The Physical Foundation of the Quark Model
The Quark Model as an approximation of the BCC Model

Jiao-Lin Xu

The Center for Simulational Physics, The Department of Physics and Astronomy
University of Georgia, Athens, GA 30602, USA.
E-mail: jxu@hal.physast.uga.edu

Abstract

From the Dirac sea concept, the BCC model infers that the quarks u and d constitute a body center cubic quark lattice in the vacuum; when a quark \( q^* \) is excited from the vacuum, the nearest primitive cell (\( u' \) and \( d' \)) is accompanying excited by the quark \( q^* \). Using the energy band theory, the model deduces the quantum numbers (I, S, C, b, and Q) and the masses of all quarks using a united mass formula. Then, it shows that the system of 3 excited quarks (\( q^*u'd' \)) is a baryon, and it deduces the baryon spectrum in terms of the sum laws. This theoretical baryon spectrum is in accordance with the experimental results. It also shows that there are only two elementary quarks (u and d), while the other quarks (s, c, b, ...) are the excited states of the elementary quarks, hence the SU(3) (u, d, and s), the SU(4) (u, d, s, and c), and the SU(5) (u, d, s, c, and b) are the natural extensions of the SU(2) (u and d). The BCC model provides the physical foundation (quarks, SU(N) groups, and that a baryon is made of 3 quarks) for the Quark Model. The Quark Model is the SU(N) approximation of the BCC model. The confinement concept is not needed in the BCC model, because it is replaced by the accompanying excitation concept. The SU(N) groups are also not necessary, as they are replaced by the body center cubic groups. We also predict some new baryons: \( \Lambda(2560) \), \( \Sigma_C(2280) \), \( \Omega^- (3720) \), \( \Lambda_C^+(6600) \), \( \Lambda_0^0 (9960) \)...
I Introduction

The Quark Model [1] has already explained the baryon spectrum and the meson spectrum in terms of the quarks. It successfully gives intrinsic quantum numbers ($I$, $S$, $C$, $b$, and $Q$) to all baryons and mesons. However, (1) it has not given a satisfactory mass spectrum of baryons and mesons in a united mass formula [2]. (2) The intrinsic quantum numbers and masses of the quarks are all entered “by hand” [3]. (3) It needs too many quarks (elementary particles: 6 flavors $\times$ 3 colors $\times$ 2 (quark and antiquark) = 36 quarks). (4) It needs too many parameters, just as T. D. Lee has already noticed [4]: “The standard model... needs $\sim$ 20 parameters: e, G, $\theta_w$, various masses for the three generations of leptons and quarks and the four weak decay angles $\theta_1$, $\theta_2$, $\theta_3$, and $\delta$”. (5) Confinement is a very plausible idea but to date its rigorous proof remains outstanding [5]. Meanwhile all free quark searches since 1977 have had negative results [6]. Therefore, although the Quark Model has a strong mathematical foundation (SU(3), SU(4), ...), its physical foundation is not sufficient. This paper attempts to provide a physical foundation for the quark model.

First, we need a mechanism.

Twenty years ago, T. D. Lee pointed out [7]:

“We believe our vacuum, though Lorentz invariant, to be quite complicated. Like any other physical medium, it can carry long-range-order parameters and it may also undergo phase transitions...”.

Recently Frank Wilczek, the J. Robert Oppenheimer Professor at the Institute for Advanced Study in Princeton, pointed out [8]:

“Empty space-the vacuum-is in reality a richly structured, though highly symmetrical, medium. Dirac’s sea was an early indication of this feature, which is deeply embedded in quantum field theory and the Standard Model. Because the vacuum is a complicated material governed by locality and symmetry, one can learn how to analyze
it by studying other such materials—that is, condensed matter.”

Professor Wilczek not only pointed out one of the most important and most urgent research directions of the modern physics—studying the structure of the vacuum, but also provided a very practical and efficient way for the study—learning from studying condensed matter.

Applying the Lee-Wilczek idea, this paper conjectures a structure of the vacuum (body center cubic quark lattice), which will be used as the mechanism to generate the quark spectrum, the baryon spectrum, and the meson spectrum.

According to Dirac’s sea concept [9], there is an electron Dirac sea, a \( \mu \) lepton Dirac sea, a \( \tau \) lepton Dirac sea, a \( u \) quark Dirac sea, a \( d \) quark Dirac sea, an \( s \) quark Dirac sea, a \( c \) quark Dirac sea, and a \( b \) quark Dirac sea... in the vacuum. All of these Dirac seas are in the same space, at any location, that is, at any physical space point. These particles will interact with one another and form the perfect vacuum material. However, some kinds of particles do not play an important role in forming the vacuum material. First, the main force which makes and holds the structure of the vacuum material must be the strong interaction, not the weak-electromagnetic interactions. Hence, in considering the structure of the vacuum material, we leave out the Dirac seas of those particles which do not have strong interactions (\( e, \mu, \text{and} \tau \)). Secondly, the vacuum material is super stable, hence we also omit the Dirac seas which can only make unstable baryons (\( s \) quark, \( c \) quark, \( b \) quark). Finally, there are only two kinds of possible particles left: the vacuum state \( u \) quarks and the vacuum state \( d \) quarks. There are super strong attractive forces between the \( u \) quarks and the \( d \) quarks which will make and hold the densest structure of the vacuum material.

According to solid state physics [10], if two kinds of particles (with radius \( R_1 < R_2 \)) satisfy the condition \( 1 > R_1/R_2 > 0.73 \), the densest structure is the body center cubic crystal [11]. We know the following: first, \( u \) quarks and \( d \) quarks are not exactly the same, thus \( R_u \neq R_d \); second, they are very close to each other (the same isospin with
different $I_z$), thus $R_u \approx R_d$. Hence, if $R_u < R_d$ (or $R_d < R_u$), we have $1 > R_u/R_d > 0.73$ (or $1 > R_d/R_u > 0.73$). Therefore, we conjecture that the vacuum state $u$ quarks and $d$ quarks construct the body center cubic quark lattice in the vacuum (in this paper, it will be regarded as the BCC model). **The BCC model will be the physical foundation of the Quark Model.**

Since the system is a multi-particle system, we cannot solve the problem exactly. We will study a series of approximations: a primitive cell [12] approximation, a periodic field approximation [13], a combined approximation of the primitive cell and periodic field, and an SU(N) symmetry (the Quark Model) approximation [14].

In the primitive cell approximation, we consider the excited quark and the primitive cell (in which the excited quark is contained) only, omitting the quark lattice. Although we can get some important results (such as: a baryon is made of three quarks, the system satisfies SU(3) symmetry...), we cannot deduce flavored quarks and flavored baryons.

In the periodic field approximation, further considering the quark lattice periodic field (with body center cubic symmetry), we obtain the baryon spectrum. However, we can not get the quark spectrum.

In the combined approximation of the primitive cell and periodic field, we can deduce the quark spectrum, the baryon spectrum, and the meson spectrum.

In the SU(N) symmetry approximation, based on the results of the combined approximation, we assume that the N ground quark excited states (with different flavors) are independent quarks. The N quarks satisfy the SU(N) symmetry and belong to the fundamental representation of the SU(N) group. We can obtain similar results with the Quark Model [1].

These approximation results show (1) quarks $u$ and $d$ are elementary particles, other quarks ($s, c, b...$) are the energy band excited states of elementary quarks; (2) $u$ quarks and $d$ quarks indeed construct the body center cubic quark lattice in the vacuum; (3) flavored quarks originate from the symmetry of the periodic field of the quark lattice; (4)
the confinement concept is unnecessary, since it can be replaced by the accompanying excitation concept; (5) the quark model is the SU(N) approximation of the BCC model; (6) and the BCC model provides the physical foundation of the quark model.

This paper is organized as follows: The fundamental hypotheses are presented in Section II. The primitive cell [12] approximation is introduced in Section III. The periodic field approximation [13] is discussed in Section IV. The combined approximation of the primitive cell and the periodic field is accomplished in Section V. The SU(N) [14] approximation (the Quark Model) is introduced in Section VI. A comparison of the results of the BCC model with the experimental results is listed in Section VII. The predictions and discussions of the model are stated in Section VIII. The conclusions are in Section IX.

II Fundamental Hypotheses

First, the BCC model assumes that u quarks and d quarks are fundamental particles and make a body center cubic quark lattice in the vacuum. Then, it deduces the spectrum of excited quarks. Finally, it finds the spectrum of baryons and mesons.

In order to explain the model accurately and concisely, we will start from the fundamental hypotheses in an axiomatic form.

Hypothesis I There are two kinds of elementary quarks in the vacuum state. They have the same baryon number $B = 1/3$, spin $s = 1/2$, and isospin $I = 1/2$. The quarks with the third component of the isospin $I_z = 1/2$ are called u quarks, and the quarks with the third component of the isospin $I_z = -1/2$ are called d quarks. There are super strong attractive interactions among the quarks. The super attractive forces make and hold an infinite body center cubic quark (u and d) lattice in the vacuum.
Hypothesis II  When a quark \((q)\) is excited from the vacuum quark lattice \((q \rightarrow q^*)\), the primitive cell (in which the excited quark is contained) is simultaneously excited by the excited quark \(q^*\) also. Since there are only two quarks, \(u\) and \(d\), in the primitive cell, there are only 2 accompanying excited quark, \(u'\) and \(d'\), in the primitive cell. We call the excitation of the primitive cell the accompanying excitation.

The accompanying excited quark \(u'\) (or \(d'\)) has not left its position in the quark lattice, so it is not free to move in the space. However, its electric charge and baryon number are temporarily excited from the vacuum state, under the influence of the excited quark \(q^*\). Because the excited energy of the electric charge is much smaller than the excited energy of the quark and because it can not be separated from the excited energy of the quark \(q^*\) in the experiments, we assume that the accompanying excitation energy is very small and can be treated as a small perturbation energy. For the zeroth-order approximation, the accompanying excited quark \(u'\) has

\[
B = 1/3, \; S = 0, \; s = I = 1/2, \; I_z = 1/2, \; Q = 2/3, \; m_{u'} = 0; \tag{1}
\]

and the accompanying excited quark \(d'\) has

\[
B = 1/3, \; S = 0, \; s = I = 1/2, \; I_z = -1/2, \; Q = -1/3, \; m_{d'} = 0. \tag{2}
\]

The accompanying excitation is temporary for a cell. When the quark \(q^*\) is excited from the vacuum, the primitive cell undergoes an accompanying excitation which is due to the excited quark \(q^*\); but when the quark \(q^*\) leaves the cell, the excitation of the cell disappears. Although the truly excited cells are quickly changed, one following another, with the motion of the excited quark \(q^*\), an excited primitive cell \((u'\) and \(d'\)) always appears to accompany the excited quark \(q^*\), just as an electric field always accompanies the originated electric charge.

Hypothesis III  Due to the effect of the vacuum quark lattice, fluctuations of energy \(\varepsilon\) and intrinsic quantum numbers (such as the strange number \(S\)) of an excited quark may
exist. The fluctuation of the Strange number, if it exists, is always $\Delta S = \pm 1$ [13]. From the fluctuation of the Strange number, we will be able to deduce new quantum numbers, such as Charmed number $C$ and Bottom number $b$.

For an excited quark $q^*$ (which is accompanied by $u'$ and $d'$) moving in the body center cubic quark lattice, the Hamiltonian can be written as

$$H = H_{q^*} + H_{Cell} + H'_{Latt} + H'_{q^*-Cell} + H'_{q^*-Latt} + H'_{Cell-Latt}.$$  \hspace{1cm} (3)

Where $H_{q^*}$ is the Hamiltonian of the excited quark $q^*$, $H_{cell}$ is the Hamiltonian of the accompanying excited cell ($u'$ and $d'$), $H'_{Latt}$ is the Hamiltonian of the vacuum quark lattice (excluding the accompanying cell), $H_{q^*-cell}$ is the interaction Hamiltonian between the excited quark and the accompanying excited primitive cell, $H_{q^*-Latt}$ is the interaction Hamiltonian between the excited quark and the vacuum quark lattice (excluding the accompanying cell), and $H'_{Cell-Latt}$ is the interaction Hamiltonian between the accompanying excited cell and the vacuum quark lattice (excluding the accompanying cell).

Since we do not know the exact form of the strong interactions, we cannot write out the expression of $H$. Furthermore, since the system is a multi-particle system, even if we have the expression of $H$, we still cannot solve the problem accurately. Thus, we have to use some approximations to attack the problem. We will study a primitive cell approximation, a periodic field approximation [13], a combined approximation of the cell and the periodic field, and an SU(N) symmetry approximation [14]. According to the BCC Model, the SU(N) approximation is the Quark Model. In other words, the Quark Model is an approximation of the BCC Model.

First, we discuss the primitive cell approximation.
III The Primitive Cell Approximation

In order to find a good approximation, we need to look at the whole system. The system is made up of the vacuum quark lattice and an excited quark \( q^* \) with an accompanying excited primitive cell \((u' + d')\). The vacuum lattice forms the physical vacuum background. The excited quark freely moves in the lattice, as an electron moves in a superconductor. An accompanying excited cell is always accompanying the excited quark, like an electric field accompanying the originated electric charge. Since the interactions between the quarks are short in range, the excited quark \( q^* \) can excite only the nearest quarks \((u' + d')\) which are inside the primitive cell. They cannot be separated, just as Coulomb’s electric field cannot be separated from the original electric charge. Therefore, an observable ‘particle’ is not only an excited quark \( q^* \), but a group of three quarks (an excited quark \( q^* \), an accompanying excited quark \( u' \), and an accompanying excited quark \( d' \)).

The simplest approximation is the primitive cell approximation. In this approximation, we consider the excited quark \( q^* \) and the accompanying excited primitive cell \((u' + d')\) only, omitting the quark lattice. Thus, there are only three quarks in the system: the excited quark \( q^* \) and the two accompanying excited quarks \( u' \) and \( d' \). We assume that the quantum numbers and energy of the system are the sums of the quantum numbers and energies of the constituent quarks \([10]\).

\[
B = \sum B_q, \quad Q = \sum Q_q, \quad I_z = \sum I_{zq}, \quad M = \sum m_q. \tag{4}
\]

A The Quarks

The system has three quarks: the excited quark \( q^*(u^* \text{ or } d^*) \), and the accompanying excited quarks \( u' \) and \( d' \). According to Hypothesis I, the excited quark \( q^*(u^* \text{ or } d^*) \)
has: for $u^*$

$$B = 1/3, \ S = 0, \ s = 1/2, \ I = 1/2, \ I_Z = 1/2, \ \text{and} \ Q = 2/3 ;$$ (5)

for $d^*$

$$B = 1/3, \ S = 0, \ s = 1/2, \ I = 1/2, \ I_Z = -1/2, \ \text{and} \ Q = -1/3.$$ (6)

From (4) and (5), we can get the quantum numbers and energies of the accompanying excited quarks $u'$ and $d'$.

**B Baryons**

Using the ‘sum formulae’ (4), we can find the quantum numbers of the three quark system ($q^*u'd'$). For $q^* = u^*$, the system ($u^*u'd'$) has $B = 1, \ I = 1/2, \ I_Z = 1/2, \ Q = 1$. Comparing it with the experimental results of a *proton* ($B = 1, \ I = 1/2, \ I_Z = 1/2, \ Q = 1$), we get

$$\ (u^*u'd') \rightarrow \text{proton}.$$ (7)

Similarly, for $q^* = d^*$, we got that the system is a neutron

$$\ (d^*u'd') \rightarrow \text{neutron}$$ (8)

with $B = 1, \ I = 1/2, \ I_Z = -1/2, \ Q = 0$.

From (4) and (2), we obtain that

$$m_{u'} = m_{d'} = 0.$$ (9)

Using $M_n \approx M_p = 939 \ Mev \approx 940 \ Mev$ and $M_p = m_{u^*} + m_{u'} + m_{d'}$, we obtain the mass of the quark $u^*$

$$m_{u^*} = 940MeV.$$ (10)
Similarly, we can get the mass of the quark $d^*$

$$m_{d^*} = 940\text{Mev}.$$  \hspace{1cm} (11)

In the three quark system $(q^*u'd')$, from Hypothesis I, quarks $u'$ and $d'$ satisfy the SU(2) symmetries. Moreover, quark $q^*$ is an excited state of $u$ or $d$. Thus, the Hamiltonia $H(q^*u'd')$ of the three quark system $(q^*u'd')$ satisfies the SU(3) symmetries.

\section*{C Mesons}

Using the sum formulae (4), we can find the quantum numbers of a quark and an antiquark system.

The system $(u^*d^*)$ has $B = 0, I = 1, I_z = 1, Q = 1$; it is a meson $\pi^+$.  

The system $(d^*u^*)$ has $B = 0, I = 1, I_z = -1, Q = 0$; it is a meson $\pi^-$.  

The system $(u^*u^*)$ has $B = 0, I = 1, 0; I_z = 0; Q = 0$.  

The system $(d^*d^*)$ has $B = 0, I = 1, 0; I_z = 0; Q = 0$.

Thus, in the primitive cell approximation, we can get the mesons $\pi = (\pi^+, \pi^0, \pi^-)$

$$\pi^+ = (u^*d^*), \quad \pi^0 = \frac{1}{\sqrt{2}}(u^*u^* - d^*d^*), \quad \pi^- = d^*u^*$$  \hspace{1cm} (12)

with $B = 0, I = 1, S = C = 0, Q = 1, 0, -1$; and the meson $\eta^0$

$$\eta^0 = \frac{1}{\sqrt{2}}(u^*u^* + d^*d^*)$$  \hspace{1cm} (13)

with $B = 0, I = 0, S = C = 0, Q = 0$.  

10
D The Resonance States

The resonance states of the baryons \((q^* u d')\) and mesons \((q^* q^\prime)\) can be found:

\[
\begin{align*}
(u^* u' d')(u^* d') &= (u^* u^* d')(d' d') \quad I_z = 3/2, Q = +2 \\
(u^* u d')(u^* u') &= (u^* u'^* d')(d' d') \quad I_z = 1/2, Q = +1 \\
(u^* u d')(d'^* d') &= (u^* u'^* d')(d' d') \quad I_z = 1/2, Q = +1 \\
((d^* u^* d')(u^* u^*)) &= (u^* u'^* d')(d' d') \quad I_z = -1/2, Q = 0 \\
((d^* u^* d')(d'^* d')) &= (u^* u'^* d')(d' d') \quad I_z = -1/2, Q = 0 \\
((d^* u^* d')(d'^* d')) &= (u^* u'^* d')(d' d') \quad I_z = -1/2, Q = 0 \\
((d^* u d')(d'^* d')) &= (u^* u'^* d')(d' d') \quad I_z = -3/2, Q = -1
\end{align*}
\]

\[
\begin{align*}
\Delta^{++} &= (u^* u u^*)(d' d') \\
\Delta^+ &= \sqrt{1/3}(u^* u^* d')(u^* u^*) + (u^* u d^*)(d' d^*) + (u^* u'^* d^*)(d' d^*) \\
\Delta^0 &= \sqrt{1/3}(d^* u^* d')(u^* u^*) + (d^* u^* d')(u^* u^*) + (d^* u'^* d^*)(d' d^*) \\
\Delta^- &= (d^* d^* d')(u^* u^*)
\end{align*}
\]

From the above, we have a 4 fold resonance state \(\Delta\). It has \(B = 1, S = 0, I = 3/2,\) and \(Q = 2, 1, 0, -1\).

E The High Energy Scattering

When the hadrons collide against each other with high energy, the accompanying excited quarks \(u'\) and \(d'\) may be excited into excited quarks \(u^*\) and \(d^*\) temporarily. Hence, there may be two or three excited quarks in the system in a very high energy scattering. **If one high energy particle collides against two quarks at the same time and with the same probability, a two-jet will be born. If the high energy particle collides against three quarks at the same time and with the same probability, a three-jet will be born.** The high energy scattering procedure is a very complex process. It is beyond the scope of this paper.
F  Summary

The primitive cell approximation looks like the Quark Model, however it is not really the Quark Model. It is only an embryo of the Quark Model. Although it is very simple, it can give many important results which are useful in the BCC model.

1. A baryon is made of three quarks from (7) and (8). This is based on the physical structure of the body center cubic quark lattice. There are only two quarks, u and d, in each primitive cell of the quark lattice. If an excited quark q* is moving in the cell, the two quarks (u and d) of the cell are accompanying excited by the excited quark q*. Thus, the system \( (q^*u'd') \) is made of three quarks now. We have already shown that the system is a baryon (7) and (8).

2. A meson is made of a quark and an antiquark from (12) and (13).

3. Any excited quark q* is always accompanied by two accompanying excited quarks, u' and d'. Since they cannot be separated, a free quark can never be seen.

4. The three quark system satisfies the SU(3) symmetry.

5. The quantum numbers of the baryons and the mesons can be determined by the sum formulae (4).

6. The three quarks are in different states. One of them (q*) is in a completely excited state from the vacuum lattice. The other two (u' and d') are in the accompanying excited states, and they are in different isospin states \( I_z(u') = +1/2, I_z(d') = -1/2 \). Thus, the three quarks are in different states, the system \( (q^*u'd') \) obeys the Pauli exclusion principle [17].

In the primitive cell approximation, the most important result is that a baryon is made of three quarks. It is based on the physical structure of the body center cubic quark lattice.

However, since the cell approximation omits the periodic field of the quark lattice,
it can not deduce the strange number, the charmed number, the bottom number, or the masses of the quarks. As we shall see in the next section, these quantum numbers are products of the periodic field of the quark lattice.

IV The Periodic Field Approximation

In the periodic field approximation, we assume:

1. The ideal quark lattice (see (3)) will be regarded as the physical vacuum background,

\[ H_{\text{Latt}} \approx 0. \]  

(16)

2. The interaction Hamiltonian between the excited quark \( q^* \) and the ideal quark lattice will be replaced by a periodic field with the body center cubic symmetries,

\[ H_{q^*-\text{Latt}} \rightarrow V(\vec{r}). \]  

(17)

3. The quantum numbers and energy of the three quark system \( (q^*u'd') \) are represented by a point particle which will be called the Lee Particle [18] in the periodic field approximation. In other words, the quantum numbers and energy of the three quark system are concentrated in the excited particle (Lee Particle). We have

\[
\begin{align*}
\bar{u}_{\text{Lee}}^* : & \quad B = 1, \; Q = 1, \\
\bar{d}_{\text{Lee}}^* : & \quad B = 1, \; Q = 0.
\end{align*}
\]  

(18)

for free Lee Particles. Therefore, in the periodic approximation, an excited quark \( q_{\text{Lee}}^* \) (Lee Particle) [18] represents a baryon.

4. In the periodic approximation, an excited quark \( q_{\text{Lee}}^* \) is regarded as a baryon. The excited quarks which are in different excited states will be different baryons.

Finally, the Hamiltonian (3) is simplified into the periodic approximation \( H_{\text{per}} \).

\[ H_{\text{per.}} = H_{q_{\text{Lee}}^*} + V(\vec{r}). \]  

(19)
Thus, in the periodic field approximation, the problem is simplified into an excited Lee Particle (the group of an excited quark $q^*$ and two accompanying excited quarks $u'$ and $d'$) moving in the periodic field. The problem has already been discussed in [18]. In that paper, we used the point particle (the Lee Particle) approximation to represent the primitive cell. We have deduced a baryon spectrum which is in agreement with the experimental results [19]. Readers can find the results in Table 1-Table 6 of that paper.

In the periodic field approximation, the most important result is that the strange numbers ($S = -1$, $S = -2$, and $S = -3$), the charmed number, and the bottom number all come from the body center cubic symmetries of the vacuum quark lattice. In fact, the strange particles ($\Lambda$, $\Sigma$, $\Xi$, and $\Omega$), the charmed particles ($\Lambda_c$), and the bottom particles ($\Lambda_b$) are not new particles which are completely different from the nucleons. They are only higher energy band excited states of the Lee Particle.

However, the periodic field approximation can not deduce the quark spectrum. Thus, it is not easy to deduce the meson spectrum. In the next section, we will see that in the combined approximation of the cell and the periodic field, we can deduce the quark spectrum, the baryon spectrum, and the meson spectrum.

V The combined approximation of the Cell and the Periodic Field

In the primitive cell approximation, we omitted the periodic field. In the periodic approximation, we united the excited quark $q^*$ and the primitive cell ($u'd'$) to get the Lee Particle (the point approximation)[18]. In the combined approximation, we will not only consider the periodic field, but also consider the accompanying excited cell.

1. The excited quark $q^*$ moves in the perfect periodic field with body center cubic
symmetries, the accompanying excited cell \((u'd')\) is always accompanying the quark \(q^*\).

2. According to the sum-formulas (4), the quantum numbers and the energy of the three quark system \((q^*u'd')\) are the sums of the quantum numbers and the energies of the constituent quarks inside the system.

In this approximation, we will deduce the spectrum of the quarks first. Then, using the sum-laws, we deduce the spectrum of the baryons.

A The Spectrum of the Quarks

First of all, we will find the energy bands of the excited quark \(q^*\). Then, we will find the quantum numbers and energies of the energy bands. Finally, we will find the spectrum of the excited quarks.

A- 1 The Motion Equation of the Quark

When a quark is excited from the vacuum, it is moving in the vacuum. Since the quark is a Fermion, its motion equation should be the Dirac equation. Taking into account that (according to the renormalization theory \([20]\)) the bare mass of the quark is much larger than the empirical values of the excited energies of the quark \(q^*\), we use the Schrödinger equation instead of the Dirac equation (our results will show that this is a very good approximation). From (19), using \(H_q = -\frac{\hbar^2}{2m_q} \nabla^2\), we have:

\[
\frac{\hbar^2}{2m_q} \nabla^2 \Psi + (\varepsilon - V(\vec{r}))\Psi = 0, \tag{20}
\]

where \(V(\vec{r})\) denotes the strong interaction periodic field of the quark lattice with body center cubic symmetries, and \(m_q\) is the bare mass of the quark \(q^*\).

A- 2 Finding the Energy Bands of the quark

Using the energy band theory \([21]\) and the free particle approximation \([22]\) (taking \(V(\vec{r}) = V_0\) constant and making the wave functions satisfy the body center cubic periodic
symmetries), we have

$$\frac{\hbar^2}{2m_q} \nabla^2 \Psi + (\varepsilon - V_0) \Psi = 0,$$  \hspace{1cm} (21)$$

where $V_0$ is a constant potential. The solution of Eq. (21) is a plane wave

$$\Psi_{k}(\vec{r}) = \exp\{-i(2\pi/a_x)[(n_1 - \xi)x + (n_2 - \eta)y + (n_3 - \zeta)z]\},$$  \hspace{1cm} (22)$$

where the wave vector $\vec{k} = (2\pi/a_x)(\xi, \eta, \zeta)$, $a_x$ is the periodic constant of the quark lattice, and $n_1, n_2, n_3$ are integers satisfying the condition

$$n_1 + n_2 + n_3 = \pm \text{even number or 0.}$$  \hspace{1cm} (23)$$

Condition (23) implies that the vector $\vec{n} = (n_1, n_2, n_3)$ can only take certain values. For example, $\vec{n}$ can not take $(0, 0, 1)$ or $(1, 1, -1)$, but can take $(0, 0, 2)$ and $(1, -1, 2)$.

The zeroth-order approximation of the energy [22] is

$$\varepsilon^{(0)}(\vec{k}, \vec{n}) = V_0 + \alpha E(\vec{k}, \vec{n}),$$  \hspace{1cm} (24)$$

$$\alpha = \frac{\hbar^2}{2m_q a_x^2},$$  \hspace{1cm} (25)$$

$$E(\vec{k}, \vec{n}) = (n_1 - \xi)^2 + (n_2 - \eta)^2 + (n_3 - \zeta)^2.$$  \hspace{1cm} (26)$$

Now we will demonstrate how to find the energy bands.

The first Brillouin zone [23] of the body center cubic lattice is shown in Fig. 1. In Fig. 1 (depicted from [21] (Fig. 1) and [23] (Fig. 8.10)), the $(\xi, \eta, \zeta)$ coordinates of the symmetry points are:

$$\Gamma = (0, 0, 0), \ H = (0, 0, 1), \ P = (1/2, 1/2, 1/2),$$

$$N = (1/2, 1/2, 0), \ M = (1, 0, 0).$$  \hspace{1cm} (27)$$
and the \((\xi, \eta, \zeta)\) coordinates of the symmetry axes are:

\[
\Delta = (0, 0, \zeta), \quad 0 < \zeta < 1; \quad \Lambda = (\xi, \xi, \xi), \quad 0 < \xi < 1/2;
\]

\[
\Sigma = (\xi, \xi, 0), \quad 0 < \xi < 1/2; \quad D = (1/2, 1/2, \xi), \quad 0 < \xi < 1/2;
\]

\[
G = (\xi, 1-\xi, 0), \quad 1/2 < \xi < 1; \quad F = (\xi, \xi, 1-\xi), \quad 0 < \xi < 1/2.
\]

For any valid value of the vector \(\vec{n}\), substituting the \((\xi, \eta, \zeta)\) coordinates of the symmetry points or the symmetry axes into Eq. (26) and Eq. (22), we can get the \(E(\vec{k}, \vec{n})\) values and the wave functions at the symmetry points and on the symmetry axes. In order to show how to calculate the energy bands, we give the calculation of some low energy bands in the symmetry axis \(\Delta\) as an example (the results are illustrated in Fig. 2(a)).

First, from (26) and (22) we find the formulae for the \(E(\vec{k}, \vec{n})\) values and the wave functions at the end points \(\Gamma\) and \(H\) of the symmetry axis \(\Delta\), as well as on the symmetry axis \(\Delta\) itself:

\[
E_{\Gamma} = n_1^2 + n_2^2 + n_3^2,
\]

\[
\Psi_{\Gamma} = \exp\{-i(2\pi/a_x)[n_1 x + n_2 y + n_3 z]\}.
\]

\[
E_{H} = n_1^2 + n_2^2 + (n_3 - 1)^2,
\]

\[
\Psi_{H} = \exp\{-i(2\pi/a_x)[n_1 x + n_2 y + (n_3 - 1)z]\}.
\]

\[
E_{\Delta} = n_1^2 + n_2^2 + (n_3 - \zeta)^2,
\]

\[
\Psi_{\Delta} = \exp\{-i(2\pi/a_x)[n_1 x + n_2 y + (n_3 - \zeta)z]\}.
\]

Then, using (29)–(34), beginning from the lowest possible energy, we can obtain the corresponding integer vectors \(\vec{n} = (n_1, n_2, n_3)\) (satisfying (23)) and the wave functions:
1. The lowest $E(\vec{k}, \vec{n})$ is at $(\xi, \eta, \zeta) = 0$ (the point $\Gamma$) and with only one value of $\vec{n} = (0, 0, 0)$ (see (29) and (30)): 
\[ \vec{n} = (0, 0, 0), \quad E_\Gamma = 0, \quad \Psi_\Gamma = 1. \] (35)

2. Starting from $E_\Gamma = 0$, along the axis $\Delta$, there is one energy band (the lowest energy band $E_\Delta = \zeta^2$, with $n_1 = n_2 = n_3 = 0$ (see (33) and (34)) ended at the point $E_H = 1$:
\[ \vec{n} = (0, 0, 0), \quad E_\Gamma = 0 \rightarrow E_\Delta = \zeta^2 \rightarrow E_H = 1, \quad \Psi_\Delta = \exp[i(2\pi/a_x)(\zeta z)]. \] (36)

3. At the end point $H$ of the energy band $E_\Gamma = 0 \rightarrow E_\Delta = \zeta^2 \rightarrow E_H = 1$, the energy $E_H = 1$. Also at point $H$, $E_H = 1$ when $n = (\pm 1, 0, 1)$, $(0, \pm 1, 1)$, and $(0, 0, 2)$ (see (31) and (32)):
\[ E_H = 1, \quad \Psi_H = [e^{i(2\pi/a_x)(\pm x)}, e^{i(2\pi/a_y)(\pm y)}, e^{i(2\pi/a_z)(\pm z)}]. \] (37)

4. Starting from $E_H = 1$, along the axis $\Delta$, there are three energy bands ended at the points $E_\Gamma = 0$, $E_\Gamma = 2$, and $E_\Gamma = 4$, respectively:
\[ \vec{n} = (0, 0, 0), \quad E_H = 1 \rightarrow E_\Delta = \zeta^2 \rightarrow E_\Gamma = 0, \quad \Psi_\Delta = \exp[i(2\pi/a_x)(\zeta z)]. \] (38)

\[ \vec{n} = (0, 0, 2), \quad E_H = 1 \rightarrow E_\Delta = (2-\zeta)^2 \rightarrow E_\Gamma = 4, \quad \Psi_\Delta = \exp[i(2\pi/a_x)(2 - \zeta z)]. \] (39)

\[ \vec{n} = (\pm 1, 0, 1)(0, \pm 1, 1), \quad E_H = 1 \rightarrow E_\Delta = 1+(1-\zeta)^2 \rightarrow E_\Gamma = 2, \quad \Psi_\Delta = e^{i(2\pi/a_x)\pm x(1-\zeta z)}, e^{i(2\pi/a_y)\pm y(1-\zeta z)}]. \] (40)
5. The energy bands with 4 sets of values $\vec{n} = (\pm 1, 0, 1), (0, \pm 1, 1)$ ended at $E_\Gamma = 2$. From (29), $E_\Gamma = 2$ also when $\vec{n}$ takes other 8 sets of values: $\vec{n} = (1, \pm 1, 0), (-1, \pm 1, 0),$ and $(\pm 1, 0, -1), (0, \pm 1, -1)$. Putting the 12 sets of $\vec{n}$ values into Eq. (30), we can obtain 12 plane wave functions:

$$E_\Gamma = 2, \Psi_\Gamma = [e^{i(2\pi/a\cdot x)(\pm x \pm y)}, e^{i(2\pi/a\cdot x)(\pm y \pm z)}, e^{i(2\pi/a\cdot x)(\pm z \pm x)}].$$ (41)

6. Starting from $E_\Gamma = 2$, along the axis $\Delta$, there are three energy bands ended at the points $E_H = 1$, $E_H = 3$, and $E_H = 5$, respectively:

$$\vec{n} = (\pm 1, 0, 1)(0, \pm 1, 1), E_\Gamma = 2 \rightarrow E_\Delta = 1 + (1-\zeta)^2 \rightarrow E_H = 1,$$ (42)

$$\vec{n} = (1, \pm 1, 0)(-1, \pm 1, 0), E_\Gamma = 2 \rightarrow E_\Delta = 2 + \zeta^2 \rightarrow E_H = 3,$$ (43)

$$\vec{n} = (\pm 1, 0, -1)(0, \pm 1, -1), E_\Gamma = 2 \rightarrow E_\Delta = 1 + (\zeta + 1)^2 \rightarrow E_H = 5.$$ (44)

Continuing the process, we can find all the energy bands and the corresponding wave functions. The wave functions are not needed for the zeroth order approximation, so we only show the energy bands in Fig. 2-4. There are six small figures in Fig. 2-4. Each of them shows the energy bands in one of the six axes in Fig.1. Each small figure is a schematic one where the straight lines (show the energy bands) should be parabolic curves. The numbers above the lines are the values of $\vec{n} = (n_1, n_2, n_3)$. The numbers under the lines are the fold numbers of the energy bands with the same energy (the zeroth-order approximation). The numbers beside both ends of an energy band (the intersection of the energy band line and the vertical lines) represent the highest and lowest $E(k, \vec{n})$ values (see Eq. (26)) of the band. Putting the values of the $E(k, \vec{n})$ into Eq. (24), we get the zeroth-order energy approximation values (in Mev).
The Quantum Numbers and Energies of the Quarks

If there were no strong interaction periodic field of the quark lattice, we would only see quarks $u^*$ and $d^*$ (inside $N$, $\Delta$, $\pi$, and $\eta$). We could not see any flavored quarks (inside flavored baryons $\Lambda$, $\Sigma$, $\Xi$, $\Omega$, $\Lambda_c$, $\Lambda_b$, ...) because they would not exist. Due to the periodic field, although the free quarks $u^*$ and $d^*$ are still the essential states, there is a slight chance that the quarks are excited to the symmetry points (see Fig. 1) of the periodic field. Once at the symmetry points, the excited quarks will show special symmetric properties. Due to the periodic field, the parabolic energy curve of the free excited quark will be changed into energy bands (see Eq. (24) and Fig. 2-5). Also, there will be energy gaps on the surfaces of the Brillouin zones which originate from the periodic field [24]. The gaps will give the excited quarks longer lifetimes and special properties, which are different from those of the free quarks $u^*$ and $d^*$. Because of these properties, physicists naturally regard them as new excited quarks which are different from the excited quarks $u^*$ and $d^*$.

There are two necessary conditions for the $q^*$ to be regarded as a new excited quark state: first, the lowest energy point of the energy band must be at a high symmetric point (the points $\Gamma$, $P$, $N$, $H$, or $M$) (see Fig. 1); second, there is an energy gap between the lowest energy point of the energy band and the ground state ($E = 0$, see Fig. 2-5).

From Fig. 2-5, we can see that all energy bands satisfy the first condition. However, the six energy bands with $\vec{n} = (0,0,0)$ of the first Brillouin zone do not satisfy the second condition. In other words, the six energy bands are all in the first Brillouin zone—a part of the parabolic energy curved surface of the free excited quark $q^*(u^* and d^*)$. Therefore, they represent the unflavored ($S = C = b = 0$) excited quark $q^*_N(940)(u^* or d^*)$.

20
| Axis | Energy Band | $\vec{n}$ | $I$ | $q^*(m)$ | $Q_u$, $Q_d$ |
|------|-------------|---------|-----|----------|-------------|
| $\Delta$ | $E_R=0 \rightarrow E_H=1$ | (0, 0, 0) | $1/2$ | $q_N^*(940)$ | $2/3$, $-1/3$ |
| $\Lambda$ | $E_R=0 \rightarrow E_P=3/4$ | (0, 0, 0) | $1/2$ | $q_N^*(940)$ | $2/3$, $-1/3$ |
| $\Sigma$ | $E_R=0 \rightarrow E_N=1/2$ | (0, 0, 0) | $1/2$ | $q_N^*(940)$ | $2/3$, $-1/3$ |
| $D$ | $E_N=1/2 \rightarrow E_P=3/4$ | (0, 0, 0) | $1/2$ | $q_N^*(940)$ | $2/3$, $-1/3$ |
| $F$ | $E_P=3/4 \rightarrow E_H=1$ | (0, 0, 0) | $1/2$ | $q_N^*(940)$ | $2/3$, $-1/3$ |
| $G$ | $E_N=1/2 \rightarrow E_M=1$ | (0, 0, 0) | $1/2$ | $q_N^*(940)$ | $2/3$, $-1/3$ |

In order to find the quantum numbers of the energy bands (except for the 6 energy bands of the first Brillouin zone) of the excited quark, we will make a new hypothesis in addition to the ones in Section II.

**Hypothesis IV**  *The quantum numbers and masses of the excited quarks are determined as follows (except for the 6 energy bands of the first Brillouin zone)*:

1. **Baryon number $B$.** When a quark is in vacuum state, $B = 0$. But if it is excited from the vacuum state, it has

   \[ B = 1/3. \]  \hspace{1cm} \text{(46)}

2. **Isospin number $I$: the isospin $I$ is determined by the energy band degeneracy $d$ \cite{21}, where**

   \[ d = 2I + 1. \]  \hspace{1cm} \text{(47)}

The concept of isospin was introduced in the early 30’s by Heisenberg \cite{25} to describe the approximate charge-independent nature of the strong interaction between protons and neutrons. For a given $I$, $I_z$ can vary from $-I$ to $I$, making a total of $2I + 1$ states. Under isospin rotations, the quantum number $I$ is preserved; however, these $2I + 1$ states of different $I_z$ transform among one another, and
therefore they are degenerate with respect to the strong interaction. Thus we can get the isospin I from the degeneracy d in terms of (17).

In some cases, the degeneracy d should be divided into sub-degeneracies before using the formula. First, if the ‘degeneracy’ energy bands are in the first and second Brillouin Zones, the ‘degeneracy’ will be divided into two sub degeneracies. Secondly, if d is larger than the rotary fold R of the symmetry axis:

\[ d > R, \]

then we assume that the degeneracy will be divided into \( \gamma \) sub-degeneracies, where

\[ \gamma = d/R. \]

For the three axes which pass through the center point \( \Gamma \) (the axis \( \Delta(\Gamma - H) \), the axis \( \Lambda(\Gamma - P) \), the axis \( \Sigma(\Gamma - N) \)), the energy bands in the same degeneracy group have symmetric \( \vec{n} = (n_1, n_2, n_3) \) values (see Fig. 2(a), 2(b) and 3(a)). Hence, if the sub-degeneracy \( d_{sub} \leq R \), it will not be divided further. About symmetric \( \vec{n} \), we give a definition: a group of \( \vec{n} = (n_1, n_2, n_3) \) values is said to be symmetric if any two \( \vec{n} \) values in the group can transform into each other by various permutation (change component order) and by changing the sign “±” (multiplied by “-1”) of the components (one, two, or three). For example, \((-2, -1, 3) \) and \((-3, 2, 1) \) are symmetric, \((-3, 0, 2) \) and \((-3, 0, 1) \) are asymmetric. For the other three symmetric axes, see Appendix A.

After finding the sub-degeneracy, \( d_{sub} \), we can use \( d_{sub} = 2I+1 \) to find the isospin I.

3. Strange number \( S \): the Strange number \( S \) is determined by the rotary fold \( R \) of the symmetry axis \[21\] with

\[ S = R - 4, \]

(50)
where the number 4 is the highest possible rotary fold number. To be specific, from Eq. (50) and Fig. 1, we get

\[ \Delta(\Gamma - H) \text{ is a 4-fold rotation axis, } R = 4 \rightarrow S = 0; \]  

(51)

\[ \Lambda(\Gamma - P) \text{ is a 3-fold rotation axis, } R = 3 \rightarrow S = -1; \]  

(52)

\[ \Sigma(\Gamma - N) \text{ is a 2-fold rotation axis, } R = 2 \rightarrow S = -2. \]  

(53)

For the other three symmetry axes \(D(P - N), F(P - H),\) and \(G(M - N)\), which are on the surface of the first Brillouin zone (see Fig. 1), we determine the strange numbers as follows:

\[ D(P - N) \text{ is parallel to axis } \Delta, \ S_D = S_\Delta = 0; \]  

(54)

\[ F \text{ is parallel to an axis equivalent to } \Lambda, \ S_F = S_\Lambda = -1; \]  

(55)

\[ G \text{ is parallel to an axis equivalent to } \Sigma, \ S_G = S_\Sigma = -2. \]  

(56)

4. Electric charge \(Q\): after obtaining \(B, S\) and \(I\), we can find the charge \(Q\) from the Gell-Mann-Nishijiman relationship [26]:

\[ Q = I_z + \frac{1}{2}(S + B). \]  

(57)

5. Charmed number \(C\) [27] and Bottom number \(b\) [28]: if a degeneracy \(d\) of an energy band is smaller than the rotary fold \(R\)

\[ d < R \text{ and } R - d \neq 2, \]  

(58)

then formula (50) will be changed as

\[ \bar{S} = R - 4. \]  

(59)
From Hypothesis III \((\Delta S = \pm 1)\), the real value of \(S\) is

\[
S = \bar{S} + \Delta S = S_{Axis} \pm 1, \quad \text{if } d < R \text{ and } R - d \neq 2. \tag{60}
\]

The “Strange number” \(S\) in (60) is not completely the same as the strange number in (50). In order to compare it with the experimental results, we would like to give it a new name under certain circumstances. Based on Hypothesis III, the new names will be the Charmed number and the Bottom number:

if \(S = +1\) which originates from the fluctuation \(\Delta S = +1\),
then we call it the **Charmed** number \(C\) \((C = +1)\);

if \(S = -1\) which originates from the fluctuation \(\Delta S = +1\),
and if there is an energy fluctuation,
then we call it the **Bottom** number \(b\) \((b = -1)\). \tag{62}

Thus, (57) needs to be generalized to

\[
Q = I_z + \frac{1}{2}(B + S_G) = I_z + \frac{1}{2}(B + S + C + b), \tag{63}
\]

where we define the generalized strange number as

\[
S_G = S + C + b. \tag{64}
\]

6. **Charmed strange baryon \(\Xi_C\) [29] and \(\Omega_C\) [30]**: if the energy band degeneracy \(d\) is larger than the rotary fold \(R\), the degeneracy will be divided. Sometimes degeneracies should be divided more than once. After the first division, the sub-degeneracy energy bands have \(S_{Sub} = S + \Delta S\). For the second division of a degeneracy bands, we have:

if the second division has fluctuation \(\Delta S = +1\),
then \(S_{Sub}\) may be unchanged and we may have
a Charmed number \(C\) from \(C = \Delta S = +1\). \tag{65}
Therefore, we can obtain charmed strange baryons $\Xi_C$ and $\Omega_C$.

7. We assume that the excited quark’s mass is the minimum of the energy band which represents the excited quark.

$$m_{q^*} = \text{Minimum} \left( \varepsilon^{(0)}(\vec{k}, \vec{n}) \right) = \text{Minimum} \left( V_0 + \alpha E(\vec{k}, \vec{n}) \right)$$

(66)

Using the static masses (static energy) $M_{nucleon}$ of the nucleons we can determine $V_0$ in formula (66). The three quark systems ($q^*u'd'$) are baryons. The mass of the baryon is determined by

$$M_B = m_{q^*} + m_{u'} + m_{d'}.$$  

(67)

Since $m_{u'} = m_{d'} = 0$ from (1) and (2), we have

$$M_B = m_{q^*} = \text{Minimum}(V_0 + \alpha E(\vec{k}, \vec{n})).$$

(68)

The lowest mass of the baryon is the static mass of the nucleons $M_{nucleon} = V_0$. From the experiments, the static mass of the nucleons $M_{nucleon} = 939$ Mev [19]. Thus, we have

$$V_0 = M_{nucleon} = 939 \text{ Mev} \approx 940 \text{ Mev}.$$  

(69)

For the sake of convenience, we take $V_0 = 940$ Mev in (69). Fitting the theoretical mass spectrum into the empirical mass spectrum of the baryons, we can also determine the $\alpha$ value in (66):

$$\alpha = \frac{h^2}{2mqa_x^2} = 360 \text{ Mev}.$$  

(70)

8. The fluctuation of the strange number will be accompanied by an energy change (Hypothesis III). We assume that the change of the energy
(perturbation energy) is proportional to \((-\Delta S)\) and a number, \(J\), representing the energy level with a phenomenological formula:

\[
\Delta \varepsilon = (-1)^{R-4} \times 100 (1 - \delta(J) + (\delta(R-4)-1) \times J)(-\Delta S),
\]

(71)

where \(R\) is the rotary number of the axis, \(\delta(J)\) is a Dirac function (when \(J = 0, \delta(J) = 1\) and when \(J \neq 0, \delta(J) = 0\). Hence, \(\delta(J)\) makes that \(J = 0 \rightarrow \Delta \varepsilon = 0\).), and \(J\) is an order number of the energy band with \(\Delta S \neq 0\). Applying (71) to the symmetry axes, we have:

for the axis \(\Delta\), \(R - 4 = 0\),

\[
\Delta \varepsilon = -100 \times \Delta S \quad J = 1, 2, ...
\]

(72)

for the axes \(\Lambda\) and \(F\), \(4 - R = 1\),

\[
\Delta \varepsilon = -100 \times (J - 1) \Delta S \quad J = 1, 2, ...
\]

(73)

for the axes \(\Sigma\), \(G\), and \(D\), \(R - 4 = 2\)

\[
\Delta \varepsilon = 100 \times (J - 1) \Delta S \quad J = 1, 2, ...
\]

(74)

Thus, the zeroth-order mass formula of the quarks (66) shall be changed to

\[
m_q = \text{Minimum}(V_0 + \alpha E(\vec{k}, \vec{n}) + \Delta \varepsilon)
\]

(75)

This formula (75) is the united mass formula which can give all masses of all quarks. It is the united mass formula which can give all masses of all baryons.

Using the above formulae for quantum numbers and energy of the quarks, we can find the quark spectrum. We will start from the axis \(\Delta\).
### A- 4 The axis $\Delta(\Gamma - H)$

From (51), we have $S = 0$. For low energy levels, there are 8 and 4 fold degenerate energy bands and single bands on the axis. Since the axis has $R = 4$, from (48) and (49), the energy bands of degeneracy 8 will be divided into two 4 fold degenerate bands.

1. The four fold degenerate bands on the axis $\Delta(\Gamma - H)$

For 4 fold degenerate bands, using (47), we get

$\mathbf{I} = \frac{3}{2}$, and using (57), we have $\mathbf{Q} = \frac{5}{3}, \frac{2}{3}, \frac{1}{3}, \frac{4}{3}$. Thus, each 4 fold degenerate band represents a 4 fold quark family $q_\Delta$ with

$B = 1/3, S = 0, I = 3/2, Q = 5/3, 2/3, -1/3, -4/3$.  \hspace{1cm} (76)

Using Fig. 2(a) and Fig. 5(a), we can get $E_{\Gamma}$, $E_{H}$, and $\vec{n}$ values. Then, putting the values of $E_{\Gamma}$ and $E_{H}$ into the energy formula (66), we can find $m_{q^*} = \varepsilon^{(0)}$. Thus, we have

\begin{align*}
E_{H} &= 1 \quad \vec{n} = (101,-101,011,0-11) \quad \varepsilon^{(0)} = 1300 \quad q_\Delta^*(1300) \\
E_{\Gamma} &= 2 \quad \vec{n} = (110,1-10,-110,-1-10) \quad \varepsilon^{(0)} = 1660 \quad q_\Delta^*(1660) \\
E_{\Gamma} &= 2 \quad \vec{n} = (10-1,-10-1,01-1,0-1-1) \quad \varepsilon^{(0)} = 1660 \quad q_\Delta^*(1660) \\
E_{H} &= 3 \quad \vec{n} = (112,1-12,-112,-1-12) \quad \varepsilon^{(0)} = 2020 \quad q_\Delta^*(2020) \\
E_{\Gamma} &= 4 \quad \vec{n} = (200,-200,020,0-20) \quad \varepsilon^{(0)} = 2380 \quad q_\Delta^*(2380) \\
E_{H} &= 5 \quad \vec{n} = (121,1-21,-121,-1-21,211,2-11,211,-2-11) \quad \varepsilon^{(0)} = 2740 \quad q_\Delta^*(2740) \\
E_{H} &= 5 \quad \vec{n} = (202,-202,022,0-22) \quad \varepsilon^{(0)} = 2740 \quad q_\Delta^*(2740) \\
E_{H} &= 5 \quad \vec{n} = (013,0-13,103,-103) \quad \varepsilon^{(0)} = 2740 \quad q_\Delta^*(2740) \\
\ldots
\end{align*}  \hspace{1cm} (77)

1. 2. The single bands on the axis $\Delta(\Gamma - H)$

For the single bands, $d = 1 < R = 4$ and $R-d = 3 \neq 2$. According to (58), we should use (50) instead of (54). Therefore, we have

$S_{\text{Single}} = \bar{S}_\Delta \pm \Delta S = 0 \pm 1$, \hspace{1cm} (78)
where $\Delta S = \pm 1$ from Hypothesis III. The best way to guarantee the validity of Eq. (59) in any small region is to assume that $\Delta S$ takes $+1$ and $-1$ alternately from the lowest energy band to higher ones. In fact, the $\vec{n}$ values are really alternately taking positive and negative values: $E_H = 1, \vec{n} = (0, 0, 2); E_T = 4, \vec{n} = (0, 0, -2); E_H = 9, \vec{n} = (0, 0, 4); E_T = 16, \vec{n} = (0, 0, -4); E_H = 25, \vec{n} = (0, 0, 6); E_T = 36, \vec{n} = (0, 0, -6) ...$

Using the fact, we can find a phenomenological formula:

$$\Delta S = -(1 + S_{axis}) \text{Sign}(\vec{n}), \quad \text{Sign}(\vec{n}) = \frac{n_1 + n_2 + n_3}{|n_1| + |n_2| + |n_3|}.$$ (79)

For the single states on the axis $\Delta$, we have $S_{axis} = 0$. Thus, from (49), we get

$$\Delta S = -(1 + S_{axis}) \text{Sign}(\vec{n}) = -\text{Sign}(\vec{n}).$$ (80)

For the single states on the axis $\Sigma$, since $S_{axis} = -2$, we have

$$\Delta S = -(1 + S_{axis}) \text{Sign}(\vec{n}) = \text{Sign}(\vec{n}).$$ (81)

At $E_T = 0$, from (45), we know the lowest energy band with $n = (0, 0, 0)$ represents the free excited quark family $q_N^*(940)$.

At $E_H = 1$, the second lowest single energy band, with $\vec{n} = (0, 0, 2)$ and $J = 1$, has the strange number $\Delta S = -1$ from (80). Thus, $\Delta \varepsilon = 100$ Mev from (72) → the energy $\varepsilon = 940 + 360 \times 1 + \Delta \varepsilon = 1400$ Mev from (75), and $I = 0, Q = -1/3$ from (57). Thus, it represents a strange quark $q_S^*(1400)$.

At $E_T = 4$, for the third lowest band with $\vec{n} = (0, 0, -2)$, we get $\Delta S = +1$ from (80). Thus, $S = S_{\Delta} + 1 = 1$. The lowest $E$ of the band is at $E_T = 4$, the energy $\varepsilon = 940 + 360 \times 4 + \Delta \varepsilon = 2380 - 100 = 2280$ from (75) and (72). Here $S = +1$ originates from the fluctuation $\Delta S = +1$ and there is an energy fluctuation of $\Delta \varepsilon = -100$. From (61), we know that the energy band has a charmed number $C = +1$. It represents a new excited state of the quark, $q_C^*(2280)$, with

$$B = 1/3, I = 0, S = 0, C = 1, Q = 2/3, m_C = 2280 \text{ Mev}. \quad (82)$$
In order to make it compatible with the results of the experiments, we will call it the Charmed quark \([27]\). It is very important to pay attention to the Charmed quark born here, on the single energy band, and from the fluctuation \(\Delta S = +1\) and \(\Delta \varepsilon = -100\text{ Mev}\).

Continuing the above procedure, from Fig. 5 (b), (75), (80) and (72), we have (notice that point H and point \(\Gamma\), the two end points of the axis \(\Delta\), have the same symmetries):

| \(n_1, n_2, n_3\) | \(\Delta S\) | \(J\) | \(\Delta \varepsilon\) | \(S\) | \(C\) | \(q^*(m)\) |
|-------------------|-------------|------|----------------|-----|-----|-------------|
| \(E_H = 1\) | 0, 0, 2 | -1 | 1 | +100 | -1 | 0 | \(q_S^*(1400)\) |
| \(E_H = 4\) | 0, 0, -2 | +1 | 2 | -100 | 0 | 1 | \(q_C^*(2280)\) |
| \(E_H = 9\) | 0, 0, 4 | -1 | 3 | +100 | -1 | 0 | \(q_S^*(4280)\) |
| \(E_H = 16\) | 0, 0, -4 | +1 | 4 | -100 | 0 | 1 | \(q_C^*(6600)\) |
| \(E_H = 25\) | 0, 0, 6 | -1 | 5 | +100 | -1 | 0 | \(q_S^*(10040)\) |
| \(E_H = 36\) | 0, 0, -6 | +1 | 6 | -100 | 0 | 1 | \(q_C^*(13800)\) |

... (83)

A- 5 The axis \(\Lambda(\Gamma - P)\)

From Fig. 2(b), we see that there is a single energy band with \(\vec{n} = (0, 0, 0)\), and all other bands are 3 fold degenerate energy bands \((d = 3)\) and 6 fold degenerate bands \((d = 6)\).

1. At \(E_\Gamma = 0\), from (45), we know the lowest energy band with \(n = (0, 0, 0)\) represents the free excited quark family \(q_N^*(940)\).

2. From (48) and (49), the 6 fold degenerate energy bands will be divided into two energy bands with 3 fold degeneracy. For the 3 fold degenerate energy band, \(R = 3\) and \(S = -1\) from (52). Using (47) and (57), we have \(I = 1\), and \(q = 2/3, -1/3, -4/3\). Thus, we get a 3 fold quark family \(q^*_C\) with \(B = 1/3\), \(S = -1\), \(I = 1\), and \(Q = 2/3, -1/3, -4/3\).
Similar to (77), using Fig. 2(b), we get

| $\vec{n}$ | $\varepsilon^{(0)}$ | $q(m)$ |
|----------|------------------|------|
| $E_\Gamma = 2$ | 0-11,10-1,-101 | 1660 | $q_\Sigma^* (1660)$ |
| $E_\Gamma = 2$ | -10-1,0-1,1-10 | 1660 | $q_\Sigma^* (1660)$ |
| $E_\Gamma = 11/4$ | (121,211,112) | 1930 | $q_\Sigma^* (1930)$ |
| $E_\Gamma = 19/4$ | (1-12,-112,21-1,2-11,12-1,-121) | 2650 | $q_\Sigma^* (2650)$ |

The axis $\Sigma$ is a 2 fold rotation axis, from (53) $S = -2$. For low energy levels, there are 2 fold degenerate energy bands, 4 fold degenerate energy bands, and single energy bands on the axis (see Fig. 3(a)).

1. The two fold degenerate energy bands on the axis $\Sigma(\Gamma - N)$

For the two fold degenerate energy bands, each of them represents a quark family $q_\Sigma^*$ with $B = 1/3, I = 1/2$ from (17), $S = -2, Q = -1/3, -4/3$ from (57). Similar to (77),
we have
\[
\begin{array}{cccccc}
E & \vec{n} = (n_1, n_2, n_3) & S & \varepsilon^{(0)} & q(m) \\
E_G = 2 & (1-10,-110) & -2 & 1660 & q_\Xi^e(1660) \\
E_N = 5/2 & (200,020) & -2 & 1840 & q_\Xi^e(1840) \\
E_G = 4 & (002,00-2) & -2 & 2380 & q_\Xi^e(2380) \\
& (-200,0-20) & -2 & 2380 & q_\Xi^e(2380) \\
E_N = 9/2 & (112,11-2) & -2 & 2560 & q_\Xi^e(2560) \\
E_G = 6 & (-1-12,-1-1-2) & -2 & 3100 & q_\Xi^e(3100) \\
\end{array}
\]
...

2. According to (49), each 4 degenerate energy band on the symmetry axis \( \Sigma \) will be divided into two 2 fold degenerate bands. From (85), each of them represents a quark family \( q_\Xi^e \) with \( B = 1/3, I = 1/2, S = -2, Q = -1/3, -4/3 \). Thus, we have
\[
\begin{array}{cccccc}
E & \vec{n} = (n_1, n_2, n_3) & q(m = \varepsilon^{(0)}) \\
E_N = 3/2 & \vec{n} = (101,10-1,011,01-1) & 2 \times q_\Xi^e(1480) \\
E_G = 2 & \vec{n} = (-101,-10-1,0-11,0-1-1) & 2 \times q_\Xi^e(1660) \\
E_N = 7/2 & \vec{n} = (121,12-1,211,21-1) & 2 \times q_\Xi^e(2200) \\
E_N = 11/2 & \vec{n} = (-121,-12-1,2-11,2-1-1) & 2 \times q_\Xi^e(2920) \\
E_G = 6 & \vec{n} = (1-12,1-1-2,-112,-11-2) & 2 \times q_\Xi^e(3100) \\
\end{array}
\]
...

3. The single energy bands on the axis \( \Sigma(\Gamma - N) \)

For the single energy bands, \( d = 1 < R = 2 \) and \( R-d = 1 \neq 2 \). According to Hypothesis IV. 5, (10), we have to use (60) instead of (57):
\[
S_{\text{Single}} = \bar{S}_\Sigma \pm \Delta S = -2 \pm 1.
\]

(87)

The strange number will take \(-1\) and \(-3\) alternately from lower to higher energy bands.

For the energy fluctuation on the axis \( \Sigma \), from (74), we have
\[
\Delta \varepsilon = 100(j - 1)\Delta S \text{ Mev, } J = 1, 2, 3...
\]

(88)

Since the end points \( \Gamma \) and \( N \) of the axis \( \Sigma \) have different symmetries, \( J \) will take 1, 2, ... from the lowest energy band to higher ones for each of the two end points respectively.
At \( E_\Gamma = 0, J_\Gamma = 0 \), from (43), the lowest energy band with \( \mathbf{n} = (0, 0, 0) \) represents the free quark family \( q_1^*(940) \).

At \( E_N = 1/2, J_N = 0 \), the second lowest energy band with \( \mathbf{n} = (1, 1, 0) \) has \( S = -1(\Delta S = +1) \) from (81). Using (73), the energy of the excited state of the quark is \( \varepsilon = 1120 \) Mev. It is very important to pay attention to the strange quark, \( q_S(1120) \), born on the single energy band of the axis \( \Sigma \) from the fluctuation \( \Delta S = +1 \). It has

\[
B = 1/3, S = -1, I = 0, Q = -1/3, \text{ and } m_S = 1120. \tag{89}
\]

At \( E_\Gamma = 2, J_\Gamma = 1 \), the third lowest band with \( \mathbf{n} = (-1, -1, 0) \) should have \( \Delta S = -1 \) from (81). Thus, it is the quark \( q_\Omega(1660) \) with \( S = -3, I = 0, Q = -4/3 \), and \( \varepsilon = 1660 \) Mev from (73).

At \( E_N = 9/2, \) the fourth one (\( \mathbf{n} = (2, 2, 0) \)) has \( \Delta S = +1 \) from (81), \( S = -2 + 1 = -1 \), and \( J_N = 1, \Delta \varepsilon = 0 \) from (85). The total energy \( \varepsilon = 2560 \) Mev from (73). Since \( \Delta \varepsilon = 0 \), from (72), we get that this energy band represents an excited quark \( q_S(2560) \) with

\[
B = 1/3, S = -1, I = 0, Q = -1/3, \text{ and } M = 2560 \text{ Mev.} \tag{90}
\]

At \( E_\Gamma = 8, \) the fifth one (\( \mathbf{n} = (-2, -2, 0) \)) has \( B = 1/3, S = -3, I = 0 \) and \( Q = -4/3 \), so it represents an excited state quark \( q_\Omega(3720) \) with \( B = 1/3, S = -3, I = 0, Q = -4/3 \).

At \( E_N = 25/2, \) the sixth one (\( \mathbf{n} = (3, 3, 0) \)) has \( J_N = 2 \) and \( S = -2 + 1 = -1 \) from (81), as well as \( \varepsilon = 5440 + 100 = 5540 \) from (75). According to Hypothesis IV. 5 (72), we know that the energy band has a bottom number \( b = -1 \). It represents an excited quark state \( q_b(5540) \) with \( I = 0, b = -1, Q = -1/3, \) and \( \varepsilon = 5540 \). In order to make it compatible with the quark model, we call it the bottom quark. It has

\[
B = 1/3, S = C = 0, b = -1, Q = -1/3, m = 5540. \tag{91}
\]
It is very important to pay attention to the **bottom quark born on the single energy band from the fluctuation** $\Delta S = +1$ and $\Delta \varepsilon = 100$ Mev. Using Fig. 5(c), we find the baryons:

| $E$ | $(n_1,n_2,n_3)$ | $S_G$ | $\Delta \varepsilon$ | $q(m)$ |
|-----|-----------------|------|----------------------|--------|
| $E_N = 1/2$ | (1,1,0) | -1 | $J_N = 0$ | 0 | $q_S^*(1120)$ |
| $E_G = 2$ | (-1,-1,0) | -3 | $J_G = 1$ | 0 | $q_{11}^*(1660)$ |
| $E_N = 9/2$ | (2,2,0) | -1 | $J_N = 1$ | 0 | $q_S^*(2560)$ |
| $E_G = 8$ | (-2,-2,0) | -3 | $J_G = 2$ | -100 | $q_{12}^*(3720)$ |
| $E_N = 25/2$ | (3,3,0) | -1 | $J_N = 2$ | +100 | $q_b^*(5540)$ |
| $E_G = 18$ | (-3,-3,0) | -3 | $J_G = 3$ | -200 | $q_{13}^*(7220)$ |
| $E_N = 49/2$ | (4,4,0) | -1 | $J_N = 3$ | +200 | $q_b^*(9960)$ |

Continuing the above procedure (see Appendix B), we can use Fig. 2-5 to find the whole excited states of the quark (the quark spectrum).
| $q^Q$ | $\hat{q}_N^q$ | $\hat{q}_V^q$ | $\hat{q}_\Delta^q$ | $\hat{q}_\Delta^q$ | $\hat{q}_\Delta^q$ | $\hat{q}_\Sigma^q$ | $\hat{q}_{\Sigma^q}$ | $\hat{q}_{\Sigma^q}$ | $\hat{q}_{\Omega^q}$ | $\hat{q}_{\Omega^q}$ |
|-------|----------------|----------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| S     | 0               | 0              | 0                   | 0                   | 0                   | -1                  | -1                  | -1                  | -1                  | -3                  |
| C     | 0               | 0              | 0                   | 0                   | 0                   | 0                   | 0                   | 0                   | 0                   | 0                   |
| b     | 0               | 0              | 0                   | 0                   | 0                   | 0                   | 0                   | 0                   | 0                   | 0                   |
| I     | 1/2             | 1/2            | 3/2                 | 3/2                 | 3/2                 | 0                   | 1                   | 1                   | 1                   | 0                   |
| $I_Z$ | 1/2             | -1/2           | 3/2                 | 1/2                 | -1/2                | -3/2                | 0                   | 1                   | 0                   | -1                  | 0                   |
| Q     | 2/3             | -1/3           | 5/3                 | 2/3                 | -1/3                | -4/3                | -1/3                | 2/3                 | -1/3                | -4/3                | -4/3                |

| $q^Q$ | $\hat{q}_{\Xi^q}$ | $\hat{q}_{\Xi^q}$ | $\hat{q}_{\Xi^q}$ | $\hat{q}_{\Xi^q}$ | $\hat{q}_{\Xi^q}$ | $\hat{q}_{\Xi^q}$ | $\hat{q}_{\Xi^q}$ | $\hat{q}_{\Xi^q}$ | $\hat{q}_{\Xi^q}$ | $\hat{q}_{\Xi^q}$ |
|-------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| S     | -2                  | -2                  | 0                   | 0                   | -2                  | -1                  | -1                  | 0                   | 0                   | 0                   |
| C     | 0                   | 0                   | 0                   | 1                   | 0                   | 1                   | 1                   | 1                   | 1                   | 1                   |
| b     | 0                   | 0                   | 0                   | -1                  | 0                   | 0                   | 0                   | 0                   | 0                   | 0                   |
| I     | 1/2                 | 1/2                 | 0                   | 0                   | 1/2                 | 1/2                 | 1                   | 0                   | -1                  | 0                   |
| $I_Z$ | 1/2                 | -1/2                | 0                   | 0                   | 1/2                 | -1/2                | 1                   | 0                   | -1                  | 0                   |
| Q     | -1/3                | -4/3                | 2/3                 | -1/3                | -1/3                | 2/3                 | -1/3                | 5/3                 | 2/3                 | -1/3                |

(93)
In (93), we list the quantum numbers (S, C, b, I, I_Z, and Q) of the quarks. In (94) and (95), we give the masses of the quarks.
B The Spectrum of the Baryons

We have already found the quantum numbers and energies of the quarks (93), (94), (95). Now we will recognize the baryons \((q'u'd')\). Using the sum laws (4), we can find the quantum numbers and energies of the three quark systems (baryons). Since the quantum numbers and energies of the accompanying excited quarks, \(u'\) and \(d'\), are already given in (3) and (4), we will focus our attention on the quantum numbers and energies of the excited quark \(q^*\).

B- 1 The energy and the quantum numbers

1. The energy of the system \((q'u'd')\) equals the energy of the excited quark \(q^*\)

\[ M_{(q'u'd')} = m_{q^*}. \]  

2. The quantum numbers of the system are the sum of the constituent quarks.

\[ B = 1, S = S_{q^*}, C = C_{q^*}, \quad b = b_{q^*}, \quad Q = \sum Q_{q}, \]  

from (3) and (2).

3. The isospin of the system \((q'u'd')\) is found by

\[ \vec{I}_B = \vec{I}_{q^*} + \vec{I}_{u'} + \vec{I}_{d'} \]

4. The top limit \(I_{\text{max}}\) of the isospin of the baryons on the symmetry axis is determined by

\[ 2I_{\text{max}}(axis) + 1 = 4 + S. \]

From(31)-(50), we have

\[ I_{\text{max}} = 3/2 \text{ for the axis } \Delta \text{ and the axis } D, \]
\[ I_{\text{max}} = 1 \] for the axis \( \Lambda \) and the axis \( F \). \hfill (101)

\[ I_{\text{max}} = 1/2 \] for the axis \( \Sigma \) and the axis \( G \). \hfill (102)

5. The top limit \( (I_{\text{max}}) \) of the isospin of the baryons at the symmetry point is determined by the highest dimension \( (D_{\text{high}}) \) of the irreducible representations of the double point group at the point:

\[ 2I_{\text{max}} + 1 = D_{\text{high}}. \hfill (103) \]

Since the highest dimension \( D_{\text{high}} = 4 \) for the double point group \( \Gamma \), the group \( H \), the group \( M \), and the group \( P \), we get \n
\[ I_{\text{max}} = 3/2, \text{ at } \Gamma, H, M, P; \hfill (104) \]

and from that the highest dimension \( D_{\text{high}} = 2 \) for the double point group \( N \), we know \n
\[ I_{\text{max}} = 1/2, \text{ at the point } N. \hfill (105) \]

B-2 The Baryons on the axis \( \Delta(\Gamma - H) \)

1. The 4 fold degenerate bands on the axis \( \Delta(\Gamma - H) \)

From (77), each 4 fold band represents a 4 fold quark family \( q_\Delta^* \) with \( S = 0, I = 3/2, \)
and \( Q = 5/3, 2/3, -1/3, -4/3 \). Adding the accompanying excited quarks \( u' \) and \( d' \), from (98), the three quark system \( (q_\Delta^*u'd') \) has \( I = 5/2, 3/2, 1/2 \). Since the top limit \( I = 3/2 \) from (100), we can have \( I = 3/2, 1/2 \) only, omitting \( I = 5/2 \). From (57), we have:

for \( I = 1/2, Q = 1, 0; \) for \( I = 3/2, Q = 2, 1, 0, -1 \); and \( S = 0, I = 1/2, Q = +1, 0 \). Thus from (77), the system has \( S = 0, I = 3/2, Q = 2, 1, 0, -1; \) or \( S = 0, I = 3/2, Q = 2, 1, 0, -1; \) and the baryon family \( N(N^+, N^0) \) has \( S = 0, I = 1/2, \) and
Q = 1, 0, we discover that the system represents the baryon families ∆ and N (i.e. for each 4 fold band, we get a ∆ family and a N family). Using (96) and (77), we have

\[
\begin{align*}
E_H &= 1 \quad \bar{n} = (101,-101,011,0-11) \quad q^{*}_\Delta(1300) \quad \Delta(1300); \quad N(1300) \\
E_H &= 2 \quad \bar{n} = (110,1-10,-110,1-10) \quad q^{*}_\Delta(1660) \quad \Delta(1660); \quad N(1660) \\
E_H &= 2 \quad \bar{n} = (101,1-10,011,0-11) \quad q^{*}_\Delta(1660) \quad \Delta(1660); \quad N(1660) \\
E_H &= 3 \quad \bar{n} = (112,1-12,-112,1-12) \quad q^{*}_\Delta(2020) \quad \Delta(2020); \quad N(2020) \\
E_H &= 4 \quad \bar{n} = (200,-200,020,0-20) \quad q^{*}_\Delta(2380) \quad \Delta(2380); \quad N(2380) \\
E_H &= 5 \quad \bar{n} = (121,1-21,-121,1-21) \quad q^{*}_\Delta(2740) \quad \Delta(2740); \quad N(2740) \\
E_H &= 5 \quad \bar{n} = (202,-202,022,0-22) \quad q^{*}_\Delta(2740) \quad \Delta(2740); \quad N(2740) \\
E_H &= 5 \quad \bar{n} = (013,0-13,103,-103) \quad q^{*}_\Delta(2740) \quad \Delta(2740); \quad N(2740)
\end{align*}
\]

2. The single bands on the axis ∆(Γ − H)

From (83), we have the excited quark q^{*}_S (with S = -1, I = 0, Q = -1/3), and the excited quark q^{*}_C (with B = 1/3, I = 0, S = 0, C = 1, Q = 2/3). Adding quarks u' and d', from (97), the three quark system (u'd'q^{*}_S) has S = -1, I = 0, Q = 0 and S = -1, I = 1, Q = +1, 0, -1. They are baryons Λ and Σ. The other three quark system (u'd'q^{*}_C) has B =1, C = +1, I = 0, Q = +1, and B = 1, C = +1, I = 1, Q = +2, 1, 0. They are baryons Λ_C and Σ_C. For example, at E_H = 4, the three quark system (u'd'q^{*}_C) is Λ_C^+(2280) and Σ_C(2280). In order to make it compatible with the results of the experiments, we will call them the Charmed baryon Λ_C^+(2280) and the charmed baryon Σ_C(2280).

Using (83) and (96), we have

\[
\begin{align*}
E_H &= 1 \quad \Delta S = -1 \quad J = 1 \quad q^{*}_S(1400) \quad \Lambda(1400) \quad \Sigma(1400) \\
E_H &= 4 \quad \Delta S = +1 \quad J = 2 \quad q^{*}_C(2280) \quad \Lambda_C^+(2280) \quad \Sigma_C(2280) \\
E_H &= 9 \quad \Delta S = -1 \quad J = 3 \quad q^{*}_S(4280) \quad \Lambda(4280) \quad \Sigma(4280) \\
E_H &= 16 \quad \Delta S = +1 \quad J = 4 \quad q^{*}_C(6600) \quad \Lambda_C^+(6600) \quad \Sigma_C(6600) \\
E_H &= 25 \quad \Delta S = -1 \quad J = 5 \quad q^{*}_S(10040) \quad \Lambda(10040) \quad \Sigma(10040) \\
E_H &= 36 \quad \Delta S = +1 \quad J = 6 \quad q^{*}_C(13800) \quad \Lambda_C^+(13800) \quad \Sigma_C(13800)
\end{align*}
\]

...
From (84), we get quarks $q'_\Sigma$. Adding quarks $u'$ and $d'$, the three quark system $(q'_\Sigma u'd')$ has the possible isospin values are $I = 2, 1, 0$, from (98). From (101), we get $I = 1, 0$ only. Using (97), for $I = 1$, we have $B = 1, S = -1, I = Q = +1, 0, -1$, the system represents a baryon family $\Sigma$; for $I = 0$, the system has $B = 1, S = -1, I = Q = 0$, it means a baryon $\Lambda$. Using (98), we get

\[
\begin{align*}
E_P &= 3/4 & \vec{n} &= (101,011,110) & q^+_\Sigma(1210) & \Sigma(1210); & \Lambda(1210) \\
E_\Gamma &= 2 & \vec{n} &= (1-10,-110,01-1,0-11,10-1,101) & q^+_\Sigma(1660) & \Sigma(1660); & \Lambda(1660) \\
E_\Gamma &= 2 & \vec{n} &= (1-10,0-1-1,-1-10) & q^+_\Sigma(1930) & \Sigma(1930); & \Lambda(1930) \\
E_P &= 11/4 & \vec{n} &= (020,002,200) & q^+_\Sigma(2380) & \Sigma(2380); & \Lambda(2380) \\
E_P &= 11/4 & \vec{n} &= (121,211,112) & q^+_\Sigma(2650) & \Sigma(2650); & \Lambda(2650) \\
E_\Gamma &= 4 & \vec{n} &= (0-20,-200,00-2) & q^+_\Sigma(3100) & \Sigma(3100); & \Lambda(3100) \\
E_P &= 19/4 & \vec{n} &= (12,-112,21-1,2-11,12-1,1-12) & q^+_\Sigma(2650) & \Sigma(2650); & \Lambda(2650) \\
E_P &= 19/4 & \vec{n} &= (202,022,220) & q^+_\Sigma(2650) & \Sigma(2650); & \Lambda(2650) \\
E_\Gamma &= 6 & \vec{n} &= (-211,2-1-1,2-1-1,11-2,-12-11-21) & q^+_\Sigma(3100) & \Sigma(3100); & \Lambda(3100) \\
\end{align*}
\]

\((108)\)

\[\ldots\]

B- 4 The baryons on the axis $\Sigma(\Gamma - N)$

1. The two fold energy bands on the axis $\Sigma(\Gamma - N)$

For the 2 fold energy bands, from (84), we get the quark families $q^+_\Xi$ with $S = -2$, $I = 1/2, Q = -1/3, -4/3$. Adding quarks $u'$ and $d'$, from (97), the three quark system
(u′d′q∗) is the baryon Ξ with S = -2, I = 1/2, Q = 0, -1. Using (96), we have

\[
\begin{align*}
E_Γ &= 2 \quad \bar{n} = (1-10,-110) \quad q_ξ^*(1660) \quad Ξ(1660) \\
E_N &= 5/2 \quad \bar{n} = (200,020) \quad q_ξ^*(1840) \quad Ξ(1840) \\
E_Γ &= 4 \quad \bar{n} = (002,0-2) \quad q_ξ^*(2380) \quad Ξ(2380) \\
E_N &= 9/2 \quad \bar{n} = (112,11-2) \quad q_ξ^*(2560) \quad Ξ(2560) \\
E_Γ &= 6 \quad \bar{n} = (-121,-12,-1-2) \quad q_ξ^*(3100) \quad Ξ(3100)
\end{align*}
\]

2. The four fold degenerate energy bands on the axis Σ(Γ − N)

From (86), each 4 fold energy band represent 2 quark families 2 × q∗ξ. Adding quarks u′ and d′, from (97), we get 2 baryon families 2 × Ξ. Using (96), we have

\[
\begin{align*}
E_N &= 3/2 \quad \bar{n} = (101,10-1,011,01-1) \quad 2 \times q_ξ^*(1480) \quad 2 \times Ξ(1480) \\
E_Γ &= 2 \quad \bar{n} = (-101,-10-1,0-11,0-1) \quad 2 \times q_ξ^*(1660) \quad 2 \times Ξ(1660) \\
E_N &= 7/2 \quad \bar{n} = (121,12-1,211,21-1) \quad 2 \times q_ξ^*(2200) \quad 2 \times Ξ(2200) \\
E_N &= 11/2 \quad \bar{n} = (-121,-12-1,2-11,2-1) \quad 2 \times q_ξ^*(2920) \quad 2 \times Ξ(2920) \\
E_Γ &= 6 \quad \bar{n} = (1-12,-1-1-2) \quad 2 \times q_ξ^*(3100) \quad 2 \times Ξ(3100)
\end{align*}
\]

3. The single energy bands on the axis Σ(Γ − N)

From (82), we get quarks q∗S and q∗Ω. Adding quarks u′ and d′, the three quark system (u′d′q∗) has I = 1, 0. Since the top limit I = 1/2 of the axis Σ from (102), we get I = 0 only, omitting I = 1. From (97), for q∗S, the system (u′d′q∗S) has B = 1, I = 0, S = -1 (Q = 0), it is a strange baryon Λ; for q∗Ω, the three quark system (u′d′q∗Ω) has B = 1, S = -3, I = 0, Q = -1, it is a baryon Ω.

At \(E_N = 25/2\), the sixth band is \(q_β^*(5540)\) with \(B = 1/3, S = C = 0, b = -1, Q = -1/3, m = 5540\). Adding quarks u′ and d′, the three quark system (u′d′q∗) has B = 1, S = C = 0, b = -1, I = 0, and Q = 0 from (97). In order to make it compatible with the results of the experiments, we call this baryon the **Bottom baryon Λ_b(5540)**
Using (96), we have:

\[
\begin{align*}
E_N &= 1/2 \quad \bar{n} = (110) \quad q_S^*(1120) \quad \Lambda(1120) \\
E_F &= 2 \quad \bar{n} = (-1-10) \quad q_{\Omega}^*(1660) \quad \Omega^- (1660) \\
E_N &= 9/2 \quad \bar{n} = (220) \quad q_S^*(2560) \quad \Lambda(2560) \\
E_F &= 8 \quad \bar{n} = (-2-20) \quad q_{\Omega}^*(3720) \quad \Omega^- (3720) \\
E_N &= 25/2 \quad \bar{n} = (330) \quad q_{b}^*(5540) \quad \Lambda_b^0 (5540) \\
E_F &= 18 \quad \bar{n} = (-3-30) \quad q_{b}^*(7220) \quad \Omega^- (7220) \\
E_N &= 49/2 \quad \bar{n} = (440) \quad q_b^*(9960) \quad \Lambda_b^0 (9960)
\end{align*}
\]

\[\text{(111)}\]

\[\text{...}\]

**B- 5 The baryons on the axis D(P-N)**

1. The 4 fold degeneracy energy bands on the axis D(P-N)

From (149), for each 4 fold degenerate energy band, we get 2 quark families \(2 \times q^*_N\) with \(B = 1/3, S = C = b = 0, I = 1/2, Q = 2/3, -1/3\). Adding quarks \(u'\) and \(d'\), the three quark system \((q^*_N u'd')\) has the possible isospin values \(I = 1/2\) and \(3/2\) at the point P from (88). But the possible isospin \(I = 1/2\) only from (105) at the point N. From (149) and (97) we can get

\[
\begin{align*}
E_N &= 5/2 \quad \bar{n} = (1-10,-110,020,200) \quad 2 q^*_N(1840) \quad 2 N(1840) \\
E_P &= 11/4 \quad \bar{n} = (-101,0-11,211,121) \quad 2 q^*_N(1930) \quad 2 N(1930) \quad 2 \Delta(1930) \\
E_N &= 7/2 \quad \bar{n} = (12-1,21-1,-10-1,0-1-1) \quad 2 q^*_N(2200) \quad 2 N(2200) \quad (112) \\
E_P &= 19/4 \quad \bar{n} = (-112,1-12,202,022) \quad 2 q^*_N(2650) \quad 2 N(2650) \quad 2 \Delta(2650)
\end{align*}
\]

\[\text{...}\]

2. The two fold energy bands on the axis \(D(P - N)\)

From (150), we get \(q_N^*, q_S^*, \text{ and } q_C^***\). Adding quarks \(u'\) and \(d'\), we get the baryons \(N\) (and \(\Delta\) at the point \(P\)), \(\Lambda\), and \(\Lambda_C\) from (96) and (97).

At \(E_P = 3/4\), the two energy bands represent a quark family \(q_N^*(1210) \Rightarrow \text{the baryon families } N(1210) \text{ and } \Delta(1210)\). The baryon \(\Delta(1210)\) is the ground state of the \(\Delta\) baryons.
At $E_N = 9/2$, $\vec{n} = (220, -1, -10)$ and $\vec{n} = (11, -2, 00, -2)$. For the first 2 energy bands they represent the quark family $q^*_N(2560) \Rightarrow$ the baryon family $N(2560)$. For the second 2 energy bands, one represents a charmed quark $q^*_C(2760)$ (with $I = 0$ and $Q = +1$) $\Rightarrow$ the Charmed baryon $\Lambda^+_C(2760)$, the other represents $q^*_S(2360) \Rightarrow$ a baryon $\Lambda(2360)$.

Therefore, using (96) and (97), we have

\begin{align*}
E_N &= 1/2 \quad \vec{n} = (000, 110) \\
J_N &= 0 \quad \vec{n} = (000) \quad q^*_N(940) \quad N(940) \\
\Delta \varepsilon &= 0 \quad \vec{n} = (110) \quad q^*_S(1120) \quad \Lambda(1120) \\
E_P &= 3/4 \quad \vec{n} = (101, 011) \quad q^*_N(1210) \quad N(1210) \quad \Delta(1210) \\
E_N &= 3/2 \quad \vec{n} = (10-1, 00-2) \quad q^*_N(1480) \quad N(1480) \\
E_P &= 11/4 \quad \vec{n} = (002, 112) \quad q^*_N(1930) \quad N(1930) \quad \Delta(1930) \\
E_N &= 9/2 \quad \varepsilon^{(0)} = 2560 \quad \vec{n} = (220, -1, -10) \quad q^*_N(2560) \quad N(2560) \\
J_N &= 1 \quad \vec{n} = (220, -1, -10) \quad q^*_N(2560) \quad N(2560) \\
J_N &= 2 \quad \vec{n} = (11-2, 00-2) \\
\Delta S &= +1 \quad q^*_C(2660) \quad \Lambda^+_C(2660) \\
\Delta S &= -1 \quad q^*_S(2460) \quad \Lambda(2460) \\
E_P &= 19/4 \quad \vec{n} = (121, 2-11) \quad q^*_N(2650) \quad N(2650) \quad \Delta(2650) \\
E_N &= 11/2 \quad \vec{n} = (2-1-1, -12-1) \quad q^*_N(2920) \quad N(2920) \\
\ldots
\end{align*}

**B- 6 The Axis $G(M - N)$**

There are 2, 4, and 6 fold energy bands on the axis (see Fig. 4(b)).

1. The two fold energy bands on the axis $G(M - N)$

   From (163), for the 2 fold energy band (see Fig. 4(b)), we have $q^*_N$, $q^*_S$, $q^*_\Xi$. Adding
quarks $u'$ and $d'$, we get the baryons $N$, $\Lambda$, and $\Xi$ from (96), (97), as follows

\[
\begin{align*}
E_N &= 1/2 & \vec{n} &= (000,110) \\
J_N &= 0 & \vec{n} &= (000) & q_N^*(940) & N(940) \\
       & & \vec{n} &= (110) & q_N^*(1120) & \Lambda(1120) \\
E_M &= 1 & \vec{n} &= (101,10-1) & q_N^*(1300) & \Xi(1300) \\
       & & \vec{n} &= (200,1-10) & q_N^*(1300) & \Xi(1300) \\
E_N &= 3/2 & \vec{n} &= (011,01-1) & q_N^*(1480) & \Xi(1480) \\
E_N &= 5/2 & \vec{n} &= (020,-110) & q_N^*(1840) & \Xi(1840) \\
E_M &= 3 & \vec{n} &= (2-11,2-1-1) & q_N^*(2020) & \Xi(2020) \\
E_M &= 5 & \vec{n} &= (3-10,2-20) & q_N^*(2740) & \Xi(2740) \\
E_N &= 11/2 & \vec{n} &= (-121,-12-1) & q_N^*(2920) & \Xi(2920) \\
\end{align*}
\]

\[\text{(114)}\]

\[
\begin{align*}
E_M &= 3 & \vec{n} &= (0-11,0-1-1, \ 211,21-1) \quad 2 \times q_N^*(2020) \quad 2 \Xi(2020) \\
E_N &= 7/2 & \vec{n} &= (-101,-10-1, \ 121,12-1) \quad 2 \times q_N^*(2200) \quad 2 \Xi(2200) \\
E_M &= 5 & \vec{n} &= (301,30-1, \ 1-21,1-2-1) \quad 2 \times q_N^*(2740) \quad 2 \Xi(2740) \\
\end{align*}
\]

\[\text{(115)}\]

Continuing the above procedure, using Fig. 2-5, from (96) and (97), we can find the whole baryon spectrum. Our results are shown in Tables 1 though 6.

C The Spectrum of Mesons

In Section V.A. we have found the quark spectrum (93), (94), (95). Using the definition that a meson = $(qq)$ and a phenomenological mass formula for the mesons, we can find the meson spectrum (see next paper: Jiao-Lin Xu and Xin Yu, The Meson Spectrum)
VI  The SU(N) Approximation (The Quark Model)

In order to find the relationship between the Quark Model and the BCC model, we study the SU(N) symmetry approximation. In the approximation, we will see that the BCC model provides the physical foundation for the Quark Model. We have found the quark spectrum (93), (94), (95) in Section V. In the SU(N) (N = 3, 4, 5) approximation, based on the quark spectrum, we assume the following:

1. The lowest energy quark excited state (ground state) of the each flavored quark excited states are regarded as the elemental particle (quark), omitting all other higher energy excited quarks of the quark spectrum (94) and (95).

There are 4 kinds of the flavored quarks in the BCC model. They are unflavored quarks, $q_N$ and $q_\Delta$; the strange quarks, $q_S$, $q_\Sigma$, $q_\Xi$, and $q_\Omega$; the charmed quark $q_C$; and the bottom quark $q_b$.

2. The N quarks satisfy the SU(N) symmetry and belong to the fundamental representations of the SU(N) group in the strong interactions. Since the N quarks are the excited states of the fundamental quarks and the fundamental quarks satisfy the SU(2) symmetry, so SU(3), SU(4), and SU(5) are the natural extensions of SU(2).

3. A baryon is made of three quarks (qqq state).
4. A meson is made of a quark and an antiquark ($q\bar{q}$ state).
5. The quantum numbers of the baryons and the mesons can be determined by the sum formulae (4).
A  The quarks

1. There are 2 kinds of unflavored quarks, $q_N$ and $q_\Delta$, with $S = C = b = 0$ from (93). According to the quark spectrum (94), the quark family $q_N(940)$ (with $B = 1/3$, $S = 0$, $s = I = 1/2$, and $Q = 2/3, -1/3$) is the ground state of the unflavored quarks.

2. For $S \neq 0$, $C = b = 0$, from (93), there are 4 kinds of strange quark excited states: the strange quark $q_S$, the strange quark family $q_{\Sigma}$, the strange quark family $q_{\Xi}$, and the strange quark $q_{\Omega}$. From (94), the strange quark $q_S(1120)$ (with $B = 1/3$, $S = -1$, $s = 1/2$, $I = 0$, and $Q = -1/3$) is their ground state.

3. For $C \neq 0$ and $S = b = 0$, from (93), there is only one kind of charmed quark excited states. The Charmed Quark $q_C(2280)$ (with $B = 1/3$, $C = +1$, $s = 1/2$, $I = 0$, $Q = 2/3$) is the ground state.

4. For $b \neq 0$ and $S = C = 0$, from (93), there is only one kind of Bottom Quark $q_b$. From (93), the Bottom Quark, $q_b(5540)$, is the ground state.

In the SU(N) approximation, since the three quarks are in the symmetry positions inside the baryon (p and n), so they have the same masses in a proton (uud) and a neutron (udd). Thus, $m_u = m_d = M_N(940)/3 = 313(Mev)$. Using the baryons $\Lambda = (uds)$ and $\Sigma = (uus, uds, dds)$, and the average mass of $\Lambda$ and $\Sigma$, $M_{\Lambda, \Sigma} = 1/2(1116 + 1193) = 1155(Mev)$, we get $m_S = 529$. Similarly, using $\Lambda_C = (udc)$ and $\Lambda_b = (udb)$, we can get $m_C = 1654(Mev)$ and $m_b = 5014(Mev)$. To sum up, the five quarks have the following quantum numbers and masses:

| $q$ | # | B  | I  | I_z | S | C | b | Q  | $M_{Mev}$ |
|-----|---|----|----|-----|---|---|---|----|----------|
| $q_u$ | $1/3$ | 1/2 | 1/2 | 0  | 0 | 0 | 2/3 | 313 |
| $q_d$ | $1/3$ | 1/2 | -1/2 | 0  | 0 | 0 | -1/3 | 313 |
| $q_s$ | $1/3$ | 0  | 0  | -1 | 0 | 0 | -1/3 | 529 |
| $q_c$ | $1/3$ | 0  | 0  | 0  | 1 | 0 | 2/3 | 1654 |
| $q_b$ | $1/3$ | 0  | 0  | 0  | 0 | -1 | -1/3 | 5014 |

Comparing (116) with Table 12.1 of [31], we find that the 5 quarks in (116) are the
same quarks in Table 12.1 of the Quark Model (in the quantum numbers $B, I, I_z, S, C, b$). Therefore, the 5 quarks ($q_u, q_d, q_s, q_c, q_b$) of the SU(N) are the same quarks as the Quark Model. In other words,

$$q_u = u, q_d = d, q_s = s, q_c = c, \text{ and } q_b = b.$$ (117)

B The fundamental representations of the SU(N) groups.

According to HYPOTHESIS I, the fundamental quarks, $u$ and $d$, satisfy the SU(2) symmetries. From the combined approximation, the quarks $q_S, q_C, \text{ and } q_b$ are the excited states of the fundamental quark $q$ ($u$ and $d$). Therefore, the SU(3) (for the quarks $u, d, \text{ and } s$), the SU(4) (for the quarks $u, d, s, \text{ and } c$), and the SU(5) (for the quarks $u, d, s, c, \text{ and } b$) symmetries are the natural extensions of the SU(2).

For the 5 quarks, we assume that their wave functions are

$$\Psi_u = \Psi(B=1/3, I=1/2, I_z=1/2, S=C=b=0, Q=2/3, m=313),$$ (118)

$$\Psi_d = \Psi(B=1/3, I=1/2, I_z=-1/2, S=C=b=0, Q=-1/3, m=313),$$ (119)

$$\Psi_s = \Psi(B=1/3, I=0, I_z=0, S=-1, C=b=0, Q=-1/3, m=529),$$ (120)

$$\Psi_c = \Psi(B=1/3, I=0, I_z=0, C=1, S=b=0, Q=2/3, m=1654),$$ (121)

$$\Psi_b = \Psi(B=1/3, I=0, I_z=0, b=-1, S=C=0, Q=-1/3, m=5014).$$ (122)

Fundamental representation 3 of the SU(3) can be written as a column matrix

$$3 = \begin{pmatrix} \Psi_u \\ \Psi_d \\ \Psi_s \end{pmatrix}.$$ (123)
Fundamental representation $4$ of the SU(4) can be written as a column matrix

$$
4 = \begin{pmatrix}
\Psi_u \\
\Psi_d \\
\Psi_s \\
\Psi_c \\
\end{pmatrix}.
$$

(124)

Fundamental representation $5$ of the SU(5) can be written as a column matrix

$$
5 = \begin{pmatrix}
\Psi_u \\
\Psi_d \\
\Psi_s \\
\Psi_c \\
\Psi_b \\
\end{pmatrix}.
$$

(125)

C The baryons (qqq states).

From the primitive cell approximation, we can draw a reasonable assumption that a baryon is made of three quarks (qqq).

The “ordinary” baryons are made up of $u$, $d$, and $s$ quarks. The three quarks satisfy SU(3) symmetry which requires that the baryons belong to the multiplets on the right side of

$$
3 \otimes 3 \otimes 3 = 10_s \oplus 8_M \oplus 8_M \oplus 1
$$

(126)

$$
10 : \Delta^{++}, \Delta^{+}, \Delta^{0}, \Delta^{-}; \Sigma^{+}, \Sigma^{0}, \Sigma^{-}; \Xi^{-}, \Xi^{0}; \Omega^{-}.
$$

$$
8 : \ p, n; \Lambda, \Sigma^{+}, \Sigma^{0}, \Sigma^{-}; \Xi^{-}, \Xi^{0}.
$$

$$
1 : \Lambda.
$$

(127)

From (127), we can see that the intrinsic quantum numbers of the SU(3) approximation are in accordance with the experimental results. However, the SU(3) approximation can
not give a mass spectrum of the baryons. It can only give some relationships inside a multiplet. Using (4), we can deduce the formula

\[ M_j - M_i = \sum q_j - \sum q_i. \]  

(128)

For an 8-multiplet, we have

\[
\begin{align*}
M_p - M_\Lambda &= [(m_u + m_u + m_d) - (m_u + m_d + m_s)], \\
M_n - M_\Lambda &= [(m_u + m_d + m_d) - (m_u + m_d + m_s)], \\
M_{\Xi^0} - M_\Lambda &= [(m_u + m_s + m_s) - (m_u + m_d + m_s)], \\
M_{\Xi^-} - M_{\Sigma^+} &= [(m_d + m_s + m_s) - (m_u + m_u + m_s)], \\
M_{\Xi^-} - M_{\Sigma^0} &= [(m_d + m_s + m_s) - (m_u + m_d + m_s)], \\
M_{\Xi^-} - M_{\Sigma^-} &= [(m_d + m_s + m_s) - (m_d + m_d + m_s)].
\end{align*}
\]  

(129)

Using \( 1/2 \ (M_p + M_n) = M_N, \ 1/2(M_{\Xi^0} + M_{\Xi^-}) = M_{\Xi}, \ \text{and} \ 1/3(M_{\Sigma^+} + M_{\Sigma^0} + M_{\Sigma^-}) = M_\Sigma, \) and \( m_u = m_d, \) we get the GMO mass relation [2] for the baryon octet

\[
1/2(M_N + M_{\Xi}) = 1/4(3M_\Lambda + M_\Sigma)
\]  

(130)

For a 10-multiplet, using \( 1/2(M_{\Xi^0} + M_{\Xi^-}) = M_{\Xi}, \ 1/3(M_{\Sigma^+} + M_{\Sigma^0} + M_{\Sigma^-}) = M_{\Sigma}, \ \text{and} \ m_u = m_d, \) we have

\[
\begin{align*}
M_\Omega - M_{\Xi} &= (m_s + m_s + m_s)(m_u + m_s + m_s) = q_s - q_u, \\
M_{\Xi} - M_\Sigma &= (m_u + m_s + m_s)(m_u + m_d + m_s) = q_s - q_u, \\
M_\Sigma - M_\Delta &= (m_u + m_d + m_s)(m_u + m_u + m_d) = q_s - q_u.
\end{align*}
\]  

(131)

Thus, we get the GMO mass relation [3] for Decuplet

\[
M_\Omega - M_{\Xi} = M_{\Xi} - M_\Sigma = M_\Sigma - M_\Delta
\]  

(132)

The addition of the c quark to the light quarks extends the flavor symmetry to SU(4).

Similar to \( 3 \otimes 3 \otimes 3, \) we have
Fig. 12.2 of [32] (a) and (b) show the SU(4) baryon multiplets. The SU(3) octet is on the ground floor of the SU(4) multiplet 20 in Fig. 12.2 (a). The SU(3) decuplet is on the ground floor of the SU(4) multiplet 20 in Fig. 12.2 (b). All discovered charmed baryons, $\Lambda_C$, $\Omega_C$, $\Xi_C$, and $\Sigma_c$, have only one charmed quark.

The addition of a bottom quark extends the flavor symmetry to SU(5). It will produce many high energy baryons which have not been discovered by the experiments.

For the “ordinary” baryons, flavor and spin may be combined in an approximate flavor-spin SU(6) in which the six basic states are $d \uparrow, d \downarrow, u \uparrow, u \downarrow, s \uparrow$, and $s \downarrow$ ($\uparrow, \downarrow = $ spin up, down). Then the baryons belong to the multiplets on the right side of

$$4 \otimes 4 \otimes 4 = 4 \oplus 20 \oplus 20 \oplus 20$$

These SU(6) multiplets decompose into flavor SU(3) multiplets as follows:

$$6 \otimes 6 \otimes 6 = 56_s \oplus 70_M \oplus 70_M \oplus 20_A.$$

where the superscript $(2s + 1)$ gives the net spins of the quarks for each baryon in the SU(3) multiplets. The $J^P = (1/2)^+$ octet containing the nucleon N(939) and $J^P = (3/2)^+$ decuplet containing $\Delta(1232)$ together make up the “ground-state” 56-plet in which the orbital angular momenta are zero. 70 and 20 require some excitation of the spacial part of the state function.
Since we have omitted the higher energy excited states of the quarks, we cannot get the higher energy baryons. Similar to the Quark Model, the states with nonzero orbital angular momenta are classified in $\text{SU}(6) \otimes \text{SO}(3)$ supermultiplets. The $\text{SU}(6) \otimes \text{SO}(3)$ provides a suitable framework for describing baryon state functions.

It is useful to classify the baryons into bands that have the same number $N$ of quanta of excitation. Each band consists of a number of supermultiplets, specified by $(D, L^P_N)$, where $D$ is the dimensionality of the SU(6) representation, $L$ is the total orbital angular momentum, $J$ is the total angular momentum, and $P$ is the total parity.

Table 12.4 of [34] shows the Quark Model (the SU(N=3) approximation) assignments for many of the known baryons in terms of a flavor-spin SU(6) basis. Only the dominant representation is listed. Assignments for some states, especially for $\Lambda(1810)$, $\Lambda(2350)$, $\Xi(1820)$, and $\Xi(2030)$ are merely educated guesses.

| $J^P$ | $(D, L^P_N)$ | S | Octet members | Singlets |
|-------|--------------|---|---------------|----------|
| 1/2$^+$ | $(56,0^+_0)$ | 1/2 | N(939), $\Lambda(1116)$, $\Sigma(1193)$, $\Xi(1318)$ | |
| 1/2$^+$ | $(56,0^+_2)$ | 1/2 | N(1440), $\Lambda(1600)$, $\Sigma(1660)$, $\Xi(?)$ | |
| 1/2$^-$ | $(70,1^-_1)$ | 1/2 | N(1535), $\Lambda(1670)$, $\Sigma(1620)$, $\Xi(?)$ | $\Lambda(1405)$ |
| 3/2$^-$ | $(70,1^-_1)$ | 1/2 | N(1520), $\Lambda(1690)$, $\Sigma(1670)$, $\Xi(1820)$ | $\Lambda(1520)$ |
| 1/2$^-$ | $(70,1^-_1)$ | 3/2 | N(1650), $\Lambda(1800)$, $\Sigma(1750)$, $\Xi(?)$ | |
| 3/2$^-$ | $(70,1^-_1)$ | 3/2 | N(1700), $\Lambda(?)$, $\Sigma(?)$, $\Xi(?)$ | |
| 5/2$^-$ | $(70,1^-_1)$ | 3/2 | N(1675), $\Lambda(1830)$, $\Sigma(1775)$, $\Xi(?)$ | |
| 1/2$^+$ | $(70,0^+_2)$ | 1/2 | N(1710), $\Lambda(1810)$, $\Sigma(1880)$, $\Xi(?)$ | $\Lambda(?)$ |
| 3/2$^+$ | $(56,2^+_2)$ | 1/2 | N(1720), $\Lambda(1890)$, $\Sigma(?)$, $\Xi(?)$ | |
| 5/2$^+$ | $(56,2^+_2)$ | 1/2 | N(1680), $\Lambda(1820)$, $\Sigma(1915)$, $\Xi(2030)$ | |
| 7/2$^-$ | $(70,3^-_3)$ | 1/2 | N(2190), $\Lambda(?)$, $\Sigma(?)$, $\Xi(?)$ | $\Lambda(2100)$ |
| 9/2$^-$ | $(70,3^-_3)$ | 3/2 | N(2250), $\Lambda(?)$, $\Sigma(?)$, $\Xi(?)$ | |
| 9/2$^+$ | $(56,4^+_4)$ | 1/2 | N(2220), $\Lambda(2350)$, $\Sigma(?)$, $\Xi(?)$ | |
If SU(3) is the main symmetry group (u, d, and s), then the baryons which are composed of the three quarks (u, d, and s) will be classed into Octets and Decuplets. However, in fact

1. There are six possible Decuplets, but only one is complete from (139).
2. Four out of the five other possible Decuplets that each needs four members (SU(3)), there is only one member in the 4 Decuplets. One member means nothing.
3. There are twelve possible Octets, but only three Octets are complete from (138).

Therefore, the above experimental results show that the SU(3) is not the most important symmetry group for the baryons which are made up of the “ordinary” quarks u(313), d(313), and s(529). Moreover, SU(4) symmetries (based on u(313), d(313), s(529), c(1654)) and SU(5) symmetries (based on u(313), d(313), s(529), c(1654), b(5014)) are very badly broken by the mass of charmed quark c and the mass of bottom quark b. Considering the fact that the Quark Model cannot deduce the mass spectrum of the baryons, we shall replace the SU(N) groups by the body center cubic groups (the space group and the point groups).

D The mesons (the \( q\bar{q} \) states)

According to the primitive cell approximation, a meson is made up of a quark and an antiquark (the \( q\bar{q} \) states). The nine possible \( q\bar{q} \) combinations containing \( q_u, q_d, \) and
q_s quarks group themselves into an octet and a singlet:

$$3 \otimes \bar{3} = 8 + 1$$

(140)

8: \(K^+, K^0, \pi^+, \pi^0, \pi^-, \bar{K}^0, K^-, \eta_8\).

1: \(\eta_1\).

(141)

A fourth quark such as Charmed quark q_c can be included by extending the symmetry to SU(4), as shown in Fig. 12.1 of [31]. Fig. 12.1 (a) shows the SU(4) 16-plets for the pseudoscalar mesons made of u, d, s, and c quarks. Fig. 12.1 (b) shows the SU(4) 16-plets for the vector mesons made of u, d, s, and c quarks. The octets and singlets of light mesons (141) occupy the central planes, to which the c\(\bar{c}\) states have been added. The neutral mesons at the center of the planes are mixtures of u\(\bar{u}\), d\(\bar{d}\), s\(\bar{s}\), and c\(\bar{c}\). The Bottom quark extends the symmetry to SU(5). Thus, we can get the same result as the Quark Model [3]. For example, we give out the mesons with the angular momentum L = 0 and the spin s = 0 (spin single S wave mesons-q\(\bar{q}\), \(\uparrow\downarrow\) = spin up, down) as follows:

\[
\begin{array}{cccccccc}
1^1S_0 & q_u \uparrow & q_d \uparrow & q_s \uparrow & q_c \uparrow & q_b \uparrow \\
q_u \downarrow & (q_u q_u)_0 & \pi^- & K^- & D^0 & B^- \\
q_d \downarrow & \pi^+ & (q_d q_d)_0 & \bar{K}^0 & D^+ & B^0 \\
q_s \downarrow & K^+ & K^0 & (q_s q_s)_0 & D_s^+ & B_s^0 \\
q_c \downarrow & \bar{D}_s^0 & D^- & D_s^- & \eta_c & (q_c q_c)_0 \\
q_b \downarrow & B^+ & \bar{B}_s^0 & (q_c q_c)_0 & (q_b q_b)_0 \\
\end{array}
\]

(q_u q_u)_0, (q_d q_d)_0, and (q_s q_s)_0 can be combined into \(\pi^0, \eta, \text{ and } \eta'\). Thus, we can get the S wave spin single (L = 0 and s = 0) mesons: \(\pi^+, \pi^0, \pi^-, \eta, \eta', \eta_c, K^+, K^0, K^-, \bar{K}^0, D^+, D^-, D^0, \bar{D}_s^0, D_s^+, D_s^-, B^+, B^-, B^0, \bar{B}_s^0, B_s^0, \bar{B}_c^+ = ((q_c q_c)_0), B_c^- = ((q_b q_b)_0), \eta_b = (q_b q_b)_0.\)

Similarly, we give out the mesons with the angular momentum L = 0 and the spin s = 1 (the spin three fold state S wave mesons-q\(\bar{q}\), \(\uparrow\uparrow\) spin up, up) as follows:

52
\[
{1^3S_1} \quad q_u \uparrow \quad q_d \uparrow \quad q_s \uparrow \quad q_c \uparrow \quad q_b \uparrow \\
\bar{q}_u \uparrow \quad (q_u \bar{q}_u)_1 \quad \rho^- \quad K^{*-} \quad D^{*0} \quad B^{*-} \\
\bar{q}_d \uparrow \quad \rho^+ \quad (q_d \bar{q}_d)_1 \quad \bar{K}^{*0} \quad D^{*-} \quad B^{*0} \\
\bar{q}_s \uparrow \quad K^{*+} \quad K^{*0} \quad (q_s \bar{q}_s)_1 \quad D_s^{*+} \quad B_s^{*0} \\
\bar{q}_c \uparrow \quad D^{*0} \quad D^{*-} \quad D_s^{*-} \quad J/\psi \quad (q_b \bar{q}_c)_1 \\
\bar{q}_b \uparrow \quad B^{*+} \quad B^{*0} \quad B_s^{*0} \quad (q_c \bar{q}_b)_1 \quad \gamma(1S)
\]

\[(143)\]

\((q_u \bar{q}_u)_1, (q_d \bar{q}_d)_1,\) and \((q_s \bar{q}_s)_1\) can be combined into \(\rho^0, \omega,\) and \(\phi.\) Thus we can get, \(\rho^+, \rho^0, \rho^-; \omega, \phi; K^{*+}, K^{*0}, K^{*-}, \bar{K}^{*0}; D^{*+}, D^{*-}, D^{*0}, \bar{D}^{*0}; D_s^{*+}, D_s^{*-}; J/\psi; B^{*+}, B^{*-}, B^{*0}, \bar{B}^{*0}, B_s^{*0}, \bar{B}_s^{*0}, B_c^{*+} = ((q_c \bar{q}_b)_1), B_c^{*-} = (q_b \bar{q}_c)_1; \gamma(1S).\)

Similarly, we can get the whole meson spectrum including the angular momentum \(L,\) the spin \(s,\) the total angular momentum \(J,\) the parity \(P,\) the isospin \(I,\) and the electric charge \(Q.\) Leaving undiscovered mesons in empty spaces, the most of the known mesons \([15]\) are listed as following \((q_u = u, q_d = d, q_s = s, q_c = c,\) and \(q_b = b.\) \([117])\)
Although the SU(N) can give out all quantum numbers of the meson spectrum, they cannot give out a mass spectrum of the mesons. If SU(4) were an accurate symmetry group, then the mesons $\pi$, $\eta$, $\eta'$, and $\eta_C$ would have the same mass, as shown in Fig.
In fact, mass ($\pi$) = 139, mass ($\eta$) = 547 $\approx$ 3.95 $\times$ 139 , mass ($\eta'$) = 958 $\approx$ 6.9 $\times$ 139 , mass($\eta_C$) = 2980 $\approx$ 21.4 $\times$ 139. If SU(5) were an accurate symmetry group, the mesons $\rho$, $\omega$, $\phi$, $J/\psi(1s)$, and $\gamma(1s)$ would have the same mass as shown in Fig. 12.1 (b) of [31]. In fact, mass ($\rho$) = 770, mass ($\omega$) = 782 $\approx$ 770 , mass ($\phi$) = 1020 $\approx$ 1.33 $\times$ 770 , mass($J/\psi(1s)$) = 3097 $\approx$ 4 $\times$ 770 , mass ($\gamma(1s)$) = 9460 $\approx$ 12.3 $\times$ 770.

The above experimental facts clearly show that the SU(N) (N = 3, 4, and 5) groups are not the best symmetry groups and that SU(N) approximations are not the best approximations.

The best approximation is the combined approximation of the cell and the periodic field. Considering all quarks, it will give the full meson spectrum. Thus, we shall replace the SU(N) group by the body center cubic groups in the classification of the mesons.

E  The top quarks

According to the BCC model, there is no excited quark with a mass about 175 Gev (about 185 times the mass of proton) [37]. It may be an energy band excited state of an electron [18].

F  The quarks decay

In the standard model [36], fermion couplings to the Higgs field not only determine their masses; they induce a misalignment of quark mass eigenstates with respect to the eigenstates of the weak charges, thereby allowing all fermions of heavy families to decay to lighter ones. The BCC model does not need the Higgs field. We know that the higher mass quarks are the higher energy excited states of the fundamental quarks (u and d), and that the lower mass quarks are the lower energy excited states of the same quarks (u and d). Under the influences of the quark lattice, the higher energy excited states
naturally decay into lower ones, and finally into the ground states, because they are not truly independent particles. In the BCC Model, the Higgs field is replaced by the periodic field of the quark lattice in the vacuum.

**Summary**

1. The SU(N) (N = 3, 4, and 5) symmetry approximations have the same quarks (u, d, s, c, and b), the same symmetry group (the SU(3), the SU(4), and the SU(5)), the same baryon and meson structure (baryons = qqq states and mesons = q̄q states), and the same sum formulae (4) (the quantum numbers and energy of the system are the sum of the quantum numbers and energies of the constituent quarks), as well as the same results as the Quark Model (5 quarks). Therefore, the SU(N) approximation is the Quark Model (with N quarks), and the Quark Model is an approximation of the BCC model.

2. The BCC model provides the quarks (with the quantum numbers and masses), the symmetry groups (SU(N)), and the baryon and meson structure molds (baryons = qqq states and mesons = q̄q states) for the SU(N) symmetry approximation. These are the physical foundations of the Quark Model. Thus, the BCC Model provides the physical foundations of the Quark Model.

3. The SU(N) approximations assume that the 5 quarks (u, d, s, c, and b) are independent particles. This explains why no one can find an united mass formula for the baryon spectrum or for the meson spectrum. At the same time, it is difficult to understand why the higher mass quarks decay into lower mass quarks.

4. The SU(N) omits all higher energy band excited quarks, and there are a lot of such excited quarks, so it cannot explain the whole baryon spectrum and meson spectrum.

5. The SU(N) omits the accompanying excited cell, it needs the confinement hypothesis to explain why free quarks cannot be discovered. Confinement is a very plausible
idea but to date its rigorous proof remains outstanding.

VII Comparing The Results

The combined approximation of the cell and the periodic field is the best approximation of the BCC model. The results of the combined approximation will be regarded as the results of the BCC model. The BCC model produces three spectra: the quark spectrum, the baryon spectrum and the meson spectrum. The theoretical quark spectrum does not have experimental results with which to compare, single free quarks have been not found in experiments yet. We will discuss the meson spectrum in the next paper (Jiao-Lin Xu and Xin Yu, The Meson Spectrum). Here, we will compare the baryon spectrum only.

Using Tables 1-6, we compare the theoretical results of the BCC model with the experimental results. In the comparison, we do not take into account the angular momenta of the experimental results. We assume that the small differences of the masses in the same group of baryons stem from their different angular momenta. If we ignore this effect, their masses would be essentially the same. In the comparison, we use the baryon name to represent the intrinsic quantum numbers as shown in the second column of Table 1.

The ground states of various kinds of baryons are shown in Table 1. These baryons have relatively long lifetimes. They are the most important experimental results of the baryons. From Table 1, we can see that all theoretical intrinsic quantum numbers (isospin $I$, strange number $S$, charmed number $C$, bottom number $b$, and electric charge $Q$) are the same as those in the experimental results. Also the theoretical mass values are in very good agreement with the experimental values.

A comparison of the theoretical results with the experimental results of the unflavored baryons $N$ and $\Delta$ is made in Table 2. From Table 2, we can see that the
intrinsic quantum numbers of the theoretical results are exactly the same as those of the experimental results. Also the theoretical masses of the baryons $N$ and $\Delta$ are in very good agreement with the experimental results. The theoretical results $N(1210)$ and $N(1300)$ are not found in the experiment. We believe that they are covered up by the experimental baryon $\Delta(1232)$ because of the following reasons: (1) they are unflavored baryons with the same $S$, $C$, and $b$; (2) the width (120 Mev) of $\Delta(1232)$ is very large, and the baryons $N(1210)$ and $N(1300)$ both fall within the width region of $\Delta(1232)$; (3) the average mass (1255 Mev) of $N(1210)$ and $N(1300)$ is essentially the same as the mass (1232 Mev) of $\Delta(1232)$ ($\Gamma = 120$ Mev).

Two kinds of the strange baryons $\Lambda$ and $\Sigma$ are compared in Table 3. Their theoretical and experimental intrinsic quantum numbers are the same. The theoretical masses of the baryons $\Lambda$ and $\Sigma$ are in very good agreement with the experimental results.

The theoretical intrinsic quantum numbers of the baryons $\Xi$ and $\Omega$ are the same as the experimental results (see Table 4). The theoretical masses of the baryons $\Xi$ and $\Omega$ are compatible with the experimental results.

The charmed and bottom baryons $\Lambda_c^+$ and $\Lambda_b^0$ can be found in Table 5. The experimental masses of the charmed baryons ($\Lambda_c^+$) and bottom baryons ($\Lambda_b^0$) coincide with the theoretical results.

Finally, we compare the theoretical results with the experimental results for the charmed strange baryons $\Omega_c$, $\Xi_c$ and $\Sigma_c$ in Table 6. Their intrinsic quantum numbers are all matched very well, and their masses are in very good agreement.

In summary, the BCC model explains all baryon experimental intrinsic quantum numbers and masses. Virtually no experimentally confirmed baryon is not included in the model. However, the angular momenta and the parities of the baryons are not included in the zeroth-order approximation of this paper. They depend on the wave functions of the energy bands. We will discuss them in the first order approximation.
VIII Predictions and Discussion

A Some New Baryons

According to the BCC model, a series of possible baryons may exist. However, when energy goes higher and higher, the energy bands will become denser and denser, and the full widths of the baryons will become wider and wider, making them extremely difficult to be separated. The following new baryons predicted by the model seem to have a better chance of being discovered in a not too distant future:

\[
\begin{align*}
I = 0 & \quad S = -1 & Q = 0 & \quad \Lambda^0(2560) & \quad (11) \\
I = 0 & \quad C = +1 & Q = +1 & \quad \Lambda_C^+(6600) & \quad \Lambda_C^+(13800) & \quad (107) \\
I = 0 & \quad S = -1 & Q = 0 & \quad \Lambda^0(4280) & \quad \Lambda^0(10040) & \quad (107) \\
I = 1 & \quad S = -1 & C = +1 & Q = 2, 1, 0 & \quad \Sigma_C(2280) & \quad (107) \\
I = 0 & \quad S = -3 & Q = -1 & \quad \Omega^-(3720) & \quad \Omega^-(7220) & \quad (111) \\
I = 0 & \quad b = -1 & Q = 0 & \quad \Lambda_b^0(9960) & \quad (111)
\end{align*}
\]

in the last column, we give the equation numbers where the baryons are first deduced in this paper.

B Discussion

1. From (147), we have

\[
m_qa_x^2 = \frac{h^2}{720 \text{ Mev.}}
\]

Although we do not know the values of \(m_q\) and \(a_x\), we find that \(m_qa_x^2\) is a constant. According to the renormalization theory [20], the bare mass of the quark should be “infinite”, so that \(a_x\) will be “zero”. Of course, the “infinite” and the “zero” are physical concepts in this case. We understand that the “infinite” means \(m_q\) is huge and the “zero” means \(a_x\) is much smaller than the nuclear radius. “\(m_q\) is huge” guarantees that we can use the Schrödinger equation instead of the Dirac
equation. Since “a is much smaller than the nuclear radius”, the vacuum material looks like a continuous media, which makes the structure of the vacuum material very difficult to be discovered.

2. In a sense, the vacuum material with the body center cubic structure works like a superconductor. There are no electric and mechanical resistances to any particle and any physical body (with or without electric charge) moving inside the vacuum material. Since the energy gaps are so large (for an electron, the energy gap is about 0.5 Mev; for a quark, the energy gap is about 940 Mev), the vacuum remains unchanged when ordinary physical phenomena occur. Although when the energy of an individual quark exceeds the gap, the vacuum state quark will be excited from the vacuum, so the vacuum material still looks unchanged.

3. If the vacuum material really exists, it shall not only be super-strong but also super-dense. As a result, even a hydrogen bomb cannot destroy it. In fact, when a hydrogen bomb explodes, an individual quark can only receive a little energy (not to exceed 5 Mev). It is much lower than the vacuum excited energy (940 Mev) of a quark. Thus, it can not break the vacuum material.

4. Since the theoretical baryon mass spectrum in the free particle approximation ($V(\vec{r}) = V_o$ and the wave functions satisfy the body center cubic periodic symmetries) is very close to the experimental mass spectrum, the amplitude (A) of the strong interaction periodic field should be much smaller than the average of the periodic field ($V_o$). According to the BCC model, Dirac’s sea concept is a complete free approximation for the vacuum periodic field ($V(\vec{r}) = V_o$ and the wave functions do not have to satisfy the body center cubic periodic symmetries). Thus, the Dirac’s sea concept is a very good approximation of the BCC Model.
IX Conclusions

1. The two quarks (u and d) are the only elemental particles in the quark family. Other quarks (s, c, b, ...) are all their energy band excited states. The BCC Model has deduced (not assumed) the intrinsic quantum numbers and masses of all quarks. Thus, the SU(3) (for the quarks u, d, and s), the SU(4) (for the quarks u, d, s, and c), and the SU(5) (for the quarks u, d, s, c, and b) symmetries are the natural extensions of the SU(2) (based on the quarks u and d). The BCC Model has shown that the excited primitive cell (the three quark system q*u'd') is a baryon. This is the physical foundation of the assumption that a baryon is made up of three quarks in the Quark Model. Therefore, the BCC Model provides a physical foundation of the quark model. The Quark Model is a approximation (SU(N)) of the BCC Model.

2. Although baryons (∆, N, Λ, Σ, Ω, ΛC, ΞC, ΣC, and ΛC) are quite different from one another in I, S, C, b, Q, and M, they have the same structure (one excited quark q* and two accompanying excised quarks u' and d'). The BCC Model has deduced all intrinsic quantum numbers and masses of all baryons.

3. The vacuum material is a super-superconductor with super-high energy gaps (proton-939 Mev, electron-0.5 Mev). It has the body center cubic periodic symmetries. We think that the body center cubic periodic symmetry may be one of the possible “have not yet been identified” symmetries [38].

4. In the BCC Model, the three excited quarks (inside a baryon) are in different states (one excited quark q*, one accompanying excited quark u’, and one accompanying excited quark d’). Hence, the system obeys the Pauli exclusion principle [17].

5. The confinement concept is not needed. It shall be replaced by the accompanying excitation concept. According to the accompanying excitation concept, any excited quark (from the vacuum) is always accompanied by two accompanying excited quarks u' and d'. They cannot be separated. Therefore, individual free quarks can never been
6. In the BCC Model, the high mass quarks are the high energy excited states of the fundamental quark q, the low mass quarks are the low energy excited states of the same quark q. Thus, the higher mass quarks will decay into the lower mass quarks.

7. Due to the existence of the vacuum material, all observable particles are constantly affected by the vacuum material (vacuum state quark lattice). Thus, some laws of statistics (such as fluctuation) cannot be ignored.

8. The SU(N) symmetries (flavor) shall be replaced by the body center cubic periodic symmetries. The body center cubic periodic symmetries of the vacuum material need to be more intensively researched.

9. We need to do first order calculations to find more accurate masses, the angular momenta, and the parities of the baryons and the mesons, using the wave functions which satisfy the symmetries of the body center cubic periodic field.

Acknowledgment

I would like to express my heartfelt gratitude to Dr. Xin Yu for checking the calculations of the energy bands and for helping to writing this paper. I sincerely thank Professor Robert L. Anderson for his valuable advice. I also acknowledge my indebtedness to Professor D. P. Landau for his help. I thank Professor W. K. Ge very much for all of his help and for recommending Wilczek’s paper [8]. I thank my friend Z. Y. Wu very much for his help in preparing this paper. I thank my classmate J. S. Xie very much for checking the calculations of the energy bands. I thank Professor Y. S. Wu, H. Y. Guo, and S. Chen [39] very much for very useful discussions. I also thank Dr. Fugao Wang very much for helping me post my paper on the web (http://arxiv.org/abs/hep-ph/0010281).

References
[1] M. Gell-Mann, Phys. Lett. 8, 214 (1964); G. Zweig, CERN Rep. 8419/TH 412 (1964); A. J. G. Hey and R. L. Kelly, Phys. Reports, 96, 71 (1983); N. Isgur, Int. Mod. Phys. E1, 465 (1992).

[2] M. Gell-Mann, Report CTSL-20 (1961 unpublished, reprinted in the book of Gell-Mann and Ne’eman, 1964), M. Gell-Mann, Y. Ne’eman, The Eightfold Way, (Benjamin, 1964); S. Okubo, Progr. Theor. Phys., 27, 949 (1962); G. Morpurgo, Phys. Rev. Lett., 68, 139 (1992).

[3] The Oecd Frum, Particle Physics (Head of Publications Service, OECD) 55 (1995).

[4] T. D. Lee, Particle Physics and Introduction to Field Theory (harwood academic, New York, 1981) 824.

[5] A. Pais, Rev. Mod. Phys., 71 No.2, S16 (1999).

[6] R. M. Barrett et al., Rev. Mod. Phys., 68, 620 (1996).

[7] T. D. Lee, Particle Physics and Introduction to Field Theory (harwood academic, New York, 1981) 826.

[8] F. Wilczek, Phys. Tod. Jan. 11(1998).

[9] P. A. M. Dirac, The Principles of Quantum Mechanics (Oxford at the Clarendon Press, Fourth Edition, 1981) 273; L. H. Ryder, Quantum Field Theory (Great Britain at the University Press, Cambridge, 1966) 45.

[10] R. C. Evans, An Introduction to Crystal Chemistry (Cambridge, 1964) 41; X. D. Xie and J. X. Fang, Solid State Physics (in Chinese, 1964) 46.

[11] R. A. Levy, Principles of Solid State Physics (Academic, New York, 1972) 41.

[12] C. Kittel, Introduction to Solid State Physics (John Wiley & Sons, Inc, New York, 1976) 6, and 16.
[13] C. Kittel, Introduction to Solid State Physics (John Wiley & Sons, Inc, New York, 1981) 183.

[14] M. Gell-Mann, Phys. Rev., 125, 1067(1962); Y. Ne'eman, Nucl. Phys., 26, 222(1961); H. Georgi, and S. L. Glaslow, Phys. Rev. Lett. 32, 438(1974).

[15] We were enlightened by the Monte Carlo methods: K. Binder, Monte Carlo Methods in Statistical Physics (Springer-Verlag, Berlin 1979); D. P. Landau, Phys. Rev. B 14, 4054 (1979).

[16] J. J. J. Kokkedee, The Quark Model (W. A. Benjamin,Inc. New York, 1969) 16.

[17] W. Pauli, Z. Physik 41, (1927) 81; M. Y. Han and Y. Nanbu, Phys. Rev. 139B, (1965)1006.

[18] J. L. Xu, [hep-ph/0010281].

[19] R. M. Barrett et al., Rev. Mod. Phys., 68, 642 (1996).

[20] S. Weinberg, The Quantum Theory of Fields (CAMBRIDGE, New York, 1995) 34, 506.

[21] J. Callaway, Energy Band Theory (Academic Press, New York and London, 1964) ; H. Jones, The Theory of Brillouin Zones and Electronic States in Crystals ( 2nd revised edition, Noth-Holanda, Amsterdam,1975) Chapter 3.

[22] P. T. Landsberg, Solid State Theory Methods and Applications (Wiley-Interscience, New York, 1969) 222.

[23] A. W. Joshi, Elements of Group Theory for Physicists (JOHN WILEY & SONS, New York, 1977) 284.

[24] G. Kittel, Introduction to Solid State Physics, fifth edition, (John Wiley & Sons, New York, 1977) 188.
[25] W. Heisenberg, Z. S. Phys., 77, 1(1932). T. D. Lee, Particle Physics and Introduction to Field Theory (harwood academic, New York, 1981) 220.

[26] M. Gell-Mann, Phys. Rev. 92, 833 (1953); K. Nishijima and T. Nakano, Prog. Theor. Phys. 10, 581 (1953); K. Nishijima, Prog. Theor. Phys. 13, (1955) 285.

[27] E. G. Cazzoli et al., Phys. Rev. Lett. 34, 1125 (1975).

[28] C. Albajar, Phys. Lett. B 273, 540 (1991)

[29] P. Coteus et al., Phys. Rev. Lett. 59, 1530 (1987); P. Avery et al., Phys. Rev. Lett, 62, 863 (1989); P. Avery et al., Phys. Rev. Lett. 75, 4364 (1995); L. Gibbons et al., Phys. Rev. Lett. 77, 810 (1996) .

[30] R. M. Barrett et al., Rev. Mod. Phys., 68, 651 (1996)

[31] R. M. Barrett et al., Rev. Mod. Phys., 68, 681 (1996).

[32] R. M. Barrett et al., Rev. Mod. Phys., 68, 683 (1996).

[33] R. H. Dalitz and L. J. Reinders, in Hadron Structure as Known from Electromagnetic and Strong Interactions, Proceedings of the Hadron '77 Conference (Veda, 1979) 11.

[34] R. M. Barrett et al., Rev. Mod. Phys., 68, 684 (1996). 

[35] R. M. Barrett et al., Rev. Mod. Phys., 68, 682 (1996).

[36] M. K. Graillard, P. D. Grannis, F. J. Sciulli, Rev. Mod. Phys., 71 No. S96 (1999).

[37] F. Abe et al., Phys. Rev. Lett., 74, 2626 (1995); S. Abachi et al., Phys. Rev. Lett., 74, 2632 (1995). B. Abbott et al, Phys. Rev. Lett. 80, 2063 (1998); F. Abe et al, Phys. Rev. Lett., 80, 2767 (1998).

[38] F. Wilczek, Rev. Mod. Phys., 71 No. 2, S85 (1999).
[39] J. L. Xu, Y. S. Wu, H. Y. Guo, and S. Chen, H. Energy and Nucl. Phys., 2, 251 (Beijing, 1980).

[40] H. Jones, *The Theory of Brillouin Zones and Electronic States in Crystals* (2nd revised edition, Noth-Holand, Amsterdam, 1975) Chapter 7.

[41] H. Jones, *The Theory of Brillouin Zones and Electronic States in Crystals* (2nd revised edition, Noth-Holand, Amsterdam, 1975) 98, 112, and 124.
Fig. 1. The first Brillouin zone of the body center cubic lattice. The symmetry points and axes are indicated. The axis \( \Delta \) is a 4 fold rotation axis, the strange number \( S = 0 \), the baryon family \( \Delta \) \((\Delta^{++}, \Delta^+, \Delta^0, \Delta^-)\) will appear on the axis. The axes \( \Lambda \) and \( F \) are 3 fold rotation axes, the strange number \( S = -1 \), the baryon family \( \Sigma \) \((\Sigma^+, \Sigma^0, \Sigma^-)\) will appear on the axes. The axes \( \Sigma \) and \( G \) are 2 fold rotation axes, the strange number \( S = -2 \), the baryon family \( \Xi \) \((\Xi^0, \Xi^-)\) will appear on the axes. The axis \( \Delta \) is parallel to the axis \( \Delta \), \( S = 0 \). And the axis is a 2 fold rotation axis, the baryon family \( N \) \((N^+, N^0)\) will be on the axis.

Fig. 2. (a) The energy bands on the axis \( \Delta \). The numbers above the lines are the values of \( \vec{n} = (n_1, n_2, n_3) \). The numbers under the lines are the fold numbers of the degeneracy. \( E_\Gamma \) is the value of \( E(\vec{k}, \vec{n}) \) (see Eq. (26)) at the end point \( \Gamma \), while \( E_H \) is the value of \( E(\vec{k}, \vec{n}) \) at other end point \( H \). (b) The energy bands on the axis \( \Lambda \). \( E_\Gamma \) is the value of \( E(\vec{k}, \vec{n}) \) (see Eq. (26)) at the end point \( \Gamma \), while \( E_P \) is the value of \( E(\vec{k}, \vec{n}) \) at other end point \( P \).

Fig. 3. (a) The energy bands on the axis \( \Sigma \). The numbers above the lines are the values of \( \vec{n} = (n_1, n_2, n_3) \). The numbers under the lines are the fold numbers of the degeneracy. \( E_\Gamma \) is the value of \( E(\vec{k}, \vec{n}) \) (see Eq. (26)) at the end point \( \Gamma \), while \( E_N \) is the value of \( E(\vec{k}, \vec{n}) \) at other end point \( N \). (b) The energy bands on the axis \( D \). \( E_P \) is the value of \( E(\vec{k}, \vec{n}) \) (see Eq. (26)) at the end point \( P \), while \( E_N \) is the value of \( E(\vec{k}, \vec{n}) \) at other end point \( N \).

Fig. 4. (a) The energy bands on the axis \( F \). The numbers above the lines are the values of \( \vec{n} = (n_1, n_2, n_3) \). The numbers under the lines are the fold numbers of the degeneracy. \( E_P \) is the value of \( E(\vec{k}, \vec{n}) \) (see Eq. (26)) at the end point \( P \), while \( E_H \) is the value of \( E(\vec{k}, \vec{n}) \) at other end point \( H \). (b) The energy bands on the axis \( G \). \( E_M \) is the value of \( E(\vec{k}, \vec{n}) \) (see Eq. (26)) at the end point \( M \), while \( E_N \) is the value of \( E(\vec{k}, \vec{n}) \) at other end point \( N \).
at other end point N.

Fig. 5. (a) The 4 fold degenerate energy bands (selected from Fig. 2(a)) on the axis $\Delta$. The numbers above the lines are the values of $\vec{n}$ ($n_1, n_2, n_3$). The numbers under the lines are the numbers of the degeneracy of the energy bands. (b) The single energy bands (selected from Fig. 2(a)) on the axis $\Delta$. The numbers above the lines are the values of $\vec{n}$ ($n_1, n_2, n_3$). (c) The single energy band (selected from Fig. 3(a)) on the axis $\Sigma$. The numbers above the lines are the values of $\vec{n}$ ($n_1, n_2, n_3$)
## Table 1. The Ground States of the Baryons.

| Theory | Quantum. No | Experiment | R | Life Time |
|--------|-------------|------------|---|-----------|
| N^+ (940) | 0, 0, 0, 1/2, 1 | p(938) | 0.2 | >10^{31} years |
| N^0 (940) | 0, 0, 0, 1/2, 0 | n(940) | 0.0 | 1.0 \times 10^8 s |
| Λ^0 (1120) | -1, 0, 0, 0, 0 | Λ^0 (1116) | 0.4 | 2.6 \times 10^{-10} s |
| Σ^+ (1210) | -1, 0, 0, 1, 1 | Σ^+ (1189) | 1.8 | 8.0 \times 10^{-10} s |
| Σ^0 (1210) | -1, 0, 0, 1, 0 | Σ^0 (1193) | 1.4 | 7.4 \times 10^{-20} s |
| Σ^- (1210) | -1, 0, 0, 1, -1 | Σ^- (1197) | 1.1 | 1.5 \times 10^{-10} s |
| Ξ^0 (1300) | -2, 0, 0, 1/2, 0 | Ξ^0 (1315) | 1.2 | 2.9 \times 10^{-10} s |
| Ξ^- (1300) | -2, 0, 0, 1/2, -1 | Ξ^- (1321) | 1.6 | 1.6 \times 10^{-10} s |
| Ω^- (1660) | -3, 0, 0, 0, -1 | Ω^- (1672) | 0.7 | 8.2 \times 10^{-10} s |
| Λ^+_c (2280) | 0, 1, 0, 0, 1 | Λ^+_c (2285) | 0.2 | 2.1 \times 10^{-12} s |
| Ξ^+_c (2550) | -1, 1, 0, 1/2, 1 | Ξ^+_c (2466) | 3.4 | 3.5 \times 10^{-12} |
| Ξ^0_c (2550) | -1, 1, 0, 1/2, 0 | Ξ^0_c (2470) | 3.3 | 1.0 \times 10^{-12} s |
| Ω_c^0 (2660) | 0, 0, -1, 0, 0 | Ω_c^0 (2704) | 1.7 | 6.4 \times 10^{-13} s |
| Λ^0_b (5540) | 0, 0, -1, 0, 0 | Λ^0_b (5641) | 1.8 | 1.1 \times 10^{-12} s |
| Δ^{++} (1210) | 0, 0, 0, 3/2, 2 | Δ^{++} (1232) | 1.8 | Γ=120 Mev |
| Δ^+ (1210) | 0, 0, 0, 3/2, 1 | Δ^+ (1232) | 1.8 | Γ=120 Mev |
| Δ^0 (1210) | 0, 0, 0, 3/2, 0 | Δ^0 (1232) | 1.8 | Γ=120 Mev |
| Δ^- (1210) | 0, 0, 0, 3/2, -1 | Δ^- (1232) | 1.8 | Γ=120 Mev |

In the fourth column, \( R = \left( \frac{\Delta M}{M} \right) \% \).
Table 2. The Unflavored Baryons $N$ and $\Delta$ ($S = C = b = 0$)

| Theory  | Experiment | $\Delta M/\%$ | Theory  | Experiment | $\Delta M/\%$ |
|---------|------------|---------------|---------|------------|---------------|
| $N(1210)$ | $N(1300)$ | | $\Delta(1210)$ | $\Delta(1300)$ | $\Delta(1232)$ |
| $\bar{N}(1255)$ | | | $\bar{N}(1255)$ | $\bar{\Delta}(1232)$ | 1.9 |
| $N(1480)$ | $N(1440)$ | $N(1520)$ | $N(1535)$ | | |
| $\bar{N}(1480)$ | $\bar{N}(1498)$ | 1.2 | | | |
| $N(1660)$ | $N(1650)$ | $N(1675)$ | $N(1680)$ | $N(1700)$ | $N(1710)$ | $N(1720)$ | $\Delta(1660)$ | $\Delta(1660)$ | $\Delta(1600)$ | $\Delta(1620)$ | $\Delta(1700)$ |
| $\bar{N}(1660)$ | $\bar{N}(1689)$ | 1.7 | $\bar{\Delta}(1660)$ | $\bar{\Delta}(1640)$ | 1.2 |
| $N(1840)$ | $N(1900)*$ | | $\Delta(1930)$ | $\Delta(1930)$ | $\Delta(1900)$ | $\Delta(1905)$ |
| $N(1930)$ | $N(1990)*$ | $N(2000)*$ | $N(2080)*$ | $N(2020)$ | | |
| $\bar{N}(1915)$ | $\bar{N}(1923)$ | 0.2 | $\bar{\Delta}(1953)$ | $\bar{\Delta}(1919)$ | 1.8 |
| $N(2200)$ | $N(2190)$ | $N(2220)$ | $N(2250)$ | | |
| $\bar{N}(2200)$ | $\bar{N}(2220)$ | 0.9 | | | |
| $N(2380)$ | | | $\Delta(2380)$ | $\Delta(2420)$ | 1.7 |
| $N(2560)$ | | | | | |
| $3N(2650)$ | | | | | |
| $\bar{N}(2628)$ | $N(2600)$ | 1.1 | $3\Delta(2650)$ | | |
| $6N(2740)$ | | | $4\Delta(2740)$ | | |

*Evidences are fair, they are not listed in the Baryon Summary Table [13].
Table 3. Two Kinds of Strange Baryons $\Lambda$ and $\Sigma$ ($S = -1$)

| Theory | Experiment | $\frac{\Delta M}{M}$ | Theory | Experiment | $\frac{\Delta M}{M}$ |
|--------|------------|----------------------|--------|------------|----------------------|
| $\Lambda(1120)$ | $\Lambda(1116)$ | 0.36 | $\Sigma(1210)$ | $\Sigma(1193)$ | 1.4 |
| $\Lambda(1400)$ | $\Lambda(1405)$ | $\Sigma(1400)$ | $\Sigma(1385)$ |
| $\bar{\Lambda}(1350)$ | $\bar{\Lambda}(1405)$ | 4.6 | $\bar{\Sigma}(1350)$ | $\bar{\Sigma}(1385)$ | 2.5 |
| $\Lambda(1660)$ | $\Lambda(1520)$ | $\Sigma(1660)$ | $\Sigma(1670)$ |
| $\Lambda(1660)$ | $\Lambda(1600)$ | $\Sigma(1660)$ | $\Sigma(1750)$ |
| $\Lambda(1660)$ | $\Lambda(1670)$ | $\Sigma(1660)$ | $\Sigma(1775)$ |
| $\Lambda(1660)$ | $\Lambda(1690)$ | $\Sigma(1660)$ | $\Sigma(1775)$ |
| $\bar{\Lambda}(1660)$ | $\bar{\Lambda}(1620)$ | 2.5 | $\bar{\Sigma}(1660)$ | $\bar{\Sigma}(1714)$ | 3.2 |
| $\Lambda(1930)$ | $\Lambda(1800)$ | $\Sigma(1930)$ | $\Sigma(1915)$ |
| $\Lambda(1930)$ | $\Lambda(1810)$ | $\Sigma(1930)$ | $\Sigma(1940)$ |
| $\Lambda(1930)$ | $\Lambda(1820)$ | $\Sigma(1930)$ | $\Sigma(1940)$ |
| $\Lambda(1930)$ | $\Lambda(1830)$ | $\Sigma(1930)$ | $\Sigma(1940)$ |
| $\Lambda(1930)$ | $\Lambda(1890)$ | $\Sigma(1930)$ | $\Sigma(1940)$ |
| $\bar{\Lambda}(1930)$ | $\bar{\Lambda}(1830)$ | 5.5 | $\bar{\Sigma}(1930)$ | $\bar{\Sigma}(1928)$ | .10 |
| $\Lambda(2020)$ | $\Lambda(2100)$ | $\Sigma(2020)$ | $\Sigma(2030)$ | .50 |
| $\Lambda(2020)$ | $\Lambda(2110)$ | $\Sigma(2020)$ | $\Sigma(2030)$ |
| $\bar{\Lambda}(2020)$ | $\Lambda(2105)$ | 4.1 | | |
| $\Lambda(2460)$ | $\Lambda(2350)$ | $\Sigma(2380)$ | $\Sigma(2250)$ | $\Sigma(2455)$* |
| $\bar{\Lambda}(2420)$ | $\bar{\Lambda}(2350)$ | 3.0 | $\bar{\Sigma}(2380)$ | $\bar{\Sigma}(2353)$ | 1.2 |
| $\Lambda(2560)$ | $\Lambda(2585)$* | 1.0 | | |
| $8\Lambda(2650)$ | | $7\Sigma(2650)$ | $\Sigma(2620)$ | 1.1 |
| $6\Lambda(2740)$ | | $6\Sigma(2740)$ |

*Evidences of existence for these baryons are only fair, they are not listed in the Baryon Summary Table [19].
Table 4. The Baryons $\Xi$ and the Baryons $\Omega$

| Theory     | Experiment | $\frac{AM}{M}$% | Theory     | Experiment | $\frac{AM}{M}$% |
|------------|------------|-----------------|------------|------------|-----------------|
| $2\Xi(1300)$ | $\Xi(1318)$ | 1.4             | $\Omega(1660)$ | $\Omega(1672)$ | 0.7             |
| $3\Xi(1480)$ | $\Xi(1530)$ | 3.3             | $\Omega(2460)$ | $\Omega(2250)$ |                 |
| $3\Xi(1660)$ | $\Xi(1690)$ | 1.8             | $\bar{\Omega}(2460)$ | $\bar{\Omega}(2367)$ | 3.9             |
| $2\Xi(1840)$ | $\Xi(1820)$ | 1.1             | $\Omega(3080)$ |           |                 |
| $\Xi(1930)$ | $\Xi(1950)$ | 1.1             | $\Omega(3720)$ |           |                 |
| $3\Xi(2020)$ | $\Xi(2030)$ | 1.0             | $\Omega(7220)$ |           |                 |
| $4\Xi(2200)$ | $\Xi(2250)^*$ | 0.5             |           |           |                 |
| $2\Xi(2380)$ | $\Xi(2370)^*$ | 2.2             |           |           |                 |
| $3\Xi(2560)$ |           |                 |           |           |                 |
| $11\Xi(2740)$ |           |                 |           |           |                 |

*Evidences of existence for these baryons are only fair, they are not listed in the Baryon Summary Table [19].

72
Table 5. Charmed $\Lambda^+_c$ and Bottom $\Lambda^0_b$ Baryons

| Theory       | Experiment | $\frac{\Delta M}{M}$% | Theory       | Experiment | $\frac{\Delta M}{M}$% |
|--------------|------------|------------------------|--------------|------------|------------------------|
| $\Lambda^+_c(2280)$ | $\Lambda^+_c(2285)$ | 0.22                   | $\Lambda^0_b(5540)$ | $\Lambda^0_b(5641)$ | 3.6                    |
| $\Lambda^+_c(2450)$ | $\Lambda^+_c(2593)$ |                        | $\Lambda^0_b(9960)$ |                        |                        |
| $\Lambda^+_c(2540)$ | $\Lambda^+_c(2625)$ |                        |              |            |                        |
| $\Lambda^+_c(2660)$ |              |                        |              |            |                        |
| $\bar{\Lambda}^+_c(2550)$ | $\bar{\Lambda}^+_c(2609)$ | 2.3                   |              |            |                        |
| $\Lambda^+_c(2970)$ |              |                        |              |            |                        |
| $\Lambda^+_c(6600)$ |              |                        |              |            |                        |
| $\Lambda^+_c(13800)$ |              |                        |              |            |                        |
Table 6. Charmed Strange Baryon Ξ_c, Σ_c and Ω_C

| Theory | Experiment | \( \Delta M \) | Theory | Experiment | \( \Delta M \) |
|--------|------------|----------------|--------|------------|----------------|
| Ξ_c(2550) | Ξ_c(2468) | Ξ_c(2645) | Σ_c(2280) | Σ_c(2455) | Σ_c(2530)* |
| Ξ_c(2557) | Ξ_c(2557) | 0.3 | Σ_c(2423) | Σ_c(2493) | 2.8 |
| Ξ_c(3170) | Σ_c(2970) | | | | |
| | | | Ω_C(2660) | Ω_C(2704) | 1.6 |
| | | | Ω_C(3480) | | |

*Evidences of existence for these baryons are only fair, they are not listed in the Baryon Summary Table [19].
Appendix A

For the three symmetry axes which are on the surface of the first Brillouin zone (the axis $D(P - N)$, the axis $F(P - H)$, the axis $G(M - N)$), the energy bands in the same degeneracy group may have asymmetric $\vec{n}$ values (see Fig. 3(b), 4(a) and 4(b)). This may indicate that they belong to different Brillouin zones. In such cases, even if a sub-degeneracy $d_{sub} \leq R$, it may still be divided further. Finding the criteria for dividing the degeneracy depends on the structures of the Brillouin zones, the irreducible representations of the single and double point groups [40], and requires a higher order approximation. Hence, it is beyond the scope of this paper. In order to simplify, we assume: (a) The degeneracy of the energy bands which are in the first and second Brillouin zones will be divided. (b) If the energy fluctuation $\Delta \varepsilon \neq 0$, an asymmetric sub-degeneracy should be divided at the point $N$ (low symmetry point, only has 8 symmetric operations [41]); at the end point $P$ (24 symmetric operations [41]), may or may not be divided with a possibility of 50%; but should not be divided at the end points $\Gamma, H$ and $M$ (high symmetry points, 48 symmetric operations [41]):

\[
\text{if } \Delta \varepsilon \neq 0, \text{ divided at the end point } N;
\]

\[
\text{not divided at end points } \Gamma, H \text{ and } M;
\]

\[
\text{divided at end point } P \text{ with possibility of } 50\%.
\]

(148)

However, if $\Delta \varepsilon = 0$, the asymmetric degeneracy should not be divided.
Appendix B

A  The axis $D(P - N)$

From (54), the axis $D$ has $S = 0$. For low energy levels, there are 4 fold degenerate energy bands and 2 fold degenerate energy bands on the axis (see Fig. 3(b)).

A- 1  The 4 fold energy bands on the axis $D(P - H)$

We can see that each 4 fold degenerate energy band has 4 asymmetric $\tilde{n}$ values. They can be divided into two groups. Each of them has 2 symmetric $\tilde{n}$ values. Using (17), for the two fold degenerate energy bands, we get $I = 1/2$, $S = 0$, $Q = 2/3$, $-1/3$ from (57). For the 4 fold energy bands, we have

| $E$  | $\tilde{n}$          | $q_N^*(m)$ |
|------|----------------------|------------|
| $E_N = 5/2$ | (1-10,-110,020,200) | 1840       |
| $\Delta S = 0$ | (1-10,-110) | $q_N^*(1840)$ |
| $\Delta S = 0$ | (020,200) | $q_N^*(1840)$ |
| $E_P = 11/4$ | (-101,0-11,211,121) | 1930       |
| $\Delta S = 0$ | (-101,0-11) | $q_N^*(1930)$ |
| $\Delta S = 0$ | (211,121) | $q_N^*(1930)$ |
| $E_N = 7/2$ | (12-1,21-1,-10-1,0-1-1) | 2200       |
| $\Delta S = 0$ | (12-1,21-1) | $q_N^*(2200)$ |
| $\Delta S = 0$ | (-10-1,0-1-1) | $q_N^*(2200)$ |
| $E_P = 19/4$ | (-112,1-12,202,022) | 2650       |
| $\Delta S = 0$ | (-112,1-12) | $q_N^*(2650)$ |
| $\Delta S = 0$ | (202,022) | $q_N^*(2650)$ |

... 

A- 2  The two fold energy bands on the axis $D(P - N)$

See Fig. 3(b). There are symmetric and asymmetric $\tilde{n}$ values in the 2 fold energy bands. The 2 fold energy bands with symmetric $\tilde{n}$ values represent the quark family $q_N^*$. 76
However, the case of 2 fold energy bands with asymmetric $\vec{n}$ values is not so simple.

At $E_N = 1/2$, $J_N = 0$ there are two energy bands with asymmetric $\vec{n} = (000, 110)$. Since the two energy bands are in different Brillouin zones, they will be divided into 2 single bands. From (45), we know that the energy band with $n = (0, 0, 0)$ represents $q_N(940)$. The energy band with $\vec{n} = (110)$ belongs to the second Brillouin zone, and it represents the excited quark $q_S(1120)$ with $S = -1, B = 1/3, I = 0, Q = -1/3, and m = 1120$ from (89).

At $E_P = 11/4$, $J_N = 1, \vec{n} = (002, 112)$, which are asymmetric $\vec{n}$ values. Using (148), the two energy bands may be divided with a possibility of 50%. However, the fluctuation of energy $\Delta \varepsilon = 0$ from (74), hence there is not enough energy of fluctuation for the two bands to be divided. Thus, the two energy bands are not be divided. They represent a quark family $q_N(1930)$.

There are two asymmetric 2 fold energy bands at $E_N = 9/2$ ($\vec{n} = (220, -1 - 10)$ and $\vec{n} = (11 - 2, 00 - 2)$). Using (74), the energy fluctuation for the first 2 energy bands ($\vec{n} = (220, -1 - 10), J_N = 1$) is $\Delta \varepsilon = 0$. Hence, they cannot be divided. Similar to $q_N(1930)$, it represents a quark family $q_N(2560)$. However, the energy fluctuation for the second 2 energy bands ($\vec{n} = (11 - 2, 00 - 2), J_N = 2$) is $\Delta \varepsilon = 100(2 - 1)\Delta S = 100\Delta S$. Using (148), the 2 fold bands should be divided into 2 single energy bands with $S = 0 + \Delta S = \pm 1$ from (64). According to (61), one of the 2 single energy bands has a charmed number $C = \Delta S = +1$ (with $I = 0$ and $Q = +1$), and it represents a Charmed quark $q^*_C$; the other has a strange number $S = -1$ (with $I = 0$ and $Q = 0$), and it represents a quark $q_S$. Thus, the second 2 energy bands, after the division, represent $q^*_C(2660)$ and $q_S(2460)$.

Therefore, we have
The axis \( F(P-H) \) is a 3 fold symmetry axis, from (3), \( S = -1 \). For low energy levels, there are 6 fold energy bands, 3 fold energy bands, and single energy bands on the axis (see Fig. 4(a)).

**B-1 The single energy bands on the axis \( F(P-H) \)**

For the single energy band, the strange number \( S = -1 \), and \( I = 0 \) from (17) and \( Q = -1/3 \) from (57). Each single energy band represents an excited quark \( q^*_s \) with \( S = -1 \), \( I = 0 \), \( Q = -1/3 \).

\[
\begin{align*}
E_P &= 3/4 & \bar{n} &= (110) & \varepsilon^{(0)} &= 1210 & S = -1 & I = 0 & Q = -1/3 & q^*_s(1210) \\
E_H &= 3 & \bar{n} &= (-1-12) & \varepsilon^{(0)} &= 2020 & S = -1 & I = 0 & Q = -1/3 & q^*_s(2020) \\
\end{align*}
\]
The three fold energy bands on the axis $F(P - H)$

See Fig. 4(a). Using (148), we know that the 3 fold degenerate energy bands (all have asymmetric $\vec{n}$ values) will not be divided at the point $H$, but may be divided at the point $P$ with a possibility of 50%. The division at point $P$ will result in a single band representing a quark $q^*_S$, and a 2 fold energy band (with symmetric $\vec{n}$ values) representing a quark family $q_N^* (\Delta S = +1)$ or $q^{*}_S (\Delta S = -1)$.

At $E_P \approx 3/4$, $\vec{n} = (000, 101, 011)$. They will be divided into a single band with $\vec{n} = (000)$ (in the first Brillouin zone) and a 2-fold energy band with $\vec{n} = (101, 011)$ (in the second Brillouin zone). From (145), the band with $\vec{n} = (0, 0, 0)$ represents $q_N^* (940)$. The 2-fold energy band with $\vec{n} = (101, 011)$ represents a quark family $q_N^* (1210)$ from (150)

At $E_P \approx 11/4$, $\vec{n} = (112, 1-10, -110)$, $J_P = 1 \rightarrow \Delta \varepsilon = 0$. Since $\Delta \varepsilon = 0$, the three energy bands will not be divided (see (148)). Thus, the energy bands represent a quark family $q^{*}_S (1930)$.

At $E_P \approx 19/4$, $J_P = 2 \rightarrow \Delta \varepsilon = -100 \Delta S$, the three energy bands with $\vec{n} = (220, 21-1, 1-2-1)$ may be divided with a possibility of 50% (see (148)). Hence the energy bands may represent $q^{*}_S (2650)$ (if not divided), or $q_s (2650)$ and $q^{*}_N (2550)$ or $q^{*}_S (2750)$ (if divided).

We have
\[ E_P = \frac{3}{4} \quad \tilde{n} = (000,011,101) \quad \varepsilon^{(0)} = 1210 \]
\[ J_P = 0 \quad \tilde{n} = (000) \quad \Delta S = 0 \quad q_N^*(940) \]
\[ \Delta \varepsilon = 0 \quad \tilde{n} = (011,101) \quad \Delta S = +1 \quad q_N^*(1210) \]
\[ E_H = 1 \quad \tilde{n} = (002,-101,0,-11) \quad \varepsilon^{(0)} = 1300 \quad q_\Sigma^*(1300) \]
\[ E_P = 11/4 \quad \tilde{n} = (112,1-10,-110) \quad \varepsilon^{(0)} = 1930 \quad q_\Sigma^*(1930) \]
\[ J_P = 1 \quad \Delta \varepsilon = 0 \]
\[ E_H = 3 \quad \tilde{n} = (-1-10,112,1-12) \quad \varepsilon^{(0)} = 2020 \quad q_\Sigma^*(2020) \]
\[ E_P = 19/4 \quad \tilde{n} = (220,21-1,12-1) \quad \varepsilon^{(0)} = 2650 \quad q_\Sigma^*(2650) \]
\[ J_P = 2 \quad \tilde{n} = (220) \quad q_s^*(2650) \]
\[ \Delta \varepsilon = -100 \quad \tilde{n} = (21-1,12-1) \quad \Delta S = +1 \quad q_N^*(2550) \]
\[ \Delta S = -1 \quad q_\Xi^*(2750) \]

\[ \vdots \]

**B-3** **The six fold energy bands \((d = 6)\) on the axis \(F(P - H)\)**

The 6 fold energy bands on axis \(F\) are a special case. The 6 asymmetric \(\tilde{n}\) values consist of three groups, each of them has 2 symmetric \(\tilde{n}\) values. However, since the axis \(F\) has a rotary \(R = 3\), so there are two ways to divide the energy bands: A) dividing the energy bands according to (48) and (49); B) dividing the energy bands according to the symmetry of \(\tilde{n}\) values.

(A). From (48) and (49), each 6 fold energy band shall be divided into two 3 fold energy bands first. They will represent two quark families \(q_\Sigma^*\). Since the 3 fold sub-degeneracy band has asymmetric \(\tilde{n}\) values, according to (48), it may be divided (second division) further at point \(P\) with a possibility of 50%, but not be divided at the point \(H\). At the point \(P\), in order to keep (59), both of the two 3 fold sub-degeneracy bands will be divided, resulting in two 2 fold bands (one with \(\Delta S = +1\), the other with \(\Delta S = -1\)) and two single bands. According to (65), the 2 fold energy band with \(\Delta S = +1\) will keep \(S\) unchanged while increasing the Charmed number \(C\) by 1. They are the 2 fold
quark excited state $q^*_\Xi_C$ with $B = 1/3$, $S = -1$, $C = +1$, $I = 1/2$, and $Q = 2/3, -1/3$.

At $E_P = 11/4$, $\vec{n} = (01-1,10-1,121,211,020,200)$, $J_P = 1$ ($E_P = 3/4, J_P = 0$), $\Delta \varepsilon = 0$ from (173). Since there is not enough fluctuation energy to divide the two 3 fold sub-degeneracies (148), they are not divided. Thus, the 6 fold band represents

$$2 \times q^*_\Sigma(1930)$$ (153)

At $E_P = 19/4$, $\vec{n} = (202,022,-121,2-11,0-1-1,-10-1)$, $J_P = 2$. According to (173), the energy fluctuation $\Delta \varepsilon = -100 \times \Delta S$. Thus, the two 3 fold bands represent (with a possibility of 50% to be divided at point $P$)

$$2 \times q^*_\Xi_C(2550) + [q^*_\Xi_C(2550) + q^*_S(2650)] + [q^*_\Xi_C(2750) + q^*_S(2650)]$$ (154)

It is very important to pay attention to the baryon $\Xi_C(2550)$ born here, on the 6 fold energy band, after the second division from the fluctuation $\Delta S = +1$ and $\Delta \varepsilon = -100$ Mev.

At $E_H = 5$, $\vec{n} = (20,200,-211,1-21,013,103)$. According to (148), the two 3 fold sub-degeneracies are not divided. Thus, the 6 fold band represents

$$2 \times q^*_\Sigma(2740)$$ (155)

At $E_H = 5$, $\vec{n} = (0,22,-202,-211,1-21,0-13,-103)$. Similarly, we have

$$2 \times q^*_\Sigma(2740)$$ (156)

At $E_P = 27/4$, $\vec{n} = (-12-1,2-1,1301,031,222,00-2)$, $J_P = 3$. Similar to the case of $E_P = 19/4$, $\Delta \varepsilon = -200 \times \Delta S$ and the 6 fold band represents (with a possibility of 50% to be divided twice at the point $P$)

$$2 \times q^*_\Xi_C(3170) + [q^*_\Xi_C(3170) + q^*_S(3370)] + [q^*_\Xi_C(3570) + q^*_S(3370)]$$ (157)
According to the symmetry values of $\vec{n}$, each 6 fold degeneracy can be divided into a 2 fold sub-degeneracy band and a 4 fold sub-degeneracy band. For the division, we get $S = \bar{S} + \Delta S = -1 \pm 1$ from $[50]$. To keep $[50]$, the 2 fold energy band may have $\Delta S = -1$, while the 4 fold band will have $\Delta S = +1 (S = 0)$. (Another possibility is that the 2 fold energy band has $\Delta S = +1 (S = 0)$, while the 4 fold band has $\Delta S = -1 (S = -2)$. However, the possibility of obtaining a 4 fold band with strange number $S = -2$ is not supported by experimental results. Hence, it will be ignored here).

Since the symmetric axis is a 3 rotary one, the 4 fold energy band will be divided further. In order to balance the 3 rotary symmetry of the axis and the 2 fold symmetry of the $\vec{n}$ values, the second division should keep the 3 rotary symmetry (may break the 2 fold symmetry of $\vec{n}$) because the first division has kept the 2 fold symmetry of the $\vec{n}$ values. Thus, the 4 fold energy band ($S = 0$) may be divided into $(q_{\Lambda_c}^* + q_{\Sigma}^*)$ or $(q_{\Sigma}^* + q_{\Sigma_c}^*)$ (if $\Delta \varepsilon \neq 0$). But if $\Delta \varepsilon = 0$, it will be divided into two 2 fold bands ($S = 0 \rightarrow 2 q_{N}^*$).

At $E_P = 19/4$, $\vec{n} = (01-1,-10-1,-121,1212,1202,202)$, $J_P = 1$, and $\Delta \varepsilon = 0$ from $[53]$. Since $\Delta \varepsilon = 0$, the 4 fold energy band will be divided into two 2 fold ($S = 0 \rightarrow 2 \times q_{N}^*(1930)$).

We get

$$q_{\Xi}^*(1930), 2 \times q_{N}^*(1930). \quad (158)$$

At $E_P = 19/4$, $\vec{n} = (01-1,-10-1,-121,1212,1202,202)$, $J_P = 2$, the energy fluctuation $\Delta \varepsilon = -100(2 - 1)\Delta S = -100 \times \Delta S$. After the first division, we have $q_{\Xi}^*(2550)$, and $q_{\Delta}^*(2550)$. For $q_{\Delta}^*(2550)$, using $[50]$, we get $S = 0 + \Delta S \equiv \pm 1$. From $\Delta S = \pm 1$, we have $q_{\Delta}^*(2550) \rightarrow [q_{\Xi}^*(2450) (\Delta S = +1) + q_{\Sigma}^*(2650) (\Delta S = -1) \text{ or } q_{\Sigma_c}^*(2450) (\Delta C = \Delta S = +1 \text{ (see (65))} + q_{\Xi}^*(2650) (S = -1))]$ to keep the 3 rotary symmetry. To sum up,
the 6 fold energy band has the possibility to represent the quark families:

\[ q_{\Xi}^*(2750), q_{\Xi}^*(2650), q_{\Sigma}^*(2450), q_{\Sigma}^*(2450), q_{5}^*(2650). \]  

(159)

At \( E_H = 5 \), \( \vec{n} = (0-20, -200, -211, 1-21, 013, 103) \), \( J_H = 1 \) the energy fluctuation \( \Delta \varepsilon = 0 \). We have:

\[ q_{\Xi}^*(2740), 2 \times q_{N}^*(2740). \]  

(160)

Also at \( E_H = 5 \), \( \vec{n} = (0-22, -202, -2-11, -1-21, 0-13, -103) \), \( J_H = 2 \), the energy fluctuation \( \Delta \varepsilon = -100 \times \Delta S \). Similarly to \( E_H = 5 \), we have:

\[ q_{\Xi}^*(2840), q_{\Sigma}^*(2740), q_{C}^*(2540), q_{\Sigma C}^*(2540), q_{S}^*(2740). \]  

(161)

At \( E_P = 27/4 \), \( \vec{n} = (-12-1, 2-1-1, 301, 031, 222, 00-2) \), \( J_P = 3 \). Similar to the case of \( E_P = 19/4 \), we have

\[ q_{\Xi}^*(3570), q_{\Sigma}^*(3700), q_{\Lambda C}^*(2970), q_{\Sigma C}^*(2970), q_{S}^*(3370) \]  

(162)

C **The axis \( G(M - N) \)**

The axis \( G(M - N) \) is a 2 fold symmetry axis. From (50), the strange number \( S = -2 \). There are 6, 4, and 2 fold energy bands on the axis (see Fig. 4(b)).

C- 1 **The two fold energy bands on the axis \( G(M - N) \)**

At \( E_N = 1/2 \), \( J_N = 0 \), the two energy bands with asymmetric \( \vec{n} = (000, 110) \) are in the first and second Brillouin zones, respectively. The energy band with \( \vec{n} = (110) \) represents \( q_{S}^*(1120) \). Another band with \( \vec{n} = (000) \) represents \( q_{N}^*(940) \).

At \( E_M = 1 \), there are two 2 fold energy bands. The 2 fold energy band with symmetric values \( \vec{n} = (101, 10 - 1) \) represents a quark family, \( q_{\Xi}^*(1300) \), similar to (53).
Another 2 fold energy band with asymmetric values $\vec{n} = (200, 1 - 10)$ will not be divided (see (148)), it represents a quark family $q^*_{\Xi}(1300)$ too.

At $E_N = 5/2$, $J_N = 1$, $\Delta\epsilon = 0$. Since $\Delta\epsilon = 0$ the 2 fold energy band with $\vec{n} = (020, 110)$ should not be divided (see (148)), it represents a quark family $q^*_{\Xi}(1300)$.

At $E_M = 5$, the 2 fold energy band with asymmetric $\vec{n} = (310, 2 - 20)$ will not be divided (see (148)), it represents quark family $q^*_{\Xi}(2740)$.

Thus, we have

\begin{align*}
E_N = 1/2 & \quad \vec{n} = (000, 110) \quad \epsilon^{(0)} = 1120 \\
J_N = 0 & \quad \vec{n} = (000) \quad S = 0 \quad q^*_N(940) \\
& \quad \vec{n} = (110) \quad S = -1 \quad q^*_S(1120) \\
E_M = 1 & \quad \vec{n} = (101, 10-1) \quad \epsilon^{(0)} = 1300 \quad q^*_S(1300) \\
& \quad \vec{n} = (200, 1-10) \quad \epsilon^{(0)} = 1300 \quad q^*_S(1300) \\
E_N = 3/2 & \quad \vec{n} = (011, 01-1) \quad \epsilon^{(0)} = 1480 \quad q^*_S(1480) \\
E_N = 5/2 & \quad \vec{n} = (020, -110) \quad \epsilon^{(0)} = 1840 \quad q^*_S(1840) \\
J_N = 1 & \quad \Delta\epsilon = 0 \\
E_M = 3 & \quad \vec{n} = (2-11, 2\cdot1-1) \quad \epsilon^{(0)} = 2020 \quad q^*_S(2020) \\
E_M = 5 & \quad \vec{n} = (310, 2-20) \quad \epsilon^{(0)} = 2740 \quad q^*_S(2740) \\
E_N = 11/2 & \quad \vec{n} = (-121, -12-1) \quad \epsilon^{(0)} = 2920 \quad q^*_S(2920) \\
\ldots
\end{align*}

C- 2 The four fold energy bands on the axis $G(M - N)$

According to (48) and (49), each 4 fold energy band will be divided into two 2 fold energy bands which represent two quark families $q^*_S (S = -2, I = 1/2, Q = -1/3, -4/3)$
similar to (86):

\[ E_M=3 \quad \vec{n}=(0-11,0-1-1, 211,21-1) \quad \varepsilon^{(0)} = 2020 \]
\[ \Delta S=0 \quad \vec{n}=(0-11,0-1-1) \quad q_\Xi^*(2020) \quad \vec{n}=(211,21-1) \quad q_\Xi^*(2020) \]
\[ E_N=7/2 \quad \vec{n}=(-101,-10-1, 121,12-1) \quad \varepsilon^{(0)} = 2200 \]
\[ \Delta S=0 \quad \vec{n}=(-101,-10-1) \quad q_\Xi^*(2200) \quad \vec{n}=(121,12-1) \quad q_\Xi^*(2200) \]
\[ E_M=5 \quad \vec{n}=(301,30-1, 1-21,1-2-1) \quad \varepsilon^{(0)} = 2740 \]
\[ \Delta S = 0 \quad \vec{n}=(301,30-1) \quad q_\Xi^*(2740) \quad \vec{n}=(1-21,1-2-1) \quad q_\Xi^*(2740) \]

\[ \ldots \]

C-3 The six fold energy bands on the axis \( G(M - N) \)

According to (148) and (149), each 6 fold energy band will be divided into three 2 fold energy bands. One of the three 2 fold sub-degeneracy energy bands has asymmetric \( \vec{n} \) values. According to (148), the 2 energy bands should be divided further at the point \( N \), but should not be divided at the point \( M \). Thus, at point \( M \), each 6 fold energy band will represent three quark families \( q_\Xi^* \) similar to (86). At the point \( N \), each 6 fold energy band will represent two \( q_\Xi^* \) and two single energy bands.

At \( E_N = 9/2 \), \( \vec{n} = (112, 11 - 2, 002, 00 - 2, 220, -1 - 10) \). First, the 6 fold energy band will be divided into: \( \vec{n} = (112, 11 - 2) \ (q_\Xi^*(2560)), \vec{n} = (002, 00 - 2) \ (q_\Xi^*(2560)), \vec{n} = (220, -1 - 10) \). Then the two energy bands with asymmetric \( \vec{n} \) values \( (\vec{n} = (220, -1 - 10)) \) will be further divided. The fluctuation of energy associated with this division is \( \Delta \varepsilon = 100 \Delta S \) Mev. (\( J_N = 2 \) at \( E_N = 9/2 \) for 2-fold asymmetric \( \vec{n} \) values, \( J_N = 0 \) at \( E_N = 1/2 \) and \( J_N = 1 \) at \( E_N = 5/2 \) (see (163))). The fluctuation of strange number associated with this division is \( \Delta S = \pm 1 \). Since this is the second division of the 6 fold energy band, according to Hypothesis IV. 6, (65), for \( \Delta S = +1 \) we can get a quark excited states \( q_\Omega^*(2660) \) with charmed number \( C = +1 \) while keeping the strange number \( S = -2 \) unchanged. Hence the 2 energy bands will be divided into:

\[ \Delta C = \Delta S = +1, \quad q_\Omega^*(2660); \quad \Delta S = -1, \quad q_\Xi^*(2460). \]

\[ (165) \]
It is very important to pay attention to the quark $q_{ΩC}^*(2660)$ born here, on the 6 fold energy band of the axis $G$, after the second division with fluctuations $ΔS = +1$ and $Δε = +100$ Mev.

At $E_M = 5$, $\vec{n} = (202, 20 - 2, 1 - 12, 1 - 1 - 2, 310, 0 - 20)$ will be divided into three 2 fold energy bands, representing 3 quark families $q_{Ξ}^*(2740)$. The 2 fold band with asymmetric $\vec{n} = (310, 0 - 20)$ will not be divided further at the point $M$ from (148). Thus, it represents a quark family $q_{Ξ}^*(2740)$.

At $E_N = 13/2$, similar to $E_N = 9/2$, the 6 fold degeneracy will be divided into two 2 fold degeneracies (two quark families $q_{Ξ}^*(2920)$), and 2 single bands. The second division (the 2 single bands) will result in a quark $q_{ΩC}^*(3220)$ and a quark $q_{Ω}^*(2620)$ ($J_N = 3$ and $Δε = 100(3 - 1)ΔS = 200ΔS$ Mev).

We have

\[
\begin{align*}
E_N &= 9/2 & \vec{n} &= (112, 11-2, 002, 00-2, 220, -1-10) & \varepsilon^{(0)} &= 2560 \\
& & & & \\
J_N &= 2 & q_{ΩC}^*(2660) & \vec{n} &= (002, 00-2) & q_{ΩC}^*(2660) \\
& \text{(divided)} & \text{single band} & & \text{single band} & \\
E_M &= 5 & \vec{n} &= (202, 20-2, 1-12, 1-1-2, 310, 0-20) & \varepsilon^{(0)} &= 2740 & 3 \ q_{ΩC}^*(2740) \\
& & & & & \\
E_N &= 13/2 & \vec{n} &= (-112, 11-2, 022, 02-2, 130, -200) & \varepsilon^{(0)} &= 3280 & \\
& & & & & \\
J_N &= 3 & q_{ΩC}^*(3280) & \vec{n} &= (022, 02-2) & q_{ΩC}^*(3280) \\
& \text{(divided)} & \text{single band} & & \text{single band} & q_{ΩC}^*(3080) \\
& & & & \ldots & \\
& & & & & (166)
\end{align*}
\]

86
The axis $\Delta$ (the axis $\Gamma$-H) is a four fold rotation axis

The axis $\Lambda$ (the axis $\Gamma$-P) is a three fold rotation axis

The axis $\Sigma$ (the axis $\Gamma$-N) is a two fold rotation axis

Figure 1
Figure 2
Figure 4
Figure 5