High Scale Mixing Unification for Dirac Neutrinos

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Starting with high scale mixing unification hypothesis, we investigate the renormalization group evolution of mixing parameters and masses for Dirac type neutrinos. Following this hypothesis, the PMNS mixing angles and phase are taken to be identical to the CKM ones at a unifying high scale. Then, they are evolved to a low scale using renormalization-group equations. The renormalization group evolution “naturally” results in a non-zero and small value of leptonic mixing angle $\theta_{13}$. One of the important predictions of this work is that the mixing angle $\theta_{23}$ is non-maximal and lies only in the second octant. We also derive constraints on the allowed parameter range for the SUSY breaking and unification scales, for which this hypothesis works. The results are novel and can be tested by present and future experiments.

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One of the most important open questions in neutrino physics is whether neutrinos are Dirac or Majorana particles. There are dedicated ongoing experiments with the sole objective to determine the nature of neutrinos [3–4]. Answering this question is essential in finding the underlying theory of neutrino masses and mixing. From a theoretical perspective, Majorana neutrinos provide an elegant explanation for the observed smallness of neutrino masses through the celebrated seesaw mechanism [5–9]. However, even for the Dirac neutrinos, there exist a number of appealing models which can explain the smallness of neutrino masses. The smallness of the masses in these models are explained in various ways such as by using extra heavy degrees of freedom, from Kähler potential of supergravity, from GUT or compactification scales etc. [10–14]. For further details on alternatives to see-saw mechanism, see [15–17].

Furthermore, from the cosmological perspective there is no compelling reason to prefer Majorana neutrinos over Dirac neutrinos. For example, Dirac neutrinos can also provide a satisfactory explanation of the observed baryon asymmetry [18–19]. Therefore, Dirac neutrinos are as plausible as Majorana ones and only experiments can settle this issue. The various experiments [14] looking for neutrinoless double beta decay, have not seen any signal so far. Hence, in view of above considerations it is important to investigate various possible scenarios for Dirac neutrinos.

The neutrino oscillation parameters are guiding light for model building. Present experimental scenario is quite exciting due to recent measurement of the mixing angle $\theta_{13}$, which is now established to be non-zero [20–24]. As a result of this rather precise measurement, many neutrino models are facing stringent constraints [25]. Therefore, it is worthwhile to explore, in details, any scenario which “naturally” predicts non-zero value of $\theta_{13}$. One such promising scenario is the ‘High Scale Mixing Unification’ (HSMU) hypothesis, which was proposed and studied in the context of Majorana neutrinos [26–28] (see, [29] for a recent analysis). The central idea of this hypothesis is that the mixing parameters of the quark sector become identical to those of neutrino sector at some unification scale, which may be the Grand Unified Theory (GUT) scale. It has been shown that, if neutrinos have a normal hierarchy and quasi-degenerate mass spectrum, the HSMU hypothesis can explain the experimentally measured neutrino mixing parameters with minimal supersymmetric standard model (MSSM) as an extension of the standard model (SM) [26–28]. At this point, we would like to point out that the inverted hierarchy of neutrino masses is not compatible with HSMU hypothesis, as it does not lead to radiative magnification for the mixing angles [22–24].

From theoretical perspective, the HSMU hypothesis appears more natural for Dirac neutrinos. In this case, unlike the Majorana neutrinos, the CKM mixing parameters can be mapped in a one-to-one correspondence with the mixing angles and phase of the PMNS matrix at the unification scale. In this letter, it is our aim to investigate the HSMU hypothesis, assuming that the nature of neutrinos is Dirac type and assay our predictions on the face of present experimental neutrino oscillation data.

The working of HSMU hypothesis requires MSSM as an extension of SM and implemented in two steps. In the first step, we follow bottom-up approach, where the CKM mixing angles ($\theta_{12}^q, \theta_{13}^q, \theta_{23}^q$) and the Dirac phase ($\delta_{CP}^q$) of quark sector are evolved through SM renormalization-group (RG) equations from a low scale ($M_Z$, mass of the $Z$ boson) to the SUSY breaking scale [31]. From the SUSY breaking scale to the unification scale, evolution is governed by MSSM RG equations [31]. After obtaining CKM mixing parameters at the unification scale, following HSMU hypothesis, we equate them to the PMNS mixing parameters at same scale, i.e. $\theta_{12}^\nu = \theta_{12}^q, \theta_{13}^\nu = \theta_{13}^q, \theta_{23}^\nu = \theta_{23}^q$ and $\delta_{CP}^\nu = \delta_{CP}^q$. The neutrino masses ($m_\nu^i$, $i = 1, 2, 3$)
$m_{2}^{0}$, $m_{3}^{0}$), at the unification scale, are taken as free parameters. Here, the superscript “0” denotes the corresponding values at the unification scale.

In the second step, we follow top-down approach. We evolve the mixing parameters and masses of neutrinos from the unification scale to the low scale. The running of mixing parameters and masses from the unification scale to SUSY breaking scale is governed by MSSM RG equations. From the SUSY breaking scale to low scale, the evolution occurs through SM RG equations. Keeping the present experimental status of SUSY searches in mind, we have chosen the scale of SUSY breaking as $2 \times 10^{16}$ GeV, the typical scale for GUT theories. These values have been taken in most part of our work, unless otherwise specified.

In order to achieve large angle magnification, the neutrino masses at unification scale are chosen to be quasi-degenerate with normal hierarchy. As mentioned before, like Majorana case, in this case also, the inverted hierarchy for neutrino masses turns out to be incompatible with HSMU hypothesis. Furthermore, we choose the neutrino masses such that all the oscillation parameters, at low scale, fall in the experimental $3\sigma$ ranges obtained from the global analysis. In our work, the RG evolution of masses and mixing parameters of quarks and neutrinos has been computed at two-loop level using a MATHEMATICA based package REAP.

1. In plotting Figure 1, the values of quark mixing parameters $\theta_{ij}$ with respect to the RG scale $\mu$, from the unification scale ($2 \times 10^{16}$ GeV) to low scale ($M_Z$). The SUSY breaking scale and $\tan \beta$ are taken as 2 TeV and 55, respectively.

The RG evolution of quark and neutrino mixing angles from the unification scale ($2 \times 10^{16}$ GeV) to low scale ($M_Z$), for certain typical values, is shown in Figure 1. In plotting Figure 1, the values of quark mixing parameters at unification scale, obtained from bottom-up running, are: $\theta_{b\bar{u}}^{0} = 13.02^\circ$, $\theta_{b\bar{d}}^{0} = 0.17^\circ$, $\theta_{b\bar{c}}^{0} = 2.03^\circ$ and $\delta_{CP}^{0} = 68.95^\circ$. According to the HSMU hypothesis, the neutrino mixing parameters at unification scale are taken to be same as those of quark mixing parameters. We choose neutrino mass $m_{\nu}^{0} = 0.1925$ eV and mass square differences $\Delta m_{21}^{2} = 2.960 \times 10^{-4}$ eV$^2$, $\Delta m_{32}^{2} = 5.718 \times 10^{-3}$ eV$^2$ at the unification scale. Starting with these initial conditions, we obtain following values of oscillation parameters, after top-down running, at low scale: $\theta_{12} = 31.20^\circ$, $\theta_{13} = 7.22^\circ$, $\theta_{23} = 50.35^\circ$, $\delta_{CP} = 28.12^\circ$, $m_{2} = 0.1746$ eV, $\Delta m_{23}^{2} = 7.917 \times 10^{-3}$ eV$^2$ and $\Delta m_{3\mu}^{2} = 2.399 \times 10^{-3}$ eV$^2$. It is clear from above data that all the low scale parameters are within their $3\sigma$ range.

The sum of neutrino masses at low scale, corresponding to the above mentioned values, turns out to be $\sum m_{i} = 0.530$ eV, where $i = 1, 2, 3$. The recent cosmological upper limit, from Planck collaboration, on the sum of neutrino masses range from 0.23 eV to 1.08 eV, depending on values chosen for priors. The sum of masses obtained from our analysis satisfy these limits except for the lowest one. Finally, the “averaged electron neutrino mass” obtained from our analysis is $m_{\nu_e} = 0.175$ eV which is slightly below the present reach of KATRIN experiment. However, it may be of interest to future experiments. A more detailed analysis of the allowed ranges for various parameters will be presented in a future work.

It is shown in Figure 2 owing to the hierarchical nature of quark masses, the quark mixing angles change very little between the two scales. In the neutrino sector, the masses have normal hierarchy pattern and because of our choice of quasi-degenerate masses, large angle magnification occurs. The small and non-zero value of $\theta_{13}$, at low scale, can be attributed to the smallness of the quark mixing angle $\theta_{13}^{0}$ which is taken as the initial value for the neutrino mixing angle $\theta_{13}^{0}$.

![Figure 1: The RG evolution of the quark and lepton mixing angles $\theta_{ij}$ (i,j = 1, 2, 3) with respect to the RG scale $\mu$, from the unification scale ($2 \times 10^{16}$ GeV) to low scale ($M_Z$). The SUSY breaking scale and $\tan \beta$ are taken as 2 TeV and 55, respectively.](image)

![Figure 2: The behaviour of leptonic mixing angle $\theta_{13}$ with $\theta_{23}$. The shaded regions lie outside the $3\sigma$ range.](image)

As clear from Figure 2, the low scale values of mixing angles $\theta_{13}$ and $\theta_{23}$ are correlated. In order to highlight this correlation, the value of $\theta_{12}$ at low scale is held fixed at 31.20°, which is near the lower end of allowed $3\sigma$
range. The purpose of this choice is only to illustrate our results. The effect of the variation of the $\theta_{12}$ is very small and will not change our conclusions. It is noteworthy to observe that the range of $\theta_{13}$, for which all the mixing parameters lie within 3$\sigma$ experimental range, is $7.19^\circ$–$8.21^\circ$ and the corresponding range for $\theta_{23}$ is $50.25^\circ$–$54.80^\circ$. The behaviour of $\theta_{13}$, beyond its above mentioned range, continues to be ‘linear’ with respect to $\theta_{23}$. It is obvious from Figure 2 that $\theta_{23}$ is non-maximal and lies only in the second octant (i.e. $> 45^\circ$) for the 3$\sigma$ allowed range of $\theta_{13}$. This is a novel and robust feature of this work. The effect of all other parameters, including $\theta_{12}$, do not change this prediction. In addition to this, the non-zero value of $\xi$, as taken because it provides access to the parameter space at low scale being within their 3$\sigma$ range. The lower bound on SUSY breaking scale is around $3 \times 10^{13}$ GeV whereas the upper bound turns out to be around $10^{18}$ GeV.

Similar analysis can also be performed for the variation of unification scale. In Figure 3, we show the variation of unification scale with respect to $\xi$. We follow exactly same procedure as in Figure 5 except, we have fixed SUSY breaking scale to be 2 TeV in this case. We observe that the lower bound on unification scale is around $3 \times 10^{13}$ GeV whereas the upper bound turns out to be around $10^{18}$ GeV.

To conclude, at present, we do not know whether neutrinos are Dirac or Majorana particles. Only the experiments can settle this long standing issue. Keeping this spirit, in this work, we have investigated the HSMU hypothesis with Dirac type neutrinos. This hypothesis assumes that the mixing parameters of quark sector become identical to those of neutrino sector at some unification scale, which could be the GUT scale. In addition to this, we also need MSSM as a natural extension of the SM. After taking quark mixing parameters equal to the neutrino mixing parameters at unification scale, we run down the neutrino MSSM RG equations from unification scale to SUSY breaking scale. From SUSY breaking scale to low scale ($M_Z$), the running is governed by SM RG equations. After RG evolution, we obtain all the neutrino oscillation parameters, at low scale, within their 3$\sigma$ range.

From this analysis, we have a clear and unambiguous prediction that $\theta_{23}$ is non-maximal and lies only in the second octant. This predictions is novel and robust. The precise determination of this angle is important to extract information about other oscillation parameters including $\delta_{CP}$. The range for $\theta_{13}$ is also tightly constrained. Moreover, the normal hierarchy and quasi-degeneracy of the neutrino mass eigenvalues, at low scale, within their 3$\sigma$ range.
trino masses, essential to our analysis, can also provide important test for this framework. These predictions can be examined in present and the future experiments like INO, T2K, NOνA, LBNE, Hyper-K, PINGU \cite{INO, T2K, NOνA, Hyper-K, PINGU}.

We have also derived constraints on the allowed range of the SUSY breaking scale and unification scale. The lower bound on SUSY breaking scale comes from the experimental SUSY searches \cite{INO, T2K, NOνA, Hyper-K, PINGU}. The upper bound on SUSY breaking scale, derived in this work, turns out to be around $4 \times 10^6$ GeV. Similarly, the lower and upper bounds on the unification scale, for HSMU hypothesis, are around $3 \times 10^{13} \sim 10^{18}$ GeV, respectively.

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