A mathematical model for the temperature field of a ballistic gravimeter is developed based on a thermal conductivity boundary value problem to be solved by the finite element method. Genetic algorithms and the Nelder-Mead method are used to develop a way for synthesizing the parameters of electric heaters to reduce the temperature gradients inside the vacuum chamber of a ballistic gravimeter and to improve its technical parameters.

**Keywords:** ballistic gravimeter, temperature field, vacuum chamber, synthesis of parameters.

Increasing the accuracy of the symmetric ballistic gravimeter (BG) which serves as the primary standard DETU 02-02-96 for the acceleration of gravity is a pressing problem [1]. One important factor that influences the accuracy of the BG is a nonuniform distribution of the temperature inside the vacuum chamber (pressure $10^{-2}$ mmHg $\approx 1.33$ Pa), within which the major optical and mechanical processes involved in measuring the acceleration of gravity take place [2]. Temperature gradients inside the vacuum chamber produce additional forces which change the way the test object with the attached optical reflector moves when it is tossed upward by the catapult electromagnet and interacts with the interferometer optical system of the BG [3].

The design of the DETU 02-02-96 ballistic gravimeter, which affects its temperature state, includes a support flange for the chamber with a vacuum cavity (Fig. 1). The space between the outer case and the chamber is filled with plastic foam insulator. The upper and lower electric heaters are wrapped around the outer surface of the vacuum chamber. The electromagnet for the catapult and an electric heater are enclosed in a housing and attached to the lower end of the flange. A fused quartz window, through which the laser optical system interacts with the reflector on the test object, is mounted at the top of the vacuum chamber. All the electric heaters are impregnated with lacquer mixed with a special paste, coated with fiberglass, and connected to a thermostat for maintaining a chamber wall temperature of $40 \pm 2^\circ$C. The dimensions of the major components of the ballistic gravimeter are indicated in Fig. 1. The temperature inside the extended vacuum chamber, especially on its central axis, along which the test object moves, is not monitored and may differ from the temperature at the walls, where it can also vary axially and affect the accuracy of the BG.

Calculating the temperature field created by heat sources is important for many electrical engineering devices [4, 5]. The task of ensuring a specified magnitude and gradient of the temperature inside the vacuum chamber is more complicated, since it is then necessary to establish the thermal parameters and spatial position of the heaters taking the thermal and geometric parameters of the BG into account.

Here we report on the development of a method for selecting parameters of the electric heaters that will minimize the gradient for a given temperature level inside the BG vacuum chamber.

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Mathematical Model for the Temperature Field of the Ballistic Gravimeter. We use the field model of Ref. 4 for determining the temperature inside the BG vacuum chamber. Including both active (the electric heaters) and passive (without heat sources) components, the temperature field is described by the following system of equations:

\[
c_n(T) \gamma_n \frac{\partial T_n}{\partial t} = \lambda_n(T) \left( \frac{\partial^2 T_n}{\partial r^2} + \frac{\partial T_n}{\partial r} + \frac{\partial^2 T_n}{\partial z^2} \right) + j_n(t)k_p(T), \tag{1}
\]

\[
c_m(T) \gamma_m \frac{\partial T_m}{\partial t} = \lambda_m(T) \left( \frac{\partial^2 T_m}{\partial r^2} + \frac{\partial T_m}{\partial r} + \frac{\partial^2 T_m}{\partial z^2} \right), \tag{2}
\]

Fig. 1. Heating system for the DETU 02-02-96 ballistic gravimeter: 1, 6) housings; 2) flange; 3) vacuum cavity; 4) chamber; 5) fused quartz window; 7, 9) electric heaters; 8) thermal insulation; 10) electromagnet; 11) lacquer; 12) fiberglass.
where \( c(T) \) is the average specific heat; \( n \) and \( m \) are the subscripts for the active and passive components; \( \gamma \) is the average density of the material; \( \lambda(T) \) is the thermal conductivity; and \( j(t), k_z, \) and \( \rho(T) \) are the current density, filling coefficient, and specific resistance of the electric heaters.

The system of equations (1) and (2) is supplemented by boundary conditions \( f \) at the surfaces. On the outside surfaces, boundary conditions of the third kind are used which describe the heat transfer process with

\[
-\partial T_f / \partial n = \alpha(T_f - T_0) / \lambda,
\]

where \( \alpha \) is the heat transfer coefficient, and \( n \) is the normal to the surface.

At the contact surfaces of the components, boundary conditions of the fourth kind are used, with

\[
T_n(r_f, z_f, t) = T_m(r_f, z_f, t); \quad \lambda_n \partial T_n / \partial n = \lambda_m \partial T_m / \partial n.
\]

On the axis of symmetry of the BG, a boundary condition of the second kind is used, with

\[
(\partial T / \partial r)_f = 0.
\]

Equations (1)–(5) have been used to develop a computer program that uses the finite element method to calculate the temperature at each point of the BG. Curves 1 of Fig. 2 show the temperature distribution in the DETU 02-02-96. In all the heating elements the specific heat release rate is \( w = 65.6 \text{ kW/m}^3 \), which yields an average temperature \( T_{\text{avg}} = 313.02 \text{ K} \) inside the vacuum chamber. Table 1 lists the average temperatures \( T_{\text{avg}} \) of the electric heaters, their heating powers \( P \), and heat fluxes in the radial \( F_r \) and axial \( F_z \) directions.

In the electromagnet with a heating power of \( P = 36.75 \text{ W} \), the average temperature is \( T_{\text{avg}} = 314.0 \text{ K} \) with radial and axial heat fluxes of \( F_r = 541.7 \text{ W/m}^2 \) and \( F_z = 2460.7 \text{ W/m}^2 \). The lowest average temperature then appears in the support flange, with \( T_{\text{avg}} = 310.25 \text{ K} \), with corresponding radial and axial heat fluxes \( F_r = 1590.7 \text{ W/m}^2 \) and \( F_z = 248.1 \text{ W/m}^2 \). This temperature field leads to radial and axial temperature gradients \( G_r = -2.52 \text{ K/m} \) and \( G_z = 8.39 \text{ K/m} \), respectively, inside the vacuum chamber.

Therefore, in the vacuum chamber of a BG with this design and an outside air temperature of 25°C, the electric heaters produce a temperature field with insignificant radial gradients but large axial gradients. The lowest temperatures occur
at the top near the quartz window and at the bottom near the steel flange, while the highest temperatures occur in the mid-section of the vacuum chamber which is surrounded by electric heaters.

**Method for Selecting the Electric Heater Parameters.** We now examine a method for ensuring a minimum gradient along the axis for a given temperature inside the vacuum chamber of a BG by choosing the parameters of the electric heaters with minimization of the indices

\[
\Theta = \frac{T_s - T}{T_s}; \quad \vartheta = \frac{T_{\text{max}} - T_{\text{min}}}{\delta}; \quad N; \quad V,
\]

where \( T_s, T_{\text{max}}, \) and \( T_{\text{min}} \) are the specified, maximum, and minimum temperatures in the volume of the vacuum chamber; \( \delta = r, z \) is a spatial coordinate; and \( N \) and \( V \) are the number and volume of the electric heaters.

Parametric and functional constraints are chosen for solving the problems:

\[
N \rightarrow \min; \quad z_1 \leq z_0 \leq z_k \leq z_k^{\text{up}} < z_k^{\text{low}}, \quad \zeta_N^{\text{low}} \geq \zeta_0; \quad w_k \leq w_{\text{max}}; \quad k \in [1, N],
\]

where \( z_0^{\text{up}}, z_0^{\text{low}}, z_k^{\text{up}}, \) and \( z_k^{\text{low}} \) are, respectively, the upper and lower limits on the position of the electric heaters and of the \( k \)th heater along the axis of the vacuum chamber, and \( w_{\text{max}} \) is the maximum specific thermal output power of the heaters.

We use an hierarchical approach in which the highest priority is assigned to the criterion characterizing the degree of disruption of the search space, i.e., going beyond the limits \( \{a_i, b_i\} \) of the permitted boundaries of the vectors of the independent variables \( x_i \):

\[
U_1(x) = \sum_{i=1}^{5} (\max\{0, a_i - x_i\} + \max\{0, x_i - b_i\}).
\]

Next in importance is a criterion characterizing the deviation of the temperature from the specified value, i.e.,

\[
U_2(x) = \min \{\Theta(x)\},
\]

and then, a criterion characterizing the spatial gradient of the temperatures,

\[
U_3(x) = \min \{\vartheta(x)\}.
\]

The criteria determining the number and dimensions of the electric heaters follow with decreasing importance:

\[
U_4(x) = \min \{N(x)\};
\]

\[
U_5(x) = \min \{V(x)\}.
\]

The solution \( x = x^* \) is regarded as found if the following conditions are satisfied:

\[
U_1(x^*) = U_2(x^*) = U_3(x^*) = 0; \quad U_4(x^*) = \min f_1; \quad U_5(x^*) = \min f_2.
\]

### TABLE 1. Parameters of the Electric Heaters in the DETU 02-02-96 Ballistic Gravimeter

| Electric heater | \( T_{\text{avg}}, K \) | \( P, W \) | \( F_r, \text{W/m}^2 \) | \( F_v, \text{W/m}^2 \) |
|-----------------|-----------------|---------|----------------|---------|
| Top             | 316.62          | 7.78    | −18.9          | 269.6   |
| Bottom          | 316.98          | 7.78    | −23.8          | −207.2  |
| Electromagnet   | 313.59          | 2.80    | 67.0           | 1813.8  |
The strategy for finding the solution for \( m \) variables in the search space involves the combined use of a global optimization method for a random search for the parameters of the electric heaters in a specified space, while keeping it from falling into a local extremum, and a local method that involves reducing the size of the parameter regions with a global extremum to a minimum [6].

For global optimization, we use a genetic algorithm based on the methods of population genetics [7]. One gene in a genotype, i.e., a bit line of fixed length, corresponds to each attribute of an object in a phenotype. A trait is broken up into a tetrad that transforms as a Grey code. When coding a binary line of \( l \) bits of the variable \( x_k \) belonging to the segment \([x_{\text{min}}, x_{\text{max}}]\), each line \( s_k \) expresses a value of the variable

\[
x_k = x_{\text{min}} + s_k(x_{\text{max}} - x_{\text{min}})/2^l,
\]

where \( s_k \) is the value of the binary number coded by this line.

Genetic algorithms can be represented in the following way:

\[
\text{GA} = (P^0, m, l, S, Q, \eta, \xi),
\]

where \( P^0 = (a_1^0, ..., a_m^0) \) is the initial population; \( a_i^0 \) is the solution of the problem in the form of a chromosome; \( i = 1, m; m \) is the size of the population; \( l \) is the length of a chromosome; \( S \) is a selection operator; \( Q \) is a mapping that defines recombination (crossover, mutation); \( \eta \) is an optimality function; and \( \xi \) is the stopping criterion.

Genetic algorithms operate as an iterative process which continues until condition (13) is satisfied to the desired accuracy. The initial population \( P^0 \) is randomly generated (initial). In each iteration cycle selection, crossover and mutation, operators are applied. The selection operator \( S \) creates an intermediate population \( R^t \) from the population \( P^t \) by selection and the generation of new copies of the elements \( P^t \): \( R^t = S(P^t) \). The optimality function \( \eta \), which provides feedback from the result of the optimization during generation \( t \), is used to select individuals from the population. The selection is based on probabilities \( p_S(a_i^t) \) calculated for each individual:

\[
p_S(a_i^t) = \eta(a_i^t) / \sum_{j=1}^{m} \eta(a_j^t).
\]

After completion of the selection, a partner from \( R^t \) is chosen for the element \( a_i^t \in R^t \) for recombination and a new chromosome is constructed.

Crossover with probability \( p_k \) takes place in the following way:

- random choice of partners for crossing (hybridization)

\[
a_1 = (a_{1,1} ... a_{1,l}) \in R^t, \quad a_2 = (a_{2,1} ... a_{2,l}) \in R^t;
\]

- random choice of the crossover point \( x \in \{1, ..., l-1\} \);

- formation of two new individuals

\[
a_1' = (a_{1,1} ... a_{1,x}a_{2,x+1} ... a_{2,l}) \quad \text{and} \quad a_2' = (a_{2,1} ... a_{2,x}a_{1,x+1} ... a_{1,l}).
\]

Mutation is a random change in a bit of a chromosome:

- random choice with probability \( p_m \) of positions \( \{x_1, ..., x_k\} \subseteq \{1, ..., l\} \) inside a bit row \( a = (a_1 ... a_l) \in R^t \) subjected to mutation;

- formation of a new individual

\[
a = (a_1 ... a_{x_i-1}\overline{a}_{x_i}a_{x_i+1} ... a_{x_i-1}\overline{a}_{x_i}a_{x_i+1} ... a_l), \quad i = 1, k.
\]
The Nelder–Mead method is used as a method for local optimization in searching for a minimum of the criterion of optimality $\Phi(X)$ in the $n$-dimensional euclidean space $R^n$:

$$\begin{align*}
\min \Phi(X) &= \Phi(X^\star) = \Phi^\star, \\
&\quad X \in R^n.
\end{align*}$$

(17)

This method implements a change in the current simplex, which consists of a convex shell of $n + 1$ points that do not lie in a single hyperplane of the $n$-dimensional euclidean space [8].

Reflection of the $k$th vertex of the simplex with vertex coordinates $X_i^r$, $i \in [1, n + 1]$ leads to formation of a simplex with vertex coordinates

$$X_i^{r+1} = X_i^r, \quad i \in [1, n + 1], \quad i \neq k, \quad X_k^{r+1} = 2X_C^r - X_k^r,$$

(18)

where

$$X_C^r = \frac{1}{n} \sum_{i=1, i \neq k}^{n+1} X_i^r,$$

is the vector of the coordinates of the center of gravity of the remaining vertices of the simplex.

As a result of reducing the vertices of the simplex $X_i^r$ to the vertex $X_k$, we obtain a simplex with vertex coordinates

$$X_i^{r+1} = X_k^r + \gamma(X_k^r - X_i^r), \quad i \in [1, n + 1], \quad i \neq k, \quad X_k^{r+1} = X_k^r,$$

(19)

where $\gamma \in (0, 1)$ with $\gamma = 0.5$ is the reduction coefficient.

Compression of the simplex $X_i^r$ in the direction $(X_k^r - X_C^r)$ yields a simplex with vertex coordinates

$$X_i^{r+1} = X_k^r, \quad i \in [1, n + 1], \quad i \neq k, \quad X_k^{r+1} = X_C^r + \beta(X_k^r - X_C^r),$$

(20)

where $\beta \in (0, 1)$ with $\beta = 0.4–0.6$ is the compression coefficient.

Stretching the simplex $X_i^r$ in the direction $(X_k^r - X_C^r)$ yields a simplex with vertex coordinates

$$X_i^{r+1} = X_k^r, \quad i \in [1, n + 1], \quad i \neq k, \quad X_k^{r+1} = X_C^r + \alpha(X_k^r - X_C^r),$$

(21)

where $\alpha = 2.8–3.0$ is the tension coefficient.

A computer program based on Eqs. (6)–(21) has been developed for choosing the parameters and axial location of heating elements that will form the required temperature distribution inside the vacuum chamber of a ballistic gravimeter.

**Calculated Parameters of the Electric Heaters for a Modernized Ballistic Gravimeter.** Based on the solution of the problem by synthesis, it was found that a minimum gradient for a given temperature level is provided by four electric heaters.
heaters EH₁–EH₄ wrapped around the vacuum chamber and one heater EH₅ wrapped around the electromagnet. Curves 2 of Fig. 2 show the temperature distribution in the modernized BG with an average temperature $T_{\text{avg}} = 313.00$ K in the vacuum chamber. The electric heaters all have different thermal and geometric (axial height $h$) parameters, which are listed in Table 2 (the electric heaters are numbered from top to bottom).

In the electromagnet of the catapult, the average temperature is $T_{\text{avg}} = 313.2$ K for a heating power of $P = 20.42$ W and radial and axial heat fluxes of $F_r = 206.1$ and $F_z = 1695.2$ W/m$^2$, respectively. Here the lowest average temperature $T_{\text{avg}} = 310.2$ K is found in the support flange, for which the fluxes are $F_r = 1452.2$ and $F_z = 177.3$ W/m$^2$. The distribution of heat within the vacuum chamber leads to radial and axial temperature gradients of $G_r = -2.77$ K/m and $G_z = 0.081$ K/m, respectively. The volume $V$ of the electric heaters in the modernized BG was 44% lower than in the DETU 02-02-96.

Thus, a temperature field with an axial gradient that is many times smaller has been created in the vacuum chamber of the modernized BG. Since the heat flux in the radial direction essentially emerges only through the support flange and is negligible through the plastic foam thermal insulation, heat flow from the vacuum chamber in the axial direction is essentially blocked by EH₁ and EH₄, while the intermediate EH₂ and EH₃ compensate the radial heat flux.

Figure 2 shows that the temperature on the axis is essentially the same as at the wall, but only in the central portion of the vacuum chamber. At the top, the on-axis temperature is significantly lower, and at the bottom, somewhat higher than at the walls. In the DETU 02-02-96, a region with an elevated temperature occurs in the central part of the vacuum chamber and a region of reduced temperature is found at the bottom and, especially, at the top; this leads to a significant axial temperature gradient. In the modernized BG with synthesized electric heaters, this effect is much (by two orders of magnitude) smaller, so that the acceleration of gravity can be measured more accurately. The axial gradient can be reduced further by installing additional thermal insulation for the support flange and fused quartz window.

**Conclusion.** A mathematical model of the temperature field in a ballistic gravimeter has been developed based on a heat conduction boundary value problem that is solved by a finite element method. It is found that there is a large axial temperature gradient in the vacuum chamber of an existing gravimeter, the state primary standard DETU 02-02-96 for the acceleration of gravity. Genetic algorithms and the Nelder–Mead method are used to develop a method of synthesizing the parameters of the electric heaters that will reduce the temperature gradients inside the vacuum chamber and improve the technical parameters of the ballistic gravimeter.

**REFERENCES**

1. W. Torge, *Gravimetry* [Russian translation], Mir, Moscow (1999).
2. V. I. Leontiev, “Laboratory study of the thermostat in the GAG-3 gravimeter,” in: *Repeated Gravimetric Observations: Coll. Sci. Papers* [in Russian], Izd. MGK, Moscow (1988), pp. 60–74.
3. V. B. Dubovskii, V. I. Leontev, and M. A. Zaionchkovskii, “Economical thermostatic control of the GAG-3 gravimeter,” in: *Measuring the Force of Gravity* [in Russian], Nauka, Moscow (1981), pp. 76–84.
4. E. I. Gurevich and A. G. Filin, “Temperature field of the windings in the stator of a high-power turbine generator during local failure of the internal water cooling system,” *Elektrichesstvo*, No. 3, 23–29 (2010).
5. Z. Radakovich and K. Feser, “A new method for the calculation of the hot-spot temperature in power transformers with ONAN cooling,” *IEEE Trans. Power Delivery*, 18, No. 4, 1284–1292 (2003).
6. V. F. Bolyukh, L. I. Lysenko, and E. G. Bolyukh, “Parameters of high-efficiency pulsed inductive electromechanical converters,” *Russ. Electr. Eng.*, Allerton Press, New York, 75, No. 12, 1–11 (2004).
7. R. Nollan, T. Pillay, and T. Hagne, “Application of genetic algorithms to motor parameter determination,” in: *Proc. IAS Meeting*, Baltimore, USA (1994), pp. 42–54.
8. E. G. Goloskokov and V. P. Severin, “Modification of the deformed polyhedron method for optimization of a hierarchical sequence of criteria,” in: *Technical Cybernetics and Applications* [in Russian], Vishcha Shkola, Kharkov (1986), pp. 27–30.