Analytical Model for the Estimation of Leak Location in Natural Gas Pipeline

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To cite this article:
Obibuike Ubanozie Julian, Ekwueme Stanley Toochukwu, Ohia Nnaemeka Princewill, Igwilo Kevin Chinwuba, Onyejekwe Ifeanyi Michael, Igbojionu Anthony Chemazu. Analytical Model for the Estimation of Leak Location in Natural Gas Pipeline. International Journal of Oil, Gas and Coal Engineering. Vol. 7, No. 4, 2019, pp. 95-102. doi: 10.11648/j.ogce.20190704.12

Abstract: Mathematical model for leak location in natural gas pipeline has been developed in this paper. The models employ an isothermal steady state approach. Leak occurrence in the pipeline divided the pipeline into two sections—the upstream and downstream sections respectively. Analyses of leak incidence was carried out in the two pipeline sections giving rise to two equations being developed to address the leak localization. The first leak equation was developed by considering the upstream section of the pipeline while the second leak equation was developed by considering the downstream section of the pipeline. The two equations were analytically developed by a slight modification of the Weymouth’s equation for gas flow in horizontal pipelines. Matlab software was used in the model simulation. Seven field data were used in the model simulation. The results from the Matlab simulation of the mathematical models developed gave the leak locations for each of the field cases. Comparison of the simulated results with actual results of leak locations determined experimentally revealed a high level of accuracy with an average error of only 0.377% which is below the minimum acceptable limit. Furthermore analyses of results shows that the two leak equations yields same results when used in the Matlab simulator. The model is highly suitable for accurate detection of leak in natural gas pipeline especially where economics and reliability is of essence.

Keywords: Leak Location, Mathematical Model, Upstream Section, Downstream Section

1. Introduction

Pipelines provide the most efficient means of transporting hydrocarbon fluids from one place to another. Pipelines provide better efficiency in fluid transportation when compared to trucks, rail or marine transportation. However, leaks, which are the main faults of gas pipelines, can cause serious problems related not only to the environment but also to economy. A leak detection system is a safety is a safety and integrity-critical component of an operating pipeline that is designed to help mitigate negative consequences following a leak occurrence in a pipeline [1, 2].

The methods for leak detection fall into two broad categories; these are externally (released fluid detection) and internally (observing hydraulic behavior) based methods. The externally based methods are also referred to as physical inspection methods and is done to identify the location and size of leaks. Physical inspection consists of gas sampling; soil monitoring; flow-rate monitoring; and acoustic-, optical-, and satellite-based hyper spectral imaging. Usually, the physical inspection can result in an accurate detection of the location and size of a leak, but this comes with the expense of production shutdown and the high cost/long time to run the physical detection, which is very crucial in a long-distance gas pipeline.

The internal methods fall into a broad category referred to as model based leak detection methods. These models are mathematical equations analytically or numerically developed to address leak phenomena in pipelines. Model based methods continually measure pressure and/or mass flow signals frequently at the intake and outtake of the pipeline [3, 4]. This Mathematical-modeling approach detects a gas leak by solving the governing mass-conservation,
momentum-conservation, and energy-balance equations, thus leading to a quick evaluation at much lower cost. It also has the advantages of monitoring the system continuously and noninterference with pipeline operations. One of the limitations of the modeling method is that it requires flow parameters, which are not always available. Leak detection from mathematical modeling also has a higher uncertainty than that from physical inspection. 

This paper presents a mathematical model to estimate leak location in a natural gas pipeline. The approach is based on the mass balance and pressure point analyses in the pipeline. Many researchers have conducted investigations on gas transient flow in pipelines to detect leaks. 

Reddy et al. [5] built a dynamic simulation model by use of a transfer-function model for online state estimation and leak detection in a gas pipeline. The model reduced the computational time, while obtaining accurate state estimation from noisy measurements. The computation required all available measurements of pressure and flow rate. 

Wang and Carroll [6] analyzed the real-time data with a transient model to detect gas- and liquid-pipeline leakage. Stochastic processing and noise filtering of the meter reading were used to reduce the impact of noise. The correlations for diagnosing the leak location and amount are derived on the basis of the online real-time observation and the readings of pressure, temperature, and flow rate at both ends of the pipeline. 

Gajbhiye and Kam [7] used a mechanism model to detect leakage in a subsea pipeline under fixed pressure boundaries. The model compared the inlet and outlet flow-rate changes with fixed-pressure boundary conditions to detect leakage. Although the model can be applied to single- and multiphase flows, it needed pressures and flow rates at both ends of the line. Elliott et al. [8] showed the efficiency of leak detection by a spherical acoustic device called a SmartBall®, which has the advantages of low cost, ease of deployment, and the ability to locate pinhole leaks immediately to within 1 m. This technique is limited by pipeline geometry. A long pipeline also requires long inspection time to detect leakage. Launching and receiving SmartBalls are necessary, which may not be applicable in some conditions. Hauge et al. [9] used an adaptive Luenberger-type estimator to locate and quantify leakage given inlet velocity, pressure, and temperature and outlet velocity and pressure. The model was built in OLGA, a commercial software from Schlumberger [10], which can handle multiphase flow and incorporate temperature dynamics. Pressure and rate at two ends of the pipeline are required for numerical calculation. Ben-Mansouri [11] developed a 3D turbulent flow model with 10cm diameter pipeline to detect small leakage in water distribution pipeline. He made use of Quadropole simulation methods leading to a much complex treatment of the leak phenomena. Balda Rivas and Civan [12] used mass-balance and transient flow models to detect leaks in liquid pipelines. The response times to the transient-flow operation were used to estimate leak location. Their model required intensive measurements of all variables. Zhou et al. [13] developed a numerical model to simulate fluid leakage from damaged submarine pipeline. In his study, the effects of oil properties, leak rate and leak sizes were examined. Hamner et al. [14] modeled transients in natural gas pipeline. He tried to optimize automatic shutdown in gas pipeline. He discovered that the influence of negative pressure wave is more complex for gas pipeline and is affected by the frictional pressure of the pipeline.  

In summary, existing methods are classified into two larger categories: physical method and mathematical model. Physical detection has the advantages of accuracy and high certainty. The online, real-time surveillance of pipelines and leak detection can be realized if monitoring equipment is installed in the pipeline. Because the physical method requires installation and maintenance of substantial levels of costly equipment on the pipeline, it may be excluded because the high operating cost is not affordable and the long time taken to detect the leak is unacceptable because of the continuous loss of revenue, damage to facilities and environment, and possible loss of life. Sometimes, a harsh environment or severe weather can make the installation of detection instruments in the pipeline and/or physical inspections impossible. In some cases, remote locations that are difficult to access make physical inspection unrealistic. The mathematical model has the advantages of low cost and quick leak detection. 

Shutdown of the operation may not be required. The continuous online, real-time monitoring of the pipeline and leak identification are possible if the required data can be measured and transmitted to the central office simultaneously. The disadvantages of the mathematical model are low accuracy and high uncertainty. High-quality and complete data sets are key factors of detecting leaks successfully [15]. In practice, the mathematical model can be used to narrow down the possible leak interval before the physical inspection is conducted. 

2. Methods 

Natural gas modelling is the representation of gas behaviour by mathematical equation. This enables prediction of behaviour at various conditions. In this case, in this case, mathematical equations are used to represent the flow behavior of natural gas as it flows in pipelines. The following assumptions were considered during the modelling.  

i. The flow is in steady state regime. 

ii. The fluid is in gaseous phase and no phase changes during flow. 

iii. The fluid is in single phase throughout the entire flow period. 

iv. The effect of environment does not affect pipeline temperature. 

v. The pipeline is horizontal and as such effect of elevation is negligible.
2.1. Basic Equations of Compressible Fluid Flow

The basic equations of compressible fluid flow are: Continuity equations, Momentum equations, Energy equations and Equations of state.

For this work the continuity equation shall be the governing equation for the determination of mathematical models that predict natural gas flow behaviour in pipeline both in the leak and no leak cases.

Consider a fluid element in a pipe of uniform cross-section.

\[ \frac{\partial}{\partial x} (\rho U_x) + \frac{\partial}{\partial y} (\rho U_y) + \frac{\partial}{\partial z} (\rho U_z) = 0 \]  

(1)

Where
- \( \rho \) = density of fluid
- \( U_x, U_y, U_z \) = volumetric velocity in 3 dimension

But for steady state,

\[ \frac{\partial}{\partial t} (\rho U_x) = 0 \]  

(2)

\[ \frac{\partial}{\partial x} (\rho U_x) + \frac{\partial}{\partial y} (\rho U_y) + \frac{\partial}{\partial z} (\rho U_z) = 0 \]  

(3)

The above equation is for three dimensional steady state flows.

Considering flow in one dimension i.e. in the x-direction

\[ \frac{\partial}{\partial x} (\rho U_x) = 0 \]  

(4)

The above holds if there is no mass accumulation Integrating w. r. t x

\[ \rho U_x = constant \]  

(5)

If there are of the flow is A, then the rate of flow is

\[ \rho AU = constant \]  

(6)

Therefore

\[ \rho_1 A_1 U_1 = \rho_2 A_2 U_2 \] (mass rate in = mass rate out)  

(7)

\[ M_{in} - M_{out} = M_{leak} \]  

(9)

2.2. Leak Localization or Location

When there is a leak, the pipeline experiencing a leak can be modelled by dividing it into three parts - exact location of leak occurrence, upstream and downstream of the leak, as schematically shown in Figure 3. The exact location of the leak is treated as a singularity point mathematically at which gas leaks out to the surrounding area (i.e., leaks from inside to outside of the pipeline).

Let us consider the diagrams below

Let \( Q_1 \) be the inlet gas flowrate of the no leak case in ft3/hr

\( Q_2 \) be the output gas flowrate of the no leak case in ft3/hr

\( Q_R \) the recorded flowrate at the output section of the pipeline in ft3/hr

\( P_R \) the recorded output pressure in psi

\[ M_{in} - M_{out} = M_{leak} \]  

(9)

Figure 2 shows a pipeline schematic for gas flow in the absence of leak. From figure 2, it is seen that the flowrate is same throughout the whole section of the pipeline since flow is in steady state.

\[ Q_1 = q = Q_2 \]  

(10)

Figure 3 shows a pipeline profile in the event of leak, the red arrow represents commodity loss from the pipeline

\[ Q_R = Q_f \]  

(11)

Natural gas flow in horizontal pipeline is modelled by the Weymouth’s equation as given below.

\[ q = 3.23 \frac{T_b}{P_b} \sqrt{\frac{1}{f}} \sqrt{\frac{(P_1^2 - P_2^2) \gamma}{\gamma + 2L}} \]  

(12)

Where \( q \) = gas flowrate in ft³/hr, \( T_b \) = base temperature (60°F or 520°F), \( P_b \) = base pressure (14.73psia), \( f \) = Fanning friction factor, \( P_1 \) = inlet pressure, psia, \( P_2 \) = outlet pressure, psia, \( \gamma \) = gas gravity, \( T \) = average gas temperature, °R, \( \bar{z} \) = gas compressibility factor, \( L \) = Length of the pipeline, miles
The Weymouth equation given above in equ (12) can be adjusted to give
\[ q = 3.23 \frac{T_b}{P_b} \sqrt{\frac{L}{g}} \left( \frac{b^5}{g^2LT} \right)^{0.5} \left[ (P_1^2 - P_2^2)^{0.5} \right] \] (13)

2.3. Leak Equation Development

The general equation for natural gas flow in pipeline is given in compact form as
\[ q = k(P_1^2 - P_2^2)^{0.5} \] (14)

This is similar to the Weymouth’s equation given above where k being the constant of proportionality represents the non-pressure terms in the RHS of the Weymouth’s equation. Thus k is given as follows
\[ k = 3.23 \frac{T_b}{P_b} \sqrt{\frac{L}{g}} \left( \frac{b^5}{g^2LT} \right)^{0.5} \] (15)

When leak occurs in the pipeline, the pipeline can be modelled by taking into consideration two sections of the pipeline. The first section before the leak point is the upstream section which starts from the pipe inlet to the point of leak. The downstream section continues from the point of leak to the outlet of the pipeline. The point of leak is called the midstream; it is not actually a section but a singularity point from which references to the two sections of the pipeline are being made.

2.3.1. Considering the Upstream Section of the Pipeline

If we consider the upstream section of the pipeline, the flow equation for gas in this section is given below
\[ Q_f = k_f(P_1^2 - P_f^2)^{0.5} \] (16)

Where
Q_f = the flowrate at the point where leak has occurred
P_f = the pressure midstream of pipeline at the instant just before leak occurs
k_f = the constant of proportionality in the leak situation similar to k above with L replaced by leak point (X)

Equation 14 describes the fluid flow equation in the pipeline in the absence of leak while equation 16 describes the fluid flow in the upstream section of the pipeline when leak has occurred.

Combining equations 14 and 15 and taking ratios
\[ \frac{Q_f}{q} = \frac{k_f(P_1^2 - P_f^2)^{0.5}}{k(P_1^2 - P_2^2)^{0.5}} \] (17)

Factoring out the k terms the equation becomes
\[ \frac{k_f}{k} = \left( \frac{q}{Q_f} \right) \left( \frac{P_1^2 - P_f^2}{P_1^2 - P_2^2} \right)^{0.5} \] (18)

\[ k_f = k \left( \frac{q}{Q_f} \right) \left( \frac{P_1^2 - P_f^2}{P_1^2 - P_2^2} \right)^{0.5} \] (19)

Where
\[ k_f = 3.23 \frac{T_b}{P_b} \sqrt{\frac{L}{g}} \left( \frac{b^5}{g^2LT} \right)^{0.5} \] (20)

Where X is the leak location
Substituting the values of k and k_f into the equation yields
\[ 3.23 \frac{T_b}{P_b} \sqrt{\frac{L}{g}} \left( \frac{b^5}{g^2LT} \right)^{0.5} = 3.23 \frac{T_b}{P_b} \sqrt{\frac{L}{g}} \left( \frac{b^5}{g^2LT} \right)^{0.5} \left( \frac{Q_f}{q} \right) \left( \frac{P_1^2 - P_f^2}{P_1^2 - P_2^2} \right)^{0.5} \] (21)

\[ 3.23 \frac{T_b}{P_b} \sqrt{\frac{L}{g}} \left( \frac{b^5}{g^2LT} \right)^{0.5} \left( \frac{1}{X} \right)^{0.5} = 3.23 \frac{T_b}{P_b} \sqrt{\frac{L}{g}} \left( \frac{b^5}{g^2LT} \right)^{0.5} \left( \frac{q}{Q_f} \right) \left( \frac{P_1^2 - P_f^2}{P_1^2 - P_2^2} \right)^{0.5} \] (22)

Eliminating same factors gives rise to
\[ \frac{1}{X^{0.5}} = \left( \frac{1}{1} \right)^{0.5} \left( \frac{q}{Q_f} \right) \left( \frac{P_1^2 - P_f^2}{P_1^2 - P_2^2} \right)^{0.5} \] (23)

\[ \frac{1}{X^{0.5}} = \frac{1}{1} \left( \frac{q}{Q_f} \right) \left( \frac{P_1^2 - P_f^2}{P_1^2 - P_2^2} \right)^{0.5} \] (24)

Taking reciprocal of both sides of equation (3.33) yields
\[ X^{0.5} = \left( \frac{q}{Q_f} \right) \left( \frac{P_1^2 - P_f^2}{P_1^2 - P_2^2} \right)^{0.5} \] (25)

There is a presence of negative pressure wave that travels forward and backwards from the leak point. That is to say the negative pressure wave travels along both the upstream and downstream sections of the pipeline. If we consider the upstream section of the pipeline, the negative pressure wave travels counterclockwise in a reverse direction. Thus the effect of this is notice in the flowrate term of equation 25. The flowrate term reverses because of the effect of negative pressure wave to equation 26
\[ X^{0.5} = \left( \frac{q}{Q_f} \right) \left( \frac{P_1^2 - P_f^2}{P_1^2 - P_2^2} \right)^{0.5} \] (26)

Squaring both sides of the equation gives
\[ X = L \left[ \left( \frac{q}{Q_f} \right)^2 \left( \frac{P_1^2 - P_f^2}{P_1^2 - P_2^2} \right) \right] \] (27)

Since the point X is the leak location and it is determined from the upstream section of the pipeline, we shall denote upstream leak location to be X1
\[ X_1 = L \left( \frac{q}{Q_f} \right)^2 \left( \frac{P_1^2 - P_f^2}{P_1^2 - P_2^2} \right) \] (28)

2.3.2. Considering the Downstream Section of the Pipeline

Considering the downstream section of the pipe. The earlier assumption is that flowrate in upstream section of the pipe is uniform while the flowrate in downstream section is also uniform and equal to the recorded output flowrate, Q_R (i.e. from equation 11).

In the downstream section of the pipeline,
\[ Q_R = k_R(P_f^2 - P_R^2)^{0.5} \]  \hspace{1cm} (29)

Where
\[ Q_R = \text{the recorded flowrate at the output section of the pipeline in ft}^3/\text{hr} \]
\[ P_R = \text{the recorded output pressure in psi} \]
\[ k_R = \text{Constant which comprise all the other terms in the Weymouth’s equation.} \]

\[ k_R = 3.23 \frac{T_b}{P_b} \sqrt{\left( \frac{D^5}{YF(L-X)} \right)^{0.5}} \]  \hspace{1cm} (30)

Comparing equation 14 with equation 16 we have that
\[ \frac{q_R}{q} = \frac{k_R(P_f^2 - P_R^2)}{k(\Delta P)^{0.5}} \]

Putting in the values of \( k_R \) and \( k \) the equation becomes
\[ \frac{q_R}{q} = \frac{3.23 \frac{T_b}{P_b} \sqrt{\left( \frac{D^5}{YF(L-X)} \right)^{0.5}}}{3.23 \frac{T_b}{P_b} \sqrt{\left( \frac{D^5}{YF(L-X)} \right)^{0.5}}} \]

Solving the equation above gives
\[ \frac{q_R}{q} = \frac{10^{0.5}(P_f^2 - P_R^2)^{0.5}}{(L-X)^{0.5}(P_f^2 - P_R^2)^{0.5}} = \left[ \frac{L(P_f^2 - P_R^2)}{(L-X)(P_f^2 - P_R^2)} \right]^{0.5} \]  \hspace{1cm} (33)

Equation 38 is used to determine the leak distance by considering flow from the downstream end of the pipe.

\[ (L-X) = \left( \frac{q}{q_R} \right)^2 \left( \frac{P_f^2 - P_R^2}{P_f^2 - P_R^2} \right) \]  \hspace{1cm} (35)

Making \( X \) the subject of formula gives
\[ X = L \left[ 1 - \left( \frac{q}{q_R} \right)^2 \left( \frac{P_f^2 - P_R^2}{P_f^2 - P_R^2} \right) \right] \]

Since the leak location \( X \) here is determined from the downstream section of the pipeline we shall denote it as \( X_2 \)
\[ X_2 = L \left[ 1 - \left( \frac{q}{q_R} \right)^2 \left( \frac{P_f^2 - P_R^2}{P_f^2 - P_R^2} \right) \right] \]  \hspace{1cm} (38)

\section*{3 Results}

The results given below shows the leak locations and the pressure in the pipeline at the point of leak, calculated from the various input variables from each case give in table 1 above.

\begin{table}  
| Table 1. Data Used for Simulations in This Work. |
|-----------------------------------------------|
| **Parameters**                             | **Input Values** |
| Pipeline length (miles) | Case 1 | Case 2 | Case 3 | Case 4 | Case 5 | Case 6 | Case 7 |
| 150 | 200 | 180 | 150 | 160 | 120 | 150 |
| 16 | 20 | 12 | 14 | 15.5 | 12.09 | 14 |
| 1020 | 1200 | 1100 | 950 | 1250 | 1000 | 1080 |
| 500 | 550 | 600 | 750 | 800 | 720 | 680 |
| 100 | 100 | 100 | 100 | 80 | 80 | 100 |
| 60 | 60 | 60 | 60 | 60 | 60 | 60 |
| 0.9 | 0.95 | 0.92 | 0.92 | 0.85 | 0.88 | 0.95 |
| 0.65 | 0.65 | 0.6 | 0.65 | 0.6 | 0.67 | 0.65 |
| 4.164 | 7.654 | 1.884 | 2.108 | 4.365 | 1.746 | 2.679 |
| 3.333 | 6 | 1 | 1.8 | 3.88 | 1.4 | 2 |
| 14.7 | 14.7 | 14.7 | 14.7 | 14.7 | 14.7 | 14.7 |
| 520 | 520 | 520 | 520 | 520 | 520 | 520 |

\begin{table}  
| Table 2. Results of simulation for various cases. |
|-----------------------------------------------|
| **Case** | **Leak Location (Miles)** | **Leak Location (Km)** | **Pressure In Pipeline at Leak Point (Pf)** |
| Case 1 | 35.05 | 56.39545 | 867.27 |
| Case 2 | 31.78 | 51.13402 | 1071.2 |
| Case 3 | 2.34 | 3.76506 | 1082 |
| Case 4 | 96.56 | 155.365 | 812.83 |

\[ L = (L-X) \left( \frac{q}{q_R} \right)^2 \left( \frac{P_f^2 - P_R^2}{P_f^2 - P_R^2} \right) \]

\[ X = L \left[ 1 - \left( \frac{q}{q_R} \right)^2 \left( \frac{P_f^2 - P_R^2}{P_f^2 - P_R^2} \right) \right] \]

\[ X_2 = L \left[ 1 - \left( \frac{q}{q_R} \right)^2 \left( \frac{P_f^2 - P_R^2}{P_f^2 - P_R^2} \right) \right] \]  \hspace{1cm} (38)
3.1. Comparison of Model with Actual Field Results

The table given below shows the result calculated from the model and the one from the actual field.

| Case | Leak Distance From Model (Km) | Actual Field Results | Absolute Error | % Error |
|------|------------------------------|----------------------|----------------|---------|
| case 1 | 56.39545 | 56.15 | 0.00437 | 0.437133 |
| case 2 | 51.13402 | 50.98 | 0.00302 | 0.302118 |
| case 3 | 3.76506 | 3.78 | 0.003952 | 0.395238 |
| case 4 | 105.293 | 104.9 | 0.00375 | 0.374604 |
| Average error | | | | 0.377273 |
| Maximum error | | | | 0.437133 |
| Minimum error | | | | 0.302118 |
| Standard deviation | | | | 0.056454 |

The average error gotten from the leak location when compared with field data is 0.377%.

3.2. Sensitivity Analyses

For each of the following cases, the pressure profile for the leak and no leak case is given in graphical form.

Case 1

Table 4. Pressure profile data for leak and no leak for case 1.

| No Leak Case | Leak Case |
|--------------|-----------|
| Length (miles) | Pressure (psi) | Length (miles) | Pressure (psi) |
| 0 | 1020 | 0 | 1020 |
| 20 | 966.9444 | 20 | 966.9444 |
| 35.047 | 925.1158 | 35.047 | 867.27 |
| 40 | 910.9096 | 150 | 603.38 |
| 80 | 786.8371 | 100 | 716.7921 |
| 120 | 639.1158 | 140 | 807.6 |
| 140 | 550.588 | 180 | 645.7 |
| 150 | 500.486 | | |

Figure 4. Pressure profile chart showing leak and no leak situations for case 1.

From figure 4, it is seen that the leak occurred at 35 miles from the inlet point. The pressure profile for the leak in the event of leak depicts reduction in flowing fluid pressure until a point 100 miles from the pipeline inlet point where the pressure has stabilized and continues to increase with increase in pipe length.

Case 2

Table 5. Pressure profile data for leak and no leak for case 2.

| No Leak Case | Leak Case |
|--------------|-----------|
| Length (miles) | Pressure (psi) | Length (miles) | Pressure (psi) |
| 0 | 1200 | 0 | 1200 |
| 20 | 1151.7 | 20 | 1151.7 |
| 31.777 | 1122.2 | 31.777 | 1071.2 |
| 40 | 1101.2 | 200 | 745.95 |
| 80 | 992.6 | 100 | 933.6 |
| 120 | 870.6 | 120 | 802.7 |
| 140 | 807.6 | 140 | 872.4 |
| 160 | 728.4 | 160 | 659.7 |
| 180 | 550.7 | 180 | 550.7 |
| 200 | 550.7 | | |

Figure 5. Pressure profile chart showing leak and no leak situations for case 2.

From figure 5 (For case 2), the leak occurred at 31.78 miles from the inlet point. This gave a sharp drop in the pressure inside the pipeline corresponding to the point of the leak. This pressure reduction decreases to a stabilization point at 100 miles. At this point the pressure in the pressure of the pipeline in the event of leak is equal to the pressure in the pipeline in the no leak situation at the same location (100 miles from inlet).

Case 3

Table 6. Pressure profile data for leak and no leak for case 3.

| No Leak Case | Leak Case |
|--------------|-----------|
| Length (miles) | Pressure (psi) | Length (miles) | Pressure (psi) |
| 0.2 | 1099.6 | 0.2 | 1099.6 |
| 0.5 | 1098.9 | 0.5 | 1098.9 |
| 1 | 1097.9 | 1 | 1097.9 |
| 2.337 | 1095 | 2.337 | 1082 |
| 10 | 1078.3 | | |
From figure 6 (case 3), the leak occurred at an early point. The leak location is at 2.34 miles from the inlet point. The pressure stabilized quickly since the leak occurred at the early point of the pipeline. The stabilization point of the leak pressure profile with the no-leak pressure profile as seen from figure 4 is at 20 miles from the inlet point.

Case 4

Table 7. Pressure profile data for leak and no leak for case 4.

| Distance (miles) | No Leak Case Pressure (Psi) | Leak Case Length (miles) | Pressure (Psi) |
|-----------------|-----------------------------|--------------------------|----------------|
| 0               | 950                         | 0                        | 950            |
| 10              | 938.0023                    | 10                       | 938.0023       |
| 20              | 925.8492                    | 20                       | 925.8492       |
| 40              | 901.0514                    | 40                       | 901.0514       |
| 60              | 875.5515                    | 60                       | 875.5515       |
| 80              | 849.2863                    | 80                       | 849.2863       |
| 96.56           | 826.9075                    | 96.56                    | 812.83         |
| 120             | 794.1541                    | 150                      | 742.33         |
| 140             | 765.0997                    |                          |                |
| 150             | 750.1506                    |                          |                |

Table 8. Summary of Leak Location Results.

| Case | Pipeline length (miles), L | Leak Location (Miles), X1 | Leak Location (Miles), X2 |
|------|----------------------------|---------------------------|---------------------------|
| Case 1 | 150                       | 35.05                     | 35.04                     |
| Case 2 | 200                       | 31.78                     | 31.77                     |
| Case 3 | 180                       | 2.34                      | 2.34                      |
| Case 4 | 150                       | 96.56                     | 96.56                     |
| Case 5 | 160                       | 139.5                     | 139.51                    |
| Case 6 | 120                       | 86.48                     | 86.48                     |
| Case 7 | 150                       | 34.94                     | 34.94                     |

4. Discussions

1. Case 1: From the table 2 above it is seen that the leak location calculated occurred at a distance of 56.395km (35.05 miles) from pipeline inlet point and pressure in the pipeline at the leak point of 867.27 psia. Pressure in the pipeline at the leak point gave a reasonable prediction by the model since the input pressure and output pressure for case 1 are 1020 psi and 500psi respectively. It is expected that the leak point pressure be lower than the input pressure and higher than the output pressure.

2. Case 2: From table 2 above case 2 predicts the leak location to be at 51.13km (31.78 miles) and pressure in the pipeline at the leak point is 1071.2psi. Also the pressure in the pipeline at the leak point is between the input pressure (1200psi) and the output pressure for no leak (550 psi).

3. Case 3: From table 2 above, the model estimated the leak location for the case 3 to be at 3.77km (2.34 miles) and pressure in the pipeline at the leak point is1082psi. The initial input and output pressures for the no-leak situation is given as 1100psi and 600psi. The pressure in the pipeline at the leak point as expected falls between the initial input and output pressures.

4. Case 4: From table 2 above, the leak location for case 4 was calculated to be at 155.365km from the inlet point. The leak point pressure is calculated to be 812.83psi. The input and output pressure for the initial no-leak situation are 950psi and 750psi respectively. The leak point pressure also falls between the input and output pressure for the no-leak case.

5. From table 3, the average percentage error from the proposed model is 0.377%. This is lower than the model proposed by Boaz et al. (2014) with average error of 3.575% which was acceptable for leak location. Thus the proposed model gives a more accurate estimation of the leak location in the event of leak in natural gas pipeline.

5 Conclusions

Mathematical models have been developed for leak location in natural gas pipeline. The models developed were in two-fold, the upstream and the downstream leak location
models. The models instantaneously estimate the location of leak upon occurrence. Either of the model can be applied without necessarily applying the two for leak location. The models have been proven to yield the same result and the results showed high accuracy when compared with data from actual leak locations from operating companies with only an average error of 0.377%. The model relies highly on the accuracy of the input data which are hydraulic parameters of the operating pipeline. This work is recommended for instant leak location in gas pipelines especially when physical inspection methods proves costly and would interrupt normal pipeline operations.

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