The Fisher Matrix is the backbone of modern cosmological forecasting. We describe the Fisher4Cast software: a general-purpose, easy-to-use, Fisher Matrix framework. It is open source, rigorously designed and tested and includes a Graphical User Interface (GUI) with automated \LaTeX file creation capability and point-and-click Fisher ellipse generation. Fisher4Cast was designed for ease of extension and, although written in Matlab, is easily portable to open-source alternatives such as Octave and Scilab. Here we use Fisher4Cast to present new 3-D and 4-D visualisations of the forecasting landscape and to investigate the effects of growth and curvature on future cosmological surveys. Early releases have been available at http://www.cosmology.org.za since May 2008 with 750 downloads in the first year. Version 2.0 is made public with this paper and includes a Quick Start guide and the code used to produce the figures in this paper, in the hope that it will be useful to the cosmology and wider scientific communities.

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I. INTRODUCTION

The need for rapid forecasts of constraints from proposed new surveys has played an important role in the approach of the cosmology community to the mysteries of dark energy. Faced with an almost total lack of understanding of the physics underlying dark energy, focus has shifted towards designing surveys that are optimal in some information theory sense. When hunting an entirely unknown animal, asking questions about its beak or tail may leave one barking up the wrong tree, so to speak. Instead a safe bet is simply to maximise the number of bits of information one gets about the animal. The Fisher Matrix formalism allows one to do this within a parameterised framework.

Despite its limitations, the Fisher Matrix formalism has become the de facto standard in cosmology for comparing surveys and for forecasting parameter constraints. It can also be easily adapted to different scenarios and has thus informed the design and funding of essentially all modern cosmological surveys.

Nevertheless, while the Fisher Matrix formalism is relatively simple in principle, students can find it challenging at first, implementations are at times buggy and usually limited – almost always assuming a flat universe, for example – and there is unnecessary repetition of code, without a clear development path accessible to the entire community to build on. Fisher4Cast was initially developed in 2007 and 2008 to address these issues with the aim of providing a free, graphical framework that would make it easy for students to learn the formalism while also simultaneously providing a rigorous, robust and general code-base for researchers to build on and extend. Version 2.0, released with this paper, continues with the same aims as Version 1.2.

We discuss the Fisher formalism in Section II and introduce the cosmology used in Fisher4Cast in Section III. The effect of including cosmic curvature as a parameter is discussed in Section IV C and the growth of structure as an observable in Section IV B. The applications of Fisher4Cast are outlined in Section IV A while new features in Versions 1.2 and 2.0 are described in Section IV D. The appendices give the general and explicit Fisher derivatives and the Quick Start guide for Fisher4Cast, as well as the samples of the Matlab

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1 For a list of over 100 recent astronomy/cosmology papers using the Fisher Matrix formalism see http://lanl.arxiv.org/find/astro-ph/1/abs:+AND+Fisher+matrix/0/1/0/all/0/1

2 An IDL codeset, iCosmo, was released subsequent to Fisher4Cast and also provides powerful routines for cosmological Fisher Matrix analysis and a lot more, with a convenient web interface and extensive online documentation as part of the Initiative for Cosmology [1].
II. FORECASTING AND THE FISHER MATRIX

Forecasting of survey constraints for a proposed survey can be achieved either through full Monte Carlo simulations of the survey which are typically time consuming and computationally intensive, or by using the much simpler and quicker Fisher Matrix technique, which we now discuss in detail and which forms the basis of Fisher4Cast. The Fisher Matrix translates errors on observed quantities measured directly in the survey into constraints on parameters of interest in the underlying model. Put more directly, it is the elegant way of doing propagation of errors in the case of multiple, correlated, measurements and many parameters. As an example, consider an arbitrary function \( y = f(z, \theta) \) of some parameter \( \theta \) and an independent variable \( z \). Assuming a perfect measurement of \( z \), the error \( \delta \theta \), for a given measured \( y \) is, by simple calculus, \( \delta \theta = (\partial y / \partial \theta)^{-1} \delta y \), or equivalently \( (\delta \theta)^{-2} = (\partial y / \partial \theta)^2 (\delta y)^{-2} \). This is perhaps the simplest example of a Fisher Matrix; with a single element, cf. Eq (2). Here \( \theta \) represents the parameter we want to measure and \( f \) the observable quantity (e.g. \( H(z) \)).

In more generality, the Fisher Matrix formalism predicts the constraints on a vector of parameters \( \theta = (\theta_1, \theta_2, ..., \theta_A, ...) \) - such as \( w_0 \) and \( w_a \) - resulting from measurements of one or more observables \( X^\alpha = X^\alpha(\theta, z) \) (such as \( H(\theta, z) \) or \( d_A(\theta, z) \)), each at a range of redshifts, \( z = (z_1, z_2, ..., z_i, ...) \) e.g. in a BAO survey one might measure \( H(\theta, z) \) and \( d_A(\theta, z) \) at a single redshift, while a Type Ia supernova (SNIa) survey may measure \( d_A(\theta, z) \) at hundreds of redshifts. There are therefore three indices in general to keep track of, \( (A, \alpha, i) \) corresponding to parameter, observable and redshift. The number of observables is arbitrary and combining results from independent observables is essentially trivial, hence we will often suppress observable index, \( X^\alpha = X \) for simplicity. Boldface indicates the entire vector, either of parameters \( \theta \), observables \( X \) or redshifts \( z \).

The Fisher Matrix estimates not only the individual errors on the parameters, \( \theta \), evaluated at a given fiducial/fiducial model \( \theta = \theta^* \), but also the correlations between them, leading to the characteristic Fisher error ellipsoids (ellipses if one considers only pairs of \( \theta_A \)). To make this clear, consider the likelihood, \( \mathcal{L} = P(d|\theta) \), for a general survey, which gives the conditional probability of observing the data \( d = (d_1, d_2, ..., d_i, ...) \) assuming the cosmological model \( \theta \) is correct. We can expand the likelihood around the fiducial model:

\[
\ln \mathcal{L}(\theta^* + \delta \theta) = \ln \mathcal{L}(\theta^*) + \sum_A \frac{\partial \ln \mathcal{L}(\theta)}{\partial \theta_A} \bigg|_{\theta = \theta^*} \delta \theta_A + \frac{1}{2} \sum_{AB} \frac{\partial^2 \ln \mathcal{L}(\theta)}{\partial \theta_A \partial \theta_B} \bigg|_{\theta = \theta^*} \delta \theta_A \delta \theta_B + \frac{1}{6} \sum_{ABD} \frac{\partial^3 \ln \mathcal{L}(\theta)}{\partial \theta_A \partial \theta_B \partial \theta_D} \bigg|_{\theta = \theta^*} \delta \theta_A \delta \theta_B \delta \theta_D + \frac{1}{12} \sum_{ABDE} \frac{\partial^4 \ln \mathcal{L}(\theta)}{\partial \theta_A \partial \theta_B \partial \theta_D \partial \theta_E} \bigg|_{\theta = \theta^*} \delta \theta_A \delta \theta_B \delta \theta_D \delta \theta_E + ..., \tag{1}
\]

where \( \partial_A = \partial \theta_A \) represents the partial derivatives with respect to the parameter \( \theta_A \). The first term in the expansion is a constant depending on the fiducial model. The fiducial model is expected (after averaging over many data realisations) to be the point of maximum likelihood, hence the first derivative of the likelihood vanishes. The third term is the curvature matrix/Hessian of the likelihood, and is the term used in the Fisher Matrix which is formally defined as the expectation value of the derivatives of the log of the likelihood with respect to the parameters \( \theta \), or

\[
F_{AB} = \left\langle \frac{\partial^2 \ln \mathcal{L}}{\partial \theta_A \partial \theta_B} \right\rangle. \tag{2}
\]

Here the angle brackets indicate the expectation value which, for an arbitrary function \( g(X) \), is defined to be \( \langle g(X) \rangle = \int_{-\infty}^{\infty} g(x)f_X(x)dx \) with \( f_X(x) \) the probability distribution function of the random variable \( x \), which here is the noise on the data, assumed to be Gaussian of mean zero.

The likelihood for a given observable \( X \) is expressed in terms of the theoretical value of the observable \( X_i \) evaluated at the redshifts \( z_i \) and data for that specific observable \( d_i \) as \( \mathcal{L} \propto \exp(-\Delta^T C^{-1} \Delta / 2) \) where \( \Delta \equiv X - d \), generalising the usual chi-squared statistic relating the theory by allowing for a general data
covariance matrix \( C \). Substituting the above expression into Eq. (2) converts the equation from derivatives of the likelihood itself into a sum over derivatives of the observable \( X \) with respect to the parameter \( \theta \):

\[
F_{AB} = \frac{\partial X^T}{\partial \theta_A} C^{-1} \frac{\partial X}{\partial \theta_B} + \frac{1}{2} \text{Tr} \left( C^{-1} \frac{\partial C}{\partial \theta_A} C^{-1} \frac{\partial C}{\partial \theta_B} \right) \tag{3}
\]

where \( \frac{\partial C}{\partial \theta_A} \) is the derivative of the data covariance matrix with respect to the parameter \( \theta_A \) which is assumed to vanish in the second equality implying that the data errors are independent of cosmological parameters. This is often the case, e.g. the errors on measurements of Type Ia supernova (SNIa) flux are independent of the dark energy parameters \( w_0, w_a \) to good accuracy. The second equality also requires that the data are uncorrelated, in which case \( C \) is diagonal with entries \( \sigma_i^2 \), with the \( \sigma_i \) the \( 1 - \sigma \) error on the \( i \)-th data point.

In the case where we have multiple independent measurements of different observables \( X^\alpha \) (e.g. \( H(z) \) and \( d_A(z) \)), the total Fisher Matrix is just the sum of the individual Fisher matrices indexed by \( \alpha \). Similarly, if we have independent prior information, this is encoded in a prior matrix between the cosmological parameters.

In this paper we will refer to the prior on a single parameter \( \theta_A \) as \( \text{Prior}(\theta_A) = (\Delta \theta_A)^{-2} \), where \( \Delta \theta_A \) is the uncertainty on the parameter as measured from prior surveys, see e.g. Figure (11). In the case where the different measurements are not independent, they must be combined with the suitable data covariance matrix. The inverse of the Fisher Matrix, \( F_{AB}^{-1} \), provides an estimate of the error covariance matrix for the parameters \( \theta_A \), as we now expand upon.

In general one often considers a \( p + n \)-dimensional likelihood which includes not only the \( p \) parameters of interest but an additional \( n \) nuisance parameters that form a natural part of the problem but are of no direct interest (such as \( H_0 \) in studies of dark energy dynamics). As a result we are usually only interested in the likelihood as a function of the \( p \) key parameters, which is obtained by marginalising over the nuisance parameters, viz.

\[
\mathcal{L}(\theta_1, \ldots \theta_p) = \int_{-\infty}^{\infty} \mathcal{L}(\theta_1, \ldots \theta_p, \ldots \theta_{p+n}) d\theta_{p+1} \ldots d\theta_{p+n}. \tag{4}
\]

For a likelihood of arbitrary shape marginalisation must be performed numerically and is well-suited to Markov Chain Monte Carlo (MCMC) methods but in the special case where it is a multivariate Gaussian,
FIG. 2: **Varying** \( w_0 \) **and** \( w_a \) **of the fiducial model** – the generated ellipses for a measurements of the Hubble parameter \( H(z) \) and angular diameter distance \( d_A(z) \) survey characterised in Table I. The values for the coefficients in the CPL parameterisation for \( w(z) \), \( w_0 \) and \( w_a \), have been varied on a grid over \(-1.3 < w_0 < -0.6, -0.7 < w_a < 1 \).

As \( w_0, w_a \) change, not only does the ellipse centre shift, but the size of the ellipse changes, as well as the slope of the degeneracy direction between the two parameters. Since the Dark Energy Task Force Figure of Merit is linked to the inverse of the area of the ellipse the value of the FoM increases for with increasing \( w_0 \) and \( w_a \).

The marginalisation can be performed analytically and simply in terms of the Fisher Matrix. Let us write the full Fisher Matrix in terms of sub-matrices, as

\[
F = \begin{pmatrix}
\Theta & \mathcal{O} \\
\mathcal{O}^T & \mathcal{N}
\end{pmatrix},
\]

where \( \Theta \) is the \( p \times p \) sub-matrix corresponding to the parameters of interest, \( \mathcal{O} \) is an \( p \times n \) matrix describing the correlation between the nuisance parameters and the parameters of interest and \( \mathcal{N} \) is the \( n \times n \) matrix representing the nuisance parameters we wish to marginalise over. The marginalised Fisher Matrix for the parameters of interest is then given by

\[
\tilde{F} = \Theta - \mathcal{O} \mathcal{N}^{-1} \mathcal{O}^T,
\]

where the first term is the matrix of the reduced parameter space of interest, the second term encodes the effects of the marginalisation over the other nuisance parameters and \( T \) represents matrix transpose.

As we mentioned before, the inverse of the Fisher Matrix provides an estimate of the parameter covariance matrix. For an unbiased estimator (that is one whose expected value of \( \theta \) is equal to the fiducial model \( \theta^* \) assumed to be correct), and in the case where one does not marginalise over any other parameters (i.e. we consider all other parameters perfectly known), the expected error on any parameter \( \theta_A \) satisfies the Cramér-Rao bound

\[
\Delta \theta_A \geq \frac{1}{\sqrt{F_{AA}}},
\]

while in the more realistic case that one wants to marginalise over all the other parameters in the problem, the bound becomes

\[
\Delta \theta_A \geq \sqrt{(F^{-1})_{AA}}
\]

\(^3\) If \( \mathcal{O} = 0 \) then the nuisance parameters have no impact on \( \Theta \).

\(^4\) For a proof see e.g. [http://en.wikipedia.org/wiki/CRLB](http://en.wikipedia.org/wiki/CRLB) or e.g. p. 426 in [1].
i.e. one first inverts the Fisher Matrix, then takes the AA component of the resulting matrix. One can show that the latter is always greater than or equal to the former, i.e. marginalisation cannot decrease the error on a parameter, and only has no effect if all other parameters are completely uncorrelated from the parameter of interest. Note that in the case where the likelihood is exactly Gaussian in the parameters, the Cramer-Rao bound becomes an equality and not just a lower bound.

Since Fisher Matrix analysis assumes the likelihood is a multivariate Gaussian (an approximation that can be made arbitrarily good by considering better and better surveys), contours of constant probability are ellipsoids within the Fisher formalism. These ellipsoids (ellipses for two parameters) are given by solving the equation

\[ \Delta \theta^T \tilde{F} \Delta \theta = \beta \]

where \( \Delta \theta = \theta - \theta^* \) is the parameter vector around the fiducial model, \( \theta^* \), and \( \beta \) is a constant determined by the desired confidence level and the number of parameters. For two parameters, the 1 and 2 \( - \sigma \) contour levels correspond to \( \beta = 2.31 \) and 6.17 respectively \([3, 5]\). The Fisher4Cast GUI allows plotting of both 1- and 2-dimensional contours and hence always marginalises the full 5-dimensional Fisher Matrix to achieve this. Marginalisation over some or all of the other parameters can be effectively switched off by making the corresponding diagonal elements of the prior matrix very large. The Fisher Matrix and the corresponding ellipses provide the Gaussian estimate for how well the parameters of the model will be constrained by a given experiment assuming the true model is that at which the Fisher Matrix was evaluated (e.g. \( \Lambda \)CDM). This is illustrated in Figure (1), which shows the 1 \( - \sigma \) error ellipse around the fiducial \( \Lambda \)CDM model with the coefficients in the Chevallier-Polarski-Linder (CPL) \([6, 7]\) parameterisation \((w_0, w_a) = (-1, 0)\), for a survey consisting of measurements of the angular diameter distance between redshifts of 0.1 and 3.

Values of \( w_0, w_a \) inside this ellipse will have expected likelihoods that differ from the fiducial model by less than 1 \( - \sigma \). The Fisher Matrix allows us to estimate which sets of parameter values we will be able to rule out at a given significance level if the fiducial cosmological model is correct. We will see in Section IV A (see Figure (2)) that changing the assumed fiducial/base model has a big effect on the ellipses for the same survey.

While the ellipses provide significant insight they do not allow immediate comparison between different surveys, a feature required if one wants to optimise or compare surveys head-to-head \([8, 9, 10, 11, 12, 13, 14, 15]\). One common way to perform such comparison is to formulate a Figure of Merit which ascribes a single real number to each survey, the simplest of which is to use the volume of the ellipsoid or of the marginalised ellipse. The volume of the \( n \)-dimensional error ellipsoid (corresponding to an \( n \)-dimensional Fisher Matrix) is:

\[ \text{Vol}_n = V_{\Sigma^n} \times \left( \frac{\beta}{\det(F)} \right)^{1/2}, \]

where \( \beta \) is defined in Eq (9), \( V_{\Sigma^n} = \pi^{n/2}/\Gamma(\frac{n}{2} + 1) \) is the volume of the \( n \)-dimensional unit sphere and \( \Gamma(u) \) is the Gamma function. For the interesting case \( n = 2 \), \( V_{\Sigma^2} = \pi \) is, of course, the area of the unit circle. Note that it is common in the literature to ignore the \( V_{\Sigma^n} \) and \( \beta \) factors and to incorrectly refer to the determinant factor alone as the area or volume of the ellipse/ellipsoid.

In fact, since the Fisher Matrix is a metric, the square root of the determinant is a natural volume element providing the Jacobian for the action of the linear mapping induced by the Fisher Matrix, i.e. one should think of the Fisher Matrix as inducing a linear mapping rather than ‘being’ an ellipse itself.

Fisher4Cast includes the standard FoMs as well as some new ones available through the GUI and command line. Those using the volume are based on Eq (10) with \( n = 2 \). Although some of the Figures of Merit are only defined for the error ellipse in the \( w_0 - w_a \) plane, where \( w_0, w_a \) are the coefficients in the CPL \([6, 7]\) parameterisation of the equation of state of dark energy (see for e.g. \([10]\)), the FoMs in Fisher4Cast are calculated by the code for any pair of cosmological parameters being considered rather than the full 5-D matrix. We briefly outline the FoMs used in Fisher4Cast:

- DETF

This Figure of Merit in the Report of the Dark Energy Task Force \([10]\) is defined to be the reciprocal of the area of the 2 \( - \sigma \) error ellipse in the \( w_0 - w_a \) plane of the CPL dark energy parameterisation\([6, 7]\). This is, via Eq (10), equal to \( \det(F^{1/2})/(\pi \sqrt{6.17}) \). Unfortunately the DETF report does not appear to use this definition, and instead quotes \( \det(F^{1/2}) \), which is the inverse of the 1 \( - \sigma \) ellipse in units of the area of the unit circle. Because of the benefits of the geometric interpretation Fisher4Cast returns the true inverse area of the 2 \( - \sigma \) ellipse. To convert from one DETF FoM to the other, one should multiply the Fisher4Cast DETF output by \( \pi \sqrt{6.17} \approx 7.8 \).
Walker (FLRW) universe. The cosmic parameters assumed for the GUI are \( \Omega_k \) measured independently (e.g. through redshift distortions - see [18, 19]).

The expansion of the dark energy equation of state [6, 7]:

\[
\frac{\Omega_k}{\Omega_m} = 1 - \Omega_m - \Omega_k
\]

\( \Omega_k \) is the curvature energy density (\( \Omega_k \)) is the curvature radius of the cosmos. Other dark energy expansions can be easily accommodated in Fisher4Cast by changing the appropriate input functions. The expansion history of a FLRW universe is described by the Hubble parameter:

\[
H^2(z) = H_0^2 F^2(z) = H_0^2 (\Omega_m(1 + z)^3 + \Omega_k(1 + z)^2 + (1 - \Omega_m - \Omega_k) f(z, w_0, w_a))
\]

where \( a_0 = a_0/(H_0 \sqrt{|\Omega_k|}) \) is the curvature radius of the cosmos. Other dark energy expansions can be easily accommodated in Fisher4Cast by changing the appropriate input functions. The expansion history of a FLRW universe is described by the Hubble parameter:

\[
H(z) = \frac{dA(z)}{dA(z)} \propto G(z) \sqrt{\Omega_m(1 + z)^3 + \Omega_k(1 + z)^2 + (1 - \Omega_m - \Omega_k) f(z, w_0, w_a)}
\]

The Fisher4Cast GUI uses the observables \( H, d_A \) and \( G \) in a general Friedmann-Lemaître-Robertson-Walker (FLRW) universe. The cosmic parameters assumed for the GUI are \( (H_0, \Omega_m, \Omega_k, w_0, w_a) \), where \( H_0 \) is the value of the Hubble constant in \( \text{km s}^{-1} \text{Mpc}^{-1} \), \( \Omega_m \) is the energy density of matter today in units of the critical density, \( \Omega_k \) is the curvature energy density (\( \Omega_k = 1 - \Omega_m - \Omega_k \)) and \( w_0 \), \( w_a \) are the coefficients in the CPL expansion of the dark energy equation of state [6, 7]:

\[
w(z) = w_0 + w_a \frac{z}{1 + z} = w_0 + w_a \left( 1 - \frac{a}{a_0} \right)
\]

III. THE COSMOLOGY OF HUBBLE, DISTANCE AND GROWTH

Although Fisher4Cast is a completely general Fisher Matrix framework at the command-line level, the GUI is coded as a cosmology interface, since this is its primary application. In the context of modern cosmological surveys, the primary observables are the expansion rate of the Universe, measured through the Hubble rate \( H(z) \), cosmological distances such as the angular diameter distance, \( d_A(z) \), and the growing mode of dark matter density perturbations, \( \delta(x, z) \propto G(z) \). \( H(z) \) and \( d_A(z) \) are provided by BAO surveys while growth can be measured using lensing or number count surveys and potentially also BAO if the bias is measured independently (e.g. through redshift distortions - see [18, 19]).

The Fisher4Cast GUI uses the observables \( H, d_A \) and \( G \) in a general Friedmann-Lemaître-Robertson-Walker (FLRW) universe. The cosmic parameters assumed for the GUI are \( (H_0, \Omega_m, \Omega_k, w_0, w_a) \), where \( H_0 \) is the value of the Hubble constant in \( \text{km s}^{-1} \text{Mpc}^{-1} \), \( \Omega_m \) is the energy density of matter today in units of the critical density, \( \Omega_k \) is the curvature energy density (\( \Omega_k = 1 - \Omega_m - \Omega_k \)) and \( w_0 \), \( w_a \) are the coefficients in the CPL expansion of the dark energy equation of state [6, 7]:

\[
w(z) = w_0 + w_a \frac{z}{1 + z} = w_0 + w_a \left( 1 - \frac{a}{a_0} \right)
\]

where \( a_0 = c/(H_0 \sqrt{|\Omega_k|}) \) is the curvature radius of the cosmos. Other dark energy expansions can be easily accommodated in Fisher4Cast by changing the appropriate input functions. The expansion history of a FLRW universe is described by the Hubble parameter:

\[
H^2(z) = H_0^2 F^2(z) = H_0^2 (\Omega_m(1 + z)^3 + \Omega_k(1 + z)^2 + (1 - \Omega_m - \Omega_k) f(z, w_0, w_a))
\]

with the evolution of the dark energy density, \( \rho_{DE}(z) \propto f(z) \) determined by

\[
f(z) = \exp \left( 3 \int_0^z \frac{1 + w(z')}{1 + z'} dz' \right)
\]

\[5\] Unless explicitly indicated elsewhere, references in this paper to \( \Omega_i \) will mean the current value of the density parameter and not its value as a function of time.
Fisher Derivatives for $H(z)$

The derivatives for growth are computed numerically since no general analytical solution for the growth function $\Lambda\text{CDM}$ model, a Taylor expansion of the derivative (Eq. B9) around $\Omega_k$ of the angular diameter distance in terms of the curvature parameter $H(\chi)$, the full set of analytical derivatives for $H(z)$ have been computed around the flat-$\Lambda\text{CDM}$ fiducial model - ($H_0$, solid dark red line), $\Omega_m$ (dot-dashed red line), $\Omega_k$ (dashed dark orange line), $w_0$ (dotted orange line) and $w_a$ (dot-dashed peach line). The full set of analytical derivatives of the $H(z)$ and $d_A(z)$ are found in Appendix [12]. In the case of the growth function $G(z)$ the derivatives of the solution to Eq. (21) are taken numerically.

For the CPL parameterisation, Eq. (11), $f(z)$ is given by

$$f(z) = (1 + z)^{3(1+w_0+w_a)} \exp \left\{ -3w_a \frac{z}{1+z} \right\}.$$  \hspace{1cm} (14)

The angular-diameter distance, $d_A(z)$ relates the angular size of an object to its known length, providing a measure of the distance to the object, and is given by:

$$d_A(z) = \frac{1}{1+z} \frac{c}{H_0 \sqrt{\Omega_k}} \sinh \left( \sqrt{\Omega_k} \chi(z) \right),$$  \hspace{1cm} (15)

where

$$\chi(z) \equiv \int_0^z \frac{dz'}{E(z')}.$$  \hspace{1cm} (16)

and $E(z)$ is as defined in Eq. (12). These forms are valid for all values of $\Omega_k$ via continuity and the trigonometric identity $\sinh(ix) = i \sin(x)$. The often-used equation for the angular diameter distance contains three equations, depending on the sign and magnitude of $\Omega_k$, however this is redundant, at least conceptually. In numerical analysis we use the Taylor series expansion for very small $\Omega_k$, Eq. (B10).

$$\frac{\partial d_A(z)}{\partial \Omega_k} \bigg|_{\Omega_k \to 0} = \frac{c}{H_0 (1+z)} \left\{ \frac{1}{6} \chi'(z,0) + \frac{\partial \chi(z,0)}{\partial \Omega_k} \right\}$$  \hspace{1cm} (17)

where

$$X(z,0) \equiv X(z)|_{\Omega_k \to 0}$$  \hspace{1cm} (18)

are the functions (for example $E(z)$, $\chi(z)$) assuming flatness.

Finally we discuss the governing equation for the growth of structure, a potentially powerful probe of dark energy [10, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29]. In general one needs to solve the differential equation for the perturbations in the matter density $\delta$ (assuming the pressure and pressure perturbations of the matter are zero - $p = \delta p = 0$) [30, 31, 32]:

$$\ddot{\delta} + 2H \dot{\delta} = 4\pi G \rho_m \delta.$$  \hspace{1cm} (19)

We discuss this equation in the context of curved universes with dynamical dark energy in Section [12]. Fisher4Cast takes as input constraints on the growth $G(z)$ which provides the temporal evolution of density perturbations, i.e. $\delta(x, z) \propto G(z)$.

Error ellipses for all the cosmic parameters in our example are shown in Figure [13], where they have been computed around the flat-$\Lambda\text{CDM}$ fiducial model - ($H_0, \Omega_m, \Omega_k, w_0, w_a) = (70, 0.3, 0, -1, 0)$ - and the full set of analytical derivatives for $H(z), d_A(z)$ are given in the Appendix [12]. In the case of the derivative of the angular diameter distance in terms of the curvature parameter $\partial d_A/\partial \Omega_k$, which will be in the flat-$\Lambda\text{CDM}$ model, a Taylor expansion of the derivative (Eq. [B9]) around $\Omega_k = 0$ is also provided in Section [12]. The derivatives for growth are computed numerically since no general analytical solution for the growth exists; see Section [IV.B].
FIG. 4: Marginalised Fisher ellipses for all parameters. Fisher error ellipses for the full range of cosmological parameters, namely $H_0, \Omega_m, \Omega_k, w_0, w_a$, (for a survey characterised by Table I with the addition of growth measurements at the same redshifts and with the same precision as the Hubble parameter) are produced by calling Fisher4Cast in a simple loop over all parameters. The dark and light filled ellipses indicate the 1$\sigma$ and 2$\sigma$ contours respectively marginalised over all other parameters. The diagonal panels show the fully marginalised one-dimensional likelihood for each parameter; 1 and 2$\sigma$ limits shown as solid and dashed vertical lines respectively. The code used to produce these plots is included in the latest release of Fisher4Cast.

IV. FISHER4CAST AND ITS APPLICATIONS

Fisher4Cast is written in Matlab\textsuperscript{6} following an object-oriented model and using standard software engineering standards for implementation and testing. The code is not specific to cosmology; Fisher matrices can be generated given any $X(\theta)$. The Fisher derivatives $\partial X / \partial \theta_A$ are computed analytically (if they are known) or numerically, as in the case of growth, allowing the code to handle complex cases without analytical formulae for $X$. The code suite includes a Graphical User Interface (GUI), which is specific to the cosmological example we discussed above in Section III. Fisher4Cast includes an automated \LaTeX report

\footnote{\url{http://www.mathworks.com}}
generating feature described in detail in the Users’ Manual[7], which automatically creates a summary of relevant data, matrices and figures.

Fisher4Cast facilitates novel research and education in two different ways. Apart from being well-tested against existing Fisher Matrix results, it is general and modular. Because of this modular nature, a natural application of Fisher4Cast is to visualisation. The code can easily be called repeatedly in large loops, enabling one to study large-scale properties of the Fisher Matrix for a wide range of surveys and cosmologies. We give examples of such studies in the following subsection. Secondly, Fisher4Cast is coded for a general FLRW universe (the curved case has rarely been studied in the literature). We study the issues of curvature and growth in dark energy Fisher analysis in the last two subsections.

FIG. 5: Figure of Merit plane – the Dark Energy Task Force Figure of Merit (FoM) for a survey consisting of one bin each of the Hubble parameter and angular diameter distance, with the fractional errors $\sigma_H / H = \sigma_{d_A} / d_A = 0.1$. The redshifts of the $H, d_A$ measurements are varied separately from $z = 0.1$ to $z = 5$, and the resulting FoM for each survey configuration is plotted as a 3-dimensional landscape (left panel), or a flat 2-dimensional plane (right panel). The colourmap in both panels reflects the value of the FoM, from low values of FoM $\sim 0$ (blue) to higher values (FoM $\sim 0.08$).

A. Visualisations

One of the key design principles behind Fisher4Cast is ease of use - the GUI was specifically created to provide users without an in-depth knowledge of Fisher Matrix theory access to the power of the formalism. Fisher4Cast can also be called from the command-line, as shown in the code examples in Appendix[9]. In this subsection we investigate applications of Fisher4Cast to probe and visualise the Fisher Matrix. Figure [2] illustrates the resulting ellipses when Fisher4Cast is called in a simple two-dimensional loop that varies the values of $w_0 - w_a$ in the cosmological model assumed to be true. The strong dependence of both the orientation and size of the resulting ellipse on the assumed model is clearly evident, as the ellipses rotate and shrink for larger values of $w_0$ and $w_a$.

Figure [3] illustrates the complementary case when the assumed cosmological model is kept fixed, but where instead the survey parameters are varied. In all cases the survey consisted of one measurement of $d_A(z)$ and $H(z)$ in a single redshift bin (with fractional errors of 10% on either observable), while the redshifts of these two bins were varied independently over the range $0.1 < z < 5$. The DETF Figure of Merit (FoM) of the survey ($\propto 1/$Area of the $w_0 - w_a$ ellipse) is plotted as a FoM “landscape”, where the colourmap is related to the value of the FoM - the higher values of the FoM are depicted in red, and the lower values in
Two interesting features are immediately apparent. First, the peak in the FoM landscape, or “FoM hotspot” occurs for a survey with measurements of $H(z)$ at $z = 1.5$ and $d_A(z)$ at $z = 0.6$, while a ridge of moderately good values of the FoM (relative to the average) emerges along the line corresponding to $H(z = 0.2)$ for measurements of $d_A$ and redshifts larger than 1.6. Secondly these two regions of higher Figure of Merit are separated by a “cold valley” of lower FoM values. This is of particular interest given the optimisation of current and future Baryon Acoustic Oscillation (BAO) surveys [8, 9, 10, 13, 34]. Future BAO surveys will measure the radial and tangential oscillation scale (see [35] for a recent review of BAO in cosmology), producing measurements of $d_A$ and $H$ at the same redshifts, i.e. along the diagonal line in this plot. For $z$ roughly between 0.5 and 3, this line intersects regions of relatively high FoM. This landscape is explained by the geometric interplay between the size and orientation of the degeneracy directions between the $H(z)$ and $d_A(z)$ ellipses, as is illustrated in detail in Appendix A. Finally we extend the $H - d_A$ landscape to include a loop over the redshift of a single growth measurement in the range $0.1 < z < 5$. In all cases we normalise the growth today; $G(z=0) = 1$. The full four dimensional surface cannot be plotted in general, we show slices through the hypersurface in Figure 6, illustrating how a measurement of the growth at high-redshift measurement leads to much higher values of the FoM overall, and opens up interesting new “hotspots”. These are only a few of the potential visualisation applications for Fisher4Cast.

**B. Dark Energy Constraints from Growth of Structure**

The growth function $G(z)$, defined as the solution to Eq. (19) is sensitive to dark energy; the Hubble parameter acts as a friction term in the differential equation, increasing or suppressing the growth of structure. While in general there is no analytical solution to Eq. (19), under the assumption of a flat universe and a
cosmological constant (or pure curvature) the growing mode satisfies the following integral form 36, 37:

\[ G(z) = \frac{5\Omega_m E(z)}{2} \int_z^\infty \frac{(1 + z') dz'}{E(z')^3}, \]

where the 5/2 coefficient is chosen to ensure that \( G(z) \to 1/(1 + z) \) during matter domination. This expression should not be used however, to compute the Fisher derivatives \( \partial G/\partial \Omega_k \), \( \partial G/\partial w_0 \) or \( \partial G/\partial w_a \) since all of these derivatives violate the validity of the equation. Instead, the growth derivatives should be computed numerically from the solution of the full differential equation for \( \delta(x) \). Rewriting the Raychaudhuri equation in terms of the Friedmann equation and the curvature density allows one to find an equation

FIG. 7: Comparing analytical derivatives to the full solution of the ordinary differential equation. The left-hand panel in this plot shows the derivatives of the growth function \( G(z) \), where analytical derivatives are taken of Eq. (20) dashed red lines, or where derivatives have been taken numerically of the solution to Eq. (21) solid brown lines. The parameters considered are \( w_0 \) (light red for the analytical derivatives and dark brown for the numerical derivatives) and \( w_a \) (dark red and light brown for analytical and numerical derivatives respectively). The right-hand panel shows the ellipses that result from a survey of the growth measured in 20 redshift bins from 0.1 to 2, with 10% error on the growth function (normalised to unity at \( z = 0 \)), and priors of \( 10^4 \) on the Hubble parameter, matter and curvature densities respectively. The ellipse that results from the analytical derivatives red dashed lines suggests (incorrectly) much tighter constraints on the dark energy parameters, whereas the ellipse from the numerical solution (solid brown line) is much more degenerate in the dark energy parameters.

| Parameter                  | Value  |
|----------------------------|--------|
| Redshifts of measurement   | \( H : z = [0.3, 0.6, 0.8, 1.0, 1.2, 3] \) |
| Percentage error \( H(z) \) | \( \sigma_H/H = [5.80, 5.19, 3.59, 2.84, 2.53, 1.48] \) |
| Percentage error \( d_A(z) \) | \( \sigma_{d_A}/d_A = [5.19, 4.30, 3.22, 2.3, 2.03, 1.19, 0.22] \) |
| Cosmological model \( (H_0, \Omega_m, \Omega_k, w_0, w_a) \) | \((70 \text{ km s}^{-1} \text{ Mpc}^{-1}, 0.3, 0, -1, 0)\) |
| Priors on model            | \((10^4, 10^4, 10^4, 0, 0)\) |

TABLE I: Survey data from the Seo & Eisenstein survey configuration 38 – used in Figures 8, 10, 2 and 4. In some cases measurements of the growth function were added, taken at the same redshifts as the Hubble parameter; in others the prior information on various parameters was changed. See the captions of the relevant figures for the specific details.
Explicitly showing the curvature and dynamical dark energy contributions to the friction term:

\[ G'' + \frac{3}{2} \left( 1 + \frac{\Omega_k(x)}{3} - w(x)\Omega_{DE}(x) \right) \frac{G'}{x} - \frac{3}{2} \Omega_m(x) \frac{G}{x^2} = 0, \tag{21} \]

where the new independent variable is \( x \equiv a/a_0 = 1/(1+z) \), \( a_0 \) is the radius of curvature and \( \Omega_k(x) = -k/(a_0^2z^2H(x)^2) \); \( \Omega_{DE}(x) = \rho_{DE}(x)/\rho_{crit}(x) \) are the fractions of the critical density in curvature and dark energy respectively. Alternatively, this can be written as a differential equation in terms of \( \ln(x) \):

\[ \frac{d^2G}{d\ln^2 x} + \frac{3}{2} \left( \frac{1}{3} + \frac{\Omega_k(x)}{2} - w(x)\Omega_{DE}(x) \right) \frac{dG}{d\ln x} - \frac{3}{2} \Omega_m(x)G = 0, \tag{22} \]

which is the equation actually solved in Fisher4Cast since it is typically more stable numerically. Appropriate initial conditions for this differential equation are set deep in the matter dominated era:

\[ G(z_i) = 1, \frac{dG}{d\ln x(z_i)} = G(z_i) \text{ for } z_i \geq 100. \]

Note that as a result, the growth solutions will be unreliable if \( w(z \rightarrow \infty) = w_0 + w_a \geq 0 \) (or even if it is close to zero from below) since then there will be significant or even dominant early dark energy. Fisher4Cast allows the user to choose the redshift where the growth is normalised to unity. The Fisher derivatives all satisfy \( \partial G/\partial \theta_i = 0 \) at the normalisation redshift.

While the growth functions Eq. (20) and Eq. (22) agree for \( \Lambda \)CDM, analytical Fisher derivatives taken of Eq. (20) will not agree with the numerical derivatives of Eq. (22), and will thus produce very different error ellipses when used incorrectly. The left-hand panel of Figure (7) shows the \( w_0, w_a \) Fisher derivatives of both solutions for \( \Lambda \)CDM. The derivatives are overestimated for both parameters, leading to (spurious) tight constraints on the parameters in the Fisher ellipse, as is illustrated in the right-hand panel of Figure (7), for a survey characterised by 20 measurements (at 10% accuracy) of the growth function between \( z = 0.1 \) and \( z = 2 \). This illustrates the danger of using derivatives based on the analytical form of \( G \), even when evaluated at the model for which the analytical form is itself valid.

As the precision of growth measurements increases, it is natural to ask how this impacts constraints on dark energy. This is easily investigated in Fisher4Cast. Consider Figure (8), which illustrates constraints on the CPL parameters around \( \Lambda \)CDM for a survey characterised by Table I with the addition of growth

\[ \sigma_{G}/G \rightarrow 0.1 - \sigma_{H}/H, \]

the FOM increases as expected. For large values of \( \alpha \), however, the FOM flattens out as there is essentially no pertinent information from the growth function.
measurements at the same redshifts as the Hubble parameter. As the error on growth relative to the error on \( H \) decreases, the Figure of Merit of the survey increases dramatically, highlighting the merit in making high-precision measurements of the growth function, as is the focus of recent interest \[21, 24, 25, 26, 27, 28, 29, 39, 40, 41, 42\].

![Graph](image)

**FIG. 9:** **Normalisation of the Growth function changes constraints** – the \( w_0 - w_a \) error ellipse for measurements of the growth combined with one measurement of \( H \) at low redshift. The normalisation redshift of the growth function is varied from \( z = 0 \) (dot-dashed line) to \( z = 4 \) (solid central black line) to illustrate the change in the degeneracy direction of the dark energy parameters with growth normalisation. Note that the constraints do not scale monotonically with redshift: the ellipse is most degenerate for normalisation of the growth at a redshift of \( z = 1 \).

As a final remark, we show how the redshift at which one normalises the growth function (normalisation redshifts \( z = 0, 1, 2, 4 \) are shown) influences dark energy constraints in Figure (9). The constraints on the dark energy parameters are plotted for 5 measurements of the growth function at \( z = (0.3, 0.6, 0.8, 1.2, 3) \) each with a 10% accuracy, and one measurement of the Hubble parameter at \( z = 0.3 \) with \( \sigma_H / H = 5.19\% \). An interesting point is that the resulting dark energy constraints do not scale monotonically with the normalisation redshift.

### C. The Effect of Cosmic Curvature On Dark Energy Constraints

The degeneracy between curvature and dark energy has been well-studied even for perfect measurements of any single observable such as \( H(z) \) \[12, 14, 15, 16, 17\], implying that marginalising over the curvature is important when performing parameter estimation and forecasting of constraints on dark energy. The degree to which curvature affects dark energy constraints is shown here as a simple example of FisherCast. Figure (10) shows Fisher error ellipses for the dark energy parameters \( w_0, w_a \), after marginalising over curvature, as the prior information on curvature is changed from Prior(\( \Omega_k \)) = 10 (weak) to Prior(\( \Omega_k \)) = 10\(^6\) (almost perfect) for the observables \( H, d_A \) and \( G \) considered separately and in combination. While uncertainty in the curvature of the universe (represented by a small prior value on \( \Omega_k \)) degrades all ellipses, this is much less pronounced when the observables are considered in combination, showing the importance of combining multiple probes of dark energy. For each of the parameters, Figure (11) shows that the constraints are eroded and the ellipse increases in size with the decrease in the prior - which expresses our confidence in the flatness of the universe. While the ellipses from the single observables such as \( H, d_A \) show a much greater increase in size with decreasing curvature prior, using a combination of parameters is more robust - since combining multiple probes helps break the curvature-dark energy degeneracy. The change in the size of the ellipse directly relates to a change in the Figure of Merit (FoM), or power, of the particular combination of observables. Various FoMs are described in Section II. In the case of the FoM of the Dark Energy Task Force (DETF) \[10\], the area of the ellipse and the DETF FoM are inversely proportional. The right-hand panel of Figure (11) shows this change in the FoM with changing curvature prior. Interestingly, the DETF FoM flattens out for both very large and very small values of the prior. This can be explained
by the fact that for very large prior values, marginalising over the curvature has no effect, since our error on the curvature is minute, whereas for very small values of Prior(Ω_k) ~ 0 the ellipse is completely degenerate with the curvature and hence reaches a maximum size.

D. New Features in Fisher4Cast

Fisher4Cast made its public debut (Version 1.1) in May 2008, with significant updates and revisions every six months since then. This work coincides with the release of Version 2.0 of the code. The main new features of versions 1.2 and 2.0 are listed below; a more comprehensive description of the changes in the new code is contained in the Readme.txt file, while the new features are described in the Fisher4Cast Users’ Manual, provided with the distribution of the code suite.

- **Report Generating Features**
  Code is included in the current version for automated generation of both \LaTeX and text reports containing information such as the input survey, fiducial cosmology, output Fisher matrices and Fisher ellipse figure.

- **Fitting formulae for Baryon Acoustic Oscillation (BAO) Surveys**
  Two extensions are provided to calculate the errors on the BAO oscillation scale according to the prescriptions of Blake et al. [48] and Seo and Eisenstein [49] (see the files in the directories called EXT_FF_Blake_etal2005 and EXT_FF_SeoEisenstein2007). These are not available via the GUI currently, and must be run directly from the command line. However, these modules combined with the rest of Fisher4Cast provide the capability of going directly from BAO survey specifications (volume, area, number density etc...) to dark energy constraints.

- **“Point-and-click” ellipse plotting**
  A new feature allows the user to click in the figure and have the ellipse automatically generated at that
FIG. 11: Curvature marginalisation degrades dark energy constraints - using the same input survey data as Figure (10), the error ellipses in the dark energy parameters for prior values on $\Omega_k$ from $10^6$ (inner curves) to 10 (outer curves) are shown. Ellipses are produced for measurements of (clockwise from top left) the Hubble parameter (dark brown curves), the angular diameter distance (orange), the growth function (light brown), a combination the Hubble parameter and the angular diameter distance (green) and a combination of all three observables (blue). The bottom right panel shows the DETF Figure of Merit (the inverse of the area of the ellipse in the $w_0 - w_a$ plane) for these ellipses as a function of the prior on $\Omega_k$. The colours correspond to the ellipses: green (dashed) is $H$ and $d_A$ combined while orange (dot-dashed) corresponds to $d_A$ alone. In all cases the shape of the curves is roughly the same, with no change in the FOM beyond a certain value of the prior, both in the case of very large and very small priors. Interestingly the curves for the Hubble parameter (brown) and angular diameter distance (orange) cross - a survey consisting only of measurements of $H$ may seem to yield tighter (or weaker) constraints on dark energy than a survey only of distance measurements depending on the curvature prior assumed.

E. User Extensions

The general philosophy of Fisher4Cast was to make it as easy as possible to mould and extend to the needs of a general user. The power of Fisher4Cast does come at a price however and to modify the code requires the user to become familiar with the underlying structures used in the design and building of Fisher4Cast. The good news is that Fisher4Cast was designed to be as logical, elegant and general as possible and quick but limiting fixes were avoided where possible in favour of flexibility. This means that once the user is...
familiar with the underlying code structure, Fisher4Cast should be easy to extend in ways that were not even conceived of when it was written.

All Fisher4Cast functions begin with the prefix FM while extensions have the EXT prefix (in addition the figure plotting code is prefixed by FIG). To use Fisher4Cast for a new application, code up your function (the X in the notation of Section [1]), as a Matlab function (or functions if you have multiple observables) and name them appropriately: FM_my_function.m. Then code the derivatives (i.e. \( \partial X / \partial \theta_A \), if you have them analytically, otherwise numerical derivatives will be used) and name appropriately, e.g. FM_my_derivs.m. Make sure all functions take in a data vector and base parameter vector (the fiducial model where the Fisher Matrix will be evaluated) as arguments and that they return a vector, as in the header documentation of FM_function_1.m and FM_analytic_deriv_1.m. Then go to the input structure and replace the entry ‘FM_analytic_deriv_1’ in the field input.function_names with ‘FM_my_function’. Then replace the field numderiv.f{1} = sprintf(‘FM_function_1’); with numderiv.f{1} = sprintf(‘FM_my_function’).

As an example consider alternative parameterisations for dark energy. Currently, the Fisher4Cast GUI is hard-coded for three cosmological observables (\( H, d_A \), and \( G \)), assuming the Chevallier-Polarski-Linder (CPL) parameterisation of dark energy with parameters (\( w_0, w_a \)), see Eq. (11). This is true of both the functions themselves, and the analytical derivatives included in the Fisher4Cast suite. The general framework of Fisher4Cast, however, means that one is not restricted to this parameterisation. As can be seen from Eq. (12), (15) and (22), the dark energy equation of state enters the cosmological observables through the evolution of the dark energy, via \( f(z) \), defined in Eq. (13). Hence for any given \( w(z) \) one only needs to specify the names of the functions (in the input structure) that will replace the current versions of FM_function_1.m (\( H(z) \)), FM_function_2.m (\( d_A(z) \)) and FM_function_3.m (\( G(z) \)). The same is true for the derivatives - either they can be coded analytically for the particular parameterisation of dark energy, or the derivatives will be evaluated numerically from the functions specified in the input structure.

As a caveat, the GUI can only be used if the new parameterisation of dark energy still contains only two coefficients. If this is not the case, Fisher4Cast must be run from the command line.

V. CONCLUSIONS

The Fisher Matrix formalism is the standard forecasting method in cosmology and assuming one is familiar with it, allows rapid and widely understood results. In principle it is easy to learn and code for oneself. In practise there is a relevance threshold below which it simply is not worthwhile.

Fisher4Cast facilitates both the acquisition of knowledge and the generation of production-quality Fisher Matrix forecasts. Written in Matlab, the Fisher4Cast Graphical User Interface (GUI) allows easy exploration of cosmological constraints and has both an interactive ‘point-and-click’ facility for automatic ellipse generation (when activated from the “F4C Extensions” drop-down menu) and an automatic \LaTeX\ summary and results file generator, making direct inclusion of any output into research documents straightforward. It doesn’t yet write the paper for you, but we are working on it.

In this paper we have focused on illustrating novel uses of the Fisher4Cast suite by exploring the landscape of Fisher Matrix cosmology, as illustrated by Figures [2-6], as well as highlighting the effects of growth and curvature on Fisher Matrix forecasts of future cosmological surveys. These illustrate a limited set of applications of Fisher4Cast which we hope stimulates members of the community to use and extend the code.

Research that has thus far used Fisher4Cast, either for forecasts or for producing plots, includes [51, 15] and [22], as well as unpublished work for the ACT survey by RH.

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FIG. 12: Varying redshift of \( H \) — examples of ellipses corresponding to particular values in the FoM landscape of Figure [5], in the particular case where the redshift bins for \( H \) are varied while keeping the bin for \( d_A \) fixed at \( z = 0.6 \). The degeneracy direction for \( d_A \) (black dashed line) and \( H \) (red solid line) are included to help see how their relative orientations contribute to the orientation and size of the combined ellipse. The degeneracy direction of \( H \) rotates anti-clockwise with the most rapid rotation experienced initially for the low redshift range of \( H \). After \( z = 1 \) the rotation of \( H \) slows down and no longer is significant. As the \( H \) and \( d_A \) degeneracy directions become more orthogonal so the resulting constraints improve, yielding a higher FoM. The ellipses are coloured corresponding to their FoM where red indicates the largest (around 0.08), and blue the smallest, FoM.

The FoM landscape in Figure [5], and in particular the valleys separating high points in FoM space warrant further investigation. We argued previously that the variation in the FoM was an interplay between the widths of the ellipses (the ellipses for a single redshift measurement are infinitely degenerate along the semi-major axis) and the orientation of the \( H \) and \( d_A \) ellipses. This is illustrated in Figure [12] where we show the two degenerate ellipses (red solid lines for \( H \) and black dashed lines for \( d_A \)) and the combined ellipse for a series where we keep the \( d_A \) bin fixed at \( z = 0.6 \) while varying \( H \) in the redshift range \( 0.1 < z < 5 \). This moves us along a line in the landscape that intersects the peak at an \( H \) bin of \( z = 1.5 \). The combined ellipses have colours that correspond to the FoM in Figure [5], the highest value of the DETF FoM (∼ 0.08).
coloured in red, and the lowest (∼0) coloured in dark blue. One can see a distinct rotation of the degeneracy direction for $H$ as the redshift changes. This anti-clockwise direction rotation is most pronounced at low redshift from $z = 0.1$ to $z = 1.0$ after which it becomes more subtle and stops rotating completely at higher redshifts.

APPENDIX B: FISHER DERIVATIVES FOR $H(z), d_A(z), G(z)$

We present the analytical Fisher derivatives of the Hubble parameter, $H(z)$, given as Eq. (12) and the angular diameter distance, $d_A(z)$, given as Eq (15) with respect to the cosmological parameters $(H_0, \Omega_m, \Omega_k, w_0, w_a)$, where $w_0, w_a$ are the CPL dark energy parameters, and assume the forms $f(z)$ and $E(z)$ as given in Eqs. (12) and (14) respectively. In all cases the derivatives are taken in a general Friedmann-Lemaître-Robertson-Walker background without assuming flatness.

1. The Hubble parameter

As the Hubble constant, $H_0$ only appears as a multiplicative term in $H(z)$, the Fisher derivative of the Hubble parameter with respect to the Hubble constant $H_0$ is simply

$$\frac{\partial H}{\partial H_0} = E(z) \quad (B1)$$

Derivatives of the function $\mathcal{E}(z) \equiv H^2(z)/H_0^2 = E^2(z)$ are found in all derivatives of both $H$ and $d_A$ and are worth defining separately:

$$\frac{\partial \mathcal{E}(z)}{\partial \Omega_m} = (1 + z)^3 - f(z)$$

$$\frac{\partial \mathcal{E}(z)}{\partial \Omega_k} = (1 + z)^2 - f(z)$$

$$\frac{\partial \mathcal{E}(z)}{\partial w_0} = (1 - \Omega_m - \Omega_k) \frac{\partial f(z)}{\partial w_0} = 3(1 - \Omega_m - \Omega_k) f(z) \ln(1 + z)$$

$$\frac{\partial \mathcal{E}(z)}{\partial w_a} = (1 - \Omega_m - \Omega_k) \frac{\partial f(z)}{\partial w_a} = 3(1 - \Omega_m - \Omega_k) f(z) \left( \ln(1 + z) - \frac{z}{1 + z} \right). \quad (B2)$$

For all the cosmological parameters we consider other than the Hubble parameter $H_0$, the derivatives with respect to the Hubble parameter can then be expressed as

$$\frac{\partial H(z)}{\partial \theta_i} = \frac{H_0 \partial \mathcal{E}(z)}{2E \partial \theta_i}, \quad \theta_i \in (\Omega_m, \Omega_k, w_0, w_a) \quad (B4)$$

2. Angular Diameter Distance

In a FLRW background, $d_A(z)$ is given by Eq (15). The function $\chi(z)$ is defined by Eq. (16). It is again useful to define first order derivatives of $\chi(z)$ with respect to the parameters, as these derivatives will occur many times in the Fisher derivatives of $d_A(z)$.

$$\frac{\partial \chi(z)}{\partial \theta_i} = - \int_0^z \frac{1}{2E^3(z')} \frac{\partial \mathcal{E}(z')}{\partial \theta_i} dz', \quad \theta_i \in (\Omega_m, \Omega_k, w_0, w_a), \quad (B5)$$
again where \( \mathcal{E}(z) \equiv \frac{H^2(z)}{H_0^2} \). The Hubble parameter appears only in the pre-factor of the angular diameter distance, and hence

\[
\frac{\partial d_A(z)}{\partial H_0} = -\frac{1}{1+z} \frac{c}{H_0^2} \frac{1}{\sqrt{\Omega_k}} \sinh \left( \sqrt{\Omega_k} \chi(z) \right) = \left( -\frac{1}{H_0} \right) d_A(z) \quad (B6)
\]

The matter density \( \Omega_m \) contributes solely to the \( \chi(z) \) term, and hence can be expressed using Eqs. \( B5 \) and \( B3 \) as

\[
\frac{\partial d_A(z)}{\partial \Omega_m} = \frac{1}{1+z} \frac{c}{H_0} \cosh \left( \sqrt{\Omega_k} \chi(z) \right) \frac{\partial \chi(z)}{\partial \Omega_m}, \quad (B7)
\]

with the derivatives of the co-moving distance given by

\[
\frac{\partial \chi(z)}{\partial \theta_i} = -\frac{1}{2} \int_0^z \frac{1}{E^3(z')} \frac{\partial \mathcal{E}(z')}{\partial \theta_i} dz' \quad \text{for } \theta_i \in (\Omega_m, \Omega_k, w_0, w_a) \quad (B8)
\]

The curvature parameter is found both in the pre-factor and the sinh term of the angular diameter distance, hence

\[
\frac{\partial d_A(z)}{\partial \Omega_k} = -\frac{1}{1+z} \frac{c}{H_0^2} \frac{1}{2\Omega_k^{3/2}} \sinh \left( \sqrt{\Omega_k} \chi(z) \right) + \frac{1}{1+z} \frac{c}{H_0} \frac{1}{\sqrt{\Omega_k}} \cosh \left( \sqrt{\Omega_k} \chi(z) \right) \left[ \frac{\chi(z)}{2\sqrt{\Omega_k}} + \sqrt{\Omega_k} \frac{\partial \chi(z)}{\partial \Omega_k} \right] = \frac{1}{2\Omega_k} d_A(z) + \frac{1}{1+z} \frac{c}{H_0} \cosh \left( \sqrt{\Omega_k} \chi(z) \right) \left[ \frac{\chi(z)}{2\Omega_k} + \frac{\partial \chi(z)}{\partial \Omega_k} \right]. \quad (B9)
\]

The Taylor series expansion of Eq \( B9 \) is used \( \Omega_k \rightarrow 0 \), namely:

\[
\frac{\partial d_A(z)}{\partial \Omega_k} \bigg|_{\Omega_k \rightarrow 0} = \frac{c}{H_0} \frac{1}{1+z} \left\{ \frac{1}{6} \chi^3(z, 0) + \frac{\partial \chi(z, 0)}{\partial \Omega_k} \right\} \quad (B10)
\]

where

\[
X(z, 0) \equiv X(z)|_{\Omega_k \rightarrow 0} \quad (B11)
\]

are the functions (for example \( E(z) \), \( \chi(z) \)) assuming flatness. Using the definitions Eqs. \( B5 \) and \( B3 \), the derivatives of the angular diameter distance with are expressed similarly for \( \theta_i \in (w_0, w_a) \) as:

\[
\frac{\partial d_A(z)}{\partial \theta_i} = \frac{1}{1+z} \frac{c}{H_0} \cosh \left( \sqrt{\Omega_k} \chi(z) \right) \frac{\partial \chi(z)}{\partial \theta_i}. \quad (B12)
\]

3. **Growth**

All derivatives related to the growth are computed numerically (in the routine FM\textunderscore num\textunderscore deriv.m) by solving Eq. \( 22 \) and using either the complex step or central finite difference algorithm, depending on the choice of the user.
APPENDIX C: FISHER4CAST FIGURE CODE

1. Code to produce Figure (3): plot of function values and derivatives

```matlab
function FIG_function_derivative_plot(deriv_flag, function_flag)
% This function generates a plot of the derivatives of the specific
% observables included in Fisher4Cast, H(data), d_A(data) and G(data).
% It must be included in the directory in which Fisher4Cast is contained,
% or that directory must be added to the Matlab path.

% The flags deriv_flag and function_flag are set to 1 (0) if you do (don't)
% want to plot the function or derivatives. If no input is given these are
% both set to 1 and you get plots of all functions and derivatives.

% As a default example it calls the Seo_Eisenstein_2003 input structure,
% but then generates a redshift vector from 0.1:10;
% The colours for line plots must be specified as 1x3 RGB vectors,
% normalised to 1 (i.e. so each entry divided by 255). See the default
% colours as an example.

close all

% Flags to control what you want to plot, either derivatives only, of
% function only, or both
if nargin == 0
    deriv_flag = 1;
    function_flag = 1;
elseif nargin == 1
    function_flag = 0;
end

% Specify the colours of the derivatives
hcolour = [147 50 0] / 255;
gcolour = [240 201 81] / 255;
dacolour = [231 109 29] / 255;
colourmat = [hcolour
gcolour
dacolour];
styless = {'-', '-.', '--', ':', '-.'};

% Generate the Input data for the derivative plot
input = Seo_Eisenstein_2003; % initialise the input structure
data = 0.1:0.1:10; % the redshift range we want to consider
data = data(:);
input.growth_zn = 0;
input.growth_zn_flag = 1;
% make sure the Growth is normalised at data = 0;
input.observable_index = [1 2 3]; % Use all three observables
input.num_observables = length(input.observable_index);
% Re-assign the redshift vectors in the input structure
input.data{1} = data;
input.data{2} = data;
```
input.data{3} = data;

% Re-assign the errors vectors in the input structure
input.error{1} = 0.1.*ones(1,length(input.data{1}));
input.error{2} = 0.1.*ones(1,length(input.data{2}));
input.error{3} = 0.1.*ones(1,length(input.data{3}));

% Use the analytical formula for H, d_A and numerical derivatives for G
input.numderiv.flag{1} = 0;
input.numderiv.flag{2} = 0;
input.numderiv.flag{3} = 1;
%--------------------------------------------------------------------------
% Run Fisher to get the parameter values and derivatives
output = FM_run(input);
close(1); % Close the figure of the Fisher Ellipse
%--------------------------------------------------------------------------
% PLOT THE DERIVATIVES
% We will plot dlnX/dtheta_i = dX/Xdtheta_i
%--------------------------------------------------------------------------
if deriv_flag == 1
    for i = 1:input.num_observables % Loop over the observable functions
        x = input.observable_index(i);
        % Set the figure properties
        figure(x*100)
        axes('FontName', 'Times', 'FontAngle', 'italic',
             'FontSize', 14 , 'XScale', 'log', 'XTickLabel', {'0.1';'1';'10'} )
        hold on
        box on
        xlabel('Redshift', 'FontName', 'Times',
             'FontAngle', 'italic', 'FontSize', 16 )
        ylabel(['Fisher Derivatives for ', input.observable_names{x},
             '(z)'], 'FontName', 'Times', 'FontAngle', 'italic', 'FontSize', 16)
        for j = 1:5
            if j==2
                % The Omega_m derivative, this is actually dlnH/dlnOm
                semilogx(data, input.base_parameters(j).*output.function_derivative{x}(:,j)/output.function_value{x},
                    'LineStyle', styles{j},'LineWidth', 2,
                    'Color', colourmat(x,:))
            else
                plot(data, (output.function_derivative{x}(:,j)/output.function_value{x}),
                    'LineStyle', styles{j},'LineWidth', 2, 'Color', colourmat(x,:))
            end
        end
        legend(legendmat{x}, 'Location','NorthWest')
    end % end the loop over the observable functions
    end % end the if loop for plotting of derivs
%--------------------------------------------------------------------------
% PLOT THE FUNCTIONS
%--------------------------------------------------------------------------

if function_flag == 1

for i = 1:input.num_observables
    % Set the figure properties
    figure(100*x + 1)
    axes( 'FontName', 'Times', 'FontAngle', 'italic', ... 
    'FontSize', 14, 'XScale', 'log', 'XTickLabel', {'0.1';'1';'10'} )
    hold on
    box on
    xlabel('Redshift', 'FontName', 'Times', 'FontAngle', ... 
    'italic', 'FontSize', 16)
    ylabel([input.observable_names{x}, '(z)'], 'FontName', ... 
    'Times', 'FontAngle', 'italic', 'FontSize', 16)

    % Plot the data
    semilogx(data, output.function_value{x}, 'LineWidth', ... 
    2, 'Color', colourmat(x,:));
end % end the loop over observables
end % end the if loop for function plotting

2. Code to produce Figure (6): volume slice plots

% --------------------------------------------------------------------------
%This function takes a three dimensional matrix, fom_vol_out, and plots a 
%slice plot for this data.
%
%fom_vol_out can be generated by calling
%>>fom_vol_out = FIG_generate_fom_volume_data(vol_res)
%If no vol_res is passed a default value of 30 is assumed.
%
%Example of using this function:
%>>FIG_plot_slice_fom_volume(fom_vol_out)
%
%if no fom_vol_out is given then the code checks to see if there is a default 
%.mat file, default_fom_vol_out.mat, to load the data from, else the 
%function FIG_generate_fom_volume_data is called with a set of default values.
%--------------------------------------------------------------------------
function FIG_plot_slice_fom_volume(fom_vol_out)

%check if a matrix of the volume space, fom_vol_out, is passed to the function
if nargin<1
    %see if the default .mat file exists and load the data
    if exist('default_fom_vol_out.mat')
        load default_fom_vol_out;
        fom_vol_out = default_fom_vol_out;
    else
        %if not then generate the fom_vol data
        fom_vol_out = FIG_generate_fom_volume_data(30);
    end
end

% This function takes a three dimensional matrix, fom_vol_out, and plots a 
% slice plot for this data.
% fom_vol_out can be generated by calling 
% >> fom_vol_out = FIG_generate_fom_volume_data(vol_res) 
% If no vol_res is passed a default value of 30 is assumed.
% Example of using this function:
% >> FIG_plot_slice_fom_volume(fom_vol_out) 
% if no fom_vol_out is given then the code checks to see if there is a default 
% .mat file, default_fom_vol_out.mat, to load the data from, else the 
% function FIG_generate_fom_volume_data is called with a set of default values.
%--------------------------------------------------------------------------
%set the colormap
colormap(jet);

%select the planes to intersect for the slice plot
%---------------------------------------------------------------------------
%this is an additional subsection of code to make the selection of redshift
%planes to plot more generic and easy to manage for a range of
%fom_vol_out's produced. Please note you must still manually specify the
%redshift range
[x_col y_col z_col] = size(fom_vol_out);
x_redshift_range = [0,5];
y_redshift_range = [0,5];
z_redshift_range = [0,5];
%calculate a relationship from column to redshift
column_to_redshift_ratio_x = x_col/x_redshift_range(end);
column_to_redshift_ratio_y = y_col/y_redshift_range(end);
column_to_redshift_ratio_z = z_col/z_redshift_range(end);
%specify the slices redshift to intersect the fom volume space
x_redshift_slice = [0.6667];
y_redshift_slice = [0.4167, 1.6667];
z_redshift_slice = [2.5];
%calculate the slices in column numbers
xslice = column_to_redshift_ratio_x.*x_redshift_slice;
yslice = column_to_redshift_ratio_y.*y_redshift_slice;
zslice = column_to_redshift_ratio_z.*z_redshift_slice;
%---------------------------------------------------------------------------
%plot using slice
s = slice(fom_vol_out,xslice,yslice,zslice);

%set the labels so they match the range of
% redshift as opposed to the column numbers
set(gca,'xtick', [0:column_to_redshift_ratio_x:x_col],...
'xticklabel', [0:x_redshift_range(end)],'ytick',...
[0:column_to_redshift_ratio_y:y_col],'zticklabel',...
[0:y_redshift_range(end)],'ztick', [0:column_to_redshift_ratio_z:z_col],...
'zticklabel', [0:z_redshift_range(end)]);

%set the x y and z labels
ylabel('H Redshift');
xlabel('d_A Redshift');
zlabel('G Redshift');

3. Code to produce Figure (11): Fisher ellipses as a function of changing curvature prior

 function FIG_vary_fom_curvature_prior
 global input plot_spec axis_spec
 % This function generates a plot of the Fisher ellipse as one changes the
 % prior value on curvature, and a corresponding plot of the Dark Energy
 % Task Force Figure of Merit (FoM) as a function of the prior.
 % See the User's Manual for definitions of the FoM.
% This code must be included in the directory in which
% Fisher4Cast is contained, or that directory must be
% added to the Matlab path.

% As a default example it calls the Seo_Eisenstein_2003 input structure,
% The Matlab 'colormap' command is used to generate the line colours,
% specific to each observable and the number of iterations is given by N.

% NOTE That this code uses getfigdata.m by M.A. Hopcroft,
% which is code from the Matlab File
% Exchange \protect\vrule width0pt\protect\href{http://www.mathworks.co.uk/matlabcentral/fileexchange/}
% It is included in this package.

close all
%--------------------------------------------------------------------------
% Choose your colour schemes for the various combinations
linecolor{1} = sort(colormap(copper), 'descend');
linecolor{2} = colormap(autumn);
linecolor{3} = colormap(copper);
linecolor{4} = colormap(winter);
linecolor{5} = colormap(summer);
input.fill_flag = 1;
valinit = 35;  % the starting colour for the plots
num_obs = [1 2];  % The vector of combinations you want:
  % 1 = Hubble
  % 2 = Angular Diameter distance
  % 3 = Growth Function
  % 4 = Hubble parameter + Angular Diameter distance + Growth
  % 5 = Hubble parameter + Angular Diameter distance
line_width = 2;
%--------------------------------------------------------------------------
% Set the Input structure for the survey you will use
input = Seo_Eisenstein_2003;
input.data{3} = input.data{1};
input.error{3} = 0.1.*ones(1,length(input.error{1}));
input.observable_index = [1 2 3];  % We will use all observables
input.fill_flag = 0;
input.numderiv.flag{3} = 1;
%--------------------------------------------------------------------------
% Set up the range you wish to consider
start_prior= 1e6;
range = 1e8;  % No of orders of magnitude in the prior
N = 20;  % Number of points
amp = (range)^(1/N);

% Initialise the priors
prior_orig = input.prior_matrix;  % this will be the default value
input.prior_matrix(3,3) = start_prior;
input.prior_matrix(2,2) = 0;
% Initialise the prior on the matter density to zero
%--------------------------------------------------------------------------
% Initialise the global FoM plot
figure(3000)
axes( 'FontName', 'Times', 'FontAngle', 'italic', 'FontSize', 14, 'XScale', 'log', 'YScale', 'log' );
hold on
box on
for ni = num_obs(1):num_obs(end)
    input.prior_matrix(3,3) = start_prior;
    if ni == 4
        % Compute the Fisher Ellipse for combination of Hubble, d_A and G
        input.observable_index = [1 2 3];
    elseif ni == 5
        % Compute the Fisher Ellipse for combination of Hubble and d_A only
        input.observable_index = [1 2 ];
    else
        input.observable_index = ni; % use index value as specified
    end

    for i=1:N
        figure(1)
        hold on
        input.prior_matrix(3,3) = input.prior_matrix(3,3)./amp;
        % modify the Prior
        output = FM_run(input); % Call Fisher4Cast
        val(i) = input.prior_matrix(3,3);
        % save the value of the prior for plotting
        outv(i,:) = output.fom;
        % Save the full FoM vector
        out(i) = outv(i,1); % Save the DETF FoM
        h = getfigdata(1);
        % call getfigdata.m to rip off the ellipse
        x{i} = h{1}.x;
        y{i} = h{1}.y;
        close(1) % close the figure
    end

    figure(1000+ni)
    axes( 'FontName', 'Times', 'FontAngle', 'italic', 'FontSize', 14 )
    hold on
    box on
    xlabel('w_0', 'FontName', 'Times', 'FontAngle', 'italic', 'FontSize', 16 )
    ylabel('w_a', 'FontName', 'Times', 'FontAngle', 'italic', 'FontSize', 16 )
    axis([-3 1 -10 10 ])
    count = 0;

    for i = 1:N
        figure(1000+ni)
        hold on
        % Use increasing or decreasing colour to get a gradient
        if i < ceil(N/2)
            count = count +1;
        else
            count = count -1;
        end

        % Plot the resulting ellipses
        ...
APPENDIX D: QUICK START GUIDE TO FISHER4CAST

1. Hardware and software requirements

This software is written to be run in Matlab (Linux, Windows and under Mac OSX, although this has not been extensively tested). The user needs Matlab installed (Tested on Version 7) to be able to run this code. Free disk space of approximately 2MB and the minimum recommended processor and memory specifications required by the Matlab version you are using is suggested.

2. Downloading Fisher4Cast

Currently the code is available for download at one of the following websites [52, 53]. Save this .zip file into the directory you want to run the Fisher4Cast suite from.

3. Getting started

The code can be run from the command line or the Graphical User Interface (GUI). We describe the command line below, and mention how to get the GUI started. More information on the GUI can be found in the Users’ Manual [33].

4. The Graphical User Interface

- Running the GUI
  The GUI can be started from the Matlab editor. The file FM_GUI.m must be opened from the directory, and once the file is opened (click on the file icon from within the Command-line interface to open it with an editor) press F5 to run the code. This will open up the GUI screen.

  »> FM_GUI

  This then functions in the same way as using FM_run in the command line (as explained in the following section).

  For more information on the technicalities of the GUI, see the full manual.

- GUI Screenshots
  We include some screenshots of the Graphical User Interface.
FIG. 13: **Plotting multiple ellipses on one axis** - using the ‘Hold on’ multiple error ellipses can be overlaid on one axis. The ‘Area Fill’ command allows you to choose the colours for the error ellipses. Also shown is the ‘Running’ window which indicates the code is running to calculate the Fisher ellipses.

FIG. 14: **Different background images and colour schemes** - the background images and colour schemes (skins) allow for a fully customisable Graphical User Interface.

FIG. 15: **Various Figures of Merit can be plotted** - the drop-down list allows for a choice between various Figure of Merit options.

5. **The Command Line**

- Running the code
Open your version of Matlab and change the working directory to be the same as where you saved Fisher4Cast in. To run the code from the command line with one of the standard test input structures supplied, type:

```matlab
>> output = FM_run(Cooray_et_al_2004)
```

This will call the code using the pre-supplied test input data (Cooray_et_al_2004) and then generate an error ellipse plot for the parameters and observables supplied in the chosen input. All the relevant generated output is written to the output structure. You can see the range of outputs to access by typing:

```matlab
>> output
```

and then examine each output individually by specifying it exactly. For example:

```matlab
>> output.marginalised_matrix
```

will access the marginalised Fisher Matrix from the output structure.

You can use the supplied input files as a template for generating new input files with your own customised parameters and values. All fields shown in the example structures must be filled in any user-defined structure.

The code can also be run from the Matlab editor. Once the code is opened (open it from inside the Matlab window), you can press F5 to run the code. Note that if the code is run from the Editor it will call the default input structure, which is the `Cooray_et_al_2004.m` file. This is an example file containing input data from the paper by Cooray et al. [54]. This output can be directly compared to that of Figure 1 of that paper. If your output compares correctly, you have a working installation of the code. Another input available is `Seo_Eisenstein_2003.m` [38].