The improved equilibrium optimization algorithm

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Abstract—Recent studies on the nature-inspired approaches are also focusing on the improvements in addition to the construction of new algorithms. Improvements on the controlling parameters are also relevant. The important control parameter balancing the exploration and exploitation ratio was always relevant to the maximum allowed iteration times. It was very difficult for engineers in order to find the solution within a given limitation. Therefore, we paid our attention to the control parameter t to the newly raised equilibrium optimization algorithm and the improvements for the engineers. Two governing equations are proposed and simulation experiments are carried out. Results show that the improvements could be used without too much influence to the final results, and sometimes the improvements on the control parameters could be forgotten.

Keywords—equilibrium algorithms; controlling parameters; simulation experiments; benchmark functions

I. INTRODUCTION

Nowadays, machine learning approaches have been the hot spot for various types of applications including the algorithm themselves. No Free Lunch Rule[1] (NFLR) was found and there indeed exists no such algorithm that could solve all problems with higher accuracy and less complexity. Therefore, despite the wandering for another algorithm, the improvements of the existed algorithm are also paid much attention to. Various types of the improvement are raised such as the hybrid algorithms which combine two or more inherent characteristics together[2], the multi-objective approaches[3] and the binary versions[4]. Some of the improvements on the controlling parameters remain the usage of the maximum allowed iteration times (labelled maxIter subsequently). However, it would be troublesome for the engineers. They want to know about how many iterations that could satisfy a restricted limitation. Accordingly, the governing equation for the control parameter must be revised to meet such needs.

In this paper, we propose two approaches to govern the control parameter t for the equilibrium optimization algorithm (EOA) we called here, which was just raised in 2019[5]. The creator claimed that EOA would perform better than others after detailed simulation experiments on benchmark functions in addition to some real engineering problems. The rest of this paper is arranged as follows: Section 2 would mainly describe on the original EOA and in section 3, the improvements on the control parameter would be shown, we would carry out some classical experiments on the representatives of benchmark functions. Discussions on the experimental results and conclusions would be made in section 5.

II. THE EQUILIBRIUM OPTIMIZATION ALGORITHM

Maybe after some careful observation on the solutions for the well-mixed dynamic balance on a control volume V, together with the well performance of the grey wolf optimization (GWO) algorithm[6]. The authors construct a new algorithm based on the analytical solutions to the equations and introduce four best individuals $C_{eq(0)}, C_{eq(1)}, C_{eq(2)}, C_{eq(3)}$ including additional averaged one $C_{ave}$ in guiding the updating concentration or position of individuals $C_i$ (for i-th individual):

$$C_i = C_{eq} + (C_{eq} - G) \cdot F + \frac{G}{AV} (1 - F) \quad (1)$$

Where $C_{eq}$ is a representative from the equilibrium optimization pool:

$$C_{eq,pool} = \{C_{eq(0)}, C_{eq(1)}, C_{eq(2)}, C_{eq(3)}, C_{ave}\} \quad (2)$$

Where:

$$C_{ave} = \frac{C_{eq(0)} + C_{eq(1)} + C_{eq(2)} + C_{eq(3)}}{4} \quad (3)$$

And for the given problems functioned as $f(x)$, the fitness values for the candidates in the equilibrium pool satisfy the following restrictions:

$$f(C_{eq(0)}) \leq f(C_{eq(1)}) \leq f(C_{eq(2)}) \leq f(C_{eq(3)}) \quad (4)$$

$F$ is an exponential parameter with the following restrictions:

$$F = a_1 \text{sign}(r_1 - 0.5)(e^{-\lambda t} - 1) \quad (5)$$

Where $a_1$ is a constant parameter controlling the exploration. $r_1$ and $\lambda$ are two random numbers. The controlling parameter $t$ is formulated as follows:
\[ t = \left(1 - \frac{it}{\text{maxIter}}\right)^{\left(\frac{it}{\text{maxIter}}\right)} \]  

(6)

G is a generation parameter as formulated:
\[ G = GCP(C_{eqs} - C_i) \cdot F \]  

(7)

\[ GCP = \begin{cases} 0.5r_2 & r_3 \geq GP \\ 0 & r_3 < GP \end{cases} \]

GP is a constant parameter balancing the ratio between exploration and exploitation.

III. THE IMPROVED EQUILIBRIUM OPTIMIZATION ALGORITHM ON THE CONTROLLING PARAMETER

The control parameter \(t\) is a function of the current iteration time \(it\) and the maximum allowed iteration time \(\text{maxIter}\). Consequently, \(\text{maxIter}\) should be known at first when EOA is applied in optimization. However, the engineers are eager to find the best solutions for the real problems within an accuracy. \(\text{maxIter}\) might be known even more, they are irrelevant to such problems. Therefore, if EOA is applied in solving such real engineering problems, the governing equation must be reformulated according to the real needs.

A. Improvements with the exponential function

The current control parameter is declined from 1 to 0 according to equation (6), as graphed in Figure 1. It was relevant to the maximum allowed iteration times. To eliminate the relevance, exponential term is usually introduced:\[ t = a_m e^{-a_n t} \]  

(8)

\(a_m\) and \(a_n\) are two constant numbers. \(a_m\) is the maximum value at the beginning while \(a_n\) controls the slope of changes. However, the exponential function has only one types of slope, seen from Figure 2.

B. Improvements with the cosine function

Considering the similar profile of equation (6) with that of cosine function. We can introduce the cosine function to construct an irrelevant equation to the maximum allowed iteration times:\[ t = 1 + \cos(a_n \cdot \pi t) \]  

(9)

\(a_n\) is another constant number. The profile of this function is shown in Figure 3. Obviously, if we cannot evaluate the maximum iteration times, cosine function would remain a fixed slope trend since the inputs for cosine function would be limited into \([0, \frac{\pi}{4}]\).

Despite the fixed slope profile, equations (8) and (9) are all irrelevant to the maximum allowed iteration times. And consequently, it would affect the performance somewhat. But with a better understanding of EOA, we could choose an approximate value at the beginning.

IV. SIMULATION EXPERIMENTS

We would carry out the simulation experiments on benchmark functions. In accordance with the optimization difficulty of them, detailed classifications are made and different setups are met. For a given restriction, 100 simulations would be carried out and a basic and simple statistical analysis would be carried on. For simplify, we would focus on the usage instead of the values, thus a universal detailed series would be restricted as \(a_1 = 2, a_2 = 2, \text{gp} = 0.5\), and the population size \(n = 30\), dimension \(d = 2\).

A. Experiments on unimodal benchmark functions

With results from various types of nature-inspired approaches, we are clearly knowing that the unimodal benchmark functions are easy to find best values with high accuracy. All of these functions are smooth and only have one global optimum, see Figure 4 with Step function.
Considering the initial objectives that designing for engineers, a fixed restriction for 1% least residual error is setup for all problems. The aim for engineers might be the final position or concentration of the best individuals. Whereas we are focusing on the minimum iteration times that used to achieve the goal.

\[ t = 1 + \cos(a_0 \pi / t) \]

Figure 3 Varying profile for the Cosine functions

For simplicity, we would label the improved EOA with the control equation (8) as IEOA and (9) IEOAS as follows. The results and their statistical analysis are shown in Table 1.

**Table 1** Results analysis for unimodal benchmark functions

| Function | Algorithms | Min | Max | Mean | Std.Dev |
|----------|------------|-----|-----|------|---------|
| Sphere   | EOA        | 5   | 17  | 9.31 | 1.6595  |
|          | EEOA       | 7   | 307 | 17.95| 29.9036 |
|          | IEOAS      | 5   | 35  | 8.73 | 3.2859  |
| Sum      | EOA        | 1   | 12  | 5.95 | 1.3955  |
| Square   | EEOA       | 2   | 16  | 7.32 | 2.4612  |
|          | IEOAS      | 2   | 11  | 5.28 | 1.5302  |
| Schwefel2.20 | EOA    | 10  | 37  | 13.9 | 3.0903  |
|          | IEOA       | 15  | 2134| 98.45| 342.4331|
|          | IEOAS      | 8   | 37  | 12.14| 3.1274s |

**B. Experiments on the multimodal benchmark functions**

The multimodal benchmark functions are more difficult to be optimized. Most of the benchmark functions are highly multimodal and consequently, the individuals of the swarms are easily trapped in local optima. We can see the complicated profiles and many peaks existed from Figure 5 for Rastrigin function and Figure 6 for Giewank function.

![Figure 5 Profile of Rastrigin function](image)

![Figure 6 Profile of Giewank function](image)

The same criterion would be met with accuracy 1%. The statistical analysis would be carried out and results are shown in Table 2.

**Table 2** Results analysis for multimodal benchmark functions

| Functions | Algorithms | Min | Max | Mean | Std.Dev |
|-----------|------------|-----|-----|------|---------|
| Rastrigin | EOA        | 6   | 148 | 28.82| 24.6322 |
|           | IEOA       | 6   | 244 | 45.98| 45.9728 |
|           | IEOAS      | 6   | 134 | 23.56| 21.5876 |
| Giewank   | EOA        | 7   | 420 | 55.12| 62.1843 |
|           | IEOA       | 2   | 654 | 129.34| 136.2408|
C. Experiments on the bottom flat-like benchmark functions

Unlike the multimodal benchmark functions, the bottom flat-like benchmark functions have basin profile near the optima. The individuals could hardly find the right direction towards the global best optimum when they are wandering on the flat. Therefore, it was worthwhile carrying on the specific experiments. For simplicity, we still restrict the accuracy to be 1% and two representatives of such benchmark functions as Quadric and Zakharrov functions are to be experimented. The Quadric function has a linear bottom (see Figure 7) while the Zakharrov has a flat-like bottom (see Figure 8).

![Figure 7 Profile of Rosenbrock function](image)

![Figure 8 Profile of Zakharrov function](image)

The results are listed in Table 3.

Table 3 Results analysis for unimodal benchmark functions

| Functions | Algorithms | Min | Max | Mean | Std.Dev |
|-----------|------------|-----|-----|------|---------|
| Quadric   | EOA        | 1   | 9   | 4.14 | 1.6126  |
|           | IEOA       | 1   | 16  | 4.43 | 1.9963  |
|           | IEOAS      | 1   | 7   | 3.86 | 1.1227  |
| Zakharrov | EOA        | 3   | 9   | 5.95 | 1.26    |

V. DISCUSSIONS AND CONCLUSIONS

It is very difficult and creative for us to construct a new algorithm, it needs better understanding and recognition and knowledge. On the contrary, there are some basic ways to improve the existed algorithms. In this paper, we propose two types of improvements on the control parameter of EOA.

EOA is proved to be significantly efficient in optimization both the benchmark functions and the real engineering problems. However, for real applications with real problems, sometimes the final accuracy is restricted and the maximum iteration times is not known. Consequently, most of the nature-inspired approaches involves the maximum iteration times parameter to carry on. In such situations, the governing equation to control parameters should be revised to meet the needs.

In this paper, we proposed two types of governing equations for the control parameter $t$ in EOA: the exponential and cosine functions. Simulation experiments were carried out for 100 separated runs. Results are statistically analysed. After careful reviewing of the results, the following conclusions could be drawn.

1. With a small accuracy of 0.1%, all of the benchmark functions involved in these experiments could be optimized within several thousands of iterations. Considering the complexity and the randomness, other classical benchmark functions would not be bothered. Therefore, most of the problems could be solved with EOA with a common used accuracy in engineering.

2. The controlling parameter would not have great influence on the performance of EOA. Results show that although the governing equations are replaced with different forms and different distributions, the final results especially the best results would not change much. Therefore, the engineers could pay more attention to the applications instead of the appearance of equations.

3. Randomness and governing equations for the updating of positions or concentrations of individuals should be paid more attention to. Further improvements could remove the focus on the control parameters, they would have little influence on the final performance.

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