Particle acceleration of two general particles in the background of rotating Ayón-Beato-García black holes

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Abstract

The rotating Ayón-Beato-García (ABG) black holes, apart from mass ($M$) and rotation parameter ($a$), has an additional charge $Q$ and encompassed the Kerr black hole as particular case when $Q = 0$. We demonstrate the ergoregions of rotating ABG black holes depend on both rotation parameter $a$ and charge $Q$, and the area of the ergoregions increases with increase in the values of $Q$, when compared with the Kerr black hole and the extremal regular black hole changes with the value of $Q$. Banados, Silk and West (BSW) demonstrated that an extremal Kerr black hole can act as a particle accelerator with arbitrarily high center-of-mass energy ($E_{CM}$) when the collision takes place at any point in the ergoregion and thus in turn provides a suitable framework for Plank-scale physics. We study the collision of two general particles with different masses falling freely from rest in the equatorial plane of a rotating ABG black hole near the event horizon and find that the $E_{CM}$ of two colliding particles is arbitrarily high when one of the particles take a critical value of angular momentum in the extremal case, whereas for nonextremal case $E_{CM}$ for a pair of colliding particles is generically divergent at the inner horizon, and explicitly studying the effect of charge $Q$ on the $E_{CM}$ for ABG black hole. In particular, our results in the limit $Q \rightarrow 0$ reduce exactly to vis-à-vis those of the Kerr black hole.

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I. INTRODUCTION

Banados, Silk and West [1] have proposed that the collision of two particles, say dark matter particles, falling from rest at infinity into the Kerr black hole [2] can produce an infinitely large center-of-mass energy ($E_{CM}$) when collision takes place in the vicinity of the event horizon, with the black hole maximally spinning, and one of the particle have critical angular momentum. This mechanism of particle acceleration by a black hole is particularly called BSW mechanism, which is interesting from the viewpoint of theoretical physics because new Planck scale physics is possible in the vicinity of the black holes. Further, the extremal Kerr black hole surrounded by dark matter could be regarded as a Planck scale collider, which might help us to explain the astrophysical phenomena, such as the the active galactic nuclei and gamma ray bursts. Hence, the BSW mechanism about the collision of two particles near a rotating black hole has attracted significant attention [3–16] (see also [17], for a review). BSW phenomena is not only done for Kerr black holes but also for Kerr-Newman black holes [18], Regular Black holes [19–22], higher dimensional black holes [23] and naked singularities [24–28]. In particular for the Kerr-Newman black hole, the $E_{CM}$ of collision depends not only on the spin $a$ but also on the charge $Q$ of the black hole. Lake [5] analyzed the $E_{CM}$ of the collision occurring at the inner horizon of the non-extremal Kerr black hole and found that $E_{CM}$ is unlimited. Grib and Pavlov [10] showed that the $E_{CM}$ for two particles collision can be unlimited even in the non-maximal rotation. Later, Zaslavskii [29, 30] demonstrated that an acceleration of the particles by BSW method is a universal property of rotating black holes. Recently, Harada and Kimura [17] generalized the BSW analysis of two colliding particles to general geodesics in the Kerr black hole to show an arbitrarily high $E_{CM}$ can occur near the horizon of maximally rotating black holes. Further, the subject of particle acceleration for two different masses and energetic particles is extended for a class of black holes [31, 32]. The analysis is also valid to the collision of particles in non-equatorial motion for Kerr black holes [33] and Kerr-Newman black holes [31]. The general explanation of BSW phenomenon is proposed in [34].

It turns out that the horizon structure of the rotating regular black hole is complicated as compared to the Kerr black hole, which depend on the mass and spin of the black hole and on an additional deviation parameter that measures potential deviations from the Kerr metric, and includes the Kerr metric as the special case if this deviation parameter vanishes. Further,
these regular black holes are very important as astrophysical black holes, like Cygnus X-1 or SgrA∗, although suppose to be like the Kerr black hole [35, 36], but the definite nature of astrophysical black hole still need to be tested, and it may deviate from the standard Kerr black hole. More recently, the BSW mechanism when applied to an extremal regular black holes [19, 20], also lead to divergence of the \( E_{CM} \). Hence, the BSW mechanism should be suitably adapted for the rotating regular black hole, which has very complicated horizon structure as compared to the Kerr black hole and can have extremal black holes depending on the additional parameter [20–22].

The rotating regular Ayón-Beato-García (ABG) black hole [37] is an exact solution of Einstein’s equations coupled to nonlinear electrodynamics. The rotating ABG black holes are axisymmetric, asymptotically flat, and depend on the mass (\( M \)) and spin (\( a \)) of the black hole as well as on a charge (\( Q \)) that measures potential deviations from the Kerr metric and includes the Kerr metric as the special case if this deviation parameter vanishes.

The main goal of this paper is to discuss the detailed behavior of the \( E_{CM} \) for two particles with different rest masses \( m_1 \) and \( m_2 \), falling freely from rest at infinity in the background of a rotating ABG black hole and calculate the \( E_{CM} \). We go on to show that the \( E_{CM} \) near the horizon of an extremal ABG black hole is arbitrarily high when one of the two particles acquire the critical angular momentum and also high at the inner horizon. We also explicitly show the effect of the parameter \( Q \) on BSW mechanism and ergoregion of the black hole.

The paper is organized as follows. In Sec. II, we shall discuss horizons of the rotating ABG spacetime and also demonstrate the effect of parameter \( Q \) on ergoregion. In Sec. III, we will discuss the equations of motion for a particle in the background of a rotating ABG black hole. The calculation of expression for \( E_{CM} \) of the collision for two general particles and their properties is the subject of Sec. IV and we conclude in the Sec. V.

II. ROTATING AYÓN-BEATO-GARCÍA BLACK HOLES

The spherically symmetric ABG black hole is an exact regular solution of general relativity with nonlinear electrodynamics field as a source, and it satisfies the weak energy condition. The gravitational field of ABG solution is described by the metric [38]:

\[
ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2d\Omega^2,
\]
with

\[ f(r) = 1 - \frac{2Mr^2}{(r^2 + Q^2)^{3/2}} + \frac{Q^2r^2}{(r^2 + Q^2)^2}, \quad (2) \]

and \( d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2 \). The parameter \( M \) and \( Q \) are respectively, mass and electric charge. The associated electric field is

\[ E = Qr^4 \left[ \frac{r^2 - 5Q^2}{(r^2 + Q^2)^4} + \frac{15}{2} \frac{M}{(r^2 + Q^2)^{7/2}} \right]. \quad (3) \]

Note that the ABG black hole is asymptotically go over to Reissner-Nordstörm black hole

\[ f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + O(1/r^3), \]

\[ E = \frac{Q}{r^2} + O(1/r^3). \]

This solution corresponds to a regular charged black hole when \(|Q| \leq 0.6M\), with the curvature invariant and electric field regular everywhere including at \( r = 0 \) [38].

The rotating ABG black hole metric obtained in [39], beginning with the metric (1) and applying the Newman-Janis transformation, they constructed a rotating ABG metric. The gravitational field of rotating ABG spacetime is described by a metric which in the Boyer-Lindquist coordinates (with \( G = c = 1 \)) [39] reads:

\[
\begin{align*}
    ds^2 &= -f(r, \theta)dt^2 - 2a\sin^2 \theta(1 - f(r, \theta))d\phi dt + \frac{\Sigma}{\Delta}dr^2 \\
    &+ \Sigma d\theta^2 + \sin^2 \theta[\Sigma - a^2(f(r, \theta) - 2)\sin^2 \theta]d\phi^2, \quad (4)
\end{align*}
\]

where, \( \Sigma = r^2 + a^2\cos^2 \theta \), \( \Delta = \Sigma f(r, \theta) + a^2 \sin^2 \theta \), \( a \) is a rotation parameter and the function \( f(r, \theta) \) is given by

\[ f(r, \theta) = 1 - \frac{2Mr\sqrt{\Sigma}}{(\Sigma + Q^2)^{3/2}} + \frac{Q^2\Sigma}{(\Sigma + Q^2)^2}. \quad (5) \]

The ABG metric (4) is a regular rotating charged black hole [20, 39], which go over to Kerr black holes [2] when \( Q = 0 \). When \( a = 0 \), it reduces to the ABG black hole [38], and for \( a = Q = 0 \), it reduces to the Schwarzschild black hole [40]. The invariant Ricci scalar \( (R_{ab}R^{ab}) \) and Kretschmann scalar \( (R_{abcd}R^{abcd}) \) are suppose to be regular everywhere including at \( r = 0 \) [20]. The metric (4) becomes singular if \( \Sigma = 0 \) or \( \Delta = 0 \) whereas \( \Sigma = 0 \) is the curvature singularity and \( \Delta = 0 \) is the coordinate singularity. \( \Delta = 0 \) gives horizons of the rotating ABG black holes [20].
FIG. 1: Plots showing the behavior of ergoregion in the xz-plane of rotating ABG black hole for $Q = 0$ and different values of $a$. The blue and red lines correspond to the static limit surface and horizons.

A. Horizons and ergoregions of rotating ABG black holes

In this section, we would like to study the properties of an ergoregion of a rotating ABG black hole which is the region between the static limit surface and the event horizon. In fact, the ergoregion plays an important role in the astrophysics for a possible observational effects, since in this region the Hawking radiation can be analyzed. The ergoregion is also important due to energetics of black holes and also Penrose process [41]. The static limit surface ($r_{\text{sls}}$) is also known as infinite redshift surface, where the time-translation Killing vector becomes null. Another property of the static limit surface is that the time-like geodesics becomes space-like after crossing the static limit surface. The static limit surface of a rotating ABG black hole can be calculated by solving the equation $g_{tt} = f(r, \theta) = 0$.

In the case of $Q = 0$, it represents the static limit surface of a Kerr black hole. The metric has a coordinate singularity at $\Delta = 0$. Horizons of the metric (4) are given by $g^{rr} = \Delta = 0$, i.e.,

$$\Sigma f(r, \theta) + a^2 \sin^2 \theta = 0. \quad (6)$$

An ergoregion is located outside an event horizon, which has an oblate shape and touches the event horizon at poles and it has a highest radius at the equator. It turns out that Eq. (6) admits two positive roots which take different values with charge $Q$. The two roots corresponding to the horizons of the black hole. Let us assume that $r_{+}^{EH}$ and $r_{-}^{CH}$ define respectively, the outer horizon (event horizon) and inner horizon of the black hole, and when they are equal $r_{+}^{EH} = r_{-}^{CH} = r_{ex}$, corresponds to an extremal black holes with degenerate horizons. Inside the ergoregion an object can not remain static, but it moves in the direction
TABLE I: Table for different values of $a$ and $Q$ for ABG black hole. Parameter $\delta^a$ is the region between static limit surface and event horizon ($\delta^a = r^\text{sls}_+ - r^E_H$).

| $Q$ | $r^E_H$ | $r^\text{sls}_+$ | $\delta^a$ | $r^E_H$ | $r^\text{sls}_+$ | $\delta^a$ | $r^E_H$ | $r^\text{sls}_+$ | $\delta^a$ | $r^E_H$ | $r^\text{sls}_+$ | $\delta^a$ |
|-----|--------|-----------------|-----------|--------|-----------------|-----------|--------|-----------------|-----------|--------|-----------------|-----------|
| 0.0 | 1.86603 | 1.93541 | 0.06938 | 1.80000 | 1.90554 | 0.10554 | 1.71414 | 1.86891 | 0.15477 | 1.60000 | 1.82462 | 0.22462 |
| 0.1 | 1.85116 | 1.92196 | 0.07080 | 1.78371 | 1.89163 | 0.10792 | 1.69558 | 1.85440 | 0.15882 | 1.57732 | 1.80932 | 0.23200 |
| 0.2 | 1.80488 | 1.88037 | 0.07549 | 1.73257 | 1.84856 | 0.11599 | 1.63641 | 1.80935 | 0.17294 | 1.50259 | 1.76161 | 0.25902 |
| 0.3 | 1.72097 | 1.80640 | 0.08543 | 1.63779 | 1.77155 | 0.13376 | 1.52186 | 1.72821 | 0.20635 | 1.33877 | 1.67476 | 0.33599 |
| 0.4 | 1.58263 | 1.68988 | 0.10725 | 1.47161 | 1.64881 | 0.17720 | 1.28210 | 1.59665 | 0.31455 | - | 1.53012 | - |

of black hole spin. In ergoregion, an object can enter and escape, and we can extract energy and mass via the Penrose process [41].

How the charge parameter $Q$ affects the shape of the ergoregion is shown in Fig. 2, when compared with the Kerr black hole’s ergoregion (cf. Fig. 1). The numerical solution of $g_{tt} = 0$ and $g^{rr} = 0$ are summarized in Table I. Fig. 2 shows that the ergoregion area of the ABG black hole increases with increase in the value of $Q$ as well as $a$. The outer horizon and static limit surface coincides at the poles (Fig. 2) as in the case of Kerr black hole (Fig. 1). It should be pointed out that when $Q > Q_C$, for a given parameter $a$, the horizons get disconnected (cf. Fig. 2). The static limit surface becomes more oblate with increase in the value of $Q$.

III. EQUATIONS OF MOTION IN THE BACKGROUND OF A ROTATING ABG BLACK HOLE

Next, we calculate the equations of motion of the particle in the background of rotating ABG black hole, which are essential to study the collision of particles. Let us consider a particle of rest mass $m$ falling from rest at infinity on the equatorial plane ($\theta = \pi/2$) of a rotating ABG black hole. The motion of a particle is determined by the Lagrangian

$$\mathcal{L} = \frac{1}{2} g_{\mu\nu} u^\mu u^\nu,$$  \hspace{1cm} (7)
FIG. 2: Plots showing the behavior of ergoregion in the xz-plane of rotating ABG black hole with charge $Q$ and for different values of rotation parameter $a$. The blue and red lines correspond to the static limit surface and horizons.
where \( u^\mu \) is the four-velocity. The metric (4) has two conserved quantities, namely, energy \( E \) and angular momentum \( L \), respectively, correspond to the timelike Killing vector \( \xi^a = (\partial/\partial t)^a \) and axial Killing vector \( \chi^a = (\partial/\partial \phi)^a \), given \([33]\)

\[
E = -g_{ab}\xi^a u^b, \quad L = g_{ab}\chi^a u^b, \tag{8}
\]

which yields

\[
E = -g_{tt}u^t - g_{t\phi}u^\phi, \quad L = g_{t\phi}u^t + g_{\phi\phi}u^\phi.
\]

We obtain the four-velocities of the particle by solving the above equations,

\[
u^t = -\frac{1}{r^2}\left[a(E - L) - \frac{r^2 + a^2}{\Delta}P\right], \tag{9}\]

\[
u^\phi = -\frac{1}{r^2}\left[aE - L - \frac{a}{\Delta}P\right], \tag{10}\]

and the normalization condition of the four-velocity \( u_\mu u^\mu = -m^2 \), yields

\[
u^r = \pm \frac{1}{r^2}\sqrt{P^2 - \Delta[m^2 r^2 + (L - aE)^2]}, \tag{11}\]

where positive and negative sign correspond to the outgoing and incoming geodesics, respectively and \( P = (r^2 + a^2)E - aL \). The above equations have same expressions that of Kerr black hole, but now \( \Delta \) is modified which includes charge \( Q \). However, to obtain the trajectories of a particle, we need the angular momentum range of the particle and it can be calculated by using the circular orbit conditions,

\[
V_{\text{eff}} = 0, \quad \text{and} \quad \frac{dV_{\text{eff}}}{dr} = 0, \tag{12}\]

where \( V_{\text{eff}} \) (effective potential) is

\[
V_{\text{eff}} = -\frac{[(r^2 + a^2)E - La]^2 - \Delta m^2 r^2 + (L - aE)^2]}{2r^4}. \tag{13}\]

It determines the allowed and prohibited regions of the particle trajectory around the ABG black hole. If \( V_{\text{eff}} \leq 0 \), then this is an allowed region and for \( V_{\text{eff}} > 0 \), the motion is prohibited. In Fig. 3, we have shown the behaviour of \( V_{\text{eff}} \) with radius \( r \). Since, the geodesics are timelike, i.e., \( dt/d\tau \geq 0 \), then using Eq. (9) we get

\[
\frac{1}{r^2}[\Delta - a(aE - L)] \geq 0, \tag{14}\]

at horizon, the above condition on simplification leads to

\[
E - \Omega_H L \geq 0, \quad \Omega_H = \frac{a}{(r_{+E}^\text{BH})^2 + a^2}. \tag{15}\]
FIG. 3: Plots showing the behavior of the effective potential $V_{\text{eff}}$ with radius $r$ for the different values of angular momentum.

TABLE II: Table for the range of the angular momentum in extremal cases of a rotating ABG black hole ($M = 1$).

| $Q$ | $\alpha = a_{\text{ex}}$ | $r = r_{\text{ex}}$ | $L_1$ | $L_2$  |
|-----|---------------------------|---------------------|-------|-------|
| 0   | 1                         | 1                   | 2.0   | -4.82842 |
| 0.15| 0.955870457               | 1.02859             | 2.06269 | -4.78476 |
| 0.25| 0.880328799               | 1.06201             | 2.16152 | -4.70709 |
| 0.35| 0.769966648               | 1.08805             | 2.30749 | -4.58729 |
| 0.45| 0.621056295               | 1.09518             | 2.55058 | -4.41379 |

The critical angular momentum of the particle is defined by $L_C = E/\Omega_H$, where $\Omega_H$ is the angular velocity at the horizon of the ABG black hole. We have shown the values of critical angular momentum for the extremal cases of rotating ABG black hole in Table II. It is known that the particle trajectory depends on the values of angular momentum which is different for different values of angular momentum. If angular momentum of the particle is larger than the critical angular momentum, i.e., $L > L_C$, then the particle will never fall into the black hole. When the angular momentum of particle is smaller then the critical angular momentum $L < L_C$, then the particle will always fall into the black hole. Moreover, when the particle’s angular momentum is equal to the critical angular momentum $L = L_C$, then the particle reaches up to the horizon and collision of two particle takes place.
IV. CENTER-OF-MASS ENERGY OF TWO PARTICLES IN THE ROTATING
ABG SPACETIME

After calculating the equations of motion of a particle, and checked the conditions for the
collision of two different mass particles. Let us consider the two particles with rest
masses $m_1$ and $m_2$ ($m_1 \neq m_2$) moving in the equatorial plane ($\theta = \pi/2$) of rotating ABG
black hole. These particles are coming from rest at infinity towards the black hole and collide
in the vicinity of an event horizon. The collision takes place in the center-of-mass frame.
The two particles which are involved in the collision have different angular momentum, i.e.,
$L_1, L_2$ and energy $E_1, E_2$. The four-momentum of the $i^{th}$ particle is given by [1]

$$ p_i^\mu = m_i u_i^\mu, \quad (16) $$

where $i = 1, 2$ and $m_i$ and $u_i^\mu$ are corresponding to the rest mass and four-velocity of the $i^{th}$
particle. The total four-momentum of two colliding particles is

$$ P_i^\mu = P_{(1)}^\mu + P_{(2)}^\mu. \quad (17) $$

Hence, the $E_{CM}$ for the collision of two different mass particles is given by [33]

$$ E_{CM}^2 = -P_i^\mu P_{i\mu} = -(P_{(1)}^\mu + P_{(2)}^\mu)(P_{(1)\mu} + P_{(2)\mu}) $$

$$ = - \left( m_1 u_{(1)}^\mu + m_2 u_{(2)}^\mu \right) \left( m_1 u_{(1)\mu} + m_2 u_{(2)\mu} \right). \quad (18) $$

After simplifying and using the condition $u_{(1)}^\mu u_{(i)\mu} = -1$, in Eq. (18), we obtain the formula
for $E_{CM}$ is

$$ \frac{E_{CM}^2}{2m_1 m_2} = 1 + \frac{(m_1 - m_2)^2}{2m_1 m_2} - g_{\mu\nu} u_{(1)}^\mu u_{(2)}^\nu. \quad (19) $$

By substituting the values of $g_{\mu\nu}, u_{(1)}^\mu$ and $u_{(2)}^\nu$ from Eqs. (4), (9), (10), and (11) into Eq. (19),
therefore the $E_{CM}$ of rotating ABG black hole takes the following form:

$$ \frac{E_{CM}^2}{2m_1 m_2} = \left( \frac{m_1 - m_2}{2m_1 m_2} \right)^2 + \frac{1}{r^2 (r^2 f(r) + a^2)} \left[ a f(r) - 1 \right] \left[ L_1 E_2 + L_2 E_1 \right] r^2 $$

$$ - a^2 (f(r) - 2 E_1 E_2 - 1) r^2 - L_1 L_2 f(r) r^2 + \left( E_1 E_2 + f(r) \right) r^4 $$

$$ - \sqrt{-r^2 \left[ (f(r) - 2 E_1^2 + m_1^2) a^2 - 2a f(r) - 1 \right] E_1 L_1 - r^2 E_1^2 + f(r) (L_1^2 + r^2 m_1^2) } $$

$$ \times \sqrt{-r^2 \left[ (f(r) - 2 E_2^2 + m_2^2) a^2 - 2a f(r) - 1 \right] E_2 L_2 - r^2 E_2^2 + f(r) (L_2^2 + r^2 m_2^2) }. \quad (20) $$
that the change. We analyze numerically the behavior of \( E \) for two different massive particles.

We have considered the motion of the particles in equatorial plane \((\theta = \pi/2)\), where \( f(r, \theta) = f(r) \). It is clear from Eq. (20), that \( E_{CM} \) depends on charge \( Q \) as well as rotation parameter \( a \). If we change the value of charge \( Q \) or rotation parameter \( a \), then the \( E_{CM} \) must be change. We analyze numerically the behavior of \( E_{CM} \) with radius \( r \). We plot Eq. (20) for different combinations of rotation parameter \( a \), electric charge \( Q \), angular momentum \( L_1 \) and \( L_2 \), and colliding particle masses \( m_1 \) and \( m_2 \). We observe from the Fig. 4 that the \( E_{CM} \) would be very large if one of the colliding particle has critical value of angular momentum and collision takes place in the vicinity of the event horizon. Keep in mind that Eq. (20) is valid for two different massive particles.

If we consider the limit \( Q \to 0 \), then it follows from Eq. (20) that

\[
\frac{E_{CM}^2}{2m_1m_2} (Q \to 0) = \frac{(m_1 - m_2)^2}{2m_1m_2} + \frac{1}{r(r^2 - 2Mr + a^2)} \left[ a^2((2M + r)E_1E_2 + r) \right. \\
-2aM(L_1E_2 + L_2E_1) - L_1L_2(-2M + r) + (-2M + r(1 + E_1E_2))r^2 \\
-\sqrt{r(r^2 + a^2)(E_1^2 - m_1^2)} + 2M(aE_1 - L_1)^2 - L_1^2r + 2Mr^2m_1^2 \\
\times \sqrt{r(r^2 + a^2)(E_2^2 - m_2^2)} + 2M(aE_2 - L_2)^2 - L_2^2r + 2Mr^2m_2^2 \right], \tag{21}
\]
ABG black hole spacetime is the generalization of colliding particles masses \( m \). Hence, we can say that the \( E \) and inner horizon (\( r_{EH}^- \)) of extremal black hole, i.e., 
\[ \frac{E^2_{CM}}{2m_0^2} = \frac{1}{r^2(r^2 f(r) + a^2)} \left[ a(f(r) - 1)(L_1 + L_2)r^2 - a^2(f(r) - 3)r^2 - L_1 L_2 f(r)r^2 \right. 
\left. + (1 + f(r))r^4 - \sqrt{-r^2[(f(r) - 1)a^2 - 2a(f(r) - 1)L_1 - r^2 + f(r)(L_1^2 + r^2)]} \times \sqrt{-r^2[(f(r) - 1)a^2 - 2a(f(r) - 1)L_1 - r^2 + f(r)(L_1^2 + r^2)]} \right], \] (22)
which is similar to the \( E_{CM} \) of rotating ABG black hole for two equal mass particles [20]. Again, if \( Q \to 0 \) and \( m_1 = m_2 = m_0 \), and \( E_1 = E_2 = E = 1 \), then Eq. (20) transform into the \( E_{CM} \) of Kerr black hole [1]:
\[ E^2_{CM}(Q \to 0) = \frac{2m_0^2}{r(r^2 - 2a^2)} \left[ 2a^2(1 + r) - 2a(L_1 + L_2) - L_1 L_2(-2 + r) + 2(-1 + r)r^2 \right. 
\left. - \sqrt{2(a - L_1)^2 - L_1^2r + 2r^2} \sqrt{2(a - L_2)^2 - L_2^2r + 2r^2} \right]. \] (23)

Hence, we can say that the \( E_{CM} \) of two different mass particles in the background of rotating ABG black hole spacetime is the generalization of \( E_{CM} \) of Kerr black hole. Now we discuss the \( E_{CM} \) of two colliding particles for extremal and nonextremal black hole.

We are interested in the near horizon collision of two different mass particles in case of extremal black hole, i.e., \( r \to r_{ex} \). We analyze the behavior of \( E_{CM} \) with radius \( r \),
FIG. 6: Plots showing the behavior of $E_{CM}$ vs $r$ for different values of $Q$ for a rotating ABG black hole with colliding particles masses $m_1 = 1, m_2 = 2$. $r = r_{ex}$ correspond to the horizon for the extremal cases.

and plot it in Fig. 4, which show that the $E_{CM}$ due to the collision of two different mass particles is infinite when one of the particles have a critical angular momentum (for a suitable choice of rotation parameter $a$ and charge $Q$). From the graphical representation of $E_{CM}$, we have seen that $E_{CM}$ is infinite for $L_1 = 2.0, 2.16152, 2.30749, 2.55058$ corresponding to $Q = 0.0, 0.25, 0.35, 0.45$ and for all other values of angular momentum, it remains finite.

After studied the near horizon collision in extremal case, now we extend it for nonextremal black holes. We can see the behavior of $E_{CM}$ vs $r$ for nonextremal black hole from Fig. 5 for different charge $Q$ and different mass. From Fig. 5, we can conclude that $E_{CM}$ would be infinite, if collision takes place at the inner horizon of the nonextremal black hole. If the particles collide at the outer horizon of the black hole, then the $E_{CM}$ will remain finite. So, for getting a very high energy, particle collision should happen at the inner horizon. Also, in Fig. 6, we have shown the effect of $Q$ on the $E_{CM}$.

V. CONCLUSION

The gravitational collapse of sufficiently massive star leads to the formation of spacetime singularities is a quite common phenomenon in general relativity as predicated by famous singularity theorems [42]. However, there is a belief that these spacetime singularities do not exist in Nature, but that they are artefact of the classical general relativity. On the other hand, the cosmic censorship conjecture asserts that these singularities can not be seen by an external observer [43]. However, the conjecture does not ruled out the possibility of
regular black holes. In this paper, we consider the rotating ABG black holes, which can be written in Kerr-like form in Boyer-Lindquist coordinates with mass $M$ and has an additional parameter charge ($Q$) that measures potential deviations from the Kerr metric and includes the Kerr metric as the special case in the absence of the charge ($Q = 0$). This special solution is a solution of general relativity coupled to nonlinear electrodynamics that is of Petrov type D and it is singularity free. We have studied in detailed the horizon properties including the ergoregion of rotating ABG black hole and also show how the charge parameter affects the ergoregion. It turns out that area of ergoregion depends both on the value of charge $Q$ and parameter $a$, and the area of ergoregion increase with either of the two parameters.

We have also studied the collision of two different rest masses particles in the equatorial plane of rotating ABG black holes and also obtained an expression for the $E_{CM}$ for the particles. We demonstrate that the $E_{CM}$ depends not only on the rotation parameter $a$ but also on the charge $Q$ of the black holes. This work is the generalization of a previous work [20], where the analysis was restricted to the particles of the same rest mass particles moving in equatorial plane. Applying this general expressions, we realize that an infinite amount of energy can be obtained when the collision of the particles takes place in the vicinity of an event horizon of an extremal black hole and also on the inner horizon for the nonextremal black hole. Furthermore, we analyze the dependence of $E_{CM}$ on charge $Q$, and explicitly show the effect of $Q$ on the $E_{CM}$. Thus, we have estimated the $E_{CM}$ of two unequal mass particles for both the extremal and the nonextremal ABG black holes when one particle has the critical angular momentum. Furthermore, we analyze the dependence of $E_{CM}$ on charge $Q$.

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[1] M. Banados, J. Silk and S. M. West, Phys. Rev. Lett. 103, 111102 (2009).
[2] R. P. Kerr, Phys. Rev. Lett. 11, 237 (1963).
[3] E. Berti, V. Cardoso, L. Gualtieri, F. Pretorius and U. Sperhake, Phys. Rev. Lett. 103, 239001 (2009).
[4] T. Jacobson and T. P. Sotiriou, Phys. Rev. Lett. 104, 021101 (2010).
[5] K. Lake, Phys. Rev. Lett. 104, 211102 (2010).
[6] S.W. Wei, Y.X. Liu, H.T. Li and F.W. Chen, J. High Energy Phys. 12, 066 (2010).
[7] P. J. Mao, R. Li, L. Y. Jia and J. R. Ren, Chin. Phys. C 41, No. 6, 065101 (2017).
[8] A. A. Grib and Y. V. Pavlov, Astropart. Phys. 34, 581 (2011).
[9] C. Liu, S. Chen, C. Ding and J. Jing, Phys. Lett. B 701, 285 (2011).
[10] A. A. Grib and Y. V. Pavlov, Grav. Cosmol. 17, 42 (2011).
[11] Y. Zhu, S.-F. Wu, Y.-X. Liu and Y. Jiang, Phys. Rev. D 84, 043006 (2011).
[12] I. Hussain, Mod. Phys. Lett. A 27, 1250068 (2012).
[13] J.L. Said, K.Z. Adami, Phys. Rev. D 83, 104047 (2011).
[14] A.A. Grib, Y.V. Pavlov, JETP letters, 92, 125 (2010).
[15] T. Igata, T. Harada and M. Kimura, Phys. Rev. D 85, 104028 (2012).
[16] O. B. Zaslavskii, Phys. Lett. B 712, 161 (2012).
[17] Harada T. and Kimura M., Phys. Rev. D 83, 084041 (2011).
[18] S.W. Wei, Y.X. Liu, H. Guo, and C.E. Fu, Phys. Rev. D 82, 103005 (2010).
[19] P. Pradhan, arXiv:1402.2748.
[20] S. G. Ghosh, P. Sheoran and M. Amir, Phys. Rev. D 90, 103006 (2014).
[21] M. Amir and S. G. Ghosh, JHEP 1507, 015 (2015).
[22] S. G. Ghosh and M. Amir, Eur. Phys. J. C 75, 553 (2015).
[23] N. Tsukamoto, M. Kimura and T. Harada, Phys. Rev. D 89, 024020 (2014).
[24] M. Patil, P.S. Joshi, Phys. Rev. D 82, 104049 (2010).
[25] M. Patil, P.S. Joshi, D. Malafarina, Phys. Rev. D 83, 064007 (2011).
[26] M. Patil, P.S. Joshi, Phys. Rev. D 86, 044040 (2012).
[27] M. Patil and P. S. Joshi, Class. Quant. Grav. 28, 235012 (2011).
[28] M. Patil, P. S. Joshi, M. Kimura and K. I. Nakao, Phys. Rev. D 86, 084023 (2012).
[29] O. B. Zaslavskii, Phys. Rev. D 82, 083004 (2010).
[30] O. B. Zaslavskii, Class. Quant. Grav. 28, 105010 (2011).
[31] C. Liu, S. Chen and J. Jing, Chin. Phys. Lett. 30, 100401 (2013).
[32] M. Amir, F. Ahmed and S. G. Ghosh, Eur. Phys. J. C 76, no. 10, 532 (2016).
[33] T. Harada and M. Kimura, Phys. Rev. D 83, 024002 (2011).
[34] O. B. Zaslavskii, Phys. Rev. D 84, 024007 (2011).
[35] C. Bambi, Mod. Phys. Lett. A 26, 2453 (2011).
[36] C. Bambi, Astron. Rev. 8, 4 (2013).
[37] C. Bambi and L. Modesto, Phys. Lett. B 721, 329 (2013).
[38] E. Ayon-Beato and A. Garcia, Phys. Rev. Lett. 80, 5056 (1998).
[39] B. Toshmatov, B. Ahmedov, A. Abdujabbarov and Z. Stuchlik, Phys. Rev. D 89, no. 10, 104017 (2014).
[40] K. Schwarzschild, Math. Phys. Tech., 189, (1916).
[41] R. Penrose and R. M. Floyd, Nature 229, 177 (1971).
[42] S.W. Hawking and G.F.R. Ellis, Cambridge University Press, Cambridge, 1973.
[43] R. Penrose, Riv. Nuovo Cimento 1, 252 (1969); in General Relativity, an Einstein Centenary Volume, edited by S. W. Hawking and W. Israel (Cambridge University Press, Cambridge, England, 1979).