Abstract

Capsule Networks ambition is to build an explainable and biologically-inspired neural network model. One of their main innovations relies on the routing mechanism which extracts a parse tree: its main purpose is to explicitly build relationships between capsules. However, their true potential in terms of explainability has not surfaced yet: these relationships are extremely heterogeneous and difficult to understand.

This paper proposes REM, a technique which minimizes the entropy of the parse tree-like structure, improving its explainability. We accomplish this by driving the model parameters distribution towards low entropy configurations, using a pruning mechanism as a proxy. We also generate static parse trees with no performance loss, showing that, with REM, Capsule Networks build stronger relationships between capsules.

1. Introduction

Capsule Networks (CapsNets) [6,9,21] were recently introduced to overcome the shortcomings of Convolutional Neural Networks (CNNs). CNNs loose the spatial relationships between its parts because of max pooling layers, which progressively drop spatial information [21]. Furthermore, CNNs are also commonly known as “black-box” models: most of the techniques providing interpretation over the model are post-hoc: they produce localized maps that highlight important regions in the image for predicting objects [22]. CapsNets attempt to preserve and leverage an image representation as a hierarchy of parts, carving-out a parse tree from the networks. This is possible thanks to the iterative routing mechanism [21] which models the connections between capsules. This can be seen as a parallel attention mechanism, where each active capsule can choose a capsule in the layer above to be its parent in the tree [21]. Therefore, CapsNets can produce explainable representations encoded in the architecture itself [21] yet can be still successfully applied to a number of applicative tasks [1, 17, 26].

However, understanding what really happens inside a CapsNet is still an open challenge. Indeed, for a given input image, there are too many active (and each-other coupled) capsules, making the routing algorithm connections still difficult to understand, as the coupling coefficients typically have each-other similar values (Fig. 1), non exploiting the routing algorithm potential and making architectures without routing performing comparably [3]. Furthermore, backward and forward passes of a CapsNet come at an enormous computational cost, since the number of trainable parameters is very high. For example, the CapsNet model deployed on the MNIST dataset by Sabour et al. [21] is composed by an encoder and a decoder part. The full architecture has 8.2M of parameters. Do we really need that amount of trainable parameters to achieve competitive results on such
a task? Recently, many pruning methods were applied to CNNs in order to reduce the complexity of the networks, enforcing sparse topologies [13, 14, 24]: is it possible to tailor one of these approaches with not only the purpose of lowering the parameters, but aiding the model’s interpretability?

This work introduces REM (Routing Entropy Minimization) for CapsNets, which aims to improve the explainability of the routing algorithm of CapsNets. Pruning can effectively reduce the overall entropy of the connections of the parse tree-like structure encoded in a CapsNet, because in low pruning regimes it removes noisy couplings which cause the entropy to increase considerably. We collect the coupling coefficients studying their frequency and cardinality, observing lower per-class conditional entropy on these: the pruned version adds a missing explicit prior in the routing mechanism, grounding the coupling of the unused primary capsules disallowing fluctuations under the same baseline performance on the validation/test set. This implies that the parse trees are more stable for the pruned models, providing higher explainability on the relevant features (per-class) selected from the routing mechanism.

This paper is organized as follows: in Section 2 we introduce some of the basic concepts of CapsNets and their related works, in Section 3 we describe our technique called REM, in Section 4 we investigate the effectiveness of our method testing it on many datasets. The last two sections discuss the limitations of our approach and the conclusion of our work.

2. Background and Related Work

This section first describes the fundamental aspects of CapsNets and their routing algorithm introduced by [21]. Then, we review the literature especially related to how introduce sparsity into CapsNets. The notation used in our work is summarized in Table 1.

Capsule Networks Fundamentals. CapsNets group neurons into capsules, namely activity vectors, where each capsule accounts for an object of one of its parts. Each element of these vectors accounts for different properties of the object such as its pose and other properties like color, deformation, etc. [21]. The magnitude of a capsule stands for the probability of existence of that object in the image [21]. Fig. 2 shows a standard architecture of a CapsNet with two capsule layers, PrimaryCaps and DigitCaps (also called OutputCaps). The poses of L-th capsules \( u_i \), called primary capsules, are built upon convolutional layers. In order to compute the poses of the capsules of the next layer \( L + 1 \), an iterative routing mechanism is performed. Each capsule \( u_i \) makes a prediction \( \hat{u}_{ij} \), thanks to a transformation matrix \( W_{ij} \), for the pose of an upper layer capsule \( j \)

\[
\hat{u}_{ij} = W_{ij} u_i. \tag{1}
\]

Then, the total input \( s_j \) of capsule \( j \) of the DigitCaps

![Figure 2. Capsule Network architecture (encoder network).](image-url)
layer is computed as the weighted average of votes \( \hat{u}_{j|i} \)

\[
s_j = \sum_i c_{ij} \hat{u}_{j|i}, \tag{2}
\]

where \( c_{ij} \) are the coupling coefficients between a primary capsule \( i \) and an output capsule \( j \). The pose \( v_j \) of an output capsule \( j \) is then defined as the normalized “squashed” \( s_j \)

\[
v_j = \text{squash}(s_j) = \frac{||s_j||^2}{1 + ||s_j||^2} s_j. \tag{3}
\]

So the routing algorithm computes the poses of output capsules and the connections between capsules of consecutive layers. The coupling coefficients are computed dynamically by the routing algorithm and they are dependent on the input \([21]\). The coupling coefficients are determined by a “routing softmax” activation function, whose initial logits \( b_{ij} \) are the log prior probabilities the \( i \)-th capsule should be coupled to the \( j \)-th one

\[
c_{ij} = \text{softmax}(b_{ij}) = \frac{e^{b_{ij}}}{\sum_k e^{b_{ik}}} \tag{4}
\]

At the first step of the routing algorithm they are equals and then they are refined by measuring the agreement between the output \( v_j \) of the \( j \)-th capsule and the prediction \( \hat{u}_{j|i} \) for a given input. The agreement is defined as the scalar product \( a_{ij} = v_j \cdot \hat{u}_{j|i} \). At each iteration, the update rule for the logits is

\[
b_{ij} \leftarrow b_{ij} + a_{ij}. \tag{5}
\]

The steps defined in (2), (3), (4), (5) are repeated for the \( t \) iterations of the routing algorithm. In order to train the network, Sabour et al. [21] replaced the cross entropy loss with the \textit{margin loss}.

\[ \text{2.1. Related work} \]

\textbf{Capsule Networks.} They were first introduced by Sabour et al. [21] and since then a lot of work has been done, both to improve the routing mechanism and to build deeper models. Regarding the routing algorithm, Hinton et al. [6] replace the dynamic routing with Expectation-Maximization, adopting matrix capsules instead of vector capsules. Wang et al. [25] model the routing strategy as an optimization problem. Li et al. [11] use master and aide branches to reduce the complexity of the routing process. Hahn et al. [5] incorporates a self-routing method such that capsule models do not require agreements anymore. De Sousa Ribeiro et al. [2] replace the routing algorithm with variational inference of part-object connections in a probabilistic capsule network, leading to a significant speedup without sacrificing performance. Since the CapsNet model introduced by Sabour et al. [21] is a shallow network, several works attempted to build deep CapsNets. Rajasegaran et al. [18] propose a deep capsule network architecture which uses a novel 3D convolution based dynamic routing algorithm aiming at improving the performance of CapsNets for more complex image datasets. Guggiferger et al. [4] introduce residual connections to train deeper capsule networks.

\textbf{Sparse Capsule Networks.} A naive solution to reduce uncertainty within the routing algorithm is to simply run more iterations. As shown by Paik et al. [16] and Gu et al. [3], the routing algorithms tends to overly polarize the link strengths, namely a simple route in which each input capsule sends its output to only one capsule and all other routes are suppressed. This is because it makes the routing algorithm more explainable, by making it possible to extract a parse tree thanks to this coupling coefficients. On the other hand, running many iterations is only useful in the case of networks with few parameters, as demonstrated by Renzulli et al. [20], otherwise the performance will drop. Rawlinson et al. [19] trained CapsNets in an unsupervised setting, showing that in unsupervised learning the routing algorithm does not discriminate among capsule anymore: the coupling coefficients collapse to the same value. Therefore, they sparsify latent capsule layers activities by masking output capsules according to a custom ranking function. Kosierok et al. [9] impose sparsity and entropy constraints into capsules, but they do not employ an iterative routing mechanism. Jeong et al. [7] introduced a structured pruning layer called ladder capsule layers, which removes irrelevant capsules, namely capsules with low activities. Kaklioglu et al. [8] solve the task of 3D object classification on point clouds with pruned Capsule Networks. Their objective was to compress robust capsule models in order to deploy them on resource-constrained devices.

The main contribution of our work relies on the fact that we regularize and prune the parameters in a CapsNet as a way to minimize the entropy of the connections computed by the routing algorithm. In fact, we show that relationships between objects and their parts in a standard CapsNets described by Sabour et al. [21] have high entropies. We minimize these entropies so that we can extract stronger parse trees. This allows us to effectively build dictionaries upon the input datasets and understand which are the shared object parts and transformations between different the entities in the images.

\[ \text{3. Routing Entropy Minimization} \]

The coupling coefficients computed by the routing mechanism model the part-whole relationships between capsules of two consecutive capsule layers, as displayed in Fig. 1. Assigning parts to objects (namely learning how
each object is composed), is a challenging task. One of the main goal of the routing algorithm is to extract a parse tree of these relationships. Given the \( \xi \)-th input of class \( j \), an ideal parse tree for a primary capsule \( i \) detecting one of the parts of the entity in the input \( \xi \) would ideally lead to

\[
\tilde{c}^\xi_{i-} = 1_{\tilde{y}^\xi}.
\]

This means that the routing process is able to carve a parse tree out of the CapsNet which explains perfectly the relationships between parts and wholes. One of the problems of this routing procedure is that there is no constraint on how strong a parse tree should be. In this section we present our technique REM, first showing how to extract a parse tree and then how to build stronger parse trees. The pipeline of our method is depicted in Fig. 3.

### 3.1. Parse trees extraction

Once we have a trained CapsNets model, in order to interpret the routing mechanism, we extract all the possible routing coupling coefficients and build a parse tree. Towards this end, we want to define a metric which helps us deciding if the relationships captured by the routing algorithm resemble or not a parse tree. Therefore, we organize the coupling coefficients into associative arrays, we can compute the number of occurrences of each coupling sequence in order to compute the entropy of the whole coupling dictionary to measure the simplicity of the parse tree. In the next paragraphs, we explain how to generate these sequences by discretizing the coupling coefficients and how to create the dictionary.

**Quantization.** During the quantization stage, we first compute the continuous coupling coefficients \( \tilde{c}^\xi_{ij} \) for each \( \xi \)-th input example. It should be noticed that \( \tilde{c}^\xi_{ij} \) are the coupling coefficients obtained after the forward pass of the last routing iteration. Then, we quantize every \( \tilde{c}^\xi_{ij} \) into \( K \) discrete levels through the uniform quantizer \( q_K(\cdot) \), obtaining

\[
\tilde{c}^\xi_{ij} = q_K(\tilde{c}^\xi_{ij}).
\]

We choose the lowest \( K \) such that the accuracy is not deteriorated. We will here on refer to CapsNet+Q as trained CapsNet where the coupling coefficients are quantized.

**Extracting the parse tree.** Given the quantized coupling coefficients of a CapsNet+Q, we can extract the parse tree (and create a dictionary of parse trees) for each class \( j \), where each entry is a string composed by the quantization indices of the coupling coefficients. We will extract the coupling coefficients \( \tilde{c}^\xi_{ij} \) between the primary capsules \( I \) and the predicted \( j \)-th output capsule. Given a dictionary for the coupling coefficients of a CapsNet+Q, we can compute the entropy for each class as

\[
H_j = -\sum_{\xi} \mathbb{P}(\tilde{c}^\xi_{ij} | y^\xi = j) \cdot \log_2 \left[ \mathbb{P}(\tilde{c}^\xi_{ij} | y^\xi = j) \right].
\]

where \( \mathbb{P}(\tilde{c}^\xi_{ij} | y^\xi = j) \) is the frequency of occurrences of a generic string \( \xi \) for each predicted class \( y^\xi \). Finally, the entropy of a dictionary for a CapsNet+Q on a given dataset is the average of the entropies \( H_j \) of each class

\[
H = \frac{1}{J} \sum_j H_j.
\]

Intuitively, the lower (9), the stronger the connections between capsules are in the parse tree-like structure carved-out from the routing algorithm. We also target to obtain the distribution of these coupling coefficients. In general, we know we have \( \Xi \times I \times J \) coupling coefficients for the full dataset (with potential redundancies). Given the \( i \)-th primary capsule, however, we are only interested to \( c^\xi_{ij} | y^\xi = j \). In this way, we reduce the coupling coefficients space to \( I \times J \). We compute then the average of all the inputs belonging to an object class in order to output just \( I \times J \) coupling coefficients.

**Preliminary result.** Let us now generate the parse trees on the CapsNet model trained on the MNIST dataset as in [21], investigating their evolution during training. To this end, we compute the distribution of the coupling coefficients on the test set for two models: the model trained...
3.2. Unconstrained routing entropy

In this subsection we are going to more-formally analyze the distribution of the coupling coefficients

$$c_{ij} = \frac{e^{b_{ij} + \sum_{r=1}^{R} v_{r} u_{r} W_{ij}}}{\sum_{k} e^{b_{ik} + \sum_{r=1}^{R} v_{r} u_{r} W_{ik}}}$$  \hspace{1cm} (10)

where $t$ indicates the target routing iterations.\footnote{For abuse of notation, in this subsection we suppress the index $\xi$.} Let us evaluate the $c_{ij}$ over a non-yet trained model: as we saw also in Section 3.1, we have

$$c_{ij} \approx \frac{1}{j} \forall i, j.$$  \hspace{1cm} (11)

When updating the parameters, following [3], we have

$$\frac{\partial L}{\partial W_{ij}} = \left[ \frac{\partial L}{\partial v_{j}} \right] c_{ij} + \sum_{m=1}^{M} \left( \frac{\partial L}{\partial v_{m}} \right) \hat{u}_{m|i} \left( \frac{\partial c_{im}}{\partial u} \right) \cdot u_{i}$$  \hspace{1cm} (12)

where we can have the gradient for $W_{ij} \approx 0$ in a potentially-high number of scenarios, despite $c_{ij} \neq \{0, 1\}$. Let us analyze the simple case in which we have perfect outputs, matching the ground truth, hence we are close to a local (or potentially the global) minimum of the loss function:

$$\left\| \frac{\partial L}{\partial v_{m}} \right\|_{2} \approx 0 \forall m.$$  \hspace{1cm} (13)

Looking at (4), we see that the right class is chosen, but given the squashing function, we have as an explicit constraint that, given the $j$-th class as the target one, we require

$$\|v_{j}\|_{2} \gg \|v_{m}\|_{2} \forall m \neq j$$  \hspace{1cm} (14)

on the $W_{ij}$, which can be accomplished in many ways, including:

- having sparse activation for the primary capsules $u_{i}$; in this case, we have constant $W_{ij}$ (typically associated to no-routing based approaches); however, we need heavier deep neural networks as they have to force sparse signals already at the output of the primary capsules. In this case, the coupling coefficients $c_{ij}$ are also constant by definition;

- having sparse votes $\hat{u}_{ij}$: this is a combination of having both primary capsules and weights $W_{ij}$ enforcing sparsity in the votes, and the typical scenario with many routing iterations.

Having sparse votes, however, does not necessarily result in having sparse coupling coefficients: according to (5), the coupling coefficients are multiplied with the votes, obtaining the output capsules. The distribution of the coupling coefficients requires (14) to be satisfied only: if $W_{ij}$ is not sparsely distributed, we can still have sparse votes. However, this is the main reason we observe high entropy in the coupling coefficients distributions (Fig. 4): as the votes $\hat{u}_{ij}$ are implicitly sparse (yet also disordered, as we are not explicitly imposing any structure in the coupling coefficients, etc.).
distribution, the model is still able to learn but it finds a typical solution where $c_{ij}$ are not sparse. However, we would like to have sparsely distributed, recurrent $c_{...}$, establishing stable relationships between the features extracted at primary capsules layer.

Minimizing explicitly the entropy term (8) is an intractable problem due to the non-differentiability of the entropy term and of the quantization step (in our considered setup) and due to the huge computational complexity to be introduced at training time. Hence, we can try to implicitly enforce routing entropy minimization by forcing a sparse and organized structure in the coupling coefficients. Towards this end, one efficient solution relies in enforcing sparsity in the $W_{ij}$ representation: enforcing a vote between the $i$-th primary capsule and the $j$-th output caps to be exactly zero for any input, according to (10)

$$c_{ij} = \frac{1}{\sum_k e^{b_{ik}} + \sum_r v_{ir} w_{rk}}.$$  

In this way, having a lower variability in the $c_{ij}$ values (and hence building more stable relationships between primary and output capsules), straightforwardly we are also explicitly minimizing the entropy of the quantized representations for the coupling coefficients. In the next subsection, we are going to tailor a sparsity technique to accomplish such a goal.

### 3.3. Enforcing REM with pruning

CapsNets are trained via standard back-propagation learning, minimizing some loss function like margin loss. Our ultimate goal is to assess to which extent a variation of the value of some parameter $\theta$ would affect the error on the network output. In particular, the parameters not affecting the network output can be pushed to zero in a soft manner, meaning that we can apply an $\ell_2$ penalty term. A number of approaches have been proposed, especially in the recent years [10, 12, 15]. One recent state-of-the-art approach, LOBSTER [23] proposes to penalize the parameters by their gradient-weighted $\ell_2$ norm, leading to the update rule

$$\theta^{t+1} = \theta^t - \eta G \left[ \frac{\partial L}{\partial \theta^t} \right] - \lambda \theta^t \text{ReLU} \left[ 1 - \left| \frac{\partial L}{\partial \theta^t} \right| \right],$$  

where $G \left[ \frac{\partial L}{\partial \theta^t} \right]$ is any gradient-based optimization update (for SGD it is the plain gradient, but other optimization strategies like Adam can be plugged) and $\eta, \lambda$ are two positive hyper-parameters.

Such a strategy is particularly effective on standard convolutional neural networks, and easy to plug in any back-propagation based learning system. Furthermore, LOBSTER is a regularization strategy which can be plugged at any learning stage, as it self-tunes the penalty introduced according to the learning phase: for this non-intrusiveness in the complex and delicate routing mechanism for CapsNets, it resulted a fair choice to enforce REM.

### 4. Experiments and Results

In this section we report the experiments and the results that we performed to test REM. We first show the results on the MNIST dataset, reporting also how the entropy and the accuracy values change during training. Then, we test REM on more complex datasets such as Fashion-MNIST, CIFAR10 and Tiny ImageNet. We also performed experiments to test the robustness to affine transformations of CapsNets+REM. Except for Tiny ImageNet, we used the same architectures configurations and augmentations described in [21].² We trained models with five random seeds. We report the classification accuracy (%) and entropy (averages and standard deviations), the sparsity (percentage of pruned parameters, median) and the number of keys in the dictionary (median). The code will be open-source released upon acceptance of the paper, to guarantee the reproducibility of the results. The experiments were run on a Nvidia Ampere A40 equipped with 48GB RAM, and the code uses PyTorch 1.9.

#### 4.1. Preliminary results on MNIST

In order to assess our REM technique, we analyze in-depth the benefits of pruning towards REM on the MNIST dataset. Nowadays, despite its outdatedness, MNIST remains an omni-present benchmark for CapsNets [3, 19, 21]. Fig. 5 shows how the entropy (red line) and classification accuracy (blue dotted line) changes as the sparsity increases during training. We can see that at the beginning of the training stage the entropy is low because the routing algorithm has not learned yet to correctly discriminate the relationships between the capsules; but at the end of the training process REM lowers the entropy of the network. In Fig. 6 we plot the distributions of the coupling coefficients for a CapsNet+Q and a CapsNet+REM following the method described in Sec. 3.1. We can see that the distributions of the CapsNet+REM model are sparser that the ones for the

²We have removed the decoder part of the network, see Sec. 4.3 for more details.
Table 4. Results for CapsNets, CapsNets+Q, CapsNets+REM on MNIST (test set).

| Model  | Accuracy       | Sparsity |
|--------|----------------|----------|
| CapsNet+Q | 99.56±0.0003 | 85.53    |
| CapsNet+REM | 99.56±0.0002 | 85.53    |

Table 5. (Quantized) CapsNets number of keys for each class on MNIST (test set).

| Model     | Sparsity | #0 | #1 | #2 | #3 | #4 | #5 | #6 | #7 | #8 | #9 | Entropy       |
|-----------|----------|----|----|----|----|----|----|----|----|----|----|---------------|
| CapsNet+Q | 85.53    | 40 | 46 | 15 | 39 | 12 | 117| 182| 14 | 310| 9  | 4.16±1.59     |
| CapsNet+REM | 85.53    | 40 | 46 | 15 | 39 | 12 | 117| 182| 14 | 310| 9  | 4.16±1.59     |

Figure 5. Accuracy and entropy curves vs pruned parameters on MNIST (test set).

Figure 6. Coupling coefficients distributions for each class of a CapsNets+Q and a CapsNet+REM on MNIST (test set).

CapsNet+Q model, namely we carve a stronger parse tree while achieving high generalization. Table 4 shows that there is no performance loss when pruning a CapsNet, even when REM is applied with its quantization levels. Table 5 shows the number of keys in each class for CapsNets+REM and a CapsNets+Q, namely a CapsNet where the quantization is applied without pruning the network during training. We also report the number of keys for each class and the entropy of the dictionary for the same quantized models shown in Table 4. We can see that the dimension of the dictionary for CapsNets+REM is lower than the one for CapsNets+Q. Also the entropy measure for CapsNets+REM is lower compared to CapsNets+Q, namely, REM has successfully built a parse tree between primary and class capsules with stronger relationships. We now show how we can improve the explainability of such connections. Each key can be seen as a cluster of images with shared features or parts. For each key of the predicted class, we take all the images of the input dataset that belong to that key, sorted according to the size of that cluster. Then, we overlap all the images in a cluster. Figure 7 depicts a visual representation of some of these clusters for a subset of object classes. We can see that a CapsNet+REM can not only predict which digit is present in an input image, but it can also show which are the shared object parts (such as different localized parts for the digit 7 and 9) and their transformations (rotation and translation for the digit 0) in a cluster.

4.2. Results

We trained and tested CapsNets+REM on more complex datasets such as:

- Fashion-MNIST, 28x28 grayscale images (10 classes);
- CIFAR10, 32x32 RGB images (10 classes);
- Tiny ImageNet, 64x64 RGB images (200 classes).
Specific setup for Tiny ImageNet. In order to test also in transfer learning scenarios, we train CapsNets on the Tiny ImageNet dataset, using in this case only a pretrained ResNet18 model as backbone and fine-tuning the CapsNet parameters only. We used 10% of the training set as validation set and the original validation set as test set.

Discussion. As we can see in Tables 6, a CapsNet+REM has a high percentage of pruned parameters with a minimal performance loss. So this confirms our hypothesis that CapsNets are over-parametrized. Furthermore, for the Fashion-MNIST dataset the entropy of the CapsNet+REM is much lower that the CapsNet+Q, while for the other datasets the differences between the two approaches is less evident. This is an expected behavior: for CIFAR10 and Tiny ImageNet, both CapsNets achieved poor classification results and this introduces noise on the coupling coefficients, leading to higher uncertainty in the classification task hence in the features selection.

Robustness to affine transformations. To test the robustness to affine transformations of CapsNets+REM, we used expanded MNIST: a dataset composed by padded and translated MNIST, in which each example is an MNIST digit placed randomly on a black background of 40x40 pixels. We used the affNIST dataset as test set, in which each example is an MNIST digit with a random small affine transformation. We tested an under-trained CapsNet with early stopping which achieved 99.22% accuracy on the expanded MNIST test set as in [3, 21]. We also trained these models until convergence. We can see in Table 7 that the under-trained networks entropies are high. Instead, a well-trained CapsNet+REM can be robust to affine transformations and have a low entropy.

4.3. Limitation

A CapsNet is typically composed of an encoder and a decoder part, where the latter is a reconstruction network with 3 fully connected layers [21]. In the previously-discussed experiments, we have removed the decoder. The main limitation of our work arise when computing the entropy of CapsNets trained with the decoder. We observed that the entropy of a CapsNets+REM is almost the same as that of a CapsNet+Q shown in Tables 5 and 6. Indeed, when the decoder is used, the activity vector of an output capsule encodes richer representations of the input. Sabour et al. [21] introduced the decoder to boost the routing performance on MNIST by enforcing the pose encoding a capsule. They also show that, when a perturbed activity vector is feeded to the decoder, such perturbation affects the reconstruction. So capsules representations are equivariant, meaning that transformations applied to the input are described by continuous changes in the output vector. In order to verify if output capsules of a trained CapsNet+REM without the decoder (so with low entropy) are still equivariant, we stacked on top of it the reconstruction network, without training the encoder. We can see in Fig. 8 that the output capsules are still equivariant to many transformations so we successfully obtained a CapsNet+REM with low entropy but with equivariant representations encoded into capsules.

5. Conclusion

This paper investigated and improved the explainability of the routing algorithm in CapsNets with REM (Routing Entropy Minimization), which drives the model parameters...
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