On invisible plasma content in radio-loud AGNs: The case of TeV blazar Markarian 421

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ABSTRACT

Invisible plasma content in blazar jets such as protons and/or thermal electron-positron ($e^{\pm}$) pairs is explored through combined arguments of dynamical and radiative processes. By comparing physical quantities required by the internal shock model with those obtained through the observed broadband spectra for Mrk 421, we obtain that the ratio of the Lorentz factors of a pair of cold shells resides in about $2 \sim 20$, which implies that the shocks are at most mildly relativistic. Using the obtained Lorentz factors, the total mass density $\rho$ in the shocked shells is investigated. The upper limit of $\rho$ is obtained from the condition that thermal bremsstrahlung emission should not exceed the observed $\gamma$-ray luminosity, whilst the lower limit is constrained from the condition that the energy density of non-thermal electrons is smaller than that of the total plasma. Then we find $\rho$ is $10^{2}-10^{3}$ times heavier than that of non-thermal electrons for pure $e^{\pm}$ pairs, while $10^{2}-10^{6}$ times heavier for pure electron-proton ($e/p$) content, implying the existence of a large amount of invisible plasma. The origin of the continuous blazar sequence is shortly discussed and we speculate that the total mass density and/or the blending ratio of $e^{\pm}$ pairs and $e/p$ plasma could be new key quantities for the origin of the sequence.

Key words:
BL Lacertae objects: general – BL Lacertae objects: individual (Mrk 421) – galaxies: active – radiation mechanisms: non-thermal

1 INTRODUCTION

The discovery of strong inverse Compton components in X and $\gamma$-ray emission from jets in active galactic nuclei (hereafter AGN) for a wide range of spatial scales (e.g., Collmar 2001 for review) enables us to probe quantitatively the energetics of relativistic jets. The kinetic power of non-thermal electrons has been estimated by various authors both for inner core jets (i.e., blazars) (e.g., Kino, Takahara and Kusunose 2002, hereafter KTK; Kusunose, Takahara and Kato 2003) and large scale jets (e.g., Tavecchio et al. 2000; Leahy and Gizani 2001, 2002; Kataoka et al. 2003). However, the material content of relativistic jets is not easily constrained by observations since the emission is dominated by that from non-thermal electrons and probably positrons and it is difficult to directly constrain thermal matter content. Hence, the plasma composition in AGN jets, whether normal proton-electron ($e/p$) plasma or electron-positron pairs ($e^{\pm}$) is a dominant composition, is still a matter of open issue (e.g., Reynolds et al. 1996; Celotti, Kuncic, Rees and Wardle 1998; Wardle et al. 1999; Hirotani et al. 1999; Sikora and Madejski 2000; Ruszkowski and Begelman 2002; Kino and Takahara 2004, hereafter KT04). This problem prevents us from estimating the total mass and energy flux ejected from a central engine.

To constrain invisible matter content such as thermal electron-positron pairs and/or protons co-existing with non-thermal electrons, dynamical considerations are indispensable. In KT04, we proposed a new procedure to constrain the invisible thermal plasma component in classical FR II radio sources. We used the fact that the mass and energy densities of the sum of thermal and non-thermal particles are larger than those of non-thermal electrons which are determined by observations. Here we apply the same technique to the inner core jets of AGNs (i.e., blazars) based on the internal shock model. The internal shock model is believed to be most plausible to explain the production of high energy photons and time variabilities in blazars. It has been widely applied also to the prompt emission of gamma-ray bursts.
(hereafter GRBs) (e.g., Rees 1978; Rees and Meszaros 1994; Kobayashi, Piran and Sari 1997; Daigne and Mochkovitch 1998; Ghisellini 1999; Spada, Ghisellini, Lazzati and Celotti 2001). It is worth to note that recently Ghisellini et al. (2005) proposed a structured jet model consisting of a fast spine surrounded by a slowly moving layer for explaining VLBI scale radio blobs. At the present, however, it is not evident where is the acceleration site of electrons in the structured jet. This is one of the prime issues which should be answered. Internal shocks are potentially the building blocks of the spine part of the structured jet. Whereas we recognize the importance of the detailed structure of jets, as a first step we focus on the physical condition of the flow based on the simple internal shock model.

The methodology of constraining the invisible plasma content in the emission region is as follows. As mentioned above, a lower limit to the total mass density (sum of non-thermal electrons and invisible plasma) is restricted by the definition that the mass density of total plasma should be smaller than that of the non-thermal electrons. The mass density of non-thermal electrons can be estimated by multi-frequency observations. For this purpose, in 
3 we review the shock dynamics of two colliding shells. Note that we do not use the simple two point-mass approximation (e.g., Piran 1999; Lazzati et al. 1999; Zhang and Mészáros 2004 for review) but employ the exact shock dynamics throughout this work. This makes outcomes more accurate. In 
4 we briefly review the previous results on the amount of non-thermal electrons based on KTK. In 
5 we constrain on the amount of total mass density. As for the upper limit, we use the constraint that bremsstrahlung emission from thermal electron (and positron) component should not exceed the observed \( \gamma \)-ray emission. We postulate synchrotron self-Compton (SSC) emission dominance in the \( \gamma \)-ray band which is supported by the observed correlations between TeV\( \gamma \)-ray and X-ray in TeV blazars (e.g., Takahashi et al. 1996, 2000; Catanese et al. 1997; Maraschi et al. 1999). We can thus bracket the amount of total mass density in the emission region from below and above. In this way we apply this method to the archetypal TeV blazar Mrk 421. In 
6 we further estimate the shock dissipation rate of the colliding cold shells. The dissipation rate is a widely discussed quantity in literatures concerning gamma-ray bursts (e.g., Lazzati, Ghisellini and Celotti 1999; Piran 1999). The shock dissipation is believed to be the ultimate source of heating and accelerating particles. Summary and discussion are in 
7.

2 KEY FEATURES OF THIS WORK

The key features of this work are briefly summarised here in advance. The existence of copious amount of invisible plasma is predicted from a qualitative consideration.

2.1 Existence of invisible plasma content

As mentioned in the introduction, we constrain on the amount of invisible plasma content by introducing the dynamical considerations. The point is that we divide total mass and energy densities into two components, i.e., those of non-thermal electrons and those of the other invisible components. The comparison of these obtained quantities enables us to constrain on the amount of invisible plasma and this is a new attempt compared with the previous works.

Bearing this in mind, next we show a quantitative consideration which derives the existence of invisible plasma in colliding shells of blazar jets. Let us discuss a collision between a pair of equal mass-density shells for instance. In the comoving frame of one shell, the particles of the other shell are coming in with a relative bulk Lorentz factor \( \Gamma_{ij} \) (see in §3 for details) of a few at most (shown in Table 1). When only pair \( e^\pm \) plasma are present and all of them are accelerated, then the average Lorentz factor of non-thermal electrons \( \langle \gamma_e \rangle \) is expected to be \( \langle \gamma_e \rangle \approx \Gamma_{ij} \). This is too small to account for observed blazar spectra \( \langle \gamma_e \rangle \approx 300 \) (this is the case of Mrk 421) obtained by KTK. Therefore only a fraction of the pair \( e^\pm \) should be accelerated, the ratio of the rest mass density of non-thermal electrons to that of total plasma is about \( \Gamma_{ij}/\langle \gamma_e \rangle \). Similarly we can discuss the case for shells with pure \( e/p \) plasma makeup. If all of the dissipated energy goes into the electron acceleration, then we have \( \langle \gamma_e \rangle \approx (m_p/m_e)\Gamma_{ij} \). This is too large to account for the spectra and it requires a limited fraction of electrons being accelerated. Thus, invisible plasma is qualitatively expected when the internal shock is responsible for the production of non-thermal electrons. In this paper, we will quantitatively explore the amount of invisible plasma in jets.

2.2 Why we use shock dynamics?

In the previous studies, the colliding shells have been approximately modeled as the simple two-point-mass collision (e.g., Piran 1999; Lazzati et al. 1999; Zhang and Mészáros 2004). The reason why we use the shock dynamics instead of the two-point-mass model is as follows. When one try to derive the mass density from the mass, one eventually needs to know lengths and velocities and they can be consistently obtained by the shock model. Hence the shock analysis is the best way for investigating the invisible plasma content in jets.

3 SHOCKS IN COLLIDING SHELLS

Here we review the relativistic shock jump conditions. We use one-dimensional shock dynamics of a pair of colliding shells to apply the standard internal shock model to blazars. Suppose the situation in which a rapid shell overtakes a previously ejected slow shell. There are four characteristic regions designated by (1) unshocked slow shell, (2) shocked slow shell, (3) shocked rapid shell, and (4) unshocked rapid shell. These regions are separated by the forward shock (FS), the contact discontinuity (CD), and the reverse shock (RS). In this paper, we use the terminology of regions \( i \) (\( i=1, 2, 3, \) and \( 4 \)) and position of discontinuity \( i \) (\( i=FS, CD, \) and \( RS \)) where FS, CD, and RS stand for the forward shock front, contact discontinuity, and reverse shock front, respectively. The fluid velocity and Lorentz factor in the region \( i \) measured in the interstellar medium (hereafter ISM) frame are expressed as \( v_i(=\beta_ic) \) and \( \Gamma_i \), respectively. The relative velocity and Lorentz factor of the fluid \( i \) measured in the frame \( j \) are denoted by \( v_{ij}(=-v_{ji} = \beta_{ij}c = -\beta_jc) \) and \( \Gamma_{ij}(=\Gamma_{ji}) \), respectively. Rest mass density, pressure, and internal energy density are expressed as \( \rho_i, P_i, \) and \( e_i, \) respectively.
respectively. As for the equation of state (EOS), we take
\( P_i = (\gamma_i - 1)(e_i - \rho_i c^2) \), where \( \gamma_i \) is the adiabatic index. We sometimes use the subscripts \( s \) and \( r \) instead of \( 1 \) and \( 4 \), such as \( \Gamma_1 = \Gamma_s \) and \( \Gamma_4 = \Gamma_r \).

In the limit of strong shock, with the assumption of cold upstream \( (P_i = 0) \), the jump conditions for the forward shock are written as follows (Blandford & McKee 1976):

\[
\Gamma_{FS1}^2 = \frac{(\Gamma_1 + 1)[\gamma_2(\Gamma_1 - 1) + 1]^2}{\gamma_2(2 - \gamma_2)(\Gamma_1 - 1) + 2},
\]

\[
e_2 = \Gamma_{12}p_2,
\]

\[
\Gamma_{FS1}^2 = \frac{(\Gamma_3 + 1)[\gamma_3(\Gamma_3 - 1) + 1]^2}{\gamma_3(2 - \gamma_3)(\Gamma_3 - 1) + 2},
\]

\[
e_3 = \Gamma_{34}p_3,
\]

where \( \Gamma_{12} = \Gamma_1 \Gamma_2(1 - \beta_1 \beta_2) \), and \( \Gamma_{FS1} \) is the Lorentz factor of forward shock measured in the rest frame of the unshocked slow shell. In the relativistic limit, the adiabatic index is \( \gamma_2 = 4/3 \). Using the same assumptions as in the forward shock, the jump conditions for the reverse shock are given by:

\[
\Gamma_{RS4}^2 = \frac{(\Gamma_4 + 1)[\gamma_2(\Gamma_4 - 1) + 1]^2}{\gamma_2(2 - \gamma_2)(\Gamma_4 - 1) + 2},
\]

\[
e_4 = \Gamma_{43}p_4,
\]

where \( \Gamma_{43} = \Gamma_3 \Gamma_4(1 - \beta_3 \beta_4) \), and \( \Gamma_{RS4} \) is the Lorentz factor of the reverse shock measured in the rest frame of the unshocked rapid shell. The equality of pressure and velocity across the contact discontinuity gives

\[
P_2 = P_3, \quad \Gamma_2 = \Gamma_3.
\]

After the shocks break out the shells, \( \Gamma_2 = \Gamma_3 \) is not satisfied because a rarefaction wave changes the density and velocity profiles (e.g., Kino, Mizuta and Yamada 2004, hereafter KMY). We do not treat the rarefaction waves for simplicity, concentrating on the major duration before shock breakout. It may be useful to rewrite the pressure balance along the CD as

\[
\frac{\rho_4}{\rho_1} = \frac{(\gamma_3 \Gamma_4 + 1)(\Gamma_4-1)}{(\Gamma_3 + 1)(\Gamma_3-1)}.
\]

In general, the number of physical quantities in each region is \( 3 \), \( \rho_i \), \( P_i \) (or \( e_i \)), and \( v_i \). Forward and reverse shock speeds (i.e., \( v_{FS} \) and \( v_{RS} \)) are two other quantities. In all, there are \( 3 \times 4 + 2 = 14 \) physical quantities. Note that \( P_i \) and \( e_i \) are connected with EOS. The total number of the jump conditions is \( 3 + 3 + 2 = 8 \). Hence, given \( 3 + 3 = 6 \) upstream quantities for each shock, we can obtain the remaining \( 8 \) downstream quantities by using \( 8 \) jump conditions. It is to be noted that the absolute value of the rest mass density is irrelevant to the shock dynamics since the shock dynamics is linear with respect to the mass density. Then, actually we need to specify \( 5 \) quantities if we give the density ratio \( \rho_4/\rho_1 \).

For a specific case for TeV blazars, we here impose the following two conditions; (i) the unshocked shells are cold, i.e., \( P_1 = P_3 = 0 \), (ii) the Lorentz factor of the shocked regions \( \Gamma_3(= \Gamma_2) \) is identified as that of the emission region obtained by the observed broadband spectra. Further, we examine the following three cases for the ratio \( \rho_4/\rho_3 \): (a) the energy of bulk motion of the rapid shell \( (E_i = \Gamma mc^2) \) equals to that of the slow one in the ISM frame (we refer to it as “equal energy (or \( E \) case)”), (b) the mass of the rapid shell \( (m = \rho \Gamma \Delta) \) equals to that of the slow one (hereafter we call it “equal mass (or \( m \) case)”), and (c) the rest mass density of the rapid shell equals to that of the slow one (hereafter we call it “equal rest mass density (or \( \rho \) case”)).

These choices are based on the conjecture that the ejecta from the “central engine” is likely to have a correlation with each other (e.g., NP02; KMY). Hereafter, we assume that the widths of two shells are the same in the ISM frame, that is \( \Delta_1/\Delta_4 = 1 \) (e.g., NP02, Spada et al. 2001). Note that in the case of \( \Delta_r = \Delta_s \) and \( \Gamma_r > \Gamma_s \), \( \rho_r \) is always larger than \( \rho_s \) for equal \( E \) and equal \( m \) cases.

Thus, we give 4 quantities, \( P_i, P_4, \Gamma_2 = \Gamma_3 \) and one relation between the rapid and slow shells, \( \rho_4/\rho_1 \) depending on cases (a) through (c) described above. As a remaining quantity, the Lorentz factor of the rapid shell \( \Gamma_4 \) is treated as a free parameter. Although we do not specify the absolute value of \( \rho \), we treat the absolute value in actual applications. It is compared with that of non-thermal electrons in the shocked regions as described in [33]. The absolute value of the rest mass density comes into play when two-body processes such as bremsstrahlung emission is used to obtain the upper limit of \( \rho \). We will properly discuss these points.

In the following sections, we focus on the values of (i) the value of \( \Gamma_1 \) and \( \Gamma_4 \), (ii) \( e_3 \) and/or \( \rho_3 \), as a tool to examine the physical quantities of invisible matter content.

4 AMOUNT OF NON-THERMAL ELECTRONS

4.1 Number and energy densities

Based on the detection of inverse Compton emission in \( \gamma \)-ray band, the number and energy densities of the non-thermal (hereafter “NT”) electrons \( n_{eNT} \) and \( e_{eNT} \) in shocked regions can be determined by the comparison of the observed broadband spectrum and the theoretical one. Although the minimum Lorentz factor of relativistic electrons is not definitely determined and affects mainly the number density \( n_{eNT} \), we regard that low energy electrons below \( \gamma_{e,\text{min}} \) constitute thermal electrons. Considering the observed flat number spectrum of electrons, fixing \( \gamma_{e,\text{min}} = 10 \) does not cause any major problem with \( n_{eNT} \).

Here, we briefly quote the resultant \( n_{eNT} \) and \( e_{eNT} \) obtained in KTK. Hereafter, we omit the subscript expressing the regions \( i(= 2, 3) \) for simplicity. For clearness of the following argument, we define that \( n_{eNT} \) and \( e_{eNT} \) also include NT positrons when they exist. The quantity \( n_{eNT} \) is written as \( n_{eNT} = \int \gamma_{e,\text{min}} n_e(\gamma_e) d\gamma_e \), while \( e_{eNT} \) is given by \( e_{eNT} = \langle \gamma_e \rangle n_{eNT} m_e c^2 \), where \( n_e(\gamma_e) \) and \( \langle \gamma_e \rangle \) are the energy spectrum and the average Lorentz factor of NT electrons, respectively. By a detailed comparison of the SSC model with observed broadband spectrum of Mrk 421, we obtained \( n_{eNT} \) as

\[
n_{eNT} \approx 11 \times \left( \frac{\gamma_{e,\text{min}}}{10} \right)^{-0.6} \text{ cm}^{-3}.
\]

Here, we adopt the index of injected electrons for Mrk 421 as \( s = 1.6 \) (e.g., Mastichiadis & Kirk 1997; Kirk & Duffy 1999) and the case of \( \gamma_{e,\text{min}} = 10 \) was examined in KTK. The best choice of the size of the emission region is \( R \approx 2.8 \times 10^{16} \text{ cm} \) with an order of magnitude uncertainty. Thus, the corresponding uncertainty of \( n_{eNT} \) amounts to two orders of magnitude; for smaller \( R \), larger \( n_{eNT} \) should be adopted.
But, as far as the the shock dynamics is concerned, only the density ratio plays a role, therefore we adopt the above value as the canonical one.

As for the average energy of NT electrons, we obtained
\[ e_{\text{NT}}^{\gamma} / n_{\text{e}}^{\gamma} = (\gamma_{e}) n_{e} c^2 \simeq 3.1 \times 10^2 n_{e} c^2. \]

Since for \( s = 1.6 \), electrons near the cooling break energy \( \sim \gamma_{e,\text{br}} \) carry most part of the kinetic energy and \( e_{\text{e}}^{\gamma} \) has a weak dependence on \( \gamma_{e,\text{br}} \) provided that \( \gamma_{e,\text{br}} \) is smaller than \( \gamma_{e,\text{br}} \sim 10^5 \). Note that the case of \( \gamma_{e,\text{br}} \sim 10^5 \) is ruled out for Mrk 421 since the case does not fit the EGRET data (KTK). Therefore, Eq. (6) is justified in any case for Mrk 421.

4.2 Forward and reverse shocks

To clarify whether the observed non-thermal emission comes mainly from FS or from RS region, the typical frequency of non-thermal synchrotron radiation and internal energy density in each region are examined here.

According to the standard diffusive shock acceleration, the acceleration time scale is estimated as \( t_{\text{acc}} = (2\pi n_{e} c \xi / e B) \), where \( \xi = \lambda / r_{g} \) is a parameter related to the amplitude of magnetic fluctuations, \( \lambda \) and \( r_{g} \) are the mean free path for the scattering of electrons and Larmor radius, respectively. Here the shock speed is taken to be \( c \). On the other hand, the synchrotron cooling time is given by \( t_{\text{syn}} = (6m_{e} c^{2}) / (\pi \gamma c B^{2}) \). The maximum Lorentz factor of the non-thermal electrons is evaluated as \( \gamma_{\text{max}} \propto B^{-1/2} \) by using the condition of \( t_{\text{acc}} = t_{\text{syn}} \) at \( \gamma_{\text{max}} \) with the assumption that \( \xi \) in FS and RS regions takes the same value. Hence, the characteristic synchrotron photon energy is given by \( h\nu_{\text{syn},\text{max}} \propto \Gamma_{s} B_{\gamma_{\text{max}}}^{2} \propto \text{const.} \). Hence, the value \( \nu_{\text{syn},\text{max}} \) in FS region and RS regions is the same.

The total internal energy of NT electrons in FS and RS regions may be discussed as follows. If \( \Gamma_{21} \gg 1 \) and \( \Gamma_{43} \gg 1 \) are satisfied, we have \( e_{2} + P_{2} \simeq e_{3} + P_{3} \). In the actual case of blazars, \( \Gamma_{ij} \) is close to order of unity and we have \( P_{2} = P_{3} \). Thus, the energy densities of regions 2 and 3 are similar and the internal energy is controlled by the comoving shell widths. Since \( \Gamma_{ij} \gg \Gamma_{21} \) is always satisfied, co-moving length of RS region is larger than that of FS region in the case of \( \Delta_{s} = \Delta_{e} \) (e.g., Kobayashi and Sari 2001; NP02; KMY). Thus, the radiation from RS dominates over that from FS region. Based on this consideration, we focus on RS dominated case in this paper. Hereafter, we omit the subscript 3 for simplicity.

5 CONSTRANTS ON THE AMOUNT OF INVISIBLE PLASMA

5.1 Lorentz factors of cold shells

It is hard to estimate the bulk Lorentz factors of cold shells simply because they are invisible. However, by using the value of Lorentz factor of shocked shell which corresponds to the beaming factor of the emission region, we can constrain on the Lorentz factors of the cold shells. Here, we consider the range from 3 to 100 for \( \Gamma_{s} \) and \( \Gamma_{e} \). Following Begelman, Rees & Sikora (1994), we consider the upper limit of the Lorentz factors of the emission region as \( \Gamma_{e,\text{max}} = 100 \), while as for the lower limit we employ \( \Gamma_{e,\text{min}} = 3 \) based on Wardle & Aaron (1997). Here, we exclude cases of very weak collisions with \( \Gamma_{e}/\Gamma_{s} < 2 \) as in NP02. As for the adiabatic index in Eq. (4), we approximate \( \gamma_{i} = 4/3 \) for \( \gamma_{ij} > 2 \), otherwise \( \gamma_{i} = 5/3 \) for simplicity (e.g., Kirk and Duffy 1999).

For the TeV blazar Mrk 421, we have already obtained \( \Gamma_{2} = \Gamma_{3} = 12 \) by the observed multi-frequency spectrum (KTK). Hence, Eq. (3) is solvable for \( \Gamma_{i} \) given \( \rho_{i} / \rho_{s} \) and \( \Gamma_{i} \). Qualitatively, a faster \( \Gamma_{i} \) requires a slower \( \Gamma_{s} \) to attain the same value of \( \Gamma_{3} \). Thus, minimum value of \( \Gamma_{i} \) corresponds to \( \Gamma_{i}/\Gamma_{s} = 2 \), while the maximum value of \( \Gamma_{i} \) corresponds to \( \Gamma_{i} = 100 \) or \( \Gamma_{s} = 3 \).

Table 1. Lorentz factors obtained by internal shock analysis for Mrk 421

| case | \( \Gamma_{s} \) | \( \Gamma_{e} \) | \( \Gamma_{12} \) | \( \Gamma_{43} \) |
|------|-----------------|-----------------|-----------------|-----------------|
| equal \( \rho \) (largest \( \Gamma_{i}/\Gamma_{s} \)) | 3 | 48.0 | 2.125 | 2.125 |
| equal \( \rho \) (smallest \( \Gamma_{i}/\Gamma_{s} \)) | 8.485 | 16.97 | 1.060 | 1.060 |
| equal \( m \) (largest \( \Gamma_{i}/\Gamma_{s} \)) | 5.12 | 100 | 1.35 | 4.22 |
| equal \( m \) (smallest \( \Gamma_{i}/\Gamma_{s} \)) | 8.983 | 17.959 | 1.042 | 1.082 |
| equal \( E \) (largest \( \Gamma_{i}/\Gamma_{s} \)) | 8.57 | 100 | 1.057 | 4.226 |
| equal \( E \) (smallest \( \Gamma_{i}/\Gamma_{s} \)) | 9.48 | 18.96 | 1.027 | 1.106 |

Notes: \( \Gamma_{\text{max}} = 100 \), \( \Gamma_{\text{min}} = 3 \), \( \Gamma_{i}/\Gamma_{s} > 2 \), and \( \Gamma_{2} = \Gamma_{3} = 12 \) are employed in this analysis.

In Table 1 we show the minimum and maximum values of \( \Gamma_{i} \) and \( \Gamma_{s} \) and the corresponding relative Lorentz factors \( \Gamma_{12} \) and \( \Gamma_{43} \) which control the shock heating of the downstreams (see Eqs. (1) and (2)). From this, we see that the range of \( \Gamma_{ij} \) lies between 1.03 and 4.2. In other words, a mildly relativistic shock is realized in the case of Mrk 421. We also note that our adopted value of \( \gamma_{e,\text{min}} = 10 \) is a reasonable choice with the assumption that \( \gamma_{e,\text{min}} \) should be a few times larger than \( \Gamma_{ij} \). The corresponding value of \( \Gamma_{i}/\Gamma_{s} \) is found as

\[
\begin{align*}
2 & \leq \Gamma_{i}/\Gamma_{s} \leq 16.0 \quad (\text{equal } \rho), \\
2 & \leq \Gamma_{i}/\Gamma_{s} \leq 19.5 \quad (\text{equal } m), \quad \text{and} \\
2 & \leq \Gamma_{i}/\Gamma_{s} \leq 11.7 \quad (\text{equal } E),
\end{align*}
\]

respectively.

5.2 Total mass density

5.2.1 Lower limit of \( \rho \)

In Eqs. (1) we show that shock is at most mildly relativistic though each shell moves at a relativistic speed. As a consequence, dissipation efficiency is relatively small and \( \gamma_{e} \gg \Gamma_{43} \) is satisfied. Therefore \( e_{\text{NT}}^{\gamma} / e = (\gamma_{e})^{\gamma} / \Gamma_{34} \rho < 1 \) gives a tighter constraint than \( \rho_{\text{NT}}^{\gamma} / \rho < 1 \). By rewriting the condition of \( e_{\text{NT}}^{\gamma} / e < 1 \), the lower limit of the total mass density is given by

\[
\rho / \rho_{\text{NT}}^{\gamma} > (\gamma_{e}) \Gamma_{34} \Gamma_{34} \approx 3.1 \times 10^{2} \Gamma_{34}.
\]

Here we omit the subscript of region number \( i = 3 \) for the various densities for thumbnail writing. From this we directly see that in order to accelerate electrons up to \( \gamma_{e} \sim 3.1 \times 10^{2} \) in the framework of standard internal shock model, where only a small available shock dissipation energy \( \Gamma_{34} \sim \text{a few} \) is realized, the invisible mass density at least
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about 100 times the rest mass density of NT electrons is definitely required. In other words, we need a loading of baryons and/or a thermal pair plasma. It is worth to note the effects of an uncertainty with \((\gamma_e)\). We estimated the uncertainty range as \(2.3 \times 10^2 < (\gamma_e) < 4.3 \times 10^2\) (KTK). The uncertainty simply leads to a shift of lower limit curve by the same factor. Since it causes only a small change on the resultant value, we focus on the best-fit case in this work for simplicity.

### 5.2.2 Upper limit of \(n_e^T\)

Here, we constrain the upper limit of the number density of thermal electrons \(n_e^T\). As mentioned in the Introduction, it is widely accepted that observed GeV and TeV \(\gamma\) -rays are SSC dominated. In the MeV range, bremsstrahlung radiation by the thermal electrons with temperature \(\Theta_e \equiv kT_e/m_e c^2 \sim 1.5 \times \text{a few MeV}\) is expected if adequate amount of thermal electrons exist in the emission region. At the moment, we do not have any observational evidence for the bremsstrahlung in the MeV band. At the same time, it is fair to note that observation in MeV range itself is a challenging area (e.g., Takahashi et al. 2003). Here we estimate the upper limit of the number density of thermal electrons by assuming the observed bolometric luminosity of bremsstrahlung \(L_{brem, o}\) should be lower than that of SSC \(L_{ssc, o}\) which is estimated as \(L_{ssc, o} = 7 \times 10^4 \text{erg s}^{-1}\) (KTK).

For \(e^+e^-\) plasma content, we employ Eqs. (21) and (22) of Svensson (1982) which express the emissivity of relativistic \(e^+e^-\) bremsstrahlung \(\epsilon_{brem}(\Theta_e, n_e^T)\) where \(n_e^T\) is the number density of thermal electrons. Note that these expressions do not include the bremsstrahlung between electron-electron and positron-positron and the limit will be severer by a factor of \(\sim 2\) if we include them. Then, the condition of \(L_{ssc, o} > L_{brem, o}\) is rewritten as

\[
n_e^T < 9.7 \times 10^2 \left(\Theta_e^{1/2}(1 + 1.7\Theta_e^{1.5})\right)^{-1/2} \text{cm}^{-3} \quad (\Theta_e < 1) \\
< 5.7 \times 10^2 \left[\Theta_e\ln(1.1\Theta_e) + 5.4\right]^{-1/2} \text{cm}^{-3} \quad (\Theta_e \geq 1).
\]

The bolometric luminosity of the optically-thin bremsstrahlung is estimated by \(L_{brem, o} = (4\pi R^2/3)\Gamma_{brem}^3\epsilon_{brem}\) with the emission size \(R = 2.8 \times 10^{16}\) cm and the Lorentz factor \(\Gamma_1\) as obtained by the broadband spectral fitting of Mrk 421 (KTK). The electron temperature is evaluated by \((\gamma_3 - 1)\Theta_e = \Gamma_{34} - 1\). The upper limit turns out to be about a thousand times larger than the mass density of non-thermal electrons. It is consistent with and relatively close to the required lower limit of the mass density by Eq. (8). This upper limit depends on the adopted value of \(R\), and it is proportional to \(R^{-3/2}\). Considering that \(n_e^T\) is roughly proportional to \(R^{-3}\), the ratio of this upper limit to \(n_e^T\) only has a weak dependence on \(R\).

Similarly, in the case of electron-proton (hereafter \(e/p\)) plasma content, we can rewrite the condition of \(L_{ssc, o} > L_{brem, o}\) as

\[
n_e^T < 9.5 \times 10^2 \left[\Theta_e^{1/2}(1 + 1.78\Theta_e^{1.34})\right]^{-1/2} \text{cm}^{-3} \quad (\Theta_e < 1) \\
< 9.6 \times 10^2 \left[\Theta_e\ln(1.1\Theta_e + 0.42) + 3/2\right]^{-1/2} \text{cm}^{-3} \quad (\Theta_e \geq 1)
\]

with Eqs. (17) and (18) of Svensson (1982). Note that electron-electron bremsstrahlung is not considered in these equations. It is clear that the upper limit of \(\rho_e/\rho_\text{e}\) in this case is \(m_\text{e}/m_\text{e}\) times larger than \(n_e^T\).

Lastly, we check the timescale of \(e^\pm\) pair annihilation \(t_{\text{ann}}\). It is estimated as \(t_{\text{ann}} \equiv \Theta_e^2/(n_e\sigma_T c) \approx 6 \times 10^6\Theta_e^2 (n_e/10^{13}\text{cm}^{-3})^{-1}\) sec. Hence we see that the annihilation time scale is much longer than the dynamical time scale \(t_{\text{dy}} \equiv \sqrt{3R}/c \approx 2 \times 10^4 (R/10^{16}\text{cm})\) sec. Therefore \(e^\pm\) pair annihilation is not effective in this situation.

### 5.2.3 Allowed range of \(\rho\)

We thus obtain the upper and lower limits on \(\rho/\rho_\text{e}\) and the results are shown in the plane of mass density of invisible plasma and \(\Gamma_1/\Gamma_3\) in the cases of “equal \(\rho^e\), “equal \(m\), and “equal \(E\)” in Figs. [1] [2] and [3] respectively. They are obtained by solving Eq. (4) and inserting \(\Gamma_{34}\) into Eqs. [8], [9], and [10]. The qualitative features are the same for these three cases, although different in quantitative detail. Summing up in advance, the most important result is that a large amount of mass density of invisible plasma is required in the emission region. As the value of \(\Gamma_1/\Gamma_3\) increases, the value of \(\Gamma_{34}\) becomes larger and the lower limit on the invisible mass density \(\rho/\rho_\text{e}\) reduces. Below we discuss two extreme cases of different plasma content. One is the case of the jet with pure \(e^\pm\) pair plasma content, whilst the other is the jet made of pure \(e/p\) plasma.

For pure \(e^\pm\) pair jet, the resultant total mass density normalized by \(\rho_\text{e}\) is

\[
2 \times 10^2 < \rho/\rho_\text{e} < 2 \times 10^3 \quad (\text{equal } \rho) \\
7 \times 10^1 < \rho/\rho_\text{e} < 2 \times 10^3 \quad (\text{equal } m) \\
6 \times 10^1 < \rho/\rho_\text{e} < 2 \times 10^3 \quad (\text{equal } E). \tag{11}
\]

For the jets consisting of pure \(e^\pm\) plasma, the predicted \(\rho/\rho_\text{e}\) is constrained in a narrow range around 100-1000 as shown in Figs. [1] [2] and [3]. The number density fractions of the shock accelerated \(e^\pm\) pairs are directly obtained as \(\rho_\text{e}/\rho = n_e/(n_e^T + n_e^N) \sim 10^{-3} - 10^{-2}\). This seems a reasonable result since the number of accelerated particles is expected to be a small fraction of the thermal pool.

In the case of pure \(e/p\) content, the allowed range of \(\rho/\rho_\text{e}\) are found to be

\[
2 \times 10^2 < \rho/\rho_\text{e} < 3 \times 10^6 \quad (\text{equal } \rho), \\
7 \times 10^1 < \rho/\rho_\text{e} < 3 \times 10^6 \quad (\text{equal } m), \quad \text{and} \\
6 \times 10^1 < \rho/\rho_\text{e} < 3 \times 10^6 \quad (\text{equal } E), \tag{12}
\]

respectively. The maximum values of \(\rho/\rho_\text{e}\) are about \(m_\text{p}/m_\text{e}\) times larger than those in the case of pure \(e^\pm\) pair content.

### 5.3 Allowed range of \(e/e_\text{e}\)

As shown above, the lower and upper limit of \(\rho/\rho_\text{e}\) have been obtained in [5.2.1] and [5.2.2] respectively. By using the obtained \(\rho/\rho_\text{e}\) shown in [5.2.3] we can estimate \(e/e_\text{e}\) of Mrk 421. For the case of pure \(e^\pm\) content, since the allowed range of \(\rho/\rho_\text{e}\) is narrow, the corresponding \(e/e_\text{e}\) is also well constrained as

\[
1 < e/e_\text{e} < 7 \quad (\text{equal } \rho), \\
1 < e/e_\text{e} < 7 \quad (\text{equal } m), \quad \text{and}
\]

\[
1 < e/e_\text{e} < 7 \quad (\text{equal } E).
\]
where we employ Eq. (11) and Table 1. Thus we find that $e/e_{NT}^e \leq 2 \times 10^3 \times 1.1/310 \approx 7$. In other words, for $e^\pm$ pair content, the total kinetic power of the shocked (emission) region is less than $L_{\text{kin}}^e \approx 7\Gamma_{\text{kin},e}^N$ where $L_{\text{kin},e}^N$ is the kinetic power of NT electrons estimated as $L_{\text{kin},e}^N = 4 \times 10^{44}\text{erg s}^{-1}$ (KTK). In the case of $e/e_{NT}^e \approx 1$, the non-linear dynamical structure of the shock (e.g., Drury and Voelk 1981; Berezhko and Ellison 1999) is required for analysing the phenomena at the vicinity of the shock front. Note that the case discussed here is consistent with our choice of $\Gamma_e/\Gamma_s > 2$.

On the contrary, for pure $e/p$ content, the energetics relevant to thermal electrons and NT and thermal protons is all quite uncertain. Based on Eq. (12) and Table 1, we can derive

$$1 < e/e_{NT}^e < 1 \times 10^4 \quad \text{(equal $\rho$)}$$
$$1 < e/e_{NT}^e < 1 \times 10^4 \quad \text{(equal $m$)}$$
$$1 < e/e_{NT}^e < 1 \times 10^4 \quad \text{(equal $E$)}.$$  

For the case of maximum values of $\rho/\rho_{NT}^e$ in Eq. (12), the total kinetic power for pure $e/p$ content reaches $L_{\text{kin}}^e \sim 10^4L_{\text{kin},e}^N$ which is extremely large and unlikely.

6 ON THE SHOCK DISSIPATION RATE

In order to examine the allocation of the bulk kinetic energy of cold shells into the thermal energy, we estimate the shock dissipation rate of bulk kinetic energy of colliding cold shells. Here we denote the thermal energy of shocked shells as $E - E_0$ where $E_0$ is the rest mass energy and $E$ is the total kinetic energy which satisfies $E \propto \Gamma e_2 + \Gamma e_3$. Then the shock dissipation rate $\epsilon_{\text{diss}}$ defined as the ratio of the thermal energy of mass elements after the collision to that of bulk kinetic energy of mass elements before the collision is given by

$$\epsilon_{\text{diss}} = \frac{\Gamma_2(E - E_0)}{E_{\text{bulk}} - E_0} = \frac{\Gamma_2[(\Gamma_2 - 1)\delta m_2 + (\Gamma_3 - 1)\delta m_3]}{(\Gamma_1 - 1)\delta m_2 + (\Gamma_4 - 1)\delta m_3},$$  

where $\delta m_2$ and $\delta m_3$ are the amount of the mass of shocked regions 2 and 3, respectively. Here, $\delta m_2$ and $\delta m_3$ are expressed as $\delta m_2 = \Gamma_2\rho_2(v_{\text{PS}} - v_{\text{CD}})\delta t$ and $\delta m_3 = \Gamma_3\rho_3(v_{\text{CD}} - v_{\text{PS}})\delta t$ where $\delta t$ is the corresponding duration time in the ISM frame. Two differences between the present work and the two-point-mass collision model (e.g., Piran 1999) are that (i) we estimate $\epsilon_{\text{diss}}$ with the shock junction conditions, and (ii) we subtract the irreducible rest mass term from the denominator. Our definition is superior to the previous one when the value of relative Lorentz factor (i.e., $\Gamma_3$ and/or $\Gamma_2$) are close to order unity and/or a small Lorentz factors for cold shells. From Eq. (15) we obtain

$$0.07 < \epsilon_{\text{diss}} < 0.44 \quad \text{(equal $\rho$)}$$
$$0.09 < \epsilon_{\text{diss}} < 0.63 \quad \text{(equal $m$)}$$
$$0.06 < \epsilon_{\text{diss}} < 0.35 \quad \text{(equal $E$).}$$  

The larger (smaller) $\Gamma_e/\Gamma_s$ becomes, the larger (smaller) $\epsilon_{\text{diss}}$ realizes. As previously mentioned (Kobayashi and Sari 2001; KMY), the case for equal mass of colliding shells realizes the largest value of $\epsilon_{\text{diss}}$ and an asymmetry of each mass reduces the value of $\epsilon_{\text{diss}}$.

The fraction of $1 - \epsilon_{\text{diss}}$ of the bulk kinetic energy of the cold shells survives and transferred to a larger scale. This is responsible for the large scale structure such as radio lobes and cocoons. Therefore the comparison with the large scale kinetic power such as an extended radio emissions of blazars (e.g., Antonucci & Ulvestad 1985) will be an important future work although this is beyond the scope of this work.

7 SUMMARY AND DISCUSSION

Invisible plasma content in blazar jets such as protons and/or thermal $e^\pm$ pairs is investigated. In this work, we divide total mass and energy densities into two components, i.e., those of non-thermal electrons and those of the other invisible components. It enable us to constrain on the amount of invisible plasma in the jet. This is a significant forward step compared with previous studies.

The methodology of constraining the invisible plasma content in the emission region is as follows. The lower limit to the mass and energy densities of total plasma is limited by the definition that the mass and energy densities of total plasma should be larger than those of the non-thermal electrons. The total mass and energy densities are constrained by the internal shock dynamics. On the other hand, the upper limit of mass and energy densities for non-thermal electrons are constrained by the condition that bremsstrahlung emission from thermal electron (and positron) component should not exceed the observed SSC $\gamma$-ray emission. We can thus bracket the amount of total mass and energy densities in the emission region from below and above.

We apply this method to the archetypal TeV blazar Mrk 421 and obtain the following results.

1. Mildly relativistic shock is realized.

   By imposing the condition of the bulk Lorentz factor of the emission region as $\Gamma_e = 12$ estimated by the multi-frequency spectrum of Mrk 421 (KTK), we explore the allowed range of $\Gamma_e/\Gamma_s$ within the framework of the standard internal shock model. Adopting the conditions of $\Gamma_e, \min > 3$, $\Gamma_e, \max < 100$, and $\Gamma_e/\Gamma_s > 2$ based on the literatures (Wardle and Aarons 1997; Begelman, Rees and Sikora 1994; NP02), we find that the values of $\Gamma_e/\Gamma_s$ for Mrk 421 are limited in the ranges of $2 < \Gamma_e/\Gamma_s < 16$ (equal $\rho$), $2 < \Gamma_e/\Gamma_s < 19.5$ (equal $m$), and $2 < \Gamma_e/\Gamma_s < 11.7$ (equal $E$), respectively. As mentioned in Kirk and Duffy (1999), a very hard injection index of $s \sim 1.6$ observed in Mrk 421 well agrees with this mildly relativistic shock regime (See Fig. 3 in their paper). Hence we conclude that mildly relativistic shocks take place in Mrk 421 from the analysis of the observed spectrum and the internal shock dynamics.

2. The mass density of invisible plasma is much heavier than that of non-thermal electrons.

   Using the condition that the mass and energy densities of non-thermal electrons should be lower than those of the total ones, we derive the lower limit of total mass density at the shocked region. Since the relative Lorentz factor between the shocked and unshocked regions is expected to be a few (in Table 1), copious amount of mass density of invisible plasma is inevitably required. The upper
limit of $n_e^T$ is constrained by the condition that the luminosity
of bremsstrahlung emission should be smaller than the observed $\gamma$-ray luminosity which is well explained by the
synchrotron-self-Compton emission. Combining them, the
allowed ranges of synchrotron-self-Compton emission. Combining them, the
limited kinetic power of the emission re-
itauet al. 1998). Hence, the leptonic components in FSRQs ejecta un-
dergo stronger radiation drag effect in larger external radia-
tion fields (e.g., Sikora et al. 1999). Therefore, the leptonic com-
ponents in FSRQs ejecta undergo stronger radiation drag effect in larger external radiation
fields (Sikora and Wilson 1981; Phinney 1982; see also

Iwamoto and Takahara 2002). However, the bulk Lorentz
factors in FSRQs are comparable to or even slightly larger
than the ones in TeV blazars in spite of being subject to
much stronger radiation drag (e.g., Kubo et al. 1998; Spada
et al. 2001; Kusunose et al. 2003). In order to realize larger
kinetic powers and larger bulk Lorentz factors against the
strong radiation drag, we may take a new conjecture that a
larger baryon loading may occur for FSRQs. Summing up,
not only the strength of the external radiation field but also
the total amount and/or blending ratio of $e^\pm$ pair and $e/p$
could be new key quantities to explore the origin of the con-
tinuous blazar sequence.

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On invisible plasma content in radio-loud AGNs: The case of TeV blazar Markarian 421
Figure 1. The allowed region of the amount of mass density of total plasma normalized by that of non-thermal electrons $\rho/\rho_{\text{eNT}}$. The gray region is allowed for pure pair plasma for “equal $\rho$” case. The line denoted by $e/p$ shows the upper limit for $e/p$ plasma. Horizontal axis shows the ratio of the Lorentz factor of a rapid shell to a slow one which lies in $2 < \Gamma_r/\Gamma_s < 16.0$.

Figure 2. The allowed region of the amount of mass density of total plasma normalized by that of non-thermal electrons $\rho/\rho_{\text{eNT}}$. The gray region is allowed for pure pair plasma for “equal $m$” case. The line denoted by $e/p$ shows the upper limit for $e/p$ plasma. Horizontal axis shows the ratio of the Lorentz factor of a rapid shell to a slow one which lies in $2 < \Gamma_r/\Gamma_s < 19.5$.

Figure 3. The allowed region of the amount of mass density of total plasma normalized by that of non-thermal electrons $\rho/\rho_{\text{eNT}}$. The gray region is allowed for pure pair plasma for “equal $E$” case. The line denoted by $e/p$ shows the upper limit for $e/p$ plasma. Horizontal axis shows the ratio of the Lorentz factor of a rapid shell to a slow one which lies in $2 < \Gamma_r/\Gamma_s < 11.7$.