Suppression of contributions from large-parton-number Fok states to T-odd distribution functions in Drell-Yan scattering involving small–x annihilating quark and antiquark.

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Abstract

T-odd distributions like Sivers and Boer-Mulders functions are normally modeled using few-body models. In the present work I want to follow the completely opposite point of view, and study a high-energy proton-proton Drell-Yan where both the annihilating partons are wee, and one (at least) comes from a high-order Fok state of the parent proton, i.e. a state where a large number $N$ of partons is present. I show that rescattering between active and spectator partons is modified, with suppression of those terms that are needed to build T-odd parton distributions.

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1 Introduction

The measurement of the $\cos(2\phi)$-asymmetry associated with the violation of the Lam-Tung rule [1] in Drell-Yan dilepton production by the collaboration E866 [2] poses the problem of the behavior of this asymmetry when the longitudinal fraction of both the annihilating partons is small. An evidently nonzero asymmetry was systematically measured in fixed-target pion-nucleus Drell-Yan experiments [3], where the value of the product $xx' \approx Q^2/s$ ($Q^2$ is the
dilepton mass, \( s \) the collision c.m. squared energy, \( x \) and \( x' \) the longitudinal fractions of the annihilating partons) always implied that one at least of the annihilating partons was a valence (anti)quark. In proton-proton Drell-Yan, sea antiquarks are necessarily involved, and the much larger c.m. energy of E866\(^2\) led statistically to a much smaller value of \( x x' \). E866 measured zero-compatible values of the asymmetry, with the possible exception of the largest—\( x \) point.

According with the modern way of seeing this asymmetry, within a leading-twist factorization formalism\(^4\) it is proportional to the convolution of two T-odd TMDF (transverse momentum dependent distribution functions), the Boer-Mulders\(^5\) functions. Models for the Boer-Mulders function, for another T-odd TMDF (the Sivers function\(^6\)), and for the related asymmetries, have been studied by many authors (see e.g. \(^7\)\(^8\)\(^9\)\(^10\)\(^11\)\(^12\)\(^13\)\(^14\)\(^15\)\(^16\)\(^17\)\(^18\)\(^19\)\(^20\)\(^21\)\(^22\)\(^23\)\(^24\)\(^25\)\(^26\)\(^27\)\(^28\)\(^29\)\(^30\)). The phenomenological models normally implement schemes where a hadron is composed by 2 or 3 constituents. This puts some limitations on the understanding of the small—\( x \) properties of T-odd TMDF and related asymmetries (see however the recent \(^31\)).

At the lowest PQCD order, the Drell-Yan cross section is proportional to the imaginary part of the amplitude \( G_0 \) of fig.1. Fig.2 shows the amplitude \( G_1 \) including part of the \( O(\alpha_s) \) corrections (for the full structure of the Drell-Yan cross section, see \(^32\) and \(^33\)). Not to violate fundamental principles, T-odd TMDF are necessarily associated with the presence of rescattering\(^9\) in the Drell-Yan process. More precisely, with the interference between rescattering and no-rescattering terms. In a leading twist factorization scheme, a model for obtaining a nonzero T-odd TMDF for the parton coming from hadron “2” needs including at least fig.1, fig.2 and its right-side specular.

Not all the rescattering terms are responsible for T-odd effects. A change \( \Delta L = 1 \) in the orbital angular momentum of the active parton is required. I will name “spin-orbit terms” those that may produce this (without violating fundamental laws), and “scalar terms” those that cannot. I expect spin-orbit rescattering terms to become ineffective when states with a large number \( N \) of partons, are involved. The qualitative arguments are:

(i) A large \( N \) state is associated with an increasing transverse radius \( R_N \) for the individual parton distribution.

(ii) The set of the rescattering spectator partons forms an overall color triplet, i.e. its total color charge does not depend on \( N \). The effectiveness of triplet-triplet \( O(\alpha_s) \)–interactions is decreased by color-charge form factors when the partons spread over a broad region.

(iii) This regards scalar and spin-orbit terms, but the latter are proportional to the transverse gradient in impact parameter space, so they are much more
Fig. 1. Cut-diagram for the Drell-Yan parton-model cross section. The relevant particles for this work are \( q_1 \) (the active quark or antiquark from proton 1) and \( p_2, p_3, ... \) (the spectator partons from proton 2). For simplicity, the virtual photon and the leptons are not drawn.

Fig. 2. Drell-Yan cross section including the active\(_1\)–spectator\(_2\) \( O(\alpha_s) \) rescattering amplitude \( A \), with detailed kinematics. \( \vec{Q}_T \) is the transverse momentum of the virtual photon, \( \vec{q}_T \) the one transported by the rescattering boson, \( \vec{k}_{1T} \) and \( \vec{k}_{2T} \) those from the initial state of the active partons.

sensitive to the shape of the color-charge distribution.

Of course other effects may suppress spin-dependent TMDF at small \( x \). Here I want to focus on the role of rescattering.
1.1 Small-\(x\) limit and general assumptions

In the present work, I assume a large but finite and fixed value of the c.m. collision energy \(s\), and consider a Drell-Yan process involving two protons where the two conditions are satisfied:

1. The active quark and antiquark have both longitudinal fraction \(x < \frac{E_0}{\sqrt{s}}\) with \(E_0 \sim 1\) GeV.

2. One of the two protons (proton “2” in fig.2) is in a high-order Fok state: the number \(N\) of its partons satisfies \(N \gg 3\).

Condition (1) for both the annihilating quark and antiquark defines a “small-\(x\) kinematic framework”, in the sense that

(i) although \(s\) is possibly large, \(Q^2 \approx xx'/s\) is at most semi-hard. The active quark and antiquark momenta have fixed upper scale \(\sim 1\) GeV. A more extensive description of the peculiarities of small-\(x\) physics in hard scattering may be found e.g. in [34]. In practice point (i) means that the conditions for an unambiguous separation of leading twist terms are not present (see also [31]). For these reasons here I will not exploit the ordinary factorization ideas, but will anyway consider factorization as the scheme according to which final results of an experiment will be rewritten.

(ii) longitudinal degrees of freedom disappear from the problem, since the active (anti)quark is not able to probe target structure details within \((1\) GeV\(^{-1}\)\(M\)\(^{-1}\) times longer than the UR-contracted target thickness). This means that the rescattering problem is 2-dimensional.

I will here rely on some qualitative results of the old multiperipheral model for hadronic collisions involving large-\(N\) Fok states. It was first proposed in [36], and later developed and branched by a countless number of authors (for a review of the basic ideas, see [35]). It organizes the (interacting) hadrons in chains of \(N\) partons whose impact-parameter wavefunction is random-walk structured: \(\psi(\vec{b}_1, \vec{b}_2, ...) \approx \prod \psi(\vec{b}_i - \vec{b}_{i-1})\). So the overall average squared transverse radius satisfies \(R_N^2 \propto N\).

I will only include \(O(\alpha_s)\) processes. Since at this order the commutativity problem is not present, I will treat color charges as abelian QED-like charges, assuming that a Drell-Yan event splits a hadron into an active (anti)quark with color charge \(-1\) and a spectator set with color charge \(+1\).
2 N-spectator rescattering

At large $E/M$ the 4 independent components of a free Dirac spinor reduce to 2. These are normally the helicity ones, but a linear combinations of these corresponding to a given transverse polarization is more comfortable when working in impact parameter space and/or with given $\vec{S}_T$ (see [26] for the technicalities of this choice, and [37] for the properties of the scattering amplitudes in transverse-spin basis). I assume a transverse spin quantization axis $\hat{x}$ and split the UR quark wavefunction into two components corresponding to $s_x = \pm \hbar/2$, and build 2-component vectors:

$$|\psi> \equiv \psi_+|+>_x + \psi_-|>_x, \rightarrow \vec{\psi}(...) \equiv \begin{pmatrix} \psi_+ (...) \\ \psi_- (...) \end{pmatrix} \quad (1)$$

where (...) means spacetime or momentum variables. In this basis a single-particle amplitude $A$ must be expressed as a 2x2 matrix operator.

I name $q_1, q_2, ...$ and $p_1, p_2, ...$ the partons from hadrons “1” and “2” respectively; $q_1$ and $p_1$ are the active quark and antiquark. $Q_\mu$ is the measured momentum of the Drell-Yan virtual photon, while $\vec{q}_T$ is the momentum exchanged between $q_1$ and $p_2, p_3, ...$ in the rescattering event. The full series of terms appearing in the Drell-Yan cross section is listed in [32]. It follows the scheme

$$\sigma \propto \text{Im}(fig.1 + fig.2 + ...) \equiv \sum d(x, x', Q_T, \{\theta\}) = \quad (2)$$

$$= \sum \int d^2k_Td_1(x, \vec{k}_T) \cdot d_2(x', \vec{Q}_T - \vec{k}_T) \quad (3)$$

where the $d(x, x', ...) \text{ structure functions are directly observable, while the tensor substructures } d_{1,2}(x, \vec{k}_T, ...) \text{ (proportional to the TMDF) assume a factorization formalism and must be somehow reconstructed. } \{\theta\} \text{ is a set of dilepton angles.}$

2.1 Multiparton rescattering amplitude $A_N$

For $x < 1 \text{ GeV}/\sqrt{s}$ the wavelength of the quark $q_1$ from hadron “1” is not able to sample the longitudinal features of the set of spectator partons $p_2, p_3, p_4, ...$ coming from hadron “2”, so $A$ may only affect the transverse degrees of freedom of $q_1$: 

\[\]
\[ d(x, x', Q_T) = \langle q_1 | \cdots (1 + \langle p_2, p_3, \ldots | A(\vec{q}_T) | p_2, p_3, \ldots \rangle) | q_1 \rangle \]
\[ \equiv \langle q_1 | \cdots (1 + A_N(\vec{q}_T)) | q_1 \rangle \]  
(4)

where \( A_N(\vec{q}_T) \equiv \langle p_2, p_3, \ldots | A(\vec{q}_T) | p_2, p_3, \ldots \rangle \) is the first order sum/average of the rescattering amplitude over \( N - 1 \) spectators, and “1” the no-rescattering term.

\( A_N \) is a 2x2 matrix acting on a \( q_1 \) wavefunction of the form \( \langle p_2, p_3, \ldots | \cdot | p_2, p_3, \ldots \rangle \) in unpolarized and single-polarized Drell-Yan, the leading twist T-odd TMDF appear in \( d_i(x, k_T) \) terms of the form (see [38])

\[ d_{i,odd}(x, k_T) = h(x, \vec{k}_{1,T})(\vec{S}_T \wedge \vec{k}_T)_z \]

where \( \vec{S} \) is the spin of one of the colliding hadrons (Sivers case) or of the active quark (Boer-Mulders case). The most general rescattering amplitude that is able to produce a term of this form (in a single-rescattering diagram, and without violating standard physical constraints including T-reversal) is:

\[ A_N(\vec{q}_T) \equiv A_{N,scalar} + A_{N,SO} = f_N(q_T^2) + g_N(q_T^2) \left( \hat{\sigma}_T \wedge \vec{q}_T \right) \]  
(5)

\[ = \int d^2\vec{b} \exp(i\vec{q}_T \cdot \vec{b}) \left\{ \hat{f}_N(b^2) + i\hat{\sigma} \wedge \frac{\partial}{\partial \vec{b}} \hat{g}_N(b^2) \right\} \]  
(6)

where “SO” means “spin-orbit”, (“spin-orbit” and “scalar” in the sense defined in the Introduction: able or not to change \( L \) by one unit). The momentum \( \vec{q}_T \) exchanged in rescattering is a loop momentum, and must neither be confused with the active quark/antiquark momenta \( \vec{k}_{1,T} \) nor with \( \vec{Q}_T = \vec{k}_{1,T} + \vec{k}_{2,T} \) (see fig.2). The “color thickness” functions \( f_N \) and \( g_N \) derive from the averaging procedure \( \langle p_2, p_3, \ldots | p_2, p_3, \ldots \rangle \) in eq[4] If \( d(x, x', \ldots) \) is organized in a factorized formalism as in eq[3] the modifications associated with fig.2 are attached to \( d_2(x', \vec{k}_{2,T}) \), that shows T-odd asymmetries \( \propto g_N(q_T^2)/[1 + f_N(q_T^2)] \) \( \approx g_N(q_T^2) \) (the “1” comes from the “1” of eq. 4 i.e. from fig.1).

Let both the scalar and the spin-orbit part of \( A_N \) be the coherent sum of single rescattering amplitudes, each one referring to boson exchange between \( q_1 \) and one of the \( p_i \):

\[ A_{N,scalar/ SO} = \sum_{i=1}^{N} c_i \int d^2\vec{b} \ e^{i\vec{q}_T \cdot \vec{b}} \prod_{j=2}^{N} d^2b_j \ \hat{O}_{scalar,SO} \ P(q_T, |\vec{b} - \vec{b}_i|) \ |\psi(b_2, b_3, \ldots)|^2 \]  
(7)

where \( c_i \) are color charges and their sum must be 1. \( \hat{O}_{scalar} \equiv \hat{I}, \ \hat{O}_{SO} \equiv \hat{\sigma}_T \wedge \partial/\partial \vec{b} \). The amplitude \( P(q_T |\vec{b} - \vec{b}_i|) \) is the scalar interaction propagator in the transverse plane (that results after integrating over the longitudinal
degrees of freedom). It has the general form

\[ P(q_T, |\vec{b} - \vec{b}_i|) = \exp\left( -i q_T |\vec{b} - \vec{b}_i| \right) P'\left( \frac{|\vec{b} - \vec{b}_i|}{R_{int}} \right) \tag{8} \]

where the strength factor \( P' \) makes the individual quark-parton interaction effective within a distance \( R_{int} \), presumably of hadronic size. I will assume \( R_{int} \) to be \( N \)-independent, or at least weakly \( N \)-dependent w.r.t. \( R_N \).

The other relevant transverse scale parameter is \( R_N \), the average transverse radius of the set of spectator partons, and for it I assume \( R_N^2 = (N - 1) R_0^2 \approx NR_o^2 \), where \( R_o \) is a hadron-size parameter.

Analytically, it is possible to study the two limiting cases \( R_N \gg R_{int} \) and \( R_N \ll R_{int} \), exploiting part of the standard procedure for describing scattering on a composite target in terms of charge and current form factors. I just outline the main steps.

2.2 case 1: The large-\( N \) limit: \( R_N \gg R_{int} \)

For \( R_N \gg R_{int} \), I may use assume a pointlike interaction range:

\[ P(q_T, |\vec{b} - \vec{b}_i|) \rightarrow \delta(|\vec{b} - \vec{b}_i|). \tag{9} \]

\[ A_{N,scalar} \rightarrow \sum_2^N c_i \prod_2^N d^2 b_j e^{i q_T \vec{b}_i} |\psi(b_2, b_3, ..)|^2 = F_c(q_T) \tag{10} \]

where \( F_c(q_T) \) is the color charge form-factor of the spectator parton set.

For the spin-orbit case one uses the fact that in eq.[7] \( \partial/\partial \vec{b} \) only acts on \( P(q_T, |\vec{b} - \vec{b}_i|) \), since all the other functions on the right of \( \partial/\partial \vec{b} \) depend on \( \vec{b}_i \) but not on \( \vec{b} \). For each \( i \),

\[ \frac{\partial}{\partial \vec{b}} P(q_T, |\vec{b} - \vec{b}_i|) K(\vec{b}_i) = - K(\vec{b}_i) \frac{\partial}{\partial \vec{b}_i} P(q_T, |\vec{b} - \vec{b}_i|), \tag{11} \]

where \( K(b_i) \) summarizes all the terms in eq.[7] that depend on \( \vec{b}_i \) but not on \( \vec{b} \). Now one may apply eq.[9] and integrate in \( d\vec{b} \). One gets

\[ A_{N,SO}(q_T) = \sigma_T \wedge q_T F_c(q_T) \tag{12} \]
2.3 Case 2: \( R_N << R_{int} \) and far away collisions: The small–\( N \) cutoff.

I examine the case where the spectators \( p_i \) are close each other and far from \( q_1 \). Rescattering may take place if \( R_N << R_{int} \). Let me first start with the scalar term.

\[
\exp(i\vec{q}_T \cdot \vec{b}) \exp(-iq_T|\vec{b} - \vec{b}_1|) \approx \exp(ibq_T(\hat{q}_T \cdot \hat{b} - 1)) \exp(ib \cdot \vec{b}_i) \quad (13)
\]

\[
\to \exp(ibq_T(\hat{q}_T \cdot \hat{b} - 1)) \exp(i\vec{q} \cdot \vec{b}_i). \quad (14)
\]

The first member joins the two exponents from eqs. 7 and 8, eq. 13 assumes \( b >> b_i \) (I put the origin in between \( p_2,..p_n \)), and I use that in further integrals \( \exp(ibq_T(\hat{q}_T \cdot \hat{b} - 1)) \) will contribute for \( \hat{q}_T \cdot \hat{b} \approx 1 \). Eq. 7 becomes

\[
A_{N,scalar}^{\vec{q}_T} \approx P(q_T) \int \prod_2^N \int_2^\infty d^2b_j \left( \sum_2^N c_i e^{i\vec{q}_T \vec{b}_i} \right) |\psi(b_2, b_3, ..)|^2 = \quad (15)
\]

\[
= P(q_T) F_c(q_T) \approx P(q_T). \quad (16)
\]

where \( P(q_T) \) is the amplitude for interacting with one pointlike constituent of charge 1. The last approximation reflects the fact that, because of the requirement \( b >> b_i \), \( P(q_T) \) is much more \( q_T \)–short-ranged than \( F_c(q_T) \).

For treating \( A_{N,SO} \) one again uses eq. 11. The final result is

\[
A_{N,SO}^{\vec{q}_T} \approx P(q_T) F_c(q_T) \sigma_T \wedge \vec{q}_T \approx P(q_T) \sigma_T \wedge \vec{q}_T \quad (17)
\]

So, we have a cutoff. For small enough \( N \) to have \( R_N \leq R_{int} \), far away rescattering is possible and likely. Then spectators appear as a unique pointlike constituent. This case corresponds to a proper use of the diquark approximation\[39\].

For large \( N \) and \( R_N > R_{int} \) the increased sensitivity to the structure of the spectator “cloud” is expressed by eqs. 10 and 12. This allows for defining the lower cutoff for the “large \( N \)” condition used in this work. It means that the equation \( \sqrt{NR_o} > R_{int} \) is satisfied.

2.4 Dependence of the observable asymmetries on \( N \)

Comparing the form-factor expressions with eq. 6, we see that

\[
\hat{f}_N(b^2) \propto F_c(q_T), \quad \hat{g}_N(b^2) \propto F_c(q_T). \quad (18)
\]
At 2nd order in $q_T R_N$

$$F_c(q_T) \approx \exp(-q_T^2 R_N^2/2) = \exp(-N q_T^2 R_o^2/2)$$  \hspace{1cm} (19)

The scalar term may be large for $q_T \approx 0$. In the spin-orbit case this possibility is forbidden since

$$A_{N,SO}(\vec{q}_T) \propto \vec{\sigma}_T \wedge \vec{q}_T \ F_c(q_T) \propto \vec{\sigma}_T \wedge \vec{q}_T \ exp(-N q_T^2 R_o^2/2).$$  \hspace{1cm} (20)

$A_N(\vec{q}_T)$ is a matrix on spin states. For a given spin, it is odd in $\vec{q}_T$, negligible for $|q_T| > 2/\sqrt{N R_o}$, and roughly linear up to its peak at $q_T \approx 1/\sqrt{N R_o}$:

$$\text{Max} |A_{N,SO}(\vec{q}_T)| \approx A_{N,SO}(q_T = 1/\sqrt{N R_o}) \propto 1/\sqrt{N}.$$  \hspace{1cm} (21)

We get observable effects from the convolution

$$A_N(\vec{Q}_T) \equiv \int d^2q_T \ G(\vec{Q}_T - \vec{q}_T) \ A_N(\vec{q}_T)$$  \hspace{1cm} (22)

where $\vec{Q}_T \equiv \vec{k}_{T,1} + \vec{k}_{T,2} + \vec{q}_T \equiv \vec{k}_T + \vec{q}_T$, and $G(\vec{k}_T)$ is a distribution for the quark/antiquark combined “intrinsic” momentum, possibly including real-gluon radiation effects.

In the “quasi-collinear” limit $\vec{k}_T$ is negligible, $\vec{Q}_T \approx \vec{q}_T$, $A_N(\vec{Q}_T) \approx A_N(\vec{q}_T)$. $A_N(Q_y) - A_N(-Q_y) \approx A_N(q_y) - A_N(-q_y) = A_{N,SO}(q_T) \sim 1/\sqrt{N}$.

However, in Drell-Yan $k_T$ receives strong contributions from hard real gluon radiation and $< k_T^2 >$ overcomes the soft hadronic momentum scales (see \[22\] and \[27\] for the behavior of T-odd TMDF at large $Q_T$). For large $N$ the average $q_T$ is $O(1/\sqrt{N})$ and it seems likely to have $k_T >> q_T$. Using eq. \[21\] and assuming a gaussian form for $F(Q_T)$,

$$A_N(Q_y) - A_N(-Q_y) = \int d^2q_T \ G(\vec{Q}_T - \vec{q}_T) \ A_{N,SO}(\vec{q}_T) \sim$$

$$\sim \left\{ [G(Q_T + 1/\sqrt{N R_o})] - [G(Q_T - 1/\sqrt{N R_o})] \right\} \left[ \int_0^{2/\sqrt{N R_o}} A_{N,SO}(q_y) dq_y \right]$$

$$\sim (2/\sqrt{N R_o})^2 \cdot (2/\sqrt{N R_o}) \frac{\partial}{\partial Q_T} F(Q_T) \propto Q_T G(Q_T)(1/\sqrt{N})^3$$  \hspace{1cm} (23)
3 Conclusions and discussion

Summarizing, if the hypotheses of this work are valid (small \( x \) and \( x' \), one hadron in a high-order Fok state with \( N \) partons) the observable T-odd effects of the rescattering amplitude in Drell-Yan are weakened by a factor ranging from \( 1/N^{1/2} \) (\( Q_T \ll 1 \) GeV/c) to \( 1/N^{3/2} \) (\( Q_T \sim 1 \) GeV/c).

The key assumption of this work is that the diagrams describing the interactions between the active (anti)quark from hadron “1” and each of the spectator partons from “2” sum coherently, weighted by coefficients that are linear in the spectator color charges. This is obvious for short-distance 1-gluon exchange, less obvious for interactions that may in principle involve large transverse distances (although the main result of this work has been obtained in the approximation of short-ranged rescattering, see eq.9).

To stress the role of color coherence in the previous results, let me consider an alternative small−\( x \) model, where I obtain a multiparton state by a chain of quasi-real and well-separated mesons, each in a pure valence state, instead of a chain of partons. This is a large−\( N \) extension of the pion-pole limit of the pion-cloud model [40]. The proton-proton Drell-Yan cross section would be the convolution between the probability of finding a \( \pi^- \) in a given place and the valence cross section of \( \pi^- \)-proton Drell-Yan. Intuition suggests that if any suppression is present, in this model it is not T-odd-selective. This extreme picture takes into account one only (“singlet \( \otimes \) singlet \( \otimes \) .... \( \otimes \) triplet”) among all the possible color space decompositions of the \( N \)-parton set. So it excludes by default amplitude cancellations with other possible color configurations. The multimeson picture is unlikely for large \( N \) (the mesons would be highly virtual, overlap and lose individuality), but it would be interesting to see how much color cancellations do/do not suppress T-odd effects in the 5-parton case of the model [40].

An important discussion hint is the widespread use, to calculate T-odd observables, of the diquark spectator model [39] (see e.g. [9,12,13,16,21,24]) or of models that only include the lowest-Fok state (e.g. [14,18,25,26,28,30]). Although large−\( N \) states are relevant at small−\( x \), my analysis says that the contribution from these states to T-odd effects is much smaller than in the T-even case. So, valence-model predictions could be better than presently imagined.

The risk however is the “opposite” error: a model that effectively includes sea properties in a valence distribution (e.g. a model that produces a valence quark distribution with typical sea magnitude at small \( x \)) will overestimate T-odd effects at small \( x \). Indeed, it will make many wee partons rescatter like a single pointlike one. In this case, I would recommend the introduction of a
gluon-spectator form factor to take finite spectator size into account.

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