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On macroscopic holes in some supercritical strongly dependent percolation models. (English)
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Summary: We consider $\mathbb{Z}^d$, $d \geq 3$. We investigate the vacant set $V^u$ of random interlacements in the strongly percolative regime, the vacant set $V$ of the simple random walk and the excursion set $E_{\geq \alpha}$ of the Gaussian free field in the strongly percolative regime. We consider the large deviation probability that the adequately thickened component of the boundary of a large box centered at the origin in the respective vacant sets or excursion set leaves in the box a macroscopic volume in its complement. We derive asymptotic upper and lower exponential bounds for these large deviation probabilities. We also derive geometric information on the shape of the left-out volume. It is plausible, but open at the moment, that certain critical levels coincide, both in the case of random interlacements and of the Gaussian free field. If this holds true, the asymptotic upper and lower bounds that we obtain are matching in principal order for all three models, and the macroscopic holes are nearly spherical. We heavily rely on the recent work by M. Nitzschner [Electron. J. Probab. 23, Paper No. 105, 21 p. (2018; Zbl 1402.60033)] and the author for the coarse graining procedure, which we employ in the derivation of the upper bounds.

MSC:
60K35 Interacting random processes; statistical mechanics type models; percolation theory
60G50 Sums of independent random variables; random walks
60F10 Large deviations
60G15 Gaussian processes
82B43 Percolation

Keywords:
random interlacements; Gaussian free field; percolation; large deviations

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References:
[1] Bodineau, T. (1999). The Wulff construction in three and more dimensions. Comm. Math. Phys. 207 197-229. · Zbl 1015.82005 · doi:10.1007/s002200050724
[2] Bricmont, J., Lebowitz, J. L. and Maes, C. (1987). Percolation in strongly correlated systems: The massless Gaussian field. J. Stat. Phys. 48 1249-1268. · Zbl 0962.82520 · doi:10.1007/BF01009544
[3] Cerf, R. (2000). Large deviations for three dimensional supercritical percolation. Astérisque 267 vi+177. · Zbl 0962.60082
[4] Černý, J. and Teixeira, A. Q. (2012). From Random Walk Trajectories to Random Interlacements. Ensaios Matemáticos [Mathematical Surveys] 23. Sociedade Brasileira de Matemática, Rio de Janeiro. · Zbl 1269.60002
[5] Chiarini, A. and Nitzschner, M. Entropic repulsion for the Gaussian free field conditioned on disconnection by level-sets. Preprint. Available at arXiv:1808.09947. · Zbl 1402.60033 · doi:10.1214/18-EJP226
[6] Deuschel, J.-D. and Stroock, D. W. (1989). Large Deviations. Pure and Applied Mathematics137. Academic Press, Boston, MA. · Zbl 0705.60029
[7] Drewitz, A., Prévost, A. and Rodriguez, P.-F. (2018). The sign clusters of the massless Gaussian free field percolate on $(\mathbb{Z}^d \setminus \text{d, geo } 3)(\text{and more}).$ Comm. Math. Phys. 362 513-546. · Zbl 1394.60099
[8] Drewitz, A., Ráth, B. and Sapozhnikov, A. (2014). An Introduction to Random Interlacements. SpringerBriefs in Mathematics. Springer, Cham. · Zbl 1304.60008
[9] Drewitz, A., Ráth, B. and Sapozhnikov, A. (2014). Local percolative properties of the vacant set of random interlacements with small intensity. Ann. Inst. Henri Poincaré Probab. Stat. 50 1165-1197. · Zbl 1319.60180 · doi:10.1214/13-AIHP540
[10] Drewitz, A. and Rodriguez, P.-F. (2015). High-dimensional asymptotics for percolation of Gaussian free field level sets. Electron. J. Probab. 20 Article ID 47. · Zbl 1321.60207
[11] Duminil-Copin, H., Raoufi, A. and Tassion, V. (2017). Sharp phase transition for the random-cluster and potts models via decision trees. Preprint. Available at arXiv:1705.03104. · Zbl 1482.82009 · doi:10.4007/annals.2019.189.1.2
Fusco, N., Maggi, F. and Pratelli, A. (2009). Stability estimates for certain Faber-Krahn, isocapacitary and Cheeger inequalities. Ann. Sc. Norm. Super. Pisa Cl. Sci. (5) 8 51-71. · Zbl 1176.49047

Lebowitz, J. L. and Saleur, H. (1986). Percolation in strongly correlated systems. Phys. A138 194-205. · Zbl 0666.60110 · doi:10.1016/0378-4371(86)90180-9

Li, X. (2017). A lower bound for disconnection by simple random walk. Ann. Probab.45 879-931. · Zbl 1421.60002 · doi:10.1214/15-AOP1077

Li, X. and Sznitman, A.-S. (2014). A lower bound for disconnection by random interlacements. Electron. J. Probab.19 Article ID 17. · Zbl 1355.60035

Lupu, T. (2016). From loop clusters and random interlacements to the free field. Ann. Probab.44 2117-2146. · Zbl 1348.60141 · doi:10.1214/15-AOP1019

Molchanov, S. A. and Stepanov, A. K. (1983). Percolation in random fields. I. Teoret. Mat. Fiz.55 246-256.

Nitzschner, M. (2018). Disconnection by level sets of the discrete Gaussian free field and entropic repulsion. Electron. J. Probab.23 1-21. · Zbl 1402.60033 · doi:10.1214/18-EJP226

Nitzschner, M. and Sznitman, A. S. Solidification of porous interfaces and disconnection. J. Eur. Math. Soc. (JEMS). To appear. Available at arXiv:1706.07229.

Popov, S. and Ráth, B. (2015). On decoupling inequalities and percolation of excursion sets of the Gaussian free field. J. Stat. Phys.159 312-320. · Zbl 1329.60342 · doi:10.1007/s10955-015-1187-z

Rodriguez, P.-F. and Sznitman, A.-S. (2013). Phase transition and level-set percolation for the Gaussian free field. Comm. Math. Phys.320 571-601. · Zbl 1269.82028 · doi:10.1007/s00220-012-1649-y

Sznitman, A.-S. (2015). Disconnection and level-set percolation for the Gaussian free field. J. Math. Soc. Japan67 1801-1843. · Zbl 1336.60194 · doi:10.2969/jmsj/06741801

Sznitman, A.-S. (2016). Coupling and an application to level-set percolation of the Gaussian free field. Electron. J. Probab.21 Article ID 35. · Zbl 1336.60194 · doi:10.1214/16-EJP4563

Sznitman, A.-S. (2017). Disconnection, random walks, and random interlacements. Probab. Theory Related Fields167 1-44. · Zbl 1365.60080 · doi:10.1007/s00440-015-0676-y

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