1. Introduction and Preliminaries

Since 1922, the pioneering fixed-point principle of Banach [1] showed exclusive interest of researchers because it has many applications, including variational linear inequalities and optimization, and applications in differential equations, in the field of approximation theory, and in minimum norm problems.

Since then, several types of contraction mappings have been introduced and many research papers have been written to generalize this Banach contraction principle.

In 1987, Guo and Lakshmikantham [2] introduced one of the most interesting concepts of coupled fixed point.

Definition 1. An element \((x, y) \in E \times E\) is said to be a coupled fixed point of the mapping \(T: E \times E \rightarrow E\) if and only if \(T(x, y) = x\) and \(T(y, x) = y\).

In 2006, Bhaskar and Lakshmikantham [3] introduced the concept of the mixed monotone property as follows.

Definition 2. Let \((E, \leq, \|\|)\) be a partially ordered normed space. Then,

(1) \(E\) is said to be sequentially lower ordered space if it fulfills the condition: If \(\{x_n\}_{n \in \mathbb{N}}\) is a nondecreasing...
sequence in $E$ such that $\{x_n\}_{n \in \mathbb{N}}$ converges strongly to $x$, then $x_n \leq x$ for all $n \in \mathbb{N}$.

(2) $E$ is said to be a sequentially upper-ordered space if it fulfills the condition: If $\{y_n\}_{n \in \mathbb{N}}$ is a nonincreasing sequence in $E$ such that $\{y_n\}_{n \in \mathbb{N}}$ converges strongly to $y$, then $y_n \geq y$ for all $n \in \mathbb{N}$.

(3) $E$ is said to be a sequentially lower-upper ordered space if it is both a lower- and upper-ordered space.

In 2006, Bhaskar and Lakshmikantham [3] proved the existence of coupled fixed points for mixed monotone mappings with weak contractivity assumption in a partially ordered Banach space $(E, \|\cdot\|, \leq)$ as follows.

**Theorem 1** (see [3]). Let $E$ be a sequentially both lower- and upper-ordered Banach space and $T: E \times E \rightarrow E$ be a mapping with mixed monotone and lower-upper properties. If there is a real number $0 \leq k < 1$ such that

$$
\|T(x, y) - T(z, w)\| \leq \frac{k}{2} (\|x - z\| + \|y - w\|),
$$

\forall x, y, z, w \in E, z \leq x, \text{ and } y \leq w, \tag{4}

then $T$ has coupled fixed points in $E$.

In 2013, Mohamed Ali [4] introduced novel contraction type of mappings and proved the following fixed-point theorem.

**Theorem 2** (see [4]). Let $(E, \|\cdot\|)$ be a Banach space and $T$ be a mapping from $E \times E$ into $E$, and we suppose there are three constants $a, b, c \in [0, 1)$ and $a + b + c < 1$ such that

$$
\|T(x, y) - T(y, z)\| \leq a\|x - y\| + b\|T(x, y) - x\| + c\|T(y, z) - y\|, \quad \forall x, y, z \in E. \tag{5}
$$

Then, there is a unique point $x_0 \in E$ such that $T(x_0, x_0) = x_0$.

There are many interesting coupled fixed-point theorems concerning some other type of contraction mappings, see [5–10].

Recently, more advanced approaches for studying coupled fixed points have been presented by the authors in [11–13].

In 2007, Huang and Zhang [14] introduced the concept of cone-metric spaces as follows: First, a subset $M$ of the real Banach space $\mathcal{E}$ is said to be a cone in $\mathcal{E}$ if and only if

1. $M$ is nonempty and $M \neq \{\emptyset\}$, where $\emptyset$ is the zero (neutral) element of $\mathcal{E}$
2. $\lambda M + \mu M \subset M$ for all nonnegative real numbers $\lambda$, $\mu$
3. $M \cap -M = \{\emptyset\}$

If $\text{int} M$ is the set of all interior points of $M$, then a cone $M$ in a normed space $\mathcal{E}$ induces the following ordered relations:

$$
u \prec \nu \iff \nu - \nu \in M, \quad u \prec \nu \iff (\nu - u \in M, \text{ and } u \neq \nu),$$

$$u \prec \nu \iff \nu - u \in \text{int}M. \tag{6}$$

If $E$ is a nonempty set, the distance $d(x, y)$ between any two elements $x, y \in E$ is defined to be a vector in the cone $M$, and the space $(E, d)$ is said to be a cone-metric space if and only if $d$ satisfies the three axioms of metric but using the ordered relation $<$ induced by $M$ for the triangle inequality instead. They studied the topological characterizations of such a defined space, and then, they applied their concept to have more generalizations of some previous fixed-point theorems for contractive type of mappings.

A mapping $T: E \rightarrow E$ is said to be a contraction if and only if there is a constant $a \in [0, 1)$ such that

$$d(T(x), T(y)) < ad(x, y), \quad \forall x, y \in E. \tag{7}$$

In 2019, Mohamed Ali Abou Bakr [15] proved the existence of a unique common fixed point of generalized joint cyclic Banach algebra contractions and Banach algebra Kannan type of mappings on cone quasimetric spaces.

In 2013, Khojasteh et al. [10] introduced the notion of $\theta$-action function, $\theta: [0, \infty) \times [0, \infty) \rightarrow [0, \infty)$, the concept of $\theta$-metric, and then, they studied the topological structures of $\theta$-metric spaces in detail. Their work led to a step-forward generalization of metric spaces.

In 2020, Mohamed Ali Abou Bakr [16] replaced $[0, \infty)$ by a cone $M$ in a normed space and used the ordered relation induced by this cone to introduce the following analogous generalization of $\theta$-action function.

**Definition 5.** Let $(\mathcal{E}, <)$ be an ordered normed space, where $<$ is an ordered relation induced by some cone $M \subset \mathcal{E}$ and $\theta: M \times M \rightarrow M$ be a continuous mapping with respect to each variable, and we denote

$$\text{Im} (\theta) = \{t: t \in M \text{ such that } \exists u_0, v_0 \in E, \quad \theta(u_0, v_0) = t\}. \tag{8}$$

Then, $\theta$ is said to be an ordered action mapping on $\mathcal{E}$ if and only if it satisfies the following conditions:

1. $\theta(\Theta, \Theta) = \Theta$ and $\theta(u, v) = \theta(v, u)$ for every $u, v \in M$
2. $\theta(u, v) < \theta(w, t)$ if either $u < w \text{ and } v < t$; or $u < w \text{ and } v < t$\tag{9}
3. For every $u \in \text{Im}(\theta)$ and every $\Theta < v < u$, there is $\Theta < w < u$ such that $\theta(v, w) = u$
4. $\theta(u, \Theta) < u$ for every $u \in (M/\Theta)$

Because $x - \Theta \in M$ for every $x \in M$, one can write instead $\Theta < x$ for every $x \in M$, $(\Theta < x$ for every $x \in (M/\Theta))$. 
In addition, Mohamed Ali Abou Bakr [16] gave further replacement, replaced the set of nonnegative real numbers \( \mathbb{R}^+ \) by a cone \( M \) in a normed space, and used \( \theta \)-ordered actions to introduce the concept of \( \theta \)-cone-metric space as follows.

**Definition 6** (see [16]). Let \((\mathcal{E}, \prec)\) be an ordered normed space, where \( \prec \) is the ordered relation induced by some cone \( M \subset \mathcal{E} \), and \( \theta \) be an ordered action on \( \mathcal{E} \). If \( E \) is a nonempty set, then the function \( d_\theta; E \times E \rightarrow M \) is said to be a \( \theta \)-cone-metric on \( E \) if and only if \( d_\theta \) satisfies the following conditions:

1. \( d_\theta(x, y) = \Theta \iff x = y \)
2. \( d_\theta(x, y) = d_\theta(y, x), \forall x, y \in E \)
3. \( d_\theta(x, y) \prec \theta(d_\theta(x, z), d_\theta(z, y)), \forall x, y, z \in E \)

The double \((E, d_\theta)\) is defined to be a \( \theta \)-cone-metric space.

The author has further given some topological characterizations of this space and then generalized some previous fixed-point theorems in this setting.

**Remark 1.** If \( \theta(u, v) = u + v \), then we have a cone-metric space.

In this paper, we extend and generalize the coupled fixed-point theorem of Baskar–Lackshmikantham (1.5) to a more general one (2.1), where the underlying space \((E, d_\theta)\) is a complete \( \theta \)-cone-metric space. On the other side, if \( T; E \times E \rightarrow E \) is a continuous mapping in the second argument and there are three constants \( a, b, c \in [0, 1) \) and \( a + b + c < 1 \) such that

\[
d_\theta(T(x, y), T(y, z)) < ad_\theta(x, y) + bd_\theta(T(x, y), x) + cd_\theta(T(y, z), y), \quad \forall x, y, z \in E, \tag{10}\]

then we proved that \( T \) has a unique fixed point in the sense that there is a unique point \( x \in E \) such that \( T(x, x) = x \).

We also claim that some results of [6–10, 17] can be proved in the case of \( \theta \)-cone-metric spaces.

### 2. Main Results

Let \((E, d_\theta, \leq)\) be a partially ordered \( \theta \)-cone-metric space. Then, the following relation defines a partial-ordered relation on \( E \times E \):

\[
(x, y) \ll (z, w) \iff x \ll z, \text{ and } w \ll y. \tag{11}\]

We have the following coupled fixed-point theorem.

**Theorem 3.** Let \((E, d_\theta, \leq)\) be a partially ordered, sequentially lower-upper ordered complete \( \theta \)-cone-metric space and \( G; E \times E \rightarrow E \) be a mapping having mixed monotone and lower-upper properties on \( E \). We assume that there exists \( r \in [0, 1) \) with

\[
d_\theta(G(x, y), G(z, w)) \leq \frac{r}{2} [d_\theta(x, z) + d_\theta(y, w)], \quad \forall (x, y) \ll (z, w). \tag{12}\]

Then, \( G \) has coupled fixed points in \( E \).

**Proof.** Since \( G \) has a lower-upper property, then there exist \( x_0, y_0 \in E \) such that

\[
x_0 \leq G(x_0, y_0) \quad \text{and} \quad y_0 \leq G(y_0, x_0) \leq y_0. \tag{13}\]

We denote \( x_1 = G(x_0, y_0) \) and \( y_1 = G(y_0, x_0) \), and then give notations for the elements of the following inductively constructed sequences:

\[
x_2 = G(x_1, y_1) = G^2(x_0, y_0), \quad y_2 = G(y_1, x_1) = G^2(y_0, x_0),
\]

\[
x_3 = G(x_2, y_2) = G^3(x_0, y_0), \quad y_3 = G(y_2, x_2) = G^3(y_0, x_0),
\]

\[\ldots\]

\[
x_n+1 = G(x_n, y_n) = G^{n+1}(x_0, y_0), \quad y_{n+1} = G(y_n, x_n) = G^{n+1}(y_0, x_0),
\]

\[\ldots\]

Using the mixed monotonicity property of \( G \) insures that each step leads to the next step in each of the following:

\[
x_0 \leq x_1 = G(x_0, y_0) \leq G(x_1, y_1) = x_2,
\]

\[
y_2 = G(y_1, x_1) \leq G(y_0, x_0) = x_1 \leq y_0,
\]

\[
x_1 \leq x_2 = G(x_1, y_1) \leq G(y_0, x_0) = y_1 \leq y_0,
\]

\[
y_3 = G(y_2, x_2) \leq G(y_1, x_1) = y_2 \leq y_1
\]

\[\ldots\]

\[
x_{n+1} = G(x_n, y_n) \leq G(x_{n-1}, y_{n-1}) = x_n,
\]

\[
y_{n+1} = G(y_n, x_n) \leq G(y_{n-1}, x_{n-1}) = y_n,
\]

\[\ldots\]

The mixed monotonicity property, the contractiveness of \( G \), and the inductive process prove the following for every \( n \in \mathbb{N} \):

\[
d_\theta(G^{n+1}(x_0, y_0), G^n(x_0, y_0)) \leq \frac{r}{2} [d_\theta(G(x_0, y_0), x_0) + d_\theta(G(y_0, x_0), y_0)],
\]

\[
d_\theta(G^{n+1}(y_0, x_0), G^n(y_0, x_0)) \leq \frac{r}{2} [d_\theta(G(y_0, x_0), y_0) + d_\theta(G(x_0, y_0), x_0)]. \tag{16}\]
Consequently, we have
$$\lim_{n \to \infty} d_{\theta}(G^{n+1}(x_0, y_0), G^n(x_0, y_0)) = \lim_{n \to \infty} d_{\theta}(G^{n+1}(x_0, y_0), G^n(x_0, y_0)) = \Theta. \quad (17)$$

Hence, we claim that both \( \{G^n(x_0, y_0)\}_{n \in \mathbb{N}} \) and \( \{G^n(y_0, x_0)\}_{n \in \mathbb{N}} \) are Cauchy sequences in \( E \). Indeed, if one of them, say \( \{G^n(x_0, y_0)\}_{n \in \mathbb{N}} \), is not Cauchy, then there exist \( v \in \text{Im}(\theta) \), \( \Theta < v \) and sequences of natural numbers \( \{i_n\}_{n \in \mathbb{N}} \) and \( \{j_n\}_{n \in \mathbb{N}} \) such that, for any \( i_n > j_n > n \),
$$v < d_{\theta}(G^{i_n}(x_0, y_0), G^{j_n}(x_0, y_0)), \quad d_{\theta}(G^{j_n-1}(x_0, y_0), G^{i_n}(x_0, y_0)) < v. \quad (18)$$

Similarly, the sequence \( \{G^n(y_0, x_0)\}_{n \in \mathbb{N}} \) is also Cauchy. Since \( E \) is a complete \( \theta \)-cone-metric space, there exist \( x, y \in E \) such that
$$\lim_{n \to \infty} d_{\theta}(G^n(x_0, y_0), x) = \Theta, \quad \lim_{n \to \infty} d_{\theta}(G^n(y_0, x_0), y) = \Theta. \quad (20)$$

\[d_{\theta}(G(x, y), x) \leq \theta\left(\frac{r}{2}\right) [d_{\theta}(x, x_0)] + d_{\theta}(y, G^n(y_0, x_0), x), \]
\[= \theta\left(\frac{r}{2}\right) [d_{\theta}(x, x_0)] + d_{\theta}(y, G^n(y_0, x_0), x)\]
\[= \theta\left(\frac{r}{2}\right) [d_{\theta}(x, x_0)] + d_{\theta}(y, G^n(y_0, x_0), x). \quad (21)\]

Taking the limit as \( n \to \infty \) with the help of equation (20), we find that
$$d_{\theta}(G(x, y), x) < \theta\left(\frac{r}{2}\right) [\Theta + \Theta], \Theta = \theta(\Theta, \Theta) = \Theta. \quad (22)$$

Hence, \( d_{\theta}(G(x, y), x) = \Theta \); therefore, \( G(x, y) = x \).

Similarly, \( G(y, x) = y \).

If the partial-ordered relation on \( E \times E \) is defined as
\[(x, y) \ll (z, w) \iff x \geq z, \text{ and } y \leq w, \quad (23)\]
then the following theorem is similarly proved.

**Theorem 4.** Let \( (E, d_{\theta}, \leq) \) be a partially ordered, sequentially lower-upper ordered complete \( \theta \)-cone-metric space and \( G \) be a sequentially both lower- and upper-ordered Banach space with \( T(x_0, y_0) \leq x_0 \) and \( y_0 \leq T(y_0, x_0) \). If there exists \( r \in [0, 1) \) with
$$d_{\theta}(G(x, y), G(z, w)) < \frac{r}{2} \left[d_{\theta}(x, z) + d_{\theta}(y, w)\right], \quad (24)$$
then \( G \) has coupled fixed points in \( E \).

**Corollary 1.** Let \( E \) be a sequentially both lower- and upper-ordered Banach space and \( T: E \times E \to E \) be a mapping with mixed monotone and lower-upper properties. If there is a real number \( 0 < k < 1 \) such that
\[ \|T(x, y) - T(z, w)\| \leq \frac{k}{2} (\|x - z\| + \|y - w\|), \]
\[ \forall x, y, z, w \in E, z \leq x, \text{ and } y \leq w, \]
(25)

then \( T \) has a coupled fixed point in \( E \).

**Proof.** We just notice that any Banach space \((E, \|\cdot\|)\) is a \( \theta \)-cone-metric space \((E, d_\theta)\), where \((\mathbb{R}, \leq) = (\mathbb{R}, |.|)\) is the Banach space of real numbers with the absolute value metric and with the usual ordered relation of real numbers, \( \theta(u, v) = u + v, \theta: [0, \infty) \times [0, \infty) \rightarrow [0, \infty) \), and the metric \( d_\theta \) is the metric induced by the norm \( \|\cdot\| \) on \( E \),
\[ d_\theta(x, y) = \|x - y\|. \]
□

**Remark 2.** Corollary 1 is Baskar–Lakshmikantham coupled fixed-point Theorem 1. This proves that Theorem 3 is a quite good generalization of the Baskar–Lakshmikantham coupled fixed-point theorem.

On the other side, we have the following results:

**Lemma 1.** Let \((E, d_\theta)\) be a \( \theta \)-cone-metric space and \( T \) be a mapping, \( T: E \times E \rightarrow E \). It is supposed that there are constants \( a, b, c \in [0, 1) \) and \( a + b + c < 1 \) such that
\[ d_\theta(T(x, y), T(y, z)) < ad_\theta(x, y) + bd_\theta(T(x, y), x) \]
\[ + cd_\theta(T(y, z), y), \quad \forall x, y, z \in E. \]
(26)

If \( x_1 \) and \( x_2 \) are arbitrary elements in \( E \), then the sequence \( \{x_n\}_{n=1}^\infty \) defined iteratively by
\[ x_n = T(x_{n-1}, x_{n-2}), \quad \forall n \in \mathbb{N}, n > 2, \]
(27)

which satisfies the following:
\[ d_\theta(x_{n+1}, x_n) < td_\theta(x_{n}, x_{n-1}), \quad \forall n > 2, \]
(28)
\[ d_\theta(x_{n+1}, x_n) < t^2d_\theta(x_{2}, x_{1}), \quad \forall n > 2, \]
(29)
where \( t = (a + c)/1 - b \). Moreover, the sequence \( \{x_n\}_{n=1}^\infty \) is a Cauchy sequence.

**Proof.** Using the contractiveness property of the given mapping gives
\[ d_\theta(x_{n+1}, x_n) = (d_\theta(T(x_n, x_{n+1})), T(x_{n-1}, x_{n-2})) \]
\[ < ad_\theta(x_n, x_{n-1}) + bd_\theta(x_{n+1}, x_n) + cd_\theta(x_{n}, x_{n-1}). \]
(30)

Hence,
\[ d_\theta(x_{n+1}, x_n) < \left( \frac{a + c}{1 - b} \right)d_\theta(x_n, x_{n-1}), \quad \forall n > 2, \]
(31)
and repeating the last step \( n - 2 \) times with the term \( d_\theta(x_n, x_{n-1}) \) proves the inequalities given in (29). To prove that the sequence (27) is Cauchy, we take the limit of both sides of (29) as \( n \rightarrow \infty \) gives \( \lim_{n \rightarrow \infty} d_\theta(x_{n+1}, x_n) = \Theta \) and suppose that \( \{x_n\}_{n=1}^\infty \) is not Cauchy; then, there exist \( v \in \text{Im}(\theta), \Theta < v \) and sequences of natural numbers \( \{i_n\}_{n=1}^\infty \) and \( \{j_n\}_{n=1}^\infty \) such that, for any \( i_n > j_n > n, \)
\[ v < d_\theta(x_{i_n}, x_{j_n}). \]
(32)

Since any subsequence of \( \{d_\theta(x_{n+1}, x_n)\}_{n=1}^\infty \) is convergent to \( \Theta \), the continuity and the properties of \( \theta \) imply the following contradiction:
\[ v < d_\theta(x_{i_n}, x_{j_n}) < \theta\left(d_\theta(x_{i_{n-1}}, x_{i_n}), d_\theta(x_{i_n}, x_{i_{n-1}})\right) \]
\[ < \theta(v, d_\theta(x_{i_n}, x_{i_{n-1}})) \rightarrow n \rightarrow \infty \theta(v, \Theta) < v. \]
□

**Theorem 5.** Let \((E, d_\theta)\) be a complete \( \theta \)-cone-metric space and \( T: E \times E \rightarrow E \) be a continuous mapping in the second argument, and we suppose there are three constants \( a, b, c \in [0, 1) \) and \( a + b + c < 1 \) such that
\[ d_\theta(T(x, y), T(y, z)) < ad_\theta(x, y) + bd_\theta(T(x, y), x) \]
\[ + cd_\theta(T(y, z), y), \quad \forall x, y, z \in E, \]
(34)

and then, \( T \) has a unique fixed point in the sense that there is a unique point \( x_0 \in E \) such that \( T(x_0, x_0) = x_0 \).

**Proof.** Since \((E, d_\theta)\) is complete, the Cauchy sequence \( \{x_n\}_{n=1}^\infty \) given in Lemma 1 is converging to some element \( x_0 \) in \( E \). We show that \( x_0 \) is fixed point of \( T \). Using the properties of \( \theta \) and the continuity of \( T \), we see that
Since $b < 1$, we get $d_\theta(T(x_n, x_0), x_0) = \Theta$; consequently, $T(x_0, x_0) = x_0$. Now, let $x$ and $y$ be two arbitrarily distinct elements in $E$ with $T(x, x) = x$ and $T(y, y) = y$, and we have

$$
\begin{align*}
\quad & d_\theta(T(x, y), x) < \theta(d_\theta(T(x, x), T(x, y)), d_\theta(T(x, x), x)) \\
& < \theta(b d_\theta(T(x, x), x) + c d_\theta(T(x, y), x), x) \\
& < \theta(b \Theta + c \Theta, x, \Theta) \\
& < c d_\theta(T(x, y), x).
\end{align*}
$$

Thus, $d_\theta(T(x, y), x) = \Theta$, that is, $T(x, y) = x$. Similarly, we get $T(y, x) = y$; therefore, $(x, y)$ is a coupled fixed point of $T$. On the other hand, we have the following contradiction:

$$
\begin{align*}
\quad & d_\theta(x, y) = d_\theta(T(x, x), T(y, y)) \\
& < \theta(d_\theta(T(x, x), T(y, y)), d_\theta(T(x, y), y)) \\
& < \theta(c d_\theta(T(x, y), x), a d_\theta(x, y) + b d_\theta(T(x, y), x)) \\
& < \theta(c d_\theta(x, y), a d_\theta(x, y) + b d_\theta(x, x)) \\
& < \theta(\Theta, a d_\theta(x, y) + \Theta) < a d_\theta(x, y).
\end{align*}
$$

Since $a < 1$, we have $d_\theta(x, y)$; consequently, $x = y$. □

We conclude the following.

**Corollary 2.** Let $(E, [\cdot, \cdot])$ be a Banach space and $T$ be a mapping from $E \times E$ into $E$, and we suppose that there are three constants $a, b, c \in [0, 1)$ and $a + b + c < 1$ such that

$$
\begin{align*}
\|T(x, y) - T(y, z)\| & \leq a \|x - y\| + b \|T(x, y) - x\| \\
& + c \|T(y, z) - y\|, \quad \forall x, y, z \in E.
\end{align*}
$$

Then, there is a unique point $x_0 \in E$ such that $T(x_0, x_0) = x_0$.

**Proof.** It can be proved in a similar way of Corollary 1 with the same notice. □

**Remark 3.** Corollary 2 is the fixed-point theorem of Mohamed Ali Abou Bakr; accordingly, Theorem 5 is a generalization of fixed-point Theorem 2 in the setting of a complete $\theta$-cone-metric space.

### 3. Conclusions

This paper gives further generalizations of some well-known coupled fixed-point theorems. Specifically, Theorem 3 generalizes the Baskar–Lackshminantham coupled fixed-point theorem [3], and Theorem 5 generalizes the Sahar Mohamed Ali Abou Bakr fixed-point theorem [4]; the underlying space $(E, d_\theta)$ is a complete $\theta$-cone-metric space, and we claim that some results of [6–10] can be proved in the case of $\theta$-cone-metric spaces.

**Data Availability**

No data were used to support this study.

**Disclosure**

This research was performed as part of the employment of Dr. Sahar Mohamed Ali Abou Bakr at Ain Shams University, Faculty of Science, Department of Mathematics, Cairo, Egypt.

**Conflicts of Interest**

The author has no conflicts of interest.
Authors’ Contributions
The sole author contributed 100% to the article. The author read and approved the final manuscript.

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