Uniform magnetization dynamics of a submicron ferromagnetic disk driven by the spin–orbit coupled spin torque

P V Bondarenko\textsuperscript{1,2} and E Ya Sherman\textsuperscript{1,3}

\textsuperscript{1} Department of Physical Chemistry, Universidad del País Vasco UPV-EHU, 48080, Bilbao, Spain
\textsuperscript{2} Institute of Magnetism, National Academy of Sciences, 03142 Kyiv, Ukraine
\textsuperscript{3} IKERBASQUE Basque Foundation for Science, Bilbao, Spain

E-mail: paulvbond@gmail.com and evgeny.sherman@ehu.es

Received 27 January 2017, revised 3 May 2017
Accepted for publication 15 May 2017
Published 12 June 2017

Abstract

A simple model of magnetization dynamics in a ferromagnet/doped semiconductor hybrid structure with Rashba spin–orbit interaction (SOI), driven by an applied pulse of the electric field, is proposed. The electric current excited by the applied field is spin-polarized, due to the SOI and therefore it induces the magnetization rotation in the ferromagnetic layer via s-d exchange coupling. Magnetization dynamics dependence on the electric pulse shape and magnitude is analyzed for realistic values of the parameters. We show that it is similar to the dynamics of a damped nonlinear oscillator with the time-dependent frequency proportional to the square root of the applied electric field. The magnetization switching properties of an elliptic magnetic element are examined as a function of the applied field magnitude and direction.

Keywords: Rashba spin–orbit interaction, macrospin, magnetization switching, spin–orbit torque

(Some figures may appear in colour only in the online journal)
studied by Wang, Bauer and Hoffmann [24], however for a different form and the physics of spin torque. Current-induced dynamics of magnetization reveal a nontrivial dependence on the exchange coupling, as has also been shown by Yokoyama [25] for a ferromagnet on the surface of a topological insulator.

The effective Hamiltonian for an electron in the planar magnetic field with the Bychkov–Rashba SOI [26–28] is presented as:

$$H = \hbar^2 k^2 / 2m + \hbar \omega_m k \cdot \hat{\sigma} + \hbar \omega / 2 [k \times \hat{\sigma}] \cdot e_z,$$  \hspace{1cm} (1)

where $m = M / M_0$ is the unit vector of the in-plane magnetization of the magnetic element, $m'$ is the effective electron mass and $e_z$ is the unit vector normal to 2DEG system. The value of the s-d exchange energy per carrier $\omega_m \approx 2 \pi \times 1.6$ THz can be estimated from the values of the kinetic-exchange coupling $J_{ex}$ between the charge carriers and local magnetic moments (see (4)) [16]. The Rashba coupling constant is $\alpha \approx 10^3$ cm$^{-1}$, as can be achieved in the structures based on the InSb [8, 29]. Thus the effective magnetic field, due to the exchange and SOI terms, is

$$B_{eff} = \frac{1}{\mu_B} \frac{\delta H}{\delta \sigma} = \frac{\hbar \omega_m}{2 \mu_B} m - \frac{\hbar \omega}{2 \mu_B} [e_z \times k].$$  \hspace{1cm} (2)

As a result of applying an in-plane electric field $E$, the electron ensemble receives a drift increment of quasi-momenta $\Delta k_{dr} \approx eE \tau_p / \hbar$, where $e$ is the electron charge. The ensemble momentum relaxation time $\tau_p \sim 10^{-11}$ ns is obtained from the experimental electron mobility at room temperature [30]. For the linear regime, when $\Delta k_{dr} \ll k_T$, where $k_T$ is the electron thermal wavevector (i.e. $E \ll \hbar k_T / |e| \tau_p \approx 10$ kV cm$^{-1}$), we can make the assumption that the spin density matrix does not change the shape and just shifts: $\hat{\rho}(k, t) \rightarrow \hat{\rho}(k + \Delta k_{dr}, t)$. As a result, in the first order approximation for $\Delta k_{dr}$, the magnetic susceptibility $\chi^{(0)}$ of the 2DEG does not change. The mean spin per electron in the applied electric field, taking into account (2), is:

$$s(m, E) \approx \chi^{(0)} B_{eff} = -s_0 \frac{m + [E \cdot e_z]}{E_0},$$  \hspace{1cm} (3)

where $E_0 = \hbar \omega_m / |e| \tau_p$ and $s_0 = \chi^{(0)} \hbar \omega_m / 2 \mu_B$. At room temperature, $E_0 \approx 7$ kV cm$^{-1}$ so $E / E_0 \ll 1$ (considering the thermodynamic limitation of the model applicability $E \ll 10$ kV cm$^{-1}$ mentioned above).

The mean spin value $s_0 \approx 0.05$ could be estimated numerically, assuming that $\hat{\rho}(r, k, t)$ is a stationary diagonal operator with elements representing spin up and spin down ($\sigma = \pm 1$) Fermi distribution functions:

$$\hat{\rho}_{0,\sigma}(k, t) = \frac{1}{1 + \exp [(e_{\sigma}(k) - \mu) / T]},$$

where $e_{\sigma}(k) = \hbar^2 k^2 / 2m + \sigma \mu_B B_{eff} / (\hbar e) [31]$. If the effective magnetic field is changed, the electron spin will relax to the value (3) during the characteristic time $\tau_c \approx 10^{-2}$ ns, which was estimated with the Elliott-Yafet mechanism [6, 32–36]. Therefore, the spin adjusting time into a new position (3) under the action of changing magnetic field can be neglected, since the spin relaxation time is considerably smaller than the minimal characteristic time of the magnetization dynamics $\sim 0.1$ ns, as will be shown below.

Magnetic energy density of a thin ferromagnetic film, in the case of uniform in-plane magnetization, is:

$$W(m) = \frac{J_0 a s_0 M_0}{2 \mu_B} m \cdot s + 2 \pi M^2 m \cdot \hat{N}(m).$$  \hspace{1cm} (4)

The first term in (4) describes the s-d exchange interaction, where we use the following numerical values: $J_{ex} = 55$ meVnm$^3$ is the kinetic-exchange coupling constant [16], $M_0 = 800$ emu cm$^{-3}$ is the permalloy saturation magnetization, and $s_{el} = 5 \cdot 10^{12}$ emu cm$^{-3}$ is the electron concentration. The second term is the dipolar shaped anisotropy energy density. The demagnetization tensor $\hat{N}$ is averaged over the ferromagnetic particle volume $V_d$. It is symmetrical and has the trace $\text{Tr} (\hat{N}) = 1$ [37]. The symmetry axes of the FM particle are the principal axes of the demagnetization tensor, i.e. it is diagonal, $\hat{N}_{ij} = \delta_{ij} N_i$, where

$$N_i = -\frac{1}{4 \pi V_d} \int_0^{V_d} \int \frac{\partial^2}{\partial r^2} \int \frac{\partial \sigma}{\partial r} \cdot \frac{d^2 r}{|r - r'|}.$$  \hspace{1cm} (5)

Here the ferromagnetic particle has the elliptic cylinder shape with the major semi-axis $a_x$, minor semi-axes $a_y$ and height $h$. Then, one can use well-known expressions for the ellipsoid demagnetization tensor coefficients [37]:

$$N_i = \frac{a_x a_x h}{2} \int_0^{\infty} \frac{dx}{(s + a_x^2)^2 \sqrt{(s + a_x^2)(s + a_y^2)(s + h^2)}}.$$  \hspace{1cm} (6)

Assuming that the uniformly magnetized element is thin ($h \approx 10^{-2} a_x$), the tensor’s in-plane components $N_{i \parallel} \propto h / a_x$ are small compared to the perpendicular component $N_{i \perp} \approx 1$. Given that $|m| = 1$ and substituting (3) into (4), the magnetic free energy density (4) is:

$$W(m) = \frac{J_0 a s_0 M_0}{2 \mu_B E_0} |E \cdot m|_1 + 2 \pi M^2 [K m_x^2 + m_y^2].$$  \hspace{1cm} (7)

One can obtain the coefficient $K$ in (7) by substituting the formula (6) in the anisotropy energy expression:

$$K = N_{i \parallel} - N_{i \perp} \approx \frac{h}{a_x} \eta^2 \sqrt{1 - \eta^2} \left(\frac{3 \pi}{16} + \frac{9 \pi}{64} \eta^2\right).$$
where $\eta$ is the eccentricity of the base of the ferromagnetic disk, $\eta^2 \equiv 1 - (a_i/a_o)^2$.

The magnetization dynamics are described by the phenomenological Landau–Lifshitz–Gilbert equation in angles $\theta$ and $\varphi$ representation ($m_1 + im_2 = e^{i\varphi} \cos \theta$ and $m_2 = \sin \theta$) [38, 39]:

$$\dot{\theta} \cos \theta = -\gamma_\perp W/m_0 + \alpha_G \varphi \cos^2 \theta$$

$$\dot{\varphi} \cos \theta = -\gamma_\parallel W/m_0 - \alpha_G \dot{\theta},$$

where $\gamma_\perp = 2\mu_0\hbar$ is the electron gyromagnetic ratio and $\alpha_G \approx 0.01$ is the dimensionless Gilbert damping constant [40, 41].

The typical value of the out-of-plane component of the magnetization unit vector is $m_z^2 \sim \omega_a/\omega_\perp \approx 0.01$, which is estimated by comparing two terms in (4). Since $|m_z| \ll 1$, one could make an appropriate linearization: $m_1 + im_2 \approx e^{i\varphi}$ and $m_z \approx \theta$. Using (7), we rewrite the equations (8):

$$\dot{\varphi} = -\omega_a \left(1 - K \sin^2 \varphi - \frac{\omega_a E}{\omega_\perp E_0} \sin(\varphi - \beta)\right) - \alpha_G \dot{\theta}$$

$$\dot{\theta} = \frac{\omega_\perp E}{E_0} \cos(\varphi - \beta) + \frac{K}{2} \sin 2\varphi + \alpha_G \varphi,$$

(9)

where $\omega_\perp = 4\pi \gamma_\perp M_0 \approx 2\pi \times 28$ GHz and $\omega_\perp = J_{ex}\hbar S_0/\hbar \approx 2\pi \times 0.35$ GHz are the effective anisotropy and exchange frequencies respectively and $\varphi - \beta$ is the angle between $m$ and $E$, as shown in the figure 1. Therefore, the in-plane rotational dynamics of magnetization is determined mainly by the out-of-plane deviation, the dynamics of which are controlled by the spin polarized current, as can be seen in equations (9).

Considering $\omega_\perp \gg \omega_\parallel$ and $\alpha_G K \ll 1$, the equations (9) can be reduced to the Hamilton-like form with dissipation:

$$\dot{\varphi} = -\omega_a \theta$$

$$\dot{\theta} = \frac{\omega_\perp E}{E_0} \cos(\varphi - \beta) + \frac{K}{2} \sin 2\varphi + \alpha_G \varphi,$$

(10)

Here $\varphi$ and $-\theta$ play the roles of the generalized coordinate and momentum, correspondingly, then $\omega_\perp^{-1}$ is the analog of the inertial mass and $\omega_\perp E/\omega_\parallel E_0$ is the amplitude of the driving force.

The resulting dynamic equation for the in-plane rotation of the magnetization angle $\varphi$ derived from equation (10) is:

$$\ddot{\varphi} + \alpha_G \omega_\perp \dot{\varphi} + \frac{K}{2} \sin 2\varphi + \omega_\perp E/\omega_\parallel E_0 \cos(\varphi - \beta) = 0.$$ (11)

This result is consistent with the approach of Bazaliy [42–44], where it was shown that planar spin transfer devices with dominating easy-plane anisotropy can be described by an effective one dimensional equation for the in-plane magnetization.

The last term in (11) is a driving rotation force that is proportional to the ratio of the drift increment of the Bychkov–Rashba SOI energy density in the electron subsystem to the anisotropy energy density in the magnetic system. It also indicates that the electric field excites the in-plane rotation of the magnetization by transferring the increment Rashba SOI via exchange interaction to the energy of the out-of-plane magnetization deviation and this effect is suppressed by the damping. Therefore, the magnetization dynamics are a superposition of the in-plane precession and out-of-plane nutation.

When we introduce dimensionless time $\tau = \alpha_G \omega_\perp t/2$, then (11) is transformed to:

$$\ddot{\varphi} + 2\dot{\varphi} + \Delta(\tau) \cos(\varphi - \beta) + \chi \sin 2\varphi = 0,$$ (12)

where $\Delta(\tau) = 4\omega_a E(\tau) \alpha_G \omega_\perp E_0$ and $\chi = 2K/\omega_\perp E_0$. The value of the dimensionless coefficient $\Delta(\tau)$ in (12), according to the typical heterostructure parameters provided above, is $\Delta(\tau) \approx 10^3 \cdot E/E_0$. It is limited due to the condition imposed above that drift increment of the electron momentum should be much smaller than the thermal one (i.e. $E \ll 10$ kV cm$^{-1}$). The value of the coefficient $\chi$ resulting from shape anisotropy is $\chi(\eta) \approx 600 \eta^2 (1 + 0.75 \eta^2)$, where $\eta$ is an eccentricity according to (7). It is relatively high, even for the element in-plane shape close to circular.

The equation (12) possesses the following symmetry: inverse in-plane magnetization rotation ($\varphi \to -\varphi$) is caused by applying the electric field at an angle $\beta \to \pi - \beta$. Generally one can change the direction angle $\beta$ of electric field $E$ by applying different alternating voltages to two perpendicular pairs of contact microstrip embedded in the hybrid structure. This can be treated as an additional possibility of magnetization switching induced by the electric field rotation, but in this work we focus on the case of constant $\beta$ for the sake of considered model simplicity.

If the ferromagnetic particle has a circular in-plane shape ($\chi = 0$), then its magnetic ground state is infinitely degenerate: the system does not have preferential in-plane direction. For constant $\beta$ we can replace $\varphi = \xi = \varphi - \beta + \pi/2$, thus (12) is transformed to:

$$\ddot{\xi} + 2\dot{\xi} + \Delta(\tau) \sin \xi = 0.$$ (13)

This equation describes the classical parametrically forced pendulum. We take the initial conditions $\xi_0 \equiv \xi(0) = \pi/2 - \beta$ and $\dot{\xi}(0) = 0$.

For constant electric field $\Delta(\tau) = \Delta_0 > 0$, one can use an approximate solution of linearized (13) (we substitute $\xi = 2 \arctan y$ and omit a nonlinear term $y^2/(1 + y^2)$):

$$\tan \xi_0 \approx e^{-\Omega t} \left(\cos \Omega t + \frac{\sin \Omega t}{\Omega}\right) \tan \xi_0 / 2,$$ (14)

where $\Omega = (\Delta_0 - 1)^{1/2}$ is a frequency of damped oscillations, shown in figure 2.

Expression (14) is also the approximate solution of (13) if $\Delta_0 < 1$. In this case, $\Omega$ is the imaginary and the trigonometric functions are replaced with hyperbolic, resulting in the aperiodic damping, shown in the figure 3.

If $\Delta_0 > 1$, i.e. the applied electric field magnitude, is high enough ($E > 7.5$ V cm$^{-1}$), the magnetization rotates to the maximal angle $|\Delta_{\varphi_{\text{max}}}| \approx \xi_0 + 2 \arctan(e^{-\pi t} \tan \xi_0/2)$ during the time $\tau_{\text{rot}} = \pi / \Omega$, which is about $\tau_{\text{rot}} \approx \pi(\omega_a \cos \beta / E_0)^{-1/2}$. Since the magnetization rotation is inertial, according to (13), the maximal rotation angle could be larger than $|\Delta_{\varphi_{\text{max}}}|$ if the applied electric field pulse time $\tau_{\text{pulse}} < 1$ so $\dot{\varphi}$ is not negligible. For the aperiodic damping ($E < 7.5$ V cm$^{-1}$), the
and GHz. Another interesting behavior.

The first $\phi_1 \phi_2$. Then, the local solution of the relevant linearized $\phi_1 \phi_2 \phi_3$. Angle $\phi$ determined only by the ratio of the damping to the restoring force coefficients.

Interesting possibilities of the magnetization controlling and parametric resonance appear for strong enough alternating electric fields ($E \gg 10$ V cm$^{-1}$) [45, 46]. According to (11) the equilibrium direction of the magnetization field is $\phi_{eq} = \beta - \pi/2$, so it reverses after the electric field reversal during the time of order of magnitude of $t_{damp}$. Therefore, in the alternating electric fields with high enough amplitude and low frequency $\omega_E$, the time profile of magnetization takes the form of rectangular pulses. We assume that the periodic alternating function $\Delta(\tau) \propto E(\tau)$ is a slowly varying function on a short time interval $[\tau, \tau + \delta \tau] (\delta \tau \ll T_E)$ and $\xi \ll 1$ ($\sin \xi \approx \xi$). Then, the local solution of the relevant linearized (13) at this interval is:

$$\xi(\tau + \delta \tau) \sim \xi(\tau) \exp \left[-(1 + \sqrt{1 - \Delta(\tau)})\delta \tau \right].$$

The requirement of damped motion is the positive logarithmic decrement:

$$\ln \left| \frac{\xi(\tau + \delta \tau)}{\xi(\tau + T_E)} \right| \sim \int_0^{T_E} \text{Re} \left[ 1 - \sqrt{1 - \Delta(\tau)} \right] d\tau > 0.$$  (16)

The estimation (16) gives the boundary values for the electric field amplitude when the solution (13) is still damped. It gives $E_{amp} < 20$ V cm$^{-1}$ ($\Delta_0 < 3$) for the case of the periodic rectangular electric field pulses $\Delta(\tau) = \Delta_0 \text{sgn}(\sin \omega_E \tau)$ and $E_{amp} < 27$ V cm$^{-1}$ ($\Delta_0 < 4$) for the harmonic applying field $\Delta(\tau) = \Delta_0 \sin \omega_E \tau$. In higher fields, the magnetization switches between two opposite directions perpendicular to $E$ and its time dependence has a rectangular pulse form, as it is shown in the figure 4.

The estimate (16) properly describes only the case of low frequencies $\omega_E \ll 2\pi \times 1$ GHz. Another interesting behavior can be seen for sufficiently large frequencies $\omega_E^2 \sim \Delta_0$. One can apply the superposition of direct and alternating electric fields and obtain a realization of the well-known classical parametrically forced pendulum with rich dynamical behavior [45, 46].

We will consider further that the particle has an elliptical in-plane shape ($\chi \neq 0$) and the electric field $E$ is a constant vector after switching on. Therefore, the ground state of magnetization is double degenerate. In the case of small deviations of the in-plane angle $\phi$, we obtain solution similar to (14):

$$\phi_{app j} = \phi_{inf} \left[ 1 - \left( \cos \Omega \tau + \frac{\sin \Omega \tau}{\Omega} \right) e^{-\delta \tau} \right].$$

Figure 2. Time dependence of magnetization direction angles in the sufficiently large applied fields: (a) $E = 70$ V cm$^{-1}$ ($\Delta_0 = 10$) and (b) $E = 0.7$ kV cm$^{-1}$ ($\Delta_0 = 100$). Angle $\phi_{app j}$ is the approximate solution given by (14).
depends on the two changeable parameters $\Delta$ and $\beta$—this defines dynamics diversity.

Let us examine the possibility of magnetization switching between two ground states ($\varphi = 0$ and $\varphi = \pi$) of the elliptically elongated magnetic particle. The analysis of the potential $V(\varphi)$ shows that $V(\varphi)$ has two local minima if $\Delta/\chi < 2$. In the vicinity of $\beta = 0$ if $\Delta/\chi > 1/(1 - \beta)$, it allows transfer from the initial state ($\varphi = 0$) to the second minimum closest to $\pi$. Then, after switching off the electric field, the in-plane magnetization will turn to $\varphi = \pi$. This case is shown in the figure 5(b). Otherwise, if both local maxima of $V(\varphi)$ are positive, the switching does not occur, as shown in the figure 5(a).

The parametric diagram shown in figure 6 depicts the final in-plane magnetization state in the elliptical particle after the electric field pulse. The duration of the applied field pulse is 8 ns that is longer than the relaxation time $2/(\alpha \omega_0) \approx 1.6$ ns and sufficient for the rotation of magnetization into a new equilibrium position. The diagram coordinates $\Delta/\chi = 2\omega_0 E_0 \omega \chi K(\eta) E_0$ and $\beta$ represent the magnitude and direction of the applied electric field. The choice of $[-\pi/2, \pi/2]$ interval for plotting $\beta$ is due to the symmetry of (12). The minimal electric field $E_{\text{min}}$ required for the switching corresponds to the $\Delta/\chi = 0.81$, as shown in the figure 6. Considering the expansion of $K(\eta)$ given in (7), the estimation for this minimal field is $E_{\text{min}} \approx 4\eta^2$ kV cm$^{-1}$ for the in-plane particle shape close to circle (the eccentricity $\eta \ll 1$).

In summary, we have studied the current-induced magnetic dynamics of the ferromagnet/doped semiconductor multilayer structure within a practical model and obtained the scaling relations for the magnetization dynamics in terms of the system parameters. We have shown that the magnetization rotates to the angle orthogonal to the electric field direction during the characteristic time $t_{\text{damp}} \approx 2\omega_0 (\omega_\chi E_0)^{-1} \approx E^{-1} \times 45$ ns·V cm$^{-1}$. The time profile of magnetization takes the ‘balanced’ rectangular waveform for the low-frequency alternating electric field. The magnetic moment of the particle with an elliptical in-plane shape with small eccentricity $\eta$ can be switched to the opposite direction by the electrical field pulse applied at a negative angle to the initial direction of magnetic moment with duration $\sim 10$ ns and magnitude $E > 4\eta^2$ kV cm$^{-1}$.

Acknowledgments

EYS acknowledges support of the University of the Basque Country UPV/EHU under program UFI 11/55, the Grant FIS2015-67161-P (MINECO of Spain/FEDER) and Grupos Consolidados UPV/EHU del Gobierno Vasco (IT-986-16). PVB acknowledges support from the Erasmus Mundus Action 2 ACTIVE project.
References

[1] Awschalom D D and Flatté M E 2007 Nat. Phys. 3 153–9
[2] Nowack K C, Koppen F H L, Nazarov Y V and Vandersypen L M K 2007 Science 318 1430–3
[3] Sarma S D, Hwang E and Kaminski A 2003 Solid State Commun. 127 99–107
[4] Papp G, Vasilopoulos P and Peeters F M 2005 Phys. Rev. B 72 115315
[5] Jungwirth T, Sinova J, Mašek J, Kučera J and MacDonald A H 2006 Rev. Mod. Phys. 78 809–64
[6] Žutić I, Fabian J and Sarma S D 2004 Rev. Mod. Phys. 76 323–410
[7] Bychkov Y A and Rashba E I 1984 J. Phys. C: Solid State 17 6039–45
[8] Winkler R 2003 Spin–Orbit Coupling Effects in Two-Dimensional Electron and Hole Systems (New York: Springer)
[9] Aronov A G and Lyanda-Geller Y B 1989 Sov. J. Exp. Theor. Phys. Lett. 50 431–4
[10] Edelstein V 1990 Solid State Commun. 73 233–5
[11] Ha J K, Hertel R and Kirschner J 2003 Phys. Rev. B 67 224432
[12] Metlov K L and Lee Y 2008 Appl. Phys. Lett. 92 112506
[13] Ralph D and Stiles M 2008 J. Magn. Magn. Mater. 320 1190–216
[14] Haney P M, Lee H W, Lee K J, Manchon A and Stiles M D 2013 Phys. Rev. B 87 174411
[15] Gambardella P and Miron I M 2011 Phil. Trans. R. Soc. A 369 3175–97
[16] Kurebayashi H et al 2014 Nat. Nanotechnol. 9 211–7
[17] Manchon A and Zhang S 2008 Phys. Rev. B 78 212405
[18] Manchon A and Zhang S 2009 Phys. Rev. B 79 094422
[19] Fukami S, Anekawa T, Zhang C and Ohno H 2016 Nat. Nanotechnol. 11 621–5
[20] Ciccarelli C et al 2016 Nat. Phys. 12 855–60
[21] Avci C O, Garello K, Ghosh A, Gabureac M, Alvarado S F and Gambardella P 2015 Nat. Phys. 11 570–5
[22] Garello K, Miron I M, Avci C O, Freimuth F, Mokrousov Y, Blügel S, Aufrert S, Boule O, Gaudin G and Gambardella P 2013 Nat. Nanotechnol. 8 587–93
[23] Miron I M, Garello K, Gaudin G, Zermatten P J, Costache M V, Aufrert S, Bandiera S, Rodmaaq B, Schuhl A and Gambardella P 2011 Nature 476 189–93
[24] Wang X, Bauer G E W and Hoffmann A 2006 Phys. Rev. B 73 054436
[25] Yokoyama T 2011 Phys. Rev. B 84 113407
[26] Eldridge P S, Leyland W J H, Lagoudakis P G, Karimov O Z, Henni M, Taylor D, Phillips R T and Harley R T 2008 Phys. Rev. B 77 125344
[27] Inoue J I, Bauer G E W and Molkenp L W 2003 Phys. Rev. B 67 0353104
[28] Wang Y, Chen W Q and Zhang F C 2015 New J. Phys. 17 053012
[29] Meier L, Salis G, Shorubalko I, Gini E, Schön S and Ensslin K 2007 Nat. Phys. 3 650–4
[30] Leyland W J H, John G H, Harley R T, Glazov M M, Ivchenko E L, Ritchie D A, Farrer I, Shields A J and Henni M 2007 Phys. Rev. B 75 165309
[31] Fert A 1969 J. Phys. C: Solid State Phys. 2 1784–8
[32] Gantmakher V F and Levinson Y B 1987 Carrier Scattering in Metals and Semiconductors (Amsterdam: Elsevier)
[33] Fishman G and Lampel G 1977 Phys. Rev. B 16 820–31
[34] Song P H and Kim K W 2002 Phys. Rev. B 66 035207
[35] Tackeuchi A, Wada O and Nishikawa Y 1997 Appl. Phys. Lett. 70 1131
[36] Khaetskii A V and Nazarov Y V 2000 Phys. Rev. B 61 12639–42
[37] Akhiezer A, Bar’yakhtar V and Peletminskii S 1968 Spin Waves (Amsterdam: North-Holland)
[38] Schryer N L and Walker L R 1974 J. Appl. Phys. 45 5406–21
[39] Hubert A and Schäfer R 1998 Magnetic Domains (Berlin, NY: Springer)
[40] Oogane M, Wakiyama T, Yakata S, Yilgin R, Ando Y, Sakuma A and Miyazaki T 2006 Japan. J. Appl. Phys. 45 3889–91
[41] Nibarger J P, Lopusnik R and Silva T J 2003 Appl. Phys. Lett. 82 2112
[42] Bazaliy Y B 2007 Phys. Rev. B 76 140402
[43] Bazaliy Y B and Arammash F 2011 Phys. Rev. B 84 134204
[44] Bazaliy Y B 2012 Phys. Rev. B 85 014431
[45] Bror H W, Hoveijn J, Noort M, Simó C and Vegter G 2004 J. Dyn. Differ. Equ. 16 897–947
[46] McLaughlin J B 1981 J. Stat. Phys. 24 375–88