RELAXING THE BIG BANG BOUND TO THE BARYON DENSITY

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In the standard picture of big-bang nucleosynthesis the yields of D, $^3$He, $^4$He, and $^7$Li only agree with their inferred primordial abundances if the fraction of critical density contributed by baryons is between $0.01h^{-2}$ and $0.02h^{-2}$ ($h$ is the present value of the Hubble constant in units of $100\text{ km s}^{-1}\text{Mpc}^{-1}$). This is the basis of the very convincing and important argument that baryons can contribute at most 10\% of critical density and thus cannot close the Universe. Nonstandard scenarios involving decaying particles,\textsuperscript{1} inhomogeneities in the baryon density,\textsuperscript{2} and even more exotic ideas\textsuperscript{3} put forth to evade this bound have been largely unsuccessful.\textsuperscript{4} We suggest a new way of relaxing the bound: If the tau neutrino has a mass of $20\text{ MeV} - 30\text{ MeV}$ and lifetime of $200\text{ sec} - 1000\text{ sec}$, and its decay products include electron neutrinos, the bound to the baryon mass density can be loosened by a about factor of 10. The key is the decay-generated electron antineutrinos: around the time of nucleosynthesis they are captured by protons to produce neutrons, thereby changing the outcome of nucleosynthesis. Experiments at $e^\pm$ colliders should soon be sensitive to a tau-neutrino mass in the required range.
Big-bang nucleosynthesis is one of the cornerstones of the hot big-bang cosmology. The successful prediction of the primordial abundances of D, $^3$He, $^4$He, and $^7$Li tests the standard cosmology back to the epoch of nucleosynthesis ($t \sim 0.01 \text{ sec} - 300 \text{ sec}$ and $T \sim 1 \text{ MeV} - 0.03 \text{ MeV}$), providing its earliest check (see Refs. 5). It also poses an important challenge: the identification of the ubiquitous dark matter that is known to account for most of the mass density and plays a crucial role in the formation of structure in the Universe.

Standard big-bang nucleosynthesis implies that the fraction of critical density contributed by baryons must be between $0.01h^{-2}$ and $0.02h^{-2}$, or less than about 10% for a generous range in the Hubble constant.\(^6\) Taken together with the fact that luminous matter contributes much less than 1% of the critical density this leads to the following possibilities: (i) $\Omega_B = \Omega_0 \lesssim 0.1$, the dark matter is baryonic and the Universe is open; (ii) $\Omega_0 \gtrsim 0.1 \gtrsim \Omega_B$ and much, if not most, of the dark matter is something other than baryons; or (iii) the standard picture of nucleosynthesis is somehow wrong or incomplete, and $\Omega_B$ is greater than 0.1. ($\Omega_0$ is the ratio of the total mass density to the critical density.)

The first possibility, an open, baryon-dominated Universe, is certainly not precluded. However, a number of lines of reasoning suggest that $\Omega_0$ is larger than 0.1, perhaps even as large as unity. Measurements of $\Omega_0$ based upon cluster dynamics, based upon the ratio of total mass to baryonic mass in clusters, and those based upon relating the peculiar motions of galaxies to the observed distribution of matter all strongly favor a value for $\Omega_0$ that is much larger than 0.1.\(^7\) Further, the most successful models for structure formation are predicated upon a flat Universe whose dominant form of matter is slowly moving elementary particles left over from the earliest moments ("cold dark matter").\(^8\) Even in the one viable model of structure formation with baryons only (PIB\(^9\)), the nucleosynthesis bound is violated by a large margin, $\Omega_B = \Omega_0 \sim 0.2-0.4$ with $h \sim 0.5-0.8$. Finally, theoretical prejudice, most especially the Dicke-Peebles timing argument\(^10\) and inflation,\(^11\) strongly favor a flat Universe.

If $\Omega_0$ does exceed 0.1, one is pushed either to option (ii), a new form of matter, or to option (iii), a modification of the standard picture of primordial nucleosynthesis. While previous attempts to circumvent the nucleosynthesis bound to $\Omega_B$ have been unsuccessful,\(^4\) the alternative for $\Omega_0 \gtrsim 0.1$ is a radical one. Thus, we believe that it is worth exploring modifications to the standard picture, especially when they are testable, as is the one discussed here.
To begin, let us review what goes wrong with the light-element abundances for large $\Omega_B$. The yields of nucleosynthesis depend upon the baryon-to-photon ratio $\eta$ which is related to $\Omega_B$ by

$$\frac{\Omega_B h^2}{0.1} = \frac{\eta}{27 \times 10^{-10}}.$$  

For the standard picture of nucleosynthesis the concordance range is $\eta \simeq 3 - 5 \times 10^{-10}$. At the time of nucleosynthesis (around 200 sec) essentially all neutrons are incorporated into $^4$He. However, because of decreasing particle densities and decreasing temperature nuclear reactions eventually cease (“freeze out”) and a small fraction of the neutrons remain in D and $^3$He. Nuclear reaction rates (per particle) depend directly upon $\eta$; for this reason the $^4$He mass fraction increases with $\eta$, though only logarithmically since the additional $^4$He synthesized is small. The D and $^3$He yields depend more sensitively upon $\eta$, decreasing as a power of $\eta$.

The $^7$Li story is more complicated; the key to understanding it involves the free-neutron fraction around the time of nucleosynthesis. When it is relatively large, as for $\eta \lesssim 3 \times 10^{-10}$, $^7$Li is produced by $^4$He(t,$\gamma$)$^7$Li and destroyed by $^7$Li(p,$\alpha$)$^4$He. The final $^7$Li abundance is determined by a competition between production and destruction and decreases with increasing $\eta$ as production decreases (fewer neutrons and less t) and destruction increases (larger $\eta$ results in faster rates). When the neutron fraction is relatively low, as for $\eta \gtrsim 3 \times 10^{-10}$, $^7$Li is produced as $^7$Be which, long after nucleosynthesis, $\beta$-decays to $^7$Li via electron capture. In this regime, the production process is $^3$He($^4$He,$\gamma$)$^7$Be, and the destruction process is $^7$Be(n,p)$^7$Li followed by $^7$Li(p,$\alpha$)$^4$He. The yield of $^7$Be increases with increasing $\eta$ because the production rate increases (larger $\eta$ leads to faster rates) and the destruction rate decreases (fewer neutrons). In the intermediate regime, $\eta \sim 3 \times 10^{-10}$, $^7$Li production achieves its minimum ($^7$Li/H $\sim 10^{-10}$) and both $^7$Li and $^7$Be processes are important.

The problem with large $\eta$ is the overproduction of $^4$He and $^7$Li and the underproduction of D. To be more specific, if we take 0.25 as an upper bound to the primordial mass fraction of $^4$He ($\equiv Y_P$), then $\eta$ must be less than $10 \times 10^{-10}$. Since $^4$He production increases very slowly with $\eta$, taking instead $Y_P \leq 0.255$ relaxes the upper bound to $\eta$ significantly, to $20 \times 10^{-10}$. Further, if one were to suppose that the tau neutrino were massive ($m_\nu \gg 1$ MeV) and disappeared before the epoch of nucleosynthesis so that the number of light
neutrino species was effectively two, then $^4\text{He}$ production constrains $\eta$ to be less than $50 \times 10^{-10}$ for $Y_P \leq 0.25$, and less than $75 \times 10^{-10}$ for $Y_P \leq 0.255$.

Deuterium is a much more sensitive “baryometer.” Since there is no plausible astrophysical source for D, big-bang production must account for at least what is observed, $D/H \gtrsim 10^{-5}$. This results in the upper bound to $\eta$ of $8 \times 10^{-10}$. Because D production decreases so rapidly with $\eta$, this upper bound to $\eta$ is relatively insensitive to the assumed lower bound for D/H.

Finally, it is believed that the $^7\text{Li}$ abundance measured in the pop II halo stars, $^7\text{Li}/H \approx 1.2 \pm 0.3 \times 10^{-10}$, accurately reflects the primordial $^7\text{Li}$ abundance.\textsuperscript{6,12} Insisting that the $^7\text{Li}$ yield be no greater than $^7\text{Li}/H = 1.5 \times 10^{-10}$ implies an upper bound to $\eta$ of $4 \times 10^{-10}$. We note that there are still uncertainties in key reaction rates for $^7\text{Li}$ and in the interpretation of the astrophysical measurements (has $^7\text{Li}$ been astrated?; different stellar atmosphere models lead to different $^7\text{Li}$ abundances for the same line strengths; and so on).\textsuperscript{6,12} Even so, $^7\text{Li}$ still poses a serious constraint: taking instead $^7\text{Li}/H \lesssim 10^{-9}$ only loosens the bound to $\eta \leq 9 \times 10^{-10}$.

In sum, the toughest challenge in relaxing the big-bang bound is simultaneously addressing the underproduction of D and the overproduction of $^7\text{Li}$. As we now describe 10 MeV – 30 MeV tau neutrino which decays to electron neutrinos and has a lifetime of order 300 sec can do just that!

Let us briefly review our recent detailed study of the effects of a massive, unstable tau neutrino on primordial nucleosynthesis.\textsuperscript{13} The abundance of a 20 MeV – 30 MeV tau neutrino (per comoving volume) ceases to decrease and freezes out when the temperature of the Universe is a few MeV; until tau neutrinos decay, their abundance per comoving volume remains constant. For this mass range the freeze-out abundance (assuming the annihilation rate predicted in the standard electroweak model) is given by $r m_\nu \sim 0.6 \text{ MeV} – 1 \text{ MeV}$, where $r$ is the abundance relative to a massless neutrino species. The quantity $r m_\nu$ serves to quantify the energy density; until tau neutrinos decay their energy density, $\rho_r(T) = r m_\nu n_\nu \simeq (r m_\nu/3T) \rho_{\nu 0}$, where $\rho_{\nu 0}$ is the energy density of a massless neutrino species.

A decaying tau neutrino can have several effects on nucleosynthesis:\textsuperscript{13} (i) the energy density it and its daughter products contribute speed up the expansion rate, tending to increase $^4\text{He}$ production; (ii) if its decay products include particles that interact electromagnetically its decays increase the entropy density and thereby reduce the baryon-to-photon ratio, which leads to decreased $^4\text{He}$ production and increased D production; (iii) if its de-
cay products include electron neutrinos (and antineutrinos) their interactions with nucleons affect the neutron-to-proton ratio and thereby the outcome of nucleosynthesis.\textsuperscript{14} In general, when the effects of a decaying tau neutrino are significant they are deleterious and large regions of the mass-lifetime plane can be excluded on this basis.\textsuperscript{15} There are exceptions; elsewhere\textsuperscript{16} we discussed the potential beneficial effects of a $1\text{ MeV} - 10\text{ MeV}$ tau neutrino for the cold dark matter scenario of structure formation; here, we discuss another.

The decay modes of interest involve electron neutrinos (and antineutrinos); e.g., $\nu_\tau \to \nu_e \phi$ or $\nu_e + \nu_e \bar{\nu}_e$ (where $\phi$ is a very light pseudoscalar particle). For the masses, $20\text{ MeV} - 30\text{ MeV}$, lifetimes, $\tau_\nu \gtrsim 200\text{ sec}$, and the abundances of interest, the energy density contributed by a massive tau neutrino is much less than that of a massless one (effectively, the number of massless neutrino species is two). The main difference then, between standard nucleosynthesis and that with a decaying tau neutrino, are the electron neutrinos and antineutrinos that are produced in equal numbers by tau-neutrino decays. Because their energies are much greater than the neutron-proton mass difference, $E_\nu = (0.33 \text{ or } 0.5) m_\nu \gtrsim 6\text{ MeV}$, the capture cross section for an antineutrino on a proton, $\bar{\nu}_e + p \to e^- + n$, is essentially equal to that for a neutrino on a neutron, $\nu_e + n \to e^- + p$. However, after the freeze out of the neutron-to-proton ratio, which occurs when $T \sim 1\text{ MeV}$ and $t \sim 1\text{ sec}$, protons outnumber neutrons by about six to one, and so the capture of decay-produced neutrinos will produce about six times as many neutrons as protons.

The probability that a nucleon captures an electron neutrino or antineutrino around the time of nucleosynthesis ($T \sim 0.1\text{ MeV}$ and $t \sim 200\text{ sec}$) is proportional to (capture cross section) $\times$ (number density of tau neutrinos) $\times$ ($t \sim 200\text{ sec}$); more precisely,

$$\mathcal{P} \approx 10^{-4} \left( \frac{r m_\nu}{0.05 \text{ MeV}} \right) \left( \frac{m_\nu}{20 \text{ MeV}} \right) \left( \frac{200 \text{ sec}}{\tau_\nu} \right),$$

where we assume that $\tau_\nu \gtrsim 200\text{ sec}$ so that the fraction of tau neutrinos that decay in a Hubble time around $t \sim 200\text{ sec}$ is $200\text{ sec}/\tau_\nu$. The upshot is that tau-neutrino decays continuously produce neutrons around the time of nucleosynthesis, amounting to a total of about $10^{-4}$ per baryon. This is the key to obtaining the “correct” D and $^7\text{Li}$ abundances for large $\eta$: the neutron fraction is increased to the value that it would have for much smaller
\( \eta \) (see Fig. 1), whereas in the standard picture, for large \( \eta \) neutrons are very inefficiently incorporated into \(^4\text{He}\), resulting in little D production and few free neutrons, which leads to the overproduction of \(^7\text{Li}\).

Because a decaying tau neutrino leads to a neutron fraction that is very similar to that in the standard picture with \( \eta \sim 3 \times 10^{-10} \), the yields of D and \(^7\text{Li}\) are very similar and vary only slowly with \( \eta \) (see Fig. 2). In the end, the maximum value of \( \eta \) consistent with the light-element abundances is controlled by the overproduction of \(^4\text{He}\) (see Fig. 3). Increasing \( rm_\nu \) allows acceptable D and \(^7\text{Li}\) abundances for larger and larger values of \( \eta \); however, it also increases the energy density contributed by the tau neutrino and its decay products which increases \(^4\text{He}\) production (from much less than that of a massless neutrino species for \( rm_\nu \ll 0.1 \text{ MeV} \) to close to that of a massless neutrino species for \( rm_\nu \sim 0.5 \text{ MeV} \)). For tau neutrino masses between 10 MeV and 30 MeV the highest values of \( \eta \) consistent with the light-element abundances are around \( 50 \times 10^{-10} \) and occur for \( rm_\nu \sim 0.03 \text{ MeV} - 0.1 \text{ MeV} \) (see Fig. 4). (Our criteria for concordance are: \( Y_P \leq 0.25 \), \( \text{D/H} \geq 10^{-5} \), \( \text{D+}^3\text{He}/\text{H} \leq 10^{-4} \), and \( 0.5 \times 10^{-10} \leq \text{Li/H} \leq 2 \times 10^{-10} \). For tau-neutrino lifetimes \( 3000 \text{ sec} \gg \tau_\nu \gg 200 \text{ sec} \) our results are relatively insensitive to \( \tau_\nu \) and depend only slightly on decay mode.)

The maximum value of the baryon density that can be allowed with a decaying tau neutrino is \( \Omega_B h^2 \approx 0.2 \); this permits closure density in baryons for a Hubble constant of slightly less than 50 km s\(^{-1}\) Mpc\(^{-1}\). This absolute bound to \( \Omega_B h^2 \) rises to about 0.3 when the constraint to the primordial mass fraction of \(^4\text{He}\) is relaxed to \( Y_P \leq 0.255 \).

Loosening the nucleosynthesis bound to the baryon density has manifold implications, especially for the formation of structure in the Universe. It makes the PIB model\(^9\) consistent with the nucleosynthesis bound to the baryon density. Or, it allows a critical Universe with no exotic dark matter. If, in addition, much of the baryon mass formed into massive objects early on, as suggested by some,\(^{17}\) such a scenario would have all the virtues of cold dark matter without the necessity of a new form of matter. Finally, increasing the baryon fraction to 20\% or 30\% of critical, but maintaining the bulk of the mass density in cold dark matter, leads to a version of “mixed dark matter” discussed a few years ago,\(^{18}\) which has the benefit of additional power on large scales.

Of course, increasing the number of baryons in the Universe also raises some serious questions. For example, where are all the baryons? Recall,
luminous matter contributes less than 1% of the critical density. While the Gunn-Petersen test tells us that essentially 100% of the baryons in the IGM must be ionized, the stringent COBE FIRAS limit to the Compton $y$ parameter tell us that there cannot be too much hot gas.$^{19}$

How plausible is a tau neutrino of mass 20 MeV to 30 MeV with a lifetime of order a few hundred seconds whose decay products include electron neutrinos? There is almost universal belief that neutrinos have mass—and almost as many neutrino mass schemes as there are particle theorists. Such a decay mode and lifetime can arise in models with “family symmetries” that relate the quarks and leptons of different generations, or models with additional $Z$ bosons. In models where tau-neutrino decays respect an $SU(2)$ symmetry, the decay width for the charged tau-lepton decay mode $\tau \to 3e$ is related directly to that for the tau-neutrino decay mode $\nu_{\tau} \to 3\nu_e$,

$$\Gamma(\tau \to 3e) = \left(\frac{m_\tau}{m_\nu}\right)^5 \Gamma(\nu_{\tau} \to 3\nu_e) \approx 6 \times 10^6 \text{sec}^{-1} \left(\frac{m_\nu}{25 \text{MeV}}\right)^5 \left(\frac{\tau_{\nu}}{300 \text{sec}}\right)^{-1}. \tag{3}$$

The current upper limit to the decay width for this mode is $\Gamma(\tau \to 3e) \leq 10^8 \text{sec}^{-1}$; improving the sensitivity by a factor of 10 or so offers a possible test of the $3\nu_e$ decay mode.

A careful reader will have noticed that the value of the relic abundance needed to loosen the nucleosynthesis bound, $r m_\nu \sim 0.03 \text{MeV} - 0.1 \text{MeV}$, is about a factor of ten smaller than that which results if the tau-neutrino abundance is determined by the freeze out of annihilations as predicted in the standard electroweak model. However, if neutrinos have mass, they necessarily have new interactions (neutrino masses are forbidden in the standard electroweak model). Additional interactions increase the annihilation cross section (a factor of about ten is required), which decreases the tau-neutrino abundance. Alternatively, entropy production after the freeze out of the tau neutrino’s abundance could have reduced its abundance. We hesitate to mention the obvious generalization of our results: if the tau neutrino can’t do the job, another particle with similar, or even greater mass, and relic abundance $r m \sim 0.3 \text{MeV} - 0.1 \text{MeV}$, could.

Finally, what are the prospects for testing this scenario? Current laboratory upper limits to the mass of the tau neutrino, based upon end-point studies of tau decays to final states with five pions, are just above 30 MeV.$^{20}$

Prospects for improving the sensitivity of these experiments, which are done
at $e^\pm$ colliders, are good. Thus, the uncertainty in the nucleosynthesis bound to the baryon density raised here should be clarified in the near future.

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Figure Captions

**Figure 1:** The neutron fraction as a function of temperature in the standard scenario with $\eta = 3 \times 10^{-10}$ (broken curve) and $\eta = 5 \times 10^{-9}$ (dotted curve), and in the decaying tau-neutrino scenario with $\eta = 5 \times 10^{-9}$, $m_\nu = 30$ MeV, $\tau_\nu = 300$ sec, and $r m_\nu = 0.03$ MeV (solid curve).

**Figure 2:** The abundance of D and $^7$Li as a function of $\eta$ in the standard scenario (solid curves) and with a decaying tau neutrino (broken curves; $r m_\nu = 0.03$ MeV, $m_\nu = 30$ MeV, and $\tau_\nu = 400$ sec).

**Figure 3:** The maximum value of $\eta$ consistent with D, $^7$Li, and $^4$He (solid: $Y_P \leq 0.25$; broken: $Y_P \leq 0.255$) production shown separately as a function of $r m_\nu$ for a tau neutrino of mass 20 MeV and lifetime $\tau_\nu = 300$ sec. The curve labeled D+$^3$He and the lower $^7$Li curve are lower limits to $\eta$.

**Figure 4:** The maximum value of $\eta$ consistent with all the light-element abundances as a function of $r m_\nu$ for $m_\nu = 10, 15, 20, 25$ MeV and $\tau_\nu = 300$ sec. The peak of the curve moves from right to left with increasing mass.
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