Application of the finite volume method for the standard ballistic model aerodynamics calculations

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Abstract. The present paper contains the results of the aerodynamics and gas dynamics calculations of HB–1 standard ballistic model performed by the UST3D code. There were the following test cases for the numerical simulations. At first, the computations were conducted for the different Mach numbers at the various angles of attack. At second, the effect of the tetrahedron grid elements number on the flow field and aerodynamics coefficients was estimated. At last, the different approaches to the decay of discontinuity investigation were applied. The comparisons between the experimental data and results of calculations are also presented.

1. Introduction

The standard ballistic models are simple configurations which have been accepted by the Supersonic Tunnel Association International (STAI) for wind tunnel flow quality estimation. This fact contributed to a comprehensive aerodynamics and heat transfer experimental investigations of these shapes in different research centers. The significant amount of the standard models computational and experimental studies have already been conducted and published over several decades. The broad range of the computer codes validation and verification tasks can be solved by the mentioned data [1–4].

The UST3D code has got its own background of the computational studies concerning compressible viscous gas flow over a complex three-dimensional bodies. There are three directions of development of the present computer code. The most considerable of them is related to the three-dimensional computational aerodynamics. The last papers were devoted to the aerodynamics characteristics definition for the different types of hypersonic vehicles, such as: X–33 and X–34 [5], X–43 [6], X–51 [7]. Expert descent space vehicle [8] and also different shapes of hypersonic waveriders [9, 10]. The simplest configurations (sphere, blunted cone and cylinder) were also considered in [5]. The second direction affects the problems of the scramjet gas dynamics accounting the mass and energy sources effects. In particular, the estimation of combustion chamber cold gas injection influence on the flow parameters in the dual mode scramjet [11]. Another important question was a determination of the aerodynamics characteristics and gas dynamics of the perspective high-speed scramjet vehicle taking into consideration energy deposition in combustion chamber [10]. It should be noted that the mentioned papers contain the results of calculations which were obtained within the framework non-reacting perfect gas model and didn’t include the questions concerning chemical reactions and radiative heat transfer in the scramjet. The more detailed investigations of this
task including also turbulence effects were presented in [12–19]. Finally, AUSM scheme was implemented in UST3D code for study of the decay of arbitrary discontinuity [20].

2. Governing equations and numerical method

The system of governing equations includes the continuity equation, the Navier-Stokes equations and the energy equation which were presented in vector form (1).

\[
\frac{\partial}{\partial t} \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \rho E \end{pmatrix} + \frac{\partial}{\partial x} \begin{pmatrix} \rho u + \rho uu + p \\ \rho u v \\ \rho u w \\ \rho uE + \rho uu + p \\ \rho E \end{pmatrix} + \frac{\partial}{\partial y} \begin{pmatrix} \rho v + \rho vv + p \\ \rho u v \\ \rho v w \\ \rho vE + \rho vv + p \\ \rho E \end{pmatrix} + \frac{\partial}{\partial z} \begin{pmatrix} \rho w + \rho ww + p \\ \rho u w \\ \rho v w \\ \rho wE + \rho ww + p \\ \rho E \end{pmatrix} = \begin{pmatrix} 0 \\ \tau_{xx} \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yy} + \nu \tau_{xy} + \nu \tau_{yz} + \tau_{zz} \end{pmatrix}
\]

(1)

The three-dimensional flow about blunted bodies is considered within the framework of the perfect gas model. The thermal and calorific perfect gas state equations (2), (3) with the specific full energy relation (4) and the Fourier law (5) closes system of equations mentioned above

\[
p = (\gamma - 1) \rho U = (\gamma - 1) \rho \left[ E - 0.5 \left( u^2 + v^2 + w^2 \right) \right]
\]

(2)

\[
U = c_v T
\]

(3)

\[
E = \frac{p}{\gamma - 1} \rho + \frac{u^2 + v^2 + w^2}{2}
\]

(4)

\[
q = -\lambda \nabla T
\]

(5)

In the listed equations \( \rho \) – density; \( u, v, w \) – velocity vector projections; \( x, y, z \) – Cartesian coordinates; \( p \) – pressure; \( E \) – specific full energy; \( \lambda \) – thermal conductivity; \( T \) – temperature; \( \mu \) – dynamic viscosity; \( \gamma \) – ratio of specific heats; \( U \) – specific internal energy; \( c_v \) – specific heat capacity. The components of the viscous tensor are given as

\[
\tau_{xx} = \mu \left( \frac{4}{3} \frac{\partial u}{\partial x} - \frac{2}{3} \frac{\partial v}{\partial y} - \frac{2}{3} \frac{\partial w}{\partial z} \right), \quad \tau_{xy} = \tau_{yx} = \mu \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)
\]

\[
\tau_{yy} = \mu \left( \frac{4}{3} \frac{\partial v}{\partial y} - \frac{2}{3} \frac{\partial u}{\partial x} - \frac{2}{3} \frac{\partial w}{\partial z} \right), \quad \tau_{yz} = \tau_{zy} = \mu \left( \frac{\partial w}{\partial x} + \frac{\partial v}{\partial z} \right)
\]

\[
\tau_{zz} = \mu \left( \frac{4}{3} \frac{\partial w}{\partial z} - \frac{2}{3} \frac{\partial u}{\partial x} - \frac{2}{3} \frac{\partial v}{\partial y} \right), \quad \tau_{zx} = \tau_{xz} = \mu \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)
\]

(6)
The UST3D code is founded on the finite volume approach to the Navier-Stokes equations integration. One of the variants of method for splitting into the physical processes [21] have been used in the computational process. There were three steps of the full gas dynamics system of equations solving. The first step implies the convective effects ignoring. The conservative variables variation occurs by the pressure influence. The mass, momentum and energy fluxes calculate on the second step. The integral conservation law are used for gas dynamics parameters final values definition on the third step.

Initially the donor cells method [22] was applied for the decay of discontinuity problem solving. This technique assumes the definition of the gas dynamics parameters on the conjugate face by simple relation

\[ f_{1/2} = \frac{1}{2}(f_L + f_R) \]

Here \( f_{1/2} \) – parameter value on the conjugate face between current and adjacent elements; \( f_L \) – parameter value corresponding to the current element; \( f_R \) – parameter value corresponding to the element bordering with current across conjugate face. Some details are shown in figure 1.

Another approach to the numerical fluxes calculation also using splitting is the AUSM scheme [23]. The foundation of the method can be explained by example of the one-dimensional Riemann problem. The flux vector can be divided on the convective part and pressure part

\[ f_c = M \begin{pmatrix} \rho c \\ \rho uc \\ (E + p)c \end{pmatrix}, \quad f_p = \begin{pmatrix} 0 \\ p \\ 0 \end{pmatrix} \]

Here \( M \) – Mach number, \( c \) – sound velocity. The convective flux splitting is implemented by means of Mach number separation

\[ M_{1/2} = M_L^+ + M_R^- \]

The components of above relation can be defined as (accounting current or neighbor element)
\[ M^\pm_{L,R} = \begin{cases} \frac{1}{2} (M \pm |M|), & \text{if } |M| > 1 \\ \pm \frac{1}{4} (M \pm 1)^2, & \text{if } |M| \leq 1 \end{cases} \]  

(10)

These relations were proposed by Van-Leer and obtained accounting several assumptions among which: functions for Mach number splitting should be continuously differentiable, monotonically increasing and also correspond to the upwind scheme.

The pressure splitting procedure is similar

\[ p_{y/2} = p^L + p^R \]  

(11)

\[ p^\pm_{L,R} = \begin{cases} \frac{p}{2} (M \pm |M|), & \text{if } |M| > 1 \\ \frac{p}{4} (M \pm 1)^2 (2 \mp M), & \text{if } |M| \leq 1 \end{cases} \]  

(12)

Thus one dimensional numerical fluxes can be presented as

\[ f^{y/2}_c = M_{y/2} \begin{pmatrix} \rho c \\ \rho u c \\ (E + p)c \end{pmatrix}, \quad f^{y/2}_p = \begin{pmatrix} 0 \\ p^L + p^R \\ 0 \end{pmatrix} \]  

(13)

The mentioned AUSM scheme was implemented within the framework the method for splitting into the physical processes. The present work also contains results concerning applying of the AUSM scheme modification known as AUSM+ scheme [24] which uses high-order polynomials for the Mach number and pressure components

\[ M^\pm_{L,R} = \begin{cases} \frac{1}{2} (M \pm |M|), & \text{if } |M| > 1 \\ \pm \frac{1}{4} (M \pm 1)^2 \pm \frac{1}{8} (M^2 - 1)^2, & \text{if } |M| \leq 1 \end{cases} \]  

(14)

\[ p^\pm_{L,R} = \begin{cases} \frac{p}{2} (M \pm |M|), & \text{if } |M| > 1 \\ \frac{p}{4} (M \pm 1)^2 (2 \mp M) \pm \frac{3p}{16} M (M^2 - 1)^2, & \text{if } |M| \leq 1 \end{cases} \]  

(15)

3. Numerical simulation results

The first part of the calculations were performed for the HB-1 standard ballistic model using tetrahedron grid including 2.4 millions of elements. The model dimensions and computational grid are shown in figures 2–3. The free-stream parameters corresponded to the hypersonic wind tunnels conditions from Arnold Engineering Development Center (AEDC) [1]. The diameter of the model was variant for various Mach number. The computations results were obtained for angle of attack range from −1° to 12°. The free-stream conditions are presented in the table 1. The donor-cells approach was employed to the decay of discontinuity problem solving.
Table 1. Free-stream conditions for the various Mach number.

|                  | $M_{\infty} = 3.01$ | $M_{\infty} = 8.09$ | $M_{\infty} = 10.16$ |
|------------------|---------------------|---------------------|---------------------|
| $p_{\infty}$, erg/cm$^3$ | 94000               | 2280                | 3230                |
| $\rho_{\infty} \cdot 10^3$, g/cm$^3$ | 0.297               | 0.023               | 0.018               |
| $T_{\infty}$, K  | 110.3               | 34.1                | 60.                 |
| $Re_{\infty} \cdot 10^6$ | 2.5                 | 2.1                 | 1.4                 |
| $d$, cm          | 10.16               | 19.05               | 19.05               |

Figure 2. HB-1 model dimensions.

Figure 3. Computational tetrahedron grid (~ 2.4 mln. Elements).

Figures 4 (a) and 4 (b) show the normal force and axial force coefficients which depend on the angle of attack. The variation of aerodynamics force coefficients have a linear behavior. There is a significant lift force decreasing at Mach number increasing. It becomes a more obvious at high angle of attack. The axial force has a weak dependence on the ballistic model orientation with respect to the free-stream. Some discrepancy between results of the calculations and experimental data for axial force prediction can be explained by the fact that there is no the estimation of surface skin friction. There is a noticeable distinction between the experimental pitching-moment coefficient distribution and numerical simulation data (see figure 4 (c)). The moment graph non-monotony is more evident at high angles of attack. The calculations results at low Mach number ($M_{\infty} = 3.01$) demonstrate most acceptable accordance with experimental data except the maximum angle of attack ($\alpha = 12^\circ$). In other cases, there is a pitching-moment coefficient increasing with angle of attack increasing at the higher...
Mach number. Thus, center of pressure location with respect to the gravity center is constant in contradistinction to the experimental data.

Figures 5 and 6 demonstrate non-dimensional density distribution about HB–1 surface for different Mach number at maximum angle of attack. There is the non-symmetric flow field with forebody shock layer and local rarefied zone about base plane and upper surface. At $M_\infty = 3.01$ shock detaching is lower relative to the maximum Mach number. The density distribution on the upper surface is non-uniform in the first case. It can particularly explain pitching-moment differences between computations and experimental data.

The flow field in the recirculation zone presents in figure 7. There are three meshes (2.4, 4.8 and 18 mln. tetrahedrons) with maximum attraction to the surface of the model. It should be noted that the vortex structures about base plane were observed only for low speed ($M_\infty = 3.01$). The quantity of the mesh elements has a slight effect on configuration of eddies, but there is some decreasing of the axial force for more detailed grids (see table 2).

Figure 4. Normal force (a) and forebody axial force (b) coefficients as well as pitching-moment coefficient (c) as functions of attack angle.
Figure 5. Non-dimensional density distribution in symmetry cross-section for $M_\infty = 3.01$ at $\alpha = 12^\circ$.

Figure 6. Non-dimensional density distribution in symmetry cross-section for $M_\infty = 10.16$ at $\alpha = 12^\circ$.

Figure 7. Longitudinal velocity distribution (at $M_\infty = 3.01$) and streamlines within the base surface for different computational grids: 2.4 mln. (a), 4.8 mln. (b) and 18 mln. (c).

The last question was a comparison between the different approaches to the arbitrary decay of discontinuity numerical simulation within the framework the method for splitting into the physical processes. The figures 8 and 9 describe the distribution of non-dimensional density and temperature along critical streamline in symmetry plane. There is a shock wave smearing which occurs both when the coarse grid is applied and when numerical method without preliminary distinguishing of the shock wave is used. The temperature at the critical point matches with stagnation temperature obtained from isentropic flow relations.
Figure 8. Non-dimensional density distribution in longitudinal direction for $M_\infty = 3.01$ (2.4 mln).

Figure 9. Temperature distribution in longitudinal direction for $M_\infty = 3.01$ (2.4 mln)

Table 2. Axial force coefficient $\alpha = 0^\circ$ for the different grids at the various Mach number.

| Mach Number | 2.4 mln. | 4.8 mln. | 18 mln. |
|-------------|----------|----------|---------|
| $M_\infty = 3.01$ | 0.493 | 0.486 | 0.485 |
| $M_\infty = 8$ | 0.467 | 0.462 | - |
| $M_\infty = 10.16$ | 0.466 | 0.460 | - |

4. Conclusions
The computational study of the HB-1 standard ballistic model aerodynamics was performed by the different UST3D code modifications. The acceptable agreement with experimental data was observed for all test cases. The results of the calculations concerning different approaches to the arbitrary decay of discontinuity problem solution were also presented.

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References
[1] Gray J D and Lindsay E E 1963 Force Tests of Standart Hypervelocity Ballistic Models HB–1 and HB–2 at Mach to 10 Arnold Engineering Development Center (AEDC) TDR-63-137
[2] Gray J D 1964 Summary Report on Aerodynamic Characteristics of Standart Models HB–1 and HB–2 Arnold Engineering Development Center (AEDC) TDR-64-137
[3] Shigeru Kuchi-Ishi, Shigeya Watanabe, Shinji Nagai, Shoichi Tsuda, Tadao Koyama, Noriaki Hirabayashi, Hideo Sekine, and Koichi Hozumi 2005 Comparative Force/Heat Flux Measurements between JAXA Hypersonic Test Facilities Using Standard Model HB–2 (Part 1: 1.27 m Hypersonic Wind Tunnel Results) JAXA Research and Development Report
[4] Adamov N P, Vasenev L G, Zvegintsev V I, Mazhul I I, Nalivaichenko D G, Novikov A V, Kharitonov A M and Shpak S I 2006 Characteristics of the AT-303 Hypersonic Wind Tunnel, Part 2. Aerodynamics of the HB–2 Reference Model Thermophysics and
Aeromechanics 13 (2) pp 157–171

[5] Surzhikov S T 2017 Validation of computational code UST3D by the example of experimental aerodynamic data J. of Physics: Conf. Series 815 012023

[6] Zheleznyakova A L and Surzhikov S T 2013 Application of the Method of Splitting by Physical Processes for the Computation of a Hypersonic Flow over an Aircraft Model of Complex Configuration High Temperature 51 (6) pp 816–829

[7] Zheleznyakova A L 2014 Numerical simulation of hypersonic external flow around model of vehicle X–51 Physical-Chemical Kinetics in Gas Dynamics 15 (2) pp 1–8 (in Russian) http://chemphys.edu.ru/issues/2014-15-2/articles/218/

[8] Kharchenko N A and Kryukov I A 2018 Aerothermodynamics calculations of the EXPERT reentry flight vehicle J. of Physics: Conf. Series 1009 012004

[9] Yatsukhno D S 2017 Numerical simulation of the flow over a hypersonic waverider using the method for splitting into physical processes J. of Physics: Conf. Series 815 012022

[10] Yatsukhno D S 2018 Computational study of the waverider aerothermodynamics by the UST3D computer code J. of Physics: Conf. Series 1009 012002

[11] Zheleznyakova A L 2014 Application of the method of splitting by physical processes for the 3D computation of flows in dual-mode scramjet combustor Physical-Chemical Kinetics in Gas Dynamics 15 (2) pp 1–9 (in Russian) http://chemphys.edu.ru/issues/2014-15-2/articles/217/

[12] Surzhikov S T and Shang J S 2013 Numerical Prediction of Convective and Radiative Heating of Scramjet Combustion Chamber with Hydrocarbon Fuels 51st Aerospace Sciences Meeting including the New Horizons Forum and Aerospace Exposition AIAA Paper 2013–1076

[13] Surzhikov S T and Shang J S 2013 Radiative Heat Exchange in a Hydrogen-Fueled Scramjet Combustion Chambers 51st Aerospace Sciences Meeting including the New Horizons Forum and Aerospace Exposition AIAA Paper 2013–0448

[14] Surzhikov S T, Zheleznyakova A L, Shang J S and Rivir R B 2013 Simulating Gasdynamic Interaction and Radiative Heating within Scramjets with Hydrocarbon Fuels 44th AIAA Thermophysics Conference AIAA Paper 2013–2642

[15] Seleznev R K 2018 History of scramjet propulsion development J. of Physics: Conf. Series 1009 012028

[16] Seleznev R K 2018 Numerical study of the flow structure in the supersonic inlet-isolator J. of Physics: Conf. Series 1009 012034

[17] Seleznev R K 2018 Validation of two-dimensional model by the example of a supersonic inlet-isolator J. of Physics: Conf. Series 1009 012030

[18] Seleznev R K 2018 Validation of 3D model by the example of a supersonic inlet-isolator J. of Physics: Conf. Series 1009 012031

[19] Seleznev R K, Surzhikov S T and Shang J 2019 A review of the scramjet experimental data base Progress in Aerospace Sciences 0376–0421

[20] Zheleznyakova A L, Surzhikov S T 2014 Calculation of a Hypersonic Flow over Bodies of Complex Configuration on Unstructured Tetrahedral Meshes Using the AUSM Scheme. High Temperature 52 (2) pp 274–284

[21] Belotserkovsky O M and Davydov Yu M 1982 Method of large particles in gas dynamics. (Moscow: Science) p 392 (in Russian)

[22] Jentry R A Martin R E and Daly B J 1966 An Eulerian Differencing Method for Unsteady Compressible Flow Problems J. of Computational Physics 1 (1) pp 87–118

[23] Liou M-S and Steffen C J Jr 1993 A New Flux Splitting Scheme J. of Computational Physics 107 pp 23–39

[24] Liou M-S 1996 A Sequel to AUSM: AUSM+. J. of Computational Physics 129 pp 364–382