Neutral Higgs Sector of the MSSM with Explicit $CP$ violation and Non-Holomorphic Soft Breaking

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(Dated: August 24, 2009)

Using the effective potential method, we computed one-loop corrections to the mass matrix of neutral Higgs bosons of the Non-Holomorphic Supersymmetric Standard Model (NHSSM) with explicit CP violation, where the radiative corrections due to the quarks and squarks of the third generation were taken into account.

We observed that the non-holomorphic trilinear couplings can compete with the holomorphic ones in CP violating issues for the mass and mixing of the neutral Higgs bosons.

PACS numbers: 12.60.-i, 12.60.Jv, 14.80.Cp

I. INTRODUCTION

In the Minimal Supersymmetric Extension of the Standard Model (MSSM) superpotential and soft breaking terms are generally considered as holomorphic functions. While the holomorphicity of the superpotential is obligatory for the MSSM, more generalized versions of the model may also include R-parity violating terms and/or non-holomorphic structures in the soft breaking sector of the theory [1, 2].

Contrary to the SM where a unique Higgs boson resides, supersymmetric models predict extra Higgs bosons via introducing different Higgs doublets. With a number of supersymmetric models and various extensions, experimental verification of Higgs bosons became one of the main objectives of the current colliders such as Tevatron [3] and LHC [4]. Compared with the SM, the allowed mass range of the lightest Higgs in the MSSM is somewhat more constrained. Indeed, the predictions related to the mass of the lightest Higgs in the MSSM give an upper bound $m_h \sim 130$ GeV [5] which can be fairly relaxed by certain extensions of the theory. In this sense, the Non-Holomorphic Supersymmetric Standard Model (NHSSM) requires the presence of the additional soft breaking parameters that can shift the upper bound to a certain extend [6] which may be required in further Higgs searches.

In addition to give a relaxation to the upper bound of lightest Higgs mass, these additional non-holomorphic soft breaking terms ensure extra degrees of freedom where, for instance, CP violating terms of the MSSM may get into trouble. It is explicitly shown in [7] that the amount of CP violation present in the SM is not adequate to explain the observed baryon asymmetry in the universe, whereas, supersymmetric models offer novel sources of CP violating terms and especially Higgs interactions can play a key role in mediating CP violation. This issue is deeply scanned in the MSSM [8]. But, this should also be probed for the extensions of the minimal model where additional sources of CP violating terms exist. In this respect, the NHSSM is an interesting model with extra sources of CP violating terms in the soft breaking part of its Lagrangian. However before doing this, precise predictions are required for the Higgs sector of the model which does not exist in the literature.

Hence, in this work, our interest focused on the neutral Higgs sector of the Non-Holomorphic Supersymmetric Standard Model (NHSSM) with R-parity conservation and explicit CP violation. We assumed that CP is explicitly violated in the Higgs sector of the NHSSM and looked for its impact on the mass and mixing of the neutral Higgs bosons, which may be important for the near future. The possibility of non-holomorphic structures in the soft breaking is realized in the literature. For a detailed list of issues ranging from $b \rightarrow s\gamma$ decay to the Renormalization Group Equations (RGEs) of the non-holomorphic supersymmetric model, we refer to [9], a possible explanation for the source of the NH structures can be found in [10].

The rest of the paper is as follows: In the following section we first described the basic low energy structure of the NHSSM. Analytical results for the mass matrix of the neutral Higgs bosons of the NHSSM are derived in the same section where the one-loop CP violating effective potential is calculated by considering only top and bottom sectors. Section III is devoted to numerical analysis where the impact of non-holomorphic trilinear terms are investigated. And we concluded in section IV.
II. NHSSM

In general the supersymmetry (SUSY) breaking sector is parameterized via holomorphic operators which must be soft i.e. the quadratic divergences must not be regenerated [11]. However, the MSSM can be extended via introducing new soft operators including R-violating and/or non-holomorphic terms in the soft breaking sector of the theory. In this sense there are different non-holomorphic models based on different approaches (see [11] and [12]). Here we follow the easiest path in which R-parity violating terms are ignored and the problematic Higgsino mass term (µ) is absent in the superpotential. Under these assumptions, the NH version of the minimal supersymmetric model can be described by the superpotential

\[ \mathcal{W} = \hat{Q} \cdot \hat{H}_u Y_u \hat{U} - \hat{Q} \cdot \hat{H}_d Y_d \hat{D} - \hat{L} \cdot \hat{H}_d Y_e \hat{E} \]

where our conventions are such that, for instance, \(\hat{\bar{Q}} \cdot \hat{Q} = \epsilon_{ij} \hat{\bar{Q}} :: \hat{Q}:: \) with \(\epsilon_{12} = -\epsilon_{21} = 1\). In the MSSM the breakdown of supersymmetry is parameterized by a number of holomorphic soft operators [11]

\[ -\mathcal{L}_{soft} = \hat{Q}^\dagger m^2_Q \hat{Q} + \hat{U}^\dagger m^2_u \hat{U} + \hat{D}^\dagger m^2_d \hat{D} + \hat{L}^\dagger m^2_L \hat{L} + \hat{E}^\dagger m^2_E \hat{E} + \frac{1}{2} (M_3 \lambda_3 \dot{\lambda}_3 + M_2 \lambda_2 \dot{\lambda}_2 + M_1 \lambda_1 \dot{\lambda}_1 + h.c.) + m^2_{H_u} \hat{H}_u^\dagger \hat{H}_u + m^2_{H_d} \hat{H}_d^\dagger \hat{H}_d + (m^2_{H_u} \hat{H}_u + \hat{H}_d + h.c.) + (\hat{Q} \cdot \hat{H}_u Y_u^A \hat{U} - \hat{Q} \cdot \hat{H}_d Y_d^A \hat{D} - \hat{L} \cdot \hat{H}_d Y_e^A \hat{E} + h.c.) \]

Here \(m^2_{Q,\ldots,E}\) are the soft mass-squareds of the scalar fermions, \(Y_{u,d,e}^A\) are their associated holomorphic trilinear couplings, and finally, \(M_1, M_2, M_3\) are, respectively, the masses of hypercharge, isospin and color gauginos. For the description of the Higgs sector soft masses \(m^2_{H_u}, m^2_{H_d}\) and \(m^2_\lambda\) are used.

In the MSSM one can introduce the CP violation through the Higgs superpotential and the soft supersymmetry breaking terms, however, as has been shown explicitly in [12, 13], in supersymmetric theories which do not have pure gauge singlets in their particle spectrum, the holomorphic supersymmetry breaking terms do not necessarily represent the most general set of soft-breaking operators. Indeed, for instance, the MSSM spectrum does not consist of any gauge singlet superfield, and thus, its soft breaking sector must necessarily include the following soft breaking terms

\[ \mathcal{L}'_{soft} = \mu' \hat{H}_u \cdot \hat{H}_d + \hat{Q} \cdot \hat{H}_u Y_u^A \hat{U} + \hat{Q} \cdot \hat{H}_d Y_d^A \hat{D} + \hat{L} \cdot \hat{H}_d Y_e^A \hat{E} + h.c. \]

in addition to those in (2). Here \(Y_{u,d,e}^A\) are non-holomorphic trilinear couplings which do not need to bear any relationship to the holomorphic ones \(Y_{u,d,e}^A\) in (2). The only exception to this can be imagined as a unique common term at very high scales but even in that case, due to renormalization group running effects it is good to assume those new trilinear couplings to be completely different from the ordinary ones. Since these non-holomorphic couplings are perfectly soft they must be taken into account when confronting the MSSM predictions with experimental data.

For possible variants of the non-holomorphic model notice that the original \(\mu\) term can be protected in the superpotential, in this case, the soft breaking \(\mu'\) can stand alone or replaced with \(\mu' = \mu\), then the \(m^2_\lambda\) term of the soft sector can be written as \(m^2_\lambda = B(\mu - \mu')\). But we have chosen to deal with only one \(\mu\) parameter for which the prime symbol will be dropped from now on.

A. Analytical Results for the Neutral Higgs Bosons

As in the MSSM, the CP violation mixes neutral Higgs bosons and hence they will be depicted as physical mass eigenstates \(h_1, h_2\) and \(h_3\). The classical potential for the neutral Higgs fields can be written for the NHSSM like in the MSSM [14] as follows

\[ V = m^2_{H_u} |H_u^0|^2 + m^2_{H_d} |H_d^0|^2 - (m^2_{H_u} H_u^0 H_u^0 + c.c.) + \frac{g_2^2}{8} (|H_u^0|^2 - |H_d^0|^2)^2 + \Delta V \]

Here \(g_2^2 = g_2^2 + g_Y^2\) and \(\Delta V\) refers to loop corrections which will be computed using the effective potential formalism and the symbols \(g_2\) and \(g_Y\) stand for the SU(2) and U(1) gauge couplings respectively. Contrary to the MSSM, in [11] \(|\mu|^2\) contributions coming from F-terms are absent which stem from the superpotential [11]. In [11] we allowed \(m^2_\lambda\) to be complex by assuming:

\[ m^2_\lambda = |m^2_\lambda| e^{i\phi} \]
This phase $\Phi$ can be set to zero at the tree level, but due to loop corrections it should be protected. Our results for Higgs masses will depend on this and another phase coming from one of the Higgs doublets. Now, the neutral components of the Higgs doublets can be expanded around their vacuum expectation values (vevs) as

$$H_0^0 = \frac{1}{\sqrt{2}} (v_d + \phi_d + ia_1), \quad H_0^0 = \frac{e^{i\theta}}{\sqrt{2}} (v_u + \phi_u + ia_2)$$

(6)

in which $v^2 = v_u^2 + v_d^2 = (246 \text{ GeV})^2$. Using the ratio of vevs we define $\tan \beta = v_u/v_d$. In the above expression, a phase shift $e^{i\theta}$ is attached to neutral part of the up Higgs doublet $H_0^0$ and this phase should be fixed by true vacuum conditions considering loop effects (see [15, 16, 17, 18] for details).

It is important to emphasize that without loop corrections the tree level Higgs potential predicts the lightest Higgs boson to be lighter than the $Z$ boson. Hence, as in the MSSM, sizeable radiative corrections are also needed in the NHSSM to satisfy the LEP bound of $m_h \sim 114 \text{ GeV}$. In this work we did not consider LEP excess events which indicate on a possibility of even a lighter Higgs [19], however, the excess events may still remain as another issue to be considered within the context of the NHSSM.

In the MSSM, the radiative corrections [15, 16] are dominated by loops of the top (s)quark, and to a lesser extent, by those of the bottom (s)quark, tau (s)lepton, charginos and neutralinos [17]. A particularly useful framework for computing the radiative corrections in the Higgs sector is effective potential approach [20]. At the one-loop level we can write the contributions of all the relevant particles (coupled to Higgs bosons) as

$$\Delta V = \frac{1}{64 \pi^2} \text{Str} \left[ M^4 \left( \ln \frac{M^2}{\Lambda^2} - \frac{3}{2} \right) \right]$$

(7)

Here, $\Lambda$ is the renormalization scale and $M$ is the field-dependent mass matrix of quarks and squarks with overall factors -12 and 6, respectively. The additional contributions coming from charginos, neutralinos, etc. are ignored in this work.

To proceed, the relevant fermion masses should be stated in a field dependent manner as in the background formed by the neutral components of the Higgs fields, for instance, the squared-mass of bottom and top quarks are given by

$$m_b^2 = |h_b|^2 \ |H_0^0|^2, \quad m_t^2 = |h_t|^2 \ |H_0^0|^2$$

(8)

and those of the scalar quarks are

$$M_b^2 = \left( m_{t_L}^2 + m_b^2 + \frac{1}{12} (3g_2^2 + g_1^2) (|H_0^0|^2 - |H_0^0|^2) \right) \ h_b \left( A_b H_0^0 - A_{b}^* H_0^0 \right), \quad m_{bR}^2 = m_b^2 + \frac{1}{3}g_2^2 (|H_0^0|^2 - |H_0^0|^2)$$

(9)

$$M_t^2 = \left( m_{t_L}^2 + m_t^2 - \frac{1}{12} (3g_2^2 - g_1^2) (|H_0^0|^2 - |H_0^0|^2) \right) \ h_t \left( A_t H_0^0 - A_{t}^* H_0^0 \right), \quad m_{tR}^2 = m_t^2 + \frac{1}{3}g_2^2 (|H_0^0|^2 - |H_0^0|^2)$$

(10)

In writing the squark mass-squared matrices we have introduced some notationally simplifying definitions such as $(m_Q^2)_{33} \equiv m_{tL}^2$ and $(Y^A_{u,d,e})_{33} \equiv h_b A_t$. In [9] and [10], the main effect of non-holomorphic trilinear couplings is to replace the $\mu$ parameter in the holomorphic MSSM in a flavor-dependent way and this shift alone tells us that the $\mu$ parameter seen by Higgsinos is completely different than what is felt by the scalar fermions. It is possible to back-transform $A_t, A_b \rightarrow \mu$ to obtain the MSSM results, but the reverse is not true. In other words, the indirect relation between scalar fermions and charginos or neutralinos over the $\mu$ parameter is completely vanished in this version of the NHSSM. From now on, bounds on the $\mu$ parameter (i.e. obtained from charginos) have no restriction on scalar fermions anymore. Besides this the mass of the Higgsinos is the same with the MSSM if we assume $\mu$ as an input parameter.

In this work, rather than providing a general analysis of $Y^A_{u,d,e}$ in regard to MSSM phenomenology (see [21] and attempts in this direction), we will focus mainly on their influence on Higgs-fermion-fermion couplings (especially for $h_i \rightarrow bb$ decay) in order to determine their distinctive features and observability in collider experiments. Concerning this class of observables, the primary objective would be to determine sensitivities of Higgs boson masses and mixings to the non-holomorphic couplings $Y^A_{u,d}$. For this purpose, to leading order, it suffices to consider only the top and bottom quark sector.

Now, for later convenience, we introduce $\Sigma$ and $\Delta$ symbols such that (s)top and (s)bottom mass eigenvalues can be written simply as

$$m_{t_{1,2}}^2 = \left( \Sigma_T \mp \sqrt{\Delta_T} \right) / 4, \quad m_{b_{1,2}}^2 = \left( \Sigma_B \mp \sqrt{\Delta_B} \right) / 4$$

(11)
and bottom sectors and hence variation of $\phi$ is never presented. Instead we concentrated on the trilinear couplings evaluated in the electroweak vacuum. The explicit form of our definitions can be found in the Appendix.

The calculation proceeds by plugging the field-dependent eigenvalues into the potential. The mass matrix of the Higgs bosons is given by the second derivatives of the potential (at vanishing external momentum). For this aim, the minimum of the potential should be obtained, which can be extracted from the first derivatives of the potential $V$. In turn, $m_{h_1}^2$, $m_{h_2}^2$ and $m_{h_3}^2$ can be expressed in terms of functions of the parameters appearing in the loop-corrected Higgs potential. We define

$$T_i = \partial V / \partial \psi_i, \quad M_{ij} = \partial^2 V / \partial \psi_i \partial \psi_j$$

for the first and second derivatives respectively with $\psi_i, \psi_j = \phi_u, \phi_d, a_1, a_2$ and both $T$ and $M^2$ should be evaluated at vacuum conditions $\psi_i, \psi_j = 0$. Among the stationary relations $T_{\phi_u}$ and $T_{\phi_d}$ are linearly independent. But $T_{a_1}$ and $T_{a_2}$ can be expressed in terms of each other. Hence we can express $m_{h_3}^2$ as follows

$$\frac{3}{16 \pi^2} \csc[\theta + \Phi] \left\{ \frac{|T_{h_3}|^2 \Delta_B}{\sqrt{\Delta_B}} \left[ \sqrt{\Delta_B} + 2m_{b_1} \ln \frac{m_{b_1}^2}{\Lambda^2} - 2m_{b_2} \ln \frac{m_{b_2}^2}{\Lambda^2} \right] + \frac{|T_{h_3}|^2 \Delta_T}{\sqrt{\Delta_T}} \left[ \sqrt{\Delta_T} + 2m_{t_1} \ln \frac{m_{t_1}^2}{\Lambda^2} - 2m_{t_2} \ln \frac{m_{t_2}^2}{\Lambda^2} \right] \right\}$$

In this equation $T_f = (A_f \epsilon^{i\theta})$ describes the amount of CP violation in the sfermion mass matrices. Notice that the combination of phases $\theta + \Phi$ is re-phasing invariant and validity of (13) can be checked from (22) in the $(A', A'_\mu) \to \mu$ limit. During the numerical analysis we fixed $\theta = -\pi/2$ and determined $\Phi$ in accordance with the input parameters.

After obtaining true tadpoles correctly, Higgs boson mass-squared matrix ($M_{ij}^2$) is acquired in the base of $\{\phi_u, \phi_d, a_1, a_2\}$ in the form of a symmetrical $4 \times 4$ matrix. The eigenvalues of this symmetric mass-squared matrix correspond to a massless Goldstone boson and three physical neutral Higgses ($m_{h_1}^2$, $m_{h_2}^2$, and $m_{h_3}^2$). They can be used for numerical purposes, but for analytical purposes it is useful to perform the following unitary transformation;

$$M^2 = S^T M^2 S \quad \text{where} \quad S = \begin{pmatrix} 1 & 0 \\ 0 & \eta \end{pmatrix} \quad \text{and} \quad \eta = \begin{pmatrix} s \beta & c \beta \\ c \beta & -s \beta \end{pmatrix}.$$  

This transformation allows us to redefine $M^2$ as a symmetric $3 \times 3$ matrix in the basis $\{\phi_u, \phi_d, a\}$ where $a$ is defined as a linear combination of $a_1$ and $a_2$ ($a = \sin \beta a_1 + \cos \beta a_2$). For instance $M_{33}^2$ component of the redefined $M^2$ matrix becomes

$$M_{33}^2 = m_{h_1}^2 + \frac{3}{32 \pi^2 v d v u} \csc[\theta + \Phi] \left\{ \frac{|T_{h_3}|^2 \Delta_B}{\sqrt{\Delta_B}} \left[ \sqrt{\Delta_B} + 2m_{b_1} \ln \frac{m_{b_1}^2}{\Lambda^2} - 2m_{b_2} \ln \frac{m_{b_2}^2}{\Lambda^2} \right] + \frac{|T_{h_3}|^2 \Delta_T}{\sqrt{\Delta_T}} \left[ \sqrt{\Delta_T} + 2m_{t_1} \ln \frac{m_{t_1}^2}{\Lambda^2} - 2m_{t_2} \ln \frac{m_{t_2}^2}{\Lambda^2} \right] \right\}$$

The explicit form of the symbols given here can be read from the Appendix. We refer to the same place for the remaining five entries of the symmetric $M$ matrix.

III. NUMERICAL ANALYSIS

In this part, based on our analytical results, our aim is to show how the interplay of the non-holomorphic couplings with holomorphic ones can change the mass and the mixing of the neutral Higgs bosons.

During the analysis, to respect the collider bounds, we require the scalar fermion masses satisfying $m_f > 100$ GeV and generally our results cover the LEP bound $m_{h_1} \sim 114$ GeV. Our basic input parameters are $M_A, m_Q, m_G, m_{\tilde{b}}$, $A_u, A_d, A'_u, A'_d$, and $\tan \beta$. During the analysis we fixed $\theta = -\pi/2$, $\Lambda = 0.5$ TeV and determined $\Phi$ in accordance with the input parameters. The true phase of CP violation can be defined as $\theta_{eff} = \arg (A_f \epsilon^{i\theta})$ for top and bottom sectors and hence variation of $\Phi$ is never presented. Instead we concentrated on the trilinear couplings and looked mainly for the mass and the mixings of the Higgs bosons under CP violating non-holomorphic trilinear couplings.
In the numerical analysis we have taken $M_A$ as one of the input parameters and forced $M_A = M_{33}$, which is slightly different from the selection of ref. [22]. Alternatively, mass of the charged Higgs boson can be used as an input parameter as it is usually done in the MSSM literature (i.e. see [8]). To make the neutral Higgs mass matrix diagonal, we also defined an orthogonal matrix $O$ such that

$$\text{diag}(m_{h_1}^2, m_{h_2}^2, m_{h_3}^2) = O^T M^2 O,$$

then, an additional parameter can be defined to describe the CP composition of the neutral Higgs bosons as [22]

$$\alpha_i = \min \left( \frac{|O_{i3}|}{\sqrt{|O_{i1}|^2 + |O_{i2}|^2}}, \frac{\sqrt{|O_{i1}|^2 + |O_{i2}|^2}}{|O_{i3}|} \right)$$

With these definitions CP composition of neutral Higgses under the influence of non-holomorphic trilinear couplings can be presented. Additionally, we have selected the decay width of neutral Higgs bosons into $b \bar{b}$ as a testing ground. Of course ($h_i \rightarrow b \bar{b}$) has massive background, but according to the SM up to $M_H \sim 130$ GeV this channel is the dominant one. So we have chosen this channel for its simplicity to show the impact of the phases. Notice in the SM that the width of this decay is $0.0035$ to $0.086$ GeV for $m_h$ between $120 - 160$ GeV [23]. But the NHSSM has potential to change it sizably, as we will see. The partial decay width of a neutral Higgs boson $h_i$ into a pair of bottom and anti-bottom quarks is given as [23]

$$\Gamma(h_i \rightarrow b \bar{b}) = \frac{3g_2^2 m_b^2 m_{h_i}}{32 \pi m_W^2} \sqrt{1 - \frac{m_b^2}{m_{h_i}^2}} \left[ O_{i3}^2 \cos^2\beta \left(1 - \frac{m_b^2}{m_{h_i}^2}\right) + \tan^2\beta O_{i3}^2 \right].$$

![FIG. 1: The lightest neutral Higgs mass $m_{h_1}$ (left panel) and its CP violating parameter $\alpha_1$ (right panel) of the MSSM (solid lines) and of the NHSSM (dashed lines), against $m_{\tilde{Q}}$ for $M_A=1$ TeV (thick lines) and $M_A=130$ GeV (thin lines). Inputs: $\mu = 500$ GeV, $A'_t = 5 m_{\tilde{Q}}$, $A'_t$ is fixed at 0, $m_{\tilde{Q}} = m_{\tilde{\chi}} = m_{\tilde{\chi}}'$, $A_t = A_b = 2 m_{\tilde{Q}}$ and $m_{\tilde{Q}}$ scans from 0.3 TeV to 1 TeV, $\tan\beta$ is fixed at 10.](image)

Now let us present our numerical results. We start with the difference of $m_{h_1}$ in the MSSM and in the NHSSM for CP violating case (see [6] for CP conserving case). For the model under concern, the $\mu$ parameter of the NHSSM is responsible for higgsino masses as given in the soft breaking sector [2], and can be bounded from chargino and neutralino masses as in the MSSM. However in the NHSSM, $\mu$ does not exist in the sfermions, so whenever we assume $A'_t \neq \mu$ we are considering the NHSSM. Thus for the differently selected $\mu$ and $A'_t$ values model dependent effects can be observed on the mass and the mixing of the Higgs bosons.

In this sense, Fig. 1 depicts the mentioned differences of the MSSM with fixed $\mu$ and of the NHSSM with different $A'_t$ and $A'_t$ values. The mass difference, as can be seen from the left panel of the figure, can be around few MeV or a few GeV and increases as the squark mass increases. This mass difference is determined by the magnitude of $M_A$ and is more visible when $M_A$ is close to $m_{h_1}$. A similar observation can be extracted from the $CP$ violating parameter of the lightest Higgs boson (right panel). Again when $M_A \sim m_{h_1}$, this parameter shows that the $CP$ composition of the lightest Higgs can be enhanced as should be expected in the MSSM [8]. In comparison to the MSSM, the $CP$ violating parameter of the NHSSM can be smaller or larger than that of the MSSM predictions, which is again determined by
$M_A$. For instance, for $M_A = 130$ GeV the CP violating parameter takes $\alpha_1^{\text{NH}} \sim 0.4$ which is approximately half of the MSSM’s prediction but for $M_A = 1$ TeV we observe $\alpha_1^{\text{NH}} \sim 0.25$ which is approximately two times larger than the prediction of the MSSM. Notice that using this parameter one can determine whether $h_1$ will behave similar to the SM’s Higgs boson or not. In sum, the effects of the extra parameter of the NHSSM ($A'_1$) is more visible when $M_A$ is close to $m_h$. This is something predictable because in this case large mixing occurs between the would-be CP-odd and CP-even Higgs bosons. On the other hand, when $M_A$ is large it may be hard to effect the mass of the lightest Higgs boson with the NH terms. But even in this case its coupling could be very different from the prediction of the MSSM, which can be read from the right panel of the figure.

In the same figure, the relaxation on the bound of sfermion masses can also be deduced for the NHSSM, i.e. as can be seen from the left panel of Fig. 1 for $m_{h_1} > 115$ GeV and $M_A = 130$ GeV the MSSM demands $m_{\text{squarks}} > 450$ GeV, but in the NHSSM this bound relaxes to $m_{\text{squarks}} > 350$ GeV, for the selected range of parameters.

![FIG. 2: Masses of all neutral Higgs bosons $m_{h_{1,2,3}}$ (left), their CP-violating mixing angles $\alpha_{1,2,3}$ (center) and their decay widths into $b\bar{b}$ pair (right) versus the argument of the top trilinear coupling arg($A'_1$). The dimensionful terms are given in GeV. Inputs: $\tan \beta = 10$, $m_\tilde{Q} = m_{\tilde{Q}} = m_{\tilde{D}} = 1$ TeV and $A_t = A_b = |A'_1| = A'_3 = 2 m_\tilde{Q}$, $M_A = 200$ GeV. Note that for this and latter figures line formats are as follows, solid line ($h_1$), dotted line ($h_2$) and dashed line ($h_3$).](image)

In Fig. 2 we present the phase dependencies of the masses of neutral Higgs bosons $h_1, h_2$ and $h_3$ in left panel, CP violating parameters ($\alpha_i$) for each of the mentioned bosons in middle panel and the corresponding decay widths $\Gamma(h_i \rightarrow b\bar{b})$ in right panel, against varying phase of the non-holomorphic trilinear coupling $A'_1$. As can be seen from the first panel of Fig. 2 all of neutral Higgs bosons are sensitive to the phase of the non-holomorphic trilinear coupling $A'_1$, with varying order. As a result of this phase the lightest Higgs boson can be made completely CP-odd or CP-even. One can easily recognize from the middle panel of Fig. 2 that CP violating mixing angles ($\alpha_i$) are suppressed for $\arg(A'_1) \sim \pi/2$ and have a sharp maximum value for $\arg(A'_1) \sim \pi/3$ and $\sim 2\pi/3$. Additionally the partial decay widths of the neutral Higgs bosons are very sensitive to the phase of $A'_1$, can exceed the prediction of the SM for each of the bosons, i.e. $\Gamma(h_1 \rightarrow b\bar{b}) \leq 0.14$ GeV is possible in the NHSSM as can be seen from the right panel of Fig. 2.

![FIG. 3: The same with Fig. 2 but now $\tan \beta = 50$, $M_A = 130$ GeV, plots are presented against varying $A'_3$. Inputs: $A_t = A_b = A'_3 = |A'_3| = 2 m_\tilde{Q}$](image)
In order to show the importance of $A'_b$ contribution, we present Fig. 3 in which all the input parameters are the same with Fig. 2 but now $\tan\beta=50$, $M_A = 130$ GeV. Here it is interesting to observe that the coupling of the lightest neutral Higgs boson is very strong, sensitive to the argument of the $A'_b$, additionally $\Gamma(h_1 \rightarrow b\bar{b})$ can be as large as $\geq 3$ GeV (solid line of the right panel) which is well above the SM prediction. A common property of the Figs. 2,3 is that when $\theta_{eff} \neq 0$, mass difference of $h_2$ and $h_3$ bosons increases.

Notice that if were to assign $\arg(A_f) = -\arg(A'_f)$ then there would be no variation for the masses, the CP compositions and the partial decay widths of Higgs bosons which can be important for CP violating issues such as Electric Dipole Moments (EDMs) of fundamental fermions. This can be seen from the definition of the CP violating parameter $I_f$.

It can be inferred from the presented figures that the NH soft breaking terms can yield sizable variations on the masses, CP compositions and partial decay widths of neutral Higgs bosons. These decay widths covers the range from $\sim$ SM values up to 3.5 GeV, CP violating parameters can be obtained from zero to one, thanks to the NH terms. Some of these results can also be simulated with a complex $\mu$ parameter of the MSSM (when $A'_b = A'_h = \mu$), but the option of replacing this parameter with the non-holomorphic ones should be seen as an attractive alternative for the continuing and coming Higgs searches. While not presented here, it is easy to guess that the couplings of Higgs bosons to vector bosons are also sensitive to the mentioned NH terms.

IV. CONCLUSION

In this paper, we studied the mass matrix of the neutral Higgs bosons in the NHSSM with explicit CP violation at the one-loop level. For doing this, we first obtained analytical expressions for the mass matrix of neutral Higgs bosons (see the Appendix) and performed a numerical study based on the new sources of CP violating trilinear terms.

In order to maximize the impact of the NH terms, among many possible parametrizations of the NH model, a special one is selected in which the indirect relation between scalar fermions and inos over the $\mu$ parameter is disappeared. In this version of the NH model, $\mu$ term is absent in the superpotential and it exists in the soft breaking part as a mass term for Higgsinos. We observed that this can heavily effect the mass and the mixings of the neutral Higgs bosons of the MSSM.

During the numerical analysis we intentionally considered beyond the MSSM scenarios (such as $A'_f \neq A'_h$) and observed that not only holomorphic soft breaking terms but also non-holomorphic terms can induce sizable amount of CP violation in the Higgs sector. In order to show this we studied numerically partial decay widths and CP-violating parameters of all the neutral Higgs bosons. We believe the selected ranges of our examples can be important for the continuing and upcoming (i.e. see [24]) Higgs searches with a generalized soft breaking MSSM. Additionally, this issue should be probed deeper because new sources of CP violating terms consisting with current collider bounds can be useful, for instance, to relax the electron and neutron EDM bounds on the CP violating terms of the MSSM [25]. Analysis of various observables ranging from $b \rightarrow s\gamma$ decay to EDM constrains can shed further light on the structure of non-holomorphic models.

V. ACKNOWLEDGMENTS

A.S gratefully acknowledges the support from the Finnish Center for International Mobility (CIMO).

APPENDIX A: DEFINITIONS AND MATRIX ELEMENTS

The definitions that appear in our calculations are as follows: For the proper treatment of the amount of CP violation we collected imaginary and real parts of frequently appearing terms as $I_b = \text{Im}(A_b A'_b e^{i\theta})$, $I_t = \text{Im}(A_t A'_t e^{i\theta})$, similarly $R_b = \text{Re}(A_b A'_b e^{i\theta})$ and $R_t = \text{Re}(A_t A'_t e^{i\theta})$. This enables one to decompose scalar fermions into parts as

$$\Sigma_B = 2m_{t_L}^2 + 2m_{b_R}^2 + v_u^2 \Sigma_{G_u} + v_d^2 (2|h_b|^2 - \Sigma_{G_h})$$
$$\Sigma_T = 2m_{t_L}^2 + v_u^2 \Sigma_{G_u} + v_d^2 (2|h_t|^2 + \Sigma_{G_t})$$

$$\Delta_B = \frac{\kappa_t^2}{\Delta_T} \Sigma_{G_h} + 8|h_b|^2 \left( |A_b|^2 v_u^2 + v_u \left( |A'_b|^2 v_u - 2v_d R_b \right) \right)$$
$$\Delta_T = \frac{\kappa_t^2}{\Delta_T} \Sigma_{G_t} + 8|h_t|^2 \left( |A'_t|^2 v_d^2 + v_u \left( |A_t|^2 v_u - 2v_d R_t \right) \right)$$

(A1) (A2)

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(A1) (A2)
Additionally, we defined the following quantities such that entries of the Higgs matrix can be expressed in simpler forms.

\[ \chi_1 = 4 \left( (4A_t^2|\kappa|^2 + \kappa_1) v_d - 4|\kappa|^2 v_u R_b \right), \chi_2 = 4 \left( (4A_t^2|\kappa|^2 + \kappa_2) v_d - 4|\kappa|^2 v_u R_i \right), \chi_3 = 4 \left( (4A_t^2|\kappa|^2 + \kappa_3) v_d - 4|\kappa|^2 v_u R_b \right), \chi_4 = 4 \left( (4A_t^2|\kappa|^2 + \kappa_4) v_d - 4|\kappa|^2 v_u R_i \right) \]

Using the definitions given above, the elements for the mass matrix of the neutral Higgs bosons due to radiative contribution of quarks and squarks are obtained as follows:

\[ \mathcal{M}_{11}^2 = \frac{g^2 v^2}{4} + m_2^2 + 3 \left( 8v_d^2 \chi_2 \Delta_T \Sigma_{G_1} + 2 \chi_2 \Delta_T \Sigma_{T} + v_d \left( \chi_2^2 - 2 \chi_4 \Delta_T \right) \Sigma_T \right) m_2^2 \]

\[ \mathcal{M}_{12}^2 = -\frac{1}{4} g^2 v_d v_u + m_2^3 (-\cos[\theta + \Phi]) + \frac{3 \chi_2 \chi_6 \Delta_B + (\chi_1 \chi_5 - 2(\chi_2 \chi_4 + \chi_6) \Delta_T)}{512 \pi^2 \Delta_B} \]

\[ \mathcal{M}_{13}^2 = m_3^2 \sin[\theta + \Phi] + \frac{3 \left( (|h_b|^2 v_d \chi_1 - 2v_d \Delta_B) \Delta_T \Sigma_{G} + |h_b|^2 \Delta_B (v_d \chi_2 - 2v_d \Delta_T) \Sigma_{T} \right)}{32 \pi^2 \Delta_B} \]

where

\[ \kappa_1 = \Delta_{G_b} \left( 2m^2_{b_r} - 2m^2_{t_L} + (v_d^2 - v_u^2) \Delta_{G_b} \right), \kappa_2 = \Delta_{G_i} \left( -2m^2_{t_L} + 2m^2_{t_R} + (v_d^2 - v_u^2) \Delta_{G_i} \right) \]

\[ \kappa_3 = \Delta_{G_b} \left( 2m^2_{b_r} - 2m^2_{t_L} + (v_d^2 - 3v_u^2) \Delta_{G_b} \right), \kappa_4 = \Delta_{G_i} \left( -2m^2_{t_L} + 2m^2_{t_R} + (v_d^2 - 3v_u^2) \Delta_{G_i} \right) \]

\[ \kappa_5 = \Delta_{G_b} \left( 2m^2_{b_r} - 2m^2_{t_L} + (3v_d^2 - v_u^2) \Delta_{G_b} \right), \kappa_6 = \Delta_{G_i} \left( -2m^2_{t_L} + 2m^2_{t_R} + (3v_d^2 - v_u^2) \Delta_{G_i} \right) \]

\[ \Delta_{G_b} = -3g_2^2 + 5g_3^2) / 12, \Sigma_{G_b} = (3g_2^2 - g_3^2) / 4, \Delta_{G_i} = (3g_2^2 - g_3^2) / 12, \Sigma_{G_i} = (g_2^2 + g_3^2) / 4 \]
\[ M_{23}^2 = m_3^2 v_3 \sin[\theta + \Phi] + \frac{3 (|h_b|^2 (v^2 \chi_5 - 2 v_6 \Delta B) \Delta_T I_b + |h_t|^2 \Delta_B (v^2 \chi_6 - 2 v_6 \Delta_T) I_t)}{32 \pi^2 v \Delta_B \Delta_T} \]
\[ - \frac{3 |h_t|^2 (4 v_2^2 v_6 \Delta_T (2 |h_t|^2 + \Sigma_{G_t}) - v_6 \chi_6 \Sigma_T + 2 v_6 \Delta_T \Sigma_T) I_b}{64 \pi^2 v \Delta_B^{3/2}} \ln \left( \frac{m_{t_1}^2}{m_{t_2}^2} \right) + \frac{3 |h_t|^2 v_6 \Sigma_T}{32 \pi^2 v} \ln \left( \frac{m_{t_1}^2 m_{t_2}^2}{v^4} \right) \]
\[ + \frac{3 |h_t|^2 (-2 v_6 \Delta_B \Sigma_B + v^2 (\chi_5 \Sigma_B - 4 v_6 \Delta_B \Sigma_{G_t})) I_b}{64 \pi^2 v \Delta_B^{3/2}} \ln \left( \frac{m_{b_1}^2}{m_{b_2}^2} \right) + \frac{3 |h_t|^2 v_6 \Sigma_B}{32 \pi^2 v} \ln \left( \frac{m_{b_1}^2 m_{b_2}^2}{v^4} \right). \]

\[ M_{22}^2 = \frac{\tilde{g}^2 v_2^2}{4} + m_3^2 v_3 \cos[\theta + \Phi] + \frac{3 (2 (\chi_5 + \chi_6) \Delta_B \Delta_T + v_6 \left( \chi_5^2 \Delta_B + (\chi_5^2 - 2 (\chi_10 + \chi_9) \Delta_B) \Delta_T \right))}{512 \pi^2 v \Delta_B \Delta_T} \]
\[ - \frac{3 |h_t|^2 v_6^2}{8 \pi^2} \ln \left( \frac{m_{t_1}^2}{m_{t_2}^2} \right) + \frac{3 (2 \chi_5 \Delta_B \Sigma_B + v_6 \left( \chi_5^2 - 2 \chi_9 \Delta_B \right) \Sigma_T - 8 v_6^2 \chi_5 \Delta_B \Sigma_{G_t})}{1024 \pi^2 v \Delta_B^{3/2}} \ln \left( \frac{m_{b_1}^2}{m_{b_2}^2} \right) \]
\[ + \frac{1024 \pi^2 v \Delta_B^{3/2}}{(2 |h_t|^2 + \Sigma_{G_t}) + 3 (2 \chi_6 \Delta_T + v_6 \left( \chi_6^2 - 2 \chi_10 \Delta_T \right)) \Sigma_T} \ln \left( \frac{m_{b_1}^2}{m_{b_2}^2} \right) \]
\[ + \frac{3 v_3^3 (\Delta_{G_t}^2 + (2 |h_t|^2 + \Sigma_{G_t})^2)}{64 \pi^2 v_3} \ln \left( \frac{m_{t_2}^2}{m_{t_1}^2} \right) + \frac{3 (v_3^3 (\Delta_{G_b}^2 + \Sigma_{G_b}^2) + 2 |h_t|^2 v_6 \Sigma_B)}{64 \pi^2 v_3} \ln \left( \frac{m_{b_1}^2 m_{b_2}^2}{v^4} \right). \]

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