Some exact periodic soliton solutions and resonance for the potential Kadomtsev-Petviashvili equation

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Abstract. In this paper, some exact periodic soliton solutions for potential Kadomtsev-Petviashvili equation are obtained using two-soliton method and bilinear form method. Singular and non-singular phenomenons of solutions are shown. In addition, resonance and non-resonance interactions between \( y - t \) periodic soliton and different line solitons are investigated.

1. Introduction
It is well known that the potential Kadomtsev-Petviashvili equation (PKP) arises in number of remarkable non-linear problems both in physics and mathematics. Finding special solutions and investigating the corresponding properties of solutions are very important in both practice and theory for understanding these problems [1-6].

The solutions of PKP equation have been studied extensively in various aspects. Using the tanh function method Senthilvelan obtained the travel wave solutions [2]. Li and Zhang [3,4] obtained some exact solutions by improving on the key steps of homogeneous balance method. Kaya and EI-Sayed [5] given numerical solutions. Inan and Kaya found out some exact linearly soliton solutions by improving tanh function method [6]. Dai Z et al. discovered singular periodic soliton and spatial-temporal bifurcation for Kadomtsev-Petviashvili equation [7,8]. Zhu and Geng [9] obtained \( N \)-soliton solution of the variable –coefficient cKP equations by using Pfaffian technique. Ma Z et al. [10] given exact multi-soliton solutions by using Painlevé-Bäcklund transformation and multilinear variable separtion approach. In [11] Xu derived soliton solutions and dromions of (3+1)-dimensions KP equation by using truncated Painlevé expansions. Wei C et al. obtained some solutions of generalized wick-type stochastic KP equation (GWSKPE) by using extended homogeneous balance method [12]. Besides these, no further results for soliton solutions were obtained.

In this work, by using two-soliton method and bilinear form method we obtain exact kink-wave solution and periodic soliton solution. Furthermore, we investigate singular and non-singular phenomenons of these solutions and explore the resonance and non-resonance interaction between \( y - t \) periodic soliton and different line solitons, respectively.

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2. Cross kink -wave and periodic soliton solutions

We consider the (2+1)-dimensional PKP equation

\[ u_{xt} + \frac{3}{2} u_{uxx} + \frac{1}{4} u_{xxxx} + \frac{3}{4} u_{yy} = 0 \]  
(2.1)

where \( u : R_x \times R_y \times R_t \rightarrow R \)

In this section, the periodic soliton are constructed by two-soliton method and bilinear form method.

Let

\[ u = (2 \ln f)_x, \]  
(2.2)

where \( f \) is an unknown function.

Substituting (2.2) into (2.1) yields

\[ (2 \ln f)_{xt} + \frac{3}{2} (2 \ln f)_{xx} \cdot (2 \ln f)_{xx} + \frac{1}{4} (2 \ln f)_{xxxx} + \frac{3}{4} (2 \ln f)_{yy} = 0. \]  
(2.3)

Integrating (2.3) with respect to \( x \), we get following equation

\[ (2 \ln f)_{xt} + \frac{3}{4} (2 \ln f)_{xx} \cdot \frac{1}{2} + \frac{1}{4} (2 \ln f)_{xxxx} + \frac{3}{4} (2 \ln f)_{yy} = C. \]  
(2.4)

Taking \( C = 0 \) and using bilinear form, we have

\[ \frac{D_x D_y f \cdot f}{f^2} + \frac{D_x^4 f \cdot f}{4 f^2} + \frac{3}{4} \frac{D_y^2 f \cdot f}{f^2} = 0, \]  
(2.5)

so

\[ (4D_x D_y + D_x^4 + 3D_y^2) f \cdot f = 0, \]  
(2.6)

where the bilinear operator \( D \) is defined as

\[ D^{\alpha} D^{\beta} f \cdot g = \left( \frac{\partial}{\partial x} - \frac{\partial}{\partial x'} \right)^{\alpha} \left( \frac{\partial}{\partial y} - \frac{\partial}{\partial y'} \right)^{\beta} f(x, y, t) \cdot g(x', y', t) \bigg|_{(x', y', t) = (x, y, t)}. \]  
(2.7)

Suppose two-soliton solution of equation (2.1) as

\[ f = 1 + e^{\eta_j} + e^{\eta_j^0} + De^{\eta_j + \eta_j^0}, \]  
(2.8)

where

\[ \eta_j = k_j x + l_j y - \Omega_j t + \eta_j^0, j = 1,2. \]  
(2.9)

Substituting (2.8), (2.9) into (2.6) leads to

\[ \Omega_j = \frac{k_j^4 + 3l_j^2}{4k_j}, j = 1,2 \]  
(2.10)

\[ D = -\frac{4(k_1 - k_2)(\Omega_1 - \Omega_2) + (k_1 - k_2)^4 + 3(l_1 - l_2)^2}{4(k_1 + k_2)(\Omega_1 + \Omega_2) - (k_1 + k_2)^4 - 3(l_1 + l_2)^2}. \]  
(2.11)

(I) Setting
\[ k_1 = -k_2 = k, l_1 = l_2 = l, \eta_1^0 = \eta_2^0 = 0, \]
then
\[ \Omega_1 = -\Omega_2 = \Omega, \quad (2.12) \]
where \( k, \Omega \) and \( l \) are all real numbers, from (2.10)-(2.12), we have
\[ \Omega = \frac{k^4 + 3l^2}{4k}, D = \frac{l^2 - k^4}{l^2}. \]

Then substituting (2.9), (2.12) into (2.8), we obtain
\[ f = 2e^y \left[ \sqrt{D} \cosh \left( ly + \ln \sqrt{D} \right) + \cosh \left( kx - \Omega t \right) \right], \quad (2.13) \]
and then
\[ f_x = 2ke^y \sinh (kx - \Omega t). \quad (2.14) \]

Substituting (2.13), (2.14) into (2.2), we obtain exact cross kink-wave solution
\[ u = 2k \frac{\sinh (kx - \Omega t)}{\sqrt{D} \cosh \left( ly + \ln \sqrt{D} \right) + \cosh (kx - \Omega t)}. \quad (2.15) \]

Notice that the denominator of solution (2.15) cannot equal zero for arbitrary \( x, y, t \). So the solution (2.15) of PKP equation is non-singular [Fig. 1]

![Figure 1: Cross kink wave](image)

(II) Setting
\[ k_1 = k_2 = ik, l_1 = l_2 = l, \Omega_1 = \Omega_2 = i\Omega, \eta_1^0 = \eta_2^0 = 0, \quad (2.16) \]
where \( k, l \) and \( \Omega \) are all real numbers. Then \( D \) and \( \Omega \) are given from (2.10), (2.11) and (2.16)
\[ \Omega = -\frac{k^4 + 3l^2}{4k}, D = \frac{l^2 - k^4}{l^2}. \quad (2.17) \]

Substituting (2.9), (2.16) into (2.8), we obtain
\[ f = 2e^y \left[ \sqrt{D} \cosh \left( ly + \ln \sqrt{D} \right) + \cos (kx - \Omega t) \right], \quad (2.18) \]
and then
\[ f_x = -2ke^y \sin (kx - \Omega t). \quad (2.19) \]
Substituting (2.18), (2.19) into (2.2), we obtain the periodic solitary wave solutions which is periodic in the $x - t$ direction

$$u = -2k \frac{\sin(kx - \Omega t)}{\sqrt{D} \cosh(ly + \ln \sqrt{D}) + \cos(kx - \Omega t)}.$$ (2.20)

We notice that $D < 1$ and the denominator of (2.20) may equal zero for some $x, y, t$. Therefore, it show this solutions is singular which is even function with regard to $Y = ly + \ln \sqrt{D}$ and one is odd function with regard to $X = kx - \Omega t$ [Fig.2].

![Figure 2: Singularly periodic soliton](image)

3. Periodic soliton resonance and non-resonance

In this section, we study the interaction between $y$-periodic soliton and line soliton for PKP equation. The solution describing the interaction between a $y$-periodic soliton and a line soliton is written as

$$u = (2 \ln f)_{\xi},$$

with

$$f = 1 - \frac{1}{\alpha^2} e^{i\xi_1} \cos \eta + \frac{M}{\alpha^4} e^{-2i\xi_1} + e^{i\xi_2} \left(1 - \frac{N}{\alpha^2} e^{i\xi_1} \cos \eta + \frac{MN^2}{\alpha^6} e^{2i\xi_1}\right),$$

where

$$\xi_1 = \alpha x - \Omega_p t + \sigma_1, \xi_2 = \beta x - \Omega_L t + \sigma_2, \eta = \delta y - \gamma t + \sigma_3.$$

By computing, we have

$$\Omega_p = \frac{\alpha^4 - 3\delta^2}{4\alpha}, \Omega_L = \frac{\beta^3}{4}, \gamma = 0,$$

$$M = \frac{\delta^2}{4\alpha^4 + 4\delta^2}, N = -\frac{(\alpha - \beta)^4}{(\alpha + \beta)^4 - 4(\alpha - \beta)(\Omega_p - \Omega_L)} - 3\delta^2.$$

On the one hand, when $\delta \neq 0$, we have $N \neq 0$ and $N \neq \infty$. That shows there is not resonance for interaction between $y - t$ periodic soliton and line soliton.
On the other hand, when $\delta = 0$, namely, the period of periodic soliton is independent of $y$, we have $N = 0$ when $\alpha = \beta$, which corresponding to the phase shift in the propagating direction for negative infinity time. This means that periodic soliton and line soliton can interact infinitely apart each other. Finally, we have $|N| = +\infty$ when $\alpha = -\beta$. This means another type of resonance interaction between the $y-t$ periodic soliton and line soliton.

4. Conclusion
By using two soliton method and bilinear form method, we obtain a new type of special solutions of PKP equation, which include cross kink-wave and periodic solitary wave solutions. We investigate singular and non-singular phenomenons of these solutions. In addition, we also study the resonance and non-resonance interaction between $y-t$ periodic soliton and different line soliton. It is interesting that there is not resonance when period of periodic soliton depends on $y$ and there is resonance when period is independent of $y$. We will study this surprising and interesting phenomena in the further work.

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