Real-space imaging of a topologically protected edge state with ultracold atoms in an amplitude-chirped optical lattice

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To describe a mobile defect in polyacetylene chains, Su, Schrieffer and Heeger formulated a model assuming two degenerate energy configurations that are characterized by two different topological phases. An immediate consequence was the emergence of a soliton-type edge state located at the boundary between two regions of different configurations. Besides giving first insights in the electrical properties of polyacetylene materials, interest in this effect also stems from its close connection to states with fractional charge from relativistic field theory. Here, using a one-dimensional optical lattice for cold rubidium atoms with a spatially chirped amplitude, we experimentally realize an interface between two spatial regions of different topological order in an atomic physics system. We directly observe atoms confined in the edge state at the intersection by optical real-space imaging and characterize the state as well as the size of the associated energy gap. Our findings hold prospects for the spectroscopy of surface states in topological matter and for the quantum simulation of interacting Dirac systems.

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Topological states of matter, as quantum Hall systems or topological insulators, cannot be distinguished from ordinary matter by local measurements in the bulk of the material. Instead, global measurements are required, revealing topological invariants as the Chern number. At the heart of topological materials are topologically protected edge states that occur at the intersection between regions of different topological orders. Ultracold atomic gases in optical lattices are promising new platforms for topological states of matter, though the observation of edge states has so far been restricted in these systems to the space state imposed by the internal atomic structure.

Here we report on the observation of an edge state between two topological distinct phases of an atomic physics system in real space using optical microscopy. An interface between two spatial regions of different topological order is realized in a one-dimensional optical lattice of spatially chirped amplitude. This novel experimental technique realizes a cold atom system described by a Dirac equation with an inhomogeneous mass term closely related to the so-called SSH model by Su, Schrieffer and Heeger. The observed edge state is characterized by measuring the overlap to various initial states, revealing that this topological state has singlet nature in contrast to the other system eigenstates, which occur pairwise. We also determine the size of the energy gap to the adjacent eigenstate doublet.

**Results**

**Background.** Figure 1a shows a schematic of the spatial variation of the relevant band structure of rubidium atoms in the one-dimensional lattice. The lattice has a spatial periodicity of \( \lambda / 4 \), where \( \lambda \) denotes the wavelength of the driving laser beams, with the corresponding potential being due to the dispersion of Raman transitions. A magnetic field gradient imprints a spatially slowly varying lattice amplitude (in comparison with the scale of the lattice periodicity) via a modification of the Raman detuning. The achieved coupling \( K(z) \) between momentum eigenstates \( \exp(\pm 2i k z) \) determining the lattice amplitude, with \( k = 2\pi / \lambda \), varies spatially along the \( z \) axis. For not too large values of \( z \) the coupling follows \( K(z) = a \cdot z \), where \( a \) is a constant, and thus changes sign from negative to positive values of \( z \). Here \( K(z) \) varies only slowly in space and it is therefore useful to discuss the bandstructure for fixed \( K(z) \). At position \( z_0 \), where \( K(z) \) is a constant, the coupling \( K \) between momentum eigenstates vanishes, a crossing of Bloch bands occurs and the ordering of bands, indicated as \( | - \rangle \) and \( | + \rangle \), respectively, in Fig. 1a, is inverted. By continuous deformation the bands cannot be transformed into each other without closing the gap between bands. For such a situation a topologically protected edge state, localized around \( z = 0 \), where the bands intersect, is expected. Formally, this can be seen with a simple model describing the system near the crossing by a one-dimensional Dirac Hamiltonian with a spatially dependent effective mass \( m_{\text{eff}}(z) = a \cdot z / c_{\text{eff}}^2 \):

\[
\hat{H}_0 = m_{\text{eff}}(z) c_{\text{eff}}^2 \sigma_x + \hat{q} c_{\text{eff}} \sigma_z
\]

where \( \hat{q} = -i \hbar \partial_z \) is the momentum operator, \( \sigma_x \) and \( \sigma_z \) are Pauli matrices, and \( c_{\text{eff}} = 2\hbar k / m \approx 1.1 \text{ cm s}^{-1} \) is an effective speed of light, with \( m \) as the rubidium atomic mass. The component Hamiltonian acts on spinors \( \psi(z) = (\Psi_1(z), \Psi_2(z)) \), with \( \Psi_1 \) and \( \Psi_2 \) corresponding to wavefunctions of atoms with momenta close to \( \pm 2\hbar k \), respectively. The total wavefunction is given by \( \phi(z) = \Psi_1(z) e^{2i k z} + \Psi_2(z) e^{-2i k z} \). The eigenenergies of the system are readily found to be \( E_0 = 0 \) and \( E_n = \pm \sqrt{n} \cdot \hbar \omega_0 \) for \( n > 0 \), with \( \omega_0 = \sqrt{2 m_{\text{eff}} a / \hbar} \). For quantum numbers \( n > 0 \) the solution comes in pairs of opposite energies, while for \( n = 0 \) there is only a singlet eigenstate, the topological edge state at the energetic position of the band crossing, which originates from the vanishing energy gap at the interface. The wavefunction of this state is

\[
\psi_0(z) = \frac{1}{\sqrt{2}} \left( \frac{a}{\pi c_{\text{eff}} \hbar} \right)^{1/4} \exp(-a z^2 / (2 c_{\text{eff}} \hbar)) \cdot \left( \begin{array}{c} 1 \\ -i \end{array} \right)
\]

which is the product of a Gaussian envelope and a coherent superposition of the two momentum eigenstates with a relative phase of \( \pi / 2 \). As suggested by the discrete nature of the eigenenergies, all system eigenstates are bound states. Figure 1b shows the spatial variation of the probability density for the topological edge state (middle) and the doublet states with \( n = 1, 2 \) and \( 3 \) (the corresponding plots on the top and bottom, respectively).

**Experimental realization.** Our experiment starts by initially cooling a dilute cloud of rubidium atoms in the \( ^{87}\text{Rb} \) in the \( m_f = 1 \) spin projection of the \( F = 1 \) hyperfine component to Bose–Einstein condensation in a combined optical dipole and magnetic trap. A magnetic field gradient of magnitude

![Figure 1](https://example.com/figure1.png)

**Figure 1 | Band structure and eigenstates.** (a) Illustration of spatial variation of the band structure of the lowest two bands denoted as \( |+\rangle \) and \( |-\rangle \), respectively, along position \( z \). The edge state, indicated by the green line at \( z = 0 \), is located between two regions of different band ordering defining the topology, with \( z < 0 \) and \( z > 0 \) respectively. (b) Probability distribution of eigenstates versus position \( z \) in units of \( \Delta z = \sqrt{c_{\text{eff}} \hbar / 2 m} \) for the states with quantum numbers \( n \leq 3 \). The middle green plot refers to the topological edge state with \( n = 0 \), which is a singlet. All other eigenstates are doublets, see the plots on the top (blue) and bottom (red), with energies \( E = \hbar \omega_0 \sqrt{n} \) and \( E = - \hbar \omega_0 \sqrt{n} \), respectively. For the explicit analytical form of the wavefunction of the states with \( n > 0 \), see Methods.
m \cdot g/|\mu_B/2| \approx 30.5 \text{ G cm}^{-1} \text{ compensates for the Earth’s gravitational acceleration. The cold atomic cloud is subsequently adiabatically expanded along the } z \text{ axis to match the size of the topological edge state, of expected width } \Delta z = \sqrt{\epsilon_{c0} \hbar / 2a} \approx 11.5 \text{ } \mu \text{m}, \text{ to obtain sufficient spatial overlap. The typical experimental width of the momentum distribution at this point is } \Delta p_z \approx 0.01 \text{hk.}

The lattice potential is realized using a rubidium atomic three-level configuration with two ground states of different spin projections and one spontaneously decaying excited state \( \left| \psi_1 \right\rangle \), \( \left| \psi_0 \right\rangle \), and \( \left| \psi_e \right\rangle \). A magnetic field gradient causes a spatial variation of the Raman detuning \( V(z) \) with envelopes \( V_{1,0}(z) = \hbar \Omega_1,0 \Omega^{\text{eff}} / \delta_0 \approx 0.36 \cdot E_r \), where \( E_r = \hbar^2 k^2 / (2m) \) denotes the recoil energy, and \( a = V_{0,0}(d B / d z) / (4 \hbar \Omega_1) \approx 19.0 \cdot E_r \text{ cm}^{-1} \). The total lattice potential can be written in the simple form \( V(z) = 2a \cdot z \cos(4kz) \), which experiences a zero crossing at \( z = 0 \). Note that for \( z > 0 \) (\( z < 0 \)) the maxima (minima) of the potential are located at integer multiples of \( \lambda / 4 \) (Fig. 2b). This phase change is reflected in the inversion of band ordering. The dynamics of atoms in such a structure near the band crossing is described by the Dirac Hamiltonian of equation (1) with good accuracy (see Methods), as the width of the topological bound state \( \Delta z \) is two orders of magnitude larger than the lattice spacing.

Characterization of the spatial variation of the band structure.

In preparatory experiments we have characterized the band structure of the one-dimensional lattice focusing on the band-inversion on sign change of \( z \). For this, the adiabatically expanded atomic cloud centred at different lateral positions \( z_0 \) along the lattice beam axis (described by a wavefunction \( \phi_n(z-z_0) \)) is transferred to the state \( \phi_1(z) = \frac{1}{\sqrt{2}} \phi_0(z-z_0)(e^{2ikz} + e^{-i2kz}) \) via two simultaneously performed Bragg pulses \( V_{0,0}(z) \), described by the spinor \( \psi(z) = \phi_1(z-z_0) \) with \( |\pm\rangle = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 1 \\ \pm 1 \end{array} \right) \) (Fig. 1). To verify whether the cloud at the corresponding position overlaps with the upper or the lower band in the lattice, respectively, the band populations following activation of the lattice are determined. As shown in Fig. 3, for the chosen initial state we find that for \( z_0 < 0 \) the loading is enhanced into the lower band, while for \( z_0 > 0 \) most atoms are transferred into the upper band, and near \( z_0 = 0 \) the curves cross. This experimentally verifies the expected spatial variation of the band structure, exhibiting a sign change of the coupling between momentum eigenstates at \( z_0 = 0 \).

Figure 2 | Experimental schematic. (a) Four-photon scheme for generation of amplitude-chirped lattice potentials \( V_1(z) \) \( (\text{a}) \) and \( V_2(z) \) \( (\text{b}) \). The scheme is based on three-level atoms with ground state sublevels \( |m_1\rangle = -1 \rangle \) and \( |m_0\rangle = 0 \rangle \); and the excited manifold \( |e\rangle \), driven by an optical field of frequency \( \omega \) and two counterpropagating fields of frequencies \( \omega \pm \Delta \omega_{1,2} \). A magnetic field gradient causes a spatial variation of the Raman-detuning \( \delta_{1,2}(z) \). (c) The plot for \( V_1(z) \) \( (\text{c}) \) and \( V_2(z) \) \( (\text{c}) \) indicates the spatial variation of the lattice depth, which increases (decreases) along \( z \) for positive (negative) values of the Raman detuning. A lattice with band structure as shown in Fig. 1a is obtained by superimposing \( V_1(z) \) and \( V_2(z) \). (d) gives a schematic of the set-up. In the real experiment, the magnetic field gradient is directed vertically.
**Imaging cold atoms at the topological interface.** We have next loaded atoms into the topological edge state. For this, using the Bragg pulses atoms are transferred to an initial state \( \phi_1(z) = \frac{1}{\sqrt{2}} \phi_0(z) (e^{ikz} + e^{-ikz}) \) with \( \phi = \pi/2 \). The atomic wavepacket is centred at \( z = 0 \), after which the lattice beams are activated. As the Bragg pulse increases the kinetic energy by more than two orders of magnitude, while the interaction energy remains largely unaffected, it turns out (see Methods) that interactions can be ignored for the subsequent time evolution.

Figure 4a (top) shows a series of atomic absorption images recorded after a variable holding time in the lattice along with a simulation (bottom). We observe that the atomic cloud remains recorded after a variable holding time in the lattice along with a residual expansion is a remaining mismatch of the initially prepared atomic wavepacket with the topological edge state, causing an admixture of eigenstates with larger values of \( n \) and correspondingly increased mode volume. This effect was accounted for in the model simulations shown in Fig. 4 (bottom), see Methods.

**Phase dependency and loading efficiency.** Figure 5a shows a series of absorption images recorded after a fixed time \( t_{\text{int}} = 1.7 \) ms for different values of the relative phase between momentum components of the initially prepared atomic wavepacket. Near a relative phase of \( \phi = \pi/2 \) we again observe a compact atomic cloud, while for \( \phi = -\pi/2 \) the cloud is split up into two components. A smooth variation between these extremes is visible for intermediate phase values. The corresponding variation of the total root mean squared (r.m.s.) width of the atomic cloud along the \( z \) axis versus the phase \( \phi \) is shown in Fig. 5b.

We have next modified the overlap of the initial state and the topological state by preparing clouds with smaller initial size and correspondingly larger momentum spread by using different final

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**Figure 3 | Spatial variation of band ordering.** Relative atomic population transferred into the upper (green circles) and lower (blue squares) band on loading from an initial state \( \phi_1(z) = \frac{1}{\sqrt{2}} \phi_0(z - z_0) (e^{ikz} + e^{-ikz}) \) with an atomic cloud centred at position \( z_0 \). The band populations were determined by accelerating the lattice away from the crossing to a relative wavevector of \( 0.5\hbar k \) so that a mapping onto the free atomic eigenstates occurs, and then applying time-of-flight imaging\(^2\). Each point corresponds to the average of three corresponding data sets, and the shown error bars are the s.d. of the mean.

**Figure 4 | Temporal evolution of atomic clouds.** Series of absorption images (top) for a relative phase of the initially prepared atomic wavepacket of (a) \( \phi = \pi/2 \) and (b) \( \phi = -\pi/2 \) for different holding times in the lattice. The temporal step size between images is 0.1 ms, and the measured optical column density is encoded in colour code. For \( \phi = \pi/2 \) we observe a trapping of atoms in the topological edge state, while for \( \phi = -\pi/2 \) the cloud splits up. Each image is the average over four realizations. The bottom plots are numerical simulations taking into account the experimental resolution of \( 4.8 \mu \text{m} \) (see Methods).
values of the dipole trapping potential during adiabatic expansion. As described above, adiabatic expansion of the condensate cloud yields atomic ensembles with down to 0.01\textit{\textmu}k momentum width, corresponding to effective temperatures in the PK regime. The dots in Fig. 5c show the relative variation of the total cloud width on the phase versus the momentum width of the atomic cloud. For a larger momentum spread of the initial wavepacket the observed phase dependency of the total cloud size reduces. This is attributed to the higher order system eigenstates with $n > 0$, which are populated when loading with a wavepacket of larger momentum width than of the topological protected edge state. The experimental results are in agreement with a simulation (solid line). The inset of Fig. 5c shows the expected variation of the loading efficiency into the edge state on the momentum width of the cloud.

FIGURE 5 | Phase and momentum width dependence of loading. (a) Series of absorption images for different relative phases of the Raman state preparation beams. (b) Corresponding variation of the r.m.s. widths along the z axis of the distribution (blue dots) along with a sinusoidal fit (solid line). Each data point is the average over four realizations, and the shown error bars correspond to the s.d. of the cloud width variation and of the mean momentum width, respectively. The solid green line is a simulation. (c) Relative variation of the cloud width along the z axis on the phase, as determined from the ratio $(\Delta \sigma_{\text{max}}(\text{int}) - \Delta \sigma_{\text{min}}(\text{int}))/((\Delta \sigma_{\text{max}}(\text{int}) + \Delta \sigma_{\text{min}}(\text{int})))$ of the sinusoidal fit, versus the momentum width of the atomic cloud. Here $\Delta \sigma_{\text{max}}$ ($\Delta \sigma_{\text{min}}$) denote the maximum (minimum) r.m.s. cloud width at the used $t_{\text{int}} = 1.7$ ms lattice interaction time. The shown error bars correspond to the s.d. of the cloud width variation and of the mean momentum width, respectively. The solid green line is a simulation. The inset shows the calculated loading efficiency into the topological edge state (for $\varphi = \pi/2$) versus the momentum width.

**Discussion**

To conclude, an edge state at the spatial interface between two regions of different topological order has been observed by real-space imaging of the cold atomic cloud. Evidence for the successful population of this topological state has been obtained from (i) the phase dependence, (ii) the dependence of the initial atomic momentum width on the loading efficiency and (iii) the vibrational frequency of the lowest excited doublet modes agreeing with expectations.

For the future, it will be important to reveal the role of interactions on the topological edge state using, for example, Feshbach resonances for sensitive control, with prospects including the simulation of interacting topological quantum matter. Other perspectives include the realization of interacting relativistic wave equation predictions, as well as novel topological Berry phase effects in phase space.
dipole trapping potential. This value corresponds to an effective kinetic temperature of 35 pK. Regarding the case of free atomic clouds, 2D matter wave imaging experiments with a rubidium Bose–Einstein condensate have achieved effective temperatures of 50 pK (ref. 28). For the data points with higher momentum spread shown in Fig. 5c, the adiabatic expansion of the condensate cloud proceeded to less low values of the trap potential, for which correspondingly also \( \phi_3 \) at the end of the ramp was higher.

The optical radiation required to generate both the optical Bragg pulses and the optical lattice is derived from a single-mode high-power diode laser operating near 783.5 nm wavelength, 3.3 nm detuned to the red of the rubidium D2-line. Its optical lattice is approximately in the experimentally relevant range (with the next term in the corresponding Taylor expansion \((x^2 + y^2)\) reaching about 25% of the linear term at positions \(z = \pm 5.0 \mu m\)).

Both during the evaporative cooling and during the time that the lattice is switched on, the magnitude of the field gradient is set by the requirement that the Stern–Gerlach force compensates the Earth’s gravitational acceleration. However, besides imprinting the periodic lattice potential, the Raman coupling induced by the lattice beams causes a small chirp of the mR0 component of the \(1S\) Zeeman state. This reduces the effective magnetic moment, and thus the Stern–Gerlach force by a corresponding amount on activation of the lattice beams. For the data shown in Figs 4 and 5, this was compensated for by a small chirp of the lattice beams.

To synthesize the amplitude-chirped lattice we use lattice potentials realized in a double-photonic lattice: Two potentials, both with a spatial periodicity of \(\lambda/4\), and by suitable choice of the phase values of the driving optical beams, the potential maxima of say \(V(x)\) are tuned to the position of the minima of \(V(y)\), see Fig. 2b of the main text. The total potential experienced by the atoms \(V(z) = V_x(z) + V_y(z)\) can be written in the form \(V(z) = K(z) \cdot 2 \cos(4\pi z)\). With the quoted experimental parameters, the coupling \(K(z)\) is approximately linear in \(z\) in the experimentally relevant range (with the next term in the corresponding Taylor expansion \((x^2 + y^2)\) reaching about 25% of the linear term at positions \(z = \pm 5.0 \mu m\)).

Detection of the atomic cloud following the interaction with the lattice was performed after extinguishing the lattice beams by activating both repumping light (tuned to the \(F = 1 \rightarrow F = 2\) component of the rubidium D2-line) and pulsing on a laser beam tuned to the \(F = 2 \rightarrow F = 3\) cycling component of this transition. This records a shadow image on a CMOS camera (model Andor Zyla 3.5s) to monitor the in situ spatial distribution \(n(x,y)\) of the atomic cloud. For the used magnification of the imaging system, the size of one pixel of the camera corresponds to 0.985 \(\mu m\) in the object plane. The size of the observed atomic cloud images along the \(x\) axis (transversely to the axis of the lattice beams) is dominated by the spatial resolution of our imaging system, in which an independent measurement was determined to 4.8 \(\mu m\) r.m.s. spatial width.

### Theoretical background

Assume that a rapidly oscillating potential \(V(z) = 2K(z) \cos(4\pi z)\) with a smooth envelop function \(K(z)\) (with \(K(z) = a - az\) in our experiment) scatters particles from momentum \(2K\hbar\) to \(-2K\hbar\). Scattering to other momenta is neglected in our treatment due to these states being energetically relatively far from resonance, see also ref. 19. Parametrizing the wavefunction by \(\phi(z, t) = (\psi(z, t) e^{i2Kz} + \psi(z, t) e^{-i2Kz})/2\), the Hamiltonian \(H_0 = \hat{H}_0 + V(z)\) can be mapped to a one-dimensional Dirac equation for the spinor \(\psi = (\psi_1, \psi_2)\) with

\[
\hat{H}_0 = -i \hbar \partial_z \sigma_3 - \hbar K(z) \sigma_z
\]

where \(\sigma_3\) are Pauli matrices and we have linearized the dispersion around \(\pm 2K\hbar\) to obtain the effective speed of light \(c\) = 2K\hbar/m. The Dirac equation possesses an emergent chiral symmetry

\[
\sigma_3 \hat{H}_0 \sigma_3 = -\hat{H}_0
\]

According to Aftalion–Zirnbauer classification scheme\(^{30}\), systems with this symmetry belong to the class BDI. In one dimension such systems possess non-trivial topological states, which can be classified with an integer winding number\(^3\). Most importantly, states with \(K(z) > 0\) and \(K(z) < 0\) belong to two different topological classes, where the difference of winding numbers is 1.
Topological enforces the existence of a bound state with energy 0 given by
\[ \psi(z) \propto e^{-i \int_{0}^{z} \sqrt{E(z')} \, dz'} \]
for \( K(z) = z + i \) to the chiral symmetry (4). For \( K(z) = z - i \) the problem simplifies (and maps to a problem, exactly equivalent to the solution of the 2D Dirac equation in a magnetic field, known from graphene\(^{23}\)). The observation that \( H_D \) maps (up to a spin-dependent shift) to a harmonic oscillator, motivates the introduction of bosonic raising and lowering operators, \( b^\dagger, b \), as used for a standard harmonic oscillator. Thus, the Dirac Hamiltonian (3) is written in the following form
\[ H_D = \hbar \omega_0 \begin{pmatrix} 0 & b^\dagger \\ b & 0 \end{pmatrix} \]
where the matrix is given in the basis \( | \alpha \rangle = (1, -i)/\sqrt{2}, | \beta \rangle = (-i, 1)/\sqrt{2} \) and \( \omega_0 = \sqrt{2 \alpha \hbar / \hbar} \), and \( H_D = (\hbar \omega_0)^2 (b^\dagger b + \frac{1}{2} - \frac{1}{2} \sigma_z) \). The eigenfunctions of \( H_D \) are analytically described by \( \psi_m = 1/\sqrt{2} (| \alpha \rangle + | \beta \rangle) + \psi_{\text{osc}} \), for \( n \geq 1 \), and \( \psi_{\text{osc}} = \phi_{\text{osc}} \). For \( n = 0 \), the bound state is described by \( \phi_{\text{osc}} = 1/\sqrt{2} (| \alpha \rangle - | \beta \rangle) \) with the characteristic frequency of the two peaks during the evolution of the \( a^\dagger a \phi_n \) is populated, and the probability distribution shows a temporal oscillation. On the timescale of the experiment interactions have no observable effect.

**Interaction effects.** Two types of interaction effects have to be considered for our set-up. First, interactions affect the initial state, that is, the shape of the wavefunction before the application of the Bragg pulse. Second, they modify the time evolution after the pulse. We will first show that the second effect is negligible while the first has to be taken into account.

To estimate the importance of interaction effects for the time evolution quantitatively, we consider a simplified one-dimensional situation, where we assume that in perpendicular direction only the lowest-energy states of a harmonic oscillator with frequencies \( \omega_0 / 2n = 28.5 \text{ Hz} \) and \( \omega_0 / 2n = 10 \text{ Hz} \) are occupied. This approximation actually overestimates interaction effects as it understimates the size of the wavefunction in perpendicular directions. After projection onto degrees of freedom described by the Dirac spinor \( \psi = \langle W_1, W_2 \rangle \), one obtains a one-dimensional Dirac equation with local interaction and a corresponding Gross–Pitaevskii–Dirac equation, which takes the form
\[ i \hbar \partial_t \psi = \mathcal{H}_0 \psi + g \mathcal{H}_{\text{int}} (3 | \psi |^2 - (\psi^+ \sigma_y \psi) \sigma_z) \psi \]
with the dimensionless interaction strength given by \( g \approx 0.1 \text{ ms}^{-1} \). Here \( \mathcal{H}_{\text{int}} \) is the scattering length of \(^{87}\text{Rb} \), the Bohr radius, and \( \omega_0, \omega_0 \), the trapping frequency in the \( x \) and \( y \) direction, respectively. We obtain for our set-up

![Figure 7 | Eigenstates and analysis.](image)

**Figure 7 | Eigenstates and analysis.** (a) Integrated atomic density distribution \( n(z) \) for atoms stored in the lattice for \( t = 1.7 \text{ ms} \) with the initial phases \( \phi = \pm \pi/2 \). In the case of \( \phi = \pi/2 \) the distribution has almost a Gaussian shape, while for \( \phi = -\pi/2 \) it has a double peak structure. Each data point correspond to the average splitting of four realizations, and the error bars are the s.d. of the mean. (b) From a sinusoidal fit to the variation of the splitting \( \Delta z \) of the two peaks during the evolution of the atoms in the lattice initially prepared with phase \( \phi = -\pi/2 \) the characteristic frequency is obtained. Each data point correspond to the average splitting of four realizations, and the error bars are the s.d. of the mean.

![Figure 8 | Validity of Dirac equation.](image)

**Figure 8 | Validity of Dirac equation.** (a) The eigenvalues of the microscopic Hamiltonian \( \mathcal{H}_0 \) (circles) close to the band crossing agree with a precision higher than 1% with the energies (crosses) of the Dirac equation. (b) Density profile of the bound state \( n = 0 \) with (dashed line, \( g = 0.5 \)) and without (solid line, \( g = 0 \)) interactions. Changes of the density profile (inset) are of the order of 1%, much smaller than the experimental resolution. (c,d) Time dependence of the density profile calculated from the solution of a Gross–Pitaevskii Dirac equation with and without interactions in the case of an initial phase of \( \phi = -\pi/2 \). Here predominantly the doublet with \( n = 1 \) is populated, and the probability distribution shows a temporal oscillation. On the timescale of the experiment interactions have no observable effect.
g > 0.5. This factor parametrizes the ratio of the interaction energy and the kinetic energy. It is important to note that the dynamics of the condensate is therefore not dominated by interactions (in contrast to the initially prepared cloud, which had a kinetic energy, which was two orders of magnitude smaller). The dominant effect of g is a shift of all energies linear in g, but all observables studied in our experiment are affected only to quadratic order in g as a consequence of the chiral symmetry of $H_0$. A strong pulse Drive shows that all interaction effects turn out to be only of the per cent level and thus negligible within our present experimental resolution.

To study the interaction effects, we have solved both the stationary and the time-dependent Gross–Pitaevskii–Dirac equation and, furthermore, calculated the Bogoliubov spectrum. Figure 8b shows that the shape of the stationary solution of the Gross–Pitaevskii–Dirac equation for $n = 0$ is almost unaffected by the interaction. Similarly, the density oscillations of the superposition of the doublet with n = 1 are indistinguishable for g = 0.5 and 0 (Fig. 8c,d). An important qualitative effect of interactions in the final state is that the system is intrinsically unstable. While for fermions Pauli blocking prohibits the decay of the topological surface state, the bosonic condensate wavefunction is not stable in the presence of interactions. As a consequence, the energies of the Bogoliubov spectrum describing the fluctuations around the stationary solution of the Gross–Pitaevskii–Dirac equation obtain imaginary contributions. An explicit calculation shows, however, that all decay rates are smaller than 1% of the typical oscillation frequency $\omega_0$.

While interaction effects thus do not play a role for the time evolution in our experiment, they do affect the initial state substantially. The main reason for this is that the Bragg pulse is applied the kinetic energy is about two orders of magnitude smaller than after the pulse. Therefore, interaction effects dominate the initial, but not the final state. To gain a quantitative theoretical description of the experiment (lower panels in Fig. 4, green curve in Fig. 5c), we therefore proceed in the following way. To obtain the initial-state wavefunction (after adiabatic expansion, before the Bragg pulse and external lattice is switched on) we first calculate the static solution $\psi_{0}(x,z)$ of the three-dimensional Gross–Pitaevskii equation using ref. 32. After the Bragg pulse (and after switching on of the lattice potential), the wavefunction is described by $\langle \psi_{0}(x,z) | e^{-iHt/C}\rangle/\sqrt{2}$. For the subsequent time evolution, we ignore interaction effects and use the non-interacting Dirac Hamiltonian (3) in combination with simple harmonic oscillator Hamiltonians for the perpendicular directions. The experiment determines the cloud from an absorption image in the $x$-$z$ plane. In the lower panel of Fig. 4 we therefore plot $|\langle \psi_{0}(x,z) | \psi_{0}(x',z') \rangle|^{2}$ convolved with a Gaussian of r.m.s. width 4.8 mm to describe the effects of the finite experimental resolution. Note that the experimental resolution has little effect for the dynamics in $z$ direction and mainly broadens the perpendicular direction. Rapidly oscillating interference fringes arising from $2Re[\langle \psi_{0}(x,z) | \psi_{0}(x',z') \rangle]^{2}$ cannot be resolved within the experimental resolution.

To obtain the green curve in Fig. 5c, we have repeated the above described calculations for several values of $\omega_0$, which gives us initial atomic distributions with different values of the momentum width. The quoted values for the momentum width are determined from $\Delta \rho = \sqrt{\langle p^{2} \rangle}$ in the initial state, the corresponding width of the final oscillating state from the value for $\sqrt{\langle z^{2} \rangle}$, as obtained after interaction with the lattice.

Data availability. The data that support the findings of this study are available from the corresponding authors on reasonable request.

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