Impact of transformer’s leakage inductance on duty cycle in isolated dc-dc converters

Rini Nur Hasanah a, Taufik Taufik b, and Onny Setyawati a

aDepartment of Electrical Engineering, Faculty of Engineering, Universitas Brawijaya, Malang 65145 Indonesia
bElectrical Engineering Department, California Polytechnic State University, San Luis Obispo, CA 93407, USA

Abstract

Switching power supplies utilizing isolated dc-dc converter are seeing increased use to power today’s consumer electronics and home appliances. However, for all the benefits of dc-dc converters such as efficiency, problems still exist with their use. One issue relates to leakage inductance of transformer used in the isolated dc-dc converters, which is often seen as one of the major contributors to degradation in performance. Leakage inductance’s role in circuit losses, changes in load regulation, and noise created by the given converter has been widely known. However, its direct impact on the operating duty cycle of the converter has not been investigated. This paper aims to examine the impact of leakage inductance to duty cycle in two popular isolated topologies, namely the single-switch forward and the flyback converters. The study used computer simulation to observe the effect of worsening the leakage inductance through transformer’s coupling coefficient. Results from computer simulation reveal a nearly linear relationship between leakage inductance and duty cycle. Under the given simulation parameters, the change in duty cycle reaches as much as 12.8% for the single-switch forward converter and 6.8% for the flyback converter.

Keywords: Duty cycle, flyback converter, leakage inductance, single-switch forward converter, transformer

1. Introduction

The role of switching power supplies in modern electrical systems has become increasingly prevalent due to their many benefits including efficiency, high energy density, fast dynamic response, bidirectional operating capability, and flexibility in providing step-up/down functions. Applications of switching power supplies encompass many industrial sectors such as consumer electronics [1], welding [2], commercial building [3], renewable energy [4], etc. For off-line applications, these switching power supplies utilize isolated dc-dc converter topologies. There are several popular topologies mainly based on required input voltage and output power ratings with the most being the forward, flyback, and bridge converters.

Forward converters are commonly used nowadays partly due to their relative simplicity and good efficiency. In spite of some disadvantages such as poor transient responses, forward converters still hold an economic edge over push-pull converters in terms of cost and space, while also being more power efficient than flyback converters. Being an isolated converter topology, one that uses a transformer to isolate the input from the output, and increase or decrease voltage through the transformers turns ratio, problems associated with the use of a transformer become an issue. While transformers allow for galvanic isolation between input and output, transformers have some shortcomings, a more obvious one being their size [5].

Flyback converters are also very popular in the world of power supplies, partly due to their relative circuit simplicity. Flyback converters, compared to other isolated dc-dc converters, have the lowest part count. As in forward and other isolated topologies, however, flyback converters are also affected by some
problems associated with the use of transformer [6].

Other than the issue with their physical size, real world transformers also have the inherent leakage inductance that affects the operation and performance of dc-dc converters, and thus switching power supplies. Leakage inductance $L_{dkg}$ in Fig. 1 results from imperfectly coupled transformer coils and represents energy that is stored in the non-magnetic regions between windings, and it is typically modelled as a small series uncoupled inductance to its associated winding. The less coupled transformer’s coils are, the more leakage inductance will exhibit from the transformer. The appearance of leakage inductance tends to be inevitable, as there will be some imperfection in a given transformer’s coupling. Although leakage inductance is not always an undesirable trait; however, in many applications it may cause major steady state and transient problems for converters such as in the single-switch forward and flyback converters. For instance, a power supply designed assuming 100% coupling runs the risk of biasing the converter toward its duty cycle limits. This means, the actual non-ideal or poor coupling could cause the converter to run with a duty cycle closer to its limit when nominal voltage values are present on the input and output. This would in turn limit the converters flexibility, which is not desirable since in general we want the power supply to have the biggest possible range of duty cycle to achieve the most robust output regulation. Therefore, isolated switching mode power supplies require at the very least well characterized transformers whose coupling coefficients are known. Additionally, there are other known issues related to the existence of leakage inductance in isolated converters such as leakage voltage spikes, additional circuit losses causing reduced efficiency, and increased level of noise generated within and outside of the converter [7].

![Transformer model showing its inherent components.](image)

This paper aims to provide some illumination and information as to how leakage inductance in single-switch forward and flyback affects the converter’s duty cycle. Through simulation using LTSpice, the goal is to gain clearer relationship between converter’s leakage inductance and duty cycle. Various articles and papers have explored leakage inductance and its effects on converters such as the forward converter. These pieces of research have focused on leakage inductance’s effect on performance parameters such as noise, regulation, and especially efficiency [7]. Topologies such as the active clamped forward converter are proven to be able to minimize the impact of leakage inductance on the switch’s voltage stress [8]. Other converter topologies make use of the circuit’s leakage inductance, and in some cases the leakage inductance provides benefits to the converter’s performance. In addition, leakage inductance in flyback converters have also been linked with the worsening of static load regulation while smoothing/improving the dynamic response [9].

Aside from the aforementioned general information on the effect of leakage inductance on converters, there is very little published information on more specific issues, such as how leakage inductance affects the converter’s duty cycle. With the forward converter being widely used, especially for powers fewer than 200 watts, a closer look at leakage inductance could prove useful and beneficial.

### 2. Problem Formulation

In order to understand the complexity of the relationship between leakage inductance and duty cycle, the derivation of the equation that links the two is presented as follows. One method that may be used is to utilize the overall converter’s efficiency equation where $\eta$ is the efficiency, $P_o$ is the average output...
power and $P_{in}$ is the average input power:

$$\eta = \frac{P_{out}}{P_{in}} \Rightarrow P_{out} = \eta \cdot P_{in} \quad(1)$$

Expressing $P_{in}$ in terms of $P_o$ and the sum of all power losses $P_{loss}$ in the converter yields:

$$P_{loss} = \frac{P_o(1-\eta)}{\eta} \quad(2)$$

The converter’s loss $P_{loss}$ typically consists of static and dynamic losses. The static loss is caused mainly from ohmic losses of parasitic resistance of components such as MOSFET’s on resistance, diode’s forward drop, inductor’s coil resistance, etc. Dynamic loss considers those losses associated with switching loss caused by the on-state and off-state transitions of current and voltage for each switching component. Thus, it is predominantly driven by the operating switching frequency of the converter, especially in pulse-width-modulated (PWM) based converters. Since the actual loss calculation is quite extensive, in this paper we will only consider those losses associated with the transformer since it contains the leakage inductance. Referring to Fig. 1, the transformer loss may be presented from the following equation, where $\tilde{i}$ and $i$ are rms and peak currents respectively, $R_{copper}$ is winding resistive loss, $P_{core}$ is the core loss, and $P_{Lmag}$ is the loss due to magnetizing inductance energy.

$$P_{loss} = \tilde{i}^2 R_{copper} + P_{core} + \frac{1}{2} L_{lkg} \tilde{i}^2 f + P_{Lmag} \quad(3)$$

Realizing that $P_{Lmag}$ is negligible, and at full load the current through the $P_{core}$ is small compared to the current through the leakage inductance, and assuming the converter has large output inductance, equation (3) reduces to:

$$P_{loss} = \left(\tilde{i} \sqrt{D}\right)^2 R_{copper} + \frac{1}{2} L_{lkg} \tilde{i}^2 f = \tilde{i}^2 \left(D \cdot R_{copper} + \frac{1}{2} L_{lkg} f\right) \quad(4)$$

$$D \cdot R_{copper} + \frac{1}{2} L_{lkg} f = \frac{P_o(1-\eta)}{\eta^2} \quad(5)$$

The parameter $D$ in the above equations represents the duty cycle of the converter. Equation (5) demonstrates the indirect relationship between duty cycle $D$ and leakage inductance $L_{lkg}$. The right-hand side of the equation must also use an assumed efficiency. This complicates even further since practically efficiency is a function of duty cycle and leakage inductance. Thus, equation (5) does not give the full mathematical description between duty cycle and leakage inductance, but it is sufficient to demonstrate the difficulty in mathematically show how the two parameters interact with each other. Therefore, a simpler alternate method to show their relationship is in order. For this study, we chose to exhibit the relationship using computer simulation with LTSpice.

In order to obtain the relationship between leakage inductance and duty cycle for single-switch forward and flyback converters, each converter was designed and simulated in LTspice. A few important parameters for each converter are coupling coefficient $k$, duty cycle $D$, and average output voltage $V_{out}$. Coupling coefficient $k$ signifies how well any two windings are coupled to each other. Its value is between 0 and 1, with 0 being the two windings are not coupled at all and 1 for perfect coupling. Thus, coupling coefficient $k$ can be used to represent leakage inductance. That is, as $k$ is larger, the leakage inductance $L_{lkg}$ is smaller. The relationship between leakage inductance and coupling coefficient can be described by the following equation.

$$L_{lkg} = (1-k) \cdot L_p \quad(6)$$

In any given isolated converter, when the transformer’s coupling changes, the operation and
performance of the converter will be affected such as the output voltage. In switched-mode power supplies such as the single-switch forward and the flyback, output voltage is related to its duty cycle. It can be deduced then that there must be a relationship between transformer’s coupling (and the resulting leakage inductance), and the converter’s duty cycle. It then makes sense to inspect the converter’s output voltage in order to get a glimpse of how leakage inductance and duty cycle are related.

Fig. 2 depicts an example of a single-switch forward converter’s output voltage changing when different coupling coefficients are applied while all other parameters remaining constant. For this particular example, the simulation starts with an ideal coupling coefficient of $k = 1.0$ leads to output voltage $V_{out}$ of 10.50V. At $k = 0.97$ and $k = 0.95$, $V_{out}$ drops to 9.73V and 9.20V respectively. As $k$ decreases, so does the converter’s output voltage.

![Fig. 2. Steady-state output voltage waveform of single-switch forward converter for $k = 0.95, 0.97, 1.0$.](image)

In Fig. 3, a flyback converter is simulated where output voltage is once again measured for $k = 1.0$, $k = 0.97$, and $k = 0.95$. At $k = 1.0$, the output voltage was observed at 20.22V, while at $k = 0.97$ and $k = 0.95$, the output voltage drops to 19.35V and 18.70V respectively. So, the behavior of the converters is similar; i.e. when the coupling coefficient is decreased, output voltage decreases as well. Through establishing this consistency, the hope is that the next simulations with the two converters yield likewise similar relationships between leakage inductance and duty cycle.

![Fig. 3. Steady-state output voltage waveform of flyback converter for $k = 0.95, 0.97, 1.0$.](image)

Tables 1 and 2 list the output voltage obtained from simulation results of a single-switch forward and flyback converters when the coupling coefficient is varied.
Table 1. Transformer’s coupling coefficient vs. average output voltage in single-switch forward converter

| Coupling coefficient $k$ | Average output voltage $V_{out} (V)$ |
|--------------------------|----------------------------------|
| 1.0                      | 10.50                            |
| 0.97                     | 9.73                             |
| 0.95                     | 9.20                             |

Table 2. Transformer’s coupling coefficient vs. average output voltage in flyback converter

| Coupling coefficient $k$ | Average output voltage $V_{out} (V)$ |
|--------------------------|----------------------------------|
| 1.0                      | 20.22                            |
| 0.97                     | 19.35                            |
| 0.95                     | 18.70                            |

3. Simulation Results

3.1. Single-switch forward converter

A single switch forward converter was constructed in LTspice for simulation, as seen in Fig. 4. The switching frequency of the converter was chosen to be at 100 kHz and the coupling coefficient $k$ was swept from 0.950 to 1.00 in increments of 0.005. Data collection started at $k = 0.950$ and output voltage 10V. The first data point required a duty cycle of 0.3993. For each successive data point, the coupling coefficient was set then the duty cycle was adjusted until output voltage reached 10V again. Table 3 shows the data gathered on the single-switch forward converter and Fig. 5 shows a plot of $D$ vs. $k$ obtained from the simulation results. The duty cycle variation due to the different values of coupling coefficient can now be calculated:

$$\%\Delta DC = \frac{0.3993 - 0.3540}{0.3540} \times 100\% = 12.8\%$$  \(7\)

![LTSpice schematic for the single-switch forward converter.](image)

Table 3. Simulation data of duty cycle and average output voltage as coupling coefficient and leakage inductance are varied in the single-switch forward converter

| Coupling coefficient $k$ | Leakage inductance $L_{Lk} (\mu H)$ | Duty cycle | Average output voltage $V_{out} (V)$ |
|--------------------------|-----------------------------------|------------|----------------------------------|
| 1.000                    | 0.0                               | 0.3540     | 10.007                           |
| 0.995                    | 0.5                               | 0.3582     | 10.006                           |
| 0.990                    | 1.0                               | 0.3620     | 10.004                           |
| 0.985                    | 1.5                               | 0.3665     | 10.001                           |
| 0.980                    | 2.0                               | 0.3712     | 10.002                           |
| 0.975                    | 2.5                               | 0.3760     | 10.001                           |
| 0.970                    | 3.0                               | 0.3806     | 10.002                           |
| 0.965                    | 3.5                               | 0.3855     | 10.008                           |
| 0.960                    | 4.0                               | 0.3900     | 10.002                           |
| 0.955                    | 4.5                               | 0.3940     | 10.007                           |
| 0.950                    | 5.0                               | 0.3993     | 10.001                           |
Fig. 5. Duty cycle values with varying coupling coefficient (left) and leakage inductance (right) in the single-switch forward converter.

3.2. Flyback converter

The flyback converter was constructed in LTSpice as depicted in Fig. 6. For consistency, the switching frequency was also chosen to be 100 kHz and the coupling coefficient $k$ was swept from 0.95 to 1.00 in increments of 0.005. Data collection started at $D = 0.2567$ and output voltage $V_{\text{out}} = 10V$. The second data point required setting coupling coefficient to 0.995 and to get the same output, and record the corresponding duty cycle which for this case is $D = 0.2586$. For each successive data point, the coupling coefficient was set, and then the duty cycle was adjusted until output voltage reached 10V again. Table 4 shows the data gathered on the flyback converter and Fig. 7 shows a plot of $D$ vs. $k$ from the simulation results. The percentage change in the duty cycle due to the variation in the coupling coefficient for flyback converter is calculated below:

$$\% \Delta DC = \frac{0.2742 - 0.2567}{0.2567} \times 100\% = 6.8\%$$

Fig. 6. LTSpice schematic for the flyback converter.

Table 4. Simulation data of duty cycle and average output voltage as coupling coefficient and leakage inductance are varied in the flyback converter

| Coupling coefficient $k$ | Leakage inductance $L_{kg}$ (µH) | Duty cycle | Average output voltage $V_{\text{out}}$ (V) |
|--------------------------|----------------------------------|------------|------------------------------------------|
| 1.000                    | 0.0                              | 0.2567     | 10.002                                   |
| 0.995                    | 0.5                              | 0.2586     | 10.012                                   |
| 0.990                    | 1.0                              | 0.2600     | 10.003                                   |
| 0.985                    | 1.5                              | 0.2615     | 10.000                                   |
| 0.980                    | 2.0                              | 0.2633     | 10.000                                   |
| 0.975                    | 2.5                              | 0.2650     | 10.003                                   |
| 0.970                    | 3.0                              | 0.2668     | 10.004                                   |
| 0.965                    | 3.5                              | 0.2685     | 10.006                                   |
| 0.960                    | 4.0                              | 0.2704     | 10.003                                   |
| 0.955                    | 4.5                              | 0.2722     | 10.000                                   |
| 0.950                    | 5.0                              | 0.2742     | 10.002                                   |
4. Conclusions

The single-switch forward converter simulation revealed a linear relationship between coupling coefficient $k$ and duty cycle $D$. As $k$ decreases $D$ needed to increase to maintain a regulated output voltage. Compared to the flyback converter, the impact of leakage inductance to duty cycle is worse as indicated numerically by the 6% (12.8%–6.8%) increase in the percent difference of $D$ change with varying $k$.

The flyback converter simulation also revealed a linear relationship between coupling coefficient and duty cycle. Similar to the single-switch forward converter, as $k$ decreases, $D$ needed to increase to maintain a regulated output voltage. The flyback converter has a shallow slope compared to the single-switch forward converter on the coupling coefficient vs. duty cycle graphs. This means the flyback converter requires a less rise in duty cycle to reach the same output; and thus as stated before, the coupling coefficient affects the flyback converter less than the single-switch forward converter. The slope of the single-switch forward converter is $-0.9111$ while the slope of the flyback converter is shown to be $-0.3465$. This shows that for our simulations, the single-switch forward is affected by the coupling coefficient 2.63 times more than that of the flyback converter.

Despite the convenience of using computer simulation to study the relationship between leakage inductance and duty cycle, future study should attempt to achieve a general mathematical description of the relationship between the two. Such mathematical representation will help us better understand the interaction between the two parameters and provide engineers with a converter design tool to select the most optimum transformer.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

The contribution of each author can be described as follows: the second author conducted and analyzed the data; all authors contributed in writing the paper and final version manuscript, the first and third authors presented the paper; all authors had approved the final version.

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