Mesoscopic Study of a Square Cavity Ventilated and heated by the bottom

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Abstract. In this paper, we present a numerical study of the laminar mixed convection in a ventilated cavity square; one of the walls is subjected to a constant temperature, while the other walls are considered adiabatic. The thermal lattice Boltzmann method with the double population model was used. A computer code was developed with the D2Q9 model for the velocity field and D2Q5 for the temperature field to determine the whole structure of the flow. The results are presented in the form of thermal fields for the Richardson number Ri = 10. Those show that each time the Richardson number increases, the number of Nusselt increases too what justifies an increase in the heat transfer.

1. Introduction

The transfer of heat by mixed convection aroused the considerable interest of several researches for technological applications such as: the ventilation of the buildings, the chemical plating of the thin layers, the cooling of the electronic parts, squanderers of heat in the solar collectors and nuclear reactors. Several simple and complex geometrical configurations were examined by report to a theoretical, digital end experimental approach.

A large number of numerical studies were interested on fixing of only one entry and exit [1–3] with an isothermal wall. Others research considered the wall heated with a heat flow [4–5], where [6] studied various configurations of the entry and exit position in order to detect the best possible position of the entry and exit opening, and to obtain a more effective cooling. Other numerical and experimental [7–9] studies treated the effect of the geometry of an obstacle like the source of heat inside the cavity in order to maximize the total conductance. The position of the air entry and the exit has a great effect of the hydrodynamic and thermal structures [10]. One cavity that has several entries [11] improves its ventilation.

The lattice Boltzmann method (LBM) being a digital method relatively recent and original which came out at the beginning of the Nineties. It is interested, either with the macroscopic quantities (celerity, pressure and density), but directly the distribution of the various particles constituting a fluid. We speak then about mesoscopic representation. What makes it competing with the other conventional methods such as finite volume, finite elements and the finite differences. It is initially resulting from the lattice gas method [12], from the automata cellular theory [13] and while being based on the physic statistical formalism [14]. It is important to be able to locate its performances compared to those of the classical.
digital methods used to simulate and reproduce the flows with the heat transfer in the ventilated enclosures. The purpose of the study presented in this work is the analysis of the phenomenon of the heat transfer with mixed flow of the convection in the laminar mode, in a square cavity provided with two openings. The interior walls are supposed to be adiabatic except for the located on the low side, it is considered isothermal. The thermal model of the lattice Boltzmann method with nine celebrities ($D_2Q_9$) is used to reproduce the dynamic field and that simplified at five celebrities ($D_2Q_5$) is used for the temperature field.

Thus, a thermal analysis will have to be carried out with LBM. That will enable us to determine the performances of this new digital method in this field.

2. Physical description of the problem
The model chosen is square cavity of coast $H$ filled with two ventilation openings, the first located in the lower left corner of $L_1$ and the second located in the upper right side $L_2 = L_1 = 20\% H$. The walls of this cavity are adiabatic except the lower wall. The walls of this cavity are adiabatic except the low wall which is maintained by a source of heat at a constant temperature $T_h$. Air entering through the left opening of the wall with a temperature $T_0$ and a uniform velocity $U_0$ as fig.1 shows it. The assumptions used are summarized in the case of a incompressible fluid, Newtonian in two dimensions, laminar, satisfying the assumption with Boussinesq, stationary with a transfer of heat by radiation and a dissipation of heat by effect of viscosity negligible.

3. Lattice Boltzmann method
In statistical physics, a gas is described by a cloud of particles obeying the Boltzmann equation. This continuous equation as it was proposed by Boltzmann in 1872 can be written as follows:

$$\frac{\partial f}{\partial t} + c \cdot \frac{\partial f}{\partial x} + F \cdot \frac{\partial f}{\partial c} = \Omega(f).$$

(1)

This equation is the basic equation of the kinetic theory of gases. It describes gas molecules distribution function which is a function of time, space and velocity of the molecule.
With the approximation Bhatnagar, Gross and Krook (BGK) [15] The collision operator is written as follows:

\[
\Omega = \omega(f^\text{eq} - f) = \frac{1}{\tau}, (f^\text{eq} - f).
\]  

(2)

The collision frequency is given by:

\[
\omega = \frac{1}{\tau}.
\]  

(3)

The local equilibrium distribution function is denoted by \( f_i^\text{eq} \); which is the distribution function of Maxwell-Boltzmann. In general the equilibrium distribution function Maxwell Boltzmann [16] is expressed by:

\[
f_i^\text{eq} (x, t) = \rho \cdot w_i \left[ 1 + \frac{e_i u}{c_s^2} + \frac{(e_i u)^2}{2 c_x^2} - \frac{u^2}{2 c_x^2} \right].
\]  

(4)

Discretizing equation (10) in space, time and compared with speeds gives the Boltzmann equation system given by the following formula:

\[
f_i(r + e_i \delta t, t + \delta t) - f_i(r, t) = -\frac{1}{\tau} \left( f_i(r, t) - f_i^\text{eq}(r, t) \right).
\]  

(5)

The Local conservation laws: the Density, The velocity and Energy are expressed as follows [17]:

\[
\rho(x, t) = \sum_{i=0}^{N} f_i(x, t) = \sum_{i=0}^{N} f_i^\text{eq}(x, t).
\]  

(6)

\[
u = u(x, t) = \frac{1}{\rho(x, t)} \sum_{i=0}^{N} e_i f_i(x, t) = \frac{1}{\rho(x, t)} \sum_{i=0}^{N} e_i f_i^\text{eq}(x, t)
\]  

(7)

\[
E = E(x, t) = \frac{1}{\rho(x, t)} \sum_{i=0}^{N} \frac{1}{2} (e_i - u)^2 f_i(x, t) = \frac{1}{\rho(x, t)} \sum_{i=0}^{N} \frac{1}{2} (e_i - u)^2 f_i^\text{eq}(x, t)
\]  

(8)

3.1. \( D_2Q_9 \) model

For the discretization of speeds, the model of the square lattice nine speeds (Fig. 2) with a unit spacing. Each node of the network is connected to eight by eight other neighbouring links. The particles can only reside on a node and move to their nearest neighbours along these links for a unit time. There are three types of particles on each node with nine different speeds.

![Figure 2. D2Q9 lattice](image2.png)

![Figure 3. D2Q5 model](image3.png)
The lattice velocities $e_i$ are:

$$
e_i = c \left( \cos \left( (i - 1) \pi \right), \sin \left( (i - 1) \pi \right) \right), \quad i = 1 - 4$$

$$
e_i = \sqrt{2}c \left( \cos \left( (i - 5) \frac{\pi}{2} + \frac{\pi}{4} \right), \sin \left( (i - 5) \frac{\pi}{2} + \frac{\pi}{4} \right) \right), \quad i = 5 - 8$$

Where $c = \frac{\Delta x}{\Delta t}$, and The corresponding weight factors are: $w_0 = 4/9, w_1 = w_2 = w_3 = w_4 = 1/9, w_5 = w_6 = w_7 = w_8 = 1/36$

The dynamic viscosity is given by the formula:

$$
\mu = \left( \tau - \frac{1}{2} \right) \frac{\rho \varepsilon^2}{3} . \quad (10)
$$

### 3.2. $D_2Q_9$ model:

For the discretization of the temperature, the square lattice pattern of five-speed (Figure 3) with a spacing unit. Each node of the network is connected to four other neighbours by four links. The particles can only reside on a node and move to their nearest neighbours along these links for a unit time.

Since there is no coupling between the celerity equation and the energy, two different distribution functions have to be solved, for example for speed $f$ and $g$ for energy. The effort provided for the establishment of a stable model for the lattice Boltzmann method thermal (TLBM) has paid off recently. The celerity function equation is the same as that of $D_2Q_9$. For the energy function, the equation is:

$$
g_i(x + \Delta x, t + \Delta t) = g_i(x, t) \cdot \left[ 1 - \omega_s \right] + \omega_x \cdot g_i^e(x, t). \quad (11)
$$

The summation of the distribution functions in each $D_2Q_9$ lattice node gives us the temperature:

$$
T = \sum_{i=1}^{5} g_i . \quad (12)
$$

The distribution equilibrium function is:

$$
g_i^e = w_i \cdot f(x, t) \cdot \left[ 1 + e_i \cdot \frac{\nu}{\lambda_s} \right]. \quad (13)
$$

$e_i$ are the lattice celerities defined by:

$$
e_i = c \left( \cos \left( \frac{2\pi(i-1)}{4} \right), \sin \left( \frac{2\pi(i-1)}{4} \right) \right), \quad i = 1, ... 5. \quad (14)
$$

The corresponding weight factors are: $w_0 = 1/3, w_1 = w_2 = w_3 = w_4 = 1/6$.

The equation of the thermal diffusion coefficient is given by

$$
\alpha = \frac{\Delta x^2}{3 \Delta t \cdot \frac{1}{\omega_0} - \frac{1}{2}}. \quad (17)
$$

### 4. Results an discussion

#### 4.1. Numerical code validation

We studied the natural convection in a square cavity closed differentially heated, where two horizontal walls of the top and bottom are considered adiabatic. It emphasizes that the right side wall of the cavity is
subjected to a cold temperature and the left at a hot temperature. We compare the results of the value of
Nusselt using the two methods with those of the literature.
In this study, we perform simulations for moderate Rayleigh numbers $10^3 \leq \text{Ra} \leq 10^5$ and Prandtl number
$Pr = 0.71$. The mesh study based on lattice Boltzmann method with dual population (TLBM) was
performed in the laboratory. Resolution mesh was satisfied between 71x71 and 151x151 for different
Rayleigh numbers.

| Ra  | Present work TLBM | De Vahl Davis [17] | Markatos & Pericleous [18] |
|-----|-------------------|-------------------|-----------------------------|
| $10^3$ | 1,115             | 1,118             | 1,108                       |
| $10^4$ | 2,221             | 2,243             | 2,201                       |
| $10^5$ | 4,461             | 4,519             | 4,430                       |

The Table 1 presents the numerical results obtains of the average Nusselt number with the TLBM code.
These are compared with the results of the literature using different numerical methods. It is clear that the
results of our TLBM model are agreed with the results of the literature in the range of Rayleigh numbers
tested. The error calculated for the average Nusselt number Nu, does not exceed 1.2% compared to the
reference results [17], 1.9% compared to [18].

4.2. Results for the ventilated cavity
The geometrical configuration treated in this study is the ventilated square cavity presented in (fig. 1).
The selected fluid inside is the air with ($Pr = 0.71$). We use the grid of 121*121 for the lattice Boltzmann
method in all our calculations. We study the influence of the Richardson number on the total structure of
the flow. For this case, the Grashof number is fixed to $Gr = 105$ and the Richardson number is varied by
taking the following values: \( \text{Ri} = 3, 5, 10, 15, 20, 30, 50 \) and \( 100 \). The results are presented in the isotherm form (figure 4). The geometrical configuration treated in this study is the ventilated square cavity presented in (figure 1). The selected fluid inside is the air with \( \text{Pr} = 0.71 \). We use the grid of \( 121 \times 121 \) for the lattice Boltzmann method in all our calculations. We study the influence of the Richardson number on the total structure of the flow. For this case, the Grashof number is fixed to \( \text{Gr} = 10^5 \) and the Richardson number is varied by taking the following values: \( \text{Ri} = 3, 5, 10, 15, 20, 30, 50 \) and \( 100 \). The results are presented in the form of the isotherms (figure 4).

We note that the distribution heat in the cavity is in conformity with the circulation of the fluid. Indeed, we note a heating of the fluid starting from the entry (low left corner) throughout the low wall to the exit (right high corner) for the eight values of the Richardson number \( \text{Ri} \). For a Richardson number \( = 3 \), we note a thermal absence of the stratification, which coincides with the beginning of the mixed convection. Starting from a Richardson number \( = 5 \), we note the appearance of a thermal stratification. From values of
Ri ≥ 10, we note a widening of temperature occupying a considerable portion of the cavity, also we observe a horizontal stratification of the layers of an air in which the natural convection is prevalent. The position of the hot wall has an influence on the heat transfer. We notice that the high temperatures are localised in narrow spaces in the vicinity of the hot wall which correspond to the thermal thickness of the boundary layer and which is largely influenced by the Richardson number. Far from the hot wall, the gradients of the temperature are weak.

In figure 5, the variation of the average Nusselt number depending on the Richardson number is presented. We note that the Nusselt number decreases with the increase of the Richardson number what justifies a decrease in the heat transfer.

5. Conclusion

This work focuses on the numerical study of laminar mixed convection in a ventilated cavity with two openings. The results obtained in this work were used to evaluate the performance and LBM's ability to reproduce the mixed convection in a ventilated cavity. The TLBM model with a double population model used has all the advantages, including good numerical stability and the ability to generally manage the overall heat transfer by convection problems.

References

[1] Zermane S, Boudebous S and Boulkroune N 2005. Etude numérique de la convection mixte laminaire dans des cavités ventilées. Sciences & Technologie B– N°23, pp. 34-44.
[2] Radhi Z K 2011. Numerical study of mixed convection heat transfer through double square cavity connected with each other. Al-Qadisiya Journal For Engineering Sciences Vol. 4 No.4.
[3] Hamini A, Bouabdallah S and Benchatti A 2012. Numerical study of Mixed Convection in a Square Cavity Containing Inlet and Outlet air. JTEMP08 EMP, (Bordj El Bahri, Algeria).
[4] Rahman M M, Alim M A, Mamun M A H, Chowdhury M K and Islam A K M S 2007. Numerical study of opposing mixed convection in a vented enclosure. Asian Research Publishing Network (ARPN). Vol. 2, N°2.
[5] Mahmoudi A H, Shahi M and Talebi F 2010. Effect of inlet and outlet location on the mixed convective cooling inside the ventilated cavity subjected to an external nanofluid. International Communications in Heat and Mass Transfer 37 - 1158–1173.
[6] Raji A, Hasnaoui M and Bahlaloui A 2008, Numerical study of natural convection dominated heat transfer in a ventilated cavity: Case of forced flow playing simultaneous assisting and opposing roles. International Journal of Heat and Fluid Flow 29 - 1174–1181.
[7] Radhakrishnan T V, Verma A K, Balaji C and Venkateshan S P 2007. An experimental and numerical investigation of mixed convection from a heat generating element in a ventilated cavity. Experimental Thermal and Fluid Science 32 - 502–520.
[8] Rahman M M, Alim M A, Saha S and Chowdhury M K 2008. Mixed convection in a vented square cavity with a heat conducting horizontal solid circular cylinder. Journal of naval architecture and marine engineering DOI: 10.3329/jname.v5iame.v5i2.2504.
[9] Benachour E, Draoui B, Rahmani L, Mebarki B, Asnoune K and Imine B 2010. Simulation Numérique de la Convection Mixte d’une pièce (type d’habitat) avec la présence d’un Corps de Chauffe. 10th International Meeting on Energetical Physics SIPE 10 (Bechar, Algeria). Journal of Scientific Research N° 0 Vol. 1.
[10] Oztop F H 2010. Influence of exit opening location on mixed convection in a channel with volumetric heat sources. International Communications in Heat and Mass Transfer 37 - 410–415.
[11] Belhi M, Boudebous S 2007. Etude numérique de la convection mixte dans une cavité carrée munie de plusieurs entrées. *Congrès Français de Thermique, SFT* (Île des Embiez, France), 29 mai - 1 juin.

[12] Dieter A. Wolf-Gladrow 2005. *Lattice-Gas Cellular Automata and Lattice Boltzmann Models - An Introduction*. Springer edition.

[13] Biggs M J and Humby S J 1998. Lattice-gas automata methods for engineering. *Chemical Engineering Research & Design*, 76 (A2):162-174.

[14] Lifshitz E M, Pitaevskii L P 1980. *Statistical Physics, Part 1*, Pergamon Press, Oxford.

[15] Bhatnagar P, Gross E, and Krook M 1954. A model for collision process in gases. I. Small amplitude process in charged and neutral one-component systems, *Phys. Rev.* 94(3), 511-525.

[16] Han K, Feng Y T, Owen D R J 2008. Modeling of thermal contact resistance within the framework of the thermal lattice Boltzmann method. *International Journal of Thermal Sciences* 47 - 1276–128.

[17] De Vahl Davis G 1983. Natural convection of air in a square cavity: a benchmark numerical solution. *Int. J. Numer. Meth, Fluids* 3, pp249.

[18] Markatos N C, Pericleous K A 1984. Laminar and turbulent natural convection in an enclosed cavity. *Int. J. Heat Mass Transfer*, 27, 755-772.