COSMOLOGICAL PARAMETER ESTIMATION FROM LARGE-SCALE STRUCTURE DEEP LEARNING

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ABSTRACT

We propose a light-weight deep convolutional neural network to estimate the cosmological parameters from simulated 3-dimensional dark matter distributions with high accuracy. The training set is based on 465 realizations of a cubic box size of 256 $h^{-1}$ Mpc on a side, sampled with $128^3$ particles interpolated over a cubic grid of $128^3$ voxels. These volumes have cosmological parameters varying within the flat ΛCDM parameter space of $0.16 \leq \Omega_m \leq 0.46$ and $2.0 \leq 10^9 A_s \leq 2.3$. The neural network takes as an input cubes with $32^3$ voxels and has three convolution layers, three dense layers, together with some batch normalization and pooling layers. We test the error-tolerance abilities of the neural network, including the robustness against smoothing, masking, random noise, global variation, rotation, reflection and simulation resolution. In the final predictions from the network we find a 2.5% bias on the primordial amplitude $\sigma_8$ that can not easily be resolved by continued training. We correct this bias to obtain unprecedented accuracy in the cosmological parameter estimation with statistical uncertainties of $\delta \Omega_m=0.0015$ and $\delta \sigma_8=0.0029$. The uncertainty on $\Omega_m$ is 6 (and 4) times smaller than the Planck (and Planck+external) constraints presented in Ade et al. (2016).

Keywords: large-scale structure of Universe — dark energy — cosmological parameters

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1. INTRODUCTION

The current standard model of cosmology has been highly successful at describing the Universe on large scales. From the anisotropic temperature fluctuations in the cosmic microwave background (CMB) to the late time clustering of galaxies, the vacuum energy dominated cold dark matter model ($\Lambda$CDM) (Weinberg 1989; Peebles & Ratra 2003; Miao et al. 2011) fits the data surprisingly well (Riess et al. 1998; Perlmutter et al. 1999; Weinberg et al. 2013; Ade et al. 2016; Alam et al. 2017).

For cosmologists, one main task would be to precisely estimate the parameters of the Universe, such as the dark matter ratio $\Omega_m$, the local expansion rate $H_0$, the amplitude and index of the primordial fluctuation $A_s$ and $n_s$, the dark energy equation of state $w_s$ together with its time dependence $w_a$, and so on.

The spatial distribution of galaxies on scales of a few hundred Megaparsecs (Mpc) forms a distinct, very complicated filamentary motif known as the ‘cosmic web’ (Bardeen et al. 1986; de Lapparent et al. 1986; Huchra et al. 2012; Tegmark et al. 2004; Guzzo et al. 2014). The distribution and clustering properties of galaxies in the cosmic web encodes information on the expansion and the structure growth history of the Universe. In the next decades, several large scale surveys (e.g., DESI\(^1\), EUCLID\(^2\), LSST\(^3\), WFIRST\(^4\)) will begin operations to map out an unprecedented large volume of the Universe with extraordinary precision. It becomes essential to develop powerful tools that can comprehensively and reliably infer the cosmological parameters from large scale structure (LSS) data.

Currently, the most widely-adopted LSS data mining methods is still the 2-point correlation function (2pCF) or power spectrum measurements, which are sensitive to the geometric and and structure growth history of the Universe (Kaiser 1987; Ballinger et al. 1996; Eisenstein et al. 1998; Blake & Glazebrook 2003; Seo & Eisenstein 2003). These methods have achieved tremendous success when applied to a series of galaxy redshift surveys such as 2-Degree Field Galaxy Redshift Survey (2dF-GRS; Colless et al. (2003)), the 6-degree Field Galaxy Survey (6dFGS; Beutler et al. (2011)), the WiggleZ survey Blake et al. (2011b,a), and the Sloan Digital Sky Survey (SDSS; York et al. (2000); Eisenstein et al. (2005); Percival et al. (2007); Anderson et al. (2012); Sánchez et al. (2012, 2013); Anderson et al. (2014); Samushia et al. (2014); Ross et al. (2015); Beutler et al. (2016); Sánchez et al. (2016); Alam et al. (2017); Chuang et al. (2017). The main caveat of this method is that, the distribution of structures and their velocities on scales of $\lesssim 40h^{-1}$ Mpc are highly affected by the non-linear processes, making it difficult to conduct a comparison between observations and theories.

Ongoing research seeks to achieve LSS data mining on non-linear scales or beyond the 2pCF. The next order correlation function, the 3-point correlation function, has been shown to add cosmological constraints beyond the 2pCF (Slepian et al. 2017) and it has also shown promise in constraining modified gravity models (Sabiu et al. 2016). The 4-point function may also lead to improved constraints if it can be modelled correctly (Sabiu et al. 2019).

Some other tests include the proposal to use the apparent stretching of cosmic voids as a probe of geometry (Ryden 1995; Lavaux & Wandelt 2012); the redshift invariance of the comoving scale information in the LSS to probe the expansion history (Li et al. 2014; Li et al. 2017); the symmetry properties of galaxy pairs to conduct an Alcock-Paczynski (AP) tests (Alcock & Paczyński 1979; Marinoni & Buzzi 2010); the redshift-dependent property of the AP effect to overcome the effect of redshift space distortion (RSD) (Li et al. 2014; Li et al. 2015) to successfully derive tight dark energy constraints from the SDSS galaxies (Li et al. 2014, 2018, 2019b; Zhang et al. 2019b). Recently, Fang et al. (2019) applied the so-called $\beta$-skeleton statistics to study LSS and proposed its application for cosmological analysis; Ramanah et al. (2019b) proposed to use the large-scale Bayesian inference framework to constrain parameters via the AP test.

To summarize, there are many alternative ideas and concepts that have been used proposed and used to extract information from the LSS, and one may refer to Weinberg et al. (2013) and the references therein for a more complete overview.

While cosmologists have obtained prominent information about the physics of the Universe via the current statistical methods, due to the extreme sophistication of the cosmic web we are still far from having a statistical method to comprehensively explore the overwhelming information encoded in the cosmic LSS. Fortunately, recent developments in machine learning techniques may allow us to capture and extract more cosmological information from the complex LSS data.

Machine learning techniques, especially the deep learning algorithms based on deep neural networks, are becoming a mainstream toolkit for modeling the relationship between complex data and the underlying

1 https://desi.lbl.gov/
2 http://sci.esa.int/euclid/
3 https://www.lsst.org/
4 https://wfirst.gsfc.nasa.gov/
variables that it corresponds to. They make it possible to extract and analyze features contained in the data, which can not be neatly extracted via traditional methods of scientific research. Recently, machine learning techniques have been applied to all domain fields of cosmology, including weak lensing (Schmelzle et al. 2017; Gupta et al. 2018; Springer et al. 2018; Flurí et al. 2019; Jeffrey et al. 2019; Merten et al. 2019; Peel et al. 2019; Tewes et al. 2019), cosmic microwave background (Caldeira et al. 2018; Rodriguez et al. 2018; Perraudin et al. 2019; Mchnmeyer & Smith 2019; Mishra et al. 2019), cosmic large scale structure (Lucie-Smith et al. 2018; Modis et al. 2018; Berger & Stein 2019; He et al. 2019; Lucie-Smith et al. 2019; Pfeffer et al. 2019; Ramanah et al. 2019a; Trstr et al. 2019; Zhang et al. 2019a), gravitational wave (Dreissigacker et al. 2019; Gebhard et al. 2019), cosmic reionization (La Plante & Ntampaka 2018; Gillet et al. 2019; Hassan et al. 2019b; Chardin et al. 2019; Hassan et al. 2019a), supernovae (Lochner et al. 2016; Moss 2018; Ishida et al. 2019; Li et al. 2019a; Muthukrishna et al. 2019), and so on. For more details, one can refer to Mehta et al. (2019); Jennings et al. 2019; Carleo et al. (2019); Ntampaka et al. (2019) and the references therein.

In a pioneering work, Ravanbakhsh et al. (2017) presented a CNN (convolutional neural network) to infer cosmological parameters from the dark matter density field. They were able to constraint $\Omega_m$ and the matter over-density variance $\sigma_8$, finding that the machine learning techniques can outperform the traditional 2pCF statistics. Mathuriya et al. (2018) presented a more sophisticated parallel calculation on tens of thousands of nodes, and simultaneously predict $\Omega_m, \sigma_8$, and the primordial power spectrum index $n_s$.

In this work we build upon that work to explore a new deep learning architecture and perform new tests to study the LSS. We show that it is possible to constrain $\Omega_m$ and $\sigma_8$ using $32^3$ voxels only as an input, a small number compared to the larger sizes of $64^3$ and $128^3$ used by Ravanbakhsh et al. (2017) and Mathuriya et al. (2018), respectively. The architecture we propose is also simpler than the ones suggested in those two works. This paper is structured as follows. In Section 2 we introduce the samples used for the training and testing, while the Section 3 we explain the architecture of our neural network. The results are presented in Section 4. We conclude in Section 5 by discussing the future of the technique and its caveats.

2. DATA

The training and testing samples are created with the COMoving Lagrangian Acceleration (COLA) code (Tassev et al. 2013; Koda et al. 2016), which is designed as a mixture of N-body and perturbation theory to simulations with fast speed and good accuracy. We choose COLA because it is hundreds of times faster than N-body simulations, while keeping a good accuracy in generating structures on non-linear scales.

We change two cosmological parameters in our simulations, the fraction of matter, $\Omega_m$, and the amplitude of the primordial power spectrum, $A_s$. Values of the other parameters are taken as $\Omega_b = 0.048206, h = 0.6777, n_s = 0.96$, the same as the MultiDark Planck N-body simulations (Klypin et al. 2016).

We vary the values of $\Omega_m$ and $A_s$ on a $31 \times 15$ grid, i.e. $0.16 \leq \Omega_m \leq 0.46$ with step size 0.01, and $2.0 \leq A_s \leq 2.3$ with step size 0.02. This parameter space is centered around the Planck 2015 best fit cosmology (Ade et al. 2016) 6. This leads to a varying $\sigma_8$ in the range of 0.4-1.1.

For all samples, we run a simulation with $128^3$ particles, in a $(256 h^{-1}\text{Mpc})^3$ box, using 40 timesteps. We output the normalized density field,
\[
\delta \rho(x) \equiv \frac{\rho(x)}{\bar{\rho}},
\]
on a grid with $128^3$ voxels at redshift $z = 0$.

To train the neural network we generate 31 $\times$ 15 samples (i.e. boxes) – one sample for an individual cosmology. The random initial conditions are different for these samples, so that our machine learning can capture the cosmic variance.

To test the neural network we generated two sets of testing samples:

- The “single-cosmology” testing samples, for which we generated 500 samples sharing the same cosmology $(\Omega_m, \sigma_8) = (0.3072, 0.8228)$. This allows us to validate the statistical error of the neural network predictions.

- The “multi-cosmology” samples, wherein we have 31 $\times$ 15 samples, using different cosmologies (on the same grid of the testing sample cosmology grid). The multi-cosmology set allows us to validate the accuracy of the parameter estimation in the whole parameter space.

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5 https://www.oreilly.com/ideas/a-look-at-deep-learning-for-science

6 Planck 2015 (TT,TE,EE+lowP+lensing) gives $\Omega_m = 0.3121 \pm 0.0087, 10^9 A_s = 2.13 \pm 0.053, \sigma_8 = 0.8150 \pm 0.0087$ in the $\Lambda$CDM framework.
The testing samples are created using initial conditions different from those of the training samples.

In Figure 1 we plot the density fields and the particle distributions of three training samples, \((\Omega_m, \sigma_8) = (0.16, 2.0.43), (0.26, 2.16, 0.72), (0.36, 2.0, 0.89)\). Obviously, the clustering strength increases when increasing \(\Omega_m\) or \(\sigma_8\), making the structures more compact. In Figure 2 we plot the cosmologies of the training and testing samples, in the \(\Omega_m\)-\(\sigma_8\) space. In contrast to the \(\Omega_m\)-\(\sigma_8\) space, here we see a strong degeneracy between the two parameters.

3. METHODOLOGY

One disadvantage of deep learning is almost impossible to design an architecture from first principles for the task at hand. Furthermore, although a precise parameter estimation is achieved, it is difficult to say what spatial scale or features are the most relevant to predict the final cosmological parameters. Here we use a large number of filters at the very beginning of convolution, based on the belief that small scale structures contain abundant information and should be convolved by many filters to extract various features.

The input of the whole network is a 32\(^3\)-voxel (i.e. \((64\ h^{-1}\ Mpc)^3\)) subcube of the original density fields that is stored in a 128\(^3\)-voxel cube. We do not feed the whole 128\(^3\) voxel cube to the neural network based on three considerations.

1. To learn a larger cube the network should have more neurons or layers and thus its training becomes much more difficult and expensive.

2. Large cubes is challenging for the memory especially for off-the-shelf GPUs.

3. In this work we want to focus on scales of \(\lesssim 50\ h^{-1}\ Mpc\)^3. On larger scales, perturbation theory and 2-point statistics of dark matter distribution has been well studied.

In the next two layers, we group these small-scale features together to extract the large-scale features. It is fair to say that, in the end, we mainly use the information of structures on scales of 6 – 64 \(h^{-1}\)Mpc.

The default architecture we describe in this section is closer to that used in Mathuriya et al. (2018) than the one used in Ravanbakhsh et al. (2017). In the next section we also discuss the effect of some changes on this default architecture.

The structure of our neural network is shown in Figure 3. It contains three convolution layers and three dense layers. In the next subsection we discuss the implementation details.

3.1. Convolution

CNNs networks are designed to be “shift/space invariant artificial neural networks”, having shared-weights architecture and translation invariance characteristics. They are especially suitable for analyzing images, videos, or any kind of structures with a large number of pixels/voxels and shift/space invariant properties.

The density field is fed to three convolutional layers. The inputs of these layers are one or many cubes. The convolutional kernels then convolve the inputs, and pass the results to the next layer.

The parameters in the convolutional kernels decide features to be extracted from the input data. They encode the prediction of cosmological parameters. For example, the first layer contains 32 3\(^2\)-filters; this means that 32 features are extracted, by conducting dot product of the kernels of filters and the \((6 \ h^{-1}\ Mpc)^3\) subcubes of the data, with a stride of 2 \(h^{-1}\)Mpc. Clearly, the information extracted here belongs to the highly non-linear clustering region. The summation of the dot-products are transformed by the activation function (to have non-linear transformation in the network), for which we use rectified linear unit (ReLU), \(f(x) = \max(x,0)\). This simple form enables fast calculation of gradients and effectively suppresses over-fitting, and we accept it in the dense layers.

In Figures 4, 5, we show how the CNN works. Step by step, the features are extracted by the three layers, and become more and more condensed. Different filters identify different features. With a large number of filters we are able to perform a very comprehensive statistics. The final outputs are 128 2\(^3\)-voxel cubes. The Figures clearly show that two different cosmologies lead to different outputs.

The parameters of filters are tuned in the training process in a way that they can extract features which are closely related to the cosmological information. By default we use the Adaptive Moment Estimation (Adam) optimization algorithm (Kingma & Ba 2014) to find the values of parameters which minimize the loss function. The optimized CNN is far more complicated than any traditional statistics (e.g., 2-point and 3-point statistics). This enables more comprehensive data mining.

3.2. Batch normalization and pooling

A batch normalization layer is placed before each convolution layer. Batch normalization is achieved through a normalization step that fixes the means and variances of each layer’s inputs. It was initially proposed to solve
Figure 1. The density field (left) and particle distribution (right) in three cosmologies \((\Omega_m, A_s, \sigma_8) = (0.16, 2.00, 0.43), (0.26, 2.16, 0.72), (0.36, 2.00, 0.89)\), selected from the training sample. We plot the 2D distribution, with the third dimension restricted to a thin slice \(0h^{-1}\text{Mpc} < z < 2h^{-1}\text{Mpc}\). The clustering strength is enhanced when increasing \(\Omega_m\) or \(A_s\), making the structures more “compact”. We train neural networks to build up connections between the density fields and their underlying cosmological parameters.
Figure 2. $\Omega_m$ and $\sigma_8$ values for the 465 training samples and single-cosmology test samples. The multi-cosmology test samples have exactly the same values of $\Omega_m$ and $\sigma_8$ as those in training samples.

“internal covariate shift” problem, and can also regularize the network such that it is easier to generalize. Now it has become a widely-accepted technique for improving the speed, performance and stability of the neural networks.

Results of each convolutional layer, are also passed to a “pooling” layer to decrease the sample size. Ravanbakhsh et al. (2017) suggests using averaging pooling for LSS data, so we adopt it as one of our default options of the network. However, for our architecture max-pooling shows better performance.

3.3. Fully Connected Layers

Outputs of the final pooling are flattened and passed to three fully connected layers with 1024, 256 and 2 neurons, respectively. They can connect the features extracted by the CNN to the values of $\Omega_m$ and $\sigma_8$. To suppress over-fitting we have a 20% dropout layer was placed before the dense layers.

4. RESULTS

In this section we present the results of the neural network.

4.1. Convergence test

The leftmost column of Figure 6 shows the learning curves of two different runs using the default architecture. Plotted are the average of the predictions from the 500 single-cosmology samples. The two runs yield very different predictions at the early stage of training, while after $\sim$200 epochs they start to converge and yield similar predictions ($n$ training epoch means the whole training samples are fed to the network by $n$-th time). After 400 epochs, their predictions are basically same.

4.2. Different architecture

Middle columns of Figure 6 show some tests on the architecture choices.

We tuned the architecture by decreasing, either the number of filters in the convolutional layers, or the neurons in the dense layers, by a fraction of 50%; yet we find no significant change in their learning curves. We also tried doubling these numbers, and still get similar learning curves.

In the middle-right panel, we present results when we 1) use max-pooling instead of average-pooling; 2) use stochastic gradient (sgd) as the optimizer (the default optimizer is Adam). These changes slightly improve the performance (especially, decreasing the bias in the estimation of $\Omega_m$).

4.3. Bias correction

We find a bias in the estimated parameters. This bias is smaller than the one reported by Ravanbakhsh et al. (2017), but larger than the apparently unbiased results of Mathuriya et al. (2018). In most cases, we underestimate $\Omega_m$ by about 0.005 (less than 2%). While $\sigma_8$ is under-estimate by about 0.02 which is about 2.5%. Increasing the training epochs to 1,500 does not reduce this bias.

We do not have a definite answer for the origin of this bias. One possibility is that it comes from the limited power of the dense layers in regressing the cosmological parameters from the 128 2$^4$-voxel features. A support to our hypothesis is, after placing another 512-neuron layer after the 256-neuron layer, to improve the ability in mapping the many voxels to the parameters, the bias is obviously decreased (see rightmost panel of Figure 6).

To have a better understanding of the bias, we plot its parameter-dependence in Figure 7. We find a clear trend of increasing bias at larger values $\Omega_m$ or $\sigma_8$. This trend is again consistent with the results in Ravanbakhsh et al. (2017).

Adding more layers/neurons in the dense layers to further decrease the bias goes against of our objective of having a simple and light convolutional network. Instead, we opt for a simpler (and possibly more accurate)
Figure 3. The architecture of our neural network. A cube having $32^3$ voxels is fed to the network. The three convolution layers have 32, 64, 128 filters, respectively. Beside each convolution layer, a batch normalization layer is added before it to normalize the distribution (so that to enhance the stability), and a pooling layer is placed after it to decrease the size of the output. After that, we got $128 \times 2^3$ voxels containing the extracted features. They are then converted to a 1-d vector by the flatten layer, and passed to three dense layers with 1028, 24, 2 neurons, to output the final predictions of $\Omega_m$ and $\sigma_8$.

Figure 4. Layer-by-layer outputs of the CNN when fed by a sample with cosmology parameters $(\Omega_m, A_s, \sigma_8) = (0.26, 2.16, 0.72)$. The many filters, determined by the 896/55,360/221,312 trainable parameters in the three convolutions layers, can capture various types of features. The final outputs of the CNN is a set of $128 \times 2^3$-boxes containing the most compressed features extracted from the data. They are passed to the dense layers (not plotted here) for parameter estimation.
Figure 5. Same as Figure 4, except that for the case of \((\Omega_m, A_s, \sigma_8) = (0.26, 2.00, 0.43)\). The features extracted are significantly different from those in the cosmology \((\Omega_m, A_s, \sigma_8) = (0.26, 2.16, 0.72)\), making it possible to distinguish these two cosmologies.
Figure 6. Learning curve using different architectures. First panel: two runs using the default options reaches convergence after 160 epochs. Second panel: decreasing the number of CNN filters or dense neurons by 50%, no significant change in the performance. Third panel: among our trials of different options, using max-pooling or sgd optimizer can notably enhance the performance. Fourth panel: an extra dense layer with 512 neurons are added before the final outputs to achieve a more accurate mapping from CNN outputs to the cosmological parameters. A good performance is detected at ≈80 epochs; more training epochs results in over-fitting.

4.4. Cosmological constraint

Figure 9 shows the final constraints derived from the 500 single-cosmology samples, where the bias has been subtracted based on the polynomial regression. To avoid self-correction, the regression is derived using the multim-cosmology samples, which have no overlapping from the single-cosmology samples.

We find the CNN accurately predicts the parameters as

\[
\Omega_m = 0.3073 \pm 0.0015, \quad \sigma_8 = 0.8178 \pm 0.0029.
\]  

They are statistically consistent with the ground truth (0.3071, 0.8228). We find the prediction of \( \sigma_8 \) still suffers from a ≈1σ bias; this can be overcome by performing a more precise bias-estimation based on larger amount of samples (e.g. a point-by-point correction on the grid).

The statistical error of \( \Omega_m \) is 6 times smaller than the Planck 2015 TT,TE,EE+lowP+lensing constraint, 4 times smaller than Planck+BAO+JLA+\( H_0 \) constraint (Ade et al. 2016). Having derived this result from a (256 \( h^{-1} \) Mpc)\(^3\), 2 \( h^{-1} \) Mpc resolution sample shows the great potential of using neural network to estimate cosmological parameters from the LSS.

\(^8\) A polynomial regression may sound arbitrary, but in principle it has no intrinsic difference from a mapping using dense layers.
Figure 7. Test of a CNN architecture (sgd) on a multi-cosmology grid. There is a strong degeneracy between $\Omega_m$ and $\sigma_8$. Left panel: Ground truth and CNN predictions of $\Omega_m$ and $\sigma_8$, in the 2-d parameter space. The black lines show the difference between them. The bias is larger at the upper-right corner of the parameter space. Right panels: Ground truth and CNN predictions for $\Omega_m$ and $\sigma_8$ panels, respectively.

Figure 8. Distribution of the systematic bias in the CNN predicted $\Omega_m$ and $\sigma_8$ (denoted as $\Delta \Omega_m$ and $\Delta \sigma_8$). Very roughly, in the parameter space we studied, there is $|\Delta \Omega_m| \lesssim 0.03$ and $|\Delta \sigma_8| \lesssim 0.05$, with mean value of $|\Delta \Omega_m| = 0.01$ and $|\Delta \sigma_8| = 0.018$. In practice one can calibrate the results by subtract the systematic bias in the CNN predictions (e.g., using the fitting formula shown in the panels), making the final estimation unbiased.

4.5. Error tolerance

So far we only apply the neural network to ideal datasets – density fields regularly sampled in a 3-d grid based on the dark matter particles. In reality, the data obtained in observations contain many sources of systematics. Here, we quantify how this noise affects the performance of the neural network.

The ET (error tolerance) tests are presented in Figure 10. For simplicity, in these tests we only use one $128^3$-voxel sample, generated using $(\Omega_m, \sigma_8) = (0.26, 0.69)$. We split the grid into 64 $32^3$-voxel subgrids to obtain 64 sets of estimated parameters. Adding different kinds of noise into the subgrids, and feed them to the neural network to predict parameters. When a certain kind of noise was added, we check whether the estimations are changed, and get some understandings about the effect of noise.

In summary, we find that:
A smoothing of the sample\textsuperscript{9} can lead to disastrous effect. Even a 1% smoothing shifts the estimation by \(\approx 2\sigma\). A 3% smoothing doubles the shifts and also doubles the statistical scattering.

In contrast, the performance of the neural network is very robust to missing voxels. We mask 1 or 4\textsuperscript{3} voxels in each of the 32\textsuperscript{3}-subgrid (by setting their values to 0), and find the predicted results almost unchanged. This ET ability is helpful, since in real observations there are always many masked regions.

The performance is not significantly improved if we conduct data enhancement (DE) via rotation and reflection. The number of the 3d subgrids can be increased by as much as 48 times after DE. No significant improvement in the predictions is detected if we feed the 48-times more samples to the neural network.

The predictions are very robust to Gaussian noise. In this test, all voxels are multiplied by a Gaussian random variable with a standard deviation of 5% or 10%. The central values and errors remain unchanged.

If we introduce a 5% or 10% global variation in the density field (linearly increased from 0% at \(x = 0\) to the maximal value at \(x = 256\, h^{-1}\, \text{Mpc}\)), notable change appears in the predicted results. Thus, when analyzing observational data, one should be careful about the factors which can globally change the survey properties in a large area.

In case that we feed the neural network using samples produced in 10\% lower/higher resolution (decreasing/increasing the number of simulation particles by 10\%), the central values are mildly shifted (\(\sim 1\sigma\)).

When handling real observational data, an observational artifact can be overcome in two ways. 1) Designing a neural network which is robust to it. 2) Adding it into the training sample, so that its effect is considered by the neural network in the training process.

5. CONCLUDING REMARKS

We used a deep convolutional neural network to estimate cosmological parameters from simulated dark matter distributions. The simulations are 128\textsuperscript{3}-voxel, \((256\, h^{-1}\, \text{Mpc})^{3}\) cubes of the dark matter density contrast field. The neural network, designed to have three convolution layers, three dense layers, including batch normalization and pooling layers, builds up a connection from the field to the cosmological parameters. It is able to yield accurate prediction of the cosmological parameters after \(\sim 200 - 300\) epochs of training. We also studied some variations on the architecture to test its convergence and overall performance.

In the estimated parameters, we find a persistent bias that can not be resolved by increasing the training epochs. We believe that this bias arises from the limited power of the dense layers, which are responsible for map-

\textsuperscript{9} Our smoothing means that each voxel is replaced by a weighted sum of itself and its six nearest neighbors. Different types of smoothing can have different effect and should be tested individually.
Figure 10. Error-tolerance tests. A 3% smoothing or 10% global variation leads to considerable change in the predicted results (\( \sim 2\sigma \) shift in central values, \( \sim 100\% \) enlarged errors). 1% smoothing, 5% global variation, and 10% change in the simulation’s resolution mildly affect the prediction (\( \sim 1\sigma \) shift in central values, errors unchanged). Other cases, including the 1 or 4 voxels removal, 5% or 8% random noise addition, rotation and reflection, does not affect the results at all.
ping the outputs of the convolution to the cosmological parameters. Using more sophisticated dense layers, or simply applying a subtraction based on polynomial regression, the bias can be suppressed. We also tested the error-tolerance abilities of the neural network, including the abilities against smoothing, masking, random noise, global variation, rotation, reflection and resolution.

We obtain precise estimations, with statistical scattering of $\delta \Omega_m = 0.0015$ and $\delta \sigma_8 = 0.0029$, from the neural network. The statistical error of $\Omega_m$ is 6 and 4 times smaller than the Planck and Planck+ext constraints presented in Ade et al. (2016). We conclude that deep neural networks are very promising in estimating cosmological parameters from the LSS.

The persistent bias in the prediction of our neural network would be the biggest caveat limiting the power of the technique. The bias was also detected in the work of Ravanbakhsh et al. (2017), yet the authors did not provide a strategy to overcome it. It seems that the bias is greatly reduced if one uses a more complex network architecture with seven convolutional layers and $128^3$ voxels as an input (Mathuriya et al. 2018).

The approach that we develop to correct the bias is a simple subtraction based on polynomial regression. This is not completely satisfactory and future work should aim to address this problem, i.e. measuring how it depends on the architecture parameters. This will allow us to design better architectures with a smaller bias, and conducting more concrete tests based on larger training samples. This study is a required prerequisite to conduct a reliable, comprehensive analysis of LSS using deep learning.

On the physical side there are at least many directions for future work.

1. In this simple work, we haven’t consider the role of redshift space distortion (RSD) in the parameter estimation. We tend to believe that the RSDs, which creates more cosmological dependent features in the matter distribution, should lead to better parameter estimation. We will test this supposition in forth-coming works.

2. In the case of a survey covering a $(512 \ h^{-1} \text{Mpc})^3$ or $(1 \ h^{-1} \text{Gpc})^3$ volume of density field, one can further decrease the statistical error by 3 or 8 times. In that case the bias and error tolerance (to systematics) of the neural network would be essentially important. Lightcone effect, selection function, galaxy bias, redshift errors, or even baryonic effects, should be tested in certain circumstances.

3. The resolution of our input sample, $2 \ h^{-1} \text{Mpc}$, is a bit high when considering the current and near-future spectroscopic surveys, which have low comoving number densities. So it will be necessary to apply the method to lower-resolution, more realistic galaxy samples. In the next step, we will apply it to dark matter halo samples, and see whether the neural network is still able to achieve precise parameter estimation in such circumstances.

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