1. Introduction

Motion control of a car with $N$ trailers (when only the car has a thruster) has been a challenging problem. The main goal of this problem is to move the car in such a way that the following trailers are placed in desired locations. This system is a nonholonomic one which is limited in movement side-wise. Since, nonholonomic systems cannot be stabilized by continuous feedback laws (Gao & Yuan, 2012; Kolmanovsky & McClamroch, 1995; Maruskin & Bloch, 2011), designing a stabilizing control law for these systems is very difficult.

First efforts in motion control of a car with $N$ trailers were based on regulating system’s states using two control inputs (Luca, Oriolo, & Samson, 1998; Sordalen, 1993). Solution of this problem has been discussed in different papers like Leng and Minor (2010), Murray and Sastry (1993), Sordalen and Wichlund (1993); however, all of these works deal with asymptotically stabilizing of state variables (not finite-time stabilization).

Recently, terminal sliding mode (TSM) technique has been developed as a finite-time control method (Daohong & Yu, 2011; Feng, Yu, & Man, 2002; Wu, Yu, & Man, 1998; Yu & Man, 1996). Applications of TSM in finite-time control of nonholonomic systems have been studied in Wang and, Wang, Lu, and Wang (2001), Yuqiang, Wang, and Zong (2005), Zhu, Dong, and Hu (2003). One of the problems of the TSM technique is its singularity for some of the regions of state space; therefore, nonsingular versions of this method have been developed in the literature (Chen & Lin, 2011; Jin, Lee, Chang, & Choi, 2009; Li & Deng, 2014). In spite of these developments, the problem of finite-time stabilization of a car with trailers is still unsolved.

This paper considers finite-time stabilizing of a tractor–trailer system based on the TSM technique. For this purpose, first, the system equations are transformed into the bilinear chained form. This transformation simplifies the design of finite-time stabilizing control laws using the TSM technique. Also, in the design procedures, the recursive sliding surfaces are mathematically derived and the finite-time control law is designed. Finally, computer simulations reveal the efficiency of the designed control law in finite-time stabilization of the tractor–trailer system.

The remainder of this paper is arranged as follows. First, the model of a tractor–trailer system is given in Section 2. In Section 3, considering necessary conditions, the state-space equations of the tractor–trailer system are transformed into the bilinear chained form. Next, in Section 4, a finite-time controller is designed based on the TSM method. Computer simulations are given in Section 5. Finally, conclusions are made in Section 6.

2. Problem definition

In this section, the equations of a tractor–trailer system are introduced. Figure 1 shows the schematic of this system. In this figure, the car and the trailer have been illustrated with two wheels and their connecting axis. In order to model this system, the position of the trailer is considered as the position of the system.

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The kinematic model of the tractor–trailer system is as follows (Sordalen, 1993):

\[
\begin{align*}
\dot{x} &= \cos \theta_1 v_1, \\
\dot{y} &= \sin \theta_1 v_1, \\
\dot{\theta}_1 &= \frac{1}{d_1} \sin(\theta_0 - \theta_1) v_0, \\
\dot{\theta}_0 &= \omega,
\end{align*}
\]  

(1)

where \((x, y)\) is the coordination of the center of the trailer, \(\theta_1\) is the angle of the trailer with respect to \(x\)-axis and \(\theta_0\) is the angle of the car. Moreover, \(d_1\) is the distance between centers of car and the trailer, \(v_0\) and \(\omega\) are linear and rotational velocities of the car, respectively, which are control input variables. Also, there are two velocities \(v_i\)'s (\(i = 0, 1\)), which are not independent from each other (Sordalen, 1993),

\[
v_i = \cos(\theta_0 - \theta_1) v_0,
\]

(2)

where all angles, \(\theta_0\) and \(\theta_1\), belong to \((-\pi/4, \pi/4)\). In order to have independent input variables, a new input variable, \(v\), is defined with respect to the velocity and the angle of the trailer as

\[
v = v_1 \cos \theta_1.
\]

(3)

Now, all \(v_i\)’s could be replaced by the new input variable, \(v\). By replacing \(v_1\) from Equation (2) into Equation (3), \(v_i\)'s may be evaluated, with respect to \(v\), as follows:

\[
\begin{align*}
v_0 &= \frac{1}{\cos \theta_1 \cos(\theta_0 - \theta_1)} v, \\
v_1 &= \frac{1}{\cos \theta_1} v.
\end{align*}
\]

(4)

Finally, if all \(v_i\)’s, from Equation (4), are replaced in Equation (1), the system equations will include, only, \(v\) and \(\omega\), as control inputs (Sordalen, 1993);

\[
\begin{align*}
\dot{x} &= v, \\
\dot{y} &= \tan \theta_1 v, \\
\dot{\theta}_1 &= \frac{1}{d_1} \tan(\theta_0 - \theta_1) v, \\
\dot{\theta}_0 &= \omega.
\end{align*}
\]

(5)

By defining the state vector as \(z = (x, y, \theta_1, \theta_0)\), the state-space equations of the system may be rewritten as

\[
\dot{z} = g_1(z) v + g_2(z) \omega,
\]

(6)

where

\[
\begin{align*}
g_1 &= \begin{bmatrix} 1 \tan \theta_1 & \frac{1}{d_1} \tan(\theta_0 - \theta_1) & 0 \end{bmatrix}^T, \\
g_2 &= \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T.
\end{align*}
\]

The task is design of control input variables \(v\) and \(\omega\) (based on the TSM method) to finite-time stabilization of the state vector to the origin. This means that for any initial condition, the trailer to be placed on the origin in a finite time and also alignment of the car and its trailer, along the \(x\)-axis, occur in a finite time.

3. **Chained form and bilinear chained form**

The TSM method uses quasi-linear structure of bilinear chained form for simplicity in its evaluations. The state-space equations of a system in the chained form are as follows:

\[
\begin{align*}
\dot{\xi}_1 &= u_1, \\
\dot{\xi}_2 &= u_2, \\
\dot{\xi}_3 &= \xi_2 u_1, \\
\vdots \\
\dot{\xi}_n &= \xi_{n-1} u_1,
\end{align*}
\]

(7)

where \(\xi_i\)'s (for \(i = 1, \ldots, n\)) are the state variables and \(u_1, u_2\) are the input variables. Also, the state-space equations of a system, in the bilinear chained form, are as follows:

\[
\begin{align*}
\dot{x}_1 &= u_1, \\
\dot{x}_2 &= u_1 x_3, \\
\vdots \\
\dot{x}_{n-1} &= u_1 x_n, \\
\dot{x}_n &= u_2,
\end{align*}
\]

(8)

where \(x_i\)'s are considered as the state variables in the bilinear chained form to be distinguished from the chained form.
3.1. Transformation to the chained form

Consider a general nonholonomic system with the following structure (which is a generalization of the system (6)):

\[ \dot{z} = g_1(z)v + g_2(z)\omega, \]  

(9)

where \( z \in \mathbb{R}^n \) is the state vector, \( g_1 \) and \( g_2 \in \mathbb{R}^n \) are linearly independent vector functions and \( v, \omega \in \mathbb{R} \) are control inputs. Defining the following distributions:

\[ \Delta_0 = \text{span}\{g_1, g_2, ad_{g_1}g_2, \ldots, ad_{g_1}^{n-2}g_2\}, \]
\[ \Delta_1 = \text{span}\{g_2, ad_{g_1}g_2, \ldots, ad_{g_1}^{n-2}g_2\}, \]
\[ \Delta_2 = \text{span}\{g_2, ad_{g_1}g_2, \ldots, ad_{g_1}^{n-3}g_2\}, \]

where \( ad_{g_1}g_2(z) = g_2(z), ad_{g_1}^k g_2(z) = [g_1, ad_{g_1}^{k-1}g_2](z) \), for \( k \geq 1 \), and \( [g_1, g_2] = (\partial g_2/\partial z)g_1(z) -(\partial g_1/\partial z)g_2(z) \). If the distribution \( \Delta_0 \), spans the space \( \mathbb{R}^n \) and the distributions \( \Delta_1 \) and \( \Delta_2 \) are involutive, then, the system (9) can be transformed to the chained form (Murray & Sastry, 1993). Now, considering \( \Delta_1 \) and \( \Delta_2 \) to transfer the system into chained form, the smooth scalar functions, \( h_1 \) and \( h_2 \), may be found by solving the following equations:

\[ dh_1, \Delta_1 = 0, \quad dh_1, g_1 = 1, \quad dh_2, \Delta_2 = 0, \]

(11)

where \( d \) is the differential operator. After finding \( h_1 \) and \( h_2 \), the following transformation may convert the system (9) into the chained form (7) (regarding to new state variables, \( \xi_i \)'s, and new input variables, \( u_i \)'s):

\[ \xi_1 = h_1, \]
\[ \xi_2 = L g_1 h_2, \]
\[ \vdots \]
\[ \xi_{n-1} = L g_1 h_2, \]
\[ \xi_n = h_2, \]
\[ u_1 = v, \]
\[ u_2 = (Lg_1 - 1)h_2) + (L g_1 - 1)h_2) \]

where the operator \( L \), is the lie derivative (Khalil, 2003).

3.2. Transformation to the bilinear chained form

The state-space equations (8), which are in the bilinear chained form, can be achieved by rearranging the state variables as follows:

\[ x_1 = \xi_1, \]
\[ x_n = \xi_2, \]
\[ x_{n-1} = \xi_3, \]
\[ \vdots \]
\[ x_2 = \xi_n. \]

(13)

3.3. Applying the bilinear transformation to the tractor–trailer system

In order to rewrite the system equations (6), in the chained form, the functions, \( h_1 \) and \( h_2 \), should be evaluated. In this case, \( h_1 = x \) and \( \Delta_2 = \text{span}\{g_2\} = [0 0 0 k_1]^T \). Therefore, solving Equations (11) to find \( h_2 \) results as,

\[ dh_2, \Delta_2 = 0 \Rightarrow \begin{bmatrix} \frac{\partial h_2}{\partial x} & \frac{\partial h_2}{\partial y} & \frac{\partial h_2}{\partial \theta_1} & \frac{\partial h_2}{\partial \theta_0} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0, \]

(14)

\[ \Rightarrow \frac{\partial h_2}{\partial \theta_0} = 0. \]

This means that the function \( h_2 \) may be any arbitrary function of \( x, y \) and \( \theta_1 \). In this case, \( h_2 = y \) may be chosen. Consequently, the following transformation, will transform Equations (5) into the chained form representation:

\[ \xi_1 = x, \]
\[ \xi_2 = \frac{\tan(\theta_0 - \theta_1)}{d_1 \cos^3 \theta_1}, \]
\[ \xi_3 = \tan \theta_1, \]
\[ \xi_4 = y, \]
\[ u_1 = v, \]
\[ u_2 = \frac{1 + \tan^2(\theta_0 - \theta_1)}{d_1 \cos \theta_1} \omega, \]
\[ + \frac{(1 + \tan^2(\theta_0 - \theta_1)) \cos \theta_1 + 3 \cos^2 \theta_1 \sin \theta_1 \tan(\theta_0 - \theta_1)}{d_1 \cos^3 \theta_1} \]
\[ \times \frac{\tan(\theta_0 - \theta_1)}{d_1 \cos \theta_1} v. \]

(15)

Rearranging the state variables, according to Equation (13), results in the bilinear chained form representation (8). The following bilinear chained form corresponds to the tractor–trailer system:

\[ \dot{x}_1 = u_1, \]
\[ \dot{x}_2 = u_1 x_3, \]
\[ \dot{x}_3 = u_1 x_4, \]
\[ \dot{x}_4 = u_2. \]

(16)

3.4. Advantage of the bilinear chained form

If the system (8) to be partitioned into two subsystems, \( \Sigma_1 \) with a state variable, \( y_1 = x_1 \), and \( \Sigma_2 \) with other state
variables, \( y_2 = [x_2 \cdots x_n]^T \); then,

\[
\begin{align*}
\Sigma_1 & : \dot{y}_1 = u_1, \\
\Sigma_2 & : \dot{y}_2 = A(u_1)y_2 + Bu_2,
\end{align*}
\]

where \( A(u_1) = \begin{bmatrix} 0 & u_1(t) & \cdots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \cdots & u_1(t) \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \quad (17)
\]

\[
B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}.
\]

The subsystem \( \Sigma_1 \) is a linear system of degree one, which may be controlled by the input \( u_1 \). Considering \( u_1 \) as a time-varying parameter in \( \Sigma_2 \), this subsystem may be considered as a linear time-varying system, with a single input \( u_2 \), which its controllability depends on the value of \( u_1 \). On the other hand, \( u_1 \) couples these two subsystems. In order to decouple them, the following control law may be supposed (Zhu et al., 2003):

\[
u_2 = \begin{cases} u_2 & \text{if } t < \tau \\ 0 & \text{if } t > \tau \end{cases}, \quad (18)
\]

where \( \tau \) is a calculable positive constant. Suppose that \( u_2(t) \) can stabilize the subsystem \( \Sigma_2 \), in a finite time, \( \tau \). Therefore, by applying \( u_2(t) = 0 \), for \( t > \tau \), in spite of changing \( u_1 \), \( y_2 \equiv 0 \) (because of the special structure of \( \Sigma_2 \)). After decoupling the subsystems, necessary conditions on the value of \( u_1 \), for the controllability of \( \Sigma_2 \), should be investigated. The following theorem determines these necessary conditions.

**Theorem 1** Consider the linear time-invariant subsystem \( \Sigma_2 \) with initial condition \( y_2(t_0) = y_{20} \). If \( u_1 \) is a differentiable function of order \( n-2 \), and there exists a time \( t_1 > t_0 \), where \( u_1(t_1) \neq 0 \), then, the subsystem \( \Sigma_2 \) is controllable at \( t_0 \).

**Proof** See Zhu et al. (2003) \( \square \)

In proceeding, considering \( u_1 \) as a time-varying function, the subsystem \( \Sigma_2 \) is stabilized in a finite time. After that, since the subsystems \( \Sigma_1 \) and \( \Sigma_2 \) are decoupled, the first subsystems may be controlled, independently.

### 4. Design of a TSM controller for the tractor–trailer system

In this section, using the TSM method, a finite-time control law for the subsystem \( \Sigma_2 \), is designed. Consider the following recursive sliding surfaces (the number of sliding surfaces is equal to number of state variables of the subsystem \( \Sigma_2 \)):

\[
\begin{align*}
S_0 &= x_2, \\
S_1 &= \dot{S}_1 + \beta_1 S_0^{q_1/p_1}, \\
S_2 &= \dot{S}_2 + \beta_2 S_1^{q_2/p_2}, \\
& \vdots \\
S_{n-2} &= \dot{S}_{n-2} + \beta_{n-2} S_{n-3}^{q_{n-2}/p_{n-2}},
\end{align*}
\]

where \( \beta_i(i = 1, 2, \ldots, n-2) \) are positive scalars, \( p_i, q_i \) are positive odd numbers and \( p_i > q_i \). In this method, the Lyapunov function, \( V = \frac{1}{2} S_n \), is considered and then the input is designed such that the state variables, first, reach to the surface, \( S_{n-2} \), then, continuing to reach to the surface, \( S_{n-3} \), and then, \( S_{n-4} \), and . . . , finally, reach to \( S_1 \) and \( S_0 \) (all in finite times). In order to avoid singularity in the control signal, \( p_i 's \) and \( q_i 's \) should satisfy \( q_i/p_i > (n-i)/n \). In the TSM method, inappropriate initial condition can make singularity in the control signal. To avoid singularity, initial conditions should belong to the following set:

\[
\Omega_{n-1} = \{ x : S_0 > 0 \} \cap \{ x : S_1 > 0 \} \cap \ldots \cap \{ x : S_{n-2} > 0 \}.
\]

Otherwise, a pre-stage control is necessary to move the state variables to \( \Omega_{n-1} \) (Feng et al., 2002). Suppose that in the time \( t_p \), the states of the subsystem \( \Sigma_2 \) have been moved to a proper position. Now, assume that \( u_1(t) = r(t) \), where \( r(t) \) is a differentiable function of order \( n-2 \), and let to define:

\[
v_i = \frac{d^{i-1} u_1(t)}{dt^{i-1}} = \frac{d^{i-1} r(t)}{dt^{i-1}} \quad i = 1, 2, \ldots, n-1. \quad (21)
\]

Then, by successive differentiations from the sliding surface, \( S_0 \), we have,

\[
\begin{align*}
\frac{dS_0}{dt} &= \dot{x}_2 = \varphi_1(v_1, x_3), \\
\frac{d^2S_0}{dt^2} &= \ddot{x}_2 = \varphi_2(x_3, x_4, v_1, v_2), \\
& \vdots \\
\frac{d^{n-2}S_0}{dt^{n-2}} &= \varphi_{n-2}(v_1, v_2, \ldots, v_{n-2}, x_3, \ldots, x_n), \\
\frac{d^{n-1}S_0}{dt^{n-1}} &= \varphi_{n-1}(v_1, \ldots, v_{n-1}, x_3 \ldots x_n) + \frac{\partial \varphi_{n-2}}{\partial x_n} \ddot{x}_n,
\end{align*}
\]

where

\[
\frac{\partial \varphi_{n-2}}{\partial x_n} = \frac{d^{n-2} r(t)}{dt^{n-2}}. \quad (23)
\]

The following discontinuous control law stabilizes the state variables of system (17) to the origin in a finite time.
vanishing of some terms in Equation (28), which results in a simpler control law.

\begin{equation}
\begin{aligned}
    u_1 &= v_1 = 5, \quad u_1 = v_2 = 0, \quad \dot{u}_1 = v_3 = 0, \\
    u_2 &= \begin{cases}
        5 & t < t_s, \\
        0 & t \geq t_s
    \end{cases}
\end{aligned}
\end{equation}

For evaluating \( u_{eq} \), it is required to calculate \( \varphi_{n-1} = \varphi_3 \), therefore, according to Equation (22),

\begin{align*}
    \frac{dS_0}{dt} &= \ddot{x}_2 = x_3 u_1 = 5x_3 = \varphi_1, \\
    \frac{d^2 S_0}{dt^2} &= \dddot{x}_2 = x_3 \ddot{u}_1 + \dddot{x}_3 u_1 = 25x_4 = \varphi_2, \\
    \frac{d^3 S_0}{dt^3} &= \frac{d^3 x_2}{dt^3} = 25 \dddot{x}_4 = \varphi_3 + \frac{\partial \varphi_2}{\partial x_4} \dddot{x}_4.
\end{align*}

Consequently, \( \varphi_3 = 0 \). Therefore, \( u_{eq} \) (referring to Equation (25)) is as follows:

\begin{equation}
    u_{eq} = -1 \left[ \frac{\varphi_{n-1}}{r_p} + \sum_{i=0}^{n-3} \beta_{p_1} \left( \frac{d^{n-i-2} |S_{n-i-2}(t)|}(d^{n-i-2} |x_{n-i-1}/(x_{n-i-1}) |) \right) \right],
\end{equation}

where \( \varphi_{n-1} \) is evaluated from recursive equations (22). The convergence time \( t_s \) of the subsystem \( \Sigma_2 \) is

\begin{equation}
    t_s = t_1 + \frac{n-2}{\beta_{p_1}} \left( p_{n-i-1} - q_{n-i-1} \right) \times |S_{n-i-2}(t_1)|^{\frac{q_{n-i-2}}{p_{n-i-2}}},
\end{equation}

and \( t_1 = t_p + |S_{n-2}(t_p)|/k_2 \). Convergence time of the system (17) named \( t_r \) is as follows:

\begin{equation}
    t_r = t_s + \frac{|x_1(t_s)|}{k_1},
\end{equation}

where \( x_1(t_s) = x_1(t_p) + \int_{t_p}^{t_s} r(\tau) \, d\tau \) (Zhu et al., 2003).

In the following section, the mentioned design procedure is applied for a car with one trailer.

### 4.1. Design of control law for a tractor-trailer system

In this case, according to Equations (19) and (21), the following three sliding surfaces are defined for system (16):

\begin{align*}
    S_0 &= x_2, \\
    S_1 &= \dot{S}_0 + \beta_1 S_0^{(q_1/p_1)} = \dot{x}_2 + \beta_1 x_2^{(q_1/p_1)} = x_3 u_1 + \beta_1 x_2^{(q_1/p_1)}, \\
    S_2 &= \dot{S}_1 + \beta_2 S_1^{(q_2/p_2)} = \ddot{x}_2 + \beta_2 x_2^{(q_2/p_2)} = x_3 v_1 + x_3 v_2 + \beta_2 x_2^{(q_2/p_2)},
\end{align*}

and according to Equations (19) and (21),

\begin{align}
    \dot{S}_0 &= \ddot{x}_2, \quad \dot{S}_0 = \dddot{x}_2, \\
    S_1 &= \dot{S}_0 + \beta_1 S_0^{(q_1/p_1)}, \\
    \dot{S}_1 &= \dot{S}_0 + \beta_1 \frac{q_1}{p_1} S_0^{(q_1/p_1)} \dot{S}_0.
\end{align}

Finally, the designed control law is as follows:

\begin{equation}
    u_1 = \begin{cases}
        5 & t \leq t_s, \\
        -k_1 \text{sgn}(x_1) & t \geq t_s
    \end{cases}
\end{equation}

\begin{equation}
    u_2 = \begin{cases}
        u_{eq} - \frac{k_2 \text{sgn}(S_1)}{25} & t \leq t_s, \\
        0 & t \geq t_s
    \end{cases}
\end{equation}

where \( S_2 = \dot{S}_1 + \beta_2 S_1^{(q_2/p_2)} \). Note that it is supposed that no pre-stage control is needed. Finally, since the real control inputs of the system (5) are \( \nu \) and \( \omega \), these inputs (usually \( \nu = u_1 \) and \( \omega \) may be evaluated, using Equations (15), with respect to \( u_1 \) and \( u_2 \).
Figure 2. Time responses of the state variables of (a) the first subsystem, $\Sigma_1$ and (b) the second subsystem, $\Sigma_2$.

Figure 3. Time responses of control signals.

Figure 4. Time responses of sliding surfaces $S_0$, $S_1$ and $S_2$. 
5. Computer simulations

In this section, computer simulations are performed to illustrate the effectiveness of the proposed control law (33) in finite-time stabilization of the tractor–trailer system. In the simulations, it is assumed that,

\[ k_1 = k_2 = 1, \]
\[ p_1 = 7, \quad q_1 = p_2 = 5, \quad q_2 = 3, \] \hspace{1cm} (34)
\[ \beta_1 = \beta_2 = 1. \]

The initial condition has been selected as follows (where there is no need to pre-stage control stage):

\[ \beta_{\theta_1, \theta_\theta_3, \theta_5} = (1, 3, 0, 0.3). \] \hspace{1cm} (35)

Figures 2–4 show simulation results. In Figure 2, finite-time convergence of the state variables has been illustrated. As it is seen, the state variables have been reached to origin in 15 s. The control signals are depicted in Figure 3. Time histories of \( S_0, S_1 \) and \( S_2 \) are also illustrated in Figure 4. As it is seen first \( S_2 \), then, \( S_1 \) and finally, \( S_0 \), have been reached to zero.

6. Conclusion

In this paper, state-space model of a tractor–trailer system (as a nonholonomic system) was introduced. Then, model of this system was transformed to the bilinear-chained form. After that, using the quasi-linear structure, a nonsingular finite-time control law, based on the TSM method, was designed. Finally, computer simulations were done and the results showed the effectiveness of the proposed method in finite-time stabilization of the state variables.

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