Strong interaction and bound states in the deconfinement phase of QCD

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Abstract

Recent striking lattice results on strong interaction and bound states above $T_c$ can be explained by the nonperturbative $Q\bar{Q}$ potential, predicted more than a decade ago in the framework of the field correlator method. Explicit expressions and quantitative estimates are given for Polyakov loop correlators in comparison with lattice data. New theoretical predictions for glueballs and baryons above $T_c$ are given.

1 Introduction

There is a growing understanding nowadays that nonperturbative dynamics plays important role in the deconfinement phase, for reviews and references see [1].

An additional part of this understanding, not contained in [1], is the realization of the fact, that at $T_c < T < 2T_c$, the colormagnetic fields are as strong as they are in the confinement phase, (where colormagnetic and colorelectric fields are of the same order) and become even stronger above $2T_c$. More than a decade ago the author has argued [2,3] that the deconfinement phase transition is the transition from the color-electric confinement phase to the colormagnetic phase, confining in 3d. This observation was supported theoretically by the calculation of $T_c$ [2,3] and on the lattice by the calculation of the spacial string tension at $T > T_c$ [4].
In 1991 the author has found [5] that colorelectric fields also survive the deconfinement transition in the form of potential $V_1(r)$, which can support $Q\bar{Q}$ bound states in some temperature interval $T_c < T < T_D$, while quarks acquire self-energy parts equal to $\frac{1}{2}V_1(\infty)$. As will be shown below this predicted picture is fully supported by recent numerous lattice experiments [6]-[13] (for a review see [10]), where $Q\bar{Q}$ bound states have been discovered. At the same time the evidence for $V_1(r)$ and selfenergies $\frac{1}{2}V_1(\infty)$ has been also obtained on the lattice in the form of Polyakov loop averages and of free and internal energies above $T_c$ [10, 11, 12]. The light quark ($m_q \sim m_s$) $q\bar{q}$ bound states have also been observed in [13].

The theory used in [5] and below is based on the powerful Method of Field Correlators MFC [14], (for a review see [15]), where the basic dynamic ingredients are the field correlators $\langle tr F_{\mu\nu}(x_1)\ldots F_{\mu\nu}(x_n) \rangle$. It was shown later [16] that the lowest quadratic (so-called Gaussian) correlator explains more than 90% of all dynamics and it will be considered in what follows. The quadratic correlator consists of two scalar form-factors, $D$ and $D_1$:

$$D_{\mu\nu,\lambda\sigma}(x,0) \equiv \frac{g^2}{N_c} tr(F_{\mu\nu}(x)\Phi(x,0)F_{\lambda\sigma}(0)) = (\delta_{\mu\lambda}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\lambda}) D(x) + \frac{1}{2} \left[ \frac{\partial}{\partial x_\mu}(x_\lambda\delta_{\nu\sigma} - x_\sigma\delta_{\nu\lambda}) + (\mu\lambda \leftrightarrow \nu\sigma) \right] D_1(x)$$

which produce the following static potential [17] between heavy quarks at zero temperature (obtained from the Wilson loop $r \times t$ with $t \to \infty$)

$$V(r) = 2r \int_0^\infty d\lambda \int_0^\infty d\tau D(\sqrt{\lambda^2 + \tau^2}) + \int_0^\infty \lambda d\lambda \int_0^\infty d\tau [-2D(\sqrt{\lambda^2 + \tau^2}) + +D_1(\sqrt{\lambda^2 + \tau^2})] = V_D(r) + V_1(r).$$

One can notice that linear confinement part of potential, $V_D = \sigma R$, is due to correlator $D(x)$, $\sigma = \frac{1}{2} \int D(x) d^2x$.

At $T > 0$ one should distinguish between electric and magnetic correlators, $D^E(x)$, $D_1^E(x)$, and $D^H(x)$, $D_1^H(x)$ and correspondingly between $\sigma^{(E)}$ and $\sigma^{(H)}$. It was argued in [2, 3] that the principle of minimality of free energy requires $D^E$ and electric confinement, $\sigma^{(E)}$, to vanish, while color-magnetic correlators, $D^H(x)$, $D_1^H(x)$ should stay roughly unchanged at least up to $2T_c$.

Several years later in detailed studies on the lattice in [18] these statements have been confirmed, and indeed magnetic correlators do not change at $1.5T_c > T > T_c$ while $D^E(x)$ vanishes in vicinity of $T_c$.

\footnote{we omit for simplicity parallel transporters $\Phi(x_i, x_j)$ necessary to maintain gauge invariance, see [15] for details.}
Not much was said about the second electric correlator $D_{1}^{E}(x)$, since in
the parametrization of [18] it was found to be smaller than $D^{E}(x)$ and hence
not so important at $T < T_{c}$.

Meanwhile a lot of information was being accumulated on the lattice.
First of all, the Polyakov loop averages already imply the presence of strong
electric fields above $T_{c}$, and the main point is that those can not be reduced
to the perturbative electric and magnetic, (see the analysis in [11, 12]).

Recently a detailed analysis of Polyakov loop correlators was done by
the Bielefeld group [9]-[12] and the singlet free energy $F_{1}(r, T)$ and internal
energy $U_{1}(r, T)$ were calculated at $T < T_{c}$ and at $T > T_{c}$. In the latter case
$F_{1}(r, T)$ was found to saturate at large $r$ at the values of the order of several
hundred MeV (e.g. for $T = 1.2T_{c}$ the value of $F_{1}(\infty, T)$ found in [12] is
around $0.7\sqrt{\sigma}$, while the internal energy is around $3T_{c}$) and this fact cannot
be explained by perturbative contributions alone – we consider it as the most
striking revelation of nonperturbative electric fields above $T_{c}$.

At the same time several groups have calculated the so-called spectral
function of heavy [6]-[10] and light quarkonia [13] at $T > T_{c}$. In all cases
sharp peaks have been observed, corresponding to the ground state levels of
c$\bar{c}$ at $L = 0, 1$ and of light $(m_{q} \approx m_{s})$ quarkonia in $V, A, S, PS$ channels. In
both heavy and light cases the peaks are possibly displaced as compared to
$T = 0$ positions and apparently almost degenerate in different $n\bar{n}$ channels.

All these facts cannot be explained in the framework of the commonly
accepted perturbative quark-gluon plasma and call for a new understanding
of the nonperturbative physics at $T > T_{c}$. In what follows we shall argue
following [5] that at $T > T_{c}$ not only nonperturbative magnetic fields, but also
strong nonperturbative electric fields are present, which can be calculated in
MFC and explain the observed data.

2 Dynamics of Polyakov loops and the correlator $D_{1}$

In this section we consider the Polyakov loop and apply to it the nonabelian
Stokes theorem and Gaussian approximation, taking first the loop as a circle
on the plane and making limiting process with the cone surface $S$ inside loop
and finally transforming cone into the cylinder by tending the vertex of the
cone to infinity\textsuperscript{2}. In doing so we are writing the nonabelian Stokes theorem and cluster expansion for the surface $S$ which is transformed from the cone to the (half) cylinder surface. As a result one has for the Polyakov loop average

$$L = \frac{1}{N_c} \text{tr} P \exp \left( -\frac{1}{2} \int_S d\sigma_{\mu\nu}(u) \int_S d\sigma_{\rho\lambda}(v) D_{\mu\nu,\rho\lambda}(u,v) \right) = \exp \left\{ -\frac{1}{4} \int_0^{1/T} d\tau \int_0^{1/T} d\tau' \int_0^\infty \xi d\xi D_1 \left( \sqrt{\xi^2 + (\tau - \tau')^2} \right) \right\}.$$  \hspace{1cm} (3)

In obtaining (3) we have omitted the contribution of $D(x)$ in $D_{\mu\nu,\rho\lambda}$, since this would cause vanishing of $L$ in the limiting process described above due to the infinite cone surface $S$. This exactly corresponds to vanishing of $L$ in the confinement region, observed on the lattice. Therefore the result (3) refers to the deconfinement phase, $T > T_c$.

As it is known from lattice \[18\] and analytic calculations, $D_1(x)$ \[19\] exponentially falls off at large $x$ as $\exp(-M_1x)$, with $M_1 \gtrsim 1$ GeV and for $T \ll M_1$ one can approximate (3) as follows\textsuperscript{3}

$$L \approx \exp \left( -\frac{1}{2} \int_0^{1/T} d\nu \int_0^\infty \xi d\xi D_1 \left( \sqrt{\xi^2 + \nu^2} \right) \right) \approx \exp \left( -\frac{1}{2T} V_1(\infty) \right), \quad V_1(r,T) = \int_0^{1/T} d\nu \int_0^r \xi d\xi D_1 \left( \sqrt{\xi^2 + \nu^2} \right).$$ \hspace{1cm} (4)

We turn now to the correlator of Polyakov loops following notations from \[11\]. Using the same limiting procedure as for one Polyakov loop, one can apply it to the correlator $\langle \tilde{\text{tr}} L_x \tilde{\text{tr}} L_y \rangle \equiv P(x - y)$, $\tilde{\text{tr}} = \frac{1}{N_c} \text{tr}$, representing the loops $L_x$ and $L_y$ as two concentric loops on the cylinder separated by the distance $|x - y|$ along its axis, the cylinder obtained in the limiting procedure from the cone with the vertex tending to infinity. One can apply in this situation the same formalism as was used in \[20\] for the case of the vacuum average of two Wilson loops. For opposite orientation of loops using Eqs. (21-28) from \[20\] one arrives at the familiar form found in \[21\]

$$P(x - y) = \frac{1}{N_c^2} \exp \left( -\frac{\tilde{F}_1(r,T)}{T} \right) + \frac{N_c^2 - 1}{N_c^2} \exp \left( -\frac{\tilde{F}_8(r,t)}{T} \right), \quad r \equiv |x - y|.\hspace{1cm} (5)$$

\textsuperscript{2}In doing so one is changing topology of the surface and as a result loses the $Z(N)$ subgroup of $SU(N)$. This however does not influence our results as long as one is remaining in the $j = 0$ sector of $Z(N)$ broken vacua (see last ref. of \[1\] for more discussion of $Z(N)$).

\textsuperscript{3}The correlators $D, D_1$ in \[13\] in principle should be taken in the periodic form, as was suggested in \[22\]. However for $T \leq 2T_c$ this modification brings additional terms of the order of $\exp(-M_1/T)$ which are neglected below.
In the Appendix two different ways of derivation of Eq.(5) are given, with the result
\[ \tilde{F}_1(r, T) = V_1(r, T) + V_D(r, T), \] (6)
\[ \exp(-\tilde{F}_8(r, T)/T) = L_{adj}(T) \exp\left\{-(V_D(r, T) - \frac{1}{8}V_1(r, T))/T\right\}. \] (7)

Here
\[ L_{adj} = \exp\left(-\left(\frac{9}{4}V_D(r^*) + \frac{9}{8}V_1(\infty, T)\right)/T\right) \] (8)
is the vacuum average of the adjoint Polyakov loop, which vanishes in the leading approximation in the confinement phase, as it is explained in the Appendix, and nonzero when gluon loops are taken into account, in which case \( \frac{9}{4}V_D(r^*, T) \sim M_1 \) and \( L_{adj} \sim \exp(-M_1/T) \ll 1. \)

The suppression of \( \exp(-\tilde{F}_8/T) \) in our approach in the confinement phase has thus the same origin as the strong damping of the adjoint Polyakov loop in that phase \cite{23} and the persistence of the Casimir scaling for adjoint static potential in the interval \( 0 \leq r < 1.2 \) fm (see \cite{24} for discussion and references).

It is clear that in the deconfinement phase with \( D \equiv 0, \ V_D \equiv 0 \) one has only \( V_1(r) \) in both \( \tilde{F}_1 \) and \( \tilde{F}_8 \), and all these quantities are finite (after the renormalization of the perturbative divergencies specific for the fixed contours, which are discussed in section 3). Thus in the deconfined phase one can write
\[ P(x - y) \equiv e^{-\tilde{F}_{q\bar{q}}/T} = \frac{1}{9}e^{-V_1(r)/T} + \frac{8}{9}e^{-\left(\frac{9}{4}V_1(\infty) - \frac{1}{8}V_1(r)\right)/T} \] (9)
where \( V_1(r) \) and \( V_1(\infty) \) are renormalized. It is clear from (9) that at small \( r \) one has \( \lim_{r \to 0} (\tilde{F}_{q\bar{q}}(r) - V_1(r)) \to T\ln N_c^2 \) as was noticed and measured in \cite{31}.

At this point one should stress the difference between the genuine free energy \( F_i(r, T), i = 1, 8 \), which is measured with some accuracy on the lattice, and the calculated above \( \tilde{F}_i(r, T) \). It is clear that \( \tilde{F}_i \) do not contain the contribution due to excitation of \( QQ \) and gluon degrees of freedom existing at finite \( T \). The latter is contained in the free energy \( F_1(r, T) \) and in the internal energy, which we denote \( U_i(r, T) = F_i + ST \) to distinguish from our \( V_i(r, T) \), since they are not equal.

In general for nonzero temperature and comparing to the lattice data on heavy-quark potential one should have in mind, that temperature effects
might be of two kinds. First, the intrinsic temperature dependence due to changing of the vacuum structure and the vacuum correlators and hence of our potentials $\tilde{F}(r, T)$. Second, the physical quantities like $F_i(r, T), U_i(r, T)$ are thermal averages over all excited states, e.g.

$$e^{-F_1(r,T)/T} = \sum_n e^{-E_n(r,T)/T}$$

(10)

$$U_1(r, T) = \sum_n E_n e^{-E_n(r,T)/T}$$

(11)

One can associate $\tilde{F}_1(r, T) = E_0(r, T)$, while the structure of excited spectrum can be traced in the temperature dependence of $F_1$ and $U_1$. E.g. assuming in the confinement phase the string-like spectrum and multiplicity for the multihybrid spectrum with two static quarks, $E_n = \sigma r + \pi n/r, n = 1, 2, \ldots$ and multiplicity $\rho(m) = \exp(m/m_0)\theta(m - m_1), m = \pi n/r$, one arrives at

$$F_1(r, T) = \sigma r + m_1(1 - T/m_0) - T \ln \left( \frac{1}{T} - \frac{1}{m_0} \right)$$

(12)

$$U_1(r, T) = \sigma r + m_1 + T/(1 - T/m_0)$$

(13)

The increase of $U_1(r, T)$ below $T_c$ in the quenched case was indeed observed in lattice calculations (see Fig.3 of [12]).

Above $T_c$ one can see in lattice data [10] the striking drop of entropy $S_1(\infty, T)$ and $U_1(\infty, T)$ in the region $T_c \leq T \leq 1.2T_c$ which can be possibly explained again by the multihybrid states occurring due to the potential $V_1(r, T) = \tilde{F}_1(r, T)$ connecting quarks and gluons, and assuming that the magnitude of $V_1(r, T)$ decreases with temperature passing at $T \approx 1.05T_c$ the critical value enabling to bind those states of high multiplicity. In this way one assumes that both below and above $T_c$ in quenched and unquenched cases the dominant (in entropy) configuration is the gluon chain connecting $Q$ and $\bar{Q}$ with gluons bound together by confining string (below $T_c$) and potential $V_1$ (above $T_c$). Thus the comparison to the lattice data on $F_1, U_1$ needs the exact knowledge of the spectrum. In what follows we shall associate our $\tilde{F}_1(r, \tau)$ with the free energy $F_1(r, T)$, since its temperature dependence is not so steep as that of $U_1(r, T)$ in this region and this discussion will be of qualitative character, leading detailed discussion of the spectrum to future publications.
3 Properties of $D_1(x)$ and $F_1(r, T)$

The correlator $D_1(x)$ was measured on the lattice [18] both below and above $T_c$, and decays exponentially with $M_1 \approx 1 \div 1.5$ GeV (in the quenched case). At the same time $D_1(x)$ can be connected to the gluelump Green’s function, and the corresponding $M_1$ for the electric correlator $D_{1E}(x)$ is $M_1 \approx 1.5$ GeV at zero $T$ [20]. Moreover in a recent paper [19] $D_{1E}(x)$ was found analytically for $T \leq T_c$, and can be represented symbolically as a sum, with perturbative part acting at small $x$,

$$D_1(x) = D^{pert}_{1}(x) + D^{(np)}_{1}(x), \quad D^{pert}_{1} = \frac{4C_2\alpha_s}{\pi x^4} + O(\alpha_s^2), \quad (14)$$

and the nonperturbative part having the asymptotic form

$$D^{(np)}_{1}(x) = \frac{A_1}{|x|}e^{-M_1|x|} + O(\alpha_s^2), \quad A_1 = 2C_2\alpha_s \sigma_{adj} M_1, \quad x \geq 1/M_1. \quad (15)$$

As will be argued below, the form of $D_1(x)$ (14) does not change for $T > T_c$, however the mass $M_1$ and $A_1$ may be there different.

Using the asymptotics (15) in the whole $x$ region for a qualitative estimate, one has

$$V^{(np)}_{1}(r, T) = A_1 \int_0^{1/T} (1 - \nu T) d\nu \int_0^r \frac{\xi d\xi e^{-M_1 \sqrt{\xi^2 + \nu^2}}}{\sqrt{\xi^2 + \nu^2}} = \frac{A_1}{M_1} \int_0^{1/T} (1 - \nu T) d\nu \left[ e^{-\nu M_1} - e^{-\sqrt{r^2 + \nu^2} M_1} \right] = V^{np}_{1}(\infty) - \frac{A_1}{M_1^2} \left[ K_1(M_1 r) M_1 r - \frac{T}{M_1} e^{-M_1 r} (1 + M_1 r) + O(e^{-M_1/T}) \right] \quad (16)$$

Finally the Polyakov loop exponent is

$$L = \exp \left( -\frac{V^{(np)}_{1}(\infty)}{2T} \right), \quad V^{(np)}_{1}(\infty) = \frac{A_1}{M_1^2} \left[ 1 - \frac{T}{M_1} (1 - e^{-M_1/T}) \right] \quad (17)$$

One can see from (17) that $V^{(np)}_{1}(\infty)$ is finite and is of the order of few hundred MeV in the interval $0 \leq T \leq 1.5 T_c$. At small $r$ from (16) $V^{(np)}_{1}(r) \approx const \cdot r^2$. The total $V_1 = V^{(np)}_{1} + V^{pert}_{1}$, contains also perturbative contribution at small $r$, which to the order $O(\alpha_s)$ is

$$V^{(pert)}_{1}(r) = \frac{2C_2\alpha_s}{\pi} \int_0^{1/T} d\nu (1 - \nu T) \left( \frac{1}{\nu^2} - \frac{1}{\nu^2 + r^2} \right) = V^{(pert)}_{1}(\infty) + V^{(C)}_{1}(r, T)$$
\[ V_1^{(C)}(r, T) = -\frac{C_2 \alpha_s}{r} f(r, T), \quad f(r, T) = 1 - \frac{2}{\pi} \arctan(rT) - \frac{rT}{\pi} \ln[1 + (rT)^{-2}] \]  

From (18) it is clear that \( V_1^{(pert)}(\infty) \) is divergent and should be renormalized, \( V_1^{(pert)}(\infty) \approx 2C_2 \alpha_s \pi \left( \frac{1}{a} - T \ln a \right), \quad a \to 0 \). Since the dominant divergent part is \( T \)-independent and \( V_1^{(np)}(r) \sim r^2 \) at small \( r \), one can renormalize matching \( V_1(r, T) \) with the Coulomb interaction at small \( r \), as it was done in [9, 11] for \( F_1(r, T) \).

As a result in the renormalized \( V_1^{(pert)}(r, T) \) the term \( V_1^{(pert)}(\infty) \) can be put equal to zero, and we shall use it in what follows.

At this point we are able to compare \( V_1(r, T) \) with the lattice data for \( F_1(r, T) \) at \( T \geq T_c \). In Fig.1 we compare the lattice data for \( F_1(r, T) \) taken from [12] for \( T = 1.05T_c, 1.2T_c \) and \( 1.5T_c \) with the potential \( V_1(r, T) \) in the form (16) parametrizing \( M_1 \) and \( a(T) \equiv \frac{\alpha(T)}{M_1} \) in it as

\[ M_1 = \text{const}, \quad a(T) = a_0 - c\frac{T - T_c}{T_c}, \quad \alpha_s = 0.3 \]  

and find that \( M_1 = 0.69 \text{ GeV} \) and \( a_0 = a(\text{conf}) = 2C_2(f)\alpha_s \sigma_{adj} \approx 0.432 \text{ GeV}^2 \), \( c = 0.36 \) provides a good agreement with the data points at \( 1.5T_c \geq T \geq T_c \), while \( a(T) \) in [19] smoothly matches at \( T = T_c \) the amplitude of the gluelump Green’s function [19]. One can see that the behaviour of the total \( V_1(r, T) = V_1^{(C)}(r, T) + V_1^{(np)}(r, T) \) which has a Coulomb part at smaller \( r \) and saturates at \( V_1 = V_1^{(np)}(\infty) \) is qualitatively very similar to the behaviour of \( F_1(r, T) \) as a function of \( r \). We also compare in Fig.2 our results with lattice data [11] for the Polyakov loop [17] and find reasonable agreement. It is clear that both Fig.1 and Fig.2 are qualitative illustrations, and for quantitative comparison one needs knowledge of excitation spectrum and analytic or lattice predictions for \( a(T), M_1(T) \) which will be given elsewhere [25, 37].
Figure 1: A comparison of behavior of $V_1(r, T) = V_1^{(np)}(r, T) + V_1^{(C)}(r, T)$ Eqs.(16),(18),(19)(solid lines, $T/T_c = 1.05; 1.2; 1.5$ from above), with the singlet free energy $F_1(r, T)$ measured in ref. [12] (filled circles.)
Figure 2: The Polyakov loop exponent $V_1(\infty, T)$ as a function of $T$ (GeV) from Eqs.(17),(19) (solid line), in comparison with lattice data for $F_1(\infty, T)$ in ref. [11] (filled circles).

4 Bound states of $q\bar{q}$ in the deconfinement region

In the recent lattice studies sharp peaks have been found in the spectral function of $c\bar{c}$ system [6]-[10], which can be associated with the quark-antiquark bound states surviving at $T \geq T_c$.

To understand qualitatively whether the interaction $V_1(r, T)$ can support bound states, one can use the Bargmann condition [27] for monotonic attractive potentials

$$2\bar{m} \int_0^\infty r dr |U(r, T)| > 1 \quad (20)$$

where $2\bar{m} = m_c$, and $U(r, T) = V_1(r, T) - V_1(\infty, T)$ which yields the condition for the bound $S$-states. Taking $m_c = 1.4$ GeV, $M_1 = 0.5$ GeV, one can deduce that $V_1^{(np)}(r, T)$ can support bound states in some interval of temperature $T_c < T < T_D$ where $T_D \sim 1.5 \div 2T_c$ and exact value depends on terms of
the order $O(e^{-M_1/T})$ and therefore is beyond the scope of the present paper. This conclusion roughly agrees with lattice calculations in [6-9] and with the calculation done in [28] and [29].

One can consider gluons and the $gg$ system in the same way as it was done for the $c\bar{c}$ system. To this end one should first multiply $V_1(r, T)$ found earlier in (16-18) by $C_2(\text{adj}) = \frac{9}{4}$, thus defining $V_1^{(\text{adj})}(r, T) = \frac{9}{4}V_1(r, T)$. This potential can be considered as the interaction kernel in the Hamiltonian of the $gg$ system as it was done in [30]. This Hamiltonian with the introduction of the einbein gluon mass $\mu_g \approx \langle \sqrt{p^2} \rangle \approx 0.6$ GeV [30], an additional (with respect to quarks) factor for the nonperturbative part in (20) is $\frac{9}{4}\mu_g m_c \approx 0.96$. Hence two-gluon glueballs should be formed in approximately the same temperature interval as charmonium states. In lattice calculations [31] scalar glueball was studied below and around $T_c$, and its width is increasing with $T$. Now we come to the baryon case and using the general formalism [32] to represent the triple Wilson loop of trajectories of 3 quarks and of the string junction in terms of field correlators $D$ and $D_1$. From [32] one has for 3 static quarks at distances $r_1, r_2, r_3$ from the string junction

$$V_{3q}(r_1, r_2, r_3; T) = \sum_{i=1}^{3} V_D(r_i, T) + \frac{1}{2} \sum_{i>j} V_1(r_i - r_j, T)$$

where $V_1(r, T)$ is given in (16), (18). In the deconfinement phase when $r_i = R$, $i = 1, 2, 3$, one obtains $V_{3q} = \frac{3}{2}V_1(\sqrt{3}R)$. For the perturbative part one has from (21) $V_{3q}^{(C)} = \frac{1}{2} \sum_{i>j} V_1^{(C)}(r_{ij}, T)|_{r_{ij}=R}$.

For the nonperturbative part from Eq.(21) it follows that $V_{3q}^{(np)}(R \to \infty) = \frac{3}{2}V_1^{(np)}(\infty)$. One can check that this prediction and the general form of $V_{3q} = \frac{3}{2}V_1(\sqrt{3}R)$ as function of $R$ is supported by the recent measurement of singlet free energy of the 3Q system in [33] at $T > T_c$. Thus it is of interest to measure the spectral functions of baryons at $T > T_c$ in the same way as it was done for mesons.

5 Summary and conclusions

Citing the 1991 paper [5] when the magnitude of $D_1$ was not exactly known 

"...Using an exponential parametrization for $D_1$, we can find $D_1^E$ with parameter values which satisfy the condition for the appearance of levels. In this
case \( \varepsilon(r) \) (our \( V_1^{(np)}(r, T) \)) is a well with a behaviour \( \varepsilon(r) \sim r^2 \) as \( r \to 0 \) and \( \varepsilon(r) \to \text{const} > 0 \) as \( r \to \infty \). The quark and antiquark are thus bound but there exists a threshold \( \varepsilon(\infty) \) above which quarks fly apart, each acquiring a nonperturbative mass increment \( \delta m = \frac{1}{2} \varepsilon(\infty) \). In the present paper this picture was further substantiated and quantified using lattice and analytic knowledge on \( D_1 \). Comparing to recent lattice data in [6]-[13] it was shown that this picture is qualitatively supported by data, and new proposals have been done for searching the glueball and baryon systems at \( T > T_c \).

The results obtained on the lattice [6]-[13] and in the present approach establish a new picture of the QCD thermodynamics at \( 1.5T_c > T > T_c \) widely discussed in [28]. As a new feature compared to the works [28] the main emphasis in this paper is done on the selfenergies \( \left( \frac{1}{2}V_1(\infty) \equiv \varepsilon_q \right. \) for quarks and \( \frac{9}{8}V_1(\infty) \equiv \varepsilon_g \) for gluons) which are large \( (V_1(\infty, T_c) > F_1(\infty, T_c) \approx 600 \text{ MeV for } n_f = 2 \) [34]) and cancel each other at small distances for white bound states, like \( q\bar{q}, (gg)_{1}, (qq\bar{q})_{1}, (qq...\bar{q})_{1} \) etc. In contrast to that colored states are higher in potential and mass by several units of \( \varepsilon_q \) and \( \varepsilon_g \) and are suppressed by the corresponding Boltzmann factors. As a result in this region white bound states of quarks and gluons are energetically preferable, while individual quarks and gluons acquire selfenergies, so that the thermodynamics of the system resembles that of the neutral gas, and for higher temperature \( T > 1.5 \div 2T_c \) a smooth transition to the "ionised" plasma of colored quarks and gluons possibly occurs.

This new state of the quark-gluon matter should be taken into account when considering ion-ion collisions. For more discussion of the thermodynamics above \( T_c \) see [28] and refs. therein.

It was noted before [35] that the behaviour of the free and internal energies above \( T_c \), with a bump around \( T \sim 1.1 \div 1.2T_c \), can be explained if gluons are supplied with the nonperturbative mass term of the order of 0.6 GeV, while for higher \( T \) this mass is less important. This can be easily understood now taking into account the value of \( \frac{1}{2}V_1(\infty, T) \) and its decreasing with growing \( T \). In this way the nonperturbative dynamics in the form of correlator \( D_1 \) can explain the observed dynamics of the deconfined QCD.

A more detailed analysis of bound states requires explicit calculation of \( Q\bar{Q} \) and \( 3Q \) bound states taking into account spin splitting in the mass of \( P \)-wave charmonia and quasi-degeneration of spectra of light \( qq \) \( V, A, S, PS \) states observed in [13]. Here spin-dependent forces are different from the confining case, since only the correlator \( D_1 \) contributes, and one can list the
corresponding terms in \[17, 36\]. It is interesting to note, that due to
the vector character of \(V_1(r, T)\), not violating chiral symmetry, bound states of
massless quarks should exhibit parity doubling. All this analysis is now in
progress [37].

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Appendix 1

Derivation of the Polyakov loop correlator

We give here two different derivations of \(\tilde{F}_i(r, T)\), \(i = 1, 8\). The first is
based on the correlator of two concentric Wilson loops, derived in [20], in
which case \(\tilde{F}_{1,8}\) are expressed in terms of surface integrals of field correlators
\(D_{\mu\nu,\rho\lambda}(u, v)\)

\[
I(S_i, S_k) \equiv \int_{S_i} d\sigma_{\mu\nu}(u) \int_{S_k} d\sigma_{\rho\lambda}(v) D_{\mu\nu,\rho\lambda}(u, v)
\]  

(A1.1)

In this way one obtains for two oppositely directed Polyakov loops from
Eqs. (29), (23), (20) of [20]

\[
\tilde{F}_1(r, T)/T = \frac{1}{2} I(S_{12}, S_{12})
\]  

(A1.2)

\[
\tilde{F}_8(r, T)/T = \frac{1}{2} I(S_1, S_1) + \frac{1}{2} I(S_2, S_2) + \frac{1}{N_c - 1} I(S_1, S_2)
\]  

(A1.3)

Here \(S_1\) is the surface on the cylinder with circumference \(1/T\) extending
from the loop 1 at coordinate \(x\) in the direction \(y\) to infinity, the surface \(S_2\)
is also infinite surface from the loop 2 at coordinate \(y\) in the same direction
(the answer does not depend on the choice of this direction).
The surface $S_{12}$ lies on the cylinder between the loops 1 and 2. Note that surface orientation in (A1.1) is fixed to be same. Calculation of $\tilde{F}_1$ according to (A1.2) reduces to that of the Wilson loop and yields

$$F_1(r, T) = V_1(r, T) + V_D(r, T) \quad (A1.4)$$

where $V_1(r, T)$ is given in [41] and $V_D$ is

$$V_D(r, T) = 2 \int_0^{1/T} d\nu (1 - \nu T) \int_0^r (r - \xi) d\xi D(\sqrt{\xi^2 + \nu^2}) \quad (A1.5)$$

Calculation of $\tilde{F}_8$ is more subtle. To this end one can use connection of $D_1(x)$ to the gluelump Green’s function $G_{\mu\nu}(x) = \delta_{\mu\nu}N_c(N_c^2 - 1)f(x^2)$ [19] $D_1(x) = -\frac{2g^2}{N_c}(N_c^2 - 1)\frac{df(x^2)}{dx^2}$, $f(x^2 \to 0) \sim \frac{1}{4\pi x^2}$ and inserting this into (4), one has

$$V_1(r, T) = \frac{g^2(N_c^2 - 1)}{N_c} \int_0^{1/T} d\nu (1 - \nu T)[f(\nu^2) - f(r^2 + \nu^2)] = V_1(\infty, T) + v_{ex}(r, T) \quad (A1.6)$$

One can see that $V_1(\infty, T) \equiv V_Q + V_{\bar{Q}}$ is the sum of equal selfenergy parts of $Q$ and $\bar{Q}$, while $v_{ex}(r, T)$ describes interaction due to one gluelump exchange between $Q$ and $\bar{Q}$.

Note that $V_1(0, T) = V_1(\infty, T) + v_{ex}(0, T) = 0$ and $v_{ex}(\infty, T) = 0$. Therefore $v_{ex}$ appears only in $I(S_1, S_2)$ in (A1.3) and one should restore there the original (opposite) orientation of loops $L_x$ and $L_y^+$ to get the correct sign of $v_{ex}$ (the same sign and factor appears in the second derivation below).

From (A1.1) one obtains for $I(S_i, S_i)$

$$\frac{1}{2}I(S_2, S_2) = V_D(r^*) + V_Q, \quad \frac{1}{2}I(S_1, S_1) = \frac{1}{2}I(S_2, S_2) + V_D(r),$$

$$I(S_1, S_2) = -v_{ex}(r, T) + I(S_2, S_2) \quad (A1.7)$$

As a result one can write using (A1.3) for $N_c = 3$

$$\tilde{F}_8(r, T) = \frac{9}{4}V_D(r^*, T) + V_Q + V_Q - \frac{1}{8}v_{ex}(r, T) +$$

$$+ V_D(r, T) = \frac{9}{8}V_1(\infty, T) - \frac{1}{8}V_1(r, T) + \frac{9}{4}V_D(r^*, T) + V_D(r, T) \quad (A1.8)$$

In (A1.8) the value of $r^*$ is infinitely large, when one neglects the valence gluon loops, as it is done everywhere above. In this case $V_D(r^*, T) \to \infty$ and
the term \( \exp(-\tilde{F}_8/T) \) vanishes in the confinement region. This is in line with the strong damping of the adjoint Polyakov loop in this region observed on the lattice \cite{23}, and with the persistence of Casimir scaling for adjoint static potential for \( 0 \leq r \leq 1.2 \) fm found on the lattice (see discussion and refs.in \cite{24}).

The correction due to the gluon determinant, producing additional gluon loops becomes important for \( r \geq 1.2 \) fm (see discussion in the second ref. in \cite{24}) and makes finite the value of \( V_D(r^*,T) \approx \sigma r^*, r^* \approx 1.2 \) fm/2 =0.6 fm.

\[
\exp(-\tilde{F}_8(r,T)/T) = L_{adj}(T) \exp\left(-\frac{V_D(r,T) - \frac{9}{8}V_1(r,T)}{T}\right),
\]

\[
L_{adj}(T) = \exp\left(-\frac{9}{8}V_1(\infty,T) - \frac{9}{4}V_D(r^*,T)\right). \tag{A1.9}
\]

In (A1.9) the effects of loop-loop interaction and of the total (adjoint) loop are separated. Physically the result (A1.9) can be easily understood: in absence of the internal interaction one has do with the adjoint Polyakov loop, which strongly changes around \( T_c \), namely \( L_{adj}(T < T_c) \) is much smaller than \( L_{adj}(T > T_c) \).

The behaviour similar to (A1.9) was observed on the lattice, see Fig. 3 of second ref. \cite{10} and ref.\cite{34}. Here one observes linear growth in \( r \) of \( \tilde{F}_8(r,T) \) below \( T_c \), and repulsive Coulomb behaviour from \( \frac{1}{8}V_1(r,T) = \frac{\alpha_s}{6r} \).

In the second derivation one is connecting two loops using parallel trans-porters and using the completeness relation

\[
\delta_{\alpha_1\beta_1}\delta_{\alpha_2\beta_2} = \frac{1}{N_c}\delta_{\alpha_1\beta_2}\delta_{\beta_1\alpha_2} + 2t^a_{\beta_2\alpha_1}t^a_{\beta_1\alpha_2} \tag{A1.10}
\]

so that the second term produces the adjoint Wilson loop on the cylinder surface.

\[
\exp(-\tilde{F}_8/T) = 2tr\langle (U(x,X;0)t^aU(X,y,0)L(y)\times \right.
\]
\[
\times U(y,X,t)t^aU(X,x,t)L^+(x) \rangle \tag{A1.11}
\]

This can be compared to the approach, suggested in \cite{38}. Our results however are different from those of \cite{38} in that both perturbative and non-perturbative interactions in \( \bar{F}_1 \) and \( \bar{F}_8 \) are different (and calculable through \( D_1 \)). Now (A1.11) can be rewritten using cluster expansion and nonabelian Stokes theorem, which finally results in the same equations as in (A1.9).
Alternatively one can use the technic exploited in [39] to separate the contributions of perturbative exchanges from the nonperturbative confining terms.

For the first ones one commutes as in [39] the color generators $t^c$ of exchanged gluon (gluelump) with $t^a$ \[ A1.11 \] according to the equality $t^c t^a t^c = -t^a / 2 N_c$, which finally gives in \[ A1.11 \] the adjoint Coulomb interaction $\frac{\alpha_s}{6\pi} \to -\frac{1}{8} V_1(r)$, while the selfenergy parts and confining terms arise from sequences $t^c t^c t^a \to \delta_{cc'} t^a$ and do not change sign. In this way one arrives at the same answer as given in \[ A1.9 \].

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