D-branes in String theory Melvin backgrounds

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Abstract

We determine the consistent D-brane configurations in type II nonsupersymmetric Melvin Background. The D-branes are analysed from three complementary points of view: the effective Born-Infeld action, the open string partition function and the boundary state approach. We show the agreement of the results obtained by the three different approaches. Among the surprising features is the existence of supersymmetric branes, some of them having a quasi-periodic direction. We also discuss the generalisation to backgrounds with several magnetic fields, some of them preserving in the closed and the open spectra some amount of supersymmetry. The case of rational magnetic flux, equivalent to freely-acting noncompact orbifolds, is also studied. It allows more possibilities of consistent D-brane configurations.

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1 Introduction and Summary of results

D-branes [1, 2] are an essential ingredient in string theory. In addition to their initial role in testing conjectures on the non-perturbative regime of supersymmetric string theories, they proved to be essential tools to study the spacetime geometry at substringy distances [3] and to holographically relate theories with gravity to Yang-Mills theories on lower dimensional spacetimes [4]. The brane world scenario in its new version represents an important phenomenological development inspired by D-branes. All these issues motivate the determination of the D-brane content of a string theory in a general background.

Many authors have studied D-branes on compact Lie groups or their quotients using the exactly solvable WZWN models [5]-[11, 13]. The D-branes in non-compact spacetime were considered for non-compact orbifolds or some group manifolds [12]. The aim of this paper is the study of D-branes on another interesting non-trivial and exactly solvable background: the Melvin spacetime [14] - [31].

The Melvin background is a flat space-time $R^9 \times S^1$ subject to the twisted identification (I) $(y, Z_0) = (y + 2\pi R, Z_0 e^{2\pi i RB})$. A constant spinor in $R^9 \times S^1$ is not invariant under (I), it picks a phase $e^{\pm i\pi RB}$ and so all the supersymmetries are broken. If we consider the identification (II) $(y, Z_0, W_0) = (y + 2\pi R, Z_0 e^{2\pi i RB_1}, W_0 e^{2\pi i RB_2})$ then the phase may be compensated when $B_1 = \pm B_2$ leaving half of the initial supersymmetries. That the Melvin background is obtained from flat spacetime by an identification strongly suggests its solvability which we review in Section II. In that section we also determine the quantization of the open strings and the various possibilities of consistent boundary conditions from the sigma model approach. One of the interesting boundary conditions is Dirichlet for $Z_0$ and Neumann for $y$. In this particular case, the intersection of this curve with the $(y, Z_0)$ Melvin space, when $BR$ is not rational, is a non-compact line. The momenta along this direction are however quantized in terms of two integers, defining a quasi-periodic space [31]. The existence of a mass gap in the spectrum in a noncompact space is a remarkable phenomenon with possible phenomenological implications. In section III, the low energy couplings of the D-branes to the closed spectrum are examined from the point of view of the Born-Infeld effective action. In Section IV we determine the cylinder partition functions from the one-loop open string amplitude for various types of D-branes, parallel or orthogonal to the relevant coordinates of the Melvin geometry. This allows the determination of the D-brane spectra. Some of them have open string Landau levels and some do not. Similarly, the boundary conditions on D-branes fix the couplings of D-branes to the Landau levels of the closed states. This is determined by the position of the D-brane with respect to the Melvin geometry. Depending on the case, supersymmetry can be completely broken, can be present in the massless spectrum or can even be present in the complete (massive and massless) spectrum of D-branes.

The results of our work may shed light on the fate of closed-string tachyons in curved backgrounds, in the presence of D-branes. Indeed, backreaction of D-branes studied in this paper on the curved (Melvin) geometry could lift in mass the closed tachyon and stabilize the background. It would be also interesting to study nonperturbative aspects of gauge theory on this type of D-branes from the dual, curved gravitational background.

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2 An example of a quasi-periodic space is an infinite length geodesic (which is not parallel to one of the cycles) on a torus.
viewpoint.

In Section V we verify the consistency of these amplitudes by determining the boundary states of the D-branes and checking the correct interpretation of the cylinder amplitude as a tree level closed string exchange. We show that there are some cases which seem consistent from the sigma model approach but which do not fulfill the requirements of a correct dual open-closed channels interpretation. In Section VI we study the case of the identification (II) which is supersymmetric. In Section VII we briefly examine the novel features which arise when \( BR \) is rational. The D-brane properties in this case are different and can be understood as freely-acting interpolations between flat-space D branes and D-branes in noncompact orbifolds. Finally, an appendix contains the definitions and modular transformation properties of the theta functions used in the text.

## 2 Closed and open strings in the Melvin background

The Melvin background is a flat space-time \( R^9 \times S^1 \) in which a \( 2\pi \) rotation in a compact coordinate \( y \) is accompanied by a rotation in a two-plane of polar coordinates \( (\rho, \phi) \)

\[
(y, \phi_0) = (y + 2\pi R, \phi_0 + 2\pi RB),
\]

where \( \phi_0 \) is an angle

\[
\phi_0 = \phi_0 + 2\pi,
\]

and \( B \) is the twist parameter, which can be seen as a closed string magnetic field. Due to (2), \( BR \) and \( BR + n \) are equivalent so we can suppose that \( BR \) is between 0 and 1. The metric is given by

\[
ds^2 = d\rho^2 + \rho^2 d\phi_0^2 + dy^2 + d\mathbf{x}^2,
\]

where \( \mathbf{x} \) denotes 7 space-time coordinates. If we define \( \phi \) by \( \phi = \phi_0 - By \), then it represents a true angular coordinate and the metric is given by

\[
ds^2 = d\rho^2 + \rho^2(d\phi + Bdy)^2 + dy^2 + d\mathbf{x}^2.
\]

Consider a sigma model with the Melvin background as a target space. The world-sheet action reads

\[
S = -\frac{1}{4\pi\alpha'} \int [d\rho \wedge^* d\rho + \rho^2(d\phi + Bdy) \wedge^* (d\phi + Bdy) + dy \wedge^* dy].
\]

The equations of motion deriving from it read

\[
\partial_+ \partial_- Z_0 = 0, \quad \partial_+ \partial_- y = 0,
\]

where we used the usual definitions \( \sigma_\pm = \tau \pm \sigma \) and we defined the free-field

\[
Z_0 = \rho \ e^{i(\phi + By)} = Z \ e^{iBy}.
\]

The solution to the field equations may be written as

\[
Z_0 = Z_{0+}(\sigma_+) + Z_{0-}(\sigma_-), \quad y = y_+(\sigma_+) + y_-(\sigma_-),
\]
in terms of right and left moving free fields. In order to get the explicit solution, one has to specify the boundary conditions. The angular nature of \( y \) and \( \phi \) imposes that after quantization, their associated momenta are quantized,

\[
P_y = \frac{k}{R}, \quad P_\phi = L ,
\]

where \( k \) and \( L \) are integers and

\[
P_\phi = \frac{1}{2\pi \alpha'} \int_0^{2\pi} d\sigma \rho^2 \left( \frac{d\phi}{d\tau} + B \frac{dy}{d\tau} \right) , \quad P_y = \frac{1}{2\pi \alpha'} \int_0^{2\pi} d\sigma \frac{dy}{d\tau} + BP_\phi . \tag{10}
\]

### 2.1 Closed strings

For closed strings the \( Z \) coordinate is periodic and the \( y \) coordinate can have winding modes \( n \) :

\[
Z(\sigma + 2\pi, \tau) = Z(\sigma, \tau) , \quad y(\sigma + 2\pi) = y(\sigma) + 2\pi n R . \tag{11}
\]

This implies for the free coordinate \( Z_0 \) the nontrivial boundary condition:

\[
Z_0(\sigma + 2\pi) = e^{2\pi i B n R} Z_0(\sigma) . \tag{12}
\]

The mode expansion of the \( y \) coordinate, due to (9) and (10), is given by

\[
y = y_0 + n R \sigma + \alpha' \left( \frac{k}{R} - BL \right) \tau + \sum_{m=1}^{\infty} \frac{1}{\sqrt{m}} \left[ y_m e^{-i m \sigma} + y_m^\dagger e^{i m \sigma} \right. \\
\left. + \tilde{y}_m e^{-i m \sigma} + \tilde{y}_m^\dagger e^{i m \sigma} \right] . \tag{13}
\]

The \( y \) contribution to the hamiltonian is

\[
N_y + \tilde{N}_y + \frac{1}{2\alpha'} [(nR)^2 + (\alpha' (k/R - BL))^2] - \frac{2}{24} . \tag{14}
\]

The hamiltonian describing the \( Z \) coordinate in the winding sector \( n \) is the one of a twisted boson. The mode expansion of \( Z_0 \) is thus given by

\[
Z_0(\sigma, \tau) = \sqrt{\alpha'} \left[ \sum_{m=1}^{\infty} \frac{a_m}{\sqrt{m - \nu}} e^{-i (m-\nu) \sigma} + \sum_{m=0}^{\infty} \frac{b_m^\dagger}{\sqrt{m + \nu}} e^{i (m+\nu) \sigma} \right. \\
\left. + \sum_{m=0}^{\infty} \frac{\tilde{a}_m}{\sqrt{m + \nu}} e^{-i (m+\nu) \sigma} + \sum_{m=1}^{\infty} \frac{\tilde{b}_m^\dagger}{\sqrt{m - \nu}} e^{i (m-\nu) \sigma} \right] , \tag{15}
\]

where \( \nu = f(BnR) \), the function \( f \) is 1-periodic and \( f(x) = x \) for \( 0 < x < 1 \). Explicitly

\[
\nu = nBR - \lfloor nBR \rfloor \text{ when } n \text{ is positive and } \nu = nBR - \lfloor nBR \rfloor + 1 \text{ when } n \text{ is negative. In (15), } a_m^\dagger \text{ and } b_m^\dagger \text{ are creation operators.}
\]

The Hamiltonian of the twisted boson, including the normal ordering constant, reads

\[
H_\nu = \frac{2}{24} - \frac{2}{8} (1 - 2\nu)^2 + \nu \left( b_0^\dagger b_0 + \tilde{a}_0^\dagger \tilde{a}_0 \right) \\
+ \sum_{m=1}^{\infty} (m-\nu) (a_m^\dagger a_m + b_m^\dagger b_m) + \sum_{m=1}^{\infty} (m + \nu) (\tilde{a}_m^\dagger \tilde{a}_m + b_m^\dagger b_m) . \tag{16}
\]
The formulae given above are valid when \( \nu \) traces in the above equation let us note that when \( \nu \)

\[ \text{Tr} \]

where \( \nu \)

modes and the Hamiltonian of the other hand, when \( \tilde{\nu} \)

zero modes involves an integration on the noncompact two-dimensional momenta. On the

In this case, the rotation generator acquires an orbital part:

\[
J + \tilde{J} = x p_y - y p_x + \sum_{m=1}^{\infty} (\tilde{a}_m^\dagger \tilde{a}_m + a_m^\dagger a_m - \tilde{b}_m^\dagger \tilde{b}_m - b_m^\dagger b_m). \tag{21}
\]

The zero modes quantum numbers when \( n \neq 0 \) are given by \((k, n, n_0, \tilde{n}_0)\), where \( n_0, \tilde{n}_0 \)

will be interpreted as Landau levels due to the presence of the magnetic field \( B \). The

level matching condition gives \(-nk = [BRn](\tilde{n}_0 - n_0)\). This state has a Hamiltonian

\((\alpha'/2)(k/R - B(\tilde{n}_0 - n_0))^2 + (nR/2)\alpha' + \nu(n_0 + \tilde{n}_0) - (1 - 2\nu)^2/4\).

On the other hand, when \( n = 0 \) the quantum numbers from the \( Z \) coordinate include

the angular momentum \( L \) and the momentum squared \((p^2 = p_1^2 + p_2^2)\). The level matching

condition is automatically satisfied and the Hamiltonian of the state is given by \(-6/24 +

\((\alpha'/2)(k/R - BL)^2 + 2\alpha'/2\).

The torus partition function, after a Poisson resummation on \( k \), is given by

\[ T = \frac{R}{\sqrt{\pi\alpha'/2}} \frac{1}{|\eta|^2} \sum_{k,n} e^{-\pi \alpha'/2 |k+\tau n|^2} Tr_{\nu} \left[ e^{2\pi i RB \hat{k}(J + \tilde{J})} q^{L_0} \bar{q}^{\bar{L}_0} \right], \tag{22} \]

where \( Tr_{\nu} \) is the trace in the sector twisted by \( \nu \). In order to calculate explicitly the

traces in the above equation let us note that when \( \nu = 0 \) and \( \hat{k} = 0 \) the trace over the

zero modes involves an integration on the noncompact two-dimensional momenta. On the

other hand, when \( \hat{k} \neq 0 \) the trace over the zero modes reads

\[
\int d^2 p < p | e^{2\pi i LB \hat{k} R} | p > (qq)^{\alpha' p^2/4} = \frac{1}{\det(1 - \theta)} = \frac{1}{4(\sin \pi BR \hat{k})^2}, \tag{23} \]

\[ \text{The complete, modular invariant torus amplitude is actually given, with this definition, by} \int (d^2 \tau / \tau_2) T. \]
where $\theta$ is a two-dimensional rotation by an angle $2\pi BR\hat{k}$. Notice that the analogous contribution for the case of a compact two-torus replacing the $(\rho, \phi)$ plane, is one. When $\nu = 0$, the oscillators contribution is given by

$$Tr_{osc}[e^{2\pi i k R \hat{J}} q^{L_0}] = 2 \sin(\pi BR\hat{k}) \frac{\eta}{\vartheta_1^{1/2+RB\hat{k}}}.$$  \hfill (24)

The contribution of the $(y, \rho, \phi)$ coordinates to the torus amplitude is thus explicitly given by

$$T = \frac{R}{\sqrt{\pi \alpha' \tau_2}} \frac{1}{|\eta|^2} \left\{ \sum_{k,n} e^{-\frac{R^2}{\alpha' \tau_2} |\hat{k}+\tau n|^2} Z(\hat{k}, w) \right\}, \hfill (25)$$

where (we suppose for the time being that $BR$ is not rational)

$$Z(0, 0) = \frac{V_2}{4\pi^2 \alpha' \tau_2} \left| \frac{\eta}{\vartheta_1^{1/2+RBn}} \right|^2, \forall (\hat{k}, n) \neq (0, 0) \hfill (26)$$

and in (26) $V_2$ is the volume of the plane $(\rho, \phi)$. There is a different and sometimes more illuminating way to obtain the partition function. Consider the free theory on $R^3$ and then perform the orbifold $R^3/Z$, where the action of the group is generated by (1). The untwisted part of the torus amplitude is obtained by the insertion in the flat space amplitude of the projector onto states invariant under (1). The latter is given by

$$\pi = \sum_{k} e^{-2\pi i k R \hat{J}} \bar{P}_{\rho} e^{2\pi i R \hat{k} \hat{J}} P_{\phi}. \hfill (27)$$

The completion of the amplitude by modular invariance gives the rest of the torus partition function.

The inclusion of the world-sheet fermions is straightforward. The world-sheet superpartners of $Z_0$, (called $\lambda$ in what follows), in the Ramond sector are twisted by $1 - \nu$ and in the NS sector are twisted by $1/2 - \nu$. The total right and left moving hamiltonians are given by

$$L_0 = N + \frac{\alpha'}{4} \left( \frac{k}{R} - B(J + \tilde{J}) + \frac{n R}{\alpha'} \right)^2 - \nu J + a,$$

$$\bar{L}_0 = \tilde{N} + \frac{\alpha'}{4} \left( \frac{k}{R} - B(J + \tilde{J}) - \frac{n R}{\alpha'} \right)^2 + \nu \tilde{J} + \bar{a}, \hfill (28)$$

where $N$ and $\tilde{N}$ include now the fermionic oscillators and

$$a(NS) = \frac{\nu - 1}{2}, \quad a(R) = 0. \hfill (29)$$

The angular momentum $J$ has in addition to the bosonic contribution (17) the fermionic one

$$J_f + \tilde{J}_f = -\frac{1}{2\pi} \int (\lambda \lambda + \tilde{\lambda} \tilde{\lambda}). \hfill (30)$$
The torus amplitude for type IIB (IIA) superstring in the Melvin background, encoding the GSO projections, is given by

\[ T = V_2 \frac{R}{\sqrt{\pi \alpha' \tau_2}} \frac{1}{(4\pi^2 \alpha' \tau_2)^{7/2} |\eta|^4} \left\{ \sum_{\tilde{k},n} e^{-\frac{\eta^2}{8\alpha' \tau_2} (\tilde{k} + \tau n)^2} Z(\tilde{k}, n) \right\}, \tag{31} \]

with

\[ Z(\tilde{k}, n) = \frac{1}{4} \left\{ \sum_{\alpha,\beta} \eta_{\alpha\beta} e^{-2\pi i \beta BRn} \frac{\tilde{\eta}^{[\alpha]}_{[\beta]}}{\eta^3} \frac{\tilde{\eta}^{[\alpha+RBn]}_{[\beta+RBk]}}{\tilde{\eta}^{1/2+RBn}_{1/2+RBk}} \right\} \left\{ \sum_{\alpha,\beta} \bar{\eta}_{\alpha\beta} e^{2\pi i \beta BRn} \frac{\bar{\eta}^{[\alpha]}_{[\beta]}}{\bar{\eta}^3} \frac{\bar{\eta}^{[\alpha+RBn]}_{[\beta+RBk]}}{\bar{\eta}^{1/2+RBn}_{1/2+RBk}} \right\}, \tag{32} \]

where \( \eta_{\alpha\beta} = (-1)^{2\alpha+2\beta+4\alpha\beta} \) and \( \bar{\eta}_{\alpha\beta} = \eta_{\alpha\beta} \) for IIB and \( \bar{\eta}_{1/2} = -\eta_{1/2} \) for IIA. Using a Jacobi identity it is possible to cast (32) in the form

\[ Z(\tilde{k}, n) = \left| \frac{\tilde{\eta}^{[\alpha]}_{[\beta]} (BR(\tilde{k} + n\tau)/2|\tau)}{\eta^3 \bar{\eta}^{[\alpha]}_{[\beta]} (BR(\tilde{k} + n\tau)|\tau)} \right|^2, \tag{33} \]

which is the form found when the Green-Schwarz form of the world-sheet action is used [21].

### 2.2 Open strings

For open strings, the consistent boundary conditions correspond to Neumann or Dirichlet for \( y \) and \( Z_0 \). They are obtained by the requirement that the boundary term arising from the variation of the sigma model action vanishes. The boundary term reads

\[ \partial_\sigma y \delta y + \partial_\sigma X_0 \delta X_0 + \partial_\sigma Y_0 \delta Y_0 = \partial_\sigma y \delta y + \partial_\sigma \rho \delta \rho + \rho^2 \partial_\sigma \phi_0 \delta \phi_0 \]
\[ = \partial_\sigma \rho \delta \rho + \rho^2 (\partial_\sigma \phi + B \partial_\sigma y) \delta \phi + (\partial_\sigma y + B \rho^2 (\partial_\sigma \phi + B \partial_\sigma y)) \delta y. \tag{34} \]

The coordinate of physical relevance is \( Z \) but, as is manifest from (34), this is not the coordinate with simple boundary conditions. The sigma model action allows uncorrelated boundary conditions for \( Z_0 \) and \( y \). In the following we examine the different possibilities.

**a-** Neumann conditions for all the coordinates. We have \( Z_{0+}(\sigma) = Z_{0-}(\sigma) = Z_{0+}(\sigma + 2\pi) \) and \( y_+ = y_- = y = \alpha'(k/R - BJ) \tau + \text{osc} \). So \( Z_0 \) represents a usual free un-twisted world-sheet boson. In particular, it has zero modes corresponding to two noncompact momenta and

\[ J = x p_y - y p_x + \sum_{m=1}^{\infty} (a_m^4 a_m - b_m^4 b_m). \tag{35} \]

The difference with respect to free bosons is that the Kaluza-Klein contribution to the Hamiltonian is replaced by \((k/R - BJ)^2\).
b- Dirichlet conditions for $y$ and $\phi_0$, and Neumann for $\rho$. This implies that $y$ and $\phi$ are Dirichlet. The expansion of $y$ reads

$$y = y_0 + 2\sigma n R + \text{osc.},$$

where $n$ is the winding mode. If $\phi(0) = \phi(\pi) = 0$, then

$$\phi_0(\pi) - \phi_0(0) = 2B\pi n R.$$  

(37)

This implies that $Z_{0+}$ and $Z_{0-}$ are related by

$$Z_{0+}(\sigma) = Z_{0-}(\sigma)^*,$$

(38)

and

$$Z_{0+}(\sigma + 2\pi) = Z_{0+}(\sigma)e^{4\pi i B n R}.$$  

(39)

The free field $Z_0$ is thus twisted by $\nu = 2BRn - [2BRn]$ and its mode expansion reads

$$Z_{0+}(\sigma_+) = \sqrt{\alpha'} \left[ \sum_{m=1}^{\infty} \frac{a_m}{\sqrt{m - \nu}} e^{-i(m - \nu)\sigma_+} + \sum_{m=0}^{\infty} \frac{b_m^\dagger}{\sqrt{m + \nu}} e^{i(m + \nu)\sigma_+} \right]$$

(40)

and the Hamiltonian for the $(\rho, \phi)$ coordinates is given by

$$H_\nu = \frac{1}{24} - \frac{1}{8}(1 - 2\nu)^2 + \nu \phi_0^\dagger \phi_0 + \sum_{m=1}^{\infty} (m - \nu)a_m^\dagger a_m + \sum_{m=1}^{\infty} (m + \nu)b_m^\dagger b_m.$$  

(41)

Notice that with these boundary conditions, the quantization condition on $P_y$ and $P_\phi$ as defined in (10) is no longer valid. The full Hamiltonian is given by $H = H_\nu + N_y + \frac{1}{\alpha'}(nR)^2 - \frac{1}{24}$.

c- Dirichlet boundary condition for $y$, $\phi_0$ and $\rho$. This corresponds to Dirichlet boundary conditions for $Z$. If $y$ has a winding mode $n$ and take for simplicity $y_0 = 0$, then $Z_0(0, \tau) = Z(0, \tau)$ but $Z_0(\pi, \tau) = e^{iB2\pi n R}Z(\pi, \tau)$. In particular if $Z(0, \tau) = Z(\pi, \tau)$ then $\phi_0(0) - \phi_0(\pi) = -2\pi B n R$ and $|Z_0(0) - Z_0(\pi)|^2 = 4\rho_0^2 \sin^2(\pi B n R)$. This gives a contribution proportional to $\sin^2(\pi BR n)$ to be added to the free Hamiltonian.

d- $y$ Neumann and $Z_0$ Dirichlet with $Z_0(0, \tau)$ and $Z_0(\pi, \tau)$ different from zero. The identification (6) leads to nontrivial zero modes for the $y$ coordinate. In fact the subspace $\phi_0 = \text{const.}$ is non-compact when $BR$ is irrational. In order to determine the allowed $y$ momenta, consider a function on the $y, \phi_0, \rho$ space. It has the expansion

$$f(y, \phi_0, \rho) = \sum_{k,l} e^{iy/k} e^{i\phi} c_{kl}(\rho),$$

(42)

where we used the fact that the variables $y, \phi$ are periodic. Now if we restrict this function to the subspace $\phi_0 = c$, we get the quasi-periodic function in $y$

$$g(y) = \sum_{k,l} e^{iy/k} e^{i(c - By)} c_{kl}(\rho),$$

(43)
that is the allowed momenta in the $y$ direction are characterised by two integers $k$ and $l$ and

$$p_y = \frac{k}{R} - lB .$$

(44)

This is a novel kind of "compactification" on quasi-periodic spaces. The length of the $y$ coordinate is not finite but its momenta are quantized with two integers. The existence of non-compact dimensions with however a mass gap in the spectrum (44) can have interesting phenomenological implications.

**e-** $y$ Neumann, $Z_0$ Dirichlet with $Z_0(0, \tau) = Z_0(\pi, \tau) = 0$. At the origin of the $(\rho, \phi_0)$ plane, the identification (44) does not lead to any twist. Indeed, the identification (44) $(y, Z_0) = (y + 2\pi, Z_0 e^{2i\pi BR})$ shows clearly the absence of any twist at the origin. The hamiltonian in this case is nearly the same as the case a) above. The difference is that $J$ does not have a zero mode component, that is

$$J = \sum_{m=1}^{\infty} (a_m^\dagger a_m - b_m^\dagger b_m) .$$

(45)

This reflects in particular in the absence of the term (23) in the open string amplitude.

**f-** $y$ Dirichlet and $Z_0$ Neumann. There are winding modes for $y$. The hamiltonian of the system is completely free.

### 3 D-Branes in the Melvin background: Born-Infeld results

A classical approach for determining properties of D-branes uses the interaction of closed string fields with the branes via the Born-Infeld action. In order to have a neat field-theory interpretation for branes with Neumann boundary conditions in $y$ in the Melvin background, we first perform a Buscher T-duality in the coordinate $y$. The resulting world-sheet action,

$$S = -\frac{1}{4\pi\alpha'} \int [d\rho \wedge^* d\rho + \frac{1}{1 + B^2 \rho^2} (\rho^2 d\phi \wedge^* d\phi + d\tilde{y} \wedge^* d\tilde{y} - 2B\rho^2 d\phi \wedge d\tilde{y})] ,$$

(46)

where $\tilde{y}$ denotes the T-dual coordinate, exhibits a curved background as well as the presence of an antisymmetric tensor $B_{\phi\tilde{y}}$. We determine in the following the spectrum of classical fluctuations around this background and compare with the closed string mass spectrum. By using these results in the Born-Infeld action, one finds the one-point functions of closed string states in the front of the D-branes, to be compared later on with the result extracted from the string brane-brane amplitudes. The classical field equation for the dilaton $\Phi$, for example, in the background (46) reads

$$\frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho \frac{\partial \Phi}{\partial \rho}) + (1 + B^2 \rho^2) \left( \frac{\partial^2 \Phi}{\partial \tilde{y}^2} + \frac{1}{\rho^2} \frac{\partial^2 \Phi}{\partial \phi^2} \right) + \Delta \Phi = 0 ,$$

(47)
where $\Delta_7$ denotes the d’Alembertian in the remaining (flat) seven spacetime coordinates $x$, which defines the mass of the particle $\Delta_7 \Phi = M^2 \Phi$. Expanding the solution into Kaluza-Klein modes along $\tilde{y}$ and angular momentum modes

$$\Phi(\rho, \phi, \tilde{y}, x) = e^{i \frac{\pi}{2} \tilde{y} + il \phi} \Phi_{k,l}(\rho, x) , \quad (48)$$

we find the Schrödinger-type equation

$$-\frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial \Phi}{\partial \rho} \right) + \left( \frac{l^2}{\rho^2} + \frac{k^2 B^2}{R^2} \rho^2 \right) \Phi = \left( M^2 - \frac{k^2}{R^2} - B^2 l^2 \right) \Phi . \quad (49)$$

The equation (49) is identical to the Schrödinger equation for a two-dimensional oscillator with frequency $\omega_k = |k| B / R$. The solution is easily found by making the change of variables

$$\Phi = f(\omega_k \rho^2) e^{-\frac{\omega_k \rho^2}{2}} . \quad (50)$$

In terms of the function $f(z = \omega_k \rho^2)$, (49) becomes

$$zf'' + (1 + |l| - z)f' = -\frac{1}{4\omega_k} \left( M^2 - \frac{k^2}{R^2} - B^2 l^2 - 2\omega_k (|l| + 1) \right) f . \quad (51)$$

The differential equation has normalizable solutions if and only if

$$M^2 = \frac{k^2}{R^2} + B^2 l^2 + 2\omega_k (2j + |l| + 1) , \quad (52)$$

for non-negative integer $j$. The mass operator (52) indeed coincides with the field-theory part of the string mass operator (18) with the following identifications

$$l = n_0 - \tilde{n}_0 , \quad 2j + |l| = n_0 + \tilde{n}_0 + 1 . \quad (53)$$

The solution of (51) gives $f(z) = A L^{|l|}_j(z)$, where $A$ is a normalization constant and the $L$’s are the Laguerre polynomials

$$L^{|l|}_j(z) = \frac{z^{-n}}{j!} e^z \frac{d^j}{dz^j} (z^{j+n} e^{-z}) . \quad (54)$$

The normalisation of the wavefunction is obtained from the closed string effective action from the requirement of a standard propagator. The normalized (in flat space) dilaton wave-functions can be then written in the form

$$\Phi_{k,l,j}(\rho, \phi, \tilde{y}) = e^{i \frac{\pi}{2} \tilde{y} + il \phi} \sqrt{\frac{\omega_k |l| + 1}{\pi (j + |l|)!}} \rho^{|l|} L^{|l|}_j(\omega_k \rho^2) e^{-\frac{\omega_k \rho^2}{2}} . \quad (55)$$

**NN boundary conditions for $Z$**

If the D-brane under consideration has Neumann boundary conditions for all relevant coordinates $y, Z$, after performing the T-duality on $y$ described above let us put the brane at the point $y = 0$ . Then the Born-Infeld action reads

$$S_{BI} = -T_p \int_{y=0} d^2 Z e^{-\Phi} \sqrt{\det(g + B)} = -T_p \int_{y=0} \rho d\rho d\phi e^{-\Phi} . \quad (56)$$
where in deriving the last equality we used explicitly the background \((46)\). The result is the same as if the space were perfectly flat. In order to compare field theory considerations with string computations it is useful to extract the one-point function \(g_{k,l,j}\) of the dilaton in front of the boundary (D-brane). By using \((48),(55)\) we find

\[
g_{k,l,j} = 2\delta_{l,0} T_p \int \rho d\rho \Phi_{k,0,j} ,
\]

and in particular the total angular momentum \(l = n_0 - \tilde{n}_0\) of closed states coupling to the brane is zero. The explicit computation of \((57)\) finally gives, by performing \(j\) integration by parts, the simple result

\[
g_{k,0,j} = T_p \sqrt{\frac{1}{\pi \omega_k}} \int_0^\infty dz \frac{L_0^0(z)}{e^{-\frac{z}{2}}} = 2(-1)^j T_p \sqrt{\frac{\pi}{\omega_k}} .
\]  

\[ \text{DD boundary conditions for } Z \]

The D-brane under consideration has Dirichlet boundary conditions for the magnetized plane \(Z\) and the D-brane is forced to sit at the origin of the plane \(Z = 0\). After performing the T-duality on \(y\) and putting the brane at \(y = 0\), the Born-Infeld action reads

\[
S_{BI} = -T_p \int_{y=0} \delta^2(Z)e^{-\Phi} \sqrt{\det(g + B)} = -T_p \left(e^{-\Phi}\right)_{y=Z=0} ,
\]

where we used again the background \((46)\) to derive the last equality. The one-point function \(g_{k,l,j}\) of the dilaton in front of the boundary (Dp-brane) is now

\[
g_{k,l,j} = T_p(\Phi_{k,j,l})|_{y=Z=0} .
\]

By using \((55)\) we find, as previously, that the non-vanishing wave-functions at \(Z = 0\) have zero total angular momentum \(l = 0\). Moreover, an explicit computation of \((60)\) finally gives the simple result

\[
g_{k,0,j} = T_p \sqrt{\frac{\omega_k}{\pi}} .
\]  

We now turn to the string theory description of D-branes and in particular to the brane-brane interactions which allow an independent check of the one-point couplings \((58)\) and \((61)\).

\section{D-Branes in the Melvin background: string amplitudes}

\subsection{D branes parallel to the Melvin space}

Let us turn to study the spectrum and interaction of D-branes in the Melvin background and start with Neumann boundary conditions for all relevant coordinates \((y,\rho,\phi)\). The
open string Hamiltonian describing strings stretched between two Dp-Dp branes separated by the distance \( r \), reads

\[
H = N + \alpha' \left( \frac{k}{R} - BJ \right)^2 + \frac{r^2}{4\pi^2\alpha'} + a ,
\]

where \( a \) vanishes in the Ramond sector and equals \(-1/2\) in the Neveu-Schwarz sector. The usual GSO projection removes the open string tachyon. The spectrum of massless open string states for a stack of \( M \) Dp branes is based on the unitary gauge group \( U(M) \). At the massless level, boson masses for the \( SO(p-2) \) Lorentz group are unaffected by the (closed) magnetic field \( B \), whereas internal vectors along the twisted plane and spacetime fermions acquire a mass proportional to \( B^2 \), thus breaking supersymmetry. Notice that the boundary conditions do not allow the presence of Landau levels in the open spectrum. The quantum numbers of a state include the value of the orbital angular momentum \( m = x p_y - y p_x \) which is an integer, as well as the norm of the momentum in the \( Z_0 \) plane and the components of the momentum \( q_\mu \) along the remaining D-brane coordinates.

Consider the lowest level vector states \( \psi_{-1/2}^{\mu} |0> \), then the mass shell condition reads

\[
q^2 = \left( k/R - B m \right)^2 ,
\]

whereas for the states \( \lambda_{-1/2}^{\mu} |0> \) it reads

\[
p^2 = \left( k/R - B (m \pm 1) \right)^2 .
\]

The one-loop cylinder amplitude encoding the Dp-Dp brane interaction is given by

\[
A_{pp} = \frac{1}{2} \int_0^\infty \frac{dt}{t} T e^{-\pi t(H + \alpha' q^2)} ,
\]

where \( t \) is the open channel cylinder modulus and the trace includes an integration over the momenta \( q \) along the worldvolume spanned by the Dp brane. As in the closed string case \((12)\), the result is easily written in terms of the Poisson-resummed momentum \( \tilde{k} \).

The final result of the computation, obtained by using the eqs. \((23)\)-(\(24\)) is

\[
A_{pp} = \frac{R}{\sqrt{4\alpha'}} \sum_{\tilde{k}} \int_0^\infty \frac{dt}{t^{3/2}} e^{-\pi \tilde{k}^2 R^2/(\alpha' t)} ,
\]

where the argument of the various modular functions is \( \tau = it/2 \). The interaction of the Dp branes with the closed string spectrum is, as usual, transparent in the closed-string channel amplitude, written in terms of the modulus \( l = 2/t \). Notice first of all that in this case \( \tilde{k} \) are actually precisely the winding states \( n \) of the closed strings exchanged by

\[\text{remember that } \lambda \text{ is the superpartner of } Z_0\]
the two Dp branes. The resulting brane-brane interaction reads

$$A_{pp} = \frac{\pi R V_p}{2(8\pi^2\alpha')^{(p+1)/2}} \int \frac{dl}{l^{p+2}} e^{-\frac{l^2}{2\alpha'}} \sum_{\alpha,\beta = 0,1/2} \frac{\partial^{\alpha\beta}[\alpha]}{\eta^{1/2}} + \frac{\pi R \alpha' V_{p-2}}{(8\pi^2\alpha')^{(p+1)/2}} \int \frac{dl}{l^{p+2}} e^{-\frac{l^2}{2\alpha'}} \sum_{n \neq 0} \frac{1}{2 \sin \pi BRn} \frac{\partial^{3\alpha\beta}[\alpha]}{i\eta^9 \partial_1(-iBRn)l} e^{-\frac{2\pi^2 R^2 l}{2\alpha'}}$$  \quad (66)

This amplitude will be recovered in the next section by using the boundary state formalism. Let us for the moment examine its low energy limit and compare with the previous Born-Infeld result [58]. The limiting case ($r >> \sqrt{\alpha'}, BR << 1$) is described by decoupling the heavy (in this limit) string oscillators in (66) and keeping only the closed string Landau levels coupling to the Dp branes. Let us discuss separately the interaction of the NS-NS and the RR closed states to the branes in this limit and check its consistency. The NS-NS exchange is described by

$$A_{pp}^{NS-NS} = A_{pp, massless}^{NS-NS} + \frac{\pi R \alpha' V_{p-2}}{(8\pi^2\alpha')^{(p+1)/2}} \int \frac{dl}{l^{p+2}} e^{-\frac{l^2}{2\alpha'}} \sum_{n \neq 0} \frac{1}{2 \sin \pi BRn} \frac{6 + 2ch(2\pi\nu l)}{sh(\pi\nu l)} e^{-\frac{\pi n^2 R^2 l}{2\alpha'}}$$  \quad (67)

where $\nu = BRn - [BRn]$.

In order to interpret the amplitude (67), it is important to account for the reflection of closed string states on the D-brane $|k, n, J, J > - > | - k, n, - J, - J >$. The set of states invariant under the reflection is $|0, n, J, - J >$. Notice that the condition

$$J|_{\text{boundary}} = -J|_{\text{boundary}} ,$$  \quad (68)

allows for one-set of Landau levels of closed strings bouncing off the brane. We therefore rediscovered at string level the vanishing of the total angular momentum of closed states coupling to the brane, first found from the Born-Infeld analysis of the previous section. The Landau levels are easily identified in (67) by performing the expansion

$$\frac{6 + 2ch(2\pi\nu l)}{sh(\pi\nu l)} = \sum_{j=0}^{\infty} \left[ 6 e^{-2\pi\nu(j+\frac{1}{2})l} + e^{-2\pi\nu(j-\frac{1}{2})l} + e^{-2\pi\nu(j+\frac{3}{2})l} \right] .$$  \quad (69)

The first term in (69), when plugged in (67), describes the Landau levels of the $(p+1)$ dim. dilaton. The second and the third term describe the components of the metric transverse to the brane, which can become tachyonic for small enough radius $R < \sqrt{2\alpha'}$.

The long-range exchange of RR states between the Dp branes is described by the amplitude

$$A_{pp}^{RR} = A_{pp, massless}^{RR} - \frac{\pi R \alpha' V_{p-2}}{(8\pi^2\alpha')^{(p+1)/2}} \int \frac{dl}{l^{p+2}} e^{-\frac{l^2}{2\alpha'}} \sum_{n \neq 0} \frac{1}{2 \sin \pi BRn} \frac{ch(\pi\nu l)}{sh(\pi\nu l)} e^{-\frac{\pi n^2 R^2 l}{2\alpha'}} .$$  \quad (70)
The interaction of Landau levels of RR states with the D-branes is obtained by performing the expansion
\[
\frac{c h(\pi \nu l)}{s h(\pi \nu l)} = \sum_{j=0}^{\infty} \left[ e^{-2\pi \nu jl} + e^{-2\pi \nu (j+1)l} \right].
\] (71)

The amplitudes (67), (70) expanded in Landau levels according to (69), (71) allow to reconstruct the hamiltonian of closed states interacting with the D-branes. In order to achieve this, we use the closed-string propagator for a canonically normalized scalar of mass \(M\) and momentum \(q\)
\[
\Delta_c = \frac{\pi \alpha'}{2} \int_0^\infty dl \ e^{-\frac{\pi \alpha'}{2} (q \cdot q + M^2)}.
\] (72)

By using (72), we finally obtain
\[
M_{\text{NS-NS}}^2 = N_{\text{osc}} + \frac{n^2 R^2}{2\alpha'} + (2j \pm 1)\nu,
\]
\[
M_{\text{RR}}^2 = N_{\text{osc}} + \frac{n^2 R^2}{2\alpha'} + 2j\nu,
\] (73)
in agreement with the argument presented in (58). Notice in (73) the coupling of the would-be tachyon \(j = 0\) in the NS-NS sector of the D-brane, corresponding to the fluctuations of the metric transverse to the brane. We are now able to extract the one-point couplings of the various closed fields and compare them with the field-theory result (58). In order to do so, notice first that by performing a T-duality on \(y\) the windings \(n\) become Kaluza-Klein states \(k\). Consider moreover the field-theory limit in (67), (70), in which (after T-duality) \( \sin(\pi B R k) \simeq \pi B R k \). The string amplitudes (67), (70) contain the tree-level propagation of closed states between the branes and therefore contain two one-point functions and a closed propagator. By a quick inspection we precisely recover the field-theory result for the one-point coupling (58). It is very interesting to go beyond the field-theory limit in (67), (70). In this case, the one-point functions show a deviation from the field-theory result which can be attributed to \(\alpha'\) corrections to the Born-Infeld action. This point could bring new insights into the higher derivative corrections to the Born-Infeld action and clearly deserves a more dedicated investigation.

4.2 D branes at the origin of the Melvin space

Here we consider the case where \(y\) is parallel to the brane which has now Dirichlet boundary conditions in the \(\rho, \phi\) plane and moreover is at the origin of the plane, anticipated in section 2.2e. The difference compared to the case where all relevant coordinates were Neumann is that there are no zero-mode (non-compact) momenta contributions from the \(Z_0\) coordinate and the the angular momentum has no zero mode contribution. The final amplitude taking into account these changes is
\[
A_{pp} = \frac{\pi R V_p}{2(4\pi^2 \alpha')^{(p+1)/2}} \int \frac{dt}{t^{p+1/2}} e^{\frac{2i}{4\alpha'} t} \sum_{\alpha, \beta = 0, 1/2} \eta_{\alpha, \beta} \frac{\vartheta^4 [\alpha]}{\eta^{12}} + \\
\frac{\pi R V_p}{(4\pi^2 \alpha')^{(p+1)/2}} \sum_{\alpha, \beta} \int \frac{dt}{t^{p+1/2}} e^{\frac{2i}{4\alpha'} t} \sum_{k \neq 0} \sin \frac{\pi B R k \eta_{\alpha, \beta} \vartheta^{3} [\alpha]}{\eta \vartheta^{1/2 + B R k}} e^{-\frac{k^2 R^2 t}{\alpha'}}.
\] (74)
The identification of the tree-level closed string exchange between two such D-branes follows closely the one described in the previous paragraph. The only difference is in the zero-mode part, which in the closed channel amounts to replace the factor $2 \sin(\pi \nu)$ in various amplitudes by its inverse $1/(2 \sin(\pi \nu))$. In the low-energy limit $\nu << 1$, the amplitude containing the square of the one-point function turns out to be very similar to that of the NN D-branes, with the notable difference that the factor $\omega_k$ is now replaced by $1/\omega_k$. This matches the field theory result (61), however the phase factor $(-1)^j$ that was found there squares to one and cannot therefore be detected by the present computation.

In order to resolve this ambiguity, we now compute the cylinder amplitude between one Dp brane with NN boundary conditions along the $Z$ plane and one D$(p-2)$ brane with DD boundary conditions, respectively. In the long-range, field-theory limit, this amplitude encodes the cross product of one-point functions of the closed fields with the two types of branes. The resulting amplitude can be easily written by quantizing properly the open strings with Neumann-Dirichlet boundary conditions along $Z$. The result is most conveniently written in the closed channel and read

$$A_{p,p-2} = \frac{\pi R V_{p-2}}{2(4\pi^2 \alpha')^{(p-1)/2}} \int \frac{dl}{l^{2-2r}} e^{-\frac{r^2}{2\pi \alpha'}} \eta_{\alpha,\beta} \int \frac{d\theta}{2\pi} \frac{\eta^{\frac{3}{2}}[\alpha]}{\eta^{\frac{3}{2}}[\beta]} \sum_{\alpha,\beta=0,1/2} \vartheta_{j}^{\frac{3}{2}}[\alpha] \vartheta_{j}^{\frac{3}{2}}[\beta + 1/2] \left(-iB R n|l\right) e^{-\frac{\omega l^2}{2\alpha'}}. \quad (75)$$

The long-range exchange of RR states between the Dp and the D$(p-2)$ brane turns out to be now given by the amplitude

$$A_{p,p-2} = \frac{\pi R V_{p-2}}{2(4\pi^2 \alpha')^{(p-1)/2}} \int \frac{dl}{l^{2-2r}} e^{-\frac{r^2}{2\pi \alpha'}} \sum_{\alpha,\beta} \eta_{\alpha,\beta} \int \frac{d\theta}{2\pi} \frac{\eta^{\frac{3}{2}}[\alpha]}{\eta^{\frac{3}{2}}[\beta]} \left(-iB R n|l\right) e^{-\frac{\omega l^2}{2\alpha'}}. \quad (76)$$

The interaction of Landau levels of RR states with the D-branes is now obtained by performing the expansion

$$\frac{sh(\pi \nu l)}{ch(\pi \nu l)} = \sum_{j=0}^{\infty} (-1)^j \left[e^{-2\pi \nu jl} - e^{-2\pi \nu (j+1)l}\right]. \quad (77)$$

Plugging in (77) in (76) we finally obtain the cross product of one-point functions of RR fields with the two different D-branes. The presence of the phase factor $(-1)^j$ and the absence of the zero mode $\sin(\pi \nu)$ factor are precisely required for the exact agreement in the field theory limit $\nu << 1$ of the one-point functions derived from (77) with the ones derived from the Born-Infeld action. Similar results hold for the couplings of the NS-NS fields to the branes.

### 4.3 D branes with fixed $Z_0$

We consider here D-branes parallel to $y$ but perpendicular to the $(\rho, \phi_0)$ plane. Here $y$ is a coordinate along the brane which has a fixed but nonzero value for $Z_0$. As explained in
section 2.2.d the \( y \) coordinate is a quasi-periodic coordinate. The cylinder amplitude is given by

\[
A_{pp} = \frac{V_p}{4(4\pi^2\alpha')^p/2} \int \frac{dt}{t^{p+2}} \sum_{k,l} e^{-\pi t \alpha'(k/R-B)l^2} \sum_{\alpha,\beta} \eta_{\alpha\beta} \frac{\varphi^{[\alpha'][\beta]}}{\eta^{[12]}} ,
\]

(78)

where the sum is the contribution of the momenta of the \( y \) coordinate. Interestingly enough, the partition function describes a supersymmetric \( D \)-brane open spectrum.

The Kaluza-Klein contribution of the \( y \) momenta can be put in the form

\[
\sum_{k,l} e^{-\pi t \alpha'(k/R-B)l^2} = \sum_l \varphi_{[0]}^{[RBl]}(0, ti t \alpha')^{1/2} \quad (79)
\]

A modular transformation allows the sum to be cast in the form

\[
\frac{R}{\sqrt{t \alpha'}} \sum_{l,k} e^{-\pi R^2 l^2 / 2t \alpha'} e^{2\pi i R Bl k} .
\]

(80)

The sum is divergent as it stands, therefore a usual finite volume regularisation is required. The simplest finite volume regularisation is to assume that \( BR \) is rational of the form \( P/Q \) where \( P \) and \( Q \) are large integers with no common divisor. The non-compact space \( Y \) space is replaced with a circle of radius \( L = QR \). The sum (79) is replaced with

\[
\sum_{k \in Z} e^{-\pi R^2 l^2 / 2t \alpha'} \sum_{l=0}^{Q-1} e^{2\pi i Plk/Q} .
\]

(81)

The sum over \( l \) can be readily calculated and gives \( Q \sum_p \delta_{k,pQ} \) which allows the regularised sum (79), in the limit of large \( L \), to be cast in the form

\[
\frac{L}{\sqrt{t \alpha'}} .
\]

(82)

### 4.4 D branes with fixed \( y \) and \( \phi \)

Here we consider open strings with Dirichlet boundary conditions for \( y \) and \( \phi_0 \) or equivalently for \( y \) and \( \phi \). The coordinate \( \rho \) is taken to be Neumann, such that the brane intersects the \( \rho, \phi \) plane with a line of constant \( \phi \). The bosonic Hamiltonian was presented in subsection 2.2b, where it was shown that the contribution from the \( Z \) coordinates is that of a twisted boson. The fermionic Hamiltonian can be deduced similarly, since the superpartners of \( Z_0 \) have the same twisting in the Ramond sector and an opposite one in the Neveu-Schwarz sector. The Hamiltonian can be put in the form

\[
H = N - \nu J + a ,
\]

(83)

where \( N \) is as usual the number operator, \( J \) is the total angular momentum and \( a \) vanishes in the Ramond sector and is given by \( (\nu - 1)/2 \) in the Neveu-Schwarz sector. The cylinder
amplitude in the direct channel encoding the GSO projection is given by

\[ A_{pp} = \frac{V_{p+1}}{2(4\pi^2\alpha')(p+1)/2} \int \frac{dt}{t^{p+3/2}} e^{-\frac{t^2}{4\alpha'}} \sum_{\alpha,\beta=0,1/2} \eta_{\alpha,\beta} \frac{\vartheta_{1}^{4}\left[\alpha\right]}{\eta_{12}^{1/2}}. \]  

Notice that this amplitude can be constructed starting from the cylinder amplitude for a D-brane in flat space and then adding the images which arise upon the identification (1). The interaction of a brane with its image is given by the amplitude of two branes distant in the y direction by \( n^2 \pi R \) and at a relative angle \( 2n\pi R \). In order to verify the consistency of the amplitude we have to consider its closed-string interpretation:

\[ A_{pp} = \frac{V_{p+1}}{2(8\pi^2\alpha')(p+1)/2} \int \frac{dl}{l^{p+1/2}} e^{-\frac{l^2}{2\pi\alpha'}} \sum_{\alpha,\beta} \eta_{\alpha,\beta} \frac{\vartheta_{1}^{4}\left[\alpha\right]}{\eta_{12}^{1/2}} + \]

\[ + \frac{V_{p}}{2(8\pi^2\alpha')/2} \int \frac{dl}{l^{p+1/2}} e^{-\frac{l^2}{2\pi\alpha'}} \sum_{k \neq 0} \sum_{\alpha,\beta} \eta_{\alpha,\beta} \vartheta_{1/2+2Rk}^{\alpha} \vartheta_{1/2+2Rk}^{\beta} e^{-2\pi l^2/l} \]  

\[ (84) \]

This is the form of the amplitude that we must recover in the next section from the closed string tree level propagation of the boundary states. Notice that, contrary to the examples discussed in sections 4.1 and 4.2, here the open sector do contain Landau levels, while the closed states interacting with the brane do not, due to the reflection on the brane.

4.5 D branes transverse to the Melvin space

Here the three coordinates \( y, \rho \) and \( \phi \) are Dirichlet coordinates. From the discussion in section 2.2.c, the partition function can be readily determined:

\[ A_{pp} = \frac{V_{p+1}}{2(4\pi^2\alpha')(p+1)/2} \sum_{n} \int \frac{dt}{t^{p+3/2}} e^{-\frac{t^2}{4\alpha'} + \frac{4n^2\sin^2(\pi R n) + (2n+R)^2}{4\pi\alpha'}} \sum_{\alpha,\beta=0,1/2} \eta_{\alpha,\beta} \frac{\vartheta_{1}^{4}\left[\alpha\right]}{\eta_{12}^{1/2}}. \]  

\[ (86) \]

The open spectrum is supersymmetric, and the sine squared term contribution to the mass is due to the fact that if the values of \( Z \) at the two ends of the open string coincide then the values of \( Z_0 \) do not.

5 Boundary states

The one loop open channel amplitudes are constrained by the requirement of their dual interpretation as tree level closed strings exchange. The boundary states \( |B> \) encode this interpretation via [32]:

\[ A_{pp} = \int dl <B|e^{-\pi(lL_0+\bar{L}_0)}|B>, \]

\[ (87) \]

where \( l \) is the closed string modular parameter related to the open one by \( l = 2/t \). In the following we shall determine the boundary states and verify the correct factorisation of the one loop amplitudes.
5.1 The \( y \) coordinate is Neumann

The boundary state verifies

\[
\partial_r y(0, \sigma)|B, \eta >= 0 , \ (\psi_+(0, \sigma) - \eta \psi_-(0, \sigma))|B, \eta >= 0 , \tag{88}
\]

where \( \psi \) is the worldsheet superpartner of \( y \). In terms of the mode expansion we get

\[
\left( \frac{k}{R} - B(J + \tilde{J}) \right)|B, \eta >= 0 , \ (y_m - \tilde{y}_m^\dagger)|B, \eta >= (y_m^\dagger - \tilde{y}_m)|B, \eta >= 0 , \tag{89}
\]

\[
(\psi_m - \eta \tilde{\psi}_m)|B, \eta >= 0 . \tag{90}
\]

Since \( BR \) is irrational we get from the first equation that both \( k \) and \( J + \tilde{J} \) must vanish. The boundary state has the form

\[
|B, \eta >= \sum_n N_n |n> \otimes \exp\{\sum_{m=1}^\infty (y_m^\dagger \tilde{y}_m + \eta \psi_m \tilde{\psi}_m)\} |0 > \otimes |\psi_n, \eta > , \tag{91}
\]

where \( n \) denotes the winding mode, \( N_n \) is a normalisation constant, and \( |\psi_n, \eta > \) is the state depending on the other coordinates which has to verify

\[
(J + \tilde{J})|\psi_n, \eta >= 0 . \tag{92}
\]

As usual, the combination which is correctly GSO projected is \( |B >= |B, + > + |B, - > \). Since the world-sheet fermions contribution has minor modifications with respect to the flat space one, in the following we shall concentrate on the nontrivial bosonic contribution which is due to the coordinates \( y \) and \( Z_0 \).

a) \( Z_0 \) Neumann. The state \( |\psi_n, \eta > \) verifies

\[
(a_m - \tilde{b}_m^\dagger)|\psi_n, \eta >= 0 , \ (\bar{a}_m - b_m^\dagger)|\psi_n, \eta >= 0 , \tag{93}
\]

\[
(a_m^\dagger - \tilde{b}_m)|\psi_n, \eta >= 0 , \ (\bar{a}_m^\dagger - b_m)|\psi_n, \eta >= 0 .
\]

When \( n \) is zero, the state \( |\psi_0, \eta > \) has zero momentum in the \( Z_0 \) directions. The state \( |\psi_n, \eta > \) is thus given by

\[
|\psi_n, \eta >= \exp\{\sum_{m=0}^\infty \bar{a}_m^\dagger b_m^\dagger + \sum_{m=1}^\infty a_m^\dagger \tilde{b}_m^\dagger\} |0 > . \tag{94}
\]

This automatically verifies the constraint (92). The normalisation constants can be determined from the comparison with the open channel amplitudes, and the result is \( N_n^2 = \frac{N}{\sin \pi BR n} \).

b) \( Z_0 \) Dirichlet. The state \( |\psi_n, \eta > \) has to satisfy the constraint

\[
(Z_0(\tau = 0, \sigma) - z_0)|\psi_n, \eta >= 0 . \tag{95}
\]

The treatment of the nonzero modes is similar to the previous case, that is

\[
(a_m + \tilde{b}_m^\dagger)|\psi_n, \eta >= 0 , \ (\bar{a}_m + b_m^\dagger)|\psi_n, \eta >= 0 , \tag{96}
\]

\[
(a_m^\dagger + \tilde{b}_m)|\psi_n, \eta >= 0 , \ (\bar{a}_m^\dagger + b_m)|\psi_n, \eta >= 0 .
\]
The main difference comes from the equations of the zero modes which depend crucially on whether \( z_0 \) vanishes or not. In fact when \( n \neq 0 \), \( Z_0 \) has no zero mode part and so if \( z_0 \neq 0 \) then necessarily we have

\[ N_n = 0 \ , \ n \neq 0 \ . \]  

(97)

This agrees with the form of the open channel amplitude (82) found in section 4.3 where it was shown that in the transverse channel there are no closed string winding or Kaluza-Klein modes propagating in the amplitude.

When \( z_0 = 0 \), the normalisation constants are determined from the open channel amplitudes found in section 4.2. We get \( N^2 = N \sin \pi BRn \).

\[ c) \ X_0 \text{ Dirichlet and } Y_0 \text{ Neumann. There is a contradiction between (92) and the conditions expressing different boundary conditions for } X_0 \text{ and } Y_0. \text{ This rules out this case as a consistent D-brane configuration.} \]

\[ 5.2 \text{ The } y \text{ coordinate is Dirichlet} \]

Here the boundary state must verify

\[ (y(0, \sigma) - y_0) |B, \eta > 0 , \ (\psi_+(0, \sigma) + \eta \psi_-(0, \sigma)) |B, \eta > 0 \ , \]  

(98)

which imply that there are no winding modes contributing to the boundary state and \( |B, \eta > \) has the form

\[ |B, \eta > = \sum_k N e^{iky_0/R} |k > \otimes \exp\{ - \sum_{m=1}^{\infty} (y_m \tilde{y}^*_m + \eta \psi^*_m \tilde{\psi}^*_m)\} |0 > \otimes |\psi_k, \eta > , \]  

(99)

where \( k \) is the Kaluza-Klein mode. Since the winding mode is zero there is no twisting of the \( Z_0 \) coordinate or its superpartner. There is no constraint analogous to (92) so it seems that all boundary conditions for \( Z_0 \) are consistent. This is the main difference with the respect to the case where the coordinate \( y \) is Neumann.

\[ a) \ Z_0 \text{ Neumann. Here the boundary state has zero momentum along the } Z_0 \text{ direction and } (J + \tilde{J}) |B, \eta > 0 , \text{ so the closed string hamiltonian acting on the boundary state is the same as the free one. This agrees with the results of section 2.2f.} \]

\[ b) \ Z \text{ Dirichlet, which implies also that } Z_0 \text{ is Dirichlet. However, if } Z \text{ has the same value at the two ends of the open string then this is not the case for } Z_0 \text{ in a } y \text{ winding sector. This gave rise in section 4.5 to a nontrivial factor } \rho^2 \sin^2(\pi BRn) \text{ in the amplitude (86). In order to retrieve the origin of this factor in the closed string let us remark that the oscillator contribution to } J + \tilde{J} \text{ vanishes on the boundary state, so it is consistent to focus on the zero modes contribution of the amplitude. The zero mode part of the boundary state reads} \]

\[ |b > = N \sum_k \int d^2 p \ e^{ip r_0} |p, k > , \]  

(100)

where \( r_0 \) denotes the position of the brane in the \( Z_0 \) plane, with \( |r_0| = \rho_0 \) and \( N \) is a normalisation constant. The zero mode contribution to the amplitude reads

\[ < b|e^{-\pi l \alpha'[(p_0 - BL)^2 + p^2]/2}|b > , \]  

(101)
where we have assumed for simplicity that $\phi|Z_0$ the Kaluza-Klein modes allows this amplitude to have the form

\[ N^2 \frac{R}{\sqrt{l\alpha'}} \sum_n \int d^2 p_1 d^2 p_2 e^{-2\pi R^2 n^2/(\alpha' l)} \langle P_1| e^{i2\pi LRln} |P_2 \rangle e^{-\pi l\alpha' p_2/2^2} e^{-i(p_1-p_2)\cdot r_0}. \]  

(102)

Let $\theta$ denote the rotation in the $Z_0$ plane with an angle $-2\pi BRn$ then the integral can be readily calculated to give

\[ N^2 \frac{R}{\sqrt{l\alpha'}} \sum_n \int d^2 p_1 d^2 p_2 e^{-2\pi R^2 n^2/(\alpha' l)} e^{-(r_0-\theta r_0)^2/(2\pi\alpha')} = N^2 \frac{R}{\sqrt{l\alpha'}} \sum_n \int d^2 p_1 d^2 p_2 e^{-2\pi R^2 n^2/(\alpha' l)} e^{-2\alpha^0 \sin^2(\pi BRn)/(2\pi\alpha')} . \]  

(103)

By using the open modulus $t = (2/l)$ we find that (103) is in perfect agreement with the open channel amplitude (85) and so the consistency of the latter is nicely verified.

**c) $\phi$ Dirichlet and $\rho$ Neumann.** Since $\phi_0$ in the direct channel satisfies a Dirichlet boundary condition, the D-brane boundary state verifies $\partial_\sigma \phi_0 |B >= 0$ which leads to $(J - \bar{J}) |B >= 0$. When acting on the boundary state the closed string Hamiltonian reduces to $H = H_0 + (\alpha'/2)(k/R - B(J + \bar{J}))^2$ and therefore

\[ \langle B| e^{-\pi(\ell L_0 + \bar{L}_0)} |B > = \sum_k \langle B| e^{-\pi(\ell H_0 + \alpha'/2(k/R - B(J + \bar{J}))^2)} |B > . \]  

(104)

Performing a Poisson resummation we get

\[ \langle B| e^{-\pi(\ell L_0 + \bar{L}_0)} |B > = \sum_k \sqrt{\frac{R^2\alpha'}{l}} \langle B| e^{-\pi H_0 e^{\pi i 2BR(J + \bar{J})k}} |B > e^{-\frac{\pi i k^2 R^2 \alpha'}{l}} . \]  

(105)

By using the form of the boundary states we can explicitly evaluate (105). In fact, the dependence of $|B>$ on the $Z_0$ coordinate is given by

\[ |b > \times \exp\{-\sum_{m=1}^{\infty} (a_m^- + b_m^+ + b_m^-)|0 > , \]  

(106)

where $|b >$ is the zero modes contribution which reads

\[ |b > = \int dp_{y_0} \left| p_{x_0} = 0, p_{y_0} > , \right. \]  

(107)

where we have assumed for simplicity that $\phi_0 = 0$ on the boundary of the open string. The bosonic contribution in the $(\rho, \phi_0)$ plane in (105), computed by using (106)-(107) and performing some standard computations turns out to be equal to

\[ \frac{e^{i\eta}}{\sin(2\pi BRk)} \prod_{m=1}^{\infty} \frac{1}{(1 - e^{4\pi i BRk} e^{-2\pi lm})(1 - e^{-4\pi i BRk} e^{-2\pi lm})} = \frac{\eta}{\Im^{1/2}} \left| q = e^{-2\pi i} \right( q \]  

(108)

where the $\sin(2\pi BRk)$ factor comes from the zero modes contribution. Collecting together the bosonic and fermionic contributions, we find indeed agreement with (85).
Supersymmetric Melvin solutions

Supersymmetric Melvin solutions, preserving one-half supersymmetry, are easily obtained by accompanying the complete translation $y \to y + 2\pi R$ around the circle with correlated rotations in the noncompact planes $(\rho_1, \phi_1), (\rho_2, \phi_2)$. More precisely, the magnetic field parameters corresponding to the two planes are $B_1 = \pm B_2$. The metric of the resulting models is

$$ds^2 = d\rho_1^2 + d\rho_2^2 + \rho_1^2(d\phi_1 + Bdy)^2 + \rho_2^2(d\phi_1 \pm Bdy)^2 + dy^2 + dx^2,$$

where $x$ denote here the resulting 5 space-time coordinates.

The torus amplitude for type IIB (IIA) in the Melvin background is given by

$$T = V_5 \frac{R}{\sqrt{\pi \alpha' \tau_2}} \left( \frac{1}{4\pi^2 \alpha' \tau_2} \right)^{5/2} \left\{ \sum_{\bar{k},n} e^{-\frac{\pi R^2}{\alpha' \tau_2} |\bar{k} + rn|^2} Z(\bar{k}, n) \right\},$$

where

$$Z(\bar{k}, n) = \frac{1}{4} \sum_{\alpha, \beta} \eta_{\alpha\beta} \frac{\vartheta^2(\alpha)}{\eta^2} \left[ \vartheta^1(\alpha + RBn) \vartheta^1(\alpha - RBn) \right]^{1/2},$$

$$Z(0, 0) = \frac{1}{4} \left( \frac{V_4}{4\pi^2 \alpha' \tau_2} \right)^{2} \left\{ \sum_{\alpha, \beta} \eta_{\alpha\beta} \frac{\vartheta^4(\alpha)}{\eta^2} \right\} \left\{ \sum_{\alpha, \beta} \bar{\eta}_{\alpha\beta} \frac{\bar{\vartheta}^4(\alpha)}{\eta^2} \right\}. $$

Using a Jacobi identity it is possible to cast (32) in the Green-Schwarz form

$$Z(\bar{k}, n) = \frac{1}{\eta^3 \vartheta(1)} \left| \frac{\vartheta^2 \vartheta^2(BR(\bar{k} + n\tau)/2\tau)}{\vartheta(1)(BR(\bar{k} + n\tau)/\tau)} \right|^2,$$

which clearly exhibits the resulting one-half supersymmetry of the closed string spectrum. The spectrum and interaction of D-branes in this supersymmetric background is a straightforward generalisation of those already presented in section 4.1. For example, in the case where all (five) relevant coordinates of this background are parallel to the D-branes under consideration, the generalization of (63) to this case is

$$A_{pp} = \frac{\pi RV_p}{2(4\pi^2 \alpha' \tau_2)^{(p+1)/2}} \int dt \frac{e^{-\frac{2\pi t}{4\pi \alpha'}}}{t^{p+1/2}} \sum_{\alpha, \beta = 0, 1/2} \eta_{\alpha\beta} \frac{\vartheta^4(\alpha)}{\eta^{12}} +$$

$$\frac{\pi R \alpha'^2 V_{p-4}}{2(4\pi^2 \alpha' \tau_2)^{(p+1)/2}} \sum_{\alpha, \beta} \int dt \frac{e^{-\frac{2\pi t}{4\pi \alpha'}}}{t^{p+1/2}} \sum_{k \neq 0} \frac{1}{2 \sin \pi BRk} \eta_{\alpha\beta} \frac{\vartheta^2(\alpha) \vartheta^2(\alpha + BRk)}{\eta^6 \vartheta^2(\alpha + BRk)} e^{-\frac{2\pi k^2 \alpha^2}{4\pi \alpha'}}.$$
7 The case of rational magnetic field

Previously the twist $BR$ was supposed to be irrational. Here, we briefly examine the case of a rational twist. We consider, for simplicity the case where

$$BR = \frac{1}{N}. \quad (114)$$

The identification (11) reads

$$(y, \phi_0) = (y + 2\pi n_1, \phi_0 + \frac{2\pi n_1}{N} + 2\pi n_2). \quad (115)$$

An equivalent description is to start with $y$ describing a circle of radius $NR$, that is

$$y = y + 2\pi RN, \quad (116)$$

and then modd out by the $Z_N$ transformations

$$(y, \phi_0) = (y + n_1 2\pi R, \phi_0 + 2\pi n_1 N), \quad n_1 = 1, \ldots N - 1. \quad (117)$$

Notice that this a freely acting orbifold with no fixed points. The torus partition function reads

$$T = V_7 \frac{R}{\sqrt{\pi \alpha' T_2}} \frac{1}{(4\pi^2 \alpha' T_2)^{7/2}} |\eta|^{12} \frac{1}{N} \sum_{K \in \mathbb{Z}, N \in \mathbb{Z}} \sum_{k,n=0}^{N-1} e^{-\frac{2\pi}{\alpha' T_2} |K N + k + \tau(N N + n)|^2} Z(\hat{k}, n), \quad (118)$$

with

$$Z(\hat{k}, n) = \frac{1}{4} \left\{ \sum_{\alpha, \beta} \eta_{\alpha\beta} e^{2\pi i (\alpha - \beta) N - \frac{\beta}{N}} \frac{\bar{\eta}^{3/2}[\beta]}{\eta^{3/2}[1/2 + \frac{\beta}{N}]} \right\} \left\{ \sum_{\alpha, \beta} \bar{\eta}_{\alpha\beta} e^{-2\pi i (\alpha - \beta) N - \frac{\beta}{N}} \frac{\bar{\eta}^{3/2}[\alpha]}{\bar{\eta}^{3/2}[1/2 + \frac{\alpha}{N}]} \right\}, $$

$$Z(0, 0) = \frac{V_2}{16 \pi^2 \alpha' T_2 |\eta|^4} \left\{ \sum_{\alpha, \beta} \bar{\eta}^{4}[\alpha] \right\} \left\{ \sum_{\alpha, \beta} \eta_{\alpha\beta} \bar{\eta}^{4}[\beta] \right\}. \quad (119)$$

In the $R \to \infty$ limit, this reduces to the 10d torus type II partition function, whereas in the $R \to 0$ limit it reduces to the $C/Z_N$ non-compact orbifold. Notice the similarity with

the generalisation to string theory of the Scherk-Schwarz [33] partition function found in [34]. There are however two important differences: $N$ is arbitrary, whereas in [34] $N$ can take only a very limited set of values and the two-dimensional space $(\rho, \phi)$ is non-compact.

7.1 Open strings

The rational magnetic case, equivalent to noncompact freely-acting orbifolds, have D-brane spectra that generalize the ones worked out in the compact Scherk-Schwarz case [33]. For example, if all coordinates are Neumann, we expect deformations of the D-brane spectra with masses proportional to $1(NR)$. On the other hand, due to the interpolation
induced by the radius $R$ in this case, the D-branes interpolate between 10d D-brane spectra and D-branes in $C/Z_N$ non-compact orbifolds.

An important difference with respect to the irrational case is that here it is possible to consistently have D-branes which were inconsistent in the irrational twist case. Consider, for instance, the case with Neumann boundary conditions for $y$ and $\rho$ and Dirichlet for $\phi_0$. To implement it, it is convenient to use the equivalent description of the background as a freely acting orbifold of a flat spacetime with a circle of radius $NR$. We start with the radius $NR$ and Dirichlet condition for $\phi_0$, then we add the $N - 1$ images accompanied by the shift in the $y$ coordinate. The resulting cylinder amplitude reads

\[
\frac{V_p}{2N(4\pi^2\alpha')^p/2} \int \frac{dt}{t^{p+1}} \left[ \sum_{k} e^{-\frac{\pi t k^2}{N^2 R^2}} \sum_{\alpha, \beta} \eta_{\alpha \beta} \frac{\vartheta^{[\alpha]}_{1/2} \vartheta^{[\beta]}_{1/2}}{\eta_{12}} \right] + \sum_{n=1}^{N-1} \sum_{k} e^{\frac{2\pi in k}{N}} e^{-\frac{\pi t k^2}{N^2 R^2}} \sum_{\alpha, \beta} \eta_{\alpha \beta} e^{-2\pi i(\beta - \frac{1}{2})} \frac{\vartheta^{[\alpha]}_{1/2} \vartheta^{[\beta + \frac{N}{2}]}_{1/2}}{\eta_{10} \vartheta^{[\frac{1}{2} + \frac{N}{2}]}_{1/2}}.
\]

Using the equivalence with the freely acting orbifold it is straightforward to examine the other D-branes in this background.

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## A Jacobi functions and their properties

For the reader’s convenience we collect in this Appendix the definitions, transformation properties and some identities among the modular functions that are used in the text. The Dedekind function is defined by the usual product formula (with $q = e^{2\pi i \tau}$)

\[
\eta(\tau) = q^{1/24} \prod_{n=1}^{\infty} (1 - q^n),
\]

whereas the Jacobi $\vartheta$-functions with general characteristic and arguments are

\[
\vartheta^{[\alpha]}_{[\beta]}(z|\tau) = \sum_{n \in \mathbb{Z}} e^{i\pi \tau(n - \alpha)^2} e^{2\pi i(z - \beta)(n - \alpha)} = q^{\alpha^2/2} e^{2\pi i \alpha(\beta - z)} \prod_{m=1}^{\infty} (1 - q^m)(1 + e^{-2\pi i(\beta - z)q^{m+\alpha-1/2}})(1 + e^{2\pi i(z - \beta)q^{m-\alpha-1/2}}).
\]

In the text we have used the definition $\vartheta_{1/2}(z|\tau) = \vartheta_{[1/2]}^{[1/2]}(z|\tau)$. The modular properties of these functions are described by

\[
\eta(\tau + 1) = e^{i\pi/12} \eta(\tau), \quad \vartheta^{[\alpha]}_{[\beta]}(z|\tau + 1) = e^{-i\pi \alpha(\alpha - 1)} \vartheta^{[\alpha]}_{[\alpha + \beta - \frac{1}{2}]}(z|\tau),
\]

(123)
\[ \eta(-1/\tau) = \sqrt{-i\tau} \eta(\tau) , \ \vartheta \left[ \begin{array}{c} \alpha \\ \beta \end{array} \right] \left( \frac{z-i}{\tau} \right) = \sqrt{-i\tau} e^{2i\pi \alpha \beta + i\pi z^2/\tau} \vartheta \left[ \begin{array}{c} \beta \\ -\alpha \end{array} \right] (z|\tau) . \] (124)

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