Direct Schmidt number measurement of high-gain parametric down conversion

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Abstract

In this work we estimate the transverse Schmidt number for the bipartite high-gain parametric down conversion state by means of second-order intensity correlation function measurement. Assuming that the number of modes is equal in both beams we determine the Schmidt number considering only one of the subsystems. The obtained results demonstrate that this approach is equally efficient over the whole propagation of the state from the near field to the far field regions of its emitter.

Keywords: entanglement, Schmidt number measurement, high-gain parametric down conversion, Bloch–Messiah decomposition

(Some figures may appear in colour only in the online journal)

1. Introduction

High-gain parametric down conversion is a basic tool applied for preparation of quadrature [1] and two-mode squeezed states [2], as well as macroscopic Bell states [3]. All these examples are considered as macroscopic quantum states of light, which are particularly appealing for providing stronger interaction with matter than microscopic (single-photon and few-photon) states of light. Gravitational wave detection [4], quantum memory [5] and super resolution [6] are the other possible applications of macroscopic states of light. However, even though huge progress has been made utilizing high-gain parametric down conversion there is still a number of issues to uncover. The current letter concerns itself with the Schmidt number of the high-gain parametric down conversion.

2. Schmidt number estimation

For the biphoton state generated via spontaneous parametric down conversion (PDC) the Schmidt decomposition of a bipartite wavefunction \(|\Psi\rangle = \sum_{n=0}^{\infty} \sqrt{\lambda_n} |\psi_n\rangle |\phi_n\rangle\) [7] is an established method to estimate the degree of entanglement represented by the Schmidt number \(K = \frac{1}{\sum_{n=0}^{\infty} \lambda_n^2}\). Here the Schmidt coefficients \(\lambda_n\) are independent on the parametric gain. Fedorov parameter measurement is a justified technique to evaluate the degree of entanglement [8, 9]. However, theory [10] predicts violation of invariance under detection apparatus translation the detection apparatus from the near field to the far field zone. Observation of two-photon interference visibility enables direct Schmidt number measurement [10]. This concept has recently been demonstrated experimentally [11]. Notably, the biphoton degree of spatial entanglement, i.e. inverse spatial coherence, can be measured by analyzing spatial intensity profiles captured in near and far fields [12].

In contrast to biphoton fields, high-gain PDC wave function description is less comprehensible. Natural description of this state arises in the Heisenberg picture. Bloch–Messiah decomposition allows one to introduce new photon creation (annihilation) operators, so called broadband modes [13]. The Schmidt number \(K\) is defined by weights \(\lambda_n\) of new broadband modes and, therefore, depends on the parametric gain [14]

\[
K = \frac{1}{\sum_{n=0}^{\infty} \lambda_n^2}.
\]

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and $\lambda_n = \frac{\sinh^2(\sqrt[N]{G_n})}{N}$. \hfill (2)

Here $G$ is a parametric gain, $N = \sum_{n=0}^{\infty} \sinh^2(\sqrt[N]{G_n})$ is the total number of photons in one beam, and $\lambda_n$ are initial Schmidt coefficients in the decomposition of the biphoton amplitude. The Schmidt number defined in (1) gives the effective number of populated modes of the infinite-dimensional bipartite bright squeezed vacuum state and is observed experimentally via correlation function measurement [15].

In this letter we demonstrate a direct high-gain PDC Schmidt number estimation method based on correlation function measurement. Our method enables accurate Schmidt number estimation in the entire region from the near to far field zone. Apart from that, we demonstrate for the first time the qualitative agreement between the theoretical approach (1), (2) and the experimental results. Our work paves the way towards a rapid and reliable technique of high-gain PDC Schmidt number estimation.

It is well known that intensity cross-correlation function $g^{(2)}$ between signal and idler beam can be expressed as

$$g^{(2)} = 1 + \frac{1}{K} + \frac{1}{N_{\text{mode}}K},$$ \hfill (3)

where $K$ is the number of populated modes and $N_{\text{mode}}$ is the number of photons per mode. Considering only signal or idler beams, one can obtain that the correlation function $g^{(2)}$ depends only on $K$ but does not depend on the brightness of the state [16]

$$g^{(2)} = 1 + \frac{1}{K},$$ \hfill (4)

i.e. it is sufficient to measure $K$ in one party to describe the whole bipartite PDC state. Robustness against detection losses makes correlation function measurement a powerful tool for the Schmidt number estimation. Surprisingly, the applicability of the well known technique (4) has never been experimentally validated with respect to invariance under translation of the detection scheme from the far to near field region. The present work demonstrates the suitability of the described approach to fill up the specified gap.

### 3. Experiment

The experimental setup is shown in figure 1. High-gain PDC beams were generated in an optical parametric amplifier (OPA) by pumping it with the 18 ps pulses of the third harmonic of a Nd:YAG laser at 355 nm wavelength, with a repetition rate 1 kHz. The laser beam diameter was reduced to 230 μm by a telescope based on a convex lens ($f = 50$ cm) and a concave lens ($f = -7.5$ cm) separated by a distance of 42.5 cm. The OPA consisted of two 5 mm thick BBO crystals cut for collinear degenerate type-II phase matching separated by a distance of 1 cm. Effect of anisotropy was compensated by orienting the optical axes in the horizontal plane at opposite angles to the pump beam [17]. Dichroic mirrors $\text{DM}_{i,2}$ filtered the pump beam out of the optical scheme. After the mirrors the pump beams were absorbed by the beam blocks. Assuming the orthogonal polarization of the PDC beams, the idler one (ordinary polarized) was eliminated by a Glan prism (GP). The width of frequency spectrum and, according to the phase-matching curve, the angular distribution was limited by a 10 nm bandwidth interference filter (IF) around 710 nm. The correlation function was measured in a Hanbury Brown–Twiss (HBT) scheme in which the beam was split in two by means of a half-wave plate ($\lambda_2/2$) and a polarization beamsplitter ($\text{PBS}_2$) and then detected by two separate $p-i-n$ diode photodetectors ($\text{D}_{1,2}$). A detailed description of the detectors and the registration part of the setup can be found in [18].

In our measurement we collected the signals per pulse over 30 000 pulses and the correlation function was calculated according to the formula $g^{(2)} = \frac{\langle S_1 S_2 \rangle}{\langle S_1 \rangle \langle S_2 \rangle}$. Here $S_{1,2}$ denote detected signals per pulse. The detection part was built on a translation stage which was transferred from the far-field to near-field zones of a lens ($L$) with a focal distance $f = 15$ cm. Len $L$ builds a 1:1 image of the output face of the second crystal. The far-field zone was at the focal plane of the lens and the near field was at $2f$ distance. The key issue in the experiment is that the whole angular spectrum was always (at any position of the registration apparatus) collected by the detectors followed by plano-convex spherical lenses ($L_0$) (focal length 3 cm).

Based on the measured average signal of the parametric down conversion as a function of the pump power the parametric gain of the optical parametric amplifier was estimated. The pump power varied in the range from 13 to 25 mW and the gain took the values within the range $5.8 < G < 8.0$.

### 4. Results and discussion

A first step was to measure a correlation function for the extraordinary polarized (signal) PDC beam versus the parametric gain. The gain was changed by adjusting pump power with a half-wave plate ($\lambda_2/2$) followed by a polarization cube ($\text{PBS}_3$). As shown in figure 2(a) we have observed the increase of $g^{(2)}$ (the reduction of the number of modes according to equation (4)) with $G$. Figure 2(b) illustrates the results of numerical calculation. The simulation was performed according to formulas (1)–(2), (4) taking into account the parameters of the experiment under the assumption of a single frequency mode. One observes a tendency of the OPA to emit single-mode radiation at high parametric gain. Assuming the presented results and sensitivity $g^{(2)}$ to the pump power, all further measurements were performed at a constant pump power $P = 20.5$ mW, which corresponds to a parametric gain of $G = 7.3 \pm 0.2$.

As a next step, we measured the number of temporal modes in a spatially filtered single-mode signal beam. The $g^{(2)}$ dependence on the aperture size placed in the focal plane of the lens ($L$) is presented in figure 3. The aperture diameter $D$ was changed in the range of $0.1 \text{ mm} < D < 25 \text{ mm}$. The total Schmidt number $K \equiv K_1 \cdot K_2$ is given by a product of the...
number of temporal $K_t$ modes and spatial $K_s$ ones. We would like to point out, that spatial filtering with a pinhole much smaller than the coherence area results in a lossy single-mode selection. As one can see in the inset of figure 3, the correlation function reaches $g^{(2)} = 1.32 \pm 0.01$ as $D \to 0$. According to (4) the effective number of temporal modes $K_t = 3.1 \pm 0.1$ was estimated. The blue solid line is a theoretical curve plotted according to (1) and (2) assuming $K_t = 3.1$.

As a confirmation of the applicability of the Schmidt number estimation technique we present $g^{(2)}$ measured at different positions in the region from the far to the near field zones. The results are shown by red circles in figure 4(a). In this measurement the pinhole was removed and the whole angular spectrum of the signal beam was collected. One can see that $g^{(2)}$ and, therefore $K_t$ does not change over the propagation from far field to the near field zone. On average the second-order correlation function took the value of $g^{(2)} = 1.052 \pm 0.001$. Hence, according to (4), we get a total Schmidt number of $K = 19.2 \pm 0.4$. Taking into account $K_t = 3.1 \pm 0.1$ as measured before, the spatial Schmidt number equals $K_s = 6.2 \pm 0.2$. For comparison, we show a theoretical expectation by a dashed line. The theoretical value of the spatial Schmidt number $K_s^{(\text{theor})} = 6.18$ was calculated for the used OPA operating at $G = 7.3$. We report good agreement between theory and experiment.

The last measurement is an auxiliary test to demonstrate the influence of spatial filtering on the applicability of the method. We have installed a 1 mm diameter pinhole in front of PBS$_2$. As shown in figure 4(b), in the Fourier conjugated zones the pinhole selected nearly a single spatial mode and, therefore, $g^{(2)}$ achieved high values. As expected, in the intermediate zone all of the transverse modes become spatially overlapped and all contribute to the signal passing through the pinhole. Hence, the aperture does not select spatial modes any more and the correlation function reaches its minimum.
In conclusion, we have shown, both theoretically and experimentally, that the Schmidt number for the high-gain PDC state decreases with increasing parametric gain. Our results demonstrate that correlation function measurement represents a useful tool for the description of the multi-dimensional quantum systems. The observed invariance of the correlation function on the position of the detection scheme is an experimental verification of the applicability of the Schmidt number estimation technique. We believe that this approach will significantly simplify the analysis of bipartite macroscopic entangled states of light.

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Figure 4. Second-order intensity correlation function measured as a function of the position of the detection scheme. (a) All of the angular spectrum was collected. (b) An aperture of diameter $D = 1$ mm was introduced in front of PBS$_2$. Red circles are the experimentally measured points in agreement with the theoretical expectation (dashed line).