Coarse-Grained Fluctuation Probabilities in the Standard Model
and Subcritical Bubbles

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Abstract

We compute systematically the probability for fluctuations of the Higgs field, averaged over a given spatial scale, to exceed a specified value, in the Standard Model. For the particular case of interest of averages over one coherence volume we show that, even in the worst possible case of taking the one-loop improved effective potential parameters, the probability for the field to fluctuate from the symmetric to the asymmetric minimum before the latter becomes stable is very small for Higgs masses of the order of those of the $W$ and $Z$ bosons, whereas the converse is more likely. As such, metastability should be satisfied dynamically at the Electroweak phase transition and its dynamics should therefore proceed by the usual mechanism of bubble nucleation with subcritical fluctuations playing no particularly relevant role in it.
The possibility of generating the baryonic asymmetry of the Universe at the Electroweak phase transition has attracted enormous enthusiasm ever since it was realised by Kusmin, Rubakov and Shaposhnikov [1] that the Standard Model potentially satisfies the three necessary Sakharov conditions [2]. Whereas it is arguable whether the \( CP \) violating terms provide the necessary magnitude for the observed baryon to entropy ratio of the Universe [3], in the minimal Standard Model, the out of equilibrium bubble growth, characteristic of a strongly first order transition, is certainly a sine qua non condition for nearly all electroweak baryogenesis scenarios [4] to thrive.

The way of determining the actual order of the electroweak transition is, in itself, a controversial matter. Arguments based on the shape of a loop expanded effective potential about a homogeneous mean field background constitute most of the evidence for a first order phase transition [5,6], but in themselves, and mostly due to unavoidable infra-red divergencies, fail to make it precise how strong it can be, i.e., whether the phase transition dynamics is dominated by a few thin-walled fast growing bubbles.

Furthermore, the presence of important non-perturbative effects seems undisputable and apart from making the perturbative expansion meaningful in the infrared, was argued by Shaposhnikov [7] could render the phase transition more strongly first order. This picture seems to be supported by lattice simulations of the three dimensional time-compactified effective theory [8].

The essential argument for the transition to be weaker, was suggested by Gleiser, Kolb and Watkins [9] and supported by Tetradis [10]. They argued that for sufficiently shallow barriers separating the two minima of the effective potential, it would be possible for the Higgs field to interpolate between them leading to approximate phase equilibrium and ousting thereby bubble nucleation as the mechanism dominating the phase transition dynamics. Instead, thermal equilibrium would approximately prevail throughout the transition making it impossible for the third Sakharov condition for baryogenesis to be fulfilled. Such fluctuations were later argued to be improbable by Dine et al. [5], following estimates for the scalar field two-point function (the variance of a Gaussian fluctuation distribution)
computed at one correlation volume, \( v = O(m(T)^{-3}) \). Subsequently Gelmini and Gleiser \([11]\) using a very different point of view based on modeling the evolution of the bubble population in both phases, rekindled the issue by finding considerable phase mixing for Higgs masses larger than about 55 GeV. Among other differences, inherent to quite distinct approaches, the nucleation rate or probability for a fluctuation, coherent over a given volume, to exceed a given amplitude seems to be at the heart of the standing discrepancies. In what follows we compute the latter quantity by following Hindmarsh and Rivers \([12]\) in showing how general fluctuating probabilities over given volumes can be systematically computed and how they are essentially characterised by a hierarchy of coarse-grained \( n \)-point correlation functions.

To achieve this we write the Higgs sector of the Standard Model, after shifting the scalar field about a homogeneous background scalar field \( \phi \), in the usual cartesian decomposition, in terms of two real scalar fields, \( \varphi_1 \) and \( \varphi_2 \), such that

\[
\Phi = \phi + \varphi_1 + i\varphi_2
\]  

(1)
as

\[
\mathcal{L}_H = -V(\phi) + \mathcal{L}_0 + \mathcal{L}_I,
\]

(2)

where

\[
\mathcal{L}_0 = \frac{1}{2}(\partial_\mu \varphi_1)(\partial^\mu \varphi_1) - \frac{1}{2}m_1^2 \varphi_1^2 \\
+ \frac{1}{2}(\partial_\mu \varphi_2)(\partial^\mu \varphi_2) - \frac{1}{2}m_2^2 \varphi_2^2
\]  

(3)

and

\[
\mathcal{L}_I = -\lambda \phi \varphi_1(\varphi_1^2 + \varphi_2^2)^2 - \frac{\lambda}{4}(\varphi_1^2 + \varphi_2^2)^2 \\
+ gA^\mu(\varphi_1 \partial_\mu \varphi_2 - \varphi_2 \partial_\mu \varphi_1) + g^2 A_\mu A^\mu [\phi \varphi_1 + \\
+ \frac{1}{2}(\varphi_1^2 + \varphi_2^2)] + \frac{gY}{\sqrt{2}} \bar{\psi}(\varphi_1 + i\gamma_5 \varphi_2) \psi.
\]  

(4)

This is usually the departure point for any perturbative computation. In the expressions above \( A_\mu \) denotes a generic gauge field with \( g \) being its coupling to the Higgs and analogously
for $g_Y$, relative to the fermionic chiral field $\psi$. The scalar excitation $\varphi_1$ is the Higgs field whereas $\varphi_2$ is the Goldstone mode, which becomes associated with the gauge field and is not observable.

The proposal in [9] can be translated into noting that, given that for Higgs masses above the experimental lower bound the barrier between the two minima of the effective potential is quite shallow, before the asymmetric one becomes energetically favoured, there would be a significant probability for coherent fluctuations in $\varphi_1$ to transpose it, leading to approximate phase equilibrium. In order to understand if this scenario is realised or excluded in the Electroweak phase transition we therefore need to be able to compute systematically the probability associated with these processes, i.e., that of the deviation from the minimum of the potential with a given amplitude $\varphi$ and over a given volume $v$, $p(\varphi_1 > \bar{\varphi})$.

To achieve this consider a scalar theory characterised by the euclidean action $S_4[\varphi]$ and its partition function

$$Z = \int_B D\varphi e^{-S_4[\varphi]},$$

(5)

where $B$ denotes the integral over periodic field configurations in imaginary time, with period $\beta = \frac{1}{T}$, ($k_B = 1$). Then (5) can be rewritten as

$$Z = \int D\varphi e^{-\beta H[\varphi]},$$

(6)

where $H[\varphi]$ is

$$H[\varphi] \simeq S_{\text{eff}}[\varphi] - \frac{\beta^2}{24} \int dx \left( \frac{\delta S_{\text{eff}}}{\delta \varphi(x)} \right)^2 + O(\beta^4).$$

(7)

Here $S_{\text{eff}}[\varphi(x)]$ is the effective three-dimensional action obtained from $S_4$ after integration over all modes, but the light $n = 0$ one, in a Matsubara field expansion. Whenever the scalar field couples to other fields their effects are included through the resulting effective self-energies and couplings for the scalar field. At high temperature and in the vicinity of a minimum of the action $S_{\text{eff}}$ all non-leading terms in the expansion (7) can be neglected and we can take $H[\varphi] \simeq S_{\text{eff}}[\varphi]$. 
Further, we will be interested in computing field fluctuations over a given spatial volume $v$, in which the field takes an average value $\varphi_v$, 

$$\varphi_v = \frac{1}{v} \int_{x \in v} dx \varphi(x). \quad (8)$$

Let $p(\varphi_v \geq \bar{\varphi})$ be the probability that the field $\varphi_v$ be larger than a given value $\bar{\varphi}$, both larger than zero. Then

$$p(\varphi_v \geq \bar{\varphi}) = \frac{1}{Z} \int_{\varphi_v \geq \bar{\varphi}} D\varphi e^{-\beta H[\varphi]}. \quad (9)$$

The Chebycheff inequality permits us to write an upper bound for the probability, as

$$p(\varphi_v \geq \bar{\varphi}) \leq \frac{1}{Z} e^{-\beta \bar{\varphi}} \int D\varphi e^{-\beta H[\varphi]+\beta \int dx J(x) \varphi(x)}, \quad (10)$$

where $J(x) = j I(x)$ and $I(x)$ is a window function restricted to the volume $v$. Next we can define a three dimensional generator of connected Greens functions $W[J]$, as

$$W[J] = \beta^{-1} \ln \left[ Z^{-1} \int D\varphi e^{-\beta H[\varphi]+\beta \int dx J(x) \varphi(x)} \right] \quad (11)$$

and an associated free energy $F[\varphi]$ through its Legendre transform

$$F[\varphi] = -W[J] + \int J \varphi \quad \text{with} \quad \varphi = \frac{\delta W}{\delta J(x)}. \quad (12)$$

Both these quantities are restricted to the volume $v$ by the convolution of the usual source with the window function $I(x)$. The bound on the probability then becomes minimised by

$$p(\varphi_v \geq \bar{\varphi}) \leq e^{\beta W[J]-\beta \int J \varphi_{|J=jI}} = e^{-\beta F[\bar{\varphi}]}, \quad (13)$$

with $j$ chosen such that

$$\int J \frac{\delta W}{\delta J(x)}|_{J=jI} = \bar{\varphi} j v. \quad (14)$$

In general a perturbative expansion for $W[J]$ can be performed, in powers of the interaction Lagrangian allowing us to compute the Green’s functions by comparison with

$$\beta W[J] = \sum_n \frac{\beta^n}{n!} \int dx_1...dx_n G^{(n)}(x_1,...,x_n) J(x_1)...J(x_n). \quad (15)$$
Imposing (14) in order to minimise the probability bound yields the polynomial equation

\[ \sum_n \frac{1}{(n-1)!} (\beta v j)^n G_v^{(n)} = \beta v j \bar{\varphi}, \]  

(16)

where \( G_v^{(n)} \) are the connected Greens functions, coarse-grained on the volume \( v \), defined as

\[ G_v^{(n)} = \frac{1}{v^n} \int dx_1 ... dx_n G^{(n)}(x_1, ..., x_n) I(x_1) ... I(x_n) \]

\[ \equiv < (\varphi_v)^n >. \]  

(17)

The upper bound on the probability is then obtained by exponentiating

\[ \sum_n \frac{(\beta v j)^n}{n!} G_v^{(n)} = \beta v j \bar{\varphi}, \]  

(18)

evaluated at the solution for (14). Given a specific theory we are then ready to compute \( p \), to any desired accuracy in the scalar effective couplings.

In order to illustrate the method consider, first, the simplest case of a free scalar field theory, with potential \( V(\varphi) = \frac{1}{2} m^2 \varphi^2 \). In this case we need only compute the two-point coarse-grained Green’s function, the probability bound becomes exact and the fluctuations are strictly Gaussian. We can proceed to obtain, for a window function \( I(x) \) with Fourier transform \( \tilde{I}(k) \),

\[ \beta v < \varphi_v^2 > = \frac{1}{v} \int \frac{d^3k}{(2\pi)^3} \frac{n(\omega)}{\omega} |\tilde{I}(k)|^2, \]  

(19)

where \( w = \sqrt{k^2 + m^2} \) and \( n(\omega) \) is the Bose-Einstein distribution. We will use the usual Gaussian window function given by

\[ I(x) = \sqrt{\frac{2}{9\pi}} e^{-\frac{x^2}{2R_0^2}} \quad \text{and} \quad \tilde{I}(k) = v e^{-\frac{k^2 R_0^2 \alpha}{2}}, \]  

(20)

where \( v \) is the spatial volume \( v = \frac{4}{3} \pi R_0^3 \). Then, from our previous discussion we will typically be interested in evaluating (18), for volumes of the order of the correlation volume for the field, \( v = \frac{4}{3} \pi R_0^3 \) with \( R_0 = O(\frac{1}{m}) \). We parameterise the number of correlation volumes by writing \( R_0 = \frac{\alpha}{m} \), where \( \alpha \) is a positive number. Taking \( \alpha \) to zero corresponds to no averaging and \( \alpha = 1 \) corresponds to one spherical correlation volume. The dependence of
the coarse-grained two-point function on \( \frac{m}{T} \), for several values of \( \alpha \) is depicted in Fig. 1. It shows that it is very sensitive to the coarse-graining scale, especially for small masses. In this regime the dependence on the mass scale is approximately linear. This can be extracted analytically as follows. Using (20) as the explicit form for the window function and noting that the integrand in (19) is only substantially different from zero for \( \alpha^2 \frac{k^2}{m^2} \leq 1 \) which can only realised at small momentum, we can expand \( \omega \), using the dispersion relation, to obtain approximately

\[
<\varphi_v^2> \simeq \int_0^\frac{m}{\alpha} \frac{k^2 dk}{(2\pi^2)} \frac{T}{m m} = \frac{1}{6\pi^2} \frac{m T}{\alpha^3}.
\]

(21)

This illustrates how the linear regime in \( m \) arises for small masses and shows that the corresponding slope varies with the inverse coarse-graining scale cubed, resulting in the sharp differences observed in Fig. 1. This will be the regime of interest when we focus our analysis on the Standard Model at its phase transition, as it was pointed out in [10], where important overall coefficients were ignored, however. The importance of the constant \( \frac{1}{12\pi^2} \) in the integral (21), in reducing the spread of the field, at one correlation volume, was pointed out in [5], and constituted the basis of their argument against a significant role played by subcritical fluctuations in the Electroweak phase transition.

We are now ready to compute fluctuation probabilities in the Standard Model.

In our previous analysis the computability of the probability bound (13) resulted from the assumption of convexity for \( H[\varphi] \) and from our ability to approximate it by \( S_3[\varphi] \). This obviously holds for a free theory as well as for an interacting theory at temperature much higher than the critical temperature, in its symmetric phase. For gauge theories such as the Standard Model, close to their critical temperature, perturbation theory is only well defined around the minima of their perturbative effective potential\(^1\). We can, nevertheless,

\(^1\)Modulo infra-red divergencies in the unbroken phase. The issue of gauge and parameterisation dependence necessarily arises, as well, affecting both the effective potential and our probability bound to the same extent. To one loop these issues probably prove immaterial, though [13].
still construct our probability bound, around each of those minima and thereby estimate the probability for a fluctuation of a given amplitude. If large amplitude fluctuations are then found to be likely, say, compared to the distance to the maximum of the effective potential separating the two minima then we will conclude that the scenario proposed in [9] is likely and unlikely or impossible if the converse happens.

To do this we have to compute the two and higher-order coarse-grained thermal Green’s functions for the Higgs field. These can be computed directly from the Higgs effective scalar theory (2) given its effective mass and couplings resulting from its interaction with the other fields in the model. The former, in the usual limit of zero momentum, is given by the square root of the second derivative of effective potential, evaluated at the corresponding minimum, 

\[ m^2_{\text{min}} = V_{\text{eff}}(\phi_{\text{min}})'' , \]

while the latter is given, in the same limit, by its fourth derivative.

Extensive calculations of the Electroweak effective potential exist in the literature up to order \( \lambda^2, g^4 \) [3]. These results, however, for their length and variety of terms are hard to manipulate. The terms that make the two loop potential result qualitatively different from the one-loop improved result of [3], are associated with logarithms of particle masses over the temperature and are relatively simple to isolate. Their total effect can be approximated by using the analogous result for the \( SU(2) \) theory, where there is no distinction between the \( W \) and \( Z \) masses, \( M \), which is given by [14],

\[
\delta V_{\text{eff}} = -\frac{51}{32\pi^2} \frac{m_W^4}{\sigma^2} \phi^2 T^2 \ln\left(\frac{M_W}{T}\right). \tag{22}
\]

Two-loop contributions arising from diagrams including the top quark are also important given its large mass. We include such corrections again as computed in [4]. In what follows we will take the top quark mass to be \( m_{\text{top}} = 170 \text{GeV} \).

To order \( \lambda \) we can, then, write the probability bound (13) as

\[
p(\varphi \geq \bar{\varphi}) \leq e^{\frac{-\bar{\varphi}^2}{2G^{(2)}_v}} \left(1 + \frac{G^{(3)}_v}{3G^{(2)}_v} \bar{\varphi}^2 + \frac{G^{(4)}_v}{12G^{(2)}_v} \bar{\varphi}^4\right), \tag{23}
\]

where \( G^{(i)}_v \) is the coarse-grained \( i^{th} \) point-function. Fig. 2 a), b) and c) show the lowest order diagrams involved in computing the two, three and four point functions respectively, with
the shaded blobs denoting averaging of the external legs over the volume $v$. Corrections of order $\lambda$ to the two point function are at most of a few percent and are unimportant for the following discussion. Higher order coarse grained Green’s functions arise at higher powers of $\lambda$ and their contribution can safely be neglected.

The probability bound, written as (23), is essentially dictated by its first term, involving the two point function alone. Figure 4 shows the dependence of the square root of the two-point function on the Higgs mass as well as the distance from the local minimum at $\phi = 0$ to the nearest inflection point, at the temperature, $T_{\text{crit}}$, when both minima are degenerate, computed for the 1-loop improved effective potential parameters $^2$. We see that there is a small probability, about 1.27%, say, for $m_{\text{Higgs}} = 70 GeV$, for the field to attain the inflection point. This translates into approximately $3 \times 10^{-3}\%$ for the fluctuations to reach the maximum separating the two local minima, which is obviously a very small value.

More important than the smallness of the this probability is perhaps to understand what happens at temperatures at which the minima are non-degenerate. Fig. 5 a) and b) display the same quantities as in Fig.3, but at a higher temperature, just below that at which the new minimum appears, for fluctuations around the symmetric and asymmetric minimum, respectively. We see that, corresponding to a significant difference in shape about each of the minima, the fluctuation probabilities are now quite different. While it is now slightly more difficult for a large fluctuation to occur for the field located at $\phi = 0$, it quite likely for the converse to happen at the asymmetric minimum $^3$. The probabilities above become

$^2$At this Temperature the 1-loop improved effective potential is symmetrical about each of the local minima so no distinction needs to be drawn between them.

$^3$In the computations of the probabilities the coherence lengths at each minimum were used. It should be noted, however, that, close to $T_{\text{min}}$ these two length scales can be quite different so that a coherence length fluctuation attaining the asymmetric minimum finds itself being of a fraction of the coherence length in the new phase and therefore has a much larger probability to fluctuate.
approximately 33% and 1.22%, respectively. This differential in the fluctuation probabilities naturally decreases from $T_{\text{min}}$ to $T_{\text{crit}}$, but until then introduces a fundamental qualitative feature. In a dynamical situation where the field fluctuates about each minimum, any field attaining the asymmetric minimum is brought back to $\phi = 0$, while the opposite process is not likely enough to replenish it. Using two-loop effective potential parameters, in the way mentioned above, only makes this statement stronger, as is shown in Fig. 6. As an example, the probabilities of the field to attain the inflection point and maximum are now of approximately 0.92%, and $7 \times 10^{-9}$% at $T_{\text{crit}}$, respectively.

Finally, we compute the change to the probability bound resulting from the three and four-point coarse-grained functions in (25). The diagrams in Fig. 2 b) and c) involve similar integrals to the usual setting-sun and basket-ball diagrams respectively\footnote{Note, however, that now there is only one vertex, so that the result is down by a power of $\lambda$ and no symmetry factors are involved.}, now convolved with window functions restricting each leg to the same volume $v$. We compute them in this way, using the integrals for their leading contributions from [15], where full account of resummation was taken care of, in the absence of coarse-graining. The resulting contribution from the four-point function to the exponent of the probability bound is of order 0.015%, and causes very little change to our previous discussion, for small amplitude fluctuations. The correction due to the three point function is more relevant but because the effective vertex is proportional to $\phi$ matters only for fluctuations at the asymmetric minimum. As can be seen in Fig. 6, it contributes to increase the probability relative to the the values considered in our previous discussion. As such, it makes fluctuations from the asymmetric minimum to the symmetric one more likely than their converse even at the critical temperature, when the effective potential is locally approximately similar for both minima. Differences to the Gaussian probability values occur mostly for fluctuations of the same size or larger than the back.
two point function and accounts for a difference of less that 5%. For larger fluctuations the effect of the four-point function starts being noticeable compensating for the increase due to three-point one. In the case of large enough fluctuations for the field to attain the nearest inflection point, at the critical temperature, this discrepancy is a fraction of a percent and thus makes very little practical difference.

From the ensemble of our calculations we can therefore safely conclude that coherence length fluctuations of the Higgs field have a very small probability of interpolating between the minima of the effective potential, even in the worst case scenario of taking 1-loop improved parameters. This should mirror the actual field dynamics provided calculations over a homogeneous mean field background and equilibrium apply. In this scenario, fluctuations of the Higgs field from the asymmetric minimum to zero are always more likely to happen than their converse, at and above the critical temperature. Together these two conditions guarantee that metastability is preserved in the Electroweak phase transition, for Higgs masses smaller or of the order of the gauge bosons and therefore the phase transition dynamics should proceed by the usual process of bubble nucleation. Finally, we note that our computations should have a natural translation into the coefficients used in the kinetic equation of [11]. It would be interesting to check, if in this context, their utilisation could lead to the reconciliation of both pictures.

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FIGURE CAPTIONS

Fig. 1 - The dependence of the coarse grained two-point function on $M/T$ for 0.01, 0.1, 0.5 and 1 correlation volumes.

Fig. 2 a) b) and c) - Diagrams involved in computing the coarse-grained two, three and four-point functions, respectively, in the probability bound (23), to lowest order. The shaded circles denote spatial integration of the external legs over the same volume $v$.

Fig. 3 - The variation of the square root of the coarse-grained two-point function with the Higgs mass, for 0.5, 1 and 2 correlation volumes as well as that of the distance between the minimum of the effective potential and the nearest inflection point, at $T_{\text{crit}}$. All quantities are calculated with 1-loop improved effective potential parameters.

Fig. 4 a) - The same quantities as in Fig. 3, computed at the symmetric minimum and at a temperature just below that at which the asymmetric minimum appears.

Fig. 4 b) - The same as Fig. 4 a), at the asymmetric minimum.

Fig. 5 - The same quantities as in Fig. 3, for the two loop effective potential parameters, at the asymmetric minimum.

Fig. 6 - The Gaussian Probability function (dotted line), obtained by neglecting the three and four-point functions in (23), and the full quantity computed to order $\lambda$ (dashed line).