Early Reionization in Cosmology

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1 Introduction

Since this is the first contribution to this meeting which is mainly involved in cosmology, I would like to put it into its cosmological perspective:

In the standard model of cosmology we assume that the universe is homogeneous and isotropic on large enough scales. This assumption, originally probably made just by reasons of simplicity, leads to a so called Friedmann Lemaitre universe which explains, e.g., the well established uniform Hubble expansion \[ H \approx 10^9 \, \text{K} \] . Within this model, the universe started out from an extremely dense, hot initial state and subsequently cooled by adiabatic expansion undergoing a series of phase transitions. At a temperature of about \[ 0.1 \, \text{MeV} \approx 10^9 \, \text{K} \], deuterons become stable and virtually all the neutrons present are bound into \[ ^4 \text{He} \]. This leads to the well established abundances of the light elements. As the universe cools further, below about \[ 3000 \, \text{o} \, \text{K} \] there are no longer enough ionizing photons around to keep the Hydrogen Helium plasma ionized. The matter in the universe recombines to neutral H/He and the universe becomes transparent to the cosmic radiation field. Radiation can then propagate freely, influenced only by the cosmic expansion and redshifted to the \[ 2.7 \, \text{o} \, \text{K} \] microwave background which we observe today. At recombination, the age of the universe was \[ t \approx 2 \times 10^5 \, \text{years} \], which corresponds to a redshift of \[ z_R \approx 1100 \].

We now want to discuss the possibility, that the cosmic plasma might have reionized again at some lower redshift \[ z_i < z_R \]. The reason one might want to investigate this idea is twofold:

- **Observationally:** The Gunn Peterson test, i.e. the absence of a Lyman-\( \alpha \) trough in all the observed quasar spectra, shows that the intergalactic medium is ionized for \[ z < 4.5 \] .

- **Theoretically:** The anisotropies in the cosmic microwave background (CMB) have turned out to represent one of the most stringent 'bottle necks' which a scenario of structure formation has to pass in order to be acceptable. As a possibility to relax this constraint, it has been proposed that early reionization can damp CMB fluctuations on small scales (\( \theta \leq 6^\circ \)) due to photon diffusion in the ionized plasma. We shall illustrate this idea in this paper.

In the next section, we want to explain the influences of reionization on the cosmic microwave background: It leads to damping of fluctuations due to photon diffusion on one hand and slightly distorts the CMB spectrum on the other hand. In Section 3, we discuss the processes in the plasma which have to be taken into account to describe reionization and formulate the system of differential equations. Finally we draw conclusions and outline future progress.

In this contribution we use the following notation:

- Greek indices run from 0 to 3 and latin ones from 1 to 3
• We assume throughout a spatially flat universe, i.e. the density parameter, $\Omega_{tot} = \rho_{tot}/\rho_c = 1$ (a possible curvature would only change angles, but no other conclusions since we are mainly interested in the regime $z >> 1$).

• We choose $t$ as conformal time coordinate and work with the metric signature $(-,+,+,+)$, so that $ds^2 = a(t)^2(-dt^2 + dx^2)$.

• Boldface characters denote 3 dimensional vectors.

• We parametrize Hubble’s constant by $H_0 = 100 \text{km/(s Mpc)}h$. The density parameter of the baryons, is denoted by $\Omega_B = \rho_B/\rho_c = \rho_B/(7.7h_5^2 \times 10^{-20} \text{g/cm}^3)$. Observations limit $0.5 < h < 0.8$ and (including nucleosynthesis calculations) $0.01 < \Omega_B < 0.1$.

2 Reionization and the cosmic microwave background

2.1 Damping of fluctuations by photon diffusion

As we shall see in this paragraph, reionization can lead to a substantial damping of fluctuations in the CMB on small angular scales. This is important for some scenarios of structure formation to overcome present limits posed by small and medium angular scale experiments (see contribution of P. Richards in this proceedings). As an example, we mention the recently investigated scenario with cold dark matter (CDM) and texture seeds [5, 6, 7, 8, 9]. There, an analysis of CMB anisotropies shows that early reionization is a crucial ingredient for this scenario [10]. Without damping, the small scale anisotropies would dominate and exceed observed limits. On the other hand, studies of the texture scenario show, that textures lead to early formation of objects [7]. At a redshift $z \approx 50$ about 1% of the baryons in the universe have collapsed and formed objects with mass $M_{nl} \leq 10^5 M_\odot$ [11]. If one assumes that, due to the formation of these objects, radiation energy of about 100keV per nucleon is emitted, this would yield a total energy density of this ionizing radiation $\rho_i = q\rho_B \approx 10^{-6} \rho_B \approx 10^3 eV \times n_B$, which is by far enough to reionize all the hydrogen in the universe.

By early reionization in this context we mean that it has to happen early enough so that Compton scattering of electrons is still effective. At late times electrons are too sparse to scatter photons effectively and the electron proton plasma is again decoupled from radiation. This decoupling time is determined by the optical depth due to Compton scattering being equal to unity:

$$\tau = \int_{t_0}^{t_{dec}} n_e \sigma_T dt = 1,$$

which leads to a decoupling redshift of

$$z_{dec} = 100 \left( \frac{0.025}{\Omega_B h} \right)^{2/3},$$

where $\sigma_T$ denotes the Thomson cross section. We further assume (for simplicity) $\Omega_{tot} = 1$, so that a substantial amount (about 90-95%) of non baryonic, dark matter has to be present. We parametrize the metric of this spatially flat Friedmann universe by using conformal time,

$$ds^2 = a^2(-dt^2 + dx^2).$$

The physical time differential is thus given by $adt$.

We now want to investigate how CMB anisotropies on scales smaller than $t_{dec}$, i.e., $\theta < 1/\sqrt{z_{dec} + 1} \approx 6^\circ$ can be damped if reionization happens substantially before $z_{dec}$.
Boltzmann’s equation for Compton scattering is given by
\[ p^\mu \partial_\mu f - \Gamma^\mu_{i\nu} p^\nu p^\rho \frac{\partial f}{\partial p^\rho} = C[f] , \] (3)
where \( f \) denotes the distribution function of the photons and \( C[f] \) is the collision integral. In a perturbed Friedmann universe we separate \( f \) into an isotropic background contribution and a small perturbation \( f = \bar{f} + \delta f \). One can then find a gauge–invariant (i.e. invariant under linearized coordinate transformations) variable \( F \), which reduces to \( \delta f \) for perturbations which are much smaller than the size of the horizon. For the energy integrated “brightness perturbation”
\[ \mathcal{M} = \frac{4\pi}{\rho} \int_0^\infty F p^3 dp \]
the perturbation of the Boltzmann equation (3) then becomes (for scalar perturbations)
\[ \dot{\mathcal{M}} + \epsilon^i \partial_i \mathcal{M} = 4\epsilon^i \partial_i (\Phi - \Psi) + a\sigma_T n_e [D^{(r)}_g - \mathcal{M} - 4\epsilon^i l \partial_i V + \frac{1}{2} \epsilon_{ij} M^{ij}] . \] (4)
Here \( \epsilon \) is a unit vector which denotes the direction of the photon momentum, \( V \) is a potential for the baryon velocity, \( \Phi, \Psi \) are the so called Bardeen potentials which parametrize scalar perturbations of the geometry and \( l \) is an arbitrary length scale introduced merely to keep the perturbation variables dimensionless. Furthermore
\[ \epsilon_{ij} = \epsilon_i \epsilon_j - \frac{1}{3} \delta_{ij} , \]
\[ D^{(r)}_g = (1/4\pi) \int \mathcal{M}(\epsilon) d\Omega \quad \text{and} \]
\[ M^{ij} = \frac{3}{8\pi} \int \mathcal{M}(\epsilon) \epsilon_{ij} d\Omega \].
The baryon equation of motion is
\[ l \partial_j \dot{V} + (\dot{a}/a) l \partial_i V = \partial_i \Psi - \frac{a\sigma_T n_e \rho_r}{3\rho_b} (M_j + 4l \partial_j V) , \] (5)
with \( M_j = \frac{3}{4\pi} \int \epsilon_j \mathcal{M} d\Omega \).

An introduction to gauge–invariant cosmological perturbation theory and a thorough derivation of eqs. (3,4) is given in [9].

In eq. (3) the first contribution to the right hand side is the gravitational force. In the second contribution, the collision integral, the first two terms are the usual smoothing, then there is a Doppler term and the final term is due to the anisotropy of the Compton cross section (it can be neglected in the limit of very many collisions). In eq. (4) we see, that on the right hand side the drag force due to the radiation drag experienced by the electrons is added to the well known gravitational acceleration term.

We now want to estimate the damping of \( \mathcal{M} \) due to photon diffusion. Setting \( t_T = (a\sigma_T n_e)^{-1} \), we investigate the limit \( t_T/\ell << 1 \). In deepest order we find
\[ \dot{D}^{(r)}_g = (4/3) l \Delta V = (4/3) \dot{D}^{(B)}_g , \]
where \( D^{(B)}_g \) is a perturbation variable for the baryon energy density perturbation, and the second equal sign is due to the baryon number conservation equation. In this limit therefore,
the entropy per baryon is conserved, $\delta s/s = \delta n_B/n_B$, i.e., baryons and radiation behave like a single perfect fluid with energy density $\rho_r + \rho_B$ and pressure $p_r = (1/3)\rho_r$. We now want to analyse the system (4,5) in second order. Let us first neglect the time dependence of the coefficients and make a plane wave ansatz

$$V \propto M \propto e^{i(k \cdot x - \omega t)}.$$ 

Inserting this ansatz into (4,5), we obtain the following dispersion relation in lowest order $kt_T$ and $\omega t_T$, i.e., for short waves;

$$\omega = \omega_0 + i\gamma, \quad \omega_0 = \frac{k}{\sqrt{3(1 + R)}}, \quad \gamma = \frac{k^2 t_T (R^2 + (4/5)(R + 1))}{6(R + 1)^2},$$

(6)

with $R = 3\rho_B/4\rho_r$. In the matter dominated regime, $R >> 1$, we have $\gamma \approx \frac{k^2 t_T}{6} \approx \frac{2\pi r}{l}$ where $l$ denotes the wavelength of the perturbation.

We then approximate the total damping by

$$e^{-f} \quad \text{with} \quad f = \int_{t_{in}}^{t_{end}} \gamma(t) dt,$$

where $t_{end}$ is defined by $t_T(t_{end}) = l/2\pi$ or $t_{end} = t_{dec}$ (whatever condition is satisfied earlier); and $t_{in}$ is the time when the perturbation enters the horizon $t_{in} \approx l/2$ or the time of reionization, $t_{in} = t_i$ (whatever happens later).

If we use this estimates for damping of fluctuations induced by a spherically symmetric collapsing texture [5, 7, 9], we obtain

$$f = \begin{cases} 
\left(\frac{1 + z_{dec}}{1 + z_{c}}\right)^{3/2} \left[\left(\frac{1 + z_{dec}}{1 + z_{c}}\right)^{15/8} - 1\right] & \text{for } z_c > z_{dec} \\
0 & \text{else} \end{cases}$$

(7)

where $z_c$ denotes the redshift of texture collapse, $t(z_c) \equiv t_c \approx l$ and we have assumed $z_i > z_c$.

This naive estimate, which leads to a factor $\approx 15$ damping for $z_c = 90z_{dec}$ ($t_c = 0.1t_{dec}$); a factor $\approx 5$ damping for $z_c = 10z_{dec}$ ($t_c = 0.3t_{dec}$) and a factor $\approx 1.6$ of damping for $z_c = 2z_{dec}$ ($t_c = (1/\sqrt{2})t_{dec}$).

In Fig. 1-4 we show the results from numerical integration of the system (4,5), with a gravitational field induced from a spherically symmetric, collapsing texture [8]. Comparisons with the naive predictions (7) show that our approximation is actually quite reasonable!

This result tells us also that we must require at least $z_i > 2z_{dec}$ to obtain damping by, say, a factor of 2.

### 2.2 Spectral distortion

In addition to damping the amplitude of fluctuations, the reionized plasma also distorts CMB spectrum: The ionizing radiation does not only ionize matter but also heats up the plasma to temperatures around typically $T_e \approx 0.3$ to a few eV. Nonrelativistic Compton scattering up scatters the low energy microwave photons. If the Plasma has a thermal (Boltzmann) distribution and the Photons are Planck distributed (as in our situation), the induced change in the spectrum can be described by a single parameter, the Compton–y parameter [12, 13]:

$$y = \sigma_T m_e \int_{t_i}^{t_0} n_e(T_e - T_{CMB}) adt \approx 0.4 \times 10^{-5} (\frac{1 + z_i}{1 + z_{dec}})^{3/2} (T_e/2eV),$$

(8)
where $T_e$ is a weighted “mean electron Temperature”:

$$T_e = (z_i + 1)^{-3/2} \int_0^{z_i} T_e(z + 1)^{1/2} dz .$$

(This approximation is excellent for $T_e >> T_{CMB}$ and $z_i >> 1$, i.e. the situation we are interested in.)

The observational limit set by the FIRAS experiment on COBE (see J. Mather, this Proceedings) is

$$y < 2.5^{-5} .$$

In the next section we shall see, that ionization can be maintained by collisions only if $T_e \geq 1.5eV$ and, on the other hand, it seems to require quite some fine tuning to maintain the plasma ionized by photoionization without heating it up to a temperature of about $(1 - 2)eV$. If we therefore assume a mean electron temperature of

$$T_e \geq 1.5eV ,$$

observation (9) already limits the ionizing redshift to

$$z_i \leq 4z_{dec} .$$

Improving the observational limit of the $y$-parameter by about a factor of 5 would therefore rule out reionization which happens early enough to lead to significant damping of CMB fluctuations ($z_i > 2z_{dec}$, say)!

### 3 Requirements for an ionizing radiation field

We now assume that at some high redshift $z >> z_{dec}$ there exists an ionizing radiation field which originates from the first nonlinear density perturbations, i.e. the first ‘macroscopic’ baryonic objects in the universe formed by some unknown mechanism, and which is capable of reionizing the universe. We want to study the properties of this radiation.

Its energy density $\rho_i$ and its spectrum $f_i$ are related by

$$\rho_i = \frac{1}{\pi^2} \int_0^\infty \omega^2 f_i(\omega) d\omega \quad (\hbar = c = k_B = 1) .$$

We set

$$\rho_i = q \rho_B$$

and assume that $q$ is slowly varying over some time period, where $\rho_B$ is the baryon energy density. For this radiation to be able to ionize the universe, we must require

$$n_i(\omega > \Delta) \geq n_B \quad (\Delta = 1 Ry = \alpha^2 m_e/2 = 13.6eV) ,$$

where

$$n_i = \frac{1}{\pi^2} \int_0^\infty \omega^2 f_i(\omega) d\omega ,$$

$\alpha$ denotes the fine structure constant and $m_e$ is the electron mass. To be specific, let us assume that $f_i$ is a Planck spectrum with chemical potential $\mu >> 1$,

$$f_i = \frac{1}{e^{\omega/T_i + \mu - 1}} .$$
\( \omega_0 [\text{eV}] \) & \( \Omega_0 \) & \( Q/10^{-7} \) & \( T_i / \Delta / \) & \text{Refs.} \\
2 \cdot 10^4 & 10^{-9} & 0.1(z + 1) & 1.5 \cdot 10^4 \cdot (z + 1) & \text{Setti '90 [14]} \\
2 \cdot 10^3 & 10^{-9} & 0.1(z + 1) & 150 \cdot (z + 1) & \text{Setti '90 [14]} \\
200 & 2 \cdot 10^{-10} & 0.02(z + 1) & 15 \cdot (z + 1) & \text{Paresce & Stern '81 [15]} \\
100 & 6 \cdot 10^{-10} & 0.06(z + 1) & 7.6(z + 1) & \text{Paresce & Stern '81 [15]} \\
20 & 7 \cdot 10^{-10} & 0.07(z + 1) & 1.5 \cdot (z + 1) & \text{Paresce & Stern '81 [15]} \\
8 & 4 \cdot 10^{-8} & 4(z + 1) & 0.6 \cdot (z + 1) & \text{Holberg '86 [16]} \\
2.8 & 2 \cdot 10^{-7} & 20(z + 1) & 0.2 \cdot (z + 1) & \text{Longair '90 [17]} \\
0.9 & 1.2 \cdot 10^{-6} & 120(z + 1) & 0.07 \cdot (z + 1) & \text{Longair '90 [17]} \\
0.5 & 3 \cdot 10^{-6} & 300(z + 1) & 0.04 \cdot (z + 1) & \text{Longair '90 [17]} \\
0.3 & 3 \cdot 10^{-6} & 30(z + 1) & 0.02 \cdot (z + 1) & \text{Longair '90 [17]} \\
0.2 & 10^{-7} & 10(z + 1) & 0.015 \cdot (z + 1) & \text{Longair '90 [17]} \\

Table 1: Limits on diffuse background radiation from X–rays down to the near infrared are listed with the corresponding references. The precipitous rise of the limits below 10eV is due to contributions from airglow and zodiacal light.

For this spectrum one finds

\[
\rho_i \approx \frac{6}{\pi^2} T_i^4 e^{-\mu} \quad \text{and thus} \quad q = \frac{6 T_i^4 e^{-\mu}}{\pi^2 \rho_B} .
\]

The requirement (11) then yields

\[
q > 0.8 \times 10^{-7} e^{(\Delta/T_i)}[(\Delta/T_i)^3 - 2(\Delta/T_i)^2 + 2(\Delta/T_i)]^{-1}
\]

This condition is shown by the heavy line in Fig. 5. It is compared with observational limits on background radiations at the wavelengths corresponding to the temperatures given in Table 1. The lower light line, which corresponds to those limits today, \( z = 0 \), shows that if there exists a unresolved ionizing radiation background today its energy has to be close to about 2eV and its intensity is only little (less than a factor of 3) below the observational limit. In the upper light line we translate the limits of Table 1 to \( z = 10 \) (multiply by \( z + 1 \)). The dashed region is the allowed parameter range if the universe is to be photoionized until \( z = 10 \), i.e., until recombination is slower than expansion and the universe remains ionized without a ionizing radiation.

3.1 Processes in the plasma

We now calculate the ionization and recombination rates and the heating and cooling functions for the cosmic medium, neglecting the Helium contribution:

**Photoionization:** The Karzas–Latter Photoionization cross section is given by

\[
\Sigma_{pi} = \frac{64 \pi}{m_i^2 \alpha \sqrt{3}} (\Delta/\omega)^3 g_{bf} ,
\]

where \( g_{bf} \) is a Gaunt factor which we set equal to 1 in the sequel [18]. The photoionization rate is correspondingly

\[
t_{pi}^{-1} = \frac{8 \pi m_i \alpha^5}{3 \sqrt{3}} (1 - x) \int_{\Delta}^{\infty} \omega^{-1} f_i \frac{d\omega}{\pi^2} ,
\]
where \( x = n_e/n_B \) is the ionization fraction. Inserting now the thermal spectrum defined before we obtain

\[
t_{pi}^{-1} \approx 6s^{-1} \times q(1-x)(\Delta/T_i)^4E_1(\Delta/T_i)(\frac{\Omega_B h^2}{0.025})(\frac{1+z}{200})^3.
\] (14)

Here \( E_1 \) denotes the usual integral exponential function

\[
E_1(x) = \int_{x}^{\infty} \frac{e^{-x}}{x} dx.
\]

The photons do not only ionize the plasma, but with their remaining energy they also heat it up. This photoionization heating, \( \Gamma_{pi} \) is given by

\[
\Gamma_{pi} = \frac{8\pi m_e \alpha^5}{3\sqrt{3}}(1-x) \int_{\Delta}^{\infty} \frac{\omega - \Delta}{\omega} f_i d\omega,
\]

\[\approx 78(eV/s) \times q(1-x)\left(\frac{\Omega_B h^2}{0.025}\right)\frac{1+z}{200}(\Delta/T_i)^3(e^{-\Delta/T_i} - (\Delta/T_i)E_1(\Delta/T_i)).\] (15)

**Recombination:** We obtain the recombination cross section from a detailed balance argument and the photoionization cross section [19]. This leads to the recombination rate

\[
t_{r}^{-1} = \frac{\Delta^{1/2}2^{13/2}\pi^{1/2}a}{m_e^{3/2}3^{3/2}} \left(\frac{3T_e}{2\Delta} + 1\right) x^2 n_B
\]

\[\approx 10^{-13} s^{-1} \times (\Delta/T_e)^{1/2} x^2 \left(\frac{\Omega_B h^2}{0.025}\right)\frac{1+z}{200}(1-x)(\Omega_B h^2/0.025)(1+z/200)\] (16)

Due to the loss of one free particle, recombination leads to a loss in kinetic energy, a contribution to the cooling function \( \Lambda \):

\[
\Lambda_r = (3/2)T_e t_{r}^{-1} = 2 \times 10^{-12} \frac{eV}{s} (T_e/\Delta)^{1/2} x^2 \left(\frac{\Omega_B h^2}{0.025}\right)\frac{1+z}{200}(1-x)(\Omega_B h^2/0.025)(1+z/200)\] (17)

**Collision:** Collisions of energetic electrons can lead to ionization and/or excitation of hydrogen. For high enough electron temperatures this is a very efficient cooling mechanism even if only a small fraction of neutral hydrogen is present. The collisional ionization rate is about [17, 21]

\[
t_{c}^{-1} = 6 \times 10^{-8} s^{-1} \times (T_e/\Delta)^{1/2} e^{-\Delta/T_e} x(1-x)\left(\frac{\Omega_B h^2}{0.025}\right)\frac{1+z}{200}(1-x)(\Omega_B h^2/0.025)(1+z/200)\] (18)

The collisional ionization cooling rate is

\[
\Lambda_c = 8 \times 10^{-7} \frac{eV}{s} (T_e/\Delta)^{1/2} e^{-\Delta/T_e} x(1-x)\left(\frac{\Omega_B h^2}{0.025}\right)\frac{1+z}{200}(1-x)(\Omega_B h^2/0.025)(1+z/200)\] (19)

Correspondingly, for the excitation cooling one obtains

\[
\Lambda_e = 10^{-6} \frac{eV}{s} e^{-3\Delta/4T_e} x(1-x)\left(\frac{\Omega_B h^2}{0.025}\right)\frac{1+z}{200}(1-x)(\Omega_B h^2/0.025)(1+z/200)\] (20)

Excitation cooling is thus always nearly \( 10^3 \) times faster than ionization cooling. Hence, the latter can be neglected.
As long as the electron temperature is substantially below 1 eV, excitations are not common enough to cool the plasma efficiently. In this situation Compton cooling of the CMB photons is the most efficient cooling mechanism.

**Compton cooling:** From the Kompaneets equation one obtains

\[
n_B \Lambda_{CMB} = \frac{T_e - T}{T} \frac{n_e \sigma_T}{\pi^2 m_e} \int_0^\infty \omega^4 f_{CMB} (f_{CMB} + 1) d\omega
\]

\[
\approx 10^{-10} eV x \left( \frac{T_e - T_{CMB}}{\Delta} \right) \left( \frac{1 + z}{200} \right)^4.
\]  

(21)

The corresponding heating process by compton scattering of the ionizing radiation can be neglected (since there are so much less ionizing photons than CMB photons).

Of course the plasma is also adiabatically cooled due to expansion and it is important to compare this with the cooling rates above. Furthermore, if the process of expansion is much faster than one of the processes calculated above, we can neglect the corresponding process in the expanding universe.

**Expansion:** In a matter dominated Friedmann universe with \( \Omega_{tot} = 1 \), the expansion rate is given by

\[
H \approx 4.6 \times 10^{-15} \text{s}^{-1} \times \left( \frac{h}{0.5} \right)^{3/2}.
\]  

(22)

Adiabatic cooling due to expansion amounts to a cooling rate of

\[
\Lambda_{exp} = (3/2)(1 + x) \left( \frac{dT_e}{dt} \right)_{ad} \approx 2 \times 10^{-13} eV x \left( \frac{T_e}{\Delta} \right) \left( \frac{1 + z}{200} \right)^3 \left( \frac{h}{0.5} \right).
\]

(23)

An additional possible cooling mechanism would be Bremsstrahlungs cooling but this turns out to be very small within the range of electron temperatures we are interested in.

### 3.2 The differential equations

The degree of ionization, \( x \), the electron temperature, \( T_e \), and the form of the ionizing spectrum, \( f_i \), are in principle determined by the following system of differential equations:

\[
\frac{dx}{dt} = -t^{-1}_r + t^{-1}_{pi} + t^{-1}_e
\]

(24)

\[
\frac{dT_e}{dt} = \frac{2}{a} T_e + \frac{2}{3(1 + x)} (\Gamma - \Lambda) - \frac{T_e}{1 + x} \frac{dx}{dt}
\]

(25)

\[
\frac{\partial f_i}{\partial t} - \frac{\dot{a}}{a} \frac{\partial f_i}{\partial \omega} = - \frac{df_i}{dt} \bigg|_{pi} + \frac{df_i}{dt} \bigg|_{rec} - \frac{df_i}{dt} \bigg|_{Kompaneets + source}.
\]

(26)

The first term on the right hand side of the second equation is adiabatic cooling due to expansion. The second term denotes the other heating and cooling mechanisms:

\[
\Gamma = \Gamma_{pi} , \quad \Lambda = \Lambda_r + \Lambda_e + \Lambda_{CMB}.
\]

The last term is due to the increase of independent particles by ionization which lowers the kinetic energy per particle and therefore the temperature. The third equation sketches the changes in the ionizing spectrum which are relevant at energies \( \omega \geq \Delta \). The second term on
the left hand side accounts for the redshift of photon energies due to expansion. The first term describes the loss of photons due to ionization. Then there comes the gain of photons by recombination, the energy losses due to Compton scattering off the lower temperature electrons and finally the source term which represents the emission of ionizing radiation from some unknown primordial objects. This term has to be guessed.

We have not yet managed to solve the system of differential equations (24,25,26) in some generality. We have just looked at the stable situation, \( \frac{dx}{dt} = \frac{dT_e}{dt} = \frac{df_i}{dt} = 0 \) and solved the resulting algebraic equations for \( x \) and \( T_e \) for a given ionization spectrum \( f_i \) which we parametrized like in Section 2 as a Planck spectrum with chemical potential. \( f_i \) is then fully determined by its temperature \( T_i \) and its chemical potential \( \mu \) or the parameter \( q \) which was introduced in Section 2. The resulting electron temperature and the degree of ionization are shown as functions of \( T_i \) in Figs. 6 and 7. Of course this is not a realistic situation since photoionization is so fast that it will quickly lead to a depletion of the spectrum above 13.6eV which is probably not readily refilled by a realistic source term. This leads to the very unphysical behavior that even for very small \( T_i \), i.e. very few ionizing photons, \( x \approx 1 \).

4 Results, Conclusions

Our preliminary results show that early reionization is still possible even though the parameter space is squeezed by the limit on the Compton \( y \) parameter and the absence of any near infrared background. The first fact tells us that the electron temperature cannot have been higher than 2eV for a substantial duration at high redshift, \( z > z_{\text{dec}} \). The second constraint reveals that, on the other hand, there was no ionizing uv–radiation present at \( z \approx 10 \) which might have kept the intergalactic medium ionized. If it was ionized at this redshift it thus was collisionally ionized, i.e., the electron temperature was above 1.5eV.

To weaken these constraints, one might still imagine the cosmic plasma to be photoionized up to a redshift \( z < z_{\text{dec}} \), say \( z \approx 50 \) and collisionally ionized later. But it is not clear if even this possibility remains, i.e. if it is possible to constantly photoionize the plasma without heating it above 1.5eV, say. To decide on this last possibility we have to fully solve the system of differential equations presented in Section 3.2 for some reasonable source models. In addition we have to take into account the clumping which must be present at a time when the first formed objects are supposed to emit ionizing radiation: Whenever a factor \( n_B^2 \) enters our equations (e.g. recombination and collisions) we have to replace the background Friedmann value of \( n_B^2 \) by

\[
    n_B^{(\text{clump})} = n_B^2(1 + \int \delta(x)^2 d^3x) = n_B^2(1 + \int P(k) dk),
\]

where \( \delta(x) \) denotes the density fluctuations in the baryon distribution and \( P(k) \) is the power spectrum, \( P(k) = (\hat{\delta}(k))^2 \).

Furthermore, if the primordial plasma is contaminated by metals from the first objects, these might contribute to the cooling rate substantially and thus influence the electron temperature. It is an important but difficult task to estimate this effect.
References

[1] A. Sandage, *Physics Today*, 34 (February 1970).

[2] J.E. Gunn & B.A. Peterson, *Ap. J.* **142**, 1633 (1965).

[3] A.C. Fabian & X. Barcons, *Rep. Prog. Phys.* **54**, 1069 (1991).

[4] P.J.E. Peebles *Large Scale Structure of the Universe*, Princeton University Press (1980).

[5] N. Turok, *Phys. Rev. Lett.* **63**, 2625 (1989).

[6] N. Turok & D.N. Spergel, *Phys. Rev. Lett.* **64**, 2736 (1990).

[7] R. Durrer, *Phys. Rev.* **D42**, 2533 (1990).

[8] D. Spergel, N. Turok, W. Press & B. Ryden, *Phys. Rev.* **D43**, 1038 (1991).

[9] R. Durrer, Fund. Cosmic Phys., in print (1993).

[10] R. Durrer, A. Howard & Z.–H. Zhou, *Phys. Rev.* **D48**, in print (1993).

[11] A. Gooding, D.N. Spergel & N. Turok *Ap. J.* **372**, L5 (1991).

[12] R.A. Sunyaev and Y.B. Zel’’dovich *Comm. Astrophys. Sp. Phys.* **4**, 173 (1973).

[13] R.A. Sunyaev & Ya.B. Zel’dovich, *Ann. Rev. Astron. Astrophys.* **18**, 537 (1980).

[14] G. Setti, in: *IAU Symp.* **139**, edt. Bowyer and Leinert, 345 (1990).

[15] F. Paresce & R. Stern, *Ap. J.* **247**, 89 (1981).

[16] J.B. Holberg *Ap. J.* **311**, 969 (1986).

[17] M.S. Longair, in: *IAU Symp.* **139**, edt. Bowyer and Leinert, 469 (1990).

[18] G.B. Rybicki & A.P. Lightman, *Radiative Processes in Astrophysics*, Wiley & Sons, New York (1979).

[19] L. Spitzer, *Physical Processes in the Interstellar Medium*, Wiley & Sons, New York (1978).

[20] W. Lotz, *Ap. J. Suppl.* **129**, 207 (1967).

[21] K.R. Lang, *Astrophysical Formulae*, Springer, Berlin (1974).

[22] P. Ehrismann, Diploma thesis, Zürich University (1992).
Figure Captions

Fig. 1  The induced microwave anisotropy for a texture collapsing at $t_c = 7.6$ is shown in the case of a reionized universe (full line) and a not ionized universe (dashed line) as a function of the impact time $\tau$. In the units chosen, the decoupling time is $t_{dec} = 74$, so that in this case $z_c/z_{dec} \sim 90$. The damping in this case is approximately a factor of 15 and the damped signal is also widened to a width of approximately $t_{dec}$.

Fig. 2  Like Fig. 1 but for $t_c = 20$, thus $z_c/z_{dec} \sim 13$. The damping factor is approximately 5 in this case.

Fig. 3  Like Fig. 1 but for $t_c = 40$, thus $z_c/z_{dec} \sim 3$. The damping factor is approximately 2 in this case.

Fig. 4  Like Fig. 1 but for $t_c = 100 > t_{dec}$. No damping occurs in this case. (The two curves overlay.)

Fig. 5  The limiting $\rho_i/\rho_B = q$ is given in units of $10^{-7}$ which leads to enough ($\geq n_B$) ionizing photons to reionize the universe for a given radiation temperature $T_i$ (plotted in units of the ionization energy $\Delta = 13.6eV$, heavy line. This limit is compared with the limits on background radiations at different wavelengths given in Table 1 (crosses connected with light lines). The lower limit applies if the ionizing radiation was emitted until today, $z = 0$. The upper limit applies if the emission of ionizing radiation seized at $z = 10$. The remaining dashed region gives the allowed range of parameters in the universe was photoionized until $z = 10$.

Fig. 6  The plasma temperature $T_e$ is given as a function of the temperature of the ionizing radiation for $q = 10$ and $f_i$ a blackbody spectrum with chemical potential for $z = 200$ and a static solution of the equations. This figure was produced for his diploma thesis by P. Ehrismann [22].

Fig. 7  The degree of ionization $x$ is given as a function of the temperature of the ionizing radiation for $q = 10$ and $f_i$ a blackbody spectrum with chemical potential for $z = 200$ and a static solution of the equations. The value $x = 1$ for small $T_i$ is unphysical. It is due to the staticity assumption. This figure was produced for his diploma thesis by P. Ehrismann [22].