Constraining the 6.05 MeV 0$^+$ and 6.13 MeV 3$^-$ cascade transitions in the $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ reaction using the Asymptotic Normalization Coefficients

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Abstract

Background: The $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ reaction plays a fundamental role in astrophysics because its cross section near 300 keV in c.m. determines the $^{12}\text{C}/^{16}\text{O}$ ratio at the end of the helium burning stage of stellar evolution. The astrophysically desired accuracy of better than 10% has not been achieved. Cascade $\gamma$ transitions through the excited states of $^{16}\text{O}$ are contributing to the uncertainty.

Purpose: To measure the Asymptotic Normalization Coefficients (ANCs) for the 0$^+$ (6.05 MeV) and 3$^-$ (6.13 MeV) excited states in $^{16}\text{O}$ and provide constraints on the cross section for the corresponding cascade transitions.

Method: The ANC$s$ were measured using the $\alpha$-transfer reaction $^{12}\text{C}(^{6}\text{Li},d)^{16}\text{O}$ performed at sub-Coulomb energies for both the entrance and exit channels.

Results: The ANC$s$ for the 0$^+$ (6.05 MeV), 3$^-$ (6.13 MeV), 2$^+$ (6.92 MeV) and 1$^-$ (7.12 MeV) states in $^{16}\text{O}$ have been measured. The contribution of the 0$^+$ and 3$^-$ cascade transitions to the $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ reaction S-factor was found to be 1.9$\pm$0.3 keV b and 0.5$\pm$0.09 keV b respectively.

Conclusions: Significant uncertainties related to the 6.05 MeV 0$^+$ and 6.13 MeV 3$^-$ cascade transitions have been eliminated. The combined contribution of the 0$^+$ and 3$^-$ cascade transitions to the $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ reaction cross section at 300 keV does not exceed 2%.
The radiative capture of $\alpha$-particles on $^{12}$C plays a fundamental role in astrophysics. The $^{12}$C($\alpha, \gamma$)$^{16}$O reaction is activated during the helium burning stages of stellar evolution. This reaction becomes important when the triple-$\alpha$ reaction, dominant during the initial stage of helium burning, produces significant abundance of carbon. The $^{12}$C($\alpha, \gamma$)$^{16}$O reaction cross section at around 300 keV determines the relative abundance of $^{12}$C/$^{16}$O in the stellar core, which is crucial for the later stellar burning stages, in particular, for the rates of the reaction $^{16}$O($\alpha, \gamma$)$^{20}$Ne. This, in turn, has important implications for the sequence of later quiescent and explosive burning stages in stars, including nucleosynthesis and production of long-lived radioactive isotopes, such as $^{26}$Al, $^{44}$Ti and $^{60}$Fe in core collapse supernova [1]. It also has direct influence on the composition of white dwarfs, and therefore plays an important role in the type Ia supernova ignition process (see Ref. [2] and references therein).

Significant progress in constraining the $^{12}$C($\alpha, \gamma$)$^{16}$O reaction rate has been achieved over the last 40 years, however, the astrophysically required precision of better than 10% [3] is still out of reach. This is because direct measurement of radiative $\alpha$-capture reaction on $^{12}$C at 300 keV is unfeasible (cross section is $\sim 10^{-17}$ b) and extrapolations from higher energy measurements have to be used. However, extrapolations are difficult because there are no resonances near 7.5 MeV excitation energy in $^{16}$O that can dominate the cross section (300 keV above the $\alpha$-decay threshold) and the $\alpha$-capture process is determined by the mixture of ground state and cascade transitions. It was assumed in the past that the ground state transition through the tails of sub-threshold states and above threshold resonances plays a dominant role and that cascade transitions are relatively unimportant. This assumption was called into question in [4] where the S-factor at 300 keV for the $0^+$ state at 6.05 MeV cascade transition was determined to be $25^{+16}_{-15}$ keV b. This is comparable to the $E2$ transition to the ground state ($53^{+13}_{-18}$ keV b) [5] and corresponds to 15% of the total S-factor. Very different conclusions were made in Refs. [6, 7], where the upper limit for the transition was set at <1 keV b. Both measurements were performed at higher energies (>2 MeV) using recoil separators and the results were extrapolated down to 300 keV. The discrepancy is caused mostly by different extrapolation approaches but it is also due to a 50% lower cross section for the 6.05 MeV transition measured in [6, 7] as compared to [4]. This discrepancy causes significant uncertainty for the $^{12}$C($\alpha, \gamma$)$^{16}$O reaction rate. In Ref. [7] the contribution of another cascade transition, the $3^-$ at 6.13 MeV, was determined to be negligibly small (0.3 keV b). The main goal of this letter is to constrain the 6.05 MeV $0^+$ and 6.13 MeV $3^-$
cascade transitions using an independent technique.

It has been shown that reliable constraints on direct proton capture transitions can be obtained if one determines the proton Asymptotic Normalization Coefficient (ANC) of the corresponding state [8]. A large number of proton-capture reactions have been investigated this way and results were benchmarked against the direct measurements (see recent review paper and references therein [9]). Application of the ANC technique for $\alpha$-capture reactions was pioneered in Ref. [10], where $\alpha$ ANCs for the $2^+$ and $1^-$ states at 6.92 and 7.12 MeV in $^{16}$O were measured using the sub-Coulomb $\alpha$-transfer reactions $^{12}$C($^6$Li,$d$)$^{16}$O and $^{12}$C($^7$Li,$t$)$^{16}$O. The advantage of using sub-Coulomb energies for $\alpha$-transfer reactions is that the extracted ANCs are practically independent of the optical model potentials. Extracting the ANC instead of the spectroscopic factor eliminates uncertainties associated with the shape of the cluster form factor potential and the number of nodes of the cluster wave function. Therefore, results of these measurements are nearly model independent and do not require any additional normalization as long as the reaction mechanism is dominated by peripheral single-step $\alpha$-capture. This experimental approach has previously been used to investigate the $^{13}$C($\alpha,n$)$^{16}$O and $^{14}$C($\alpha,\gamma$)$^{18}$O reactions [11,12]. More recently, a benchmark experiment was performed in Ref. [13] where the validity of the sub-Coulomb $\alpha$-transfer approach was demonstrated by measuring the ANC of the $1^-$ state at 5.9 MeV in $^{20}$Ne and comparing it to the well known width of this state.

The ANCs for the $2^+$ at 6.92 MeV and $1^-$ at 7.12 MeV states in $^{16}$O have been previously measured using sub-Coulomb energies in Ref. [10] and above barrier energies in Refs. [14,15]. However, the ANCs of the $0^+$ at 6.05 MeV and $3^-$ at 6.13 MeV states have not been measured. The 6.05-MeV $0^+$ transition could not be studied in previous sub-Coulomb $^{12}$C($^6$Li,$d$)$^{16}$O measurements [10] because de-excitation $\gamma$-rays were detected and the 6.05 MeV $0^+$ state decays by monopole ($E0$) transition (mostly electron-positron pair creation). The experimental technique employed in the present work does not suffer from this limitation and the ANCs for all relevant sub-threshold states were measured simultaneously: 6.05 MeV $0^+$, 6.13 MeV $3^-$, 6.92 MeV $2^+$ and 7.12 MeV $1^-$. The experiment was carried out at the John D. Fox Superconducting Accelerator Laboratory, at Florida State University. The cross section for $^6$Li($^{12}$C,$d$)$^{16}$O was measured at three energies of $^{12}$C beam: 5, 7 and 9 MeV. Inverse kinematics (heavy beam and light target) was chosen to achieve the lowest energies in the center of mass (c.m.) and still be able to detect...
the recoil deuterons. The $^{12}$C beam was delivered by an FN Tandem Van de Graaff accelerator using a SNICS-II cesium-sputter ion source. Several $^6$Li targets of thicknesses of about 35 $\mu$g/cm$^2$ were used. The $^6$Li targets were prepared under vacuum and transported to the chamber in a vacuum container in order to prevent oxidation. For the identification of the reaction products two $\Delta E$-$E$ telescopes were mounted on remotely controlled rotating rings placed to the right and left of the beam axis. Each of the $\Delta E$-$E$ telescopes was constructed with a position sensitive proportional counter and four pin diode $2\times2$ cm$^2$ silicon detectors, contained in a box filled with a P10 gas (10% methane and 90% Ar mixture). A Kapton foil of 7.5 $\mu$m thickness was used as the entrance window separating the P10 gas inside the detector from the chamber vacuum. This setup allows the measurement and identification of deuterons down to an energy of 1 MeV when 150 Torr of P10 pressure is used and also to observe the backscattered $^6$Li ions when the pressure in the proportional counters is reduced to 50 Torr. The intensity of the incoming beam was measured using a Faraday cup placed at the end of the chamber.

The target thickness was measured using $^6$Li ions elastically backscattered by the $^{12}$C beam at 9 MeV and the $^{16}$O beam at 10 MeV. Two different beams were used to evaluate the systematic uncertainty of the target thickness measurements which turned out to be 10%. Elastic scattering of $^6$Li was also used to monitor target integrity and effective thickness. The control measurements were performed every time a new target was used and after about three hours of use. It was found that after 3-5 hours of target usage the energy of the elastically backscattered $^6$Li was decreased slightly which was attributed to carbon buildup on the surface of the target. Since the sub-Coulomb $\alpha$-transfer cross section is very sensitive to the energy of the beam, the targets were changed frequently and corresponding corrections were implemented. Details are given in Refs. [13, 16]. After the carbon deposition effect was corrected for and the energy loss in half of the target was considered, the beam energies of 8.7 MeV, 6.75 MeV and 4.7 MeV were used for all the calculations.

The two-dimensional $\Delta E$ vs $E$ scatter plot is shown in Fig. 1 where it can be seen that deuterons are clearly identified. A strong proton peak around 1 MeV is seen in Fig. 1. This peak corresponds to $^{12}$C+$p$ elastic scattering due to the hydrogen contained in the target and has a much higher intensity than the events of interest. The $\Delta E$ tail from these protons leaks into the deuteron cut and prevents deuteron identification below 1 MeV. For the 9 MeV and 7 MeV data this proton background does not overlap with the deuterons of
interest. However, for the 5 MeV data the deuterons from the $2^+$ and $1^-$ states overlap with the proton background. Therefore, only the $0^+$ and $3^-$ states were studied in the lowest energy 5-MeV dataset.

![Figure 1](image1.png)

**FIG. 1.** $\Delta E$ vs $E$ scatter plot with the cut on the deuterons using the 9-MeV data for a pin detector at 30° in the laboratory frame.

The spectrum of deuterons from $^{12}$C($^6$Li,$d$)$^{16}$O reaction at sub-Coulomb energy is shown in Fig. 2. As can be seen in Fig. 2, the energy resolution of the experiment is good enough to resolve the $2^+$ and $1^-$ states that are 200 keV apart, but insufficient for clean separation between the $0^+$ and $3^-$ states that are only 80 keV apart. The $3^-$ state manifests itself as a shoulder toward the higher excitation energy visible in the peak that is dominated by the $0^+$ state. We used two overlapping Gaussians fit to determine strengths of the $0^+$ and $3^-$ states. This fit has only two free parameters (amplitudes of the two Gaussians) since the excitation energies of the states are well known and the experimental resolution is set by the widths of the resolved states ($2^+$ and $1^-$). A reliable fit could only be achieved for the 9- and 7-MeV dataset since the 5-MeV dataset had limited statistics. For the 5-MeV data all the events in the 6 MeV peak were used to calculate the cross section of the $0^+$ and $3^-$ states combined. Then, the values of the ANCs and cross section for each of the states were calculated based on the ANCs ratios of the $0^+$ and $3^-$ states obtained with the 9- and 7-MeV data.

Angular distributions for deuterons from the $^6$Li($^{12}$C,$d$)$^{16}$O reaction, performed at the $^{12}$C beam energies of 9, 7 and 5 MeV, populating $0^+$, $3^-$, $2^+$ and $1^-$ states at 6.05 MeV, 6.13 MeV, 6.92 MeV and 7.12 MeV respectively are shown in Fig. 3 together with the corresponding DWBA calculations. For beam energy of 5 MeV, all the data points (corrected to the $0^+$
FIG. 2. Spectrum of deuterons from the $^{12}$C($^6$Li,$d$)$^{16}$O reaction. The $^{12}$C effective beam energy is 8.7 MeV (energy in the middle of the $^6$Li target) and the deuteron scattering angle is 119° in the center of mass.

cross section) are shown in Fig. 3 (a) and only the calculated DWBA cross section is shown in Fig. 3 (b) for the 3$^-$ state. The computer code fresco (version FRES 2.9) [17] was used to perform finite range DWBA calculations with the full complex remnant term. The potential for $^6$Li+$^{12}$C is obtained from Ref. [18], where energy dependent parameters are obtained for energy range from 4.5 to 156 MeV ($^6$Li beam energy). It was observed that changing the value of $V_0$ from 174 to 167 MeV produces better fit to the shape of the 0$^+$ angular distribution. The $d+^{16}$O optical potential parameters were obtained from [19]. The potential parameters for $\alpha+d$ form factor were taken from Ref. [20]. By normalizing the DWBA calculations to the experimental data and using the equations provided in Refs. [10, 21] together with the known value for $^6$Li $\alpha$-ANC ($\langle C_{\alpha d}^{^6}{\text{Li}}\rangle^2 = 5.3 \pm 0.5$ fm$^{-1}$) [22], the ANC values for the 0$^+$ (6.05 MeV), 3$^-$ (6.13 MeV), 2$^+$ (6.92 MeV) and 1$^-$ (7.12 MeV) states were determined. The obtained square ANCs are shown in Table I and compared to previous measurements for the 2$^+$ (6.92 MeV) and 1$^-$ (7.12 MeV) states.

The total uncertainty of the extracted ANCs is a combination of statistical uncertainties, normalization uncertainties and uncertainties in the parameters used for the DWBA calculations such as the optical potential parameters and the number of nodes (see Refs. [13, 16]). Due to the fact that the reaction is performed at near and sub-Coulomb energies the uncertainty related to the optical potential parameters is small with one excep-
FIG. 3. Experimental data and DWBA cross section as a function of center-of-mass angle for the $0^+$ at 6.05 MeV (a), $3^-$ at 6.13 MeV (b), $2^+$ at 6.92 MeV (c) and $1^-$ at 7.12 MeV (d) in $^{16}$O using the $^{12}$C beam energies of 5 MeV (dashed-dotted line), 7 MeV (solid line) and 9 MeV (dashed line).

It was found that for the highest energy dataset (9 MeV) the angular distribution for the 6.05 MeV $0^+$ state is somewhat sensitive to the optical model parameters because the exit channel ($d+^{16}$O) is above the Coulomb barrier. The ANCs for each state and the corresponding uncertainties were determined for each beam energy datasets and then com-
TABLE I. Square ANCs for the \(0^+\) (6.05 MeV), \(3^-\) (6.13 MeV), \(2^+\) (6.92 MeV) and \(1^-\) (7.12 MeV) sub-threshold states in \(^{16}\text{O}\), compared to previous measurements.

| \((C_{^{16}\text{O}(0^+)}^{12}\text{C})^2\) | \((C_{^{16}\text{O}(3^-)}^{12}\text{C})^2\) | \((C_{^{16}\text{O}(2^+)}^{12}\text{C})^2\) | \((C_{^{16}\text{O}(1^-)}^{12}\text{C})^2\) | Ref. |
|---------------------------------|---------------------------------|---------------------------------|---------------------------------|-----|
| \((10^6 \text{ fm}^{-1})\)     | \((10^4 \text{ fm}^{-1})\)     | \((10^{10} \text{ fm}^{-1})\)  | \((10^{28} \text{ fm}^{-1})\)  |     |
| -                              | -                              | -                              | -                              |     |
| 2.07±0.80 4.00±1.38            | 1.29±0.23 4.33±0.84            | 1.96±1.41 3.48±2.0             | This work                       |     |

The ANCs for the \(2^+\) (6.92 MeV) and \(1^-\) (7.12 MeV) sub-threshold states have been measured in [10, 14, 15]. There is excellent agreement between all measurements with sub-Coulomb \(\alpha\)-transfer ([10] and this work) providing the most precise values. The contribution of the \(2^+\) (6.92 MeV) and \(1^-\) (7.12 MeV) sub-threshold states to the astrophysical S-factor have been evaluated in [10] using ANCs that are nearly identical to the results of the present work and therefore there is no need to repeat the R-matrix analysis already performed in [10]. However, the ANC of the 0\(^+\) and 3\(^-\) states have been measured for the first time and it is paramount to evaluate the contribution of the direct capture to these excited states to the total \(^{12}\text{C}(\alpha,\gamma)^{16}\text{O}\) reaction rate. The S-factor for direct \(\alpha\)-capture to the 0\(^+\) and 3\(^-\) states was calculated using the R-matrix formalism described in [23] and implemented in the code AZURE [24]. The \(E2\) transition dominates the direct \(\alpha\)-capture to the 0\(^+\) and 3\(^-\) states \((E1 \text{ and } M1 \text{ transitions were evaluated and were found to be negligible for both cascade transitions})\). The S-factors for the direct \(E2\) transitions to the 0\(^+\) and 3\(^-\) states are shown in Fig. [4]. They are completely determined by the measured ANCs with uncertainties related to the choice of channel radius being very small compared to the experimental uncertainties.
of the ANCs. The corresponding S-factors at 300 keV are 3.1±0.4 keV b and 1.2±0.2 keV b for direct capture to the 0\(^+\) and 3\(^-\) states respectively.

The radiative capture to the first excited state of \(^{16}\)O (0\(^+\) at 6.05 MeV) state is contributed by the interfering direct capture and the capture through the sub-threshold resonance 2\(^+\) at 6.92 MeV. The amplitude of the capture through the sub-threshold resonance contains the product of the \(\alpha\) partial width amplitude in the entrance channel and the radiative width amplitude for the decay of the sub-threshold resonance to the first excited state 0\(^+\). A priori, the interference sign between the direct capture and the capture through the sub-threshold resonance is unknown but it can be fixed using the ANC method [25].

The radiative width amplitude can be decomposed into the internal and external (channel) parts leading to the corresponding splitting of the sub-threshold resonance amplitude. The channel radiative width amplitude is proportional to the ANC of the sub-threshold state [21]. Hence the sign of the channel sub-threshold resonance amplitude is synchronized with the sign of the non-resonant capture amplitude, which is also peripheral. This reveals another important role of the ANC in the analysis of the radiative processes: The ANC controls the relative sign of the sub-threshold (or real) resonance channel amplitude and direct capture amplitude. The sign of the internal radiative width amplitude can be found from the microscopic calculations. However, in the case under consideration the transition is \(E2\), the matrix element determining the radiative width contains \(r^2\), where \(r\) is the distance between the \(\alpha\)-particle and \(^{12}\)C. The presence of \(r^2\) significantly suppresses the internal contribution to the radiative width amplitude. An additional suppression of the internal part is caused by the oscillations of the \(\alpha\)-\(^{12}\)C cluster radial wave function. The internal radiative width amplitude is fixed because the experimental (total) radiative width for the 2\(^+\) (6.92 MeV) \(\rightarrow\) 0\(^+\) (6.05 MeV) transition is known, the external radiative width is determined by the ANC and the three values are related as \(\Gamma_\gamma = |\Gamma_{\text{int}}^{1/2} \pm \Gamma_{\text{ext}}^{1/2}|^2\). The choice of negative sign in this relation would lead to the internal radiative width amplitude that is always greater than the external one regardless of the choice of channel radius and that contradicts to the physical picture. If channel radius is chosen to be sufficiently small the external amplitude should become dominant. For this specific example, the ratio of internal to external amplitudes at the channel radius 5.2 fm is 4.45 for positive sign and 6.45 for negative sign, but it becomes 0.33 for positive sign at channel radius 4 fm and remains large (2.33) for the case of negative sign. The corresponding S-factor is almost independent of the channel radius, but
it depends strongly on the choice of the interference sign. From the considerations above we can reject the negative sign between the internal and external radiative widths amplitudes as unphysical. The positive sign leads to destructive interference between non-resonant and resonance capture and accounts for 39\% reduction of the S-factor for transition to the 0\(^+\) state at 6.05 MeV. We find \(S(300)=1.9\pm0.3\) keV b for the 0\(^+\) cascade transition. Similar considerations lead to destructive interference assignment between the direct capture to the 3\(^-\) state and capture through the sub-threshold 1\(^-\) state at 7.2 MeV. This reduces the 3\(^-\) cascade transition S-factor by 60\%. Then, the contribution of the 3\(^-\) state cascade transition to the S-factor is \(S(300)=0.5\pm0.09\) keV b. Fig. 4 shows the contribution to the S-factor for the 0\(^+\) cascade transition without interference and interference with the 2\(^+\) at 6.92 MeV, and for the 3\(^-\) cascade transition without interference and interference with the 1\(^-\) state at 7.12 MeV.

![S-factor graph](image)

**FIG. 4.** S-factor of the \(^{12}\text{C}(\alpha, \gamma)^{16}\text{O}\) reaction for the direct \(E2\) transition to the sub-threshold 0\(^+\) at 6.05 MeV without interference (dashed-dotted line) and interference with the 2\(^+\) at 6.92 MeV (dashed-double-dotted line), and for the 3\(^-\) at 6.13 MeV without interference (dashed line) and interference with the 1\(^-\) state at 7.12 MeV (dotted line) in \(^{16}\text{O}\).

This result is in obvious disagreement with [4] where it was found that the 0\(^+\) cascade transition contributes 25 keV b to the total S-factor. That experiment assumed \(E1\) as a dominant component for the transition. Angular distribution of \(\gamma\)-rays was measured in Ref. [6] and it was shown that in fact \(E2\) dominates. We find that contribution from the 0\(^+\) cascade transition is one order of magnitude smaller. However, we also disagree with the
results of [7], where the $0^+$ cascade transition was found to contribute only 0.3 keV. The origin of this disagreement is easy to point out. It turned out that the cross section for the $0^+$ cascade transition at 300 keV is dominated by a single parameter - the ANC for the $0^+$ state at 6.05 MeV. This is largely due to the $\alpha$-cluster nature of this state. Based on the measured ANC (and assuming a channel radius of 5.2 fm), this state has an $\alpha + ^{12}\text{C}(\text{g.s.})$ spectroscopic factor of around 40%. This is not surprising, since this state has long been identified as a bandhead of an $\alpha$-cluster inversion doublet quasi-rotational band [26]. The $\alpha$ reduced width calculated from the measured ANC (using equations provided in Ref. [21]) is $\gamma_{6.05} = 0.48 \pm 0.08 \text{ MeV}^{1/2}$. The reduced width amplitude used in Refs. [6, 7] for this state was $\gamma_{6.05} = 0.01\pm 0.05 \text{ MeV}^{1/2}$. This accounts for one order of magnitude difference in the S-factor. Such small value was based on the $^{12}\text{C}+\alpha$ elastic scattering data of [27]. However, a recent extensive R-matrix analysis of all available $^{16}\text{O}$ compound nucleus reactions has been carried out in Ref. [28], including the elastic scattering data from [27], and the small $\alpha$ reduced width amplitude for the 6.05 MeV state was not confirmed. On the contrary, the ANCs for the $0^+$ and $3^-$ suggested in [28] ($3.2 \times 10^6 \text{ fm}^{-1}$ and $2.3 \times 10^4 \text{ fm}^{-1}$, no uncertainties are given) are in surprisingly good agreement with the direct measurements reported here.

In summary, we have investigated the important astrophysical reaction $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ through the $\alpha$-transfer reaction $^{12}\text{C}(^6\text{Li},d)^{16}\text{O}$ at sub-Coulomb energies. The ANCs for the sub-threshold states $0^+$ (6.05 MeV), $3^-$ (6.13 MeV), $2^+$ (6.92 MeV) and $1^-$ (7.12 MeV) in $^{16}\text{O}$ have been determined. The extracted ANCs for the $2^+$ and $1^-$ states are in very good agreement with previous measurements [10, 14, 15]. The ANCs of the $0^+$ (6.05 MeV) and $3^-$ (6.13 MeV) states were directly measured for the first time. The uncertainties related to the contribution of the $0^+$ (6.05 MeV) and $3^-$ (6.13 MeV) cascade transitions to the total S-factor at energies near 300 keV are now eliminated. The cascade transitions to the $0^+$ and $3^-$ states were found to be determined by the interference of the $E2$ direct capture amplitude with the amplitude for the sub-threshold resonance capture through the $2^+$ state at 6.92 MeV and $1^-$ state at 7.12 MeV respectively. The ANCs of the corresponding states, all of which were measured in this work, determine both the direct capture amplitude and the interference sign. This allowed us to eliminate uncertainties related to the corresponding transitions. The combined contribution of these cascade transitions does not exceed 2% of the total $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ S-factor.

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