Possible polarisation and spin dependent aspects
of quantum gravity

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We argue that quantum gravity theories that carry a Lie algebraic modification
of the Poincaré and Heisenberg algebras inevitably provide inhomogeneities that
may serve as seeds for cosmological structure formation. Furthermore, in this class
of theories one must expect a strong polarisation and spin dependence of various
quantum-gravity effects.

I. Introduction— Quantum gravity proposals often come with a modification of the
Heisenberg, and Poincaré, algebras. Confining ourselves to Lie algebraic modifications,
we argue that the underlying physical space of all such theories must be inhomogeneous.
In order to establish this result, we first review how, within a quantum framework, the
homogeneity and continuity of physical space lead inevitably to the Heisenberg algebra.
We then review general arguments that hint towards algebraic modifications encountered
in quantum gravity proposals. Next, we argue that a natural extension of physical laws
to the Planck scale can be obtained by a Lie algebraic modification of the Poincaré and
Heisenberg algebras in such a way that the resulting algebra is immune to infinitesimal
perturbations in its structure constants. With the context so chosen, we establish the
main thesis: that quantum gravity theories of the aforementioned class inevitably provide
inhomogeneities that may serve as seeds for structure formation; and that quantum
gravity induced effects may carry a strong polarisation and spin dependence.

The established results are not restricted to the chosen algebra but may easily be
extended to all Lie algebraic modifications that alter the Heisenberg algebra.\(^1\)

2. Homogeneity and continuity of physical space, and its imprint in the Heisenberg algebra— In order to understand the fundamental origin of primordial inhomogeneities we will first review the fundamental connection between the homogeneity and continuity of physical space and the Heisenberg algebra. It is in this spirit that we remind our reader of an argument that is presented, for example, by Isham in [1, Section 7.2.2]. There it is shown that, in the general quantum mechanical framework, and under the following two assumptions,

— physical space is homogeneous,
— any spatial distance \( r \) can be divided into two equal parts, \( r = r/2 + r/2 \),

it necessarily follows that the operator \( x \) associated with position measurements along the \( x \)-axis, and the generator of displacements \( d_x \) along the \( x \)-direction, satisfy \([x, d_x] = i\). If one now requires consistency with the elementary wave mechanics of Heisenberg, one must identify \( d_x \) with \( p_x / \hbar \) (\( p_x \) is the operator associated with momentum measurements along the \( x \)-direction). This gives, \([x, p_x] = i\hbar\). Without any additional assumptions, the argument easily generalises to yield the entire Heisenberg algebra \([x_j, p_k] = i\hbar \delta_{jk}, \ [p_j, p_k] = 0, \ [x_j, x_k] = 0\), where \( x_j, j = 1, 2, 3 \), are the position operators associated with the three coordinate axes, where the observer is assumed to be located at the origin of the coordinate system.

Thus it is evident that a quantum description of physical reality, with spatial homogeneity and continuity, inevitably leads to the Heisenberg algebra.

3. On the need to go beyond the Heisenberg and Poincaré algebraic-based description of physical reality— From an algebraic point of view much of the success of modern physics can be traced back to the Poincaré and Heisenberg algebras. Had the latter algebra been discovered before the former, the conceptual formulation and evolution of theoretical physics would have been significantly different. For instance, it is a direct implication of Heisenberg’s fundamental commutator \([x_i, p_j] = i\hbar \delta_{ij}\) (with \( i, j = 1, 2, 3 \)), that events should be characterised not only by their spatiotemporal location \( x_\mu \), but also...

\(^1\)A slightly weaker argument can be constructed for non-Lie algebraic proposals when we confine ourselves to probed distances significantly larger than the length scale associated with the loss of spatial continuity.
by the associated energy momentum $p_\mu$; and that should be done in a manner consistent with the fundamental measurement uncertainties inherent in the formalism. The reader may wish to come back to these remarks in the context of Eq. (16) where one shall find that in a specific sense the physical space that underlies the conformal algebra does indeed combine the notions of spacetime and energy momentum. Furthermore, as will be seen from Eq. (18) and the subsequent remarks, this interplay becomes increasingly important as we consider the early universe above $\approx 100$ GeV.

In the mentioned description the interplay of the general relativistic and quantum mechanical frameworks becomes inseparably bound. To see this, consider the well-known thought experiment to probe spacetime at spatial resolutions around the Planck length $\ell_P \equiv \sqrt{\hbar G/c^3}$. If one does that, one ends up creating a Planck mass $m_P \equiv \sqrt{\hbar c/G}$ black hole. This fleeting structure carries a temperature $T \approx 10^{30}$K and evaporates in a thermal explosion in $\approx 10^{-40}$ seconds. This, incidentally, is a long time – about ten thousand fold the Planck time $\tau_P \equiv \sqrt{\hbar G/c^5}$. The formation and evaporation of the black hole places a fundamental limit on the spatiotemporal resolution with which spacetime can be probed.

The authors of [2, 3] have argued that once gravitational effects associated with the quantum measurement process are accounted for, the Heisenberg algebra, and in particular the commutator $[x_j, p_k]$, must be modified. The role of gravity in the quantum measurement process was also emphasised by Penrose [4].

*From the above discussion, we take it as suggestive that an operationally-defined view of physical space (or, its generalisation) shall inevitably ask for the length scale, $\ell_P$ to play an important role.*

In the context of the continuity of physical space we will take it as a working hypothesis that, just as a lack of commutativity of the $x$ and $p_x$ operators does not render the associated eigenvalues discrete, similarly the existence of a non-vanishing $\ell_P$ does not necessarily make the underlying space lose its continuum nature. This is a highly non-trivial issue requiring a detailed discussion from which we here refrain; yet, an element of justification shall become apparent below.

From a dynamical point of view, as early as late 1800’s, the symmetries of Maxwell’s equations were already suggesting a merger of space and time into one physical entity, spacetime [5]. Algebraically, these symmetries are encoded in the Poincaré algebra. The emergent unification of space and time called for a new fundamental invariant, $c$, the speed of light (already contained in Maxwell’s equations). From an empirical point
of view, the Michelson-Morley experiment established the constancy of the speed of light for all inertial observers, and thus re-confirmed, in the Einsteinian framework, the implications of the Poincaré spacetime symmetries.

Concurrently, we note that while in classical statistical mechanics it is the volume that determines the number of accessible states and hence the entropy, the situation is dramatically different in a gravito-quantum mechanical setting. One example of this assertion may be found in the well-known Bekenstein-Hawking entropy result for a Schwarzschild black hole, $S_{BH} = (k/4)(A/\ell^2_P)$; where $k$ is the Boltzmann constant, and $A$ is the surface area of the sphere contained within the event horizon of the black hole. Thus quantum mechanical and gravitational realms conspire to suggest the holographic conjecture [6, 7, 8]. The underlying physics is perhaps two fold: (a) contributions from higher momenta in quantum fields to the number of accessible states is dramatically reduced because these are screened by the associated event horizons; and (b) the accessible states for a quantum system are severely influenced by the behaviour of the wave function at the boundary.

From this discussion, we take it as suggestive that in quantum cosmology/gravity the new operationally-defined view of physical space shall inevitably ask for a cosmological length scale, $\ell_C$.

These observations prepare us to reach the next trail in our essay.

In the immediate aftermath of cosmic creation with the big bang, the physical reality knew of no inertial frames of Einstein. This is due to the fact that massive particles had yet to appear on the scene. The spacetime symmetries at cosmic creation are encoded in the conformal algebra. So, whatever new operational view of spacetime emerges, it must somehow also incorporate a process by which one evolves from the “conformal phase” of the universe at cosmic creation to the present (see Fig. 1).

Algebraically, we take it to suggest that there must be a mechanism that describes how the present day Poincaré-algebraic description relates to the conformal-algebraic description of the universe at its birth.

We parenthetically note that in the conformal phase, where leptons and quarks were yet to acquire mass (through the Higg’s mechanism, or something of that nature), the operationally-accessible symmetries are not Poincaré but conformal. This is so because to define rest frames, so essential for operationally establishing the Poincaré algebra, one
needs massive particles. In the transition when massive particles come to exist, the local algebraic symmetries of general relativity suffer an operational change. Consequently, for the cosmic epoch before \( \approx 100 \text{ GeV} \) general relativistic description of physical reality might require modification.

4. A new algebra for quantum gravity and the emergent inhomogeneity of physical space—Mathematically, a Lie algebra incorporating the three italicised items in Sec. 3 already exists. It was inspired by Faddeev’s mathematical analysis of the quantum and relativistic revolutions of the last century [9] and was followed up by Vilela Mendes in his 1994 paper [10]. The uniqueness of the said algebra was then explored through a Lie-algebraic investigation of its stability by Chryssomalakos and Okon, in 2004 [11]. Some of the physical implications were subsequently explored in Refs. [12, 13], and its Clifford-algebraic representation was provided by Gresnigt et al. [14]. Its importance was further noted in *CERN Courier* [15].

However, its candidacy for the algebra underlying quantum cosmology/gravity has been difficult to assert. This is essentially due to a perplexing observation made in Ref. [11] regarding the interpretation of the operators associated with the spacetime events. In this essay we overcome this interpretational hurdle and argue that it contains all the desired features for such an algebra.

To this end we first write down what has come to be known as the Stabilised Poincaré-Heisenberg Algebra (SPHA) and then proceed with the interpretational issues. The SPHA contains the Lorentz sector (we follow the widespread physics convention which takes the \( J_{\mu\nu} \) as dimensionless and \( P_{\nu} \) as dimensionful)

\[
\left[ J_{\mu\nu}, J_{\rho\sigma} \right] = i \left( \eta_{\nu\rho} J_{\mu\sigma} + \eta_{\mu\sigma} J_{\nu\rho} - \eta_{\mu\rho} J_{\nu\sigma} - \eta_{\nu\sigma} J_{\mu\rho} \right) \tag{1}
\]

This remains unchanged (as is strongly suggested by the analysis presented in [16]), as does the commutator

\[
\left[ J_{\mu\nu}, P_{\lambda} \right] = i \left( \eta_{\nu\lambda} P_{\mu} - \eta_{\mu\lambda} P_{\nu} \right) \tag{2}
\]

These are supplemented by the following modified sector

\[
\left[ J_{\mu\nu}, X_{\lambda} \right] = i \left( \eta_{\nu\lambda} X_{\mu} - \eta_{\mu\lambda} X_{\nu} \right) \tag{3}
\]
\[
\left[ P_{\mu}, P_{\nu} \right] = iq \alpha_1 J_{\mu\nu} \tag{4}
\]
\[
\left[ X_{\mu}, X_{\nu} \right] = iq \alpha_2 J_{\mu\nu} \tag{5}
\]
\[
\left[ P_{\mu}, X_{\nu} \right] = iq \eta_{\mu\nu} I + iq \alpha_3 J_{\mu\nu} \tag{6}
\]
\[ [P_\mu, I] = i\alpha_1 X_\mu - i\alpha_3 P_\mu \] (7)
\[ [X_\mu, I] = i\alpha_3 X_\mu - i\alpha_2 P_\mu \] (8)
\[ [J_{\mu\nu}, I] = 0 \] (9)

The metric \( \eta_{\mu\nu} \) is taken to have the signature \((1, -1, -1, -1)\). The SPHA is stable, except for the instability surface defined by \( \alpha_3^2 = \alpha_1 \alpha_2 \) (see Fig. 2). Away from the instability surface the SPHA is immune to infinitesimal perturbations in its structure constants. This distinguishes SPHA from many of the competing algebraic structures because a physical theory based on such an algebra is likely to be free from “fine tuning” problems. This is essentially self evident because if an algebraic structure does not carry this immunity, one can hardly expect the physical theory based upon such an algebra to enjoy the opposite.

The SPHA involves three parameters \( \alpha_1, \alpha_2, \alpha_3 \). The \( c \) and \( \hbar \) arise in the process of the Lie algebraic stabilisation that takes us from the Galilean relativity to Einsteinian relativity, and from classical mechanics to quantum mechanics. Their specific values are fixed by experiment. Similarly, \( \alpha_1, \alpha_2, \alpha_3 \) owe their origin to a similar stabilisation of the combined Poincaré and Heisenberg algebra.

Except for the fact that \( \alpha_1 \) must be a measure of the size of the observable universe (here assumed to be operationally determined from the Hubble parameter), the Lie algebraic procedure for obtaining SPHA does not determine \( \alpha_1, \alpha_2, \alpha_3 \). Dimensional and phenomenological considerations, along with the requirement that we obtain physically viable limits, suggest the following identifications\(^2\)

\[ \alpha_1 := \frac{\hbar}{\ell_C} \] (10)

where \( \ell_C \) is of the order of the Hubble radius, and therefore it depends on the cosmic epoch. The introductory remarks, and existing data suggest that \( \ell_C \)

\[ \alpha_2 = \frac{\ell_P^2}{\hbar} \] (11)

In the limit \( \ell_P \to 0, \ell_C \to \infty, \beta \to 0, I \to I \), the identity operator, the SPHA splits into Heisenberg and Poincaré algebras. In that limit, the symbols \( X_\mu \to x_\mu, P_\mu \to p_\mu, J_{\mu\nu} \to J_{\mu\nu} \), and \( I \to I \). Thus \( x_\mu, p_\mu, J_{\mu\nu}, I \) acquire their traditional meaning, while

\(^2\)In making the identifications it is understood that these may be true up to a multiplicative factor of the order of unity.
\( X_\mu, \mathcal{P}_\mu, \mathcal{J}_{\mu\nu}, \mathcal{I} \) are to be considered their generalisations. In particular, \( x_\mu \) should then be interpreted as the generator of energy-momentum translation. The latter parallels the canonical interpretation of \( p_\mu \) as the generator of spacetime translation. This interpretation, we believe, removes the problematic interpretational aspects associated with \( X_\mu \) in the analysis of Ref. [11].

The identification of \( q \) with \( \bar{\hbar} \) is dictated by the demand that we recover the Heisenberg algebra. It also suggests that at the present cosmic epoch \( \alpha_3 \) should not allow the second term in the right hand side of equation (6) to have a significant contribution. It will become apparent below that \( \alpha_3 \) is intricately connected to the conformal algebraic limit of SPHA. With these identifications, and with \( \alpha_3 \) renamed as the dimensionless parameter \( \beta \), the SPHA takes the form

\[
\begin{align*}
\left[ \mathcal{J}_{\mu\nu}, \mathcal{J}_{\rho\sigma} \right] &= i \left( \eta_{\nu\rho} \mathcal{J}_{\mu\sigma} + \eta_{\mu\sigma} \mathcal{J}_{\nu\rho} - \eta_{\mu\rho} \mathcal{J}_{\nu\sigma} - \eta_{\nu\sigma} \mathcal{J}_{\mu\rho} \right) \quad (12) \\
\left[ \mathcal{J}_{\mu\nu}, \mathcal{P}_\lambda \right] &= i \left( \eta_{\nu\lambda} \mathcal{P}_\mu - \eta_{\mu\lambda} \mathcal{P}_\nu \right), \quad \left[ \mathcal{J}_{\mu\nu}, \mathcal{X}_\lambda \right] = i \left( \eta_{\nu\lambda} \mathcal{X}_\mu - \eta_{\mu\lambda} \mathcal{X}_\nu \right) \quad (13) \\
\left[ \mathcal{P}_\mu, \mathcal{P}_\nu \right] &= i \left( \hbar^2 / \ell_C^2 \right) \mathcal{J}_{\mu\nu}, \quad \left[ \mathcal{X}_\mu, \mathcal{X}_\nu \right] = i \ell_C^2 \mathcal{J}_{\mu\nu}, \quad \left[ \mathcal{P}_\mu, \mathcal{X}_\nu \right] = i \hbar \eta_{\mu\nu} \mathcal{I} + i \beta \mathcal{J}_{\mu\nu} \quad (14) \\
\left[ \mathcal{P}_\mu, \mathcal{I} \right] &= i \left( \hbar / \ell_C^2 \right) \mathcal{X}_\mu - i \beta \mathcal{P}_\mu, \quad \left[ \mathcal{X}_\mu, \mathcal{I} \right] = i \beta \mathcal{X}_\mu - i \left( \ell_C^2 / \hbar \right) \mathcal{P}_\mu, \quad \left[ \mathcal{J}_{\mu\nu}, \mathcal{I} \right] = 0 \quad (15)
\end{align*}
\]

Since cosmic creation began with massless particles, it should be encouraging if in some limit SPHA reduced to the conformal algebra. This is indeed the case. It follows from a somewhat lengthy, though simple, exercise. Towards examining this question we introduce two new operators

\[
\begin{align*}
\tilde{\mathcal{P}}_\mu &= a \mathcal{P}_\mu + b \mathcal{X}_\mu, \quad \tilde{\mathcal{X}}_\mu &= a' \mathcal{X}_\mu + b' \mathcal{P}_\mu \quad (16)
\end{align*}
\]

and find that if the introduced parameters \( a, b, a', b' \) satisfy the the following conditions

\[
a = \frac{\ell_P^2}{b' \hbar}, \quad b = \frac{1 - \beta}{b'}, \quad a' = \frac{b' \hbar}{\ell_C^2 (1 - \beta)} \quad (17)
\]

with \( \beta^2 \) restricted to the value \( 1 + (\ell_P^2 / \ell_C^2) \), then SPHA written in terms of \( \tilde{\mathcal{P}}_\mu \) and \( \tilde{\mathcal{X}}_\mu \) satisfies the conformal algebra [17, Sec. 4.1].

Using these results, we can re-express \( \tilde{\mathcal{P}}_\mu \) and \( \tilde{\mathcal{X}}_\mu \) in a fashion that supports the view taken in the opening paragraph of this section

\[
\begin{align*}
\tilde{\mathcal{P}}_\mu &= a \left( \mathcal{P}_\mu + \frac{\hbar}{\ell_P^2} (1 - \beta) \mathcal{X}_\mu \right), \quad \tilde{\mathcal{X}}_\mu &= a' \left( \mathcal{X}_\mu + \frac{\ell_C^2}{\hbar} (1 - \beta) \mathcal{P}_\mu \right) \quad (18)
\end{align*}
\]
with

$$\beta^2 = 1 + \frac{\ell_P^2}{\ell_C^2}. \quad (19)$$

Near the big bang, $\ell_C \approx \ell_P$ and thus $\beta \to \pm \sqrt{2}$ (see, Fig. 1). This results in a significant mixing of the $X_\mu$ and $P_\mu$ in the conformal algebraic description in terms of $\tilde{X}_\mu$ and $\tilde{P}_\mu$.

In contrast, hypothetically, had we been on the conformal surface at present then taking $\ell_C \gg \ell_P$ makes $\beta \to \pm 1$. Consequently, for $\beta \to +1$, $\tilde{P}_\mu$ becomes identical to $P_\mu$ up to a multiplicative scale factor $a$. Similarly, $\tilde{X}_\mu$ becomes identical to $X_\mu$ up to a multiplicative scale factor $a'$. As is evident from Eq. (17), the multiplicative scale factors $a$ and $a'$ are constrained by the relation $aa' = \ell_P^2/(\ell_C^2(1 - \beta))$. We expect that similar modifications to spacetime symmetries would occur if we were to explore it at Planckian energies in the present epoch. For $\beta \to -1$ ($\ell_C \gg \ell_P$), one again obtains significant mixing of the $X_\mu$ and $P_\mu$.

By containing $\ell_P$ and $\ell_C$, the SPHA unifies the extreme microscopic with the extreme macroscopic, i.e., the cosmological. In the early universe it allows for the existence of conformal symmetry. The significant departure from the Heisenberg algebra at big bang, yields primordial inhomogeneities in the underlying physical space and the quantum fields that it supports. The latter is an unavoidable consequence of the discussion presented in Sec. 2.

5. Polarisation and spin dependence of the cosmic inhomogeneities and other quantum gravity effects— A careful examination of SPHA presented in equations (12-15) reveals a strong $J_{\mu\nu}$ dependence of the modifications to the Heisenberg algebra. Physically, this translates to the following representative implications

— The induced primordial cosmic inhomogeneities are dependent on spin and polarisation of the fields for which these are calculated.

— The operationally-inferred commutativity/non-commutativity of the physical space depends on the spin and polarisation of the probing particle.

— The just enumerated observation implies that a violation of equivalence principle is inherent in the SPHA based quantum gravity.

3 Any one of the other suggestions in quantum gravity that modify the Heisenberg algebra (see, e.g., references [18]-[28]) carry similar implications for homogeneity and isotropy of the physical space.
Since Heisenberg algebra uniquely determines the nature of the wave particle duality \[\lambda = \frac{h}{p}\] (including the de Broglie result “\(\lambda = \frac{h}{p}\)”), it would undergo spin and polarisation dependent changes in quantum gravity based on SPHA.

All these results carry over to any theory of quantum gravity that modifies the Heisenberg algebra with a \(J_{\mu\nu}\) dependence.

6. Conclusion — In this essay we have motivated a new candidate for the algebra which may underlie a physically viable and consistent theory of quantum cosmology/gravity. Besides yielding an algebraic unification of the extreme microscopic and cosmological scales, it generalises the notion of conformal symmetry. The modifications to the Heisenberg algebra at the present cosmic epoch are negligibly small; but when \(\ell_C\) and \(\ell_P\) are of the same order (i.e., at, and near, the big bang), the induced inhomogeneities are intrinsic to the nature of physical space. These can then be amplified by the cosmic evolution and result in important back reaction effects \[29\ [30, 31, 32].\] An important aspect of the SPHA-based quantum gravity is that it inevitably provides inhomogeneities that may serve as an important ingredient for structure formation \[33.\] Furthermore, in this class of theories one must expect a strong polarisation and spin dependence of various quantum-gravity effects.

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Figure 1: This figure is a cut, at $\ell_P = 1$ (with $\hbar$ set to unity), of Fig. 2 and it schematically shows the cosmic evolution along two possible scenarios. For this purpose, only $\beta \geq 0$ values have been taken. The $\beta < 0$ sector can easily be inferred from symmetry consideration. In one of the scenarios the conformal symmetry of the early universe is lost without crossing the instability surface, while in the other it crosses that surface. In the latter case the algebra changes from $\mathfrak{so}(2,4)$ to $\mathfrak{so}(1,5)$. This crossover, we speculate, may be related to the mass-generating process of spontaneous symmetry breaking (SSB) of the standard model of high energy physics. The big bang is here identified with $\ell_C \approx \ell_P$. 
Figure 2: The unmarked arrow is the $\ell_P = \hbar\alpha_2$ axis. The Poincaré-Heisenberg algebra corresponds to the origin of the parameters space, which coincides with the apex of the instability cone. In reference to Eq. (10), note that $\ell_C^2 = \hbar/\alpha_1$. Here, $\beta$ is a dimensionless parameter that corresponds to a generalisation of the conformal algebra. The SPHA lives in the entire $(\ell_C, \ell_P, \beta)$ space except for the surface of instability. The SPHA becomes conformal for all values of $(\ell_C, \ell_P, \beta)$ that lie on the “conformal surface”.

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