Local Search algorithm for two-stage problem of radio communication systems planning

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Abstract. The development of the Arctic territories is one of the priority strategies of the Russian Federation. In this regard, there is a large number of questions in various areas that need to be discussed. The article deals with the problem of improving radio communication in the region. One of the problems related to the location of communication stations is formulated. Its mathematical model is constructed, approximation algorithms of its decision are proposed, the first results of numerical experiments are discussed.

1. Introduction

The development of the Russian Arctic territories is one of the important directions of the modern Russian economy. Interest is associated with various issues, including: ensuring sustainable socio-economic development of the country, the organization of new sources of natural resources and others. Along the entire coast of the seas of the Arctic ocean lies the Northern sea route, which is the shortest route between the European part of Russia and the Far East region. The settlement of these territories is associated with many problems that can be solved using mathematical models and methods. One of these problems is the organization of modern information and telecommunication infrastructure that allows for the provision of communication services. Due to the low population density of the Arctic region of the country, as well as severe climatic conditions, it is impractical to use a wired connection. In addition, the trajectories of most of the satellites do not cross the Arctic regions, which does not allow the use of satellite communications. Therefore, shortwave radio is used for communication. Shortwave radio is radio transmission using the high frequency band (HF), its advantage is the possible signal transmission distance. The paper is devoted to development of radio communication systems. One of the problems arising in this area is formulated, the mathematical model of two-stage facility location problem is constructed, one approach to its solution is proposed.

2. Problem Formulation and Mathematical Model

It is assumed that modern radio communication should be fully automated and able to adapt to changing conditions [1]. The complexity of its creation is associated with the peculiarity of the propagation of radio waves and many factors that affect it. The difficulty of adaptation is a complex problem that consists of a wide range of different tasks, including questions of structural reliability, ensuring high quality of communication, rational use of equipment and others. In particular, it is necessary to take into account the rational location of points of contact, which allows to save capital investments and operating costs. Studying the process of creating...
communication networks, we can identify various situations that can be formulated as discrete location problems. In this paper we consider the case when High frequency (HF) transmitters are far in the continental territory. The signal from them comes to the the radio stations with HF receiver and Medium frequency (MF) transmitters, which provide radio communication to customers. Thus, the radio signal will be distributed in two stages: from the transmitter to the radio station, then from the radio station to the customer. Such problems are called multi-stage, various formulations of them can be found, for example, in [2, 3].

So, this work is devoted to the following problem of designing radio communication systems using HF transmitters. There are a set of points of possible HF transmitters location and a set of points for the location of radio stations, receiving HF signal and retransmitting it on the medium-wave range (MF) to a variety of end users of communication. The working HF transmitter forms the coverage area of the HF signal, and the radio station – the area of the MF signal. It is necessary to locate HF transmitters and radio stations in such way that all customers are in the coverage area, all open radio stations are included into the coverage area of the MF signal, and the total cost of location was minimal.

This problem can be described using the model of two-stage discrete location problem. We introduce it using the following notation:

- $I$ is the set of the possible location points for the HF transmitters (first stage facilities);
- $J$ is the set of the possible location points for the radio stations (second stage facilities);
- $L$ is the set of the radio customers;
- $A = (a_i)$ is the HF transmitters cost, $i \in I$;
- $B = (b_j)$ is the radio stations cost, $j \in J$;
- $D = (d_{ij})$ is the matrix of connections between the HF transmitters and stations, $d_{ij} = 1$ if point $j$ is in the coverage area of the transmitter located in point $i$ and 0 otherwise, $i \in I, j \in J$;
- $C = (c_{jl})$ is the matrix of connections between the radio stations and customers, $c_{jl} = 1$ if customer $l$ is in the coverage area of the radio stations located in point $j$ and 0 otherwise, $j \in J, l \in L$.

Let us introduce the variables $X = (x_i), Y = (y_j)$ of the mathematical model. Variables of first stage facility location $x_i = 1$, if the HF transmitter is located in the point $i \in I$, $x_i = 0$ otherwise. Variables of the second stage of facility location $y_j = 1$, if the radio station is located in the point $j \in J$, $y_j = 0$ otherwise. Let us call a pair $(X, Y)$ the solution of the problem and denote a value of objective function by $F(X, Y)$.

Based on above notation, the mathematical model is as follows:

$$\min \left( \sum_{i \in I} a_i x_i + \sum_{j \in J} b_j y_j \right)$$  \hspace{1cm} (1)

s.t.:\n
$$\sum_{i \in I} d_{ij} x_i \geq y_j, \; j \in J;$$  \hspace{1cm} (2)

$$\sum_{j \in J} c_{jk} y_j \geq 1, \; k \in N;$$  \hspace{1cm} (3)

$$x_i, \; y_j \in \{0, 1\}, \; i \in I, \; j \in J.$$  \hspace{1cm} (4)

The objective function (1) minimizes the total cost of location of HF transmitters and radio stations. Constraint (2) allows locating radio stations only at points in the HF communication coverage area. Constraint (3) indicates that all customers should be provided with radio communication.
3. Solution Algorithms

For the last two decades, approximate solution methods have been actively developed. They are successfully used to solve many difficult applied problems, (see, for example [4, 6–9]). One of them is a Local Search algorithm based on the move from one solution to another solution close to it in some local neighborhood. Modern local search includes Simulating Annealing, Tabu Search, Variable Neighborhood Search, Genetic Algorithm and others. This approach has been applied to many combinatorial optimization problem, for example Traveling Salesman problem, Machine scheduling, Vehicle routing, Location problem and so on [5, 7, 8].

To solve this problem, a Local Search algorithm based on special Greedy algorithm was proposed. The idea of the Greedy algorithm is as follows. First, we locate the facilities (radio stations) of the first stage until all the opened radio stations of the second stage are covered by radio communication. Then we open the facilities (transmitters) of the first stage until all customers are served by radio. Then we open the facilities (transmitters) of the first stage until all the opened radio stations of the second stage are covered by radio communication.

It should be noted that the problem (1)-(4) can be infeasible if it is impossible to transmit the signal from the first stage transmitters to the customers. It depends on location structure and can be detected before the algorithm starts analysing matrices $D$ and $C$.

To describe the algorithm’s scheme, we introduce the following notation:

- $U$ is the set of points of the current location of the transmitters, $U \subseteq I$, point $i \in U$ iff $x_i = 1$, $i \in I$;
- $V$ is the set of points of the current location of the radio stations, $V \subseteq J$, point $j \in V$ iff $y_j = 1$, $j \in J$;
- $T_i$ is the service area of the transmitter $i \in I$, $T_i \subseteq J$ is the set such that $j \in T_i$ iff $d_{ij} = 1$;
- $S_j$ is the service area of radio station $j \in J$, $S_j \subseteq L$ is the set such that $l \in S_j$ iff $c_{jl} = 1$;
- $G$ is a set of stations which are not provided with radio communication at the current location $U$ of transmitters;
- $G_i$ is a subset of stations that are included in the coverage area when the transmitter is located in the point $i$, $G_i \subseteq G$, $i \in I$.
- $K$ is a set of customers which are not provided with radio communication at the current location $V$ of communication stations;
- $K_j$ is a subset of customers that are included in the coverage area when the communication station is located in the point $j \in J$, $K_j \subseteq K$.

Consider the scheme of the Greedy algorithm for the radio communication systems (GRC).

1. Let the initial sets $K := L, V := \emptyset, K_j := K \cap S_j, j \in J$.
2. Choose point $j'$ from the set $J \setminus V$ such that $j' = \arg \min_{j \in J \setminus V} \frac{b_j}{|K_j|}$. Put $V := V \cup \{j\}', K := K \setminus K_j'$; for all $j \in J$, redefine $K_j = K \setminus S_j$. If $K = \emptyset$ then go to Step 3. Else go to Step 2.
3. Let the sets $G := V, U := \emptyset, G_i := G \cap T_i$.
4. Choose point $i'$ from $I \setminus U$ such that $i' = \arg \min_{i \in I \setminus U} \frac{g_i}{c_{i1}}$. Put $U := U \cup \{i\}', G := G \setminus G_{i'}$; for all $i \in I$, redefine $G_i = G \setminus T_i$. If $G = \emptyset$ then go to Step 5. If $G \neq \emptyset$ then go to Step 4.
5. Output $U, V$.

Note that it is easy to construct vectors $X$ and $Y$ by sets $U$ and $V$. Solution, found by GRC algorithm is used as an initial solution for the Local Search.
To build a Local Search algorithm, it was necessary to develop a special problem-specified neighborhood. Let us call the neighborhood $N_{RC}(X, Y)$ of solution $(X, Y)$ of the problem a set $N_{RC}(X, Y)$ of vectors $(X', Y')$ that is obtained by the following steps:

1. Construct a neighborhood $N_{1\text{-swap}}(Y)$ of vector $Y$;
2. Construct a vector $X'$ using the Greedy algorithm GRC for each $Y'$ from the neighborhood $N_{1\text{-swap}}(Y)$.

A set of $(X', Y')$ is a neighborhood $N_{RC}$. Neighborhood 1-Swap is a set of Boolean vectors that differ from a given Boolean vector by two components: one 1 is replaced by 0, and one 0 is replaced by 1.

The scheme of the Local Search Algorithm (LS) is the following.

1. Set the initial solutions $(X, Y)$; let the best known solution $(X, Y)^* := (X, Y)$; calculate the best known value of objective function $F^* := F(X, Y)^*$.
2. Until the stopping criterion is met do the steps (3)–(5).
3. Choose solution $(X', Y')$ from the neighborhood $N_{RC}$ of solution $(X, Y)$ randomly.
4. Apply the Drop algorithm [11] to the solution $(X', Y')$, a new local minima $(X'', Y'')$ will be obtained.
5. Set $(X, Y) := (X'', Y'')$: if $F(X, Y) < F^*$ then $F^* := F(X, Y)$ & $(X, Y)^* := (X, Y)$.

The most common criteria for stopping are the fixed computing time, given number of iterations, or the number of steps without improving the value of the objective function. In this paper, the last one is selected. The number of steps without improvement was chosen during a special numerical experiment. Information about the computational experiment is contained in the following section.

4. Experimental research

The algorithm has been coded and successfully verified for the set of test instances. The set of test instances consists of 10 problems, were built according to the following rules:

- $|I| = 20$, $|J| = 40$, $|L| = 100$;
- matrices $D$ and $C$ are filled randomly, the probability $\varphi$ of the existence of connections $\varphi = P(d_{ij} = 1) = P(c_{jl} = 1) = 0.3$ for $i \in I$, $j \in J$, $l \in L$;
- HF transmitters costs $a_i$ are integer selected randomly from 15 to 20;
- radio stations costs $b_j$ are integer selected randomly from 5 to 15.

During the development of Local Search algorithm achieving the maximum number of iterations without improvement of objective function was chosen as the stopping criterion, and the maximum number of iterations was set $w = |J|$. Every time after $r = (|J|/5)$ steps without improvement of the solution the Local Search returns to the best found solution $(X, Y)^*$, at the same time counting steps without improvement continues. If the solution is improved, the counter starts from the beginning. This is to allow the algorithm to continue its search. The values of the parameters $\varphi, w, r$ were chosen experimentally at the preliminary stage of experimental studies.

Implementation is done on Intel Core Processor i7 with 2.4 GHz, 8 gigabyte RAM. At the main stage of experimental research the Local Search algorithm was run 50 times for each test instance. The next table shows minimum (“min”), average (“avg”) and maximum (“max”) values of relative errors w.t.r. the exact solutions obtained by CPLEX solver of system GAMS [10], relative errors $\Delta_{GRC}$ of solutions obtained by GRC, algorithm, number of values equal to
the exact value ("exact") found in 50 runs for each instance. Relative error $\Delta = \frac{F_A - F^*}{F^*} \cdot 100\%$ where $F_A$ is the objective function value found by algorithm, $F^*$ is the optimal objective function value. The table shows that the minimal average relative error is 0.3% and the maximal relative error not exceed 9%. It should be noted that for each test instance the algorithm has found the optimal solution. The number of times the optimal solutions found by the algorithm reaches 43 times out of 50. In the rows of the table one can see how the initial solution is improved in the process of local search. Computational time for every run was less then 1 sec, as well as computational time of CPLEX. At the same time, we would like to reduce the average values of the relative error. This can be achieved by changing the neighborhood type, alternating them, using the information about the search history and etc. Research in this direction should be continued.

5. Conclusions
In this paper a new model of Two-stage Facility Location Problem was build. The model describes the situation that arises in the process of solving the problem planning of radio communication system. To solve it, a variant of Local Search algorithm based on problem-specific Greedy algorithm was suggested. Test examples are constructed and numerical experiments are carried out. The computational behaviour of the algorithm confirms the validity and perspectivity of work in this direction. The Local Search algorithm finds good quality solutions in terms of the value of the objective function in a time not exceeding 1 second. Thus, for multiple run, the algorithm finds the optimal solution for each test instance. At the same time, it should be noted that this is the first experience of solving problems arising in the development of radio networks in the Arctic region. It is possible to formulate other types of location problems. In addition, there are other factors to be considered such as the peculiarities of radio wave propagation, reflection from the ionosphere, the influence of the earth landscape, etc. It is interesting to improve these studies, research certainly needs to be continued.

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