{\psi}_B(0) \gamma_\mu \psi_A(0)] \right]$$

$$+ p_1 \mu \left[ [\bar{\Xi}_1(x) \gamma_\nu \psi_B(x)] [\bar{\psi}_B(0) \gamma_\mu \psi_A(0)] \right] \frac{2N_c}{s^2}$$

(6.30)

where we used $\bar{\Xi}_1 = - (\bar{\psi}_A \frac{1}{\alpha}) \gamma^4 B_i \frac{\vec{p}_2}{s}$. The most important terms are those proportional to $p_2 \mu$. Using Fierz transformation and separating color singlets, they can be rewritten as (cf. eq. (6.17))

$$\frac{N_c}{s} \langle A, B | [\bar{\Xi}_1(x) \gamma_\mu \psi_B(x)] [\bar{\psi}_B(0) \gamma_\nu \psi_A(0)] + \mu \leftrightarrow \nu | A, B \rangle$$

$$= \frac{p_2 \mu}{s^3} \left\{ \frac{1}{\alpha} \left[ \langle \bar{\psi}(0) \gamma^4 B_i(0) \psi(x) \rangle_B \right]$$

$$+ \langle \bar{\psi}(0) \gamma^4 B_i(0) \psi(x) \rangle_B \right\}$$

(6.31)

After some algebra with $\gamma$-matrices this can be transformed to

$$W^{(1)}_{2\mu\nu}(x) = \frac{N_c}{s} \langle A, B | [\bar{\Xi}_1(x) \gamma_\mu \psi_B(x)] [\bar{\psi}_B(0) \gamma_\nu \psi_A(0)] + \mu \leftrightarrow \nu | A, B \rangle + x \leftrightarrow 0$$

$$= \frac{p_2 \mu}{s^3} \left\{ \frac{1}{\alpha} \left[ \langle \bar{\psi}(0) \gamma^4 \gamma^\nu \psi(0) \rangle_B \right]$$

$$- i \langle \bar{\psi}(0) \gamma^4 \psi(0) \rangle_B \right\}$$

(6.32)
plus terms small in comparison to $\frac{p_{\perp\mu} p_{\perp\nu}}{q^2}$. Using eq. (C.2) we can transform $(\bar{\psi}_{\perp\alpha}(x))^2$ to

\[
\int dx_\bullet d^2x_\perp e^{-i\alpha x_\bullet + i(k, x)_\perp} \langle \left( \bar{\psi}_{\perp\alpha}(x) \Gamma \psi(0) \right)_A \\
= i \int dx_\bullet \int_{-\infty}^{\infty} dx_\perp' d^2x_\perp e^{-i\alpha x_\bullet + i(k, x)_\perp} \langle \bar{\psi}(x_\bullet', x_\perp) \Gamma \psi(0) \rangle_A \\
= -\frac{1}{\alpha_q} \int dx_\bullet d^2x_\perp e^{-i\alpha x_\bullet + i(k, x)_\perp} \langle \bar{\psi}(x_\bullet, x_\perp) \Gamma \psi(0) \rangle_A
\]

(6.33)

Using QCD equation of motion and other formulas from sections B and C one gets

\[
\frac{1}{16\pi^4} \frac{1}{s} \int dx_\bullet dx_\perp d^2x_\perp e^{-i\alpha x_\bullet - i\beta x_\perp + i(q, x)_\perp} \\
\times \langle A, B | [\Xi_{1}(x) \gamma_\mu \psi_B(x)] [\bar{\psi}_B(0) \gamma_\nu \psi_A(0)] + \mu \leftrightarrow \nu | A, B \rangle \\
= \frac{p_{\perp\mu} p_{\perp\nu}}{\alpha_q N_c} \left[ (q - k)_\nu f_1^f (\alpha_q, k_\perp) f_1^f (\beta_q, (q - k)_\perp) \\
- k_\nu (q - k)^2 m^2 \frac{H_{f}^f (\alpha_q, k_\perp) H_{f}^f (\beta_q, (q - k)_\perp)}{m^2} \right] + \mu \leftrightarrow \nu
\]

(6.34)

so the contribution of eq. (6.28) is effectively doubled. Again, the term with $x \leftrightarrow 0$ exchange leads to eq. (6.34) with $f_1 \leftrightarrow f_1$ and $h_1^f \leftrightarrow h_1^f$ replacement.

Thus, the sum of first and second terms in eq. (6.2) leads to twice eq. (6.28)

\[
W^{(1)\mu\nu}_{1+2} = \frac{2p_{\perp\mu}^2}{\alpha_q s} \frac{1}{N_c \phi} \int d^2k_\perp \left[ (q - k)^\perp \Gamma_f (q, k_\perp) - k_\perp^\nu \frac{(q - k)^2}{m^2} \frac{H_f^f (q, k_\perp)}{m^2} \right] + \mu \leftrightarrow \nu
\]

(6.35)

6.1.2 Term with $\Xi_2$

In this section we calculate the third term in eq. (6.2).

\[
\tilde{W}^{(1)}_{3\mu\nu} = \frac{N_c}{s} \langle A, B | \bar{\psi}_A(x) \gamma_\mu \Xi_2(x) | \bar{\psi}_B(0) \gamma_\nu \psi_A(0) \rangle + \mu \leftrightarrow \nu | A, B \rangle + x \leftrightarrow 0
\]

(6.36)

Again, main contribution correspond to one index (e.g. $\mu$) being longitudinal and the other transverse so we need

\[
\left( \frac{2p_{\perp\mu}^2}{s} + \mu \leftrightarrow \mu' \right) \frac{N_c}{s} \langle A, B | \bar{\psi}_A(x) \gamma_\mu \Xi_2(x) | \bar{\psi}_B(0) \gamma_\nu \psi_A(0) \rangle + \mu' \leftrightarrow \nu | A, B \rangle \\
= \langle A, B | p_{\perp\mu} \left[ \bar{\psi}_A(x) \gamma_\mu \Xi_2(x) \right] [\bar{\psi}_B(0) \gamma_\nu \psi_A(0)] + [\bar{\psi}_A(x) \gamma_\nu \psi_A(0)] [\bar{\psi}_B(0) \gamma_\mu \Xi_2(x)] \left[ \bar{\psi}_A(0) \gamma_\mu \psi_A(0) \right] \rangle \\
+ p_{\perp\mu} \left[ [\bar{\psi}_A(x) \gamma_\mu \Xi_2(x)] [\bar{\psi}_B(0) \gamma_\nu \psi_A(0)] | A, B \rangle \right] \frac{2N_c}{s^2}
\]

(6.37)

(recall that $\Xi_2(x) = -\frac{1}{2} \gamma^i A_i \frac{1}{\gamma^j} \psi_B(x)$ so $\bar{\psi}_1 \Xi_2 = 0$).
Let us consider first the term proportional to $p_{1\mu}$. Performing Fierz transformation (A.1) and sorting out the color-singlet contributions we get (cf. eq. (C.2))

$$
\frac{2p_{1\mu}}{s} \frac{p_{2\nu}^t}{s} N_c \langle A, B | [\bar{\psi}_B(0) \gamma_{\mu} \psi_A(0)] [\bar{\psi}_A(x) \gamma_{\nu} \Xi_2(x)] + \mu' \leftrightarrow \nu | A, B \rangle
= \frac{p_{1\mu}}{s^2} \left\{ \left. \frac{1}{2} \left( \langle \bar{\psi} \gamma_\nu A \bar{A}(x) \psi(0) \rangle_B \langle \bar{\psi}(0) \gamma^t_1 \beta \psi(x) \rangle_B \right) + (\psi(0) \otimes \psi(x) \leftrightarrow \gamma_5 \psi(0) \otimes \gamma_5 \psi(x) \right) \\
- \frac{1}{s} \left. \langle \bar{\psi} p_{2\nu} \bar{A}_\nu(x) \psi(0) \rangle_B \langle \bar{\psi}(0) \gamma^t_1 \beta \psi(x) \rangle_B \right) + (\psi(0) \otimes \psi(x) \leftrightarrow \gamma_5 \psi(0) \otimes \gamma_5 \psi(x) \right) \\
- \frac{1}{s} \left. \langle \bar{\psi} A^i(x) \sigma_{\nu \mu} \psi(0) \rangle_B \langle \bar{\psi}(0) \gamma^t_1 \beta \psi(x) \rangle_B \right) - \frac{2i}{s^2} \left. \langle \bar{\psi} A^i(x) \sigma_{\nu \mu} \psi(0) \rangle_B \langle \bar{\psi}(0) \gamma^t_1 \beta \psi(x) \rangle_B \right) \right\}
+ \frac{i}{s} \left. \langle \bar{\psi}(0) \sigma_{\nu \mu} \gamma^t_1 \beta \psi(x) \rangle_B \langle \bar{\psi} A^i(x) \sigma_{\nu \mu} \gamma^t_1 \beta \psi(x) \rangle_B \right) + \frac{i}{s} \left. \langle \bar{\psi}(0) \sigma_{\nu \mu} \gamma^t_1 \beta \psi(x) \rangle_B \langle \bar{\psi} A^i(x) \sigma_{\nu \mu} \gamma^t_1 \beta \psi(x) \rangle_B \right)
$$

(6.38)

It is clear that matrix elements in the first line in the r.h.s. can produce only transverse factors $\sim q_\perp$ so the corresponding contribution $\sim \frac{p_{1\mu} q_\perp^2 m^2}{s q_\perp^2}$ can be neglected. Also, matrix element $\langle \bar{\psi}(0) \gamma^t_1 \beta \psi(x) \rangle_B$ vanishes as seen from eq. (B.5). The remaining terms can be rewritten as

$$
\frac{2p_{1\mu}}{s} \frac{p_{2\nu}^t}{s} N_c \langle A, B | [\bar{\psi}_B(0) \gamma_{\mu} \psi_A(0)] [\bar{\psi}_A(x) \gamma_{\nu} \Xi_2(x)] + \mu' \leftrightarrow \nu | A, B \rangle
= \frac{p_{1\mu}}{s^3} \left\{ \left. \langle \bar{\psi} \gamma_\nu A \bar{A}(x) \psi(0) \rangle_B \langle \bar{\psi}(0) \gamma^t_1 \beta \psi(x) \rangle_B \right) - \langle \bar{\psi} A^i(x) \sigma_{\nu \mu} \psi(0) \rangle_B \langle \bar{\psi}(0) \gamma^t_1 \beta \psi(x) \rangle_B \right\}
+ \frac{i}{s} \langle \bar{\psi}(0) \sigma_{\nu \mu} \gamma^t_1 \beta \psi(x) \rangle_B \langle \bar{\psi} A^i(x) \sigma_{\nu \mu} \gamma^t_1 \beta \psi(x) \rangle_B
$$

(6.39)

where we have used eq. (A.4). It is clear that only the first line in the r.h.s. can give the non-negligible contribution to $W_{\mu \nu}$. Indeed, matrix element $\langle \bar{\psi}(x) A_{\nu \perp}(x) \sigma_{\nu \mu} - A_j(x) \sigma_{\nu \mu} \rangle \psi(0) \rangle_A$ vanishes for unpolarized hadrons due to parity, see eq. (6.21). In the third line in the r.h.s., neither matrix element can produce $s$ so the corresponding contribution is again $\sim \frac{p_{1\mu} q_\perp^2 m^2}{s q_\perp^2}$ while contribution from the last line is even smaller, of order of $\frac{p_{1\mu} q_\perp^2 m^4}{s^2 q_\perp^2}$. Thus, we get

$$
\frac{2p_{1\mu}}{s} \frac{p_{2\nu}^t}{s} N_c \langle A, B | [\bar{\psi}_B(0) \gamma_{\mu} \psi_A(0)] [\bar{\psi}_A(x) \gamma_{\nu} \Xi_2(x)] + \mu' \leftrightarrow \nu | A, B \rangle
= \frac{p_{1\mu}}{s^3} \left\{ \left. \langle \bar{\psi} \gamma_\nu A \bar{A}(x) \psi(0) \rangle_B \langle \bar{\psi}(0) \gamma^t_1 \beta \psi(x) \rangle_B \right) + i \langle \bar{\psi} A^i(x) \psi(0) \rangle_B \langle \bar{\psi}(0) \sigma_{\nu \mu} \gamma^t_1 \beta \psi(x) \rangle_B \right\}
$$

(6.40)

It remains to prove that the last term in eq. (6.37) proportional to $p_{2\mu}$ is small. One can rewrite that term similarly to eq. (6.26) with replacement $p_1 \leftrightarrow p_2$ and (projectile matrix elements) $\leftrightarrow$ (target ones). After that, the proof repeats arguments after eq. (6.26) and
one obtains the estimate
\[
\frac{2p_2\mu}{s^2} N_c \langle A, B | [\bar{\psi}_A(x)\gamma_\mu \Xi_2(x)] [\bar{\psi}_B(0) p_1\psi_A(0)] | A, B \rangle \sim p_2 \mu q_{\perp} \frac{m_2^2}{\beta_\| s^2} \tag{6.41}
\]

Similarly, by repeating arguments from section 6.1.1 with replacement \( p_1 \leftrightarrow p_2 \) and projectile matrix elements \( \leftrightarrow \) target ones, one can demonstrate that terms in eq. (6.36) with \( \mu, \nu \) both longitudinal or both transverse are small in comparison to terms listed in eq. (5.11).

Thus,
\[
\vec{W}^{(1)}_{3\mu\nu}(x) = \frac{N_c}{s} \langle A, B | [\bar{\psi}_A(x)\gamma_\mu \Xi_2(x)] [\bar{\psi}_B(0)\gamma_\nu \psi_A(0)] + \mu \leftrightarrow \nu | A, B \rangle + x \leftrightarrow 0 \\
= \frac{p_1\mu}{s^2} \left[ \langle \bar{\psi}_A(x, x_\perp) p_2\gamma_\nu \psi(0) | A \rangle \left( \bar{\psi}(0) \frac{1}{\beta} \psi(x_\perp, x_\perp') \right)_B \right. \\
+ i \langle \bar{\psi}_A(x, x_\perp) p_2\psi(0) | A \rangle \left( \bar{\psi}(0) \sigma_{\mu\nu} \frac{1}{\beta} \psi(x_\perp, x_\perp') \right)_B + \mu \leftrightarrow \nu \left. \right] + x \leftrightarrow 0 \tag{6.42}
\]

Using QCD equation of motion and formulas from appendix, we obtain the corresponding contribution to \( W_{\mu\nu} \) in the form
\[
\frac{1}{16\pi^2} \frac{1}{s} \int dx_\bullet dx_{s\perp} d^2x_{\perp} e^{-i\alpha x_\bullet - i\beta x_{s\perp} + i(q, x)_\perp} \\
\times \langle A, B | [\bar{\psi}_A(x)\gamma_\mu \Xi_2(x)] [\bar{\psi}_B(0)\gamma_\nu \psi_A(0)] | A, B \rangle + \mu \leftrightarrow \nu \\
= \frac{1}{64\pi^6 N_c} \frac{p_1\mu}{s^3} \int dx_\bullet dx_{s\perp} d^2k_{\perp} \int dx_\bullet d^2x_{\perp} e^{-i\alpha x_\bullet + i(k, x)_\perp} \int dx_{s\perp} d^2x'_{\perp} e^{-i\beta x_{s\perp} + i(q-k, x')_\perp} \\
\times \left[ \langle \bar{\psi}_A(x_\bullet, x_{s\perp}) p_2\gamma_\nu \psi(0) | A \rangle \left( \bar{\psi}(0) \frac{1}{\beta} \psi(x_{s\perp}, x'_{\perp}) \right)_B \right. \\
+ i \langle \bar{\psi}_A(x_\bullet, x_{s\perp}) p_2\psi(0) | A \rangle \left( \bar{\psi}(0) \sigma_{\mu\nu} \frac{1}{\beta} \psi(x_{s\perp}, x'_{\perp}) \right)_B + \mu \leftrightarrow \nu \left. \right] \\
= \frac{p_1\mu}{\beta_\| s N_c} \int d^2k_{\perp} \left[ k_\nu F_{\parallel}(\alpha_q, k_{\perp}) F_{\perp}(\beta_q, (q-k)_\perp) \right. \\
- (q-k)_\nu \frac{k_\perp^2}{m_2^2} H_{\parallel}(\alpha_q, k_{\perp}) H_{\perp}(\beta_q, (q-k)_\perp) + \mu \leftrightarrow \nu \tag{6.43}
\]

Same as in previous section, the term with \( x \leftrightarrow 0 \) exchange leads to eq. (6.43) with \( f_1 \leftrightarrow \tilde{f}_1 \) and \( h_1^+ \leftrightarrow \tilde{h}_1^+ \) replacement so we get
\[
W^{(1)}_{3\mu\nu} = \frac{p_1\mu}{\beta_\| s N_c} \int d^2k_{\perp} \left[ k_\nu F_{\parallel}(q, k_{\perp}) - (q-k)_\nu \frac{k_\perp^2}{m_2^2} H_{\parallel}(q, k_{\perp}) \right] + \mu \leftrightarrow \nu \tag{6.44}
\]

Repeating arguments from previous section, it is possible to show that the contribution of the fourth term
\[
\vec{W}^{(1)}_{4\mu\nu}(x) = \frac{N_c}{s} \langle A, B | [\bar{\psi}_A(x)\gamma_\mu \Xi_2(x)] [\bar{\psi}_B(0)\gamma_\nu \psi_A(0)] + \mu \leftrightarrow \nu | A, B \rangle + x \leftrightarrow 0 \\
= \frac{p_1\mu}{s^2} \left[ \langle \bar{\psi}_\gamma \gamma_\nu p_2 A(0) \psi(0) | A \rangle \left( \bar{\psi}(0) \frac{1}{\beta} \psi(x_\perp, x_\perp') \right)_B \right. \\
- i \langle \bar{\psi}(x, x_\perp) p_2 A(0) \psi(0) | A \rangle \left( \bar{\psi}(0) \sigma_{\mu\nu} \psi(x, x_\perp') \right)_B + \mu \leftrightarrow \nu \left. \right] + x \leftrightarrow 0 \tag{6.45}
\]
doubles that of the third term so we get the full contribution of the terms with one quark-antiquark-gluon operators in the form

\[
W^{(1)}_{\mu
u} = \frac{2}{N_c} \int d^2 k_\perp \left[ \left( \frac{p_{1\mu} k_{\perp \nu}}{\beta q s} + \frac{p_{2\mu}(q-k)_{\perp \nu}}{\alpha q s} \right) F^f(q, k_\perp) \right. \\
- \left. \left( \frac{p_{1\mu}(q-k)_{\perp \nu}}{\beta q s} \frac{k_{\perp \nu}^2}{m^2} + \frac{p_{2\mu} k_{\perp \nu}^2}{\alpha q s} \frac{(q-k)_{\perp \nu}^2}{m^2} \right) H^f(q, k_\perp) \right] + \mu \leftrightarrow \nu
\]  

(6.46)

This result agrees with the corresponding $1/Q$ terms in ref. [20].

6.2 Term with two quark-quark-gluon operators coming from $\Xi_1$ and $\Xi_2$

Let us start with the first term in the r.h.s. of eq. (6.3). Performing Fierz transformation (A.1) we obtain

\[
\frac{N_c}{s} \langle A, B | (\bar{\psi}_A(x) \gamma_{\mu} \Xi_2(x)) (\bar{\psi}_B(0) \gamma_{\nu} \Xi_1(0)) + \mu \leftrightarrow \nu | A, B \rangle + x \leftrightarrow 0 = g_{\mu\nu} \tilde{V}_1 + \tilde{V}_{2\mu\nu} + \tilde{V}_{3\mu\nu}
\]  

(6.47)

where

\[
\tilde{V}_1 = \frac{N_c}{2 s} \langle A, B | -[\bar{\psi}_A(x) \Xi_1^\mu(0)][\bar{\psi}_B(0) \Xi_2^\mu(x)] + [\bar{\psi}_A(x) \gamma_5 \Xi_1^\mu(0)][\bar{\psi}_B(0) \gamma_5 \Xi_2^\mu(x)] + \mu \leftrightarrow \nu | A, B \rangle + x \leftrightarrow 0,
\]

(6.48)

\[
\tilde{V}_{2\mu\nu} = \frac{N_c}{2 s} \langle A, B | -[\bar{\psi}_A(x) \gamma_{\mu} \Xi_1^\nu(0)][\bar{\psi}_B(0) \gamma_{\nu} \Xi_2^\mu(x)] \\
- [\bar{\psi}_A(x) \gamma_{\mu} \gamma_5 \Xi_1^\nu(0)][\bar{\psi}_B(0) \gamma_\nu \Xi_2^\mu(x)] + \mu \leftrightarrow \nu | A, B \rangle + x \leftrightarrow 0,
\]  

(6.49)

and

\[
\tilde{V}_{3\mu\nu} = \frac{N_c}{2 s} \langle A, B | -[\bar{\psi}_A(x) \gamma_{\mu} \Xi_1^\nu(0)][\bar{\psi}_B(0) \gamma_{\mu} \Xi_2^\nu(x)] + \mu \leftrightarrow \nu \\
- g_{\mu\nu}[\bar{\psi}_A(x) \sigma_{\alpha\beta} \Xi_1^\nu(0)][\bar{\psi}_B(0) \sigma_{\alpha\beta} \Xi_2^\nu(x)] | A, B \rangle + x \leftrightarrow 0
\]  

(6.50)

It is convenient to define $\tilde{V}_{3\mu\nu}$ to be traceless. In next sections, we will consider these terms in turn.

6.2.1 Term propotional to $g_{\mu\nu}$

Using $\Xi_1 = -\frac{g_F}{s} \gamma^i B_i \frac{1}{\alpha + i \epsilon} \psi_A$ and $\Xi_2 = -\frac{g_F}{s} \gamma^i A_i \frac{1}{\beta + i \epsilon} \psi_B$ from eq. (4.12) and extracting color-singlet contributions one obtains

\[
\tilde{V}_1 = \frac{1}{2 s^3} \left\{ -\left[ \langle \bar{\psi}_A(x) \gamma_k \frac{1}{\alpha} \psi(0) \right] \langle \bar{\psi}_B(0) \gamma_{1/\beta} \psi(x) \rangle \langle \psi(0) \otimes (x) \leftrightarrow g_5 \psi(0) \otimes g_5 \psi(x) \right] \\
+ \left[ \langle \bar{\psi}_A(x) \gamma_k \frac{1}{\alpha} \psi(0) \right] \langle \bar{\psi}_B(0) \gamma_{1/\beta} \psi(x) \rangle \langle \psi(0) \otimes (x) \leftrightarrow g_5 \psi(0) \otimes g_5 \psi(x) \right] \\
+ \frac{2}{s} \left[ \langle \bar{\psi}_A(x) \gamma_k \frac{1}{\alpha} \psi(0) \rangle \langle \bar{\psi}_B(0) \gamma_{1/\beta} \psi(x) \rangle \langle \psi(0) \otimes (x) \leftrightarrow g_5 \psi(0) \otimes g_5 \psi(x) \right] \\
+ \langle \psi(0) \otimes (x) \leftrightarrow g_5 \psi(0) \otimes g_5 \psi(x) \right] \right\} + x \leftrightarrow 0
\]  

(6.51)
Let us start with the first term. Using eq. (A.21) and the fact that \( \langle \bar{\psi}(x)[A_k \sigma_{s j} - A_j(x) \sigma_{s k}] \psi(0) \rangle_A = 0 \) (see the footnote 11), we obtain

\[
- \frac{1}{2s^3} \left\langle \bar{\psi}(x) \gamma_\mu \frac{1}{\alpha} \psi(0) \right\rangle_A \left\langle \bar{\psi}(x) \gamma_\nu \frac{1}{\beta} \psi(0) \right\rangle_B - \frac{1}{4s^2} \left\langle \bar{\psi}(x) \gamma_\mu \frac{1}{\alpha} \psi(0) \right\rangle_A \left\langle \bar{\psi}(x) \gamma_\nu \frac{1}{\beta} \psi(0) \right\rangle_B
\]

\[
- \frac{1}{8s^2} \left[ \left\langle \bar{\psi}(x) \sigma_{j k} \frac{1}{\alpha} \psi(0) \right\rangle_A \left\langle \bar{\psi}(x) \sigma_{j k} \frac{1}{\beta} \psi(0) \right\rangle_B \right]
\]

\[
= - \frac{1}{2s^3} \left\langle \bar{\psi}(x) \gamma_\mu \frac{1}{\alpha} \psi(0) \right\rangle_A \left\langle \bar{\psi}(x) \psi(0) \right\rangle_B \left[ 1 + O\left( \frac{q_1^2}{s} \right) \right]
\]

where we used the fact that projectile and target matrix elements in the two last terms in the l.h.s. cannot produce factor of \( s \).

Next, consider second term in eq. (6.51). Using eqs. (A.17) and (A.21), one can rewrite is as

\[
\frac{1}{2s^3} \left[ \left\langle \bar{\psi}(x) \gamma_\mu \frac{1}{\alpha} \psi(0) \right\rangle_A \left\langle \bar{\psi}(x) \gamma_\nu \frac{1}{\beta} \psi(0) \right\rangle_B + \psi(0) \otimes \psi(x) \leftrightarrow \gamma_5 \psi(0) \otimes \gamma_5 \psi(x) \right]
\]

\[
= \frac{1}{s^3} \left\langle \bar{\psi}(x) \gamma_\mu \frac{1}{\alpha} \psi(0) \right\rangle_A \left\langle \bar{\psi}(x) \gamma_\nu \frac{1}{\beta} \psi(0) \right\rangle_B
\]

Similarly, from eq. (A.21) we get the third term in the form

\[
\frac{1}{s^3} \left[ \left\langle \bar{\psi}(x) \gamma_\mu \frac{1}{\alpha} \psi(0) \right\rangle_A \left\langle \bar{\psi}(x) \gamma_\nu \frac{1}{\beta} \psi(0) \right\rangle_B + \psi(0) \otimes \psi(x) \leftrightarrow \gamma_5 \psi(0) \otimes \gamma_5 \psi(x) \right]
\]

\[
= \frac{1}{4s^2} \left[ \left\langle \bar{\psi}(x) \gamma_\mu \frac{1}{\alpha} \psi(0) \right\rangle_A \left\langle \bar{\psi}(x) \gamma_\nu \frac{1}{\beta} \psi(0) \right\rangle_B + \psi(0) \otimes \psi(x) \leftrightarrow \gamma_5 \psi(0) \otimes \gamma_5 \psi(x) \right]
\]

Since both projectile and target matrix elements cannot give factor \( s \) this contribution is \( O\left( \frac{q_1^2}{s} \right) \) in comparison to that of the two first terms. Thus, we get

\[
\tilde{V}_1 = \frac{1}{s^3} \left[ \frac{1}{2} \left\langle \bar{\psi}(x) \gamma_\mu \frac{1}{\alpha} \psi(0) \right\rangle_A \left\langle \bar{\psi}(x) \gamma_\nu \frac{1}{\beta} \psi(0) \right\rangle_B + \left\langle \bar{\psi}(x) \gamma_\mu \frac{1}{\alpha} \psi(0) \right\rangle_A \left\langle \bar{\psi}(x) \gamma_\nu \frac{1}{\beta} \psi(0) \right\rangle_B \right] \left[ 1 + O\left( \frac{q_1^2}{s} \right) \right] + x \leftrightarrow 0
\]

Next, using QCD equations of motion (C.11), (C.14) and formulas from appendix B, we obtain the contribution to \( W_{\mu \nu} \) in the form

\[
g_{\mu \nu} V_1(x) = \frac{g_{\mu \nu}}{16 \pi^4 N_c} \int d x_\perp d x_\perp d^2 x_\perp e^{-i q_\perp x_\perp - i q_\perp x_\perp + i(q, x) \perp} \tilde{V}_1(x)
\]

\[
= \frac{g_{\mu \nu}}{\alpha_s \beta_0 \beta_0 s N_c} \int d^2 k_\perp \left[ \left( k, q - k \right) \perp F^j(q, k_\perp) - \frac{1}{2 m^2} k_\perp^2 \left( q - k \right) \perp F^j(q, k_\perp) \right]
\]

where replacements \( f^j_1 \leftrightarrow \tilde{f}^j_1 \) and \( h^j_1 \leftrightarrow \tilde{h}^j_1 \) come from \( x \leftrightarrow 0 \) term.
6.2.2 Term with TMD’s $f_1$

Separating color-singlet contributions one can rewrite eq. (6.49) as

$$
\tilde{V}_{2\mu\nu} = -\frac{1}{2s^3} \left\{ \langle \bar{\psi} A_i(x) \gamma_{\mu\perp} \slashed{p}_2 \gamma^j \frac{1}{\alpha} \psi(0) \rangle_A \langle \bar{\psi} B_j(0) \gamma_{\nu\perp} \slashed{p}_1 \gamma^i \frac{1}{\beta} \psi(x) \rangle_B + \psi(0) \otimes \psi(x) \leftrightarrow \gamma_5 \psi(0) \otimes \gamma_5 \psi(x) + \mu \leftrightarrow \nu \right\} + x \leftrightarrow 0
$$

(6.57)

We need to consider three cases: both $\mu$ and $\nu$ are transverse, both of them are longitudinal, and $\mu$ is longitudinal and $\nu$ transverse (plus vice versa).

In the first case we can use formula (A.18) and get

$$
\tilde{V}_{2\mu\perp\nu\perp} = -\frac{1}{2s^3} \left\{ \langle \bar{\psi} A_i(x) \gamma_{\mu\perp} \slashed{p}_2 \gamma^j \frac{1}{\alpha} \psi(0) \rangle_A \langle \bar{\psi} B_j(0) \gamma_{\nu\perp} \slashed{p}_1 \gamma^i \frac{1}{\beta} \psi(x) \rangle_B + \psi(0) \otimes \psi(x) \leftrightarrow \gamma_5 \psi(0) \otimes \gamma_5 \psi(x) + \mu \leftrightarrow \nu \right\} + x \leftrightarrow 0
$$

(6.58)

which gives the contribution to $W_{\mu\nu}$ in the form

$$
V_{2\mu\perp\nu\perp} = \frac{1}{16\pi^2 N_c} \int dx_\perp dx_\perp d^2x_\perp e^{-i\alpha q x_\perp - i\beta q x_\perp + i(q,x)_\perp} \tilde{V}_{2\mu\perp\nu\perp}(x)
$$

(6.59)

where we again used formulas from appendices B and C.

Next, if both $\mu$ and $\nu$ are longitudinal, we get

$$
\tilde{V}_{2\mu\nu} = -\frac{2}{s^3} \langle p_{1\mu} p_{2\nu} + \mu \leftrightarrow \nu \rangle \left\{ \langle \bar{\psi} A_i(x) \slashed{p}_1 \slashed{p}_2 \gamma^j \frac{1}{\alpha} \psi(0) \rangle_A \langle \bar{\psi} B_j(0) \gamma^i \frac{1}{\beta} \psi(x) \rangle_B + \psi(0) \otimes \psi(x) \leftrightarrow \gamma_5 \psi(0) \otimes \gamma_5 \psi(x) \right\} + x \leftrightarrow 0
$$

(6.60)

Using formula (A.20) we rewrite r.h.s. of eq. (6.57) as follows

$$
\tilde{V}_{2\mu\nu} = -\frac{1}{2s^3} \langle p_{1\mu} p_{2\nu} + \mu \leftrightarrow \nu \rangle \left\{ \langle \bar{\psi} A_i(x) \gamma^j \frac{1}{\alpha} \psi(0) \rangle_A \langle \bar{\psi} B_j(0) \gamma^i \frac{1}{\beta} \psi(x) \rangle_B + \psi(0) \otimes \psi(x) \leftrightarrow \gamma_5 \psi(0) \otimes \gamma_5 \psi(x) \right\} + x \leftrightarrow 0
$$

(6.61)

Since matrix elements in the r.h.s. cannot give factor $s$, the contribution of this term to $W_{\mu\nu}$ is $\sim \frac{q^2}{s}$ times that of eq. (6.59).
Finally, let us consider the case when one index is longitudinal and the other transverse. Using eq. (A.23) we get

\[ V^\|_{2\mu\nu} = -\frac{1}{s^4} \left\{ p_{2\mu} \left\langle \bar{\psi} A_i(x) \gamma^1 \frac{1}{\alpha} \psi(0) \right\rangle_A \left\langle \bar{\psi} B_j(0) \gamma_{\nu \perp} \gamma^1 \frac{1}{\beta} \psi(x) \right\rangle_B \right. 
\]

\[ + p_{1\mu} \left\langle \bar{\psi} A_i(x) \gamma_{\nu \perp} \gamma^1 \frac{1}{\alpha} \psi(0) \right\rangle_A \left\langle \bar{\psi} B_j(0) \gamma_{\nu} \gamma^1 \frac{1}{\beta} \psi(x) \right\rangle_B 
\]

\[ + \bar{\psi}(0) \otimes \psi(x) \leftrightarrow \gamma_5 \psi(0) \otimes \gamma_5 \psi(x) + \mu \leftrightarrow \nu \bigg\} + x \leftrightarrow 0 
\]

\[ = -\frac{1}{2s^3} \left\{ p_{2\mu} \left\langle \bar{\psi}(x) \gamma^1 A_\nu(x) \frac{1}{\alpha} \psi(0) \right\rangle_A \left\langle \bar{\psi} B(0) \gamma^1 \frac{1}{\beta} \psi(x) \right\rangle_B \right. 
\]

\[ + p_{1\mu} \left\langle \bar{\psi} A(x) \gamma^1 \frac{1}{\alpha} \psi(0) \right\rangle_A \left\langle \bar{\psi}(0) \gamma^1 B_\nu(0) \frac{1}{\beta} \psi(x) \right\rangle_B 
\]

\[ + \bar{\psi}(0) \otimes \psi(x) \leftrightarrow \gamma_5 \psi(0) \otimes \gamma_5 \psi(x) + \mu \leftrightarrow \nu \bigg\} + x \leftrightarrow 0 \quad (6.62) \]

It is clear that \( \langle \bar{\psi} A(x) \gamma^1 \frac{1}{\alpha} \psi(0) \rangle_A \) and \( \langle \bar{\psi} B(0) \gamma^1 \frac{1}{\beta} \psi(x) \rangle_B \) bring one factor \( s \) so

\[ \tilde{V}^\|_{2\mu\nu} \sim \frac{p_{1\mu} q_{\mu\nu} + \mu \leftrightarrow \nu m^2}{\alpha q \beta_s^2} \text{or} \sim \frac{p_{2\mu} q_{\mu\nu} + \mu \leftrightarrow \nu m^2}{\alpha q \beta_s^2} \quad (6.63) \]

which is \( \frac{1}{s^4} \) or \( \frac{1}{2s^3} \) correction in comparison to eq. (6.46). Thus, the contribution to \( W_{\mu\nu} \) is given by eq. (6.59)

\[ V_{2\mu\nu} = -\frac{g_{1\mu}}{\alpha q \beta_s^2 N_c} \int d^2 k(k, q - k) \perp F^f(p, k) \quad (6.64) \]

### 6.2.3 Term with TMD’s \( h_1^\perp \)

Let us consider now

\[ \tilde{V}^\perp_{3\mu\nu} = \frac{N_c}{2s} \left\langle A, B \right| (\bar{\psi}_A^m(x) \sigma_{\mu \alpha} \Xi_1^m(0)) [\bar{\psi}_B^m(0) \sigma_{\nu \beta} \Xi_1^n(0)] A, B \rangle + \mu \leftrightarrow \nu + x \leftrightarrow 0 \quad (6.65) \]

(the trace will be subtracted after the calculation). Separating color-singlet contributions, we get

\[ \tilde{V}^\perp_{3\mu\nu} = \frac{1}{2s^3} \left\langle \bar{\psi} A_i(x) \sigma_{\mu \perp} \gamma^1 \frac{1}{\alpha} \psi(0) \right\rangle_A \left\langle \bar{\psi} B_j(0) \sigma_{\nu \perp} \gamma^1 \frac{1}{\beta} \psi(x) \right\rangle_B + \mu \leftrightarrow \nu + x \leftrightarrow 0 \quad (6.66) \]

First case is when \( \mu \) and \( \nu \) are transverse

\[ \tilde{V}^\perp_{3\mu\perp \nu \perp} = -\frac{1}{2s^3} \left\langle \bar{\psi} A_i(x) \sigma_{\mu \perp} \gamma^k \frac{1}{\alpha} \psi(0) \right\rangle_A \left\langle \bar{\psi} B_j(0) \sigma_{\nu \perp} \gamma^1 \frac{1}{\beta} \psi(x) \right\rangle_B \quad (6.67) \]

\[ -\frac{1}{s^4} \left\langle \bar{\psi} A_i(x) \sigma_{\mu \perp} \gamma^k \frac{1}{\alpha} \psi(0) \right\rangle_A \left\langle \bar{\psi} B_j(0) \sigma_{\nu \perp} \gamma^1 \frac{1}{\beta} \psi(x) \right\rangle_B + \mu \leftrightarrow \nu + x \leftrightarrow 0 \]
With the help of eq. (A.4) the first term in the r.h.s. turns to

\[
\frac{1}{2s^3} \left\langle \tilde{\psi} A^i(x)[g_{\mu j}\sigma_{sk} - g_{jk}\sigma_{s\mu}] \frac{1}{\alpha} \psi(0) \right\rangle_A \left\langle \tilde{\psi} B^j(0)[g_{\nu l}\sigma_{k^*} - \delta^k_{\nu}\sigma_{k^*\nu}] \frac{1}{\beta} \psi(x) \right\rangle_B + \mu \leftrightarrow \nu + x \leftrightarrow 0
\]

\[
= \frac{1}{2s^3} \left\langle \tilde{\psi} A_\nu(x) \sigma_{sk} \frac{1}{\alpha} \psi(0) \right\rangle_A \left\langle \tilde{\psi} B_\mu(0) \sigma_{k^*} \frac{1}{\beta} \psi(x) \right\rangle_B
\]

\[
- \left\langle \tilde{\psi} A_\nu(x) \sigma_{s\mu} \frac{1}{\alpha} \psi(0) \right\rangle_A \left\langle \tilde{\psi} B_j(0) \sigma_j \frac{1}{\beta} \psi(x) \right\rangle_B
\]

\[
+ \left\langle \tilde{\psi} A^i(x) \sigma_{s\mu} \frac{1}{\alpha} \psi(0) \right\rangle_A \left\langle \tilde{\psi} B_i(0) \sigma_{s\nu} \frac{1}{\beta} \psi(x) \right\rangle_B + \mu \leftrightarrow \nu + x \leftrightarrow 0
\]  \hspace{1cm} (6.68)

After some algebra, it can be rewritten as

\[
- \frac{g_{\mu \nu}}{2s^3} \left\langle \tilde{\psi} A_i(x) \sigma_{si} \frac{1}{\alpha} \psi(0) \right\rangle_A \left\langle \tilde{\psi} B^j(0) \sigma_j \frac{1}{\beta} \psi(x) \right\rangle_B
\]

\[
+ \frac{1}{s^3} \left\{ \tilde{\psi} \left( A_k \sigma_{s\mu} - \frac{1}{2} g_{mk} \sigma_{s\nu} A^j(x) \frac{1}{\alpha} \psi(0) \right) A \right\}
\]

\[
\times \left\langle \tilde{\psi} \left( B_j B_i - \frac{1}{2} \delta_{m^*} \sigma_{m^*} B^j \right) (0) \frac{1}{\beta} \psi(x) \right\rangle_B + \mu \leftrightarrow \nu + x \leftrightarrow 0
\]  \hspace{1cm} (6.69)

where again we used property (6.21). Using QCD equations of motion (C.12), (C.14) and parametrization (C.18) one can write the corresponding contribution to \( W_{\mu\nu} \) as

\[
V_{3\mu\nu} = \frac{g_{\mu\nu}}{2\alpha q\beta q s N_c} \int d^2k_1 \frac{k_1^2}{m^2} \left( q - k_1 \right)^2 \frac{H^f(q, k_1)}{m^2}
\]

\[
+ \frac{1}{\alpha q\beta q s N_c} \int d^2k_1 \frac{k_1^2}{m^2} \left[ \frac{k_1^2}{m^2} (q - k_1 \nu) + \mu \leftrightarrow \nu \right] \left( k, q - k \right)_\perp \left( q - k \right)_\perp \nonumber
\]

\[
- (q - k_1)^2 k_1^2 \frac{1}{m^2} \left( q - k_1 \right)_\perp \left( q - k_1 \right)_\perp \right] H_A^f(q, k_1)
\]  \hspace{1cm} (6.70)

where we introduced the notation

\[
H_A^f(q, k_1) \equiv h_A^f(\alpha_q, \beta_q, q - k_1) + h_A^f \leftrightarrow \tilde{h}_A^f
\]  \hspace{1cm} (6.71)

The second term in eq. (6.67) can be rewritten as

\[
\tilde{V}_{3\mu\nu} = -\frac{1}{4s^2} \left\langle \tilde{\psi} A_i(x) \left( g_{\mu j} - i e_{\mu j} \gamma_5 - i e_{\mu j} \frac{2i}{s} g_{\mu j} \gamma_{15} \right) \frac{1}{\alpha} \psi(0) \right\rangle_A
\]

\[
\times \left\langle \tilde{\psi} B_j(0) \left( g_{\nu k} - i e_{\nu k} \gamma_5 + i e_{\nu k} \frac{2i}{s} g_{\nu k} \gamma_{15} \right) \frac{1}{\beta} \psi(x) \right\rangle_B + \mu \leftrightarrow \nu + x \leftrightarrow 0
\]  \hspace{1cm} (6.72)

where we used eq. (A.4). It is clear that neither projectile no target matrix element in the r.h.s. can bring factor \( s \) so

\[
\tilde{V}_{3\mu\nu} \sim \frac{m_1^4}{\alpha q\beta q s^2}
\]  \hspace{1cm} (6.73)

which is \( O\left( \frac{m_2^2}{s} \right) \) in comparison to eq. (6.70).
Next, consider the case when both $\mu$ and $\nu$ are longitudinal. The non-vanishing terms are

$$V^l_{3\mu\nu} = \frac{4p_{1\mu}p_{2\nu}}{s^2} \int d^2k_\perp \frac{k_\perp^2 (q-k)^2}{m^2} H(q,k_\perp)$$

The corresponding contribution to $W_{\mu\nu}$ has the form

$$V^l_{3\mu\nu} = -\frac{g_{\mu\nu}}{\alpha_s^2 q_s N_c} \int d^2k_\perp k_\perp^2 \frac{q_\perp (q-k)^2}{m^2} H(q,k_\perp)$$

where again we used QCD equations of motion (C.12), (C.14) and parametrization (C.18).

Next, it is easy to see that the third term in eq. (6.74) is small in comparison to eq. (6.76):

$$-\frac{4p_{2\mu}p_{1\nu}}{s^2} \int d^2k_\perp \frac{k_\perp^2 (q-k)^2}{m^2} H(q,k_\perp) + x \leftrightarrow 0$$

because neither projectile no target matrix element can bring factor $s$.

Finally, take one of the indices (say, $\mu$) longitudinal and the other transverse. From eq. (6.66) we get

$$V_{3\mu\nu} = \frac{p_{\mu\nu}}{s^2} \int d^2k_\perp \frac{k_\perp^2 (q-k)^2}{m^2} H(q,k_\perp) + x \leftrightarrow 0$$
Using formulas (A.4) this can be rewritten as follows

\[ V_{3\mu\nu\perp} = \frac{ip_{1\mu}}{2s^3} \left\langle \psi A^i(x) \left[ g_{jk} + i\epsilon_{jk}\gamma_5 + i\sigma_{jk} - \frac{2i}{s} g_{jk}\sigma_{*} \right] \frac{1}{\beta} \psi(0) \right\rangle_A \]
\[ \times \left\langle \psi B^j(0) \left[ g_{j\perp} + i\epsilon_{j\perp}\gamma_5 + i\sigma_{j\perp} - \frac{2i}{s} g_{j\perp}\sigma_{*} \right] \frac{1}{\beta} \psi(0) \right\rangle_B \]
\[ - \frac{ip_{1\mu}}{2s^3} \left\langle \psi A^i(x) \left[ g_{j\perp} + i\epsilon_{j\perp}\gamma_5 + i\sigma_{j\perp} - \frac{2i}{s} g_{j\perp}\sigma_{*} \right] \frac{1}{\beta} \psi(0) \right\rangle_A \]
\[ \times \left\langle \psi B^j(0)\sigma_{*} \frac{1}{\beta} \psi(x) \right\rangle_B + \mu \leftrightarrow \nu + \nu \leftrightarrow 0 \]  

(6.79)

As we discussed above, projectile matrix elements in the r.h.s. like \( \langle \psi B^j(0)\sigma_{*} \frac{1}{\beta} \psi(x) \rangle_B \) can bring factor \( s \) but the target matrix elements cannot so the corresponding contribution to \( W_{\mu\nu} \) is of order

\[ V_{3\mu\nu\perp} \sim (p_{1\mu} q_\perp^i + \mu) \frac{m^2}{\alpha_q^3 s^2} \]  

(6.80)

which is \( O\left(\frac{q^2}{s}\right) \) in comparison to eq. (6.46).

Next, the sum of eqs. (6.70) and (6.76) is

\[ V'_{3\mu\nu} = \frac{g_{1\mu} - g_{1\nu}^{||}}{2\alpha_q^3 s N_c} \int d^2k_\perp \frac{k_\perp^2 (q-k)^2}{m^2} H_1^f(q,k_\perp) \]
\[ + \frac{1}{\alpha_q^3 s N_c} \int d^2k_\perp \frac{1}{m^2} \left\{ (k_\perp^2 (q-k)^2 + \mu \leftrightarrow \nu)(k,q-k)_\perp - k_\perp^2 (q-k)^2 \right\} H_1^f(q,k_\perp) \]
\[ - (q-k_\perp)^2 k_\perp^2 k_\perp^2 - \frac{g_{1\mu} - g_{1\nu}^{||}}{2} (q-k_\perp)^2 - g_{1\mu}^{||} (k,q-k)_\perp^2 - \frac{1}{2} k_\perp^2 (q-k_\perp)^2 \right\} \}

\[ \times H_1^f(q,k_\perp) \]  

(6.81)

so subtracting trace we obtain

\[ V_{3\mu\nu} = V'_{3\mu\nu} - \mu \nu V_{3\xi} \]  

(6.82)

\[ = \frac{g_{1\mu} - g_{1\nu}^{||}}{2\alpha_q^3 s N_c} \int d^2k_\perp \frac{1}{m^2} k_\perp^2 (q-k)^2 H_1^f(q,k_\perp) \]
\[ + \frac{1}{\alpha_q^3 s N_c} \int d^2k_\perp \frac{1}{m^2} \left\{ (k_\perp^2 (q-k)^2 + \mu \leftrightarrow \nu)(k,q-k)_\perp - k_\perp^2 (q-k)^2 \right\} H_1^f(q,k_\perp) \]
\[ - (q-k_\perp)^2 k_\perp^2 k_\perp^2 + g_{1\mu}^{||} (k,q-k)_\perp^2 - \frac{1}{2} k_\perp^2 (q-k_\perp)^2 \right\} \}

As we will see in section 8, cancellation of terms \( \sim g_{1\mu}^{||} \) proportional to \( h_A \) in the r.h.s. of this equation is actually a consequence of (EM) gauge invariance.

Let us now assemble the contribution of terms (6.47) to \( W_{\mu\nu} \). Summing eq. (6.56),
\[
V_{\mu\nu}(q) = \frac{1}{32\pi^2} \int dx_\perp dx_\perp d^2 x_\perp e^{-i\alpha x_\perp - i\beta x_\perp + i(q,x)_\perp} \\
\quad \left[ \langle A, B | \tilde{\psi}_\mu \gamma_\nu \Xi_2^0(x) \rangle (\tilde{\psi}_\mu^0(0) \gamma_\nu \Xi_1^0(0)) + \mu \leftrightarrow \nu | A, B \rangle + x \leftrightarrow 0 \right] \\
= \frac{g_{\mu\nu}}{\alpha_q \beta_q s N_c} \int d^2 k_\perp \left\{ (k, q - k)_\perp F^f(q, k_\perp) - \frac{1}{m^2} k^2_\perp (q - k)_\perp^2 H^f(q, k_\perp) \right\} \\
+ \frac{1}{\alpha_q \beta_q s N_c} \int d^2 k_\perp \frac{1}{m^2} \left\{ [k^2_\perp (q - k)_\perp^2 + \mu \leftrightarrow \nu] (k, q - k)_\perp - k^2_\perp (q - k)_\perp^2 \right\} H^f_A(q, k_\perp)
\]

(6.83)

Finally, to get \(W_{\mu\nu}^{(2a)}(q)\) of eq. (6.3) we need to add the contribution of the term \([\Xi_1(x) \gamma_\mu \psi_\nu(0)] [\Xi_2(0) \gamma_\nu \psi_A(0)]\). Similarly to the case of one quark-quark-gluon operator considered in section 6.1, it can be demonstrated that this contribution doubles the result (6.83) so we get

\[
W_{\mu\nu}^{(2a)}(q) = \frac{1}{32\pi^2} \int dx_\perp dx_\perp d^2 x_\perp e^{-i\alpha x_\perp - i\beta x_\perp + i(q,x)_\perp} \langle A, B | \tilde{\psi}_\mu \gamma_\nu \Xi_2^0(x) \rangle \Xi_2(0) \gamma_\nu \psi_A(0) + \mu \leftrightarrow \nu | A, B \rangle + x \leftrightarrow 0 \rangle \\
= \frac{2 g_{\mu\nu}}{Q^2 \alpha_q \beta_q s N_c} \int d^2 k_\perp \left\{ (k, q - k)_\perp F^f(q, k_\perp) - \frac{1}{m^2} k^2_\perp (q - k)_\perp^2 H^f(q, k_\perp) \right\} \\
+ \frac{2}{Q^2 \alpha_q \beta_q s N_c} \int d^2 k_\perp \frac{1}{m^2} \left\{ [k^2_\perp (q - k)_\perp^2 + \mu \leftrightarrow \nu] (k, q - k)_\perp - k^2_\perp (q - k)_\perp^2 \right\} H^f_A(q, k_\perp)
\]

(6.84)

where \(Q^2 \equiv \alpha_q \beta_q s\)

6.3 Term with two quark-quark-gluon operators coming from \(\Xi_2\) and \(\Xi_2\)

Let us start with the first term in eq. (6.4).

\[
\tilde{W}_{1,\mu\nu}^{(2b)} = \frac{N_c}{s} \langle A, B | [\tilde{\psi}_\mu \gamma_\nu \Xi_2^0(x)] \Xi_2(0) \gamma_\nu \psi_A(0) + \mu \leftrightarrow \nu | A, B \rangle + x \leftrightarrow 0 \rangle 
\]

(6.85)

After Fierz transformation (A.1) we obtain

\[
\tilde{W}_{1,\mu\nu}^{(2b)} = -\frac{N_c}{2s} (\delta_\mu^\alpha \delta_\nu^\beta + \delta_\mu^\beta \delta_\nu^\alpha - g_{\mu\nu} \delta_\alpha^\beta) \langle A, B | [\tilde{\psi}_\mu \gamma_\alpha \psi_A(0)] [\Xi_2(0) \gamma_\beta \Xi_2^0(x)] \\
+ \gamma_\alpha \otimes \gamma_\beta \leftrightarrow \gamma_\alpha \otimes \gamma_\beta \otimes \gamma_3 \rangle A, B \rangle \\
+ \frac{N_c}{2s} (\delta_\mu^\alpha \delta_\nu^\beta + \delta_\mu^\beta \delta_\nu^\alpha - \frac{1}{2} g_{\mu\nu} \delta_\alpha^\beta) \langle A, B | [\tilde{\psi}_\mu^0(x) \gamma_\alpha \psi_A(0)] [\Xi_2^0(0) \gamma_\beta \Xi_2^0(x)] A, B \rangle + x \leftrightarrow 0
\]

(note that \(\Xi_2 \Xi_2 = \Xi_2 \Xi_2 = 0\)). Using explicit expressions (4.12) for quark fields and separating color-singlet terms we get

\[
\tilde{W}_{1,\mu\nu}^{(2b)} = V_{\mu\nu} + \tilde{V}_{\mu\nu}^5
\]

(6.87)
where
\[
\hat{V}^{-4}_{\mu\nu} = -\frac{1}{s^3} (\delta^\alpha_\mu p_{1\nu} + \delta^\alpha_\nu p_{1\mu} - g_{\mu\nu} p^\alpha \sigma^0 \sigma^1) \left( \langle \bar{\psi}(x) A_j(x) \gamma^\alpha A_i(0) \psi(0) \rangle_A \right)
\times \left\{ \left( \frac{-1}{\beta} \right) (0) \gamma^i \gamma^j \frac{1}{\beta} \psi(x) \right\}_B + \psi(0) \otimes \psi(x) \leftrightarrow \gamma_5 \psi(0) \otimes \gamma_5 \psi(x) + x \leftrightarrow 0 \quad (6.88)
\]
and
\[
\hat{V}^{5}_{\mu\nu} = \frac{1}{s^3} (\delta^\alpha_\mu \delta^\beta_\nu + \delta^\alpha_\nu \delta^\beta_\mu - \frac{1}{2} g_{\mu\nu} g^\alpha\beta)
\times \left\{ - p_{1\beta} \langle \bar{\psi}(x) A_j(x) \sigma_{\alpha k} A_i(0) \psi(0) \rangle_A \left( \left( \frac{-1}{\beta} \right) (0) \gamma^i \sigma^{k} \gamma^j \frac{1}{\beta} \psi(x) \right) \right\}_B
+ \langle \bar{\psi}(x) A_j(x) \sigma_{\alpha \bullet} A_i(0) \psi(0) \rangle_A \left( \left( \frac{-1}{\beta} \right) (0) \gamma^i \sigma_{\bullet \gamma} \gamma^j \frac{1}{\beta} \psi(x) \right) \right\}_B + x \leftrightarrow 0 \quad (6.89)
\]
We will consider them in turn.

### 6.3.1 Term proportional to \( f_1 \tilde{f}_1 \)

Let us start with the \( g_{\mu\nu} \) term in eq. (6.88).

\[
\frac{g_{\mu\nu}}{s^3} \langle \bar{\psi}(x) A_j(x) \sigma_{\mu \nu} A_i(0) \psi(0) \rangle_A \left( \left( \frac{-1}{\beta} \right) (0) \gamma^i \gamma^j \frac{1}{\beta} \psi(x) \right) \right\}_B + \phi_1 \otimes \gamma_j \leftrightarrow \phi_1 \gamma_5 \otimes \gamma_j \gamma_5 \quad (6.90)
\]

It is obvious that the target matrix element can bring factor \( s \). On the contrary, as we discussed above, the projectile matrix element cannot produce \( s \) since

\[
\langle \bar{\psi}(x) A_j(x) \gamma^\alpha A_i(0) \psi(0) \rangle_A \sim \frac{P_{2\alpha}}{P_{1 \cdot P_2}} \times [g_{ij} \phi(x^2_\perp) + x_i x_j \xi(x^2_\perp)] + \ldots \quad (6.91)
\]

Indeed, since projectile matrix elements know about \( p_2 \) only through the direction of Wilson lines, the l.h.s. can be proportional only to factor \( \frac{P_{2\alpha}}{P_{1 \cdot P_2}} \) that does not change under rescaling of \( p_2 \). Also, due to eq. (C.2) \( \langle \bar{\psi}(x) \psi(x) \rangle_B \) can be replaced by \(- \frac{1}{p^2} \langle \bar{\psi}(0) \otimes \psi(x) \rangle_B \).

Consequently, the r.h.s. of eq. (6.90) is \( \sim g_{\mu\nu} \frac{m^4_{f_1}}{2q^2} \)

\[
\frac{g_{\mu\nu}}{s^3} \langle \bar{\psi}(x) A_j(x) \sigma_{\mu \nu} A_i(0) \psi(0) \rangle_A \left( \left( \frac{-1}{\beta} \right) (0) \gamma^i \gamma^j \frac{1}{\beta} \psi(x) \right) \right\}_B + \phi_1 \otimes \gamma_j \leftrightarrow \phi_1 \gamma_5 \otimes \gamma_j \gamma_5 \sim g_{\mu\nu} \frac{m^4_{f_1}}{2q^2} \quad (6.92)
\]

which is \( \mathcal{O}(\frac{m^2_{f_1}}{\beta q^2}) \) in comparison to eq. (6.84).

We get
\[
\hat{V}^{-4}_{\mu\nu} = \frac{P_{1\mu}}{s^3} \left( \langle \bar{\psi}(x) A_j(x) \gamma^\alpha A_i(0) \psi(0) \rangle_A \left( \left( \frac{-1}{\beta} \right) (0) \gamma^i \gamma^j \frac{1}{\beta} \psi(x) \right) \right)_B
+ \psi(0) \otimes \psi(x) \leftrightarrow \gamma_5 \psi(0) \otimes \gamma_5 \psi(x) + x \leftrightarrow 0 \quad (6.93)
\]

If the index \( \nu \) is transverse, the contribution of this equation to \( W_{\mu\nu} \) is of order of

\[
\hat{V}^{-4}_{\mu\nu} \sim p_{1\mu} q_{\nu} \frac{m^2_{f_1}}{\beta q^2} \quad (6.94)
\]
which is \( \mathcal{O}(\frac{m^2_{f_1}}{\beta q^2}) \) in comparison to eq. (6.46).
For the longitudinal indices $\mu$ and $\nu$ we get
\[
\hat{V}_{\mu\nu}^A = -\frac{4p_{1\mu}p_{1\nu}}{s^4}\left(\langle \bar{\psi}(x)A_j(x)\not{p}_2A_i(0)\psi(0)\rangle_A\left\langle \left(\frac{1}{\beta}\right)(0)\gamma^i\not{p}_1\gamma^j\frac{1}{\beta}\psi(x)\right\rangle_B + \psi(0) \otimes \psi(x) \leftrightarrow \gamma_5\psi(0) \otimes \gamma_5\psi(x)\right)
-\frac{g_{\mu\nu}}{s^3}\left(\langle \bar{\psi}(x)A_j(x)\not{p}_2A_i(0)\psi(0)\rangle_A\left\langle \left(\frac{1}{\beta}\right)(0)\gamma^i\not{p}_1\gamma^j\frac{1}{\beta}\psi(x)\right\rangle_B + \psi(0) \otimes \psi(x) \leftrightarrow \gamma_5\psi(0) \otimes \gamma_5\psi(x)\right) + x \leftrightarrow 0
\] (6.95)

Similarly to eq. (6.92), the contribution of the second term to $W_{\mu\nu}$ is
\[
\sim \frac{g_{\mu\nu}m_1^4}{\beta_s^2s^2} = O\left(\frac{\alpha_s m_1^2}{\beta_s^2 s^s}\right) \times \text{r.h.s. of eq. (6.84)}
\] (6.96)

so we are left with the first term in the r.h.s. of eq. (6.95). Using eq. (A.8) it can be rewritten as
\[
\hat{V}_{\mu\nu}^A = -\frac{4p_{1\mu}p_{1\nu}}{s^4}\left(\langle \bar{\psi}(x)A_j(x)\not{p}_2A_i(0)\psi(0)\rangle_A\left\langle \left(\frac{1}{\beta}\right)(0)\gamma^i\not{p}_1\gamma^j\frac{1}{\beta}\psi(x)\right\rangle_B + \psi(0) \otimes \psi(x) \leftrightarrow \gamma_5\psi(0) \otimes \gamma_5\psi(x)\right) + x \leftrightarrow 0
\] (6.97)

The corresponding contribution to $W_{\mu\nu}$ is obtained from QCD equation of motion (C.15) and formula (C.2) from appendix C:
\[
V_{\mu\nu}^A(q) = \frac{4p_{1\mu}p_{1\nu}}{\beta_s^2s^2N_c}\int d^2k_\perp k_\perp^2 F^J(q,k_\perp)
\] (6.98)

### 6.3.2 Term proportional to $\tilde{h}_1^+\tilde{h}_1^+$

Let us start with $g_{\mu\nu}$ term in eq. (6.89).
\[
\frac{g_{\mu\nu}}{s^3}\langle \bar{\psi}(x)A_j(x)\sigma_{\mu k}A_i(0)\psi(0)\rangle_A\left\langle \left(\frac{1}{\beta}\right)(0)\gamma^k\sigma_{\nu}\not{p}_1\gamma^j\frac{1}{\beta}\psi(x)\right\rangle_B + x \leftrightarrow 0
\] (6.99)

The target matrix element is proportional to $s$ while the projectile one cannot bring $s$ due to eq. (6.91), so the contribution of the r.h.s of eq. (6.99) to $W_{\mu\nu}$ is of order
\[
\sim \frac{g_{\mu\nu}}{\beta_s^2s^2}m_1^4 = O\left(\frac{m_1^4}{s\beta_s^2}\right) \times \text{r.h.s. of eq. (6.84)}
\] (6.100)

similarly to eq. (6.96). We get
\[
\hat{V}_{\mu\nu}^A = \frac{1}{s^3}\left\{-p_{1\mu}\langle \bar{\psi}(x)A_j(x)\sigma_{\mu k}A_i(0)\psi(0)\rangle_A\left\langle \left(\frac{1}{\beta}\right)(0)\gamma^k\sigma_{\nu}\not{p}_1\gamma^j\frac{1}{\beta}\psi(x)\right\rangle_B + \psi(0) \otimes \psi(x) \leftrightarrow \gamma_5\psi(0) \otimes \gamma_5\psi(x)\right) + (\bar{\psi}(x)A_j(x)\sigma_{\mu k}A_i(0)\psi(0))_A\left\langle \left(\frac{1}{\beta}\right)(0)\gamma^k\sigma_{\nu}\not{p}_1\gamma^j\frac{1}{\beta}\psi(x)\right\rangle_B + x \leftrightarrow 0
\] (6.101)
Let us start now consider the second term in this formula:

\[
\frac{1}{s^4} \langle \bar{\psi}(x) A_j(x) \sigma_{\mu_1 \nu_1} A_i(0) \psi(0) \rangle_A \left( \left( \frac{1}{\beta} \right) (0) \gamma^i \sigma_{\nu_2} \gamma^j \frac{1}{\beta} \psi(x) \right)_B + \frac{2 p_{\mu_1}^4}{s^4} \langle \bar{\psi}(x) A_j(x) \sigma_\nu A_i(0) \psi(0) \rangle_A \left( \left( \frac{1}{\beta} \right) (0) \gamma^i \sigma_{\nu_2} \gamma^j \frac{1}{\beta} \psi(x) \right)_B + \mu \leftrightarrow \nu \quad (6.102)
\]

Similarly to eq. (6.91), projectile matrix elements cannot give factor \( s \) so the corresponding contribution to \( W_{\mu \nu} \) is of order of

\[
(a q^\perp_\mu q^\perp_\nu + b q_\perp^2 g^\perp_{\mu \nu}) \frac{m_1^2}{\beta^2 q^2 s^2} \quad \text{or} \quad \frac{2 p_{\mu_1}^4 m_1^4}{s^3 \beta^2 q^2 s^2} \quad (6.103)
\]

that are \( O \left( \frac{m^2}{q^2 s^2} \right) \) in comparison to eqs. (6.46) and (6.84), respectively.

We are left with

\[
\tilde{V}^5_{\mu \nu} = - \frac{p_{\mu_1} p_{\nu_1}}{s^4} \langle \bar{\psi}(x) A_j(x) \sigma_{\mu k} A_i(0) \psi(0) \rangle_A \left( \left( \frac{1}{\beta} \right) (0) \gamma^i \sigma^k \gamma_j \frac{1}{\beta} \psi(x) \right)_B + \mu \leftrightarrow \nu + x \leftrightarrow 0
\]

\[
= - \frac{4 p_{\mu_1} p_{\nu_1}}{s^4} \langle \bar{\psi}(x) A_j(x) \sigma^k A_i(0) \psi(0) \rangle_A \left( \left( \frac{1}{\beta} \right) (0) \gamma^i \sigma^k \gamma_j \frac{1}{\beta} \psi(x) \right)_B
\]

\[
- \frac{g_{\mu \nu}}{s^4} \langle \bar{\psi}(x) A_j(x) \sigma_{\nu k} A_i(0) \psi(0) \rangle_A \left( \left( \frac{1}{\beta} \right) (0) \gamma^i \sigma^k \gamma_j \frac{1}{\beta} \psi(x) \right)_B
\]

\[
- \frac{p_{\mu_1}}{s^3} \langle \bar{\psi}(x) A_j(x) \sigma_{\nu k} A_i(0) \psi(0) \rangle_A \left( \left( \frac{1}{\beta} \right) (0) \gamma^i \sigma^k \gamma_j \frac{1}{\beta} \psi(x) \right)_B + x \leftrightarrow 0
\]

(6.104)

First, note that the two last terms are small, of order of eq. (6.103), for the same reason as eq. (6.101) above. As to the first term in r.h.s. of eq. (6.104), using eq. (A.7) it can be rewritten as

\[
\tilde{V}^5_{\mu \nu} = - \frac{4 p_{\mu_1} p_{\nu_1}}{s^4} \langle \bar{\psi}(x) A_j(x) \sigma_{\nu k} A_i(0) \psi(0) \rangle_A \left( \left( \frac{1}{\beta} \right) (0) \sigma^k \gamma_j \frac{1}{\beta} \psi(x) \right)_B + x \leftrightarrow 0
\]

(6.105)

so the corresponding contribution to \( W_{\mu \nu} \) takes the form

\[
V^5_{\mu \nu} = - \frac{4 p_{\mu_1} p_{\nu_1}}{\beta^2 s^2 N_c} \int d^2 k_\perp \frac{1}{m^2} k^2_\perp (k, q - k) \perp H_f(q, k_\perp)
\]

(6.106)

where we used eqs. (C.2) and (C.16).

The full result for \( W^{(2b)}_{1 \mu \nu} \) is given by the sum of eqs. (6.98) and (6.106)

\[
W^{(2b)}_{1 \mu \nu} = \frac{4 p_{\mu_1} p_{\nu_1}}{\beta^2 s^2 N_c} \int d^2 k_\perp \left[ k^2_\perp F_f(q, k_\perp) - \frac{1}{m^2} k^2_\perp (k, q - k) \perp H_f(q, k_\perp) \right]
\]

(6.107)

6.3.3 Second term in eq. (6.4)

Let us start now consider the second term in eq. (6.4).

\[
\tilde{W}^{(2b)}_{2 \mu \nu} = \frac{N_c}{s} \langle A, B \rangle \left[ \bar{\Xi}_1(x) \gamma_\mu \psi_B(x) \right] [\bar{\psi}_B(0) \gamma_\nu \Xi_1(0)] + \mu \leftrightarrow \nu |A, B \rangle + x \leftrightarrow 0
\]

(6.108)
After Fierz transformation (A.1) we obtain

\[
\hat{W}^{(2b)}_{2\mu\nu} = -\frac{N_c}{2s} (\delta^\alpha_\mu \delta^\beta_\nu + \delta^\alpha_\nu \delta^\beta_\mu - g_{\mu\nu} g^{\alpha\beta}) \langle A, B | [\bar{\Xi}_1^m(x) \gamma_\alpha \Xi_1^m(0)] [\bar{\psi}_B^0(0) \gamma_\beta \psi_B^m(x)] \rangle + \epsilon_{\gamma\alpha_5 \gamma_\beta} \langle A, B | \gamma_\alpha \bar{\psi}_B(0) \gamma_\beta \psi_B^m(x) \rangle |A, B\rangle
\]

(6.109)

\[+ \gamma_\alpha \gamma_\beta \leftrightarrow \gamma_\alpha \gamma_5 \otimes \gamma_\beta \gamma_5 |A, B\rangle
\]

\[+ \frac{N_c}{2s} (\delta^\alpha_\mu \delta^\beta_\nu + \delta^\alpha_\nu \delta^\beta_\mu - \frac{1}{2} g_{\mu\nu} g^{\alpha\beta}) \langle A, B | [\bar{\Xi}_1^m(x) \sigma_\alpha \Xi_1^m(0)] [\bar{\psi}_B^0(0) \sigma_\beta \psi_B^m(x)] \rangle |A, B\rangle
\]

\[+ x \leftrightarrow 0
\]

Sorting out color-singlet terms, we get similarly to sum of eqs. (6.88) and (6.89)

\[
\hat{W}^{(2b)}_{2\mu\nu} = -\frac{1}{s} (\delta^\alpha_\nu p_{2\mu} + \delta^\alpha_\mu p_{2\nu} - g_{\mu\nu} p^2) \langle \bar{\psi}(x) B_1(x) \gamma_\alpha B_1(0) \psi(0) \rangle
\]

\[\times \left\langle \left( \frac{1}{2} \bar{\psi}(x) (0) \gamma^\beta \gamma^\gamma \gamma^\gamma \bar{\psi}(x) \right)_A + \psi(0) \otimes \psi(x) \leftrightarrow \gamma_5 \psi(0) \otimes \gamma_5 \psi(x) \right\rangle + x \leftrightarrow 0
\]

\[+ \frac{1}{s} (\delta^\alpha_\mu \delta^\beta_\nu + \delta^\alpha_\nu \delta^\beta_\mu - \frac{1}{2} g_{\mu\nu} g^{\alpha\beta})
\]

\[\times \left\{- p_2 \alpha \left\langle \left( \frac{1}{2} \bar{\psi}(x) (0) \gamma^\beta \sigma_\alpha \gamma^\gamma \gamma^\gamma \bar{\psi}(x) \right)_A \langle \bar{\psi}(x) B_1(x) \sigma_\beta \gamma_5 B_1(0) \psi(0) \rangle \right\}_B
\]

\[+ \left\langle \left( \frac{1}{2} \bar{\psi}(x) (0) \gamma^\beta \sigma_\alpha \gamma^\gamma \gamma^\gamma \bar{\psi}(x) \right)_A \langle \bar{\psi}(x) B_1(x) \sigma_\beta \gamma_5 B_1(0) \psi(0) \rangle \right\}_B \right\} + x \leftrightarrow 0
\]

(6.110)

Starting from this point, all calculations repeat those of sections 6.3.1 and 6.3.2 with replacements of \( p_1 \leftrightarrow p_2, \alpha_q \leftrightarrow \beta_q \) and exchange of projectile matrix elements and the target ones. The result is eq. (6.111) with these replacements so we finally get

\[
W^{(2b)}_{\mu\nu} = \frac{4p_{1\mu} p_{1\nu}}{\beta_q s N_c} \int d^2 k_\perp \left[ \frac{k_\perp^2}{m^2} \left[ f_f^i (\alpha_q, k_\perp) F_f^i (q, k_\perp) - \frac{1}{m^2} (q-k)^2 (q-k)_\perp H_f^i (q, k_\perp) \right] \right.\]

\[+ \frac{4p_{2\mu} p_{2\nu}}{\alpha_q s^2 N_c} \int d^2 k_\perp \left[ (q-k)^2 F_f^i (q, k_\perp) - \frac{1}{m^2} (q-k)^2 (q-k)_\perp H_f^i (q, k_\perp) \right]\]

(6.111)

### 6.4 Third term with two quark-quark-gluon operators

Let us consider the first term in the r.h.s. of eq. (6.5). After Fierz transformation it turns to

\[
\frac{1}{2} [\bar{\Xi}_1^m(x) \gamma_\mu \Xi_1^m(x)] (\bar{\psi}_B^0(0) \gamma_\nu \psi_A^m(0)) + \mu \leftrightarrow \nu
\]

\[= -\frac{g_{\mu\nu}}{4} (\bar{\Xi}_1^m(x) \gamma_\mu \psi_A^m(0)) (\bar{\psi}_B^0(0) \Xi_1^m(x)) + \frac{g_{\mu\nu}}{4} (\bar{\Xi}_1^m(x) \gamma_5 \psi_A^m(0)) (\bar{\psi}_B^0(0) \gamma_5 \Xi_1^m(x))
\]

\[+ \frac{g_{\mu\nu}}{4} (\bar{\Xi}_1^m(x) \gamma_\mu \psi_A^m(0)) (\bar{\psi}_B^0(0) \gamma_5 \Xi_1^m(x)) + \frac{g_{\mu\nu}}{4} (\bar{\Xi}_1^m(x) \gamma_5 \psi_A^m(0)) (\bar{\psi}_B^0(0) \gamma_\mu \Xi_1^m(x))
\]

\[+ \frac{1}{4} [\bar{\Xi}_1^m(x) \gamma_\mu \psi_A^m(0)) (\bar{\psi}_B^0(0) \gamma_5 \Xi_1^m(x)) + \mu \leftrightarrow \nu]
\]

\[+ \frac{1}{4} [\bar{\Xi}_1^m(x) \gamma_5 \psi_A^m(0)) (\bar{\psi}_B^0(0) \gamma_\mu \Xi_1^m(x)) + \mu \leftrightarrow \nu]
\]

\[+ \frac{1}{4} [\bar{\Xi}_1^m(x) \gamma_\mu \psi_A^m(0)) (\bar{\psi}_B^0(0) \sigma_{\mu\nu} \Xi_1^m(x)) + \mu \leftrightarrow \nu]
\]

\[-\frac{g_{\mu\nu}}{8} (\bar{\Xi}_1^m(x) \sigma^{\alpha\beta} \psi_A^m(0)) (\bar{\psi}_B^0(0) \sigma_{\alpha\beta} \Xi_1^m(x))
\]

(6.112)
Let us demonstrate that after sorting out color-singlet matrix elements the contribution \( W^{(2c)}_{\mu \nu} \) is \( O \left( \frac{1}{N_c^2} \right) \) in comparison to \( W^{(2a)}_{\mu \nu} \) (and \( W^{(2b)}_{\mu \nu} \)). Consider a typical term in the r.h.s. of eq. (6.112)

\[
\frac{N_c}{s} \langle A, B | \bar{\Xi}_1^m(x) \Gamma_1 \psi_A^n(0) (\bar{\psi}_B^n(0) \Gamma_2 \Xi_2^m(x)) | A, B \rangle \tag{6.113}
\]

\[
= \frac{N_c}{s} \langle A, B \rangle \left( \bar{\psi}_A^k \frac{1}{\alpha} \right) (x) A^{ml}_{j}(x) \gamma^j \frac{\not{p}_2}{s} \Gamma_1 \psi_A^n(0) \left( \bar{\psi}_B^n(0) \Gamma_2 \frac{\not{p}_1}{s} \gamma^j B_{km}(x) \frac{1}{\beta} \bar{\psi}_B(x) \right) | A, B \rangle
\]

After separation of color singlet contributions

\[
\langle A, B | (\bar{\psi}_A^k (A_i)_{nm} \psi_A^n) (\bar{\psi}_B(B_j)_{km} \psi_B^n) | A, B \rangle
\]

\[
= \langle \bar{\psi}_A^k (A_i)_{nm} \psi_A^n \rangle_A \langle \bar{\psi}_B^l (B_j)_{km} \psi_B^n \rangle_B - i f^{abc} \langle \bar{\psi}_A^k t_{ia} A_i^a \psi_A^n \rangle_A \langle \bar{\psi}_B^l t_{jd} B_j^d \psi_B^n \rangle_B
\]

\[
= \frac{1}{N_c} \langle \bar{\psi}_A A_i \psi_A \rangle_A \langle \bar{\psi}_B B_j \psi_B \rangle_B - 2 i f^{abc} (\bar{\psi}_A^k t_{ia} \psi_A) A (B_j^b t_{jd} \psi_B) B
\]

\[
= \frac{1}{N_c} \langle \bar{\psi}_A A_i \psi_A \rangle_A \langle \bar{\psi}_B B_j \psi_B \rangle_B - 2 i f^{abc} (\bar{\psi}_A^k t_{ia} \psi_A) A (B_j^b t_{jd} \psi_B) B
\]

we get

\[
\frac{N_c}{s} \langle A, B | \bar{\Xi}_1^m(x) \Gamma_1 \psi_A^n(0) (\bar{\psi}_B^n(0) \Gamma_2 \Xi_2^m(x)) | A, B \rangle \tag{6.115}
\]

\[
= \frac{1}{(N_c^2 - 1) s^4} \left( \frac{1}{\alpha} \right) A_j(x) \gamma^j \frac{\not{p}_2}{s} \Gamma_1 \psi(0) \left( \bar{\psi}(0) \Gamma_2 \frac{\not{p}_1}{s} \gamma^j B_i(0) \frac{1}{\beta} \bar{\psi}(x) \right) \tag{6.116}
\]

Since projectile and target matrix elements can bring \( s \) each and \( \frac{1}{\alpha} \) and \( \frac{1}{\beta} \) convert to \( \frac{1}{\alpha_s} \) and \( \frac{1}{\beta_s} \), the typical contribution of (6.115) to \( W_{\mu \nu}(q) \) is

\[
\sim \frac{1}{N_c^2} \times \left( g_{\mu \nu}^q \frac{q_1^2}{\alpha_s q_s} \frac{q_1^2}{\alpha_s q_s} \frac{q_1^2}{\alpha_s q_s} \right)
\]

In ref. [28] we calculated the sum of these structures corresponding to convolution of \( \mu \) and \( \nu \). In principle, one can repeat that calculation and find contribution to these structures separately. However, since the corresponding matrix elements of quark-quark-gluon operators are virtually unknown, in this paper we will disregard such \( \frac{1}{N_c^2} \) terms.

Thus, the contribution of eq. (5.6) to \( W_{\mu \nu}(q) \) is given in the leading order in \( N_c \) by the sum of equations (6.46), (6.84), and (6.111).
7 Power corrections from $J_A^\mu(x)J_B^\nu(0)$ terms

Power corrections of the second type come from the terms

$$\bar{\Psi}_1(x)\gamma_\mu \Psi_1(x)\bar{\Psi}_2(0)\gamma_\nu \Psi_2(0) + x \leftrightarrow 0$$  \hspace{2cm} (7.1)

where $\Psi_1$ and $\Psi_2$ are given by eq. (4.12).\footnote{In the appendix 8.3.2 to [28] it is demonstrated that higher-order terms in the expansion eq. (4.11) (denoted by dots) are small in our kinematical region $s \gg Q^2 \gg q_1^2$.}

We get

$$\begin{align*}
&\left[ (\bar{\psi}_A + \bar{\Xi}_1)(x)\gamma_\mu (\psi_A + \Xi_1)(x) \right] \left[ (\bar{\psi}_B + \bar{\Xi}_2)(0)\gamma_\nu (\psi_B + \Xi_2)(0) \right] + x \leftrightarrow 0 \\
= & \left[ \bar{\psi}_A(x)\gamma_\mu \psi_A(x) \right] [\bar{\psi}_B(0)\gamma_\nu \psi_B(0)] \\
&+ \left[ \bar{\Xi}_1(x)\gamma_\mu \psi_A(x) \right] [\bar{\psi}_B(0)\gamma_\nu \psi_B(0)] + \left[ \bar{\psi}_A(x)\gamma_\mu \Xi_1(x) \right] [\bar{\psi}_B(0)\gamma_\nu \Xi_2(0)] \\
&+ \left[ \bar{\Xi}_1(x)\gamma_\mu \psi_A(x) \right] [\bar{\psi}_B(0)\gamma_\nu \Xi_2(0)] + \left[ \bar{\psi}_A(x)\gamma_\mu \Xi_1(x) \right] [\bar{\psi}_B(0)\gamma_\nu \Xi_2(0)] \\
&+ \left[ \bar{\Xi}_1(x)\gamma_\mu \psi_A(x) \right] [\bar{\Xi}_2(0)\gamma_\nu \psi_B(0)] + \left[ \bar{\psi}_A(x)\gamma_\mu \Xi_1(x) \right] [\bar{\Xi}_2(0)\gamma_\nu \psi_B(0)] + x \leftrightarrow 0.
\end{align*}$$  \hspace{2cm} (7.2)

First, let us demonstrate that contributions to $W_{\mu\nu}$ from the second to fifth lines in eq. (7.2) vanish. Obviously, matrix element of the operator in the second line vanishes. Formally,

$$\int dx_\bullet e^{-i\alpha A_0} \langle p_A|\tilde{\psi}(x_\bullet, x_\perp)\gamma_\mu \hat{\psi}(x_\bullet, x_\perp)|p_A\rangle = \delta(\alpha_q)\langle p_A|\hat{\psi}(0)\gamma_\mu \hat{\psi}(0)|p_A\rangle,$$

$$\int dx_\bullet e^{-i\beta B_0} \langle p_B|\hat{\psi}(0)\gamma_\nu \hat{\psi}(0)|p_B\rangle = \delta(\beta_q)\langle p_B|\hat{\psi}(0)\gamma_\nu \hat{\psi}(0)|p_B\rangle$$  \hspace{2cm} (7.3)

and, non-formally, one hadron cannot produce the DY pair on its own.

It is easy to see that contributions to $\tilde{W}_{\mu\nu}$ from the third and the fourth lines in eq. (7.2) vanish due to the absence of color-singlet structure. Indeed, let us consider for example the term

$$\left[ \bar{\Xi}_1(x)\gamma_\mu \psi_A(x) \right] [\bar{\psi}_B(0)\gamma_\nu \psi_B(0)] = - \left[ \bar{\psi}_A^m \frac{1}{\alpha} \right] (x)\gamma^i B_i^{mn} \frac{\beta_2}{s} \gamma_\mu \psi_A^m(x) \right] [\bar{\psi}_B(0)\gamma_\nu \psi_B^l(0)]$$  \hspace{2cm} (7.4)

The corresponding term in $\tilde{W}_{\mu\nu}$ is

$$- \frac{N_c}{s} \left< \left( \bar{\psi}_A^m \frac{1}{\alpha} \right) (x)\gamma^i \frac{\beta_2}{s} \gamma_\mu \psi_A^m(x) \right> \left< \bar{\psi}_B^l(0)B_i^{mn}(0)\gamma_\nu \psi_B^l(0) \right>_{\mu, \nu} + \mu \leftrightarrow \nu$$  \hspace{2cm} (7.5)

which obviously does not have color-singlet contribution. Similarly, other three terms in the third and fourth lines in eq. (7.2) vanish.
Next, let us demonstrate that the contribution of the fifth line in eq. (7.2) vanishes for the same reason as in eq. (7.3). Let is consider for example the first term in the fifth line

\[
[\tilde{\Xi}_1(x) a_{\mu} \tilde{\Xi}_1(x)] [\bar{\psi}_B(0) \gamma_\mu \psi_B(0)]
\]

where we separated color-singlet contribution in the last line. The corresponding term in \(W_{\mu \nu}\) is

\[
\frac{1}{64 \pi^2 N_c} \int d^2 k_\perp \int d^2 x_\perp e^{i \alpha x_\perp + i (k,x)_\perp} \int d^2 x_\perp' e^{-i \beta x_\perp' + i(q,x')_\perp} \left( \left( \frac{1}{2} \right) \gamma_\mu \psi \right)_A \left( B_{\mu}^a(x,x') B_{\nu}^a(x,x') \bar{\psi}(0) \gamma_\nu \psi(0) \right)_B
\]

\[
= \frac{\delta(\alpha_q)}{32 \pi^5 N_c} \int d^2 x_\perp e^{i \alpha x_\perp + i (q,x)_\perp} \left( \left( \frac{1}{2} \right) \gamma_\nu \psi \right)_A \left( B_{\mu}^a(x,x') B_{\nu}^a(x,x') \bar{\psi}(0) \gamma_\nu \psi(0) \right)_B = 0
\]

Similarly, the contribution of the second term in the fifth line of eq. (7.2) will be proportional to \(\delta(\beta_q)\) and hence vanish.

Let us now discuss the non-vanishing contributions coming from last two lines in eq. (7.2). For example, the first term in the sixth line is

\[
[\tilde{\Xi}_1(x) a_{\mu} \tilde{\Xi}_1(x)] [\bar{\psi}_B(0) \gamma_\nu \tilde{\Xi}_2(0)] = \left( \frac{1}{2} \right) \gamma_\nu \psi_A(x) [\bar{\psi}_B(0) \gamma_\nu \tilde{\Xi}_2(0)] = \left[ \left( \frac{1}{2} \right) \gamma_\nu \psi_A(x) \right] [\bar{\psi}_B(0) \gamma_\nu \tilde{\Xi}_2(0)]
\]

\[
= \left[ \left( \frac{1}{2} \right) \gamma_\nu \psi_A(x) \right] [\bar{\psi}_B(0) \gamma_\nu \tilde{\Xi}_2(0)]
\]

Separating color-singlet contributions with the help of the formula

\[
\langle \bar{\psi}_m A_i^a \psi_n \rangle = \frac{2 f_{mn}}{N_c^2 - 1} \langle \bar{\psi}_A \psi \rangle
\]

we get the corresponding term in \(W_{\mu \nu}\) in the form

\[
\frac{N_c}{N_c^2 - 1} \int \frac{2}{s^3} \left( \left( \frac{1}{2} \right) \gamma_\nu \psi_A(x) \right) [\bar{\psi}_B(0) \gamma_\nu \tilde{\Xi}_2(0)] [\bar{\psi}_B(0) \gamma_\nu \tilde{\Xi}_2(0)]
\]

which is similar to eq. (6.57) with exception of extra color factor \(\frac{N_c}{N_c^2 - 1} \simeq \frac{1}{N_c}\). Consequently, as discussed in section 6.2.2, non-negligible contributions come from transverse \(\mu\) and \(\nu\) only. We calculate them in next section.

7.1 Last two lines in eq. (7.2)

In this section we calculate the traceless part of sixth and seventh lines eq. (7.2). Since we consider only transverse \(\mu\) and \(\nu\), to simplify notations we will call them \(m\) and \(n\) in
this section. Using eq. (4.12) and separating color-singlet matrix elements with the help of eq. (7.9), we rewrite the traceless part of sixth and seventh lines in eq. (7.2) as

\[
\begin{align*}
\frac{1}{2} & \left( \left[ \Xi_1(x) \gamma_m \psi_A(x) \right] [\bar{\psi}_B(0) \gamma_n \Xi_2(0)] + [\bar{\psi}_A(x) \gamma_m \Xi_1(x)] \left[ \Xi_2(0) \gamma_n \psi_A(0) \right] \\
\right. \\
+ & \left[ \Xi_1(x) \gamma_m \psi_A(x) \right] \left[ \Xi_2(0) \gamma_n \psi_B(0) \right] + [\bar{\psi}_A(x) \gamma_m \Xi_1(x)] \left[ \bar{\psi}_B(0) \gamma_n \Xi_2(0) \right] + m \leftrightarrow n \\
= & \frac{1}{2(N_c^2 - 1)s^2} \left( \left[ \left( \psi_A \frac{1}{\alpha} \right)(x) \gamma^\rho \not{p_2} \gamma_m \not{A}_k(0) \psi_A(x) \right] \left[ \bar{\psi}_B(0) \gamma_\mu \not{p_1} \not{B}_j(x) \frac{1}{\beta} \psi_B(0) \right] \\
+ & \left[ \bar{\psi}_A(x) \gamma_m \not{p_2} \gamma^\rho \not{A}_k(0) \psi_A(x) \right] \left[ \left( \bar{\psi}_B \frac{1}{\beta} \right)(0) \gamma_\mu \not{p_1} \gamma^\lambda \not{B}_j(x) \psi_B(0) \right] \\
+ & \left[ \bar{\psi}_A(x) \gamma_m \not{p_2} \gamma^\rho \not{A}_k(0) \psi_A(x) \right] \left[ \left( \bar{\psi}_B \frac{1}{\beta} \right)(0) \gamma_\mu \not{p_1} \gamma^\lambda \not{B}_j(x) \psi_B(0) \right] \\
+ & \left[ \bar{\psi}_A(x) \gamma_m \not{p_2} \gamma^\rho \not{A}_k(0) \psi_A(x) \right] \left[ \left( \bar{\psi}_B \frac{1}{\beta} \right)(0) \gamma_\mu \not{p_1} \gamma^\lambda \not{B}_j(x) \psi_B(0) \right] + m \leftrightarrow n \right) \tag{7.11}
\end{align*}
\]

To save space, hereafter we do not display subtraction of trace with respect to \(m, n\) indices but it is always assumed. Using formulas (A.22) we can write down the contribution to \(\hat{W}_{\mu
u}\) from sixth and seventh lines in eq. (7.2) in the form

\[
\begin{align*}
\hat{W}^{6+7th}_{mn} = & \frac{N_c}{(N_c^2 - 1)s^3} \left\langle \left( \psi \frac{1}{\alpha}(x) \not{p_2} \not{A}_m(0) \psi(x) \right) A \langle \bar{\psi}(0) \not{B}_n(x) \not{p_1} \frac{1}{\beta} \psi(0) \rangle \right\rangle_B \\
+ & \left\langle \bar{\psi}(x) \not{A}_{m}(0) \not{p_2} \frac{1}{\alpha} \psi(x) \right\rangle A \left\langle \left( \bar{\psi} \frac{1}{\beta} \right)(0) \not{p_1} \not{B}_n(x) \psi(0) \right\rangle_B \\
+ & \left\langle \left( \bar{\psi} \frac{1}{\alpha} \right)(x) \not{p_2} \not{A}_{m}(0) \psi(x) \right\rangle A \left\langle \left( \bar{\psi} \frac{1}{\beta} \right)(0) \not{p_1} \not{B}_n(x) \psi(0) \right\rangle_B \\
+ & \left\langle \bar{\psi}(x) \not{A}_{m}(0) \not{p_2} \frac{1}{\alpha} \psi(x) \right\rangle A \left\langle \bar{\psi}(0) \not{B}_n \not{p_1} \frac{1}{\beta} \psi(0) \right\rangle B + m \leftrightarrow n \right) + x \rightarrow 0, \\
= & \frac{N_c}{(N_c^2 - 1)s^3} \left\langle \left( \bar{\psi} \frac{1}{\alpha} \right)(x) \not{p_2} \not{A}_{m}(0) \psi(x) + \bar{\psi}(x) \not{A}_{m}(0) \not{p_2} \frac{1}{\alpha} \psi(x) \right\rangle_A \\
\times & \left\langle \left( \bar{\psi} \frac{1}{\beta} \right)(0) \not{p_1} \not{B}_n(x) \psi(0) \right\rangle_B + \bar{\psi}(0) \not{B}_n \not{p_1} \frac{1}{\beta} \psi(0) \right\rangle_B + m \leftrightarrow n \right) + x \rightarrow 0
\end{align*}
\]

Let us now consider corresponding matrix elements. It is easy to see that

\[
\begin{align*}
\frac{1}{8\pi^3 s^2} & \int d^2x \cdot dx \cdot e^{-i\alpha x \cdot + j(k,x) \cdot} \left\langle \left( \bar{\psi}(x, x \perp) \not{A}_i(0) \not{p_2} \frac{1}{\alpha} \psi(x, x \perp) \right) \right\rangle_A \\
= & \frac{1}{4\pi^3 s^2} \int d^2x \cdot dx \cdot e^{-i\alpha x \cdot + j(k,x) \cdot} \int_{-\infty}^\infty dx' \langle \bar{\psi}(x, x \perp) \not{p_2} [F_{si} + i\gamma_5 \tilde{F}_{si}] (0) \psi(x', x \perp) \rangle_A \\
= & \frac{k_i}{\alpha} j_1(\alpha, k \perp), \\
\frac{1}{8\pi^3 s^2} & \int d^2x \cdot dx \cdot e^{-i\alpha x \cdot + j(k,x) \cdot} \left\langle \left( \bar{\psi} \frac{1}{\alpha}(x, x \perp) \not{p_2} \not{A}_i(0) \psi(x, x \perp) \right) \right\rangle_A \\
= & -\frac{1}{4\pi^3 s^2} \int d^2x \cdot dx \cdot e^{-i\alpha x \cdot + j(k,x) \cdot} \int_{-\infty}^\infty dx' \langle \bar{\psi}(x', x \perp) \not{p_2} [F_{si} - i\gamma_5 \tilde{F}_{si}] (0) \psi(x, x \perp) \rangle_A, \\
= & -\frac{k_i}{\alpha} j_1(\alpha, k \perp), \tag{7.13}
\end{align*}
\]
\[ \frac{1}{8\pi^3 s} \int dx_\cdot e^{-iax_\cdot} \left\langle \bar{\psi}(0) \tilde{A}_1(x_\cdot, x_\perp) \frac{1}{\alpha} \psi(0) \right\rangle_A \]
\[ = -\frac{1}{\alpha} \frac{1}{4\pi^3 s^2} \int d^2 x_\perp dx_\cdot e^{-iax_\cdot+i(k_x)\perp} \int_{-\infty}^0 dx_\cdot' \left\langle \bar{\psi}(0) \gamma_2 \frac{1}{\alpha} \psi(0) \right\rangle_A \]
\[ = -k_i \frac{1}{\alpha} j_1^*(\alpha, k_\perp), \]
\[ = \frac{1}{8\pi^3 s} \int d^2 x_\perp dx_\cdot e^{-iax_\cdot+i(k_x)\perp} \left\langle \bar{\psi}(x_\cdot, x_\perp) \tilde{A}_1(0) \frac{1}{\alpha} \psi(x_\cdot, x_\perp) \right\rangle_B = k_i \frac{1}{\alpha} j_1^*(\alpha, k_\perp), \]
\[ \left\langle \bar{\psi}(0) \gamma_2 \frac{1}{\alpha} \psi(0) \right\rangle_A = -k_i \frac{1}{\alpha} j_1^*(\alpha, k_\perp), \]
\[ \left\langle \bar{\psi}(x_\cdot, x_\perp) \tilde{A}_1(0) \frac{1}{\alpha} \psi(x_\cdot, x_\perp) \right\rangle_B = k_i \frac{1}{\alpha} j_1^*(\alpha, k_\perp), \]
\[ \left\langle \bar{\psi}(0) \gamma_2 \frac{1}{\alpha} \psi(0) \right\rangle_A = -k_i \frac{1}{\alpha} j_1^*(\alpha, k_\perp), \]
\[ \left\langle \bar{\psi}(x_\cdot, x_\perp) \tilde{A}_1(0) \frac{1}{\alpha} \psi(x_\cdot, x_\perp) \right\rangle_B = k_i \frac{1}{\alpha} j_1^*(\alpha, k_\perp), \] (7.14)

where we used parametrization (D.2). For the target matrix elements, we obtain
\[ \frac{1}{8\pi^3 s} \int d^2 x_\perp dx_\cdot e^{-iax_\cdot+i(k_x)\perp} \left\langle \bar{\psi}(x_\cdot, x_\perp) \tilde{B}_1(0) \frac{1}{\beta} \psi(x_\cdot, x_\perp) \right\rangle_B = k_i \frac{1}{\beta} j_1^*(\alpha, k_\perp), \]
\[ \left\langle \bar{\psi}(0) \gamma_2 \frac{1}{\beta} \psi(0) \right\rangle_A = k_i \frac{1}{\beta} j_1^*(\alpha, k_\perp), \]
\[ \left\langle \bar{\psi}(x_\cdot, x_\perp) \tilde{B}_1(0) \frac{1}{\beta} \psi(x_\cdot, x_\perp) \right\rangle_B = k_i \frac{1}{\beta} j_1^*(\alpha, k_\perp), \] (7.15)

The corresponding contribution to (traceless) \( W(\alpha_q, \beta_q, x_\perp) \) takes the form
\[ W^{6+7\text{th}}_{mn}(q) - \text{trace} = \frac{s/2}{(2\pi)^4 N_c} \int d^4 x e^{-iax} (\bar{W}^{6+7\text{th}}_{mn}(x_\perp) - \text{trace}), \]
\[ = \frac{1}{(N_c^2 - 1) \alpha_q \beta_q s} \int d^2 k_\perp [(j_1 - j_1)(\alpha_q, k_\perp)(j_1^* - j_1^*)(\beta_q, (q - k)_\perp) + \text{c.c.}] \]
\[ \times |k_m(q - k)_n + m \leftrightarrow n + g_{mn}(k, q - k)_\perp| \] (7.16)

where we have recovered the subtraction of trace.

The trace part can be obtained in a similar way. Using eq. (A.21) one gets
\[ g^{mn} \bar{W}^{6+7\text{th}}_{mn} = \frac{2N_c}{(N_c^2 - 1)s^3} \left( \left\langle \left\langle \bar{\psi}(0) \gamma_2 \tilde{A}_m(0) \psi(x) \right\rangle_A \left\langle \bar{\psi}(0) \gamma_2 \tilde{B}_m(0) \frac{1}{\beta} \psi(0) \right\rangle_B \right. \right. \]
\[ \left. + \left\langle \bar{\psi}(0) \gamma_2 \tilde{A}_m(0) \psi(x) \right\rangle_A \left\langle \bar{\psi}(0) \gamma_2 \tilde{B}_m(0) \frac{1}{\beta} \psi(0) \right\rangle_B \right) \]
\[ + \left\langle \bar{\psi}(0) \gamma_2 \tilde{A}_m(0) \psi(x) \right\rangle_A \left\langle \bar{\psi}(0) \gamma_2 \tilde{B}_m(0) \frac{1}{\beta} \psi(0) \right\rangle_B \]
\[ + \left\langle \bar{\psi}(0) \gamma_2 \tilde{A}_m(0) \psi(x) \right\rangle_A \left\langle \bar{\psi}(0) \gamma_2 \tilde{B}_m(0) \frac{1}{\beta} \psi(0) \right\rangle_B + m \leftrightarrow n \right) + x \leftrightarrow 0, \]
\[ = \frac{2N_c}{(N_c^2 - 1)s^3} \left( \left\langle \left\langle \bar{\psi}(0) \gamma_2 \tilde{A}_m(0) \psi(x) + \bar{\psi}(x) \gamma_2 \tilde{A}_m(0) \frac{1}{\alpha} \psi(x) \right\rangle_A \right. \right. \]
\[ \left. \times \left\langle \left\langle \bar{\psi}(0) \gamma_2 \tilde{B}_m(0) \frac{1}{\beta} \psi(0) \right\rangle_B + \bar{\psi}(0) \gamma_2 \tilde{B}_m(0) \frac{1}{\beta} \psi(0) \right\rangle_B \right) + x \leftrightarrow 0 \] (7.17)
The corresponding contribution to trace part of \( W(\alpha_q, \beta_q, x_\perp) \) takes the form

\[
g^{mn} W^{6+7th}_{mn}(q) = \frac{s/2}{(2\pi)^4 N_c} \int d^4 x \ e^{-i q x} g^{mn} W^{6+7th}_{mn}(x_\perp) \tag{7.18}
\]

\[
= - \frac{2}{(N_c^2 - 1) \alpha_q \beta_q s} \int d^2 k_\perp (k, q - k)_\perp [(j_2 - \tilde j_2)(\alpha_q, k_\perp)(j^*_2 - \tilde j^*_2)(\beta_q, (q - k)_\perp) + \text{c.c.}]
\]

which agrees with eq. (6.2) from ref. [28] after replacements \( j_2 = j^\text{tw3}_2 - i j^{\text{tw3}}_2\) and \( \tilde j_2 = j^\text{tw3}_1 + i j^\text{tw3}_1\). It should be noted that the difference between \( j_1 \) and \( j_2 \) in traceless vs trace part is due to difference in formulas (A.22) and (A.21).

Thus, the result is the sum of eqs. (7.16) and (7.18)

\[
W^{6+7th}_{\mu \nu}(q) = \frac{s/2}{(2\pi)^4 N_c} \int d^4 x \ e^{-i q x} \tilde W^{6+7th}_{mn}(x_\perp)
\]

\[
= \frac{1}{(N_c^2 - 1) \alpha_q \beta_q s} \int d^2 k_\perp \left( [(j_1 - \tilde j_1)(\alpha_q, k_\perp)(j^*_1 - \tilde j^*_1)(\beta_q, (q - k)_\perp) + \text{c.c.}]ight.
\]

\[
\times [k_\mu(q - k)_\nu + \mu \leftrightarrow \nu + g^1_{\mu \nu}(k, q - k)_\perp]
\]

\[
- g^{1\nu}_{\mu \nu}(k, q - k)_\perp [(j_2 - \tilde j_2)(\alpha_q, k_\perp)(j^*_2 - \tilde j^*_2)(\beta_q, (q - k)_\perp) + \text{c.c.}] \tag{7.19}
\]

As we mentioned in the Introduction, in this paper we will take into account only leading and sub-leading terms in \( N_c \) and leave the \( \frac{1}{N_c^2} \) corrections discussed above for future publications.

Finally, as proved in appendix E, we can neglect contributions proportional to the product of quark and gluon TMDs.

### 8 Results and estimates

#### 8.1 Results

Assembling eqs. (5.10), (6.46), (6.84), (6.111), and (7.19) we get the result for \( W_{\mu \nu}(q) \) that consists of two parts:

\[
W_{\mu \nu}(q) = W^{1\nu}_{\mu \nu}(q) + W^{2\nu}_{\mu \nu}(q) \tag{8.1}
\]

The first, gauge-invariant, part is given by

\[
W^{1\nu}_{\mu \nu}(q) = W^{1F}_{\mu \nu}(q) + W^{1H}_{\mu \nu}(q),
\]

\[
W^{1F}_{\mu \nu}(q) = \sum_f e_f^2 W^{1F}_{\mu \nu}(q),
\]

\[
W^{1H}_{\mu \nu}(q) = \sum_f e_f^2 W^{1H}_{\mu \nu}(q),
\]

\[
W^{1F}_{\mu \nu}(q) = \frac{1}{N_c} \int d^2 k_\perp F^f(q, k_\perp) W^{E}_{\mu \nu}(q, k_\perp),
\]

\[
W^{1H}_{\mu \nu}(q) = \frac{1}{N_c} \int d^2 k_\perp H^f(q, k_\perp) W^{H}_{\mu \nu}(q, k_\perp) \tag{8.2}
\]

where \( F^f \) and \( H^f \) are given by eq. (6.29) and

\[
W^{E}_{\mu \nu}(q, k_\perp) = -q^\perp_{\mu \nu} + \frac{1}{Q^2} (q^\parallel_\mu q^\perp_\nu + q^\parallel_\nu q^\perp_\mu) + \frac{q^2}{Q^4} q^\parallel_\mu q^\parallel_\nu + \frac{\hat q^\perp q_\nu}{Q^2} [q^2 - 4(k, q - k)_\perp]
\]

\[
- \left[ \frac{\hat q^\perp}{Q^2} \left( \frac{g^\perp_{\nu \perp} - \frac{q^\perp q_\mu}{Q^2}}{q^\perp} \right) (q - 2k)_\perp + \mu \leftrightarrow \nu \right] \tag{8.3}
\]
\[ m^2 W_{\mu\nu}^H(q, k_{\perp}) \]
\[ = -k^\perp_{\mu}(q-k)^\perp_{\nu} - k^\perp_{\nu}(q-k)^\perp_{\mu} - g^\perp_{\mu\nu}(k, q - k)_{\perp} + 2 \tilde{q}^\perp_{\mu} \tilde{q}^\perp_{\nu} \frac{q^\parallel q^\parallel}{Q^2} k^2_{\perp}(q-k)^2_{\perp} \]
\[ - \left( \frac{q^\parallel}{Q^2} k^2_{\perp}(q-k)^\perp_{\nu} + k^\perp_{\nu}(q-k)^2_{\perp} \right) + \tilde{q}^\perp_{\mu} \frac{q^\parallel}{Q^2} \left[ k^2_{\perp}(q-k)^\perp_{\nu} - k^\perp_{\nu}(q-k)^2_{\perp} \right] + \mu \leftrightarrow \nu \]
\[ - \frac{\tilde{q}^\perp_{\mu} q^\perp_{\nu} + q^\perp_{\mu} q^\perp_{\nu}}{Q^2} [q^2_{\perp} - 2(k, q - k)_{\perp}] (k, q - k)_{\perp} - \frac{\tilde{q}^\perp_{\mu} q^\perp_{\nu} + \tilde{q}^\perp_{\mu} q^\perp_{\nu}}{Q^2} (2k - q, q)_{\perp} (k, q - k)_{\perp} \]

where \( q^\parallel = \alpha_q p_1 + \beta_q p_2 \) and \( \tilde{q}^\perp_{\mu} \equiv \alpha_q p_1 - \beta_q p_2 \). These are the same expressions as in eq. (2.3) if one identifies \( x_A \) with \( \alpha_q \) and \( x_B \) with \( \beta_q \) and neglects \( O(\frac{m^2}{Q^2}) \) terms in \( p_A \) and \( p_B \) and \( O(\frac{Q^4}{Q^2}) \) corrections due to difference between \( Q^2 \) and \( Q^2_{\perp} \). It is easy to see that \( q^\mu W_{\mu\nu}^F = 0 \) and \( q^\mu W_{\mu\nu}^H = 0 \). Note that \( q^\mu W_{\mu\nu}^F \) and \( q^\mu W_{\mu\nu}^H \) are exactly zero without any \( \frac{q^2}{Q^2} \) corrections. This is similar to usual “forward” DIS, but different from off-forward DVCS where the cancellations of right-hand sides of Ward identities involve infinite towers of twists [42–44].

The second part is
\[ W_{\mu\nu}^2(q) = \frac{1}{N_c} \sum_f e^2_f \int d^2 k_{\perp} \left[ \frac{1}{m^2} \left( [k^\perp_{\mu}(q-k)^\perp_{\nu} + \mu \leftrightarrow \nu] (k, q - k)_{\perp} \right. \right. \]
\[ - k^\perp_{\nu}(q-k)^\perp_{\mu} - (q-k)_{\perp} k^\perp_{\mu} k^\perp_{\nu} + g^\perp_{\mu\nu}(k, q - k)_{\perp} - g^\perp_{\mu\nu} k^2_{\perp}(q-k)_{\perp} \right] H_F \frac{f(q, k_{\perp})}{(N^2_c - 1)} \]
\[ + \frac{N_c}{N^2_c - 1} \left[ (k^2_{\perp}(q-k)^\perp_{\nu} + \mu \leftrightarrow \nu + g^\perp_{\mu\nu}(k, q - k)_{\perp}) J_{\perp} \frac{J_{\perp}(q, k_{\perp})}{(N^2_c - 1)} \right] \]
\[ - g^\perp_{\mu\nu}(k, q - k)_{\perp} J_{\perp} \frac{J_{\perp}(q, k_{\perp})}{(N^2_c - 1)} \right] + O \left( \frac{1}{N^2_c} \right) \right] + O \left( \frac{Q^4}{Q^2} \right) \]
(8.5)

where \( H_A \) is given by eq. (6.71) and
\[ J_{\perp} \frac{J_{\perp}(q, k_{\perp})}{(N^2_c - 1)} \right] + O \left( \frac{1}{N^2_c} \right) \right] + O \left( \frac{Q^4}{Q^2} \right) \]
(8.6)

These terms are not gauge invariant: \( q^\mu W_{\mu\nu}^2(q) \neq 0 \). The reason is that gauge invariance is restored after adding terms like \( \frac{m^2}{Q^2} \times \) eq. (5.11) which we do not calculate in this paper. Indeed, for example,
\[ q^\mu W_{\mu\nu}^2(q) \sim \frac{q^2}{\alpha_q \beta_q s} \text{ and } q^\mu \times \frac{q^2}{\alpha_q \beta_q s} \]

They are of the same order so one should expect that gauge invariance is restored after calculation of the terms \( \sim \frac{q^2}{\alpha_q \beta_q s} \) which are beyond the scope of this paper. For the same reason we see that all structures in eq. (5.11) except \( \frac{q^2}{\alpha_q \beta_q s} \) and \( \frac{q^2}{\alpha_q \beta_q s} \) are determined by leading-twist TMDs \( f_1 \) and \( h_1 \).

Sometimes it is convenient to represent hadronic tensor in transverse coordinate space. Introducing
\[ f_{\ell}(\alpha, b_{\perp}) \equiv \int \frac{d^2 k_{\perp}}{4\pi^2} e^{i(k, b)_{\perp}} \left\{ f(\alpha, k_{\perp}) \right\} \]

(8.8)
which agrees with eq. (6.2) from ref. [28]. This equation gives the sum of structures production. From eq. (8.1) we get

\[ W^{1F}_{\mu\nu}(\alpha_q, \beta_q, b_{\perp}) \]

\[
= 4\pi^2 \sum_f e_f^2 \left\{ -g_{\mu\nu} + \frac{i}{Q_\|}(q_{\|}\partial_{\nu} + q_{\|}\partial_{\mu}) - \frac{q_{\|}q_{\|} + \bar{q}_{\|}\bar{q}_{\|}}{Q_\|^2} f_1 - \frac{4\bar{q}_{\|}\bar{q}_{\|}}{Q_\|^2}(\partial_{\mu}f_1')(\partial_{\nu}f_1') \right\}
\]

\[
- \left[ \frac{\bar{q}_{\|}}{Q_\|^2} \left( \partial_{\mu}f_1 - \bar{f}_1\partial_{\mu}f_1' \right) + \mu \leftrightarrow \nu \right] + f_1 \leftrightarrow f_1 \} + O(\alpha_s)
\]

where \( f_1 = f_1(\alpha_q, b_{\perp}), \bar{f}_1 \equiv \bar{f}_1(\beta_q, b_{\perp}) \) everywhere except \( f \leftrightarrow \bar{f} \) terms where it is opposite (the question about rapidity cutoffs for TMDs will be addressed in section 9).

Similarly, we can write down \( W^H \) contribution in coordinate space. For future use, however, it is convenient to define Fourier transform in a slightly different way. Introduce

\[ h_i(k_{\perp}) \equiv k_i h_i^+(k), \bar{h}_i(k_{\perp}) \equiv \bar{k}_i \bar{h}_i^+(k) \]

and then \( W^{1IH}_{\mu\nu} \) can be represented as

\[
m^2 W^{1IH}_{\mu\nu}(\alpha_q, \beta_q, b_{\perp}) = 4\pi^2 \sum_f e_f^2 \left\{ \frac{g_{\mu\nu} h_i f_j - h_i f_j - h_i \bar{h}_j + 2 \frac{q_{\|}q_{\|} - \bar{q}_{\|}\bar{q}_{\|}}{Q_\|^4} \delta_{\mu\nu} h_i f_j + \frac{q_{\|}}{Q_\|^2} \left[ (i\partial^\| h_i^\|) h_j^\| - h_j^\| i\partial^\| h_i^\| \right] + \mu \leftrightarrow \nu \right\} - \frac{\bar{q}_{\|}\bar{q}_{\|}}{Q_\|^4} \frac{\bar{q}_{\|} q_{\|}}{Q_\|^2} \frac{\bar{q}_{\|} q_{\|}}{Q_\|^2} \left[ (i\partial^\| h_j^\|) h_i^\| - h_i^\| i\partial^\| h_j^\| \right] + h \leftrightarrow \bar{h}
\]

where \( h_i = h_i(\alpha_q, k_{\perp}) \) and \( \bar{h}_i \equiv \bar{h}_i(\beta_q, (q - k)_{\perp}) \) everywhere except \( h \leftrightarrow \bar{h} \) terms where it is opposite, cf. eq. (8.9).

### 8.2 Four Lorentz structures of hadronic tensor

The four Lorentz structures of hadronic tensor in Collins-Soper frame are given by eq. (1.2) where \( Q_{\perp} \equiv |Q_{\perp}| \)

\[
Z = \frac{\bar{q}}{Q_\|} \frac{1}{Q_{\perp}} (\alpha_q p_1 - \beta_q p_2), \quad X = \left[ \frac{Q_{\perp}}{Q_{\perp} Q_\|} (\alpha_q p_1 + \beta_q p_2) + \frac{q_{\|}}{Q_{\perp} Q_\|} q_{\perp} \right]
\]

such that \( q \cdot X = q \cdot Z = X \cdot Z = 0 \) and \( X^2 = Z^2 = -1 \).

First, let us check the structure corresponding to the total cross section of DY pair production. From eq. (8.1) we get

\[ W^\mu_{\mu}(q) = -\frac{2}{N_c} \sum_f e_f^2 \int d^2 k_{\perp} \left\{ \left[ 1 - 2 \frac{(k, q - k)_{\perp}}{Q_\|^2} \right] F^f(q, k_{\perp}) \right\}
\]

\[
+ 2 \frac{k_\|^2 (q - k)_{\perp}^2}{m_N Q_\|^2} H^f(q, k_{\perp}) + 4 \frac{N_c}{N_c^2 - 1} \left( k_{\perp} (q - k)_{\perp} + 2M^f(q, k_{\perp}) \right) \left[ 1 + O\left( \frac{1}{N_c^2} \right) \right] + O\left( \frac{q_{\|}^4}{Q_\|^4} \right)
\]

which agrees with eq. (6.2) from ref. [28]. This equation gives the sum of structures \( W^\mu_{\mu} = -(2W_T + W_L) \).
8.2.1 $W_L$

The easiest structure to get is $W_L$. Multiplying eq. (8.1) by $Z_\mu Z_\nu$ and comparing to eq. (1.1) we get

$$W_L(q) = Z^\mu Z^\nu W^{1\mu\nu}(q) = \sum_f e_f^2 \frac{1}{Q^2 N_c} \int dk_\perp \{ (q - 2k_\perp)^2 F^f(q, k_\perp)$$

$$+ \frac{1}{m^2} (2k_\perp^2 (q - k_\perp)^2 - [k_\perp^2 + (q - k_\perp)^2] (k, q - k_\perp) H^f(q, k_\perp) \} [1 + O\left(\frac{q^2}{Q^2}\right) + O\left(\frac{1}{N_c^2}\right)]$$

(8.14)

Thus, one may say that $W_L$ is known at LHC energies at $q_\perp^2 \ll Q^2$ as far as $f_1$ and $h_1^\perp$ are known.

8.2.2 $W_\Delta$

Using formula $q_\perp^2 Z^\mu Z^{\mu\nu} = (X \cdot q)_\perp W_\Delta = -\frac{Q_\perp Q_{\perp\perp}}{Q} W_\Delta$ we get

$$W_\Delta = \frac{Q}{Q_\parallel Q_{\perp\perp} N_c} \sum_f e_f^2 \int d^2 k_\perp \{ (q, q - 2k_\perp)^2 F^f(q, k_\perp)$$

$$- (q, q - 2k_\perp) \frac{(k, q - k_\perp)}{m^2} H^f(q, k_\perp) \} [1 + O(\frac{q^2}{Q^2}) + O(\frac{1}{N_c^2})]$$

(8.15)

Again, we see that $W_\Delta$ is expressed via $f_1$ and $h_1^\perp$ with great accuracy.

8.2.3 $W_T$

Next, from eqs. (8.13), (8.14) and $W^{\mu}_\mu = -2W_T + W_L$ one easily obtains

$$W_T(q) = \frac{1}{N_c} \sum_f e_f^2 \int d^2 k_\perp \{ \left[ 1 - \frac{q_\perp^2}{2Q_\parallel^2} \right] F^f(q, k_\perp)$$

$$+ \frac{1}{2m^2 Q_\parallel^2} (2k_\perp^2 (q - k_\perp)^2 + [k_\perp^2 + (q - k_\perp)^2] (k, q - k_\perp) H^f(q, k_\perp)$$

$$+ \frac{N_c}{N_c^2 - 1} (k, q - k_\perp) J^f_2(q, k_\perp) \} [1 + O\left(\frac{1}{N_c^2}\right) + O\left(\frac{q_\perp^2}{Q^2}\right)]$$

(8.16)

8.2.4 $W_\Delta^\Delta$

Finally, the easiest way to pick out $W_\Delta^\Delta$ is to multiply $W^{\mu\nu}$ by $q_\perp^2 q_\perp^2 / q_\perp^4$. One obtains from eq. (1.1) $q_\perp^2 q_\perp^2 W^{\mu\nu}(q) = \frac{q_\perp^2}{Q^2}(W_T - W_\Delta^\Delta)$. On the other hand, from eqs. (8.3) and (8.4) one gets

$$\frac{q_\perp^2 q_\perp^2}{q_\perp^4} W^{1\mu\nu}(q)$$

$$= \frac{1}{N_c} \sum_f e_f^2 \int d^2 k_\perp \{ F^f(q, k_\perp) + \left[ \frac{(k, q - k_\perp)^2}{m^2} - \frac{2(q, k_\perp)(q, q - k_\perp)}{m^2 q_\perp^2} \right] H^f(q, k_\perp) \}$$

(8.17)
and from eq. (8.5)

\[
\frac{q_\parallel^4 q_\perp^4 W_{\mu\nu}(q)}{q_\perp^2} = \frac{1}{N_c} \sum_f e_f^2 \frac{1}{Q_f^2 q_\perp^2} \int d^2 k \left\{ \frac{1}{m^2} \{2(q, k)_\perp (q - k)_\perp (k, q - k)_\perp \right. (8.18) \\
- k_1^2 (q - k)_\perp^2 - (q - k)_\perp^2 (q, k)_\perp^2 - q_1^2 (k, q - k)_\perp^2 + q_2^2 k_2^2 (q - k)_\perp^2 \} H_A(q, k_\perp) \\
+ \frac{N_c}{N_c^2 - 1} \left\{ [2(q, k)_\perp (q - k)_\perp - q_1^2 (k, q - k)_\perp] J_1^f(q, k_\perp) + q_2^2 (k, q - k)_\perp J_2^f(q, k_\perp) \right\}.
\]

Thus, we get

\[
W_{\Delta \Delta} = W_T - \frac{Q^2 q_\parallel^4 q_\perp^4 W_{\mu\nu}}{Q_f^2 q_\perp^2} = \frac{1}{N_c} \sum_f e_f^2 \frac{1}{Q_f^2 q_\perp^2} \int d^2 k \left\{ \frac{q_\parallel^2}{2Q_f^2} F^f(q, k_\perp) \\
+ \left( \frac{2(q, k)_\perp (q - k)_\perp}{q_\perp^2} - (k, q - k)_\perp \right) + \frac{1}{Q_f^2} \left\{ \frac{q_\parallel^2}{2Q_f^2} F^f(q, k_\perp) \right. (8.19) \\
+ \frac{1}{m^2} k_1^2 (q - k)_\perp^2 + q_1^2 (k, q - k)_\perp^2 - 2(q, k)_\perp (q - k)_\perp^2 \} \left\{ \frac{1}{m^2} H^f(q, k_\perp) \\
+ \frac{1}{m^2} Q_f^2 \left( \frac{k_2^2}{q_\perp^2} (q - k)_\perp^2 + \frac{q - k)_\perp^2}{q_\perp^2} - k_2^2 (q - k)_\perp^2 + (k, q - k)_\perp^2 \right) \right\} J_1^f(q, k_\perp) + O\left( \frac{1}{N_c^2} \right) + O\left( \frac{q_\perp^4}{Q^4} \right)
\]

This is the only function which has a \( O\left( \frac{1}{Q^2} \right) \), leading-\( N_c \) contribution proportional to twist-three TMD \( H_A \) not related to leading-twist TMDs by equations of motion. The functions \( W_T, W_L \), and \( W_{\Delta \Delta} \) do not have such contributions (although they have such contributions at the \( \frac{1}{N_c} \) level).

### 8.3 Estimates of \( W_i(q) \) at \( q_\perp^2 \gg m^2 \)

#### 8.3.1 Order-of-magnitude estimates

Following the analysis in ref. [28], let us estimate the relative strength of Lorentz structures \( W_i \) at \( q_\perp^2 \gg m^2 \). First, we assume that \( \frac{1}{k_1^2} \) is a good parameter and leave only terms leading in \( N_c \). Second, at \( q_\perp^2 \gg m^2 \) we probe the perturbative tails of TMD’s \( f_1 \approx \frac{1}{k_1^2} \) and \( h_1^+ \approx \frac{1}{k_1^2} \) [45]. So, as long as \( Q^2 \gg q_\perp^2 \gg m^2 \) we can approximate

\[
f_1(\alpha, k_\perp^2) \approx \frac{f(\alpha)}{k_\perp^2}, \quad h_1^+(\alpha, k_\perp^2) \approx \frac{m_N^2 h(\alpha)}{k_\perp^2}, \quad \bar{f}_1 \approx \frac{\bar{f}(\alpha)}{k_\perp^2}, \quad \bar{h}_1^+ \approx \frac{m_N^2 \bar{h}(\alpha)}{k_\perp^2} \quad (8.20)
\]

(up to logarithmic corrections). Similarly, for the target we can use the estimate

\[
f_1(\beta, k_\perp^2) \approx \frac{f(\beta)}{k_\perp^2}, \quad h_1^+(\beta, k_\perp^2) \approx \frac{m_N^2 h(\beta)}{k_\perp^2}, \quad \bar{f}_1 \approx \frac{\bar{f}(\beta)}{k_\perp^2}, \quad \bar{h}_1^+ \approx \frac{m_N^2 \bar{h}(\beta)}{k_\perp^2} \quad (8.21)
\]
as long as \( k_\perp^2 \ll Q^2 \). Thus, we get an estimate

\[
F^f(q, k_\perp) \simeq \frac{F^f(\alpha_q, \beta_q)}{k_\perp^2(q - k)^2}, \quad F^f(\alpha_q, \beta_q) \equiv f^f(\alpha_q)h^f(\beta_q) + f^f \leftrightarrow \tilde{f}^f, \\
H^f(q, k_\perp) \simeq m^2 \frac{H^f(\alpha_q, \beta_q)}{k_\perp^2(q - k)^2} , \quad H^f(\alpha_q, \beta_q) \equiv h^f(\alpha_q)\tilde{h}^f(\beta_q) + h^f \leftrightarrow \tilde{h}^f
\]

(8.22)

Note that due to the “positivity constraint” [46]

\[
h^f_+(x, k_\perp^2) \leq \frac{m}{|k_\perp|} f^f_+(x, k_\perp^2)
\]

(8.23)

we can safely assume that the functions \( f(x) \) and \( h(x) \) defined in eqs. (8.20) and (8.21) are of the same order of magnitude. Moreover, both theoretical [47] and phenomenological [48, 49] analysis indicate that \( h^f_+ \) is several times smaller than \( f^f_+ \) so in numerical estimates we will disregard the contribution of \( h^f_+ \).

### 8.3.2 Power corrections for total DY cross section

Substituting the above approximations to eq. (8.13) we get the following estimate of the strength of power corrections for total DY cross section [28]

\[
W^\mu_\mu(q) = -\frac{2}{Nc} \sum \epsilon^2 \int d^2k_\perp \left\{ \left[1 - \frac{2(k, q - k)_\perp}{Q^2}\right] F^f(q, k_\perp) + \frac{2k_\perp^2(q - k)^2}{m^2Q^2} H^f(q, k_\perp) \right\} \\
\simeq -\frac{2}{Nc} \sum \epsilon^2 \int d^2k_\perp \left\{ \left[1 - \frac{2(k, q - k)_\perp}{Q^2}\right] \frac{F^f(\alpha_q, \beta_q)}{k_\perp^2(q - k)^2} + \frac{2m^2 H^f(\alpha_q, \beta_q)}{Q^2 k_\perp^2(q - k)^2} \right\} \\
\simeq -\frac{2}{Nc} \sum \epsilon^2 \int d^2k_\perp \left[1 - \frac{2(k, q - k)_\perp}{Q^2}\right] \frac{F^f(\alpha_q, \beta_q)}{k_\perp^2(q - k)^2}
\]

(8.24)

where we used estimates (8.22) and the fact that \((k, q - k)_\perp \sim q_\perp^2 \gg m^2\). Thus, the relative weight of the leading term and power correction is determined by the factor \(1 - \frac{2(k, q - k)_\perp}{Q^2}\).

Due to eqs. (8.20) and (8.21), the integrals over \( k_\perp \) are logarithmic and should be cut from below by \( m^2N \) and from above by \( Q^2 \) so we get an estimate

\[
\frac{1}{k_\perp^2(q - k)^2} \simeq \frac{2\pi}{q_\perp^2} \ln \frac{q_\perp^2}{m^2}, \quad \int d^2k_\perp \frac{(k, q - k)_\perp}{k_\perp^2(q - k)^2} \simeq -\pi \ln \frac{Q^2}{q_\perp^2}
\]

(8.25)

where we assumed that the first integral is determined by the logarithmical region \( q_\perp^2 \gg k_\perp^2 \gg m^2N \) and the second by \( Q^2 \gg k_\perp^2 \gg q_\perp^2 \). Taking these integrals to eq. (8.24) one obtains

\[
W^\mu_\mu(q) = -\frac{4\pi}{Nc} \sum \epsilon^2 \left[ \frac{1}{q_\perp^2} \ln \frac{q_\perp^2}{m^2N} + \frac{1}{Q^2} \ln \frac{Q^2}{q_\perp^2} \right] F^f(\alpha_q, \beta_q)
\]

(8.26)

By this estimate, the power correction reaches the level of few percent at \( q_\perp \sim \frac{Q}{4} \).
8.3.3 Power corrections for $W_T$

Let us now consider estimates described in section 8.3.1 for $W_T$ given by eq. (8.16). At large $N_c$, we can omit the third line so

$$W_T(q) = \frac{1}{N_c} \sum_f e_f^2 \int d^2k_\perp \left[ 1 - \frac{q^2}{2Q^2} \right] F^j(q,k_\perp)$$

$$+ \frac{1}{2m^2Q^2} (2k^2_\perp(q-k)^2 + [k^2_\perp + (q-k)^2]_\perp)(q-k_\perp)H^j(q,k_\perp)$$

$$\simeq \frac{1}{N_c} \sum_f e_f^2 \int d^2k_\perp \left[ 1 - \frac{q^2}{2Q^2} \right] \frac{F^j(\alpha_q, \beta_q)}{k^2_\perp(q-k)^2_\perp}$$

$$+ \frac{m^2}{Q^2} \left( 1 + [k^2_\perp + (q-k)^2_\perp]_\perp \right) \frac{H^j(\alpha_q, \beta_q)}{k^2_\perp(q-k)^2_\perp}$$  \hspace{1cm} (8.28)

Again, due to $q^2_\perp \gg m^2$ the second term in braces can be neglected and we get

$$W_T(q) \simeq \frac{2\pi}{N_c} \left[ 1 - \frac{1}{2Q^2} \right] \ln \frac{q^2_\perp}{m^2} \sum_f e_f^2 F^j(\alpha_q, \beta_q)$$  \hspace{1cm} (8.29)

Thus, for $W_T$ the power correction reaches 10% level at $q_\perp \sim \frac{Q}{2}$.

8.3.4 Estimate of $W_L$

Again, using estimates from section 8.3.1 one obtains

$$W_L(q) = \sum_f e_f^2 \frac{1}{Q^2N_c} \int \frac{dk_\perp}{k^2_\perp(q-k)^2_\perp} \left\{ (q - 2k)^2_\perp F^j(\alpha_q, \beta_q) \right. $$

$$ + m^2 \left( 2 - [k^2_\perp + (q-k)^2_\perp]_\perp \right) H^j(q,k_\perp) \right\}$$  \hspace{1cm} (8.30)

which gives approximately

$$W_L(q) \simeq \frac{2\pi}{Q^2N_c} \left[ \ln \frac{q^2_\perp}{m^2} + 2 \ln \frac{Q^2}{q^2_\perp} \right] \sum_f e_f^2 F^j(\alpha_q, \beta_q)$$  \hspace{1cm} (8.31)

in agreement with eqs. (8.26) and (8.29). The estimate of the ratio of $W_L/W_T$ is

$$\frac{W_L(q)}{W_T(q)} \simeq \frac{q^2_\perp}{Q^2} \left[ 1 + 2 \frac{\ln Q^2/q^2_\perp}{\ln q^2_\perp/m^2} \right]$$  \hspace{1cm} (8.32)

8.3.5 Magnitude of $W_\Delta$

It is easy to see that $W_\Delta$ vanishes if one uses the estimates (8.20) and (8.21). Indeed, with these formulas $F(q,k_\perp)$ and $H(q,k_\perp)$ are symmetric under replacement $k_\perp \leftrightarrow (q-k)_\perp$ whereas $(q,q-2k)_\perp$ in the integrand in eq. (8.15) is antisymmetric. Moreover, this vanishing of $W_\Delta$ will occur for any factorizable model of TMDs $f_1$ and $h^\perp_1$: if $f_1(\alpha,k_\perp) = f(\alpha)\phi(k_\perp)$ and $h^\perp_1(\alpha,k_\perp) = h(\alpha)\psi(k_\perp)$ the integral (8.15) vanishes. On the other hand, $W_\Delta$ is only $\sim \frac{Q}{q} W_T$ so without better knowledge of TMDs it is impossible to tell whether $W_\Delta$ is smaller or bigger than, say, $W_L$. Also, if the parameter $\alpha_q \frac{Q}{q^2_\perp}$ is not negligible, to compare $W_\Delta$ and $W_L$ one needs to take into account $O(\alpha_q)$ corrections to $W_\Delta$ defined by TMDs other than $f_1$ and $h^\perp_1$. 

\[\text{Page } 47\]
8.3.6 Estimate of $W_{\Delta\Delta}$

Let us consider the relative weight of the terms in the r.h.s. of eq. (8.19). As we mentioned, we assume that $\frac{1}{k_0^2}$ is a valid small parameter so we can omit the last $J_1$ term. Also, it is natural to assume that $H_A^f(q,k_\perp)$ is of the same order of magnitude as $H^f(q,k_\perp)$ and, since the term with $H_A$ is a power correction, it is not unreasonable to neglect this term in the first approximation. Using now estimates (8.22) and the integrals

$$\int d^2k \frac{k_i(q-k)_j}{k_\perp^2(q-k)_\perp^4} \theta(k_\perp^2 - m^2) \theta((q-k)_\perp^2 - m^2) \simeq \frac{\pi}{2q_\perp^2} (g_{ik} + 2\frac{q_kq_i}{q_\perp^2}) \ln \frac{q_\perp^2}{m^2}$$

$$\int d^2k \frac{k_i(q-k)_j}{k_\perp^2(q-k)_\perp^4} \theta(k_\perp^2 - m^2) \theta((q-k)_\perp^2 - m^2) \simeq \frac{\pi}{q_\perp^2} \left( g_{ik} + 4\frac{q_kq_i}{q_\perp^2} \right) \ln \frac{q_\perp^2}{m^2}$$

one gets an estimate

$$W_{\Delta\Delta} \simeq \frac{1}{N_c} \sum_f e_f^2 \int d^2k_\perp \left\{ \frac{q_\perp^2}{2q_\perp^2} F^f(q,k_\perp) + \left( \frac{2(q_k)_\perp(q,q-k)_\perp}{q_\perp^2} - (q,k)_\perp + \frac{1}{Q^2} \left[ k_\perp^2(q-k)_\perp^2 \right. \right. \\
+ \left. \frac{1}{2} (k,q-k)_\perp^2 + (q-k)_\perp^2 \right] \frac{q_j^2}{q_\perp^2} - 2(q_k)_\perp^2(q,k)_\perp \right) \frac{1}{m^2} H^f(q,k_\perp) \right\}$$

$$\simeq \frac{\pi}{Q^2 N_c} \ln \frac{q_\perp^2}{m^2} \sum_f e_f^2 \left[ F^f(\alpha_q,\beta_q) + 4m^2Q^2 \left( 1 - \frac{q_\perp^2}{2Q^2} \right) H^f(\alpha_q,\beta_q) \right]$$

(8.34)

8.3.7 Lam-Tung relation

It is easy to see that if one neglects $H$ in eq. (8.14) the ratio of $W_L$ and $2W_{\Delta\Delta}$ is approximately

$$\frac{W_L}{2W_{\Delta\Delta}} \simeq 1 + 2 \ln \frac{Q^2}{q_\perp^2}$$

(8.35)

It seems like the Lam-Tung relation works better if we move closer to the domain of collinear factorization $Q^2 \sim Q_\perp^2 \gg m^2$.

8.3.8 Estimates of asymmetries

The differential cross section of DY process is parametrized as

$$\left( \frac{d\sigma}{d^4q} \right)^{-1} \frac{d\sigma}{d\Omega d^3q} = \frac{3}{4\pi(\lambda + 3)} \left[ 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi \right]$$

(8.36)

where $\Omega$ is the solid angle of the lepton in terms of its polar and azimuthal angles in the center-of-mass system of the lepton pair. The angular coefficients $\lambda$, $\mu$, and $\nu$ can be expressed in terms of the hadronic tensor:

$$\lambda = \frac{W_T - W_L}{W_T + W_L}, \quad \mu = \frac{W_\Delta}{W_T + W_L}, \quad \nu = \frac{2W_{\Delta\Delta}}{W_T + W_L}$$

(8.37)
For an estimate, let us take $s=8\text{ TeV}$ and $Q=90\text{ GeV}$ so that $x_A \sim x_B \sim 0.1$ in central region of rapidity. Although we did not include the contribution of $Z$-boson, we can compare our order-of-magnitude estimates with experimental data at this kinematics [50, 51]. Let us take $Q_\perp=20\text{ GeV}$ so the power corrections $\sim Q_\perp^2$ are small but sizable, of order of few per cent. At this kinematics, we obtain

$$1 - \lambda = 2 \frac{W_L}{W_T + W_L} \approx 2 - \frac{1 + 2 \ln Q^2/q_\perp^2}{2\ln q_\perp^2/m^2} \approx 0.19$$

from eq. (8.31) which agrees with estimates in ref. [52]. Next, in our kinematics the expression in square brackets in the r.h.s. in eq. (8.34) is approximately $F + 0.17H$. Since the Boer-Mulders function seem to be of order of few percent of $f^1$ (see the discussion in section 8.3.1), the term with $H$ can be safely neglected and we get

$$\nu = \frac{2W_{\Delta\Delta}}{W_T + W_L} \approx \frac{1}{2 + 2\ln Q^2/q_\perp^2} \approx 0.05$$

As to $\mu$ coefficient, as we mentioned, we cannot estimate it since with factorization hypothesis for TMDs it vanishes. Reversing the argument, if $\mu$ will be measured to be much smaller than $\nu$, it will be an argument in favor of factorization hypothesis for TMDs $f^1$ and $h^1_\perp$. Actually, there are experiments at much lower $q_\perp$ and $Q \sim \text{few GeV}$ which indicate that $\mu$ is very small [53].

Last but not least, let us estimate Lam-Tung relation. With our approximation in the above kinematics we get

$$\frac{W_L}{2W_{\Delta\Delta}} \approx 1 + 2\ln Q^2/q_\perp^2 \approx 2.0$$

so it seems to be violated at this kinematics. Again, these order-of-magnitude estimates do not include the contribution to DY cross section mediated by the $Z$-boson.

9 Coefficient functions and matching of rapidity cutoffs

The result (8.2) is a tree-level formula and to fully understand eq. (1.3) we should specify the rapidity cutoffs for $f^1$’s and $h^1_\perp$’s. As we discussed in section 3, the rapidity cutoff for longitudinal momenta in $f^1(\alpha_q, k_L)$ is $\beta \leq \sigma_p$ and for $f^1(\beta_q, k_L)$ $\alpha \leq \sigma_t$, where $\sigma_p$ and $\sigma_t$ are rapidity bounds for central fields. To avoid double counting, the region where both $\alpha < \sigma_t$ and $\beta < \sigma_p$ should give only small power corrections. This is achieved if one takes $\sigma_p, \sigma_t \sim \sqrt{Q_\perp}$ so power corrections from double counting are $Q_\perp^2$. In this case, the region $\alpha_q > \alpha > \sigma_t$, $\beta_q > \beta > \sigma_p$ gives Sudakov double-log factor

$$C(q, k, \frac{\alpha_q}{\sigma_t}, \frac{\beta_q}{\sigma_p}) \sim e^{\frac{a_{\text{cusp}}}{\pi} \ln \frac{\alpha_q}{\sigma_t} \ln \frac{\beta_q}{\sigma_p}}$$

where the coefficient $a_{\text{cusp}}/\pi$ is two times $\gamma_{\text{cusp}}$ for quarks. A more precise formula can be obtained from the requirement that the product of two TMDs and the coefficient function (9.1) does not depend on the “rapidity divides” $\sigma_p$ and $\sigma_t$. For simplicity, let us start
with the leading-twist term $\sim g_{\mu\nu}^+ F \gamma$. Rapidity evolution of the function $f_1(\alpha_q, b_\perp; \gamma_p)$ was found in ref. [54]\textsuperscript{13}

$$
\frac{d}{d\ln \gamma_p} f_1(\alpha_q, b_\perp; \gamma_p) = \frac{\alpha_s C_F}{\pi} \left[ -\ln \alpha_q \gamma_p - 1/2 \ln \frac{b_\perp^2 s}{4} - \gamma_E + O(\alpha_s) \right] f_1(\alpha_q, b_\perp; \gamma_p)
$$

$$
\frac{d}{d\ln \gamma_t} f_1(\beta_q, b_\perp; \gamma_t) = \frac{\alpha_s C_F}{\pi} \left[ -\ln \beta_q \gamma_t - 1/2 \ln \frac{b_\perp^2 s}{4} - \gamma_E + O(\alpha_s) \right] f_1(\beta_q, b_\perp; \gamma_t)
$$

(9.2)

where $\gamma_p = \sigma_p b_\perp \sqrt{s}$, $\gamma_t = \sigma_t b_\perp \sqrt{s}$ are $b_\perp$-dependent cutoffs providing conformal invariance of the leading-order TMD rapidity evolution (in the coordinate space) and $\gamma_E$ is Euler’s constant. Similar equation holds true for $\tilde{f}_1$ since it is obtained from the evolution of the same operator.

Looking at eqs. (9.1) and (9.2) one can guess that the coefficient function $\sim g_{\perp}^{\mu\nu}$ times two TMDs $f_1$ in the coordinate space has the form

$$
W_{g_\perp}(\alpha_q, \beta_q, b_\perp) \sim M(\alpha_q, \beta_q, b_\perp; \gamma_p, \gamma_t) \left[ f_1(\alpha_q, b_\perp; \gamma_p) \tilde{f}_1(\beta_q, b_\perp; \gamma_t) + f_1 \leftrightarrow \tilde{f}_1 \right]
$$

(9.3)

where

$$
M(\alpha_q, \beta_q, b_\perp; \gamma_p, \gamma_t) = e^{-\frac{\alpha_s C_F}{\pi} \ln \left( \frac{\alpha_q b_\perp \sqrt{s}}{\gamma_p} e^{\gamma_E} \right) \ln \left( \frac{\beta_q b_\perp \sqrt{s}}{\gamma_t} e^{\gamma_E} \right) + \frac{\alpha_s C_F}{2\pi} \ln^2 \gamma_p \gamma_t} \left[ 1 + O(\alpha_s) \right]
$$

(9.4)

It is easy to check that with $M$ given by eq. (9.4) we have $\frac{d}{d\gamma_p}(\text{r.h.s. of eq. (9.3)}) = 0$ and $\frac{d}{d\gamma_t}(\text{r.h.s. of eq. (9.3)}) = 0$ so our guess (9.4) for the coefficient function is correct up to $O(\alpha_s)$ terms.\textsuperscript{14}

To write precise matching for other parts of $W^{1F}_{\mu\nu}$ is a more complicated task. Let us start with $W^{1F}_{1\perp}$ terms considered in the next section.

### 9.1 Matching for $W^{1F}$ terms

We need to multiply eq. (8.9) in coordinate space by $M(\alpha_q, \beta_q, b_\perp; \gamma_p, \gamma_t)$. First, recall that

$$
M(\alpha_q, \beta_q, b_\perp; \gamma_p, \gamma_t) \left[ f_1(\alpha_q, b_\perp; \gamma_p) \tilde{f}_1(\beta_q, b_\perp; \gamma_t) + f_1 \leftrightarrow \tilde{f}_1 \right]
$$

(9.5)

does not actually depend on the “rapidity divides” $\gamma_p$ and $\gamma_t$. However, the differentiation $\frac{\partial}{\partial \gamma_p}$ affects evolution equations (9.2). In this case we modify the derivative with respect to $b_i$ as follows

$$
\frac{\partial}{\partial b_i} f_1(\alpha_q, b_\perp; \gamma_p) \equiv \left( \partial_i - \frac{\alpha_s C_F}{\pi} \frac{b_i}{b_\perp} \ln \gamma_p \right) f_1(\alpha_q, b_\perp; \gamma_p),
$$

$$
\frac{\partial}{\partial b_i} f_1(\beta_q, b_\perp; \gamma_t) \equiv \left( \partial_i - \frac{\alpha_s C_F}{\pi} \frac{b_i}{b_\perp} \ln \gamma_t \right) f_1(\beta_q, b_\perp; \gamma_t)
$$

(9.6)

\textsuperscript{13}As noted in ref. [54], the factor $\sim \gamma_E$ depends on the exact way to cut integrals over $\alpha$ and $\beta$. Here the factor $-\gamma_E$ corresponds to “smooth” cutoffs $e^{-\frac{\gamma_E}{2\pi}}$ and $e^{-\frac{\gamma_E}{\pi}}$, see the discussion in ref. [54].

\textsuperscript{14}The eq. (9.4) is obtained in the leading order in $\alpha_s$ so the argument of coupling constant is left undetermined. One should expect Sudakov formula with running coupling constant [11, 55] at the NLO level.
(and similarly for \(\hat{f}'s\)) so that the l.h.s. of these equations satisfy eqs. (9.2). Note also that \(\partial_t(M\hat{f}) = M(f\hat{f}_t + \hat{f}f_t)\). With these definitions, one can write \(W_{11}^{1F}\) in the double-log approximation in the form

\[
W_{11}^{1F}(\alpha_q, \beta_q, b_\perp) = \sum_{\text{flavors}} e_I^2 W_{IJ}^{1F}(\alpha_q, \beta_q, b_\perp) \quad (9.7)
\]

\[
W_{11}^{1F}(\alpha_q, \beta_q, b_\perp) = 4\pi^2 \left\{ - g_{\mu\nu} + \frac{i}{Q_\parallel} (q_\parallel^\mu \partial_\nu + q_\parallel^\nu \partial_\mu) - \frac{\bar{q}_\mu q_\nu + \bar{q}_\nu q_\mu}{Q_\parallel^2} M_1 f_1 - \frac{4\bar{q}_\mu q_\nu}{Q_\parallel^2} M(\bar{\partial}_t f_1)(\partial_t f_1) \right\}
\]

\[
- \left[ \frac{\bar{q}_\mu}{Q_\parallel^2} \left( \delta_\nu^i - \frac{q_\nu}{Q_\parallel^2} i\partial_t^i \right) M(f_1 \bar{\partial}_t f_1 - \bar{f}_1 \partial_t f_1) + f_1 \leftrightarrow \bar{f}_1 \right] + O(\alpha_s)
\]

where \(M = M(\alpha_q, \beta_q, b_\perp; \varsigma_p, \varsigma_t)\) and \(f_1 = f_1(\alpha_q, b_\perp, \varsigma_p), \bar{f}_1 \equiv \bar{f}_1(\beta_q, b_\perp, \varsigma_t)\) everywhere except \(f \leftrightarrow \bar{f}\) terms where it is opposite.

It is easy to check gauge invariance: \((\alpha_q p_1^\mu + \beta_q p_2^\mu + i\partial_\mu)W_{11}^{1F}(\alpha_q, \beta_q, b_\perp) = 0\).

9.2 Matching for \(W_{11}^{1H}\) terms

First, with our definitions (8.10) the eq. (B.6) reads

\[
\frac{1}{8\pi^2 s} \int dx_\perp^2 \left[ e^{-\text{perp}(\alpha_q, b_\perp; \varsigma_p)} \langle A_{\alpha} \bar{\psi}(x_\parallel, x_\perp) \psi(0) | A \rangle = \frac{1}{m_N} h_{\parallel}^{(1)}(\alpha, k_\perp) \right]
\]

\[
\frac{1}{8\pi^2 s} \int dx_\perp^2 \left[ e^{-\text{perp}(\alpha_p, b_\perp; \varsigma_p)} \langle A_{\beta} \bar{\psi}(x_\parallel, x_\perp) \psi(0) | A \rangle = \frac{1}{m_N} h_{\parallel}^{(1)}(\beta, k_\perp) \right] \quad (9.8)
\]

and similarly for the target matrix elements. With such definition, the evolution equation for \(h_{\parallel}(\alpha, b_\perp; \varsigma_p) \equiv \frac{1}{4\pi^2} \int d^2 k_\perp e^{i(k, x)_\perp} h_2(\alpha, k_\perp)\) is the same as eq. (9.2)

\[
\frac{d}{d\ln s_p} h_{\parallel}(\alpha_q, b_\perp; \varsigma_p) = \frac{\alpha_s C_F}{2} \left[ -\ln \alpha_q s_p - \ln \left( \frac{b_\perp s_p}{2} - \gamma_E \right) \right] h_{\parallel}(\alpha_q, b_\perp; \varsigma_p)
\]

\[
\frac{d}{d\ln s_t} h_{\parallel}(\beta_q, b_\perp; \varsigma_t) = \frac{\alpha_s C_F}{\pi} \left[ -\ln \beta_q s_t - \frac{1}{2} \ln \left( \frac{b_\perp s_t}{4} - \gamma_E \right) \right] h_{\parallel}(\beta_q, b_\perp; \varsigma_t) \quad (9.9)
\]

and similarly for \(\hat{h}_t\). The reason is that one-loop rapidity evolution for \(\hat{\psi}(x_\parallel, x_\perp) \Gamma \hat{\psi}(0)\) in the Sudakov region is the same for all matrices \(\Gamma\) between \(\hat{\psi}(x_\parallel, x_\perp)\) and \(\hat{\psi}(0)\) due to the fact that the “handbag” diagram in figure 3c is small and in two other diagrams (as well as self-energy corrections) the matrix \(\Gamma\) between \(\hat{\psi}(x_\parallel, x_\perp)\) and \(\hat{\psi}(0)\) just multiplies the result of calculation.

Next, to write down the product of \(m^2 W_{11}^{1H}\) and the coefficient function we need modified derivatives of \(h_t\)'s of eq. (9.6) type:

\[
\tilde{\partial}_t h_{\parallel}(\alpha_q, b_\perp; \varsigma_p) \equiv \left( \partial_t - \frac{\alpha_s C_F}{\pi} \frac{b_\perp}{\mu} \ln s_p \right) h_{\parallel}(\alpha_q, b_\perp; \varsigma_p),
\]

\[
\tilde{\partial}_t h_{\parallel}(\beta_q, b_\perp; \varsigma_t) \equiv \left( \partial_t - \frac{\alpha_s C_F}{\pi} \frac{b_\perp}{\mu} \ln s_t \right) h_{\parallel}(\beta_q, b_\perp; \varsigma_t) \quad (9.10)
\]

\footnote{Strictly speaking, the difference between \(\tilde{\partial}_t f\) and \(\partial_t f\) is \(\sim O(\alpha_s)\) but since our matching is correct at single-log limit (see eq. (9.4)) we keep \(\partial_t f\) to avoid corrections \(\sim O(\alpha_s \ln \varsigma)\) in our matching formulas (8.9) (and (8.11) below).}
At this point, the result of Sudakov evolution is that, as discussed in ref. [54], one can use the double-log Sudakov evolution (9.2) until down factorization of the amplitude in projectile, target and central fields at $M$ except $M$ (and similarly for $\bar{M}$’s) so that $\partial_i h_j$ will satisfy same evolution equations (9.9) as $h_j$. We get then

$$W_{\mu\nu}^{1H}(\alpha_q, \beta_q, b_\perp) = \sum_{\text{flavors}} e_2^2 W_{\mu\nu}^{fH}(\alpha_q, \beta_q, b_\perp),$$

$$\frac{m^2}{4\pi^2} W_{\mu\nu}^{fH}(\alpha_q, \beta_q, b_\perp) = M \left( \frac{g_{\mu\nu}(h^f_{ij}) - h^f_{ij} - h^f_{ij} - h^f_{ij}}{Q^4_{\parallel}} + \frac{2 g_{\parallel\parallel} - \bar{q}_{\parallel \nu} - \bar{q}_{\parallel \nu} - \bar{q}_{\parallel \nu} - \bar{q}_{\parallel \nu}}{Q^4_{\parallel}} \partial_i h^f_{ij} \partial_j h^f_{ij} \right)$$

$$+ \frac{q_{\parallel \nu} [ (\bar{q}_{\parallel \nu} h^f_{ij}) - h^f_{ij} - h^f_{ij} - h^f_{ij} ] + \bar{q}_{\parallel \nu} [ (\bar{q}_{\parallel \nu} h^f_{ij}) - h^f_{ij} - h^f_{ij} - h^f_{ij} ] + \mu \leftrightarrow \nu}$$

$$- \frac{\bar{q}_{\parallel \nu} - q_{\parallel \nu} - \bar{q}_{\parallel \nu} - \bar{q}_{\parallel \nu}}{Q^4_{\parallel}} \partial_i (M h^f_{ij} h^f_{ij} - h^f_{ij} h^f_{ij}) + h \leftrightarrow \bar{h}$$

(9.11)

where $M = M(\alpha_q, \beta_q; \varsigma_p, \varsigma_t)$ and $h_i = h_i(\alpha_q, k_\perp; \varsigma_p), \tilde{h}_i = \tilde{h}_i(\beta_q, (q-k)_\perp; \varsigma_t)$ everywhere except $h \leftrightarrow \bar{h}$ terms where it is opposite, cf. eq. (8.9).

Let us comment on the choice of “rapidity divides” $\varsigma_p$ and $\varsigma_t$ in the product $M(\alpha_q, \beta_q; \varsigma_p, \varsigma_t) f_1(\alpha_q, \beta_q, b_\perp; \varsigma_p) f_1(\beta_q, b_\perp; \varsigma_t)$ (and in similar $M_{\bar{h}}$ product). As we mentioned in the beginning of this section, in order to avoid double counting one should write down factorization of the amplitude in projectile, target and central fields at $\varsigma_p, \varsigma_t \sim 1$. After that, as discussed in ref. [54], one can use the double-log Sudakov evolution (9.2) until

$$\varsigma_p \geq \tilde{\varsigma}_p, \quad \varsigma_t \equiv \frac{1}{\alpha_q b_\perp \sqrt{s}}, \quad \varsigma_t \geq \tilde{\varsigma}_t, \quad \varsigma_t \equiv \frac{1}{\beta_q b_\perp \sqrt{s}}$$

(9.12)

At this point, the result of Sudakov evolution is

$$M(\alpha_q, \beta_q, b_\perp; \varsigma_p, \varsigma_t) [f_1(\alpha_q, b_\perp; \varsigma_p) f_1(\beta_q, b_\perp; \varsigma_t) + f_1 \leftrightarrow \bar{f}_1]$$

$$= e^{-\frac{2 \varsigma_p^2}{\pi \mu} \ln^2 \frac{Q^2 b_\perp^2}{2}} [f_1(\alpha_q, b_\perp; \varsigma_p) f_1(\beta_q, b_\perp; \varsigma_t) + f_1 \leftrightarrow \bar{f}_1]$$

(9.13)
so in the final result (8.9) one should take

\[ M \bar{f} f \rightarrow e^{-\frac{N_c \alpha_s}{2 \pi} \ln^2 Q^2 b_\perp^2} \left[ 1 + O(\alpha_s) \right] f_1(\alpha_q, b_\perp; \zeta_p) \bar{f}_1(\beta_q, b_\perp; \zeta_t) \]  

(9.14)

Similarly, for \( W_{\mu\nu}^{1H}(\alpha_q, \beta_q, b_\perp) \) one should take

\[ M \bar{h}_i h_j \rightarrow e^{-\frac{N_c \alpha_s}{2 \pi} \ln^2 Q^2 b_\perp^2} \left[ 1 + O(\alpha_s) \right] h_i(\alpha_q, b_\perp; \zeta_p) \bar{h}_j(\beta_q, b_\perp; \zeta_t) \]  

(9.15)

at the end of Sudakov evolution (9.9). It should be emphasized that since factor \( M \) is universal for (9.14) and (9.15), our estimates of asymmetries in section 8.3.8 are not affected by summation of Sudakov double logs.

### 9.2.1 Rapidity-only cutoff for TMD

As discussed in refs. [28, 29], from the rapidity factorization (3.8) we get TMDs with rapidity-only cutoff \( |\alpha| < \sigma_t \) or \( |\beta| < \sigma_p \) (or with modifications (9.12)). Such cutoff, relevant for small-\( x \) physics, is different from the combination of UV and rapidity cutoffs for TMDs used by moderate-\( x \) community, see the analysis in two [56–58] and three [59] loops. For the tree-level formulas of section 8, this difference in cutoffs does not matter, but if one uses the formulas from section 9 and integrates models for TMDs with Sudakov factor \( M \) of eq. (9.4), one has to relate TMDs with rapidity-only cutoffs to the TMD models with conventional cutoffs. This requires calculations at the NLO level which are in progress.

### 10 Conclusions and outlook

Main result of this paper is eq. (8.1) which gives the DY hadronic tensor for electromagnetic current at small \( x \) with gauge invariance at the \( 1/Q^2 \) level. The part (8.2), determined by leading-twist TMDs \( f_1 \) and \( h_\perp^1 \), is manifestly gauge invariant. The only non-gauge invariant term at the \( 1/Q^2 \) level is eq. (8.6) with transverse \( \mu \) and \( \nu \) which is \( \sim \frac{\nu_{\perp} \nu_{\perp}}{Q^2} \) times twist-3 TMDs. Also, in the leading-\( N_c \) approximation the only structure affected by those terms is \( W_{\Delta \Delta} \), all other structures are calculated up to \( O(\frac{Q^2}{Q^4}) \) terms. It is interesting to note that \( \frac{1}{Q^2} \) terms necessary for gauge invariance are calculated more than two decades after the calculation of \( \frac{1}{Q^2} \) corrections in ref. [20].

It should be emphasized that, as discussed above, our rapidity factorization is different from the standard factorization scheme for particle production in hadron-hadron scattering, namely splitting the diagrams in collinear to projectile part, collinear to target part, hard factor, and soft factor [9]. Here we factorize only in rapidity and the \( Q^2 \) evolution arises from \( k^2_{\perp} \) dependence of the rapidity evolution kernels, same as in the BK (and NLO BK [60]) equations. Also, since matrix elements of TMD operators with our rapidity cutoffs are UV-finite [61, 62], the only UV divergencies in our approach are usual UV divergencies absorbed in the QCD running coupling. For the tree-level result (8.1) this does not matter, but if one intends to use the result like (8.9) with Sudakov logarithms for conventional TMDs with double UV and rapidity cutoffs, one needs to relate our TMDs with rapidity-only cutoff to
conventional TMDs. Needless to say, the gauge-invariant tree-level result (8.2) should be correct for TMDs with any cutoffs.

An obvious outlook is to extend these results to the “real” DY process involving $Z$-boson contributions which are relevant for our kinematics. Another outlook is the one-loop calculations in this rapidity-based factorization and comparison to resummations of large $\ln x$ and $\ln Q^2/Q^2_\perp$ based on usual collinear factorization, see e.g. refs. [34, 63]. The study is in progress.

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A Formulas with Dirac matrices

A.1 Fierz transformation

First, let us write down Fierz transformation for symmetric hadronic tensor

\[
\frac{1}{2}[(\bar{\psi}\gamma_\mu\chi)(\bar{\chi}\gamma_\nu\psi) + \mu \leftrightarrow \nu] \quad (A.1)
\]

\[
= -\frac{1}{4}(\delta^\alpha_\mu\delta^\beta_\nu + \delta^\alpha_\nu\delta^\beta_\mu - g_{\mu\nu}g^{\alpha\beta})[(\bar{\psi}\gamma_\alpha\gamma_\mu\chi)(\bar{\chi}\gamma_\beta\chi) + (\bar{\psi}\gamma_\alpha\gamma_5\psi)(\bar{\chi}\gamma_5\chi)]
\]

\[
+ \frac{1}{4}(\delta_\mu^\alpha\delta_\nu^\beta + \delta_\nu^\alpha\delta_\mu^\beta - \frac{1}{2}g_{\mu\nu}g^{\alpha\beta}) (\bar{\psi}\gamma_\alpha\gamma_\mu\xi\psi)(\bar{\chi}\gamma_5\beta\chi) - \frac{g_{\mu\nu}}{4}(\bar{\psi}\gamma_5\psi)(\bar{\chi}\chi) + \frac{g_{\mu\nu}}{4}(\bar{\psi}\gamma_5\psi)(\bar{\chi}\gamma_5\chi)
\]

A.2 Formulas with $\sigma$-matrices

It is convenient to define\(^{16}\)

\[
\epsilon_{ij} \equiv \frac{2}{s}\epsilon_{ij} = \frac{2}{s}p^\mu_1 p^\nu_2 \epsilon_{\mu\nu ij}
\]

(A.2)

such that $\epsilon_{12} = 1$ and $\epsilon_{ij}\epsilon_{kl} = g_{ik}g_{jl} - g_{il}g_{jk}$. The frequently used formula is

\[
\sigma_{\mu\nu}\sigma_{\alpha\beta} = (g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha}) - i\epsilon_{\mu\nu\alpha\beta}\gamma_5 - i(g_{\mu\alpha}\sigma_{\nu\beta} - g_{\mu\beta}\sigma_{\nu\alpha} - g_{\nu\alpha}\sigma_{\mu\beta} + g_{\nu\beta}\sigma_{\mu\alpha}) \quad (A.3)
\]

with variations

\[
\frac{2}{s}\sigma_{\mu\nu}\sigma_{\alpha\beta} = g_{ij} - i\epsilon_{ij}\gamma_5 - i\sigma_{ij} - \frac{2i}{s}g_{ij}\sigma_{\mu\nu}, \quad \frac{2}{s}\sigma_{\alpha\beta}\sigma_{\mu\nu} = g_{ij} + i\epsilon_{ij}\gamma_5 - i\sigma_{ij} + \frac{2}{s}g_{ij}\sigma_{\alpha\beta},
\]

\[
\sigma_{ij}\sigma_{sk} = -\sigma_{sk}\sigma_{ij} = -ig_{ik}\sigma_{sj} + ig_{jk}\sigma_{si}, \quad \sigma_{ij}\sigma_{sk} = -\sigma_{sk}\sigma_{ij} = -ig_{ik}\sigma_{sj} + ig_{jk}\sigma_{si} \quad (A.4)
\]

\(^{16}\)We use conventions from Bjorken & Drell where $\epsilon^{0123} = -1$ and $\gamma^\mu\gamma^\nu\gamma^\lambda = g^{\mu\nu}\gamma^\lambda + g^{\nu\lambda}\gamma^\mu - g^{\mu\lambda}\gamma^\nu - i\epsilon^{\mu\nu\lambda\rho}\gamma^\rho\gamma_5$. Also, with this convention $\bar{\sigma}_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\lambda\rho}\sigma^{\lambda\rho} = i\sigma_{\mu\nu}\gamma_5$. 

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We need also the following formulas with $\sigma$-matrices in different matrix elements

$$
\bar{\sigma}_{\mu\nu} \otimes \bar{\sigma}_{\alpha\beta} = -\frac{1}{2}(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha})\gamma_{\xi\eta} \otimes \gamma^{\xi\eta}
$$
(A.5)

$$
+ g_{\mu\alpha}\sigma_{\beta\xi} \otimes \sigma_{\nu\xi} - g_{\nu\alpha}\sigma_{\beta\xi} \otimes \sigma_{\mu\xi} - g_{\mu\beta}\sigma_{\alpha\xi} \otimes \sigma_{\nu\xi} + g_{\nu\beta}\sigma_{\alpha\xi} \otimes \sigma_{\mu\xi} - \sigma_{\alpha\beta} \otimes \sigma_{\mu\nu}
$$
and

$$
\bar{\sigma}_{\mu\xi} \otimes \bar{\sigma}_{\nu} = -\frac{g_{\mu\nu}}{2}\sigma_{\xi\eta} \otimes \sigma_{\xi\eta} + \sigma_{\mu\xi} \otimes \sigma_{\nu} \quad \sigma_{\xi\eta} \otimes \sigma_{\xi\eta} = \bar{\sigma}_{\xi\eta} \otimes \sigma_{\xi\eta}
$$
(A.6)

$$
\sigma_{\mu\alpha\gamma_5} \otimes \sigma_{\nu\alpha\gamma_5} + \mu \leftrightarrow \nu - \frac{g_{\mu\nu}}{2}\sigma_{\xi\eta}\gamma_5 \otimes \sigma_{\xi\eta}\gamma_5 = -\left[\sigma_{\mu\xi} \otimes \sigma_{\nu} + \mu \leftrightarrow \nu - \frac{g_{\mu\nu}}{2}\sigma_{\xi\eta} \otimes \sigma_{\xi\eta}\right]
$$

$$
\sigma_k^i \otimes \gamma_1 \sigma_k \gamma_j = \hat{p}_2 \gamma_k^i \otimes \hat{p}_1 \gamma_1 \gamma_k \gamma_j = \hat{p}_2 \gamma_k \otimes \hat{p}_1 (g_{ik} \gamma_j + g_{jk} \gamma_i - g_{ij} \gamma_k)
$$

$$
= \hat{p}_2 (g_{ik} \gamma_j + g_{jk} \gamma_i - g_{ij} \gamma_k) \otimes \hat{p}_1 \gamma^k = (\gamma_2 \sigma_k^i \gamma_1) \otimes \sigma_k
$$
(A.7)

We will need also

$$
\hat{p}_2 \otimes \gamma_1 \hat{p}_1 \gamma_j + \hat{p}_2 \gamma_5 \otimes \gamma_1 \hat{p}_1 \gamma_5 \gamma_5 = \gamma_j \hat{p}_2 \gamma_1 \otimes \hat{p}_1 + \gamma_j \hat{p}_2 \gamma_5 \gamma_5 \otimes \hat{p}_1 \gamma_5
$$
(A.8)

### A.3 Formulas with $\gamma$-matrices and one gluon field

In the gauge $A_\mu = 0$ the field $A_i$ can be represented as

$$
A_i(x_\mu, x_\perp) = \frac{2}{s} \int_{-\infty}^{\infty} dx' \cdot A_i(x', x_\perp)
$$
(A.9)

(see eq. (4.3)). We define “dual” fields by

$$
\bar{A}_i(x_\mu, x_\perp) = \frac{2}{s} \int_{-\infty}^{\infty} dx' \cdot \bar{A}_i(x', x_\perp), \quad \bar{B}_i(x_\mu, x_\perp) = \frac{2}{s} \int_{-\infty}^{\infty} dx' \cdot \bar{B}_i(x', x_\perp),
$$
(A.10)

where $\bar{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\lambda\rho} F^{\lambda\rho}$ as usual. With this definition we have $\bar{A}_i = -\epsilon_{ij} A_j$ and $\bar{B}_i = \epsilon_{ij} B_j$ so

$$
\hat{p}_2 \bar{A}_i = -\bar{A} \hat{p}_2 \gamma_i, \quad \bar{A}_i \hat{p}_2 = -\gamma_i \hat{p}_2 \bar{A}, \quad \hat{p}_1 \bar{B}_i = -\bar{B} \hat{p}_1 \gamma_i, \quad \bar{B}_i \hat{p}_1 = -\bar{B} \hat{p}_1 \gamma_i
$$
(A.11)

where

$$
\bar{A}_i \equiv A_i - i \bar{A}_i \gamma_5, \quad \bar{B}_i \equiv B_i - i \bar{B}_i \gamma_5
$$
(A.12)

We also use

$$
A^j \hat{p}_2 \otimes \gamma_n \hat{p}_1 \gamma_j + A^j \hat{p}_2 \gamma_5 \otimes \gamma_n \hat{p}_1 \gamma_5 \gamma_5 = -\hat{p}_2 \bar{A}_n \otimes \hat{p}_1 - \hat{p}_2 \bar{A}_n \gamma_5 \otimes \hat{p}_1 \gamma_5
$$

$$
A^j \hat{p}_2 \otimes \gamma_1 \hat{p}_1 \gamma_n + A^j \hat{p}_2 \gamma_5 \otimes \gamma_1 \hat{p}_1 \gamma_5 \gamma_5 = -\bar{A}_n \hat{p}_2 \otimes \hat{p}_1 - \bar{A}_n \hat{p}_2 \gamma_5 \otimes \hat{p}_1 \gamma_5
$$

$$
\gamma_n \hat{p}_2 \gamma^i \otimes \hat{p}_1 B_i + \gamma_n \hat{p}_2 \gamma^i \gamma_5 \otimes \hat{p}_1 \gamma_5 B_i = -\hat{p}_2 \otimes \hat{p}_1 \bar{B}_i - \hat{p}_2 \gamma_5 \otimes \hat{p}_1 \bar{B}_i \gamma_5
$$

$$
\gamma^i \hat{p}_2 \gamma_\gamma \otimes \hat{p}_1 B_i + \gamma^i \hat{p}_2 \gamma_5 \gamma_5 \otimes \hat{p}_1 \gamma_5 B_i = -\hat{p}_2 \otimes \hat{p}_1 \bar{B}_i \gamma_5 - \hat{p}_2 \gamma_5 \otimes \hat{p}_1 \bar{B}_i \gamma_5
$$
(A.13)

and

$$
\frac{2}{s} \left[ \hat{p}_1 \hat{p}_2 \gamma_\gamma \otimes \gamma_\gamma + \hat{p}_1 \hat{p}_2 \gamma_5 \gamma_5 \otimes \gamma_\gamma \gamma_5 \right] = \gamma_\gamma \otimes \gamma_\gamma \bar{B}_i + \gamma_\gamma \gamma_5 \otimes \gamma_\gamma \bar{B}_i \gamma_5
$$

$$
\frac{2}{s} \left[ \gamma_\gamma \hat{p}_1 \hat{p}_2 \gamma_\gamma \otimes \gamma_\gamma + \gamma_\gamma \hat{p}_1 \hat{p}_2 \gamma_5 \gamma_5 \otimes \gamma_\gamma \gamma_5 \right] = \gamma_\gamma \otimes \gamma_\gamma \bar{B}_i \gamma_5 + \gamma_\gamma \gamma_5 \otimes \gamma_\gamma \bar{B}_i \gamma_5
$$
A.4 Formulas with $\gamma$-matrices and two gluon fields

With definition (A.10), we have the following formulas

$$A_i \otimes \tilde{B}_j = g_{ij} \hat{A}_k \otimes B^k - \hat{A}_j \otimes B_i, \quad \hat{A}_i \otimes B_j = g_{ij} A_k \otimes \tilde{B}^k - A_j \otimes \tilde{B}_i \quad (A.14)$$

$$\tilde{A}_i \otimes \tilde{B}_j = -g_{ij} A_k \otimes B^k + A_j \otimes B_i \quad \Rightarrow \quad \hat{A}_i \otimes \tilde{B}^i = -A_i \otimes B^i, \quad \hat{A}_i \otimes B_i = A_i \otimes \tilde{B}^i$$

Using these formulas, after some algebra one obtains

$$\gamma_m p_2^2 \gamma_j A^i \otimes \gamma_n p_1^1 \gamma_i B^j + \gamma_m p_2^2 \gamma_j A^i \otimes \gamma_n p_1^1 \gamma_i B^j \gamma_5 = \hat{p}_2 \hat{A}_n \otimes \hat{p}_1 \hat{B}_m + \hat{p}_2 \hat{A}_n \gamma_5 \otimes \hat{p}_1 \hat{B}_m \gamma_5$$

$$\gamma_j p_2^2 \gamma_m A^i \otimes \gamma_n p_1^1 \gamma_i B^j + \gamma_j p_2^2 \gamma_m A^i \otimes \gamma_n p_1^1 \gamma_i B^j \gamma_5 = \hat{p}_2 \hat{A}_n \otimes \hat{B}_m \gamma_1 - \hat{p}_2 \hat{A}_n \gamma_5 \otimes \hat{B}_m \gamma_1$$

$$\gamma_m p_2^2 \gamma_j A^i \otimes \gamma_n p_1^1 \gamma_i B^j + \gamma_m p_2^2 \gamma_j A^i \otimes \gamma_n p_1^1 \gamma_i B^j \gamma_5 = \hat{A}_n \otimes \hat{p}_2 \hat{B}_m + \hat{A}_n \gamma_5 \otimes \hat{p}_2 \hat{B}_m \gamma_5$$

$$\gamma_j p_2^2 \gamma_m A^i \otimes \gamma_n p_1^1 \gamma_i B^j + \gamma_j p_2^2 \gamma_m A^i \otimes \gamma_n p_1^1 \gamma_i B^j \gamma_5 = \hat{A}_n \otimes \hat{B}_m \gamma_1 + \hat{A}_n \gamma_5 \otimes \hat{B}_m \gamma_1 \gamma_5 \quad (A.15)$$

and

$$\hat{p}_2 \hat{A}_m \otimes \hat{p}_1 \hat{B}_n + \hat{p}_2 \hat{A}_n \gamma_5 \otimes \hat{p}_1 \hat{B}_m \gamma_5 = g_{mn} \hat{p}_2 \hat{A}_k \otimes \hat{p}_1 \hat{B}^k$$

$$\hat{p}_2 \hat{A}_m \otimes \hat{B}_n \hat{p}_1 + \hat{p}_2 \hat{A}_n \gamma_5 \otimes \hat{B}_m \hat{p}_1 = g_{mn} \hat{p}_2 \hat{A}_k \otimes \hat{B}^k \hat{p}_1$$

$$\hat{A}_m \hat{p}_2 \otimes \hat{p}_1 \hat{B}_n + \hat{A}_n \hat{p}_2 \otimes \gamma_5 \hat{A}_p \hat{p}_1 = g_{mn} \hat{A}_k \hat{p}_2 \otimes \hat{B}^k \hat{p}_1$$

$$\hat{A}_m \hat{p}_2 \otimes \hat{B}_n \hat{p}_1 + \gamma_5 \hat{A}_n \hat{p}_2 \otimes \hat{B}_m \hat{p}_1 = g_{mn} \hat{A}_k \hat{p}_2 \otimes \hat{B}^k \hat{p}_1 \quad (A.16)$$

The corollary of eq. (A.16) is

$$\hat{p}_2 \hat{A}_k \gamma_5 \otimes \hat{p}_1 \hat{B}^k \gamma_5 = \hat{p}_2 \hat{A}_k \otimes \hat{p}_1 \hat{B}^k \gamma_5$$

$$\gamma_5 \hat{A}_k \hat{p}_2 \otimes \hat{p}_1 \hat{B}^k \gamma_5 = \hat{A}_k \hat{p}_2 \otimes \hat{p}_1 \hat{B}^k$$

$$\gamma_5 \hat{A}_k \hat{p}_2 \otimes \gamma_5 \hat{B}^k \hat{p}_1 = \hat{A}_k \hat{p}_2 \otimes \hat{B}^k \hat{p}_1 \quad (A.17)$$

From eqs. (A.15) and (A.16) one easily obtains

$$\gamma_m p_2^2 \gamma_j A^i \otimes \gamma_n p_1^1 \gamma_i B^j + \gamma_m p_2^2 \gamma_j A^i \gamma_5 \otimes \gamma_n p_1^1 \gamma_i B^j \gamma_5 + m \leftrightarrow n = 2 g_{mn} \hat{p}_2 \hat{A}_k \otimes \hat{p}_1 \hat{B}^k \quad (A.18)$$

and

$$\gamma_m p_2^2 \gamma_j A^i \otimes \gamma_n p_1^1 \gamma_i B^j + \gamma_m p_2^2 \gamma_j A^i \gamma_5 \otimes \gamma_n p_1^1 \gamma_i B^j \gamma_5 - m \leftrightarrow n$$

$$= 2 \hat{p}_2 \hat{A}_n \otimes \hat{p}_1 \hat{B}_m - m \leftrightarrow n,$$

$$\gamma_j \hat{p}_2^2 A^i \otimes \gamma_n \hat{p}_1 \gamma_i B^j + \gamma_j \hat{p}_2^2 A^i \gamma_5 \otimes \gamma_n \hat{p}_1 \gamma_i B^j \gamma_5 - m \leftrightarrow n$$

$$= 2 A_n \otimes \hat{B}_m \hat{p}_1 - m \leftrightarrow n \quad (A.19)$$

We need also formulas

$$4 \hat{A}^i \hat{p}_1 \hat{p}_2 \gamma_j \otimes B^j \hat{p}_1 \hat{p}_2 \gamma_i$$

$$= A^i \gamma_j \otimes B^j \gamma_i - i A^i \gamma_j \gamma_5 \otimes B^j \gamma_i + i A^i \gamma_j \otimes B^j \gamma_5 \gamma_5 + A^i \gamma_j \gamma_5 \otimes B^j \gamma_5 \gamma_5,$$

$$4 \hat{A}^i \hat{p}_1 \hat{p}_2 \gamma_j \otimes B^j \hat{p}_1 \hat{p}_2 \gamma_i + A^i \hat{p}_1 \hat{p}_2 \gamma_j \gamma_5 \otimes B^j \hat{p}_1 \hat{p}_2 \gamma_5$$

$$= \gamma_j \hat{A}_j \otimes \gamma_5 \hat{B}_j + \gamma_j \hat{A}_j \gamma_5 \otimes \gamma_5 \hat{B}_j \gamma_5,$$

$$\gamma_i \hat{A}^i \otimes \gamma_j \hat{B}_j \gamma_5 = \gamma_i \hat{A}^i \otimes \gamma_5 \hat{B}_j + \gamma_i \hat{A}_j \gamma_5 \otimes \gamma_5 \hat{B}_j \gamma_5 \quad (A.20)$$
and

\[ A_k \gamma_i \bar{p}_2 \gamma^j \otimes B_j \gamma^i p_1 \gamma^k = p_2 \bar{A}_i \otimes p_1 \bar{B}^i = A \bar{p}_2 \gamma_i \otimes B \gamma^j, \]
\[ A_k \gamma_i \bar{p}_2 \gamma^j \otimes B_j \gamma^k \gamma^i = \bar{A}_p \gamma_i \otimes B \gamma^j, \]
\[ A_k \gamma_i \bar{p}_2 \gamma^j \otimes B_j \gamma^k \gamma^i = p_2 \bar{A}_i \otimes B^i \gamma^j = A \bar{p}_2 \gamma_i \otimes \gamma^j \gamma^k, \]
\[ A_k \gamma_i \bar{p}_2 \gamma^j \otimes B_j \gamma^i p_1 = A \bar{p}_2 \gamma_i \otimes B \gamma^j, \]
\[ A_k \gamma_i \bar{p}_2 \gamma^j \otimes B_j \gamma^i p_1 = A \bar{p}_2 \gamma_i \otimes B \gamma^j, \]
\[ (A.21) \]
\[ A_k \gamma_m \bar{p}_2 \gamma^j \otimes B_j \gamma^m \gamma^k + m \leftrightarrow n - g_{mn} A_k \gamma_i \bar{p}_2 \gamma_j \otimes B^j \gamma^k \gamma_i = \bar{A}_m \gamma_i \otimes \bar{B} \gamma^k \gamma_i, \]
\[ A_k \gamma_m \bar{p}_2 \gamma^j \otimes B_j \gamma^k \gamma^i = \bar{A}_m \gamma_i \otimes \bar{B} \gamma^k \gamma_i, \]
\[ A_k \gamma_m \bar{p}_2 \gamma^j \otimes B_j \gamma^i \gamma^k = \bar{A}_m \gamma_i \otimes \bar{B} \gamma^k \gamma_i. \]
\[ (A.22) \]

\[ (B.1) \]

B Parametrization of leading-twist matrix elements

Let us first consider matrix elements of operators without \( \gamma_5 \). The standard parametrization of quark TMDs reads (see e.g. ref. [64])

\[ \frac{1}{16 \pi^3} \int dx_s d^2 x_{\perp} e^{-i \alpha x_s + i(k,x)} \langle A | \bar{\psi}_f(x_s, x_{\perp}) \gamma^\mu \psi_f(0) | A \rangle = p_i^\mu f_i^f(\alpha, k_{\perp}), \]
\[ \frac{1}{16 \pi^3} \int dx_s d^2 x_{\perp} e^{-i \alpha x_s + i(k,x)} \langle A | \bar{\psi}_f(x_s, x_{\perp}) \psi_f(0) | A \rangle = m_N e_f(\alpha, k_{\perp}) \]

for quark distributions in the projectile and

\[ \frac{1}{16 \pi^3} \int dx_s d^2 x_{\perp} e^{-i \alpha x_s + i(k,x)} \langle A | \bar{\psi}_f(0) \gamma^\mu \psi_f(x_s, x_{\perp}) | A \rangle = \]
\[ -p_i^\mu f_i^f(\alpha, k_{\perp}) - k_{\perp}^\mu f_i^f(\alpha, k_{\perp}) - p_i^\mu m_N f_i^f(\alpha, k_{\perp}), \]
\[ \frac{1}{16 \pi^3} \int dx_s d^2 x_{\perp} e^{-i \alpha x_s + i(k,x)} \langle A | \bar{\psi}_f(0) \psi_f(x_s, x_{\perp}) | A \rangle = m_N e_f(\alpha, k_{\perp}) \]

for the antiquark distributions.\(^{17}\)

\(^{17}\)In an arbitrary gauge, there are gauge links to \(-\infty\) as displayed in eq. (5.9).
The corresponding matrix elements for the target are obtained by trivial replacements $p_1 \leftrightarrow p_2$, $x_* \leftrightarrow x_*$ and $\alpha \leftrightarrow \beta$:

\[
\frac{1}{16\pi^3} \int dx_+ d^2 x_\perp \ e^{-i\beta x_\perp + i(k, x_\perp)} \ (B| \bar{\psi}_f(x_*, x_\perp) \gamma^\mu \psi_f(0)|B)
\]

\[
= p_2^\mu f_1^\mu (\beta, k_\perp) + k_\perp^\mu f_1^\mu (\beta, k_\perp) + p_1^\mu \frac{2m_N^2}{s} f_3^\mu (\beta, k_\perp),
\]

\[
\frac{1}{16\pi^3} \int dx_+ d^2 x_\perp \ e^{-i\beta x_\perp + i(k, x_\perp)} \ (B| \bar{\psi}_f(x_*, x_\perp) \psi_f(0)|B) = m_N e^f (\beta, k_\perp),
\]

and

\[
\frac{1}{16\pi^3} \int dx_+ d^2 x_\perp \ e^{-i\beta x_\perp + i(k, x_\perp)} \ (B| \bar{\psi}_f(0) \gamma^\mu \psi_f(x_*, x_\perp)|B) = m_N \bar{e}^f (\beta, k_\perp).
\]

Matrix elements of operators with $\gamma_5$ are parametrized as follows:

\[
\frac{1}{16\pi^3} \int dx_+ d^2 x_\perp \ e^{-i\alpha x_\perp + i(k, x_\perp)} \ (A| \bar{\psi}_f(x_*, x_\perp) \gamma^\mu \gamma_5 \psi_f(0)|A) = -i\epsilon_{\mu, j} k^j g^f_1 (\alpha, k_\perp),
\]

\[
\frac{1}{16\pi^3} \int dx_+ d^2 x_\perp \ e^{-i\alpha x_\perp + i(k, x_\perp)} \ (A| \bar{\psi}_f(0) \gamma^\mu \gamma_5 \psi_f(x_*, x_\perp)|A) = -i\epsilon_{\mu, j} k^j \bar{g}^f_1 (\alpha, k_\perp)
\]

The corresponding matrix elements for the target are obtained by trivial replacements $p_1 \leftrightarrow p_2$, $x_* \leftrightarrow x_*$ and $\alpha \leftrightarrow \beta$ similarly to eq. (B.4).

The parametrization of time-odd Boer-Mulders TMDs are

\[
\frac{1}{16\pi^3} \int dx_+ d^2 x_\perp \ e^{-i\alpha x_\perp + i(k, x_\perp)} \ (A| \bar{\psi}_f(x_*, x_\perp) \sigma^\mu \omega \psi_f(0)|A)
\]

\[
= \frac{1}{m_N} (k_\perp^\mu p_1^\nu - \mu \leftrightarrow \nu) h^f_1 (\alpha, k_\perp) + \frac{2m_N}{s} (p_1^\mu p_2^\nu - \mu \leftrightarrow \nu) h_f (\alpha, k_\perp)
\]

\[
+ \frac{2m_N}{s} (k_\perp^\mu p_2^\nu - \mu \leftrightarrow \nu) h^f_3 (\alpha, k_\perp),
\]

\[
\frac{1}{16\pi^3} \int dx_+ d^2 x_\perp \ e^{-i\alpha x_\perp + i(k, x_\perp)} \ (A| \bar{\psi}_f(0) \sigma^\mu \omega \psi_f(x_*, x_\perp)|A)
\]

\[
= -\frac{1}{m_N} (k_\perp^\mu p_1^\nu - \mu \leftrightarrow \nu) h^f_1 (\alpha, k_\perp) - \frac{2m_N}{s} (p_1^\mu p_2^\nu - \mu \leftrightarrow \nu) h_f (\alpha, k_\perp)
\]

\[
- \frac{2m_N}{s} (k_\perp^\mu p_2^\nu - \mu \leftrightarrow \nu) h^f_3 (\alpha, k_\perp)
\]

and similarly for the target with usual replacements $p_1 \leftrightarrow p_2$, $x_* \leftrightarrow x_*$ and $\alpha \leftrightarrow \beta$.

Note that the coefficients in front of $f_3$, $g^f_1$, $h$ and $h^f_3$ in eqs. (B.1), (B.3), (B.5), and (B.6) contain an extra $\frac{1}{2}$ since $p_2^\mu$ enters only through the direction of gauge link so the result should not depend on rescaling $p_2 \rightarrow \lambda p_2$. For this reason, these functions do not contribute to $W(q)$ in our approximation.

Last but not least, an important point in our analysis is that any $f(x, k_\perp)$ may have only logarithmic dependence on Bjorken $x$ but not the power dependence $\sim \frac{1}{x}$. Indeed,
the low-x behavior of TMDs is determined by pomeron exchange with the nucleon. The interaction of TMD with BFKL pomeron is specified by so-called impact factor and it is easy to check that the impact factors for all leading-twist TMDs are similar and do not give extra $\frac{1}{x}$ factors. The only $\frac{1}{x}$ may had come from some unfortunate definition of TMD which includes factor $s$ artificially, but from power counting (5.11) we see that all definitions of leading-twist TMDs do not have such factors.

C Matrix elements of quark-quark-gluon operators

In this section we will demonstrate that matrix elements of quark-antiquark-gluon operators from section 6 can be expressed in terms of leading-power matrix elements from section B.

First, let us note that operators $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ in eqs. (4.13) are replaced by $\pm \frac{1}{\alpha_q}$ and $\pm \frac{1}{\beta_q}$ in forward matrix elements. Indeed,

$$\int dx_\bullet e^{-i\alpha x^\bullet} \left\langle \Phi(x_\bullet, x_{\perp}) \Gamma \frac{1}{\alpha + i\epsilon} \psi(0) \right\rangle_A$$

(C.1)

$$= \frac{1}{i} \int dx_\bullet \int_{-\infty}^{0} dx'_\bullet e^{-i\alpha x^\bullet} \left\langle \Phi(x_\bullet, x_{\perp}) \Gamma \psi(x'_\bullet, 0) \right\rangle_A = \frac{1}{\alpha_q} \int dx_\bullet e^{-i\alpha x^\bullet} \langle \Phi(x_\bullet, x_{\perp}) \Gamma \psi(0) \rangle_A$$

where $\Phi(x_\bullet, x_{\perp})$ can be $\bar{\psi}(x_\bullet, x_{\perp})$ or $\tilde{\psi}(x_\bullet, x_{\perp}) A_i(x_\bullet, x_{\perp})$ and $\Gamma$ can be any $\gamma$-matrix. Similarly,

$$\int dx_\bullet e^{-i\alpha x^\bullet} \left\langle \frac{1}{\alpha - i\epsilon} \psi(0) \right\rangle_A = \frac{1}{\alpha_q} \int dx_\bullet e^{-i\alpha x^\bullet} \langle \bar{\psi}(x_\bullet, x_{\perp}) \Gamma \psi(0) \rangle_A$$

(C.2)

$$\int dx_\bullet e^{-i\alpha x^\bullet} \left\langle \frac{1}{\alpha - i\epsilon} \psi(0) \Gamma \Phi(x_\bullet, x_{\perp}) \right\rangle_A = -\frac{1}{\alpha_q} \int dx_\bullet e^{-i\alpha x^\bullet} \langle \bar{\psi}(0) \Gamma \Phi(x_\bullet, x_{\perp}) \rangle_A$$

$$\int dx_\bullet e^{-i\alpha x^\bullet} \langle \bar{\psi}(0) \Gamma \psi(x_\bullet, x_{\perp}) \rangle_A = -\frac{1}{\alpha_q} \int dx_\bullet e^{-i\alpha x^\bullet} \langle \bar{\psi}(0) \Gamma \psi(x_\bullet, x_{\perp}) \rangle_A$$

(C.3)

The corresponding formulas for target matrix elements are obtained by substitution $\alpha \leftrightarrow \beta$ (and $x_\bullet \leftrightarrow x_\bullet$).

Next, we will use QCD equation of motion to reduce quark-quark-gluon TMDs to leading-twist TMDs (see ref. [20]). Let us start with matrix element

$$\int dx_\bullet dx_{\perp} e^{-i\alpha x^\bullet + i(k, x)_{\perp}} \langle A | \bar{\psi}(x_\bullet, x_{\perp}) \gamma_\perp \gamma_2 \tilde{A}_i(x_\bullet, x_{\perp}) \psi(0) | A \rangle$$

(C.4)

$$= - \int dx_\bullet dx_{\perp} e^{-i\alpha x^\bullet + i(k, x)_{\perp}} \langle A | \bar{\psi}(x_\bullet, x_{\perp}) \gamma_\perp \gamma_2 \gamma_i \psi(0) | A \rangle$$

$$= \int dx_\bullet dx_{\perp} e^{-i\alpha x^\bullet + i(k, x)_{\perp}}$$

$$\times [ \langle A | \bar{\psi}(x_\bullet, x_{\perp}) \gamma_\perp \gamma_j \gamma_2 \gamma_i \psi(0) | A \rangle + i \langle A | \bar{\psi}(x_\bullet, x_{\perp}) \gamma_j \gamma_2 \gamma_i \psi(0) | A \rangle ]$$.
Using QCD equations of motion \((4.1)\) we can rewrite the r.h.s. of eq. \((C.4)\) as

\[
\int dx_\bullet dx_{\perp} e^{-i\alpha_q x_\bullet + i(k,x)_\perp} \left( \langle A | \tilde{\psi}(x_\bullet, x_{\perp}) \tilde{\gamma}_5 \gamma_i \tilde{\psi}(0) | A \rangle + \alpha_q \langle A | \tilde{\psi}(x_\bullet, x_{\perp}) \tilde{p}_2 \gamma_i \tilde{\psi}(0) | A \rangle \right) = \int dx_\bullet dx_{\perp} e^{-i\alpha_q x_\bullet + i(k,x)_\perp} \left( -k_i \langle A | \tilde{\psi}(x_\bullet, x_{\perp}) \tilde{p}_2 \gamma_i \tilde{\psi}(0) | A \rangle + \alpha_q s \langle A | \tilde{\psi}(x_\bullet, x_{\perp}) \gamma_i \tilde{\psi}(0) | A \rangle \right)
\]

\[
= -k_i 8\pi^3 s f_1(\alpha_q, k_{\perp}) + 8\pi^3 s \alpha_q k_i \left[ f_\perp(\alpha_q, k_{\perp}) + \gamma^\perp(\alpha_q, k_{\perp}) \right],
\]

where we used parametrizations \((B.1)\) and \((B.5)\) for the leading power matrix elements.

Now, the second term in eq. \((C.5)\) contains extra \(\alpha_q\) with respect to the first term,\(^{18}\) so it should be neglected in our kinematical region \(s \gg Q^2 \gg q_0^2\) and we get

\[
\frac{1}{8\pi^3 s} \int dx_\bullet dx_{\perp} e^{-i\alpha_q x_\bullet + i(k,x)_\perp} \langle A | \tilde{\psi}(x_\bullet, x_{\perp}) \tilde{p}_2 \tilde{A}_i(0) \tilde{\psi}(0) | A \rangle = -k_i f_1(\alpha_q, k_{\perp}).
\]

By complex conjugation

\[
\frac{1}{8\pi^3 s} \int dx_{\perp} dx_\bullet e^{-i\alpha_q x_\bullet + i(k,x)_\perp} \langle A | \tilde{\psi}(x_\bullet, x_{\perp}) \tilde{A}_i(0) \tilde{p}_2 \tilde{\psi}(0) | A \rangle = -k_i f_1(\alpha_q, k_{\perp}).
\]

For the corresponding antiquark distributions we get

\[
\frac{1}{8\pi^3 s} \int dx_\bullet dx_{\perp} e^{-i\alpha_q x_\bullet + i(k,x)_\perp} \langle A | \tilde{\psi}(0) \tilde{A}_i(0) \tilde{p}_2 \tilde{\psi}(x_\bullet, x_{\perp}) | A \rangle = -k_i f_1(\alpha_q, k_{\perp}).
\]

and

\[
\frac{1}{8\pi^3 s} \int dx_\bullet dx_{\perp} e^{-i\alpha_q x_\bullet + i(k,x)_\perp} \langle A | \tilde{\psi}(0) \tilde{p}_2 \tilde{A}_i(0) \tilde{\psi}(x_\bullet, x_{\perp}) | A \rangle = -k_i f_1(\alpha_q, k_{\perp}).
\]

The corresponding target matrix elements are obtained by trivial replacements \(x_\bullet \leftrightarrow x_\bullet, \alpha_q \leftrightarrow \beta_q\) and \(\tilde{p}_2 \leftrightarrow \tilde{p}_1\).

Next, let us consider

\[
\frac{1}{8\pi^3 s} \int dx_\bullet dx_{\perp} e^{-i\alpha_q x_\bullet + i(k,x)_\perp} \langle A | \tilde{\psi}(x_\bullet, x_{\perp}) \tilde{p}_2 A(x_\bullet, x_{\perp}) \tilde{\psi}(0) | A \rangle
\]

\[
= \frac{1}{8\pi^3 s} \int dx_\bullet dx_{\perp} e^{-i\alpha_q x_\bullet + i(k,x)_\perp}
\]

\[
\times \left[ \langle A | \tilde{\psi}(x_\bullet, x_{\perp}) \tilde{\gamma}_5 \gamma_i \tilde{\psi}(0) | A \rangle + i \langle A | \tilde{\psi}(x_\bullet, x_{\perp}) \tilde{p}_2 \tilde{\psi}(0) | A \rangle \right].
\]

\(^{18}\)As discussed in the end of section B, all leading-twist TMDs can have only logarithmic dependence on Bjorken \(x\) (which is here either \(\alpha_q\) for the projectile or \(\beta_q\) for the target matrix elements).
Using QCD equation of motion and parametrization (B.6), one can rewrite the r.h.s. of this equation as

\[
\frac{1}{8\pi^3 s} \int dx_\bot dx_\bot \ e^{-i\alpha_q x^\bot + i(k,x)_\bot} \left[A|\bar{\psi} (x_\bot, x_\bot) \hat{k}_\bot \hat{p}_2 \bar{\psi}(0)|A\right] + \alpha_q \langle A|\bar{\psi} (x_\bot, x_\bot) \hat{p}_1 \hat{p}_2 \bar{\psi}(0)|A\rangle
\]

\[
= i \frac{k^2}{m_N} h^\perp (\alpha_q, k_\perp) + \alpha_q m_N \left[e(\alpha, k_\perp) + ih(\alpha, k_\perp)\right].
\]  

(C.11)

Again, only the first term contributes in our kinematical region so we finally get

\[
\frac{1}{8\pi^3 s} \int dx_\bot dx_\bot \ e^{-i\alpha_q x^\bot + i(k,x)_\bot} \langle A|\bar{\psi}^f (x_\bot, x_\bot) \hat{p}_2 \mathcal{A}(x_\bot, x_\bot) \bar{\psi}^f (0)|A\rangle = i \frac{k^2}{m} h^\perp (\alpha_q, k_\perp).
\]  

(C.12)

By complex conjugation we obtain

\[
\frac{1}{8\pi^3 s} \int dx_\bot dx_\bot \ e^{-i\alpha_q x^\bot + i(k,x)_\bot} \langle A|\bar{\psi}^f (x_\bot, x_\bot) \hat{p}_2 \mathcal{A}(0) \bar{\psi}^f (0)|A\rangle = i \frac{k^2}{m} h^\perp (\alpha_q, k_\perp).
\]  

(C.13)

For corresponding antiquark distributions one gets in a similar way

\[
\frac{1}{8\pi^3 s} \int dx_\bot dx_\bot \ e^{-i\alpha_q x^\bot + i(k,x)_\bot} \langle A|\bar{\psi}^f (0) \hat{p}_2 \mathcal{A}(x_\bot, x_\bot) \bar{\psi}^f (x_\bot, x_\bot)|A\rangle = i \frac{k^2}{m} h^\perp (\alpha_q, k_\perp),
\]

\[
\frac{1}{8\pi^3 s} \int dx_\bot dx_\bot \ e^{-i\alpha_q x^\bot + i(k,x)_\bot} \langle A|\bar{\psi}^f (0) \hat{p}_2 \mathcal{A}(0) \bar{\psi}^f (x_\bot, x_\bot)|A\rangle = i \frac{k^2}{m} h^\perp (\alpha_q, k_\perp).
\]  

(C.14)

The target matrix elements are obtained by usual replacements \(x_\ast \leftrightarrow x_\ast\), \(\alpha_q \leftrightarrow \beta_q\) and \(\hat{p}_2 \leftrightarrow \hat{p}_1\).

Finally, we need

\[
\frac{1}{8\pi^3 s} \int dx_\bot dx_\bot \ e^{-i\alpha_q x^\bot + i(k,x)_\bot} \langle A|\bar{\psi} (x_\bot, x_\bot) \mathcal{A}(x_\bot, x_\bot) \hat{p}_2 \mathcal{A}(0) \bar{\psi}(0)|A\rangle
\]

\[
= \frac{1}{8\pi^3 s} \int dx_\bot dx_\bot \ e^{-i\alpha_q x^\bot + i(k,x)_\bot} \langle A|\bar{\psi} (x_\bot, x_\bot) (\hat{k}_\bot + i \hat{D}) \hat{p}_2 (\hat{k}_\bot - i \hat{D}) \bar{\psi}(0)|A\rangle
\]

\[
= \frac{k^2}{16\pi^3} f_1 (\alpha_q, k_\perp) + O(\alpha_q, \beta_q)
\]  

(C.15)

and similarly

\[
\frac{1}{8\pi^3 s} \int dx_\bot dx_\bot \ e^{-i\alpha_q x^\bot + i(k,x)_\bot} \langle A|\bar{\psi} (x_\bot, x_\bot) \mathcal{A}(x_\bot, x_\bot) \sigma_{si} \mathcal{A}(0) \bar{\psi}(0)|A\rangle
\]

\[
= \frac{1}{8\pi^3 s} \int dx_\bot dx_\bot \ e^{-i\alpha_q x^\bot + i(k,x)_\bot} \langle A|\bar{\psi} (x_\bot, x_\bot) (\hat{k}_\bot + i \hat{D}) \sigma_{si} (\hat{k}_\bot - i \hat{D}) \bar{\psi}(0)|A\rangle
\]

\[
= \frac{1}{16\pi^3} \frac{k_i k_j}{m} h^\perp (\alpha_q, k_\perp) + O(\alpha_q, \beta_q)
\]  

(C.16)
For corresponding antiquark distributions we get

\[
\frac{1}{8\pi^3 s} \int dx_\bullet d^2 x_\perp e^{-i\alpha_q x_\bullet + i(k,x)_\perp} \langle A|\bar{\psi}(0)\bar{A}(0)\bar{q}^\bullet A(x_\bullet, x_\perp)\psi(x_\bullet, x_\perp)|A\rangle
\]

\[
= -\frac{k^2}{16\pi^3} f_1(\alpha_q, k_\perp) + O(\alpha_q, \beta_q)
\]

\[
\frac{1}{8\pi^3 s} \int dx_\bullet d^2 x_\perp e^{-i\alpha_q x_\bullet + i(k,x)_\perp} \langle A|\bar{\psi}(x_\bullet, x_\perp)A(x_\bullet, x_\perp)\sigma_{ij}\bar{A}(0)\psi(0)|A\rangle
\]

\[
= -\frac{1}{16\pi^3} \frac{k_1k^2}{m} h^j_\perp(\alpha_q, k_\perp) + O(\alpha_q, \beta_q)
\]

(C.17)

Also, as we saw in section 6.2.3, at the leading order in N_c there is one quark-antiquark-gluon operator that does not reduce to twist-2 distributions. It can be parametrized as follows (cf. eq. (C.14))

\[
\frac{1}{16\pi^3} \frac{2}{s} \int dx_\bullet d^2 x_\perp e^{-i\alpha_x x_\bullet + i(k,x)_\perp} \langle A|\bar{\psi}_j(x_\bullet, x_\perp) [A_i(x)\sigma_{ij} - \frac{1}{2} g_{ij} A^k \sigma_{jk}(x)] \psi_f(0)|A\rangle
\]

\[
= -\left( k_i k_j + \frac{1}{2} g_{ij} k^2 \right) \frac{1}{m} \tilde{h}^i_\perp(\alpha, k_\perp),
\]

\[
\frac{1}{16\pi^3} \frac{2}{s} \int dx_\bullet d^2 x_\perp e^{-i\alpha_x x_\bullet + i(k,x)_\perp} \langle A|\bar{\psi}_j(0)[A_i(0)\sigma_{ij} - \frac{1}{2} g_{ij} A^k \sigma_{jk}(0)]\psi_f(x_\bullet, x_\perp)|A\rangle
\]

\[
= -\left( k_i k_j + \frac{1}{2} g_{ij} k^2 \right) \frac{1}{m} \tilde{h}^i_\perp(\alpha, k_\perp)
\]

(C.18)

and similarly for the target matrix elements.

D Parametrization of TMDs from section 7.1

We parametrize TMDs from section 7.1 as follows

\[
\frac{1}{8\pi^3 s} \int d^2 x_\perp dx_\bullet e^{-i\alpha_x x_\bullet + i(k,x)_\perp} \int_{-\infty}^{\infty} dx'_\bullet \langle A|\bar{\psi}(x_\bullet, x_\perp) [F_{\psi}(0) + i\gamma_5 \bar{F}_{\psi}(0)] \psi(x'_\bullet, x_\perp)|A\rangle
\]

\[
= k_i j_1(\alpha, k_\perp),
\]

\[
\frac{1}{8\pi^3 s} \int d^2 x_\perp dx_\bullet e^{-i\alpha_x x_\bullet + i(k,x)_\perp} \int_{-\infty}^{\infty} dx'_\bullet \langle A|\bar{\psi}(x_\bullet, x_\perp) [F_{\psi}(0) - i\gamma_5 \bar{F}_{\psi}(0)] \psi(x'_\bullet, x_\perp)|A\rangle
\]

\[
= k_i j_2(\alpha, k_\perp),
\]

\[
\frac{1}{8\pi^3 s} \int d^2 x_\perp dx_\bullet e^{-i\alpha_x x_\bullet + i(k,x)_\perp} \int_{-\infty}^{\infty} dx'_\bullet \langle A|\bar{\psi}(x'_\bullet, x_\perp) [F_{\psi}(0) - i\gamma_5 \bar{F}_{\psi}(0)] \psi(x_\bullet, x_\perp)|A\rangle
\]

\[
= k_i j_1(\alpha, k_\perp),
\]

\[
\frac{1}{8\pi^3 s} \int d^2 x_\perp dx_\bullet e^{-i\alpha_x x_\bullet + i(k,x)_\perp} \int_{-\infty}^{\infty} dx'_\bullet \langle A|\bar{\psi}(x'_\bullet, x_\perp) [F_{\psi}(0) + i\gamma_5 \bar{F}_{\psi}(0)] \psi(x_\bullet, x_\perp)|A\rangle
\]

\[
= k_i j_2(\alpha, k_\perp)
\]

(D.1)
By complex conjugation we get

\[
\frac{1}{8\pi^4 s} \int d^2 x \cdot d x^\prime e^{-i \alpha x \cdot \epsilon (k, x) \cdot} \int_{-\infty}^{0} d x^\prime \left\langle A | \bar{\psi}(x^\prime, 0) \frac{2 \not{p} \not{k}}{s} [F_{\ast i}(x) - i \gamma_5 \hat{F}_{\ast i}(x)] \psi(0) | A \right\rangle, \\
= k_i j^i_i (\alpha, k_\perp),
\]

\[
\frac{1}{8\pi^4 s} \int d^2 x \cdot d x^\prime e^{-i \alpha x \cdot \epsilon (k, x) \cdot} \int_{-\infty}^{0} d x^\prime \left\langle A | \bar{\psi}(x^\prime, 0) \frac{2 \not{p} \not{k}}{s} [F_{\ast i}(x) + i \gamma_5 \hat{F}_{\ast i}(x)] \psi(0) | A \right\rangle, \\
= k_i j^i_2 (\alpha, k_\perp),
\]

\[
\frac{1}{8\pi^4 s} \int d^2 x \cdot d x^\prime e^{-i \alpha x \cdot \epsilon (k, x) \cdot} \int_{-\infty}^{0} d x^\prime \left\langle A | \bar{\psi}(0) \frac{2 \not{p} \not{k}}{s} [F_{\ast i}(x) + i \gamma_5 \hat{F}_{\ast i}(x)] \psi(x^\prime, 0) | A \right\rangle, \\
= k_i j^i_1 (\alpha, k_\perp),
\]

\[
\frac{1}{8\pi^4 s} \int d^2 x \cdot d x^\prime e^{-i \alpha x \cdot \epsilon (k, x) \cdot} \int_{-\infty}^{0} d x^\prime \left\langle A | \bar{\psi}(0) \frac{2 \not{p} \not{k}}{s} [F_{\ast i}(x) - i \gamma_5 \hat{F}_{\ast i}(x)] \psi(x^\prime, 0) | A \right\rangle, \\
= k_i j^i_2 (\alpha, k_\perp).
\]

(D.2)

Note that unlike two-quark matrix elements, quark-quark-gluon ones may have imaginary parts.

Target matrix elements are obtained by usual substitutions \( \alpha \leftrightarrow \beta, \not{p}_2 \leftrightarrow \not{p}_1, \not{x} \leftrightarrow \not{x}_\ast, \) and \( \hat{F}_{\ast i} \leftrightarrow \hat{F}_{\ast i} \).

For completeness let us present the explicit form of the gauge links in an arbitrary gauge:

\[
\bar{\psi}(x^\prime, x_\perp) F_{\ast i}(0) \psi(x_\ast, x_\perp) \rightarrow \bar{\psi}(x^\prime, x_\perp) [x^\prime_\ast, -\infty ] \cdot [x_\perp, 0_\perp] \cdot \infty_\perp \\
\times [-\infty, 0] F_{\ast i}(0) [0, -\infty]_0 \cdot [0_\perp, x_\perp] \cdot [-\infty, x_\perp] \cdot \psi(x_\ast, x_\perp).
\]

(E.3)

E Gluon power corrections from \( J^\mu_A(x)J^\mu_A(0) \) terms

There is one more type of contributions proportional to the product of quark and gluon TMDs

\[
J^\mu_A(x)J^\mu_A(0) = \sum_{\text{flavors}} \left( [\bar{\Xi}_1(x) \gamma^\mu \psi_A(x)] [\bar{\psi}_A(0) \gamma^\nu \Xi_1(0)] + [\bar{\psi}_A(x) \gamma^\mu \Xi_1(x)] [\bar{\Xi}_1(0) \gamma^\nu \psi_A(0)] \\
+ [\bar{\Xi}_1(x) \gamma^\mu \psi_A(x)] [\bar{\Xi}_1(0) \gamma^\nu \psi_A(0)] + [\bar{\psi}_A(x) \gamma_\mu \Xi_1(x)] [\bar{\psi}_A(0) \gamma^\nu \Xi_1(0)] \right),
\]

where we neglected terms which cannot contribute to \( W \) due to the reason discussed after eq. (7.3), i.e. that one hadron (“A” or “B”) cannot produce the DY pair on its own.

Let us consider the first term in the r.h.s. of this equation

\[
W^\mu_{\nu, \ast}(x) = \left\langle A, B | [\bar{\Xi}_1(x) \gamma^\mu \psi_A(x)] [\bar{\psi}_A(0) \gamma_\nu \Xi_1(0)] | A, B \right\rangle \\
= -\frac{g^2}{s(N_c^2 - 1)} \left\langle \left( \frac{1}{a} \right)(x) \gamma_\mu \not{p}_2 \gamma_\nu \psi(0) \gamma_\nu \not{p}_2 \gamma_\mu \bar{\psi}(0) \right\rangle_A \langle A^{a\ast}(x) A^{a\ast}(0) \rangle_B.
\]

(E.2)
To estimate the magnitude of this contribution, first note that
\[
\int dx_e e^{-i\beta q x_e} \langle B | A_i^a(x) A_j^a(0) | B \rangle
\]
\[= \frac{4}{s^2} \int dx_e e^{-i\beta q x_e} \int_{-\infty}^{x_e} dx'_e \int_{-\infty}^{0} dx''_e \langle B | F_{i}^{a}(x'_e, x_\perp) F_{j}^{a}(x''_e, 0_\perp) | B \rangle
\]
\[= \frac{4}{\beta q s^2} \int dx_e e^{-i\beta q x_e} \langle B | F_{i}^{a}(x_e, x_\perp) F_{j}^{a}(0) | B \rangle
\]
\[= -\frac{1}{\beta q} 8\pi^2 \alpha_s \left[ D_g(\beta_q, x_\perp) + \frac{1}{m^2} (2\partial_i\partial_j + g_{ij} \partial_\perp^2) H_g(\beta_q, x_\perp) \right]
\]
where we used parametrization (3.26) from ref. [29]. Since the gluon TMDs $D_g(x_B, x_\perp)$ and $H_g(\beta_q, x_\perp)$ behave only logarithmically as $x_B \to 0$ [62], the contribution of eq. (E.2) to $W(q)$ is of order of $\frac{m^2}{Q^2} \ll \frac{m^2}{Q^2}$. (As discussed in ref. [29], the projectile TMD in the r.h.s. of eq. (E.2) does not give $\frac{1}{\alpha_q}$ after Fourier transformation). Also, this contribution is $\sim \frac{1}{N_c}$ with respect to our leading terms.

Similarly, all other terms in eq. (E.1) are either $\frac{m^2}{\alpha_q}$ or $\frac{m^2}{\alpha_q}$ times $\frac{1}{N_c}$ so they can be neglected.\(^{19}\)

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\(^{19}\)It is worth mentioning that if the DY pair is produced in the region of rapidity close to the projectile, the contribution (E.3) may be the most important since gluon parton densities at small $x_B$ are larger than the quark ones.
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