Nuclear dependence asymmetries in direct photon production

Hong Shen$^{a,b}$ and Ma Luo$^{a,b}$

$^a$CCAST(World Lab.), P.O.Box 8730, Beijing 100080, China
$^b$Department of Physics, Nankai University, Tianjin 300071, China

Abstract

We study the nuclear dependences of high-$p_T$ jet cross sections in one photon and one jet production in proton-nucleus collisions. We find that there exist asymmetries between the outgoing jets and photons. A convincing reason responsible for those asymmetries are demonstrated in perturbative QCD. Significant nuclear enhancements are also found in the inclusive jet cross sections.

Anomalous nuclear dependences are observed in hadron-nucleus scatterings \cite{1}, as well as in some extensive experimental data \cite{2, 3}. It is shown that the inclusive cross sections for single high-$p_T$ particles rise rather faster than proportional to the atomic number $A$ of the nuclear target, while a linear $A$ dependence is naively expected in single scattering model. This behavior is often described as due to multiple scatterings of the partons in the nuclear matter, primarily due to double scatterings \cite{4, 5, 6, 7}.

In ref.\cite{5}, M. Luo, J. Qiu and G. Sterman showed that the anomalous nuclear enhancements in deeply inelastic scatterings and photoproductions could be described naturally in perturbative QCD, in term of a higher twist formalism. They presented the details of the use of factorization at higher twist to describe multiple scattering in nuclei. The contribution from

\footnotesize

$^1$Supported in part by National Natural Science Foundation of China
$^2$email address: songte@public1.tpt.tj.cn
multiple scattering was factorized into a calculable short-distance partonic part multiplied by a multiparton correlation function. The double scattering must have one hard scattering to produce the high-$p_T$ observables along with another scattering which, in principle, could be either soft or hard. It was also pointed that the one hard plus one soft scatterings would dominate the nuclear dependences. Moreover the soft scattering could occur either before or after the hard scattering (referred to below as initial-state or final-state multiple scattering). The initial-state multiple scatterings, however, are absent in deeply inelastic scattering and photoproduction because of no direct interaction between photons and gluons, therefore only the final-state multiple scatterings were discussed in ref.[5].

In order to study the contributions from the initial-state multiple scatterings, X. Guo and J. Qiu extended the higher twist perturbative QCD treatment to high-$p_T$ direct photon productions in hadron-nucleus scatterings [6]. The direct photon productions provide excellent tests of initial-state multiple scatterings, since the final-state multiple scatterings are simply ruled out due to the fact that none of gluons from the nuclear target could interact with the final photon. They calculated the nuclear dependences of the direct photon inclusive cross sections in hadron-nucleus collisions. Small nuclear enhancements contributed from initial-state multiple scatterings were obtained and discussed in ref.[6], which were also compared with the data from Fermilab experiment E706 [3].

In this paper, we work on the same reaction ($P + A \rightarrow \gamma + jet + X$). We concentrate, however, on the inclusive cross section of the final jet instead of the direct photon. We demonstrate that the final jet receives contributions from both the initial-state and final-state multiple scatterings, while the direct photon receives contribution from the initial-state multiple scattering only, which results in the asymmetry between the distributions of the final jet and the direct photon. We calculate the final jet inclusive cross section, and reproduce the direct photon inclusive cross section which has been derived in ref.[6]. In order to compare with ref.[5] and [6], we adopt the same multiple correlation functions and similar notations for the formalism in the following discussion.

The process we study here is that one photon and one jet with high-$p_T$ produced in proton-nucleus collision, $P + A \rightarrow \gamma + jet + X$. The inclusive cross section for either the final jet or the
Direct photon can be expressed as a sum of contributions from single and double scatterings, while contributions from triple and even higher multiple scatterings are ignored,

\[ E_l \frac{d\sigma_i}{d^3l} = E_l \frac{d\sigma_i^{(S)}}{d^3l} + E_l \frac{d\sigma_i^{(D)}}{d^3l}, \quad (1) \]

where we use the index \( i \) to identify the inclusive cross section in which the \( i \)-particle (\( i = \text{jet} \) or \( \gamma \)) is detected with momentum \( l \), the superscripts \( (S) \) and \( (D) \) represent the single and double scatterings, respectively. In this paper we emphasize the difference between the inclusive cross sections \( E_l \frac{d\sigma_{\text{jet}}(l)}{d^3l} \) and \( E_l \frac{d\sigma_{\gamma}(l)}{d^3l} \), and its anomalous nuclear enhancement.

The single scattering cross section is proportional to \( A \) as

\[ E_l \frac{d\sigma_i^{(S)}}{d^3l} = A \sum_{a,b} \int dx' f_{a/P}(x') \int dx f_{b/N}(x) \delta \left( x - \frac{-xt}{xs + u} \right) \times \alpha_{em} \alpha_s \left( \frac{1}{s} \right) \left( \frac{1}{x's + u} \right) |\bar{M}_{ab \rightarrow \gamma \text{jet}}|^2, \quad (2) \]

where \( f_{a/P}(x') \) and \( f_{b/N}(x) \) are regular parton distribution functions with the \( P \) incident proton and the \( N \) effective nucleon in the nuclear target, the \( a \) and \( b \) run over all gluon, quark, and antiquark flavors by \( \sum_{a,b} \). The matrix elements of the subprocesses in eq.(2) are given by

\[ |\bar{M}_{gg \rightarrow \gamma q}|^2 = e_q^2 \left( \frac{1}{6} \right) 2 \left( \frac{-\hat{u}}{\hat{s}} + \frac{\hat{s}}{-\hat{u}} \right), \quad (3) \]

\[ |\bar{M}_{gg \rightarrow \gamma q}|^2 = e_q^2 \left( \frac{1}{6} \right) 2 \left( \frac{-\hat{t}}{\hat{s}} + \frac{\hat{s}}{-\hat{t}} \right), \quad (4) \]

\[ |\bar{M}_{qg \rightarrow \gamma q}|^2 = e_q^2 \left( \frac{4}{9} \right) 2 \left( \frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} \right), \quad (5) \]

where \( e_q \) is the fractional quark charge, and the invariants \( \hat{s}, \hat{t}, \) and \( \hat{u} \) are

\[ \hat{s} = x'x(p' + p)^2, \quad \hat{t} = x'(p' - l)^2, \quad \hat{u} = x(p - l)^2, \quad (6) \]

with the \( p' \) and \( p \) are the momenta of the incident proton and the effective nucleon in nuclear target, respectively. For the single scattering term in eq.(1) we simply have

\[ E_l \frac{d\sigma_{\text{jet}}^{(S)}}{d^3l} = E_l \frac{d\sigma_{\gamma}^{(S)}}{d^3l}, \quad (7) \]

and the asymmetry of the \( E_l \frac{d\sigma_i^{(D)}}{d^3l} (i = \text{jet or } \gamma) \) majorly comes from their double scattering terms, \( E_l \frac{d\sigma_i^{(D)}}{d^3l} \).
The double scattering cross section of the final jet, \( E_l \frac{d\sigma^{(D)}_{\text{jet}}(l)}{d^3l} \), is contributed from both final-state and initial-state interactions,

\[
E_l \frac{d\sigma^{(D)}_{\text{jet}}(l)}{d^3l} = E_l \frac{d\sigma^{(D-f)}_{\text{jet}}(l)}{d^3l} + E_l \frac{d\sigma^{(D-i)}_{\text{jet}}(l)}{d^3l},
\]

while the double scattering cross section of the direct photon receives contribution from the initial-state interaction only \( 9 \),

\[
E_l \frac{d\sigma^{(D)}(l)}{d^3l} = E_l \frac{d\sigma^{(D-i)}(l)}{d^3l}.
\]

Since the initial-state interaction occurs before the hard scattering, which produce the final jet and the direct photon equally, the initial-state interaction term in the double scattering cross section of jet, \( E_l \frac{d\sigma^{(D-i)}_{\text{jet}}(l)}{d^3l} \), has the same formalism as that of the direct photon, \( E_l \frac{d\sigma^{(D-i)}(l)}{d^3l} \), which has been given in ref.[6] in detail. As a result, in the following discussion we only need to work on the derivation of the term \( E_l \frac{d\sigma^{(D-f)}_{\text{jet}}(l)}{d^3l} \).

A regular factorization procedure can separate the incident proton part out, and turn the jet cross section to

\[
E_l \frac{d\sigma^{(D-f)}_{\text{jet}}(l)}{d^3l} = \sum_a \int dx' f_a(p'(x')) E_l \frac{d\sigma^{(D-f)}_{aA\rightarrow\text{jet}}(x', l)}{d^3l}.
\]

In this topic the cross sections \( E_l \frac{d\sigma^{(D-f)}_{aA\rightarrow\text{jet}}(x', l)}{d^3l} \) can be classified in three groups shown in fig.1(1)-fig.1(3). As a result of collinear expansion \( 3, 3 \), each individual sort of these cross sections in turn can be decoupled to a product of a hard process cross section multiplied by a corresponding four operator matrix element, such as for fig.1(1),

\[
E_l \frac{d\sigma^{(D-f)}_{aA\rightarrow\text{jet}}(x', l)}{d^3l} = -\frac{1}{16x's} \int \frac{dy_1}{2\pi} \frac{dy_2}{2\pi} \frac{dy_3}{2\pi} \langle A|\bar{\psi}_q(0)\gamma^+ F_{\perp}(y_2) F_+(y\gamma)\psi(y_1)|A\rangle
\]

\[
\times \frac{\partial^2}{\partial k_\perp \partial k_\perp} \left\{ \int dx dx' d_k e^{ixp\cdot y_1} e^{ixp\cdot y_2} e^{-i(x_k-x_{k'}) p\cdot y_1} \right\} |_{k_\perp=0},
\]

where the partonic hard scattering function \( H_{a\beta}^{(1)}(x', x, x_k, x_{k'}, k_\perp, l) \) is specified by the lower part in fig.1(1); similarly, for fig.1(2),

\[
E_l \frac{d\sigma^{(D-f)}_{aA\rightarrow\text{jet}}(x', l)}{d^3l} = -\frac{1}{16x's} \int \frac{dy_1}{2\pi} \frac{dy_2}{2\pi} \frac{dy_3}{2\pi} \langle A|A^\perp(0)\gamma^+ F_{\perp}(y_2) F_+(y\gamma)\gamma\cdot p p^\alpha p^\beta|A\rangle
\]

\[
\times \frac{\partial^2}{\partial k_\perp \partial k_\perp} \left\{ \int dx dx' d_k e^{ixp\cdot y_1} e^{ixp\cdot y_2} e^{-i(x_k-x_{k'}) p\cdot y_1} \right\} |_{k_\perp=0};
\]
and for fig.1(3),
\[ E_{1} \sigma^{(D-f)(3)}_{AA \rightarrow jet}(x', l) = - \frac{1}{16x's} \int \frac{dy_{1}^{+}}{2\pi} \frac{dy_{2}^{-}}{2\pi} \frac{dy_{2}^{-}}{2\pi} \langle A|\bar{\psi}_{q}(0)\gamma^{+}F_{+\perp}(y_{2}^{-})F_{+\perp}(y_{2}^{-})\psi_{q}(y_{1}^{-})|A \rangle \]
\[ \times \frac{\partial^{2}}{\partial k_{\perp} \partial k_{\perp}} \left\{ \int dx_{k} dx_{k'} e^{ix_{k}p^{+}y_{1}^{-}} e^{ix_{k'}p^{+}y_{2}^{-} - i(x_{k} - x_{k'})p^{+}y_{2}^{-}} \right. \]
\[ \times Tr \left[ H^{(3)}_{\alpha\beta}(x', x, x_{k}, x_{k'}, k_{\perp}, l)\gamma^{\cdot}p \cdot p^{a}p^{a} \right] \left|_{k_{\perp} = 0} \right. \]

Apparently, two more extra gluons occur in the double scattering as compared with single scattering case. They go through the four operator matrix element to form a virtual loop, so that the variables \( x_{k} \) and \( x_{k'} \) carried by these extra gluons, in principle, can run over from \(-\infty \) to \(+\infty \). In order to do the integrals \( \int_{-\infty}^{+\infty} dx_{k} dx_{k'} \) we extend the \( x_{k} \) and \( x_{k'} \) to their complex planes, then by using the contour integral technic carry the integrals \( \int_{-\infty}^{+\infty} dx_{k} dx_{k'} \) out. As a result, the \( x_{k} \) and \( x_{k'} \) are fixed at zero. The vanish \( x_{k} \) and \( x_{k'} \) in turn set the relation,
\[ e^{ix_{k}p^{+}y_{1}^{-}} e^{-i(x_{k} - x_{k'})p^{+}y_{2}^{-}} = 1. \]

In this way, we eventually factorize the four operator matrix elements out of the convolution along with the partonic hard scattering functions \( H^{(i)}_{\alpha\beta} \) with \( i = 1 - 3 \). These factorized four operator matrix elements are responsible for the twist-4 partonic distribution functions, they are read as: for fig.1(1) and fig.1(3) we have
\[ T_{q}(x, A) = \int \frac{dy_{1}^{+}}{2\pi} \frac{dy_{2}^{-}}{2\pi} \frac{dy_{2}^{-}}{2\pi} e^{ix_{k}p^{+}y_{1}^{-}} Tr \left[ \langle A|\bar{\psi}_{q}(0)\gamma^{+}F_{+\perp}(y_{2}^{-})F_{+\perp}(y_{2}^{-})\psi_{q}(y_{1}^{-})|A \rangle \theta(y^{-} - y_{1}^{-})\theta(y_{2}^{-}) \right], \]
\[ \text{for fig.1(2),} \]
\[ T_{g}(x, A) = \int \frac{xp^{+}dy_{1}^{+}}{2\pi} \frac{dy_{2}^{-}}{2\pi} \frac{dy_{2}^{-}}{2\pi} e^{ix_{k}p^{+}y_{1}^{-}} \langle A|A^{1}(0)F_{+\perp}(y_{2}^{-})F_{+\perp}(y_{2}^{-})A_{\perp}(y_{1}^{-})|A \rangle \theta(y^{-} - y_{1}^{-})\theta(y_{2}^{-}), \]
\[ \text{where the factor} \theta(y^{-} - y_{1}^{-})\theta(y_{2}^{-}) \text{results from the contour integrals} \left[ \int \int \right]. \text{Since the factor} \theta(y^{-} - y_{1}^{-})\theta(y_{2}^{-}) \text{can not confines the integral} \int_{-\infty}^{+\infty} dy^{+} \text{going to infinity, extra contributions proportional to the nuclear size, or} \ A^{1/3} \text{, occur in these double scattering processes, which physically result in the anomalous nuclear enhancements} \left[ \int \int \right]. \text{Alongside figuring these long distance partonic distributions out, we obtain the double scattering cross sections,} \]
contributions, cut left and right, to construct a restriction the rest part of the cut middle contribution in each diagram combine all of the other cut. It may be noticed that there are three cuts in each diagram in fig. 1. The long distance partonic effect, hence do not contribute the nuclear enhancement.

\[ E_l \frac{d\sigma^{(D-f)}_{aA \rightarrow \text{jet}}(x', l)}{d^3l} = \alpha_{em}(4\pi\alpha_s)^2 e_q^2 \left( \frac{1}{2x's} \right) \left( \frac{1}{x's + u} \right) 2H_q \]
\[ \times \left\{ \frac{\partial^2}{\partial x^2} \left( \frac{T_q(x, A)}{x} \right) \right\} \left( \frac{x's l_T}{u(x's + u)} \right)^2 + \left[ \frac{\partial}{\partial x} \left( \frac{T_q(x, A)}{x} \right) \right] \left( \frac{-x's}{u(x's + u)} \right) \right\} \] (17)

\[ E_l \frac{d\sigma^{(D-f)}_{aA \rightarrow \text{jet}}(x', l)}{d^3l} = \alpha_{em}(4\pi\alpha_s)^2 e_q^2 \left( \frac{1}{2x's} \right) \left( \frac{1}{x's + u} \right) 2H_g \]
\[ \times \left\{ \frac{\partial^2}{\partial x^2} \left( \frac{T_g(x, A)}{x} \right) \right\} \left( \frac{x's l_T}{u(x's + u)} \right)^2 + \left[ \frac{\partial}{\partial x} \left( \frac{T_g(x, A)}{x} \right) \right] \left( \frac{-x's}{u(x's + u)} \right) \right\} \] (18)

\[ E_l \frac{d\sigma^{(D-f)}_{aA \rightarrow \text{jet}}(x', l)}{d^3l} = \alpha_{em}(4\pi\alpha_s)^2 e_q^2 \left( \frac{1}{2x's} \right) \left( \frac{1}{x's + u} \right) 2H_{q\bar{q}} \]
\[ \times \left\{ \frac{\partial^2}{\partial x^2} \left( \frac{T_{q\bar{q}}(x, A)}{x} \right) \right\} \left( \frac{x's l_T}{u(x's + u)} \right)^2 + \left[ \frac{\partial}{\partial x} \left( \frac{T_{q\bar{q}}(x, A)}{x} \right) \right] \left( \frac{-x's}{u(x's + u)} \right) \right\} \] (19)

where the partonic parts \( H_q, H_g \) and \( H_{q\bar{q}} \) are given by

\[ H_q = \left( \frac{1}{36} \right) \left( \frac{x's}{u} - \frac{u}{x's} \right), \] (20)

\[ H_g = \left( \frac{1}{36} \right) \left( \frac{x's}{x's + u} + \frac{x's + u}{x's} \right), \] (21)

\[ H_{q\bar{q}} = \left( \frac{1}{6} \right) \left( \frac{-u}{x's + u} + \frac{x's + u}{-u} \right). \] (22)

It may be noticed that there are three cuts in each diagram in fig. 1. The long distance partonic distribution functions shown in eq.(15) and (16) come from the middle cut diagrams, and the rest part of the cut middle contribution in each diagram combine all of the other cut contributions, cut left and right, to construct a restriction \( |y_1^-| > |y^-| > |y_2^-| > 0 \), which rule out the long distance effect, hence do not contribute the nuclear enhancement.

The total contribution of the jet double scattering cross section with final state interaction is

\[ E_l \frac{d\sigma_{\text{jet}}^{(D-f)}(l)}{d^3l} = \alpha_{em}(4\pi\alpha_s)^2 \int dx'dx \ \delta \left( x - \frac{-x't}{x's + u} \right) \left( \frac{1}{x's + u} \right) \left( \frac{1}{x's + u} \right) \sum_q e_q^2 \] (23)
\[ \times [f_{g/P}(x')\Phi_q(x, x', A)H_q + f_{q/P}(x')\Phi_q(x, x', A)H_g + f_{q/P}(x')\Phi_q(x, x', A)H_{q\bar{q}}], \]
where $\sum_q$ runs over all quark and antiquark flavors. The functions $\Phi_i$ with $i = q, g$ are related to the twist-4 matrix elements $T_i$ defined in eq.(15) and (16) as

$$\Phi_i = \left[ \frac{\partial^2}{\partial x^2} \left( \frac{T_i(x, A)}{x} \right) \right] \left( \frac{x's l_T}{u(x's + u)} \right)^2 + \left[ \frac{\partial}{\partial x} \left( \frac{T_i(x, A)}{x} \right) \right] \left( \frac{-x's}{u(x's + u)} \right).$$  \hspace{1cm} (24)

We adopt the same approximation for the twist-4 matrix elements as used in ref.[4] and [5]:

$$T_i(x, A) = \lambda^2 A^{4/3} f_{i/N}(x),$$  \hspace{1cm} (25)

where $i = q, \bar{q},$ and $g$, with the $f_{i/N}$ are the regular parton distributions in a nucleon. We observe that the EMC effect in these parton distributions can be neglected for the nuclear enhancement, which is pointed in ref.[6].

In this paper, we are actually working on parton double scatterings in nuclei. We naturally assume that one initial parton (e.q. the $q$ in fig.3) comes from one nucleon (e.q. the $N_a$ in fig.3) and the other parton (e.q. the $g$ in fig.3) from another nucleon (e.q. the $N_b$ in fig.3). In other words, both the $g$ and $g'$ in fig.3 stand for the same gluon from the nucleon $N_b$, hence, the distance between $g$ and $g'$ is physically zero. Therefore, the pole structure of the propagator $A$ in fig.3 is eliminated by the zero distance between $g$ and $g'$. Namely, this diagram, which might contribute to the cross section a little, is not responsible for the long distance nuclear effect. Similarly the pole structure of the propagator $B$ is also dismissed. In this way, we can ignore the interference between the initial-state and final-state multiple scatterings.

In fig.4 and fig.5 we show the nuclear dependence of the final jet as compared with that of the direct photon within the 515 GeV proton beam on $Cu$ target. The parameter for the nuclear dependence, $\alpha_i$ with $i = jet$ and $\gamma$, is defined by

$$A^{\alpha_i-1} E_i \frac{d\sigma_i^{(S)}(l)}{d^3l} = E_i \frac{d\sigma_i^{(S)}(l)}{d^3l} + E_i \frac{d\sigma_i^{(D)}(l)}{d^3l}. \hspace{1cm} (26)$$

We compare $\alpha_{jet}$ shown by solid curve with $\alpha_\gamma$ shown by dashed curve, and found that the jet inclusive cross section has a large nuclear enhancement while the photon inclusion cross section has rather little nuclear dependence. It is clear that there exists an asymmetry between the jet and the photon inclusive cross sections in the direct photon production process. This is because that the direct photon receives contribution from only initial-state multiple scattering, while the jet are influenced by both the initial-state and the final-state multiple scatterings. In fig.4,
we plot the nuclear dependence parameter $\alpha_i$ as a function of $x_F$ defined by $x_F = -2l_T^3/\sqrt{s}$ at $l_T = 6$ GeV. It is found that the asymmetry increase as $x_F$ decrease at fixed transverse momentum $l_T$. In fig.5, we show the $l_T$ dependence of $\alpha_i$ at $x_F = 0$.

In summary, we discussed in this paper that there exists an asymmetry between the jet and the photon inclusive cross sections in the reaction $P + A \rightarrow \gamma + jet + X$. We also show that the jet inclusive cross section has observable nuclear enhancement, which is mainly contributed from final-state multiple scatterings.

References

[1] J. W. Cronin et al., Phys. Rev. D 11 (1975) 3105.

[2] D. Naples, in Proceedings of the Seventh Meeting of the American Physical Society Division of the Particles and Fields, Fermi National Accelerator Laboratory, Batavia. IL, edited by C. H. Albright et al. (World Scientific, Singapore, 1993), p.948; D. Naples et al., Phys. Rev. Lett. 72 (1994) 2341.

[3] E706 Collaboration, M. Zielinski, in proceeding of the Conference on the Intersections of Nuclear and Particle Physics, St. Petersburg, Florida, 1994.

[4] M. Luo, J. Qiu, and G. Sterman, Phys. Lett. B 279 (1992) 377.

[5] M. Luo, J. Qiu, and G. Sterman, Phys. Rev. D 50 (1994) 1951.

[6] X. Guo and J. Qiu, Phys. Rev. D 53 (1996) 6144.

[7] X. Guo, Phys. Rev. D 58 (1998) 036001.
Figure captions

**Fig.1.** Feynman diagrams for the three types of leading order partonic subprocesses contributed to the final-state double scattering. The first hard collision occurs between (1) gluon from proton and quark(antiquark) from nucleus. (2) quark(antiquark) from proton and gluon from nucleus; (3) quark(antiquark) from proton and antiquark(quark) from nucleus;

**Fig.2.** Graphical representation of double scattering contributions from the parton-nucleus collisions.

**Fig.3.** Graphical representation of interference between initial-state and final-state multiple scatterings.

**Fig.4.** Behavior of $\alpha_i$, defined in eq.(26), as a function of $x_F$ at $l_T = 6$ GeV, with 515 GeV proton beam on Cu target used. The solid curve denotes the $\alpha_{jet}$, while the dashed curve for $\alpha_\gamma$.

**Fig.5.** Behavior of $\alpha_i$, as in fig.4, but as a function of $l_T$ at $x_F = 0$. 
fig. 1

\begin{align*}
\frac{1}{x_p} & \quad \frac{1}{x_{p'}} \\
\text{(1)} & \\
\frac{1}{x_p+k_T} & \quad \frac{1}{x_{p'}+k_T} \\
\text{(2)} & \\
\frac{1}{x_p} & \quad \frac{1}{x_{p'}} \\
\text{(3)} & \\
\frac{1}{x_p+k_T} & \quad \frac{1}{x_{p'}+k_T}
\end{align*}
fig. 2
Beam: 515 GeV proton
Target: Cu
$l_T = 6$ GeV
Beam: 515 GeV proton
Target: Cu
$x_F = 0$