Three-dimensional flow in a shear-driven cube

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The flow in a cubic cavity is studied when a constant shear stress is imposed on one of its square faces. The three-dimensional basic flow undergoes a first steady, symmetry-breaking, pitchfork bifurcation. On an increase of the Reynolds number the symmetry-broken flow becomes time-dependent via a Hopf bifurcation. Even though the basic flow is similar to the one in the lid-driven cube, the sequence of bifurcations differs significantly.

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1 Introduction

The flow of an incompressible fluid with kinematic viscosity $\nu$ and density $\rho$ confined to a cubic cavity with edge length $L$ is considered. The flow is driven by imposing a constant shear stress $\tau$ on one of its faces. The system represents a basic model for thermocapillary flow in a differentially heated cavity in the limit of small Prandtl numbers [1] and it is related to the shear flow over an open cavity [2]. Interest in the shear-driven cubic cavity (SDC) also derives from its similarity to the well-known lid-driven cubic cavity (LDC) which has been thoroughly investigated during the last decade [3–6]. Thus one may inquire about similarities of and differences between both systems.

The Navier–Stokes equations are made dimensionless using the scales $L$, $L^2/\nu$, $\nu/L$, $\rho \nu^2/L^2$ for length, time, velocity and pressure, respectively. The stress-based Reynolds number is defined as $Re_\tau = L^2 \tau / \rho \nu^2$. To compute the steady flow in the cube $V = [-1/2, 1/2]^3$ the BoostConv algorithm [7] is employed. It can be implemented around any unsteady Navier–Stokes solver. The algorithm is based on a smart recombination of residuals to accelerate convergence to steady state by erasing the most amplified linear perturbation modes. In particular, the method allows to recover an unstable steady solution even if the most dangerous perturbation modes are not oscillatory. Here, the spectral element solver NEK5000 is employed, using a spatial discretization of $12^3$ elements of order 6 and a third-order Adams–Bashforth scheme (BDF-3) in time.

Fig. 1: (a) Schematic representation of the cavity with the main vortex (arrows) indicated in the symmetry plane (shaded). (b) Spanwise velocity component $w$ on the mid plane $z = 0$ at $Re_\tau = 53500$. The dashed lines indicate $w = 0$ and the marker ($\times$) shows the monitoring point $x_p$.

2 Results

As the problem is symmetric with respect to the midplane $z = 0$ (Fig. 1a), a three-dimensional flow exists at low Reynolds numbers exhibiting the same symmetry, \textit{i.e.} $w(z = 0) = 0$. However at $Re_\tau = 53500$, the symmetric steady basic state is not stable anymore and the dynamical system settles on another steady state with broken symmetry for which $w(z = 0) \neq 0$ as shown in Fig. 1b. At $Re_\tau = 53500$ the velocity on the free surface takes its maximum at $(x, z) = (-0.382, 0.448)$ with

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(u, w) = (−1852.5, 35.9). The magnitude of u indicates that the strength of the flow is comparable with that of the LDC at Re = UL/ν ≈ 1900 with lid velocity U.

The loss of stability is also recovered by a linear stability analysis: a real eigenvalue of the linear stability problem crosses the imaginary axis at Reτ,c = 53486 (● in Fig. 2a). As the Reynolds number increases, the amplitude of the symmetry-breaking flow, measured by \( w(x_p) \) at the monitoring point \( x_p = (-0.4, 0, 0) \) on the midplane (cross in Fig. 1b), increases with the square root of the distance from the critical point \( (Reτ - Reτ,c)^{1/2} \), the footprint of a pitchfork bifurcation.

The spanwise velocity squared \( w^2(x_p) \) is shown in Fig. 2(a) as function of Reτ. The extrapolation to \( w^2(x_p) = 0 \) yields a critical Reynolds number Reτ,c = 53485 which agrees with the value obtained by the linear stability analysis.

On a further increase of Reτ, the linear stability analysis shows that the unstable symmetric basic flow also becomes unstable to an oscillatory mode at Reτ,c2 = 54100 with frequency \( \omega_2 \approx 689.4 \) (● in Fig. 2a). On the other hand, the finite-amplitude symmetry-breaking solution bifurcating from the basic state at Reτ,cl becomes unstable to an oscillatory flow at Reτ,c = 55700 (▲ in Fig. 2a) with \( \omega = 764.16 \), where the superscript ‘a’ indicates the asymmetric flow. This frequency is close to \( \omega_2 \), despite of the different flows from which the oscillatory modes bifurcate. The energy budgets of the oscillatory neutral modes, as defined in [8], is shown in Fig. 2b. The integral contributions to the different destabilization processes [8] is nearly the same for both modes. This suggests the Hopf bifurcations from the symmetric and from the asymmetric solution branches are caused by the same type of instability mechanism with the flow symmetry delaying the oscillatory instability.

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