To mathematical modeling of deformation of micropolar thin bodies with two small sizes

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Abstract. We consider some problems of modeling the deformation of micropolar thin bodies with two small sizes. Using the three-dimensional equations of motion, the constitutive relations and the boundary conditions of the micropolar elasticity theory \cite{1, 2}, we got the equations of motion, the constitutive relations and the boundary conditions of the micropolar theory of thin bodies under the parametrization of the thin body domain with two small sizes \cite{3}. The boundary conditions and various representations of the system of equations of motion and the constitutive relations of physical content in moments with respect to the Legendre polynomials are obtained. Note that using this method of construction the thin bodies theory with two small sizes, we get an infinite system of ordinary differential equations. This system contains quantities which depends on one variable, namely, depends on the parameter of the base line. Thus, decreasing the number of independent variables from three to one we increase the number of equations to infinity, which, of course, has its obvious practical inconveniences. In this regard, we reduce an infinite system to the finite system. The initial-boundary value problems are formulated. To satisfy the boundary conditions on the front surfaces we constructed correcting terms \cite{4}. As a special case, we considered a prismatic body. We used the tensor calculus to do this research \cite{5–8}.

1. To parametrization of a thin body domain with two small sizes

Let the thin body have a cross-section in the form of a rectangle, which sides are much smaller than its length (third dimension). Let the considered domain does not have the line of symmetry. The position vector of an arbitrary point of the thin body domain is represented as \cite{3}

\[
\hat{r}(x', x^3) = r(x^3) + \sum_{K=1}^{2} \left[ \bar{h}^K(x^3) + x^K h^K(x^3) \right] e^K(x^3) = r(x^3) + \sum_{K=1}^{2} h^{-1}_K \left[ \bar{h}^K(x^3) + x^K h^K(x^3) \right] r^K(x^3), \quad -1 \leq x^I \leq 1, \tag{1}
\]

where \( r = r(x^3) \) is the vector parametric equation of the baseline, \( x^3 \equiv s \) is a natural parameter, \( \bar{h}^I = (h^+_I - h^-_I)/2, \) \( h_I = (h^+_I + h^-_I)/2, \) \( r_I = h_I e_I, \) \( < I = 1, 2 >, \) \( r_3 = \partial_3 r \) is the unit tangent vector to the baseline, \( e_1 \) and \( e_2 \) are the unit vectors of the principal normal and binormal to
the baseline, respectively. So, \( \mathbf{e}_1, \mathbf{e}_2, \mathbf{r}_3 \) is the natural trihedral. Obviously, the ratio (1) for
\(-1 \leq x^l \leq 1 \) and \( x^3 \in [0, l) \), where \( l \leq \infty \), is the vector parametric equation of a thin body with
two small sizes. Moreover, if \( l < \infty \), then we have a thin body of finite length, and if \( l = \infty \),
then we have a thin body of infinite length.

Since the number of pages is limited, we will not dwell on the issues of parameterization of
the thin body domain. All calculations can be found in [3]. We also will not discuss problems of
the theory of moments of \((m,n)\)-th order. We only give a definition of this moment with respect
to the system of Legendre polynomials. The calculation of the moments of some functions under
this parametrization can be found in [3].

Let \( \{ u_k \}_{k=0}^{\infty} \) be some orthogonal system of the Legendre polynomials on the segment \([-1, 1]\),
and \( F(x^1, x^2, x^3) \) is a tensor field.

**Definition** Moment \( \bar{M}(F) \) of \((m,n)\) order of the quantity \( F(x', x^3) \) with respect to
the system of polynomials \( \{ u_k \}_{k=0}^{\infty} \) is the integral

\[
\frac{(m,n)}{\bar{M}(F)} = \frac{2m + 2n + 1}{2} \int_{-1}^{1} \int_{-1}^{1} F(x^1, x^2, x^3) P_m(x^1) P_n(x^2) dx^1 dx^2. \tag{2}
\]

Thereafter, we will call \( M(F) \) the operator of moments of \((m,n)\)-th order with respect to
the Legendre polynomials. In this case, the first index \( m \) refers to the variable \( x^1 \), and
the second index \( n \) refers to the variable \( x^2 \). Taking into account the orthogonality of the system
of Legendre polynomials and the definition of (2), it is easy to see that we will have

\[
F(x^1, x^2, x^3) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(m,n)}{M(F)} P_m(x^1) P_n(x^2) = \sum_{m=0}^{\infty} \frac{(m)}{M(F)} P_m(x^1) = \sum_{n=0}^{\infty} \frac{(n)}{M(F)} P_n(x^2). \tag{3}
\]

2. Various representations of the system of equations of motion in moments
with respect to the system of Legendre polynomials

Taking into account representations of gradient and divergence under this parametrization
(see [3]) we can write the equations of motion of the moment theory in the following forms:

\[
g^3_{33} N_3 P^3 + \partial_p P^p + \rho F = \rho \partial^2 \varphi, \quad \partial^3_{33} N_3 \mu^3 + \partial_p \mu^p + C \hat{\otimes} P + \rho m = J \cdot \partial^2 \varphi, \tag{4}
\]

\[
\nabla^3_3 P^3 + \nabla_p P^p + \rho F = \rho \partial^2 \varphi, \quad \partial \mu^3 + \nabla_p \mu^p + C \hat{\otimes} P + \rho m = J \cdot \partial^2 \varphi, \tag{5}
\]

where \( C \) is the 3d rank discriminant tensor, \( P^p \) and \( \mu^p \) are the components of stress tensor
and couple stress tensor, respectively, \( u \) is the displacement vector, \( \varphi \) is the rotation vector, \( \rho \) is the
density, \( J \) is the 2d rank inertia tensor, \( \hat{\otimes} \) is the inner 2-product [3, 5, 7, 9].

Applying the moment operator of \((m,n)\)th-order to (5), we get

\[
\nabla^{(m,n)}_3 M(P^3) + \nabla^{(m)}(P^1) + \nabla^{(m,n)} M(P^2) + \rho \frac{(m,n)}{M(F)} = \rho \frac{(m,n)}{M(F)} \partial^2 \varphi, \tag{6}
\]

\[-(\delta^1 + \delta^2) \leq x^l \leq \delta^1 + \delta^2, \quad m, n \in \mathbb{N}_0.\]
3. On the boundary conditions under the parametrization with an arbitrary base line of a thin body domain with two small sizes

Let \( S_1 \) (\( S_2 \)) and \( S_2 \) (\( S_2 \)) be the front surfaces of a thin body with two small sizes, which are determined by \( (1) \) when \( x^3 = x^3 = 1 \) and by arbitrary \( x^2 \) and \( x^3 \), and \( x^2 = -1 \) \( x^2 = 1 \) and arbitrary \( x^4 \) and \( x^3 \), respectively. Let \( S_1 \) (\( S_2 \)) be the left (right) element of a covering determined as \( (1) \) when \( x^3 = x^3 = 1 \) (\( x^3 = x^3 = 1 \)) and \( -1 \leq x^l \leq 1 \). \( \mathbf{P} \) \((1) \)(\( x^2 \), \( x^3 \), \( t \)) and \( \mathbf{\mu} \) \((1) \)(\( x^2 \), \( x^3 \), \( t \)) \( \mathbf{P} \) \((2) \)(\( x^2 \), \( x^3 \), \( t \)) and \( \mathbf{\mu} \) \((2) \)(\( x^2 \), \( x^3 \), \( t \)) are known vectors of stress and couple stress on the surfaces \( \hat{S}_1 \) (\( \hat{S}_2 \)) \( \hat{S}_2 \) (\( \hat{S}_2 \)). Besides, let us introduce \( \mathbf{P} \) \((1) \)(\( x^1 \), \( x^2 \), \( t \)) and \( \mathbf{\mu} \) \((1) \)(\( x^1 \), \( x^2 \), \( t \)) \( \mathbf{P} \) \((2) \)(\( x^1 \), \( x^2 \), \( t \)) and \( \mathbf{\mu} \) \((2) \)(\( x^1 \), \( x^2 \), \( t \)) as known vectors of stress and couple stress on the end-faces \( \hat{S}_1 \) (\( \hat{S}_2 \)).

The boundary conditions of physical content on the front surfaces of the thin body can be represented as

\[
\begin{align*}
\mathbf{P}^{(1)}_1 + h_1^{-1} (\partial_3 h_1 - k_2^2 h_2) g_3^{-1} \mathbf{P}_3^{(1)} - h_1^{-1} h_2 k_2 (x^2 g_3^{-1} \mathbf{P}_3^{(1)}) &= -\mathbf{P}^{(1)}_1 (x^2, x^3, t), \\
\mathbf{P}^{(2)}_1 - h_1^{-1} (\partial_3 h_1 + k_2^2 h_2) g_3^{-1} \mathbf{P}_3^{(2)} - h_1^{-1} h_2 k_2 (x^2 g_3^{-1} \mathbf{P}_3^{(2)}) &= \mathbf{P}^{(2)}_1 (x^2, x^3, t), \\
\mathbf{P}^{(2)}_2 + h_2^{-1} (\partial_3 h_2 + k_2^2 h_1) g_3^{-1} \mathbf{P}_3^{(2)} + h_2^{-1} h_1 k_2 (x^1 g_3^{-1} \mathbf{P}_3^{(2)}) &= -\mathbf{P}^{(2)}_2 (x^1, x^3, t), \\
\mathbf{P}^{(2)}_2 - h_2^{-1} (\partial_3 h_2 - k_2^2 h_1) g_3^{-1} \mathbf{P}_3^{(2)} + h_2^{-1} h_1 k_2 (x^1 g_3^{-1} \mathbf{P}_3^{(2)}) &= \mathbf{P}^{(2)}_2 (x^1, x^3, t), \quad \mathbf{P} \to \mathbf{\mu}.
\end{align*}
\]

Here we introduced notations

\[
\mathbf{P}^{(\alpha)}_\alpha = \sqrt{g^{\alpha \alpha}} \mathbf{P}^{(\alpha)}_\alpha, \quad \mathbf{P}^{(\alpha)}_\alpha = \sqrt{g^{\alpha \alpha}} \mathbf{P}^{(\alpha)}_\alpha, \quad \mathbf{P} \to \mathbf{\mu}.
\]

We can see that based on \((7)\) the boundary conditions of physical content on the front surfaces in moments will be written in the form

\[
\begin{align*}
\mathbf{M} \mathbf{P}^{(1)} + h_1^{-1} (\partial_3 h_1 - k_2^2 h_2) \mathbf{M} \mathbf{P}_3^{(1)} - h_1^{-1} h_2 k_2 \mathbf{M} (x^2 \mathbf{P}_3^{(1)}) &= -\mathbf{M} \mathbf{P}^{(1)}_1, \\
\mathbf{M} \mathbf{P}^{(1)}_1 - h_1^{-1} (\partial_3 h_1 + k_2^2 h_2) \mathbf{M} \mathbf{P}_3^{(2)} - h_1^{-1} h_2 k_2 \mathbf{M} (x^2 \mathbf{P}_3^{(2)}) &= \mathbf{M} \mathbf{P}^{(2)}_1, \\
\mathbf{M} \mathbf{P}^{(2)}_2 + h_2^{-1} (\partial_3 h_2 + k_2^2 h_1) \mathbf{M} \mathbf{P}_3^{(2)} + h_2^{-1} h_1 k_2 \mathbf{M} (x^1 \mathbf{P}_3^{(2)}) &= -\mathbf{M} \mathbf{P}^{(2)}_2, \\
\mathbf{M} \mathbf{P}^{(2)}_2 - h_2^{-1} (\partial_3 h_2 - k_2^2 h_1) \mathbf{M} \mathbf{P}_3^{(2)} + h_2^{-1} h_1 k_2 \mathbf{M} (x^1 \mathbf{P}_3^{(2)}) &= \mathbf{M} \mathbf{P}^{(2)}_2, \quad \mathbf{P} \to \mathbf{\mu}.
\end{align*}
\]

Applying the moment operator of \((m, n)\)th-order to boundary conditions, we will obtain the required boundary conditions of the physical content in moments on the end-faces of a thin body in the form

\[
\mathbf{P}^{(1)}_3 = -\mathbf{M} \mathbf{\hat{P}}^{(1)}_1 \text{ when } x^3 = x_1, \quad \mathbf{P}^{(2)}_3 = -\mathbf{M} \mathbf{\hat{P}}^{(2)}_2 \text{ when } x^3 = x_2, \quad \mathbf{P} \to \mathbf{\mu}.
\]

Note that we can also present \((10)\) in the form

\[
\mathbf{M} \mathbf{P}^{(1)}_3 = -\mathbf{M} \mathbf{P}^{(1)}_1 \text{ when } x^3 = x_1, \quad \mathbf{M} \mathbf{P}^{(2)}_3 = \mathbf{P}^{(2)}_2 \text{ when } x^3 = x_2, \quad \mathbf{P} \to \mathbf{\mu}.
\]
The boundary conditions of the thermal content can be considered in the same way as (9)–(11).

Now it is not difficult to write down some of the required representations of the system of motion equations in moments with respect to the system of Legendre polynomials. Indeed, from (6) we get

\[ \{ \nabla M_{(m,n)} (P^\beta) \} = \frac{2m+1}{2} \sum_{p=m}^{\infty} \left[ 1 - (-1)^{m+p} \right] M_{(p,n)} (P^1) + h_{1}^{-1} \partial_3 h_{1} (m + 1) M_{(m,n)} (P^3) - \]

\[ - \frac{2m+1}{2} \sum_{p=m}^{\infty} h_{1}^{-1} \left( (\partial_3 h_{1} + k_{2} \tilde{h}_{2}) + (-1)^{m+p} (\partial_3 h_{1} - k_{2} \tilde{h}_{2}) \right) M_{(p,n)} (P^3) - \]

\[ - \frac{2m+1}{2} h_{1}^{-1} k_{2} \sum_{p=m}^{\infty} \left[ 1 - (-1)^{m+p} \right] \left[ \frac{n}{2m-1} M_{(p,n)} (P^3) + \frac{n+1}{2m+3} M_{(p,n+1)} (P^3) \right] + \]

\[ + \frac{2n+1}{2} \sum_{p=n}^{\infty} \left[ 1 - (-1)^{n+p} \right] M_{(m,p)} (P^2) + h_{2}^{-1} \partial_3 h_{2} (n + 1) M_{(m,n)} (P^3) - \]

\[ - \frac{2n+1}{2} \sum_{p=n}^{\infty} h_{2}^{-1} \left( (\partial_3 h_{2} - k_{2} \tilde{h}_{1}) + (-1)^{n+p} (\partial_3 h_{2} + k_{2} \tilde{h}_{1}) \right) M_{(m,p)} (P^3) + \]

\[ + \frac{2n+1}{2} h_{2}^{-1} k_{2} \sum_{p=n}^{\infty} \left[ 1 - (-1)^{n+p} \right] \left[ \frac{m}{2m-1} M_{(m,p)} (P^3) + \frac{m+1}{2m+3} M_{(m+1,p)} (P^3) \right] \]

\[ + \rho M (F) = \rho M (\partial^2_t u), \]

\[ \{ P \to \mu \} + C \otimes M (P) + M = J \cdot M (\partial^2_t \varphi), \quad m, n \in \mathbb{N}_0, \]

Further, from (6) we will have the following required representation of the system of motion equations in moments:

\[ \left\{ \nabla M_{(m,n)} (P^\beta) \right\} - \frac{2m+1}{2} \sum_{p=0}^{m} \left[ 1 - (-1)^{m+p} \right] M_{(p,n)} (P^1) - h_{1}^{-1} \partial_3 h_{1} m M_{(m,n)} (P^3) + \]

\[ + \frac{2m+1}{2} \sum_{p=0}^{m} h_{1}^{-1} \left( (\partial_3 h_{1} + k_{2} \tilde{h}_{2}) + (-1)^{m+p} (\partial_3 h_{1} - k_{2} \tilde{h}_{2}) \right) M_{(p,n)} (P^3) + \]

\[ + \frac{2m+1}{2} h_{1}^{-1} k_{2} \sum_{p=0}^{m} \left[ 1 - (-1)^{m+p} \right] \left[ \frac{n}{2m-1} M_{(p,n)} (P^3) + \frac{n+1}{2m+3} M_{(p,n+1)} (P^3) \right] - \]

\[ - \frac{2n+1}{2} \sum_{p=0}^{n} \left[ 1 - (-1)^{n+p} \right] M_{(m,p)} (P^2) - h_{2}^{-1} \partial_3 h_{2} n M_{(m,n)} (P^3) + \]

\[ + \frac{2n+1}{2} \sum_{p=0}^{n} h_{2}^{-1} \left( (\partial_3 h_{2} - k_{2} \tilde{h}_{1}) + (-1)^{n+p} (\partial_3 h_{2} + k_{2} \tilde{h}_{1}) \right) M_{(m,p)} (P^3) - \]

\[ - \frac{2n+1}{2} h_{2}^{-1} k_{2} \sum_{p=0}^{n} \left[ 1 - (-1)^{n+p} \right] \left[ \frac{m}{2m-1} M_{(m,p)} (P^3) + \frac{m+1}{2m+3} M_{(m+1,p)} (P^3) \right] \]

\[ + \Phi = \rho M (\partial^2_t u), \]

\[ \{ P \to \mu \} + C \otimes M (P) + M = J \cdot M (\partial^2_t \varphi), \quad m, n \in \mathbb{N}_0, \]

where we introduced notations

\[ \Phi (x^3,t) = \left\{ \frac{2m+1}{2} \left[ M_{(m)} (P^1) + (-1)^{m} M_{(m)} (P^1) \right] + \right\] 

\[ + \frac{2n+1}{2} \left[ M_{(n)} (P^1) + (-1)^{n} M_{(n)} (P^1) \right] \]

\[ + \rho M (F), \]

\[ M (x^3,t) = \{ P \to \mu \} + \rho M (m). \]
The relations (12) – (13) are different representations of the system of motion equations in moments with respect to the system of Legendre polynomials of the thin body with two small sizes. Using (4) we can get other representations of the system of equations in the moments [3,10]. But we will not dwell on this.

Note that each equation (without taking into account the boundary conditions) (12), with fixed values of m and n, contains an infinite number of terms. And each equation (using the boundary conditions) (13) contains the finite number of terms. Moreover, each equation contains moments of the contravariant component $P^3 = g_3^3 P^3 \ (\mu^3 = g_3^3 \mu^3)$ of the stress tensor (couple stress tensor). Introducing $g_3^3 = \sum_{s=0}^{r} [k_1 (h_1 + x^1 h_1)]^s$ and replacing $P^3$ and $\mu^3$ with $P^3_{(r)}$ and $\mu^3_{(r)}$ into (12) and (13) respectively, where

$$P^3_{(r)} = g_3^3 P^3, \quad \mu^3_{(r)} = g_3^3 \mu^3,$$

we will obtain the representations of the system of motion equations in moments of $r$th approximation. In addition, let us fix some non-negative integers $M$ and $N$. Then, choose the first $(M+1)(N+1)$ equations ($m = 0, M, n = 0, N$) from the system of equations. In equations containing an infinite number of terms, we neglect the moments of unknown quantities whose order is greater than $M$ (by the first index) and $N$ (by the second index). Thus, we obtain representations of the system of motion equations in moments of $(M,N)$th approximation. If the representations of the system of motion equations in moments are obtained from the corresponding representations of the $r$th approximation, then such representations are called representations of the system of motion equations in moments of $(r, M,N)$th approximation. Above mentioned applies to the representations of the heat inflow equation and the constitutive relations.

4. Presentations of constitutive relations of micropolar theory of thermoelasticity in moments for a thin body with two small sizes

The constitutive relations (CR) for non-isothermal processes [1,3,11,12] have the form

$$\mathbf{P} = \mathbf{A} \otimes \nabla \mathbf{u} + \mathbf{B} \otimes \nabla \varphi - \mathbf{A} \otimes C \cdot \varphi - b \varphi, \quad \mathbf{\mu} = C \otimes \nabla \mathbf{u} + D \otimes \nabla \varphi - C \otimes C \cdot \varphi - \beta \varphi,$$

where $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$ are the fourth-rank tensors, called tensors of elastic moduli, $b = \mathbf{A} \otimes a + \mathbf{B} \otimes d$, $\beta = \mathbf{C} \otimes a + \mathbf{D} \otimes d$ are the tensors of thermomechanical properties, $\mathbf{u}$ is the displacement vector, $\varphi$ is the internal rotation vector, $\vartheta = T - T_0$ is temperature drop, $\mathbf{a}$ and $\mathbf{d}$ are the tensors of thermal expansion. Particular cases of CR can be viewed in [2,13].

Therefore, the relations (15) can be written in the appropriate form and then presented in moments. From (15) it is clear that to represent them in moments it is sufficient to write down the gradient of a vector in moments. If the body is homogeneous with respect to $x^I, I = 1, 2$, then applying the $(m,n)$th-order moment operator to (15), we will have

$$\mathbf{P}^{(m,n)} = \mathbf{A}^{(m,n)} \otimes \mathbf{M} (\nabla \mathbf{u}) + \mathbf{B}^{(m,n)} \otimes \mathbf{M} (\nabla \varphi) - \mathbf{A}^{(m,n)} \otimes C^{(m,n)} \cdot \varphi - b^{(m,n)} \varphi, \quad -1 \leq x^I \leq 1,$$

$$\mathbf{\mu}^{(m,n)} = C^{(m,n)} \otimes \mathbf{M} (\nabla \mathbf{u}) + D^{(m,n)} \otimes \mathbf{M} (\nabla \varphi) - C^{(m,n)} \otimes C^{(m,n)} \cdot \varphi - \beta^{(m,n)} \varphi, \quad m, n \in \mathbb{N}_0.$$

Note that (16) are valid for any system of polynomials (Legendre, Chebyshev) on given segment.
Solving, for example, some boundary-value problem of \((0,M,N)\) approximation, we will obtain approximate expressions for the fields of displacement vector and stress tensor, respectively, in the following form [3]

\[
\mathbf{u}(M,N) = \sum_{m=0}^{M} \sum_{n=0}^{N} (m,n) P_m(x^1) P_n(x^2), \quad \mathbf{P}(0,M,N) = \sum_{m=0}^{M} \sum_{n=0}^{N} (m,n) P_m(x^1) P_n(x^2).
\] (17)

Consequently, the expressions (17) satisfy the boundary conditions on the end-faces. For example, if the displacement field is given on the end-faces, then the constructed approximate solution (the first relation (17)) is consistent with the kinematic boundary conditions \((m,n)\)

\[ u = \tilde{f}(1) \]

when \(x^3 = x_1^3\) and \((m,n)\)

\[ u = \tilde{f}(2) \]

when \(x^3 = x_2^3\), where \(\tilde{f}(I), I = 1, 2\), are moments of given vector fields on the end-faces \(S_1\) and \(S_2\), respectively.

Obviously, the issue arises how accurately the relations (17) satisfy the boundary conditions of physical content on the front surfaces. Generally, these relations will not be fulfilled with the required accuracy. Thus, we add the term \(U_0(x^1, x^3, t)\) to the approximate expression for the displacement vector \(u_{(M,N)}(x^1, x^3, t)\) (corrective factor) for satisfying the boundary conditions of physical content on the front surfaces. These problems are considered in detail in [3].

Note that, similarly to the above, we can consider various representations of the heat influx equation, the Fourier thermal conductivity law, and the boundary conditions for the thermal content in moments. We also note that different methods for reducing the infinite system to finite in case of thin bodies with one small size can be viewed in [3,11], and for thin bodies with two small sizes can be seen in [3,10,14]. It is not difficult to obtain initial conditions for the kinematic and thermal contents in moments, and to formulate the statements of problems in moments [3,10]. In order to shorten the text we will not dwell on these issues.

We also note that applying the canonical representations of material tensors and tensor-block matrices, the above relations can be represented in terms of eigenvalues and eigenvectors (eigentensor columns) of material tensors (tensor-block matrices). The application of eigenvalue problems of tensors and tensor-block matrices in mechanics is an upcoming trend, which deserves the attention of researchers. Some issues related to this direction are presented in the papers [7,9,15–17]. Finally, we note that from the point of practice the next works [18–21] have merited attention. Note also, that the authors studied the dynamics and design of the power unit with hydraulic piston drive in last two papers.

**Conclusion.** Some problems of modeling the deformation of thin bodies with two small sizes are considered. The equations of motion and the constitutive relations, and also the boundary conditions are obtained. Definitions of the moment \((m,n)\)th order of a certain quantity are given with respect to the system of Legendre polynomials. The boundary conditions and the various representations of the system of equations of motion and the constitutive relations in moments are obtained.

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