Investigation into First-Year College Students' Misconceptions about Limit Concept: A Case Study Based on Cognitive Style

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Abstract The cognitive style has an important role in determining the characteristics of students on the conceptualization. One of the two types of cognitive styles are field independent (FI) and field dependent (FD). The purpose of this study was to identify the form of conceptual misconceptions about limit and their causes. This type of research is a case study. The data was obtained through interviews based on the conceptual problem-solving tasks. Correspondents used in this study were from first-year college students (95 people) who were studied at Universitas Jambi. The results showed that (1) FI forms of students' misconceptions, namely: misconceptions about interpreting the concept of function limit and function conditions which have limits, such as: left limit and right limit. This problem occurred due to incomplete reasoning and wrong intuition. The other result of this research was to obtain (2) FD forms of students' misconception, namely, (a) misconceptions about prerequisite material: drawing graphs of functions, misconceptions about determining the domain of a function, misconceptions about definitions and forms of function notations, misconceptions about determining function values on function graphs (b) misconceptions about the concept of limits: misconceptions about interpreting the concept of limits in a function, the function requirements (left limit, right limit) and misconceptions about understanding the meaning which is close to the limit function (limit-x) approaching infinity.

Keywords Cognitive Style, Field Independent and Field Dependent, Limit Concept, Misconceptions, Mathematics Education

1. Introduction

Mathematics is the science of logic regarding form, structure, quantity, and concepts that interconnected between one to another. Thus, one of the competencies needed by students in mastering mathematics is a deep understanding of mathematical concepts. The theory of constructivism stated that students have to find and transform complex information, check new information with old rules, and revise them if the rules are no longer appropriate by themselves [1]. The theory was the basis of learning activities. It also emphasized the fact that if students do a cognitive process, it will fulfill students' cognition in the form of structuring concepts from the learning environment delivered by the teacher. When students constructed a concept understanding in the cognitive structure which was obtained from interactions with the learning environment, it would highly produce errors in interpreting concepts. Mistakes in interpreting mathematical concepts are often called misconceptions [2].

Misconception was referred to a concept that was not followed by a scientific understanding or expert agreement in the field [3]. Furthermore, [4] and [5] defined misconception as a cognitive structure in the form of a strong understanding that was different from what should be according to scientific principles in general. Besides, it would disturbed the acceptance of new understandings. Misconceptions in understanding mathematical concepts have been found in many studies [5], [6], [15], [16], [7]–[14].

First-year college student need to adapt to entering university level [17]–[23]. Usually, they still carry the previous concept of learning, namely, high school level [18]. Early detection related to misconceptions could facilitate teachers to provide appropriate learning instructions. Limit concept underlay all concepts in
calculus, thus it made limit become extremely fundamental in this learning [5]. If not properly being prepared, then limit concept could be difficult to be understood in calculus [24].

Specifically, a misconception about the limit concept has been carried out by [5] for secondary school level. They also explained how cognitive style affects the misconceptions. Misconceptions would occurred when students received information about a concept. Each students has a unique way or fondness in obtaining, organizing information (cognition) and processing information (conceptualization), which we usually called cognitive style [25]–[30]. Thus, it could be presumed that the cognitive style would play an important role in making students’ misconceptions. The research had the same results with Lawson & Thompson (1988) who mentioned that one of the factors which caused students’ misconceptions was the field-independent (FI) and field-dependent (FD) cognitive styles.

2. Method

2.1. Research Design

This study identified forms of misconceptions about the limit concept in first-year college students based on their cognitive style. Therefore, this research used a qualitative paradigm with a case study approaching.

2.2. Subject Research

This research was conducted in The Mathematics Education Study Program in Universitas Jambi. The research subjects were first-year college students (95 people) who took calculus course by considering the following reasons for research subjects selection: (1) registered first-year college students who had to take calculus courses, (2) first-year college students who had a cognitive style of FI and FD, (3) first-year college students who had misconceptions about the concept of limit based on the Certainty of Response Index (CRI) category with CRI value > 2.5 and wrong answer category, but high CRI value, which could be interpreted as experiencing misconception.

2.3. Data Collection

The data collection is through a pretest before learning is carried out. This means that students’ knowledge of the concept of limits is inherited from high school. Research data collection was carried out by the following stages: (1) A diagnostic was conducted through a multiple Certainty of Response Index (CRI) test. The aim of the test was to obtain data on students experiencing misconceptions [4]. (2) Cognitive style tests that using GEFT was carried out [30], [32]to determine the type of congressional style of research subjects based on CRI tests that were distinguished based on cognitive style. (3) After finding the research subject that experienced misconceptions based on the classification of the positive force, then the limit concept understanding test would be performed in the form of an essay test (as example in Figure 1). Based on the results of the third stage of the test, further identification of the form of misconception was carried out. (4) The forms of misconception from the concept understanding test were also explored to find out the forms and causes of misconceptions that occurred, through semi-structural interviews and video records. This activity was a form of data triangulation aimed at testing the validity and reliability of the data that had been obtained.

![Figure 1. Problem about understanding the concept of limit](image-url)
2.4. Data Analysis

The data consisted of: (1) misconception classification tests using CRI TEST. The misconception criteria adopted from [33] are explained in Table 1 as follows.

| Answer Criteria | Low CRI Value (<2.5)                                           | High CRI Value (>2.5)                                           |
|-----------------|----------------------------------------------------------------|----------------------------------------------------------------|
| Correct answer  | The answer is correct, but the CRI value is low, meaning it doesn't know the concept (Lucky guess) | Correct answer and high CRI value mean mastering the concept well. |
| Incorrect answer| Wrong answers and low CRI values mean it doesn't know the concept. | Incorrect answer but high CRI value mean misconception.          |

The CRI scores in Table 1 were taken from the average CRI scores for every student. There were groups of students who knew the concept (TK), did not know the concept (TTK) and those who had misconceptions (MK); (2) data from the interview activity was recorded and then mapped based on the misconception indicator code to the concept of limit, as shown in Table 2 below.

| Domain Concept | Indicator                                                                 |
|----------------|---------------------------------------------------------------------------|
| The concept of prerequisite limit of functions | Understanding the definition of a function |
|               | Understanding the meaning of notation in a function                        |
|               | Drawing a graph of a function                                              |
|               | Determining the value of a function                                        |
|               | Determining the area of origin of a function.                              |
|               | Determining the result area of a function.                                 |
| The concept of limit of functions                 | Understanding the definition of a limit of function                        |
|               | Understanding the meaning of notation in the limit of function             |
|               | Understanding the meaning “close” to the limit of function                |
|               | Determining the right limit of a function                                  |
|               | Determining the left limit of a function                                   |
|               | Understanding the properties continuity of a function                      |
|               | Determining the conditions for the existence a limit of function           |
|               | Determining the value limit of function                                    |

3. Research Result

3.1. Forms of Misconception about the Concept of Limit Experienced by FI Students

3.1.1. Misconception about Understanding the Concept Definition of the Limit on a Function

We know that the formal definition of a \( \lim_{x \to c} f(x) = L \) is function \( f \) approaches the limit \( L \) as \( x \) approaches \( c \). thinking about limits is related to the behavior of a function near \( c \) not at \( c \). And the function \( f \) does not require to be defined in \( c \) [34]. FI students understood the concept of \( \lim_{x \to c} f(x) = L \), as \( x \) approached \( c \), the value of \( x \) did not approach to \( c \). Then, misconceptions occured when understanding the value of \( L \), where subjects understood that the value of \( L \) is right at \( f(x) \) and required the function \( f \) to be defined in \( c \).
FL students understood that the value of left limit was the same as the right limit, which was 1, but due to an empty circle at the graph function, \( \lim_{x \to 1} f(x) \) did not have a limit value. So, it could be interpreted that FL students’ understanding of \( \lim_{x \to c} f(x) = L \), required a value in \( f(x) \), or in other words, the value of \( L \) was exact on \( f(x) \).

### 3.1.2. Determining the Conditions for the Existence of a Limit of Function (Left and Right-Hand Limit)

| Questions                                                                 | Student’s Answers                                                                 |
|---------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
| Explain the definition of \( \lim_{x \to c} f(x) = L \)                   | Student: So, the function of limit is \( f(x) \) as \( x \) approaches \( c \), then \( f(x) = L \). Teacher: Furthermore, if \( x \) approaches 1, is it very close to 1 or equal to 1? Student: I think Sir, limit \( f(x) \) is not equal to 1 as \( x \) approaches 1. Teacher: How about the value \( f(x) = L \), is the value exact \( L \) or close to \( L \). Student: Yes, same as \( L \) (exact) |
| Analysis of \( \lim_{x \to 1} f(x) \) base on the left and right-hand limits | Student FI considers that the “empty sphere” on the graph he made states that it has no limits. It can be seen from the following graph function made by Student FI. |

Table 3. Misconceptions in interpreting the concept of limit on a function

### 3.1.3. Understanding the Meaning "Close" to the Limit of Function

The misconception that occurred in the next FL student was interpreting the close meaning of a function. Students understood that in an empty circle, \( f(x) \) is undefined when \( x = c \), and then it would define that the function has no limits. In question number 2 part “c”, limit \( f(x) \) when \( x \) approaches \( -2 \), students answered “its limit does not exist” for the reason that the left limit and right limit are the same (found). But because \( f(-2) \) is an empty round, then the limit does not exist. So, based on the previous description, it can be interpreted that FL students understand that a limit must be exact when \( x = c \), \( f(c) \) exists. The concept of a limit that uses language approaches, does not have to be right at \( x = c \) not owned by FL student.
Table 5. Understanding the meaning “close” to the limit of function

| Questions                                                                 | Student’s Answers                                                                 |
|---------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------|
| Determine limit value of \( \lim_{x \to -2} f(x) \) or state if it does not have the limit value. | Teacher: Why \( \lim_{x \to -2} f(x) \) does not have a limit?  
Student: The limit does not exist because the circle is empty, but has the left and right limits. |
| Determine limit value of \( \lim_{x \to \infty} f(x) \)                      | Teacher: Why \( \lim_{x \to \infty} f(x) \) does not have a limit?  
Student: Because I don’t know where it ends.                                                                                     |

FI students stated that there was no limit approaching infinity because they did not know where it ends.

3.2. Forms of Misconception about the Concept of Limits Experienced by FD Students

3.2.1. Misconception about Sketching Function Graphs

Misconceptions that occurred in FD students was found in sketching graphs of functions. Students understood that in drawing a graph of a conditional function, the closed and opened intervals at the intervals/limits of a given function did not affect (continuous and non-continuous functions at \( x = c \)) the graph functions. This could be seen in the sketch that drawn by FD students as follows,

![Figure 2. Sketch of a function graph by FD students](image)

3.2.2. Misconception about Understanding the Concept Definition of the Limit on a Function

The researcher asked the FD students about both of their knowledge or their understanding about the concept definition of the limit on a function. The FD students replied that the function is like a sequence. The value of \( L \) is the limit or the limit value. Like a sequence, if there is no limit the sequence has no limit. The students’ statement could be seen in Table 6.

Table 6. Misconception about understanding the concept definition of the limit on a function

| Questions                                                                 | Student’s Answers                                                                 |
|---------------------------------------------------------------------------|----------------------------------------------------------------------------------|
| \( \lim_{x \to c} f(x) = L \)                                            | Teacher: How do you understand \( \lim_{x \to c} f(x) = L \)?  
Student: This states that the value (L) of the limit of a function \( f(x) \) when \( x \) approaches \( c \) |
| Define \( \lim_{x \to c} f(x) = L \)                                      | Teacher: Explain what is your understand about the value of L                     
Student: The value of L is the limit or the limit value. Like a sequence, if there is no limit the sequence has no limit. |

3.2.3. The Misconception about Determining the Function Value of a Function Graph

In determining the function value of a function graph, FD students experienced misconceptions about determining the function value of a function that had an asymptote line. Students understood that the asymptote line was the value of a function. Misconceptions could be seen from the answer sheet FD in question number 2 part "e". FD students answered that the value of \( f(3) \) was 3. Based on the interview in Table 7, it appeared that FD students understood that the asymptote line was \( x = c \), so \( f(x) = c \).
Table 7. The misconception about determining the function value of a function graph

| Questions | Student's Answers |
|-----------|-------------------|
| Determine the value of \( f(3) \) | Teacher: Why is your answer \( f(3) = 3 \)? Student: Actually I'm confused, but my feeling is the dotted line at 3, so I answered |

Then the researchers assumed if at \( f(x) \), \( x \) would approached -4 from the right limit with a full round whether the limit had a value, and then the FD students answered no value for it. The researcher asked the value of limit \( f(x) \), when \( x \) approached -4, and FD1 replied 2 and 4.

Table 8. Determination of the conditions for the existence a limit function (left and right-hand limits)

| Questions | Student's Answers |
|-----------|-------------------|
| Determine the limit value of \( \lim_{x \to -4} f(x) \) or state if it does not have the limit value | Teacher: Why is your answer \( \lim_{x \to -4} f(x) = 2 \)? Student: Because there is a full circle, that is \( f(-4) = 2 \) |

4. Discussion, Conclusions and Suggestion

This study discusses the forms of misconceptions experienced by FD and FI students about understanding the concept of limit functions. [35] stated that it was not enough to know only about the misconceptions, but it must be understood in detail and strategies were needed to solve it. Based on the results, FD students experienced misconceptions starting from the learning materials delivery from the the teachers in learning activities. FD students experienced not only misconceptions of concept of limits, but also misconceptions about the prerequisite concept, such as: sketching graphs of functions. [36] stated that individual knowledge, related to understanding the problem situation, would cause variations in mental models that were formed and were constructed. The misconceptions stemmed from a wrong understanding of a concept. Schema Cognitive structure that has been formed into an understanding would continue to be used in the formation of further cognitive structure schemes.

The term cognitive was used to refer the processes that occured in the individual brain, which assisted in the process, manipulation, storing, and retrieving information about the outside world [37]. Piaget in [38] explained that schemata (schemas) were a cognitive structure. An individual could bind, understood, and responded to a stimulus due to the operation of this scheme. In other words, cognitive skills were the processes or skills that could help us think, solve problems, collaborate, and create schemes.

Incorrect understanding of the concept of prerequisites for FD students made an understanding scheme. When an external stimulus (new matrices schemes) was given, FD students would tend to assimilate it. Assimilation is the process of directly integrating new stimuli into established schemes[38], which means that FD students do not modify the schemes they have. For example, FD students understand the value of a function in \( c \) or \( f(c) \), and there is a new concept that is given that is the limit of infinite
functions. The understanding of the concept of infinite functions formed into misconceptions in which FD students tended to assimilate the prior knowledge schemes that they had before, namely the concept of function values. FD students understood that \( \lim_{x \to c} f(x) = L \) then the function had no limit, and students assumed that the limit existed if the function was defined \( x = c \). Misconceptions of FD students occurred because they were confused about the term "close" to the function limit. [39] stated that the delivery of ideas (ways of conveying concepts) from the teacher in learning that was beyond the ability of students' thinking could cause misconceptions about understanding the concept of limits. This is an important factor for teachers to know what students' abilities and characteristics are in learning. The characteristics of FD students who tended to be influenced by complex contexts and tended to view a concept as a whole made acceptance of student concepts become unobstructed, so that understanding concepts could form misinterpretation because of the limitations of the appropriate concept structure. For example, one of the causes of FD students' misconceptions was the students' cognitive style, which occurred in the concept of the definition of the limit function. FD students understood that \( \lim_{x \to c} f(x) = L \), required using limits, and if there were no restrictions, then it was infinite and had no limit on that function. This indicated that in the understanding of a complex nature, cognitive style influenced students on understanding the concept of "close" to the limit, thus causing misconceptions. Misconceptions about the limit function concept also occurred in FI students. However, misconceptions about prerequisite material did not occur in FI students, and it appeared due to incomplete reasoning and wrong intuition [31]. It is possible to happen because the FI students tend to be good at understanding complex concepts. As explained by [30], FI students were not easily fooled by elements that were not relevant to the context. They also could determine the part - simple parts separate from the original context.

With the understanding of teachers related to detecting and preventing misconceptions, it was expected to minimize the occurrence of misconceptions in students. [40] stated that teachers would be able to arrange their instructions if they knew possible misunderstandings and misconceptions that students might have. Misconceptions could be minimized by providing visual displays and animations when learning limit functions [41]. However, [5] stated that the use of single animation might not be effective, because an explanation should be given while using it. Visual learning method alone was not sufficient in developing the concept of limits.

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