Sensitivity of single crystals to the circular polarization of high-energy $\gamma$-quanta

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Abstract

It is shown that single crystals are sensitive to the initial circular polarization of $\gamma$-quanta with energies in tens GeV and more. The possibility of measurement of $\gamma$-beam polarization is discussed. The obtained results may be useful for creation of polarimeters for high energy beams of $\gamma$-quanta.

Key words: circular polarization, single crystal, polarimeter, propagation
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1 Introduction

The birefringence of $\gamma$-quanta with energies $> 1$ GeV propagating in single crystals was predicted in [1]. The main process by which $\gamma$-quanta are absorbed in single crystals is the electron-positron pair production [2, 3]. The cross section of the process depends on the direction of linear polarization of the $\gamma$-quanta relative to the crystallographic planes. As a result of interaction with the electric field of the single crystal, a monochromatic, linearly polarized beam of $\gamma$-quanta comprises two electromagnetic waves with different refractive indices, so that linear polarization is transformed into circular polarization or vice versa. This polarization phenomenon should be observed for symmetric orientations of single crystals with respect to the direction of motion of $\gamma$-quanta. Similar effects for the production or analysis of polarized $\gamma$-quanta have been discussed in [4]-[11].

The anisotropic medium is determined as medium, whose optical properties can be described using a symmetric permittivity tensor [12]. The single crystals serve as examples of the similar medium for $\gamma$-quanta with energies in tens GeV and more. The general case of the propagation of high energy $\gamma$-quanta in the anisotropic medium was considered in [13]. This process was investigated in detail for propagation in a dichromatic laser wave, which is the simplest sample of the anisotropic medium of general type. (The dichromatic wave is a superposition of the two linearly polarized laser waves with different frequencies moving in the same direction and, broadly speaking, nonzero angle between directions of polarization of these waves.) The cited paper shows that in the general case the unpolarized beam of $\gamma$-quanta obtains some degree of circular polarization (in contrast to the case in [4]).

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In this paper we examine the propagation of high energy $\gamma$-quanta in single crystals oriented in some regions of coherent $e^\pm$-pair production. We show that single crystals are sensitive to the circular polarization of propagating $\gamma$-quanta.

2 Propagation of $\gamma$-quanta in single crystals.

The permittivity tensor in single crystals oriented in the regions corresponding to coherent $e^\pm$-pair production was obtained with the help of dispersion relations in [14]. The components of this tensor are complex values and have the following form:

$$\varepsilon_{ij} = \varepsilon'_{ij} + i\varepsilon''_{ij}, \quad (i, j = 1, 2)$$

The process is determined by the transverse part of the tensor. It is the symmetrical tensor (i.e., $\varepsilon_{12} = \varepsilon_{21}$) with components depending on the energy of $\gamma$-quanta and two orientation angles of the single crystal with respect to the direction of $\gamma$-beam motion.

It is well-known that in the general case the symmetric complex tensor is not reduced to principal axes (i.e., there does not exist a coordinate system in which the tensors $\varepsilon'_{ij}$ and $\varepsilon''_{ij}$ are both diagonal). It is really, this situation is realized in single crystals at orientations when some "strong" planes act together on propagating $\gamma$-quanta [14]. Obviously, these orientations take a place, when a beam of $\gamma$-quanta move under relatively small angles with respect to one of crystallographic axes.

Knowing the permittivity tensor $\varepsilon_{ij}$, one can find the refractive indices of $\gamma$-quanta

$$\tilde{n}^2 = (\varepsilon_{11} + \varepsilon_{22})/2 \pm \sqrt{(\varepsilon_{11} - \varepsilon_{22})^2/4 + \varepsilon_{12}\varepsilon_{21}},$$

Consequently, a beam of $\gamma$-quanta propagates in the single crystals as a superposition of two electromagnetic waves, which have different refractive indices in general. One can describe the polarization state either (of the two) wave by the use of the Stokes parameters $X_i, Y_i, \quad (i = 1, 3)$ ($X_i$-values correspond to one wave and $Y_i$ to another). These parameters are determined by the following relations [13]

$$X_1 = \frac{\kappa + \kappa^*}{1 + \kappa\kappa^*},$$

$$X_2 = \frac{i(\kappa - \kappa^*)}{1 + \kappa\kappa^*},$$

$$X_3 = \frac{\kappa\kappa^* - 1}{1 + \kappa\kappa^*},$$

$$X_1 = -X_2, X_2 = X_1, X_3 = -X_3, X_1^2 + X_2^2 + X_3^2 = 1,$$

where $\kappa$ is the ratio of components of electric induction vector $D(D_1, D_2, 0)$ in the coordinate system, in which one axis is parallel to the wave vector of $\gamma$-quanta:

$$\frac{D_1}{D_2} = \kappa = \frac{\tilde{n}^2 - \varepsilon_{22}}{\varepsilon_{21}}$$

We call attention to relation: $X_2 = Y_2$ (i.e., waves have the same value of circular polarization). These waves, described here, are the eigenfunctions of problem, and they named as normal electromagnetic waves [12]. In general these waves are elliptically polarized at propagation of $\gamma$-quanta in single crystals [14].
Representing the beam of $\gamma$-quanta by a superposition of two normal electromagnetic waves with previously determined refractive indices and polarization characteristic, we obtain relations describing the variation of the intensity and Stokes parameters of the $\gamma$-quanta propagating in the single crystal [13]:

\begin{align}
J_\gamma(x) &= J_a(x) + J_b(x) + 2J_c(x), \\
\xi_1(x) &= (X_1J_a(x) + Y_1J_b(x) + p_1J_d(x))/J_\gamma(x), \\
\xi_2(x) &= (X_2J_a(x) + Y_2J_b(x) + p_2J_c(x))/J_\gamma(x), \\
\xi_3(x) &= (X_3J_a(x) + Y_3J_b(x) + p_3J_d(x))/J_\gamma(x), \\
p_1 &= -\frac{2X_3}{X_2}, \quad p_2 = \frac{2}{X_2}, \quad p_3 = \frac{2X_1}{X_2},
\end{align}

where $J_\gamma(x), \xi_1(x), \xi_2(x), \xi_3(x)$ are the intensity and Stokes parameters of $\gamma$-quanta on the single crystal thickness equal to x. Besides, we assume that $J_\gamma(0) = 1$. The partial intensities $J_i(x), (i = a, b, c, d)$ have the following form:

\begin{align}
J_a(x) &= J_a(0) \exp(-2\text{Im}(\bar{n}_1)\omega x/c), \\
J_b(x) &= J_b(0) \exp(-2\text{Im}(\bar{n}_2)\omega x/c), \\
J_c(x) &= \exp(-\text{Im}(\bar{n}_1 + \bar{n}_2)\omega x/c)\{J_c(0)\cos(\text{Re}(\bar{n}_1 - \bar{n}_2)\omega x/c) + J_d(0)\sin(\text{Re}(\bar{n}_1 - \bar{n}_2)\omega x/c)\}, \\
J_d(x) &= -\exp(-\text{Im}(\bar{n}_1 + \bar{n}_2)\omega x/c)\{J_c(0)\sin(\text{Re}(\bar{n}_1 - \bar{n}_2)\omega x/c) - J_d(0)\cos(\text{Re}(\bar{n}_1 - \bar{n}_2)\omega x/c)\},
\end{align}

where $\omega$ is the frequency of $\gamma$-quanta and $c$ is the speed of light. One can understand the physical meaning of $J_i$-values, if we write the matrix $\mathbf{D}\mathbf{D}^*$ in the component-wise form, where $\mathbf{D} = \mathbf{D}_1 + \mathbf{D}_2$ is a superposition of the normal waves. The initial partial intensities are defined from relations:

\begin{align}
J_a(0) &= \frac{1 + X_1\xi_1(0) - X_2\xi_2(0) + X_3\xi_3(0)}{2(X^2_1 + X^2_3)}, \\
J_b(0) &= \frac{1 - X_1\xi_1(0) - X_2\xi_2(0) - X_3\xi_3(0)}{2(X^2_1 + X^2_3)}, \\
J_c(0) &= \frac{X_2\xi_2(0) - X_2^2}{2(X^2_1 + X^2_3)}, \\
J_d(0) &= \frac{X_2(X_1\xi_3(0) - X_3\xi_1(0))}{2(X^2_1 + X^2_3)}.
\end{align}

For small thickness one can obtain the following relation

\begin{equation}
J_\gamma(x) = 1 - N\sigma_\gamma(\xi_1(0), \xi_3(0))x,
\end{equation}

where $\sigma_\gamma$ is the total cross section [14] of $e^\pm$-pair production by $\gamma$-quanta with the initial $\xi_1(0), \xi_3(0)$-Stokes parameters and N is the number of atoms per unit volume. Notice that the dependence on $\xi_2(0)$ appears only at $x^2$-terms (and higher) in the expansion (if $X_2 \neq 0$).
One can see that the initially unpolarized beam of $\gamma$-quanta ($\xi_i(0) = 0$, $i = 1−3$) obtains some degree of linear and circular polarization at propagation in the single crystal. However, this beam gets the circular polarization only in the case when $X_2 \neq 0$ (i.e., normal waves are elliptically polarized). Taking into account Eq.(9), one can expect that the circular polarization may be appreciable only on a significant thickness.

The value of intensity of $\gamma$-quanta on any thickness $x$ depends on their initial polarization. It is convenient to define the following value of asymmetry:

$$A_P(x|P) = \frac{J_\gamma(x|P) - J_\gamma(x|0,0,0)}{J_\gamma(x|0,0,0)},$$

where the matrix $P = (\xi_1(0), \xi_2(0), \xi_3(0))$. It easy to see that the $A_P$-asymmetry is equal to the relative variation of intensity between the two cases; in one case the $\gamma$-quanta is initially unpolarized, and, in second case the $\gamma$-quanta have the polarization, which described by the matrix P. Any asymmetry $A_P$ can be expressed with the use of the three basic asymmetries $A_1, A_2$ and $A_3$ in the following form:

$$A_P(x) = A_1(x)\xi_1(0) + A_2(x)\xi_2(0) + A_3(x)\xi_3(0).$$

The basic asymmetries $A_1(x), A_2(x)$ and $A_3(x)$ are determined correspondingly by the next P-matrices: $(1,0,0), (0,1,0), (0,0,1)$.

Besides, the $A_i(x)$-parameters connect with the corresponding Stokes parameters $P_i(x)$ (i=1-3) for initially unpolarized $\gamma$-quanta on the same thickness by the following relations

$$P_1(x) = A_1(x), \ P_2(x) = -A_2(x), \ P_3(x) = A_3(x).$$

Note that the positive quantity of $\xi_2$ corresponds to the right circular polarization.

### 3 Calculations and discussion

We have considered the process of 50-GeV $\gamma$-quanta propagation in the silicon single crystal at room temperature. The case, when the beam of $\gamma$-quanta move under small angle (at some milliradians) with respect to $<001>$ axis of the single crystal, was selected for computations. In calculations the Moliere atomic form factor was employed.

One can determine the direction of $\gamma$-quanta motion with the use of angle $\theta$ with respect to $<001>$-axis and the azimuth angle $\alpha$ around of this axis ($\alpha = 0$ when the momentum of $\gamma$-quanta lies in the (110) plane). However, another angles are more convenient to use $\theta_H = \theta \cos \alpha, \theta_V = \theta \sin \alpha$.

Fig.1 illustrates the calculations of the $|X_2|$-value as a function of the two angles $\theta_H$ and $\theta_V$. The numbers on Fig.1 show the degree of circular polarization of normal waves. The number 0 corresponds to $|X_2|$-values from 0 to 0.1, number 1 corresponds to ones from 0.1 to 0.2 and so on. One can see that the widths of both orientation angles where $|X_2| > 0.5$ is about 0.3-0.4 mrad. The same picture takes a place also at different energies of $\gamma$-quanta. However, in this case the angles of orientation depend inversely proportional on the energy.
Fig. 2 illustrates the calculations of the absolute values of basic asymmetry parameters for the two orientations of the single crystal as a function of the thickness $x$. One can see that $|A_2(x)| << |A_3(x)|$ if thickness $x < 10 - 15$ cm. Note that intensity of initially unpolarized $\gamma$-quanta is equal to 0.0082 at $x=25$ cm (0.000078 at $x=50$ cm) for the first orientation (solid lines) and correspondingly 0.0058 (0.000034) for the second orientation (dashed lines). The $|X_2|$-values are equal to 0.88 and 0.48 for these orientations. Note, we use in calculations the Cartesian coordinate system with one axis along the momentum of $\gamma$-quanta and with the other two axes lying in the (110) and (110) crystallographic planes.

The polarization state of propagating $\gamma$-beam become same as the state of one normal wave begining from some thickness. However, it takes a place at the very large values of $x > 500$ cm (for curves on fig.2), and it is out of significance to the practicability. The reason is the small value of $\tilde{n}_1 - \tilde{n}_2$ for high circular polarizations of normal waves ($\tilde{n}_1 - \tilde{n}_2 \rightarrow 0$ at $X_2 \rightarrow \pm 1$).

The sensitivity of single crystals to the circular polarization can be used for its measurements. In the general case initial beam of $\gamma$-quanta has the linear and circular polarizations. As follows from Eq.(22) two measurements of asymmetry $A_P(x)$ make possible determination of the circular polarization of $\gamma$-quanta. Let $A_P(x|\xi_1(0), \xi_2(0), \xi_3(0))$ be the measured asymmetry in the first measurement. In the second measurement, it needs to change the direction of linear polarization on $90^\circ$ with respect to the axis of single crystal (the $\theta_H, \theta_V$ are the same in both cases). It is possible to make by rotating of the single crystal around the momentum of $\gamma$-quanta. As a result we get

$$A_P((x|\xi_1(0), \xi_2(0), \xi_3(0)) + A_P(x| - \xi_1(0), \xi_2(0), -\xi_3(0)) = 2A_2(x)\xi_2(0).$$

(24)

One can rewrite this equation in the following form:

$$J_\gamma(x|\xi_1(0), \xi_2(0), \xi_3(0)) + J_\gamma(x| - \xi_1(0), \xi_2(0), -\xi_3(0)) =$$

$$= 2J_\gamma(x|0,0,0) + 2J_\gamma(x|0,0,0)A_2(x)\xi_2(0).$$

(25)

Knowing the $J_\gamma(x|0,0,0)$ and $A_2(x)$-values, one can determine the circular polarization of $\gamma$-quanta if the relative intensities $J_\gamma(x| \pm \xi_1(0), \xi_2(0), \pm \xi_3(0))$ were found experimentally.

### 4 Conclusion

We have considered the general case of propagation of $\gamma$-quanta in single crystals. It was shown that there exist some regions of orientations where single crystals are sensitive to circular polarization of $\gamma$-beam, and the possibility of its measurements is discussed. Note that there are few in number of processes in the condensed medium sensitive to the circular polarization of $\gamma$-quanta with energies in tens GeV.

The obtained results may be useful for creation of polarimeters for high energy beams of $\gamma$-quanta.
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Figure 1: The $|X_2|$-value as a function of orientation angles $\theta_H$ and $\theta_V$ for 50-GeV-$\gamma$-quanta in the silicon single crystal. Further explanations are given in the text.
Figure 2:  The absolute values of basic asymmetries $A_i, (i = 1 - 3)$ as functions of the single crystal thickness. The calculated values of asymmetries for angles $\theta_H = 1.6\, mrad, \theta_V = 1.8\, mrad$ are presented by the solids curves, and for $\theta_H = 1.0\, mrad, \theta_V = 1.8\, mrad$ - by the dashed curves.