Kalaj, David

Gaussian curvature of minimal graphs in $M \times \mathbb{R}$. (English) [Zbl 07739945]

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Summary: In this paper, we consider minimal graphs in the three-dimensional Riemannian manifold $M \times \mathbb{R}$. We mainly estimate the Gaussian curvature of such surfaces. We consider the minimal disks and minimal graphs bounded by two Jordan curves in parallel planes. The key to the proofs is the Weierstrass representation of those surfaces via $\mathcal{P}$-harmonic mappings. We also prove some Schwarz lemma type results and some Heinz type results for harmonic mappings between geodesic disks in Riemannian surfaces.

MSC:

31C05 Harmonic, subharmonic, superharmonic functions on other spaces
53A10 Minimal surfaces in differential geometry, surfaces with prescribed mean curvature

Keywords:

harmonic mappings; minimal surfaces

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