MAXIMAL-ACCELERATION PHASE SPACE RELATIVITY FROM CLIFFORD ALGEBRAS

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Abstract
We present a new physical model that links the maximum speed of light with the minimal Planck scale into a maximal-acceleration Relativity principle in the spacetime tangent bundle and in phase spaces (cotangent bundle). Crucial in order to establish this link is the use of Clifford algebras in phase spaces. The maximal proper-acceleration bound is \( a = c^2/\Lambda \) in full agreement with the old predictions of Caianiello, the Finslerian geometry point of view of Brandt and more recent results in the literature. We present the reasons why an Extended Scale Relativity based on Clifford spaces is physically more appealing than those based on kappa-deformed Poincare algebras and the inhomogeneous quantum groups operating in quantum Minkowski spacetimes. The main reason being that the Planck scale should not be taken as a deformation parameter to construct quantum algebras but should exist already as the minimum scale in Clifford spaces.

I . Introduction

Relativity in C-spaces (Clifford manifolds) [1] is a very natural extension of Einstein’s relativity and Nottale’s scale relativity [2] where the impassible speed of light and the minimum Planck scale are the two universal invariants. An event in C-space is represented by a polyvector, or Clifford-aggregate of lines, areas, volumes. which bear a one-to-one correspondence to the holographic shadows/projections (onto the embedding spacetime coordinate planes) of a nested family of p-loops (closed p- branes of spherical topology) of various dimensionalities: \( p = 0 \) represents a point; \( p = 1 \) a closed string, \( p = 2 \) a closed membrane, etc.

The invariant “line” element associated with a polyparticle is:

\[
d\Sigma^2 = dX.dX = d\Omega)^2 + \Lambda^{2D-2}(dx_\mu dx^\mu) + \Lambda^{2D-4}(dx_{\mu\nu})(dx^{\mu\nu}) + ... \quad (1.1)
\]
the Planck scale appears as a natural quantity in order to match units and combine p-branes (p-loops) of different dimensions. The fact that the Planck scale is a minimum was based on the real-valued interval $dX$ when $dX.dX > 0$. The analog of photons in C-space are tensionless branes: $dX.dX = 0$. Scales smaller than $\Lambda$ yield "tachyonic" intervals $dX.dX < 0$ [1]. Due to the matrix representation of the gamma matrices and the cyclic trace property, it can be easily seen why the line element is invariant under the C-space Lorentz group transformations:

$$\text{Trace } X'^2 = \text{Trace } [RX^2R^{-1}] = \text{Trace } [RR^{-1}X^2] = \text{Trace } X^2 , \quad (1.2)$$

where a finite polydimensional rotation that reshuffles dimensions is characterized by the C-space "rotation" matrix:

$$R = \exp[i(\theta I + \theta^\mu \gamma_\mu + \theta^{\mu\nu} \gamma_{\mu\nu} + \ldots)]. \quad (1.3)$$

The parameters $\theta, \theta^\mu, \theta^{\mu\nu}$ are the C-space extension of the Lorentz boost parameters and for this reason the naive Lorentz transformations of spacetime are modified to be:

$$x'^\mu = L^\mu_{\nu} [\theta, \theta^\mu, \theta^{\mu\nu}, \ldots] x^\nu . \quad (1.4)$$

Due to the presence of all the C-space boost parameters in the modified Lorentz matrix $L^\mu_{\nu}$ one induces an effective Lorentz transformation with an effective boost parameter of the form $\xi_{eff} = z(\xi)$ discussed in [12] in order to ensure that the minimal Planck scale is never surpassed at infinite momentum. For details we refer to [1].

It was argued in [1] that the extended Relativity principle in C-space may contain the clues to unravel the physical foundations of string and M-theory since the dynamics in C-spaces encompass in one stroke the dynamics of all p-branes of various dimensionalities. In particular, how to formulate a master action that encodes the collective dynamics of all extended objects.

For further details about these issues we refer to [1,3,16] and all the references therein. Like the derivation of the minimal length/time string/brane uncertainty relations; the logarithmic corrections to the black-hole area-entropy relation; the origins of a higher derivative gravity with torsion; the construction of the p-brane propagator; the role of supersymmetry; the emergence of two times; the reason behind a running Planck constant and the
variable fine structure constant; the way to correctly pose the cosmological constant problem as well as other results.

In this letter, we will present another physical that links the maximum speed of light and the minimal Planck scale into a maximal-acceleration principle in the spacetime tangent bundle, and consequently, in the phase space (cotangent bundle). Crucial in order to establish this link is the use of Clifford algebras in phase spaces. The maximal proper acceleration bound is $a = c^2/\Lambda$ in full agreement with [4] and the Finslerian geometry point of view in [6].

In section II, we show how to derive the Nesterenko action [5] associated with a sub-maximally accelerated particle in spacetime directly from phase-space Clifford algebras and present a full-fledged C-phase-space generalization of the Nesterenko action associated with the multi-symplectic geometry of a polyparticle.

In III, we present a series of reasons why we believe C-space Relativity is more physically appealing than all the others proposals based on kappa-deformed Poincare algebras and other quantum algebras [10-13,17]. We also argue why the theories based on kappa-deformed Poincare algebras may in fact be related to a Moyal star-product deformation of a classical Lorentz algebra whose deformation parameter is precisely the Planck scale $\Lambda = 1/\kappa$.

II. Maximal-Acceleration from Clifford algebras

We will follow closely the procedure described in the book [3] to construct the phase space Clifford algebra. For simplicity we shall begin with a two-dimensional phase space, with one coordinate and one momentum variable and afterwards we will generalize the construction to higher dimensions.

Let $e_p e_q$ be the Clifford basis elements in a two-dimensional phase space obeying the following relations:

$e_p e_q \equiv \frac{1}{2}(e_q e_p + e_p e_q) = 0. \quad e_p e_p = e_q e_q = 1. \quad (2.1)$

The Clifford product of $e_p, e_q$ is by definition the sum of the scalar product and wedge product furnishing the unit bivector:

$e_p e_q \equiv e_p e_q + e_p \wedge e_q = e_p \wedge e_q = j. \quad j^2 = e_p e_q e_p e_q = -1. \quad (2.2)$
due to the fact that $e_p, e_q$ anticommute, eq. (2.1).

In this fashion, using Clifford algebras one can justify the origins of complex numbers without introducing them ad-hoc. The imaginary unit $j$ is $e_p e_q$. For example, a Clifford vector in phase space can be expanded, setting aside for the moment the issue of units, as:

$$Q = q e_q + p e_p, \quad Q e_q = q + p e_q e_p = q + j p = z. \quad e_q Q = q + p e_q e_p = q - j p = z^*, \quad (2.3)$$

which reminds us of the creation/annihilation operators used in the harmonic oscillator case and in coherent states.

The analog of the action for a massive particle is obtained by taking the scalar product:

$$dQ.dQ = (dq)^2 + (dp)^2 \Rightarrow S = m \int \sqrt{dQ.dQ} = m \int \sqrt{(dq)^2 + (dp)^2}. \quad (2.4)$$

One may insert now the appropriate length and mass parameters in order to have consistent units:

$$S = m \int \sqrt{(dq)^2 + (\frac{\Lambda}{m})^2(dp)^2}. \quad (2.5)$$

where we have introduced the Planck scale $\Lambda$ and the mass $m$ of the particle to have consistent units, $\hbar = c = 1$. The reason will become clear below.

Extending this two-dimensional action to a higher $2n$-dimensional phase space requires to have $e_{p\mu}, e_{q\mu}$ for the Clifford basis where $\mu = 1, 2, 3...n$. The action in this $2n$-dimensional phase space is:

$$S = m \int \sqrt{dq^\mu dq_\mu} + (\frac{\Lambda}{m})^2(dp^\mu dp_\mu) = m \int \sqrt{1 + (\frac{\Lambda}{m})^2(dp^\mu / d\tau)(dp_\mu / d\tau)} \quad (2.6)$$

in units of $c = 1$, one has the usual infinitesimal proper time displacement $d\tau^2 = dq^\mu dq_\mu$.

One can easily recognize that this action (2.6), up to a numerical factor of $m/a$, is nothing but the action for a sub-maximally accelerated particle given by Nesterenko [5]. It is sufficient to rewrite: $dp^\mu / d\tau = md^2 x^\mu / d\tau^2$ to get from eq. (2.6):

$$S = m \int \tau \sqrt{1 + \Lambda^2 (d^2 x^\mu / d\tau^2)(d^2 x_\mu / d\tau^2)}. \quad (2.7)$$
Using the postulate that the maximal-proper acceleration is given in a consistent manner with the minimal length principle (in units of $c = 1$):

$$a = c^2/\Lambda = 1/\Lambda \Rightarrow S = m \int \tau \sqrt{1 + \frac{1}{a^2}(d^2 x^\mu/d\tau^2 (d^2 x_\mu/d\tau^2)} ,$$  \hspace{1cm} (2.1)

which is exactly the action of [5], up to a numerical factor of $m/a$, when the metric signature is $(+,-,-,-)$.

The proper acceleration is *orthogonal* to the proper velocity as a result of differentiating the timelike proper velocity squared:

$$V^2 = \frac{dx^\mu}{d\tau} \frac{dx_\mu}{d\tau} = 1 = V^\mu V_\mu > 0 \Rightarrow \frac{dV^\mu}{d\tau} V_\mu = \frac{d^2 x^\mu}{d\tau^2} V_\mu = 0 ,$$  \hspace{1cm} (2.9)

which means that if the proper velocity is timelike the proper acceleration is spacelike so that:

$$g^2(\tau) \equiv -(d^2 x^\mu/d\tau^2 (d^2 x_\mu/d\tau^2)) > 0 \Rightarrow S = m \int \tau \sqrt{1 - \frac{g^2}{a^2}} \equiv m \int d\omega ,$$  \hspace{1cm} (2.10)

where we have defined:

$$\omega \equiv \sqrt{1 - \frac{g^2}{a^2}}d\tau.$$  \hspace{1cm} (2.11)

The dynamics of a submaximally accelerated particle in Minkowski spacetime can be reinterpreted as that of a particle moving in the spacetime *tangent* – *bundle* background whose Finslerian-like metric is:

$$d\omega^2 = g_{\mu\nu}(x^\mu, dx^\mu)dx^\mu dx^\nu = (d\tau)^2 1 - \frac{g^2}{a^2}.$$  \hspace{1cm} (2.12)

For uniformly accelerated motion, $g(\tau) = g = constant$ the factor:

$$\frac{1}{\sqrt{1 - \frac{g^2}{a^2}}}$$  \hspace{1cm} (2.13)

plays a similar role as the standard Lorentz time dilation factor in Minkowski spacetime.
The action is real valued if, and only if, \( g^2 < a^2 \) in the same way that the action in Minkowski spacetime is real valued if, and only if, \( v^2 < c^2 \). This explains why the particle dynamics has a bound on proper-accelerations. Hence, for the particular case of a \textit{uniformly} accelerated particle whose trajectory in Minkowski spacetime is a hyperbola, one has an Extended Relativity of \textit{uniformly} accelerated observers whose proper-acceleration have an upper bound given by \( c^2/\Lambda \). Rigorously speaking, the spacetime trajectory is obtained by a canonical projection of the spacetime tangent bundle onto spacetime. The invariant time, under the pseudo-complex extension of the Lorentz group [8], measured in the spacetime tangent bundle is no longer the same as \( \tau \), but instead, it is given by \( \omega(\tau) \).

This is similar to what happens in C-spaces, the truly invariant evolution parameter is not \( \tau \) nor \( \Omega \), the Stuckelberg parameter [3], but it is \( \Sigma \) which is the world interval in C-space and that has units of \textit{length}^D. The \textit{group} of C-space Lorentz transformations preserve the norms of the Polyvectors and these have units of hypervolumes; hence C-space Lorentz transformations are volume-preserving.

Another approach to obtain the action for a sub-maximally accelerated particle was given by [8] based on a pseudo-complexification of Minkowski spacetime and the Lorentz group that describes the physics of the spacetime tangent bundle. This picture is not very different form the Finslerian spacetime tangent bundle point of view of Brandt [6]. The invariant group is given by a pseudo-complex extension of the Lorentz group acting on the extended coordinates \( X = ax^\mu + I\nu^\mu \) with \( I^2 = 1 \) (pseudo-imaginary unit) where both position and velocities are unified on equal footing. The invariant line interval is \( a^2d\omega^2 = (dX)^2 \).

A C-phase-space generalization of these actions (for sub-maximally accelerated particles, maximum tidal forces) follows very naturally by using polyvectors:

\[
Y = q^\mu e_{q_\mu} + q^{\mu\nu} e_{q_\mu} \wedge e_{q_\nu} + q^{\mu\nu\rho} e_{q_\mu} \wedge e_{q_\nu} \wedge e_{q_\rho} + \ldots \\
+ p^\mu e_{p_\mu} + p^{\mu\nu} e_{p_\mu} \wedge e_{p_\nu} + \ldots ,
\]

(2.14)

where one has to insert suitable powers of \( \Lambda \) and \( m \) in the expansion to match units.

The C-phase-space action reads then:
This action is the C-phase-space extension of the action of Nesterenko and involves quadratic derivatives in C-spaces which from the spacetime perspective are effective higher derivatives theories [16] where it was shown why the scalar curvature in C-spaces is equivalent to a higher derivative gravity. One should expect a similar behaviour for the extrinsic curvature of a polyparticle motion in C-spaces. This would be the C-space version of the action for rigid particles [7]. Higher derivatives are the hallmark of \( W \)-geometry (higher conformal spins).

Born-Infelds have been connected to maximal-acceleration [8]. Such models admits an straightforward formulation using the geometric calculus of Clifford algebras. In particular one can rewrite all the nonlinear equations of motion in precise Clifford form [9]. This lead the author to propose the nonlinear extension of Dirac’s equation for massless particles due to the fact that spinors are nothing but right/left ideals of the Clifford algebra: i.e., columns, for example, of the Maxwell-Field strength bivector \( F = F_{\mu\nu}\gamma^\mu \wedge \gamma^\nu \).

Actions with higher derivatives may exhibit tachyonic behaviour and may contain ghosts. In C-spaces the actions written for the Clifford polyvector variables do not involve higher derivatives than the quadratic ones. For this reason it is very compelling to suggest that well-behaved physical theories in C-spaces may appear from the ordinary spacetime perspective as tachyonic, since quadratic holographic derivatives involving areas, volumes can be translated as higher derivatives in ordinary spacetime variables [16]. Therefore, the question of whether or not tachyons are unphysical depends on which space perspective one is taking. See [3] for an interesting discussion on this issue from the point of view of the many worlds interpretation of QM.

To sum up, we have now linked the maximal proper acceleration Relativity of the spacetime tangent bundle with the minimal Planck scale Relativity in C-spaces by a simple use of Clifford algebras in phase spaces. We obtained in one stroke the action of Nesterenko compatible with the results of Brandt, Schuller, Caianiello. Moreover, one could naturally generalize these actions by working with the polyvector coordinates associated with a polyparticle in a full-fledged Clifford-phase-space. In this fashion one will have the starting point for a phase space Relativity theory based on Clifford algebras.
Therefore, Extended Scale Relativity in C-spaces admits a natural extension to phase spaces that will allow us to construct connections, curvatures in C-phase spaces in the same way it was achieved for curved C-spaces. The scalar curvature in C-spaces was given by sums of products of ordinary curvature with torsion [16]. The Einstein-Hilbert action in C-spaces is a higher derivative gravity with torsion in the ordinary spacetime. It is unnecessary to insert these higher derivative terms by hand since they follow from the geometry of C-spaces. This agrees with the low energy effective string action obtained from non-linear $\sigma$ s. Recent results to construct a phase space Relativity appeared in [14], however these authors are not invoking an extended Relativity principle based on a minimal Planck scale nor a maximal proper acceleration and the use of Clifford algebras.

### III. Why C-space Relativity over all the others

There are several physical reasons why we believe that the Extended Scale Relativity in C-spaces based on Clifford algebras is more physically appealing than the construction of relativity theories based on kappa-deformed Poincare algebras. The first reason is based on the fact that quantum group symmetries (algebras) do not act on classical spacetimes. Whereas the C-space Lorentz group and the Extended Relativity principle already operates in a classical Clifford space: i.e., the quantization procedure is not responsible for the Extended Relativity principle. One can then proceed to quantize our physical using quantum deformation of Clifford algebras, in particular, Braided Hopf Quantum Clifford algebras.

The Magueijo-Smolin, a particular class of Doubly Special Relativity, was shown recently to be unphysical [15]. This corresponds to a particular basis of kappa-deformed Poincare algebras. Whether these unphysical results will also affect the outcome in other bases is not clear yet. If so this raises questions about the validity of constructing Relativity theories based on such quantum algebra.

Secondly, there is no reason why the Relativity theory based on kappa-deformed Poincare algebras is more advantageous than the ones based on the inhomogeneous Lorentz quantum groups acting in quantum-Minkowski spacetimes [17]. Evenfurther, it was pointed out by Castellani that kappa-deformed Poincare algebras are not bicovariant which is a problem if one wishes to construct physical theories, whereas the quantum algebras used in
his $q$- Gravity construction are bicovariant [17].

If the scale $\Lambda = 1/\kappa$ was identified with the deformation parameter one could equally as well relate the $q$-deformation parameter of the inhomogeneous Lorentz quantum group to the minimum Planck scale. The classical limit is recovered when $q \to 1$; $\Lambda \to 0$.

Furthermore, there exists multiparametric quantum deformations of classical algebras and hence one could have assigned physical meanings (like the minimal invariant scale) to each single one of those quantum parameters.

We may notice that $q$ could be written in terms of an upper and lower scale as: $q = e^{\Lambda/L}$. The classical limit $q = 1$, is obtained when $\Lambda = 0$ and also when $L = \infty$. This entails that there could be two dual quantum gravitational theories with the same classical limit. Castellani has also pointed out that a sort of a “large/small” duality exists in some of these s based on those quantum algebras: there is a symmetry $q \leftrightarrow 1/q$ reminiscent of the $T$-duality in string theory. One can see that by simply changing the signs of $\Lambda$ or of $L$ this is equivalent to replacing $q$ for $1/q$. This large/small duality is just another manifestation of the ultraviolet/infrared entanglement of QFT defined over Noncommutative spaces (geometries). Nottale postulated that if there is a minimum Planck scale, by duality, there should be an upper impassible invariant scale $L$ in Nature. This was his proposal for the resolution of the cosmological constant problem [2].

These arguments raises the possibility that the Double Special Relativity theories based on the kappa-deformed Poincare algebra [10-13] might be obtained by a Moyal deformation quantization procedure in phase spaces where the Moyal deformation parameter is precisely $\Lambda = 1/\kappa$.

Vasiliev [18] has constructed a consistent infinite-component higher spin field theory in Anti de Sitter spaces based on similar star products whose deformation parameter is the inverse size of the Anti de Sitter throat. The “classical” flat spacetime limit is recovered when $l \to \infty$ so $1/l \to 0$. This is an identical situation in constructing the Poincare algebra from the de Sitter algebra by a Wigner- Inonü contraction, when $l = \infty$ the commutators

$$[P^\mu, P^\nu] = \frac{1}{l^2} [J^5_\mu, J^5_\nu] = 0.$$  \hspace{1cm} (3.1)

It is true that Hopf quantum algebras are richer than a naive Moyal deformation of ordinary algebras. In particular, in order to write down the phase space algebra consistent with the Hopf quantum algebra of spacetime it
is necessary to use the Heisenberg-double prescription based on the interplay between the algebraic sector of the Hopf algebra and the co-algebraic sector [10].

Put it simply, one cannot naively use the phase space algebra \([x, p]\) in order to generate the kappa-deformed Poincare algebra in the bicrossproduct basis, or the Snyder basis, for example, by naively writing the boosts-momentum commutator:

\[
[N_j, p_k] = [x_0 p_j - x_j p_0, p_k] = [x_0, p_k] p_j - [x_j, p_k] p_0 , \quad (3.2)
\]
due to the fact that the deformed boost generator has no longer the same form as in the classical case:

\[
N_j^{(\kappa)} = N_j + O(1/\kappa) = x_0 p_j - x_j p_0 + O(1/\kappa) . \quad (3.3)
\]

This is the reason why one must use the Heisenberg-double prescription in order to extract the phase space commutator algebra from the kappa-deformed boosts-momentum commutators and the remaining ones.

Nevertheless, one alternative is not to separate the configuration space from the momentum space but could be by starting to work directly in the phase space and writing down the Moyal star product:

\[
X(q, p, \kappa) \ast P(q, p, \kappa) = \sum_0^{\infty} \frac{(1/\kappa)^n}{n!} \omega^{A_1B_1} \omega^{A_2B_2} \omega^{A_nB_n} (\partial_{A_1} \partial_{A_2} ... \partial_{A_n} X(q, p, \kappa))(\partial_{B_1} \partial_{B_2} ... \partial_{B_n} P(q, p, \kappa)) , \quad (3.4)
\]

using the \(1/\kappa\) as the deformation parameter in order to evaluate the Moyal brackets. The nondegenerate invertible symplectic form \(\omega_{AB}\), associated with the \(2n\)-dimensional phase space is what defines the standard Poisson brackets:

\[
y^A \equiv q^0, q^1, q^2, ..., q^n; p^0, p^1, p^2, ..., p^n \Rightarrow \{y^A, y^B\}_{PB} = \omega^{AB} . \quad (3.5)
\]

The Moyal bracket with respecto to the \((q, p)\) variables is defined in terms of the star product as:

\[
\{X, P\}^{MB} \equiv \frac{X \ast P - P \ast X}{(1/\kappa)} \leftrightarrow \frac{1}{i}[X, P] . \quad (3.6)
\]
Therefore, the following Moyal bracket must obey:

\[
\{N_i, P_j\}_{MB} = \{X_0 P_i - X_i P_0, P_j\}_{MB} = F_{ij}[q(X, P, \kappa), p(X, P, \kappa), \kappa] = \\
\delta_{ij}\left[\frac{\kappa}{2}(1 - e^{-2P_0/\kappa}) + \frac{1}{2\kappa}\vec{P}^2\right] - \frac{1}{\kappa}P_i P_j ,
\]

(3.7)
after re-expressing the bracket back into the original \(X, P\) variables, in order for the Moyal bracket to be isomorphic to the commutator \([\hat{N}_j, \hat{P}_k]_\kappa\) of the kappa-deformed Poincare algebra in the bicrossproduct basis, for example. For this to occur one must find if, and only if, there is a one-to-one and invertible analytic map obeying such conditions:

\[
X = X(q, p, \kappa) = \sum X_{(n)}(q, p)(1/\kappa)^n , \\
P = (q, p, \kappa) = \sum P_{(n)}(q, p)(1/\kappa)^n
\]

and thus \(\Leftrightarrow q = q(X, P, \kappa), \ p = p(X, P, \kappa)\), with the provision that

\[
X_{(o)}^\mu = q^\mu, \quad P_{(o)}^\mu = p^\mu.
\]

(3.9)

It is fairly clear that the classical basis of kappa-deformed Poincare is given precisely by \(X = q, P = p\) so the Moyal bracket algebra collapses to the ordinary Poisson bracket algebra and one recovers the undeformed classical Poincare algebra.

It is essential to notice that the map (3. is noncanonical, i.e., it does not preserve the symplectic form! For this reason, the quantum algebra is not unique in so far that the commutators change under a noncanonical change of basis; i.e., the quantum algebra is not invariant under a noncanonical basis change of coordinates.

If one wishes to reproduce the kappa-deformed Poincare algebra in the Snyder basis, for example, from the classical algebra, one will require a different phase space transformation than before

\[
X' = X'(q, p, \kappa), \quad P' = P'(q, p, \kappa) ,
\]

(3.10)
to ensure that \(\{N_j, P_k\}_{MB}\) is isomorphic to the commutator \([\hat{N}_j, \hat{P}_k]\) in the Snyder basis. Of course, one still has to check that the undeformed sector of the quantum algebra remains intact. We don’t know if this procedure will
reproduce the kappa-deformed Poincare algebra in any basis. Perhaps this is an empty academic exercise because one must be forced to always use the Heisenberg-double prescription, which is deeply ingrained in the algebra and co-algebra sector of the Hopf algebra, in order to read-off the phase space algebra and which does not have a naive Moyal star product interpretation.

In four dimensions we have the choice of eight functions $X(q, p, \kappa), P(q, p, \kappa)$ to match the kappa-deformed Poincare algebra in a given basis. We must see whether or not one can find indeed eight functions that reproduce all the independent commutators of the quantum algebra. Not all commutators are independent due to the constraints imposed by the Jacobi identities. This is not an easy task.

The reason we raise this possibility is because it is plausible that the role of the Planck scale $\Lambda = 1/\kappa$ in kappa-deformed Poincare Relativity might be identical to the deformation parameter of the noncommutative Moyal star product construction in phase spaces. This is in sharp distinction to the natural role of the Planck scale in C-space Relativity: it must be there on pure dimensional grounds to combine objects (strings, branes) of different dimensionalities and consistent with the postulated minimum scale principle. To have real valued intervals $dX.dX > 0$ in C-space requires that the variable scales which encode the magnitudes of the generalized (holographic) velocities cannot be smaller than $\Lambda$ [1].

One main advantage of using Clifford algebras versus the kappa-deformed Poincare algebras is that the C-space Lorentz transformations form a group in a very natural fashion! Recently it has been argued [13] that the kappa-deformed algebra forms a group in the bicrossproduct basis but unfortunately there was a caveat because the infinite series expansion obtained from the Baker-Cambell-Hausdorff formulae might not converge. Even if this were the case, it is not true that one has the required group structure in the other infinite bases which is very unphysical since there is no reason why one basis should be more “physical” than the others, i.e., it questions the minimal length Relativity principle based on kappa-deformed Poincare algebras.

In addition, there still remains the serious problem of how to add momenta. The momenta-addition law is nonabelian in kappa-deformed Poincare due to the nontrivial nature of the co-product and poses problems for the physics of many-particle systems. The nonabelian addition law contradicts well known experimental facts. In C-space, polyvectors are added in a linear and abelian fashion. Finally, there is no need to add an extra discrete
dimension to explain the Snyder noncommutative algebra for the spacetime coordinates [1] nor to work in six-dimensions in order to derive the algebra of the conformal group $SO(4, 2)$. All can be explained naturally from the Clifford algebra of the four-dimensional spacetime [16]. These results are also shared by the pseudo-complexified Minkowski spacetime [8] approach to the maximal acceleration.

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