Thermal rounding of the depinning transition

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Abstract – We study thermal effects at the depinning transition by numerical simulations of driven one-dimensional elastic interfaces in a disordered medium. We find that the velocity of the interface, evaluated at the critical depinning force, can be correctly described with the power law $v \sim T^\psi$, where $\psi$ is the thermal exponent. Using the sample-dependent value of the critical force, we precisely evaluate the value of $\psi$ directly from the temperature dependence of the velocity, obtaining the value $\psi = 0.15 \pm 0.01$. By measuring the structure factor of the interface we show that both the thermally rounded and the $T=0$ depinning, display the same large-scale geometry, described by an identical divergence of a characteristic length with the velocity $\xi \propto v^{-\nu/\beta}$, where $\nu$ and $\beta$ are, respectively, the $T=0$ correlation and depinning exponents. We discuss the comparison of our results with previous estimates of the thermal exponent and the direct consequences for recent experiments on magnetic domain wall motion in ferromagnetic thin films.

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Understanding the physics of disordered elastic lines has a direct impact on a large variety of experimental systems. Indeed such systems are realized for isolated lines by magnetic [1–4] or ferroelectric [5,6] domain walls, contact lines [7] and fractures [8,9] and for periodic systems by vortex lattices [10,11], charge density waves [12] or Wigner crystals [13]. In these systems, one particularly important question is to understand the response of the interface to an externally applied force, such response being directly measurable in all the above systems.

In all these systems the competition between the disorder and the elasticity of the lines leads to pinned configurations. In order to set the system in motion it is thus necessary to apply a force $F$ exceeding a critical force $F_c$. At zero temperature the average velocity $V$ remains zero for $F < F_c$, while the system moves for $F > F_c$. In addition to their experimental relevance, the understanding of the above properties constitutes a considerable theoretical challenge [14]. A very fruitful approach was to draw on the analogies between such a phenomenon and a standard critical phenomenon to predict scaling properties of the various physical observables close to the depinning transition [15]. In that respect an important question is how to extend the above results to the case of finite temperature. This is of course directly relevant for the experimental systems. A finite temperature allows the system to move even if $F < F_c$ leading to a thermal rounding of the depinning transition. Indeed, such rounding was recently experimentally observed [4], paving the way to an experimental determination of its scaling. A natural way to analyze such thermal effect is to draw further on the analogy with equilibrium critical phenomena. This is however not so simple since the depinning is an out-of-equilibrium feature and it is unclear how far such analogy should be carried out. Recently, serious differences between depinning and static critical phenomenon were indeed pointed out. In particular, there are drastic differences between statics and depinning due to the inherent irreversibility of the quasi-static process as the system approach $F_c$ from above. This yields corrections to the critical exponents [16,17]. In addition, contrarily to what happens in a static standard critical phenomenon, it was numerically shown that no divergent correlation length exists in the steady-state geometry by approaching the depinning threshold from below [18,19]. Therefore, it is now clear that the analogy between depinning and static standard critical phenomena should be carefully interpreted, and the thermal rounding is thus still a considerable theoretical challenge.

Therefore, both for its experimental relevance and its intrinsic theoretical interest, it is important to understand
the thermal rounding of the depinning transition. Unfortunately, the analytical methods able to tackle such an out-of-equilibrium situation are rare. A very successful method to describe the zero-temperature depinning is the functional renormalization group (FRG), see refs. [20–22] and [16]. It allowed to obtain an expansion in $4 - d$, $d$ being the dimension of the interface, of the depinning exponents for $T = 0$. Recently, one-loop FRG equations describing the full dynamics of the interface at finite temperature have been derived [22]. Such equations were proven to be very efficient to obtain the dynamics at very small force (the so-called creep behavior [10]). Unfortunately, solving them around $F_c$ at $T > 0$ is considerably more complicated, and no complete solution exists to date, even if some crude prediction for the rounding exponent can be made [22,23].

In addition to the intrinsic problem of solving the equations, the fact that they are obtained in a $4 - d$ expansion makes them poorly suited to quantitatively describe the important case of the one-dimensional interface. Alternative methods, such as numerical studies, are thus clearly needed. Indeed, there were various attempts to obtain from numerical simulations the thermal exponent which characterizes the power law growth of the velocity with temperature at the critical force, $V(F_c) \sim T^\nu$. In ref. [24] a semi-infinite medium, which is numerically simpler than a system with periodic boundary conditions, but induces extremal critical-force fluctuations, was used to extract the thermal rounding exponent, assuming the two systems were in the same universality class. Similarly, a finite-temperature study of the depinning transition in a model of extremal activated dynamics [25] was reported. Other works had computed the thermal exponent in various dimensions by analyzing the domain wall generated in simulations of the random-field Ising model [26–28]. In these works, in addition to the intrinsic numerical limitations or questions on the universality class of the model, the authors used the average critical values for the pinning force, obtained by fitting the zero-temperature velocity-force characteristics. Unfortunately, this force has severe sample-to-sample fluctuations, which indeed strongly depend on the size of the system [29–31]. This made the determination of the thermal rounding exponent obtained by such methods relatively imprecise. More importantly, it made it difficult to check whether one had the proper scaling to even define such an exponent, an important point in view of the existing differences between depinning and a standard critical phenomena.

In this paper we analyze the finite-temperature dynamics of driven elastic interfaces evolving in a two-dimensional random media. We adopt a conceptually different approach by using the pinning force for each disorder realization [29] as was done in [32] for analyzing the zero-temperature case. We obtain a precise value of the thermal exponent, $\psi = 0.15 \pm 0.01$, directly from the velocity vs. temperature relation at the critical force, and we show that it is related to the existence of a temperature-dependent correlation length, growing with decreasing velocity in the same way as for the zero-temperature case. The existence of such velocity-dependent correlation length is directly proved by the analysis of the finite-temperature structure factor. This shows that the large-scale geometry of the interface is only controlled by the velocity regardless of whether one approaches the critical point from positive forces at zero temperature or from the temperature axis at the critical force.

We consider an elastic interface described by a single valued function $u(z,t)$, giving its transverse position $u$ in the $z$-axis. The interface evolves with time $t$ according to

$$\gamma \partial_t u(z,t) = c \partial^2_z u(z,t) + F_p(u,z) + F + \eta(z,t),$$

where $\gamma$ is the friction coefficient and $c$ the elastic constant.

The pinning force $F_p(u,z) = -\partial_u U(u,z)$ represents the effects of a random-bond disorder described by the potential $U(u,z)$, whose sample-to-sample fluctuations are given by $[\overline{U(u,z) - U(u',z')}]^2 = \delta(z-z') R(u-u')$, where $R(u)$ stands for a correlator of range $r_f$ [22], and the overline indicates average over disorder realizations. The thermal noise $\eta(z,t)$ satisfies $\langle \eta(z,t) \rangle = 0$ and $\langle \eta(z,t) \eta(z',t') \rangle = 2\gamma T \delta(t-t') \delta(u-u')$. Finally, the force $F$ corresponds to a uniform and constant external field.

In order to numerically solve eq. (1) we discretize the $z$-direction in $L$ segments of size $\delta z = 1$, i.e. $z \rightarrow j = 0, \ldots, L - 1$, while keeping $u_j(t)$ as a continuous variable. The equation is integrated using the Euler method with a time step $\delta t = 0.01$. The continuous random potential is modeled by a cubic spline passing through $M$ regularly spaced uncorrelated Gaussian numbers points [29,33]. The numerical simulations are performed using $\gamma = 1$, $c = 1$, $r_f = 1$, and with $R(0) = 1$ giving the strength of the disorder. Periodic boundary conditions are used in both spatial directions, thus defining an $L \times M$ system.

Figure 1 shows typical velocity-force characteristics for a single sample of size $L = 1024$, obtained by computing the steady-state velocity $V = \langle \partial_z u(z,t) \rangle$ through numerical integration of eq. (1). The sample-dependent critical force is also quoted, which can be obtained for each realization of the disorder configuration [29]. We can observe that the sharp depinning transition is rounded by temperature as expected. The straight dashed line indicates the fast-flow limit $V = F$, which is reached at high force. The left inset shows the velocity-force curve for $T = 0.001$, for a smaller system size $L = 128$, and for two different values of the critical force, as indicated. At large forces both curves asymptotically approach the fast-flow limit $V = F$. However, as is shown in the left inset, near the depinning transition the velocity strongly depends on the precise value of the critical force. Therefore, in order to properly analyze the rounding of the depinning transition, averages of small velocities must be carefully defined. To avoid this problem we exploit the access we have to a high-precision value of $F_c$ for each sample [29], as was previously done for the $T = 0$ case [32]. We thus define the reduced velocity and force variables, $v = \langle \partial_z u(z,t) \rangle / F_c$.

26005-p2
that one sweep is also good enough to properly account for the procedure and how to choose the size $M$ during one sweep over the lateral size $L$. We start with a flat initial condition and let the system evolve during one sweep over the lateral size $L = 128$. The right inset shows the same data as in the left inset but in reduced variables $v = V/F_c$ and $f = (F - F_c)/F_c$.

For a given disorder realization we first compute the critical force of the sample and then average the reduced velocity $v$ for a fixed value of the control parameters $f$ and $T$, both set with high precision. Since thermal and disorder averages must be taken in the steady state, we start with a flat initial condition and let the system evolve during one sweep over the lateral size $M$ which is large enough to assure the steady-state equilibration for all the temperatures analyzed. We then compute $v$ during a second sweep over the lateral size $M$. We find that one sweep is also good enough to properly account for the thermal fluctuations of $v$. The justification of this procedure and how to choose the size $M$ will be given below. Disorder average is finally performed by repeating this procedure for $N$ samples of a given system size. In the following $v$ represents the final, thermal and disorder, averaged value of the steady-state velocity.

Since the critical-force distribution and the geometry of the critical configuration depends on the relation $L/M$ for a periodic system [30], it is important to properly set the aspect ratio of the system before attempting any finite-size analysis of the data. An efficient choice [30,31] is to work with $M = L^{\zeta_{dep}}$, where $\zeta_{dep}$ is the roughness of the line at $T = 0$ depinning. We take the value $\zeta_{dep} = 1.25$ determined numerically [32,34]. This choice ensures, on the one hand, that $M$ is large enough for periodicity effects to be absent and decorrelation at the largest length scales to take place in one sweep as will be explained below, and, on the other hand, that $M$ is small enough to have no more than one dominant configuration [18] controlling the small-velocity regime in each sample. Otherwise this would complicate drastically the analysis of thermal effects at low temperatures. In the following, we will present results with $L = 128, 256, 512, 1024$ and $2048$, with $M = 430, 1024, 2436, 5792$ and $13770$, respectively.

Figure 2 shows $v$ against $T$ at $F_c$, i.e., $f = 0$, for different system sizes. This data is averaged over $N = 1000$ samples for $L = 128, 256, N = 500$ samples for $L = 512, 1024$, and $N = 100$ for $L = 2048$. Strong finite-size effects can clearly be seen at the smallest temperatures, where the motion becomes increasingly more correlated. Moreover, we find that finite-size effects are appreciable up to $L = 512$ in all the temperature range analyzed in fig. 2. However, curves with $L = 1024$ and $L = 2048$ collapse very well for $T > 5 \cdot 10^{-4}$. Although a system-size–dependent saturation value of the velocity is expected for $T \ll L^{-\beta/\psi}$, the temperature range $T > 5 \cdot 10^{-4}$ is independent of the system size for $L \geq 1024$. We thus use this data free of finite-size effects to determine the thermal rounding exponent $\psi$. At high temperatures, the fast-flow regime, $v = 1 + f$, should be observed. Well below this regime, the range $5 \cdot 10^{-4} < T < 5 \cdot 10^{-2}$ corresponds to the power law behavior $v \sim T^\psi$. The best-fitted value of the thermal exponent is $\psi = 0.15 \pm 0.01$, with the error bar larger than

\[ v(T) = |v_T| (1 - T/T_c)^\psi \]

where $|v_T| = |v(x, L, M)|_{T=0}$, $v_T$ is the transient value of $v$ at a given temperature, and $T_c$ is the critical temperature for a given disorder realization.
the pure statistical one and taking into account the fact that we tested different fitting temperature ranges.

In analogy with critical phenomena, one can assume that the steady-state velocity, which represents the order parameter of the depinning transition [15], is a homogeneous function of the “state variables”, f and T, although only for positive f [18]. This is consistent with the existence of a growing correlation length controlled by the velocity. With this assumption, scaling arguments lead to universal functions allowing to describe finite-size regimes.

At the critical force f = 0 such scaling relation can be written in terms of the system size and temperature as [20,35]

\[ v \sim L^{-\beta/\nu} h \left( T^{\beta/(\nu \psi)} \right), \]

(2)

with a scaling function h(x) behaving as h(x) \sim 1 for x \ll 1 while h(x) \sim x^z for x \gg 1. Due to strong corrections-to-scaling effects, which go beyond relation (2), the scaling of the data in fig. 2 using this relation is only satisfactory for the two largest system sizes, but with the finite-size regime at low temperature not well developed. However, the scaling relation (2) strongly suggests the existence of a growing dynamical correlation length \( \xi_T \) at depinning, such that \( \xi_T \sim T^{\nu z/\beta} \) at the critical force. We test this hypothesis by measuring the temperature-dependent structure factor \( S(q) \) of the interface, which is defined by

\[ S(q) = \left[ \frac{1}{L} \sum_{j=0}^{L-1} u_j(t) e^{-i q j} \right]^2, \]

(3)

where \( q = 2\pi n/L \), with \( n = 1, \ldots, L - 1 \). From simple dimensional analysis one infers that for small q, \( S(q) \sim q^{-(1+2z)} \) for a line with a roughness exponent \( z \).

FRG calculations show [22] that the large-scale motion of an interface at \( T = 0 \) and finite velocity (i.e. \( f > 0 \)) can be described by the Edwards-Wilkinson model with an effective temperature determined by the velocity and by the disorder strength. At \( T = 0 \) the corresponding crossover length \( \xi_T \) diverges as \( f \to 0^+ \) and separates two roughness regimes [22,32]: at length scales larger than \( \xi_f \), the roughness scales with the depinning exponent \( \zeta_{\text{dep}} \), while at length scales smaller than \( \xi_f \) it scales with a purely thermal roughness exponent \( \zeta_T = 1/2 \), with \( \zeta_T < \zeta_{\text{dep}} \). It is thus natural to check whether the correlation length \( \xi_T \) which controls the rounding of the depinning transition at \( f = 0 \) and \( T > 0 \) has the same geometrical interpretation. In such a case, the structure factor would scale as

\[ S(q) \sim T^{-\psi(1+2\zeta_{\text{dep}})/\beta} s \left( q T^{\nu z/\beta} \right). \]

(4)

where the scaling function \( s(x) \) behaves as \( s(x) \sim x^{-1+2z(\nu z/\beta)} \) for \( x \ll 1 \) and \( s(x) \sim x^{-1+2z\zeta_{\text{dep}}} \) for \( x \gg 1 \).

Figure 3 shows the rescaling of the structure function \( S(q) \) according to eq. (4) for different temperatures (the unscaled data is shown in the inset). The system size used is \( L = 1024 \) and the range of temperatures analyzed covers well the power law regime of \( v \) from which we have measured the thermal exponent \( \psi \). To rescale \( S(q) \) we have used the thermal exponent measured here, \( \psi = 0.15 \), together with \( \beta = 0.33 \) [32], \( \nu = 1.33 \) [19], \( \zeta_T = 0.5 \) and \( \zeta_{\text{dep}} = 1.25 \) [32,34] from \( T = 0 \) depinning simulations. As can be observed, the collapse of curves is excellent in the three temperature decades analyzed. Figures 2 and 3 thus give strong support to the view that the scaling behavior at the depinning transition, both for \( f > 0 \), \( T = 0 \) and the \( f = 0 \), \( T > 0 \) cases, is controlled by the same velocity-dependent correlation length, having identical geometrical interpretations. This length diverges with the velocity as \( \xi \sim v^{-\nu z/\beta} \) regardless of how the \( v = 0 \) critical point is approached (\( f \to 0^+ \) at \( T = 0 \) or \( T \to 0 \) at \( f = 0 \)).

Let us note as well that the characteristic time \( \tau \sim \xi^2 \sim v^{-\nu z/\beta} \), with \( z \) the dynamical exponent, is identical to the zero-temperature case. A short-time dynamics analysis [19,36] at the depinning transition shows that this is also the time needed for an initially flat configuration to reach the steady state: for times smaller than \( \tau \) the initial velocity decreases as a power law and crossovers, for times larger than \( \tau \), to an exponential relaxation towards the steady-state velocity. This means that the time to perform one sweep in our system is \( M/v \sim L^2/\psi v \sim v^{-\nu z/\beta} \), thus justifying our criterion for steady-state equilibration for any temperature \( T \) such that \( \xi \ll L \), and showing the convenience of simulating an \( M \times L \) system with \( M = L^2 \).

Let us now compare our results with previous analysis of the thermal rounding. In ref. [24], the authors gave a first estimate for the thermal exponent of an elastic string in a random medium. Instead of dealing with a finite-size system, they simulated the driven interface in a semi-infinite medium with \( M \to \infty \) by dynamically generating
disorder in a small region of the sample around the moving system. By fitting the resulting time-averaged velocity-force characteristics they have calculated the critical force, and the critical exponents. Their value for the thermal exponent, \( \psi = 0.16 \) [24], is very close to our present estimate. However, their numerical procedure also gives \( \beta \approx 0.24 \), which is much lower than the one obtained in more recent numerical simulations, \( \beta \approx 0.33 \) [19,32] and analytical calculations [16,22], prompting for questions on the accuracy of the obtained critical exponents. More recently, a finite-temperature study of the depinning transition [25] was reported. Analyzing the geometry of the line at depinning with this artificial dynamics gives a characteristic length \( l \sim T^{-0.95} \) which separates the \( \zeta_{\text{dep}} = 1.25 \) and \( \zeta_T = 0.5 \) regimes of roughness and which can be also associated with the distribution of subcritical forces along the front. If \( l \) were the velocity-dependent dynamical correlation length of the depinning transition \( \xi \), this result would imply \( \psi = 0.24 \), which is much higher than ours and previous reported values. So most likely this artificial dynamics does not allow to infer the velocity for the depinning problem.

For systems with the proper dynamics, the thermal exponent \( \psi \) has been also determined for domain wall motion in the random-field Ising model, by fitting various parameters of the \( V(F,T) \) curves in order to obtain universal functions. The reported value for this indirect measure is \( \psi \approx 0.2 \) in \( (1 + 1) \) dimensions [26], higher than our value. Although our study rests on random-bond disorder, it has been shown [17,22,37] that for the \( T = 0 \) dynamics random-bond disorder and random field are in the same universality class, contrarily to the statics. One would thus naively expect the same thermal exponent as the one found in ref. [26], assuming that anharmonic corrections to the elasticity, present in the random-field Ising model but absent in our model, cannot change the value of \( \psi \). If that is the case, the difference in the thermal exponent can be ascribed to the numerical limitations in [26] compared to our method, where the control parameter \( F - F_c \) can be determined with high accuracy, for obtaining this exponent.

One of the interests in trying to obtain an accurate determination of the rounding exponent is to determine which scaling law governs it and whether it is an independent exponent from the \( T = 0 \) depinning exponents or not. In the simpler case of a particle in a one-dimensional potential, the value of the thermal exponent is related to the first-passage-time problem of overcoming, by thermal fluctuations, the vanishing barrier at the depinning or saddle-node bifurcation [38,39]. This leads to the relation \( \psi = \beta/(2 - \beta) = 1/3 \) with \( \beta = 1/2 \) for any analytical potential with non-vanishing third derivative at the critical point. For the case of the interface no solid determination of the rounding exponent exists. One proposal [23] for the rounding exponent is \( \psi = \beta/(1 + 2\beta) \). This relation, with \( \beta = 1/3 \) would lead to \( \psi = 1/5 \) which would be much higher than our numerical estimate, although it would be compatible with the value found in ref. [26]. It would also work poorly for the \( d = 2 \) case of the random-field Ising model (using \( \beta = 2/3 \) it would give \( \psi = 2/7 \) instead of the measured [27] \( \psi = 0.42 \) and seems thus to be ruled out.

For charge density waves it was proposed [15] using a mean-field approximation that \( \psi = 2\beta/3 \). This law agreed with numerical simulations for finite-dimension charge density waves models [35] and was compatible with the data of [24]. Our value of \( \psi \) rules out this relation for domain walls. Using the analogy with standard critical phenomena, (viewing \( F - F_c \) as \( T - T_c \), \( V \) as a magnetization, and \( T \) as an external magnetic field) a scaling relation for the thermal exponent can be obtained by using the standard hyperscaling relation [27], leading to \( \psi = \beta/[(d + 1)\nu - 2] \), where \( d \) is the internal dimension of the interface (\( d = 1 \) in the present case). Such an estimate gives good results for the \( d = 2 \) rounding exponents for the random-field Ising model [27]: using \( \beta = 2/3 \) and \( \nu = 3/4 \) it predicts \( \psi = 8/19 \approx 0.421 \). For our one-dimensional case, it would predict the value \( \psi = 1/7 \approx 0.143 \), lower but very close to our numerical value. This relation seems thus empirically quite good. Of course, this approach is purely phenomenological as there is not any clear justification for using the equilibrium scaling relations for non-equilibrium dynamical transitions.

A rigorous derivation is thus clearly needed. In principle the finite-temperature FRG calculation [22] allows to reach the thermal exponent. A brute force expansion in the flow [22] gives back directly the relation \( \psi = \beta/(1 + 2\beta) \) [23], but the equations show clearly that there are corrections that should be taken into account and potentially modify the relation. Unfortunately, due to the interplay between velocity and temperature the equations are quite complicated and the involved analytical approach has not been accomplished so far. In that respect our results provide an important clue by showing that the scaling properties near the critical point \( f = T = 0 \) are controlled by the same velocity-dependent correlation length as for the \( T = 0, f > 0 \) case. The large-scale geometry is thus the same in both cases, regardless of the origin of the steady-state velocity. In the FRG this implies that the flow of the friction/velocity (the parameter \( \lambda \) in [22]) is identical to the zero-temperature case, providing an independent confirmation of the hypothesis used to study the \( T = 0 \) flow, that the rounding of the cusp occurs at a scale \( \rho_A \) which is negligible compared to \( \lambda \) close to the depinning [22]. Note that since \( v \sim T^\psi \) and \( \psi < 1 \) the velocity is indeed large compared to the temperature. This makes it likely that the whole rounding of the cusp is still controlled by the velocity, although in principle it could be also possible that, even if the temperature is smaller than \( \lambda \), it is larger that the velocity cusp-rounding scale, \( \rho_A \). Our result thus urges for a reexamination and a further analysis, either analytical or numerical, to check these possible scenarios and to extract the rounding exponent directly from the FRG calculation [36]. Finally, let us point out that, although the results here presented correspond strictly to the steady-state evolution of the
driven interface, we have also found that the value of $\psi$ agrees well with the ones obtained from the analysis of the non-steady short-time relaxation of the interface at $f = 0$ and $T > 0$ [36].

Our results are directly relevant for recent experiments on the domain wall motion in ferromagnetic thin films [4]. Indeed, in these experiments both the thermal exponent $\psi$ and the geometrical properties such as the dynamical correlation length $\xi$ and the roughness exponents $\zeta_T$ and $\zeta_{dep}$ could be obtained by imaging the structure of the moving domain walls as for the creep case [1]. This would allow to check the current picture of the depinning transition as a collective phenomenon. It is also worth stressing that our study, and in particular our finite-size effects analysis, is relevant for other model systems with system size limitations, such as the recently reported simulations on the temperature dependence of the flux lines dynamics in high-temperature superconductors [40,41].

In conclusion we have studied the thermal rounding of the depinning transition by analyzing both the steady-state velocity and geometry of a one-dimensional interface in a two-dimensional random medium. We have obtained the thermal exponent $\psi = 0.15 \pm 0.01$ directly from the temperature dependence of the velocity at the critical force, $v \sim T^{\psi}$. As was recently done in ref. [32] for the $T = 0$ case, we have exploited the access to the exact critical force for each disorder realization by using the powerful Rosso-Krauth algorithm [29,34]. This allowed us to eliminate the statistical uncertainty in the control parameter induced by the sample-to-sample fluctuations of the critical force, which is present in all previous numerical approaches to the thermal-rounding problem. Moreover, we have shown explicitly that the value of $\psi$ is consistent with the existence of a velocity-dependent correlation length $\xi$ separating two regimes of roughness, and we find that $\xi$ diverges as $v \to 0$ the same way, regardless of whether we approach the depinning threshold $f = T = 0$ from positive forces, or from the temperature axis. Our results are relevant for recent experiments on domain wall motion in ferromagnetic thin films [4].

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