QUANTUM COSMOLOGICAL ORIGIN OF UNIVERSES

V.N. Pervushin, V.A. Zinchuk

Joint Institute for Nuclear Research,
141980, Dubna, Russia.

Abstract

A direct pathway from Hilbert’s “Foundation of Physics” to Quantum Gravity is established through Dirac’s Hamiltonian reduction of General Relativity and Bogoliubov’s transformation by analogy with a similar pathway passed by QFT in 20th century. The cosmological scale factor appears on this pathway as a zero mode of the momentum constraints treated as a global excitation of the Landau superfluid liquid type. This approach would be considered as the foundation of the well–known Lifshitz cosmological perturbation theory, if it did not contain the double counting of the scale factor as an obstruction to the Dirac Hamiltonian method. After avoiding this “double counting” the Hamiltonian cosmological perturbation theory does not contain the time derivatives of gravitational potentials that are responsible for the CMB “primordial power spectrum” in the inflationary model.

The Hilbert – Dirac – Bogoliubov Quantum Gravity gives us another possibility to explain this “spectrum” and other topical problems of cosmology by the cosmological creation of both universes and particles from Bogoliubov’s vacuum. We listed the set of theoretical and observational arguments in favor of that the CMB radiation can be a final product of primordial vector W-, Z- bosons cosmologically created from the vacuum when their Compton length coincides with the universe horizon. The equations describing longitudinal vector bosons in SM, in this case, are close to the equations of the inflationary model used for description of the “power primordial spectrum” of the CMB radiation.

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1. Introduction

It is now accepted that quantum General Relativity does not exist. In the present paper, we give a set of theoretical and observational arguments in favor of the opposite opinion: General Relativity (GR) \[^{[1, 2]}\] has a consistent interpretation only in the form of quantum theory of the type of the microscopic theory of superfluidity \[^{[3, 4, 5]}\].

Our hopes for an opportunity to construct a realistic quantum theory for GR are based, on the one hand, on the existence of the Hilbert geometric formulation \[^{[1, 6, 7]}\] of Special Relativity (SR) \[^{[8, 9]}\] considered as the simplest model of GR and, on the other hand, on the contemporary quantum field theory (QFT) \[^{[10, 11]}\] based on the dynamic version of SR \[^{[8, 9]}\] that appears after resolution of constraints \[^{[6, 7]}\].

The geometric formulation of SR is an action of the type of the action in GR, which Hilbert has reported on 20th November 1915 in his talk in Göttingen mathematical society \[^{[1]}\].
We shall try here to reconstruct a direct pathway from geometry of a relativistic particle in SR to the causal operator quantization of fields of these particles and their quantum creation from vacuum in order to formulate a similar direct way from geometry of GR [1] to the causal operator quantization of universes and to their quantum creation from vacuum considered as a state with the minimal “energy”. This formulation includes the Wheeler – DeWitt definition of “field space of events” [13]; the separation of gauge transformations from ones of the frames of references using the Fock simplex of reference [12]; the choice of the Dirac specific frame of reference [14]; resolving the energy constraint in the class of functions of the gauge transformations established by Zel’manov [15], where the cosmological scale factor appears as a zero mode of the momentum constraints [6, 7]; the calculation of values of the geometric action and interval onto the resolutions of the energy constraint [16] in order to get the dynamic “reduced” action in terms of gauge-invariant variables and to define the notions “energy”, “time”, “particle” and “universe”, “number” of “particles” and “universes” by the low-energy expansion of this “reduced” action following Einstein’s correspondence principle with nonrelativistic theory [8, 9].

In the present paper the cosmological evolution [17, 18, 19] is treated as independent “superfluid” dynamics [4]. One of attributes of the phenomenon of superfluidity is the application of Bogoliubov’s transformations [5, 20] to obtain a set of integrals of motions and calculate the distribution functions of cosmological creation of both “universes” and “particles” from “vacuum”. All these attributes: London’s unique wave function [3], Landau’s independent “superfluid” dynamics [4], and Bogoliubov’s transformations [5] are accompanied by a set of physical consequences that can be understood as only pure quantum effects.

We shall show how this “superfluid” dynamics of GR gives us possible solutions of the topical astrophysical problems, including horizon, homogeneity, cosmological singularity, arrow of time, Dark Matter, and Dark Energy.

In Section 2, the direct way from Hilbert’s geometric formulation of SR to QFT is established. Section 3 is devoted to a similar Hilbert’s foundation of quantum cosmology. In Section 4, GR is considered as a microscopic theory of superfluidity.

2. Hilbert’s Foundation of Quantum Field Theory

2.1. Hilbert’s version of Special Relativity

The Hilbert geometric formulation of a relativistic particle [1, 6, 7] is based on the action:

$$S_{1915}^{SR} = -\frac{m}{2} \int_{\tau_1}^{\tau_2} d\tau \left[ \frac{\dot{X}_\alpha^2}{e(\tau)} + e(\tau) \right],$$

(1)

and an “geometric interval”

$$ds = e(\tau)d\tau \quad \mapsto \quad s(\tau) = \int_0^{\tau} d\tau e(\tau),$$

(2)

where $\tau$ is the “coordinate evolution parameter” given in a one-dimensional Riemannian manifold with a single component of the metrics $e(\tau)$ and the variables $X_\alpha$ form the Minkowskian “space of events”, where $(X_\alpha)^2 = X_0^2 - X_i^2$.

The action (1) and interval (2) are invariant with respect to reparametrizations of the “coordinate evolution parameter” (treated as “gauge transformations”)

$$\tau \quad \mapsto \quad \tilde{\tau} = \tilde{\tau}(\tau);$$

(3)
therefore, the theory given by (1) and (2) can be considered as the simplest model of GR.

A single component of the metrics \( e(\tau) \) (known as a “lapse-function”) plays the role of the Lagrange multiplier in the Hamiltonian form of the action (1):

\[
S_{1915}^{SR} = \int_{\tau_1}^{\tau_2} d\tau \left[ -P_\alpha \partial_\tau X^\alpha + \frac{e(\tau)}{2m} \left( P_\alpha^2 - m^2 \right) \right].
\] (4)

Varying action (4) over lapse-function \( e(\tau) \) defines the “constraint”:

\[
(P_\alpha)^2 - m^2 = 0.
\] (5)

Varying action (4) over dynamical variables \((P_\alpha, X_\alpha)\) gives the equations of motion:

\[
P_\alpha = m \frac{dX_\alpha}{ds}, \quad \frac{dP_\alpha}{ds} = 0,
\] (6)

taking into consideration \( ds = e(\tau) d\tau \). Solutions of equations (6) in terms of gauge-invariant “geometric interval” (2) take the form

\[
X_\alpha(s) = X_\alpha(0) + \frac{P_\alpha(0)}{m} s.
\] (7)

2.2. Dirac’s Hamiltonian reduction

The physical meaning of this solution is revealed in a specific “frame of reference”. In particular, solutions of energy constraint (5) with respect to a temporal component \( P_0 \) of momentum \( P_\alpha \)

\[
P_{0\pm} = \pm \sqrt{m^2 + P_i^2}
\] (8)

are considered as the “reduced Hamiltonian” in the “reduced phase space” \( \{X_i, P_j\} \) that becomes the energy \( E(P) = \sqrt{m^2 + P_i^2} \) onto a trajectory [8, 9]. The time component of solution (7)

\[
s = \frac{m}{P_{0\pm}} [X_0(s) - X_0(0)]
\] (9)

shows us that the “time-like variable” \( X_0 \) is identified with the time measured in the rest frame of reference, whereas an interval \( s \) is the time measured in the comoving frame.

The dynamic version of SR [8, 9] can be obtained as values of the geometric action (4) onto solutions of the constraint (5)

\[
S_{1915}^{SR} |_{P_0=P_{0\pm}} = S_{1905}^{SR} = \int_{X_{0j}}^{X_0} dX_0 \left[ P_i \frac{dX_i}{dX_0} - P_{0\pm} \right].
\] (10)

Just the values of the “geometric interval” (9) and action (10) onto resolutions (8) of constraint (5) in the specific frame of reference will be called the “Hamiltonian reduction” of Hilbert’s geometric formulation of SR given by Eqs. (1) and (2) (see [8, 10]).
2.3. Dynamic version of Special Relativity of 1905

The “Hamiltonian reduction” leads to action (10) of the dynamic theory of a relativistic particle of “1905” [8, 9] that establishes a correspondence with the classical mechanic action by the low-energy decomposition

\[ E(P) = \sqrt{m^2 + P_i^2} = m + \frac{P^2}{2m} + ... \]  

(11)

It gives us the very important concept of particle “energy” \( E(0) = mc^2 \). We can see that relativistic relation (9) between the “time as the variable” and the “time as the interval” appears in the geometric version of “1915” [1] as a consequence of the variational equations (8), whereas in the dynamic version of “1905” [8, 9] the same relativistic relation in the form of a kinematic Lorenz relativistic transformation is supplemented to variational equations following from the dynamic action (10).

2.4. Quantum geometry of a relativistic particle

The next step forward to QFT is the primary quantization of particle variables: \( i\{\hat{P}_\mu, X_\nu\} = \delta_{\mu\nu} \), that leads to the quantum version of the energy constraint (5) \( \Box + m^2 \psi(X_0, X_i) = 0 \) known as the Klein – Gordon equation of the wave function. The general solution of this equation

\[ \partial_0^2 \psi_p + E_p^2 \psi_p = 0 \]  

(12)

for a single p-Fourier harmonics \( \psi_p(X_0) = \int d^3X \exp iP_jX^j \psi(X_0, X_i) \) takes the form of the sum of two terms

\[ \psi_p = \frac{1}{\sqrt{2E_p}} \{ a_p^+(X_0) + a_p^-(X_0) \}, \]

where \( a_p^+(X_0), a_p^-(X_0) \) are solutions of the equations

\[ (i\partial_0 + E_p)a_p^+(X_0) = 0, \quad (i\partial_0 - E_p)a_p^-(X_0) = 0. \]  

(13)

They are treated as the Shrödinger equations of the dynamic theory (10) for the case of positive and negative particle “energies” [3] revealed by resolving energy constraint (5).

QFT is formulated as the secondary quantization of a relativistic particle \( [a_p^-, a_p^+] = 1 \) [11]. In order to remove the negative “energy” \(-E_p\) and to provide the quantum system with stability, the field \( a_p^+ \) is considered as the operator of creation of a particle and \( a_p^- \) as the operator of annihilation of a particle, both with positive “energy”. The initial datum \( X_I(0) \) is treated as a point of this creation or annihilation. This interpretation means postulating vacuum as a state with minimal “energy” \( a_p^-|0\rangle = 0 \), and it restricts the motion of a particle in the space of events, so that a particle with \( P_{0+} \) moves forward and with \( P_{0-} \) backward.

\[ P_{0+} \rightarrow X_I(0) \leq X_(0); \quad P_{0-} \rightarrow X_I(0) \geq X_(0). \]  

(14)

As a result of such a restriction the interval (3) becomes

\[ s_{(P_{0+})} = \frac{m}{E_p} [X_0(s) - X_0(0)]; \quad X_I(0) \leq X_(0), \]  

(15)

\[ s_{(P_{0-})} = \frac{m}{E_p} [X_0(0) - X_0(s)]; \quad X_I(0) \geq X_(0). \]  

(16)
One can see that in both cases the geometric interval is positive. In other words, the stability of quantum theory and the vacuum postulate as its consequence lead to the absolute reference point of this interval \( s = 0 \) and its positive arrow. The last means violation of the symmetry of classical theory with respect to the transformation \( s \rightarrow -s \). Recall that the violation of the symmetry of classical theory by their quantization is called the quantum anomaly \([21,22,23]\). The quantum anomaly as the consequence of the vacuum postulate was firstly discovered by Jordan \([24]\) and then rediscovered by a lot of authors (see \([21]\)).

2.5. Creation of particles

Creation of particles is described by QFT obtained by quantization of classical fields with masses depending on time \( m = m(X_0) \). Classical field equation \((12)\) can be got by varying the action

\[
S_p = \int dX_0 \left\{ P_p \partial_0 \Psi_p - H_p \right\},
\]

where \( H_p = \frac{1}{2} \left[ P_p^2 + E_p^2(X_0)\Psi_p^2 \right] \) is the field Hamiltonian, here we kept only one \( p \)-harmonics. The holomorphic representation of the fields \([20,29]\)

\[
\Psi_p = \frac{1}{\sqrt{2E_p(X_0)}} \left\{ a_p^{(+)}(X_0) + a_p^{(-)}(X_0) \right\},
\]

\[
P_p = i \sqrt{\frac{E_p(X_0)}{2}} \left\{ a_p^{(+)}(X_0) - a_p^{(-)}(X_0) \right\}.
\]

allows us to express the field Hamiltonian in action \((17)\) in terms of observable quantities — the one-particle energy \( E_p(X_0) \) and “number” of particles \( N_p(X_0) = [a_p^+ a_p^-] \):

\[
H_p = \frac{1}{2} \left[ P_p^2 + E_p^2(X_0)\Psi_p^2 \right] = E(X_0) \left[ N_p(X_0) + \frac{1}{2} \right].
\]

While the canonical structure \( P_p \partial_0 \Psi \) in \((17)\) takes the form:

\[
P_p \partial_0 \Psi_p = \left[ \frac{i}{2} (a_p^+ \partial_0 a_p^- - a_p^- \partial_0 a_p^+) - \frac{i}{2} (a_p^+ a_p^- - a_p^- a_p^+) \frac{\partial_0 E(X_0)}{2E(X_0)} \right].
\]

The corresponding Bogoliubov equations of diagonalization expressed in terms of the distribution function of “particles” \( N_p(X_0) \) and the rotation function \( R_p(X_0) \)

\[
N_p(X_0) = |\beta|^2 \equiv p < a_p^+ a_p^- >_p \equiv \sinh r^2, \quad R_p(X_0) = i(\alpha^* \beta - \alpha \beta^*) \equiv -\sin(2\theta) \sinh 2r
\]
take the form \[20\]

\[
\begin{align*}
\frac{dN_p}{dX_0} &= \frac{\partial_0 E(X_0)}{2E(X_0)} \sqrt{4N_p(N_p + 1) - R_p^2}, \\
\frac{dR_p}{dX_0} &= -2E(X_0)\sqrt{4N_p(N_p + 1) - R_p^2},
\end{align*}
\]

(24)

\[E_B(X_0) = E_p(X_0) - \partial_0 \theta \cosh 2r.\]

(25)

These equations supplemented by the quantum geometric interval \[15\] and \[16\] are the complete set of equations for description of the phenomenon of particle creation.

Thus, the direct way from Hilbert’s geometric formulation of any relativistic theory to the corresponding “quantum field theory” goes through Dirac’s Hamiltonian reduction and Bogoliubov’s transformations. As a result, we have the description of creation of a relativistic particle in the space of events at the absolute reference point of geometric interval \(s\) of this particle. The physical meaning of this interval is revealed in the Quantum Cosmology considered below.

3. Quantum Cosmology

3.1. Hilbert’s “Foundation of Physics” of 1915 \[1\]

Einstein’s GR \[2\] is based on the dynamic action

\[S_{GR} = \int d^4x \sqrt{-g} \left[ -\frac{\varphi_0^2}{6} R(g) + \mathcal{L}_{\text{matter}} \right], \quad \varphi_0^2 = \frac{3}{8\pi} M_{\text{Planck}}^2 \]

(26)

proposed by Hilbert in his talk on 20th November 1915 in Göttingen mathematical society \[1\]; this action is given in a Riemannian space-time manifold with “geometric interval”

\[ds = g_{\mu\nu} dx^\mu dx^\nu.\]

(27)

Both the action and interval are invariant with respect to general coordinate transformations

\[x^\mu \rightarrow \tilde{x}^\mu(x^0, x^1, x^2, x^3).\]

(28)

GR \[20\], \[27\] is similar to the geometric version of SR considered in the previous section. Therefore, we can repeat the pathway of SR to QFT of particles considered in Section 2.

3.2. Foundation of Quantum Cosmology of 2005

In order to demonstrate to a reader the direct pathway from Hilbert’s “Foundation of Physics” to QFT of universes through Dirac’s Hamiltonian reduction \[6\], we consider GR in the homogeneous approximation of the interval

\[ds^2 \simeq ds_{\text{WDW}}^2 = a^2(x^0)[(N_0(x^0)dx^0)^2 - (dx^i dx^i)].\]

where Hilbert’s action \[20\] and interval \[27\] take the form

\[S_{\text{cosmic–1915}} = V_0 \int dx^0 N_0 \left[ - \left( \frac{d\varphi}{N_0 dx^0} \right)^2 - \rho_0(\varphi) \right], \]

(29)

\[d\eta = N_0(x^0)dx^0 \rightarrow \eta = \int_0^{x^0} dx^0 N_0(x^0),\]

(30)
here $a(x^0)$ is the cosmological scale factor, $\varphi(x^0) = \varphi_0 a(x^0)$, $\rho_0(\varphi)$ is the energy density of the matter in a universe, $V_0$ is a spatial volume, and $N_0$ is the lapse function. This homogeneous approximation keeps the symmetry of the action and the interval with respect to reparametrizations of the coordinate evolution parameter $x^0 \rightarrow \tilde{x}^0(x^0)$. Recall that similar transformations in the case of SR play the role of gauge transformations; gauge invariance determines observable quantities of the type of energy, time-like variable and number of particles.

In the WDW cosmology this gauge symmetry means that the scale factor $\varphi$ is the "time-like variable" in the "field space of events" introduced by Wheeler and DeWitt in [13], and the canonical momentum $P_\varphi$ is the corresponding Hamiltonian that becomes the energy $E_\varphi$ into equations of motion.

The direct pathway from Hilbert’s SR to QFT of particles considered in Section 2 shows us a similar direct pathway from Hilbert’s “Foundation of Cosmology” [29], [30] to “QFT” of universes. This pathway includes:

1) Hamiltonian approach:

$$S_{\text{cosmic-1915}} = \int dx^0 \left[ -P_\varphi \partial_0 \varphi - \frac{N_0}{4V_0} ( -P_\varphi^2 + E_\varphi^2 ) \right],$$

where

$$E_\varphi = 2V_0 \sqrt{\rho_0(\varphi)}$$

is treated as the "energy of a universe”.

2) constraining: $P_\varphi^2 - E_\varphi^2 = 0$,

3) primary quantization: $[\hat{P}_\varphi^2 - E_\varphi^2] \Psi = 0$, here $\hat{P}_\varphi = -id/d\varphi$;

4) secondary quantization: $\Psi = ([A^+ + A^-]/\sqrt{2E_\varphi})$;

5) Bogoliubov’s transformation: $A^+ = \alpha B^+ + \beta^* B^-$,

6) postulate of Bogoliubov’s vacuum: $B^-|0_{V} = 0$, and

7) cosmological creation of the "universes” from the Bogoliubov vacuum.

Let us carry out this programme in detail.

### 3.3. “Hamiltonian reduction”

In the cosmological model [31], there are two independent equations: the one of the lapse function $\delta S_{\text{cosmic-1915}}/\delta N_0 = 0$:

$$P_\varphi^2 = E_\varphi^2,$$

(33)

treated as the energy constraint, and the equation of momentum $\delta S_{\text{cosmic-1915}}/\delta P_\varphi = 0$

$$P_\varphi = 2V_0 \varphi',$$

(34)

where $\varphi' = \frac{d\varphi}{d\eta}$. The constraint [33] has two solutions

$$P_\varphi^\pm = \pm E_\varphi = \pm 2V_0 \sqrt{\rho(\varphi)},$$

(35)

where $E_\varphi$, given by Eq. [32], is identified with the “one-universe energy”. The substitution of these solutions into action [31] and interval [30] gives us their values

$$S_{\text{cosmic-1915}}|_{P_\varphi = P_\varphi^\pm} = S_{\text{cosmic-1905}}^\pm = \mp 2V_0 \int_{\varphi_0}^{\varphi_0} d\varphi \sqrt{\rho_0(\varphi)},$$

(36)
and

\[ \eta(\varphi|\varphi_0) = 2V_0 \int_{\varphi_0}^{\varphi} \frac{d\bar{\varphi}}{P_{\bar{\varphi}}} = \pm \int_{\varphi_0}^{\varphi} \frac{d\bar{\varphi}}{\sqrt{\rho_0(\bar{\varphi})}} \equiv \pm (\eta_0 - r), \] (37)

respectively. We called these values the "Hamiltonian reduction" of the geometric system [31]. Eq. (37) is treated, in the observational cosmology [25, 26, 27, 28], as the conformal version of the Hubble law. This law describes the relation between the redshift \( z + 1 = \varphi_0/\varphi(\eta) \) of spectral lines of photons (emitted by atoms on a cosmic object at the conformal time \( \eta(\varphi|\varphi_0) = \eta \) and the coordinate distance \( r = \eta_0 - \eta \) of this object, where \( \eta_0 \) is the present-day moment. Thus, we see that WDW cosmology coincides with the Friedmann one \( \varphi'^2 = \rho_0 \). Our task is to consider the status of this Friedmann cosmology in QFT of universes obtained by the first and the secondary quantization of the constraint (33).

### 3.4. QFT of universes

After the primary quantization of the cosmological scale factor \( \varphi \): \( i[P_\varphi, \varphi] = 1 \) the energy constraint [33] transforms to the WDW equation

\[ \partial^2 \Psi + E^2 \Psi = 0. \] (38)

This equation can be obtained in the corresponding classical WDW field theory for universes of the type of the Klein – Gordon one:

\[ S_U = \int d\varphi \left\{ \frac{1}{2} (\partial_\varphi \Psi)^2 - E^2 \Psi^2 \right\} = \int d\varphi L_U. \] (39)

Introducing the canonical momentum \( P_\Psi = \partial L_U/\partial (\partial_\varphi \Psi) \), one can obtain the Hamiltonian form of this theory

\[ S_U = \int d\varphi \{ P_\Psi \partial_\varphi \Psi - H_U \}, \] (40)

where

\[ H_U = \frac{1}{2} \left[ P^2_\Psi + E^4 \Psi^2 \right]. \] (41)

is the Hamiltonian. The concept of the one-universe “energy” \( E_\varphi \) gives us the opportunity to present this Hamiltonian \( H_U \) in the standard forms of the product of this “energy” \( E_\varphi \) and the “number” of universes

\[ N_U = A^+ A^-, \] (42)

\[ H_U = E_\varphi \frac{1}{2} [A^+ A^- + A^- A^+] = E_\varphi [N_U + \frac{1}{2}] \] (43)

by means of the transition to the holomorphic variables

\[ \Psi = \frac{1}{\sqrt{2E_\varphi}} \{ A^{(+)} + A^{(-)} \}, \quad P_\Psi = i \sqrt{E_\varphi / 2} \{ A^{(+)} - A^{(-)} \}. \] (44)

The dependence of \( E_\varphi \) on \( \varphi \) leads to the additional term in the action expressed in terms the holomorphic variables

\[ P_\Psi \partial_\varphi \Psi = \left[ \frac{i}{2} (A^+ \partial_\varphi A^- - A^- \partial_\varphi A^+) - \frac{i}{2} (A^+ A^- - AA) \Delta(\varphi) \right], \] (45)

where

\[ \Delta(\varphi) = \frac{\partial_\varphi E_\varphi}{2E_\varphi}. \] (46)

The last term in (45) is responsible for the cosmological creation of “universes” from “vacuum”.

9
3.5. Creation of universes

In order to define stationary physical states, including a “vacuum”, and a set of integrals of motion, one usually uses the Bogoliubov transformations \[5, 20\] of the holomorphic variables of universes \((A^+, A^-)\):

\[
A^+ = \alpha B^+ + \beta^* B^-,
A^- = \alpha^* B^- + \beta A^+ \quad (|\alpha|^2 - |\beta|^2 = 1),
\]

so that the classical equations of the field theory in terms of universes

\[
(i\partial_\varphi + E_\varphi)A^+ = iA^- \Delta(\varphi), \quad (i\partial_\varphi - E_\varphi)A^- = iA^+ \Delta(\varphi),
\]

take a diagonal form in terms of “quasiuniverses” \(B^+, B^-\):

\[
(i\partial_\varphi + E_B(\varphi))B^+ = 0, \quad (i\partial_\varphi - E_B(\varphi))B^- = 0.
\]

The diagonal form is possible, if the Bogoliubov coefficients \(\alpha, \beta\) in Eqs. \(47\) satisfy to equations

\[
(i\partial_\varphi + E_\varphi)\alpha = i\beta \Delta(\varphi), \quad (i\partial_\varphi - E_\varphi)\beta^* = i\alpha^* \Delta(\varphi).
\]

For the parametrization

\[
\alpha = e^{i\theta(\varphi)} \cosh r(\varphi), \quad \beta^* = e^{i\theta(\varphi)} \sinh r(\varphi),
\]

where \(r(\varphi), \theta(\varphi)\) are the parameters of “squeezing” and “rotation”, respectively, Eqs. \(51\) become

\[
(i\partial_\varphi \theta - E_\varphi) \sinh 2r(\varphi) = -\Delta(\varphi) \cosh 2r(\varphi) \sin 2\theta(\varphi), \quad \partial_\varphi r(\varphi) = \Delta(\varphi) \cos 2\theta(\varphi),
\]

while “energy” of “quasiuniverses” in Eqs. \(49\) is defined by expression

\[
E_B(\varphi) = \frac{E_\varphi - \partial_\varphi \theta(\varphi)}{\cosh 2r(\varphi)}.
\]

Due to Eqs. \(49\) the “number” of “quasiuniverses” \(N_B = (B^+ B^-)\) is conserved

\[
\frac{dN_B}{d\varphi} = d(B^+ B^-) = 0.
\]

Therefore, we can introduce the “vacuum” as a state without “quasiuniverses”:

\[
B^- |0 >_U = 0.
\]

A number of created “universes” from this Bogoliubov vacuum is equal to the expectation value of the operator of the “number of universes” \(\|2\) over the Bogoliubov vacuum

\[
N_U(\varphi) = U < A^+ A^- >_U \equiv |\beta|^2 = \sinh^2 r(\varphi),
\]

where \(\beta\) is the coefficient in the Bogoliubov transformation \(47\), and \(N_U(\varphi)\) is called the “distribution function”. Introducing the “rotation function”

\[
R_U(\varphi) = i(\alpha \beta^* - \alpha^* \beta) \equiv U < P_\Psi \Psi >_U,
\]

one can rewrite the Bogoliubov equations of the diagonalization \(50\) in the form of \(51\)

\[
\begin{align*}
\frac{dN_U}{d\varphi} &= \Delta(\varphi) \sqrt{4N_U(N_U + 1) - R^2_U}, \\
\frac{dR_U}{d\varphi} &= -2E_\varphi \sqrt{4N_U(N_U + 1) - R^2_U}.
\end{align*}
\]

It is natural to propose that at the moment of creation of the universe \(\varphi(\eta = 0) = \varphi_I\) both these functions are equal to zero \(N_U(\varphi = \varphi_I) = R_U(\varphi = \varphi_I) = 0\). This moment of the conformal time \(\eta = 0\) is distinguished by the vacuum postulate \(55\) as the beginning of a universe.
3.6. Quantum anomaly of conformal time

As we have seen in the case of a particle in Section 2.4., the postulate of a vacuum as a state with minimal “energy” restricts the motion of a “universe” in the space of events, so that a “universe” with $P_{\varphi^+}$ moves forward and with $P_{\varphi^-}$ backward.

$$P_{\varphi^+} \rightarrow \varphi_I \leq \varphi_0; \quad P_{\varphi^-} \rightarrow \varphi_I \geq \varphi_0. \quad (59)$$

If we substitute this restriction into the interval (37)

$$\eta(P_{\varphi^+}) = \int_{\varphi_0}^{\varphi_I} \frac{d\varphi}{\sqrt{\rho_0(\varphi)}}, \quad \varphi_I \leq \varphi_0,$$

$$\eta(P_{\varphi^-}) = \int_{\varphi_0}^{\varphi_I} \frac{d\varphi}{\sqrt{\rho_0(\varphi)}}, \quad \varphi_I \geq \varphi_0, \quad (60)$$

one can see that the geometric interval in both cases is positive. In other words, the stability of quantum theory as the vacuum postulate leads to the absolute point of reference of this interval $s = 0$ and its positive arrow. In QFT the initial datum $\varphi_I$ is considered as a point of creation or annihilation of universe. One can propose that the singular point $\varphi = 0$ belongs to antiuniverse. In this case, a universe with a positive energy goes out of the singular point $\varphi = 0$.

In the model of rigid state $\rho = p$, where $E_{\varphi} = Q/\varphi$ Eqs. (58) have an exact solution

$$N_U = \frac{1}{4Q^2 - 1} \sin^2 \left[ \sqrt{Q^2 - \frac{1}{4}} \ln \frac{\varphi}{\varphi_I} \right] \neq 0, \quad (62)$$

where

$$\varphi = \varphi_I \sqrt{1 + 2H_I \eta} \quad (63)$$

and $\varphi_I, H_I = \varphi'_I / \varphi_I = Q/(2V_0\varphi^2_I)$ are the initial data.

We see that there are results of the type of the arrow of time and absence of the cosmological singularity (60), which can be understood only on the level of quantum theory [21, 22, 23].

4. General Relativity as a microscopic theory of superfluidity

4.1. “Foundation of Physics” in terms of Fock’s simplex of reference

General Relativity (GR) [2, 1] is given by two fundamental quantities: the “dynamic” action

$$S[\varphi_0|F] = \int d^4x \sqrt{-g} \left[ -\frac{\varphi_0^2}{6} R(g) + L_\text{(M)}(\varphi_0|g,f) \right], \quad (64)$$

where $\varphi_0^2 = \frac{3}{8\pi} M^2_{\text{Planck}}$ is the Newton constant, $L_\text{(M)}$ is the Lagrangian of the matter field $f$, $F = (g, f)$, and “geometric interval” [21]

$$g_{\mu\nu} dx^\mu dx^\nu \equiv \omega_{(a)} \omega_{(a)} = \omega^{(0)} \omega^{(0)} - \omega^{(1)} \omega^{(1)} - \omega^{(2)} \omega^{(2)} - \omega^{(3)} \omega^{(3)}, \quad (65)$$

where $\omega_{(a)}$ linear differential forms introduced by Fock [12] as components of an orthogonal simplex of reference.
Hilbert’s “Foundation of Physics” in terms of Fock’s simplex \( [53], [65] \) contains two principles of relativity: the “geometric” — general coordinate transformations
\[
x^\mu \rightarrow \tilde{x}^\mu = \tilde{\omega}(x^\mu), \quad \omega(\alpha)(x^\mu) \rightarrow \tilde{\omega}(\alpha)(\tilde{x}^\mu) = \omega(\alpha)(x^\mu)
\] (66)
and the “dynamic” principle formulated as the Lorentz transformations of an orthogonal simplex of reference
\[
\omega(\alpha) \rightarrow \tilde{\omega}(\alpha) = \mathbf{L}(\alpha)\beta \omega(\beta).
\] (67)
The latter are considered as transformations of a frame of reference.

Fock’s separation of the frame transformations (67) from the gauge ones (66) \([12]\) allows us to consider GR and SR on equal footing. Therefore, we shall try to reconstruct a direct pathway from Hilbert’s geometric formulation of GR to Quantum Gravity through Dirac’s Hamiltonian reduction and Bogoliubov transformations:

\[
\begin{array}{ccc}
\text{GR-1915} & \text{SR-1915} & \text{reduction} \\
\downarrow & \downarrow & \leftarrow \\
\text{GR-1905} & \text{SR-1905} & \text{quantization} \\
\downarrow & \downarrow & \leftarrow \\
\text{QFT of universes} & \text{QFT of particles} & \\
\end{array}
\]

Let us consider this pathway that was almost passed by Fock, Dirac, and other physics.

4.2. The Dirac – ADM frame

The Hamiltonian approach to GR is formulated in the frame of reference given by Fock’s simplex of reference in terms of the Dirac variables \([14]\)
\[
\omega(0) = \psi^6 N^i dx^0, \quad \omega(b) = \psi^2 e_{(b)i}(dx^i + N^i dx^0); \quad (68)
\]
here triads \( e_{(a)i} \) form the spatial metrics with \( \det |e| = 1 \).

4.3. Wheeler-DeWitt relativistic universe

A “universe” is a solution of the Einstein equations that describes a hypersurface in the “field space of events” identified with the set of all field variables \( F = (g,f) \) \([13]\) in a specific frame of reference \([63]\). There are two types of the variational equations: six equations of motion
\[
\frac{\delta S}{\delta \psi} = 0, \quad \frac{\delta S}{\delta e_{(a)i}} = 0 \quad (69)
\]
and four constraints
\[
\frac{\delta S}{\delta N^0} = 0, \quad \frac{\delta S}{\delta N^k} = T_{kl}^{(\nu_0)}F = 0. \quad (70)
\]
The general solution of these equations should be given in the class of functions of the gauge transformations in terms of the Dirac gauge-invariant observables in each frame of reference, so that the “dynamic principle” is formulated as independence of equations of motion (but not their solutions) on a choice of a frame of reference, and the “geometric principle” is treated in \([6, 16]\) as the gauge invariance of observables.
4.4. Zel’manov’s class of functions of the gauge transformations

Zel’manov established in [15] that a frame of reference determined by forms [68] is invariant with respect to transformations

$$x^0 \rightarrow x^0 = \tilde{x}^0(x^0); \quad x_i \rightarrow \tilde{x}_i = \tilde{x}_i(x^0, x_1, x_2, x_3),$$  \hspace{1cm} (71)

$$\tilde{N}_d = N_d \frac{dx^0}{d\tilde{x}^0}; \quad \tilde{N}^k = N^k \frac{\partial \tilde{x}^k}{\partial x^i} \frac{dx^0}{d\tilde{x}^0} - \frac{\partial \tilde{x}^k}{\partial x^i} \frac{\partial x^j}{\partial \tilde{x}^0}.$$  \hspace{1cm} (72)

This group of transformations conserves a family of hypersurfaces $$x^0 = \text{const.}$$, and it calls the “kinemetric” subgroup of the group of general coordinate transformations. The “kinemetric” subgroup contains reparametrizations of the coordinate evolution parameter.

The reparametrization of the coordinate evolution parameter ($x^0$) means that this specific frame of reference [68] should be redefined by pointing out two “Dirac observables”: “time-like variable” in the “field space of events” and the “time” as a “geometric interval” [6 7 20].

4.5. Cosmological scale factor as zero mode of the momentum constraints

One of the main problems of the Hamiltonian approach to GR is to pick out the global variable which can play the role of “time-like variable”. There is a lot of speculations on this subject [6 7 20 30 31]. In [6 7 20] one proposed to identify this “internal evolution parameter” with the cosmological scale factor $a(x_0)$ considered as a zero mode of the momentum constraints [70] $T_{k_1}^0(F) = 0$ given in the class of functions of Zel’manov’s gauge transformations [71] that includes homogeneous functions $a(x_0)$.

Separation of real dynamical variables from nondynamical ones is the crucial step in extracting the relevant physical information from the gauge theories. The usual method for achieving this purpose — by imposing a gauge-fixing condition, might not be always adequate to the dynamical content of the classical equations of motion. Another possibility, offered by Dirac [16], consists in introduction of gauge invariant dynamical variables through an explicit solution of the Gauss equation. Such an explicit solution might contain some additional physical information which is implicitly lost by the gauge fixing. This can be seen even on the simplest example of the two–dimensional QED [23], where the Gauss constraint $\partial_1 E(x^0, x^1) = 0$ has a nontrivial homogeneous solution $E(x^0, x^1) = E_0(x^0)$ that determines the topological structure and energy spectrum of the theory; and this solution is called “zero mode”. There is a similar nontrivial homogeneous solution of the constraint (70) $T_{k_1}^0(\varphi_0|F) = 0$ in GR.

Constraints $T_{k_1}^0(\varphi_0|F) = 0$ are invariant with respect to the Lichnerowicz scale transformation [32]: $F^{(n)} = a^n(x_0)F^{(n)}$, where $(n)$ is the conformal weight of a field: $T_{k_1}^0(\varphi|F) = T_{k_1}^0(\varphi_0|F)$, like the Gauss constraint in two–dimensional QED [23] $\partial_1 E(x^0, x^1) = 0$ is invariant with respect to the transformation $E(x^0, x^1) = E_0(x^0) + F(x^0, x^1)$. Therefore, in the Hamiltonian approach to GR, the scale $a(x^0)$ is considered as a “zero mode” of the momentum constraints in the definite frame of reference.

Thus, a general solution of constraints $T_{k_1}^0(\varphi_0|F) = 0$ in the class of functions [71] can be written in the form of the Lichnerowicz scale transformation [32]. In particular, the spatial metric determinant takes the form $\psi^2 = a(x_0)\overline{\psi}^2$. The last equation can be treated as the decomposition of the logarithm of spatial metric determinant in the form of a sum of zero-Fourier harmonics $\langle \log \psi^2 \rangle \equiv \int d^3x \log \psi^2/V_0$ (where $V_0 = \int d^3x < \infty$ is a finite volume) and nonzero ones distinguished by the identity

$$\int d^3x \log \overline{\psi}^2 = \int d^3x \left[ \log \psi^2 - \langle \log \psi^2 \rangle \right] \equiv 0.$$  \hspace{1cm} (73)
4.6. Separation of the zero mode by Lichnerowicz’s transformation

In order to find values of the action \( \mathcal{L}_N(\phi_0) \) into resolutions of constraints \( g_{\mu\nu} = a^2(x^0)\overline{g}_{\mu\nu} \), we redefine a specific frame of reference \( \mathcal{L}_N(\phi_0) \) by the Lichnerowicz scale transformation \( F^{(n)} = a^n(x_0)\overline{F}^{(n)} \). The Einstein – Hilbert action \( \mathcal{L}_N(\phi_0) \) after the scale transformation takes form

\[
S[\varphi|F] = S[\varphi|\overline{F}] + \int dx^0 \varphi \partial_0 \left[ \frac{\partial_0 \varphi}{N_0} \right],
\]

where \( N_0(x_0)^{-1} = V_0^{-1} \int V_0 \, d^3x \overline{N}_d^{-1}(x_0, x') \equiv \langle \overline{N}_d^{-1} \rangle \) is the averaging of the inverse lapse function \( \overline{N}_d \) over spatial volume \( V_0 = \int d^3x \),

\[
S[\varphi|\overline{F}] = \int d^4x \left[ K[\varphi|\overline{g}] - P[\varphi|\overline{g}] + S[\overline{g}] + \mathcal{L}_{(\phi)}(\varphi|\overline{F}) \right]
\]

is the action \( \mathcal{L}_N(\phi_0) \), where \( [\varphi_0|F] \) is replaced by \( [\varphi|\overline{F}] \); here \( \varphi(x^0) = \varphi_0 a(x^0) \) is the running scale of all masses of the matter field,

\[
K[\varphi|\overline{g}] = \varphi^2 N_d \left[ -4 v_\psi^2 + \frac{v_{(ab)}^2}{6} \right],
\]

\[
P[\varphi|\overline{g}] = \frac{\varphi^2}{6} \left[ (3) R(e) \psi + 8 \Delta \psi \right],
\]

\[
S[\varphi|\overline{g}] = 2 \varphi^2 \left[ \partial_0 v_\psi \partial_1 (N^t v_\psi) \right] - \frac{v_\psi^2}{3} \partial_1 \bar{\psi} \partial^i (\bar{\psi} \bar{N}_d) \]

are the kinetic, potential, and “quasi–surface” terms respectively,

\[
v_{(ab)} = \frac{1}{2} \left( e_{(a)i} e^i_{(b)} + e_{(b)i} e^i_{(a)} \right), \quad v_{(a)i} = \frac{1}{N_d} \left[ (\partial_0 - N^t \partial_i) e^i_{(a)} + \partial_0 N^i - \frac{e^i_{(a)}}{3} \partial_i N^l \right]
\]

are velocities of triads \( e_{(a)i} \), \( (3) R(e) \) is the curvature of the triads, and

\[
v_\psi = \frac{1}{N_d} \left[ (\partial_0 - N^t \partial_i) \ln \bar{\psi} - \frac{1}{6} \partial_i N^l \right]
\]

is the trace of the “second form”.

The last term in Eq. (74) determines the “time as interval” in Dirac’s frame (78)

\[
d\zeta = N_0(x^0)dx^0; \quad \zeta(x^0) = \int d^3x_0 N_0(x^0).
\]

4.7. Avoiding double counting of canonical momenta

Neglecting total time derivatives, we keep in Eq. (74) only the part of Lagrangian describing the spatial metric determinant:

\[
L_{SD} = - \int d^3x N_d \left[ 4 \varphi^2 (v_\psi)^2 + 4 \varphi v_\varphi + (v_\varphi)^2 \right],
\]

where \( v_\varphi = \partial_0 \varphi / N_d \), the first term in the Lagrangian arises from the kinetic part \( K[\varphi|\overline{g}] \) in Eq. (75), the second goes from the “quasi–surface” one (78), and the third term goes from the second
one in Eq. (74). The canonical momentum of the scale factor can be obtained by variation of Lagrangian (82) with respect to the time derivative of scale factor $\partial_0 \phi$:

$$P_\phi \equiv \frac{\partial L_{SD}}{\partial (\partial_0 \phi)} = - \int d^3x [4\phi v_\psi + 2v_\phi] \equiv - [4\phi V_\psi + 2V_\phi],$$

while the zero Fourier harmonics of a canonical momentum of the spatial metric determinant is

$$P_\psi \equiv - \int d^3x \frac{\partial L_{SD}}{\partial (\partial_0 \log \psi)} = - \int d^3x \bar{p}_\psi = \int d^3x \left[ 8\phi^2 v_\psi + 4\phi v_\phi \right] \equiv - 2\phi [4\phi V_\psi + 2V_\phi],$$

where $V_\phi = \int d^3x v_\phi, V_\psi = \int d^3x v_\psi$. These two equations have no solutions, as the matrix of the transition from “velocities” to momenta has the zero determinant. This means that the “velocities” $[V_\phi, V_\psi]$ could not be expressed in terms of the canonical momenta $[P_\phi, P_\psi]$ and the Dirac Hamiltonian approach becomes a failure. To be consistent with identity (73) and to keep the number of variables of GR, we should impose the strong constraint

$$V_\psi \equiv \int d^3x v_\psi \equiv 0,$$

otherwise we shall have the double counting of the zero-Fourier harmonics of spatial metric determinant.

What is double counting? A “double counting” is replacement of $L_1 = (\dot{x})^2/2$ by $L_2 = (\dot{x} + \dot{y})^2/2$. The second theory is not mathematically equivalent to the first. The test of this nonequivalence is the failure of the Hamiltonian approach to $L_2 = (\dot{x} + \dot{y})^2/2$. Therefore, the replacement $L_1 \rightarrow L_2$ is nonsense in the context of the Hamiltonian approach.

The next example is Lifshitz’s perturbation theory given by Eq. (3.21) p. 217 in [19]

$$ds^2 = a^2(\eta)((1 + 2\Phi)d\eta^2 - (1 - 2\Psi)\gamma_{ij}dx^idx^j)$$

This formula contains the double counting of the zero Fourier-harmonics of the spatial metrics determinant presented by two variables: the scale factor $a$ and $<\Psi> = \int d^3x \Psi(\eta, x_i)$ instead of one. If we impose the strong constraint

$$<\Psi> = \int d^3x \Psi(\eta, x_i) \equiv 0; \quad <P_\psi> = \int d^3x \partial L_{(2)}/\partial \Psi(\eta, x_i) \equiv 0,$$

in order to remove the “double counting”, we shall return back to the Einstein theory (64). In the Einstein theory, instead of the equations of $\Psi$ and $\Phi$ (4.15) on p. 220 in [19]

$$-3H(\dot{\Phi} + \Psi') + \triangle \Psi = 4\pi G \delta T^0_0$$

$$3[(2H^2 + H\dot{\Phi})\Phi + H\dot{\Phi}' + \Psi'' + 2H\Psi'] + \triangle (\Phi - \Psi) = - 4\pi G \delta T^i_i,$$

where $4\pi G = 3/(2\varphi^2)$, $H = a'/a$, and $\triangle = \partial^2_t$, we shall obtain in Section 4.10 the equations

$$\triangle \Psi = 4\pi G \delta T^0_0$$

$$\triangle (\Phi - \Psi) = - 4\pi G \delta T^i_i.$$
4.8. Hamiltonian approach

The Hamiltonian action of GR \((74), (75)\) in the field space \([\varphi|F]\) takes the form \([6, 7]\)

\[
S = \int dx^0 \left[ -P_\varphi \partial_0 \varphi + N_0 \frac{P_F^2}{4V_0} + \int d^3x \left( \sum_F P_F \partial_0 F + C - N_d H_t \right) \right],
\]

where \(P_F\) is the set of the field momenta \(p_\varphi, p_{(ab)}, p_F\); the sum of constraints \(C = N(a) T^0(\alpha) + C_0 p_\psi + C_\alpha \partial_\alpha e_a\) contains the weak Dirac constraints of transversality \(\partial_\alpha e_a \approx 0\) and the minimal space-like surface \([14]\)

\[
\mathcal{H}_t = \frac{1}{\varphi^2} \left[ 6p_{(ab)} p_{(ab)} - 16(\partial_0 \varphi)^2 \right] + \frac{\varphi^2 \psi^7}{6} \left[ \langle 3 R(e) \rangle \psi + 8\Delta \psi \right] + \psi^4 T^0(0(M)),
\]

is the Dirac Hamiltonian density \([14]\) in terms of \(p_{(ab)} = \left[ p^a_{(a)} e_{(b)i} + p^b_{(b)} e_{(a)i} \right] / 2\),

\[
T^0(\alpha) = -\nabla_0 \partial_\alpha \nabla_0 \varphi + \frac{1}{6} \partial_\alpha (\nabla_0 \nabla_0) - \partial_\beta \partial_{(0a)} p_{(ab)} - p_{(ab)} e_{(a)\beta} (\partial_\beta e^i_{(a)} - \partial_\alpha e^i_{(a)}) + T^0(0(M)),
\]

and \(T^0(0(M)), T^0(0(M))\) are components of the energy–momentum tensor in terms of the York conformal fields \(F^n = (\varphi^2)^n F_\varphi^n\) \([30]\).

The gauge-invariant lapse function \(N(d)/N_0 = N\) and the spatial metric determinant \(\nabla_0 \varphi \) can be determined by their equations for both the zero Fourier harmonic \(\langle F \rangle\) and the nonzero ones \(\bar{F} = F - \langle F \rangle\):

\[
\left< \frac{N_0}{\delta S[\varphi]} \right> = 0 \implies \varphi^2 = \rho_t, \quad \left< \frac{N_0}{\delta S[\varphi]} \right> = 0 \implies \frac{\rho_t}{N} = \mathcal{N} H_t,
\]

\[
\left< \frac{\psi}{2 \delta \varphi} \right> = 0 \implies (\varphi^2)^\prime = 3(\rho_t - \rho_1), \quad \left< \frac{\psi}{2 \delta \varphi} \right> = 0 \implies \frac{\rho_0}{\mathcal{A} N} = 0,
\]

where \(\varphi' \equiv d\varphi / d\zeta, \rho_t \equiv \langle N H_t \rangle \) and \(p_t = \langle N \psi^4 T^0(0(M)) / 3 \rangle \) are the energy density and pressure of all fields, respectively; and \(\mathcal{A}\) is a differential operator:

\[
\mathcal{A} N \equiv \frac{2 \varphi^2}{3} \left[ \langle 3 R(e) \rangle \varphi^8 + 8 \varphi^7 \Delta \varphi \rangle N + \partial_j [\varphi^2 \partial^j (\psi^6 N)] \right] + \psi^4 \left[ 3 T^0(0(M)) - T^0(0(M)) \right] N.
\]

Equations \([30]\) and \([31]\) show us that their zero harmonics coincide with the conformal version of the Friedmann equations with the scale factor \(a = \varphi / \varphi_0\) \([4]\). In this case the Hamiltonian cosmological perturbation theory does not require its convergence to be proved because the perturbations are in a different class of functions (with nonzero Fourier harmonics) than the cosmological dynamics described by the equations in the zero harmonic sector.

4.9. Dirac’s Hamiltonian reduction

The energy constraints \([30]\) have solutions

\[
P_{\varphi(\pm)} = \pm 2 V_0 \varphi' = \pm 2 V_0 \langle \sqrt{H}_t \rangle, \quad N = \langle \sqrt{H}_t \rangle / \sqrt{H}_t
\]

(92)
expressing the reduced Hamiltonian $P_\varphi(\pm)$ and lapse function $\mathcal{N}$ through the field energy density $\mathcal{H}_t$ given by Eq. (88). If we substitute these solutions into the action (87) and solve the first equation of $P_\varphi$ with respect to the time $\zeta$, we obtain the “reduced action” and “interval” (81):

$$S_{\pm}[\varphi|\varphi_0]|_{\text{constraint}} = \int d\varphi \left\{ \int d^3x \left[ \sum_F P_F \partial_\varphi F + \tilde{C} \mp 2\sqrt{\mathcal{H}_t} \right] \right\},$$

$$\zeta_{\pm}[\varphi|\varphi_0]|_{\text{constraint}} = \pm \int \frac{d\varphi}{\sqrt{\mathcal{H}_t}},$$

where $\tilde{C} = C/\partial_0 \varphi$ and $\varphi_0$ is the present-day datum. Action (93) determines the evolution of fields directly in terms of the redshift parameter connected with the scale factor $\varphi$ by the relation $\varphi = \varphi_0/(1+z)$ and interval (94) gives the Friedmann-like cosmic evolution. The Dirac constraint $p_\psi = 0$ in Eq. (88) leads to the Hermitian reduced Hamiltonian. The Dirac Hamiltonian “reduction” of the GR action (26) onto its values (93) obtained by the explicit resolution of the energy constraints determines main concepts of the primary quantization and secondary one, whereas the “reduced interval” (94) gives us the opportunity to clear up the status of the Hubble evolution in the Hamiltonian theory. As we have seen in Section 3, the corresponding quantum theory describing the cosmological creation of a universe from the Bogoliubov stable vacuum explains the absolute beginning of time and removes a cosmological singularity. In other words, there are reasons to treat the evolution of the cosmological scale factor as a quantum collective motion of a system of all fields as the whole of the type of the phenomenon of “superfluidity” [3, 4, 5].

### 4.10. Hamiltonian cosmological perturbation theory

The Hamiltonian cosmological perturbation theory [33] is defined using the decomposition of the forms (68)

$$\omega(0) = a(1 + \Phi) d\eta, \quad \omega(a) = a(1 - \Psi) (dx(a) + h_{(a)i}^{(TT)} dx^i + N(a) d\eta),$$

where $a = \varphi/\varphi_0$ is the cosmological factor, $N(a) = \partial(a) \sigma + N^{(T)}_{(a)}$. We take into account the Dirac constraints of transversality $\partial_i h_{(a)i}^{(TT)} = 0$ and minimal surface $p_\psi = 8 \varphi v_\psi \simeq 0$, where $v_\psi$ is given by Eq. (80). The latter defines the longitudinal shift vector (96):

$$N^{(\perp)}_{(a)} = \partial(a) \sigma; \quad \Delta \sigma = -\frac{3}{4} \Psi'. \quad (96)$$

Therefore, there are six independent components: two scalars $\Phi$ and $\Psi$, two vector ones $N^{(T)}_{(a)}$, and two tensor ones $h_{(a)i}^{(TT)}$.

The cosmological perturbations of the metric components are defined in the class of functions with the nonzero Fourier harmonics $\hat{\Phi}(k) = \int d^3x \Phi(x) e^{ikx}$ (satisfying the strong constraint $\int d^3x \Phi(x) = 0$).

In the approximation

$$\varphi^2 k^2 \gg \rho_s = \langle T_{0(M)}^0 \rangle \gg \delta T_{0}^{0} = (T_{0(M)}^{0} - \langle T_{0(M)}^{0} \rangle);$$

$$\varphi^2 k^2 \gg 3p_s = \langle T_{k(M)}^{k} \rangle \gg \delta T_{k}^{k} = (T_{k(M)}^{k} - \langle T_{k(M)}^{k} \rangle),$$
six equations of the theory for the scalar, vector, and tensor components take form

\[ \triangle \Psi = 4\pi G \delta T^0_0, \]
\[ \triangle \Phi = 4\pi G (\delta T^0_0 - \delta T^j_j), \]
\[ \frac{1}{4} \triangle N^T_j = -4\pi G \delta T^0_0(T), \]
\[ \frac{1}{8} \left[ -\triangle h^{TT}_{ij} + \left( \frac{\varphi^2 h^{TT}_{ij}}{\varphi^2} \right) \right] = 4\pi G \delta T^{TT}_{ij}, \]

where \( \partial_i \delta T^{TT}_{ij} = 0, \ \delta T^{TT}_{ii} = 0, \ \partial_j \delta T^0_0(T) = 0, \) and \( 4\pi G = 3/(2\varphi^2) \) is running Newton coupling “constant”.

Eqs. \( 97 \) and \( 98 \) differ from the standard cosmological perturbation theory \( 17, 19 \) by the absence of the time derivative of deviations of the linear spatial metric determinant \( \Psi \) and lapse function \( \Phi \) (see Eqs. (4.15) in \( 19 \)). These time derivatives, in the Lifshitz perturbation theory \( 17, 19 \), go from the kinetic term \( K \) and from the zero Fourier harmonics of the trace of second form. In the Hamiltonian perturbation theory (with the Dirac minimal surface \( \overline{p}_\varphi = 0 \) that leads to the Hermitian Hamiltonian in the field space of events \( [\varphi|F] \)), the kinetic term \( K \) in action \( 75 \) contributes only to the next orders of the perturbation theory, while the “quasi–surface” term \( (\int d^3 x S) \) was removed from action \( 75 \) in order to formulate the Hamiltonian approach to GR without the double counting of the scale factor velocity. The potential term \( P \) leads only to the spatial derivatives.

The solutions of \( 97 \) and \( 98 \) take the form of standard classical solutions with the Newton gravitational constant \( G = 3/8\pi \varphi^2 \) (see Eqs. \( 85 \) and \( 86 \)):

\[ \bar{\Psi} = -\frac{4\pi G}{k^2} \delta T^0_0; \quad \bar{\Phi} = -\frac{4\pi G}{k^2} \left[ \delta T^0_0 - \delta T^k_k \right]. \]

The minimal surface \( \overline{p}_\varphi = 0 \) \( 96 \) gives the shift of the coordinate origin in the process of evolution, in particular, in the case of a point source \( \delta T^0_0 = M_J \delta^3(x - y_J) - 1/V_0 \), we got the shift vector:

\[ N^i = -\frac{3(GM_J)(x - y_J)^i}{4|x - y_J|}. \]

Interval \( 85 \) determines an equation for the photon momenta

\[ p_\mu p_\nu g^{\mu\nu} \simeq (p_0 + N^i p_i)^2(1 - 2\Phi) - p_j^2(1 + 2\Psi) = 0, \]

from which we obtain a photon energy

\[ p_0 \simeq -N^i p_i + [1 + (\Phi + \Psi)]|p|; \quad |p| = \sqrt{p_j^2}. \]

This formula shows us the relative magnitude of spatial fluctuations of a photon energy in terms of the metric components \( p_0 - |p|/|p| = -[N^i n^i + (\Phi + \Psi)], \ n^i = p_i/|p|. \) The appearance of the spatial anisotropy \( 102 \) in the flow of the photon energy is the consequence of the minimal surface \( \overline{p}_\varphi = 0, \) and this anisotropy \( 103 \) can be taken account in order to describe the spectrum of CMB temperature fluctuation.

4.11. Bogoliubov’s quantum Gravity

Quantum theory for GR is based, on the one hand, on the existence of the Hilbert geometric formulation \( 1 \) of Special Relativity considered in Section 2 as the simplest model of GR and, on
the other hand, on the contemporary quantum field theory appearing as a result of resolution of the energy constraint and its primary and secondary quantizations.

Resolution of the energy constraint leads to the “Hamiltonian reduction” of the Hilbert geometric action \( S^0 \), and it determines the “reduced energy” \( E_\varphi = 2V_0(\sqrt{\mathcal{H}_I}) \) as the central concept of the primary quantization \( \hat{P}_\varphi \Psi = -i\partial_\varphi \Psi \). The primary quantization converts the energy constraint \( P_\varphi^2 - E_\varphi^2 = 0 \) into the WDW equation: \( \partial^2 \Psi + E_\varphi^2 \Psi = 0 \) \[13\].

The next step is the secondary quantization, where \( \Psi = [A^+ + A^-]/(2E_\varphi) \) is considered as a “quantum field” with “one-universe energy” \( E_\varphi \). We have seen in Section 3 that the Bogoliubov transformation \( A^+ = \alpha B^+ + \beta B^- \) \[5\] of operators of “universe” \( A^+ \), \( A^- \) to ones of “quasiuniverse” \( B^+, B^- \) allows us to postulate the vacuum as a stable state of the minimal energy \( B^-|0 >_U = 0 \), to find conserved numbers of “quasiuniverses” \( N_B = (B^+ B^-) \) and calculate a distribution function of creation of the “universe” \( N_U(\varphi) = U < A^+ A^- >_U \equiv |\beta|^2 \) and the “rotation function” \( R_U = i(\alpha^* \beta - \alpha \beta^*) \) satisfying equations \[20\]:

\[
\frac{dN_U}{d\varphi} = -\frac{\partial_\varphi E_\varphi}{4E_\varphi^2} dR_U, \quad \frac{dR_U}{d\varphi} = -2E_\varphi \sqrt{4N_U(N_U + 1) - R_U^2} \tag{105}
\]

with the initial data \( N_U(\varphi = \varphi_I) = R_U(\varphi = \varphi_I) = 0 \).

The “vacuum” postulate restricts the motion of the universe in the field space of events \([\varphi|F]\): a universe moves forward \( \varphi > \varphi_I \) for positive energy \( P_\varphi \geq 0 \) (creation of a universe), and a universe moves backward \( \varphi < \varphi_I \) for negative \( P_\varphi \leq 0 \) (annihilation of a universe), where \( \varphi_I \) is the initial data. This restriction leads to positive arrow of the “interval” \[51\] \( \zeta_\pm \geq 0 \) and its absolute beginning \[6\].

### 4.12. Einstein’s correspondence principle and relative units

Einstein’s correspondence principle \[6\] as the low-energy expansion of the “reduced action” \[8\] over the field density \( T_{s_0}^0 \)

\[
d\varphi \sqrt{\mathcal{H}_I} = d\varphi \sqrt{\rho_0(\varphi) + T_{s_0}^0} = d\varphi \left[ 2\sqrt{\rho_0(\varphi) + T_{s_0}^0/\sqrt{\rho_0(\varphi)}} \right] + ...
\]

gives the sum: \( S^{(+)}|\varphi_I|\varphi_0|_{\text{constraint}} = S^{(+)}_{\text{cosmic}} + S^{(+)}_{\text{field}} + ... \), where \( S^{(+)}_{\text{cosmic}}|\varphi_I|\varphi_0| = -2V_0 \int d\varphi \sqrt{\rho_0(\varphi)} \)

is the reduced cosmological action \[80\], and

\[
S^{(+)}_{\text{field}} = \int_{\eta_I}^{\eta_0} d\eta \int d^3x \left[ \sum_F P_F \partial_\eta F + \bar{C} - T_{s_0}^0 \right] \tag{106}
\]

is the standard field action in terms of the conformal time: \( d\eta = d\varphi/\sqrt{\rho_0(\varphi)} \), in the conformal flat space–time with running masses \( m(\eta) = a(\eta)m_0 \) that describes the cosmological particle creation from vacuum \[20\].

This expansion shows us that the Hamiltonian approach identifies the “conformal quantities” with the observable ones including the conformal time \( d\eta \), instead of \( dt = a(\eta)d\eta \), the coordinate distance \( r \), instead of Friedmann one \( R = a(\eta)r \), and the conformal temperature \( T_c = Ta(\eta) \), instead of the standard one \( T \). Therefore the correspondence principle distinguishes the conformal cosmology (CC) \[25\ \[26\], instead of the standard cosmology (SC). In this case the red shift of the spectral lines of atoms on cosmic objects

\[
\frac{E_{\text{emission}}}{E_0} = \frac{m_{\text{atom}}(\eta_0 - r)}{m_{\text{atom}}(\eta_0)} = \frac{\varphi(\eta_0 - r)}{\varphi_0} = \frac{a(\eta_0 - r)}{1+z} = \frac{1}{1+z}
\]
Figure 1: The Hubble diagram \cite{27, 28} in cases of the \textit{“absolute”} units of standard cosmology (SC) and the \textit{“relative”} ones of conformal cosmology (CC). The points include 42 high-redshift Type Ia supernovae \cite{34} and the reported farthest supernova SN1997ff \cite{35}. The best fit to these data requires a cosmological constant $\Omega_\Lambda = 0.7 \Omega_{\text{ColdDarkMatter}} = 0.3$ in the case of SC, whereas in CC these data are consistent with the dominance of the rigid (stiff) state.

is explained by the running masses $m = a(\eta)m_0$ in action \cite{106}.

The conformal observable distance $r$ loses the factor $a$, in comparison with the nonconformal one $R = ar$. Therefore, in the case of CC, the reduced interval \cite{37} describing the redshift – coordinate-distance relation \cite{26} corresponds to a different equation of state than in the case of SC. The best fit to the data, including Type Ia supernovae \cite{34, 35}, requires a cosmological constant $\Omega_\Lambda = 0.7$, $\Omega_{\text{ColdDarkMatter}} = 0.3$ in the case of the Friedmann \textit{“absolute quantities”} of standard cosmology. In the case of \textit{“conformal quantities”} in CC, the Supernova data \cite{34, 35} are consistent with the dominance of the stiff (rigid) state, $\Omega_{\text{Rigid}} \approx 0.85 \pm 0.15$, $\Omega_{\text{Matter}} = 0.15 \pm 0.15$ \cite{26, 27, 28}. If $\Omega_{\text{Rigid}} = 1$, we have the square root dependence of the scale factor on conformal time $a(\eta) = \sqrt{1 + 2H_0(\eta - \eta_0)}$. Just this time dependence of the scale factor on the measurable time (here – conformal one) is used for description of the primordial nucleosynthesis \cite{28, 36}.

This stiff state is formed by a free scalar field when $E_\varphi = 2V_0\sqrt{\varphi_0} = Q/\varphi$. Just in this case there is an exact solution of Bogoliubov’s equations \cite{62}

$$N_U(\varphi_0) = \frac{1}{4Q^2 - 1} \sin^2 \left[ \sqrt{Q^2 - \frac{1}{4}} \ln \frac{\varphi_0}{\varphi_I} \right] \neq 0,$$  \hspace{1cm} (107)

where the Planck mass $\varphi_0 = \varphi_I\sqrt{1 + 2H_I\eta_0}$ belongs to the present-day data $\eta = \eta_0$ and $\varphi_I, H_I = \varphi_I'/\varphi_I = Q/(2V_0\varphi_I^2)$ are the initial data.

4.13. Cosmological creation of matter

These initial data $\varphi_I$ and $H_I$ are determined by the parameters of matter cosmologically created from the Bogoliubov vacuum at the beginning of a universe $\eta \simeq 0$. 
Figure 2: Longitudinal ($N_z(x)$) components of the boson distribution versus the dimensionless time $\tau = 2\eta H_I$ and the dimensionless momentum $x = q/M_I$ at the initial data $M_I = H_I (\gamma_v = 1)$ [37, 38].

The Standard Model (SM) density $T_{s0}$ in action (106) shows us that W-, Z- vector bosons have maximal probability of this cosmological creation due to their mass singularity [37]. One can introduce the notion of a particle in a universe if the Compton length of a particle defined by its inverse mass $M_I^{-1} = (a_I M_W)^{-1}$ is less than the universe horizon defined by the inverse Hubble parameter $H_I^{-1} = a_I^2 (H_0)^{-1}$ in the stiff state. Equating these quantities $M_I = H_I$ one can estimate the initial data of the scale factor $a_I^2 = (H_0/M_W)^{2/3} = 10^{29}$ and the primordial Hubble parameter $H_I = 10^{29} H_0 \sim 1\text{mm}^{-1} \sim 3K$. Just at this moment there is an effect of intensive cosmological creation of the vector bosons described in [37]; in particular, the distribution functions of the longitudinal vector bosons demonstrate us a large contribution of relativistic momenta, as it was shown in Fig. 2. Their conformal (i.e. observable) temperature $T_c$ (appearing as a consequence of collision and scattering of these bosons) can be estimated from the equation in the kinetic theory for the time of establishment of this temperature $\eta_{relaxation}^{-1} \sim n(T_c) \times \sigma \sim H$, where $n(T_c) \sim T_c^3$ and $\sigma \sim 1/M^2$ is the cross-section. This kinetic equation and values of the initial data $M_I = H_I$ give the temperature of relativistic bosons

$$T_c \sim (M_I^2 H_I)^{1/3} = (M_0^2 H_0)^{1/3} \sim 3K$$

as a conserved number of cosmic evolution compatible with the Supernova data [26, 34, 55]. We can see that this value is surprisingly close to the observed temperature of the CMB radiation $T_c = T_{CMB} = 2.73 K$.

The primordial mesons before their decays polarize the Dirac fermion vacuum (as the origin of axial anomaly [21, 22, 23, 24]) and give the baryon asymmetry frozen by the CP – violation. The value of the baryon–antibaryon asymmetry of the universe following from this axial anomaly was estimated in [37] in terms of the coupling constant of the superweak-interaction

$$n_b/n_\gamma \sim X_{CP} = 10^{-9}.$$ 

The boson life-times $\tau_W = 2H_I \eta_W \simeq \left(\frac{2}{\tau_a}\right)^{2/3} \simeq 16$, $\tau_Z \sim 2^{2/3} \tau_W \sim 25$ determine the present-day visible baryon density

$$\Omega_b \sim \alpha_g = \alpha_{QED}/\sin^2 \theta_W \sim 0.03.$$
This baryon density as a final product of the decay of bosons with momentum $q$ and energy
\[
\omega(\eta) = (M^2(\eta) + q^2)^{1/2}
\]
oscillates as \[\cos \left( \int_0^{\eta} \frac{d\eta}{\omega(\eta)} \right) \] \[37\]. One can see \[38\] that the number of density oscillations of the primordial bosons during their life-time for the momentum $q \sim M_I$ is of order of 20, which is very close to the number of oscillations of the visible baryon matter density recently discovered in researches of large scale periodicity in redshift distribution \[39, 40\]
\[H_0 \times 128 \text{ MPc}^{-1} \sim 20 \div 25 \sim (\alpha_g)^{-1}. \] \[111\]
The results \[108, 109, 110, 111\] testify to that all visible matter can be a product of decays of primordial bosons with the oscillations forming a large-scale structure of the baryonic matter.

4.14. The Dark Matter problem

In the considered model, galaxies and their clusters are formed by the Newton Hamiltonian with running masses $E(\eta) = p^2/2m(\eta) - r_g(\eta)m(\eta)/2r$, where the Newton coupling $r_g(\eta)m(\eta)/2 = r_g(\eta_0)m(\eta_0)/2$ is a motion constant. One can see that the running masses lead to the effect of the capture of an object by a gravitational central field at the time when $E(\eta_{\text{capture}}) = 0$. After the capture the conformal size of the circle trajectories decreases as $r(\eta) = R_0/a(\eta)$, $R_0 = \text{const}$.

The running masses change the orbital curvatures \[11\]
\[v_{\text{orbital}}(R_0) = \sqrt{\frac{r_g}{2R_0} + \gamma(R_0H)^2}, \] \[112\]
where
\[\gamma = 2 - \frac{3}{2} \Omega_{\text{Matter}} - 3\Omega_{\Lambda}\]
is determined by the equation of state. We can see that in the case of the stiff state of the conformal cosmology, when $\Omega_{\text{Matter}} = \Omega_{\Lambda} = 0$ and $\gamma = 2$, the cosmological evolution plays the role of the Dark Matter, and it can explain the deficit of the luminous matter $M/M_L \sim 10^2$, where $M_L$ stands for the mass of luminous matter, in superclusters with a mass $M \geq 10^{15}M_\odot$, $R \gtrsim 5\text{Mpc} \[11\]$, where the Newton velocity becomes less than the cosmic one. In the case of standard cosmology: $\Omega_{\text{Matter}} = 0.3$, $\Omega_{\Lambda} = 0.7$, the last term is negative $\gamma = -1/2$. Thus, the standard cosmology requires one more Dark Matter in contrast to the conformal cosmology \[23\].

However the cosmological modification of the Newton dynamics given by Eq. \[112\] is not sufficient to explain the constant orbital velocities in spiral galaxies\(^1\).

5. Conclusion

We have seen that GR could pass along the pathway of quantum field theory through the Dirac reduction and the Bogoliubov transformation (see the table on p. \[12\]), in order to describe the cosmological creation of universes \[105\]. This pathway includes the zero mode of a general resolution of constraints in the class of functions of gauge transformations (as the global homogeneous excitation of the type of Landau superfluid liquid), the WDW unique wave function as \[1\]
The cylindric symmetry of matter sources in spiral galaxies points out that their gravitational potential can be the two-dimensional Newton one $\Delta_{(2)} \Phi = (1/2\ell^2)\delta^2(x)$ with the length of an axis $2\ell \[11\]$. This potential leads to the corresponding orbital velocity $v_{\text{orbital}}(R_0) = r_g/(2\sqrt{L^2 + R_0^2})$, and it can explain the constant rotational curves for spiral galaxies in the region $R_0 \lesssim l$ in the case of $\Omega_b \sim 5\%$.

\(^1\)The cylindric symmetry of matter sources in spiral galaxies points out that their gravitational potential can be the two-dimensional Newton one $\Delta_{(2)} \Phi = (1/2\ell^2)\delta^2(x)$ with the length of an axis $2\ell \[11\]$. This potential leads to the corresponding orbital velocity $v_{\text{orbital}}(R_0) = r_g/(2\sqrt{L^2 + R_0^2})$, and it can explain the constant rotational curves for spiral galaxies in the region $R_0 \lesssim l$ in the case of $\Omega_b \sim 5\%$. 

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London’s attribute of superfluidity, the postulate of the quantum Bogoliubov vacuum leading to the absolute beginning of geometric time, the Einstein correspondence principle identifying the conformal quantities with the “measurable” ones, and the uncertainty principle for establishing the point of the beginning of the cosmological creation of the primordial W-, Z- bosons from vacuum due to their mass singularity.

The Hamiltonian approach revealed the double counting of the cosmological scale factor in the standard Lifshitz perturbation theory. It means that this standard perturbation theory does not coincide with the Einstein theory. Avoiding this double counting, in order to return back to GR, we have obtained new Hamiltonian equations. These equations do not contain the time derivatives that are responsible for the “primordial power spectrum” in the inflationary model [19]. However, Dirac’s Hamiltonian approach to GR gives us another possibility to explain this “spectrum” and other topical problems of cosmology by the cosmological creation of the primordial W-, Z- bosons from vacuum when their Compton length coincides with the universe horizon.

The equations describing the longitudinal vector bosons in SM, in this case, are close to the equations of the inflationary model used for description of the “power primordial spectrum” of the CMB radiation. We listed the set of theoretical and observational arguments in favor of that the CMB radiation can be a final product of primordial vector W-, Z- bosons cosmologically created from Bogoliubov vacuum.

This pathway of quantization points out that in GR and SM there is a new principle of relativity - a relativity of units of measurements. It means that equations of motion do not depend not only on the data but also on the units of measurement of these data. In context of this principle of relativity one can propose that the Higgs potential is not necessarily [42].

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