Foundations and Trends ® in Robotics

A Survey on the Integration of Machine Learning with Sampling-based Motion Planning

Suggested Citation: Troy McMahon, Aravind Sivaramakrishnan, Edgar Granados and Kostas E. Bekris (2022), “A Survey on the Integration of Machine Learning with Sampling-based Motion Planning”, Foundations and Trends ® in Robotics: Vol. 9, No. 4, pp 266–327. DOI: 10.1561/2300000063.

Troy McMahon*
Rutgers University
tm799@rutgers.edu

Aravind Sivaramakrishnan*
Rutgers University
as2578@rutgers.edu

Edgar Granados
Rutgers University
eg585@rutgers.edu

Kostas E. Bekris
Rutgers University
kb572@rutgers.edu

This article may be used only for the purpose of research, teaching, and/or private study. Commercial use or systematic downloading (by robots or other automatic processes) is prohibited without explicit Publisher approval.
Contents

1 Introduction 268

2 Sampling-based Motion Planning 273

3 Learning Primitives of Sampling-based Motion Planning 278
   3.1 Sampling Sequences 278
   3.2 Collision Checking 283
   3.3 Distance Metrics, Heuristics and Constraints 288
   3.4 Steering Functions and Local Planners 294
   3.5 Adaptive Selection of Primitives and Meta-Reasoning 296

4 Learning-based Pipelines 301
   4.1 Retrieve and Repair Methods 301
   4.2 Neural Motion Planning 302
   4.3 Learning a lower dimensional planning space 304

5 SBMP with Learned Models 307
   5.1 Modeling Robot Uncertainty 308
   5.2 Modeling Uncertainty in Obstacle Dynamics 309
6 Discussion

6.1 Computational Efficiency ......................................... 312
6.2 Path Quality ............................................................. 313
6.3 Ease of Use ................................................................. 313
6.4 Limitations of Existing Methods .................................... 314
6.5 Potential Future Work .................................................. 315
6.6 Alternative Planning Frameworks ................................. 316

References ..................................................................... 317
A Survey on the Integration of Machine Learning with Sampling-based Motion Planning

Troy McMahon\textsuperscript{1,*}, Aravind Sivaramakrishnan\textsuperscript{1,*}, Edgar Granados\textsuperscript{1} and Kostas E. Bekris\textsuperscript{1}

\textsuperscript{1}Department of Computer Science, Rutgers University, NJ, USA; kb572@rutgers.edu, as2578@rutgers.edu
*Equal contribution.

ABSTRACT

Sampling-based methods are widely adopted solutions for robot motion planning. The methods are straightforward to implement, effective in practice for many robotic systems. It is often possible to prove that they have desirable properties, such as probabilistic completeness and asymptotic optimality. Nevertheless, they still face challenges as the complexity of the underlying planning problem increases, especially under tight computation time constraints, which impact the quality of returned solutions or given inaccurate models. This has motivated machine learning to improve the computational efficiency and applicability of Sampling-Based Motion Planners (SBMPs). This survey reviews such integrative efforts and aims to provide a classification of the alternative directions that have been explored in the literature. It first discusses how learning has been used to enhance key components of SBMPs, such as node sampling, collision detection, distance or nearest neighbor computation, local
planning, and termination conditions. Then, it highlights planners that use learning to adaptively select between different implementations of such primitives in response to the underlying problem’s features. It also covers emerging methods, which build complete machine learning pipelines that reflect the traditional structure of SBMPs. It also discusses how machine learning has been used to provide data-driven models of robots, which can then be used by a SBMP. Finally, it provides a comparative discussion of the advantages and disadvantages of the approaches covered, and insights on possible future directions of research. An online version of this survey can be found at: https://prx-kinodynamic.github.io/
Introduction

Motion planning is the problem of finding valid paths, expressed as sequences of configurations, or trajectories, expressed as sequences of controls, which move a robot from a given start state to a desired goal state while avoiding obstacles. It has applications in problems ranging from mobile robotics (Barraquand and Latombe, 1991), manipulation planning (Oriolo and Mongillo, 2005), graphics and animation (Kallmann et al., 2008), protein folding (Song and Amato, 2001) to crowd simulation (Bayazit et al., 2002; Sud et al., 2008; Toll et al., 2012) and multi-robot applications (Svestka and Overmars, 1995; Clark and Rock, 2001). Variations of the motion planning problem can include dynamic constraints, which can be important in autonomous driving and aerial vehicles (Figure 1.1).

The motion planning problem is PSPACE-Complete (Reif, 1979; Canny, 1988; Latombe, 1991), and its complexity depends exponentially on the number of degrees of freedom of the robotic system. This makes traditional, complete methods difficult to apply for problems with more than 4 or 5 degrees of freedom. This has motivated work on developing sampling-based methods, which often use random samples to explore the underlying configuration space. Examples of popular Sampling-
Based Motion Planners (SBMPs) include the Probabilistic Roadmap (PRM) (Kavraki et al., 1996), the Rapidly-exploring Random Tree (RRT) (Lavalle, 1998) and the Expansive Spaces (Hsu et al., 1997) algorithms. Sampling-based motion planners give up on the traditional notion of completeness and instead aim for probabilistic completeness, which means that they are guaranteed to eventually discover a solution if one exists but cannot confirm solution non-existence. Progress in the field has also allowed the development of methods, such as $\text{RRT}^*$ and $\text{PRM}^*$ (Karaman and Frazzoli, 2011), which are also asymptotically optimal, i.e., they guarantee convergence to an optimal solution if one exists.

Beyond their algorithmic properties, these methods have proven
Introduction

quite effective in finding solutions for relatively high-dimensional challenges, where the traditional approaches do not scale. Their popularity also stems from the fact that they provide flexible frameworks, which are rather straightforward to implement and adapt for a large variety of robotic systems. Nevertheless, they may still face challenges as the complexity of the underlying planning problem increases:

1. Some of the challenges relate to computational efficiency, which may be hindered from the exploration of the underlying configuration space via sampling, especially when the key primitives of these planners, such as collision checking or forward propagation of the system’s dynamics, are computationally expensive.

2. Other issues relate to path quality. Despite the progress in understanding the conditions for asymptotic optimality, convergence to high-quality solutions may be hindered in practice when naive exploration primitives are employed, such as the random sampling of controls.

3. Furthermore, SBMPs like most motion planning methods, typically assume the availability of an accurate, complete model. Traditional, engineered models may be inaccurate or unable to express all critical physical aspects of the problem or not predict how a dynamic environment may evolve. Furthermore, sensing constraints may introduce partial observability and uncertainty about the environment. These factors can limit the applicability of SBMPs.

These challenges motivate the use of machine learning to improve the computational efficiency of SBMPs, accelerate their practical convergence to high-quality solutions, and provide access to accurate-enough, data-driven models, which adapt to varying environmental conditions and sensing input.

Machine learning enables the autonomous derivation of solutions to problems based on prior experience and data. It promises to constantly improve performance by incorporating new data and identifying solutions that engineered approaches may not be able to achieve. This makes machine learning especially useful for an application like robotics, where a robotic agent must contend with an endless variety of tasks and environments. There are also many scenarios where learned agents can
approximate costly computations. In these cases, the learned agent can be trained to model the computations in a pre-processing step, then used during run-time in place of the computation or as a heuristic for it. This is especially useful in robotics, where the robot must act and react to situations in real-time.

Fundamentally, machine learning algorithms operate by building a model of observed data, that can predict and generalize to new examples. This data can come from various sources: an existing dataset, through human training or demonstration, or it can be accumulated from the results of previous predictions that the model has made. For model-based agents, learning is done by fitting the model’s parameters to data, which can be done using methods such as regression or reinforcement learning. During query time, the model is queried to give predictions based on patterns observed in the data.

There is a large body of literature on the application of machine learning algorithms to improve the efficiency of robotic systems in general (Kober et al., 2013; Kroemer et al., 2019). Recently, there has been a lot of attention on the progress of deep learning methods, which has resulted in many efforts to utilize the corresponding tools in robotics (Sünderhauf et al., 2018). This survey focuses specifically on integrating machine learning tools to improve the efficiency, convergence, and applicability of SBMPs.

This survey covers a wide breadth of robotic applications, including, but not limited to, mobile robot navigation, manipulation planning, and planning for systems with dynamic constraints. In particular, this monograph first reviews the attempts to use machine learning to improve the performance of individual primitives used by SBMPs (Section 3). It also studies a series of planners that use machine learning to adaptively select from a set of motion planning primitives. It then proceeds to study a series of integrated architectures that learn an end-to-end mapping of sensor inputs to robot trajectories or controls (Section 4). Finally, it studies how SBMPs can operate over learned models of robotic system that account for noise and uncertainty (Section 5).

The survey concludes with a comparative discussion of the different approaches covered in Sections 3 - 5. It evaluates these approaches in terms of their impact on computational efficiency of the planner,
quality of the computed paths, and their overall applicability. It then outlines the broad difficulties and limitations of these methods, as well as potential directions of future work.
Figure 2.1: Solving a motion planning problem using a batch version of the *Probabilistic Road-Map (PRM)* method. (a) The problem is transformed from the workspace where the robot has geometry (top) to the *configuration space* (bottom), where the robot is a point. (b) After *sampling* the configuration space and *validity checking* each sample in the workspace, valid samples (blue) are retained and invalid samples (orange) are discarded. (c) For every sample, local edges are *constructed* to nearby samples and *validity checked*. Valid edges (black) are retained and invalid edges (orange) are discarded. (d) Once the final roadmap is obtained, each edge is weighted by a *cost* or a *distance function*. (e) The final path is obtained using a graph search technique (such as A*), typically aided by an admissible *heuristic function*. 

273
An instance of a motion planning problem consists of a mobile object in a known environment. The object could be a geometric shape, a rigid body, or a set of rigid bodies that are connected by joints, such as a robotic arm or articulated linkage. The environment consists of a geometric space (called the \textit{workspace}), as well as a notion of \textit{validity} that determines if paths within the environment are valid. Normally this is accomplished by dividing the environment into valid (i.e. free) regions and invalid regions (i.e. obstacles). A path is considered to be valid if it is totally contained in the free regions. For problems with constraints (e.g. dynamics constraints), the path must also satisfy these constraints. A solution to the motion planning problem is a valid path by which the object traverses from a given start configuration to a given goal region.

The basic version of the motion planning problem assumes that the environment is fully known and that the motions of the robot are exact. It does not take into account uncertainty in the environment due to issues such as noise in the robot’s sensors or actuators. It also assumes that the robot’s location in the environment is always known and as such does not address the problem of robot localization.

The object’s motion is defined in terms of degrees of freedom, which include translational and rotational degrees of freedom as well as internal degrees of freedom (e.g. joint angles). A \textit{configuration} consists of a setting for each of the degrees of freedom of the object and uniquely defines a \textit{state} of the robot. The space of all possible configurations for a system is defined as the \textit{configuration space} or $\mathbb{C}$. A configuration is defined as valid or invalid based on if it satisfies a problem-specified validity condition. This condition usually requires that there are no collisions between the object and any obstacles in the environment. For robots that consist of multiple bodies (e.g. articulated linkages), validity conditions usually also require that there is no collisions between the bodies of the robot (i.e. self-collision). Configurations can be tested for collision using a closed-box collision checker.

A path or a trajectory is a continuous sequence of configurations. A path/trajectory is valid if all of the configurations along it are valid, otherwise it is invalid. For systems with constraints (e.g. dynamics), trajectories must also adhere to these constraints of the system in order
to be considered valid. As stated previously, a solution to the motion planning problem is a valid path/trajectory between the start and goal configuration.

For some applications, it is also important to consider path quality. The quality of a path is indicated by a given cost function which the problem wishes to minimize (e.g. distance traveled or energy expended by the robot). A cost function takes as input a path and returns a real number indicating the cost associated with the path.

The complexity of motion planning in $C$ grows exponentially with respect to the number of degrees of freedom, which make modeling $C$ directly computationally challenging. This has led to the development of sampling-based methods, such as the PRM (Kavraki et al., 1996), RRT (Lavalle, 1998) and RRT* (Karaman and Frazzoli, 2011), which construct an abstract representation of $C$ via sampling. The sampled configurations are classified as valid (free) or invalid (obstacle) using a validity checker that tests if the sample is in the valid portion of $C$. Valid samples provide a representation of the free portion of $C$ ($C_{free}$) and the invalid samples provide a representation of the invalid portion ($C_{obs}$). SBMPs have been applied to a wide variety of problems including in mobile robot navigation, robotic manipulation, molecular simulations and animation.

Sampling-based motion planning methods search for paths by constructing a roadmap or a tree that consists of a set of sampled states (nodes) that are connected by a path/trajectory that is defined by a reproducible function (e.g. straight-line interpolation). These paths / trajectories are referred to as local plans. Local plans can be tested for validity by stepping through the path/trajectory at a small interval and testing configurations for validity.

The PRM algorithm (Kavraki et al., 1996) constructs a roadmap by randomly sampling configurations across $C$. It then tests the samples for validity using a validity checker (e.g., a collision detector) and removes any samples that are invalid. It then constructs a roadmap in which there is a node for each valid sample. It then attempts to connect each sample to its $k$ nearest neighbors according to a distance metric (e.g. Euclidean distance). If the connection is successful, it adds an edge between the nodes of the roadmap that correspond to the sample and
the neighbor. This roadmap can then be queried by inserting a given start and goal node into the roadmap and using a search procedure like Dijkstra’s algorithm to find a path from the start node to the goal node. An illustration of this procedure is provided in Figure 2.1.

The RRT algorithm (Lavalle, 1998) builds a tree by incrementally expanding existing nodes. This method initializes the tree to contain the specified start node. It then incrementally expands the tree by randomly sampling in $\mathbb{C}$, locating the node in the tree that is closest to the sample according to some distance metric and expanding that node by $\epsilon$ in the direction of the sample. If the local path between the nearest node and the expanded node is valid then the expanded node is added to the tree. The algorithm then tests if the local path terminates in the goal region. If so, it extracts a path from the start to the goal by backtracking.

Recently there has been a great deal of focus on developing methods that produce good quality paths and in particular on methods that are guaranteed to converge to a good quality path. This has lead to the advent of asymptotically optimal motion planners planner RRT*, PRM* (Karaman and Frazzoli, 2011), RRT-# (Arslan and Tsiotras, 2013), Stable Sparse RRT (Li et al., 2016), RRT-Blossom (Kalisiak and van de Panne, 2006), Fast Marching Trees (FMT*) (Janson et al., 2015), AO-X (Hauser and Zhou, 2016), Dominance-Informed Region Trees (DIRT) (Littlefield and Bekris, 2018), AO-RRT2 (Kleinbort et al., 2020), and Batch-Informed Trees (BIT*) (Gammell et al., 2020). Asymptotically optimal methods are guaranteed to converge to an optimal solution as the number of samples approaches infinity (see related reviews on the subject: Bekris and Shome, 2020; Gammell and Strub, 2021).

RRT* (Karaman and Frazzoli, 2011) is an incremental approach that constructs a tree rooted at the start configuration, similar to RRT. At every iteration, the nearest tree node from a sample is used to steer towards the sample to generate a node to add. For each such new node, the best parent is selected from a valid neighborhood and steering is again used to connect the best parent to the new node. The best parent is decided in terms of the cost from the root of the tree to the new node via the candidate parent. The neighborhood is also rewired by checking connections from the new node to existing nodes inside the
neighborhood. This ensures that the algorithm maintains the *optimal tree* in terms of cost from the start for the set of generated nodes. The solution can be returned by tracing the tree backwards from a node inside the goal to the root.
Sampling-based motion planners, such as PRM (Kavraki et al., 1996) and RRT (Lavalle, 1998), are comprised of a set of primitive operations such as sample generation, collision detection and distance computation. This section discusses how machine learning has been applied to each of these primitive operations. It also discusses a set of adaptive methods that use learning to select between different combinations of primitive operations.

3.1 Sampling Sequences

One of the most intuitive ways to apply learning to SBMPs is to use it to improve the quality and efficiency of sampling. Learned models can be used to produce samples that are more likely to be valid and/or occur in critical regions such as narrow passages (Figure 3.1).

Categories of work on Sampling Sequences

- **Modeling the C-space** (Arslan and Tsiotras, 2015; Burns and Brock, 2005; Chamzas et al., 2019; Sutanto et al., 2020; Liu et al., 2020; Kingston et al., 2019)
3.1. Sampling Sequences

Figure 3.1: Given a planning problem: start (red), goal (green) and obstacles (grey), a learned sampling strategy predicts samples inside the blue region, characterized by parts of $C_{\text{free}}$ that lie along a solution path. Some of these samples (purple) lie inside critical regions, such as narrow passages.

- **Biasing samples along likely shortest paths** (Baldwin and Newman, 2010; Zucker *et al.*, 2008; Ichter *et al.*, 2018; Kumar *et al.*, 2019; Wang *et al.*, 2020; Ichter *et al.*, 2020; Huh and Lee, 2018)

3.1.1 Modeling the C-space

This class of approaches learn a representation of a problem’s $C$ and query the learned model to generate samples that are likely to be in relevant regions, or to filter out samples that are likely to be invalid.

One such method learns two probability distributions that model $C_{\text{free}}$ and $C_{\text{obs}}$ of a planning problem via a kernel density estimator function (Arslan and Tsiotras, 2015). The estimator approximates the likelihood that a drawn sample will be in $C_{\text{free}}$ or $C_{\text{obs}}$. The model is trained on previously drawn samples that have been collision-checked. During runtime, a Bayesian classifier is used to predict if a new sample lies in $C_{\text{free}}$ or not.

*Model-predictive Motion Planning* (Burns and Brock, 2005) uses
locally-weighted regression to build a model that predicts if a given sample is in $C_{free}$ or $C_{obs}$. This approximate model is constructed incrementally as a solution to the motion planning problem is computed. The sampling strategy associated with the learned model adapts sampling densities in proportion to an area’s complexity, allowing it to explore new regions efficiently.

The Global-Local Sampler (GL-Sampler) (Chamzas et al., 2019) uses a learned database of local samplers to discover the connectivity of configuration space. It decomposes the workspace into a set of local primitives. During the offline phase, it computes local samplers for these primitives and saves them to a database. During the online phase, the GL-sampler maps primitives to local samplers from the database, and then uses these samplers to synthesize a global sampler.

Learned models of implicit manifold configuration space (IMACS) generate tangent spaces that represent manifolds (Kingston et al., 2019). These tangent spaces provide piecewise-linear approximations of manifolds and can generate manifold samples and perform local planning on manifolds.

Equality Constraint Manifold Neural Network (ECoMaNN) (Sutanto et al., 2020) is a neural network that models manifold constraints that are commonly found in tasks like grasping and manipulation. It takes a configuration as input and outputs a predicted value that indicates the distance from the manifold. ECoMaNN is trained to minimize the errors between the tangents and normals of the manifold and those predicted by the model. The training data is augmented using off-manifold points. ECoMaNN is used in combination with sequential motion planning methods to generate paths on these manifolds.

Learned sampling distributions have also extended sampling-based planners to unknown environments (Liu et al., 2020). The distributions are derived from both geometric representations (e.g. occupancy grids) and object-level representations (e.g. contextual cues), and trained using a dataset of example trajectories. A sampling-based planner samples vertices from the learned distribution and scores roadmap edges using a proposed objective function.
3.1.2 Biasing samples along likely shortest paths

A second class of approaches learn a model that identifies paths in the environment, then uses the model to bias sampling along likely paths. Such methods can be trained to identify higher quality paths quickly.

Given paths produced by an expert, it is possible to use Learning from Demonstrations (LfD) to produce similar paths (Baldwin and Newman, 2010). This is accomplished by learning a non-parametric representation of the distribution of samples along the expert paths, modeled using an Infinite Gaussian Mixture Model (IGMM).

Early work included learning a sample distribution function over a discretization of the workspace that is then used to bias sampling towards critical regions such as narrow passages (Zucker et al., 2008). Sampling is performed by obtaining an instance from the probability distribution, then randomly generating a sample from the distribution of samples given this instance. The sample distribution function is obtained by reinforcement learning. The resulting motion plans are then evaluated using a reward function, and the parameters of the sample distribution are adjusted towards the direction of the approximate gradient of the reward function.

Conditional Variational Autoencoders (CVAEs) have been used to effectively learn sample distributions (Ichter et al., 2018). CVAEs are advantageous because they can represent complex, high-dimensional distributions that can be conditioned on arbitrary problem inputs. The CVAE is trained from demonstrations (examples of successful motion plans, human demonstrations, etc.) on similarly structured environments. In the online operation of the motion planner, samples are generated via the latent space of the CVAE, conditioned on problem-specific variables, such as start, goal and obstacle information (for e.g., an obstacle occupancy map). Theoretical properties are maintained by also generating samples using an auxiliary (uniform) sampler over the state space.

While the above approach is shown to be effective on many problems, it can potentially fail in the face of complex obstacle configurations, or mismatch between training and testing environments. Leveraging Experience with Graph Oracles (LEGO) (Kumar et al., 2019) addresses
these issues by predicting samples that belong only to bottleneck regions, and which are spread out across diverse regions to maximize the likelihood of a feasible path existing.

The *Neural RRT* (Wang et al., 2020) uses a convolutional neural network (CNN) to bias sampling towards predicted optimal paths. It formulates the problem of generating good samples as an image-to-image mapping problem where the network’s input is an RGB image of the workspace and robot-specific attributes such as the step size and clearance. The network outputs a probability density as a pixelation over the workspace image, where the probability associated with each pixel value indicates the likelihood that the pixel will contain the optimal path. The architecture is trained from a large dataset of optimal paths generated using an A* algorithm to minimize the cross-entropy between the predicted optimal-path probabilities and the ground truth. The predicted probability distribution is then used to generate samples to be used by an RRT* planner. In order to maintain probabilistic completeness, a proportion of samples are generated using uniform sampling.

The *Critical PRM* (Ichter et al., 2020) identifies critical states along solution trajectories using graph-theoretic techniques, and builds a dataset of critical states and local environment features (such as an occupancy grid) for different planning problems. Then, in a new environment, a criticality prediction network is trained to predict the criticality of a new sample. In the proposed Critical PRM method, a proportion of states are sampled proportional to their criticality, while the rest are sampled uniformly. Critical PRMs are demonstrated to achieve up to three orders of magnitude improvement over uniform sampling, while preserving the guarantees and complexity of SBMP.

*Q-function Sampling RRT* (QS-RRT) (Huh and Lee, 2018) learns a state-action-value function (Q-function) over $\mathbb{C}$, where a state corresponds to a node on the tree, and an action is an extension of the tree from that node. The approach makes use of a Radial Basis Feature (RBF) representation in $\mathbb{C}$ to improve the effectiveness of the Q-function learning. The Q-function is integrated using a softmax node selection procedure inside RRT, and the selection of the best tree expansion is performed using numerical optimization.
3.2 Collision Checking

Collision detection is often the most computationally expensive step of solving a motion planning problem (Kleinbort et al., 2016). It is therefore a promising direction to use machine learning to reduce the cost of this step (Figure 3.2). In order to avoid collisions in the real world, prediction errors in the form of false positives and false negatives must be suitably mitigated.

A practical strategy to minimize the number of collision checks while retaining theoretical properties of the motion planner is the idea of lazy collision checking (Bohlin and Kavraki, 2000; Hauser, 2015; Mandalika et al., 2019). Under this strategy, edges are not immediately checked for collision, but rather are checked only when a candidate path to the goal is found. Feasible edges are marked and infeasible ones are deleted. The
process repeats until a feasible path to the goal is found. This strategy avoids checking the vast majority of edges that have no chance of being on an optimal path.

**Categories of work on Collision Checking**

- **Identifying samples that are guaranteed to be valid** (Bialkowski et al., 2016; Bialkowski et al., 2013)
- **Using a learned model in place of a collision detector** (Burns and Brock, 2005; Huh and Lee, 2016; Das and Yip, 2020; Kew et al., 2020; Yu and Gao, 2021)
- **Determining the order in which to collision check nodes or edges** (Pan et al., 2013; Bhardwaj et al., 2019; Choudhury et al., 2017; Choudhury et al., 2018; Hou et al., 2020)
3.2. Collision Checking

3.2.1 Identifying samples that are guaranteed to be valid

This area of work uses learned models to identify samples that are guaranteed to be valid, eliminating the need to test them using an expensive collision checker. The Safety Certificate approach (Bialkowski et al., 2013; Bialkowski et al., 2016) proposes an adaptive sampling distribution that provably converges to a uniform distribution. When a sample is generated and collision-checked traditionally, this method stores a lower bound on the sample’s distance to the nearest obstacle. This distance, referred to as the safety certificate, defines a region of the search space that is guaranteed to be collision-free. Subsequent samples that are generated within a safety certificate are guaranteed to be valid and do not need to be collision-checked. As the number of samples goes to infinity the safety certificates asymptotically cover the entire search space and the amortized cost of collision checking becomes negligible with respect to the overall running time of the algorithm.

3.2.2 Using a learned model in place of a collision detector

The most common approach to learned collision checking is to replace a traditional collision checker with a learned model.

Model-based Motion Planning (Burns and Brock, 2005) also helps perform predictive edge validation rather than calling a dedicated collision detector. As discussed in Section 3.1, this method uses locally weighted regression to build an approximate model of $C$. This model can be used to predict if unsampled regions of $C$ are valid or invalid, and if edges are valid.

Gaussian mixture models (GMMs) have been used for collision checking in RRTs (Huh and Lee, 2016). The GMM is trained online during execution of the RRT using Incremental Expectation Maximization (EM). The training examples are generated by a traditional collision checker. By using biased sampling from the learned GMM distribution, the number of collision checks required for future iterations is shown experimentally to be reduced.

The Fastron-RRT (Das and Yip, 2020) uses a kernel perceptron to create a model of $C$ that can be used as a proxy kinematic-based collision detector. The kernel perceptron generates a set of support
points that form a separating hyperplane between two classes (in this case, $C_{\text{free}}$ and $C_{\text{obs}}$). Samples are collision-checked by querying which side of the hyperplane they lie on. *Fastron-RRT* can also operate under an active learning setting that updates the model’s hyperplane in order to reflect changes in the environment.

*ClearanceNet-RRT* (Kew et al., 2020) is a highly parallelizable extension of Fastron-RRT that uses a neural network (ClearenceNet) to predict obstacle clearance in order to obtain a heuristic for collision checking. The neural network obtains as input the poses of the robot and the obstacles in the environment, and outputs the distance to the nearest obstacle. This allows for the construction of an RRT where collision-checking can be deferred. Once a path has been found, ClearanceNet-RRT fixes errors in the path by performing gradient descent based path repair to move the path away from obstacles.

A recent approach to learned collision checking employs graph neural networks (GNNs) to reduce the number of collision checks on a Random Geometric Graph (RGG) (Yu and Gao, 2021). Given the vertices and edges of the graph, as well as problem-specific information, such as obstacle information and goal location, an attention-based neural network outputs vertex and edge embeddings, which are then used by the GNN. The GNN consists of a path exploration component that iteratively predicts collision-free edges to prioritize their exploration, and a path smoothing component that optimizes the obtained paths.

### 3.2.3 Determining the order in which to collision check nodes/edges

Another class of approaches use a learned model to determine the order in which nodes/edges are tested (collision checked) in order to more efficiently identify a valid path.

Instance-based learning can be used to store previous local planning queries and their outcomes (i.e. whether they were in collision or collision-free) (Pan et al., 2013). When a new sample is generated, this method identifies the $k$ nearest neighbors (or approximate nearest neighbors) from the nodes stored in a hash table. Given the neighbor set, the probability that the node is in collision is computed using a softmax function. This probability is then used as part of a collision filter for
local planning queries. It is also used as part of a method for exploring \( C \) around an existing sample in order to identify new samples for applications like RRT expansion. Finally, it is used to defer collision detection to query time in order to perform collision detection in a lazy manner.

**Search Though Oracle Learning and Laziness (STROLL)** (Bhardwaj et al., 2019) uses a neural network to determine which edges to collision-check expensively when performing a lazy search on a roadmap. It formulates the problem of edge selection as a Markov Decision Process (MDP) over the state of the search problem, and uses Q-learning to solve this MDP.

The problem of path validity prediction can be formulated as a Decision Region Determination (DRD) problem (Choudhury et al., 2017; Choudhury et al., 2018). This method first creates a training set from a sampled set of possible worlds (environments), that form a set of hypotheses. Then, it generates a roadmap without testing the edges and obtains a set of possible paths. Each path is valid in some of the sampled worlds and invalid in others. The set of worlds in which the path is valid forms the decision region for that path. This method then uses the \textit{DIRECT} algorithm to obtain a decision tree that indicates what edges should be tested in order to prune inconsistent worlds and identify if a path is valid.

**Posterior Sampling for Motion Planning (PSMP)** (Hou et al., 2020) formulates anytime search on graphs as an instance of Bayesian Reinforcement Learning (Bayesian RL). Unlike prior work, PSMP aims for anytime performance by leveraging learned posteriors on edge collisions to quickly discover an initial feasible path and progressively yield shorter paths.

**Discussion on Collision Checking**

The most straightforward way of applying a learned collision detector is to model it as a classification problem. For a static environment, an offline dataset of robot configurations in \( \mathcal{C}_{\text{free}} \) and \( \mathcal{C}_{\text{obs}} \) can be collected to train an appropriate machine learning model that predicts if an input configuration is in collision. It can also be
beneficial to use a learned model that outputs a confidence score about its prediction (e.g., a Bayesian model that outputs class probabilities). For predictions that have a confidence score below a fixed threshold the planner can query a more expensive, traditional collision checker.

If traditional collision checking, including with lazy evaluation, proves to be relatively expensive, it makes sense to use a learned model to instead determine the order in which to test nodes/edges.

3.3 Distance Metrics, Heuristics and Constraints

In SBMP, distance metrics are used to estimate the viability of connecting samples and as a cost function for optimization. A common choice of metric is the Euclidean distance, under the intuition that the further the robot has to travel between states, the more likely that it will encounter an invalid state along its path. However, this metric does not take into consideration the topology of the environment, the complex motions performed by the rigid bodies of the robot, or the robot dynamics. For applications where a more accurate distance metric may be available, it may be computationally prohibitive to compute. Moreover, for many applications it is not immediately obvious what an accurate distance metric would look like.

Many sampling-based motion planners also use an estimate of the cost-to-go from a node to the goal in order to bias the selection of nodes that are closer to the goal, and as a heuristic to speed up the search for the best solution. If the cost-to-go metric used is admissible, then it can be added to the cost of getting from the start node (the cost-to-come) to obtain a lower bound on the possible cost of paths that go through the node (Figure 3.3). This can be used to give preference to expanding nodes that may yield shorter paths to the goal, while pruning nodes that cannot.

Categories of work on Distance Metrics, Heuristics & Constraints

- Approximating distance metrics
Figure 3.3: The cost-to-go is a heuristic estimate of the cost to move the robot from its current configuration to the goal. In this example, the Euclidean distance of the sample to the goal (orange line) is used to approximate the cost-to-go.

(Chiang et al., 2018; Bharatheesha et al., 2014; Palmieri and Arras, 2015; Li and Bekris, 2011; Knobloch et al., 2018)

- **Approximating cost-to-go values** (Arslan and Tsiotras, 2015; Phillips-Grafflin and Berenson, 2015; Ye and Alterovitz, 2017; Huh et al., 2021)
- **Planning for systems with dynamics** (Kalisiak and van de Panne, 2007; Li et al., 2018)
- **Modelling the effect of a heuristic** (Hauser et al., 2005; Bhardwaj et al., 2017; Yuan et al., 2021; Wells et al., 2019; Kim et al., 2019; Zhao et al., 2020)

### 3.3.1 Approximating the distance metric

A straightforward approach is to use a machine learning model to learn an approximation of a complex distance metric. The learned model can then be used in place of a computationally expensive distance computation step in the planning phase.
For instance, the *swept volume* is the volume in the workspace, that the local planner sweeps over while passing between two configurations. It is generally considered to be an ideal distance metric, however it is very expensive to compute online. Deep neural networks can be used to estimate the swept volume between two points (Chiang *et al.*, 2018). The network is trained in an obstacle-free environment from examples of pairs of configurations and the swept volume between them. The trained network, alongside a hierarchical neighborhood search procedure, is shown to be able to approximate swept-volume distance effectively and efficiently.

Parametric models like locally-weighted projection regression can be used to approximate distances (Bharatheesha *et al.*, 2014). The model is trained by approximating the optimal cost between pairs of states using an iterative Linear Quadratic Regulator (iLQR). The learned model approximates the original cost with a reasonable tolerance and gives a speed-up of a factor of 1000 over computing the actual cost. It is also possible to use a simpler non-linear parametric model to approximate distances (Palmieri and Arras, 2015). The model is trained using a novel steering function that solves the two-point boundary value problem (BVP) between two configurations of a wheeled mobile robot.

The distance between two states can also be approximated by constructing a densely sampled roadmap offline and querying the roadmap during planning time (Li and Bekris, 2011). The graph is generated by sampling a large number of states in an obstacle-free environment and then applying controls to the states. Two states \((s_i, s_j)\) are considered to be connected if there is a control that extends the state \(s_i\) to a state that is close to \(s_j\). To reduce the online cost, the sampled states are mapped into a higher-dimensional Euclidean space through multi-dimensional scaling (MDS) that retains the relative distances represented by the sampled graph.

Orthogonal to the other methods discussed above, the *Distance-Aware Dynamic Roadmap* (DA-DRM) (Knobloch *et al.*, 2018) learns a voxel distance grid which is updated using perceptual information from a humanoid robot’s perception system. The learned distance information is used to estimate costs during roadmap search, and for path smoothing.
3.3. Distance Metrics, Heuristics and Constraints

3.3.2 Approximating the cost-to-go value

Informed SBMPs typically exploit the structure of the planning problem via an approximate cost-to-go value (or a heuristic) to improve the practical efficiency of the search process (Gammell and Strub, 2021). It may thus be beneficial to learn a function that approximates the problem-specific cost-to-go.

Similar to early work in approximating distance metrics, early work models the problem of learning an approximate cost-to-go as a supervised regression problem (Arslan and Tsiotras, 2015). It uses a training dataset where each data point consists of a randomly drawn point along with its locally minimum cost-to-come estimate (lmc value). Distance-weighted regression is used to fit a surface from points in the training set and to estimate estimate the cost-to-go of new points. See Section 3.1 for additional information on the proposed method.

For problems where the cost metric may be difficult to mathematically represent (like robot surgery), it can be learned from demonstrations through active learning and Inverse Optimal Control (IOC) (Phillips-Grafflin and Berenson, 2015). Similarly, Demonstration-Guided Motion Planning (DGMP) (Ye and Alterovitz, 2017) learns a cost function that encodes task constraints for household tasks (for e.g., a full glass of water must be held upright) from expert demonstrations of the task. The learned cost function extracts time-dependent task constraints by learning low variance aspects of the demonstrations, which are correlated with the task constraints. The learned cost metric is optimized using Multi-Component Rapidly-Exploring Roadmaps (MC-RRM), a SBMP technique.

A neural network architecture, c2g-hof (Huh et al., 2021), takes as input a point cloud representation of the workspace and a goal configuration to output a cost-to-go estimate. It is trained on a dataset of costs between every pair of configurations in randomly generated workspaces. During execution, the gradient of the output cost-to-go is followed to yield continuous, collision-free trajectories.
3.3.3 Planning for systems with dynamics

Planning with *viability filtering* (Kalisiak and van de Panne, 2007) learns a viability model of an agent’s *perceptual space* that is used to direct planning. This method uses a virtual range finder sensor to model the robot’s perception at a given state. It trains a viability classifier on a large set of motions along with their associated virtual perceptions that can be obtained from previous solutions or from simulation. This method uses the classifier online to limit the planner’s search to motions that lead to viable regions. It improves planning efficiency by preventing the planner from exploring nonviable regions, which will unavoidably lead to failure.

Near-Optimal RRT (NoD-RRT) (Li et al., 2018) uses a neural network to predict the cost between two given states considering nonlinear constraints. The neural network is trained on examples where the two point BVP is solved between pairs of states. The authors show theoretically that NoD-RRT is asymptotically optimal.

3.3.4 Modelling the effect of a heuristic

In some cases, it may be easier to model the effect of using a good heuristic, such as the order in which nodes must be expanded during search.

In applications like rock climbing where multiple queries (locomotion, manipulation, contacts) must be processed to solve a specific planning problem, it is imperative to decide quickly if a query is solvable or not. Towards this objective, it is possible to train a classifier (Hauser et al., 2005) to predict if a query is solvable. The classifier is trained using data obtained from many iterations of Basic-MSP (multi-step planning) in randomly generated terrains. This classifier can then be integrated with a multi-step planning algorithm to avoid spending computational effort in solving infeasible queries.

*Search As Imitation Learning* (SAIL) (Bhardwaj et al., 2017) uses a heuristic policy that is trained to imitate an oracle that has perfect knowledge of the world and is capable of making decisions that minimize the required search. SAIL posits that the manner in which a tree-based planner explores an environment can be seen as a wave-front and that
it is desirable to keep the wave-front as small as possible while focusing
the search towards the direction of the goal. It formulates the problem
of selecting nodes for expansion as a sequential decision making problem
in which information extracted from the wavefront is used to decide
what nodes to expand. It then uses dynamic programming to compute
the heuristic values for all states to determine the order of exploration.

Piecewise Linear Regression Complex (PLRC*) (Zhao et al., 2020) is
a data structure that approximates the costs of crossing local regions of
a configuration space using piecewise linear regression. It is constructed
by decomposing $C$ into a set of cells. For each cell, pairs of points
are sampled along the boundary, and the distance between them is
calculated and stored. Then, this data is used to build a regression
model that can return the costs between boundary points of every cell.
For simple problems, the memory required to store this representation
compares favorably to that required for standard discrete vertex-and-
edge models, while achieving similar path quality.

There has been recent interest in methods that encode prior informa-
tion about the planning problem in the form of a sparse data structure.
Yuan et al., 2021 constructs a topological feature tree and computes
a heuristic path on the tree. The computed path is then integrated
with an SBMP to improve the success rate in environments with dynamic
obstacles.

Learned models have also been used to improve the efficiency of Task
and Motion Planning by learning a classifier for feasible motions, and
using the output of this classifier as a heuristic for guiding the search
(Wells et al., 2019). The classifier is trained on minimal exemplar scenes.
On more complex scenarios, principled approximations are applied so
as to minimize the effect of errors. Even in the presence of classification
errors, properly biasing the heuristic ensures that the search is still
orders of magnitude faster than an uninformed search.

An algorithm for learning to guide a planner (Kim et al., 2019)
proposes learning to predict constraints on the solution. A new type
of problem-instance representation called score space is proposed. Its
main advantage is that it relays information about the similarity of
problem instances. To transfer knowledge from past experience, the
algorithm uses the correlation information in the problem score space
representation of a given instance. This limits planning to the constrained subspace, which reduces the size of the space that needs to be searched.

**Discussion on Distance Metrics & Heuristics**

Learned models, such as those based on neural networks, can be used to approximate a more expensive to compute distance / cost-to-go metric, if a considerably large dataset of pairs of states and the true distance / cost-to-go between them can be collected. Access to the aforementioned offline dataset is not possible for some systems because it is not clear what a good distance / cost-to-go metric would look like. Approximating the distance metric computation using a learned model opens up interesting avenues for speeding up the nearest neighbor search component as well. There is a caveat, however. Due to the inherent bias-variance trade-off in any machine learning model, it is possible for the predicted distance / cost-to-go to have some error, which must be taken into consideration.

It is also possible to use learning to simulate the effect of using a good heuristic, that biases the sampling of nodes to find higher-quality solutions more quickly - similar to some of the models discussed in Section 3.1. These methods, however, are not directly applicable to systems with constraints on their motion (such as dynamics), or where the cost function being optimized encodes task-specific attributes (such as grasping).

### 3.4 Steering Functions and Local Planners

One of the requirements of many state-of-the-art SBMPs is the availability of a local planner (or a **steering function**) that connects two states of the system being planned for. For some problems, such as those with constraints on the robot dynamics, a steering function corresponds to a boundary value problem (BVP) solver, which may not be available, or may be computationally expensive to query. This has led to the development of learned models that can be used in place of local planners.
Categories of work on Steering Functions & Local Planners

- Learned steering functions for kinodynamic systems
  (Faust et al., 2017; Faust et al., 2014; Faust et al., 2018; Chiang et al., 2019; Sivaramakrishnan et al., 2021; Zheng and Tsiotras, 2021)

Reinforcement learning offers many promising avenues to learn a local planner (or a policy). It has been used to enable aerial robots to quickly navigate an environment and deliver payloads while complying with the dynamic constraints introduced by these loads (Faust et al., 2017; Faust et al., 2014). The proposed method learns a policy for the system, that minimizes the residual oscillations in the load. The large size of the action space associated with planning for aerial vehicles makes direct policy learning impractical for this system; so the policy is trained on a simplified problem space. During the planning stage, a PRM is used to generate collision-free paths. The learned policy is then used to transform these paths into trajectories that satisfy the constraints imposed by the suspended load.

**PRM-RL** (Faust et al., 2018) is a long-distance navigation method that uses reinforcement learning to obtain a policy for point-to-point navigation. The policy is first trained in a simple environment where it learns dynamics of the system, along with any task constraints and sensor noise. The trained policy is then used during edge connection to obtain paths between candidate neighbors. Neighbors are connected if the learned policy can consistently find a path between them. Similar to PRM-RL, **RL-RRT** (Chiang et al., 2019) uses a learned obstacle-avoidance policy to perform local planning between two states, along with a learned reachability estimator to bias tree growth. The reachability estimator is trained via supervised learning to predict the time required for the obstacle-avoidance policy to reach a state given the obstacles that are present.

Recent work (Sivaramakrishnan et al., 2021) has proposed training a goal-reaching controller in an obstacle-free environment and applying this controller for planning in environments with obstacles. The node
expansion procedure makes use of an informed local goal generation strategy for input to the learned controller. The expansion procedure is integrated inside the AO DIRT (Littlefield and Bekris, 2018; Granados et al., 2021–2021) kinodynamic planner for improving success rate and solution quality in vehicular navigation problems.

If optimal trajectories between two states can be computed offline, a supervised learning controller can be trained to steer the system from points in an initial state set to points in a final goal set. Learning-based kinodynamic RRT* (Zheng and Tsiotras, 2021) integrates this learned controller along with a learned cost-to-go metric.

**Discussion on Steering Functions and Local Planners**

In order to improve the efficiency of sampling-based kinodynamic planning via a learned model, it is possible to either learn a function that predicts the cost-to-go between two states, or a local planner than can viably connect two states, or both. Recent advances in deep reinforcement learning have focused on learning value functions and policies for a variety of different robotics tasks, but require a large dataset of interactions with the environment. It is a promising area of research to integrate these RL advances into SBMPs, which allow for planning over longer horizons, while maintaining theoretical properties and sample efficiency considerations.

### 3.5 Adaptive Selection of Primitives and Meta-Reasoning

Many planning algorithms have been developed that use different combinations of sampling methods, neighborhood finders, expansion methods and connection strategies. It is often difficult to ascertain what combination of methods is best for a particular problem. One way of addressing this issue is to use learning to predict what methods or combinations of methods work best for a new planning problem. In some cases, it is useful to determine when to stop computational effort for a given planning problem. This form of reasoning is known as *meta-reasoning*, and recent work has proposed machine learning-based solutions.
3.5. Adaptive Selection of Primitives and Meta-Reasoning

Categories on Adaptive Selection & Meta-Reasoning

- **Planner selection** (Morales et al., 2005; Choudhury et al., 2015)
- **Connection strategy selection** (Ekenna et al., 2013; Uwacu et al., 2016)
- **Sampler selection** (Hsu et al., 2005)
- **Paths & parameter selection** (Chamzas et al., 2021a; Moll et al., 2021)
- **Meta-reasoning** (Li and Dantam, 2021; Sung et al., 2021)

3.5.1 Planner Selection

One adaptive selection method uses hand-crafted features in the environment to determine which combination of primitive methods to use (Morales et al., 2005). It first partitions the environment by recursively dividing $C$ until it obtains regions that are classified as homogeneous according to a set of features, which are obtained by constructing a simple PRM in each region and collecting statistics such as free node ratio, number of connected components, etc. Then, it uses a decision tree classifier which labels each region as free, cluttered, narrow passage or non-homogeneous based on the features observed in that region. The decision tree is constructed using feature data that is collected from known examples of free, cluttered, narrow passage, and non-homogeneous environments. The planner that is selected for each region type is specified by the user (for e.g. PRM in the free regions and OBPRM in cluttered, narrow passage, and non-homogeneous environments).

The Planner Ensemble (Choudhury et al., 2015) learns a mapping from applications to appropriate methods. From a set of complementary planners (including both sampling-based planners and trajectory optimization methods), it selects which planner to use for a specific application by learning priors on the planners’ performance. The priors are then used to estimate the probability that each method will be able to solve a new problem in order to select the ensemble of planners that has the maximum likelihood of finding a feasible solution. The model is trained on a dataset consisting of 10k planning problems that are
Learning Primitives of SBMP

generated by taking permutations and combinations of various primitive scenarios.

### 3.5.2 Neighborhood Connection Strategy Selection

The *Adaptive Neighbor Connection* (ANC) strategy (Ekenna et al., 2013) uses a learned model to select from a set of neighborhood connection strategies based on their cost and how well they perform. A neighborhood connection strategy consists of a candidate neighbor selection method (e.g. *k*-closest, *k*-Rand, etc.) and a distance metric (e.g. Euclidean, swept volume). ANC maintains a selection probability for each method, which is initially the same for all methods. ANC selects a method according to the probability distribution, and after the connection step, it applies a reward or penalty to the selected method based on how many connections were successful and the number of collision detection calls made. It then updates the selection probability of the methods based on this penalty/reward. An extension of ANC (Ekenna et al., 2015) uses a *local* learning method that selects appropriate connection methods for roadmap construction. As with ANC, this method dynamically divides the environment into regions. It then selects what connection policy to use in each region using an adaptive probability distribution. Extensions of this work have shown that introducing randomness into neighbor selection during training improves learning efficiency (Uwacu et al., 2016).

### 3.5.3 Sampler Selection

Another approach uses machine learning to adaptively combine multiple samplers (such as a Gaussian or Uniform sampler) by observing the performance of each sampler, and selecting the samplers that perform well more frequently (Hsu et al., 2005). This method maintains a probability for each sampler based on its cost and performance. To compute this probability, it maintains a weight for each sampler that captures its past performance. In order to ensure that all samplers have a reasonable chance of being tried, the weights for each sampler are initially assigned to be equal. These weights are then used to compute a *cost-insensitive* probability, which is scaled by the inverse of the cost of that sample to
obtain a selection probability for each sampler. At each timestep, the method selects a sampler based on this probability.

### 3.5.4 Paths & Parameter selection

*SPARK* and *FLAME* (Chamzas et al., 2021a) are experience-based frameworks for complex manipulators in 3D environments. Both *SPARK* and *FLAME* build an experience database of local primitives, which are created through a decomposition of the environment to capture local workspace features. Each primitive is associated with a local sampler, which, given previous experiences, is able to generate samples in critical regions of the primitive. This method solves motion queries by retrieving similar primitives from the database and combining their local samplers. A follow-up work, *FIRE* (Chamzas et al., 2022) extracts local representations of various planning problems to learn the similarity function over them.

*HyperPlan* (Moll et al., 2021) formulates the problem of motion planning algorithm selection and parameter optimization as a hyperparameter optimization problem. It defines a loss function that captures the planning speed, combined planning and execution time, as well as the convergence to optimal solutions. Bayesian Optimization can then be used to optimize this loss function.

*ALEF* (Kingston et al., 2021) uses experience from similar planning queries to efficiently solve constrained motion planning for multi-modal problems, such as the ones that arise during manipulation when an object is grasped, or the end effector is constrained to move along a straight-line path. ALEF builds a sparse roadmap within an augmented, manifold-constrained state space which aggregates experience gathered from different single-mode problems within a mode family. Upon a new query, paths from this roadmap are retrieved and used to bias sampling in an SBMP.

### 3.5.5 Meta-Reasoning

Given the intuition that search trees rooted at the start and goal of a planning problem form two separate classes in $C$, recent work has proposed learning a manifold that separates these classes (Li and
Dantam, 2021). Points are then sampled on the manifold, and a closed polytope is constructed to approximate it. Under some assumptions, the output is either an infeasibility proof of the planning problem, or a motion plan that connects the start and goal states.

Data-driven learning methods for *meta-reasoning* have been used to model the relationship between solution quality and planning time in order to determine when to stop planning (Sung et al., 2021). Data for this method is generated by running an anytime planner on training problems for a long enough time to obtain an optimal solution with a high probability. A model-based meta-reasoning method is proposed, which formulates the problem as a MDP, along with a model-free variation, which uses supervised learning in the form of a recurrent neural network (RNN).

### Discussion on Adaptive Selection & Meta-Reasoning

The application of machine learning tools to adaptively select sampling-based motion planning primitives offers many promising avenues for exploration. Existing methods estimate the utility of different primitives or their combination by constructing a probability distribution that reflects their relative utility. These distributions are conditioned on features extracted from the environment, which can either be handcrafted or learned through a more expressive model. It is also interesting to consider learning the parameters of this distribution in a reinforcement learning (RL) setting, rather than a supervised setting.

Another set of approaches accommodate heterogeneous problems by dividing them into *regions*, and maintaining a separate probability distribution for each region based on the performance of different primitives in that region. Given the advances made in the expressive power of models like deep neural networks, it would be interesting to see if the classification of these regions can be learned automatically from prior planning experience, rather than through handcrafted features.
This chapter looks at machine learning models that learn from prior experience to help solve a sampling-based motion problem, but do not explicitly focus on learning a single primitive.

---

**Categories of work on Learning-based Pipelines**

- **Retrieve and repair methods** (Berenson *et al.*, 2012; Coleman *et al.*, 2014)
- **Neural motion planning** (Qureshi *et al.*, 2019; Jurgenson and Tamar, 2019; Strudel *et al.*, 2020; Johnson *et al.*, 2020; Li *et al.*, 2021; Qureshi *et al.*, 2021; Qureshi *et al.*, 2020)
- **Learning a lower dimensional planning space** (Ichter and Pavone, 2019; Chen *et al.*, 2019; Ichter *et al.*, 2021)

---

### 4.1 Retrieve and Repair Methods

This class of models use a database of paths and solve queries by retrieving these paths and adjusting them to generate solutions to new problems (Figure 4.1). The *Lightning* framework (Berenson *et al.*, 2012) incorporates a *Retrieve-Repair* module that retrieves a path from a
library stored on the cloud that includes experience (e.g. paths) collected from multiple robots. *Lightning* selects a path from the library using two heuristics: one that quickly selects a set of candidate paths using the distance between their endpoints and the start configuration, and one that selects the path with the least constraint violation among these candidates.

Similarly, the *Thunder* framework (Coleman *et al.*, 2014) is an experience-based planner that stores paths obtained by probabilistic sampling in a SPArse Roadmap Spanner (SPARS). To solve a new problem, it searches the database for a solution that was able to solve a similar problem and uses this solution as a basis to solve the new problem. It identifies parts of the solution that are no longer feasible given the new problem and “repairs” them to form a valid solution. This approach is well suited for large configuration spaces and spaces that include invariant constraints.

### 4.2 Neural Motion Planning

This class of methods uses a dataset of solved motion planning problems to learn a neural network model that improves the efficiency of motion planning on different, yet similar problems (Figure 4.2). *Motion Planning Networks (MPNet)* (Qureshi *et al.*, 2019) is a deep neural network architecture which learns to approximate the computation of a sampling-based motion planner. MPNets are comprised of an encoder network and a planning network. The encoder network takes as input a point-cloud
Figure 4.2: Architecture of a neural motion planner (NMP) (Qureshi et al., 2019). The NMP takes as input the current state $x_i$, the goal state $x_G$ and a representation of the environment such as a point cloud. (Left) The NMP outputs $\hat{x}_{i+1}$, the predicted next state of the robot at the next timestep, and attempts to connect to it. (Middle) If the connection fails, a path from the goal is attempted. (Right) If the connection succeeds, the predicted state is added to the solution path. A hybrid approach uses the NMP alongside a traditional SBMP like RRT*.

representation of a workspace and learns to encode this point-cloud into a latent space. The planning network takes as input the latent space, the robot’s configuration at the current time-step and a goal configuration, and it is trained to predict the robot’s configuration at the next time-step. The planning network is used along with a bi-directional iterative search algorithm to generate trajectories that are feasible given the problem’s constraints.

Deep Deterministic Policy Gradient for Motion Planning (DDPG-MP) (Jurgenson and Tamar, 2019) investigates an alternative approach to training a motion planner approximated as a neural network (referred to as a neural motion planner). The approach is based on reinforcement learning rather than supervised learning, and achieves higher accuracy by actively exploring the problem domain.

An integral component of neural motion planners is the learned representation of the problem’s obstacles from sensory input. Recently, the PointNet architecture (Qi et al., 2017) has been used to learn an efficient and effective representation of point clouds for neural motion planning (Strudel et al., 2020). This representation is then concatenated with the goal configuration and passed to a neural network which outputs actions. This network uses the Soft Actor-Critic (SAC) reinforcement learning algorithm to learn the control policy online.

Dynamic MPNet (Johnson et al., 2020) extends the MPNet framework to non-holonomic robots. The network is trained on example solution trajectories produced by an RRT* planner to predict the next
Learning-based Pipelines

state that the robot must steer to. The network receives as input the goal state and a costmap of the local area surrounding the robot’s current state. The neural planner is called iteratively at every state until it produces a feasible solution trajectory. If no such trajectory is found, the method falls back to classical motion planning.

MPC-MPNet (Li et al., 2021) extends the MPNet framework to deal with the challenges of kinodynamic planning. The network is trained on example solution trajectories produced by the AO SST algorithm (Li et al., 2016) to predict a batch of candidate next states (waypoints) given a batch of environment encodings as well as current and goal states. Parallelized Model Predictive Control (MPC) is used to steer the robot from its current state to the predicted waypoints. The procedure is repeated either in a greedy or tree-based search framework to obtain kinodynamically feasible solution trajectories in new planning environments.

Constrained MPNets (CoMPNet) (Qureshi et al., 2021) and its extension, CoMPNet X (Qureshi et al., 2020) extend the MPNet framework to the problem of motion planning in the presence of task constraints. CoMPNet takes as input a task and observation encoding and outputs an intermediate configuration for path planning. The demonstrated trajectories are collected using a bi-directional RRT planner. CoMPNetX introduces neural-gradient-based projections to generate informed, implicit manifold configurations that can speed up any SBMP.

4.3 Learning a lower dimensional planning space

Under this relatively new framework, a learned model maps a high-dimensional motion planning problem into a lower dimensional state space (Figure 4.3), and uses SBMP techniques to solve the lower dimensional problem. Learned Latent RRT (L2RRT) (Ichter and Pavone, 2019) learns this mapping via an autoencoder. It constructs a tree directly in this lower dimensional space by propagating randomly sampled controls in the lower dimensional space using a learned latent dynamics model, and checking for collisions in the lower dimensional space using a learned collision checking neural network. The autoencoder and latent dynamics model are trained using trajectories obtained by iteratively
4.3. Learning a lower dimensional planning space

Figure 4.3: Example of using learned lower dimensional space. A simulated quadruped robot (left) with a 111-dimensional state space and 8-dimensional control space must navigate the maze to reach the goal (red circle). It may be practically infeasible to apply a SBMP directly to this problem, so a learned encoder model transforms the problem into a lower-dimensional space (right) where a SBMP can be applied to find a solution. Once a solution has been found, it can be applied directly to the high-dimensional problem via a learned decoder.

propagating random samples from the control space for a fixed number of time steps. The collision checking network is trained using randomly samples from low dimensional states (obtained via the trained encoder) that are in $C_{\text{free}}$ and $C_{\text{obs}}$. The collision checking network outputs the probability of a latent state $z_t$ being in collision or not, and L2RRT makes use of a pre-defined confidence threshold to decide whether to trust the learned collision checker or not.

The Neural EXploration-EXploitation Tree (NEXT) (Chen et al., 2019) is a meta neural motion planner that learns generalizable problem structures from previous planning experiences. An attention-based neural network architecture is trained to extract a planning representation from demonstrated paths on multiple random problems. The learned tree expansion procedure can balance exploration and exploitation by using an Upper Confidence Bound (UCB) style algorithm.

Broadly-Exploring, Local-policy Trees (BELT) (Ichter et al., 2021)
is an RRT-like algorithm for long-horizon task planning that plans over an abstract task space, which is learned from demonstrations. The exploration of the RRT is guided by a task-conditioned, learned policy capable of performing general short-horizon tasks. Once a solution is found by using search, the local task-conditioned policy is capable of executing the trajectory on a real robotic platform.

**Discussion on Learning-based Pipelines**

There is a growing body of work in collecting prior motion planning experience, and using learned models to imitate the motion planner’s performance on tasks from a similar domain. These pipelines may also project a higher dimensional problem into a reduced space, where it may be easier to find a feasible path. These pipelines can result in practical efficiency by reducing expensive SBMP operations to a few inference calls to a learned model. Nevertheless, there are considerations about the size of the datasets required to train these models, their generalization capabilities, as well as theoretical guarantees on path quality, that remain open directions for future research.
This section looks into how machine learning has been used to handle, and plan under, uncertainty. Noisy sensors, actuators and external agents are potential sources of uncertainty. Sampling-based planners have been applied to planning under uncertainty through belief space planning. As with kinodynamic planning, many belief-space planning problems do not have access to a steering function, and thus planning must be done using only forward propagation. Uncertainties can also arise from the unavailability to measure parameters like wheel-ground friction, air friction, spring constants etc.

**Categories on SBMP with Learned Models**

- **Modeling robot uncertainty** (Kimmel et al., 2019; Sintov et al., 2020; McConachie et al., 2020; Malone et al., 2012; Wang et al., 2021; Curtis et al., 2022; Bähnemann et al., 2017; Burri et al., 2018; Granados et al., 2022)

- **Modeling uncertainty in obstacle dynamics** (Aoude et al., 2013; Zhu, 1991; Fulgenzi et al., 2008; Fulgenzi et al., 2010; Zhang et al., 2020)
5.1 Modeling Robot Uncertainty

Learned models have been used for systems where precisely modeling the motion of the objects being manipulated is infeasible (for instance, manipulation with underactuated hands) (Kimmel et al., 2019). This method learns a stochastic model of the system’s dynamics from a dataset of trajectories with randomly sampled actions. Two choices of stochastic models - a Bayesian neural network and a Gaussian Process (GP) are explored. The trained dynamics model is used with a sampling-based planner that represents beliefs as a particle cloud over the state space. The model forward propagates these particle clouds in order to construct a planning tree. Critic models have also been applied in this domain to estimate the error in the learned transition model (Sintov et al., 2020). The output of the critic model is used as a cost function by an AO planner to direct planning towards regions where there is less uncertainty in the learned dynamics.

Learned models have also been applied to problems where the underlying system may be difficult and/or time consuming to model correctly, and thus the planner acts over a reduced state space (McConachie et al., 2020). Training data for this model is collected by generating plans in the reduced state space, and then applying these plans to the true system. This data is used to train a classifier that labels plans generated in the reduced state space as reliable or unreliable. The learned model is incorporated into an RRT-based planner in order to bias sampling away from transitions that are labeled as unreliable.

The Brain-Emulating Cognition and Control Architecture (BECCA) (Malone et al., 2012) uses online reinforcement learning to exploit task structure as well as to address environmental noise and hardware imperfection inherent to manipulation tasks. At every time step, BECCA makes an observation in the world, extracts features from the sensory input, performs an action in response to the input, and receives a reward. The feature extractor identifies patterns and correlations in the input vector in an unsupervised manner. The action selector learns, via online reinforcement learning, a mapping from features to actions that lead to the highest recorded reward. The architecture is trained on a simulated system until it obtains satisfactory performance, and
is allowed to update its model after being deployed. It is mapped to a PRM by reducing the agent’s state space to roadmap nodes, and limiting the agent’s actions to edges between the nodes.

Conditions for pouring or scooping skills are learned using GPs to enable risk-aware predictions (Wang et al., 2021). Alongside active learning, this approach reduces the model’s uncertainty. The learned skills are then used by a RRT-Connect (Kuffner and LaValle, 2000) to produce geometric paths for the gripper. A related approach (Curtis et al., 2022), explores ways to implement perceptual learned modules for shape estimation and grasp generation that are then used by a SBMP.

Hybrid data-driven approaches have been used to learn models for robots with imprecise or uncertain dynamics. These techniques combine physics models of a robot with machine learning methods to self-tune the physics models, in order to accommodate robot design imprecision. A crucial component is generating valuable data in order to reduce the tuning time. For this purpose, a modified RRBT planner is used to search for informative trajectories, those which maximize the information gain of the specified unknown parameters (Bähnemann et al., 2017). This method is used for calibrating a micro aerial vehicle in a constrained environment with minimal user interaction, and is later applied in Burri et al., 2018 to produce trajectories for accurate parameter estimation based on maximum likelihood. Another system identification approach is to, given the dynamical model of the system, learn the unknown friction coefficients. The learned model is used by a SBMP to generate more reliable trajectories (Granados et al., 2022).

5.2 Modeling Uncertainty in Obstacle Dynamics

RR-GP (Aoude et al., 2013) applies learned Gaussian Processes (GPs) for probabilistic safe motion planning with dynamic obstacles and uncertain motion patterns. RR-GP combines GPs with RRT-Reach to build a learned motion pattern model that can be used to predict obstacle trajectories. These predictions are conditioned on feasible paths which are identified using reachability analysis. RR-GP uses a chance-constrained RRT to identify probabilistically feasible paths.

Hidden Markov Models (HMMs) have also been used to model the
possible motion of dynamic obstacles (Zhu, 1991). Obstacles are modeled as a stochastic process within the HMM, which is queried to obtain obstacle motions and potential variations of the motion states. Planning is performed using a trajectory-guided path-planning algorithm which samples and evaluates a set of candidate trajectories.

Learning has been used to predict motions of dynamic obstacles, such as pedestrians or vehicles, in order to reduce the risk of collision (Fulgenzi et al., 2008; Fulgenzi et al., 2010). Motions of obstacles are modeled using GPs, which are learned by observing an environment to capture common behaviors and patterns. This model provides predictions, which are continuous over space and time, and when applied to planning gives better performance than kinematic-based predictions.

In a similar approach, Zhang et al., 2020 uses SBMP in the context of autonomous driving. This work proposes labeling and training models to more accurately predict a vehicle’s intention, in order to sample states leading to a high-quality collision-free trajectory. Prediction of surrounding vehicles and imitation learning are used to generate collision-free samples near the human-driving trajectory that are used by the planning algorithm.

**Discussion on Planning with Learned Models**

Machine learning models, such as Bayesian Neural Networks and Gaussian Processes (GPs), allow to reason about uncertainty in a learned model. When SBMP techniques are applied to problems involving robots in the real world, they have to often deal with physical processes that cannot be modeled perfectly. A promising area of research is to use these stochastic machine learning models to improve the efficiency and accuracy of sampling-based planners in a way that allows for the underlying uncertainty of the model.
This monograph studied how machine learning has been used to improve the performance of SBMPs. This can be achieved by using machine learning models to approximate the primitive operations of a sampling-based planner or to select between different choices of primitive operations. It also discussed a relatively new area of research, where machine learning models have been proposed that approximate the operation of an optimal sampling-based planner. Finally, it briefly covered some of the literature where sampling-based planning can be performed over a learned robot and/or environment model.

Many SBMPs exhibit desirable theoretical properties, such as probabilistic (or resolution) completeness and asymptotic optimality, but may be slow to converge to high quality solutions in practice. One promising direction to improve the quality of solutions is to work within the SBMP framework and provide desirable guarantees, while improving the practical convergence rate by using efficient operations based on machine learning models. This monograph concludes by briefly evaluating the successes (and failures) of the different proposed integration techniques, in the context of the practical challenges described in Section 1.
6.1 Computational Efficiency

The first form of computational efficiency that bears consideration is that of offline vs online computation. For a machine learning model to capture relevant features of a motion planning problem instance, either in a single workspace or across workspaces, it must have access to a high-quality dataset that does not lead to a significant statistical bias.

Some works discussed in this survey trade an environment-specific, offline cost, to obtain better performance on new planning problems in the same environment. Alternatives trade a higher offline cost to obtain better performance on new environments (that are similar to the ones seen during training). Deciding which of these trade-offs to use depends on the application where the planner is being deployed, such as whether the environment the robot will be deployed is known or not.

For some problems, such as learning a sampler via supervised learning or learning a collision-checker inside a workspace, it is straightforward to annotate high-quality datasets via significant offline computation. The trained model rewards this offline effort by speeding up the runtime of the planner on a new problem instance. Nevertheless, this survey discussed applications, such as approximating a distance function or sampling strategies for manipulation planning, where obtaining such a dataset is computationally more challenging, which motivates additional research effort in these areas.

A relatively unexplored area of research is the idea of an SBMP that learns from previous planning experience on similar, yet different, problems, and manages to transfer the experience across environments. Such a planner would spend more computational time at the beginning of its life cycle but would eventually gather enough planning experience to find high-quality solutions to new problems quickly.

A form of computational efficiency that must also be taken into account is online inference time, e.g., when the learned module inside an SBMP uses a large machine learning model, such as a deep neural network. Such an integration may lead to an improvement over an alternative planner in terms of the number of iterations to find a solution. For most practical purposes, however, the metric of interest is overall planning time, where the integration of an SBMP with a learned model may under-
perform due to inference time. Specialized hardware, such as GPUs, can perform model inference for large machine learning architectures to obtain significant gains. This may not be practical, however, in all applications.

6.2 Path Quality

For graph-based SBMPs, such as the PRM, the most significant determiners of path quality are the sampling method and the nearest neighbor selection, which is performed according to a distance metric. Learned samplers discussed in Section 3.1 produce samples along the shortest path and show the most promise for improving path quality. Learning accurate distance metrics, and integrating them efficiently into nearest neighbor search subroutines, remains an open area of research.

For tree-based SBMPs, such as the RRT, in addition to node selection, node expansion majorly impacts path quality. Expansion typically involves randomly sampling a state, and then querying a local planner to make progress towards the selected state. This opens the door for learned models to both identify good states to sample (i.e., are likely to lie along a shortest path), and good local plans that bias the final path towards lower costs.

Irrespective of the underlying SBMP being used, it would be interesting to explore the application of machine learning to perform post-processing on paths computed by an SBMP. Just like machine learning models have been used to identify the order in which edges must be collision-checked, a model could be trained to identify the segments of a tree or a graph that are good candidates for smoothing, or to apply the smoothing directly. Potentially, this can be achieved in conjunction with an optimization process.

6.3 Ease of Use

One consideration during the deployment of SBMPs is that they involve a set of parameters that need to be tuned. Furthermore, the sampling strategy, distance metric, collision detection method, and connection strategies, are all exchangeable components. The selection of these prim-
itives as well as parameter values is non-trivial, and often requires expert knowledge of motion planning for the specific application. Inappropriate selection can handicap the performance of the planner.

Adaptive methods that automatically select combinations of primitive operations and parameter settings offer a solution, which allows non-expert users to make use of SBMPs effectively. Previous work includes adaptive methods for selecting samplers, connection strategies and parameter settings. It would be interesting and useful to combine these methods to create universal planning frameworks that use a learned model to adaptively select combinations of primitive operations and parameter settings.

6.4 Limitations of Existing Methods

Successful motion plans are a rich source of data about multiple facets of the environment or that particular planning instance, including distance between robot configurations, and the validity of different parts of the workspace. The majority of the approaches discussed in Section 3, however, deal with only a single primitive replaced with a learned module. On the other side of the spectrum, the neural motion planners in Section 4 replace the entire SBMP with learned models. There has not been enough exploration in the literature that focuses on the interaction of multiple learned components within an SBMP and for which purpose they offer the most benefits.

Different machine learning models expose different types of inductive bias, i.e., the ability of the learning algorithm to predict outputs given inputs that it has not encountered during the training process. Relatively new architectures, such as Graph Neural Networks (GNNs) (Bruce et al., 2014) and Transformers (Vaswani et al., 2017), have been used in a wide variety of applications in computer vision and natural language processing. These models may enable similar advancements in improving the performance of SBMPs.

Poorly designed and inadequately trained models will significantly handicap planning performance. Thus, it becomes increasingly important to develop motion planning algorithms that are robust to inaccurate models. A learned model can also potentially adversarially impact the
6.5 Potential Future Work

probabilistic guarantees of SBMPs. Many works discussed in this survey do not discuss the impact of machine learning on the properties (probabilistic completeness or asymptotic optimality) of the SBMP framework. For example, if a learned sampler only generates samples over a subset of $C$, then any SBMP that uses this sampler will not be complete. It would be interesting to develop learning-based methods that provide theoretical guarantees.

6.5 Potential Future Work

In most robotic applications, there is the consideration of integrating the planning algorithm with the robot’s perception. Although machine learning promises end-to-end pipelines that output low-level control policies for a given task, given raw sensory input, these are often data-hungry solutions, and they do not easily generalize across domains and tasks. Nevertheless, there are a lot of opportunities in using machine learning models to learn high-level, task-specific, and interpretable representations of a robot’s sensory input, which a downstream SBMP can use.

As there are many avenues for integrating machine learning tools with SBMPs, it is also useful to define clear benchmarks that can evaluate the performance, and highlight the strengths and weaknesses of a new motion planner across a variety of problems. There has been preliminary work in this area (Chamzas et al., 2021b).

Many successful algorithmic solutions that integrate machine learning tools with SBMPs exploit the inherent structure present in specific applications, and design a machine learning model that can solve a problem component. Examples in this survey include the design of a learned local planner for systems with significant dynamics, and learning stochastic dynamics models for planning under uncertainty. These solutions are often handcrafted to specific applications, but they offer promising avenues for motion planning researchers with domain expertise to investigate.
6.6 Alternative Planning Frameworks

Although outside the scope of this survey, there are other approaches to the motion planning problem beyond SBMPs. These include search-based approaches, which search for paths from start to goal configurations by combining a series of motion primitives (short, kinematically feasible motions). They also include trajectory optimization methods (Ratliff et al., 2009), that assign a cost to each robot trajectory and incorporate constraints, such as path smoothness and collisions. They can use a broad class of optimization techniques, such as sequential convex optimization, to obtain a good solution trajectory. There are also efforts that apply machine learning to these alternative planning methodologies (Vemula et al., 2020; Mukadam et al., 2018).
References

Aoude, G., B. Luders, J. Joseph, N. Roy, and J. How. (2013). “Probabilistically safe motion planning to avoid dynamic obstacles with uncertain motion patterns”. *Autonomous Robots*. 35: 51–76.

Arslan, O. and P. Tsiotras. (2015). “Machine learning guided exploration for sampling-based motion planning algorithms”. In: *IROS*.

Arslan, O. and P. Tsiotras. (2013). “Use of relaxation methods in sampling-based algorithms for optimal motion planning”. In: *ICRA*.

Bähnemann, R., M. Burri, E. Galceran, R. Siegwart, and J. Nieto. (2017). “Sampling-based motion planning for active multirotor system identification”. In: *ICRA*. 3931–3938.

Baldwin, I. and P. Newman. (2010). “Non-parametric learning for natural plan generation”. In: *IROS*.

Barraquand, J. and J.-C. Latombe. (1991). “Nonholonomic multibody mobile robots: controllability and motion planning in the presence of obstacles”. In: *ICRA*.

Bayazit, O. B., J.-M. Lien, and N. M. Amato. (2002). “Better Group Behaviors in Complex Environments Using Global Roadmaps”. In: *Proceedings of the Eighth International Conference on Artificial Life*.

Bekris, K. E. and R. Shome. (2020). “Asymptotically Optimal Sampling-Based Planners”. In: *Encyclopedia of Robotics*. Ed. by M. H. Ang, O. Khatib, and B. Siciliano. 1–12.
Berenson, D., P. Abbeel, and K. Goldberg. (2012). “A robot path planning framework that learns from experience”. In: ICRA.

Bharatheesha, M., W. Caarls, W. J. Wolflag, and M. Wisse. (2014). “Distance metric approximation for state-space RRTs using supervised learning”. In: IROS.

Bhardwaj, M., S. Choudhury, and S. Scherer. (2017). “Learning Heuristic Search via Imitation”. In: CoRL.

Bhardwaj, M., S. Chowdhury, B. Boots, and S. Srinivasa. (2019). “Leveraging Experience in Lazy Search”. In: RSS.

Bialkowski, J., M. Otte, and E. Frazzoli. (2013). “Free-configuration biased sampling for motion planning”. In: IROS.

Bialkowski, J., M. Otte, S. Karaman, and E. Frazzoli. (2016). “Efficient collision checking in sampling-based motion planning via safety certificates”. IJRR. 35: 767–796.

Bohlin, R. and L. E. Kavraki. (2000). “Path planning using lazy PRM”. In: ICRA.

Bruce, J., K. Caluwaerts, A. Iscen, A. P. Sabelhaus, and V. SunSpiral. (2014). “Design and evolution of a modular tensegrity robot platform”. In: ICRA.

Burns, B. and O. Brock. (2005). “Sampling-Based Motion Planning Using Predictive Models”. In: ICRA.

Burri, M., M. Bloesch, Z. Taylor, R. Siegwart, and J. Nieto. (2018). “A framework for maximum likelihood parameter identification applied on MAVs”. Journal of Field Robotics. 35(1): 5–22.

Canny, J. (1988). The complexity of robot motion planning. MIT press.

Chamzas, C., A. Cullen, A. Shrivastava, and L. E. Kavraki. (2022). “Learning to Retrieve Relevant Experiences for Motion Planning”. ICRA.

Chamzas, C., Z. Kingston, C. Quintero-Peña, A. Shrivastava, and L. E. Kavraki. (2021a). “Learning Sampling Distributions Using Local 3D Workspace Decompositions for Motion Planning in High Dimensions”. ICRA.

Chamzas, C., C. Quintero-Pena, Z. Kingston, A. Orthey, D. Rakita, M. Gleicher, M. Toussaint, and L. E. Kavraki. (2021b). “MOTION-BENCHMAKER: A Tool to Generate and Benchmark Motion Planning Datasets”. RA-L. 7(2): 882–889.
Chamzas, C., A. Shrivastava, and L. E. Kavraki. (2019). “Using local experiences for global motion planning”. In: ICRA.
Chen, B., B. Dai, Q. Lin, G. Ye, H. Liu, and L. Song. (2019). “Learning to Plan in High Dimensions via Neural Exploration-Exploitation Trees”. In: ICLR.
Chiang, H.-T. L., J. Hsu, M. Fiser, L. Tapia, and A. Faust. (2019). “RL-RRT: Kinodynamic Motion Planning via Learning Reachability Estimators From RL Policies”. RA-L. 4(4): 4298–4305.
Chiang, L., A. Faust, S. Sugaya, and L. Tapia. (2018). “Fast Swept Volume Estimation with Deep Learning”. In: WAFR.
Choudhury, S., S. Arora, and S. Scherer. (2015). “The planner ensemble: Motion planning by executing diverse algorithms”. ICRA.
Choudhury, S., S. Javdani, S. Srinivasa, and S. Scherer. (2017). “Near-Optimal Edge Evaluation in Explicit Generalized Binomial Graphs”. In: NeurIPS.
Choudhury, S., S. Srinivasa, and S. Scherer. (2018). “Bayesian Active Edge Evaluation on Expensive Graphs”. IJCAI.
Clark, C. and S. Rock. (2001). “Randomized Motion Planning for Groups of Nonholonomic Robots”. In: International Symposium on Artificial Intelligence, Robotics, and Automation in Space.
Coleman, D., I. Sucan, M. Moll, K. Okada, and N. Correll. (2014). “Experience-Based Planning with Sparse Roadmap Spanners”. ICRA.
Curtis, A., X. Fang, L. P. Kaelbling, T. Lozano-Pérez, and C. R. Garrett. (2022). “Long-horizon manipulation of unknown objects via task and motion planning with estimated affordances”. In: ICRA.
Das, N. and M. Yip. (2020). “Learning-Based Proxy Collision Detection for Robot Motion Planning Applications”. T-RO.
Ekenna, C., S. A. Jacobs, S. Thomas, and N. M. Amato. (2013). “Adaptive neighbor connection for PRMs: A natural fit for heterogeneous environments and parallelism”. In: IROS.
Ekenna, C., S. Thomas, and N. M. Amato. (2015). “Adaptive local learning in sampling based motion planning for protein folding”. In: BIBM.
Faust, A., K. Oslund, O. Ramirez, A. Francis, L. Tapia, M. Fiser, and J. Davidson. (2018). “PRM-RL: Long-range Robotic Navigation Tasks by Combining Reinforcement Learning and Sampling-Based Planning”. In: ICRA.

Faust, A., I. Palunko, P. Cruz, R. Fierro, and L. Tapia. (2017). “Automated aerial suspended cargo delivery through reinforcement learning”. *AI*. 247: 381–398.

Faust, A., P. Ruymgaart, M. Salman, R. Fierro, and L. Tapia. (2014). “Continuous Action Reinforcement Learning for Control-Affine Systems with Unknown Dynamics”. *IEEE/CAA Journal of Automatica Sinica*. 1(3): 323–336.

Fulgenzi, C., A. Spalanzani, C. Laugier, and C. Tay. (2010). “Risk based motion planning and navigation in uncertain dynamic environment”. *Research Report*. 14. URL: https://hal.inria.fr/inria-00526601.

Fulgenzi, C., C. Tay, A. Spalanzani, and C. Laugier. (2008). “Probabilistic navigation in dynamic environment using rapidly-exploring random trees and gaussian processes”. In: IROS.

Gammell, J. D., T. D. Barfoot, and S. S. Srinivasa. (2020). “Batch Informed Trees (BIT*): Informed asymptotically optimal anytime search”. *IJRR*. 39(5): 543–567.

Gammell, J. D. and M. P. Strub. (2021). “Asymptotically Optimal Sampling-Based Motion Planning Methods”. *Annual Review of Control, Robotics, & Autonomous Systems*. 4: 295–318.

Granados, E., A. Boularias, K. Bekris, and M. Aanjaneya. (2022). “Model Identification and Control of a Low-Cost Wheeled Mobile Robot Using Differentiable Physics”. *ICRA*.

Granados, E., A. Sivaramakrishnan, T. McMahon, Z. Littlefield, and K. E. Bekris. (2021–2021). “Machine Learning for Kinodynamic Planning (ML4KP)”. https://github.com/PRX-Kinodynamic/ML4KP.

Hauser, K. (2015). “Lazy collision checking in asymptotically-optimal motion planning”. In: ICRA.

Hauser, K., T. Bretl, and J. Latombe. (2005). “Learning-Assisted Multi-Step Planning”. In: ICRA.
Hauser, K. and Y. Zhou. (2016). “Asymptotically Optimal Planning by Feasible Kinodynamic Planning in a State–Cost Space”. *T-RO*. 32(6): 1431–1443.

Hou, B., S. Choudhury, G. Lee, A. Mandalika, and S. S. Srinivasa. (2020). “Posterior sampling for anytime motion planning on graphs with expensive-to-evaluate edges”. In: *ICRA*.

Hsu, D., G. Sanchez-Ante, and Zheng Sun. (2005). “Hybrid PRM Sampling with a Cost-Sensitive Adaptive Strategy”. In: *ICRA*.

Hsu, D., J.-C. Latombe, and R. Motwani. (1997). “Path planning in expansive configuration spaces”. In: *ICRA*.

Huh, J., V. Isler, and D. D. Lee. (2021). “Cost-to-Go Function Generating Networks for High Dimensional Motion Planning”. *ICRA*.

Huh, J. and D. D. Lee. (2018). “Efficient sampling with Q-learning to guide rapidly exploring random trees”. *RA-L*. 3(4).

Huh, J. and D. D. Lee. (2016). “Learning high-dimensional Mixture Models for fast collision detection in Rapidly-Exploring Random Trees”. In: *ICRA*.

Ichter, B., J. Harrison, and M. Pavone. (2018). “Learning Sampling Distributions for Robot Motion Planning”. In: *ICRA*.

Ichter, B., E. Schmerling, T. -. E. Lee, and A. Faust. (2020). “Learned Critical Probabilistic Roadmaps for Robotic Motion Planning”. In: *ICRA*.

Ichter, B. and M. Pavone. (2019). “Robot motion planning in learned latent spaces”. *RA-L*. 4(3): 2407–2414.

Ichter, B., P. Sermanet, and C. Lynch. (2021). “Broadly-Exploring, Local-Policy Trees for Long-Horizon Task Planning”. In: *5th Annual Conference on Robot Learning*.

Janson, L., E. Schmerling, A. Clark, and M. Pavone. (2015). “Fast marching tree: A fast marching sampling-based method for optimal motion planning in many dimensions”. *IJRR*. 34(7): 883–921.

Johnson, J. J., L. Li, F. Liu, A. H. Qureshi, and M. C. Yip. (2020). “Dynamically Constrained Motion Planning Networks for Non-Holonomic Robots”. In: *IROS*.

Jurgenson, T. and A. Tamar. (2019). “Harnessing Reinforcement Learning for Neural Motion Planning”. In: *RSS*.
Kalisiak, M. and M. van de Panne. (2006). “RRT-blossom: RRT with a local flood-fill behavior”. In: *ICRA*.

Kalisiak, M. and M. van de Panne. (2007). “Faster Motion Planning Using Learned Local Viability Models”. In: *ICRA*.

Kallmann, M., A. Aubel, T. Abaci, and D. Thalmann. (2008). “Planning Collision-Free Reaching Motions for Interactive Object Manipulation and Grasping”. In: *ACM SIGGRAPH 2008 Classes*.

Karaman, S. and E. Frazzoli. (2011). “Sampling-based algorithms for optimal motion planning”. *IJRR*. 30(7): 846–894.

Kavraki, L. E., P. Svestka, J. .-. Latombe, and M. H. Overmars. (1996). “Probabilistic roadmaps for path planning in high-dimensional configuration spaces”. *T-RO*. 12(4): 566–580.

Kew, J. C., B. Ichter, M. Bandari, T.-W. E. Lee, and A. Faust. (2020). “Neural Collision Clearance Estimator for Fast Robot Motion Planning”. In: *WAFR*.

Kim, B., Z. Wang, L. P. Kaelbling, and T. Lozano-Pérez. (2019). “Learning to guide task and motion planning using score-space representation”. *IJRR*. 38(7): 793–812.

Kimmel, A., A. Sintov, J. Tan, B. Wen, A. Boularias, and K. E. Bekris. (2019). “Belief-Space Planning using Learned Models with Application to Underactuated Hands”. *ISRR*.

Kingston, Z., C. Chamzas, and L. E. Kavraki. (2021). “Using Experience to Improve Constrained Planning on Foliations for Multi-Modal Problems”. In: *IROS*.

Kingston, Z., M. Moll, and L. E. Kavraki. (2019). “Exploring implicit spaces for constrained sampling-based planning”. *IJRR*. 38(10-11): 1151–1178.

Kleinbort, M., E. Granados, K. Solovey, R. Bonalli, K. E. Bekris, and D. Halperin. (2020). “Refined Analysis of Asymptotically-Optimal Kinodynamic Planning in the State-Cost Space”. In: *ICRA*.

Kleinbort, M., O. Salzman, and D. Halperin. (2016). “Collision Detection or Nearest-Neighbor Search? On the Computational Bottleneck in Sampling-based Motion Planning”. In: *WAFR*.

Knobloch, A., N. Vahrenkamp, M. Wächter, and T. Asfour. (2018). “Distance-Aware Dynamically Weighted Roadmaps for Motion Planning in Unknown Environments”. *RA-L*. 3(3): 2016–2023.
Kober, J., J. Bagnell, and J. Peters. (2013). “Reinforcement Learning in Robotics: A Survey”. *IJRR*. 32: 1238–1274.

Kroemer, O., S. Niekum, and G. D. Konidaris. (2019). “A Review of Robot Learning for Manipulation: Challenges, Representations, and Algorithms”. *CoRR*. abs/1907.03146.

Kuffner, J. J. and S. M. LaValle. (2000). “RRT-connect: An efficient approach to single-query path planning”. In: *ICRA*.

Kumar, R., A. Mandalika, S. Choudhury, and S. Srinivasa. (2019). “LEGO: Leveraging Experience in Roadmap Generation for Sampling-Based Planning”. In: *IROS*.

Latombe, J.-C. (1991). “Robot Motion Planning”.

Lavalle, S. M. (1998). “Rapidly-Exploring Random Trees: A New Tool for Path Planning”. *Tech. rep.*

Li, L., Y. Miao, A. H. Qureshi, and M. C. Yip. (2021). “MPC-MPNet: Model-Predictive Motion Planning Networks for Fast, Near-Optimal Planning under Kinodynamic Constraints”. *RA-L*. 6(3): 4496–4503.

Li, S. and N. T. Dantam. (2021). “Learning Proofs of Motion Planning Infeasibility”. In: *RSS*.

Li, Y. and K. E. Bekris. (2011). “Learning approximate cost-to-go metrics to improve sampling-based motion planning”. In: *ICRA*.

Li, Y., R. Cui, Z. Li, and D. Xu. (2018). “Neural Network Approximation Based Near-Optimal Motion Planning With Kinodynamic Constraints Using RRT”. *IEEE Transactions on Industrial Electronics*. 65(11): 8718–8729.

Li, Y., Z. Littlefield, and K. E. Bekris. (2016). “Asymptotically optimal sampling-based kinodynamic planning”. *IJRR*. 35(5): 528–564.

Littlefield, Z. and K. Bekris. (2018). “Efficient and Asymptotically Optimal Kinodynamic Motion Planning via Dominance-Informed Regions”. In:

Liu, K., M. Stadler, and N. Roy. (2020). “Learned Sampling Distributions for Efficient Planning in Hybrid Geometric and Object-Level Representations”. In: *ICRA*.

Malone, N., B. Rohrer, L. Tapia, R. Lumia, and J. Wood. (2012). “Implementation of an embodied general reinforcement learner on a serial link manipulator”. In: *ICRA*.
References

Mandalika, A., S. Choudhury, O. Salzman, and S. Srinivasa. (2019). “Generalized lazy search for robot motion planning: Interleaving search and edge evaluation via event-based toggles”. In: *Proceedings of the International Conference on Automated Planning and Scheduling*. Vol. 29. 745–753.

McConachie, D., T. Power, P. Mitrano, and D. Berenson. (2020). “Learning When to Trust a Dynamics Model for Planning in Reduced State Spaces”. *RA-L*. 5(2): 3540–3547.

Moll, M., C. Chamzas, Z. Kingston, and L. E. Kavraki. (2021). “HyperPlan: A Framework for Motion Planning Algorithm Selection and Parameter Optimization”. In: *IROS*.

Morales, M., L. Tapia, R. Pearce, S. Rodriguez, and N. Amato. (2005). “A Machine Learning Approach for Feature-Sensitive Motion Planning”. In: *WAFR*.

Mukadam, M., J. Dong, X. Yan, F. Dellaert, and B. Boots. (2018). “Continuous-time Gaussian process motion planning via probabilistic inference”. *IJRR*. 37(11): 1319–1340.

Oriolo, G. and C. Mongillo. (2005). “Motion Planning for Mobile Manipulators along Given End-effector Paths”. In: *ICRA*.

Palmieri, L. and K. O. Arras. (2015). “Distance metric learning for RRT-based motion planning with constant-time inference”. In: *ICRA*.

Pan, J., S. Chitta, and D. Manocha. (2013). “Faster Sample-Based Motion Planning Using Instance-Based Learning”. *WAFR*.

Phillips-Grafflin, C. and D. Berenson. (2015). “Reproducing Expert-Like Motion in Deformable Environments Using Active Learning and IOC”. In: *ISRR*.

Qi, C. R., H. Su, K. Mo, and L. J. Guibas. (2017). “Pointnet: Deep learning on point sets for 3d classification and segmentation”. In: *CVPR*.

Qureshi, A. H., A. Simeonov, M. J. Bency, and M. C. Yip. (2019). “Motion planning networks”. In: *ICRA*.

Qureshi, A. H., J. Dong, A. Choe, and M. C. Yip. (2020). “Neural Manipulation Planning on Constraint Manifolds”. *RA-L*. 5(4): 6089–6096.

Qureshi, A. H., J. Dong, A. Baig, and M. C. Yip. (2021). “Constrained Motion Planning Networks X”. *T-RO*: 1–19.
Ratliff, N., M. Zucker, J. A. Bagnell, and S. Srinivasa. (2009). “CHOMP: Gradient optimization techniques for efficient motion planning”. In: ICRA.

Reif, J. H. (1979). “Complexity of the mover’s problem and generalizations”. In: 20th Annual Symposium on Foundations of Computer Science (SFCS 1979). 421–427.

Sintov, A., A. Kimmel, K. E. Bekris, and A. Boularias. (2020). “Motion Planning with Competency-Aware Transition Models for Underactuated Adaptive Hands”. In: ICRA.

Sivaramakrishnan, A., E. Granados, S. Karten, T. McMahon, and K. E. Bekris. (2021). “Improving Kinodynamic Planners for Vehicular Navigation with Learned Goal-Reaching Controllers”. In: IROS.

Song, G. and N. M. Amato. (2001). “Using Motion Planning to Study Protein Folding Pathways”. In: Proceedings of the Fifth Annual International Conference on Computational Biology. 287–296.

Strudel, R., R. Garcia, J. Carpentier, J.-P. Laumond, I. Laptev, and C. Schmid. (2020). “Learning Obstacle Representations for Neural Motion Planning”. CoRL.

Sud, A., R. Gayle, E. Andersen, S. Guy, M. Lin, and D. Manocha. (2008). “Real-Time Navigation of Independent Agents Using Adaptive Roadmaps”. In: ACM SIGGRAPH 2008 Classes.

Sünderhauf, N., O. Brock, W. Scheirer, R. Hadsell, D. Fox, J. Leitner, B. Upcroft, P. Abbeel, W. Burgard, M. Milford, et al. (2018). “The limits and potentials of deep learning for robotics”. IJRR. 37(4-5): 405–420.

Sung, Y., L. P. Kaelbling, and T. Lozano-Pérez. (2021). “Learning When to Quit: Meta-Reasoning for Motion Planning”. IROS.

Sutanto, G., I. M. R. Fernández, P. Englert, R. K. Ramachandran, and G. S. Sukhatme. (2020). “Learning Equality Constraints for Motion Planning on Manifolds”. CoRL.

Svestka, P. and M. Overmars. (1995). “Coordinated motion planning for multiple car-like robots using probabilistic roadmaps”. In: ICRA.

Toll, W. van, A. IV, and R. Geraerts. (2012). “Real-Time Density-Based Crowd Simulation”. Computer Animation and Virtual Worlds. 23( Feb.): 59–69.
Uwacu, D., C. Ekenna, S. L. Thomas, and N. Amato. (2016). “The Impact of Approximate Methods on Local Learning in Motion Planning”. In: RLP.

Vaswani, A., N. Shazeer, N. Parmar, J. Uszkoreit, L. Jones, A. N. Gomez, L. Kaiser, and I. Polosukhin. (2017). “Attention is all you need”. NeurIPS.

Vemula, A., Y. Oza, J. Bagnell, and M. Likhachev. (2020). “Planning and Execution using Inaccurate Models with Provable Guarantees”. In: R:SS.

Wang, J., W. Chi, C. Li, C. Wang, and M. Q. Meng. (2020). “Neural RRT*: Learning-Based Optimal Path Planning”. T-ASE.

Wang, Z., C. R. Garrett, L. P. Kaelbling, and T. Lozano-Pérez. (2021). “Learning compositional models of robot skills for task and motion planning”. IJRR. 40(6-7): 866–894.

Wells, A. M., N. T. Dantam, A. Shrivastava, and L. E. Kavraki. (2019). “Learning feasibility for task and motion planning in tabletop environments”. IEEE RA-L. 4(2): 1255–1262.

Ye, G. and R. Alterovitz. (2017). “Demonstration-Guided Motion Planning”. In: ISRR.

Yu, C. and S. Gao. (2021). “Reducing Collision Checking for Sampling-Based Motion Planning Using Graph Neural Networks”. In: NeurIPS.

Yuan, Y., J. Liu, J. Wang, W. Chi, G. Chen, and L. Sun. (2021). “A Knowledge-Based Fast Motion Planning Method Through Online Environmental Feature Learning”. ICRA.

Zhang, Y., J. Zhang, J. Zhang, J. Wang, K. Lu, and J. Hong. (2020). “A novel learning framework for sampling-based motion planning in autonomous driving”. In: Proceedings of the AAAI Conference on Artificial Intelligence. Vol. 34. No. 01. 1202–1209.

Zhao, L., J. Putman, W. Wang, and D. Balkcom. (2020). “PLRC*: A piecewise linear regression complex for approximating optimal robot motion”. In: IROS.

Zheng, D. and P. Tsiotras. (2021). “Sampling-based Kinodynamic Motion Planning Using a Neural Network Controller”. In: AIAA Scitech 2021 Forum. 1754.

Zhu, Q. (1991). “Hidden Markov model for dynamic obstacle avoidance of mobile robot navigation”. T-RO. 7(3): 390–397.
Zucker, M., J. Kuffner, and J. A. Bagnell. (2008). “Adaptive workspace biasing for sampling-based planners”. In: IROS.