Universal Deformations

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Confinement in 2 dimensions

Start with a generic 2d gauge theory (w/ matter)

**Q:** Does it confine?

Only a sharp question if there is a **1-form symmetry**

**Q:** Is the 1-form symmetry spontaneously broken?

**Coleman-Mermin-Wagner:**

No discrete 0-form SSB in QM

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No discrete 1-form SSB in 2d

Gaiotto, Kapustin, Seiberg, Willett '14

**Deconfined counter-examples:**

- Massless Schwinger models
- Massless 2d adjoint QCD*
- other QCD-like theories*

Casher, Kogut, Susskind '71
Gross, Klebanov, Matytsin, Smilga '95
Komargodski, Ohmori, Roumpedakis, Seifnashri '20
Dempsey, Klebanov, Pufu '21
Delmastro, Gomis, Yu '21
Wilson loops and Universes

Wilson loops separate 2d spacetime into two regions:

If a 1-form symmetry acts on Wilson loops, there is a distinct sector for each 1-form charge.

Each sector is called a ‘Universe’

Full path integral decomposes: $Z = \sum_{\text{universes}} Z_k$

e.g. $\mathbb{Z}_N$ 1-form symmetry in $SU(N)$ adj QCD: $k = 0, \ldots, N - 1$ labels $N$-ality

Hellerman, Henriques, Pantev, Sharpe, Ando ’06
Seiberg ’10
Anber, Poppitz ’18
Tanizaki, Ünsal ’19
Komargodski et. al. ’20
Cherman, Jacobson ’20
Wilson loops and Universes

Area law: confined

Perimeter law: deconfined
Universes and Local Topological Operators

- $p$-form symmetry $\iff$ codimension-$(p + 1)$ topological operator
- 1-form symmetry in 2d $\iff$ local topological operators (LTOs)

**WLOG:** $\mathbb{Z}_N$ symmetry

$U_k(x), \; k = 0, \ldots, N-1$

$$U_k(x) \cdot W_q(C) = e^{\frac{2\pi ik}{N}} q$$

Expectation values $\langle U_k \rangle$ of LTOs label universes

$$\langle U_k \cdot \rangle_q = \langle U_k \cdot \rangle_0 = e^{\frac{2\pi ik}{N}} q$$
**Universal Deformations**

Operators are topological

\[ \langle \underbrace{\cdots \cdots \cdots} \rangle^n = \langle \cdot \cdot \cdot \rangle \]

- \( U_k(x) \) has scaling dimension 0
- \( \Rightarrow \) always a relevant deformation

\[ \Delta S = \int d^2x \Lambda^2 \sum_k (c_k U_k + \text{h.c.}) \]

Effect can be computed exactly: shifts relative vacuum energies of universes

\[ \mathcal{E}_q \rightarrow \mathcal{E}_q + \Lambda^2 \sum_k c_k \cos \left( \frac{2\pi k}{N} q \right) \]
Example: Charge-$N$ Schwinger Model

\[ S = \int d^2 x \frac{1}{4e^2} (da)^2 + \bar{\psi} (\partial \psi - Ni\phi - m_\psi) \psi \]

$\mathbb{Z}_N$ 1-form symmetry generated by $U_k(x)$

- $m_\psi = 0$, $\mathbb{Z}_N^{(0)} \times \mathbb{Z}_N^{(1)}$ anomaly
  \[ \Rightarrow \text{deconfined} \]
- $m_\psi \neq 0$ confined

Universal deformation:

\[ \Delta S_k = \int d^2 x \Lambda^2 (U_k + U_k^\dagger) \]

- breaks chiral symmetry
  - but does not generate a fermion mass!
- triggers confinement/deconfinement
Ex: charge-2 Schwinger model at $\theta = 0$

$m_\psi = 0, \Lambda^2 < 0$

$m_\psi = 0, \Lambda^2 = 0$

$m_\psi > 0, \Lambda^2 < 0$

$m_\psi > 0, \Lambda^2 = 0$
General Picture for Confinement in 2d

Universal deformations in the UV are always relevant.

Consider these deformations in the context of IR EFT.

Unifying perspective suggested by M. Ünsal and M. Nguyen.

UV \hspace{2cm} 2d gauge theory

\hspace{2cm} IR

Long-distance 2d EFT with 1-form symmetry

Expectation value of large Wilson loop?

Which is generic/robust? Confinement or deconfinement?
General Picture for Confinement in 2d

Suppose $\mathbb{Z}_N$ 1-form symmetry is spontaneously broken:

Simplest low-energy EFT: 

$$S_{BF} = \frac{iN}{2\pi} \int_{M_2} \varphi \, da$$

Why deconfined? $\exists$ 0-form symmetry $\varphi \rightarrow \varphi + \frac{2\pi}{N}$ w/ mixed anomaly

Unlike in $d > 2$, this symmetry (and anomaly) is not robust:

deforations by $e^{ik\varphi}$ are relevant, and will be generated*

Generic low-energy EFT: 

$$S_{BF} + S_{U.D.} = \frac{iN}{2\pi} \int_{M_2} \varphi \, da + \int_{M_2} d^2x \Lambda^2 \cos(\varphi)$$

lifts degeneracy, string tension $\sim \Lambda^2$
Confinement is Generic in 2d

2d gauge theory with 1-form symmetry

universal deformations generated

fine-tuning

confined

deconfined

exact 0-form symmetry with mixed anomaly