Torsion and the gravitational interaction

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Abstract. By using a nonholonomic-frame formulation of the general covariance principle, seen as an active version of the strong equivalence principle, an analysis of the gravitational coupling prescription in the presence of curvature and torsion is made. The coupling prescription implied by this principle is found to be always equivalent with that of general relativity, a result that reinforces the completeness of this theory, as well as the teleparallel point of view according to which torsion does not represent additional degrees of freedom for gravity, but simply an alternative way of representing the gravitational field.

1. Introduction

In general relativity torsion is assumed to vanish from the very beginning. In teleparallel gravity§, on the other hand, instead of torsion, curvature is assumed to vanish. In spite of this fundamental difference, the two theories are found to yield equivalent descriptions of the gravitational interaction [2]. An immediate implication of this equivalence is that curvature and torsion might be simply alternative ways of describing the gravitational field, and consequently related to the same degrees of freedom of gravity. This property is corroborated by the fact that the symmetric energy-momentum tensor appears as source in both theories: as the source of curvature in general relativity, and as the source of torsion in teleparallel gravity. On the other hand, more general gravity theories, like for example Einstein-Cartan and gauge theories for the Poincaré and the affine groups [3], consider curvature and torsion as representing independent degrees of freedom. In these models, differently from teleparallel gravity, torsion becomes relevant only when spins are important [4]. This situation could be achieved either at the microscopic level or near a neutron star, for example. According to this point of view, torsion might represent additional degrees of freedom in relation to curvature, and consequently new

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§ The name teleparallel gravity is commonly used to denote the general three-parameter theory [1]. Here, however, we use it as a synonymous for the teleparallel equivalent of general relativity, a theory obtained for a specific choice of these parameters.
physics phenomena might be associated with it. Now, the above described difference
rises a conceptual question on the actual role played by torsion in the description of the
gravitational interaction. The basic purpose of this paper will be to provide a possible
answer to this problem.

The approach we are going to use consists in studying the gravitational coupling
prescription in the presence of curvature and torsion, independently of the theory
governing the dynamics of the gravitational field. This can clearly be done because
the equation of motion of test particles, as well as the field equation of any matter
field coupled to gravity, can be obtained independently of the gravitational theory. One
has only to consider a gravitational field presenting curvature and torsion, and use it
to obtain, from an independent variational principle, the particle or field equations in
the presence of gravity. Since our interest here will be to study these particle and
field equations—or equivalently, the coupling prescription of particles and fields to
gravitation—we can simply consider this general gravitational field without specifying
the gravitational theory from which it is obtained as solution.

Our main problem, therefore, will be to obtain the gravitational coupling prescrip-
tion in the presence of curvature and torsion. This, however, is not an easy task.
The basic difficulty is that, differently from all other interactions of nature, where
the requirement of covariance does determine the gauge connection, in the presence
of curvature and torsion, covariance—seen as a consequence of the strong equivalence
principle [5]—is not able to determine the form of the gravitational coupling prescription.
The reason for this indefiniteness is that the space of Lorentz connections is an affine
space [6], and consequently one can always add a tensor to a given connection without
destroying the covariance of the theory. As a result of this indefiniteness, there will exist
infinitely many possibilities for the gravitational coupling prescription. Notice that in
the specific cases of general relativity and teleparallel gravity, characterized respectively
by a vanishing torsion and a vanishing curvature, the above indefiniteness is absent since
in these cases the connections are uniquely determined—and the corresponding coupling
prescriptions completely specified—by the combined use of covariance and the strong
equivalence principle. Notice furthermore that in the case of internal (Yang-Mills) gauge
theories, where the concept of torsion is absent∥, the above indefiniteness is not present
either.

To deal with the above problem, we are going to use a strategy based on the
equivalence principle. We begin by noticing that, due to the intrinsic relation of
gravitation with spacetime, there is a deep relationship between covariance (either
under general coordinate or local Lorentz transformations) and the strong equivalence
principle. In fact, an alternative version of this principle is the so called principle of
general covariance [5]. It states that a special relativity equation can be made to hold in
the presence of gravitation if it is generally covariant, that is, it preserves its form under
a general transformation of the spacetime coordinates. The process runs as follows. In

∥ We remark that absence of torsion, which happens in internal gauge theories, is different from the
presence of a vanishing torsion, which happens in general relativity ¶.
order to make an equation generally covariant, a connection is always necessary, which is in principle concerned only with the inertial properties of the coordinate system under consideration. Then, by using the equivalence between inertial and gravitational effects, instead of representing inertial properties, this connection can equivalently be assumed to represent a true gravitational field. In this way, equations valid in the presence of gravitation are obtained from the corresponding special relativity equations. Of course, in a locally inertial coordinate system, they must go back to the corresponding equations of special relativity. The principle of general covariance, therefore, can be seen as an active version of the equivalence principle in the sense that, by making a special relativity equation covariant, and by using the strong equivalence principle, it is possible to obtain its form in the presence of gravitation. The usual form of the equivalence principle, on the other hand, can be interpreted as its passive version in the sense that the special relativity equation must be recovered in a locally inertial coordinate system, where the effects of gravitation are absent. It should be emphasized that general covariance by itself is empty of physical content as any equation can be made generally covariant. Only when use is made of the strong equivalence principle, and the inertial compensating term is interpreted as representing a true gravitational field, the principle of general covariance can be seen as an alternative version of the strong equivalence principle [8].

The above description of the general covariance principle refers to its usual holonomic version. An alternative, more general version of the principle can be obtained in the context of nonholonomic frames. The basic difference between these two versions is that, instead of requiring that an equation be covariant under a general transformation of the spacetime coordinates, in the nonholonomic-frame version the equation is required to transform covariantly under a local Lorentz rotation of the frame. Of course, in spite of the different nature of the involved transformations, the physical content of both approaches are the same [9]. The frame version, however, is more general in the sense that, contrary to the coordinate version, it holds for integer as well as for half-integer spin fields.

The crucial point now is to observe that, when the purely inertial connection is replaced by a connection representing a true gravitational field, the principle of general covariance naturally defines a covariant derivative, and consequently also a gravitational coupling prescription. For the cases of general relativity and teleparallel gravity, the nonholonomic-frame version of this principle has already been seen to yield the usual coupling prescriptions of these theories [10]. The basic purpose of this paper will then be to determine, in the general case characterized by the simultaneous presence of curvature and torsion, the form of the gravitational coupling prescription implied by the general covariance principle. As we are going to see, in addition to explaining why there are infinitely many covariant coupling prescriptions for gravitation, all of them physically equivalent, the resulting coupling prescription will also provide an alternative interpretation for the role played by torsion in the description of the gravitational interaction. We begin by reviewing, in the next section, the nonholonomic-frame formulation of the general covariance principle.
2. Nonholonomic general covariance principle

2.1. Passage to a general nonholonomic frame

The usual holonomic general covariance principle is quite well known. Here, our interest here will be its more general nonholonomic-frame version. Let us consider the Minkowski spacetime of special relativity, endowed with the Lorentzian metric \( \eta \). We are going to use the Greek alphabet \( \mu, \nu, \rho, \ldots = 0, 1, 2, 3 \) to denote holonomic spacetime indices, and the Latin alphabet \( a, b, c, \ldots = 0, 1, 2, 3 \) to denote nonholonomic indices related to each local tangent Minkowski spaces. If \( \{x^\mu\} \) are inertial Cartesian coordinates in flat spacetime, the basis of (coordinate) vector fields \( \{\partial_\mu\} \) is then a global orthonormal coordinate basis for the flat spacetime. The frame \( \delta_a = \delta_a^\mu \partial_\mu \) can then be thought of as a trivial (holonomous) tetrad, with components \( \delta_a^\mu \). Consider now a local, that is, point-dependent Lorentz transformation \( \Lambda_a^b = \Lambda_a^b(x) \). It yields the new frame

\[
e_a = e_a^\mu \partial_\mu,
\]

with components

\[
e_a^\mu(x) = \Lambda_a^b \delta_b^\mu.
\]

Notice that, on account of the locality of the Lorentz transformation, the new frame \( e_a \) is nonholonomous,

\[
[e_a, e_b] = f_{ab}^\ c e_c,
\]

with \( f_{ab}^\ c \) the coefficient of nonholonomy. Now, making use of the orthogonality property of the tetrads, we see from Eq. (2) that the Lorentz group element can be written in the form \( \Lambda_b^d = e_b^\rho \delta_\rho^d \). Using this expression, it follows that

\[
(e_a \Lambda_b^d) \Lambda^c_d = \frac{1}{2} (f_{ab}^\ c + f_{ba}^\ c - f_{ba}^\ a).
\]

On the other hand, the action describing a free particle is

\[
S = -mc \int ds,
\]

with \( ds = (\eta_{\mu\nu} dx^\mu dx^\nu)^{1/2} \) the spacetime invariant interval. Seen from the holonomous frame \( \delta_a \), the corresponding equation of motion is given by

\[
dv^a ds = 0,
\]

where \( v^a = \delta^a_\mu v^\mu \), with \( v^\mu = (dx^\mu / ds) \) the holonomous particle four-velocity. Seen from the nonholonomous frame \( e_a \), a straightforward calculation shows that the equation of motion \( \Box \) is

\[
dV^c ds + \frac{1}{2} (f_{ab}^\ c + f_{ba}^\ c - f_{ba}^\ a) V^a V^b = 0,
\]

where \( V^c = \Lambda_c^d v^d = e_a^d v^a \), and use has been made of Eq. (1). It is important to emphasize that, although we are in the flat spacetime of special relativity, we are free to choose any tetrad \( \{e_a\} \) as a moving frame. The fact that, for each \( x \in M \), the frame
e_a can be arbitrarily rotated introduces the compensating term \( \frac{1}{2} (f^c_{ab} + f^c_{ba} - f_{ba}^c) \) in the free-particle equation of motion. This term, therefore, is concerned only with the (inertial) properties of the frame. It is also important to remark that the equation of motion \( (7) \) can be obtained from the action \( (5) \) provided it is written in the nonholonomous frame, in which case it assumes the form

\[
S = -mc \int (\eta_{ab} e^a e^b)^{1/2},
\]

where, owing to the fact that the Lorentz transformation is an isometry, the metric \( \eta_{ab} = \Lambda_a^c \Lambda_b^d \eta_{cd} \) is kept fixed, and \( e^a = (\Lambda^a_d \delta^d \mu) dx^\mu \).

2.2. Using the equivalence between inertial and gravitational effects

According to the general covariance principle, the equation of motion valid in the presence of gravitation can be obtained from the corresponding special relativistic equation by replacing the inertial compensating term by a connection \( \Gamma_{abc} \) representing a true gravitational field. We consider here only Lorentz-valued connections, and consequently vanishing nonmetricity. In this case, in the presence of both curvature and torsion, such a connection satisfies \[11\]

\[
\Gamma_{ba} c - \Gamma_{ab} c = T_{ab} c + f_{ab} ^c,
\]

with \( T_{ba} ^c \) the torsion of the connection \( \Gamma_{abc} \). Use of this equation for three combination of indices gives

\[
\Gamma_{ab} c = \frac{1}{2} (f^c_{ab} + f^c_{ba} - f_{ba}^c) + \frac{1}{2} (T^c_{ab} + T^c_{ba} - T_{ba} ^c).
\]

Accordingly, the compensating term of Eq. \[7\] can be written in the form

\[
\frac{1}{2} (f^c_{ab} + f^c_{ba} - f_{ba}^c) = \Gamma_{ab} c - K_{ab} c,
\]

where

\[
K_{ab} c = \frac{1}{2} (T^c_{ab} + T^c_{ba} - T_{ba} ^c)
\]

is the contortion tensor. Equation \[11\] is actually an expression of the equivalence principle. In fact, whereas its left-hand side involves only inertial properties of the frames, its right-hand side contains purely gravitational quantities. Using this expression in Eq. \[7\], we get

\[
\frac{dV^c}{ds} + \Gamma_{ab} c V^a V^b = K_{ab} c V^a V^b.
\]

This is the particle equation of motion in the presence of curvature and torsion that follows from the principle of general covariance. It entails a very peculiar interpretation for contortion, which appears playing the role of a gravitational force \[12\]. Because of the identity

\[
\Gamma_{ab} c - K_{ab} c = \Gamma_{ab} c,
\]

with \( \Gamma_{ab} c \) the spin connection of general relativity, the equation of motion \[13\] is found to be equivalent with the geodesic equation of general relativity.
2.3. Gravitational coupling prescription

The equation of motion (13) can be written in the form

\[ V^\mu D_\mu V^c \equiv V^\mu \left[ \partial_\mu V^c + (\Gamma^c_{\mu b} - K^c_{\mu b}) V^b \right] = 0, \]  

(15)

with \( D_\mu \) a covariant derivative. Applied on a general vector field \( X^c \), it assumes the form

\[ D_\mu X^c = \partial_\mu X^c + (\Gamma^c_{\mu b} - K^c_{\mu b}) X^b. \]  

(16)

Using the vector representation of the Lorentz generators \[ \mathbf{13}, \]

\[ (S_{ab})^c_d = i(\delta^c_a \eta_{bd} - \delta^c_b \eta_{ad}), \]  

(17)

it becomes

\[ D_\mu X^c = \partial_\mu X^c - \frac{i}{2} (\Gamma^c_{\mu b} - K^c_{\mu b}) (S_{ab})^c_d X^d. \]  

(18)

Now, although obtained in the case of a Lorentz vector field (four-velocity), the compensating term (4) can be easily verified to be the same for any field. In fact, denoting by \( g \equiv g(\Lambda) \) the element of the Lorentz group in an arbitrary representation, it can be shown that

\[ (e_a g)^{-1} = -\frac{i}{4} (f_{ab} + f_{ba} - f_{ba} J^{bc}), \]  

(19)

with \( J^{bc} \) denoting the corresponding Lorentz generator. In this case, the covariant derivative \[ \mathbf{18} \] will have the form

\[ D_\mu = \partial_\mu - \frac{i}{2} (\Gamma^c_{\mu b} - K^c_{\mu b}) J_{ab}. \]  

(20)

Consequently, the coupling prescription—in the presence of curvature and torsion—of fields carrying an arbitrary representation of the Lorentz group, will be

\[ \partial_a \equiv \delta^a_\mu \partial_\mu \rightarrow D_a \equiv e^a_\mu D_\mu. \]  

(21)

Of course, due to the relation \[ \mathbf{14}, \] it is clearly equivalent with the coupling prescription of general relativity.

3. Universality of the general relativity prescription

3.1. The connection space: characterizing the affinity

A general connection space is an infinite, homotopically trivial affine space \[ \mathbf{14}. \] In the specific case of Lorentz connections, a point in this space will be a connection

\[ A = A^{bc}_{\mu} J_{bc} dx^\mu, \]  

(22)

presenting simultaneously curvature and torsion, 2-forms defined respectively by

\[ R = dA + AA \equiv \nabla_A A \]  

(23)

and

\[ T = de + Ae \equiv \nabla_A e, \]  

(24)
where \( e = e^a_\mu dx^\mu \partial_a \), and \( \nabla_A \) denotes the covariant differential in the connection \( A \). Under a local Lorentz transformation, a Lorentz connection transforms according to
\[
A \rightarrow A' = gAg^{-1} + gdg^{-1}.
\] (25)

The curvature and torsion 2-forms are covariant under these transformations:
\[
R' = gRg^{-1} \quad \text{and} \quad T' = gTg^{-1}.
\] (26)

Given two connections \( A \) and \( \tilde{A} \), the difference
\[
k = \tilde{A} - A
\] (27)
is also a 1-form assuming values in the Lorentz Lie algebra, but transforming covariantly:
\[
k = gkg^{-1}.
\] (28)

Its covariant derivative is consequently given by
\[
\nabla_A k = dk + \{ A, k \}.
\] (29)

It is then easy to verify that, given two connections such that \( \tilde{A} = A + k \), their curvature and torsion will be related by
\[
\tilde{R} = R + \nabla_A k + k k
\] (30)
and
\[
\tilde{T} = T + k e.
\] (31)

The effect of adding a covector \( k \) to a given connection \( A \), therefore, is to change its curvature and torsion 2-forms [15].

Let us now rewrite Eq. (27) in components:
\[
A_{ab}^c \equiv e^\mu_a A_{\mu b}^c = \tilde{A}_{ab}^c - k_{ab}^c.
\] (32)

Since \( k_{ab}^c \) is a Lorentz-valued covector, it is necessarily anti-symmetric in the last two indices. We notice that the presence of nonmetricity would spoil the anti-symmetry in the last two indices, and consequently the connection would not be Lorentz valued. Separating \( k_{ab}^c \) in the symmetric and anti-symmetric parts in the first two indices, we get
\[
k_{ab}^c = \frac{1}{2} (k_{ab}^c + k_{ba}^c) + \frac{1}{2} (k_{ab}^c - k_{ba}^c).
\] (33)

If we call
\[
k_{ab}^c - k_{ba}^c = t_{ab}^c,
\] (34)
we see that \( t_{ab}^c \) will automatically satisfy \( t_{ba}^c = -t_{ab}^c \). It is then easy to verify that the symmetric part turns out to be
\[
k_{ab}^c + k_{ba}^c = t_{ab}^c + t_{ba}^c.
\] (35)

Therefore, \( k_{ab}^c \) can always be written in the form
\[
k_{ab}^c = \frac{1}{2} (t_{ab}^c + t_{ba}^c).
\] (36)

This means essentially that the difference between any two Lorentz-valued connections, that is, the affinity covector, has the form of a contortion tensor.
3.2. Equivalence under translations in the connection space

As already discussed, due to the affine character of the connection space, one can always add a tensor to a given connection without spoiling the covariance of the derivative (20). Since adding a tensor to a connection corresponds just to redefining the origin of the connection space, this means that covariance does not determine a preferred origin for this space. Let us then analyze the physical meaning of translations in the connection space. To begin with, we take again the connection appearing in the covariant derivative (20), which is given by

\[ A_{ab}^c \equiv \Gamma_{ab}^c - K_{ab}^c, \]  

where we have used the relation \( A_{abc} = e^a \Gamma_{ab}^c \). A translation in the connection space with parameter \( k_{ab}^c \) corresponds to

\[ \bar{A}_{ab}^c = A_{ab}^c + k_{ab}^c \equiv \Gamma_{ab}^c - K_{ab}^c + k_{ab}^c. \]  

Now, since \( k_{ab}^c \) has always the form of a contortion tensor, as given by Eq. (36), the above connection is equivalent to

\[ \bar{A}_{ab}^c = A_{ab}^c - \bar{K}_{ab}^c, \]  

with \( \bar{K}_{ab}^c = K_{ab}^c - k_{ab}^c \) another contortion tensor.

Let us then consider a few particular cases. First, we choose \( t_{ba}^c \) as the torsion of the connection \( \Gamma_{ab}^c \), that is, \( t_{ba}^c = T_{ba}^c \). In this case, the last two terms of Eq. (38) cancel each other, yielding \( \bar{K}_{ab}^c = 0 \). This means that the torsion of \( \bar{A}_{ab}^c \) vanishes, and we are left with

\[ \bar{A}_{ab}^c = \bar{\Gamma}_{ab}^c, \]  

with \( \bar{\Gamma}_{ab}^c \) denoting the torsionless spin connection of general relativity. On the other hand, if we choose \( t_{ba}^c \) such that

\[ t_{ba}^c = T_{ba}^c - f_{ba}^c, \]  

the connection \( \Gamma_{ab}^c \) vanishes, which characterizes teleparallel gravity. In this case, the resulting connection has the form

\[ \bar{A}_{ab}^c = -\bar{K}_{ab}^c, \]  

where \( \bar{K}_{ab}^c \) is the Weitzenböck contortion, that is, the contortion tensor written in terms of the Weitzenböck torsion \( \bar{T}_{ab}^c = -f_{ab}^c \). There are actually infinitely many choices for \( t_{ba}^c \), each one defining a different translation in the connection space, and consequently yielding a connection with different curvature and torsion. All of them, however, are ultimately equivalent with the coupling prescription of general relativity as for all cases we have the identity

\[ \bar{A}_{ab}^c = A_{ab}^c - \bar{K}_{ab}^c \equiv \bar{\Gamma}_{ab}^c. \]  

It is important to emphasize that, despite yielding physically equivalent coupling prescriptions, the physical equations are not covariant under a translation in the
connection space. For example, under a particular such translation, the geodesic equation of general relativity becomes the force equation of teleparallel gravity, which are completely different equations. These two equations, however, as well as any other obtained through a general translation in the connection space, are equivalent in the sense that they describe the same physical trajectory.

4. Final remarks

The general covariance principle, seen as an active version of the usual (passive) strong equivalence principle, naturally defines a coupling prescription of any field to gravitation. By considering the case of a Lorentz connection presenting simultaneously curvature and torsion, we have shown that the gravitational coupling prescription implied by the general covariance principle is such that it preserves the equivalence with the coupling prescription of general relativity. This result reinforces the completeness of general relativity, as well as gives support to the point of view of teleparallel gravity, according to which torsion does not represent additional degrees of freedom of gravity, but simply an alternative way of representing the gravitational field. As a consequence, it becomes a matter of convention to describe gravitation by curvature, torsion, or by a combination of them.

The above result can be better understood if we remember that gravitation presents two alternative descriptions. In fact, according to the gauge approach provided by teleparallel gravity, the gravitational interaction is described by a force equation similar to the Lorentz force equation of electrodynamics, with contortion playing the role of force. On the other hand, due to the universality of free fall, it is also possible to describe gravitation, not as a force, but as a geometric deformation of flat Minkowski spacetime. This is the approach used by general relativity, in which the gravitational field is supposed to produce a curvature in spacetime, and its action on (structureless) particles is described by letting them follow the geodesics of the curved spacetime. In this approach, therefore, geometry replaces the concept of gravitational force, and the trajectories are determined, not by force equations, but by geodesics. In the general case, characterized by a connection presenting both curvature and torsion, the gravitational interaction is described by a mixture of force and geometry. Since all cases are ultimately equivalent, how much of the interaction is described by curvature (geometry), and how much is described by contortion (force), is a matter of convention. All these equivalent cases are related through translations in the connection space, whose affinity allows the addition of a tensor (actually, a Lorentz-valued covector) to a given connection without destroying the covariance of the theory.

Sometimes, the autoparallel curves are considered as describing the motion of a spinless particle in the presence of curvature and torsion\cite{17}. Such curves are characterized by parallel transporting the tangent vector itself, and by the fact that they do not represent the shortest line between two points of spacetime. However, it has already been shown that autoparallels cannot be obtained from a Lagrangian
formalism [18], which means that a spinless particle following such a trajectory does not have a Lagrangian. Taking into account that the energy-momentum is defined as the functional derivative of the Lagrangian with respect to the metric tensor (or equivalently, to the tetrad field), it would not be possible to define an energy-momentum density for such particle. Despite the non-existence of experimental data, the results presented here can be considered as a theoretical evidence favoring the fact that, even in the presence of curvature and torsion, a spinless particle will always follow a trajectory that can ultimately be represented by a geodesic of the underlying Riemannian spacetime.

The new interpretation for torsion here described differs from the usual one used in theories of the Einstein-Cartan type, for example. It is, however, self-consistent, and presents several conceptual advantages in relation to the Einstein-Cartan interpretation: it preserves the role played by torsion in teleparallel gravity; it is consistent with the general covariance principle, that is, with the strong equivalence principle; the corresponding gravitational coupling prescription can be applied in the Lagrangian or in the field equations with the same result; when applied to describe the interaction of the electromagnetic field with gravitation, it does not violate the U(1) gauge invariance of Maxwell’s theory¶. In addition, considering that, at least up to now, there is no compelling experimental evidence for new physics associated with torsion, we could say that our results agree with the available experimental data. For example, no new gravitational physics has ever been reported near a neutron star, where torsion would become important according to the Einstein-Cartan type theories. Of course, it should be remarked that, because of the weakness of the gravitational interaction, no experimental data exist on the coupling of the spin of the fundamental particles to gravitation.

From the classical point of view, as we have seen, two connections differing by a Lorentz-valued covector yield the same physical result. On the other hand, from the point of view of a prospective connection-based quantum theory for gravitation [20], these two connections might produce different observable effects [21]. If, however, the connection-space translation could somehow be gauged in the quantum theory, the choice of the connection would then correspond to a kind of “gauge choice”, and the final quantum theory should naturally yield the same physical equivalence of the classical theory. The connection choices (40) and (42) could accordingly be named respectively the Einstein and the Weitzenböck gauges. Of course, there exist infinitely more physically equivalent gauge choices, each one characterized by a different proportion between curvature and torsion, and differing by a translation in the connection space. It should be noted finally that, if gravitation eventually loses its universal character at the quantum level+, the general relativistic geometrical description (in terms of curvature) of the gravitational interaction would break down, and due to the fact that teleparallel gravity does not require the validity of the equivalence principle [23], the Weitzenböck

¶ This comes from the equivalence with general relativity, whose coupling prescription does not violate the U(1) gauge invariance of Maxwell’s theory. For the specific case of teleparallel gravity, see [19].
+ There are some controversies concerning this point; for a discussion, see e.g. [22].
Torsion and the gravitational interaction

gauge may eventually become mandatory.

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