THERMALIZATION IN RELATIVISTIC OUTFLOWS AND THE CORRELATION BETWEEN SPECTRAL HARDNESS AND APPARENT LUMINOSITY IN GAMMA-RAY BURSTS

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ABSTRACT

We present an interpretation of the phenomenological relations between the spectral peak, isotropic luminosity, and duration of long gamma-ray bursts that have been discovered by Amati and coworkers, Ghirlanda and coworkers, Firmani and coworkers, and Liang & Zhang. In our proposed model, a jet undergoes internal dissipation which prevents its bulk Lorentz factor from exceeding $1/\beta$ (\(\beta\) being the jet opening angle) until it escapes from the core of its progenitor star, at a radius of order $10^{12}$ cm; dissipation may continue at larger radii. The dissipated radiation will be partially thermalized, and we identify its thermal peak (Doppler boosted by the outflow) with $E_{pk}$. The radiation comes, in effect, from within the jet photosphere. The nonthermal, high-energy part of the GRB emission arises from Comptonization of this radiation by relativistic electrons and positrons outside the effective photosphere. This model can account naturally not only for the surprisingly small scatter in the various claimed correlations, but also for the normalization, as well as the slopes. It then has further implications for the jet energy, the limiting jet Lorentz factor, and the relation of the energy, opening angle and burst duration to the mass and radius of the stellar stellar progenitor. The observed relation between pulse width and photon frequency can be reproduced by inverse Compton emission at $\sim 10^{12}$ cm from the engine, but there are significant constraints on the energy distribution and isotropy of the radiating particles.

Subject headings: gamma rays: bursts — radiation mechanisms: nonthermal — supernovae: general

1. INTRODUCTION

The mechanism by which gamma rays are created in the transient events known as gamma-ray bursts (GRBs) has remained surprisingly resistant to interpretation. Two key ideas have guided theoretical efforts. The first is that thermalization must be rapid and nearly complete close to the engine. A fireball is created, which could reach a temperature as high as $\sim 1$ MeV within a region the size of a neutron star or stellar-mass black hole (Paczynski 1986; Goodman 1986). The second idea is that strongly nonthermal particle distributions are expected at large distances from the central engine, e.g., when the relativistic ejecta collide with the ambient medium (Rees & Meszaros 1992). Continuing dissipation within the outflow itself can also be expected, due to the formation of internal shocks (Rees & Meszaros 1994), or the release of magnetic energy by reconnection (Thompson 1994; Drenkhahn & Spruit 2002) or global current-driven instabilities (Giannios & Spruit 2006; Lyutikov & Blandford 2003).

The prompt emission of a GRB appears at first sight to be highly nonthermal. A high-energy cutoff is seen only in a modest fraction of GRB spectra (Pendleton et al. 1997; Ryde 2004). Often the low-energy spectrum appears to be harder than would be consistent with optically thin synchrotron emission (Crider et al. 1997; Ghirlanda et al. 2003). A discrete thermal bump is usually not apparent in the usual spectral analyses, although Ryde (2004) has shown that up to 30% of long GRB can be interpreted as a combination of a thermal peak plus a power-law spectrum. Intriguing evidence has, furthermore, emerged that indicates that the gamma-ray emission is seeded by radiation with a nearly black-body spectral distribution. This evidence comes from measurements of the photon energy $E_{pk}$ at which the GRB energy spectrum $E_{\gamma}^2 dN_{\gamma}/dE_{\gamma}$ has a maximum.

There has been longstanding interest in the distribution of $E_{pk}$ values in GRBs. BATSE measured a distribution that is clustered around 200 keV (e.g., Preece et al. 2000). Observational selection might explain the high-energy cutoff to this distribution; the GRB spectrum is harder below the peak and so a lower flux is measured within the BATSE band from bursts with large $E_{pk}$ (Piran & Narayan 1996). Nonetheless, the existence of a substantial population of GRBs with $E_{pk}$ higher than $\sim 1$ MeV remains an open question.

Our knowledge of the $E_{pk}$ distribution has improved dramatically with the localizations provided by BeppoSAX and HETE-2, which have allowed a direct measurement of the distances to bursts (see Piran 2005; Mészáros 2006, for detailed references). The value of $E_{pk}$ in the cosmological rest frame of the burst source correlates well the isotropic gamma-ray energy (Amati et al. 2002; Lamb et al. 2005),

$$E_{pk} = 100 \left( \frac{E_{iso}}{10^{52} \text{ erg}} \right)^{0.5} \text{ keV.} \quad (1)$$

Bursts with lower peak energies are therefore substantially dimmer. Related correlations have been obtained by correcting $E_{iso}$ for the beaming angle, as inferred from the measurement of breaks in the afterglow light curve (Ghirlanda et al. 2004; Liang & Zhang
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2006), and by measuring the joint correlation of $E_{pk}$ with $E_{iso}$ and the burst duration (Firmani et al. 2006).

The breadth of the intrinsic $E_{pk}$ distribution provides valuable information about the emission physics. In the sample of bursts with measured redshifts, $E_{pk}$ generally lies in the range 100 keV–1 MeV (Amati et al. 2006). These values are generally lower than what would be expected from thermalization close to the engine. Although a broader distribution of $E_{pk}$ can result from prompt thermalization followed by adiabatic cooling (e.g., Thompson 1994; Mészáros & Rees 2000; Mészáros et al. 2002), a more plausible explanation is that thermalization takes place at greater distances from the engine than was suggested by the original fireball models. The relation between $E_{iso}$ and $E_{pk}$ that results from this type of distributed heating has been investigated recently (Rees & Mészáros 2005; Thompson 2006; Pe’er et al. 2006). The peak energy can remain quite high at large distances from the engine, after allowance is made for beaming and bulk relativistic motion. Continuing dissipation in the jet is a natural consequence of its interaction with the core and envelope of a Wolf-Rayet star.

Bursts with peak energies less than 50 keV (X-ray flashes) have been detected by Ginga and HETE-2 (Strohmayer et al. 1998; Sakamoto et al. 2005), but few have measured redshifts. Their $T_{90}$ distribution is, nonetheless, remarkably similar to that of the harder spectrum bursts, which suggests that the low $E_{pk}$ is not primarily due to a large cosmological redshift. Since the formation of a nearly blackbody spectrum is possible only out to some distance from the engine, one expects that the thermal peak energy must cover only a limited range. In fact, as we will see, quasi-thermal models of the prompt GRB emission over ~10 s cannot easily be extended beyond $E_{pk} \sim 50$ keV without invoking strong adiabatic cooling. Peak energies much higher than 1 MeV also cannot easily be obtained without invoking upscattering by relativistic particles.

It has been noted (Nakar & Piran 2005) that among long bursts in the BATSE catalog whose redshift is unknown, there may be up to 25%–30% whose low luminosity and hard spectra are incompatible with the Amati et al. (2002) relation. The implication is that this relation may mark out a limiting value of the spectral peak energy at a given apparent luminosity. For the quasi-thermal model discussed here, we point out that in fact low luminosities a departure from the Amati et al. relation is expected, which works in the sense of the above discrepancy; relativistic jets with large opening angles and relatively low isotropic luminosities should produce spectrally harder bursts.

In addition to providing seeds for the prompt gamma-ray emission, a thermal photon field also can play an important role in accelerating the outflow (Paczyński 1990; Shemi & Piran 1990). Phenomenological scaling relations such as the Amati et al. (2002) relation therefore provide crucial information about the relative sequence of thermalization and acceleration. If thermalization occurs first, at some characteristic radius, then a tight relation between $E_{pk}$ and $E_{iso}$ follows naturally. Moreover, it has been suggested that the slope of the $E_{pk}$–$E_{iso}$ relation can be reproduced most easily if the internal heating is driven by nonradial shear instabilities near the base of the jet, where its Lorentz factor $\Gamma$ is comparable to the inverse of the opening angle $\theta$, as opposed to radial instabilities in a relativistic flow with $\Gamma \gg 1/\theta$ (Thompson 2006). This raises the possibility that components of the jet which emerge from a Wolf-Rayet star with low baryon loading but also low entropy (e.g., the jet core) will attain lower terminal Lorentz factors and may manifest themselves in the afterglow phase.

The approach adopted in this paper is twofold. After an initial outline in §2 of the gist of our thermal interpretation of the Amati et al. (2002) relation, we begin by working backward from the observed correlation between spectral peak energy and isotropic burst energy. In §2 we show that thermalization must continue out to a large distance ($\sim 10^{10}$ cm) from the engine in order to reproduce the normalization of the Amati et al. relation. We show that thermalization at this radius is reasonable and derive a condition for the outflow to generate enough photons to establish a blackbody spectral distribution (§2.1). In particular, the magnetic field must carry at least ~10% of the outflow luminosity. The acceleration of the outflow is addressed in §2.2, where it is shown that the blackbody photons can effectively accelerate the entrained baryons and magnetic field up to a limiting Lorentz factor of 100–500. In §2.3 we examine the effect on the $E_{pk}$–$E_{iso}$ relation of having thermalization occur before or after the acceleration of the jet material. In the second case, $E_{pk}$ has a much weaker dependence on $E_{iso}$ than is observed.

The alternative works much better. The normalization and slope of the Amati et al. (2002) relation follow immediately from two simple assumptions: first, that the total jet energy is regulated to a value close to the net gravitational binding energy of the Wolf-Rayet core, and, second, that the GRB-emitting component of the jet has $\Gamma \sim 1/\theta$ at the core boundary. This simple model predicts the joint correlation of $E_{pk}$ with $E_{iso}$ and the burst duration $t_j$ derived from the data by Firmani et al. (2006). The possibility of a range of jet energies is examined in §2.4, where it is shown that the Ghirlanda et al. (2004) relation is consistent with an approximate scaling $E_{pk} \sim \theta_j^{-1}$. Thermalization can also continue out to the photosphere in jets with heavier baryon loadings, and a minimum thermal peak energy of 50 keV (without adiabatic cooling) is derived in §2.5. The implications of the $E_{pk}$–$E_{iso}$ relation of short GRBs for our model are outlined in §2.6, where it is shown that a large thermalization radius, similar to that of the long GRBs ($R_0 \sim 10^{10}$ cm) is implied. The pulses of GRBs are generally narrower at higher frequencies (Fenimore et al. 1995), and it is argued in §3.1 that this is nicely consistent with inverse Compton (IC) scattering of the seed thermal photons by a power-law distribution of electrons (and positrons) at $\sim 10^{14}$ cm from the engine. However, this explanation for the pulse duration has an important implication; the IC emission by particles with relativistic energies must be beamed in the bulk frame. In addition, the distribution of particle energies must have a lower cutoff at transrelativistic energies ($\gamma_{min} \sim 1$). We consider the implications for models of particle heating (by turbulence and shocks) in §3.2. The paper concludes with some summary observations in §4.

2. QUASI-THERMAL MODEL OF BURST EMISSION

We assume here that the flow can be characterized by a mean Lorentz factor $\Gamma(r)$ and opening angle $\theta(r)$ at each radius $r$. The opening angle can vary with radius if a nonradial magnetic field plays a role in collimation and acceleration (e.g., Lynden-Bell 2003; Vlahakis & Königl 2003). An observer viewing the outflow face-on sees a luminosity

$$L_{iso}(r) = \frac{2L_j}{\theta_j^2(r)} \quad \text{for } \theta_j \ll 1,$$  

where $L_j$ is the total output of the engine and $\theta_j$ is the half-opening angle of the outflow.

A fraction $\epsilon_{br}$ of the outflow energy is assumed to be completely thermalized at a radius $R_0$, where the outflow Lorentz factor is $\Gamma_0$. In the present context, we will find that the temperature $T'$ in the rest frame of the outflow is low enough that thermally created pairs do not contribute significantly to the specific heat.
The bulk frame thermal energy density is then \( \propto a T^4 \), where \( a \) is the Stefan-Boltzmann constant. An observer at rest with respect to the engine sees a temperature

\[
T_{\text{obs}} = \frac{4}{3} \Gamma_0 T_0^\prime = \varepsilon_{bb}^{1/4} \left( \frac{\Gamma_0}{R_0} \right)^{1/2} \left( \frac{16 L_{\text{iso}}}{27 \pi a c} \right)^{1/4}.
\]

(3)

It is reasonable to assume that strong thermalization takes place only at radii comparable or less than the radius of the stellar progenitor, \( R_0 \approx R_* \), e.g., that being the limiting radius where oblique shocks can cause dissipation. At this radius, one would expect the bulk Lorentz factor \( \Gamma_0(R_0) \approx 1/2 \sqrt{3 \theta} \), where \( \theta \) is the jet opening angle at the surface, since above this Lorentz factor shear-driven modes in the jet will have no time to grow on a lengthscale \( \sim R, \theta \) (in the bulk frame). Thus, using equations (2) and (3) and assuming \( \Gamma(R) \approx \theta^{-1} \) and \( L_j = E_j/\theta \) \( \propto \) constant (as motivated by Fialf et al. 2001, who find \( E_j \approx \) constant) one obtains (Thompson 2006)

\[
E_{pk} \propto R_0^{-1/2} \Gamma \varepsilon_{iso}^{1/2}.
\]

(4)

This is essentially the relation discovered by Amati et al. (2002), if we consider that \( R_0 \approx R_* \) \( \propto \) constant (e.g., all progenitors of long GRBs are lacking hydrogen envelopes) and that the dispersion in \( \Gamma \) \( \propto 10^{4.2}, \) (approximately 10 2 decades).

This simple derivation of the Amati et al. (2002) relation involves making some reasonable physical assumptions. However, it is more illuminating to work backward and use the observed relation (1) to derive implications for the physics of the model and the progenitor. Combining the blackbody law (3) with equation (1) gives \( \Gamma_0 \)

\[
R_0 \approx 6 \times 10^{9} \varepsilon_{bb}^{1/2} \varepsilon_{iso}^{-1/2} \Gamma^{1/2} \text{ cm}.
\]

(5)

One observes that the Amati et al. relation can be reproduced, but only if thermalization takes place at a radius \( R_0 \) that is much larger than the engine radius (in the conventional picture where the engine is an accreting black hole or possibly a rapidly spinning magnetar). If the motion of the jet is only mildly relativistic at this point then \( R_0 \) is comparable to the core radius of a Wolf-Rayet star.

2.1. Thermalization in a Plasma of Moderate Scattering Optical Depth

Consider now the consequences of heating an ionized plasma that is optically thick to scattering, but optically thin to free-free absorption. Under what circumstances does the radiation field relax to a nearly blackbody distribution? To get a sense of the Thomson optical depth \( \tau_T \) that is required, we first assume that the radiation spectrum is nearly blackbody, and then examine the self-consistency of this assumption.

Synchrotron emission by relativistic electrons (and positrons) is generally a copious source of soft photons. In spite of this, a minimal magnetic field is required for synchrotron emission to supply enough photons to establish a blackbody gas. We suppose that a fraction \( \varepsilon_{\text{sa}} \) of the energy density \( U' \) of the outflow is transferred to electrons that radiate at the synchrotron self-absorption frequency (at a Lorentz factor \( \gamma_{\text{sa}} \)). A fraction \( \varepsilon_B/(\varepsilon_B + \varepsilon_\gamma) \) of the energy of these relativistic electrons is released to synchrotron photons (as opposed to IC scattering of the ambient radiation field, with a total energy density \( U' = \varepsilon_\gamma U' \) from all sources). The number of (new) synchrotron photons is

\[
n_{\gamma, \text{synch}} \approx \left( \frac{\varepsilon_B}{\varepsilon_B + \varepsilon_\gamma} \right) \frac{\varepsilon_{\text{sa}} U'}{0.3 \gamma_{\text{sa}}^2 \hbar c B'/m_e c}.
\]

(6)

Let us compare this expression with the number density of photons in a blackbody gas (of temperature \( T_{\text{sa}} \)) that carries a fraction \( \varepsilon_{bb} \) of the outflow energy. The characteristic self-absorption frequency can be obtained from

\[
\gamma_{\text{sa}} \approx \frac{\tau_T}{\alpha_{em}} \left( \frac{B'}{B_{\text{QED}}} \right)^{-1}.
\]

(7)

where \( \tau_T = \sigma_T n_e' (\gamma_{\text{sa}}) c' \) is the Thomson depth through the electrons of energy \( \gamma_{\text{sa}} m_e c^2 \). The net output in synchrotron photons at the self-absorption frequency is given by

\[
\varepsilon_{\text{sa}} U' \approx n_{\gamma} (\gamma_{\text{sa}}) \times \left( \frac{4}{3} \gamma_{\text{sa}}^2 \gamma_{\text{bb}} \gamma_{\text{sa}} B^2}{8 \pi} c'.
\]

(8)

The optical depth \( \tau_T \) is therefore regulated to a value

\[
\frac{4}{3} \gamma_{\text{sa}}^2 \gamma_{\text{bb}} \gamma_{\text{sa}} \approx \frac{\varepsilon_{\text{sa}}}{\varepsilon_B}.
\]

(9)

Combining this expression with equation (7) gives

\[
\gamma_{\text{sa}} \approx \left( \frac{\varepsilon_{\text{sa}}}{\varepsilon_B} \right)^{1/7} \left( \frac{B'}{B_{\text{QED}}} \right)^{-1/7}.
\]

(10)

The relative numbers of synchrotron and blackbody photons are obtained by substituting equation (10) and the relation \( k_B T_{\text{bb}}'/m_e c^2 (B'/B_{\text{QED}})^{-1/2} = 1.7 (\varepsilon_{bb}/\varepsilon_B)^{1/4} \) into equation (6),

\[
n_{\gamma, \text{synch}}^{\text{bb}} \approx 10^{7/14} \varepsilon_{\text{sa}}^{9/14} \varepsilon_B^{8/14} \left( \gamma_{\text{bb}} + \varepsilon_B \right) \left( \gamma_{\text{bb}} m_e c^2 \right)^{-3/7}.
\]

(11)

Recall that \( \varepsilon_{\text{sa}} \) labels the net energy that is injected into relativistic particles of energy \( \gamma_{\text{sa}} m_e c^2 \). For example, if the energy injected into relativistic electrons is distributed uniformly over Lorentz factor and comprises a fraction \( \varepsilon_{\text{nh}} \) of the total, then \( \varepsilon_{\text{sa}} = \varepsilon_{\text{nh}}/\ln (\gamma_{\text{max}}/\gamma_{\text{min}}) \approx 0.1 \varepsilon_{\text{nh}} \). The right-hand side of equation (11) can be larger than unity, but only if

\[
\frac{\varepsilon_B}{\varepsilon_{bb}} \geq \left( \frac{\varepsilon_{\text{nh}}}{\varepsilon_{bb}} \right)^{10/13} \left( \frac{k_B T_{\text{bb}}'}{m_e c^2} \right)^{6/13}.
\]

(12)

We conclude that photon creation in the outflow can be rapid enough to create a blackbody photon gas, but only if the magnetic field carries more than \( \approx 10\% \) of the total dissipated energy.

Soft photons injected into the outflow are redistributed in frequency by multiple Compton scatterings. When the electrons and the photons have the same temperature, there is no net transfer of energy from electrons to photons, but soft photons will increase their energy, at a rate \( \dot{E}_z/E_z \approx 4 k_B T_{\text{bb}}'/m_e c^2 \sigma_T n_e' c \). The efficient redistribution of photons in frequency via Compton scattering requires that

\[
\left( \frac{k_B T_{\text{bb}}'}{m_e c^2} \right)^{1/7} \tau_T \geq 1,
\]

(13)

1 Throughout, we use the shorthand \( X \approx X_0 \times 10^x \), where the quantity \( X \) is measured in cgs units.
where $\tau_T$ is the Thomson depth through the warm electrons with a bulk-frame temperature $T'_b$. From equation (13), one obtains a (conservative) lower bound on the Thomson depth that guarantees efficient thermalization of the soft seed photons at an effective temperature $T'_{bb}$.

$$\tau_T \gtrsim \left( \frac{k_B T'_b}{m_e c^2} \right)^{-1}. \quad (14)$$

We will adopt this value of the optical depth when evaluating the temperature of the radiation that emerges from outflows with a relatively high baryon loading.

At energies above the thermal peak, a power component can arise in the usual manner from shocks and synchrotron/IC radiation above the photosphere, and/or a comptonized tail in the photosphere itself. This approximation of the spectral peak determined by a blackbody temperature $E_{bb} \sim 3 k_B T'$ is only slightly changed if one considers the Compton equilibrium in the dissipating photosphere which is optically thick to scattering, $E_{bb} \simeq 3 k_B T(1 + A^{-1})$, where $A \sim 1$ is the photon to electron energy density ratio (Pe’er et al. 2005).

2.2. Adiabatic Cooling and Radiative Acceleration of the Outflow

This fireball radiation may be cooled adiabatically if it is generated close enough to the engine, so that the fireball Lorentz factor saturates at a value $\eta = L_j/M_c c^2$ before the photons and the electrons are decoupled from each other. A condition for the neglect of adiabatic cooling is obtained by comparing the saturation radius $R_{sat}$ with the photospheric radius. For the moment, we do not specify how outflow is accelerated and take

$$\Gamma(r) = \left( \frac{r}{R_0} \right)^\alpha \Gamma_0 \quad \text{for } R_\tau > r > R_0. \quad (15)$$

The saturation radius is then

$$R_{sat} = \left( \frac{\eta}{\Gamma_0} \right)^{1/\alpha} R_0. \quad (16)$$

If the electron-ion photosphere sits outside $R_{sat}$, then its position is determined by setting

$$\sigma_T \frac{R_\tau}{2 \Gamma^2} \left( \frac{L_{iso}}{4 \pi \eta \mu c^3 R_\tau^2} \right) = 1. \quad (17)$$

Here $\mu$ is the mean mass per scattering charge. It is useful to defining the compactness

$$\tilde{\epsilon}_0 = \frac{\sigma_T L_{iso}}{8 \pi m_e c^3 \Gamma_0^3 R_0}$$

at the thermalization radius, and the “reduced” compactness

$$\tilde{\epsilon}_0 = \frac{m_e}{\mu} \tilde{\epsilon}_0.$$

Combining equations (16) and (17) gives

$$\frac{R_{sat}}{R_{\tau}} = \left( \frac{\eta}{\Gamma_0} \right)^{(1+3\alpha)/(1+2\alpha)} \tilde{\epsilon}_0^{-1/(1+2\alpha)}. \quad (20)$$

Adiabatic cooling can be neglected if $R_{sat} > R_\tau$, which corresponds to

$$\eta > \eta_{cool} = \tilde{\epsilon}_0^\alpha/(1+3\alpha) \Gamma_0, \quad (21)$$

or equivalently to

$$\frac{R_0}{\Gamma_0^{\alpha/(1+3\alpha)}} > \frac{L_{iso} \sigma_T}{8 \pi \mu c^4 \eta^2/(1+3\alpha)}. \quad (22)$$

This expression simplifies to

$$\frac{R_0}{\Gamma_0} > 6 \times 10^6 \frac{L_{iso} \sigma_T}{\Gamma_0} \eta^2 \text{ cm} \quad (23)$$

in the case of a ballistically expanding outflow ($\alpha = 1$). Comparing this expression with equation (5), one sees that adiabatic cooling can be neglect beyond the radius where the thermal peak energy is established, as long as $\eta = L_j/M_c c^2 \gtrsim 10^2$.

There is a close correspondence between the condition that adiabatic cooling be absent and the condition that the thermal photon flux be strong enough to push the entrained baryons to a high Lorentz factor the electron-ion photosphere (see Grimsrud & Wasserman 1998). One requires a mechanism for creating very high entropies in order to reach a terminal Lorentz factor as high as $\sim 50–100$. The photon field is quasi-isotropic in a frame that moves with a Lorentz factor

$$\Gamma_\gamma(r) = \left( \frac{r}{R_\tau} \right) \Gamma(R_\tau), \quad (24)$$

is the combined Lorentz factor of the matter and photons at the photosphere. At $r > R_\tau$, the radiation field has an anisotropic component in the rest frame of the matter, and the resulting force keeps $\Gamma \simeq \Gamma_\gamma$ out to a radius where

$$\sigma_T \frac{R_\tau}{2 \Gamma} \left( \frac{L_{iso}}{4 \pi \Gamma^2 \eta \mu c^3} \right) \sim 1. \quad (26)$$

The matter therefore reaches a maximum Lorentz factor

$$\Gamma_{max} = \tilde{\Gamma}_0^{1/4} \Gamma_0 R_0^{1/4 \Gamma_0} \Gamma_\tau \quad (27)$$

which is

$$\frac{\Gamma_{max}}{\eta_{cool}} = \left( \frac{\eta}{\eta_{cool}} \right)^{(1-\alpha)/(1+2\alpha)} \text{ for } \eta > \eta_{cool}. \quad (28)$$

When $\alpha = 1$, one recovers

$$\Gamma_{max} = \tilde{\epsilon}_0^{1/4} \Gamma_0 = 90 L_{iso} \sigma_T \tilde{\epsilon}_0^{1/4} \left( \frac{R_0}{10^{10} \text{ cm}} \right)^{-1/4} \left( \frac{\mu}{m_p} \right)^{-1/4}. \quad (29)$$

The terminal Lorentz factor is smaller if the thermalization and acceleration are offset from the engine by a distance $R_0 \gg 10^6$ cm. When $\alpha < 1$, the terminal Lorentz factor is generally smaller than equation (29) and saturates this bound only if $\eta$ is large enough.
that the photosphere has contracted to $R_e \sim R_0$. Note also that the entrained magnetic field does not limit acceleration by the thermal photons as long as $\Gamma \gg (B^2/8\pi pc^2)^{1/3}$ (Thompson 2006). Here $\rho$ is the baryon density as measured in the rest frame of the central engine.

This estimate of the terminal Lorentz factor allows for pair creation in the outflow outside the baryonic photosphere. The conversion of a modest fraction of the outflow energy to pairs would significantly decrease the inertia per scattering charge $\mu$, and therefore increase the terminal Lorentz factor. In the absence of pair creation, the compactness at the electron-ion scattering photosphere is $\ell \sim (m_e/m_\nu)(\gamma/\Gamma)$, which is easily large enough to allow effective collisions between photons of an energy $\sim m_e c^2$ in the bulk frame. Only a tiny fraction $\sim 3/\ell$ of the outflow energy must be converted to such photons to increase the number of light charges in the outflow. One can expect the relativistic ejecta to reach a terminal Lorentz factor as high as $\sim 500$ due to this effect.

2.3. Implications of and Models for the Amati and Firmani Relations

One can consider two basic solutions to equation (5) for the thermalization radius $R_0$, as deduced from the Amati et al. (2002) relation. In the first, the acceleration of the outflow is delayed, so that $\Gamma_0 \sim 1/\theta_0 \sim$ few at a radius $R_0$. In the second case, the outflow has already attained a substantial fraction of its terminal speed, $\Gamma \sim \eta$, when thermalization occurs.

2.3.1. Case 1

When acceleration occurs after thermalization, one finds

$$R_0 = 1 \times 10^{10} (\Gamma_0 \theta_0)^{-1/2} E_{\text{iso}}^{1/2} \frac{\ell_1/2}{E_j^{1/2} I_j^{1/2}} \text{ cm.} \quad (30)$$

Here we have rewritten equation (5) by substituting $E_{\text{iso}} = 2E_j/\theta_j^2$.

A characteristic value of the thermalization radius follows from the following considerations. A certain fraction of the jet energy is mixed with the material of the Wolf-Rayet star, thereby forming a cocoon structure. When the energy in this cocoon becomes comparable to the binding energy of the Wolf-Rayet star, the star will explode and the rate of mass accretion onto the engine (the central black hole) must slow dramatically.

After the jet head has reached the edge of the Wolf-Rayet core, relativistic material will continue to flow through the created opening. The Lorentz factor of this material generally increases away from the engine, but at a much slower rate than the Bernoulli rate for adiabatic flow. (This effect is observed in the simulations of an MHD jet propagating through a stellar envelope by McKinney 2006.) Two relativistic components of the jet can be distinguished: a central core which moves with a Lorentz factor $\Gamma > 1/\theta$ and an annular region sandwiched between the core and cocoon which is susceptible to strong nonradial shear instabilities. (These instabilities can involve several possible processes: oblique shocks, turbulent shear viscosity, and magnetic reconnection, the details of which are not addressed here.)

This last jet component moves with a Lorentz factor $\Gamma \sim 1/\theta$; it is much hotter than the core but entrains only a small mass in baryons. Its width will be comparable to the distance that a signal can propagate in the bulk frame, over the radial flow time $R_0/2\Gamma_0 c$. This gives $\theta_0 \Gamma_0 = 1/2\sqrt{3}$, where we have taken the signal speed to be the sound speed $c/\sqrt{3}$ in an isotropic relativistic fluid. Substituting this expression and $E_j \approx 10^{51}$ ergs (the binding energy of the core of a Wolf-Rayet star of initial mass $\sim 25 M_\odot$; Woosley & Weaver 1995) into equation (30) gives

$$R_0 = 3 \times 10^9 \frac{E_{\text{iso}}^{1/2}}{E_j^{1/2} I_j^{1/2}} \text{ cm.} \quad (31)$$

The peak energy scales with other quantities as (Thompson 2006)

$$E_{\text{pk}} \propto \frac{E_{\text{iso}}^{1/2}}{E_j^{1/4} I_j^{1/4} \Gamma_0^{1/2}}. \quad (32)$$

This interpretation of the Amati et al. (2002) relation has several interesting consequences:

1. The thermalization radius of equation (31) corresponds closely to the radius of the CO core of a Wolf-Rayet star if the total jet energy is set equal to the core binding energy. The binding energy just before the collapse is approximately $1 \times 10^{51}(M_{\text{ZAMS}}/M_\odot)$ in a star with a zero-age main sequence mass of 20–30 $M_\odot$ (Woosley & Weaver 1995). The CO core binding energy and radius depend relatively weakly on details of the evolution of the progenitor (e.g., binary interaction, and the loss of a hydrogen or helium envelope). The typical duration of a long GRB is comparable to the collapse time of the core, whereas the light travel time across the core is only $\sim 0.1$ s.

2. The temperature of the thermal radiation that is seen outside the photosphere is also insensitive to the baryon loading, as long as $\eta$ lies below a critical value where $\gamma \tau \gg (\kappa_B T/m_e c^2)^{-1}$ at the thermalization radius. The electron-ion photosphere also lies well beyond the thermalization radius,

$$R_e = 80 \frac{L_{\text{iso}}^{3/2} I_j^{1/2}}{\eta_2^{1/3}} \left( \frac{R_0}{10^{10} \text{ cm}} \right)^{-1/3} \text{ cm for } L_{\text{iso}} \gtrsim 10^{51} \text{ ergs s}^{-1}, \quad (33)$$

as long as the matter loading is high enough that the assumption of complete thermalization is justified.

3. The relation between temperature and isotropic energy is insensitive to continuing dissipation over some distance outside the thermalization radius, if the outflow Lorentz factor grows linearly with radius ($\alpha = 1$)

$$\Gamma(r) = \Gamma_0 \left( \frac{r}{R_0} \right). \quad (34)$$

The scattering depth through the outflow decreases rapidly with radius, $\tau_{\gamma \gamma} \propto r^{-1} \gamma^{-2} \propto r^{-3}$, so that the surface $\tau_{\gamma \gamma} = (\kappa_B T_{\text{iso}}/m_e c^2)^{-1}$ sits at a radius $\sim 0.2$–0.4 $R_e$. In other words, dissipation can continue over a factor 20–30 in radius beyond $R_0$ and still result in a nearly blackbody spectrum; but continuing dissipation between this point and the photosphere will result in a broader “graybody” spectrum (Pe’er et al. 2006).

4. The thermal photons effectively accelerate the outflow even outside the electron-ion photosphere. As a result, portions of the jet that achieve higher entropies inside the Wolf-Rayet envelope may reach higher terminal Lorentz factors. A jet may contain a cooler core that flows with a Lorentz factor $\Gamma > 1/\sqrt{3}\theta$ and is subject only to weak shear-driven instabilities. This feedback between heating and acceleration provides an explanation for why a thermal radiation field should generically be present in the parts of the outflow that produce the prompt GRB emission.

2.3.2. Case 2

Now let us examine the other case, where acceleration occurs before thermalization. The temperature is now more sensitive to
continuing dissipation in the outflow, and the relation between $E_{pk}$ and $E_{iso}$ is softer than implied by either the Amati et al. (2002) or Ghirlanda et al. (2004) relations. Suppose that the outflow Lorentz factor has saturated at $\Gamma \approx \eta$. The thermal peak energy that results from dissipating a fraction $\epsilon_{bb}$ of the outflow energy at an optical depth $\tau_{\epsilon} \approx (\eta^2 T^*/m_{e}c^2)^{-1}$ is easily found to be

$$E_{pk}^{t} = 2.7 \left(\frac{4}{3} \eta\right) k_{B} T^* = 0.5 \epsilon_{bb}^{1/6} \eta^{5/3} \left(\frac{\mu}{m_{p}}\right)^{1/3} \text{keV.} \quad (35)$$

In this case, the peak energy depends strongly on the baryon loading. In some models of the engine, e.g., those in which the outflow is driven by a magnetic field threading the horizon of a black hole (Blandford & Znajek 1977; Mészáros & Rees 1997), it is not obvious why $\eta$ should correlate strongly with $L_{iso}$.

Recently, Firmani et al. (2006) have pointed out that a rather tight correlation exists between three observed quantities that are derived from measurements of the prompt emission, namely, $L_{iso}$, $E_{pk}$, and $t_{b,45}$, where $L_{iso} \propto E_{pk}^{1/4} t_{b,45}$ and $t_{b,45}$ is essentially a measure of the duration of the prompt emission above a certain level (originally used for measuring the variability or spikiness of the prompt emission, by Ramirez-Ruiz & Fenimore 2000 and Reichert et al. 2001). Assuming that $t_{b,45} \sim t_{b}$ and that the energy output rate is $L_{iso} \propto E_{iso}/t_{b}$, the Firmani relation can be written as $E_{iso} \propto E_{pk}^{2/3} t_{b}^{1/2}$. The assumptions of a characteristic jet energy and thermalization radius, which were previously used to motivate the Amati et al. (2002) relation, imply (see eq. [32])

$$E_{iso} \propto E_{pk}^{2/3} t_{b}^{1/2} R_{0}^{1/2}. \quad (36)$$

This essentially reproduces the phenomenological Firmani et al. (2006) relation.

2.4. Correlation of $E_{pk}$ with Jet Opening Angle: Ghirlanda Relation

Ghirlanda et al. (2004) derived a relation connecting the observed spectral peak energy $E_{pk}$ to the collimation-corrected jet energy $E_{j}$. The observed quantities are $E_{pk}$, $E_{iso}$, and the break time $t_{b}$ of the afterglow light curve. They are related by

$$E_{pk} \propto E_{iso}^{1/2} t_{b}^{1/2}, \quad (37)$$

(Liang & Zhang 2006, hereafter LZ06; also Nava et al. 2006).

A narrower distribution of burst energies results from the jet collimation correction in Ghirlanda et al. (2004), but a spread of about 1 order of magnitude in energy remains. In this section, we relax our assumption of a single burst energy and consider the implications of the Ghirlanda et al. relation for the jet properties within the Wolf-Rayet envelope.

From $t_{b}$ and $E_{iso}$ one can obtain the opening angle $\theta_{j}$ of the jet at the time of the afterglow break. We assume that the ambient medium has a power-law density profile,

$$\rho_{ext} \propto r^{-k}. \quad (38)$$

The radius at which the break occurs is

$$r(t_{b}) \propto \left[ E_{iso}/\Gamma^{2}(t_{b}) \right]^{1/(3-k)} \propto E_{j}^{1/(3-k)}, \quad (39)$$

where $\Gamma(t_{b}) \sim 1/\theta_{j}$ and $E_{j} \sim (\theta^{2}/2) E_{iso}$ were used. Substituting

$$t_{b} \sim r(t_{b})/c \Gamma(t_{b}) \quad (40)$$

into the observed LZ06 relation $E_{pk}(E_{iso}, t_{b})$ (eq. [37]), one finds that $\eta$ cancels out and

$$E_{pk} \propto E_{j}^{(4-k)/(6-2k)} \propto \begin{cases} E_{j}^{2/3} \text{ for } k = 0, \\ E_{j} \text{ for } k = 2. \end{cases} \quad (41)$$

For a uniform external medium, $k = 0$, this yields the original Ghirlanda et al. (2004) relation $E_{pk} \propto E_{iso}^{2/3}$, whereas for a wind, $k = 2$ it yields $E_{pk} \propto E_{j}$ (a form which, as pointed out by Nava et al. 2006, is Lorentz invariant, since both quantities depend in the same manner on the bulk Lorentz factor).

Taking the above correlation at face value, one can work backward and deduce a relation between the total jet energy and the opening angle of the jet. This relation is not unique, in that it depends on the index $k$ of the density profile in the medium outside the progenitor. We focus, as before, on the interaction of the jet with the core of the Wolf-Rayet star, and fix the thermalization radius at the core radius $R_{0}$. Combining relation (41) with the blackbody relation $E_{pk} \propto [\Gamma(R_{0})/R_{0}]^{1/2} (E_{iso}/t_{b})^{1/4}$ and assuming the jet to be heated by shear-driven instabilities $[\Gamma(R_{0}) \sim \theta^{-1}]$, one finds

$$E_{j} \propto (\theta^{2} t_{j}^{1/2} R_{0})^{-2(3-k)/(5-k)}. \quad (42)$$

This gives $E_{j} \propto \theta^{-12.5}$ for $k = 0$ and $E_{j} \propto \theta^{-4.5}$ for $k = 2$. The same scaling would, of course, hold if we chose some other fixed radius for the formation of the thermal spectrum.

The Amati et al. (2002) and Firmani et al. (2006) relations both involve quantities that refer only to the prompt gamma-ray emission. The Ghirlanda et al. (2004) relation combines quantities referring to both the prompt emission and the afterglow, which introduces an additional layer of model dependence. For example, the jet opening angle that is derived from the jet break time is assumed to be the same as the opening angle in the prompt emission phase. A narrow range of outflow energies that is assumed in the derivation of the Amati et al. relation in § 2.3, whereas jet break correction suggests a somewhat broader range of energies (a factor $\sim 10$ for a wind medium). The true range of jet energies is as yet undetermined.

2.5. Heavier Baryon Loading and Possible Relation to X-Ray Flashes

Heavy baryon loading has two effects on an outflow; it tends to reduce the temperature of the thermal photons emerging from the flow, and it can also lengthen the duration of the outflow and hence of the photon signal. The first effect is generally encountered at lower baryon loadings than the second.

X-ray flashes and GRBs are observed to have a similar distribution of durations (Sakamoto et al. 2005). Although the X-ray flashes are fainter and so could be observed from a smaller cosmological redshift, their intrinsic durations cannot differ by more than a factor 2–3 from those of the GRBs. One thereby obtains a strong constraint on the the baryon loading in the outflows that emit X-ray flashes.

Consider an outflow in which the Lorentz factor has saturated at a value $\Gamma \approx \eta$ and most of the energy is in the kinetic energy of the baryons. The photon signal is lengthened considerably if $\eta$ smaller than the critical value where

$$\eta_{c} \approx \frac{R_{\gamma}}{2\eta^{2}c}. \quad (43)$$
namely,
\[ \eta_L = \left( \frac{\sigma_T E_{\text{iso}}}{8\pi \rho c^2 T_{\text{iso}}^2} \right)^{1/5} = 20 \frac{E_{\text{iso}}^{1/5}}{m_p} \left( \frac{\rho}{\rho_0} \right)^{-2/5} \left( \frac{T_{\text{iso}}}{T_0} \right)^{-1/5}. \] (44)

The photosphere generally sits outside the saturation radius (eq. [16]) when \( \eta \approx \eta_r \). Substituting equation (44) into equation (20) and taking \( \Gamma = r/R_0 \) gives
\[ \frac{R_\tau}{R_{\text{sat}}} = 10^{1/15} \left( \frac{E_{\text{iso}}^{1/5}}{m_p} \right)^{1/15} \left( \frac{\rho}{\rho_0} \right)^{-1/15}. \] (45)

This means that dissipation must continue beyond the saturation radius if a significant thermal photon energy flux is to be observed. The peak energy resulting from the dissipation of a fraction \( \varepsilon_{bb} = \frac{1}{4} \) of the energy flux at a scattering depth \( \tau_T \sim (k_T/m_c c^2)^{-1} \) is obtained by substituting equation (44) into equation (35),
\[ E_{\text{pk}}' = 2.7 \left( \frac{4}{3} \eta \right) k_T T_{\text{iso}}^{1/6} 50 E_{\text{iso}}^{1/5} \left( \frac{\rho}{\rho_0} \right)^{-1/15} \text{ keV}. \] (46)

Note that the dependence of \( E_{\text{pk}}' \) on \( E_{\text{iso}} \) in this expression is weaker than in a blackbody. We conclude that values of \( E_{\text{pk}}' \) as low as \( \approx 50 \) keV are consistent with burst durations of \( \approx 10^5 \) s.

Lower thermal photon energy fluxes and temperatures are possible if the dissipation occurs deeper in the outflow and the thermal photons are adiabatically cooled. A simple relation between \( E_{\text{pk}}' \) and \( E_{\text{iso}} \) is not expected in this regime, due to the sensitivity of the output spectrum to the location of the dissipation and the level of baryon loading. It should be emphasized that the adiabatically softened thermal photons can still act as the dominant coolant for relativistic electrons farther out in the outflow, even if their energy density is lower than that of the ambient magnetic field, in some models of electron heating (e.g., Thompson 2006). In this case, the thermal photons must be upscattered in frequency to absorb a significant portion of the outflow energy, and the effects of adiabatic cooling will be reversed.

2.6. Implications for Long, Dim, Hard Outliers to the Amati Relation

As pointed out by Nakar & Piran (2005), a significant number of long, dim, and hard BATSE bursts fall, for any value of their unknown redshift, above and to the left of the nominal \( E_{\text{pk}} \) versus \( E_{\text{iso}} \) Amati et al. (2002) relation. A relevant point of the present thermal model (Thompson 2006) is that the predicted \( E_{\text{pk}}' \)-\( E_{\text{iso}} \) relation must transition from the nominal \( E_{\text{pk}} \propto E_{\text{iso}}^{1/2} \) law to a flatter blackbody type law \( E_{\text{pk}} \propto E_{\text{iso}}^{1/4} \) at low values of \( E_{\text{iso}} \), when \( \theta_c \) gets larger than a critical value (corresponding to \( \Gamma (R_0) \sim (1/2) \sqrt{3 \theta_c} \sim 1 \), or \( \theta_c \sim 20^\circ \) [from the Frail et al. 2001 relation \( \theta_{\text{iso}}(\theta_c/2) = 5 \times 10^{50} \) ergs, which leads to \( \Gamma (R_0) = (1/2) \sqrt{3 \theta_c} = 0.9 \theta_{\text{iso}}(\theta_c/2)^{1/2} \)]. In fact, the redshift-localized bursts appear to cut off, in the Nakar and Piran (2005) simulations, at about the point where one would expect to see the transition, at \( E_{\text{iso}} \sim 10^{52} \) ergs.

However, there could be other factors complicating the \( E_{\text{pk}}' \)-\( E_{\text{iso}} \) relation below \( 10^{52} \) ergs. There could be a mix of different types of events, such off-axis, dirty fireballs that are adiabatically cooled, and off-center fireballs in which the reheating of soft bremsstrahlung photons is limited by pair annihilation and the saturation of the temperature at \( 30 - 50 \) keV (SN 1998bw type events). Thus, one should be cautious in interpreting the \( E_{\text{pk}}' \) values of the couple of very soft \( \text{HETE}-2 \) bursts that are frequently plotted on the Amati et al. (2002) curve.

2.7. Low-Energy Bursts Similar to GRB 980425/SN 1998bw

Long bursts associated with detected supernovae (GRB/SNe) are a subclass whose conformity (or not) to the Amati et al. (2002) relation is complicated. Some of these events satisfy this relation (e.g., GRB 060218/SN 2006aj; Campana et al. 2006) while others do not (e.g., GRB 980425/SN 1998bw and GRB 031203/SN 2003gw). The gamma-ray output of many GRB/SNe is intrinsically low, and there is evidence pointing toward the presence of a transrelativistic (\( \Gamma \sim 1 \)) outflow, e.g., GRB 060218/SN 2006aj, GRB 980425/SN 1998bw, and GRB 031203/SN 2003gw. Other GRB/SNe which are not particularly dim, such as GRB 030329/SN 2003dh, may also have a transrelativistic component.

This component could be identified with the ejection of a thin, fast shell during the breakout of a shock across the Wolf-Rayet photosphere (Colgate 1974; Tan et al. 2001), which can supply up to \( 10^{46} \) ergs in transrelativistic material, or, in more energetic events, with a jet cocoon (Ramirez-Ruiz et al. 2002; Pe’er et al. 2006). A third possibility is a shell of Wolf-Rayet material that is entrained at the head of a relativistic jet (Waxman & Mészáros 2003). Such a “breakout shell” is susceptible to fragmentation, and in the case of a wide jet (opening angle tens of degrees) with a relatively short duration (\( \tau_j < 10 \) s), its rest energy can approach the total jet energy (Thompson 2006). In this case, the two components will intermix within \( \approx 10^{11} - 10^{12} \) cm from the engine, and may not develop a large Lorentz factor. (The breakout shell is lighter and accelerates more easily in more focused jets with \( E_{\text{iso}} \approx 10^{52} \) ergs.)

The radius at which gamma rays are emitted from mildly relativistic ejecta depends on the ejecta mass and the density of the progenitor wind. There are various possibilities; the emission radius could be identified with the deceleration radius of the relativistic shell (especially if the ejecta shell is light and the wind is dense; Tan et al., 2001), with the photosphere of the wind (Wang et al. 2007), or with the photosphere of the ejecta themselves. In the first case, the emission occurs at a time
\[ t_{\text{em}} \sim \frac{E_{\text{ej}}}{2 \Gamma_{\text{ej}}^4 V_{\text{w}}} \left( \frac{M_{\text{w}}}{M_{\text{ej}}} \right)^{-3} \] (for a wind composed of alpha elements, \( Y_e = \frac{1}{3} \) electrons per baryon). In the third case, the ejecta themselves become transparent to scattering at
\[ t_{\text{em}} \approx \frac{Y_e \sigma_T E_{\text{ej}}}{16 \pi \Gamma_{\text{ej}}^3 m_p c^4} \] (again neglecting pair creation and assuming \( Y_e = \frac{1}{2} \) within the ejecta. (Note that in this last case, the ejecta are too heavy to be decelerated significantly by the Wolf-Rayet wind before they become optically thin, if the ejecta energy is as large as \( \approx 10^{50} - 10^{51} \) ergs.)
These timescales can be compared with the durations of GRB 980425 ($t_{\text{em}} \sim 30$ s) and GRB 060218 ($t_{\text{em}} \sim 3 \times 10^3$ s for the prompt thermal X-ray component).

Pair creation could evidently be important in the emission of GRB 980425, since $E_{\text{pk}} \sim 100$ keV was above the threshold for thermal pair creation ($\sim 30$ keV in the bulk frame). We consider the simplest case of pairs in chemical equilibrium with a Wien gas of photons (density $n'$, and mean energy $3 k_B T'$). The pair density is related to the photon density by

$$n'_e + n'_\gamma = \left( \frac{\pi}{2} \right)^{1/2} \left( \frac{k_B T'}{m_e c^2} \right)^{-3/2} \exp \left( - \frac{m_e c^2}{k_B T'} \right). \quad (47)$$

Setting $T' \sim (k_B T'/m_e c^2)^{-1}$ (see eq. [14]) for the limiting radius of thermalization gives an implicit relation for the rest frame temperature,

$$\left( \frac{k_B T'}{m_e c^2} \right)^{3/2} \exp \left( \frac{m_e c^2}{k_B T'} \right) \approx 0.3 \ell',$$  

where

$$\ell' = \frac{\sigma_T L_{\text{ej}}}{8 \pi \Gamma_{\text{ej}}^5 c m_e c^2 r_F} = 6 \times 10^{-2} E_{\text{pk}} \left( \frac{t_j}{30 \text{ s}} \right)^{-2} \left( \frac{\Gamma_{\text{ej}}}{2} \right)^{-5}. \quad (49)$$

is the compactness in the emission zone. When the temperature is lower than this critical value, it is not possible for the thermal pairs to upscatter soft keV photons that are advected with the ejecta from the Wolf-Rayet photosphere. The energy of the Wien peak cannot be pushed any lower by sharing the thermal energy among a larger number of photons.

In the case of GRB 980425, equations (48) and (49) give $k_B T' \simeq 50$ keV and $E_{\text{pk}} \approx 4 \times 10^{52}$ ergs, which is about a factor of 3 too hard for $\Gamma_{\text{ej}} = 2$, but it should be recalled that upscattering of soft keV photons freezes out at an optical depth $\tau_\gamma \sim 10$. After the heating rate slows down, the photons entrained by the optically thick pair cloud would cool adiabatically.\(^2\) The temperature could drop by as much as $\tau_\gamma^{2/3} = 0.2$ if pairs did not annihilate and $\Gamma$ remained constant; in fact, some deceleration and annihilation will take place, which have opposing effects on the output temperature. A burst duration of $\sim 30$ s is obtained if the initial ejecta Lorentz factor is $\Gamma_{\text{ej}, 0} \simeq 2 M_{\odot}^{-1/4}$.

The X-ray spectrum of GRB 060218 was much too soft for pairs to contribute significantly to the optical depth. (The compactness $\ell'$ is inferred to be only $\sim 10$ in the emission zone if the ejecta are mildly relativistic.) The high-energy tail of the X-ray spectrum could be explained by the Fermi acceleration of soft keV photons at the forward shock as it passes through the photosphere of the Wolf-Rayet wind, if $\Gamma_{\text{ej}} \sim 1-2$ (Wang et al. 2007). This has the interesting implication that the preburst mass-loss rate is quite high, $M \sim 10^{-2} M_{\odot}$ yr$^{-1}$, in order to place the wind photosphere at a distance $c t_{\text{em}} \sim 10^{14}$ cm from the engine (see also Dai et al. 2006).

It also turns out that the radius $c t_{\text{em}}$ in GRB 060218 is close to the scattering photosphere of the ejecta themselves, if they are mildly relativistic and have a kinetic energy of $\sim 10^{51}$ ergs.

\(^2\) The specific heat of the photons dominates that of the pairs at temperatures well below $m_e c^2/k_B$. We are interested in the regime where the baryonic rest energy is comparable to that of the photons, so the baryons provide inertia. They do not, however, provide a significant optical depth if $\ell' \lesssim (m_p/m_e) \Gamma_{\text{ej}}/1)$. The scattering depth through the neutralizing electrons can be smaller than unity at the point where the pairs freeze out, if the pairs are heated continuously starting at a large optical depth. Therefore a related possibility is that the keV photons are warmed up by turbulence within the ejecta (which would be excited by the differential motion of the relativistic ejecta with respect to the fragments of the breakout shell; see § 3.1.1). It should be emphasized that the blackbody radius of $\sim 10^{12}$ cm for the keV thermal emission (Campana et al. 2006) is much smaller than the photospheric radius that is inferred from the burst duration. This is consistent with the calculation of photon creation in § 2.1.

It is possible that all GRB/SNe combine a transrelativistic outflow with a relativistic jet, the relative strengths of the two components contributing in varying ratios to the nonthermal gamma-ray emission. However, the Amati et al. (2002) relation clearly relates to the relativistic component, whose presence is inferred in the great majority of classical bursts, whereas in events where the transrelativistic component dominates it appears that the Amati et al. relation is generally not satisfied. Our analysis in §§ 2.3 and 2.4 applies to bursts where the relativistic jet component dominates.

### 2.8. Implications for the $E_{\text{pk}}-E_{\text{iso}}$ Relation of Short GRBs

The short GRB population offers a nice test of the idea that the peak energy is fixed by thermalization inside the outflow photosphere. Although the two burst populations have different progenitors, the outflow that produces them must have a very high compactness in both cases, and may be driven by essentially the same mechanism (e.g., a MHD jet). The short GRBs have systematically higher peak energies than is implied by the Amati et al. (2002) relation (Donaghy et al. 2007), but so far only GRB 050709 has both a well-defined redshift and measured spectral peak energy (Fox et al. 2005; Villasenor et al. 2005).

One should be kept in mind that the blackbody temperature depends more directly on the outflow luminosity than the total burst energy. To obtain the relevant thermalization radius, we can make use of the scaling (eq. [3]),

$$R_0 \propto \frac{\Gamma_{\text{ej}} E_{\text{iso}}^{1/2}}{t_j^{1/2} E_{\text{pk}}^{2/3}}. \quad (50)$$

The isotropic energy of GRB 050709 is $\sim 1 \times 10^{50}$ ergs and its peak energy is $\sim 100$ keV; some 10 times higher than what would be expected based on the value of $E_{\text{pk}}$ alone. However, the burst’s $T_0$ duration is $\sim 0.07$ s, more than 100 times shorter than a typical long GRB. Combining equation (50) with equation (5) one obtains a thermalization radius of $R_0 \sim 1 \times 10^{10}$ cm, similar to what we deduced for a long GRB with $E_{\text{iso}} \sim 10^{52}$ ergs s$^{-1}$ and $t_j \sim 10$ s.

A much smaller thermalization radius is deduced for the giant flare of 2004 December 27 from SGR 1806–20. Here the peak energy was $\sim 500$ keV and the isotropic luminosity was $4 \times 10^{47}$ ergs s$^{-1}$ (Hurley et al. 2005; Palmer et al. 2005). The thermalization radius is therefore inferred to be $\sim 50$ km, about the Alfvén radius expected for that luminosity and for a dipolar magnetic field of $10^{15}$ G.

### 3. IMPLICATIONS FOR THE EMISSION MECHANISM

#### 3.1. Peak Width-Photon Frequency Correlation

The light curves of most gamma-ray bursts contain multiple pulses. A pulse is typically narrower when observed in a higher frequency wave band; one observes the scaling (Fenimore et al. 1995)

$$\delta t \propto E_{\gamma}^{-0.5}. \quad (51)$$

A pulse results from dissipation within some part of the outflow. Its duration is limited by particle cooling as well as by the
differential propagation time of the radiation across the emitting volume. We consider both of these effects in turn.

A characteristic pulse width resulting from dissipation at radius $r$ and Lorentz factor $\Gamma$ is $\tau \sim r/2\Gamma^2c$. Multiple pulses of this width would naturally result from dissipation well inside the radius at which the reverse shock wave passes through the ejecta shell (e.g., Sari & Piran 1997). This broadening effect is geometrical, however, and the resulting pulse width is to a first approximation independent of frequency.

3.1.1. Implications for Beaming

Our interpretation of the $E_{\text{pk}} - E_{\text{iso}}$ relation implies that photons above the spectral peak of a GRB are thermal photons upscattered in energy. The existence of a direct mapping between the spectral peak energy of a GRB and the temperature of the seed thermal photons then implies that photons near the spectral peak must be emitted by material moving in the direction of the observer. The same conclusion applies to higher energy photons. The frequency dependence of the pulse profile could also, in principle, depend on the orientation of the emitting material with respect to the line of sight (with softer photons being preferentially emitted off-axis). Our model suggests that such orientation effects are of secondary importance in explaining the correlation (eq. [51]).

This observation has important consequences for the emission mechanism. The scaling (eq. [51]) is suggestive of the cooling of relativistic particles; the characteristic frequency of the synchrotron or inverse Compton photons emitted by an electron of energy $\gamma_e m_e c^2$ is proportional to $\gamma^2_e$, and the cooling time is inversely proportional to $\gamma_e$. The pulse width is controlled by cooling (over some range of frequencies) only if the light travel time across the radiating plasma is shorter than the cooling time at each (observed) frequency. This implies that the size of the cooling region is $L/cn' \lesssim (\tau \gamma^2_e)^{-1}$, where $\tau \gtrsim 1$ is the compactness in the bulk frame.

One can consider dividing up each causally connected patch in the outflow (of size $c' \sim r/2\Gamma$) into cells of size $c/\tau \gamma^2_e$. A single causal patch then comprises $\gamma^2_e$ such cells. If the emission is nearly isotropic in each cell, then the emission from any of them will be detectable. On the other hand, if the emission is beamed into a solid angle $\sim 1/\gamma^2_e$, then only $1/\gamma_e$ cells are visible to any observer. In this case, the pulse duty cycle is given simply by the fraction of cells that experience strong dissipation.

Strong beaming of this type is not expected if the cooling particles are accelerated at a shock. Two mechanisms for beaming have been suggested in the case where the outflow is magnetically dominated: bulk relativistic motion of the magnetofluid (due to, e.g., current-driven instabilities; Lyutikov & Blandford 2003) and heating of the motion of the light charges parallel to the background magnetic field, due to Landau damping of high-frequency Alfvénic turbulence (Thompson & Blaes 1998; Thompson 2006).

An additional form of beaming is expected if the outflow carries a slower and denser component. The fragmentation of the heavy component occurs over a range of angular scales as it is accelerated outward by the momentum flux of the lighter relativistic fluid. At a distance of $\sim 10^{14}$ cm from the engine, the differential Lorentz factor between the two components has been reduced to $\Delta \Gamma \sim 2$ (Thompson 2006). A fraction of the seed thermal radiation will side-scatter off the heavy component (which remains optically thick out to $\sim 10^{14}$ cm from the engine) and can provide an enhanced coolant for relativistic particles in the relativistic fluid. (See Sikora et al. [1994] for related considerations in the context of blazar jets.) The inverse Compton radiation so produced will be preferentially beamed with respect to the side-scattered thermal radiation by a modest Doppler factor $\sim \Delta \Gamma$.

3.1.2. Inverse Compton Cooling and Other Cooling Mechanisms

A simple relation between cooling time and photon frequency is expected if each high-energy photon results from a single upscattering. The cooling time in the bulk frame of the outflow is $t_{\text{cool}}^{\text{IC}} = 3m_e c/4\gamma_e \sigma_T U'_1$, and the observed cooling time $t_{\text{cool}}$ is shorter by a factor $1/2\Gamma$. Setting

$$E_\gamma = \gamma_e^2 E_{\text{pk}},$$

and making use of the Amati et al. (2002) relation (eq. [1]) gives

$$t_{\text{cool}}^{\text{IC}} = 0.8 \frac{E_{\text{iso}}^{3/4} T_1^{3/4} f_{15}}{E_{\text{iso}}^{52}} \left( \frac{E_\gamma}{100 \text{ keV}} \right)^{-1/2} \text{s},$$

where $E_{\text{iso}}$ is the isotropic energy of the prompt emission. Pulses of a width of $\sim 10^{-2}$ s must be emitted within $\sim 10^{15}$ cm from the engine, if the outflow Lorentz factor is close to the limiting value of $\sim 10^2$ (eq. [29]). Requiring that the cooling time (eq. [53]) be shorter than $r/2\Gamma^2c$ implies

$$r < 2 \times 10^{15} \frac{E_{\text{iso}}^{3/4} T_1^{3/4} f_{15}}{E_{\text{iso}}^{52}} \left( \frac{E_\gamma}{100 \text{ keV}} \right)^{1/2} \text{cm}.$$  

Pair creation in the GRB outflow just outside the baryonic photosphere has a strong influence on the pulse duration. The prefaCTOR $\gamma^2_e$ could be as small as $\sim 10^{-2}$ if the outflow became heavily pair loaded while being accelerated by the anisotropic pressure of the fireball photons (eq. [29]). Substituting the upper bound (eq. [54]) on $r$ back into equation (53) for $t_{\text{cool}}^{\text{IC}}$, one deduces

$$t_{\text{cool}}^{\text{IC}} < 0.01 \frac{E_{\text{iso}}^{3/4} T_1^{3/4} f_{15}}{E_{\text{iso}}^{52}} \left( \frac{E_\gamma}{100 \text{ keV}} \right)^{1/2} \text{s}.$$  

Broader pulses can, of course, result from dissipation in the outflow outside the radius (eq. [54]), but their duration is limited by causality and not by IC cooling.

A more energetic particle is required to emit a synchrotron photon of a given energy, which means that the synchrotron cooling time is correspondingly shorter. The equivalent result for synchrotron emission may be obtained by setting $B'^2/8\pi = (E_{\text{iso}}/E_{\gamma}) U'_1$ and $E_{\gamma} = 0.3\gamma^2_e h \epsilon B/m_e c$. One finds

$$\frac{t_{\text{cool}}^{\text{synch}}}{t_{\text{cool}}^{\text{IC}}} = 2 \times 10^{-3} \frac{E_{\text{iso}}^{7/8} T_1^{1/8} f_{15}^{1/4}}{E_{\text{iso}}^{52} T_1^{3/4} I_{15}^{1/4} \gamma^2_e}.$$  

Pulses as broad as $\sim 1$ s, with the frequency-dependent width observed in GRB light curves, cannot easily be explained by synchrotron emission.
A frequency-dependent pulse width would also result from the damping of bulk relativistic motion by Compton drag, but in that case the scaling with photon frequency would be different. Suppose that a small blob of speed $\Delta \beta_0$ [Lorentz factor $\Delta \Gamma_0 = (1 - \Delta \beta_0^2)^{-1/2}$] is created in the outflow (e.g., by relativistic reconnection). As it slows down to a speed $\Delta \beta < \Delta \beta_0$, the size of the blob increases to $\sim (1 - \Delta \beta)/c t_{\text{drag}}$, where $t_{\text{drag}}$ is the drag time at Lorentz factor $\Delta \Gamma$. The duration of the emission is therefore $\delta t \propto t'/\Delta \Gamma^3$. Since the energy of the inverse Compton (IC) photons is $E_{\gamma} \propto \Delta \Gamma^{-2}$, one finds the scaling $\delta t \propto E_{\gamma}^{-3/2}$, much stronger than is observed. The frequency dependence of the pulse width at high and low frequencies deserves special scrutiny, for two reasons:

1. The size of the emitting cells in the outflow is more or less independent of the maximum Lorentz factor $\gamma_{\text{eq}}$ of the heated particles. This means that observations of gamma-ray pulses at energies well above $\sim 1$ MeV may provide a direct diagnostic of the cell size; above some characteristic value of $E_{\gamma}$, the observed pulse width should saturate at some minimum value $\sim L/c \Gamma$.

2. Photons detected below the spectral peak have not been upscattered by particles with relativistic energies in the bulk frame. The pulse width should, as a result, have a weaker frequency dependence below the spectral peak.

We can summarize our conclusions as follows. First, the frequency-dependence of the pulse width arises most naturally from IC cooling of random particle motion. Since the cooling timescale is much shorter than the flow time in the emitting frame, the dissipation producing each pulse must be localized at some radius. This implies that the motion of the heated particles is strongly anisotropic; otherwise, the emission on shorter timescales (at higher frequencies) would be smeared out by the differential propagation delay associated with shell curvature. Some additional beaming of the gamma-ray-emitting material will result from the fragmentation of the breakouts (as described at the beginning of this section) and would produce some broadening of individual pulses at low frequencies.

### 3.2. Implications for the Mechanism of Particle Heating

The temperature of the seed thermal photons is independent of radius (in the absence of adiabatic cooling). The peak energy resulting from IC scattering of the thermal photons therefore depends most directly on the mechanism by which the particles are heated: it is sensitive to the distribution of particle energies. When the heated particles have a power-law energy distribution that is cut off from below at $\gamma_{\min} m_e c^2$, it is clear that $E_{\text{pk}}$ will remain close to the thermal peak energy only if $\gamma_{\min} \sim 1$.

A value of $\gamma_{\min}$ close to unity is easily achieved if the particles are heated continuously, and the outflow is at least moderately optically thick to scattering. Stochastic acceleration by turbulence (Dermer et al. 1996; Thompson 2006) or by electrostatic acceleration in reconnection layers (Romanova & Lovelace 1992; Larrabee et al. 2003) are possible heating mechanisms. This implies that the high-energy tail of the prompt GRB spectrum is generated near the scattering photosphere.

The synchrotron self-Compton process (Ghisellini & Celotti 1999, Stern & Poutanen 2004) requires $\gamma_{\min} \gg 1$. If the particle heating is truly continuous, then this condition can only be satisfied outside the scattering photosphere. In that case, $\gamma_{\min}^2 \sim \Gamma_1^2 \propto \eta L_{\text{iso}}$. The self-absorption frequency is given by $\nu_{\text{sa}} = \Gamma_{\text{iso}} \gamma_{\min} \nu_{\text{iso}}$ (10). The peak of the Comptonized synchrotron spectrum is at

$$E_{\text{pk}} \sim \gamma_{\min}^2 \nu_{\text{sa}} \propto \eta \Gamma_{1/6} L_{\text{iso}}^{9/14} E_{\gamma}^{2/7}. \quad (57)$$

The explicit dependence on radius nearly scales out here, but the exponent of $L_{\text{iso}}$ has the wrong sign. (In this model, it is not clear how $\Gamma$ and $L_{\text{iso}}$ are related, since the seed thermal radiation field must be assumed to be absent.)

The continuous heating of seed thermal photons by a second-order Compton process has also been considered (Thompson 1994; Giannios 2006), but this mechanism does not provide an obvious explanation for the correlation (eq. [51]) between pulse width and photon frequency, or for the lags of soft photons with respect to hard photons that are observed in some GRBs (especially those with broad pulses, e.g., Norris et al. 2005).

#### 3.2.1. Shock Acceleration and $\gamma_{\min} \sim 1$

Shock acceleration is a rapid process; particles reach an energy $\gamma_{\min} m_e c^2$ on a timescale comparable to the gyroperiod $\gamma_{\min} m_e c$ of the particle. Only modest Lorentz factors ($\gamma_{\min} \sim 1$–10) are needed to create an IC spectrum extending up to $\sim 10^2 E_{\gamma}$. At such low energies, the particle distribution is modified by cooling only at a large distance downstream of the shock.

Under what conditions can shock acceleration generate a particle distribution with $\gamma_{\min} \sim 1$, with a significant fraction of the outflow energy deposited in the nonthermal particles? Three basic requirements must be satisfied: first, a large fraction of the particle inertia in the outflow must be in light charges (electrons and positrons); second, a significant fraction of the light charges which encounter a shock must undergo Fermi acceleration; and, third, the momentum distribution of the accelerated particles must have an index close to $-2$ (or harder). Although the behavior of a pair-dominated, relativistic shock is not fully understood (see Hoshino et al. 1992 for particle-in-cell simulations of electron-positron ion shocks), these considerations are largely independent of such details.

Pair creation in the outflow occurs primarily through collisions between photons (Cavallo & Rees 1978; Baring & Harding 1997; Lithwick & Sari 2001). These pair-creating photons have a characteristic energy $\gamma_{\min} m_e c^2$ in the bulk frame and are a byproduct of the cooling of charges of an energy $\gamma_{\min} \sim (\Gamma m_e c^2/E_{\gamma})^{1/2}$. Cooling is faster than pair creation, and so the equilibrium pair density can be determined by balancing the rates of pair creation and annihilation. Essentially the entire energy of the injected nonthermal particles is converted to a nonthermal photon spectrum by rapid IC cooling.

To simplify matters, we assume that this process occurs over some range of radius in the outflow, so that the pair density is already close to its equilibrium value. For a high-energy photon index $\beta \geq 2$, the positron creation rate is $n_{p+}' \approx (0.1 \sigma_T n_{\gamma}'(m_e c^2)^2$ (Svensson 1987), where $n_{\gamma}'(m_e c^2)$ is the photon density at the threshold energy $E_{\gamma}' = m_e c^2$. Balancing this with the annihilation rate $n_{p+}' = -(3/8) \sigma_T c n_{p+}'(m_e c^2)^2$ in a warm pair plasma gives

$$\frac{(n_{p+}' + n_{\gamma}'(n_{p+}c^2)}{U_{\gamma}} \zeta \nu_{\text{sa}} \approx \varepsilon_{\text{nth}} (\beta - 2) \left( \frac{E_{\gamma}'}{m_e c^2} \right)^{\beta - 2}$$

for rest energy density in the created pairs. The total photon energy density $U_{\gamma}'$ can be divided into a nonthermal tail carrying a fraction $\varepsilon_{\text{nth}}$ of the total, and a thermal peak that has been boosted in energy by the cooling of the remaining thermal pairs. Now $\gamma_{\min}$ is minimized if a significant fraction of the pairs are converted to a nonthermal distribution at the shock. The net energy...
density deposited in nonthermal pairs with particle index $p$ is then

$$\gamma_{\text{min}} \approx (p-2) m_e c^2 (n_e^*+n_v^*) \simeq \varepsilon_{\text{mb}} U_e^*.$$ (59)

Combining this expression with equation (58) and the relation $p-2=2(\beta-2)$ for rapidly cooling particles gives

$$\gamma_{\text{min}} \approx 2 \left( \frac{E_{pk}}{\Gamma m_e c^2} \right)^{-(\beta-2)}.$$ (60)

For example, if $\beta=2.2$, $E_{pk}=100$ keV, and $\Gamma=10^2$ then $\gamma_{\text{min}} \approx 7$. Even a modest departure of the photon spectrum from a $-2$ index implies that $\gamma_{\text{min}}$ is large enough to give a considerable IC boost to the energy of the seed thermal photons.

Softer high-energy spectra are observed in many GRBs (e.g., Preece et al. 2000). These spectra could still be consistent with a hard IC spectrum in the emission zone, if the high-energy photon flux were degraded by pair creation at a larger radius (e.g., in a dense Wolf-Rayet wind; Thompson & Madau 2000; Mészáros et al. 2001; Beloborodov 2002).

4. CONCLUSIONS

Most models of GRB outflows assume that they contain a single dominant component, e.g., baryons with a variable Lorentz factor (in the case of internal shock models) or a nonradial magnetic field with a reversing sign (in the case of reconnection models). Our central argument in this paper is that a second component is essential for understanding the prompt emission of GRBs: blackbody radiation that is emitted where the GRB jet forces its way through the core of a Wolf-Rayet star. Its thermal peak (Doppler boosted by the outflow) is identified with $E_{pk}$. The non thermal high-energy part of the GRB emission arises from Comptonization of this radiation by relativistic electrons outside the effective photosphere.

This model accounts naturally for the small scatter in the Amati et al. (2002) and Firmani et al. (2006) relations. It should be reiterated that both these relations can be tied directly to the jet properties in the thermalization zone and are reproduced without free parameters by a very simple model. (Indeed, the relative dependence of $E_{pk}$ on the jet energy and duration that was measured by Firmani et al. 2006 was anticipated in the analysis of this model in Thompson 2006.) If we assume that the radius of prompt thermalization is fixed at the progenitor core radius (or some other characteristic radius), then it is possible to reproduce the Ghirlanda et al. (2004) relation for a particular dependence of jet energy on opening angle, but not one that is motivated by a simple argument. A key, unresolved question centers on how the relativistic jet material builds up in the core of a Wolf-Rayet star; how wide is the outflow from the central engine, and how tightly regulated is the total energy that is deposited in the relativistic fluid with respect to the core binding energy?

This quasi-thermal model for the observed spectral correlations has interesting implications for the limiting jet Lorentz factor, and the relation of the jet energy, opening angle and burst duration to the mass and radius of the stellar stellar progenitor. The observed relation between pulse width and photon frequency can be explained by Compton cooling, but one requires that the relativistic particles in the outflow have an energy distribution with a low-energy (transrelativistic) cutoff, and that the IC emission is beamed. The relation between $E_{pk}$ and $E_{\text{iso}}$ becomes more complicated for bursts with isotropic energies less than $\sim 10^{52}$ ergs; the spectrum can be harder than is implied by the Amati et al. (2002) relation if the jet is clean but covers a large solid angle, or if the ejecta mass and energy are low.

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