Academic performance and mathematical processes under a causal model. Correlations found in three public institutions

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Abstract. This research implemented a model of structural equations aimed at determining the possible causal relationship between the mathematical processes mentioned in the national teachers council mathematics and academic performance in mathematics. The level of development of the mathematical processes promoted by the teacher was analyzed, in students of the fourth to seventh grades of basic education in three public educational institutions of the San José de Cúcuta, Colombia, characterized by obtaining levels of performance (respectively) in the Saber 11 tests of 2018. The sample size was 400 students enrolled in 2019. A questionnaire was proposed and validated that included sociodemographic information of students and then determined the presence and development of the various mathematical processes in classroom work. The results highlight that, of the five mathematical processes, reasoning, representation and problem solving are presented as determining variables of academic performance in students, while the processes of communication and connections are little work in the classroom by teachers. From the results obtained arises as future research, determine what teachers understand by mathematical problems? How do you promote problem solving in the classroom?

1. Introduction

The technological advances experienced in the last three decades by our society, has generated a snowball effect that has permeated all aspects of human performance, to as stated by García Moreno" the dependence by technology within our society is reflected in the fact that we live in a medium modelled by it and in which it sets the standards of social life and the rhythm of progress" [1], thus showing that the educational process is no stranger to this condition of influence.

Today's society requires education a new teaching proposal, which considers the motivations, needs and expectations of students in their curriculum planning process. An education that responds to the needs of the new millennium characterized by constant change, by the trend towards the globalization of economies and societies, in line with the quality of education that should be delivered in the various schools regardless of their public or private character.

In this regard and from the education sciences a wide number of research has been led, some focused on students seeking to identify learning difficulties, while others have focused on teachers and in some aspects associated with the teaching process they carry out, such as their pedagogical practices, teaching processes and their evaluative practices; finally, a small number of them, have considered the regulations that regulate the educational process in Colombia and how efficient it could become in the pedagogical exercise that is ahead of the interior of the institutions.
Among the characterization of research mentioned, many are highlighted in which the memoristic process that has dominated the teaching processes is criticized, regardless of the level of schooling that is considered. It is suggested from many of these works, which must be educated for life, in order to ensure true learning of knowledge, where the student must demonstrate in his daily life the field of application of those knowledges addressed in school.

For the particular case of the study of mathematics, various researches are mentioned in the works of [2–5], among many others; they demonstrate the excessive process of instrumentalization of mathematical concepts, which have led to a reductionist practice of mathematical processes and that in the end produce undesired results that do not go directly to quantity, each increasing research in the field of mathematics education [6].

For educational institutions, [7] suggests for educational institutions the development of a more rigorous, focused and coherent curriculum where the teaching and evaluation of mathematics promotes understanding mathematical reasoning, as well as fluency in mathematical skills. Only in this way could you help ensure that everyone, or at least a good number of students, are ready for college or the job market once their vocational middle education training cycle is completed, for Colombia.

To ensure compliance with this purpose, the [7] proposes five mathematical processes that must be developed in the student as part of the pedagogical process of training during its cycle in the educational system:

- **Problem solving.** It is expected that the student from the concepts he already possesses, will begin to build new mathematical knowledge but generated from the solution of problem situations presented in various problems presented in various application contexts.
- **Reasoning and testing.** This process requires students to investigate, propose and evaluate various conjectures in such a way that the results obtained propose logical arguments supported by various types of reasoning.
- **Communications.** Since the human being is a sociable being then the student in the classroom is required to develop appropriate communication processes both informally and from the math’s point of view; connections. The student is expected to understand how mathematical ideas relate and articulate in a whole in a coherent way, while being applied in various non-mathematical contexts as it is from their daily perspective or from the other areas of knowledge that are part of their school curriculum; representations. Support the processes of modelling and interpretation of phenomena in various fields such as the field of application of mathematics, using the use and transformation of various records of semiotic representation in order to offer the student sufficient tools for mathematical argumentation and decision-making.

As evidenced to obtain better results in learning mathematics, changes in the way we teach, in the teaching resources implemented in the classroom and in evaluative practices, among many other aspects, could only be the case ensure that the quality of mathematics education is improved to meet society's demand for problem-solving [8–10].

2. **Method**

2.1. **Focus and design**

The research carried out is part of the mixed approach in which two stages are developed, starting with the quantitative phase in which a series of correlations are determined that allow to arrive at the construction of a causal model based on the technique called structural equations and that is articulated in a way consistent with some research work that precedes this process; subsequently progress to the qualitative phase in which the qualitative phase is made by the conducting a series of interviews is intended to carry out the process of validating the proposed model.

The design of the research conforms to the characteristics of the non-experimental cross-section in which three phases distributed sequentially are distinguished, starting from a descriptive report to
advance the modelling of relationships and correlations, to close with the analysis of the relevance of the model built.

2.2. Population and sample
The population was made up of all students enrolled by 2019 in three public educational institutions located in the city of San José de Cúcuta, Colombia. The three educational institutions offer education to students from the first grade of primary basic education to the eleven degree in vocational middle education. The three institutions work in a two-time manner and serve students from socioeconomic strata between 1 and 3, with a wide range of options in terms of home composition, economic activities, access to professional and technological resources that support their academic performance, among many other important aspects. For the selection of the sample, non-probabilistic sampling was used under the intentional sampling technique, complying as with the following selection criteria:

(i) being enrolled and actively participating throughout the year 2019 in one of the selected educational institutions;
(ii) be studying between the fourth and seventh grades;
(iii) have parental consent. Based on the fulfillment of these selection criteria, a sample size of 400 students were consolidated.

2.3. Instrument for data collection
The survey was used as a data collection technique in this investigation. The instrument consisted of two sections distributed as follows: in the first section some descriptive were incorporated both of the students (age, grade, gender, taste for mathematics and note obtained in the last academic period) and of the institution (type of institution and geographical location); later in the second section, 37 single-response multiple-selection items were incorporated, with a Likert scale with five levels (two negative perception, two positive perception and one intermediate option).

These items analyzed the various aspects associated with each of the five mathematical processes defined by the [7] thus distributed:
- Problem solving 7 items.
- Reasoning and testing with 8 items.
- Communication with 9 items.
- Representation with 7 items.
- Connections with 8 items.

The instrument was initially applied to a group of 40 students from another institution with characteristics similar to those of the sample institutions. From this pilot application is derived the reliability report which it produced at the general level of the whole instrument, a value of 0.887 in the coefficient Alpha of Cronbach which according to [11] is admissible as valid in the instrument. Reliability values for each of the mathematical processes are shown in Table 1.

| Scale              | Reliability Value |
|--------------------|-------------------|
| Problem solving    | 0.689             |
| Reasoning and testing | 0.778          |
| Communication      | 0.800             |
| Representation     | 0.785             |
| Connections        | 0.801             |
| General            | 0.887             |

2.4. Data collection process
Following the design and validation of the instrument, the approach to the educational institutions in which the research would be advanced, for which the director of each school was contacted, the project was presented, the scopes and pursued objectives. Subsequently, the parents of each student were asked
to have informed consent that their children were informants of the investigation, for which they were given an office explaining what was intended to be done together with the authorization. This process was carried out for two weeks and thus the group of informants for the investigation was completed.

2.5. Data processing and analysis

SPSS software was used to contrast hypotheses and determine correlations between each of the mathematical processes, sociodemographic variables and academic performance of students in the area of mathematics. For the degree of association (correlation coefficient) between the variables under study, the Spearman Rho coefficient was used which is used to determine the independence or dependence of two random variables [12]. Spearman's Rho correlation coefficient is the coefficient used when the variables are ordinal and/or the presumption of normality is not met.

This coefficient is very useful when the number of pairs of associated subjects is less than 30, but it is also useful with large sample sizes (as is our case). Its value ranges in the range of $-1$ to $1$. The value of zero indicates that there is no correlation between the analyzed variables, while a value of implies a perfect inverse correlation between the variables and the full evidence value direct correlation between them $1$ [13].

3. Results

The description of the characteristics of the sample begins with the determination of a slight predominance of the male gender in 51% of the sample members. The average age was 10.7 years with a minimum of 8 years and a maximum of 13 years. With respect to the total sample size, its distribution by degree was 20% in Fourth, 28% in Quinto, 30% in Sixth and 22% in Seventh. 68% of students say they like math and the average grade earned in the last period was 6.8 on a scale of 0 to 10. Regarding the location of the institution, two of them are classified as urban and one is rural.

Of the five mathematical processes, problem solving, and reasoning and testing achieved 40% favorable perception regarding their development in the classroom. Connections were identified as the least enhanced mathematical process in the classroom, although Colombian regulations in [14–16] invite mathematical concepts presented in the classroom to be developed in the context of mathematics, other sciences and student daily life.

[17] It is stated that the educational process is part of the social sciences, so any research that is carried out must create knowledge about the social reality of the informant, considering its structure, the relationship between its components, its system changes to such components or in their entirety.

Since they are phenomena with many variables to consider, obeying multiple causes and are often measured with a certain level of error, adequate multivariate methods are required to identify the origin of such variability. The reason is that modelling is used by structural equations since it is a modelling technique that combines multiple regression, combinatory factorial analysis and trail analysis, acting all of them in order to examine several relationships simultaneously. Among the advantages that can be highlighted from the structural equations models, it is emphasized that:

- It allows to work with latent variables or constructs that are measured through indicators.
- Incorporates multiple endogenous and exogenous variables.
- Allows to evaluate the effects of latent variables with each other, without contamination resulting from the measurement error.
- The researcher introduces theoretical knowledge into the model specification.

Within the process of modelling with structural equations, the measurement model must first be established, in which the relationship between the latent variables and their respective indicators is identified. These relationships are shown in Table 2.

Table 3 shows the value obtained from Rho de Spearman among the variables that were significantly correlated, highlighting that as influential variables directly in the academic performance of students in mathematics there are the processes of problem solving, reasoning and testing, and the representation. connections turned out to be directly related to problem solving, but not to academic performance, while
communication is directly related to representation and problem solving. Four of the five latent variables determined to be correlated with the liking for mathematics, while the gender and grade it pursues proved insignificant within the model.

**Table 2.** Relationships defined in the measurement model.

| Latent variables or Constructs | Indicators                      |
|-------------------------------|--------------------------------|
| Problem Solving               | Approach                       |
|                               | Resolution                     |
|                               | Contextualization              |
| Reasoning and Testing          | Guess-up formulation           |
|                               | Strategies solution            |
|                               | Process feedback               |
| Communication                 | Communication in the classroom |
|                               | Mathematical communication      |
| Representation                | From the concrete              |
|                               | From the semiotic representation|
| Connections                   | Everyday                       |
|                               | Mathematics                    |
|                               | Other areas                    |

**Table 3.** Summary of significant correlations between the variables analyzed.

| Latent variables                          | Rho of Spearman |
|-------------------------------------------|-----------------|
| Problem solving - academic performance    | 0.731           |
| Reasoning and testing – academic performance | 0.853          |
| Representation - academic performance     | 0.781           |
| Problem solving – reasoning and testing   | 0.725           |
| Problem solving – communication           | 0.834           |
| Reasoning and testing – representation    | 0.698           |
| Representation – communication            | 0.842           |
| Connections – problem solving             | 0.768           |
| Taste for mathematics – problem solving   | 0.793           |
| Taste for mathematics – reasoning and testing | 0.804         |
| Taste for mathematics – representation    | 0.816           |
| Taste for mathematics - communication     | 0.829           |

**Figure 1.** Structural equation model proposal.
After the verification of the particular correlations, progress is made in the construction of the proposed model which can be visualized in Figure 1 by using the analysis of moment structures (AMOS) which is an application offered by SPSS. The next step is the estimation of parameters for which the traditional method of maximum verisibility is discarded because the assumption of multivariate normality between the variables involved in the proposed model is not met.

Due to non-compliance with the assumption of normality of variables, the Asymptotic Free Distribution Criterion – ADF was selected as an alternative method since its application is possible when the assumption of normality is violated. Once you can estimate the measurement model you will be evaluated the shape in each indicator measured with each latent variable, and then estimate the parameters of the structural model in which each of the proposed causal relationships is studied.

Finally, an important aspect of structural models is the fact that within the measurement model you can quantify the level of impact of the measurement error associated with each variable and indicator.

4. Conclusions
From the results obtained in this research, a background is being generated to corroborate the relationships that have been mentioned in various works but had not been quantified under the technique of structural equations in the regional context of the “Departamento de Norte de Santander, Colombia”.

The results differ from some works in that they found gender and the degree of gender as a significant factor in academic performance. It was determined that the liking for mathematics directly affects four of the five mathematical processes, with Connections being the ones that were little influenced, while directly affecting problem solving, reasoning and testing, representations and communication.

It was possible to verify that there are processes that are strongly linked to each other as is the case of connections with problem resolution which is obvious since the student must solve problems not only of mathematics but of other areas of knowledge and preferably of the daily life of the student, allowing this connection to act as a motivating factor for his study.

With respect to latent variables that report direct correlation with academic performance, it was identified that problem solving, representations, communication and reasoning and testing are significantly related. Within the latent variable communication, it was possible to show that the indicator of communication in the classroom proved not significant, which allows us to conclude that in the study of mathematics is important in the student is to manage the disciplinary language, but their relationships with everyday partners don't turn out to directly affect.

Finally, it can be corroborated that the representations play an important role in learning mathematics, then it is an invitation to teachers to be considered within their classroom planning.

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