Low energy constituent quark and pion effective couplings to external electromagnetic field and a weak magnetic field

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In this work, a compilation of effective electromagnetic field couplings to pions and constituent quarks and their effective interactions derived previously, including corrections to the NJL model, is presented. The particular case of a weak external magnetic field along the $\hat{z}$ direction is considered shortly and effective coupling constants are redefined to incorporate the weak-$B_0$ dependence. They correspond to corrections to well known pion-constituent quark couplings and to the NJL and vector NJL effective couplings that break isospin and chiral symmetries.

I. INTRODUCTION

Low energy effective models for hadrons are usually based on phenomenology and also general theoretical results and symmetries from QCD. Nambu-Jona-Lasinio-type NJL model are emblematic so that they are expected to describe important qualitative effects from QCD such as Dynamical Chiral Symmetry Breaking DChSB and the emergence of the chiral condensate in the QCD phase diagram. Effective field theories (EFT) have been developed and strengthened and they contribute for establishing these conceptual and calculational gaps between the two levels in the description of strong interactions systems. A large $N_c$ EFT that copes the large $N_c$ expansion and the constituent quark model was proposed [1] in Ref. [2]. This EFT is composed by the leading large $N_c$ terms for constituent quarks coupled to pions and constituent gluons, besides the leading terms of chiral perturbation theory. However, in spite of the phenomenological successes, they do not provide microscopic first ground numerical predictions for the low energy coefficients. Whereas the light mesons sector have been investigated within global color-type models (GCM) and NJL models in the 1980’s and 1990’s [3–5], the baryon interactions to mesons have faced more difficulties. The constituent quark model framework assumes these baryon effective interactions are equivalent to the constituent quark effective interactions. In [6–10] we have proposed a QCD mechanism by which these baryons-light mesons interactions emerge. The method will not be explained in the present work and the reader will find all details in these references. It considers a Fierz transformation from a gluon mediated quark interaction to make possible to exploit the whole flavor structure with the introduction of the light mesons fields by means of auxiliary field method. In the present work, a set of resulting interactions are shown mainly for the case when the system undergoes interactions with a weak background external electromagnetic field. The recent interest on magnetic field effects on hadron dynamics [11] lead us to show these interactions can reduce to those with a weak magnetic field in a simple way when considering only the first Landau orbit. The resulting terms can be calculated perturbatively for increasing strength of the magnetic field as discussed in [12].

All the effective couplings presented below are derived from the following leading term of QCD effective action:

$$Z = N \int D[\bar{\psi}, \psi] e^{i \int \bar{\psi} \left( iD - m \right) \psi - e^2 \int \bar{\psi} j^a_\mu \left( x \right) R_{\mu \nu}^{bc} \left( x - y \right) \bar{\psi} \gamma_\nu \left( y \right) + \bar{\psi} J \psi } ,$$

where the color quark current is $j^a_\mu = \bar{\psi} \lambda^a \gamma_\mu \psi$, and: $D_\mu = \partial_\mu - ieQ A_\mu$ with the diagonal matrix $\hat{Q} = diag(2/3, -1/3)$. In several gauges the dressed gluon kernel is written in terms of $R_T(k)$, $R_L(k)$ In particular it will be assumed and required that this dressed gluon propagator provides enough strength for generating DChSB, so that a chiral condensate and the corresponding large effective constituent quark mass appear. This can only be achieved by incorporating to some extent the non Abelian gluon dynamics.

II. LIGHT MESONS AND CONSTITUENT QUARKS

The sea quark determinant is obtained in terms of the light mesons fields and constituent quark currents. By neglecting light vector mesons that are considerably heavier for the low energy regime, and by performing a chiral
rotation that eliminates the scalar degree of freedom, the determinant is given by:

\[ S_{\text{eff}} = i \text{Tr} \ln \left\{ i \left( S_c^{-1}(x-y) + \sum_q a_q \Gamma_q j_q(x,y) \right) \right\}, \tag{2} \]

where \( \text{Tr} \) stands for traces for all discrete internal indices and integration of spacetime coordinates and the quark kernel can be written as

\[ S_c^{-1}(x-y) = S_0^{-1}(x-y) + \Xi(x-y), \tag{3} \]

where \( S_0^{-1}(x-y) = (i\partial - M^*) \delta(x-y) \) and \( M^* \) is the resulting effective quark mass from the gap equation for the DChSB solution, \( M^* = m + \langle S \rangle \). The following quantity was used:

\[ \Xi(x,y) = F(P_R U + P_L U^\dagger) \delta(x-y) = F(P_R e^{i\vec{r} \cdot \vec{s}} + P_L e^{-i\vec{r} \cdot \vec{s}}) \delta(x-y), \]

where \( P_R/L \) are the chiral right/left hand projectors. The constituent quark degrees of freedom appear in terms of quark flavor currents \( j_q(x,y) \), and with the Pauli matrices for SU(2) isospin and Dirac matrices can be written as:

\[ \sum_q a_q \Gamma_q j_q(x,y) = 2 R(x-y) \left[ \bar{\psi}(y)\psi(x) + i\gamma_5\sigma_i \bar{\psi}(y)i\gamma_5\sigma_i \psi(x) + \bar{\psi}(y)\psi(x) + i\gamma_5\bar{\psi}(y)i\gamma_5\psi(x) \right] - \bar{R} \left[ \gamma_\mu \sigma_i \left[ \bar{\psi}(y)\gamma_\mu \sigma_i \psi(x) + i\gamma_5\bar{\psi}(y)i\gamma_5\gamma_\mu \sigma_i \psi(x) \right] - \gamma_\mu \left[ \bar{\psi}(y)\gamma_\mu \psi(x) + i\gamma_5\bar{\psi}(y)i\gamma_5\gamma_\mu \psi(x) \right] \right]. \tag{4} \]

Saddle point equations are calculated for each of the auxiliary field and only the scalar auxiliary field develops a classical counterpart by neglecting the eventual magnetic field.

### III. FIRST ORDER CONSTITUENT PHOTON-QUARK-PION EFFECTIVE COUPLINGS

From a very quark and gluon effective mass expansion the leading effective constituent quark-pion terms and their leading (dipolar type) couplings with the external photon field arise in the large quark mass and local very longwavelength limit:

\[ \mathcal{L}_{Q\pi} = g_2 F F_F Z_+ j_s + g_1 F F_F (\sigma_i Z_-) j_{ps}^i + 2g_V \partial_{\mu} F_F (\sigma_i \partial_{\mu} Z_-) j_{i,\mu}^V + 2g_A \partial_{\mu} F_F (\sigma_i \partial_{\mu} Z_-) j_{A,\mu}^i \tag{5} \]

\[ \mathcal{L}_{Q\pi A} = M_F F_{\mu\nu} F_{\mu\nu} j_s + g_{vmd} A_\mu j_{i,\mu}^A + g_{F-J} F F_{\mu\nu} (Q, Z_-) j_{\mu\nu} \tag{6} \]

\[ + g_{F-J} F F_{\mu\nu} (Q, Z_-) j_{\mu\nu}^A \]

where \( Z_{\pm} = \frac{1}{2}(U \pm U^\dagger) \). In the expression \([5] \ g_1, g_2\) correspond respectively to the usual pseudoscalar and scalar couplings of one and two pions to a pseudoscalar/scalar quark currents. \( g_V \) the two pion coupling to a vector quark current and \( g_A \) the usual axial coupling. It has been found \( g_V = g_A \) at this level and the Goldberger Treiman relation \([15] \) is satisfied \([8] \). In all these expressions \( F_F \) stands for the trace in isospin indices. In expression \([6] \) the leading couplings to an external photon field are shown. The canonical normalization of the pion field requires a multiplicative factors \( 1/F \) to redefine coupling constants. The expresions for the effective coupling constants in terms of the components of quark and gluon kernels for the case of zero external magnetic field can be found in \([6] [9] \) and they are not presented here.

If the above external photon correspond to a weak external magnetic field \( A_\mu = B_0/2(0,-y,x,0) \) the above expression \([6] \) except the VMD (vector meson dominance) term, reduce to the following weak magnetic field dependent expression:

\[ \mathcal{L}_{Q\pi-A} = M_B j_s + \bar{g}_{F_{J\pi}} F_F \{ Q, Z_+ \} j_s + i\bar{g}_{F_{ps}} F_F \{ Q, Z_- \} j_{ps}^i \tag{7} \]

where the expressions for the effective coupling constants \( \bar{g} \) by accounting the leading Landau orbit were given in \([9] \) and the stronger magnetic field case will be analysed elsewhere.

The traces in flavor indices of the Pauli matrices with the matrix \( Q \) given after expression \([1] \) were computed for the leading weak pion field: \( U \simeq 1 + i\vec{r} \cdot \vec{p} + \ldots \) and \( U^\dagger \simeq 1 - i\vec{r} \cdot \vec{p} + \ldots \). The leading terms of the effective couplings
The following isospin coefficients:

\[ T \int \text{terms depending on } \delta_{ij} \delta_{kj} \delta_{ik} \]

where the coupling constants \( g_{s,B} \), \( g_{ps,B} \), \( g_{4v,B} \), \( g_{4v,B} \), \( g_{4v,B} \), \( g_{4v,B} \), \( g_{4v,B} \), \( g_{4v,B} \), \( g_{4v,B} \), \( g_{4v,B} \), \( g_{4v,B} \) are magnetic field dependent coupling constants expressed as functions of components of the quark and gluon kernels given in \( [8] \). The effective couplings with \( g_{s,B}, g_{ps,B} \) are chiral symmetry breaking ones and they were discussed in \( [7] \). The last term represents the photon coupling to a neutral vector meson rho coupling \( \Gamma_{\mu} \). The following notation was adopted in the terms depending on the coefficients \( c_i \) with operators \( \Gamma_i \): \( c_1 (\bar{\psi} \Gamma_1 \psi)^2 = c_1 (\bar{\psi} \Gamma_1 \psi)^2 + c_2 (\bar{\psi} \Gamma_2 \psi)^2 + c_3 (\bar{\psi} \Gamma_3 \psi)^2 \), being defined the following isospin coefficients: \( c_1 = -\frac{3}{5}, c_2 = \frac{4}{5} \) and \( c_3 = \frac{5}{8} \). These expressions also make clear the different electromagnetic couplings of currents of charge quarks, although it also presents different channels of the interaction in which neutral quark currents also appear.
TABLE I: In the first column the following set of values are displayed $M^*$ for given $\Lambda$, and $\alpha$, being that $\alpha$ is a factor representing quark gluon coupling constant. This factor was chosen to reproduce the value of the pion vector or axial coupling constant $g_v, \alpha_a = 1$ and it multiplies the gluon propagator. The gluon propagator taken from Ref. [12]. From the second to the last columns, values for some of the effective coupling constants and parameters from the expressions presented in [9] for the usual pion field definition in terms of the functions $U, U^\dagger$. The last two columns show two of the quark-quark effective coupling constant correction due to a very weak magnetic field divided by an estimate for the $g_4$ NJL model coupling constant obtained from the same method. (e.v.) in the last line stands for some experimental or expected values. In this, it was assumed the constituent quark mass of pion should be half of the pion mass 140MeV and the constituent quark mass one third of the nucleon mass 939MeV. The larger values of $M^*$ are obtained as consequence of the magnetic catalysis due to $B_0$.

| $M^*$ (GeV) | $h_a$ (GeV) | $\Lambda$ (MeV) | $M_3h_a$ (MeV) | $g_vh_a$ (GeV) | $g_ah_a$ (GeV) | $g_{vmd}h_a$ (GeV) | $\frac{g_{vmd}h_a}{M_3^2}$ (MeV$^{-2}$) |
|------------|-------------|-----------------|----------------|----------------|----------------|-----------------|------------------|
| 0.45 $\frac{1}{2\pi}$ | 0.600 | 2760 | 0.9 | 1 | 1556 | 4.4 | 1.1 | 3.0 | $1.0 \times 10^{-4}$ | $1.4 \times 10^{-4}$ |
| 0.41 $\frac{1}{2\pi}$ | 0.600 | 2672 | 1 | 1 | 1289 | 3.9 | 1.2 | 2.9 | $1.0 \times 10^{-4}$ | $1.3 \times 10^{-4}$ |
| 0.30 $\frac{1}{2\pi}$ | 0.575 | 1752 | 2.6 | 1 | 628 | 1.8 | 1.1 | 0.2 | $1.1 \times 10^{-4}$ | $1.5 \times 10^{-4}$ |
| 0.07 $\frac{1}{2\pi}$ | 0.450 | 335 | 9.3 | 1 | 18 | 0.2 | 0.6 | 0.2 | $-2.0 \times 10^{-5}$ | $1.0 \times 10^{-6}$ |
| e.v. | 0.07 | 1 | - | - | - | - | - | - | - | - | - |

V. SUMMARY AND FINAL REMARKS

A collection of the leading electromagnetic couplings to pions and constituent quarks was presented extracted from [8, 9]. A large effective quark mass expansion for the sea quark determinant yielded different known pion effective couplings to quarks: vector, axial, pseudoscalar and scalar. The corresponding couplings to the electromagnetic field explicitly break chiral and isospin symmetries, and they have been considered for $(eB_0/M^2) << 1$. It is interesting to note that, in the leading order terms, the weak magnetic field does not mix the contribution of each of the gluon propagator components, transversal or longitudinal, what has been shown by considering only the leading contribution from the leading Landau orbit according to [12]. It is remarkable that the best agreement for the known effective parameters in the Table were obtained for a quark effective mass 70MeV that is half of the pion mass. This is the effective mass obtained from the gap equation of the type of the GCM or NJL models, and it was associated to sea quarks. The constituent quark effective mass was associated rather to $M_3$ that is an effective parameter in the resulting effective model. A more complete account of the Landau orbits for the stronger magnetic field cases will be presented elsewhere.

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