Neutron stars, $\beta$-stable ring-diagram equation of state and Brown-Rho scaling

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Neutron star properties, such as its mass, radius, and moment of inertia, are calculated by solving the Tolman-Oppenheimer-Volkov (TOV) equations using the ring-diagram equation of state (EOS) obtained from realistic low-momentum NN interactions $V_{\text{low}-k}$. Several NN potentials (CDBon, Nijmegen, Argonne V18 and BonnA) have been employed to calculate the ring-diagram EOS where the particle-particle hole-hole ring diagrams are summed to all orders. The proton fractions for different radial regions of a $\beta$-stable neutron star are determined from the chemical potential conditions $\mu_n - \mu_p = \mu_e = \mu_\pi$. The neutron star masses, radii and moments of inertia given by the above potentials all tend to be too small compared with the accepted values. Our results are largely improved with the inclusion of a Skyrme-type three-body force based on Brown-Rho scalings where the in-medium meson masses, particularly those of $\omega$, $\rho$ and $\pi$, are slightly decreased compared with their in-vacuum values. Representative results using such medium corrected interactions are maximum neutron-star mass $M \sim 1.8M_\odot$ with radius $R \sim 9\text{ km}$ and moment of inertia $I \sim 60M_\odot k\text{m}^2$, values given by the four NN potentials being nearly the same. The effects of nuclei-crust EOSs on properties of neutron stars are.

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I. INTRODUCTION

Neutron stars are very interesting physical systems and their properties, such as masses and radii, can be derived from the equation of state (EOS) of the nuclear medium contained in them. In carrying out such derivation, there is, however, a well-known difficulty, namely the EOS is not fully known. Determination of the EOS for neutron stars is an important yet challenging undertaking. As reviewed in [1, 2, 3, 4, 5], this topic has been extensively studied and much progress has been made. Generally speaking, there are two complementary approaches to determine the EOS. One is to deduce it from heavy-ion collision experiments, and crucial information about the EOS has already been obtained [1, 2, 3, 4, 5]. Another approach is to calculate the EOS microscopically from a many-body theory. (See, e.g. [2, 9, 10] and references quoted therein.) As is well-known, there are a number of difficulties in this approach. Before discussing them, let us first briefly outline the derivation of neutron-star properties from its EOS. One starts from the Tolman-Oppenheimer-Volkov (TOV) equations

$$\frac{dp(r)}{dr} = -\frac{GM(r)\epsilon(r)}{c^2 r^2} \left[ 1 + \frac{p(r)}{\epsilon(r)} \right] \left[ 1 + \frac{4\pi r^3 \rho(r)}{M(r) r^2} \right],$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \epsilon(r)$$

(1)

where $p(r)$ is the pressure at radius $r$ and $M(r)$ is the gravitational mass inside $r$. $G$ is the gravitational constant and $\epsilon(r)$ is the energy density inclusive of the rest mass density. The solutions of these equations are obtained by integrating them out from the neutron-star center till its edge where $p$ is zero. (Excellent pedagogical reviews on neutron stars and TOV equations can be found in e.g. [3, 11].) In solving the above equations, an indispensable ingredient is clearly the nuclear matter EOS for the energy density $\epsilon(n)$, $n$ being the medium density. As the density at the neutron star center is typically very high (several times higher than normal nuclear saturation density of $n_0 \approx 0.16 fm^{-3}$), we need to have the above EOS over a wide range of densities, from very low to very high.

In the present work we shall calculate the nuclear EOS directly from a fundamental nucleon-nucleon (NN) interaction $V_{NN}$ and then use it to calculate neutron star properties by way of the TOV equations. There have been neutron-star calculations using a number of EOSs, most of which empirically determined, and the mass-radius trajectories given by them are widely different from each other (see, e.g. Fig. 2 of [1]). To determine the EOS with less uncertainty would certainly be desirable. There are a number of different NN potential models such as the CDBonn [12], Nijmegen [12], Argonne V18 [14] and BonnA [15] potentials. These potentials all possess strong short-range repulsions and to use them in many-body calculations one needs first take care of their short-range correlations by way of some renormalization methods. We shall use in the present work the recently developed renormalization group method which converts $V_{NN}$ into an effective low-momentum NN inter-
action $V_{\text{low}-k}$ \cite{16,17,18,19,20,21}. An advantage of this interaction is its near uniqueness, in the sense that the $V_{\text{low}-k}$s derived from different realistic NN interactions are nearly the same. Also $V_{\text{low}-k}$ is a smooth potential suitable for being directly used in many-body calculations. This $V_{\text{low}-k}$ will then be used to calculate the nuclear EOS using a recently developed low-momentum ring-diagram approach \cite{22}, where the particle-particle hole-hole ($\text{pphh}$) ring diagrams of the EOS are summed to all orders. The above procedures will be discussed in more detail later on.

We shall also study the effects of Brown-Rho (BR) scaling \cite{20,21,22,23,24} on neutron star properties. As discussed in \cite{22}, low-momentum ring diagram calculations using two-body NN interactions alone are not able to reproduce the empirical properties for symmetric nuclear matter; the calculated energy per particle ($E_0/A$) and saturation density ($n_0$) are both too high compared with the empirical values of $E_0/A \approx 16$ MeV and $n_0 \approx 0.16 \text{ fm}^{-3}$. A main idea of the BR scaling is that the masses of in-medium mesons are generally suppressed, because of their interactions with the background medium, compared with mesons in free space. As a result, the NN interaction in the nuclear medium can be significantly different from that in free space, particularly at high density. Effects from such medium modifications have been found to be very helpful in reproducing the empirical properties of symmetric nuclear matter \cite{22}. Dirac-Brueckner-Hartree-Fock (DBHF) nuclear matter calculations have been conducted with and without BR scaling \cite{23,24,25}. In addition, BR scaling has played an essential role in explaining the extremely long life time of $^{14}\text{C}$ $\beta$-decay \cite{33}. As mentioned earlier, the central density of neutron stars is typically rather high, $\sim 8n_0$ or higher, $n_0$ being the saturation density of normal nuclear matter. At such high density, the effect of BR scaling should be especially significant. Neutron stars may provide an important test for BR scaling.

In the following, we shall first describe in section II the derivation of the low-momentum NN interaction $V_{\text{low}-k}$, on which our ring-diagram EOS will be based. Our method for the all order summation of the ring diagrams will also be addressed. Previously such summation has been carried out for neutron matter \cite{31} and for symmetric nuclear matter \cite{22}. In the present work we consider $\beta$-stable nuclear matter composed of neutrons, protons, electrons, and muons. Thus we need to calculate ring diagrams for asymmetric nuclear matter whose neutron and proton fractions are different. An improved treatment for the angle averaged proton-neutron Pauli exclusion operator will be discussed. In section III we shall outline the BR scaling for in-medium NN interactions. We shall discuss that the effect of BR scaling can be satisfactorily simulated by an empirical three-body force of the Skyrme type. In section IV, we shall present and discuss our results of neutron star calculations based on ring-diagram pure-neutron EOSs, ring-diagram $\beta$-stable EOSs consisted of neutrons, protons, electrons and muons, and the well-known EOS of Baym, Pethick and Sutherland (BPS) \cite{32} for the nuclei crust of neutron stars.

II. RING-DIAGRAM EOS FOR ASYMMETRIC NUCLEAR MATTER

In our calculation of neutron star properties, we shall employ a nuclear EOS derived microscopically from realistic NN potentials $V_{NN}$. Such microscopic calculations would provide a test if it is possible to derive neutron star properties starting from an underlying NN interaction. In this section, we shall describe the methods for this derivation. A first step in this regard is to derive an effective low-momentum interaction $V_{\text{low}-k}$ by way of a renormalization procedure, the details of which have been described in \cite{16,17,18,19,20,21}. Here we shall just briefly outline its main steps. It is generally believed that the low-energy properties of physical systems can be satisfactorily described by an effective theory confined within a low-energy (or low-momentum) model space \cite{20}. In addition, the high-momentum (short range) parts of various $V_{NN}$ models are model dependent and rather uncertain \cite{20}.

Motivated by the above two considerations, the following low-momentum renormalization (or model-space) approach has been introduced. Namely one employs a low-momentum model space where all particles have momentum less than a cut-off scale $\Lambda$. The corresponding renormalized effective NN interaction is $V_{\text{low}-k}$, which is obtained by integrating out the $k > \Lambda$ momentum components of $V_{NN}$. This “integrating-out” procedure is carried out by way of a $T$-matrix equivalence approach. We start from the full-space $T$-matrix equation

$$T(k', k, k^2) = V_{NN}(k', k) + \mathcal{P} \int_0^\infty dq \frac{V_{NN}(k', q)T(q, k, k^2)}{k^2 - q^2},$$

where $\mathcal{P}$ denotes the principal-value integration. Notice that in the above equation the intermediate state momentum $q$ is integrated from 0 to $\infty$. We then define an effective low-momentum $T$-matrix by

$$T_{\text{low}-k}(p', p, p^2) = V_{\text{low}-k}(p', p) + \mathcal{P} \int_0^\Lambda dq \frac{V_{\text{low}-k}(q, p, p^2)T_{\text{low}-k}(q, p, p^2)}{p^2 - q^2},$$

where the intermediate state momentum is integrated from 0 to $\Lambda$, the momentum space cutoff. The low momentum interaction $V_{\text{low}-k}$ is then obtained from the above equations by requiring the $T$-matrix equivalence condition to hold, namely

$$T(p', p, p^2) = T_{\text{low}-k}(p', p, p^2); \ (p', p) \leq \Lambda.$$
phase shifts of $V_{NN}$ are preserved by $V_{low-k}$ and so is the deuteron binding energy. As we shall discuss later, for neutron star calculations we need to employ a cut-off scale of $\Lambda \sim 3 \, fm^{-1}$.

For many years, the Brueckner-Hartree-Fock (BHF) [13, 34, 35, 36, 53] and the DBHF [38] methods were the primary framework for nuclear matter calculations. (DBHF is a relativistic generalization of BHF). In both BHF and DBHF the G-matrix interaction is employed. This G-matrix interaction is energy dependent. This energy dependence adds complications to calculations. In contrast, the above $V_{low-k}$ is energy independent which facilitates the calculation of nuclear EOS.

We shall use $V_{low-k}$ to calculate the EOS of asymmetric nuclear matter of total density $n$ and asymmetric parameter $\alpha$

$$n = n_n + n_p; \quad \alpha = \frac{n_n - n_p}{n_n + n_p}$$

where $n_n$ and $n_p$ denote respectively the neutron and proton density and they are related to the respective Fermi momentum by $k_{F_n}^3/(3\pi^2)$ and $k_{F_p}^3/(3\pi^2)$. The proton fraction is $\chi = (1-\alpha)/2$.

![Diagram](image)

**FIG. 1**: Diagrams included in the all-order $pphh$ ring-diagram summation for the ground state energy shift of nuclear matter. Each dashed line represents a $V_{low-k}$ vertex.

In our ring-diagram EOS calculation, the ground-state energy shift $\Delta E_0$ is given by the all-order sum of the $pphh$ ring diagrams as illustrated in Fig. 1, where (a), (b) and (c) are respectively 1st-, 4th- and 8th-order such diagrams. ($\Delta E_0$ is defined as $(E_0 - E_0^{free})$ where $E_0$ is the true ground state energy and $E_0^{free}$ that for the non-interacting system.) Following [22], we have

$$\Delta E_0(n, \alpha) = \int_0^1 d\lambda \sum_{m} \sum_{ijkl<\Lambda} Y_m(ij, \lambda) \times Y_m^*(kl, \lambda) \langle ij|V_{low-k}|kl \rangle,$$

where the transition amplitudes are given by the RPA equation

$$\sum_{kl} [(\epsilon_i + \epsilon_j) \delta_{ij,kl} + \lambda(\bar{f}_i \bar{f}_j - f_i f_j)(ij|V_{low-k}|kl)]$$

$$\times Y_n(kl, \lambda) = \omega_n Y_n(ij, \lambda); \quad (i,j,k,l) \leq \Lambda. \quad (7)$$

In the above, the single particle (s.p.) indices $(i,j,...k,l)$ denote both protons and neutrons. The s.p. energies $\epsilon$ are the Hartree-Fock energies given by

$$\epsilon_k = \hbar^2 k^2/2m + \sum_{h<k} \langle kh|V_{low-k}|kh \rangle,$$

where $k_F(h)$ is $k_{Fn}$ if $h$ is neutron and $k_{Fp}$ if it is proton. Clearly $\epsilon_k$ depends on the nuclear matter density $n$ and the asymmetric parameter $\alpha$. The occupation factors $f_i$ and $f_j$ of Eq.(7) are given by $f_a = 1$ for $k \leq k_F(a)$ and $f_a = 0$ for $k > k_F(a)$; also $f_a = (1-f_0)$. Again $k_F(a) = k_{Fn}$ if $a$ is a neutron and $k_{Fp}$ if it is a proton.

The factor $(\bar{f}_i \bar{f}_j - f_i f_j)$ is clearly also dependent on $n$ and $\alpha$. Note that the normalization condition for $Y_m$ in Eq.(6) is $\langle \bar{Y}^*_m,\bar{Y}^*_m \rangle = -1$ and $Q(k_i, k_j) = (\bar{f}_i \bar{f}_j - f_i f_j)$ [39]. In addition, $\sum_m$ in Eq.(6) means we sum over only those solutions of the RPA equation (7) which are dominated by hole-hole components as indicated by the normalization condition. Note that there is a strength parameter $\lambda$ in the above, and it is integrated from 0 to 1.

The amplitudes $Y$ in Eq.(6) actually represent the overlap matrix elements

$$Y_m^*(kl, \lambda) = \langle \Psi_m(\lambda, \lambda - A - 2)f_0|\Psi_0(\lambda, A)\rangle$$

where $\Psi_0$ denotes the true ground state of nuclear matter which has $A$ particles while $\Psi_m$ denotes the $m$th true eigenstate of the $(A-2)$ system. If there is no ground-state correlation (i.e. $\Psi_0$ is a closed Fermi sea), we have $Y_m^*(kl, \lambda) = f_0 f_1$ and Eq.(6) reduces to the HF result. Clearly our EOS includes effect of ground-state correlation generated by the all-order sum of $pphh$ ring diagrams.

A computational aspect may be mentioned. We shall solve ring-diagram equations on the relative and center-of-mass (RCM) coordinates $\bar{k}$ and $\bar{K}$ [40]. $(\bar{k} = (\bar{k}_i - \bar{k}_j)/2$ and $\bar{K} = \bar{k}_i + \bar{k}_j$). In so doing, the treatment of the Pauli operator $Q(\bar{k}_i, \bar{k}_j) = (\bar{f}_i \bar{f}_j - f_i f_j)$ of Eq.(7) plays an important role. This operator is defined in the laboratory frame, with its value being either 1 or -1 (for $pphh$ ring diagrams). In our calculation, however, we need $Q$ in the RCM coordinates. Angle-average approximations are commonly used in nuclear matter calculations, and with them we can obtain angle-averaged $Q(k, K)$. For symmetric nuclear matter, detailed expressions for $Q$ have been given, see Eqs.(4.9) and (4.9a) of [40]. The results for this case, where the Fermi surfaces for neutron and proton are equal, are already fairly complicated. Derivation of the angle-averaged Pauli operator for asymmetric nuclear matter has been worked out in [41]. Their derivation and results are both considerably more complicated than the symmetric case. It would be desirable if they can be simplified. We have found that this can be attained by way of a scale transformation. Namely we introduce new s.p. neutron and proton momentum coordinates $\bar{k}_n'$ and $\bar{k}_p'$ defined by

$$\bar{k}_n' = \bar{k}_n \sqrt{k_{Fp}/k_{Fn}}; \quad \bar{k}_p' = \bar{k}_p \sqrt{k_{Fn}/k_{Fp}}. \quad (10)$$
On this new frame, the Fermi surfaces for neutron and proton become equivalent, both being \((k_{Fn}, k_{FP})^{1/2}\); asymmetric nuclear matter can then be treated effectively as a symmetric one as far as the Pauli operator is concerned. We have found that this transformation largely simplifies the derivation and calculation of the angle-averaged asymmetric Pauli operator \(Q\) and will be employed in the present work.

III. IN-MEDIUM NN INTERACTIONS BASED ON BROWN-RHO SCALING

A main purpose of the present work is to study whether neutron star properties can be satisfactorily described by EOS microscopically derived from NN interactions. Before proceeding, it is important to also check if such EOS can satisfactorily describe the properties of normal nuclear matter. The EOS and NN interaction one uses for neutron star should also be applicable to normal nuclear matter. As discussed in \[22\], many-body calculations for normal nuclear matter using two-body NN interactions alone are generally not capable in reproducing empirical nuclear matter saturation properties. To remedy this shortcoming one needs to consider three-body forces and/or NN interactions with in-medium modifications \[22, 23, 33\].

A central result of the BR scaling is that the masses of mesons in nuclear medium are suppressed (dropped) compared to those in free space \[24, 27, 22, 29\]. Nucleon-nucleon interactions are mediated by meson exchange, and clearly in-medium modifications of meson masses can significantly alter the NN interaction. These modifications could arise from the partial restoration of chiral symmetry at finite density/temperature or from traditional many-body effects. Particularly important are the vector mesons, for which there is now evidence from both theory \[42, 43, 44\] and experiment \[45, 46\] that the masses may decrease by approximately \(10 \sim 15\%\) at normal nuclear matter density and zero temperature. For densities below that of nuclear matter, a linear approximation for the in-medium mass decrease has been suggested \[42\], namely

\[
\frac{m_V^*}{m_V} = 1 - C \frac{n}{n_0},
\]

where \(m_V^*\) is the vector meson mass in-medium, \(n\) is the local nuclear matter density and \(n_0\) the nuclear matter saturation density. \(C\) is a constant of value \(\sim 0.10 \sim 0.15\). BR scaling has been found to be very important for nuclear matter saturation properties in the ring-diagram calculation of symmetric nuclear matter of \[22\].

It is of interest that the effect of BR scaling in nuclear matter can be well represented by an empirical Skyrme three-body force. \[22\] The Skyrme force has been a widely used effective interaction in nuclear physics and it has been very successful in describing the properties of both finite nuclei and nuclear matter \[47\]. It has both two-body and three-body terms, namely

\[
V_{Skyrme} = \sum_{i<j} V(i, j) + \sum_{i<j<k} V_{3b}(i, j, k). \quad (12)
\]

Here \(V(i, j)\) is a momentum dependent zero-range interaction. Its three-body term is also a zero-range interaction

\[
V_{3b}(i, j, k) = t_3 \delta(\vec{r}_i - \vec{r}_j) \delta(\vec{r}_j - \vec{r}_k) \quad (13)
\]

which is usually expressed as a density-dependent two-body interaction of the form

\[
V_n(1, 2) = \frac{1}{6} (1 + x_3 P_\sigma) t_3 \delta(\vec{r}_1 - \vec{r}_2) n(\vec{r}_av), \quad (14)
\]

where \(P_\sigma\) is the spin-exchange operator and \(\vec{r}_{av} = \frac{1}{2}(\vec{r}_1 + \vec{r}_2)\). \(t_3\) and \(x_3\) are parameters determined by fitting certain experimental data. The general structure of \(V_{Skyrme}\) is rather similar to the effective interactions based on effective field theories (EFT) \[23\], with \(V(i, j)\) corresponding to \(V_{low-k}\) and \(V(i, j, k)\) to the EFT three-body force. The Skyrme three-body force, however, is much simpler than that in EFT.

IV. RESULTS AND DISCUSSIONS

A. Symmetric nuclear matter and BR scaling

When an EOS is used to calculate neutron-star properties, it is important and perhaps necessary to first test if the EOS can satisfactorily describe the properties of symmetric nuclear matter such as its energy per particle \(E_0/A\) and saturation density \(n_0\). In principle, only those EOSs which have done well in this test are suitable for being used in neutron star calculation. In this subsection, we shall calculate properties of symmetric nuclear matter using the low-momentum ring-diagram EOS which we will use in our neutron star calculations, to test if it can meet the above test. As described in section II, we first calculate the \(V_{low-k}\) interaction for a chosen decimation scale \(\Lambda\). Then we calculate the ground state energy per particle \(E_0/A\) using Eq.(6) \((E_0 = E_0^{free} + \Delta E_0)\) with the ppbh ring diagrams summed to all orders.

In the above calculation, the choice of \(\Lambda\) plays an important role. As discussed in \[22\], the tensor force is important for nuclear saturation and therefore one should use a sufficiently large \(\Lambda\) so that the tensor force is not integrated out during the derivation of \(V_{low-k}\). Since the main momentum components of the tensor force has \(k \sim 2fm^{-1}\), one needs to use \(\Lambda \sim 3fm^{-1}\) or larger. There is another consideration concerning the choice of \(\Lambda\). The density of neutron star interior is very high, several times larger than \(n_0\). To accommodate such high density, it is necessary to use sufficiently large \(\Lambda\), suggesting a choice of \(\Lambda\) larger than \(\sim 3fm^{-1}\). As discussed in \[20\], a nice feature of \(V_{low-k}\) is its near uniqueness: The \(V_{low-k}\)s derived from various different realistic
NN potentials are practically identical to each other for $\Lambda < \sim 2.1 \text{fm}^{-1}$, while for larger $\Lambda$s the resulting $V_{\text{low-k}}$ begin to have noticeable differences but are still similar to each other for $\Lambda$ up to about $3.5 \text{fm}^{-1}$. This and the above considerations have led us to choose $\Lambda$ between $\sim 3$ and $\sim 3.5 \text{fm}^{-1}$ for our present study. The dependence of our results on the choice on $\Lambda$ will be discussed later on.

As discussed in section III, the above situation can be largely improved by way of using a $V_{NN}$ with BR scaling. In $^{23}$ authors have carried DBHF calculations for symmetric matter using a BR-scaled BonnB NN potential, and they obtained results in good agreement with the empirical values, largely improved over those from the unscaled potential. In $^{22}$, ring-diagram EOS calculations for symmetric nuclear matter have been performed using the Nijmegen potential without and with BR scaling, the latter giving highly improved results for nuclear matter saturation. It should be useful if the above effect on nuclear saturation from BR scaling also holds for other NN potentials. To study this, we use in the present work a different potential, the BonnA potential $^{13}$, for investigating the effect of BR scaling on ring-diagram calculations for symmetric nuclear matter. In Fig.3, results of such ring-diagram calculations for symmetric nuclear matter with and without BR scaling are presented. For the scaled calculation, the mesons $(\rho, \omega, \sigma)$ of the BonnA potential are slightly scaled according to Eq.(11) with the choice of $C_\rho = 0.113, C_\omega = 0.128$ and $C_\sigma = 0.102$. These values are chosen so that the calculated $E_0/A$ and $k_F^0$ are in satisfactory agreement with the empirical values. The EOS given by the above BR-scaled potential is shown by the top curve of Fig. 3 (labelled as ‘$BR_1$’), and it has $E_0/A=-15.3 \text{MeV}$ and $k_F^0=1.33 \text{fm}^{-1}$, in good agreement with the empirical values. In addition, it has compression modulus $\kappa=225 \text{MeV}$. The result using $V_{\text{low-k}}$ alone is also shown in Fig.3 (bottom curve). Clearly BR scaling is also important and helpful for the BonnA potential in reproducing empirical nuclear matter saturation properties.

We shall now discuss if the above effect of BR scaling can be simulated by an empirical three-body force of the Skyrme type. It is generally agreed that the use of two-body force alone can not satisfactorily describe nuclear saturation, certain three-body forces are needed to ensure nuclear saturation $^{23}$. There are basic similarities between three-body force and BR scaling. To see this, let us consider a meson exchanged between two interacting nucleons. When this meson interacts with a third spectator nucleon, this process contributes to BR scaling or equivalently it generates the three-body interaction. In $^{22}$ it was already found the ring-diagram results of BR-scaled $V_{\text{low-k}}$ derived from the Nijmegen potential can be well reproduced by the same calculation except the use of the interaction given by the sum of the $V_{\text{low-k}}$ plus the empirical three-body force (TBF) $V_{3b}$ of Eq.(13). (Note that $V_{3b}$ is calculated using Eq.(14) with $n$ being the local nuclear matter density.) Here we repeat this calculation.
using a different potential, namely the BonnA potential. The strength parameter \( t_3 \) is adjusted so that the low-density \((< \sim n_0)\) EOS given by the \( (V_{\text{low-k}}+V_{3b}) \) calculation are in good agreement with that from the BR-scaled \( V_{\text{low-k}} \). (We fix the parameter \( x_3 \) of Eq.(14) as zero, corresponding to treating the \( 1S_0 \) and \( 3S_1 \) channels on the same footing.) Results for such a calculation, with \( t_3 \) chosen as 2000 MeVfm\(^6\) are presented as the middle curve of Fig. 3 (labelled as \( V_{\text{low-k}+TBF} \)). As shown, for \( k_F \leq 1.4 \text{fm}^{-1} \) they agree very well with the results from the BR-scaled \( V_{\text{low-k}} \). The above \( (V_{\text{low-k}}+V_{3b}) \) calculation gives \( E_0/A=14.7 \text{ MeV} \) and \( k_F^0 = 1.40 \text{fm}^{-1} \) in satisfactory agreement with the BR-scaled results given earlier. Its compression modulus is \( \kappa=140 \text{ MeV} \).

It should be noticed, however, for \( k_F \geq 1.4 \text{fm}^{-1} \) the curve for \('BR_1' \) rises much more rapidly (more repulsive) than the \( 'V_{\text{low-k}+TBF}' \) one. The compression modulus given by them are also quite different, 225 versus 140 MeV. These differences may be related to the linear BR-scaling adopted in Eq.(11). This scaling is to be used for density less than \( n_0 \). For density significantly larger than \( n_0 \), such as in the interior of neutron star, this linear scaling is clearly not suitable.

To our knowledge, how to scale the mesons at high densities is still an open question \([26, 27, 28, 29, 42]\). In the present work, we have considered two schemes for extending the BR scaling to higher densities: One is the above Skyrme-type extrapolation; the other is an empirical modification where in the high density region a nonlinear scaling is assumed, namely \( m^*/m = (1 - C(n/n_0)^B) \) with \( B \) chosen empirically. The exponent \( B \) is 1 in the linear BR scaling of Eq.(11). As seen in Fig. 3, the linear BR-scaled EOS agrees well with the \( 'V_{\text{low-k}+TBF}' \) EOS only in the low-density \(< \sim n_0 \) region, but not so for densities beyond. Can a different choice of \( B \) give better agreement for the high-density region? As seen in Fig. 3, to obtain such better agreements we need to use a scaling with weaker density dependence than \( BR_1 \). Thus we have considered \( B < 1 \), and have found that the EOSs with \( B \) near 1/3 have much improved agreements with the Skyrme EOS in the high density region. To illustrate, we have repeated the \('BR_1'\) EOS of Fig. 3 with only one change, namely changing \( B \) from 1 to 0.3. (The scaling parameters \( C \)s are not changed, for convenience of comparison.) The new results, labelled as \('BR_2'\), are also presented in Fig.3. As seen, \('BR_2'\) and \('V_{\text{low-k}+TBF}'\) are nearly identical in a wide range of densities beyond \( \sim n_0 \). This is an interesting result, indicating that below \( \sim n_0 \) the \('V_{\text{low-k}+TBF}'\) EOS corresponds to the linear \( BR_1 \) scaled EOS while beyond \( \sim n_0 \) the nonlinear \( BR_2 \) one. The \( BR_1 \) and \( BR_2 \) EOSs have a small discontinuity (in slope) at \( n_0 \), and the above EOS with TBF is practically a continuous EOS with good fitting to both. As we shall discuss later, the above three-body force is also important and desirable for neutron-star calculations involving much higher densities. Possible microscopic connections between the Skyrme three-body force and BR scalings are being further studied, and we hope to report our results soon in a separate publication.

The ring-diagram nuclear matter EOSs using the \('V_{\text{low-k}+TBF}'\) interaction are in fact rather insensitive to the choice \( \Lambda \). As discussed earlier, a suitable range for \( \Lambda \) is from \( 3 \) to \( 3.5 \text{ fm}^{-1} \). So in carrying out the above calculations, one first chooses a \( \Lambda \) within the above range. Then \( t_3 \) is determined by the requirement that the low-density \(< \sim n_0 \) ring-diagram EOS given by \( BR_1 \)-scaled \( V_{\text{low-k}} \) is reproduced by that from \( (V_{\text{low-k}}+V_{3b}) \). In Fig. 4 we present some sample results for \( \Lambda = 3 \) and \( 3.5 \text{ fm}^{-1} \) with CDBonn and BonnA potentials, all using \( t_3=2000 \text{ MeVfm}^6 \). Note that this \( t_3 \) value is for \( \Lambda = 3.5 \text{ fm}^{-1} \) and BonnA potential; for convenience in comparison it is here used also for the other three cases. It is encouraging to see that within the above \( \Lambda \) range our results are remarkably stable with regard to the choice of both \( \Lambda \) and \( t_3 \). The four curves of Fig. 4 are nearly overlapping, and their \( (E_0/A, k_F^0, \kappa) \) values are all close to \((-15 \text{ MeV}, 1.40 \text{fm}^{-1}, 150 \text{ MeV}) \). We have repeated the above calculations for the Nijmegen and Argonne V18 potentials, and have obtained highly similar results. As we shall discuss in the next subsection, the inclusion of \( V_{3b} \) is also important in giving a satisfactory neutron-matter ring-diagram EOS. Calculations of neutron star properties using the above \((V_{\text{low-k}}+V_{3b})\) interaction will also be presented there. Unless otherwise specified, we shall use from now on \( \Lambda = 3.5 \text{ fm}^{-1} \) for the decimation scale and \( t_3=2000 \text{ MeVfm}^6 \) for the three-body force \( V_{3b} \).

![FIG. 4: Ring-diagram EOS for symmetric nuclear matter with the interaction being the sum of \( V_{\text{low-k}} \) and the three-body force of Eq.(13). Four sets of results are shown for CDBonn and BonnA potentials with \( \Lambda = 3 \) and \( 3.5 \text{ fm}^{-1} \). A common three-body force of \( t_3=2000 \text{ MeVfm}^6 \) is employed.](image)

### B. Neutron star with neutrons only

As a preliminary test of our ring-diagram EOS, in this subsection we shall consider neutron stars as composed of pure neutron matter only. This simplified structure is
convenient for us to describe our methods of calculation. In addition, this also enable us to check how well can the properties of neutron stars be described under the pure-neutron matter assumption. Realistic neutron stars have of course more complicated compositions; they have nuclei crust and their interior composed of neutrons as well as other elementary particles [1, 2]. We shall study the effects of using β-stable and nuclei-crust EOSs in our neutron star calculations in the next subsection. In the present work we consider neutron stars at zero temperature.

Using the methods outlined in section II, we first calculate the ground-state energy per particle $E_0/A$ for neutron matter. Then the energy density $\varepsilon$, inclusive of the rest-mass energy, is obtained as

$$\varepsilon(n) = n\left(\frac{E_0}{A} + m_nc^2\right) \quad (15)$$

where $c$ is the speed of light and $m_n$ the nucleon mass. By differentiating $E_0/A$ with density, we obtain the pressure-density relation

$$p(n) = n^2\frac{d(E_0/A)}{dn}. \quad (16)$$

From the above two results, the EOS $\varepsilon(p)$ is obtained. It is the EOS $\varepsilon(p)$ which is used in the solution of the TOV equations.

To accommodate the high densities in the interior of neutron stars, we have chosen $\Lambda=3.5 \text{ fm}^{-1}$ for our present neutron star calculation. Our ring-diagram EOS for neutron matter is then calculated using the interaction $(V_{\text{low-}k} + V_{3b})$ with the parameter $t_3 = 2000 \text{ MeV fm}^6$. Note this value was determined for symmetric nuclear matter, as discussed in subsection A. Is this $t_3$ also appropriate for the neutron matter EOS? We shall address this question here. In Fig. 5 we present results from the above neutron matter EOS calculations for four interactions (CDBon, Nijmegen, Argonne V18, BonnA). It is seen that the EOSs given by them are quite close to each other, giving a nearly unique neutron-matter EOS. Friedman and Pandharipande (FP) [49] have carried out variational many-body calculations for neutron matter EOS using the two- and three-nucleon interactions $V_{14}$ and TNI respectively, their EOS results also shown in Fig. 5. Brown [50] has carried out extensive studies of neutron matter EOS, and has found that the FP EOS can be reproduced by the EOS given by certain empirical Skyrme effective interactions (with both two- and three-body parts). As seen in Fig 5, our results agree with the FP EOS impressively well. For comparison, we present in Fig. 5 also the CDBonn EOS without the inclusion of $V_{3b}$ (i.e. $t_3 = 0$). It is represented by the dotted-line, and is much lower than the FP EOS, particularly at high densities. For $n < n_0/2$ the effect of $V_{3b}$ is rather small, and in this density range one may calculate the EOS using $V_{\text{low-}k}$ alone. Clearly the inclusion of $V_{3b}$ with $t_3 = 2000 \text{ MeV fm}^6$ is essential for attaining the above good agreement between our EOSs and the FP one. It is of interest that the $t_3$ value determined for symmetric nuclear matter turns out to be also appropriate for neutron matter.

In Fig. 6, our results for the EOS $\varepsilon(p)$ are presented, where the EOSs given by various potentials are remarkably close to each other. The inclusion of $V_{3b}$ is found to be also important here. As also shown in Fig. 6, the EOS given by $V_{\text{low-}k}$ alone (without $V_{3b}$) lies considerably higher than those with $V_{3b}$. It is of interest that for a given pressure, the inclusion of $V_{3b}$ has large effect in reducing the energy density. We have chosen to use $\Lambda = 3.5 \text{ fm}^{-1}$, and this limits the highest pressure $p_A$ which can be provided by our ring-diagram EOS calculation. As shown in the figure, the highest pressure there is about $650 \text{ MeV fm}^3$. But in neutron star calculation we need EOS at higher pressure such as $1000 \text{ MeV fm}^3$ (or $\sim 4 \times 10^{-4}M_0c^2/km^3$). The EOS at such high pressure is indeed uncertain, and some model EOS has to be employed. In the present work we shall adopt a polytrope approach, namely we fit a section of the calculated EOS near the maximum-pressure end by a polytrope $\varepsilon(p) = \alpha p^\gamma$ and use this polytrope to determine the energy density for pressure beyond $p_A$. (In our fitting the section is chosen as (0.8 to 1)pA.) The polytrope EOS has been widely and successfully used in neutron star calculations [11, 12]. In fact we have found that our calculated EOS, especially the section near its high-pressure end, can be very accurately fitted by a polytrope. In Table I we list the polytropes obtained from the above fitting for four NN interactions. It is seen that the four polytropes are close to each other. The exponent $\gamma$ plays an important role in determining the neutron-star maximum mass.

![Fig. 5: Ring-diagram neutron matter EOS obtained from four realistic NN potentials. The interaction ‘V_{low-k} plus TBF’ is used. The solid line with filled small circles presents the results from the variational many-body calculation of Friedman-Pandharipande [41]. The dotted line denotes the EOS using CDBonn-V_{low-k} only (TBF suppressed).](image)

In obtaining the neutron star properties, we numerically solve the TOV equations (1) by successive integrations. In so doing, we need to have the pressure $P_c$ at
the center of the neutron star to begin the integration. As we shall see soon, different \( P_c \)s will give, e.g. different masses for neutron stars. We also need the EOS \( \varepsilon(p) \) for a wide range of pressure. As discussed above, we shall use the ring-diagram EOS for pressure less than \( p_{\Lambda} \) and the fitted polytrope EOS for larger pressure. In Table II, we list some typical results for the neutron star mass \( M \) and its corresponding radius \( R \) and static moment of inertia \( I \). (The calculation of \( I \) will be discussed later.) They were obtained with four different center pressures \( P_c \), and as seen these properties of the neutron star vary significantly with \( P_c \).

We present some of our calculated results for the mass-radius trajectories of neutron stars in Fig. 7. They were obtained using the CD-Bonn \( V_{\text{low}-k} \) (\( \Lambda=3.5 \text{ fm}^{-1} \)) with and without the three-body force \( V_{3b} \) (\( l_3=2000 \text{ MeV fm}^6 \)) discussed earlier. As seen, the inclusion of \( V_{3b} \) significantly increases both the maximum neutron star mass \( M \) and its corresponding radius \( R \); the former increased from \( \sim 1.2 \) to \( \sim 1.8M_\odot \) and the latter from \( \sim 7 \) to \( \sim 9 \)km. The above results are understandable, because \( V_{3b} \) makes the EOS stiffer and consequently enhances both \( M \) and \( R \). Note that our results are within the causality limit. We have repeated the above calculations using the Nijmegen, Argonne and BonnA potentials, with results quite similar to the CD-Bonn ones. In Fig. 8, we present the density profiles corresponding to the maximum-mass neutron stars of Fig. 7. It is clearly seen that the inclusion of the three-body force TBF has important effect in neutron-star’s density distribution, reducing the central density and enhancing the outer one.

We have also performed calculations using the \( BR_3 \)-scaled \( V_{\text{low}-k} \) interaction (BonnA and \( \Lambda=3.5 \text{ fm}^{-1} \)) without \( V_{3b} \). The resulting maximum mass and its radius given are respectively \( \sim 3.2M_\odot \) and \( \sim 12 \)km, both being considerably larger than the values of Fig. 7. This

The upper-left thin line denotes the \( \varepsilon(p) \) from CD-Bonn-\( V_{\text{low}-k} \) only (TBF suppressed).

**TABLE I: Fitted polytrope \( \alpha \gamma \) for high pressure region. See text for other explanations.**

| Potentials  | \( \alpha \)  | \( \gamma \)  |
|-------------|--------------|--------------|
| CD-Bonn     | 69.69 ± 1.01 | 0.4876 ± 0.0022 |
| Nijmegen    | 69.99 ± 1.01 | 0.4885 ± 0.0021 |
| BonnA       | 72.30 ± 1.01 | 0.4779 ± 0.0021 |
| Argonne V18 | 67.71 ± 1.01 | 0.4887 ± 0.0021 |

* unit of \( \alpha \) is \( (\text{MeV/fm}^3)^{-\gamma} \).

**TABLE II: Neutron stars with different center pressures.**

| \( P_c [M_\odot \text{C}^2/\text{km}^3] \) | \( M [M_\odot] \) | \( R [\text{km}] \) | \( I [M_\odot \text{km}^2] \) |
|-----------------|--------------|-------------|--------------|
| \( 8.07 \times 10^{-6} \)  | 0.101        | 11.58       | 3.78         |
| \( 7.18 \times 10^{-6} \)  | 0.347        | 10.12       | 13.51        |
| \( 5.38 \times 10^{-5} \)  | 1.037        | 10.10       | 50.02        |
| \( 2.33 \times 10^{-4} \)  | 1.597        | 10.00       | 70.69        |

FIG. 6: Neutron matter \( \varepsilon(p) \) obtained from four realistic NN potentials. The upper-left thin line denotes the \( \varepsilon(p) \) from CD-Bonn-\( V_{\text{low}-k} \) only (TBF suppressed).

FIG. 7: Mass-radius trajectories of pure neutron stars from ring-diagram EOSs given by the CD-Bonn \( V_{\text{low}-k} \) interaction with and without the three-body force (TBF) \( V_{3b} \). Only stars to the right of maximum mass are stable against gravitational collapse. Causality limit is indicated by the straight line in the upper left corner.

FIG. 8: Density profiles of maximum-mass neutron stars of Fig. 7.
is also reasonable, because, as was shown in Fig. 3 the $BR_1$-scaled EOS is much stiffer than the 'V$_{low-k}$ plus TBF' one. It may be mentioned that if the neutron-matter EOS given by the BR-scaled interaction is plotted in Fig. 5, it would be very much higher, especially in the high density region, than the FP EOS shown there. However, the 'V$_{low-k}$ plus TBF' ones are very close to the FP one as shown earlier. We feel that the above comparison is a further indication that the linear $BR_1$ scaling of Eq.(11) is not suitable for high density. It is suitable only for density up to about $\sim n_0$.

Moment of inertia is an important property of neutron stars. Here we would like to calculate this quantity using our $V_{low-k}$ ring diagram formalism. Recall that we have used the TOV equations (1) to calculate neutron star mass and radius, and in so doing we also obtain the density distribution inside neutron stars. From this distribution, the moment of inertia $I$ of neutron stars is readily calculated. It may be noted that the TOV equations are for spherical and static (non-rotating) neutron stars, and the $I$ so obtained is the static one for spherical neutron stars. The moment of inertia for rotating stars are more complicated to calculate, but for low rotational frequencies (less than $\sim 300$Hz) they are rather close to the static ones. In Fig. 8, we present our results for two calculations, the interactions used being the same as in Fig. 7. It is seen that the the inclusion of our three-body force $V_{3b}$ (TBF) largely enhances the moment of inertia of maximum-mass neutron star.

The measurement of neutron-star moment of inertia is still rather uncertain, and the best determined value so far is that of the Crab pulsar ($97\pm 38\ M_\odot km^2$). For $M \geq 1.0M_\odot$, Lattimer and Schutz have determined an empirical formula relating the moment of inertia $I$ of neutron stars to their mass $M$ and radius $R$, namely

$$I \approx (0.237 \pm 0.008)MR^2 
\times (1 + 4.2 \frac{M \ km}{M_\odot R} + 90(\frac{M \ km}{M_\odot R})^4).$$

(17)

To check if our calculated ($M$, $R$, $I$) are consistent with this empirical relation, we have computed $I$ using our calculated $M$ and $R$ values (with TBF as in the top curve of Fig. 9) as inputs to Eq.(17). Results of this computation are also shown in Fig. 9. As shown, they are in good general agreement with the empirical formula. Especially our moment of inertia at maximum mass agrees remarkably well with the corresponding empirical value. We have also repeated the above computation with other potentials and obtained similar results.

C. Effects from $\beta$-stable and nuclei-crust EOSs

In the preceding subsection, we considered neutron star as composed of neutrons only, and we have obtained rather satisfactory results. Would the quality of them be significantly changed when we use a more realistic com-

position? As a small-step improvement, in this subsection we shall first carry out calculations using the ring-diagram $\beta$-stable EOS composed of neutrons, protons, electrons and muons only. The results of them will be briefly compared with those obtained with neutrons only. Calculations using a combination of the BPS EOS inside the nuclei crust and our $\beta$-stable EOS for the interior will also be carried out. The crust of the neutron star is composed of two parts, the outer and inner crust. The choice of the density regions defining these crusts and how to match the EOSs at the boundaries between different regions will be discussed.

Let us first discuss our $\beta$-stable EOS, where the composition fractions of its constituents are determined by the chemical equilibrium equations

$$\mu_n = \mu_p + \mu_e$$

(18)

$$\mu_e = \mu_\mu$$

(19)

together with the charge and mass conservation conditions

$$n_p = n_e + n_\mu$$

(20)

$$n = n_n + n_p + n_e + n_\mu.$$  

(21)

In the above, $\mu_n, \mu_p, \mu_e$ and $\mu_\mu$ are the chemical potentials for neutron, proton, electron and muon respectively, and their densities are respectively $n_n, n_p, n_e$ and $n_\mu$. The total density is $n$. For a given $n$, the composition fractions of these constituents are determined by the above equations. Note that these equations are solved self-consistently, since the chemical potentials and densities are inter-dependent. We have used iteration methods for this solution. Clearly the composition fractions are not uniform inside the $\beta$-stable neutron star; they depend on the local density. For example, the proton fractions $\chi = n_p/n$ in different density regions of $\beta$-stable neutron star are generally different. In solving the
above equations, we have calculated the chemical potentials $\mu_n$ and $\mu_p$ using the HF approximations as indicated by Eq.(8). Since the interactions involving electrons and muons are much weaker than the strong nucleon ones, we have treated them as free Fermi gases and in this way their chemical potentials are readily obtained.

Results for the proton fractions $\chi$ calculated from the above equations are displayed in Fig. 10; they were obtained using four $V_{\text{low-k}}(\Lambda=3.5 \text{ fm}^{-1})$ interactions, all with the three-body force $V_{3b}(t_3=2000 \text{ MeV fm}^6)$. We note that our calculated proton fractions are all quite small, the maximum proton fraction given by BonnA potential being $\sim 7\%$ and that by both Argonne V18 and Nijmegen potentials being $\sim 2\%$. Also they exhibit a saturation behavior, being near maximum at density between $\sim 0.6$ and $\sim 0.8 \text{ fm}^{-3}$ and diminishing to near zero on both sides. Our results suggest that $\beta$-stable neutron star has small proton admixtures only within an intermediate layer; the neutron star’s core and surface layer are both essentially pure neutron matter. Proton fractions in $\beta$-stable neutron stars are an important topic and have been extensively studied [58, 59]. They are closely related to the density dependence of symmetry energy, which is being determined in several laboratories [58]. There have been a number of calculations for these fractions using different many-body methods and different interactions; their results are, however, widely different from each other (see Fig.1 of [58]), some of them being close to ours. Further studies of the proton fractions would certainly be in need and of interest.

With our calculated proton fractions, we proceed to calculate the properties of $\beta$-stable neutron stars whose energy-density EOS is $\varepsilon(n) = \varepsilon_{np} + \varepsilon_e + \varepsilon_\mu$, where $n$ is the total density of Eq.(21). Electrons and muons are treated as free Fermi gas and their energy densities are readily obtained. $\varepsilon_{np}$ is the neutron-proton energy density to be evaluated using the ring-diagram method described in section II. This energy density is in fact $\varepsilon_{np}(n_{np}, \alpha)$ where $n_{np} = n_n + n_p$, the combined nucleon density, and $\alpha$ is the asymmetric parameter of Eq.(5). Calculations for $\beta$-stable neutron stars are more complicated than the pure neutron matter case of subsection B, for which $\alpha = 1$ independent of the total density $n$. In contrast, here we need to calculate $\varepsilon_{np}$ for many $(n_{np}, \alpha)$ values since they are dependent on $n$ (see Eqs.(18-21)). Then the EOS $\varepsilon_{np}(p)$, which expresses energy density in terms of pressure $p$, is obtained by density differentiations of $\varepsilon_{np}(n_{np}, \alpha)$, similar to what we did in subsection A. Then by solving the TOV equations, the various properties of $\beta$-stable neutron stars are obtained.

![FIG. 10: Proton fraction of $\beta$-stable neutron star from realistic NN potentials. Symbols are BonnA(●), CDBonn(○), Argonne V18 (□) and Nijmegen (×). The interaction ‘$V_{\text{low-k}}$ plus TBF’ is used.](image)

![FIG. 11: Mass-radius trajectories of neutrons stars obtained using only the ring-diagram $\beta$-stable EOS (‘$\beta$-alone’) and a combination of the ring and nuclei-crust EOSs with inner-crust boundary $n_i=0.04$ (‘$\beta$-crust$_1$’) and $0.05 \text{ fm}^{-3}$ (‘$\beta$-crust$_2$’). The ‘CDBonn-$V_{\text{low-k}}$ plus TBF’ potential is used for the ring EOS. See the caption of Fig. 7 for other explanations.](image)
the outer-, inner-crust and core regions. These regions refer to the density regions \( n < n_{\text{out}} \), \( n_{\text{out}} < n < n_t \) and \( n > n_t \) respectively, with \( n_{\text{out}} = 2.57 \times 10^{-4} \, \text{fm}^{-3} \) \( [54, 56] \), \( n_t \) is the transition density separating the inner crust and the homogeneous core, and several models have been employed to determine its value \( [54, 56] \). For the outer-crust region we use the well-known BPS nuclear-EOS \( [52] \). For the core region, our \( \beta \)-stable ring-diagram EOS will be employed. The EOS in the inner-crust region is somewhat uncertain, and so is the transition density separating the inner crust and core. We shall use in our calculations \( n_t = 0.05 \) and \( 0.04 \, \text{fm}^{-3} \) \( [54, 55] \) to illustrate the effect of the nuclei crust. Following \( [54, 57] \), we use in the inner-crust region a polytropic EOS, namely \( p = a + b n^{4/3} \) with the constants \( a \) and \( b \) determined by requiring a continuous matching of the three EOSs at \( n_{\text{out}} \) and \( n_t \).

### TABLE III: Maximum mass and the corresponding radius and moment of inertia of \( \beta \)-stable neutron stars with nuclei-crust boundary \( n_t = 0.04 \, \text{fm}^{-3} \). The three-body force \( V_{3b} \) is included for the results in the first four rows, but is not in the last.

| Potentials | \( M[M_\odot] \) | \( R[\text{km}] \) | \( I[M_\odot\,\text{km}^2]\) |
|------------|----------------|----------------|---------------------|
| CDBonn \( V_{3b}=0 \) | 1.24 | 7.26 | 24.30 |
| CDBonn | 1.80 | 8.94 | 60.51 |
| Nijmegen | 1.76 | 8.92 | 57.84 |
| BonnA | 1.81 | 8.86 | 61.09 |
| Argonne V18 | 1.82 | 9.10 | 62.10 |

In Fig. 11, our results for the mass-radius trajectories using the above three EOSs with \( n_t=0.04 \) and \( 0.05 \, \text{fm}^{-3} \), labelled \( \beta \)-crust\(_1\) and \( \beta \)-crust\(_2\) respectively, are compared with the trajectory given by the \( \beta \)-stable alone (namely \( n_t=0 \)). As seen, the effect from the nuclei-crust EOS on the maximum neutron-star mass and its radius is rather small, merely increasing the maximum mass by \( \sim 0.02M_\odot \) and its radius \( \sim 0.1\text{km} \) as compared with the \( \beta \)-alone ones. However, its effect is important in the low-mass large-radius region, significantly enhancing the neutron-star mass there. That the maximum mass is not significantly changed by the inclusion nuclei-crust EOS is consistent with Fig. 8 which indicates the mass of maximum-mass neutron stars being confined predominantly in the core region. It may be mentioned that our ring-diagram EOS is microscopically calculated from realistic NN interactions, while the crust EOSs are not. So there are disparities between them. It would be useful and of much interest if the crust EOSs can also be derived from realistic NN interactions using similar microscopic methods. Further studies in this direction are needed.

In Fig. 12 we present our mass-radius results using the above three EOSs with \( n_t=0.04 \, \text{fm}^{-3} \). Four NN potentials are employed, and they give similar trajectories, especially in the high- and low-mass regions. A corresponding comparison for the moment of inertia is presented in Fig. 13; again the results from the four potentials are similar. In Table III, our results for the maximum neutron mass and its radius and moment of inertia using the above combined EOSs are presented, and as seen the results for the maximum-mass neutron star given by the four potentials are indeed close to each other. It is also seen that the effect of the three-body force is quite important for \( M, R \) and \( I \), as illustrated by the CDBonn case.

### V. SUMMARY AND CONCLUSION

We have performed neutron-star calculations based on three types of EOSs: the pure-neutron ring-diagram, the \( \beta \)-stable \( (n, p, e, \mu) \) ring-diagram and the BPS nuclei-crust EOS. The ring-diagram EOSs, where the pp\(hh\) ring diagrams are summed to all orders, are microscopically

![Fig. 12: Mass-radius trajectories of neutron stars calculated with a combination of \( \beta \)-stable ring-diagram EOS for the core and the nuclei-crust EOSs for the crusts. Ring-diagram EOSs given by four NN potentials, all with the CDBonn three-body force, \( \Lambda=3.5 \, \text{fm}^{-1} \) and \( t_3=2000 \, \text{MeV} \, \text{fm}^6 \) are used. \( n_t=0.04 \, \text{fm}^{-3} \) is used for the inner crust boundary. See the caption of Fig. 7 for other explanations.](image1)

![Fig. 13: Moments of inertia of neutron stars of Fig. 12.](image2)
derived using the low-momentum interaction $V_{\text{low-}}^k$ obtained from four realistic NN potentials (CDbonn, Nijmegen, Argonne V18, BonnA). We require that the EOS used for neutron stars should give satisfactory saturation properties for symmetric nuclear matter, but this requirement is not met by our calculations using the above potentials as they are. Satisfactory nuclear matter saturation properties can be attained by using the above potentials with the commonly used linear BR scaling ($BR_1$) where the masses of in-medium mesons are slightly suppressed compared with their masses in vacuum. But this linear scaling is not suitable for neutron stars; the maximum mass of neutron star given by our $BR_1$ ring-diagram calculation is $\sim 3.2M_\odot$ which is not satisfactory. $BR_1$ is suitable only for low densities; it needs some extension so that it can be applied to the high densities inside the neutron star. We have used an extrapolation method for this extension, namely we add an empirical Skyrme-type three-body force $V_{3b}$ to $V_{\text{low-}}^k$. We have found that the EOS given by this extrapolation agrees well with the EOS obtained from linear $BR_1$ scaling for low densities, but for high densities it agrees well with that from a nonlinear $BR_2$ scaling. The EOS using the above extrapolation gives satisfactory saturation properties for symmetric nuclear matter, and for neutron matter it agrees well with the FP EOS for neutron matter.

The effects from $V_{3b}$ have been found to be both important and desirable. Compared with the results given by the unscaled $V_{\text{low-}}^k$, it increases the maximum mass of the neutron star and its radius and moment of inertia by $\sim 40\%$, $\sim 20\%$ and $\sim 150\%$ respectively. The proton fractions are found to be generally small ($<7\%$), making our neutron-star results using the pure-neutron EOS and those using the $\beta$-stable EOS being nearly the same. We have estimated the effect from the nuclei-crust EOSs by using a combination of three EOSs: the BPS EOS for the outer crust, a fitted polytropic EOS for the inner crust and our $\beta$-stable ring-diagram EOS for the core region. The effect from the nuclei-crust EOSs on the maximum neutron-star mass and its radius is found to be rather small, as compared with those given by the calculation where the $\beta$-stable EOS is used throughout. However, its effect is important in the low-mass large-radius region, significantly enhancing the neutron-star mass there. Using the above combined three EOSs, our results for neutron star’s maximum mass and its radius and moment of inertia are, respectively, $\sim 1.8M_\odot$, $\sim 9km$ and $\sim 60M_\odot km^2$, all in good agreement with accepted values.

How to extend the BR scaling to high densities is still an open question. Although we have obtained satisfactory results by using a nonlinear scaling for the high-density region, or equivalently a Skyrme-type three-body force, for the extension, it would still be certainly useful and interesting to explore other ways for doing so. Further studies in this direction would be very helpful in determining the medium dependent modifications to the NN potentials in the high-density region.

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