VARIABLE-MASS DARK MATTER AND THE AGE OF THE UNIVERSE*

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Models with variable-mass particles, called VAMPs, have been proposed as a solution to the cosmic coincidence problem that plagues dark energy models. In this contribution we make a short description of this class of models and explore some of its observational consequences. In particular, we show that fine tuning is still required in this scenario and that the age of the Universe is considerably larger than in the standard Λ-CDM model.

1. Introduction

The Wilkinson Microwave Anisotropy Probe (WMAP) satellite ¹, along with other experiments, has confirmed that the universe is very nearly flat, and that there is some form of dark energy (DE) that is the current dominant energy component, accounting for approximately 70% of the critical density. Non-baryonic cold dark matter (DM) contributes around 25% to the total energy density of the universe and the remaining 5% is the stuff we are made of, baryonic matter.

DE is smoothly distributed throughout the universe and its equation of state with negative pressure is causing its present acceleration. It is generally modelled using a scalar field, the so-called quintessence models, either

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slowly rolling towards the minimum of the potential or already trapped in this minimum. 

An intriguing possibility is that DM particles could interact with the DE field, resulting in a time-dependent mass. In this scenario, dubbed VAMPs (VARIable-Mass Particles), the mass of the DM particles evolves according to some function of the dark energy field \( \phi \). In this case, the DM component can have an effective equation of state with negative pressure that could present the same behaviour as DE.

We studied a model with exponential coupling between DM and DE, since it presents a tracker solution where the effective equation of state of DM mimics the effective equation of state of DE and the ratio between DE and DM energy density remains constant after this attractor is reached. This behavior could solve the “cosmic coincidence problem”, that is, why are the DE and DM energy densities similar today.

This type of solution also appears when the DE field with a exponential potential is not coupled to the other fluids. In fact, Liddle and Scherrer showed that for a non-coupled DE, the exponential potential is the only one that presents stable tracker solutions. In this case, however, it is not able to explain the current acceleration of the universe and other observational constraints, unless we assume that the field has not yet reached the fixed point regime.

In this contribution, we review our results presented in 14.

2. A simple exponential VAMP model

In the exponential VAMP model, the potential of the DE scalar field \( \phi \) is given by

\[
V(\phi) = V_0 \, e^{\beta \phi / m_p},
\]

where \( V_0 \) and \( \beta \) are positive constants and \( m_p = M_p / \sqrt{8 \pi} = 2.436 \times 10^{18} \) GeV is the reduced Planck mass in natural units, \( \hbar = c = 1 \). Dark matter is modelled by a scalar particle \( \chi \) of mass

\[
M_\chi = M_{\chi 0} \, e^{-\lambda (\phi - \phi_0) / m_p},
\]

where \( M_{\chi 0} \) is the current mass of the dark matter particle (hereafter the index 0 denotes the present epoch, except for the potential constant \( V_0 \)) and \( \lambda \) is a positive constant.
In this case we can show that
\[
\dot{\rho}_\chi + 3H\rho_\chi(1 + \omega^{(e)}_\chi) = 0, \tag{3}
\]
\[
\dot{\rho}_\phi + 3H\rho_\phi(1 + \omega^{(e)}_\phi) = 0, \tag{4}
\]
where
\[
\omega^{(e)}_\chi = \frac{\lambda\dot{\phi}}{3Hm_p} = \frac{\lambda\phi'}{3m_p}, \tag{5}
\]
\[
\omega^{(e)}_\phi = \omega_\phi - \frac{\lambda\dot{\phi}}{3Hm_p} \frac{\rho_\chi}{\rho_\phi} = \omega_\phi - \frac{\lambda\phi'}{3m_p} \frac{\rho_\chi}{\rho_\phi}, \tag{6}
\]
are the effective equation of state parameters for dark matter and dark energy, respectively. Primes denote derivatives with respect to \( u = \ln(a) = -\ln(1 + z) \), where \( z \) is the redshift, and \( a_0 = 1 \).

At the present epoch the energy density of the universe is divided essentially between dark energy and dark matter. In this limit, there is a fixed point solution for the equations of motion of the scalar field such that
\[
\Omega_\phi = 1 - \Omega_\chi = \frac{3}{(\lambda + \beta)^2} + \frac{\lambda}{\lambda + \beta} \tag{7}
\]
\[
\omega^{(e)}_\chi = \omega^{(e)}_\phi = -\frac{\lambda}{\lambda + \beta}. \tag{8}
\]

The equality between \( \omega_\chi \) and \( \omega_\phi \) in the attractor regime comes from the tracker behavior of the exponential potential \(^7,^{11}\) in this regime.

However, this is only valid in the fixed point regime. If we want to know what happens before, the equations must be solved numerically. The density parameters for the components of the universe and the effective equations of state for the DE and DM for a typical solution are shown in figure 1. Notice that the transition to the tracker behavior in this example is occurring presently.

3. Cosmological constraints and the age of the universe in the exponential VAMP model

We have calculated the age of the universe for the models that satisfy the Hubble parameter and the dark energy density observational constraints \((h = 0.72 \pm 0.08, 0.6 \leq \Omega_{\rho_0} \leq 0.8)\). We have used step sizes \( \Delta \lambda = \Delta \beta = 0.2 \) and \( \Delta V_0 = 0.05 \rho_c \) for the region \( \lambda = [0.01, 20], \beta = [0.01, 20], V_0 = [0.1 \rho_c, 0.8 \rho_c] \), generating \( 1.4 \times 10^5 \) models.
Figure 1. Top panel: Density parameters of the components of the universe as a function of $u = -\ln(1 + z)$ for $\lambda = 3$, $\beta = 2$ and $V_0 = 0.1\rho_c$. After a transient period of baryonic matter domination (dot-dashed line), DE comes to dominate and the ratio between the DE (solid line) and DM (dashed line) energy densities remains constant. Bottom panel: Effective equations of state for DE (solid line) and DM (dashed line) for the same parameters used in top panel. In the tracker regime both equations of state are negative.

Fitting the distribution of models as function of ages, the age of the universe in this VAMP scenario was found to be

$$t_0 = 15.3^{+1.3}_{-0.7} \text{ Gyr} \quad 68\% \text{ C.L.,}$$

which is considerably higher than the age of models of non-coupled dark energy. This seems natural, since in these models the CDM also has an effective negative equation of state and accelerates the universe. This result is very conservative, since it relies only on the well established limits.
on the Hubble constant and the dark energy density today.

4. Conclusion

VAMP scenario is attractive since it could solve the problems of exponential dark energy, giving rise to a solution to the cosmic coincidence problem. However, we found that in order to obtain solutions that can provide realistic cosmological parameters, the constant \( V_0 \) has to be fine tuned in the range \( V_0 = [0.25\rho_c, 0.45\rho_c] \) at 68% C. L. This implies that the attractor is being reached around the present epoch. In this sense, the model is not able to solve the coincidence problem.

A generic feature of this class of models is that the universe is older than non-coupled dark energy models. Better model independent determination of the age of the universe could help to distinguish among different contenders to explain the origin of the dark energy.

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References

1. D. N. Spergel et al., Astrophys. J. Supp. 148, 175 (2003).
2. See, e.g., P. J. E. Peebles, B. Ratra, Rev. Mod. Phys. 75, 559 (2003) and references therein.
3. G. W. Anderson, S. M. Carrol, astro-ph/9711288.
4. J. A. Casas, J. García-Bellido, M. Quirós, Class. Quantum Grav. 9, 1371 (1992).
5. G. R. Farrar, P. J. E. Peebles, Astrophys. J. 604, 1 (2004).
6. M. B. Hoffman, astro-ph/0307350.
7. L. Amendola, Phys. Rev. D 62, 043511 (2000).
8. L. Amendola, D. Tocchini-Valentini, Phys. Rev. D 64, 043509 (2001).
9. L. Amendola, Mon. Not. Roy. Astron. Soc. 342, 221 (2003).
10. M. Pietroni, Phys. Rev. D 67, 103523 (2003).
11. D. Comelli, M. Pietroni, A. Riotto, Phys. Lett. B 571, 115 (2003).
12. A. R. Liddle, R. J. Scherrer, Phys. Rev. D 59, 023509 (1998).
13. U. Franca, R. Rosenfeld, JHEP 10, 015 (2002).
14. U. Franca, R. Rosenfeld, Phys. Rev. D 69, 063517 (2004).
15. M. Kunz, P. S. Corasaniti, D. Parkinson, E. J. Copeland, Phys. Rev. D 70, 041301 (2004).