We study the relaxation dynamics of an extended Fermi-Hubbard chain with a strong Wannier-Stark potential tilt coupled to a bath. When the system is subjected to dephasing noise, starting from a pure initial state the system’s total von Neumann entropy is found to grow monotonously. The scenario becomes rather different when the system is coupled to a thermal bath of finite temperature. Here, for sufficiently large field gradients and initial energies, the entropy peaks in time and almost reaches its largest possible value (corresponding to the maximally mixed state), long before the system relaxes to thermal equilibrium. This entropy peak signals a prethermal memory loss and a very different behavior in space and time. By comparing the system’s dynamics to that of a simplified model, the underlying mechanism is found to be related to the localization property of the Wannier-Stark system, which favors dissipative coupling between eigenstates that are close in energy.

This effect implies a prethermal memory loss and a universal non-equilibrium dynamics. It is reminiscent of the universal scaling behavior recently observed in isolated quantum gases \[73, 74\], which was attributed to non-thermal fixed points \[75, 76\].

We consider a one-dimensional extended Hubbard chain half filled with spin-polarized fermions and subjected to a linear potential gradient. It is described by the Hamiltonian

\[
H = -J \sum_{i=1}^{M-1} \left( c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i \right) + \sum_{i=1}^{M} W_i n_i + V \sum_{i=1}^{M-1} n_i n_{i+1}. \tag{1}
\]

Here \( c_i^\dagger \) and \( n_i = c_i^\dagger c_i \) are the creation and number operator for a fermion on lattice site \( i \). Moreover, \( J \) is the tunneling parameter, \( W_i = -\imath \) captures the potential gradient \( r \), and \( V \) quantifies nearest-neighbor interactions. The single-particle (bulk) eigenstates, known as Wannier-Stark states, are centered at the potential gradient \( r \), and \( V \) quantifies nearest-neighbor interactions. The single-particle (bulk) eigenstates, known as Wannier-Stark states, are centered at the potential gradient \( r \), and \( V \) quantifies nearest-neighbor interactions.

On the other hand, there is an increased recent interest in the non-equilibrium properties of open many-body quantum systems \[37, 69\]. While the coupling to an environment of finite temperature constitutes a natural situation, it is rather cumbersome to simulate. Therefore, the impact of dissipation on the dynamics of many-body systems is often treated by using dephasing noise \[70\] as a simpler bath model. Even though dephasing noise will eventually drive the system into an infinite-temperature state, it is assumed to qualitatively capture the effect of weak coupling to a bath on the transient evolution. Understanding, under which circumstances this assumption breaks down is an important question for the simulation of open many-body systems (see also Refs. \[71, 72\]).

Here we report on a surprising phenomenon in the relaxation dynamics of an interacting Wannier-Stark system coupled to a thermal bath of finite temperature. It sheds light on both of the above questions, since it can neither be observed for a disorder-localized system nor for dephasing noise. In a large parameter regime, we find that long before the system reaches thermal equilibrium, it transiently approaches the maximally mixed state.
FIG. 1. (a), (b) Time evolution of the von Neumann entropy for the tilted ladder [1] initialized in a Fock state with the left half of the chain occupied and coupled to (a) a dephasing bath [Eq. (3)] or (b) a thermal bath [Eq. (2)]. The dotted line marks the largest possible entropy \( S_\infty \). (c) Normalized peak entropy for initial Fock states of various energies \( E_{\text{ini}} \). The vertical dashed line marks \( E_\infty \). (d) Evolution of the mean occupation of the third lattice site relative to the time \( t_{\text{peak}} \), where the entropy peak is reached. The line colors mark different initial Fock states corresponding to colored bullets in (c). (e) Density profile \( \langle n_i \rangle \) at equidistant times during the time window marked by the shaded area in (b). The black line marks \( t = t_{\text{peak}} \). The inset shows the collapse of all the curves by rotating them by an angle proportional to the corresponding time. (f) Diagonal elements of the density matrix \( \rho_{kk} \) at three points in time [marked by (i)-(iii) in (b)] (solid lines) compared to effective thermal states of identical average energy (dashed lines). The parameters are \( M = 8 \), \( V = 1 \), \( \gamma = \eta = 0.1 \), and \( T \) so that \( S_T = S_\infty /2 \). The field gradient for (c)-(f) is \( r = 4 \).

\[
g(E) = \frac{J(E)}{(e^{E/T} - 1)}
\]

where we assume an Ohmic spectral density \( J(E) = E \).

In the high-temperature limit, we have \( g(E) \approx T \) and the transition rate \( R_{\text{qpd}} \) becomes independent of energy. Thus, the master equation reduces to

\[
\frac{d\rho}{dt} = -i[H, \rho] + \gamma \sum_{i=1}^{M} \left( n_i \rho_m - \frac{1}{2} \eta^2 \rho - \frac{1}{2} m_i^2 \right)
\]

with \( \gamma = \eta T \), describing dephasing noise.

Figure 1 shows the time evolution of the von Neumann entropy \( S = -\text{tr} \{ \rho \log(\rho) \} \) of the total system, when coupled to (a) a dephasing bath or (b) a finite-temperature bath. It is calculated by numerically integrating Eqs. (3) and (2), respectively, starting from a Fock state with the left half of the chain occupied. The temperature \( T \) of the thermal bath is chosen such that the corresponding equilibrium entropy, \( S_T \) (obtained for the Gibbs state \( \rho_T = Z_T^{-1} \sum_k e^{-\varepsilon_k/T} |k\rangle \langle k| \) with \( Z_T = \sum_k e^{-\varepsilon_k/T} \), is equal to half the largest possible entropy \( S_\infty = S_{T=\infty} = \log(\mathcal{D}) \), with Hilbert space dimension \( \mathcal{D} = M!/(\langle M/2 \rangle)!^2 \) (see Fig. S7 of the Supplemental Material [8] for the results of other temperatures). When the system is under dephasing noise [Fig. 1(a)], the entropy grows monotonously to the maximum value \( S_\infty \), being insensitive to the sign of the potential gradient \( r \). In turn, when the system is coupled to a finite-temperature bath [Fig. 1(b)], the entropy approaches its equilibrium value rather differently for negative and positive \( r \). While in the former case the entropy grows monotonously to \( S_T \) (except for small \( |r| \)), in the latter case it first reaches a peak value \( S_{\text{peak}} \) well above \( S_T \), before relaxing to equilibrium. As will become apparent in the following, this difference can be attributed to the different mean energies of the initial states.

Remarkably, we can observe in Fig. 1(b) that for large positive gradients \( r \), the peak entropy almost reaches the largest possible entropy \( S_\infty \) (dotted line), which uniquely corresponds to the maximally mixed state \( \rho_\infty = \rho_{T=\infty} = \mathcal{D}^{-1} \sum_k |k\rangle \langle k| \). This effect can be observed for a wide range of initial conditions: In Fig. 1(c) we plot the peak entropies \( S_{\text{peak}} \) reached during the evolution starting from various initial Fock states, versus their mean energy \( E_{\text{ini}} \) (scaled between 0, for the ground-state energy \( \varepsilon_{\text{min}} \), and 1, for the energy \( \varepsilon_{\text{max}} \) of the most excited state). Peak entropies close to \( S_\infty \) are found as long as \( E_{\text{ini}} \) lies well above the energy \( E_\text{median} = \text{tr}\{\rho_\infty H\} \) of the maximally mixed state (dashed line).

Since the maximally mixed state is unique, reaching an entropy peak with \( S_{\text{peak}} \approx S_\infty \) indicates that the memory of the initial condition is approximately lost already long before the system reaches thermal equilibrium [79]. Thus, we can expect the system dynamics to show universal behavior near and after approaching the peak entropy. This prethermal memory loss is confirmed in Fig. 1(d), where we plot the evolution of the site occupation \( \langle n_i \rangle \) relative to the time \( t_{\text{peak}} \) at which the entropy peak is reached. The different curves, which correspond to different initial states [labeled by line colors corresponding to the colored bullets in Fig. 1(c)], clearly converge near \( \eta(t - t_{\text{peak}}) = 0 \) and subsequently show almost identical behavior. Similar behavior can also be observed for other site occupations \( \langle n_i \rangle \) as well as for larger systems (see Figs. S3, S4, and Figs. S6, S9 of Ref. [8], respectively).

Moreover, we also find that the way the system approaches the maximally mixed state shows universal scaling behavior. In Fig. 1(e) we plot the density distribution, \( \langle n_i \rangle \), at various times near \( t_{\text{peak}} \) within
Further insight on how the system approaches the maximally mixed state is gained by looking at the probability distribution $p_k = \langle k | \rho | k \rangle$ for occupying many-body energy eigenstates $| k \rangle$. In Fig. 1(f) we plot the distribution $p_k$ (solid lines) for $r = 4$ at three times: (i) slightly before, (ii) at, and (iii) slightly after $t_{\text{peak}}$ [as indicated in Fig. 1(b) relative to the green curve]. Interestingly, these distributions agree rather well to those for thermal states $\rho_{T=\text{eff}}$ (dashed lines) with the effective temperature $T_{\text{eff}}$ determined by the instantaneous energy, $\text{tr} \{ \rho_{T=\text{eff}} H \} = E = \text{tr} \{ \rho H \}$. This observation suggests that the prethermal memory loss and the associated universal non-equilibrium dynamics are due to a dissipative form of prethermalization, where the system rapidly approaches a Gibbs state, whose effective temperature $T_{\text{eff}}$ then slowly relaxes to the equilibrium temperature $T$. This scenario immediately explains that an infinite temperature state is approached in a universal fashion as long as the initial energy $E_{\text{ini}}$ lies well above the infinite-temperature energy $E_{\infty}$. Namely, in this case the system has enough time to approach a prethermal state $\rho_{T=\text{eff}}$ before $1/T_{\text{eff}}$ passes through zero from below at the time when $E$ drops below $E_{\text{ini}}$. We will now argue that the mechanism for such dissipative prethermalization is rather different from standard prethermalization in isolated systems, which can be understood as a form of eigenstate thermalization to a generalized Gibbs state constrained by approximate integrals of motion $[80,81]$.

Although it is straightforward to see that a two-level system prepared in its excited state passes through the maximum entropy state while equilibrating with a thermal bath $[78]$, such behavior is highly nontrivial in the case of an interacting many-body system. In order to figure out the condition for a close-to-maximum peak entropy, let us focus on the weak-coupling limit, where the secular approximation $[37,38]$ is justified. Then, the master equation (2) reduces to a Lindblad form,

$$\frac{d\rho}{dt} = -i[H,\rho] + \gamma \sum_{k,q=1}^M R_{kq} \left( L_{kq} \rho L_{kq}^\dagger - \frac{1}{2} \{ L_{kq}^\dagger L_{kq} , \rho \} \right),$$

with rates $R_{kq} = R_{qk}$ that obey $R_{kq}/R_{qk} = e^{-(\varepsilon_k - \varepsilon_q)}/T$, so that low energy states are favored and the system is driven to the Gibbs state $\rho_T$. The matrix elements $\rho_{kq} \equiv \langle k | \rho | q \rangle$ obey $\dot{\rho}_{kq} = -i(\varepsilon_k - \varepsilon_q)\rho_{kq} + \eta \sum_p [R_{kp}\rho_{pp}\rho_{kq} - \frac{1}{2}(R_{pk} + R_{qp})\rho_{kq}]$, where diagonal and off-diagonal elements decouple from each other. The latter decay with rates $\Gamma_{kq} = \frac{1}{2}\eta \sum_p (R_{pk} + R_{qp})$ and have to be negligible already when the peak entropy is reached, to allow for the observed transient approach of the maximally mixed state (this is indeed the case, see Fig. S2 and discussion in Ref. [78]).
\( p_k \equiv \rho_{kk} \) is determined by the rate matrix \( R_{kj} \) through the Pauli rate equation \( \dot{p}_k = \eta \sum_{q} \left[ R_{kq} p_q - R_{qk} p_k \right] \) [87].

In Fig. 2 (a) and (b) we compare rate matrices for \( r = 0 \) and \( r = 4 \). Without potential gradient, one finds long-range coupling with respect to energy. In contrast, at a large field gradient (\( r = 4 \)) the transition rates predominantly close states that are close by in energy. The latter is a consequence of Stark localization, where eigenstates that are close in space, so that they are coupled by the bath via the densities \( n_i \), are close also with respect to energy. Note that for a disorder-localized Fermi-Hubbard chain without this spatio-energetic correlation we find non-local rate matrices and no prethermal memory loss (see Fig. S10 of Ref. [78]). This constitutes an even more drastic difference between the relaxation dynamics of Stark and disorder-induced MBL than the one observed for the system size \( N \), respectively. Scaling time with \( \tau = \ln(D/\alpha) + O(\alpha) \) [Fig. 3(d)].

To address the question, why the appearance of a close-to-maximum entropy \( S_{\text{peak}} \approx S_\infty \), we investigate the rate matrices for rather different on-site potentials \( W_i \) [see Fig. 2(c)] that (were optimized to) equally give rise to large peak entropies [see Fig. 2(d)]. It turns out that, indeed, they also show near-neighbor coupling [see Figs. 2(c)-(f)]. Moreover, also the tilted bosonic Hubbard chain [given by Eq. (1) with bosonic annihilation operators \( \epsilon_i \) and the last term replaced by on-site interactions \( \frac{1}{2} V \sum_i n_i (n_i - 1) \)] shows \( S_{\text{peak}} \approx S_\infty \) together with an energy local rate matrix [see Figs. 2(g)-(h)]. This, together with results for simplified models shown below (and in Fig. S13 of Ref. [78]) indicates also clearly that the prethermal memory loss discussed here is a very robust phenomenon.

To address the question, why the appearance of a close-to-maximum entropy is associated with local coupling between energy states, let us now consider a simplified rate model. Here the energy eigenstates have equally spaced non-degenerate energies \( \varepsilon_k = k \), with \( k = 0, 1, \cdots D - 1 \), and are coupled by thermal rates that are homogeneous and local with respect to energy. We define \( R_{k+n,k} \equiv R_n \), where \( R_n = 0 \) for \( |n| > n_m \) and \( R_n = g(nr) \) for \( |n| \leq n_m \), so that (as a property of the bath correlation function \( g \)) \( R_n/R_{-n} = (R_+/R_-)^n = e^{-nr/\tau} \), with \( R_\pm = R_{\pm1} \).

Let us first study nearest-neighbor coupling, \( n_m = 1 \). By defining \( \nabla^2 p_k = p_{k+1} + p_{k-1} - 2p_k \) and \( \nabla p_k = (p_{k+1} - p_{k-1})/2 \), we can cast the Pauli rate equation into the form of a discrete drift-diffusion equation, \( \dot{p}_k = \eta(\nabla^2 p_k + 5R\nabla p_k) \) [82], where diffusion and drift are quantified by \( R = (R_+ + R_-)/2 \) and \( \delta R = R_- - R_+ \), respectively. Scaling time with \( \langle gR \rangle^{-1} \), the model is completely characterized by the ratio \( \alpha = \delta R/\langle g \rangle \) and the system size \( D \).

Starting from the highest excited state, \( p_k(0) = \delta_k, k_{ini} \) with \( k_{ini} = D - 1 \), and setting \( D = 50 \), in Figs. 3(a) and (b) we plot the time evolution of \( p_k \) for \( \alpha = 0.1 \) and \( \alpha = 1 \), respectively (solid lines). While for \( \alpha = 0.1 \), a rather uniform distribution is found at a time \( t_{\text{peak}} \) [indicated by the fat line in Fig. 3(a)], approximating the maximum entropy state with \( p_k = 1/D \), this is not the case for larger drift, \( \alpha = 1 \). Before reaching thermal equilibrium, we find the distribution well approximated by a Gaussian of standard deviation \( \sigma = \sqrt{2\langle g \rangle t} \) centered at \( k_0 = k_{ini} - \delta R t \) [82] (dashed lines). The condition for reaching an almost flat distribution is, thus, given by the intuitive requirement that the drift time needed to reach \( k_0 = D/2, \tau_F = (k_{ini} - D/2)/\delta R \), is larger than the diffusion time giving rise to \( \sigma = D/2 \), \( \tau_D = D^2/(8\langle g \rangle) \). Thus, for \( k_{ini} = D - 1 \) we expect \( S_{\text{peak}} \approx S_\infty \) as long as \( \alpha \lesssim 4D^{-1} \), which is confirmed in Fig. 3(c) [see also Fig. S12(a) of Ref. [78]]. However, \( S_{\text{peak}} \approx S_\infty \) is a non-trivial result only as long as the thermal entropy \( S_T \), plotted in Fig. 3(d), lies well below \( S_\infty \). For \( D \tau \gg T \) we can neglect the upper bound of the spectrum and \( S_T \) approaches the value for an harmonic oscillator with frequency \( \omega_r \), \( S_T \approx \pi^{-1} \log(x) - \log(1-x) \) with \( x = e^{-r/T} = R_+/R_- = (2 - \alpha)/(2 + \alpha) \), so that for \( \alpha < 1 \) one has \( S_T \approx \log(1/\alpha) + \bigO(\alpha) \) [Fig. 3(d)]. Thus, \( S_T/S_\infty < s \) as long as \( \alpha \gtrsim eD^{-\alpha} \). While for \( D \rightarrow \infty \) this requirement is incompatible with the one for large peak entropies, \( \alpha \lesssim 4D^{-1} \), it turns out that the different prefactors appearing in both conditions (whose values can deviate from our estimates \( e \) and \( 4 \)) still give rise to a large non-trivial regime for finite \( D \), as can be inferred from Figs. 3(c) and (d).

The simplified rate model with \( n_m = 1 \) roughly corresponds to the case of a single particle in a tilted lattice. While it describes peak entropies \( S_{\text{peak}} \approx S_\infty \) and prethermal memory loss, it gives rise to a Gaussian rather than an exponential prethermal distribution. This

![FIG. 3. Simplified rate model with nearest-neighbor coupling](image-url)
FIG. 4. Evolution of the simplified rate model starting from the most excited state for $D = 50$, $\eta = 0.1$, and $\ell$ so that $S_{T} = S_{\infty}/2$. (a) Entropy evolution for different $n_m$. Inset: Normalized peak entropies versus $n_m$. (b) Distribution $p_k$ (solid lines) for $n_m = 11$ at the times marked in the inset compared to effective Gibbs states (dashed lines).

suggests that the formation of a prethermal Gibbs state requires more complex rate matrices as they are found for the many-particle case. In Fig. [4] we investigate, what happens when increasing the coupling range $n_m$, and thus the complexity, of the simplified rate model [using $D = 50$ and $\eta = 0.1$]. We observe that $S_{\text{peak}}$ first increases with $n_m$ before, after reaching a maximum at $n_m = 11$, it decreases again. The first increase with $n_m$ might be explained by the prethermal distribution becoming more Gibbs like and thus flatter at $t = t_{\text{peak}}$ than the Gaussian (which always retains a finite $\sigma$). This is confirmed in Fig. [4(b)], where we plot the distribution $p_k$ at different times for $n_m = 11$ and find rather good agreement with an effective Gibbs state. The subsequent decrease of $S_{\text{peak}}$ can, in turn, be attributed to an increase of the drift velocity with $n_m$, which is clearly visible also in Fig. [4(a)] and which reduces the time available for reaching a prethermal distribution. This mechanism explains, why large transient peak entropies $S_{\text{peak}} \approx S_{\infty}$, are found for rate matrices that are local in energy.

In conclusion, we have shown that the non-equilibrium relaxation dynamics of interacting Wannier-Stark ladders coupled to a finite-temperature environment can feature prethermal memory loss and universal scaling behavior. The effect is found to rely on a dissipative form of prethermalization. In experiment with ultracold atoms, a thermal environment could be provided, for instance, by the coupling to a second atomic species [33].

We acknowledge discussions with Markus Oberthaler. This research was funded by the Deutsche Forschungsgemeinschaft (DFG) via the Research Unit FOR 2414 under Project No. 277974659.

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A TWO-LEVEL SYSTEM

Consider a two-level system with eigenstates $|+\rangle$, $|-\rangle$ and the corresponding eigenenergies $\varepsilon_{\pm}$. According to the Lindblad master equation introduced in the main text, the dynamics of the corresponding occupation probabilities $p_{\pm}$ is governed by the Pauli master equation

$$\dot{p}_{+} = \eta(R_{+}p_{-} - R_{-}p_{+}) = -\dot{p}_{-}. \quad (S1)$$

Its solution is given by

$$p_{+} = \alpha - (\alpha - p_{+}(0))e^{-\eta R_{0}t} = 1 - p_{-}, \quad (S2)$$

where $R_{0} = R_{+} + R_{-}$ and $\alpha = R_{+}/R_{0}$. While $\alpha = 1/2$ for dephasing noise, for a thermal bath of finite temperature we find $\alpha = (1 + e^{2\Omega/T})^{-1}$, with $\Omega = \varepsilon_{+} - \varepsilon_{-}$ being the energy splitting.

The off-diagonal terms of the density matrix

$$\rho = \begin{pmatrix} p_{+} & \rho_{+-} \\ \rho_{-+} & 1 - p_{+} \end{pmatrix}, \quad (S3)$$

are described by

$$\rho_{\pm\mp} = e^{i\Omega t - \eta R_{s}t/2}\rho_{\pm\mp}(0), \quad (S4)$$

with $R_{s} = \sum_{k,q=\pm} R_{kq}$.

To quantify the difference between thermal bath and dephasing noise, we investigate the purity. It is calculated as $f = \sum_{k} \lambda_{k}^{2}$, with $\lambda_{k}$ being the eigenvalues of the density matrix. It shows similar behavior as the von Neumann entropy $S = -\sum_{k} \lambda_{k} \log(\lambda_{k})$, but is easier to handle analytically. For the two-level system, we can verify that the purity is given by

$$f = \frac{1}{2} + 2|p_{+-}|^{2} + 2(p_{+} - 1/2)^{2}$$

$$= A_{1}e^{-\eta R_{0}t} + A_{2}e^{-2\eta R_{0}t} + Be^{-\eta R_{s}t} + C, \quad (S5)$$

with

$$A_{1} = 2(1 - 2\alpha)(\alpha - p_{+}(0)),$$

$$A_{2} = 2(\alpha - p_{+}(0))^{2},$$

$$B = 2|p_{+-}(0)|^{2},$$

$$C = 2\alpha(\alpha - 1) + 1. \quad (S6)$$

Except for $A_{1}$, all the other coefficients are non-negative. For dephasing noise, we have $\alpha = 1/2$, thus $A_{1} = 0$. Hence, the purity $f$ decays monotonously to its steady state value $C = 1/2$. While for a finite-temperature bath, $A_{1}$ can be negative, depending on the parameters of the system and the initial state. When $A_{1} < 0$, there will be a competition between the first term and the others, leading to a non-monotonous behavior in the purity. An obvious example is when we start from the excited state. Then the purity will degrade from its initial value 1 (for a pure state with $p_{+} = 1$) to the minimal value 1/2 (uniquely corresponding to the maximally mixed state with $p_{+} = p_{-} = 1/2$), and then increase again until it approaches the steady state value (for the finite-temperate thermal state with $p_{+} < 1/2$). In contrast, if the initial state is the ground state, the purity decreases monotonously, both for thermal bath and dephasing noise.
1D SPINLESS FERMION CHAIN SUBJECTED TO A LINEAR POTENTIAL

Dependence of the entropy on temperature

Figure S1(a) shows the dynamics of the von Neumann entropy $S$ for a half-filling spinless Fermion chain described by Hamiltonian (1) in the main text coupled to a thermal bath at different temperatures. The peak entropy during the evolution shows a weak dependence on the temperature, with a larger entropy at a higher temperature, as shown in Fig. S1(b).

FIG. S1. (a) Time evolution of the von Neumann entropy $S$ for a half-filling spinless Fermion chain whose dynamics is governed by Eq. (2). The dotted line marks the maximum entropy $S_\infty$ for the maximally mixed state. The initial state is a Fock state with the left side of the chain occupied. (b) Peak entropy $S_{\text{peak}}$ as a function of the temperature of the thermal bath. The parameters are $M = 8$, $V = J$, $r = 4J$, $\eta = 0.1J$.

Dependence of the entropy on the initial state

FIG. S2. (a),(c) Time evolution of the von Neumann entropy $S$ for a half-filling spinless Fermion chain coupled to a thermal bath described by Eq. (2). The dotted line marks the maximum entropy $S_\infty$ for the maximally mixed state. The temperature of the thermal bath is chosen such that $S_T = S_\infty/2$. (b),(d) Peak entropy $S_{\text{peak}}$ (normalized by $S_\infty$) as a function of the averaged energy of the initial state $E_{\text{ini}}$ (scaled between 0, for the ground-state energy $\epsilon_{\text{min}}$, and 1, for the energy $\epsilon_{\text{max}}$ of the most excited state). The dotted line marks the average energy of the system, which is also the mean energy of the maximally mixed state (infinity-temperature state), $E_\infty = \text{tr}(\rho_\infty H)$. The initial states for (a), (b) are Fock states, and the initial states for (c), (d) are many-body energy eigenstates. The dashed lines in (c) are the results of (a). The parameters are $M = 8$, $V = J$, $r = 4J$, $\eta = 0.1J$.

In Fig. S2 we compare the time evolution of the entropy for different initial states. While (a) and (b) correspond to initial Fock states, (c) and (d) capture the dynamics starting from many-body energy eigenstates. While for the
former scenario the initial density matrix possesses off-diagonal elements in energy representation, this is not the case
for the latter one. Thus, the fact that both scenarios show almost identical behavior, indicates that the off-diagonal
elements of the initial Fock states have decayed before reaching the peak entropy.

Universal behavior of mean occupation in real space

Figure S3 shows time evolution of the mean occupation in real space \( \langle n_i \rangle \) for a half-filling chain under a linear
potential described by Hamiltonian (1) starting from different initial Fock states. The dashed lines denote the time
\( t_{\text{peak}} \) when the peak entropy is reached. By plotting the evolution of the site occupation relative to the time
\( t_{\text{peak}} \), the different curves clearly converge near \( t = t_{\text{peak}} \), as shown in Fig. S4.

![Figure S3](image1)

**FIG. S3.** Time evolution of the mean occupation in real space \( \langle n_i \rangle \) for a half-filling Fermion chain [with Hamiltonian (1)]
coupled to a thermal bath described by Eq. (2). The vertical dashed lines marks the time when the entropy gets maximal. The
initial states are 7 Fock states with the highest energies. The parameters are \( M = 8, r = 4J, V = J, \eta = 0.1J \).

![Figure S4](image2)

**FIG. S4.** The same as Fig. S3 except that the time \( t \) is shifted by the time to get the peak entropy \( t_{\text{peak}} \). The black dashed
line marks \( t = t_{\text{peak}} \).
FIG. S5. (a) Time evolution of the von Neumann entropy $S$ for a half-filling spinless Fermion chain [with Hamiltonian (1)] coupled to a thermal bath described by Eq. (2) starting from three different initial Fock states. (b)-(d) Mean occupation in real space for the time window marked by shading area in (a). The inset shows the collapse of the curves by rotating them by an angle proportional to the corresponding time. The axis in the inset is $i' = (i - i_0) \cos[\chi(t - t_0)] + (\langle n_i \rangle - y_0) \sin[\chi(t - t_0)]$, and $\langle n_i \rangle' = ((\langle n_i \rangle - y_0) \cos[\chi(t - t_0)] - (i - i_0) \sin[\chi(t - t_0)]$ with the parameters $x_0, y_0, t_0$ and $\chi$ adjusted to get the optimal collapse. The parameters are $M = 8, V = J, r = 4J, \eta = 0.1J, T$ is chosen to make $S_T = S_\infty/2$.

In Fig. S5 we show the mean occupation in real space starting from three different initial Fock states [(b)-(d)] for the time window marked by shading area in (a). The density profiles at different times in the vicinity of peak entropy are found to share similar shape. As shown in the inset, by rotating these curves by an angle proportional to the corresponding time, they collapse onto each other.

Results for larger systems

Pauli rate equation

According to the Lindblad master equation introduced in the main text, the dynamics of the diagonal and off-diagonal elements of the density matrix are decoupled. By neglecting the off-diagonal elements (which allows us to study larger system), the density matrix is given by $\rho = \sum_k p_k |k\rangle \langle k|$, with the diagonal elements $p_k$ governed by the Pauli rate equation

$$\dot{p}_k = \eta \sum_q (R_{kq} p_q - R_{qk} p_k). \tag{S7}$$

Figure S6 shows time evolution of the mean occupation in real space $\langle n_i \rangle$ for a half-filling Fermion chain with $M = 16$ sites by solving Eq. S7. We find similar universal dynamics as in small system [see Fig. S4].
FIG. S6. Time evolution of the mean occupation in real space $\langle n_i \rangle$ starting from different initial states with high energies. The density matrix is approximated by $\rho = \sum_k p_k |k\rangle \langle k|$ with $p_k$ governed by Eq. (S7). The parameters are $M = 16$, $V = J$, $r = 4J$.

**Kinetic theory**

The equations of motion for the mean occupations in single-particle eigenstates are given by

$$\langle \dot{n}_k \rangle = \sum_q \left[ \tilde{R}_{kq} \langle n_q \rangle - \tilde{R}_{qk} \langle n_k \rangle \right],$$

(S8)

where $\tilde{R}_{kq} = \sum_i |\psi_{ik}|^2 |\psi_{iq}|^2$ and $\psi_{iq}$ is the single particle eigenstate. In order to obtain a closed set of equations in terms of the mean occupations, we employ the mean-field approximation,

$$\langle n_k n_q \rangle \simeq \langle n_k \rangle \langle n_q \rangle,$$

(S9)

for $k \neq q$. Then we obtain a set of nonlinear kinetic equations of motion

$$\langle \dot{n}_k \rangle = \sum_q \left[ \left( \tilde{R}_{kq} \langle n_q \rangle - \tilde{R}_{qk} \langle n_k \rangle \right) - \left( \tilde{R}_{kq} - \tilde{R}_{qk} \right) \langle n_k \rangle \langle n_q \rangle \right],$$

(S10)

which can be solved numerically.

The mean-field approximation is equivalent to a Gaussian ansatz for the density matrix

$$\rho = \frac{1}{Z} \exp(-\sum_k \alpha_k n_k),$$

(S11)
where $Z$ is the partition function and $\alpha_k$ is determined by the $M$ mean occupations $\langle n_k \rangle$, as $\alpha_k = \ln(\langle n_k \rangle^{-1} - 1)$. Figure S7 compares the time evolution of entropy for a half-filling chain of $M = 8$ sites from exact diagonalization (solid lines) and from mean-field theory (dashed lines) for different initial Fock states. Figure S8 compares the time evolution of the corresponding mean occupation in real space. The deviations that are visible in the steady state result from the fact that Eq. (S11) corresponds to the grand canonical ensemble, rather than to a canonical Gibbs state. Figure S9 shows the time evolution of the mean occupation in real space for a half-filling chain of $M = 20$ sites from mean-field theory starting from different initial Fock states.

**FIG. S7.** Comparison of time evolution of entropy from numerically solving Eq. (2) (solid lines) and from kinetic theory (dashed lines) for different initial Fock states. The parameters are $M = 8$, $V = 0$, $r = 4J$. 
FIG. S8. Comparison of time evolution of the mean occupation in real space $\langle n_i \rangle$ from numerically solving Eq. (2) (solid lines) and from kinetic theory (dashed lines) for different initial Fock states. The parameters are $M = 8, V = 0, r = 4J$. 
FIG. S9. Time evolution of the mean occupation in real space \( \langle n_i \rangle \) from kinetic theory for a half-filling chain under a linear potential described by Hamiltonian (1). The vertical dashed lines marks the time of reaching peak entropy. The initial states are different Fock states with high energies. The parameters are \( M = 20, \; r = 4J, \; \eta = 0.1J \).

DISORDERED POTENTIAL

Fig. [S10(a)] shows the rate matrix for a Fermi-Hubbard chain under a disordered potential with random on-site energies uniformly drawn from the interval \([-W, W]\) coupled to a thermal bath. It has a non-local structure, which is different from that of the Stark model in the main text [see Fig. 2(b)]. Such a rate matrix can not give rise to a close-to-maximum peak entropy, as shown in Fig. [S10(b)].

OPTIMIZED POTENTIAL MODEL

Figure [S11] shows time evolution of entropy \( S = - \sum_k p_k \log(p_k) \) for a half-filling chain (described by Hamiltonian (1) with the on-site potential \( W \), shown in the inset) coupled to a thermal bath. The dynamics of \( p_k \) is governed by rate equation. The distribution \( p_k \) at the three different times marked by vertical lines in (a) is shown by solid lines in
FIG. S10. (a) Rate matrix for a Fermi-Hubbard chain under a disordered potential with disorder strength $W = 20J$ coupled to a thermal bath. (b) Time evolution of entropy. The initial state is the highest excited state. The results are for one disorder realization. The parameters are $M = 8, V = J, \eta = 0.1J, T$ is chosen to make $S_T = S_\infty/2$.

(b). It is found to be close to a thermal distribution, analogous to the Stark model [see Fig. 1(f) in the main text].

FIG. S11. (a) Time evolution of entropy $S = -\sum_k p_k \log(p_k)$ for a half-filling chain (described by Hamiltonian (1) with the on-site potential $W_i$ shown in the inset) coupled to a thermal bath. The dynamics of $p_k$ is governed by rate equation (S7). (b) Distribution of $p_k$ at three moments marked in (a) are shown in solid lines, which is close to the distribution of the corresponding thermal states with the same average energy shown in dashed lines. The parameters are $M = 8, V = J, \eta = 0.1J, T$ is chosen to make $S_T = S_\infty/2$.

SIMPLIFIED MODELS

Figure S12 shows the normalized peak entropy $S_{\text{peak}}$ and thermal entropy $S_T$ for the simplified model where $p_k$ is governed by $p_k = \eta(R \nabla^2 p_k + \delta R \nabla p_k)$ starting from the highest excited state. In (a)-(c), the dependence of $S_{\text{peak}}$ and $S_T$ on $\alpha = \delta R/R$ for three different system dimensionality $D$ is shown. In (d)-(f), the dependence of $S_{\text{peak}}$ and $S_T$ on $D$ for three different $\alpha$ is shown. The results indicate the existence of a non-trivial parameter regime with both $S_{\text{peak}} \approx S_\infty$ and $S_T$ well below $S_\infty$.

In Fig. S13(a) we show the time evolution of entropy $S = -\sum_k p_k \log(p_k)$ with $p_k$ governed by rate equation under two different rate matrices shown in (b) and (c). The former ($R_1$) is the real rate matrix of the noninteracting Stark model. It depends on both the bath correlation function and the overlap of wave functions, i.e., $R_{kq} = \pi v_{kq} g(\varepsilon_k - \varepsilon_q)$ with $v_{kq} = \sum_i |\langle k | n_i | q \rangle|^2$, which endows it a complex texture. The rate matrix $R_2$ used in (c) is obtained from $R_1$ by replacing $v_{kq}$ by the binary values $\epsilon$ or 0, depending on whether $|v_{kq}| > 0.1$ or $< 0.1$, respectively. As shown in Fig. S13(a), the entropies for these two rate matrices are found to be almost the same. In (d)-(f), we show the corresponding results for interacting system with $V = J$. 
FIG. S12. The peak entropy ($S_{\text{peak}}$) and the thermal entropy ($S_T$) (normalized by the maximum entropy $S_\infty = \log(D)$) as a function of (a)-(c) $\alpha \equiv \delta R/\bar{R}$ and (d)-(f) $D$. The entropy is calculated as $S = -\sum_k p_k \log(p_k)$, with $p_k$ governed by rate equation (S7). The initial state is $p_k = \delta_{k,D-1}$ for $0 \leq k \leq D - 1$. The dotted lines are from the expression $S_T = \frac{x}{x-1} \log(x) - \log(1-x)$ with $x = (2 - \alpha)/(2 + \alpha)$.

FIG. S13. (a) Time evolution of the entropy $S = -\sum_k p_k \log(p_k)$ with $p_k$ governed by rate equation (S7) under two different rate matrices shown in (b) and (c). The initial state is the highest excited state. The parameters are $M = 8$, $V = 0$, $\epsilon J = 4$, $\eta = 0.1 J$, $\epsilon = 0.15$. (d)-(f) show the corresponding results for interacting system with $V = J$. The fitting parameter is $\epsilon = 0.22$ in this case.