Zitterbewegung of electrons in quantum wells and dots in the presence of an in-plane magnetic field

Tutul Biswas and Tarun Kanti Ghosh

Department of Physics, Indian Institute of Technology-Kanpur, Kanpur-208 016, India
E-mail: tutulb@iitk.ac.in

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Abstract
We study the effect of an in-plane magnetic field on the zitterbewegung (ZB) of electrons in a semiconductor quantum well (QW) and in a quantum dot (QD) with the Rashba and Dresselhaus spin–orbit interactions (SOIs). We obtain a general expression of the time-evolution of the position vector and current of the electron in a semiconductor QW. The amplitude of the oscillatory motion is directly related to the Berry connection in momentum space. We find that in presence of the magnetic field the ZB in a QW does not vanish when the strengths of the Rashba and Dresselhaus SOIs are equal. The in-plane magnetic field helps to sustain the ZB in QWs even at a low value of $k_0d$ (where $d$ is the width of the Gaussian wavepacket and $k_0$ is the initial wavevector). The trembling motion of an electron in a semiconductor QW with high Landé $g$-factor (e.g. InSb) is sustained over a long time, even at a low value of $k_0d$. Further, we study the ZB of an electron in QDs within the two sub-band model numerically. The trembling motion persists in time even when the magnetic field is absent as well as when the strengths of the SOI are equal. The ZB in QDs is due to the superposition of oscillatory motions corresponding to all possible differences of the energy eigenvalues of the system. This is an another example of multi-frequency ZB phenomenon.

1. Introduction
In recent years there has been a growing interest in the field of spin based electronic devices. There has been a lot of study in this field after the proposal by Datta and Das of the spin field effect transistor [1]. The charge carriers carry spin in addition to their charges. The ultimate goal in this field is to control the spin degree of freedom of the charge carriers to produce and detect spin-polarized current in semiconductor nanostructures [2]. One may be able to develop device technology [3] and quantum information processing [4] in future on the basis of the manipulation of the spin degree of freedom. The coupling between the intrinsic spin of an electron with its orbital angular momentum constitutes the intrinsic spin–orbit interaction (SOI) in low-dimensional semiconducting systems. There are two kind of SOI present in low-dimensional semiconductor structures. One is the Rashba [5] spin–orbit interaction (RSOI) and another is the Dresselhaus [6] spin–orbit interaction (DSOI). The RSOI arises mainly from the inversion asymmetry of the confining potential in semiconductor heterojunctions. The strength of the RSOI is proportional to the externally applied electric field which can be tuned by an external bias [7, 8] or internally generated due to the crystal potentials. On the other hand the DSOI is present in bulk materials and semiconductor heterostructures which lack bulk inversion symmetry. The form of the DSOI term strongly depends on the growth direction of the semiconductor quantum well (QW) [9–11]. The strength of the DSOI depends on the properties of the material and the crystal structure.

In 1930, Schrödinger [12] predicted that a free particle described by the relativistic Dirac equation will perform an oscillatory motion which is known as the zitterbewegung (ZB). The free relativistic Dirac particle has two energy branches: $\epsilon(p) = \pm \sqrt{m_0^2c^4 + c^2p^2}$. It is well understood that this oscillatory motion results from interference between these two energy branches [13]. The large oscillation frequency $\omega_c \simeq 10^{21}$ Hz and the small oscillation amplitude $\lambda_c \simeq 10^{-13}$ m are not accessible to modern experimental
techniques. Most of the studies on the ZB of electrons used plane waves to describe the electrons. It was pointed out by Lock [14] that a plane wave is not a localized state, and therefore rapid oscillations on the average position of a plane wave have some limitations. He also demonstrated that when an electron is described by a wavepacket, the ZB oscillation has a transient character.

In 2005, Zawadzki et al [15] studied ZB in narrow gap semiconductor (NGS) by using the analogy between \( k \cdot p \) theory of the energy bands in NGS and the free Dirac relativistic equation for electrons. They found a much more favorable amplitude and frequency of the oscillation than those in a vacuum for a free electron. At the same time, Schliemann et al [16, 17] studied the ZB of an electron in III–V zincblende semiconductor QWs in the presence of the SOI. The above studies initiated intense theoretical research on the ZB of electrons in various condensed matter systems [18–25].

A proposal of an experimental scheme for observing ZB in ultra-cold [26] atomic gases was also made. The trembling motion was proposed for photons in a two-dimensional photonic crystal and for Ramsey interferometry [27]. It was reported in [28] that an acoustic analog of ZB in a macroscopic two-dimensional sonic crystal was observed. A general theory of the ZB of a multi-band Hamiltonian was studied by David and Cserti [29]. Recently, Vaseghi et al [30] studied the effect of an external perpendicular magnetic field on the ZB for both a quantum wire and quantum dot (QD) within the two sub-band model.

The dimensionless parameter \( p_0 = k_0 d \) dictates whether the motion will be oscillatory or not. It was shown in [14] that the motion will be oscillatory when \( p_0 \gg 1 \). It was also shown that the ZB vanishes when the strengths of the RSOI and DSOI equal. This is due to an additional conserved quantity.

In this work we study the effect of the in-plane magnetic field on the ZB of electrons in a semiconductor QW and in a QD with the RSOI and DSOI. We obtain an analytical expression for the time-evolution of the position vector of an electron in a QW by using the Schrödinger picture. The advantage of using the Schrödinger picture is to see the direct relation between the amplitude of the oscillatory motion and the Berry connection in momentum space. We find that in the presence of a magnetic field the ZB in a QW does not vanish even when the strengths of the RSOI and DSOI equal. The ZB in a QW is sustained even at very low value of \( k_0 d \) due to the presence of the magnetic field. The trembling motion of an electron in a semiconductor QW with high Landé \( g \)-factor (e.g. InSb) lasts for a long time, even at very low values of \( k_0 d \). The time-period of the oscillation decreases as \( k_0 d \) increases. Next, we study the ZB of an electron in a QD within the two sub-band model numerically. The trembling motion does not fade away in time even when the magnetic field is absent as well as when the strengths of the RSOI and DSOI are equal.

This paper is organized as follows. In section 2, we review the Hamiltonian and eigenstates of a two-dimensional electron gas (2DEG) with both type of SOI in the presence of an in-plane magnetic field. In section 3, we derive time-evolution of the electron position vector in the Schrödinger picture. Numerical results and a discussion are given in section 2.2. Section 2.2.1 contains the calculation and result of the ZB of electrons in a GaAs/AlGaAs QD. The conclusion is presented in section 4.

2. Two-dimensional electron gas with spin–orbit interactions

We consider a 2DEG with the RSOI and DSOI in the presence of an in-plane magnetic field \( \mathbf{B} = \hat{B}_x + \hat{y} B_y \). The single-particle Hamiltonian of this system is given by

\[
H = \frac{\mathbf{p}^2}{2m^*} + \frac{\alpha}{\hbar} (\sigma_x p_x - \sigma_y p_y) + \frac{\beta}{\hbar} (\sigma_y p_x - \sigma_x p_y)
+ \frac{g_s}{2} \mu_B (B_x \sigma_x + B_y \sigma_y),
\]

(1)

where \( \mathbf{p} \) and \( m^* \) are the momentum and the effective mass of an electron, respectively. Here \( \alpha \) and \( \beta \) are the strengths of the RSOI and DSOI, respectively. The last term in the Hamiltonian is the Zeeman term due to the application of the in-plane magnetic field \( \mathbf{B} \), \( g_s \) is the effective Landé \( g \)-factor and \( \mu_B \) is the Bohr magneton. The energy eigenvalues and the corresponding eigenstates of the system [31, 32] are, respectively, given by

\[
E_\pm(\mathbf{k}) = \frac{\hbar^2 (k_x^2 + k_y^2)}{2m^*} \pm \sqrt{C^2 + D^2},
\]

(2)

where \( C = \alpha k_x + \beta k_y + g_s \mu_B B_y/2 \), \( D = \alpha k_y - \beta k_x - g_s \mu_B B_x/2 \), and the spin eigenstates are

\[
|k, +\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -ie^{-i\theta_k} \end{pmatrix},
\]

\[
|k, -\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ ie^{-i\theta_k} \end{pmatrix},
\]

(3)

with \( \theta_k = \tan^{-1}(C/D) \).

In the absence of a magnetic field, a new conserved quantity \( \Sigma = \sigma_x - \sigma_y \) exists for \( \alpha = \beta \) [33, 34]. In this situation, the spin eigenstates \( |k, \pm\rangle \) become independent of the wavevector \( k \) and therefore the ZB does not exist due to the absence of spin randomization [16, 17]. The situation is completely different when an in-plane magnetic field is present. It is clear from equations (3) that the spin states are always function of \( k \) even at \( \alpha = \beta \) as long as \( \mathbf{B} \neq 0 \). Therefore, spin randomization occurs and we would expect to see the ZB, as shown in section 2.1.

2.1. Time-evolution of the wavepacket and the Zitterbewegung

We shall use the Schrödinger picture to analyze the time-evolution of the electron position vector. We represent the initial wavefunction of an electron by a Gaussian wavepacket with initial spin polarization along the \( z \)-axis as given by

\[
\psi(r, 0) = \frac{1}{2\pi} \int d^2k a(k, 0)e^{i\mathbf{k} \cdot \mathbf{r}} \begin{pmatrix} 1 \\ 0 \end{pmatrix},
\]

(4)
where \( a(\mathbf{k}, 0) = d/(\sqrt{\pi}) e^{-\frac{1}{4}d^2 (\mathbf{k} - \mathbf{k}_0)^2} \). Here, \( d \) and \( \mathbf{k}_0 \) are the initial width and the initial wavevector of the wavepacket, respectively. The time-evolution of the initial wavepacket in the Schrödinger representation can be obtained in the usual manner as \(\psi(\mathbf{r}, t) = U(t) \psi(\mathbf{r}, 0)\), where \( U(t) = e^{-iH_t/\hbar}\) is the time-evolution operator. After doing some straightforward algebra we obtain

\[
\psi(\mathbf{r}, t) = \frac{1}{\pi} \int d^2 k a(\mathbf{k}, 0)e^{i\mathbf{k}\cdot\mathbf{r}} e^{-\frac{1}{4}d^2 (\mathbf{k} - \mathbf{k}_0)^2} \times \left\{ \cos[\omega(\mathbf{k})]t \left( \begin{array}{c} 1 \\ 0 \end{array} \right) - e^{i\phi_k} \sin[\omega(\mathbf{k})]t \left( \begin{array}{c} 0 \\ 1 \end{array} \right) \right\},
\]

(5)

where \( \omega(\mathbf{k}) = |E_+ (\mathbf{k}) - E_-(\mathbf{k})|/(2\hbar) \).

Equation (5) is nothing but the Fourier transformation of the following function:

\[
\phi(\mathbf{k}, t) = a(\mathbf{k}, 0)e^{-\frac{i\alpha^2}{\sin^2 \theta}} \left( \frac{\cos[\omega(\mathbf{k})]t}{-e^{i\phi_k} \sin[\omega(\mathbf{k})]t} \right).
\]

The expectation value of position operator is then simply given by

\[
\langle \mathbf{r}(t) \rangle = i \int d^2 k \phi^* (\mathbf{k}, t) \nabla_n \phi(\mathbf{k}, t).
\]

(6)

Now it is straightforward to show that

\[
\langle \mathbf{r}(t) \rangle = \langle \mathbf{r}(0) \rangle + \frac{h \mathbf{k}_0}{m^*} t
\]

\[ - \frac{1}{2} \int d^2 k |a(\mathbf{k}, 0)|^2 (\nabla_n \phi_k) [1 - \cos[\omega(\mathbf{k})]t] \].

(7)

The last oscillatory term \( \cos[\omega(\mathbf{k})]t \) in equation (7) indicates the ZB. Two important observations can be made by analyzing equation (7). First, the amplitude of the ZB is directly proportional to the Berry connection \( \{ k, \pm |\frac{d^2}{dk^2} |k, \pm \} = \nabla_n \phi_k \). Second, for \( \alpha = \beta \) the amplitude (or the Berry connection) of the oscillation does not vanish when the in-plane magnetic field is present, but it vanishes if the magnetic field is absent.

After substituting the expressions for \( a(\mathbf{k}, 0) \), \( \nabla_n \phi_k \) in equation (7) and considering the initial velocity of the wavepacket along y-direction (i.e. \( \mathbf{k}_0 = 0, \mathbf{k}_y = \mathbf{k}_0 \)) one can easily obtain the following expressions

\[
\langle x(t) \rangle = \frac{d}{2\pi} e^{-\frac{d^2}{\sin^2 \theta}} \int_{0}^{2\pi} \int_{0}^{2\pi} dq e^{-q^2 + 2d \phi \sin \phi} \times \frac{q^2 (1 - \eta^2) \sin \phi + q(\epsilon_\phi + \eta \epsilon_y)}{Q^2} \times \left[ 1 - \cos \left( \frac{2a \phi}{\hbar d} Q \right) \right],
\]

(8)

and

\[
\langle y(t) \rangle = \frac{h \mathbf{k}_0}{m^*} + \frac{d}{2\pi} e^{-\frac{d^2}{\sin^2 \theta}} \int_{0}^{2\pi} \int_{0}^{2\pi} dq e^{-q^2 + 2d \phi \sin \phi} \times \frac{q^2 (\eta^2 - 1) \cos \phi + q(\eta \epsilon_\phi + \epsilon_y)}{Q^2} \times \left[ 1 - \cos \left( \frac{2a \phi}{\hbar d} Q \right) \right].
\]

(9)

where \( Q^2 = (q \cos \phi + \eta q \sin \phi - \epsilon_y)^2 + (q \sin \phi + \eta q \cos \phi + \epsilon_y)^2 \), \( \eta = \beta/\alpha \), \( q = kd \), \( \epsilon_x = g^* \mu_B B_x d/(2a) \) and \( \epsilon_y = g^* \mu_B B_y d/(2a) \).

From equations (8) and (9) one can infer that ZB is absent in the limit \( d \to 0 \). On the other hand, as \( d \to \infty \) the Gaussian approaches to a delta function, i.e. \( a(\mathbf{k}, 0) = \delta(\mathbf{k} - \mathbf{k}_0) \), and in this limit we find an analytical expression for the ZB given by

\[
\langle x(t) \rangle = \frac{1}{4\pi} \left( 1 - \eta^2 \right) k_0^2 + \frac{\epsilon^2}{4a} (B_x + \eta B_y) \times \left[ 1 - \cos[\omega(\mathbf{k}_0)]t \right],
\]

(10)

and

\[
\langle y(t) \rangle = \frac{h \mathbf{k}_0}{m^*} + \frac{1}{4\pi} \left( 1 - \eta^2 \right) k_0^2 + \frac{\epsilon^2}{4a} (\eta B_x + B_y) \times \left[ 1 - \cos[\omega(\mathbf{k}_0)]t \right].
\]

(11)

We also calculate the expectation values of the velocity of electron in the x and y directions. They are given by

\[
\langle v_x(t) \rangle = \frac{\partial \langle x \rangle}{\partial t} = \frac{\nu_R \cos \phi}{2} \int_{0}^{2\pi} d\phi \times \int_{0}^{\infty} dq e^{q^2 + 2d \phi \sin \phi} \times \frac{q^2 (1 - \eta^2) \sin \phi + q(\epsilon_\phi + \eta \epsilon_y)}{Q^2} \sin \left( \frac{2a \phi}{\hbar d} Q \right).
\]

(12)

and

\[
\langle v_y(t) \rangle = \frac{h \mathbf{k}_0}{m^*} + \frac{\nu_R \cos \phi}{2} \int_{0}^{2\pi} d\phi \times \int_{0}^{\infty} dq e^{q^2 + 2d \phi \sin \phi} \times \frac{q^2 (\eta^2 - 1) \cos \phi + q(\eta \epsilon_\phi + \epsilon_y)}{Q^2} \sin \left( \frac{2a \phi}{\hbar d} Q \right).
\]

(13)

Here, \( \nu_R = \Omega d = 2\alpha/\hbar \) is the velocity corresponding to the SOI. The corresponding currents are simply given by \( \langle j_x(t) \rangle = e \langle v_x(t) \rangle \) and \( \langle j_y(t) \rangle = e \langle v_y(t) \rangle \), where \( e \) is the electronic charge. Equations (9) and (13) tell us that the ZB along the y direction vanishes when the in-plane magnetic field and the DSOI are absent simultaneously [16] because in this case \( \int_{0}^{2\pi} e^{2d \phi \cos \phi} \sin \phi d\phi = 0 \). But when there is a finite in-plane magnetic field present the ZB in the y direction does not vanish even at \( \beta = 0 \). So this is an effect of the in-plane magnetic field on the ZB.

2.2. Numerical results and discussion

In this section we evaluate the time-evolution of the observables—the position vector and current density—for different values of the parameters like magnetic field, \( k_0d \), \( \beta \) etc.

2.2.1. GaAs/AlGaAs QW. We consider a GaAs/AlGaAs QW for which the effective Landé g-factor is \( g^* = -0.44 \). The value of the Rashba coefficient is taken to be \( \alpha = 1.0 \times 10^{-11} \) eV m. We set the condition \( dk_0 = 5 \) in all cases. To
investigate the time dependence of the expectation values of position and current of an electron, we numerically evaluate equations (8), (9), (12) and (13). Here it should be mentioned that equations (9) and (11) contain two parts: the first depends linearly on time and the second one is oscillatory in time 

\[ \langle x(t) \rangle/d, \langle j_x(t) \rangle/evR \] and \( \langle j_x(t) - j_0 \rangle/evR \) as a function of \( \Omega t \). In all the cases we set \( \beta = 0 \) for GaAs/AlGaAs QW. Solid line, \( B_x = 0, B_y = 0 \); dotted line, \( B_x = 1/\sqrt{2} T, B_y = 1/\sqrt{2} T \); dashed line, \( B_x = 6 T, B_y = 8 T \).

\[ \langle x(t) \rangle/d, \langle j_x(t) \rangle/evR \] and \( \langle j_x(t) - j_0 \rangle/evR \) as a function of \( \Omega t \). In the cases we set \( \beta = 0.5\alpha \) for GaAs/AlGaAs QW. Solid line, \( B_x = 0, B_y = 0 \); dotted line, \( B_x = 1/\sqrt{2} T, B_y = 1/\sqrt{2} T \); dashed line, \( B_x = 6 T, B_y = 8 T \).

\[ \langle x(t) \rangle/d, \langle j_x(t) \rangle/evR \] and \( \langle j_x(t) - j_0 \rangle/evR \) as a function of \( \Omega t \). In all cases we set \( \beta = 0.5\alpha \) for GaAs/AlGaAs QW. Solid line, \( B_x = 1/\sqrt{2} T, B_y = 1/\sqrt{2} T \); dotted line, \( B_x = 3 T, B_y = 4 T \); dashed line, \( B_x = 6 T, B_y = 8 T \).

\[ \alpha = 0.9 \times 10^{-11} \text{ eV m. We set here } \beta = 0.5\alpha \text{ and } dk_0 = 5. \]

In figure 4 \( \langle x(t) \rangle/d \) is plotted with respect to \( \Omega t \) for different values of the magnetic field. But the situation is different from the GaAs/AlGaAs QW case. The amplitude decreases, but the number of oscillations contained in the ZB within the same time range is quite large as we increase the magnetic field. Since the magnitude of \( g^* \) is large the coupling between the electron’s spin and magnetic field is strong.

One can obtain more oscillation in the ZB by increasing the magnitude of the parameter \( dk_0 \), and we have shown that when \( dk_0 \gg 1 \) the pattern is completely oscillatory as evident from equation (10). In all these cases it is observed that the ZB is transient in nature, i.e. its amplitude decreases with time, a direct consequence of Lock’s [14] prediction that the ZB of

2.2.2. InSb QW. Here, we consider InSb QW for which the effective Landé g-factor is very high (e.g. \( g^* = -50 \)) as compared to GaAs/AlGaAs. The RSOI strength is taken to be
electron will not be persistent in time if it is represented by a wavepacket.

3. Zitterbewegung in a quantum dot

In this section we would like to study the ZB of electrons in a semiconductor QD. We consider a 2DEG confined by an isotropic harmonic oscillator potential \( V(x, y) = (1/2)\hbar \omega (x^2 + y^2) \). In this context our Hamiltonian reads as

\[
H = \frac{p^2}{2m} + V(x, y) + \frac{\alpha}{\hbar} (\sigma_y p_x - \sigma_x p_y) + \frac{\beta}{\hbar} (\sigma_x p_x - \sigma_y p_y) + \frac{g^*}{2} \mu_B (B_x \sigma_x + B_y \sigma_y) + \frac{\hbar^2}{2} \omega (l^2 / (\hbar \omega)) \sigma_z.
\]

We introduce the conventional harmonic oscillator creation and annihilation operators as

\[
a_x = (x/l + i p_x/\hbar)/\sqrt{2}, \quad a_y = (y/l + i p_y/\hbar)/\sqrt{2}, \quad a_x^\dagger = (x/l - i p_x/\hbar)/\sqrt{2}, \quad a_y^\dagger = (y/l - i p_y/\hbar)/\sqrt{2},
\]

where \( l = \sqrt{\hbar / (\omega \hbar)} \) is the harmonic oscillator length.

This Hamiltonian can be re-written as

\[
H = \hbar \omega (a_x^\dagger a_x + a_y^\dagger a_y + 1) + \frac{i \alpha}{\sqrt{2l}} [(a_x - a_x^\dagger) \sigma_y - (a_y - a_y^\dagger) \sigma_x]
- \frac{i \beta}{\sqrt{2l}} [(a_x^\dagger - a_x)(a_y - a_y^\dagger)(\sigma_x - (a_x^\dagger - a_x)(\sigma_y)]
+ \frac{g^*}{2} \mu_B (B_x \sigma_x + B_y \sigma_y).
\]

We consider only two lowest occupied energy states (the ground state and first excited state) of a two-dimensional harmonic oscillator potential. This approximation is known as the ‘two sub-band model’. Within this approximation the Hilbert space is spanned by the following six basis vectors:

\[
|00\rangle, |01\rangle, |10\rangle, |11\rangle, |0\uparrow\rangle, |1\uparrow\rangle.
\]

Here, \( \uparrow \) and \( \downarrow \) represent the \( z \)-component of the electron’s spin vector.

Within six basis vectors one can write the Hamiltonian in a matrix form as

\[
H = \begin{pmatrix}
H_{11} & H_{12} & 0 & H_{14} & 0 & H_{16} \\
H_{21} & H_{22} & H_{23} & 0 & H_{25} & 0 \\
0 & H_{32} & H_{33} & 0 & 0 & 0 \\
H_{41} & 0 & H_{43} & H_{44} & 0 & 0 \\
0 & H_{52} & 0 & 0 & H_{55} & H_{56} \\
0 & 0 & 0 & 0 & H_{65} & H_{66}
\end{pmatrix}.
\]

The matrix elements are as follows:

\[
H_{11} = H_{22} = \hbar \omega = \epsilon_0,
H_{33} = H_{44} = H_{55} = H_{66} = 2\hbar \omega = \epsilon_1,
H_{12} = H_{21} = H_{14} = H_{41} = -H_{23} = -H_{32} = (\alpha - i \beta)/(l\sqrt{2}),
H_{25} = H_{52} = -H_{61} = H_{25}^* = -H_{76} = -i(\beta + i\alpha)/(l\sqrt{2}).
\]

Here \( \epsilon_0 = \hbar \omega \) is the zero-point energy and \( \epsilon_1 = 2\hbar \omega \) is the first excited state energy of the two-dimensional harmonic oscillator.

We want to determine the expectation value of the time-dependent position operator in this system. Let us consider that at \( t = 0 \) the system is in the ground state and the spin is oriented along the positive \( z \)-direction. This initial state is given by \( |\psi(0)\rangle = |00\rangle \uparrow \rangle \), so the expectation value of the position operator is given by

\[
\langle x_{1H}(t) \rangle = \langle \psi(0)|x_{1H}(t)|\psi(0)\rangle = \langle \psi(0)|e^{iHt/\hbar}|\psi(0)\rangle = \langle 0|V^\dagger e^{-iHt/\hbar}V\psi(0)|0\rangle
= \langle 0|V^\dagger e^{-iHt/\hbar}V\psi(0)|0\rangle = \sum_{i=1}^{6} \sum_{j=1}^{6} V_{ij} \hbar \omega e^{-i(\lambda_i - \lambda_j)\sigma_z/\hbar} V^\dagger_{ij},
\]

where \( V \) is the diagonalization matrix which diagonalizes the Hamiltonian \( H \) and \( V^\dagger V = I \). Also, \( \omega_{ij} = (\lambda_i - \lambda_j)/\hbar \) is the beating frequency.

The \( V \) matrix also diagonalizes \( U(t) \) and becomes \( U_{diag}(t) = VU(t)V^\dagger = e^{-iHt/\hbar}I \) and \( X = VxV^\dagger \). The position operator is given by \( x = \lambda (a + a^\dagger)/\sqrt{2} \). In the above-mentioned basis this can be written in a matrix form as

\[
\begin{pmatrix}
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0
\end{pmatrix}.
\]

3.1. Numerical results and discussion

The ZB of a GaAs/AlGaAs QD in the presence of an in-plane magnetic field with SOI is investigated here. The value of the Rashba strength is taken as \( \alpha = 1.0 \times 10^{-11} \text{ eV m} \) and the zero-point energy of the harmonic oscillator potential is fixed to the value \( \epsilon_0 = 5 \text{ meV} \). We find the expectation values of the position coordinate as a function of time, which are plotted in figures 5 and 6. In figure 5 we fix \( \beta = 0 \)
different values of magnetic field. Here (a) ($B_x = 0.1 \text{T}$, $B_y = 0.1 \text{T}$), (b) ($B_x = 1/\sqrt{2} \text{T}$, $B_y = 1/\sqrt{2} \text{T}$), and (c) ($B_x = 6 \text{T}$, $B_y = 8 \text{T}$). We consider only RSOI, i.e. $\beta = 0$.

Figure 5. Plots of $\langle x(t) \rangle/\lambda$ versus $\omega t$ for GaAs/AlGaAs QD for different values of $\beta$. The value of the magnetic field is kept fixed to $(B_x, B_y) = (0.1, 0.1) \text{T}$; (a)–(c) Correspond to $\beta = 0$, $\beta = 0.5 \alpha$, and $\beta = \alpha$, respectively.

and vary the magnetic field strengths. The magnetic field is kept constant and $\beta$ is varied in figure 6. The ZB in QDs is similar to the beating effect in the classical wave mechanics with different frequencies. The oscillatory motion is due to the superposition of individual oscillatory motions with frequencies corresponding to all the possible energy eigenvalue differences of the Hamiltonian. In this case, there are six non-degenerate eigenvalues and we have six values of the energy differences. The number of beating frequencies is six.

4. Conclusion

In this work we have investigated the effect of an external in-plane magnetic field on the ZB of an electron in semiconductor QW and QD with RSOI and DSOI. For QW a general expression of the expectation values of position coordinate and current due to ZB within the Gaussian wavepacket is obtained. For the QW case, the oscillatory quantum motion of electron which is represented by a wavepacket shows transient behavior, and this signature is a proof of Lock’s argument. Another important point is that ZB does not vanish even at $\alpha = \beta$ when a finite in-plane magnetic field is present. The $y$-component of the current also performs ZB motion with a finite magnetic field. We study the same problem for a high Landé $g$-factor QW like InSb in comparison with low Landé $g$-factor QW like GaAs–AlGaAs. We have also studied the problem of ZB in a GaAs/AlGaAs QD numerically. The ZB in GaAs/AlGaAs QD is persistent in time. The ZB in quantum dots show a beating-like pattern which it is similar to the beating effect in the classical wave mechanics.

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