Time dependent correlators of holographic EPR pairs

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Abstract

We study the field correlators between the quark and anti-quark EPR pair in the super Yang-Mills theory using the holographic description, which is a string in AdS space with its two ends anchored on the boundary. We consider the cases that the end points of the string are static, and that end points are uniformly accelerated in opposite direction where the exact solutions for the string’s profiles are available. In both cases, the two-points correlators of the boundary fields which are described by the linearized perturbations in the worldsheet can also be derived exactly and the all-time evolution of the correlators are obtained. In the case of the accelerating string, the induced geometry on the string worldsheet has the causal structure of a two-sided AdS black hole with a wormhole connects two causally disconnected boundaries, which is a realization of the ER=EPR conjecture. We find that the causality plays an crucial role in determining the nature of the dispersion relation of the particles anchoring at the end points of the string, and the feature of the induced mutual interaction between two particles. In the case of the causally disconnected two ends, the induced effect from the field gives the dissipative dynamics of each particles with no dependence of the distance between two ends, and the induced mutual coupling between two particles vanishes in the late times, following a power law. When two ends are causally connected, the induced dispersion relation becomes non-dissipative at the late times. Here we will also comment on the implication of our findings in the entangled particle dynamics and the ER=EPR conjecture.

PACS numbers:
I. INTRODUCTION

Time dependent field correlators give a qualitative description of how fast the information spreads out in a quantum theory. This is also related to how fast two entangled subsystems get disentangled. In the content of the AdS/CFT correspondence, ER = EPR is a conjecture stating that two entangled particles, so-called Einstein-Podolsky-Rosen or EPR pair, can be associated in a gravity theory by a wormhole background (or Einstein-Rosen bridge) [1]. One realization is the eternal AdS black hole which is dual to the thermal field double state [2]. In this case, the perturbed field state leads to a power law decaying time-dependent correlator between two points in the causally disconnected boundaries, and this is in tension with the information conservation for a quantum black hole [2]. Another realization of the ER=EPR conjecture is the model consists of a string in AdS space with its two ends located at the AdS boundary, which is due to the entangled pair of quark and anti-quark in the super Yang Mills theory. As the two ends are uniformly accelerating in the opposite directions, the exact worldsheet profile was found in [3]. Moreover, it was proposed in [4] and [5], that the EPR pair of quark and anti-quark is holographically encoded in the induced wormhole geometry on the string worldsheet. For example, the entanglement entropy of the EPR pair is dual to the entropy in the two-sided black hole of the worldsheet, and the gravity action is dual to the EPR pair production rate.

How fast the information spreads out is also important in the application in quantum information processing. The viability of quantum computing depends on how long we can keep the system in coherent or entangled states without losing information to environment. Thus it is interesting to study how the quantum field influences the entanglement of the system, starting from the initial density matrix of the system and environment. The effective theory for the system can be described by the reduced density matrix, which is obtained by tracing out environmental variables in the full density matrix, and becomes an essential quantity for studying the dynamics of entanglement [6]. In [7, 8], the entanglement of two objects interacting with the common free quantum field is studied when two objects either undergo the uniform acceleration or are at rest. They found that the entanglement between two objects can be created through the coupling with the quantum field and decays to zero in some finite duration (called the “sudden death” of quantum entanglement [9]).

The idea of holographic duality has also been applied for the study of strong coupling
problems in condensed matter systems and the hydrodynamics of the quark-gluon plasma (see [10, 11] for reviews). There were considerable efforts by employing the holographic duality to explore the dissipation behavior of a particle moving in a strongly coupled environment. In these studies, the end point of the string on the boundary of the AdS black hole serves as a probe particle. A review on the application to non-equilibrium Brownian motion can be found in [12]. We have used this approach to study various behaviors of Brownian particles in the strongly coupled fields, which can potentially be verified experimentally [13–15].

In this work we would like to obtain the time dependent correlators of the strongly coupled fields from the holographic approach when there are two probed entangled particles which are either undergoing uniform acceleration or are static. The dual gravity description is the probed string in AdS space with its two ends anchoring at the boundary. In the case of the uniform accelerating pair, the setting is exactly the same as in [3]. We found that causality in the string worldsheet plays a crucial role in determining the nature of the dispersion relation of the particles but also the feature of the induced mutual interaction between them. In the case of the causally disconnected two ends, the induced effect from the field gives the dissipative dynamics for the particles with no dependence of the distance between them. Moreover, the induced mutual coupling between two ends vanishes in the late times, following a power law. This is the case for both the accelerating string during all time evolution and the static string, at the early enough time, when two ends are not in the causal contact with each other. In contrary, when two ends are causally connected in the static case, the induced dispersion relation of each ends becomes non-dissipative.

The remaining part of the paper is organized as follow. In section II we will review the exact solution of the accelerating string, and then solve the linearized equation for the perturbations in this worldsheet background. The all-time two-point correlators of the fields that quark and anti-quark couple to are obtained in contrast to the earlier works where only the late time behavior of the correlators was found [3]. As a comparison, we also find the exact solution of the linearized equation of string worldsheet in the case when its two ends are static in section III. We then conclude and comment in section IV.
II. FLUCTUATIONS OF TRANSVERSE MODES IN THE ACCELERATING STRING

We first review the key equations and their exact solutions of the accelerating string in $AdS_{d+1}$ space \cite{3}. Then we exactly solve the linearized equation for perturbations in these string backgrounds, and give a prescription to obtain the generating function for the boundary fields coupled to the string end points.

A. The exact solutions of the string background and perturbations

We consider the $AdS_{d+1}$ metric in the Poincare coordinates given by

$$ds^2 = \frac{R^2}{z^2} (-dt^2 + dz^2 + dx^2 + \sum_{i=1}^{d-2} dy_i^2),$$

where $R$ is the curvature radius. The string in this background is described by the Nambu-Goto action

$$S = -T_0 \int d\tau d\sigma \sqrt{-\dot{h}},$$

where $T_0$ is the string tension and $h = \det h_{ab}$ is the determinant of the induced metric. We choose a gauge such that $(\tau, \sigma) = (t, z)$ and the embedding of the string $X^\mu(t, z) = (t, z, x(t, z), 0, \cdots, 0)$ with $\mu = 1, 2, 3, \cdots, d$. Then we have $\sqrt{-h} = \frac{R^2}{z^2} \sqrt{1 - \dot{x}^2 + x'^2}$ with the classical equation of motion given by

$$\frac{\partial}{\partial t} \left( \frac{\dot{x}}{z^2 \sqrt{1 - \dot{x}^2 + x'^2}} \right) - \frac{\partial}{\partial z} \left( \frac{x'}{z^2 \sqrt{1 - \dot{x}^2 + x'^2}} \right) = 0,$$

where $' = \frac{\partial}{\partial z}$ and $\cdot = \frac{\partial}{\partial t}$. This equation has an exact solution as found in \cite{3},

$$x_b(t, z) = \pm \sqrt{t^2 + b^2 - z^2},$$

where $b$ is a real constant. In the solution, the trajectory of the end-points at $z = 0$ describes the motion of two particles along $x$-direction with uniform deceleration $\frac{1}{b}$, which head toward each other from $x = \pm \infty$ when $t = -\infty$, subsequently stop at $x = \pm b$ when $t = 0$ and then turn around in the opposite directions, moving away from each other. We label the position of the particle moving in the positive (negative) $x$ region by $q_R(q_L)$ respectively. The induced metric on the worldsheet with the embedding coordinates is

$$ds^2_{ws} = \frac{R^2}{(t^2 + b^2 - x^2)^2} ((x^2 - b^2) dt^2 - 2tx dt dx + (t^2 + b^2) dx^2).$$
As noticed in [4], this metric has the same casual structure of the eternal two-sided $AdS$ back hole (see Fig.1), and it has bifurcate horizons located at $t = \pm x$, the time-like boundaries at the particle trajectories $x^2 - t^2 = b^2$, and the space-like "singularity" at $x^2 - t^2 \to -\infty$. However this space-like singularity is not a physical singularity, it just reflects the coordinate singularity in the background $AdS$ as $z \to \infty$ as approaching the Poincare Killing horizon. Due to the analogue between the EPR pair and the wormhole geometry [1], it was proposed in [4, 5] that this metric (5) gives the gravity description of the entangled quark and anti-quark pair.

![FIG. 1: Few sample light rays (solid black lines) in the worldsheet(5). Here $x$ and $t$ are in unit of $b$. The hyperbolic curves (blue) are trajectories of quark and anti-quark. Two quarks are causally disconnected with the horizons at $t = \pm x$ (dash lines).](image)

We now consider the fluctuations of a string in the direction transverse to $x$, say $y_1$, with the embedding in [1] as $X^\mu = (t, z, x_b(t, z), q(t, z), 0, \cdots, 0)$ where $q$ is considered to be small as compared to $x_b$. The quadratic action for $q$ is obtained as

$$S_q = -R^2 T_0 \int dt \, dz \, \frac{1}{z^2} \left( \frac{q'^2 - \dot{q}^2 - \dot{x}_b^2 q'^2 - \dot{x}_b^2 q^2 + 2\dot{x}_b x_b' \dot{q}q'}{2 \sqrt{1 - \dot{x}_b^2 + x_b'^2}} \right). \quad (6)$$

As can be seen in the action, in this coordinate system, there is no separable solution in the variables $t$ and $z$. However in the comoving frame of one of the accelerating quarks, say the
\[ R \text{ branch in (4), defined as,} \]
\[
x = \sqrt{b^2 - r^2} e^{\frac{\alpha}{2}} \cosh \frac{\tau}{b},
\]
\[
t = \sqrt{b^2 - r^2} e^{\frac{\alpha}{2}} \sinh \frac{\tau}{b},
\]
\[
z = re^{\frac{\alpha}{2}},
\]
with \(0 < r < b\), the metric of (1) becomes
\[
ds^2 = \frac{R^2}{r^2} \left(-\left(1 - \frac{r^2}{b^2}\right) d\tau^2 + \frac{dr^2}{1 - \frac{r^2}{b^2}} + d\alpha^2 + e^{-\frac{2\alpha}{b}} \sum_{i=1}^{d-2} d\gamma_i^2 \right).
\]
As in [3], this metric has the event horizon at \(r = b\) with the Hawking temperature \(T = \frac{1}{2\pi b}\), which shows no causal connection between two particles. However, on the worldsheet (5), it can be seen that two particles are connected by a wormhole. Thus we also expect that in the particle rest frame, the particles will experience quantum fluctuations at finite temperature \(T\) as we will verify later. Note that this temperature \(T\) can also be interpreted as Unruh temperature that an accelerating quark observes. In this coordinate system, the background worldsheet solution in (4) is
\[
\alpha(\tau, r) = 0.
\]
And the quadratic action for the perturbations in \(y_1\) direction on the worldsheet becomes
\[
S_{qc} = -\frac{R^2 T_0}{2} \int d\tau dr \frac{1}{r^2} \left(1 - \frac{r^2}{b^2}\right) \dot{q}^2 - \frac{q^2}{1 - \frac{r^2}{b^2}}
\]
with \(\dot{'} = \frac{\partial}{\partial r}\) and \(\cdot = \frac{\partial}{\partial \tau}\) that gives the equation of motion in terms of the Fourier basis defined to be
\[
q(\tau, z) = \int \frac{d\omega}{2\pi} y_\omega(z) e^{-i\omega \tau},
\]
as
\[
y''_\omega - \frac{2}{z(1 - \frac{z^2}{b^2})} y'_\omega + \frac{\omega^2}{(1 - \frac{z^2}{b^2})^2} y_\omega = 0.
\]
Notice that on the worldsheet, the comoving coordinate \(r\) is equal to static coordinate \(z\) and now \(\dot{'} = \frac{\partial}{\partial z}\). To get the finite action, we introduce a cutoff scale by putting the boundary brane at the finite \(z = z_m\). Then, with the normalized condition \(y_\omega(z_m) = 1\), the equation (14) has two independent solutions
\[
Y_\omega(z) = \frac{(1 - i\omega z)e^{i\omega b \tanh^{-1} \frac{z}{b}}}{(1 - i\omega z_m)e^{i\omega b \tanh^{-1} \frac{z_m}{b}}} \quad \text{and} \quad Y_\omega^*(z),
\]
where \( Y_\omega(z) \) \( (Y^*_\omega(z)) \) is the incoming (outgoing) wave near the horizon \( z = b \). We would like to stress that these are the exact solutions, whereas in [3] only the approximate solutions in the small-\( \omega \) expansion are given.

**B. The generating function**

Using the holographic prescription, the retarded Green’s function for the one end of the string is

\[
G_R(\omega) = \frac{R^2 T_0}{b^2} \left( \frac{b^2}{z^2_m} - 1 \right) Y'_\omega(z_m)Y^*_\omega(z_m) = \frac{R^2 T_0}{z_m b^2 \left( 1 + \omega^2 z^2_m \right)} \left( \omega^2 (b^2 - z^2_m) + i (z_m \omega + b^2 z_m \omega^3) \right) = i \gamma_T \omega + M_T \omega^2 + \mathcal{O}(\omega^3),
\]

which is analytic in the upper half complex \( \omega \)-plane. Its small \( \omega \) limit agrees with the results in [3] and also our previous papers [13] with the temperature dependent damping term \( \gamma_T = R^2 T_0 / b^2 \) and the effective mass term \( M_T = M_0 + \Delta M_T \) giving the large temperature independent mass \( M_0 = R^2 T_0 / z_m \) and the small temperature dependent mass correction \( \Delta M_T = -R^2 T_0 z_m / b^2 \) in the limit of \( z_m / b \rightarrow 0 \). As far as we know, via a bilinear coupling between the particle and environmental fields, the temperature dependent damping term can only be obtained from the holographic approach, whereas in the free field theory the same type of coupling leads to the state-independent back reaction from the field to the particle[18]. It is also worth mentioning that when two ends of the string undergo the acceleration, they are never in causal contact during their journeys. In this case, the energy transferred to the field does not have chance to come back and this gives the dissipative dynamics of the particles. Moreover, based upon the causality consideration, the retarded Green’s function of the particle has no dependence on the distance between two particles. This will be contrary to the static string case where two ends are initially causally disconnected and then become connected at the time scale of the distance between two particles, giving a rather different dispersion relation of the particle after two ends reach the causal contact.

To obtain the force-force correlations between two ends of string, which encode the information of the wormhole, we need to obtain the generating functional for the fields that couple to the two ends of the string. For this purpose, we transform the solution in (15) back to the coordinates in [1]. The \( R(L) \) branch corresponds to \(+(-)\) background solution
in \(^4\). Also notice that the \(R\) and \(L\) branches have different relations between the proper time \(\tau\) and the coordinate time \(t\). Thus, from (7) \(\tau_{R/L} = \mp \tanh^{-1} \frac{t}{\sqrt{b^2+t^2-z^2}}\). Then we can write the general solutions as

\[
q^{R/L}(t,z) = \int \frac{d\omega}{2\pi} e^{\mp i\omega b \tanh^{-1} \frac{t}{\sqrt{b^2+t^2-z^2}}} \left( f^{R/L}(\omega) Y_\omega(z) + g^{R/L}(\omega) Y_\omega^*(z) \right).
\]

The boundary values of \(q^{R/L}(t,z)\) are interpreted holographically as the sources for the quark and anti-quark, \(q^{R/L}(t,z_m) \equiv q_0^{R/L}(t)\). And their "Fourier transform" is defined as

\[
q_0^{R/L}(t) = \int \frac{d\omega}{2\pi} e^{\mp i\omega u(t)} \tilde{q}_0^{R/L}(\omega),
\]

where \(u(t) \equiv b \tanh^{-1} \frac{t}{\sqrt{b^2+t^2-z_m^2}}\). Thus we have two boundary conditions at \(z = z_m\) as

\[
f^{R/L}(\omega) + g^{R/L}(\omega) = \tilde{q}_0^{R/L}(\omega).
\]

The other two boundary conditions for \(q^{R/L}(t,z)\) are imposed at the horizon \(z = b\). As in \([16]\), the analyticity boundary conditions for the mode functions cross the horizon of maximally extended black hole give the Schwinger-Keldysh correlators for a single particle. In our previous work \([14]\), we also obtained the same set of correlators by imposing the constrains from the unitarity and periodicity of the finite temperature boundary correlators, and this turns out to be equivalent to the analyticity of mode functions cross the horizon. In the setting of this paper, the maximally extended black hole does not describe the double copy of a single particle action giving the Schwinger-Keldysh correlators, but the action of the two coupled particles. To achieve it, we will modify the definition of the Kruskal coordinates in \([16]\). We will also give the prescription from the field theory constrains similar to the one in \([14]\).

Near the horizon in \((10)\), the mode functions can be simplified when we define the Kruskal coordinates, say in the \(R\) branch, as

\[
V = -e^{-\frac{r^*+r^*}{b}}, \quad U = e^{\frac{r-r^*}{b}},
\]

where \(r^* = b \tanh^{-1} \frac{r}{b}\). Then we can write the mode function as

\[
q^R \simeq f^R(\omega)(U)^{-ib\omega} + g^R(\omega)(-V)^{ib\omega}.
\]

Notice that \(UV = \frac{r+b}{r+b}\). So, we can even define the mode functions inside the horizon as \(UV > 0\). The similar definition of the mode functions for the \(L\) branch is adopted, where
now \( r^* = -b \tanh^{-1} \frac{r}{b} \). So, the value of \( r^* \) is negative in the \( L \) branch and becomes positive in the \( R \) branch. Also now

\[
V = e^{\frac{r^*}{b}}, \quad U = -e^{-\frac{r^*}{b}}. \tag{22}
\]

Note that \( \tau \) in the \( L \) branch has an opposite sign relative to the \( R \) branch. Then we can write the mode function in the \( L \) branch as

\[
q^L \approx f^L(\omega)(V)^{ib\omega} + g^L(\omega)(-U)^{-ib\omega}. \tag{23}
\]

To match \( q^R \) with \( q^L \), we require them to be analytic on the real \( U \) and \( V \) axes and lower half of complex \( U \) and \( V \) planes as in [16]. Thus, the matching conditions are given by

\[
f^R(\omega) = e^{-\pi b\omega}g^L(\omega), \quad g^R(\omega) = e^{\pi b\omega}f^L(\omega), \tag{24}
\]

and together with (19), they uniquely fix the bulk mode functions \( q^R/L(t, z) \) with

\[
f^R(\tilde{\omega}) = \frac{\tilde{q}^R_0(\omega)}{1 - e^{2\pi \omega}} - \frac{e^{b\pi \omega}q^L_0(\omega)}{1 - e^{2\pi \omega}}, \tag{25}
\]

\[
g^R(\tilde{\omega}) = \frac{e^{b\pi \omega}q^L_0(\omega)}{1 - e^{2\pi \omega}} - \frac{e^{2b\pi \omega}\tilde{q}^R_0(\omega)}{1 - e^{2\pi \omega}}, \tag{26}
\]

\[
f^L(\tilde{\omega}) = \frac{\tilde{q}^L_0(\omega)}{1 - e^{2\pi \omega}} - \frac{e^{b\pi \omega}q^R_0(\omega)}{1 - e^{2\pi \omega}}, \tag{27}
\]

\[
g^L(\tilde{\omega}) = \frac{e^{b\pi \omega}q^R_0(\omega)}{1 - e^{2\pi \omega}} - \frac{e^{2b\pi \omega}\tilde{q}^L_0(\omega)}{1 - e^{2\pi \omega}}. \tag{28}
\]

The bulk onshell action for this solution, which contains only the boundary terms, is a functional of \( q^R/L_0(t) \) and is identified as a generating function for the boundary field correlators coupled to the quark anti-quark pair. Leaving out the contact terms that should be related to the renormalization of the particle self-energy, we have

\[
S[q^R_0, q^L_0] = S_q(q^R_0) + S_q(q^L_0) \tag{29}
\]

\[
= \frac{1}{2} \int \frac{d\omega}{2\pi} \{q^R_0(\omega)[\text{Re}G^R(\omega) + \frac{e^{\tau}}{e^{\tau} - 1}i\text{Im}G^R(\omega)]q^R_0(-\omega) \tag{30}
\]

\[
+ q^R_0(\omega)[\frac{-2ie^{\bar{\tau}}}{e^{\bar{\tau}} - 1}\text{Re}G^R(\omega)]q^L_0(-\omega) \tag{31}
\]

\[
+ q^L_0(\omega)[\frac{-2ie^{\bar{\tau}}}{e^{\bar{\tau}} - 1}\text{Im}G^R(\omega)]q^R_0(-\omega) \tag{32}
\]

\[
+ q^L_0(\omega)[\text{Re}G^R(\omega) + \frac{e^{\bar{\tau}} + 1}{e^{\bar{\tau}} - 1}i\text{Im}G^R(\omega)]q^L_0(-\omega) \}. \tag{33}
\]
with \( G_R(\omega) \) given in (16). Then we can read off the correlators
\[
G^{ij}(\omega) = \frac{\delta G^{ij}}{\delta q_0^i} \frac{\delta G^{ij}}{\delta q_0^j}
\]
with \( i, j = R, L \), and the real time correlators:
\[
G^{ij}(t, t') = (-1)^i b \sqrt{\frac{b^2 + t^2 - \frac{z^2}{m}}{b^2 + t'^2 - \frac{z^2}{m}}} \int \frac{d\omega}{2\pi} G^{ij}(\omega) e^{-i\omega(\tau^i(t) - \tau^j(t'))},
\]
(34)
when \( i = R, \tau^i(t) = u(t), (-1)^R = 1 \) and when \( i = L, \tau^i(t) = -u(t), (-1)^L = -1 \).

From the field theory point of view, we consider the particles \( q_{0}^{R/L} \) coupled to the common quantum field at finite temperature through a bilinear coupling with the same coupling strength. Integrating out the field degree of freedom leads to the effective action of the particles. Thus the generating function of \( \tilde{q}_{0}^{R/L} \) requires to be symmetric under the exchange of \( R \) and \( L \), leading to \( G_{RL} = G_{LR} \) and \( G_{RR} = G_{LL} \), and also the Green’s functions \( G_{RR} \) and \( G_{LL} \) should have the form of the finite temperature time-ordered Green’s functions at temperature \( T = \frac{1}{2\pi b} \). The above requirements turn out to be equivalent to (24) from the bulk analyticity condition.

Notice that \( G_{RR}(\omega) = G_{LL}(\omega) \) has the same form of the finite temperature order correlators. The Fourier transform back to real time is not well defined, but as in [13, 19], the late time particle correlators can be estimated to be \( \langle q_{0}^{R}(t)q_{0}^{R}(0) \rangle = \langle q_{0}^{L}(t)q_{0}^{L}(0) \rangle \propto t^{-2} \), the correlation between two entangled particles, which is encoded in \( G^{LR}(t, 0) = G^{RL}(0, t) \).

In Fig.2, the numerical plot for the cross correlators of the fields is drawn.

**Fig. 2:** The time evolution of the \( G^{LR}(t, 0) = G^{RL}(0, t) \) correlator. Here \( t \) is in unit of \( b \) and \( G^{LR} \) is in unit of \( \frac{4R^2T_0}{b^4 \sqrt{1 - \frac{z^2}{m^2}}} \). The curves correspond to the chosen parameters as \( \frac{z_m}{b} = 0.9 \) (blue), 0.7 (orange) and 0.1 (green) respectively.

To see the late time and early time analytic behaviors of the \( G^{LR}(t, 0) = G^{RL}(0, t) \) cor-
relators, the retarded Green’s function, \( G_R(\omega) \) has the pole at \( \omega = -\frac{i}{z_m} \) and the Boltzmann factor gives the poles at \( \omega = \frac{in}{b} \) with non-zero integer. We consider \( b \gg z_m \), where \( b \) plays a role of an IR cut-off and \( z_m \) an UV cut-off. Thus, in the limit \( t \ll z_m \) with the proper time \( u(t) \simeq t \), the time evolution of \( G^{LR}(t, 0) = G^{RL}(t, 0) \) is dominated by the pole at \( \omega = -\frac{i}{z_m} \) giving

\[
G^{LR}(t, 0) = G^{RL}(t, 0) \propto e^{-\frac{t}{z_m}} \simeq 1 - \frac{t}{z_m} + \cdots .
\]

As for \( t \gg b \) with \( u(t) \simeq b \ln(t/b) \), the pole at \( \omega = -\frac{i}{b} \) dominates to the correlators \( G^{ij}(t, 0) \) resulting in the power-law decay

\[
G^{LR}(t, 0) = G^{RL}(t, 0) \propto \left( \frac{t}{b} \right)^{-2} .
\]

This can be compared with the correlators of the free scalar fields on the spacetime points of two observers following the trajectories along say, the \( x \)-direction with the uniform acceleration \( 1/b \). The worldlines of two observers are specified by \( x^\mu_L = (b \sinh \frac{\tau}{b}, -b \cosh \frac{\tau}{b}, 0, 0) \) and \( x^\mu_R = (b \sinh \frac{\tau}{b}, b \cosh \frac{\tau}{b}, 0, 0) \). In particular, the correlator of the free scalar field at the spacetime point \( x^\mu_L \) and \( x^\mu_R \) is given by

\[
G(x^\mu_L, x^\mu_R) = \frac{1}{4\pi^2} \frac{1}{|\vec{x}_R - \vec{x}_L|^2 - |x^0_R - x^0_L|^2} .
\]

Thus, when \( t \ll b \) giving the proper time \( \tau \simeq t \),

\[
G(t, 0) \propto \left( 1 - \frac{t^2}{2b^2} + \ldots \right) ,
\]

whereas for \( t \gg b \) giving \( \tau \simeq b \ln(t/b) \),

\[
G(t, 0) \propto \left( \frac{t}{b} \right)^{-1} ,
\]

instead. The strongly coupled fields induce the couplings between two particles, which make the correlation between them dies out more quickly than the one from the free field, and this potentially gives a relatively short time scale for particles to sustain quantum entanglement between them.

### III. Fluctuations of Transverse Modes in the Static String

In this section we consider the static string in the same \( AdS_{d+1} \) background in [1]. We also solve the linearized equation of perturbations in this string background exactly and
obtain the generating functional for the fields that couple to the two ends of the string. In
the same gauge choice as in (3) with $x$, which is independent of time, the classical equation
of motion for the static string with its two ends at the boundary $z = 0$ has a solution with
two-branch joint at $z = z_0$,[20],

$$x_0(z) = \pm \int_{z_0}^{z} \sqrt{\frac{y^4}{z_0^4 - y^4}} dy.$$  (40)

The induced metric on this worldsheet (parametrized by $t$ and $x$) has the causal structure
as in Fig. 3. As compare with the case of accelerating string (in Fig. 1), we see that two
quarks are in causal contact in this case.

![Fig. 3](image)

**FIG. 3:** Few sample light rays (black curves) in the worldsheet[40]. Here $x$ and $t$ are in unit of $z_0$. Two vertical lines (blue) are trajectories of quark and anti-quark. Light rays emitted from one quark reach the other in some finite times. So two quarks are causally connected.

Similar to the accelerating string case, we consider the linearized perturbations
in a direction transverse to the worldsheet, with the embedding in [1] as $X^\mu = (t, z, x_0(z), p(t, z), 0, 0...)$ where $p$ is considered to be small as compared to $x_0$. The Fourier transform of the transverse modes is defined as

$$p(t, z) = \int \frac{d\omega}{2\pi} \tilde{h}_\omega(z) e^{-i\omega t},$$  (41)
which has the linearized equation of motion on both $R$ and $L$ branches as,

$$h''_{\omega} - \frac{2}{\rho(1-\rho^4)}h'_{\omega} + \frac{\tilde{\omega}^2}{1-\rho^4}h_{\omega} = 0,$$

(42)

with the dimensionless variables $\rho = \frac{z}{z_0}$ and $\tilde{\omega} = z_0\omega$. There are two exact independent solutions with normalized boundary condition at $\rho = \rho_m \equiv \frac{z_m}{z_0}$,

$$F_{\tilde{\omega}}(\rho) = e^{A_{\tilde{\omega}}(\rho) - A_{\tilde{\omega}}(\rho_m)}, \quad H_{\tilde{\omega}}(\rho) = e^{B_{\tilde{\omega}}(\rho) - B_{\tilde{\omega}}(\rho_m)},$$

(43)

where

$$A_{\tilde{\omega}}(\rho) = \frac{1}{2} \ln(1 + \tilde{\omega}^2 \rho^2) + \frac{\sqrt{1 - \tilde{\omega}^4}}{\tilde{\omega}}(E_{\tilde{\omega}}(\rho) - E_{\tilde{\omega}}(\rho_m)), \quad B_{\tilde{\omega}}(\rho) = \frac{1}{2} \ln(1 + \tilde{\omega}^2 \rho^2) - \frac{\sqrt{1 - \tilde{\omega}^4}}{\tilde{\omega}}(E_{\tilde{\omega}}(\rho) - E_{\tilde{\omega}}(\rho_m)),$$

(44)

(45)

with

$$E_{\tilde{\omega}}(\rho) = \int_{0}^{\rho} \frac{dy}{\sqrt{1 - y^4}}, \quad E_{\tilde{\omega}}(\rho) = \int_{0}^{\rho} \frac{dy}{(1 + \tilde{\omega}^2 y^2)\sqrt{1 - y^4}}.$$ 

(46)

Then the general solutions of the perturbations on the $R$ and $L$ branches are written as

$$p^{R/L}(t, \rho) = \int \frac{d\omega}{2\pi} \left(C^{R/L}_1(\tilde{\omega})F_{\tilde{\omega}}(\rho) + C^{R/L}_2(\tilde{\omega})H_{\tilde{\omega}}(\rho)\right) e^{-i\omega t}. \quad (47)$$

The boundary values of the perturbations are interpreted as the sources for the quark pair defined as

$$p^{R/L}(t, \rho_m) = p^{R/L}_0(t) = \int \frac{d\omega}{2\pi} p^{R/L}_0(\omega) e^{-i\omega t}.$$ 

(48)

Thus we have the boundary conditions

$$C^{R/L}_1(\tilde{\omega}) + C^{R/L}_2(\tilde{\omega}) = p^{R/L}_0(\tilde{\omega}).$$

(49)

The other two boundary conditions are imposed at the tip of the string ($\rho = 1$). We require that the perturbations to be smooth across the tip, namely $p^R(t, 1) = p^L(t, 1)$ and $\partial_\rho p^R(t, 1) = -\partial_\rho p^L(t, 1)$, which give

$$C^{R}_1(\tilde{\omega})F_{\tilde{\omega}}(1) + C^{R}_2(\tilde{\omega})H_{\tilde{\omega}}(1) = C^{L}_1(\tilde{\omega})F_{\tilde{\omega}}(1) + C^{L}_2(\tilde{\omega})H_{\tilde{\omega}}(1), \quad (50)$$

$$C^{R}_1(\tilde{\omega})F_{\tilde{\omega}}(1) - C^{R}_2(\tilde{\omega})H_{\tilde{\omega}}(1) = -C^{L}_1(\tilde{\omega})F_{\tilde{\omega}}(1) + C^{L}_2(\tilde{\omega})H_{\tilde{\omega}}(1).$$

(51)
Note that to obtain (51), we have factored out a common divergence factor. Together with (49), the unique solution can be obtained as

\[
C_1^R(\tilde{\omega}) = \frac{\tilde{p}_0^R(\tilde{\omega})}{1 - e^{2\kappa(\tilde{\omega})}} - \frac{e^{\kappa(\tilde{\omega})} \tilde{p}_0^L(\tilde{\omega})}{1 - e^{2\kappa(\tilde{\omega})}},
\]

\[
C_2^R(\tilde{\omega}) = \frac{e^{\kappa(\tilde{\omega})} \tilde{p}_0^L(\tilde{\omega})}{1 - e^{2\kappa(\tilde{\omega})}} - \frac{e^{2\kappa(\tilde{\omega})} \tilde{p}_0^R(\tilde{\omega})}{1 - e^{2\kappa(\tilde{\omega})}},
\]

\[
C_1^L(\tilde{\omega}) = \frac{\tilde{p}_0^L(\tilde{\omega})}{1 - e^{2\kappa(\tilde{\omega})}} - \frac{e^{\kappa(\tilde{\omega})} \tilde{p}_0^R(\tilde{\omega})}{1 - e^{2\kappa(\tilde{\omega})}},
\]

\[
C_2^L(\tilde{\omega}) = \frac{e^{\kappa(\tilde{\omega})} \tilde{p}_0^R(\tilde{\omega})}{1 - e^{2\kappa(\tilde{\omega})}} - \frac{e^{2\kappa(\tilde{\omega})} \tilde{p}_0^L(\tilde{\omega})}{1 - e^{2\kappa(\tilde{\omega})}},
\]

where \( \kappa(\tilde{\omega}) = \frac{2\sqrt{1 - \tilde{\omega}^2}}{\tilde{\omega}}(E_0(1) - E_\tilde{\omega}(1) - E_0(\rho_m) + E_\tilde{\omega}(\rho_m)) \). Moreover, the on-shell action, which can be identified as the generating function for the quantum fluctuations of the pair of quarks becomes

\[
S[\tilde{p}_0^R, \tilde{p}_0^L] = -\frac{R^2 T_0}{2z_0^3} \int_{\rho \to \rho_m} dt \sqrt{\frac{1 - \rho^4}{\rho^2}} \left( p^R(t, \rho) \partial_{\rho} p^R(t, \rho) + p^L(t, \rho) \partial_{\rho} p^L(t, \rho) \right) \] 

\[
- \frac{R^2 T_0}{2z_0^3} \int d\omega \frac{1}{2\pi} \left\{ \tilde{p}_0^R(\tilde{\omega}) \left[ \frac{e^{\kappa(\tilde{\omega})}}{1 - e^{2\kappa(\tilde{\omega})}} (\tilde{A}_\omega'(\rho_m) - e^{2\kappa(\tilde{\omega})} \tilde{B}_\omega'(\rho_m)) \right] \tilde{p}_0^R(-\tilde{\omega}) \right\} 
\]

\[
+ \tilde{p}_0^R(\tilde{\omega}) \left[ \frac{e^{\kappa(\tilde{\omega})}}{1 - e^{2\kappa(\tilde{\omega})}} (\tilde{B}_\omega'(\rho_m) - \tilde{A}_\omega'(\rho_m)) \right] \tilde{p}_0^L(-\tilde{\omega})
\]

\[
+ \tilde{p}_0^L(\tilde{\omega}) \left[ \frac{1}{1 - e^{2\kappa(\tilde{\omega})}} (\tilde{A}_\omega'(\rho_m) - e^{2\kappa(\tilde{\omega})} \tilde{B}_\omega'(\rho_m)) \right] \tilde{p}_0^R(-\tilde{\omega}) 
\]

Similar to the case of the accelerating string, based upon the analytical property of the mode functions we can identify the single particle retarded Green’s function as

\[
G^{(0)}_R(\omega) = -\frac{R^2 T_0}{z_0^3} \sqrt{1 - \rho_m^4} F_\omega'(\rho_m) F_\omega(\rho_m)
\]

\[
= -\frac{R^2 T_0}{z_0^3} \sqrt{1 - \rho_m^4} A_\omega(\rho_m) = -\frac{R^2 T_0}{z_0^3} \sqrt{1 - \rho_m^4} \frac{\tilde{\omega} \rho_m(\tilde{\omega} + \rho_m \sqrt{1 - \tilde{\omega}^2})}{1 + \tilde{\omega}^2 \rho_m^2},
\]

which is analytic in the upper half complex \( \omega \) plane. Similarly the advanced Green’s function, analytic in the upper half complex \( \omega \) plane, can also be identified as

\[
G^{(0)}_A(\omega) = -\frac{R^2 T_0}{z_0^3} \sqrt{1 - \rho_m^4} B_\omega'(\rho_m) = -\frac{R^2 T_0}{z_0^3} \sqrt{1 - \rho_m^4} \frac{\tilde{\omega} \rho_m(\tilde{\omega} - \rho_m \sqrt{1 - \tilde{\omega}^2})}{1 + \tilde{\omega}^2 \rho_m^2}.
\]
Thus from the generating function we can read off the field correlators arising from the field vacuum fluctuations as

\[ G^{(0)RR}(\omega) = G^{(0)LL}(\omega) = \frac{1}{1 - e^{2\kappa(\tilde{\omega})}} \left( G^{(0)}_R(\omega) - e^{2\kappa(\tilde{\omega})} G^{(0)}_A(\omega) \right), \]

\[ G^{(0)RL}(\omega) = G^{(0)RL}(\omega) = \frac{e^{\kappa(\tilde{\omega})}}{1 - e^{2\kappa(\tilde{\omega})}} \left( G^{(0)}_R(\omega) - G^{(0)}_A(\omega) \right). \]

(64)

(65)

To see that these Green’s functions are reasonable, we consider the limits of \( \omega^{-1} \ll z_0 \) and \( z_0 \gg z_m \), leading to \( \tilde{\omega} \gg 1 \) and \( \rho_m \ll 1 \), where the single particle time-order correlators is expectedly recovered due to the fact that the scale \( z_0 \) is pushed to infinity. With \( \kappa(\tilde{\omega}) \simeq 2i\gamma_1 \tilde{\omega} \) and \( \gamma_1 \equiv E_0(1) = \frac{\sqrt{\pi} \Gamma\left(\frac{5}{4}\right)}{\Gamma\left(\frac{3}{4}\right)} \), \( G^{(0)}_R(\omega) = \left( G^{(0)}_A(\omega) \right)^* \), giving the Green’s functions of the form

\[ G^{(0)RR}(\omega) = G^{(0)LL}(\omega) \simeq \text{Re} G^{(0)}_R(\omega) + \frac{1 + e^{4i\gamma_1 \tilde{\omega}}}{1 - e^{4i\gamma_1 \tilde{\omega}}} \text{Im} G^{(0)}_R(\omega). \]

(66)

Note that this Green’s function is not well defined, since there are poles on the real \( \omega \) values. So, the \( i\varepsilon \) prescription is needed to replace \( \omega \) by \( \omega(1 - i\varepsilon) \) and to take \( \varepsilon \to 0 \) only in the end of the calculations. Then in the large \( \tilde{\omega} \) limit, the Green’s functions can further reduce to

\[ G^{(0)RR}(\omega) = G^{(0)LL}(\omega) \simeq \text{Re} G^{(0)}_R(\omega) + \text{sign}(\omega)i\text{Im} G^{(0)}_R(\omega), \]

(67)

which is precisely the Feynmann Green’s function at zero temperature probing the field vacuum fluctuations. In particular, in the limit \( \omega \gg z_0^{-1} \) with the relatively large distance between two particles but \( \omega \ll z_m^{-1} \) for this small \( \omega \) compared with the UV cutoff scale \( z_m^{-1} \), the retarded Green’s function is approximated by

\[ G^{(0)}_R(\omega) \simeq -\frac{R^2 T_0}{z_m^3} \frac{z_m \omega^2 + i z_m^3 \omega^3}{1 + z_m^2 \omega^2} \]

\[ \simeq -M_0 \omega^2 - i\gamma \omega^3 + \cdots. \]

(68)

The induced mass \( M_0 \simeq R^2 T_0 / z_m \) has the same form as in the above case. The damping term of the \( \omega^3 \) dependence is a typical self-force effect for the Brownian motion with the cutoff-independent damping coefficient \( \gamma \simeq R^2 T_0 \). This also agrees with our previous results [13]. Furthermore, this damping term can be compared with the one in the Abraham-Lorentz-Dirac equation of the charged particle with the self-force effects from the coupling to the electromagnetic field [17]. And, as expected, the \( z_0 \) dependence drops off in this limit where two particle are far apart. Moreover, with the needed \( i\varepsilon \) prescription, the cross correlators \( G^{(0)RL}(\omega) \) and \( G^{(0)LR}(\omega) \) vanish in this limit.
In the other limits of $\tilde{\omega} \ll 1$ and $\rho_m \ll 1$, the wavelength of the perturbations is longer than the $z_0$ scale where the late time regime when two ends are causally connected is probed. In this case, we have

$$\kappa(\tilde{\omega}) = 2\gamma_2 \tilde{\omega}$$

(69)

with $\gamma_2 = \frac{\sqrt{\pi} \Gamma(\frac{7}{4})}{3\Gamma(\frac{3}{4})}$. As $\omega < z_0^{-1}$ both the retarded and advanced Green’s functions become real-valued. This implies that there exists the threshold time scale $z_0^{-1}$, after which the particle dynamics becomes non-dissipative and depends on $z_0$. Thus, the strongly coupled fields induce striking different effects on particles as compared with those given by the free field theory [17]. In this late time limit, the retarded and advanced Green’s functions can be approximated by

$$G_R^0(\omega) \approx -\frac{R^2 T_0}{z_0^3} \left( \tilde{\omega} + \frac{\tilde{\omega}^2}{\rho_m} - \rho_m^2 \tilde{\omega}^3 + O(\tilde{\omega}^4) \right),$$

(70)

$$G_A^0(\omega) \approx -\frac{R^2 T_0}{z_0^3} \left( -\tilde{\omega} + \frac{\tilde{\omega}^2}{\rho_m} + \rho_m^2 \tilde{\omega}^3 + O(\tilde{\omega}^4) \right).$$

(71)

Then the correlators become

$$G^{(0)RR}(\omega) = G^{(0)LL}(\omega) \approx \frac{1}{2}(G_R^0(\omega) + G_A^0(\omega)) + \frac{1}{2} \left[ 1 + e^{4\gamma_2 \tilde{\omega}} - 1 \right] \left( G_R^0(\omega) - G_A^0(\omega) \right)$$

(72)

$$\approx -M_{z_0} \omega^2 + \frac{R^2 T_0}{2\gamma_2} \frac{1}{z_0^3} + O(\omega^4),$$

(73)

and also

$$G^{(0)RL}(\omega) = G^{(0)LR}(\omega) \approx \frac{e^{2\gamma_2 \tilde{\omega}}}{1 - e^{4\gamma_2 \tilde{\omega}}} \left( G_A^0(\omega) - G_R^0(\omega) \right)$$

(74)

$$\approx -\frac{R^2 T_0}{2\gamma_2} \left( \frac{1}{z_0^3} - \frac{z_m^2}{z_0^3} \omega^2 + O(\omega^4) \right).$$

(75)

Thus we find that the induced mass $M_{z_0} = M_0 + \Delta M_{z_0}$ now have a small correction $\Delta M_{z_0} = \frac{R^2 T_0}{2\gamma_2} \frac{z_m^2}{z_0^3}$ of the $z_0$ dependent. Even though we have not derived the Langevin equations for the two entangled particles, if we interpret $G^{(0)RL}$ and $G^{(0)RL}$ as the fluctuation force acting on the particles, the dominant constant term in the correlators can lead to the decay of the particle correlators $\langle p_R^R(t)p_L^R(0) \rangle = \langle p_L^L(t)p_R^L(0) \rangle \propto t^{-1}$, which may potentially lead to the same decay rate for the entanglement between two particle as in [19]. This can be compared to the accelerating string case, where the decay rate of the cross correlators is $\propto t^{-2}$, potentially giving the faster disentangled rate. This will be in our next work.
IV. CONCLUSION

In this paper we study the holographic dual of the EPR pair of the quark and anti-quark in the SYM theory. We calculate the two-points field correlators exactly, which are described by the linearized perturbations in the worldsheet of the string that either undergoes uniform acceleration or remains static. We find that the causality between two ends crucially determine the nature of the induced dispersion relation of the particles anchoring at the end points and also the feature of the mutual coupling of two particles. For the causally disconnected two ends, the induced dispersion relation of each particles is dissipative and the their mutual correlation dies out in a power law of time as $t^{-2}$. In contrary, for the casually connected two ends, the dispersion of relation becomes non-dissipative and the mutual correlation decays in a power law of time as $t^{-1}$. There are some interesting related problems to pursue for further understanding of the underlying physics through the entanglement entropy between two particles. In the future work, we plan to explore the distance $z_0$ between two particles to the time evolution of the entanglement entropy of the Brownian particle anchoring at two ends of a string as compare with the field correlators obtained in the paper [14].

Another interesting problem is to consider the time evolution of the entanglement entropy between the entangled quark and anti-quark in [3] identifying the entanglement entropy associated with the accelerating string solution, which may be subject to the time evolution due to field fluctuation as considered in this paper. In comparison to the weak coupling analysis in [7, 8], we would obtain some insights on the time evolution of entanglement in strongly coupled theories. In particular, it is of interest to know whether or not the sudden death of the entanglement entropy also occurs in strongly coupled theory. If yes, since the entanglement is geometrically realized as a fundamental string connecting quark-antiquark pair, the time evolution may involve some nonperturbative effects leading to the change of the background solutions. However for this purpose we should extend the formula to that of the holographic influence functional in [16] and [14] for finding the dynamics of a pair of the entangled particles.

In [2], the power law decay of the correlation between two entangled boundary regions in the eternal AdS black hole implies the lost of information of black hole behind the horizon. In the case of the accelerating string, the induced worldsheet background also has the
causal structure exactly the same as an eternal AdS black hole. However the decay of the correlation between two boundary particles is expected as they couple to the SYM fields and the information can leak to the environments. If the ER=EPR conjecture also works in this case, the disentanglement of two particles can lead to the breakdown of the wormhole geometry, which is beyond the probed string approach we consider in this paper. We do not have a solution for this problem yet. And how this solution can be related to the black hole information problem is also an interesting problem to pursue.

**Acknowledgments**

The work of S. K. was supported in part by MOST109–2811–M–007–558 and MOST109–2112–M–007–018–MY2. The work of D.S. L. was supported in part by MOST-110-2112-M-259 -003. The work of C.P. Y. was supported in part by MOST-110-2112-M-259 -004 and MOST-109-2112-M-259 -006.

[1] J. Maldacena and L. Susskind, “Cool horizons for entangled black holes,” Fortsch. Phys. 61, 781 (2013), arXiv:1306.0533 [hep-th].

[2] J. M. Maldacena, “Eternal black holes in anti-de Sitter,” JHEP 04 (2003) 021, arXiv:hep-th/0106112 [hep-th].

[3] B.-W. Xiao, “On the exact solution of the accelerating string in AdS(5) space ,” PLB 665, 173 (2008), arXiv: 0804.1343 [hep-th].

[4] K. Jensen and A. Karch, “Holographic Dual of an Einstein-Podolsky-Rosen Pair has a Wormhole,” Phys. Rev. Lett. 111, no. 21, 211602 (2013), arXiv:1307.1132 [hep-th].

[5] J. Sonner, “Holographic Schwinger Effect and the Geometry of Entanglement,” Phys. Rev. Lett. 111, no. 21, 211603 (2013), arXiv:1307.6850 [hep-th].

[6] Wei-Can Syu, Da-Shin Lee, Chen-Pin Yeh, “Entanglement of quantum oscillators coupled to different heat baths,” J. Phys. B: At. Mol. Opt. Phys. 54 055501 (2021).

[7] Shih-Yuin Lin, B. L. Hu, “Entanglement creation between two causally-disconnected objects,” Phys. Rev. D 81, 045019 (2010).

[8] Shih-Yuin Lin, B. L. Hu, “Temporal and Spatial Dependence of Quantum Entanglement from
a Field Theory Perspective,” Phys. Rev. D 79, 085020 (2009).

[9] T. Yu and J.H. Eberly, Phys. Rev. Lett. 93, 140404 (2004).

[10] S. Hartnoll, Class. Quant. Grav. 26, 224002 (2009).

[11] M. Rangamani, Class. Quant. Grav. 26, 224003 (2009).

[12] J. Boer, V. Hubeny, M. Rangamani and M. Shigemori, JHEP 07, 094 (2009); V. Hubeny and M. Rangamani, Adv. High Energy Phys. 2010, 297916 (2010).

[13] C.-P. Yeh, J.-T. Hsiang, and D.-S. Lee, “Holographic Approach to Nonequilibrium Dynamics of Moving Mirrors Coupled to Quantum Critical Theories,” Phys.Rev.D 89, 066007 (2014), arXiv:1310.8416 [hep-th].

[14] C.-P. Yeh, J.-T. Hsiang, and D.-S. Lee, “Holographic influence functional and its application to decoherence induced by quantum critical theories,” Phys.Rev.D 91, 046009 (2015), arXiv:1410.7111 [hep-th].

[15] Chen-Pin Yeh, Da-Shin Lee, “Subvacuum effects in Quantum Critical Theories from Holographic Approach,” Phys. Rev. D 93, 126006 (2016), arXiv:1510.05778 [hep-th].

[16] C. P. Herzog and D. T. Son, “Schwinger-Keldysh propagators from AdS/CFT correspondence,” JHEP 03, 046 (2003), arXiv:hep-th/0212072 [hep-th].

[17] Jen-Tsung Hsiang, Tai-Hung Wu, Da-Shin Lee, “Stochastic Lorentz forces on a point charge moving near the conducting plate,” Phys. Rev. D 77, 105021 (2008).

[18] Jen-Tsung Hsiang, Tai-Hung Wu, Da-Shin Lee, “Brownian motion of a charged particle in electromagnetic fluctuations at finite temperature,” Found Phys 41, 77 (2011).

[19] D.-S. Lee, and C.-P. Yeh, “Time evolution of entanglement entropy of moving mirrors influenced by strongly coupled quantum critical fields,” JHEP 06, 068 (2019), arXiv: 1904.06831 [hep-th].

[20] J. M. Maldacena, “Wilson loops in large N field theories,” Phys. Rev. Lett. 80 (1998), 4859-4862