Triviality and the Precision Bound on the Higgs Mass

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Abstract

The triviality of the scalar sector of the standard one-doublet Higgs model implies that this model is only an effective low-energy theory valid below some cut-off scale Λ. For a heavy higgs this scale must be relatively low (10 TeV or less). Additional interactions coming from the underlying theory, and suppressed by the scale Λ, give rise to model-dependent corrections to precisely measured electroweak quantities. Dimension six operators arising from the underlying physics naturally contribute to the $S$ and $T$ parameters, and their effects should be included in a global fit to the precision data that determines any limit on the Higgs mass. Using dimensional analysis, we estimate the expected size of these corrections in a custodially-symmetric strongly-interacting underlying theory. Taking these operators’ coefficients to be of natural size gives sufficiently large contributions to the $T$ parameter to reconcile Higgs masses as large as 400-500 GeV with the precision data.

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The standard one-doublet Higgs model is, at first sight, a fully consistent, renormalizable, quantum field theory. Order by order in perturbation theory, all experimentally measurable quantities are completely calculable (see, for example, ref. 1 and references therein) in terms of the gauge coupling constants, the weak scale \( v \approx 250 \text{ GeV} \), the quark and lepton masses and mixing angles (most importantly, the top-quark mass), and the Higgs boson mass. Conversely, using sufficiently many measurements one may test the consistency of the data with the standard Higgs model, and extract the best-fit values of the parameters used to define the theory. This method results in an indirect determination of the Higgs boson mass which is relatively low, \( m_H < 262 \text{ GeV} \) (95\% C.L.) \([2]\). This bound is only relevant for the standard one-doublet Higgs model in the absence of new physics at higher energies \([3]\).

However, the triviality \([4]\) of the scalar sector of the standard one-doublet Higgs model implies that the model is only an effective low-energy theory valid below some cut-off scale \( \Lambda \). Additional interactions coming from the underlying theory\([1]\), and suppressed by the scale \( \Lambda \), give rise to model-dependent corrections to precisely measured electroweak quantities. In this sense the standard model, for a given Higgs boson mass, is not a single theory but rather a class of theories. The most important corrections from the underlying theory are encoded in dimension six operators \([8]\) which contribute to the Peskin-Takeuchi \( S \) and \( T \) parameters \([9]\). Their contribution should be included in a global fit to the precision data that determines a limit on the Higgs mass \([3]\). Here we emphasize the role that triviality bounds have in this context, compare the precision bounds with the triviality bound, and discuss the natural size of the couplings and the scale \( \Lambda \) for a strongly-interacting underlying theory.

Clearly, if \( \Lambda \) can be taken to be arbitrarily high, the corrections from the underlying theory will be irrelevant. Because of triviality, however, for a given Higgs boson mass the scale \( \Lambda \) cannot be arbitrarily high \([10]\). An estimate of the upper bound on the cut-off can be taken from lowest order perturbation theory. Integrating the lowest order beta function for the Higgs self coupling \( \lambda \)

\[
\beta(\lambda) = \mu \frac{d\lambda}{d\mu} = \frac{3}{2\pi^2} \lambda^2 + \ldots
\]

one finds

\[
\frac{1}{\lambda(\mu)} - \frac{1}{\lambda(\Lambda)} = \frac{3}{2\pi^2} \log \frac{\Lambda}{\mu}.
\]

Using the relation \( m_H^2 = 2\lambda(m_H)v^2 \) then implies

\[
m_H^2 \log \frac{\Lambda}{m_H} \leq \frac{4\pi^2 v^2}{3}.
\]

\(^1\)Examples of such theories include top-condensate \([3, 4, 5]\) and composite Higgs models \([7]\).
The resulting upper bound on $\Lambda$ as a function of $m_H$ is shown in Figure 1. If the underlying theory comes close to saturating the upper bound, then $\lambda(\Lambda)$ is large and the underlying theory must be strongly interacting. A theory with a Higgs mass above 500 GeV, for example, must have a cut off below about 12 TeV. This derivation is based on perturbation theory and would appear suspect, but non-perturbative [11] studies on the lattice using analytic and Monte Carlo techniques confirm that the estimate in eqn. (3) is reasonably accurate.

To estimate the sizes of effects from the underlying physics, we will rely on dimensional analysis [12]. In brief, a theory with light scalar particles depends on two parameters: $\Lambda$, the energy scale of the underlying physics, and $\kappa$, a measure of the size of dimensionless couplings (in the chiral lagrangian in QCD $\kappa = \mathcal{O}(\Lambda_m^{\chi_{\text{SB}}}/f_\pi)$ [13]). For a strongly-coupled underlying theory, $\kappa$ is expected of order $4\pi$. Starting from the kinetic energy term (which is bilinear in the scalar field), the sizes of operators in the effective low-energy theory can be estimated by including an extra value of $\kappa$ for each scalar field (beyond the two present in the kinetic energy) and making up the mass-dimension by using the appropriate power of $\Lambda$. The leading operators in the effective lagrangian [8] which contribute to electroweak measurements are then,

$$-\frac{a}{2! \Lambda^2} \{[D_\mu, D_\nu] \phi \}^\dagger [D^\mu, D^\nu] \phi + \tilde{b} \kappa^2 \frac{2!}{2! \Lambda^2} (\phi^\dagger \tilde{D}_\mu \phi)(\phi^\dagger \tilde{D}_\mu \phi), \quad (4)$$

where $a$ and $\tilde{b}$ are expected to be of order one [14, 15].

The second term in eqn. (1) violates custodial symmetry [16], and if it were possible for

$$...$$
Figure 2: The oval demarks the area of the S-T plane compatible with the precision
electroweak measurements at the 95% confidence level. The line is the trajectory of Higgs
mass in the standard model from 76 GeV to 1 TeV. The black rectangles show the natural
size of corrections from the underlying physics for different scales $\Lambda$ and varying $a$ and $b\kappa$
between $\pm 1$ (they should be centered on the point on the Higgs line corresponding to the
Higgs mass of interest).

the underlying theory to respect this symmetry $\hat{b}$ would be zero. There must, however, be
sufficient custodial violation to give rise to the top-quark Yukawa coupling. In the absence
of custodial symmetry, dimensional analysis predicts a top Yukawa coupling of order $\kappa$. The
violation of custodial symmetry, therefore, introduces the small parameter $y_t/\kappa \simeq 1/\kappa$
[14]. The second term in eqn. (4) is then suppressed by this amount and in a custodially-
symmetric strongly-interacting underlying theory we expect the low energy operators

$$ - \frac{a}{2! \Lambda^2} \{[D_\mu, D_\nu]\phi\}^\dagger [D^\mu, D^\nu]\phi + \frac{b \kappa}{2! \Lambda^2} (\phi^\dagger \not{D}_\mu \not{D}_\mu \phi) (\phi^\dagger \not{D}_\mu \not{D}_\mu \phi), \quad (5) $$

where $a$ and $b$ are both $O(1)$. These operators give rise to the corrections

$$ \Delta S = \frac{4\pi a v^2}{\Lambda^2}, \quad \Delta T = \frac{b\kappa v^2}{\alpha \Lambda^2} \quad (6) $$

where $\alpha$ is the electromagnetic coupling.

In figure 2, we display a fit to current electroweak data [17] from $Z^0$ measurements at
LEP and SLD and the $W$ mass measurements from LEP II and the Tevatron, allowing

\footnote{In principle, since the top-quark mass operator “transforms” as a custodial isospin $I = 1$ operator and the second operator in eqn. (4) as $I = 2$, the suppression could be as much as $(y_t/\kappa)^2 \simeq 1/\kappa^2$. In practice, by varying $b\kappa$ between 1 and $4\pi$ in eqn. (5) we expect (6) will yield a reasonable estimate of the size of these effects.}
for the presence of flavor-universal “oblique” contributions to the gauge-boson self-energies \([9, 18]\). The fit was done by incorporating the Peskin-Takeuchi \(S\) and \(T\) parameters \([9]\), defined using a reference Higgs boson mass of 76 GeV, into the prediction of the electroweak observables \([19]\). The 95\% confidence contour in this plane encloses the point \(S = T = 0\), displaying the agreement of the data with the standard model with a light Higgs boson.

Changing Higgs mass can be viewed as parametrically changing \(S\) and \(T\). The \(S\) and \(T\) dependence on the higgs mass has been calculated in \([20]\). The curve giving the standard model prediction varying the Higgs mass from 76 GeV to 1 TeV is also shown in figure 2. These considerations yield, at 95\% confidence level, a limit of 230 GeV on the Higgs mass (120 GeV at 67\%) which agrees well with the fit of ref. \([2]\). The dependence of \(S\) and \(T\) on the higgs mass is only logarithmic and the bounds are thus very sensitive to the data (for example removing the SLD measurement of the left right asymmetry increases the 95\% bound to of order 400 GeV \([2]\)). The bounds are also very sensitive to the inclusion of new physics \([3]\) as well.

In figure 2 we also show the natural size of the corrections from the underlying physics (the operators in eqn. \([3]\)) as error boxes (which should be centered on the appropriate point on the Higgs mass curve to see the effect of the corrections for a given Higgs mass) for different scales \(\Lambda\) and varying \(a\) and \(b\kappa\) between ±1. Note that the error boxes are rather narrow – it is the shifts in \(T\) induced by the underlying physics that are the most relevant. For \(\Lambda\) of order 5 TeV to 10 TeV, which are the upper bounds on \(\Lambda\) for Higgs bosons with masses of order 500 to 600 GeV, corrections to the \(T\) parameter are of sufficient size that they are not negligible in the context of the Higgs mass bounds. If we allow larger values of \(a\) and \(b\kappa\), then lower scales become disfavored because the natural contribution to \(T\) is too large (this is the familiar problem found in accommodating the top mass in technicolor type models \([21]\)) but the corrections from the underlying physics remain important up to a scale of order 50 TeV! To reconcile a heavy higgs with the precision data requires positive contributions to the \(T\) parameter. In fact, generically, isospin breaking does give rise to positive contributions to \(T\) (e.g. mass splitting in heavy electroweak doublets, mixing with additional \(U(1)\) gauge bosons etc) and negative contributions are much harder to achieve. Alternatively, negative contributions to \(S\) could reconcile a heavy higgs but the natural size of corrections from the underlying physics does not seem compatible with this choice.

If we include the corrections \([4]\) we may derive bounds in the higgs mass - \(\Lambda\) plane. These are shown in figure 1 for varying values of \(b\kappa\) (we set \(a = -b\kappa\) to provide the most conservative bound but the dependence on the \(S\) parameter is small). For Higgs masses above 500 GeV the triviality bound itself is in fact a stronger bound than the precision data including corrections from the underlying physics.
Figure 3: The value of $b\kappa$ compatible with the precision bounds as a function of $\Lambda$ for different values of $m_h$. The blocked off area in the last plot is forbidden by triviality. The values of cut off on the x axis varies logarithmically between 2500 – 25000 GeV.
One may worry that $bk$ must be very finely adjusted to make heavier Higgs bosons consistent with precision electroweak measurements, and that this situation is therefore not generic. In figure 3 we plot, for various Higgs boson masses, the values of $bk$ which are allowed by precision electroweak measurements as a function of $\Lambda$. We see that, for Higgs masses up to 500 GeV and scales $\Lambda$ of order 10 TeV or less, no unnatural adjustment of parameters is required.

That $\Lambda$ is naturally of order 10 TeV or higher, with smaller cut-offs requiring a greater degree of fine tuning of $bk$, corresponds to the result of ref. [14]. In [15] it was argued that if the fundamental physics at the scale $\Lambda$ does not respect flavor symmetries then neutral meson mixing gives a constraint on the scale $\Lambda$ of order 20 TeV. Higgs masses above 460 GeV would then be ruled out by triviality alone and to reconcile higgs mass between 230-460 GeV would require larger values of $bk$. This stronger constraint is more speculative than that from the $T$ parameter because, whilst isospin is known to be broken by the top Yukawa, a GIM-type mechanism suppressing flavor-changing neutral-currents could arise from the underlying physics.

The triviality of the higgs sector of the standard model requires that the standard model with a heavy higgs is only an effective field theory that must break down at relatively low scales of $\mathcal{O}(10 \text{ TeV})$ or less. Even if the higgs is lighter, a low cut-off may exist in nature. Higher dimension operators in the effective higgs theory suppressed by the cut-off scale will contribute to the $S$ and $T$ parameters. Using dimensional analysis, we have estimated the expected size of these corrections in a custodially-symmetric strongly-interacting underlying theory. Taking these operators’ coefficients to be of natural size gives sufficiently large contributions to the T parameter to reconcile Higgs masses as large as 400-500 GeV with the precision data.

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