On the design of multiple-relay cooperative MIMO networks with partial channel state information

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Abstract—This paper deals with the problem of jointly designing the source precoder, the relaying matrices, and the destination equalizer in a multiple-relay amplify-and-forward (AF) cooperative multiple-input multiple-output (MIMO) wireless network, when partial channel-state information (CSI) is available. Specifically, the considered approaches are based on the knowledge of instantaneous CSI of the first-hop channel matrix, whereas only statistical CSI of the second-hop channels is assumed. In such a scenario, with respect to the case when instantaneous CSI of both the first- and second-hop MIMO channel matrices is exploited, existing network designs exhibit a significant performance degradation. Relying on a relaxed minimum-mean-square-error (MMSE) criterion, we show that the design based on the potential activation of all possible antennas for all available AF relays leads to a mathematically intractable optimization problem. Therefore, we develop a joint relay-antenna selection procedure that determines the best subset of the available antennas possibly belonging to different relays. Monte Carlo simulations show that, compared to designs based on the selection of the best relay, the proposed strategy offers a significant performance gain, by also outperforming other recently proposed relay/antenna selection schemes.

Index Terms—Amplify-and-forward relays, multiple-input multiple-output (MIMO), partial channel state information.

I. INTRODUCTION AND SYSTEM MODEL

A
mplify-and-forward (AF) relaying is an effective way to improve transmission reliability over fading channels, by taking advantage of the broadcast nature of wireless communications [1]. Further performance improvements can be achieved by equipping the cooperative network with multiple-input multiple-output (MIMO) terminals [2], [3].

The reference model for a cooperative MIMO network that aims at transmitting a symbol block $b \in \mathbb{C}^{N_b}$ from a source to a destination is the following one. Let us assume that there is no direct link between the source and a destination, due to high path loss values or obstructions, and that the transmission between the source and the destination is assisted by $N_R$ half-duplex relays. The numbers of antennas at the source, relays, and destination are $N_S$, $N_R$, and $N_D$, respectively. The received signal at the destination can compactly be expressed as

$$r = Cb + v$$

where $C \triangleq GFH_0 \in \mathbb{C}^{N_b \times N_b}$ is the dual-hop channel matrix and $v \triangleq GFw + n$ is the equivalent noise vector. The composite matrices $H \triangleq [H_1^T, H_2^T, \ldots, H_{N_R}^T]^T \in \mathbb{C}^{(N_c \times N_b) \times N_b}$ and $G \triangleq [G_1, G_2, \ldots, G_{N_R}] \in \mathbb{C}^{N_b \times (N_c \times N_b)}$ collect the first- (backward) and second-hop (forward) MIMO channel coefficients of all the relays, respectively, whereas the diagonal blocks $F_i \in \mathbb{C}^{N_b \times N_b}$ of $F \triangleq \text{diag}(F_1, F_2, \ldots, F_{N_R})$ denote the relaying matrices, and $F_0 \in \mathbb{C}^{N_c \times N_b}$ represents a source precoding matrix. Finally, $w \in \mathbb{C}^{N_c N_b}$ and $n \in \mathbb{C}^{N_b}$ gather the noise samples at all the relays and at the destination, respectively. The vector $r$ is subject to linear equalization at the destination through the equalizing matrix $D \in \mathbb{C}^{N_b \times N_b}$, hence yielding an estimate $\hat{b} \triangleq Dr$ of the source block $b$, whose entries are then subject to minimum-distance (in the Euclidean sense) detection.

With reference to the aforementioned system model, to achieve the expected gains, accurate channel state information (CSI) is required at the MIMO nodes, i.e., source, AF relays, and destination. Full CSI (F-CSI) is invoked in many papers (see, e.g., [4]–[12]) dealing with the optimization of cooperative MIMO networks. Specifically, accounting for (1), F-CSI is tantamount to requiring: (i) instantaneous knowledge of the first-hop channel matrix $H$; (ii) instantaneous knowledge of the second-hop channel matrix $G$; (iii) instantaneous knowledge of dual-hop channel matrix $C$. While the dual-hop channel matrix $C$ can be directly estimated at the destination by training, separate acquisition of the first- and second-hop matrices $H$ and $G$ is more burdensome in communication resources and signal overhead, especially in networks with multiple relays.

Relay selection allows one to reduce signaling overhead and system design complexity in single-input single-output (SISO) cooperative networks [13]–[15]. The design of SISO relay selection procedures that provide diversity gains - even when F-CSI is not available - has been addressed in [16]–[20]. Such methods rely on partial CSI (P-CSI) since selection of the best relay is based only on the instantaneous knowledge of the source-to-relay channels. However, it can be seen from these works that the developed methods do not achieve full diversity: irrespective of the number of relays, the diversity order of such schemes is limited to only one (or two if the destination is within the coverage range of the source). For SISO nodes, a P-CSI relay selection scheme has been
proposed in [21] that yields full diversity order $N_C$. However, besides the instantaneous knowledge of the source-to-relay channels, such a method requires that the selected relay sends instantaneous CSI of the corresponding source-to-relay channel to the destination for optimal decoding. Moreover, the optimization in [21] does not admit a closed-form solution and is solved numerically by using a line search algorithm.

It has been shown in [22] that P-CSI relay selection approaches based only on the instantaneous knowledge of $H$ suffer from diversity loss even for MIMO nodes. In addition to the instantaneous knowledge of $H$, statistical CSI of the second-hop matrix $G$ is used in [11, 23] to perform relay/antenna selection for a MIMO AF cooperative network. However, with respect to F-CSI designs, the solutions developed in [11, 23] still exhibit a significant performance degradation.

In this paper, we study optimization methods for multiple-relay cooperative MIMO networks with instantaneous knowledge of $H$ and statistical CSI of $G$. In this P-CSI scenario, we consider a relaxed joint minimum-mean-square-error (MMSE) optimization of the source precoder $F_0$, the AF relaying matrices in $F$, and the destination equalizer $D$, with power constraints at the source [24] and at destination [2], [4]. Specifically, capitalizing on our preliminary results in [11], three additional contributions are reported in this paper. First, we show that the design attempting to activate all possible antennas for all available AF relays leads to a mathematically intractable optimization problem. Second, we provide the proofs of the main results reported in [11], by enlightening that the recruitment of the best relay is a suboptimal design. Third, we develop a joint antenna-and-relay selection algorithm that can significantly outperform the relay/antenna selection approaches proposed in [11], [18], [23] in terms of average symbol error probability (ASEP).

II. BASIC ASSUMPTIONS AND PRELIMINARIES

The symbol block $b$ in [1] is modeled as a circularly symmetric complex random vector, with $E[|b b^H|] = I_{N_b}$. The entries of $H$ and $G$ are assumed to be unit-variance circularly symmetric complex Gaussian (CSCG) random variables. Additionally, the noise vectors $w$ and $n$ are modeled as mutually independent CSCG random vectors statistically independent of $(b, H, G)$, with $E[|w w^H|] = I_{N_C N_b}$ and $E[|n n^H|] = I_{N_0}$, respectively. Hereinafter, we assume that $C$ in [1] and the conditional covariance matrix

$$K_{vv} \triangleq E[|v v^H| G] = G F F^H G^H + I_{N_0}$$

of $v$, given $G$, have been previously acquired at the destination during a training session. In this case, it is well-known (see, e.g., [24]) that, for fixed matrices $F_0$ and $F$, the matrix $D$ minimizing the trace of the conditional mean square error (MSE) matrix $E(F_0, F, D) \triangleq E[(b - b)(b - b)^H]$, given $H$ and $G$, is the Wiener filter $D_{\text{mmse}} = C_H(C C_H + K_{vv})^{-1}$.

Optimization of $F_0$ and $F$ is herein carried out under the assumption that only P-CSI is available at the source and at the relays: the source and the relays perfectly know the first-hop channel matrix $H$, but the $i$th relay has only knowledge of the second-order statistics (SOS) of its own second-hop channel matrix $G_i$, for each $i \in \{1, 2, \ldots, N_C\}$. In some systems, the relays may be able to exchange information among themselves before transmission [25]. In this case, knowledge of $H$ at the relays is realistic [4], [11], [13]–[17]. Moreover, since the SOS of $G_i$ vary much more slowly than the instantaneous values of $G_i$, the feedback overhead from the destination to the relays is significantly reduced, compared to [21].

III. THE PROPOSED DESIGN WITH P-CSI

In order to obtain $F_0$ and $F$, we propose to minimize the statistical average of the trace of

$$E(F_0, F) \triangleq E(F_0, F, D_{\text{mmse}}) = (I_{N_b} + C_H K_{vv}^{-1} C)^{-1}$$

with respect to $G$, under appropriate power constraints. To this aim, we assume that: a1) $F_0$ is full-column rank, i.e., rank($F_0$) $= N_b \leq N_s$; a2) $G F H$ is full-column rank, i.e., rank($G F H$) $= N_s \leq N_0$. It noteworthy that assumption a2) necessarily requires that the matrices $F H$ and $H$ are full-column rank, i.e., rank($F H$) $= N_s \leq N_C N_b$. Such assumptions ensure that $C$ is full-column rank. Specifically, we consider the following optimization problem:

$${\min_{F_0, F}} \mathbb{E}_G \left\{ \right. \left. \text{tr} \left[ (I_{N_b} + C_H K_{vv}^{-1} C)^{-1} \right] H \right\} \text{ subject to (s.to)}$$

$$\text{tr}(F_0 F_0^H) \leq P_S \quad \text{and} \quad \mathbb{E}_G \left\{ \right. \left. \text{tr}(G F K_{zz} F^H G^H) \right\} |H| \leq P_D$$

where $K_{zz} \triangleq E[z z^H | H] = H F_0 F_0^H H^H + I_{N_s N_b}$ is the conditional covariance matrix of the vector $z \in C^{N_s N_b}$ collecting the signals received by all the relays, given $H$, whereas $P_S > 0$ and $P_D > 0$ limit the average transmit power of the source [24] and the average received power at the destination [2], [4], respectively. Since problem (4) is nonconvex, we consider its relaxed version

$${\min_{F_0, F}} \mathbb{E}_G \left\{ \right. \left. \text{tr} \left[ (I_{N_b} + F_0^H H^H F^H G^H G F F_0)^{-1} \right] H \right\} \text{ s.to} \text{ tr}(F_0 F_0^H) \leq P_S \quad \text{and} \quad \mathbb{E}_G \left\{ \right. \left. \text{tr}(G F F^H G^H) \right\} |H| \leq P_D$$

where we have used the expression of $C$ and the inequalities $\text{tr}(I_{N_s} + C_H K_{vv}^{-1} C) \geq \text{tr}(I_{N_s} + C_H C)^{-1}$ and $\text{tr}(G F K_{zz} F^H G^H) \leq \text{tr}(G F G^H) \text{tr}(K_{zz})$ [26], [27]. Closed-form evaluation of the objective function in (5) is cumbersome; however, under a1) and a2), a manageable expression can be obtained by observing that

$$\mathbb{E}_G \left\{ \right. \left. \text{tr} \left[ (I_{N_b} + F_0^H H^H F^H G^H G F F_0)^{-1} \right] H \right\} \approx \mathbb{E}_G \left\{ \right. \left. \text{tr} \left[ (F_0^H H^H F^H G^H G F F_0)^{-1} \right] H \right\}$$

when the minimum eigenvalue of $F_0^H H^H F^H G^H G F F_0$ is approximation (6) is very accurate in the high signal-to-noise ratio (SNR) region, even for small values of $N_C$. The right-hand side (RHS) of (6) can be evaluated in closed-form as stated by the following lemma.
Lemma 1. Let us assume that: a3) $N_D > N_B$. Then, under a1), a2), and a3), it results that
\[
\mathbb{E}_G \left\{ \text{tr} \left[ (F_0^H H^H G F H F_0)^{-1} \right] | H \right\} = \frac{\text{tr}(R^{-1})}{N_D - N_B}
\]
where $R \triangleq F_0^H H^H F H F_0 \in \mathbb{C}^{N_D \times N_B}$. 

Proof: See the Appendix.

At this point, evaluation of the expectation in the second constraint of (5) is in order. In this respect, one has
\[
\mathbb{E}_G \left[ \text{tr}(F^H G^H G F) | H \right] = \mathbb{E}_G \left[ (G^H G) F^H F \right] = \text{tr}(F^H F)
\]
where we have also used the cyclic property of the trace operator. Therefore, under a1), a2), and a3), the optimization problem (5) can be simplified as follows
\[
\min_{F_0,F} \text{tr} \left[ (F_0^H H^H F H F_0)^{-1} \right] \\
\text{s.t.} \quad \text{tr}(F_0^H F_0) \leq \mathcal{P}_S \quad \text{and} \quad \text{tr}(F^H F) \leq \mathcal{P}_D. \tag{9}
\]
To solve (9), we use the following lemma.

Lemma 2. For positive definite matrix $A \in \mathbb{C}^{n \times n}$, the following inequality holds
\[
\text{tr}(A^{-1}) \geq \sum_{\ell=1}^m \frac{1}{\{A\}_{\ell\ell}} \tag{10}
\]
where $\{A\}_{\ell\ell}$ is the $\ell$th diagonal entry of $A$ and the inequality is achieved if $A$ is diagonal.

Proof: See [29] p. 65.

As a consequence of Lemma 2 the minimum value of the cost function in (9) is achieved if $F_0^H A F_0$ is diagonal, with $A \triangleq H^H F^H F H \in \mathbb{C}^{N_D \times N_D}$. In the following subsections, we consider three different approaches to obtain the diagonalization of $F_0^H A F_0$: the first one is based on the SVD of the composite matrix $H = [H_1^T, H_2^T, \ldots, H_{N_C}^T]^T$ and it results in a (possible) selection of all the relays; the second one relies on the SVDs of the single matrices $H_1, H_2, \ldots, H_{N_C}$, thus leading to a single-relay selection; the last one exploits the SVDs of row-based partitions of $H$ and it can be interpreted as a joint antenna-and-relay selection.

A. Design based on the SVD of $H$

One can attempt to recruit all the relays in the second phase of the cooperative scheme by diagonalizing $F_0^H A F_0$ through the SVD $H = U_h (C_{N_C \times (N_{C}N_B - N_C)} A_h)^T V_h^H$ of $H$, where the matrices $U_h \in \mathbb{C}^{N_D \times (N_{C}N_B - N_C)}$ and $V_h \in \mathbb{C}^{N_D \times N_{C}N_B}$ are unitary, and $A_h \triangleq \text{diag} [\lambda_h(1), \lambda_h(2), \ldots, \lambda_h(N_{C}N_B)]$ gathers the corresponding nonzero singular values arranged in increasing order. By substituting the SVD of $H$ in $A$, it follows by direct inspection that $F_0^H A F_0$ is diagonal if
\[
F_0 = V_{h,\text{right}} \Omega^{1/2} \tag{11}
\]
\[
F = Q_i \Delta_i^{1/2} U_{h,\text{right},i} \uparrow \tag{12}
\]
where $V_{h,\text{right}} \in \mathbb{C}^{N_D \times N_B}$ contains the $N_B$ rightmost columns from $V_h$, the matrices $\Omega \triangleq \text{diag}[\omega(1), \omega(2), \ldots, \omega(N_B)]$ and $\Delta_i \triangleq \text{diag}[\delta_i(1), \delta_i(2), \ldots, \delta_i(N_S)]$ are determined in a second step, for $i \in \{1, 2, \ldots, N_C\}$, the arbitrary matrix $Q_i \in \mathbb{C}^{N_{C}N_B \times N_{C}N_B}$ obeys $Q_i^H Q_i = I_{N_{C}N_B}$, provided that $N_S \leq N_R$, $U_{h,\text{right}} \equiv [U_{h,\text{right},1}^T, U_{h,\text{right},2}^T, \ldots, U_{h,\text{right},N_C}^T]^T \in \mathbb{C}^{(N_{C}N_B - N_C) \times N_S}$ contains the $N_S$ rightmost columns from $U_h$, with the matrix $U_{h,\text{right},i} \in \mathbb{C}^{N_{C}N_B \times N_S}$ being full-column rank, and the superscript $\dagger$ denotes the Moore-Penrose inverse of $Q_i$.

Using (11) and (12), problem (9) ends up to
\[
\min_{\omega, \{\delta_i\}_{i=1}^{N_C}} f_0 (\omega, \{\delta_i\}_{i=1}^{N_C}) \quad \text{s.t.} \quad \sum_{\ell=1}^{N_B} \omega(\ell) \leq \mathcal{P}_S, \omega(\ell) > 0, \tag{13}
\]
and
\[
\sum_{i=1}^{N_C} \sum_{\ell=1}^{N_S} \delta_i(\ell) (U_{h,\text{right},i} U_{h,\text{right},i}^H)^{-1} \leq \mathcal{P}_D, \delta_i(\ell) > 0
\]
with $\Delta_N \triangleq N_S - N_B \geq 0$. All the inequality constraints in (13) are linear. However, it is shown in Appendix B that, when $N_C > 1$, the cost function (14) is the sum of $N_B$ functions that are neither strictly convex nor strictly concave on $\mathbb{R}^{+1}$. Hence, trying to solve (13) with available optimization tools leads to poor performance in multiple-relay networks.

B. Design based on the SVD of $H_i$

A simple design can be developed by setting $F_1 = O_{N_B \times N_{C}N_B}$, for each $i \in \{1, 2, \ldots, N_C\} - i^*$. Basically, such a choice leads to a single-relay selection scheme [11], which imposes that only one relay (i.e., that for $i = i^*$) is recruited to transmit and all the remaining ones keep silent in the cooperative phase.

Herein, we assume that $H_i$ is full-column rank, i.e., rank($H_i$) = $N_S \leq N_R$, for each $i \in \{1, 2, \ldots, N_C\}$. Let $U_{h,\text{right}} [O_{N_{C}N_B \times (N_{C}N_B - N_C)} A_h] \uparrow V_{h,i}^H$ be the SVD of $H_i$, where $A_{h,i} \triangleq \text{diag}[\lambda_{h,i}(1), \lambda_{h,i}(2), \ldots, \lambda_{h,i}(N_{C}N_B)]$ contains the singular values of $H_i$, arranged in increasing order, and the unitary matrices $U_{h,i} \in \mathbb{C}^{N_D \times N_{C}N_B}$ and $V_{h,i} \in \mathbb{C}^{N_D \times N_{C}N_B}$ collect the corresponding left and right singular vectors, respectively.

In this case, it results that $A = H_i^H F_i^H F_i H_i$ and, by substituting the SVD of $H_i$ in this matrix equation, one has that the diagonalization of $F_0^H A F_0$ is ensured by
\[
F_0 = V_{h,i^*,\text{right}} \Omega^{1/2} \tag{15}
\]
\[
F = Q_i \Delta_i^{1/2} U_{h,i^*,\text{right}} \tag{16}
\]
where $U_{h,i^*,\text{right}} \in \mathbb{C}^{N_D \times N_B}$ and $V_{h,i^*,\text{right}} \in \mathbb{C}^{N_D \times N_{C}N_B}$ contain the $N_S$ and $N_B$ rightmost columns from $U_{h,i^*}$ and $V_{h,i^*}$, respectively. $Q_i \in \mathbb{C}^{N_{C}N_B \times N_{C}N_B}$ is an arbitrary matrix obeying $Q_i^H Q_i = I_{N_{C}N_B}$ and $\Delta_i \triangleq \text{diag}[\delta_i(1), \delta_i(2), \ldots, \delta_i(N_S)]$. To fully specify the solution of (9) in the case of single-relay selection, optimization of $\Omega$, $\Delta_i$, and $i^*$ is accomplished in two steps.
First, for a given \(i^* \in \{1, 2, \ldots, N_C\}\), by substituting (13) and (16) in (9), one obtains the scalar optimization problem with linear inequality constraints:

\[
\min_{\omega, \delta} f_1(i^*, \omega, \delta) \quad \text{s.t.} \quad \sum_{\ell=1}^{N_\ell} \omega(\ell) \leq P_S, \quad \omega(\ell) > 0, \\
\quad \text{and} \quad \sum_{\ell=1}^{N_\ell} \delta(\ell) \leq P_D, \quad \delta(\ell) > 0 \quad (17)
\]

with

\[
f_1(i^*, \omega, \delta) \triangleq \sum_{\ell=N_\ell-N_{\ell+1}}^{N_\ell} \frac{1}{\lambda^2_{h,i^*}(\ell) \omega(\ell) \delta(\ell)} \quad (18)
\]

where \(\omega\) has been previously defined in Subsection III-A and \(\delta \triangleq [\delta(1), \delta(2), \ldots, \delta(N_\ell)]^T \in \mathbb{R}^{N_\ell}\). Since \(f_1(i^*, \omega, \delta)\) is a convex function (see Appendix B), the optimization problem (17) is convex and, thus, its solution \(\omega_{\text{opt}}(i^*)\) and \(\delta_{\text{opt}}(i^*)\) can be found by using efficient numerical techniques [30].

Second, the optimal value \(i_{\text{opt}}\) of \(i^*\) is obtained as

\[
i_{\text{opt}} \triangleq \arg \min_{i \in \{1, 2, \ldots, N_C\}} f_1(i^*, \omega_{\text{opt}}(i^*), \delta_{\text{opt}}(i^*)) \quad (19)
\]

which can be numerically evaluated by exhaustive search. In a nutshell, the solution of (19) allows one to single out the best relay among the \(N_C\) available ones.

In the SISO configuration, i.e., when \(N_B = N_S = N_R = N_D = 1\), and when there is no precoding at the source, i.e., \(F_0 = F = \sqrt{P_S}\), it can be verified that \(F_{i^*} = \text{diag}(0, 0, 0, 0, 0, 0, 0)\) and (19) boils down to \(i_{\text{opt}} \triangleq \arg \max_{i \in \{1, 2, \ldots, N_C\}} |h_i|^2\), with \(h_i\) denoting the channel coefficients between the source and the \(i\)th relay. According to [31], such a scheme has a full diversity order equal to \(N_C\). However, as we will see in Section IV such a design suffers from a diversity loss in a MIMO setting, i.e., when \(N_S, N_R, N_D > 1\).

C. Design based on the SVDs of row-based partitions of \(H\)

In the considered cooperative MIMO network, there are \(N_C\) relays equipped with \(N_R\) antennas, which amounts to a total number of \(N_C N_R\) distributed antennas. Here, we propose to choose the best \(N_B = N_S\) antennas out of the \(N_C N_R\) ones.\(^1\) Such antennas can either be physically located on a single relay or be spatially distributed over different relays, thus accomplishing a joint antenna-and-relay selection scheme.

Let \(S \triangleq \{(n, i), \forall n \in \{1, 2, \ldots, N_R\}, \forall i \in \{1, 2, \ldots, N_C\}\}\) collect all the \(N_C N_R\) antenna elements in the network, with the generic (ordered) pair \((n, i)\) uniquely identifying the \(n\)th antenna located on the \(i\)th relay. The number of distinct subsets of \(S\) that have exactly \(N\) elements is given by the binomial coefficient \(Q \triangleq \binom{N_C N_R}{N}\). By excluding the trivial choice \(\emptyset\) and the degenerate case \(S\) (discussed in Subsection III-A), we denote with \(S(q) \triangleq \{(n_1^q, i_1^q), (n_2^q, i_2^q), \ldots, (n_{N(q)}^q, i_{N(q)}^q)\}\) the selected subset of \(S\), with \(q \in \{1, 2, \ldots, Q - 2\}\), obeying \(S(q) \neq S(q')\) for \(q_1 \neq q_2\). Additionally, we use the notation \(N_i(q) \in \{0, 1, \ldots, N_B\}\) to indicate the number of pairs of \(S(q)\) having the same second entry: in other words, \(N_i(q)\) represents the number of antennas activated on the \(i\)th relay according to the \(q\)th selection. It results that \(\sum_{i=1}^{N_C} N_i(q) = N_B\). The selected relaying antennas generate a first-hop channel matrix \(H(q) \triangleq [(H_1(q))^T, (H_2(q))^T, \ldots, (H_N(q))^T]^T \in \mathbb{C}^{N_B \times N_n}\) and a relaying matrix \(F(q) \triangleq \text{diag}(F_1(q), F_2(q), \ldots, F_N(q)) \in \mathbb{C}^{N_B \times N_n}\), with \(H(q) \in \mathbb{C}^{N_B \times N_n}\) and \(F(q) \in \mathbb{C}^{N_B \times N_n}\). By convention, if \(N_i(q) = 0\), then \(H(q)\) and \(F(q)\) are empty matrices.

With reference to the \(q\)th selection, we formulate a new optimization problem, for \(q \in \{1, 2, \ldots, Q - 2\}\), which is formally obtained from (9) by replacing \(H\) and \(F\) with \(H(q)\) and \(F(q)\), respectively, whose cost function achieves its minimum value if \(F(q)^H A(q) F_{0} \) is diagonal (see Lemma 2), with \(A(q) \triangleq (H(q))^H (F(q))^H F(q) H(q)\). For \(i \in \{1, 2, \ldots, N_C\}\), let \(H_i(q) = U_{h,i}(q) [O_{N_{\ell}(q)\times(N_B-N_{\ell}(q))}, A_{h,i}(q)] (V_{h,i}(q))^H\) be the SVD of the (nonempty) matrix \(H_i(q)\), which is assumed to be full-row rank, i.e., \(\text{rank}(H_i(q)) = N_{\ell}(q)\), where \(U_{h,i}(q) \in \mathbb{C}^{N_{\ell}(q)\times N_{\ell}(q)}\) and \(V_{h,i}(q) \in \mathbb{C}^{N_B \times N_{\ell}(q)}\) are unitary, and the diagonal matrix \(A_{h,i}(q) \triangleq \text{diag}([\lambda_{h,i}(1), \lambda_{h,i}(2), \ldots, \lambda_{h,i}(N_{\ell}(q))])\) collects the corresponding non-zero singular values arranged in increasing order. In this case, the diagonalization of \(F(q)^H A(q) F_{0}\) can be obtained by resorting to the following structures

\[
F_{0} = [(V_{h,\text{right}}(q))^H]^{-1} \Omega_{\text{right}}^{-1/2} \quad (20)
\]

\[
F_{i}^{(q)} = Q_{i} \Delta_{i}^{-1/2} (U_{h,i}(q))^H \quad (21)
\]

where \(V_{h,\text{right}}(q) \triangleq [V_{h,\text{right},1}(q), V_{h,\text{right},2}(q), \ldots, V_{h,\text{right},N_C}(q)] \in \mathbb{C}^{N_B \times N_{\ell}(q)}\) with \(V_{h,\text{right},i}(q) \in \mathbb{C}^{N_B \times N_{\ell}(q)}\) gathering the \(N_{\ell}(q)\) rightmost columns from \(V_{h,i}(q)\), \(Q_{i} \in \mathbb{C}^{N_{\ell}(q)\times N_{\ell}(q)}\) is an arbitrary unitary matrix, and \(\Omega\) and \(\Delta_{i}\) have been defined in Subsection III-A.

To optimize \(\Omega, \Delta_{i}\), and \(q\), we resort to a two-step procedure as in the previous subsection. By substituting (20) and (21) in (9) (with \(H(q)\) and \(F(q)\) in lieu of \(H\) and \(F\), respectively), for a given \(q \in \{1, 2, \ldots, Q - 2\}\), one gets the convex optimization problem (see Appendix B) with linear inequality constraints:

\[
\min_{\omega, \{\delta_i\}_{i=1}^{N_C}} f_2(q, \omega, \{\delta_i\}_{i=1}^{N_C}) \quad \text{s.t.} \quad \sum_{\ell=1}^{N_\ell} \omega(\ell) \left[ V_{h,\text{right}}(q)^H V_{h,\text{right}}(q) \right]^{-1}_{\ell \ell} \leq P_S, \quad \omega(\ell) > 0, \\
\quad \text{and} \quad \sum_{\ell=1}^{N_\ell} \delta(\ell) \leq P_D, \quad \delta(\ell) > 0 \quad (23)
\]

with

\[
f_2(q, \omega, \{\delta_i\}_{i=1}^{N_C}) \triangleq \sum_{i=1}^{N_C} \sum_{\ell=1}^{N_\ell} \frac{1}{\lambda^2_{h,i}(\ell) \omega(\ell) \delta(\ell)} \quad (24)
\]

where \(\omega(\ell) \triangleq \omega \left( \sum_{m=1}^{\ell-1} N_{m}(q) + \ell \right)\), whereas \(\omega\) and \(\delta\) have been defined in Subsection III-A. Similarly to the problem (17), the solution \(\omega_{\text{opt}}(q)\) and \(\{\delta_{\text{opt}}(q)\}_{i=1}^{N_C}\) of (23) can be
found by using convex numerical methods [30]. Finally, the best value \( q_{\text{opt}} \) of \( q \) is found by numerically solving the optimization problem

\[
q_{\text{opt}} \triangleq \arg \min_{q\in\{1,\ldots,Q-2\}} f_2\left(q, \omega_{\text{opt}}(q), \{\delta_{i,\text{opt}}(q)\}_{i=1}^{N_C}\right).
\]

(25)

When \( N_B = N_S = N_R = 1 \), the optimization problems \([17]-[19] \) and \([23]-[25] \) yield the same solution and, thus, the design \([25] \) exhibits full diversity order, too. However, we will show in the next section that, when \( N_B, N_S, N_R > 1 \) (MIMO network), the proposed joint antenna-and-relay selection scheme ensures a significant performance improvement with respect to single-relay selection, in terms of both diversity order and coding gain.

IV. NUMERICAL RESULTS

In this section, to assess the performance of the considered P-CSI designs, we present the results of Monte Carlo computer simulations, aimed at evaluating the ASEP of the corresponding cooperative systems, transmitting quadrature phase-shift-keying symbols. We considered two different system configurations: in the former one, all the nodes have a single antenna, i.e., \( N_B = N_S = N_R = N_D = 1 \) (single-antenna nodes); in the latter one, source and relays are equipped with \( N_B = N_S = N_R = 3 \) antennas (multiple-antenna nodes), respectively. We also assumed that \( P_S = P_D = P_{\text{tot}} \). Consequently, the SNR was defined as \( \text{SNR} \triangleq P_{\text{tot}} \), which measures the per-antenna link quality of both the first- and second-hop transmissions. Besides the single-relay selection method described in Subsection III-B referred to as “1-R Selection”, and the joint antenna-and-relay selection scheme developed in Subsection III-C referred to as “JAR Selection”, we also reported the performance of [18] CSI Assumptions I and II in the case of single-antenna nodes and that of [23] for both single- and multiple-antennas nodes. As a reference lower bound, we additionally included in all the plots the ASEP curves of the F-CSI design proposed in [12], whose design however relies on the additional knowledge of the \( i \)th second-hop channel matrix \( G_i \) at the \( i \)th relay, for \( i \in \{1, 2, \ldots, N_C\} \). This F-CSI method exhibits a theoretical diversity order \( N_C N_R - N_B + 1 \) [12].

The ASEP was evaluated by carrying out \( 10^3 \) independent Monte Carlo trials, with each run using independent sets of channel realizations and noise, and an independent record of \( 10^9 \) source symbols.

A. Example 1: Single-antenna nodes

We reported in Figs. 1 and 2 the ASEP performance of the considered schemes as a function of the SNR, for single-antenna nodes and two different values of the number of relays \( N_C \in \{2, 3\} \). We remember that, in the case of \( N_B = N_S = N_R = N_D = 1 \), the two approaches “1-R Selection” and “JAR Selection” are equivalent and, thus, we only depicted the performance of the “1-R Selection” method.

Results clearly show that the diversity order of [18] (CSI Assumption II corresponding to P-CSI) and [23] is limited to only one, irrespective of the number of relays. On the other hand, the “1-R Selection” scheme exhibits the same diversity order of the F-CSI methods proposed in [18] (CSI Assumption I) and [12], which linearly increases with \( N_C \). This fact allows the “1-R Selection” design to significantly outperform both [18] (P-CSI) and [23], which rely on the same amount of CSI. Remarkably, the “1-R Selection” scheme performs comparably to [13] (F-CSI) in the case of \( N_C = 2 \) relays. Compared to single-relay selection, the performance improvement of the F-CSI approaches - arising from the additional instantaneous knowledge of the second-hop matrix \( G \) - becomes more and more evident when the number of relays \( N_C \) increases.

B. Example 2: Multiple-antenna nodes

Figs. 3 and 4 show the ASEP performance of the considered designs as a function of the SNR, for multiple-antenna nodes and two different values of the number of relays \( N_C \in \{2, 3\} \).

It is apparent from Figs. 3 and 4 that the “1-R Selection” approach and [23] perform comparably, by exhibiting an evident diversity loss with respect to the F-CSI design [12]. As announced, especially in the high SNR regime, the proposed
“JAR Selection” design significantly outperforms both the “1-R Selection” scheme and [23], under the same amount of P-CSI. Remarkably, the diversity order of the “JAR Selection” scheme is $N_C = 2$, as in the F-CSI case [12].

V. CONCLUSIONS

We studied the problem of designing multi-relay AF cooperative networks, which involve the knowledge of the instantaneous values of the first-hop MIMO channel matrix and only the SOS of the second-hop one. In such a scenario, antenna/relay selection is necessary to formulate design problems, which can be solved by using standard convex optimization tools. We showed that, in a MIMO setting, the selection of the best relay is suboptimal and much better performance can be obtained by selecting the best antennas distributed over multiple relays. Numerical simulations showed that the proposed joint antenna-and-relay selection approach outperforms significantly existing schemes, which rely on the same amount of P-CSI.

APPENDIX A

PROOF OF LEMMA [1]

Preliminary, we remember that $C = G F H F_0$ is full-column rank if $a1$ and $a2$ hold. It can be shown (see, e.g., [22]) that, conditioned on $H$, the $k$th diagonal entry of the matrix $(C^H C)^{-1} k_{kk}$ follows an inverse-Gamma distribution, with shape parameter $\alpha \triangleq N_D - N_B + 1$ and scale parameter $\beta_k \triangleq 1/(R^{-1})_{kk}$, where $R$ is defined in the lemma statement. Thus, the probability density function (pdf) of the random variable $(C^H C)^{-1} k_{kk}$, given $H$, reads as

$$p_k(x) = \frac{1}{\Gamma(\alpha) \beta_k^\alpha} x^{-\alpha-1} e^{-\frac{x}{\beta_k}}$$  \hspace{1cm} (26)$$

where the gamma function $\Gamma(\alpha) = (\alpha - 1)!$ since $N_D - N_B$ is a non-negative integer number [33]. Therefore, relying on (5) and using (26), one has

$$\mathbb{E}_G \left[ \text{tr} \left( (C^H C)^{-1} \right) \left| H \right. \right] = \sum_{k=1}^{N_B} \mathbb{E}_G \left[ (C^H C)^{-1} k_{kk} \left| H \right. \right]$$

$$= \frac{1}{\Gamma(\alpha)} \sum_{k=1}^{N_B} \frac{1}{\beta_k^\alpha} \left( \lim_{\delta \to 0} \int_{\delta}^{\infty} x^\alpha e^{-\frac{x}{\beta_k}} \hspace{0.2cm} dx \right).$$ \hspace{1cm} (27)$$

After some calculations, eq. (27) can be rewritten as

$$\mathbb{E}_G \left[ \text{tr} \left( (C^H C)^{-1} \right) \left| H \right. \right] = \sum_{k=1}^{N_B} \frac{\beta_k^\alpha}{\Gamma(\alpha)} \lim_{\delta \to 0} \gamma(\alpha - 1, (\delta \beta_k)^{-1})$$

$$= \frac{\Gamma(\alpha - 1)}{\Gamma(\alpha)} \sum_{k=1}^{N_B} \frac{1}{\beta_k} = \frac{1}{\alpha - 1} \sum_{k=1}^{N_B} (R^{-1})_{kk} = \frac{\text{tr}(R^{-1})}{N_D - N_B}$$ \hspace{1cm} (28)$$

where we have exploited the definition of the incomplete gamma function $\gamma(s, x) \triangleq \int_0^x t^{s-1} e^{-t} \hspace{0.2cm} dt$ [33] and its asymptotic property $\Gamma(s) = \lim_{x \to +\infty} \gamma(s, x)$. 

APPENDIX B

HESSIAN OF THE COST FUNCTION [13]

Let us check convexity of a generic summand of the cost function (14). To this end, it is sufficient to study the multivariate function

$$f(x, y_1, y_2, \ldots, y_n) \triangleq \frac{1}{Ax (y_1 + y_2 + \cdots + y_n)}$$ \hspace{1cm} (29)$$

with $A > 0$, $x > 0$, and $y_i > 0$ for each $i \in \{1, 2, \ldots, n\}$. The domain of $f$ is therefore given by $\mathbb{R}^{n+1}_+$, which is a convex set. The function $f$ is twice differentiable over its domain. It is noteworthy that, when $n = 1$, the function (29) ends up to a generic summand of (17) or [23].

Let us calculate the Hessian matrix $\nabla^2 f \in \mathbb{R}^{(n+1) \times (n+1)}$, whose entries are the second-order partial derivatives of $f$ at $(x, y_1, y_2, \ldots, y_n) \in \mathbb{R}^{n+1}_+$, i.e.,

$$\{ \nabla^2 f \}_{ij} = \begin{cases} \frac{\partial^2 f}{\partial x^2}, & \text{for } i = j = 1; \\ \frac{\partial^2 f}{\partial x \partial y_i}, & \text{for } i = 1 \text{ and } j \in \{2, 3, \ldots, n\} \\ \frac{\partial^2 f}{\partial y_i \partial y_j}, & \text{for } j = 1 \text{ and } i \in \{2, 3, \ldots, n\}; \\ \frac{\partial^2 f}{\partial y_i \partial y_j}, & \text{for } i, j \in \{2, 3, \ldots, n\}. \end{cases}$$ \hspace{1cm} (30)$$
We recall that the function $f$ is convex [concave] if and only if the Hessian matrix $\nabla^2 f$ is positive [negative] semidefinite for all the points belonging to its domain.

Using standard calculus concepts, it can be verified that

$$\frac{\partial^2}{\partial x^2} f = \frac{2}{A x^2 (y_1 + y_2 + \cdots + y_n)}$$

$$\frac{\partial^2}{\partial x \partial y_j} f = \frac{1}{A x^2 (y_1 + y_2 + \cdots + y_n)^2}$$

$$\frac{\partial^2}{\partial y_i \partial y_j} f = \frac{2}{A x (y_1 + y_2 + \cdots + y_n)^3}.$$ 

We note that all the entries of $\nabla^2 f$ are nonnegative on $\mathbb{R}^{n+1}$.

In the particular case of $n = 1$, it is readily seen that the determinant of $\nabla^2 f \in \mathbb{R}^{2 \times 2}$ is given by

$$\det(\nabla^2 f) = \frac{3}{A^2 x^4 y_1^2} > 0$$

which shows that, when $n = 1$, $f$ is a strictly convex function on $\mathbb{R}_+$. Therefore, since the sum of convex functions is convex [30], the cost functions [17] or [23] are convex.

On the other hand, when $n > 1$, by resorting to the Laplacian determinant expansion by minors, it results that

$$\det(\nabla^2 f) = \sum_{j=1}^{n+1} (-1)^{j+1} \left\{ \nabla^2 f \right\}_{1j} M_{1j}$$

where $M_{1j} \in \mathbb{R}^{n \times n}$ is a so-called minor of $\nabla^2 f$, obtained by taking the determinant of $\nabla^2 f$ with row 1 and column $j$ crossed out. It can be verified that $M_{1j}$ is zero, for each $j \in \{1, 2, \ldots, n+1\}$. Thus, the determinant of $\nabla^2 f$ is zero at each point belonging to the domain of $f$ if $n > 1$. This is sufficient to infer that $\nabla^2 f$ is neither positive nor negative definite, which implies in its turn that, when $n > 1$, $f$ is neither strictly convex nor strictly concave on $\mathbb{R}^{n+1}$.

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