P-MODE LINE-SHAPES IN THEORY AND PRACTICE

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ABSTRACT
The asymmetries of solar p-mode lines provide important diagnostic clues as to the nature and location of the source of mode excitation. At the same time, they can also be responsible for systematic errors in the determination of mode frequencies.

I present a matched wave asymptotic analysis which leads to an expression for the spectrum valid for arbitrary stratifications and arbitrary source types. From this it is possible to deduce the correct asymmetric line profile to use in fitting the spectra, along with the physical interpretation of the fitted parameters.

I apply this procedure to low-degree MDI doppler data from which I am able to deduce corrections to the mode frequencies. A preliminary comparison of the asymmetry parameter with numerical calculations yields an estimate of the source depth.

Key words: solar oscillations; line asymmetry; excitation.

1. INTRODUCTION
The discovery of the asymmetry of solar p-mode lines dates back to Duvall et al. (1993), but it is only with recent SOHO and GONG data that we can now hope to see the asymmetry clearly in individual lines. Consequently it is only recently that we have begun to worry about possible systematic errors in frequency determinations which might arise as a result of ignoring the asymmetries. Essentially there are three questions in which we are interested: 1) What is the correct line profile to use in fitting asymmetric p-mode lines? 2) What can we learn about the source of excitation from the asymmetries?, and 3) Why do the asymmetries appear oppositely signed in velocity and intensity data? In this paper I will concentrate on (1) and (2) and comment only briefly on (3).

There have been numerous models of the asymmetry (Gabriel 1992; Gabriel 1995; Abrams & Kumar 1996) from which it is clear that solar oscillations driven near the upper turning point will inevitably have asymmetric line-profiles whose properties will be sensitive to the location and type of the source. The discussion of Rast & Bogdan (1998) is particularly illuminating because they show that the minima or troughs in the spectrum are (at least in some circumstances) eigenfrequencies of a solar model truncated from above at the location of the source, and calculated with an upper boundary condition determined by the source type. The relative location of the peaks and troughs is then responsible for the asymmetry of the lines. This suggests that, if one could determine the locations of these troughs, one could analyse the frequencies of these minima using standard helioseismic techniques. For example, the trough-frequencies must satisfy a Duvall Law and a variational principle. This conceptual framework provides the basis for much of the following discussion.

2. GENERAL ASYMPTOTIC THEORY
The general asymptotic theory is described in Rosenthal (1998) (in what follows, R98). The basic problem is the solution of the oscillation equation for a plane-parallel stratified layer in the form

$$\frac{d^2 \Psi}{dz^2} + K^2 \Psi = A_{\omega,k}(z)$$

where the local vertical wavenumber is given by

$$K^2 = \frac{1}{c^2} \left[ \omega^2 - \omega_c^2 - c^2 k^2 \left( 1 - \frac{N^2}{\omega^2} \right) \right]$$

and the source is assumed to have the form

$$A_{\omega,k}(z) = a_0 |K(z_0)|^{\frac{3}{2}} \delta(z-z_0) + a_1 |K(z_0)|^{-\frac{1}{2}} \delta'(z-z_0).$$

Note that the solution with $a_1 = 0$ is the Green’s Function and can therefore be used to generate the solution for a truly arbitrary source. For brevity I here consider only the low-frequency case in which the equation has two turning points and the source lies wholly above the upper turning point. The intermediate and high frequency cases, and the solution for an arbitrary source variation with depth, are discussed in R98. In the low-frequency case the Fourier spectrum in the region above the source is found to be

$$F = \frac{1}{2} (-a_0 - a_1) + \frac{1}{4} (-a_0 + a_1) e^{-2\Delta} \tan \phi$$

where

$$\phi \equiv \int_{z_t}^{z_0} K dz.$$
and
\[ \Delta \equiv \int_{z_0}^{z_t} |K|^2 |\tau| \, dz \tag{6} \]
and \( z_0 \) and \( z_t \) are the lower and upper turning points, respectively.

2.1. Properties of the Peaks and Troughs

The spectrum diverges at frequencies satisfying
\[ \int_{z_t}^{z_0} K \, dz = \left( n - \frac{1}{2} \right) \pi. \tag{7} \]
This is a well-known expression for the asymptotic frequencies of p modes and, as one would expect, it does not depend on the source properties.

If the ratio \( a_0 + a_1/a_0 - a_1 \) is real, for example if either \( a_0 \) or \( a_1 \) is zero, then the spectrum also has zeroes whose location is given by
\[ \int_{z_t}^{z_0} K \, dz \approx \left( n - \frac{1}{2} \right) \pi + 2e^{-2\Delta} a_0 + a_1/a_0 - a_1. \tag{8} \]
If \( \Delta \) is large, which is to say that the source lies well inside the upper evanescent region, the troughs will lie close to the corresponding peaks and the asymmetry will be large. The sign of the asymmetry clearly depends on the source type. A source containing only \( a_0 \) (a monopole source) will have a zero lying immediately above the mode frequency, and will consequently have a negative asymmetry, as is seen in the MDI and GONG doppler data. For a dipole source the sign of asymmetry is reversed.

If \( \Delta \) and the source amplitudes are functions only of frequency then equations (6) and (8) are both in the form of Duvall Laws, \( F_D(\omega/k) = \pi(n + \alpha(\omega)/\omega) \) with the same Duvall function, \( F_D \), but differing phase functions \( \alpha \):
\[ \delta \alpha \equiv \alpha_t - \alpha_m = \frac{2}{\pi} e^{-2\Delta} \left( \frac{a_0 + a_1}{a_0 - a_1} \right), \tag{9} \]
where \( \alpha_t \) and \( \alpha_m \) are the phase functions describing the troughs and peaks respectively. Evidently the sign of \( \delta \alpha \) is positive for a pure monopole source and negative for a pure dipole, while its magnitude depends quite strongly on the source location. The higher the source is placed, the smaller the value of \( \delta \alpha \), the closer together the peaks and troughs, and the greater the asymmetry. I will therefore use \( \delta \alpha \) as a diagnostic for the source.

2.1.1. A Solar-Cycle Dependence in the Asymmetry?

Formally the WKB theory predicts that the mode frequencies depend on the structure between the turning points, while the trough frequencies depend on the structure between the lower turning point and the source location. However, while the former statement is only an approximation, the latter statement can actually be made exact. If the phase of the source is constant across its depth, and the phase of the oscillation is constant across the source, there will exist a family of nulls corresponding to frequencies at which the amplitude of the solution is identically zero in the region above the source. The location of these nulls is (obviously) strictly independent of the stratification in the region above the source, since the amplitude is identically zero there. Thus any changes to the atmospheric structure as a result, say, of solar-cycle effects, would change the mode frequencies but leave the trough frequencies fixed. There therefore exists the possibility of a solar-cycle dependence in the asymmetry. Any such effect is likely to be extremely small as the asymmetry is significant only for low-frequency modes whose frequencies vary only very slightly over the solar cycle.

2.1.2. Intensity and Velocity Data

Equation (4) is the solution for the acoustic wave field. Nigam et al (1998), Nigam & Kosovichev (1998) and Roxburgh & Vorontsov (1997) all explain the reversal of asymmetry between doppler and intensity measurements as being due to the addition of correlated noise, by which they mean that the actual measured signal, in at least one of the measurements, is contaminated by the source. This is equivalent to adding terms proportional to \( a_0 \) and/or \( a_1 \) to the right hand side of equation (4). It is clearly therefore extremely difficult to tell the difference between a change of source type and the addition of correlated noise. In the data analysis below I will essentially assume that the doppler data represents the true acoustic wave field and that correlated noise is absent in the doppler signal. At present it is not clear how that assumption should be validated from the data.

3. Application to Data

3.1. Asymmetric Line Profiles

Equation (4) suggests the following form for the power in the asymmetric peaks:
\[ P_1(\nu) = \alpha_2 + \alpha_3 \times \left| 1 + 2i\alpha_5 \right|^2 \tag{10} \]
which approximates to
\[ P_1(\nu) \approx \alpha_2 + \alpha_3 \frac{(\nu - \alpha_4)^2 + (\alpha_3 - \alpha_5)^2}{\alpha_1^2} \tag{11} \]
where
\[ x \equiv \frac{\pi}{\delta} (\nu - \nu_0). \tag{12} \]
In practice I have found that \( \alpha_5 \), which is non-zero only if there is a phase-variation across the source, is underconstrained by the data, so the final preferred form for fitting is
\[ P_2(\nu) = \alpha_2 + \alpha_3 + \frac{\alpha_2^2 \alpha_4}{(\nu - \nu_0)^2 + (\nu - \nu_0)^2} \left[ 1 - 2\frac{\nu - \nu_0}{\alpha_4} \right] \tag{13} \]
where the parameters to be fitted are $\alpha_2$, $\alpha_3$, $\alpha_4$, the eigenfrequency $\nu_0$ and the full-width at half-maximum, $\Gamma$. The asymmetry is evidently due entirely to $\alpha_4$. These various forms for the asymmetric profile are all broadly similar to that proposed by Nigam et al. (1998). However, attention should be drawn to the fact that the profiles proposed here are based on a general solution to the excitation problem valid for an arbitrary stratification and source profile, and asymptotic forms for the various parameters (in particular $\alpha_4$) can be easily deduced by comparison with equation (6), to yield a ready theoretical interpretation of the fitted values.

### 3.2. Data Analysis

The data analysed consists of 360 day time series of MDI doppler-velocity spherical-harmonic coefficients for degrees $\ell \leq 10$. (The gap-filled and detrended data were provided by Jesper Schou.) The decision to look at low-degree modes was based on the fact these are less affected by leakage from modes of nearby degree. The remaining problem to be addressed is therefore the m-leakage from modes of differing azimuthal order within the same multiplet. For this preliminary analysis, the m-leakage issue was sidestepped by looking only at sectoral modes for which the effect is negligible. Moreover, the effect of m-leakage on the asymmetries of prograde and retrograde modes should be oppositely directed. Hence if the fitted values of $\alpha_4$ agree between prograde and retrograde modes this provides an a posteriori justification for neglecting m-leakage.

The time series were divided up into ten equal months and FFT power spectra calculated for each month. The results were then averaged over the ten months. Fitted $a$-coefficients were used to estimate mode frequencies and all peaks of a particular $m/l$ value in a given bin in degree and frequency were averaged to find a mean line profile. Figure 1 shows a typical example for a low-frequency bin. The different modes have very similar line profiles and show a clear negative asymmetry.

The peaks were then averaged to obtain a mean line profile, which is then fit to the theoretically derived profile (13). Because each peak is an average of several modes over ten monthly spectra I assume that Gaussian statistics are valid and the fitting was therefore carried out using a chi-squared minimisation with a weighting proportional to the inverse-square of the power. Figure 2 shows the result of the fitting for the same case as in Figure 1. Evidently the asymmetric profile provides a much better fit to the data than the symmetric Lorentzian.

### 3.3. Frequency Corrections

Figure 3 shows the difference in fitted frequencies between fits made using symmetric and asymmetric profiles. For the sectoral modes, the difference is small at low frequencies, where the line is so narrow that the asymmetry makes little difference to the fitted frequency. At higher frequencies, the lines are sufficiently symmetric that the asymmetry coefficient is difficult to determine. The frequency error appears to decrease again above 2.5mHz, but there is a large scatter. The maximum error is about 0.1µHz in the 2 – 2.5mHz range. There is no obvious systematic variation in the frequency error as a function of degree for these low-degree modes.

### 3.4. Asymmetry Parameter

Figure 4 shows the asymmetry parameter $\alpha_4/2$ which is a measure of the separation of the peak and trough. The results for prograde and retrograde sectoral modes are extremely similar (see also Figure 5), providing an a posteriori justification for the assumption that they are unaffected by m-leakage. Once again, there is no clear systematic variation with degree.
3.5. Comparison with Theory

Numerical peak and trough frequencies are calculated as follows. Eigenmode frequencies for a standard solar model are determined in the usual way by solving the oscillation equations with the appropriate causal upper boundary condition. The model is then truncated at some interior mesh point, corresponding to the source location, and the oscillation equations are solved with the upper boundary condition $\delta p = 0$ applied at that point. The frequencies obtained correspond to the nulls for the case $a_1 = 0$, which Rast & Bogdan (1998) refer to as a “$\delta p$ source” and which Rosenthal (1998) calls a monopole source. However it should be noted that this is not the same usage of the term “monopole” as that of Goldreich & Kumar (1990).

The frequency differences between modes and troughs for various source depths are converted into phase-function differences by multiplying by an empirically-determined Duvall function. The measured peak-trough differences $\alpha_4/2$ are also converted to phase-function differences by multiplying by the same Duvall function. Since each peak fitted is an average over several peaks, error estimates can be made by bootstrap resampling of the peaks over which the averaging is performed. The results shown in Figure 5 are broadly consistent with a fairly shallow source, located not more than about 250 km below the solar surface, but evidently the results at this stage are somewhat preliminary.

3.6. Line Widths

For completeness, Figure 6 shows the fitted line widths. The widths are consistent with previously known results, particularly the dip in linewidth between 2.5 mHz and 2.8 mHz, providing an extra validation of the fitting procedure. The inclusion of asymmetry has no measurable effect on the fitted widths.

4. Conclusions

The general asymptotic theory of the excitation of solar oscillations demonstrates the relationship between the physics of the excitation and the line-shape of the mode spectrum. Using this information, I have shown how MDI data can be used to determine the form and location of the excitation source. The reanalysis also provides corrections to the mode frequencies. The consequences of these corrections for...
Figure 6. Linewidths for prograde sectoral modes in the bin $0 \leq \ell \leq 10$. The solid curve is for the asymmetric fit and the dashed curve is for the symmetric fit. Error bars are 70% confidence limits.

our understanding of solar structure are discussed in more detail in Christensen-Dalgaard et al. (1998).

In the future, this analysis can be greatly refined by the use of more data. In particular, by taking into account the $m$-leakage, it will be possible to use information from whole multiplets, not just sectoral modes as used here. This will allow much tighter limits to be placed on the asymmetry parameter and its variation with frequency and degree. Crucially it will also then be possible to determine any effect the asymmetry may have on the determination of multiplet splittings.

We also need to consider how to implement the fitting of these asymmetric profiles into the standard analysis packages used to determine helioseismic frequencies from the data. An important issue is likely to be the modelling of the background. A linearly sloping background could, in theory, mimic an asymmetric line profile, which is why I have here chosen a flat background. Standard fitting procedures assume backgrounds with various power-law forms and it will be important to determine if the fitted parameters depend significantly on assumptions about the background (Kumar & Basu 1998).

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