Goldstino Decoupling in Spontaneously Broken Supergravity Theories

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Abstract

If the gravitino is sufficiently light and stable it will behave as an effective massless neutrino species at the time of nucleosynthesis. Depending on the temperature at which it decouples from the thermal bath in the early universe, the gravitino mass will be bounded by the primordial $^4\text{He}$ abundance. Assuming a conservative estimate that the number of neutrino families, $N_{\nu} < 3.6$, superlight gravitinos with a mass $m_{3/2} \lesssim 10^{-6}\text{eV}$ are ruled out. This bound is weaker than previous estimates because the Goldstino annihilation cross section was overestimated.

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I. INTRODUCTION

If supersymmetry is to describe the real world then it must be broken at some scale \( \Lambda \) and give rise to supersymmetry breaking mass splittings not greater than \( \tilde{m} \sim \mathcal{O}(\text{TeV}) \). The signal for local supersymmetry breaking occurs when the superpartner of the graviton, the gravitino, receives a mass via the superHiggs mechanism. There is an upper bound of \( \mathcal{O}(\text{TeV}) \) on the gravitino mass which comes from requiring that supersymmetry provides the solution to the naturalness problem. However, the gravitino mass is not necessarily constrained to be \( \mathcal{O}(\text{TeV}) \) because the gravitino only interacts gravitationally and so is weakly coupled (compared to the gauge forces). This is not the case for the superpartners of the particles in the standard model which experience gauge forces. Their soft masses are constrained from experiment to be in the range \( \mathcal{O}(10^2 - 10^3) \text{GeV} \).

The relationship between the gravitino mass and the soft masses depends on the messenger sector. The messenger sector is responsible for communicating the spontaneous breakdown of supersymmetry from a hidden sector, where the supersymmetry breaking dynamics occurs, to the observable sector. Normally in hidden sector scenarios of spontaneous supersymmetry breaking, the messenger sector is gravitational and gives rise to a gravitino mass \( m_{3/2} \approx \tilde{m} \). A light gravitino is precluded because it would also mean light gauginos, sleptons and squarks. This typically assumes generic choices of the Kahler potential, \( \mathcal{G}(z, z^*) \) and the gauge kinetic function \( f_{ab}(z) \) where \( z \) is the hidden sector chiral superfield. However, in no-scale supergravity models particular choices of \( \mathcal{G}(z, z^*) \) and \( f_{ab}(z) \) make it possible for the gravitino mass \( m_{3/2} \ll \tilde{m} \).

A hierarchy between \( m_{3/2} \) and \( \tilde{m} \) is also possible if the messenger sector responsible for communicating supersymmetry breaking to the visible sector is not gravitational. Recently there has been a renewed interest in dynamical supersymmetry breaking models \( [2–4] \) where gauge forces communicate the breakdown of supersymmetry to the squarks, sleptons and gauginos. In these models the soft masses \( (\tilde{m}) \) are generated by radiative corrections and are proportional to the gauge couplings squared. The gauge particles will still receive soft masses from gravitational interactions, but these contributions are much smaller than the contributions from the gauge-mediated messenger sector. Since the gravitinos do not carry gauge quantum numbers they do not receive any mass from the gauge messenger sector but still receive mass from gravitational interactions. Consequently, in these models the gravitino is naturally light \( (\ll \mathcal{O}(\text{TeV})) \) and becomes the lightest supersymmetric particle (LSP). This can have interesting cosmological implications, such as contributing to the dark matter component of the universe \( [5] \).

Given the possibility of non-minimal kinetic terms or a non-gravitational messenger sector one can ask how light can the gravitino mass be? A naive estimate of this mass comes from assuming that the supersymmetry breaking scale \( \Lambda \sim 10^2 \text{GeV} \), which gives \( m_{3/2} \sim \Lambda^2/M_{Pl} \sim 10^{-6} \text{eV} \) where \( M_{Pl} \) is the Planck mass. If the gravitino were this light then it would essentially behave as a massless neutrino species during nucleosynthesis. This is because at energies \( E \gg m_{3/2} \) the longitudinal component of the gravitino dominates during interactions, which is just a statement of the Equivalence Theorem \( [6] \). The longitudinal components of the gravitino are the helicity \( \pm 1/2 \) modes which come from absorbing the spin-1/2 Goldstone fermion (or Goldstino) during the spontaneous breakdown of supersymmetry. The Goldstino component of the gravitino couples with a strength proportional to \( \kappa/m_{3/2} \),
where $\kappa = 1/M_{pl}$. If the gravitino mass is light this coupling can be much stronger than naively expected from the gravitational force. This means that as the temperature of the universe cools, the Goldstino will remain in thermal contact with the heat bath longer because of its enhanced coupling. Since the expansion rate of the universe depends on the number of degrees of freedom, the universe will expand faster with Goldstinos present at temperatures $T \lesssim \mathcal{O}(100 \text{ MeV})$ and cause the neutrinos to decouple earlier. This will affect the production of neutrons (via the charged current weak interactions) by causing more neutrons to survive and ultimately increases the primordial $^4\text{He}$ abundance. Thus the observed primordial $^4\text{He}$ abundance sets a lower bound on the gravitino mass, which in turn is related to how late the Goldstino decouples from the thermal bath.

A lower bound on the gravitino mass based on the nucleosynthesis argument was originally discussed by Fayet [7], who considered the effect of the Goldstino together with a light photino on the effective number of massless neutrino species. The bound derived by Fayet ($m_{3/2} \gtrsim 10^{-2} \text{eV}$) is not effective anymore because the photino is now believed to be of order the electroweak scale and the gravitino (if it is light enough) can only thermally interact with leptons and photons below the quark-hadron phase transition. More recently Moroi et al [8] have also considered this bound and obtain an estimate $m_{3/2} \gtrsim 10^{-4} \text{eV}$. However the cross section used by them to recalculate the Fayet bound is only valid for $T \gg \tilde{m}, m_{3/2}$. In this work we will calculate the Goldstino decoupling temperature using the cross section for Goldstino annihilation in the limit $\tilde{m} \gg T \gg m_{3/2}$, where we assume that the supersymmetry breaking mechanism gives rise to a superlight gravitino. The gravitino mass bound will be shown to be much weaker than has previously been estimated.

The outline of this paper is as follows. We begin in Sec.2 with a discussion of the effective interaction Lagrangian for the Goldstino. This Lagrangian is used to calculate the Goldstino annihilation cross section into leptons and photons. In Sec.3 the thermally averaged Goldstino annihilation rate is obtained. This will be used to calculate the Goldstino decoupling temperature in Sec.4. A lower bound on the gravitino mass and the low energy supersymmetry breaking scale will be derived from the Goldstino contribution to the effective number of massless neutrino species. Final comments and the conclusion will be presented in Sec.5.

II. EFFECTIVE GOLDSTINO LAGRANGIAN

The effective Lagrangian for the gravitino is obtained from the N=1 supergravity theory [4], irrespective of the mechanism employed in the messenger sector to communicate supersymmetry breaking to the observable sector. Denoting the chiral matter supermultiplets by $(\phi, \chi)$ and the gauge vector supermultiplets by $(\lambda, A_\mu)$ the supergravity Lagrangian for the interaction of the gravitino with the vector and chiral multiplets is given by

$$\mathcal{L} = \frac{\kappa}{4} \lambda^{\mu}{\phi^\alpha}\Phi_\mu F_{\alpha\beta} + \frac{\kappa}{4} h_{\mu\nu} \left[ \eta^{\alpha\beta}(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho}) F_{\rho\alpha} F_{\sigma\beta} - \frac{1}{2} \eta^{\mu\nu} F_{\alpha\beta} F_{\alpha\beta} \right]$$

$$+ \left[ \frac{\kappa}{\sqrt{2}} \bar{\Omega}_{\mu} \gamma^\mu \eta^\phi_i \psi_i D_\mu \phi^i - \frac{\kappa}{16} \bar{\Omega}_{\mu} \gamma^\mu \eta^\phi_i \psi_i D_\mu \phi^i + \bar{\Omega}_{\mu} \gamma^\mu \eta^\phi_i \psi_i D_\mu \phi^i \right]$$

$$+ i \frac{\kappa}{8} h_{\mu\nu} \left[ \bar{\psi}_i (\gamma^\mu \partial^\mu + \gamma^\mu \partial^\nu) \psi_i - (\partial^\mu \bar{\psi}_i \gamma^\mu + \partial^\nu \bar{\psi}_i \gamma^\mu) \psi_i \right]$$

3
The symmetric tensor for an on-shell gravitino \( \gamma \) limit. If we substitute (3) into the supergravity Lagrangian (1) and make use of the fact that the standard electroweak model behaving as Nambu-Goldstone bosons in the high energy

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Fig. 1. The first contribution is due to the

t-

channel exchange of the photino, where

\( \langle f_{ab} \rangle = \langle f \rangle \delta_{ab} \) and we have adopted the normalising conventions used in Ref. [10]. In the Lagrangian (1) we have neglected all terms which are not relevant for the calculation of the gravitino annihilation cross section.

The Lagrangian (1) may be used to calculate scattering processes involving all the helicity components of the massive gravitino. However, in the limit that the energy scale of the gravitino \( E \gg m_{3/2} \), the longitudinal component of the gravitino will dominate and the gravitino effectively behaves as a spin-1/2 Goldstino

where \( m_{\tilde{g}} \) is the gaugino mass, \( \langle f_{ab} \rangle = \langle f \rangle \delta_{ab} \) and we have adopted the normalising conventions used in Ref. [10]. In the Lagrangian (1) we have neglected all terms which are not relevant for the calculation of the gravitino annihilation cross section.

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where \( \tilde{G} \) denotes the Goldstino and the factor \( \sqrt{3} \) is a Clebsch-Gordon coefficient. This is just a statement of the Equivalence Theorem and is analogous to longitudinal W bosons in the standard electroweak model behaving as Nambu-Goldstone bosons in the high energy limit. If we substitute (3) into the supergravity Lagrangian (1) and make use of the fact that for an on-shell gravitino \( \gamma^\mu \Psi_\mu = 0 \) then we obtain an effective Lagrangian for the interaction of the Goldstino with gauge and matter multiplets.

The annihilation channels that will interest us are Goldstino scattering to leptons and photons. We will assume that all other particles have decoupled and are not important for the calculation of the Goldstino decoupling temperature. Consider first Goldstino annihilation into photons. There are three different contributions to this amplitude which are depicted in Fig. 1. The first contribution is due to the \( t \) and \( u \)-channel exchange of the photon, where for simplicity we ignore neutralino mixing. In addition there are \( s \)-channel annihilation diagrams which result from graviton exchange and hidden sector scalar exchange. These diagrams are important for the cancellation of the leading order energy dependence in the Equivalence Theorem limit [11]. The Lagrangian relevant for these processes is given by

\[
\mathcal{L} = \frac{1}{2 \sqrt{6} m_{3/2}} \partial_\mu \tilde{\lambda} \gamma^\mu [\gamma^\nu \gamma^\rho] \tilde{G} \partial_\nu A_\rho + \frac{\kappa}{4} h_{\mu\nu} \left[ \eta^{\alpha\beta} (\eta^{\rho\sigma} \eta^{\nu} + \eta^{\mu\sigma} \eta^{\rho}) \right] F_{\rho\alpha} F_{\sigma\beta} - \frac{1}{2} \eta^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}
\]
FIG. 1. Feynman diagrams for the scattering process $\tilde{G}\tilde{G} \rightarrow \gamma\gamma$.

\begin{equation}
- \frac{1}{6} \frac{\kappa}{m_{3/2}} (\eta^{\mu\nu}\eta^{\rho\sigma} + \eta^{\lambda\nu}\eta^{\rho\mu} - \eta^{\mu\nu}\eta^{\lambda\rho}) \partial_\mu \overline{G} \partial_\nu \tilde{G} h_{\lambda\rho} \\
+ \frac{i}{6} \frac{\kappa}{m_{3/2}^2} \epsilon^{\mu\nu\rho\sigma} \partial_\mu \overline{\gamma_5} \gamma_5 \left[ \gamma_\sigma, \gamma_\rho \right]_+ \partial_\nu \tilde{G} \partial_\tau h_{\lambda\rho} \\
+ \frac{\kappa}{2} \sum_i c_i (\partial_\nu A_\mu \partial^\nu A^\mu - \partial^\nu A_\mu \partial^\mu A_\nu) S_i + \frac{\kappa}{3 m_{3/2}} \sum_i d_i \partial_\mu \overline{G} \partial^\mu \gamma_5 \tilde{G} S_i \\
+ \frac{\kappa}{2} \sum_i c_i \epsilon^{\mu\nu\rho\sigma} \partial_\mu A_\nu \partial_\rho A_\sigma P_i - \frac{1}{6} \frac{\kappa}{m_{3/2}^2} \sum_i d_i \partial_\mu \overline{\gamma_5} \gamma_5 \partial^\mu \gamma_5 \tilde{G} \partial_\nu P_i.
\end{equation}

Assuming that $\sqrt{s} \gg m_{3/2}$ the Goldstino annihilation cross section into photons is calculated using the above Lagrangian (4) to be

\begin{equation}
\sigma (\tilde{G}\tilde{G} \rightarrow \gamma\gamma) = \frac{1}{1728 \pi} \frac{\kappa^4}{m_{3/2}^4} m_\gamma^4 s \left[ 1 + \frac{6x(1 - 2x - 4x^2)}{1 + x} + \frac{6x(-x + 4x^2 + 8x^3)}{1 + 2x} \log(1 + \frac{1}{x}) \right].
\end{equation}

where $x = m_\gamma^2/s$. This agrees with the result quoted in Ref. [11]. In the limit that $\sqrt{s} \gg m_\gamma$ the cross section is in agreement with the Equivalence Theorem, namely

\begin{equation}
\sigma (\tilde{G}\tilde{G} \rightarrow \gamma\gamma) \simeq \frac{1}{1728 \pi} \frac{\kappa^4}{m_{3/2}^4} m_\gamma^4 s \simeq f^4 \frac{\kappa^4 \sqrt{s}}{1728 \pi}
\end{equation}

where $f$ generically denotes the photino "Yukawa" coupling. On the other hand in the intermediate limit $m_\gamma \gg \sqrt{s}$ the cross section (3) becomes
FIG. 2. Comparison of the various limits for the cross section $\sigma(\tilde{G}\tilde{G} \rightarrow \gamma\gamma)$ where only the quantity in square brackets ($\equiv \sigma_x$) in Eq. (5) is plotted. The solid curve represents the exact expression for $\sigma_x$, while the dot-dashed line depicts $\sigma_x$ in the Equivalence Theorem limit ($\sqrt{s} \gg \tilde{m}$) and the dashed line represents $\sigma_x$ in the intermediate limit ($\tilde{m} \gg \sqrt{s}$).

$$\sigma(\tilde{G}\tilde{G} \rightarrow \gamma\gamma) = \frac{1}{576\pi} \frac{\kappa^4}{m_{3/2}^4} \frac{\tilde{m}^2}{m_{3/2}} \frac{s^2}{m_{\tilde{g}}}$$  \hspace{1cm} (7)

which has a different dependence on the energy scale $\sqrt{s}$ than in (6). This will be the limit that is needed to accurately calculate the Goldstino decoupling temperature. In Fig. 2 these various limits are compared with the exact result. It is clear from the figure that the cross section (6) is a poor approximation in the limit $m_{\tilde{g}} \gg \sqrt{s}$.

Notice also that the cross section (6) is proportional to $E^2$. This is due to the non-renormalisibility of the supergravity Lagrangian and can lead to unitarity violation above a critical energy [10],

$$E_{cr} = \sqrt{\frac{288\pi m_{3/2}}{\kappa m_{\tilde{g}}}}.$$  \hspace{1cm} (8)

If the critical energy is taken to be the Planck scale, then it is difficult to reconcile a light gravitino mass with (8). However it was pointed out in Ref. [12] that one can interpret the critical energy as the scale of new physics at which the gaugino mass, $m_{\tilde{g}}$ becomes effective. This scale need not necessarily be anywhere near the Planck scale. For a gravitino mass $m_{3/2} \sim \mathcal{O}(10^{-6}\text{eV})$ the critical energy turns out to be $E_{cr} \sim 3\text{TeV}$. This unitarity
FIG. 3. Feynman diagrams for the scattering process \( \tilde{G}\tilde{G} \to f\bar{f} \).

The limit is safely above the energy scales that we will be concerned with, which are below the quark-hadron phase transition (\( \sim \mathcal{O}(100\,\text{MeV}) \)).

The Goldstino annihilation amplitude into fermions is similarly obtained from the following Lagrangian:

\[
\mathcal{L} = i \frac{\kappa^2}{\sqrt{3} m_{3/2}} (\partial_\mu \tilde{G} f_L \partial^\mu \tilde{f}_L - \tilde{f}_L \partial_\mu \tilde{G} \tilde{f}^\mu L + \tilde{f}_R \partial_\mu \tilde{G} \tilde{f}^\mu R - \partial_\mu \tilde{G} f_R \partial^\mu \tilde{f}_R) \\
+ \frac{\kappa}{6 m_{3/2}} \tilde{f} \gamma_5 \gamma_\alpha f \partial^\alpha \tilde{G} \gamma_5 \gamma^\beta \partial_\beta \tilde{G} + i \frac{\kappa}{4} [\tilde{f} (\gamma^\nu \partial^\mu + \gamma^\mu \partial^\nu) f - (\partial^\mu \tilde{f} \gamma^\nu + \partial^\nu \tilde{f} \gamma^\mu) f] h_{\mu\nu} \\
- \frac{\kappa}{6 m_{3/2}} (\eta^{\lambda\mu} \eta^{\rho\nu} + \eta^{\lambda\nu} \eta^{\rho\mu} - \eta^{\mu\nu} \eta^{\lambda\rho}) \partial_\mu \tilde{G} \partial_\nu \tilde{G} h_{\lambda\rho} \\
+ \frac{i \kappa}{6 m_{3/2}} \epsilon^{\mu\nu\lambda(\gamma_5 [\gamma_\alpha, \sigma^\rho)]}_+ \partial_\mu \tilde{G} \partial_\nu \tilde{G} \partial_\lambda \tilde{G} \\
- \frac{\kappa}{2} \sum_i \tilde{d}_i \tilde{f} \gamma_\mu f \partial_\mu p_i - \frac{\kappa}{6 m_{3/2}^2} \sum_i \tilde{d}_i \partial_\mu \tilde{G} \gamma_5 \gamma_\beta \partial^\mu \tilde{G} \partial_\beta p_i (9)
\]

where \( f \) denotes the fermion Dirac spinor. Note that due to gauge invariance there is no fermion coupling to the scalar \( S_i \). The relevant Feynman diagrams are depicted in Fig. 3. Besides the sfermion, graviton and hidden sector scalar exchange diagrams, there is also a four-Fermi interaction term due to the nonrenormalisability of the supergravity Lagrangian. Assuming that the fermions are relativistic the cross section is calculated to be

\[
\sigma(\tilde{G}\tilde{G} \to f\bar{f}) = \frac{1}{108\pi m_{3/2}^4 s} \left[ 1 + \frac{3x(-1 + 2x + 4x^2)}{1 + x} - 12x^3 \log(1 + \frac{1}{x}) \right] (10)
\]
FIG. 4. Comparison of the various limits for the cross section $\sigma(\tilde{G}\tilde{G} \to f\bar{f})$ where only the quantity in square brackets ($\equiv \sigma_x$) in Eq. (10) is plotted. The solid curve represents the exact expression for $\sigma_x$, while the dot-dashed line depicts $\sigma_x$ in the Equivalence Theorem limit ($\sqrt{s} \gg \tilde{m}$) and the dashed line represents $\sigma_x$ in the intermediate limit ($\tilde{m} \gg \sqrt{s}$).

where $x = m_f^2/s$ and the contribution from the pseudoscalar $P_i$ diagram turns out to be zero. In the limit that $\sqrt{s} \gg m_f$ one can easily check that (11) agrees with the Equivalence Theorem. If instead one considers the intermediate limit $m_f \gg \sqrt{s}$ then the annihilation cross section (10) becomes

$$\sigma(\tilde{G}\tilde{G} \to f\bar{f}) = \frac{1}{180\pi} \frac{\kappa^4}{m_f^{5/2}} s^3$$

where again the energy dependence is different than that in the high energy limit $\sqrt{s} \gg m_f$. The various limiting behaviours are shown in Fig. 4. One can clearly see in the figure that the Equivalence Theorem limit is not a good approximation when $\sqrt{s} \ll \tilde{m}$ and infact the cross section is considerably smaller.

To obtain the cross section for Goldstino decay into massless neutrinos one simply neglects the right handed components in the Lagrangian (9). In this case one finds

$$\sigma(\tilde{G}\tilde{G} \to \nu_L\bar{\nu}_L) = \frac{1}{2} \sigma(\tilde{G}\tilde{G} \to f\bar{f}).$$

The total Goldstino annihilation cross section is given by the sum of (10) and (11) and will be used to calculate the thermally averaged annihilation rate in the early universe.
III. THERMALLY AVERAGED ANNIHILATION RATE

The annihilation cross sections obtained in the previous section were calculated at a temperature \( T = 0 \). In the early universe we need to average over the statistical distributions of the colliding particles in the thermal heat bath. The thermally averaged annihilation cross section times velocity for the scattering process \( 1 + 2 \rightarrow F \) is given by

\[
\langle \sigma v_{\text{Mol}} \rangle = \frac{\int \frac{d\sigma}{dn_1^{eq}dn_2^{eq}} \sigma v_{\text{Mol}}}{\int \frac{d\sigma}{dn_1^{eq}dn_2^{eq}}}
\]

where

\[
dn_i^{eq} = f(E_i, t) g_i \frac{d^3 p_i}{(2\pi)^3}
\]

and \( f(E_i, t) \) is the statistical distribution function and \( g_i \) is the number of internal degrees of freedom for the particle species \( i \). The factor \( v_{\text{Mol}} = \sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2 / (E_1 E_2)} \) is known as the Møller velocity (see Ref. [13]) and \( \sigma \) is the sum over all possible annihilation channels of particles 1 and 2. We have neglected the Pauli blocking factors \( 1 - f \) for final state particles and antiparticles in (13), which typically amounts to a 10% correction in the determination of the decoupling temperature [14].

In the case of Goldstino annihilation we are assuming that \( E \gg m_{3/2} \) so that the Goldstinos will be relativistic. Note that for Goldstinos which obey Fermi-Dirac statistics, \( f(E, t) = 1/(e^{E/T} + 1) \) the equilibrium number density is

\[
n_{\tilde{G}} = \frac{3^{2/3}}{2^{2/3} \zeta(3)} T^3
\]

where \( \zeta(3) \approx 1.202 \). In the early universe we will be interested in the role of the Goldstino at temperatures \( T \sim O(100 \text{ MeV}) \). At these times \( \sqrt{s} \ll \bar{m} \) and in fact \( x = 10^6 \) for \( \bar{m} \sim O(100 \text{ GeV}) \). If we parametrise the annihilation cross section as \( \sigma = \sum_i \tilde{\sigma}_i s^{ni} \) where \( s \) is the Mandelstam variable and \( \tilde{\sigma}_i \) is a constant, then the thermally averaged annihilation cross section times velocity for the Goldstino in the limit \( T \ll \bar{m} \) is given by

\[
\langle \sigma v_{\text{Mol}} \rangle = \frac{1}{9 \zeta(3)^2} \sum_i \frac{2^{2n_i+3}}{(n_i + 2)^2} I(n_i)^2 \tilde{\sigma}_i T^{2n_i}
\]

where \( I(n_i) = \int_0^\infty dy y^{n_i+2} / (e^y + 1) \), \( n_i \) are integers and the sum is over all annihilation channels.

Using the expressions (7) and (11) for the Goldstino annihilation cross sections derived in the previous section we obtain

\[
\sigma_A = \sigma(\tilde{G}\tilde{G} \rightarrow \gamma\gamma) + \sum_f \sigma(\tilde{G}\tilde{G} \rightarrow f\bar{f}) = \tilde{\sigma}_{\gamma\gamma} s^2 + \sum_f \tilde{\sigma}_{ff} s^3,
\]

where we have summed over all possible fermion pairs in the final state. Assuming a photino mass \( m_{\tilde{\gamma}} \sim O(100 \text{ GeV}) \), the dominant part of the total cross section actually comes from the Goldstino annihilation into photons so that the thermally averaged cross section times velocity is approximately
\langle \sigma_A v_{M\text{ol}} \rangle \simeq 1800 \frac{\zeta(5)^2}{\zeta(3)^2} \hat{\sigma}_{\gamma\gamma} T^4, \quad (18)

where \( \zeta(5) \simeq 1.037 \). The average Goldstino annihilation rate which is defined to be \( \Gamma_A = n_{G} \langle \sigma_A v_{M\text{ol}} \rangle \) is then given by

\[ \Gamma_A \simeq \frac{75}{16\pi^3} \frac{\zeta(5)^2}{\zeta(3)} \frac{\kappa^4}{m_{3/2}^4} m_\gamma^2 T^7 \simeq 0.135 \frac{\kappa^4}{m_{3/2}^4} m_\gamma^2 T^7 \quad (19) \]

where \( \hat{\sigma}_{\gamma\gamma} \) is determined from Eq. (7). This rate will be used to calculate the Goldstino decoupling temperature.

**IV. GOLDSTINO DECOUPLING TEMPERATURE**

During the radiation dominated era of the universe, the energy density which is dominated by relativistic particles, can be expressed in terms of the photon energy density \( \rho_\gamma(T) \) as

\[ \rho(T) = \frac{1}{2} g_\rho(T) \rho_\gamma(T) = g_\rho(T) \frac{\pi^2}{30} T^4, \quad (20) \]

where the effective number of relativistic degrees of freedom of the bosons \((B)\) and fermions \((F)\) present is

\[ g_\rho(T) = \sum_B g_B \left( \frac{T_B}{T} \right)^4 + \frac{7}{8} \sum_F g_F \left( \frac{T_F}{T} \right)^4. \quad (21) \]

In Eq. (21) \( g_B(g_F) \) is the number of internal degrees of freedom for each boson (fermion) and \( T_{B,F} \) represent the possibility of the decoupled particles having a temperature which differs from the photon temperature, \( T \). The Hubble expansion rate during this era will be

\[ H = \sqrt{\frac{4\pi^3}{45} g_\rho^{1/2} \kappa T^2}. \quad (22) \]

The Goldstinos thermally decouple from the heat bath when their annihilation rate \( \Gamma_A \lesssim H \). Using (19) and (22) one finds that the Goldstino decoupling temperature is

\[ T_D \simeq \frac{5}{3} g_\rho^{1/10} m_{3/2}^{4/5} m_\gamma^{-2/5} \kappa^{-3/5}. \quad (23) \]

This equation shows that as the gravitino mass becomes lighter, it causes the Goldstino to decouple later in the evolution of the universe. The later the Goldstino decouples, the greater the possibility of the Goldstino interfering with nucleosynthesis. Clearly there exists a lower bound on the gravitino mass for which the predictions of nucleosynthesis are not affected.

During nucleosynthesis the energy density of new massless particles, \( i \) is equivalent to an effective number \( \Delta N_\nu \) of additional doublet neutrinos:
\[
\Delta N_\nu = f_{B,F} \sum_i \frac{g_i}{2} \left[ \frac{g_\rho(T_\nu)}{g_\rho(T_{D,i})} \right]^{4/3}
\]

(24)

where \( f_B = 8/7 \) for bosons, \( f_F = 1 \) for fermions and \( g_i \) is the number of internal degrees of freedom of the particle species \( i \) [14]. At neutrino decoupling the only known particles which can contribute to \( g_\rho \) are \( \gamma, e^\pm, \nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \) and \( \bar{\nu}_\tau \). This means that the effective number of degrees of freedom at neutrino decoupling are, using Eq. (21)

\[
g_\rho(T_\nu) = 2 + \frac{7}{8}(4 + 3 \times 2) = \frac{43}{4}.
\]

(25)

Assuming the conservative estimate that \( \Delta N_\nu < 0.6 \) [16], places a lower bound on the Goldstino decoupling temperature, \( T_D \). For example, if \( T_D = T_\nu \simeq \mathcal{O}(\text{MeV}) \) then according to (24), \( \Delta N_\nu = 1 \) and the Goldstino would behave like an extra neutrino family. This would mean that \( T_D > T_\nu \). A stronger bound can be obtained by supposing that the Goldstino decouples during the temperature range \( T_\nu < T_D < T_\mu \), where \( T_\mu \) is the muon decoupling temperature. When the Goldstino decouples the effective number of degrees of freedom would be

\[
g_\rho(T_D) = 2 + \frac{7}{8}(4 + 3 \times 2 + 2) = \frac{25}{2}.
\]

(26)

Using Eq. (24) this will contribute an amount \( \Delta N_\nu = 0.82 \), which means that the Goldstino decoupling temperature \( T_D > T_\mu \simeq \mathcal{O}(100\text{MeV}) \). Imposing this condition on the expression (23) leads to the lower bound on the gravitino mass

\[
m_{3/2} \gtrsim 10^{-6}\text{eV} \left( \frac{m_\tilde{\gamma}}{100\text{GeV}} \right)^{1/2}.
\]

(27)

This bound is much weaker than that quoted by Moroi et al [8] because the Goldstino annihilation cross section is not as large as assumed by those authors.

The mass bound (27) may be strengthened slightly by supposing that muons are also in thermal equilibrium when the Goldstino decouples. In this case one obtains \( g_\rho(T_D) = 16 \) and \( \Delta N_\nu = 0.59 \). Assuming that the primordial \(^4\text{He} \) abundance rules this contribution out as well, causes the the lower bound on the Goldstino decoupling temperature to increase up to the pion mass \( T \sim m_\pi \) and the lower bound (27) to increase slightly. Clearly, the higher the Goldstino decoupling temperature becomes the less it will contribute to \( N_\nu \) as many more particles and resonances contribute to \( g_\rho(T_D) \). In order to significantly increase the bound in future a more accurate estimate of \( \Delta N_\nu \) would be needed.

Of course it is likely that there are other beyond the standard model particles which could contribute to \( \Delta N_\nu \). In this case it may be more difficult to accommodate the Goldstino, which would cause the bound (27) to increase further. This could happen for example if neutrinos have Dirac masses. It is known that neutrino Dirac masses can contribute a large amount to \( \Delta N_\nu \) [17]. Accounting for the Goldstino and neutrino masses could place further constraints on the mass parameters.
V. CONCLUSION

If a spontaneously broken N=1 supergravity theory produces a sufficiently light gravitino then it will have consequences during the nucleosynthesis era of the early universe. This may be possible in no-scale supergravity theories or in theories of low energy dynamical supersymmetry breaking with a gauge-mediated messenger sector. The gravitino will obtain a mass via the superHiggs effect by absorbing the Goldstino. The enhanced coupling of the Goldstino causes it to interact more strongly with chiral and vector supermultiplets, which means that the Goldstino can decouple just prior to the nucleosynthesis era.

The primordial $^4$He abundance critically depends on the number of massless neutrino families. If we require that the Goldstino not contribute significantly to the number of massless neutrino families, a lower bound on the gravitino mass can be obtained. Previous lower bounds have ranged from $10^{-4} - 10^{-2}$eV \cite{7,8}. By calculating the Goldstino annihilation cross section into leptons and photons in the limit $\sqrt{s} \ll \tilde{m}$ we were able to show that this bound is considerably weaker than previous estimates. If $N_\nu < 3.6$ then typically $m_{3/2} \gtrsim 10^{-6}$eV for $m_\gamma \simeq \mathcal{O}(100$ GeV). This bound complements previous gravitino mass bounds derived from collider experiments \cite{18,19} and astrophysics \cite{20}.

In addition a bound on the supersymmetry breaking scale, $\Lambda$ can also be obtained for various scenarios of supersymmetry breaking dynamics. Assuming that $m_{3/2} \simeq \Lambda^2/M_{pl}$, the bound on the gravitino mass implies that the scale of supersymmetry breaking, $\Lambda \gtrsim 100$GeV. In particular this would set a lower bound (100 GeV) on the scale of the supercolour sector in the recent gauge mediated models. These bounds are right at the forefront of existing collider energy scales.

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