Dark Matter from the vector of SO(10)

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Abstract

SO(10) grand unified theories can ensure the stability of new particles in terms of the gauge group structure itself, and in this respect are well suited to accommodate dark matter (DM) candidates in the form of new stable massive particles. We introduce new fermions in two vector 10 representations. When SO(10) is broken to the standard model by a minimal \(45 + \mathbf{126} + \mathbf{10}\) scalar sector with \(SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}\) as intermediate symmetry group, the resulting lightest new states are two Dirac fermions corresponding to combinations of the neutral members of the \(SU(2)_L\) doublets in the 10’s, which get splitted in mass by loop corrections involving \(W_R\). The resulting lighter mass eigenstate is stable, and has only non-diagonal \(Z_{L,R}\) neutral current couplings to the heavier neutral state. Direct detection searches are evaded if the mass splitting is sufficiently large to suppress kinematically inelastic light-to-heavy scatterings. By requiring that this condition is satisfied, we obtain the upper limit \(M_{W_R} \lesssim 25\) TeV.

Keywords: Dark Matter, Grand Unified Theories

1. Introduction

A plethora of astrophysical and cosmological observations have firmly established that non-baryonic dark matter (DM) must exist in our Universe, and contribute to the overall cosmological energy density about five times more than ordinary matter. However, none of the particles of the standard model (SM) can account for the DM, which therefore constitutes a clear hint of new physics. Colorless, electrically neutral and weakly interacting massive particles with mass in the GeV-TeV range are ubiquitous in new physics models, and appear to be well suited to reproduce quantitatively the measured DM energy density if their stability on cosmological time scales can be ensured.

From the model-building point of view, DM stability is most commonly enforced by assuming some suitable symmetry that forbids its decay into lighter SM particles. For example, in supersymmetric models this role is played by R-parity that stabilizes the lightest supersymmetric state, in universal extra dimensional models conservation of Kaluza-Klein parity ensures that the lightest Kaluza-Klein state remains stable [1], T-parity stabilizes the lightest T-odd particle in the littlest Higgs model [2], suitable \(Z_2\) parities play the same role e.g. in the scotogenic model [3, 4], in the inert doublet model [5–7], and in several other cases. Often
these stabilizing symmetries are just imposed by hand on the low energy Lagrangian, and it is certainly more satisfactory when their origin can be traced back to some high energy completion of the model in question. A plausible way to generate unbroken discrete \( Z_N \) symmetries relies on assuming extra gauged \( U(1) \) Abelian factors which are only broken by order parameters carrying \( N \) units of the \( U(1) \) charge [8] (see also [9–12]). Such a mechanism renders grand unified theories (GUTs) based on gauge groups of rank larger than four particularly interesting, since they contain extra Cartan generators besides the \( 2 + 1 + 1 \) of the \( SU(3)_C \times SU(2)_L \times U(1)_Y \) SM gauge group that, when broken by vacuum expectation values (vevs) of scalars in appropriate representations, yield discrete \( Z_N \) symmetries which inherit all the good properties of the parent local gauge symmetry. In particular, this type of symmetries remain protected from gravity induced symmetry breaking effects [13–16] which, although suppressed by the Planck scale, could jeopardize DM stability [17, 18].

One of the most interesting GUT groups that allows to preserve at low energies an unbroken discrete gauge parity is the rank five group \( SO(10) \). As is well known, \( SO(10) \) has many theoretically appealing properties: it unifies all SM fermions in a single \( 16 \) dimensional irreducible representation including one right handed (RH) neutrino, it can explain the suppression of neutrino masses via the seesaw mechanism [19–23], it allows for gauge coupling unification at a sufficiently high scale to account for proton stability, and is automatically free from gauge anomalies. In this paper we focus on a \( SO(10) \) GUT model in which the breaking to the SM gauge group is driven by vevs of scalars in the \( 45_H \oplus 126_H \oplus 10_H \) representation. It has been recently shown that this model is compatible with unification [24], and that it can fit all charged fermion masses and mixings as well as the low energy neutrino data [25, 26], while simultaneously explaining the cosmological baryon asymmetry via leptogenesis [27]. It is therefore interesting to see if this framework can also accommodate automatically stable DM candidates in the fundamental \( 10 \) dimensional representation of the group.

2. Motivations and general considerations

\( SO(10) \) is a rank five group and thus with respect to the SM model it contains one additional Cartan generator that, upon breaking of the unified group, can give rise to a new \( U(1) \) gauge group factor. The \( U(1) \) charges of the component fields are conventionally normalized by setting the smallest charge equal to one. Then, if \( U(1) \) is further broken by vevs of scalars carrying \( n_1, n_2, \ldots \) units of charge with \( N > 1 \) as their greatest common divisor, a discrete center \( Z_N \in U(1) \) remains unbroken [28–30]. \(^1\) In our setup \( SO(10) \) is broken to \( U(1)_Q \times SU(3)_C \) by vevs of scalars in \( 45_H \oplus 126_H \oplus 10_H \), which are all \( SO(10) \) tensor representations. With respect to the non-SM \( U(1) \) factor singled out in the maximal subgroup \( SU(5) \times U(1) \), which is the one whose breaking gives rise to the gauge discrete symmetry, all \( SO(10) \) tensor representations branch to \( SU(5) \times U(1) \) fragments which have even values of the \( U(1) \) charge. The lowest charge value for the fragments acquiring vevs is 2 [e.g.: \( 10 \rightarrow 5(2) \oplus 5(-2) \)] and therefore a \( Z_2 \) parity survives, which can guarantee the stability of the lightest particles belonging to appropriately chosen representations.

\(^1\)The fact that breaking \( SO(10) \) with vevs in tensor representations can result in an unbroken \( Z_2 \) parity was already pointed out in [31], in relation to the possible appearance of extended topological structures of cosmological relevance. We thank Q. Shafi for bringing this reference to our attention.
Restricting to dimensions $D < 320$ we have that, depending if they are fermions or bosons, stable particles appear in the following representations [30, 32–34]:

$$\begin{align*}
\text{Fermions:} & \quad 10, 45, 54, 120, 126, 210, 210', \\
\text{Bosons:} & \quad 16, 144.
\end{align*}$$

(1) (2)

For example, fermions in the vector $10$ cannot decay into SM fermions in the $16$ since this is a spinorial representation for which all fragments under $SU(5) \times U(1)$ carry odd $U(1)$ charges and upon $U(1)$ breaking then acquire odd $Z_2$ parity. Various proposals for $SO(10)$ DM candidates that are stabilized by the $Z_2$ parity of gauge origin have been put forth in the recent literature: a dedicated analysis of scalar DM in the $16$ was carried out in [32, 33], while the possibility of fermionic DM in the $45$ was addressed in [34] (see also [35] where the $45$ is allowed to mix with a $10$). Other more general studies regarding possible embeddings of DM in $SO(10)$ can be found in [36, 37]. Indeed, so far a special attention has been devoted to DM in the scalar $16$ and in the fermionic $45$, and a possible reason for this might be the fact that both these representations contain SM singlets. Needless to say, identifying DM candidates with SM singlets can naturally explain why all experimental direct detection (DD) searches have been eluded so far. In contrast, the $10$ dimensional vector representation of $SO(10)$ has not attracted much attention, although this could well be considered as the minimal choice. Perhaps this is due to the fact that the $10$ does not contain SM singlets, and in particular all its states carry hypercharge, and thus couple to the $Z$ boson, which might led to the conclusion that this possibility is excluded by DD limits.

In this letter we argue that fermionic DM in the $10$ of $SO(10)$ is instead a viable possibility. Our main observation is that in a scenario in which fermions in a vectorlike $10_L \oplus 10_R$ acquire tree level masses via a Yukawa coupling with the (antisymmetric) $45_H$, loop diagrams involving an insertion of $W_L-W_R$ mixing produce a mass splitting between the two lightest mass eigenstates, which (in our minimal realization) are two neutral Dirac fermions. We show that the neutral $Z_{L,R}$ gauge bosons couple non-diagonally the light eigenstate to the heavier one and, as a result, at the leading order only inelastic neutral current scatterings of DM off target nuclei is allowed. If the mass splitting between the light and heavy mass eigenstates is larger than the typical DM kinetic energy $E_K \sim 200 \text{keV}$, then the scattering is kinematically forbidden and DD bounds are automatically evaded. Since the loop-induced splitting is suppressed by the RH gauge boson mass, the previous requirement can be translated into an upper-bound on $M_{W_R}^2$ which, combined with the lower bounds from flavour and CP violating processes in the $K$ and $B$ meson systems [38] and from direct searches at the LHC [39–41], results in $2.9 \text{TeV} \lesssim M_{W_R} \lesssim 25 \text{TeV}$. The fact that the null result of DD DM searches constrains $M_{W_R}$ to lie at a relatively low scale, can reinforce the hope that a rich phenomenology could be within the reach of the LHC.

### 3. The $SO(10)$ framework

We assume that $SO(10)$ is broken at the unification scale to the intermediate group $G_I = SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ by the vev of a $45_H$. $G_I$ is then broken at an intermediate scale $\Lambda_I$ to the SM gauge
group $G_{SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ by the vev of $\mathbf{126}_H$, and finally $G_{SM}$ is broken by electroweak doublet vevs in a (complexified) $\mathbf{10}_H$ down to $SU(3)_c \times U(1)$:

$$
\begin{align*}
SO(10) & \rightarrow (\mathbf{45}_H) \rightarrow 3C2L2R1B-L \\
& \rightarrow (\mathbf{126}_H) \rightarrow 3C2L1Y \otimes Z_2 \\
& \rightarrow (\mathbf{10}_H) \rightarrow 3C1Q \otimes Z_2.
\end{align*}
$$

In eq. (3) we have introduced for the gauge groups the short-hand notation e.g. $G_I = 3C2L2R1B-L$, and together with the unbroken continuous gauge symmetries we have also written down the discrete $\mathbb{Z}_2$ factor which survives down to the last breaking step. Although the GUT symmetry breaking triggered by an adjoint $\mathbf{45}_H$ together with a $\mathbf{126}_H$ develops instabilities at the tree level, it has been recently shown that the inclusion of quantum corrections can solve this problem and make the model viable [24, 42, 43]. The first symmetry breaking is achieved via the $3C2L2R1B-L$ singlet contained in the $\langle \mathbf{45}_H \rangle$. The second step is driven by a $\langle \mathbf{126}_H \rangle$ vev in the SM singlet direction which also provides Majorana masses for the RH neutrinos, and the last step is driven by the vevs of electroweak doublets contained in the $\mathbf{10}_H$. Note that while a real $\mathbf{10}_H$ is sufficient to drive the $G_I \rightarrow G_{SM}$ symmetry breaking, a $\mathbf{10}_H$ containing complex fields is needed to reproduce realistic fermion masses [44, 45] and in particular to accommodate the $m_t/m_b$ mass ratio. Moreover, to reproduce the complete charged fermion mass spectrum accounting also for Yukawa non-unification of the lepton and down-type quarks of the first two generations, a contribution from the vevs of the electroweak doublets appearing in the $\mathbf{126}_H$ is also necessary [44]. These vevs are unavoidably induced when $SU(2)_L \times U(1)_Y$ is broken by the $\langle \mathbf{10}_H \rangle$ [44, 45]. All in all, the masses of the SM fermions are generated from the following Yukawa terms:

$$
-L_{SM} = 16_i \left( h_{ij} \mathbf{10}_H + g_{ij} \mathbf{10}_H^* + f_{ij} \mathbf{126}_H \right) 16_j,
$$

where $h$, $g$ and $f$ are $3 \times 3$ symmetric matrices in flavour space and $i,j$ are family indices. The fermion couplings to $\mathbf{10}_H$ can be forbidden by assigning to the fields a global $U(1)$ Pececi-Quinn charge [44–46]. In practice this sets $g_{ij} \rightarrow 0$, simplifying the Yukawa structure of the model, and providing a DM candidate for non-supersymmetric $SO(10)$ models in the form of axions. In the spirit of avoiding the introduction of additional symmetries, and given that we are interested in a weakly interacting DM candidate, we will not follow this route, and we allow for $g_{ij} \neq 0$. Possible FCNC arising from coupling quarks of the same type to two different Higgs doublets, as it would happen in this situation, can be kept under control in various ways e.g. by assuming a hierarchy $g \ll f$.

**Adding fermions in the vector representation**

Let us now add to the $SO(10)$ model outlined above a pair of fundamentals $\mathbf{10}_c \oplus \mathbf{10}_\pi$ containing new fermions.\(^2\) The tensor product of two vectors of $SO(10)$ is:

$$
\mathbf{10} \otimes \mathbf{10} = \mathbf{1}^a \oplus \mathbf{45}^a \oplus \mathbf{54}^a,
$$

\(^2\)We denote $L$ and $R$ chiralities with calligraphic subscripts ($c, \pi$), while normal subscripts ($L,R$) label the $SU(2)$ gauge group factors.
where the superscripts denote symmetric and antisymmetric representations. Although our model does not contain a 54 of fundamental scalars, loop corrections can generate mass contributions that mimic the coupling to (effective) representations, as long as these couplings are allowed by the symmetries of the model. To keep as general as possible it is then convenient to write down all the allowed gauge invariant Yukawa couplings, which are:

\[-L_{\text{DM}} = \sum_{a = \xi, \chi} 10_a (M_a + \lambda_a 54_H) 10_a \]

\[+ [10_c (M + y 45_H + \lambda 54_H) 10_{\chi} + \text{H.c.}], \hspace{1cm} (6)\]

where, in order not to over-clutter the expressions, we have left understood the usual spinor notations. It is instructive to analyze these couplings in terms of representations of the \(SU(5) \subset SO(10)\). The branching rule for the \(SO(10)\) vector is \(10 = 5 + \bar{5}\) so that in the first line the invariant mass term \(M_a\) multiplies the \(5 \otimes \bar{5}\) singlet from the product of the same \(10_a\). The second term involves the symmetric \(54 = 15 + \bar{15} + 24\) and, besides containing a \(5 \otimes 24 \otimes \bar{5}\) coupling involving the \(SU(5)\) adjoint, it also includes couplings of the symmetric \(15\) to a pair of fundamentals: \(5 \otimes 15 \otimes 5 + \text{c.c.}\). Let us note at this point that if the colorless \(SU(2)\) triplet contained in the \(15\) \((15)\) acquires a small vev, these terms would generate a Majorana mass for the neutral components of the fermion doublets in the \(5 (\bar{5})\). However, the same is not true for the analogous term in the second line since it contains only terms that couple two different \(10\)'s. Finally, since the \(45_H\) is antisymmetric, it must couple different representations, and thus it appears only in the second line.

The model we will now study is specified by the following ingredients: (i) the \(54_H\) is absent; (ii) the adjoint vev \(\langle 45_H \rangle\) which can be written as:

\[\langle 45_H \rangle = \text{diag}(a, a, a, b, b) \otimes (\frac{0}{1} - \frac{1}{0}) , \hspace{1cm} (7)\]

acquires a Dimopoulos-Wilczek structure \([47, 48]\) with \(a \sim \Lambda_{\text{GUT}}\) and \(b/a \approx 0\) (since \(b \neq 0\) breaks \(SU(2)_L \times SU(2)_R\) we require \(b \lesssim \Lambda_T\) in order to respect the symmetry breaking pattern eq. (3)); (iii) we set \(M_a \rightarrow 0\). This can be viewed as technically natural since in this limit a global \(U(1)\) symmetry \(10_{\xi, \chi} \rightarrow e^{i\alpha_{\xi, \chi}} 10_{\xi, \chi}\) arises (we will briefly comment below on the consequences of relaxing this assumption); (iv) finally, we will also work with \(M \rightarrow 0\). This is just a simplification: a term proportional to \(M\) preserves the global \(U(1)\) symmetry obtained for \(\alpha_L = -\alpha_R\) which eventually ensures the Dirac nature of the DM states (see next section), and as long as \(M \sim y b \ll a\) a non-vanishing \(M\) would not change the analysis.

4. Fermion spectrum and neutral current couplings

In the following we label \(SU(2)_L \otimes SU(2)_R \otimes SU(3)_C \otimes U(1)_{B-L}\) representations as \((d_L d_R d_C)_{B-L}\) where \(d_{L,R,C}\) denote the dimensions of the multiplets under the respective symmetry factor, and \(B - L\) gives the value of the \(U(1)_{B-L}\) charge. Each one of the two \(10\)'s contains one \(2_L 2_R 1_C\) bi-doublet. We denote the bi-doublet contained in \(10_{\xi, \chi}\) as \(\xi_{\xi, \chi}\), with components:

\[\xi_{\xi, \chi} = \left( \begin{array}{c|c} \xi_{\xi, \chi}^- & \xi_{\xi, \chi}^+ \\ \hline \xi_{\xi, \chi}^- & \xi_{\xi, \chi}^+ \end{array} \right), \hspace{1cm} (8)\]
where the superscripts carried by the component fields denote the $SU(2)_L \otimes SU(2)_R$ isospin eigenvalues of $T_{3L,3R}$ in units of $\pm \frac{1}{2}$. Given that the $10$ has vanishing $U(1)_{B-L}$ quantum number, the electromagnetic charge for this representation is simply $Q = T_{3L} + T_{3R}$. Thus, $\xi^{\pm}_{\rho, \pi}$ and $\xi^{\mp}_{\rho, \pi}$ are neutral fields, while $\xi^{\pm}_{\rho, \pi}$ and $\xi^{\mp}_{\rho, \pi}$ have electric charge $Q = \pm 1$. The mass term for the neutral states arising from $(45_H)$ in Eq. (7) reads:

$$- \mathcal{L}_m = m_b \left[ (\xi^{\mp*}_{\rho})^\dagger \xi^{\mp}_{\pi} + (\xi^{\pm*}_{\rho})^\dagger \xi^{\pm}_{\pi} + \text{H.c.} \right]$$

(9)

with $m_b = y_b$. A similar mass term can be written also for the charged components $\xi^{\pm}_{\rho, \pi}$ which at lowest order are degenerate in mass with the neutral states. However, electromagnetic corrections from loops involving SM gauge bosons lift this degeneracy inducing a charged-neutral mass difference $m^+ - m^0 \simeq 340 \text{ MeV}$ [49], thus ensuring that the lightest states are the neutral ones.³

Eq. (9) describes a pair of neutral Dirac fermions $\xi^{+-} \equiv (\xi^{+-}_{\rho}, \xi^{+-}_{\pi})^T$ and $\xi^{--} \equiv (\xi^{--}_{\rho}, \xi^{--}_{\pi})^T$ with degenerate masses equal to $m_b$. However, these states get mixed via loop diagrams (depicted in figure 2) involving two external $(10)_H$ vevs, which generate a mass term:

$$- \mathcal{L}_\delta = \delta_m \left[ (\xi^{+-*}_{\rho})^\dagger \xi^{+-}_{\pi} + (\xi^{--*}_{\rho})^\dagger \xi^{--}_{\pi} + \text{H.c.} \right].$$

(10)

The tree level eq. (9) and the loop induced mass term eq. (10) are sketched in figure 1. Upon diagonalization of the mass matrix (see the Appendix) we end up with one heavier ($\chi_h$) and and one lighter ($\chi_l$) Dirac fermions, respectively with masses

$$m_{h,l} = m_b \pm \delta_m.$$  

(11)

The important point is that the couplings of $\chi_{h,l}$ to the neutral gauge bosons $Z_{L,R}$ are off-diagonal: both $Z_{L,R}$ couple (with an opposite overall sign) to the vectorlike neutral current

$$J^{nc}_\mu = \frac{1}{2} \bar{\chi}_h \gamma_\mu \chi_l + \text{H.c..}$$

(12)

Two ingredients are crucial for this result: (i) a complex $10_H$: this is in any case needed to reproduce the SM fermion mass pattern. The tensor product of two $10$’s contains in its symmetric part the $54$, which can be regarded here as an effective $54_H$ coupled to the fermions via the loop diagram. Moreover, since $10_H$ develops vevs in the $L$-$R$ doublet components, the effective $54_H \subset 10_H \otimes 10_H$ contains a non-vanishing $SU(2)_L \times SU(2)_R$ breaking vev in $(3_L, 3_R, 1_C)_0$, which provides an independent mass term to the fermion bi-doublets. (ii) a tree level mass coupling between $10_\rho$ and $10_\pi$: a non-vanishing loop mass contribution can only appear if a chirality flip can be inserted inside the loop diagram. In our simplified setup with $M_{\rho,\pi}, M = 0$ this can only be provided by $m_b$ (a contribution from $M \neq 0$ which also couples $10_\rho$ to $10_\pi$ would act in the same way with the replacement $m_b \rightarrow m_b + M$). Note that the global $U(1)$ symmetry mentioned at the end of section 3 remains unbroken also at the loop level.⁴

³We have checked that loops of RH gauge bosons do not contribute. This is because the $10$ has vanishing $B-L$ charge, and $|T_{3R}| = \frac{1}{2}$ for both charged and neutral states, so that the corresponding loop contributions cancel in the mass difference. Therefore the SM result [49] for the charged-neutral mass difference for fermion doublets holds also in this case.

⁴ Allowing for $M_{\rho,\pi} \neq 0$ would instead break this $U(1)$. As a result, we can expect that the two Dirac states will split into four Majorana fermions with a spectrum determined by the relative sizes of $m_b$, $\delta m$ and $M_{\rho,\pi}$. 

6
5. Constraints from direct detection

The mass splitting $m_h - m_l = 2\delta_m$ is an important quantity, since for $m_{DM} \sim 1$ TeV, the $Z_{L,R}$ mediated inelastic DM scattering off target nuclei $\chi_l + N \rightarrow \chi_h + N$ is kinematically forbidden only if $2\delta_m \geq 200$ keV [50], and only in this case $\chi_l$ could have escaped DD DM searches. Let us then proceed to estimate its value.

We denote the bi-doublet scalar contained in $10_H$ as $\phi$, with components:

$$\phi = \begin{pmatrix} \varphi^+ \varphi^- \\ \varphi^{++} \varphi^{--} \end{pmatrix}, \quad (13)$$

where the superscripts have the same meaning as for the fermions eq. (8). Assuming for simplicity that there is only one scalar bidoublet and that its vevs are real, we can write

$$\langle \phi \rangle = \begin{pmatrix} v_u \\ v_d \end{pmatrix}, \quad (14)$$

where $v_u^2 + v_d^2 = v^2$ is the electroweak breaking vev. One diagram contributing to $\delta_m$ is depicted in figure 2. The crossed diagrams should also be added, and a similar pair of diagrams can be drawn for external fermions with exchanged LR isospin labels $(+-) \leftrightarrow (-+)$. Taking $M_{W_R}$ as the largest mass scale in the loop, the diagram can be estimated as

$$\frac{1}{2} \delta_m \sim \frac{g_L^2 g_R^2}{16\pi^2} \frac{v_u v_d}{M_{W_R}^2} m_b \sim \frac{2\alpha}{4\pi s_W^2} \frac{v_u v_d}{v_R^2} m_b = 5 \times 10^{-3} \vartheta_{LR} m_b, \quad (15)$$

where we have used $M_{W_R} = g_R v_R / \sqrt{2}$ with $v_R$ the $G_I$ breaking vev of $126_H$, $s_W = \sin \theta_W$ with $\theta_W$ the Weinberg angle, and $m_b \gg \delta_m$ is to a very good approximation the DM mass. In the last expression we have introduced the $W_R-W_L$ mixing parameter $\vartheta_{LR} = \frac{v_u v_d}{v_R^2}$ which is experimentally bounded in various ways. Electroweak precision data set the upper limit $\vartheta_{LR} < 0.013$ [51, 52]. On the other hand, $\vartheta_{LR}$ is bounded from above also by the ratio of gauge boson masses squared:

$$\vartheta_{LR} = \frac{2v_u v_d}{4\pi s_W^2} \frac{M_{W_L}^2}{M_{W_R}^2} \lesssim \frac{M_{W_L}^2}{M_{W_R}^2}, \quad (16)$$

where in the first equality we have approximated $g_R \approx g_L$. Flavour and CP violating processes in the $K$ and $B$ meson systems provide an absolute lower bound on the $SU(2)_R$ gauge bosons mass $M_{W_R} > 2.9$ TeV [38].
Similar bounds on $M_{W_R}$ have been also obtained from direct searches at the LHC [39–41]. Using these figures we obtain the conservative upper bound $\vartheta_{LR} < 7.7 \times 10^{-4}$. The mass splitting $2\delta_m$ between the light and heavy neutral states can then be bounded from above as:

$$2\delta_m \lesssim 15 \left( \frac{2.9 \text{ TeV}}{M_{W_R}} \right)^2 \left( \frac{m_b}{1 \text{ TeV}} \right) \text{ MeV}. \quad (17)$$

The constraint from DM non-observations in DD experiments [50] $2\delta_m \gtrsim 200 \text{ keV}$ can then be translated in the following upper limit on the $SU(2)_R$ gauge boson masses:

$$M_{W_R} \lesssim 25 \left( \frac{m_b}{1 \text{ TeV}} \right)^{1/2} \text{ TeV}. \quad (18)$$

Indeed this result suggests the possibility of a non trivial interplay between DM searches in DD experiments, and searches for new physics at the LHC or at a future 100 TeV hadron collider.\footnote{Let us recall at this point that a certain number of anomalies have been recently reported by both ATLAS [53] and CMS [54, 55] which could be explained by a low-scale L-R model with $M_{W_R} \sim 2$ TeV, see e.g., [56–64].}

Before concluding this section, let us note that non-vanishing DD cross sections will appear at the loop level, with leading contributions involving the exchange of pairs of $SU(2)_L$ gauge bosons ($W_L W_L$ or $Z_L Z_L$). The quantitative effects of the corresponding diagrams have been studied for example in refs. [65–68], and it was found that the resulting cross sections do not exceed $\sim O(10^{-47})$ cm$^2$, which is far below the current experimental bounds [69, 70]. In the relevant mass range ($m_\chi > \sim$ TeV) this remains also below the reach of next generation DD experiments [71] and close to the neutrino scattering background.

6. Neutrino masses

The relatively low value of the intermediate symmetry breaking scale implied by eq. (18) could be of some concern for what regards the light neutrino masses. In general, the fact that the seesaw mechanism [19–23] can be automatically embedded within $SO(10)$ provides an elegant way to explain why the neutrino masses are so suppressed. $SO(10)$ unification implies relations between the light neutrino, RH neutrinos, and up-type quark masses, which generically require rather heavy RH neutrinos ($M_{N_R} \sim m^2_u/m_\nu$). The natural
range for $M_{N_R}$ is then loosely determined by the up and top quark masses as $10^4 \text{GeV} \lesssim M_{N_R} \lesssim 10^{14} \text{GeV}$. On the other hand $N_R$'s acquire their masses from the same $\mathbf{126}_H$ vev that breaks $G_I$ and concurs to determine the value of $M_{W_R}$, and thus we would expect their masses to be of the order of $M_{W_R}$, which remains bounded from above by eq. (18). Such a relatively low mass scale for the RH neutrinos does not provide enough suppression. One might then be tempted to appeal to a different suppression mechanism. For example the inverse seesaw \cite{72} is an elegant option that allows to suppress neutrino masses even when $M_{W_R} \sim \text{TeV}$. However, implementing the inverse seesaw requires the addition of a $SO(10)$ singlet with a (small) Majorana mass term, which couples to $N_R$ and to some scalar representation with non-vanishing vev. Given that $N_R \in \mathbf{16}$ the only option for writing down a renormalizable coupling is a scalar multiplet $\mathbf{16}_H$. However, $\langle \mathbf{16}_H \rangle \neq 0$ would break the $Z_2$ parity thus allowing DM decays. A similar conclusion can be reached also in the case the new fermion is not a $SO(10)$ singlet but is assigned to some suitable $SO(10)$ representation. We must then conclude that in our framework the inverse seesaw does not provide a viable alternative to explain the neutrino mass suppression.

A straightforward, although not so elegant, way out, is to appeal to cancellations in the neutrino Dirac mass matrix. This relies on the fact that, when projected onto SM multiplets, the fermion mass matrices originating from eq. (4) acquire non trivial Clebsch-Gordan coefficients that weight the various vev contributions and that are different for different fermion species. For the up-type quark and Dirac neutrino masses we have \cite{27,46,73}:

$$m_u = +(hv_u + gv_d) + \sqrt{3}f\kappa_u,$$

$$m_D^{\nu} = -(hv_u + gv_d) + 3\sqrt{3}f\kappa_u,$$

where $h, g, f$ are the symmetric Yukawa matrices introduced in eq. (4) and $\kappa_u$ is the up-type induced doublet vev from $\mathbf{126}_H$. If we take e.g. $M_{N_R} \sim 10^{\text{TeV}}$, no particular cancellation is needed for the mass entries related to the up quark mass, while for those related to the heavy third generation, a tuning in the cancellation of up to one part in $10^5$ is required. While this is certainly unpleasant, we should not forget that non-supersymmetric $SO(10)$ suffers a naturalness problem from the start, which already requires a tuning in the theory at a much higher level than $10^{-5}$.

7. Relic density

We now turn to the calculation of the relic density of the DM candidate $\chi_l$. Right above the intermediate scale (of order TeV) our $SO(10)$ model corresponds essentially to a low-scale L-R extension of the SM \cite{23,74–78} with the addition of two (massless) fermion bi-doublets. After $SU(2)_R$ breaking, two degenerate Dirac $SU(2)_L$ doublets with mass $m_b$ appear. Below the electroweak breaking scale the neutral members of these doublets combine into two neutral fermions $\chi_b$ and $\chi_l$ with masses $m_b \pm \delta m$. Let us recall that $\delta m \sim \mathcal{O}(\text{MeV})$ while the mass difference with the heavier charged partners is of about $340 \text{MeV}$ (see the discussion in section 4) so that both mass splittings are much smaller than the typical DM freeze-out temperature, and have negligible effects on the determination of the DM relic abundance. This is because in the limit of unbroken LR symmetry the fermion states are degenerate members of gauge multiplets, and
Figure 3: DM relic density as a function of $m_b$ for $M_{W_R} = 3 \text{ TeV}$ ($M_{Z_R} \sim 5 \text{ TeV}$) and $M_{W_R} = 10 \text{ TeV}$ ($M_{Z_R} \sim 17 \text{ TeV}$).

‘cohannihilation’ channels can be simply taken into account by including appropriate factors of gauge multiplicities both in the annihilation process and in the counting of particle degrees of freedom. However, in our numerical study we have kept track of the small mass splittings induced by symmetry breaking thus differentiating between annihilation and cohannihilation, but the effects of this more refined treatment remain irrelevant. Of course, eventually the charged states and the heavier neutral state $\chi_h$ will all decay to $\chi_I$, thus adding their contribution to the DM relic abundance.

Before tackling the calculation of the DM relic density in our model, let us first recall the generic features of the analogous computation in the case of a DM SM doublet. The relic density of the neutral component of an $SU(2)_L$ doublet of mass $m_{DM} > M_{Z_L}$ can be cast as [49]:

$$\Omega_{DM} h^2 \approx 0.1 \frac{4.2 \times 10^{-3} \text{ TeV}^{-2}}{\langle \sigma v \rangle} \approx 0.1 \left( \frac{m_{DM}}{1.1 \text{ TeV}} \right)^2,$$

where $\langle \sigma v \rangle$ is the thermally averaged annihilation cross-section times the relative velocity, and includes SM gauge annihilation and co-annihilation processes. The scaling of $\Omega_{DM}$ with the square of $m_{DM}$ follows from the scaling of the annihilation cross section as $m_{DM}^{-2}$, when the DM mass is larger than the SM gauge boson masses. Clearly, in the presence of two quasi-degenerate doublets with mass $m_b$, the same value of $\Omega_{DM}$ would be reproduced for $m_b = m_{DM}/\sqrt{2}$. However, in our case in addition to the SM gauge interactions we have new annihilation channels mediated by $Z_R$ and $W_R$. For $m_b \sim M_{W_R}$ the cross section does not scale simply as $m_b^{-2}$, and in particular for $m_b \sim M_{W_R}/2$ and $m_b \sim M_{Z_R}/2$ annihilation in the s-channel proceeds via gauge boson resonances which drastically lowers the otherwise large relic density implied by Eq. (21). Therefore, in addition to the solution at 0.77 TeV, which approximately holds for $M_{Z_L} < m_b \ll M_{W_R}$ we
expect other phenomenologically viable values of $m_b$ which will depend on the values of the RH gauge boson masses.

In order to explore quantitatively the mass parameter space for DM, including also the effects of resonant annihilation and co-annihilation via $Z_R, W_R$, we have carried out a numerical analysis by implementing our model in MICROMEGAS [79]. The results are summarized in figure 3 where the dependence of the relic density on the DM mass $m_b$ is shown for two benchmark values of $M_{W_R}$: 3 TeV and 10 TeV (the corresponding $Z_R$ mass values are respectively 5 TeV and 17 TeV). The first solution corresponds to the expected value $m_b \simeq 0.77$ TeV. However, other values of $m_b$ become viable close to the resonance regions $m_b \sim M_{W_R}/2$ and $m_b \sim M_{Z_R}/2$. The width of these resonances is large enough to allow for a clear separation of the two crossings that are present for each resonance. All in all, we have five phenomenologically viable values of $m_b$ for each value of $M_{W_R}$. Eventually, for very large DM masses $m_b \gg M_{W_R}$ we recover the scaling of Eq. (21). However, in this region the relic densities are at least one order of magnitude larger than the observed $\Omega_{DM}$.

As we have seen in section 4, DM does not couple diagonally to the neutral $Z_{L,R}$ gauge bosons. Then the leading annihilation channel for indirect detection searches is into $W_L W_L$ and $Z_L Z_L$ (respectively via $t$-channel exchange of $\chi^\pm$ and $\chi_h$) with comparable branching ratios for the two diboson final states (for the DM solutions corresponding to $m_b \sim M_{Z_R}/2$, see figure 3, $W_L W_R$ final states are also kinematically accessible). The velocity-averaged cross-section for $\chi_l \bar{\chi}_l \rightarrow W_L W_L$ can be estimated as $\langle \sigma v \rangle \sim \pi \alpha_s^2/(32m_l^2) \sim 3 \times 10^{-28} (2 \text{TeV}/m_l)^2 \text{ cm}^3/\text{s}$ with $\alpha_s$ the $SU(2)$ fine structure constant. A more accurate estimate, including non-relativistic Sommerfeld corrections, gives for the same mass range an enhancement up to one order of magnitude [82]. In spite of this the signal remains well below the present limits $\langle \sigma v \rangle \lesssim (10^{-25} - 10^{-24}) \text{ cm}^3/\text{s}$ for the mass range 1 TeV $< m_l < 4$ TeV [83].

As regards collider limits, at the LHC the most sensitive searches in our scenario, in which $\chi_\alpha = \chi_{l,b}, \chi^\pm$ are quasi-degenerate, are monojet signatures from processes like $pp \rightarrow \chi_\alpha \chi_{l,b} j$, where the pair of $\chi$’s are produced via the $s$-channel exchange of a SM gauge boson. However, this signal is accompanied by large backgrounds from $Z, W$+jets, which render the experimental search particularly difficult. A dedicated analysis of signatures of quasi-degenerate Higgsino like DM, which closely resembles our scenario, indicates a surprisingly low reach $m_l \sim 250$ GeV even for LHC-13 [84]. We have no reason to expect that this limit could be largely exceeded in our case, so that we can conclude that even the non-resonant (lowest mass) DM solution with $m_l \sim 0.77$ TeV remains unconstrained by collider searches.

Before concluding this section, one additional remark is in order. Our Dirac DM candidates carry hypercharge $Y = T_{3R}$, which can be used to distinguish particles (e.g. $\chi^{++}, \chi^{-+}$) from antiparticles (respectively $\chi^{--}, \chi^{++}$). During their thermal history, the $\chi$’s will unavoidably enter in chemical equilibrium with the thermal bath, inheriting an asymmetry similar to that of all SM hypercharged states. For example, in the temperature range $T_R > T > T_L$ ($T_{R,L}$ denote the temperatures at which $SU(2)_{R,L}$ get broken) that is the relevant range in which DM annihilates efficiently when the DM mass is above a few TeV, processes like

\footnote{The model file was generated with FEYNRULES [80] by modifying the model of Ref. [81].}
$\xi^{+}\xi^{-} \leftrightarrow W_R^{0,+1}$ which occur as long as $T \gtrsim M_{W_R}$, or $2 \leftrightarrow 2$ scatterings mediated by $D = 6$ effective operators like $\frac{g_R}{M_{W_R}} (\xi^{-}\gamma_\mu \xi^{-}) (\bar{u}_R \gamma^\mu d_R)$ which occur when $T < M_{W_R}$, will enforce chemical equilibrium between the $\chi$ system and the SM thermal bath, and thus an asymmetry will develop in the DM sector as well. The issue whether such an asymmetry could play any relevant role in determining the final DM relic density by quenching the annihilations when the DM density becomes of the order of the density asymmetry was recently studied in [85] for the general case of stable relics belonging to scalar and fermion hypercharged multiplets of dimension $D \geq 2$. It was found that for fermion doublets, as for most of the other cases, the effects of the DM asymmetry on the surviving relic density are generally negligible. However, the results of the analysis in [85] rely on certain assumptions, among which: (i) there are no new hypercharged particles besides the multiplet in question; (ii) the same operator responsible for the transfer of the asymmetry is also responsible for the mass splitting between the neutral hypercharged states.

In the present case however, these two conditions are not satisfied: (i) besides the new fermions in the bi-doublets, also the charged gauge bosons $W_R$ carry hypercharge $Y = T_{3R}$; (ii) the loop operator that induces the mass splitting (figure 2) is generated only after $SU(2)_L$ breaking, while the asymmetry transfer is mediated by tree level interactions with real or virtual $W_R$ bosons, and is most efficient well above $T_L$. Moreover, resonantly enhanced annihilation and co-annihilation, that were not present in the scenario analyzed in [85], here play a very important role, and many of the solutions for the correct DM relic density are found in DM mass regions close to the gauge boson resonances. Nevertheless, in spite of all these differences, given that co-annihilation via $W_R$ exchange, that is one of the dominant annihilation processes, does not suffer from any asymmetry-related quenching, it is reasonable to expect that in most of the parameter space the DM asymmetry will be largely uninfluential in determining the final value of $\Omega_{DM}$.\(^7\)

8. Conclusions

Breaking the $SO(10)$ GUT group to the SM via the intermediate group $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ by means of vacuum expectation values in $45_H \oplus 126_H \oplus 10_H$ preserves an exact discrete gauge symmetry $Z_2$, which ensures the stability of the lightest among new fermions belonging to $10$-dimensional vector representations. We have added to the $SO(10)$ model two fermionic $10$’s, and we have argued that the lightest stable states belonging to these representations correspond to the four neutral members of the $SU(2)_L \otimes SU(2)_R$ bi-doublets contained in the two $10$’s. Thus, a first requirement for viable DM candidates, namely electric charge neutrality, is satisfied. After $SU(2)_R$ breaking the four neutral states arrange into two Dirac spinors describing two types of fermions that are degenerate in mass, and after $SU(2)_L$ breaking loop corrections involving the $W_R$ gauge bosons mix these states in such a way that: (i) the mass eigenstates get splitted by an amount $\delta m$; (ii) both the $Z_{L,R}$ neutral gauge bosons couple non-diagonally the lighter neutral fermion to the heavier one. Requiring that $\delta m$ is large enough to forbid neutral current inelastic scatterings of DM into the heavier fermions allows to evade all the limits from DD DM searches, in spite of the fact that DM is constituted by weakly interacting Dirac fermions. We have shown that the quantitative requirement $\delta m \gtrsim 200 \text{keV}$ can be satisfied only if the scale of $SU(2)_R$ breaking is not too large, and we have derived

\(^7\)The special cases in which the relic density is determined by annihilations close to the $Z_R$ resonance could be an exception.
an upper limit on the RH gauge bosons mass \( M_{W_R} \lesssim 25 \text{ TeV} \). This upper bound strengthen the possibility that the LHC might detect signatures of a low scale LR symmetry, and this is particularly exciting in view of the anomalies recently reported by ATLAS [53] and CMS [54, 55] which can all be explained with a RH gauge boson with mass \( M_{W_R} \sim 2 \text{ TeV} \).

Before concluding, we should also point out some issues that within our minimal scenario are left open, and that might deserve further studies. In section 6 we have pointed out that, as in all \( SO(10) \) derived low scale LR models, there is not enough suppression for the light neutrino masses from the seesaw mechanism, and to accommodate the neutrino mass scale we had to invoke some amount of tuning in the Yukawa sector. A related issue is the fact that the scale of the RH neutrino masses is too low to allow for an explanation of the cosmological baryon asymmetry via the standard leptogenesis scenario [86] (see [87] for a review). This is because the RH neutrinos are too light to provide sufficiently large CP violating asymmetries [88]. Finally, we have verified that with the minimal particle content that we have assumed, gauge coupling unification does not occur; however, we expect that it can be recovered by adding new particles in suitable incomplete \( SO(10) \) representations (see for example [89] for different ways to recover gauge coupling unification in low scale LR models).

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Appendix

A. Mass eigenstates and gauge interactions

The two \( (2_L 2_R 1_C)_0 \) bi-doublet contained in the \( 10 \) of \( SO(10) \) can written in components as:

\[
\xi_{\nu, \bar{\nu}} = \begin{pmatrix}
\xi_{\nu, \bar{\nu}}^+ \\
\xi_{\nu, \bar{\nu}}^-
\end{pmatrix}
\]  

(A.1)

\( \xi_{\nu, \bar{\nu}} \) are multiplets of Weyl fermions coupled to \( SU(2)_L \times SU(2)_R \) gauge fields through the gauge-invariant kinetic Lagrangian:

\[
\mathcal{L}_K = \xi_{\nu, \bar{\nu}}^+ i \sigma_\mu D^\mu \xi_{\nu, \bar{\nu}} + \xi_{\nu, \bar{\nu}}^+ i \sigma_\mu D^\mu \xi_{\nu, \bar{\nu}},
\]  

(A.2)

where \( \sigma_\mu = (I, -\vec{\sigma}) \) and \( \sigma_\mu = (I, \vec{\sigma}) \) with \( \vec{\sigma} \) the Pauli matrices acting in Lorentz space, and

\[
D^\mu = \partial^\mu - ig_L \bar{\nu}_L \gamma^\mu \frac{\tau_L}{2} - ig_R \bar{\nu}_R \gamma^\mu \frac{\tau_R}{2},
\]  

(A.3)
where $\bar{\tau}_L$ are Pauli matrices acting in $SU(2)_L$ group space, i.e. on the first superscript labels of the multiplet components in eq. (A.1), while $\bar{\tau}_R$ of $SU(2)_R$ act on the second labels, and the reversed vector sign reminds that the action is from the right: $\xi \to \xi' = \bar{\tau}_L \bar{\tau}_R \xi$. Let us now define

$$\zeta_c = \sigma_2 \xi^*_\pi, \quad \zeta^\dagger_c = \xi^T \sigma_2.$$  \hspace{1cm} (A.4)

From the free Weyl equation for R-chirality spinors $i\sigma_\mu \partial^\mu \xi^\pi = 0$ and using the relation $\sigma_2 \bar{\tau}^+ \sigma_2 = -\bar{\tau}$ it is easily seen that $\zeta_c$ satisfies $i\bar{\sigma}_\mu \partial^\mu \zeta_c = 0$ and thus it contains $L$-chirality spinors. In terms of $\zeta_c$ the second term in eq. (A.2) can be rewritten as:

$$\zeta^\dagger_c i\sigma_\mu D^\mu \xi^\pi = \zeta^\dagger_c i\sigma_\mu \partial^\mu \zeta_c$$

$$+ \zeta^\dagger_c i\bar{\sigma}_\mu \left[ ig_\mu \tilde{W}^{\mu T}_L \frac{\bar{\tau}_L}{2} + i g_\mu W^{\mu T}_R \frac{\bar{\tau}_R}{2} \right] \zeta_c$$  \hspace{1cm} (A.5)

where we have integrated by parts the derivative term (neglecting a 4-divergence) and an overall change of sign is due to anticommutation of the fermion fields. Eq. (A.5) shows explicitly that $\zeta_c$ transforms in the $SU(2)_L \otimes SU(2)_R$ conjugate representation $(\bar{2}, 2)$ with generators $\overline{\sigma} = -\bar{\tau}^\pi = \tau_2 \bar{\tau}_2$, where the last relation expresses the pseudoreality of $SU(2)$ representations. It is then convenient to define new $\zeta_c$ multiplets transforming similarly to $\xi_c$ in $(2, 2)$:

$$\zeta_c = \begin{pmatrix} \zeta^+_c & \zeta^{++}_c \\ \zeta^-_c & \zeta^{+\pi}_c \end{pmatrix} = \tau_{2L} \zeta_c \tau_{2R}$$

$$= \begin{pmatrix} \sigma_2 (\bar{\xi}^{+\pi}_c) \sigma_2 (\bar{\xi}^{-\pi}_c) \\ -\sigma_2 (\bar{\xi}^{-\pi}_c) \sigma_2 (\bar{\xi}^{+\pi}_c) \end{pmatrix}.$$  \hspace{1cm} (A.6)

Focusing now on the neutral fermions, let us define:

$$\Psi_1 = \begin{pmatrix} \xi^+_c \\ \xi^+_{\pi} \end{pmatrix}, \quad \Psi_2 = \begin{pmatrix} \xi^-_c \\ \xi^-_{\pi} \end{pmatrix}.$$  \hspace{1cm} (A.7)

The neutral current interactions for $\Psi_i$ ($i = 1, 2$) read:

$$\mathcal{L}^{ne}_\mu = J^{ne}_\mu \cdot (g_\mu W^{\mu}_{3L} - g_{3R} W^{\mu}_{3R}) ,$$

$$J^{ne}_\mu = - \sum_{i=1,2} \Psi^\dagger_i i \bar{\sigma}_\mu T_3 \Psi_i,$$  \hspace{1cm} (A.8)

with $T_3 = \text{diag}(+\frac{1}{2}, -\frac{1}{2})$. In order to write $J^{ne}_\mu$ in terms of the mass eigenstates, let us study how the mass terms are rewritten from the original basis of LH and RH Weyl spinors $\xi_c, \pi$ to our basis of LH spinors $\xi_c$ and $\zeta_c$. The fermion bilinears multiplying the tree level mass term for charged and neutral states eq. (9) are rewritten as:

$$m_6 \text{Tr} (\xi^\dagger_c \xi + \xi^\dagger_{\pi} \xi_{\pi}) = -m_6 \text{Tr} (\zeta^T \sigma_2 \xi_c + \xi^\dagger_{\pi} \sigma_2 \xi^*_{\pi}).$$  \hspace{1cm} (A.9)

Written explicitly:

$$\text{Tr} (\zeta^T \sigma_2 \xi_c) = - (\zeta^+_c)^T \sigma_2 \xi^+_{c} - (\zeta^{++}_c)^T \sigma_2 \xi^{+\pi}_c$$

$$+ (\zeta^-_c)^T \sigma_2 \xi^{-\pi}_c + (\zeta^{+\pi}_c)^T \sigma_2 \xi^{++}_c.$$  \hspace{1cm} (A.10)
Similarly, the $SU(2)_L \otimes SU(2)_R$ breaking loop induced mass term for the neutral states eq. (10) reads:

$$\delta_m \left[ (\zeta^+_c)^T \sigma_2 \xi^+_c + (\zeta^-_c)^T \sigma_2 \xi^-_c \right], \quad (A.11)$$

so that the Lagrangian for the masses of the neutral states is:

$$-\mathcal{L}_m^0 = \Psi_2^T \sigma_2 \mathcal{M} \Psi_1 + H.c. \quad (A.12)$$

with

$$\mathcal{M} = \begin{pmatrix} \delta & -m_b \\ -m_b & \delta \end{pmatrix}. \quad (A.13)$$

Being $\mathcal{M}$ symmetric it can be factorized as $\mathcal{M} = V^T M V$ with $V$ unitary and $M$ diagonal with real and positive eigenvalues. The two eigenvalues are $m_{h,l} = m_b \pm \delta$, and the heavy and light mass eigenstates $\chi_1 = (\chi_{1h}, \chi_{1l})^T$ and $\chi_2 = (\chi_{2h}, \chi_{2l})^T$ are given by $\chi_{1,2} = V \Psi_{1,2}$ with

$$V = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & 1 \\ i & i \end{pmatrix}. \quad (A.14)$$

By redefining now $\chi_1 = \chi_c$ and $\chi_\pi = -\sigma_2 \chi_2^*$, the mass term can be written as:

$$-\mathcal{L}_m^0 = \overline{\chi}_l m_l \chi_l + \overline{\chi}_h m_h \chi_h, \quad (A.15)$$

where we have introduced the four spinor $\chi_l = [(\chi_{l})_{\ell}, (\chi_{l})_{\gamma}]^T$ and a similar one for $\chi_h$, and we have adopted the usual convention $\overline{\chi}_l = \chi_l^T \gamma_0$ with $\gamma_0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$ in the chiral basis. Thus, upon diagonalization of the mass matrix the new fermions organize into two Dirac mass eigenstates splitted in mass by $2\delta$.

Coming back to the neutral current gauge interactions, after rotating the interaction eigenstates $\Psi_i$ in eq. (A.8) onto mass eigenstates, the neutral current reads:

$$J_{\mu}^{nc} = -\sum_{i=1,2} \overline{\chi}_i i \sigma_\mu \left( V T_3 V^\dagger \right) \chi_i$$

$$= \frac{1}{2} \overline{\chi}_h \gamma_\mu \chi_l + \text{h.c.}, \quad (A.16)$$

with $\gamma_0 \gamma_\mu = \begin{pmatrix} \sigma_\mu & 0 \\ 0 & \sigma_\mu \end{pmatrix}$. We see that the neutral gauge bosons couple the light mass eigenstates to the heavy ones, a result that follows from the fact that $V T_3 V^\dagger = -\frac{1}{2} \tau_2$ is anti-diagonal ($\tau_2$ denotes the second Pauli matrix, but with no relation here with gauge group factors or spinors). Obviously, since $V \partial_\mu V^\dagger = \partial_\mu$ the purely kinetic term for the mass eigenstates remains diagonal.

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