Blind Quantum Computation without Trusted Center

Shih-Min Hung and Tzonelih Hwang*

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Abstract

Blind quantum computation (BQC) protocol allows a client having partially quantum ability to delegate his quantum computation to a remote quantum server without leaking any information about the input, the output and the intended computation. Recently, many BQC protocols have been proposed with the intention to make the ability of client more classical. In this paper, we propose two BQC protocols, in which the client does not have to generate photons, but only has to perform either rotation or reorder on the received photons.

Keywords: Blind Quantum Computation; Quantum Cryptography.

1 Introduction

Quantum computation is one based on the principle of quantum mechanics. Compared with classical computation, it provides the advantage on calculation speed [1]. For example, quantum computer can simulate the property of quantum mechanics, and it is very difficult for classical computer to do that [2]. Shor’s algorithm [3] offers an exponential speedup over the best-known classical solution for factorizing big integers and solving discrete logarithm problems, and Grover’s algorithm [4] are also much faster than the best-known classical search algorithms.

However, realization of quantum computer is still an enormous challenge now. Although quantum computer appears to be very promising, it still has a long way to go before it becomes popular. Consequently,
in the near future only a few expensive quantum servers can be accessed by clients who have to perform quantum computation, but with only a limited quantum ability. Hence, the clients may have to delegate his problem to a quantum server without revealing his/her information, including the input, the output and the intended computation revealed to the server. Blind quantum computation (BQC) protocol is particularly suitable to satisfy this requirement.

Based on quantum circuit model, Childs [5] proposed the first BQC protocol, where clients need quantum memory and the ability to perform SWAP gate. Arrighi et al. [6] also proposed a BQC protocol, in which the client needs to prepare entanglement states and measure them. However, these protocols are not universal protocols, in the sense that they only work on certain classical function, and even the server can reveal partial information of the client. Broadbent et al. [7] then presented the first universal BQC protocol, where a client does not have any quantum computation ability and quantum memory except generating rotated single photon and the private information of the client can be unconditionally secure. After that, many BQC protocols have been proposed with the intention to make the ability of client more classical. Li et al. [8] proposed a triple-server BQC protocol using entanglement state and Xu et al. [9] proposed a single-server BQC protocol based on Li et al.’s protocol. Both Li et al. and Xu et al. claimed that in their protocol the client only needs to have a quantum channel to receive and resend photons. However, these protocols have to assume the existence of a trusted center, which is not practical in reality. Besides, Hung et al. [10] pointed out that both Li et al.’s and Xu et al.’s protocols are not secure because server can get the information of the client.

This paper intends to design two secure BQC protocols without trusted center. The clients in the new BQC protocols only have to perform either rotation operation or reorder the particles.

The rest of this article is organized as follows. Section 2 reviews Broadbent et al.’s BQC protocol. Section 3 proposes two BQC protocols. Section 4 analyzes the security of two proposed protocols. Finally, a concluding remark is given in Section 5.
2 Review Broadbent et al.’s BQC protocol

Before presenting our protocol, let us briefly review Broadbent et al.’s BQC protocol first. Suppose that a client Alice with limited quantum capability wants to delegate a quantum problem to a quantum server Bob on the \( m \)-qubit graph states corresponding to the graph \( G \) without revealing any information about the input, the output and the intended computation. The Broadbent et al.’s protocol can be briefly described as follows.

**Step 1.** Alice prepares \( m \) qubits and sends them to Bob. The state of each qubit is
\[
|\theta_i\rangle = |0\rangle + e^{i\theta_i}|1\rangle \quad (i = 1, 2, ..., m),
\]
where \( \theta_i \) is selected randomly from the set \( S = \{k\pi/4|k = 0, 1, ..., 7\} \).

**Step 2.** Alice asks Bob to generate a brickwork state according to the graph \( G \) specified by her.

**Step 3.** According to the graph \( G \), Bob produces a brickwork state \( |G(\theta)\rangle \) by applying CTRL-Z gates between the qubits sent from Alice.

**Step 4.** Alice sends \( \delta_i = \theta_i + \phi'_i + r_i\pi \) to Bob, where \( r_i \in \{0, 1\} \) is randomly selected by Alice and \( \phi'_i \) is a modification of \( \phi_i \) that depends on the previous measurement outcomes. Then Bob can measure the \( ith \) qubits \( (i = 1, 2, ..., m) \) of \( |G(\theta)\rangle \).

**Step 5.** Bob performs a measurement on the \( ith \) qubit \( (i = 1, 2, ..., m) \) in the basis \( \{|\pm\delta_i\rangle\} \) and sends Alice the measurement result.

**Step 6.** Alice can get the computation output from the measurement result.

In Broadbent et al.’s protocol, Alice needs the ability of generating rotated single photon. However, the ability of generating rotated single photon for a client is still considered to be very difficult now.

3 Proposed BQC protocols

This section proposes two BQC protocols. Each reduces the client’s quantum ability to only rotation operation on the particles or to reorder the particles. The details of these two protocols are described in Sec. 3.1 and Sec. 3.2 respectively.
3.1 The first proposed protocol

**Step 1.** Bob prepares $m$ qubits and sends them one-by-one to Alice. The state of each qubit is $|+\rangle$.

**Step 2.** Alice preforms rotation operation $Z_{\theta_i}$ on each qubit received from Bob, where $\theta_i$ is selected randomly from the set $S$.

**Step 3.** Alice sends back this qubit to Bob.

**Step 4.** Since Bob has the $m$ qubit graph state $\otimes_{i=1}^m |\theta_i\rangle (i = 1, 2, ..., m)$, and only Alice knows the values of $\theta_i$, Alice can run Broadbent et al.’s single-server BQC protocol from Step 2, Bob’s preparation, to delegate the quantum problem to Bob.

3.2 The second proposed protocol

**Step 1.** Bob generates $m$ Bell pairs $|\psi_{0,0} (B_k, A_k)\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) (k = 1, 2, ..., 2n)$ and sends the particle $A_k$ of each Bell state to Alice.

**Step 2.** After Alice receives all particles sent from Bob, she reorders those particles and sends them back to Bob.

**Step 3.** Alice sends $m$ classical message $\{\theta_i\}_{i=1}^m$ to Bob, where $\theta_i$ is selected randomly form the set $S$.

**Step 4.** Bob measures his $m$ particles $B_k$ in the basis $\{\pm \theta_k\}_{k=1}^m$ and sends the measurement results $\{b_k\}_{k=1}^m$ to Alice.

**Step 5.** Upon receiving $\{b_k\}_{k=1}^m$ form Bob, she knows the state of each $A_k$ Bob kept by the measurement results $\{b_k\}_{k=1}^m$ and the reorder information.

**Step 6.** Since Bob has the qubit graph state $\otimes_{i=1}^m |\theta_i + b_i\pi\rangle (i = 1, 2, ..., m)$, and only Alice knows the values of $\theta_i$ and $b_i$, Alice can run Broadbent et al.’s single-server BQC protocol from Step 2, Bob’s preparation, to delegate the quantum problem to Bob.
4 Security Analysis and Comparison

4.1 Security Analysis

In this section, we discuss the security analysis about both proposed protocols. Since the security of Broadbent et al.’s BQC protocol has been proved, we only focus on the privacy of \( \{\theta_i\}_{i=1}^n \); if Bob obtains the \( \theta \) of each particle, then he can calculate the input, the output and the intended computation.

In the first proposed protocol, since Alice performs the rotation operation \( Z_{\theta} \) on the particles in private and then sends them back to Bob, only Alice knows the \( \theta \) of each particle. Hence, this protocol is as secure as Broadbent et al.’s protocol.

In the second proposed protocol, because Bob knows the measurement basis \( \pm \theta \) and the measurement result for each particle \( B_k \), he can calculate the state of \( A_k \). However, since only Alice knows the new order of the particles \( \{A_k\}_{k=1}^m \) sent from Alice to Bob, Bob cannot find which two particles have entanglement. Hence, Bob cannot know the state of each \( A_k \), and hence this protocol is as secure as Broadbent et al.’s protocol.

4.2 Comparison

In this sub-section, we give a comparison of Broadbent et al.’s BQC protocol and two proposed BQC protocols (see also Table 1). In Broadbent et al.’s protocol, the client needs the ability to generate rotated single qubits; in the first proposed protocol, the client’s ability is reduced to preforming rotation operation; in the second proposed protocol, the client only needs to reorder the particles. However, to achieve this reduction on the client’s ability, the second proposed protocol pays the cost: it needs some devices to prevent Trojans horse attack, and the qubit efficiency of second proposed protocol is lower than Broadbent et al.’s protocol and the first proposed protocol.

5 Conclusions

This paper has proposed two BQC protocols for a client to delegate a quantum computation to a remote quantum server without revealing the input, the output and the intended computation. In both proposed
Table 1: Comparison table

|                      | Broadbent’s protocol | The first proposed protocol | The second proposed protocol |
|----------------------|----------------------|----------------------------|-----------------------------|
| Client ability       | generate qubits      | rotation operation         | reorder                     |
| Qubit efficiency     | 1/1                  | 1/1                        | 1/2                         |
| Trojans horse attack | Automatically prevent | Automatically prevent       | Need device to prevent      |

protocols, the client needs less quantum ability than in Broadbent et al.’s protocol. Whereas the client in the first protocol needs to perform rotation operation on the particles, the client in the second protocol only needs the ability to reorder the particles. We have shown that both proposed protocols are as secure as Broadbent et al.’s protocol. Yet, how to further reduce a client’s quantum ability in a BQC without revealing any client’s information will be a promising future work.

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