Gluon correlations in the glasma

T. Lappi

Physics Department, P.O. Box 35, 40014, University of Jyväskylä, Finland and
Helsinki Institute of Physics, P.O. Box 64, 00014 University of Helsinki, Finland
E-mail: tuomas.lappi@jyu.fi

Abstract. The physics of the initial conditions of heavy ion collisions is dominated by the nonlinear gluonic interactions of QCD. These lead to the concepts of parton saturation and the Color Glass Condensate (CGC). We discuss recent progress in calculating multi-gluon correlations in this framework, prompted by the observation that these correlations are in fact easier to compute in a dense system (nucleus-nucleus) than a dilute one (proton-proton).

1. Introduction

Bulk particle production in relativistic collisions around midrapidity originates from small $x$ degrees of freedom, predominantly gluons, in the wavefunctions of the colliding hadrons or nuclei. At large energies these gluons form a dense system characterized by a saturation scale $Q_s$. The degrees of freedom with $p_T \lesssim Q_s$ are fully nonlinear Yang-Mills fields with large field strength $A_\mu \sim 1/g$ and occupation numbers $\sim 1/\alpha_s$; they can therefore be understood as classical fields radiated from the large $x$ partons. Because of their large longitudinal momentum, the large $x$ degrees of freedom are effectively “frozen” during the interaction. They can be described as random color charges drawn from a classical probability distribution $W_y[\rho]$ that depends on the rapidity cutoff $y = \ln 1/x$ separating the large and small $x$ degrees of freedom. The dependence of $W_y[\rho]$ on $y$ is described by a Wilsonian renormalization group equation known by the acronym JIMWLK. Note that while this description is inherently nonperturbative, it is still based on a weak coupling argument, because the classical approximation requires $\alpha_s(Q_s)$ to be small and therefore $Q_s \gg \Lambda_{QCD}$. The Color Glass Condensate (CGC) is a systematic effective theory (effective because the large $x$ part of the wavefunction is integrated out) description of the classical small $x$ degrees of freedom.

The term glasma [1] refers to the coherent, classical field configuration resulting from the collision of two such objects CGC. The glasma fields are initially longitudinal, whence the “glasma flux tube” [2, 3] picture. More importantly for computing multigluon correlations, they are boost invariant (to leading order in the QCD coupling) and depend on the transverse coordinate with a characteristic correlation length $1/Q_s$. There are several signals in the RHIC data [4] that point to strong correlations originating from the initial stage of the collision. The glasma fields provide a natural framework for understanding these effects, although much work is still left to do in understanding the interplay with purely geometrical effects from the fluctuating positions of the nucleons in the colliding nuclei [5].

We shall first describe some general observations on computing multigluon correlations in the glasma, arguing in Sec. [2] that they are in some sense simpler to compute in a collision of two dense, saturated nuclear wavefunctions than in the dilute limit (see Ref. [6] for a more formal
2. Multigluon correlations in the glasma

The gluon fields in the glasma are nonperturbatively strong, $A_\mu \sim 1/g$. This means that the gluon multiplicity is $N \sim 1/\alpha_s$. For a fixed configuration of the classical color sources it is well known that the multiplicity distribution of produced gluons is Poissonian, i.e. $\langle N^2 \rangle - \langle N \rangle^2 = \langle N \rangle$. In this case the correlations and fluctuations in the gluon multiplicity are all quantum effects that appear only starting from the one-loop level, i.e. suppressed by a power of the coupling constant $\alpha_s$. The computation in the CGC framework does not end here, however. To calculate the moments of the gluon multiplicity distribution one must first calculate the gluon spectra for fixed configuration of the color charges $\rho$ and then average over the probability distribution $W[y[\rho(x_\perp)]]$. For the $n$th moment of the multiplicity distribution, i.e. an $n$-gluon correlation, the leading order result is

$$
\left\langle \frac{dN}{d^3p_1} \cdots \frac{dN}{d^3p_n} \right\rangle = \left\langle \int W[\rho_1(y)] W[\rho_2(y)] \frac{dN}{d^3p_1}_{LO} \cdots \frac{dN}{d^3p_n}_{LO} \right\rangle
$$

(1)

This averaging, even after the subsequent subtraction of the appropriate disconnected contributions, introduces a correlation already at the leading order in $\alpha_s$, i.e. enhanced by an additional $1/\alpha_s$ compared to the quantum correlations. A natural example is the negative binomial distribution that we shall discuss below, with variance $\langle N^2 \rangle - \langle N \rangle^2 = \langle N \rangle^2/k + \langle N \rangle$. One must emphasize here that although these contributions arise as formally classical correlations in the effective theory that is the CGC, they are physically also quantum effects, where the weak coupling is compensated by a large logarithm of the energy that has been resummed into the probability distribution $W[y[\rho(x_\perp)]]$. In this sense the leading correlations are present already in the wavefunctions of the colliding objects.

3. Multiplicity distribution

We can then apply this formalism to the calculation of the probability distribution of the number of gluons in the glasma [7]. We shall assume the “AA” power counting of sources that are parametrically strong in $g$, but nevertheless work to the lowest nontrivial order in the color sources. Formally this would correspond to a power counting $\rho \sim g^{\epsilon - 1}$ with a small $\epsilon > 0$. In this limit, as we have discussed, the dominant contributions to multiparticle correlations come from diagrams that are disconnected for fixed sources and become connected only after averaging over the color charge configurations. The corresponding two gluon correlation function was computed in Ref. [2] and generalized to a three gluons in Ref. [8]. We shall here sketch the derivation [7] of the general $n$-gluon correlation in this simplified limit.

Working with the MV model Gaussian probability distribution

$$
W[\rho] = \exp \left[ - \int d^2x_\perp \frac{\rho^a(x_\perp) \rho^a(x_\perp)}{g^4 \mu^2} \right]
$$

(2)

computing the correlations and the multiplicity distribution in the linearized approximation is a simple combinatorial problem. It can be expressed in terms of two parameters, the mean multiplicity $\bar{n}$, and a parameter $k$ describing the width of the distribution. The result of the combinatorial exercise is that the $q$th factorial moment $m_q$ (defined as $\langle N^q \rangle$ minus the
corresponding disconnected contributions) is, with $S_\perp = \pi R^2$:

$$m_q = (q - 1)! k \left( \frac{\bar{n}}{K} \right)^q \quad \text{with} \quad k \approx \frac{(N_c^2 - 1)Q_s^2 S_\perp}{2\pi} \quad \text{and} \quad \bar{n} = fN \frac{1}{\alpha_s} Q_s^2 S_\perp. \quad (3)$$

These moments define a negative binomial distribution with parameters $k$ and $\bar{n}$, known as a phenomenological observation in high energy hadron and nuclear collisions already for a long time. In terms of the glasma flux tube picture this result has a natural interpretation. The transverse area of a typical flux tube is $1/Q_s^2$, and thus there are $Q_s^2 S_\perp = N_{FT}$ independent ones. Each of these radiates particles independently into $N_c^2 - 1$ color states in a Bose-Einstein distribution (see e.g. [9]). A sum of $k \approx N_{FT}(N_c^2 - 1)$ independent Bose-Einstein-distributions is precisely equivalent to a negative binomial distribution with parameter $k$. A numerical evaluation [10] of the second moment of the distribution, parametrized in terms of

$$\kappa_2(p_\perp, q_\perp) = Q_s^2 S_\perp \left( \frac{d^2N}{d^2p_\perp d^2q_\perp} - \frac{dN}{d^2p_\perp} \frac{dN}{d^2q_\perp} \right) / \frac{dN}{d^2p_\perp} \frac{dN}{d^2q_\perp} \quad (4)$$

is shown in Fig. 1. It confirms the expectations of [7] that this ratio is of order one and depends only weakly on the momenta $p_\perp, q_\perp$.

4. Rapidity dependence

The general discussion of Sec. 2 on the different nature of multigluon correlations in the “AA” case applies also to the rapidity dependence. Until now we have only been discussing gluon production in a rapidity interval smaller than $1/\alpha_s$. For this we needed only the correlations between the color charges $\rho(x_\perp)$ measured at this same rapidity. To understand the rapidity dependence of the correlations one needs also the correlation between color charges at different rapidities, $\langle \rho_q(x_\perp) \rho_{q'}(y_\perp) \rangle$. Also this information is contained in the JIMWLK renormalization group evolution, at least to leading ln $1/x$ accuracy [12]. An intuitive description of the resulting correlations is provided by the formulation of JIMWLK as a Langevin equation in the space of Wilson lines formed from the color charges. In this picture the evolution proceeds in individual trajectories along an increasing rapidity. A first attempt of a realistic estimate of the rapidity dependence of two-gluon correlations is performed in Ref. [11]. Evaluating the two gluon correlation in a dilute limit in a $k_\perp$-factorized approximation, but keeping the general structure
resulting from the JIMWLK evolution leads to the following expression:

\[ C(p, q) \sim \int_{k_{\perp}} \Phi_{A_1, y_1}^2 (k_{\perp}) \Phi_{A_2, y_2} (p_{\perp} - k_{\perp}) \Phi_{A_2, y_2} (q_{\perp} + k_{\perp}) + |k_{\perp} \leftrightarrow -k_{\perp}| + |A_1 \leftrightarrow A_2| \]  

(5)

Note the very different structure of this correlation compared to one where the gluons would be produced from the same diagram for fixed sources. The two gluon correlation function is proportional to the product of four unintegrated gluon distributions, with three of them evaluated at the rapidity of one of the produced gluons and only one at the other. This structure is a direct consequence of the nature of JIMWLK evolution. The resulting correlation is compared to PHOBOS data in fig. 2.

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