Negative Domain Wall Resistance in Ferromagnets

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Abstract

The electrical resistance of a diffusive ferromagnet with magnetic domain walls is studied theoretically, taking into account the spatial dependence of the magnetization. The semiclassical domain wall resistance is found to be either negative or positive depending on the difference between the spin-dependent scattering life-times. The predictions can be tested experimentally by transport studies in doped ferromagnets.

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A domain wall (DW) is the region between two ferromagnetic domains in which the direction of the magnetization rotates. A number of experiments have been conducted which show either an increase [1–3] or a decrease [4–8] of the resistance due to DWs compared to the resistance of a single domain ferromagnet. These experiments have been done on thin films, structured thin films, and membranes, in the diffusive transport regime where the electron mean free path is shorter than the typical system size.

In the diffusive limit, Cabrera and Falicov [9] calculated an increase of the resistance caused by the back-reflection of electrons by the domain wall. The reflection probability was found to be exponentially small in the ratio of the DW width to the Fermi wavelength. An increase of the semiclassical resistance has also been predicted by Tatara and Fukuyama [10] by linear response calculations assuming spin-independent relaxation times. Levy and Zhang [11] obtained the DW resistance from a Boltzmann equation. They showed that spin-dependent relaxation times can enhance this positive DW resistance, depending on the ratio of relaxation times of the majority and minority spin electrons. Brataas et al. [12] calculated the domain wall resistance generalizing the approach of Tatara and Fukuyama to include spin-dependent lifetimes with qualitively similar results to Levy and Zhang.

The only intrinsic mechanism which explains a decrease of the resistance has been proposed by Tatara and Fukuyama [10], viz. the destruction of electron weak localization by the dephasing caused by the domain wall decreases the resistance. However, experimentally, the negative domain wall resistance persists up to relatively high temperatures [5,6] where localization does not play a role. Kent et al. [6] explain the negative DW resistance by an extrinsic effect: reduced surface scattering.

It is the purpose of this Letter to show that the semiclassical DW resistance of diffusive ferromagnets can be negative as well as positive when the electronic structure of the domain wall is taken into account semiclassically. The experimental results [1–8] may thus originate from one and the same intrinsic semiclassical effect.

Let us first describe the elementary physics of electron transport in a domain wall. The
Drude resistivity of a single-domain ferromagnet reads in the 2-band Stoner model \[13\]

\[
\rho = \frac{m}{e^2} \frac{1}{n_+ \tau_+ + n_- \tau_-}
\]

(1)

where \(m\) is the mass of the electron, \(e\) is the charge of an electron, \(n_+ (n_-)\) is the density of spin-up (down) electrons and \(\tau_+ (\tau_-)\) is the scattering relaxation time for the spin-up (down) electrons which at low temperatures depends on the (spin-dependent) impurity potential and (spin-dependent) density of states. A redistribution of the electrons between the spin-up and down bands (i.e. a change in magnetization) modifies the resistivity when \(\tau_+ \neq \tau_-\).

With \(n = n_+ + n_-\), \(n_\pm = n_0^\pm + \delta n_\pm\) and \(\delta n_+ = -\delta n_-\), the change in resistivity is found to be:

\[
\delta \rho \approx -\rho_0^2 \frac{e^2}{m} \delta n_+(\tau_+ - \tau_-)
\]

(2)

where \(\rho_0\) is the resistivity of a single domain ferromagnet. We see that a modified magnetization causes the resistivity to either increase or decrease, depending on the relaxation times. The relaxation times in a ferromagnet depend on the type of impurities that are present in the material \[14\]. The sign and the magnitude of the resistivity change are therefore impurity specific. In the following we show that the magnetization is modified in a domain wall and contributes to the DW resistance on top of the DW scattering mechanisms discussed in literature \[9–12\].

We approximate the domain wall by a local constant rotation along the \(z\)-direction \[11\]

\[
\partial_z \phi(z) = \frac{\pi}{\lambda_w} = a_0
\]

where \(\lambda_w\) is the length of the domain wall, as indicated in Fig. 1. This is allowed when the domain wall is much wider than the Fermi wave length. The DW resistance can in this limit be calculated by interpreting the DW as a finite slice of a so-called spin-spiral ferromagnet. The total resistance of a ferromagnet is determined by simply adding the resistivities of the DWs and the domains. The relative change in resistance due to the domain walls is \((R - R_0)/R_0 = (L \rho_0 + \lambda_w (\rho_0 + \delta \rho) - (L + \lambda_w) \rho_0)/(L + \lambda_w) \rho_0\) = \(\lambda_w \delta \rho / ((L + \lambda_w) \rho_0)\), where \(L\) is the length of the domain.

The magnetization is a result of the exchange interaction. We follow the common procedure to express the exchange energy by the continuum limit of the mean-field Heisenberg
model

\[ E_{\text{ex}} = \int dr \int dr' K(|r - r'|) \langle m(r') \rangle \cdot m(r), \tag{3} \]

where \( K(|r - r'|) \) is the exchange interaction between electrons in a volume element \( r \) and \( r' \), \( m(r) = \mu_B s(r) \) is the magnetization operator and \( \langle m(r) \rangle \) denotes the thermal ensemble average of the magnetization. The range \( \lambda_{\text{ex}} \) of the exchange interaction is of the order of the Fermi wavelength. In a DW neighboring spins are canted. Hence the exchange energy is reduced. Using \( \langle m(r') \rangle \cdot m(r) = \cos(\pi(z - z')/\lambda_w) \langle m(r') \rangle \cdot m(r) \) and taking into account that \( \lambda_w \gg \lambda_{\text{ex}} \), the exchange energy becomes \( E_{\text{ex}} = \int dr H_{\text{ex}}(r) \cdot m(r) \), where the exchange field is

\[ H_{\text{ex}}(r) \approx K \left[ 1 - \frac{1}{2} \left( \frac{\pi \lambda_{\text{ex}}}{\lambda_w} \right)^2 \right] \langle m(r) \rangle, \tag{4} \]

and \( K \) is the total exchange integral. The exchange field and therefore the splitting decrease with decreasing \( \lambda_w \) and increasing \( \lambda_{\text{ex}} \). The effect of the reduced magnetization on the resistivity seems small, since the domain wall is much wider than the Fermi wavelength. However, we will show that the effect of the reduced magnetization on the DW resistance is of the same order as that of other mechanisms studied previously \([10-12]\). Expressing the exchange splitting as \( 2\Delta(r) = 2J s(r) \), we have \( J = \mu_B K (1 - (\pi \lambda_{\text{ex}}/\lambda_w)^2/2) \) (the thermal average spin-density is \( s(r) = |\langle s(r) \rangle| \)). The relative change in the effective coupling constant \( J \) due to the domain wall can be written as \( \delta J/J = -\kappa 2E_w/((\epsilon^+_{F} + \epsilon^-_{F})) \), where \( E_w = \hbar^2 a_0^2/(2m) \) is an energy parameter of the rotation of the magnetization, and \( \epsilon^\pm_{F} = \hbar^2 (k^\pm_{F})^2/(2m) \) is related to the (spin dependent) Fermi wavevectors. Using an estimate for \( \lambda_{\text{ex}} \approx \lambda_F/2 = \pi/k_F \) we find \( \kappa = \pi^2/2 \). A longer range of the exchange interaction \([3]\) will have even larger effects. Note that \( \Delta \) decreases even faster than \( \delta J/J \) because in the self-consistent mean field approximation \( |s| \) is also reduced in the DW, as will be shown below. Other effects that may change the magnetization and therefore also the resistivity are magnetostriction and the internal dipolar magnetic field. The former can change the exchange integral due to a change in lattice constant, caused by the domain wall. The latter directly affects the splitting. However we expect these effects to be smaller than those discussed here.
The electronic structure of a two band ferromagnet with non-collinear magnetization can be found from the Stoner Hamiltonian

\[ H = -\frac{\hbar^2}{2m} \nabla^2 + \mu B \mathbf{H}_{\text{ex}}(\mathbf{r}) \cdot \mathbf{\sigma}, \]  

(5)

where the three components of \( \mathbf{\sigma} \) are the Pauli spin matrices \( (\sigma_x, \sigma_y \text{ and } \sigma_z) \) and \( \mathbf{H}_{\text{ex}}(\mathbf{r}) \) is the exchange field as described above. We disregard the spin-orbit interaction and the Lorentz force due to the internal magnetization, because the DW magnetoresistance can experimentally be separated from the anisotropic magnetoresistance (AMR) and the ordinary magnetoresistance (OMR) \[6\].

We solve the eigenvalue problem for the Hamiltonian Eq. (5) by a local gauge transformation, assuming translational symmetry in the \( x \) and \( y \) directions and introducing the Fourier transform of the gradient of the magnetization direction \( \partial_z \phi(z) = \sum q \exp(iqz)a_q \) \[10,12\], where \( \phi(z) \) is the angle of magnetization in the rotation plane. After the gauge transformation the Hamiltonian (5) becomes

\[ \tilde{H} = H_0 + V, \]

where \( H_0 = \sum_{k\sigma} (\epsilon_{k\sigma} - \mu) c_{k\sigma}^\dagger c_{k\sigma} \) and the interaction \( V \) with the DW is specified in Ref. \[10,12\].

We proceed by calculating \( \delta \rho \), the difference in resistivity of the spin spiral compared to the single domain state. In the spin-spiral \( \partial_z \phi \) is constant everywhere. The Hamiltonian (5) is diagonalized in spin space by \( u_\pm = N_\pm [1, i(1 \mp \sqrt{1 + \alpha^2})/\alpha]^T \), where \( N_\pm \) is a normalization constant, \( \alpha = k_z a_0/p^2 \), and \( p^2 = 2m\Delta/\hbar^2 \) \[12\]. The eigenvalues are:

\[ E^\pm_k = \frac{\hbar^2}{2m} \left( k^2 + a_0^2 \pm \sqrt{k_z^2 a_0^2 + p^4} \right). \]

Due to the rotation of the direction of the spin-quantization axis

\[ u_\pm^\dagger \sigma_z u_\pm = \pm \frac{1}{\sqrt{1 + \alpha^2}}, \]

(7)

the spin-density in the direction of the local magnetization becomes

\[ s_\pm = \pm \frac{1}{V} \sum_k \frac{1}{\sqrt{1 + \alpha^2}} f(E^\pm_k - \mu), \]

(8)

whereas the electron densities of the spin-up and down eigenstates remain as \( n_\pm = (1/V) \sum_k f(E^\pm_k - \mu) \). The total spin-density is \( s = s_+ + s_- \).
At $T = 0 \text{K}$ we find

$$n_{\pm} = \frac{1}{6\pi^2} (k_{1\pm}^*)^3 \left(1 \pm \frac{E_w}{4\Delta}\right)$$

(9)

and

$$s_{\pm} = \pm \left( n_{\pm} - \frac{1}{30\pi^2} (k_{1\pm}^*)^5 \frac{h^2 E_w}{\Delta^2 4m} \right),$$

(10)

where $k_{1\pm}^* = (2m/h^2)^{1/2} \left(\mu \pm \Delta - \frac{1}{4} E_w \right)^{1/2}$. Without spin rotation ($a_0 \to 0$), $n$ and $s$ reduce to the familiar form $n_0^\pm = s_0^\pm = (1/6\pi^2)(2m/h^2)^{3/2}(\epsilon_F^\pm)^{3/2}$, where $\epsilon_F^\pm = \mu \pm \Delta$.

The numbers of spin-up and spin-down electrons have to be calculated self-consistently, since the effective spin splitting depends on the spin densities. A wide domain wall only weakly modifies the electronic structure. Therefore the reduced magnetization can be calculated by perturbation theory. Charge neutrality is taken into account as in Ref. [12] introducing a shift in the chemical potential $\mu = \mu_0 + \delta \mu$. The spin densities are denoted as $s_{\pm} = \pm n_0^\pm + \delta s_{\pm}$, and the splitting of the bands becomes $\Delta = \Delta_0 + \delta \Delta$, where $\mu_0$ is the single-domain chemical potential, $\delta \mu$ is the change in chemical potential caused by the DW, $\delta s_+$ ($\delta s_-$) is the change in spin density in the spin-up (down) band, and $\Delta_0 = (n_0^+ - n_0^-)J_0$.

The exchange splitting is modified as

$$\delta \Delta = (|\delta s_+| - |\delta s_-|)J_0 + (n_0^+ - n_0^-)\delta J,$$

(11)

where the first term is due to the reduced spin density in the spin spiral, and the second term reflects the reduced exchange interaction. We obtain:

$$\delta n_{\pm} = N_{\pm} \left( \delta \mu \pm \delta \Delta - \frac{1}{4} E_w \pm \frac{E_w}{6\Delta} \epsilon_F^\pm \right)$$

(12)

and

$$\delta s_{\pm} = \pm \left( \delta n_{\pm} - \frac{E_w}{15\Delta^2} N_{\pm} (\epsilon_F^\pm)^2 \right),$$

(13)

where $N_{\pm}$ ($N_\mp$) is the density of states of the spin-up (down) band at the Fermi energy.

With $n_{\pm} = 2N_{\pm}\epsilon_F^\pm/3$, which holds for parabolic bands, and Eqs. (11), (12), and (13), we can find the electron densities, which substituted into Eq. (2) yield:
\[\delta \rho = \frac{e^2 \rho_0^2}{m} \frac{E_w}{J_0 D \Delta_0} (\tau_+ - \tau_-) \times \left( \frac{\kappa \Delta_0^2}{\epsilon_F^+ + \epsilon_F^-} - \frac{\epsilon_F^+ + \epsilon_F^-}{24} + \frac{J_0}{\Delta_0} \frac{N_+(\epsilon_F^+)^2 - N_-(\epsilon_F^-)^2}{30} \right), \]

which is our main result. The first term is directly due to the reduced exchange interaction, the second term reflects the change in dispersion of the spin-up and spin-down band (last term in Eq. (6)), and the third term is due to the reduced spin-density (Eq. (7)). The first term is almost always bigger than the other two terms. Eq. (14) shows that the change in resistivity due to the reduced magnetization is of the same order as the effect of the spin-flip scattering [12], both scaling linearly with \(E_w\). The dimensionless denominator \(D = (N_+ + N_-)/(4N_+N_-J_0) - 1\) appears as a result of the self-consistency and is always positive (see below). \(D\) vanishes when the spontaneous magnetization disappears with decreasing \(J_0\). In that case the non-degenerate perturbation is not valid anymore, because \(\delta \rho \to \infty\).

In order to estimate the importance of the effect, we introduce dimensionless variables: \(\gamma \equiv k_+/k_-\) is the ratio of the Fermi wavevectors and \(\beta \equiv \tau_+ / \tau_-\) is the ratio of relaxation times. The dimensionless denominator then becomes \(D = (\gamma + \gamma^{-1} - 2)/3 > 0\). Microscopic theory [12] reveals that additional spin flip terms, which always increase the resistance, have to be added to the result derived here [16]. Fig. 2 shows \(\delta \rho / \rho_0\), as calculated from Eq. (14) (fine dashed line), due to the spin-flip scattering (taken from Ref. [12]) (coarse dashed line) and the sum of both effects (solid line) as a function of the ratio of relaxation times \(\beta\), for two different ratios of the Fermi wavelengths \(\gamma\). \(E_{w}/\epsilon_F = 4.3 \times 10^{-4}\) for Co has been estimated from \(\lambda_w = 15\) nm and \(k_F = 1\) \(\text{Å}^{-1}\). The exchange length is equal to \(\lambda_{ex} = 2.8\) \(\text{Å}\). In Fig. 2 we can see that the DW resistivity is between -5% and 10% for the parameters chosen, i.e. depending on the value of the exchange integral, the impurities, and the band structure of the material. Smaller spin splittings and larger asymmetries in the relaxation times increase the domain wall resistance.

Our results agree with the experimental finding that the DW resistance can be negative [4–8]. We have shown that the sign of the DW resistance depends on the difference of scat-
tering relaxation times, which can be positive or negative. This difference is to a large extent determined by the kind of impurities present in the sample, e.g. theoretical calculations for Cr impurities in Fe give $\beta = 0.11$, i.e. $\tau_+ - \tau_- < 0$ whereas for Cu impurities $\beta = 3.68$, i.e. $\tau_+ - \tau_- > 0$. Also for Ni the DW resistance can have both signs, since $0.2 < \beta < 30$. Experimentally the parameter $\beta$ is not available, but the ratio of the resistivities has been determined yielding for Co $0.1 < \rho_+ / \rho_- < 5$, Ni $0.05 < \rho_+ / \rho_- < 8$, and Fe $0.1 < \rho_+ / \rho_- < 10$, where $\rho_+ / \rho_- = \tau_- n_- m_+ / (n_+ \tau_+ m_-)$. (Only $\tau_+$ and $\tau_-$ depend on the type of impurities). Therefore, a reasonable agreement exists between experimental and theoretical values of the scattering relaxation times. This means that our predictions for the DW resistance can be experimentally tested by intentionally doping samples with different impurities and measuring the DW resistance as a function of type and concentration.

For realistic band structures the ballistic contribution to the domain wall resistivity from the spin flip terms is enhanced due to the near degeneracy of the different bands at the Fermi energy. Similarly, we expect an enhancement of the present effect, because the exchange splitting is more sensitive to the gradient of the magnetization when the bands are nearly degenerate.

In conclusion we have shown that a negative as well as a positive domain wall resistance is possible in the semiclassical regime, due to spin-dependent relaxation times and the spatial dependence of the magnetization.

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FIGURES

FIG. 1. A schematic picture of the magnetization angle as a function of position. The dashed line sketches a typical magnetization and the solid line the piece-wise spin-spiral approximation.

FIG. 2. The relative change in resistivity due to the reduced magnetization calculated here, the DW spin flip scattering [12], and the sum of both effects [16] as a function of the ratio of relaxation times. (a) is for the ratio of Fermi wave vectors $\gamma = 1.1$ and (b) $\gamma = 2$. 
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\[ \frac{\delta \rho}{\rho_0} (\%) \]

\( \beta = \frac{\tau_+}{\tau_-} \)

(a) \( \gamma = 1.1 \)

(b) \( \gamma = 2 \)

- Reduced M
- Spin-flip
- Total