Skin values of $^{208}$Pb and $^{48}$Ca determined from reaction cross sections

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Background: The PREX and the CREX group reported skin values, $r_{\text{skin}}^{208}$ (PREX2) = 0.283 ± 0.071 fm and $r_{\text{skin}}^{48}$ (CREX) = 0.121 ± 0.026 (exp) ± 0.024 (model) fm, respectively. Using the Love-Franey (LF) t-matrix folding model with the neutron and proton densities scaled to the neutron radius $r_n^{208}$ (PREX2) and the proton radius of the electron scattering, we found that the reaction cross sections $\sigma_R$ reproduce the data for $p+$ $^{208}$Pb scattering at $E_{\text{lab}} = 534.1, 549, 806$ MeV. Zenihiro et al. deduce neutron radii $r_n^{48,40}$ (exp) from the angular distributions of the cross sections and analyzing powers of proton elastic scattering, whereas we determine matter radius $r_m^{40}$ (exp) = 3.361 ± 0.075 fm from measured $\sigma_R$ for $^4\text{He}+$ $^{40}$Ca scattering.

Aim: Our first aim is to determine $r_{\text{skin}}^{208}$ from measured $\sigma_R$ of $p+$ $^{208}$Pb scattering at $E_{\text{lab}} = 534.1, 549, 806$ MeV by using the Love-Franey (LF) t-matrix folding model. Our second aim is to determine $r_{\text{skin}}^{48}$ from $r_{\text{skin}}^{40}$ (exp) and the difference $\Delta \equiv r_{\text{skin}}^{48} - r_{\text{skin}}^{40}$ (exp) that is evaluated from the $r_n^{48,40}$ (exp) and the $r_p^{48,40}$ (exp) calculated with the isotope shift method based on the electron scattering.

Method and results: For the first aim, we use the Love-Franey t-matrix model with the densities scaled from the D1S-GHFB+AMP neutron density, where D1S-GHFB+AMP stands for D1 S Gogny HFB (GHFB) with the angular momentum projection (AMP). The D1M-GHFB+AMP is also used to estimate a theoretical error. The resulting skin values are $r_{\text{skin}}^{208} = 0.324 \pm 0.047$ fm for D1S and $r_{\text{skin}}^{208} = 0.333 \pm 0.047$ fm for D1M. The difference $\Delta = 0.109$ fm and $r_{\text{skin}}^{208} = 3.470 \pm 0.075$ fm, leading to $r_{\text{skin}}^{208} = 0.144 \pm 0.075$ fm.

Conclusion: We conclude that $r_{\text{skin}}^{48} (\text{exp}) = 0.324 \pm 0.047 (\text{exp}) \pm (0.009)_{\text{th}}$ fm for p scattering at $E_{\text{lab}} = 534.1, 549, 806$ MeV. Our skin value $r_{\text{skin}}^{48} = 0.144 \pm 0.083 = 0.061 \sim 0.227$ fm is consistent with $r_{\text{skin}}^{208} (\text{CREX}) = 0.071 \sim 0.171$ fm.

Background: Horowitz et al. [1] proposed a direct measurement for neutron skin $r_{\text{skin}}$. The measurement consists of parity-violating weak scattering and elastic electron scattering. The neutron radius $r_n$ is determined from the former experiment, whereas the proton radius $r_p$ is from the latter.

The direct measurement was applied for $^{208}$Pb and $^{48}$Ca. As for $^{208}$Pb, the PREX collaboration presented $r_{\text{skin}}^{208} (\text{PREX2}) = 0.283 \pm 0.071$ fm, (1)

combining the original Lead Radius EXperiment (PREX) result with the updated PREX2 result [2,4]. As for $^{48}$Ca, the CREX group presented [5]

$$r_{\text{skin}}^{48} (\text{CREX}) = 0.121 \pm 0.026 (\text{exp}) \pm 0.024 (\text{model}) = 0.071 \sim 0.171 \text{ fm.} \quad (2)$$

The $r_{\text{skin}}^{208} (\text{PREX2})$ and the $r_{\text{skin}}^{48} (\text{CREX})$ are most reliable at the present stage, and provide crucial tests for the equation of state (EoS) of nuclear matter [6-10] as well as nuclear structure.

Reed et al. [11] reported a value of the slope parameter of the EoS and examine the impact of such a stiff symmetry energy on some critical neutron-star observables. The $r_{\text{skin}}^{208} (\text{PREX2})$ value is considerably larger than the other experimental values that are model-dependent [12-15]. Meanwhile, the nonlocal dispersive-optical-model (DOM) analysis of $^{208}$Pb yields $r_{\text{DOM}}^{208} = 0.25 \pm 0.05$ fm [16]. The value is consistent with $r_{\text{skin}}^{208} (\text{PREX2})$.

Using the chiral (Kyuushu) g-matrix folding model, we determine $r_{\text{skin}}^{208} (\text{exp}) = 0.278 \pm 0.035$ fm from reaction cross section $\sigma_R$ in $30 \sim E_{\text{lab}} \leq 100$ MeV [17]. In addition, for $^4\text{He}+$ $^{208}$Pb scattering, we determine $r_{\text{skin}}^{208} (\text{exp}) = 0.416 \pm 0.146$ fm from measured $\sigma_R$ in $E_{\text{lab}} = 30 \sim 50$ MeV [18]. These values are consistent with $r_{\text{skin}}^{208} (\text{PREX2})$.

For $^{12}$C scattering on $^4\text{He}, ^{12}$C, $^{37}$Al targets, we tested reliability of the Kyushu g-matrix folding model and found that the folding model is reliable in $30 \leq E_{\text{lab}} \leq 100$ MeV and $250 \leq E_{\text{lab}} \leq 400$ MeV [19]. Furthermore, we mentioned that the difference between the t-matrix and the g-matrix is small in $E_{\text{lab}} \geq 400$ MeV. Since the cutoff of the chiral nucleon-nucleon (NN) is 550 MeV, the chiral NN t-matrix is useful in $400 \leq E_{\text{lab}} \leq 500$ MeV. For $E_{\text{lab}} \geq 400$ MeV, the most famous t-matrix is Love-Franey (LF) t-matrix [20].

As for $^{208}$Pb, it is possible to determine reliable neutron radius $r_n (\text{PREX2}) = 5.727 \pm 0.071$ fm and matter radius $r_m (\text{PREX2}) = 5.617 \pm 0.044$ fm from $r_p (\text{exp}) = 5.444$ fm [21] of electron scattering and $r_{\text{skin}}^{208} (\text{PREX2})$. The $r_p$ calculated with D1S-Gogny-HFB (D1S-GHFB) with the angular momentum projection (AMP) agrees with $r_p (\text{exp})$. The neutron density calculated with D1S-GHFB+AMP is scaled so as to $r_n^{\text{scaling}} = 5.727$ fm. In Ref. [22], we showed that the LF t-matrix folding model with the scaled neutron density and the D1S-GHFB+AMP proton one reproduces the data $\sigma_R (\text{exp})$ at $E_{\text{lab}} = 534.1, 549, 806$ MeV within total error bars.

Nevertheless, we do not determine $r_{\text{skin}}^{208}$ from the data at $E_{\text{lab}} = 534.1, 549, 806$ MeV.

As for $^{48}$Ca, an indirect measurement is made with the high-resolution $E1$ polarizability experiment (E1Pe) [25]. The skin value $r_{\text{skin}}^{48} (\text{E1Pe}) = 0.14 \sim 0.20$ fm is consistent with $r_{\text{skin}}^{48} (\text{CREX})$. Using $^4\text{He}+$ $^{40}$Ca scattering in $E_{\text{lab}} = 30 \sim 50$ MeV, we determine matter radius $r_m^{40} (\text{exp})$ from measured $\sigma_R$ [18], whereas Zenihiro et al. deduce neutron radii $r_n^{48,40} (\text{exp})$ from the angular distributions of the cross sections and analyzing powers of polarized proton elas-
tic scattering at $E_{\text{lab}} = 295$ MeV [26]. The $r_{48}\text{ skin}(\exp)$ = 0.168$^{+0.025}_{-0.029}$ fm determined by Zenihiro et al. is consistent with $r_{48}\text{ skin}(\text{CREX})$.

**Aim:** The first aim is to determine $r_{208}\text{ skin}(\exp)$ from the data [23,24] on $\sigma_T$ of $p + 208\text{ Pb}$ scattering at $E_{\text{lab}} = 534.1, 549, 806$ MeV by using the LF $t$-matrix folding model.

The second aim is to determine $r_{48}\text{ skin}(\exp)$ with the result $r_{48}\text{ m}\text{(}\exp\text{)} = 3.361 \pm 0.075$ fm [18] of $^{4}\text{He} + ^{40}\text{Ca}$ scattering in $E_{\text{lab}} = 30 \sim 50$ MeV and the difference $\Delta = r_{48}\text{ m}(\exp) - r_{48}\text{ m}(\exp)$, since there is no data on $\sigma_T$ for $^{4}\text{He} + ^{40}\text{Ca}$ scattering. The derivation of $\Delta$ is shown below.

**Method for determining $r_{48}\text{ skin}(\exp)$:** Zenihiro et al. determine neutron radii $r_{40}\text{ m}(\exp) = 3.375^{+0.022}_{-0.023}$ fm and $r_{48}\text{ m}(\exp) = 3.555^{+0.026}_{-0.028}$ fm from the angular distributions of the cross sections and the analyzing powers of proton elastic scattering [26]. We can obtain the proton radii for $^{40,48}\text{Ca}$ with the isotope shift method based on the electron scattering [27], i.e., $r_{40}\text{ p}(\exp) = 3.378$ fm and $r_{48}\text{ p}(\exp) = 3.385$ fm. Using these values, we can obtain $r_{40}\text{ m}(\exp) = 3.777^{+0.023}_{-0.025}$ fm, $r_{48}\text{ m}(\exp) = 3.485^{+0.025}_{-0.026}$ fm.

From the central values of $r_{40}\text{ m}(\exp)$ and $r_{48}\text{ m}(\exp)$, we obtain the difference $\Delta = r_{48}\text{ m}(\exp) - r_{40}\text{ m}(\exp) = 0.109$ fm. In Ref. [18], meanwhile, we determined $r_{40}\text{ m}(\exp)$ = 3.361 $\pm$ 0.075 fm from measured $\sigma_T$ of $^{4}\text{He} + ^{40}\text{Ca}$ scattering in $E_{\text{lab}} = 30 \sim 50$ MeV. We can then obtain $r_{48}\text{ m}(\exp)$ = 3.470 $\pm$ 0.075 fm from $r_{40}\text{ m}(\exp)$ = 3.361 $\pm$ 0.075 fm and $\Delta$. The $r_{48}\text{ m}(\exp)$ = 3.470 $\pm$ 0.075 fm and $r_{48}\text{ m}(\exp)$ = 3.385 fm lead to $r_{48}\text{ skin}(\exp) = 0.144 \pm 0.075$ fm, respectively.

**Method for determining $r_{208}\text{ skin}(\exp)$:** We use the folding model based on Lovey-dovey (LF) $t$-matrix [20] to determine $r_{208}\text{ skin}(\exp)$ from data $\sigma_T(\exp)$ [23,24] at $E_{\text{lab}} = 534.1, 549, 806$ MeV. We have already applied the LF $t$-matrix folding model for $p + ^{4,6,8}\text{He}$ scattering at 700 MeV to determine matter radii $r_{m}(\exp)$ from the high-accuracy data [28]. The results are $r_{4m}(\exp) = 2.483 (3), 2.53 (2)$ fm and $r_{4\text{ skin}} = 0.78 (3), 0.82 (2)$ fm for $^{6,8}\text{He}$ [29].

Now we show the formulation on the LF $t$-matrix folding model below. For proton-nucleus scattering, the potential $U(\mathbf{R})$ between an incident proton ($p$) and a target ($T$) has the direct and exchange parts, $U^{\text{DR}}$ and $U^{\text{EX}}$, as

$$U^{\text{DR}}(\mathbf{R}) = \sum_{\mu,\nu} \rho^{\mu}_T(\mathbf{r}_T) \hat{t}^{\text{DR}}_{\mu\nu}(s; \rho_{\mu\nu}) d\mathbf{r}_T,$$

$$U^{\text{EX}}(\mathbf{R}) = \sum_{\mu,\nu} \int \rho^{\mu}_T(\mathbf{r}_T, \mathbf{r}_T + s) \times \hat{t}^{\text{EX}}_{\mu\nu}(s; \rho_{\mu\nu}) \exp[-i \mathbf{K}(\mathbf{R}) \cdot \mathbf{s}/M] d\mathbf{r}_T,$$

where $\mathbf{R}$ is the relative coordinate between $p$ and $T$, $s = -\mathbf{r}_T + \mathbf{R}$, and $\mathbf{r}_T$ is the coordinate of the interacting nucleon from $T$. Each of $\mu$ and $\nu$ denotes the $z$-component of isospin. The non-local $U^{\text{EX}}$ has been localized in Eq. (3b) with the local semi-classical approximation [30,32] where $\mathbf{K}(\mathbf{R})$ is the local momentum between $p$ and $T$, and $M = A/(1 + A)$ for the mass number $A$ of $T$; see Ref. [33] for the validity of the localization.

The direct and exchange parts, $t^{\text{DR}}_{\mu\nu}$ and $t^{\text{EX}}_{\mu\nu}$, of the $t$ matrix are described by

$$t^{\text{DR}}_{\mu\nu}(s) = \frac{1}{4} \sum_{S} \hat{S}^{2} \hat{S}^{1}_{\mu\nu}(S) \text{ for } \mu + \nu = \pm 1,$$

$$t^{\text{DR}}_{\mu\nu}(s) = \frac{1}{8} \sum_{S,T} \hat{S}^{2} \hat{T}^{S}_{\mu\nu}(S) \text{ for } \mu + \nu = 0,$$

$$t^{\text{EX}}_{\mu\nu}(s) = \frac{1}{4} \sum_{S} \hat{S}^{2} \hat{T}^{S}_{\mu\nu}(S) \text{ for } \mu + \nu = \pm 1,$$

$$t^{\text{EX}}_{\mu\nu}(s) = \frac{1}{8} \sum_{S,T} \hat{S}^{2} \hat{T}^{S}_{\mu\nu}(S) \text{ for } \mu + \nu = 0,$$

where $\hat{S} = \sqrt{2S + 1}$ and $\hat{T}^{S}_{\mu\nu}$ are the spin-isospin components of the $t$-matrix interaction.

As proton and neutron densities, $\rho_{p} = -1/2$ and $\rho_{n} = 1/2$, we use D1S-GHFB+AMP; see Ref. [34] for the formulation. As a way of taking the center-of-mass correction to the densities, we adapt the method of Ref. [35].

We scale the D1S-GHFB+AMP neutron density so that the radius $r_{n}(\text{scaling})$ of the scaled density can reproduce $\sigma_{T}(\exp)$, since the $r_{p}$ calculated with the D1S-GHFB+AMP density agrees with $r_{p}(\exp) = 5.444$ fm [21] of electron scaling. The same procedure is taken the D1M-GHFB+AMP neutron density, where D1M [36,37] is an improved version of D1S and the proton radius calculated with D1M-GHFB+AMP agrees with $r_{p}(\exp) = 5.444$ fm.

Our scaling procedure is explained below. The scaled density $\rho_{\text{scaling}}(r)$ is determined from the original (D1S-GHFB+AMP or D1M-GHFB+AMP) one $\rho(r)$ as

$$\rho_{\text{scaling}}(r) = \frac{1}{\alpha^3} \rho(r/\alpha), \quad r_{\text{scaling}} = r/\alpha$$

with a scaling factor

$$\alpha = \sqrt{\langle r^{2}\rangle_{\text{scaling}} / \langle r^{2}\rangle}.$$
FIG. 1. $E_{\text{lab}}$ dependence of reaction cross sections $\sigma_R$ for $p+^{208}\text{Pb}$ scattering. Open circles stand for the results of the LF $t$-matrix folding model with the D1S-GHFB+AMP densities, whereas open triangles correspond to that with the D1M-GHFB+AMP densities. The data are taken from Refs. [23,24].

Now we scale the D1S-GHFB+AMP neutron density so that the result of the LF $t$ matrix folding model agrees with the data [23,24]. In the present case, the neutron scaling factor is $\alpha = 1.017$. Since the resulting $r_n(\exp)$ depends on $E_{\text{lab}}$, we take the weighted mean and its total error for $E_{\text{lab}} = 534.1, 549, 806$ MeV. Neutron and matter radii thus obtained are $r_n(\exp) = 5.768 \pm 0.047$ fm and $r_m(\exp) = 5.643 \pm 0.047$ fm, leading to $r_{\text{skin}}^{208}(\exp) = 0.324 \pm 0.047$ fm.

The same procedure is taken for D1M-GHFB+AMP. This leads to $r_{\text{skin}}^{208}(\exp) = 0.333 \pm 0.047$ fm, where the neutron scaling factor is $\alpha = 1.038$. The theoretical error is evaluated with the difference between the central values of D1S-GHFB+AMP and D1M-GHFB+AMP. The value is $0.009$ fm. The result of D1S-GHFB+AMP yields better agreement with the data than that of D1M-GHFB+AMP. We then obtain $r_{\text{skin}}^{208}(\exp) = 0.324 \pm (0.047)_{\text{exp}} \pm (0.009)_{\text{th}}$ fm.

Discussions: Finally, the uncertainties of our results are listed.

1. Ambiguity of original densities taken: As for proton and neutron densities for $^{48}\text{Ca}$, we used D1S and D1M in Ref. [38]. Our result is $r_{\text{skin}}^{48}(\exp) = 0.158 \pm (0.023)_{\exp} \pm (0.012)_{\text{th}}$ fm; the theoretical error $(0.012)_{\text{th}}$ fm is evaluated with D1S and D1M. The same procedure is taken for $^{208}\text{Pb}$. Our result is $r_{\text{skin}}^{208}(\exp) = 0.324 \pm (0.047)_{\exp} \pm (0.009)_{\text{th}}$ fm.

2. Experimental ambiguity: Our present result $r_{\text{skin}}^{48} = 0.144 \pm 0.075$ fm based on $\Delta$ is consistent with $r_{\text{skin}}^{48}(\exp) = 0.158 \pm (0.023)_{\exp} \pm (0.012)_{\text{th}}$ fm of Ref. [38]. The central values are different from each other. The difference comes from the data used.

Conclusion: Our final values are $r_{\text{skin}}^{208}(\exp) = 0.324 \pm (0.047)_{\exp} \pm (0.009)_{\text{th}}$ fm and $r_{\text{skin}}^{48} = 0.144 \pm 0.075$ fm. Our results are consistent with $r_{\text{skin}}^{208}$ (PREX2) and $r_{\text{skin}}^{48}$ (CREX), respectively. These values are tabulated in Table I.

| $r_{\text{skin}}^{208}(\exp)$ or $r_{\text{skin}}^{48}(\exp)$ | Value |
|-----------------------------------------------|-------|
| PREX2                                        | $0.283 \pm 0.071$ |
| TW ($^{208}\text{Pb}$)                       | $0.324 \pm (0.047)_{\exp} \pm (0.009)_{\text{th}}$ |
| CREX                                         | $0.121 \pm 0.026(\exp) \pm 0.024(\text{model})$ |
| TW ($^{48}\text{Ca}$)                        | $0.144 \pm 0.075$ |

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