Short-time decoherence of Josephson charge qubit nonlinearly coupling with its environment

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November 15, 2018

Abstract
At first, we generally investigate the short-time decoherence of a qubit nonlinearly coupling with a bath. The measure of the decoherence is chosen as the maximum norm of the deviation density operator. Then we concretely investigate the Josephson charge qubit (JCQ) model. It is shown that at the temperature \( T \sim 30 \text{mK} \), the loss of fidelity (due to decoherence) of the JCQ is bigger than the DiVincenzo low decoherence criterion. The decoherence will decrease with the decrease of the experimental temperature. When the temperature decreases to \( T \sim 0.3 \text{mK} \) the DiVincenzo low decoherence criterion can be satisfied.

PACS numbers: 03.65.Ta, 03.65.Yz, 85.25.Cp

Keywords: Short-time decoherence, Josephson junction, quantum computation

1 Introduction

David DiVincenzo put forward a low decoherence criterion for the candidates of quantum computing hardware to be satisfied [1]. An approximate benchmark of the criterion is a fidelity loss no more than \( \sim 10^{-4} \) per elementary quantum gate operation. In general, the loss of the fidelity results from quantum leakage as well as decoherence. The Josephson junction is considered to be a promising physical realization of the qubit. It has been shown that the quantum leakage of Josephson charge qubit (JCQ) is not serious [2]. So it is very interesting to investigate the decoherence of the JCQ. To perform the quantitative study of the decoherence for a qubit, in general one needs to solve the quantum dynamical problem of the qubit coupling with its environment. However, the solutions of the coupling system can not be obtained easily, usually a kind of approximation must be appealed. The Markovian approximation [3] is the most familiar one. This approximation has been used to study the decoherence of qubits in

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last years [4, 5]. But it is not a suitable tool for researching the short-time decoherence of the qubits [6]. However, the short-time processes is very important because almost all of the gate operations of the quantum devices are short time. Fortunately, a short-time approximation scheme for the reduced density matrix has been developed recently [6]. Another core of the issue to investigate the decoherence of qubit is to make certain what the model of the environment and the coupling form of the qubit with its environment are. In general, according to the characteristic of the environment we set it a bosonic (or fermi) bath. The coupling of the qubit-bath can be supposed linear because the interaction between the qubit and the modes of the bath is very weak. Under these suppositions (linear coupling and short-time approximation), V. Privman et al. [6, 7, 8] investigated the decoherence of many kinds of open systems and we also investigated a concrete JCQ model in Ref. [9]. However, the nonlinearly coupling forms of qubit-bath certainly exists [13]. In this Letter, we firstly introduce a model that JCQ nonlinearly couples with its bath and then use the short-time approximation to investigate the decoherence of the JCQ in the qubit-bath model.

2 Nonlinearly coupling qubit-bath model

Though the combined effects of the bath modes on the qubit may be large enough, the coupling of the qubit with each of the modes of the bath is very weak and the qubit-bath can be considered a linear coupling. However, due to the development of low temperature technique and fabrication technique of high quality quantum systems the nonlinearly coupling qubit-bath models can not be excluded in the devices of quantum. The linearly coupling qubit-bath model is analogous to the amplitude damping model. In this Letter, we introduce a nonlinearly coupling qubit-bath model which corresponds to the phase damping model [10]. We begin our investigation by considering a general open quantum system

\[ H = H_S + H_B + H_I. \] (1)

Here, \( H_S \) is the Hamiltonian of the investigated quantum system; \( H_B \) is the Hamiltonian of the bath; and \( H_I \) describes the interaction of the system and the bath. The bath is traditionally modeled by a large number of uncoupled harmonic oscillators (with ground energy shifted to zero),

\[ H_B = \sum_j \omega_j b_j^\dagger b_j. \] (2)

If the interaction is linear, in the rotating-wave approximation the interaction Hamiltonian is

\[ H_I = \Lambda \sum_j g_j b_j^\dagger + \Lambda^\dagger \sum_j g_j^* b_j, \] (3)

with the interaction constants \( g_j \). If the investigated system is a two-level system then \( \Lambda \to \sigma_-, \Lambda^\dagger \to \sigma_+ \). The short-time decoherence of this model has been
investigated in Ref.[7]. Suppose the bath be ohmic then
\[ g^2(\omega) D(\omega) = \eta \omega \exp(\omega/\omega_c), \]
where \( D(\omega) \) is the density of states of the bath and \( \omega_c \) is the cutoff frequency. For the model of JCQ coupling with the bath (see Fig.1 of Ref.[12]), the damping coefficient can be calculated with
\[ \eta = \frac{R}{R_Q} \left( \frac{C_t}{C_J} \right)^2, \]
where \( R_Q = (2e)^2/h \) is the (superconducting) resistance quantum. When we investigate the short-time decoherence of this model, we can directly use \( g^2(\omega) D(\omega) \) and do not need solving \( D(\omega) \) then \( g(\omega) \).

As well known that the interaction Hamiltonian of a oscillator in the phase damping bath is \[ H_I = \sum_j \left( g_j b_j^\dagger b_j \right) a^\dagger a, \]
which is a nonlinear coupling. If we replace the oscillator with a two-level system, namely, \( a^\dagger \rightarrow \sigma_+, a \rightarrow \sigma_- \), then the total Hamiltonian becomes
\[ H = H_S + \sigma_z \sum_j g'_j b_j^\dagger b_j + \sum_j \omega'_j b_j^\dagger b_j, \]
where \( g'_j = g_j/2 \) and \( \omega'_j = g_j/2 + \omega_j \). This Hamiltonian models a kind of nonlinear coupling of the bath to the exposed qubit. This bath can be considered resulting from the fluctuations of the Josephson nanocircuits and the surrounding circuits. A more general model which includes other sources of decoherence, such as quasiparticle tunneling, fluctuating background charges and flux noise has been put forward \[13 \] \[14 \]. Suppose the bath is also ohmic then Eq.(4) will be held. But in this case, in order to investigate the decoherence of the qubit we must find out \( g(\omega_j) \) through solving \( D(\omega_j) \). We suppose that the initial state of the environment state is \( \Theta \) which is the product of the bath modes density matrices
\[ \theta_j = \frac{e^{-\beta M_j}}{\text{Tr}_j \left( e^{-\beta M_j} \right)}, \]
where \( M_j = \omega_j b_j^\dagger b_j \). Because the degrees of freedom of the bath are much larger than that of the qubit, in the short time we can take the bath’s states unchanged and consider the bath keeping in their thermal equilibrium state. We take the bath to be a mix of infinite harmonic oscillators at temperature \( T \). Thus, the density of state of the bath reads
\[ D(\omega_j) = \frac{1 - \exp (-\hbar \omega_j)}{\exp (\beta \hbar \omega_j)} \approx \hbar \omega_j, \]
for $kT \ll \hbar \omega_j$. Here $\beta = 1/kT$, $k$ is the Boltzmann constant. By using Eq. (4) we have

$$g'(\omega_j) = \frac{1}{2} \sqrt{\frac{\hbar \eta \omega_j}{D(\omega_j)}} \exp(\omega_j/2\omega_c)$$

$$\approx \frac{1}{2} \sqrt{\eta} \exp(\omega_j/2\omega_c).$$

(10)

3 Short-time dynamics of the qubit in the nonlinear qubit-bath model

In this section we will investigate the general expression for the time evolution operator of the qubit in the nonlinear qubit-bath model within the short time. The definition of the short-time concept has been stated in Ref. [6] [7] [8]. The evolution operator is

$$U = e^{-iH\tau/\hbar} = e^{-i(H_s+H_I+H_B)\tau/\hbar}.$$  

(11)

In the following we set $t = \tau/\hbar$. Due to non-conservation of $H_s$ in this system, the evolution operator cannot be in a general way expressed as $e^{-iH_s t} e^{-i(H_I+H_B) t}$. But in the sort-time approximation, the operator can be approximately expressed as [15] [16]

$$U = e^{-iH_s t/2} e^{-i(H_I+H_B) t} e^{-iH_s t/2} + o(t^3).$$  

(12)

It has been proved that the expression is accurate enough for the time being short to the characteristic time. So the density matrix elements of the reduced density matrix $\rho(t)$ in the basis of operator $H_s$ can be expressed as

$$\rho_{mn}(t) = \text{Tr}_B \langle \phi_m | e^{-iH_s t/2} e^{-i(H_I+H_B) t} e^{-iH_s t/2} R(0) e^{iH_s t/2} e^{i(H_I+H_B) t} e^{iH_s t/2} | \phi_n \rangle.$$  

(13)

Here, we suppose that the initial state of the system is $R(0) = \rho(0) \otimes \Theta$, where $\rho(0)$ is the initial state of the qubit and $\Theta = \prod \theta_j$. By using the completeness relation of the eigenstates of $H_s$, $\sum |\rangle \langle | = 1$, we have

$$e^{\pm iH_s t/2} = \sum_{j=0,1} e^{\pm it\lambda_j} |\varphi_j \rangle \langle \varphi_j|,$$

(14)

where $\lambda_j (j = 0, 1)$ and $|\varphi_j \rangle$ are eigenvalues and eigenstates of Hamiltonian $H_s$. Similarly, we have

$$e^{\pm i(H_I+H_B)t} = \sum_{j=0,1} \exp \left\{ \sum_j \left[ \pm i (g'_j \chi_j + \omega'_j) b_j^\dagger b_j t \right] \right\} |\psi_j \rangle \langle \psi_j|,$$

(15)

where $\chi_j (j = 0, 1)$ and $|\psi_j \rangle (j = 0, 1)$, are the eigenvalues and eigenstates of operator $\sigma_z$. So we have

$$\rho_{mn}(t) = \text{Tr}_B \left\{ \langle \varphi_m | \sum_{\alpha=0,1} e^{-it\lambda_\alpha} |\varphi_\alpha \rangle \langle \varphi_\alpha| \right\}.$$
\[
\sum_{\xi=0,1} \exp \left\{ \sum_j \left[ -i \left( g'_j \chi_\xi + \omega'_j \right) b_j^\dagger b_j t \right] \right\} |\psi_\xi \rangle \langle \psi_\xi |
\]
\[
\sum_{\beta=0,1} e^{-i t \lambda_\beta} |\varphi_\beta \rangle \langle \varphi_\beta | \sum_{p,q=0,1} |\varphi_p \rangle \langle \varphi_q |
\]
\[
\times \rho_{pq}(0) \prod_j \theta_j \sum_{\mu=0,1} e^{it \lambda_\mu} |\varphi_\mu \rangle \langle \varphi_\mu |
\]
\[
\sum_{\nu=0,1} e^{it \lambda_\nu} |\varphi_\nu \rangle \langle \varphi_\nu | \}
\]
\[
\sum_{\nu=0,1} e^{it \lambda_\nu} |\varphi_\nu \rangle \langle \varphi_\nu |
\]

namely,
\[
\rho_{mn}(t) = \sum_{\alpha,\beta,\xi,\mu,p,q,\nu=0,1} e^{it(\lambda_n + \lambda_\nu - \lambda_\alpha - \lambda_\beta)} \langle \varphi_m | \varphi_\alpha \rangle \langle \varphi_\xi | \varphi_\beta \rangle \langle \varphi_p | \varphi_q \rangle \langle \varphi_\mu | \varphi_\nu \rangle \langle \psi_\xi | \psi_\alpha \rangle \langle \psi_\mu | \psi_\nu \rangle \langle \varphi_n | \varphi_\mu \rangle \rho_{pq}(0) \operatorname{Tr}_B \left\{ \exp \left\{ \sum_j \left[ -i \left( g'_j \chi_\xi + \omega'_j \right) b_j^\dagger b_j t \right] \right\} \right\} \prod_j \theta_j \exp \left\{ \sum_j \left[ i \left( g'_j \chi_\xi + \omega'_j \right) b_j^\dagger b_j t \right] \right\}.
\]

Taking the \( j \) term in \( \operatorname{Tr}_B \{ \ldots \} \), and denoting it by \( \operatorname{Tr}_B \{ \ldots \} \), we have
\[
\operatorname{Tr}_B \{ \ldots \} = \operatorname{Tr}_B \left\{ \exp \left\{ -i \left( g'_j \chi_\xi + \omega'_j \right) b_j^\dagger b_j t \right\} \theta_j \exp \left\{ i \left( g'_j \chi_\xi + \omega'_j \right) b_j^\dagger b_j t \right\} \right\} = \operatorname{Tr}_B \left\{ \exp \left\{ -i \left( g'_j \chi_\xi + \omega'_j \right) b_j^\dagger b_j t \right\} \exp \left( -\beta \omega_j b_j^\dagger b_j \right) \left[ \operatorname{Tr} \exp \left( -\beta \omega_j b_j^\dagger b_j \right) \right]^{-1} \exp \left\{ i \left( g'_j \chi_\xi + \omega'_j \right) b_j^\dagger b_j t \right\} \right\}.
\]
Because
\[
\exp \left\{ x b_j^\dagger b_j t \right\} =: \exp \left\{ (e^x - 1) b_j^\dagger b_j \right\}:,
\]
where \( : \) : denotes the normally ordering form. By using the relationship of the completeness of the coherent states
\[
\int \frac{d^2 z}{\pi} |z \rangle \langle z | = 1,
\]
\[
\int \frac{d^2 \xi}{\pi} |\xi \rangle \langle \xi | = 1,
\]
\[
\int \frac{d^2 \varsigma}{\pi} |\varsigma \rangle \langle \varsigma | = 1,
\]
(20)
we have

\[
TRB \left[ \ldots \right]_j = \frac{1}{Z_j} \int \frac{d^2 z}{\pi} |z \rangle : \exp \left[ \left( e^{-i \phi_j \chi + \omega_j^t} - 1 \right) b_j^\dagger b_j \right] : \exp \left[ \exp \left[ \left( e^\frac{i}{\pi} \sum_j \left( e^{-\beta \omega_j} - 1 \right) b_j^\dagger b_j \right) : \exp \left[ \left( e^{-i \phi_j \chi + \omega_j^t} - 1 \right) b_j^\dagger b_j \right] : |z \rangle \right] \right.
\]

\[
= \frac{1}{Z_j} \int \frac{d^2 z}{\pi} \int \frac{d^2 \xi}{\pi} \int \frac{d^2 \varsigma}{\pi} \exp \left( -|z|^2 - |\xi|^2 - |\varsigma|^2 \right) \exp \left[ e^{-i \phi_j \chi + \omega_j^t} z^* \xi + e^{-\beta \omega_j} \xi^* \varsigma + e^{i \phi_j \chi + \omega_j^t} \varsigma^* z \right],
\]

(21)

where \( Z_j = (1 - e^{-\beta \omega_j})^{-1} \). By using the formula

\[
\int \frac{d^2 z}{\pi} \exp \left( a |z|^2 + bz + cz^* + f z^2 + g z^* z \right) = \frac{1}{\sqrt{a^2 - 4fg}} \exp \left[ \frac{-abc - b^2 g - c^2 f}{\sqrt{a^2 - 4fg}} \right],
\]

(22)

we can obtain the result of \( \rho_{mn}(t) \) as

\[
\rho_{mn}(t) = \sum_{\alpha, \beta, \xi, \nu, \mu, \rho, \varphi, \sigma} e^{it(\lambda_\varphi - \lambda_\sigma)} \langle \phi_m | \varphi_\alpha \rangle \langle \varphi_\alpha | \psi_\xi \rangle \langle \psi_\xi | \varphi_\beta \rangle \langle \varphi_\beta | \varphi_\rho \rangle \langle \varphi_\rho | \varphi_\mu \rangle \langle \varphi_\mu | \psi_\nu \rangle \langle \psi_\nu | \varphi_n \rangle \rho_{pq}(0) \prod_{j=1}^N \frac{1 - e^{-\beta \omega_j}}{1 - e^{i \phi_j \chi + \omega_j^t} e^{-\beta \omega_j}},
\]

(23)

It should be noted that it is not necessary to take \( N \) too large in the following numerical simulations because the \( \omega_j \) increase with \( j \) and the \( e^{-\beta \omega_j} \) decrease with \( \omega_j \) exponentially. By using Eq. (23) we can investigate various characteristics of the nonlinear qubit-bath systems. By using it, in the following we shall study the short-time decoherence of the JCQ.

4 Measure of decoherence for a qubit

In order to investigate the decoherence of the qubits, one must designate the measure to be used. There are many choices for this purpose. When the evolution time is very long, the qubit interacting with the large bath will fall into the thermal equilibrium at temperature \( T \). In this case, Markovian type approximation can be used to quantify the decoherent processes and it usually yields the exponential decay of the density matrix elements in the energy basis of the Hamiltonian \( H_s \). In this time scale the measures of entropy and the first entropy can be used to quantify the decoherence. But the decoherence of the qubit gates cannot be characterized by this methods because the time of the elementary quantum gate operations are much shorter than the thermal relaxation time. It has been shown that the norm \( \| \sigma \|_\lambda \) is useful for describing the
decoherence of the short-time evolutions. In this section, we will review the measure according to [6]. We set $\sigma$ is the deviation operator defined as

$$\sigma (t) = \rho (t) - \rho^i (t),$$

(24)

where $\rho (t)$ and $\rho^i (t)$ are density matrixes of the “real” evolution (with interaction) and the “ideal” one (without interaction) of the investigated system. $||\sigma||_\lambda$ is defined as

$$||\sigma||_\lambda = \sup_{\psi \neq 0} \sqrt{\langle \psi | \sigma | \psi \rangle \langle \psi | \psi \rangle}.$$  

(25)

For a qubit, the norm can be given by

$$||\sigma||_\lambda = \sqrt{|\sigma_{10}|^2 + |\sigma_{11}|^2}.$$  

(26)

In general, the norm $||\sigma||_\lambda$ increases with time, reflecting the decoherence of the system. However, it is oscillating at the system’s internal frequency. Thus, the decohering effect of the bath is better quantified by the maximal operator norm, $D (t)$ which is defined as

$$D (t) = \sup_{\rho(0)} (||\sigma (t, \rho (0))||_\lambda).$$

(27)

It has been shown that the measure can rightly describe the short-time decoherence of qubits. In the following we will use this measure to investigate the short-time decoherence of JCQ in nonlinearly coupling JCQ-bath model.

## 5 Short-time decoherence of the JCQ

In this section we firstly review the JCQ model. The single JCQ Hamiltonian is

$$H_R = E_{ch} (n - n_g)^2 - E_J \cos \varphi.$$  

(28)

Here, $E_{ch} = 2e^2 / (C_g + C_J)$ is the charging energy; $E_J = \Phi_0 I_c / 2\pi$ is the Josephson coupling energy, where $I_c$ is the critical current of the Josephson junction, $\Phi_0 = h c / 2 e$ denotes the flux quantum; $n_g = C_g V_g / 2e$ is the dimensionless gate charge, where $C_g$ is the gate capacitance, $V_g$ the controllable gate voltage. When the Josephson coupling energy $E_J$ is much smaller than the charging energy $E_{ch}$, and both of them are much smaller than the superconducting energy gap $\Delta$, the Hamiltonian Eq.(28) can be parameterized by the number of the Cooper pairs $n$ on the island as

$$H_R = \sum_n \left\{ E_{ch} (n - n_g)^2 |n\rangle \langle n| - \frac{1}{2} E_J (|n\rangle \langle n| + |n+1\rangle \langle n+1|) \right\}.$$  

(29)
When the temperature $T$ is low enough the system can be reduced to a two-state system (qubit) because all other charge states have much higher energy and can be neglected. So the Hamiltonian of the system can \textit{approximately} reads

$$H_s = -\frac{1}{2} B_z \sigma_z - \frac{1}{2} B_x \sigma_x.$$ (30)

Here, $B_z = E_{ch} (1 - 2n_g)$ and $B_x = E_J$. Eq. (30) is similar to the ideal single qubit model [18], but it can be modulated only by changing one parameter $B_z$. However, changing the parameter $B_z$ (through switching the gate voltage) one can perform the desirable one-bit operations. If, for example, one chooses the idle state far to the left from the degeneracy point, the eigenstate lose to $|0\rangle$ and $|1\rangle$. Then switching the system suddenly to the degeneracy point for a time $t$ and back produces a rotation in spin space [18],

$$U_J = \exp \left( \frac{iE_J t}{2} \sigma_z \right) = \begin{pmatrix} \cos \frac{tE_J}{2} & \sin \frac{tE_J}{2} \\ \sin \frac{tE_J}{2} & \cos \frac{tE_J}{2} \end{pmatrix}. \quad (31)$$

This can be obtained by modulating the gate voltage and making $B_z = E_{ch} (1 - 2n_g) = 0$. Thus, $H_S = -\frac{B_x}{2} \sigma_x$, and Eq. (7) becomes

$$H = \frac{B_x}{2} \sigma_x + \sigma_z \sum_j g_j^{\dagger} b_j b_j + \sum_j \omega_j^{\dagger} b_j b_j. \quad (32)$$

In the following, we will calculate the $\| \sigma (t) \|$ and the decoherence $D(t)$ of JCQ in the system $\{18\}$. In the calculation, three pure initial states are chosen, they are $|\phi_0\rangle = (1, 0)^T$, (corresponding to points in the following Figs.); $|\phi_1\rangle = \left( \frac{\sqrt{3}}{2}, \frac{1}{2} \right)^T$, (corresponding to above lines); $|\phi_2\rangle = \left( \frac{1}{2}, \frac{\sqrt{3}}{2} \right)^T$, (corresponding to below lines), [because it has been shown that evaluation of the supremum over the initial density operators in order to find $D(t)$, see Eq. (27) one can do over only pure-state density operators [6]]. Here, the eigenstates and eigenvalues of $H_s$ are

$$|\varphi_0\rangle = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle), \quad \lambda_0 = \frac{B_x}{2},$$

$$|\varphi_1\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle), \quad \lambda_1 = -\frac{B_x}{2}. \quad (33)$$

The eigenstates and eigenvalues of $\sigma_z$ are

$$|\psi_0\rangle = |0\rangle, \quad \chi_0 = 1,$$

$$|\psi_1\rangle = |1\rangle, \quad \chi_0 = -1. \quad (34)$$

Thus, by using Eq. (28) we can obtain

$$\sigma_{11} = \frac{1}{4} (\rho_{00} - \rho_{11}) (2 - W_1 - W_2) + \frac{1}{2} \rho_{10} \sin \frac{tE_J}{2} (W_2 - W_1), \quad (35)$$
\[
\sigma_{10} = \frac{1}{4} (\rho_{00} - \rho_{11}) \, e^{\frac{i}{2} E_J t} (W_1 - W_2) \\
+ \frac{1}{4} \rho_{10} \left[ 2 \left( 1 + e^{i E_J t} \right) + (W_1 + W_2) \left( e^{i E_J t} - 1 \right) \right],
\]
(36)

where
\[
W_1 = \prod_{j=N_0}^{N} \frac{-1 + e^{-\beta \omega_j}}{-1 + e^{i 2 g_j^* t - \beta \omega_j}}, \quad W_2 = \prod_{j=N_0}^{N} \frac{-1 + e^{-\beta \omega_j}}{-1 + e^{-i 2 g_j^* t - \beta \omega_j}}.
\]
(37)

In the following, we numerically investigate the decoherence of the J CQ in this model. We choose \(E_J = 51.8 \mu ev\) according to [19], and \(T = 30 mK\). Suppose the frequencies of the harmonic oscillators made up of bath are range from \(\omega_{\text{low}} = 1 \text{MHz}\) to \(\omega_{\text{high}} = 20 \text{MHz} = \omega_c\) (corresponding \(N_0 = 1, N = 20\)). Thus, we can plot the norms \(\|\sigma\|_\lambda\) versus time \(t\) in Fig.1. It is shown that when the initial state is \(\rho(0) = |\phi\rangle_0 \langle \phi|\), \(\|\sigma(t)\|_\lambda\) becomes maximum and it equals to \(D(t)\) (plotted by points in the Figs.). We denote the low decoherence \((D(t) \leq 10^{-4})\) time \(t_{ld}\). From Fig.1b we obtain \(t_{ld}^1 \approx 8 \times 10^{-5} \times 6.582 \times 10^{-10} s = 5.266 \times 10^{-2} ps\).

We denote the elementary gate operation time, the characteristic time \(t_g\). In this case, \(t_g^1 = \hbar/E_J = 1.27 \times 10^3 ps\). Thus, \(t_{ld}^1 \ll t_g^1\). It shows that at the temperature \(T = 30 mK\) the present setup of the JCQ cannot be taken as the qubit for quantum computation because it do not satisfy the DiVincenzo low decoherence criterion [1]. However, if we make the temperature \(T\) be lowered, the decoherence will decreases. For example, setting \(T = 0.3 mK\), and keeping \(E_J = 51.8 \mu ev\), we can plot the norms \(\|\sigma\|_\lambda\) versus time \(t\) in Fig.2. In this case we can obtain \(t_{ld}^2 \approx 2 \times 10^{-2} \times 6.582 \times 10^{-10} s = 13.16 ps\) and \(t_g^2 = t_g^1 = 12.7 ps\), where \(t_{ld}^2 \gg t_g^2\). It is shown that when the temperature decreases to \(T = 0.3 mK\), within the whole time of elementary gate operation, \(D(t) \leq 10^{-4}\). Theoretically, in the lower temperature the JCQ in nonlinearly coupling JCQ-bath becomes an ideal qubit for making quantum computer because it satisfy the DiVincenzo low decoherence criterion.

**Fig.1a, Fig.1b**

**Fig.1:** Norms \(\|\sigma\|_\lambda\) versus time \(t\), (a) in a longer time comparing to the elementary gate operation time; (b) in the low decoherence \((D(t) \leq 10^{-4})\) time. Here, the points and lines correspond to different initial states (see the explanation in text), \(E_J = 51.8 \mu ev, T = 30 mK\) and \(N_0 = 1, N = 20\). The unit of the time is \(6.582 \times 10^{-10} s\).
Fig. 2: Norms $\|\sigma\|_\lambda$ versus time $t$, (a) in a longer time comparing to the elementary gate operation time; (b) in the low decoherence ($D(t) \leq 10^{-4}$) time. Here, the values and meanings of the parameters are same as Fig. 1 except for $T = 0.3\text{mk}$.

6 Conclusions

In this Letter we firstly constructed a qubit-bath model where we supposed that the qubit nonlinearly couples with its environment. Then using the model we investigated the short-time decoherence of the JCQ. The decoherence is described by the norm of the deviation density operator, $\|\sigma(t)\|_\lambda$. We define the biggest $\|\sigma(t)\|_\lambda$ the quantity of decoherence $D(t)$. In the previous paper [9] by using the same setup of the JCQ and bath’s parameters we calculated the decoherence of the JCQ, where the JCQ is supposed linearly coupling with its bath. In the linearly coupling model, the decoherence is not larger than the DiVincenzo low decoherence criterion so the JCQ can be taken as the qubit for quantum computations. However, the results of this Letter show that at the same experimental temperature $T = 30\text{mk}$, when the JCQ nonlinearly couples with its bath, it is not a ideal block to build the quantum computer because the decoherence is larger than the DiVincenzo low decoherence criterion. But when the temperature decreases to $T = 0.3\text{mk}$ the decoherence decreases to a endurable grade. It suggests that in order to make the JCQ become a block of quantum computing hardware developing the technique of low temperature may be important. It should be noted that this is a pilot study to the decoherent problem of JCQ. The frequencies of the harmonic oscillators of the bath modes are chosen tentatively. They may only be obtained by virtue of some ingenious experiments. A further numerical simulation shows that when the frequencies are larger than our setting the JCQ will adapt a higher temperature. But, no matter what they are, we can find out a proper temperature for the JCQ in low decoherence by using our method.

Acknowledgement: This work was supported by the National Natural Science Foundation of China (NSFC), grant Nos. 10347133, 10347134 and Ningbo Youth Foundation, grant Nos. 2004A620003 and 2003A620005. X. T. L thanks Jiang Wei (Univ. of Washington) for his help.

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The diagram illustrates the norm of a function $\|\sigma\|_\lambda$ over time $t$. The graph shows oscillatory behavior with peaks and valleys, indicating a periodic or oscillatory nature of the function. The $y$-axis represents the norm of the function, and the $x$-axis represents time.
