Transverse Momentum Distribution in $b \rightarrow s\gamma$

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We present the complete calculation of the transverse momentum distribution for the decay $b \rightarrow s\gamma$. The contributions of the leading operator $\hat{O}_7$ are computed: infrared logarithms are resummed with next-to-leading accuracy according to usual techniques of resummation. Non logarithmic terms are evaluated to $O(\alpha_S)$ by calculating one loop diagrams.

1. Introduction

The rare decay $b \rightarrow s\gamma$ has been widely studied in the last years, both experimentally and theoretically, for several reasons: being loop mediated even at the lowest order, it was used to search signals of new physics. Moreover it is a way to measure CKM matrix elements, which is one of the main goals of beauty physics to constraint the Standard Model.

Perturbative QCD has been applied to $b$ physics since a long time, because the QCD coupling constant is assumed to be small at $b$ quark mass

$$\alpha_S(m_b^2) \sim 0.21$$

(1)

However a direct check of perturbative QCD is difficult to perform, because several poorly known parameters are involved in theoretical predictions (for example CKM matrix elements and the beauty quark mass). On the contrary one usually prefers to assume that perturbative QCD is reliable in $b$ physics and can be used to extract these parameters.

In our opinion, it is not clear whether the perturbative approach is viable at $b$ mass energies, both in inclusive quantities and, a fortiori, in semi-inclusive distributions.

In the latter case a powerful tool is the resummation of the large logarithms appearing at the border of phase space. This approach had a wide application in past years in processes such as $e^+e^-, \, \bar{p}p$, deep inelastic scattering, ..., for semi-inclusive distributions and in recent years has become an ordinary technique in $b$ physics too.

It is well known that the resummation of large logarithms for a generic distribution $D(x)$ gives the general result [2]:

$$D(x) = K(\alpha_S)\Sigma(x; \alpha_S) + R(x)$$

(2)

where

- $\Sigma(x; \alpha_S)$ is a universal, process independent function resumming the infrared logarithms in exponentiated form. In particular

$$\log \Sigma(x; \alpha_S) = L g_1(\alpha_S L) + g_2(\alpha_S L) + \ldots$$

(3)

being $L$ a large logarithm. The functions $g_i$ have an expansion as

$$g_i(z) = \sum_{n=1}^{\infty} g_{i,n} z^n$$

(4)

and resum logarithms of the same size: in particular $g_1$ resums leading logs $\alpha_S^n \log^{n+1}$ and $g_2$ the next-to-leading ones $\alpha_S^n \log^n$;

- $K(\alpha_S)$ is the coefficient function, a process dependent function which can be calculated in perturbation theory by evaluating the constant terms at the required order

$$K(\alpha_S) = 1 + \frac{C_F \alpha_S}{\pi} k_1 + \ldots$$

(5)
• \( R(x) \), the remainder function, is process-dependent and satisfies the condition

\[
R(x) \rightarrow 0 \quad \text{for} \quad x \rightarrow 0
\]

(6)

It takes into account hard contributions and can be calculated in perturbation theory as an \( \alpha_S \) expansion.

\[
R(x) = \frac{\alpha_S C_F}{\pi} r_1(x) + \ldots
\]

(7)

For the previous reasons, a precise calculation of the terms involved in this approach is needed to compare theoretical predictions and experimental data.

For \( b \rightarrow s\gamma \) the resummation has been applied in the photon spectrum distribution \[3\], which is sensitive to longitudinal degrees of freedom. We have extended the analysis to the transverse momentum distribution of the strange quark with respect to the photon direction, which is sensitive to transverse degrees of freedom \[4\].

The resummation of large logarithms seems to be required in the transverse momentum distribution for \( b \rightarrow s\gamma \), because the double logarithmic correction becomes large near the border of phase space, compared to 1 (lowest order contribution to the rate):

\[
- \frac{1}{4} \frac{C_F \alpha_S (m_b^2)}{\pi} \log^2 \frac{\Lambda^2}{m_b^2} \sim -0.7
\]

(8)

if we push the transverse momentum to its physical limit, \( \Lambda \sim 300\text{MeV} \), before the appearance of Landau pole effects.

The single logarithm becomes large too, due to its large numerical coefficient

\[
- \frac{5}{4} \frac{C_F \alpha_S (m_b^2)}{\pi} \log \frac{\Lambda^2}{m_b^2} \sim 0.6
\]

(9)

However hard contributions as the function \( R(x) \) should be carefully evaluated, being at these energies more relevant than in processes such as \( e^+e^- \), deep inelastic scattering, due to the size of \( \alpha_S \) \[1\].

2. Transverse Momentum Distribution for \( b \rightarrow s\gamma \)

Let us consider the decay \( b \rightarrow s\gamma \) in the \( b \) rest frame and define the photon direction as the \( z-axis \). Let us consider the transverse momentum \( p_\perp \) of the strange quark with respect to the photon direction and, in particular, the partially integrated distribution

\[
D(x) = \int_0^x \frac{1}{\Gamma_0} \frac{d\Gamma}{dx'} dx'
\]

(10)

where \( x \) is an adimensional variable defined as

\[
x = \frac{p^2_{\perp}}{m_b^2}
\]

(11)

being \( m_b \), the \( b \) quark mass, the hard scale of the process. \( \Gamma_0 \) is the Born amplitude, as defined in \[3\].

According to \[2\], an accurate calculation of \( D(x) \) requires two steps:

• the resummation of large logarithms arising at the border of phase space (infrared region) \[4\];

• a fixed order calculation to extract non logarithmic contributions, such as constants, contained in \( K(\alpha_S) \), and functions vanishing for \( x \rightarrow 0 \), contained in \( R(x) \) \[1\].

2.1. Resummation of infrared logarithms

The resummation of infrared logarithms with next-to-leading accuracy requires the knowledge of the coefficients of double and single logarithms at one loop and the coefficient of leading logarithms at two loops.

At one loop the distribution takes the look

\[
D(x) = 1 - \alpha_S A_1 \log^2 x + \alpha_S B_1 \log x + \frac{\alpha_S C_F}{\pi} \frac{1}{\Lambda_1} + \frac{\alpha_S C_F}{\pi} r_1(x)
\]

(12)

The coefficients \( A_1 \) and \( B_1 \) can be calculated by general properties of QCD, applying the perturbative evolution for the light quark, described by the Altarelli-Parisi kernel, and the eikonal approximation for the massive quark \[6\].

The coefficients turns out to be \[4\]

\[
A_1 = \frac{C_F}{\pi}, \quad B_1 = -\frac{5}{4} \frac{C_F}{\pi}
\]

(13)

Let us notice that the coefficient \( B_1 \) is larger than in processes involving light quarks only, due to gluon emissions from the heavy quark and this
produces an enhancement of the single logarithmic term. The two loop coefficient $A_2$ is known from literature [7]

$$A_2 = \frac{G_F}{2\pi^2} [C_A \left(\frac{67}{18} - \frac{\pi^2}{6} - \frac{5}{9} n_f\right)]$$  \hspace{1cm} (14)

The resummed distribution of transverse momentum is performed in the space of impact parameter $b$

$$\tilde{D}(b) = \int_{-\infty}^{+\infty} dp_\perp e^{ip_\perp \cdot \vec{b}} \frac{1}{b^2} \frac{d\Gamma}{d^2p_\perp} (p_\perp)$$  \hspace{1cm} (15)

which allow the factorization of the phase space and of the kinematical constraint.

The result for the functions $g_1$ and $g_2$ appearing in [3] is

$$g_1(\omega) = \frac{A_1}{2b_0} \omega \log(1-\omega + \omega)$$  \hspace{1cm} (16)

$$g_2(\omega) = -\frac{A_2}{2b_0} \left[ \frac{\omega}{1-\omega} + \log(1-\omega) \right] + \frac{A_1}{2b_0} \log(1-\omega)$$

with

$$+ \frac{1}{2} \log^2(1-\omega) + \frac{1}{\beta_0} \log(1-\omega)$$  \hspace{1cm} (17)

being $\omega = \beta_0 \alpha_S \log(\frac{Q^2}{b_0})$ and $b_0 = 2e^{-2\gamma_E}$.

Let us notice that the resummed distribution diverges for $\omega \rightarrow 1$ and this signals the appearance of non perturbative effects, related to the Landau pole of the theory. They appear when the distribution approaches the limit of the phase space in the infrared region, that is, roughly speaking, for $p_\perp \sim \Lambda$.

The resummed distribution is more reliable than the fixed order calculation near the border of phase space, nevertheless it breaks down for small value of the transverse momentum.

This divergence should be factorized in a non perturbative structure function: for the photon spectrum an approach based on an effective theory has been used, by introducing a shape function [8].

2.2. Fixed order calculation

A complete resummed distribution with next-to-leading accuracy requires the calculation of the constant $k_1$ in [12]. The first term in the expansion of the remainder function, $r_1(x)$, should be relevant too, due to the low value of the hard scale. They can be extracted by an explicit calculation of real and virtual diagrams at one loop. The process $b \rightarrow s\gamma$ is loop mediated in the Standard Model: an effective Hamiltonian has been developed to integrate out the heavy particles in the loop [5].

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{j=1}^8 C_j(\mu) \hat{O}_j(\mu)$$  \hspace{1cm} (18)

where $C_j$ are Wilson coefficients and $\hat{O}_j(\mu)$ are effective operators ($i = 1, \ldots, 8$)[9]. $\hat{O}_7$ is the most relevant operator because it is affected by logarithmic enhancement in the infrared region, while the other operators contribute to constants [10].

Real diagrams are calculated in dimensional regularization in $n = 4 + \epsilon$ dimensions.

$$D_R(x) = \int_0^1 d\Phi \mathcal{M}_{4+\epsilon}(\omega, t) \theta[x - \omega^2 t (1-t)]$$  \hspace{1cm} (19)

being $\Phi$ the 3-body phase space, $\omega$ the gluon energy fraction and $t = (1 - \cos \theta)/2$, where $\theta$ is the angle between the gluon and the direction $-\vec{z}$. $\mathcal{M}_{4+\epsilon}(\omega, t)$ is the matrix element with the insertion of $\hat{O}_7$ in the vertex, in dimensional regularization.

The integration may be analytically performed by introducing harmonic polylogarithms as in [11], satisfying the properties

$$H(a; y) = \int_0^y dy' g(a; y') \quad a \neq 0$$

$$H(0; y) = \log y$$

$$H(\vec{m}_w; y) = \int_0^y dy' g(a; y') H(\vec{m}_{w-1}; y')$$

$$\frac{d}{dy} H(\vec{m}_w; y) = g(a; y) H(\vec{m}_{w-1}; y)$$  \hspace{1cm} (20)

where the basis of functions $g(a; y)$ is

$$g[0; y] = \frac{1}{y}$$

$$g[-1; y] = \frac{1}{y+1}$$
Details of the calculation and the explicit result for the first term in the expansion of the remainder function are shown in [1]. Virtual diagrams are calculated with ordinary techniques: they cancel the infrared poles arising from real diagrams and don’t depend on the kinematics, giving contribution to the coefficient function only. In particular the vertex correction is calculated by reducing the diagram to simpler topologies and using integration by parts identities: details of the calculation are discussed in [1]. Summing real and virtual diagrams the first term of the perturbative expansion of the coefficient function for the operator $O_7$ turns out to be

$$k_1 = \frac{C_F}{\pi} \left( - \frac{11}{4} - \frac{\pi^2}{12} + \log \frac{m_b}{\mu} \right)$$

An improved result for the coefficient function, taking into account the other operators of the basis [18], can be found in [1].

3. Conclusions and Outlook

$b \rightarrow s \gamma$, though a rare decay, seems to be a very clean process to study hadronic physics in $b$ decays and to apply resummation techniques to check whether they are reliable in $b$ physics or not. A very accurate calculation of all the ingredients involved in the resummation “recipe” is needed to this aim. Otherwise, it would be impossible to conclude whether discrepancies from experimental data may be attributed to this approach or to other sources of uncertainty typical of $b$ physics, such as an imperfect knowledge of CKM matrix elements or the $b$ quark mass.

A more interesting application should concern the transition $b \rightarrow c$, which is phenomenologically more relevant, but whose calculation is complicated by the presence of the $c$ quark mass. From this point of view the study of $b \rightarrow s \gamma$ may be seen as a first step to apply resummation techniques to $b$ physics, before going to more complicated but more important processes.

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