SEARCHES FOR $D^0$-$\bar{D}^0$ MIXING:
FINDING THE (SMALL) CRACK IN THE STANDARD MODEL

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Abstract

We review results from searches for mixing and $CP$ violation in the $D^0$-$\bar{D}^0$ system. No evidence for mixing or $CP$ violation is found, and limits are set for the mixing parameters $x$, $y$, $x'$, $y'$, and several $CP$-violating parameters.

1 Introduction

Despite numerous searches, mixing between $D^0$ and $\bar{D}^0$ flavor eigenstates has not yet been observed. Within the Standard Model (SM), the short-distance "box" diagram (which plays a large role in $K^0-\bar{K}^0$ and $B^0-\bar{B}^0$ mixing) is doubly-Cabibbo-suppressed (DCS) and GIM-suppressed; since the $D^0$ decay width is dominated by Cabibbo-favored (CF) amplitudes, $D^0$-$\bar{D}^0$ mixing is expected to be a rare phenomenon. Observing mixing at a rate significantly above the SM expectation could indicate new physics.
The formalism describing $D^0$-$\bar{D}^0$ mixing is given in several papers.\textsuperscript{1, 2)} The parameters used to characterize mixing are $x = \Delta m/\Gamma$ and $y = \Delta \Gamma/(2\Gamma)$, where $\Delta m$ and $\Delta \Gamma$ are the mass and decay width differences between the two mass eigenstates, and $\Gamma$ is the mean decay width. Within the SM, $x$ and $y$ are difficult to calculate as there are long-distance contributions. For $m_q \gg \Lambda_{\text{QCD}}$, these contributions can be estimated using the heavy-quark expansion; however, $m_c$ may not be large enough for this calculation to be reliable. Current theoretical predictions\textsuperscript{3)} span a wide range: $|x| \sim |y| \sim (10^{-7} \text{ to } 10^{-2})$, with the majority being $<10^{-3}$.

For decay times $t \ll 1/\Delta m, 1/\Delta \Gamma$, which is well-satisfied for charm decay, the time-dependent $D^0(t) \to f$ and $\bar{D}^0(t) \to \bar{f}$ decay rates are

\begin{equation}
R_{D^0} = |A_f|^2 e^{-\Gamma t} \left[ 1 + |y| \text{Re}(\lambda) - x \text{Im}(\lambda) \right] (\Gamma t) + |\lambda|^2 \frac{(x^2 + y^2)}{4} (\Gamma t)^2 \right] (1)
\end{equation}

\begin{equation}
R_{\bar{D}^0} = |\bar{A}_f|^2 e^{-\Gamma t} \left[ 1 + |y| \text{Re}(\bar{\lambda}) - x \text{Im}(\bar{\lambda}) \right] (\Gamma t) + |\bar{\lambda}|^2 \frac{(x^2 + y^2)}{4} (\Gamma t)^2 \right] (2)
\end{equation}

where $\lambda = (q/p)(\bar{A}_f/A_f)$, $\bar{\lambda} = (p/q)(A_f/\bar{A}_f)$, $q$ and $p$ are complex coefficients relating flavor eigenstates to mass eigenstates, and $A_f (\bar{A}_f)$ and $\bar{A}_f (A_f)$ are amplitudes for a pure $D^0 (\bar{D}^0)$ state to decay to $f$ and $\bar{f}$, respectively.

In this paper we discuss five methods used to search for $D^0$-$\bar{D}^0$ mixing and CP violation (CPV). These methods use the following decay modes:\textsuperscript{1) semileptonic $D^0 \to K^+\ell^\pm\nu$ decays, decays to $CP$-eigenstates $K^+K^-$ and $\pi^+\pi^-$, DCS $D^0 \to K^+\pi^-$ decays, $D^0 \to K^0_S\pi^+\pi^-$ decays, and multi-body DCS $D^0 \to K^+n(\pi)$ decays. A newer method based on quantum correlations\textsuperscript{4)} in $e^+e^- \to \psi''(3770) \to D^0\bar{D}^0$ production is not discussed here. The flavor of a $D^0$ when produced is determined by requiring that it originate from a $D^{*+} \to D^0\pi^+_s$ decay; the charge of the low momentum ("slow") $\pi^+_s$ determines the charm flavor at $t = 0$. As the kinetic energy released in $D^{*+} \to D^0\pi^+_s$ decays is only 5.8 MeV (very near threshold), requiring that $Q \equiv M_{K\pi\pi_s} - M_{K\pi} - m_\pi$ be small greatly reduces backgrounds.

\textsuperscript{1}Charge-conjugate modes are implicitly included throughout this paper unless noted otherwise.
2 $D^0(t) \rightarrow K^{(*)+}\ell^−\nu$ Semileptonic Decays

Because the $K^{(*)+}\ell^−\nu$ final state can only be reached from a $D^0$ decay, observing $D^0(t) \rightarrow K^{(*)+}\ell^−\nu$ would provide clear evidence for mixing. In Eq. (1) only the third term is nonzero; integrating this term over all times and assuming $|q/p| = 1$ (i.e., neglecting CPV in mixing) gives

$$\frac{\int R(D^0 \rightarrow K^+\ell\nu) \, dt}{\int R(D^0 \rightarrow K^-\ell\nu) \, dt} \approx \frac{x^2 + y^2}{2} \equiv r_D.$$ (3)

Several experiments $^5, \ 6)$ have used this method to constrain $r_D$; the most stringent constraint is from the Belle experiment using $253 \text{ fb}^{-1}$ of data. $^6)$

Due to the neutrino, the final state is not fully reconstructed; however, at an $e^+e^-$ collider there are enough kinematic constraints to infer the neutrino momentum. Specifically, momentum conservation prescribes $P_\nu = P_{CM} - P_{\pi_sK\ell}$, where $P_{CM}$ is the four-momentum of the $e^+e^-$ center-of-mass (CM) system, $\pi_s$, $K$, and $\ell$ are daughters from $D^* \rightarrow D^0\pi_s \rightarrow \pi_sK\ell\nu$, and $P_{\text{rest}}$ is the four-momentum of the remaining particles in the event. In the Belle analysis the magnitude $|P_{\text{rest}}|$ is rescaled to satisfy $(P_{CM} - P_{\text{rest}})^2 = m_{D^*}^2$, and after this rescaling the direction of $\vec{P}_{\text{rest}}$ is adjusted to satisfy $P_{\nu}^2 (= m_{\nu}^2) = 0$.

The $\Delta M \equiv M_{\pi_sK\ell\nu} - M_{K\ell\nu}$ distributions for “right-sign” (RS) $D^0 \rightarrow K^-\ell^+\nu$ and “wrong-sign” (WS) $D^0 \rightarrow K^+\ell^−\nu$ samples are shown in Fig. 1. Sensitivity to mixing is improved by utilizing information on the decay time, which is calculated by projecting the $D^0$ flight distance onto the (vertical) $y$ axis: $t = (M_{D^0}/c) \times (y_{\text{vtx}} - y_{\text{IP}})/p_y$. This projection has superior decay time resolution, as the beam profile is only a few microns in $y$ and thus the interaction point ($y_{\text{IP}}$) is well-determined. Events satisfying $t > \tau_{D^0}$ are divided into six $t$ intervals, and the event yields $N_{RS}^{(t)}$ and $N_{WS}^{(t)}$, acceptance ratio $\varepsilon_{WS}^{(t)}/\varepsilon_{RS}^{(t)}$, and resulting mixing parameter $r_D^{(t)}$ are calculated separately for each. $N_{RS}^{(t)}$ and $N_{WS}^{(t)}$ are obtained from fitting the $\Delta M$ distributions. Doing a $\chi^2$ fit to the six $r_D^{(t)}$ values gives an overall result $r_D = [0.20 \pm 0.47 \text{ (stat)} \pm 0.14 \text{ (syst)}] \times 10^{-3}$, or $r_D \leq 0.10\%$ at 90% C.L. No evidence for mixing is observed. The total number of signal candidates in all $t$ intervals is $90601 \pm 372$ RS events and $10 \pm 80$ WS events.
Figure 1: $\Delta M$ distributions for RS $D^0 \rightarrow K^-\ell^+\nu$ candidate decays (left) and WS $D^0 \rightarrow K^+\ell^-\nu$ candidate decays (right), from Belle using 253 $fb^{-1}$ of data. The WS plot shows no visible signal above background.

3 $D^0(t) \rightarrow K^+K^-, \pi^+\pi^- \ CP$-Eigenstate Decays

When the final state is self-conjugate, e.g., $K^+K^-$, there is no strong phase difference between $\bar{A}_f$ and $A_f$. Assuming $|\bar{A}_f| = |A_f|$ (no direct CPV), $\lambda = -|q/p|e^{i\phi}$ and $\bar{\lambda} = -|p/q|e^{-i\phi}$, where $\phi$ is a weak phase difference and the leading minus sign is due to the phase convention $CP|D^0⟩ = -|\bar{D}^0⟩$. Inserting these terms into Eqs. (1) and (2) and dropping the very small last term gives

$$R(D^0 \rightarrow K^+K^-) = |A_{K+K^-}|^2 e^{-\frac{\Gamma}{2} t} \left[ 1 - \frac{|q/p|}{|1 + |q/p|(y \cos \phi - x \sin \phi)|} \right]$$

(4)

$$R(\bar{D}^0 \rightarrow K^+K^-) = |A_{K+K^-}|^2 e^{-\frac{\Gamma}{2} t} \left[ 1 - \frac{|p/q|}{|1 + |p/q|(y \cos \phi + x \sin \phi)|} \right]$$

(5)

Eqs. (4) and (5) imply that the measured $D^0$ and $\bar{D}^0$ inverse lifetimes are $\frac{1}{\Gamma}[1 + |q/p|(y \cos \phi - x \sin \phi)]$ and $\frac{1}{\Gamma}[1 + |p/q|(y \cos \phi + x \sin \phi)]$, respectively. We define $y_{CP} \equiv \frac{\tau_{K^-\pi^+}}{\tau_{K^+\pi^-}} - 1$, which equals $|q/p|(y \cos \phi - x \sin \phi)$ for $D^0$ decays and $|p/q|(y \cos \phi + x \sin \phi)$ for $\bar{D}^0$ decays. For $|q/p| = 1$, i.e., no CPV in mixing, $y_{CP} = y \cos \phi$ for equal numbers of $D^0$ and $\bar{D}^0$ decays together. If also $\phi = 0$ (no CPV), $y_{CP} = y$. The observable $y_{CP}$ is measured by fitting the $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow K^-\pi^+$ decay time distributions.

To date, five experiments have measured $y_{CP}$; the most precise
value is from BaBar using 91 fb$^{-1}$ of data.\textsuperscript{8} To increase statistics, BaBar used both $K^+K^-$ and $\pi^+\pi^-$ decays, and, in addition, the $D^0\to K^+K^-$ analysis used both a large inclusive $D^0$ sample and a smaller, higher purity sample in which the $D^0$ was required to originate from $D^{\ast+}\to D^0\pi^+$. The respective decay time distributions are shown in Fig. 2. Doing an unbinned maximum likelihood fit to each sample, combining results for $K^+K^-$ and $\pi^+\pi^-$, and taking the ratio of lifetimes gives $y_{CP} = [0.8 \pm 0.4 \text{ (stat)} \pm 0.5 \text{ (syst)}]\%$. This value is consistent with, but smaller than, the relatively large value measured by FOCUS:\textsuperscript{9} $y_{CP} = [3.4 \pm 1.4 \text{ (stat)} \pm 0.7 \text{ (syst)}]\%$.

BaBar also measures $\Delta Y \equiv (\tau^+ - \tau^-)/(\tau^+ + \tau^-) \times \tau_{K^-\pi^-}/\tau$, where $\tau^+$ ($\tau^-$) is the lifetime for $D^0\to K^+K^-$ ($D^0\to K^-\pi^+$) and $\tau = (\tau^+ + \tau^-)/2$. For $|q/p| = 1$, $\Delta Y = x\sin \phi$. The result is $\Delta Y = [-0.8 \pm 0.6 \text{ (stat)} \pm 0.2 \text{ (syst)}]\%$, which indicates that either $x$ is small or $\phi$ is small.

### 4 $D^0(t) \to K^+\pi^-$ Doubly-Cabibbo-Suppressed Decays

For $D^0 \to K^+\pi^-$, $A_f$ is DCS, $\bar{A}_f$ is CF, and thus $|A_f| \ll |\bar{A}_f|$. In addition, there may be a strong phase difference ($\delta$) between the amplitudes. Defining $R_D \equiv |A_f/\bar{A}_f|^2$ and $\overline{R_D} \equiv |\bar{A}_f/A_f|^2$, $\lambda = |q/p|R_D^{1/2}e^{i(\phi+\delta)}$ and $\overline{\lambda} = |p/q|\overline{R_D}^{1/2}e^{i(-\phi+\delta)}$. Inserting these terms into Eqs. (1) and (2) gives

\begin{equation}
R(D^0\to K^+\pi^-) \propto e^{-\overline{T}_t} \left[ R_D + \left| \frac{q}{p} \right| \sqrt{R_D} \left[ y' \cos \phi - x' \sin \phi \right] (\overline{T}_t) + \left| \frac{q}{p} \right|^2 \frac{(x'^2 + y'^2)}{4} (\overline{T}_t)^2 \right] \end{equation}

\begin{equation}
R(\overline{D^0} \to K^-\pi^+) \propto e^{-\overline{T}_t} \left[ \overline{R_D} + \left| \frac{p}{q} \right| \sqrt{\overline{R_D}} \left[ y' \cos \phi + x' \sin \phi \right] (\overline{T}_t) + \left| \frac{p}{q} \right|^2 \frac{(x'^2 + y'^2)}{4} (\overline{T}_t)^2 \right] , \end{equation}

where $x' \equiv x \cos \delta + y \sin \delta$ and $y' \equiv -x \sin \delta + y \cos \delta$. These “rotated” mixing parameters absorb the unknown strong phase difference $\delta$. CPV enters Eqs. (6) and (7) in three ways: $|q/p| \neq 1$ (CPV in mixing), $R_D \neq \overline{R_D}$ (CPV in the DCS amplitude), and $\phi \neq 0$ (CPV via interference between the DCS and mixed amplitudes). Assuming no CPV gives the simpler expression

\begin{equation}
R \propto e^{-\overline{T}_t} \left[ R_D + \sqrt{R_D} y' (\overline{T}_t) + \frac{(x'^2 + y'^2)}{4} (\overline{T}_t)^2 \right] . \end{equation}
Figure 2: Decay time distributions for $CF \ D^0 \rightarrow K^-\pi^+$ (upper left), $D^0 \rightarrow K^+K^-$ (upper right), $D^0 \rightarrow \pi^+\pi^-$ (lower left), and $D^0 \rightarrow K^+K^-$ selected without using a $D^*^+$ tag (lower right), from BaBar using $91 \ fb^{-1}$ of data. \cite{8}) The shaded histograms show the signal component obtained from the fit; residuals from the fit are plotted below each distribution.

To date, six experiments \cite{10, 11, 12}) have done a time-dependent analysis of $D^0 \rightarrow K^+\pi^-$ decays; the most stringent constraints on $x^2$ and $y'$ are from Belle using $400 \ fb^{-1}$ of data. \cite{12}) The reconstructed $M_{K\pi}$ and $Q$ distributions after all selection criteria are shown in Fig. 3; fitting these distributions yields $1073993 \pm 1108$ RS signal events and $4024 \pm 88$ WS signal events. Those events satisfying $|M_{K\pi} - M_{D^0}| < 22 \ MeV/c^2$ and $|Q - 5.8 \ MeV| < 1.5 \ MeV$ ($4\sigma$ intervals) have their decay times fitted for $x^2$, $y'$, and $R_D$. The results are listed in Table 1; projections of the fit are shown in Fig. 4(left).

A 95\% C.L. region in the $x^2$-$y'$ plane is obtained using a frequentist technique based on “toy” Monte Carlo (MC) simulation. For points $\vec{\alpha} = (x^2, y')$, 

\begin{align*}
D^0 \text{Candidates / 0.20 ps} \\
-4 & -2 0 2 4 6 \\
-2 & 2 2 \\
+ p & - K^+ \ K^-
\end{align*}
Table 1: Limits on mixing parameters obtained from fitting the decay time distribution of WS $D^0 \rightarrow K^+\pi^-$ decays, from Belle using 400 fb$^{-1}$ of data.\cite{12}

| Fit Case | Parameter | Fit Result | 95% C.L. interval |
|----------|-----------|------------|------------------|
|          | $x'^2$    | $0.18_{-0.23}^{+0.21}$ | $<0.72$          |
|          | $y'$      | $0.6_{-3.9}^{+4.0}$     | $(-9.9, 6.8)$    |
|          | $R_D$     | $3.64 \pm 0.17$         | $(3.3, 4.0)$     |
|          | $R_M$     | $-$                      | $(0.63 \times 10^{-5}, 0.40)$ |
| CPV allowed | $x'^2$    | $-$                      | $<0.72$          |
|          | $y'$      | $-$                      | $(-28, 21)$      |
|          | $R_M$     | $-$                      | $<0.40$          |
|          | $A_D$     | $23 \pm 47$              | $(-76, 107)$     |
|          | $A_M$     | $670 \pm 1200$           | $(-995, 1000)$   |
| No mixing/CPV | $R_D$     | $3.77 \pm 0.08$ (stat)  | $\pm 0.05$ (syst) |

one generates ensembles of MC experiments and fits them using the same procedure as that used for the data. For each experiment, the difference in likelihood $\Delta L \equiv \ln L_{\text{max}} - \ln L(\vec{\alpha})$ is calculated, where $L_{\text{max}}$ is evaluated for $x'^2 \geq 0$. The locus of points $\vec{\alpha}$ for which 95% of the ensemble has $\Delta L$ less than that of the data is taken as the 95% C.L. contour. This contour is shown in Fig. 4(right); projections of the contour are listed in the right-most column of Table 1.

CPV is accounted for by fitting the $D^0 \rightarrow K\pi$ and $\bar{D}^0 \rightarrow K\pi$ samples separately; this yields six values: $x'^{\pm}$, $y'^{\pm}$, and $R_{D}^{\pm}$. Defining $R_{M}^{\pm} \equiv (x'^{\pm 2} + y'^{\pm 2})/2$ and $A_{M} \equiv (R_{M}^{+} - R_{M})/(R_{M}^{+} + R_{M})$, one finds

$$x'^{\pm} = \left(\frac{1 \pm A_{M}}{1 + A_{M}}\right)^{1/4} (x' \cos \phi \pm y' \sin \phi)$$

$$y'^{\pm} = \left(\frac{1 \pm A_{M}}{1 + A_{M}}\right)^{1/4} (y' \cos \phi \mp x' \sin \phi),$$

where there is an implicit sign ambiguity in $x'^{\pm}$ due to Eqs. (6) and (7) being quadratic in $x'$. To allow for CPV, one obtains separate $1-\sqrt{0.05}=77.6\%$ C.L. contours for $(x'^{+ 2}, y'^{+})$ and $(x'^{- 2}, y'^{-})$; points on the $(x'^{+ 2}, y'^{+})$ contour are
then combined with points on the \((x'^2, y')\) contour and the combination used to solve Eqs. (9) and (10) for \(x'^2\) and \(y'\). Because the relative sign of \(x'\) and \(x'^{-}\) is unknown, there are two solutions (one for each sign); Belle plots both in the \((x'^2, y')\) plane and takes the outermost envelope of points as the 95% C.L. contour allowing for CPV. This contour has a complicated shape [see Fig. 4(right)] due to the two solutions. Projections of the contour are listed in the right-most column of Table 1. In the case of no CPV, the no-mixing point \(x'^2 = y' = 0\) lies just outside the 95% C.L. contour; this point corresponds to 3.9% C.L. with systematic uncertainty included.

Figure 3: WS \(D^0 \to K^+\pi^-\) decays: \(M_{K\pi}\) spectrum for events satisfying \(Q \in (5.3, 6.5)\) MeV (left), and \(Q\) spectrum for events satisfying \(M_{K\pi} \in (1.845, 1.885)\) GeV/c\(^2\), from Belle using 400 fb\(^{-1}\) of data. \([12]\)

5 \(D^0(t) \to K^0_S \pi^+\pi^-\) Dalitz Plot Analysis

In this method one considers a self-conjugate final state that is not a CP eigenstate, e.g., a three-body decay that can have either \(L = 0\) (CP-even) or \(L = 1\) (CP-odd). If CPV is negligible, CP-eigenstates (denoted \(D_-, D_+\)) are mass eigenstates (denoted \(D_1, D_2\)), and the amplitude for \(D^0(t) \to K^0_S \pi^+\pi^-\) is:

\[
A_{K^0\pi\pi} = \frac{1}{2p} \left( \langle K^0_S \pi^+\pi^- | H | D_-(t) \rangle + \langle K^0_S \pi^+\pi^- | H | D_+(t) \rangle \right)
\]

\[
\equiv A_- e^{-(\Gamma_1/2 + i\gamma_1) t} + A_+ e^{-(\Gamma_2/2 + i\gamma_2) t} \tag{11}
\]
\[ R_{K^0\pi\pi} = |A_-|^2 e^{-\frac{\Gamma(1-y)}{2}t} + |A_+|^2 e^{-\frac{\Gamma(1+y)}{2}t} + 2e^{-\frac{\Gamma}{2}t} \left[ \text{Re}(A_+ A_-^*) \cos(\Delta m t) + \text{Im}(A_+ A_-^*) \sin(\Delta m t) \right], \]  

where $A_{+, -}$ is the amplitude for $D_{+, -}\rightarrow K_S^0 \pi^+ \pi^-$ multiplied by $1/(2p)$. Note that $x = (m_2 - m_1)/\Gamma$ and $y = (\Gamma_2 - \Gamma_1)/(2\Gamma)$. For a three-body final state, one can distinguish the $A_+$ and $A_-$ components via a Dalitz plot analysis; i.e., a $K_S^0 f_0(980)$ intermediate state is CP-even and contributes to $A_+$, $K_S^0 \rho^0$ is CP-odd and contributes to $A_-$, $K^+(890)^\pi^-$ is a flavor-eigenstate and contributes to both $A_+$ and $A_-$, etc. Thus one models $A_{+, -}$ by separate sums of amplitudes $\sum_j a_j e^{i\delta_j} A_j$, where $A_j$ is the Breit-Wigner amplitude for resonance $j$ and is a function of the Dalitz plot position $M_{K^0\pi\pi}^2, M_{K^0\rho\pi}^2$. Using the probability density function of Eq. (12), one does an unbinned maximum likelihood fit to $M_{K^0\pi\pi}^2, M_{K^0\rho\pi}^2$, and the decay time $t$ to determine $a_j, \delta_j, x$, and $y$. There is systematic uncertainty arising from the decay model, i.e., one must decide which intermediate states to include in the fit. Unlike Eq. (6), Eq. (12) depends linearly on $x$ ($x \ll 1$) and is therefore sensitive to its sign.

This analysis was developed by CLEO, and their result based on 9.0 fb$^{-1}$ has not yet been superseded. To minimize backgrounds, the $D^0$ can-
didate is required to originate from $D^+ \to D^0 \pi^+$. The final Dalitz plot sample (Fig. 5) contains 5299 events with only $(2.1 \pm 1.5)\%$ background. 15)

The decay model used consists of $D^0 \to K^{*}(890)^- \pi^+$, $K^{*}(1430)^0 \pi^+$, $K^{*}(1680)^- \pi^+$, $K_S^0 \rho$, $K_S^0 \omega$, $K_S^0 f_0(980)$, $K_S^0 f_2(1270)$, $K_S^0 f_0(1370)$, WS $D^0 \to K^{*}(890)^+ \pi^-$, and a nonresonant component. The fit results are listed in Table 2; the 95% C.L. intervals correspond to the values at which $-2 \ln L$ rises by 3.84 units, where $L$ is the likelihood function. CPV is included in the fit by introducing parameters $\varepsilon \equiv (p - q)/(p + q)$ (in analogy with $K^0$ decays) and $\phi$, the weak phase difference between $A_{K^0\pi\pi}$ and $A_{K^{*0}\pi\pi}$. The results listed are consistent with no mixing or CPV.

![Figure 5: Dalitz plot (lower right) and projections (lower left, upper plots) for $D^0 \to K_S^0 \pi^+\pi^-$ decays, from CLEO using 9.0 fb$^{-1}$ of data. 15)](attachment:figure5.png)
Table 2: Limits on mixing and CPV parameters from a t-dependent fit to the $D^0 \to K_S^0 \pi^+\pi^-$ Dalitz plot, from CLEO using 9.0 fb$^{-1}$. The errors are statistical, experimental systematic, and modeling systematic, respectively.

| Fit          | Param. | Fit Result (%) | 95% C.L. Inter. (%) |
|--------------|--------|----------------|---------------------|
| No CPV       | $x$    | $1.8_{-3.2}^{+3.4} \pm 0.4 \pm 0.4$ | $(-4.7, 8.6)$      |
|              | $y$    | $-1.4_{-2.4}^{+2.5} \pm 0.8 \pm 0.4$ | $(-6.3, 3.7)$      |
| CPV Allowed  | $x$    | $2.3_{-3.4}^{+3.5} \pm 0.4 \pm 0.4$ | $(-4.5, 9.3)$      |
|              | $y$    | $-1.5_{-2.4}^{+2.5} \pm 0.8 \pm 0.4$ | $(-6.4, 3.6)$      |
|              | $\epsilon$ | $1.1 \pm 0.7 \pm 0.4 \pm 0.2$ | $(-0.4, 2.4)$      |
|              | $\phi$ | $(5.7 \pm 2.8 \pm 0.4 \pm 1.2) \degree$ | $(-0.3^\circ, 11.7^\circ)$ |

6 $D^0(t) \to K^+\pi^-\pi^0$ and $K^+\pi^-\pi^+\pi^-$ Multibody Decays

Mixing has also been searched for in WS multibody final states $^{10, 16, 17}$ $K^+\pi^-\pi^0$ and $K^+\pi^-\pi^+\pi^-$; the most recent measurement is from Belle using 281 fb$^{-1}$ of data. $^{17}$ The final signal yields are $1978 \pm 104$ $D^0 \to K^+\pi^-\pi^0$ decays and $1721 \pm 75$ $D^0 \to K^+\pi^-\pi^+\pi^-$ decays. For this analysis no decay time information is used, i.e., Belle measures the time-integrated ratio of WS to RS decays:

$$R_{WS} = \frac{\int R[D^0 \to K^+\pi^-(n\pi)] dt}{\int R[D^0 \to K^-\pi^+(n\pi)] dt} \approx R_D + \sqrt{R_D y'} + \frac{x'^2 + y'^2}{2}, \quad (13)$$

where $R_D$ is the ratio of the DCS rate to the CF rate as previously defined for $D^0 \to K^+\pi^-$ decays. The results are $R_{WS} = [0.229 \pm 0.015 \, \text{(stat)} \, ^{+0.013}_{-0.009} \, \text{(syst)}] \%$ for $K^+\pi^-\pi^0$ and $[0.320 \pm 0.018 \, \text{(stat)} \, ^{+0.018}_{-0.013} \, \text{(syst)}] \%$ for $K^+\pi^-\pi^+\pi^-$. Inserting these values into Eq. (13) allows one to determine $R_D$ as a function of $x'$ or $y'$. Assuming $x' = 0$ and $|x'| = 0.027$ gives the curves shown in Fig. 6; the latter $|x'|$ value corresponds to Belle’s 95% C.L. upper limit from $D^0 \to K^+\pi^-$ decays (see Table 1). However, the value of $x'$ from $D^0 \to K^+\pi^-$ may differ from that from $D^0 \to K^+\pi^-\pi(\pi)$ due to the strong phase differences ($\delta$) being different.

7 Summary

The 95% C.L. allowed ranges for $x'$ and $y'$ are plotted in Fig. 7; for simplicity we assume negligible CPV. The most stringent constraints are $|x'| < 2.7\%$
and $y' \in (-1.0\%, 0.7\%)$. These ranges are projections of the two-dimensional 95\% C.L. region for $x'^2, y'$ from Belle [Fig. 4(right)].

The results for $y_{CP}$ are plotted in Fig. 8. Here the central values and 1\% errors are shown; combining the results assuming the errors uncorrelated gives $y_{CP} = (1.09 \pm 0.46)\%$. This value differs from zero by 2.4\% and indicates a nonzero decay width difference $\Delta \Gamma$. Assuming negligible CPV, one can combine this value with Belle’s central value for $y'$, $(0.06^{+0.40}_{-0.39})\%$. The result is $y'/y = \cos \delta - (x/y) \sin \delta = 0.05^{+0.30}_{-0.37}$, where the error is obtained from an MC calculation as the fractional errors on $y$ and $y'$ are large. This small central value (albeit with a large error) implies $\tan \delta \approx y/x$; i.e., if $x \ll y$, then $\delta$ is near 90\%. Such a strong phase difference would be much larger than expected.

References

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Figure 7: 95% C.L. allowed ranges for $x'$ (top) and $y'$ (bottom) from various experiments assuming no CPV. The CLEO Dalitz results are for $x$ and $y$.

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Figure 8: $\gamma_{CP}$ central values and 1$\sigma$ errors measured by various experiments, and the combined result assuming the individual errors uncorrelated. The Belle 2002 data sample ($23 \text{ fb}^{-1}$) has some overlap with the Belle 2003 data sample ($158 \text{ fb}^{-1}$), and thus this result is not included in the average.

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