Gravitational wave detection through microlensing?

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ABSTRACT

It is shown that accurate photometric observations of a relatively high-magnification microlensing event (A ≫ 1), occurring close to the line of sight of a gravitational wave (GW) source, represented by a binary star, can allow the detection of subtle gravitational effects. After reviewing the physical nature of such effects, it is discussed to what extent these phenomena can actually be caused by GWs. Expressions for the amplitude of the phenomena and the detection probability are supplied.

Key words: gravitational lensing – gravitational waves.

1 INTRODUCTION

Gravitational waves (GWs) are predicted by general relativity (GR) and their existence has been indirectly proven by binary pulsar timing (Hulse & Taylor 1975; Taylor, Fowler & McCulloch 1979). GWs, as received on Earth from any astrophysical source, produce extremely small effects and no GW has been detected, as yet.

The effect of GWs on some astrophysical measurable quantities have been the subject of studies and proposals: but, as a result, only upper bounds on the strength of GWs have been posed. Scintillation of starlight by GW focusing of the electromagnetic radiation toward Earth (Labeyrie 1993; Bracco 1997) has been proposed (Zipoy 1966) to place an upper limit on the theoretically predicted GW background, in analogy with the cosmic microwave background radiation. To detect the same stochastic background of GWs, the deflection of electromagnetic beams has been studied by several authors (Linder 1988; Bar-Kana 1996; Pyne et al. 1996) giving, again, only upper bounds. The same has been done in studies on the time delay in lensed quasar images (Frieman, Harari & Surpi 1994). It is worth noting a proposal to find GWs from supernovae (Fakir 1993, 1994b) and to discriminate between different gravitational theories (Faraoni 1996; Bracco & Teyssandier 1998), observing light deflection from the GWs themselves. Most of these techniques rely on the basic idea that the strength of a GW, while extremely low on Earth, would be noticeably larger if detected closer to the source and that the deflection angle of a light beam, interacting with such a GW is of the order of the metric perturbation in the region of closest approach. However, it has recently been pointed out that, at least in the standard GR framework and up to a certain degree of approximation, such a statement is incorrect (Bracco 1998; Damour & Esposito-Farese 1998; Kopeikin et al. 1999; for a brief discussion see also Crosta, Lattanzi & Spagna 1999).

In our Galaxy the most noticeable (and predictable) sources of GWs are binary stars (Lipunov, Postnov & Prokhorov 1987), in particular, W-UMa stars (Mironovskii 1966).

A microlensing event occurs when, by chance, a precise alignment between a background source and a deflecting mass is experienced by the observer (Paczyński 1986). The gravitational perturbation generated by a binary star very close to the line of sight of the event, slightly deflects light ray trajectories (Durrer 1994), introducing a small distortion in the microlensing alignment. This effect translates into a modulation of the light amplification that is synchronous with the binary star.

While we note that a similar idea has been proposed to describe some light variation in quasar microlensing (Schild & Thomson 1997; Larson & Schild 2000), here we intend to focus our attention to the particular case of the perturbation by a binary star of a microlensing event, inside our Galaxy.

The technique that will be described in the following sections could lead, in our opinion, to an unambiguous detection of such subtle gravitational effects from a stable source. Even if such a detection has to occur just during a microlensing alignment, the GW source can be studied and observed well outside the microlensing time-span. It is remarkable that the perturbation of the microlensing is not the most sensitive-to-misalignment astronomical phenomenon one can conceive. In fact, microlensing, including light interference effects (Deguchi & Watson 1986; Jaroszyński & Packzyński 1995; Ulmer & Goodman 1995) could lead to an even greater detection probability of such elusive phenomena.

The role of the approximations employed to obtain the solutions for the light ray propagation equations is crucial, in order to determine to what extent the deflection angle may be directly ascribed to genuine GWs. Correspondingly, how the measurements of such small deflection angles and their time behaviour could lead to new
Let us consider (see Fig. 1) a double star, the components of which are close to the line of sight. These stars are separated by a distance \( d = r_1 + r_2 \). Special care is to be given, here, to the relativistic definition of the angular speed \( \omega \) and to the binary frequency \( f \) and the body motion is characterized by an angular speed \( \omega \). One can also define the velocity components of the two bodies, along the approaching light beam direction, as \( V_{\parallel} = v_\parallel \cdot \hat{k} \), so that

\[
V_{\parallel} = \omega r_1 \cos \omega t
\]

Formally, equation (1) can be recovered from the equations of classical mechanics, just by replacing all the masses with their relativistic counterparts:

\[
m_i^* = \frac{m_0}{\sqrt{1 - v_i^2/c^2}}, \quad i = 1, 2,
\]

where \( m_0 \) denote the rest masses of the two component stars. For interacting bodies, however, in equation (1), a binding energy

\[
W_b = -\frac{Gm_1m_2}{2r}, \quad (3)
\]

has to be taken into account. Imitating what we have just done in equation (2), one has to replace the mass terms in the classical formula with the more complicated versions (see, for instance, Landau & Lifshitz 1971, problem 2, Section 106):

\[
m_i = m_i^* + W_b, \quad i = 1, 2.
\]

Setting the origin of our reference frame to the centre of mass, by definition

\[
m_1r_1 = m_2r_2 \quad (5)
\]

and one can define the time-dependent impact parameters of the two bodies as follows:

\[
d_1 = d + r_1 \cos \omega t
\]

\[
d_2 = d - r_2 \cos \omega t, \quad (6)
\]

where it is assumed that the eccentricities of the orbits are zero (i.e. a circular orbit) and the body motion is characterized by an angular speed \( \omega \). One can also define the velocity components of the two bodies, along the approaching light beam direction, as \( V_{\parallel} = v_\parallel \cdot \hat{k} \), so that

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V_{\parallel} = \omega r_1 \cos \omega t
\]

\[
V_{\parallel} = \omega r_2 \cos \omega t, \quad (7)
\]

where \( \hat{k} \) is a unit vector aligned with the unperturbed light propagation direction and the body motion is characterized by an angular speed \( \omega \). One can also define the velocity components of the two bodies, along the approaching light beam direction, as

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V_{\parallel} = v_\parallel \cdot \hat{k}, \quad \text{so that}
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\]
\[
\begin{align*}
\frac{w}{\text{c}} &= \frac{c^3}{16\pi G} \left(\frac{\partial h}{\partial t}\right)^2, \\
\text{where } h &\text{ is the dimensionless amplitude of the GW perturbation of the metric (Weinberg 1972; Hawking & Israel 1979; Derouelle & Piran 1983). In the particular case of a monochromatic GW with phase angular speed } \omega_G \text{ we have} \\
h &= h_0 \sin(\omega_G t) \\
\text{and the quadratic term appearing on the right-hand side of equation (13) becomes} \\
\left(\frac{\partial h}{\partial t}\right)^2 &= \frac{h_0^2 \omega_G^2}{2} \{1 + \cos(2\omega_G t)\}. \\
\text{After averaging it for a time-span much larger than } 1/f_\alpha \text{ one obtains} \\
\left\langle\left(\frac{\partial h}{\partial t}\right)^2\right\rangle &= \frac{h_0^2 \omega_G^2}{2} \\
\text{and substituting it for the binary rotation angular speed} \\
\left\langle\left(\frac{\partial h}{\partial t}\right)^2\right\rangle &= 2h_0^2 \omega_G^2, \\
\text{so that equation (13) becomes} \\
w &= \frac{c^3 h_0^2 \omega_G^2}{8\pi G}. \\
\text{Assuming an isotropic energy distribution around the binary star, the energy density at a distance } d \text{ from the GW source is} \\
w &= \frac{W}{4\pi d^2} \\
\text{and, using equations (12) and (18) one finally obtains} \\
h_0 &= \frac{8}{\sqrt{3}} \frac{G\mu^2}{c^4 d}. \\
\text{From the third Kepler law} \\
\rho^2 &= \frac{G^2 M^2}{\omega^4}, \\
\text{hence} \\
h_0 &= \frac{8}{\sqrt{3}} \frac{G^2 \omega^2 \mu M^2}{\omega^4} c^2 d. \\
\text{A numerical approximation of equation (22) can be given in MKS units as} \\
h_0 &\approx 4.85 \times 10^{-5} \frac{\omega^2 \mu M^2}{d} \Omega^{2/3} \\
\text{and in astrophysical ones as} \\
h_0 &\approx 1.77 \times 10^{-14} \frac{\mu M}{M_\odot} (P/d)^{2/3} (d/\text{au})^{1/3}. \\
\text{At one GW wavelength } \Lambda \text{ is given by} \\
\Lambda &= \frac{2\pi c}{\omega_G} = \frac{\pi c}{\omega}, \\
\text{the metric perturbation } h_0 \text{ becomes} \\
h_0(\Lambda) &= \frac{8}{\pi \sqrt{3}} \frac{G^2 \omega \mu M^2}{c^5}. \\
\end{align*}
\]

### 2.2 Calculation technique

Hereafter, for the sake of simplicity, let us assume that the deflection of a light ray due to a GW source is confined to a very small region, close to the point of minimum distance between the light ray and the GW source. Consequently, such a perturbation is characterized by the deflection angle, since we can assume that the interaction is essentially concentrated at the minimum impact point. In what follows let us denote such an angle by \(\tilde{\alpha}\) with various subscripts or special signs, in order to distinguish the various degrees of approximation used to derive such a value.

We rewrite equation (68) of Kopeikin & Schafer (1999) for the bending angle of the light ray \(\tilde{\alpha}\), under the assumption that the impact parameter \(d\) is negligible with respect to the distance of the deflecting masses from the observer, obtaining

\[
\tilde{\alpha} = -\frac{4G}{c^2} \sum_{i=1}^{2} m_i \frac{(1 - V_{\Delta i})}{d_i}. \\
\]

We stress that such a result is the same as that obtained by Pyne & Birkinshaw (1993) in their equation (45). Furthermore, let us rewrite the total bending angle \(\Delta \phi\) as the sum of several deflection angle contributions:

\[
\Delta \phi = \alpha_{\text{PN}} + \alpha_{\text{PM}} + \alpha_{\text{PPN}} + \alpha_{\text{F}} + \cdots, \\
\]

where \(\alpha_{\text{PN}}\) refers to the post-Newtonian formalism, that is to the linearized Einstein field equation, in the slow motion approximation or, in other words, approximating the light deflection angle in equation (27) for \(V_{\Delta i} \to 0\) and, with a certain abuse of language:

\[
\alpha_{\text{PM}} = \tilde{\alpha} - \alpha_{\text{PN}}, \\
\]

since we are mainly interested in the new effects arising from the motion of the stars in the binary system under consideration. Actually, the term post-Minkowskian usually refers to the whole amount given by \(\tilde{\alpha}\) (for a thorough discussion see Thorne 1987). Furthermore, a post-post-Newtonian term \(\alpha_{\text{PPN}}\) is produced by a second-order expansion term in \(G\), while \(\alpha_{\text{F}}\) refers to the deflection effect claimed by Fakir (1994a) and criticized by Kopeikin et al. (1999) and by Damour & Esposito-Farese (1998).

In order to gain an idea of the order of magnitude for the terms in equation (27), we just note that, by replacing \(m\) and \(d\) by the solar values, one obtains for the leading term the well-known deflection angle at the edge of the Sun, namely \(\approx 1.75\) arcsec.

The approach used here is to compute the deflection angle by writing the impact parameters \(d_i\) and \(d_f\) of the two masses in the binary star, as a function of the average common impact parameter \(d\) (see equation 6). The result is then expanded in a series of \(1/d\) terms. The following relations will be used throughout this paper:

\[
\frac{1}{1 \pm \varepsilon} \approx 1 \mp \varepsilon \mp \varepsilon^2 \mp \varepsilon^3 + \cdots \\
\left(1 \pm \varepsilon\right)^2 \approx 1 \mp 2\varepsilon + 3\varepsilon^2 \mp 4\varepsilon^3 + 5\varepsilon^4 \mp 6\varepsilon^5 + \cdots, \\
\]

where, for truncating the expansion up to a certain degree in \(\varepsilon\), some hypotheses on the smallness of \(\varepsilon\) (in comparison with unity) are required.

In this way, terms of the type \(m_1 r_1^2 \pm m_2 r_2^2\) will appear.

Finally, it is convenient to consider as a basic geometric configuration, the one shown in Fig. 1, where the plane of the binary star orbit includes the straight line joining the source of the ray of light under scrutiny and the observer. That very configuration is the one that allows the maximum effect.

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After each of the following two subsections, dedicated to the most relevant approximation schemes, we shall briefly discuss to what extent our statements have to be weakened for a generic geometric configuration. On the contrary, in Section 2.7 hereafter, devoted to an overview of the results, we will neglect such dependences, since we are interested in estimating the order of magnitude of light ray deflection. Of course, these considerations should properly be taken into account for any specific case.

2.3 The quasi-static field

We can now write down the overall deflection of the light beam due to the two masses as the linear superposition of the light deflection produced by each body. We write such an angle as $\alpha_{\text{ps}}$, to distinguish from other sources of deflection that we shall examine in the paper. The angle will be given by

$$\alpha_{\text{ps}} = -\frac{4G}{c^2} \left( \frac{m_1}{d_1} + \frac{m_2}{d_2} \right)$$

which, using equation (6), translates into

$$\alpha_{\text{ps}} = -\frac{4G}{c^2} \left[ \frac{m_1}{1 + (r_1/d) \cos \alpha} + \frac{m_2}{1 - (r_2/d) \cos \alpha} \right].$$

Adopting the first of the expansions in equation (30), the previous relation can be rewritten as

$$\alpha_{\text{ps}} \approx -\frac{4G}{c^2} \left[ \frac{m_1}{d} + \left( m_1 r_1^2 + m_2 r_2^2 \right) \frac{\cos^2 \omega \alpha}{d^3} \right]$$

or in full form

$$\alpha_{\text{ps}} = -\frac{4G}{c^2} \left[ \frac{m_1 + m_2}{d} + \sum_{n=1}^{\infty} \left( m_1 r_1^{2n} + m_2 r_2^{2n} \right) \frac{\cos^{2n} \omega \alpha}{d^{2n+2}} \right]$$

We just stress that the first term of the first series expansion in brackets is not time dependent. It simply gives the deflection due to the whole mass of the binary, as concentrated in its centre of mass. We also note that in the case of a perfectly symmetric binary, only terms depending upon odd powers of $d$ are non-zero, with the $d^{-1}$ term not depending upon the time.

As pointed out by Kaiser & Jaffe (1997), on some occasions the simple Schwarzschild metric perturbation (the same effect giving the $\approx 1.75$-arcsec deflection of light at the edge of the Sun) can be of a similar order of magnitude. It is clear, however, that the latter statement can be proven only under some particular conditions. In fact, the larger the separation of a binary star is, the weaker the GW strength and the stronger the Schwarzschild metric perturbation. It has also to be pointed out that while for an oscillating mass the Schwarzschild perturbation drops as $d^{-2}$, in the case of a binary source the perturbation will drop as a much faster $d^{-3}$ law.

The first time-varying term of equation (34) can be expressed in astrophysical units as

$$\alpha'_{\text{ps}} \approx 1.75 \times 10^{-7} \frac{(m_1/M_\odot)(\rho_1/R_\odot)^2 + (M_2/M_\odot)(\rho_2/R_\odot)^2}{(d/\text{au})^3}.$$  

With reference to Fig. 2, we note that this effect scales with the cosine of the angle $\xi$. In fact, the above-mentioned deflection disappears when the impact parameter, as seen by the observer, during motion remains perpendicular to the line joining the stars of the binary.

2.4 A post-Minkowskian treatment

The deflection angle of an approaching light beam by a single mass is given by the usual term $4Gm/c^2$ times $d^{-1}$ in the perturbing mass reference frame. If the mass is moving with an arbitrary speed $v$, one further term appears, where only the speed component $v_i$ along the line of sight is relevant. One can think of it as an additional deflection angle $\alpha_{\text{PM}}$ given by the linear superposition of the two moving masses in the binary star:

$$\alpha_{\text{PM}} = \frac{4G}{c^3} \left( \frac{m_1 v_{1i}}{d_1} + \frac{m_2 v_{2i}}{d_2} \right).$$

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With the perturbative approach so far described, the latter equation can be rewritten as
\[
\alpha_{\text{PM}} = -\frac{4G\omega \cos \omega t}{d c^3} \left[ \frac{m_1 r_1}{1 + (r_1/d) \cos \omega t} - \frac{m_2 r_2}{1 - (r_2/d) \cos \omega t} \right]
\] (37)
and expanded as
\[
\alpha_{\text{PM}} \approx \frac{4G\omega}{c^3} \left[ (m_1 r_1^2 + m_2 r_2^2) \frac{\cos^2 \omega t}{d^2} - (m_1 r_1^3 - m_2 r_2^3) \frac{\cos^3 \omega t}{d^3} \right.
+ \left. (m_1 r_1^4 + m_2 r_2^4) \frac{\cos^4 \omega t}{d^4} + \cdots \right].
\] (38)

The resulting relations can be rewritten in the following compact and exact form:
\[
\alpha_{\text{PM}} = \frac{4G\omega}{c^3} \left[ \sum_{n=1}^{\infty} \left( m_1 r_1^{2n+1} + m_2 r_2^{2n+1} \right) \frac{\cos^{2n+1} \omega t}{d^{2n+1}} \right]
- \sum_{n=1}^{\infty} \left( m_1 r_1^{2n+1} - m_2 r_2^{2n+1} \right) \frac{\cos^{2n+1} \omega t}{d^{2n+1}} \right].
\] (39)

In contrast to what happens for the post-Newtonian contribution, the above effect depends on the cosine of the angle \( \psi \). This means that, while for the configuration shown in Fig. 1 both the PN and PM terms amount to the full figure worked out here, for a generic configuration of the orbital plane (see Fig. 2 for two particular sets of cases, where only one of the two angles is different from zero), these two terms are, in general, attenuated, but they cannot vanish simultaneously.

2.5 Post-post-Newtonian relativistic deflection

Post-Newtonian relativistic deflection by a mass is obtained from the first term expansion of the Schwarzschild metric in the impact distance \( d \). Of course, it is possible to go further and write down the deflection angle up to the \( d^{-2} \) term as in Epstein & Shapiro (1980), see also Ebina et al. (2000). Let us write this additional contribution to the deflection angle, considering, as before, the linear superposition of the effect due to the two masses in the binary:
\[
\alpha_{\text{PPN}} \approx -\frac{15\pi G^2}{4c^4} \left[ \left( \frac{m_1}{d_1} \right)^2 + \left( \frac{m_2}{d_2} \right)^2 \right].
\] (40)

Following the same approach we used in the previous sections, the latter equation can be rewritten as
\[
\alpha_{\text{PPN}} = -\frac{15\pi G^2}{4c^4 d^2} \left[ \left( \frac{m_1^2}{1 + r_1/d \cos \omega t} \right)^2 + \left( \frac{m_2^2}{1 - r_2/d \cos \omega t} \right)^2 \right]
\] (41)
and, after substituting the expansion given in equation (30), one obtains
\[
\alpha_{\text{PPN}} \approx \frac{15\pi G^2}{4c^4 d^2} \left[ (m_1^2 + m_2^2) \frac{1}{d^2} - 2 \left( m_1^2 r_1^2 - m_2^2 r_2^2 \right) \frac{\cos \omega t}{d^3} \right.
+ \left. 3 \left( m_1^2 r_1^2 + m_2^2 r_2^2 \right) \frac{\cos^2 \omega t}{d^4} + \cdots \right].
\] (42)

Contrary to what one could expect, it is remarkable that the first non-vanishing, time-varying term is of the same order as it occurs in the post-Newtonian approach, at least when a non-symmetric binary is considered. As before, we also give the complete expression for

\[
\alpha_{\text{PPN}} \approx \frac{15\pi G^2}{4c^4 d^2} \left[ \frac{m_1^2}{d^2} - \sum_{n=1}^{\infty} \left( 2n (m_1^2 r_1^{2n-1} - m_2^2 r_2^{2n-1}) \frac{\cos^{2n-1} \omega t}{d^{2n+1}} \right. \right.
+ \left. \left. \sum_{n=1}^{\infty} \left( 2n + 1 \right) (m_1^2 r_1^{2n} + m_2^2 r_2^{2n}) \frac{\cos^{2n} \omega t}{d^{2n+2}} \right].
\] (43)

2.6 Radiative term cancellation in GR

The first estimate for the light deflection angle due to GWs is likely to be due to Fakir (1994a). Under some specific assumptions, he obtains that, at an impact parameter \( d \) equal to one GW wavelength, a light ray is deflected by an angle \( \alpha_{\ell} \) given by
\[
\alpha_{\ell,d=\lambda} = 3 \frac{\pi^2}{\lambda^2} h_{\ell,d=\lambda}.
\] (44)

This result and its possible extensions to different values of \( d \) were examined by many authors (Linet & Tourrenc 1976; Durrer 1994; Fakir 1995; Kaiser & Jaffe 1997), with results only partially in agreement among themselves.

In particular, Durrer (1994) claims \( \alpha_{\ell} \propto h \) at any distance \( d \). Under the above assumption, replacing \( h \) in equation (44) with \( h_0 \), expressed in equation (22), one obtains
\[
\alpha_{\ell,d=\lambda} = \frac{12\pi G^5 c^6 \alpha_0^5 M_2^{5/3}}{\sqrt{5}}
\] (45)
and, expressing the relationship in MKS units
\[
\alpha_{\ell,d=\lambda} \approx 7.63 \times 10^{-9} \alpha_0^{5/3} M_2^{5/3},
\] (46)
where \( \alpha_0 \) is given in radians, while in astrophysical units
\[
\alpha_{\ell,d=\lambda} \approx 6.28 \times 10^{-10} \frac{(\mu/M_2)(M/M_2)^{2/3}}{P/d^{5/3}}
\] (47)
for deflection angle expressed in arcsec units. However, Fakir (1994a) pointed a slightly faster decrease of the light deflection with the distance \( d (h \propto d^{-1}) \). Kaiser & Jaffe (1997) confirm Fakir’s result for a range of \( d \) of the order of \( \Lambda \) but are unable to confirm Durrer’s claim. In more recent times Fakir’s results have been strongly criticized. Bracco (1998) was the first to point out that the dependence on \( d^{-1} \) in the deflection angle is too optimistic, although he still considered the \( d^{-3} \) term he found in its place, as a radiative one, or, in other words, intimately linked to the GW nature of the perturbation.

Later, Damour & Esposito-Farese (1998) and Kopeikin et al. (1999), while pointing out the same result shown by Bracco, also claimed that the \( d^{-3} \) contribution is of a quasi-static nature and has nothing to do with the radiative nature of GWs emitted by the binary. They pointed out a radiative term of the deflection, but only due to the value of the metric perturbation at the observer and at the location of the source (called edge effects), which, of course, prevents one from sensing GW fields in positions significantly closer to the GW source.

While the limits of these findings are going to be briefly discussed in the next subsection, we want to point out that both Damour & Esposito-Farese (1998) and Kopeikin stress that their results are due to a perfect cancellation of terms in \( d^{-1} \) in the GR framework, so that any discrepancy in the latter can translate into a renaissance of the \( \alpha_{\ell} \) term. In particular, scalar GWs are likely to introduce
radiative deflection angles, which, under particular circumstances, could become comparable to the mentioned one, so that one can imagine using a measure of $\alpha$ in order to establish the existence of a term of the $\alpha_3$ type (Faraoni 1996; Bracco & Teyssandier 1998; Liu & Overduin 2000; Will 2001).

### 2.7 Are we neglecting a relevant term?

Let us now define the average mass $m = (m_1 + m_2)/2$ and the mass asymmetry $\Delta m = |m_2 - m_1|$ of the binary star. Even if almost all real binaries, such as that reported in our example (Section 5), are not close to being symmetric, a series expansion in $\Delta m/m$ is still possible, provided that it is not stopped at the first terms. Notwithstanding, the first terms can be useful for identifying the different $d^{-n}$ dependences in the deviation angles computed according to the different approximation schemes we examined. Thus, for the reader’s benefit, we have collected the above results in Table 1.

| $n$ | $d^{-1}$ | $d^{-2}$ | $d^{-3}$ | $d^{-4}$ |
|-----|--------|--------|--------|--------|
| PN  | Static | $-$    | $\omega m^2/r$ | $4\Delta m/r^2$ |
| PM  | Static | $-$    | $4\Delta m/r^2$ | $\omega m^2/r^2$ |
| PPN | $[15G/(16c^2)]^{-1}$ | $-$    | $3m^2/2r$ | $3m^2/2r$ |
| F   | $\approx (3\pi^2\omega^2)/(\sqrt{3}c^2) \times m^2$ | $-$    | $?$    | $?$    |

(i) we consider an expansion both in the GW perturbation $h$ (see equation 20) and in the ratio $\Lambda/d$ between the characteristic waveform $\Lambda$ (see equation 25) and the impact parameter $d$;

(ii) just to help the reader in grasping the dependence structure of such an expansion, we intentionally omit all the trigonometric functions appearing in its time-dependent terms;

(iii) the static (not depending on time) terms are neglected.

As a result, one obtains

$$\Delta \phi = \frac{\sqrt{3}}{\pi} h \left\{ \frac{3\pi^3}{2\sqrt{5}} \eta p \left( \frac{\Lambda}{d} \right) \left[ 1 + \frac{8\Delta m r}{m d} + \left( \frac{r}{d} \right)^2 + \cdots \right] \right. \right.$$  

$$\left. \frac{-1}{\pi} \left( \frac{\Lambda}{d} \right)^2 \left[ 1 + \frac{2\Delta m r}{m d} + \left( \frac{r}{d} \right)^2 + \cdots \right] + \cdots \right\},$$

(48)

where $\eta_p$ is just a coefficient related to Fakir’s claim that turn out to be zero in the strict GR framework.

The validity of the further expansion in $r/d$, carried outside each term, relies on the fact that $d \approx \Lambda$ for the cases of interest here [those where the various terms in $(\Lambda/d)^n$ are comparable for various values of $\chi$] and that $r$ can be obtained through equation (25), recalling that $v = \omega \rho$. In fact, it turns out that the $r/d$ series expansion given here is equivalent to a series expansion in $v/c$.

Such an expression can easily be rewritten in the approximation of $r/d \rightarrow 0$ (small binary star approximation, where the star dimension $r$ is negligible compared with the impact parameter $d$) as

$$\lim_{\delta \rightarrow 0} [\Delta \phi] = \frac{3\pi^3}{2\sqrt{5}} h \eta p \left( \frac{\Lambda}{d} \right) \left[ 1 + \frac{8\Delta m r}{m d} + \cdots \right],$$

(49)

which may be clearly interpreted as a series expansion in $\Lambda/d$. Furthermore, we point out a fact that, in our opinion, holds true in all the calculations reported here and previously by Fakir (1993), Faraoni (1996), Bracco & Teyssandier (1998), Bracco (1998), Damour & Esposito-Farese (1998) and Kopeikin et al. (1999).

As one can see from equation (48), when $\Lambda$ becomes of the same order as $d$ (i.e. in the limit $\Lambda/d \rightarrow 1$), all the terms deriving from the different approaches (PM, PN, etc.) to solve the Einstein field equations become comparable in strength. Just some numerical coefficients appear in the relative ratios, such as the factor between the $d^{-2}$ and $d^{-1}$ terms. This means that, in our opinion, further theoretical developments could lead to terms in $\Lambda/d$ of the order of greater than 2, but still relevant in all the cases where $d$ is of the same order of $\Lambda$. We recall again that $h \propto d^{-1}$ with a consequent dependence upon $d$ with a power law steeper than a cubic one. Our position is just that if a certain approximation technique makes a radiative term disappear up to a certain power, it is necessary to take into account the first non-vanishing term (if it exists).

That is why it could become particularly important to find an astrophysical case for probing the situation described above, which is when higher-order terms become relevant.
3 PERTURBATION ON A MICROLENSING EVENT

As pointed out in a similar situation by Bracco (1997), some bending of light of a given angle does not necessarily translate into effects that are simply proportional to such an angle; they have to be rescaled according to the geometry involved.

Let us consider a microlensing event, where the source S, the lens L and Earth as the observing point, O, lie approximately on a straight line. Let us denote by $D_{SL}$ the distance between the source and the lens, by $D_{LO}$ that between the lens and the observer, and by $D_{SO} = D_{SL} + D_{LO}$ the distance between the observer and the source. The impact parameter, $r$, is the separation between the lens and the straight line joining the observer and the source: it is measured on the lens plane orthogonal to the line of sight. The GW crosses the line of sight transversally at a distance $D_{GW}$ from the observer. Let us also assume, hereafter, that the GW wavelength, denoted by $\lambda$, is much greater than both $r$ and $\rho$, the impact parameter and the Einstein radius of the microlensing event, respectively. In this way, any differential deflection between different rays focused by the lens toward the observer becomes negligible.

3.1 GW source between the lens and the observer

According to the notation explained in Fig. 3, let us define an auxiliary distance $p$ on the plane defined by the observer and perpendicular to the microlensing alignment line. Since the deflection angle is very small, we write

$$ p = \Delta \phi D_{GW} $$

(50)

and, using $\alpha = p/D_{SO}$ and $\Delta r = \alpha D_{SL}$, one obtains

$$ \Delta r = \Delta \phi D_{GW} D_{SL} / D_{SO}, $$

(51)

where $\Delta r$ is the variation of the impact parameter $r$, due to the GW. Of course such a parameter oscillates, in the case of a monochromatic GW, between $r - \Delta r$ and $r + \Delta r$. We just note that when $D_{GW} = 0$, $\Delta r$ vanishes, so that any GW source close to the observer does not produce any effect. The maximum effect holds when $D_{GW} = D_{LO}$, that is when the GW is generated in the neighbourhood of the lens. In such a case, the following equation holds true:

$$ \Delta r = \Delta \phi D_{LO} D_{SL} / D_{SO}. $$

(52)

When $D = D_{LO} = D_{SL}$ we obtain $\Delta r = \Delta \phi D / 2$, so the maximum optical lever is equal to one-quarter of the whole distance between the source and the observer. It is worth noting that, in the case of

Figure 3. A binary star as a source of GWs is between the lens and the observer during a microlensing event. GWs will perturb the alignment leading to some noticeable signature in the observed light curve. In order to make the figure clearer the deflection due to the lens is not shown.

Galactic measurements, this condition poses an upper limit on the maximum lever, given by roughly half the distance of the Sun from the Galactic Centre.

3.2 GW source between the source and the lens

With reference to Fig. 4 it is useful, in this case, to introduce the displacement $q$, given by

$$ q = \Delta \phi (D_{SO} - D_{GW}) $$

(53)

as in the previous subsection, let us define the angle $\beta \approx q/D_{SO}$ and the variation $\Delta r \approx \beta D_{LO}$, thus obtaining

$$ \Delta r = \Delta \phi D_{SO} - D_{GW} D_{LO}. $$

(54)

It is easy to see that the above relation is close to equation (51). The behaviour of the optical lever is similar, provided that one replaces the distance of the GW from the observer with the same distance, but measured from the source. It is also evident that equations (54) and (51) give the same value when the GW is located close to the lens. The same considerations described in Section 3.1 hold true here.

With reference to Fig. 5, one can introduce a lever length $l_{GW}$, such that $\Delta r = \Delta \phi l_{GW}$. The behaviour of $l_{GW}$, measured on the microlensing straight line connecting the source and the observer is the following one: starting from the zero value at the source, it grows linearly since it reaches its maximum at the lens position, then

Figure 4. As in Fig. 3, but with the source of the GW between the source and the lens. Again, the deflection due to the lens is not shown.

Figure 5. The ratio between the effect on the impact parameter, $\Delta r$, and the deflection angle generated by the GW perturbation, $\Delta \phi$, is defined here as a lever length $l_{GW}$, which is maximum near the lens and drops linearly to zero both toward the source and the observer. For a hypothetical event, where the lensing object is located in the bulge of our Galaxy (see the upper right-hand inset), the search for a GW signature as described in the text is equivalent to probing for GW sources within a biconical volume.

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it decreases linearly to zero, approaching the observer. Provided that a certain sensitivity to \( \Delta r \) is accomplished, a bi-conic volume of the Galaxy is probed in searching for GWs larger than a given threshold (see the upper right-hand inset in Fig. 5).

### 4 PHOTOMETRIC EFFECTS ON MICROLENSING

Following Paczyński (1986) we use the Einstein radius \( r_E \) defined as

\[
r_E = \sqrt{\frac{4GM_1 D_{LO} D_A}{c^2 D_L}}.
\]

where \( M_1 \) is the mass of the lensing object. Denoting by \( u = r/r_E \), a dimensionless impact parameter, the amplification factor \( A(u) \) is given by

\[
A(u) = \frac{u^2 + 2}{u\sqrt{u^2 + 4}}.
\]

The GW within the line of sight of a microlensing event introduces a tiny perturbation in \( u \) and a corresponding modulation of the amplification, which can be estimated as

\[
\frac{\partial A}{\partial u} = -\frac{8}{u^2(u^2 + 4)\sqrt{u^2 + 4}}.
\]

We point out that in the cases of interest here, \( u \ll 1 \) holds true, hence the previous relations simplify into \( A \approx u^{-1} \) and \( \partial A/\partial u \approx -A^2 \).

As a result of the perturbation, \( \Delta r \) the dimensionless impact parameter is perturbed by an amount \( \Delta u = \Delta r/r_E \) and the magnification \( A \) will exhibit, in the approximation \( A \gg 1 \):

\[
\Delta A \approx -A^2 \Delta u.
\]

Such a perturbation leads to a hopefully measurable brightness variation \( \Delta I \) of the observed flux \( I \) (see also Fig. 6). Let us define \( \Delta m \) as the maximum photometric magnitude difference between several measurements affected by an intensity \( I \pm \Delta I \), i.e.

\[
\Delta m = \frac{5}{2} \log\left(1 + 2\frac{\Delta I}{I}\right),
\]

so that one can write the approximation

\[
\Delta m \approx \frac{5}{2} \frac{\Delta A}{\ln 10 A}.
\]

Using equation (58) and the concept of effective length \( l_{GW} \) defined in the previous section, one can write

\[
\Delta m \approx \frac{5}{2} \frac{\Delta \Phi l_{GW}}{\ln 10 r_E}.
\]

Taking into account equations (22), (44) and (61) it is possible to estimate the maximum projected distance \( d_{\text{max}} \), around which a binary star produces a perturbation, leading to a given photometric amplitude \( \Delta m \):

\[
d_{\text{max}} = \frac{12\sqrt{57\pi^2} G^{5/3} A l_{GW} \omega^{2/3}\mu M_{\odot}^{2/3}}{\ln 10 c^2 r_E \Delta m},
\]

where we have grouped on the right-hand side of equation (62) the numerical coefficients, the physical constants, the microlensing parameters and that of the binary star into four different fractions. The linear dependence upon \( l_{GW} \) we have found deserves, however, some specific comments. We just note, in fact, that equation (55) can be rewritten as

\[
r_E = \sqrt{r_s l_{GW}},
\]

where \( r_s \) is the Schwarzschild radius of the lens and \( l_s \) is the effective length of the microlensing setup, defined similarly to \( l_{GW} \). While \( l_{GW} \) and \( l_s \) are a priori fully uncorrelated, it is clear that by the selection effect, the largest \( \Delta m \) can be obtained when \( l_{GW} \approx l_s \). Under this condition, the dependence in equation (62) from such a characteristic length becomes weaker, i.e. a square root. Also the dependence upon the mass of the lensing object becomes an inverse square root.

### 5 AN EXAMPLE WITH A W-UMA BINARY

Following Mironovskii (1966) we take a typical W-UMa binary as the average of those listed by Kopal (1959), obtaining \( m_1 = 1.46 M_\odot, m_2 = 0.78 M_\odot \) and \( P = 0.3 \) d, leading to \( \omega \approx 2.42 \times 10^{-4} \) s\(^{-1}\). This kind of double star has a typical size of \( \approx 10^{-2} \) au and a radial velocity \( v \) such that \( v/c \approx 10^{-4} \). A GW wavelength \( \Lambda \approx 26 \) au is obtained. We just note that such a figure is at least an order of magnitude larger than a typical \( r_E \) for microlensing in the Galaxy, so that the approximations used in the calculation reported in this work appear reasonable.

At a distance \( d = \Lambda, h_0 \approx 1.33 \times 10^{-15} \) is obtained. Assuming that this W-UMa is in the bulge of our Galaxy, the same perturbation on Earth will be lowered to a mere \( h \approx 1.68 \times 10^{-23} \).

According to equation (49), one estimates a light deflection of \( \Delta \Phi \approx 9.5 \times 10^{-16} \), equivalent to \( \Delta \Phi \approx 0.20 \) nano-arcsec (mainly from the PM term). Here and in the following considerations, we assume that what we called Fakir’s contribution vanishes: \( \eta = 0 \); none the less, we note that our results could increase by a factor of \( \approx 20 \) in the case where \( \eta = 1 \).

Let us suppose that a microlensing event occurs and the corresponding magnification is relatively high \( (A = 100) \); let us also suppose that the lens determines a value \( r_E = 0.1 \) au and the average W-UMa under investigation is located near the Galactic Centre, at roughly 10 kpc from the Sun. With the further assumption that the source is situated 10 kpc far away from the lensing mass, \( l_{GW} = 5 \) kpc is obtained. Combining these figures in equation (61) one can estimate \( \Delta m \approx 2.2 \times 10^{-3} \). Nevertheless, one should note that a precision of the order of 1/100 of magnitude should be enough to detect such a GW event, provided that one takes the average of several GW-induced oscillations around the time corresponding to the magnification peak.

The assumption we made concerning \( r_E \) is roughly one order of magnitude smaller than a typical one. While we note that this is just a factor of 3 smaller than the typical mass for microlensing, we point out that similar results can be obtained with a smaller impact factor \( d \). In particular, the PN term, growing with \( d^{-3} \), can make a similar perturbation on the microlensing event when, with a more typical \( r_E = 1 \) au, the impact factor drops to \( d \approx 5.8 \) au, while the PM term, much larger by a factor of at one GW wavelength distance, behaving just as a \( d^{-2} \) power, becomes of the same magnitude at \( d \approx 5.1 \) au. All the calculations reported in this paper are carried out under the assumption that the rays focused by the microlensing are approximately subject to the same gravitational deflection. Even if this condition fits the astrophysical cases described here very well, of course it is no longer true when the binary itself is responsible for microlensing. It is easy to realize that in this latter case, the effect should be much smaller, because the focusing deflection would change slightly and the relative variation in the amplification \( A \) should be of the same order of magnitude as the ratio between the deflection due to the time-dependent terms and the deflection caused by the static contributions. Nevertheless, since such a possibility is beyond the scope of our paper, we have not investigated...
it in more detail, so that we cannot exclude that further interesting results could be obtained in this way.

6 DETECTION PROBABILITY

From an inspection of the MACHO project microlensing alert Web page\(^1\) we found an average detection rate of events with \(A > 8\) of the order of \(\approx\)five events per year, with an average light amplification \(A \approx 20\); we dropped from our estimate caustic-crossing events, the treatment of which is beyond the scope of this paper. In the MACHO programme \(N_\alpha \approx 4.3 \times 10^4\) stars are observed photometrically (Alcock et al. 1995), hence we can define an event detection rate (hereafter denoted by EDR) \(\eta\), expressed in detected events per star, per year, for such a type of high-magnification events, as

\[
\eta = \frac{N(A > 8)}{N_\alpha} \approx 1.2 \times 10^{-5}.
\]

Because \(A \approx u^{-1}\) one can easily use the estimation of \(\eta\) to evaluate the EDR corresponding to the case in which another star (the lens) falls within the line of sight of a given star in the bulge (the source) at a certain distance \(d_{\text{lensing}}\), given by

\[
d_{\text{lensing}} \approx \frac{r_\gamma}{A} \approx 0.05\text{ au}.
\]

In fact, such an EDR for a diameter of the order of \(d_{\text{max}}\) will be obtained by simply scaling \(\eta\) by the ratio of the cross-sections defined by the two distances under study (see Fig. 7), leading to

\[
\mathcal{R} = \eta \left( \frac{d_{\text{max}}}{d_{\text{lensing}}} \right)^2.
\]

The EDR to have both a microlensing event (such as those considered here) and, simultaneously, a third object within a much larger distance \(d_{\text{max}}\) can be written as the product of the two terms:

\[
\mathcal{R} \approx \gamma \eta \left( \frac{d_{\text{max}}}{d_{\text{lensing}}} \right)^2,
\]

where \(\gamma\) is a coefficient that takes into account that the two events should occur simultaneously. Indeed, a given star spends a fraction \(\gamma \eta\) of the time undergoing high-magnification events, where \(\gamma\) is the average duration of a high-magnification event, expressed in years. A typical value for \(\gamma\) is of the order of \(10^{-2}\).

Let us estimate \(d_{\text{max}}\) for a typical Galactic event, where the lens is located in the bulge: in this case we assume \(l_{\text{GW}} \approx 5\text{ kpc}\) and \(r_\gamma \approx 1\text{ au}\). In order to achieve the highest possible photometric accuracy, it is to be recalled that one can average a few GW periods in the time-span covered during the high-magnification event. Assuming a final error of the order of \(\Delta m \approx 10^{-3}\) mag (Gilliland & Brown 1992; Frandsen 1993) one can obtain, using equations (61) and (67), together with the average W-UMa as from the previous section, and the above-mentioned estimates, a figure for \(d_{\text{max}} \approx 6\text{ au}\). We just note that this is significantly smaller than \(A\) so that, in such a regime, some still unvetted terms in higher powers of \(A/d\) can dominate.

Finally, one has to further select only the cases where the second star is of W-UMa type. The population density of W-UMa is almost constant all the way to the Galactic bulge, amounting to roughly \(\rho_{\text{W-UMa}} \approx 1/280\) (Rucinski 1994, 1997). Then, one can combine all of these EDR parameters into a single relationship, giving the average GW detection time interval \(\tau \approx \mathcal{R}^{-1}\) between two successive events:

\[
\tau = \frac{d_{\text{lensing}}^2}{\gamma \eta^2 d_{\text{max}}^2 \rho_{\text{W-UMa}} N_\alpha} \approx 4.3 \times 10^4\text{ yr}.
\]

This figure could make any reasonable search for such an event, truly hopeless. It has been obtained assuming that the W-UMa and the lensing star position are fully uncorrelated (which translates into a square power dependence upon \(\eta\) and to the appearance of \(\gamma\)). However, one should also consider the possibility of having a triple star system comprising by a W-UMa and a third companion at a much larger distance, responsible for the microlensing. Because \(d_{\text{max}}\) is so small, we found that the triple star case, in spite of its low probability, has an even higher EDR compared with the simpler case discussed previously.

In the triple star case, in fact, the EDR of a microlensing event on the companion of a W-UMa, with high magnification rate, is simply

\[
1 \text{http://www.macho.mcmaster.ca}
\]
Given by
\[ R' \approx \eta \eta_{\text{W-UMa}} \eta_{\text{3}}, \]
where \( \eta_{3} \) is the fraction of W-UMa exhibiting a third companion at a projected distance of the order of magnitude similar to \( d_{\text{max}} \). Such a number turns out to be of the order of \( \eta_{3} \approx 0.2 \) (Herczeg 1988; Tokovinin 1997), leading to a typical time interval between two different GW potential detections (or at least of the gravitationally induced effects discussed in the text) of the order of
\[ \tau' = \frac{1}{\eta \eta_{\text{W-UMa}} \eta_{\text{3}}} \approx 273 \text{ yr}, \]
that is a factor of roughly 200 times smaller than in the case of a free floating W-UMa and microlensing star. We note that in the triple star case the condition \( L_{\text{GW}} = l_{\mu} \) is automatically fulfilled.

Care should be taken in considering such figures. Most of the parameters involved are rough estimates and several uncertainties of the order of at least a factor of 2 can be considered. This leads, in our opinion, to a significant diminishing in the order of magnitude for \( \tau' \).

One should also consider that such a time will be lowered by future improved photometric capabilities, and that it has been devised for an average W-UMa: in principle, one cannot exclude that, by chance, some stronger event may occur. Moreover, we point out that the simultaneity condition discussed in the case of no correlation between the W-UMa and the lensing star, is well verified for the more optimistic assumptions. Notwithstanding, it can introduce some further augmentation of \( \tau \) for the more pessimistic calculations. The relatively low time-scale, however, also suggests that there is a small chance that a gravitational effect signature (to be ascribed to a genuine GW effect or mainly to a post-Minkowskian contribution is beyond our purpose here) could even be hidden in some of the microlensing photometric runs already collected from the ground.

7 CONCLUSIONS

In the opinion of the authors, a critical review of the light deflection contributions due to the gravitational field of a binary star shows that terms directly linked to a GW (or radiative ones) cannot be excluded by current approximations, available in the literature. When the impact parameter is of the same order of magnitude as the wavelength of the GW, such terms could be as important as the leading, non-radiative ones. Moreover, a non-quasi-static term depending upon the inverse square of the impact parameter is found. Such deflection angles are so small that one would be tempted to leave them in the academic rather than the experimental realm. However, we have shown that such a deflection, occurring close to the line of sight of a microlensing event, can give rise to detectable effects. The probability of such an alignment is, however, extremely small, except for the case of microlensing by a third companion of a close binary star. In that case, from the probability to detect such an event, one can reasonably expect that those effects could be observable in the near future, or even that one or a few of them are hidden in the existing literature data.

We avoided speculating, in the previous sections, on some extreme cases where the gravitational effects could be very large. One could, in principle, conceive a geometry where a small lens (for instance, with a mass of the order of Jupiter) produces a highly amplified (\( A \approx 100 \)) microlensing event for a duration of several days, with a massive binary such as \( \mu\text{-Sco} \), close to the line of sight. In this case, the binary star could form with the microlensing event an angle sufficient to study photometrically the binary star well separated from the microlensing event, for a reasonable number of binary periods. Even if such an event appears to be very unlikely, it could determine many insights on gravitational and GW studies, including the possibility of probing gravitation theories that do not lead to a perfect cancellation of the term claimed by Fakir.

On the other hand, we believe it to be remarkable that ordinary events, where a gravitational or GW signature can be clearly identified, have a significant chance of popping-up in current surveys. Post-microlensing studies should permit the eventual confirmation of the GW nature of the waviness detected during microlensing. Such approaches include spectroscopic and photometric studies (to ensure in detail the nature of the binary star responsible for the GWs) and, in future, astrometric studies carried out with some high angular resolution tool (such as speckle or adaptive optics) in order to establish the exact geometry of the three objects involved (source, lens and binary stars) at the moment of microlensing peak magnification.

A positive detection, such as that described here, can be seen as a sort of astrometric detection of a GW effect using a Galaxy-sized telescope for which the objective is formed by the gravitational lens responsible for the microlensing (Labeyrie 1994).

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