Studying of the stability of the flow rates wells unto errors permeability field identification under conditions of single-phase stationary fluid filtration

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Abstract. The problem of calculating the producing wells flow rates in a two-dimensional reservoir is considered under conditions of stationary single-phase fluid filtration. The permeability field of the reservoir is assumed to be unknown and is determined from the solution of the inverse coefficient problem. The identification of the permeability field is carried by the known values of bottomhole pressure and is reduced to the problem of minimizing the residual function.

1. Introduction
To solve the prediction problems arising in the development of hydrocarbon field requires knowledge of the reservoir parameters, in particular, the permeability field. In this paper the model problems of the permeability field identification of a two-dimensional reservoir by the known values of bottomhole pressure in a single-phase stationary fluid filtration are considered. The permeability field is determined from the solution of the inverse coefficient problem, which is reduced to the problem of the residual function minimization [1]-[4]. The residual function is the sum of the squared differences between the calculated and known bottomhole pressure values. The minimization of residual function is carried by Levenberg-Marquardt method. At solving the inverse problem the permeability field is searched as a spline function. The coordinates of interpolation nodes are coincided with the coordinates wells. The calculated permeability field is used to calculate the producing wells flow rates when the flow rates of injection wells change and when adding new injection wells. A comparison is made with the results obtained using the true permeability field and the stability to identification errors is estimated. The calculation results are also influenced by the bottomhole pressure measurement errors. To study this effect, random errors are introduced into the bottomhole pressure measurements.

2. Problem of the permeability field identification
In a two-dimensional reservoir, a single-phase stationary fluid filtration, obeying the Darcy law, is described by the equation [5]

\[ \nabla \left( \frac{k_h}{\mu} \nabla p \right) = \sum_{i=1}^{M} Q_i \delta (x_i, y_i), \tag{1} \]
where \( k = k(x, y) \) is the coefficient of permeability, \( h = h(x, y) \) is the thickness of reservoir, \( \mu \) is the coefficient of fluid viscosity, \( p \) is the pressure, \( Q_i, (x_i, y_i) \) are the flow rate and coordinates of \( i \)-th wells, \( \delta(x_i, y_i) \) is the delta function, \( M \) is the number of wells. For the equation (1) the following initial conditions is assigned

\[
p |_{\Gamma} = p_{\Gamma},
\]

where \( \Gamma \) is the boundary of reservoir \( \Omega \).

To determine the pressure field from the system (1)-(2), the coefficients of equation (1) must be known. In this paper \( h = h(x, y) \) and \( \mu \) are considered known, and the permeability coefficient \( k(x, y) \) is considered unknown. To determine it, the inverse coefficient problem is solved, which is reduced to the problem of the residual function minimization. The residual function is constructed on a known bottomhole pressure measurements in the wells \( p_{wi}^*(i = 1, M) \) and has the form

\[
J(k) = \frac{1}{2} \sum_{i=1}^{M} (p_{wi} - p_{wi}^*)^2,
\]

where \( p_{wi} \) are the bottomhole pressure values obtained as a result of the solution equation (1). The Levenberg-Marquardt method is used for the residual function minimization [6]. Stopping of the minimization process is carried out according to two criteria: achieving of the given accuracy of bottomhole pressure measurements \( \max_{j=1,M} |p_{wi} - p_{wi}^*| < 0.01 \) MPa or slow convergence of minimization process \( J^n - J^{n+1} < 0.01 J^n \) during 3 iterations, where \( n \) is the iteration number.

In this paper the permeability field parameterization is used in the form of a spline function [7]

\[
k(x, y) = \sum_{i=1}^{N} c_i r_i^2 \ln r_i^2 + c_{N+1} + c_{N+2} x + c_{N+3} y,
\]

where \( r_i^2 = (x - x_i)^2 + (y - y_i)^2 \), \( (x_i, y_i) \) are the coordinates of interpolation nodes. The number of interpolation nodes \( N \) is corresponded with number of identified parameters. The coefficients \( c_i, i = 1, N + 3 \), are determined from the solution of the equations system

\[
k(x_i, y_i) = k_i, i = 1, N,
\]

\[
\sum_{i=1}^{N} c_i = 0, \sum_{i=1}^{N} c_i x_i = 0, \sum_{i=1}^{N} c_i y_i = 0,
\]

where \( k_i \) are the permeability values in the interpolation nodes. The using of the spline function avoids the construction of an additional grid for calculating permeability values. Interpolation nodes can be placed arbitrarily and for a single solution to exist, it is sufficient that at least three interpolation nodes do not lie on the same line. In [8, 9] two approaches to the choice of interpolation nodes were used. In this paper the coefficients of interpolation nodes are coincided with the wells coordinates.

### 3. Model tasks

The reservoir \( \Omega \) (5000 m \( \times \) 8000 m, thickness of reservoir is 10 m) has the impermeable faults (figure 1). The reservoir \( \Omega \) opens up 12 injection and 123 production wells (figure 1). The flow rate of 100 m\(^3\)/day is assigned at injection wells, at production wells flow rate varies from 1.7 m\(^3\)/day up to 350 m\(^3\)/day. The radius of wells is 0.1 m. At the boundary of \( \Omega \) is assigned pressure of 20 MPa. The coefficient of fluid viscosity is 10 mPa-c. The model problem of the permeability field identification is constructed as follows. The reservoir \( \Omega \) is covered with a square grid (5 \( \times \) 9). In the nodes of this grid the permeability values are assigned from 0.1 mkm\(^2\) up to 10 mkm\(^2\). The true permeability field \( k(x, y) \) is constructed by kriging method.
using these values (figure 2). Then the pressure field $p^{tr}$ and the bottomhole pressure values $p^*_{wi}, i = 1, M$ are calculated from the solution of the equations system (1)-(2) for the field $k^{tr}$.

The equations system (1)-(2) is solved numerically. For approximation by spatial variables of the equation (1) the method of control volumes is used. The reservoir $\Omega$ is covered a square grid with a step of 50 m ($L = 16000$ is number of control volumes). The resulting system of linear algebraic equations is solved by the method of conjugate gradients in the form approximate Cholesky factorization [10, 11]. The values of bottomhole pressure are calculated by using Peaceman formula [12].

Further the permeability field $k(x, y)$ is considered unknown and is restored as a spline function by known bottomhole pressure values. In real bottomhole pressure measurements there is always some error, so the bottomhole pressure measurements $p^*_{wi}$ are randomly entered errors, and their influence on the results of calculations are investigated. For comparison of the calculated permeability fields $k^c(x, y)$ and pressure $p(x, y)$ with true fields are given values of the deviations $\Delta K_{av} = \sqrt{\frac{1}{L} \sum_{i=1}^{L} (k^{tr}_i - k^c_i)^2}$ and $\Delta r_{av} = \sqrt{\frac{1}{L} \sum_{i=1}^{L} (p^{tr}_i - p^*_i)^2}$, the maximum deviations $\Delta K_{max} = \max_{i=1,L} |k^{tr}_i - k^c_i|$ and $r_{max} = \max_{i=1,L} |p_i - p^*_i|$ in all control volumes of the grid, the maximum residuals for wells $r_{well_{max}} = \max_{j=1,M} |p_{wj} - p^*_{wj}|$. The results obtained without errors in bottomhole pressure measurements and with six variants errors are shown in table 1. The solutions of all tasks are obtained with the given accuracy on the pressure. The permeability fields obtained without errors in bottomhole pressure measurements and with -0.1 MPa $\leq \varepsilon_i \leq 0.1$ MPa, 0 MPa $\leq \varepsilon_i \leq 0.1$ MPa, -0.1 MPa $\leq \varepsilon_i \leq 0$ MPa errors are shown in figures 3-6.
**Table 1.** Comparison the calculated permeability fields with the true permeability field.

| Errors on Measurements | $\Delta K_{\text{av}}$ [mkm$^2$] | $\Delta K_{\text{max}}$ [mkm$^2$] | $r_{\text{av}}$ [MPa] | $r_{\text{max}}$ [MPa] | $r_{\text{well max}}$ [MPa] |
|-------------------------|---------------------------------|-------------------------------|------------------|------------------|------------------|
| Without errors          | 0.1932                          | 1.7412                        | 0.0379           | 0.3501           | 0.0047           |
| 0 MPa $\leq \varepsilon_i \leq$ 0.1 MPa | 0.8391                          | 10.262                        | 0.04364          | 0.3165           | 0.0096           |
| 0 MPa $\leq \varepsilon_i \leq$ 0.01 MPa     | 0.1998                          | 1.7228                        | 0.0376           | 0.3425           | 0.0048           |
| 0 MPa $\leq \varepsilon_i \leq$ 0.001 MPa    | 0.2150                          | 3.3940                        | 0.0611           | 0.3291           | 0.0087           |
| 0 MPa $\leq \varepsilon_i \leq$ 0 MPa        | 0.1952                          | 1.7652                        | 0.0389           | 0.3478           | 0.0048           |
| -0.01 MPa $\leq \varepsilon_i \leq$ 0 MPa    | 0.3556                          | 2.8233                        | 0.0426           | 0.3021           | 0.0086           |
| -0.01 MPa $\leq \varepsilon_i \leq$ 0 MPa    | 0.1945                          | 1.6989                        | 0.03684          | 0.3448           | 0.0047           |
| -0.1 MPa $\leq \varepsilon_i \leq$ 0 MPa     | 0.3556                          | 2.8233                        | 0.0426           | 0.3021           | 0.0086           |

**Figure 5.** The permeability field $k_c$, with errors 0 MPa $\leq \varepsilon_i \leq$ 0.1 MPa.

**Figure 6.** The permeability field $k_c$, with errors -0.1 MPa $\leq \varepsilon_i \leq$ 0 MPa.

Further, the calculated permeability fields are used to calculate the flow rates of production wells when changing the flow rates of injection wells and when adding new injection wells. At solution of these tasks are assigned the flow rates for the injection wells and the bottomhole pressures in production wells. The bottomhole pressures of production wells were taken to be the same in all variants of the task. Three variants of the debits calculation task are considered.

(i) On all injection wells the flow rate is increased to 150 m$^3$/day (instead of 100 m$^3$/day, as it was in the initial task).

(ii) The nine production wells were transferred to injection wells, all injection wells have a flow rate of 100 m$^3$/day, a total of 21 injection wells (figure 7).

(iii) One production well is transferred to the injection well, on all injection wells the flow rate is 100 m$^3$/day, the location of the new injection well is determined from the condition of the maximum total flow rate of all production wells.

All tasks are solved with a true permeability field and with calculated permeability fields. The found location of the new injection well for the third variant of the task is shown in figure 8 (△ - for the permeability field in case -0.1 MPa $\leq \varepsilon_i \leq$ 0.1 MPa measurements error, ○ - for the true permeability field and for other variants of the calculated permeability fields). To compare the results the total flow rate of all production wells is given in table 2. The flow rates of production wells for first variant of task are shown in figure 9.
Figure 7. The location of wells and faults, (■ - production wells, × - injection wells).

Figure 8. The location of wells and faults, (■ - production wells, × - injection wells).

Table 2. The total flow rate, m³/day.

| Variant | Variant 1 | Variant 2 | Variant 3 |
|---------|-----------|-----------|-----------|
| True    | 8653.23   | 8815.53   | 8330.20   |
| Without errors | 8650.65 | 8810.87   | 8330.01   |
| -0.1 MPa ≤ ε_i ≤ 0.1 MPa | 8653.51 | 8814.42   | 8331.89   |
| -0.01 MPa ≤ ε_i ≤ 0.01 MPa | 8649.92 | 8810.20   | 8329.21   |
| 0 MPa ≤ ε_i ≤ 0.1 MPa | 8801.40 | 8960.38   | 8481.06   |
| 0 MPa ≤ ε_i ≤ 0.01 MPa | 8665.18 | 8825.28   | 8344.58   |
| -0.1 MPa ≤ ε_i ≤ 0 MPa | 8504.26 | 8666.24   | 8182.50   |
| -0.01 MPa ≤ ε_i ≤ 0 MPa | 8635.40 | 8795.81   | 8314.66   |

Figure 9. Projected flow rates of production wells are in m³/day.

4. Conclusions
On model tasks the problem stability of calculation of producing wells flow rates is studied in the presence of error permeability field identification. The permeability field is determined
in the form of a spline function from the solution of the inverse coefficient problem by known bottomhole pressure measurements. In the identification of permeability field in the bottomhole pressure measurements is entered different error. The results show a good agreement between the predicted production wells flow rates calculated from the true permeability field and the identified permeability fields.

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