Finite frequency noise spectroscopy for fractional Hall states at $\nu = 5/2$

A Braggio$^{1,2}$, M Carrega$^1$, D Ferraro$^3$ and M Sassetti$^{1,4}$

1 SPIN-CNR, Via Dodecaneso 33, 16146 Genova, Italy
2 I.N.F.N. Sezione di Genova, Via Dodecaneso 33, 16146 Genova, Italy
3 Département de Physique Théorique, Université de Genève, 24 quai Ernest Ansermet, CH-1211 Geneva, Switzerland
4 Dipartimento di Fisica, Università di Genova, Via Dodecaneso 33, 16146 Genova, Italy
E-mail: alessandro.braggio@spin.cnr.it

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Abstract. We investigate the finite frequency noise of a quantum point contact at filling factor $\nu = 5/2$ using a weakly coupled resonant LC circuit as a detector. We show how one could spectroscopically address the fractional charged excitations inspecting separately their charge and scaling dimensions. We thus compare the behaviour of the Pfaffian and the anti-Pfaffian non-Abelian edge states models in order to give possible experimental signatures to identify the appropriate model for these fractional quantum Hall states. Finally we investigate how the temperature of the LC resonant circuit can be used in order to enhance the sensibility of the measurement scheme.

Keywords: fractional QHE (theory), Luttinger liquids (theory)
Among the extremely remarkable properties of the fractional quantum Hall (FQH) effect [1] a major role is played by the emergence of anyonic excitations carrying fractional charge and statistics [2]. In particular, quasiparticle (qp) excitations for states belonging to the Laughlin [3] and Jain sequence [4] are predicted to have Abelian exchange statistics. More intriguingly, some of the proposed models for the filling factor $\nu = 5/2$ [5] predict the emergence of excitations with charge $e^* = e/4$, and multiples with possible non-Abelian properties [6]. These predictions paved the way for possible applications of non-Abelian anyons in fault-tolerant topological quantum computation (see [7] and the references therein). Unfortunately, as far as we know, a direct confirmation of fractional statistics is still lacking even if different proposals are reported in the literature [8–11]. So far, evidence of the fractional statistics is indirect, essentially based on the evidence of the existence of fractional charges [3].

A lot of experimental effort has been devoted in recent years to access fractional charges through shot noise measurements in quantum point contact (QPC) geometry starting with the seminal works of [12, 13]. In this direction, composite states (for example $\nu = 2/5, 2/3$) have shown quite a universal phenomenology leading to a crossover between two different values of the effective charge as a function of temperature or bias [14–16]. This behaviour has been explained in terms of the competition of two charge carriers: the agglomerates and the single-qp [17–19]. Similar arguments also hold for the state at $\nu = 5/2$ [20]. Other interpretations based on edge state reconstruction [21], local filling factor effects [22–24] or tunnelling amplitude non-linearities [25, 26] have also been proposed. Unfortunately, in the discussed measurements, the contributions associated with the various excitations are typically mixed because these studies are conducted at very low frequencies (almost dc). Therefore, it is useful to find alternative methods to address the excitations separately.
A possible way is to consider the noise at finite frequency (f.f.) [27]. Indeed, this quantity presents resonant singular behaviour (such as peaks or dips) in correspondence of the Josephson frequency $\omega_j = qV/\hbar$ associated with each charge carrier $q$ with $V$ the applied bias. This is an independent method to measure the charge of the fractional excitations in the system that has not yet be experimentally explored for FQH states so far. Indeed, for sufficiently low temperatures $k_B T \ll qV$, it allows the separation of the different charge contributions, realising a sort of qp spectroscopy. Intriguingly enough, f.f. noise could also efficiently address other properties like the scaling dimensions associated with each qp excitation using only a bias scan at fixed frequency, as we will show later.

This combination of information has the potential to give further constraints on the edge state model [28, 29], and finally to address the topological order of the bulk ground state [30]. The first theoretical steps in this direction were made on symmetrised noise [31, 32], a typical quantity considered at low frequencies. In this case the expected features associated with the state at $\nu = 5/2$ [33] as well as the possibility of accessing the contribution associated with the different qps predicted by theoretical models [34] have been investigated. However, at high frequencies $\omega \gg k_B T$, quantum effects become relevant and the symmetrised f.f. noise is only one possible choice among the different experimental quantities addressed by different protocols. Indeed, in such a regime, one has to identify the relevant quantity measured with the specific setup under investigation [35]. Hereafter, we get our inspiration from the proposal of Lesovik and Loosen [36], where a model based on a resonant LC circuit is discussed in order to extract non-symmetrised current–current correlations. Recent experiments carried out for a two-dimensional electron gas QPC in the absence of a magnetic field fully agree with theoretical predictions [37]. Since the resonant circuit coupled to a QPC is the prototypical measurement scheme in FQH experiments [12, 13] it appears quite obvious to explore the same physics in this context. We have recently investigated the same setup for Abelian FQH states [38], and here we consider it for the case $\nu = 5/2$ analysing the signatures of the different non-Abelian phases (Pfaffian and anti-Pfaffian). The goal of the present paper is to analyse in detail the expected f.f. measured noise for a realistic situation. The effects associated with the temperature of the FQH QPC system and the LC detector on the visibility are also carefully considered.

The paper is divided as follows: in section 2 we discuss the non-Abelian models for edge states pointing out similarities and differences in terms of the most dominant fractionally charged excitations. In section 3 we discuss the definition of the noise properties for a QPC in the weak-backscattering regime and the definition of the noise power measured in the proposed setup. In section 4 we discuss the results for the measured noise power obtained for two considered non-Abelian models and we also compare them with the well-known symmetrised noise. Finally, we look at the effects of changing the detector temperature $T_c$.

2. Model

We start by recalling the two more accredited models of composite edge states at filling factor $\nu = 5/2$ [5]: the Pfaffian (P) [6, 39] and the disorder dominated anti-Pfaffian
The associated Lagrangian densities are given by the sum of charged ($L_c$) and neutral contributions ($L_n$), namely $L = L_c + L_n$, with $(\hbar = 1)$

$$L_c = -\frac{1}{2\pi} \partial_x \varphi_c (\partial_t \varphi_c + v_c \partial_x \varphi_c)$$

and

$$L_n = -i \psi (\xi \partial_t \psi + v_n \partial_x \psi) - \frac{\alpha}{4\pi} \partial_x \varphi_n (\xi \partial_t \varphi_n + v_n \partial_x \varphi_n).$$

They both describe a Hall fluid at filling factor $\nu = 1/2$ with two additional filled Landau levels, playing the role of the vacuum of the theory, according to the conventional decomposition $5/2 = 1/2 + 2$. The charged bosonic field $\varphi(x)$ is related to the electron number density through $\rho(x) = \partial_x \varphi(x)/2\pi$, while $\varphi_n(x)$ is a bosonic neutral field and $\psi(x)$ represents a neutral Majorana fermion in the Ising sector [9]. The parameter $\xi = \pm 1$ denotes the direction of propagation of neutral modes with respect to the charged ones. In particular, for $\xi = +1$ the modes are co-propagating, while for $\xi = -1$ they are counter-propagating.

The two models differ in the neutral sector $L_n$ [42], with $\alpha = 0$ and $\xi = 1$ for P, $\alpha = 1$ and $\xi = -1$ for AP. The propagation velocities of the charged and neutral modes are indicated with $v_c$ and $v_n$, respectively. Due to the hidden symmetry of the neutral sector of AP model [41] in the disorder dominated phase, one has the same velocity $v_n$ both for the bosonic $\varphi_n$ and the fermionic neutral modes $\psi$. Moreover, one may reasonably assume a larger charge velocity $v_c \gg v_n$ [16, 20, 34, 43]. The quantisation of the above bosonic fields is given by the commutation relation

$$[\varphi_{c/n}(x), \varphi_{c/n}(y)] = i\pi \nu_{c/n} \text{sgn}(x - y),$$

with $\nu_c = 1/2$ and $\nu_n = \xi$, while the Majorana fermion commutes with both.

### 2.1. Quasiparticle operators and scaling dimensions

Operators destroying an excitation along the edge can be written as [2, 39, 41]

$$\Psi^\chi_m(x) = \chi(x)e^{im/4 \varphi(x)}$$

$$\Psi^\chi_n(x) = \chi(x)e^{\left[m/2 \varphi(x) + n/2 \varphi_n(x)\right]},$$

with $m$, $n$ integer numbers and where the operator in the Ising sector $\chi(x)$ can be the identity operator $I$, the Majorana fermion $\psi(x)$ or the spin operator $\sigma(x)$. They are associated with the fact that the excitation charge is an even or odd multiple of the fundamental charge of the model $e^* = (e/4)$. Indeed, all the excitations described by previous operators have charge $(m/4)e$ and we call them $m$-agglomerates [17, 20]. The single-valuedness properties of the phase acquired by an $m$-agglomerate with respect to the operation of encircling an electron in the bulk$^5$, force $m$ and $n$ to be: even integers for $\chi = 1$ or $\psi$, and odd integers for $\chi = \sigma$ [9, 44]. Note that the presence of $\sigma$ in the operator leads to non-Abelian statistical properties important for fault-tolerant quantum computation, as determined by the fusion rules [2, 7].

$^5$ The holographic principle [30] imposes that the same restriction applies to the edge theory.
The zero temperature time-dependent Green’s functions associated with the operators of the Ising sector and the charged and neutral bosonic fields are \([18, 19, 45–49]\)

\[
\langle \chi(0, t)\chi(0, 0) \rangle = (1 + i\omega_n t)^{-\delta_I},
\]

\[
\langle \varphi_s(0, t)\varphi_s(0, 0) \rangle = -|\nu_s|\ln(1 + i\omega_s t) \quad s = c, n
\]

with \(\delta_I = 0, \delta_c = 1\) and \(\delta_n = 1/8\) the conformal weights of the field in the Ising sector \([2, 7, 45]\) and we introduced the energy bandwidths \(\omega_{c/n} = a^{-1}v_{c/n}\), with \(a\) a finite length cut-off. In the following we will assume \(\omega_c\) as the largest energy scale of the model. From the long-time behaviour of the imaginary time two-point Green’s function \([6]\)

\[
\langle T_\tau \Psi_l^{(m)}(\tau)\Psi_l^{(m)}(0) \rangle \propto |\tau|^{-2\Delta_l^{(m)}} \quad l = P, AP
\]

we can extract the scaling dimensions \([50]\) of the \(m\)-agglomerates

\[
\Delta_P^{(m)} = \frac{1}{2}\delta_\chi + \frac{1}{16}m^2; \quad \Delta_{AP}^{(m)} = \frac{1}{2}\delta_\chi + \frac{1}{16}m^2 + \frac{1}{8}n^2
\]

which depends on the model considered. Therefore, for the single-qp with minimal charge \(e^* = e/4\) \((m = 1, \chi = \sigma\) and only for AP \(n = \pm 1\) one has, respectively

\[
\Delta_P^{(1)} = \frac{1}{8}; \quad \Delta_{AP}^{(1)} = \frac{1}{4},
\]

while the 2-agglomerate excitation with charge \(e/2\) \((m = 2, n = 0, \chi = I)\), with a scaling dimension driven by the charged mode contribution only with

\[
\Delta_P^{(2)} = \Delta_{AP}^{(2)} = \frac{1}{4}.
\]

These values indicate the single-qp as the most dominant excitation at low energy in the P case, while in the AP case single-qp and 2-agglomerate have equal relevance with the same scaling dimensions \([41]\). The latter situation is quite general and valid for all anti-Read–Rezayi states \([51]\). All other excitations with higher charges have higher scaling dimensions and can be safely neglected in what follows. It is worth mentioning that interactions with the external environment can lead to renormalisations of the scaling parameters with remarkable consequences on the transport properties (see \([52]\) for a better discussion). In the following, for the sake of simplicity, we will focus on the unrenormalised case only, despite the fact that method may be generalised to the renormalised case.

### 3. Noise properties in QPC at finite frequency

Having characterised the excitations of the considered models for \(\nu = 5/2\), we can investigate the associated f.f. backscattering noise in the QPC geometry, as shown in

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6 Note that, for notational convenience, we have omitted any reference to the neutral sectors in the upper index in operator of the vertex \(\Psi_l^{(m)}\) and in the scaling \(\Delta_l^{(m)}\). Indeed, from now on we will consider for any \(m\)-agglomerate only the qp operators with the minimal scaling dimension compatible with the single-valuedness requirement.

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A similar measurement scheme was proposed for the first time by Lesovik and Loosen in [36]. Here, the QPC was subjected to a bias voltage $V$ and coupled to a resonant LC circuit, playing the role of the detector (with measurement frequency $\omega = 1/LC$), via an impedance matching circuit (see the dashed box in figure 1). With a strong magnetic field the impedance matching in the system is a challenging technological problem [53, 54], therefore it is advantageous to suppose working at fixed resonant frequency $\omega$ assuming a very high-quality factor of the detector.

We focus on the simple two terminal geometry in the weak backscattering limit, where $m$-agglomerate tunnelling processes can be treated separately. In real systems a four-terminal version of this setup is required, although this does not change the main result obtained for this simplified version.

The point-like tunnelling of a generic $m$-agglomerate between the right-propagating (+) and the left-propagating (−) edge can be described through the tunnelling Hamiltonian ($l = P, AP$)

$$\mathcal{H}_{T,l}^{(m)} = t_m \Psi_{l,+}^{(m)}(0)\Psi_{l,-}^{(m)}(0) + \text{h.c.},$$

where $t_m$ is the $m$-agglomerate tunnelling amplitude (assumed energy independent). The finite bias $V$ between the two edges can be included in our formalism through the gauge transformation $t_m \rightarrow t_m e^{im\omega_0 t}$, where, $\omega_0 = e^*V/h$ is the Josephson frequency associated with the fundamental charge $e^*$ [55]. From the tunnelling Hamiltonian in (11) one can easily derive the backscattering current operator associated with the $m$-agglomerate [17–20]

$$I_{B,l}^{(m)}(t) = i me^{*}(t_m e^{im\omega_0 t}\Psi_{l,+}^{(m)}(0, t)\Psi_{l,-}^{(m)}(0, t) - \text{h.c.}).$$

The contribution to the averaged backscattering current of the $m$-agglomerate at lowest order in the tunnelling can be easily written in terms of the tunnelling rates $\Gamma_i^{(m)}$ [34]
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$$\langle I_{B,l}^{(m)} \rangle = me^*(1 - e^{-m\omega_0/T})\Gamma_l^{(m)}(\omega_0),$$  \hspace{1cm} (13)

with the average $\langle ... \rangle$ taken over the quantum statistical ensemble. The rates can be also evaluated analytically at low temperatures $T \ll \omega_n, \omega_c$ \cite{17, 34}

$$\Gamma_l^{(m)}(\omega_0) = \frac{|t_m|^2}{(2\pi)^2} \frac{(2\pi)^{n_0 + \mu_{l,\alpha}}}{\omega_m^{n_0 + \mu_{l,\alpha}}} T^{n_0 + \mu_{l,\alpha} - 1} \frac{m\omega_0}{e^{2\pi T}} \times B\left(\eta_m + \mu_{l,\alpha} - \frac{1}{2\pi T}, \eta_m + \mu_{l,\alpha} + \frac{1}{2\pi T}\right)$$ \hspace{1cm} (14)

where $B(x, y)$ is the Euler beta function. Here, $\eta_m = m^2/4$ and $\mu_{l,\alpha} = 2\delta\chi + \alpha n^2/2$ depend on the variables $m, \chi$ and $n$ which characterise the tunnelling excitation for the specific P or AP model considered (see discussion around (9) and (10)). Note that for $k_B T \ll \omega_0 \ll \omega_n, \omega_c$ asymptotic expansion shows that $\Gamma_l^{(m)}(\omega_0) \propto |\omega_0|^{4\Delta_l^{(m)} - 1}$, with the expected power law dependences of the rates from the bias energy $\omega_0$ and the scaling dimension $\Delta_l^{(m)}$, as usually happens in the Luttinger liquid theory. For higher bias value $\omega_n \ll \omega_0 \ll \omega_c$ the power law no longer depends on the neutral components and one finds $\Gamma_l^{(m)}(\omega_0) \propto |\omega_0|^{n^2/4 - 1}$, where the power-law scaling is determined only by the charge of the $m$-agglomerates. The low-energy analytical result presented corresponds to the standard golden rule rate for the tunnelling processes and one may eventually also calculate it with numerical methods following the prescription of [34].

The proper quantity to consider in order to investigate the current fluctuations of the QPC coupled to the resonant circuit is the non-symmetrised noise \cite{36, 56–60}

$$S_+^{(m)}(\omega) = \frac{1}{2} \int_{-\infty}^{+\infty} dt \ e^{i\omega t} \langle \delta I_{B,l}^{(m)}(0) \delta I_{B,l}^{(m)}(t) \rangle,$$ \hspace{1cm} (15)

where we have introduced the backscattering current fluctuation $\delta I_{B,l}^{(m)} = I_{B,l}^{(m)} - \langle I_{B,l}^{(m)} \rangle$\footnote{For notational convenience we have omitted the index $l = P, AP$ on the noise power since its definition is exactly the same for the two models.}. This quantity represents, for $\omega > 0$, the noise power emitted by the system into the detector. The corresponding absorptive part is given by

$$S_-^{(m)}(\omega) = \frac{1}{2} \int_{-\infty}^{+\infty} dt \ e^{i\omega t} \langle \delta I_{B,l}^{(m)}(t) \delta I_{B,l}^{(m)}(0) \rangle = S_+^{(m)}(-\omega).$$ \hspace{1cm} (16)

With these quantities it is easy to calculate the f.f. symmetrised noise \cite{27, 31, 32, 34, 61} usually considered in the literature

$$S_{\text{sym}}^{(m)}(\omega) = \frac{1}{2} \int_{-\infty}^{+\infty} dt \ e^{i\omega t} \langle \{ \delta I_{B,l}^{(m)}(t), \delta I_{B,l}^{(m)}(0) \} \rangle = S_+^{(m)}(\omega) + S_-^{(m)}(\omega)$$ \hspace{1cm} (17)

having indicated with $\{ \cdot, \cdot \} \delta$ the anticommutator. At the lowest order in the tunnelling amplitudes and using standard Keldysh formalism, the non-symmetrised noise can also be expressed in terms of the QPC tunnelling rates [38]

$$S_+^{(m)}(\omega; \omega_0) = \frac{(me^*)^2}{2} [\Gamma_l^{(m)}(-\omega + m\omega_0) + \Gamma_l^{(m)}(-\omega - m\omega_0)],$$ \hspace{1cm} (18)
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The detector of figure 1 represents a concrete measurement scheme to access current fluctuations at high frequencies. In the following we will focus on the regime where the QPC temperature $T$ is lower than the frequency (quantum limit) and the bias (shot noise limit), i.e. $k_B T \ll e^* V, \omega$. This allows us to investigate the fractional qp contributions via a sort of spectroscopy.

The measurable quantity is the spectral power measured in the amplifier chain (the grey area in figure 1), which is proportional to the variation of the energy stored in the LC before and after the switching on of the LC-QPC coupling. From now on we will indicate it as measured noise $S_{\text{meas}}(\omega, \omega_0)$, where with $\omega$ we indicate the frequency of the LC circuit and with $\omega_0$ the QPC bias. At the lowest order in the coupling $K \ll 1$ it can be expressed as

$$S_{\text{meas}}^{(m)}(\omega, \omega_0) = K\{S_+^{(m)}(\omega, \omega_0) + n_B(\omega)[S_+^{(m)}(\omega, \omega_0) - S_-^{(m)}(\omega, \omega_0)]\},$$

where the non-symmetrised noise QPC spectra for the $m$-agglomerate are $S_\pm^{(m)}(\omega, \omega_0)$ of (18). Here, $n_B(\omega) = 1/[e^{\beta \omega} - 1]$ the Bose distribution describing the equilibrium state of the LC detector and $\beta = 1/k_B T_c$ the detector inverse temperature. In general $T_c$ can be different from the system temperature $T$ since the system and detector are weakly coupled. Finally, we wish to recall that this quantity can also be investigated using a strategy similar to the definition of the excess noise, which further simplifies the impedance matching problem at the level of the LC-QPC coupling (see [38] for details).

To consider all contributions due to the tunnelling of different $m$-agglomerates, in weak tunnelling one can directly sum them ($i = \text{meas, sym}$)

$$S_i(\omega_0) = \sum_m S_i^{(m)}(\omega_0),$$

where from now on we suppress the explicit dependence on the LC frequency $\omega$ since in the following discussion it is always kept fixed. We conclude this part by noting that these results suggest that f.f. noise is a spectroscopy tool for different tunnelling charges.

4. Results and discussion

The results concerning the measured f.f. noise $S_{\text{meas}}(\omega_0)$ in (19) and (20) at $\nu = 5/2$ are shown in figure 2 (upper panels) as a function of the QPC bias $\omega_0$ for different QPC temperatures $T$, keeping fixed the resonant circuit temperature $T_c$. We discuss only the behaviour at positive bias $\omega_0 > 0$ since the noise is a symmetric function of the QPC bias $\omega_0$. Analogies and differences between the P (upper left panel) and AP (upper right panel) models become evident from a direct comparison. Starting from the lowest temperature case ($T = 0.1$ mK, black curves) we observe that both models show a flat behaviour at $\omega_0/\omega \approx 0$, which is a clear signature of the lack of contribution of ground state fluctuations in the considered measurement scheme [36]. The slight deviations from zero are associated with the mismatch between the system and detector.
temperature, which can be always cancelled when $T = T_c$ [38]. Steep jumps associated with the 2-agglomerate contribution appear at $|\omega_0|/\omega = 1/2$ showing an identical profile in both models, which reflects the same scaling dimension of the two models for that excitation (see (10)). The spike associated with the single-qp occurring at $|\omega_0|/\omega = 1$ is different. Indeed, they are much higher and sharper in the P case with respect to the AP reflecting a lower scaling dimension of qp excitation for the P model. This feature could quite clearly distinguish between the two models. However, the temperature should be kept quite low, since on increasing it the differences are progressively less marked. Eventually some signatures survive only by considering the bias derivative of this quantity (see the bottom panels in figure 2).

The previous behaviours can be explained in a simple way in the quantum limit ($k_B T_e \ll \omega$) for the detector and the shot-noise limit ($k_B T \ll \omega_0$) for the system. In this case one has the contributions of single and double excitations for the two models ($l = P, AP$) [38]

$$S_{\text{meas}}(\omega_0) \approx \alpha_1 \Gamma_l^{(1)}(\omega_0 - \omega) + \alpha_2 \Gamma_l^{(2)}(2\omega_0 - \omega)$$

with $\alpha_1$ and $\alpha_2$ constant prefactors and the explicit expression of the rates are reported in (14) for $\omega, \omega_0 \ll \omega_{Q1}, \omega_c$. This result confirms the same scaling for the 2-agglomerate in the two models, but different behaviours for the single-qp contributions (see (9)

Figure 2. (Upper panels) Measured f.f. noise $S_{\text{meas}}$ (in units of $S_0 = K e^2|t_1|^2/(2\pi\alpha)^2\omega_c$) for the P (left) and the AP (right) model at $\nu = 5/2$ as a function of the bias $\omega_0 = eV$ and at fixed frequency $\omega$. The QPC temperature associated with each curve is indicated in the legend (bottom panels). The corresponding derivatives $\partial S_{\text{meas}}/\partial \omega_0$. The curve at the lowest temperature ($T = 0.1$ mK) has been omitted for better visibility. The other parameters expressed in temperature scale are: $T_c = 15$ mK, $\omega = 60$ mK, $\omega_0 = 50$ mK, $\omega_c = 1$ K and $|t_1|^2/|t_2|^2 = 1$. 

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and (10)). As we observed after (14) for high biases $\omega_0 \gg \omega_n$ the scaling dimensions are determined only by the charged part that is the same for the two models. This is another reason why conventional scaling analysis, which is typically done in asymptotic regime, would fail to detect the differences between the two models, especially when the neutral mode bandwidth $\omega_n$ is quite small.

By increasing the system temperature $T$, keeping the detector ($T_c = 15$ mK) fixed, the peaked structure become progressively smoother due to a rounding of the singularities. In this regime, the differences in the measured f.f. noise become clear only by looking at the derivative with respect to the QPC bias $\partial S_i/\partial \omega_0$, as shown in the bottom panels in figure 2. By focusing on the blue curves, and keeping in mind the different scale in the ordinates between the two panels, one can observe again the similarity of peaks associated with the 2-agglomerate ($\omega_0/\omega = 1/2$). However, concerning the single-qp ($\omega_0/\omega = 1$), the difference in the scaling leads to a pronounced peak followed by a stronger dip in the P case with respect to the AP case.

Until now all the plots have been done for fixed ratio $k = |t_2|^2/|t_1|^2 = 1$. However, this parameter is unknown and may change for any specific experimental realisation. For this reason we present in figure 3 with dashed lines $S_{\text{meas}}(\omega_0)$ (right panel) and its bias derivative $\partial S_{\text{meas}}/\partial \omega_0$ (left panel) as a function of bias with changing $k$ values. We concentrate mainly on the AP model but similar considerations can be repeated in the P case. As expected, increasing this parameter progressively enhances the 2-agglomerate contribution with respect to the single-qp but still leaves both visible at different bias values. This is particularly true when looking at the bias derivative. This result shows the convenience of the proposed setup in order to address the presence of the two different charged excitations, as well as when eventually one of the contributions is deeply suppressed compared to the other due to non-universal effects.

In order to make this statement more quantitative, in figure 3 we compare $S_{\text{meas}}(\omega_0)$ with the f.f. symmetrised noise $S_{\text{sym}}(\omega_0)$ (solid lines) in (17) and their bias derivatives.

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Figure 3. (Left panel) Comparison for the AP model between the measured f.f. noise $S_{\text{meas}}$ (dashed lines) and the symmetrised noise $S_{\text{sym}}$ (solid lines) for different values of the ratio $k = |t_2|^2/|t_1|^2$ (indicated in the legend). (Left panel) The f.f. noises $S_i/S_0$ with $i = \text{meas,sym}$ in units of $S_0 = K e^2 |t_1|^2 / (2\pi \alpha)^2 \omega_0$. Note that $K = 1$ for the symmetrised noise. (Right panel) The bias derivatives $\partial S_i/\partial \omega_0$ in units $S_0/\omega_0$. The other parameters are: $T = 5$ mK, $T_c = 5$ mK, $\omega_n = 50$ mK and $\omega_c = 1$ K.

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8 According to the condition $v_n \ll v_c$. 

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Here, again, we keep the frequency $\omega$ fixed, changing the bias $\omega_0$, which is by far the most convenient protocol in the investigated GHz range. We see that the $\omega_0$ dependence of the symmetrized $S_{\text{sym}}(\omega_0)$ is unable to detect the two singularities associated with the two fractional charges even in the bias derivatives. In particular, by changing the tunnelling amplitude ratio $k$, the quantity seems only affected by a common multiplicative factor demonstrating that the signature of the two excitations is mainly mixed in that quantity. This supports the idea that in this bias-dependent protocol $S_{\text{sym}}(\omega_0)$ is not useful, especially in comparison with $S_{\text{meas}}(\omega_0)$. This statement can be easily verified by looking at both the left and right panels in figure 3.

Finally, in order to find a signature of the charged excitations without identifying their scaling dimension, we could vary the LC detector temperature $T_c$ in order to increase the sensitivity for charge detection. In figure 4 we show that this approach works for both the two non-Abelian edge models. Increasing $T_c$ increases the height of the jumps in $S_{\text{meas}}(\omega_0)$ (top panels) which also corresponds to an increase in the height of the peaks in the derivatives (bottom). Note that since the coupling with the detector is assumed weak (no poisoning from the detector) the width of the peaks is only slightly influenced by the detector temperature, preserving the resolving power of the discussed bias spectroscopy. Then the crucial limiting factor to the bias spectroscopy is the QPC temperature $T_c$.

Figure 4. (Upper panels) Measured f.f. noise $S_{\text{meas}}$ (in units of $S_0 = Ke^2|t|^2/(2\pi\alpha)^2\omega_c$) for the P (left) and the AP (right) model for the Hall state at $\nu = 5/2$ as a function of the voltage and at fixed frequency ($\omega_0/\omega$) varying the detector temperature (see legend). (Bottom panels) Corresponding derivatives $\partial S_{\text{meas}}/\partial \omega_0$. The other parameters are (in temperature units where necessary): $T = 5$ mK, $\omega = 60$ mK, $\omega_n = 50$ mK, $\omega_c = 1$ K and $|t_2|^2/|t_1|^2 = 1$.

The impedance-matching condition is much easier to obtain at a fixed frequency.
As shown in figure 4, the power to distinguish between the AP and P model is not essentially compromised by the detector temperature since all the features characterising the model in terms of $S_{\text{meas}}(\omega_0)$ seem to be mutually amplified. Increasing $T_c$ is an interesting resource in order to increase the detection efficiency in this perspective.

5. Conclusions

We have investigated the behaviour of the f.f. emitted power $S_{\text{meas}}(\omega_0)$ of a resonant LC circuit weakly coupled to a QPC built in a quantum Hall bar at filling factor $\nu = 5/2$. We have shown that the emitted power is represented in terms of the non-symmetrised noise components of the quantum Hall QPC weighted by the bosonic distribution of the resonant LC circuit. We have inspected the different predictions of the Pfaffian and anti-Pfaffian non-Abelian edge states models for this quantity. We have shown that this setup can detect and discriminate between the dominant and sub-dominant fractionally charged excitations looking at the bias dependence at fixed GHz frequencies. We have also discussed the advantage of using this measurement protocol in comparison with the f.f. symmetrised noise. Finally, we have demonstrated how the sensibility of the proposed setup can be increased varying the LC detector temperature $T_c$.

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References

[1] Tsui D C 1999 Rev. Mod. Phys. 71 891
[2] Stern A 2008 Ann. Phys. 323 204
[3] Laughlin R B 1983 Phys. Rev. Lett. 50 1395
[4] Jain J K 1989 Phys. Rev. Lett. 63 199
[5] Willett R, Eisenstein J P, Stormer H L, Tsui D C, Gossard A C and English J H 1987 Phys. Rev. Lett. 59 1776
[6] Moore G and Read N 1991 Nucl. Phys. B 360 362
[7] Nayak C, Simon S H, Stern A, Freedman M and Das Sarma S 2008 Rev. Mod. Phys. 80 1083
[8] Bishara W, Bonderson P, Nayak C, Shenggel K and Slingerland J K 2009 Phys. Rev. B 80 155303
[9] Stern A, Rosenov B, Iian R and Halperin B I 2010 Phys. Rev. B 82 085321
[10] Willett R L, Pfeiffer L N and West K W 2010 Proc. Natl Acad. Sci. 106 8853
[11] Rosenov B and Simon S H 2012 Phys. Rev. B 85 201302
[12] de Picciotto R, Reznikov M, Heiblum M, Umansky V, Bumin G and Mahalu D 1997 Nature 389 162
[13] Saminadayar L, Glattli D C, Jin Y and Etienne B 1997 Phys. Rev. Lett. 79 2526
[14] Chung Y C, Heiblum M and Umansky V 2003 Phys. Rev. Lett. 91 216804
[15] Bid A, Ofek N, Heiblum M, Umansky V and Mahalu D 2009 Phys. Rev. Lett. 103 236802
[16] Dolev M, Gross Y, Chung Y C, Heiblum M, Umansky V and Mahalu D 2010 Phys. Rev. B 81 161303

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[17] Ferraro D, Braggio A, Merlo M, Magnoli N and Sassetti M 2008 Phys. Rev. Lett. 101 166805
[18] Ferraro D, Braggio A, Magnoli N and Sassetti M 2010 Phys. Rev. B 82 085323
[19] Ferraro D, Braggio A, Magnoli N and Sassetti M 2010 New J. Phys. 12 013012
[20] Carrega M, Ferraro D, Braggio A, Magnoli N and Sassetti M 2011 Phys. Rev. Lett. 107 146404
[21] Wang J, Meir Y and Gefen Y 2013 Phys. Rev. Lett. 111 246803
[22] Roddaro S, Pellegrini V, Beltram F, Biasiol G and Sorba L 2004 Phys. Rev. Lett. 93 046801
[23] Roddaro S, Pellegrini V, Beltram F, Pfeiffer L N and West K W 2005 Phys. Rev. Lett. 95 156804
[24] Hashisaka M, Ota T, Muraki K and Fujisawa T 2015 Phys. Rev. Lett. 114 056802
[25] Shtanko O, Snizhko K and Cheianov V 2014 Phys. Rev. B 89 125104
[26] Smits O, Slingerland J K and Simon S H 2014 Phys. Rev. B 89 045308
[27] Rogovin D and Scalapino D J 1974 Ann. Phys. 86 1
[28] Radu I P, Miller J B, Marcus C M, Kastner M A, Pfeiffer L N and West K W 2008 Science 320 899
[29] Yang G and Feldman D E 2014 Phys. Rev. B 90 161303
[30] Susskind L 1995 J. Math. Phys. 36 6377
[31] Chamon C, Freed D E and Wen X G 1995 Phys. Rev. B 51 2363
[32] Chamon C, Freed D E and Wen X G 1996 Phys. Rev. B 53 4033
[33] Bena C and Nayak C 2006 Phys. Rev. B 73 155335
[34] Carrega M, Ferraro D, Braggio A, Magnoli N and Sassetti M 2012 New J. Phys. 14 023017
[35] Bednorz A, Bruder C, Renulet B and Belzig W 2013 Phys. Rev. Lett. 110 250404
[36] Lesovik G B and Loosen R 1997 JETP Lett. 65 295
[37] Zakka-Bajjani E, Segala J, Portier F, Glattli D C, Cavanna A and Jin J 2007 Phys. Rev. Lett. 99 236803
[38] Ferraro D, Carrega M, Braggio A and Sassetti M 2014 New J. Phys. 16 043018
[39] Fendley P, Fisher M P A and Nayak C 2007 Phys. Rev. B 75 045317
[40] Lee S-S, Ryu S, Nayak C and Fisher M P A 2007 Phys. Rev. Lett. 99 236807
[41] Levin M, Halperin B I and Rosenow B 2007 Phys. Rev. Lett. 99 236806
[42] Boyarsky A, Cheianov V and Froehlich J 2009 Phys. Rev. B 80 233302
[43] Hu Z-X, Rezayi E H, Wan X and Yang K 2009 Phys. Rev. B 80 235330
[44] Bishara W, Fiete G A and Nayak C 2008 Phys. Rev. B 77 245106
[45] Ginsparg P 1989 Applied Conformal Field Theory (Les Houches Lectures) ed E Brezin and J Zinn-Justin (Amsterdam: North Holland)
[46] Weiss U 1999 Quantum Dissipative System (Singapore: World scientific)
[47] Cuniberti G, Sassetti M and Kramer B 1996 J. Phys.: Condens. Matter 8 L21
[48] Braggio A, Grifoni M, Sassetti M and Napoli F 2000 Europhys. Lett. 50 236
[49] Braggio A, Sassetti M and Kramer B 2001 Phys. Rev. Lett. 87 146802
[50] Kane C L and Fisher M P A 1992 Phys. Rev. Lett. 68 1220
[51] Braggio A, Ferraro D and Magnoli N 2012 Phys. Scr. T 151 014052
[52] Braggio A, Ferraro D, Carrega M, Magnoli N and Sassetti M 2012 New J. Phys. 14 093032
[53] Altimiras C, Parlavecchio O, Joyez P, Vion D, Roche P, Esteve D and Portier F 2013 Appl. Phys. Lett. 103 212601
[54] Altimiras C, Parlavecchio O, Joyez P, Vion D, Roche P, Esteve D and Portier F 2014 Phys. Rev. Lett. 112 236803
[55] Martin T 2005 Course 5 Noise in Mesoscopic Physics (France: Les Houches)
[56] Gavish U, Levinson Y and Imry Y 2000 Phys. Rev. B 67 10637
[57] Aguado R and Kouwenhoven L P 2000 Phys. Rev. Lett. 84 1986
[58] Creux M, Crépieux A and Martin T 2006 Phys. Rev. B 74 115323
[59] Zazunov A, Creux M, Paladino E, Crépieux A and Martin T 2007 Phys. Rev. Lett. 99 066601
[60] Chevallier D, Jonckheere T, Paladino E, Falci G and Martin T 2010 Phys. Rev. B 81 205411
[61] Blanter Y M and Büttiker M 2000 Phys. Rep. 336 1
[62] Gavish U, Yurke B and Imry Y 2004 Phys. Rev. Lett. 93 250601
[63] Gavish U, Levinson Y and Imry Y 2001 Phys. Rev. Lett. 87 216807

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