The Application of Linear and Nonlinear Water Tanks Case Study in Teaching of Process Control

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Abstract: In the traditional process control teaching, the importance of passing knowledge is emphasized while the development of creative and practical abilities of students is ignored. Traditional teaching methods are not very helpful to breed a good engineer. Case teaching is a very useful way to improve students’ innovative and practical abilities. In the traditional case teaching, knowledge points are taught separately based on different examples or no examples, thus it is very hard to setup the whole knowledge structure. Though all the knowledge is learned, how to use the knowledge to solve engineering problems keeps challenging for students. In this paper, the linear and nonlinear tanks are taken as illustrative examples which involves several knowledge points of process control. The application method of each knowledge point is discussed in detail and simulated. I believe the case-based study will be helpful for students.

1. Introduction
Process control is an important branch of automation technology. In recent years, the process control technology and its application field has been developed rapidly. At present, a large number of engineering talents with automation expertise is urgently needed. Therefore, how to make students understand and master the process control technology has become more and more important. The traditional curriculum teaching cannot meet the needs of modern education and teaching.

The traditional curriculum teaching cannot meet the needs of modern education and teaching. Enough attention is not paid to the development of innovative and practical ability. Instruction-driven learning model is developed and it is not conducive to innovative thinking and innovation ability of creative talent growth [1]. The student must be an active learner and not be a passive audience in this process. The student should be the main actor in the learning/teaching arena and he/she should be engaged in doing something besides listening to a lecturer and taking notes. This implies not only a precise definition of objectives and capabilities to be acquired, but also the reformulation of strategies and teaching methodologies [2]. Traditional teaching methods cannot make students adapt to modern requirements.

Case based teaching has attracted many teachers’ attention.

Through the teaching materials in recent years, some good teaching methods are proposed. Firstly, simulation is considered to be a good choice. In the teaching process, graphical and interactive of the main teaching contents maybe realized easily through the organic combination of MATLAB language, Simulink and the multimedia courseware of modern control theory [3][4]. A new simulation software PISim (PI simulation) is applied into process control teaching [5]. In [6], the use of simulation platform in the teaching process in the UK has been a great success in robotics teaching.
Experiment-Based teaching is another good way to study. Some educators have tried some new method to teach students. Experiment-Based teaching has been put forward to teach an advanced control engineering syllabus [7]. Many of the core concepts in controls are introduced and key theorems are proved using only high school algebra through a variety of practical and interesting cases [8]. Controllers are designed for a reduced-order model of a synchronous generator to teach Modern control system analysis and design [9]. The application of case based teaching in power electronics has improved the quality of teaching, explained the knowledge points through theory and practice, and obtained the students’ positive response [10]. Case teaching has been used to introduce the concept of modern electrical energy systems with different types of sustainable energy sources [11].

Process control systems involve dynamic behavior, physical and empirical modeling, computer simulation, measurement and control technology, basic control concepts and advanced control strategies. Different examples help much. Quadruple-Tank Process case is used to Multivariable control teaching [12]. Water tanks are applied to teach Fuzzy Control [13]. Double water tank system is introduced to environment communication and control systems [14].

Existing teaching methods and examples are not very suitable to process control. Process control needs specific example to chain the points of knowledge so as to make undergraduates have a systematic study. Lack of specific examples would lead to insufficient understanding. In this paper, linear and nonlinear tanks are introduced to illustrate knowledge of Process Control.

2. Describe the case

With the needs of the research and progress of technology, water tank experiment systems become the mainstream of the process control experiment devices. The water tank system involving typical process variables, such as liquid level, flow rate, pressure and temperature, can be used to simulate systems of different order, linear or non-linear, single or multi-capacitive and large time constants. Control devices could be PLC, PC, or a distributed control system or field bus control system. And the experimental platform is more open and convenient to achieve a variety of conventional or advanced control strategy.

Water tanks involves most knowledge of the process control, which are very suitable to be illustrative examples. Fig 1 and Fig 2 show the diagrams of linear and nonlinear tanks.

![Figure 1. Linear water tank](image1)

![Figure 2. Nonlinear water tank](image2)

2.1. Knowledge points

A. Modelling Based on Theoretical Conservation Equation

Theoretical Dynamic Models is based on relevant balance equation. The important condition of the Theoretical Dynamic Models is to have a good understanding of the principles of the production process. It is useful for exploring a wide range of operating conditions. Studies using theoretical models can be performed quickly, safely and inexpensively. They can also form the basis of a training simulator. First-principles (theoretical) dynamic models result from conservation equations. Conserved variables include mass and energy. Balances are created by defining a boundary around the
process and then computing the following:
\[
\text{Accumulation} = \text{In} - \text{Out} + \text{Generation} - \text{Consumption}
\]

B. Linearization of Nonlinear Equations

Linearization is a method used to approximate nonlinear equations with non-constant coefficients as linear equations with constant coefficients. In practice, almost all components or systems of dynamical equations are nonlinear equations, but in a relatively small range these relationships are linear.

The linear approximation is around at one point, named steady point, and has good agreement in a narrow range around that point. Steady point is usually chosen as the point where the process spends most of its time, the design level of operation [15].

The method for linearizing a nonlinear equation is to approximate with a Taylor series expansion around the point:
\[
F(x) = \sum_{n=0}^{\infty} \frac{F(x)^{(n)}}{n!} (x - x_0)^n = F(x_0) + F(x_0)^{(1)}(x - x_0) + \{ \sum_{n=2}^{\infty} \frac{F^n}{n!} (x - x_0)^n \}
\]

Only the linear terms of the expansion are kept and the Higher Order Terms (HOT) of the Taylor series are ignored. The individual nonlinear terms of a complicated equation should be isolated and linearized and then substituted back into the original form.

C. Tuning PID Parameters

PID controller is expressed as:
\[
u(t) = K(e(t) + \frac{1}{T_i} \int_0^t e(\tau) d\tau + T_d \frac{de(t)}{dt})
\]

The control signal is thus a sum of three items: P-item (which is proportional to the error), I-item (which is proportional to the integral of the error), and D-item (which is proportional to the derivative of the error). The controller parameters include proportional gain \(K\), integral time \(T_i\), and derivative time \(T_d\). The integral, proportional and derivative items can be interpreted as control actions based on the past, the present and the future. The derivative part can also be interpreted as prediction. The action of the different items can be illustrated by the following figures showing the response to step signals in set point.

There are kinds of PID parameter tuning methods, such as continuous cycling method, step test method and empirical knowledge method. The PID parameter tuning method described in this paper is based on continuous cycling method. The steps of continuous cycling method are as follows.

1) puts the integrator's integration time constant \(T_i\) to the maximum \((T_i = \infty)\), the differential time constant is zero \((T_d = 0)\), the controller is kept proportional to error.

2) reduces the proportional band \(\delta (\delta = \frac{1}{k})\) and obtains the equal amplitude oscillation response. The critical proportional band \(\delta_k\) and oscillation period \(T_k\) is recorded.

3) according to the values of \(\delta_k\) and \(T_k\), the empirical formula is used to calculate the parameters \(\delta, T_i\) and \(T_d\).

| \(P\) | 2\(\delta_k\) | ----- | ----- |
| \(PI\) | 2.2\(\delta_k\) | \(T_k/1.2\) | ----- |
| \(PID\) | 1.6\(\delta_k\) | 0.5\(T_k\) | 0.25\(T_k\) |

There is much more to PID than is revealed. A faithful implementation of the equation will actually not result in a good controller. To obtain a good PID controller it is also necessary to consider [16].

In the following Section, the water tanks are introduced. The mathematical model and simulation model are also established. Then PID controller is designed in the water tanks.
3. Process modelling

3.1. Case 1
The water tank shown below has liquid flowing in the top and freely draining out of a outlet pipeline in the bottom. If the inflow and outflow are the same, the level of the tank will keep the same. Once the balance of the inflow and outflow is broken, the level of the tank will rise or fall, and the balance between input flow and output flow can be reestablished. The constant volumetric flow rate $Q_1$ is controlled by the control valve opening coefficient $\Delta \mu$ and $K_\mu$ is the valve flow coefficient. Then $Q_1 = K_\mu \Delta \mu$.

Figure 3. The model of linear water tank

According to the mass balance, the difference between the liquid inflow and outflow of the tank within the unit time should be equal to the rate of change of the liquid accumulates in the tank. Thus

$$\frac{dV}{dt} = Q_1 - Q_2$$

(1)

$$V = Ah(t)$$

(2)

The relationship between the output water flow $Q_2$ and the liquid level $h$ is $Q_2 = k\sqrt{h}$, which is non-linear. We linearize it near the equilibrium point $(h_0, Q_0)$.

$$Q_2 = k\sqrt{h} = \sum_{n=0}^{\infty} \frac{Q_0^n}{n!} (h - h_0)^n = k\sqrt{h_0} + \frac{k}{2\sqrt{h_0}}(h - h_0) + \{\sum_{n=2}^{\infty} \frac{Q_0^n}{n!} (h - h_0)^n\} (HOT)$$

(3)

$$\Delta Q_2 = \frac{\Delta h}{R}, (R = \frac{2\sqrt{h}}{k}, \Delta Q_2 = Q_2 - Q_0, \Delta h = h - h_0)$$

(4)

According to (1)(2)(3)(4).

$$T \frac{d\Delta h}{dt} + \Delta h = K \Delta \mu, (T = RA, K = K_\mu R)$$

(5)

After the Laplace transform

$$\begin{align*}
H(s) &= L[\Delta h(t)] \\
\mu(s) &= L[\Delta \mu(t)]
\end{align*}$$

(6)

$$G_s(s) = \frac{H(s)}{\mu(s)} = \frac{K}{T_s + 1} = \frac{K_\mu R}{ARS + 1} = K_\mu \frac{1}{1 + \frac{1}{RAS}}$$

(7)

$T$ is the time constant of the liquid level process, which is related to tank storage capacity and the
water resistance of the outlet valve. $T$ increases as $R$ or $S$ increases. $K$ is the magnification factor of the liquid level process, which is related to the flow rate coefficient of the inlet valve and the water resistance of the outlet valve. $K$ increases as $K_\mu$ or $R$ increases. [17]

![Flow chart of linear water tank](image)

Figure 4. Flow chart of linear water tank

### Table 2. NOMINAL PARAMETERS

| Parameter | Value |
|-----------|-------|
| $A$       | $10m^2$ |
| $K_\mu$   | 10    |
| $h(0)$    | 2m    |
| $k$       | 10    |

Then the PID controller is used to control the process, the close loop is built up in Simulink Model:

![Simulink model of linear water tank](image)

Figure 5. Simulink model of linear water tank

The parameter selection of this controller is determined according to the method of continuous cycling method. When $K_p = 10, K_i = 10, K_d = 10$ the results are more ideal, the simulation results are as follows:
3.2. Case 2
In general, the mathematical model of the system is established by modeling and testing. A liquid of constant density is fed at a constant volumetric flow rate $Q_1$ into a conical tank of height $H_{max}$ and maximal radius $R_{max}$. The constant volumetric rate $Q_1$ is controlled by the control valve opening coefficient $\Delta \mu$ and $K_{\mu}$ is the valve flow coefficient, or $Q_1 = K_{\mu} \Delta \mu$. In Figure 7, the outlet flow rate of the tank is $Q_2$ or $K_v \sqrt{h(t)}$, where $h$ is the height of the liquid in the tank and $K_v$ is the valve coefficient. Following good engineering practice, we begin with a picture, followed by conservation equations details of the derivation.

According to the material balance, the difference between the inflow and outflow of the tank within the unit time should be equal to the rate of change of the liquid accumulated in the tank. So

$$\frac{dV}{dt} = Q_1 - Q_2$$

(1)
\[ V = \frac{1}{3} \pi r(t)^2 h(t) \]  

the geometry of the tank indicates that

\[ \frac{r(t)}{R_{\text{max}}} = \frac{h(t)}{H_{\text{max}}} \]  

Substituting (2) (3) into equation (1)

\[ \frac{dV}{dt} = \pi R_{\text{max}}^2 h(t)^2 \frac{dh}{dt} = Q_1 - Q_2 \]  

\[ Q_1 = K_\mu \Delta \mu, Q_2 = K_v \sqrt{h(t)} \]  

Assuming for very small time \( dt \), \( h(t) \) is considered to be constant, volume liquid accumulated into the tank is

\[ dV(t) = (Q_1 - Q_2) dt = (K_\mu \Delta \mu - K_v \sqrt{h(t)}) dt \]  

Therefore

\[ \left( K_\mu \Delta \mu - K_v \sqrt{h(t)} \right) dt = \pi \left( \frac{R_{\text{max}}^2 h(t)}{H_{\text{max}}} \right)^2 dh \]  

\[ \dot{h} = \frac{dh}{dt} = \frac{K_\mu \Delta \mu - K_v \sqrt{h(t)}}{\pi \left( \frac{R_{\text{max}}^2 h(t)}{H_{\text{max}}} \right)^2} \]  

Condition for steady state: \( \dot{h}/dt = 0 \)

\[ H_{\text{max}}^2 \left( K_\mu \Delta \mu - K_v \sqrt{h(t)} \right) = 0 \]  

\[ K_\mu \Delta \mu = K_v \sqrt{h(t)} \]  

Simulate the system in order to verify the predictions using the following nominal parameters:

**Table 3. NOMINAL PARAMETERS**

| Parameter | Value |
|-----------|-------|
| \( H_{\text{max}} \) | 5 m |
| \( R_{\text{max}} \) | 2 m |
| \( K_v \) | 0.2 \( m^2/s \) |
| \( h(0) \) | 0.4 m |
| \( K_\mu \) | 0.8 \( m^3/s \) |
| \( \Delta \mu \) | 50% |
It can be seen from the simulation results that the time required for the system to settle is longer. Therefore, the PID regulation is added during the tank level control. The simple control system consists of four basic components, the controlled process, the measurement transmission device, the controller and the actuator. PID regulator combines the advantages of a variety of regulation, both to improve the stability of the system, but also to eliminate the offset [18]. The flow chart is
Figure 10. Flow chart of nonlinear water tank

Figure 11. Simulation model of nonlinear water tank with PID

According to continuous cycling method, $K_p = 4.5, K_i = 1.6, K_d = 0$, the results are as follows:
3.3. The Result of Case Teaching

1) Improve students’ practical ability. Through case studies, students’ understanding of textbook knowledge is reflected in the actual case, which enhances the students’ practical application of knowledge and improves the students’ practical ability. The case teaching method is applied to process control in order to enable students to use their own knowledge, through in-depth and meticulous analysis to solve the practical problems in the work. Students can give full play to their own imagination, combined with reality, and find the right answer.

2) Inspired students’ interest in learning. The case chosen in this paper comes from the actual production work. Non-linear water tank case can be directly described by the teacher, and then through analysis and discussion, we take the initiative to explore ways to solve the problem, active classroom atmosphere. In the course of the implementation of the case teaching, students’ interest in learning is stimulated to a higher level, which is conducive to the study and understanding of theoretical knowledge. At the same time, students are no longer passive knowledge receiver. This way promotes the students to master the relevant problems in the case, and let students think seriously to propose solutions. With the enrichment and development of teaching content, students’ initiative learning ability will be better.

3) Help to improve the level of teachers. First of all, teachers must collect and organize appropriate cases, and have a more profound understanding of the case involved in the relevant knowledge. In addition to the content of the teaching of a comprehensive understanding of the content, while the other content of the knowledge of the corresponding extension of the actual knowledge should be understood more profoundly. Secondly, in the specific teaching process, teachers must seize the opportunity to organize a good coordination. Finally, teachers should have a strong ability to synthesize, so as to be able to summarize and give comment on the analysis of the case.

4. Conclusion

The following conclusions can be drawn from the cases of water tanks. First, students have two basic methods of establishing process mathematical model, namely, mechanism method and experimental test method. Second, students know that the establishment of controlled process mathematics. The process of the model is not only derived from the formula, but also through the Matlab simulation software for intuitive and comprehensive analysis. Third, through the simulation software to explain the process of knowledge points for students to consolidate the previously learned mathematical
modeling and automatic control. Principles related to knowledge, and improve the ability of students to use the knowledge to solve practical problems. Fourth, this case teaching has played a positive role in the cultivation of their engineering practice, and they can easily enter the industry and engage in some relevant professional work.

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