Analytical study and parameter-sensitivity analysis of catalytic current at a rotating disk electrode

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Abstract

A convective-diffusion equation with semi-infinite boundary conditions for rotating disk electrodes under the hydrodynamic conditions is discussed and analytically solved for electrochemical catalytic reactions. The steady-state catalytic current of the rotating disk electrode is derived for various possible values of parameters by using a new approach of the homotopy perturbation method. The theoretical approach in this paper is described, for the first time, on the basis of convection–diffusion equations for the kinetics of Fenton’s reagent using a platinum rotating disk electrode. The obtained approximate analytical expression for the concentrations of ferric and ferrous ions for steady-state conditions are shown to be highly accurate when compared to numerical results and other approximations found in the literature. A sensitive analysis of parameters of the current and concentration is presented.

Nomenclature

δ Thickness of hydrodynamic boundary layer (cm)
η Potential
B Dimensionless parameter defined in equation (9)
F Dimensionless parameter defined in equation (9)
ω Rotation of the RDE (s⁻¹)
ψ Dimensionless bulk concentration of ethanol
ψSV Dimensionless bulk concentration of ethanol
θA Dimensionless concentration of ferric ion
θB Dimensionless concentration of ferrous ion
Ci,i = A, B, C, D Concentration of species i (M)
Ci,i = A, C Concentration of species i (M)
Di,i = A, C Diffusivity of species i (cm² s⁻¹)
E Electrode potential
E0 Standard potential of the electrode reaction
F Faraday constant
iL Limiting current (A cm⁻²)
K Equilibrium constant
Kb second order rate constant in the backward direction (M⁻¹ s⁻¹)
Kf Product of forward rate constant and hydrogen ion concentration

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second order rate constant in the forward direction (M$^{-1}$s$^{-1}$)  
$m$ parameter defined following equation (33) (cm$^{-1}$)  
$n$ Number of moles of electrons transferred in the balanced equation  
$R$ Gas constant  
$T$ Temperature  
$v_x$ velocity (cm s$^{-1}$)  
$x$ Distance from the RDE (cm)  
y dimensionless distance from the surface of electrode

1. Introduction

The rotating disc electrode (RDE), also known as the hydrodynamic working electrode, is the hydrodynamic electroanalytical technique used to limit the diffusion layer thickness. RDE, whose steady-state current is determined by a solution flow rather than a diffusion, is becoming one of the most powerful methods for studying both diffusions in electrolytic solutions and kinetics of fast electrode reaction [1, 2].

Since the sixties of the last century, RDE has been an effective tool for investigating electrochemical reactions when Levich [3] suggested an empirical relationship between diffusion limiting current and rotation rate ($\omega$) by solving the convective-diffusion equation. Nikolic et al [4] calculated an important hydrodynamic parameter in RDE, namely the diffusion coefficient. Compton et al [5] applied the Hale approach for the calculation of the chronoamperometric responses arising from potential-step experiments at rotating disk electrodes. Newman [6], Gregory et al [7], and Diard and Montella [8] obtained different expressions of the convection–diffusion equation in rotating disc electrode for finite and infinite Schmidt number (Sc) using symbolic and numerical computation procedures. Rajendran et al [9] derived a two-point Padé approximation of mass-transfer rate for rotating disc electrodes for various values of Schmidt numbers. Rani et al [10] solved the mathematical problem corresponding to a one-electron reversible electron transfer at a rotating disk electrode under transient and steady-state conditions by using the homotopy perturbation method. Daniel Okuonghae et al [11] employed the discontinuous Galerkin finite element method for solving the rotating disk electrode for various reaction mechanisms.

To our knowledge, analytical expressions for the concentrations of the ferric ion and ferrous ion, and the corresponding current response are not available for all possible values of parameters under steady-state conditions. In addition, the theory and experimental results of rotating disk electrodes are available only for the case of boundary conditions in a finite domain. However, the discussion in this manuscript is concerned with infinite domain boundary condition, that when the concentration of both ferric ion and ferrous ion are known when the distance from the surface of electrode tends to infinity. Approximate analytical expressions for the steady-state concentrations and the current for rotating disk electrodes for the catalytic reaction mechanism will be driven.

2. Mathematical formulation of the problem

The kinetics of catalytic reactions occurring at the rotating disk electrode can be written as follows:

$$A + R \rightarrow C,$$  \hspace{1cm} (1)

$$C + B \xrightleftharpoons[k_b]{k_f} A + D,$$  \hspace{1cm} (2)

where in the case of Fenton’s reaction, $A$, $R$, $C$, $B$ and $D$ respectively denote ferric ion, reactant, ferrous ion, hydrogen peroxide and radical. $k_f$ and $k_b$ are, respectively, the forward and backward reaction rate constants.

This catalytic model is considered under the following assumptions [12]:

I. Second order rate constant in the forward direction is greater than the second order rate constant in the backward direction.

II. Concentration of the catalyzing species B is much larger than that of the reactant and product of the electrochemical reaction, A and C.

III. All other species have uniform concentration throughout the diffusion layer.

IV. Diffusion coefficients of reacting species are equal ($D_A = D_C = D$).
V. Mass transfer of any electroactive species takes place only by means of diffusion and convection, and that concentration changes due to any other effects such as migration are negligible.

Figure 1 shows a schematic diagram of the rotating disk electrode for the catalytic reaction mechanism [13]. The mass balance equations for the above reaction scheme may be written as follows [12]

\[
D \frac{d^2 C_A}{dx^2} - \nu_v \frac{dC_A}{dx} + k_f C_b C_C - k_h C_A C_D = 0
\]  

(3)

\[
D \frac{d^2 C_C}{dx^2} - \nu_v \frac{dC_C}{dx} - k_f C_b C_C + k_h C_A C_D = 0
\]  

(4)

where \( C_A \), \( C_C \), \( C_b (H_2O_2) \), \( C_D \) and \( k_f \) respectively denote ferric ion, ferrous ion, the hydrogen peroxide, radical and the product of the forward rate constant, respectively. In addition, the velocity of the solution is \( \nu_v = -0.51 \omega^{3/2} x^{-1/2} \). In the bulk solution, the ferric and ferrous ion concentrations are both assumed constants. At the electrode surface \( (x = 0) \), the ferric ion concentration is zero because of the limiting current condition. Also the flux of ferric and ferrous ion near the disk electrode are assumed equal. Therefore, we have the following set of boundary conditions:

\[
\begin{align*}
 & \quad \text{at } x = 0, \quad C_A = 0, \quad \frac{dC_A}{dx} + \frac{dC_C}{dx} = 0, \quad (5) \\
 & \quad \text{at } x = \infty, \quad C_A = C_A^b, \quad C_C = C_C^b = \frac{B}{F}, \quad (6)
\end{align*}
\]

3. Dimensionless form of convection–diffusion equations

In dimensionless form, equations (3) and (4) take the form

\[
\frac{d^2 \Theta_A}{dy^2} + 2.13 y^2 \frac{d\Theta_A}{dy} + \mathcal{F} \Theta_C - B \Theta_A = 0,
\]  

(7)

\[
\frac{d^2 \Theta_C}{dy^2} + 2.13 y^2 \frac{d\Theta_C}{dy} - \mathcal{F} \Theta_C + B \Theta_A = 0,
\]  

(8)

where

\[
y = \frac{x}{\delta}, \quad \Theta_i = \frac{C_i}{C_A^b}, \quad \mathcal{F} = \frac{k_f C_A^b \delta^2}{D}, \quad B = \frac{k_h C_D \delta^2}{D},
\]  

(9)
and $\delta = 1.61D^{1/3}\gamma^{1/6}u^{-1/2}$ is the hydrodynamic boundary layer thickness. The dimensionless boundary conditions become:

$$\text{at } y = 0, \theta_A = 0, \frac{d\theta_A}{dy} + \frac{d\theta_C}{dy} = 0 \quad (10)$$

$$\text{at } y = \infty, \theta_A = 1, \theta_C^b = \frac{B}{\epsilon} \quad (11)$$

### 4. Analytical expressions of concentrations and current

Many analytical methods have been developed in recent years to find approximate solutions for nonlinear boundary value problems that arise in various fields of sciences and engineering such as fluid mechanics, plasma physics, optical fibre, solid-state physics, and biological and chemical sciences. These methods include, but not limited to, the residual method [14], series method [15], hyperbolic function method [16], variation iteration method [17, 18], The Adomian decomposition method [19], the differential transformation method [20], Green’s function based methods [21–23] and Taylor series method [24].

One of the methods that has received a great deal of attention is the homotopy perturbation method (HPM), which uses the embedding parameter as a small parameter. The HPM often reaches an asymptotic solution with a few iterations. First proposed by Ji-Haun He [25], HPM has been intensively used by researchers, in all fields of sciences and engineering, for solving various nonlinear problems. In recent years, HPM has gone through many modifications to make its implementation easier and to achieve faster convergence even for strong nonlinear differential equations [26–30]. In this paper, a modified form of the HPM is employed to obtain the following concentrations of ferric ion ($\theta_A$) and ferrous ion ($\theta_C$) (see details in appendix):

$$\left(1 - p\right) \left(\frac{d^2\theta_A}{dy^2} + \mathcal{F}\theta_C - B\theta_A\right) + p \left(\frac{d^2\theta_A}{dy^2} + 2.13\gamma^2\frac{d\theta_A}{dy} + \mathcal{F}\theta_C - B\theta_A\right) = 0 \quad (12)$$

$$\left(1 - p\right) \left(\frac{d^2\theta_C}{dy^2} - \mathcal{F}\theta_C + B\theta_A\right) + p \left(\frac{d^2\theta_C}{dy^2} + 2.13\gamma^2\frac{d\theta_C}{dy} - \mathcal{F}\theta_C + B\theta_A\right) = 0 \quad (13)$$

The analytical homotopy perturbation solution for equations (7) and (8) is expressed in the following series forms

$$\theta_A = p^0\theta_{A0} + p\theta_{A1} + p^2\theta_{A2} + \cdots, \quad (14)$$

$$\theta_C = p^0\theta_{C0} + p\theta_{C1} + p^2\theta_{C2} + \cdots. \quad (15)$$

Substituting equations (14) and (15) respectively into equations (12) and (13), then equating like powers of $p$ lead to the nonlinear system for $p^0$

$$p^0 : \frac{d^2\theta_{A0}}{dy^2} + \mathcal{F}\theta_{C0} - B\theta_{A0} \quad (16)$$

$$p^0 : \frac{d^2\theta_{C0}}{dy^2} - \mathcal{F}\theta_{C0} + B\theta_{A0} \quad (17)$$

with boundary conditions

$$\text{at } y = 0, \theta_{A0} = 0, \frac{d\theta_{A0}}{dy} + \frac{d\theta_{C0}}{dy} = 0 \quad (18)$$

$$\text{at } y = \infty, \theta_{A0} = 1, \theta_{C0} = \theta_{C0}^b = \frac{B}{\epsilon}, \quad (19)$$

and the nonlinear system for $p^1$

$$p^1 : \frac{d^2\theta_{A1}}{dy^2} + 2.13\gamma^2\frac{d\theta_{A0}}{dy} + \mathcal{F}\theta_{C1} - B\theta_{A1} \quad (20)$$

$$p^1 : \frac{d^2\theta_{C1}}{dy^2} + 2.13\gamma^2\frac{d\theta_{C0}}{dy} + \mathcal{F}\theta_{C1} - B\theta_{A1} \quad (21)$$

with boundary conditions

$$\text{at } y = 0, \theta_{A1} = 0, \frac{d\theta_{A1}}{dy} + \frac{d\theta_{C1}}{dy} = 0 \quad (22)$$
and so on up to the desired power of \( p \). For equations (7) and (8), we will find a series solution up to the second power of \( p \). So, by solving system (16)–(23), we obtain

\[
\theta_{\lambda}(y) = 1 - \exp(-y\sqrt{B + F})
\]

\[
\theta_{c}(y) = \frac{B}{F} - \exp(-y\sqrt{B + F})
\]

\[
\theta_{\lambda}(y) = 2.13 \exp(-y\sqrt{B + F}) \left( \frac{y}{4(B + F)} + \frac{y^2}{4\sqrt{B + F}} + \frac{y^3}{6} \right)
\]

\[
\theta_{c}(y) = -2.13 \exp(-y\sqrt{B + F}) \left( \frac{y}{4(B + F)} + \frac{y^2}{4\sqrt{B + F}} + \frac{y^3}{6} \right)
\]

The two-term HPM solution is now obtained from the relations

\[
\theta_{\lambda}(y) = \lim_{p^{-1}} (\theta_{\lambda 0} + p\theta_{\lambda 1}) = \theta_{\lambda 0} + \theta_{\lambda 1}
\]

\[
\theta_{c}(y) = \lim_{p^{-1}} (\theta_{c 0} + p\theta_{c 1}) = \theta_{c 0} + \theta_{c 1}
\]

That is,

\[
\theta_{\lambda}(y) = 1 - \exp(-y\sqrt{B + F}) \left( 1 - 2.13 \left( \frac{y}{4(B + F)} + \frac{y^2}{4\sqrt{B + F}} + \frac{y^3}{6} \right) \right)
\]

\[
\theta_{c}(y) = \frac{B}{F} + \exp(-y\sqrt{B + F}) \left( 1 - 2.13 \left( \frac{y}{4(B + F)} + \frac{y^2}{4\sqrt{B + F}} + \frac{y^3}{6} \right) \right)
\]

The limiting current of ferric ion reduction at the RDE is computed by

\[
\psi = \left. \frac{i}{nFAD_{\lambda}^2} \frac{d\theta_{\lambda}}{dy} \right|_{y=0}
\]

Algebraic manipulation of equations (30) and (32) gives the

\[
\psi = \delta m + \frac{0.5325}{\delta^2 m^2}
\]

where \( m = \sqrt{\frac{k_0 D_{\lambda} + k_{1\lambda} D_{\lambda}}{k}} \). The current for steady-state voltammetry is computed by the formula

\[
\psi_{SSV} = \frac{1}{1 + e^{\eta}} \left( \delta m + \frac{0.5325}{\delta^2 m^2} \right),
\]

where \( \eta = \frac{n\nu}{RT}(E - E^0) \) is the potential.

5. Previous analytical results

The theory and experimental results of rotating disk electrodes that are available in the literature are only for boundary conditions in the finite domain. For example, a system of a convection–diffusion equation in a finite domain modeling an EC-catalytic mechanism at the rotating disk electrode [31] and a system of convection–diffusion equations representing a pseudo-first-order EC-catalytic process at a rotating disc electrode in a finite domain [32].

Lin et al [12] used the perturbation technique to derive the following approximate distributions of ferric ion and ferrous ion, respectively

\[
\theta_{\lambda}(y) = 1 - \exp(-y\sqrt{B + F}),
\]

\[
\theta_{c}(y) = \frac{B}{F} + \exp(-y\sqrt{B + F}).
\]

This result is valid for low rotation rates or when hydrodynamic boundary layer thickness \( \delta \) is large. In addition, Haberland et al [33] discussed the same nonlinear diffusion equations for the case when \( y^2 \frac{d\theta_{c}}{dx} \ll |F/R| \).

Equations (35) and (36) are, however, valid when the convective terms (second terms of the left hand side of equations (7) and (8)) are dropped from the equation, assuming very small velocities of electrolytic solutions. Note that the first two terms of equations (30) and (31) are identical to the right hand sides of equations (35) and
and in this case, the current becomes [12]
\[
\psi = \sqrt{B + \mathcal{F}} = \frac{\delta}{\sqrt{D}} \sqrt{k_f C_B + k_b C_D}.
\]

6. Results and discussion

In this section, we test the accuracy of the obtained analytical expressions by the means of numerical simulations. The derived equations (30) and (31)) represent new simple analytical expressions for the concentrations of ferric ion and ferrous ion for all possible values of parameters. These concentrations are dependent on the dimensionless parameters \( B \) and \( \mathcal{F} \), which in return are dependent on the parameters and \( k_f, k_b, C_B, C_D, D \) and \( \delta \). The MATLAB function pdepe is used to solve the boundary value problems for the two convection–diffusion equations (7) and (8). In tables 1–6, the analytical expressions for the concentrations of ferric ion and ferrous ion are compared with the numerical results.
ferric and ferrous ions, obtained in section 4, are compared with numerical simulation results and other analytical results in the literature for various values of the dimensionless parameters $\overline{F}$ and $\overline{E}$. In tables 1–3, it is observed that the concentration of the ferric ion is depleted at the surface and hence a higher cathode current is noticed. Equation (33) represents the obtained analytical expression of the current for all possible values of parameters. Figures 2(a)–(d) show the variation of the dimensionless current for different values of diffusivity $D$, concentration of $\text{HO}_2^-$, hydrodynamic boundary layer thickness $\delta$ and parameter $m$. From figure 2(a), it is observed that the current begins its course decreasing until it reaches a minimum value then increase over the domain for all values of $D$. This behaviour of the current seems to be identical for all other parameters as evident in figures 2(c)–(d).

### Table 3. Comparison between the results of the proposed HPM and perturbation method [12] for the concentration of ferric ion $\theta_A$ assuming $\overline{B} = 2$ and $\overline{F} = 1$.  

| $y$ | Numerical Result | Proposed HPM equation (30) | Perturbation equation (35) [12] | Proposed HPM equation (30) | Perturbation equation (35) [12] |
|-----|------------------|---------------------------|-------------------------------|---------------------------|-------------------------------|
| 0.00 | 0.0000           | 0.0000                    | 0.0000                        | 0.00                      | 0.00                          |
| 0.05 | 0.0919           | 0.0918                    | 0.0829                        | 0.11                      | 0.97                          |
| 0.15 | 0.2554           | 0.2556                    | 0.2268                        | 0.08                      | 10.42                         |
| 0.25 | 0.3952           | 0.3963                    | 0.3514                        | 0.28                      | 11.08                         |
| 0.35 | 0.5147           | 0.5173                    | 0.4546                        | 0.51                      | 11.68                         |
| 0.45 | 0.6163           | 0.6214                    | 0.5413                        | 0.83                      | 12.17                         |
| 0.55 | 0.7020           | 0.7106                    | 0.6143                        | 1.23                      | 12.49                         |
| 0.65 | 0.7734           | 0.7868                    | 0.6756                        | 1.73                      | 12.65                         |
| 0.75 | 0.8318           | 0.8515                    | 0.7272                        | 2.37                      | 12.58                         |
| 0.85 | 0.8786           | 0.9062                    | 0.7706                        | 3.14                      | 12.29                         |
| 0.95 | 0.9151           | 0.9539                    | 0.8071                        | 4.02                      | 11.80                         |
| 1.05 | 0.9427           | 0.9930                    | 0.8405                        | 5.34                      | 10.84                         |
| 5.00 | 1.0000           | 1.0090                    | 0.9998                        | 0.90                      | 0.02                          |
| 10   | 1.0000           | 1.0000                    | 1.0000                        | 0.00                      | 0.00                          |

Average relative error (%) 1.47 9.13

### Table 4. Comparison between the results of the proposed HPM and perturbation method [12] for the concentration of ferrous ion $\theta_C$ assuming $\overline{B} = 1$ and $\overline{F} = 3$.  

| $y$ | Numerical Result | Proposed HPM equation (31) | Perturbation equation (36) [12] | Proposed HPM equation (31) | Perturbation equation (36) [12] |
|-----|------------------|---------------------------|-------------------------------|---------------------------|-------------------------------|
| 0.00 | 1.3333           | 1.3333                    | 1.3333                        | 0.00                      | 0.00                          |
| 0.05 | 1.2310           | 1.2311                    | 1.2378                        | 0.01                      | 0.55                          |
| 0.15 | 1.0520           | 1.0537                    | 1.0738                        | 0.16                      | 2.07                          |
| 0.25 | 0.9045           | 0.9059                    | 0.9395                        | 0.15                      | 3.87                          |
| 0.35 | 0.7819           | 0.7827                    | 0.8296                        | 0.10                      | 6.10                          |
| 0.45 | 0.6806           | 0.6801                    | 0.7396                        | 0.07                      | 8.67                          |
| 0.55 | 0.5977           | 0.5950                    | 0.6659                        | 0.45                      | 11.41                         |
| 0.65 | 0.5305           | 0.5247                    | 0.6055                        | 1.09                      | 14.14                         |
| 0.75 | 0.4769           | 0.4670                    | 0.5561                        | 2.08                      | 16.61                         |
| 0.85 | 0.4351           | 0.4200                    | 0.5157                        | 3.47                      | 18.52                         |
| 0.95 | 0.4033           | 0.3822                    | 0.4826                        | 5.23                      | 19.66                         |
| 1.05 | 0.3797           | 0.3521                    | 0.4555                        | 7.27                      | 19.96                         |
| 5.00 | 0.3333           | 0.3333                    | 0.3333                        | 0.00                      | 0.00                          |
| 10.0 | 0.3333           | 0.3333                    | 0.3333                        | 0.00                      | 0.00                          |

Average relative error (%) 1.43 8.68
Figure 3 shows the current versus the hydrodynamic boundary layer thickness for stationary and rotating disk electrodes ($\delta$ is directly proportional to $\omega^{-1/2}$). It is inferred from figure 3 that the current decreases to its minimum value (approx. 1.531 82) before it starts to increase again whereas the current for stationary electrode attains the steady-state value. Figures 4(a) and (b) confirm that the catalytic current for steady-state voltammetry tends to a limiting value at a high potential value, which increases dramatically as the thickness of boundary layer $\delta$ is raised. The dependence of voltammetry current on rotation rate or boundary layer thickness is an evidence that a convection and diffusion control the process.

By the sensitivity analysis, we study the effect of each parameter on the current density. By evaluating the slope of the current density with respect to a parameter, we can determine the influence of that parameter on the current. The percentage of change in current with respect to $D$, $\delta$ and $m$ are found to be 99.9%, 0.08% and 0.02%, respectively. As depicted in figure 5, it is concluded that the diffusion coefficient almost has the entire effect on the current density compared to $\delta$ and $m$.
Figure 2. Analytical expression of the current (equation (33)).

(a) Effect of diffusivity

(b) Effect of $\text{H}_2\text{O}_2$ concentration

(c) Effect of Thickness of hydrodynamic boundary layer

(d) Effect of $m$

Figure 3. Current versus thickness of boundary layer for stationary and rotating disk electrode.
7. Conclusion

In this paper, the system of steady-state nonlinear convection–diffusion equations in the kinetics of the catalytic reaction system is solved analytically. The model investigated the influence of parameters on the response of catalytic current. Approximate closed-form analytical expressions pertaining to the concentrations of the ferric ion and ferrous ion in addition to the current for all possible values of the diffusion and kinetic parameters are obtained using a modified approach of the HPM. The obtained analytical results are believed to be useful in describing the fundamental feature of hydrogen peroxide action by the ferric and ferrous ion concentration. The ideas and discussions in this paper can be extended to other nonlinear problems in rotating disc electrodes and rotating ring-disc electrodes.

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Appendix. The basic idea of the homotopy perturbation method

Consider the nonlinear differential equation

\[ A(u) - f(r) = 0, \quad r \in \Omega, \tag{A.1} \]
with the boundary condition

$$B\left( u, \frac{du}{dr} \right) = 0, \quad r \in \Gamma,$$  \hspace{1cm} (A.2)

where $A$, $B$, and $f(r)$ are a general differential operator, a boundary operator, a known analytical function and the boundary of the domain $\Omega$, respectively. Expressing $A(u)$ as the sum of linear ($L$) and nonlinear ($N$) parts, equation (A.1) becomes

$$L(u) + N(u) - f(r) = 0.$$  \hspace{1cm} (A.3)

The homotopy technique begins by defining $v(r, p)$: $\Omega \times [0, 1] \rightarrow R$, such that

$$H(v, p) = (1 - p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0,$$  \hspace{1cm} (A.4)

where $p \in [0,1]$ is an embedding parameter and $u_0$ is an initial approximation of equation (A.1) that satisfies boundary conditions (A.2). Evidently, equation (A.4) implies that

$$H(v, 0) = L(v) - L(u_0) = 0,$$  \hspace{1cm} (A.5)

$$H(v, 1) = A(v) - f(r) = 0.$$  \hspace{1cm} (A.6)

As $p$ changes from 0 to 1, $v(r, p)$ changes from $u_0$ to $u$, a process known as a homotopy. The solution of equation (A.4) may be expressed in terms of a power series in the form:

$$v = v_0 + p v_1 + p^2 v_2 + \cdots.$$  \hspace{1cm} (A.7)

With $p = 1$, an approximate solution to equation (A.4) is given by:

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + \cdots.$$  \hspace{1cm} (A.8)

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**References**

[1] Bard A J and Faulkner L R 2000 Electrochemical Methods: Fundamentals and Applications 2nd edn (New York: John Wiley and Sons)
[2] Rieger P H 1994 Electrochemistry 2nd edn (Englewood Cliffs, NJ: Prentice-Hall)
[3] Levich V 1962 Physicochemical Hydrodynamics (Englewood Cliffs, NJ: Prentice-Hall)
[4] Nikolic J, Exposito E, Iniesta J, González-García J and Montiel V 2000 J. Chem. Ed. 77 9
[5] Compton R, Laing M, Mason D, Northing R and Unwin P 1988 Rotating disc electrodes: the theory of chronoamperometry and its use in mechanistic investigations Proc. R. Soc. Lond. A 418 113–54
[6] Newman J 1966 Schmidt number correction for the rotating disk J. Phys. Chem. 70 1327
[7] Gregory D P and Raddadfòrd A C 1956 Transport to the surface of a rotating disc J. Chem. Soc. Part III 1756–64
[8] Diard J P and Montella C 2013 Re-examination of steady-state concentration profile near a uniformly accessible rotating disk electrode J. Electroanal. Chem. 703 52–5
[9] Rajendran L, Subbiah A and Vasudevan T 2003 A two-point Padé approximation for the mass-transfer rate at rotating disc electrodes J. Elect. Chem. 547 173–7
[10] Jansi Rani P, Kirthiga M, Molina A, Laborda E and Rajendran L 2017 Analytical solution of the convection-diffusion equation for uniformly accessible rotating disk electrodes via the homotopy perturbation method I. Elect. Chem. 799 175–80
[11] Okaounghae D 2006 Application of hp-discontinuous Galerkin finite element methods to the rotating disk electrode problems in electrochemistry J. Appl. Math. Comp. 22 1–20
[12] Lin Y-P, Wu H and Selman J R 1990 Perturbation analysis of catalytic current at a rotating disk electrode Ind. Eng. Chem. Res. 29 1189–94
[13] Litter M I and Słodowicz M 2017 An overview on heterogeneous Fenton and photo-Fenton reactions using zerovalent iron materials J. Adv. Oxid. Technol. 20 19
[14] Saranya K, Mohan V and Rajendran L 2020 Steady-state concentrations of carbon dioxide absorbed into phenyl glycidyl ether solutions by residual method J. Math. Chem. 58 1250–46
[15] Joy Salomi R, Vinolyn Sylvia S, Rajendran L and Abukhaled M 2020 Electric potential and surface oxygen ion density for planar, spherical and cylindrical metal oxide grains Sensors Actuators B 321 128576
[16] Devi M C, Pirabaharan P, Rajendran L and Abukhaled M 2020 An efficient method for finding analytical expressions of substrate concentrations for different particles in an immobilized enzyme system React Kinet Mech Cat 130 35–53
[17] He J H 2007 Variational iteration method–some recent results and new interpretations J. Comput. Appl. Math. 207 3–17
[18] Abukhaled M 2013 Variational iteration method for nonlinear singular two-point boundary value problems arising in human physiology J. Math. 2013 1–4
[19] Wazwaz A 1999 A reliable modification of Adomian decomposition method Appl. Math. Comput. 102 77–86
[20] Chen C L and Liu Y C 1998 Solution of two-point boundary-value problems using the differential transformation method J. Optimisation Theory and Appl. 99 23–35
[21] Abukhaled M 2017 Green’s function iterative method for Solving a class of boundary value problems arising in heat transfer Appl. Math Inform Sci. 11 229–34
[22] Abukhaled M and Khuri S 2020 Efficient numerical treatment of a conductive-radiative fin with temperature-dependent thermal conductivity and surface emissivity Int J Comput Meth Eng Sci Mech 21 159–68
[23] Abukhaled M and Khuri S 2019 An efficient semi-analytical solution of a one-dimensional curvature equation that describes the human corneal shape Math. Comput. Appl. 24 8
[24] He J H and Ji F Y 2019 Taylor series solution for Lane–Emden equation J. Math. Chem. 57 1932–4
[25] He J-H 1999 Homotopy perturbation technique Comput. Methods Appl. Mech. Engrg. 178 257–62
[26] He J H 2005 Homotopy perturbation method for bifurcation of nonlinear problems Int. J. Non-linear Sci. Num. Sim. 6 207–8
[27] Visuvasam J, Molina A, Laborda E and Rajendran L 2018 Mathematical models of the infinite porous rotating disk electrode Int. J. Elect. Sci. 13 9999–10022
[28] Saravankumar R, Pirabaharan P, Muralikannan M and Rajendran L 2018 Transient current for a rotating disk electrodes produced by a potential step Russ. J. Elect. Chem. 54 863
[29] Wu Y and He J-H 2018 Homotopy perturbation method for nonlinear oscillators with co-ordinate dependent mass Results in Physics 10 270–1
[30] Thamizh Suganya S, Balaganesan P, Rajendran L and Abukhaled M 2020 Analytical discussion and sensitivity analysis of parameters of magnetohydrodynamic free convective flow in an inclined plate EJPAM 13 631–44
[31] Angel Joy R, Rajendran V and Rajendran L 2018 Reaction diffusion processes at rotating disk electrode International Journal of Research 7 988–97
[32] Devi M C, Pirabaharan P, Abukhaled M and Rajendran L 2020 Analysis of the steady-state behavior of pseudo-first-order EC-catalytic mechanism at a rotating disk electrode Electrochim. Acta 345 136175
[33] Haberland D and Landsberg R 1966 Zur Anwendung der rotierenden Scheibenelektrode beim Studium achgelagerter chemischer Reaktionen Berichte der Bunensgesellschaft für physikalische Chemie 70 724–7