Cosmic Rays

I. The cosmic ray spectrum between $10^4$ GeV and $3 \times 10^9$ GeV

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Summary. Based on a conjecture about the diffusion tensor of relativistic particles perpendicular to the magnetic field at a shock, and considering particle drifts, I develop a theory to account for the Cosmic Ray spectrum between $10^4$ GeV and $3 \times 10^9$ GeV. The essential assumption is that the free mean path perpendicular to the magnetic field is independent of energy and has the scale of the thickness of the shocked layer. I then use the basic concept, that the energetic Cosmic Ray particles are accelerated in a Supernova shock that travels down the density gradient of a stellar wind; as an example I use a Wolf Rayet star wind. Physically important ingredients beside the presence of a strong shock are diffusion, drifts, convection, adiabatic cooling, the injection history, and the topology of the magnetic field, assumed to behave similarly to the solar wind. The result is a spectrum, which for strong shocks and negligible wind speeds in a gas with adiabatic index 5/3 yields a spectrum of $E^{-7/3}$. Discussion of the latitude dependence of the acceleration leads to a knee energy which is determined by an expression of which the functional form leads to a suggestion on the physical origin of the mechanical energy of Supernova explosions, namely the gravitational potential energy mediated by the angular momentum and the magnetic field. Interstellar turbulence with a Kolmogorov spectrum then leads by losses from the galactic disk to a spectrum, which is $E^{-8/3}$ below the knee, as observed in Cosmic Rays, and as deduced from radio observations of the nonthermal emission of our Galaxy as well as that of all other well observed galaxies. At the knee the particles segregate with particle energy according to their charge, with H dropping off first, then CNO elements, then Mg, Si etc., and finally iron nuclei. Further consideration of the energy gain due to drifts at high particle energies leads to a spectrum beyond the knee. This spectrum is $E^{-29/11}$ at injection, and, corrected for diffusive transport through the Galaxy, very close to $E^{-3}$, as observed. Beyond the knee, iron and other heavy nuclei dominate out to the highest energies of galactic Cosmic Ray particles.

Key words: Acceleration of particles – Cosmic Rays – Plasmas – Supernovae: general – Shockwaves

1. Introduction

The origin of Cosmic Rays is still not completely understood. There are few well accepted arguments: a) The Cosmic Rays below about $10^4$ GeV are predominantly due to the explosion of stars into the normal interstellar medium (Lagage and Cesarsky 1983). b) The Cosmic Rays from near $10^4$ GeV up to the knee, at $5 \times 10^8$ GeV are very likely predominantly due to explosions of massive stars into their former stellar wind (Böck and Biermann, 1988). Clearly, there is some overlap between the contributions from normal Supernova explosions into the interstellar medium and explosions into a wind cavity. The consequences of this concept have been checked by calculating the Cosmic Ray abundances and comparing them with observations (Silberberg et al. 1990); the comparison suggests that up to the highest energy where abundances are known, this concept successfully explains the data. It is especially interesting, that no direct mixing from the Supernova ejecta is required to account for the known Cosmic Ray abundances, winds from red and blue supergiants as well as Wolf Rayet winds as sources are all that is needed at present in the high energy range below the knee. c) For the energies beyond the knee there is no consensus; Jokipii and Morfill (1987) argue that a galactic wind termination shock might be able to provide those particles, while Protheroe and Szabo (1992) argue for an extragalactic origin, although in either case the matching of the flux at the knee from two different source populations remains somewhat difficult. d) For the Cosmic Rays beyond the ankle at about $3 \times 10^9$ GeV an extragalactic origin is required because of the extremely large gyroradii of such particles.

Biermann (1992), Rachen (1992), Rachen and Biermann (1992) have proposed that these particles arise from hot spots in nearby radio galaxies; this hypothesis leads to a successful and nearly parameterfree explanation (Rachen 1992) of the intensity and spectrum of these particles with the important proviso that the mean free path in intergalactic space should be not much smaller than the characteristic distances between the sources and us, and may be similar to the scale of the large scale bubbles in the universe.

In all such arguments (also, e.g., Bogdan and Völk, 1983, Drury et al. 1989, Markiewicz et al. 1990) the spectrum of the Cosmic Rays remains largely unexplained. And yet the observations of the Cosmic Rays themselves, and of the nonthermal radioemission from our Galaxy as well as from all other well observed galaxies (Golla 1989) strongly suggests, that in all
galactic environments studied carefully, the Cosmic Rays have an universal spectrum of very nearly \( E^{-8/3} \) below the knee at \( 5 \times 10^6 \) GeV (electrons only < 10 GeV); direct air shower experiments show the spectrum beyond the knee to be well approximated by \( E^{-3} \) (Stanek 1992). This overall spectrum is clearly influenced by propagation effects, since particles at different energies have different chances to escape from the disk of the galaxy. It appears to be a reasonable hypothesis to approximate the interstellar diffusion coefficient proportional to \( E^{1/3} \). Such an energy dependence then requires a source spectrum of Cosmic Rays of approximately \( E^{-7/3} \) below the knee, and approximately \( E^{-8/3} \) above the knee. In this paper I propose to derive such a spectrum.

I note, that Ormes and Freier (1978) have argued already for such a source spectrum. The basic hypothesis is, again, that I consider explosions into a stellar wind with a Parker spiral topology. I remind the reader that in such a wind in the asymptotic regime, where the magnetic field decreases with the radius \( r \) as \( 1/r \) and the wind velocity is constant, the Alfvén velocity is also constant with radius.

I also note that the wind in massive stars has a similar energy integrated over the main sequence life time as the subsequent supernova explosion. The wind bubble is large and is itself surrounded by a dense shell. Hence I can expect the shock of the supernova to disperse this shell and to mix the energetic particle population produced in the shock running through the wind directly into the interstellar medium. Thus, there is no additional energy dependence introduced here to go from the spectrum which I calculate below to the injection spectrum of cosmic rays.

In parallel and following papers I will test and explore the consequences of the model proposed.

2. The interstellar diffusive transport of Cosmic Rays

There are arguments from the secondary to primary Cosmic Ray ratio that at moderate energies the leakage time from the galactic disk is about \( 10^{17} \) years and has an energy dependence which varies as \( E^{0.6 \pm 0.1} \). The higher energy data, however, suggest (Engelmann et al. 1985, 1990) that the source spectrum is close to \( E^{-2.4} \), so that the leakage time varies with an energy exponent near \( 1/3 \). Data from 1 GeV/amu to 1 TeV/amu (Swordy et al. 1990) suggest that the leakage is down by one power of ten over these three decades in particle energy, and so suggests again a leakage time energy dependence at these energies of power \( 1/3 \). The question of interest in our context is how this time scale varies with high energy and how its energy dependence can be related to the turbulence in the interstellar medium.

The turbulence in the interstellar medium is known from pulsar scintillation data and direct velocities of interstellar clouds to be remarkably close to a Kolmogorov law (Larson 1979, 1981, Rickett 1990), which would translate to an \( 1/3 \) power for the leakage time energy dependence. The high energy dependence of the diffusion coefficient and the leakage time has to fulfill the condition that the leakage time scale should not fall below the light travel time across the disk because then, obviously, no transport is possible, and, in addition, one should observe Cosmic Ray anisotropies. Such anisotropies are not observed and so, even at the highest energies of those Cosmic Rays which I believe to come from sources in the galactic disk, the leakage time has to be of order a few thousand years or longer. This condition is readily fulfilled by an energy dependence of the leakage time of an exponent of 0.4 or smaller, if the entire energy range can be covered by one powerlaw. On the other hand, if the entire energy range is not covered by one single law, then there should be a characteristic energy at which the energy dependence of the diffusion changes; such a characteristic energy in turn corresponds to a characteristic length: There is no evidence of any special length scale in the interstellar medium between the dissipation and the Larmor radius of thermal particles on the one hand, and the thickness of the hot disk on the other hand, except for the possibility of a characteristic length associated with giant molecular clouds, as discussed further below. Clearly, the simplest hypothesis is to assume that the diffusive transport is governed by basically one powerlaw; it is obvious that at low energies there are likely to be variations on this due to source structure and cross sections which almost certainly influence the secondaries energy dependence.

Plasma simulations (Matthaeus and Zhou 1989) suggest that in a plasma, where the energy density of the thermal gas and that of the magnetic field are similar, the typical turbulence spectrum is indeed Kolmogorov. Thus basing my case on 1) observations of Cosmic Rays and the interstellar medium as well as as on 2) theoretical work in plasma physics which was developed for our physical understanding of the solar wind, I assume that the leakage time has a \( 1/3 \) powerlaw over the entire energy range of interest. Then the leakage time scale even at \( 3 \times 10^9 \) GeV is much longer than the light travel time and so no anisotropy is expected. With such a concept it becomes obvious that I have to look for an explanation of a source spectrum of Cosmic Rays which is approximately \( E^{-7/3} \) below the knee and \( E^{-8/3} \) above the knee.

There is one scale in the interstellar medium which may be relevant: The size of giant molecular clouds. Giant molecular clouds are assemblies of cloudlets that aggregate in a spiral arm. With this aggregation they can trap Cosmic Ray particles through the magnetic fields, which permeate the clouds. For particles with a mean free path shorter than the size of the giant molecular clouds, trapping occurs. This limit defines a critical particle energy. This trapping leads to a production of secondaries (see also Dogiel and Sharov 1990) which constitute an additional source term over and above that of the normal interstellar medium. Here the secondaries have an injection spectrum from the giant molecular cloud steeper than the normal primaries by the energy dependence of the diffusion coefficient. Assuming a Kolmogorov law also inside the cloud, which seems well substantiated by observations (Larson 1981), this leads to a spectrum steeper by \( 1/3 \) than the standard Cosmic Ray spectrum. This population is then injected into the average interstellar medium upon dissolution of the giant molecular cloud. I emphasize that here the time dependence of the process is important, since an equilibrium is never established in this picture. Since then at these low energies these secondaries in the average interstellar medium are once more subject to diffusive leakage from the disk, their spectrum is once more steepened by another \( 1/3 \), and so below the critical particle energy defined above the secondaries are expected to show a spectrum steeper by \( 2/3 \) than the primaries.
3. Definition of the task

I consider the acceleration of particles in a shock that travels down a steady stellar wind. In this paper I will assume that the shock speed is very much larger than the wind velocity; in Paper II (Biermann and Cassinelli 1992), where those slower shocks in Wolf Rayet and OB star winds are considered which cause nonthermal radio emission through the acceleration of electrons, I will include the effects of finite wind speeds.

The acceleration of particles is governed by the standard theory (Parker 1965)

\[
\frac{\partial N}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \kappa_{rr} \frac{\partial N}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \mu} \left( (1 - \mu^2) \kappa_{\phi\phi} \frac{\partial N}{\partial \mu} \right) - (V_r + V_{d,r}) \frac{\partial N}{\partial r} + \frac{V_{d,\theta}}{r} \frac{\partial N}{\partial \theta} + \frac{V_{d,equ}}{r} \frac{\partial N}{\partial \theta} + Q
\]

(1)

where \( N \) is the particle distribution function, and \( Q \) is the source; both are functions of the coordinates radial distance \( r \), colatitude \( \theta \) or \( \mu = \cos \theta \), time \( t \) and particle energy \( E \), which I have taken to be relativistic. The terms in the above equation are first time change, then radial diffusion, latitude diffusion, radial drift, latitude drift, then compression and finally sources. The two drift terms have a different sign, because I use the cosine of the colatitude as my coordinate along \( \theta \).

The components of the diffusion tensor of interest here are the radial diffusion term \( \kappa_{rr} \) and the latitude diffusion term \( \kappa_{\phi\phi} \). The drifts are also important with the radial component \( V_{d,r} \) and the latitude component \( V_{d,\theta} \). Outside the wind acceleration region stellar winds are likely to be similar to the solar wind, and so I will assume a Parker spiral topology of the magnetic field (e.g. Jokipii et al. 1977):

\[
(B_r, B_\phi) = B_s \left( \frac{r^2}{r^*} - \frac{r^2}{r_{\text{W}}} (1 - \mu^2)^{1/2} \right).
\]

(2)

Here \( B_s \) is the surface magnetic field of the star, assumed to be radial, \( r_s \) is the surface radius of the star, and \( r_W = v_W / \Omega_s \) with \( v_W \) and \( \Omega_s \) the wind velocity and the angular rotation rate of the star. I consider only distances \( r \) much larger than either \( r_s \) or \( r_W \).

This leads then to drift velocity components of

\[
V_{d,r} = \frac{2}{3} c \frac{E}{Ze B_s} \frac{r^3_W}{r^2_{\text{W}} (1 - \mu^2)^2} \frac{r}{r^*}
\]

(3)

and

\[
V_{d,\theta} = \frac{2}{3} c \frac{E}{Ze B_s} \frac{r W}{r^2} \frac{1}{(1 - \mu^2)^{1/2}}
\]

(4)

I note that, using the expression for the magnetic field above, this can also be written as

\[
V_{d,\theta} = -\frac{2}{3} c r g / r,
\]

(5)

where \( r_g \) is the Larmor radius of the particle under consideration, and \( r_g < 0 \) and \( V_{d,\theta} > 0 \), i.e. the drift is towards the equator, for \( Z B_s > 0 \) (see also Section 10). This drift here is just due to the unperturbed structure of the stellar wind magnetic field; I will consider the consequences of additional curvature from turbulence below. At the equator I also have a radial drift in the equatorial plane (Jokipii et al. 1977) which can be written as

\[
V_{d,r,\text{equ}} = -\frac{2}{3} c \delta (\theta - \pi/2) r g / r
\]

(6)

with the same sign convention as in Eq. (5). This drift will not become important, however.

My boundary conditions are the usual: I inject particles at some low particle energy which is assumed to be independent of all relevant properties of the problem, i.e. not dependent on radial distance, magnetic field strength or latitude. The injection density depends, of course, on the density of the wind medium. Downstream I assume the flow to take particles out of the system with the normal probability 4, where Drury (1983), where \( U_2 \) is the downstream velocity relative to the shock.

I propose to derive the essential properties of the particle distribution function by analytic means, using heuristic arguments. Key will be the form of the diffusion tensor, especially the radial component \( \kappa_{rr} \).

4. Conjecture: Diffusion perpendicular to the overall magnetic field

I consider the propagation of a shock wave into a stellar wind which has the standard Parker spiral magnetic-field structure. If the radial diffusion coefficient increases linearly with \( r \) (as assumed, for example, by Völk and Biermann, see also, below) adiabatic loss time and acceleration time in a first order Fermi theory have the same radial dependence leading to the preservation of the highest energy reached by particles until the shocks runs into the stellar wind termination shell, well deep in the interstellar medium (Völk and Biermann, 1988). This can lead to much larger particle energies than in alternative pictures, such as in an explosion into a homogeneous interstellar medium in the Sedov expansion phase (Lagage and Cesarsky 1983). Thus I am faced with the difficulty considering a shock which propagates perpendicular to the magnetic field over almost all \( 4\pi \) steradians. In such a case particle drifts at the shock and in the upstream and downstream regions are important (see, e.g., Jokipii 1987).

Observations can be a guide here: The explosions into the interstellar medium are also explosions where the magnetic field is nearly perpendicular to the shock direction over most of \( 4\pi \) steradians. Radio polarization observations of supernova remnants yield clear evidence what the local structure of these shocked plasmas typically is. The observational evidence (Milne 1971, Downs and Thompson 1972, Reynolds and Chevalier 1982, Milne 1987, Dickel et al. 1988) has been summarized
by Dickel et al. (1991) in the statement that all shell type supernova remnants less than 1000 years age show dominant radial structure in their magnetic fields near their boundaries. There are several possibilities to explain this: Rayleigh-Taylor instabilities between ejected and swept up material can lead to locally radial differential motion and so produce a locally radial magnetic field (Gull 1973). It could also be due to strong radial velocity gradients of various ejecta, or due to overrun clouds that now evaporate and cool the surrounding material.

The important conclusion for us here is that observations suggest the existence of strong radial differential motions in perpendicular shocks which in turn suggest that particles get convected parallel to the shock direction. I emphasize that convective motion at a given scale entails that particle diffusion is independent of energy. I assume this convective turbulence with associated particle transport to be a diffusive process, for which I have to derive a natural velocity and a natural length scale, which can be combined to yield a diffusion coefficient. A classical prescription is the method of Prandtl (1925) whose line of argument is nicely reviewed and discussed by Stanisic (1988): In Prandtl’s argument an analogy to kinetic gas theory is used to derive a diffusion coefficient from a natural scale and a natural velocity of the system. Despite many weaknesses of this generalization Prandtl’s theory has held up remarkably well in many areas of physics far beyond the original intent. I will use a similar prescription here.

Consider the structure of a layer shocked by a Supernova explosion into a stellar wind in the case, that the adiabatic index of the gas is $5/3$ and the shock is strong. Then there is an inherent length scale in the system, namely the thickness of the shocked layer, in the spherical case for a shock velocity much larger than the wind speed and in the strong shock limit $r/4$. There is also a natural velocity scale, namely the velocity difference of the flow with respect to the two sides of the shock. Both are the smallest dominant scale, in velocity and in length; I will use the assumption that the smallest dominant scale is the relevant scale several times in the course of this paper in order to derive diffusion coefficients and other scalings.

My basic conjecture, Postulate 1, based on observational evidence, is that the convective random walk of energetic particles perpendicular to the magnetic field can be described by a diffusive process with a downstream diffusion coefficient $\kappa_{rr,2}$ which is given by the thickness of the shocked layer and the velocity difference across the shock, and is independent of energy:

$$\kappa_{rr,2} = \frac{1}{3} \frac{U_2}{U_1} r (U_1 - U_2)$$

(7)

The upstream diffusion coefficient can be derived in a similar way, but with a larger scale. I make here the second critical assumption, Postulate 2, namely that the upstream length scale is just $U_1/U_2$ times larger, and so is $r$. This, obviously, is the same ratio as the mass density and the ratio of the gyroradius of the same particle energy. Since the magnetic field is lower by a factor of $U_1/U_2$ upstream, that means that the upstream gyroradius of the maximum energy particle that could be contained in the shocked layer, is also $r$. Hence the natural scale is just $r$. And so the upstream diffusion coefficient is

$$\kappa_{rr,1} = \frac{1}{3} r (U_1 - U_2)$$

(8)

It immediately follows that the diffusive scales relative to $r$ are

$$\kappa_{rr,1} = \frac{ \kappa_{rr,2} }{ U_1 } = \frac{1}{3} \left( 1 - \frac{U_2}{U_1} \right)$$

(9)

For these diffusion coefficients, it also follows that the residence times (Drury 1983) on both sides of the shock are equal and are

$$\frac{4 \kappa_{rr,1}}{U_1 c} = \frac{4 \kappa_{rr,2}}{U_2 c} = \frac{4}{3} \frac{r}{c} \left( 1 - \frac{U_2}{U_1} \right).$$

(10)

Adiabatic losses then cannot limit the energy reached by any particle since they run directly with the acceleration time, both being independent of energy, and so the limiting size of the shocked layer limits the energy that can be reached to that where the gyroradius just equals the thickness of the shocked layer, provided the particles can reach this energy. I assume here that the average of the magnetic field $\langle B \rangle$ is not changed very much by all this convective motion, but leave the possibility open that the root mean square magnetic field $\langle B^2 \rangle^{1/2}$ is increased; this implies that a magnetic dynamo does not work as fast as the time scales given by the shock (see, e.g., Galloway and Proctor 1992 for arguments on the shortest possible dynamic time scale). This then leads to a maximum energy of

$$E_{max} = \frac{U_2}{U_1} Z e B_2 = Z e B_1$$

(11)

where $Ze$ is the particle charge and $B_{1,2}$ is the magnetic field strength on the two sides of the shock. This means, once again, that the energy reached corresponds to the maximum gyroradius the system will allow on both sides of the shock. It also means that I push the diffusive picture right up its limit where on the downstream side the diffusive scale becomes equal to the mean free path and the gyroradius of the most energetic particles.

Jokipii (1987) has derived a general condition for possible values of the diffusion coefficient: Its value has to be larger than the gyroradius multiplied by the shock speed. This condition is fulfilled here, for the maximum energy particles only by a factor of $1 - U_2/U_1 < 1$, since here the shock speed and the radial scale of the system give both the largest gyroradius as well as the diffusion coefficient.

There is an important consequence of this picture for the diffusion laterally: From the residence timescale and the velocity difference across the shock I find a distance which can be traversed in this time of

$$\frac{4}{3} \frac{r}{c} \left( 1 - \frac{U_2}{U_1} \right) (U_1 - U_2).$$

(12)

Since the convective turbulence in the radial direction also induces motion in the other two directions, with maximum velocity differences of again $U_1 - U_2$, this distance is also the typical lateral length scale. From this scale and again the residence time I can construct an upper limit to the diffusion coefficient in lateral directions of

$$\kappa_{\theta \theta, max} = \frac{4}{9} \left( 1 - \frac{U_2}{U_1} \right) \frac{U_1}{c} r c.$$

(13)
which is for strong shocks equal to

$$\kappa_{\theta, \text{max}} = \frac{1}{3} \left( \frac{3}{4} \frac{U_1}{c} \right)^2 r c.$$  \hspace{1cm} (14)

Again in the spirit of the idea, that the smallest dominant scale wins, this then will begin to dominate as soon as the $\theta$-diffusion coefficient reaches this maximum at a critical energy. As long as the $\theta$-diffusion coefficient is smaller, it will dominate particle transport in $\theta$ and the upper limit derived here is irrelevant. When the $\theta$-diffusion coefficient reaches and passes this maximum, then the particle in its drift will no longer see an increased curvature due to the convective turbulence due to averaging and the part $(1/3)$ of total for strong shocks) of drift acceleration due to increased curvature is eliminated. This then reduces the energy gain, and the spectrum becomes steeper from that energy on. The critical particle energy thus implied will be identified below with the particle energy at the knee of the observed cosmic ray spectrum.

### 5. Particle Drifts

Consider particles which are either upstream of the shock, or downstream; as long as the gyrocenter is upstream I will consider the particle to be there, and similarly downstream.

In general, the energy gain of the particles will be governed primarily by their adiabatic motion in the electric and magnetic fields. The expression for the energy gain is then (Northrop, 1963, equation 1.79), for an isotropic angular distribution

$$dE = Ze V_d \frac{U \times B}{c} + \frac{pw \partial ln B}{c} \frac{\partial t}{\partial t}$$  \hspace{1cm} (15)

where the first term arises from the drifts and the second from the induced electric field. This equation is valid in any coordinate frame. I explicitly work in the shock frame, separate the two terms above and consider the drift term first. The second term is accounted for further below, in Section 7.

The $\theta$-drift velocity in a normal stellar wind is given above, in Section 3. The $\theta$-drift can be understood as arising from the asymmetric component of the diffusion tensor, the $\theta r$-component. The natural scales there are the gyroradius and the speed of light, and so I note that for (Forman et al. 1974)

$$\kappa_{\theta r} = \frac{1}{3} r_g c,$$  \hspace{1cm} (16)

the exact limiting form derived from ensemble averaging, I obtain the drift velocity by taking the proper covariant divergence (Jokipii et al. 1977); this is not simply (spherical coordinates) the $r$-derivative of $\kappa_{\theta r}$. The general drift velocity is given by (see, e.g., Jokipii 1987)

$$V_{d, \theta} = c \frac{E}{3ze} \text{curl}_i \frac{B}{B^2}.$$  \hspace{1cm} (17)

The $\theta$-drift velocity is thus:

$$V_{d, \theta} = \frac{2}{3} c r_g/r$$  \hspace{1cm} (18)

where $r_g$ is now taken to be positive (see Eq. 5). This drift velocity is just that due to the gradient as well as the curvature, and in fact both effects contribute here equally. However, it must be remembered that there is a lot of convective turbulence which increases the curvature: The characteristic scale of the turbulence is $r/4$ for strong shocks, and thus the curvature is $4/r$ maximum. Taking half the maximum as average I obtain then for the curvature a factor of $2/r$ which is twice the curvature without any turbulence; this increases the curvature term by a factor of two thus changing its contribution from $1/3$ to $2/3$ in the numerical factor in the expression above. Hence the total drift velocity, combining now again the curvature (2/3) and gradient (1/3) terms, is thus

$$V_{d, \theta} = \frac{1}{3} (1 + \frac{U_1}{2U_2}) c r_g/r.$$  \hspace{1cm} (19)

now written for arbitrary shock strength. It is easily verified that the factor in front is unity for strong shocks where $U_1/U_2 = 4$.

The energy gain associated with such a drift is given by the product of the drift velocity, the residence time, and the electric field. Upstream this energy gain is given by

$$\Delta E_1 = \frac{4}{3} \frac{E}{c} \frac{U_1}{f_d} (1 - \frac{U_2}{U_1}),$$  \hspace{1cm} (20)

where

$$f_d = \frac{1}{3} (1 + \frac{U_1}{2U_2}).$$  \hspace{1cm} (21)

Thus, $f_d = 1$ for strong shocks. The corresponding expression downstream is

$$\Delta E_2 = \frac{4}{3} \frac{E}{c} \frac{U_2}{f_d} (1 - \frac{U_2}{U_1})$$  \hspace{1cm} (22)

giving a total energy gain of

$$\Delta E/E = \frac{4}{3} \frac{U_1}{ZeB_1} f_d (1 + \frac{U_2}{U_1})(1 - \frac{U_2}{U_1}).$$  \hspace{1cm} (23)

The drift energy gain averages over the magnetic field strength during the gyromotion. I emphasize that this energy gain is independent of this average magnetic field, so that even variations of the magnetic field strength do not change this energy gain.

It is of interest to note here, that the net distance travelled (i.e. drifted) by the particle, e.g. upstream, is given by

$$l_{\perp 1} = \frac{4}{3} \frac{E}{ZeB_1} f_d (1 - \frac{U_2}{U_1})$$  \hspace{1cm} (24)

which is the gyroradius itself for $U_2/U_1 = 1/4$, corresponding to a strong shock. This then says that one is at a gyroradius limit for the drift distance, just as in isotropic turbulence the gyroradius is a lower limit to the mean free path for particle scattering parallel to the magnetic field in a turbulent plasma, suggesting that it may be useful to think of the plasma also as maximally turbulent perpendicular to the flow and perpendicular to the magnetic field. I emphasize that during
this drift the particle makes many gyromotions. It is also important to note that the magnetic field structure in the shocked region - as discussed in Section 4 on the basis of observations - will contain local regions of opposite magnetic field and so the drift itself will be erratic and be the sum of many single element drift movements. What I have derived is the average net energy gain due to drifts, with the drift distance corresponding to the average magnetic field strength.

6. The energy gain of particles

Let us consider then one full cycle of a particle remaining near the shock and cycling back and forth from upstream to downstream and back. The energy gain just due to the Lorentz transformations in one cycle can then be written as

$$\frac{\Delta E}{E} = \frac{4}{3} \frac{U_1}{c} \left(1 - \frac{U_2}{U_1}\right).$$  \hspace{1cm} (25)

Adding the energy gain due to drifts I obtain

$$\frac{\Delta E}{E} = \frac{4}{3} \frac{U_1}{c} \left(1 - \frac{U_2}{U_1}\right)x$$  \hspace{1cm} (26)

where

$$x = 1 + \frac{1}{3} \left(1 + \frac{U_1}{2U_2}\right) \left(1 + \frac{U_2}{U_1}\right)$$  \hspace{1cm} (27)

which is $9/4$ for a strong shock when $U_1/U_2 = 4$.

Allowing for a general form of the diffusion coefficient, but keeping here $\kappa_{rr,1}/U_1 = \kappa_{rr,2}/U_2$ for simplicity, I can also write this result as

$$x = 1 + \frac{3\kappa_{rr,1}}{rU_1} \tilde{f}_d \left(1 + \frac{U_2}{U_1}\right)/\left(1 - \frac{U_2}{U_1}\right),$$  \hspace{1cm} (28)

where now the generalized factor $\tilde{f}_d$ is given by

$$\tilde{f}_d = \frac{1}{3} \left(1 + \frac{1}{6} \frac{rU_1}{\kappa_{rr,1}} \frac{U_1}{U_2} \left(1 - \frac{U_2}{U_1}\right)\right)$$  \hspace{1cm} (29)

This expression demonstrates how the effect of drifts gets smaller but does not go to zero with a smaller diffusion coefficient. On the other hand, obviously, given a geometry where the magnetic field is parallel or nearly parallel to the shock normal, the extra energy gain due to drifts no longer plays a role as, e.g., in the polar cap.

It is easy to show that the additional energy gain flattens the particle spectrum by

$$\frac{3U_2}{U_1 - U_2} \left(1 - \frac{1}{x}\right).$$  \hspace{1cm} (30)

7. Expansion and injection history

Consider how long it takes a particle to reach a certain energy:

$$\frac{dt}{dE} = \left(\frac{8\kappa_{rr,1}}{U_1c}\right) \left(\frac{4}{3} \frac{U_1}{c} \left(1 - \frac{U_2}{U_1}\right)xE\right).$$  \hspace{1cm} (31)

Here I have used that $\kappa_{rr,1}/U_1 = \kappa_{rr,2}/U_2$. Since I have

$$r = U_1 t$$  \hspace{1cm} (32)

this leads to

$$\frac{dt}{t} = \frac{dE}{E} \frac{3U_1}{U_1 - U_2} \frac{2\kappa_{rr,1}}{rU_1}$$  \hspace{1cm} (33)

and so to a dependence of

$$t(E) = t_o \left(\frac{E}{E_o}\right)^{\beta}$$  \hspace{1cm} (34)

with

$$\beta = \frac{3U_1}{U_1 - U_2} \frac{2\kappa_{rr,1}}{rU_1}$$  \hspace{1cm} (35)

which is a constant independent of $r$ and $t$.

Particles that were injected some time ago were injected at a different rate, say, proportional to $r^b$. This then leads to a correction factor for the abundance of

$$\left(\frac{E}{E_o}\right)^{-b\beta}. \hspace{1cm} (36)$$

However, in a $d$-dimensional space, particles have $r^d$ more space available to them than when they were injected, and so I have another correction factor which is

$$\left(\frac{E}{E_o}\right)^{-d\beta}. \hspace{1cm} (37)$$

The combined effect is a spectral change by

$$\frac{3U_1}{U_1 - U_2} \frac{2}{r^d + b} \frac{\kappa_{rr,1}}{rU_1}. \hspace{1cm} (38)$$

Thus I have a density correction factor, which depends on the particle energy, and so changes the spectrum. This expression can be compared with a limiting expansion derived by Drury (1983; Eq. 3.58), who also allowed for a velocity field; Drury (1983) generalized earlier work on spherical shocks by Krymskii and Petukhov (1980) and Prishchep and Ptuskin (1981). Drury's expression agrees with the more generally derived expression given here for $x = 1$. The comparison with Drury's work clarifies that for $\kappa \sim r$ the inherent time dependence drops out except, obviously, for the highest energy particles, discussed further below; the same comparison shows that the statistics of the process are properly taken into account in my simplified treatment. If the expansion is linear, as is the case here, then the $r^d$-term also describes the adiabatic losses in their effect on the spectrum, due to the general expansion of
the shock layer and thus accounts for the second term in Eq. (15) above in Section 5. Hence the total spectral difference, as compared with the plane-parallel case, is given by
\[
\frac{3U_1}{U_1 - U_2} \left( \frac{U_2}{U_1} \frac{1}{x} - 1 \right) + \frac{2}{x} (b + d) \frac{k_{rr,1}}{r U_1}.
\]  
(39)

Here I use the following sign convention: For this expression positive the spectral index of the particle distribution is steeper than without this correction; this then takes the minus sign in Eqs. (36 - 38) properly into account.

This expression Eq. (39) together with Eq. (28) constitutes the basic result of this paper; variants of this equation will be used below and in subsequent communications for different modes or sites of acceleration. For a wind I have \( b = -2 \) and \( d = -3 \), and so \( b + d = 1 \). The total spectral change is then for \( U_1/U_2 = 4 \) given by 1/3, so that the spectrum obtained is

\[
\text{Spectrum (source)} = E^{-7/3}.
\]  
(40)

This is what I wanted to derive. After correcting for leakage from the galaxy the spectrum is

\[
\text{Spectrum (earth)} = E^{-8/3}.
\]  
(41)

very close to what is observed near earth at particle energies below the knee.

Such an injection spectrum of \(-7/3\) of relativistic particles in strong and fast shocks propagating through a stellar wind leads to an unambiguous radio synchrotron emission spectrum of \(\nu^{-2/3}\) (compare, e.g., the nonthermal radio emission of OB stars, WR stars, novae, radio supernovae, and supernova remnants, which I plan to discuss in subsequent communications).

It is of interest to note, that an injection spectrum of \(-7/3\) can also lead via pp-collisions in a synchrotron dominated regime to a spectrum of pair-secondaries of \(E^{-10/3}\), which translates in the synchrotron spectrum to a \(-7/6\) flux density spectrum and a \(-13/6\) photon number spectrum, very close to that observed by GRO for the Crab pulsar (Schönfelder 1992, seminar in Bonn). Such a speculative interpretation would place the origin of the pulses in periodically excited shocks travelling down a perpendicular magnetic field configuration as considered here.

8. The maximum energy of particles

The maximum energy particles can reach is given in Section 4, and depends linearly on the magnetic field. Thus, I require estimates for the magnetic field in the stellar winds of Wolf Rayet stars and other massive stars, that explode as supernova, like red and blue supergiants. Comparing at first the corresponding estimates that Völk and Biermann (1988) used, I note that the energies implied here are larger by approximately \(c/U_1\) for the same given magnetic field strength, since their expression for the maximum energy that particles could reach contains an additional factor of approximately \(U_1/c\) as compared with my Eq. (11).

Cassinelli (1982), Maheswaran and Cassinelli (1988, 1992) have argued that Wolf Rayet stars have very much larger magnetic fields than Völk and Biermann used, in order to drive their winds. The magnetic fields given by Cassinelli and coworkers are of order a few thousand Gauss on the surface of the star. I introduce the conjecture here, discussed in more detail in Paper II (Biermann and Cassinelli 1992), that the Alfvén radius of the stellar wind is close to the stellar surface itself. Then it follows that the product \(Br\) has approximately the same value on the surface as in the wind, and is of order \(3\times10^{14}\) cm. From this number I infer a maximum energy of particles of

\[
E_{\text{max}}(\text{protons}) = 9 \times 10^7 \text{ GeV}
\]  
(42)

and

\[
E_{\text{max}}(\text{iron}) = 3 \times 10^9 \text{ GeV}.
\]  
(43)

It follows that the highest energy particles from the acceleration process discussed here are mostly iron or other heavy nuclei. The chemical composition should change abruptly to mostly protons again when the extragalactic component takes over (Rachen and Biermann 1992) somewhere near \(3\times10^9\) GeV. If this mechanism provides the largest particles energies, then obviously other contributions are not excluded, by pulsars, neutron star binaries, or even from a hypothetical termination of the galactic wind.

9. The knee in the Cosmic Ray spectrum

I wish to discuss here the bend in the spectrum of Cosmic Rays at the knee, near \(5 \times 10^6\) GeV.

Let us consider the structure of the wind through which the supernova shock is running. The maximum energy a particle can reach is proportional to \(\sin^2 \theta\), since the space available for the gyromotion from a particular latitude is limited in the direction of the pole by the axis of symmetry. Hence, clearly the maximum energy attainable is lowest near the poles. Then, consider the pole region itself, where the radial dependence of the magnetic field is \(1/r^2\), and the magnetic field is mostly radial. I can make two arguments here: Either I put the upstream diffusive scale \(\kappa_{rr,1}/(c U_1)\) equal to \(r/c\) in the strong shock limit, or I can put acceleration time and flow time equal to each other. Both arguments lead to the same result. Using here the Bohm limit in the diffusion coefficient \(\kappa_{rr,1} = 1/ZeB(r)\), since I have a shock configuration near the pole, where the direction of propagation of the shock is parallel to the magnetic field – often referred to as a parallel shock configuration, then leads to a maximum energy for the particles of

\[
E = \frac{3}{4} \frac{ZeB(r) r U_1}{c},
\]  
(44)

which is proportional to \(1/r\) near the pole, where the magnetic field is parallel to the direction of shock propagation; the corresponding gyroradius is then given by \(r_{\text{gyr}} = c/(ZeB(r))\). Putting this equal to the gyroradius of particles that are accelerated further out at some colatitude \(\theta\), where the magnetic field is nearly perpendicular to the direction of shock propagation, gives the limit where the latitude-dependent acceleration breaks down. This then gives the critical angle as
\[
\sin \theta_{\text{crit}} = \frac{3U_1}{4c},
\]
(45)

The angular range of \( \theta < \theta_{\text{crit}} \) I refer to as the polar cap below. The energy at that location is then given by

\[
E_{\text{knee}} = ZeB(r)r \left( \frac{3U_1}{4c} \right)^2.
\]
(46)

I identify this energy with the knee feature in the Cosmic Ray spectrum, since all latitudes outside the polar cap contribute the same spectrum up to this energy; from this energy to higher particle energies a smaller part of the hemisphere contributes and also, the energy gain is reduced, as argued below. This is valid in the region where the magnetic field is nearly perpendicular to the shock, and thus this knee energy is independent of radius.

All this immediately implies that the chemical composition at the knee changes so, that the gyroradius of the particles at the spectral break is the same, implying that the different nuclei break off in order of their charge \( Z \), considered as particles of a certain energy (and not as energy per nucleon).

In the polar cap the acceleration is a continuous mix between the regime where the diffusion coefficient is determined by the thickness of the shell, and the regime where it is dominated by turbulence parallel to the magnetic field; this latter regime is rather small in angular extent. Thus, \( B \) near \( r \) might be quite a bit smaller than \( 1 \) GeV. Hence the polar cap will have a spectrum which is determined by a range of

\[
0 < \frac{\zeta_{\text{rr},1}}{\dot{U}_1} < \frac{1}{3} \left( 1 - \frac{U_2}{U_1} \right),
\]
(47)
as well as by a rather reduced role for the extra energy gain due to drifts. This clearly corresponds (see Eqs. 28 and 39) to a spectral index – here again for simplicity in the strong shock regime – in the range 2 to 7/3. Thus the spectrum of the particles below the knee is likely to be flatter than \( E^{-7/3} \) at injection, or flatter than \( E^{-8/3} \) after leakage from the galaxy. The spectrum is harder in the polar cap region, because I am close to the standard parallel shock configuration, for which the particle spectrum is well approximated by \( E^{-2} \). On the other hand, the polar cap is small relative to 4\( \pi \) with about \((U_2/U_1)^2\)  and only a spectrum much flatter than \( E^{-7/3} \) like, e.g., indeed \( E^{-2} \) will make it possible for the polar cap to contribute appreciably near the knee energy, because then the spectral flux near the knee is increased relative to 1 GeV by \((E_{\text{knee}}/m_p c^2)^{1/3}\), which approximately compensates for its small area. During an episode with drift towards the poles, a larger part of the sphere can contribute for larger energy particles, and so there is an additional tendency to flatten the spectrum of the polar cap contribution. The combination of the polar cap with the rest of the stellar hemisphere might lead to a situation where up to, say, 10\( ^4 \) GeV the entire hemisphere excluding the polar cap dominates, while from 10\( ^3 \) GeV up to the knee the polar cap begins to contribute appreciably. Near the knee energy the polar caps might thus contribute equally to the rest of the 4\( \pi \) steradians. Because of spatial limitations most of the hemisphere has to dominate again above the knee, although with a fraction of the hemisphere that decreases with particle energy. The superposition of such spectra for different chemical elements will be tested elsewhere.

The expression for the particle energy at the knee also implies by the observed relative sharpness of the break of the spectrum that the actual values of the combination \( B(r)r \dot{U}_1^2 \) must be very nearly the same for all supernovae that contribute appreciably in this energy range. Please note that \( B(r)r \) is evaluated in the Parker regime, and so is related to the surface magnetic field by \( B(r)r = B_s r_s^2 \Omega_s / v_W \), where the values with index \( s \) refer to the surface of the star and \( v_W \) is the wind velocity. Thus the expression

\[
B_s r_s^2 \Omega_s / v_W \dot{U}_1^2
\]
(48)
is approximately a universal constant for all stars that explode as supernova after a Wolf-Rayet phase. It may also hold for all massive stars of lower mass that explode as supernovae.

This then implies that I have found a functional relationship for the mechanical energy of exploding stars connecting the magnetic field, the angular momentum and the ejection energy. Such a relationship could be fortuitous, since all massive stars become very similar to each other near the end of their evolution. But it could also be an indication for an underlying physical cause. Related ideas have been expressed and discussed by Kardashev (1970), Bisnovatyi-Kogan (1970), LeBlanc and Wilson (1970), Ostriker and Gunn (1971), Ammel et al. (1972), Bisnovatyi-Kogan et al. (1976), and Kundt (1976), with Bisnovatyi-Kogan (1970) the closest to the argument below. All this leads to the following interesting suggestion:

Consider a Wolf-Rayet star before it explodes as a supernova. Given my conjecture on the structure of the wind, i.e. that the Alfvén radius is near the stellar surface, it is plausible to expect that WR stars rotate not very far from critical at their surface. On the other hand, since WR stars represent the former inner convective cores of OB stars, and in fact still have convective interiors, the magnetic dynamo mechanism can be expected to operate and will produce maximum magnetic fields of order 10\( ^7 \) Gauss for maximum rotation (see Paper II). Such magnetic fields make it plausible to assume that the stars also rotate as a near-rigid body. The time scale of the transition from a Wolf-Rayet star to the final explosion is very short, and is accompanied by the formation of further chemical abundance gradients which slow any mixing. Hence it is plausible to assume that the specific angular momentum distribution is not changed anymore in the transition from the assumed solid body rotation in the Wolf-Rayet phase to the final explosion. Using the models for Wolf-Rayet stars of Langer (1989) with \( Y = 1 \), i.e. complete Helium composition, the angular momentum \( J_W \) of a Wolf-Rayet star can then be written as

\[
J_W = 5.3 \times 10^{52} \left( \frac{M_W}{5 M_\odot} \right)^{1.792} j_W 10^4 \Omega_W \text{ g cm}^2 \text{ sec}^{-1}
\]
(49)

where \( j_W \) is the correction factor for the density structure which enters the moment of inertia, and \( \Omega_W \) is the fraction of critical rotation at the surface, while \( M_W \) is the mass of the Wolf-rayet star. I have also scaled the properties from interpolating between the 5\( M_\odot \) and 20\( M_\odot \) mass models of Langer (1989) representing the lower mass range for the most abundant Wolf.
Rayet stars. The angular momentum of that part of the stars which will form a neutron star later, the innermost 1.4 \( M_\odot \) presumably, is given by

\[
J_{n,s} = 6.7 \times 10^{50} \left( \frac{M_W}{5 M_\odot} \right)^{0.066} j_{n,s} \alpha_W \text{ g cm}^2 \text{ sec}^{-1} \tag{50}
\]

where \( j_{n,s} \) is again a structural parameter for the density distribution which enters the moment of interia. When the star then implodes, the different mass shells halt their contraction when they locally reach virial equilibrium between rotation and gravitational energy. The innermost region of the Wolf Rayet star, that region destined to become a neutron star, collapses most, but also reaches a halt in collapse due to virial equilibrium; this is conceptually rather similar to the collapse most, but also reaches a halt in collapse due to virial equilibrium of the core to the outer shells. Because these outer shells are in virial equilibrium before receiving this additional energy, the additional energy will explode them. The scale of the energy is easily shown to be of order \( 10^{51} \) ergs for an initial state of rotation not far from breakup at the surface:

\[
E_b = 1.7 \times 10^{51} \left( \frac{M_W}{5 M_\odot} \right)^{0.132} \left( \frac{j_{n,s} \alpha_W}{j_{n,b}} \right)^{-2} \text{ ergs,} \tag{53}
\]

I note, that in this scenario the radius at which core collapse is halted briefly, is of order \( 10^3 \) the final neutron star radius, and so densities are still comparatively low. This radius is very close to other estimates of the final pre-collapse stellar configurations, and is here nearly independent of the initial Wolf Rayet stars mass; the structural parameters \( j_{n,s,a} \) and \( j_{n,s,b} \) enter as ratios and so also cancel to a large degree, and only the rotation parameter \( \alpha_W \) enters as a square, and hence the assumption of near critical solid body rotation for the Wolf Rayet star is the most important one in this argument (only relevant for the innermost part of the star). The final energy depends on the assumed mass of the neutron star, here 1.4 \( M_\odot \), as \( M_W^{5/3} \).

In this suggestion then, the source of the mechanical energy observable in supernova explosions is then the gravitational energy at a scale determined by angular momentum and mediated by the magnetic field. I leave a discussion how this argument may lead to a surface magnetic field strength to later.

10. The latitude distribution of the particles

Consider Eq. (1) for the derivation of the spectrum beyond the knee. Since the maximum energy particles can attain is a strong function of colatitude, the spectrum beyond the knee requires a discussion of the latitude distribution, which I have to derive first. The latitude distribution is established by the drift of particles which builds up a gradient which in turn leads to diffusion down the gradient. Hence it is clear that drifts towards the equator lead to higher particle densities near the equator, and drifts towards the poles lead to higher particle densities there. Thus the equilibrium latitude distribution is given by the balancing of the \( \theta \)-diffusion and the \( \theta \)-drift.

The diffusion tensor component \( \kappa_{\theta \theta} \) can be derived similar to my heuristic derivation of the radial diffusion term \( \kappa_{rr} \), again by using the smallest dominant scales. The characteristic velocity of particles in \( \theta \) is given by the erratic part of the drifting, corresponding to spatial elements of different magnetic field direction, and this is on average the value of the drift velocity \( |V_{d,\theta}| \), possibly modified by the locally increased values of the magnetic field strength, and the characteristic distance is the distance to the symmetry axis \( r \sin \theta \); this is the smallest dominant scale as soon as the thickness of the shocked layer is larger than the distance to the symmetry axis, i.e. \( \sin \theta < U_2/U_1 \). Thus I can write in this approximation, Postulate 3,

\[
\kappa_{\theta \theta,1} = \frac{1}{3} \left| V_{d,\theta} \right| r (1 - \mu^2)^{1/2}. \tag{54}
\]

Here \( \mu \) is again, see Eq. (1), the cosine of the colatitude on the sphere I consider for the shock in the wind. Interestingly, this can also be written in the form

\[
\frac{1}{3} r_g c (1 - \mu^2)^{1/2},
\]

where \( r_g \) is taken as positive; I also note that \( c (1 - \mu^2)^{1/2} \) is the maximum drift speed at a given latitude, valid for the local maximum particle energy. This suggests that the latitude diffusion might be usefully thought of as diffusion with a length scale of the gyroradius, and the particle speed, to within the angular factor which just cancels out the latitude dependence of the magnetic field strength in the denominator of the gyroradius.

I assume then for the colatitude dependence a powerlaw \((1 - \mu^2)^{-a}\) and first match the latitude dependence of the diffusion term and the drift, and then use the numerical coefficients to determine the exponent in this law. The diffusion term and the drift term have the same colatitude dependence since the double derivative and the internal factor of \((1 - \mu^2)\) lead to a \((1 - \mu^2)^{-a-1}\) for the diffusive term, while the drift term is just the simple derivative giving the same expression. For \((1 - \mu^2) \ll 1\) the condition then is

\[
\frac{2}{3} a^2 = \pm a.
\]

It is important to remember the sign of these terms. The diffusive term is always positive, while the \( \theta \)-drift term is negative for \( Z B_\theta \) negative. This means for positive particles and a magnetic field directed inwards the \( \theta \)-drift is towards
the pole. In that case then the exponent $a$ is either zero or $a = 3/2$. Since the drift itself clearly produces a gradient, the case with $a = 0$ is of no interest here. It follows that for positive particles and an inwardly directed magnetic field the latitude distribution is strongly biased towards the poles, emphasizing in its integral the lower energies, and thus making the overall spectrum steeper beyond the knee energy. The radial drift in this case is directed outwards, which means that particles drift ahead of the shock by a small amount only to be caught up again by the diffusive region ahead of the shock. For the magnetic field directed outwards and positive particles the radial drift is inwards, taking particles out of the system at an slightly increased rate and thus steepening the overall spectrum by a small amount.

When the magnetic field is directed outwards and the particles are positive, the drift is towards the equator with then a positive gradient with $(1 - \mu^2)^{3/2}$, again in the limit $(1 - \mu^2) \ll 1$.

I note that this exponent $3/2$ is reduced in the case, when the erratic part of the drift is increased over the steady net drift component.

11. The range of the energy gain with drifting

Consider for simplicity the case when the shock is strong, so that $U_1/U_2 = 4$.

Ignoring at first the diffusive part of the drifting I find the following: Then the combination of drift, acceleration and downstream convection is given per cycle $\Delta n = 1$

$$\Delta \mu = \pm \frac{5}{4} \frac{E}{E_{\text{max}}} \Delta n$$

while

$$\frac{\Delta E}{E} = \frac{3}{4} \frac{U_1}{c} \Delta n$$

with the losses downstream given by

$$N = N_0 (1 - \frac{U_1}{c})^n.$$  \hspace{1cm} (57)

This obviously is constructed to give the overall spectrum derived earlier of power $-7/3$.

The latitude drift can be integrated to give

$$\mu = \mu_0 \pm (E/E_o - 1)/b_E$$  \hspace{1cm} (58)

with

$$b_E = \frac{3}{5} \frac{U_1}{c} \frac{E_{\text{max}}}{E_o}.$$  \hspace{1cm} (59)

It follows that even particles that drift down from the region near the pole will not reach the maximum particle energy possible in direct drifting, but are limited to

$$E_* = \frac{3}{5} \frac{U_1}{c} E_{\text{max}},$$  \hspace{1cm} (60)

which is formally very nearly the same as the maximum energy attainable in the polar cap region, but only after allowing for the different radial behaviour of the magnetic field there. This corresponds to the potential drop between the pole and the equator and characterizes the anomalous component of the Cosmic Rays near the Sun.

However, the diffusive part of the drifting, responsible for the $\theta$ diffusion, leads to additional energy gain - independent of the strength of the magnetic field -, without a large net motion in latitude, and so it is possible for particles to go to higher energies. The purely diffusive case leads in an analogous integration to a maximum energy of

$$E_{\text{crit}} = \left( \frac{3}{4} \frac{U_1}{c} \right)^2 E_{\text{max}} = E_{\text{knee}}.$$  \hspace{1cm} (63)

I emphasize that two different basically geometric arguments lead to the same critical energy, $E_{\text{knee}}$. This means, Postulate 4, that for

$$E > E_{\text{knee}}$$

the drift energy gain is down by $2/3$ to the level what the pure gradient and curvature drift yields, Eq. (5), which results in

$$x = 11/6.$$  \hspace{1cm} (65)

The reduced drift energy gain reduces this value of $x$ below the limiting value derived earlier, of $9/4$. This then leads to an overall spectrum of

$$E^{-29/11},$$  \hspace{1cm} (66)

before taking leakage into account, and

$$E^{-98/33} \approx E^{-3},$$  \hspace{1cm} (67)

with leakage accounted for. This is what I wanted to derive.
13. Summary

In the spirit of the Prandtl mixing length theory, often found to be useful far beyond its original purpose, I make four postulates, all defining components of the diffusion tensor for energetic particles near perpendicular shocks in stellar winds:

Postulate 1 and 2 stipulate that the radial diffusion coefficient is composed from the thickness of the shocked layer and the velocity difference across the shock downstream and is larger by the ratio of velocities upstream. These diffusion tensor components are thus independent of particle energy. Using in addition the well established connection between the transverse diffusion and drifts I derive a) the spectrum of energetic particles below the knee. Using geometrical arguments I derive furthermore b) the maximum energy of particles, c) the knee energy, d) the change of the chemical composition of the energetic particle population at the knee, and e) a suggestion of the physical origin of the mechanical energy of supernova explosions of Wolf Rayet stars, possibly applicable to a larger range of massive stars with winds.

Postulate 3 stipulates that the latitude diffusion coefficient is composed from the drift speed and the distance to the symmetry axis for particle energies below the knee energy, at which the drift speeds reach the velocity difference across the shock at a critical latitude. This leads to the latitude distribution of energetic particles (there are two extreme cases depending on the orientation of the magnetic field). For particle energies above the knee the latitude drift is dominated by the lateral convective motions induced by the radial convection. This leads, Postulate 4, to an elimination of any contribution by increased curvature to the drift and thus, in the wind case, to a reduction by a factor 2/3 of the drift associated energy gain. From this postulate I derive f) the spectrum of energetic particles beyond the knee, out to the maximum particle energies.

The relevant numbers all depend on adopting the high magnetic field strengths proposed by Cassinelli et al. for Wolf Rayet stars; I suggest that these high magnetic fields are common and argue that they can be understood as the result of a dynamo working in the convection zones in the interior of massive stars (for a detailed derivation of the maximum magnetic field strength expected from dynamo theory arguments, see Paper II). I argue specifically about Wolf Rayet stars, but would like to suggest that all massive stars with winds, red and blue supergiants, undergo a similar fate, and thus may contribute the dominant part of the Cosmic Ray spectrum from about $10^4$ GeV onwards, while at lower particle energies explosions of massive stars also contribute. But it is not clear at present whether a) explosions into a homogeneous interstellar medium, b) pulsar driven explosions or c) explosions into stellar winds dominate at particle energies below $10^3$ GeV. What I have shown here is that the observed Cosmic Ray spectra also below $10^3$ GeV can be matched with the acceleration process in winds as described here. I plan to make the detailed comparison of these three sites for particle acceleration in another communication.

I note that my approach relates the spectrum of accelerated particles to the curvature of the shock, and thus, everything else being equal, might help to disentangle – or confuse – inferences about the shock from spectral indices observed in nonthermal radio emission.

It is clear that the analytic and heuristic arguments made here can only approximate the complexities inherent in particle acceleration in perpendicular shocks, in which the direction of shock propagation is perpendicular to the magnetic field; on the other hand, before a fully developed numerical treatment of the same processes might become possible on highly parallel machines or with newly developed sophisticated codes, I hope that analytic and semianalytic progress to refine the theory and its concepts presented here will be possible.

In this first paper of a series on the process of Cosmic Ray acceleration I have introduced a basic conjecture (Section 4) on the diffusion of particles in a shock perpendicular to the magnetic field. In a similar vein I have introduced a number of heuristic arguments which require testing against observations. I plan to test all implications of the model proposed here against available observations. I will further explore the consequences of this concept in a following paper (with J.P. Cassinelli) on the nonthermal radioemission of Wolf-Rayet stars and demonstrate that my concept can produce the proper radio spectral indices, luminosities and temporal behaviour. In further communications I plan to apply this model to explosions into the homogeneous interstellar medium, to the radioemission of novae, and I will test in detail the predictions of this model as regards the chemical abundances of Cosmic Rays. What I have shown here, is that the entire Cosmic Ray spectrum from $10^4$ GeV out to $3 \times 10^9$ GeV, with the knee energy and the chemical abundances, can be understood with the same physical ingredients already well tested in the solar wind shock, namely a strong shock, diffusion, drifts, convection, adiabatic cooling, injection history and the familiar topology of the magnetic field in a realistic Parker spiral. The main requirements on the magnetic fields are that they as strong as already implied by independent arguments from Wolf Rayet star wind models, that help drive the wind with the rotating magnetic field.

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