Stable Solutions of the Double Compactified D=11 Supermembrane Dual

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Abstract

The hamiltonian formulation of the supersymmetric closed 2-brane dual to the double compactified D=11 closed supermembrane is presented. The formulation is in terms of two $U(1)$ vector fields related by the area preserving constraint of the SUSY 2-brane. Stable solutions of the field equations, which are local minima of the hamiltonian, are found. In the semiclassical approximation around the stable solutions the action becomes the reduction of D=10 Super-Maxwell to the worldvolume. The solutions carry RR charges as a type of magnetic charges associated with the worldvolume vector field. The geometrical interpretation of the solution in terms of $U(1)$ line bundles over the worldvolume is obtained.
The interest in D=11 supermembranes [1] has been renewed by the realization that 11-dimensional supergravity may describe the long distance behaviour of M-theory [2, 3, 4, 5]. The spectrum of the supermembrane in D=11 Minkowski target space has been shown to be continuous when the SU(N) regularization is used [6]. The analysis of the spectrum in the case of a compactified target space has not been completed. Meanwhile, the search for massless states, corresponding to the states of 11-dimensional supergravity, still continues [3, 7].

In this work we analyze the closed $D = 11$ supermembrane in a target space $M_9 \times S^1 \times S^1$ [8]. We consider the dual formulation in terms of two vector fields. We perform the complete canonical analysis, determine its physical hamiltonian and found its minimal configurations. We show that the semiclassical approximation of the action around the minimal configurations describes the reduction of $D = 10$ Super-Maxwell theory to the world volume, showing that the configurations are stable ones. The minimal configurations carry a type of magnetic charge that may be interpreted as a RR charge which couples to the Kaluza-Klein vector field.

The 2-brane action in the form which includes an independent auxiliary metric may be expressed as,

$$S(\gamma, X) = -\frac{1}{2} \int d^3 \xi \sqrt{-\gamma} \left( \gamma^{ij} \partial_i X^\mu \partial_j X^\nu \eta_{\mu\nu} - 1 \right)$$

(1)

where $i, j$ denote the world volume indices while $\mu, \nu$ denote the 11 dimensional indices. $X^\mu$ define maps from the 3-dimensional world volume $M_3$, which will be assumed to be $M_3 = \Sigma \times R$ where $\Sigma$ is a compact Riemann surface of genus g, to the 11-dimensional space-time which is assumed to be $M_{11-q} \times [S^1]^q$ where $q$ is the number of compactified coordinates. Each compactified coordinates $X$ is a map from $\Sigma \times R$ to $S^1$. It then satisfies

$$\int_c dX = 2\pi q_c$$

(2)

where $c$ is a basis of homology of dimension 1 elements, while $q_c$ is an integral number associated to each element of the basis. The set of $q_c$ for a basis of homology defines the winding of the maps over $S^1$.

We will be interested in particular in the case $q \geq 2$ where the situation of irreducible winding may arise [8].
The dual formulation to (1) may be obtained by considering

\[ S = -\frac{1}{2} \int_{M_3} d^3\xi \sqrt{-\gamma} \left( \gamma^{ij} \gamma^{kl} F^r_{ik} F^r_{jl} - 1 \right) \]

where \( r, s = 1, ..., \hat{q} \) and \( m = 0, ..., (10 - \hat{q}) \). \( A^r_k \) are \( \hat{q} \) connection 1-forms which locally ensure that \( L^r_i \) satisfy the right constraints in order to recover (1), where \( \hat{q} \leq q \), that is not all compactified directions need be dualized. To implement the global condition (2) one has to impose the condition

\[ \oint_{M_2} F(A^r) = 2\pi p^r \]

where \( M_2 \) is a basis of homology of dimension 2 and summation over all \( p^r \) must be performed [9]. The equivalence between formulation (1) and (3) is ensured provided summation in all winding is consider in (1) and summation in all \( p^r \) is taken in (3) [9], [10]; although this is strictly true for a compact euclidean world volume.

The geometrical interpretation of (4) is given by the Weil theorem [11] which ensures the existence of a U(1) principle bundle over \( M_3 \) (compact, euclidean) and a connection 1-form over it whose curvature is identical to \( F(A^r) \).

Functional integration over \( L^r \) leads to the action

\[ S(\gamma, X, A) = -\frac{1}{2} \int d^3\xi \sqrt{-\gamma} \left( \gamma^{ij} \gamma^{kl} F^r_{ik} F^r_{jl} + \frac{1}{2} \gamma^{ij} \gamma^{kl} F^r_{ik} F^r_{jl} - 1 \right) \] (5)

The action (3) for \( q = 1 \) and a gauge vector field corresponding to a trivial line bundle was first obtained in [3] following previous work in [12] and [13].

In order to analyze (3), one has to distinguish the case of an open membrane from the closed one. In the former, a solution to the field equations arises by considering the time and two spatial coordinates in the target space to coincide with the local coordinates on the worldvolume. The semiclassical approximation of the action around this solution yields the reduction of D=10 Maxwell action to the worldvolume. The situation for the case of a closed membrane is different. A solution where the time and two spatial coordinates in the target space coincide with the coordinates on the worldvolume requires three compactified dimensions of the target, two of them identified with the worldvolume spatial coordinates and the third one dualized to achieve a closed membrane with a U(1) gauge vector. There is however
another interesting scenario, which will be the point to be analyzed in this paper, where \( \hat{q} = q = 2 \). We will show that there are stable solutions to the field equations arising from the sector of the two U(1) gauge vectors, and not from the metric on the worldvolume. The solutions are characterized by a quantized type of magnetic charge in distinction to the above discussed solutions. Furthermore the action around that solution corresponds to the reduction of D = 10 Super-Maxwell to the worldvolume. It is interesting to mention that in the case \( q = 0, 1 \) the solutions corresponding to the minima of the hamiltonian are unstable. It is required at least two compactified directions, \( q = 2 \), in order to have stable solutions [3].

If the metric \( \gamma^{ij} \) is eliminated by using perturbative field equations, it was shown in [3] for \( \hat{q} = q = 1 \), that the Born-Infeld theory is obtained. See also [4].

To obtain the hamiltonian version of the theory we consider, in the usual way, the ADM decomposition of the metric

\[
\begin{align*}
\gamma_{ab} &= \beta_{ab} \\
\gamma^{0a} &= N^a N^{-2} \\
\gamma^{ab} &= \beta^{ab} - N^a N^b N^{-2} \\
\gamma^{00} &= -N^{-2}
\end{align*}
\]

(6)

where

\[
\beta^{ab} \beta_{bc} = \delta^a_e
\]

(7)

and

\[
\sqrt{-\gamma} = N \sqrt{\beta}
\]

(8)

a,b are the 2 spatial indices.

The canonical action may then be rewritten as

\[
S = \int_{M_3} d^3 \xi \left( P_m \dot{X}^m + \Pi^a_r \dot{A}_a^r - \mathcal{H} \right)
\]

(9)

where the hamiltonian density may be expressed in the following way

\[
\mathcal{H} = \frac{N}{2} \left( \frac{P_m P_m}{\sqrt{\beta}} + \frac{\beta^{ab} \Pi^a_r \Pi^b_r}{\sqrt{\beta}} + \sqrt{\beta} \beta^{ac} \beta^{bd} F_{ac}^r F_{bd}^r \right)
\]

\[
+ \Pi^a_r \partial_a A^0_r + N^a \left( \partial_a X^m P_m + \Pi^b_r F_{ab}^r \right)
\]

(10)
where $N$ and $N^a$ are Lagrange multipliers and $\beta_{ab}$ must satisfy

$$\beta_{ab} = \left(1 + \frac{1}{2}F^r F^r - \beta^{-1}\Pi_r \Pi_r \right)^{-1} \left(\partial_a X^m \partial_b X_m + \beta^{cd} F^r_{ac} F^r_{bd} - \beta^{-1}\Pi_{ra} \Pi_{rb} \right)$$

(11)

where $\beta$ is the determinant of the matrix $\beta_{ab}$. $F^r F^r$ and $\Pi_r \Pi_r$ are contracted with the metric $\beta_{ab}$.

We now consider the light-cone gauge fixing condition:

$$X^+ = P^+_0 T \quad P^+ = P^+_0 \sqrt{W}$$

(12)

where $T$ is the time coordinate on the world volume and $W$ the determinant of the metric over $\Sigma$. Some authors take $W$ to be 1, however this condition cannot be imposed globally over any compact riemann surface. The metric over $\Sigma$ only appears in the action through its determinant.

In the usual way one obtains

$$\partial_a X^- = -\frac{\partial_a X^M P_M - \Pi^b_{ra} F^r_{ab}}{P_-}$$

(13)

giving rise to the integrability constraint

$$\epsilon^{ab} \partial_b \left[ \frac{\partial_a X^M P_M + \Pi^c_{ra} F^r_{ac}}{\sqrt{W}} \right] = 0$$

(14)

$M$ denote the transverse indices.

(14) is a necessary integrability condition for $X^-$. In addition there are also global conditions on the right hand side member of (13) in order to have a well defined solution for $X^-$. They are

$$\oint_c \left( \frac{\partial_a X^M P_M + \Pi^b_{ra} F^r_{ab}}{P_-} \right) d\xi^a = 2\pi n_c$$

(15)

where $c$ is a basis of homology of dimension one. $P^-$ is obtained algebraically from the constraint associated to the lagrange multiplier $N$.

The final form of the physical hamiltonian density is then

$$\mathcal{H} = \frac{1}{2} \frac{1}{\sqrt{W}} \left( P^M P_M + \beta + \frac{1}{2} \beta F^r F^r \right) - A^r_0 \partial_r \Pi^a + \Lambda \epsilon^{ab} \partial_b \left( \frac{\partial_a X^M P_M + \Pi^c F^r_{ac}}{\sqrt{W}} \right)$$

(16)
where the first class constraint (14) has been left as a restriction to the phase space variables. Its associated gauge symmetry has not been fixed by the LC gauge condition, as occurs in the canonical analysis for the membrane theory. The first class constraint generates the algebra of the area preserving diffeomorphisms in the extended phase space given by $X, P$ and $A, \Pi$. The matrix $\beta_{ab}$ is an auxiliary variable satisfying (11)

Any 2x2 invertible matrix $\beta_{ab}$ satisfies

$$\beta^{cd} \epsilon_{ac} \epsilon_{bd} = \beta^{-1} \beta_{ab}$$

consequently (11) may be simplify to

$$\beta_{ab} = \left(1 - \beta^{-1}\Pi_r \Pi_r\right)^{-1} \left(\partial_a X^M \partial_b X^M - \beta^{-1}\Pi_{ra} \Pi_{rb}\right)$$

that is $\beta_{ab}$ may be expressed as a function of $X^M$ and $\Pi_{ra}$ only. We now observe that the contraction

$$F_r F_r = F_{ab} F_{cd} \beta^{ac} \beta^{bd}$$

may be rewritten as

$$F_r F_r = \frac{1}{2} \beta^{-1} W (\ast F_r)^2$$

where

$$\ast F_r = \frac{e_{ab}}{\sqrt{W}} F_{ab}$$

The first parenthesis of the hamiltonian density in (16) may then be expressed as

$$\frac{1}{2} \frac{1}{\sqrt{W}} \left(P_M P_M + \beta\right) + \frac{1}{8} \sqrt{W} (\ast F_r)^2$$

showing that the terms $\frac{1}{2} \frac{1}{\sqrt{W}} P_M P_M + \frac{1}{8} \sqrt{W} (\ast F_r)^2$ formally decouple from $\frac{1}{2} \frac{1}{\sqrt{W}} \beta$ which depends on $X$ and $\Pi$ only. However $X^M, P_M, \ast F$ and $\Pi_r$ are related by the area preserving constraint.

We will now study the local minimal configurations of the hamiltonian (16). We will be in particular interested in the stable local minima. To determine such configurations we need to introduce a global geometric condition in the phase space. The straightforward minima of the hamiltonian arise over field configurations which are infinite dimensional and are unstable, i.e. when $\Pi^a_r = 0$ we have infinite valleys configurations as in the case of the supermembrane.
The minimal configurations are obtained for $\hat{P}$ and $\hat{A}$ satisfying

$$\hat{P}_M = 0 \quad (23)$$

$$d^* \hat{F}^r = 0 \quad (24)$$

and

$$\hat{\Lambda} = 0 \quad (25)$$

(24) imply

$$^*\hat{F}^r = cte \quad (26)$$

When the area of $\Sigma$ is normalized to $2\pi$, the minimal configuration are obtained for

$$^*\hat{F}^r = m^r \quad (27)$$

where $m^r$ are integral numbers, characterizing the $U(1)$ principle bundle associated to each connection 1-forms $A^r$

We will consider the configurations for $\Pi_r^a$, denoted by $\hat{\Pi}_r^a$, satisfying

$$\hat{\Pi}_r^a\hat{\Pi}_r^b \epsilon_{ab} \epsilon^{rs} = n \sqrt{W} \quad n \neq 0 \quad (28)$$

and the transverse coordinates

$$\hat{X}^M = 0 \quad M = 1, ..., 7. \quad (29)$$

For this particular configuration we obtain from (18):

$$\hat{\beta}^{ab} = \left(1 - \hat{\beta}^{-1} \hat{\Pi}_r^a \hat{\Pi}_r^b \right)^{-1} \left(-\hat{\beta}^{-1} \hat{\Pi}_r^a \hat{\Pi}_r^b \right) \quad (30)$$

consequently

$$\hat{\beta}^{-1} \hat{\Pi}_r^a \hat{\Pi}_r^b = 2 \quad (31)$$

and then

$$\hat{\beta}^{ab} = \hat{\beta}^{-1} \hat{\Pi}_r^a \hat{\Pi}_r^b \quad (32)$$

where, using (28)

$$\hat{\beta} = \det \left(\hat{\Pi}_r^a \hat{\Pi}_r^b \right) = \frac{1}{2} \hat{\Pi}_r^a \hat{\Pi}_r^b \hat{\Pi}_s^c \hat{\Pi}_s^d \epsilon_{ac} \epsilon_{bd} = \frac{n^2}{4} W \quad (33)$$
We will now consider the stability around these solutions. We take
\[ \Pi^a_r = \hat{\Pi}^a_r + \delta \Pi^a_r \]  
and expand \( \beta \) up to second terms in \( \delta X^M \) and \( \delta \Pi^a_r \). We then obtain from \( (18) \)
\[ 1 - \beta^{-1} \Pi_r \Pi_r = -1 + \partial X^M \partial X_M = -1 + \partial \delta X^M \partial \delta X_M \]  
which may then be substituted into \( (18) \) to get explicit expressions for the variations of \( \beta \). The terms linear in the variations are
\[ 2 \left( \hat{\Pi}^a_r \hat{\Pi}^d_r \epsilon_{bd} \right) \left( \delta \Pi^c_s \hat{\Pi}^c_s \epsilon_{ac} - \delta \Pi^c_s \hat{\Pi}^c_r \epsilon_{ac} \right) \]  
the first factor being proportional to \( \sqrt{W} \) from \( (28) \). The other factor may be expressed as a total derivative and integrated out to give zero contribution. We conclude then that \( (28), (29) \) is a stationary configuration of \( (22) \).

The terms quadratic in the variations are
\[ \beta = \hat{\beta} + \beta \left( \partial_a \delta X^M \partial_b \delta X_M \hat{\beta}^{ab} \right) + \frac{1}{2} \left( \delta \Pi^c_s \hat{\Pi}^c_s \epsilon_{ac} - \delta \Pi^c_s \hat{\Pi}^c_r \epsilon_{ac} \right)^2 + O(\delta^3) \]  
showing that \( (28), (29) \) is indeed a minimal configuration of the Hamiltonian.

We have, because of the constraint on the momenta,
\[ \delta \Pi^a_r = \epsilon^{ad} \partial_d \delta \Pi_r, \]  
then
\[ \delta \Pi^a_r \hat{\Pi}^c_s \epsilon_{ac} \epsilon^{rs} = \partial_c \delta \Pi_r \hat{\Pi}^c_s \epsilon^{rs} = \epsilon^{cb} \partial_c \delta \Pi_r \partial_b \hat{\Pi}^c_s \epsilon^{rs} \]  
It is convenient to define
\[ A_b = \delta \Pi_r \partial_b \hat{\Pi}^c_s \epsilon^{rs} \]  
\[ F_{cb} = \partial_c A_b - \partial_b A_c, \]  
\[ *F = \frac{\epsilon^{cb}}{\sqrt{W}} F_{cb} \]  
In terms of them \( (37) \) may be rewritten as
\[ \beta = \hat{\beta} + \beta \left( \partial_a \delta X^M \partial_b \delta X_M \hat{\beta}^{ab} \right) + \frac{W}{4} (\ast F)^2 + O(\delta^3) \]
We then conclude that
\[ \langle \beta \sqrt{W} \rangle = \langle \hat{\beta} \sqrt{W} + O(\delta^3) \rangle \] (43)
if and only if
\[ \partial_a \delta X^M = 0 \]
\[ F_{cb} = 0 \] (44)
i.e. the minimal configurations are strict up to closed 1-forms. It is interesting that the interpretation of \( \mathcal{A}_b \) as a 1-form connection may be extended to consider \( \tilde{\Pi}^b_{r} F^r \) as its conjugate momenta.

In fact, the kinetic term in the action
\[ \langle \Pi^a_{r} \dot{A}^r_a \rangle \] (45)
may be rewritten as
\[ \langle \Pi^a_{r} \hat{A}^r_a \rangle = \langle \hat{A}_b \Pi^b \rangle \] (46)
where
\[ \Pi^b = \frac{1}{n} \tilde{\Pi}^b_{r} F^r \] (47)

The area preserving generator, the first class constraint in (16) may then be rewritten as
\[ \partial_b \Pi^b + O(\delta^2) = 0, \] (48)
This is, up to second order in the variations in the action, the Gauss constraint on the conjugate momentum to \( \mathcal{A} \).

The hamiltonian (16), in the semiclassical approximation, becomes then
\[ \mathcal{H} = \dot{\mathcal{H}} + \frac{1}{2} \hat{\mathcal{H}} \left( P_M P^M + \frac{n}{2} \Pi^a \Pi^b W_{ab} \right) + \]
\[ \frac{1}{4} \sqrt{W} \left( \mathcal{F}_{ab} \mathcal{F}_{cd} W^{ac} W^{bd} + n \partial_a \delta X^M \partial_b \delta X_M W^{ab} \right) \] (49)
where
\[ W_{ab} = \frac{2}{n} \hat{\beta}_{ab} \quad detW_{ab} = W \] (50)
and
\[ \hat{\mathcal{H}} = \frac{1}{8} \sqrt{W} \left( [m^r]^2 + n^2 \right) \] (51)
We thus conclude that the Hamiltonian, in the semiclassical approximation around the minimal configurations we found, exactly describes the 10 dimensional Maxwell theory dimensionally reduced to the world volume of the 2-brane. $X^M M = 1, \ldots, 7$ represent the transverse coordinates to the 2-brane, while $A_b$ and $\Pi^b$ describe the Maxwell theory over the world volume. This field content corresponds to a 2-Dbrane in 10 dimensions [17], [18]. We notice that in order to obtain this description the metric in the world volume is however completely specified. It is given by (50) in terms of the minimal configuration. The metric (50) is precisely the canonical metric which naturally appears associated to the monopole configurations [15].

This structure of the Hamiltonian in the quadratic approximation ensures that the local minima are stable solutions to the field equations. In distinction to the case $n = 0$, in which the minimal configurations expand an infinite dimensional subspace, as in the D=11 Supermembrane theory over Minkowski space time.

Following (40) we may introduce

$$\hat{A}_b = \frac{1}{2} \hat{\Pi}_r \partial_b \hat{\Pi}_s \epsilon^{rs} \quad (52)$$

and interpret (28) as

$$\ast \hat{\mathcal{F}} = n \quad (53)$$

These solutions describe then monopole connections over the Riemann surface $\Sigma$, generalizing the Dirac monopole over $S^2$. The integer $n$ in (53) classifies all non-trivial $U(1)$ line bundles over $\Sigma$. The general explicit expression for the gauge vectors was obtained in [17] in terms of $U(1)$ connections 1-forms with non-trivial transitions over $\Sigma$. See also [16]. A subset of these solutions are dual to the ones obtained in [8], the interpretation is now natural in terms of the $U(1)$ gauge vector field, even when the global aspects of dualization are strictly valid on compact worldvolumes.

The generalization of the above result to the super 2-brane in the semiclassical approximation arises in the same way. Instead of (40), we may consider the supermembrane action [1], and perform the double dualization procedure as in the pure bosonic case.

The light cone gauge may be imposed by taking

$$X^+ = P_0^+ \mathcal{T}$$

$$P^+ = P^+_0 \sqrt{W}$$

$$\Gamma^+ \psi = 0 \quad (54)$$

10
We end up with the hamiltonian density

$$\mathcal{H} = \frac{1}{2} \frac{1}{\sqrt{W}} \left( P_M P^M + \beta + \frac{1}{2} \beta^{ac} \beta^{bd} F_{ab}^r F_{cd}^r \right) + \Pi^a_\alpha \partial_a A_0^\alpha + \Lambda \Phi$$

$$-\epsilon^{ba} \bar{\psi} \Gamma_M \partial_a \psi \partial_b X^M - \bar{\psi} \Gamma^r \partial_b \psi \Pi^b_r$$

(55)

\(\beta_{ab}\) satisfies the same constraint (53) and the generator of the area preserving diffeomorphisms \(\Phi\) has the expression

$$\Phi = \epsilon^{ab} \left( \partial_b \bar{\psi} \Gamma_\alpha \partial_a \psi + \frac{1}{P_0^+} \partial_b \left[ \frac{\partial_a X^M P_M + F_{ac}^r \Pi^c_r}{\sqrt{W}} \right] \right)$$

(56)

To quadratic order the only new contribution to \(\mathcal{H}\) with respect to (19) is given by

$$-\bar{\psi} \Gamma^r \partial_b \psi \hat{\Pi}^b_r$$

(57)

The worldvolume canonical action arising from (55) may be analyzed as in the bosonic case. The result is that the configuration (28) together with \(\psi = 0\), describe the minimal configurations of (55). The semiclassical approximation around that minima describe the D=10 Maxwell supermultiplet dimensionally reduced to three dimensions, in terms of \(X^M, M = 1, ..., 7\) scalar fields (the transverse coordinates to the world volume of the 2D-brane), the world volume vector potential \(A\) and eight \(Sl(2,R)\) spinors arising from the Majorana spinors \(\psi\) satisfying the LCG condition (54).

The main property of the above minimal configurations is that they describe isolated local minima (up to closed 1-forms over the Riemann surface \(\Sigma\)) of the hamiltonian. That is, they are locally stable solutions. This property distinguish the double compactified closed D=11 supermembrane from the single compactified closed one as discussed in [8].

The results in this paper extend then the construction obtained in [8] for the bosonic membrane to the supermembrane case. The interpretation of the solution in terms of the \(U(1)\) vector field of the 2D-brane becomes then natural. In fact, the solution carry a type of magnetic charge given by

$$Q = \int_{\Sigma} \mathcal{F} = \int d^2 \xi \sqrt{W} * \mathcal{F} = Vol \Sigma \cdot n$$

(58)

The topological coupling of the \(U(1)\) vector field on the worldvolume to the RR 1-form connection \(B\) of the IIA superstring is naturally given by

$$\int_{M_3 = \Sigma \times R} \mathcal{F} \wedge B$$

(59)
We may then interpret $Q$ as the RR charge carried by the closed 2D-brane.

In [3] it was argued how a classical closed membrane configuration could be identified with a 0-brane which is needed to describe the dynamics of the centre of mass motion of the supermembrane. Moreover, it was argued that a closed membrane may carry the 0-brane RR charge as a type of magnetic charge associated with its world volume vector field, and that its centre of mass motion should be described by the 0-brane $U(1)$ supersymmetric quantum gauge mechanics. This was interpreted as an evidence that the 0-brane is included in the supermembrane spectrum and hence that massless states are included in it.

One further requirement should be added to the argument in [3]. The solutions carrying the charge $Q$ should be stable solutions of the membrane field equations. The solutions we found have precisely the required stability property.

We have thus, constructed the physical hamiltonian of the supersymmetric closed 2-brane dual to the double compactified D=11 closed supermembrane with target space $M_9\times S^1\times S^1$. The formulation is originally in terms of two $U(1)$ gauge fields over the worldvolume, related by the area preserving constraint. The target space being $M_9$. We found the stable minimal configurations of the hamiltonian and showed that the semiclassical approximation of the action around those minimal configurations correspond to the reduction of 10 dimensional Super Maxwell to the worldvolume of the 2-brane. In that approximation, the original formulation in terms of two gauge field is equivalent to a theory in terms of only one $U(1)$ Maxwell potential, the area preserving constraint being the Gauss constraint for it.

The minimal configurations are classified by three integer $m^r, r = 1, 2, \text{ and } n$. The integer $n$ was interpreted as a RR charge carried by the 2-brane and corresponds geometrically to the integral number classifying the principle bundles over which the $U(1)$ Maxwell connection is defined. When $m^r = 0, r = 1, 2$, we then have from (13) and (29)

$$
\partial_a \hat{X}^- = 0, \quad \partial_a \hat{X}^M = 0
$$

that is, there is a solutions $\hat{X}^- = 0, \hat{X}^M = 0$. This solution may be interpreted as a massless 0-brane moving in the light cone of $M_9$ carrying a quantized RR charge. When $M^r \neq 0$, it follows from (13) that $\hat{X}^-$ is proportional to $m^r\Pi_r$. The target space time coordinate $X^0$ becomes then a multivalued function over the worldvolume.
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