Algebraic form Problem Solving Based on Student Abstraction Ability

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Abstract. The purpose of this research is to describe the ability of problem solving algebraic form based on abstraction ability of Junior High School students in Pacitan. This research used qualitative method. The problem of this research is how the ability of problem solving on algebraic form based on students abstraction ability. The results showed that students were able to pass all four Polya problem solving steps on the subject of algebraic form well but only reached the third level of abstraction ability (structural abstraction), because the students did not know the reason for every answer that was found. The conclusion of this study is that students went through the four problem-solving steps on the subject of the algebraic form, and reached the third level of abstraction abilities. This research is expected to give information about things that can support the problem solving ability of student algebra form.

1. Introduction
Problem solving abilities are an absolute ability to possess by students when learning math. Rodney at all. defined problem solving as the use of knowledge and ability of individuals who have been previously owned, to face new and different situations [1]. Problem solving needs to be presented in a wide variety of subjects in mathematics, one of which is the algebraic form. Problem solving and algebraic are two aspects which are interrelated to high way of thinking. So it needs to be studied about ability problem solving algebraic form in terms of abstraction ability of student. Herskowitz at all suggests that abstraction is an activity where students reorganize mathematical structures that have been constructed prior to the new mathematical structures [2].

Drager conducted research on the relationship between abstract reasoning abilities and performance on the subject of algebraic. The control variables from this study were age, motivation, and previous achievement. The results showed that abstract reasoning ability became the most important variable in achieving algebraic learning success. However, the tool for measuring success is learning algebraic final exam test results in the form of multiple choice, so it is not known about the settlement process by the students [3]. Ferrari conducted research on abstractions in the mathematical perspective of history and learning. Ferrari also examined abstraction as generalization, abstraction and the emergence of new objects, abstraction in language, and abstraction and mathematics learning. However, Ferrari studies focus more on the realm of abstraction and not specifically in the realm of mathematical essence [4]. Based on these studies, there are problems that the mathematical tool does not use a problem that can describe the process of completion of students, and not the focus of studies on the essence of mathematics. So it is necessary to conduct research that examines the process of problem solving by students on the subject of mathematics in particular.
Based on the background, the purpose of this research is to describe the ability of problem solving algebraic form based on abstraction ability of middle school students in Pacitan. The research was conducted by using qualitative method, with research aids instrument in the form of description test. The results showed that students were able to pass all four Polya problem solving steps on the subject of algebraic form well but only reached the third level of abstraction ability, because the students did not know the reason for every answer that was found. This research is expected to provide teacher information about the need to improve students’ math problem solving abilities.

1.1 Problem solving Algebraic Form

Problem solving becomes one of the essential essences of mathematics. Polya states that there are four steps in problem solving, namely:

a. Understanding the problem, at this stage the student must be able to point out the core or important parts of the problem, such as the "known," "asked" and "conditions" in the problem.

b. Devising a plan, students think of ideas or ideas plan obtained from previous experience.

c. Carrying out the plan, in accordance with the previously prepared plan

d. Looking back, students re-examine the results that have been obtained and apply problem-solving solutions to other situations. [5]

Problem solving can be applied in various subjects in mathematics lessons. One is the algebraic form. The algebraic form is a mathematical form which in its representation contains letters to represent unknown numbers. Algebra contains elements in it, namely variables, coefficients, and constants. Consider the algebraic form \(5x + 3y + 8x – 6y + 9\). In the algebraic form, the letters \(x\) and \(y\) are called variables. The variable is a substitute symbol of an unknown number of values clearly. Variables are usually denoted by lowercase \(a, b, c, \ldots, z\). The number 9 in the algebraic form is called a constant. Constants are terms of an algebraic form of numbers and do not contain variables. The coefficient is a constant factor of a term in algebraic form. Consider the coefficient of each tribe in algebraic form \(5x + 3y + 8x – 6y + 9\). The coefficient on the \(5x\) is 5, the \(3y\) is 3, the \(8x\) is 8, and the \(-6y\) is -6. The tribe is a variable along with its coefficients or constants on algebraic forms separated by operating quantities or differences. Similar tribes are terms that have variables and rank of each of the same variables. Example: \(5x\) and \(-2x\), \(3a^2\) and \(a^2\). Unlike the tribe is a tribe that has the variables and rank of each variable is not the same. Example: \(2x\) and \(3x^2\), \(-y\) and \(-x^3\).

In this study the assessment of algebraic forms is presented in the problem solving model. It is intended so that researchers can know in depth the ability to solve the student's algebraic form. Clements, Murphy, Stacey and Macgregor stated that at this time, the assessment in algebra still tends to get the correct answer, the symbol of manipulation, rote skill and does not refer to conceptual financing or problem solving [6, 7, 8].

1.2 Abstraction Ability

Problem solving can be affected by many factors. As Chrysostomou discloses that factors that affect student problem solving skills include intelligence, logical thinking, creativity, cognitive style, personality, values, attitudes, and interests [9]. Johnson & Rising reported mathematical problem solving is a complex mental process, requiring visualization, imagination, manipulation, analysis, abstraction and idea statement. Based on these two expert opinions, it is clear that abstraction is the focus of problem solving, while problem solving is one of the essence of learning mathematics [10]. Herskowitz at all suggests that abstraction is an activity in which students reorganize previously structured mathematical structures into new mathematical structures [11]. Piaghet argues that abstraction is divided into three parts, the first empirical abstraction that focuses on the way students construct the properties of objects. Second, pseudo empirical abstraction focuses on the way students construct the meaning of action properties on objects. Third, the reflective abstraction that focuses on the idea of action and operation becomes the thematic object of thought [12]. Based on the three sections of the abstraction, reflective abstraction is a part of abstraction that is closely related to the problem solving ability. So the discussion will be focused on a reflective abstraction.

The levels in Cifarelli's reflective abstraction activity are as follows: first level is recognition, at this level problem solver faces a new situation, and recalls or identifies activity from previous
situations related to the problem at hand. The second level is representation, at this level indicates that in solving the problem and to assist reflection, the student uses a diagram. Reflection at this level acts as a tool to describe a certain degree of flexibility and control of the student over previously performed activities. The third level is structural abstraction, this level indicates that the student is able to project and reorganize the structure created from the activity and interpretation of the previous student into a new situation, thus increasing the student's own knowledge. The fourth level is structural awareness, in this level students will demonstrate an ability to anticipate the results of the activities undertaken, without having to solve in advance all the activities that are considered. Structural awareness refers to the awareness of student metacognition and organizing in its cognitive structure [13]. Goodson-Espy states that these abstraction level abilities can help identify whether a problem solver uses a previous settlement method or whether he or she uses a new troubleshooting method [14].

2. Experimental Method
This research used qualitative method with descriptive research type. The strategy chosen is a case study strategy. As revealed by Stake in Denzin and Lincoln that the case study is one strategy that many do in qualitative research, although not all use this case study is a qualitative research [15]. The subject of research is one Junior High School student in Pacitan with high academic ability. The subject is female. The data were collected in Grade VII of Junior High School in the academic year 2016/2017. The data used in this study is the primary data. This means that data obtained from the results of data mining by the researchers themselves. The data in this research is the result of writing test about problem solving ability based on Polya step on the subject of algebraic form, based on abstraction ability level and interview result. Technique of data collecting done by giving write test to student, then continued by interview so that can be explored more in abstraction ability and problem solving ability of student. The data analysis begins by reviewing the overall data consisting of written test result data and interview transcripts. The final stage is to summarize the problem solving abilities of students' algebraic abilities based on abstraction abilities.

3. Result and Discussion
Question:
Mrs. Ratna is a cake entrepreneur. Once Mrs. Ratna gets orders to make a wide variety of cakes. The material Mrs needs to buy. Ratna is 2 sacks of flour, one sack of coconut, and five egg buckets. Express in algebra the price of all the things Mrs. Ratna!
Answer:
Figure 1 below is a Photograph of the student's first student writing test answer.

![Figure 1. Photograph of Answer Writing Test in the First Activity](image)

Figure 1 show that students write down what is known in the matter of 2 sacks of rice, 1 sack of coconut, and 5 egg buckets. The student also writes the question asked of the algebraic form of the price of all the items Mrs. Ratna. The results of the researcher's interview with the subject on the first activity are listed in Table 1 below:
Table 1. The Result of First Activity Interview

| Code | Interview Result |
|------|------------------|
| R    | "Try to read the question!"  |
| SI1  | "Alright Mom" (students read question) |
| R    | "What information is contained in that question?" |
| SI2  | "There are things that are known and asked in it" |
| R    | "What is known and what is asked?" |
| SI3  | "There is a cake making material, namely 2 sacks of flour, 1 sack of coconut, and 5 egg buckets. And the question is the algebraic form of the price of all the things that will buy Mrs. Ratna." |
| R    | "According to you, what method will be used to solve the problem?" |
| SI4  | "Addition and multiplication" |

The student's first activity remembers and identifies the previous activity related to the problem. First the students read the questions as SI1 and understand the information contained in the problem (SI2). There are two main points in the matter that are the things that are known and asked as Figure 1 and the results of the interview on SI3. The activity is an activity on the level of recognition [12]. At this stage students also go through the first process of problem solving according to Polya, that is understanding the problem well [5]. The student is able to recall that the problem is related to the addition and multiplication operation. Students also plan that to solve the problem will apply the addition and multiplication operations that have been previously studied and change the problem into the form of mathematics (SI4). The planning activity is the second activity of problem solving based on Polya steps [5]. Next activity was students starting to solve the problems given based on the methods planned in the previous stage. Figure 2 shows the activities of both students in solving the problem.

Figure 2. Photograph of Answer Writing Test in the Second Activity

It can be seen in Figure 2 that students change the known points from matter to mathematical form. Students give flour as \(x\), coconut sebaagi \(y\), egg as \(z\), and price of all goods as \(c\). The results of interviews with students on the first activity are listed in Table 2 below:

Table 2. Results of Second Activity Interview

| Code | Interview Result |
|------|------------------|
| R    | "what steps will you use to solve the problem?"  |
| SI5  | "first I put the flour as \(x\), coconut as \(y\), egg as \(z\), and the price of all goods as \(c\)" |
| R    | "why do you think so?" |
| SI6  | "to make it easier to do" |

In activity Table 2 students solve the problem by running the planned method at SI4 stage. Students will enter the level of representation. At the level of representation, students perform activities representing problems into mathematical form and running pre-planned solutions [12]. Students are able to reach the level of representation well. Furthermore, the students try to represent the problem in the form of addition and multiplication, the price of all goods is the sum of the total goods to be purchased as shown in Figure 3 below:
The results of interviews with students on the third activity are listed in Table 3 below:

| Code | Interview Result                                                                 |
|------|----------------------------------------------------------------------------------|
| R    | “after that what are you doing?”                                                 |
| SI7  | “I use the sum and multiplication method, that the price of all the goods is the sum of the total goods to be purchased” |

At this stage the student enters the third step of problem solving that is to carry out the planned method [5]. In Figure 3 the student runs the addition and multiplication method as shown in Table 3. It shows that the student has known the relationship between the known and asked questions. The student continues the activity by reflecting the summation in Figure 3 into the algebraic problem. As Cifarelli pointed out this reflecting activity is a structural abstraction activity [12]. The student continues the activity by reflecting the summation in Figure 3 into the algebraic problem as shown in Figure 4 below.

The activity in Figure 4 shows that the student reflects the summation in Figure 3 into the algebraic problem. The student states that the price of all goods is the sum of the price of 2 sacks of flour + the price of one sack of coconut + the price of 5 eggs, and the algebraic form is $2x + y + 5z = c$. The results of the interview with the students on the fourth activity are listed in Table 4 below:

| Code | Interview Result                                                                 |
|------|----------------------------------------------------------------------------------|
| R    | “What is the sum and multiplication method you mean?”                             |
| SI8  | “so the price of all items purchased is the sum of the price Two bags of flour, plus the price of a single sack of coconut, plus the price of five eggs. And the algebraic form is $2x + y + 5z = c$” |

In Figure 4 and Table 4 the student used method by removing attributes other than mathematics like the name of the object, and only works on the mathematical context only. With the completion of the stage in Figure 4 means the student has found a solution to the given problem, but still need to know the reason students change the problem into a mathematical form. Activity shows the reason students change the problem into a mathematical form is a structural awareness activity [12]. Figure 5 below shows students’ ability to solve problems at the structural awareness stage.
In Figure 5 the student shows that $x$, $y$, and $z$ used as variables, 2, 1, and 3 as coefficients, and $c$ as constants. The results of the interview on Figure 5 are listed in Table 5 below.

### Table 5. Results of the fifth Interview Activity

| Code | Interview Result to S1 Subject |
|------|--------------------------------|
| R    | "Is the problem solving already completed?" |
| SI9  | "Yes Mom" |
| R    | "Then what do you mean by writing the variables, coefficients and constants?" |
| SI10 | "Because you asked me to write in algebraic form" |
| R    | "Well, what variables are there?" |
| SI11 | "x, y, and z" |
| R    | "Why did you call them variable?" |
| SI12 | "Because of letter" |
| R    | "Next, what coefficients are there?" |
| SI13 | "2, 1, 5" |
| R    | "Why did you call them coefficient?" |
| SI14 | "Because of number" |
| R    | "Then, what constants are there?" |
| SI15 | "c" |
| R    | "Why was c called as constant?" |
| SI16 | "because of c doesn’t have number" |

Figure 5 shows that students know which point used as variables, coefficients, and constants (SI11, SI13, and SI15). However, the student has not been able to provide an appropriate argument about the selection of points that used as variables, coefficients, and constants (SI12, SI14, and SI16). Student has not been able to reach the structural awareness level well. Then the students are given a question to find out the students’ ability in the final step of problem solving according to Polya as Table 6 below:

### Table 6. Interview Result of the Sixth Activity

| Code | Interview Result to S1 Subject |
|------|--------------------------------|
| R    | "Are you sure that all of your answers are true?" |
| SI17 | "Sure Mom" |
| R    | "Later, can we use problem solving this algebraic form to finish other problems?" |
| SI18 | "Of course mom, for example we buy more than one item in same shop, by using this way, we can know exactly price for each item" |

Table 6 shows that the student performed the final step of problem solving which is to review the results obtained. Student was able to provide arguments about the usefulness of problem solving solution for different situation [5].

4. Conclusion

Based on the analysis on the result of the research, the researcher concluded that student was able to solve problem in algebraic form by doing four steps of Polya’s problem solving and reached third level of abstraction ability (structural abstraction).

This research is expected to provide teacher information that students’ math problem solving skills need to be improved, as well as the need to examine matters that can support math problem solving skills. That’s because the ability to solve mathematical problems is very influential on the mindset of student.

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