Lepton flavor violating decays of mesons to lepton-pairs in a gauge group $SU_L(2) \times U_Y(1) \times SU_X(2)$

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Abstract: The lepton flavor conserving and lepton flavor violating decays of $K$ and $B$ mesons to lepton pairs in the gauge group $G = SU_L(2) \times U_Y(1) \times SU_X(2)$ are discussed. The quark-lepton transitions mediated by the lepto-quark bosons $X_{\mu}^{3/2}$ of the group $SU_X(2)$ provide a framework to construct an effective Hamiltonian for these decays. The effective coupling constant $\frac{G_X}{\sqrt{2}} = \frac{g_X^2}{m_X^2}$, $m_X$ is the mass at which the group $G$ is broken to the SM group. The upper bound on $G_X/G_F$ is obtained from the most stringent experimental limit on the $B.R(K^0_L \rightarrow \mu^+\mu^-)$. Several cases of pairing three generations of leptons and quarks in the representation $(2,2)$ of the group are analyzed. For some pairing, the upper bound on $(G_X/G_F)$ is of the order $(6 - 9) \times 10^{-6}$ and is compatible with the upper limits on various LF violating $K$-decays. In particular for these cases, we find the upper limit on the branching ratio $B.R(K^0_L \rightarrow \mu^+\mu^-) \sim (1.9 - 8.3) \times 10^{-9}$. It is shown that for LF violating $B$-decays to lepton pairs, the time integrated decay rate $B^0_{u,s} \rightarrow \ell^-\ell^+(\tau^-\ell^+)$ is a promising area to test the model.
1 Introduction

In weak decays, mediated by charged vector bosons $W^\pm$, the hadronic flavor changes but lepton flavor is conserved. The relative strength of weak decays in hadronic sector is determined by CKM matrix. However, the weak neutral current is flavor conserving. The flavor changing neutral current (FCNC) induced weak decays are not allowed at tree level. These decays are highly suppressed and occur through loop diagrams in the standard model. FCNC induced weak decays are lepton flavor conserving in the standard model; there are stringent experimental limits on the lepton flavor changing (LFC) decays.

The electromagnetic unification scale is 260 Gev. The Higgs mass cannot be predicted in the electroweak theory spontaneously broken by the scalar fields. The discovery of a new particle 125 Gev, most likely to be Higgs boson has implication for the "hierarchy problem". The "Higgs mechanism" of the electroweak set the scale for all known particles. The "hierarchy problem" is why it is so small compared to another mass, the Planck mass ($10^{19}$ Gev) which is the fundamental unit of mass in theory of gravitation.

It is of interest to explore the intermediate mass scales, between the Higgs mass and the Planck mass. One way to do it is to extend the electroweak group to a higher group \cite{1}; the extra vector boson of the extended group provide the intermediate mass scales.

In this paper, we consider extension of the group $SU_L(2) \times U_Y(1)$ to $SU_L(2) \times U_{Y_1}(1) \times SU_X(2)$. The three generations of left-handed fermions

$\begin{pmatrix}
  u_i \\
  d_i \\
  e_i
\end{pmatrix}_{L}$, \quad $i = e, \mu, \tau (\nu_e, \nu_\mu, \nu_\tau)$

$= d, s, b (u, c, t)$

($i$: is the generation index; the color index is suppressed) are assigned to the representation

$(2, 2)_{Y_1}: \quad Y_1 = 0$ for leptons

$= -2/3$ for quarks

Note that $I_X = 1/2, -1/2$ for the quarks and lepton respectively. The charge $Q$ is given by

$Q = \frac{1}{2}(\tau_L + \tau_X + Y_1) = \frac{1}{2}(\tau_L + Y)$
where

\[ Y = \tau X + Y_1 \]

\[ = -1 \text{ for leptons} \]
\[ = 1/3 \text{ for quarks} \]

as in the standard model. The right-handed fermions are singlet:

\[(1, 1) : Y = Y_1\]
\[= 4/3 \text{ for up quarks} \]
\[= -2/3 \text{ for down quarks} \]
\[= -2 \text{ for charged leptons} \]
\[= 0 \text{ for neutrinos} \]

The left-right symmetric gauge group \(SU_L(2) \times SU_R(2) \times U_Y(1)\) likewise can be extended.

\[
\begin{pmatrix} u_i & \nu_i \\ d'_j & e_i \end{pmatrix}_L : (2, 1, \bar{2}) Y_1
\]
\[
\begin{pmatrix} u_i & \nu_i \\ d'_j & e_i \end{pmatrix}_R : (1, 2, \bar{2}) Y_1
\]

Note that

\[
\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V \begin{pmatrix} d \\ s \\ b \end{pmatrix}
\]

where \(V\) is the usual CKM matrix.

A salient feature of the model is quark-lepton transitions mediated by the lepto-quarks \(X_{\mu}^{-2/3}, X_{\mu}^{2/3}\) carrying baryon and lepton numbers \((1/3, -1), (-1/3, 1)\) respectively. These transitions generate the flavor changing current, containing both lepton flavor conserving and lepton flavor violating.

We first consider the group \(SU_L(2) \times U_Y(1) \times SU_X(2)\); the left-right symmetric model is discussed in the last section.

2 Interaction Lagrangian

It is straightforward to write the Lagrangian. The interaction Lagrangian is given by

\[
L_{\text{int}} = -g \sin \theta_W J_{\text{em}}^\mu \epsilon^\mu A_\mu - \frac{g}{2\sqrt{2}} \left[ \tilde{\nu}_i \Gamma^\mu_{L} \nu_i + \bar{u}_i \Gamma^\mu_{L} d'_i \right] W^+ \mu + h.c
\]
\[
- \frac{g}{\cos \theta_W} J_{Z\mu} Z_\mu - \frac{g}{\sqrt{1 - g^2/g_X^2}} J_{Z\mu} Z'_\mu
\]
\[
- \frac{g}{2\sqrt{2}} \left\{ \sum_i \sum_j \left[ \bar{\nu}_i (DU)^{\dagger}_{ij} \Gamma^\mu_{L} u_j + \bar{e}_i (DU)^{\dagger}_{ij} \Gamma^\mu_{L} d'_j \right] X_{\mu}^{-2/3} + h.c \right\}
\]

where \(\Gamma^\mu_{L} = \gamma^\mu (1 - \gamma^5), J_{Z\mu}\) is the weak neutral current coupled to \(Z_\mu\) and

\[
J_{Z\mu} = \frac{1}{4} \left[ (\bar{u}_i \Gamma^\mu_{L} u_i + \bar{d}_i \Gamma^\mu_{L} d_i - \bar{e}_i \Gamma^\mu_{L} e_i - \bar{\nu}_i \Gamma^\mu_{L} \nu_i)
\]
\[- \frac{g^2}{g_X^2} \left( 4 J_{\text{em}}^\mu - (\bar{u}_i \Gamma^\mu_{L} u_i - \bar{d}_i \Gamma^\mu_{L} d_i + \bar{\nu}_i \Gamma^\mu_{L} \nu_i - \bar{e}_i \Gamma^\mu_{L} e_i) \right) \]

(2.2)
If we take \( g_X = g \),

\[
\frac{g_X}{\sqrt{1 - g^2/g_X^2}} = \frac{g}{\sqrt{1 - \tan^2 \theta_W}} \tag{2.3}
\]

\[
J^{Z'} = \tan^2 \theta_W J^Z - \frac{1}{4} \left[ 4 \sin^2 \theta_W J^\mu_{em} - (\bar{u}_i \Gamma^\mu_L u_i + \bar{d}_i \Gamma^\mu_L d_i) + (\bar{\nu}_i \Gamma^\mu_L \nu_i + \bar{e}_i \Gamma^\mu_L e_i) \right] \tag{2.4}
\]

The matrix \( D \) in the last term of Eq. (2.1) is the 3 \times 3 matrix associated with the permutation group \( S_3 \):

\[
D(e) = D(123) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\]

\[
D(13) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad D(23) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}
\]

\[
D(231) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad D(12) = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.5}
\]

\( D(13) \) interchanges first and third generation, \( D(23) \) interchanges second and third generation, \( D(12) \) interchanges first and second generation and \( D(231) \) interchange \((123) \rightarrow (231)\) of leptons respectively in the representation \((2, \bar{2})\). \( U \) in Eq. (2.1) is a 3 \times 3 unitary matrix for lepton mixing. In introducing these matrices, we have utilized the freedom of permutation and mixing of three generations of leptons in the representation \((2, \bar{2})\). For \( D = D(e) \) and \( U = 1 \), we have the standard assignment \( \text{viz.} \):

\[
\begin{pmatrix} u \nu_e \\ d \tau e \end{pmatrix}_L , \quad \begin{pmatrix} c \nu_\mu \\ s t \mu \end{pmatrix}_L , \quad \begin{pmatrix} t \nu_\tau \\ b \tau \end{pmatrix}_L
\]

The physical neutral vector bosons \( A_\mu, Z_\mu \) and \( Z'_\mu \) are related to neutral vector bosons \( W_{3\mu}, B_{1\mu} \) and \( X_{3\mu} \) of the extended group as follows

\[
W_{3\mu} = \frac{1}{\sqrt{g^2 + g'^2}} (g' A_\mu + g Z_\mu)
\]

\[
X_{3\mu} = \frac{1}{\sqrt{g_1^2 + g_X^2}} (g_1 B_\mu + g_X Z'_\mu)
\]

\[
B_{1\mu} = \frac{1}{\sqrt{g_1^2 + g_X^2}} (g_X B_\mu - g_1 Z'_\mu) \tag{2.6a}
\]

where vector boson

\[
B_\mu = \frac{1}{\sqrt{g^2 + g'^2}} (g A_\mu - g' Z_\mu) \tag{2.6b}
\]

is coupled to the Abelian group \( U_Y(1) \) of the standard model. The gauge coupling \( g' \) of \( U_Y(1) \) and the electromagnetic coupling are related to the gauge couplings \( g, g_1 \) and \( g_X \) as follows

\[
\frac{1}{g'^2} = \frac{1}{g_1^2} + \frac{1}{g_X^2}
\]

\[
\frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2} \tag{2.7}
\]
The gauge group $SU_L(2) \times U_Y(1) \times SU_X(2)$ is spontaneously broken to the group $SU_L(2) \times U_Y(1)$ by introducing a scalar doublet $\eta$ of $SU_X(2)$:

$$\eta = (\eta^{2/3}, \eta^0); \quad Y_1 = 1/3 \left(1 - \frac{2}{3}\right), 1$$

The relevant term in the Lagrangian of the scalar doublet $\eta$:

$$\left[ \partial^\mu \eta + \frac{i}{2} g_X \eta \bar{\tau}_X^\mu \cdot \vec{X} \right] + \frac{i}{2} g_Y \eta B'^\mu - \frac{i}{2} g_Y \eta^{2/3} B'^1_1$$

$$\times \left[ \partial^\mu \bar{\eta} - \frac{i}{2} g_X \bar{\tau}_X^\mu \cdot \vec{X} \bar{\eta} - \frac{i}{2} g_Y B_1 \bar{\eta} + \frac{i}{2} g_Y B_1 \eta^{-2/3} \right]$$

Now $\eta$ can be put in the form:

$$\eta = (\eta^{2/3}, \eta^0) = \left(\eta^{2/3}, \frac{\sqrt{2} \eta^0 + \sqrt{2} \eta^0}{\sqrt{2}}\right)$$

by spontaneous symmetry breaking the group $SU_L(2) \times U_Y(1) \times SU_X(2)$ to the standard model group $SU_L(2) \times U_Y(1)$. Note that the scalars $\eta^{2/3}$ and $h_X$ have been absorbed to give masses to $X_{\mu}^{\pm 2/3}$ and $Z'$ respectively. The vector bosons mass term is given by

$$- \frac{1}{4} \left(0, \frac{\sqrt{2} \eta^0 + \sqrt{2} \eta^0}{\sqrt{2}}\right) \left(g_X \bar{\tau}_X^\mu \cdot \vec{X} \eta + g_Y B'^\mu \eta\right) \left(0, \frac{\sqrt{2} \eta^0 + \sqrt{2} \eta^0}{\sqrt{2}}\right)$$

$$= - \frac{1}{4} \left(\frac{\sqrt{2} \eta^0 + \sqrt{2} \eta^0}{\sqrt{2}}\right)^2 \left[2g_X^2 X_{\mu}^{2/3} X_{\mu}^{-2/3} + (g_Y^2 + g_X^2) Z'' Z'_\mu\right]$$

Hence we have

$$m_X^2 = \frac{1}{4} g_X^2 \eta^0$$

$$m_{Z'}^2 = \frac{1}{4} (g_Y^2 + g_X^2) \eta^0 = \frac{1}{4} \frac{g_X^2 \eta^2}{1 - g_Y^2 / g_X^2}$$

$$= \frac{m_X^2}{1 - g_Y^2 / g_X^2}$$

For $g_X = g$,

$$m_X^2 = \frac{1}{4} g^2 \eta^0 = m_X^2 \frac{\eta^0}{\eta^0}$$

$$m_{Z'}^2 = \frac{m_X^2}{(1 - \tan^2 \theta_W)}$$

By the symmetry breaking mechanism discussed above, the standard model is decoupled from the extended group at a mass scale $m_X$. The vector bosons $X_{\mu}^{\pm 2/3}$ and $Z''_\mu$ acquire super heavy masses and give the intermediate mass scale. The stringent experimental limits on lepton flavor violating decays generated by the exchange of bosons $X_{\mu}^{\pm 2/3}$ provide a lower bound on the intermediate mass scale.
3 Effective Hamiltonian for FCNC

The flavor changing current coupled to \( X^{±2/3}_μ \) are given by

\[
J^{Xμ} = \bar{e}_i (U^†)_{ij} (D^†)_{jk} \Gamma^μ_L d_k \\
= \bar{e}_i (U^† D^†)_{ik} V_{ka} \Gamma^μ_L d_a \\
= \bar{e}_i (U^† D^† V)_{iα} \Gamma^μ_L d_a = C_{iα} \bar{e}_i \Gamma^μ_L d_a
\] (3.1)

where

\[
C_{iα} = (U^† D^† V)_{iα} = (DU)^† V_{iα}
\]

\[
C^∗_{iα} = (C^†)_{αi}
\] (3.2)

Similarly

\[
J^{Xμ}(ν) = (C_ν)_{iα} \bar{ν}_i \Gamma^μ_L u_α
\] (3.3)

where

\[
(C_ν)_{iα} = (DU)^†_{iα} = (U^† D^†)_{iα}
\] (3.4)

Hence the effective Hamiltonian involving the current \( J^{Xμ}(e) \) is given by

\[
\mathcal{H} = \frac{g^2}{g m_X^2} |C_{iα} \bar{e}_i \Gamma^μ_L d_α| [d_β \Gamma_L μ e_j] C^∗_{jβ}
\] (3.5)

After Fierz reshuffling

\[
\mathcal{H} = \frac{G_X}{\sqrt{2}} C_{iα} C^∗_{jβ} [d_β \Gamma^μ_L d_α] [\bar{e}_i \Gamma_L μ e_j]
\] (3.6)

where

\[
\frac{G_X}{\sqrt{2}} = \frac{g^2}{g m_X^2} G_F = \left( \frac{g X}{g} \right)^2 \frac{m^2_μ}{m^2_X} G_F
\] (3.7)

Similarly the effective Hamiltonian involving \( J^{Xμ}(ν) \) is given by

\[
\mathcal{H} = \frac{G_X}{\sqrt{2}} (C_ν)_{iα} C^∗_{jβ} [d_β \Gamma^μ_L u_α] [\bar{ν}_i \Gamma_L μ e_j]
\]

The effective Hamiltonian given in Eq.(3.6) is the basic Hamiltonian for FCNC induced processes. The most stringent experimental limits on the LF violating decays are for the \( K \) mesons decays[2]:

\[
\text{B.R. } (K^0_L \to e^- μ^+) < 4.7 \times 10^{-12}
\] (3.8a)

\[
\text{B.R. } (K^0_L \to μ^- μ^+) = (6.84 ± 0.11) \times 10^{-9}
\] (3.8b)

\[
\text{B.R. } (K^- \to π^- μ^- e^+) < 1.3 \times 10^{-11}
\] (3.9a)

\[
\text{B.R. } (K^- \to π^- e^- μ^+) < 1.5 \times 10^{-10}
\] (3.9b)
For $K$ mesons, the effective Hamiltonian is given by

$$\mathcal{H} = \frac{G_X}{\sqrt{2}} \left\{ [\bar{d}\Gamma^d_L s] [(C_{es} C_{ed}^*) \bar{e} \Gamma_{L\mu} e] + (C_{es} C_{ed}^*) (\bar{e} \Gamma_{L\mu} e) + (C_{es} C_{ed}^*) (\bar{e} \Gamma_{L\mu} e) + (C_{es} C_{ed}^*) (\bar{e} \Gamma_{L\mu} e) \right\}$$

(3.10)

The above Hamiltonian is relevant for the decays

$$K_L^0 \rightarrow e^- e^+$$
$$\rightarrow \mu^- \mu^+$$
$$\rightarrow e^- \mu^+, \mu^- e^+$$

and

$$K^- \rightarrow \pi^- e^+$$
$$\rightarrow \pi^- \mu^+(\mu^- e^+)$$
$$\rightarrow \pi^- \mu^+$$

shown in Figs (1) and (2).

From the Hamiltonian (3.10), we obtain the following branching ratios

B.R. ($K_L^0 \rightarrow e^\pm \mu^\mp$) $\approx \left( \frac{G_X}{G_F} \right)^2 \left( 1 - \frac{m_e^2}{m_K^2} \right) \frac{1}{2} \left[ \frac{G_F^2}{8\pi} \int_{K}^2 2m_e^2 m_K \right] \times |C_{es} C_{ed}^* + C_{cs} C_{sd}^*|^2$ (3.11)

B.R. ($K_L^0 \rightarrow \mu^- \mu^+$) $\approx \left( \frac{G_X}{G_F} \right)^2 \left( 1 - \frac{m_{\mu}^2}{m_K^2} \right) \frac{1}{2} \left[ \frac{G_F^2}{8\pi} \int_{K}^2 2m_{\mu}^2 m_K \right] \times |C_{es} C_{ed}^*|^2$ (3.12)

B.R. ($K^- \rightarrow \pi^- \mu^- e^+$) $\approx \left( \frac{G_X}{G_F} \right)^2 \left[ \frac{C_{es} C_{ed}^*}{V_{\mu s}} \right]^2 \times \text{B.R.}(K^- \rightarrow \pi^0 \mu^- \bar{\nu}_{\mu})$ (3.13)

B.R. ($K^- \rightarrow \pi^- e^+ \mu^+$) $\approx \left( \frac{G_X}{G_F} \right)^2 \left[ \frac{C_{es} C_{ed}^*}{V_{\mu s}} \right]^2 \times \text{B.R.}(K^- \rightarrow \pi^0 \mu^- \bar{\nu}_{\mu})$ (3.14)
In the above expressions, we have neglected $m_2^2$ as compared to $m_0^2$.

Using the following values:

$$\tau_{K_L} = 5.116 \times 10^{-8}s, \, G_F = 1.166 \times 10^{-5}\text{GeV}^{-2}$$

$$f_K = 0.160\text{GeV}, \, m_K = 0.497\text{GeV}, \, m_\mu = 0.106\text{GeV}$$

we obtain from Eqs.(3.11) and (3.12):

$$\text{B.R.} \, (K_L^0 \rightarrow e^\pm \mu^\mp) \approx 27 \left( \frac{G_F}{G_X} \right)^2 |C_{\mu s}C_{ed}^* + C_{es}C_{\mu d}|^2$$  \hspace{1cm} (3.15)

$$\text{B.R.} \, (K_L^0 \rightarrow \mu^\pm \tau^\mp) \approx 216 \left( \frac{G_F}{G_X} \right)^2 |C_{\mu s}C_{\mu d}|^2$$  \hspace{1cm} (3.16)

The multiplets $(2, 2)$ follows the patterns:

$$D(e) : \left( \begin{array}{c} u \nu_e \\ d' \nu_e \\ e \mu \\ s' \mu \\ t \nu_e \end{array} \right), \left( \begin{array}{c} c \nu_e \\ s' \nu_e \\ e \mu \\ s' \mu \\ t \nu_e \end{array} \right)$$

$$D(12) : \left( \begin{array}{c} u \nu_\mu \\ d' \nu_\mu \\ e \mu \\ s' \mu \\ t \nu_\mu \end{array} \right), \left( \begin{array}{c} c \nu_\mu \\ s' \nu_\mu \\ e \mu \\ s' \mu \\ t \nu_\mu \end{array} \right)$$

$$D(23) : \left( \begin{array}{c} u \nu_\tau \\ d' \nu_\tau \\ e \mu \\ s' \mu \\ t \nu_\tau \end{array} \right), \left( \begin{array}{c} c \nu_\tau \\ s' \nu_\tau \\ e \mu \\ s' \mu \\ t \nu_\tau \end{array} \right)$$

$$D(231) : \left( \begin{array}{c} u \nu_\mu \\ d' \nu_\mu \\ e \mu \\ s' \mu \\ t \nu_\tau \end{array} \right), \left( \begin{array}{c} c \nu_\mu \\ s' \nu_\mu \\ e \mu \\ s' \mu \\ t \nu_\tau \end{array} \right)$$

The matrix $U = U_{PMNS}$, involves the parameters $(c_{12}, s_{12})$, $(c_{23}, s_{23})$, $(c_{13}, s_{13}e^{i\theta})$[2, 3], where

$$c_{12} = \cos \theta_{12} \approx 0.835, \, s_{12} = \sin \theta_{12} \approx 0.551,$$  

$$c_{23} = \cos \theta_{23} \approx 0.707, \, s_{23} = \sin \theta_{23} \approx 0.707,$$  

$$c_{13} = \cos \theta_{13}, \, s_{13} = \sin \theta_{13}$$

We take $\theta_{13} \approx 0$, although a recent experiment [4] gives $\theta_{13} \approx 9^\circ$. With these values

$$U_{PMNS} = \begin{pmatrix} 0.835 & 0.551 & 0 \\ -0.389 & 0.590 & 0.707 \\ 0.389 & -0.590 & 0.707 \end{pmatrix}$$  \hspace{1cm} (3.17)

For the CKM matrix $V$, we use the following values [2]

$$V = \begin{pmatrix} 0.974 & 0.225 & 3.89 \times 10^{-3} e^{-i\gamma} \\ -0.223 & 1 & 4.06 \times 10^{-2} \\ 8.4 \times 10^{-3} e^{i\alpha} & -3.87 \times 10^{-2} & 1 \end{pmatrix}$$  \hspace{1cm} (3.18)

In this paper, the coefficients $C_{\alpha\beta}$ are obtained for the following two cases I and II

**Case I:** $D(e)$, $D(23), D(12), D(231)$ (i) $U = 1$ (ii) $U = U_{PMNS}$. Using these coefficients and Eq.(3.15), upper limit on $(G_X/G_F)$ is obtained from the inequality (3.8a). Then using Eqs. (3.16), (3.13) and (3.14), we get upper limits on the branching ratios $K_L^0 \rightarrow \mu^\pm \tau^\mp, K^- \rightarrow \pi^- \mu^+ e^+ (\pi^- e^- \mu^+)$. The results are given in Table1.

**Case II:** $D = 1$

(i) $U = \begin{pmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$, (ii) $U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & -s_{23} \\ 0 & s_{23} & c_{23} \end{pmatrix}$  \hspace{1cm} (3.19)
The results are given in Table II.

From Tables I and II, it is clear that for the case I, $D = D(23)$, $D(12)$, $U = U_{PMNS}$ and for the case II, $D = 1$, $U = U(i)$, the upper bound on $\left< \frac{G_F}{m_l} \right>$ is the lowest and the branching ratios are compatible with the experimental bounds except for the B.R. $(K^0_L \rightarrow \mu^+\mu^-)$ which exceeds the experimental value for case I: $D(12), U = U_{PMNS}$.

Finally from the effective Hamiltonians (3.6) and (3.7), we obtain the branching ratios for L.F. violating $\tau$ decays and for $\pi^+ \rightarrow \mu^+\nu_e(\bar{\nu}_e)$, $n \rightarrow p\bar{e}^{-}\bar{\nu}_\mu(\bar{\nu}_\mu)$:

\[
B.R.(\tau^- \rightarrow K^{0}\bar{l}^-) = \left( \frac{G_X}{G_F} \right)^2 |C_{\tau X}C_{\tau X}|^2 \cdot B.R.(\tau^- \rightarrow K^{-}\bar{\nu}_\tau) \tag{3.20}
\]

\[
B.R.(\tau^- \rightarrow \rho^{0}l^-) = \left( \frac{G_X}{G_F} \right)^2 |C_{\tau X}C_{\tau X}|^2 \cdot B.R.(\tau^- \rightarrow \rho^{-}\bar{\nu}_\tau) \tag{3.21}
\]

\[
B.R.(\pi^+ \rightarrow \mu^+\nu_e,\tau) = \left( \frac{G_X}{G_F} \right)^2 \frac{C_{\mu X}C_{\mu X}}{V_{ud}} |C_{\mu X}C_{\mu X}|^2 \cdot B.R.(\pi^+ \rightarrow \mu^+\nu_\mu) \tag{3.22}
\]

\[
B.R.(n \rightarrow p\bar{e}^{-}\bar{\nu}_\mu,\tau) = \left( \frac{G_X}{G_F} \right)^2 \frac{C_{\mu X}C_{\mu X}}{V_{ud}} |C_{\mu X}C_{\mu X}|^2 \cdot B.R.(n \rightarrow p\bar{e}^{-}\bar{\nu}_e) \tag{3.23}
\]

\[
B.R.(\tau^- \rightarrow K^{*}l^- e) = \left( \frac{G_X}{G_F} \right)^2 |C_{\tau X}C_{\tau X}|^2 \cdot B.R.(\tau^- \rightarrow K^{-}\bar{\nu}_\tau) \tag{3.24}
\]

\[
B.R.(\tau^- \rightarrow \rho l^- e) = \left( \frac{G_X}{G_F} \right)^2 |C_{\tau X}C_{\tau X}|^2 \cdot B.R.(\tau^- \rightarrow \rho^{-}\bar{\nu}_\tau) \tag{3.25}
\]

where $l = e, \mu$. In Eqs. (3.20-3.25), we have neglected the terms of order $m_l^2/m_\tau^2$. From these equations, we get the following values for the branching ratios:
4 Lepton Flavor (LF) violating B decays

The effective Hamiltonian for LF violating B decays is given by

$$\mathcal{H}_{\text{eff}} = \frac{G_X}{\sqrt{2}} [C_{ib}C_{jd}^* |(d\Gamma^m_{Lb})(\bar{v}, \Gamma_{L\mu}, e_j| + (C_{ib}C_{jd}^*)(|s\Gamma^m_{Lb})(\bar{v}, \Gamma_{L\mu}, e_j|)$$

(4.1)

In this paper, only the decays $B^0 \rightarrow \ell^- \ell^+$ (i.e., $B^0 \rightarrow \ell^- \ell^+$) (4.2)

are analysed.

The branching ratio is given by

$$B.R.(B^0 \rightarrow \ell^- \ell^+) = \frac{\tau_{B^0}}{\tau_{B^0}} \left( \frac{G_X}{G_F} \right)^2 \left( 1 - m_{\ell 1}^2 / m_{B^0}^2 \right) \left( 1 - m_{\ell 2}^2 / m_{B^0}^2 \right) \left[ \frac{C_{i\beta}^2}{8\pi} m_{B^0} f_{B^0}^2 (m_{\ell 1}^2 + m_{\ell 2}^2) \times |g_{ji}|^2 \right]$$

(4.2)

where

$$g_{ji} = C_{i\beta} C_{jd}^*$$

Since branching ratio is proportional to $m_{\ell 1}^2$, we will take $i = \tau, j = e, mu, \mu, g_{ji} = g_{1\tau}, l = e, mu, \mu$. To obtain the branching ratios, we use the following values:

$$\tau_{B^0} = 1.52 \times 10^{-12}s, \quad m_{B^0} = 5.279\text{Gev}, \quad f_{B^0} = 0.220\text{Gev}$$

$$\tau_{B^*} = 1.47 \times 10^{-12}s, \quad m_{B^*} = 5.366\text{Gev}, \quad f_{B^*} = 0.234\text{Gev}$$

Using above values, we get from Eq.(4.2)

$$B.R.[B^0 \rightarrow \ell^- \tau^+ (\tau^- \ell^+)] = \left( \frac{G_X}{G_F} \right)^2 F \left[ |g_{1\tau}|^2 \right]$$

(4.3)

where

$$F = 7.87, \quad B^0 = 8.87, \quad B^*$$

(4.4)

For $B^0 \rightarrow \tau^- \tau^+, \mu^- \mu^+$

$$F = 15.48, \quad 0.071 \quad B^0$$

(4.5)

Now

$$\left( \frac{G_X}{G_F} \right)^2 < (5.71 \times 10^{-13}), \quad (8.23 \times 10^{-11}), \quad (3.76 \times 10^{-11})$$

(4.6)

for $D(e), D(23), D(12), U = U_{PMNS}$ respectively. The factors $g_{1\tau}$, $(g_{\tau\tau})$ in Eq.(4.3) for the three cases (i), (ii), (iii) are given below

(i) $B^0$:

$$g_{1\tau}(g_{\tau\tau}) \approx 0.602 (-0.051), \quad g_{\mu\tau}(g_{\tau\mu}) \approx 0.259 (0.077)$$

$$g_{\tau - \tau} \approx -0.101, \quad g_{\mu - \mu} \approx -0.197$$
With above values for $g_B$, the branching ratios can be calculated. The upper limit on the branching ratios for various decay channels are given in Table III. The experimental limits on

$$
\begin{array}{ccc}
\text{Case} & \text{(i)} & \text{(ii)} \\
B_{d,s}^\pm & e^- \tau^+ & 1.60 \times 10^{-12} \\ & & (7.78 \times 10^{-14}) \\
& \tau^- e^+ & 1.17 \times 10^{-14} \\ & & (3.04 \times 10^{-13}) \\
& \mu^- \tau^+ & 3.01 \times 10^{-13} \\ & & (1.17 \times 10^{-12}) \\
& \tau^- \mu^+ & 2.66 \times 10^{-14} \\ & & (7.01 \times 10^{-13}) \\
& \tau^- \tau^+ & 9.02 \times 10^{-14} \\ & & (2.37 \times 10^{-12}) \\
& \mu^- \mu^+ & 1.57 \times 10^{-15} \\ & & (5.52 \times 10^{-15})
\end{array}
$$

$$
\begin{array}{ccc}
\text{B}_\tau^\pm: & \ \\
g_{\tau\tau}(g_{\tau e}) \approx -0.124(0.245), & g_{\mu\tau}(g_{\tau\mu}) \approx 0.495(-0.372) \\
g_{\tau^- \tau^+} \approx 0.488, & g_{\mu^- \mu^+} \approx -0.350
\end{array}
$$

(ii) $B_{d'}^\pm$:

$$
\begin{array}{ccc}
\text{B}_\tau^\pm: & \ \\
g_{\tau\tau}(g_{\tau e}) \approx 0.520(0.058), & g_{\mu\tau}(g_{\tau\mu}) \approx 0.492(-0.089) \\
g_{\tau^- \tau^+} \approx -0.115, & g_{\mu^- \mu^+} \approx 0.379
\end{array}
$$

(iii) $B_{d'}^\pm$:

$$
\begin{array}{ccc}
\text{B}_\tau^\pm: & \ \\
g_{\tau\tau}(g_{\tau e}) \approx -0.408(0.291), & g_{\mu\tau}(g_{\tau\mu}) \approx 0.327(-0.389) \\
g_{\tau^- \tau^+} \approx 0.498, & g_{\mu^- \mu^+} \approx -0.257
\end{array}
$$

With above values for $g_{\tau\ell}$, the branching ratios can be calculated. The upper limit on the branching ratios for various decay channels are given in Table III. The experimental limits on

$$
\begin{array}{ccc}
\text{B}_\tau^\pm: & \ \\
g_{\tau\tau}(g_{\tau e}) \approx 0.530(0.056), & g_{\mu\tau}(g_{\tau\mu}) \approx 0.511(-0.075) \\
g_{\tau^- \tau^+} \approx 0.095, & g_{\mu^- \mu^+} \approx 0.420
\end{array}
$$
L.F. violating $B^o \rightarrow \ell \tau^\pm (j \neq i)$ decays are not stringent as for the $K-$decays. For $B^o$ decays, experimental limits are [2, 5, 7]

\[
\text{B.R.}(B^o_\ell \rightarrow e^\mp \tau^\pm) < 2.8 \times 10^{-5}
\]

\[
\text{B.R.}(B^o_\ell \rightarrow \mu^\mp \tau^\pm) < 2.2 \times 10^{-5}
\]

\[
\text{B.R.}(B^o_\ell \rightarrow \tau^\mp \tau^\pm)
\]

\[
\text{B.R.}(B^o_\ell \rightarrow \mu^\mp \tau^\pm)_{\text{exp}} = (3.2 \pm 1.5 \pm 1.2) \times 10^{-9}
\]

\[
\text{B.R.}(B^o_\ell \rightarrow \mu^\mp \tau^\pm)_{\text{SM}} = (3.32 \pm 0.17) \times 10^{-9}
\]

\[
\text{B.R.}(B^o_\ell \rightarrow \tau^- \tau^+)^{\tau}_{\text{exp}} < 0.8 \times 10^{-9}
\]

\[
\text{B.R.}(B^o_\ell \rightarrow \mu^- \tau^+)_{\text{SM}} = (0.10 \pm 0.01) \times 10^{-9}
\]

It is clear from the above Table III, the upper limits on branching ratios for $B^o_{s,d} \rightarrow \mu^- \mu^+$ are much below than the experimental upper limits for these decays.

However, a distinctive feature of this model is violation of charge symmetry for the decays $B^o_{\ell, s,d} \rightarrow e^- (\mu^-) \tau^+$ and $B^o_{\ell, s,d} \rightarrow \tau^- e^+ (\mu^+)$ and $B^o_{\ell, s,d} \rightarrow e^- (\mu^-) \tau^+$ and $B^o_{\ell, s,d} \rightarrow \tau^- e^+ (\mu^+)$ for (ii) and (iii) respectively. The branching ratios are significantly different for the channels $e^- (\mu^-) \tau^+$ and $\tau^- e^+ (\mu^+)$; the branching ratios are much smaller for the channels $\tau^- e^+ (\mu^+)$ than for $e^- (\mu^-) \tau^+$.

Finally for the case II, $D = 1, U = U(i)$; $(G_X/G_F) < 9.5 \times 10^{-6}$. For this case:

\[
B^o_d : g_{\tau e}(g_{\tau \tau}) \approx 0.69(0); \quad g_{\mu \tau}(g_{\tau \mu}) \approx 0.70(0)
\]

\[
B^o_s : g_{\tau e}(g_{\tau \tau}) \approx 0.74(0); \quad g_{\mu \tau}(g_{\tau \mu}) \approx 0.77(0)
\]

The upper limitations on the branching ratios for the case II are

\[
\text{B.R.}(B^o_d \rightarrow e^- \tau^+ (\mu^- \tau^+)) < 3.4 \times 10^{-10}(3.7 \times 10^{-10})
\]

\[
\text{B.R.}(B^o_s \rightarrow e^- \tau^+ (\mu^- \tau^+)) < 4.4 \times 10^{-10}(4.7 \times 10^{-10})
\]

The branching ratios for $B^o_{d,s} \rightarrow \tau^- e^+ (\tau^- \mu^+)$ are much smaller. For this case charge symmetry is badly broken.

The time integrated decay rates due to quantum interference of $B^o$ and $\bar{B}^o$ for the LF violating decays of $B^o$ to lepton pairs are a promising area to test the model based on the extended electroweak unification group $SU_L(2) \times U_Y \times SU_X(2)$.

Now [3]

\[
\Gamma_f(t) = \frac{1}{4} e^{-\Gamma(t)} \left( \left| g_{\tau \ell} \right|^2 + \left| g_{\ell \tau} \right|^2 \right)(1 + \cos(\Delta m t))
\]

where $f = \ell^-, \ell^+; \bar{\ell} = e, \mu$

First we note that $\beta = 0$, for $B^o_{s,d}$, so sin $\Delta m t$ term vanishes for $B^o_{s,d}$. For the decays for which $g_{\tau \ell} \ll g_{\ell \tau}$, we have [3]

\[
\Gamma_f(t) = \frac{1}{4} e^{-\Gamma(t)} \left| g_{\ell \tau} \right|^2 (1 + \cos(\Delta m t))
\]

\[
\Gamma_f(t) = \frac{1}{4} e^{-\Gamma(t)} \left| g_{\ell \tau} \right|^2 (1 - \cos(\Delta m t))
\]
The interaction Lagrangian is given by

\[ L_{\text{int}}(\tau^-\ell^+) = \sum_{i,j} \left( \bar{u}_i \Gamma_{iL}^\mu \ell_j + \bar{\nu}_i \Gamma_{iR}^\mu \ell_j + \bar{e}_i \Gamma_{iL}^\mu \ell_j + \bar{\nu}_i \Gamma_{iR}^\mu \ell_j + \bar{\tau}_i \Gamma_{iL}^\mu \ell_j + \bar{\nu}_i \Gamma_{iR}^\mu \ell_j \right) \]

where we have used the experimental values \( x_d \approx 0.77, x_s \approx 26 \) for \( B^3_d \) and \( B^3_s \) respectively.

Hence for time integrated decay rates for \( B^3_R \), the \( \tau^-/\tau^+ = e^-/e^+ \approx 0.23 \) i.e. depletion of \( \tau^- \) or \( e^- \), compared to \( \tau^+ \) or \( e^+ \) i.e. charge symmetry is badly violated for the time integrated decay rates \( B^3_d \rightarrow \tau^-\ell^+ \), \( \ell^-\tau^+ \).

However, for time integrated decay rates \( B^3_s \rightarrow \tau^-\ell^+ \), \( \ell^-\tau^+ \), there is only extremely small deviation from 1, although \( B.R.(B^3_s \rightarrow \tau^-\ell^+)/B.R.(B^3_s \rightarrow \tau^-\ell^+) \approx 1 \).

5  Left-Right symmetric group \( SU_L(2) \times SU_R(2) \times U_Y(1) \times SU_X(2) \)

The left-handed and right-handed fermions are assigned to following representation of the group:

\[ \begin{pmatrix} u_i & u' \cr d_i & e_i \end{pmatrix}_L : (2,1,2) \]

\[ \begin{pmatrix} u_i & N_i \cr d'_i & e_i \end{pmatrix}_R : (1,2,2) \]

\[ Q = \frac{1}{2} (\tau_L + \tau_R + Y_1 + \tau_{X_s}) \]

\[ Y_1 = \begin{cases} 0 & \text{for leptons} \\ -1/3 & \text{for quarks} \end{cases} \]

The interaction Lagrangian is given by

\[ L_{\text{int}} = -g \sin \theta_W J_{\mu}^\mu \mu \frac{g}{2\sqrt{2}} \sum_{i,j} \left[ \left( \bar{u}_i \Gamma_{iL}^\mu \ell_j + \bar{\nu}_i \Gamma_{iR}^\mu \ell_j + \bar{e}_i \Gamma_{iL}^\mu \ell_j + \bar{\nu}_i \Gamma_{iR}^\mu \ell_j + \bar{\tau}_i \Gamma_{iL}^\mu \ell_j + \bar{\nu}_i \Gamma_{iR}^\mu \ell_j \right) \right] \]

where \( J_{\mu}^\mu \) is the neutral current of the standard model

\[ J_{\mu}^\mu = \frac{1}{4} \left( \tan^2 \theta_W \left( \bar{u}_i \Gamma_{iL}^\mu \nu_i + \bar{e}_i \Gamma_{iL}^\mu \ell_i + \bar{\nu}_i \Gamma_{iR}^\mu \nu_i + \bar{e}_i \Gamma_{iR}^\mu \ell_i + \bar{\tau}_i \Gamma_{iL}^\mu \nu_i + \bar{\nu}_i \Gamma_{iR}^\mu \nu_i \right) - 4 J_{\mu R}^\mu \right) \]

\[ J_{\mu R}^\mu = \frac{1}{4} \left( \tan^2 \theta_W \left( \bar{u}_i \Gamma_{iL}^\mu \nu_i + \bar{\nu}_i \Gamma_{iR}^\mu \nu_i + \bar{\tau}_i \Gamma_{iL}^\mu \nu_i + \bar{\nu}_i \Gamma_{iR}^\mu \nu_i \right) + 2 \left( \bar{e}_i \gamma^\mu e_i - \bar{\nu}_i \gamma^\mu \nu_i - \bar{\nu}_i \gamma^\mu \nu_i \right) \right) \]

\[ J_{\mu L}^\mu = \frac{1}{4} \left( \left( 1 - \frac{g^2}{g_X^2} \right) \left( \bar{u}_i \Gamma_{iL}^\mu \nu_i + \bar{\nu}_i \Gamma_{iR}^\mu \nu_i \right) + 2 \left( \bar{\nu}_i \gamma^\mu \nu_i - \bar{\nu}_i \gamma^\mu \nu_i - \bar{\nu}_i \gamma^\mu \nu_i \right) \right) \]

\[ J_{\mu=0}^\mu = \frac{1}{4} \left( \frac{g^2}{g_X^2} \left( \bar{u}_i \gamma^\mu u_i + \bar{\nu}_i \gamma^\mu \nu_i \right) + 2 \left( \bar{\nu}_i \gamma^\mu \nu_i - \bar{\nu}_i \gamma^\mu \nu_i - \bar{\nu}_i \gamma^\mu \nu_i \right) \right) \]
The gauge group $SU_L(2) \times SU_R(2) \times U_Y(1) \times SU_X(2)$ is spontaneously broken to the group $SU_L(2) \times SU_R(2) \times U_Y(1)$ by introducing a scalar doublet of $SU_X(2)$ as in the section 2. The vector bosons $m_X^2$ and $Z'' \mu$ acquire the masses
\[
m_X^2 = \frac{1}{4} g_X^2 \sqrt{2}, \quad m_{Z''}^2 = \frac{1}{4} (g_I^2 + g_X^2) \sqrt{2} = \frac{1}{4} \frac{g_X^2 \sqrt{2}}{1 - g''/g_X^2}
\] (5.4)

By above symmetry breaking mechanism, the group $SU_L(2) \times SU_R(2) \times U_Y(1)$ is decoupled from the extended gauge group at mass scale $m_X$.

The group $SU_L(2) \times SU_R(2) \times U_Y(1)$ is spontaneously broken to $U_{em}(1)$ in the standard way [6], by introducing the scalar doublets
\[
\Phi_L : (2,1)_{Y=1}, \quad \Phi_R : (1,2)_{Y=1}, \quad \Delta_R : (1,3)_{Y=2}
\] (5.5)
The weak vector bosons, $W^\pm_{L \mu}$, $W^\pm_{R \mu}$, $Z'_{\mu}$ acquire masses:
\[
L_{mass}(W) = -\frac{1}{8} \sqrt{\frac{g}{\sqrt{2}}} \left[ 2 g^2 W^{-}_{L \mu} W^{+}_{L \mu} + \left( g W^\mu_{3 L} - g' B^\mu \right) \left( g W^\mu_{3 L} - g' B^\mu \right) \right]
\]
\[
-\frac{1}{8} \sqrt{\frac{g}{\sqrt{2}}} \left[ 2 g^2 W^{-}_{R \mu} W^{+}_{R \mu} + \left( g W^\mu_{3 R} - g' B^\mu \right) \left( g W^\mu_{3 R} - g' B^\mu \right) \right]
\]
\[
-\frac{1}{8} \sqrt{\frac{g}{\sqrt{2}}} \left[ 2 g^2 W^{-}_{R \mu} W^{+}_{R \mu} \right]
\] (5.6)
where $B_{\mu}$ is the vector boson associated with $U_Y(1)$, with coupling constant $g'$. We get
\[
m^2_{W_L} = \frac{1}{4} g^2 \sqrt{2}, \quad m^2_{W_R} = \frac{1}{4} g^2 (\sqrt{2} + V^2)
\]
\[
m^2_{Z'} \approx \frac{1}{4} \frac{g^2}{\cos^2 \theta_W} v^2, \quad m^2_{Z'} \approx \frac{1}{4} \frac{g^2}{1 - \tan^2 \theta_W} v^2
\] (5.7)
for $V_L^2/\sqrt{2} \ll 1$. We note
\[
g W^\mu_{3 L} - g' B^\mu = -\frac{g}{\cos \theta_W} Z^\mu - \frac{g \tan^2 \theta_W}{\sqrt{1 - \tan^2 \theta_W}} Z''_{\mu}
\]
\[
g W^\mu_{3 R} - g' B^\mu = -\frac{g}{\sqrt{1 - \tan^2 \theta_W}} Z''_{\mu}
\] (5.8)
From Eq.(59), it follows that the effective Hamiltonian for the FCNC involving $e^-$, $\mu^-$, $\tau^-$ is given by
\[
\mathcal{H}_{eff} = \frac{g_X^2}{m_X} \left[ C_{\alpha \beta} e_i \left( \Gamma^\mu_L + \Gamma^\nu_R \right) d_{\alpha} \right] \left[ d_{\beta} \left( \Gamma_{L \mu} + \Gamma_{R \mu} \right) \right] C^*_{j \beta}
\] (5.9)
After Fierz reshuffling:
\[
\mathcal{H}_{eff} = \frac{G_X}{\sqrt{2}} C_{\alpha \beta} C^*_{j \beta} \left[ \left( d_{\beta} \Gamma^\mu_L d_{\alpha} \right) \left( e_i \Gamma_{L \mu} e_j \right) + 2 \left( d_{\beta} (S - P) d_{\alpha} \right) \left( e_i (S - P) e_j \right) \right]
\] (5.10)
In this paper, the effective Hamiltonian given in Eq. (5.10) for FCNC induced $K$, $B$ meson
decays is not further discussed.
6 Summary and conclusion

The lepton flavor violating decays are strictly forbidden in the standard model. The gauge group 
\( G = SU_L(2) \times U_Y(1) \times SU_X(2) \) beyond the SM provides a framework to derive the effective 
Hamiltonian for FCNC induced decays of \( K \) and \( B \) mesons to lepton pairs. The effective coupling 
constant \( \frac{g}{\sqrt{2} m_X} = \frac{g}{\sqrt{2} m_X} \), where \( m_X \), the mass of lepto-quark bosons \( X^{\pm 2/3} \) of \( SU_X(2) \) gives the mass scale at which the group \( G \) is broken to \( SU_L(2) \times U_Y(1) \). The effective Hamiltonian gives the 
lepton flavor conserving and LF violating decays of the same order. The upper bound on \( (G_X/G_F) \) 
is obtained from the most stringent experimental limits on the BR\( X \)\( Y \)\( Z \). For some cases the upper bound on \( (G_X/G_F) \) is analyzed. For some cases the upper bound on \( (G_X/G_F) \) is of order of \( (6 - 9) \times 10^{-6} \) and compatible with the upper limits on various LF violating \( K \)-decays. In particular for these cases, we find the \( B.R.(K_L^0 \rightarrow \mu^- \mu^+) < (1.95 - 8.28) \times 10^{-9} \) to be compared with the experimental value 
\( B.R.(K_L^0 \rightarrow \mu^- \mu^+) = (6.84 \pm 0.11) \times 10^{-9} \). It is tempting to take \( (G_X/G_F) \) equal to \( (6 - 9) \times 10^{-6} \); 
this gives \( (g/g_X)(m_\mu/m_W) \approx (3 - 4) \times 10^2 \).

In the SM [5, 7],

\[
B.R.(B_S^0 \rightarrow \mu^- \mu^+) = \frac{\tau_{B_S^0}}{\tau_{B_S^0}} \frac{G_F^2 \alpha^2}{16\pi^3} |V_{ts}|^2 \left( 1 - \frac{2m_\mu^2}{m_B^2} \right) \times m_B, f_{B_S^0} m_\mu |C_{10}|^2 \tag{6.1}
\]

Using the experimental values for the masses and \( \tau_{B_S^0}, f_{B_S} \approx 0.234, |V_{ts}| = 3.87 \times 10^{-2} \) and for 
the Wilson coefficient \( C_{10}^{\text{eff}}(m_b) = -4.13 \), one gets \( B.R.(B_S^0 \rightarrow \mu^- \mu^+) = 3.1 \times 10^{-9} \). More accurate 
calculation gives its value \((3.2 \pm 0.2) \times 10^{-8}(\text{Exp. value} = (3.2 \pm 1.3) \times 10^{-9})\).

From Eq. (6.1), we obtain

\[
B.R.(B_S^0 \rightarrow \tau^+ \tau^-) = (1 - 2m_\tau^2/m_B^2)/(1 - 2m_\mu^2/m_B^2) \times (m_\tau/m_\mu)^2 \times \frac{B.R.(B_S^0 \rightarrow \mu^- \mu^+)}{B.R.(B_S^0 \rightarrow \mu^- \mu^+)} \times \frac{\tau_{B_S^0}}{\tau_{B_S^0}} \frac{G_F^2 \alpha^2}{16\pi^3} |V_{ts}|^2 \approx (6.9 \pm 0.4) \times 10^{-7} \tag{6.2}
\]

\[
B.R.(B_S^0 \rightarrow \mu^- \mu^+) = \frac{\tau_{B_S^0}}{\tau_{B_S^0}} \frac{f_{B_S^0} m_B}{f_{B_S^0} m_B}|V_{ts}|^2 \approx 0.13 \times 10^{-9} \tag{6.3}
\]

For \( K_L^0 \rightarrow \mu^+ \mu^- \), Eq. (6.1) is modified to

\[
B.R.(K_L^0 \rightarrow \mu^+ \mu^-) = 2(\tau_{K_L^0}/\tau_{K_L^0}) \frac{G_F^2 \alpha^2}{16\pi^3} |V_{ts}|^2 \left( 1 - \frac{2m_\mu^2}{m_K^2} \right) m_K f_{K^0}^2 |C_{10}|^2 \approx 5.92 \times 10^{-11}(C_{10}^{\text{eff}}(m_b))^2 \tag{6.4}
\]

which is much below the experimental value, unless the last factor on the right hand side of 
Eq. (6.4) is of order of \( 10^2 \).

For LF violating \( B \)-decays, the experimental limits on the branching ratios are not very stringent. 
However the time integrated decay rates of \( B_{d,s}^0 \rightarrow \ell^- \ell^+ \) due to quantum interference 
of \( B^0 \) and \( \bar{B}^0 \) is a promising area to test the model. We find for time integrated decay rates for 
\( B_{d,s}^0 \rightarrow \ell^- \ell^+ \) the ratio \( \tau^-/\tau^+ = \ell^+/\ell^- \approx 23 \% \), i.e. depletion of \( \tau^- \) or \( \ell^+ \) compared to 
\( \tau^+ \) and \( \ell^- \) i.e. violation of charge symmetry.

With more precise experimental data on the LF violating on the branching ratio for \( K_L^0 \) and \( B_{d,s}^0 \) decay to lepton pairs, it may be possible to test the model.
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