Analysis of time-varying meshing stiffness of helical gear pair with tooth surface spalling

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Abstract. Time-varying meshing stiffness (TVMS) provides significant information about the health of gear systems. Tooth spalling will change the TVMS. Considering the contact state of gear pair, the purpose of this study is to investigate the influence of tooth surface spalling on the TVMS of helical gears. Firstly, the meshing stiffness analysis algorithm of helical gear pair with tooth surface spalling is proposed, based on the potential energy method, slicing method and numerical integration formula. Then the influence of different spalling parameters on the TVMS of helical gear pair is analysed by considering spalling length, width and the oblique cylinder bottom shape. Finally, the results show that the contact line falls into the defect area, the spalling has the most significant effect on the stiffness. The influence of oblique cylinder shape at the bottom of spalling is mainly affected after the contact line passes through the spalling area. This study improve the theoretical basis for spalling fault of helical geared mesh.

1. Introduction

Tooth surface spalling is a common failure of gear transmission. In the study of time-varying meshing stiffness of gears, the potential energy method is adopted by most researchers. Liu Wen et al. [1] used the potential energy method to calculate the meshing stiffness of helical gears, and Feng et al. [2] proposed an improved calculation method for the meshing stiffness of helical gears. Researchers established different shapes of spalling features. Liang et al. [3] studied circular spalling at equal depths, and Luo et al. [4] analyzed ellipsoidal spalling areas with arc-shaped bottom features and calculated the meshing stiffness of spur gears. Han et al. [5] calculated the meshing stiffness of helical gears with parallelograms and equal depth spalling.

The bottom of the spalling of the gear surface material is usually not a plane [4]. Simulation of tooth spalling with equal depth may lead to meshing stiffness error, especially bending stiffness and shear stiffness which are sensitive to tooth cross-sectional area and moment of inertia. In this paper, the calculation model of the helical gear pair stiffness of the helical pit tooth surface peeling is established. The analytical formula of meshing stiffness of helical gear pair with tooth surface peeling is deduced. The factors affecting meshing stiffness is analyzed.

2. Calculation of time-varying meshing stiffness of helical gears

The slicing method is used to calculate the meshing stiffness of the unilateral helical gear. The stiffness calculation model and formula refer to literature [5].
According to actual datum, spalling defect usually arises near pitch line. In this work, the shape of spalling is assumed by an oblique cylindrical pit, which symmetric line is parallel to the contact line and the bottom of the oblique cylinder is parallel to the end surface, as shown in fig. 1. Given that the change of the involute tooth surface of the peeling area is much smaller than that of the radius of the peeling area, the tooth surface of the peeling area is equivalent to a small plane. The intersection line of the oblique cylinder and the tooth surface is a parallelogram. The spalling size and location of the tooth surface spalling model can be defined by five variables: the distance from the spalling center to the pitch line $x_{sp}$, and to the AB end face $y_{sp}$, length $l_{sp}$, width $b_{sp}$ and the cylindrical surface of the spalling area pit radius $R_{sp}$. Where AB and CD are gear end faces.

$N_{start}$ and $N_{end}$ represent the range of contact numbers corresponding to the contact line at different moments of gear meshing. The expressions of $N_{start}$ and $N_{end}$ are as follows:

$$N_{start} = \begin{cases} 
0 & 0 \leq vt < \varepsilon_1 P_{bl} \\
\text{round}\left[\left(vt - \varepsilon_1 P_{bl}\right)N/(B \tan \beta_b)\right] & \varepsilon_1 P_{bl} \leq vt < \varepsilon_2 P_{bl}
\end{cases}$$

$$N_{end} = \begin{cases} 
\text{round}\left[vtn/(B \tan \beta_b)\right] & 0 \leq vt < \varepsilon_2 P_{bl} \\
N & \varepsilon_2 P_{bl} \leq vt < \varepsilon_3 P_{bl}
\end{cases}$$

where $\varepsilon_1 = \max(\varepsilon_a, \varepsilon_b)$, $\varepsilon_2 = \min(\varepsilon_a, \varepsilon_b)$, round is a random rounding function.

According to the analysis of the meshing process in Figure 1, the expressions of $t_1$, $t_2$, $t_3$, $t_4$ are given as:

$$t_1 = \left[(B - y_{sp}) \tan \beta_b + L_{pc} - x_{sp} - l_{sp}/2\right]/v$$

$$t_2 = \left[(B - y_{sp}) \tan \beta_b + L_{pc} - x_{sp} + l_{sp}/2\right]/v$$

$$t_3 = \left[(B - y_{sp} - b_{sp} \cos \beta_b / 2) \tan \beta_b + \varepsilon_3 P_{bl}\right]/v$$

$$t_4 = \left[(B - y_{sp} + b_{sp} \cos \beta_b / 2) \tan \beta_b + \varepsilon_3 P_{bl}\right]/v$$

where $t_1$ is the time when the contact line passes $N_{start}$, and $t_4$ is the time when the contact line passes $N_{end}$. Where $r_a$ is the addendum radius, $r'$ is the pitch radius.

$$L_{pc} = \sqrt{\left(r^2 - r_a^2\right)} - \sqrt{\left(r'^2 - r_a^2\right)}$$

Considering the spalling contact line length $l_{sp}(t)$, $b(t)$is $l(t)-b_{sp}$ when $t_1 < t < t_2$. The Hertzian contact stiffness $k_0$ of the helical gear with spalling defects is expressed as:

$$k_0 = \frac{\pi E b_0(t)}{4(1 - \nu^2)}$$

Spalling will cause the decrease of effective cross-sectional area $A'$ and the moment of inertia $I'$ of the gear teeth, which affect the meshing rigidity of the gear. The cross-sectional schematic diagram of
a gear cantilever beam model with spalling defects of oblique cylindrical pits is shown in Fig. 2. The distance between any point in the spalling area and the center surface of the gear tooth is $h(x)$. 

![Model of tooth with spalling defects](image)

Fig.2 Model of tooth with spalling defects

According to the geometric relationship, the expressions of $\theta_{sp}$ and $d_{sp}$ are as follows:

$$\theta_{sp} = \arcsin \left( \frac{(h_{start} - h_{end})}{l_{sp}} \right)$$  \hspace{1cm} (9)

$$d_{sp} = \begin{cases} \sqrt{R_b^2 - (l_{sp}^2 / 2 - vt - d_{sp}\tan\theta_{sp})^2} & \text{for } vt_1 \leq vt < vt_1 + l_{sp} / 2 \\ \sqrt{R_b^2 - (vt - l_{sp} / 2 + d_{sp}\tan\theta_{sp})^2} & \text{for } vt_1 + l_{sp} / 2 \leq vt < vt_2 \end{cases}$$  \hspace{1cm} (10)

where $h_{start}$ is the distance between start point of spalling area and the center line of gear tooth, $h_{start} = R_b[(\alpha_2 - \alpha_{spi})\cos(\alpha) + \sin(\alpha)]$, $h_{end}$ is the distance from the end point of spalling area to the center line of tooth, $h_{end} = R_b[(\alpha_2 - \alpha_{spo})\cos(\alpha) + \sin(\alpha)]$.

The expression of $\Delta_{sp}$ can be obtained as follows:

$$\Delta_{sp} = d_{sp} / \cos(\theta_{sp})$$  \hspace{1cm} (11)

The $I'_{x}$ and $A'_{x}$ of the spalling area are obtained as follows:

$$I'_{x} = \frac{1}{12} \left[ h(x) + h'(x) \right] dy$$  \hspace{1cm} (12)

$$A'_{x} = \left[ h(x) + h'(x) \right] dy$$  \hspace{1cm} (13)

Considering the influence of spalling on each meshing moment, the bending stiffness $k_b$, shear stiffness $k_s$ and axial compression stiffness $k_a$ of helical gears with spalling defects can be obtained:

$$k_j = \begin{cases} \sum_{i=N_{start}}^{N_{end}} \frac{1}{l_{x}} \int_{t_{i-1}}^{t_i} A d\alpha & t < t_1 \\ \sum_{i=N_{start}}^{N_{end}} \frac{1}{l_{x}} \int_{t_{i-1}}^{t_i} A d\alpha + \sum_{i=N_{start}+N_{j}}^{N_{end}} \frac{1}{l_{x}} \int_{t_{i-1}}^{t_i} A d\alpha & t_1 \leq t < t_2 \\ \sum_{i=N_{start}}^{N_{end}} \frac{1}{l_{x}} \int_{t_{i-1}}^{t_i} A d\alpha - \sum_{i=N_{start}}^{N_{end}} \frac{1}{l_{x}} \int_{-\alpha_{a_{spo}}}^{\alpha_{a_{spo}}} A d\alpha + \sum_{i=N_{start}+N_{j}}^{N_{end}} \frac{1}{l_{x}} \int_{t_{i-1}}^{t_i} A d\alpha & t_2 \leq t < t_3 \\ \sum_{i=N_{start}}^{N_{end}} \frac{1}{l_{x}} \int_{t_{i-1}}^{t_i} A d\alpha - \sum_{i=N_{start}}^{N_{end}} \frac{1}{l_{x}} \int_{-\alpha_{a_{spo}}}^{\alpha_{a_{spo}}} A d\alpha + \sum_{i=N_{start}+N_{j}}^{N_{end}} \frac{1}{l_{x}} \int_{t_{i-1}}^{t_i} A d\alpha & t_3 \leq t < t_4 \\ \sum_{i=N_{start}}^{N_{end}} \frac{1}{l_{x}} \int_{t_{i-1}}^{t_i} A d\alpha & t \geq t_4 \end{cases}$$  \hspace{1cm} (14)

In the formula, $\alpha_{a_{j}}$ is the angle between the meshing force $F$ at the contact line position of each tooth and the vertical direction of the gear center line [4].

When $j=b$, $k_j$ means bending stiffness,
\[ A = \frac{3(\alpha_z - \alpha) \cos \alpha}{\sin \alpha + (\alpha_z - \alpha) \cos \alpha} \times \frac{1 + \cos \alpha'[(\alpha_z - \alpha) \sin \alpha - \cos \alpha]}{2E\Delta y} \]  
\[ A' = \frac{3(\alpha_z - \alpha) \cos \alpha}{\sin \alpha + (\alpha_z - \alpha) \cos \alpha - 0.5\Delta_{sp}/r_{ns}} \times \frac{1 + \cos \alpha'[(\alpha_z - \alpha) \sin \alpha - \cos \alpha]}{2E\Delta y} \]

When \( j=s \), \( k_j \) means shear stiffness,
\[ A = \frac{1.2(1 + v)(\alpha_z - \alpha) \cos \alpha \cos^2 \alpha'}{E[\sin \alpha + (\alpha_z - \alpha) \cos \alpha] \Delta y} \]
\[ A' = \frac{1.2(1 + v)(\alpha_z - \alpha) \cos \alpha \cos^2 \alpha'}{E[\sin \alpha + (\alpha_z - \alpha) \cos \alpha - 0.5\Delta_{sp}/r_{ns}] \Delta y} \]

When \( j=a \), \( k_j \) means axial compression stiffness,
\[ A = \frac{(\alpha_z - \alpha) \cos \alpha \sin^2 \alpha}{2E[\sin \alpha + (\alpha_z - \alpha) \cos \alpha] \Delta y} \]
\[ A' = \frac{(\alpha_z - \alpha) \cos \alpha \sin^2 \alpha}{2E[\sin \alpha + (\alpha_z - \alpha) \cos \alpha - 0.5\Delta_{sp}/r_{ns}] \Delta y} \]

3. Results and discussion

This study solves the integration by developing a program. A pair of helical gears with spalling on one tooth of pinion is assumed to investigate the influences on time-varying mesh stiffness. The number of driving gear and driven gear is 34 and 31 respectively. Module \( m_n \) is 2mm and pressure angle \( \alpha_n \) is 22.5°. The helix angle \( \beta \) is 30° and the tooth width \( B \) is 58mm. The input speed \( n \) is 7500r/min and the input power \( P_{in} \) is 400kW. The spalling shape is an oblique cylinder, and the center of the spalling area is set at the midpoint of the pitch line.

3.1. Effect of spalling length

In this section, effect of spalling length is investigated. The radius of the spalling inclined cylindrical surface is taken as \( R_{sp}=1.5mm \). The width \( b_{sp} \) is set to 4mm and \( l_{sp}=2, 2.5, 3mm \). The results are shown in Fig.3.

As the length of the peeling area \( l_{sp} \) increases, when the contact line passes through the peeling area, the time \( t_2-t_1 \) of the gear in the peeling area will become longer, resulting in a wider area where the meshing stiffness decreases. The increase of \( l_{sp} \) will result in the decrease of \( \alpha_{spi}, \alpha_{spo} \) and \( \Delta_{sp} \), which leads the changing of \( I_x \) and \( A_x \). The decrease of meshing stiffness becomes larger.

3.2. Effect of spalling width

The different spalling width are assumed in this simulation with \( l_{sp}=2.5mm \) and \( R_{sp}=1mm \). Results are given in Fig.4. With the increase of \( b_{sp} \), \( t_2-t_1 \) does not change. Since the decrease in the length of the contact line increases in the period, the weakening effect on the meshing stiffness increases.

3.3. Effect of spalling depth

Effect of spalling depth is revealed in this section. Assuming that the size of the intersection line between the spalling area and the tooth surface remains unchanged, and the diameter of the bottom surface of the oblique cylinder is greater than the spalling area length \( l_{sp} \), the spalling depth increase with spalling inclined cylindrical surface radius \( R_{sp} \) decreasing. \( R_{sp} \) takes 1.5mm, 2mm, 2.5mm. The width \( b_{sp} \) is set to 4mm and \( l_{sp}=2.5mm \). Fig.5 shows the results.

The bending stiffness and shear stiffness are sensitive to the change of the tooth cross-sectional area and the section moment of inertia, which are obviously affected by \( R_{sp} \). \( R_{sp} \) has weakly effect on the contact stiffness. With the decrease of \( R_{sp} \), the change of meshing stiffness in \( t_1-t_2 \) period is negligible, and the meshing stiffness in \( t_2-t_4 \) period decreases.
4. Conclusion

An analytical method is proposed in this study to analyze the influence of spalling on TVMS of helical gears. The conclusions are as follows:

The contact line length, effective cross-sectional area and cross-sectional inertia moment of the gear pair are affected after the tooth surface spalling, which reduces the Hertzian contact stiffness, bending stiffness, shear stiffness and axial compression stiffness of the gear teeth. The decrease of the contact line length will greatly weaken the Hertzian contact stiffness, resulting in the abrupt change of the meshing stiffness.
When the contact line falls into the defect area, the spalling has the most significant effect on the stiffness. Even if the contact line passes through the spalling area, its stiffness will be affected to some extent, especially by the spalling depth.

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