Analysis of correction of asteroid Apophis’ orbit providing its collision with the Moon

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Abstract. In case of possible asteroid Apophis’ collision with the Earth, we estimate characteristics of such orbit correction, which will provide asteroid’s deflection from the Earth and its collision with the Moon. By this way, we will avoid any danger from this asteroid’s close approaches with the Earth in the future. We developed a method for searching these trajectories providing collision with the Moon depending on date of correction. Such trajectories were found for next two decades. Parameters for one-impulse correction were estimated. Additionally an optimization of correction impulse was performed based on the time of collision with the Moon. Parameters of Apophis’ collision with the Moon were calculated.

1. Introduction

According to the observation data (for example, [1]), in 2029 asteroid Apophis will pass close to the Earth, at the distance of ~ 38 000 km from its centre. As a result, asteroid’s major semi-axes and orbital period will change significantly. This approach also will change its class from Aten to Apollo. (figure 1). Also the dispersion tube of asteroid trajectories will scatter dramatically after 2029 (figure 2) and there are still probabilities of future collisions with Earth afterwards.

![Figure 1](image-url)

**Figure 1.** Heliocentric orbits of the Earth (II), and asteroid Apophis - before (I) and after (III) its approach with the Earth in 2029.
Figure 2. Dispersion tube of Apophis’ trajectories (A) during the approach with the Earth (E) in 2029. Energy of the Apophis-Earth impact is estimated to be over 500 megatons. Such collision would result in total destruction of an area about a 100 km wide and provide other devastating consequences. Thus it’s still important tusk to monitor the asteroid’s orbit and to prevent this possible collisions.

2. One-impulse lunar correction
Correction of asteroid’s orbit is an effective method to prevent its collision with the Earth. But there is no confidence that it will not come back later through the resonance trajectory. One of the possible ways to avoid that risk is to choose via correction a specific trajectory which will provide collision of a dangerous asteroid with another planet or moon and will lead to asteroid’s destruction. The main candidate for such body in case of dangerous NEAs is the Moon. Correction scheme is presented at figure 3.

Figure 3. Scheme of one-impulse lunar correction.

For one-impulse correction, small velocity impulse $\Delta V_c$ is given to the asteroid at a certain moment of time, which provides an orbit’s change. Magnitude and direction of that impulse must correspond to the terminal orbit to the Moon. This problem usually has a solution as a result of the search of the asteroid’s velocity vector at its fixed position that leads to the end position vector equals to the Moon
center. Varying the time of collision with the Moon, we can optimize (minimize) the magnitude of the velocity impulse for the fixed time of the correction. Moreover, after that we can optimize the correction, changing the time of the correction. Such problem can be solved numerically because of complexity of asteroid’s motion equations. A developed method of analysis is presented further.

3. **Perilune distance as a function of the velocity impulse** $\Delta V_c$

Let $r_0$, $V_0$ be the initial vectors of position and velocity, corresponding time $t_0$ is the correction time $t_C$. We will specify initial state vector of the asteroid as $(r_0, V_0)$ at a certain initial time $t_0$, assuming that trajectory corresponding to that initial condition will result in collision with the Earth at the moment $t_1$. At some time $t_2$ near $t_1$ perilune distance $r_\pi$ of asteroid’s orbit will be reached. Small variation of asteroid’s velocity $\Delta V_c$ at the time $t_0$ will result in change of perilune distance $r_\pi$ (and Moon collision - destination - time $t_2$):

$$r_\pi = r_\pi (\Delta V_c).$$

(1)

Correction velocity for the asteroid can be found as solution vector $\Delta V_c$ of a minimization problem for $r_\pi(\Delta V_c)$. Iterative minimization process can be stopped when the next condition is fulfilled:

$$r_\pi < R_M,$$

(2)

here $R_M$ is the mean Moon radius. Function $r_\pi (\Delta V_c)$ determined by that way tends to be extremely pathological. Because of this, standard linear methods of gradient descend work with it very poorly [2]. We developed combined method to improve its efficiency. At figure 4 a sketchy example of its one iteration is shown.

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**Figure 4.** Combined minimization method.

G – gradient descend step,
A – accidental direction step,
R1, R2, R3 – points at the ravine bottom.

At first, from starting point, method descends to the bottom of ravine in the direction of anti-gradient and stops when the function reduction becomes relatively small. Then it stops and fixes point R1. Afterwards a random direction is picked up (by step A, figure 4) for initial step of searching the second point R2. Length of this step corresponds with dimensions of ravine borders. Then the method applies gradient descent again and fixes R2 point (figure 4). R1R2 vector roughly matches the bottom of the ravine and sets direction for the next step. The method analyses a side of reduction for $r_\pi$ in that direction by taking two steps in both directions, descending to the bottom and then comparing results. That is how R3 point is chosen on figure 4. According to values of $r_\pi$ in the points R1, R2 n R3 method decides:
- either to continue an iteration process from point R3, if $r_\pi$ (R3) is minimal;
- or to modify R3, if $r_\pi$ (R3) is neither maximal nor minimal. In that case the point with maximum value of $r_\pi$ is ignored for next calculation of bottom direction;
- or to modify R2, if value of $r_\pi$ in R3 is maximal.

Steps in the directions of A and R1R2 are modified when method’s convergence become very bad due to the curvature of the ravine. And if values at ravine’s bottom become less then the mean Moon radius or comparative to the values after A or R1R2 step, then the method stops.

4. Parameters for asteroid’s orbits which terminate at the Moon
Nominal trajectory for Apophis was chosen from the set of the asteroid’s trajectories with the Earth-collision in 2036 [3, 4]. For this trajectory the correction time $t_0$ is taken once in a month. The virtual one-impulse lunar correction was performed with resulting collision with the Moon in 2036.

![Figure 5. Lunar correction velocity depending on the moment of correction.](image)

Modeling results shown that the correction velocity impulse $\Delta V_c$ for lunar correction before 2029 (figure 5) is big enough. It is considerably (4-5 orders) bigger than one for simple one-impulse
correction which is required for avoiding collision with the Earth, Earth-correction [4]. It seems that such lunar correction requires orbit's form and also plane changing which demand considerable energy expenses. After 2029 the magnitude of lunar correction velocity impulse increases but slower than one for simple earth-correction, so that energy expenses for both of them become comparable. Usually, an angle between correction velocity impulse and current velocity of an asteroid (figure 6) remains near 90°, which means that magnitude of velocity (and also mean motion) remains relatively undisturbed and matches that main energy expenses goes to change orbital form and plain.

![Figure 6. Angle between current velocity of an asteroid and vector of lunar correction velocity.](image)
Figure 7. Impact velocity for Apophis with the Moon in 2036 depending on moment of correction. Velocity of the impact with the Moon fluctuates near 6 km per second (Figure 7). For the asteroids’ mass of \( \sim 2.7 \times 10^{10} \) kg, it corresponds to impact energy of \( \sim 110 \) megaton. Such collision will not significantly disturb the Moon’s orbit and will create final crater with diameter of \( \sim 1 \) km.

5. Minimization of correction velocity impulse magnitude by variation of the time of collision with the Moon

If asteroid’s initial position and velocity are defined and the time of collision with the Moon is not fixed then we have an one-parameter set of collision trajectories depending on the time of collision. It is reasonable to choose one that corresponds to the minimal value of correction velocity impulse. In that case we need to solve a problem of conditional minimization of \( r_n (\Delta V_c) \) with minimal possible value of \( \Delta V_c \).

This problem requires new objective function

\[
F(X_0, \Delta V) = |\Delta V| = \sqrt{\Delta V_x^2 + \Delta V_y^2 + \Delta V_z^2}
\]

Which need to be minimized with the additional condition \( r_n(t_2) = 0, \quad t_2 \in (t_{\text{min}}, t_{\text{max}}) \) for at least one time \( t_2 \) near \( t_1 \). The Lagrange function for our problem would be

\[
G(X_0, \Delta V, L) = F(X_0, \Delta V) + L \cdot r_n(X_0, \Delta V)
\]

Its conditional extremum was analyzed with the previously mentioned method.
Figure 8 represents optimal lunar correction velocity impulse $\Delta V_c$. With optimization, it became clear that there are two sets of initial conditions on asteroid’s orbit that provide most effective optimal correction through years. Further analysis had shown that they are grouped near two points of an orbit which lie at a maximum distance from the Apophis’ orbital nodes line (figure 9). These sets corresponds to correction velocity of 2.5 m/sec and 3.4 m/sec respectively for correction performed before 2029. Such momentum can be provided for example by nuclear blast with energy of 1 megaton [4].

![Figure 9](image)

**Figure 9.** Two sets of optimal positions of Apophis for lunar correction on its orbit (Apophis orbit is green). 2.5 m/sec (blue dots), 3.4 m/sec (red dots). The Earth orbit is grey.

### 6. Conclusion

Lunar correction for asteroid Apophis was studied in this paper. Method for searching the Lunar correction of the Apophis trajectories with the Earth-collision was developed. The Apophis’trajectory, which has collision with the Earth, after this Lunar correction is deflected from the Earth and directed to the collision with the Moon. This method developed worked well both with the problem of unconditional and conditional minimization of objective function. The optimal correction velocity impulse value was estimated at $\sim 2.5$ m/sec. Such velocity impulse can be provided for example by nuclear blast with energy of 1 megaton. It can provide a final solution of a problem of dangerous asteroid more easily than its direct total destruction.

### References

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