Abstract. We consider the Regge-Teitelboim model for a relativistic extended object embedded in a fixed background Minkowski spacetime, in which the dynamics is determined by an action proportional to the integral of the scalar curvature of the worldvolume spanned by the object in its evolution. In appearance, this action resembles the Einstein-Hilbert action for vacuum General Relativity: the equations of motion for both are second order; the difference is that here the dynamical variables are not the metric, but the embedding functions of the worldvolume. We provide a novel Hamiltonian formulation for this model. The Lagrangian, like that of General Relativity, is linear in the acceleration of the extended object. As such, the model is not a genuine higher derivative theory, a fact reflected in the order of the equations of motion. Nevertheless, as we will show, it is possible as well as useful to treat it as a ‘fake’ higher derivative system, enlarging the phase space appropriately. The corresponding Hamiltonian on this phase space is constructed: it is a polynomial. The complete set of constraints on the phase space is identified. The fact that the equations of motion are of second order in derivatives of the field variables manifests itself in the Hamiltonian formulation through the appearance of additional constraints, both primary and secondary. These new constraints are second class. In this formulation, the Lagrange multipliers implementing the primary constraints get identified as accelerations. This is a generic feature of any Lagrangian linear in the acceleration possessing reparametrization invariance.

1. Introduction

The idea that spacetime may be viewed as the trajectory of an extended object, or brane, embedded in a background spacetime was proposed by Regge and Teitelboim [RT] back in 1977 [1], inspired by the Nambu-Goto [NG] model for relativistic strings [2]. Over the years, this idea has been revisited [3, 4, 5, 6, 7, 8, 9, 10]. Regge and Teitelboim’s proposal was to consider four-dimensional spacetime as the worldvolume of some three-dimensional spacelike object embedded in a fixed ten-dimensional flat Minkowski background; ten being the minimum number of dimensions necessary to capture General Relativity [GR] within this framework. The worldvolume is described by its embedding functions into the background spacetime. The dynamics of the extended object is determined by an action proportional to the worldvolume integral
of the worldvolume scalar curvature. If the metric itself is the dynamical variable, this
coincides with the Einstein-Hilbert action for vacuum GR. In the RT model, however,
the dynamical variables are the embedding functions themselves; the worldvolume
metric is thus a derived geometric quantity. This leads to important differences, both
of a technical and a conceptual nature, between the RT model and vacuum GR.
To consider a fixed background spacetime is anathema to many relativists, but it is
standard in string theory. For example, the NG action is proportional to the area of
the worldvolume spanned by a relativistic string in a fixed background, the point of
departure, in its various forms, of string theory [11].

Besides the fact that the RT action is what would occur naturally to a relativist
when looking at relativistic extended objects—indeed the title of the RT paper is
Gravity à la String— the RT action possesses a key feature in common with the NG
action: they both lead to equations of motion that are of second order in derivatives
of the field variables, the embedding functions. While this is obvious for the NG action,
it is not for the RT action. Among candidate actions for extended objects that are
local, reparametrization invariant and invariant under rigid motions of the ambient
spacetime they are quite special in this sense‡—most actions one can construct which
satisfy these requirements in general will involve derivatives higher than second; this is
the case, for instance, for the action proportional to the integral of the squared mean
curvature proposed by Polyakov as an effective model in QCD [12].

An additional motivation for re-examining the RT action comes from recent
developments in brane world scenarios [13, 14]. It should be emphasized, however, that
such scenarios are much more complicated, as they involve a non-trivial background
spacetime. With this motivation in mind, Karasik and Davidson recently proposed
a Hamiltonian formulation of the RT model [10]. With the addition of a Lagrange
multiplier, they adapt the ADM canonical formulation of GR to the RT model, so that
it is written as a system that depends only on the velocities of the extended object.
The price they pay is that the Legendre transformation yielding the Hamiltonian is
not simple. Moreover, the explicit form of the Hamiltonian, as well as the constraints
that generate reparametrization invariance, are not polynomials in the phase space
variables. Despite these difficulties, they proceed to include possible matter fields, they
consider both canonical and path integral quantizations of the system, and analyze a
minisuperspace model.

In this paper we provide an alternative Hamiltonian formulation of the RT model
yielding a Hamiltonian functional that is polynomial in the phase space variables.
How this is accomplished is by extending the phase space to include, in addition to
the position of the brane at a fixed time and its conjugate momentum, also the velocity
of the brane at a fixed time and its conjugate momentum. In other words, we treat the
RT model as a higher derivative theory. This may seem perverse, in light of what we
said above: the RT model is not a higher derivative theory. Its action does, however,
depend on the brane acceleration; why the model is not a higher derivative one is
because the dependence on the acceleration is linear. As we will show, this means
that the extra degrees of freedom we introduce are pure gauge, and thus harmless.
We will rely heavily on the formalism developed in [15] for truly higher derivative
models of relativistic extended objects. The simplification of the functional form of
the Hamiltonian comes with a price. As was to be expected from reparametrization

‡ Another model that shares this property is the Tolman model, the worldvolume integral of the
mean extrinsic curvature, which exists however only for hypersurfaces.
Hamiltonian dynamics of extended... invariance, there are constraints. We find that there are second class constraints and that the constraint algebra is quite complicated. Nevertheless, we do possess at our disposal Dirac’s powerful algorithm to deal with this difficulty in a systematic way (see e.g. [16]). Even if the model itself is not completely compelling, it does provide an excellent test-bed for the Dirac framework within the context of higher derivative theories.

The paper is organized in the following way: In Sect. 2, we introduce some basic notions of the worldvolume geometry, and the RT action together with its equations of motion. We also recall how the equations of motion can be written as a conservation laws in terms of a stress-tensor, and its relation with the conserved linear momentum associated with invariance under background translation. In Sect. 3, we perform a $d+1$ decomposition of the worldvolume geometry, in the spirit of the ADM canonical formulation of GR. This involves focusing on the geometry of the brane $\Sigma$. The $d+1$ decomposition of the RT action allows us to identify the appropriate Lagrangian functional for the model. In Sect. 4, we construct the Hamiltonian formulation of the model. Firstly, we obtain the appropriate phase space. Secondly, via a Legendre transformation, we arrive at the canonical Hamiltonian. Next, we identify the primary and secondary constraints associated with the reparametrization invariance of the model and we discuss their content. Finally, we look briefly at Hamilton’s equations. We conclude in Sect. 5 with a few brief remarks.

2. Regge-Teitelboim model

We consider the $(d+1)$-dimensional worldvolume $m$ spanned by the evolution of a $d$-dimensional spacelike extended object, or brane, $\Sigma$ in a fixed Minkowski background spacetime of dimension $D$. The worldvolume $m$ is described by the embedding $x^\mu = X^\mu(\xi^a)$, where $x^\mu$ are local coordinates for the background spacetime, $\xi^a$ local coordinates for the worldvolume $m$, and $X^\mu$ the embedding functions $(\mu, \nu = 0, 1, \ldots, N-1; a, b = 0, 1, \ldots, d)$. We denote by $X'_a = \partial_a X^\mu = \partial X^\mu / \partial \xi^a$ the tangent vectors to $m$. Their inner product gives the induced metric on $m$,

$$g_{ab} = \eta_{\mu\nu} X'_a X'_b = X_a \cdot X_b,$$

where $\eta_{\mu\nu}$ is the Minkowski metric with one minus sign. (To avoid notational clutter, wherever possible we will not write the spacetime indices.) We denote by $g^{ab}$ the inverse of $g_{ab}$. The $D-d-1$ unit spacelike normals $n^a_i$ to $m$ are defined implicitly by $n_i \cdot X_a = 0$, $n_i \cdot n_j = \delta_{ij}$ ($i, j = 1, \ldots, D-d-1$).

The extrinsic curvature of $m$ is $K_{ab} = g^{ij} K_{ab}^i$. The scalar curvature $R$ of $m$ can be obtained either directly from the induced metric $g_{ab}$, or, in terms of the extrinsic curvature, via the contracted Gauss-Codazzi equation:

$$R = K^i K_i - K_{ab} K^{ab}.$$

The RT action for a $d$-dimensional brane is given by

$$S_{RT}[X^\mu] = \frac{\alpha}{2} \int_m d^{d+1}\xi \sqrt{-g} R,$$

where the constant $\alpha$ has dimensions $[L]^{(1-d)}$. Note that for a relativistic string (with $d = 1$), this action is a topological invariant because of the Gauss-Bonnet theorem, so that it has empty equations of motion.

The equations of motion that follow from this action are

$$\alpha G^{ab} K_{ab}^i = 0,$$

where $G^{ab}$ is the Gauss-Codazzi tensor.
where $G_{ab} = \mathcal{R}_{ab} - (1/2)\mathcal{R}g_{ab}$ is the worldvolume Einstein tensor, with $\mathcal{R}_{ab}$ the Ricci tensor. These equations of motion are of second order in derivatives of the embedding functions because of the presence of the extrinsic curvature. There are only $D - d - 1$ equations, along the normals; the remaining $d + 1$ tangential components are satisfied identically, as a consequence of the reparametrization invariance of the action. If $d = 3$ and $D = 10$, there are six equations: this is the same as the number of Einstein equations for a four-dimensional spacetime modulo the Bianchi identities.

It is of interest to compare the RT model to the NG model, proportional to the area of the worldvolume spanned by the extended object in its evolution:

$$S_{NG}[X^\mu] = -\mu \int_m d^{d+1}\xi \sqrt{-g},$$

with equations of motion

$$\mu g^{\alpha\beta} K_{\alpha\beta}^i = 0,$$

the mean curvature vanishes. Comparison with the RT equations of motion shows that the Einstein tensor plays the same role there as here the induced metric.

The equations of motion (4) can be written in terms of a stress-tensor $f^{\mu\alpha}$ as

$$\alpha G^{ab} K_{ab}^i n^i = \nabla_a f^{\mu\alpha},$$

where the stress tensor is given by

$$f^{\mu\alpha} = -\alpha G^{ab} X_\mu^a,$$

and (5) follows from the Bianchi identity $\nabla_a G^{ab} = 0$ and the definition of the extrinsic curvature, $\nabla_a X^a_b = -K_{ab}^i n_i$. The stress-tensor is purely tangential, and this is related to the fact that the equations of motion are of second order in derivatives of the embedding functions. Moreover, if we let $\eta^a$ denote the timelike unit normal from $\Sigma$ into $m$, then we can construct the quantity

$$\pi^\mu = \eta_\alpha f^{\mu\alpha} = -\alpha \eta_\alpha G^{ab} X^\mu_b,$$

which is the conserved linear momentum density associated with the invariance of the action (2) with respect to background translations (17).

3. ADM decomposition

We split the $d + 1$ worldvolume coordinates $\xi^a$ into an arbitrary evolution parameter $t$ and $d$ coordinates $u^A$ that parametrize, at fixed $t$, the spacelike brane $\Sigma (A, B, \ldots = 1, \ldots, d)$, in the sense that $\Sigma$ is represented by the embedding $x^\mu = X^\mu(t = \text{const.}, u^A)$. Alternatively, $\Sigma$ can be described by its embedding in the worldvolume itself, $\xi^a = X^a(u^A)$. These two descriptions are related by composition. (Details can be found in [15].) The tangent vectors to $\Sigma$ are denoted by $X^\mu_A = \partial X^\mu/\partial u^A$, and the induced metric on $\Sigma$ is $h_{AB} = X_A \cdot X_B$, with inverse $h^{AB}$, and determinant $h$. We denote with $\mathcal{D}_A$ the $\Sigma$ covariant derivative compatible with $h_{AB}$. The unit timelike normal to $\Sigma$ into $m$ is $\eta^\mu$, defined by $\eta \cdot X_A = 0$, $\eta \cdot \eta = -1$.

The velocity $\dot{X} = \partial_t X$ is a vector tangent to the worldvolume $m$. It can be expanded in components with respect to the worldvolume basis $\{\eta, X_A\}$ as

$$\dot{X} = N \eta + N^A X_A,$$

where $N$ and $N^A$ are the lapse function and the shift vector, respectively.
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We decompose now the geometry of the worldvolume along the basis \( \{ \dot{X}, X_A \} \), adapted to the evolution of \( \Sigma \). The decomposition of the induced metric \( g_{ab} \) with respect to this basis is

\[
g_{ab} = \begin{pmatrix}
-N^2 + N^A N^B h_{AB} & N^B h_{AB} \\
N^A h_{AB} & h_{AB}
\end{pmatrix},
\]

and it follows that its determinant takes the simple form

\[
g = -N^2 h.
\]

We will also need the decomposition of the inverse metric:

\[
g^{ab} = \frac{1}{N^2} \begin{pmatrix}
-1 & N^A \\
N^B (N^2 h^{AB} - N^A N^B)
\end{pmatrix}.
\]

For the extrinsic curvature tensor, we have

\[
K_{ab} = -\left( n^i \cdot \ddot{X} n^i \cdot D_A \dot{X} n^i \cdot D_B \dot{X} \right).
\]

In order to find the decomposition of the scalar curvature \( R \), we use the decompositions (12), (13), and the Gauss-Codazzi equation (11). We find

\[
R = \frac{1}{N^2} \left[ -2(n^i \cdot D^A D_A X)(n_i \cdot \ddot{X}) + 2 J_R \right],
\]

where we have isolated the part of \( R \) that does not depend on the acceleration \( \ddot{X} \):

\[
J_R(X, \dot{X}) = \frac{1}{N^2} [h^{AB}(n^i \cdot D_A \dot{X})(n_i \cdot D_B \dot{X}) + 4N^{[C} h^{B]} A(n^i \cdot D_A D_B X)(n_i \cdot D_C \dot{X}) \\
+ 2(h^{A[C} N^{B]} N^D + h^{A[B} C[D})(n^i \cdot D_A D_B X)(n_i \cdot D_C D_D X)].
\]

We are now in a position to rewrite the RT action (2) in terms of quantities defined with respect to the brane \( \Sigma \) as

\[
S_{RT}[X^\mu] = \int dt L[X, \dot{X}, \ddot{X}],
\]

where we identify the Lagrangian functional

\[
L[X, \dot{X}, \ddot{X}] = \int_{\Sigma} d^d u L(X, \dot{X}, \ddot{X}),
\]

\[
= \int_{\Sigma} d^d u \frac{\alpha \sqrt{h}}{N} \left[ -(n^i \cdot D^A D_A X)(n_i \cdot \ddot{X}) + J_R \right],
\]

and \( L(X, \dot{X}, \ddot{X}) \) denotes the Lagrangian density. The important thing to note is that the dependence of the Lagrangian on the acceleration \( \ddot{X} \) is linear. The dependence on the velocity \( \dot{X} \) is somewhat complicated: it enters through the lapse function \( N \), the normal vectors \( n^i \), as well as the quantity \( J_R \). We note that the dependence on the position vectors \( X \) comes in only through its derivatives with respect to the coordinates on \( \Sigma, u^A \).
4. Hamiltonian formulation

The Lagrangian functional \( \mathcal{L} \) is the starting point of the Hamiltonian formulation of the model defined by the action \( \mathcal{S} \). Although we know from the outset that the model is not a higher derivative theory, we will treat it as though it were. As will become clear below, although additional degrees of freedom are introduced, there is no inconsistency in the strategy: the acceleration does not need to be specified as an initial condition.

The phase space involves two conjugate pairs \( \{ X, p; \dot{X}, P \} \), where \( p \) and \( P \) denote the momenta conjugate to the position functions \( X \) and the velocities \( \dot{X} \), respectively. They are defined by
\[
P = \frac{\delta \mathcal{L}}{\delta \dot{X}},
\]
\[
p = \frac{\delta \mathcal{L}}{\delta X} - \partial_t \left( \frac{\delta \mathcal{L}}{\delta \dot{X}} \right).
\]
For the momenta \( P \) we obtain immediately from \( \mathcal{L} \):
\[
P = -\alpha \sqrt{h} \frac{N}{N} (n_i \cdot \nabla A X) n^i.
\]
They are normal to the worldvolume. Note that the right hand side is a function only of \( X \) and \( \dot{X} \); it does not involve \( \ddot{X} \). The factor of \( \sqrt{h} \) tells us that the momenta are spatial densities. For \( p \) we obtain:
\[
p = \sqrt{h} \pi + \partial_A \left[ N A P - \alpha \sqrt{h} (n_i \cdot \nabla A \dot{X}) n^i \right],
\]
where \( \pi \) is the conserved linear momentum density obtained from Noether’s theorem. \( p \) and \( \pi \) differ only by a spatial divergence; their integral over a closed geometry is the same.

We have thus identified the appropriate phase space for the RT model. We emphasize that a truly higher derivative model would have qualitatively different momenta: \( P \) would continue to be normal to the worldvolume but it would depend on the acceleration \( \ddot{X} \); the \( \pi \) contribution to \( p \) would possess a part normal to the worldvolume and \( p \) would depend on \( X \) with three dots.

The canonical Hamiltonian is obtained via a Legendre transformation of the Lagrangian functional \( \mathcal{L} \) with respect to both \( X \) and \( \dot{X} \) (see e.g. \[15\]):
\[
H_0[X, p; \dot{X}, P] = \int_{\Sigma} d^4u \left( p \cdot \dot{X} + P \cdot \ddot{X} \right) - L[X, \dot{X}, \ddot{X}].
\]
Since the Lagrangian is linear in the acceleration, and the term \( p \cdot \dot{X} \) is already in canonical form, we obtain immediately
\[
H_0[X, p; \dot{X}] = \int_{\Sigma} d^4u \left[ p \cdot \dot{X} - \alpha \sqrt{h} J_R(X, \dot{X}) \right],
\]
where the quantity \( J_R(X, \dot{X}) \) is given by \[14\]. Note that this Hamiltonian is independent of the higher momentum \( P \), linear in \( p \), that comes in only in the first term, \( p \cdot \dot{X} \), and highly non-linear in its dependence on the canonical variables \( \dot{X}, X \).

Since the Lagrangian \( \mathcal{L} \) is linear in the acceleration, its Hessian vanishes identically,
\[
\mathcal{H}_{\mu \nu} = \frac{\delta \mathcal{L}}{\delta \dot{X} \mu \delta \dot{X} \nu} = 0.
\]
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This implies, as expected, that there are constraints on the phase space variables. The first set of (primary) constraints is readily identified as the definition of the higher momentum $P$, as given by (20). Therefore we have the $D$ primary constraints

$$C = P + \frac{\sqrt{h}}{N} (n_i \cdot D_A D^A X) n^i = 0.$$  

(25)

A word of caution is perhaps useful: it is tempting to square this constraint, just as one does in the Hamiltonian formulation of the NG model (see e.g. [18]). However, as argued convincingly by Nesterenko in his Hamiltonian formulation of a geometrical model of a relativistic particle, this would lead to error [19]. One would be throwing away $D-1$ primary constraints.

The total Hamiltonian that generates evolution in the phase space is given by adding to the canonical Hamiltonian (23) the primary constraints (25). This results in the Hamiltonian:

$$H[X, p; \dot{X}, P] = H_0[X, p; \dot{X}, P] + \int_{\Sigma} d^d u \lambda \cdot C.$$  

(26)

where $\lambda$ are Lagrange multipliers enforcing the constraints.

At this point, let us recall that the Poisson bracket appropriate for a higher derivative theory is, for two arbitrary phase space functionals, $f$ and $g$,

$$\{f, g\} = \int_{\Sigma} \left[ \frac{\delta f}{\delta X} \frac{\delta g}{\delta p} + \frac{\delta f}{\delta \dot{X}} \frac{\delta g}{\delta P} - (f \leftrightarrow g) \right],$$  

(27)

and that the time derivative of any phase space function is given by its Poisson bracket with the total Hamiltonian (26):

$$\partial_t f = \dot{f} = \{f, H\}.$$  

(28)

We have identified the $D$ primary constraints (26). This is not the whole story, however. Consistency requires that their conservation in time vanishes as well. In this case, this produces a set of secondary constraints. Setting $H_0 = \int_{\Sigma} d^d u H_0$, we find the $D$ secondary constraints

$$S_0 = H_0 = 0,$$  

(29)

$$S_A = p \cdot X_A + P \cdot \partial_A \dot{X} = 0,$$  

(30)

$$S_i = p \cdot n_i - n_i \cdot \partial_A \left[ N^A P - \alpha \sqrt{h} \left( n_j \cdot D^A \dot{X} \right) n^j \right].$$  

(31)

There are no other, tertiary constraints. The first secondary constraint (29) is the vanishing of the canonical Hamiltonian, the hallmark of reparametrization invariance. It generates diffeomorphisms out of the extended object $\Sigma$ onto the worldvolume $m$, and not necessarily normal to it. The second constraint (30) is the generator of diffeomorphisms tangential to $\Sigma$. The structure of these two constraints is the same found in the Hamiltonian analysis of genuine higher derivative brane models in [15].

The third constraint (31) is, however, characteristic of a model linear in acceleration; it expresses the fact that the momentum density (8) is tangential just as it is in the case of the NG model. In general, if the action depends on accelerations in a non-linear way, the momentum picks up a normal component [15].

Let us consider briefly the constraint algebra. (A detailed treatment is outside the scope of this paper and will be considered elsewhere [20].) For this, it is convenient
to project the primary constraints along the basis $\{\dot{X}, X_A, n^i\}$ to obtain the equivalent set of constraints:

\[ C_0 = P \cdot \dot{X} = 0, \quad (32) \]
\[ C_A = P \cdot X_A = 0, \quad (33) \]
\[ C_i = P \cdot n_i + \frac{\sqrt{h}}{N} (n_i \cdot D_A D^A X) = 0. \quad (34) \]

Again, the first two constraints are the same found in [15] for truly higher derivatives brane models, the third is particular to this model. A lengthy computation shows that the constraints $\{S_0, S_A, C_0, C_A\}$ are first class, i.e. in involution among themselves; the constraints $\{S_i, C_i\}$, however, are second class.

The counting of physical degrees of freedom goes as follows: number = (1/2) \( \left[ \text{dimension of the phase space} \right] \times (\text{number of first class constraints} - \text{number of second class constraints}) \right. \]. In this case we obtain: \( \left(1/2\right)[4D - 4(d + 1) - 2(D - d - 1)] = D - d - 1: \) one physical degree of freedom along each normal, just as is the case for the NG model [18].

Let us consider briefly the structure of the Hamilton’s equations that follow from the Hamiltonian (26). We will not write them down explicitly since their form is complicated and not particularly illuminating. In any case, one can find the needed variational tools in [15]. The first equation is apparently a trivial identity

\[ \partial_t X = \frac{\delta H}{\delta p} = \dot{X}, \quad (35) \]

since the only dependence on $p$ in the Hamiltonian is through the linear term $p \cdot \dot{X}$ in (26); however, this equation serves to identify the canonical variable $\dot{X}$ with the time derivative of the position functions $X$. The second Hamilton equation is given by

\[ \partial_t \dot{X} = \ddot{X} = \frac{\delta H}{\delta P} = \lambda, \quad (36) \]

and it identifies the Lagrange multipliers $\lambda$ with the acceleration $\ddot{X}$. This is the hallmark of a theory linear in acceleration. One can verify that is completely analogous to the situation one encounters when the dynamics of a non-relativistic free particle is formulated using the Lagrangian $L(x, \ddot{x}) = -x\ddot{x}/2$, which is equivalent to the quadratic in velocities form, modulo a boundary term. In other words, despite appearances $\ddot{X}$ does not need to be specified among the initial data which, happily, is consistent with the fact that the equations of motion are of second order in derivatives of the field variables. The third Hamilton equation is

\[ \partial_t P = \dot{P} = -\frac{\delta H}{\delta \dot{X}}, \quad (37) \]

and its role is to identify the form of the momenta $p$ in terms of $X, \dot{X}, P$ and $\dot{P}$. Using the constraints, it reproduces (21). Finally, the fourth Hamilton equation is

\[ \partial_t p = \ddot{p} = -\frac{\delta H}{\delta X}, \quad (38) \]

and it is the equation of motion [8] in its canonical disguise.
5. Concluding remarks

In this paper we have presented a Hamiltonian formulation of the RT model for an extended object, treated as a ‘fake’ higher derivative theory. Within our framework, the Hamiltonian function that determines evolution is polynomial in the phase space variables. We have identified the phase space constraints: there is a set of first class constraints, associated with the symmetry of reparametrization invariance, and a set of secondary constraints. One is now in a position to canonically quantize the RT model based on this Hamiltonian formulation. It would be interesting to compare the result with the formulation of the theory described by Karasik and Davidson in [10]. In our formulation, the canonical quantization of the model will require the secondary constraints to be analyzed using the Dirac algorithm [16]. Work on this is in progress. Finally, we note also that our formulation is not changed substantially by the addition of a cosmological constant term and/or matter fields. Terms get added to the momenta and the canonical Hamiltonian in a way which leaves the structure unchanged.

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