Achieving Delay Rate-function Optimality in OFDM Downlink with Time-correlated Channels

Zhenzhi Qian*, Bo Ji†, Kannan Srinivasan*, Ness B. Shroff‡

*Department of Computer Science and Engineering, The Ohio State University, Columbus 43210, OH
†Department of Computer and Information Sciences, Temple University, Philadelphia 19122, PA
‡Department of Electrical and Computer Engineering, The Ohio State University, Columbus 43210, OH

Abstract—There have been recent attempts to develop scheduling schemes for downlink transmission in a single cell of a multi-channel (e.g., OFDM-based) cellular network. These works have been quite promising in that they have developed low-complexity index scheduling policies that are delay-optimal (in a large deviation rate-function sense). However, these policies require that the channel is ON or OFF in each time-slot with a fixed probability (i.e., there is no memory in the system), while the reality is that due to channel fading and doppler shift, channels are often time-correlated in these cellular systems. Thus, an important open question is whether one can find simple index scheduling policies that are delay-optimal even when the channels are time-correlated. In this paper, we attempt to answer this question for time-correlated ON/OFF channels. In particular, we show that the class of oldest packets first (OPF) policies that give a higher priority to packets with a large delay is delay rate-function optimal under two conditions: 1) The channel is non-negatively correlated, and 2) The distribution of the OFF period is geometric. We use simulations to further elucidate the theoretical results.

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a digital multi-carrier modulation method that has been widely used in wideband digital communications. A practical and important application is the downlink phase of a single cell of OFDM-based cellular networks, where the wideband can be divided into a large number of orthogonal sub-carriers, which can be used to carry data for different users. In this system, the Base Station (BS) maintains a separate queue to store data packets requested by each user. When the sub-carrier seen by a user is in good channel condition, the sub-carrier can successfully transmit a packet to the user from its designated queue. We will focus on the setting of a single-hop multi-user multi-channel system and study the delay performance of this system from a large-deviations perspective.

In wireless networks, a key problem that has been extensively studied is the design of high-performance scheduling policies. It is well known from the seminal work [1] that the MaxWeight policy is throughput-optimal, in the sense that it can stabilize the system under any feasible arrival rates. However, it has been shown in [2] that the MaxWeight policy sacrifices the delay performance (and may lead to very large queue lengths) for better throughput. This fact has motivated researchers to look for policies that can improve the delay performance measured by a queue-length-based metric. In [3], the authors showed that the maximum-throughput and load-balancing (MTLB) policy can achieve delay optimality for two special cases of ON/OFF channels with a two-user system or a system that allows fractional server allocation. However, this problem becomes much harder in general cases. On the other hand, in cellular networks, minimizing average delay may cause a large delay for certain users that have stringent delay requirements.

Another line of works focus on designing scheduling policies that maximize the rate-function of the steady-state probability that the largest queue length exceeds a given threshold when the number of channels and users both go to infinity. In [4] and [5], the authors showed that their proposed policy can achieve both throughput optimality and queue length rate-function optimality. However, simulations in [6] - [8] show that good queue length performance does not necessarily imply good delay performance. In fact, queue-length-based policies usually suffer from the so called “last packet” problem, which occurs in the situation where a certain queue has a very small number of packets. Hence, this queue is rarely scheduled by the queue-length-based policies, resulting in large packet delays.

To that end, a delay-based metric has been investigated in recent works in [7], [10] and [11]. The authors developed several policies that achieve both throughput optimality and delay rate-function optimality (or near-optimality). Although the results hold for general arrivals (e.g., time-correlated arrivals are allowed), the channels are assumed to be i.i.d. over time. In practice, the current channel condition could depend on past channel conditions. Therefore, the following important question remains: How do we design a low-complexity scheduling policy that achieves provably good throughput and delay performance in the OFDM downlink system with time-correlated channels?

While it is relatively straightforward to develop throughput optimal policies even for time-correlated channels, developing policies that are delay-optimal or delay-efficient for time-correlated channels remains an open problem.

To that end, we are motivated to consider the following question: Can we find index scheduling policies that are delay-optimal even when the channels are time-correlated? In this paper, we provide a positive answer in some cases. Specifically, we analyze the delay rate-function of the class of oldest packets first (OPF) policies which give a higher priority to packets with a large delay and present two conditions under which delay rate-function optimality can be achieved by any
The key contributions of this paper are summarized as follows. We use an alternating renewal process to model a general ON/OFF time-correlated channel. We first prove an upper bound on the delay rate-function for any scheduling policy. Then, we analyze the delay rate-function of the class of OPF policies, which give a higher priority to older packets. We present two conditions and show that if both conditions are satisfied, delay rate-function optimality can be achieved by any OPF policy. The first condition requires that the channel condition is non-negatively correlated over time. This is often observed in practical time-correlated channels. The second condition requires that the “OFF” period distribution has the memoryless property, whereas the “ON” period distribution could be arbitrary.

The rest of the paper is organized as follows. In Section II, we describe the system model and the performance metric. In Section III, we derive an upper bound on the rate-function for any possible policy, and in Section IV, we obtain an achievable rate-function of the class of OPF policies. Then in Section V, we propose two conditions that imply delay rate-function optimality of the class of OPF policies. We conduct simulations to validate our theoretical results in Section VI and make concluding remarks in Section VII.

II. SYSTEM MODEL

We use a time-slotted multi-queue multi-server system to model the downlink phase of a single cell OFDM system. In particular, we assume that there are $n$ servers which stand for frequency sub-carriers. Furthermore, we assume the number of users is equal to the number of channels for ease of presentation [10]. The Base Station maintains a queue/buffer to store packets requested by each user, hence there are also $n$ queues in the queueing system. (We use terms “server” and “channel”, “queue” and “user” interchangeably throughout this paper.) Next, we present several notations that will be used later in this paper. We use $Q_i$ to denote the queue associated to the $i$-th user, and use $S_j$ to denote the $j$-th server for $1 \leq i, j \leq n$. We use $A_i(t)$ to denote the queue length of queue $Q_i$ at the beginning of time-slot $t$ immediately after new packet arrivals. All queues are assumed to have infinite buffer size. Further, we use $W_i(t)$ to denote the head-of-line (HOL) delay of queue $Q_i$ at the beginning of time-slot $t$ and use $W(t) = \max_{1 \leq i \leq n} W_i(t)$ to denote the largest packet delay in the system at the beginning of time-slot $t$. Finally, we use $\mathbb{I}_A$ to denote the indicator function that indicates whether event $A$ occurs or not.

A. Arrival Process

The arrival process to each queue is assumed to be stationary and ergodic. We also assume the arrivals are i.i.d. across all users, but could be correlated over time. Let $A_i(t)$ denote the number of packet arrivals to queue $Q_i$ in time-slot $t$. Let $A(t) = \sum_{i=1}^{n} A_i(t)$ denote the total packet arrivals coming into the system in time-slot $t$, and let $A(t_1,t_2) = \sum_{\tau=t_1}^{t_2} A(\tau)$ denote the cumulative packet arrivals to the system from time-slot $t_1$ to time-slot $t_2$.

Next, we will introduce several assumptions on the arrival process for purpose of rate-function delay analysis.

Assumption 1: The number of arrivals are bounded, i.e., there exists a finite number $L$ such that $A_i(t) \leq L$ for any $i$ and $t$. Also, we assume $\mathbb{P}(A(s,s+t-1) = Lnt) > 0$ for any $s$, $t$ and $n$.

Assumption 2: The arrival process are i.i.d across all users, and the mean arrival rate is $\lambda$ (we assume $\lambda < 1$, otherwise the system could not be stable under any scheduling policy) for every user. Given any $\epsilon > 0$ and $\delta > 0$, there exists a positive function $I_B(\epsilon, \delta)$ independent of $n$ and $t$ such that

$$\mathbb{P}\left( \frac{\sum_{\tau=1}^{t} \mathbb{1}_{\{|A(\tau) - \mu| > \epsilon\}}}{t} > \delta \right) < \exp(-\epsilon t I_B(\epsilon, \delta)). \quad (1)$$

for all $t \geq T_B(\epsilon, \delta)$ and $n \geq N_B(\epsilon, \delta)$.

Assumptions 1 and 2 are mild. Packet arrivals per time-slot are typically bounded in practice. In addition, it has been shown in [7] that Assumption 2 is a general result of the statistical multiplexing effect of a large number of sources and holds for both i.i.d. arrivals and Markov chain driven arrivals.

B. Stochastic Connectivity

We assume that each channel has unit capacity and changes between “ON” state and “OFF” state from time to time. We use $C_{i,j}(t)$ to indicate the connectivity between queue $Q_i$ and server $S_j$ in time-slot $t$: $C_{i,j}(t) = 1$ when the channel is “ON” and $C_{i,j}(t) = 0$ when the channel is “OFF”. We define “ON” period to be the number of time-slots between the last time the channel was “OFF” until the next time-slot it becomes “OFF” again. “OFF” period is defined in the similar way. From time to time, the channel state alternates between “ON” periods and “OFF” periods. We use an alternating renewal process to model the stochastic connectivity. In other words, the channel
is initially "ON" for a time period $U_1$ and then "OFF" for a time period $D_1$, followed by another "ON" period $U_2$ and so on. In particular, the sequences of "ON" times $\{U_n : n \geq 1\}$ and "OFF" times $\{D_n : n \geq 1\}$ are independent sequences of i.i.d. positive random variables. Let $U$ be a generic "ON" time and $D$ be a generic "OFF" time. We use $F_U(\cdot)$ and $F_D(\cdot)$ to denote the CDF of random variable $U$ and $D$, respectively.

**Assumption 3:** The sum of "ON" and "OFF" periods $U+D$ is aperiodic with $\mathbb{E}[U] < \infty$ and $\mathbb{E}[D] < \infty$.

It is well known (e.g. [9]) that under Assumption 3, we have:

$$
\lim_{t \to \infty} \mathbb{P}(C_{i,j}(t) = 0) = \frac{\mathbb{E}[D]}{\mathbb{E}[U] + \mathbb{E}[D]} = \pi_0.
$$

for any $i,j$.

**Remark 1:** If $U + D$ is periodic with period $d$, the above result is true if $t$ is an integral multiple of $d$. For simplicity, we only focus on the aperiodic case.

Note that this is a general model that can capture the time-correlation of a channel. If $U$ and $D$ are geometrically distributed with parameters $1-q$ and $q$, respectively, it degenerates to a static i.i.d. channel model with channel "ON" probability $q$. Similarly, if $U$ and $D$ have a geometric distribution with parameter $p_{10}$ and $p_{01}$, respectively, it becomes the Markovian channel model with transition matrix

$$
T = \begin{bmatrix}
1-p_{01} & p_{01} \\
p_{10} & 1-p_{10}
\end{bmatrix}.
$$

In each time-slot, a scheduling policy allocates servers to serve packets from user queues. We further assume that a server can only serve one queue in a time-slot, however, a queue can get service from multiple servers simultaneously in one time-slot. In addition, one packet from queue $Q_i$ can be served if an "ON" channel is allocated to queue $Q_i$.

**C. Problem Formulation**

In this paper, the metric we use to measure the delay performance is the large deviation rate-function of the steady-state probability that the largest packet delay exceeds a given threshold $b$. Assume the system starts at minus infinity, then $W(0)$ is the largest packet delay over all the queues in the steady-state. We define the rate-function $I(b)$ as the asymptotic decay-rate of the probability that $W(0) > b$ for a given threshold $b$:

$$
I(b) \triangleq \lim_{n \to \infty} \frac{-1}{n} \log \mathbb{P}(W(0) > b).
$$

Note that by the definition of rate-function $I(b)$, we can estimate the order of delay violation probability (i.e., $\mathbb{P}(W(0) > b)$ by $\exp(-nI(b))$). It is obvious that a larger rate-function implies a smaller delay violation probability and a better delay performance. In this paper, our objective is to maximize the rate-function $I(b)$.

**III. AN UPPER BOUND ON THE RATE-FUNCTION**

In this section, we derive an upper bound on the best achievable delay-rate function. Later, we will use this upper bound as a baseline to evaluate the delay performance of the OPF policies.

First, as in [6], [7], we define quantity $I_A(t,x)$ for any integer $t > 0$ and any real number $x \geq 0$:

$$
I_A(t,x) \triangleq \sup_{\theta > 0} \{\theta(t+x) - \lambda_{A_t(-t+1,0)}(\theta)\},
$$

where $\lambda_{A_t(-t+1,0)}(\theta) = \log \mathbb{E}[e^{\theta A_t(-t+1,0)}]$ is the cumulant-generating function of $A_t(-t+1,0) = \sum_{\tau=-t+1}^{0} A_t(\tau)$.

From Cramer’s Theorem, $I_A(t,x)$ is equal to the asymptotic decay-rate of the probability that in any interval of $t$ time-slots, the total number of packet arrivals to the system is no smaller than $n(t+x)$ as $n$ tends to infinity, i.e.,

$$
\lim_{n \to \infty} \frac{-1}{n} \log \mathbb{P}(A(-t+1,0) \geq n(t+x)) = I_A(t,x).
$$

We define $t_x$ for $L > 1$ and non-negative integer $x$:

$$
t_x \triangleq \frac{x}{L-1}.
$$

Then we define an integer set $\Psi_b \triangleq \{c \in \{1,2,\cdots,b\} | t_{b-c} \in \mathbb{Z}^+\}$. For any integer $b \geq 0$, let

$$
I_U(b) \triangleq \min \left\{ \log \left( \frac{1}{\max_{\tau \in \{0,1,\cdots\}} \frac{1}{1-F_D(\tau+b)}} - \log \pi_0, \right. \right.
\left. \left. \min \left\{ \inf_{t \geq t_b} I_A(t,b), \min_{1 \leq c \leq b} \inf_{t \geq t_{b-c}} I_A(t,b-c) - \log \pi_0 + \log \left( \max_{\tau \in \{0,1,\cdots\}} \frac{1}{1-F_D(\tau+c-1)} \right) \right),
\left. \min_{c \in \Psi_b} I_A(t_{b-c},b-c) - \log \pi_0 + \log \left( \max_{\tau \in \{0,1,\cdots\}} \frac{1}{1-F_D(\tau+c)} \right) \right\}. \right.
$$

In addition, we define:

$$
I_U(b) = \left\{ \begin{array}{ll}
-\log \pi_0 + \log \left( \max_{\tau \in \{0,1,\cdots\}} \frac{1}{1-F_D(\tau+b)} \right), & L=1 \\
I_U^L(b), & L>1
\end{array} \right.
$$

The following theorem shows that for any integer $b \geq 0$, $I_U(b)$ is an upper bound on the delay rate-function for any feasible scheduling policy.

**Theorem 1:** For any integer threshold $b \geq 0$ and any scheduling policy, we have:

$$
\lim_{n \to \infty} \frac{-1}{n} \log \mathbb{P}(W(0) > b) \leq I_U(b).
$$

**Proof:** We consider two cases $L > 1$ and $L = 1$. For the case $L > 1$, we will consider three types of events: $\chi_1$, $\chi_2$ and $\chi_3$, which are subsets of the delay-violation event $\{W(0) > b\}$. Note that bursty arrivals and sluggish services both cause large packet delay in the system. In particular, $\chi_1$ is the event with sluggish services while $\chi_2$ and $\chi_3$ are events with bursty arrivals and sluggish services.
**Event \( \chi_1 \):** Suppose there is a packet that arrives to the network at the beginning of time-slot \(-b - 1\). Since the arrival process is independent across all queues, without loss of generality, we assume that this packet arrives to \( Q_1 \). Furthermore, \( Q_1 \) is assumed to be disconnected from all \( n \) servers from time-slot \(-b - 1\) to \(-1\). As a result, at the beginning of time-slot 0, this packet is still in the network and has a delay of \( b + 1 \), which violates the delay threshold \( b \). Therefore, \( \chi_1 \subseteq \{ W(0) > b \} \).

First we want to calculate the probability that \( Q_1 \) is disconnected from an arbitrary server \( S_j \) from time-slot \(-b - 1\) to \(-1\), i.e., \( C_{1,j}(t) = 0 \) for all \(-b - 1 \leq t \leq -1\). In this case, time-slot \(-b - 1\) must fall in some “OFF” period and this “OFF” period must cover the time from time-slot \(-b - 1\) to time-slot \(-1\). Let \( \tilde{T}(-b, C_{i,j}) \) be the length of the time period from the beginning of channel \( C_{i,j} \)’s last state-change (“ON” to “OFF”) time-slot before time-slot \(-b\) to the beginning of time-slot \(-b - 1\). Thus, we have:

\[
\begin{align*}
\mathbb{P}(C_{1,j}(t) = 0 \text{ for all } -b - 1 \leq t \leq -1) & = \sum_{\tau = 0}^{\infty} \mathbb{P}(C_{1,j}(-b - 1) = 0, \tilde{T}(-b, C_{1,j}) = \tau, D \geq \tau + b + 1) \\
& = \sum_{\tau = 0}^{\infty} \mathbb{P}(C_{1,j}(-b - 1) = 0) \cdot \mathbb{P}(\tilde{T}(-b, C_{1,j}) = \tau, D \geq \tau + b + 1) \\
& \leq \pi_0 \sum_{\tau = 0}^{\infty} \mathbb{P}(D \geq \tau + b + 1 | \tilde{T}(-b, C_{1,j}) = \tau, C_{1,j}(-b - 1) = 0) \\
& = \pi_0 \sum_{\tau = 0}^{\infty} \mathbb{P}(D \geq \tau + b + 1 | \tilde{T}(-b, C_{1,j}) = \tau, C_{1,j}(-b - 1) = 0) \\
& \quad \cdot \mathbb{P}(\tilde{T}(-b, C_{1,j}) = \tau | C_{1,j}(-b - 1) = 0) \\
& \geq \pi_0 \sum_{\tau = 0}^{\infty} \mathbb{P}(D \geq \tau + b + 1 | \tilde{T}(-b, C_{1,j}) = \tau, C_{1,j}(-b - 1) = 0) \\
& \geq \pi_0 \min_{\tau \in \{0,1,\ldots\}} \frac{1 - F_D(\tau + b)}{1 - F_D(\tau)} \\
& \geq \pi_0 \cdot \min_{\tau \in \{0,1,\ldots\}} \frac{1 - F_D(\tau + b)}{1 - F_D(\tau)}.
\end{align*}
\]

and thus

\[
\limsup_{n \to \infty} -\frac{1}{n} \log \mathbb{P}(W(0) > b) \leq -\log \pi_0 + \log \left( \max_{\tau \in \{0,1,\ldots\}} \frac{1 - F_D(\tau + b)}{1 - F_D(\tau)} \right).
\]

**Event \( \chi_2 \):** Consider any fixed \( c \in \{0,1,2,\ldots,b\} \) and any \( t > t_{b-c} \). Recall that \( t_{b-c} = \frac{b-c + \delta}{\epsilon} \). Then, for all \( t > t_{b-c} \), we have \( b - c < (L - 1)t \), and thus \( I_A(t, b-c) = I_A^+(t, b-c) \) from Lemma 11 (Right continuity of function \( I_A(t, x) \)) in [10]. Hence, for any fixed \( c > 0 \), there exists a \( \delta > 0 \) such that \( I_A(t, b-c + \delta) \leq I_A^+(t, b-c) + \epsilon = I_A(t, b-c) + \epsilon \). Suppose that from time-slot \(-t-b\) to \(-b-1\), the total number of packet arrivals to the system is greater than or equal to \( nt+n(b-c+\delta) \), and let \( p_{b-c+\delta} \) denote the probability that this event occurs. Then, from Cramer’s Theorem, we have

\[
\lim_{n \to \infty} -\frac{1}{n} \log \mathbb{P}(W(0) > b) \leq I_A(t, b-c) + \epsilon.
\]

Clearly, the total number of packets that are served in any time-slot is no greater than \( n \). For any fixed \( \delta \), we have \( n \delta \geq 1 \) for large enough \( n \) (when \( n \geq \frac{1}{\delta} \)). Hence, if the above event occurs, at the end of time-slot \(-c-1\), the system contains at least one packet that arrived before time-slot \(-b\).

**Case 1:** \( c = 0 \)

In this case, the probability that event \( \chi_2 \) occurs can be computed by:

\[
\mathbb{P}(\chi_2^0) = p_{b+\delta}.
\]

And since \( \chi_2^0 \subseteq \{ W(0) > b \} \), we have:

\[
\limsup_{n \to \infty} -\frac{1}{n} \log \mathbb{P}(W(0) > b) \leq I_A(t, b) + \epsilon.
\]

**Case 2:** \( c \geq 1 \)

Applying the same method we used to bound the event \( \chi_1 \), we have:

\[
\mathbb{P}(C_{1,j}(t) = 0 \text{ for all } 1 \leq j \leq n, -c \leq t \leq -1) \geq \left( \pi_0 \cdot \min_{\tau \in \{0,1,\ldots\}} \frac{1 - F_D(\tau + c) - 1}{1 - F_D(\tau)} \right)^n.
\]

Note that the channel condition is independent from the arrival process, and the probability \( \chi_2^c \) occurs can be bounded as:

\[
\mathbb{P}(\chi_2^c) \geq p_{b-c+\delta} \left( \pi_0 \cdot \min_{\tau \in \{0,1,\ldots\}} \frac{1 - F_D(\tau + c - 1)}{1 - F_D(\tau)} \right)^n.
\]
Hence, we have:

\[
\limsup_{n \to \infty} \frac{-1}{n} \log \mathbb{P}(W(0) > b) \\
\leq I_A(t, b - c) + \epsilon - \log \pi_0 \\
+ \log \left( \max_{\tau \in \{0, 1, \ldots\}} \frac{1 - F_D(\tau)}{1 - F_D(\tau + c - 1)} \right).
\]

Since inequality (23) or (24) holds for any \( c \in \{0, 1, 2, \ldots, b\} \), for any \( t > t_{b-c} \), and any \( \epsilon > 0 \), applying the results we have in (23) and (24) and by letting \( \epsilon \) tend to 0, taking the infimum over all \( t > t_{b-c} \), and taking the minimum over all \( c \in \{0, 1, 2, \ldots, b\} \), we have:

\[
\limsup_{n \to \infty} \frac{-1}{n} \log \mathbb{P}(W(0) > b) \\
\leq \min \left\{ \inf_{t > t_{b-c}} I_A(t, b - c), \inf_{c \in \{0, 1, 2, \ldots, b\}} I_A(t, b - c) \right\} \\
- \log \pi_0 + \log \left( \max_{\tau \in \{0, 1, \ldots\}} \frac{1 - F_D(\tau)}{1 - F_D(\tau + c - 1)} \right).
\]

**Event \( \chi_2 \):** Consider any fixed \( c \in \Psi_b \subseteq \{ c \in \{1, 2, \ldots, b\} | t_{b-c} \in \mathbb{Z}^+ \} \). Suppose that from time-slot \(-t_{b-c} - b \to -b - 1\), the total number of packet arrivals to the system is equal to \( n(t_{b-c}) + n(b - c) = nLt_{b-c} \), and let \( p_{(b-c)} \) denote the probability that this event occurs. Note that the total number of packet arrivals to the system from time-slot \(-t_{b-c} - b \to -b - 1\) can never exceed \( nLt_{b-c} \). Then, from Cramer’s Theorem, we have

\[
\lim_{n \to \infty} \frac{-1}{n} \log p_{(b-c)} = I_A(t_{b-c}, b - c).
\]

Clearly, the total number of packets that can be served during the interval \([-t_{b-c} - b, -c - 1]\) is no greater than \( n(t_{b-c}) + n(b - c) = nLt_{b-c} \). Suppose that there exists one queue, say \( Q_1 \) that is disconnected from all the servers in time-slot \(-c - 1\). Then, at the end of time-slot \(-c - 1\), the system contains at least one packet that arrived before time-slot \(-b\). Further, if queue \( Q_1 \) is disconnected from all the \( n \) servers from time-slot \(-c - 1\) to \(-1\). Then, at the beginning of time-slot \(0\), we have a packet in the system that arrived before time-slot \(-b\). Thus, in this case we have \( W(0) > b \), and \( \chi_2 \subseteq \{ W(0) > b \} \). Note that the probability that event \( \chi_2 \) occurs can be bounded as

\[
\mathbb{P}(\chi_2) \geq p_{(b-c)} \left( \min_{\tau \in \{0, 1, \ldots\}} \frac{1 - F_D(\tau + c)}{1 - F_D(\tau)} \right)^n.
\]

Combining events \( \chi_1 \), \( \chi_2 \), \( \chi_3 \), the delay rate-function is upper bounded by \( I_U(b) \):

\[
\limsup_{n \to \infty} \frac{-1}{n} \log \mathbb{P}(W(0) > b) \\
\leq \min \left\{ \log \left( \max_{\tau \in \{0, 1, \ldots\}} \frac{1 - F_D(\tau)}{1 - F_D(\tau + b)} \right), \log \pi_0, \right\} \\
- \log \pi_0 + \log \left( \max_{\tau \in \{0, 1, \ldots\}} \frac{1 - F_D(\tau)}{1 - F_D(\tau + c - 1)} \right).
\]

Next, we consider the case where \( L = 1 \). In this case, we only need to consider event \( \chi_1 \), combining the case \( L = 1 \) and \( L > 1 \), we have shown that \( I_U(b) \) is an upper bound on the delay rate-function.

**IV. ACHIEVABLE RATE-FUNCTION OF OPF POLICIES**

In this section, we aim to derive a non-trivial achievable delay rate-function of the class of OPF policies. First, we state the definition of the class of OPF policies.

**Definition 1:** A scheduling policy \( P \) is said to be an OPF (oldest packets first) policy if in any time-slot, policy \( P \) can serve the \( k \) oldest packets in the system for the largest possible value of \( k \in \{0, 1, 2, \ldots, n\} \).

We want to show that the achievable rate-function of any OPF policy \( P \) is no smaller than \( I_0(b) \), defined as:

\[
I_0(b) = \begin{cases} 
- \log \pi_0 + b \cdot \log \frac{1}{1 - q}, & L = 1, \\
I_0^*(b), & L > 1.
\end{cases}
\]

where the parameter \( q \) is defined to be

\[
\hat{q} \triangleq \min \left\{ \min_{k \in \{0, 1, \ldots\}} \frac{1 - F_D(k)}{1 - F_D(k + 1)}, \min_{k \in \{0, 1, \ldots\}} \frac{1 - F_U(k + 1)}{1 - F_U(k)} \right\}.
\]

and

\[
I_0^*(b) \triangleq \min \left\{ b \cdot \log \frac{1}{1 - q} - \log \pi_0, \right\} \\
- \log \pi_0 + \min \left\{ \inf_{t > t_k} I_A(t, b - c), \inf_{1 \leq c \leq b, t > t_{b-c}} I_A(t, b - c) \right\} \\
- \log \pi_0 + (c - 1) \cdot \log \frac{1}{1 - q} \right\}.
\]

The analysis of delay rate-function follows a similar line of argument as in the case of \( i.i.d. \) channels. Specifically, we analyze the rate-function of the Frame Based Scheduling (FBS) policy and the perfect-matching policy and exploit the dominance property of the OPF policies over both of them. However, in the case of time-correlated channels, it becomes more challenging to derive a good lower bound on the achievable rate-function. Since the channel has different behaviors (distributions) for state-change and state-keeping. To
address this key challenge, we prove two important properties of the FBS policy and the perfect-matching policy (Section IV.A), which will play a key role in the proof. We start by briefly describing the operations of the FBS policy and the perfect-matching policy.

Under the FBS policy, packets are served in unit of frames. Each frame is constructed according to a given operating parameter $h$, such that: 1) the difference of the arrival times of any two packets within a frame must be no greater than $h$; and 2) the total number of packets in each frame is no greater than $n_0 = n - Lh$. In each time-slot, the packets arrived at the beginning of this time-slot are filled into the last frame until any of the above two conditions are violated, in which case a new frame will be opened. In each time-slot, the HOL frame can be served only if there exists a matching that can serve all the packets in the HOL frame. Otherwise, no packet will be served. In any time-slot, the FBS policy serves the HOL frame that contains the oldest (up to $n_0$) packets with high probability for a large $n$. Under the perfect-matching policy, if a perfect matching can be found, i.e., every queue can be matched with a different server that is connected to this queue, the HOL packet of every queue will be served by the respective server determined by the perfect matching. Otherwise, none of the packets will be served. It has been shown in [10] that any OPF policy dominates the FBS policy and the perfect-matching policy, i.e., given the same packet arrivals and channel realization, any OPF policy will serve every packet that the FBS policy has served up to time $t$; and the same for the perfect-matching policy. Therefore, the FBS and perfect-matching policy will provide lower bounds on the delay rate-function that any OPF policy can achieve.

A. Properties of FBS and Perfect Matching Policy

In this subsection, we derive the following properties of FBS and perfect matching policy, which will later be used for the rate-function analysis. For ease of presentation, we define function $X_F(t)$ as:

$$X_F(t) = \begin{cases} 
1 & \text{if a frame can be served in time-slot } t \\
0 & \text{otherwise.}
\end{cases}$$

We have the following lemma that gives a lower bound on the probability that $X_F(t) = 1$.

**Lemma 1:** Consider an $n \times n$ bipartite graph $G$, where the time-varying connectivity has the general time-correlation property described in Section II. Then, there exists an $N_F > 0$, such that for all $n \geq N_F$, the conditional probability that $X_F(t) = 1$ is bounded by:

$$
\mathbb{P}(X_F(t) = 1 | S(t_1), \cdots, S(t_d), S(t-1)) \geq 1 - \left( \frac{n}{1 - \frac{d}{q}} \right)^{2H} e^{n \log \frac{1}{1-q}}.
$$

We omit the proof here, as the same technique used in the proof of Lemma 1 can be applied.

**Proof:** We provide the proof in APPENDIX A.

Lemma 1 shows that given the past channel state information, a frame can be successfully served with high probability. We are interested in finding an upper bound on the probability that during the time interval $[-t-b, -1]$, exactly $t+a$ frames can be successfully served by the FBS scheduling policy. We have the following lemma:

**Lemma 2:** For all $a \leq b - 1$, we have:

$$
\mathbb{P}(\sum_{\tau=-t-b}^{t-a} X_F(\tau) = t + a) \leq 2^{t+b} \left( \frac{n}{\pi_0} \right)^{2H} \left( \frac{n}{1-q} \right)^{2H} e^{-n \left(- \log \pi_0 + (b-a-1) \log \frac{1}{1-q} \right)}.
$$

**Proof:** We provide the proof in APPENDIX B.

Similarly, we have the following lemma:

**Lemma 3:** Consider an $n \times n$ bipartite graph $G$, where the time-varying connectivity has general time-correlation property. There exists an $N_{PM} > 0$, for all $n \geq N_{PM}$ the probability that $G$ has no perfect matching can be bounded as:

$$
\mathbb{P}(X_{PM}(t) = 0 | S(t_1), \cdots, S(t_d), S(t-1)) \leq 3n e^{-n \log \frac{1}{1-q}}.
$$

**Proof:** We omit the proof here, as the same technique used in the proof of Lemma 1 can be applied.

Similarly, it can be shown that for all $a \leq b - 1$:

$$
\mathbb{P}(\sum_{\tau=-t-b}^{t-a} X_{PM}(\tau) = t + a) \leq 2^{t+b} n^e e^{-n \left(- \log \pi_0 + (b-a-1) \log \frac{1}{1-q} \right)}.
$$

The above inequality holds for sufficiently large $n \geq N_{PM}$. Note that the R. H. S. of inequalities (15) and (18) are both monotonically increasing with respect to $a$.

B. Achievable Rate-function

We first consider the case where $L > 1$. We need to pick an appropriate choice for the value of parameter $h$ for FBS based on the statistics of the arrival process. We fix $\delta < \frac{3}{4}$ and $\epsilon < p/2$. Then, from Assumption 2, there exists a positive function $I_B(\epsilon, \delta)$ such that for all $n \geq N_B(\epsilon, \delta)$ and $t \geq T_B(\epsilon, \delta)$, we have

$$
\mathbb{P}\left(\sum_{\tau=l+1}^{l+t} \mathbb{1}_{|A(\tau) - p_m| > m} > \delta \right) < \exp(-nt I_B(\epsilon, \delta)).
$$

where $l$ is any arbitrary integer. Choose parameter $h$ to be:

$$
h = \max\left\{ T_B(\epsilon, \delta), \left[ \frac{1}{(p-\epsilon)(1-\frac{3\delta}{n})}, \frac{2I_B(0)}{I_B(\epsilon, \delta)} \right] \right\} + 1.
$$
and define $H = Lh$.

The reason for choosing this value of $h$ will later become clearer. Note that in Assumption 2, the maximum number of arrivals in a time-slot is $L$.

Let $L(-b)$ be the last time before time-slot $-b$, when the backlog is empty, i.e., all the queues have a queue-length of zero. Also, let $E_t$ be the set of sample paths such that $L(-b) = -t - b - 1$ and $W(0) > b$ under policy $P$. Then, we have

$$\Pr(W(0) > b) = \sum_{n=1}^{\infty} \Pr(E_t).$$ (34)

Let $E_t^F$ and $E_t^{PM}$ be the set of sample paths such that given $L(-b) = -t - b - 1$, the event $W(0) > b$ occurs under the FBS policy and the perfect-matching policy, respectively. Recall that policy $P$ dominates both the FBS policy and the perfect-matching policy. Since each packet not served by the OPF policy is also not served by the FBS policy or perfect matching policy, then for any $t > 0$ we have

$$E_t \subseteq E_t^F \cap E_t^{PM}. \ (35)$$

Recall that $p$ is the mean arrival rate to a queue. Now, we choose any fixed real number $\hat{p} \in (p, 1)$, and fix a finite time $t^*$ as

$$t^* = \max \left\{ T_1, \left[ \frac{I_B(b)}{I_{BX}} \right], \max \{ t_{b-c} | c \in \Psi_b \} \right\}, \ (36)$$

where $T_1$ is defined as:

$$T_1 = \max \left\{ T_B(\hat{p} - p, \frac{1 - \hat{p}}{6(L + 2)}), \left[ \frac{6}{1 - \hat{p}} \right] \right\}. \ (37)$$

and

$$I_{BX} = \min \left\{ \left( \frac{1 - \hat{p}}{9} \right) \log \frac{1}{\frac{1}{3} I_B(\hat{p} - p, \frac{1 - \hat{p}}{6(L + 2)})} \right\}. \ (38)$$

Hence, if we let 

$$P_1 = \sum_{t=1}^{t^*} \Pr(E_t^F \cap E_t^{PM}), \ (39)$$

and

$$P_2 = \sum_{t=t^*}^{\infty} \Pr(E_t^F \cap E_t^{PM}). \ (40)$$

From the relation in (22), we can bound $\Pr(E_t)$ as:

$$\Pr(E_t) \leq P_1 + P_2. \ (41)$$

Hence, we can divide the rate-function analysis into two parts. In part 1, we show that there exists a finite $N_1 > 0$ such that for all $n \geq N_1$, we have

$$P_1 \leq C_1 n^{7(b+1)}H e^{-nI_0(b)}. \ (42)$$

Then, in part 2, we show that there exists a finite $N_2 > 0$ such that for all $n \geq N_2$,

$$P_2 \leq 4e^{-nI_0(b)}. \ (43)$$

By combining part 1 and part 2, there exists a finite $N \neq \max\{N_1, N_2\}$, such that for all $n \geq N$,

$$\Pr(W(0) > b) \leq \left(C_1 n^{7(b+1)}H + 4\right)e^{-nI_0(b)}. \ (44)$$

If we take logarithm and limit as $n$ goes to infinity, we obtain

$$\lim_{n \to \infty} \inf_{b} \frac{1}{n} \log \Pr(W(0) > b) \geq I_0(b),$$

which is the desired result.

C. Part 1

In this section, we want to show that there exists a finite value $N_1$, such that for all $n \geq N_1$, we have:

$$P_1 \leq C_1 n^{7(b+1)}H e^{-nI_0(b)}. \ (45)$$

First we consider the case when $t < t^*$, let $E_t^\alpha$ denote the set of sample paths in which there are at least $n$ arrivals seen by every $h - 1$ time-slots in the time interval $[-t - b, -b - 1]$. Let $E_t^\beta$ be the set of sample paths such that $\frac{A(-t - b, -b - 1)}{n_0} - \sum_{\tau = -t - b} X_F(\tau) > 0$. According to [7], due to the choice of parameter $h$, we have

$$E_t^\alpha \subseteq (E_t^\alpha)^c \cup E_t^\beta \ (46)$$

and also there exists $N_3 > 0$ and $C_2 > 0$ such that for all $n \geq N_3$,

$$\Pr(E_t^\alpha) > 1 - C_2 te^{-nI_0(b)}. \ (47)$$

For the probability of $E_t^\beta$ for each $t$, we can derive an upper bound on the probability of a large burst of arrivals during an interval of $t$ time-slots [10].

$$\Pr(A(-t + 1, 0) > n_0(t + x)) \leq \Pr(A(-t + 1, 0) \geq (n - H)(t + x) + 1) \leq e^{-nI_A(t + x)e^{(H(t + x) + 1)\theta^*}}. \ (48)$$

for all $x \in [0, (L - 1)t]$ and $\theta^* = \max \{\theta_1, \theta_2, \cdots, \theta_t\}$ where

$$\theta_i = \arg\max_{\theta} \theta(t + x) - \lambda_{A_i}(-t + 1, 0)(\theta).$$

Recall that $t_* = \frac{x}{L-1}$. We first consider any $t \in \{1, 2, \cdots, t^*\}\{t_{b-c'} \in \Psi \}$. Let $c_t \in \mathbb{Z}$ be the smallest integer such that $t_{b-c_t} < t$ (i.e., $t_{b-c_t} < t < t_{b-c_t+1}$ or $c_t = 0$ if $t > t_b$). Then for $z \in \{c_t, c_t + 1, \cdots, b\}$, we have $t_{b-z} < t$, thus $t + b - z < Lt$ and for all $z' < c_t$ we have $t_{b-z'} > t$ and thus $t + b - z' > Lt$. From
Applying the results in section IV-A here, and we know that $z_{a×b} = 1 ≤ \max ≤ t(\tau_{a×b} - 1) ≥ a_{0}n_{0})$

Next, we need to deal with any $t_{b-c} ∈ \{t_{b-c} | c' ∈ \Psi_{b}\}$. In this case, we can use the dominance property over FBS and perfect matching policy, i.e., $\mathcal{E}_{l} \subseteq \mathcal{E}_{l}^{F}$ and $\mathcal{E}_{l} \subseteq \mathcal{E}_{l}^{PM}$. Note that we have $t_{b-c} = \frac{b-c}{\tau_{b-c}} > 0$, thus

$$t_{b-c} + b - c = Lt_{b-c} \tag{53}$$

If we define

$$K_{1} = \mathbb{P}(\mathcal{E}_{t_{b-c}}^{F} \cap \mathcal{E}_{t_{b-c}}^{PM}, A(-t_{b-c} - b, -b - 1) < (t_{b-c} + b - c)n_{0})$$
$$K_{2} = \mathbb{P}(\mathcal{E}_{t_{b-c}}^{F} \cap \mathcal{E}_{t_{b-c}}^{PM}, A(-t_{b-c} - b, -b - 1) ≥ (t_{b-c} + b - c)n_{0}) \tag{54}$$

According to union bound, we have:

$$\mathbb{P}(\mathcal{E}_{t_{b-c}}^{F} \cap \mathcal{E}_{t_{b-c}}^{PM}) ≤ K_{1} + K_{2} \tag{55}$$

In particular,

$$K_{1} ≤ \mathbb{P}(\mathcal{E}_{t_{b-c}}^{F}, A(-t_{b-c} - b, -b - 1) < (t_{b-c} + b - c)n_{0})$$
$$≤ 1 - \mathbb{P}(\mathcal{E}_{t_{b-c}}^{F} \supseteq) + K_{1}' \tag{56}$$

where $K_{1}' = \mathbb{P}(\mathcal{E}_{t_{b-c}}^{F}, A(-t_{b-c} - b, -b - 1) < (t_{b-c} + b - c)n_{0})$.

We have the similar result for $K_{1}'$, $n ≥ N_{F}$:

$$K_{1}' = \mathbb{P}(A(-t_{b-c} - b, -b - 1) ≥ \sum_{a=0}^{t_{b-c} + b - c - 1} X_{F}(\tau) > 0, \quad A(-t_{b-c} - b, -b - 1) < (t_{b-c} + b - c)n_{0}$$
$$≤ \sum_{a=0}^{t_{b-c} + b - c - 1} \mathbb{P}(\sum_{\tau=t_{b-c} - b}^{t_{b-c} - b} X_{F}(\tau) = a) \times \mathbb{P}(A(-t_{b-c} - b, -b - 1) > an_{0}))$$

$$≤ C_{3} n^{(b+1)H} \exp \left\{-n \min_{z ∈ \{c_{1}, c_{1} + 1, \ldots, b\}} \{I_{A}(t, b - z) - \log \pi_{0} + (z - 1) \log \frac{1}{1 - q}, \min \frac{1}{1 - q} \right\} \tag{51}$$

where $C_{3} = (t + b + 1)^{2+H} \left(\frac{\pi_{0}(1-q)^{b}}{1-(1-q)^{b}}\right) e^{(H(t+b)-1)q}$, and we let $z = b - a$ in the last step. Hence, for all $n ≥ N_{b} \max\{N_{a}, N_{F}\}$, we have

$$\mathbb{P}(\mathcal{E}_{l}^{F} \cap \mathcal{E}_{l}^{PM}) ≤ \mathbb{P}(\mathcal{E}_{l}^{F})$$

$$≤ 1 - \mathbb{P}(\mathcal{E}_{l}^{F}) + \mathbb{P}(\mathcal{E}_{l}^{\beta})$$

$$≤ C_{4} n^{(b+1)H} e^{-nI_{A}(b)} \tag{52}$$

where $C_{4} \triangleq \max\{C_{2} t^{*}, C_{3}\}$ and $t^{*} \in \{1, 2, \ldots, t\} \backslash \{t_{b-c} | c' ∈ \Psi_{b}\}$.
For all $n \geq N_{PM}$

$$K_{2} \leq \mathbb{P}(E_{t_{b-c}}^{PM}|A(-t_{b-c} - b, -b - 1) \geq (t_{b-c} + b - c)n_{0})$$

$$\times \mathbb{P}(A(-t_{b-c} - b, -b - 1) \geq (t_{b-c} + b - c)n_{0})$$

$$\leq \mathbb{P}(E_{t_{b-c}}^{PM}|A(-t_{b-c} - b, -b - 1) = L_{n_{0}}(t_{b-c}))$$

$$\times e^{-nI_{A}(t_{b-c}+b-c)\epsilon(H(t_{b-c}+b-c)-1)\theta_{*}}$$

$$\leq \mathbb{P}\left(\sum_{\tau=t_{b-c}-b}^{t_{b-c}+b-c-1} X_{PM}(\tau) < t_{b-c} + b - c\right)$$

$$\times e^{-nI_{A}(t_{b-c}+b-c)\epsilon(H(t_{b-c}+b-c)-1)\theta_{*}}$$

$$\leq (t_{b-c} + b - c)$$

$$\times \max_{a \in \{0, \ldots, t_{b-c}+b-c-1\}} \mathbb{P}(\sum_{\tau=t_{b-c}-b}^{t_{b-c}+b-c-1} X_{PM}(\tau) = a)$$

$$\times e^{-nI_{A}(t_{b-c}+b-c)\epsilon(H(t_{b-c}+b-c)-1)\theta_{*}}$$

$$\leq (t_{b-c} + b - c)2^{t_{b-c}+b-c+1}e^{(H(t_{b-c}+b-c)-1)\theta_{*}}b^{n_{b}}$$

$$\times e^{-nI_{A}(t_{b-c}+b-c)\log n_{0} + c\log \frac{1}{t_{b-c}+b-c}}$$

$$\leq C_{5}n_{b}e^{-nI_{A}(t_{b-c}+b-c)\log n_{0} + c\log \frac{1}{t_{b-c}+b-c}} \tag{59}$$

where $C_{5} \triangleq (t_{b-c} + b - c)2^{t_{b-c}+b-c+1}e^{(H(t_{b-c}+b-c)-1)\theta_{*}}b^{n_{b}}$.

Combining results in (28) and (29), we have for any $t_{b-c} \in \{t_{b-c} | c \in \Psi_{b}\}$ and $n \geq N_{5} \triangleq \max\{N_{3}, N_{P}, N_{PM}\}$,

$$\mathbb{P}(E_{b-c}^{F} \cap E_{t_{b-c}}^{PM}) \leq C_{6}\min\{t_{b-c}+b-c\}^{-1}e^{-nI_{0}(b)} \tag{60}$$

where $C_{6} \triangleq \max\{C_{2}t_{*}, C_{3}, C_{5}\}$.

Summing over $t = 1$ to $t = t_{*}$, we have

$$P_{1} = \sum_{t=1}^{t_{*}} \mathbb{P}(L(-b) = -t_{b-c} - b - 1, E_{t}) \tag{61}$$

$$\leq C_{1}n_{0}^{-t_{b-c}}e^{-nI_{0}(b)} \tag{62}$$

for all $n \geq N_{1} \triangleq \max\{N_{4}, N_{5}\}$, where $C_{1} \triangleq C_{6}t_{*}$.

**D. Part 2**

In this section, we want to show that there exists an $N_{2} > 0$, such that for $n \geq N_{2}$,

$$P_{2} \leq 4e^{-nI_{0}(b)} \tag{63}$$

Let $R_{0}$ be the empty space in the end-of-frame at the end of time-slot $t_{1}$. Then, let $A_{F}^{R_{0}}(t_{1}, t_{2})$ denote the number of new frames created from time-slot $t_{1}$ to $t_{2}$, including any partially-filled frame in time-slot $t_{2}$, but excluding the partially-filled frame in time-slot $t_{1}$. Also, let $A_{F}(t_{1}, t_{2}) = A_{F}^{R_{0}}(t_{1}, t_{2})$, if $R_{0} = 0$. As in the proof for Theorem 2 of [7], for any fixed real number $\hat{p} \in (0, 1)$, we consider the arrival process $\hat{A}(\cdot)$, by adding extra dummy arrivals to the original arrival process $A(\cdot)$. The resulting arrival process $\hat{A}(\cdot)$ is simple, and has the following property:

$$\hat{A}(\tau) = \begin{cases} \hat{p}n & \text{if } A(\tau) \leq \hat{p}n \\ Ln & \text{if } A(\tau) > \hat{p}n \end{cases} \tag{64}$$

Since $\hat{A}(\tau) \geq A(\tau)$, if we can find an upper bound on $\hat{A}_{F}(-t-b,-b-1)$, then it is also an upper bound on $A_{F}(-t-b,-b-1)$.

Consider any $t \geq t_{*}$. Let $B = \{b_{1}, b_{2}, \ldots, b_{|B|}\}$ be the set of time-slots in the interval from $-t-b$ to $-b-1$ when $\hat{A}(\tau) = Ln$. Given $L(-b) = -t-b-1$, from Corollary 2 of [6] and the proof of Theorem 2 in [10], we have

$$\hat{A}_{F}(-t-b,-b-1) \leq \frac{n}{n_{0}}(\hat{p}t + (L + 2)|B| + 1) \tag{65}$$

From Assumption 2, we know that $|B|$ could be arbitrary small for large enough $t$ and $n$. Suppose for $n \geq \frac{(2+\hat{p})t_{*}}{1-p}$,

$$A_{F}(-t-b,-b-1) \leq \hat{A}_{F}(-t-b,-b-1)$$

$$\leq \frac{n}{n_{0}}(\hat{p}t + (L + 2)|B| + 1)$$

$$\leq \frac{2 + \hat{p}}{3}t$$

$$\leq \frac{2 + \hat{p}}{3}t \tag{66}$$

In fact, for $n \geq N_{6} \triangleq \max\{N_{B}(\hat{p} - p, \frac{1-p}{6(L+2)}), \frac{(2+\hat{p})H}{1-p}\}$ and $t \geq T_{1}$,

$$\mathbb{P}(A_{F}(-t-b,-b-1)) \geq \frac{2 + \hat{p}}{3}t, L(-b) = -t-b-1$$

$$= 1 - \mathbb{P}(A_{F}(-t-b,-b-1) < \frac{2 + \hat{p}}{3}t)$$

$$L(-b) = -t-b-1$$

$$\leq 1 - \mathbb{P}(|B| \leq \frac{1 - \hat{p}}{6(L + 2)t}) \tag{67}$$

$$\leq e^{-nI_{B}(\hat{p} - p, \frac{1-p}{6(L+2)t})} \tag{68}$$

where the last inequality is from Assumption 2.

Next, let’s restate Lemma 1 in [7].

**Lemma 4:** Let $X_{i}, i = 1, 2, \ldots$ be a sequence of binary random variables such that for all $i$,

$$\mathbb{P}(X_{i} = 0|X_{i'}, i' < i) \leq c(n)e^{-nb} \tag{69}$$

where $c(n)$ is a polynomial in $n$ of finite degree. Let $N_{9}$ be such that $c(n) < e^{\frac{n}{2}}$ for all $n > N_{9}$. Then, for any $0 < a < 1$,

$$\mathbb{P}\left(\sum_{i=1}^{t} X_{i} < (1-a)t\right) < e^{-\frac{tn_{ab}}{a}} \tag{70}$$

for all $n > N_{10} := \max\{\frac{12}{a^{2}}, N_{9}\}$.

**Proof:** Due to the choice of $N_{9}$, we have for all $n > N_{9}$,

$$\mathbb{P}(X_{i} = 0|X_{i'}, i' < i) \leq e^{-\frac{a}{2}}. \tag{71}$$
From Markov’s inequality we have for any $r > 0$:
\[
\mathbb{P}\left(\sum_{i=1}^{t} X_i \leq (1 - a)t\right) = \mathbb{P}\left(e^{-r\sum_{i=1}^{t} X_i} \geq e^{-(1-a)t}\right) \\
\leq \frac{\mathbb{E}[e^{-r\sum_{i=1}^{t} X_i}]}{e^{-(1-a)t}}
\]
(72)

From the definition of conditional expectation, we have:
\[
\mathbb{E}[e^{-r\sum_{i=1}^{t} X_i}] = \mathbb{E}[\mathbb{E}[e^{-r\sum_{i=1}^{t} X_i} | X_1, X_2, \ldots, X_{t-1}]] \\
= \mathbb{E}[e^{-r\sum_{i=1}^{t-1} X_i} \mathbb{E}[e^{-rX_i} | X_1, X_2, \ldots, X_{t-1}]] \\
\leq \mathbb{E}[e^{-r\sum_{i=1}^{t-1} X_i} \left((1 - e^{-\frac{nt}{2}}) + e^{-r\frac{nt}{2}}\right)]
\]
(73)

The last inequality comes from the condition (77), continuing for $t-1, t-2, \cdots$ down to 1, we have:
\[
\mathbb{E}[e^{-r\sum_{i=1}^{t} X_i}] \leq \left((1 - e^{-\frac{nt}{2}}) + e^{-r\frac{nt}{2}}\right)^{t}
\]
(74)

Applying the proof technique used in [12], we can get:
\[
\mathbb{P}\left(\sum_{i=1}^{t} X_i \leq (1 - a)t\right) < e^{-tD(a)|e^{-\frac{nt}{2}}|}
\]
(75)

From the proof of Lemma 1 in [7], we know that the result holds for sufficiently large $n > N_{10}$.

From Lemma 2, there exists $N_7 \triangleq \max\{N_0, N_F\}$, such that for all $n \geq N_7$,
\[
\mathbb{P}(X_F(-t - b, -b - 1) < \frac{2 + \hat{\rho}}{3}t, L(-b) = -t - b - 1) \\
\leq \mathbb{P}(X_F(-t - b, -b - 1) < \frac{2 + \hat{\rho}}{3}(t + b), L(-b) = -t - b - 1) \\
\leq e^{-\frac{nt}{2}(1 - \frac{\hat{\rho}}{3})\log \frac{1}{1 - \frac{\hat{\rho}}{3}}} \\
\leq e^{-\frac{nt}{2}(1 - \frac{\hat{\rho}}{3})\log \frac{1}{1 - \frac{\hat{\rho}}{3}}}
\]
(76)

From (77) (77), for all $n \geq N_8 \triangleq \max\{N_0, N_7\}$ and $t \geq T_1$
\[
\mathbb{P}\left(A_F(-t - b, -b - 1) - X_F(-t - b, -1) > 0, L(-b) = -t - b - 1\right) \\
\leq 1 - (1 - e^{-ntL_B}(\frac{1 + \rho}{1 - b + \rho}))\left(1 - e^{-nt\frac{(1 - \frac{\hat{\rho}}{3})\log \frac{1}{1 - \frac{\hat{\rho}}{3}}}}\right) \\
\leq 2e^{-ntL_B}
\]
(77)

Summing over all $t \geq t^*$, we have for $n \geq N_2 \triangleq \max\{N_8, \lfloor \frac{\log 2}{T_B} \rfloor\}$.
\[
P_2 = \sum_{t=t^*}^{\infty} (\mathcal{E}_t^F \cap \mathcal{E}_t^{PM}) \\
\leq \sum_{t=t^*}^{\infty} \mathbb{P}(\mathcal{E}_t^F) \\
\leq \sum_{t=t^*}^{\infty} \mathbb{P}(A_F(-t - b, -b - 1) - X_F(-t - b, -1) > 0, L(-b) = -t - b - 1) \\
\leq \sum_{t=t^*}^{\infty} 2e^{-ntL_B} \\
\leq 4e^{-nt^*L_B} \\
\leq 4e^{-nI_0(b)}
\]
(78)

where the last two inequalities are from the choice of $N_2$ and $I_B$.

Combining part 1 and part 2, it is easy to verify
\[
\liminf_{n \to \infty} \frac{1}{n} \log \mathbb{P}(W(0) > b) \geq I_0(b).
\]

E. Delay Rate-Function Analysis for $L = 1$

In this case, we want to show that for any fixed integer $b > 0$, the rate-function achieved by OPF policy is greater than $b \log \frac{1}{1 - \frac{\hat{\rho}}{3}} - \log \pi_0$.

Similarly, we can choose $t'$ as:
\[
t' \triangleq \max\left\{T_1, \left[\frac{b \log \frac{1}{1 - \frac{\hat{\rho}}{3}} - \log \pi_0}{I_B}\right]\right\}
\]
(79)

If we define probabilities
\[
P_1' \triangleq \sum_{t=1}^{t'} (\mathcal{E}_t^F \cap \mathcal{E}_t^{PM}) \\
P_2' \triangleq \sum_{t=t'}^{\infty} (\mathcal{E}_t^F \cap \mathcal{E}_t^{PM})
\]
(80) (81)

Due to the dominance property over FBS policy and the perfect-matching policy, we can split the delay violation probability $\mathbb{P}(W(0) > b)$ as:
\[
\mathbb{P}(W(0) > b) \leq P_1' + P_2'.
\]
(82)

Likewise, we still divide the proof into two parts. In part 1, we need to show that for all $n \geq N_1'$
\[
P_1' \leq C\pi_0\pi^n\pi_0^{(b + \frac{\log 2}{T_B})}n^2
\]
(83)

Consider $t \leq t'$, the total packet arrivals during the interval of $[-t - b, -b - 1]$ cannot exceed $nt$ when $L = 1$, hence if $\sum_{t=-t-b}^1 X_{PM}(t) \geq t$ then all packets that arrive before $-b$ have been served at time-slot 0, event $\mathcal{E}_t^{PM}$ does not occur. We have $\mathcal{E}_t^{PM} \subseteq \{L(-b) = -t - b - 1, \sum_{t=-t-b}^1 X_{PM}(t) < t\}$
Thus, we have
\[
P(E'_t \cap E'_t^{PM}) \\
\leq P(E'_t^{PM}) \\
\leq P\left( \sum_{\tau = t-b}^{t-1} X_{PM}(\tau) < t \right) \\
\leq t^2 e^{t^2 + 3b} n^{-b} e^{-n\left( \frac{b \log \frac{1}{\log \pi_0}}{\log \pi_0} \right)}
\]
where \( C'_3 \triangleq t^2 + 3b \).
Let \( C'_4 \triangleq t C'_3 \), summing over all \( t \leq t' \), we have
\[
P'_1 \leq C'_4 n^{-b} e^{-n\left( \frac{b \log \frac{1}{\log \pi_0}}{\log \pi_0} \right)}
\]
(84)

For part 2, by applying the same argument as in the case of \( L > 1 \), we can show that there exists a finite \( N_2' \) such that for all \( n \geq N_2' \), we have \( P'_2 \leq 4e^{-n\left( \frac{b \log \frac{1}{\log \pi_0}}{\log \pi_0} \right)} \). Combining both parts, we complete the proof for the case of \( L = 1 \).

Finally, combining both cases of \( L > 1 \) and \( L = 1 \), we show that the OPF policy achieves delay rate-function at least \( I_0(b) \) in general time-correlated channel.

V. THE RELATIONSHIP BETWEEN \( I_U(b) \) AND \( I_0(b) \)

We have already shown that \( I_U(b) \) is an upper bound on the delay rate-function under any possible scheduling policies. Also, we show that the delay rate-function that can be achieved by any OPF policy is no smaller than \( I_0(b) \). In this section, we investigate the relationship between the values of these two rate-functions. We show that if the channel is non-negatively correlated (Condition A) and the distribution of the OFF period is memoryless (Condition B), any OPF policy can achieve the optimal delay rate-function, i.e., \( I_U(b) = I_0(b) \) for any fixed integer \( b \geq 0 \).

1) Condition A: Any vector of finite channel states satisfies non-negative correlation condition:

In statistics, two random variables \( X, Y \) are non-negatively correlated if \( \text{cov}(X, Y) = \mathbb{E}[XY] - \mathbb{E}[X] \mathbb{E}[Y] \geq 0 \). The following definition from [13] is a reasonable generalization of non-negative correlation to a set of random variables.

Definition 2: (Non-negative Correlation Condition) Let \( X = (X_1, \cdots, X_n) \) be a vector of random variables. Then the random vector \( X \) satisfies non-negative correlation condition if the conditional expectation \( \mathbb{E}[X_i, i \in I | X_j = t_j, \forall j \in J] \) is non-decreasing in each \( t_j, j \in J \) for any disjoint index set \( I, J \subseteq \{1, n\} \).

Lemma 5: If condition A holds, then the class of OPF policies can achieve a delay rate-function of \( I_0(b) \) with parameter \( \bar{q} \) replaced by \( \bar{q} \), which is given by:

\[
\bar{q} = \min_{k \in \{0, 1, \cdots\}} \frac{P(D = k + 1)}{1 - F_D(k)}
\]
(85)

Proof: Note that condition A implies:

\[
\begin{align*}
&\mathbb{E}[C_{i,j}(t)] | C_{i,j}(t_1) = c_1, \cdots, C_{i,j}(t_p) = c_p, C_{i,j}(t - 1) = 1] \\
&\geq \mathbb{E}[C_{i,j}(t)] | C_{i,j}(t_1) = c_1, \cdots, C_{i,j}(t_p) = c_p, C_{i,j}(t - 1) = 0
\end{align*}
\]
(86)

for any \( c_1, \cdots, c_p \in \{0, 1\} \).

Since \( C_{i,j}(t) \) is a binary random variable, we can rewrite the inequality as follows:

\[
\begin{align*}
P(C_{i,j}(t) = 1 | S(t_1) = s_1, \cdots, S(t_p) = s_p, C_{i,j}(t - 1) = 1) \\
\geq P(C_{i,j}(t) = 1 | S(t_1) = s_1, \cdots, S(t_p) = s_p, C_{i,j}(t - 1) = 0)
\end{align*}
\]
(87)

where \( s_1, \cdots, s_p \) are any arbitrary binary \( n \times n \) matrices.

Substituting inequality (87) into (37), the result follows.

2) Condition B: Distribution \( D \) has a memoryless property:

When the distribution \( D \) has a memoryless property, namely \( D \) is geometrically distributed, we have:

\[
\frac{1 - F_D(k + n - 1)}{1 - F_D(k + n - 1)} = \frac{1 - F_D(k + n - 2)}{1 - F_D(k + n - 1)} = \cdots = \frac{1 - F_D(k)}{1 - F_D(k + 1)}.
\]
(88)

for any \( k \geq 1 \) and \( 1 \leq n \leq b \). Multiplying all these \( n \) fractions, we can obtain the following equation:

\[
\left( \frac{1 - F_D(k)}{1 - F_D(k + 1)} \right)^n = \frac{1 - F_D(k)}{1 - F_D(k + n)}.
\]
(89)

Finally, if the above two conditions are both satisfied, we have the following theorem:

Theorem 2: The class of OPF policies achieve optimal delay rate-function performance under the general time-correlated channel model if conditions A and B both hold:

Proof: Condition A ensures that \( \bar{q} \) is related to the distribution of random variable \( D \) and does not depend on \( U \). If we substitute the value of \( \bar{q} \) into \( I_0(b) \), it is easy to see that the expression for \( I_0(b) \) is very similar to \( I_U(b) \), except for the terms related to the CDF of \( D \). Applying condition B, we can obtain \( I_0(b) \geq I_U(b) \) directly. Since \( I_0(b) \) is an lower bound on the delay rate-function that can be achieved by any OPF policy and \( I_U(b) \) is an upper bound on the delay rate-function under any possible scheduling policies, we can conclude that the class of OPF policies achieve delay rate-function optimality in general correlated channel model.

In fact, i.i.d. channel and non-negatively correlated Markovian channel are two special cases, in which both conditions A and B are satisfied, and thus, the optimal rate-function is achieved.

Remark 2: Under i.i.d. channel model with channel “ON” probability \( q \), conditions A and B always hold. In this case, \( U \) has a geometric distribution with parameter \( 1 - q \), and \( D \) has a geometric distribution with parameter \( q \).

\[
\mathbb{E}[C_{i,j}(t)] | C_{i,j}(t_1) = c_1, \cdots, C_{i,j}(t_k) = c_k] = \mathbb{E}[C_{i,j}(t)] = q.
\]

Since the conditional expectations remain the same for any \( c \), the non-negative correlation condition (condition A) holds.
On the other hand, since random variable $D$ is geometrically distributed, $D$ has a memoryless property, i.e., condition B holds.

**Remark 3:** Under Markovian channel model with transition matrix $T$, condition A is equivalent to the standard notion of non-negative correlation for a two-state Markov chain. In this case, $U$ has a geometric distribution with parameter $p_{10}$, and $D$ has a geometric distribution with parameter $p_{01}$. Substituting the PMF of the geometric distribution, we have

$$
\mathbb{E}[\{C_{i,j}(t)\} | C_{i,j}(t_1) = c_1, \ldots, C_{i,j}(t-1) = 1] = 1 - \frac{\log p_{10}}{\log(1-p_{01})}, \quad (90)
$$

and

$$
\mathbb{E}[\{C_{i,j}(t)\} | C_{i,j}(t_1) = c_1, \ldots, C_{i,j}(t-1) = 0] = 1 - \frac{\log p_{10}}{\log(1-p_{01})}, \quad (91)
$$

Hence, condition A is equivalent to:

$$
1 - p_{10} \geq p_{01} \iff p_{01} \geq 1 - p_{10} \leq 1, \quad (92)
$$

which is the condition for non-negative correlation in a two-state Markov chain. Similarly, condition B is satisfied because $D$ is also geometrically distributed.

**Theorem 3:** Under negatively correlated Markovian channel model, i.e., $p_{01} + p_{10} > 1$, the class of OPF policies can achieve a delay rate-function that is no smaller than $\frac{\log p_{10}}{\log(1-p_{01})}$ fraction of the optimal value, where $p_{01}$ and $p_{10}$ come from the transition probability.

**Proof:** Since $p_{01} + p_{10} > 1$, the conditional probability $\mathbb{P}(C_{i,j}(t) = 1 | C_{i,j}(t-1) = 0)$ is lower bounded by $1 - p_{10}$. Thus, by using the same proof technique, we can show that the same results hold for $\log \frac{1}{p_{01}}$ replaced by $\log \frac{1}{p_{10}}$. Note that the upper bound still remains the same, therefore, it is easy to see that the delay rate-function achieved by the OPF policies is no smaller than $\frac{\log p_{10}}{\log(1-p_{01})}$ fraction of the optimal value. $\blacksquare$

## VI. Numerical Results

In this section, we conduct simulations to compare scheduling performance under different channel settings. Among all the OPF policies such as delay weighted matching (DWM) [6, 7], DWM-n and hybrid policy [10], we choose DWM in our simulations as DWM has the best empirical performance in various scenarios [7]. The DWM policy considers at most $n$ oldest packets from each queue, i.e., a total of at most $n^2$ packets and chooses the schedule that maximizes the sum of the delays in each time-slot. We consider 0-5 i.i.d. arrivals i.e.,

$$
A_i(t) = \begin{cases} 
5, & \text{with probability } \mu, \\
0, & \text{with probability } 1 - \mu,
\end{cases} \quad (93)
$$

for all $i$. The arrival processes are assumed to be independent across all the queues. For the channel model, we assume that all the channels are homogeneous and consider the following seven channel settings, channel settings 1 and 2 are i.i.d. ON/OFF channels with “ON” probability $q_1 = 0.6$ and $q_2 = 0.5$, respectively, and channel settings 3, 4, 5, 6 and 7 are Markovian channels with transition matrix $T = \begin{bmatrix} 0.94 & 0.06 \\
0.04 & 0.96 \\
0.85 & 0.15 \\
0.1 & 0.9 \\
0.01 & 0.99 \end{bmatrix}$, respectively. Note that channel settings 1, 2, 3, and 4 are non-negatively correlated, while channel settings 5, 6, and 7 are negatively correlated. In addition, we fix the channel/server number to 10, i.e., $n = 10$.

![Fig. 3. Performance comparison under different channel settings with $\mu = 0.15$, $n = 10$. Channels are i.i.d. in channel setting 1 and from channel setting 4 to 3, channels become more positively correlated.](image)

![Fig. 4. Performance comparison under different channel settings, with $\mu = 0.13$, $n = 10$. Channels are i.i.d. in channel setting 1 and from channel setting 4 to 3, channels become more positively correlated.](image)
neighborhood of the mean value $nq$ with high probability. Thus, the service variation under Markovian channels should be larger than the counterpart under the i.i.d. channels.

Given the same mean service rate, we know from basic queueing theory that the Markovian channel setting should have a larger delay. Moreover, if we further lower the arrival rate (e.g., decrease $\mu$ from 0.15 to 0.13), the simulation results show that as the channels become more positively correlated the delay gap increases further.

Next, we would like to explore the story under negatively correlated channels. As before, we plot the delay violation probability against different $b$ under channel settings 2, 5, 6, and 7 for $\mu = 0.15$. As we can see from Fig. 5, when channel becomes more negatively correlated, the system has a smaller delay. An extreme example is channel setting 5, where alternating ON-OFF-ON... will be observed with high probability. Once the initial state is determined, the service rate of the system is almost deterministic. According to basic queueing theory, smaller service variation should give us a smaller delay. An extreme example is channel setting 5, channels become more negatively correlated.

**VII. Conclusions**

In this paper, we considered the scheduling problem of an OFDM downlink system with multiple users and multiple subcarriers with time-correlated channels. Our theoretical result shows that the class of oldest packets first (OPF) policies, which give a higher priority to large delay packets, is delay rate-function optimal when two conditions are both satisfied: 1) The channel is non-negatively correlated, and 2) The distribution of “OFF” period has a memoryless property. An open problem for future work is to consider multi-rate channels rather than ON/OFF channels with a unit capacity. In this multi-rate channel model, a lexicographically-optimal algorithm that makes the HOL delays most balanced over all the queues is expected to achieve good delay performance. However, the channel-rate heterogeneity introduces a new trade-off between maximizing instantaneous throughput and balancing delays. Nonetheless, we believe that the results in this paper will provide useful insights for designing high-performance scheduling policies for more general scenarios.

**Appendix A**

**Proof of Lemma 1**

Applying the law of total probability to different values of $C_{i,j}(t-1)$, we have:

$$\mathbb{P}(C_{i,j}(t) = 1|\mathcal{S}(t_1), \mathcal{S}(t_2), \cdots, \mathcal{S}(t_d), S(t-1))$$

where

$$k = \sum_{t=0}^{\infty} \left( \mathbb{P}(C_{i,j}(t) = 1|\mathcal{S}(t_1), \cdots, \mathcal{S}(t_d), C_{i,j}(t-1) = 1, \hat{T}(t, C_{i,j}) = k) \times \mathbb{P}(\hat{T}(t, C_{i,j}) = k|\mathcal{S}(t_1), \cdots, \mathcal{S}(t_d), C_{i,j}(t-1) = 1) \right).$$

(95)

Recall that $\hat{T}(t, C_{i,j})$ is the length of the time period from the beginning of its last state-change (“ON” to “OFF” or “OFF” to “ON”) time-slot before time-slot $t$ to the beginning of time-slot $t-1$. Summing up all possible values for $\hat{T}(t, C_{i,j})$, we have:

$$\mathbb{P}(C_{i,j}(t) = 1|\mathcal{S}(t_1), \mathcal{S}(t_2), \cdots, \mathcal{S}(t_d), C_{i,j}(t-1) = 1)$$

$$= \sum_{k=0}^{\infty} \left( \mathbb{P}(C_{i,j}(t) = 1|\mathcal{S}(t_1), \cdots, \mathcal{S}(t_d), C_{i,j}(t-1) = 1, \hat{T}(t, C_{i,j}) = k) \times \mathbb{P}(\hat{T}(t, C_{i,j}) = k|\mathcal{S}(t_1), \cdots, \mathcal{S}(t_d), C_{i,j}(t-1) = 1) \right).$$

(94)

Note that $C_{i,j}(t)$ only depends on the last known state (here is $C_{i,j}(t-1)$) and the last state-change time-slot before time-slot $t$, thus, we can simplify the above equation as:

$$\mathbb{P}(C_{i,j}(t) = 1|\mathcal{S}(t_1), \mathcal{S}(t_2), \cdots, \mathcal{S}(t_d), C_{i,j}(t-1) = 1)$$

$$= \sum_{k=0}^{\infty} \left( \mathbb{P}(C_{i,j}(t) = 1|\mathcal{S}(t_1), \cdots, \mathcal{S}(t_d), C_{i,j}(t-1) = 1, \hat{T}(t, C_{i,j}) = k) \times \mathbb{P}(\hat{T}(t, C_{i,j}) = k|\mathcal{S}(t_1), \cdots, \mathcal{S}(t_d), C_{i,j}(t-1) = 1) \right)$$

$$= \sum_{k=0}^{\infty} \mathbb{P}(U \geq k + 2) \mathbb{P}(U \geq k + 1) \mathbb{P}(\hat{T}(t, C_{i,j}) = k|\mathcal{S}(t_1), \cdots, \mathcal{S}(t_d), C_{i,j}(t-1) = 1)$$

$$\geq \min_{k \in \{0, 1, \cdots \}} \mathbb{P}(U \geq k + 1) \mathbb{P}(\hat{T}(t, C_{i,j}) = k|\mathcal{S}(t_1), \cdots, \mathcal{S}(t_d), C_{i,j}(t-1) = 1)$$

(96)

Applying the same method, we have:

$$\mathbb{P}(C_{i,j}(t) = 1|\mathcal{S}(t_1), \mathcal{S}(t_2), \cdots, \mathcal{S}(t_d), C_{i,j}(t-1) = 0)$$

$$\geq \min_{k \in \{0, 1, \cdots \}} \mathbb{P}(D = k + 1) \mathbb{P}(\hat{T}(t, C_{i,j}) = k|\mathcal{S}(t_1), \cdots, \mathcal{S}(t_d), C_{i,j}(t-1) = 0).$$

(97)
\[ \mathbb{P}(C_{i,j}(t) = 1 | S(t_1), S(t_2), \ldots, S(t_d), S(t-1)) \]
\[ \geq \min_{k \in \{0, 1, \ldots\}} \frac{1 - F_U(k+1)}{1 - F_U(k)} \cdot \mathbb{P}(C_{i,j}(t-1) = 1 | S(t_1), \ldots, S(t_d), S(t-1)) \]
\[ + \min_{k \in \{0, 1, \ldots\}} \frac{1 - P_D(k+1)}{1 - P_D(k)} \cdot \mathbb{P}(C_{i,j}(t-1) = 0 | S(t_1), \ldots, S(t_d), S(t-1)) \]
\[ = \min \left\{ \min_{k \in \{0, 1, \ldots\}} \frac{1 - F_U(k+1)}{1 - F_U(k)}, \min_{k \in \{0, 1, \ldots\}} \frac{1 - P_D(k+1)}{1 - P_D(k)} \right\} = \hat{q}. \quad (98) \]

The above result gives us the lower bound on the conditional probability that \( C_{i,j}(t) = 1 \) given \( S(t_1), \ldots, S(t_d), S(t-1) \), thus, by simply replacing \( q \) with \( \hat{q} \) in the proof of Lemma 6 in [7], the result stated in the lemma follows.

**APPENDIX B**

**Proof of Lemma 2**

From Lemma 1, there exists an \( N_F > 0 \), for all \( n > N_F \), the probability that \( X_F(t) = 0 \) occurs given the connectivity at time-slots \( t_1, \ldots, t_d, t-1 \) can be bounded as,

\[ \mathbb{P}(X_F(t) = 0 | S(t_1), \ldots, S(t_d), S(t-1)) \]
\[ \leq \left( \frac{n}{1 - q} \right)^{\frac{\tau}{\log q}} e^{-n \log \frac{1}{1 - q}}. \quad (99) \]

Now, we are seeking an upper bound on the probability that there are exactly \( t + a \) time-slots that satisfy \( X_F(t) = 1 \) among all \( t + b \) time-slots during the time interval \([-t-b, -1]\).

\[ \mathbb{P} \left( \bigcup_{\tau = -t-b}^{t-a} X_F(\tau) = t + a \right) \]
\[ \leq \mathbb{P} \left( \bigcup_{t_1 < t_2 < \ldots < t_{a-n}} X_F(t_1) = 0, \ldots, X_F(t_{b-a}) = 0 \right) \]
\[ \leq \left( \frac{t + b}{t + a} \right)^{\max_{t_1 < t_2 < \ldots < t_{a-n}}} \mathbb{P} \left( X_F(t_1) = 0, \ldots, X_F(t_{b-a}) = 0 \right). \quad (100) \]

Applying the chain rule of conditional probability, we have:

\[ \mathbb{P} \left( X_F(t_1) = 0, \ldots, X_F(t_{b-a}) = 0 \right) \]
\[ = \mathbb{P}(X_F(t_1) = 0) \mathbb{P}(X_F(t_2) = 0 | X_F(t_1) = 0) \times \cdots \times \mathbb{P}(X_F(t_{b-a}) = 0 | X_F(t_1) = 0, \ldots, X_F(t_{b-a-1}) = 0). \quad (101) \]

Next, we consider the R. H. S. of (44). The upper bound on the first term is quite obvious: substituting \( q \) by the stationary probability \( 1 - \pi_0 \) in lemma 6 in [7], we have:

\[ \mathbb{P}(X_F(t_1) = 0) \leq \left( \frac{n}{\pi_0} \right)^{\frac{\tau}{\log \pi_0}} e^{-n \log \pi_0}. \quad (102) \]

For the \( d^{th} (d > 1) \) term, it is the probability of \( \{ X_F(t_d) = 0 \} \) happens given \( \{ X_F(t_1) = 0, \ldots, X_F(t_{d-1}) = 0 \} \) occurs.

Now, we want to obtain a bound for \( \mathbb{P}(X_F(t_d) = 0 | X_F(t_1) = 0, \cdots, X_F(t_{d-1}) = 0) \):

\[ \mathbb{P}(X_F(t_d) = 0 | X_F(t_1) = 0, \cdots, X_F(t_{d-1}) = 0) \]
\[ = \mathbb{P}(X_F(t_d) = 0 | X_F(t_1) = 0, \cdots, X_F(t_{d-1}) = 0) \]
\[ \times \mathbb{P}(X_F(t_{d-1}) = 0 | X_F(t_1) = 0, \cdots, X_F(t_{d-1}) = 0) \]
\[ + \mathbb{P}(X_F(t_d) = 0 | X_F(t_1) = 0, \cdots, X_F(t_{d-1}) = 0) \]
\[ \times \mathbb{P}(X_F(t_{d-1}) = 0 | X_F(t_1) = 0, \cdots, X_F(t_{d-1}) = 0) = 0. \]

(103)

We use \( S \) to represent the connectivity \( S(\cdot) \) at each time-slot, and define \( F \) to be a collection of all possible vectors \( S \) such that \( X_F(t_1) = 0, \cdots, X_F(t_{d-1}) = 0, X_F(t_d - 1) = 0 \).

Then we evaluate the following term:

\[ \mathbb{P}(X_F(t_d) = 0 | X_F(t_1) = 0, \cdots, \]
\[ X_F(t_{d-1}) = 0, X_F(t_d - 1) = 0) \]
\[ = \sum_{S \in F} \mathbb{P}(X_F(t_d) = 0 | S) \cdot \mathbb{P}(S | F) \]
\[ = \sum_{S \in F} \mathbb{P}(X_F(t_d) = 0 | S(t_1), \ldots, S(t_{d-1}), S(t_d - 1)) \cdot \mathbb{P}(S | F) \]
\[ \leq \left( \frac{n}{1 - q} \right)^{\frac{\tau}{\log q}} \sum_{S \in F} \mathbb{P}(S | F) \]
\[ = \left( \frac{n}{1 - q} \right)^{\frac{\tau}{\log q}} \sum_{S \in F} \mathbb{P}(S | F) \]. \quad (104)

where the inequality comes from Lemma 1. Similarly, we have the same result for \( \mathbb{P}(X_F(t_d) = 0 | X_F(t_1) = 0, \cdots, X_F(t_{d-1}) = 0, X_F(t_d - 1) = 0) \), thus we have:

\[ \mathbb{P}(X_F(t_d) = 0 | X_F(t_1) = 0, \ldots, X_F(t_{d-1}) = 0) \]
\[ \leq \left( \frac{n}{1 - q} \right)^{\frac{\tau}{\log q}} \sum_{S \in F} \mathbb{P}(S | F) \]. \quad (105)

Combining what we have already derived in (45) and (48), we have:

\[ \mathbb{P}(X_F(t_1) = 0, \ldots, X_F(t_{b-a}) = 0) \]
\[ \leq \left( \frac{n}{\pi_0} \right)^{\frac{\tau}{\log \pi_0}} \left( \frac{n}{1 - q} \right)^{\frac{\tau}{\log q}} e^{-n (\log \pi_0 + (b-a-1) \log \frac{1}{1 - q})}. \quad (106) \]

This inequality holds for any \( t_1, t_2, \ldots, t_{b-a} \), hence,

\[ \max_{t_1, t_2, \ldots, t_{b-a}} \mathbb{P}(X_F(t_1) = 0, \ldots, X_F(t_{b-a}) = 0) \]
\[ \leq \left( \frac{n}{\pi_0} \right)^{\frac{\tau}{\log \pi_0}} \left( \frac{n}{1 - q} \right)^{\frac{\tau}{\log q}} e^{-n (\log \pi_0 + (b-a-1) \log \frac{1}{1 - q})}. \quad (107) \]

Thus, we have for all \( a \leq b - 1 \):

\[ \mathbb{P}( \bigcup_{\tau = -t-b}^{t-a} X_F(\tau) = t + a ) \]
\[ \leq \left( \frac{t + b}{t + a} \right)^{\max_{t_1, t_2, \ldots, t_{b-a}}} \mathbb{P}(X_F(t_1) = 0, \ldots, X_F(t_{b-a}) = 0) \]
\[ \leq 2^{t+b} \left( \frac{n}{\pi_0} \right)^{\frac{\tau}{\log \pi_0}} \left( \frac{n}{1 - q} \right)^{\frac{\tau}{\log q}} e^{-n (\log \pi_0 + (b-a-1) \log \frac{1}{1 - q})}. \quad (108) \]
ACKNOWLEDGMENT

This work is funded in part by NSF grants CNS-1446582, CNS-1421576, and CNS-1518829, ONR grant N00014-15-1-2166, and ARO grant W911NF-14-1-0368.

REFERENCES

[1] L. Tassiulas and A. Ephremides, “Stability properties of constrained queueing systems and scheduling policies for maximum throughput in multihop radio networks,” IEEE Trans. Automatic Control, vol. 37, no. 12, pp. 1936-1948, 1992.

[2] S. Bodas, S. Shakkottai, L. Ying, and R. Srikant, “Scheduling in Multi-Channel Wireless Networks: rate-function Optimality in the Small Buffer Regime,” in Proceedings of ACM SIGMETRICS, 2009.

[3] S. Kittipiyakul and T. Javidi, “Delay-optimal server allocation in multiqueue multiserver systems with time-varying connectivities,” IEEE Transactions on Information Theory, vol. 55, no. 5, pp. 2319-2333, 2009.

[4] S. Bodas, S. Shakkottai, L. Ying, and R. Srikant, “Scheduling in multi-channel wireless networks: rate-function optimality in the small-buffer regime,” in ACM Proceedings of the eleventh international joint conference on Measurement and modeling of computer systems (SIGMETRICS), 2009, pp. 121-132.

[5] S. Bodas and T. Javidi, “Scheduling for multi-channel wireless networks: Small delay with polynomial complexity,” in 2011 International Symposium on Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks (WiOpt). IEEE, 2011, pp. 78-85.

[6] M. Sharma and X. Lin, “OFDM downlink scheduling for delay-optimality: Many-channel many-source asymptotics with general arrival processes,” IEEE Information Theory and Applications Workshop (ITA), 2011.

[7] M. Sharma and X. Lin, “OFDM downlink scheduling for delay-optimality: Many-channel many-source asymptotics with general arrival processes,” Purdue University, Tech. Rep., 2011. [Online]. Available: https://engineering.purdue.edu/%7elinx/papers.html

[8] B. Ji, C. Joo, and N. B. Shroff, “Delay-Based Back-Pressure Scheduling in Multihop Wireless Networks,” IEEE/ACM Transactions on Networking, vol. 21, no. 5, pp. 1539-1552, 2013.

[9] V. G. Kulkarni, Modeling and Analysis of Stochastic Systems, CRC Press, 1996

[10] B. Ji, G. Gupta, X. Lin and N. B. Shroff, “Low-Complexity Scheduling Policies for Achieving Throughput and Asymptotic Delay Optimality in Multi-Channel Wireless Networks,” IEEE/ACM Transactions on Networking, vol. 22, no. 6, pp. 1911-1924, 2014.

[11] B. Ji, G. Gupta, M. Sharma, X. Lin and N. B. Shroff, “Achieving Optimal Throughput and Near-Optimal Asymptotic Delay Performance in Multi-Channel Wireless Networks with Low Complexity: A Practical Greedy Scheduling Policy,” IEEE/ACM Transactions on Networking, vol. 23, no. 3, pp. 880-893, 2015.

[12] D. P. Dubhashi and A. Panconesi, Concentration of Measure for the Analysis of Randomized Algorithms, Cambridge University Press, 2009.

[13] D. Dubhashi, D. Ranjan, “Balls and bins: A study in negative dependence,” BRICS Report Series 3, no. 25, 1996.