Multiscale damage constitutive model of jointed rock masses based on the influence of crack initiation

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Abstract: A rock mass has a large number of macroscopic and microscopic defects, such as joint cracks and microcracks, which greatly affect its physical and mechanical properties. In this study, a multiscale elastoplastic damage model was proposed to describe the influence of the existence and evolution of two types of cracks on rocks. The macroscopic damage variable expressed by the stress intensity factor was derived on the basis of damage mechanics and fracture mechanics and regarded as a piecewise function with initiation stress as the dividing point. The rock was divided into two parts based on micromechanics: matrix and microcracks. The macroscopic damage was considered the deterioration of the mechanical properties of the rock matrix. Under the framework of thermodynamics, a constitutive model that could reflect the growth of macrocracks and the friction–damage coupling of microcracks was constructed. This model could reflect the physical mechanism of macroscopic and microscopic damage of jointed rock masses, so the physical parameters of the model had clear meanings. Lastly, the rationality of the model was verified via uniaxial compression tests of rock-like materials with a single flaw of different inclination angles.

1. Introduction

A rock mass is a type of heterogeneous and noncontinuous natural composite medium containing joints and cracks of different scales [1]. The expansion and penetration of macroscopic and microscopic fissures greatly affect the damage and destruction of rocks, which in turn determine the stability and safety of rock mass engineering. The constitutive relationship is an important theoretical basis for studying the mechanical behavior of materials [2]. If a reasonable constitutive model can be constructed on the basis of the macroscopic and microscopic crack evolution mechanism, the multiscale mechanical properties of rock masses can be described, and the stress–strain relationship can be calculated quantitatively. Furthermore, this model is a prerequisite for the numerical calculation of jointed rock masses.

Laboratory tests and constitutive models are two basic methods used to study the multiscale mechanical properties of jointed rock masses [3]. A large number of experiments have been performed to explore the crack initiation and propagation state of jointed rock materials under different loading conditions; studies have also analyzed the influence of different macroscopic and microscopic crack occurrence states on stress–strain curves [4, 5]. Thus, they have provided a guarantee for the derivation of a multiscale constitutive model of jointed rock masses.
Joints are numerous and discontinuous inside a rock mass, but they cannot be analyzed as contact elements. Some scholars regarded them as damages inside a rock and determined their status through fracture mechanics. Chen W et al. [6] suggested that the increment of additional strain energy caused by fracture is equal to the release of damage strain energy and deduced the damage variable of nonpenetrating fracture expressed by the fracture strength factor. A statistical damage constitutive organically combines continuum and statistical strength theories [7], which can describe the random distribution of the strength and elastoplastic characteristics of rock microelements [8]. Considering the microstructural characteristics and local mechanical properties of rocks, Zhu Q [3] established a series of multiscale damage constitutive models that can demonstrate the micromechanical mechanism and macromechanical characteristics based on homogenization and thermodynamics. To consider the comprehensive influence of macroscopic and microscopic defects, Yang G et al. [9] proposed the idea of macroscopic and microscopic damage coupling based on the Lemaitre strain equivalence assumption. Furthermore, other scholars have made a variety of improvements. Liu H et al. [10] and Chen S et al. [11] proposed the damage coupling expressions based on the energy equivalence method and the statistical damage method, respectively. In summary, numerous studies on the initiation and extension mechanism of macroscopic and microscopic fracture in rocks have been carried out. Damage mechanics and fracture mechanics are widely applied to constitutive models of jointed rock masses. Micromechanics-based models can reflect the physical mechanism of damage and failure to a greater extent than statistical damage constitutive models. However, studies that simultaneously consider micromechanics and the effect of macrodamage have yet to be performed.

In this study, the influence of macrocrack initiation on macrodamage was considered, and the macrodamage variable was regarded as a piecewise function, which was introduced into a model based on micromechanics. The proposed model could describe the initiation of macrocracks and the friction–damage coupling of microcracks during loading. Therefore, it could demonstrate the physical mechanism of damage and failure process of jointed rock masses.

2. Derivation of the constitutive model

A rock contains a large number of randomly distributed microcracks. In the analysis of its mechanical properties, it can be simplified as a heterogeneous material composed of a rock matrix and microcracks. Its mechanical properties are further reduced because of the existence of macroscopic joints and their expansion during loading. The deterioration of the rock matrix is considered macroscopic damage, which ultimately affects the overall mechanical properties (Figure 1).

![Figure 1. Distribution of macroscopic and microscopic cracks in jointed rock masses.](image_url)

2.1. Free energy function

The free energy of a system expressed in Eq. (1) is divided into two parts: the elastic free energy of the matrix and the free energy related to the friction and sliding of a microcrack surface [12]:

\[
\text{Free energy} = \text{Elastic free energy of matrix} + \text{Energy related to friction and sliding of microcrack surface}
\]
\[ \psi = \frac{1}{2} (\varepsilon - \varepsilon^p) : C_0 : (\varepsilon - \varepsilon^p) + \frac{1}{2} \varepsilon^p : \mathbb{C}_p : \varepsilon^p, \]  

where \( \psi \) is the free energy of the system, \( \varepsilon \) is the total strain, \( \varepsilon^p \) is the plastic strain caused by microcrack friction, \( C_0 \) is the elastic stiffness tensor of the matrix phase, and \( \mathbb{C}_p \) is a fourth order tensor related to the friction–damage coupling mechanism of microcracks. \( C_0 \) and \( \mathbb{C}_p \) are expressed as:

\[ C_0 = 3k_s (D) \mathbb{J} + 2\mu_s (D) \mathbb{K}, \]  

\[ \mathbb{C}_p = \frac{1}{d} \left( \frac{1}{3} 3k_s (D) \mathbb{J} + \frac{1}{\beta} 2\mu_s (D) \mathbb{K} \right), \]

where \( \mathbb{J} \) and \( \mathbb{K} \) are the introduced fourth-order tensor operators; \( J_{ijkl} = \frac{1}{3} \delta_{ij} \delta_{kl} ; K_{ijkl} = I_{ijkl} - J_{ijkl} ; I_{ijkl} = \frac{1}{2} [\delta_{ij} \delta_{kl} + \delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk} + \delta_{ij} \delta_{kl}] ; d \) is the damage variable related to the microcracks; \( k_s (D) \) and \( \mu_s (D) \) are the bulk modulus and shear modulus of the rock matrix affected by the macroscopic damage \( D \), respectively; \( \alpha = \frac{16}{9} \frac{1 - v_s^2}{1 - 2v_s} ; \beta = \frac{32}{45} (1 - v_s) (5 - v_s) \); and \( v_s \) is the Poisson's ratio of the matrix, which is a constant under different stress states.

According to the principle of strain equivalence [13], the elastic modulus of the rock matrix \( E_s \) is affected by macroscopic damage \( D \), that is, \( E_s = E_s (1 - D) \). Thus, the bulk modulus and shear modulus of the rock matrix can be expressed as:

\[ k_s (D) = k_s (1 - D) \]  

\[ \mu_s (D) = \mu_s (1 - D) \]

Thermodynamic forces associated with damage variables and plastic strain can be expressed as [3]:

\[ F = -\frac{\partial \psi}{\partial d} = \frac{1}{2d^2} \varepsilon^p : \mathbb{C}^d : \varepsilon^p, \]  

\[ \sigma^p = -\frac{\partial \psi}{\partial \varepsilon^p} = \sigma - \mathbb{C}^p : \varepsilon^p, \]

where \( F \) is the thermodynamic force associated with the microdamage variable, \( \mathbb{C}^d = \frac{1}{d} \mathbb{C}^p \), and \( \sigma^p \) is the local stress.

2.2. Macroscopic damage

In damage mechanics, if the rock volume is \( V \), the change in elastic strain energy due to the existence of macroscopic cracks can be expressed as [10]:

\[ \Delta U^E = V \left( U^E - U_0^E \right) = V \left[ \frac{\sigma^2}{2E_0 (1 - D)} - \frac{\sigma^2}{2E_0} \right], \]

where \( \Delta U^E \) is the amount of change in the elastic strain energy of rock masses, \( U^E \) is the elastic strain energy per unit volume of an ideal complete rock, \( U_0^E \) is the strain energy per unit volume of jointed rock masses, and \( E_0 \) is the elastic modulus of an intact rock.

According to fracture mechanics, the additional strain energy \( U_1 \) caused by the existence of cracks in the plane stress state is expressed as follows [10]:
\[ U_i = \int_0^A GdA = \frac{1}{E_i} \int_0^A \left( K_{Ii}^2 + K_{IIi}^2 \right) dA, \]  

where \( G \) is the energy release rate; \( K_{Ii}, K_{II} \) are I and II stress intensity factors at the tip of the wing crack, respectively; and \( A \) is the joint surface area, where \( A = 2Ba \), \( B \) is the joint depth, and \( a \) is half the length of the joint.

When the uniaxial compressive stress is greater than the crack initiation stress \( \sigma_{cr} \), the crack propagates, and additional strain energy is generated (Figure 2). The elastic strain energy \( U_2 \) released by crack propagation can be expressed as [11]:

\[ U_2 = 2\int_0^l Gdl = \frac{2}{E_0} \int_0^l \left( K_{Ii}^2 + K_{IIi}^2 \right) dl, \]

where \( l \) is the crack growth length.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{crack_initiation.png}
\caption{Crack initiation state under different conditions.}
\end{figure}

Under uniaxial compression, \( \Delta U^E = U_1 \) when the pressure does not reach the crack initiation stress, i.e., \( \sigma < \sigma_{cr} \); otherwise, \( \Delta U^E = U_1 + U_2 \). When the influence of the rock sample size is considered, the stress intensity factor can be expressed as follows:

\[ K_i = \left[ \frac{2\tau_{eff} \sin \theta}{\sqrt{\pi(l + l_i)}} + \sigma_n (\sigma, \alpha + \theta) \sqrt{\pi l_i} \right] \sqrt{\sec \left( \frac{\pi a}{w} \right)} , \]

\[ K_{II} = \left[ \frac{2\tau_{eff} \cos \theta}{\sqrt{\pi(l + l_i)}} - \tau_n (\sigma, \alpha + \theta) \sqrt{\pi l_i} \right] \sqrt{\sec \left( \frac{\pi a}{w} \right)} , \]

where \( \sigma_n, \tau_n \) are the shear stress and shear stress on the joint surface, respectively; \( \tau_{eff} \) is the slip driving force on the fracture surface; \( w \) is the width of the flat specimen; \( \varphi \) is the friction angle of the fracture surface; and \( \alpha \) is the inclination. Then, \( l_i = 0.27l_0 \) is introduced. When the crack finally propagates, the propagating direction is parallel to the loading direction, i.e., \( \alpha + \theta = \frac{\pi}{2} \). Thus, the specific expression of the macroscopic damage variable can be derived:
2.3. Macroscopic and microscopic damage constitutive model

In the derivation of the constitutive model based on micromechanics, local stress is used to describe the evolution of the inelastic deformation caused by the friction slip of microcracks [3]:

$$f(\mathbf{s}^p) = \|\mathbf{s}^p\| + k\sigma_m^p \leq 0,$$

where \( \mathbf{s}^p = \mathbf{K} : \mathbf{\sigma}^p \) is the deviatoric stress part of the local stress, \( \|\mathbf{s}^p\| = \sqrt{\mathbf{s}^p : \mathbf{s}^p} \), \( k \) is the friction coefficient, and \( \sigma_m^p \) is the spherical stress part of the local stress. Therefore, the plastic strain of rock material can be expressed as:

$$\mathbf{e}^p = \Lambda^p \mathbf{D},$$

where \( \lambda^p \) is the plasticity multiplier; \( \Lambda^p = \int \lambda^p \mathbf{D} \) ; and \( \mathbf{D} \) is the tensor indicating the direction of plastic strain flow, i.e., \( \mathbf{D} = \frac{\partial f}{\partial \mathbf{s}^p} = \frac{\mathbf{s}^p}{\|\mathbf{s}^p\|} + \frac{1}{3} \kappa \mathbf{I} \). Then, the relationship between local stress and overall stress can be obtained. The damage criterion based on the strain energy release rate is [3]:

$$g(F,d) = F - R(d) \leq 0,$$

where \( R(d) \) is the damage resistance function, \( R(d) = r_c \frac{4d / d_c}{(1 + d / d_c)^2} \), \( d_c \) is the critical damage value corresponding to the peak stress, and \( r_c \) is the maximum value of damage resistance. \( \chi \) is defined as

$$\chi = \frac{1}{2} \mathbf{D} : \mathbf{C}^0 : \mathbf{D} = \frac{k_1(D)k^2}{2\alpha} + \frac{\mu(D)}{\beta}.$$  

The criterion for the overall stress expression considering the macroscopic and microscopic damage can be obtained:

$$f(\mathbf{\sigma}) = \|\mathbf{s}\| + k\sigma_m - 2\sqrt{R(d)}\chi \leq 0.$$

Under uniaxial compression, \( \|\mathbf{s}\| = \sqrt{6} \sigma \), and \( \sigma_m = \frac{\sigma}{3} :$

$$\sigma = \frac{6\sqrt{\chi R(d)}}{\sqrt{6} - k}.$$

The macroscopic strain can be expressed as:

$$\epsilon = \mathbf{C}^0 : \mathbf{\sigma} + \lambda^p \mathbf{D}$$

The component of the plastic flow direction tensor is

$$D_{11} = \frac{2}{\sqrt{6}} + \frac{k}{3}.$$
\[ \varepsilon = \begin{cases} \frac{\sigma}{E_s(1-D)}, & f < 0 \\ \frac{\sigma}{E_s(1-D)} + \frac{(\sqrt{6-k})dD_{ii}\sigma}{6\gamma}, & f = 0 \end{cases} \] (19)

3. Test verification

3.1. Test procedure

The model was verified with the experimental data published by the research team in the early stage [14, 15]. Sample preparation, basic mechanical parameters, and loading methods are illustrated in Figure 3. During loading, the strain field of the sample was observed through digital image correlation (DIC), and stress–strain curves under different crack inclination angles were obtained. In addition, the initiation stresses of cracks with different inclination angles were recorded. Preliminary analysis found that the stress–strain curve of the 45° inclination angle was obviously incompatible with other angles; therefore, it is ignored in the calculation [15].

![Figure 3](image)

**Figure 3.** Basic parameters of the specimen [14, 15].

3.2. Parameter identification

The macroscopic and microscopic parameters should be clarified in the model. The macroscopic parameters were related to sample size and prefabricated cracks, including sample volume \( V \), crack length \( 2a \), crack width \( B \), sample width \( w \), crack inclination \( \alpha \), and joint surface friction angle \( \phi \). A pre-existing fissure was fabricated with a piece of a thin aluminum sheet, which was slowly pulled out during sample solidification, so the friction angle of the fissure surface was taken as 0°. The model parameters related to microscopic damage were the elastic modulus of the matrix phase \( E_s \), Poisson’s ratio \( v_s \), the generalized friction coefficient \( k \), the critical damage resistance \( r_c \), and the critical damage value \( d_c \). The specific processes for determining the micromechanical parameters were described in a previous study [16]. When the stress was greater than the crack initiation stress, \( f \) changed with crack evolution. For the convenience of calculation, the crack initiation length was taken as a fixed value of 0.1 mm.

| Table 1. Parameters of the microdamage model. |
|---|---|---|---|---|
| \( E_s \) (MPa) | \( v_s \) | \( k \) | \( r_c \) (J/m²) | \( d_c \) |
| 28700 | 0.23 | 2.0987 | 1.5066/10⁴ | 0.82 |
3.3. Result analysis

The proposed macroscopic and microscopic damage constitutive model was used to simulate the stress–strain curves of the specimens with inclination angles of 15° and 75°. The proposed damage constitutive model can properly simulate test results and reflect the influence of crack initiation (Figure 4). At the initial stage of loading, only the damage caused by the existence of macroscopic cracks should be considered, and the specimen is in the elastic stage. As the load increases, the specimen gradually reaches its yield state, and a microdamage begins to develop, eventually leading to the failure of the specimen. During loading, the two ends of the pre-existing crack expand to form wing cracks, which have a greater impact on the stress–strain curves. At the point of initiation stress, the stress decreases and forms a step because of the sudden increase in macroscopic damage. Nevertheless, the proposed model can subtly reflect this detail. From the perspective of energy dissipation, the “step” of the stress–strain curve is the energy released by the initiation of macroscopic cracks.

![Figure 4](image-url)  
**Figure 4.** Simulation and test results. Inclination angles of (a) 15° and (b) 75°

![Figure 5](image-url)  
**Figure 5.** Macrodamage and peak strength.

![Figure 6](image-url)  
**Figure 6.** Macrodamage and maximum strain.

The macrodamage value initially increases and subsequently decreases as the inclination angle increases, reaching the peak at 45° (Figure 5). The macrodamage is symmetrically distributed around the peak. After the crack propagates, the macroscopic damage value increases correspondingly, but the overall distribution pattern is consistent with that before the crack propagates. The peak strength initially decreases and subsequently increases as the inclination angle increases. The increase and decrease trend of the peak strength is opposite to that of the macrodamage. Therefore, the model can reflect the degradation degree of the mechanical properties of rocks with different crack inclination angles.
The maximum strain concentration value observed via DIC in a previous study [14] is compared with the macroscopic damage variables (Figure 6). The results reveal that the crack inclination angle affects the distribution of the strain field, and the cracks have different degrees of opening and expansion. The maximum strain value turns at 45°, and the macroscopic failure peak appears at 45°. Therefore, changes in macroscopic damage are related to the deformation characteristics around the cracks. In future research, local strain should be considered to analyze damage evolution.

4. Conclusions

Macroscopic damage variables expressed by the stress intensity factor were derived on the basis of damage mechanics and fracture mechanics. The microscopic damage caused by microcracks was analyzed on the basis of micromechanics by considering the friction and damage coupling mechanism of the microcrack surface. Under the framework of thermodynamics, a multiscale damage elastoplastic constitutive model of jointed rock masses was proposed by considering the influence of crack initiation. The main conclusions were as follows:

The macroscopic damage value is a piecewise function with initiation stress as the boundary point, which can reflect the “step” of the stress–strain curve caused by pre-crack initiation.

The macroscopic damage initially increases and subsequently decreases as the inclination angle increases. It is also symmetrically distributed, which can reflect the pattern of the peak strength and the changes in the maximum strain concentration with inclination angle.

The proposed model considers the expansion of macroscopic cracks and the friction–damage coupling of microscopic cracks. It can also reflect the physical mechanism of damage and destruction of jointed rock masses. Therefore, the parameters in the model have clear physical meanings.

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