QCD SUM RULE ANALYSIS OF LEADING TWIST NON–SINGLET OPERATOR MATRIX ELEMENTS

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Abstract

We use QCD sum rules to determine the difference between moments of the non–singlet structure functions. This combination decouples from the singular behaviour of the structure functions near \( x = 1 \) as calculated in the quark–gluon basis and thus should lead to improved sum rule predictions. However, we find there are still very large errors due to higher order corrections. In order to refine the error analysis, we study the effect of renormalon ambiguities on our QCD sum rules results.

1 Introduction

The application of QCD sum rules to deep inelastic scattering has been pioneered by Belyaev and Ioffe [1, 2] and subsequently developed further by Balitsky et al [3]. In [4] we used QCD sum rules to determine the leading twist-two non-singlet operator matrix elements (OMEs) contributing to deep inelastic scattering from nucleon targets and an error analysis was carefully performed. The comparison with the measured values was disappointing and we investigated the source of errors in detail. We found large sources of error from both the neglect of higher order terms in the expansion in powers of \( 1/p^2 \) where \( p \) is the momentum flowing through the nucleon source (“higher–dimension” terms) and the neglect of higher order terms in the perturbative expansion in powers of the QCD coupling. The phenomenological analysis is very sensitive to the singular contributions in the structure functions at \( x = 1 \) corresponding to unphysical quark–gluon kinematics. The authors of [5] have suggested that this problem can be evaded if one uses the so–called “Ioffe–time coordinate space distributions”, as a suitable alternative to the conventional parton momentum distributions. Here we examine another way of suppressing the singular contributions at \( x = 1 \). To do this, we consider specific combinations of moments which eliminate the effects of the singular corrections at \( x = 1 \). Unfortunately, there is not a great improvement in the QCD sum rule predictions. Again there seem to be anomalously large corrections at \( O(\alpha_s \langle \bar{q}q \rangle^2/(p^2)^2) \) coming from the radiative correction to the quark condensate term. We also investigate further corrections due to renormalon ambiguities giving power law corrections which cannot be absorbed in the terms arising from quark and gluon condensates. Indeed, renormalon singularities and their implications for theoretical predictions have been studied for some time and it has now become clear that one cannot sensibly talk of power–suppressed corrections if the ambiguities in the leading term are not under control [6, 7, 8]. The reason the renormalon ambiguity arises is that the QCD sum rule provides an estimate for an OME where the operator is normalised at the scale \( p^2 \). Estimating the OMEs’ dependence on \( p^2 \) involves perturbative calculations that introduce the renormalon ambiguities. There is a
simple correspondence between renormalon positions and the power corrections to fixed–order perturbative predictions evaluated with an infrared cutoff [10]. Following Webber’s approach [11], we impose such a cutoff by introducing a small mass $m_g^2$ into the gluon propagator and examine the effect on the QCD sum rules determination of the structure functions moments and whether it is process dependent or not, i.e. whether the effect is important for some moments while it is negligible for others.

2 QCD sum rules for the difference of moments

Our starting point is the result obtained in [1] for the quark distributions (valid for intermediate values of $x$):

$$xu_v(x, Q^2) + M^2 A^{vp}(x, Q^2) = \frac{M^6}{2\lambda_N^2} e^{m^2/M^2} L^{-\frac{4}{9}} \left\{ 4E_2 \left( \frac{W^2}{M^2} \right) x(1-x)^2(1+8x) + \frac{b}{M^4} \left( -\frac{4}{27} + \frac{7}{6} - \frac{19}{12} x + \frac{97}{108} x^2 \right) E_0 \left( \frac{W^2}{M^2} \right) + \frac{8}{9} \alpha_s a^2 \left[ \frac{46}{9} x - \frac{38}{9} x^2 - \frac{2}{9} \frac{x}{1-x} (1+14x) \right] \right\}$$

(1)

$$xd_v(x, Q^2) + M^2 A^{vp}(x, Q^2) = \frac{M^6}{2\lambda_N^2} e^{m^2/M^2} L^{-\frac{4}{9}} \left\{ 4E_2 \left( \frac{W^2}{M^2} \right) x(1-x)^2(1+2x) + \frac{b}{M^4} \left( -\frac{4}{27} + \frac{7}{6} - \frac{11}{54} x \right) E_0 \left( \frac{W^2}{M^2} \right) + \frac{16}{9} \alpha_s a^2 \frac{1}{M^6} \ln \frac{Q^2}{M^2} + C - 1 \right\}$$

(2)

where $m$ is the nucleon mass, $C$ is the Euler constant,

$$a = -(2\pi)^2 \langle 0 | \bar{\psi} \psi | 0 \rangle$$

$$b = (2\pi)^2 \langle 0 | \frac{\alpha_s}{\pi} G_{\mu\nu} G^{\mu\nu} | 0 \rangle$$

$E_0(z) = 1 - e^{-z}$, $E_2(z) = 1 - e^{-z}(1 + z + \frac{z^2}{2})$, $W$ is the continuum threshold while $L = \ln(M/\Lambda) / \ln(\mu/\Lambda)$ takes account of the anomalous dimension of the currents. The dependence of $\bar{\lambda}_N^2$ on the Borel parameter $M^2$ is expressed via the mass sum rule [12]

$$\bar{\lambda}_N^2 e^{-m^2/M^2} = M^6 L^{-\frac{4}{9}} E_2 \left( \frac{W^2}{M^2} \right) + \frac{4}{3} a^2 L^\frac{4}{9}.$$  

(3)

In equations (1) and (2), the first term on the LHS comes from the resonant saturation with a nucleon double pole while the second parametrises some of the effects of the non–resonant

\footnote{In [12], the sign of the anomalous dimension of baryonic currents is wrong resulting in a wrong expression for $\bar{\lambda}_N^2(M^2)$ in [4]. Taking account of this fact affects only slightly the results and conclusions of [4].}
background corresponding to a single pole term. The RHS comes from an evaluation of the deep–inelastic scattering process in perturbative QCD.

In [4], these relations were used to estimate the quark distribution moments \( M_n^q \) defined by

\[
M_n^u = \int_0^1 x^n u(x, Q^2) \\
M_n^d = \int_0^1 x^n d(x, Q^2)
\]

which are related to the OMEs \( A_{n}^{u,d} = \langle N| \hat{O}_{n}^{u,d}| N \rangle \) and the Wilson coefficients \( C_n \) in the operator product expansion (OPE) via

\[
M_n(Q^2, \alpha_s(Q^2)) = \sum_\tau A_\tau^r(\mu_0^2, \alpha_s(\mu_0^2)) C_\tau^r(Q^2, \mu_0^2, \alpha_s(\mu_0^2))
\]

where the sum runs over the twist \( \tau \) of the operators \( \hat{O}_n \) and \( \mu_0^2 \) is the scale at which the operator \( \hat{O}_n \) is renormalised.

A phenomenological study was performed for the OMEs and it revealed a large discrepancy when comparing with experiment. In [4] we argued that this discrepancy was due to a breakdown in the perturbative analysis used in the QCD sum rules. In particular, the \( O(\alpha_s a^2) \) radiative corrections give contributions to the structure functions proportional to \( \frac{1}{1-x} \) (which must be regulated at \( x = 1 \)) and these terms gave very large contributions to the moments suggesting that the perturbation series in \( \alpha_s \) does not converge. Further, there are terms proportional to \( \delta(1-x) \) (not displayed in eqs (1) and (2) which apply at intermediate \( x \)) which give very large higher dimension terms in the moments and suggest the expansion in powers of \( \frac{1}{p^2} \) does not converge either.

One point of view promoted by Belyaev and Ioffe is that the QCD sum rule predictions should be used only for the structure functions at intermediate \( x \), keeping away from the troublesome singularities at \( x = 1 \). However we find this unconvincing as such terms come from unphysical quark and gluon singularities which must be averaged over when making comparison with data. This averaging procedure will feed the effects of the singular terms at \( x = 1 \) to intermediate \( x \) causing the same discrepancy with data as is found in the moment relations (which represent a particular form of averaging). Another possibility that has recently been proposed is that the problem can be evaded if one uses the so–called “Ioffe–time coordinate space distributions”, as a suitable alternative to the conventional parton momentum distributions.

In this paper, we wish to explore a more straightforward way of eliminating the singularities at \( x = 1 \) for it is easy to do so even in the conventional picture. In order to achieve this, one considers the difference of two consecutive moments. To see that this eliminates the effect of singularities at \( x = 1 \) we note that the structure function has the form (c.f. eqs (1) and (2)):

\[
q(x) = f_1(x) + f_2(x)\delta(1-x) + \frac{f_3(x)}{1-x}
\]

where \( f_1, f_2 \) and \( f_3 \) are non–singular functions. By integrating, one obtains the moments:

\[
M_n = \int_0^1 x^n q(x) dx = \int_0^1 f_1(x) x^n dx + f_2(1) + \int_0^1 \frac{f_3(x)}{1-x} x^n dx .
\]

As mentioned in [4], in order to find the “physical” moments one needs to regulate the third term:

\[
A_n = \int_0^1 f_1(x) x^n dx + f_2(1) + \int_0^1 \frac{f_3(x) x^n - f_3(1)}{1-x} dx .
\]
Figure 1: Twist–two non–singlet OMEs’ difference following from QCD sum rules together with the error estimates. Also shown are the experimental determinations of the OMEs’ difference (diamonds).

Now we see that if we subtract $A_{n}$ from $A_{n+1}$ the effect of the singularity $\delta(1-x)$ disappears while that of $\frac{1}{1-x}$ is reduced to an integral of a regular function (the effect of the regulating term $\frac{f_3(1)}{1-x}$ cancels).

$$K_n = A_{n+1} - A_n = \int_0^1 f_1(x)x^{n+1}dx - \int_0^1 f_1(x)x^n dx - \int_0^1 f_3(x)x^n dx$$  \hspace{0.5cm} (9)

Using eqs (1) and (2) we may write down the QCD sum rules for this difference in moments and arrive at:

$$K_n + \frac{M^2}{m^2}D_n^u = \frac{M^6}{2\lambda_N^2} e^{m^2/M^2} L^{-\frac{1}{4}} \left\{ \right.$$  

$$4E_2 \left( \frac{W^2}{M^2} \right) $$  

$$\left( \frac{1}{n+2} + \frac{6}{n+3} - \frac{15}{n+4} + \frac{8}{n+5} - \frac{1}{n+1} - \frac{6}{n+2} + \frac{15}{n+3} - \frac{8}{n+4} \right)$$  

$$+ \frac{b}{M^4} E_0 \left( \frac{W^2}{M^2} \right) $$  

$$\left( \frac{4}{27} \frac{1}{n} + \frac{7}{6n+1} - \frac{19}{12n+2} + \frac{97}{108n+3} + \frac{4}{27n-1} - \frac{71}{6n} + \frac{19}{12n+1} - \frac{97}{108n+2} \right)$$  

$$+ \frac{8\alpha_s}{9\pi M^6} \left[ \frac{46}{9n+1} + \frac{38}{9n+2} - \frac{2}{n+3} - \frac{46}{9n} + \frac{38}{9n+1} + \frac{2}{n+2} \right.$$  

$$+ \frac{2}{9n+1} + \frac{28}{9n+2} \right\}$$  \hspace{0.5cm} (10)
Figure 2: Analysis of the importance of the various contributions to the QCD sum rules to the OMEs’ difference. In (a) we consider the up quark operators. The stars show the results neglecting the gluon condensate and the $O(\alpha_s a^2)$ corrections. The dots show the results including the gluon condensate. The triangles show the complete result. (b) shows the equivalent quantities for the down quark.

$$K_n^d + \frac{M^2}{m^2} D_n^d = \frac{M^6}{2\lambda^2 N} e^{m^2/M^2} L^{-\frac{1}{2}} \left\{ \right.$$  

$$4E_2 \left( \frac{W^2}{M^2} \right) \left( \frac{1}{n+2} - \frac{3}{n+4} + \frac{2}{n+5} - \frac{1}{n+1} + \frac{3}{n+3} - \frac{2}{n+4} \right)$$  

$$+ \frac{b}{M^4} E_0 \left( \frac{W^2}{M^2} \right) \left( -\frac{4}{27} \frac{1}{n} + \frac{7}{6} \frac{1}{n+1} - \frac{11}{12} \frac{1}{n+2} - \frac{7}{54} \frac{1}{n+3} + \frac{4}{27} \frac{1}{n-1} - \frac{7}{6} \frac{1}{n} + \frac{11}{12} \frac{1}{n+1} + \frac{7}{54} \frac{1}{n+2} \right)$$  

$$+ \frac{-16 \alpha_s a^2}{9 \pi M^6} \left[ -2 (\psi'(n+2) + \psi'(n+4)) + 2 (\psi'(n+1) + \psi'(n+3)) \right]$$  

$$+ \frac{-8}{9} \frac{1}{n} + \frac{13}{9} \frac{1}{n+1} + \frac{247}{36} \frac{1}{n+2} + \frac{-6}{n+3}$$  

$$+ \left( \ln \left( \frac{\mu_0^2}{M^2} \right) + C - 1 \right) \left( \frac{1}{n+1} + \frac{1}{n+3} \right) \right\}.$$  

(11)

The same phenomenological analysis as in [4] can be done for these sum rules and the results, for the same numerical values of the parameters as in [4], ($a = 0.55 \pm 0.20 GeV^3$, $b = 0.45 \pm 0.10 GeV^4$, $\mu = 0.5 GeV$, $\mu_0^2 = 1 GeV^2$, $\Lambda = 125 \pm 25 MeV$, $m = 1.00 \pm 0.15 GeV$ and $W^2 = 2.3 \pm 0.23 GeV^2$) are shown in Figs. 1 and 2.

One may see from Fig. 1 that there is still a discrepancy between the prediction of QCD sum rules and the experimental measurement. To analyse why, we show in Fig. 2 the effect of including various contributions. One may see that the $O(\alpha_s a^2)$ contribution is still very large suggesting that the perturbative series is not convergent.
Figure 3: Diagrams involved in the calculation of the OME. The solid lines represent quarks, the wavy lines represent gluons and the circle represents the operator.

3 Renormalon effects

Given that the QCD sum rule predictions fail to reproduce well either the moments or the difference of moments it is perhaps appropriate to consider other possible sources for the discrepancy beyond the immediate (and depressing) possibility that the expansion in higher dimension terms and higher order radiative terms is not convergent. We have identified one possible further error coming from renormalons.

To see how renormalons affect our analysis of the moments, we look again at the OPE form for the moments, eq (5). There are two scales in our QCD sum rules analysis: \( p^2 \) and \( Q^2 \). The sensitivity to nucleon momentum scale \( p^2 \), which we assume to be large and euclidean in order to justify the perturbative evaluations of the graphs, resides in the OME \( A_n \) in eq (5), while the dependence on the momentum transfer scale, \( Q^2 \), resides in the coefficient function \( C_n \). By going to large \( Q^2 \) the coefficient functions may be reliably calculated, as discussed in [4]. However, the \( p^2 \) dependence cannot be reliably calculated in perturbation theory because the matching condition of QCD sum rules requires values of \( p^2 \approx 1 \text{ GeV}^2 \) where non–perturbative corrections are important. Once the \( Q^2 \)–dependence is removed using the OPE the only scale other than \( p^2 \) is \( \mu^2 \), the operator renormalisation scale. The latter is in principle arbitrary, the operator \( \mu^2 \)–dependence being cancelled by the \( \mu^2 \)–dependence of the coefficient functions. In practice, however, in evaluating the OME by sum rules, the normalisation scale must be chosen of the same order as the only other scale \( p^2 \) to avoid the appearance of large radiative corrections (which have not been calculated and thus do not show up explicitly in the analysis). Thus, one should interpret the result of the QCD sum rule predictions as being for the operator renormalised at a scale \( p^2 \). Then, in order to calculate the OME at a fixed scale \( \mu_0^2 \) appropriate to the comparison with the matrix elements extracted from deep inelastic scattering processes, one must determine the \( \mu^2 \)–dependence of the OME \( A_n(\mu^2) \)

\[
\langle N(p)|\hat{O}_{\beta\alpha_1...\alpha_n}(\mu^2)|N(p)\rangle = A_n(\mu^2) \ p^\beta p^{\alpha_1}...p^{\alpha_n} + \ldots.
\] (12)

To allow \( A_n(p^2) \) to be related to \( A_n(\mu_0^2) \).

To estimate the \( \mu^2 \)–dependence of the OME, one should evaluate the graphs in Fig. 3. The “renormalon” contributions are described by graphs such as those in Fig. 4 and in order to take account of their effects, one should introduce an infrared cutoff in the integrals to be done when evaluating the graphs. Equivalently one may introduce a mass \( m_g \) into the gluon propagator and expand in powers of \( \frac{m_g^2}{p^2} \), a procedure which has the advantage of maintaining Lorentz invariance [11]. Using this technique after some calculation one arrives at the following results:
anomalous dimension of the operator $\hat{A}$ so we can express the OME at different scales involves corrections of $O(\frac{1}{p^2})$ corresponding to the mixing of operators of $SU(3)$. The singular terms when $\epsilon \to 0$ determine the anomalous dimension of the operator $\hat{O}_n$, while the other terms are finite.

We now introduce the renormalisation scale $\mu^2$ and we get, up to order $g^2$,

$$\frac{1}{A_n(\mu^2)} = 1 - \frac{g^2C_F}{8\pi^2}F_1(n)\ln(-\mu^2) - \frac{g^2C_F}{8\pi^2}F_2(n) - \frac{g^2C_F}{8\pi^2}F(n)\frac{m_q^2}{-\mu^2} - \frac{g^2C_F}{8\pi^2}(1 - 2n)\frac{m_q^2}{-\mu^2}\ln\frac{m_q^2}{-\mu^2},$$

so we can express the OME $A_n$ as

$$A_n(p^2) = A_n(\mu_0^2) \left[ 1 + \frac{g^2C_F}{8\pi^2}F_1(n)\frac{p^2}{\mu_0^2} + \frac{g^2C_F}{8\pi^2}F(n)\left(\frac{m_q^2}{p^2} + \frac{m_q^2}{\mu_0^2}\right) \right. $$

$$\left. + \frac{g^2C_F}{8\pi^2}(1 - 2n)\left(\frac{m_q^2}{-p^2}\ln\frac{m_q^2}{-p^2} + \frac{m_q^2}{\mu_0^2}\ln\frac{m_q^2}{-\mu_0^2}\right) \right].$$

We see from this equation that the relation between matrix elements for operators renormalised at different scales involves corrections of $O(\frac{1}{p^2})$ corresponding to the mixing of operators of
different twist. This difference gives a further source of error in the QCD sum rule predictions which we now consider. As we argued above, one should interpret the QCD sum rule predictions as giving \( A_n(p^2) \) and so the left hand side of the equation has the form \( \frac{A_n(p^2)}{(p^2 - m^2)^2} \) for the nucleon pole contribution. The \( \ln(\mu_0^2 / \mu^2) \) and the \( \ln(\frac{m_0^2}{p^2}) \) terms give, when Borel transformed, terms proportional to \( \ln(\frac{M^2}{\mu_0^2}) \) and \( \ln(\frac{M^2}{m_0^2}) \) respectively. These terms can be neglected if, for the purpose of getting a rough estimate of the renormalon effects, we choose \( m_0^2 \sim \mu_0^2 \sim 1 GeV^2 \) to be in the QCD sum rules’ “overlapping window” \( (M^2 \sim 1 GeV^2) \). For this choice \( (m_0^2 \sim \mu_0^2) \), we can also ignore the \( \ln \frac{m_0^2}{\mu_0^2} \) term. That leaves us with the terms proportional to \( \frac{m_0^2}{p^2} + \frac{m_1^2}{p^2 - m_2^2} \). The former contributes additional terms corresponding to different poles on the phenomenological side of the sum rule

\[
\frac{1}{-p^2(p^2 - m^2)^2} = \frac{-1}{m_0^2} + \frac{-1}{m_1^2} + \frac{1}{m_2^2} \quad \text{(18)}
\]

The last term corresponds to a single pole contribution so it can be absorbed by the unknown non–resonant background effect. The first term is also expected to be small since the structure of eq (17) applies only for \( p^2 \) Euclidean and far from 0. To enforce this we introduce a threshold cutoff in the Borel integration of this term, leading to a vanishing contribution. Finally, the contribution of the second term is of the same form as that of the term proportional to \( \frac{m_1^2}{p^2} \) in eq (17). Combining the two with \( -\mu_0^2 \sim + m^2 \sim 1 GeV^2 \) gives

\[
A_n(p^2 = O(\mu_0^2)) \approx A_n(\mu_0^2) \left( 1 + 2\frac{\alpha_s(\mu_0^2)C_F}{2\pi} F(n) \frac{m_2^2}{\mu_0^2} \right). \quad \text{(19)}
\]

Physically, we view the gluon mass \( m_0^2 \) as an infrared matching parameter which represents the scale below which we switch from the perturbative to the non–perturbative domain and which is in principle much greater than \( \Lambda_{QCD} \). We shall take it to be a non–perturbative and process–independent parameter (c.f. [11]). If we denote by \( A_n^{QCDSR} \) the QCD sum rules predictions of [H] then, to first order in \( \alpha_s \), we have

\[
A_n(\mu_0^2) = A_n^{QCDSR} \left( 1 - 2\frac{\alpha_s(\mu_0^2)C_F}{2\pi} F(n) \frac{m_2^2}{\mu_0^2} \right). \quad \text{(20)}
\]

4 Discussion

Eq (20) shows that renormalon effects lead to a correction of the QCD sum rule estimate of the OME as measured in deep inelastic scattering. Following [11], we shall assume that the \( n \)–dependence given by the perturbative calculation with a gluon mass correctly gives the relative magnitude of the renormalon corrections, the only unknown being the ratio \( \frac{m_2^2}{\mu_0^2} \). We compare this renormalon effect with the deviation \( d_n^A \) of the \( A_n^{QCDSR} \) from the experimental data obtained by fitting the relation

\[
A_n^{data}(\mu_0^2) = A_n^{QCDSR} \left( 1 + d_n^A \right) \quad \text{(21)}
\]

between our QCD sum rules predictions \( A_n^{QCDSR} \) and our experimental data \( A_n^{data}(\mu_0^2) \) for the OME. The results are shown in Table 1 for the u–quark and Table 2 for the d–quark where we have taken the values \( \frac{\alpha_s(\mu_0^2 = 1 GeV^2)}{2\pi} \approx 0.054 \), \( \mu_0^2 \sim - 1 GeV^2 \) and \( m_2^2 \sim 1 GeV^2 \).
Table 1: Results for the u-quark for the numerical values $\mu_0^2 \simeq -1 GeV^2$ and $m_\gamma^2 \simeq 1 GeV^2$. $K(n) = A(n+1) - A(n)$; $A_{\text{data}}(n)$ and $K_{\text{data}}(n)$ are the experimental data while $A_{\text{QCDSR}}(n)$ and $K_{\text{QCDSR}}(n)$ represent the QCD sum rules predictions. $d^n A(n)$ represents the deviation between data and QCD sum rules predictions, $\text{Ren} A(n) = -2\frac{a_s(\mu_0^2)}{2\pi} C_F F(n) \frac{m_\gamma^2}{\mu_0^2}$ represents the “renormalon” correction, and $d^n A(n)/\text{Ren} A(n)$ represents the proportion of these two quantities. $d^n K(n)$, $\text{Ren} K(n)$ and $d^n K(n)/\text{Ren} K(n)$ represent the same quantities but for the difference of moments $K(n)$.

| n  | 2   | 3    | 4    | 5    | 6    | 7    | 8    | 9    |
|----|-----|------|------|------|------|------|------|------|
| $F(n)$ | 0.5 | -0.83 | -2.75 | -5.12 | -7.85 | -10.89 | -14.20 | -17.75 |
| $A_{\text{data}}(n)$ | 0.11 | 0.043 | 0.021 | 0.011 | 0.0064 | 0.0039 | 0.0025 | 0.0017 |
| $A_{\text{QCDSR}}(n)$ | 0.32 | 0.31 | 0.32 | 0.34 | 0.35 | 0.36 | 0.37 | 0.38 |
| $d^n A(n)$ | -0.67 | -0.86 | -0.94 | -0.97 | -0.98 | -0.99 | -0.995 | -0.995 |
| $\text{Ren} A(n)$ | 0.072 | -0.12 | -0.40 | -0.74 | -1.13 | -1.57 | -2.04 | -2.56 |
| $d^n A(n)/\text{Ren} A(n)$ | -9.26 | 7.18 | 2.36 | 1.31 | 0.87 | 0.63 | 0.49 | 0.39 |
| $K_{\text{data}}(n)$ | -0.062 | -0.023 | -0.0098 | -0.0047 | -0.0025 | -0.0014 | -0.0008 |
| $K_{\text{QCDSR}}(n)$ | -0.0035 | 0.0088 | 0.012 | 0.013 | 0.013 | 0.012 | 0.011 |
| $d^n K(n)$ | 16.49 | -3.54 | -1.78 | -1.35 | -1.19 | -1.11 | -1.07 |
| $\text{Ren} K(n)$ | 17.04 | -10.20 | -9.58 | -11.22 | -13.58 | -16.34 | -19.46 |
| $d^n K(n)/\text{Ren} K(n)$ | 0.97 | 0.35 | 0.19 | 0.12 | 0.088 | 0.068 | 0.055 |

Table 2: The same explanation as for the previous table but for the d–quark.

| n  | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    |
|----|------|------|------|------|------|------|------|------|
| $A_{\text{data}}(n)$ | 0.031 | 0.011 | 0.0047 | 0.0023 | 0.0012 | 0.0007 | 0.0004 | 0.0003 |
| $A_{\text{QCDSR}}(n)$ | 0.70 | 0.61 | 0.55 | 0.50 | 0.47 | 0.44 | 0.41 | 0.39 |
| $d^n A(n)$ | -0.95 | -0.98 | -0.991 | -0.995 | -0.997 | -0.998 | -0.999 | -0.999 |
| $\text{Ren} A(n)$ | 0.072 | -0.12 | -0.40 | -0.74 | -1.13 | -1.57 | -2.04 | -2.56 |
| $d^n A(n)/\text{Ren} A(n)$ | -13.25 | 8.18 | 2.50 | 1.35 | 0.88 | 0.64 | 0.49 | 0.39 |
| $K_{\text{data}}(n)$ | -0.020 | -0.0064 | -0.0024 | -0.0011 | -0.0005 | -0.0003 | -0.0001 |
| $K_{\text{QCDSR}}(n)$ | -0.087 | -0.061 | -0.045 | -0.036 | -0.029 | -0.025 | -0.021 |
| $d^n K(n)$ | -0.77 | -0.89 | -0.95 | -0.97 | -0.98 | -0.99 | -0.995 |
| $\text{Ren} K(n)$ | 1.42 | 2.37 | 3.40 | 4.43 | 5.44 | 6.43 | 7.38 |
| $d^n K(n)/\text{Ren} K(n)$ | -0.54 | -0.38 | -0.28 | -0.22 | -0.18 | -0.15 | -0.13 |
One can do the same analysis for the difference of moments $K_n = A_{n+1} - A_n$, to obtain

$$K_n(\mu_0^2) = K_n^{QCDSR} \left( 1 - 2\frac{\alpha_S(\mu_0^2) C_F}{2\pi} \frac{A_{n+1}^{QCDSR} F(n+1) - A_n^{QCDSR} F(n) m_g^2}{F_n^{QCDSR} - A_n^{QCDSR} m_g^2} \right)$$

(22)

$$K_n^{\text{data}}(\mu_0^2) = K_n^{QCDSR} \left( 1 + d_n^K \right)$$

(23)

where $K_n^{QCDSR}$ denotes the QCD sum rules predictions for the difference of moments $K_n$, and $K_n^{\text{data}}$ are the corresponding experimental values. These results are also shown in Tables 1 and 2.

We see from both these tables that, for higher moments ($n \geq 5$), the magnitude of the cutoff–dependent “renormalon” contribution for the OME $A_n$ is comparable to, or greater than, the experimental deviation $d_n^A$. This situation extends even to lower moments in the case of the difference of moments $K_n$. This means that renormalon corrections cannot be neglected in the QCD sum rule evaluation of the higher moments. It is possible to account for some of the discrepancy between the QCD sum rules and experiment via the renormalon term through a suitable choice of $m_g^2$. However, at best this only alleviates the problem slightly. A more cynical view is that the new corrections identified here are just another source of large higher dimension corrections to QCD sum rules which render the whole method of doubtful use in determining the nucleon properties. As we have seen, the problem persists for the subtracted moments so we cannot blame the unphysical quark and gluon singularities at $x = 1$ for the failure of the sum rules. The disappointing conclusion of this is that in determining the leading twist OMEs, where good experimental measurements are available to check the results, the QCD sum rule method fails largely because of uncontrollable higher dimension corrections. In our opinion, this casts doubt on the QCD sum rule predictions for other nucleon properties such as higher twist matrix elements (where the troublesome higher order corrections have not been computed).

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