Modeling Stock Market Based on Genetic Cellular Automata

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An artificial stock market is established with the modeling method and ideas of cellular automata. Cells are used to represent stockholders, who have the capability of self-teaching and are affected by the investing history of the neighboring ones. The neighborhood relationship among the stockholders is the expanded Von Neumann relationship, and the interaction among them is realized through selection operator and crossover operator. Experiment shows that the large events are frequent in the fluctuations of the stock price generated by the artificial stock market when compared with a normal process and the price returns distribution is a Lévy distribution in the central part followed by an approximately exponential truncation.

Keywords: Complex Systems; Artificial Stock Market; Cellular Automata; Genetic Operator; Multi-Agent.

Financial markets are typical complex systems in which the large-scale dynamical properties depend on the evolution of a large number of nonlinear-coupled subsystems. The efficient market hypothesis (EMH) based on rational expectation assumption considers the price of financial markets a random walk, thus the variety of price is unpredictable. In the recent years, the EMH suffers the impugnation on rational expectation assumption and the challenge of actual financial data\textsuperscript{1}, and some financial markets models are established, including behavior-mind model\textsuperscript{2,3}, dynamic-games model\textsuperscript{4}, multi-agent model\textsuperscript{5,6,7,8,9}, and so on.

Cellular automata (CA) model is a special multi-agent model in which the topological structure is fixed. It is widely applied in both natural science and social science\textsuperscript{10,11}. In this paper, an artificial stock market based on genetic cellular automata is established. Cells are used to represent stockholders, who has the capability of self-teaching and are affected by the investing history of the neighboring ones. The topological structure of CA in this paper is a two-dimensional square lattice with periodic boundary conditions, which can be considered as the expanded Von Neumann relationship. The information flow will not be cut off by the array-edge.
Before a trade, each stockholder should choose the trading strategies: to buy, to sell or to ride the fence. The stockholder’s decision includes two steps: first, each stockholder works out a preparatory decision according to the history of its investment and the stock price. The stockholders of different risk-properties have different decision methods. The risk-neutral individuals directly inherit the last decision. The risk-aversed individuals’ investing strategy is to buy at a low price and to sell at a high price. For an arbitrary risk-aversed individual \( A \), if the average price of \( A \)’s shares in hand is \( \langle p \rangle \) (if \( A \) haven’t any shares in hand, we let \( \langle p \rangle \) be the mean price of the stock) and the present stock price is \( s(t) \), then \( A \)’s decision is to depend on the parameter \( x = (s(t) - p)/p \in (-1, +\infty) \). She will chose to buy at the probability \( \sqrt{1 - (x + 1)^2} \) when \( x \leq 0 \) and to sell at the probability \( \frac{2}{\pi} \arctan(2 + \sqrt{3})x \) when \( x > 0 \), otherwise, she will hold shares. The risk-taking individuals tend to buy at a up-going price and to sell when the price is down-going. According to the present up-going range of stock price \( c = (s(t) - s(t - 1))/s(t - 1) \in (-1, +\infty) \), they will chose to buy at the probability \( \frac{2}{\pi} \arctan(10\sqrt{3}x) \) when \( x \geq 0 \), and to sell at the probability \( \frac{2}{\pi} \arctan(-10\sqrt{3}x) \) when \( x < 0 \), otherwise, they will do nothing.

The individuals’ risk-properties are given randomly in initializing process, and can change along with the evolvement of the stock market. If an individual chose to buy or to sell, she should determine the price and amount of the trading-application. The buying-price and selling-price will be chosen completely randomly in the interval \( [s(t), 1.1s(t)] \) and \( [0.9s(t), s(t)] \) respectively. The trading-amount is proportional to the quantity of capital owned by swapper. After that, each stockholder starts to investigate its neighbors and change its decision (even its risk-property) at a certain probability. That is, the next risk-property and the final decision of an individual is obtained by choosing neighboring individual to carry on genetic operation. Considering that stockholders are always inclined to listen to the winners, the individual beneficial coefficient \( \kappa \) is set as the ratio of current capitalization to initial capitalization. Then the individual’s fitness is \( F = \kappa^h (h \geq 0) \), where \( h \) is called the influence factor. The gap between winners’ and losers’ influence will grow larger if \( h \) grows bigger. Among the four neighbors, the central individual will select one to run the crossover operation at the probability, between whom and the individual’s fitness there’s a direct proportion. The risk-property and the decision of each individual are naturally divided into four types of genes logically, including risk-property, deal-decision, price-decision and amount-decision. In this paper, the four genes’ crossing over operation is independent from each other. The offspring draws its final decision or changes its risk-property by selecting the genes of the central individual at the probability of \( \lambda \), and of the neighboring individual at the probability of \( 1 - \lambda \). The parameter \( \lambda \in [0, 1] \) is called individual independence degree, which describes the affection of neighboring ones upon the individual. It should be emphasized that the stockholder’s risk-property and final decision may be changed
at a very small mutation probability. The buyer with higher price and the seller with lower price will trade preferentially, and the trading-price is the average of seller’s and buyer’s price. The stock price is the weighted average of trading-price according to the trading-amount.

When proper initial condition and parameters have been chosen, the artificial stock market can generate its stock price whose trend and fluctuations are rather similar to that of real stock market. The figure 1 gives a simulating experimental result. In the experiment we set the market size as $40 \times 40$ (i.e. 1600 stockholders), the initial stock price as 2.30, the total quantity of shares as 5 million units and the total quantity of fund as 10 million. The initial quantity of fund and shares owned follows normal distribution, and the mutation probability is 0.02. The initial risk-properties of individuals are drawn randomly.

**Fig. 1.** Time series of the typical evolution of the stock price, where $h = 1$ and $\lambda = 0.4$

Let $P(t)$ denote the stock price time series, the price returns $Z(t)$ are defined as the difference between two successive logarithms of the price:

$$P(t) : Z(t) = \log P(t + \Delta t) - \log P(t).$$

The corresponding price returns as $\Delta t = 1$ are shown in figure 2.

**Fig. 2.** The corresponding price returns as $\Delta t = 1$
Mandelbrot proposed that the distribution of returns is consistent with a Lévy stable distribution. In 1995, Mantegna and Stanley analyzed a large set of data of the S&P500 index. It has been reported that the central part of the distribution of S&P500 returns appears to be well fitted by a Lévy distribution, but the asymptotic behavior of the distribution shows faster decay than that predicted by a Lévy distribution. The similar characteristic of the distribution of returns is also found in Heng Seng index. Figure 3 shows the probability distributions of price returns for $\Delta t=1,2,4,8,16,32,64$.

In figure 2 and figure 3, it can be seen that large events are frequent in the fluctuations of the stock price generated by the artificial stock market when compared with a normal process. We also studied the peak values at the center of the distributions, figure 4 shows the central peak value versus $\Delta t$ in a log – log plot. It can be seen that all the data can be well fitted by a straight line with a slope -0.5632. This observation agrees with theoretical model leading to a Lévy distribution.

In this article, a stock market model is established based on genetic cellular automata, who has some key characteristics according with the real-life stock market. Some other experiments (not include in this paper) indicate that the interaction among individuals will give rise to clusters and herd behaviors, which may be the possible mechanism that lead to the existence of large events in our model. In addition, the mutation is very important too. Further researches can reveal the multi-level of this system, the process of the forming and damaging of the self-similar structure in cellula space, etc. Since the main goal of this article is to establish and describe the model itself, we won’t give detailed experiment results and analyzing, which will be given elsewhere.
Fig. 4. The central peak value as a function of $\Delta t$

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