Soft X-ray transients in the Hertzsprung gap

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ABSTRACT
We apply the disc instability model for soft X-ray transients to identify system parameters along evolutionary sequences of black hole X-ray binaries (BHXBs) that are consistent with transient behaviour. In particular, we focus on the hitherto neglected group of BHXBs with intermediate-mass giant donor stars. These spend a significant fraction of their X-ray active phase crossing the Hertzsprung gap.

Three case B binary sequences with a black hole accretor and 2.5–3.5 M⊙ initial donor mass are presented in detail. We formulate rules which summarizes the behaviour of these sequences and provide an approximate description for case B mass-transfer in intermediate-mass BHXBs. Chiefly, the time-scale of the overall radius expansion is given by the initial donor mass, while the surface appearance is determined by the current donor mass.

With these rules we obtain a general overview of transient and persistent behaviour of all intermediate-mass BHXBs by just considering single star sequences of different mass. We find that although systems in the process of crossing the Hertzsprung gap are in general persistently bright, with Eddington or super-Eddington transfer rates, there is a narrow instability strip where transient behaviour is possible. This strip extends over a secondary mass range 2.0 ≤ M2 ≤ 3.5 M⊙. GRO J1655–40 might be such a system. We predict that there are no BHXB transients with (sub)giant donors more massive than 3.5 M⊙, and no neutron star transients in the Hertzsprung gap.

Key words: accretion, accretion discs – black hole physics – binaries: close – stars: evolution – stars: individual: GRO J1655–40.

1 INTRODUCTION
Low-mass X-ray binaries (LMXBs) are semidetached compact binaries with a black hole (BH) or neutron star (NS) primary that accretes mass from an accretion disc. A Roche lobe filling low-mass main-sequence or (sub)giant star feeds the disc. LMXBs appear in two main varieties: they are either persistently bright or have outbursts and long phases of quiescence (e.g. White & van Paradijs 1996). The latter, known as soft X-ray transients (SXTs: cf. Tanaka & Shibazaki 1996 and Chen, Shradler & Livio 1997 for reviews), are believed to have an unstable accretion disc which alternates between a hot and cool state (Cannizzo, Wheeler & Ghosh 1982; Lin & Taam 1984; Cannizzo, Chen & Livio 1995; King & Ritter 1998), similar to dwarf novae among cataclysmic variables.

A simple criterion for stable disc accretion is that the temperature Td at the outer disc edge is larger than the hydrogen ionization temperature THI = 6500 K (e.g. King, Kolb & Burderi 1996a, hereafter KKB). As Td increases with the mass-transfer rate M, a system is persistently bright if M is larger than the critical rate Mcrit where Td = THI. The system is transient if M < Mcrit.

An important difference between dwarf novae and SXTs is the dominant role that irradiation of the accretion disc by the central accreting source plays in LMXBs. Van Paradijs (1996) showed that the observed critical transfer rate separating transient from persistent neutron star LMXBs is much lower than the critical rate separating dwarf novae from persistently bright (nova-like) cataclysmic variables. This can be understood quantitatively by assuming that irradiation dominates over viscous heating at the outer disc rim, hence stabilizing the disc than in the absence of irradiation at a smaller mass-transfer rate. Similarly, irradiation naturally explains the observed large ratio of optical to X-ray flux from the disc and the observed long, quasi-exponential decline after an SXT outburst (King & Ritter 1998).

Models for the evolution of LMXBs predict the mass-transfer rate and thus, by comparison with Mcrit, the appearance of the system as a transient or persistently bright X-ray source (assuming that the instantaneous transfer rate is close to the evolutionary mean transfer rate). This is a powerful diagnostic tool for testing evolutionary models, or, in turn, the disc instability model for SXTs. King, Kolb & Szuszkiewicz (1997b, hereafter KKS), King et al.
(1997a) and KKB investigated short-period systems with essentially unevolved donors, and long-period systems having low-mass giant donors with degenerate helium cores. In the following we extend this work, and present evolutionary models for black hole X-ray binaries (BHXBs) with intermediate-mass donors in the process of crossing the Hertzsprung gap, a hitherto largely neglected parameter range. We explore to what extent such systems appear as SXs. A major motivation for this study is provided by recent optical observations of the transient X-ray source GRO J1655–40 (Orosz & Bailyn 1997), well-known from the apparent superluminal motion in its radio jets (Hjellming & Rupen 1995). System parameters derived for GRO J1655–40 place this BH binary just in the above parameter range discussed here. Some of the results presented below have already been applied to a discussion of the evolutionary state of GRO J1655–40 (Kolb et al. 1997).

We put intermediate-mass BHXBs into context by reviewing the main concepts of LMXB evolution briefly in Section 2. In Section 3 we present in detail three evolutionary sequences, with parameters close to the ones observed in GRO J1655–40. We generalize the results of these calculations in Section 4 in the form of three rules which approximately describe the general evolution of intermediate-mass BHXBs. These rules allow one to obtain a general overview of transient and persistent behaviour of all intermediate-mass BHXBs by just considering single star sequences of different mass. Section 5 concludes with a critical discussion.

## 2 EVOLUTION OF LMXBs

The mechanism driving mass-transfer naturally divides LMXBs into two distinct classes. In the first group, nuclear expansion of the secondary maintains the semidetached state, while in the second group, orbital angular momentum losses such as gravitational radiation and magnetic braking drive mass-transfer. For convenience we denote neutron star or black hole LMXBs in the two groups by n-driven or j-driven systems.

Generally, n-driven LMXBs are long-period systems with (sub)-giant donor stars on the first giant branch. Hydrogen burns in a shell source above a nuclearly inactive helium core. The orbital period increases until mass-transfer terminates when core helium burning ignites, or when the hydrogen-rich envelope is fully transferred to the primary, whichever happens earlier. The final state is a wide, detached binary with a white dwarf secondary and a BH or NS primary (possibly a millisecond pulsar). The LMXB lifetime depends on the initial secondary mass and orbital distance and can be as long as $10^9$ yr if both are small, but is typically $10^7$–$10^8$ yr.

Conversely, j-driven systems are short-period binaries with main-sequence donors and evolve to shorter orbital period. If the j-driven evolution is the same as for cataclysmic variables (see e.g. Kolb 1996, King 1988 for reviews) the bright phase where magnetic braking operates ($P \geq 3$ h) typically lasts $10^8$ yr. For $P \leq 3$ h mass-transfer continues indefinitely at a lower rate, but the period increases again when the secondary becomes degenerate, i.e. when its mass is $\leq 0.06 M_\odot$.

A critical bifurcation period $P_b = 1.2$ d (Pilyser & Savonije 1988, 1989) separates j-driven and n-driven LMXBs. Systems born with $P < P_b$ evolve towards shorter periods, and those born with $P > P_b$ towards longer $P$. Similarly, the initial donor mass $M_2$ plays a role in determining the group membership. If $M_2 \leq 0.8 M_\odot$ the system is j-driven, as the main-sequence lifetime of the donor is longer than a Hubble time. If $M_2 > 1.5 M_\odot$ the system cannot evolve to shorter orbital periods as magnetic braking does not operate in such massive stars, and gravitational radiation alone is too weak to dominate the evolution. In the intermediate range $0.8 \leq M_2 \leq 1.5$ both groups exist. Mass-transfer stability defines an upper limit for the mass ratio $q = M_2/M_1$ ($M_1$ is the donor mass, $M_1$ the primary mass), hence for $M_2$ and $M_3$. This limit depends on the response of the donor to mass loss (e.g. Hjellming 1989), i.e. it is a non-trivial function of the evolutionary state. As a rule, the limit is roughly $M_2 \leq M_1$ in the case of conservative mass-transfer, but can be somewhat larger with mass loss from the system (cf. Kalogera & Webbink 1996, where the NS case is discussed in detail). If the donor is more massive the system suffers a short, violent phase of mass-transfer/loss and is unlikely to appear as an X-ray source. Obviously, neutron star X-ray binaries with Roche lobe filling donors more massive than $2–3 M_\odot$ do not exist, while there is no such upper limit for BHXBs; hence no clear separation between low-mass and high-mass BHXBs. The importance of BHXBs with intermediate-mass donors was first pointed out by Romani (1994).

Recently, two limiting cases of LMXB evolution have been considered to determine the incidence of transient behaviour among LMXBs. KKB and KKS described the j-driven evolution of completely unevolved zero-age main sequence (ZAMS) donors. They found that BH systems are always transient, consistent with observations, and that donor stars in NS systems need to be very close to the end of core H burning in order to be transient, consistent with the observed small fraction of transient NS LMXBs. King et al. (1997a) and KKS studied n-driven LMXBs with low-mass donor stars well-established on the first giant branch, and found that essentially all of them are transient. They considered giant donors with a thin H burning shell source above a degenerate He core where the stellar radius $R$ and luminosity $L$ are unique functions, $R \propto M_2^{1/4}, L \propto M_2^{11/4}$, of the slowly growing core mass $M_2$, essentially independent of the total stellar mass (e.g. Webbink, Rappaport & Savonije 1983). Stars with mass $\leq 2 M_\odot$ indeed establish such a structure soon after the end of central H burning and spend a considerable time ascending the first giant branch, before the ignition of helium burning in the centre terminates their radius expansion. Most n-driven NS LMXBs are well described in this way, but not BHXBs with more massive donor stars ($M \geq 2.0–2.5 M_\odot$). These ignite core helium burning before the core becomes highly degenerate, i.e. before the first giant branch evolution governed by the above core mass relations takes hold. Simplified descriptions for the evolution of these stars fail, and full stellar models are needed.

We consider these n-driven intermediate-mass BHXBs in the next sections.

## 3 ILLUSTRATIVE CASE B MASS-TRANSFER SEQUENCES WITH BLACK HOLE ACCRETORS

The binary evolutionary phase we consider here is case B mass-transfer: the donor star fills its Roche lobe after termination of core H burning but before the ignition of core He burning. Case B mass-transfer is well-studied (e.g. Kippenhahn & Weigert 1967; Pilyser & Savonije 1988; Kolb & Ritter 1990; De Greve 1993) in the context of Algol evolution. All these studies assume that the donor is more massive than the accretor, simply because the faster evolving, more massive component always fills its Roche lobe first. In BHXBs, with a more complicated evolutionary history, case B mass-transfer begins with an already inverted mass ratio, i.e. the donor is less massive than the BH. Hence there is no initial rapid phase with extremely high transfer rate.

In the following we present calculations of three case B mass-transfer sequences on to a BH primary from a secondary with
initial mass close to the transition region between degenerate and non-
degenerate helium ignition. We report briefly on the numerical
technique and associated problems (Section 3.1), describe in detail
the evolution of the secondary star (Section 3.2), and then focus on the
resulting appearance in X-rays along the sequences in Section 3.3.

3.1 Input parameters and computational technique

We used Mazziotti’s stellar evolution code in a version as described
by Mazziotti (1989; see also references therein) with pre-OPAL
opacities. The computations started from chemically homogenous
ZAMS models with a Population I mixture (X = 0.70, Y = 0.28).
Convection is treated by the standard mixing-length theory; a
calibration to a solar model determines the mixing length parameter
to be 1.4 (no overshooting was allowed).

To allow the application to binary evolution the mass-transfer
rate $\dot{M}$ was calculated for each time-step according to

$$\dot{M} \propto \exp \left( \frac{\Delta R}{kH} \right)$$

(cf. Kolb & Ritter 1990), where $\Delta R = R - R_{\text{L}}$ is the difference
between the radius $R$ of the secondary and the Roche lobe radius
$R_{\text{L}}$, $H$ the atmospheric pressure scaleheight, and $k$ is a numerical
constant, usually taken as $k = 1$. Although (1) is strictly valid only
for $\Delta R < 0$ and has to be replaced by an expression with an
approximate power-law dependence on $\Delta R/H$ for $\Delta R > 0$, we use
(1) for any $\Delta R$. Furthermore, to avoid numerical instabilities at
high transfer rates we set $k = 10$ for $\dot{M} \geq 10^{-7} \, M_\odot \, \text{yr}^{-1}$. This
procedure is justified as long as $\Delta R/R < 1$ (Kolb & Ritter 1990), i.e.
as long as the stellar radius is close to the critical Roche radius.
For the sequences presented here this is always the case. A high-
resolution reference sequence with $k = 1$ was calculated to check
the validity of sequences obtained with $k = 10$ explicitly in the case
of sequence S3 (see below). Attempts to retain the damping factor
$k = 10$ also in later phases (where $\Delta R < 0$) failed as $H/R$ becomes
non-negligible for extended giants. A damped evolution would
proceed qualitatively different from the true evolution with $k = 1$,
leading to higher mass-transfer rates and a correspondingly earlier
termination of mass transfer.

3.2 Detailed description of the sequences

All three binary sequences, S1–S3, begin mass-transfer with a 2.5-
$M_\odot$ donor star and a 6.8-$M_\odot$ black hole. The initial orbital
separation is $a_i = 13.2$ (S1), 15.2 (S2) and 18.5 $R_\odot$ (S3); see the
summary in Table 1. Mass transfer is conservative, i.e. the total
binary mass and orbital angular momentum are constant. Sequence
S3 follows the donor star until ignition of central helium burning.
The subsequent evolution is detached, and we evolved the second-
ary further through central helium burning and the brief subsequent
asymptotic giant-branch phase with weak thermal pulses. In
contrast, sequences S1 and S2 have been terminated once the
secondary was established on the first giant branch.

In Figs 1–5 we show characteristic parameters along the
sequences S1–S3, sometimes together with the evolution of a
2.5-$M_\odot$ single star (for convenience referred to as sequence S0).

Fig. 1 (upper panel) reveals that the overall radius evolution with
time is very similar in all sequences S0–S3, except for the ‘plateau
phase’ soon after mass-transfer turn-on, which corresponds to a
brief radius-contraction phase at the red end of the Hertzsprung gap
of S0. This phase occurs at a characteristic radius, the ‘plateau
radius’ $R_p$, and will be examined in more detail below. The
similarity shows that significant mass loss from the outer envelope

| sequence | $M_1/M_\odot$ | $M_2/M_\odot$ | $a/R_\odot$ | int |
|----------|---------------|---------------|-------------|-----|
| S1 (initial) | 6.8 | 2.5 | 13.2 | b860 |
| S2 (initial) | 6.8 | 2.5 | 15.2 | b849 |
| S3 (initial) | 6.8 | 2.5 | 18.5 | b830/32 |
| S3 (final) | 8.726 | 0.574 | 213.1 |

Figure 1. Radius $R$ (upper panel) and effective temperature $T_{\text{eff}}$ (lower panel) of a 2.5-$M_\odot$ single star (dashed line) and the secondary star in the binary sequence S1 (dash–dotted line), S2 (thick solid line) and S3 (thin solid line), cf. Table 1, as a function of time elapsed since the (donor) star left the main sequence.

Figure 2. Surface luminosity $L$ (top), nuclear luminosity $L_{\text{nuc}}$ (middle panel) and absolute value of the gravothermal luminosity $L_g = L - L_{\text{nuc}}$ (bottom) of a 2.5-$M_\odot$ single star (dashed line) and the secondary star in binary sequences S1–S3, as a function of time elapsed since the (donor) star left the main sequence. The line style is as in Fig. 1.
sprung gap, where hydrogen burning changes from a thick to a thin
imposed on the luminosity-evolution pattern of S0 in the Hertz-
siness and S3 (upper panel). The line style is broken when the mass-transfer rate is
left the main sequence) for sequences S1 (bottom panel), S2 (middle panel)
with largest $M_c$ along the single star sequence coincide with the
governed by the nuclear evolution of the core region. The overall
has little effect on the overall expansion of the donor star, which is
governed by the nuclear evolution of the core region. The overall
temporal evolution of the surface luminosity $L$ is systematically
different for sequences with and without mass-transfer (Fig. 2): in
S1–S3 mass loss causes a steep build-up of gravothermal lumin-
sity $L_{\text{g}}$ ($L_{\text{g}} < 0$) and a slight reduction of the nuclear luminosity
from the hydrogen shell source. This behaviour is super-
imposed on the luminosity-evolution pattern of S0 in the Hertz-
sprung gap, where hydrogen burning changes from a thick to a thin
shell. The net effect is that thin hydrogen-shell burning is estab-
ilished earlier, at a slightly smaller core mass. Once thin H burning is
fully established the luminosity grows with further growing core
mass. The corresponding effective core mass–luminosity relation
is much steeper than the standard relation $L \propto M_c^{3.3}$ for fully degener-
ate cores, and meets the latter one only immediately before core
helium ignition (Fig. 3). As a result, the secondary in S1–S3 begins
its ascent along the first giant branch at a significantly smaller
surface luminosity than the 2.5-M$_\odot$ single star. The drop of $L_c$
as a function of time (elapsed since the donor
temperature
the secondary in the mass-loss sequence crosses the Hertzsprung gap
even faster than the 2.5-M$_\odot$ single star (Fig. 1, lower panel).
Roughly, at any time along S1–S3 in the Hertzsprung gap, $L$ and
$T_{\text{eff}}$ are the same as for a single star with the same mass and radius
as the secondary star at that time.

With the ignition of helium burning the radius of the secondary
decreases sharply and mass-transfer terminates. At this point the
binary parameters of S3 are $M_1 = 8.726$ M$_\odot$, $M_2 = 0.574$ M$_\odot$
(with core mass 0.320 M$_\odot$) and $P = 118.3$ d, $a = 213.1$ R$_\odot$ (see
Table 1). For the next 270 Myr the secondary burns helium in the
centre and is well inside its Roche lobe ($R = 8–10$ R$_\odot$ versus
$R_L = 34.6$ R$_\odot$). Subsequently, He burning moves to a shell source
and the radius increases again, at a core mass of $\approx 0.420$ M$_\odot$.
The ensuing double-shell burning is unstable and leads to thermal
pulses. We find that these pulses are rather weak, with decreasing
radius in maximum. The radius maximum in subsequent pulses is
always smaller than the Roche radius (though not very much), and
quickly decreasing. Eventually the secondary contracts towards the
white dwarf stage. If there are no significant wind losses during the
thermal pulses the final state of S3 is a wide, detached binary with a
$\approx 9$ M$_\odot$ black hole primary, a $\approx 0.55$ M$_\odot$ carbon–oxygen white
dwarf companion and $\approx 5.6$ month orbital period.

### 3.3 Transient versus persistent accretion

We distinguish transient from persistent BHXBs by comparing the
mass-transfer rate $\dot{M}$ with the critical rate $\dot{M}_{\text{crit}}$ for which hydrogen

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**Figure 3.** Surface luminosity $L$ as a function of core mass $M_c$ (defined as the
mass enclosed by the shell where the H burning energy-generation rate is
maximal). Full triangles: selected models along sequence S3. The triangles
with smallest and largest $M_c$ correspond to models at the luminosity
minimum and close to central He ignition, respectively. The open triangle
marks a 2.5-M$_\odot$ single star at its luminosity minimum. The positions of
models with larger $M_c$ along the single star sequence coincide with the
positions of sequence S3. The solid curve is the core-mass luminosity
relation for low-mass ($< 1.5$ M$_\odot$) giant stars, cf. King et al. (1997a).

**Figure 4.** Mass-transfer rate $\dot{M}$ as a function of time (elapsed since the donor
left the main sequence) for sequences S1 (bottom panel), S2 (middle panel)
and S3 (upper panel). The line style is broken when the mass-transfer rate is
smaller than the critical rate $\dot{M}_{\text{crit}}$ for disc instability (taken from KKS).

**Figure 5.** Mass-transfer rate $\dot{M}$ as a function of effective temperature $T_{\text{eff}}$
for sequences S1 (dash–dotted line), S2 (thick solid line) and S3 (thin solid
line). Also shown is the critical rate for disc instabilities (from KKS) for
sequence S1.
just ionizes at the outer disc edge. A system is transient if $M < M_{\text{crit}}$. In Fig. 4 we plot $M$ as a function of time along sequences S1–S3. The linestyle is broken if $M < M_{\text{crit}}$ with $M_{\text{crit}}$ taken from KKS. Obviously, the accretion disc is stable for about $=4–5$ Myr when the secondary crosses the Hertzsprung gap, and the transfer rate is close to (or slightly above) the Eddington limit

$$M_{\text{edd}} = 2 \times 10^{-8} M_\odot \text{ yr}^{-1} \left( \frac{M_i}{M_\odot} \right)$$  \hspace{1cm} (2)

(assuming an effective accretion efficiency $\eta = L_{\text{acc}}/M c^2 = 0.1$). The system becomes a transient source soon after it begins to ascend along the Hayashi line, and remains transient until helium ignites in the centre $=12$ Myr later. Significantly, there is also a short transient phase (lasting $=1$ Myr) right in the Hertzsprung gap, centred around $\log T_{\text{eff}}/K = 3.73$. This phase is caused by the transition from thick- to thin-shell source burning, the same effect that leads to a brief radius-contraction phase when the 2.5-$M_\odot$ single star crosses the Hertzsprung gap. The associated change of the internal structure of the star is highly non-linear and complex. Just as the ultimate cause for the expansion to the red giant state cannot be easily derived from a simple analysis of the stellar structure equations, there is no easy way to understand this temporary halt in the expansion either.

The Hertzsprung-gap transient phase in S1–S3 and the radius-contraction phase of S0 all occur at the same range of $T_{\text{eff}}$ (Fig. 5), while the corresponding plateau radius $R_p$ is largest for S0, being progressively smaller the earlier mass transfer begins in the Hertzsprung gap. The secondary mass in the plateau phase is $1.9, 2.05$ and $2.25$ $M_\odot$ for sequences S1, S2 and S3, respectively. In fact, $R_p(M_2)$ is practically the same as the plateau radius of a single star with the mass of the secondary in the plateau phase.

Details of the exact location (e.g. the radius $R_p$), duration and depth of the radius-contraction phase of a single star must necessarily depend on details of the stellar input physics, in particular on opacities and treatment of convection. As an example, the 2.5-$M_\odot$ single star track obtained by Salaris et al. (1997) with OPAL opacities shows a much less pronounced radius-contraction phase at a slightly higher effective temperature ($\log T_{\text{eff}}/K = 3.75$) than our sequence S0. Hence we do not expect our models to reproduce the precise location of the real transient phase in the Hertzsprung gap and the actual mass-transfer rate in this phase. Nevertheless, our sequences certainly show the differential change of that phase with varying initial orbital distance: the earlier mass-transfer starts in the Hertzsprung gap, the less pronounced the decrease of $\dot{M}$ in the transient phase. If the secondary is already very close to the plateau phase when it fills its Roche lobe for the first time, the system might almost detach in the transient phase and not appear as an X-ray source at all; but if the system begins mass transfer early in the Hertzsprung gap it certainly will appear as an X-ray source.

## 4 Generalization: Intermediate-Mass BHXB Evolution

We formulate three rules which summarize the behaviour of the above evolutionary sequences and provide an approximate description for the case B evolution of intermediate-mass BHXBs.

(R1) The overall time evolution $R(t)$ of the donor star radius in a binary sequence with initial donor mass $M_2$ is the same as the time evolution of the radius of a single star with mass $M_0 = M_2$, while

(R2) the plateau phase (the brief contraction phase or slow-down of the expansion phase) in the Hertzsprung gap occurs at a donor radius $R_p$ equal to the plateau radius of a single star with the mass of the donor and

(R3) a donor star with mass $M_2$ and radius $R$ has the same effective temperature $T_{\text{eff}}$ (and surface luminosity $L$) as a single star with mass $M_0 = M_2$ and radius $R$. In other words, the time-scale of the overall radius expansion is given by the initial donor mass, while the surface appearance and the plateau phase are given by the current donor mass.

In the following we use these rules to investigate the incidence of transient and persistent behaviour in intermediate-mass BHXBs with arbitrary initial mass and initial separation, simply by considering the evolution of single stars in this mass range.

In order to do so we note that in the case of stationary mass transfer $(\dot{R}/R = \dot{R}_i/R_i)$ the transfer rate is given by

$$\dot{M} = M_2 \frac{K}{\dot{\gamma}_{\text{eq}} - \dot{\gamma}_R}$$  \hspace{1cm} (3)

(cf. e.g. Ritter 1996). Here $K = (d \ln R/dt)_{M\text{const}}$ is the radius expansion in the absence of mass loss (e.g. due to nuclear evolution), $\dot{\gamma}_{\text{eq}}$ is the thermal equilibrium mass–radius exponent and $\dot{\gamma}_R$ is the Roche-lobe index ($\dot{\gamma}_R = 2M_2/M_1 - 5/3$ for conservative evolution). We set $\dot{\gamma}_{\text{eq}} = 0$ and, using (R1), approximate $K$ by $d \ln R/dt$ along the single star sequence. It is clear that these secondaries are not in thermal equilibrium (Fig. 2), and their radius expansion is not purely nuclear. However, the mass loss perturbs the overall radius evolution only slightly.

Then we obtain the approximate binary evolution for given initial donor mass $M_2$, by integrating $M(t)$, i.e. $K(t)$ of a single star with mass $M_0 = M_2$, over time. To decide the stability of the accretion disc we need to know the mass-transfer rate $\dot{M}$ as a function of orbital period $P$. The main characteristics of the real evolutionary tracks $M(P)$ are already evident from Fig. 6 where we plot the mass-transfer rate $\dot{M}$, soon after turn-on of mass transfer, estimated from.

![Figure 6. Transfer rate $\dot{M}$ versus orbital period $P$, at turn-on of mass transfer, for three different donor masses (full line: $3.5\, M_\odot$; dashed line: $2.5\, M_\odot$; dotted line: $1.3\, M_\odot$), estimated from equation (3) and assuming a 8-$M_\odot$ primary. The horizontal line indicates the corresponding Eddington transfer rate, see (2). The thick solid line is the critical mass accretion rate $\dot{M}_{\text{crit}}$ separating transient ($\dot{M} < \dot{M}_{\text{crit}}$) from persistent ($\dot{M} > \dot{M}_{\text{crit}}$) systems, taken from KKS.](https://academic.oup.com/mnras/article-abstract/297/2/419/988364)
For super-Eddington mass transfer; note that because \( i^* \) (assuming that the character of disc irradiation does not change

Figure 7. Orbital period–secondary mass \((P-M_2)\) plane for BHXBs, showing the exclusion zone for transients (unhatched). A BH mass of 8 \( M_\odot \) was assumed. Systems in the unhatched region are always persistent; systems in the narrow hatched instability strip are always transient. Systems that are born in the large hatched area are always transient, but systems that have evolved from the unhatched into the large hatched area will remain persistent for some time before they, too, become transient. Evolutionary tracks for sequences S1–S3 are also shown (solid where persistent, dashed where transient). The heavy dot along each sequence marks the location of the transient phase in the Hertzsprung gap. The dash–dotted line is the transient). The heavy dot along each sequence marks the location of the transient phase in the Hertzsprung gap. The dash–dotted line is the.

Figure 8. As Fig. 7, but with the critical boundary between transient and persistent systems for different BH masses short-dashed line: 5 \( M_\odot \); solid line: 10 \( M_\odot \); dotted line: 15 \( M_\odot \). The cross indicates the observed system parameters of GRO J1655–40 (Orosz & Bailyn 1997). The solid curves represent evolutionary tracks assuming conservative evolution (full line) and evolution with constant black hole mass (the mass leaving the system carries the specific orbital angular momentum of the black hole; long-dashed line).

case of conservative evolution the evolutionary tracks follow curves with \((M-M_2)^2 M\dot{P} = \text{constant} (M)\) (the total binary mass); see the tracks of sequences S1–S3. In binaries with larger initial donor mass \((M_2 \approx 2.5 \ M_\odot)\) the transfer rate is super-Eddington and the evolution non-conservative. The primary accretes at \( M_{\text{crit}} \) and surplus material leaves the system at a rate \( M - M_{\text{crit}} \), carrying some specific angular momentum (of order the specific orbital angular momentum of the BH) determined by details of how and where the material is accelerated. In this case the mass-transfer rate is slightly lower and the period increases significantly faster than for conservative evolution, i.e. the evolutionary tracks would have a slightly larger slope \(|dP/dM_2|\) (cf. the example in Fig. 8).

A simple corollary of (R2) is that a system will reach the transient plateau phase when its evolutionary track crosses the instability strip in Fig. 7. This is because \( P_0 \) is a unique function of the donor mass, independent of the initial donor mass. In contrast, as the initial donor mass dictates the mass-transfer rate outside the plateau phase, we can expect that the critical period where the system changes from a persistently bright to a transient X-ray source is close to \( P_{\text{crit}}(M_2) \). Then, as a consequence of the opposite slopes of evolutionary tracks and the curve \( P_{\text{crit}}(M_2) \) in Fig. 7, a system remains persistently bright when its track leaves the unhatched region, and becomes transient only later when its period roughly equals \( P_{\text{crit}}(M_2) \). The sequences S1–S3 (solid where persistent, broken where transient; heavy dots mark the plateau phase) and actual integrations of (3) confirm this. Hence no system in the unhatched region is transient; it is a ‘transient exclusion zone’, bisected by the Hertzsprung gap instability strip. The hatched region, on the other hand, may contain both persistently bright and transient systems.

Fig. 8 shows how the transient exclusion zone depends on the BH mass \( M_1 \). As \( M_1 \) increases, the steep branch of \( P_{\text{crit}}(M_2) \) moves to larger \( M_2 \), while the flat branch moves to longer \( P \). The BH mass does not affect the location of the instability strip.
5 SUMMARY AND DISCUSSION

In extension of the work by KKB, KKS and King et al. 1997a we studied the evolution of BHXBs with intermediate-mass giant donor stars in the context of the disc instability model for soft X-ray transients. These represent a hitherto somewhat neglected group of systems with very high mass-transfer rates and donors spending a significant fraction of the X-ray active time in crossing the Hertzsprung gap. We find that although systems in the process of crossing the Hertzsprung gap are in general persistently bright with Eddington or super-Eddington transfer rates, there is a narrow instability strip where transient behaviour is possible. This strip extends over a secondary mass range $2.0 \leq M_2/M_\odot \leq 3.5$ and is roughly given by $P/d = 4.3 M_2/M_\odot - 6.2$, but the precise location is subject to uncertainties in the stellar input physics. We predict that BHXBs in the Hertzsprung gap are not transient if the donor is more massive than $\approx 3.5 M_\odot$, and that neutron star LMXB transients in the Hertzsprung gap do not exist. The latter is a consequence of a number of factors: in neutron star LMXBs the upper (stability) limit to the Hertzsprung gap do not exist. The latter is a consequence of a number of not very well determined quantities. Most notably the hydrogen ionization temperature in the disc, the relative disc thickness, the accretion efficiency $h$, the nuclear time-scale of the donor, and the albedo. A major justification for the adopted normalization comes from the observed spread of the critical transfer rate in this phase. (Ritter, Zhang & Kolb 1995; King et al. 1996b). In LMXBs this mechanism does not work as the irradiating flux on the secondary is too large (cf. King 1998). Hence it seems not unlikely that the actual transfer rate is close to the evolutionary mean.

The theoretical value of the critical rate $M_{\text{crit}}$ depends on a number of not very well determined quantities. Most notably these are the hydrogen ionization temperature in the disc, the relative disc thickness, the accretion efficiency $\eta$ and the albedo. A major justification for the adopted normalization comes from the fact that it matches the observed location of the critical rate separating transient from persistent sources (van Paradis 1996). The remaining uncertainty in the normalization of $M_{\text{crit}}$ does not affect the main results of this study, i.e. the existence and location of the transient instability strip in the Hertzsprung gap. This is a result of the steep gradient $dM/dP$ of the transfer rate in this phase.

The transient X-ray source GRO J1655–40 is the first confirmed member of the class of intermediate-mass BHXBs considered in this paper. GRO J1655–40 is also close to the SXT instability strip and, given the uncertainties in both the theoretical and observed values, might actually be a system right in the instability strip, consistent with its transient nature (e.g. Harmon et al. 1995; Tavani et al. 1996; Levine et al. 1996). Implications of this interpretation are discussed elsewhere (Kolb et al. 1997). Particular problems arise from the apparent lack of persistently bright systems with similar parameters in the Hertzsprung gap. It was suggested that these are not seen in X-rays as their effective photosphere might radiate at much longer wavelengths. Alternatively, GRO J1655–40 could be one of the predicted systems in the Hertzsprung gap with a stable accretion disc. It is by no means clear if such systems do appear as persistently bright sources. Instabilities in the accretion flow might cause dramatic changes in the effective photosphere, causing variability in a given waveband. Kolb et al. (1997) also suggested such a mechanism for GRS 1915+105, the other X-ray source with apparent superluminal motion.

A second possible intermediate-mass BHXB is 4U 1543–47. Recently, Orosz et al. (1998) determined its orbital period as 1.123 d and found that the components are likely to be a $\approx 7 M_\odot$ BH and a $2.1–2.5 M_\odot$ main-sequence donor. This places the system below the turn-off main-sequence period ($P_{\text{TMS}}$ line) in Fig. 7 (dashed–dotted line; note that $P_{\text{TMS}}$ as shown in the equivalent fig. 2 of Kolb et al. 1997 is slightly too small due to a calibration error). Therefore 4U 1543 – 47 is, unlike GRO J1655–40, in a phase of case A mass transfer where the transfer rate is determined by the nuclear time-scale of the donor. With $M_2 = 2.3 M_\odot$ and replacing $K$ in (3) by the nuclear expansion d ln $R/dt$ on the main sequence we estimate $M_2 = 4 \times 10^{-8} M_\odot$ yr$^{-1}$, well below the critical rate KKS find for transient behaviour in BHXBs (but slightly above the critical rate claimed for neutron star systems, cf. KKB). A more detailed account of case A mass transfer in BHXBs is in preparation.

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REFERENCES

Cannizzo J. K., Wheelock J. C., Ghosh P., 1982, in Cox J. P., Hansen C. J., eds, Pulsations in Classical and Cataclysmic Variable Stars. Univ. of Colorado Press, Boulder, p. 13
Cannizzo J. K., Chen W., Livio M., 1995, ApJ, 454, 880
Chen W., Shriver C. R., Livio M., 1997, ApJ, 491, 312
De Greve J. P., 1993, A&A, 97, 527
Harmon B. A. et al., 1995, Nat, 374, 703
Hjellming M. S., 1989, PhD thesis, Univ. Illinois
Hjellming R. M., Rupen M. P., 1995, Nat, 375, 464
Kalogera V., Webbink R. F., 1996, ApJ, 458, 301
Kipping P., 1988, QJRAS, 29, 1
King A. R., 1998, in Howell S. B., Kuulkers E., Woodward C., eds, ASP Conf. Ser. Vol 137, Proc. 13th North American Workshop on Cataclysmic Variables and Related Objects: Wild Stars in the Old West. Astron. Soc. Pac., San Francisco, in press
King A. R., Ritter H., 1998, MNRAS, 293, L42
King A. R., Kolb U., Burderi L., 1996a, ApJ, 464, L127 (KKB)
King A. R., Frank J., Kolb U., Ritter H., 1996b, ApJ, 467, 761
King A. R., Frank J., Kolb U., Ritter H., 1997a, ApJ, 484, 844
King A. R., Kolb U., Szeszkiwitzcz E., 1997b, ApJ, 488, 89 (KKS)
Kippenhahn R., Weigert A., 1967, Z. Astrophysik, 65, 251
Kolb U. 1996, in Evans A., Wood J. H., eds, Proc. IAU Colloq. 158, Cataclysmic Variables and Related Objects. Kluwer, Dordrecht, p. 433
Kolb U., Ritter H., 1990, A&A, 236, 385
Levine A. M., Bradt H., Cui W., Jermain J. G., Morgan E. H., Remillard R., Shirley R. E., Smith D. A., 1996, ApJ, 469, L33
Lin D. N. C., Taam R. E., 1984, in Woosley S. E., ed., AIP Conf. Proc. 115, High Energy Transients in Astrophysics. Am. Inst. Phys., New York, p. 83

Mazzitelli I., 1989, ApJ, 340, 249

Orosz J. A., Bailyn C. D., 1997, ApJ, 477, 876

Orosz J. A., Jain R. K., Bailyn C. D., McClintock J. E., Remillard R. A., 1998, ApJ, in press

Pylyser E. H. P., Savonije G. J., 1988, A&A, 191, 57

Pylyser E. H. P., Savonije G. J., 1989, A&A, 208, 52

Ritter H., 1996, in Wijers R. A. M. J., Davies M. B., Tout C. A., eds, NATO ASI Series C, Vol. 477, Evolutionary Processes in Binary Stars. Kluwer, Dordrecht, p. 223

Ritter H., Zhang Z., Kolb U., 1995, in Bianchini A., Della Valle M., Orio M., eds, Astrophysics and Space Science Library 205, Cataclysmic Variables. Kluwer, Dordrecht, p. 479

Romani R. W., 1994, in Shafter A., ed., ASP Conf. Ser. Vol. 56, Interacting Binary Stars. Astron. Soc. Pac., San Francisco, p. 196

Salaris M., Domínguez I., García-berro E., Hernanz M., Isern J., Mochkovitch R., 1997, ApJ, 486, 413

Tanaka Y., Shibazaki N., 1996, ARA&A, 34, 607

Tavani M., Fruchter A., Zhang S. N., Harmon B. A., Hjellming R. N., Rupen M. P., Bailyn C., Livio M., 1996, ApJ, 473, L103

Van Paradijs J., 1996, ApJ, 464, L139

Warner B., 1987, MNRAS, 227, 23

Webbink R. F., Rappaport S., Savonije G. J., 1983, ApJ, 270, 678

White N. E., van Paradijs J., 1996, ApJ, 473, L25

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