Study on gamma spectrum wavelet noise reduction based on scintillation detector

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Abstract. Wavelet transform is a time-scale signal analysis method, which holds the characteristics of the same window size, arbitrary shape change and multi-resolution analysis. Most applications of wavelet transform need to be based on signal decomposition and reconstruction, that is, through the combination of low-pass and high-pass filters to achieve frequency division, the signal is decomposed into signal detail components and large-scale approximate components. In the measurement process of the original energy spectrum data, the denoting process plays a role in reducing the incidence of statistical fluctuations to a certain extent, which is of profound significance to improve the data accuracy. In this paper, wavelet transform is used as the theoretical basis to denoise the actual energy spectrum, at the same time based on the MATLAB platform, simulation of signal with noise, using the different parameters on the signal processing. Firstly, the signal-to-noise ratio (SNR) of the hard and soft thresholds to the signal processing under the same conditions was compared, and then the change of the SNR was observed when the number of decomposition layers was changed under the wavelet basis of db3 (soft threshold). The experimental results show that: 1) under the same conditions, the soft threshold method has higher SNR than hard threshold; 2) The number of decomposition layers and the elimination moment affect the signal-to-noise ratio and smoothness of the processed signal respectively, and when the number of decomposition layers is 7-8 layers, the signal-to-noise ratio is about 29, the signal processing effect is the best. This study has shown that Wavelet method is an effective way to process energy spectrum.

1. Introduction
Gamma ray spectrum measurement can be used for nuclear radiation detection, scintillation detector is one of the most commonly used gamma ray detection instruments. Currently the most commonly used gamma ray spectrum scintillation detector is NaI (Ti) spectrometer. In actual gamma ray detection, statistical fluctuation is an inevitable phenomenon in original energy spectrum measurement. In order to ensure the accuracy of subsequent processing, it is necessary to eliminate the influence of statistical fluctuations, that is, it is necessary to denoise the original data and retain effective information to improve the accuracy of analysis. According to the development of denoising algorithms in gamma spectrum, the most representative methods include least square fitting, NASVD, eMNF (enhanced Maximum Noise Fraction) and wavelet transform. Among them, the least square method has not been put forward for many years, only as a reference to evaluate other methods.
Many attempts have been made in order to improve the visual quality of gamma spectrum data. Especially around 2000, this field has achieved rapid development, and the NASVD method, MNF method and eMNF method have been proposed one after another, all of which normalize the variance of the spectrum. These methods can obtain the important components of gamma spectrum data through principal component analysis, and use the processed noise to reconstruct the spectrum [1]. Wavelet method is a mathematical method proposed in the 1980s, which is mainly used to solve the noise problem in signal analysis. Compared with the traditional Fourier transform, wavelet transform can better solve the difficult problems in data compression, image processing and adaptive filtering [3]. In 2001, Xiao Gang used the adaptive wavelet method and the traditional polynomial fitting smoothing method to smooth the gamma spectrum. The results show that the wavelet analysis method can completely eliminate the statistical fluctuations and reduce the peak shape distortion, which is of great significance for the discrimination of overlapping peaks and weak peaks in the high background spectrum [4]. Luo Yaoyao put forward the idea of using Wiener filter in 2012. In the process of spectral line noise reduction, combined with the use of double wavelet basis, it can be found that the standard deviation of window count rate has changed, and the error of the wavelet method using Wiener filter is less than the least square method and the wavelet soft threshold method. It is found that the appropriate threshold function can better improve the signal smoothing effect [2]. In 2016, Li Huailiang further improved the wavelet transform denoising algorithm, which has the threshold translation invariant of high-resolution gamma spectrum, optimized the threshold function of wavelet transform, and is suitable for large-sample and high-continuity non-uniform gamma scanning system caused by Compton scattering [8]. In 2019, SPCSHRINK was proposed. Inspired by the application of control charts, it inherited the mathematical justification of statistical process control based on wavelet shrinkage. Singular value decomposition (SVD) based on improved curvature spectrum is used for processing. The improved singularity value selection method of curvature spectrum can better retain the echo signal and avoid the loss of effective signal. Simulation and experimental results show that the noise reduction effect of wavelet transform can be better improved by selecting the optimized threshold value. Compared with the common singular value decomposition method and wavelet threshold denoising method, the two-layer framework can better suppress noise and extract echo signals [9]. Since the birth of wavelet analysis is in the early 1980s, it has been widely used in many fields of science and technology, although it has only been 40 years.

2. **Principles and algorithms**

Wavelet transform is a time-scale signal analysis method with the same window size, arbitrary shape change and multi-resolution analysis algorithm, which has a high adaptability in the process of signal processing, and is called mathematical microscope (J. Morlet, 1974). It combines the characteristics of time domain and frequency domain on the basis of the traditional δ function and Fourier function, inherits and develops the concept of the original method, and overcomes the shortcomings of the original processing method, such as the window size does not change with frequency, the lack of discrete orthogonal basis and so on. The local information can be displayed in both time domain and frequency domain. Wavelet method combines a low-pass filter and a series of bandpass filters (high-pass filters), which can reduce noise by combining the characteristics of noise signal, such as retaining the original characteristics of signal at low frequency and eliminating irrelevant noise information at high frequency. In addition, this method can have higher frequency resolution and lower time resolution at low frequencies, while it has opposite characteristics at high frequencies [7].

The acquired signal will contain many non-fixed components, such as deviation, trend, etc., from which the main information of the signal can be obtained during processing. Since wavelet inherits the idea of fast Fourier transform, it is very suitable to find and detect the transient anomaly of the signal and display its component.

Let \( \phi (t) \in L^2 \mathbb{R} \) (\( L^2 \mathbb{R} \) represents the square integrable real number space, namely the signal space with finite energy), and its Fourier transform is \( \Phi (\omega) \). \( \Phi (\omega) \) is in the Admissible Condition.
\[ C_\phi = \int \frac{\phi(\omega)^2}{|\omega|} d\omega < \infty \]  

(1)

\( \phi(t) \) is a parent wave, and a wavelet sequence can be obtained after stretching and transforming the parent function \( \phi(t) \).

For any function, the continuous wavelet transform is:

\[ W_f(a,b) = \langle f, \phi_{a,b} \rangle = \left| a \right|^{-1/2} \int_R f(t) \phi^\ast\left(\frac{t-b}{a}\right) dt \]  

(2)

In Formula (2), \( \phi^\ast\left(\frac{t-b}{a}\right) \) is the conjugate function of \( \phi\left(\frac{t-b}{a}\right) \), and the inverse transformation is:

\[ f(t) = \frac{1}{C_\phi} \int_R \int_R \frac{1}{a^2} W_f(a,b) \phi\left(\frac{t-b}{a}\right) da db \]  

(3)

In practical application, discrete wavelet transform is often used, and its corresponding wavelet sequence is as follows:

\[ \phi_{j,k}(t) = 2^{-j/2} \phi(2^{-j} t - k) \cdots j, k \in Z \]  

(4)

3. Experimental design

In the process of noise reduction by the actual energy spectrum, no noise spectrum can be obtained, so it is difficult to use the formula of SNR, and it is impossible to compare the advantages and disadvantages of the methods. Therefore, this paper constructs the noiseless spectrum and noise signal artificially, and then calculates the SNR of different parameters after processing.

A model with noise can usually be expressed by the following formula:

\[ f(t) = s(t) + \sigma e(t), t = 0,1 \cdots n-1 \]  

(5)

In Equation (5), \( s(t) \) is the real signal, \( e(t) \) is the noise, and \( \sigma \) is the noise level coefficient.

The waveform in the actual energy spectrum is similar to the Gauss distribution, so different Gauss distributions are used to superimpose in the simulation signal to simulate the noiseless signal. Noise signal is white noise signal, which is a kind of noise whose power spectral density is a constant at all frequencies. Matlab gives the normal distribution probability density function formula and the standard normal distribution probability density function formula, in the simulation signal directly call \texttt{normpdf} function to generate the standard normal distribution probability density function signal, and finally use \texttt{plot} function to draw the standard normal distribution probability density function graph.

The use method of function \texttt{normpdf} is as follows:

\[ Y = \text{normpdf}(X, \text{mu}, \text{sigma}) \]  

(6)

In Equation (6), the variable \( \text{mu} \) in the function is the mean, the third term is the standard deviation, and the value of the function is the value of the normal probability density function at x-coordinate X.

Use the \texttt{normpdf} function for many times and multiple Gauss functions are used for superposition. As shown in Figure 1, the energy spectrum is formed by the superposition of multiple functions, which can be regarded as noiseless energy spectrum (This is shown in Figure 1).
In order to get the actual simulated signal of energy spectrum, it is also necessary to add noise signal to the original noiseless energy spectrum. Here, the noise signal conforming to the standard Gauss distribution is added. In this Matlab simulation, Gauss white noise is added to the original Gauss superimposed signal. After the appropriate signal is selected, the energy spectrum after the noise is loaded is shown in Figure 2.
Then try to use different parameters for noise reduction to observe the effect of noise reduction.
The formula used for \( \text{wdencmp()} \) function is:
\[
\text{wdencmp('gbl',X,'wname',N,THR,SORH,KEEPAPP)}
\] (7)

In Formula (8), the first variable in the function is the abbreviation of global, which means that each layer adopts the same threshold for processing. If the variable is LVD method, the threshold value of each layer is different. The third variable represents the name of the wavelet function used; The fourth variable represents the layer of wavelet decomposition; THR is the threshold vector, and the length of the THR is N; The sixth variable indicates whether to choose soft threshold or hard threshold (s and h, respectively). When the value of the last parameter is 1, threshold quantization is not carried out for the low-frequency coefficient; otherwise, threshold quantization is carried out for the low-frequency coefficient [5].

When wavelet transform is applied to signal denoising, the selection of threshold has a great influence on the denoising effect. Threshold method can select a dynamic threshold at different scales, which is a threshold shrinkage method based on wavelet transform coefficient. Among them, soft threshold denoising method and hard threshold denoising method are most commonly used [6].

Therefore, the number of decomposition layers was selected by comparing the signal processing effects of soft threshold and hard threshold. This time, 5 layers were selected and DB3 wavelet basis was selected. The influence of soft and hard thresholds on the processing results was shown in Figure 3, and results of hard threshold was shown in Figure 4.

![Figure 3. Soft threshold processing results.](image-url)
Figure 4. Results of hard threshold processing.

The SNR is 27.1 after soft threshold processing, and the SNR is 19.7 after hard threshold processing. Therefore, in the process of wavelet denoising, the appropriate threshold value can be selected to retain the signal information to the maximum extent while denoising, and under this condition, soft threshold processing has a higher signal-to-noise ratio.

Secondly, in order to compare the impact of decomposition layers on the processing results, hard and soft thresholds and DB3 wavelet bases were selected first, and then the signal processing effects of decomposition layers were compared, as shown in the figure. Figure 5 represents the decomposition results of 3 layers, and Figure 6 represents the decomposition results of 8 layers.

Figure 5. Results of 3-layer decomposition.
Figure 6. Results of 8-layer decomposition.

The signal noise of signals processed by different decomposition layers is shown in Table 1:

| Decomposition layers | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  |
|----------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Signal to noise ratio| 16.8| 19.5| 22.2| 25.1| 26.8| 27.5| 29.2| 29.9| 29.1| 28.2|

As can be seen from the table, when the number of decomposition layers is within 8 layers, the SNR gradually increases with the increase of the number of decomposition layers, and the maximum value is 29.9. However, when the number of decomposition layers is more than 8 layers, the SNR begins to decrease. The change trend of SNR with the number of decomposition layers is shown in Figure 7.

Figure 7. Relationship between different decomposition layers and SNR.
4. Conclusions
This paper mainly studies the principle and steps of the realization of the wavelet threshold and
different processing of the analog signal. This paper mainly compares with soft threshold and hard
threshold. It is found that when the number of decomposition layers is 5 and the wavelet basis is DB3,
the soft threshold processing has a greater signal-to-noise ratio. Then the soft threshold method with
wavelet basis of DB3 was selected and the SNR was compared by changing the number of
decomposition layers. The comparative experimental results show that: 1) When the number of
decomposition layers is less than 8, the SNR will increase with the increase of the number of
decomposition layers; When the number of decomposition layers is too high (such as when the
number exceeds 8 layers in the experiment in this paper), the SNR will decrease. 2) The number of
decomposition layers mainly affects the signal-to-noise ratio of the signal processing, and the selection
of vanishing moment mainly affects the smoothness of the signal.

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