Effects of the induced magnetic field, thermophoresis, and Brownian motion on mixed convective Jeffrey nanofluid flow through a porous channel

Adigoppula Raju | Odelu Ojjela

1Department of Mathematics, Sumathi Reddy Institute of Technology for Women, Warangal, India
2Department of Applied Mathematics, Defence Institute of Advanced Technology (Deemed University), Pune, India

Correspondence
Odelu Ojjela, Department of Applied Mathematics, Defence Institute of Advanced Technology (Deemed University), Pune-411 025, India. Email: odelu@diat.ac.in; odelu3@yahoo.co.in

The main purpose of this research is to explore a comparative study of viscous and Jeffrey nanofluid flows through a parallel channel embedded in a porous medium under the influence of an induced magnetic field, Brownian motion, and thermophoresis. The convective boundary conditions are employed to study the heat and mass transfer at the lower plate. The system of transport constituent relations is reduced into coupled nondimensional ordinary differential equations through similar variables with appropriate boundary conditions. The resulting equations are analyzed for flow characteristics, heat and mass transfers, and magnetic diffusivities throughout the channel with various physical nondimensional parameters via shooting technique along with Runge-Kutta fourth-order scheme. It is observed that the temperature and concentration decrease with increasing Brownian motion parameters for both the viscous and Jeffrey fluid. The velocities decrease with increasing of the inverse Darcy parameter for Jeffrey fluid whereas velocities increase for a viscous fluid. The profiles of temperature and concentration for both fluids decrease with increasing of the Brownian motion parameter. The velocity profiles rise with suction/injection parameter for the both fluids. Finally, the numerical results of the present method are compared with a work available in the literature for the Newtonian case. An excellent agreement is found between the present and published numerical results.

KEYWORDS
convective boundary condition, induced magnetic field, Jeffrey fluid, shooting method, thermophoresis and Brownian motion

1 | INTRODUCTION

A non-Newtonian fluid model in which the relation between the shear stress and the shear rate is not linear. These fluids are classified into a differential, rate, and integral types. The Jeffrey model is a limiting case of the viscoelastic model, which is a subclass of rate type. Moreover, it does not provide with a single constitute relation. In addition, it has a stress relaxation properties. Such behavior is broadly observed in many materials, such as polymer solutions, molten, ketchup, custard, toothpaste, starch suspensions, paint, blood, and shampoo. The combined effects of momentum and
heat and mass transfer in a non-Newtonian fluid through porous boundaries have received great interest due to their applications in engineering and industry, such as electrostatic precipitation, aerodynamic heating, dying of paper and textile, solidifications of liquid crystals, food preservation, petroleum industries, cooling of metallic sheet in the bath, grain regression, magnetohydrodynamic (MHD) pumps, and magnetohydrodynamic generators. A steady three-dimensional flow of an incompressible Jeffrey fluid past bidirectional stretching surface have reported by Hayat et al. Ahmed et al discussed the dilating and the squeezing porous channel flow of an incompressible Jeffrey fluid and the flow is generated due to suction/injection at the walls. Shehzad et al have disclosed the nature of the thermophoresis and Brownian motion on a three-dimensional channel flow of an incompressible, MHD Jeffrey nanofluid in the presence of thermal radiation. An investigation explores the characteristics of thermophoresis and Brownian motion with the MHD Jeffrey fluid flow over a stretching surface by Khan et al. Murthy et al surveyed the significance of thermophoresis and Brownian motion effects on MHD slip flow of Casson nanofluid through an exponentially stretching surface. Shah et al inspected the influence of thermophoresis and Brownian motion over a rotating third grade fluid flow between two parallel plates. Khan et al have discussed of MHDs and radiative heat transportation in convectively heated stratified flow of Jeffrey nanofluid.

A laminar incompressible flow through the parallel channel with suction or injection at the wall have been attending many researchers for the last few decades. Berman was the first investigator for the laminar incompressible flow of a viscous fluid with suction/injection at the uniform porous walls with different permeability, and later, the work was extended with consideration MHD and heat and mass transfers by Terrill, Walker and Davies, Terrill and Shresth, Nigam and Singh, Cox, Bujurke et al, and Ganesh and Krishnambal. A study on viscous fluid through a parallel channel with bottom injection and top suction was explained by Hafeez and Ndikilar. Rao and Moizuddi analyzed a mathematical model with a perturbation technique for a steady, incompressible flow of micropolar fluid through a parallel channel with suction/injection at the walls. The MHD flow of upper-convected Maxwell fluid through a parallel channel that is embedded in a porous medium has been reported by Hayat et al.

In recent years, most of the researchers have been modeling theoretically to investigate the heat enhancement process by suspending high thermal conductivity nanoparticle in the base fluid. Choi and Eastman was first who proposed the word nanofluids. From there onwards, many researchers have extended this work most recent, and Dogonchi et al explored the heat transfers of MHD squeezing flow of viscous nanofluid between two parallel plates in the presence of thermal radiation. Sheikholeslami et al examined the influence Brownian and thermophoresis characteristics over a steady laminar an incompressible nanofluid flow in a rotating horizontal channel. Hayat et al investigated the impact of Cattaneo-Christov heat flux model in flow and stagnation point flows, respectively. MHD stagnation point flow of Casson fluid toward a stretching sheet is addressed by Khan et al.

A numerical solution is obtained to reveal effective thermal conductivity of the copper-water nanofluid through a parallel channel with consideration of Brownian motion, which was studied by Khan et al. Makinde and Aziz analyzed the significance of Brownian motion and thermophoresis diffusivity on a steady two-dimensional boundary layer flow of nanofluid over a stretching sheet. The effects of thermophoresis and Brownian motion of a nanofluid flow between two vertical plates embedded in a porous medium are reported by Matin and Ghanbari. Babu et al have discussed a comparative study on thermophoresis and Brownian motion on a steady, incompressible flow of MHD fluid over three different geometries via Runge-Kutta and Newton's method. The thermophoresis and Brownian motion effects have been studied by Hayat et al with thermal radiation on a laminar incompressible flow of Jeffrey nanofluid over stretching surface subject to the heat and mass flux boundary conditions. Hayat et al have examined the entropy generation in flow of viscous nanofluid by rotating disk. Khan et al and Rashid et al have analyzed of entropy generation of nanofluid flow by employing nonlinear thermal radiation. Khan et al addressed the significance of nonlinear radiation in mixed convection flow of magnetor Walter-B nanoliquid. Hayat et al considered the significance of heat generation/absorption and Soret effects on peristalsis flow of pseudoplastic fluid in an inclined channel. Hayat et al explored the physical aspects of irreversibility in radiative flow of viscous fluid with chemical reaction. The analysis carried out the effect of convective heat and mass conditions on a steady laminar MHD flow of Jeffrey nanofluid with an accounting of thermophoresis and Brownian motion by Shehzad et al. Ramzan et al have examined deliberately with the thermal radiation, thermophoresis, and Brownian motion on an incompressible flow of Jeffrey nanofluid flow along stretching sheet. Waqas et al analyzed for magnetic dipole impact in nonlinear thermally radiating Carreau nanofluid flow subject to heat generation. An investigation carried out the attribute of Brownian motion and thermophoresis on an MHD flow of incompressible Jeffrey nanofluid over a bidirectional stretching sheet reported by Hayat et al. Sadeep et al have studied comparative on MHD flow of Jeffrey, Maxwell, and Oldroyd-B nanofluids flow over a stretching sheet. Dalir et al have discussed with the numerical technique of MHD flow of Jeffrey nanofluid through a stretching sheet with consideration.
of Brownian motion and thermophoresis effects. Mahmoodi and Kandelousi have examined the significant effects of Brownian motion and thermophoresis phenomenon on an incompressible flow of kerosene-alumina nanofluid through the rotating parallel channel.

Many authors have considered the effect of the induced magnetic field on an incompressible laminar peristaltic flow of nanofluids, and very few reports are available other than the study of the induced magnetic field with peristaltic flow. Denno and Fouad have investigated the impact induced magnetic field due to strong nonuniform applied magnetic field on an inviscid flow through the parallel channel. Singh and Singh addressed the effect of induced magnetic fluid on an incompressible, MHD flow of viscous fluid in a rotating channel bounded by nonconducting plates. A numerical report by Ibrahim delights that the impact of induced magnetic field on an MHD flow of an incompressible nanofluid toward a stagnation point over a stretching sheet with Brownian movement and thermophoresis and furthermore utilizing convective boundary conditions. This numerical report was extended by Sandeep et al with employing chemical reaction. Raju and Ojjela have studied with the numerical technique significance of the induced magnetic field and variable thermal conductivity on convective Jeffrey fluid flow with nth-order chemical reaction.

The main objective of this article is to address the comparative study of viscous and Jeffrey nanofluids flow between two parallel plates with periodic injection/suction at the walls with convective boundary conditions embedded in porous medium under the influence of magnetic field, Brownian, and thermophoretic diffusivity. The flow is assumed to be a laminar, incompressible, and unsteady. The governing flow field equations are reduced into highly nonlinear coupled equations by using similarity transformation, and then, the results are elaborated with the graphs and tables through the numerical technique, namely, shooting method.

## 2 MATHEMATICAL FORMULATION

Consider an unsteady laminar, incompressible MHD flow of Jeffrey fluid through a porous medium between two parallel porous plates at \( y = 0 \) and \( y = h \). A Cartesian coordinate system is chosen such that the axial (\( u \)) and transverse (\( v \)) velocity components are along \( X \)- and \( Y \)-directions, respectively, as shown in Figure 1. Here, assume that, at the lower plate, the fluid injected into the channel with the velocity of \( V_1 e^{i\omega t} \) and the suctioning fluid with the velocity \( V_2 e^{i\omega t} \) from the upper plates and subject to the condition \( |V_2| \geq |V_1| \). The lower and upper plates are maintained at two different temperatures \( T_1 e^{i\omega t} \) and \( T_2 e^{i\omega t} \) concentrations \( C_1 e^{i\omega t} \) and \( C_2 e^{i\omega t} \), respectively. A uniform magnetic field is applied of strength \( B_0 \) along \( Y \)-direction, which leads to an induces the magnetic field \( B_x \) and \( B_y \) in \( X \)- and \( Y \)-directions, respectively. Therefore, the total magnetic field vector becomes \( B(B_x, B_0 + B_y, 0) \).

Under the above assumptions, the governing equations for conservative momentum, energy, concentration, and induction equations with thermophoresis and Brownian motion are

\[
\begin{align*}
\rho (u_1 + uu_x + vu_y) &= -p_x + \frac{\mu}{1 + \lambda_1} (u_{xx} + u_{yy}) + \frac{\mu k_2}{1 + \lambda_1} \left\{ (\partial_t + u \partial_x + v \partial_y) (u_{xx} + u_{yy}) ight. \\
&\quad + 2u_x u_{xx} + 2v_y u_{xy} + (u_x \partial_x + v_y \partial_y)(u_x + v_y) \left. \right\} + \mu \left\{ B_x(B_x)_x ight. \\
&\quad + (B_0 + B_y)(B_x)_y - \frac{1}{2} \partial_x \left( B_x^2 + (B_0 + B_y)^2 \right) \left. \right\} - \frac{\mu u}{k_1} \\
&\quad + \rho g \beta_T \left( T - T_1 e^{i\omega t} \right) + \rho g \beta_C \left( C - C_1 e^{i\omega t} \right)
\end{align*}
\]
\[ \rho \left( v_x + uw_x + vv_x \right) = -p_y + \frac{\mu}{1 + \lambda_1} \left[ v_{xx} + v_{yy} \right] + \frac{\mu \lambda_2}{1 + \lambda_1} \left\{ \left( \partial_t + u \partial_x + v \partial_y \right) \left( v_{xx} + v_{yy} \right) + 2v_x v_{yy} + 2u_y v_{xy} + \left( u_x \partial_x + v_y \partial_y \right) \left( u_y + v_x \right) \right\} + \mu_i \left\{ B_x \left( B_y \right)_x + \left( B_0 + B_y \right) \left( B_y \right)_y - \frac{1}{2} \partial_y \left( B_0^2 + \left( B_0 + B_y \right)^2 \right) \right\} - \frac{\mu_{i}}{k_1} v \]  

\[ \rho c \left( T_i + u T_x + v T_y \right) = k \left[ T_{xx} + T_{yy} \right] + \frac{\mu}{1 + \lambda_1} \left[ 2(u_x)^2 + 2(v_y)^2 + \left( u_y + v_x \right)^2 \right] + \frac{\mu_{i} k_1}{k_1} \left( u^2 + v^2 \right) \left\{ \frac{\mu \lambda_2}{1 + \lambda_1} \left\{ 2u_x \left( u_{xx} + uu_{xx} + vv_{xy} \right) + 2v_y \left( v_{st} + vv_{xy} + vv_{yy} \right) + \left( u_y + v_x \right) \left( \partial_t + u \partial_x + v \partial_y \right) \left( u_y + v_x \right) \right\} \right\} + \tau_i \rho c \left\{ D_B \left( C_x T_x + C_y T_y \right) + \frac{D_T}{T_2} \left( T_x^2 + T_y^2 \right) \right\} \]  

\[ C_i + u C_x + v C_y = D_B \left[ C_{xx} + C_{yy} \right] + \frac{D_T}{T_2} \left\{ T_{xx} + T_{yy} \right\}. \]  

The Maxwell equations and Ohm’s law by neglecting the displacement current are

\[ \nabla \times \vec{E} = \left( \vec{B} \right)_t, \quad \nabla \times \vec{B} = \mu_0 \vec{J}, \quad \nabla \cdot \vec{B} = 0, \quad \vec{J} = \sigma \left( \vec{E} + \vec{q} \times \vec{B} \right). \]  

From Equation (6), we reduced into an induction equation as

\[ \begin{aligned} (B_x)_t + u (B_x)_x + v (B_x)_y &= B_x u_x + \left( B_0 + B_y \right) u_y + \frac{1}{\sigma_{mu}} \left[ (B_x)_{xx} + (B_x)_{yy} \right], \\
(B_y)_t + u (B_y)_x + v (B_y)_y &= B_y v_x + \left( B_0 + B_y \right) v_y + \frac{1}{\sigma_{mu}} \left[ (B_y)_{xx} + (B_y)_{yy} \right] \end{aligned} \]  

where \( \mu \) is the dynamic viscosity, \( k \) is the thermal conductivity, \( k_1 \) is the wall permeability, \( \mu_i \) is the magnetic permeability, \( \rho \) is the density of the fluid, \( \beta_T \) and \( \beta_c \) are coefficients of thermal and solutal expansions, respectively, \( g \) is the acceleration due to gravity, \( c \) is the specific heat at constant temperature, \( \sigma \) is the electric conductivity of the fluid, \( \vec{B} = B_0 \vec{j} + \vec{b} \) magnetic field vector, and \( \vec{J} = B_0 \vec{j} + \vec{B} \) is an induced magnetic field vector.

The above model has been reduced for the viscous nanofluid by assuming \( \lambda_1 = \lambda_2 = 0 \).

The associated boundary conditions are given as follows at the lower and upper plates:

\[ \begin{aligned} & \text{at } y = 0: \quad u(x, y, t) = 0, \quad v(x, y, t) = V_1 e^{i \omega t}, \quad -k T_y = h_1 \left( T - T_1 e^{i \omega t} \right), \quad -D_B C_y = h_2 \left( C - C_1 e^{i \omega t} \right), \\
& (B_x)_y = 0, \quad B_y (x, y, t) = 0 \end{aligned} \]  

\[ \begin{aligned} & \text{at } y = h: \quad u(x, y, t) = 0, \quad v(x, y, t) = V_2 e^{i \omega t}, \quad T(x, y, t) = T_2 e^{i \omega t}, \quad C(x, y, t) = C_2 e^{i \omega t}, \\
& B_x (x, y, t) = B_0 e^{i \omega t}. \end{aligned} \]
Let us choose the similarity transformations as follows\textsuperscript{1,12,14,18,47}:

\[
\begin{align*}
  u(x, \lambda, t) &= \left( \frac{U_0}{a} - \frac{V_0 x}{h} \right) f^I(\lambda) e^{i\omega t}, \quad v(x, \lambda, t) = V_2 f(\lambda) e^{i\omega t} \\
  T(x, \lambda, t) &= \left( T_1 + \frac{\mu V_0}{\rho c} \left[ \phi_1(\lambda) + \left( \frac{U_0}{a V_2} - \frac{x}{h} \right) \phi_2(\lambda) \right] \right) e^{i\omega t} \\
  C(x, \lambda, t) &= \left( C_1 + \frac{n_a}{h} \right) g_1(\lambda) + \left( \frac{U_0}{a V_2} - \frac{x}{h} \right) g_2(\lambda) \right) e^{i\omega t} \\
\end{align*}
\]

\[B_x = B_0 \left( \frac{U_0}{a V_2} - \frac{x}{h} \right) \psi^I(\lambda) e^{i\omega t}, \quad B_y = B_0 \psi(\lambda) e^{i\omega t}\]

where \( \lambda = y/h \) and \( f, \phi_1, \phi_2, g_1, g_2, \) and \( \psi \) are the dimensionless unknown functions to be computed.

The skin friction coefficient and the heat and mass transfer rates are also calculated through the Nusselt and Sherwood numbers at upper and lower plates

\[
C_f = \frac{2 \tau_{xy}}{\rho V_1^2}, \quad \text{where} \quad \tau_{xy} = \frac{\mu}{1 + \lambda_1} \left[ (u_y + v_x) + \frac{\mu \lambda_2}{1 + \lambda_1} [u_y + v_x + uu_{xy} + vv_{xy} + uu_{yy} + vv_{yy}] \right] \tag{11}
\]

\[
Nu = \frac{\text{Heat transfer rate}}{\text{Diffusion rate}} = \frac{-T_y}{T_1-T_y} = -\left( \phi_1(\lambda) + \xi^2 \phi_2(\lambda) \right)_{\lambda=0,1} \tag{12}
\]

\[
Sh = \frac{\text{Mass transfer rate}}{\text{Diffusion rate}} = \frac{-C_y}{(C_1-C_2)/h} = -\left( g_1(\lambda) + \xi^2 g_2(\lambda) \right)_{\lambda=0,1} \tag{13}
\]

Substitute the above similar variable Equation (10) into the governing of momentum, energy, concentration, and induction Equations (2) to (5) and then eliminating the pressure gradient. We obtain a system of the nondimensional coupled nonlinear ordinary differential Equations (14)

\[
\begin{align*}
  f' &= \frac{-1}{f} \left( f f^{IV} - 2 f^I f^{III} \right) + \frac{Re (1 + \lambda_1)}{\beta f} \left( f f^{III} - f^I f^{II} \right) - \frac{1}{\beta f \cos \theta} f^{IV} \\
  &- \frac{St^2 \text{Re} (1 + \lambda_1)}{\beta f} \left( \psi f^{III} - \psi f^{II} \right) - \frac{St^2 \text{Re} (1 + \lambda_1)}{\beta f \cos \theta} \psi^{III} - \frac{EcGr (1 + \lambda_1)}{\xi \beta f \cos \theta} \left( \phi_1 + \xi^2 \phi_2 \right) \\
  &- \frac{ShGm (1 + \lambda_1)}{\xi \beta f \cos \theta} \left( g_1 + \xi^2 g_2 \right) + \frac{Da^{-1} (1 + \lambda_1)}{\beta f \cos \theta} f^{II} \\
  \phi_1' &= \text{RePr} f \phi_1' \cos \theta - 2 \phi_1 - \frac{4 \text{RePr}}{(1 + \lambda_1)} \left[ \left( f^{I} \right)^2 \cos \theta + \beta f f^I f^II \cos 2\theta \right] \\
  &- \frac{Da^{-1} \text{RePr} f^2 \cos \theta - Sh Nbg_1 \phi_1' \cos \theta - Ec Nt \left( \phi_1' \right)^2 \cos 2\theta} \\
  \phi_2' &= \text{RePr} \left( f^I f^II - 2 f^I \phi_2 \right) \cos \theta - \frac{1}{(1 + \lambda_1)} \left[ \left( f^{II} \right)^2 \cos \theta - \beta f f^I f^{III} \cos 2\theta \right] - Da^{-1} \left( f^{I} \right)^2 \cos \theta \\
  &+ Sh Nb \left( g_2 \phi_2 + g_1' \phi_1 + g_2' \phi_1 + \xi^2 g_2' \phi_2 \right) \cos \theta + Ec Nt \left( 4 \phi_2^2 + \xi^2 \left( \phi_1' \right)^2 + 2 \phi_1' \phi_2' \right) \cos \theta \\
  g_1' &= -2g_2 + \text{ReSc} f g_1' \cos \theta - \frac{Ec Nt}{Nb Sh} \left( 2 \phi_2 + \phi_1' \right) \\
  g_2' &= \text{ReSc} \left( f g_2' - 2g_2 f' \right) \cos \theta - \frac{Ec Nt}{Nb Sh} \phi_2' \\
  \psi^{III} &= R_m^2 \left[ \left( f^2 \psi^{I} - \psi f^{II} \right) \cos \theta - f f^{II} \cos \theta - 4R_m f^II \cos \theta - R_m f^{II} \right]. \tag{14}
\end{align*}
\]

Nondimensional skin friction is given by

\[
\text{Re} C_f = \frac{2 \xi}{1 + \lambda_1} \left[ f^{II} \cos \theta - \beta \left( f^I f^{II} - f f^{III} \right) \cos 2\theta \right], \tag{15}
\]
where the prime represents the differentiation of unknown function with respects to the nondimensional variable.

From Equation (10), the nondimensional temperature and concentrations are given by

\[
T^* = \frac{T - T_1e^{\text{int}}}{(T_2 - T_1)e^{\text{int}}} = Ec \left( \phi_1 + \xi^2 \phi_2 \right), \quad C^* = \frac{C - C_1e^{\text{int}}}{(C_2 - C_1)e^{\text{int}}} = Sh \left( g_1 + \xi^2 g_2 \right). \tag{16}
\]

The corresponding dimensionless form of the boundary conditions in terms of unknown function \( f, \phi_1, \phi_2, g_1, g_2, \) and \( \psi \) are

\[
\begin{align*}
  f(0) &= 1 - a, \quad f'(0) = 0, \quad f''(0) = 0, \quad \phi_1(0) = -Bi_1 \phi_1(0), \quad \phi_2(0) = -Bi_2 \phi_2(0), \\
  g_1'(0) &= -Bi_1 g_1(0), \quad g_2'(0) = -Bi_2 g_2(0), \quad \psi(0) = 0, \quad \psi'(0) = 0, \\
  f(1) &= 1, \quad f'(1) = 0, \quad \phi_1(1) = 1/Ec, \quad \phi_2(1) = 0, \\
  g_1(1) &= 1/Sh, \quad g_2(1) = 0, \quad \psi'(1) = 1/\xi.
\end{align*}
\tag{17}
\]

3 | NUMERICAL SOLUTION OF THE PROBLEM

The model that has been transformed into a highly nonlinear coupled ordinary differential Equations (14) with an associate dimensionless boundary conditions (17) is not possible to solve analytically so that we adopt a numerical technique, namely, shooting technique along with the Runge-Kutta fourth-order integration scheme. In order to solve Equations (14), we have converted the system of equations into sixteen first order simultaneous ordinary differential equations with sixteen unknowns. In this context, we required 16 initial conditions, but we have nine initial conditions that are known and the rest of the unknowns are calculated with generalized Newton-Raphson technique with Runge-Kutta fourth-order scheme that satisfies the end conditions and repeated the technique until to get the results within the tolerance limit \(10^{-5}\). The whole calculation has done with MATLAB software by taking step size as \( \lambda = 0.01 \).
$g_1 = x_{10}, g'_1 = x_{11}$

$g''_1 = -2x_{12} + \text{RePr}x_1 x_{11} \cos \theta - \frac{EcNt}{NbSh} \left( 2x_8 + \phi''_1 \right)$

$g_2 = x_{12}, g'_2 = x_{13}$

$g''_2 = \text{ReSc} \left[ x_1 x_{13} - 2x_{12} x_2 \right] \cos \theta - \frac{EcNt}{NbSh} \phi''_2$

$\psi = x_{14}, \psi' = x_{15}, \psi'' = x_{16}$

$\psi''' = Rm^2 \left[ \left( x_1^2 x_{15} - x_{14} x_1 x_2 \right) \cos \theta - x_1 x_2 \right] \cos \theta - 4Rm x_3 \cos \theta - Rm x_3$

4 | NUMERICAL RESULTS AND DISCUSSION

In this discussion, we have studied comparatively for viscous and Jeffrey nanofluids with various dimensionless involved key parameters, namely, Eckert number (Ec), inverse Darcy parameter (Da$^{-1}$), Brownian motion parameter (Nb), suction/injection ratio (a), and Reynolds number (Re) on flow, heat and mass transfer characteristics, and axial induced magnetic field form throughout the domain [0, 1]. In addition, we also calculated the skin friction coefficients, heat and mass transfer rates at the plates by varying different parameters. The numerical results of the present method are compared with the existing literature. Figures 2 to 6 reveal the influence of Ec on axial velocity ($f'$), transverse velocity ($f$), temperature ($T^*$), concentration distributions ($C^*$), and axial induced magnetic field ($\psi'$) for Re = 0.5, $\beta = 0.8$, St = 0.5,

![FIGURE 2](image1)  Influence of Ec on axial velocity

![FIGURE 3](image2)  Influence of Ec on transverse velocity
It is noticed that, when raising the value of $Ec$, the velocity and axial induced magnetic field profiles for viscous and Jeffrey fluids are in opposite nature. Meanwhile, the profile $T^*$ decreases and profile $C^*$ increases in both cases of nanofluids. Physically, $Ec$ is the inverse proposition dissipation term in the equation of energy; therefore, larger values of $Ec$ should lead to decreases the quantity of heat being produced by the shear forces in the fluid and as
a result decline the fluid temperature. The variation of the nondimensional profiles of $f', f$, $T'$, $C'$, and $\psi'$ for different values of the inverse Darcy parameter $Da^{-1}$ are illustrated in Figures 7 to 11 for $Re = 0.02$, $St = 0.5$, $\beta = 0.5$, $Ec = 0.5$, $Sh = 0.5$, $\lambda_1 = 16$, $Gr = 0.0004$, $Gm = 0.0004$, $Pr = 7$, $Nb = 0.5$, $Nt = 0.5$, $Sc = 0.22$, $\xi = 0.6325$, $a = 0.6$, $\theta = 0.2$, $Bi_1 = 0.09$, $Bi_2 = 0.02$, $Rm = 2$. It can be seen clearly that increasing value of the inverse Darcy parameter, the velocities (axial and transverse direction) of the flow are decreasing for Jeffrey fluid, whereas for the viscous fluid, these are increasing. It
is due to the resistance offered by the porosity of the medium is much more than the resistance due to the magnetic lines of force. The concentration profile of the fluid $C^*$ increases with increasing of $Da^{-1}$, whereas the temperature of the fluid $T^*$ decreasing for both viscous and Jeffrey fluids also the profiles of axial induced magnetic field are in the opposite nature for both cases. Fixing the parameter values as $Re = 0.5$, $\beta = 0.5$, $St = 0.1$, $\lambda_1 = 15$, $Ec = 0.1$, $Sh = 0.1$, $Gr = 0.25$, $Gm = 0.25$, $Da^{-1} = 2$, $Pr = 10$, $Nt = 0.5$, $Sc = 0.2$, $\xi = 0.6325$, $a = 0.01$, $\theta = 2.4$, $Bi_1 = 0.6$, $Bi_2 = 0.2$, $Rm = 0.006$, Figures 12
FIGURE 13  Effect of Nb on transverse velocity

FIGURE 14  Effect of Nb on temperature

FIGURE 15  Effect of Nb on concentration

to 16 depict the parameter Nb on $f'$, $f$, $T^*$, $C^*$, and $\psi'$. It is declared for the Figures 12 and 13 the axial and transverse velocities are increasing with increasing of Nb for Jeffrey case and it is opposite for the viscous case. Figures 14 and 15 represent the temperature and concentration of the fluid are decreasing with increasing of Nb for both viscous and Jeffrey nanofluids. It is interesting to note that the Brownian motion of nanoparticles at molecular and nanoscale levels is a key nanoscale mechanism governing their thermal and solute behaviors, and Figure 16 explores the $\psi'$ on Nb and we noticed that the profile of $\psi'$ is increases with increasing of Nb for Jeffrey fluid, whereas it is decreasing for viscous fluid.
Figures 17 and 18 demonstrate the effect of the suction/injection ratio ($a$) with velocities of the fluid for $Re = 0.5$, $\beta = 0.5$, $St = 0.1$, $\lambda_1 = 10$, $Ec = 0.1$, $Sh = 0.1$, $Gr = 0.25$, $Gm = 0.25$, $Da^{-1} = 1$, $Pr = 10$, $Nb = 0.5$, $Nt = 0.5$, $Sc = 0.2$, $\zeta = 0.6325$, $a = 0.1$, $\theta = 2.4$, $Bi_1 = 0.6$, $Bi_2 = 0.2$, $Rm = 0.3$. It is clear that axial and transverse velocities of the fluid increase with the
increasing of $a$ for both the cases. It is due to the fact that the suction velocity increases when “$a$” increases; therefore, the velocity has enhanced. For different values of the Renold’s number (Re), the axial and transverse velocities are plotted in Figures 19 and 20 for Re = 0.2, $\beta = 0.5$, St = 0.1, $\lambda_1 = 10$, Ec = 0.07, Sh = 0.07, Gr = 0.04, Gm = 0.04, Da$^{-1}$ = 2, Pr = 10, Nb = 0.5, Nt = 0.5, Sc = 0.2, $\xi = 0.6325$, a = 0.9, $\theta = 2.4$, $B\lambda_1 = 0.2$, $B\lambda_2 = 0.2$, Rm = 0.03. It is displayed that, in raising the value, Re parameter depreciates the velocity profile for Jeffrey fluid, whereas the reverse trend is observed for viscous fluid. The variations of skin frictions and the heat and mass transfer rates at the plates are shown in Table 1 for different values the parameters $a$, Nb, Nt, and St. It is witness that, by varying Nb, the skin friction and mass transfer rate are decreased at both the plates, whereas heat transfer rate is increasing at upper plate and decreasing at lower plate. For different values of the suction/injection ratio ($a$), the mass transfer rates are increasing, whereas heat transfer rates are decreasing at the lower and upper plates. However, the skin friction at upper plate increases, whereas it decreases at the lower plate.

Table 2 shows that the validation of the present shooting method with perturbation series solution for the Newtonian case. It is clear that the present shooting technique along with Runge-Kutta fourth-order scheme has good agreement with the analytical method to the skin friction values at lower and upper plates by neglecting the $St = Pr = Ec = Gr = Sh = Gm = \beta = \lambda_1 = Nb = Nt = 0.15$. 

**FIGURE 19** Effect of Re on axial velocity

**FIGURE 20** Effect of Re on transverse velocity
TABLE 1  The numerical values of skin friction, heat transfer, and mass transfer rates at the lower and upper plates varying $a$, $N_b$, $N_t$, and $St$ for $Re = 0.5$, $\beta = 0.5$, $\lambda_1 = 1.2$, $Ec = 0.1$, $Sh = 0.1$, $Gr = 0.25$, $Gm = 0.25$, $Da^{-1} = 0.6$, $Pr = 10$, $Sc = 0.2$, $\xi = 0.6325$, $\theta = 2.4$, $B_i = 0.6$, $B_i = 0.2$, $Rm = 0.3$

| $a$ | $N_b$ | $N_t$ | $St$ | Skin friction $\lambda = 0$ | Heat Transfer rate $\lambda = 0$ | Mass Transfer rate $\lambda = 0$ |
|-----|-----|-----|-----|-----------------|-----------------|-----------------|
| $0.1$ | $0.5$ | $0.5$ | $0.1$ | $-0.4582$ | $6.7044$ | $-0.6105$ |
| $0.2$ | $0.5$ | $0.5$ | $0.1$ | $-0.9601$ | $7.8660$ | $-1.0254$ |
| $0.3$ | $0.5$ | $0.5$ | $0.1$ | $-1.4832$ | $11.8236$ | $-1.6022$ |
| $0.4$ | $0.5$ | $0.5$ | $0.1$ | $-2.0265$ | $15.8638$ | $-2.9777$ |
| $0.5$ | $0.1$ | $0.5$ | $0.1$ | $-0.4122$ | $4.3550$ | $-0.6177$ |
| $0.6$ | $0.2$ | $0.5$ | $0.1$ | $-0.4410$ | $4.1190$ | $-0.6139$ |
| $0.7$ | $0.3$ | $0.5$ | $0.1$ | $-0.4506$ | $4.0403$ | $-0.6123$ |
| $0.8$ | $0.4$ | $0.5$ | $0.1$ | $-0.4554$ | $4.0009$ | $-0.6113$ |
| $0.9$ | $0.5$ | $0.1$ | $0.1$ | $-0.4674$ | $3.9022$ | $-0.6199$ |
| $1.0$ | $0.5$ | $0.2$ | $0.1$ | $-0.4651$ | $3.9209$ | $-0.6177$ |
| $1.1$ | $0.5$ | $0.3$ | $0.1$ | $-0.4628$ | $3.9397$ | $-0.6154$ |
| $1.2$ | $0.5$ | $0.4$ | $0.1$ | $-0.4605$ | $3.9584$ | $-0.6130$ |
| $1.3$ | $0.5$ | $0.5$ | $0.1$ | $-0.4582$ | $3.9772$ | $-0.6105$ |
| $1.4$ | $0.5$ | $0.5$ | $0.2$ | $-0.4584$ | $3.9760$ | $-0.6105$ |
| $1.5$ | $0.5$ | $0.5$ | $0.3$ | $-0.4587$ | $3.9741$ | $-0.6105$ |
| $1.6$ | $0.5$ | $0.5$ | $0.4$ | $-0.4591$ | $3.9713$ | $-0.6104$ |

TABLE 2  Comparison of the present nondimensional skin friction values at lower and upper plates with those in the work of Terrill and Shrestha15 for Newtonian fluid for mixed injection case

| $Re$ | $a$ | Numerical15 | $f''(0)$ Series sol.15 | Present | Numerical15 | $f''(1)$ Series sol.15 | Present |
|-----|-----|----------------|-----------------|------|----------------|-----------------|------|
| $0.23842$ | $1.70503$ | $10.19902$ | $10.19906$ | $10.1991$ | $-10.3218$ | $-10.3217$ | $-10.3218$ |
| $0.81110$ | $1.93519$ | $11.6917$ | $11.6918$ | $11.6917$ | $-11.8032$ | $-11.8027$ | $-11.8032$ |
| $6.30190$ | $1.00863$ | $3.065$ | $3.065$ | $3.1928$ | $-12.45$ | $-11.65$ | $-12.4534$ |
| $6.61552$ | $1.62204$ | $7.022$ | $7.688$ | $7.0171$ | $-15.59$ | $-14.23$ | $-15.6076$ |
| $7.02870$ | $1.66842$ | $7.23$ | $8.11$ | $7.2149$ | $-16.22$ | $-14.60$ | $-16.252$ |
| $-0.77394$ | $1.24575$ | $7.7782$ | $7.7786$ | $7.7787$ | $-7.089$ | $-7.091$ | $-7.0899$ |
| $-1.20075$ | $1.7750$ | $7.5594$ | $7.5597$ | $10.7132$ | $-6.485$ | $-6.466$ | $-10.2869$ |
| $-8.80710$ | $1.56536$ | $10.547$ | $10.547$ | $10.547$ | $-7.39$ | $-8.76$ | $-7.3989$ |
| $0.42766$ | $0.47233$ | $2.68709$ | $2.68704$ | $2.6871$ | $-2.99927$ | $-2.9991$ | $-2.9993$ |
| $0.84194$ | $0.51425$ | $2.7861$ | $2.7857$ | $2.7861$ | $-3.4456$ | $-3.4449$ | $-3.4456$ |

5  CONCLUDING REMARKS

In the present article, a comparative analysis has been performed to investigate the effects of the induced magnetic field, Brownian motion and thermophoresis on an incompressible, and laminar flow of viscous and Jeffrey nanofluids through a porous medium between two parallel porous walls with convective boundary conditions. The governing flow field equations are transformed into a set of nonlinear ordinary differential equations through similarity variables. The results are numerically analyzed for various nondimensional functions, which govern the flow, energy, mass, and magnetic diffusivity with certain nondimensional parameters. The shooting technique along with the Runge-Kutta fourth-order algorithm has been employed. Therefore, the following observations can be drawn.

- The profiles of temperature and concentration for both fluids decrease with increasing of the Brownian motion parameter.
- The velocity profiles raise with suction/injection parameter for the both fluids.
- The tendency of axial induced magnetic field is enhanced with the $Ec$ and $Nb$ nondimensional parameter for Jeffrey fluid, and at the same time, it decreases for the viscous fluid.
- The velocities decrease with increasing of inverse Darcy parameter for Jeffrey fluid, whereas for viscous fluid, it is increasing.
- A comparison of a limiting case as made for the present numerical method with published article in the literature has better compatibility.15

The results have many feasible applications in engineering, applied science, and biological flows such as transpiration cooling, electrostatic precipitation, aerodynamic heating, polymer technology, preservation of food, filtration, boundary
List of symbols

- \( a \): Suction/injection ratio (mixed suction case \(|V_2| \geq |V_1|\) \(1 - \frac{V_1}{V_2} \))
- \( B \): Total magnetic field vector
- \( B_0 \): Applied magnetic field strength
- \( B_s \): Axial induced magnetic field
- \( B_r \): Radial induced magnetic field
- \( Bi_1 \): Biot number \( \frac{h_1 h}{k} \)
- \( Bi_2 \): Biot number \( \frac{h_2 h}{k} \)
- \( C \): Concentration
- \( C_{1e} \): Concentrations at the lower plate
- \( C_{2e} \): Concentrations at the upper plate
- \( C^* \): Nondimensional concentration
- \( D_B \): Mass diffusion coefficient
- \( Da^{-1} \): Inverse Darcy number \( h^2/k_1 \)
- \( E \): Electric field
- \( Ec \): Eckert number, \( \frac{\mu V_2}{\rho c (T_2 - T_1)} \)
- \( Gm \): Solutal Grashof number, \( \frac{\rho g \beta_c (C_2 - C_1) h^2}{\mu V_2^3} \)
- \( Gr \): Thermal Grashof number, \( \frac{\rho g \beta_T (T_2 - T_1) h^2}{\mu V_2^3} \)
- \( h \): Width of the channel
- \( h_1 \): Convection heat transfer coefficient
- \( h_2 \): Convection mass transfer coefficient
- \( J \): Current density
- \( \dot{n}_A \): Mass transfer rate
- \( Nb \): Brownian motion parameter, \( \frac{\tau_1 D_B (C_2 - C_1)}{a} \)
- \( Nt \): Thermophoresis parameter, \( \frac{\tau_1 D_t (T_2 - T_1)}{T_2 a} \)
- \( P \): Pressure of the fluid
- \( Pr \): Prandtl number, \( \frac{\nu c}{k} \)
- \( \bar{q} \): Velocity of the fluid
- \( R_1 \): Suction Reynolds number
- \( Re \): Suction/Injection Reynolds number, \( \frac{\rho V_h h}{\mu} \)
- \( Rm \): Magnetic Reynolds number, \( \sigma \mu_c h V_2 \)
- \( Sc \): Schmidt number, \( \frac{\mu}{\rho D} \)
- \( Sh \): Sherwood number, \( \frac{h_3}{m(c_2 - c_1)} \)
- \( St \): Magnetic force number (Strommer’s number), \( \frac{R_t}{V_2} \sqrt{\frac{\rho}{\mu}} \)
- \( t \): Time
- \( T^* \): Nondimensional temperature
- \( T \): Temperature
- \( T_{1e}, T_{2e} \): Temperatures at the bottom and top plates, respectively
- \( 1/\sigma \mu_c \): Magnetic diffusivity
- \( U_0 \): Arbitrary velocity, at \( x = 0 \)
- \( u \): Axial velocity component
- \( v \): Transverse velocity component
- \( V_1e \): Injection velocity
- \( V_2e \): Suction velocity
Greek letters:

\[ \begin{align*}
\alpha & : \text{Thermal diffusivity, } k/\rho c \\
\beta & : \text{Deborah number, } \frac{kV_2}{h} \\
\theta & : \text{Frequency, } \omega t \\
\xi & : \text{Dimensionless axial variable, } \left( \frac{U_0 a V_2}{V_2 h} \right) \\
\lambda & : \text{Nondimensional variable, } y/h \\
\lambda_1 & : \text{Ratio of relaxation to retardation times} \\
\lambda_2 & : \text{Relaxation time} \\
\tau & : \text{Shear stress} \\
\tau_1 & : \text{Ratio of heat capacity of nanoparticles to heat capacity of fluid } (\rho c_p)/(\rho c_f) \\
v & : \text{Kinematic viscosity}
\end{align*} \]

**CONFLICT OF INTEREST**

The authors declare that there is no conflict of interests regarding the publication of this article.

**ORCID**

Odelu Ojjela https://orcid.org/0000-0001-6577-6051

**REFERENCES**

1. Krishnan JM, Deshpande AP, Kumar PBS. *Rheology of Complex Fluids*. New York, NY: Springer; 2010.
2. Chhabra RP, Richardson JF. *Non-Newtonian Flow and Applied Rheology Engineering Applications*. Oxford, UK: Butterworth-Heinemann; 2008.
3. Rudolph N, Osswald TA. *Rudolph Natalie, Polymer Rheology: Fundamentals and Applications*. Munich, Germany: Hanser Publications; 2015.
4. Ojjela O, Naresh Kumar N. Unsteady MHD flow and heat transfer of micropolar fluid in a porous medium between parallel plates. *Can J Phys*. 2015;93(8):880-887.
5. Hayat T, Awaïs M, Alsaedi A, Alhothuali MS. 3-D flow of Jeffery fluid in a channel with stretched wall. *Eur Phys J Plus*. 2012;127:128.
6. Ahmed N, Khan U, Mohyud-Din ST. Two-dimensional flow of a Jeffery fluid in a dilating and squeezing porous channel. *World J Model Sim*. 2016;12:59-69.
7. Shehzad SA, Hayat T, Alsaedi A, Obid MA. Nonlinear thermal radiation in three-dimensional flow of Jeffrey nanofluid: a model for solar energy. *Appl Math Comp*. 2014;248:273-286.
8. Khan MI, Hayat T, Waqas M, Alsaedi A, Khan MI. Effectiveness of radiative heat flux in MHD flow of Jeffrey-nanofluid subject to Brownian and thermophoresis diffusions. *Int J Hydrog Energy*. 2019;31:421-427.
9. Murthy MK, Sreenadh S, Lakshminarayana P, Sucharitha G, Rushikumar B. Thermophoresis and Brownian motion effects on three dimensional magnetohydrodynamics slip flow of a Casson nanofluid over an exponentially stretching surface. *J Nanofluids*. 2019;8:1267-1272.
10. Shah Z, Gul T, Islam S, et al. Three dimensional third grade nanofluid flow in a rotating system between parallel plates with Brownian motion and thermophoresis effects. *Results Phys*. 2018;10:36-45.
11. Khan MI, Waqas M, Hayat T, Alsaedi A, Khan MI. Significance of nonlinear radiation in mixed convection flow of magneto Walter-B nanoliquid. *Int J Hydrog Energy*. 2017;42:26408-26416.
12. Berman AS. Laminar flow in channels with porous walls. *J Appl Phys*. 1953;24:1232-1235.
13. Terril RM. Laminar flow in a uniformly porous channel with large injection. *Aero J*. 1964;15:299-310.
14. Walker G, Davies T. Mass transfer in laminar flow between parallel permeable plates. *AIChE Journal*. 1974;20:881-889.
15. Terril RM, Shrestha GM. Laminar flow through parallel and uniformly porous walls of different permeability. *Zeitschrift Angewandte Math Physik ZAMP*. 1965;16:470-482.
16. Nigam SD, Singh SN. Heat transfer by laminar flow between parallel plates under the action of transverse magnetic field. *Q J Mech Appl Math*. 1960;13:85-97.
17. Cox SM. Two-dimensional flow of a viscous fluid in a channel with porous walls. *J Fluid Mech*. 1991;227:1-33.
18. Bujurke NM, Madalli VS, Mullimani BG. Long series analysis of laminar flow through parallel and uniformly porous walls of different permeability. *Comp Methods Appl Mech Eng*. 1998;160:39-56.
19. Ganesh S, Krishnambal S. Magnetohydrodynamic flow of viscous fluid between two parallel porous plates. *J Appl Sci*. 2006;6:2420-2425.
20. Hafeez HY, Ndikilar CE. Flow of viscous fluid between two parallel porous plates with bottom injection and top suction. *Prog Phys*. 2014;10:49-51.
21. Rao GT, Moizuddi M. Steady flow of micropolar incompressible fluid between two parallel porous plates. *Def Sci J*. 2014;30:105-112.
22. Hayat T, Sajjad R, Abbas Z, Sajid M, Hendi AA. Radiation effects on MHD flow of Maxwell fluid in a channel with porous medium. *Int J Heat Mass Transf*. 2011;54:854-862.

23. Choi SUS, Eastman JA. Enhancing thermal conductivity of fluids with nano-particles. *ASME Publ Fed*. 1995;231:99-106.

24. Dogonchi AS, Divsalar K, Ganji DD. Flow and heat transfer of MHD nanofluid between parallel plates in the presence of thermal radiation. *Comp Methods Appl Mech Eng*. 2016;310:58-76.

25. Sheikholeslami M, Rashidi MM, Al Saad DM, Firozii F, Rokni HB, Domairry G. Steady nanofluid flow between parallel plates considering thermophoresis and Brownian effects. *J King Saud Univ Sci*. 2016;28:380-389.

26. Hayat T, Khan MI, Farooq M, Alsaedi A, Waqas M, Yasmeen T. Impact of Cattaneo–Christov heat flux model in flow of variable thermal conductivity fluid over a variable thicked surface. *Int J Heat Mass Transf*. 2016;99:702-710.

27. Hayat T, Khan MI, Farooq M, Yasmeen T, Alsaedi A. Stagnation point flow with Cattaneo-Christov heat flux and homogeneous-heterogeneous reactions. *J Mol Liq*. 2016;220:49-55.

28. Khan MI, Waqas M, Hayat T, Alsaedi A. Colloidal study of Casson fluid with homogeneous-heterogeneous reactions. *J Colloid Inter Sci*. 2017;498.

29. Khan U, Ahmed N, Mohyud-Din ST. Analysis of magnetohydrodynamic flow and heat transfer of Cu–water nanofluid between parallel plates for different shapes of nanoparticles. *Neural Comp Appl*. 2018;29:695-703.

30. Makinde OD, Aziz A. Boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition. *Int J Therm Sci*. 2011;50:1326-1332.

31. Matin MH, Ghanbari B. Effects of Brownian motion and thermophoresis on the mixed convection of nanofluid in a porous channel including flow reversal. *Transp Porous Media*. 2014;101:115-136.

32. Babu MJ, Sandeep N, Saleem S. Free convective MHD Cattaneo-Christov flow over three different geometries with thermophoresis and Brownian motion. *Alex Eng J*. 2017;56:659-669.

33. Hayat T, Khan MI, Qayyum S, Alsaedi A. Entropy generation in flow with silver and copper nanoparticles. *Colloids Surf A Physicochem Eng Asp*. 2018;539:335-346.

34. Hayat T, Qayyum S, Khan MI, Alsaedi A. Entropy generation in magnetohydrodynamic radiative flow due to rotating disk in presence of viscous dissipation and joule heating. *Phys Fluids*. 2018;2018(30):017101.

35. Hayat T, Khan SA, Khan MI, Alsaedi A. Optimizing the theoretical analysis of entropy generation in the flow of second grade nanofluid. *Physica Scripta*. 2019;94(8):085001.

36. Khan MWA, Khan MI, Hayat T, Alsaedi A. Entropy generation minimization (EGM) of nanofluid flow by a thin moving needle with nonlinear thermal radiation. *Phys B Condens Matter*. 2018;534:113-119.

37. Rashid M, Khan MI, Hayat T, Alsaedi A. Physical significance of heat generation/absorption and Soret effects on peristalsis flow of pseudoplastic fluid in an inclined channel. *J Mol Liq*. 2019;275:599-615.

38. Hayat T, Javed S, Khan MI, Khan MI, Alsaedi A. Physical aspects of irreversibility in radiative flow of viscous material with cubic autocatalysis chemical reaction. *Eur Phys J Plus*. 2019;134(4):1-24.

39. Shehzad SA, Hayat T, Alsaedi A. MHD flow of Jeffrey nanofluid with convective boundary conditions. *J Braz Soc Mech Sci Eng*. 2015;37:873-883.

40. Ramzan M, Bilal M, Chung JD, Mann AB. On MHD radiative Jeffery nanofluid flow with convective heat and mass boundary conditions. *Neural Comp Appl*. 2017;30:2739-2748.

41. Waqas M, Jabeen S, Hayat T, Khan MI, Alsaedi A. Modeling and analysis for magnetic dipole impact in nonlinear thermally radiating Carreau nanofluid flow subject to heat generation. *J Magn Magn Mater*. 2019;485:197-204.

42. Hayat T, Muhammad T, Shehzad SA, Alsaedi A. A mathematical study for three-dimensional boundary layer flow of Jeffrey nanofluid. *Zeitschrift für Naturforschung a*. 2015;70:225-233.

43. Sandeep N, Kumar BR, Kumar MSJ. A comparative study of convective heat and mass transfer in non-Newtonian nanofluid flow past a permeable stretching sheet. *J Mol Liq*. 2015;212:585-591.

44. Dalir N, Dehsara M, Nourazar SS. Entropy analysis for magnetohydrodynamic flow and heat transfer of a Jeffrey nanofluid over a stretching sheet. *Energy*. 2015;79:351-362.

45. Mahmoodi M, Kandelousi S. Effects of thermophoresis and Brownian motion on nanofluid heat transfer and entropy generation. *J Mol Liq*. 2015;211:15-24.

46. Noreen S. Mixed convection peristaltic flow of third order nanofluid with an induced magnetic field. *PLOS ONE*. 2013;8:e78770.

47. Akbar NS, Raza M, Ellahi R. Interaction of nanoparticles for the peristaltic flow in an asymmetric channel with the induced magnetic field. *Euro Phys J Plus*. 2014;129:155.

48. Mustafa M, Hina S, Hayat T, Ahmad B. Influence of induced magnetic field on the peristaltic flow of nanofluid. *Meccanica*. 2014;49:521-534.

49. Hayat T, Bibi A, Yasmin H, Alsaedi FE. Magnetic field and thermal radiation effects in peristaltic flow with heat and mass convection. *J Therm Sci Eng Appl*. 2018;10:051018.

50. Sheikholeslami M, Zia QM, Ellahi R. Influence of induced magnetic field on free convection of nanofluid considering Koo-Kleinstreuer-Li (KKL) correlation. *Applied Sciences*. 2016;6:324.

51. Hasona WM, El-Shekhipy AA, Ibrahim MG. Combined effects of magneto-hydrodynamic and temperature dependent viscosity on peristaltic flow of Jeffrey nanofluid through a porous medium: applications to oil refinement. *Int J Heat Mass Transf*. 2018;126:700-714.
53. Denno KI, Fouad AA. Effects of the induced magnetic field on the magnetohydrodynamic channel flow. *IEEE Trans Electron Devices*. 1972;19:322-331.

54. Singh NP, Singh AK. MHD effects on flow of viscous fluid with induced magnetic field. *Indian J Pure Appl Phys*. 2001;39(4):240-245.

55. Ibrahim W. The effect of induced magnetic field and convective boundary condition on MHD stagnation point flow and heat transfer of nanofluid past a stretching sheet. *IEEE Trans Nanotechnol*. 2015;14:178-186.

56. Sandeep N, Sulochana C, Animasaun IL. Stagnation-point flow of a Jeffrey nanofluid over a stretching surface with induced magnetic field and chemical reaction. *Int J Eng Res Afr*. 2016;20:93-111.

57. Raju A, Ojjela O. Combined effects of variable thermal conductivity and induced magnetic field on convective Jeffrey fluid flow with nth order chemical reaction. *Heat Transf Asian Res*. 2019;48:663-683.

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