Anomalous three-dimensional refraction in the microwave region by ultra-thin high efficiency metalens with phase discontinuities in orthogonal directions

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Abstract
An ultrathin flat metalens that experimentally realizes three-dimensional microwave manipulation has been demonstrated as able to approach the theoretical limit of cross-polarization (cross-pol) conversion efficiency of the transmission, as predicted by Monticone et al (2013 Phys. Rev. Lett. 110 203903). The helicity-dependent phase change is introduced to the transmission and can be engineered by assembling the spatial orientation of each Pancharatnam–Berry phase element. By realizing the constant phase gradients in orthogonal directions, an anomalous non-coplanar refraction is unanimously demonstrated in the three-dimensional space under the circular-polarized incidence, and the refraction angle is well predicted with the generalized Snell’s law, derived with phase gradients in orthogonal directions. More importantly, the maximum conversion efficiency of the cross-pol transmission is as high as 24\%, which approaches the upper-bound of the theoretical limit. The proposed metalens has only a single layer as thin as 0.001\(\lambda\), which massively reduces the thickness of the microwave lens along the wave propagation direction. With the great improvements in efficiency and the thickness reduction, as well as the
excellent non-coplanar refraction, our design provides a promising approach to miniaturize, planarize and integrate multiple microwave components.

Keywords: metasurface, Pancharatnam–Berry phase, microwave

1. Introduction

The manipulation of electromagnetic (EM) waves has always been a hot topic. The traditional means, which includes using dielectric materials (lenses) and metallic surfaces (antennae), show their abilities to control the transmission of EM waves; however, they both hold inherent limitations in the microwave region. For lenses, the phase accumulation, which is the origin for the specific refraction effect, comes from either the specific surface topography or the spatial variation of the refractive index of the lens. Thickness is therefore an essential factor for realizing specific functions [1]. For antennae, both the size of a single antenna and the distance between the antenna cells for the array are of the same order of magnitude as the working wavelength, which means either a single antenna array cannot be treated as effective media [2–4]. Furthermore, the microwave-phased array antenna, which is composed of a great number of bulky antennae, feeding networks and sources, can hardly be integrated with other equipment. Although the rise of metamaterials and transform optics provides new approaches for the design of lenses and antennae [5–7], the inherent limitations still cannot be conquered.

The concept of an abrupt phase change at the interface provides a powerful solution to overcome the limitations mentioned above [8]. By introducing abrupt phase changes with sub-wavelength unit cells into the cross-polarized (cross-pol) wave, the phase accumulation in the traditional lens can be substituted by phase discontinuities on the interface, which provide possibilities for constructing the ultrathin metalenses. Based on the plasmonic response of the gold V-shaped nano-antenna, anomalous refraction and reflection are observed in both simulation and experiment, as predicted by the generalized Snell’s law (GSL) [8]. Until now, the phase discontinuities have been introduced in the mid-infrared region [8], the near-infrared region [9], the optical region [10] and the terahertz region [11]. The Y-shaped nano-antenna, which could tailor two eigenmodes more directly [12], is proposed and analyzed rigorously, and the complementary structure of the V-shaped antenna is also used in ultrathin planar metalenses with extraordinarily strong focusing abilities [13]. Graphene-loaded plasmonic antennae have been introduced to provide the possibility of biased control of optical devices [14].

By tailoring the profile of the phase distributions, extraordinary EM wave manipulation has been demonstrated in the transmission area, such as the optical vertex plate [8, 12], dual-polarity plasmonic metalenses [10], axicons [13], etc. In recent work, the non-coplanar refraction has been obtained through rotating the metasurface by an arbitrary angle with respect to the plane of incidence in the mid-infrared region [15, 16]. All of these results may excite new technologies in the fields of energy harvesting and on-chip optical devices, as predicted in [17].

The conversion efficiency, defined as the ratio of the energy of the transmitted cross-pol wave (or the reflected co-polarized wave) to that of the total incident wave for a transmission-type (or reflection-based) metasurface, is a key factor for the widespread use of the metasurface device based on the phase discontinuity. For reflection-type metalenses with a ground plane on the backside, a relatively high efficiency of 27% is achieved in the planar mirror to reflect and
focus the linearly polarized incident beam, while the theoretical value can be as high as 78\% [18]. The conversion between the propagating wave (PW) and the surface wave (SW) is also investigated [19, 20]. However, the transmission-type metalens, which particularly operates upon the cross-pol wave, faces the challenge of extremely low conversion efficiency. The maximum efficiency achieved was only a few per cent [8–10, 14–16, 21], which could be a great obstacle for the applications, although it was predicted theoretically that the maximum attainable cross-coupling is 25\% based on the S-parameters of the four-port network [22, 23]. The thinner-than-wavelength Huygens’ surface is also put forward to fulfil the manipulation of the wavefront, which exhibits high transmission efficiency, and 30\% in the infrared region [24] and 90\% in the microwave region [25] can be achieved owing to impedance matching by both the electric and magnetic response. However, it is important to note that [24] and [25] do not deal with the cross-pol conversion and are therefore free from the imposed theoretical limit at 25\% [22, 23]. On the other hand, the transverse magnetic currents require loops perpendicular to the screen, which implies that a finite minimum thickness (0.28\,\lambda in [24] and 0.2\,\lambda in [25]) is a must. Therefore, despite their efficiency limitations, ultrathin single-layer designs are still very appealing for practical applications.

In this paper, we propose and demonstrate for the first time an ultrathin planar metalens experimentally with phase discontinuities in orthogonal directions at the microwave region. The phase modification based on the Pancharatnam–Berry phase is adopted to achieve a specifically designed phase profile. With the contributions of the phase gradients in two orthogonal directions, we therefore demonstrate the anomalous non-coplanar refraction in a three-dimensional space for the circular-polarized incidence. Moreover, both the theoretical and experimental results show that the manipulation efficiency of our design is capable of approaching the theoretical upper limit. In addition, due to the Pancharatnam–Berry phase design, the proposed metalens only has a thickness as thin as 0.001\,\lambda for the operating frequency. Therefore, our ultrathin metalens provides a promising approach to miniaturize and planarize microwave devices.

2. High efficiency and the wide-band unit cell

High transmission efficiency and a wide working frequency band are two significant factors for realistic applications. However, extraordinary EM wave manipulation with the previously reported cross-pol effect always suffers from a very low efficiency. In this study, the complementary structure of the U-slot shown in figure 1(a) is adopted to achieve a high coupling efficiency in a broad frequency band, which has found applications in coplanar imaging [26], transformation-optics metasurface lenses [27] and bifunctional Luneburg-fisheye lenses [28]. For an aperture cell, it has been proven to be a band-pass cell [21, 29]. The extraordinary optical transmission (EOT) effects are supported by localized waveguide resonances of the slot [30], which occur at the first and second resonances of the unit cell; this corresponds with the x-polarized and y-polarized incidence, respectively. When the incident wave is circular-polarized, a transmitted field can be described by [31, 32]:

\[
\begin{pmatrix}
\mathbf{E}_{\text{out}} \\
\mathbf{E}_{\text{in}}
\end{pmatrix} = \sqrt{\eta_R} \begin{pmatrix}
\mathbf{E}_{\text{in}} \\
\mathbf{R}
\end{pmatrix} + \frac{1}{\sqrt{\eta_R}} e^{\pm 2i\theta} \begin{pmatrix}
\mathbf{E}_{\text{in}} \\
\mathbf{L}
\end{pmatrix}
\]

(1)
\[ \eta_E = \left( \frac{1}{2} (t_x + t_y e^{i\phi}) \right)^2, \quad \eta_R = \left( \frac{1}{2} (t_x - t_y e^{i\phi}) \right)^2 \]

where \( \eta_E \) and \( \eta_R \) are the polarization order coupling efficiencies, \( \langle \cdot | \cdot \rangle \) denotes the inner product and \( |R| \) represents the right-handed (left-handed) circular polarization components. \( t_x \) and \( t_y \) are amplitudes of the transmission coefficients for two linear polarizations, and \( \phi \) is the phase difference between the transmission coefficients. For circularly polarized incidence, the transmitted electric field has two components. One keeps the original helicity; the other is transformed into the cross-pol field, which is the foundation for the construction of a metalens with phase discontinuities. Here, it can be noted that the coupling efficiency for the cross-pol wave is related with \( t_x, t_y \) and \( \phi \). With \( t_x(t_y) \) tending to 1 and \( t_y(t_x) \) tending to zero, the maximum of the coupling efficiency for the cross-pol wave can be obtained. For a band-pass structure such as the aperture, the maximum of the transmitted field for the circular-polarized incidence occurs at the EOT points, according to equation (1); however, the bandwidth for the effective conversion efficiency (>20%) is obviously very narrowly restricted by the resonance [21, 32], while for the corresponding complementary structure (see the strip shown in figure 1(a)), it behaves as a band-stop cell. One can see in figure 1(b) that for the normal incident plane wave polarized along the x- and y-axis, the y-polarized incidence can pass through the unit cell completely in a broadband frequency range, while the x-polarized component is suppressed around 9.5 GHz. It is important to notice that a relatively wide working frequency band can be achieved by a single band-stop cell, according to equation (1), without using the combination of unit cells with the gradient geometrical parameters, such as in the research of metamaterials.

Then, the coupling efficiency for the cross-pol wave can be calculated, based on equation (1), to be above 20% for the frequency range from 8.9 GHz to 10.2 GHz (the light grey region in figure 1(b), 13.6% relative bandwidth). The result is also in accordance with the transmission coefficient of the left-handed circular polarization (LCP) under the right-handed circular polarized (RCP) incidence, which is relatively high compared with the current results and almost reaches the theoretical limit [22, 23].
To achieve the phase discontinuities on the metalens, the Pancharatnam–Berry (P–B) phase is taken into consideration for circular-polarized incidence. In the microwave region, the metal just behaves as a perfect electric conductor (PEC) and shows no surface plasmonic effects, so the phase discontinuities cannot be generated through the cross-pol resonance as they can in the infrared or optical region [8–16]. However, we can still obtain phase discontinuities in the process of the circular-polarized incidence transformed into its cross-pol wave [31, 32], and the phenomenon of the P–B geometric phase arises from space-variant polarization modifications. Here, a phase factor (PF) is obtained when the polarization changes from the initial state to the final state, which can be conveniently visualized on the Poincaré sphere. As shown in figure 2(a), the two poles on the sphere indicate the RCP and LCP states, while the equator corresponds to the linear polarization states. In other words, when the incident beam undergoes a space-variant polarization manipulation, its phase also changes in space, as this modification

![Figure 2](image-url)

**Figure 2.** The Pancharatnam–Berry phase of the unit cell. (a) Schematic illustration of the polarization states and phase difference on the Poincaré sphere. (b) The phase and (c) amplitude characteristics of the cross-pol wave under the circular-polarized incidence, when the unit cell is rotated around the shape’s center at the step of $\pi/6$. The dots are experimental data. (d) The transmission coefficient and phase response of the cross-pol wave under the circular polarized incidence with different incident angle ($\theta_i$). The dots and squares are the experimental data of the phase and amplitude, respectively.

3. **Pancharatnam–Berry (P–B) phase**

To achieve the phase discontinuities on the metalens, the Pancharatnam–Berry (P–B) phase is taken into consideration for circular-polarized incidence. In the microwave region, the metal just behaves as a perfect electric conductor (PEC) and shows no surface plasmonic effects, so the phase discontinuities cannot be generated through the cross-pol resonance as they can in the infrared or optical region [8–16]. However, we can still obtain phase discontinuities in the process of the circular-polarized incidence transformed into its cross-pol wave [31, 32], and the phenomenon of the P–B geometric phase arises from space-variant polarization modifications. Here, a phase factor (PF) is obtained when the polarization changes from the initial state to the final state, which can be conveniently visualized on the Poincaré sphere. As shown in figure 2(a), the two poles on the sphere indicate the RCP and LCP states, while the equator corresponds to the linear polarization states. In other words, when the incident beam undergoes a space-variant polarization manipulation, its phase also changes in space, as this modification
is of a purely geometric nature [31]. To quantify the relation between the space-variant polarization modification and the phase change, it has been shown that the PF is equal to half of the area that is encompassed by the loop on the Poincaré sphere, and its absolute value can be calculated as \( \text{PF} = \frac{1}{2} (\theta_1 - \theta_0) \), assuming that the polarization changes between the two poles of the sphere (from purely RCP to LCP, or vice versa), as shown in figure 2(a). With arbitrary polarization rotation, any desired phase modification in the range of \([0, 2\pi]\) can thus be achieved. When the incident wave is purely RCP (or LCP), the efficiency \( \eta_R \) (or \( \eta_L \)) vanishes, and equation (1) indicates that the transmitted field from a P–B element comprises two polarization orders. One maintains the phase and the original polarization state of the incident wave, while the other exhibits opposite handedness and a phase modification of \( \pm 2\theta \), where the plus or minus signs correspond to both the rotating direction of the unit cells and the helicity of the incidence. The value of the phase delay is twice that of the rotation angle of the unit cell. Then, one can see that the value of the phase delay can be controlled by the rotation angle of the individual unit cell in the entire range from 0 to \(2\pi\), which can be easily used to construct the discretized spatial phase function that corresponds to the specific refraction or reflection effects predicted by the GSL [8].

Then, the P–B phase of the unit cell is simulated and measured. It can be observed in figure 2(b) that when the unit cell is rotated around the optical axis with a step of \(\pi/6\), the phase difference between the cross-pol component in the transmitted field agrees with the theoretical value of the P–B phase exactly, with a maximum error of 0.02 \(\pi\). Both the simulation and the measurement show that the phase delay of the transmitted cross-pol component is twice that of rotation angle \(\theta\), while amplitudes of the cross-pol wave in the transmitted fields are almost invariant during the rotation, as shown in figure 2(c). The experimental results of the transmission coefficient at some frequency points are a little higher than the simulation results, which may be caused by diffraction, since the test is conducted using two horn antennae. Furthermore, the phase and magnitude responses of the unit cell with the change of the incident angle are also taken into consideration. For the symmetrical structure of the unit cell, we just simulate and test the oblique incidence here in the range of 0° to 45° instead of \([-45°, 45°]\). It can be seen in figure 2(d) that although the transmission coefficient of the cross-pol wave reduces from 0.5 to 0.4 when the incident angle changes from 0° to 45°, the phase characteristics of the cross-pol wave show little change, which will keep the phase gradient stable on the metalens.

4. Derivation of anomalous non-coplanar refraction and reflection in three-dimensional space

By supposing the plane wave incident from the XOZ plane, the wave vector can be expressed as \(\hat{k}_i\), and the incident angle between \(\hat{k}_i\) and the z-axis is denoted as \(\theta_i\). Here, we consider the scenario that there exists angle \(\alpha\) between the plane of incidence and the direction of the phase discontinuities for generality, as shown in figure 3(a). Obviously, the phase discontinuities on the metalens in both the \(x'\)- and \(y'\)-axis (\(\frac{\partial \Phi}{\partial x'}\) and \(\frac{\partial \Phi}{\partial y'}\), respectively) will contribute to the tangential wave vector in the \(x\)- and \(y\)-axis [8, 15, 16]. Combined with the wavevector of the incidence, there will obviously be non-coplanar refraction and reflection in three-dimensional (3D) space. Compared with [15] and [16], there is one more degree of freedom in our design to control the reflection and refraction. We will demonstrate that a metalens with phase
discontinuities in orthogonal directions exhibits an enhancement in the control of anomalous refraction and reflection.

According to Fermat’s principle, the variation of the accumulated phase is zero when the variation of the optical path tends to be infinitesimal. Then, we can derive the pitch angle \( \theta_{t(r)} \) and the azimuth angle \( \phi_{t(r)} \) in the spherical coordinate for anomalous refraction (reflection) by the metalens with phase discontinuities in both the \( x' \)- and \( y' \)-axis:

\[
\begin{align*}
\sin \theta_{t} \sin \phi_{t} &= \frac{1}{n_{t}k_{0x}} \frac{d\Phi_{\theta}}{dx} \quad (2a) \\
\sin \theta_{r} \sin \phi_{r} &= \frac{1}{n_{r}k_{0y}} \frac{d\Phi_{\phi}}{dy} \quad (2b) \\
\sin \theta_{r} \cos \phi_{r} - \sin \theta_{t} &= \frac{1}{n_{t}k_{0x}} \frac{d\Phi_{\theta}}{dx} \quad (2c) \\
\sin \theta_{r} \cos \phi_{r} - \sin \theta_{t} &= \frac{1}{n_{t}k_{0y}} \frac{d\Phi_{\phi}}{dy} \quad (2d)
\end{align*}
\]

where \( \frac{d\Phi_{\theta}}{dx} \) (\( \frac{d\Phi_{\phi}}{dy} \)) is the combined phase gradient along \( k_{x}(k_{y}) \), and \( n_{t(r)} \) is the refraction index of incident (transmission) space. As shown in figure 3(b), when there are phase gradients in both the \( x' \)- and \( y' \)-directions, both of the phase gradients will contribute to the changes of the wave vectors, which are related with rotation angle \( \alpha \) of the metalens with respect to the \( k \)-space. The phase gradients in equation (2) can be further expressed as:

\[
\frac{d\Phi_{\theta}}{dx} = \frac{d\Phi_{\theta'}}{dx'} \cdot \cos \alpha + \frac{d\Phi_{\phi'}}{dy'} \cdot \sin \alpha \quad (3a)
\]
It can be seen in equation (3) that there is one more degree of freedom in our design to control the reflection and refraction compared with [15] and [16]. Here, we suppose that the directions of phase changes on the metasurface follow the positive direction of the \(x'\) and \(y'\)-axis, respectively. One can note that for any numerical values of phase changes along the \(x'\) or \(y'\)-axis, the 3D refraction and reflection turn into the in-plane scenario [8] when \(\alpha\) equals \(\Phi_2\), namely the synthetic phase discontinuity lies only along the \(x\) or \(y\) direction. Furthermore, for the case in which the phase gradient along the \(x'\)-axis is the same as that along the \(y'\)-axis, the combined phase gradient in the \(x\)-axis reaches the maximum value when \(\alpha = \pi/4\), which can be calculated to be \(\sqrt{2} \frac{d\Phi_y}{dx'}\). This means that we can get the same phase gradient from 0 to \(2\pi\) within a smaller distance, which provides a powerful approach for the miniaturization of a metasurface in a theoretical manner, in addition to using different kinds of unit cells with minimized geometrical sizes and fewer discrete values to mimic the phase change from 0 to \(2\pi\).

Furthermore, it can be seen that the change of Snell’s law results in a new critical angle. Due to the nonlinear property of equation (2), there exist two critical angles for both refraction and reflection as expressed below, which corresponds to \(k_{\text{t}}\), \(z = 0\) and \(k_{\text{r}}\), \(z = 0\), respectively:

\[
\theta_{\text{c},t} = \arcsin \left( \pm \sqrt{\frac{n_z}{n_t} \left( \frac{1}{n_t k_{\text{t}} dy} \right)^2 - \frac{1}{n_t k_{\text{t}} dx} \frac{d\Phi_y}{dy}} \right) \tag{4a}
\]

\[
\theta_{\text{c},r} = \arcsin \left( \pm \sqrt{1 - \left( \frac{1}{n_t k_{\text{t}} dy} \right)^2 - \frac{1}{n_t k_{\text{t}} dx} \frac{d\Phi_y}{dx}} \right) \tag{4b}
\]

In equation (4), it can be noted that the existence of the critical angle depends on the numerical value of the phase changes along the \(x\)- and \(y\)-axis. In other words, we can eliminate the critical angle by choosing proper phases gradients on the metasens, even for the case in which the wave transmits from one medium to the other medium with a smaller refraction index.

5. Results and discussions

Based on the high coupling efficiency of the U-shaped strip cell, the metasens with phase discontinuities in orthogonal directions is proposed, simulated and measured. Here, we choose the same phase gradient along the \(x'\)-axis and the \(y'\)-axis on the metasens, and the unit cell is rotated in the step of \(\pi/12\) to achieve the phase gradient of \(\frac{d\phi_y}{dy'} = \frac{d\phi_{x'}}{dy'} = \frac{2\pi}{12a}\), where \(a\) is the lattice constant of the unit cell. Here, it should be noted that the induced currents on the unit cells will also produce phase discontinuities, but the simulation results and measurement results demonstrate that all of the unit cells with different rotating angles work at the same frequency under the circular-polarized incidence, as shown in figure 2(c), and the induced currents on the unit cells basically follow the same distribution. Then, it can be concluded that the induced
current on each unit cell contributes the same to the phase discontinuities and to the function of the spatial phase distribution. Since only the phase gradient is involved in the application of the GSL, the phase discontinuities produced by the induced currents on the unit cells will not affect the application of the GSL.

The metalens is modeled and simulated using CST Microwave Studio, the commercial software based on the finite integration technique (FIT). Here, we first set the normal incidence of the RCP plane wave of 10 GHz that impinges onto the metalens with $\alpha = 0$. From the far-field pattern in figure 4(a), we can see that in addition to the normal refraction and reflection, there are non-coplanar refraction and reflection in three-dimensional space, and the refraction angle agrees with the theoretical result of $\phi_t = 45^\circ$ and $\theta_t = 27^\circ$. Then, the metalens is fabricated on the microwave composite substrate, as depicted in the inset of figure 4(b). Here, one should note that the U-shaped copper strip cells are the functional parts, while the substrate just functions as the mechanical support. The thickness of the metasurface is only 0.035 mm ($0.001 \lambda_0$ for the working frequency of 10 GHz), which can be considered as an ultrathin metalens. Then, the

Figure 4. Anomalous non-coplanar refraction in three-dimensional space. (a) The simulation result of the normalized far-field pattern under RCP incidence. (b) A photograph of the measurement set-up, and the inset is a photograph of the fabricated metalens. (c) The measured normalized far-field pattern of the metalens at 10 GHz under RCP incidence. (d) The anomalous refraction angle changing with $\theta_i$ in three-dimensional space; here, $\theta_t$ and $\phi_t$ represent the pitch angle and azimuth angle of the refraction, respectively. The light gray region represents the negative refraction, and the dark gray region is the region where the incident angle is beyond the critical angle. (e) and (f) are the measured normalized far-field patterns under incident angles of $+45^\circ$ and $-45^\circ$, respectively.
The performance of the metalens is measured in StarLab, as shown in Figure 4(b). The circular-polarized horn antenna is chosen as the source of the incident plane wave, and the metalens is fixed on the top of the antenna by nylon rods; the rods’ length can be adjusted to achieve specific incident angle $\theta_i$, as shown by the dashed line in Figure 4(b). It can be observed in Figure 4(c) that the measured 3D far-field pattern for the anomalous refraction at 10 GHz agrees with the simulation quite well. The non-coplanar reflection here has not been measured, since there is a feeding network, circular polarization adjuster and supporting rod at the backside of the horn antenna, which are all metallic and can affect measurement significantly. Then, the anomalous refraction angle of the metalens is measured in the frequency band of 9.5–10.5 GHz under the incident angle, which ranges from $-45^\circ$ to $+45^\circ$. Here, we should point out that in the measurement, the horn antenna is fixed, and the metalens is rotated to achieve a specific incident angle according to the experimental requirement, which is different with the condition of the theoretical calculation that the metalens is fixed, and the incident plane wave is rotated. So, the measured refraction angle is recalculated through rotating the coordinate system by an angle $\theta_i$ in order to achieve the straightforward depiction of the anomalous refraction as the theoretical results. It can be seen in Figure 4(d) that the experimental data show close agreement with the theoretical results. Meanwhile, one can observe that negative refraction is achieved in the light gray region in Figure 4(d), where the anomalous refractions are in the same side as the incidence. The deep gray region in Figure 4(d) represents the region where the incident angle is beyond the critical angle predicted in equation (4), so the cross-pol field in this scenario no longer exists in the refraction, and the transmitted field only contains the co-polarized component. The far-field patterns with the incident angle of $+45^\circ$ (Figure 4(e)) and $-45^\circ$ (Figure 4(f)) are both measured to verify this. It can be seen that there is only a co-polarized component in the refraction for the incident angle of $+45^\circ$, which is beyond the critical angle for the frequency of 10 GHz, according to equation (4), while there are both a co-polarized component and a cross-pol component in the refraction for the $-45^\circ$ incidence. The theoretical results and the experiments coincide with each other very well.

Second, the decomposition and re-combination of the phase gradients on the metalens are measured. According to equation (3), the actual phase gradients can be calculated through the phase gradients on the metalens along the $x'$-axis, the $y'$-axis and the rotating angle $\alpha$ of the metalens. Since the phase gradients along the $x'$-axis and $y'$-axis possess the same numerical value in this research, two special cases of $\alpha=45^\circ$ and $\alpha=-135^\circ$ are taken into consideration here to achieve the maximum value of the synthesized phase gradient. It can be seen in equation (3) that when $\alpha=45^\circ$ or $\alpha=-135^\circ$, the synthesized phase gradient only lies in the $x$-axis, which will convert the non-coplanar refraction into a coplanar one. The far-field patterns with $\alpha=45^\circ$ and $\alpha=-135^\circ$ are measured, and the case with $\alpha=0^\circ$ is taken for comparison. One can observe in Figures 5(a)–(c) that with the rotation of the metalens, the non-coplanar refraction ($\alpha=0^\circ$, Figure 5(a)) of the cross-pol wave in the transmitted wave turns into a coplanar refraction ($\alpha=45^\circ$, Figure 5(b) and $\alpha=-135^\circ$, Figure 5(c), and obviously the anomalous refraction beam turns from one side of the normal line to another, with the change of the phase gradient from positive to negative. Furthermore, the detailed refraction angles for $\alpha=45^\circ$ and $\alpha=-135^\circ$ are also tested and shown in Figure 5(d) for the plane of $\phi=0^\circ$. The measured refraction angle is in good agreement with the theoretical value based on equations (2) and (3).

Finally, the coupling efficiency of the metalens for the transmitted cross-pol wave is tested. Here, both the normal incidence and the oblique incidence are taken into consideration. For the normal incidence, we also test the coupling efficiency when the metalens is rotated. It can be
seen in figure 6 that the peak value of the coupling efficiency is as high as 24%, and the measured coupling efficiency under normal incidence is around 20% in the frequency band of 9.5–10.5 GHz. The rotation of the metalens does not show great influence on the coupling efficiency of the cross-pol wave. For the oblique incidence, the transmission coefficient of the unit cell is reduced with the increase of the incident angle, as shown in figure 2(d), which will no doubt result in a reduction of the coupling efficiency. However, the coupling efficiency under the −45° oblique incidence is still around 15%, which is much higher than the published results that work under the cross-pol wave [8–10, 14–16, 21].

6. Conclusion

In this paper, an ultrathin high efficiency metalens with phase discontinuities in two orthogonal directions is proposed, designed, fabricated and measured. Non-coplanar anomalous refraction
is achieved in three-dimensional space for the first time in the microwave region, and the measured efficiency of the cross-pol conversion approaches the theoretical limit. The experimental results of the bending angle for the anomalous refraction show excellent agreement with the theoretical results, which also verifies the decomposition and recombination of the phase gradients. The planar metalens proposed in this paper enables a significant reduction in thickness, versatile beam behavior and high transmission efficiency simultaneously, which provides a great alternative for microwave lenses. With the sub-wavelength control of the phase and amplitude, it is a promising technology for feeding-network free microwave systems.

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