On the Masses of the Universal hypermultiplets in heterotic M-theory

Nasr Ahmed
Astronomy Department, National Research Institute of Astronomy And Geophysics, Helwan, Cairo, Egypt.

1 abstract
The reduced 5D Heterotic M-theory has a deeply rich structure. For every Calabi-Yau compactification, there exists a gravitational hypermultiplet \((g_{\mu\nu}, \psi_\mu, A_\mu)\) and a universal hypermultiplet. In this paper we derive the formulae for the masses of the scalar sector of the universal hypermultiplet \((V, \sigma, \zeta, \bar{\zeta})\) in the framework of 5D Heterotic M-theory.

2 Introduction
In the original formulation of M-theory \cite{1, 2}, all the standard model fields are trapped on two 9-branes located at the end points of an \(S^1/Z_2\) orbifold. The 6 extra dimensions on the branes are compactified. A 5 dimensional reduction of the original Hořava-Witten theory and the corresponding braneworld cosmology is given in \cite{3, 4, 5}.

In the 11-dimensional theory, the supergravity multiplet consists of the graviton field or the metric \(g\), gravitino field \(\psi_I\) and a three index gauge field \(C_{IJK}\) with a field strength \(G_{IJKL}\). The total bulk field content of this 5 dimensional theory is given by the gravity multiplet \((g_{\alpha\beta}, A_\alpha, \psi^I_\alpha)\) together with the universal hypermultiplet \((V, \sigma, \zeta, \bar{\zeta})\). \(A_\alpha\) is a gauge field with field strength \(F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha\). \(\zeta\) is a background complex field, \(V\) is the Calabi-Yau volume and \(\psi^I_\alpha\) is the gravitino field. After the dualization, the three-form \(C_{\alpha\beta\gamma}\) produces a scalar field \(\sigma\). The 5 dimensional effective action can be written as \cite{3}

\[
S_5 = S_{bulk} + S_{bound} \tag{1}
\]

where

\[
S_{bulk} = \frac{-1}{2\kappa_5^2} \int_{M_5} \sqrt{-g} \left[ R + \frac{3}{2} F_{\alpha\beta} \tilde{F}^{\alpha\beta} \right] \tag{2}
\]

\[
+ \frac{1}{\sqrt{2}} \epsilon^{\alpha\beta\gamma\delta\epsilon} A_\alpha \tilde{F}_{\beta\gamma} \tilde{F}_{\delta\epsilon} + \frac{1}{2V^2} \partial_\alpha V \partial^\alpha V
\]

\[
+ \frac{1}{2V^2} \left[ (\partial_\alpha \sigma - i(\zeta \partial_\alpha \bar{\zeta} - \bar{\zeta} \partial_\alpha \zeta) - 2\alpha \epsilon(x^{11}) A_\alpha)\right] + \frac{2}{V} \partial_\alpha \zeta \partial^\alpha \zeta + \frac{\alpha^2}{3V^2} \right]
\]
and
\[
S_{\text{bound}} = \sqrt{\frac{2}{\kappa_5^2}} \left[ \int_{M_4^{(1)}} \sqrt{-g} V^{-1} \alpha - \int_{M_4^{(2)}} \sqrt{-g} V^{-1} \alpha \right] - \frac{1}{16\pi \alpha_{\text{GUT}}} 
\]
\[
\sum_{i=1}^{2} \int_{M_4^{(i)}} \sqrt{-g} \left( V \text{tr} F_{\mu \nu}^{(i)} F^{(i) \mu \nu} - \sigma \text{tr} F^{(i)} \tilde{F}^{(i) \mu \nu} \right). 
\]
where \( \tilde{F}^{(i) \mu \nu} = \frac{1}{2} \epsilon^{\mu \nu \rho \sigma} F_{\rho \sigma}^{(i)} \) and the expansion coefficients \( \alpha_i \) are
\[
\alpha_i = \frac{\pi}{\sqrt{2}} \left( \frac{\kappa}{4\pi} \right)^{2/3} \frac{1}{v^{2/3} \beta_i}, 
\]
\[
\beta_i = -\frac{1}{8\pi^2} \int_{C_i} \text{tr}(\mathcal{R} \wedge \mathcal{R}). 
\]
with the Calabi-Yau volume \( V \) defined as
\[
V = \frac{1}{v} \int_X \sqrt{g^{(6)}} 
\]
where \( g^{(6)} \) is the determinant of the Calabi-Yau metric.

3 The Non-linear Sigma model Lagrangian for the background \( V \) and \( \zeta \) fields

In gaugino condensates the gaugino acquires non-zero vacuum expectation value which breaks the supersymmetry. The gaugino condensates lead to a background \( \zeta \) field. So, now we have a background \( V \) field (represents the size of Calabi-Yau space), a background \( \zeta \) field and a background metric. We take the following nonlinear sigma model lagrangian
\[
L = -\frac{(\partial V)^2}{4V^2} - \frac{1}{V} (\partial \zeta)(\partial \bar{\zeta}) - U 
\]
\[
U = \frac{\alpha^2}{6V^2}. 
\]
Where \( U = \frac{\alpha^2}{6V^2} \). We define the metric as
\[
dh^2 = \frac{dV^2}{4V^2} + \left( \frac{d\sigma - i(\zeta d\bar{\zeta} - \bar{\zeta} d\zeta)}{4V^2} \right)^2 + \frac{d\zeta d\bar{\zeta}}{V} 
\]
and introduce the one-forms \( u \) and \( v \) with their complex conjugate \( \bar{u} \) and \( \bar{v} \)
\[
w^a = (u, \bar{u}, v, \bar{v}) 
\]
\[
\text{Where} 
\]
\[
u = \frac{1}{2V} (dV + id\sigma + \zeta d\bar{\zeta} - \bar{\zeta} d\zeta) 
\]
This leads to
\[
du = -\frac{1}{2}(v + \bar{v}) \wedge u 
\]
\[
dv = -\bar{v} \wedge v - \bar{\bar{u}} \wedge u 
\]
and the connection two-forms
\[
w_{\nu v} = \bar{v} - v 
\]
\[
w_{\mu u} = \frac{1}{2}(\bar{v} - v) 
\]
\[
w_{\nu v} = -u 
\]
Cartan’s structure equation are
\[
T^a = d\theta^a + w_b^a \wedge \theta^b 
\]
\[
\Omega_b^a = dw_b^a + w_c^a \wedge w_b^c 
\]
For the torsion and curvature 2-form respectively. The curvature 2-form gives (when expressed locally)

\[ \Omega^a_b = \frac{1}{2} R^a_{bcd} \theta^c \wedge \theta^d \]  

(18)

Where the components \( R^a_{bcd} \) are in orthonormal basis. The affine connection form satisfies

\[ w_{ab} + w_{ba} = dg_{ab}, \quad w_{ab} = g_{ac} w^c_b \]  

(19)

Since for the orthonormal metric \( g_{ab} = \eta_{ab} \) is constant, we have For the curvature 2-form we have

\[ \Omega_{ab} = g_{ac} \Omega^c_b = -\Omega_{ba} \]  

(20)

For a free torsion space, Cartan’s first structure equation is

\[ d\theta^a = -w^a_b \wedge \theta^b \]  

(21)

So, the key formulas we are going to use to derive the connection coefficients and the corresponding curvature tensor are

\[ w_{ab} = -w_{ba} \]  

(22)

\[ d\theta^a = -w^a_b \wedge \theta^b \]  

(23)

\[ \frac{1}{2} R^a_{bcd} \theta^c \wedge \theta^d = dw^a_b + w^a_c \wedge w^c_b \]  

(24)

For the Ricci tensor we find

\[ R^u_u = \frac{1}{2} R^u_{a\beta} w^a \wedge \theta^\beta = (\bar{v} \wedge v) + (\bar{u} \wedge u) \]  

(25)

And

\[ R^u_v = (\bar{u} \wedge u) - (\bar{v} \wedge v) \]  

(26)

The components of the curvature tensor are \( (g_{\alpha\bar{u}} = \frac{1}{2})\):

\[ R_{\alpha\beta\bar{u}\bar{u}} = 1, \quad R_{\alpha\bar{u}v\bar{v}} = \frac{1}{2}, \quad R_{v\bar{v}v\bar{v}} = 1, \]

\[ R_{\bar{u}\bar{u}v\bar{v}} = \frac{1}{2}, \quad R_{\bar{u}\bar{u}v\bar{v}} = -1, \quad R_{\bar{u}\bar{u}v\bar{v}} = -1, \]

\[ R_{\bar{v}\bar{v}v\bar{v}} = -1, \quad R_{\bar{v}\bar{v}v\bar{v}} = -\frac{1}{2}, \quad R_{\bar{v}\bar{v}v\bar{v}} = -\frac{1}{2}, \]

\[ R_{\bar{v}\bar{v}v\bar{v}} = -1, \quad R_{\bar{v}\bar{v}v\bar{v}} = 1, \quad R_{\bar{v}\bar{v}v\bar{v}} = \frac{1}{2}, \]

\[ R_{\bar{v}\bar{v}v\bar{v}} = \frac{1}{2}, \quad R_{\bar{v}\bar{v}v\bar{v}} = \frac{1}{2}, \quad R_{\bar{v}\bar{v}v\bar{v}} = -1, \]

\[ R_{\bar{v}\bar{v}v\bar{v}} = \frac{1}{2}, \quad R_{\bar{v}\bar{v}v\bar{v}} = \frac{1}{2}, \quad R_{\bar{v}\bar{v}v\bar{v}} = -1. \]

The grad of the potential \( U \) is

\[ \nabla U = -\frac{\alpha^2}{3V^2}(v + \bar{v}) \]  

(27)

\[ \nabla^2 U = \frac{2\alpha^2}{3V^2}(v + \bar{v}) \otimes (v + \bar{v}) - \frac{\alpha^2}{3V^2} \]  

(28)

\[ (v - \bar{v}) \otimes (v - \bar{v}) + \frac{2\alpha^2}{3V^2} u \otimes \bar{u} \]

Now we would like to express the lagrangian (7) in terms of the one-forms \( u \) and \( v \). We make use of the general form of the nonlinear sigma model lagrangian with a background field \( \zeta \)

\[ L = -\frac{1}{2} g_{ij}(D_u \zeta^i)(D^j \zeta^j) \]

(29)

\[ + \frac{1}{2}(\partial_i \phi^j)(\partial^i \phi^j)R_{ijkl} \zeta^k \zeta^l - \frac{1}{2} U;ij \zeta^i \zeta^j. \]

Where \( R_{ijkl} \) is the curvature of \( g_{ij} \). After some manipulations we get the lagrangian in the form

\[ L = L_1 + L_2 \]  

(30)
Where

\[ L_1 = -(D_\alpha \zeta^u)(D^\alpha \zeta^u)-(D_\alpha \zeta^v)(D^\alpha \zeta^v) \]  
(31)

And

\[ L_2 = \frac{\alpha^2}{2} V^{-2} \left( [(\zeta^u - \bar{\zeta}^\bar{u}) + C(\zeta^u - \bar{\zeta}^\bar{u})] \right)^2 \]  
(32)

\[ = \frac{\alpha^2}{2} V^{-2} (\zeta^u - C\zeta^v)(\bar{\zeta}^\bar{u} - C\bar{\zeta}^\bar{v}) \]

\[ = \frac{\alpha^2}{2} V^{-2} (\zeta^u + \zeta^v)^2 - \frac{\alpha^2}{3} V^{-2} \zeta^u \zeta^v \]

The first part \( L_1 \) is diagonalized and we need to diagonalize the second part \( L_2 \) to get the eigen Values. To do that we make the following change of variables

\[ \zeta^v = \frac{1}{\sqrt{2}} (X + iY) , \quad \zeta^u = \frac{1}{\sqrt{2}} (Z + iW) \]  
(33)

That means we have 4 fields \( X, Y, Z, \) and \( W \). In terms of the new fields, the lagrangian \( L_2 \) becomes

\[ L_2 = -\alpha^2 V^{-2} (Y + CW)^2 \]

\[ - \frac{\alpha^2}{4} V^{-2} \left( (Z - CX)^2 + (W - CY)^2 \right) \]

\[ - \frac{\alpha^2}{3} V^{-2} \left( 2X^2 + Y^2 + \frac{1}{2} Z^2 + \frac{1}{2} W^2 \right) \]

Which could be written in a matrix form as

\[ L_2 = \frac{\alpha^2}{V^2} \begin{pmatrix} 2 & 0 & -C & 0 \\ 0 & \frac{1}{3} + \frac{C^2}{4} & 0 & 2C \\ -\frac{C}{2} & 0 & \frac{5}{12} & 0 \\ 0 & -\frac{C}{2} & 0 & \frac{5}{12} + C^2 \end{pmatrix} \]

After diagonalization, the eigen values that represent the masses of the scalar sector of the universal hypermultiplet \((V, \sigma, \zeta, \bar{\zeta})\) are

\[ \frac{\alpha^2}{24V^2} \left( -21 - 15C^2 + \sqrt{121 - 774C^2 + 81C^4} \right) \]  
(35)

\[ \frac{\alpha^2}{24V^2} \left( -21 - 15C^2 + \sqrt{121 - 774C^2 + 81C^4} \right) \]  
(36)

\[ \frac{\alpha^2}{24V^2} \left( -13 - 3C^2 + 3\sqrt{1 + 18C^2 + C^4} \right) \]  
(37)

\[ \frac{\alpha^2}{24V^2} \left( -13 - 3C^2 - 3\sqrt{1 + 18C^2 + C^4} \right) \]  
(38)

4 Conclusion

Making use of the non-linear sigma model Lagrangian of the background fields, we derived the formulae for the masses of the scalar sector of the universal hypermultiplet \((V, \sigma, \zeta, \bar{\zeta})\) in the reduced 5D Heterotic M-theory.

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