Fluctuations of conserved charges in strong magnetic fields

Jun-Hong Liu (刘俊宏)
Central China Normal University (华中师范大学)

H.-T. Ding, S.-T. Li, Q. Shi, X.-D. Wang, Eur.Phys.J.A 57 (2021) 6, 202
H.-T. Ding, S.-T. Li, J.-H. Liu, X.-D. Wang, arXiv:2208.07285

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Strong magnetic fields in heavy-ion collisions

\[ eB_{\tau=0} \sim 3M_{\pi}^2 \text{ in RHIC} \]
\[ eB_{\tau=0} \sim 40M_{\pi}^2 \text{ in LHC} \]

V. Skokov et al. Int. J. Mod. Phys. A24 (2009) 5925
Wei-Tian Deng et al. Phys.Rev.C 85 (2012) 044907

Anping Huang et al. Phys.Lett.B 777 (2018) 177-183
A. Bzdak et al. Physics Reports 853 (2020) 1–87
Isospin symmetry breaking at $eB \neq 0$ manifested in chiral condensates

$$\Sigma_l = \frac{2m_l}{(M^2_{\pi})} \times [\bar{\psi}_l(B) - \bar{\psi}_l(0)] + 1$$

$T = 0$

A clear effect but Not accessible in HIC experiments!

$H.-T.Ding, S.-T. Li, A. Tomiya, X.-D. Wang and Y. Zhang, PRD 126 (2021) 082001$

See also in reviews e.g. M. D’Elia, Lect.NotesPhys.871(2013)181
Fluctuations of net baryon number, electric charge and strangeness

Taylor expansion of the QCD pressure:

\[
\frac{p}{T^4} = \frac{1}{VT^3} \ln \mathcal{Z}(T, V, \hat{\mu}_u, \hat{\mu}_d, \hat{\mu}_s) = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{BQS}}{i!j!k!} \left( \frac{\mu_B}{T} \right)^i \left( \frac{\mu_Q}{T} \right)^j \left( \frac{\mu_S}{T} \right)^k
\]

Taylor expansion coefficients at \( \mu = 0 \) are computable in LQCD

\[
\hat{\chi}_{ijk}^{uds} = \left. \frac{\partial^{i+j+k} p/T^4}{\partial (\mu_u/T)^i \partial (\mu_d/T)^j \partial (\mu_s/T)^k} \right|_{\mu_{u,d,s}=0}
\]

\[
\hat{\chi}_{ijk}^{BQS} = \left. \frac{\partial^{i+j+k} p/T^4}{\partial (\mu_B/T)^i \partial (\mu_Q/T)^j \partial (\mu_S/T)^k} \right|_{\mu_{B,Q,S}=0}
\]

\[
\mu_u = \frac{1}{3} \mu_B + \frac{2}{3} \mu_Q
\]
\[
\mu_d = \frac{1}{3} \mu_B - \frac{1}{3} \mu_Q
\]
\[
\mu_s = \frac{1}{3} \mu_B - \frac{1}{3} \mu_Q - \mu_S
\]

At \( eB \neq 0 \) a lot more need to be explored

**HRG:** G. Kadam et al., JPG 47 (2020) 125106, Ferreira et al., PRD 98(2018)034003, Fukushima and Hidaka, PRL117 (2016)102301, Bhattacharyya et al., EPL115(2016)62003

**PNJL:** W.-J. Fu, Phys. Rev. D 88 (2013) 014009

**See recent reviews:**
LQCD: H.-T.Ding, F. Karsch, S.Mukherjee, Int. J. Mod. Phys. E 24 (2015) no.10, 1530007
Exp.: X.-F. Luo & N. Xu, Nucl. Sci. Tech. 28 (2017) 112
Ratio for 2nd order diagonal fluctuations

$N_{f=2+1}$ QCD, $M_\pi(eB = 0) \approx 135$ MeV, $T_{pc}(eB = 0) \approx 156$ MeV, with HISQ action.

$X \left( eB, T_{pc}(eB) \right) \over X \left( 0, T_{pc}(0) \right)$: $R_{cp}$ like observable

At $eB \approx 9M_\pi^2$: $\sim 1.3-1.4$
$N_{t=2+1}$ QCD, $M_\pi(eB=0) \approx 135$ MeV, $T_{pc}(eB=0) \approx 156$ MeV, with HISQ action

$$X(eB, T_{pc}(eB)) \over X(0, T_{pc}(0)) : R_{cp} \text{ like observable}$$

At $eB \simeq 9M_\pi^2$: $\sim 2-2.4$
Ratio for other 2nd order fluctuations

At $eB \simeq 9M_{\pi}^2$:

- Ratio of $\chi^S_2 \sim 1.1$
- Ratio of $\chi^Q_2 \sim 1.07$
- Ratio of $\chi^{BS}_{11} \sim 1.25$
- Ratio of $\chi^{QS}_{11} \sim 1.03$
Lattice QCD meets experiment

Lattice QCD

Proxy of $\chi_2^B$

\[ C_2 = \frac{\chi_2^B(eB, T_{pc}(eB))}{\chi_2^B(0, T_{pc}(0))} \]

- $N_T = 8$
- $N_T = 12$
- cont. est.

Smaller $eB$ \quad Larger $eB$

ALICE: Nucl. Phys. A 982 (2019) 851

Volume!
Lattice QCD meets experiment

Proxy of $\chi_{11}^{BS}/\chi_{2}^{S}$

\[ \frac{\chi_{11}^{BS}(eB, T_{pc}(eB))}{\chi_{11}^{BS}(0, T_{pc}(0))} \]

Data
UrQMD

$C_{p,k}(=\sigma_{p,k}^{1,1}/\sigma_{k}^{2})$

STAR, Phys.Rev.C 100 (2019) 1, 014902
Contributions from Individual hadrons in Hadron resonance gas model

The results of HRG model are consistent with LQCD up to $eB \sim 6M^2_\pi$

$\Delta^{++}(1232)$ and $\Delta^{--}(1232)$ give most of the contributions of magnetic field dependence of $\chi^{BQ}_{11}$

$\Delta^{++}(1232) \rightarrow p + \pi^+$: almost 100% branching ratio!
Proxy construction based on the HRG

\[ \frac{p_c^{MB}}{T^4} = \frac{|q_i| B}{2\pi^2 T^3} \sum_{s_z=-s_i} \sum_{l=0}^{\infty} \varepsilon_0 \sum_{k=1}^{\infty} (\pm 1)^{k+1} \frac{e^{k\mu/T}}{k} K_1 \left( \frac{k\varepsilon_0}{T} \right) \]

\[ \varepsilon_0 = \sqrt{m_i^2 + 2 |q_i| B (l + 1/2 - s_z)} \]

\[ \chi^{BQS}_{ijk}(T, \hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S) = \sum_R B^i_R Q^j_R S^k_R \frac{\partial^2 p_R/T^4}{\partial \hat{\mu}_R^2} \]

fluctuations expressed in terms of stable hadronic states

\[ \chi^{BQ}_{11}(T, \hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S) = \sum_R (P_{R\rightarrow\text{net-B}}) (P_{R\rightarrow\text{net-Q}}) \frac{\partial^2 p_R/T^4}{\partial \hat{\mu}_R^2} \]

where \( P_{R\rightarrow i} = \sum_{\alpha} N_{R\rightarrow i}^\alpha n_{i,\alpha} \)

In our case:

\[ \tilde{p}\tilde{p} + \tilde{p}\tilde{\pi}^+ + \tilde{p}\tilde{K}^+ \] as the proxy for \( \chi^{BQ}_{11} \)
Proxy for $\chi_{11}^{BQ}/\chi_{11}^{QS}$

\[
\frac{\chi_{11}^{BQ}/\chi_{11}^{QS}(eB, T_{pc}(eB))}{\chi_{11}^{BQ}/\chi_{11}^{QS}(0, T_{pc}(0))}
\]

$\sigma_{Q^{PID}, p}^{1,1}/\sigma_{Q^{PID}, K}^{1,1}$: A good proxy of $\frac{X(eB, T_{pc}(eB))}{X(0, T_{pc}(0))}$, here $X = \chi_{11}^{BQ}/\chi_{11}^{QS}$

\[
\sigma_{Q^{PID}, p}^{1,1}(\chi_{11}^{BQ}) : \tilde{p}\tilde{p} + \tilde{p}\tilde{\pi}^+ + \tilde{p}\tilde{K}^+
\]

\[
\sigma_{Q^{PID}, K}^{1,1}(\chi_{11}^{QS}) : \tilde{K}^+\tilde{p} + \tilde{K}^+\tilde{\pi}^+ + \tilde{K}^+\tilde{K}^+
\]
Summary and outlook

- QCD benchmarks are provided for the 2nd order fluctuations of conserved charges based on LQCD computation on Nt=8 and 12 lattices.

- $\chi_{11}^{BQ}$ is strongly affected by $eB$.

- $R_{cp}$ like quantity could be useful to detect the existence of the magnetic field in HIC.

- Computation of 4th order fluctuations is on the way.
Thank you for your attention!
Backup
B pointing to the z direction

\[
u_x(n_x, n_y, n_z, n_\tau) = \begin{cases} 
\exp[-ia^2BN_xn_y] & (n_x = N_x - 1) \\
1 & \text{(otherwise)} 
\end{cases}
\]

\[
u_y(n_x, n_y, n_z, n_\tau) = \exp[ia^2Bn_x]
\]

\[
u_z(n_x, n_y, n_z, n_\tau) = u_t(n_x, n_y, n_z, n_\tau) = 1
\]

Quantization of the magnetic field

\[q_u = 2/3e\]
\[q_d = -1/3e\]
\[q_s = -1/3e\]

\[eB = \frac{6\pi N_b}{N_xN_y}a^{-2}\]
Second order fluctuation in $N_{\tau} = 8$ case

$N_{f}=2+1$ QCD, $M_{\pi}(eB = 0) \approx 135$ MeV, $T_{pc}(eB = 0) \approx 156$ MeV, with HISQ action
Isospin symmetry breaking in $N_{\tau} = 8$ lattice

Due to $\chi_{11}^{us} = \chi_{11}^{ds}$ at $eB = 0$ case, we get:

\[ 2\chi_{11}^{QS} - \chi_{11}^{BS} = \chi_{2}^{S}, \]
\[ 2\chi_{11}^{BQ} - \chi_{11}^{BS} = \chi_{2}^{B}. \]
Transition line on $T - eB$ plane

\[
\Sigma = \frac{1}{f_K^4} \left[ m_s \langle \bar{u}u + \bar{d}d \rangle - (m_u + m_d) \langle \bar{s}s \rangle \right]
\]

\[
\chi^\Sigma = m_s \left( \frac{\partial}{\partial m_u} + \frac{\partial}{\partial m_d} \right) \Sigma
\]

Finding the peak location of $\chi^\Sigma$ at each $eB$ value
\[ m_s = m_s^{\text{phy}}, m_l = m_l^{\text{phy}}, m_\pi \sim 135\text{MeV} \]

The \( N_\sigma \) is fixed to 32, 48; \( N_\sigma = N_x = N_y = N_z \)

The \( N_\tau \) is fixed to 8, 12

\( T \) window: \((144\text{MeV}, 165\text{MeV})\) around \((0.9T_{pc}, 1.1T_{pc})\)

\( a \) is changed to get the targeted \( T \), \( T = \frac{1}{aN_\tau} \)

\( eB \) window: \((0.9m_\pi^2)\)