Development an empirical formula to calculate the scour depth at different shapes of non-uniform piers

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Abstract. The study of local scour around foundation of bridge piers is very important for safe design of piers and other hydraulic structures. Various studies about local scour around uniform bridge piers but a few studied non-uniform bridge piers. Non-uniform bridge pier is one for which the cross-sectional dimension varies over the length of pier (piers with footings). In this study, the effect of main parameters (pier shapes, foundations shapes, level of foundation, flow intensity, and Froude number) on local scour with different shapes of non-uniform piers were experimentally investigated with five different velocities (0.16, 0.18, 0.21, 0.24, and 0.27) m/sec. The tests were occurred using laboratory flume, operated under the clear water condition using sand as a bed material. The test program was done on five different piers Rectangular, Chamfered, oblong, Octagonal, and Hexagonal, foundation shapes were rectangular, oblong, and hexagonal. The results showed that the rectangular pier with rectangular foundation gives the maximum scour depth about 6.5 cm when foundation is placed above bed level, while the hexagonal pier with hexagonal foundation gives the minimum scour depth 2 cm, when foundation is placed below bed level.

1. Introduction
Scour is happened as effect of the passing of wave at the seaboar regions and sweep of sediment around and near the structures which placed in flowing water this procures to run-down the bed level by the water erosions and unrobes the footing of the structure [1]. Scour also define as a natural phenomenon that occur from the erosive impact of passing water, carving and carrying away materials from the banks and trend bed and from around the abutments and piers of bridges [2].

The sweep of sediments from around piers and abutments that confirming to bridge collapse and loss the life, merit of the structure. The footing of bridge structure should be given high importance in design and analysis in comparison with other parts of the structure, because the failure of footing would break down the all structure.

Scouring occurs naturally as a part of the morphologic changes of rivers and as a result of man-made structures. Generally, any events may cause the scour in the river bed. There are two main types of scour, the general scour and localized scour. Unlike general scour, the combined of local and contraction scour together called localized scour, due to the presence of the bridge. Other subdivisions types of scour are shown in Figure 1.
This type of scour is lowering the streambed across the stream caused by natural changes in the catchment or by human activities. General scour develops regardless of the presence of the bridge structure and being divided to short-term or long-term scour. These two types are distinguished by the time taken for the development of scour. Short-term type occurs during a single flood event or several consecutive floods. In contrast to short-term general scour, long-term type has an extremely longer time scale, normally within several years, and includes gradually degradation of the river bed and lateral bank erosion [3].

Contraction scour happens directly because of the existence of the bridge structure as shown in Figure 2 [4]. This type of scour places as a result of flow contraction due to the bridge or its road approaches exceed onto the floodplain of a river. The flow at a bridge usually shrinks within the bridge opening, and converges when approaching from the bridge, accelerating the narrowest section between the abutments and piers. This accelerated flow induces scour at the contracted section thus called the contraction scour.

![Figure 1. Classification of Scour [3].](image1)

![Figure 2. Features that create a contracted section in a channel [4].](image2)
2. Dimensional analysis

In this study use dimensional analysis technique to develop an empirical formula to calculate the scour depth at the different shape of piers. A dimensional analysis is made below for the parameters that affect the local scour mechanism. The parameters are classified in terms of mass (M), length (L) and time (T).

The classification of parameters that affect the local scour mechanism are:

1- Parameters characterizing the flow: $V_c=$Critical mean approach flow, $v=$Approach flow velocity, $y=$Approach flow depth, $\rho=$Density of the fluid, $\nu=$Kinematic viscosity of the fluid.

2- Parameters characterizing the flume: $B=$width of the channel, $S=$longitudinal slope of flume and neglect in this study because the flume has flats slope.

3- Parameters characterizing the pier: $L=$Pier length, $b=$Pier diameter or pier width, $K_s=$Pier shape factor, $\beta=$Angle of approach flow to the axis

4- Parameters characterizing the foundation: $L_f=$length of foundation, $b_f=$diameter or width of footing, $K_{sf}=$shape factor of foundation, $Z=$elevation of foundation with bed level.

5- Parameters characterizing the bed material: $d_{50}=$Median sediment size, $\rho_s=$Density of the sediment

The variables, which influence on mechanism of local scour are abstracted by this relationship:

$$F(y/b, v, b, y, \mu, g, \rho, K_s, z, \beta, B, L_f, \rho_s, d_{50}, v_c, b_f, L_f, L_f)=0$$

(1)

By using Buckingham $\pi$-theorem and after simplification of the formula and removal of the parameters with negligible and constant values, the following consideration were applied to Eq. (1):

1- term $y/b$ is ignored because it is constant for all tests
2- the term $\mu/\rho vb$ is ignored according to (Ettema et al., 1998) [5] the flow is fully turbulent around the pier.

3- All footings are used have the same length and projected width for flow, so the ratio $L_f/b$ has no influence on equilibrium of scour depth.

4- The ratio of flume width to maximum pier diameter is more than 6.25 to Shun the influence of the side wall on the local scour [6]; therefore, the term $B/b$ is neglected.

5- the term of relative density $\rho_s/\rho$ is not affected because the density of sediment and fluid is constant.

6- The pier diameter $b=45$ mm and the median particle size $d_{50}=0.71$ mm According to (Melville and Sutherland, 1988) [7], if $b/d_{50}>25$ the influence of this term is neglected , in this study $b=4$ cm and $d_{50}=0.4mm$ ;therefore , this term is ignored .

7- the term $b_f/b$ is dropped because its constant for all models.

8- the term $L/b$ is not affected because its constant for all shapes of pier in this study.

After limitation and omission process, the $\pi$-terms that governed the depth of scour around non-uniform pier can be written as below:

$$y_s/b=f(v/\sqrt{gb}, v_c/v, z/b, \mu, g, \rho_s, K_f)$$

(2)

The term $v/\sqrt{gb}$ is defined as Froude number of pier $Fr_p$.

The experimental run depends on the parameter in equation (2).

3. The experimental work

Tests were occurred in a 0.615m wide, 0.97 m height and 5.72 m long rectangular flume at the Hydraulic Laboratory, of university of Al-Basra college. Uniform sediment with a mean size diameter $d_{50}=0.4$ mm. All tests were carried out with condition of clear water, steady flow, subcritical flow and plain bed. Screens are located inlet of the flume in order to repel any trouble and smoother the flow to ban any undesired forms of bed (dune or ripple) at the working section.
The flow is controlled by regulating valve which open and close manually by hands. A sharp crested rectangular weir is used to measured discharge, its located at upstream of the flume. The flow depth is kept by an adjustable tail gate located at the downstream of the flume. A point gauge is used to measure flow depth which have an accuracy of (± 0.1 mm), the point gauge is ridded on a carriage which can be shifted to any portion above the flume by a pair of parallel rails reposed on walls of the flume. The test conditions for each shape of non-uniform piers are summarized in table 1.

### Table 1. Flow condition.

| Fr  | 0.431 | 0.383 | 0.335 | 0.288 | 0.256 |
|-----|-------|-------|-------|-------|-------|
| y(cm)| 3     | 3     | 3     | 3     | 3     |
| V (m/s)| 0.27  | 0.24  | 0.21  | 0.18  | 0.16  |
| Q (m³/s)| 0.0048 | 0.0043 | 0.0038 | 0.0033 | 0.0028 |

The piers and footing as shown in "figures 3 and 4" are manufactured of wood and greased to avoid swelling when they are put in the water for a long time. The ratio of flume width to maximum pier diameter is more than 6.25 to avoid the influence of the side wall on the local scour [6].

### Figure 3. Different shapes of piers.
Figure 4. different shapes of foundation.

4. Bed material
A uniform sand bed was used in all tests in laboratory with median grain sizes of (0.4mm), sieving analysis was taking out at soil laboratory in order to find the characteristics of the sand bed. The geometric standard deviation of sediment size equal to 1.292, which refer that the sand is of a uniform size distribution.

The ratio of pier width to grain size is more than 25, the influence of sediment size can be neglected [12].

The top level of foundation of non-uniform bridge pier was located at different location Z with respect to initial bed level.

In this study, the following three cases are considered,

Case 1. Top level of foundation is taken as 1cm above the initial bed level.
Case 2. Top of foundation is taken as at initial bed level.
Case 3. Top of foundation is taken as 1cm below initial bed level

5. Result and discussion
Development of New formula
As shown perversely, scour depth is a function of some variables. The dimensionless functional relationship of equation (2) should be progressed as an empirical formula by using a multi-regression analysis. The trust of proposed relationship found according to the coefficient of determination (R2).

The IBM SPSS Statistics v14 software is employed to make analysis for the equation through a nonlinear regression analysis. The following relationships are suggested to find the scour depth around non-uniform bridge piers.

\[ y_s/b = (c_1 \times (F_p)^{c_2} \times (K_y)^{c_3} \times (v/v_c)^{c_4}) + (c_5 \times (Z/b)^{c_6}) \]  

(3)

About 80% of data used to analysis and obtained this formula, 20% of remain data used to test equation.

5.1 Development of a New Formula for Rectangular footing
After analyzing the data in SPSS software, It is obtained on the following
So, the equation becomes:

\[ y_s/b = \{(209.903 \times (F_{rp})^{5.246} \times (K_s)^{1.96} \times (v/v_c)^{-3.682}) + (-1.686 \times (Z/b)^{1.64}\}\} \tag{4} \]

The coefficient of determination \((R^2)\) for this formula is 0.868.

Statistical compression of equation is used to show convergence of the predict and the observed value, as shown in Figure 5.

![Figure 5](image-url)

**Figure 5.** Calculated scour depth (equation 4 versus observed scour depth).

### 5.2 Development of a New Formula for oblong footing

After analyzing the data in SPSS software, it is obtained on the following cases:

\[ c_1 = 1425.063, c_2 = 7.287, c_3 = 2.136, c_4 = -5.431, c_5 = -0.847, c_6 = 1.786 \]

So, the equation becomes:

\[ y_s/b = \{(1425.063 \times (F_{rp})^{7.287} \times (K_s)^{2.136} \times (v/v_c)^{-5.431}) + (-0.847 \times (Z/b)^{1.786}\}\} \tag{5} \]

The coefficient of determination \((R^2)\) for this formula is 0.869.

Statistical compression of equation is used to show convergence of the predict and the observed value, as shown in Figure 6.
Figure 6. Calculated scour depth (equation 5 versus observed scour depth).

5.3 Development of a New Formula for hexagonal footing

After analyzing the data in SPSS software, it is obtained on the following

\[ c_1 = 2507.652, c_2 = 7.935, c_3 = 2.087, c_4 = -5.880, c_5 = -0.678, c_6 = 1.284 \]

So, the equation becomes:

\[ \frac{y_s}{b} = \left(2507.652 \times (R_{tp})^{7.935} \times (K_s)^{2.087} \times \left( \frac{v}{v_c} \right)^{5.88} \right) + \left(-0.678 \times \left( \frac{Z}{b} \right)^{1.284} \right) \tag{6} \]

The coefficient of determination \( R^2 \) for this formula is 0.891.

Statistical compression of equation is used to show convergence of the predict and the observed value as shown in Figure 7".
From equations, (4), (5), (6) found that the most influential parameters for each shape of foundation were Froude number, flow intensity, shape factor and foundation level position.

6. Conclusions
This study leads to find important parameter that influenced on local scour depth when used non-uniform piers and to develop an empirical formula to calculate the scour depth at the different shapes of non-uniform piers.

The position level of foundation relative to bed level has significant impact on scour depth around non-uniform piers, it was found that the reduction and limiting the scour due to the footing happen when it is located below the bed. The scour depth increases with footing level when the footing located above the initial bed level.

The best level of footing for various footing shapes is different. The result displays rectangular, oblong and hexagonal footing when located below the bed level gives a maximum reduction in scour depth equal to (47%, 49%, 56%) respectively as compared of the scour depth without footing scour depth.

Non-uniform pier shape has important effect on equilibrium depth of scour and initial scour rate. Rectangular pier with rectangular foundation has a maximum scour depth of (6.5) cm, it is higher than others shapes, while the scour depth for hexagonal pier with hexagonal foundation was (2) cm because it has a minimum exposed area. According to this study results show that the hexagonal pier shape with hexagonal foundation can be considered as the best shape of non-uniform piers where it reduces the maximum scour depth by 69% as compared with rectangular pier shape with rectangular foundation.

The best footing shape which gives minimum scour around it, was hexagonal shape, oblong, and rectangular shape, respectively.

7. Definition of symbols
\[ b = \text{pier width or pier diameter (L)} \]
\[ b_{f} = \text{diameter or width of footing (L)} \]
\[ Fr = \text{Froude number of pier} \]
\[ d_{50} = \text{median size of sand (L)} \]
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\( g = \text{Gravitation acceleration (L}T^{-2}\)  \\
\( K_F = \text{shape factor of foundation} \)  \\
\( K_d = \text{Pier shape factor} \)  \\
\( L = \text{Pier length (L)} \)  \\
\( L_f = \text{length of foundation (L)} \)  \\
\( Q = \text{Calculated discharge (L}^3/T\)  \\
\( U = \text{Approach flow velocity (L}T^{-1}\)  \\
\( y = \text{Approach flow depth (L)} \)  \\
\( y_s = \text{Scour depth (L)} \)  \\
\( Z = \text{Top elevation of foundation measured from initial bed level (L)} \)  \\
\( \beta = \text{Angle of approach flow to the axis} \)  \\
\( \mu = \text{Dynamic viscosity of water (MLT}^{-2}\)  \\
\( \rho = \text{Mass density of water (M/L}^3\)  \\
\( \rho_s = \text{Mass density of water (ML}^{-3}\)  

8. References

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