The disordered Fermi-liquid fixed point and its instabilities

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Abstract. The concept of a disordered Fermi-liquid fixed point is introduced and used to understand various properties of disordered metals within a unifying framework. Corrections to scaling near this fixed point give what are commonly called weak-localization effects. Various instabilities of the disordered Fermi-liquid phase are discussed. These include two distinct types of superconducting-to-normal-metal quantum phase transitions. First, the quantum phase transition from a disordered metal to a conventional superconductor in bulk materials is considered. Second, a quantum phase transition in two dimensions from metallic-like behavior to a novel type of disorder induced, spin-triplet, even-parity, superconductivity is treated. The paper is concluded with a discussion of the nature of the ground state of two-dimensional, disordered electron systems.

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1 Introduction

The many-fermion problem has a long history due to its importance in condensed matter physics. Historically, this problem has been studied using either many-body perturbation theory [1], or Landau’s Fermi-liquid theory [2]. In recent years various aspects of the many-fermion problem have been examined using renormalization group (RG) and field theoretic techniques. Much of this work has been done on clean fermion systems, applying the RG approach either directly to the Grassmann field theory for fermions [3], or to a composite field theory that describes boson-like excitations in a fermion system [4]. Among the results of this new approach are a RG derivation of Fermi-liquid theory starting from a microscopic theory, and the introduction of the Fermi-liquid fixed point notion. Other results include RG derivations of the Cooper instability problem and of the random-phase approximations that lead to, e.g., screening.

For disordered electronic systems the RG approaches developed for clean Fermi systems are not directly applicable, because the sets of soft modes are very different for the two cases. However, a field-theoretic description based on composite variables that are bilinear in the basic fermionic fields, and hence effectively bosonic in nature,
has been used for some time. This approach has been pioneered by Wegner \cite{6}, who showed that, for noninteracting electrons, the effective field theory takes the form of a nonlinear sigma model. This was generalized by Finkel'stein \cite{7} to the case of interacting electrons. The key idea underlying these effective theories is to keep explicitly only those degrees of freedom that are likely to be relevant for the problem under consideration, and to integrate out all others in some simple approximation.

In this paper we show how field theoretic and RG methods can be used to describe the disordered fermion problem. In particular we first review how Finkel'stein’s theory can be derived from a microscopic model with the aid of RG ideas. To this end we introduce the notion of a disordered Fermi-liquid fixed point (FP), and show that corrections to scaling near this FP give what are commonly called weak-localization effects. We then discuss various instabilities of the disordered Fermi-liquid fixed point. Physically, these instabilities indicate phase transitions from the disordered Fermi-liquid phase to various magnetic and superconducting metallic phases, as well as to magnetic and nonmagnetic insulator phases.

2 Fermionic field theory

2.1 Grassmannian field theory

Since the description of fermions involves anticommuting variables, any field-theory description for electrons must be formulated in terms of anticommuting or Grassmann variables \cite{7}. For simplicity we consider here a model for a homogeneous electron fluid subject to a random potential that models the quenched disorder. The partition function can then be written,

$$Z = \int D[\bar{\psi}, \psi] \exp[S] \ .$$

(1)

Here the functional integration is with respect to Grassmann valued fields, $\bar{\psi}$ and $\psi$, and the action $S$ is given by

$$S = S_0 + S_{\text{dis}} + S_{\text{int}} \ .$$

(2)

$S_0$ describes free electrons

$$S_0 = \int dx \sum_\sigma \bar{\psi}_\sigma(x) \left[ -\partial_\tau + \frac{\nabla^2}{2m} + \mu \right] \psi_\sigma(x) \ ,$$

(3)

with $x = (x, \tau)$, and $\int dx = \int dx \int_0^{1/T} \int_0^1 d\tau$. $x$ denotes position, $\tau$ imaginary time, $\sigma$ a spin label, $\mu$ is the chemical potential, and $m$ is the electron mass. $S_{\text{dis}}$ describes a static random potential, $u(x)$, that couples to the fermionic number density,

$$S_{\text{dis}} = -\int dx \ u(x) \sum_\sigma \bar{\psi}_\sigma(x) \psi_\sigma(x) \ ,$$

(4)

and $S_{\text{int}}$ denotes a spin-independent two-particle interaction.
For disordered systems, in contrast to clean ones, single-particle momentum eigenstates are not long lived, but decay exponentially on a time scale given by the elastic mean-free time for electron-impurity interactions. The important physics on the longest length and times scales is controlled by two-particle excitations that are soft either because they are related to conserved quantities or because of a mechanism related to Goldstone’s theorem. For a detailed discussion of this point we refer the reader to Ref. [7]. It turns out that of all the excitations whose interactions are described by $S_{\text{int}}$, the dominant soft modes are those that involve fluctuations of either the particle number density $n_n$, or the spin density $n_s$, or density fluctuations in the Cooper channel, $n_c$. In Fourier space, $q = (q_\text{v},\omega_n)$, with $\omega_n$ a Matsubara frequency, these densities can be easily written in terms of fermion variables. The interaction, $S_{\text{int}}$, can be written in terms of these fluctuations as,

$$S_{\text{int}} = S^{(s)}_{\text{int}} + S^{(t)}_{\text{int}} + S^{(c)}_{\text{int}},$$

with singlet, triplet, and Cooper channel terms denoting interactions between the densities discussed above,

$$S^{(s)}_{\text{int}} = -\frac{\Gamma^{(s)}}{2} \sum_q n_n(q) n_n(-q), \quad S^{(t)}_{\text{int}} = \frac{\Gamma^{(t)}}{2} \sum_q n_s(q) \cdot n_s(q), \quad S^{(c)}_{\text{int}} = -\frac{\Gamma^{(c)}}{2} \sum_q n_c(q) n_c(q).$$

Here $\Gamma^{(i)}$, $i = s, t, c(s)$, are interaction amplitudes in the particle-hole spin-singlet and triplet, and in the particle-particle or Cooper spin-singlet interactions channels, respectively. Notice that there is no Cooper spin-triplet interaction term. The reason is that for models with static, point-like, interaction amplitudes, as we have assumed here, such a term is forbidden by the Pauli principle. We will come back to this in Sec. 3.2.2 below.

2.2 Nonlinear sigma model

The idea behind the nonlinear sigma model description of disordered electron systems is that the long-distance (long-wavelength) and long-time (low-frequency) properties of the system are determined by the massless modes or excitations. The derivation of an effective theory then becomes a two-step process. First, the massless modes in the system need to be identified. For example, in a simple Fermi liquid with no long-range magnetic order, the only soft modes are diffusive modes representing, e.g., number density, energy density, and spin density diffusion. Second, the microscopic field theory, Eqs. (2) - (5), must be transformed into an effective one that keeps only these soft modes, while all other degrees of freedom are integrated out in some approximation that respects crucial features of the full theory like, e.g., conservation laws. All of this can be done, and has been discussed in great detail in Ref. [7]. The resulting effective field theory is called a generalized nonlinear sigma model. Within this theory the partition function can be written in terms of matrix fields $Q$,

$$Z = \int D[Q] \ e^{A[Q]}.$$
The matrix elements of $Q$ are isomorphic to bilinear products of the elements of $\bar{\psi}$ and $\psi$. The action $\mathcal{A}$ is given by,

$$\mathcal{A} = -\frac{1}{2G} \int dx \text{tr} (\nabla Q(x))^2 + 2H \int dx \text{tr} (\Omega Q(x)) + \mathcal{A}_{\text{int}}$$

with $\mathcal{A}_{\text{int}}$ given by Eqs. (6,7) written in terms of $Q$-matrices. The explicit derivation yields the coupling constants as $G = 8/\pi \sigma_0$, with $\sigma_0$ the conductivity in the self-consistent Born approximation, and $H = \pi N_F/8$ which can be interpreted as the quasiparticle density of states ($N_F$ is the bare single-particle density of states at the Fermi level). $G$ is a measure of the disorder in the system. $\Omega$ is a diagonal matrix whose elements are the fermionic Matsubara frequencies $\omega_n$. $Q$ is subject to the constraints

$$Q^2(x) = 1 \quad , \quad Q^\dagger = Q \quad , \quad \text{tr} Q(x) = 0 \quad .$$

A standard way to enforce these constraints is to write

$$Q = \left( \begin{array}{cc} (1 - q q^\dagger)^{1/2} & q \\ q^\dagger & -(1 - q^\dagger q)^{1/2} \end{array} \right)$$

where the matrix $q$ has elements $q_{nm} \equiv q(\omega_n, \omega_m)$ with $n \geq 0$, $m < 0$.

The effective action given above is the generalized nonlinear sigma model that was first proposed by Finkelstein as a model for disordered interacting electrons near a metal-insulator transition.

### 2.3 The disordered Fermi-liquid fixed point

Insight into a disordered metal can be gained from studying the RG and fixed point properties of the generalized sigma model field theory given by Eq. (9). We begin our discussion of the model by performing a momentum-shell RG procedure. For the rescaling part of this transformation, we need to assign a scale dimension to the field $q$. Choosing the scale dimension of a length $L$ to be $[L] = -1$, we write

$$[q(x)] = \frac{1}{2} (d - 2 + \eta) \quad ,$$

which defines the exponent $\eta$. The stable Fermi-liquid FP of the theory is characterized by the choice

$$\eta = 0 \quad .$$

Physically this corresponds to diffusive correlations of $q$ in the disordered Fermi-liquid phase. In addition, we must specify the scale dimension of the frequency or temperature, i.e., the dynamical scaling exponent $z = [\omega] = [T]$. In order for the FP to

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We are suppressing some technicalities here. For instance, the $q$ also carry replica indices, and the matrix elements are quaternion-valued.

We adopt the RG philosophy of S.-K. Ma, where FPs are selected by choosing the values of certain scale dimensions or critical exponents.
be consistent with diffusion, that is with frequencies that scale as the square of the wavenumber, we must choose

$$z = 2$$.

(14)

Now we expand the sigma model action in powers of $q$. In a symbolic notation that leaves out everything not needed for power counting purposes, we write

$$A = -\frac{1}{G} \int \! dx (\nabla q)^2 + H \int \! dx \, \omega q^2 + \Gamma T \int \! dx \, q^2 + O(\nabla^2 q^4, \omega q^4, T q^3)$$,

(15)

with $G \sim 1/\sigma_0$ and $H \sim N_F$, and $\Gamma$ denotes any of the interaction amplitudes. Power counting shows that with the above choices for the exponents, all these coupling constants have vanishing scale dimensions with respect to our FP:

$$[G] = [H] = [\Gamma] = 0$$.

(16)

These terms (together with some others that we can suppress for our present purposes) therefore make up our FP action.

Now consider the leading corrections to the FP action, as indicated in Eq. (15). Power counting shows that all these terms are irrelevant to the Fermi-liquid FP as long as $d > 2$, and they become marginal in $d = 2$ and relevant for $d < 2$. All terms that were neglected in deriving the sigma model can be shown to be even more irrelevant than the ones considered here. We conclude that Eq. (15) is a FP action and that any of the leading irrelevant operators (which we collectively denote by $u$) have dimension $[u] = -(d-2)$. We can use these results to discuss the leading corrections to scaling near the disordered Fermi-liquid FP for physical quantities such as the conductivity, the specific heat, and various susceptibilities.

Let us first consider the dynamical conductivity $\sigma(\omega)$. Its bare value is proportional to $1/G$, and according to Eq. (16) its scale dimension is zero. We therefore have the scaling law

$$\sigma(\omega) = \sigma(\omega b^z, ub^{-(d-2)})$$,

(17)

where $b$ is an arbitrary RG scale factor. By putting $b = 1/\omega^{1/z}$, and using $z = 2$, Eq. (14), as well as the fact that $\sigma(1, x)$ is an analytic function of $x$, we find that the conductivity has a singularity at zero frequency, or a long time tail, of the form

$$\sigma(\omega) = \text{const} + \omega^{-(d-2)/2}$$.

(18)

This nonanalyticity is well known from perturbation theory for both noninteracting and interacting electrons. Note that these results were derived basically without doing any calculations, illustrating the power of the RG. The RG derivation also proves that the $\omega^{-(d-2)/2}$ is the exact leading nonanalytic behavior.

Similarly, the low-frequency (or low-temperature, or small-wavenumber) behavior of other physical quantities can be obtained. For example, the single-particle density of states behaves as

$$N(\omega \to 0) = \text{const} + \omega^{(d-2)/2}$$

(19)

and, as a function of wavenumber, the static spin susceptibility scales as

$$\chi_s(q, T = 0) = \text{const} + |q|^{(d-2)}$$.

(20)

Here we have omitted the prefactors of the nonanalyticities.
2.4 Metal-insulator transitions

In addition to the stable Fermi-liquid FP discussed in the previous subsection, there also is a critical FP that describes a metal-insulator transition. This is the FP first discussed by Finkel’stein [6], and later analyzed in greater detail by others. For lack of space we cannot discuss this here, and refer the reader to the reviews [10] and [11].

3 Instabilities of the Fermi liquid

The Fermi-liquid FP discussed in Sec. 2, and the phase it describes, are perturbatively stable for weak interactions. However, if the electron-electron interaction in some channel becomes large, the Fermi-liquid FP becomes unstable, and the system undergoes a transition to a different phase. We will be concerned with magnetic and superconducting phases. The properties of quantum phase transitions to magnetic phases are discussed elsewhere in these proceedings [12, 13]. However, before we turn to superconducting instabilities, we will discuss some soft-mode induced properties of itinerant ferromagnets that are akin to the weak-localization effects discussed in Sec. 2 above.

3.1 Magnons at zero temperature

An important manifestation of quantum long-range order in a ferromagnet is the existence of spin waves. In conventional Heisenberg ferromagnets the damping of the spin waves is negligible, and in the long-wavelength limit the spin-wave dispersion is

$$\Omega = D(m) q^2 .$$

(21)

The coefficient $D(m)$ depends on the dimensionless magnetization $m$. In the conventional theory for clean ‘weak’ ferromagnets [14], $D(m \to 0) = D_0 m$, with $D_0$ a constant. In general this result is not correct for itinerant ferromagnets, because the same effects that lead to the weak-localization nonanalyticities discussed above also lead to a nonanalytic dependence of $D(m)$ on $m$ at $m = 0$.

To see this we use a scaling argument. For more details, as well as explicit calculations, we refer the reader to Ref. [15]. First we note that the coefficient $D$ in Eq. (21) can be related to $m$ times the coefficient of the $q^2$ term in the wavenumber expansion of a spin susceptibility. In the paramagnetic state, Eq. (20) shows that the leading small wavenumber term is actually $q^d |q|^{d-4} = |q|^{d-2}$. (Here we consider disordered systems, we will come back to the clean case later.) In the ferromagnetic state, on the other hand, the small wavenumber singularity is cut off by the magnetization $m$. In particular, a finite magnetization acts like a magnetic field in this respect and supplies a frequency scale that is proportional to $m$ (viz., the cyclotron frequency). For diffusive dynamics, the wavenumber scales like the square root of the frequency. This implies a length scale cutoff, $\ell_m$, due to the magnetization, that scales as $\ell_m \sim m^{-1/2}$. The net result is that for disordered systems, for $2 < d < 4$, $D(m)$ behaves as

$$D(m \to 0) \propto m \ell_m^{4-d} \propto m^{(d-2)/2} ,$$

(22)
This leads to the striking experimental prediction that in bulk systems, \( D(m \to 0) \propto m^{1/2} \) compared to the linear \( m \)-dependence predicted by conventional theories. The net result is that the ‘speed’ of spin waves in ‘weak’ \((m \to 0)\) ferromagnets is larger than the conventional theory predicts.

For clean itinerant ferromagnets a similar, albeit weaker, effect has been predicted [13]. For bulk systems, \( D(m \to 0) \propto m \ln(1/m) \), while in two-dimensions, \( D(m \to 0) \propto m^0 = \text{const.} \).

3.2 Superconducting phases and phase transitions

A large interaction in the Cooper channel triggers an instability of the Fermi liquid in favor of a superconducting state. We first discuss the phase transition to a conventional (BCS) superconducting state at zero temperature. Then we discuss how disorder can lead to more exotic types superconductivity, especially in \( d = 2 \).

3.2.1 Superconducting-to-normal-metal transition at zero temperature

The standard starting point to describe any phase transition is to construct a Landau-Ginzburg-Wilson (LGW) functional for the order parameter fluctuations. The basic idea is that these fluctuations are the important ones to describe the large scale changes associated with the phase transition. In a disordered metal the situation is more complicated because, in addition to the order parameter fluctuations being soft, the diffusive modes discussed in Sec. 2 are also soft, and in general they couple to the order parameter fluctuations. The usual procedure of integrating out all modes other than the order parameter fluctuations therefore in particular integrates out some soft modes, which leads to a nonlocal LGW theory. Nevertheless, the critical behavior of the resulting theory can still be determined using RG ideas. Furthermore, since the nonlocality of the LGW functional corresponds to a long-ranged effective interaction between the order parameter fluctuations, the critical behavior can be determined exactly from a Gaussian FP, as has been explained in more detail in Ref. [16].

To illustrate this, we denote the LGW functional by \( \Phi \) and the superconducting order parameter by \( \Psi \). Gauge invariance implies that only even powers of \( \Psi \) appear in \( \Phi \). The coefficients in this expansion, i.e., the vertex functions of the effective field theory, are connected correlation functions of the anomalous or Cooper channel density \( n_c \), see Eq. (7), in a system away from any phase transition point [17]. This auxiliary system we will refer to as a reference ensemble. In particular, the Gaussian vertex is determined by the anomalous density-density correlation in the reference ensemble. Denoting the latter by \( C(q) \), the Gaussian term in the LGW functional is,

\[
\Phi^{(2)}[\Psi] = \int dq \Psi^*(q) \left[ 1 / |\Gamma_c| - C(q) \right] \Psi(q). \tag{23}
\]

\( C(q) \) is a complicated correlation function. However, since the reference ensemble is a disordered Fermi liquid, its structure is known [14]. RG arguments show that the structure of \( C \) at low frequencies and long wavelengths in the limit \( T \to 0 \) is

\[
C(q) = \frac{Z}{h} \frac{\ln(\Omega_0/(D q^2 + |\Omega_n|))}{1 + (\delta \Gamma^{c(s)}/h) \ln(\Omega_0/(D q^2 + |\Omega_n|))}. \tag{24}
\]
Here $\Omega_0$ is a frequency cutoff on the order of the Debye frequency (for phonon-mediated superconductivity), and $\delta \Gamma^{c(s)}$ is a repulsive interaction in the Cooper channel that is generated in perturbation theory even if its bare value is zero. $D$ is the diffusion coefficient of the electrons in the reference system, $h$ is the renormalized value of $H$ (see Eq. (3)), and $Z$ is a wavefunction renormalization. Using these results in Eq. (23) we obtain a Gaussian LGW functional that has the form

$$\Phi^{(2)} = \int dq \Psi^*(q) \left[ t + \frac{1}{\ln(\Omega_0/(Dq^2 + |\Omega_n|))} \right] \Psi(q),$$

(25)

where $t = -(Z |\Gamma^{c(s)} - \delta \Gamma^{c(s)}|)$ is the bare distance from the phase transition.

From the structure of the Gaussian vertex $\Gamma^{(2)}$, i.e., the term in brackets in Eq. (24), we read off the values of the exponents $\eta$, $\gamma$, and $z$, defined as $\Gamma^{(2)}(q, \Omega = 0) \sim |q|^{-\eta}$, $\Gamma^{(2)}(q = 0, \Omega = 0) \sim t^\gamma$, and $\xi_r \sim \xi_z$, with $\xi_r$ the relaxation time. They are

$$\eta = 2, \quad \gamma = 1, \quad z = 2.$$

(26)

By scaling $|q|$ with the correlation length $\xi$, we also obtain the scaling behavior of the latter,

$$\xi \sim \exp(1/2|t|).$$

(27)

The exponent $\nu$ as usually defined therefore does not exist, $\nu = \infty$.

In order to determine how the order parameter behaves near the critical point, one needs the quartic term $\Phi^{(4)}$ in the LGW functional. One finds that the exponent $\beta$ also does not exist, $\beta = \infty$, but that the ratio $\beta/\nu$ is finite and is equal to the expected (from scaling) value

$$\beta/\nu = 2.$$

(28)

Finally, it is also possible to determine the behavior of the conductivity and specific heat near the quantum transition point. For these details we refer to Ref. [16].

3.2.2 Novel superconducting states: disordered-induced spin-triplet even-parity superconductivity

As mentioned after Eq. (6), the Pauli principle does not allow for a static, point-like interaction amplitude, $\Gamma^{c(t)}$, in the particle-particle spin-triplet channel. More generally, any point-like $\Gamma^{c(t)}$ must be an odd function of frequency. In general, this means that if $\Gamma^{c(t)}$ exists, then it vanishes at zero frequency, and is therefore not important at long times or low temperatures. This seems to preclude spin-triplet even-parity superconductivity triggered by $\Gamma^{c(t)}$. However, this argument is fallacious for low-dimensional disordered electron systems. The basic point is that in the presence of electronic interactions the diffusive modes that lead to weak-localization effects, and to nonlocal theories for magnetic and other superconducting phase transitions, also lead to a $\Gamma^{c(t)}$ with a nonanalytic frequency dependence that approaches a discontinuity as $d \to 2$. As a result, the presence of both disorder and electron-electron interactions seem to inevitably lead to exotic superconductivity in $d = 2$!

To understand physically how this happens let us first consider the effect of the spin-triplet interaction amplitude in the particle-hole channel, $\Gamma^{(t)}$. By means of $\Gamma^{(t)}$,
an electron spin polarizes its environment. This polarization is ferromagnetic. Suppose an electron has created a spin polarization cloud and then moves away. In a clean system the polarization cloud at a given point will decay quickly in time. The polarization cloud therefore essentially moves with the electron that creates it, giving rise to the standard Fermi-liquid renormalization of the spin susceptibility. In a disordered system, a spin density fluctuation $\delta n_s$ will decay algebraically like $\delta n_s \sim t^{-d/2}$, because of the long-time tail effects already discussed (see Eq. (18)). The polarization cloud will therefore persist even after the electron that created it has diffused away. A second electron moving into the region at a later time will still see the remains of the polarization cloud. It will get attracted to it if the two electrons form a spin triplet. We therefore expect an attractive contribution of $\Gamma^{(t)}$ to $\Gamma^{(t)}$.

Next consider the effect of the spin-singlet interaction, $\Gamma^{(s)}$. By means of $\Gamma^{(s)}$, an electron charge polarizes its environment. Again, while in a clean system this leads only to Fermi-liquid renormalizations, in a disordered system the resulting charge polarization cloud will decay as $\delta n_c \sim t^{-d/2}$. A second electron will be attracted to this region regardless of spin. The conclusion is that both $\Gamma^{(t)}$ and $\Gamma^{(s)}$ will lead to an attractive $\Gamma^{(c)(t)}$. Further, since the Fourier transform of $t^{-d/2}$ is proportional to $\ln \omega$ in $d = 2$, we expect these contributions to be singular at low frequencies, or low temperature. Finally, since there is no bare repulsive $\Gamma^{(c)(t)}$ to overcome, the conclusion is that in $d = 2$ there is a tendency toward a novel type of superconductivity at $T = 0$.

Explicit calculations confirm the above physical picture [10]. Further, both the critical properties near this quantum phase transition and the physical properties of the novel superconducting state have been studied in some detail. For the phase transition properties we refer to Ref. [18]. Here we restrict ourselves to briefly discussing some interesting properties of the superconducting state.

The first relevant question concerns the (mean-field) critical temperature for reasonable parameter values. As noted above, there are two mechanisms leading to superconductivity. For realistic MOSFET parameters, the charge fluctuation mechanism leads to $T_c \approx 10 \text{ mK}$. The spin fluctuation mechanism, on the other hand, can lead to a mean-field $T_c$ that is a fraction of the Fermi temperature, i.e., about $1 \text{ K}$. Keeping in mind that mean-field estimates generally overestimate $T_c$, we conclude that in at least some systems, this type of superconductivity is a possibility. Other quantities of interest are the tunneling density of states, the diamagnetic susceptibility, and the conductivity. Model calculations yield a density of states that vanishes logarithmically as the bias voltage approaches zero, so the predicted superconducting state has a pseudo-gap. Calculations have also established that there is a conventional Meissner effect, and that the electrical conductivity is identical in form to that in a conventional BCS superconductor. These results answer some questions that had been raised about the stability of odd-gap superconductivity. For details, see Ref. [18].

4 The two-dimensional ground state problem

Our discussion so far implies that the nature of the ground state of a generic 2-$d$, interacting, disordered electron system is not known, and that there are a number of possibilities. Indeed, this is one of the most fundamental open problems in quantum
many-body physics, and we call it the two-dimensional ground state problem.

For noninteracting electrons, at zero magnetic field, the ground state is known to be an Anderson insulator \[9\]. When repulsive electron-electron interactions are added, a number of distinct possibilities arise. First, it could still be an insulator. Indeed, this was the conventional view until very recently. Second, the ground state could be the novel disorder-induced superconducting state discussed above. Third, it has been known for some time that if there are no magnetic impurities or impurity spin orbit scattering, there is a tendency toward some type of magnetic state that may (or may not) be conducting \[10\]. Fourth, there have been speculations that some other, even more exotic, state may occur \[19\].

Experimentally, the situation is very interesting. There is mounting evidence that in some 2-d systems, especially low-electron density MOSFETs, there is some kind of a conducting phase at very low temperatures \[20\]. Other 2-d systems are surely insulators. These observations are consistent with the notion that there are numerous possibilities for the 2-d ground state, and that the ultimate choice depends on microscopic details and cannot be universally determined. A detailed free energy calculation will be needed to determine what ground state is the thermodynamically stable one for a specific system.

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