Higher Order Stability of a Radiatively Induced 220 GeV Higgs Mass

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The effective potential for radiatively broken electroweak symmetry in the single Higgs doublet Standard Model is explored to four sequentially subleading logarithm-summation levels (5-loops) in the dominant Higgs self-interaction couplant $\lambda$. We augment these results with all contributing leading logarithms in the remaining large but sub-dominant Standard Model couplants ($t$-quark, QCD and $SU(2) \otimes U(1)$ gauge couplants) as well as next to leading logarithm contributions from the largest of these, the $t$-quark and QCD couplants. Order-by-order stability is demonstrated for earlier leading logarithm predictions of an $\mathcal{O}(220 \text{ GeV})$ Higgs boson mass in conjunction with fivefold enhancement of the value for $\lambda$ over that anticipated from conventional spontaneous symmetry breaking.

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Radiative symmetry breaking, as proposed by S. Coleman and E. Weinberg \cite{1}, embraced the premise that the Standard Model (SM) Lagrangian was protected by some symmetry from a tree level mass term for the Higgs field. In the absence of large destabilizing Yukawa couplings (heavy fermions), Coleman and Weinberg were able to show that $\lambda$, the scalar self-coupling within that Lagrangian, was of the same order as the $SU(2) \otimes U(1)$ gauge coupling constants to the fourth power. However, such a small magnitude for $\lambda$ no longer occurs in the presence of the large $t$-quark Yukawa coupling, which requires an even larger value of $\lambda$ to stabilize the SM effective potential. Very recent work, \cite{2, 3} based upon full consideration of all contributing leading-logarithm (LL) terms in the effective potential for the single-Higgs-doublet SM effective potential that devolves from a Coleman-Weinberg (mass-term-protected) tree potential, has predicted a Higgs boson mass (218 GeV) well within indirect-measurement bounds \cite{4}, in conjunction with a Higgs-doublet SM effective potential that devolves from a Coleman-Weinberg (mass-term-protected) tree potential, [conventionally, $y = m_H^2/(8\pi^2 \phi^2) = 0.01$, an enhancement directly measurable in processes such as $WW \rightarrow HH$ \cite{5}. The purpose of the present article is to ascertain whether such clear phenomenological signatures for radiative electroweak symmetry breaking based upon such a conformally invariant tree potential (as may arise from a protective higher symmetry) persist upon inclusion of subsequent-to-leading logarithm contributions to the effective potential.

In ref. \cite{2}, the summation of LL contributions to the SM effective potential is expressed in terms of its dominant three couplants $x = g_t^2(v)/4\pi^2 = 0.0253$, $y = \lambda(v)/4\pi^2$, $z = \alpha_s(v)/\pi = 0.0329$, where the momentum scale $v = \langle \phi \rangle = 246.2 \text{ GeV}$ is the vacuum expectation value (vev) of electroweak symmetry breaking. This $LL$ effective potential may be expressed as a power series in the logarithm $L = \log(\phi^2/v^2)$:

$$ V_{\text{eff}} \equiv \pi^2 \phi^4 S = \pi^2 \phi^4 \left( A + BL + CL^2 + DL^3 + EL^4 + \ldots \right). \tag{1} $$

The constant $A = y + K$, where $K$ includes all finite $\phi^4$ counterterms remaining after divergent contributions from $\phi^4$ graphs degree-2 and higher in couplant powers are cancelled. Coefficients $\{B, C, D, E\}$ are explicitly obtained in refs. \cite{2, 3} via the renormalization group (RG) equation,

$$ \left[ ( -2 - 2\gamma ) \frac{\partial}{\partial L} + \beta_x \frac{\partial}{\partial x} + \beta_y \frac{\partial}{\partial y} + \beta_z \frac{\partial}{\partial z} - 4\gamma \right] S = 0, \tag{2} $$

as degree $\{2, 3, 4, 5\}$ polynomials in the couplants $x, y, z$. The unknown couplant $y(v)$ and finite counterterm $K\phi^4$ are numerically determined by the simultaneous application of Coleman and Weinberg’s renormalization conditions \cite{1}:

$$ V_{\text{eff}}' (v) = 0 \implies K = -B/2 - y; \quad V_{\text{eff}}^{(4)} (v) = V_{\text{tree}}^{(4)} (v) \implies y = \frac{11}{3} B + \frac{35}{3} C + 20D + 16E. \tag{3} $$

Given the $LL$ expressions for $\{B, C, D, E\}$ in Eqs. (8)–(11) of ref. \cite{2}, one finds that $y = 0.05383, K = -0.05794$, in which case coefficients $\{A, B, C, D, E\}$ in the potential \cite{1} are numerically determined. The “running Higgs boson
mass$^*$ [6] at the vev-momentum-scale is found from the second derivative of the effective potential

$$m^2_H = V''_{eff}(v) = 8\pi^2 v^2 (B + C),$$

(4)

to be $m_H = 216$ GeV at LL level [2]. Incorporation of LL contributions to $\{A - E\}$ from the much smaller electroweak gauge couplings $r = g_3^2/4\pi^2 = 0.0109$, $s = g^2/4\pi^2 = 0.00324$ modifies the value of $m_H$ to be 218 GeV and $y(v)$ to be 0.0545 [3].

We consider here whether this large value of the self-interaction coupling $y(v)$ is still sufficiently small for the LL Higgs boson mass (218 GeV) to be subject to controllable corrections from those subsequent-to-leading-logarithm contributions to the effective potential that are dominated by higher powers of $y$. To address stability when $y(v)$ is large, we first consider the scalar field theory projection (SFTP) of the SM effective potential, obtained by setting all SM couplings except the dominant coupling $y (y > z, x, r, s)$ to zero. Indeed, focusing on the large-coupling subtheory and then taking into account subdominant couplings is analogous to the usual treatment of processes in which QCD and electroweak corrections both occur (e.g. $e^+e^- \rightarrow$ hadrons).

When supplemented by scalar-field kinetic terms, the SFTP of the SM electroweak effective potential is equivalent to a globally O(4) symmetric massless scalar field theory for which $\beta_y$ and $\gamma$ have been calculated in $\overline{\text{MS}}$ to five-loop order. [The coupling constant in ref. [7] is $g = 3\sqrt{2}/(8\pi^2) = 3y/2.]$ The LL SFTP of the SM effective potential is just the $x = 0$ limit of Eq. (6.1) of ref. [3]:

$$V_{\text{SFTP}}^{LL} = \pi^2 \phi^4 \left[ \frac{y}{1 - 3yL} + K' \right].$$

(5)

The constant $K'$ represents the contribution of all finite $\phi^4$ counterterms degree-2 and higher in $y$. Curiously, if $v$ retains its 246 GeV SM value, the running Higgs mass and scalar coupling $y(v)$ extracted from this potential are not very different from those of the full LL series. By applying conditions [3] to the SFTP LL values $\{B, C, D, E\} = \{3y^2, 9y^3, 27y^4, 81y^5\}$ obtained from Eq. (5), one finds that $y(v) = 0.05414$ and $K' = -y - \frac{3}{2}y^2 = -0.05853$. Substituting this value for $y$ into $B$ and $C$ [Eq. (1)], we obtain a running Higgs boson mass of 221 GeV at the vev-momentum-scale, only a small departure from the 216 GeV result [2] when Standard Model couplings $x$ and $z$ are assigned physical vev-momentum-scale values instead of the value zero. [Note that the SFTP is not scale-free: a physical vev scale $v = (\sqrt{2}G_F)^{-1/2}$ arises from the SM gauge sector.] These results suggest that the SFTP of the SM dominates radiative electroweak symmetry breaking, subject to manageably small corrections from the other smaller SM interaction couplings ($x, z, etc.$)

In the absence of an explicit mass term, the SFTP all-orders potential takes the form of a perturbative field theoretic series ($y = \lambda/4\pi^2$, $\mathcal{L} = \log(\phi^2/\mu^2)$)

$$V_{\text{SFTP}} = \pi^2 \phi^4 S_{\text{SFTP}}, \quad S_{\text{SFTP}} = y + \sum_{n=1}^{\infty} \sum_{m=0}^{n} T_{n,m} y^{n+1} L^m.$$  

(6)

LL contributions to this series involve coefficients $T_{n,n} = 3^n$ from Eq. (5); NLL contributions correspond to coefficients $T_{n,n-1}$; and so forth. The invariance of $V_{\text{SFTP}}$ under changes in the renormalization scale $\mu$ implies that $S_{\text{SFTP}}$ satisfies the $\overline{\text{MS}}$ renormalization-group (RG) equation [2] with $\beta_y = \beta_z = 0$, and with [7]

$$\gamma = \frac{3}{8}y^2 - \frac{9}{16}y^3 + \frac{55}{128}y^4 - 49.8345y^5 + \ldots$$

(7)

The series $S_{\text{SFTP}}$ in the full potential [3] may be rewritten in terms of sums of leading ($S_0$) and successively subleading ($S_1, S_2, ...$) logarithms:

$$S_{\text{SFTP}} = yS_0(yL) + y^2 S_1(yL) + y^3 S_2(yL) + y^4 S_3(yL) + \ldots; \quad S_k(u) = \sum_{n=k}^{\infty} T_{n,n-k} u^{n-k}.$$  

(8)

Given $u = yL$, we employ the methods of ref. [8] to obtain successive differential equations for $S_k(u)$, first by substituting Eq. (8) into the RG equation [2] with nonzero RG functions [1], and then by organizing the RG equation in powers of $y$:

$$\mathcal{O} \left( y^2 \right): \quad 2 \left( 1 - 3u \right) \frac{dS_0}{du} - 6S_0 = 0, \quad S_0 (0) = 1;$$

(9)

$$\mathcal{O} \left( y^3 \right): \quad 2 \left( 1 - 3u \right) \frac{dS_1}{du} - 12S_1 = -21S_0 - \frac{39}{2} u \frac{dS_0}{du}, \quad S_1 (0) = T_{1.0};$$

(10)

$$\mathcal{O} \left( y^4 \right): \quad 2 \left( 1 - 3u \right) \frac{dS_2}{du} - 18S_2 = \frac{41823}{220} S_0 - \frac{3}{4} u \frac{dS_0}{du} + \frac{10332}{55} u \frac{dS_0}{du} - \frac{81}{2} S_1 - \frac{39}{2} u \frac{dS_1}{du}, \quad S_2 (0) = T_{2.0}.$$  

(11)
| $n^\alpha$ LL SFTP | SFTP + $\alpha$ LL in $\{x,z\}$ | SFTP + $\alpha$ LL in $\{x,z,r,s\}$ |
|------------------|---------------------|---------------------|
| \(n\) \(y(v)\) | \(m_H\) \(T_{\alpha,0}\) | \(y(v)\) \(m_H\) \(T_{\alpha,0}\) | \(y(v)\) \(m_H\) \(T_{\alpha,0}\) |
| 0 0.05414 221.2 | 1 | 0.05383 215.8 | 1 | 0.05448 218.3 | 1 |
| 1 0.05381 227.0 | 2.5521 | 0.05351 221.7 | 2.5533 | 0.05415 224.4 | 2.5603 |
| 2 0.05392 224.8 | -8.1770 | 0.05362 219.5 | -8.1744 | 0.05426 222.1 | -8.1773 |
| 3 0.05385 226.2 | 83.211 | 0.05355 221.3 | 83.190 | 0.05419 224.0 | 83.195 |
| 4 0.05391 225.6 | 114.18 | 0.05338 223.6 | -982.24 | 0.05406 225.5 | -1191.8 |

Table I: Perturbative stability of results inclusive of $N^\alpha$ LL contributions from the dominant couplant $y(v) = (\lambda(v)/4\pi)^2$ to the SFTP of the SM effective potential (columns 2-4). Columns 5-7 show the effect of augmenting this projection with prior determinations of $LL$ contributions to the effective potential from $t$-quark ($x = g_T^2(v)/4\pi^2$) and from QCD ($z = \alpha_s(v)/\pi$). Columns 8-10 further augment this projection with $LL$ contributions from electroweak $SU(2)$ $(r = g_2^2(v)/4\pi^2)$ and $U(1)$ $(s = g^2/4\pi^2)$ gauge couplants. $m_H$ denotes the vev-referenced running Higgs boson mass $[V_{eff}(v)]^{1/2}$ in GeV units ($v = 246.2$ GeV).

Equations for $S_3$ and $S_4$ (not displayed) are also straightforward to obtain from the 5-loop RG functions (7). One can thus obtain exact solutions to the sums of $LL$, $NLL$, $N^2LL$, $N^3LL$, and $N^4LL$ contributions to the series (5).

As before we choose $\mu = v$, the scalar field vev, in which case $L \rightarrow L = \log(\phi^2/v^2)$ (5). Recall that the counterterm $K'$ in the $LL$ potential (16) ($K' = -0.05853$) and the couplant $y = (0.05414)$ are comparable in magnitude; $K'$ is not a single higher-order counterterm coefficient. The SFTP $\phi^4$ counterterm coefficient $K'$ can accommodate contributions of any logarithm-free terms $T_{n,0}y^{n+1}$ ($n > 1$) in the complete series (6). The all-orders SFTP of the effective potential

$$V_{SFTP} = \pi^2 \phi^4 \sum_{n=0}^{\infty} T_{n,m} y^n L^m = \pi^2 \phi^4 \sum_{k=0}^{\infty} y^k S_k(yL),$$

is then approached by the following successive approximations to Eq. (12), which incorporate the summations of successively subleading logarithms contributing to the complete series (5):

$$V_{LL} = \pi^2 \phi^4 \left[ \sum_{n=0}^{\infty} T_{n,n} (yL)^n + yT_{1,0} + y^2 T_{2,0} + y^3 T_{3,0} + \cdots \right] \equiv \pi^2 \phi^4 [yS_0(yL) + K'],$$

$$V_{NLL} = \pi^2 \phi^4 \left[ \sum_{n=0}^{\infty} T_{n,n} (yL)^n + y \sum_{n=1}^{\infty} T_{n,n-1} (yL)^{n-1} + y^2 T_{2,0} + \cdots \right]$$

$$= \pi^2 \phi^4 \left[ yS_0(yL) + y^2 S_1(yL) + (K' - y^2 T_{1,0}) \right],$$

$$V_{N^2LL} = \pi^2 \phi^4 \left[ \sum_{q=0}^{\infty} y^q S_q(yL) \left( K' - \sum_{q=1}^{\infty} y^{q+1} T_{q,0} \right) \right] =_{\text{lim } p \rightarrow \infty} V_{N^2LL} = V_{SFTP}. \quad \text{(15)}$$

Note from Eq. (13) that $K'$ is numerically inclusive of all finite $\phi^4$ counterterms. Thus for the $NLL$ case with $K'$ already determined from application of Eq. (5) to Eq. (5), we now find from Eq. (3) that $T_{1,0} = -[4K' - 21y^3 + 6y^2 + 4y]/12y^3 = 2.5521, y = 0.05381$, and from Eq. (5) that $[V_{SFTP}(v)]^{1/2} = 227$ GeV.

One can continue this procedure through $N^4LL$ order, applying conditions (5) on potentials (15) to determine $T_{n,0}$ and $y$ while making use of the information $(T_{1,0}, \ldots, T_{n-1,0}, K')$ from preceding orders. The results, as summarized in columns 2-4 of Table I, show remarkable order-by-order stability in the values obtained both for the couplant $y(v)$ and the running Higgs boson mass. Note also that the SFTP potential (12) is compatible by construction with the $\overline{MS}$ renormalization scheme, since the summations $S_k(yL)$ of $N^kLL$’s are obtained from differential equations [e.g. Eqs. (9)-(11)] derived from $\overline{MS}$ RG functions (7).

We now augment the SFTP with the smaller subdominant SM couplants $\{x, z, r, s\}$ (3). If only the $LL$ from $\{x, z\}$, the Yukawa interaction sector, are included, $K$ is found from the minimization condition (3) to be $K = -y - 3y^2/2 + 3x^2/8 = -0.05793$, where $y$ is the $LL$ value 0.05383 obtained [via Eq. (3)] in ref. (3). If $LL$ contributions from $\{r, s\}$, the $SU(2) \otimes U(1)$ gauge couplants, are also included, the constant $K$ is now $K = -y - 3y^2/2 + 3x^2/8 - 3rs/64 - 9r^2/128 - 3s^2/128 = -0.058704$, where $y$ is the $LL$ value for $y(v) = 0.05481$ obtained [via Eq. (5)] in ref. (5). These results are listed in columns 5-10 of Table I. The order-by-order stability of improved predictions $y(v) = 0.054$, $m_H = 220–227$ GeV is quite striking. Moreover, we have also found that this stability persists when contributions
from subdominant couplings $x$ and $z$ are considered to NLL order via use of two-loop $\overline{\text{MS}}$ RG functions in Eq. 11. The all-orders effective potential analogous to Eq. (6) but now inclusive of all three dominant SM couplings $x$, $y$, $z$ is of the form

$$V_{xyz} = \pi^2 \phi^4 \sum_{n=0}^{\infty} x^n \sum_{k=0}^{\infty} y^k \sum_{\ell=0}^{n+k+\ell-1} z^\ell \sum_{p=0}^{\ell} L^p D_{n,k,\ell,p} = \pi^2 \phi^4 S_{xyz} \quad (D_{0,1,0,0} = 1, \, D_{1,0,0,0} = D_{0,0,1,0} = 0), \quad (16)$$

where the series $S_{xyz}$ can be expressed either as summations of LLs ($p = n + k + l - 1$), NLLs ($p = n + k + l - 2$), etc, as in Eq. 12, or as power series in the logarithm $L$, as in Eq. 11. To NLL order, only the $y^2 \quad (D_{0,2,0,0} = T_{1,0} \equiv a)$ and $x^2 \quad (D_{2,0,0,0} \equiv b)$ finite counterterms from divergent one-loop $\phi^4$ graphs contribute to the coefficients $\{B, C, D, E\}$. The NLL contributions to $\{B, C, D, E\}$ from Eq. 10 are

$$B = \left[ 3y^2 - \frac{3}{4} \right]_{LL}$$

$$\quad \quad + \left[ \left( -\frac{27}{4} + \frac{3a}{2} \right) xy^2 + \left( \frac{3}{2} + \frac{3b}{4} \right) x^3 - \left( 1 + 4b \right) x^2 z + \left( 6a - \frac{21}{2} \right) y^3 + \left( \frac{3}{4} - \frac{3a}{2} \right) x^2 y \right]_{NLL},$$

$$C = \left[ 9y^3 + \frac{9}{4} xy^2 - 9x^2 y + \frac{3}{2} x^2 z - \frac{9}{32} x^3 \right]_{LL}$$

$$\quad \quad + \left[ \left( 27a - \frac{621}{8} \right) y^4 + \left( \frac{27a}{2} - \frac{225}{4} \right) xy^3 + \left( \frac{3a}{2} + \frac{21}{2} \right) x^2 y^2 + \left( \frac{3a - 9}{2} \right) x^2 y z + \left( -\frac{225a}{32} + \frac{27}{8} \right) x^2 y^2 \right]_{NLL},$$

$$D = \left[ 27y^4 + \frac{27}{2} xy^3 - \frac{3}{2} xy^2 z + 3x^2 y z - \frac{225}{32} x^2 y^2 - \frac{23}{8} x^2 z^2 + \frac{15}{16} x^3 z - \frac{45}{16} x^3 y + \frac{99}{256} x^4 \right]_{LL}$$

$$\quad \quad + \left[ \left( -\frac{801}{2} + 108a \right) y^5 + \left( \frac{11547}{32} + 81a \right) xy^4 + \left( -12a + \frac{147}{2} \right) x y^3 z + \left( \frac{15a}{8} - \frac{291}{16} \right) x y^2 z^2 \right]_{NLL},$$

$$\quad \quad + \left[ \left( -\frac{23a}{4} + \frac{75}{4} \right) x^2 y^2 + \left( \frac{45}{32} - \frac{45a}{2} \right) x^2 y^3 + \left( \frac{177a}{16} - \frac{33}{8} \right) x^2 y^2 z + \left( -\frac{877}{32} - \frac{115b}{4} \right) x^2 z^3 \right]_{NLL},$$

$$\quad \quad + \left[ \left( \frac{3125}{128} + \frac{201b}{16} \right) x^3 z^2 + \left( \frac{69a}{8} - \frac{615}{16} \right) x^3 y z + \left( \frac{19323}{256} - \frac{2781a}{128} \right) x^3 y^2 \right]_{NLL},$$

$$\quad \quad + \left[ \left( -\frac{1023}{128} - \frac{135b}{32} - \frac{9a}{4} \right) x^4 z + \left( \frac{3825}{256} + \frac{45a}{128} \right) x^4 y + \left( -\frac{1035}{512} + \frac{45b}{64} + \frac{81a}{64} \right) x^5 \right]_{NLL}.$$
\[
E = \left[ 81y^5 + \frac{243}{4}xy^4 - 9xy^3z + \frac{45}{32}xy^2z^2 - \frac{69}{16}x^2yz - \frac{135}{8}x^2y^3 + \frac{531}{64}x^2y^2 + \frac{345}{64}x^2z^3 - \frac{603}{256}x^3z^2 \\
+ \frac{207}{32}x^3yz - \frac{8343}{512}x^3y^2 - \frac{459}{512}x^4z + \frac{135}{512}x^4y + \frac{837}{1024}x^5 \right]_{LL} \\
+ \left[ \left( -\frac{55539a}{4096} + \frac{1081377}{8192} \right)y^2x^4 + \left( \frac{2187a}{256} - \frac{29133}{2048} \right)yx^5 + \left( -\frac{125793}{64} + 405a \right)y^6 \right]_{NLL} \\
+ \left[ \left( \frac{1035b}{64} + \frac{105c}{16} + \frac{111633}{4096} \right)x^4z + \left( -\frac{1215a}{32} - \frac{195939}{1024} \right)y^4x^2 + \left( \frac{452b}{64} + \frac{38613}{512} \right)x^2z^4 \right]_{NLL} \\
+ \left[ \left( -\frac{315b}{64} + \frac{207a}{32} + \frac{17703}{2048} \right)x^5z + \left( \frac{459b}{128} - \frac{75315}{1024} \right)x^3z^3 + \left( -\frac{28323}{16} + 405a \right)y^6 \right]_{NLL} \\
+ \left[ \left( \frac{4581}{64} + \frac{855a}{32} \right)y^3x^2z + \left( -\frac{31455a}{256} + \frac{227529}{512} \right)y^3x^3 + \left( \frac{3231}{8} - \frac{135a}{2} \right)y^4xz \right]_{NLL} \\
+ \left[ \left( -\frac{3807}{32} + \frac{225a}{16} \right)y^3x^2z + \left( -\frac{28197}{128} + \frac{7191a}{128} \right)y^2x^2z^2 + \left( \frac{14847}{512} - \frac{1215a}{64} \right)y^2x^2z^2 \right]_{NLL} \\
+ \left[ \left( \frac{3603}{128} + \frac{165a}{64} \right)y^2x^3z + \left( -\frac{35145}{512} + \frac{621a}{256} \right)yx^4z + \left( \frac{7323}{64} - \frac{2643a}{128} \right)yx^3z^2 \right]_{NLL} \\
+ \left[ \left( -\frac{93}{2} + \frac{345a}{32} \right)y^2x^3z + \left( \frac{208629}{32768} + \frac{1485b}{2048} + \frac{1269a}{1024} \right)x^6 \right]_{NLL}.
\]

Analogous to \( A = y + K \) in the series expansions of Eqs. (14)-(15), the leading term \( A \) in the power series \( \tilde{A} \) is just \( y + K \), where the constant \( K \) is inclusive of all degree-2 and higher purely \( \phi^4 \) terms (\( p = 0 \)) in the full potential \( V \). Given our previous determination (in the absence of gauge couplant s \( r \) and \( s \) of \( K = -0.057353 \)) and our NLL SFTP + \( \{ x, z \}_{LL} \) result \( a = T_{1,0} = 2.5533 \) [Table II] we find upon application of conditions \( \theta \) that \( b = -17.306 \) and \( y(v) = 0.05311 \). Substituting these results into Eq. \( \Gamma \) [via Eqs. (17)-(20)], we find that \( V_{eff} = (227.8 \text{GeV})^2 \). This result, involving a full NLL treatment of dominant \( \{ x, y, z \} \) contributions to \( V_{eff} \), is a full next-order extension of the LL contributions of \( \{ x, y, z \} \) to \( V_{eff} \) presented in [2]. If we further augment this NLL result with LL contributions from electroweak gauge couplants \( \tilde{V} \), \( T_{1,0} = 2.5603 \) [Table II], \( b = -17.857 \), \( y(v) = 0.05374 \), and \( V_{eff} = (230.7 \text{GeV})^2 \).

In radiative electroweak symmetry breaking, the next order relationship between the physical Higgs boson mass and \( V_{eff} \) has been worked out in principle [1]. In particular, the next order Higgs inverse propagator mass term must remain \( V_{eff}(v) \) because of the absence of a primitive \( \phi^2 \) term in the original Lagrangian. The kinetic term for the inverse propagator (as seminally discussed for the massless gauge boson propagator in [12]) must retain consistency with the relation \( m_{H}^2 = -\gamma(x) \phi \) implicit within the RG equation [2], in which case the next-order inverse propagator for the Higgs field at \( \mu = v \) may be expressed as

\[
\Gamma(p^2, v) = \left[ 1 - \left( \frac{3}{4}x(v) - \frac{9}{16}r(v) - \frac{3}{16}s(v) \right) \log \left( \frac{p^2}{v^2} \right) \right] p^2 - \frac{v^2}{V_{eff}(v)}.
\]

We have included only those SM contributions to \( \gamma(v) \) that are linear in the couplants \( \{ x, y, z, r, s \} \) [10]. The zero of \( \Gamma \) is the NLL prediction for the physical Higgs boson mass \( \Gamma \left( m_H^2, v \right) = 0 \), which is found to be reduced by only 0.2–0.3 GeV from values respectively below 231 GeV and above 220 GeV extracted from \( V_{eff}(v) \) past LL order. We therefore conclude that the 220–230 GeV Higgs boson mass and a factor of five enhancement of the scalar-field self-interaction coupling are indeed signature predictions for radiative SM electroweak symmetry breaking. This enhancement should be particularly evident in WW → HH cross-sections [13] accessible in the not too distant future.

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