Analytical Modelling and FEM Simulation of Capacitive Pressure Sensor for Intraocular Pressure Sensing

Rishabh Bhooshan Mishra, S Santosh Kumar, Ravindra Mukhiya
Smart Sensor Area, CSIR – Central Electronics Engineering Research Institute, Pilani, Rajasthan, India – 333031
Email: santoshkumar.ceeri@gmail.com

Abstract. The paper presents analytical modelling and Finite Element Method (FEM) based simulation of clamped circular capacitive pressure sensors for Intraocular Pressure (IOP) measurement. The parallel plate capacitive pressure sensor consists of a fixed backplate and a circular shaped clamped thin diaphragm made of silicon material. The plates are separated by a vacuum media. MATLAB® and COVENTORWARE® are used for analytical and FEM simulations, respectively, and a comparison of maximum deflection in the diaphragm, capacitance after pressure application and sensitivity is carried out. A pressure range of 0-60 mmHg is used for IOP measurement.

1. Introduction

Over conventional technique of Pressure measurement, MEMS pressure sensor offers several advantages such as small size, less weight, low power consumption, lower cost, and higher reliability. In these days, the research in piezoresistive and capacitive type pressure sensors are in trend. In comparison to piezoresistive sensors, the capacitive sensors offer several advantages like low power consumption, higher sensitivity to pressure, low thermal sensitivity and large dynamic range. The capacitive pressure sensors are generally used for automotive, biomedical, oceanography, aerospace application, consumer electronics etc. But miniaturization of MEMS devices for biomedical applications is very popular research area [1 - 2].

A lot of people are suffering from the irreversible vision loss eye disease i.e. glaucoma. The normal eye pressure is 1.6 to 2.8 kPa. More than normal eye pressure does not definitely cause of glaucoma disease but if the IOP is more than normal eye pressure and there are no signs of glaucoma then the problem may be ocular hypertension. The excessive pressure of fluid “aqueous humour” within eye is the major reason for the damage of the optical nerves which introduces glaucoma disease because it fills interior eye chamber. IOP increases due to several reasons like occlusion of drain ducts, closer or narrow angle between cornea and iris. The Goldmann Applanation Tonometer has been used for IOP measurement for past several decades but accuracy of this technique depends on the central corneal stiffness. For IOP measurement, the telemetry sensing technique plays important role. The measurement of IOP helps in glaucoma disease diagnosis and monitoring using telemetry principles, in which the data is monitored by implanted sensing device using wireless technique, so that further steps can be taken for medical treatment to patients [3 - 5].
2. **Analytical Modeling of Capacitive Pressure sensor**

The design and simulation of a circular diaphragm pressure sensor is carried out in this work for IOP measurement. Such a sensor may be fabricated using a standard fabrication process for capacitive pressure sensor.

2.1 **Analytical modeling of deflection in circular shaped clamped diaphragm**

The diaphragm deflection follows Hook’s Law i.e. the diaphragm deflection varies linearly when pressure is applied on the diaphragm. Before defining the deflection in diaphragm, following four basic assumptions have been made for small deflection plate theory [6]:

- The diaphragm is flat and made of elastic, homogeneous and isotropic material.
- In comparison to diaphragm thickness, deflection in diaphragm is very small (deflection must be less than 1/5th of thickness in case of small deflection theory). Hence slope of deflection is very much small and the square of deflection slope is neglected.
- The normal stress to the middle plane is neglected.
- After bending, the middle surface of diaphragm remains unstrained and the strain line remains perpendicular and straight to middle surface.

The uniform pressure \( P \) is applied on edge clamped thin circular plate which is made of elastic, isotropic and homogeneous material, and then plate equation can be represented by [7]:

\[
\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial w}{\partial r}\right) = \frac{P}{2D}
\]

or

\[
\frac{1}{r}\frac{\partial^3 w}{\partial r^3} + \frac{1}{r}\frac{\partial^2 w}{\partial r^2} - \frac{1}{r^2}\frac{\partial w}{\partial r} = \frac{P}{2D}
\]

Here, \( D = \frac{Er^3}{(12 - \nu^2)} \) is flexural rigidity of the diaphragm, \( w \) is deflection at radius \( r \) from centre of diaphragm, \( t \) is diaphragm thickness, \( \nu \) is Poisson’s Ratio and \( E \) is Young’s modulus using which the diaphragm is made.

The following boundary conditions are used to solve the plate equation:

\[
w(r = a) = 0
\]

\[
\frac{\partial w(r = 0)}{\partial r} = 0
\]

\[
\frac{\partial w(r = a)}{\partial r} = 0
\]

Here, \( a \) is diaphragm radius.

After using boundary conditions, from Eq. (3), (4) and (5), the diaphragm deflection can be given by:

\[
w(r) = \frac{Pa^4}{64D}\left[1 - \left(\frac{r}{a}\right)^2\right]^2
\]

2.2 **Analytical modeling of capacitance of circular capacitive pressure sensor**

In this presented paper, we propose an equation of capacitance of circular capacitive pressure sensor after pressure application on the top of diaphragm. If \( A, \varepsilon \) and \( d \) be overlapping area between plates, permittivity of medium and separation gap between parallel plates, respectively. Then base capacitance can be given by [2]:
\[ C_b = \frac{\epsilon A}{d} \]  

After pressure application on the diaphragm, the change in capacitance can be given by [2]:

\[ C_w = \int_0^{2\pi} \int_0^a \frac{\epsilon \, rdr \, d\theta}{d - w(r)} \]  

From Eq. (6) and (8), we get:

\[ C_w = \int_0^{2\pi} \int_0^a \frac{\epsilon \, rdr \, d\theta}{d - \frac{Pa^4}{64D} \left[ 1 - \left(\frac{r}{a}\right)^2 \right]^2} \]  

After solving double integral, following solution is obtained:

\[ C_w = 4\pi \epsilon \frac{D}{Pd} \ln \left[ \frac{a^2\sqrt{P} + 8\sqrt{dD}}{a^2\sqrt{P} - 8\sqrt{dD}} \right] \]  

2.3 Sensitivity of clamped circular shaped capacitive pressure sensor

The sensitivity of sensor can be defined by ratio of change in capacitance in defined pressure range and change in pressure. The sensor’s sensitivity can be defined by:

\[ S = \frac{C_{\text{max}} - C_{\text{min}}}{P_{\text{max}} - P_{\text{min}}} \]  

3. Simulation Results

3.1 Deflection in Diaphragms

Three different diaphragm thicknesses (4 μm, 5 μm and 6 μm) for 3 μm separation gap are chosen to obtain three different designs in the present work. A gap of 3 μm is chosen keeping in view the ease of fabrication. The sensors are designed for a pressure range from 0 to 8 kPa (0 to 60 mmHg) in order to measure the IOP. In each design, the maximum deflection of diaphragm at maximum pressure (8 kPa) is kept less than 1/4th of the separation gap to ensure linear characteristics. Based on these considerations and the small deflection theory, the diaphragm radius of each sensor model is determined. The deflection results of FEM simulation are performed using MemMech module of COVENTORWARE® and compared with the analytical results obtained using MATLAB® code which is based on Eq. (6) and (10).

The simulation of diaphragm deflection using FEM based tool COVENTORWARE® for diaphragm thickness of 4 μm is shown in Fig. 1. Similarly the 5 μm and 6 μm thick diaphragms are simulated. The comparison of deflection in each 4 μm, 5 μm and 6 μm thick diaphragms using analytical equations and FEM simulations are presented in Figs. 2, 3 and 4 for specific diaphragm radiiuses. The radius is obtained for separation gap of 3 μm for each diaphragm thickness. Diaphragm of particular thickness and radius has less than 0.75 μm (1/4th of 3 μm) deflection at 8 kPa pressure. The diaphragm deflection increases as radius of diaphragm increases. In all simulations, silicon is used as the diaphragm material and the following properties are used: E = 169.8 GPa and ν = 0.066.
3.2 Capacitance variation with pressure

In the present work, the capacitance variation with pressure for different diaphragm thicknesses are obtained using MemElectro module and graph between capacitance versus pressure for three sensor designs using CoSolveEM module of COVENTORWARE®. The comparison of results obtained by analytical modelling and FEM are shown in Figs. 5 to 7.

Figure 1. Maximum deflection in 4µm thick diaphragms using COVENTORWARE®.

Figure 2. Maximum deflection in 4 µm thick diaphragm.

Figure 3. Maximum deflection in 5 µm thick diaphragm.

Figure 4. Maximum deflection in 6 µm thick diaphragm.

Figure 5. Capacitance vs. Pressure for 4 µm thick diaphragm.
3.3 Sensitivity

The sensitivity of all the three designs is shown in Table I.

Table 1 Sensitivity of different designs

| Thickness (µm) | Radius (µm) | Sensitivity (fF/kPa) [Analytical] | Sensitivity (fF/kPa) [FEM] |
|---------------|-------------|----------------------------------|---------------------------|
| 4             | 265         | 7.122                            | 7.422                     |
| 5             | 310         | 9.285                            | 10.160                    |
| 6             | 360         | 10.325                           | 15.389                    |

4. Discussion and Conclusion

In this paper, capacitive pressure sensor is utilised for IOP sensing application using small deflection theory. The difference between the value of diaphragm deflection obtained using analytical and FEM simulation are due to certain assumptions which are practically not possible to achieve. The deviation in results in the values of capacitance obtained is due to the fringing field effect which is not considered in the analytical solution. We can conclude that the design having a diaphragm thickness of 6 µm, separation gap of 3 µm and diaphragm radius of 360 µm has the maximum sensitivity amongst all the three designs. It sensitivity is 10.325 fF/kPa and 15.389 fF/kPa using analytical equations and FEM, respectively. Therefore, this design is best suited for IOP measurement.

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