Symmetry Tests within the Standard Model and Beyond
from Nuclear Muon Capture

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Abstract

Precision measurements in nuclear muon capture on the proton and $^3$He allow for tests of the Standard Model for the strong and electroweak interactions, complementary to those achieved in high energy experiments. The present situation and future prospects are reviewed, emphasizing where renewed efforts could prove to be rewarding in exploring ever further beyond the confines of the Standard Model.

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1 Introduction

With the availability of intense muon beams of well defined characteristics, such as those at the Paul Scherrer Institute, as well as much improved and new experimental techniques, muonic physics has regained much of its past impetus. Precision tests of the Standard Model of the strong and electroweak interactions have become a reality in recent years, with rare or dominant muon decay modes \[1\] and nuclear capture processes contributing to possibly unveiling the new physics which is lurking beyond the confines of the Standard Model, in ways complementary to those of experiments at much higher energies.

In the cases of nuclear muon capture on \(^{3}\)He and the proton, achieved \[2, 3\] or foreseen \[2, 4\] precisions allow for specific tests of symmetries both of the hadronic sector of the Standard Model and the electroweak interaction \[3, 5, 6, 7\]. For instance, muon capture on hydrogen within the Standard Model involves directly the nucleon matrix elements of the vector and axial quark current operators, which, by virtue of Lorentz covariance, are parametrised according to the expressions

\[
\begin{align*}
\langle n|\bar{d}\gamma_{\mu}u|p\rangle &= \bar{n}\left[ g_{V}\gamma_{\mu} + ig_{M}\sigma_{\mu\nu}\frac{q^{\nu}}{2M_{N}} + g_{S}\frac{q_{\mu}}{2M_{N}} \right] p, \\
\langle n|\bar{d}\gamma_{\mu}\gamma_{5}u|p\rangle &= \bar{n}\left[ g_{A}\gamma_{\mu}\gamma_{5} + g_{P}\gamma_{5}\frac{q_{\mu}}{m_{\mu}} + ig_{T}\sigma_{\mu\nu}\gamma_{5}\frac{q^{\nu}}{2M_{N}} \right] p,
\end{align*}
\]

where the quantities \(g_{V}, g_{M}, g_{S}, g_{A}, g_{P}\) and \(g_{T}\) are form factors which are functions of the momentum transfer invariant \(q^{2}\) with \(q^{\mu} = p_{n}^{\mu} - p_{p}^{\mu}\) \((u, d, p\) and \(n\) stand for the Dirac quantum spinor field operators of massive spin 1/2 relativistic particles, \(M_{N}\) for the average nucleon mass and \(m_{\mu}\) for the muon mass). These form factors provide for a phenomenological parametrisation of the non perturbative quark bound state structure of the nucleons, to be determined from experimental observables and symmetry considerations. Requiring invariance of the above matrix elements under time reversal implies all these form factors to be real under complex conjugation. Imposing exact \(G\)-parity invariance, \textit{i.e.} exact isospin and charge conjugation symmetry, implies that the second-class form factors \(g_{S}\) and \(g_{T}\) vanish identically for all \(q^{2}\) values (isospin breaking effects are such that \(|g_{S}/g_{V}|\) and \(|g_{T}/g_{A}|\) are expected \[8\] not to exceed 0.01 to 0.02). Likewise, in the limit of the exact conservation of the vector current—the CVC hypothesis—, the remaining vector current form factors \(g_{V}\) and \(g_{M}\) are related to those of the electromagnetic current, which are probed through electron scattering experiments. From the latter data \[9\], one deduces \(g_{V}(q_{0}^{2}) = 0.9755 \pm 0.0005\) and \(g_{M}(q_{0}^{2}) = 3.582 \pm 0.003\), \(q_{0}^{2} = -0.88\) \(m_{\mu}^{2}\) being the invariant momentum transfer relevant to muon capture on the proton. The value for \(g_{A}(q_{0}^{2})\) follows
from $g_A(q^2 = 0) = 1.2601 \pm 0.0025$ [10] and the nucleon axial charge radius [11], so that $g_A(q_0^2) = 1.238 \pm 0.003$. Finally, the value for $g_P(q^2)$ is related to that of $g_A(q^2)$ through the partial conservation of the axial current (PCAC) hypothesis, which in modern terms is embodied in the approximate chiral symmetries of the underlying theory for the strong interactions among quarks, namely quantum chromodynamics (QCD). Chiral perturbation theory leads to a value for $g_P(q_0^2)$ which depends in particular on the pion-nucleon coupling constant. The latest prediction [12] is precise to 2.7%,

$$g_P(q_0^2) = 8.44 \pm 0.23, \quad \frac{g_P(q_0^2)}{g_A(0)} = 6.70 \pm 0.18. \quad (3)$$

This result is in fair agreement with the present experimental value stemming from ordinary muon capture on the proton [13], $g_P(q_0^2)/g_A(0) = 6.9 \pm 1.5$, precise to 22%. However, it is in flagrant conflict with a recent radiative muon capture measurement [14] precise to 8%, namely $g_P(q_0^2)/g_A(0) = 9.8 \pm 0.8$, which thus disagrees with the theoretical prediction by a large 4.2 $\sigma$ margin.

Clearly, such a situation in the hadronic sector of the Standard Model calls for a renewed effort in a precision measurement of an observable in muon capture on hydrogen which is sensitive to $g_P$, both to reach the precision level of the theoretical prediction as an important test of our understanding of the chiral symmetry properties of non perturbative QCD, as well as to dispel the present conflict within the experimental situation. As the discussion which is to follow will illustrate, this is but one instance of a precision measurement in nuclear muon capture which offers the potential for testing underlying symmetries of the strong and electroweak interactions.

More specifically, we shall concentrate on three types of observables. First, a measurement of the statistical capture rate on $^3$He to the triton channel [2, 3], $\lambda_{\text{stat}}^{\exp} = 1496 \pm 4$ s$^{-1}$, precise to 0.3%, which agrees remarkably well with the theoretical prediction [15] of $\lambda_{\text{stat}}^{\text{theor}} = 1497 \pm 12$ s$^{-1}$. Second, the triton asymmetry for capture in a polarised $\mu^-^3$He system, whose vector analysing power $A_v$ is predicted [15] to be $A_v^{\text{theor}} = 0.524 \pm 0.006$, to be compared to the preliminary experimental value $A_v^{\exp} = 0.63 \pm 0.09^{+0.11}_{-0.14}$ [16]. And third, the foreseen 1% precision in the measurement of the singlet capture rate on hydrogen [4, 1], which should allow for a determination of $g_P(q_0^2)$ to better than 6%. The prospects offered by these different observables will be considered first within the hadronic sector of the Standard Model, and next, within a phenomenological context beyond that Model. The presentation thus follows that same outline.
2 Muon Capture within the Standard Model

Since the \((p, n)\) and \((^3\text{He}, ^3\text{H})\) systems are both spin 1/2 isospin doublets, a phenomenological description of muon capture on either hydrogen or \(^3\text{He}\) proceeds in a similar manner. Thus within the Standard Model, the effective Hamiltonian for this semi-leptonic process reads

\[
H_{\text{eff}}^L = \frac{g_2^L}{8M_W^2} V_{ud}^L J^\mu_{\text{lept}} \Gamma J^\mu_{\text{hadr}} , \quad \frac{g_2^L}{8M_W^2} = \frac{G_F}{\sqrt{2}} . \tag{4}
\]

Here, \(G_F/\sqrt{2}\) represents the Fermi coupling strength, \(V_{ud}^L = 0.9751 \pm 0.0006\) \[10\] the Cabibbo-Kobayashi-Maskawa up-down quark flavour mixing matrix element, and \(J^\mu_{\text{lept}}\), \(J^\mu_{\text{hadr}}\) the leptonic and hadronic charged currents, respectively. For the muon leptonic flavour, \(J^\mu_{\text{lept}} = \bar{\mu} \gamma^\mu (1 - \gamma_5) \nu_\mu\), while the hadronic current is of the \((V-A)\) form, \(J^\mu_{\text{hadr}} = V^\mu_{\text{hadr}} - A^\mu_{\text{hadr}}\), with matrix elements \(V^\mu_{\text{hadr}}\) and \(A^\mu_{\text{hadr}}\) of the quark vector and axial current operators parametrised as in \[1\] and \[2\] (in the case of muon capture on \(^3\text{He}\), the relevant form factors are denoted rather as \(F_V, F_M, F_S, F_A, F_P\) and \(F_T\)). This description corresponds to the so called “elementary particle model” (EPM) approach \[17\], in which the underlying bound state structure of nuclei is represented through phenomenological form factors.

For capture on hydrogen, the relevant momentum transfer is \(q_{01}^2 = -0.88 m_\mu^2\), while for capture on \(^3\text{He}\), it is \(q_{11}^2 = -0.954 m_\mu^2\).

Values for these form factors for the \((p, n)\) system have been discussed above. For the \((^3\text{He}, ^3\text{H})\) system, the authors of \[15\] performed a very careful assessment of these values, with the following conclusions. For the vector current, one has the first-class form factors \(F_V(q_{11}^2) = 0.834 \pm 0.011\) and \(F_M(q_{11}^2) = -13.969 \pm 0.052\). For the axial current, \(F_A(q_{11}^2) = -1.052 \pm [0.005 - 0.010]\) stems for the \(\beta\)-decay rate of \(^3\text{H}\) and an educated guess as to the \(q^2\)-dependence of this form factor, which attempts at including mesonic exchange current corrections. The uncertainty in this dependency leads to the range \([0.005 - 0.010]\) in the error given for \(F_A(q_{11}^2)\), with the truth lying somewhere in between \[14\]. Consequently, uncertainties of results to be quoted hereafter will include this range of values for \(F_A(q_{11}^2)\). The value for \(F_P(q_{11}^2)\) is again determined from the PCAC hypothesis, which implies \(F_{P,\text{PCAC}}(q^2) = 2m_\mu M F_A(q^2)/(m_\pi^2 - q^2)\), \(M\) being the average mass value of the initial and final nuclear states (strictly speaking, this relation assumes that the \(q^2\) dependencies of \(F_A(q^2)\) and the \(\pi-^3\text{He}^3\text{H}\) coupling constant are identical \[13\]). Finally, in the limit of exact \(G\)-parity invariance, \(F_S\) and \(F_T\) vanish identically, with isospin breaking corrections being at most of a few percent.

Given these values, the statistical capture rate \(\lambda_{\text{stat}}\) on \(^3\text{He}\) to the triton channel, as well as the triton vector analysing power \(A_v\), are predicted to be \[13\]
\[ \lambda_{\text{theor}} = 1497 \pm [12 - 21] \text{ s}^{-1} \text{ and } A_{v \text{ theor}} = 0.524 \pm [0.006 - 0.006], \]

with sensitivities to \( F_A \) and \( F_P \) given by \( F_A / \partial \sigma / \partial F_A = (1.521, -0.134) \text{ and } F_P / \partial \sigma / \partial F_P = (-0.116, -0.377) \) where \( \sigma = (\lambda_{\text{stat}}, A_v) \) in the same order. Note the rather large sensitivity of the capture rate to \( F_A \), whose uncertainty thus dominates that of \( \lambda_{\text{theor}} \), a situation which is opposed to that for \( A_v \), the latter observable being also over three times more sensitive to \( F_P \) than is \( \lambda_{\text{stat}} \). Consequently, a combined precision measurement of both \( \lambda_{\text{stat}} \) and \( A_v \) would enable a model independent determination of \( F_A \) and \( F_P \), and thereby a convincing test of nuclear PCAC. A very precise value for \( \lambda_{\text{stat}} \) is indeed available \([3]\), but the preliminary result \([16]\) for \( A_v \) is not to the required standard.

More specifically, given the result \( \lambda_{\text{exp stat}} = 1496 \pm 4 \text{ s}^{-1} \) \([3]\), and fixing the values for all form factors as explained above with the exception of \( F_P \), the nuclear PCAC test is \([3, 5, 6]\) \( F_P / F_{PCAC} = 1.004 \pm [0.076 - 0.132]\) [exp : 0.023], where the first two numbers in brackets include all theoretical and experimental uncertainties and correspond to the range of values associated to the uncertainty in \( F_A \), while the number indicated with “exp” only includes the uncertainty stemming from the experimental error on \( \lambda_{\text{exp stat}} \) alone. Clearly, this test of nuclear PCAC precise to about 10% could be improved to some extent were a better value for \( F_A \) to be available independently. Within an impulse approximation nuclear model calculation including mesonic exchange corrections \([18]\), the same experimental result leads to a 18% precise PCAC test at the nucleon level, \( g_P / g_{PCAC} = 1.05 \pm 0.19 \) \([18]\). Even though the precision of the theoretical prediction \([3]\) is yet to be attained, these conclusions show agreement with QCD chiral perturbation theory, as opposed to the result of \([14]\) (incidentally, note that the argument may be turned around, and used to determine a rather precise value for the \( \pi - ^3\text{He} - ^3\text{H} \) coupling constant \([19]\)).

Similarly, given the same purpose, let us consider the triton vector analysing power \( A_v \), assuming a value precise to 1% centered onto the theoretical prediction of \( A_{v \text{ theor}} = 0.524 \). All other form factors being fixed at their specified values, the nuclear PCAC test for \( F_P \) would then be precise to 3.9%, irrespective of the uncertainty on \( F_A \) in the range \([0.005 - 0.010]\), while including only the experimental error of 1% on \( A_v \) would provide a PCAC test for \( F_P \) to 2.7%. It may also be shown that extracting combined values for \( F_A \) and \( F_P \) from both \( \lambda_{\text{stat}} \) and \( A_v \), with a precision on \( F_A \) at least as good as the present range of \([0.005 - 0.010]\), requires a measurement of \( A_v \) to at least 1% relative precision, no small feat by any means, but a worthy experimental challenge indeed!

Similar considerations may be developed for the second-class form factors \( F_S \) and \( F_T \), assuming all other form factors set to their specified values (in particular, it may be shown that by letting \( F_P \) vary within 10% of its value, the values for
and assuming either one of the factors

$$F_S$$ and $$F_T$$ also vary within their respective uncertainties. Here again, it is $$A_v$$ which offers the better prospects for improvement, with sensitivities such that

$$\frac{1}{\mathcal{O} d\mathcal{O}/dF_S} = (0.007, 0.017)$$ and $$\frac{1}{\mathcal{O} d\mathcal{O}/dF_T} = (-0.006, -0.019)$$ for $$\mathcal{O} = (\lambda_{\text{stat}}, A_v)$$ in the same order. Specifically, with the result $$\lambda_{\text{stat}}^{\exp} = 1496\pm4 \text{ s}^{-1},$$ and assuming either one of the factors $$F_S$$ or $$F_T$$ to vanish in turn, one obtains $$F_S = -0.062 \pm [1.18 - 2.02] [\exp : 0.38], F_T = 0.075 \pm [1.43 - 2.45] [\exp : 0.46].$$ These results improve on the existing situation for these second-class form factors

$$\lambda_{\text{stat}}^{\exp} = 1496\pm4 \text{ s}^{-1},$$ and much improving the limits on second-class currents in the muon semi-leptonic sector.

$$\lambda_{\text{stat}}^{\exp} = 1496\pm4 \text{ s}^{-1},$$

On the other hand, given a measurement of $$A_v$$ precise to 1% in the manner assumed above, the corresponding uncertainties would be $$[0.9 - 0.9] [\exp : 0.58]$$ and $$[0.8 - 0.8] [\exp : 0.54]$$ for $$F_S$$ and $$F_T$$, respectively. Hence here again, one would gain both from a better theoretical knowledge of $$F_A$$, as well as from a 1% precise measurement of $$A_v$$.

Turning to muon capture on hydrogen, a similar analysis may be applied. Sensitivities to form factors of the singlet capture rate $$\lambda_S$$ are as follows, $$g_X/\lambda_S d\lambda_S/dg_X = (0.47, 0.15, 1.57, -0.18)$$ for $$g_X = (g_V, g_M, g_A, g_P),$$ and $$1/\lambda_S d\lambda_S/dg_X = (0.023, 0.024)$$ for $$g_X = (g_S, g_T).$$ With regards to the first-class form factors, the situation is comparable to that for $$^3\text{He,}$$ with the important difference however, that the value for $$g_A(q_0^2)$$ is known to much better precision. With regards to the second-class form factors $$g_S$$ and $$g_T$$ as well, the sensitivity is also much improved.

Specifically, a 1% precise measurement of $$\lambda_S$$ centered at its theoretical value implies $$g_P = 8.44 \pm [0.50] [\exp : 0.46]$$—namely a PCAC test at the nucleon level precise to 5.9% (exp: 5.5%)—, as well as $$g_S = 0.0 \pm [0.51] [\exp : 0.43]$$ and $$g_T = 0.0 \pm [0.50] [\exp : 0.42],$$ thereby zooming into the theoretically expected range of values for these form factors, and much improving the limits on second-class currents in the muon semi-leptonic sector.

### 3 Muon Capture beyond the Standard Model

Any new physics contribution beyond the Standard Model may phenomenologically be parametrised according to the following effective Hamiltonian

$$\mathcal{H}_{\text{eff}} = \frac{g^2}{\sqrt{M}} V_{ud} \sum_{\eta_1, \eta_2 = +, -} \left[ (h^Y_{\eta_1, \eta_2})^* \bar{\nu}_\mu \gamma^\mu (1 + \eta_1 \gamma_5) \mu \bar{d} \gamma_\mu (1 + \eta_2 \gamma_5) u + + (h^S_{\eta_1, \eta_2})^* \bar{\nu}_\mu (1 + \eta_1 \gamma_5) \mu \bar{d} (1 - \eta_2 \gamma_5) u + + \frac{1}{2} (h^T_{\eta_1, \eta_2})^* \bar{\nu}_\mu \sigma^{\mu\nu} (1 + \eta_1 \gamma_5) \mu \bar{d} \sigma_{\mu\nu} (1 - \eta_2 \gamma_5) u \right].$$

(5)
Here, $g$, $V_{ud}$ and $M^2$ are arbitrary parameters, which in the Standard Model coincide with the quantities introduced in (1), while the coefficients $h^{S,V,T}_{\eta_1 \eta_2}$ are effective couplings constants parametrising any possible contribution from physics beyond the Standard Model in the charge exchange form, the upper index characterizing the tensor property of the interaction, and the lower indices $\eta_1$ and $\eta_2$ the $\mu$ and $d$ chiralities, respectively. This effective parametrisation is analogous to the by now standard one used for muon decay \cite{10} in terms of coefficients $g_0^{S,V,T}$, while a similar one may be considered also for $\beta$-decay processes in terms of coefficients $f_0^{S,V,T}$. Clearly in the Standard Model, all these coefficients vanish identically, except for $h^V_{1,+} \equiv h^V_{-} = 1$. Given such general parametrisations, the effective values for $g^2/8M^2$ and $V_{ud}$ have to be determined accordingly from the muon decay rate and the $0^+-0^+$ superallowed $\beta$-decay rates, respectively.

The above interactions contribute to muon capture through the hadronic matrix elements of the corresponding quark operators. For the vector and axial currents, the parametrisation in terms of form factors has been introduced in (1) and (2). Likewise for the scalar, pseudoscalar and tensor operators $\bar{d}\sigma_{\mu\nu}u$, respectively, the associated nuclear matrix elements may be parametrised in terms of form factors $g_0^{S}, g_0^{V}$ and $g_0^{T}$, or $G_S, G_P$ and $G_T$, for the $(\mu)$ systems respectively, when ignoring possible recoil order corrections which are subdominant in any case.

Sensitivities of the statistical capture rate $\lambda_{\text{stat}}$ in the case of $^3\text{He}$, and of the singlet one $\lambda_{S}$ in the case of hydrogen, to the coefficients $h^{S,V,T}_{\eta_1 \eta_2}$ are as follows (assuming that all form factors just introduced, beyond the vector and axial current ones, are set to the value unity, and also that all the $h^{S,V,T}_{\eta_1 \eta_2}$ coefficients are real under complex conjugation, and thus do not lead to potential new sources of CP violation). One has $1/\lambda_{\text{stat}} d\lambda_{\text{stat}}/dh^X = (2.0,-0.81,0.38,-0.0056,5.82)$ as well as $1/\lambda_{S} d\lambda_{S}/dh^X = (2.0,-0.76,0.41,0.022,-5.53)$, with $h^X = (h^V_{-},h^V_{-},h^S_{+},h^P_{+},h^T_{+}/2)$ in the same order, and the definitions $h^S_+ = h^S_{++} + h^S_{+-}$ and $h^P_+ = h^S_{++} - h^S_{+-}$. Hence, these sensitivities are comparable in both cases, except for a possible pseudoscalar interaction.

More explicitly, consider the case of $^3\text{He}$, with $\lambda^{\text{exp}}_{\text{stat}} = 1496 \pm 4 \text{ s}^{-1}$ \cite{3}. Assuming that only $h^V_{1,+}$ is induced with a value different from unity, as well as $f^V_{LL}$ in the electronic sector, one establishes the $e-\mu$ universality test

$$|h^V_{1,+}/f^V_{LL}|^2 = 0.9996 \pm 0.0083 - 0.0140 \text{ [exp : 0.0023]} ,$$

$$|h^V_{1,+}/f^V_{LL}| = 0.9998 \pm 0.0042 - 0.0071 \text{ [exp : 0.0013]} ,$$

(6)

to be compared to the usual $e-\mu$ universality test from $\pi$ decay, $|h^V_{1,+}/f^V_{LL}|^2 = 1.0040 \pm 0.0033$ \cite{21}. Here again, were the value of $F_A$ to be improved, a genuinely
significant independent test of $e-\mu$ universality would become feasible. Otherwise, assuming now that $h_{L L}^V = 1$ and that only one new effective coupling $h_{n_1 n_2}^{S,V,T}$ takes a non-vanishing value, one infers the following constraints,

$$
\begin{align*}
 h_{V+}^T &= 0.0005 \pm [0.0102 - 0.0176] \ [\exp : 0.0033] , \\
h_S^G S &= -0.0012 \pm [0.022 - 0.038] \ [\exp : 0.0071] , \\
h_P^G P &= -0.078 \pm [1.49 - 2.56] \ [\exp : 0.48] , \\
\frac{1}{2} h_{T+}^T G_T &= -0.0008 \pm [0.00143 - 0.00245] \ [\exp : 0.00046] .
\end{align*}
$$

In particular, the constraints on the scalar $h_+^S$ and tensor $h_{+ -}^T$ interactions are very stringent, and provide a genuine improvement on the existing situation by a large margin, also when compared with the electronic sector stemming from $\beta$-decay. In addition, here again, it may be shown that when $F_P(q_1^2)$ is left to vary within 10% of its value, the above results are in fact quite robust, since they remain within their uncertainties. Also note that these limits once again would gain from a better knowledge of $F_A(q_1^2)$.

The above constraints are valid quite independently of any model for physics beyond the Standard Model. Nevertheless, it proves useful to also consider specific model extensions to assess in clearer physical terms the reach of these limits. Thus for example, within the context of so-called left-right symmetric SU(2)$_L \times$SU(2)$_R \times$U(1)$_{B-L}$ gauge models [22], the constraint on $h_{V+}^T$ implies $g_R/g_L \text{Re} \left( e^{i\omega} V_{udR}/V_{udL} \right) \tan \frac{\pi}{2} = -0.0005 \pm [0.0102 - 0.0176] \ [\exp : 0.0033]$, where $g_{R,L}$ and $V_{udR,L}$ are gauge coupling constants and Cabibbo-Kobayashi-Maskawa matrix elements associated to the sectors of right- and left-handed chiralities, while $\omega$ is the mixing angle for massive charged gauge bosons and $\omega$ is a CP violating phase also following from the diagonalisation of the charged gauge boson mass matrix. In particular for so-called manifestly left-right symmetric models with $g_R = g_L$, $V_{udR} = V_{udL}$ and $\omega = 0$, the ensuing constraint on the mixing angle $\omega$ is competitive with limits stemming from $\beta$-decay [10], and would also gain from an improvement on $F_A(q_1^2)$.

Another quite popular model extension of the Standard Model are so-called contact interactions, in which a specific energy scale $\Lambda$ is associated to a possible substructure of quarks and leptons [10]. By way of example, and using the customary parametrisation of contact interactions specified in [10], the value for $\lambda_{\text{stat}}^\exp$ translates for instance into the following limits for such a compositeness scale, $\Lambda_{V+} > [4.9 - 3.8] \ [\exp : 8.4] \ \text{TeV} \ (90\% \ C.L.)$ as well as $\Lambda_{T+} > [9.3 - 7.2] \ [\exp : 15.9] \ \text{TeV} \ (90\% \ C.L.)$ (the latter values assume $G_T = 1$). These constraints, of
application to interactions coupling the second leptonic generation to the first quark generation, are complementary to existing ones [10], which often involve rather the first generation leptons. In fact, these limits are a genuine competition for high energy experiments in the case of charged current electroweak contact interactions, for which the energy scale \( \Lambda \) is typically in the 2-4 TeV range.

Yet another model extension of the Standard Model are so called lepto-quark interactions [23, 24] (for which the notations of [24] will be used to refer to leptoquarks and their Yukawa couplings). Note however that the constraints to be given presently apply to Yukawa couplings and leptoquark masses coupling the second lepton generation to the first quark generation. Focusing again onto the more competitive limits, the scalar and tensor effective interactions provide for the followings constraints. In the scalar case, the combination

\[
\left| \frac{\lambda_{L_0} \lambda_{R_0}}{M^2(S_0)} + \frac{\lambda_{L_{1/2}} \lambda_{R_{1/2}}}{M^2(S_{1/2})} + 4 \frac{\lambda_{V_0} \lambda_{V_0}}{M^2(V_0)} + 4 \frac{\lambda_{V_{1/2}} \lambda_{V_{1/2}}}{M^2(V_{1/2})} \right| ,
\]

is bounded above by \([2.2 - 3.8] \text{ TeV}^{-2} \) (90% C.L.), or equivalently by

\[
\left[ \frac{0.023}{(100 \text{ GeV})^2} - \frac{0.038}{(100 \text{ GeV})^2} \right] \left[ \exp : \frac{0.008}{(100 \text{ GeV})^2} \right] \text{ (90\% C.L.) ,}
\]

when normalising the leptoquark mass scale to 100 GeV/c\(^2\). Similarly in the tensor case, the combination

\[
\left| \frac{\lambda_{L_0} \lambda_{R_0}}{M^2(S_0)} - \frac{\lambda_{L_{1/2}} \lambda_{R_{1/2}}}{M^2(S_{1/2}(-2/3))} \right| G_T \right| ,
\]

is bounded above by \([0.29 - 0.50] \text{ TeV}^{-2} \) (90% C.L.), or equivalently by

\[
\left[ \frac{0.003}{(100 \text{ GeV})^2} - \frac{0.005}{(100 \text{ GeV})^2} \right] \left[ \exp : \frac{0.001}{(100 \text{ GeV})^2} \right] \text{ (90\% C.L.) .}
\]

These limits are quite competitive with, and complementary to existing ones from high energy experiments [24, 10], especially for the last set of constraints stemming from tensor type effective interactions. Once more, note how these results would also gain from an improved knowledge of \( F_A(q_0^2) \).

The potential for similar tests of physics beyond the Standard Model from a precision measurement of the singlet capture rate \( \lambda_S \) on hydrogen is as follows. Assuming again a 1% precise result centered onto the theoretically expected value,
the corresponding constraints are (taking $g_0^S$, $g_0^P$ and $g_0^T$ all equal to unity),

\[
\begin{align*}
    h_V^- &= 1.0 \pm 0.0060 \text{[exp : 0.0050]} \\
    h_V^+ &= 0.0 \pm 0.0156 \text{[exp : 0.0131]} \\
    h_S^+ &= 0.0 \pm 0.0287 \text{[exp : 0.0242]} \\
    h_P^+ &= 0.0 \pm 0.55 \text{[exp : 0.46]} \\
    \frac{1}{2}h_T^- &= 0.0 \pm 0.0023 \text{[exp : 0.0019]},
\end{align*}
\]

(12) to be compared to those given in (5) and (7). Thus also for muon capture on hydrogen, it is the tensor effective coupling coefficient $h_T^-$ which would be subjected to the most stringent constraint, while those for the vector $h_V^\pm$ and scalar $h_S^+$ coefficients remain also of much interest.

4 Conclusions

As this contribution has demonstrated, precision measurements in muon capture on hydrogen and $^3$He provide for important symmetry tests of the Standard Model—both in its hadronic as well as in its electroweak sector—which are competitive with, and complementary to experiments at high energies.

The potential for such tests has already been established for the statistical capture rate on $^3$He to the triton channel, given the recent 0.3% precise measurement of [3]. The physics reach of the ensuing contraints could be improved still further through a better knowledge of the nuclear axial form factor $F_A(q^2)$, a problem for which a chiral perturbation approach at the nuclear level, including isospin breaking effects, could be envisaged.

Further progress is to be made through a forthcoming measurement of the singlet capture rate $\lambda^S$ on hydrogen, to a precision better than 1% [4]. The physics reach of such result is complementary to that for $^3$He, since it is much less affected by theoretical uncertainties, while on the other hand a 0.5% precision measurement now seems feasible [8], thereby improving by almost a factor two the uncertainties on the symmetry tests discussed in this contribution. In particular, this would bring the uncertainty of an experimental test of the chiral symmetry prediction for $g_P$ down to the same level of precision as the theoretical value [12].

Finally, renewed efforts in a precision measurement of the vector analysing power for the triton asymmetry in capture on $^3$He [16]—or any other polarisation observable for that matter—, would provide for additional stringent symmetry
tests as well [7], which would be complementary to those stemming from the capture rate result [3] and be more independent of model assumptions. The required precisions are quite demanding however, but the experimental challenge is certainly to the standard of its possible physics rewards.

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