The temperature in Hawking radiation as tunneling

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Abstract

The quasi-classical method of deriving Hawking radiation under the consideration of canonical invariance is investigated. We find that the horizon should be regarded as a two-way barrier and the ingoing amplitude should be calculated according to the negative energy particles tunneling into the black hole because of the whole space-time interchange and thus the standard Hawking temperature is recovered. We also discuss the advantage of the Painlevé coordinates in Hawking radiation as tunneling.

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I. INTRODUCTION

A classical black hole can only absorb and not emit particles. When considering quantum effect, however, Hawking discovered [1] that black hole emits thermal radiation with a temperature \( T = \frac{\kappa}{2\pi} \), where \( \kappa \) is the surface gravity of black hole. The physical reason of radiation was explained [2] as coming from vacuum fluctuations tunneling through the horizon of the black hole. But some original derivation based on the Bogoliubov transformation [1] or other methods [2, 3, 4] didn’t have the direct connection with the view of tunneling. Moreover, these methods, in which the background geometry is considered fixed, didn’t enforce the energy conservation during the radiation process. Recently Parikh and Wilczek suggested [5] a method based on energy conservation by calculating the particle flux in Painlevé coordinates from the tunneling picture. Their result recovered the Hawking’s original result in leading order and gave the consistent temperature expression and the entropy relation. The method had also been discussed generally in different situations [6, 7, 8, 9, 10] and showed the formula was self consistent even checked by using thermodynamic relation [11, 12, 13]. Another importance is to give the non-thermal spectrum which implies there may exist the information-carrying correlation in the radiation.

Another method called Hamilton-Jacobi method [14] had also been proposed to obtain the tunneling probability besides the radial null geodesic method [5]. But by using the Hamilton-Jacobi equation, the Ref. [15] gave the temperature twice as large as the Hawking temperature. Although the double temperature didn’t affect the connection of black hole radiation with the thermodynamic law as long as the proportional relation between the temperature and the surface gravity is held [16], it will also cause the change of entropy and radiation temperature which is observable by astrophysical method or at LHC. So determining whether the temperature is twice or whether there exists the factor of 2 problem is important. There had been two methods suggested to solve this problem and concluded the temperature was the same as the Hawking temperature. But the two methods look as if they were different completely. One of them [17] explained that the standard temperature could be obtained by using the detailed balancing formula that is the ratio of the outgoing and incoming probabilities, in which the canonical invariance or the tunneling dependent on direction of move was not considered. The other one [18, 19] pointed out that one must take into account the ignored temporal contribution of the action in order to recover the
original Hawking temperature, in which the normalization of the ingoing probability was not involved and the temporal rotation at the Schwarzschild horizon is periodic and indefinite. So it is necessary to find another method which not only overcomes these deficiencies but also recovers the standard Hawking temperature.

In our paper we discuss the property of the horizon and regard the horizon as a two way barrier when one considers virtual particle pairs inside and outside the horizon. According to the propagator theory, we find that when one treats the black hole radiation as tunneling, the ingoing amplitude should be calculated as tunneling of negative energy particles. Thus we can recover the standard Hawking temperature by using the canonically invariant tunneling transmission rate, \( \Gamma = e^{-\text{Im}\left(\oint p\,dx\right)} \). We show that the Painlevé coordinates is more convenient than Schwarzschild coordinates for calculating the Hawking temperature in the picture of tunneling. In the end we also discuss the other two methods [17, 18, 19] and compare them with our method.

In this paper we take the unit convention \( k = \hbar = c = G = 1 \).

II. TUNNELING, HORIZON AND PROPAGATOR

Tunneling is a quantum mechanical phenomenon to happen when the initial and final states are separated by a barrier which cannot be classically crossed because the system does not have enough energy. Generally speaking, there are two kinds of tunneling which can be described as one when the barrier is insensitive to the direction of motion and the other one when the barrier is sensitive to the direction of motion. For the former there are two equivalent expressions for the tunneling transmission coefficient, \( \Gamma = e^{-2\text{Im}\left(\oint p\,dx\right)} = e^{-\text{Im}\left(\oint p\,dx\right)} \); for the latter only one of the expressions is applicable under invariance of canonical transformation, \( \Gamma = e^{-\text{Im}\left(\oint p\,dx\right)} \). Generally we do not have this problem in the usual examples of tunneling for the form, \( \Gamma = e^{-2\text{Im}(f\,p\,dx)} \), which is because for those situations the tunneling in both direction is equally suppressed [20]. But for other situations such as the tunneling through black hole horizon, we have to notice this problem because the tunneling through black hole horizon is sensitive to the direction of motion. The classical infalling particles face no barrier at all and cross the horizon freely but the classical outgoing
particles are forbidden or cannot cross the horizon. When considering the quantum effect, however, the vacuum fluctuation can lead to generate the virtual pairs of negative-positive energy particles and makes the tunneling possible. The tunneling includes two parts, one of which is the positive energy particles tunnel out of the horizon for the virtual pair inside the horizon and the other one of which is the negative energy particles tunnel inward through the horizon for the virtual pair outside the horizon. Thus the horizon represents a two way barrier when one considers virtual particle pairs inside and outside the horizon. So we should choose $\Gamma = e^{-\text{Im}\left(\oint p dx\right)}$ as proper observable for the tunneling through black hole horizon \[15\].

On the other hand, the propagator as a transition amplitude \[21\] is symmetric under interchange of space-time coordinates. The propagator can be written as $K(x_f t_f; x_i t_i) = \langle x_f, t_f | x_i, t_i \rangle = e^{-iS}$, where the quantum system is transferred from the initial place and time $x_i, t_i$ to the final place and time $x_f, t_f$, $t_f > t_i$ and $S$ is the action of the system. So we can gain the propagator under interchange of space-time coordinates as

$$K(x_i t_i; x_f t_f) = \langle x_i, t_i | x_f, t_f \rangle = e^{iS^+} = e^{i\text{Re} S} e^{\text{Im} S},$$

and

$$K(x_f t_f; x_i t_i) = \langle x_f, t_f | x_i, t_i \rangle = e^{iS} = e^{-i\text{Re} S} e^{\text{Im} S}.$$  

This shows that the propagator is equivalent under interchange of space-time coordinates up to a pure imaginary phase. When the Hamiltonian is independent on the time, the action is separable for the time and space coordinates, $S = Et + S_0(x)$. And when the system can be treated reliably in the short-wavelength limit, the WKB approximation can be used. For the Hawking radiation as tunneling, the conditions of WKB approximation are satisfied because the Schwarzschild space-time is stationary and the particles or the short wavelength limit is supported due to the infinite blueshift of the outgoing wave-packet near the horizon \[22\]. The propagator can be also be written as $K(x_f t_f; x_i t_i) = \langle x_f \exp(-iH(t_f - t_i)) | x_i \rangle$, where $H$ is the Hamiltonian of the particles tunneling outward in black hole radiation. Since the particle with energy $E$ is considered, we have

$$K(x_f t_f; x_i t_i) = \langle x_f | e^{-iH(t_f - t_i)} | x_i \rangle = \langle x_f | x_i \rangle e^{-iE \Delta t},$$

and

$$K(x_i t_i; x_f t_f) = \langle x_i | e^{-iH(t_i - t_f)} | x_f \rangle = \langle x_i | x_f \rangle e^{iE \Delta t} = \langle x_i | x_f \rangle e^{-i(-E) \Delta t}.$$
where the amplitude $\langle x_f|x_i \rangle$ and $\langle x_i|x_f \rangle$ can be calculated in the semiclassical approximation and $\Delta t = t_f - t_i > 0$. The amplitude can be used to describe the tunneling probability, $\Gamma = \langle x_f|x_i \rangle \langle x_i|x_f \rangle = \exp \left( \text{Im} \left( \oint pdx \right) \right)$. However, when $\langle x_f|x_i \rangle$ and $\langle x_i|x_f \rangle$ are regarded as tunneling amplitude and their time is going on according to $t_i \rightarrow t_f$, it is noted that for ingoing amplitude $\langle x_i|x_f \rangle$ the energy of the tunneling particles must be treated as negative. Actually the ingoing amplitude is obtained along the reversed time, so according to Feynman’s idea that negative-energy particles can only travel backward in time, the energy of the tunneling particles for ingoing amplitude should also be negative. Therefore, when we calculate the closed contour integral, the outgoing amplitude and ingoing amplitude have to be calculated as tunneling of particles with the opposite energy. Noticed that the horizon as barrier is single-directional for particles with the same energy. Here the horizon as barrier can be treated as both-directional for particles with the opposite energy due to the consideration of the temporal interchange of outgoing and ingoing amplitude, but this doesn’t mean the both direction is equally suppressed. We have to calculate them respectively. Along this line we will recover the temperature of black hole radiation in the next section.

III. THE TEMPERATURE

For a particle, of mass, $m$, the Hamilton-Jacobi equation is

$$g^{\mu \nu} \partial_{\mu} \partial_{\nu} S + m^2 = 0,$$

(5)

where $g^{\mu \nu}$ is the inverse metric of the background space-time and $S$ is the action of the particle. Thus one can express the scalar field as $\phi(x) = \exp[-\frac{i}{\hbar} S + \cdots]$. In the picture of Hawking radiation as tunneling, the Painlevé coordinates is considered as appropriate because it, unlike Schwarzschild coordinates, is not singular at the horizon. The barrier is created by the outgoing particles themselves, which is ensured by the energy conservation \[22\]. We can express the Painlevé coordinates as

$$ds^2 = -(1 - \frac{2M}{r})dt_p^2 + 2\sqrt{\frac{2M}{r}}dr dt_p + dr^2 + r^2 d\Omega^2.$$

(6)

Since the metric is stationary and has a time-like Killing vectors, we can split the action into a time and spatial part, $S = Et_p + S_0(r)$, where $E$ is the energy of particle. We use Eq.
and obtain

\[
S_0(r) = - \int \frac{dr}{1 - \frac{2M}{r}} \sqrt{\frac{2M}{r}} E \pm \int \frac{dr}{1 - \frac{2M}{r}} \sqrt{E^2 - m^2(1 - \frac{2M}{r})},
\]

(7)

where the positive and negative sign indicates that the particle is ingoing and outgoing. Note that the contour integral includes a singularity at \( r = 2M \) and it has to be made by going around the pole at singularity. In Ref. [15], the result is obtain as \( \text{Im} S_0(r) = 0 \) for the ingoing particles (which corresponds to the plus sign in Eq. (7)) since the first and second terms have the same magnitude and \( \text{Im} S_0(r) = -4\pi ME \) for the outgoing particles and the authors conclude that the temperature \( T = \frac{1}{4\pi M} \) can be obtained by comparing the tunneling probability \( \Gamma = \exp(\text{Im} \oint pdr) = e^{-4\pi ME} \) with a Boltzmann factor \( \Gamma = \exp(-\frac{E}{T}) \). The temperature is twice as large as the original Hawking temperature and it is the same for the other coordinates (Schwarzschild, isotropic and so on), which seems to imply that one should discard the Hawking’s original calculation. It is not the case, however. We note that the calculation above for the ingoing particles is concerned about the positive energy particles and according to our analysis in the last section when considering the temporal interchange the energy of ingoing particles should be treated as negative. So \( \text{Im} S_0(r) = 4\pi ME \) for the ingoing particles, which is consistent with that of the negative energy particles tunneling inward calculated in Ref. [5]. It should be stressed that here \( E \) is always larger than zero and when we consider the negative energy particle, the minus sign before the first term in Eq. (7) has to be changed to plus sign and so the calculation becomes the addition of the two equivalent terms. Thus we can obtain the tunneling probability as

\[
\Gamma = e^{\text{Im} \left( \oint pdr \right)} = e^{\text{Im} \left( \int p^{\text{out}} dr - \int p^{\text{in}} dr \right)} = e^{-8\pi ME}. \tag{8}
\]

And in the same way we associate it with a Boltzmann factor \( \Gamma = \exp(-\frac{E}{T}) \), so the temperature \( T = \frac{1}{8\pi ME} \), which is the standard temperature obtained by Hawking.

If the Schwarzschild metric is used, this yields \( \text{Im} S_0(r) = \pm 2\pi ME \) [15]. But we observed that when the plus sign is taken, the ingoing amplitude is not a decay but is an amplification. Thus in the classical limit (\( \hbar \to 0 \)) the tunneling will not disappear and trends to infinity [17], which is inconsistent with our experiential fact that the tunneling is a kind of quantum effect and doesn’t occur in the classical field. Therefore one must use the integral constant to adjust the amplitude and for the positive energy particle tunneling outward through the
horizon, we have

$$\text{Im } S^\text{out}_\text{pe} = -2\pi ME + C,$$

$$\text{Im } S^\text{in}_\text{pe} = 2\pi ME + C,$$

where the label $\text{pe}$ means the tunneling energy is positive and $C$ is a constant. In order to avoid the infinity problem in the classical limit and ensure that the amplitude is unity in the classical limit where everything is absorbed, we have to take $C = -2\pi ME$. That is to say that the amplitude $\langle \text{in}|\text{out} \rangle_\text{pe} = \exp(i\theta)$, where $\theta$ is an arbitrary phase and may be related to the horizon when there existed the quantum fluctuation. Thus $\text{Im } S^\text{out}_\text{pe} = -4\pi ME$ and so the amplitude is gotten as

$$\langle \text{out}|\text{in} \rangle_\text{pe} = \exp(-i\theta) \exp(-4\pi ME).$$

The constant $C$ occurred may be due to the “badness” of the Schwarzschild coordinates near the horizon. It is noted if we choose the Painlevé coordinates to calculate, the result is $\text{Im } S^\text{out}_\text{pe} = -4\pi ME$ and $\text{Im } S^\text{in}_\text{pe} = 0$ and so the normalization is not needed. This implies that the Painlevé coordinates indeed is a good choice which not only behaves well at the horizon but also is convenient to obtain the temperature even by Hamilton-Jacobi method.

Similarly for the negative energy particle tunneling inward through the horizon, we have

$$\text{Im } S^\text{out}_\text{ne} = 2\pi ME + D,$$

$$\text{Im } S^\text{in}_\text{ne} = -2\pi ME + D,$$

where the label $\text{ne}$ means the tunneling energy is negative and $D$ is a constant. The constant introduced is the same reason as that above and if we choose the Painlevé coordinates, the constant is not necessary. Here suppose that $E$ is positive and so for the negative energy we have to replace $E$ by $-E$. The same reason makes us take $D = -2\pi ME$. It shows that in classical limit the negative energy particles can only move out of the black hole and so the mass or area of black hole never decrease which is consistent with the second law of classical black hole thermodynamics [23]. Therefore the amplitude $\langle \text{out}|\text{in} \rangle_\text{ne} = \exp(-i\theta)$. On the other hand, $\text{Im } S^\text{in}_\text{ne} = -4\pi ME$ and so the ingoing amplitude is gotten as

$$\langle \text{in}|\text{out} \rangle_\text{ne} = \exp(i\theta) \exp(-4\pi ME).$$

From the analysis above one knows that $\langle \text{in}|\text{out} \rangle^* \neq \langle \text{out}|\text{in} \rangle$ if we only consider the positive or negative energy particles tunneling. However, it is noted that the calculation of $S^\text{in}$ and
$S^{out}$ only includes the space coordinates change, that is to say the integral is made from $r^{out} \rightarrow r^{in}$ and $r^{in} \rightarrow r^{out}$, but the time coordinates change is not considered. According to our analysis in the last section, when calculating the tunneling probability by the closed contour integral we have to take the amplitude $\langle out|in \rangle_{pe}$ and $\langle in|out \rangle_{ne}$ and so we gain

$$\Gamma = \langle out|in \rangle_{pe} \langle in|out \rangle_{ne} = \exp(-8\pi ME).$$

And the temperature is $T = \frac{1}{8\pi M}$ which is also consistent with the Hawking’s original result. On the other hand, it is noted that when considering the problem of tunneling inward we can obtain the probability $\Gamma = \langle in|out \rangle_{pe} \langle out|in \rangle_{ne} = 1$, which is consistent with the fact the ingoing particles face no barrier.

IV. DISCUSSION

We also notice that there are two methods suggested to solve this problem.

One suggestion [17] is made along the line of path integral in the complex time analysis [2] in which the amplitudes for particle emission is related to that for particle absorption with the result that the ratio of emission and absorption probabilities for energy $E$ is given by

$$P_{emission} = \exp(-\frac{E}{T_H})P_{absorption}.$$  \hspace{1cm} (16)

This formula is used to obtain the Hawking radiation in a new path integral method and at the same time it also gives the same temperature as Hawking’s original result. In Ref. [17], it is applied to solve the factor of 2 problem about black hole temperature and the problem of the absorption probability which tends to be greater than unity and goes to infinity in the classical limit has been pointed out. After normalization, one can find that $\langle in|out \rangle_{pe} = 1$ and $\langle out|in \rangle_{pe} = -4\pi ME$. It is noticed that the author obtains the emission and the absorption probability directly from the modulus square of the amplitude while doesn’t consider that the tunneling is sensitive to the direction of motion. According to our suggestion, the emission probability should be calculated as

$$P_{emission} = \langle out|in \rangle_{pe} \langle in|out \rangle_{ne} = -8\pi ME;$$ \hspace{1cm} (17)

and the absorption probability is

$$P_{absorption} = \langle in|out \rangle_{pe} \langle out|in \rangle_{ne} = 1.$$ \hspace{1cm} (18)
Thus the tunneling probability is gotten as
\[ \Gamma = \frac{P_{\text{emission}}}{P_{\text{absorption}}} = \exp(-8\pi ME), \] (19)
and the temperature is \( T = \frac{1}{8\pi M} \). It seems that such treatment gives the same result as that in Ref. [17]. But this doesn’t mean that the tunneling is not dependent on the direction of move. Especially whether the amplitude \( \langle \text{in}|\text{out} \rangle_{ne} \) is equal to \( \langle \text{out}|\text{in} \rangle_{pe} \) for all situations has to be proven further, but here they are the same.

The second suggestion [18, 19] is that not only the spatial part but also the temporal part contributes to the imaginary part of action. In Schwarzschild background, the spatial contribution to the action is \( \oint p\,dr = -4\pi ME \) and the temporal contribution to the action is seen by transferring the Schwarzschild coordinates into Kruskal-Szekeres coordinates. The transformation is given as
\[
T = \left( \frac{r}{2M} - 1 \right)^{1/2} e^{r/4M} \sinh\left( \frac{t}{4M} \right),
\]
\[
R = \left( \frac{r}{2M} - 1 \right)^{1/2} e^{r/4M} \cosh\left( \frac{t}{4M} \right),
\] (20)
for the region exterior to the black hole \( (r > 2M) \) and
\[
T = \left( 1 - \frac{r}{2M} \right)^{1/2} e^{r/4M} \cosh\left( \frac{t}{4M} \right),
\]
\[
R = \left( 1 - \frac{r}{2M} \right)^{1/2} e^{r/4M} \sinh\left( \frac{t}{4M} \right),
\] (21)
for the interior of the black hole \( (r < 2M) \). To connect these two patches across the horizon at \( r = 2M \) one needs to “rotate” the Schwarzschild \( t \) as \( t \to t - 2i\pi M \) (together with the change \( r - 2M \to 2M - r \)). So the temporal contribution is \( \text{Im}(E\Delta^{\text{in, out}}) = \pm 2\pi ME \). By adding the temporal and spatial contribution, the Hawking temperature is recovered as \( T = \frac{1}{8\pi M} \). This is indeed an ingenious solution. However there are still some subtle places to be noticed. At first, the time transformation is periodic (the period is \( 8i\pi M \)) and no necessary reason demands the “rotation” is \( 2i\pi M \) (maybe it is \( 6i\pi M \), but this is indefinite). In other words, one can also suppose that for the outgoing amplitude the rotation is \( 2i\pi M \) and for the ingoing amplitude the rotation is \( 6i\pi M \) because they exist within one period, so the total rotation is \( 8i\pi M \). In fact, the time axial can be extended from real axial to virtual axial in Kruskal-Szekeres coordinates, \( t = -i\tau \). Thus in the the region exterior to the black hole the virtual \( \tau \) is a coordinate with period \( 8\pi M \) and such character satisfies “thermal Green function”, \( G_T(x, t; x_0, t_0) \sim G_T(x, t + i\beta; x_0, t_0) \) where \( \beta = 8\pi M = \frac{1}{T} \). We can relate the path integral propagator with thermal propagator and thus to the observer in static frame it will seems as if he is in a bath of blackbody radiation at the above
temperature [24, 25]. Secondly, the rotation described in Ref. [19] may be only applicable to the Schwarzschild coordinates, for other coordinates this has to be treated carefully. For example, for the Schwarzschild and Painlevé coordinates, \( \int E dt + \oint p dr = \int E dt_p + \oint p_p dr \)
where \( \oint p dr = \oint p_p dr \) because of canonical invariance. So \( \int E dt = \int E dt_p \), which shows the Painlevé time coordinates have to exist the same virtual rotation as the Schwarzschild time coordinates, but this is not seen clearly in Painlevé coordinates because on one hand, the Painlevé coordinates behave well at horizon and on the other hand, the transformation from the Painlevé coordinates to Kruskal-Szekeres coordinates doesn’t have the same form as [20] and [21]. Thirdly, there is no reasonable explanation about the temporal contribution depending on which direction the horizon is crossed. The momentum is directional but the energy is not. Otherwise, we also noticed that the rotation \( t \to t - 2i\pi M \) will lead to the same result as that \( t \to t + 2i\pi M \) for the aim at connecting these two patches. So whether the temporal contribution from ingoing and outgoing particles is dependent on the direction has to be considered carefully. However the temporal contribution indeed exists. Just as we have pointed out in the last section, the factor of 2 problem is because the calculation of ingoing amplitude \( \langle \text{in} | \text{out} \rangle \) is made only for the spatial change while not included the temporal change. The Refs. [18, 19] have presented the temporal change clearly, but we inclined to think the temporal contribution depending on which direction the horizon is crossed is due to the interchange between positive energy and negative energy while the imaginary rotation value is the same in the process. If so, the ingoing negative energy particles can be regarded as tunneling inward and the amplitude can be calculated as in Eq. [14]. Such explanation is reasonable because the temporal contribution can also be calculated by treating the ingoing wave amplitude properly, as pointed out in Refs. [2] that the propagator or amplitude at a certain complex value of \( t \) can be obtained by solving the Hamilton-Jabobi equations. Indeed we find that when calculating the ingoing amplitude one can obtain the temporal contribution by changing the positive energy into negative energy in calculation (the normalization included here). So the calculation in our method is consistent with that in Refs. [18, 19] based on the third point of discussion.

Thus, the method suggested in the present paper, which can recover the standard Hawking temperature by calculating the ingoing and outgoing amplitude afresh and noting that the ingoing amplitude should be calculated according to the negative energy particles tun-
neling inward, is consistent with the other two methods discussed above. But in our method we use the canonical invariant tunneling probability and analyze the ingoing and outgoing amplitude respectively and this can not only related the two other methods but also supplement or avoid their some deficiencies.

V. CONCLUSION

We have showed that the standard Hawking temperature can be recovered by using the canonically invariant tunneling probability. In our treatment we find that the ingoing amplitude should be calculated according to the negative energy particles tunneling into the black hole and this is because when we change the spatial direction to calculate the ingoing amplitude, the temporal transformation have to be considered. In our method, the horizon as two-way barrier and the Painlevé coordinates that is proper for discussing the temperature Hawking radiation as tunneling can be presented clearly. In the end we also discuss the other two methods and compare them with our method, which show indeed the radiation temperature \( T = \frac{1}{8\pi M} \).

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