Nearly Degenerate dS Horizons from a 2-D Perspective

A.J.M. Medved

Department of Physics and Theoretical Physics Institute
University of Alberta
Edmonton, Canada T6G-2J1
E-Mail: amedved@phys.ualberta.ca

Abstract

Some recent studies have implied that quantum fluctuations will prevent a near-extremal black hole from ever attaining a state of precise extremality. In this paper, we consider the analogous situation for the scenario of a nearly degenerate Schwarzschild-de Sitter black hole (of arbitrary dimensionality). For this purpose, we utilize a holographic type of duality between the solutions of interest and the near-massless sector of (two-dimensional) Jackiw-Teitelboim theory. After explicitly demonstrating this duality, we go on to argue that, on the basis of one-loop considerations, a similar censorship applies in this de Sitter context. That is, quantum back-reaction effects will conspire to prohibit a non-degenerate Schwarzschild-de Sitter spacetime from continuously evolving into a degenerate (i.e., Nariai) black hole solution.

I. INTRODUCTION

From a theoretical perspective, black holes provide us with an intriguing “laboratory” for examining many aspects of quantum and semi-classical gravity. With regard to such investigations, the thermodynamic properties associated with black hole horizons have played an especially prominent role. In particular, black holes are commonly attributed a temperature that is proportional to the horizon surface gravity and an entropy that is proportional to the horizon surface area [1]. The initial motivation for these identifications was an observed analogy between black hole horizon mechanics and the thermodynamic first and second laws [2,3]. Nonetheless, this thermodynamic correspondence is generally given a physical status on the basis of Hawking radiation [4]. That is, it has been shown that, when quantum effects are considered, black holes emit thermal radiation and, remarkably, at the exact value of temperature that the analogy suggests.

On the other hand, in a statistical mechanical sense, there are still conceptual difficulties with this thermodynamic picture. For instance, “what is the microscopic origin of the black hole entropy?” has become a popular slogan (paraphrasingly speaking) among gravitational researchers [5]. However, putting this controversial issue aside, one can see that the entropy (and not only the temperature) does, indeed, have a clear physical interpretation. To elaborate, the black hole horizon is essentially a membrane that obstructs the flow of information.
That is, an external observer will be unable to access any degrees of freedom from behind the horizon, and just such an obstruction will naturally induce, via coarse-graining effects, an entropy of entanglement [6]. Indeed, direct calculations of this entanglement entropy (e.g., [7]) have been known to reproduce the Bekenstein-Hawking entropic area law [2,3]. (Albeit, up to some ambiguity in the proportionality factor. However, it is expected that the correct factor will be reproduced once renormalization effects have properly been accounted for [8].)

Moreover, there is substantial evidence that any horizon (i.e., any null hypersurface that prevents the accessibility of information) can be attributed a temperature and entropy in the same manner as for a black hole (e.g., [9]). Although intuitively sensible, this may not be a priori clear for horizons that are observer dependent; most notably, Rindler horizons (which are manifestations of an accelerating observer [10]) and de Sitter cosmological horizons (which are defined in terms of a given observer’s causal patch of de Sitter space [11]). On the other hand, the principle of black hole complementarity [12] implies that the existence (or lack thereof) of a black hole horizon is also an objective feature of the spacetime; for instance, a free-falling observer will detect no horizon at all. Hence, observer dependence should not, after further consideration, be that significant of an issue.

Furthermore, our understanding of horizon thermodynamics is expected to persist in spacetimes with multiple horizons (e.g., charged and/or spinning black holes and Schwarzschild-de Sitter spacetimes), provided that each horizon is treated locally [9] or as a distinct physical system [13]. There is, however, a very important caveat to this statement when two (or more) otherwise distinct horizons become degenerate. For such an occurrence, the thermodynamic properties typically become ambiguous and sometimes even ill-defined (for further discussion and references, see [15]).

This formal breakdown in degenerate-horizon thermodynamics brings us to the problem of how to interpret the degenerate limit of relevant spacetimes; most notably, the extremal limit of a charged and/or spinning black hole and the Nariai limit [16] of a Schwarzschild-de Sitter spacetime. (Since a degenerate limit naively coincides with a limit of vanishing temperature, this problem can alternatively be viewed as an ambiguity in formulating the third law of horizon thermodynamics [17].) That such a problem exists is re-enforced when one considers these degenerate limits from a topological perspective. That is, the degenerate solutions (as indicated above) have non-trivial qualitative differences in topology from their non-degenerate counterparts (e.g., [18,19]).

The resolution of this degenerate-limit problem is, in some (perhaps naive) sense, really quite simple. One needs only to conjecture that a non-degenerate solution is prohibited from continuously evolving into a degenerate one (and vice versa) [18]. That this should intuitively be the case follows from the topological distinctions mentioned above. (This still does not resolve the related problem of how should the thermodynamic properties in a degenerate spacetime be defined. We will not, however, be addressing this issue in the current paper. Rather, our focus is on the “near-degenerate” sector of a particular theory.)

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1It has recently been conjectured, however, that the temperature in a Schwarzschild-de Sitter spacetime can be given a global meaning [14].

2But for further discussion and references in an extremal black hole context, see [15].
To physically motivate this conjecture, one still requires a mechanism for enforcing the proposed censorship. Fortunately, it appears that quantum effects efficiently and naturally serve this purpose. Indeed, recent studies in the context of black hole spectroscopy have directly implied that quantum fluctuations will prevent a charged and/or spinning black hole from ever reaching a precise state of extremality [20,21]. Also recently, a similar outcome has been obtained by the current author via independent means [22,23]. As this latter approach is central to the present analysis, let us give a point-by-point recount of the relevant steps and results.

(i) We started off with a Reissner-Nordstrom (i.e., charged and static) black hole in an asymptotically flat spacetime. (The number of dimensions was initially four [22] but later arbitrary [23].) The focus was on the near-extremal sector; in particular, the deviations (from extremality) in the black hole entropy and temperature were determined as functions of the deviation in the mass (with the charge regarded as a fixed parameter).

(ii) Subjecting the “fundamental” action to a procedure of reduction and field reparametrization, we obtained a two-dimensional effective model. (The motivation being that quantum back-reaction effects, which are ultimately of interest, can be much more easily addressed in the lower-dimensional theory.) Then following [24,25], we expanded the two-dimensional action about the extremal configuration. The resulting theory, which should effectively describe the Reissner-Nordstrom near-extremal sector, turned out to be a two-dimensional anti-de Sitter action; that is, the Jackiw-Teitelboim model [26].

(iii) A duality was re-established [24,25] between the near-extremal Reissner-Nordstrom black hole and the near-massless sector of Jackiw-Teitelboim theory. That is, after appropriate identifications, the thermodynamic deviations (as discussed above) were shown to coincide exactly with the thermodynamics of the two-dimensional black hole solutions.

(iv) The next phase of the analysis concentrated on first-order quantum effects. Here, we began by incorporating the simplest possible matter field having a higher-dimensional pedigree; namely, a massless scalar field that is minimally coupled to the fundamental gravity theory. The same procedure of dimensional reduction, field reparametrization and expansion was then repeated for the revised theory. Interestingly, we found that the revised effective action mimics the theory of a dimensionally reduced BTZ black hole with minimally coupled matter.

(v) Eventually, attentions were focused on the quantum-corrected form of the surface gravity or (equivalently) the temperature. The desired result, at the one-loop level, could be directly extracted from a prior study on the thermodynamics of dimensionally reduced BTZ black holes [15] (also, [29]).

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3The BTZ black hole refers to special solutions of 2+1-dimensional anti-de Sitter gravity that exhibit all the properties of black holes [27]. The dimensionally reduced BTZ model was first discussed in [28].

4Actually, [15] used a form of dimensionally reduced (one-loop) action that has recently been criticized on technical grounds [30,31]. If the action in question does require (yet unknown) corrections, then it is certainly clear that any quantitative results would have to be closely scrutinized. However, we suggest that the coarse, qualitative features of the spacetime would be less sensitive
a non-negative surface gravity [32], we were able to establish a finite lower bound on the black hole mass. That is, quantum back-reaction effects will prevent the (two-dimensional) black hole from completely evaporating; rather, the system will “freeze” at a non-vanishing value of mass.

\[ (vi) \text{In view of the prior outcome and the previously established duality, we ultimately argued that quantum back-reaction effects will prevent the Reissner-Nordstrom black hole from ever reaching a state of precise extremality. That is, a non-extremal Reissner-Nordstrom black hole will be unable to continuously evolve into an extremal solution (and vice versa).} \]

The objective of the current paper is to extend the above treatment to the scenario of a nearly degenerate Schwarzschild-de Sitter black hole spacetime. In this case, the two horizons represent the (inner) black hole horizon and the (outer) cosmological horizon of an asymptotically de Sitter spacetime. Also of interest, the point of degeneracy corresponds to the so-called Nariai black hole solution [16], which can equivalently be regarded as the Schwarzschild-de Sitter solution of maximal mass. For motivation, let us point out that asymptotically de Sitter spacetimes have experienced a recent surge in interest; thanks to, for instance, astronomical observations [33], possible holographic dualities [34] and quasi-de Sitter inflationary scenarios [35]. (For a general overview, see [19].)

The rest of the paper is organized as follows. Section 2 introduces the “fundamental” theory of interest: a Schwarzschild-de Sitter spacetime of arbitrary dimensionality. Here, the focus is on the thermodynamic properties of near-degenerate solutions. In Section 3, we employ a suitable reduction ansatz, as well as other manipulations, to derive an effective two-dimensional model of the near-degenerate sector. We go on to demonstrate a clear thermodynamic duality between this effective description and the originating theory. (Some of this section follows parts of [24], although the perspective of the cited paper differs substantially from the present one.) In Section 4, we consider one-loop corrections and present an argument as to why back-reaction effects would prevent a degenerate state from ever being reached. Finally, Section 5 contains a brief summary and discussion.

\section*{II. THE FUNDAMENTAL THEORY}

Let us formally begin by writing down the de Sitter gravitational action for a spacetime of arbitrary dimensionality \((d = n + 2 > 3)\):

\[
I^{(n+2)} = \frac{1}{16\pi l^n} \int d^{n+2}x \sqrt{-g^{(n+2)}} \left[ R^{(n+2)} - 2\Lambda \right], \tag{1}
\]

where \(l^n\) is the \(n+2\)-dimensional Newton constant (with \(l\) being the analogue to the Planck length) and \(\Lambda\) is the positive cosmological constant. Note that

\[
\Lambda = \frac{n(n + 1)}{2L^2}, \tag{2}
\]

where \(L\) is the curvature radius of de Sitter space.

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to any such corrections.
If we assume no electrostatic charges, then the unique static solution of Eq.(1) can be written as follows:

\[ ds^2 = -h(r)dt^2 + \frac{1}{h(r)}dr^2 + r^2d\Omega^2_n, \]  

(3)

\[ h(r) = 1 - \frac{16\pi l^n M}{n V_n r^{n-1}} - \frac{r^2}{L^2}, \]  

(4)

where \( M \) represents the conserved mass \( [36] \) and \( d\Omega^2_n \) is an \( n \)-dimensional spherical hypersurface with volume \( V_n = 2\pi^{\frac{n+1}{2}}/\Gamma \left( \frac{n+1}{2} \right) \).

The above solution is that of a (non-degenerate) Schwarzschild-de Sitter black hole provided that the mass observable is constrained according to

\[ 0 < M < M_0 \equiv \frac{n}{(n+1)} V_n \left[ \left( \frac{n-1}{n+1} \right) L^2 \right]^{\frac{n+1}{2}}. \]  

(5)

For \( M \) within these limits, one can readily verify the existence of exactly two positive, real and distinct horizons; which can be located by solving the polynomial \( h(r = r_H) = 0 \). The smaller of this pair (to be denoted as \( r_H = r_- \)) is a Schwarzschild-like black hole horizon, while the larger is the de Sitter cosmological horizon (which will be denoted as \( r_H = r_+ \)).

Of special interest are the upper and lower limits in Eq.(5). Firstly, when \( M = 0 \), there is no black hole to speak of and the spacetime corresponds to pure de Sitter space. In this event, there is a single (cosmological) horizon at \( r_+ = L \). Secondly, when \( M = M_0 \), the two horizons coincide and this solution of maximal mass is often referred to as the Nariai \([16]\) black hole. More explicitly, by solving for \( h(r_H; M = M_0) = 0 \), one finds that \( r_- = r_+ = r_0 \), where

\[ r_0 \equiv \sqrt{\frac{n-1}{n+1} L}. \]  

(6)

Let us also point out that either \( M < 0 \) or \( M > M_0 \) results in a naked singularity,\(^5\) but these awkward scenarios can (and will) be disregarded on the grounds of cosmological censorship \([37]\).

Since our current focus is on the case of nearly degenerate horizons, we will now assume that \( M \) is close to, but always less than, the Nariai value of \( M_0 \). To put this notion on a more quantitative level, let us introduce the following measures of deviation from degeneracy:

\[ \Delta M \equiv M - M_0, \]  

(7)

\[ \Delta r_{\pm} \equiv r_{\pm} - r_0, \]  

(8)

while insisting that \( |\Delta M| << M_0 \) and \( |\Delta r_{\pm}| << r_0 \). It should be kept in mind that \( \Delta M < 0 \) must always be imposed. On the other hand, it should be clear that one of the horizon shifts

\(^5\)To elaborate, the black hole horizon disappears when \( M < 0 \) and both horizons vanish if \( M > M_0 \).
will be positive and the other, negative. By our prior convention, this means that $\Delta r_+ > 0$ and $\Delta r_- < 0$. Moreover, to lowest order in $\sqrt{\Delta M}$, one can employ the horizon condition (i.e., $h(r = r_H) = 0$) and readily show that

$$\Delta r_\pm = \pm \frac{L}{n+1} \sqrt{\frac{2|\Delta M|}{M_0}}. \quad (9)$$

Let us now consider the thermodynamic properties - specifically, the Bekenstein-Hawking entropy [2,3] and Hawking temperature [4,38] - of these horizons. (For the generalization of these properties to a cosmological horizon, see [39,11].) Here, we will employ the standard prescriptions for the entropy and temperature (respectively):

$$S_H = \frac{A_H}{4\hbar l^n} = \frac{\mathcal{V}_n r^n_H}{4\hbar l^n}, \quad (10)$$

$$T_H = \frac{\hbar}{2\pi} |\kappa_H| = \frac{\hbar}{4\pi} \left| \frac{dh}{dr} \right|_{r=r_H} = \frac{\hbar}{\pi} \left| \frac{4(n-1)\ln M}{n\mathcal{V}_n r^n_H} - \frac{r_H}{2\pi L^2} \right|, \quad (11)$$

where $A_H$ is the surface area and $\kappa_H$ is the surface gravity with respect to the horizon at $r = r_H$.\(^6\)

Some straightforward calculations reveal that, to lowest order in $\sqrt{|\Delta M|}$ (as appropriate for the near-degenerate regime of interest),

$$\Delta S_\pm \equiv S_\pm(M_0 + \Delta M) - S_H(M_0)$$

$$= \pm \frac{1}{\hbar} \left[ \frac{n\pi \mathcal{V}_n}{(n-1)} \frac{r_0^{n+1}}{ln |\Delta M|} \right], \quad (12)$$

$$\Delta T_\pm \equiv T_\pm(M_0 + \Delta M) - T_H(M_0)$$

$$= \frac{2\hbar}{(n+1)} \left[ \frac{(n-1)}{n\pi \mathcal{V}_n} \frac{ln}{r_0^{n+1} |\Delta M|} \right]. \quad (13)$$

Note that $S_H(M_0) = \mathcal{V}_n r_0^n / 4\hbar l^n$ and $T_H(M_0) = 0$.\(^7\)

It may be bothersome that the first law of thermodynamics is apparently violated at the cosmological horizon, $r_H = r_+$, given that the entropy is increasing ($\Delta S_+ > 0$) while the mass is decreasing ($\Delta M < 0$). However, this paradoxical feature of cosmological horizons

\(^6\)Note that, here and throughout, the speed of light and Boltzmann’s constant have been set to unity.

\(^7\)Actually, rigorous analysis indicates that $T = \hbar / 2\pi r_0$ for a Nariai black hole [40]. Nevertheless, our interest is in the near-degenerate regime (rather than the Nariai solution per se), and the naive extrapolation of non-degenerate thermodynamics does indeed suggest a limit of vanishing temperature.
can be anticipated and is, in fact, well understood [11]. (Reassuringly, there is no such violation for the black hole horizon, \( r_H = r_- \).) Speaking of the first law, one finds that the expected relation of

\[
\frac{\partial |\Delta S_H|}{\partial |\Delta M|} = \frac{1}{\Delta T_H},
\]

is not quite obtained; rather, it is off by a simple numerical factor. This slight discrepancy can be viewed as a consequence of treating each of the horizons as an isolated system. Nonetheless, this effect can be accommodated for with a simple rescaling: \( \Delta M \to \Delta M(n - 1)/(n + 1) \). Hence, we now have (for either horizon)

\[
|\Delta S_H| = \frac{1}{\hbar} \sqrt{\frac{n + 1}{n - 1}} \left[ \frac{n\pi V_n}{r_0^{n+1}} \right] \frac{l^n}{V_n} |\Delta M|,
\]

\[
\Delta T_H = 2\hbar \sqrt{\frac{n - 1}{n + 1}} \left[ \frac{(n - 1)}{n\pi V_n} \right] \frac{l^n}{V_n} |\Delta M|.
\]

**III. THE EFFECTIVE THEORY**

The next step in our procedure is the dimensional reduction of the fundamental action (1) to an effective two-dimensional theory [29]. The reduction that we have in mind can be obtained by imposing a spherical ansatz:

\[
ds_{n+2}^2 = ds_2^2(t, x) + \phi^2(x, t) d\Omega_n^2,
\]

which transforms Eq.(1) into the following functional (also see [41,24]):

\[
I = \frac{\mathcal{V}_n}{16\pi l^n} \int d^2x \sqrt{-g} \phi^n \left[ R + n(n - 1) \left( \frac{\nabla \phi}{\phi^2} + \frac{1}{\phi^2} \right) - \frac{n(n + 1)}{L^2} \right].
\]

Here, the “dilaton”, \( \phi \), can be identified with the radius of the symmetric \( n \)-sphere and all geometric quantities are now defined with respect to the resultant 1+1-dimensional manifold.

It proves to be convenient if the dilaton is redefined in accordance with

\[
\psi(x, t) = \left[ \frac{\phi}{l} \right]^{\frac{2}{n-1}}.
\]

The reduced action (18) can then be reformulated as follows:

\[
I = \frac{1}{2G} \int d^2x \sqrt{-g} \left[ D(\psi) R + \frac{1}{2} (\nabla \psi)^2 + \frac{1}{l^2} V_\lambda(\psi) \right],
\]

where the following definitions have been invoked:

\[
\frac{1}{2G} \equiv \frac{8(n - 1)\mathcal{V}_n}{16\pi n},
\]
The above action is an particularly convenient form, as it is now suitable for the implementation of a field reparametrization that eliminates the kinetic term [42]. Following the prescribed methodology, let us redefine the dilaton, metric and “dilaton potential” as follows:

\[ D(\psi) = \frac{n}{8(n-1)} \psi^2, \]  
\[ V_\Lambda(\psi) = \frac{n^2}{8} \psi \frac{2n-4}{n} \left( \frac{n+1}{n} \frac{n^2 L^2}{8(n-1) L^2} \psi^2. \right) \]  

where

\[ \Omega^2(\psi) \equiv \exp \left[ \frac{1}{2} \int \frac{d\psi}{(dD/d\psi)} \right] = C \left[ \frac{8(n-1)\psi}{n} \right]^{n+1}. \]  

In the last line, \( C \) represents an arbitrary constant of integration that can (in a general sense) often be fixed via physical arguments. Here, we will follow the fifth section of [41] (which similarly considers the spherical reduction of an arbitrary-dimensional action) and set \( C = n^2/8(n-1) \).

With the above reparametrizations, the reduced action (20) transforms into the following expression:

\[ I = \frac{1}{2G} \int d^2x \sqrt{-g} \left[ \nabla_R(\overline{g}) + \frac{1}{l^2} \nabla_\Lambda(\overline{\psi}) \right]. \]  

It can readily be seen that the degenerate-horizon limit of the higher-dimensional theory is precisely recovered when \( \nabla_\Lambda(\overline{\psi}) = 0 \). Denoting this extremal value of the dilaton as \( \overline{\psi} = \overline{\psi}_0 \), we find that (cf, Eqs.(26,23,24))

\[ \overline{\psi}_0 = \frac{n}{8(n-1)} \left[ \frac{(n-1)L^2}{(n+1)L^2} \right]^{n/2}. \]  

With the above discussion in mind, let us define \( \tilde{\psi} \equiv \overline{\psi} - \overline{\psi}_0 \) and appropriately expand the action (28) about the extremal configuration. To first order in \( \tilde{\psi} \), such an expansion yields

\[ I = \frac{1}{2G} \int d^2x \sqrt{-g} \left[ \tilde{\psi} R(\overline{g}) + \frac{1}{l^2} \tilde{\psi}_0(\overline{\psi}) \right], \]  

where
where

\[ \tilde{V}_\Lambda(\tilde{\psi}) \equiv \left. \frac{dV_\Lambda}{d\psi} \right|_{\tilde{\psi}} \tilde{\psi} \]
\[ = -16 \frac{(n-1)^2}{n^2} \left[ \frac{(n+1)l^2}{(n-1)L^2} \right]^{(n+1)/2} \tilde{\psi} \]
\[ \equiv -2\lambda \tilde{\psi}. \] (31)

Note that Eqs. (23, 24, 26, 27, 29) were applied in attaining the second to last line and \( \lambda > 0 \).

We now drop the various tildes and bars and thus consider the following action:

\[ I = \frac{1}{2G} \int d^2x \sqrt{-g} \psi \left[ R(g) - 2\frac{\lambda}{l^2} \right]. \] (32)

We could, in principle, continue on with this form of effective action; essentially, a two-dimensional de Sitter theory of constant curvature [43]. However, it is substantially easier to work with the anti-de Sitter analogue, for which the static solutions are described by the very well-known Jackiw-Teitelboim (JT) black hole [26]. This JT analogue can be obtained with a simple analytic continuation of the length parameter (i.e., \( l^2 \to -l^2 \)), and so we will subsequently work with

\[ I = \frac{1}{2G} \int d^2x \sqrt{-g} \psi \left[ R(g) + 2\frac{\lambda}{l^2} \right], \] (33)

where \( \lambda \) is (still) a positive quantity. Actually, if the reduced action described a fundamental theory, then such a continuation would have to be treated very carefully; especially, in any attempt to extract physically meaningful results about the spacetime. However, in the current analysis, where the reduced theory serves as only an effective description of a reduced phase space, such a procedure should be perfectly viable.

The general solution of this effective JT action (33) can be conveniently expressed in a static, Schwarzschild-like gauge:

\[ ds^2 = -(\lambda \frac{x^2}{l^2} - 2lGm)dt^2 + (\lambda \frac{x^2}{l^2} - 2lGm)^{-1}dx^2, \] (34)

\[ \psi = \frac{x}{l}. \] (35)

Here, \( m \) represents the conserved mass of the JT black hole. Moreover, we can apply the formalism of [44] (which is applicable to a generic theory of two-dimensional dilaton gravity), to obtain the other thermodynamic properties of relevance; namely, the entropy and the temperature:

\[ S_{JT} = \frac{2\pi}{\hbar G} \psi_+, \] (36)

\[ T_{JT} = \frac{\hbar \lambda}{2\pi l} \psi_+, \] (37)

9
where \( \psi_+ = x_+/l = \sqrt{2lGm/\lambda} \) is the horizon value of the dilaton field.

Substituting for \( \lambda \) (31) and \( G \) (21) into the above expressions, as well as applying the defining relation for \( r_0 \) (6), we are able to derive the following results:

\[
S_{JT} = \frac{1}{\hbar} \sqrt{\frac{n\pi \mathcal{V}_n}{(n - 1)}} \frac{r_0^{n+1}}{n^n} m, \tag{38}
\]

\[
T_{JT} = 2\hbar \sqrt{\frac{(n - 1)}{n\pi \mathcal{V}_n}} \frac{l^n}{r_0^{n+1}} m. \tag{39}
\]

Let us now make a reasonable identification between the JT black hole mass, \( m \), and the mass deviation (from degeneracy) of the higher-dimensional model, \( |\Delta M| \). It then becomes quite evident that the two theories are closely related. To be explicit (cf, Eqs.(15,16)),

\[
S_{JT} = K|\Delta S_H|, \tag{40}
\]

\[
T_{JT} = \frac{1}{K} |\Delta T_H|, \tag{41}
\]

where \( K = \sqrt{n - 1/n + 1} \) is a dimensionless numerical factor. Clearly, we can eliminate the numerical factor, \( K \), by rescaling an appropriate parameter of the effective theory (such as the effective Planck’s constant). The important point is that \( T_{JT}S_{JT} = \Delta T_H|\Delta S_H| \) and all of the dimensional quantities do indeed coincide precisely. Hence, we have established the anticipated thermodynamic duality: the nearly degenerate sector of Schwarzschild-de Sitter black holes with the near-massless sector of JT theory.

Before proceeding to the next section, let us point out an important property of the JT black hole: as the mass, \( m \), goes to zero, so does the temperature, \( T_{JT} \), and the entropy, \( S_{JT} \). Although this relation is intuitively expected in conventional thermodynamic systems, it is, nevertheless, very unusual in a black hole context (where, typically, a vanishing temperature corresponds to both a finite mass and entropy). It is, in fact, this property of JT black holes that makes them an ideal framework for studying the low-temperature regime of dually related theories.

\[
\text{IV. QUANTUM EFFECTS}
\]

In analogy to our prior studies on near-extremal black holes [22,23], we will now invoke one-loop considerations to argue the following: a nearly degenerate Schwarzschild-de Sitter solution will be unable to evolve continuously into a degenerate, Nariai spacetime (and \textit{vice versa}).

The essence of our argument goes as follows (for a more detailed discussion, see [22]). We start by considering the simplest possible matter having a higher-dimensional pedigree: namely, a massless scalar field \( f \) that is minimally coupled to the \( n+2 \)-dimensional de Sitter theory. The revised (total) action can now be expressed as
\[ I_{TOT}^{(n+2)} = I^{(n+2)} - \frac{\hbar}{16\pi l^n} \int d^{n+2}x \sqrt{-g^{(n+2)}(\nabla^{(n+2)} f)^2}, \]  
(42)

where \( I^{(n+2)} \) is the classically defined action of Eq.(1). Again imposing the reduction ansatz of Eq.(17), as well as constraining \( f = f(t,x) \), we obtain the following reduced form:

\[ I_{TOT} = I - \frac{\hbar \mathcal{V}_n}{16\pi l^n} \int d^2x \sqrt{-g} \bar{\psi}^n (\nabla f)^2, \]  
(43)

where \( I \) is the reduced action of Eq.(18).

Following the same pattern of field reparametrization and expansion as described in Section 3, we can eventually write

\[ I_{TOT} = I_{JT} - \frac{\hbar (n-1) \mathcal{V}_n}{2\pi n} \int d^2x \sqrt{-g} \tilde{\psi} (\nabla f)^2, \]  
(44)

where \( I_{JT} \) is the JT action of Eq.(33), and the tilde and bar notation has been resurrected for maximal clarity.

The above result informs us that the dilaton-matter coupling is precisely that obtained in the dimensional reduction (from three to two dimensions) of a BTZ black hole; under the assumption of minimally coupled matter in the three-dimensional theory [27,28]. This observation is important because, as discussed on a rigorous level in [22], the dimensionally reduced BTZ black hole suffers a formal breakdown as the limit of zero mass is approached (i.e., as \( m \to 0 \)). More specifically, at some small but finite value of \( m \), the surface gravity (or, equivalently, the temperature) will take on a negative value and become increasingly more negative as the \( m = 0 \) limit is approached. Significantly, it has been argued that a non-negative surface gravity is a necessary criteria for a consistent black hole solution [32]. On this basis, we have further argued that, because of quantum back-reaction effects, the reduced BTZ black hole will be unable to attain a state of vanishing mass [22]. Obviously, the same conclusion (if valid) must apply to the JT black hole when it is coupled to matter with a higher-dimensional pedigree.

Let us now remind ourselves of the main outcome of Section 3; namely, the near-massless limit of the JT sector is dual (at least at the level of thermodynamics) to the near-degenerate sector of a Schwarzschild-de Sitter black hole. It therefore stands to reason that the censorship of massless JT black holes will translate into the following dual statement: a Schwarzschild-de Sitter black hole will be unable to reach the limit of horizon degeneracy (i.e., the limit \( |\Delta M| \to 0 \)). To put it another way, quantum back-reaction effects will prohibit a nearly degenerate Schwarzschild-de Sitter black hole to ever reaching a precise state of degeneracy. Hence, a non-degenerate Schwarzschild-de Sitter spacetime should never be able to evolve continuously into a Nariai black hole and vice versa.

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8The unorthodox factor in front of the above matter action is of no relevance, as this constant can always be absorbed in a rescaling of \( f \).
V. CONCLUSION

In summary, we have been considering a holographic type of duality [24] between a nearly degenerate Schwarzschild-de Sitter spacetime (of arbitrary dimensionality) and the near-massless sector of Jackiw-Teitelboim theory [26]. We started out by rigorously demonstrating this dual relationship at the level of horizon thermodynamics. We then went on to argue that, on the basis of this duality and one-loop considerations, the higher-dimensional spacetime will never be able to continuously evolve into a Nariai solution [16]; that is, a Schwarzschild-de Sitter black hole of maximal mass and perfectly coincident horizons. Let us emphasize that our conclusions in no way undermine the Nariai black hole as a viable solution of a spacetime with a positive cosmological constant. Moreover, the type of censorship discussed here does not apply to more exotic mechanisms, such as quantum tunneling between Nariai and non-degenerate solutions [19].

It should again be pointed out that the latter part of the paper was based (vicariously through [22,23]) on the outcomes of a one-loop analysis of the dimensionally reduced BTZ black hole. This analysis was based, in turn, on a form of dimensionally-reduced action that has been the subject of some recent criticism [30,31]. In spite of the potentially disturbing implications, we expect that our qualitative outcomes will not be in any jeopardy. To motivate this claim, let us take note of some independent studies in the context of black hole spectroscopy [20,21]. In these treatments, it was shown, quite generically, that quantum fluctuations will prevent a charged or spinning black hole from ever reaching a state of exact extremality. Although these studies are not directly applicable to the current analysis, their generality does insinuate a similar censorship in any spacetime with potentially degenerate horizons.

Finally, one might wonder if this discontinuity between non-degenerate and degenerate Schwarzschild-de Sitter black holes could have any ramifications in quantum gravity. In this regard, let us take note of the conjectured duality between time evolution in an asymptotically de Sitter spacetime and a suitably defined renormalization-group flow [45]. (This duality can be viewed as a manifestation of the so-called “dS/CFT” holographic correspondence [34].) Perhaps significantly, it has been suggested that the Nariai solution should correspond to the infrared fixed point of the holographic flow [46]. It is not obvious to us, at the current time, if our findings could play a role in such a context; nonetheless, it could yet prove to be a worthwhile direction of investigation.

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REFERENCES

[1] See, for a review, R.M. Wald, Living Rev. Rel. 4, 6 (2001) [gr-qc/9912119].
[2] J.D. Bekenstein, Lett. Nuovo. Cim. 4, 737 (1972); Phys. Rev. D7, 2333 (1973); Phys. Rev. D9, 3292 (1974).
[3] S.W. Hawking, Comm. Math. Phys. 25, 152 (1972); J.M. Bardeen, B. Carter and S.W. Hawking, Comm. Math. Phys. 31, 161 (1973).
[4] S.W. Hawking, Comm. Math. Phys. 43, 199 (1975).
[5] See, for an overview, S. Carlip, Nucl. Phys. Proc. Suppl. 88, 10 (2000) [gr-qc/9912118].
[6] J.D. Bekenstein, “Do We Understand Black Hole Entropy”, gr-qc/9409015 (1994).
[7] M. Srednicki, Phys. Rev. Lett. 71, 666 (1993) [hep-th/9303048].
[8] L. Susskind and J. Uglum, Phys. Rev. D50, 2700 (1994) [hep-th/9401070].
[9] T. Padmanabhan, Class. Quant. Grav. 19, 5387 (2002) [gr-qc/0204019].
[10] N.D. Birrell and P.C.W. Davies, Quantum Fields in Curved Space, (Cambridge University Press, Cambridge, 1982).
[11] M. Spradlin, A. Strominger and A. Volovich, “Les Houches Lectures on de Sitter Space”, hep-th/0110007 (2001).
[12] L. Susskind, L. Thorlacius and J. Uglum, Phys. Rev. D48, 3743 (1993) [hep-th/9306069].
[13] C. Teitelboim, “Gravitational Thermodynamics of Schwarzschild-de Sitter Space”, hep-th/0203258 (2002).
[14] S. Shankaranarayanan, “Temperature and Entropy of Schwarzschild-de Sitter Spacetime”, gr-qc/0301090 (2003).
[15] A.J.M. Medved and G. Kunstatter, Phys. Rev. D63, 104005 (2001) [hep-th/0009050].
[16] H. Nariai, Sci. Rep. Tohoku Univ. 34, 160 (1950); ibid 35, 62 (1951).
[17] R.M. Wald, Phys. Rev. D56, 6467 (1997) [gr-qc/9704008].
[18] S.W. Hawking, G.T. Horowitz and S.F. Ross, Phys. Rev. D51, 4302 (1995) [gr-qc/9409013].
[19] R. Bousso, “Adventures in de Sitter Space”, hep-th/0205177 (2002), and references therein.
[20] A. Barvinsky, S. Das and G. Kunstatter, Class. Quant. Grav. 18, 4845 (2001) [gr-qc/0012066]; Phys. Lett. B517, 415 (2001) [hep-th/0102061].
[21] G. Gour and A.J.M. Medved, “Kerr Black Hole as a Quantum Rotator”, gr-qc/0211089 (2002); “Quantum Spectrum for a Kerr-Newman Black Hole”, gr-qc/0212021 (2002).
[22] A.J.M. Medved, “Reissner-Nordstrom Near Extremality from a Jakiw-Teitelboim Perspective”, hep-th/0111091 (2001).
[23] A.J.M. Medved, “Near-Extremal Spherically Symmetric Black Holes in an Arbitrary-Dimensional Spacetime”, hep-th/0112056 (2001).
[24] D.J. Navarro, J. Navarro-Salas and P. Navarro, Nucl. Phys. B580, 311 (2000) [hep-th/9911091].
[25] A. Fabbri, D.J. Navarro and J. Navarro-Salas, Nucl. Phys. B595, 381 (2001) [hep-th/0006035].
[26] R. Jackiw in, Quantum Theory of Gravity, ed. S. Christensen (Hilger, Bristol) (1984), p.403; C. Teitelboim, ibid, p.327; R. Jackiw, Nucl. Phys. B252, 343 (1985).
[27] M. Banados, C. Teitelboim and J. Zanelli, Phys. Rev. Lett. 69, 1849 (1992) [hep-th/9204099].
[28] A. Achucarro and M.E. Ortiz, Phys. Rev. D48, 3600 (1993) [hep-th/9304068].
[29] See, for reviews on two-dimensional gravity with higher-dimensional origins, S. Nojiri and S.D. Odintsov, Int. J. Mod. Phys. A16, 1015 (2001) [hep-th/0009202]; D. Grumiller, W. Kummer and D.V. Vassilevich, Phys. Rept. 369, 327 (2002) [hep-th/0204253].
[30] V.P. Frolov, P.J. Sutton and A.I. Zelnikov, Phys. Rev. D61, 024021 (2000) [hep-th/9909086].
[31] Y.V. Gusev and A.I. Zelnikov, Phys. Rev. D61, 084010 (2000) [hep-th/9910198].
[32] P.R. Anderson, W.A. Hiscock and B.E. Taylor, Phys. Rev. Lett. 85, 2438 (2000) [gr-qc/0002007].
[33] N. Bahcall, J.P. Ostriker, S. Perlmutter and P.J. Steinhardt, Science 284, 1481 (1999) [astro-ph/9812133].
[34] A. Strominger, JHEP 0110, 034 (2001) [hep-th/0106113].
[35] U.H. Danielsson, “Holography, Inflation, and Quantum Fluctuations in the Early Universe”, hep-th/0301182 (2003).
[36] V. Balasubramanian, J. de Boer and D. Minic, Phys. Rev. D65, 123508 (2002) [hep-th/0110108].
[37] See, for instance, R.M. Wald, General Relativity, (University of Chicago Press, 1984).
[38] G.W. Gibbons and S.W. Hawking, Phys. Rev. D15, 2752 (1977).
[39] G.W. Gibbons and S.W. Hawking, Phys. Rev. D15, 2738 (1977).
[40] R. Bousso and S.W. Hawking, Phys. Rev. D54, 6312 (1996) [gr-qc/9606052].
[41] G. Kunstatter, R. Petryk and S. Shelemy, Phys. Rev. D57, 3537 (1998) [gr-qc/9709043].
[42] D. Louis-Martinez, J. Gegenberg and G. Kunstatter, Phys. Lett. B321, 193 (1994) [gr-qc/9309018]; D. Louis-Martinez and G. Kunstatter, Phys. Rev. D49, 5227 (1994).
[43] M. Cadoni, P. Carta, M. Cavaglia and S. Mignemi, Phys. Rev. D66, 065008 (2002) [hep-th/0205211].
[44] J. Gegenberg, G. Kunstatter and D. Louis-Martinez, Phys. Rev. D51, 1781 (1995) [gr-qc/9408015].
[45] A. Strominger, JHEP 0111, 049 (2001) [hep-th/0110087].
[46] E. Halyo, JHEP 0203, 009 (2002) [hep-th/0112093].