A CRITICAL LOOK AT MASSIVE SCALAR FIELD COLLAPSE

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We present the findings of an investigation of critical behavior in the collapse of spherically symmetric distributions of massive scalar field. Two distinct types of phase transition are observed at the verge of black hole formation and a criterion for determining when each type of transition will occur is given.

1 Introduction

Critical behavior in gravitational collapse is a fascinating area of research within classical General Relativity and exemplifies the role played by non-linear dynamics at the verge of black hole formation. Since Choptuik’s discovery of critical point behavior in the collapse of spherically symmetric distributions of real, massless scalar field, similar phenomenology has been observed in other models of gravitational collapse. Here we summarize the results of an investigation of critical behavior in the collapse of massive spherically symmetric distributions of real scalar field described by the Einstein-Klein-Gordon (EKG) system of equations. Its attractiveness, as another model exhibiting critical point behavior, is further enhanced by the two following properties:

1. The characteristic length associated with the mass $\mu$ destroys the scale invariance of the EKG equations.

2. The EKG system admits unstable soliton-like solutions.

These observations suggest that the qualitative picture of critical point behavior in massive scalar field collapse could differ from the massless limit and might be similar to that found by Choptuik et al. in their study of the collapse of a Yang-Mills field.

2 Results

We find two distinct types of phase transition occur in the collapse of a massive scalar field:

Type I: Black hole formation turns on at finite mass and the critical solutions are unstable soliton stars with masses $M_s \lessapprox 0.6\mu^{-1}$.

Type II: Black hole formation turns on at infinitesimal mass and the critical solution is identical to that found by Choptuik in the collapse of massless scalar fields.
We also formulate a criterion for determining when each type of phase transition will occur, and which helps to clarify the role that intrinsic scales play in critical collapse. If $\lambda$ is the radial extent of the field in its initial configuration, then

- Type I behavior occurs when $\lambda\mu \gtrsim 1$,
- Type II behavior occurs when $\lambda\mu \lesssim 1$.

Intuitively, this selection rule seems reasonable since one expects the massive results to differ from the massless results only if the Compton wavelength of the field $\mu^{-1}$ is smaller than the radial extent of the initial field pulse. Indeed, Fig. 1 (a) shows the Bondi mass of the initial field pulses, described by data sets (i) and (ii) of Table 1, and the resulting black hole masses, at the critical point $\phi_0 = \phi_0^*$. Type I behavior is clearly evident when $\lambda\mu \gtrsim 1$, whilst Type II behavior is observed for $\lambda\mu \lesssim 1$. Figure 1 (a) also indicates that the interface between Type I and Type II phase transitions occurs when the Bondi mass of the initial pulse, $M_B \sim 0.4\mu^{-1}$.

![Figure 1: (a) The Bondi mass of the initial field pulse, and the black hole mass at criticality, versus the radial extent of the field, for the initial data sets (i) and (ii) of Table 1. (b) The black hole mass spectrum for supercritical Type I evolutions.](image)

For generic initial data it is not possible to define the radial extent of the field. However, for initial data sets (i) and (ii) in Table 1 we define $\lambda = 2\sigma$. 

Figure 1 (b) shows the black hole mass spectrum, $M_{\text{BH}}$ vs. $\log|\phi_0 - \phi_0^*|$, for supercritical, Type I, evolutions with $\mu = 1$. The results displayed are for initial data set (iii) of Table 1. Here we can see the mass gap at the threshold of black hole formation is $M_{\text{gap}} \sim 0.51\mu^{-1}$. In general the mass gap lies in the range $0.35 \lesssim \mu M_{\text{gap}} \lesssim 0.59$, with the upper limit being fixed by the maximum mass a soliton star can have. The inset of Fig. 1 (b) demonstrates the care that must be taken when setting the tolerance that decides if, and when, a black hole forms.
For Type II behavior, a tolerance of $10^4$ is sufficient to achieve accurate results. However, for Type I transitions, if the tolerance is too low, we observe spurious discontinuities in the mass spectrum. The inset shows the mass spectrums for black hole formation tolerances of $10^4$ and $10^{10}$, obtained under identical evolutions. While a tolerance of $10^4$ exhibits widespread discontinuities, raising the tolerance to $10^{10}$ reveals them to be purely numerical effects. This notwithstanding, the oscillation imposed on the mass spectrum is not an artifact of the numerics but is similar to the fine structure found by Hod and Piran in the massless results.

Table 1: Three typical initial data sets considered in the collapse of a massive scalar field. The parameters of the set, which may be varied, are shown under Parameters while the types of phase transition which may occur are shown under Type.

| Set | $\phi(u = 0, r)$ | Parameters | Type |
|-----|-----------------|------------|------|
| (i) | $\phi_0 r^2 \exp[-(r - r_0)^2/\sigma^2]$ | $\sigma, \phi_0$ | I, II |
| (ii) | $\phi_0 (1 - \tanh[(r - r_0)/\sigma])$ | $\sigma, \phi_0$ | I, II |
| (iii) | $\phi_0 r(r + r_0) - \sigma/(1 + \exp[r])$ | $\sigma, \phi_0$ | I, II |

3 Conclusions

We find the presence of an intrinsic length scale changes the nature of critical phenomenon in the collapse of a scalar field and speculate that unstable, confined solutions could act as critical solutions in other matter models.

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An explanation of the tolerance and the discontinuities in the black hole mass spectrum is given by Brady et al.