About one synthesis method for adaptive control systems with reference models

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Abstract. This article describes the synthesis technique of static and recurrent algorithms for adaptive parametric and structurally optimized controllers for systems with a reference model. As models, systems are used that are described in the form of a state space, transfer functions, and vector difference equations. And also considered the task of developing adaptive-interval algorithms for the synthesis of control systems for technological objects.

1. Introduction

In the practice of designing automatic control systems for technological objects, a situation often arises when the real values of individual parameters of control objects are unknown and there are no static descriptions of them. Uncertainties in the parameters can appear due to various reasons. Various methods are used to solve problems of managing objects with uncertain parameters. One of these methods is the interval analysis apparatus. With the selected structure of the system and the main control loop, the central problem is the synthesis of adaptation algorithms. The principles of building a searchless self-tuning system with frequency characteristics control and a system with a model implemented in the time domain were proposed. [1] In the world literature such systems have come to be called adaptive systems with a reference model.

2. Problem statement

Consider the main circuit of an adaptive system with reference models (ASRM). The main circuit (MC) scheme is shown in Figure 3.1. It is formed by a generalized object (GO), including a drive, a real dynamic object and sensors, and adaptive direct and feedback controllers AR1 and AR2.

The generalized object through the channel of passage of the disturbance (CPD) is affected by the noise θ, which in adaptive combined control systems is measured and fed to the input AR1 (dashed line in the figure). The control command g is input MC, while the output signal of the generalized object y is input AR2, and the output signal of the latter in the form of a feedback signal is fed to input AR1. The structural blocks of GO and CPD are combined in object O.

Suppose that the mathematical models of GO, CPD are known up to unknown parameters Q1, Q2. In (ASRM), two subgoals of management can be distinguished:

- coincidence of the MC movement in one or more coordinates with some desired movement;
- the coincidence of the control coefficients MC, connecting a specific output coordinate with one or more input actions, with the coefficients of the equation of the desired motion, described by the reference model of RM. In turn, the desired movement is often called the reference path. To perform the second subgoal in the adaptive control system, it is necessary to configure the parameters R1, R2 of the adaptive regulators AR1, AR2.
3. Research questions

As a rule, only a valid area of the object's variable parameter space is known, within which they change in an unknown way in advance. Adaptive controllers AR1, AR2 must be designed in such a way that:

a) in the nominal mode (with fixed values of the variable parameters Q1, Q2, R1, R2), provide the desired dynamic characteristics;

b) in any arbitrary mode of the object (if any laws change variables of the object Q1, Q2 in time and subject to their being inside the feasible region) there were such laws, changes of controller parameters R1, R2 for which the operator main circuit coincides with the operator of the reference model.

Obviously, the second condition physically means that it is possible to transfer the managed adaptive system from some initial mismatch of the parameters of the MC and RM operators to zero, which is similar to the physical meaning of the object's controllability conditions, or the main contour in relation to coordinate mismatch at the initial time. The specified property of existence (existence and uniqueness) of the vector of "ideal" settings of the controllers AR1, AR2 in relation to an arbitrary acceptable vector of object parameters is called the property of adaptability of the main contour. The full adaptability property of the MC guarantees the possibility of absolute parametric invariance and serves as a structural characteristic of the MC (or adaptive regulator) that expresses the limiting possibilities of compensating for the influence of parametric perturbations on its dynamic characteristics [1,2,5-7].

4. Purpose of the study

Usually, the behavior of the control system set by its technical requirements is associated with the reference model of the control system or its parts.

It is under the reference model of a closed window that the Adjuster adjusts the regulator of the control system, and the parameters of the latter (existing only in the mind of the Adjuster) are determined by the selected values of the regulator settings in the nominal mode.

The presence or absence of the adaptability property of an MC or part of it is used in solving problems of analysis and synthesis of the following classes of adaptive systems [3,7]:

- direct or indirect adaptive control with a reference model;
- with a standard observer status;
- optimal adaptive systems for direct and indirect adaptive control with a stationary and trainable reference model.

In all these classes of adaptive systems, indirect adaptive control algorithms are directly generated using adaptability matrices.

Let there be a \([n + 2l \times 2]^{st}\) order model for an extended open system:

\[
\begin{align*}
Z(k+1) &= A'Z(k) + Bu(k) + G_1'g(k+1) + G_2'g(k) + F_1'f(k+1) + F_2'f(k), \\
y(k) &= C'Z(k) + f(k),
\end{align*}
\]  

(1)

where:

\[Z^*(k) = \begin{bmatrix} x^T(k) & W_1^T(k) & W_2^T(k) & V_1^T(k) & V_2^T(k) \end{bmatrix} \]

\[
\begin{bmatrix}
A & 0 & 0 & 0 & 0 \\
0 & I & 0 & 0 & 0 \\
CA & 0 & I & 0 & 0 \\
CA-G & 0 & 0 & 0 & 0
\end{bmatrix}B^* = col[B:O:CB:CB]C^* = col[C:O:O:O:O],
\]

(2)

\[G_1^* = col[O:1:O:1:O] \quad G_2^*(k) = col[O:O:O:-1:O], \quad F_1^* = col[O:O:1:O:1] \quad F_2^*(k) = col[O:O:O:-1] \]

In the class of adaptive control systems with a reference model, the goal of adaptive control is to approximate the dynamic characteristics of the system described by equations (1) - (3), in which instead of matrices \(A, B, C, G, K\), their values are substituted in nominal mode \(A_\mu, B_\mu, C_\mu, G_\mu, K_\mu\), and all variables except input actions \(g, f\) are marked index "m".

Such a choice of the structure and parameters of the MC reference model should be linked to its feasibility in terms of stability and quality of regulation, which imposes certain restrictions on the set of model values of the PID controller settings. The synthesis of parameter matrices \(G_\mu, K_\mu\) \((\nu = -1, 1)\) can be performed by various methods of the theory of stationary control systems [1,2,4-7]. Equation RM MC closed system has the form:

\[
\begin{align*}
Z_\mu(k+1) &= \overline{A_\mu}Z_\mu(k) + G_{\mu1}g(k+1) + \overline{G_{\mu2}}g(k) + F_{\mu1}f(k+1) + \overline{F_{\mu1}}g(k), \\
y_\mu(k) &= C_{\mu}Z_\mu(k) + f(k),
\end{align*}
\]  

(3)

where the matrices \(\overline{A_\mu}, \overline{G_{\mu1}}, \overline{G_{\mu2}}, \overline{F_{\mu1}}, \overline{F_{\mu2}}\), are calculated by formulas (1), (2) with the substitution of the matrices \(A_\mu, B_\mu, C_\mu, G_\mu, K_\mu\) in them.

The goal of adaptive management is formulated in terms of management sub-goals, respectively:

\[
y(k) \equiv y_\mu(k), \quad u(k) \equiv u_\mu(k), \quad \overline{A} \equiv \overline{A_\mu}, \quad \overline{G_1} \equiv \overline{G_{\mu1}}, \quad \overline{G_2} \equiv \overline{G_{\mu2}},
\]

or (and):
5. Research methods

The tasks of stabilizing the output quantities, tracking reference trajectories, and program control can be formulated as a goal (5).

Identities (5), (6) mean the requirement of parametric invariance of the operator connecting a certain output vector \( y \) with certain input vectors \( q \) or \( f \), or simultaneously with \( q \) and \( f \) [4-7].

As already noted, the concept of the indirect adaptive control method is based on estimating the unknown parameters of the controlled object from the signals at its input and output, followed by using these estimates to reconfigure the controller parameters. Suppose that the values of the matrices of the parameters of the MC are determined in adaptive systems using an identifier of the appropriate type, which guarantees the non-bias and consistency of the parameter estimates. Then identities (5), (6) (or part of them) express the laws of static adaptation of the PID controller, written in implicit form (3), provided that these identities are compatible.

Applying the \( z \) - transformation to equation (3), assuming that the quasistationarity hypothesis and the initial conditions are satisfied, we obtain:

\[
y(z) = H(z)u(z) + f(z) \tag{7}
\]

\[
H(z) = \tilde{C}^*(zI - \tilde{A}^*)^{-1}\tilde{B}^*z^{-b} \tag{8}
\]

where \( H(z) \) is the transfer function of the op-amp taking into account a zero-order extrapolator, and \( \tilde{A}^* \), \( \tilde{B}^* \), \( \tilde{C}^* \) is the matrix of his model in the state space.

We introduce the notation

\[
\hat{G}_{\rho}(z) = \sum_{n=1}^{3} \hat{G}_{\rho n}z^{-(\eta-1)}, \quad \tilde{K}_{\rho}(z) = \sum_{n=1}^{3} \tilde{K}_{\rho n}z^{-(\omega-1)},
\]

\[
W(z) = H(z)\hat{G}_{\rho}(z),
\]

\[
\hat{G}_{i} = [G_{i} + G_{o} + G_{i}], \quad \hat{G}_{i} = [-G_{o} - 2G_{i}], \quad \hat{G}_{i} = [-2G_{i}],
\]

\[
\tilde{K}_{i} = [K_{o} + K_{i} + K_{o}], \quad \tilde{K}_{i} = [-K_{o} - 2K_{i}], \quad \tilde{K}_{i} = [-2K_{i}],
\]

where \( \hat{G}_{\rho}(z), \tilde{K}_{\rho}(z), W(z) \) is the transfer function of the feedback and direct connection of the PID controller and the open main circuit.

The reference model of MC, which sets the desired quality of adaptive regulation, is determined by the parameters of MC in some fixed, nominal mode:

\[
A_{M}^*, B_{M}^*, C_{M}^*, \hat{G}_{\rho}(G_{M}), \tilde{K}_{\rho}(K_{M}), \eta = 1,3, \quad \nu = -1,1
\]

Using the notation introduced and taking into account equation (3.18), the MC control system (3) - (4) in the \( z \)-plane is defined in the form:

\[
y(z) = \left[ I + H(z)\hat{G}_{\rho}(z) \right]^{-1} H(z)K_{\rho}(z)q(z) + \left[ I + H(z)G_{\rho}(z) \right]^{-1} f(z) \tag{10}
\]

We obtain the RM MC equations in the \( z \)-plane, assigning to all variables the equations (10), except for external influences, as well as to the parameter matrices of the object control and the matrix of settings of the index controller "m".
If the output vector is y, and the input q and f, then the goals of adaptive control in the z-plane are determined by the identities:

\[ y(z) \equiv y_M(z) \]  \hspace{1cm} (11)
\[ W(z) \equiv W_M(z), \quad K_p(z) \equiv \tilde{G}_p(z) \]  \hspace{1cm} (12)

It is required:
1) synthesize such algorithms for tuning the matrix of parameters of the PID controller, \( G_c = G_c(A, B, C), \quad K_c = K_c(A, B, C) \), where \( \{A, B, C\} \) are the estimates of the matrix of parameters of the object obtained using the identifier, so that identities (11), (12) are satisfied;
2) to identify the conditions for the realizability of identities (11), (12) in the adaptive controller;
3) if it is impossible to implement identities (11), (12) within the given controller structure and features of a multiply connected op amp, synthesize matrix adjustment algorithms \( G_c, K_c \) that would best approximate the transfer function \( MC \) to the transfer function of the reference model \( MC \).

In the synthesis of controls that ensure stability and a given level of quality of systems with limited uncertainty of parameters, two approaches can be used that are widely used in interval analysis \([2,5,8]\).

The first approach involves the consideration of interval mathematical objects (matrices, functions, equations, etc.) as sets whose elements are the corresponding “point” objects (objects with real coefficients). In this case, the necessary actions are performed on point objects with the subsequent combination of the results.

This approach is used at the first stage of synthesis - the stage of isolating many stabilizing regulators. The second stage of the synthesis - determination of the parameters of the adaptive controller - is connected with the solution of the interval matrix equation. At this stage, the approach associated with the consideration of an interval object as a set of point objects requires solving a continuum of algebraic equations with real coefficients. To overcome the difficulties that arise along this path, the problem of finding a pointwise or combined set of solutions of the interval equation is transformed into the problem of determining an approximating set of solutions, which is solved in the traditional framework of the interval calculus by constructing two-sided inequalities.

The coefficients of the real managed object will differ from the calculated ones due to the inevitable measurement error, inaccurate calculations, parameter changes over time and a number of other factors \([3,6,7]\). Therefore, it will be more realistic to assume that in the synthesis of adaptive controllers we are dealing with interval uncertainty of the parameters of the control object.

By the interval uncertainty of the control object parameters it is understood that it is not the exact values of the parameters that are known, but their belonging to a certain a priori specified numerical range:

\[ \tilde{a}_{ij} \in \tilde{a}_{ij} = [a_{ij}, \overline{a}_{ij}], \quad i = 1, n, \quad j = 1, n, \]
\[ \tilde{b}_{ik} \in \tilde{b}_{ik} = [b_{ik}, \overline{b}_{ik}], \quad k = 1, \overline{m}, \]
\[ \tilde{c}_{pi} \in \tilde{c}_{pi} = [c_{pi}, \overline{c}_{pi}], \quad p = 1, \overline{I}, \]  \hspace{1cm} (13)

where \( \tilde{a}_{ij}, \tilde{b}_{ik}, \tilde{c}_{pi} \) are, respectively, elements of the matrices \( \tilde{A}, \tilde{B}, \tilde{C} \) of the multiply connected op amp.

We assume that the identified object control parameters belong to the indicated ranges, and for any set of coefficients from certain intervals, the MC does not lose its full controllability properties \([8]\).
6. Conclusion

The formulation of the problem of adaptive control of technological objects in the conditions of interval parameter uncertainty is considered. The adaptive control problem is formulated in adaptive control systems with interval parameter uncertainty, and the conditions for the existence of its solution are obtained. Interval adaptive algorithms with a reference model of controlled processes are proposed.

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