THE IMPACT OF THE UNCERTAINTY IN SINGLE-EPOCH VIRIAL BLACK HOLE MASS ESTIMATES ON THE OBSERVED EVOLUTION OF THE BLACK HOLE–BULGE SCALING RELATIONS

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ABSTRACT
Recent observations of the black hole (BH)–bulge scaling relations usually report positive redshift evolution, with higher redshift galaxies harboring more massive BHs than expected from the local relations. All of these studies focus on broad line quasars with BH mass estimated from virial estimators based on single-epoch spectra. Since the sample selection is largely based on quasar luminosity, the cosmic scatter in the BH–bulge relation introduces a statistical bias leading to an average overestimation of the true BH masses at high redshift. We here emphasize a previously underappreciated statistical bias resulting from the uncertainty of single-epoch virial BH mass estimators and the shape of the underlying (true) BH mass function, which leads to an overestimation of the true BH masses at the high-mass end. We demonstrate that the latter virial mass bias can contribute a substantial amount to the observed excess in BH mass at fixed bulge properties, comparable to the Lauer et al. bias. The virial mass bias is independent of the Lauer et al. bias; hence if both biases are at work, they can largely (or even fully) account for the observed BH mass excess at high redshift.

Key words: black hole physics – galaxies: active – quasars: general – surveys

Online-only material: color figures

1. INTRODUCTION

The redshift evolution of the local scaling relations between galaxy bulge properties and the mass of the central supermassive black hole (SMBH) has important clues to the establishment of these tight relations across cosmic time. There is currently a huge effort in measuring this evolution, either in terms of the BH mass–bulge velocity dispersion (\(M_\star–\sigma\)) relation, or the BH mass–bulge stellar mass/luminosity (\(M_\star–M_{\text{bulge}}, M_\star–L_{\text{bulge}}\)) relation (e.g., Shields et al. 2003; Peng et al. 2006a, 2006b; Salviander et al. 2007; Shen et al. 2008b; Woo et al. 2006, 2008; Treu et al. 2007; Jahnke et al. 2009; Greene et al. 2010; Merloni et al. 2010; Decarli et al. 2010; Bennert et al. 2010). With a few exceptions, most of these studies report a strong positive evolution in BH mass for fixed bulge properties, which reaches as high as \(\sim 0.6\) dex offset in BH mass from the local relation at the highest redshift probed (\(z \sim 2\)).

This observed strong evolution is difficult to understand in two aspects. For the \(M_\star–\sigma\) relation, numerical simulations based on self-regulated BH growth (e.g., Robertson et al. 2006) and other theoretical arguments (e.g., Shankar et al. 2009) favor a mild evolution in the normalization of the \(M_\star–\sigma\) relation, in conflict with the otherwise claimed strong evolution. For the \(M_\star–M_{\text{bulge}}\) or \(M_\star–L_{\text{bulge}}\) relation, many of the observed hosts at earlier epochs are already bulge-dominated; hence, unless their bulges continue to grow a considerable amount over time, it is difficult to understand how these systems would have migrated to the local relation.

An alternative explanation for this observed strong evolution is that it is caused, at least partly, by the systematics involved in deriving both host properties and BH masses. All these studies mentioned above focus on broad line quasar samples, with host properties measured from the quasar-light-subtracted images or spectra. The systematics in subtracting the quasar light as well as in converting observables (such as photometric colors or spectral properties) to physical quantities (such as bulge stellar mass) is a potential contamination to the final results. More importantly, the samples are predominately selected in quasar luminosity and hence are not unbiased samples for studies of the BH scaling relations.

One statistical bias resulting from using quasar-luminosity-selected samples was discussed in detail in Lauer et al. (2007). Because there is cosmic scatter in the BH scaling relations and because the galaxy luminosity (or stellar mass, velocity dispersion, etc.) function is bottom heavy, there are more smaller galaxies hosting the same mass BHs than the more massive galaxies. Hence, when selecting samples based on quasar luminosities, low-mass BHs are underrepresented, leading to an apparent excess in BH mass at fixed host properties. If the cosmic scatter in BH scaling relations increases with redshift and reaches \(\gtrsim 0.5\) dex, it can in principle account for all the excess in BH mass observed for the most luminous quasars (e.g., Lauer et al. 2007; Merloni et al. 2010).

A second statistical bias, resulting from the uncertainty in the BH mass estimates, was pointed out in Shen et al. (2008b); also see Kelly et al. (2009a). In all these studies, the BH masses are estimated using the so-called virial method based on single-epoch spectra. In this method, one assumes that the broad line region (BLR) is virialized, and the BH mass can be estimated as \(M_{\text{vir}} \approx G^{-1} R V^2\), where \(R\) is the BLR radius and \(V\) is the virial velocity; one further estimates the BLR radius using a correlation between \(R\) and the continuum luminosity \(L\).

In this way, one can estimate a virial BH mass using single-epoch spectra: \(\log M_{\text{vir}} = \log R + 2 \log(FWHM) + \text{const} = C_1 \log L + 2 \log(FWHM) + \text{const}\). The coefficients are calibrated empirically using RM samples or inter-calibrations between various lines (e.g., McLure & Jarvis 2002; Vestergaard & Peterson 2006; McGill et al. 2008).
Even though the virial method is widely used, these BH mass estimates based on several lines (usually Hα, Hβ, Mg II, and C iv) have a non-negligible uncertainty \( \sigma_{\text{vir}} \gtrsim 0.4 \text{ dex} \), when compared to RM masses or masses derived from the \( M_{\star} - \sigma \) relation (e.g., Vestergaard & Peterson 2006; Onken et al. 2004). This uncertainty must come from the imperfection of using luminosity and line width as proxies for the BLR radius and virial velocity, i.e., there are substantial uncorrelated variations \( \sigma_L \) in luminosity and variations \( \sigma_{\text{FWHM}} \) in line width, which together contribute to the overall uncertainty \( \sigma_{\text{vir}} \), i.e.,

\[
\sigma_{\text{vir}} = \sqrt{(C_1 \sigma_L)^2 + (2\sigma_{\text{FWHM}})^2}. \tag{1}
\]

One can then imagine that a statistical bias will arise if the underlying active black hole mass function (BHMF) is bottom heavy. In particular, the variations (uncompensated by the variations in FWHM) of luminosity at fixed true BH mass, \( \sigma_L \), will scatter more lighter BHs into a luminosity bin than heavier BHs, and bias the mean BH mass in that bin. This is the Malmquist-type bias (or Eddington bias) emphasized in Shen et al. (2008a, Section 4.4), which has received little attention in the studies on the evolution of BH scaling relations to date.

In this paper, we examine the impact of this mass bias on the observed evolution in BH scaling relations with realistic models for the underlying true BHMF and quasar luminosity function (LF). In Section 2, we review the statistical mass bias discussed in Shen et al. (2008a) and demonstrate its effects with simple models; in Section 3, we consider more realistic intrinsic BHMF and quasar LF, and estimate the mass bias as functions of luminosity and redshift; we discuss the impact of this mass bias and conclude in Section 4. Throughout the paper, we use cosmological parameters \( \Omega_0 = 0.3 \), \( \Omega_\Lambda = 0.7 \), and \( H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \). Luminosities are in units of erg s\(^{-1}\) and BH masses are in units of \( M_\odot \). Unless otherwise specified, “luminosity” refers to the bolometric luminosity, and we are only concerned with the active BH population.

### 2. THE BLACK HOLE MASS BIAS

Denoting \( \lambda \equiv \log L \), where \( L \) is quasar luminosity (bolometric or in a specific band), \( m \equiv \log M_\star \) the true BH mass, and \( m_e \equiv \log M_{\text{vir}} \) the virial BH mass, the probability distribution of virial mass \( m_e \) given true mass \( m \) is

\[
p_0(m_e|m) = \left(2\pi \sigma_{\text{vir}}^2\right)^{-1/2} \exp \left[-\frac{(m_e - m)^2}{2 \sigma_{\text{vir}}^2}\right], \tag{2}
\]

where \( \sigma_{\text{vir}} \) is the uncertainty of the virial BH mass.

The probability distribution of true BH mass \( m \) given virial mass \( m_e \) is

\[
p_1(m|m_e) = p_0(m_e|m) \Psi_M(m) \left( \int p_0(m_e|m) \Psi_M(m) \text{d}m \right)^{-1}, \tag{3}
\]

where \( \Psi_M(m) \equiv \text{dn/dm} \) is the true BHMF. If the sample is selected irrespective of luminosity (i.e., no flux limit) and so we can see all BHs with arbitrary \( m \), then for a power-law true BHMF \( \Psi_M(m) \propto 10^{m/m_\ast} \) and at fixed \( m_e \), the statistical bias between the expectation value \( \langle m \rangle \) and virial mass \( m_e \) is simply

\[
\Delta \log M_\star = m_e - \langle m \rangle = -\ln(10) \gamma_M \sigma_{\text{vir}}^2. \tag{4}
\]

In reality the sample is usually selected in quasar luminosity. If the distribution of luminosity at a given true BH mass for broad line quasars is \( g(\lambda|m) \) where \( \int g(\lambda|m) \text{d}\lambda = 1 \), the expectation value of \( m \) at fixed luminosity \( \lambda \) is

\[
\langle m \rangle_\lambda = \frac{\int m g(\lambda|m) \Psi_M(m) \text{d}m}{\int g(\lambda|m) \Psi_M(m) \text{d}m}, \tag{5}
\]

and the luminosity-weighted average of \( m \) in the quasar sample is

\[
\langle m \rangle = \frac{\int_{\lambda_1}^{\lambda_2} \Psi_L(\lambda) \langle m \rangle_\lambda \text{d}\lambda}{\int_{\lambda_1}^{\lambda_2} \Psi_L(\lambda) \text{d}\lambda}, \tag{6}
\]

where \( \lambda_1 \) and \( \lambda_2 \) are the luminosity limits in the sample, and the quasar LF \( \Psi_L \equiv \text{dn/d}\lambda \) is

\[
\Psi_L(\lambda) = \int \Psi_M(m) g(\lambda|m) \text{d}m. \tag{7}
\]

On the other hand, the virial BH masses depend on luminosities, and the sample averaged virial mass is

\[
\langle m_e \rangle = \frac{\int_{\lambda_1}^{\lambda_2} \Psi_L(\lambda) \int_{\lambda_1}^{\lambda_2} \frac{f(m_e|m_\gamma)}{f(m_\gamma|m_\gamma)} \frac{d\lambda}{\Psi_L(\lambda)} d\lambda}{\int_{\lambda_1}^{\lambda_2} \Psi_L(\lambda) d\lambda}, \tag{8}
\]

where \( f(m_e|m) \) is the probability distribution of \( m_e \) given luminosity \( \lambda \) and \( \int f(m_e|m) \text{d}m_e = 1 \). In the absence of a theoretical model for \( f(m_e|m) \), we adopt here an empirical recipe for \( f(m_e|m) \): recall that the virial BH mass is expressed in terms of luminosity and FWHM. Neglecting the scatter from converting bolometric luminosity to continuum luminosity, which is typically \( \sim 0.1 \text{ dex} \) (e.g., Richards et al. 2006a), we have

\[
m_e = C_1 \lambda + 2 \log(\text{FWHM}) + C_2. \tag{9}
\]

Here, \( C_1 \approx 0.5-0.7 \) is the slope in the measured luminosity–radius relation for BLRs and \( C_2 \) is calibrated empirically (e.g., Kaspi et al. 2005, 2007; Bentz et al. 2006, 2009; McLure & Jarvis 2002; Vestergaard & Peterson 2006). For statistical quasar samples, the distribution of FWHMs does not depend much on luminosity (e.g., Shen et al. 2008a; Fine et al. 2008), i.e., for any given luminosity, the FWHM distribution has a constant mean value and a log-normal scatter \( \sigma_{\text{FWHM}} \) around the mean. Therefore at fixed luminosity, the value of \( m_e \) is independent of the true mass \( m \), i.e.,

\[
f(m_e|m) = \left(2\pi \sigma_{\text{line}}^2\right)^{-1/2} \exp \left[-\frac{(m_e - (C_1 \lambda + C_2))^2}{2 \sigma_{\text{line}}^2}\right]. \tag{10}
\]

where \( \sigma_{\text{line}} = 2\sigma_{\text{FWHM}} \) is the scatter in virial mass resulting from the scatter in FWHM \( \sigma_{\text{FWHM}} \) at fixed luminosity, and the constant mean value of FWHM is absorbed in \( C_2 \).

Equations (6)–(8) have analytical results for specific forms of \( \Psi_M(m) \) and \( g(\lambda|m) \). For a power-law true BH mass function \( \Psi_M(m) \propto 10^{m/m_\ast} \), and a power-law relation between true mass and luminosity \( \lambda = am + b \) with a constant log-normal scatter, \( \sigma_L, \) i.e.,

\[
g(\lambda|m) = \left(2\pi \sigma_L^2\right)^{-1/2} \exp \left[-\frac{(\lambda - (am + b))^2}{2 \sigma_L^2}\right]. \tag{11}
\]

Equations (6)–(8) yield

\[
\Psi_L(\lambda) \propto 10^{\lambda g/a} \tag{12}
\]
\[ \langle m \rangle = \frac{\gamma_M \ln(10) \sigma^2_L}{a^2} - \frac{1}{\gamma_M \ln(10)} + \frac{\lambda_2 10^{\gamma_M/a} - \lambda_1 10^{\gamma_M/a}}{a(10^{\gamma_M/a} - 10^{\gamma_M/a})} ; \]

(13)

\[ \langle m_e \rangle = C_2 - \frac{C_1 a}{\gamma_M \ln(10)} + \frac{C_1 (\lambda_2 10^{\gamma_M/a} - \lambda_1 10^{\gamma_M/a})}{10^{\gamma_M/a} - 10^{\gamma_M/a}}. \]

(14)

Note throughout this paper we assume that this constant scatter, \( \sigma_L \), is solely the variations that are uncompensated by the corresponding variations in FWHM, i.e., it is one of the two sources of the virial uncertainty \( \sigma_{\text{vir}} \) (Equation (1)).

There are eight parameters that determine the bias \( \Delta \log M_\bullet = \langle m_e \rangle - \langle m \rangle; \sigma_L, \gamma_M, a \) and \( b \) are for the underlying true BH mass function and luminosity (or Eddington ratio) distributions at fixed true mass, which are model assumptions and must be tuned to produce the observed quasar LF; \( C_1 \) and \( C_2 \) are determined from the empirically calibrated virial estimators and measured FWHM distributions in statistical quasar samples; \( \lambda_1 \) and \( \lambda_2 \) are observational windows determined from the specific quasar sample under investigation. A further constraint comes from the assumption that virial mass is an unbiased estimator of the true sample under investigation. A further constraint comes from the assumption that virial mass is an unbiased estimator of the true sample under investigation.

Thus the bias reduces to

\[ \Delta \log M_\bullet = -\gamma_M \ln(10) \frac{\sigma^2_L}{a^2} = \] (15)

from Equations (13) and (14) (see also Section 4.4 in Shen et al. 2008a), which is independent of luminosity.

For demonstration purposes here we take a model with parameters \( \sigma_L = 0.4, \gamma_M = -4, a = 2, \) and \( b = 28.4 \). These parameters produce a slope of \( \gamma_M = -2 \) in the quasar LF, close to the observed bright-end slope (e.g., Richards et al. 2006b; Hopkins et al. 2007), and constants \( C_1 = 1/a = 0.5 \) and \( C_2 = -b/a = -14.2 \), which are consistent with the commonly used virial mass calibrations and the observed FWHM distributions (e.g., McLure & Jarvis 2002; Vestergaard & Peterson 2006; McGill et al. 2008; Shen et al. 2008a). With these parameters, the difference between the sample-averaged true and virial BH masses is \( \langle m_e \rangle - \langle m \rangle \approx 0.37 \) dex.

It is possible to incorporate an additional correlated variation term in luminosity at fixed true mass, which is compensated by the variations in FWHM and hence does not add to the virial uncertainty, but this will make the mass bias discussed in Section 3 even more severe (see later discussions).

where the BH scaling relation is \( \langle m \rangle = C_3 s + C_4 \) with a cosmic scatter \( \sigma_M \), where \( C_3 \) and \( C_4 \) are constants. Given the power-law distribution of \( s \) (Equation (17)), the distribution of true BH mass \( m \) is then also a power-law \( \Psi_M(m) \propto 10^{m/a} \) where \( \gamma_M = \gamma_s/C_3 \). Assuming the distribution of luminosity \( \lambda \) at fixed \( m \) is given by Equation (11) we can derive the distribution of \( \lambda \) at fixed \( s \) as

\[ p_0(\lambda|s) = \int g(\lambda|m)p_0(m|s)dm \]

\[ = \left( 2\pi \sigma^2_\lambda + a^2 \sigma^2_\mu \right)^{-1/2} \exp \left\{ -\frac{[\lambda - (aC_3 s + b + aC_4)]^2}{2(\sigma^2_\lambda + a^2 \sigma^2_\mu)} \right\} \]

(19)

Therefore the distribution of \( s \) at fixed luminosity \( \lambda \) is

\[ p_1(s|\lambda) = p_0(\lambda|s)\Psi_s(s) \left( \int p_0(\lambda|s)\Psi_s(s)ds \right)^{-1} \]

\[ = \left( 2\pi \sigma^2_\lambda \right)^{-1/2} \exp \left\{ -\frac{[s - (C_{\lambda} + C_6)]^2}{2\sigma^2_\lambda} \right\} \]

(20)

where

\[ \sigma^2_\lambda = \frac{\sigma^2_L + a^2 \sigma^2_\mu}{a^2 C_3^2}, \quad C_5 = \frac{1}{aC_3}, \quad C_6 = \ln(10)\gamma_s \sigma^2_\lambda - \frac{b + aC_4}{aC_3}. \]

(21)

Similarly, the distribution of \( m \) at fixed \( \lambda \) is

\[ p_1(m|\lambda) = g(\lambda|m)\Psi_M(m) \left( \int g(\lambda|m)\Psi_M(m)dm \right)^{-1} \]

\[ = \left( 2\pi \sigma^2_\mu \right)^{-1/2} \exp \left\{ -\frac{[m - (C_{\lambda} + C_6)]^2}{2\sigma^2_\mu} \right\} \]

(22)

where

\[ \sigma^2_\mu = \frac{\sigma^2_L}{a^2 C_3^2}, \quad C_7 = \frac{1}{a}, \quad C_8 = \frac{\ln(10)\gamma_s \sigma^2_L - b}{a^2 C_3}. \]

(23)

Hence, the Lauer et al. bias, i.e., the excess in true BH mass at fixed luminosity is simply

\[ \Delta \log M_{\bullet, \text{Lauer}} = \langle m \rangle|_\lambda - (C_3 (s)|_\lambda + C_4) \]

\[ = -\ln(10)\gamma_s \sigma^2_\lambda / C_3. \]

(24)

Since \( \gamma_s \) is negative, the Lauer et al. bias states that at fixed luminosity, the average true BH mass is already biased high with respect to the expectation from the mean BH scaling relation. On top of that, the virial mass bias (Equation (16)) states that the average BH mass estimate is further biased high with respect to the true mass.

### 3. More Realistic Models

The simple estimate of the BH mass bias discussed above neglects the curvature in the intrinsic BHMF and quasar LF, and hence is only valid at the bright end of the LF. In particular, since the quasar LF flattens below the break luminosity \( \lambda_{\text{break}} \sim 46 \), we expect that the mass bias becomes less severe toward fainter luminosities. It is important to realize that in essentially all of the studies mentioned in Section 1, larger offsets usually occur...
at higher redshift, when their samples are sampling the highermass end of the intrinsic BHMF. It is conceivable then that a false evolution will arise simply from the increasing mass bias toward the high-mass tail.

To estimate the BH mass bias at various redshifts and luminosity sampling ranges, we assume an underlying BHMF as constraints. This forward-modeling procedure is similar to the simulations done in Shen et al. (2008a) and Kelly et al. (2009a, 2009b).

We start with a broken power-law model for the true BHMF:

$$\Psi_M(m) \propto \frac{1}{10^{-\gamma_1(m-m_\ast)} + 10^{-\gamma_2(m-m_\ast)}}.$$

where $\gamma_1$ and $\gamma_2$ are the low-mass and high-mass end slope, and $m_\ast$ is the break BH mass. For the luminosity distribution $g(\lambda|m)$, we still use the Gaussian form in Equation (11). For simplicity, we assume that the set of six parameters, $\gamma_1$, $\gamma_2$, $m_\ast$, $a$, $b$, and $\sigma_z$, are only functions of redshift. At a fixed redshift, a set of the six parameters determines the shape of the quasar LF (7), which is to be constrained by observations. Assuming that the commonly used virial BH estimators are unbiased, we fix $a = 2$ and $b = 28.4$, which corresponds to a BLR radius–luminosity relation $R \propto L^{0.5}$ and virial coefficients and FWHM distributions consistent with previous work (e.g., Shen et al. 2008a; Fine et al. 2008). We also fix the value of $\sigma_z$ during model fitting. Therefore, there are four free parameters: $\gamma_1$, $\gamma_2$, $m_\ast$, and the normalization of the BHMF, $\Phi_0$ (in units of Mpc$^{-3}$dex$^{-1}$). We list the best-fit model parameters in Table 1 for different values of $\sigma_z$ and at several redshifts. We note that these models are mainly used to demonstrate the effects of the virial mass bias and should not be interpreted as our attempt to constrain the true active BHMF.

Given the model true BHMF and luminosity distribution, we can now estimate the bias $\Delta \log M_\ast = \langle m_\ast \rangle - \langle m \rangle$ as a function of the sampled luminosity range using Equations (6) and (8). Figure 2 shows $\Delta \log M_\ast$ for luminosity-limited quasar samples with $\lambda > \lambda_0 \equiv \log L_0$ at several redshifts and for fixed $\sigma_z = 0.3, 0.4$, and 0.5 dex. The bias generally rises when luminosity increases and approaches the asymptotic value $-\gamma_2 \ln(10) \sigma_z^2/a^2$ at the bright end. Because we are fitting our model against the bolometric LF data at individual redshifts, we do not expect identical results since there will be systematics involved in deriving the bolometric LF data from band conversions and from the assumed quasar spectral energy distribution (see details in Hopkins et al. 2007). Nevertheless, this analysis demonstrates that the mass bias $(m_\ast - m)$ could be substantial for bright luminosities and for large dispersion $\sigma_z$ in the luminosity distribution at fixed true mass.

4. DISCUSSION

The virial mass bias discussed here depends on the amplitude of the scatter $\sigma_z$. This is the variation in luminosity at fixed true BH mass which is not compensated by the variation in FWHM. How large is $\sigma_z$? For the best-studied local RM samples and for H$\beta$ only (e.g., Kaspi et al. 2005; Bentz et al. 2006, 2009), the scatter of luminosity around fixed BLR size is on the order of
of $\lesssim 0.2$ dex. However, this is for the best cases. We generally expect larger $\sigma_L$ for statistical quasar samples with single-epoch spectra for the following reasons: (1) single-epoch spectra have larger scatter than averaged spectra in RM samples; (2) these samples usually cover a luminosity range that is poorly sampled by current RM data; (3) the UV regime of the spectrum (in particular for C IV) has more peculiarities and larger scatter in the $R-L$ relation than the H$\beta$ regime, but is usually used at high redshift due to the drop off of H$\beta$ in the optical spectrum; (4) even if some variations in luminosity truthfully trace the variations in BLR radius, $R$, they may still be uncompensated by FWHM if, for instance, there is a non-virial component in the broad line which does not respond to BLR radius variations (especially for C IV; see discussions in Shen et al. 2008a); (5) in many statistical quasar samples, the spectra have low signal-to-noise ratios, which lead to increased uncorrelated scatter between the measured luminosity and FWHM, and bias the virial mass estimates more. Hence it is very plausible that $\sigma_L \gtrsim 0.4$ in most cases, which then accounts for $\gtrsim 0.2$ dex in $\sigma_M$ (e.g., Equation (1)).

Therefore, we conclude that the mass bias discussed here contributes at least 0.2–0.3 dex at $L_\text{bol} \gtrsim 10^{46}$ erg s$^{-1}$, comparable to the statistical bias resulting from the cosmic scatter in the BH scaling relations (e.g., Lauer et al. 2007). In Figure 2, we also show the “observed” BH mass offset (excess at fixed galaxy properties) from the literature. We note that the amount of the uncorrelated scatter $\sigma_L$ might increase with redshift both due to the switch from the H$\beta$ line to the more problematic UV lines and due to the often decreased spectral quality at high redshift. Since the virial mass bias is independent of the Lauer et al. bias, the two statistical biases together can account for a large (or even full) portion of the BH mass excess seen in the data, hence no exotic scenarios are needed to explain this strong redshift evolution. It is possible, however, that there is still a mild evolution in these BH scaling relations, which is inherent to the cosmic co-evolution of SMBHs and their hosts. But given these statistical biases, and given other systematics with single-epoch virial estimators (not discussed here, but see, e.g., Denney et al. 2009), it is premature to claim a strong positive evolution in the BH scaling relations.

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Figure 2. Estimated BH mass bias using our best-fit models of the true BHMF and luminosity distributions, for luminosity-limited quasar samples with $L > L_\text{bol}$ at various redshifts. Shown here are the results for three fixed values of $\sigma_L = 0.3, 0.4,$ and 0.5 dex. The bias increases when the luminosity or $\sigma_L$ increases. We also plot the observed BH mass excess at fixed host properties from Woo et al. (2006, 2008, filled square), Salvianer et al. (2007, open circles), Merloni et al. (2010, filled triangles), and Decarli et al. (2010, open squares) at the corresponding redshifts. Error bars are the standard deviations of objects in their samples. Note that the data from Salvianer et al. (2007) are already binned, hence the dispersion is substantially smaller.

(A color version of this figure is available in the online journal.)