Generation of the Scalar Field and Anisotropy at Quantum Creation of the Closed Universe

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Abstract

The behaviour of the wave function of the Universe under the barrier for anisotropic cosmological Bianchi type IX model with account of influence of the scalar field is explored. In view of known difficulties with interpretation of multidimensional wave functions the method of reduction of such problems to one-dimensional is offered. For this purpose in frameworks of semiclassical approach the system of characteristics equations relative to one variable is written out. This system describe a bundle of the characteristics along which the multidimensional problem is reduced to one-dimensional one that allows to utilize the standard interpretation of the wave function as well as for usual Schrödinger equation. The obtained results for Bianchi type IX model are reduced to the following statement: the Universe tunnels through the barrier from an isotropic state with zero initial value of the scalar field and appear in classically allowed region with small anisotropy that is necessary for providing of long-lived inflation for deriving the Universe such as ours.
1 Introduction

One of the basic problems of a classic cosmology is the presence of an initial singularity. For its overcoming is considering the process of the quantum creation of the Universe. Such approach was suggested by DeWitt [1] and Misner [2] and further was developed by series of the authors. Ya. Zel’dovich [3] and A. Vilenkin [4] have offered the mechanism of the spontaneous creation of the Universe from ”nothing” where ”nothing” means an absence of classic spacetime. The cosmological wave function can be used for calculation of probability distribution of initial configurations of born Universes.

However, the quantum cosmology being based on quantum gravity maintains all of its problems, in particular the inherent divergences. In addition there are some other problems related to process of quantization of the closed Universe. First of them follows from the fact that the wave function of the Universe $\psi$ does not depend on time. It should be understood [1, 5] in the sense that the wave function should describe everything, including the clocks. In other words, time is a some interior parameter intrinsically given configuration and expressed through the material or geometrical variables. The important requirement imposed on this parameter is the monotony of its change. The second problem is the difficulties emergent at attempt of correct determination of a probability density current. We could attempt using of the conservation law of this current [1]

$$j = \frac{i}{2} (\psi^* \nabla \psi - \psi \nabla \psi^*) , \quad \nabla j = 0,$$

but here we conflict with the same problem as in a case of the relativistic Klein-Gordon equation: the probability density current obtained by this way is not positive-definite. However, though we do not know how to solve these problems in general it is possible to do at semiclassical statement of the problem [6].

On the other hand, it is very difficult to find physical interpretation of wave functions even in a semiclassical approximation in case of minisuperspace with several degrees of freedom. Thereupon some authors offers the methods of reduction of multidimensional problems to one-dimensional [7, 8].

The old problem that our Universe even in early youth ”did not want to be” anisotropic and inhomogeneous is only partially resolved in models of the early Universe. It is seems probable that the causality of the Universe at the moment of creation allows to smooth out inhomogeneities of an energy distribution. However, we have no such answer for anisotropy of the Universe. The same problem is of interest for models of quantum cosmology with creation of the Universe from ”nothing”. As is known, the given scenario requires a finiteness of Euclidean action for the Universe
and consequently the Wheeler-DeWitt equation (WDW) for Bianchi type IX models is explored. Closed Friedmann cosmological model is rather special case of this ensemble.

The relevant WDW equation for Bianchi type IX was considered in other statement of problems earlier. In particular, Del Campo and Vilenkin calculated the wave function of the Universe in some extreme cases when the anisotropy is either small or large \[9\]. Amsterdamski \[10\] calculated the wave function in semiclassical approximation at assumption that anisotropy is small.

The peculiar behaviour of the wave function under the barrier clarified by the authors in \[11\] allows to consider the indicated problem in another statement. In instantonic treatment the transition to imaginary time is similar to considering of the WDW equation under the barrier. Thus, as is noted in \[11\] , equation for physical parameters, in particular for the scalar field changes a sign of a kinetic energy under the barrier. It gives in appearance of a principle opportunity of increase of the field during tunneling. This feature was utilised for the scenario of creation of the Universe from "nothing" with small initial energy density of the scalar field. The quantity of last one starts to grow under the barrier that gives on boundary of exterior classically allowed region the value is necessary for sufficiently long period of inflation and further transition to the standard hot Universe. The indicated value of field is determined by the "easiest" way of tunneling.

The similar problem is considered for Bianchi type IX model in this paper.

2 WDW equation for Bianchi type IX model

The metric for Bianchi type IX has the following form (in this paper we use Planck unities \(c = G = \hbar = 1\) \[12\]):

\[
ds^2 = dt^2 - a^2 e^{2\beta_{ij}} \sigma^i \sigma^j,
\]

where \(\sigma^i\) represents the dual 1-forms satisfying to relations \(d\sigma^i = \eta^i_{jk} \sigma^j \wedge \sigma^k\) and

\[
\begin{align*}
\sigma^1 &= \cos \psi d\theta + \sin \psi \sin \theta d\phi, \\
\sigma^2 &= \sin \psi d\theta - \cos \psi \sin \theta d\phi, \\
\sigma^3 &= d\psi + \cos \theta d\phi,
\end{align*}
\]

\(0 \leq \theta \leq \pi, 0 \leq \phi \leq 2\pi, 0 \leq \psi \leq 4\pi\), and the diagonal tensor \(\beta_{ij}\) is parametrized as follows:

\[
\beta_{ij} = \text{diag} \left( \beta_+ + \sqrt{3}\beta_-, \beta_+ - \sqrt{3}\beta_-, -2\beta_+ \right).
\]
As is known, the basic equation describing the quantum evolution of the Universe is the WDW equation [1]. For its construction we shall consider the theory of the scalar field $\phi$ with Lagrangian

$$L = -\frac{R}{2k} + \frac{1}{2} (\partial_\mu \phi)^2 - V(\phi),$$  \hspace{1cm} (5)$$

here $k = 8\pi$ is Einstein gravitational constant. From here, by taking advantage (2), (3) and (4) we obtain:

$$L = 6\pi^2 \left\{ \frac{a^2 \dot{a}^2}{k} - \frac{a^3}{k} \left( \beta_+^2 + \beta_-^2 \right) + \frac{a}{k} (U[\beta_+, \beta_-] - 1) - \frac{a^3}{3} \left[ \frac{1}{2} \beta^2 - V(\phi) \right] \right\},$$  \hspace{1cm} (6)$$

$$U[\beta_+, \beta_-] = 1 + \frac{2}{3} e^{4\beta_+} \left[ \cosh \left( 4\sqrt{3} \beta_- \right) - 1 \right] + \frac{1}{3} e^{-8\beta_+} - \frac{4}{3} e^{-2\beta_-} \cosh \left( 2\sqrt{3} \beta_- \right),$$

where through dot is designated derivative relative to time $t$. Further, we shall find the momenta conjugate with $a, \phi, \beta_+, \beta_-:

$$p_a = \frac{12\pi^2 a^2 \dot{a}}{k}, \quad p_\phi = -2\pi^2 a^3 \dot{\phi}, \quad p_{\beta_+} = -\frac{12\pi^2 a^3 \beta_+}{k}, \quad p_{\beta_-} = -\frac{12\pi^2 a^3 \beta_-}{k}. \hspace{1cm} (7)$$

The Hamiltonian of the system is:

$$H = -\frac{1}{4\pi^2 a^3 p_\phi^2} + \frac{k}{24\pi^2 a^3} p_a^2 - \frac{k}{24\pi^2 a^3} (p_{\beta_+}^2 + p_{\beta_-}^2) - 6\pi^2 \left[ \frac{a}{k} (U[\beta_+, \beta_-] - 1) + \frac{a^3}{3} V(\phi) \right].$$  \hspace{1cm} (8)$$

Quantizing (7) by replacement of momenta $p_a, p_\phi, p_{\beta_+}, p_{\beta_-}$ on the operators $-i\partial/\partial a, -i\partial/\partial \phi, -i\partial/\partial \beta_+, -i\partial/\partial \beta_-$ accordingly and using the rescaling $\phi \rightarrow \sqrt{3/\pi}/2 \Phi$ we write the WDW equation:

$$-\frac{1}{a^3} \frac{\partial}{\partial a} \left( a^3 \frac{\partial \Psi}{\partial a} \right) + \frac{1}{a^2} \left( \frac{\partial^2 \Psi}{\partial \Phi^2} + \frac{\partial^2 \Psi}{\partial \beta_+^2} + \frac{\partial^2 \Psi}{\partial \beta_-^2} \right) +$$

$$+ \frac{9}{4} \pi^2 a^2 \left[ 1 - m^2 a^2 \Phi^2 - U[\beta_+, \beta_-] \right] \Psi = 0. \hspace{1cm} (9)$$

In this equation potential of the scalar field is taken as $V(\phi) = m^2 \phi^2/2$ where $m$ is a mass of quantum of the scalar field. So-called factor ordering $p$ is also introduced. The usage of latter is stipulated by ambiguity of commutation properties of $a$ and $p_a$ [2].
Transformation of the wave function

\[ \psi = a^{-p/2} \Psi \]

allows to eliminate first derivative in (9)

\[ - \frac{\partial^2 \psi}{\partial a^2} + \frac{1}{a^2} \frac{\partial^2 \psi}{\partial \Phi^2} + \frac{1}{a^2} \left( \frac{\partial^2 \psi}{\partial \beta_+^2} + \frac{\partial^2 \psi}{\partial \beta_-^2} \right) - \frac{p}{2} \left( 1 - \frac{p}{2} \right) \frac{1}{a^2} + \frac{9}{4} \pi^2 a^2 \left[ 1 - m^2 a^2 \Phi^2 - U [\beta_+, \beta_-] \right] \psi = 0, \tag{10} \]

or

\[ \left( \nabla^2 - W \right) \psi = 0, \]

where \( \nabla^2 = \frac{1}{\sqrt{g}} \partial_a \left( \sqrt{g} g^{a \beta} \partial_{\beta} \right) \), \( g = |\det g_{\alpha \beta}| \) and \( g_{\alpha \beta} \) is the "metric" in minisuperspace of variables \( a, \Phi, \beta_+, \beta_- \) and the "superpotential" has the form:

\[ W [a, \Phi, \beta_+, \beta_-] = -\frac{p}{2} \left( 1 - \frac{p}{2} \right) \frac{1}{a^2} + \frac{9}{4} \pi^2 a^2 \left[ 1 - m^2 a^2 \Phi^2 - U [\beta_+, \beta_-] \right]. \tag{11} \]

3 Semiclassical approximation

3.1 Deriving of the system of characteristics equations

Let’s take advantage of the obtained WDW equation for examination of a problem of quantum creation of the closed Universe from "nothing". For an isotropic case such problem was considered in [11]. As the total energy of the closed Universe is zero it is clear from form of superpotential (11) that all space is parted into three regions:

1) interior, representing classically unavailable area which we conventionally call "nothing". In this region classical space and time does not exist that is similar to that there is no classical trajectory of a particle in forbidden area at \( \alpha \)-decay of atomic nucleus. The metric in this area experiences strong quantum fluctuations. Its size is defined by the parameter of ordering \( p \);

2) classically forbidden region under the barrier;

3) classically allowed region.

The evolution of the wave function of the Universe represents tunneling through the barrier from "nothing" and output in classically allowed region. In this case the quantities of all physical parameters on which the wave function depends are determined by process of tunneling instead of their arbitrary choice on boundary of classically forbidden and allowed regions.
It is well known that the WDW equation (10) has no the exact solution. Therefore, we shall search for its approximate solution within the framework of the semiclassical approach. Let’s consider evolution of the wave function under the barrier. For this purpose we search for the solution of (10) as $\psi_c = e^{-S}$. The relevant equation for action $S(a, \varphi, \beta_+, \beta_-)$ shall be:

$$- \left( \frac{\partial S}{\partial a} \right)^2 + \frac{1}{a^2} \left[ \left( \frac{\partial S}{\partial \Phi} \right)^2 + \left( \frac{\partial S}{\partial \beta_+} \right)^2 + \left( \frac{\partial S}{\partial \beta_-} \right)^2 \right] + W[a, \varphi, \beta_+, \beta_-] = 0. \quad (12)$$

This equation represents an analog of the Hamilton-Jacobi equation from classical mechanics. For finding of the solution of this nonlinear differential equation it is possible to reduce it to system of the ordinary differential equations called characteristic system of the given partial equation. Utilizing this system it is possible to construct an integrated surface of equation (12) consisting from the characteristics. The required system of the characteristics written relative to arbitrary parameter has a form:

$$\frac{d\Phi}{da} = -\frac{q}{a^2 F}, \quad \frac{d\beta_+}{da} = -\frac{v}{a^2 F}, \quad \frac{d\beta_-}{da} = -\frac{w}{a^2 F},$$

$$\frac{dq}{da} = -\frac{9 \pi^2 m^2 a^4 \Phi}{4 F}, \quad \frac{dv}{da} = -\frac{9 \pi^2 a^2 \partial U[\beta_+, \beta_-]/\partial \beta_+}{2 F},$$

$$\frac{dw}{da} = -\frac{9 \pi^2 a^2 \partial U[\beta_+, \beta_-]/\partial \beta_-}{2 F},$$

$$\frac{dS}{da} = -\frac{1}{4a^2} + \frac{9}{4} \pi^2 a^2 \left[ 1 - m^2 a^2 \Phi^2 - U[\beta_+, \beta_-] \right]/F.$$  

Here the designations are introduced:

$$q = \partial S/\partial \Phi, \quad v = \partial S/\partial \beta_+, \quad w = \partial S/\partial \beta_-,$$

$$F = \sqrt{\frac{1}{a^2} (q^2 + v^2 + w^2) + \left[ \frac{1}{4a^2} + \frac{9}{4} \pi^2 a^2 \left[ 1 - m^2 a^2 \Phi^2 - U[\beta_+, \beta_-] \right] \right]},$$

and the quantities $q, v, w$ represent the "momenta" of the given system and factor ordering $p = 1$. In this case, the role of an arbitrary parameter is played by the scale factor $a$. The obtained system of equations describes an one-dimensional motion of a "particle" along a characteristic. In this case, monotonically varying parameter $a$ can play a role of time \[13\] to which there is an evolution of the Universe. As the momentum of a "particle" is equal to a gradient of action, its possible trajectories are orthogonal to surfaces $S = \text{const}$, i.e. surfaces of a constant phase of the wave function. Thus, at transition to a classical mechanics "beams" associated with function $\psi_c$ (the trajectories orthogonal to surfaces of a constant phase) represent possible trajectories of a motion of "particle" \[6, 14\].
3.2 The analysis of the system of the characteristics

Let's examine directly the system of the characteristics (13). In view of equality to zero of a total energy of the Universe the boundary between classically forbidden and allowed regions is defined from requirement $W[a, \Phi, \beta_+, \beta_-] = 0$. At $\Phi, \beta_+, \beta_- = 0$ the Universe can exist infinitely long time in the interior area posed in an interval

$$0 \leq a \leq a_0,$$

where $a_0 = (1/9\pi^2)^{1/4}$.

For finding of the solutions of system of the characteristics (13) it is necessary to set boundary conditions which look like:

$$a = a_0, \Phi = 0, \beta_+ = 0, \beta_- = 0, q = q_0, v = v_0, w = w_0, S = 0.$$  \hspace{1cm} (15)

Let's specify that we shall start from a limiting isotropic state that follows from equality to zero of parameters of an anisotropy $\beta_+$ and $\beta_-$. Requirements on $q, v, w$ means that a bundle of the characteristics with different initial values of "momenta" transits under the barrier. The given bundle of the characteristics is restricted by the upper limiting characteristic defined as $q_0, v_0, w_0 \to -\infty$ and ground one which is coincident with axis $a$ at $q_0, v_0, w_0 \to 0$ (fig. 1). We are finding the values of quantities $a_*, \Phi_*, \beta_+, \beta_-, S_*$ at an output of each characteristic on boundary of the exterior classically allowed area $a = a_*$ (the last is defined from requirement $W[a_*, \Phi_*, \beta_+, \beta_-] = 0$).

It is clear from numerical analysis of system of the characteristics (13) that the quantity of action $S$ is great at small values of initial "momenta" $q, v, w$. The relevant penetrability of the barrier for such characteristics will be exponentially small since last one will be defined from

$$D = \exp(-2S).$$  \hspace{1cm} (16)

Therefore, the basic contribution to penetrability of the barrier will be given by the characteristics for which $|q, v, w| > 10$. On the other hand, after escaping under the barrier a necessary stage of the evolution of the Universe is inflationary stage. For latter guarantee two important requirements are imposed on scalar field $\Phi$ [3]: 1) value of the field should be about Planck; 2) it should vary enough slowly with the purpose of guarantee of a long-lived period of inflation that is necessary for an stretching of the size of the Universe from Planck to macroscopic. For sufficing these requirements it is necessary that the initial $q$ were at list on the order more than $v$ and $w$. Otherwise, anisotropy suppress the increase of the scalar field and the requirements of guarantee of inflation are disturbed.
It is possible significantly simplify system of the characteristics (13) based on condition of large value of parameters \( q, v, w \)

We note that in this case quantities \( q, v, w \) remains practically constant with increasing of \( a \).

Then (13) accept form:

\[
\frac{d\Phi}{da} = -\frac{q}{a^2 F} \frac{d\beta_+}{da} = -\frac{v}{a^2 F} \frac{d\beta_-}{da} = -\frac{w}{a^2 F},
\]

\[
q = \text{const}, \ v = \text{const}, \ w = \text{const},
\]

(17)

where \( F \approx \sqrt{(q^2 + v^2 + w^2)/a^2} = A/a, \ A^2 = q^2 + v^2 + w^2 \). From (15) we have:

\[
\Phi = \frac{q}{A} \ln \left( \frac{a}{a_0} \right), \ \beta_+ = \frac{v}{A} \ln \left( \frac{a}{a_0} \right), \ \beta_- = \frac{w}{A} \ln \left( \frac{a}{a_0} \right),
\]

(18)

\[
\frac{dS}{da} = \frac{1}{A} \left[ -\frac{1}{4a} + \frac{9}{4} \pi^2 a^3 \left[ 1 - m^2 a^2 \Phi^2 - U[\beta_+, \beta_-] \right] \right].
\]

(19)

For calculation of action \( S \) we shall note that the quantities of parameters of an anisotropy remain always rather small (see fig. 1) that enables to expand potential \( U[\beta_+, \beta_-] \) in series whence, utillizing (18) we have

\[
U[\beta_+, \beta_-] = 8 \left( \beta_+^2 + \beta_-^2 \right) = \frac{8}{A^2} \ln^2 \left( \frac{a}{a_0} \right) (v^2 + w^2).
\]

(20)

Utilizing the last expression integrate (19) that yields

\[
S = \int_{a_0}^{a_*} \frac{1}{A} \left[ -\frac{1}{4a} + \frac{9}{4} \pi^2 a^3 \left[ 1 - m^2 a^2 \Phi^2 - U[\beta_+, \beta_-] \right] \right] da =
\]

\[
= \frac{\pi^2}{48 A^3} - \frac{12 A^2}{\pi^2} \ln \left( \frac{a}{a_0} \right) + 27 q^2 \left( a_*^4 - a_0^4 \right) - m^2 q^2 \left( a_*^6 - a_0^6 \right) +
\]

\[
+ 6 a_*^4 \left[ a^2 m^2 q^2 + 18 (v^2 + w^2) \right] \ln \left( \frac{a}{a_0} \right) - 18 a_*^4 \left[ a^2 m^2 q^2 + 12 (v^2 + w^2) \right] \ln^2 \left( \frac{a}{a_0} \right).
\]

(21)

As it was already noted, the given formula usable only at large values \( q, v, w \). But thus it is visible from (21) that the quantity of action is small and accordingly evaluation of a penetrability of the barrier with help of (16) is impossible as the precision of WKB-approach is considerably decreased. Therefore, in this case it is necessary to use the formula of generalized WKB for evaluation of penetrability of the barrier (13):

\[
D = \frac{\exp(-2S)}{1 + \exp(-2S)}.
\]

(22)
For calculation of average penetrability of the barrier it is necessary to integrate (22) on all $q$, $v$ and $w$. Utilizing (21) we are obtaining:

$$
D = \lim_{q,v,w \to \infty} \frac{1}{q\,v\,w} \int_0^q \int_0^v \int_0^w \frac{\exp(-2S(q,v,w))}{1 + \exp(-2S(q,v,w))} dq\,dv\,dw = \frac{1}{2}.
$$

(23)

4 Conclusions

The solution of a WDW equation is inevitably conjugates to difficulties of interpretation of physical sense of the wave function even in a semiclassical approximation. Similarly to Klein-Gordon equation, even for two variables such wave functions have no positive definiteness of the probability distribution. If in the case of the Klein-Gordon equation this problem is solved because of existence of antiparticles, so for the WDW equation this problem remains un-closed. In this case, the unique intelligent variant is the one-dimensional treatment of the indicated equation with positive definiteness of the probability distribution. This difficulty was mentioned by many authors (see [6] and references inside) at exceeding the limits of the one-dimensional solutions. The consideration of the multidimensional WDW solutions is inevitably connect with using of set of the one-dimensional solutions. Problems numbered in the given paper is inevitably exceed the limits of frameworks of the one-dimensional solutions. In this connection, for tunneling of the wave function under the barrier in semiclassical approximation we use the relevant characteristics equations permitting to analyse the problem on a bundle of the characteristics with the subsequent averaging of results on a selected bundle. Because of the real of wave functions under the barrier such procedure doesn’t conjugate to an interference of wave functions. It does not give in interference problems during an averaging on what the obtained above results are based.

Usage of the characteristics allows to avoid one more difficulty associated with interpretation of multidimensional wave functions. It is often use an analogy between the WDW equation (10) and Schrödinger equation. However, there is an essential difference between them: in the Schrödinger equation the kinetic energy $-\nabla^2$ is positive definite quantity and region where $W > 0$ is really classically forbidden. In the WDW equation, generally speaking, the operator $-\nabla^2$ is non-positive definite that is associated with critical difference of signs of second derivatives. Therefore the classic trajectories can penetrate ”classically forbidden” region with non-zero values of corresponding ”momenta”. In our formulation this problem is solved by a natural way: at using of the characteristics the problem becomes one-dimensional and any problems with signs of ”momenta” do not arise as there is a unique momentum $dS/da$. 
from (13) along the characteristic.

In the interior classically forbidden region ("nothing"), as well as in [1], the existence of the wave function of the Universe relevant to a homogeneous isotropic Universe is supposed. Similar to [11] during tunneling under the barrier the opportunity of increase of the scalar field together with parameters of an anisotropy of model is supposed. The relevant analysis of such scenario, shown above, displays that the large anisotropy of the model collected under the barrier does not ensure necessary increasing of an energy of the scalar field and, therefore, the enough long-lived inflation in the early Universe. The density of energy of the scalar field generated under the barrier which is sufficient for deriving of the Universe such as ours automatically ensures a small anisotropy at transition of the wave function of the Universe in classically allowed region. Thus, the result of the carried out examination is the following statement: the cosmological model created from "nothing" from as much as possible symmetric state (see, e.g. [13]) and ensuring the inflation which is requisite for deriving of the Universe such as ours has a small anisotropy with necessity.

This result, in principle, is in the consent with [17] where the problem on naturalness of inflation in Bianchi type IX model with positive cosmological constant is explored. There it was shown that in case of small parameters of an anisotropy the inflation is inevitable otherwise it is not obvious.

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Fig. 1. The diagrams are constructed subject to boundary conditions (15). The quantities $q, v, w$ are assumed accordingly equal to -1000, -100, -100. a) One-dimensional superpotential $W(a)$. The process of tunneling from "nothing" starts in the point $a_0$ and is ended in $a^*$. b), c), d) Evolution of the scalar field $\Phi$ and parameters of an anisotropy $\beta, \beta^\perp$ accordingly. A dashed line is the boundary of the classically allowed and forbidden areas.