Non-locally Regularized Field-Antifield Quantization of the Chiral Schwinger Model

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Abstract

The non-local regularization is a powerful method to regularize theories with an action that can be decomposed in a kinetic and an interacting part. Recently it was shown how to regularize the Batalin-Vilkovisky field-antifield formalism of quantization of gauge theories with the non-local regularization. We compute precisely the anomaly of the Chiral Schwinger model with this extended non-local regularization.

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1 Introduction

The method developed by Batalin-Vilkovisky (BV)\cite{1} showed itself to be a very powerful way to quantize the most difficult field theories. A two dimensional gauge theory, the string theory, is one of these examples. For a review see\cite{2, 3, 4}.

The BV, or field-antifield, formalism provides, at lagrangian level, a general framework for the covariant path integral quantization of gauge theories. This formalism uses very interesting mathematical objects like a Poisson-like bracket (the antibracket), canonical transformations, ghosts for the BRST transformations, etc. The most important object of this method at the classical level is an equation called classical master equation (CME).

The fundamental idea of the formalism is BRST invariance. The fields $\Phi^A$, i.e., the classical fields of the theory together with the ghosts, and the auxiliary fields, have their canonically conjugated, the antifields $\Phi^*_A$. With all this elements we construct the so called BV action. At the classical level, the BV action becomes the classical action when all the antifields are put to be zero. A gauge-fixed action can be obtained by a canonical transformation. At this time we can say that the action is in a gauge-fixed basis. The other way to fix the gauge is towards a choice of a gauge fermion and making the antifields to be equal to the functional derivative of this fermion. The method can be applied to gauge theories which have an open algebra (the algebra of gauge transformations closes only on shell), to closed algebras, to gauge theories that have structure functions rather than constants (soft algebras), and to the case where the gauge transformations may or may not be independent, reducible or irreducible algebras respectively.

Zinn-Justin introduced the concept of sources of BRST-transformations\cite{5}. These sources are the antifields in the BV formalism. It was shown also that the geometry of the antifields have a natural origin\cite{6}.

At quantum level, the field-antifield formalism also works at one-loop anomalies\cite{7, 8}. There, with the addition of extra degrees of freedom, causing an extension of the original configuration space, we have a solution for a regularized quantum master equation (QME) at one-loop, that has been obtained as a part independent of the antifields in the anomaly. When the Wess-Zumino terms, which cancel the anomaly, can not be found, the theory can be said to have a genuine anomaly. Recently, a method was developed to handle with global anomalies\cite{9}, when a quantity that is conserved classically is not conserved at quantum level.

However, the solution of the QME is not easily obtained because there is a divergence when the $\Delta$ operator, a two order differential operator defined below, is applied on local functionals, a $\delta(0)$-like divergence. Therefore, a regularization method has to be used to cut the divergence in the QME. One of them is the Pauli-Villars (PV) regularization\cite{10, 11, 12}, where new fields, the PV fields, and an arbitrary mass matrix are introduced. But this method is very useful only at one-loop level. At higher orders, the PV method is still misterious. Very recently, a BPHZ renormalization\cite{13} of the BV formalism was formulated\cite{14, 15}. A dimensional regularization method in the quantum aspect of the field-antifield quantization has been studied in ref.\cite{16}.

The non-local regularization (NLR)\cite{17, 18, 19} gives a consistent way to com-
pute anomalies at higher order levels of $\hbar$. The main ideas were based in Schwinger’s proper time method \[21\]. The preliminary results \[21, 22\] were very well received. The NLR separates the original divergent loop integrals in a sum over loop contribution in such a way that the loops, now composed of a set of auxiliary fields, contain the original singularities. To regularize the original theory one has to eliminate these auxiliary fields by putting them on shell. In this way the theory is free of the quantum fluctuations. An extension of the NLR method to the BV framework has been recently formulated by J. Paris \[23\].

In this work we regularize the chiral Schwinger model (CSM), which has been constructed and completely solved by Jackiw and Rajaraman \[24\], within the context of this extended non-local BV regularization. The anomaly at one-loop is computed precisely. In section 2 a brief review of the field-antifield formalism has been made. In section 3 the original NRL is depicted. The extended non-local regularization is described in section 4. The computation of the CSM anomaly at one-loop has been calculated in section 5. The conclusions and final remarks were accomplished in section 6.

2 The Field-Antifield Formalism

Let’s construct the complete set of fields, including in this set the classical fields, the ghosts for all gauge symmetries and the auxiliary fields. This complete set will be denoted by $\Phi^A$. Now, let’s extend this space with the same number of fields, but at this time, one will define the antifields $\Phi^*_A$, which is the canonical conjugated variables with respect to the antibracket structure. This is constructed like

$$(X, Y) = \frac{\delta_Y X}{\delta \phi} \frac{\delta_Y Y}{\delta \phi^*} - (X \leftrightarrow Y),$$

where the indices $r$ and $l$ denote right and left derivation respectively.

By means of antibrackets, one can write the canonical conjugation relations

$$(\Phi^A, \Phi^*_A) = \delta^A_B, \quad (\Phi^A, \Phi^B) = (\Phi^*_A, \Phi^*_B) = 0. \quad \text{(2)}$$

The antifields $\Phi^*_A$ have opposite statistics than their conjugated fields $\Phi^A$. The antibracket is a fermionic operation so that the statistics of the antibracket $(X, Y)$ is opposite to that of $XY$. The antibracket also satisfies some graded Jacobi relations:

$$(X, (Y, Z)) + (-)^{\epsilon_X \epsilon_Y + \epsilon_X + \epsilon_Y} (Y, (X, Z)) = ((X, Y), Z). \quad \text{(3)}$$

where $\epsilon_X$ is the statistics of $X$, i.e. $\epsilon(X) = \epsilon_X$.

We define a quantity named ghost number to fields and antifields. These are integers such that

$$gh(\Phi^*) = -1 - gh(\Phi). \quad \text{(4)}$$

One can then construct an action of ghost number zero so that it is an extended action, the so called BV action, also called classical proper solution, so that

$$S(\Phi, \Phi^*) = S_0(\Phi) + \Phi^*_A R^A(\Phi) + \frac{1}{2} \Phi^*_A \Phi_B R^{AB}(\Phi) + \ldots + \frac{1}{n!} \Phi^*_A \ldots \Phi^*_n R^{A_n \ldots A_1} + \ldots \quad \text{(5)}$$
This equation contains all the algebra of the theory, the gauge invariances of the classical action ($S_{cl} = S_{BV}(\Phi^A, \Phi^{*A} = 0)$), Jacobi identities,... Gauge fixing is obtained either by a canonical transformation or by choosing a fermion $\Psi^A$ and writing

$$\Phi^* = \frac{\delta \Psi^A}{\delta \Phi^A}$$  \hspace{1cm} (6)

At quantum level the quantum action can be defined by:

$$W = S + \sum_{p=1}^{\infty} \hbar^p M_p,$$  \hspace{1cm} (7)

where the $M_i$ are the quantum corrections, the Wess-Zumino terms, to the quantum action. The expansion (7) is not the only one, but is the usual one. An expansion in $\sqrt{\hbar}$ [26] can be made. This will originate the so called backgroun charges, that is usefull in conformal field theory [27].

The quantization of the theory is made by the Green function’s generating functional:

$$Z(J, \Phi^*) = \int D\Phi \exp \frac{i}{\hbar} \left[W(\Phi, \Phi^*) + J_A \Phi^{*A}\right].$$  \hspace{1cm} (8)

But the definition of a path integral properly lacks on a regularization framework, which can be seen as a way to define the measure. Anomalies represent the non conservation of the classical symmetries at quantum level.

For a theory to be free of anomalies, the quantum action $W$ has to be a solution of the QME,

$$(W, W) = 2 i \hbar \Delta W$$  \hspace{1cm} (9)

where

$$\Delta \equiv (-1)^{A+1} \frac{\partial_r}{\partial \Phi^A} \frac{\partial_r}{\partial \Phi^{*A}}.$$  \hspace{1cm} (10)

In the equation (9) one can see that:

$$\mathcal{A} \equiv \left[\Delta W + \frac{i}{2\hbar} (W, W)\right](\Phi, \Phi^*).$$  \hspace{1cm} (11)

And computing a $\hbar$ expansion,

$$\mathcal{A} = \sum_{p=0}^{\infty} \hbar^{p-1} M_p$$  \hspace{1cm} (12)

one have the form of the $p$-loop BRST anomalies:

$$\mathcal{A}_0 = \frac{1}{2} (S, S) = 0$$  \hspace{1cm} (13)

$$\mathcal{A}_1 = \Delta S + i (M_1, S)$$  \hspace{1cm} (14)

$$\mathcal{A}_2 = \Delta M_{p-1} + i \sum_{q=1}^{p-1} (M_q, M_{p-q}) + i (M_p, S), \quad p \geq 2$$  \hspace{1cm} (15)

The first equation is the known CME. The second one is an equation for $M_1$. If the second equation does not have a solution for $M_1$ then $\mathcal{A}$ is called anomaly. The
anomaly is not uniquely determined since $M_1$ is arbitrary. The anomaly satisfy the Wess-Zumino consistency condition \cite{25}:

\[(A, S) = 0.\] \hspace{1cm} (16)

\section{The Non-local Regularization}

As we explained at the introduction, the non-local regularization can be applied only to theories which have a perturbative expansion, i.e. for actions that can be decomposed into a free and an interacting part. For much more details, including the diagramatic part, the interested reader can see the references \cite{17, 18, 19, 23}.

Let’s define an action $S(\Phi)$ where $\Phi$ is the set $\Phi^A$ of the fields, $A = 1, \ldots, N$, and with statistics $\epsilon(\Phi^A) \equiv \epsilon_A$.

\[S(\Phi) = F(\Phi) + I(\Phi),\] \hspace{1cm} (17)

$F(\Phi)$ is the kinetic part and $I(\Phi)$ is the interacting part.

Then one can write

\[F(\Phi) = \frac{1}{2} \Phi^A \mathcal{F}_{AB} \Phi^B\] \hspace{1cm} (18)

and $I(\Phi)$ is an analytic function in $\Phi^A$ around $\Phi^A = 0$. $\mathcal{F}_{AB}$ is called the kinetic operator.

We have now to introduce a cut-off or regulating parameter $\Lambda^2$. An arbitrary and invertible matrix $T_{AB}$ has to be introduced too. With the combination between $\mathcal{F}_{AB}$ and $(T^{-1})^{AC}$ we can define a second order derivative regulator:

\[R_{AB}^A = (T^{-1})^{AC} \mathcal{F}_{AB}.\] \hspace{1cm} (19)

The construction of two important operators can be made with these objects. The first is the smearing operator

\[\epsilon_B^A = \exp \left( \frac{R_{AB}^A}{2\Lambda^2} \right),\] \hspace{1cm} (20)

and the second is the shadow kinetic operator

\[\mathcal{O}_{AB}^{-1} = T_{AC}(\tilde{\mathcal{O}}^{-1})_C^B = \left( \frac{\mathcal{F}}{\epsilon^2 - 1} \right)_{AB},\] \hspace{1cm} (21)

with $(\tilde{\mathcal{O}})_B^A$ defined as

\[\tilde{\mathcal{O}}_B^A = \left( \frac{\epsilon^2 - 1}{\mathcal{R}} \right)_B^A = \int_0^1 \frac{dt}{\Lambda^2} \exp \left( t \frac{R_{AB}^A}{\Lambda^2} \right).\] \hspace{1cm} (22)

Expanding our original configuration space, for each field $\Phi^A$ an auxiliary field $\Psi^A$ has been constructed, the shadow field, with the same statistics. A new auxiliary action couple both sets of fields

\[\tilde{S}(\Phi, \Psi) = F(\tilde{\Phi}) - A(\Psi) + I(\tilde{\Phi} + \Psi).\] \hspace{1cm} (23)

\footnote{For convenience we are using the same notation of the reference \cite{23}.}
The second term of this auxiliary action is called kinetic term, and is constructed by
\[ A(\Psi) = \frac{1}{2} \Psi^A (\mathcal{O}^{-1})_{AB} \Psi^B. \] (24)

The fields \( \hat{\Phi}^A \), the smeared fields, which make part of the auxiliary action are defined by
\[ \hat{\Phi}^A \equiv (\epsilon^{-1})^A_B \Phi^B. \] (25)

It can be proved that, to eliminate the quantum fluctuations associated with the shadow fields at the path integral level one has to accomplish this by putting the auxiliary fields \( \Psi \) on shell. So, the classical shadow fields equations of motion are
\[ \frac{\partial_r \tilde{S}(\Phi, \Psi)}{\partial \Psi} = 0 \implies \Psi^A = \left( \frac{\partial_r I}{\partial \Phi^B(\Phi + \Psi)} \right) \mathcal{O}^{RA}. \] (26)

These equations can be solved in a perturbative fashion. The classical solutions \( \bar{\Psi}_0(\Phi) \) can now be substituted in the auxiliary action (23). This substitution modifies the auxiliary action so that a new action, the non-localized action appear,
\[ S_{\Lambda}(\Phi) \equiv \tilde{S}(\Phi, \bar{\Psi}_0(\Phi)). \] (27)

The action (27) can be expanded in \( \bar{\Psi}_0 \). As a result, we see the appearance of the smeared kinetic term \( F(\hat{\Phi}) \), the original interaction term \( I(\Phi) \) and an infinite series of new non-local interaction terms. But all these interaction terms are of \( O \left( \frac{1}{\Lambda^2} \right) \) and when the limit \( \Lambda^2 \rightarrow \infty \) will be made, then we will have that \( S_{\Lambda}(\Phi) \rightarrow S(\Phi) \), and the original theory is obtained. Equivalently to this limit, the same result can be acquired with the limits
\[ \epsilon \rightarrow 1, \quad \mathcal{O} \rightarrow 0, \quad \bar{\Psi}_0(\Phi) \rightarrow 0. \] (28)

With all this framework, when we introduce the smearing operator, any local quantum field theory can be made ultraviolet finite. But a question about symmetry can appear. Obviously this form of non-localization destroy any kind of gauge symmetry or its associated BRST symmetry. The final consequence is the damage of the corresponding Ward identities at the tree level.

Let’s make an analysis of what happen. If the original action (17) is invariant under the infinitesimal transformation
\[ \delta \Phi^A = R^A(\Phi) \] (29)
so, the auxiliary action is invariant under the auxiliary infinitesimal transformations
\[ \hat{\delta} \Phi^A = \left( \epsilon^2 \right)^A_B R^B(\Phi + \Psi), \]
\[ \hat{\delta} \Psi^A = \left( 1 - \epsilon^2 \right)^A_B R^B(\Phi + \Psi). \] (30)

However, the non-locally regulated action (27) is invariant under the transformation
\[ \delta_{\Lambda}(\Phi^A) = \left( \epsilon^2 \right)^A_B R^B(\Phi + \bar{\Psi}_0(\Phi)), \] (31)
remembering that \( \Psi_0(\Phi) \) are the solutions of the classical equations of motions (26).

Hence, any of the original continuous symmetries of the theory are preserved at the tree level, even the BRST transformations, and consequently, the original gauge symmetry. The reader can see [17, 18, 19] for details.

4 The Extended (BV) Non-local Regularization

As had been said before, the fundamental principle of the field-antifield formalism is BRST invariance. Therefore, it is simple to realize that the connection between the NLR method and the BV formalism is possible. Using the above construction of the NLR and the BV results, one can build a regulated BRST classical structure of a general gauge theory from the original one. Consequently, a non-locally regularized BV formalism comes out.

We are now in the BV environment. Hence, the configuration space has to be enlarged introducing the antifields \( \{ \Psi_A, \Psi^*_A \} \). Note that the shadow fields have antifields too. Then, an auxiliary proper solution, which incorporates the auxiliary action (23), corresponding to the gauge-fixed action \( S(\Phi) \), its BRST symmetry (30) and the unknown associated higher order structure functions. The auxiliary BRST transformations (30), are modified by the presence of the term \( \Phi^*_A R^A(\Phi) \) in the original proper solution. Then it can be written that the BRST transformations are

\[
\left[ \Phi^*_A (\epsilon^2)^A_B + \Psi^*_A (1 - \epsilon^2)^A_B \right] R^B (\Phi + \Psi) \tag{32}
\]

which are originated from the substitution

\[
\Phi^*_A \rightarrow \left[ \Phi^*_A (\epsilon^2)^A_B + \Psi^*_A (1 - \epsilon^2)^A_B \right] \equiv \Theta^*_A \\
R^A \rightarrow R^A (\Phi + \Psi) \equiv R^A (\Theta). \tag{33}
\]

For higher orders, the natural way would be

\[
R^{A_n...A_1}(\Phi) \rightarrow R^{A_n...A_1}(\Phi + \Psi) = R^{A_n...A_1}(\Theta) \tag{34}
\]

and an obvious ansatz for the auxiliary proper solution is

\[
\tilde{S}(\Phi, \Phi^*; \Psi, \Psi^*) = \tilde{S}(\Phi, \Psi) + \Theta^*_A R^A(\Theta) + \Theta^*_A \Theta^*_B R^{AB}(\Theta) + \\
+ \Theta^*_{A_1} \cdots \Theta^*_{A_n} R^{A_n...A_1}(\Phi) + \ldots \tag{35}
\]

It is intuitive to see that the same canonical conjugation relations, equations (2), should be obtained, i.e.

\[
\left( \Theta^A, \Theta^*_B \right) = \delta^A_B. \tag{36}
\]

Consequently, we have to construct a new set of fields and antifields \( \{ \Sigma^A, \Sigma^*_A \} \) defined by

\[
\Sigma^A = \left[ (1 - \epsilon^2)^A_B \Phi^B - (\epsilon^2)^A_B \Psi^B \right], \tag{37}
\]

7
and

$$\Sigma_A^* = \Phi_A^* - \Psi_A^*. \quad (38)$$

Now we have that the linear transformation

$$\{ \Phi^A, \Phi_A^*; \Psi^A, \Psi_A^* \} \rightarrow \{ \Theta^A, \Theta_A^*; \Sigma, \Sigma_A^* \} \quad (39)$$

is canonical in the antibracket sense. And the auxiliary action $\Omega$ is the original proper solution (5) with arguments $\{ \Theta^A, \Theta_A^* \}$.

The elimination of the auxiliaries fields in the BV method is the next step. The shadow fields have to be substituted by the solutions of their classical equations of motion. At the same time, their antifields goes to zero. In this way we can write

$$S_A(\Phi, \Phi^*) = \tilde{S}(\Phi, \Phi^*; \bar{\Psi}, \Psi^* = 0), \quad (40)$$

and the classical equations of motion are

$$\frac{\delta_r \tilde{S}(\Phi, \Phi^*; \Psi, \Psi^*)}{\delta \Psi_A} = 0 \quad (41)$$

with solutions $\bar{\Psi} = \bar{\Psi}(\Phi, \Phi^*)$, which explicitly read

$$\bar{\Psi}^A = \left[ \frac{\delta_r I}{\delta \Phi_B} (\Phi + \Psi) + \Phi^*_C (\epsilon^2)^C_D R^D_B (\Phi + \Psi) + O((\Phi^*)^2) \right] \quad (42)$$

with

$$R^A_B = \frac{\delta_r R^A (\Phi)}{\delta \Phi_B}. \quad (43)$$

The lowest order of equation (42) is,

$$\bar{\Psi}^A = \left( \frac{\delta_r I}{\delta \Phi_B} (\Phi + \Psi) \right) O^{BA} \quad (44)$$

and one can obtain an expression for $\bar{\Psi}(\Phi, \Phi^*)$ at any desired order in antifields $\mathcal{M}$.

To quantize the theory, it is necessary to add extra counterterms $M_p$ to preserve the quantum counterpart of the classical BRST scheme. It is the same as to substitute the classical action $S$ by a quantum action $W$. In the original papers $\mathcal{M}$, $\mathcal{N}$, $\mathcal{O}$ the quantization of the theory was already analyzed, but it seems that only one-loop $M_1$ corrections acquired BRST invariance. It can be proved that in the field-antifield framework, in general, two and higher order loop corrections should also be considered $\mathcal{P}$.

The complete interaction $I(\Phi, \Phi^*)$ of the original proper solution can be written as

$$I(\Phi, \Phi^*) \equiv I(\Phi) + \Phi_A^* R^A(\Phi) + \Phi^*_A \Phi^*_B R^{AB}(\Phi) + \ldots \quad (45)$$

The non-localization of this interaction part furnishes a way to regularize interactions from counterterms $M_p$. To construct the auxiliary free and interactions parts

$$\tilde{F}(\Phi + \Psi) = F(\Phi) - A(\Psi), \quad I(\Phi, \Phi^*; \Psi, \Psi^*) = I(\Theta, \Theta^*) \quad (46)$$

with $\{ \Theta, \Theta^* \}$ already known.
Now one have to put the auxiliary fields on shell and its antifields to zero, so that

\[ \begin{align*}
F_A (\Phi, \Phi^*) &= \tilde{F} (\Phi, \tilde{\Psi}_0), \\
\mathcal{I}_A (\Phi, \Phi^*) &= \tilde{\mathcal{I}} (\Phi + \tilde{\Psi}_0, \Phi^* \epsilon^2),
\end{align*} \]

then \( S_A = F_A + \mathcal{I}_A \).

The quantum action \( W \) can be expressed by

\[ W = F + \mathcal{I} + \sum_{p=1}^{\infty} \hbar M_p \equiv F + \mathcal{Y} \]

where \( \mathcal{Y} \) now is the generalized quantum interaction.

An analogous procedure of the previous section can be applied to the quantum action \( W \). We will omit all the formal steps here.

A decomposition in its divergent part and its finite part when \( \Lambda^2 \to \infty \) can be accomplished in the regulated QME.

It can be shown that the expression of the anomaly is the value of the finite part in the limit \( \Lambda^2 \to \infty \) of

\[ A = \left[ (\Delta W)_R + \frac{i}{2\hbar} (W, W) \right] (\Phi, \Phi^*) \]

and the regularized value of \( \Delta W \) defined as

\[ (\Delta W)_R \equiv \lim_{\Lambda^2 \to \infty} \Omega_0 \]

where

\[ \Omega_0 = \left[ S^A_A (\delta A)_C^B (\epsilon^2)_C^B \right]. \]

\( (\delta A)_B^A \) is defined by

\[ (\delta A)_B^A = \left( \delta B^A - O^{AC} \mathcal{I}_{CB} \right)^{-1} = \delta B^A + \sum_{n=1}^{\infty} \left( O^{AC} \mathcal{I}_{CB} \right)^n, \]

with

\[ S^A_B = \frac{\delta_r \delta_l S}{\delta \Phi^B \delta \Phi^A}, \]

\[ \mathcal{I}_{AB} = \frac{\delta_r \delta_l \mathcal{I}}{\delta \Phi^A \delta \Phi^B} \]

Applying the limit \( \Lambda^2 \to \infty \) in (50), it can be shown that

\[ (\Delta S)_R \equiv \lim_{\Lambda^2 \to \infty} [\Omega_0]_0 \]

And finally that

\[ A_0 \equiv (\Delta S)_R \]

\[ = \lim_{\Lambda^2 \to \infty} [\Omega_0]_0 \]

All the higher orders loop terms of the anomaly can be obtained from equation (49), but this will not be analyzed in this paper.
5 The Chiral Schwinger Model Extended Non-locally Regularized

The classical action for the chiral Schwinger model is

\[ S = \int d^2 x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} i \gamma^\mu \partial^\mu (1 - \gamma_5) A^\mu \psi \right], \]

which obviously has a perturbative expansion.

This action is invariant for the following gauge transformations:

\[
\begin{align*}
A^\mu(x) &\rightarrow A^\mu(x) + \partial^\mu \theta(x) \\
\psi(x) &\rightarrow \exp \left[ i e (1 - \gamma_5) \theta(x) \right] \psi(x)
\end{align*}
\]

The kinetic part of the action (56) is given by

\[ F = \int d^2 x \left[ \frac{1}{2} \bar{\psi} i \gamma^\mu \partial^\mu \psi + \frac{1}{2} \bar{\psi} i \partial^i \psi \right] \]

Integrating by parts the second term we have that

\[ F = \int d^2 x \left[ \frac{1}{2} \bar{\psi} i \gamma^\mu \partial^\mu \psi - \frac{1}{2} (i \gamma^\mu \partial^\mu \psi) \right] \]

The kinetic term has the form

\[ F = \frac{1}{2} \Psi^A F_{AB} \Psi^B \]

So,

\[ \Psi = \begin{pmatrix} \bar{\psi} \\ \psi \end{pmatrix} \]

and

\[ F = \frac{1}{2} \left( \bar{\psi} \psi \right) \begin{pmatrix} 0 & i \partial^t \\ i \partial^t & 0 \end{pmatrix} \begin{pmatrix} \bar{\psi} \\ \psi \end{pmatrix} \]

and we have that the kinetic operator \( F_{AB} \) is

\[ F_{AB} = \begin{pmatrix} 0 & i \partial^t \\ i \partial^t & 0 \end{pmatrix} \]

The regulator, a second order differential operator, is

\[ \mathcal{R}_{\beta}^\alpha = (T^{-1})^{\alpha\gamma} F_{\gamma\beta}, \]

where \( T \) is an arbitrary matrix, and one can make the following choice:

\[ \mathcal{R}^\alpha_{\beta} = -\partial^2 \]
Using the definition of the smearing operator,

\[ \epsilon^A_B = \exp \left( \frac{-\partial^2}{2A^2} \right), \]  

(67)

and the smeared fields are defined by

\[ \tilde{\Phi}^A = (\epsilon^{-1})^A_B \Phi^B \]  

(68)

In the NLR scheme the shadow kinetic operator is

\[ O^{-1}_{\alpha\beta} = \left( \frac{\mathcal{F}}{\epsilon^2 - 1} \right)_{\alpha\beta} \]  

(69)

then

\[ O = \begin{pmatrix} 0 & -iO' \rho' \\ -iO' \rho' & 0 \end{pmatrix} \]  

(70)

where

\[ O' = \frac{\epsilon^2 - 1}{\rho \rho'} \]
\[ = \int_0^1 dt \exp \left( t \frac{\rho' \rho}{\Lambda^2} \right) \]  

(71)

The interacting part of the action (56) is

\[ I[A_\mu, \psi, \bar{\psi}] = e \bar{\psi} \gamma_\mu (1 - \gamma_5) A^\mu \psi \]  

I[A_\mu, \psi + \Phi, \bar{\psi} + \bar{\Phi}] = e (\bar{\psi} + \bar{\Phi}) \gamma_\mu (1 - \gamma_5) A^\mu (\psi + \Phi) \]  

(72)

(73)

where \( \Phi \) are the shadow fields.

The BRST transformations are given by

\[ \delta A_\mu = \partial_\mu c, \]
\[ \delta \psi = i(1 - \gamma_5) \psi c, \]
\[ \delta \bar{\psi} = -i\bar{\psi}(1 + \gamma_5)c, \]
\[ \delta c = 0 \]  

(74)

Making the substitution (33) where the antifields are functions of the auxiliary fields,

\[ \psi^* \rightarrow \left[ \psi^* \epsilon^2 + \Phi^*(1 - \epsilon^2) \right] \]
\[ \bar{\psi}^* \rightarrow \left[ \bar{\psi}^* \epsilon^2 + \bar{\Phi}^*(1 - \epsilon^2) \right]. \]  

(75)

The generator of BRST transformations are

\[ R(\psi) \rightarrow R(\psi + \Phi) = i(1 - \gamma_5)(\psi + \Phi)c \]
\[ R(\bar{\psi}) \rightarrow i(\bar{\psi} + \bar{\Phi})(1 + \gamma_5)c \]
\[ R(c) = 0 \]  

(76)
We are able now to construct the non-local auxiliary proper action. It will be given in general by

\[ S_\Lambda(\Phi, \Phi^*) = \tilde{S}_\Lambda(\Phi, \Phi^*; \psi_s, \psi^* = 0) \]  

(77)

where \( \psi_s \) are the solutions of the classical equations of motion.

After an algebraic manipulation, one can write the non-localized action as

\[
\tilde{S}_\Lambda(\psi, \psi^*) = \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} + \bar{\psi} i \not{D} \psi + A_\mu^* \partial^\mu c + \frac{e i (i \not{D}) [\bar{\psi} \gamma^\mu (1 - \gamma_5) A_\mu \psi]}{i \not{D} + e \gamma^\mu (1 - \gamma_5) A_\mu (\epsilon^2 - 1)} + \frac{i \psi^* e^2 c (-i \not{D}) [(1 - \gamma_5) \psi]}{i \not{D} + e \gamma^\mu (1 - \gamma_5) A_\mu (\epsilon^2 - 1)} + \frac{i \psi^* e^2 c (i \not{D}) [\bar{\psi} (1 - \gamma_5)]}{i \not{D} + e \gamma^\mu (1 - \gamma_5) A_\mu (\epsilon^2 - 1)}. 
\]  

(78)

It can be seen easily that when one take the limit \( \epsilon^2 \to 1 \), the original proper solution of the CSM, shown below, is obtained.

The final part is the computation of the one-loop anomaly of the Chiral Schwinger model. Firstly, we have to construct some very important matrices,

\[ S_B^A = \frac{\delta_i \delta_l}{\delta \Phi^B} \frac{S_{BV}}{\delta \Phi^A} \]  

(79)

with the proper solution, the BV action, given by

\[
S_{BV} = \int d^2 x \left\{ - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} i \not{D} \psi + e \frac{\gamma^\mu (1 - \gamma_5) A_\mu \psi}{2} + A_\mu^* \partial^\mu c \right\} + i \psi^* (1 - \gamma_5) \psi c - i \bar{\psi} \psi (1 + \gamma_5) c \right\} \]  

(80)

then

\[ S_B^A = \left( \begin{array}{cc}
-ic(1 - \gamma_5) & 0 \\
0 & ic(1 + \gamma_5)
\end{array} \right) \]  

(81)

The operator \( \mathcal{I}_{AB} \) in this case is,

\[ \mathcal{I}_{AB} = \frac{\delta_i \delta_r [I(\Phi) + \Phi^*_r R^c(\Phi)]}{\delta \Phi^A \delta \Phi^B} \]  

(82)

and the result is,

\[ \mathcal{I}_{AB} = \left( \begin{array}{cc}
0 & -\frac{\gamma^\mu (1 - \gamma_5) A_\mu}{2} \\
\frac{\gamma^\mu (1 - \gamma_5) A_\mu}{2} & 0
\end{array} \right) \]  

(83)

The one-loop anomaly is given by:

\[
\mathcal{A} \equiv (\Delta S)_R \quad (\Delta S)_R = \lim_{\Lambda^2 \to \infty} [\Omega_0]_0 \]  

(84)

\[
\Omega_0 = \left[ e^2 S^A_A \right] + \left[ e^2 S^A_B O^{BC} \mathcal{I}_{CA} \right] + O \left( \frac{(\Phi^*)^2}{\Lambda^2} \right) \]  

(85)

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For the first term
\[
\epsilon^2 S_A^A = \epsilon^2 tr S_B^A = 0 \quad (87)
\]
and we have that
\[
(\Delta S)_R = \lim_{\Lambda^2 \to \infty} tr \left[ \epsilon^2 S_B^A O^{BC} I_{CA} \right] \quad (88)
\]
Using the $\gamma$ matrix representation
\[
\gamma^0 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \quad (89)
\]
\[
\gamma^1 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad (90)
\]
and
\[
\gamma^5 = -i \gamma_1 \gamma_0 \quad (91)
\]
in this representation we have that $\gamma_5^5 = \gamma_5$.

Finally, after a little algebra
\[
(\Delta S)_R = \lim_{\Lambda^2 \to \infty} tr \left[ \epsilon^2 (-ec) \frac{\epsilon^2 - 1}{\partial^2} (\partial_\mu A^\mu - \epsilon^{\mu\nu} \partial_\mu A_\nu) \right] \quad (92)
\]
But we know that
\[
\lim_{\Lambda^2 \to \infty} tr \left[ \epsilon^2 F \partial^n \frac{\epsilon^2 - 1}{\partial^2} \partial G \partial^m \right] = \quad (93)
\]
\[
= -\frac{i}{2\pi} \left[ \sum_{k=0}^{m} \binom{m}{k} \frac{(-1)^k}{n + m + 1 - k} \left( 1 - \frac{1}{2n+m+1-k} \right) \right] \int d^2 x \, F \partial^{n+m+1} G
\]
In our case
\[
n = m = 0 \quad F = 2ec \quad \partial G = \partial_\mu A^\mu - \epsilon^{\mu\nu} \partial_\mu A_\nu \quad (94)
\]
and the final result is
\[
A = (\Delta S)_R = \frac{i\epsilon}{2\pi} \int d^2 x \, c (\partial_\mu A^\mu - \epsilon^{\mu\nu} \partial_\mu A_\nu) \quad (95)
\]
which is the one-loop anomaly of the CSM.
6 Conclusions

The non-local regularization formalism is a new and a quite powerfull method to regularize theories with a perturbative expansion which have higher loop order divergences. The field-antifield framework exhibits a divergence on the application of the \( \Delta \) operator. Hence it needs a regularization. The connection between both generates an extended non-locally regularized BV quantization method. The quantization of anomalous gauge theories can be computed exactly. The one-loop anomaly of the chiral Schwinger model has been calculated.

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