Period ratios in multiplanetary systems discovered by *Kepler* are consistent with planet migration

Hanno Rein

Institute for Advanced Study, 1 Einstein Drive, Princeton, NJ 08540, USA

ABSTRACT

The *Kepler* planet candidates are an interesting test bed for planet formation scenarios. We present results from *N*-body simulations of multiplanetary systems that resemble those observed by *Kepler*. We add both smooth (Type I/II) and stochastic migration forces. The observed period ratio distribution is inconsistent with either of those two scenarios on its own.

However, applying both stochastic and smooth migration forces to the planets simultaneously results in a period ratio distribution that is similar to the observed one. This is a natural scenario if planets form in a turbulent protoplanetary disc where these forces are always present. We show how the observed period ratio and eccentricity distribution can constrain the relative strength of these forces, a parameter which has been notoriously hard to predict for decades.

We make the source code of our simulations and the initial conditions freely available to enable the community to expand this study and include effect other than planetary migration.

Key words: methods: numerical.

1 INTRODUCTION

The number of discovered extrasolar planets is increasing rapidly. At the time when this Letter was submitted, the number of planets has reached 784.1 Of those, 275 (35 per cent) are in multiplanetary systems with two or more planets orbiting the same star. These systems are of particular interest to theorists as they can provide valuable information about their formation history.

The existence of mean motion resonances (MMRs) has been confirmed in multiple systems. The most studied planetary system in an MMR is Gliese 876. It consists of three gas giants which are locked in a tight 1:2:4 Laplace resonance (Marcy et al. 2001; Rivera et al. 2010). A large number of studies (e.g. Lee & Peale 2001, 2002; Snellgrove, Papaloizou & Nelson 2001; Nelson & Papaloizou 2002; Beaugé & Michtchenko 2003; Veras 2007) suggest that migration driven by a variety of mechanisms has played an important role in shaping this system.

This is not surprising as planet migration is a natural outcome of the interaction of a planet with the protoplanetary disc that it forms in (Goldreich & Tremaine 1980). Both *N*-body and hydrodynamical models can easily reproduce the observed period ratio, the eccentricities and the libration pattern even though the precise speed at which migration occurs is still up for debate. Furthermore, it is possible to place limits on the strength of additional stochastic forces which might be present in a turbulent protoplanetary disc (e.g. Rein & Papaloizou 2009).

Most of these planets have been discovered with the radial velocity method. This method is biased towards finding heavy planets on close-in orbits. Since 2009, the *Kepler* spacecraft is monitoring over one hundred thousand stars. *Kepler* has discovered thousands of planet candidates which now await confirmation (Batalha et al. 2012). These planets have on average a much smaller mass than those discovered by radial velocity. *Kepler* therefore opens a new window to test planet formation scenarios.

In this Letter, we apply a model of smooth migration (i.e. Type I or Type II depending on disc and planet properties) as well as stochastic migration forces to each candidate system with multiple planets. We show that even a small amount of smooth migration produces a distribution of period ratio that is inconsistent with observations. At each resonance, planets pile up, leading to distinct features in the cumulative distribution function of period ratios.

When stochastic migration forces are added, these features are smeared out. By adding just the right amount, one can retain some of the features, leading to a period ratio distribution similar to the observed one. Our simulations show a pile-up just outside of the exact commensurability. This is also seen in the *Kepler* data. However, it is less apparent in our simulations. This could either be resolved by fine-tuning several parameters or by adding additional physics such as tidal damping or gap edges.

2 SET-UP

We use the *REBOUND* code (Rein & Liu 2012) to simulate the orbital evolution of planetary systems. We add dissipative forces to the equations of motion which mimic the interactions of a planet...
with a protoplanetary disc (Lee & Peale 2001). This set-up allows us to choose two-dimensional parameters for each planet, the migration time-scale $\tau_a$ and the eccentricity damping time-scale $\tau_e$. Most Kepler planets are not massive enough to open a gap in a minimum mass solar nebula (Weidenschilling 1977; Crida, Morbidelli & Masset 2006; Crida 2009) and migrate in the Type I regime (Ward 1986). We therefore choose $\tau_a$ to be a typical value of the Type I migration rate for these planets, $10^3-10^4$ yr. Note that many effects such as a non-isothermal disc may change the Type I migration rate (Paardekooper et al. 2010). We set the eccentricity damping time-scale to be 10 times shorter than the migration time-scale in all our simulations, $\tau_e = \tau_a/10$. This choice has been adopted by many authors in the past and has been justified by hydrodynamical simulations (Kley, Peitz & Bryden 2004). Tests have shown that the results do not strongly depend on this precise value. We use a 15th order RADAU integrator with a time-step set to $10^{-3}$ times the innermost planet’s period. Further tests have shown that the results do not depend on the choice of integrator or time step.

To study the effects of migration, we set up systems that closely resemble those observed by Kepler. In fact, we initialize the entire set of 364 multiplanetary systems and take the stellar mass, the planet periods and the planet radii directly from the published Kepler Objects of Interest (KOI) tables. From transit observations only the planet radius is known. We follow Fabrycky et al. (2012) and assume a simple mass–radius power-law to get a reasonably accurate mass for each planet. We adopt $M_r = 0.01 M_\odot$ for Jupiter and $0.005 M_\odot$ for Saturn. Almost all systems are stable for at least $10^7$ yr when initialized this way and we ignore those few ($<1$ per cent) that are unstable. We also do not take into account that several KOI objects have already been confirmed and now have improved orbital fits. We are confident that this procedure sets up systems that are indeed similar to the real planetary systems, at least in an average sense. We have furthermore tested that a perturbation of the initial orbital parameters (such as starting with a more hierarchical period ratio distribution) does not change the outcome of our simulations.

### 3 SMOOTH PLANETARY MIGRATION

We add smooth, dissipative migration forces to the equations of motions for the outermost planet in each system. All other planets only feel the gravitational forces from the star and the other planets. This set-up naturally leads to convergent migration. As the outer planet moves in, it captures the other planets in resonance. Which resonance is chosen depends on the initial position, the migration speed and the planet masses (Mustill & Wyatt 2011). We stop the integrations after $10^5$ periods of the outer planet at it’s initial location. By that time it has moved in significantly. We tested removing the migration forces after a different amount of time but did not see any qualitative difference.

In Fig. 1 we show the cumulative distribution of period ratios of neighbouring planets in all observed KOI systems as a solid (red) curve. Note that there are surprisingly few features near integer ratios. However, by closely inspecting the curve near 3:2 and 2:1 period ratios, one can see a deficit of planets exactly in the commensurability and a slight pile-up just outside.

The dashed curves show the final period ratios in our integrations. We plot the results for two different migration rates: $\tau_a = 10^3$ and $10^4$ yr. There are now very clear and sharp features that can be associated with resonances. Most planets end up in the 3:2 resonance followed by the 2:1 and 4:3 resonance. This is clearly not consistent with the observed distribution. By closely inspecting each of these locations, one can see a tiny asymmetry favouring larger period ratios over the exact commensurability. However, this is nowhere near the very apparent deficit of planets in the observed period ratio distribution.

### 4 STOCHASTIC PLANETARY MIGRATION

It is very likely that some kind of stochastic force was acting on planets at least during some parts of their past. These forces could result form Magneto Rotational Instability (MRI) turbulence in the protoplanetary disc (e.g. Rein & Papaloizou 2009; Gressel, Nelson & Turner 2011). Another possibility is the gravitational interaction with a remnant planetesimal disc. This scenario also leads to migration containing both a smooth and stochastic component (Ormel, Ida & Tanaka 2012). Even the interaction with other planets can be described as a random walk in certain cases (see e.g. Zhou, Lin & Sun 2007; Wu & Lithwick 2011). Each of these processes is not completely explored and it is hard to estimate the precise amplitude (or the diffusion coefficient) of the stochastic forces in each of those scenarios. We therefore parametrize the forces in this study.

Stochastic forces are modelled following the procedure described in Rein & Papaloizou (2009). The radial and azimuthal components of the stochastic force are modelled independently as a Markov process. The forces have an exponentially decaying autocorrelation function with a finite correlation time. This mimics the forces of a turbulent accretion disc. We set the correlation time to be half of the orbital period of the outer planet. The strength of the forces is measured relative to the gravitational force from the central star by the dimensionless parameter $\sigma$. Rein & Papaloizou (2009) estimate the value for $\sigma$ to be $\sim 5 \times 10^{-6}$ for small mass planets that are embedded in a fully MRI turbulent disc (see their section 3.1). It is

---

2 Rein et al. (2012) present a similar plot for radial velocity system.
Period ratios in multiplanetary systems

5 ECCENTRICITY DISTRIBUTION

When planets migrate and capture into resonances, their eccentricities rise. Eventually the excitation from the convergent migration and the eccentricity damping from the disc reach an equilibrium (Papaloizou 2003). Stochastic forces also cause the planet's eccentricities to grow (Rein & Papaloizou 2009).

If the combination of the smooth and stochastic migration scenarios, as presented in Section 4, is indeed responsible for the observed period ratio distribution, then we can make a prediction for the eccentricity distribution. The mean eccentricity in our simulation with parameters $\tau_a = 10^4$ yr and $\alpha = 10^{-6}$ is $\langle e \rangle \sim 0.01$. In the simulation with parameters $\tau_a = 10^4$ yr and $\alpha = 10^{-5}$ we have $\langle e \rangle \sim 0.05$.

In Fig. 3 we plot the final cumulative eccentricity distribution in our simulations. The run with a short migration time-scale, $\tau_a = 10^3$ yr, produces high eccentricity planets. Planets are captured into resonance earlier and eccentricities grow faster. The simulations without stochastic forcing, illustrated by a short and medium dashed curve (blue), show signs of a bimodal distribution. This is because planets that do (do not) capture in resonance have a high (low) eccentricity. Finally, note that the run with $\alpha = 10^{-5}$ also leads to high eccentricities. This is due to the random excitation and not due to resonances.

Unfortunately eccentricities are not known for most of the *Kepler* candidates. Only radial velocity follow-up observations or transiting variations allow a measurement of the eccentricities. However, the eccentricity distribution of multiplanetary systems can be estimated statistically. Tremaine & Dong (2012) and Moorhead et al. (2011) report values of $e$ between $\sim 0.1$ and 0.25.

It is unfortunate that the simulation which gives the best fit to the period ratio distribution is not the best solution in terms of the mean eccentricity. However, it is worth pointing out that observed eccentricities have the tendency to be overestimated and the actual eccentricities might be lower. This is a well-known effect in radial velocity observations (Zakamska, Pan & Ford 2011) and most likely also present in the *Kepler* sample. For example, noise in the data or non-transiting planets creating additional transit timing variation might lead to an overestimate of the mean eccentricity. The underlying reason for this is that the eccentricity is a positive definite quantity. In addition to that, there are multiple possible physical solutions to this discrepancy. First, one can fine-tune the eccentricity damping and migration time-scales. The ratio was fixed in our simulations. Further experiments have shown that this ratio indeed has

---

**Figure 2.** Cumulative distribution of the period ratios in KOI systems with multiple planets – solid (red) line: observed period ratios; long dashed line: period ratios after migration with stochastic forces with $\alpha = 10^{-6}$; and short dashed line: period ratios after migration with stochastic forces with $\alpha = 10^{-5}$.

**Figure 3.** Cumulative distribution of the final eccentricities in runs with $\alpha = 0$, $10^{-6}$ and $10^{-5}$, and $\tau_a = 10^3$ and $10^4$. 

---

© 2012 The Author, MNRAS 427, L21–L24

Monthly Notices of the Royal Astronomical Society © 2012 RAS
an influence on the final eccentricity distribution, but it is not possible to push the mean eccentricity all the way up to \( \sim 0.2 \). Secondly, the length of our integrations was rather short. Longer integration times lead to higher eccentricities due to secular interactions and the still active stochastic force. Thirdly, massive, non-transiting planets in the systems might also alter the eccentricity distribution. All of these effects tend to increase the final eccentricity, thus going in the right direction.

A complete model that takes all these effects into account will be able to constrain both smooth and stochastic migration forces. This is interesting because Type I migration time-scales are hard to predict as they depend strongly on local disc properties (see e.g. Paardekooper et al. 2010). A similar problem exists with stochastic forces. Dead zones might lower the amplitudes in some regions of the disc. However, a precise modelling of these effects goes beyond the scope of this Letter.

## 6 DISCUSSION

Our results show that neither smooth nor stochastic planetary migration alone can reproduce the observed period ratio distribution of multiplanetary systems in the Kepler sample. However, a combination of those two effects can create a period ratio distribution which is similar to the observed one. If this scenario is true, then we can use the eccentricity distribution of Kepler planets to constrain the relative strength of stochastic and smooth migration forces.

Other ideas have been brought up to explain the observed period ratios. Terquem & Papaloizou (2007) show that migration of multiple planets in a disc with an inner edge, together with orbital circularization, causes strict commensurability to be lost. Similar scenarios involving tidal interactions have been studied by Delisle et al. (2012), Batygin & Morbidelli (2012) and Lithwick & Wu (2012). There is one important difference to the migration scenario presented here. The inclusion of tides leads a strong radial dependence of the effect. Such a dependency is currently not observed in the Kepler data set. This can be verified in Fig. 4 where the cumulative distribution is divided into two bins with almost equal number of planets. The two bins contain systems where the innermost planet’s period is shorter/longer than 5 d. If tides were an important factor in shaping this distribution, one would expect a bigger effect for the bin that contains close-in planets. However, both distributions are identical.

Although this Letter focused on the migration scenario, we are interested in seeing other ideas being tested in a framework similar to the one presented in this Letter. We therefore decided to make the initial conditions, the integrator and all plotting routines freely available. They can be downloaded as a tar-file from https://github.com/hannorein/rebound/tree/resonancelocation. The relevant files for this project are located in the directory problems/resonancelocation. We hope that this enables the community to further investigate these extremely interesting planetary systems and come up with new and maybe even better ideas.

## ACKNOWLEDGMENTS

This Letter benefited greatly from comments by the reviewer, Alexander Mustill. HR was supported by the Institute for Advanced Study and the NFS grant AST-0807444. HR would like to thank Willy Kley, Dave Spiegel, Subo Dong and Scott Tremaine for helpful comments. Dave Spiegel and Subo Dong helped to create a consistent set of initial conditions including the stellar mass and the planet radii for each KOI object.

## REFERENCES

Batalha N. M. et al., 2012, preprint (arXiv:1202.5852)
Batygin K., Morbidelli A., 2012, preprint (arXiv:1204.2791)
Beaugé C., Michtchenko T. A., 2003, MNRAS, 341, 760
Crida A., 2009, ApJ, 698, 606
Crida A., Morbidelli A., Masset F., 2006, Icarus, 181, 587
Delisle J.-B., Laskar J., Correia A. C. M., Boué G., 2012, preprint (arXiv:1207.3171)
Fabrycky D. C. et al., 2012, preprint (arXiv:1207.6328)
Goldreich P., Tremaine S., 1980, ApJ, 241, 425
Gressel O., Nelson R. P., Turner N. J., 2011, MNRAS, 415, 3291
Kley W., Peitz J., Bryden G., 2004, A&A, 414, 735
Lee M. H., Peale S. J., 2001, BAAS, 33, 1198
Lee M. H., Peale S. J., 2002, ApJ, 567, 596
Lithwick Y., Wu Y., 2012, ApJ, 756, L11
Marcy G. W., Butler R. P., Fischer D., Vogt S. S., Lissauer J. J., Rivera E. J., 2001, ApJ, 556, 296
Moorhead A. V. et al., 2011, ApJS, 197, 1
Mustill A. J., Wyatt M. C., 2011, MNRAS, 413, 554
Nelson R. P., Papaloizou J. C. B., 2002, MNRAS, 333, L26
Ormel C., Ida S., Tanaka H., 2012, preprint (arXiv:1207.7104)
Paardekooper S., Baruteau C., Crida A., Kley W., 2010, MNRAS, 401, 1950
Papaloizou J. C. B., 2003, Celest. Mech. Dyn. Astron., 87, 53
Rein H., Liu S.-F., 2012, A&A, 537, A128
Rein H., Papaloizou J. C. B., 2009, A&A, 497, 595
Rein H., Payne M. J., Veras D., Ford E. B., 2012, preprint (arXiv:1204.0974)
Rivera E. J., Laughlin G., Butler R. P., Vogt S. S., Haghighipour N., Meschiari S., 2010, ApJ, 719, 890
Sanchis-Ojeda R. et al., 2012, preprint (arXiv:1207.5804)
Snellgrove M. D., Papaloizou J. C. B., Nelson R. P., 2001, A&A, 374, 1092
Terquem C., Papaloizou J. C. B., 2007, ApJ, 650, 1110
Tremaine S., Dong S., 2012, AJ, 143, 94
Veras D., 2007, Celest. Mech. Dyn. Astron., 99, 197
Ward R. W., 1986, Icarus, 67, 164
Weidenschilling S. J., 1977, Ap&SS, 51, 153
Wu Y., Lithwick Y., 2011, ApJ, 735, 109
Zakamska N. L., Pan M., Ford E. B., 2011, MNRAS, 410, 1895
Zhou J.-L., Lin D. N. C., Sun Y.-S., 2007, ApJ, 666, 423

This paper has been typeset from a \LaTeX\ file prepared by the author.