Collective modes of asymmetric nuclear matter in Quantum HadroDynamics

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(March 30, 2022)

We discuss a fully relativistic Landau Fermi liquid theory based on the Quantum HadroDynamics (QHD) effective field picture of Nuclear Matter (NM). From the linearized kinetic equations we get the dispersion relations of the propagating collective modes. We focus our attention on the dynamical effects of the interplay between scalar and vector channel contributions. A beautiful “mirror” structure in the form of the dynamical response in the isoscalar/isovector degree of freedom is revealed, with a complete parallelism in the role respectively played by the compressibility and the symmetry energy. All that strongly supports the introduction of an explicit coupling to the scalar-isovector channel of the nucleon-nucleon interaction.

In particular we study the influence of this coupling (to a δ-meson-like effective field) on the collective response of asymmetric nuclear matter (ANM). Interesting contributions are found on the propagation of isovector-like modes at normal density and on an expected smooth transition to isoscalar-like oscillations at high baryon density.

Important “chemical” effects on the neutron-proton structure of the mode are shown. For dilute ANM we have the isospin distillation mechanism of the unstable isoscalar-like oscillations, while at high baryon density we predict an almost pure neutron wave structure of the propagating sounds.

PACS: 24.10.Cn, 24.10.Jv, 21.30.Fe, 24.30.Cz

I. INTRODUCTION

The QHD effective field model represents a very successful attempt to describe, in a fully consistent relativistic picture, equilibrium and dynamical properties of nuclear systems at the hadronic level [1–3]. Very nice results have been obtained for the nuclear structure of finite nuclei [4–6], for the NM Equation of State and liquid-gas phase transitions [7] and for the dynamics of nuclear collisions [8,9]. Relativistic Random-Phase-Approximation (RRPA) theories have been developed to study the nuclear collective response [10–15].

In this paper we present a relativistic linear response theory with the aim of a transparent connection between the collective dynamics and the coupling to various channels of the nucleon-nucleon interaction. In particular we will focus our attention on the dynamical response of asymmetric nuclear matter since one of the main points of our discussion is the relevance of the coupling to a scalar isovector channel, the virtual δ[α₀(980)] meson, not considered in the usual dynamical studies. Another point of interest is the dynamical treatment of the Fock terms, neglected in the usual Relativistic Mean Field (RMF) scheme of the papers cited before.

A relativistic extension of the Landau linear response theory of Fermi Liquids has been considered before just starting from the relativistic form of the Landau parameters [16–18]. We will show that the full dispersion relations obtained from the relativistic kinetic equations present some interesting corrections that cannot be neglected.

The main physics results are:

• The important effect of a δ-meson coupling on the isovector collective mode at saturation baryon density. This is of interest for the relativistic study of the Giant Dipole Resonance in heavy finite nuclei. It is important to note that the inclusion of Fock terms is acting in the same direction.

• The presence of noticeable “chemical effects” in the propagating collective oscillations, i.e. the charge symmetry of the “waves” is quite different from the asymmetry of the initial equilibrium matter. The effect is opposite for the unstable modes present at low densities, more proton rich and leading to the isospin distillation effect, and for the stable propagating sounds at high baryon density which appear mostly like “pure neutron waves”.

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A stimulating “mirror” structure of the isoscalar and isovector linear response, with the restoring forces given by the potential part respectively of the compressibility and the symmetry energy. The interplay between the scalar and vector meson effective fields in the dynamics is very similar for the two degrees of freedom, as already observed for static properties \[^{[13]}\]. The conclusion is that a consistent relativistic effective field model has to include on the same footing isoscalar and isovector meson fields, both scalar and vector.

Our results around normal density can be used as general guidelines in predicting the behaviour of volume collective modes in finite $\beta$–unstable nuclei. Similar study for asymmetric NM have been performed in Ref. \[^{[21]}\] using Skyrme–like interactions. Apart the difference in the used interactions, in particular for the symmetry terms, we will see similar results and interesting new relativistic effects.

In Sect.II we derive the kinetic equations in the general case of non-linear self-interacting terms, including the Fock corrections. We discuss the inclusion of the $\delta$-meson channel, also on the model parameters. In Sect.III we present the relativistic linear response equations. In Sect.IV we have a general discussion on the formal structure of the dispersion relations, the role played by the scalar/vector mesons and the comparison to non-relativistic cases. Results for isovector(-like) collective modes are presented in Sect.V, in particular for the asymmetry and baryon density effects. The isoscalar(-like) response is analysed in Sect. VI. Conclusions and outlooks can be found in Sect.VII.

II. KINETIC EQUATIONS FROM A QHD EFFECTIVE THEORY

We start from the QHD effective field picture of the hadronic phase of nuclear matter \[^{[1,3]}\]. In order to include the main dynamical degrees of freedom of the system we will consider the nucleons coupled to the isoscalar $\sigma$ and vector $\omega$ mesons and to the isovector scalar $\delta$ and vector $\rho$ mesons.

The Lagrangian density for this model, including non-linear isoscalar/scalar $\sigma$-terms \[^{[2]}\], is given by:

$$
\mathcal{L} = \bar{\psi} \left[ i \partial^{\mu} - g_\omega \gamma^{\mu} - g_{\rho} B_\mu \tau \right] \psi - \left( M - g_\sigma \phi - g_\delta \delta \right) \psi + \\
\frac{1}{2} \left( \partial_\mu \phi \partial^\mu \phi - m^2_\phi \phi^2 \right) - \frac{a}{4} \phi^4 - \frac{b}{4} \phi^4 - \frac{1}{4} W_{\mu\nu} W^{\mu\nu} + \frac{1}{4} m^2_\omega \phi^2 \phi^2 + \\
\frac{1}{2} \left( \partial_\mu \delta \cdot \partial^\mu \delta - m^2_\delta \delta^2 \right) - \frac{1}{4} G_{\mu\nu} \cdot G^{\mu\nu} + \frac{1}{2} m^2_\rho B_\mu \cdot B^\mu
$$

(1)

where $W^{\mu\nu}(x) = \partial^{\mu} \phi(x) - \partial^{\nu} \phi(x)$ and $G^{\mu\nu}(x) = \partial^{\mu} B(x) - \partial^{\nu} B(x)$.

Here $\psi(x)$ is the nucleon fermionic field, $\phi(x)$ and $\phi(x)$ represent neutral scalar and vector boson fields, respectively. $\delta(x)$ and $b(x)$ are the charged scalar and vector fields and $\tau$ denotes the isospin matrices.

From the Lagrangian Eq.(1) with the Euler procedure a set of coupled equations of motion for the meson and nucleon fields can be derived. The basic approximation in nuclear matter applications consists in neglecting all the terms containing derivatives of the meson fields, with respect to the mass contributions. Then the meson fields are simply connected to the operators of the nucleon scalar and current densities by the following equations:

$$
\hat{\Phi}/f_\sigma + A\hat{\Phi}^2 + B\hat{\Phi}^3 = \bar{\psi}(x)\psi(x) \equiv \hat{\rho}_{\bar{S}}
$$

(2)

$$
\hat{\phi}^\mu(x) = f_\omega \bar{\psi}(x)\gamma^\mu \psi(x) \equiv f_\omega \hat{J}_\mu ,
$$

$$
\hat{B}_\mu(x) = f_\rho \bar{\psi}(x)\gamma^\mu \tau \psi(x) ,
$$

$$
\hat{\delta}(x) = f_\delta \bar{\psi}(x) \tau \psi(x)
$$

(3)

where $\Phi = g_\sigma \phi$, $f_\sigma = (g_\sigma/m_\sigma)^2$, $A = a/g_\phi^2$, $B = b/g_\phi^4$, $f_\omega = (g_\omega/m_\omega)^2$, $f_\rho = (g_\rho/2m_\rho)^2$, $f_\delta = (g_\delta/m_\delta)^2$.

For the nucleon fields we get a Dirac-like equation. Indeed after substituting Eqs.(23) for the meson field operators, we obtain an equation which contains only nucleon field operators. All the equations can be consistently solved in a Mean Field Approximation (RMF), where most applications have been performed, in particular in the Hartree scheme \[^{[13]}\].

The inclusion of Fock terms is conceptually important \[^{[22,23]}\], since it automatically leads to contributions to various channels, also in absence of explicit coupling terms. We will discuss this point later. A thorough study of the Fock contributions in a QHD approach with non-linear self-interacting terms has been recently performed \[^{[24]}\], in particular for asymmetric matter \[^{[23]}\].

The present approximation implies that retardation and finite range effects in the exchange of mesons between nucleons are neglected. Nevertheless, thanks to the small Compton wave–lengths of the mesons $\sigma$, $\omega$, and $\rho$,...
with the effective coupling functions given by Eqs. [3] are quite reasonable. For light mesons such as pions this approximation is not justified. However in this case a perturbative expansion in the pion-nucleon coupling constant seems to be reasonable [22]. Moreover it has been shown that the inclusion of pions does not change qualitatively the description of nuclear matter around normal conditions [22].

We remark that the kinetic approach discussed here is fully consistent with the previous approximation. We are concerned with a semiclassical description of nuclear dynamics, so that the nuclear medium is supposed to be in states for which the nucleon scalar and current densities are smooth functions of the space–time coordinates.

Within a mean field picture of the QHD model we focus our analysis on a description of the many-body nuclear system in terms of one–body dynamics. This is enough for the scope of the paper. Correlation effects can be effectively included at the level of coupling constants, as noted in the discussion of the results.

We will perform the many-body calculations in the quantum phase-space introducing the Wigner transform of the one-body density matrix for the fermion field [24,25].

The one–particle Wigner function is defined as:

$$[\tilde{F}(x,p)]_{\alpha\beta} = \frac{1}{(2\pi)^3} \int d^4Re^{-ip\cdot R} \langle \bar{\psi}_\beta(x + \frac{R}{2})\psi_\alpha(x - \frac{R}{2}) \rangle,$$

where $\alpha$ and $\beta$ are double indices for spin and isospin. The brackets denote statistical averaging and the colons denote normal ordering. The Wigner function is a matrix in spin and isospin spaces; in the case of asymmetric NM it is useful to decompose it into neutron and proton components. Following the treatment of the Fock terms in non-linear asymmetric NM it is useful to decompose it into neutron and proton components. Following the treatment of the Fock terms in non-linear QHD introduced in Ref. [24,25], we obtain for the Wigner function the following kinetic equation:

$$\frac{i}{2} \partial_\mu \gamma^\mu \tilde{F}^{(i)}(x,p) + \gamma^\mu p^*_\mu \tilde{F}^{(i)}(x,p) - M^*_i \tilde{F}^{(i)}(x,p) + \frac{i}{2} \Delta \left[ \tilde{f}_\omega j_\mu(x) \gamma^\mu \hat{\pm} \hat{f}_\sigma j_\mu(x) \gamma^\mu - \hat{f}_\sigma \rho S(x) \mp \hat{f}_\delta \rho S^3(x) \right] \tilde{F}^{(i)}(x,p) = 0, \quad i = n, p$$

(4)

where $\Delta = \partial_x \cdot \partial_p$, with $\partial_x$ acting only on the first term of the products. Here $\rho S^3 = \rho S_p - \rho S_n$ and $j_\mu(x) = j_\mu^S(x) - j_\mu^n(x)$ are the isovector scalar density and the isovector baryon current, respectively. We have defined the kinetic momentum and effective masses, as:

$$p^*_\mu = p_\mu - \hat{f}_\omega j_\mu(x) \pm \hat{f}_\sigma \rho S(x) \mp \hat{f}_\delta \rho S^3(x),$$

$$M^*_i = M - \hat{f}_\sigma \rho S(x) \pm \hat{f}_\delta \rho S^3(x),$$

(5)

with the effective coupling functions given by:

$$\tilde{f}_\sigma = \frac{\Phi}{\rho S} - \frac{1}{8} \frac{d\Phi(x)}{d\rho S(x)} - \frac{1}{2} \frac{Tr\tilde{F}^2(x) d^2\Phi(x)}{d\rho S^2(x)} + \frac{1}{2} \hat{f}_\omega + \frac{3}{2} \hat{f}_\rho - \frac{3}{8} \hat{f}_\delta,$$

$$\tilde{f}_\omega = \frac{1}{8} \frac{d\Phi(x)}{d\rho S(x)} + \frac{5}{4} \hat{f}_\omega + \frac{3}{4} \hat{f}_\rho + \frac{3}{8} \hat{f}_\delta,$$

$$\tilde{f}_\delta = - \frac{1}{8} \frac{d\Phi(x)}{d\rho S(x)} + \frac{1}{2} \hat{f}_\omega - \frac{1}{2} \hat{f}_\rho + \frac{9}{8} \hat{f}_\delta,$$

$$\tilde{f}_\rho = \frac{1}{8} \frac{d\Phi(x)}{d\rho S(x)} + \frac{1}{4} \hat{f}_\omega + \frac{3}{4} \hat{f}_\rho - \frac{1}{8} \hat{f}_\delta,$$

(6)

where $8 \; Tr\tilde{F}^2(x) = \rho S^2 + \rho S j_\mu^S j^\mu + \rho S^3 j_\mu^S j_\mu^S$. We remind that we are dealing with a transport equation so the currents and densities, in general, are varying functions of the space–time, at variance with the case of nuclear matter at equilibrium.

The expression of Eq. [6] for the effective mass, embodies an isospin contribution from Fock terms also without a direct inclusion of the $\delta$ meson in the Lagrangian. The usual RMF approximation (Hartree level) is covered by the Hartree-Fock results, one has has only to change the coupling functions $\tilde{f}_i (i = \sigma, \omega, \rho, \delta)$, Eqs. [6], with the coupling constants $f_i$.

**Equilibrium properties: the nuclear Equation of State**

In the following we will study the collective modes. In order to analyze the results it is essential to relate them to the equation of state (EOS), that we will briefly discuss in the following. In particular for the collective modes in asymmetric nuclear matter it is important the behaviour of the symmetry energy $E_{sym}$. 


The energy density and pressure for symmetric and asymmetric nuclear matter and the \(n,p\) effective masses can be self-consistently calculated just in terms of the four boson coupling constants, \(f_i = \frac{g_i^2}{M_i}\), \(i = \sigma, \omega, \rho, \delta\), and the two parameters of the \(\sigma\) self-interacting terms, \(A = \frac{a_\sigma^2}{M_\sigma}\) and \(B = \frac{b_\sigma}{g_\sigma^2}\), see ref. [24,25].

The isoscalar meson parameters are fixed from symmetric nuclear matter properties at \(T = 0\): saturation density \(\rho_0 = 0.16 fm^{-3}\), binding energy \(E/A = -16 MeV\), nucleon effective mass \(M^* = 0.7M_N\) (\(M_N = 939 MeV\)) and incompressibility \(K_V = 240 MeV\) at \(\rho_0\). The fitted \(f_\sigma, f_\omega, A, B\) parameters are reported in Table I. They have quite standard values for these minimal non-linear RMF models. \(Set I\) and \(Set II\) correspond to the best parameters within a non-linear Hartree calculation, respectively with the \(\rho - (Set I, NLH + \rho)\) and with the \(\rho + \delta\) \((Set II, NLH + (\rho + \delta))\) couplings in the isovector channel (see the discussion in ref. [19]). \(NLHF\) stands for the non-linear Hartree-Fock scheme described before.

**Table I.** Parameter sets.

| parameter | \(f_\sigma\) \((fm^2)\) | \(f_\omega\) \((fm^2)\) | \(f_\rho\) \((fm^2)\) | \(f_\delta\) \((fm^2)\) | \(A\) \((fm^{-1})\) | \(B\) |
|-----------|----------------|----------------|----------------|----------------|----------------|-------|
| \(Set I\) | 11.27 | same | 9.15 | 15.73 | 0.022 | -0.0039 |
| \(Set II\) | 6.48 | same | 3.22 | 10.55 | 0.00 | 0.00 |

In the table we report also the \(NL3\) parametrization, widely used in nuclear structure calculations [28]. We remind that the \(NL3\)-saturation properties for symmetric matter are chosen as \(\rho_0 = 0.148 fm^{-3}\), \(M^* = 0.6M_N\), \(K_V = 271.8 MeV\). The symmetry parameter is \(a_4 = 37.4 MeV\).

The symmetry energy in \(ANM\) is defined from the expansion of the energy per nucleon \(E(\rho_B, \alpha)\) in terms of the asymmetry parameter \(\alpha\) defined as

\[
\alpha \equiv -\frac{\rho_B}{\rho_B} = \frac{\rho_B - \rho_{Bn}}{\rho_B} = \frac{N - Z}{A}.
\]

We have

\[E(\rho_B, \alpha) = \frac{\epsilon(\rho_B, \alpha)}{\rho_B} = E(\rho_B) + E_{sym}(\rho_B)\alpha^2 + O(\alpha^4) + ...\]  

and so in general

\[E_{sym} = \frac{1}{2} \frac{\partial^2 E(\rho_B, \alpha)}{\partial \alpha^2} |_{\alpha = 0} = \frac{1}{2} \frac{\partial^2 \epsilon}{\partial \rho_B^2} |_{\rho_B = 0} \]  

In the Hartree case an explicit expression for the symmetry energy can be easily derived [24,19]

\[E_{sym}(\rho_B) = \frac{1}{6} \frac{k_F^2}{E_F^4} + \frac{1}{2} \frac{f_\rho}{E_F} - \frac{1}{2} \frac{f_\delta}{E_F^2} \left[ 1 + \frac{f_\delta}{E_F} A(k_F, M^*) \right] \equiv E_{sym}^{kin} + E_{sym}^{pot} \]  

where \(k_F\) is the nucleon Fermi momentum corresponding to \(\rho_B\), \(E_F^p \equiv \sqrt{(k_F^2 + M^{*2})}\) and \(M^*\) is the effective nucleon mass in symmetric \(NM\), \(M^* = M_N - g_\sigma \phi\).

The integral

\[A(k_F, M^*) = \frac{4}{(2\pi)^3} \int d^3k \frac{k^2}{(k^2 + M^*2)^{3/2}} = 3 \left( \frac{\rho_S}{M^*} - \frac{\rho_B}{E_F} \right) \]

We remark that \(A(k_F, M^*)\) is certainly very small at low densities, and actually it can be still neglected up to a baryon density \(\rho_B \simeq 3 \rho_0\) (see ref. [19]).
Then in the density range of interest here we can use, at the leading order, a much simpler form of the symmetry energy, with transparent $\delta$-meson effects:

$$E_{sym}(\rho_B) = \frac{1}{6} \frac{k_F^2}{E_F} + \frac{1}{2} \left[ f_\rho - f_\delta \left( \frac{M^*}{E_F} \right)^2 \right] \rho_B$$

We see that, when the $\delta$ is included, the observed $a_4$ value actually assigns the combination $[f_\rho - f_\delta (\frac{M^*}{E_F})^2]$ of the $(\rho, \delta)$ coupling constants. If $f_\delta \neq 0$ we have to increase the $\rho$-coupling (see Fig. 1 of ref. [29]). In our calculations we use the value $a_4 = 32$ MeV.

In Table I the Set I corresponds to $f_\delta = 0$. In the Set II $f_\delta$ is chosen as $2.0 fm^2$. Although this value is relatively well justified, [19], we stress that aim of this work is just to show the main qualitative new dynamical effects of the $\delta$-meson coupling. In order to have the same $a_4$ we must increase the $\rho$-coupling constant of a factor three, up to $f_\rho = 2.8 fm^2$. Now the symmetry energy at saturation density is actually built from the balance of scalar (attractive) and vector (repulsive) contributions, with the scalar channel becoming weaker with increasing baryon density [19]. This is indeed the isovector counterpart of the saturation mechanism occurring in the isoscalar channel for the symmetric nuclear matter. From such a scheme we get a further strong fundamental support for the introduction of the $\delta$-coupling in the symmetry energy evaluation.

![FIG. 1. Total (kinetic+potential) symmetry energy as a function of the baryon density. Long dashed: Hartree ($NLH + \rho$). Dotted: Hartree ($NLH + \rho + \delta$). Solid: Hartree-Fock ($NLHF$).](image)

![FIG. 2. Baryon density variation of the isovector effective coupling when the Fock terms are included.](image)

Details of the calculation can be found in Refs. [25,19], here we show only the result corresponding to the parametrizations (Sets I, II and NLHF) that we will use to investigate the dynamical response. In Fig. 1 we report the symmetry energy as a function of the baryon density in the Hartree case ($\rho$ and $\rho + \delta$) and with the Fock terms ($\rho + \delta$ case). When the $\delta$ “channel” is included the behaviour is stiffer for the relativistic mechanism discussed before, see also ref. [19].

When the Fock terms are evaluated the new “effective” couplings Eqs. (1) naturally acquire a density dependence. This is shown in Fig. 2 for the isovector terms. The decrease of the “effective” $\rho$ coupling at
functions become

\[
\frac{i}{2} \partial_\mu \gamma^\mu \hat{G}_1(x,p) + (\Pi_\mu - \hat{f}_\rho b_\rho) \gamma^\mu \hat{G}_1(x,p) - M_1^2 \hat{G}_1(x,p) = \left(1 - \frac{i}{2} \Delta\right) (\hat{F}(x) + \hat{F}_3(x)) \hat{H}_1(p),
\]

for protons, and

\[
\frac{i}{2} \partial_\mu \gamma^\mu \hat{G}_2(x,p) + (\Pi_\mu + \hat{f}_\rho b_\rho) \gamma^\mu \hat{G}_2(x,p) - M_2^2 \hat{G}_2(x,p) = \left(1 - \frac{i}{2} \Delta\right) (\hat{F}(x) - \hat{F}_3(x)) \hat{H}_2(p),
\]

for neutrons, where \( M_1 = M - \hat{f}_\rho \rho S - \hat{f}_\delta \rho S_3 \) and \( M_2 = M - \hat{f}_\sigma \rho S + \hat{f}_\delta \rho S_3 \). The quantities \( \hat{F}(x) \) and \( \hat{F}_3(x) \) are the isoscalar and the isovector components of the self–consistent field:

\[
\hat{F}(x) = -8 \hat{f}_\sigma G(x) + 8 \hat{f}_\rho \gamma_\mu G^\mu(x) - 8 \frac{\partial \hat{f}_\sigma}{\partial \rho S} \rho S G(x) - 8 \frac{\partial \hat{f}_\rho}{\partial \rho S} \rho S G^\mu(x)
- 8 \frac{\partial \hat{f}_\rho}{\partial \rho S} \rho S G_3^\mu(x) + 8 \frac{\partial \hat{f}_\sigma}{\partial \rho S} \gamma_\mu j^\mu G(x),
\]

\[
\hat{F}_3(x) = -8 \hat{f}_\delta G_3(x) + 8 \hat{f}_\rho \gamma_\mu G^\mu_3(x) - 8 \frac{\partial \hat{f}_\rho}{\partial \rho S} \rho S_3 G(x) + 8 \frac{\partial \hat{f}_\delta}{\partial \rho S} \gamma_\mu j^\mu_3 G(x).
\]

III. LINEAR RESPONSE EQUATIONS

In this section we study collective oscillations that propagate in cold nuclear matter due to the mean field dynamics. In some sense we follow a relativistic extension of the method introduced by Landau to study liquid–\(^3\)He [30–32] and recently applied to investigate stable and unstable modes in nuclear matter [20,33,34].

The starting point is the kinetic transport equation Eq. (9). We look for solutions corresponding to small oscillations of \( \hat{F}(x,p) \) around the equilibrium value. Therefore we put

\[
\hat{F}(x,p) = \hat{H}(p) + \hat{G}(x,p)
\]

where \( \hat{H}(p) \) is the Wigner function at equilibrium (see Appendix) and \( \hat{G}(x,p) \) represent its fluctuations.

For the equilibrium state the Wigner function contains only the isoscalar term and the third component of the isovector term. We limit ourselves to study excited states in these channels. Therefore we consider isovector density fluctuations with \( m_T = 0 \) only, i.e. we do not study processes where a neutron converts into a proton or vice versa. In a linear response scheme oscillations in the aforementioned channels are decoupled from the remaining ones (\( m_T = \pm 1 \)).

In the linear approximation, i.e. neglecting terms of second order in \( \hat{G}(x,p) \), the equations for the Wigner functions become

\[
\frac{i}{2} \partial_\mu \gamma^\mu \hat{G}_1(x,p) + (\Pi_\mu - \hat{f}_\rho b_\rho) \gamma^\mu \hat{G}_1(x,p) - M_1^2 \hat{G}_1(x,p) = \left(1 - \frac{i}{2} \Delta\right) (\hat{F}(x) + \hat{F}_3(x)) \hat{H}_1(p),
\]

for protons, and

\[
\frac{i}{2} \partial_\mu \gamma^\mu \hat{G}_2(x,p) + (\Pi_\mu + \hat{f}_\rho b_\rho) \gamma^\mu \hat{G}_2(x,p) - M_2^2 \hat{G}_2(x,p) = \left(1 - \frac{i}{2} \Delta\right) (\hat{F}(x) - \hat{F}_3(x)) \hat{H}_2(p),
\]

for neutrons, where \( M_1 = M - \hat{f}_\sigma \rho S - \hat{f}_\delta \rho S_3 \) and \( M_2 = M - \hat{f}_\sigma \rho S + \hat{f}_\delta \rho S_3 \). The quantities \( \hat{F}(x) \) and \( \hat{F}_3(x) \) are the isoscalar and the isovector components of the self–consistent field:

\[
\hat{F}(x) = -8 \hat{f}_\sigma G(x) + 8 \hat{f}_\rho \gamma_\mu G^\mu(x) - 8 \frac{\partial \hat{f}_\sigma}{\partial \rho S} \rho S G(x) - 8 \frac{\partial \hat{f}_\rho}{\partial \rho S} \rho S G^\mu(x)
- 8 \frac{\partial \hat{f}_\rho}{\partial \rho S} \rho S G_3^\mu(x) + 8 \frac{\partial \hat{f}_\sigma}{\partial \rho S} \gamma_\mu j^\mu G(x),
\]

\[
\hat{F}_3(x) = -8 \hat{f}_\delta G_3(x) + 8 \hat{f}_\rho \gamma_\mu G^\mu_3(x) - 8 \frac{\partial \hat{f}_\rho}{\partial \rho S} \rho S_3 G(x) + 8 \frac{\partial \hat{f}_\delta}{\partial \rho S} \gamma_\mu j^\mu_3 G(x).
\]

The Hartree approximation is recovered by vanishing all the derivatives inside the quantities \( \hat{F}(x) \) and \( \hat{F}_3(x) \), except \( \frac{\partial \hat{f}_\rho}{\partial \rho S} \), since still \( \hat{f}_\sigma = \Phi(\rho S)/\rho S \).

In order to obtain the equations for the collective oscillations we multiply Eqs. (13) and (14) by \( \gamma \). After performing the traces, and we equate to zero both the real and imaginary parts of the result. Furthermore, by Fourier transforming and integrating over four–momentum, we get the set of equations for the scalar and vector fluctuation of each species (\( i = 1, 2 \) for proton, neutron respectively):

\[
\sum_{j=1}^{2} \left\{ \delta_{ij} + \left( \frac{\rho S_i}{M_i} - 4 \frac{C^{(i)}(k)}{k} \right) D_{ij}^S + 4 \frac{C^{(i)}(k)}{k} B_{ij}^V \right\} G_{(j)}(k) + \left[ 4 \frac{C^{(i)}(k)}{k} D_{ij}^V + \left( \frac{\rho S_i}{M_i} - 4 \frac{C^{(i)}(k)}{k} \right) B_{ij}^S \right] G_{(j)}^\mu(k) \right\} = 0,
\]
the Eqs. (19, 20) both proton/neutron densities and currents are appearing. isovector components are mixed in the normal modes. Here this can be argued by the fact that in each of (normal modes) can be derived. It should be remarked that in asymmetric nuclear matter isoscalar and vanishes, i.e. the dispersion relations. In correspondence the neutron/proton structure of the eingenvectors plainer structure of the equations due to the constant value of all couplings, except \( \tilde{\sigma} \) usual Hartree approximation (RMF) the isoscalar and isovector densities and currents (for details, see [24]), then it is even more general.

The explicit expressions of the coefficients \( C^{(i)}(k), C^{(j)}(k), \) and \( C^{(i)}_{\lambda}(k) \), together with some of their properties can be found in the Appendix.

We notice, however, that the formalism developed in Hartree-Fock approximation allows to achieve the set of equations valid also for an approach to QHD based on a dependence on the scalar density of all couplings [25]. In particular in our case the coupling functions to the scalar-isoscalar channel depend on all the isoscalar and isovector densities and currents (for details, see [24]), then it is even more general. The set of equations developed in the Hartree-Fock approximation recover the ones corresponding to the usual Hartree approximation (RMF). As already mentioned, it is easily obtained by considering the coupling \( f_1 \) to each channel equal to the coupling constant of the corresponding meson. The result is an appreciable plainer structure of the equations due to the constant value of all couplings, except \( \tilde{f}_\sigma \) [36]. The Eqs. (17) and (18) can be reduced to:

\[
\sum_{j=1}^{2} \left\{ \delta_{i,j}g^i_\lambda + \left( \frac{\rho S_{i}}{M_i}g^i_\lambda + 4 C^{(i)}_{\lambda}(k) \right) D^S_{ij} - 4 C^{(i)}(k) B^S_{\mu ij} \right\} G^\mu_{(j)}(k) = 0 , \tag{19}
\]

\[
\sum_{j=1}^{2} \left\{ \delta_{i,j}g^i_\mu + \left( \frac{\rho S_{i}}{M_i}g^i_\mu + 4 C^{(i)}_{\mu}(k) \right) D^V_{ij} - 4 C^{(i)}(k) B^V_{\mu ij} \right\} G^\mu_{(j)}(k) = 0 , \tag{20}
\]

with \( D^S_{ij} = \frac{\partial f_\sigma}{\partial \rho S} + (-1)^{i+j} f_\delta, \ D^V_{ij} = f_\omega + (-1)^{i+j} f_\rho \).

The physics effects appear more transparent and we will follow the Hartree scheme in the next sections keeping well in mind that the Fock contributions can be easily included. We expect to have some extra contributions in the various interaction channels without qualitative modifications of the physical response.

The normal collective modes are plane waves, characterized by the wave vector \( (k^i = (k^0, 0, 0, | k |)) \), they are determined by solving the set of homogeneous Eqs. (13-20). The solutions correspond only to longitudinal waves and do not depend on \( k^0 \) and \( | k | \) separately, but only on the ratio

\[ v_s = \frac{k^0}{| k |} . \]

The sound velocities are given by values of \( v_s \) for which the relevant determinant of the set (Eqs. (17-18)) vanishes, i.e. the dispersion relations. In correspondence the neutron/proton structure of the eingenvectors (normal modes) can be derived. It should be remarked that in asymmetric nuclear matter isoscalar and isovector components are mixed in the normal modes. Here this can be argued by the fact that in each of the Eqs. [13-20] both proton/neutron densities and currents are appearing.

However we remind that one can still identify isovector-like excitations as the modes where neutrons and protons move out of phase, while isoscalar-like modes are characterized by neutrons and protons moving in phase [20] [37].
IV. THE ROLE OF SCALAR/VECTOR FIELDS IN THE DYNAMICAL RESPONSE

Before showing numerical results for the dynamical response of asymmetric nuclear matter, in various baryon density regions and using the different effective interactions, we would like to analyse in detail the structure of the relativistic linear response theory in order to clearly pin down the role of each meson coupling.

Isovector Response

One may expect that once $a_4$ is fixed, the velocity of sound is also fixed [10]. On the other hand our results clearly indicate a different dynamical response with or without the $\delta$-meson channel, for interactions which give exactly the same $a_4$ parameter, see Figs.(3,4) in the following. In order to get a clear understanding of this effect we will consider the case of symmetric nuclear matter in the Hartree scheme, where the dispersion relations are assuming a transparent analytical form.

For symmetric $NM$ the densities, the effective masses and the coefficients $C^{(i)}(k)$, $C_\lambda^{(i)}(k)$ and $C_\rho^{(i)}(k)$ are equal for protons and neutrons. Now it is also possible to decouple the collective modes into isovector modes, [20]. After a straightforward rearrangement of Eqs. (19), (20), we have for the isovector modes:

$$
\delta \rho_3 + \left[ \frac{\rho_S}{M} - 8 \frac{C_{00}(k)}{M^*} (1 - v_s^2) \right] f_\rho \delta \rho_3 - 8 f_\delta C_0(k) \delta \rho_{S3} = 0
$$

$$
\delta \rho_{S3} + \left[ \frac{\rho_S}{M^*} - 8 C(k) \right] f_\delta \delta \rho_{S3} + 8 f_\rho C_0(k) (1 - v_s^2) \delta \rho_3 = 0 .
$$

(21)

We stress that the structure is the same for the isoscalar excitations, of course one has to change the isovector fluctuations with the isoscalar ones ($\delta \rho_B, \delta \rho_S$), and the coupling constants of isovector mesons with those of the isoscalar mesons [30].

Note that in this case to find the zero–sound velocity one has to evaluate determinant of a $2 \times 2$ matrix (and not a $4 \times 4$), hence the condition for having a solution can be written as

$$
1 + N_F \left[ f_\rho (1 - v_s^2) - f_\delta M^2 E_F^{-2} \left( 1 - f_\delta A(k_F, M^*) - f_\rho \frac{\rho_S}{M^*} v_s^2 \right) \right] \varphi(s) = 0
$$

(22)

here $N_F = \frac{2K_F E_F^*}{\pi}$ is the density of states at the Fermi surface and $s \equiv v_s/v_F$. To get Eq.(22) we have used the expression for $C(k)$, $C_0(k)$ and $C_{00}(k)$ in terms of the Lindhard function $\varphi(s)$ (see Appendix). The quantity $A(K_F, M^*)$ is the same integral discussed in Eqs. (9,10,11).

At this point we can make the following approximation

$$
v_s^2 \approx v_F^2 = \frac{k_F^2}{E_F^2},
$$

to evaluate the expression inside the square brackets. Looking at Fig.s (3,4,5) this is a good approximation within a 3%. The Eq.(22) assumes a quite clear form:

$$
1 + \frac{6 E_F^*}{k_F^2} \left[ E_{sym}^{pot} - f_\rho \frac{k_F^2}{2 E_F^*} \left( 1 - f_\delta \frac{M^*}{E_F^2} \rho_B \right) \rho_B \right] \varphi(s) = 0 .
$$

(23)

where the potential part of the symmetry energy explicitly appears in the dispersion relations, but joined to an important correction term which shows a different $f_\rho, f_\delta$ structure with respect to that of $E_{sym}^{pot}$, Eqs.(3, 4). We can easily have interactions with the same $a_4$ value at normal density but with very different isovector response. E.g. when we include the $\delta$ channel we know that we have to increase the $f_\rho$ coupling in order to have the same $a_4$, see the discussion of the Eqs.(3, 4), but now the “restoring force” (coefficient of the Lindhard function in the Eq.(23)) will be strongly reduced.

Equation (23) suggests to define an effective symmetry energy like

$$
E_{sym}^{*} = E_{sym}^{pot} - f_\rho \frac{k_F^2}{2 E_F^*} \left( 1 - f_\delta \frac{M^*}{E_F^2} \rho_B \right) \rho_B .
$$

(24)

which acts as a restoring force for the isovector mode. We can see that once the symmetry energy is fixed its effect on the dynamical response depend on the strength of each isovector field. In particular we can easily
verify that the $f_\delta$ factor inside bracket in Eq.(24) is a second order correction and the “leading contribution” to the reduction of $\Delta E_{sym} = E_{sym}^\star - E_{sym}$ is essentially given by the coupling of the $\rho$-meson field. On the other hand we know [23] that once the symmetry energy at saturation density $a_4$ is fixed, the change of $f_\rho$ only due to the strenght of $f_\delta$. We have seen from Table I that $f_\rho$ can go from 1 $fm^2$, if we switch off the $\delta$-channel, to $f_\rho = 2.8$ $fm^2$. In terms of the effective symmetry energy this means (if we consider the dynamical response at $\rho_0$), $\Delta E_{sym}^\star \approx 4$ $MeV$ if $f_\delta \approx 2.0$ $fm^2$. This “softening” of the restoring force easily accounts for the decrease of the sound velocity ($v_s/v_{Fn}$) seen in Fig.3 for symmetric nuclear matter, $\alpha = 0$, when we pass from $NLH - \rho$ and $NLH - (\rho + \delta)$.

**Isoscalar Response**

As already remarked, we like to note that for symmetric $NM$ there is a tight analogy between the isoscalar and the isovector response in the RMF approach. In the isoscalar degree of freedom the compressibility will play the same role of the symmetry energy in the dispersion relation equations. Also in this case we will have an important correction term coming from the interplay of the scalar and vector channel.

The Eq.(22) now becomes [36]:

$$1 + N_F \left[ f_\omega(1 - v_s^2) - f_\sigma M^* F_M \left( 1 - f_\sigma A(k_F, M^*) - f_\omega \frac{\rho S}{M^*} v_s^2 \right) \right] \varphi(s) = 0 \quad (25)$$

that can be reduced to the isoscalar equivalent of the Eq.(23):

$$1 + \frac{E_F^*}{3 k_F^2} \left[ K_{NM}^{pot} - 9 f_\omega k_F^2 \frac{k_F^2}{E_F^*} \left( 1 - f_\sigma \frac{M^*}{F_M^*} \rho B \right) \rho B \right] \varphi(s) = 0 \quad (26)$$

where the $K_{NM}^{pot}$ is the potential part of the nuclear matter compressibility that in the Hartree scheme has the simple structure [26] (see also Eq.(16) of ref. [10])

$$K_{NM}(\rho_B) = \frac{3 k_F^2}{E_F^*} + 9 \left[ f_\omega - f_\sigma \left( \frac{M^*}{F_M^*} \right)^2 \right] \rho_B \equiv K_{NM}^{kin} + K_{NM}^{pot} \quad (27)$$

By means of such an analogy, the previous discussion can be extended to isoscalar oscillations with the role of $E_{sym}$ now “played” by the compressibility. In this case however one always takes into account both the scalar and vector channel in any RMF models. However the coupling constant $f_\omega$ can assume very different values depending on the required value for effective masses $M^*_0$. This is easy to understand since in the RMF limit the saturation binding energy has the simple form

$$E/A(0) = E_F^* + f_\omega \rho_B(0) - M_N$$

where $M_N$ is the bare nucleon mass. So we see that the same saturation values of $\rho_B$, $E/A$ when decrease $M^*_0$ we have to increase $f_\omega$. Just to get an idea, we mention that among the most common used RMF parametrizations, $f_\omega$ can go from $3.6$ $fm^2$ of NL2 [3] to $10.2$ $fm^2$ of NL3 [12], just decreasing the effective mass at saturation from $0.82 M_N$ to $0.6 M_N$. In some sense this is obvious because if two EOS have different effective masses even if the compressibility is equal the dynamical behaviour is expected to be different. This is a very general feature present also in non-relativistic approaches.

From studies on monopole resonances in finite nuclei with RMF it seems that a higher value of compressibility is required respect to non-relativistic calculations. Many authors state that this certainly demands for a clarification [13][14]. Even if the monopole resonance is not directly connected to the isoscalar collective mode in nuclear matter, our discussion nicely suggests to look at the interplay between effective mass and compressibility. For example we can estimate by means of Eq.(27) that we can have a shift between the compressibility and the “effective compressibility” of the order of $\sim 100$ $MeV$ among different parametrization with the same $K_{NM}$. Therefore model with $K \sim 300$ $AMeV$ can reproduce the same frequencies of other models with $K \sim 200$ $AMeV$ (and a slightly larger $M^*_0$).

**Landau Parameters**

We would like to briefly discuss the relativistic equations for collective modes in terms of the Landau parameters. Interesting features will appear from the comparison to the non-relativistic analogous case. We
will focus first on the isovector response, but, as already clearly shown before, the structure of the results will be absolutely similar in the isoscalar channel.

The general non–relativistic expression for the isovector modes can be found in Ref. [31]:

\[
1 + \left[F^a_0 + \frac{F^a_1}{1 + 1/3 F^2_i} s^2 \right] \varphi(s) = 0
\]  

(28)

where \( F^a_0 \) is the “isovector” combination of the Landau \( F_0 \) parameters for neutrons and protons \( F^a_0 = F^{nn}_a - F^{np}_a \), that can be expressed in terms of density variations of the chemical potentials:

\[
F_{qq}^a \equiv \frac{\partial \mu_q}{\partial \rho_{qq}} N_q - \delta_{qq'} , \quad N_q \equiv \frac{k_{F_q} E^{*}_{F_q}}{\pi^2} , \quad q = n, p
\]  

(29)

\( F^a_1 \) are the equivalent for the momentum dependent part of the mean field. In the relativistic approach, for symmetric nuclear matter, we get:

\[
F^a_0 = F_0^\rho - F_0^\delta \frac{M^*2}{E^{*2}_{F_i}} \frac{1}{1 + f^a_{\delta} A(k_F, M^*)}
\]

\[
F^a_1 = -F_0^\rho \frac{v^2_{F_i}}{3 F_0^\rho v^2_{F_i}} ,
\]  

(30)

where \( F_i = N_F f_i (i = \rho, \delta) \) with \( N_F = 2N_{n,p} \). Note that the \( F^a_0 \) contribution comes only from the vector coupling. By using the expression \( E_{sym}^{pot} \), Eq.(10), we can write Eq.(28) in the same form of Eqs.(22, 23). The result is a similar expression but with the lack of the term in \( f_\delta \) inside the brackets in Eq.(23). As said, this is not the leading term, however around saturation density it amounts to about a 10% of the total correction.

Moreover turning to the analogy with isoscalar channel the coupling of the \( \sigma \) field is now much larger and this purely relativistic contribution could be up to a 20%. We underline this point because generally the linear response in \( RMF \) is discussed calculating the Landau parameters and then using these estimations directly into the non–relativistic expression for collective modes [16,18].

In conclusion from the analysis in terms of the Landau parameters, we can describe the effect of the scalar-vector coupling competition previously discussed in the following way. The symmetry energy fixes the \( F^a_0 \), in fact:

\[
E_{sym} = \frac{k^2_{F_i}}{6 E^*_{F_i}} (1 + F^a_0)
\]  

(31)

but in the dynamical response enters also the \( F^a_1 \), linked to the momentum dependence of the mean field, mostly given by the vector meson coupling. The results are completely analogous in the isoscalar channel, with the compressibility given by

\[
K_{NM} = \frac{3 k^2_{F_i}}{E^*_{F_i}} (1 + F^a_0)
\]  

(32)

with “isoscalar” combination \( F^a_1 = F^{nn}_0 + F^{np}_0 \). The relativistic forms of the isoscalar Landau parameters are exactly the same as in Eq.(19), just substituting the \( \delta, \rho \) coupling constants with the \( \sigma, \omega \) ones [36].

V. ISOVECTOR COLLECTIVE MODES IN ASYMMETRIC NUCLEAR MATTER

In this section we discuss results for the isovector collective oscillations which are driven by the symmetry energy terms of the nuclear \( EOS \). The aim is mainly to investigate the effect of the scalar-isovector channel. This is normally not included in studying the isovector modes and in general the properties of symmetric matter in a relativistic approach, while it should be naturally present on the basis of the analysis shown in the previous section (and in ref. [19] for equilibrium properties). Moreover we stress again that Hartree-Fock scheme embodies in any case the presence of a scalar-isovector channel, even without the inclusion of the \( \delta \)-meson field [23].

We will first show results obtained in the Hartree scheme (\( NLH \)) including either both the isovector \( \rho \) and \( \delta \) mesons or only the \( \rho \) meson. Even if the Hartree approximation has a simpler structure, it contains all the physical effects we want to point out. Finally from the complete Hartree-Fock (\( NLHF \)) calculations we will confirm the dynamical contribution of the scalar isovector channel.
For NLH calculations we use the parametrizations of Table I, Set I \((\rho)\) and Set II \((\rho + \delta)\). In the Hartree-Fock case the coupling constant \(f_\delta\) is adjusted to the value \(f_\delta(\rho_0) = 2.0 fm^2\) of the NLHF model Eq. (3). We note that this value is smaller that the prediction of recent Dirac-Brueckner-Hartree-Fock calculations \([8]\), therefore the effects due to the \(\delta\)-channel presented in the following could be even underestimated in the NLHF case.

### Hartree Results

Let us start by considering isovector-like excitations. In Fig. 3a we show the sound velocities in the Hartree approximation, as a function of the asymmetry parameter \(\alpha\) for different baryon densities. We actually plot the sound velocities in units of the neutron Fermi velocities. This is physically convenient: when the ratio is approaching 1.00 we can expect that this “zero” sound will not propagate due to the strong coupling to the “chaotic” single particle motions (“Landau damping”). This quantity then will also directly give a measure of the “robustness” of the collective mode we are considering.

Dotted lines refer to calculations including \((\rho + \delta)\) mesons, long-dashed lines correspond to the case with only the \(\rho\) meson. Calculations are performed at \(\rho_B = \rho_0\) and \(\rho_B = 2 \rho_0\). We stress that the results of the two calculations differ already at zero asymmetry, \(\alpha = 0\). At normal density \((\rho_0\) curves), in spite of the fact that the symmetry energy coefficient, \(a_4 = E_{\text{sym}}(\rho_0)\), is exactly the same in the two cases, significant differences are observed in the response of the system. From Fig. 3a we can expect a reduction of the frequency for the bulk isovector dipole mode in stable nuclei when the scalar isovector channel \((\delta - \text{like})\) is present. Moreover we note that, in the NLH - \(\rho\) case, the excitation of isovector modes persists up to higher asymmetries at saturation density.

![Fig. 3. Isovector-like modes: (a) Ratio of zero sound velocities to the neutron Fermi velocity \(V_{F_n}\) as a function of the asymmetry parameter \(\alpha\) for two values of baryon density. Long dashed line: NLH - \(\rho\); Dotted line: NLH - \((\delta + \rho)\). (b) Corresponding ratios of proton and neutron amplitudes. All lines are labelled with the baryon density, \(\rho_0 = 0.16 fm^{-3}\). The full circles in panel (b) represent the trivial behaviour of \(- (\rho_p/\rho_n)\) vs. \(\alpha\).]

These are non-trivial features, related to the different way scalar and vector fields are entering in the dynamical response of the nuclear system. Such behaviours are therefore present in both collective responses, isoscalar and isovector. We have devoted the whole previous section (Sec.IV) to a complete discussion of this effect.

Differences are observed even at \(\rho_B = 2 \rho_0\), where however also the symmetry energy is different. A larger \(E_{\text{sym}}\) is obtained in the case including the \(\delta\) meson (see Fig. 3a) and this leads to a compensation of the effect observed at normal nuclear density. In particular, at higher asymmetries \(\alpha\) the collective excitation becomes more robust for NLH - \((\rho + \delta)\). Differences are observed also in the ”chemical” structure of the mode, represented by the ratio \(\delta \rho_p/\delta \rho_n\), plotted in Fig. 3(b). The ratio of the out of phase \(n,p\) oscillations is not following the ratio of the \(n,p\) densities for a fixed asymmetry, given by the full circles in the figure. We systematically see a larger amplitude of the neutron oscillations. The effect is more pronounced when the \(\delta\) (scalar-isovector) channel is present (dotted lines).
Hartree-Fock Results

We have also performed the calculation in the more general case of the Hartree-Fock approximation, \textit{NLHF}, whose formalism has been presented before, Eqs.(17, 18). We have fitted the same properties of symmetric \textit{NM} at the saturation density as for Hartree case, \textit{NLH}. In particular at \(\rho_0\) the value of the isovector coupling is fixed in order get the same symmetry energy (the \(a_4\) parameter) of the \(NLH - (\rho + \delta)\) case.

FIG. 4. The same as Fig.3 but for \textit{NLHF} (solid lines) and \(NLH - (\delta + \rho)\) (dotted lines).

In Fig.4 we can see that quite similar results are obtained in Hartree-Fock calculations, with respect to the Hartree results including \(\rho\) and \(\delta\) mesons, especially at the normal density. This can be understood by considering that in Hartree-Fock calculation the effective density dependent couplings associated with the isovector channels are tuned in such a way to roughly reproduce, at normal density, the values of the coupling constants \(f_\rho\) and \(f_\delta\) of the Hartree scheme: then not only \(a_4\) is the same but also its internal structure. Since such a tuning can be done only at a given density value, some differences are observed at \(\rho_B = 2\rho_0\), due to the density dependence of the effective coupling constants of the \textit{NLHF} scheme. Therefore the greater value of sound velocity in the Hartree case can be linked to the greater value of \(E_{\text{sym}}\), see Fig.4.

Disappearance of the Isovector Modes

For asymmetric matter we have found that, in all the calculation schemes, with increasing baryon density the isovector modes disappear: we call such densities \(\rho_B^{\text{cross}}\). E.g. from Figs.3(b),4(b) we see that the ratio \(\delta\rho_p/\delta\rho_n\) tends very quickly to zero with increasing baryon density, almost for all asymmetries. Around this transition density we expect to have an almost pure neutron wave propagation of the sound. Here we show the results of the \(NLH + \rho\) case, see Figs.5 and 6, but the effect is clearly present in all the models.

For symmetric matter we have a real crossing of the two phase velocities, isoscalar and isovector, as shown in Fig.5(a). Above \(\rho_B^{\text{cross}}\) the isoscalar mode is the most robust.
VI. ISOSCALAR COLLECTIVE MODES IN ASYMMETRIC NUCLEAR MATTER

So far we have focussed our discussion on the isovector-like response of the asymmetric nuclear matter. However, it is well known that in asymmetric nuclear matter can exist also isoscalar-like modes, see [20,37] and ref.s therein.

Exotic High Baryon density modes

From the previous analysis we have seen the isoscalar-like excitations to become dominant at high baryon density, above the $\rho_B^{\text{cross}}$ introduced before.
Some results are shown in Fig. 7. It should be noticed that the frequency of the isoscalar-like modes is essentially related to the compressibility of the system at the considered density. In Fig. 7(a) we display the sound velocity obtained in Hartree and Hartree-Fock calculations at $\rho_B = 3.5 \rho_0$, as a function of the asymmetry $\alpha$. The differences observed among calculations performed within the Hartree or Hartree-Fock scheme are due to a different behaviour of the associated equation of state at high density.

At $\alpha = 0$ the two Hartree models have exactly the same isoscalar mean fields, but for asymmetric nuclear matter the different behaviour of the symmetry energy leads to a different compressibility. The case $NLH - (\rho + \delta)$ which has the stiffer $E_{sym}$ (resulting in a greater incompressibility for $\alpha > 0$) with respect to $NLH - (\rho)$ shows also a greater increase of $v_s/v_{Fn}$ with density. Instead, $NLHF$ even if it has the same compressibility $K_{NM}$ at saturation density, shows a different $v_s$. This should be due to the density dependence of the coupling function arising from exchange terms which leads to different values of $K_{NM}$ out of $\rho_0$ (even for $\alpha = 0$).

Some differences are observed also in the chemical composition of the mode (Fig. 7(b)). The black spots show the behaviour of $\rho_p/\rho_n$ vs. $\alpha$. Note the pure neutron wave structure of the propagating sound, since the oscillations of protons appear strongly damped ($\delta \rho_p/\delta \rho_n \ll \rho_p/\rho_n$); unfortunately this is an effect not experimentally accessible (at present), see also the discussion at the end of the previous Section.

Before closing this discussion we have to remark that the isoscalar-like modes at high baryon density are vanishing if the nuclear EOS becomes softer. This is indeed the results of two recent models, ref.s [18,39], where the nuclear compressibility is decreasing at high baryon density for a reduction of the isoscalar vector channel contribution. In [18] this is due to self-interacting high order terms for the $\omega$ meson, while in ref. [39] to a reduced $f_\omega$ coupling with increasing baryon density.

Finally we note that all causality violation problems (superluminal sound velocities) observed in the non relativistic results at high baryon density, see [16] and Fig.3c in ref. [20], are completely absent in the relativistic approach, see the high density trends in Fig.5.

Isospin Distillation in Dilute Matter

We have also investigated the response of the system in the region of spinodal instability associated with the liquid–gas phase transition, which occurs at low densities. It is known that in this region an isoscalar unstable mode can be found, with imagnary sound velocity, that gives rise to an exponential growth of the fluctuations. The latter can represent a dynamical mechanism for the multi–fragmentation process observed in heavy–ion collisions. We have found this kind of solution in the present approach. In Fig. 8 we show the ratio $\delta \rho_p/\delta \rho_n$ as function of the initial asymmetry for such a collective mode. For all the interactions this ratio is different from the corresponding $\rho_p/\rho_n$ of the initial asymmetry $\alpha$. This is exactly the chemical effect associated with the new instabilities in dilute asymmetric matter [7,37].
FIG. 8. Isoscalar–like unstable modes at $\rho_B = 0.4 \rho_0$: Imaginary sound velocity (a) and ratio of proton and neutron amplitudes (b) as a function of the asymmetry $\alpha$. Solid line: $NLHF$; Dotted line: $NLH - (\rho + \delta)$; Long Dashed line: $NLH - \rho$. The full circles in panel (b) represent the behaviour of $\delta \rho_p / \delta \rho_n$ vs. $\alpha$.

In particular it is found that, when isoscalar-like modes become unstable, the ratio $\delta \rho_p / \delta \rho_n$ becomes larger than the ratio $\rho_p / \rho_n$ (at variance with the stable modes at high densities, see Fig.7). Hence proton oscillations are relatively larger than neutron oscillations leading to a more symmetric liquid phase and to a more neutron rich gas phase, during the disassembly of the system. This is the so-called isospin distillation effect in fragmentation, and signatures of this effect could be searched by looking at the ratio $N/Z$ of fragments produced in dissipative heavy ion collisions [40,41].

We note here that in dilute asymmetric $NM$ we can distinguish two regions of instability, mechanical (cluster formation) and chemical (component separation). There is however no discontinuity in the structure of the unstable modes which are developing. For all realistic effective nuclear interactions (relativistic and non) the nature of the unstable normal modes at low densities is always isoscalar-like, i.e. with neutrons and protons oscillating in phase, although with a distillation effect discussed before, see ref. [37] for a fully detailed study of this important property of asymmetric nuclear matter.

In Fig.8 we observe that Hartree results (with and the without $\delta$ meson) are very similar and, indeed, at low density the symmetry energy behaviour is nearly the same in the two cases. On the other hand, differences are observed with respect to the Hartree-Fock case. In fact, the $NLHF$ symmetry energy presents a softer behaviour (around $\rho_B = 0.4 \rho_0$), that leads to a smaller distillation effect. We remark that the equality between $NLH$ with and without $\delta$ is in agreement with the analysis in terms of the generalized Landau parameters associated to normal modes developed in Ref. [19]. We can conclude that there are essentially no effects of the scalar isovector channel on isospin distillation in the spinodal decomposition.

VII. CONCLUSIONS AND OUTLOOK

We have developed a linear response theory starting from a relativistic kinetic equations deducted within a Quantum-Hadro-Dynamics effective field picture of the hadronic phase of nuclear matter. In the asymmetric case we consider as the main dynamical degrees of freedom the nucleon fields coupled to the isoscalar, scalar $\sigma$ and $\omega$, and to the isovector, scalar $\delta$ and vector $\rho$, mesons.

Using the Landau procedure we derive the dispersion relations which give the sound phase velocity and the internal structure of the normal collective modes, stable and unstable. We have focussed our attention on the effect of the isovector mesons on the collective response of asymmetric (neutron-rich) matter. In order to better understand the dynamical role of the different mesons, the results are obtained in the Hartree approximation, which has a simpler and more transparent form. The contribution of Fock terms is also discussed.

We have singled out some qualitative new effects of the $\delta$-meson-like channel on the dynamical response of ANM. Essentially, our investigation indicates that even if the symmetry energy is fixed, the dynamical response is affected by its internal structure, i.e. the presence or not of an isovector-scalar field. This is implemented by the explicit introduction of an effective $\delta$-meson and/or by the Fock term contributions.
The interest comes from the fact that both mechanisms are absent in the usual relativistic RPA calculations for finite nuclei.

It is important to stress that the same interplay between scalar (σ-meson) and vector (ω-meson) contributions can be seen in the dynamical isoscalar response. In general we clearly show a close analogy in the structure of the linear response equations:

- Same form of the dispersion relations, cfr. Eq.\((22)\) and Eq.\((23)\).
- Parallel role of \(E_{\text{sym}}^{\text{pot}}\) and \(K_{NM}^{\text{pot}}\) in the determination of the restoring force, Eqs.\((23)\) and \((27)\).
- Parallel structure of the corrections due to the scalar-vector meson competition, Eqs.\((23)\) and \((27)\).

This appears to be a beautiful “mirror” structure of the relativistic approach that strongly support the introduction of a \(δ\)-meson-like coupling in the isovector channel. We like to remind that the same “mirror” structure of the relativistic picture has been recently stressed in ref. \([19]\) for equilibrium properties, saturation binding and symmetry energy, the \(a_1\) and \(a_4\) parameters of the Weizsaecker mass formula.

The relativistic dispersion relations have been compared with the non-relativistic ones of the Landau Fermi Liquid theory expressed in terms of Landau parameters evaluated in a relativistic scheme \([18]\). The corrections appear to be not negligible, particularly for the isoscalar response.

From the numerical results on the collective response of \(ANM\) some general features are qualitatively present in all the effective interactions in the isovector channel:

- In asymmetric matter we have a mixing of pure isoscalar and pure isovector oscillations which leads to chemical effect on the structure of the propagating collective mode: the ratio of the neutron/proton density oscillations \(\delta \rho_n/\delta \rho_p\) is different from the initial \(\rho_n/\rho_p\) of the matter at equilibrium. However we can still classify the nature of the excited collective motions as isoscalar – like (when neutron and protons are oscillating in phase) and isovector – like (out of phase). To note that similar effects can be obtained also using non-relativistic effective forces \([34,35]\).

- For a given asymmetry the isovector-like mode is the most robust at low baryon density, always showing a larger neutron component in the oscillations. With increasing baryon density we observe a smooth transition, at a \(\rho_B^{\text{cross}} \approx 2−3\rho_0\), to an isoscalar-like branch, still with a dominant \(\delta \rho_n\). In the region of the transition we predict a propagation of almost pure neutron waves. For relatively large asymmetries \((\alpha \equiv \frac{N−Z}{N+Z} = 0.5, N = 3Z)\) this behaviour is present in a wide interval of densities around \(\rho_B^{\text{cross}}\). All that seems to suggest the possibility of an experimental observation of related effects in intermediate energy heavy ion collisions with exotic beams. If the compressibility of nuclear matter is decreasing at high baryon density also these exotic isoscalar-like mode will disappear. This could be a nice signature of the softening of nuclear EOS at high densities.

- The isoscalar-like motions become unstable at sub-saturation densities still with a strong chemical effect, now in the opposite direction with respect to the one discussed before, present in the stable high density modes. Now the unstable oscillation is more proton-rich, eventually leading to the formation of more symmetric clusters vs. a very neutron-rich gas phase. This is the neutron distillation effect \([7,20,34,37,40,41]\), a new important feature of the liquid-gas phase transition in asymmetric nuclear systems.

A. Acknowledgements

We warmly thank Hermann H. Wolter and Stephan Typel for several pleasant and stimulating discussions.

VIII. APPENDIX

The Wigner matrix \(\hat{H}(p)\) for matter at equilibrium saturated in spin has the following form:

\[
\hat{H}(p) = H(p) + \gamma_\mu H^\mu(p).
\]

From the kinetic equation one obtain the relation for nuclear matter at equilibrium between the scalar and vector parts:

\[
H^\mu(p) = \frac{p^\mu}{M} H^\mu(p).
\]
where the zero component of the vector part is proportional to the Fermi Dirac distribution function:

\[ H_0^{(i)}(p) = \frac{1}{4} \frac{1}{(2\pi)^3} \Theta(E^*_{F_i} - E_{k_i}) \delta(p_0^{(i)} - E_{k_i}) \]

where \( i = n, p \) and \( E^*_{k_i} = (k^2 + M_i^*)^{1/2} \).

The coefficients \( C^{(i)}(k), C_\lambda^{(i)}(k) \) and \( C_{\lambda\mu}^{(i)}(k) \) introduced in Sec. II are given by the integrals

\[
C^{(i)}(k) = M_i^* \int d^4p \frac{H^\lambda_{(i)}(p)k_\lambda}{p^i_k p^\rho_k} , \tag{A1a}
\]

\[
C_\lambda^{(i)}(k) = \int d^4p \frac{H^\mu_{(i)}(p)k_\mu p^\lambda_i}{p^i_k p^\rho_k} , \tag{A1b}
\]

\[
C_{\lambda\mu}^{(i)}(k) = \int d^4p \frac{H^\rho_{(i)}(p)k_\rho p^\lambda_i p^\mu_i}{p^i_k p^\rho_k} , \tag{A1c}
\]

where \( H^\lambda_{(i)}(p) = \partial^\lambda_{(p)} H_{(i)}(p) \). The index \( i \) specifies the kind of nucleon: \( i = 1 \) for protons and \( i = 2 \) for neutrons. The frequency \( k^0 \) includes an imaginary part \( i\epsilon \) with \( \epsilon \) positive infinitesimal.

By using the definitions (A1) it can be easily checked that

\[
k^\lambda C_\lambda^{(i)}(k) = 0, \quad k^\lambda C_{\lambda\mu}^{(i)}(k) = -k_\mu \frac{\rho_{Si}}{4} , \quad \sum_{\mu} C_{\rho\mu}^{(i)}(k) = M_i^* C^{(i)}(k) - \frac{1}{2} \rho_{Si} . \tag{A2}
\]

In order to be more specific we choose the \( z \) axis in the direction of the wave vector \( k \). As a consequence, the following coefficients identically vanish:

\[
C_1^{(i)}(k), \quad C_2^{(i)}(k), \quad C_{10}^{(i)}(k), \quad C_{20}^{(i)}(k), \quad \text{and} \quad C_{1m}^{(i)}(k)
\]

for \( l \neq m \) (\( l \) and \( m \) are space indices). In addition, for symmetry reasons,

\[
C_{11}^{(i)}(k) = C_{22}^{(i)}(k) .
\]

The integrals in Eqs. (A1) can be evaluated analytically. They give

\[
C^{(i)}(k) = -\frac{1}{2} M_i^* \rho_{Si} + \frac{3}{4} \frac{\rho_{Bi}}{E^*_{F_i}} - \frac{1}{4} N_i M_i^* E^*_{F_i} \varphi(s_i) , \tag{A3a}
\]

\[
C_0^{(i)} = -\frac{1}{4} N_i \varphi(s_i) , \tag{A3b}
\]

\[
C_{00}^{(i)} = -\frac{1}{4} \rho_{Si} + \frac{1}{4} N_i M_i^* \varphi(s_i) , \tag{A3c}
\]

\[
C_{11}^{(i)}(k) = \frac{1}{4} \rho_{Si} - \frac{3}{8} \frac{M_i^*}{E^*_{F_i}} \rho_{Bi} - \frac{3}{8} \frac{M_i^*}{E^*_{F_i}} (s_i^2 - 1) \varphi(s_i) , \tag{A3d}
\]

where \( v_{F_i} \) is the Fermi velocity, \( s_i = k^0/(v_{F_i}|k|) \), \( N_i \) are the density of states at Fermi surface and

\[
\varphi(s_i) = 1 - \frac{s_i}{2} \ln \left| \frac{s_i + 1}{s_i - 1} \right| + \frac{i}{2} \pi s_i \theta(1 - s_i)
\]

is the Lindhard function. The remaining coefficients \( C_{33}^{(i)}(k), C_{33}^{(i)}(k) \) and \( C_{33}^{(i)}(k) \) can be evaluated by means of the relations (A2).
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