On the Mellin Transform of the Coefficient Functions of $F_L(x, Q^2)$

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Abstract
The Mellin-transforms of the next-to-leading order Wilson coefficients of the longitudinal structure function are evaluated.
1 Introduction

For the study of the scaling violations of deep-inelastic scattering structure functions different techniques were developed [1, 3]. In most of the approaches the evolution equations of the parton densities are solved as integro-differential equations in $x$-space [1]. In the case of twist-2 operators the Mellin-transform $M[f](N)$, given by

$$M[f](N) = \int_0^1 dx x^{N-1} f(x),$$

maps the convolutions between the parton densities, splitting functions, and coefficient functions into ordinary products, which leads to a considerable simplification of the problem [2]. Here $N$ denotes the integer moment-index. The analytic continuation [3] of the functional $M[f](N)$ into the complex plane is later used to perform the inverse transformation to $x$-space. It appears useful to apply the Mellin-representation not only for the evolution equations of the parton densities but also to the structure functions themselves. Their representation in $x$-space is then obtained directly by a single inverse Mellin transform. For various next-to-leading order studies which were performed so far, as e.g. of the structure functions $F_2(x, Q^2)$, $g_1(x, Q^2)$, and $xF_3(x, Q^2)$, the coefficient functions were requested only in $O(\alpha_s)$ and the above procedure has been followed already. In the case of the structure function $F_L(x, Q^2)$ the NLO expressions [4–6] contain as well the Wilson coefficients in $O(\alpha_s^2)$. They were calculated in the $x$-representation in ref. [4] for the quarkonic and in ref. [3, 5] for the gluonic coefficient functions, where also the results for the quarkonic coefficient functions of ref. [4] were confirmed. A careful check of all results was performed in ref. [6] where a series of integer moments was evaluated both numerically and analytically and compared to the results of an independent calculation, ref. [7]. Previously the $x$-space expressions were used in various studies, see e.g. [8].

It is the aim of this note to shortly present the representation of the coefficient functions in Mellin-$N$ space and their analytic continuation to allow also to evaluate the longitudinal structure function at next-to-leading order directly within the approach used in ref. [4].

2 The Wilson Coefficients in $x$ space

The longitudinal structure function $F_L(x, Q^2)$ has the representation

$$F_L(x, Q^2) = x \left( C_{NS}(x, Q^2) \otimes f_{NS}(x, Q^2) + \delta_f \left[ C_s(x, Q^2) \otimes \Sigma(x, Q^2) + C_g(x, Q^2) \otimes G(x, Q^2) \right] \right)$$

in the case of pure photon exchange. The symbol $\otimes$ denotes the Mellin convolution

$$A(x, Q^2) \otimes B(x, Q^2) = \int_0^1 dx_1 \int_0^1 dx_2 \delta(x - x_1 x_2) A(x_1, Q^2) B(x_2, Q^2).$$

The combinations of parton densities are

$$f_{NS}(x, Q^2) = \sum_{i=1}^{N_f} e_i^2 \left[ q_i(x, Q^2) + \bar{q}_i(x, Q^2) \right],$$

$$\Sigma(x, Q^2) = \sum_{i=1}^{N_f} \left[ q_i(x, Q^2) + \bar{q}_i(x, Q^2) \right].$$
\(G(x, Q^2)\) denotes the gluon density, \(e_i\) the electric charge, and \(\delta_f = (\sum_{i=1}^{N_f} e_i^2)/N_f\), with \(N_f\) the number of active flavors.

The coefficient functions \(C_i(x, Q^2)\) are given by

\[
C_{NS}(z, Q^2) = a_s c_{L,q}^{(1)}(z) + a_s^2 c_{L,q}^{(2),NS}(z)
\]

\[
C_{S}(z, Q^2) = a_s^2 c_{L,q}^{(2),PS}(z)
\]

\[
C_{g}(z, Q^2) = a_s c_{L,q}^{(1)}(z) + a_s^2 c_{L,q}^{(2)}(z),
\]

where \(a_s = \alpha_s(Q^2)/(4\pi)\). For convenience we list as well the coefficient functions in \(x\)-space, since we will give the Mellin transforms of the individual contributing functions separately below. The leading order coefficient functions are given by

\[
c_{L,q}^{(1)}(z) = 4C_F z
\]

\[
c_{L,g}^{(1)}(z) = 8N_f z(1 - z).
\]

In the \(\overline{\text{MS}}\) scheme the NLO coefficient functions read [4–6]

\[
c_{L,q}^{(2),NS}(z) = 4C_F (C_A - 2C_F) z \left\{ \frac{4}{15z^2} \left[ 6 - 3z + \frac{47}{2} z^2 - \frac{9}{2} z^3 \right] \ln z \\
- 4 \ln z \left[ \ln z - 2 \ln(1 + z) \right] - 8 \zeta(3) - 2 \ln^2 z \left[ \ln(1 + z) + \ln(1 - z) \right] \\
+ 4 \ln z \ln^2(1 + z) - 4 \ln z \ln(1 + z) + \frac{2}{5} (5 - 3z^2) \ln^2 z \\
- \frac{1}{5z^2} \left[ 2 \ln z - \ln(1 + z) \right] \\
+ \frac{3}{3z} \ln z + \frac{6 - 25z}{6z} \ln(1 - z) - \frac{2}{3} \ln z \ln(1 - z) \right\}
\]

\[
c_{L,q}^{(2),PS}(z) = 16 \left\{ \frac{z}{9z} C_F N_f \left[ 3(1 - 2z - 2z^2)(1 - z) \ln(1 - z) + 9z^2 \ln z - \zeta(2) \right] \\
+ 9z(1 - z - 2z^2) \ln z - 9z^2 (1 - z) - (1 - z)^3 \right\}
\]

\[
c_{L,g}^{(2)}(z) = C_F N_f \left\{ 16z [\ln z + \ln z \ln(1 - z)] \\
+ \left( \frac{32}{3} - \frac{64}{5} z^2 + \frac{32}{15z^2} \right) \ln z \right\}
\]

\[
+ \left( \frac{32}{3} + \frac{32}{5} z^2 \right) \ln^2 z + \frac{1}{15} \left[ -104 - 624z + 288z^2 - \frac{32}{z} \right] \ln z
\]

\[
+ \left( \frac{32}{3} + \frac{64}{5} z^2 \right) \zeta(2) - \frac{128}{15} - \frac{304}{5} z + \frac{336}{5} z^2 + \frac{32}{15z} \right\}
\]

\[
+ C_A N_f \left\{ -64 \ln z - (32z + 32z^2) \ln z \ln(1 + z) \right\}
\]
+ (16z - 16z^2) \ln^2(1 - z) + (-96z + 32z^2) \ln z \ln(1 - z)
+ \left(-16 - 144z + \frac{464}{3}z^2 + \frac{16}{3z} \right) \ln(1 - z) + 48z \ln^2 z + (16 + 128z - 208z^2) \ln z
+ 32z^2 \zeta(2) + \frac{16}{3} + \frac{272}{3}z - \frac{848}{9}z^2 - \frac{16}{9z} \right) ,
\end{equation}
with $C_A = N_c = 3, C_F = (N_c^2 - 1)/(2N_c) = 4/3$. The corresponding expressions in the DIS scheme are given in \[6\]. The class of basic functions is the same for both schemes.

3 The Mellin Transform

For most of the functions $f_i(x)$ contributing to eqs. (7–11) the Mellin transforms may be evaluated straightforwardly. They are listed in Table 1 for the individual functions. In these cases infinite (or finite) sums occurring may be traced back to representations being based on the functions

$$\psi^{(k)}(z) = \frac{1}{\Gamma(z)} \frac{d^k}{dz^k} \Gamma(z) ,$$
which represent the sums

$$S_n(N) = \sum_{k=1}^{N} \frac{1}{k^n} ,$$
with

$$S_1(N) = \psi(N + 1) + \gamma_E ,$$
and

$$S_n(N) = \frac{(-1)^{n-1} \psi^{(n-1)}(N + 1)}{\Gamma(n)} + \zeta(n) , \quad n > 1 .$$
Here $\gamma_E = -\psi(1)$ is the Mascheroni constant and $\zeta(n) = \sum_{k=1}^{\infty} (1/k^n)$ denotes the Riemann $\zeta$-function. One further may use Euler’s relation

$$\text{Li}_2(1 - z) = -\text{Li}_2(z) - \ln z \ln(1 - z) + \zeta(2)$$

(16)

to reduce the number of functions in the above expressions. Similarly it is convenient to combine the functions $\text{Li}_2(-x) + \ln x \ln(1 + x)$ whenever they occur with the same coefficient to

$$\text{Li}_2(-x) + \ln x \ln(1 + x) = -\frac{1}{2} \Phi(x) + \frac{1}{4} \ln^2 x - \frac{\zeta(2)}{2} ,$$

(17)
since the Mellin transform of the function

$$\Phi(x) = \int_{x/(1 + x)}^{1/(1 + x)} \frac{dz}{z} \ln \left( \frac{1 - z}{z} \right)$$

(18)
turns out to be simpler than that of either $\text{Li}_2(-x)$ or $\ln x \ln(1 + x)$. All these functions can be traced back to expressions which are built solely out of the functions $\psi^{(k)}(z)$, see Table 1.

Useful relations can be found in refs. [10, 11].

3
| No. | \( f(z, r) \) | \( M[f](N) \) |
|-----|----------------|------------------|
| 1   | \( z^r \)      | \( \frac{1}{N+r} \) |
| 2   | \( z^r \ln^n z \) | \( \frac{(-1)^n}{(N+r)^{n+1}} \Gamma(n+1) \) |
| 3   | \( z^r \ln(1 - z) \) | \( \frac{S_1(N+r)}{N+r} \) |
| 4   | \( z^r \ln(1 + z) \) | \( \frac{(-1)^{N+r-1}}{N+r} \left[ -S_1(N+r) + \frac{1 + (-1)^{N+r-1}}{2} S_1 \left( \frac{N+r-1}{2} \right) 
+ \frac{1 - (-1)^{N+r-1}}{2} S_1 \left( \frac{N+r}{2} \right) \right] 
+ \frac{1 + (-1)^{N+r-1}}{N+r} \ln(2) \) |
| 5   | \( z^r \ln^2(1 - z) \) | \( \frac{S_1^2(N+r) + S_2(N+r)}{N+r} \) |
| 6   | \( z^r \ln z \ln(1 - z) \) | \( \frac{S_1(N+r)}{(N+r)^2} + \frac{1}{N+r} \left[ S_2(N+r) - \zeta(2) \right] \) |
| 7   | \( z^r \ln^2 z \ln(1 - z) \) | \( \frac{2}{N+r} \left[ \zeta(3) + \frac{\zeta(2)}{N+r} - \frac{S_1(N+r)}{(N+r)^2} - \frac{S_2(N+r)}{N+r} - S_3(N+r) \right] \) |
| 8   | \( z^r \ln^2 z \ln(1 + z) \) | \( \frac{2(-1)^{N+r}}{N+r} \left[ \frac{S_1(N+r)}{(N+r)^2} + \frac{S_2(N+r)}{N+r} + S_3(N+r) - \frac{\zeta(2)}{2(N+r)} - \frac{3\zeta(3)}{4} \right] 
+ \frac{1 + (-1)^{N+r-1}}{2(N+r)} \left[ \frac{2}{(N+r)^2} S_1 \left( \frac{N+r-1}{2} \right) 
+ \frac{1}{N+r} S_2 \left( \frac{N+r-1}{2} \right) 
+ \frac{1}{2} S_3 \left( \frac{N+r-1}{2} \right) 
+ \frac{4 \ln 2}{(N+r)^2} \right] 
- \frac{1 - (-1)^{N+r-1}}{2(N+r)} \left[ \frac{2}{(N+r)^2} S_1 \left( \frac{N+r}{2} \right) 
+ \frac{1}{2} S_2 \left( \frac{N+r}{2} \right) 
+ \frac{1}{2} S_3 \left( \frac{N+r}{2} \right) \right] \) |
| 9   | \( z^r \text{Li}_2(z) \) | \( \frac{1}{N+r} \left[ \zeta(2) - \frac{S_1(N+r)}{N+r} \right] \) |
| 10  | \( z^r \text{Li}_2(z) \ln z \) | \( \frac{1}{(N+r)^2} \left[ -2\zeta(2) + \frac{2S_1(N+r)}{N+r} + S_2(N+r) \right] \) |

Table 1: Mellin transforms of basic functions contributing to the Wilson coefficients of \( F_L(x, Q^2) \) in NLO.
| No. | $f(z, r)$ | $M[f](N)$ |
|-----|---------|-----------|
| 11  | $z^r \Phi(z)$ | \[
\frac{1}{(N + r)^3} + 2 \frac{(-1)^{N+r}}{N + r} \left[ S_2(N + r) - \zeta(2) \right] \\
- \frac{1 + (-1)^{N+r}}{2(N + r)} \left[ S_2 \left( \frac{N + r}{2} \right) - \zeta(2) \right] \\
+ \frac{1 - (-1)^{N+r}}{2(N + r)} \left[ S_2 \left( N + r - 1 \right) \right] - \zeta(2) \right] \\
\] |
| 12  | $z^r \text{Li}_2(-z) \ln z$ | \[
\frac{(-1)^{N+r-1}}{(N + r)^2} \left[ \frac{2S_1(N + r)}{N + r} + S_2(N + r) \right] \\
- \frac{1 + (-1)^{N+r-1}}{2(N + r)^2} \left[ \frac{2}{N + r} S_1 \left( \frac{N + r - 1}{2} \right) + \frac{1}{2} S_2 \left( \frac{N + r - 1}{2} \right) + \frac{4 \ln 2}{N + r} \right] \\
+ \frac{1 - (-1)^{N+r-1}}{2(N + r)^2} \left[ \frac{2}{N + r} S_1 \left( \frac{N + r}{2} \right) + \frac{1}{2} S_2 \left( \frac{N + r}{2} \right) + \zeta(2) \right] \\
\] |
| 13  | $z^r \text{Li}_3(z)$ | \[
\frac{1}{N + r} \left[ \zeta(3) - \zeta(2) \frac{N + r}{N + r} + \frac{S_1(N + r)}{(N + r)^2} \right] \\
\] |
| 14  | $z^r \text{Li}_3(-z)$ | \[
(-1)^{N+r-1} \frac{S_1(N + r)}{(N + r)^3} - \frac{1 + (-1)^{N+r-1}}{2(N + r)^3} \left[ S_1 \left( \frac{N + r - 1}{2} \right) + 2 \ln 2 \right] \\
+ \frac{1 - (-1)^{N+r-1}}{2(N + r)^3} \left[ S_1 \left( \frac{N + r}{2} \right) + \frac{\zeta(2)}{2(N + r)^2} - \frac{3 \zeta(3)}{4(N + r)} \right] \\
\] |

Table 1 continued

To the non-singlet coefficient function also the functions

$$
\ln(x) \ln^2(1 + x), \quad S_{1,2}(-x), \quad \text{and Li}_2(-x) \ln(1 + x)
$$

contribute, which lead to different sums. The Mellin transform of $\ln(x) \ln^2(1 + x)$ can be obtained by a serial expansion

$$
\int_0^1 dx x^{N-1} \ln x \ln^2(1 + x) = \frac{2 \gamma_E}{N^2} \left[ \ln 2 - \frac{1}{2} \psi \left( \frac{N}{2} + 1 \right) + \frac{1}{2} \psi \left( \frac{N + 1}{2} \right) \right] \\
+ \frac{\gamma_E}{2N} \left[ \psi' \left( \frac{N}{2} + 1 \right) - \psi' \left( \frac{N + 1}{2} \right) \right] - 2 \sum_{k=1}^{\infty} \frac{(-1)^k \psi(k)}{k(N + k)^2}.
$$

The infinite sum in eq. (20) is easily evaluated by a recursive algorithm. For numerical applications one may use as well representations of $\log(1 + x)$ in the range $x \in [0, 1]$, cf. [12], as, e.g.,

$$
\ln(1 + x) \simeq \sum_{k=1}^{8} a_k x^k,
$$

which are as accurate as $10^{-8}$. The coefficients $a_k$ read

$$
a_1 = 0.9999964239 \quad a_2 = -0.4998741238 \quad a_3 = 0.331790258 \quad a_4 = -0.2407338084 \\
a_5 = 0.1676540711 \quad a_6 = -0.0953293897 \quad a_7 = 0.0360884937 \quad a_8 = -0.0064535442.
$$
The Mellin transform of \( \ln x \ln^2(1 + x) \) is then given by

\[
\int_0^1 dx x^{N-1} \ln x \ln^2(1 + x) \simeq -\sum_{k=2}^{16} \frac{b_k}{(N+k)^2},
\]

where the coefficients \( b_k \) are obtained by taking the square of the polynomial in eq. (21).

We furthermore observe that \( S_{1,2}(-x) \) and \( \text{Li}_2(-x)\ln(1 + x) \) may be combined using the integral representation, cf. [11].

\[
F_1(x) = \frac{1}{x}\{\ln(1 + x)\text{Li}_2(-x) - \zeta(2) + S_{1,2}(-x) - 2\text{Li}_3(-x)\} = \int_0^1 dy \frac{\ln(y)\ln(1 - y)}{1 + xy}.
\]

Since the functions \( S_{1,2}(-x) \) and \( \text{Li}_2(-x)\ln(1 + x) \) occur with the same weight factor they can be dealt with together with the help of the representation eq. (23). In evaluating the Mellin-transform the \( x \)-integral yields

\[
\sum_{k=0}^{\infty} (-1)^k \frac{y^k}{N+k} \equiv \Phi(-y, 1, N).
\]

Here the function \( \Phi(y, a, b) \), ref. [13], is related to the generalized Riemann \( \zeta \)-function \( \zeta(c, n) \) [14]. Calculating the \( y \)-integral by using eq. (6) of table 1, one finally obtains

\[
M[F_1](N) = \sum_{k=0}^{\infty} \frac{(-1)^k}{N+k} \left[ \frac{\psi(2+k) + \gamma_E}{(k+1)^2} - \frac{\psi'(2+k)}{k+1} \right].
\]

The representation eq. (23) is fastly converging since for large values of \( k \), \( \psi(k) \sim \ln(k) \) and \( \psi'(k) \sim 1/k \). As in the case of the Mellin transforms of other functions emerging in the \( x \)-space representation of the different coefficient and splitting functions the poles in eqs. (20) and (23) are situated at the non-positive integers.

We finally would like to comment on the analytic continuation of the sums

\[
\tilde{S}_n(N) = \sum_{k=1}^{N} \frac{(-1)^k}{k^n} S_1(k), \quad n \geq 2.
\]

which emerge in some of the Mellin-transforms. \( \tilde{S}_2(N) \) contributes, e.g., to the NLO anomalous dimensions. By using the relation

\[
c_{k,n} = \sum_{l=1}^{k} \frac{(-1)^l}{l^n} = -\left\{ (1 - \frac{1}{2^{n-1}}) \zeta(n) + \frac{(-1)^{k+n-1}}{2^n\Gamma(n)} \left[ \psi^{(n-1)}\left( \frac{k+1}{2} \right) - \psi^{(n-1)}\left( \frac{k}{2} + 1 \right) \right] \right\}
\]

one may express \( \tilde{S}_k(N) \) as

\[
\tilde{S}_n(N) = \frac{(-1)^N S_1(N)}{N^n} - \sum_{l=2}^{\infty} (-1)^{l} \zeta(l) \left\{ (1 - \frac{1}{2^{n-l}}) \zeta(n - l + 1) - \frac{(-1)^{n-l}}{2^{n-l}1^n} \left[ \psi^{(n-l)}\left( \frac{N}{2} \right) - \psi^{(n-l)}\left( \frac{N+1}{2} \right) \right] \right\} - (-1)^n \left[ \zeta(n) \ln 2 - (-1)^N \int_0^1 dx x^{N-1}\text{Li}_n(x) \right. - \left. \int_0^1 dx \frac{\text{Li}_n(x)}{1 + x} \right],
\]

\(^2\text{Note a misprint in eq. (3.12.22) of ref. [11]. The factor in front of } \text{Li}_3(-b/a) \text{ should be 2.}\)
where the Mellin-transform of \( \text{Li}_n(x)/(1 + x) \) is given by

\[
M[F_2](N) = \int_0^1 dx \frac{\text{Li}_n(x)}{1 + x} x^{N-1} = \sum_{k=1}^{\infty} \frac{(-1)^k}{N + k} c_{k,n}.
\] (29)

The series eq. (29) converges rather fast for \( n > 2 \). For the special case \( n = 2 \), being dealt with before in ref. [15], one obtains from (28)

\[
\tilde{S}_2(N) = -\frac{5}{8}\zeta(3) + (-1)^N \left[ \frac{S_1(N)}{N^2} - \frac{\zeta(2)}{2} G(N) + \int_0^1 dx x^{N-1} \frac{\text{Li}_2(x)}{1 + x} \right],
\] (30)

with

\[
\int_0^1 dx \frac{\text{Li}_2(x)}{1 + x} = \zeta(2) \ln 2 - \frac{5}{8}\zeta(3) \text{ and } G(N) = \psi \left( \frac{N+1}{2} \right) - \psi \left( \frac{N}{2} \right).
\] (31)

For \( n = 2 \) the series eq. (29) converges slowly since the modulus of the expansion coefficients \( c_{k,2} \) approach \( \zeta(2)/2 = \pi^2/12 \) as \( k \to \infty \) and the series is essentially logarithmic. One may, however, rewrite \( M[F_2](N) \) as

\[
M[F_2](N) = \zeta(2) \ln 2 - \int_0^1 dx x^{N-2} \ln(1 + x) [(N-1)\text{Li}_2(x) - \ln(1 - x)].
\] (32)

Again the function \( \ln(1 + x) \) can be represented by eq. (21) yielding

\[
M[F_2](N) = \zeta(2) \ln 2 - \sum_{k=1}^{8} a_k \left\{ (N-1) \left[ \frac{\zeta(2)}{N + k - 1} - \frac{\psi(N + k) + \gamma_E}{(N + k - 1)^2} \right] + \frac{\psi(N + k) + \gamma_E}{N + k - 1} \right\}.
\] (33)

which holds at an accuracy of better than \( 4 \cdot 10^{-7} \) for \( N \in [1, 20] \). Another approximate expression for \( M[F_2](N) \) was given in [15].

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