Effects of a Magnetic Field on Superconductivity and Quantum Criticality in Quasi-Two-Dimensional Systems with Dirac Electrons

E. C. Marino a and Lizardo H. C. M. Nunes a

Abstract

We study the effects of an external magnetic field on the superconducting phase diagram of a quasi-two-dimensional system of Dirac electrons at an arbitrary temperature. At zero temperature, there is a quantum phase transition connecting a normal and a superconducting phase, occurring at a critical line that corresponds to a magnetic field dependent critical coupling parameter, which should be observed in planar materials containing Dirac electrons, such as Cu$_x$TiSe$_2$. Moreover, the superconducting gap is obtained as a function of temperature, magnetic field and coupling parameter ($\lambda_R$). From this, we extract the critical magnetic field $B_c$ as a function of the temperature. For small values of $B_c$, we obtain a linear decay of the critical field, which is similar to the behavior observed experimentally in the copper doped dichalcogenide Cu$_x$TiSe$_2$ and also in intercalated graphite.

Key words: Dirac electrons, superconductivity, quantum criticality

PACS: 11.10.Wx, 74.25.Ha, 74.78.Fk

1. Introduction

Among the materials presenting Dirac electrons as their elementary excitations, there are a few, which have been intensely focused lately. These are high-Tc cuprates [1], graphene [2], carbon nanotubes [3], and transition metal dichalcogenides [4].

In the present work, we study the effects of an applied constant magnetic field, perpendicular to a quasi-two-dimensional superconducting system containing Dirac electrons.

The Lagrangian describing a quasi-two-dimensional superconducting electronic system, containing two Dirac points, in the presence of the external magnetic field along the c-axis, $Bz$, is given by

$$
\mathcal{L} = \overline{\psi} \sigma \left[ i \psi \left( \frac{\sqrt{2} e}{c} A_i \right) \right] \psi - g \left( \psi_{1\uparrow}^+ \psi_{2\downarrow}^+ \sigma_{1\downarrow} + \psi_{1\downarrow} \psi_{2\uparrow} \sigma_{1\uparrow} \sigma_{2\downarrow} \sigma_{1\downarrow} + \psi_{1\uparrow} \psi_{2\downarrow} \sigma_{1\downarrow} \right) - \frac{1}{2} \sigma_0 \left( g B \cdot \sigma \right) \psi_{1\uparrow}^+ \psi_{2\downarrow} \psi_{1\downarrow}^+ \psi_{2\uparrow} \psi_{1\uparrow} \psi_{2\downarrow} \psi_{1\downarrow} \psi_{2\uparrow},
$$

where the electron creation operator will be $\psi_{i\sigma,a}^+$, where $i = 1, 2$ denotes the Dirac point and $\sigma = \uparrow, \downarrow$, the z-component of the spin, $a = 1, ..., N$ is an extra label, identifying the plane to which the electron belongs, $A_i$ is the vector potential corresponding to $B$, $\sigma$ are Pauli matrices and $\mu_B$ is the Bohr magneton. The second and third terms, respectively, contain the coupling of the magnetic field to orbital and spin degrees of freedom. As in [5], we assume there is an effective superconducting interaction whose origin will not influence the results of this work. $g$ is the superconducting coupling constant, which is supposed to depend on some external control parameter. In order to make the lagrangian smooth, we define $g = \lambda/N$. We use the same convention for the Dirac matrices as in [5].

Introducing a Hubbard-Stratonovich complex scalar field $\sigma = -g (\psi_{2\uparrow} \psi_{1\downarrow} + \psi_{1\uparrow} \psi_{2\downarrow})$, where $\sigma^\dagger$ is a Cooper pair creation operator, the Lagrangian, in terms of $\sigma$ becomes,

$$
\mathcal{L}_{\Psi,\sigma} = -\frac{1}{g} \sigma^* \sigma + \Psi_{a \sigma}^\dagger A \Psi_a \tag{2}
$$

where

$$
A = \begin{pmatrix} 
\hat{\sigma}_0 & -\hat{\sigma}_+ & 0 & 0 \\
-\hat{\sigma}_+ & \hat{\sigma}_0 & \sigma & 0 \\
0 & \sigma^* & \hat{\sigma}_0 & \hat{\sigma}_+ \\
\sigma^* & 0 & \hat{\sigma}_- & \hat{\sigma}_0 
\end{pmatrix}
$$

* This work has been supported in part by CNPq and FAPERJ. ECM has been partially supported by CNPq.
with $\tilde{\partial}_0 \equiv i (h\partial_0 + \mu_B B)$, $\tilde{\partial}_\pm \equiv i v_F (h\partial_\pm + i(e/c)A_\pm)$ and $\partial_\pm = \partial_2 \pm i \partial_1$. The fermions are in the form of a Nambu field $\Psi^\dagger = (\psi_{1\sigma}^\dagger \psi_{2\sigma}^\dagger \psi_{1\sigma} \psi_{2\sigma})$.

Integrating on the fermion fields, we obtain the effective action per plane for $\sigma$, namely

$$S_{\text{eff}} (|\sigma|, B) = \int d^2x \left( -\frac{N}{\lambda} |\sigma|^2 \right) - iN \ln \text{Det} \left[ \frac{A[|\sigma|, B]}{A[|\sigma|=0, B=0]} \right].$$  

At $T=0$, the renormalized effective potential per plane is

$$V_{\text{eff},R} (|\sigma|, B) = \frac{|\sigma|^2}{\lambda_R} - f(B, \sigma_0) |\sigma|^2 + \frac{2}{3\alpha} (|\sigma|^2 + B\kappa) \sigma,$$

where $\kappa$ and $\alpha$ denote $v_F^2 e/c$ and $2\pi v_F^2$, respectively, $\lambda_R$ is the renormalized coupling and $\lambda_c = 2\alpha / 3\sigma_0$.

Studying the minima of the previous expression, the superconducting gap $\Delta \equiv |\langle 0 | \sigma | 0 \rangle| = 0$ exists only for

$$\lambda_R < \lambda_c(B) = \lambda_c \frac{\sqrt{1 + \frac{\mu_B B}{k_B T}}} {1 + \frac{2\kappa}{3\sqrt{B(1 + \frac{\mu_B B}{k_B T})}}},$$

where $\tilde{B} = B(\kappa / \sigma_0^2)$ and $\sigma_0$ is an arbitrary finite scale, and a quantum phase transition connecting a normal and a superconducting phase occurs at the magnetic field dependent quantum critical point $\lambda_c(B)$, given by the above expression.

We turn now to finite temperature effects. Using a large $N$ expansion and evaluating (4) at $T \neq 0$, we find the effective potential, whose minima provide a general expression for the superconducting gap $\Delta(T, B)$, as a function of the temperature and of the magnetic field,

$$\Delta^2(T, B) = \left\{ k_B T \cosh^{-1} \left[ \frac{e^{\frac{\sqrt{\frac{\sigma_0^2}{\kappa^2} + \frac{\mu_B B}{k_B T}}}{2}} - \cosh \left( \frac{\mu_B B}{k_B T} \right)} {2} \right] \right\}^2 - B\kappa,$$

where $\Delta_0 \equiv \Delta(T = 0)$.

From (7), and using the fact that $\Delta(T, B_c) = 0$ when the critical magnetic field $B_c$ is applied to the system, we can obtain $B_c$ vs. $T$ phase diagram for the quasi-two-dimensional superconducting Dirac electronic system. This is represented in Fig. 1. Particularly interesting is the linear behavior of the critical magnetic field for $B \gtrsim 0$, which is explicitly derived from (7) in the small $B$ region:

$$B_c(T) \sim \frac{8 \ln 2 k_F^2}{A \kappa} T_c^2(0) \left( 1 - \frac{T}{T_c(0)} \right),$$

where $A = 1 - (3/4\ln 2)[1 - (\lambda_c / \lambda_R)]$ and $k_B T_c(0) = (3\sigma_0 / 4\ln 2)[1 - (\lambda_c / \lambda_R)]$.

In Fig. 1, the linear behavior of the critical field is indicated by the dotted lines for different values of the dimensionless parameter $x \equiv \lambda_R / \lambda_c$ (this is actually valid for $\lambda_R < 13\lambda_c$, when $A$ is positive). This differs from the quadratic behavior predicted by BCS theory.

A linear decay of the critical field with the temperature similar to the one obtained here has been experimentally observed in intercalated graphite compounds [6] and also in the copper-doped dichalcogenide Cu$_x$TiSe$_2$ [7]. Since graphene and also the transition metal dichalcogenides are well-known to possess Dirac electrons in their spectrum of excitations, one is naturally led to wonder whether the presence of such electrons could explain such a behavior of the critical field.

![Fig. 1. B vs. T phase diagram. The critical magnetic field $B_c$ as a function of the temperature for several values of the dimensionless coupling parameter $x \equiv \lambda_R / \lambda_c$. The dotted lines in the figure indicate the linear behavior of $B$ given by (8) as $B \to 0$.](image)

References

[1] A.C. Durst and P.A. Lee, Phys. Rev. B 62 (2000) 1270; E.J. Ferrer, V.P. Gusynin and V. de la Incera, Mod. Phys. Lett. B 16 (2002) 107; F. Herbut, Phys. Rev. Lett. 88 (2002) 047006; M. Franz and Z. Tesanović, Phys. Rev. Lett. 84 (2000) 554, Phys. Rev. Lett. 87 (2001) 257003

[2] Y. Zhang et al., Nature 438 (2005) 201; K.S. Novoselov et al., Nature 438 (2005) 197; J. Gonzalez, F. Guinea and M.A.H. Vozmediano, Phys. Rev. B 63 (2001) 134421; N.M.R. Peres, F. Guinea and A.H. Castro Neto, Ann. Phys. 321 (2006) 1559; J. Nilsson et al., Phys. Rev. B 73 (2006) 214418; N.M.R. Peres, A.H. Castro Neto and F. Guinea, Phys. Rev. B 73 (2006) 239902; ibid. 241403; 245426; 195411; 205408; 125411; V.M. Pereira et al. Phys. Rev. Lett. 96 (2006) 036801

[3] L. Balents and M.P.A. Fisher, Phys. Rev. B 55 (1997) R 11973

[4] A.H. Castro Neto, Phys. Rev. Lett. 86 (2001) 4382; B. Uchoa, A.H. Castro Neto and G.G. Cabrera, Phys. Rev. B 69 (2004) 145112; B. Uchoa, G.G. Cabrera and A.H. Castro Neto, Phys. Rev. B 71 (2005) 184509

[5] E.C. Marino and L.H.C.M. Nunes, Nucl. Phys. B741 [FS] (2006) 404; E.C. Marino and L.H.C.M. Nunes, cond-mat/0703184 (to appear in Nuclear Physics B)

[6] N. Emery et al. Phys. Rev. Lett. 95 (2005) 087003; T.E. Ellerby et al. Nature Physics 1 (2005) 39; M. Ellerby et al., Physica B - Condensed Matter, 378-380 (2006) 636

[7] E. Morosan et al., cond-mat/0606529 (2006) (to appear in Nature Physics)