Carrier Lifetimes and Polaronic Mass Enhancement in the Hybrid Halide Perovskite CH$_3$NH$_3$PbI$_3$ from Multiophonon Fröhlich Coupling

Martin Schlipf,$^1$ Samuel Poncè,$^1$ and Feliciano Giustino$^{1,2,*}$

$^1$Department of Materials, University of Oxford, Parks Road, Oxford OX1 3PH, United Kingdom
$^2$Department of Materials Science and Engineering, Cornell University, Ithaca, New York 14853, USA

We elucidate the nature of the electron-phonon interaction in the archetypal hybrid perovskite CH$_3$NH$_3$PbI$_3$ using ab initio many-body calculations and an exactly solvable model. We demonstrate that electrons and holes near the band edges primarily interact with three distinct groups of longitudinal-optical vibrations, in order of importance: the stretching of the Pb–I bond, the bending of the Pb–I–Pb bonds, and the libration of the organic cations. These polarons induce ultrafast intraband carrier relaxation over timescales of 6–30 fs and yield polaron effective masses 28% heavier than the bare band masses. These findings allow us to rationalize previous experimental observations and provide a key to understanding carrier dynamics in halide perovskites.

Recently, hybrid organic-inorganic lead-halide perovskites like CH$_3$NH$_3$PbI$_3$ emerged as promising materials for high-performance solar cells [1, 2]. These systems are unique semiconductors, insofar as solution-processed thin films exhibit optoelectronic properties on par with monocrystalline inorganic semiconductors [3, 4]. This exceptional performance originates from the direct gap near the Schockley-Queisser limit, the low and balanced carrier effective masses, the long recombination lifetimes, the tolerance to defects, and the comparatively high carrier mobilities for solution-processed semiconductors [5–7]. Several of these attributes are connected with the electron-phonon interaction (EPI) [8–13]: the blueshift of the optical absorption onset and the photoluminescence peak with temperature [11–14] [15]; presumably the long recombination lifetimes [16–18] (combined with the Rashba-Dresselhaus effect, EPIs may induce spin-forbidden transitions near the band extrema); and a hotphonon bottleneck possibly originating from an imbalance between electron thermalization via optical phonon emission and heat transport [19–21].

The nature of EPIs in CH$_3$NH$_3$PbI$_3$ is being intensely debated.Transient photoluminescence studies indicate that the EPI in this compound is dominated by the Fröhlich coupling of the carriers with one longitudinal-optical (LO) phonon [22] identified by infrared spectroscopy [23]. Yet some studies suggest acoustic phonons playing an important role [24] and limiting charge transport via scattering [25, 26]. Recent theoretical modeling debated whether LO [22] or acoustic [28] phonon scattering limits the carrier mobility. By offering an atomic-scale perspective on the role of each phonon, ab initio calculations can contribute to elucidating the fundamental mechanisms underpinning EPIs in CH$_3$NH$_3$PbI$_3$.

In this work, we study the EPIs from first principles by employing state-of-the-art many-body calculations. We demonstrate that CH$_3$NH$_3$PbI$_3$ is unique among polar semiconductors in that not a single but three distinct groups of LO phonons dominate the EPI: two associated with the PbI$_6$ octahedra and one with the librations of the organic cations, while acoustic phonons hardly contribute. This unusual multiphonon Fröhlich coupling is a direct cause of the structural complexity of hybrid perovskites and is responsible for the ultrafast relaxation of photoexcited carriers and for a moderate polaronic enhancement of their masses.

We perform ab initio calculations within the low-temperature orthorhombic Pnma phase [29] of CH$_3$NH$_3$PbI$_3$ shown in Supplemental Material Fig. S1 [30]. These calculations are unusually challenging due to the necessity of including spin-orbit coupling [51] and quasiparticle GW corrections [52, 53], the large unit cell consisting of 48 atoms, and the sensitivity of EPI calculations to the Brillouin-zone integration. Various approximations have been employed in order to circumvent some of these obstacles: Kawai et al. [54] calculated EPIs in the simpler model system CsPbI$_3$ by neglecting spin-orbit effects. In Refs. [55] and [56], the authors investigated the cubic, 12-atom unit cell, aligning the organic cations in a ferroelectric configuration. In Ref. [22], the authors studied the orthorhombic phase considering only the long-range coupling to polar phonons.

Here we go beyond these previous studies by calculating [30] and analyzing the complete ab initio self-energy arising from EPIs in many-body perturbation theory. Since CH$_3$NH$_3$PbI$_3$ is in the weak-coupling regime [57], we use perturbation theory within the retarded Feynman-Migdal electron self-energy $\Sigma_{nk}(\varepsilon)$ [60, 62]:

$$
\Sigma_{nk}(\varepsilon) = \sum_{mnq} \int \frac{dq}{\Omega_{BZ}} |g_{mnq}(k, q)|^2 \\
\times \left( \frac{1}{\varepsilon - \varepsilon_{nk+q}} + (1 \mp 2f_{nk+q} - 1)/2 \right) \pm \hbar \omega_{q}, \pm i\eta. \quad (1)
$$

Here $\varepsilon_{nk}$ denotes the Kohn-Sham eigenvalue for the band $n$ and wave vector $k$, $\omega_{q}$ is the vibrational frequency
for the phonon branch \( \nu \) and wave vector \( \mathbf{q} \), \( g_{\mu
u
\nu}(\mathbf{k}, \mathbf{q}) \) are the electron-phonon matrix elements, and the integral is performed over the Brillouin zone of volume \( \Omega_{BZ} \).

The temperature is included via the Fermi-Dirac \( f_{\mu k + \mathbf{q}} \) and Bose-Einstein \( n_{\nu q} \) occupations. Using the self-energy in Eq. (1), we determine the quasiparticle lifetimes and effective mass renormalization within Brillouin-Wigner perturbation theory as \( E_{nk} + i\Gamma_{nk} = \varepsilon_{nk} + Z_{nk}(\varepsilon_{nk}) \) with \( \Gamma_{nk} = |1 - \text{Re}(\partial \Sigma_{nk}/\partial \varepsilon)|^{-1} \), where \( E_{nk} \) is the quasiparticle and \( \Gamma_{nk} \) its associated broadening [62, 63].

The conventional Rayleigh-Schrödinger approach is obtained by setting \( Z_{nk} = 1 \). The quasiparticle lifetime is obtained via \( \tau_{nk} = \hbar/2\Gamma_{nk} \) [62, 63]. The renormalization of the effective masses is calculated from the \( k \) derivatives of \( E_{nk} \) [30]. The quasiparticle mass can be expressed in terms of the bare band mass as \( m_{nk}^{QP,\nu} = (1 + \lambda_{nk})m_{nk}^* \), where \( \lambda_{nk} \) is analogous to the mass-enhancement parameter in metals [48].

Figure 1(a) shows the calculated quasiparticle broadening near the valence and conduction band edges at 1 K. For completeness, the band structure and phonon dispersion are shown in Supplemental Material Fig. S2 [30]. A similar trend is found for a range of temperatures. The broadening is essentially the same along different high-symmetry lines, indicating that the EPI in CH\(_3\)NH\(_3\)PbI\(_3\) is isotropic. At the band edges, we have sharp quasiparticle bands with vanishing broadening. Here the absorption and emission of phonons are forbidden, because there are no thermally excited phonons to be absorbed and phonon emission would send the carriers inside the band gap. Away from the band edges, the density of final electron or hole states available for scattering increases, and we see a finite broadening. The red symbols in Fig. 1(a) show the contribution arising from acoustic phonons and from low-energy transverse-optical (TO) phonons (\( \hbar\omega \lesssim 2 \text{ meV} \)). Their vanishingly small contributions indicate that the EPI is dominated by polar modes (see Supplemental Material Fig. S3 for the same plot at 300 K [30]). The steplike feature seen in Fig. 1(a) arises primarily from LO phonons: the steep increase is due to the activation of multiple electron-LO phonon scattering channels at increasingly higher energy, while the quasiparticle broadening is determined by the band velocity rather than the density of states as in metals [43], leading to the plateau at higher energies. The characteristic energy scale of this step is set by the most energetic LO phonon around 22 meV, as we discuss below. The quasiparticle broadening at the plateau corresponds to an intrinsic, carrier lifetime for electrons and holes of approximately 30 fs at 1 K and 6 fs at 300 K.

Figure 1(b) shows that the lifetimes decrease rapidly with increasing temperature. Our calculations explain the differences between the experimental photoconductivity (PC) data of Ref. [10] and the photoluminescence (PL) linewidth of Ref. [22] In the PL experiment, carriers recombine close to the band edges and experience a smaller EPI than carriers above the most energetic LO phonon at 22 meV probed by the PC experiment. In fact, the temperature dependence of the PC scattering rate follows a \( T^{-3/2} \) law; precisely what we observe if we calculate the average scattering time (see Supplemental Ma-
These observations highlight the importance of understanding the energy distribution of the charge carriers probed by different experiments, which report scattering times between 4 and 15 fs at 300 K. The mismatch between measured and calculated phonon frequency [40] may lead to up to 25% smaller scattering times (see Supplemental Material Figure S4 [39]). The femtosecond timescale of the EPI is too small for optical phonons to contribute to the hot-phonon bottleneck, which takes place on picosecond timescales. Therefore our calculations support the recent proposal of Ref. [21] that the bottleneck must relate to the up-conversion of acoustic phonons.

Figure 1(c) shows the calculated electron-phonon renormalization of the electron and hole masses of CH$_3$NH$_3$PbI$_3$. At 300 K, the EPI enhances both electron and hole masses by 28%. Importantly the effective mass is mostly independent of temperature, increasing by only 9% from 1 to 300 K. This result is counterintuitive, as electron-phonon effects are usually expected to become more pronounced as more phonons become available to dress the carriers. What happens here is that the increase of the quasiparticle broadening with temperature offsets the increase in the real part of the self-energy. This nontrivial effect mirrors an analogous mechanism that the bottleneck must relate to the up-conversion of acoustic phonons.

The mismatch between measured and calculated phonon energies and couplings of the compact, three-phonon Fröhlich model [30] after using the residue theorem to evaluate the integral [30]. Eq. (1) becomes

$$
\Sigma_{nk}(\varepsilon) = -\sum_{\nu, \pm} \frac{n_{\nu} + (1 \pm \sigma_n)/2}{\Omega_{\nu} \varepsilon_{nk}/(2\pi^2 g_{\nu}^2 k)} \arcsin \left[ \frac{1 - \varepsilon \pm \hbar \omega_{\nu}}{\varepsilon_{nk}} \right]^{-1/2},
$$

where $\sigma_n = \pm 1$ for valence and conduction, respectively.

The EPI strength $g_{\nu}^2$ appearing in this expression is obtained by averaging the ab initio Fröhlich matrix elements [43] over a small sphere around $q = 0$ [30]. The distribution of coupling strengths is shown in Fig. 2(a) as a function of phonon energy, in analogy with the standard Eliashberg function. We see that the Fröhlich interaction is most pronounced for three distinct groups of LO phonons. Using the mode analysis of Ref. [23] we assign these features to the bending motion of the Pb–I–Pb bonds (yellow), the stretching motion of the Pb–I bonds (blue), and the librations of the organic cations (red). Representative atomic displacements for these modes are shown in Figs. 2(b)–(d), respectively. It is useful to condense the information presented in Fig. 2(a) into three “effective” phonons carrying the same total EPI strength. By averaging these results we obtain $\hbar \omega_{B} = 3.9 \text{ meV}$, $\hbar \omega_{L} = 98.9 \text{ meV}^2 \text{Å}^{-2}$; $\hbar \omega_{L} = 20.4 \text{ meV}$, $\hbar \omega_{L} = 67.2 \text{ meV}^2 \text{Å}^{-2}$ (the subscripts stand for bending, stretching, and libration, respectively).

The coupling of electrons to multiple polar phonons, may explain why previous attempts at fitting experimental data using the Fröhlich model yielded LO phonon energies ranging from 11.5 [24] to 23.5 meV [14]. Furthermore, the multi-LO coupling offers a possible explanation for the observation of two distinct stages in the relaxation.
of hot electrons \[19\].

Figure 1(a) shows a comparison between our model self-energy from Eq. (2) (solid line) and the \textit{ab initio} calculation with EPW. The model is very accurate near the band edges, and starts deviating from the EPW data for carrier energies \(> 60 \text{ meV}\) from the edges, cf. Supplemental Material Fig. S3 \[30\]. This deviation arises from the assumption of parabolic bands employed in the model, that progressively breaks down as we move away from the band edges. In fact, if we recalculate the self-energy using the \textit{first-principles} density of states, we reproduce the \textit{ab initio} data very accurately, as shown in Supplemental Fig. S5 \[30\]. Similar comparisons are shown in Figs. 1(b) and 1(c) for the lifetimes and mass enhancement. Also in these cases the model captures the essential features of the \textit{ab initio} calculations.

The vertical bars in Fig. 1 show a decomposition of scattering rates and mass enhancement parameter in the contributions associated with each of the three polar modes. At low temperature, the dominant contributions arise from stretching and libration, while the bending modes are only weakly coupled to electrons. With increasing temperature the bending modes become more important due to their larger Bose-Einstein occupation factor. In the orthorhombic structure the CH\(_3\)NH\(_3\)\(^+\) cations vibrate around their equilibrium sites, but do not spin around the C-N axis as in the tetragonal phase (between 160 and 330 K) \[66–68\]. In the high-temperature cubic phase (above 330 K) \[66–68\] the organic cations are fully disordered and we expect that the librational modes will not contribute to the EPI. To assess the validity of the calculated electron-phonon scattering rates across a wider temperature range, we simulate orientational disorder of the organic cations by computing the EPI of CsPbI\(_3\) in its cubic phase. In this case the scattering is reduced by about 10\%–20\% as compared to the orthorhombic phase of CH\(_3\)NH\(_3\)PbI\(_3\) due to the absence of librational modes; however, the energy dependence of the scattering rate is qualitatively similar to what we obtained for CH\(_3\)NH\(_3\)PbI\(_3\) (see Supplemental Material Fig. S3 \[30\]).

Since the model self-energy in Eq. (2) captures the main trends of our complete \textit{ab initio} calculations, we can use this model to estimate the change in the electron lifetimes and mass enhancement resulting from quasiparticle \textit{GW} corrections to the band structures. Filip \textit{et al.} \[12\] showed that \textit{GW} calculations increase the electron and hole effective masses of CH\(_3\)NH\(_3\)PbI\(_3\) by 90\% and 80\%, respectively. At 300 K, using the \textit{GW} effective masses for the calculation of the self-energy in Eq. (2) has the effect of decreasing the relaxation times at the band edges by about 7\% and of increasing the mass enhancement factor by 10\% as compared to the values shown in Fig. 1. This indicates that \textit{GW} corrections only induce small quantitative changes to the picture, but do not change the essence of the present analysis.

Figure 3 shows the quasiparticle spectral function calculated for CH\(_3\)NH\(_3\)PbI\(_3\) using the self-energy in Eq. (2). The spectral function is obtained from the self-energy via

\[ A(k,\varepsilon) = -\pi^{-1} \sum_n \text{Im} \left[ \varepsilon - \varepsilon_n(k) - \Sigma_n(k) \right]^{-1} \]

and represents the many-body \(k\)-resolved density of states, i.e., the many-body band structure of the system. In the absence of EPI the bands are sharp and parabolic near the band edges. When EPI is taken into account, we find strongly renormalized but weakly damped carriers for energies below \(\sim 22 \text{ meV}\), and weakly renormalized but heavily damped carriers above this threshold. In addition we find replica bands that are reminiscent of the polaronic satellites recently observed in oxide perovskites and other transition metals oxides \[73,78\]. The features in the valence bands should be observable via high-resolution angle-resolved photoelectron spectroscopy experiments on single-crystal CH\(_3\)NH\(_3\)PbI\(_3\) samples.

We note that, even though our calculations of the EPI matrix elements include spin-orbit coupling, we do not observe a dynamic Rashba-Dresselhaus spin splitting at the band edges. The absence of spin splitting is consistent with the inversion symmetry of the crystal.

Since polar modes dominate the EPI in CH\(_3\)NH\(_3\)PbI\(_3\), we can discuss its polaronic properties starting from the mass enhancement parameter. The calculated parameter \(\lambda = 0.28\) is relatively small; therefore, we can determine the polaron coupling strength \(\alpha\) using the weak-coupling expansion of Feynman’s polaron mass \[59\]:

\[ 1 + \alpha/6 + 0.025\alpha^2 = 1 + \lambda \]

We obtain \(\alpha = 1.4\), which
falls in the weak-coupling regime, as anticipated. This polaron coupling strength can be used to determine the polaron binding energy $E_p$ and its radius $r_p$. From Feynman’s model we have $E_p = \alpha(1 + 0.0123\alpha)\hbar\omega$ [81] and $r_p = (3.4 h/m^* \alpha \omega)^{1/2}$ [80].\footnote{feliciano.giustino@materials.ox.ac.uk} Using $\hbar\omega = 13-22$ meV from the above analysis and $m^* = 0.22$ [42] we find $E_p = 19 - 31$ meV and $r_p = 62 - 81$ Å. These results indicate that the electron-phonon coupling in CH$_3$NH$_3$PbI$_3$ leads to the formation of large polarons extending over more than 20 PbI$_6$ octahedra. The binding energy of these quasiparticles is comparable to the vibrational energy of the polar modes at room temperature; therefore, we do not expect any localization or self-trapping under standard operating conditions. On the other hand, we note that the polaron binding energy is similar to the exciton binding energy $E_x = 20 \pm 2$ meV in this compound [81], therefore polarons may play a role in the excitonic physics of halide perovskites.

In conclusion, we presented the first complete many-body investigation of electron-phonon physics in CH$_3$NH$_3$PbI$_3$. We found evidence for a novel multi-phonon Fröhlich coupling, and used this to rationalize a number of experimental observations. We established that the EPI leads to ultrafast carrier relaxation near the band edges, and a moderate renormalization of the effective masses. Our analysis indicates that this system is in the weak polaronic regime. More generally, the multi-phonon Fröhlich model that we developed to examine our ab initio data can be used to investigate electron-phonon physics in the broader family of halide perovskites, and to establish design rules for engineering carrier dynamics in this promising class of semiconductors.

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