Multi-item multi-constraint supply chain integrated inventory model with multi-variable demand under the effect of preservation technology

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Abstract: Today’s every firms have realized that managing supply chain (SC) and deciding their integrated scheduling policy is substantial. Decision-makers of all the firms always keep searching different policies such as trade credit, preservation technology. By adopting these policies they help in strengthen the relationship among different players of supply chain. It is also observed that decision-makers develop policies for integrated system under some restrictions such as budget constraint. Infeasible solutions of many real world problems are obtained by ignoring such restriction. On focusing all these issues, in this paper, a multi-item integrated supply chain inventory model is formulated by considering one manufacturer and one retailer over a finite planning horizon. Manufacturer provides trade credit period to its retailer to strengthen the supply chain. It is assumed that the demand is multi-variable which depends on trade credit period and selling price of the items. In a real-life integrated supply chain inventory system, limitations on available budget and storage space are always faced by the manufacturer and the retailer. So, model is formulated to minimize the total integrated cost of the system subject to space and budget constraints. Optimal values of decision variables and objective function is obtained by using Lagrangian Multiplier Method (LMM). Proposed model is illustrated with the help of numerical and sensitive analysis is carried out with respect to different parameters.

Keywords: integrated model; multi-variable demand; budget and space constraint; trade credit; Lagrangian multiplier method

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PUBLIC INTEREST STATEMENT
Aim of this paper is to develop a multi-item integrated supply chain inventory model by considering one manufacturer and one retailer over a finite planning horizon. Manufacturer provides trade credit period to its retailer to strengthen the supply chain. It is assumed that the demand is multi-variable which depends on trade credit period and selling price of the items. In practical situation, limitations on available budget and storage space are always faced by the manufacturer and the retailer. So, model is formulated to minimize the total integrated cost of the system subject to space and budget constraints.
1. Introduction

Now-a-days, people try to utilize their maximum time to manage their professional life and their target is to invest minimum time for each and every non professional work. Therefore, everyone prefers to shop for their necessary commodities in a single roof and save their time. Due to this, importance of shopping mall or a supermarket type retailing shop in modern civilization is increasing slowly but surely with the passage of time. Its basic appeal is the availability of variety of goods under a single roof. Consequently, manufacturing and marketing policies are also changed with time. Manufacturer of different industries like electrical goods industries, food product industries, automobile industries etc. not focused to manufacturer only single item but their target is to manufacture multi-item under a single roof. Now, it is a big challenge to the inventory practitioners and researchers to coordinate multi-item manufacturing industry and retailing shops in a supply chain. Inventory practitioners also start practicing in the direction that when retailer/manufacturer of a factory starts the business/manufacturing has a fixed budget to purchase the items and a godown of finite area to store the items.

Miller (1971) proposed a mathematical model to control multi-item inventory problem with back-order where the objective function is to minimize subjected to a budget constraint. Page and Paul (1976) formulated the basic lot size inventory problem when there was a multi-item for which inventories must be maintained. They optimized their objective function subjected to the space constraints and budget constraints. Rosenblatt and Rothblum (1990) developed mathematical model for multi-item inventory problem subjected to a single resource capacity constraints where the capacity of storage space was considered as decision variables. Das, Roy, and Maiti (2000) formulated a fully backlogged multi-item inventory model with constant demand subjected to storage area, total average inventory investment cost and total average shortage cost. They considered that inventory costs are directly proportional to the respective quantities and the unit production cost is inversely related to the demand. Das, Roy, and Maiti (2004) formulated fully backlogged stochastic and fuzzy-stochastic multi-item inventory problems with demand dependent unit cost under storage-space and budgetary constraints. They assumed that inventory costs were dependent on their respective quantities. They considered demand and both the resources i.e. budgetary amount and storage area as random variable for the stochastic model. Maity and Maiti (2007) developed a mathematical model for multi-item dynamic production-inventory control problem. They obtained the optimal values of objective function subjected to the imprecise budget and storage capacity constraints which are of possibility and/or necessity types respectively. Guichhait, Maiti, and Maiti (2010) proposed a multi-item inventory model of breakable items with stock dependent demand. They considered that breakability rates increases nonlinearly with time and linearly with stock. Mandal, Maity, Maity, Mondal, and Maiti (2011) first time proposed a stock dependent multi-item multi-period production problem without and with the preparation time for a fixed time horizon. They considered holding and shortage costs and available space as fuzzy-random parameters. They solved the problem by using GA. Pal, Sana, and Chaudhuri (2012) studied a multi-item deterministic economic order quantity model for a vendor when the demand rate increase exponentially with increasing level of price break and decreases quadratically with increasing sales price. They considered that when the revenue of the vendor crosses the level of price break, price discount is offered to the customers. Jana, Das, and Maiti (2014) developed partially backlogged multi-item inventory models for deteriorating items in a random planning horizon with exponential distribution under the effect of inflation and time value money with budget and space constraints. They assumed that the demand rate depends on the stock. They developed two models. In first they considered that inventory parameters other than planning horizon are deterministic while in the other deterioration and net value of the money are fuzzy, available budget and space are fuzzy and random fuzzy respectively. Saha, Kar, and Maiti (2015) developed a fuzzy-stochastic multi-item multi-objective supply chain models having budget and risk constraints for long-term contract with a profit sharing scheme. They assumed that the demands of the items were random at each period and the manufacturing costs of the items were fuzzy. To reduce the multi-objective problems into corresponding single objective problems, they used fuzzy compromise programming method, global criteria method and weighted sum method. Nia, Farb, and Niaki (2015) proposed a multi-item economic order quantity model with
shortage for a single-supplier single-buyer supply chain under green vendor managed inventory (VMI) policy. They explicitly included the VMI contractual agreement between the vendor and the buyer such as warehouse capacity and delivery constraints, bounds for each order, and limits on the number of pallets. Topan, Bayındır, and Tan (2017) considered a multi-item two-echelon inventory system. They considered that central warehouse operates under a \((Q, R)\) policy and local warehouses operates \((S−1, S)\) policy. The objective of this study was to find the policy parameters minimizing system-wide inventory holding and fixed ordering under aggregate and individual response time constraints.

Traditionally, integrated supply chain inventory models (ISCIM) are developed in infinite planning horizon because it is assumed that during the course of planning including the various inventory parameters, demand, etc. remain unchanged over the future infinite time. In practice, it is not so due to several reasons such as variation in inventory costs, changes in item specifications and designs, technological changes due to environmental conditions, availability of item, etc. Moreover, for seasonal products like warm garments, fruits, etc. business period is finite. Das, Kar, Roy, and Kar (2012) supported this idea.

In practical situation, manufacturer offer a trade credit period to retailers to settle outstanding amount of the purchasing costs to boost the demand of their products. During trade credit period, the retailer can earn interest from selling items; otherwise, the retailer has to pay interest to the manufacturer for late payment. This policy reduces the purchasing cost of the retailer indirectly and motivates the retailer to buy more. Jaber and Osman (2006) proposed a two level supply chain coordination approach in order to minimize the cost of the system, by incorporating a permissible delay in the payment strategy. Kin Chan, Lee, and Goyal (2010) developed an integrated single-vendor and multi-buyer supply chain model. In order to integrate the system they synchronizing the ordering and production cycles, where delayed payments based on the buyers’ ordering intervals. Tsao (2011) determined the optimal ordering and pricing policy in order to maximize the profit of integrated system when the supplier offers a cash discount to a specific retailer and a credit period to another. Shastri, Singh, Yadav, and Gupta (2014) developed an inventory model for a retailer under two-level of trade credit to reflect the supply chain management. Moussawi-Haidar, Dbouk, Jaber, and Osman (2014) investigated an integrated supply chain, which comprised a capital-constrained supplier, a retailer, and a financial intermediary (bank). The objective of supply chain is to minimize the total supply chain costs while the supplier allowed a trade credit period payment to the retailer. Cárdenas-Barrón and Sana (2014) proposed an integrated supply chain (manufacturer-retailer) model that includes sales teams’ initiatives, variable procurement costs, sensitive demand, variable production rate, production lot size, and backordering where backordering occurs only at the retailer. Chakraborty, Jana, and Ray (2015) developed multi-item integrated (supplier-retailer) supply chain model with constant rate of deterioration with stock dependent demand. They considered that supplier’s production cost as non-linear function depending of production rate, supplier’s transportation cost (TC) as a nonlinear function of the amount of quantity ordered by retailer and retailer’s procurement cost exponentially depends on the trade credit period.

Deterioration is natural phenomenon that occurs for most items in the world. It means that the worth of the product gradually decreases with respect to time. However, investigation on preservation technology to reduce deterioration rate has received little attention in the past years. The consideration of preservation technology is important due to the fact that preservation technology can reduce the deterioration rate significantly.

Deterioration of items present in inventory had been studied in the past decades (Chakraborty et al., 2015; Dye, Ouyang, & Hsieh, 2007; Lin, Tan, & Lee, 2000; Wee, 1995; Widyanada et al., 2011; Yadav, Singh, & Kumari, 2013). However, investigation on preservation technology to reduce deterioration rate has received little attention in the past years. Murr and Morris (1975) showed that a lower temperature will increase the storage life and decrease decay. Moreover, drying or vacuum technology are introduced to reduce the deterioration rate of medicine and foodstuff. Zauberman, Ronen,
Akerman and Fuchs (1990) developed a method for color retention of Litchi fruits with SO2 fumigation. Quyang, Wu, and Yang (2006) found that if the retailer can reduce effectively the deteriorating rate of item by improving the storage facility, the total annual relevant inventory cost will be reduced.

Incorporating the above mentioned real life phenomenon, in this paper, a multi-item integrated supply chain inventory model is formulated by considering one manufacturer and one retailer over a finite planning horizon. Here, unit manufacturing cost, directly proportional to the corresponding raw material cost, development cost due to the system’s reliability and labour, energy, cost of technology, design, complexity, resources etc. The development costs and wear-tear costs are inversely and directly proportional to production rate, respectively. Manufacturer provides trade credit period to its retailer to strengthen the supply chain. It is assumed that the demand is multi-variable which depends on trade credit period and selling price of the items. Demand is increasing function with respect to trade credit period and decreasing with respect to selling price of items. In a real-life integrated supply chain inventory system, limitations on available budget and storage space are always faced by the manufacturer and the retailer. So, model is formulated to minimize the total integrated cost of the system subject to space and budget constraints. Optimal values of decision variables and objective function is obtained by using Lagrangian Multiplier Method (LMM).

2. Assumptions and notations

2.1. Assumption
The mathematical model developed in this paper is based on the following assumptions:

(1) Consider a supply chain in which a manufacturer produces ‘n’ different items and distributes to a retailer.
(2) Manufacturing rate is assumed to be greater than the demand rate.
(3) In this manufacturing-inventory system the whole business period is assumed to be finite.
(4) The replenishment rate is infinite and there is no lead time.
(5) Shortages are not allowed.
(6) Deterioration is considered at the retailer end only.
(7) Retailer invests in the preservation technology (PT) to reduce the deterioration rate.
(8) It is assumed that the trade credit period \( (M) \) offered by the manufacturer must be within each replenishment period \( (T) \) for the \( i \)th item. This assumption is considered for the convenience of mathematical representation. This assumption is practically true for the developing countries where manufacturer do not take much risk.
(9) Demand rate \( D(s, M) \) is the function of selling price and the credit period offered by the manufacturer to the retailer. Demand rate is increasing function with respect of trade credit and decreasing function with respect to selling price.
(10) Retailer’s procurement cost depends on the trade credit period offered by the manufacturer.
(11) Manufacturer transportation cost depends on the order size ordered by the retailer.
(12) Unit manufacturing cost is related to the manufacturing rate.

2.2. Assumptions
Following notations has been used for the development of mathematical model developed in this paper:

\[
A_i \quad \text{Retailer ordering cost per order for the } i\text{th item}
\]
\[
S_{mi} \quad \text{Manufacturer’s setup cost per production run for the } i\text{th item}
\]
3. Formulation of multi-item integrated supply chain mathematical model

In this work, a two-echelon supply chain with one manufacturer and one retailer is considered. Manufacturer producing n different items and distribute to a retailer from where customer demands are satisfied. The production period of the manufacturer starts at time \( t = 0 \) at the manufacturing rate \( P_m \) (for \( i \)th item). At the end of the replenishment period, retailer receives the amount \( Q_i \) (for the \( i \)th item) from the manufacturer. During the period of \( T_r \), retailer satisfies the demand of the customer from the stock. At the end of period retailer again placed the order to the manufacturer of quantity \( Q_i \) (for the \( i \)th item). Manufacturer again delivered the ordered quantity \( Q_i \) (for the \( i \)th item) to the retailer. This process continues up to time \( nT_r \). At the end of the time \((n + 1)T_r\) the inventory
The behavior of the inventory level of manufacturer and retailers is presented in Figure 1.

### 3.1. Formulation of retailer mathematical model

Inventory level $I_i(t)$ for the $i$th item for each replenishment cycle of retailer can be describe by the following differential equation:

$$\frac{dI_i(t)}{dt} + (1 - f_i(u_i)) \theta_i I_i(t) = -D_i(s_i, M_i), \quad 0 \leq t \leq T_i \tag{1}$$
With the boundary condition $I_i(0) = Q_i$ and $I_i(T_i) = 0$.

On solving the differential Equation (1) by using the boundary condition, we get:

$$I_i(t) = \frac{D_i(s_i, M_i)}{(1 - f_i(u_n)) \theta_i} \left[ e^{(1-f(u_n))\theta_i(T_i-t)} - 1 \right]$$

(2)

We have $I_i(0) = Q_i$, so we get:

$$Q_i = \frac{D_i(s_i, M_i)}{(1 - f_i(u_n)) \theta_i} \left[ e^{(1-f(u_n))\theta_i T_i} - 1 \right]$$

Now, we calculate different cost associated with retailer one by one.

Retailer’s average ordering cost for the $i$th item is:

$$ROC_i = \frac{A_i}{T_i}$$

Retailer’s average holding cost for the $i$th item is:

$$RHC_i = \frac{C_i h_i}{T_i} \int_0^{T_i} I_i(t) dt = \frac{C_i h_i}{T_i} \frac{D_i(s_i, M_i)}{(1 - f_i(u_n)) \theta_i} \int_0^{T_i} \left[ e^{(1-f(u_n))\theta_i(T_i-t)} - 1 \right] dt$$

$$= \frac{C_i h_i D_i(s_i, M_i)}{T_i (1 - f_i(u_n)) \theta_i} \left[ e^{(1-f(u_n))\theta_i T_i} - 1 \right] \left( \frac{1 - e^{(1-f(u_n))\theta_i T_i}}{1 - f_i(u_n) \theta_i} - T_i \right)$$

Retailer’s average deterioration cost for the $i$th item is:

$$RDC_i = \frac{C_i d_i}{T_i} \left[ Q_i - T_i D_i(s_i, M_i) \right] dt = \frac{C_i d_i}{T_i} \left[ Q_i - D_i(s_i, M_i) T_i \right]$$

Retailer’s average preservation technology investment cost for the $i$th item is:

$$RPTC_i = (1 - \phi_n) g_n u_n$$

Retailer’s average interest charged for the $i$th item is:

$$RIC_i = \frac{C_i I_{ci}}{T_i} \int_0^{T_i} I_i(t) dt$$

$$RIC_i = \frac{C_i I_{ci}}{T_i} \frac{D_i(s_i, M_i)}{(1 - f_i(u_n)) \theta_i} \int_0^{T_i} e^{(1-f(u_n))\theta_i(T_i-t)} dt$$

$$= \frac{C_i I_{ci} D_i(s_i, M_i)}{T_i (1 - f_i(u_n)) \theta_i} \left[ e^{(1-f(u_n))\theta_i T_i - 1} \right] \left[ 1 - e^{(1-f(u_n))\theta_i T_i} \right] \left( 1 - (T_i - M_i) \right)$$

Retailer’s average interest earned for $i$th items is:

$$RIE_i = \frac{C_i q_i d_i}{T_i} \int_0^{T_i} D_i(s_i, M_i) (T_i - t) dt$$

$$RIE_i = \frac{C_i q_i d_i D_i(s_i, M_i) T_i}{2}$$
Therefore, the retailer’s total average cost for the \( i \)th item is:

\[
TR_i(M_i, T_i) = ROC_i + RHC_i + RDC_i + RPTC_i + RIC_i - RIE_i
\]

Hence, the retailer’s total average cost is given by:

\[
TR(M_i, T_i) = \sum_{i=1}^{n} TR_i(M_i, T_i) = \sum_{i=1}^{n} [ROC_i + RHC_i + RDC_i + RPTC_i + RIC_i - RIE_i]
\]

\[
= \sum_{i=1}^{n} \left[ A_n \frac{T_i}{T_i} + \frac{C_m h_m D_i(s_i, M_i)}{T_i(1 - f_i(u_i)) \theta_n} \left[ e^{(1 - f_i(u_i)\theta_n)T_i} - 1 \right] \frac{T_i}{(1 - f_i(u_i)) \theta_n} - T_i \right] + \frac{C_n}{T_i} [Q_i - D_i(s_i, M_i)T_i]
\]

\[
+ (1 - \varphi_n) g_n u_n + \frac{C_m L_o D_i(s_i, M_i)}{T_i(1 - f_i(u_i)) \theta_n} \left[ e^{(1 - f_i(u_i)\theta_n)\left(T_i - M_i\right)} - 1 \right] \left( T_i - M_i \right) - \frac{C_o Q_i^2 D_i(s_i, M_i)T_i}{2}
\]

(3)

### 3.2. Formulation of manufacturer mathematical model

The manufacturer starts manufacturing process from time \( t = 0 \) and whole of its inventory vanish at time \( t = n_i T_i \) for the \( i \)th item. It is the time when the retailer receives last replenishment of the \( i \)th item. The manufacturing process of \( i \)th item continues up to that period of time so that total demand of \( i \)th item of retailer i.e. \( n_i Q_i \) can be satisfied. Inventory level of manufacturer increase that period of time and after that every cycle of time \( T_i \), the inventory level of manufacturer decreases by the amount \( Q_i \) due to the demand of retailer for the \( i \)th item as shown in Figure 1.

Manufacturer’s average holding cost for the \( i \)th item is:

\[
MHC_i = \frac{h_m}{n_i T_i} \left[ n_i Q_i - \frac{1}{2} n_i Q_i \int \frac{n_i Q_i}{p_m} - Q_i \left( T_i + 2T_i + 3T_i + \ldots + (n_i - 1)T_i \right) \right] = \frac{h_m Q_i}{2} \left( n_i + 1 \right) - \frac{n_i Q_i}{T_i} \frac{1}{p_m}
\]

Manufacturer’s average setup cost for \( i \)th item is:

\[
MSC_i = \frac{S_m}{n_i T_i}
\]

Manufacturer’s average manufacturing cost for the \( i \)th item is:

\[
MMC_i = \frac{M(P_m)Q_i}{T_i}
\]

Manufacturer’s average opportunity interest loss for the \( i \)th item is:

\[
MOIL_i = \frac{q_d^m C_n M_i Q_i}{T_i}
\]

Manufacturer’s average transportation cost for the \( i \)th item is:

\[
MTC_i = \frac{TC_m}{T_i}
\]

Therefore, the manufacturer’s total average cost for the \( i \)th item is:
\[ TM_i(M_i, T_i) = MHC_i + MSC_i + MOIL_i + MTC_i \]

Hence, the manufacturer's total average cost is given by:

\[
TM(M_i, T_i) = \sum_{i=1}^{n} TM_i(M_i, T_i) = \sum_{i=1}^{n} [MHC_i + MSC_i + MOIL_i + MTC_i]
\]

\[
= \sum_{i=1}^{n} \left[ \frac{h_m Q_i}{2} (n_i + 1) - \frac{n_i Q_i}{T_i P_{mi}} + S_{mi} T_i + M(P_{mi})Q_i + \frac{q_{mi}^m C_h M_i Q_i}{T_i} + \frac{TC_{mi}}{T_i} \right] \tag{4}
\]

3.3. Formulation of integrated supply chain model

Our objective is to minimize total integrated cost of the integrated supply chain system under the space constraints and budget constraints.

\[
TIC(M_i, T_i) = \sum_{i=1}^{n} TR_i(M_i, T_i) + \sum_{i=1}^{n} TM_i(M_i, T_i)
\]

\[
= \sum_{i=1}^{n} \left[ A_{mi} T_i + \frac{C_{mi} h_m D_i(s_i, M_i)}{T_i (1 - f_i(u_n))} \theta_m \left[ \frac{e^{(1-f_i(u_n))\theta_m T_i} - 1}{(1 - f_i(u_n)) \theta_m} - T_i \right] + \frac{C_{mi} Q_i}{T_i} - D_i(s_i, M_i) T_i \right]
\]

\[
+ (1 - \theta_m) g_s u_j + \frac{C_{mi} I_o D_i(s_i, M_i)}{T_i (1 - f_i(u_n))} \theta_m \left[ \frac{e^{(1-f_i(u_n))\theta_m (T_i - M_i)} - 1}{(1 - f_i(u_n)) \theta_m} - (T_i - M_i) \right] - \frac{C_{mi} q_{mi}^m D_i(s_i, M_i) T_i}{2}
\]

\[
= \sum_{i=1}^{n} \left[ \frac{h_m Q_i}{2} (n_i + 1) - \frac{n_i Q_i}{T_i P_{mi}} + S_{mi} T_i + M(P_{mi})Q_i + \frac{q_{mi}^m C_h M_i Q_i}{T_i} + \frac{TC_{mi}}{T_i} \right] \tag{5}
\]

Here the objective function is to determine the optimal value of trade credit period \( M_i \) and optimal value of replenishment period \( T_i \) for each item in such a way that total integrated cost of the system is optimum.

Hence

Minimize \( TIC(M_i, T_i) \)

Subject to

\[
\sum_{i=1}^{n} (W_i + W_m) Q_i \leq W \]

\[
\sum_{i=1}^{n} (M(P_{mi})Q_i + C_h M_i Q_i) \leq B \tag{6}
\]

where \( TIC(Q_i, T_i) \) is given by the Equation (5).

4. Formulation of non-deteriorating multi-item integrated supply chain mathematical model

The non-deteriorating multi-item integrated supply chain model can be obtained from above model by taking \( \theta_m \to 0 \) for the \( i \)th item.

4.1. Formulation of non-deteriorating retailer mathematical model

Retailer’s average ordering cost for the \( i \)th item is:

\[ NDROC_i = \lim_{\theta_m \to 0} ROC_i = \frac{A_i}{T_i} \]
Retailer's average holding cost for the ith item is:

\[ NDRHC_i = \lim_{\theta_i \to 0} RHC_i = \lim_{\theta_i \to 0} \frac{C_n h_n D_i(s_n, M_i)}{T_i (1 - f_i(u_n)) \theta_i} \left[ e^{(1-f_i(u_n))\theta_i T - 1} - 1 \right] = \frac{C_n h_n D_i(s_n, M_i) T_i}{2} \]

Retailer's average deterioration cost for the ith item is:

\[ NDRDC_i = \lim_{\theta_i \to 0} RDC_i = \lim_{\theta_i \to 0} \frac{C_i D_i(s_i, M_i)}{T_i} \left[ \frac{D_i(s_i, M_i)}{(1 - f_i(u_n)) \theta_i} \left[ e^{(1-f_i(u_n))\theta_i T - 1} - 1 \right] - D_i(s_i, M_i) T_i \right] = \frac{C_i D_i(s_i, M_i) T_i}{2} \]

Retailer's average preservation technology investment cost for the ith item is:

\[ NDRPTC_i = \lim_{\theta_i \to 0} RPTC_i = 0 \text{ (as there is no deterioration so there is no need to invest in preservation technology).} \]

Retailer's average interest earned for ith items is:

\[ NDRIC_i = \lim_{\theta_i \to 0} RIC_i = \lim_{\theta_i \to 0} \frac{C_i I_i D_i(s_i, M_i)}{T_i (1 - f_i(u_n)) \theta_i} \left[ e^{(1-f_i(u_n))\theta_i T - 1} - 1 \right] = \frac{C_i I_i D_i(s_i, M_i) (T_i - M_i)^2}{2T_i} \]

Retailer's average interest charged for the ith item is:

\[ NDRIE_i = \lim_{\theta_i \to 0} RIE_i = \lim_{\theta_i \to 0} \frac{C_i q_i^d D_i(s_i, M_i)}{2} = \frac{C_i q_i^d D_i(s_i, M_i) T_i}{2} \]

Therefore, the retailer's total average cost for the ith item is:

\[ NDTR_i(M_i, T_i) = NDROC_i + NDRHC_i + NDRIC_i + NDRPTC_i + NDRDC_i - NDRIE_i \]

Hence, the retailer's total average cost is given by:

\[ NDTR = \sum_{i=1}^{n} NDTR_i(M_i, T_i) = \sum_{i=1}^{n} [NDROC_i + NDRHC_i + NDRDC_i + NDRPTC_i + NDRIC_i - NDRIE_i] \]

\[ = \sum_{i=1}^{n} \left[ A_i T_i + \frac{C_n h_n D_i(s_n, M_i) T_i}{2} + \frac{C_i I_i D_i(s_i, M_i) (T_i - M_i)^2}{2T_i} \right] \]

\[ = \sum_{i=1}^{n} \left[ A_i T_i + \frac{h_n D_i}{2} \left[ (n_i + 1) - \frac{n_i Q_i}{P_{mi}} \right] - \frac{h_n D_i T_i}{2} \left[ (n_i + 1) - \frac{n_i D_i}{P_{mi}} \right] \right] \]

4.2. Formulation of non-deteriorating manufacturer mathematical model

Manufacturer's average holding cost for the ith item is:

\[ NDMHC_i = \lim_{\theta_i \to 0} MHC_i = \lim_{\theta_i \to 0} \frac{h_{mi} Q_i}{2} \left[ (n_i + 1) - \frac{n_i Q_i}{P_{mi}} \right] = \frac{h_{mi} D_i T_i}{2} \left[ (n_i + 1) - \frac{n_i D_i}{P_{mi}} \right] \]

Manufacturer's average setup cost for ith item is:

\[ NDMSC_i = \lim_{\theta_i \to 0} MSC_i = \frac{S_{mi}}{n_i T_i} \]
Manufacturer’s average manufacturing cost for the \( i \)th item is:

\[
NDMMC_i = \lim_{\epsilon_i \to 0} MMC_i = \lim_{\epsilon_i \to 0} \frac{M(P_{m_i})Q_i}{T_i} = M(P_{m_i})D_i
\]

Manufacturer’s average opportunity interest loss for the \( i \)th item is:

\[
NDMOIL_i = \lim_{\epsilon_i \to 0} MOIL_i = \lim_{\epsilon_i \to 0} \frac{q_d^m C_i M_i Q_i}{T_i} = q_d^m C_i M_i D_i
\]

Manufacturer’s average transportation cost for the \( i \)th item is:

\[
NDMTC_i = \lim_{\epsilon_i \to 0} MTC_i = \lim_{\epsilon_i \to 0} \frac{TC_{mi}}{T_i} = \frac{TC_{mi}}{T_i}
\]

Therefore, the manufacturer’s total average cost for the \( i \)th item is:

\[
NDT_i(M_i, T_i) = NDMHC_i + NDMSG_i + NDMOIL_i + NDMTC_i
\]

Hence, the manufacturer’s total average cost is given by:

\[
NDT(M_i, T_i) = \sum_{i=1}^{n} NDT_i(M_i, T_i) = \sum_{i=1}^{n} [NDMHHC_i + NDMSG_i + NDMOIL_i + NDMTC_i]
\]

\[
= \sum_{i=1}^{n} \left[ \frac{h_m D_i T_i}{2} \left( n_i + 1 \right) - \frac{n_i D_i}{P_{m_i}} \right] + \frac{S_{mi}}{n_i T_i} + M(P_{m_i})D_i + q_d^m C_i M_i D_i + \frac{TC_{mi}}{T_i} \right]
\]

**4.3. Formulation of non-deteriorated integrated supply chain model**

Our objective is to minimize total integrated cost of non-deteriorating integrated supply chain system under the space constraints and budget constraints.

Now, the total cost of integrated system is given by:

\[
NDTIC(M_i, T_i) = \sum_{i=1}^{n} NDTR_i(M_i, T_i) + \sum_{i=1}^{n} NDT_i(M_i, T_i)
\]

\[
= \sum_{i=1}^{n} \left[ \frac{A_{di}}{T_i} + \frac{C_{ni} h_m D_i (s_i, M_i) T_i}{2} \left( n_i + 1 \right) - \frac{n_i D_i}{P_{m_i}} \right] + \frac{C_{ni} I_{ci} D_i (s_i, M_i) (T_i - M_i)}{2T_i} - \frac{C_d q_d^m D_i (s_i, M_i) T_i}{2T_i}
\]

\[
+ \frac{h_m D_i T_i}{2} \left( n_i + 1 \right) - \frac{n_i D_i}{P_{m_i}} \right] + \frac{S_{mi}}{n_i T_i} + M(P_{m_i})D_i + q_d^m C_i M_i D_i + \frac{TC_{mi}}{T_i} \right]
\]

Here the objective function is to determine the optimal value of trade credit period \( M_i \) and optimal value of replenishment period \( T_i \) for each item in such a way that total integrated cost of the system for non-deteriorated items is optimum.

Hence

\[
\text{Minimize} \quad NDTIC(M_i, T_i)
\]

Subject to

\[
\begin{align*}
\sum_{i=1}^{m} (W_{ri} + W_{mi}) Q_i & \leq W \\
\sum_{i=1}^{m} (M(P_{mi}) Q_i + C_i Q_i) & \leq B
\end{align*}
\]

where \( NDTIC(M_i, T_i) \) is given by the Equation (9).
5. Solution procedure

From Equation (6) it is observed that the objective function is non-linear. So it is a non-linear problem model which cannot be solved easily by the exact method. Hence, a simple Lagrangian multiplier (similar to Uthayakumar and Priyan, (2013)) is used to solve the problem.

In the present problem, there are two constraints imposed simultaneously in inventory system. To solve such type of problem, we can follow the following steps:

**Step 1:** First we obtain the optimal value \((M^*_i, T^*_i)\) of decision variables by ignoring space constraints and budget constraints.

Objective function is

\[
TIC(M_i, T_i) = \sum_{i=1}^{n} \left[ A_{ii} \frac{T_i}{T_i^*} + C_i h_i D_i(s_i, M_i) \left( \frac{e^{(1-f_i(u_i))n_iT_i}}{(1-f_i(u_i))n_iT_i^*} - 1 \right) + \frac{C_i}{T_i^*} [Q_i - D_i(s_i, M_i)T_i] \right. \\
+ \left. \left( 1 - \varphi_i \right) g_i u_i + \frac{C_i I_i D_i(s_i, M_i)}{T_i^* (1-f_i(u_i))n_iT_i} \left( \frac{e^{(1-f_i(u_i))n_i(T_i - M_i)}}{(1-f_i(u_i))n_i(T_i - M_i)} - 1 \right) - (T_i - M_i) \right] \\
+ \sum_{i=1}^{n} \left[ \frac{h_m Q_i}{2} \left( n_i + 1 \right) - \frac{n_i Q_i}{T_i^* P_m} \right] + \frac{S_m}{n_i T_i} + \frac{M(P_m) Q_i}{T_i} + \frac{q_m^d C_i M_i Q_i}{T_i} + \frac{T C_m}{T_i^*}
\]

Objective function \(TIC(M_i, T_i)\) is a function of 2n variables \(M_1, M_2, ..., M_n, T_1, T_2, ..., T_n\).

**Necessary conditions:** A necessary conditions for the minimum of \(TIC(M_i, T_i)\) (where \(i = 1, 2, ..., n\)) give

\[
\frac{\partial TIC(M_i, T_i)}{\partial T_i} = 0 \quad \frac{\partial TIC(M_i, T_i)}{\partial M_i} = 0 \quad \frac{A_{ii}}{T_i^*} - \frac{C_i h_i D_i(s_i, M_i)}{T_i^* (1-f_i(u_i))n_iT_i} \left( \frac{e^{(1-f_i(u_i))n_iT_i}}{(1-f_i(u_i))n_iT_i^*} - 1 \right) - T_i \\
+ \frac{C_i h_i D_i(s_i, M_i)}{T_i^* (1-f_i(u_i))n_iT_i} \left( \frac{e^{(1-f_i(u_i))n_i(T_i - M_i)}}{(1-f_i(u_i))n_i(T_i - M_i)} - 1 \right) - (T_i - M_i) \\
+ \frac{C_i I_i D_i(s_i, M_i)}{T_i^* (1-f_i(u_i))n_iT_i} \left( \frac{e^{(1-f_i(u_i))n_i(T_i - M_i)}}{(1-f_i(u_i))n_i(T_i - M_i)} - 1 \right) - (T_i - M_i) \\
+ \frac{C_i I_i D_i(s_i, M_i)}{T_i^* (1-f_i(u_i))n_iT_i} \left( \frac{e^{(1-f_i(u_i))n_i(T_i - M_i)}}{(1-f_i(u_i))n_i(T_i - M_i)} - 1 \right) - (T_i - M_i) \\
+ \frac{h_m Q_i}{2} \left[ \frac{S_m}{n_i T_i} + \frac{M(P_m) Q_i}{T_i} + \frac{q_m^d C_i M_i Q_i}{T_i} + \frac{T C_m}{T_i^*} \right] \\
+ \frac{D_i(s_i, M_i)e^{(1-f_i(u_i))n_iT_i}}{T_i} = 0
\]  

\(11\)
\[
\frac{h_n}{T_i(1-f(u_n))\theta_n}\left[\frac{e^{(1-f(u_n))\theta_nT_i} - 1}{(1-f(u_n))\theta_n} - T_i \right]\left(C_i^c\epsilon_i^M D_i(s_n, M) + C_n\left(-\alpha_n(s_n) (1-r_c)^M \log (1-r_c) + \beta_n(s_n) (1-r_c)^M \log (1-r_c)\right)\right]
\]

(12)

\[
+ \frac{C_i}{T_i}\left[Q_i - \left(-\alpha_n(s_n) (1-r_c)^M \log (1-r_c) + \beta_n(s_n) (1-r_c)^M \log (1-r_c)\right)T_i\right]
\]

\[
+ \frac{C_i^c}{T_i}\left[C_i^c\epsilon_i^M D_i(s_n, M) + C_n\left(-\alpha_n(s_n) (1-r_c)^M \log (1-r_c) + \beta_n(s_n) (1-r_c)^M \log (1-r_c)\right)\right]
\]

\[
\frac{e^{(1-f(u_n))\theta_nT_i} - 1}{(1-f(u_n))\theta_n} - (T_i - M_i) \right]
\]

\[
P_i^d D_i(s_n, M) \frac{e^{(1-f(u_n))\theta_nT_i} - 1}{(1-f(u_n))\theta_n} - (T_i - M_i)
\]

\[
\frac{q_i^d Q_i}{T_i} \left(C_i + M_i\epsilon_i^M\right) = 0
\]

On solving Equations (11) and (12) simultaneously we get the value of \(M_i\) and \(T_i\).

**Sufficient condition:** A sufficient condition for \(TIC(M_i, T_i)\) to have a relative minimum at \((M_i^*, T_i^*)\) is that the following Hessian matrix is positive definite.

\[
\begin{bmatrix}
\text{TIC}_{M_i, M_i} & \text{TIC}_{M_i, M_2} & \ldots & \text{TIC}_{M_i, M_n} & \text{TIC}_{M_i, T_1} & \ldots & \text{TIC}_{M_i, T_n} \\
\text{TIC}_{M_2, M_i} & \text{TIC}_{M_2, M_2} & \ldots & \text{TIC}_{M_2, M_n} & \text{TIC}_{M_2, T_1} & \ldots & \text{TIC}_{M_2, T_n} \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
\text{TIC}_{M_n, M_i} & \text{TIC}_{M_n, M_2} & \ldots & \text{TIC}_{M_n, M_n} & \text{TIC}_{M_n, T_1} & \ldots & \text{TIC}_{M_n, T_n} \\
\text{TIC}_{T_1, M_i} & \text{TIC}_{T_1, M_2} & \ldots & \text{TIC}_{T_1, M_n} & \text{TIC}_{T_1, T_1} & \ldots & \text{TIC}_{T_1, T_n} \\
\text{TIC}_{T_2, M_i} & \text{TIC}_{T_2, M_2} & \ldots & \text{TIC}_{T_2, M_n} & \text{TIC}_{T_2, T_1} & \ldots & \text{TIC}_{T_2, T_n} \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
\text{TIC}_{T_n, M_i} & \text{TIC}_{T_n, M_2} & \ldots & \text{TIC}_{T_n, M_n} & \text{TIC}_{T_n, T_1} & \ldots & \text{TIC}_{T_n, T_n}
\end{bmatrix}
\]

If optimal value \((M_i^*, T_i^*)\) satisfies both the constraints, then obtained values of decision variables are optimal solutions such that the integrated total cost is minimum and go to step 5.

**Step 2:** Else solve the optimization problem subject to space constraint and ignore the budget constraint. The Lagrange function, \(L\), can be defined as follows by introducing one Lagrangian multiplier \(\lambda\) to the space constraint.

\[
L(M_i, T_i, \lambda) = TIC(M_i, T_i) + \lambda \left[W - \sum_{j=1}^{m} (W_{M_j} + W_{M_m})Q_j\right]
\]

Lagrangian function \(L(M_i, T_i, \lambda)\) is a function of \(2n + 1\) variables \(M_i, M_2, \ldots, M_n, T_1, T_2, \ldots, T_n, \lambda\). 

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Necessary conditions: A necessary conditions for the minimum of \( L(M_i, T_i, \lambda) \) (where \( i = 1, 2, ..., n \)) give:

\[
\frac{\partial L(M_i, T_i, \lambda)}{\partial M_i} = 0, \quad \frac{\partial L(M_i, T_i, \lambda)}{\partial T_i} = 0, \quad \frac{\partial L(M_i, T_i, \lambda)}{\partial \lambda} = 0
\]

\[
\begin{align*}
&- \frac{A_i}{T_i^2} - \frac{C_i h_1 D_i(s_i, M_i)}{T_i^2(1 - f_i(u_n))} \theta_n \left[ \frac{e^{(1 - f_i(u_n))u_n T_i} - 1}{(1 - f_i(u_n))} - T_i \right] + \frac{C_i h_1 D_i(s_i, M_i)}{T_i} \left[ \frac{e^{(1 - f_i(u_n))u_n T_i} - 1}{(1 - f_i(u_n))} - T_i \right] \\
&- \frac{C_i [Q_i - D_i(s_i, M_i)T_i] + C_i \left[ D_i(s_i, M_i) e^{(1 - f_i(u_n))u_n T_i} - D_i(s_i, M_i) \right]}{T_i} \left[ \frac{e^{(1 - f_i(u_n))u_n (T_i - M_i)} - 1}{(1 - f_i(u_n))} - (T_i - M_i) \right] + \frac{C_i h_1 D_i(s_i, M_i)}{T_i} \left[ \frac{e^{(1 - f_i(u_n))u_n (T_i - M_i)} - 1}{(1 - f_i(u_n))} - (T_i - M_i) \right] \\
&- \frac{C_i q_i d_i D_i(s_i, M_i)}{2} + \frac{h_m D_i(s_i, M_i) e^{(1 - f_i(u_n))u_n T_i}}{2} \left[ (n_i + 1) - \frac{n_i Q_i}{T_i P_{mi}} \right] \\
&+ \frac{h_m Q_i}{2} \left[ \frac{n_i Q_i - n_i D_i(s_i, M_i) e^{(1 - f_i(u_n))u_n T_i}}{T_i P_{mi}} \right] - \frac{1}{T_i} \left[ \frac{S_{mi}}{n_i} + M(P_{mi}) Q_i + q_m d_i Q_i + q_m c M_i Q_i + T_c_{mi} \right] \\
&+ \frac{D_i(s_i, M_i) e^{(1 - f_i(u_n))u_n T_i}}{T_i} \left[ M(P_{mi}) + q_m d_i + q_m c M_i \right] - \lambda \sum_{i=1}^{m} (W_i + W_{mi}) D_i(s_i, M_i) e^{(1 - f_i(u_n))u_n T_i} = 0
\end{align*}
\]

(13)

\[
\frac{h_n}{T_i(1 - f_i(u_n))} \theta_n \left[ \frac{e^{(1 - f_i(u_n))u_n T_i} - 1}{(1 - f_i(u_n))} - T_i \right] \left( C_i e^{M_i} D_i(s_i, M_i) + C_n \left( -\alpha_n(s_i) (1 - r_i)^M \log (1 - r_i) \\
+ \beta_n(s_i) (1 - r_i)^M \log (1 - r_i) \right) \right) + \frac{C_i h_1 D_i(s_i, M_i)}{T_i} [Q_i - D_i(s_i, M_i) T_i] \\
+ \frac{C_i h_1 D_i(s_i, M_i)}{T_i} \left[ Q_i - \left( -\alpha_n(s_i) (1 - r_i)^M \log (1 - r_i) + \beta_n(s_i) (1 - r_i)^M \log (1 - r_i) \right) T_i \right] \\
+ \frac{C_i h_1 D_i(s_i, M_i)}{T_i} \left[ Q_i - \left( -\alpha_n(s_i) (1 - r_i)^M \log (1 - r_i) + \beta_n(s_i) (1 - r_i)^M \log (1 - r_i) \right) T_i \right] \\
- \frac{C_i q_i d_i D_i(s_i, M_i) T_i}{2} - \frac{C_i q_i d_i \left( -\alpha_n(s_i) (1 - r_i)^M \log (1 - r_i) + \beta_n(s_i) (1 - r_i)^M \log (1 - r_i) \right) T_i}{2} \\
+ \frac{q_m d_i Q_i}{T_i} (C_m + M_i C_i e^M) = 0
\]

(14)

\[
W - \sum_{i=1}^{m} (W_i + W_{mi}) Q_i = 0
\]

(15)

On solving Equations (13)–(15) simultaneously we get the value of \( M_i, T_i \), and \( \lambda \).

Sufficient condition: A sufficient condition for \( TIC(M_i, T_i) \) to have a relative minimum at \( (M_i^*, T_i^*) \) is that the following bordered Hessian matrix is positive definite.
If optimal value \((M^*, T^*)\) satisfies budget constraints, then obtained values of decision variables are optimal solutions such that the integrated total cost is minimum and go to step 5.

**Step 3:** Else solve the optimization problem subject to budget constraint and ignore the space constraint. The Lagrange function, \(L\), can be defined as follows by introducing one Lagrangian multiplier \(\lambda\) to the space constraint.

\[
L(M_i, T_j, \mu) = TIC(M_i, T_j) + \mu \left[ B - \sum_{i=1}^{m} (M(P_{mi})Q_i + C_iQ_i) \right]
\]

Lagrangian function \(L(M_i, T_j, \mu)\) is a function of \(2n+1\) variables \(M_1, M_2, ..., M_n, T_1, T_2, ..., T_n, \mu\).

In this step whole of the computational process is same as step-2.

If optimal value \((M^*, T^*)\) satisfies space constraints, then obtained values of decision variables are optimal solutions such that the integrated total cost is minimum and go to step 5.

**Step 4:** If none of the above three step is applicable, solve the optimization problem subject to both space constraint and the budget constraint. The Lagrange function, \(L\), can be defined as follows by introducing one Lagrangian multiplier \(\lambda\) to the space constraint.

\[
L(M_i, T_j, \lambda, \mu) = TIC(M_i, T_j) + \lambda \left[ W - \sum_{i=1}^{m} (W_{pi} + W_{mi})Q_i \right] + \mu \left[ B - \sum_{i=1}^{m} (M(P_{mi})Q_i + C_iQ_i) \right]
\]

Lagrangian function \(L(M_i, T_j, \lambda, \mu)\) is a function of \(2n+2\) variables \(M_1, M_2, ..., M_n, T_1, T_2, ..., T_n, \lambda, \mu\).

**Necessary conditions:** The necessary conditions for the minimum of \(L(M_i, T_j, \lambda, \mu)\) (where \(i = 1, 2, ..., n\)) give:
\[
\frac{\partial L(M, \lambda)}{\partial M_i} = 0, \quad \frac{\partial L(M, \lambda)}{\partial \lambda_i} = 0, \quad \frac{\partial L(M, \lambda)}{\partial \lambda_i} = 0, \quad \frac{\partial L(M, \lambda)}{\partial \lambda_i} = 0
\]  
(16)

\[
- \frac{A_{n_i}}{T_i} - C_n h_n D(s_n, M_i) \left[ \frac{e^{(1-f(u_n))\theta_n T_i} - 1}{(1 - f(u_n))\theta_n} - T_i \right] + C_n h_n D(s_n, M_i) \left[ \frac{e^{(1-f(u_n))\theta_n T_i} - 1}{(1 - f(u_n))\theta_n} - T_i \right] - C_n h_n D(s_n, M_i) \left[ \frac{e^{(1-f(u_n))\theta_n T_i} - 1}{(1 - f(u_n))\theta_n} - T_i \right]
\]

\[
\times \left[ \frac{e^{(1-f(u_n))\theta_n (T_i - M_i)}}{(1 - f(u_n))\theta_n} - (T_i - M_i) \right] + C_n I_0 D(s_n, M_i) \left[ \frac{e^{(1-f(u_n))\theta_n (T_i - M_i)}}{(1 - f(u_n))\theta_n} - (T_i - M_i) \right]
\]

\[
- C_n q_{n_i}^d D(s_n, M_i)
\]

\[
\times \frac{h_m D(s_m, M_m) e^{(1-f(u_m))\theta_m T_m}}{2} \left[ (n_i + 1) - \frac{n_i Q_i}{T_i P_m} \right] + \frac{h_m Q_i}{2} \left[ \frac{n_i Q_i}{T_i^2 P_m} - \frac{n_i D(s_m, M_m) e^{(1-f(u_m))\theta_m T_m}}{T_i P_m} \right]
\]

\[
- \frac{1}{T_i} \left[ \frac{S_{n_i}}{\eta_i} + M(P_m) Q_i + q_{n_i}^d Q_i + q_{n_i}^d C_m Q_i \right]
\]

\[
+ \frac{TC_m}{T_i} + \frac{D(s_m, M_m) e^{(1-f(u_m))\theta_m T_m}}{T_i} \left[ M(P_m) + q_{n_i}^d + q_{n_i}^d C_m M \right]
\]

\[
- \left[ \lambda \sum_{i=1}^{m} (W_i + W_{m_i}) + \mu \sum_{i=1}^{m} (M(P_m) + C_n) \right] D(s_m, M_m) e^{(1-f(u_m))\theta_m T_m} = 0
\]

\[
\frac{h_m S_{n_i}}{T_i(1 - f(u_n))\theta_n} \left[ \frac{e^{(1-f(u_n))\theta_n T_i} - 1}{(1 - f(u_n))\theta_n} - T_i \right]
\]

\[
\times \left[ C_n e^{M_i} D(s_n, M_i) + C_n (-\alpha_n(s_n)(1 - r_j)^M \log(1 - r_j) + \beta_n(s_n)(1 - r_j)^M \log(1 - r_j)) \right]
\]

\[
+ C_n I_0 D(s_n, M_i) \left[ \frac{e^{(1-f(u_n))\theta_n (T_i - M_i)}}{(1 - f(u_n))\theta_n} - (T_i - M_i) \right] + C_n I_0 D(s_n, M_i) \left[ \frac{e^{(1-f(u_n))\theta_n (T_i - M_i)}}{(1 - f(u_n))\theta_n} - (T_i - M_i) \right]
\]

\[
\times \frac{h_m D(s_m, M_m) e^{(1-f(u_m))\theta_m T_m}}{2} + C_n q_{n_i}^d (\alpha_n(s_n)(1 - r_j)^M \log(1 - r_j) + \beta_n(s_n)(1 - r_j)^M \log(1 - r_j)) \right]
\]

\[
+ C_n q_{n_i}^d \left[ C_n e^{M_i} D(s_n, M_i) - \mu \sum_{i=1}^{m} (C_n e^{M_i} Q_i) = 0 \right]
\]  
(17)

\[
\left[ W - \sum_{i=1}^{m} (W_i + W_{m_i}) Q_i \right] = 0
\]  
(18)

\[
\left[ B - \sum_{i=1}^{m} (M(P_m) Q_i + C_n Q_i) \right] = 0
\]  
(19)

On solving Equations (16)–(19) simultaneously we get the value of \( M, T_i \) and \( \lambda, \mu \).
**Sufficient condition:** A sufficient condition for $TIC (M_i, T_i)$ to have a relative minimum at $(M^*_i, T^*_i)$ is that the following bordered Hessian matrix is positive definite.

\[
\begin{bmatrix}
TIC_{M_1 M_1} & TIC_{M_1 M_2} & \ldots & TIC_{M_1 M_n} & TIC_{M_1 T_1} & \ldots & TIC_{M_1 T_n} & g_{M_1} & f_{M_1} \\
TIC_{M_2 M_1} & TIC_{M_2 M_2} & \ldots & TIC_{M_2 M_n} & TIC_{M_2 T_1} & \ldots & TIC_{M_2 T_n} & g_{M_2} & f_{M_2} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
TIC_{M_n M_1} & TIC_{M_n M_2} & \ldots & TIC_{M_n M_n} & TIC_{M_n T_1} & \ldots & TIC_{M_n T_n} & g_{M_n} & f_{M_n} \\
TIC_{T_1 M_1} & TIC_{T_1 M_2} & \ldots & TIC_{T_1 M_n} & TIC_{T_1 T_1} & \ldots & TIC_{T_1 T_n} & g_{T_1} & f_{T_1} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
TIC_{T_n M_1} & TIC_{T_n M_2} & \ldots & TIC_{T_n M_n} & TIC_{T_n T_1} & \ldots & TIC_{T_n T_n} & g_{T_n} & f_{T_n} \\
g_{M_1} & g_{M_2} & \ldots & g_{M_n} & g_{T_1} & \ldots & g_{T_n} & 0 & 0 \\
f_{M_1} & f_{M_2} & \ldots & f_{M_n} & f_{T_1} & \ldots & f_{T_n} & 0 & 0 \\
\end{bmatrix}
\]

Values of $(M^*_i, T^*_i)$ are such that the objective function integrated system is optimum and satisfies both the constraints and go to step 5.

**Step 5:** Stop.

6. Numerical analysis

The purposes of numerical analysis are as follows:

1. Obtain the optimal solution to illustrate the developed model.
2. Through sensitive analysis, we try to highlights how the different parameters effect the optimal solution.

6.1. Numerical example

In order to illustrate the developed model, let us consider the following study.

**XYZ** is a telecommunication firm in India which was established in 2010. It is a joint venture of two companies named as **X** and **Y** to manufacture smartphones. **XYZ** Mobiles India pvt Ltd. has a strategic tie-up between **X** and **Y**. The role of company **X** (which is known as manufacturer) is to manufacture three different smartphones (S1, S2, S3) of different range whereas company **Y** (which is known as retailer) deals the marketing and sales department.

Manufacturing rate of S1, S1, S3 are respectively 1,500, 1,300, 1,250 units per year. Different component of unit manufacturing cost of phones are described in Tables 1 and 2.

Different component of transportation cost from manufacturer to retailer are described in Table 3.

From survey it is observed that different cost component regarding retailer are described in Table 4.

Deterioration occurs at the end of retailer. So retailer uses preservation technology to reduce the rate of deterioration as much as possible. Different cost component and other parameters are described in Table 5.

Here, $f_i(u_{ri}) = 1 - e^{-\gamma u_{ri}}$ where $\gamma = 0.5$
To strengthen the integrated system, manufacturer provides the trade credit period to the retailer. Regarding this values of the parameters are described in Table 6.

Space constraints and budget constraints are imposed to obtain solution of optimal solution. Total space available to store items is 6,000 sq. ft. and $98,000 is available to invest in business. Regarding this values of the parameters are described in Table 7.

Here it is assumed that demand of customer at the end retailer depends on selling price and trade credit offered by manufacturer to the retailer (see Table 8).

Now, problem is to determine optimal value of trade credit period offered by manufacturer and replenishment period, so the total integrated inventory cost of the supply chain is minimum.

Using the solution procedure discussed in Section 3.5, we can get the optimal values of decision variables and objective function (see Table 9).

### Table 1. Different parameters of unit manufacturing cost

| For S1 | For S2 | For S3 |
|-------|-------|-------|
| $M_{m1}$ | $G_{m1}$ | $B_{m1}$ | $y_1$ | $M_{m2}$ | $G_{m2}$ | $B_{m2}$ | $y_2$ | $M_{m3}$ | $G_{m3}$ | $B_{m3}$ | $y_3$ |
| 15 | 60 | 5 | 0.0006 | 18 | 70 | 3 | 0.0005 | 20 | 55 | 4 | 0.0007 |

### Table 2. Holding cost and setup cost for manufacturer

| For S1 | For S2 | For S3 |
|-------|-------|-------|
| Holding cost ($) | Setup cost ($) | Holding cost ($) | Setup cost ($) | Holding cost ($) | Setup cost ($) |
| $h_{m1}$ | $s_{m1}$ | $h_{m2}$ | $s_{m2}$ | $h_{m3}$ | $s_{m3}$ |
| 6 | 3,000 | 5 | 3,500 | 7 | 3,300 |

### Table 3. Different parameters of transportation cost

| For S1 | For S2 | For S3 |
|-------|-------|-------|
| $TC_{0}^{s1}$ | $TC_{1}^{s1}$ | $TC_{0}^{s2}$ | $TC_{1}^{s2}$ | $TC_{0}^{s3}$ | $TC_{1}^{s3}$ |
| 150 | 0.04 | 160 | 0.02 | 155 | 0.03 |

### Table 4. Retailer’s ordering cost, procurement cost, holding cost, and selling price

| For S1 | For S2 | For S3 |
|-------|-------|-------|
| $A_{r1}$ | $C_{r1}^{0}$ | $C_{r1}^{1}$ | $h_{r1}$ | $s_{r1}$ | $A_{r2}$ | $C_{r2}^{0}$ | $C_{r2}^{1}$ | $h_{r2}$ | $s_{r2}$ | $A_{r3}$ | $C_{r3}^{0}$ | $C_{r3}^{1}$ | $h_{r3}$ | $s_{r3}$ |
| 130 | 25 | 0.8 | 0.6 | 70 | 140 | 35 | 0.7 | 0.5 | 60 | 160 | 40 | 0.9 | 0.5 | 55 |

### Table 5. Parameters related to preservation technology cost and rate of deterioration

| For S1 | For S2 | For S3 |
|-------|-------|-------|
| $\theta_{r1}$ | $u_{r1}$ | $\varphi_{r1}$ | $g_{r1}$ | $\theta_{r2}$ | $u_{r2}$ | $\varphi_{r2}$ | $g_{r2}$ | $\theta_{r3}$ | $u_{r3}$ | $\varphi_{r3}$ | $g_{r3}$ |
| 0.05 | 10 | 0.02 | 8 | 0.10 | 13 | 0.03 | 7 | 0.15 | 12 | 0.02 | 9 |
6.2. Sensitivity analysis

In this section, the sensitivity analysis on the equilibrium strategies with respect to different key parameters is carried out. This analysis is carried out by changing one parameter and remaining the others fixed.

6.2.1. Sensitivity w.r.t holding cost

From Figure 2 it is observed that as the holding cost of the retailer and manufacturer increases the total inventory cost of retailer, manufacturer and integrated system also increases. But change is cost higher in case of retailer whereas it is least in case of manufacturer. As the holding cost increases from −50 to +50%, the ordering quantity decreases as a result total inventory cost of integrated system increases.

6.2.2. Sensitivity w.r.t selling price

Figure 3 shows that as the selling price increases total inventory cost of retailer, manufacturer and integrated system decreases. This is due to as selling price increases demand of items decreases therefore ordering quantity decreases and hence total inventory cost decreases.

6.2.3. Sensitivity w.r.t trade credit period

Figure 4 shows the effect of trade credit period on the optimal solution. It is observed that as the trade credit period increases retailer’s, manufacturer’s and integrated inventory cost is also increases. As trade credit period increases, demand increases, ordering quantity increases therefore holding cost increases and hence total inventory cost of the integrated system increases.
6.2.4. Sensitivity w.r.t procurement cost

Figure 5 reflects the effect of procurement cost on the inventory cost of retailer, manufacturer and integrated system. As procurement cost increases the holding cost increases and interest charged by the manufacturer increases and hence total inventory cost of retailer, manufacturer and integrated system increases. Retailer's inventory cost is highly affected by the procurement cost but its effect is least in case of manufacturer.
7. Conclusion
In this paper, a more practical multi-item integrated inventory system involving one manufacturer and one retailer is introduced. Two models are developed here one with deterioration and the other without deterioration. Multivariable demand is considered here which increases due to increase in trade credit period and decreases due to increase in selling price. Whole of the analysis is carried out over finite planning horizon. Addition cost to use the preservation technology in order to maintain the worth of items as long as possible is considered here. Long term bonding between manufacturer and retailer is developed by offering trade credit period to the retailer by the manufacturer. Cost minimization methodology is used here to find the optimal solution of the proposed model. Lagrangian Multiplier Method (LMM) is used to obtain the optimal value of objective function subject to space constraints and budget constraints. Validity of proposed model is illustrated with the help of numerical example. Managerial outcomes of numerical results are obvious and provide a suitable framework to reduce the integrated cost of the supply chain. Sensitive analysis is carried out with respect to holding cost, procurement cost, selling price and trade credit. Inventory cost of retailer is highly affected by procurement cost and holding cost. The proposed model incorporates practical features that are likely to relate with some kinds of inventory. Some of the advantages of the proposed model are as follows:

1. There is hardly any supply chain which focused on only single item. So considering multi-item problem is more practical approach.
2. Consideration of budget constraint and space constraint make the model realistic as ignorance of these restriction results in infeasible solution.
3. Making provision of some budget for preservation technology increase the goodwill of the firm in the market.
4. Providing trade credit strengthens the supply chain coordination.

This analysis will help the inventory practitioners of seasonal products such as fashionable goods, warm cloths, domestic goods and medicines. There are several ideas to extend the present model to make it more practical. In this study we have considered constant rate of deterioration. But in future, it may be taken as time dependent. Shortages may also be considered to make it more useful. This analysis can be carried out under effect of inflation in future.
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