$t\bar{t}$ Production Rates at the Tevatron and the LHC in Topcolor-Assisted Multiscale Technicolor Models

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Abstract

We study the contributions of the neutral pseudo Goldstone bosons (technipions and top-pions) to the $t\bar{t}$ production cross sections at the Tevatron and the LHC in topcolor-assisted multiscale technicolor (TOPCMTC) models via the gluon-gluon fusion process from the loop-level couplings between the pseudo Goldstone bosons and the gluons. The MRS set $A'$ parton distributions are used in the calculation. It is shown that the new CDF datum on the $t\bar{t}$ production cross section gives constraints on the parameters in the TOPCMTC models. With reasonable values of the parameters in TOPCMTC models, the cross section at the Tevatron is larger than that predicted by the standard model, and is consistent with the new CDF data. The enhancement of the cross section and the resonance peaks at the LHC are more significant, so that it is testable in future experiments.

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1. Introduction

Among the yet discovered fermions, the top quark has the strongest coupling to the electroweak symmetry breaking (EWSB) sector. So that processes with top quarks are good places for probing the EWSB mechanism. Experimental measurements of the top quark mass \( m_t \) and the \( t \bar{t} \) production cross section \( \sigma_{t\bar{t}} \) at the Fermilab Tevatron have been improving. In the new 1996 CDF data \(^2\), \( m_t = 175.6 \pm 5.7 (\text{stat}) \pm 7.1 (\text{syst}) \) GeV and \( \sigma_{t\bar{t}} = 7.5^{+1.9}_{-1.6} \) pb, the error bars are well reduced relative to the 1995 data by the CDF and D0 Collaborations \(^1\). The above experimental value of \( \sigma_{t\bar{t}} \) is slightly larger than the standard model (SM) predicted value (taking into account of resummation of soft gluon contributions) which is around 5 pb \(^3\). Of course, one should wait for further improved data to see whether this really means something. But, as the study of the EWSB mechanism, it is interesting to study the \( t \bar{t} \) production cross section in EWSB mechanisms other than the SM Higgs sector, and see if the present experimental data can give constraints on the parameters in the EWSB models.

Technicolor (TC) \(^4\) is an interesting idea for naturally breaking the electroweak gauge symmetry to give rise to the weak-boson masses. It is one of the important candidates for the mechanism of electroweak symmetry breaking. Introducing extended technicolor (ETC) \(^5\) provides the possibility of generating the masses of ordinary quarks and leptons. The original ETC models suffer from the problem of predicting too large flavor changing neutral currents. It has been shown, however, that this problem can be solved in walking technicolor (WTC) theories \(^6\). The electroweak parameter S in WTC models is smaller than that in the simple QCD-like ETC models and its deviation from the experimental central value

\(^2\)The 1996 D0 data still contains rather large error bars.
may fall within current experimental bounds \[7\]. To explain the large hierarchy of the quark 
masses, multiscale WTC (MWTC) model was further proposed \[8\]. However, even in this 
model, it is difficult to generate such a large top-quark mass as what is measured at the 
Tevatron \[2\] without exceeding the experimental constraint on the electroweak parameter 
\( T \) \[9\] even with “strong” ETC \[10\]. In addition, this model generates too large corrections 
to the \( Z \rightarrow b\bar{b} \) branching ratio \( R_b \) compared with the LEP data \[11\] due to the smallness of 
the decay constant \( F_Q \), and a consistent value of \( R_b \) can be obtained \[12\] by combining this 
model with the topcolor interactions for the third generation quarks \[13\] at the energy scale of 
about 1 TeV. Similar to QCD, topcolor-assisted multiscale technicolor (TOPCMTC) theory 
predicts certain pseudo Goldstone bosons (PGB’s) including technipions and top-pions \[8\] 
\[15\] \[16\] which can be the characteristics of this theory.

In the SM, \( t\bar{t} \) production at the Tevatron energy is dominated by the sub-process \( q\bar{q} \rightarrow t\bar{t} \) 
\[3\]. However, in a recent interesting paper, Eichten and Lane \[17\] showed that, in TC 
theories, color-octet technipions \( \Pi^{0a} \) could make important contributions to \( t\bar{t} \) production 
at the Tevatron via the gluon-gluon fusion sub-process \( gg \rightarrow \Pi^{0a} \rightarrow t\bar{t} \) due to the large 
triangle-loop gluon-gluon-PGB coupling [cf. Fig. 1(a)], and such PGB could be tested 
by measuring the differential cross section \[4\]. Considering the total \( t\bar{t} \) production cross 
section, the color-singlet technipion \( \Pi^0 \) also contributes. Furthermore, apart from the 
technifermion-loop contributions considered in Ref. \[17\] [fig. 1(a)], the isospin-singlet PBG’s 
\( \Pi^{0a} \) and \( \Pi^0 \) can also couple to the gluons through the top-quark triangle-loop \[20\], and make 

\[3\] It has been shown that ETC models without exact custodial symmetry may give rise to consistent 
values of \( R_b \) \[14\], but such models may make the electroweak parameter \( T \) too large.

\[4\] The contribution of color-octet technirhos to the \( t\bar{t} \) production has been considered in Refs. \[8\] 
\[18\] \[19\].
contributions shown in Fig.1(b). In the TOPCMTC theory, the top-pion $\Pi_0^t$, as an isospin-triplet, can couple to the gluons through the top-quark triangle-loop in an isospin-violating way similar to the coupling of $\pi^0$ to the gluons in the Gross-Treiman-Wilczek formula \cite{21}, and the large isospin violation \( \frac{m_t-m_b}{m_t+m_b} \approx 1 \) makes its contribution to the $t\bar{t}$ production cross section important as well [cf. fig. 1(b)]. In this paper we study all these contributions to the production cross section of the sub-process $gg \rightarrow t\bar{t}$, and use the MRS set $A'$ parton distributions \cite{22} to calculate the cross sections at both the Tevatron and the LHC. The results of the total production cross sections show that, with these contributions, the cross section at the Tevatron is consistent with the new CDF datum for a certain range of the parameters, and the new CDF datum does give constraints on the parameters in TOPCMTC models. The cross section at the 14 TeV LHC is significantly larger than the SM prediction. The results of the differential cross sections show clear resonances of the PGB $\Pi^{0a}$ if its mass is in the reasonable range $400 - 500$ GeV. Therefore, this kind of model can be clearly tested by future experiments.

This paper is organized as follows: Sec. 2 is devoted to the calculation of the $gg \rightarrow t\bar{t}$ amplitude contributed by the PGB’s $\Pi^{0a}, \Pi^0$, and $\Pi^t_0$. In Sec. 3, we present the numerical results of the total contributions of $\Pi^{0a}, \Pi^0$ and $\Pi^t_0$ to the $t\bar{t}$ production cross sections at the Tevatron and the LHC in TOPCMTC models considering all fermion loops in Fig. 1(a)-(b). The conclusions are given in Sec. 4.

2. The $gg \rightarrow t\bar{t}$ amplitude contributed by $\Pi^{0a}, \Pi^0$, and $\Pi^t_0$

In the topcolor-assisted multiscale technicolor theory, there are a lot of PGB’s. What are relevant to the $t\bar{t}$ production process are the neutral technipions $\Pi^{0a}, \Pi^0$, and the neutral top-pion $\Pi^t_0$. In the MWTC sector, the masses of $\Pi^{0a}$ and $\Pi^0$ have been estimated to be
\( M_{\Pi^a} \approx 200 - 600 \text{ GeV} \) and \( M_{\Pi^0} \approx 100 - 300 \text{ GeV} \), and the decay constants are \( F = F_Q = F_L \approx 30 - 50 \text{ GeV} \) [3]. In the topcolor sector, if the topcolor scale is of the order of 1 TeV, the mass of \( \Pi^0_t \) is around 200 GeV and its decay constant is \( F_t \approx 50 \text{ GeV} \) [13]. Since these PGB masses are not far from the \( tt \) threshold and \( F, F_t \) are all small, they may give important contributions to the \( tt \) production rates. In this section, we give the formulae for calculating the production amplitudes \( gg \rightarrow \Pi^{0a} \rightarrow tt, gg \rightarrow \Pi^0 \rightarrow tt, \) and \( gg \rightarrow \Pi^0_t \rightarrow tt \) shown in Fig. 1(a) and fig. 1(b). These concern the couplings of the PGB’s to fermions and to gluons, and the PGB propagators.

In the TOPCMTC theory, the top- and bottom-quark masses \( m_t \) and \( m_b \) come from both the top-quark condensate and the ETC sector. It can be made that the large \( m_t \) is mainly contributed by the top-quark condensate, so that the ratio between the ETC contributed top- and bottom-quark masses \( m'_t \) and \( m'_b \) is about the the same as the ratio between the charm- and strange-quark masses, i.e. \( (m'_t/m'_b) \approx (m_c/m_s) \approx 10 \). This makes the value of the electroweak parameter \( T \) not too large in this theory. The value of \( m'_t (m'_b) \) depends on the parameters in the TOPCMTC model. For reasonable values of the parameters, \( m'_t \sim 20 - 50 \text{ GeV} \) [13].

We first consider the couplings of the PGB’s to \( tt \). At the relevant energy scale, the PGB’s can be described by local fields. In the MWTC theory, the coupling of technipions to fermions are induced by ETC interactions and hence are model dependent. However, it has been generally argued that the couplings of the PGB’s to the quark \( q \) and antiquark \( \bar{q} \) are proportional to \( m'_q/F \) [17] [23] [18], where \( m'_q \) is the part of the quark mass acquired from the ETC. The PGB-\( q-q \) vertices are of the following forms [17] [18]:

\[
\frac{C_q m'_q}{\sqrt{2}F} \Pi^0 (q \gamma^5 \bar{q}), \quad \frac{C_q m'_q}{F} \Pi^{0a} (q \gamma^5 \lambda^a \bar{q}), \quad (1)
\]

where \( \lambda^a \) is the Gell-Mann matrix of the color group, \( C_q \) is a model dependent coupling
constant which is expected to be typically of $O(1)$ \[17\] \[23\] \[18\]. In the topcolor sector, by similar argument, we can obtain the interactions of the top-pions with the top and bottom quarks by replacing $m'_q$ by $m_q - m'_q$, and $F$ by $F_t$ in (1), i.e. \[16\]

$$\frac{m_t - m'_t}{\sqrt{2}F_t} t\gamma_5 t\Pi^0 + \frac{i}{\sqrt{2}} \bar{t}(1 - \gamma_5)b\Pi^+ + \frac{1}{\sqrt{2}}\bar{b}(1 + \gamma_5)t\Pi^-,$$

(2)

$$\frac{m_b - m'_b}{\sqrt{2}F_t} \bar{b}\gamma_5 b\Pi^0.$$

(3)

Next we consider the couplings of the PGB’s to the gluons. Consider a general formula for the coupling of a PGB to two gauge fields $B_{1\mu}$ and $B_{2\nu}$. As far as the PGB’s are described by local fields, the triangle fermion loops coupling the PGB’s to $B_1$ and $B_2$ can be evaluated from the Adler-Bell-Jackiw anomaly. The general form of the effective PGB-$B_1$-$B_2$ interaction is \[24\] \[18\]

$$\frac{1}{(1 + \delta_{B_1,B_2})} \left( \frac{S_{\Pi B_1 B_2}}{4\pi^2 F} \right) \Pi \epsilon_{\mu\nu\lambda\rho} (\partial^\mu B_1^\nu)(\partial^\lambda B_2^\rho),$$

(4)

where $\Pi$ stands for $\Pi^0$, $\Pi^{0a}$ or $\Pi^0_t$; and when $B_1$ and $B_2$ are gluons, the factors $S_{\Pi gg}$ in different cases are as follows.

For $\Pi^0$ and $\Pi^{0a}$ with technifermion triangle-loop \[24\],

$$S_{\Pi^0 gg g}^{(Q,L)} = \sqrt{2}g_s^2 N_{TC} \delta_{bc}, \quad S_{\Pi^0 gg g}^{(Q,L)} = \sqrt{2}g_s^2 N_{TC} d_{abc}. \quad (5)$$

For $\Pi^0$ and $\Pi^{0a}$ with top-quark triangle-loop \[20\],

$$S_{\Pi^0 gg g}^{(t)} = \frac{C_t}{\sqrt{2}} g_s^2 J(R_{\Pi^0}) \delta_{bc}, \quad S_{\Pi^{0a} gg g}^{(t)} = \frac{C_t}{2} g_s^2 d_{abc} J(R_{\Pi^{0a}}), \quad (6)$$

with

$$J(R_\Pi) = -\frac{m'_t}{m_t} \frac{1}{R_\Pi^2} \int_0^1 \frac{dx}{x(1-x)} \ln[1 - R_\Pi^2 x(1-x)], \quad (7)$$

where $R_\Pi \equiv \frac{M_\Pi}{m_t}$. 

5
The coupling of $\Pi_0$ to gluons via the top-quark triangle-loop is isospin-violating similar to the coupling of $\pi^0$ to gluons in the Gross-Treiman-Wilczek formula [21]. It can also be calculated from the formula in Ref. [20] which gives

$$S_{\Pi_0g_0g_0} = \frac{1}{\sqrt{2}} g_0^2 \delta_{bc} J(R_{\Pi_0}),$$  \hspace{1cm} (8)$$

with

$$J(R_{\Pi_0}) = -\frac{m_t - m_t'}{m_t} \frac{1}{R_{\Pi_0}^2} \int_0^1 \frac{dx}{x(1-x)} \ln[1 - R_{\Pi_0}^2 x (1-x)],$$ \hspace{1cm} (9)$$

where $R_{\Pi_0} \equiv \frac{M_{\Pi_0}}{m_t}$.

Finally the $\Pi$ ($\Pi^0$, $\Pi^{0a}$, or $\Pi^0_t$) propagator in Fig. 1 takes the form

$$i \frac{\hat{s} - M_{\Pi}^2 + i M_{\Pi} \Gamma_{\Pi}}{\hat{s} - M_{\Pi}^2 + i M_{\Pi} \Gamma_{\Pi}},$$ \hspace{1cm} (10)$$

where $\sqrt{\hat{s}}$ is the c.m. energy and $\Gamma_{\Pi}$ is the total width of the PGB $\Pi$. The $i M_{\Pi} \Gamma_{\Pi}$ term in (11) is important when $\hat{s}$ is close to $M_{\Pi}^2$. The widths $\Gamma_{\Pi^0}$, $\Gamma_{\Pi^{0a}}$, and $\Gamma_{\Pi^0_t}$ can be obtained as follows.

From (1) and (4) we see that the dominant decay modes of $\Pi^0$ are $\Pi^0 \rightarrow b\bar{b}$ and $\Pi^0 \rightarrow gg$. So that

$$\Gamma_{\Pi^0} \approx \Gamma(\Pi^0 \rightarrow b\bar{b}) + \Gamma(\Pi^0 \rightarrow g_ag_b).$$ \hspace{1cm} (11)$$

From (1) and (5), we can obtain

$$\Gamma(\Pi^0 \rightarrow b\bar{b}) = \frac{3C_b}{16\pi} \frac{m_b^2 M_{\Pi}}{F^2} \sqrt{1 - \frac{4m_b^2}{M_{\Pi}^2}},$$ \hspace{1cm} (12)$$

and

5 It is proportional to the isospin-violating factor $\frac{m_t - m_b}{m_t + m_b} \approx 1$. 

6
\[ \Gamma(\Pi^0 \rightarrow g_a g_b) = \Gamma^{(Q,L)}(\Pi^0 \rightarrow g_a g_b) + \Gamma(t)(\Pi^0 \rightarrow g_a g_b) = \Gamma^{(Q,L)}(\Pi^0 \rightarrow g_a g_b) 1 + \frac{C_t J(R_{\Pi^0})^2}{2 N_{TC}} \]

\[ = \frac{\alpha_s^2 N_{TC}^2 M_{\Pi^0}^3}{16\pi^3 F^2} \left| 1 + \frac{C_t J(R_{\Pi^0})}{2 N_{TC}} \right|^2, \quad (13) \]

where \( \Gamma^{(Q,L)} \) and \( \Gamma(t) \) are the \( \Pi^0 \rightarrow gg \) rates contributed by the technifermion loop and top-quark loop, respectively.

It has been shown \cite{20} \cite{25} that \( \Pi^0a \) decays dominantly into \( t\bar{t}, \, gg, \) and \( gZ \). So that

\[ \Gamma_{\Pi^0a} \approx \Gamma(\Pi^0a \rightarrow b\bar{b}) + \Gamma(\Pi^0a \rightarrow g_a g_b) + \Gamma(\Pi^0a \rightarrow t\bar{t}) + \Gamma(\Pi^0a \rightarrow gZ). \quad (14) \]

From (1), (4) and the value of \( S(\Pi^0a gZ) \) given in Ref. \cite{23} \cite{18}, we can obtain

\[ \Gamma(\Pi^0a \rightarrow q\bar{q}) = \frac{C_q}{16\pi} \frac{m_q^2 M_{\Pi^0a}}{F^2} \left| 1 - \frac{4m_q^2}{M_{\Pi^0a}^2} \right|, \quad q = t, b, \quad (15) \]

\[ \Gamma(\Pi^0a \rightarrow g_a g_b) = \Gamma^{(Q,L)}(\Pi^0a \rightarrow g_a g_b) 1 + \frac{C_t J(R_{\Pi^0a})^2}{2 \sqrt{2} N_{TC}} \]

\[ = \frac{5\alpha_s^2 N_{TC}^2 M_{\Pi^0a}^3}{384\pi^3 F^2} \left| 1 + \frac{C_t J(R_{\Pi^0a})}{2 \sqrt{2} N_{TC}} \right|^2, \quad (16) \]

\[ \Gamma(\Pi^0a \rightarrow gZ) = \frac{\alpha_{\alpha_s}}{144\pi^3} \left( \frac{N_{TC}}{4} \right)^2 \tan^2\theta_W \frac{M_{\Pi^0a}^2}{F^2}. \quad (17) \]

Since the top-pion mass is around 200 GeV, it decays mainly into \( b\bar{b} \) and \( gg \). Thus

\[ \Gamma_{\Pi^0_t} \approx \Gamma(\Pi^0_t \rightarrow b\bar{b}) + \Gamma_g(\Pi^0_t \rightarrow g_a g_b). \quad (18) \]

From (1) and (4) we obtain

\[ \Gamma(\Pi^0_t \rightarrow b\bar{b}) = \frac{3}{16\pi} \frac{(m_b - m_t')^2}{F_t^2} M_{\Pi^0_t} \sqrt{1 - \frac{4m_t^2}{M_{\Pi^0_t}^2}}, \quad (19) \]

and
\[
\Gamma_g(\Pi^0_t \rightarrow g_a g_b) = \frac{\alpha_s^2}{64 \pi^3} \frac{M_{H_0}^3}{F_t^2} |J(R_{\Pi^0})|^2. \tag{20}
\]

With the above formulae, we can obtain the following production amplitudes.

\[
\mathcal{A}(g_b g_c \rightarrow \Pi^0 \rightarrow t\bar{t}) = \mathcal{A}^{(Q,L)}(g_b g_c \rightarrow \Pi^0 \rightarrow t\bar{t}) + \mathcal{A}^{(t)}(g_b g_c \rightarrow \Pi^0 \rightarrow t\bar{t})
= \frac{C_t m_t^2 g_s^2 [N_{TC} + C_t J(R_{\Pi^0})/(2\sqrt{2})]}{4\pi^2 \sqrt{2} F_t^2 [\hat{s} - M_{\Pi^0} + i M_{\Pi^0} \Gamma_{\Pi^0}]} (\hat{r}_5 t) \gamma^\mu \gamma^\lambda \gamma^\rho \epsilon_{\mu\nu\lambda\rho} k_1^\mu k_2^\rho \epsilon_1^\nu \epsilon_2^\lambda, \tag{21}
\]

\[
\mathcal{A}(g_b g_c \rightarrow \Pi^0 \rightarrow t\bar{t}) = \mathcal{A}^{(Q,L)}(g_b g_c \rightarrow \Pi^0 \rightarrow t\bar{t}) + \mathcal{A}^{(t)}(g_b g_c \rightarrow \Pi^0 \rightarrow t\bar{t})
= \frac{1}{\sqrt{2} 4\pi^2 \sqrt{2} F_t^2 [\hat{s} - M_{\Pi^0} + i M_{\Pi^0} \Gamma_{\Pi^0}]} (\hat{r}_5 t) \gamma^\mu \gamma^\lambda \gamma^\rho \epsilon_{\mu\nu\lambda\rho} k_1^\mu k_2^\rho \epsilon_1^\nu \epsilon_2^\lambda, \tag{22}
\]

\[
\mathcal{A}(g_b g_c \rightarrow \Pi^0 \rightarrow t\bar{t}) = \frac{1}{2} \frac{(m_t - m_t^t) g_s^2 J(R_{\Pi^0}) \delta_{bc}}{\hat{s} - M_{\Pi^0} + i M_{\Pi^0} \Gamma_{\Pi^0}} (\hat{r}_5 t) \gamma^\mu \gamma^\lambda \gamma^\rho \epsilon_{\mu\nu\lambda\rho} k_1^\mu k_2^\rho \epsilon_1^\nu \epsilon_2^\lambda. \tag{23}
\]

It is easy to obtain the SM tree-level \( t\bar{t} \) production amplitudes

\[
\mathcal{A}^{SM}_{tree}(q\bar{q} \rightarrow t\bar{t}) = \frac{i g_s^2 v(p_q) \gamma^\mu \gamma^\lambda \gamma^\rho \epsilon_{\mu\nu\lambda\rho} \hat{s}}{\hat{s}} u(p_\bar{q}) u(p_t) \gamma_\mu \gamma_\lambda \gamma_\rho \epsilon_{\mu\nu\lambda\rho} (p_t), \tag{24}
\]

and

\[
\mathcal{A}^{SM}_{tree}(gg \rightarrow t\bar{t}) = \mathcal{A}^{SM(s)}_{tree}(gg \rightarrow t\bar{t}) + \mathcal{A}^{SM(t)}_{tree}(gg \rightarrow t\bar{t}) + \mathcal{A}^{SM(u)}_{tree}(gg \rightarrow t\bar{t}), \tag{25}
\]

with

\[
\mathcal{A}^{SM(s)}_{tree}(gg \rightarrow t\bar{t}) = -i g_s^2 [(k_2 - k_1)^\mu (\epsilon_2 \cdot \epsilon_1) + (k_2 + 2k_1) \cdot \epsilon_2 \epsilon_1^\mu - (2k_2 + k_1) \cdot \epsilon_1 \epsilon_2^\mu] 
\times \frac{1}{\hat{s}} \hat{u}(p_t) \gamma_\mu (i f_{abc} \frac{\lambda^c}{2}) \hat{v}(p_\bar{t}), \tag{26}
\]

\[
\mathcal{A}^{SM(t)}_{tree}(gg \rightarrow t\bar{t}) = -i g_s^2 \frac{\hat{u}(p_t) \hat{f}_1 (\frac{q - m_t}{q^2 - m_t^2}) \frac{\lambda^a}{2} \hat{v}(p_\bar{t})}{q^2 - m_t^2}, \quad q \equiv p_t - k_1. \tag{27}
\]
$$A_{\text{tree}}^{SM(u)}(gg \rightarrow t\bar{t}) = A_{\text{tree}}^{SM(t)}(gg \rightarrow t\bar{t})[1 \leftrightarrow 2, \ a \leftrightarrow b],$$  \hspace{1cm} (28)

where \( k_1, k_2 \) are the momenta of the two initial-state gluons, \( p_t \) is the momentum of the top-quark.

Adding all these amplitudes together, we obtain the total \( t\bar{t} \) production amplitude.

3. The \( t\bar{t} \) production cross sections at the Tevatron and the LHC

Once we have the cross section at the parton level \( \hat{\sigma} \), the cross section at the hadron collider is obtained by convoluting it with the parton distributions

$$\sigma(pp(\bar{p}) \rightarrow t\bar{t}) = \sum_{ij} \int dx_i dx_j f_i^{(p)}(x_i, Q)f_j^{(p(\bar{p}))}(x_j, Q) \hat{\sigma}(ij \rightarrow t\bar{t}),$$ \hspace{1cm} (29)

where \( i, j \) stand for the partons \( g, q \) and \( \bar{q} \); \( x_i \) is the fraction of the longitudinal momentum of the proton (antiproton) carried by the \( i \)-th parton; \( Q^2 \approx \hat{s} \); and \( f_i^{(p(\bar{p}))} \) is the parton distribution functions in the proton (antiproton). In this paper, we take the MRS set \( A' \) parton distrbution for \( f_i^{(p(\bar{p}))} \). Taking into account of the QCD corrections, we shall multiply the obtained \( \sigma \) by a factor 1.5 \([3]\) as what was done in Ref. [17].

The main purpose of Ref. [17] is to show the signal of \( \Pi^{0a} \) at the Tevatron, so that they only calculated the technifermion-loop contributions and neglected the interference between \( A_{\text{tree}}^{SM}(gg \rightarrow t\bar{t}) \) and \( A(gbgc \rightarrow \Pi^{0a} \rightarrow t\bar{t}) \) as a first investigation. In this section, we present the cross sections at the Tevatron and the LHC in TOPCMTC models considering the contributions of \( \Pi^{0a}, \Pi^0 \) and \( \Pi^0_i \) from Fig. 1(a) and Fig. 1(b) with the interferencess taken into account. In our calculation, we take the more updated parton distribution functions MRS set \( A' \) instead of EHLQ set 1 taken in Ref. [17]. The fundamental SM parameters in our calculation are taken to be \( m_t = 176 \text{ GeV}, \ \sin^2 \theta_W = 0.231 \), and \( \alpha_s(\sqrt{s}) \) the same as that in the MRS set \( A' \) parton distributions. For the parameters in the TOPCMTC models,
we simply take $C_t = C_b = 1$ and take the reasonable values $F = 40$ GeV, $F_t = 50$ GeV in this calculation. For the technipion masses, we fix $M_{\Pi_0} = 150$ GeV, and vary $M_{\Pi_0a}$ from 400 GeV to 500 GeV. The values of $\Pi^0_t$ and $m'_t$ depend on the parameters in the TOPCMTC models. To see how these values affect the cross sections, we take, some reasonable values for each of them, namely $M_{\Pi_0} = 150$ GeV and 350 GeV, $m'_t = 20$, 35 and 50 GeV.

The results of the cross sections at the 1.8 TeV Tevatron are listed in Table 1, in which $\Delta \sigma_{t\bar{t}}^{(i)}$ is the TOPCMTC correction [including the interferences between the TOPCMTC amplitudes (21)-(23) and the tree-level SM amplitudes (24)-(28)] to the tree-level SM cross section in the total cross section $\sigma_{t\bar{t}}^{(i)}$, with $i = 1, 2, 3$ corresponding to $m'_t = 20$ GeV, 35 GeV, and 50 GeV. We see that for most values of the parameters the cross sections $\sigma_{t\bar{t}}$ are consistent with the new CDF data except that the cross sections are too large for $m_{\Pi_0a} = 400$ GeV with $m'_t \geq 35$ GeV. Therefore the CDF data does give constraints on the values of $m_{\Pi_0a}$ and $m'_t$ which depends on the specific model. To see the constraints more precisely, we plot the cross section versus $m'_t$ in Fig. 2 (with $m_{\Pi_0} = 150$ GeV) and Fig. 3 (with $m_{\Pi_0} = 350$ GeV), in which the solid, dashed and, dotted lines stand for $m_{\Pi_0a} = 400$, 450, and 500 GeV, respectively. Comparing with the new CDF data (the shaded band), we see that there are parameter ranges outside the band of the CDF data, especially for $m_{\Pi_0} = 150$ GeV, the range of parameters $m_{\Pi_0a} = 400$ GeV with $m'_t > 30$ GeV is disfavored; for $m_{\Pi_0} = 350$ GeV, the range of parameters $m_{\Pi_0a} = 400$ GeV with all $m'_t > 20$ GeV is disfavored.

In Figs. 4-6, we plot the differential cross sections $\frac{d\sigma_{t\bar{t}}}{dm_{t\bar{t}}}$ versus the $t\bar{t}$ invariant mass $m_{t\bar{t}}$ at the $\sqrt{s} = 1.8$ TeV Tevatron for various values of the parameters. We see that clear peak of the $\Pi^{0a}$ resonance emerges when $m_{\Pi_0a}$ lies in the range of 400 to 500 GeV. The larger the value of $m'_t$, the clearer the signal. This is because that the coupling in (1) is
proportional to $m'_t$. For the case of $m_{\Pi^0} = 350 \text{ GeV}$, the $\Pi^0_t$ peak can also be seen. Thus the model can be tested by the differential cross section for certain values of the parameters.

In Table 2, we list the values of $\Delta \sigma_{t\bar{t}}$ and $\sigma_{t\bar{t}}$ at the $\sqrt{s} = 14 \text{ TeV LHC}$. We see that the cross sections are much larger than those at the Tevatron due to the fact that at the LHC $t\bar{t}$ production is dominated by gluon-fusion. The obtained cross section is significantly larger than the SM predicted value, so that it can be easily tested by the future experiment. In Figs. 7-9, we plot the differential cross sections at the LHC for various values of the parameters. We see that differential cross sections are similar to those at the Tevatron but the peaks are more significant due to the same reason. So that the models can be better tested at the LHC.

4. Conclusions

In this paper, we studied the $t\bar{t}$ production cross sections at the $\sqrt{s} = 1.8 \text{ TeV Tevatron}$ and the $\sqrt{s} = 14 \text{ TeV LHC}$ in the TOPCMTC models. The TOPCMTC contributions are mainly via the $s$-channel PGB’s $\Pi^0_a$, $\Pi^0$, and $\Pi^0_t$ through gluon-fusion. We calculated both the diagrams in Fig.1(a) and Fig.1(b), and took into account the interferences between the tree level SM amplitudes [(24)-(28)] and the TOPCMTC amplitudes [(21)-(23)]. The MRS set $A'$ parton distribution functions are taken in this calculation. In the study, we take $m_{\Pi^0} = 150 \text{ GeV}$ and vary other parameters in the models. Our results show that the production cross sections are enhanced by the TOPCMTC contributions. The present CDF datum on the production cross section gives constraints on the model-dependent parameters $m_{\Pi^0_a}$ and $m'_t$, i.e. $m_{\Pi^0_a} = 400 \text{ GeV}$ with large $m'_t$ is disfavored. In the differential cross
sections, clear peaks of the $\Pi^0_{1a}$ and $\Pi^0_{1t}$ can be seen for reasonable range of the parameters, so that the models are experimentally testable at the Tevatron and the LHC. The cross section at the LHC is significantly larger than the SM predicted value, and the peaks are more significant at the LHC than at the Tevatron due to the fact that $t\bar{t}$ production at the LHC is dominated by gluon-fusion. Therefore the models can be better tested at the LHC.

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Table 1. $t\bar{t}$ production cross section $\sigma(gg \rightarrow \pi^0(a)\pi^0, \pi^0 \rightarrow t\bar{t})$ at the $\sqrt{s} = 1800$ GeV Tevatron in the topcolor-assisted multiscale walking technicolor model with $m_{\pi^0} = 150$ GeV. $\Delta\sigma^{(i)}$ is the TOPCMTC correction to the tree-level SM cross section and $\sigma^{(i)}_{t\bar{t}}$ is the total cross section, where $i = 1, 2, 3$ correspond to $m'_{t} = 20$ GeV, 35 GeV, and 50 GeV, respectively. A factor 1.5 of QCD corrections has been taken into account.

| $M_{\pi^0}$ (GeV) | $M_{\pi^0}$ (GeV) | $\Delta\sigma^{(1)}$ (pb) | $\sigma^{(1)}_{t\bar{t}}$ (pb) | $\Delta\sigma^{(2)}$ (pb) | $\sigma^{(2)}_{t\bar{t}}$ (pb) | $\Delta\sigma^{(3)}$ (pb) | $\sigma^{(3)}_{t\bar{t}}$ (pb) |
|-------------------|-------------------|--------------------------|-----------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| 150               | 400               | 2.750                    | 7.861                       | 5.678                    | 10.789                   | 7.829                    | 12.940                   |
| 150               | 450               | 1.350                    | 6.461                       | 2.688                    | 7.798                    | 3.618                    | 8.729                    |
| 150               | 500               | 0.632                    | 5.743                       | 1.245                    | 6.356                    | 1.680                    | 6.791                    |
| 350               | 400               | 4.638                    | 9.749                       | 7.127                    | 12.238                   | 8.834                    | 13.945                   |
| 350               | 450               | 2.970                    | 8.081                       | 3.965                    | 9.076                    | 4.571                    | 9.682                    |
| 350               | 500               | 2.279                    | 7.390                       | 2.480                    | 7.591                    | 2.598                    | 7.709                    |

Table 2. $t\bar{t}$ production cross section $\sigma(gg \rightarrow \pi^0(a)\pi^0, \pi^0 \rightarrow t\bar{t})$ at the $\sqrt{s} = 14$ TeV LHC in the topcolor-assisted multiscale walking technicolor model with $m_{\pi^0} = 150$ GeV. $\Delta\sigma^{(i)}$ is the TOPCMTC correction to the tree-level SM cross section and $\sigma^{(i)}_{t\bar{t}}$ is the total cross section, where $i = 1, 2$ correspond to $m'_{t} = 20$ GeV and 35 GeV, respectively. A factor 1.5 of QCD corrections has been taken into account.

| $M_{\pi^0}$ (GeV) | $M_{\pi^0}$ (GeV) | $\Delta\sigma^{(1)}$ (nb) | $\sigma^{(1)}_{t\bar{t}}$ (nb) | $\Delta\sigma^{(2)}$ (nb) | $\sigma^{(2)}_{t\bar{t}}$ (nb) |
|-------------------|-------------------|--------------------------|-----------------------------|--------------------------|--------------------------|
| 150               | 400               | 2.753                    | 3.493                       | 5.577                    | 6.317                    |
| 150               | 450               | 2.073                    | 2.813                       | 4.167                    | 4.907                    |
| 150               | 500               | 1.596                    | 2.336                       | 3.180                    | 3.920                    |
| 350               | 400               | 3.791                    | 4.531                       | 6.293                    | 7.033                    |
| 350               | 450               | 3.182                    | 3.922                       | 5.006                    | 5.746                    |
| 350               | 500               | 2.568                    | 3.308                       | 3.978                    | 4.718                    |
Figure Captions

**Fig. 1.** Feynman diagrams for the TOPCMTC contributions to the $t\bar{t}$ productions at the Tevatron and the LHC.

(a). Techniquark loop contributions. (b). Top-quark loop contributions.

**Fig. 2.** The plot of $\sigma_{t\bar{t}}$ versus $m_t'$ for $m_{\Pi^0_t} = 150$ GeV at the Tevatron. The solid, dashed, and dotted lines stand for $m_{\Pi^{0_a}} = 400$, 450, and 500 GeV, respectively. The CDF data is indicated by the shaded band.

**Fig. 3.** Same as **Fig. 2** but for $m_{\Pi^0_t} = 350$ GeV.

**Fig. 4.** Differential cross section $\frac{d\sigma_{t\bar{t}}}{dm_{t\bar{t}}}$ (in logarithmic scale) versus the $t\bar{t}$ invariant mass $m_{t\bar{t}}$ at the Tevatron for $m_{\Pi^{0_a}} = 400$, 450, and 500 GeV with $m_{\Pi^0} = 150$ GeV, $m_t' = 20$ GeV, and $m_{\Pi^0_t} = 150$ GeV.

**Fig. 5.** Same as **Fig. 3.** but with $m_t' = 35$ GeV and $m_{\Pi^0_t} = 150$ GeV.

**Fig. 6.** Same as **Fig. 3.** but with $m_t' = 20$ GeV and $m_{\Pi^0_t} = 350$ GeV.

**Fig. 7.** Same as **Fig. 3.** but at the LHC.

**Fig. 8.** Same as **Fig. 4.** but at the LHC.
Fig. 9. Same as Fig. 5 but at the LHC.
Fig. 1
Fig. 2
Fig. 3

\[ \sigma(\bar{p}p \rightarrow t\bar{t}) \] (pb) vs. \[ m_t (GeV) \]
Fig. 4

$\sigma_d/dm_t (pb/GeV)$

$m_{\Pi_0a} = 400$ GeV

$m_{\Pi_0a} = 450$ GeV

$m_{\Pi_0a} = 500$ GeV

$m_t = 20$ GeV

$m_{\Pi_t} = 150$ GeV
Fig. 5

\[ \frac{d\sigma_{tt}}{dm_{tt}} \text{ (pb/GeV)} \]

- $m_{\Pi 0a} = 400$ GeV
- $m_{\Pi 0a} = 450$ GeV
- $m_{\Pi 0a} = 500$ GeV

$m_t = 35$ GeV

$m_{\Pi_t} = 150$ GeV
\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig6.png}
\caption{\textbf{d}\sigma_{tt}/d_m (pb/GeV)}
\end{figure}

- \( m_{\Pi_0a} = 400 \text{ GeV} \)
- \( m_{\Pi_0a} = 450 \text{ GeV} \)
- \( m_{\Pi_0a} = 500 \text{ GeV} \)

- \( m_t = 20 \text{ GeV} \)
- \( m_{\Pi_t} = 350 \text{ GeV} \)
Fig. 7
Fig. 8

\[ \sigma_{t \bar{t}} / (\text{nb}/\text{GeV}) \]

- \( m_{\Pi_0a} = 400 \text{ GeV} \)
- \( m_{\Pi_0a} = 450 \text{ GeV} \)
- \( m_{\Pi_0a} = 500 \text{ GeV} \)

\( m_t = 35 \text{ GeV} \)
\( m_{\Pi_t} = 150 \text{ GeV} \)
Fig. 9

$\sigma_d/\Delta m$ (nb/GeV)

$m_\Pi 0a = 400$ GeV

$m_\Pi 0a = 450$ GeV

$m_\Pi 0a = 500$ GeV

$m_i^0 = 20$ GeV

$m_{\Pi_i} = 350$ GeV