MASS GENERATION FOR GAUGE FIELDS
WITHOUT SCALARS

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ABSTRACT

We present an alternative to the Higgs mechanism to generate masses for non-abelian gauge fields in (3+1)-dimensions. The initial Lagrangian is composed of a fermion with current-current and dipole-dipole type self-interactions minimally coupled to non-abelian gauge fields. The mass generation occurs upon the fermionic functional integration. We show that by fine-tuning the coupling constants the effective theory contains massive non-abelian gauge fields without any residual scalars or other degrees of freedom.

1. Introduction

The standard model has been widely accepted as the theory of electro-weak interactions. It has successfully accounted for all experiments to date, making it perhaps one of the greatest successes of modern theoretical physics. However, apart from the unknown value of the top quark mass, one of the present mysteries in the standard model is the absence of the Higgs particle in present experiments. This fundamental particle is a scalar, which relegates it as the only one in this category. Perhaps, we should see these arguments as indications for looking for alternatives to the Higgs mechanism for generating masses for the elementary fermions and vector bosons.

Over the last twenty years, other mechanisms to account for mass generation in the standard model have been proposed such as technicolor theories \textsuperscript{1}, and dynamical symmetry breaking via a top-quark condensate \textsuperscript{2} in analogy with BCS theory of superconductivity and the Nambu-Jona-Lasinio mechanism in nuclear structure \textsuperscript{3}. The latter mechanism generates the Higgs particle (not as a fundamental particle) and its consequences with a four-fermion interaction.

In (2+1)-dimensions, it is well-known that the addition of a topological Chern-Simons term to the gauge field kinetic part provides a mechanism for gauge fields
mass generation without spoiling gauge invariance. In a relatively similar spirit, in (3+1)-dimensions, attempts to reproduce the Chern-Simons term involves a product of field strength tensors $\epsilon^{\mu\nu\alpha\beta} F_{\mu
u} F_{\alpha\beta}$. However, since this expression can be written as a total derivative, it cannot bring any modifications to physics. It is therefore necessary to introduce another field if we want to mimic the Chern-Simons mechanism in (3+1)-dimensions. For instance, Freedman and Townsend (F-T) and others have developed theories containing an antisymmetric tensor field coupled to an abelian gauge field.

Recently, a mechanism for photon mass generation in (3+1)-dimensions has been suggested, which consists of a functional integration over fermions minimally coupled to a low-energy abelian gauge field. The fermions self-interacts via two types of contact interactions: current-current and dipole-dipole terms. An antisymmetric tensor field is introduced via the Hubbard-Stratonovich transformation to perform the fermion’s integration. After imposing conditions on the coupling constants of the theory, it is possible to write the low-energy effective action as the abelian model discussed in Ref. [5,6,7,8], which reproduces a massive abelian gauge field.

Since the massive mediators of forces present in the weak interactions are known to be of non-abelian nature, we generalize here the above argument to the case of non-abelian gauge fields.

### 2. Non-Abelian high energy model

We begin with the following non-renormalizable but SU(N) gauge invariant Lagrangian at the high-energy scale $\Lambda$ in which a fermion field is minimally coupled to non-abelian gauge fields

$$
\mathcal{L} = -\frac{1}{2} \text{tr} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\slashed{D} - m)\psi - g_1 \text{tr} j_\mu j^\mu - g_2 \text{tr} j_\mu j^{\mu\nu} + \bar{\psi} \gamma^\mu T^a \psi \bar{\psi} \gamma^\mu T^a \psi - g_2 \text{tr} (\bar{\psi} \gamma^{\mu\nu} T^a \psi)(\bar{\psi} \gamma^{\mu\nu} T^a \psi)$$

where $D_\mu = \partial_\mu - igA_\mu$ is the covariant derivative. The non-abelian gauge fields are defined by $A_\mu = A^a_\mu T^a$ where $T^a$ are the Lie algebra generators obeying the commutation relations $[T^a, T^b] = iC^{abc} T^c$ and the trace relation $\text{tr}(T^a T^b) = \frac{1}{2} \delta^{ab}$.

The last two quantities in (1) are the current-current and dipole-dipole fermionic self-interactions. The four-vector current and the dipole current are given respectively by $j_\mu = j^a_\mu T^a$ and $j_{\mu\nu} = j^a_{\mu\nu} T^a$ with components

$$
\begin{align*}
\bar{\psi} j^a_\mu \psi &= \bar{\psi} \gamma^\mu T^a \psi \\
\bar{\psi} j^a_{\mu\nu} \psi &= \bar{\psi} \gamma^{\mu\nu} T^a \psi
\end{align*}
$$

both of which transform in the adjoint representation of the SU(N) gauge group.

In components, the Lagrangian (1) can be written as

$$
\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \bar{\psi}(i\slashed{D} - m)\psi - g_1 (\bar{\psi} \gamma^\mu T^a \psi)(\bar{\psi} \gamma^\mu T^a \psi) - g_2 (\bar{\psi} \gamma^{\mu\nu} T^a \psi)(\bar{\psi} \gamma^{\mu\nu} T^a \psi)
$$

(3)
where the field strength is given by

\[ F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g C^{abc} A^b_\mu A^c_\nu. \]  

(4)

This model is interesting because of the form of the dipole-dipole interaction, which makes it different from the one studied in Ref. [2].

From now on, for definiteness and due to obvious potential applications, we will consider only the SU(2) gauge group with the usual su(2) Lie algebra given by the Pauli matrices \( T^a = \frac{\tau^a}{2} \), \( a = 1, 2, 3 \) and structure constants given by \( C^{abc} = \epsilon^{abc} \).

We next apply the Hubbard-Stratonovich transformation by introducing auxiliary non-abelian antisymmetric tensor fields \( b_{\mu\nu}^a \), which belong also in the su(2)-Lie algebra, transform according to the adjoint representation and have mass dimension \( [b_{\mu\nu}^a] = 1 \). Their introduction permit us to rewrite the dipole-dipole interaction as

\[
-\frac{g_2}{2} (\bar{\psi} \sigma^{\mu\nu} \gamma_5 \frac{\tau^a}{2} \psi)^2 \rightarrow -\frac{1}{2g_2} b_{\mu\nu}^a b^{a\mu\nu} + i b_{\mu\nu}^a (\bar{\psi} \sigma^{\mu\nu} \gamma_5 \frac{\tau^a}{2} \psi)
\]

(5)

since one can regain the original Lagrangian by solving the equation of motion for \( b_{\mu\nu}^a \) and substituting the result in the Lagrangian. As noted in Ref.[9], we could also transform, in a similar way, the current-current interaction by introducing other auxiliary gauge fields \( a^a_\mu \). In what follows, we choose instead to consider only the introduction of auxiliary antisymmetric tensor fields and treat perturbatively the remaining four-fermion term.

### 3. Fermionic functional integration

We are interested in evaluating the behavior of the theory at low energy for the non-abelian gauge fields and antisymmetric tensor fields. We compute the effective action keeping the gauge and antisymmetric tensor fields external and integrating out the fermions :

\[
e^{i\Gamma_{\text{eff}}[A^a, b^a]} = \int [D\bar{\psi}[D\psi] \exp i \int d^4 x \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} - \frac{1}{2g_2} b_{\mu\nu}^a b^{a\mu\nu} + (\bar{\psi}(i\partial_\mu - m + g A^a_\mu \frac{\tau^a}{2} + b^a_\mu \frac{\tau^a}{2})\psi - \frac{g_1}{2} (\bar{\psi} \gamma^\mu \frac{\tau^a}{2} \psi)^2 \right) \]

(6)

where \( b^a_\mu \equiv i\sigma^{\mu\nu} \gamma_5 b_{\mu\nu}^a \).

In order to perform the integration over the fermion fields, we expand the last term in power series and use the vacuum dominance approximation to write

\[
(\bar{\psi} \gamma^\mu \frac{\tau^a}{2} \psi)^2 \rightarrow \langle 0 | (\bar{\psi} \gamma^\mu \frac{\tau^a}{2} \psi)^2 | 0 \rangle \simeq | \langle 0 | (\bar{\psi} \gamma^\mu \frac{\tau^a}{2} \psi) | 0 \rangle |^2 \equiv (j^{a\mu})^2
\]

(7)

where \( \langle j^{a\mu} \rangle \) is the current expectation value. Using (7) in (6), we obtain the effective action at order \( g_1 \) given by

\[
e^{i\Gamma_{\text{eff}}[A^a, b^a]} = \exp \left\{ i \int d^4 x \left( \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} - \frac{1}{2g_2} b_{\mu\nu}^a b^{a\mu\nu} - \frac{g_1}{2} (j^{a\mu})^2 \right) \right\} e^{i\Gamma_0^{(f)}}
\]

(8)
where

$$\Gamma_0^{(f)} = \int [\mathcal{D}\bar{\psi}] [\mathcal{D}\psi] \exp \left\{ i \int d^4x \bar{\psi} \left( \slashed{D} - m + i g^a \frac{\sigma^a}{2} + b^a \frac{\sigma^a}{2} \right) \psi \right\}$$

(9)

is the contribution coming from the integration of the bilinear part in fermions. \(\Gamma_0^{(f)}\) may be rewritten as

$$\Gamma_0^{(f)} = -i \text{Tr} \log \left( \slashed{D} - m + i g^a \frac{\sigma^a}{2} + b^a \frac{\sigma^a}{2} \right)$$

$$= \sum_{n=1}^{\infty} \frac{i}{n} (-1)^n \text{Tr} \left( \frac{1}{\slashed{D} - m} \left( g^a \frac{\sigma^a}{2} \right)^n \right)$$

(10)

and will be evaluated in a derivative expansion up to two derivatives on fields. In the last equality the trace is taken over spinor indices, group indices and momenta.

For reasons that will become clear in the course of the discussion, we need to compute the effective action up to four-point functions \((n=4\) in (10)). In part this is guided by the presence of the SU(2) gauge invariance and also by noticing that the self-coupling of the non-abelian gauge fields already contains terms with four fields due to the nonlinearity of the theory. Competition against those expressions will result in our model.

Regularization is achieved using a cut-off and Pauli-Villars methods when gauge invariance has to be preserved. Note that as stated by Faddeev and Slavnov \(^{11}\), Pauli-Villars methods may be extended to regularize in a gauge invariant way in the present theory.

Once the above contributions are evaluated, we compute the expectation value of the current given by \(\langle j^{a\lambda} \rangle = \delta \Gamma_0^{(f)} / \delta A_a^{\lambda} \) evaluated at \(A_a^{\lambda} = 0\). Readers interested in details of the computation are referred to ref. [10]. We stress however here two important results coming from the evaluation: 1) a \(b \wedge F\)-type term arises from the combination of \(\Gamma_{\text{eff}}[A^a, b^b] \) and \(\Gamma_{\text{eff}}[A^a, A^b, b^c] \) at order \(g_1^0\); 2) the effective current calculated from the \(b \wedge F\)-type term gives, when squared, the kinetic part for the antisymmetric fields.

4. Low energy effective action

After integrating out the fermions and evaluating the effective current, we obtain the following low energy effective Lagrangian for the gauge and antisymmetric tensor fields

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \frac{1}{12} H_{\mu
u\rho}^a H^{a\mu\nu\rho} + \frac{g}{4\sqrt{g_1}} \epsilon^{\alpha\beta\mu\nu} b^a_{\alpha\beta} F^a_{\mu\nu}$$

$$+ \frac{1}{2g_2} \left( \frac{16\pi^4}{g_1 m^2 \log(\Lambda^2/m^2)} \right) \left( \frac{m^2}{4\pi^2} g_2 \log \frac{\Lambda^2}{m^2} - 1 \right) b^a_{\mu\nu} b^{a\mu\nu}$$

$$- \frac{1}{3} \left( \frac{g_2}{g_1 m^2 \log(\Lambda^2/m^2)} \right) b^{a\mu
u} \left( g_{\mu\nu} g^2 - 4 \partial^\mu \partial^\nu \right) b^a_{\mu\nu}$$

$$- \frac{1}{3} \left( \frac{g_2}{g_1 m^2 \log(\Lambda^2/m^2)} \right) \epsilon^{abc} \left\{ 2 A^{a\mu} [2 b^b_{\mu\nu} (\partial^\alpha b^c_{\alpha\nu}) + (\partial^\alpha b^b_{\mu\nu}) b^c_{\alpha\nu}] + \ldots \right\}$$

$$+ \left( \frac{g_2^2}{g_1 m^2 \log(\Lambda^2/m^2)} \right) \Delta^{abcd} \left\{ 2 A^{a\mu} A^{b\nu} (b^c_{\nu\alpha} b^d_{\alpha\mu} + b^c_{\alpha\mu} b^d_{\alpha\nu}) - A^{a\mu} A^{b\nu} b^c_{\alpha\mu} b^d_{\alpha\nu} \right\}$$

(11)
where the ellipsis represents contributions to the current of the same form as the term displayed in the corresponding square bracket but with shuffled indices and we have redefined the antisymmetric tensor fields as

$$\sqrt{g_1} \frac{m}{4\pi^2} \log(\Lambda^2/m^2) \, b_{\mu\nu}^a \rightarrow b_{\mu\nu}^a.$$  \hfill (12)

We now proceed with approximations that will help us to clean up the expression of Eq. (11) in order to recover the claims stated in the abstract. To eliminate the mass term for the antisymmetric tensor fields (fourth term in (11)), we tune the parameters in such a way that the following equation is satisfied

$$m^2 = \Lambda^2 e^{-4\pi^2/m^2} g_2^2$$  \hfill (13)

which is consistent with our perturbative analysis; for large $\Lambda^2$ we have small coupling constant $g_2$. We can take care of the other unwanted terms (fifth to last term in Eq. (11)) by assuming a small ratio of coupling constants

$$\frac{\pi^2}{g_1 m^2 \log(\Lambda^2/m^2)} \sim \frac{g_2}{g_1} \ll 1.$$  \hfill (14)

Note, there are other terms arising from squaring the effective current that we have not displayed here for aesthetical reasons. These terms may be dropped in the same way as stated above. However, we need to be a bit caution in taking care of them since it is necessary to show more clearly the energy scale of the derivatives expansion approximation made when we compute the effective action. Qualitatively, we may estimate the energy scale of the derivative as $[\partial] \sim [A, b] \sim M_{\text{bosons}} = \frac{g}{\sqrt{g_1}}$ [see below]. This qualitative evaluation combined with the approximation (14) made on the coupling constants and the fact that we consider a weak theory ($g < 1$) give us a sufficient argument for dropping these undesirable terms.

Finally, we are left with the following low energy gauge invariant effective Lagrangian

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \frac{1}{12} H_{\mu\nu\rho}^a H^{a\mu\nu\rho} + \frac{1}{4\sqrt{g_1}} g^a e^{\alpha\beta\mu\nu} b_{\alpha\beta} F_{\mu\nu}^a$$  \hfill (15)

valid up to energy $m$, which belongs to a class of non-Abelian $B^\wedge F$-type model \textsuperscript{12} and describes two transverse and one longitudinal propagating modes of mass $g/\sqrt{g_1}$ which consist of the three degrees of freedom of massive non-Abelian gauge bosons with mass $g/\sqrt{g_1}$ for each group color indices\textsuperscript{10}.

5. Conclusions

We have succeeded in functionally integrating the four-Fermi theory to end up with an effective $b \wedge F$-theory in agreement with the model proposed by Lahiri \textsuperscript{12} but different than the one proposed by Freedman and Townsend \textsuperscript{5}. It is interesting to note that we did not reproduce Freedman-Townsend’s model because the non-linearities in the antisymmetric tensor fields are suppressed by the cutoff $\Lambda$. This
is perhaps unfortunate because the F-T model has higher reducible vector gauge invariance and behaves properly for renormalization purpose \(^{13}\). However, it has been shown to have unitarity problems in tree-level scattering \(^{14}\).

The degree of freedom count of the theory reveal that the Lagrangian of Eq. (15) describes indeed, for each color, two massive transverse modes combined with a massive longitudinal mode of the same mass, which is necessarily interpreted as the third degree of freedom for the non-abelian gauge fields. The mechanism described here appears as a valuable dynamical mass generation process for non-abelian (and abelian) gauge fields.

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