Optimal configuration of voltage sag monitors considering sensitive areas

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Abstract. Voltage sags are unavoidable and cause serious harm to sensitive loads. Reasonable configuration of limited monitors can reduce monitoring costs and provide data support for sag management and reduction of sag hazards. Therefore, the optimal configuration of sag monitors is of great significance. Aiming at the traditional method that ignores the inconsistency of the degree of sag damage in different areas, an optimal configuration model of sag monitor considering the reliability of monitoring in sag sensitive areas and the location of phasor measurement unit (PMU) is proposed. The model takes sag observability as a constraint, takes the smallest number of monitors and covers the widest range of sag sensitive areas as the goal, and further configures PMU to assist in monitoring voltage sags. In addition, the Zeroin method is proposed to improve the accuracy of the exposed area calculation. The IEEE-30 system simulation shows that the proposed method can ensure the minimum number of monitor and high redundancy coverage of sensitive loads, ensuring its economic benefits. The proposed method overcomes the difficulty of balancing the monitoring cost with the monitoring capability of sensitive areas.

1. Introduction
With the development of semiconductor digitization and other technologies, new-type electrical equipment is playing an increasingly important role in all walks of life. As the most prominent problem in power quality, voltage sag has become the main threat to these devices [1]. Monitoring the sag is helpful to analyze its causes, evaluate its disturbance level, and provide data support for customers to select the best access point or grid transformation [2-3]. Therefore, a reasonable configuration of the sag monitor has important practical significance.

At present, for the research on the optimal configuration of sag monitors, the most important model is based on the Monitor Reach Area (MRA) method [4,5]. The model aims at the minimum number of voltage sag monitors, and takes the observability of the voltage sag as a constraint condition [6]. Algorithms such as genetic, particle swarm, integer linear programming can be used to solve [6-8], but this model usually has multiple solutions and it is difficult to determine the optimal solution. On the one hand, this is due to the lack of new optimization goals, resulting in insufficient optimization conditions. On the other hand, traditional methods regard all nodes in the system as equally important, so the configuration of monitors is lack of pertinence. Subsequent studies determine the optimal solution of the configuration plan by introducing new optimization goals, such as: the largest sag observability index [9], the largest sag severity index [10], the largest sag weight coefficient [11], the smallest uncertainty area index [12], and the largest anti-interference index [13], etc. These studies can uniquely determine the monitoring plan, but some have increased the number of monitors, and some still need to
further improve the ability to monitor the sag sensitive areas. In addition, PMU can collect current and voltage information at high frequency, and can record recorded data under disturbances such as sags [20], which can theoretically be used to assist in monitoring voltage sags.

Based on the above analysis, this article believes that the sag sensitive area in the system is more important, and proposes an optimization model of sag monitors that considers the reliability of the sag sensitive area monitoring and PMU placement. First, it is proposed to use Zeroin iteration method to improve the accuracy of exposed area calculation. Then a risk area matrix is defined on the basis of the exposed area to refine the sag sensitivity of different positions in the system. Then, with the minimum number of monitors and the maximum coverage of the sag sensitive area as the goals, and the entire network observability of the voltage sag as the constraint, the priority factor is introduced to simplify the formation of a multi-objective optimization configuration model. On the basis of the model, PMU is considered to be used to assist in monitoring voltage sag to further reduce the number of sag monitors and improve the reliability of monitoring voltage sag sensitive areas. From the simulation results of IEEE-30 system, the proposed model can only obtain the scheme that meets the configuration economy and the monitoring requirements of sensitive areas, and has practical application value.

2. Calculation of the exposed area and the monitor reach area

2.1 Calculation of exposed area

The area where a short-circuit fault in the power system can cause the sag amplitude of the busbar connected to the sensitive load to be lower than the set sag threshold is the exposed area of the busbar under the set amplitude. When the fault point moves on a certain line in the system, the sag amplitude at the bus bar where the sensitive load is located is a unimodal function with an opening downwards, and is approximate to a quadratic function. Use the sag amplitude at the three positions of 0, 0.5 and 1 on the line to perform quadratic interpolation, and form an equation with the sag threshold to solve the critical point [14]. In order to improve the calculation accuracy of the critical value, literature [15] proposed a piecewise quadratic interpolation technique.

The short circuit of the line in the system is the main reason for the sag of the sensitive load bus. Therefore, to solve the exposed area, it is first necessary to obtain the residual voltage of the load point under different short circuit types. The short-circuit calculation model of the power system is as follows:

\[
\text{Power System Grid}
\]

\[
\text{Fig. 1. Model of power system short-circuit calculation.}
\]

Suppose \( m \) is the sensitive bus, the fault at \( j \) in the line \( i-j \) between nodes \( i \) and \( j \), \( R_f \) is the fault resistance, and \( p \) is the normalized distance from the fault location to node \( i \). First carry out the power flow calculation, and then form each sequence impedance, and then obtain the three-phase voltage amplitude of the sensitive bus \( m \) under different fault types and take the smallest absolute value as the sag amplitude, namely \( |V_{\text{fault}}^{m}| = \min(|V_{A,m}^{\text{fault}}|, |V_{B,m}^{\text{fault}}|, |V_{C,m}^{\text{fault}}|) \), the calculation formula can refer to literature [16].

After the short-circuit calculation is completed, the discriminant matrices \( B_{\text{sag}} \) and \( L_{\text{sag}} \) need to be formed according to the short-circuit calculation results to determine the inclusion of different bus nodes and lines in the sensitive bus exposed area. First calculate the sag amplitude vector \( V_{\text{sag}} \) of the bus node \( m \):

\[
V_{\text{sag}} = \begin{bmatrix} |V_{\text{fault}}^{m,u,1}| & |V_{\text{fault}}^{m,u,2}| & |V_{\text{fault}}^{m,u,n}| \end{bmatrix}^T
\]

Among them, \( |V_{\text{fault}}^{m,u}| \) refers to the sag amplitude at bus \( m \) when bus \( u \) fails, and \( n \) is the total number of buses in the system. Then the discriminant matrices \( B_{\text{sag}} \) and \( L_{\text{sag}} \) are formed to determine the
calculation lines required by the Zeroin method. The element $b_{\text{sag}, i}$ of the $B_{\text{sag}}$ matrix is determined by the difference $\mathbf{A}V_{\text{sag}}$ between the sag amplitude vector $V_{\text{sag}}$ of the bus $m$ and the sag threshold vector $V_{\text{th}}$:

$$\mathbf{A}V_{\text{sag}} = [\Delta V_{\text{sag}, 1}, \Delta V_{\text{sag}, 2}, \ldots, \Delta V_{\text{sag}, s}]^T = V_{\text{sag}} - V_{\text{th}}$$

(2)

$$b_{\text{sag}, i} = \begin{cases} 1; & \text{if } \Delta V_{\text{sag}, i} \leq 0, \text{in the exposed area} \\ 0; & \text{if } \Delta V_{\text{sag}, i} > 0, \text{not in the exposed area} \end{cases}$$

(3)

Suppose the number of lines is $v$, and the parameter matrix $L_{\text{sag}}$ of the line can be further determined according to the matrix $B_{\text{sag}}$ as follows:

$$L_{\text{sag}} = \begin{bmatrix} l_{\text{sag}, 1} \\ \vdots \\ l_{\text{sag}, v} \end{bmatrix} = \begin{bmatrix} b_{\text{sag}, 1} \\ \vdots \\ b_{\text{sag}, j} \end{bmatrix} + \begin{bmatrix} b_{\text{sag}, j} \\ \vdots \\ b_{\text{sag}, j} \end{bmatrix}$$

(4)

Among them, the subscripts $i$ and $j$ of $b_{\text{sag}, i}$ and $b_{\text{sag}, j}$ indicate the bus node numbers at both ends of the corresponding line in the row where they are located, and the element in $L_{\text{sag}}$ is 0 means that the corresponding line $i-j$ is not in the $m$ exposed area, and no subsequent calculation is required; a value of 1 indicates that the line $i-j$ contains a critical point, and a value of 2 means that there are two critical points. After determining the number of critical points of the line, the sag amplitude can be fitted by appropriate interval and endpoint values to calculate the critical points in the line [15].

2.2 Calculation of monitor reach area

The sag caused by a short circuit is random, and the key to the configuration of the sag monitor is whether it can accurately identify the sag event caused by any short circuit fault. The observability of the voltage sag of the system can be reflected by the voltage sag observability matrix, also known as the MRA matrix. Assuming that the number of nodes in the system is $n$ and the number of line segments is $s$, the MRA matrix $M_w$ under any short-circuit fault type can be expressed as follows:

$$M_w = \begin{bmatrix} m_{11, w} & m_{12, w} & \ldots & m_{1w, w} \\ m_{21, w} & m_{22, w} & \ldots & m_{2w, w} \\ \vdots & \vdots & \ddots & \vdots \\ m_{s1, w} & m_{s2, w} & \ldots & m_{sw, w} \end{bmatrix}$$

(5)

Among them, $w$ represents four types of faults, the values 0, 1, 2, and 3 respectively represent three-phase short-circuit, single-phase grounding short-circuit, two-phase short-circuit and two-phase grounding short-circuit; and $M_w$ is a binary matrix, and its elements are as follows Rule-based value.

$$m_{ab, w} = \begin{cases} 1 & V_{ab} \leq V_{\text{th}} \\ 0 & V_{ab} > V_{\text{th}} \end{cases}$$

(6)

Among them, $a = 1, 2, \ldots, n$, $b = 1, 2, \ldots, s$, $V_{ab}$ represents the sag amplitude at node $b$ when a short circuit occurs in line segment $a$; $V_{\text{th}}$ represents the sag threshold set by the sag monitor. When the element $m_{ab}=1$ in the matrix, it means that a short circuit in the line segment $a$ will cause the node $b$ to sag and be monitored by the monitor; on the contrary, a value of 0 means that it cannot be monitored. Since the rows in the $M_w$ matrix represent the sag monitoring range of the corresponding nodes, if some nodes can be determined and the union of their monitoring range can cover the whole power system, the whole network observability of sag can be achieved without configuring monitors at all nodes.

3. Optimized configuration of voltage sag monitor

3.1 Calculation of exposed area based on zeroin method

Due to the complexity of the power system, the relationship between the sag amplitude and the line cannot be completely described by the fitted quadratic function, and the quality of the fitting is directly affected by the initial selected value. In fact, the essence of determining the exposed area of the bus is
to find the critical point in each line, that is, to solve the root of the equation \( V_{\text{fault}}^m - V_{\text{th}} = 0 \). In this paper, the equation is transformed into the objective function shown in equation (7), and the accuracy and speed of the exposed area calculation are ensured through a suitable iterative method.

\[
\min f = |V_{\text{fault}}^m - V_{\text{th}}|	ag{7}
\]

The critical point on each line can be determined according to \( L_{\text{ sag}} \). When \( L_{\text{ sag}} = 0 \), the line \( v \) is outside the exposed area, so there is no need to calculate the critical point. When \( L_{\text{ sag}} = 1 \), there is only one critical point on the line at this time, which can be obtained by using the Zeroin iteration method of equation (7). When \( L_{\text{ sag}} = 2 \), there are two critical points on the line. At this time, we first use the zeroen method to solve the point \( p_{\text{max}} \) in the line that makes the sag amplitude \( V_{\text{fault}}^m \) maximum, and then use the zeroen iterative method to solve the two critical points in the interval of \([0, p_{\text{max}}]\) and \([p_{\text{max}}, 1]\). The specific steps of using Zeroin algorithm to solve the line critical point are as follows:

- **Step 1:** Select the initial values \( a, b \) so that the signs of \( f(a) \) and \( f(b) \) are opposite, and the initial value can be the endpoint value.
- **Step 2:** Assign the value of \( a \) to \( c \).
- **Step 3:** If the signs of \( f(b) \) and \( f(a) \) are the same, assign \( c \) to \( a \).
- **Step 4:** If \( |f(a)| < |f(b)| \), assign the value of \( b \) to \( c \), and then reverse the values of \( a \) and \( b \).
- **Step 5:** If \( c \neq a \), use \( a, b, c \) and their function values to do inverse quadratic interpolation and assign them to \( c \); if \( c = a \), use \( a, b \) and their function values to do secant iteration and assign them to \( c \).
- **Step 6:** If the iterative result \( c \in [a, b] \) in step 5, assign \( c \) to \( b \), otherwise take the average of \( a \) and \( b \) as the value of \( c \), and assign \( c \) to \( b \).
- **Step 7:** Repeat steps 3–6 until \( f(b) = V_{\text{th}} \) or \( b - a < \epsilon |b| \), where \( \epsilon \) is the error control threshold. Traverse the lines in the system and repeat steps 1–7 to get the exposed area of bus node \( m \).

3.2 Consider the economical configuration of monitors in sensitive areas of sag

Assuming that the system has a total of \( n \) nodes, the decision vector for configuring the sag monitor can be expressed as equation (8):

\[
X = [x_1, x_2, \ldots, x_n] \tag{8}
\]

\( X \) is a binary vector whose elements are taken according to formula (9):

\[
x_i = \begin{cases} 
1; & \text{if monitor is needed at } i \\
0; & \text{if monitor is not needed at } i 
\end{cases} \tag{9}
\]

Among them, \( i = 1, 2, \ldots, n \). According to the configuration principle, the constrained condition is constructed with the entire network observability of the sag, which is to meet the basic principle that the sag caused by any fault can be recorded by at least one monitoring device, as shown in equation (10):

\[
\sum_{i=1}^{n} x_i m_i \geq 1 \tag{10}
\]

Considering the high cost of monitors in practical applications, the configuration should aim at the minimum number of monitors, as shown in formula (11):

\[
\min f_i = \sum_{i=1}^{n} x_i \tag{11}
\]

The above configuration method defaults that all nodes are equally important. In addition, insufficient optimization conditions also result in a non-unique configuration scheme. Failures in the sag sensitive area will cause greater harm to the system and customers. In order to improve the safety and reliability of system operation, it can be considered that the sag sensitive area is more important. On the basis of considerable sag, it is helpful to improve the monitoring redundancy of sag sensitive areas, take relevant treatment measures in time when the sag is caused by short circuit, reduce the harm caused by sag as much as possible, and improve the reliability of system operation. In addition, the division of node importance also avoids the blindness of configuration. The exposed area can present the sag
sensitive area in the system, but it is a concept for a bus node. In the actual operation of the power system, the equipment must be distributed on multiple bus bars. Based on this, this paper proposes a risk area matrix as shown in equation (12) to further characterize the sag sensitivity of different areas in the system:

\[
R = \begin{bmatrix}
  r_{11} & r_{12} & \cdots & r_{1l} \\
  r_{21} & r_{22} & \cdots & r_{2l} \\
  \vdots & \vdots & \ddots & \vdots \\
  r_{n1} & r_{n2} & \cdots & r_{nl}
\end{bmatrix}
\tag{12}
\]

The subscript \( n \) represents the number of system nodes, and \( l \) represents the total number of system lines. When the matrix element \( r_{cd} \) is not equal to 0, its value is the total number of times that line \( d \) is included in the exposed area of all nodes, and only line \( d \) is at node \( c \), \( r_{cd} \) is not 0 when in the exposed area. When \( r_{cd} > 1(\# = 1, 2, \ldots, n) \), it indicates that the line \( d \) is in the exposed area of more than one node, and the larger the \( r_{cd} \) value is, the more sensitive it is to the sag. Equation (12) is the risk area matrix under a single failure type. In order to comprehensively consider the four failure types, the following corrections are needed:

\[
R = \lambda_1 R_1 + \lambda_2 R_2 + \lambda_3 R_3 + \lambda_4 R_4
\tag{13}
\]

Among them, \( R_1 \), \( R_2 \), \( R_3 \), and \( R_4 \) represent the risk area matrix when three-phase short-circuit, single-phase short-circuit, two-phase short-circuit and two-phase grounding short-circuit, respectively, and its coefficients \( \lambda_1 \), \( \lambda_2 \), \( \lambda_3 \) and \( \lambda_4 \) represent the probability of occurrence of several types of faults, respectively, 0.7, 0.15, 0.1, 0.05 [21]. Take formula (11) as the first-level target; then use the risk area matrix shown in formula (13) to take the wider and more sensitive weak links of the monitor as the second-level target, as shown in formula (14):

\[
\max f_2 = \sum_{i=1}^{n} x_i \cdot r_{i*}, \# = 1, 2, \ldots, l
\tag{14}
\]

In order to facilitate the solution, it needs to be deformed according to the actual characteristics of the two objective functions. Introducing the priority factor \( \alpha \) to equation (11), the two goals can be transformed into equation (15):

\[
\min f = \alpha \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} x_i \cdot r_{i*}, \# = 1, 2, \ldots, l
\tag{15}
\]

In order to determine the value of \( \alpha \), sensitivity analysis is performed on equation (14). Let the number of sag monitors change by one, namely:

\[
\Delta \sum_{i=1}^{N} x_i = 1
\tag{16}
\]

Then the secondary target change amount is:

\[
\Delta f_2 = \left( \Delta \sum_{i=1}^{n} x_i \cdot r_{i*} \right) \in [0, n \cdot l], \# = 1, 2, \ldots, l
\tag{17}
\]

Therefore, the change in formula (15) is:

\[
\Delta f = \alpha \Delta \sum_{i=1}^{n} x_i - \Delta \sum_{i=1}^{n} x_i \cdot r_{i*} = \alpha - \beta
\tag{18}
\]

Among them, \( \beta \in [0, n \cdot l] \), to ensure that the first-level target is satisfied before the second-level target, it needs to satisfy \( \alpha \gg \beta \), that is, \( \alpha \gg n \cdot l \). The optimal model can be expressed as equation (19) by dividing goal two by \( n \cdot l \).

\[
\begin{aligned}
\min f &= \alpha \sum_{i=1}^{n} x_i - \frac{\sum_{i=1}^{n} x_i \cdot r_{i*}}{n \cdot l}, \alpha \gg 1 \\
\text{s.t.} \quad \sum_{j=1}^{m} x_j \cdot m_j &\geq 1
\end{aligned}
\tag{19}
\]
3.3 Optimal configuration of voltage sag monitor for auxiliary monitoring of PMU

PMU can be widely used in many professional fields such as power system state estimation, transient stability analysis and prediction, fault location, system protection and so on, and has a very important position [17-18]. Therefore, a lot of work has been done in the unified planning and deployment of PMU in China. On the one hand, the PMU can sample the node voltage vector and the branch current flowing through the node with high frequency and high resolution [19], which fully meets the conditions for monitoring voltage sags [20]. On the other hand, the voltage sag monitor is the same as the PMU, and it is difficult to fully lay it out in the short term due to the high cost. There is a high need for PMU laying, and theoretically it can also monitor voltage sags. Using the PMU installed in the power grid to assist in monitoring voltage sags can further reduce the installation cost of sag monitors.

The primary goal of the optimized configuration of the voltage sag monitor combined with PMU is to ensure the minimum number of sag monitors installed, and the second is to improve the monitoring redundancy in the sag sensitive area. First, use the optimized configuration models shown in equations (10) and (11) to obtain all the configuration schemes that can guarantee a considerable sag and the minimum number of monitors, and then determine whether the existing PMU laying nodes appear in these configuration schemes. If it appears, select a group of configuration schemes that maximize Eq. (14) from the configuration schemes containing PMU nodes. If it does not appear, it indicates that the number of sag monitors can not be reduced due to the constraints of sag observability when the existing PMU is used to assist in monitoring voltage sag. The optimized configuration of the sag monitor with the highest redundancy for monitoring sag sensitive areas can be obtained by improving the optimization model shown in equation (19) and using the PMU to assist in monitoring voltage sags. The improved optimized configuration model is as shown in equation (20) Shown:

\[
\begin{align*}
\min f &= \alpha \sum_{i=1}^{n} x_i \sum_{j=1; j \neq k}^{n} \frac{x_j r_{ij}}{n!}, \alpha \gg 1 \\
\text{s.t. } x_i m_{ij} + \sum_{j=1; j \neq k}^{n} x_j m_j &\geq 1
\end{align*}
\]

In the formula, \(x_i\) represents one or more nodes where the PMU is installed, and the subscript \(k\) is the number of the node where the PMU is installed.

4. Simulation verification

4.1 Analysis of accuracy of exposed area recognition

In order to verify the feasibility and effectiveness of the method proposed in this article, the IEEE-30 node is tested. The system consists of 37 lines connected to 30 bus nodes. Assuming that the sag threshold at bus No. 20 is 0.841p.u., the golden section method [14], the dichotomy method [15] and the Zeroin method are used to obtain the critical point of the line \(l_{2,4}\). The results are shown in the figure:

Fig. 2. Comparison chart of calculation results of line critical points.
The two dotted lines in the figure represent the results of using quadratic interpolation to fit the sag amplitude in the literature [14] and [15]. The solid line in the figure represents the actual sag amplitude when the fault point moves with the line, which is also the result of the proposed method. The projection of the intersection of the three lines and the sag threshold to the horizontal axis represents the critical point in the line. The exact values of the two critical points in the line are 0.1488 and 0.3701, and the solutions obtained using literature [14] and [15] are 0.3056, 0.3761 and 0.2132, 0.4934, respectively. This paper uses Zeroin method to iteratively solve the sag amplitude equation. Therefore, the error between the results of 0.1487 and 0.3700 and the actual value is small, indicating that the calculation method of exposed area described in this paper can effectively improve the accuracy of solving the exposed area.

In order to illustrate the calculation efficiency of different methods, take the single-phase short-circuit fault of the line between nodes 2 and 4 as an example to calculate the critical point. Set the convergence condition as $\varepsilon = 0.0001$, and Figure 4 shows the number of iterations required for each method to reach the set threshold.

![Fig. 3. Comparison chart of the relationship between error and the number of iterations.](image)

The error in Fig. 3 is the error between the iterative value of different methods and the solution conforming to the convergence condition of the method itself, not the error between the real value and the iterative value shown in Fig. 2. In addition, it can be seen from the figure that the Zeroin method also has great advantages in calculation speed.

**4.2 Analysis of monitoring redundancy in sag sensitive area**

First, by solving the traditional configuration model shown in equations (10) and (11), the configuration scheme to achieve 0.9p.u. sag observability of the whole network is obtained, as shown in Table 1. It can be seen from the table that under the constraint of the observability of the whole network of voltage sag, the scheme only meeting the goal of minimum number of monitoring devices is not unique.

| Scheme No | Node number | Scheme No | Node number |
|-----------|-------------|-----------|-------------|
| 1         | 2, 24       | 10        | 5, 27       |
| 2         | 2, 25       | 11        | 5, 29       |
| 3         | 2, 26       | 12        | 5, 30       |
| 4         | 2, 27       | 13        | 7, 24       |
| 5         | 2, 29       | 14        | 7, 25       |
| 6         | 2, 30       | 15        | 7, 26       |
| 7         | 5, 24       | 16        | 7, 27       |
| 8         | 5, 25       | 17        | 7, 29       |
| 9         | 5, 26       | 18        | 7, 30       |

In order to use the multi-objective optimization model proposed in this paper and obtain the optimal configuration plan, first obtain the exposed areas of all buses in the system, and then count the total...
number of times the line appears in different bus exposed areas. The higher the number, the more the line needs to be focus on monitoring. The relevant statistical results are shown in Table 2:

| S | E | C | S | E | C | S | E | C |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 0 | 10 | 20 | 19 | 19 | 20 | 16 |
| 1 | 3 | 2 | 10 | 21 | 21 | 21 | 22 | 19 |
| 2 | 4 | 2 | 10 | 22 | 21 | 22 | 24 | 19 |
| 2 | 5 | 0 | 12 | 13 | 15 | 23 | 24 | 18 |
| 2 | 6 | 6 | 12 | 14 | 16 | 24 | 25 | 16 |
| 3 | 4 | 13 | 12 | 15 | 19 | 25 | 26 | 6 |
| 4 | 6 | 22 | 12 | 16 | 20 | 25 | 27 | 13 |
| 5 | 7 | 2 | 14 | 15 | 17 | 27 | 29 | 6 |
| 6 | 7 | 10 | 15 | 18 | 18 | 27 | 30 | 6 |
| 6 | 8 | 12 | 15 | 23 | 18 | 29 | 30 | 6 |
| 9 | 10 | 21 | 16 | 17 | 19 | 8 | 28 | 11 |
| 9 | 11 | 14 | 18 | 19 | 16 | 6 | 28 | 20 |
| 10 | 17 | 20 |   |   |   |   |   |   |

The letters S, E, and C in Table 2 respectively indicate: the line start node number, the line end node number, and the total number of the line in the exposed area. In addition, the inclusion of lines in different exposed areas is shown in Table 3:

| Node number | Number of included lines | Node number | Number of included lines |
|-------------|--------------------------|-------------|--------------------------|
| 1           | 1,2                      | 16          | 3,5~7,9~30,32,36,37      |
| 2           | 1~5                      | 17          | 3,5~7,9~30,32,36,37      |
| 3           | 1~3,5~7,9~11,19,36,37    | 18          | 3,5~7,9~30,32,36,37      |
| 4           | 1~3,5~7,9~11,13,15~17,   | 19          | 3,5~7,9~30,32,36,37      |
|             | 19,20,36,37              |             |                          |
| 5           | 1,2,4,8                  | 20          | 3,5~7,9~30,32,36,37      |
| 6           | 3,5~11,13,15,16,36,37    | 21          | 3,5~7,9~32,36,37         |
| 7           | 1~10,36,37               | 22          | 3,5~7,9~33,36,37         |
| 8           | 5,7,10,36,37             | 23          | 3,5~7,9~34,36,37         |
| 9           | 3,6,7,9~16,19,20,22~30,36,37 | 24          | 3,5~7,9~37          |
| 10          | 3,5~7,9~30,32,36,37      | 25          | 3,5~7,9~37          |
| 11          | 12                       | 26          | 3,5~7,9~37          |
| 12          | 3,5~7,9~11,13~30,37      | 27          | 3,5~7,9~37          |
| 13          | 17,19                    | 28          | 3~7,9~11,13,15,16,27,28,30,32~34,36,37 |
| 14          | 3,5~7,9~30,32,36,37      | 29          | 3,5~7,9~37          |
| 15          | 3,5~7,9~30,32,36,37      | 30          | 3,5~7,9~37          |

Based on the statistical results, formula (12) is used to form a risk area matrix, and then the multi-objective optimization model shown in formula (19) is used to obtain the optimal solution: bus 7 and bus 30 are equipped with one sag monitor respectively. Finally, a group of schemes such as 5 and 26 are randomly selected from the configuration schemes obtained by the traditional method and compared with schemes 7 and 30 obtained by the method in this paper. Based on table 2 and table 3, the total number of times of the lines covered by the monitors in each exposed area of the system in the traditional and the proposed monitoring scheme is 573 and 804 respectively, which indicates that the proposed method can detect more sag sensitive lines in the system. In order to intuitively show the monitoring redundancy of the two monitoring schemes for the sag sensitive area, through the calculation results of the exposed area, the schematic diagram of the monitoring range of the two schemes is drawn, as shown in Fig. 4.
Fig. 4. Comparison of the monitoring scope of the two programs.

Fig. 4. only lists the situation of a single-phase short-circuit fault, and the results are similar under other faults. In the figure above, the areas with different colors indicate the monitoring range when the monitor is located at different nodes. MS in the legend indicates the monitoring range, CMS represents the common monitoring range. It can be seen from the figure that the union of the monitoring range of different monitors in the two monitoring schemes can cover the whole system, indicating that the two schemes can achieve the whole network observability of voltage sag. However, the sensitive area covered by two monitors in the traditional scheme is smaller. Therefore, when a single monitor fails, the monitoring capability of the remaining monitor to the sensitive area is insufficient compared with the scheme proposed in this paper. The results show that the proposed method can improve the monitoring reliability of the area with high sag risk.

4.3 Analysis of monitoring redundancy of sag sensitive area considering PMU
Assuming that the PMU monitor is installed on node 9 and the scheme shown in Table 1 does not include this node, the number of sag monitors is still 2. According to the optimal configuration model shown in equation (20), the laying nodes of the monitor are No. 7 and No. 30. At this time, the PMU at node 9 auxiliary monitors the voltage sag, and the monitoring range under joint monitoring is as follows:
As the PMU assists in monitoring the voltage sag, the total number of times that the line covered by
the monitor appears in different exposed areas of the system at this time is 1188. Compared with the
above two monitors, the scheme can further improve the monitoring reliability of sag sensitive areas,
but does not increase the number of sag monitors.

5. Conclusion
This paper builds a risk area matrix based on the concept of exposed area, depicts the degree of harm to
the system caused by sags caused by different line faults, and classifies important levels for different
areas. This division principle also conforms to the basic requirements of ensuring safe, reliable and high-
quality power supply for the operation of the power system, and avoids the blindness of configuration
caused by the traditional method in the installation of the sag monitor that all nodes are equally important.
When solving the exposed area of the system, the ZeroIn iteration is used to combine the advantages of
the stability of the dichotomy with the rapid convergence of the parabolic method and the secant method,
which improves the solution speed and greatly improves the accuracy compared with the fitting method.
This paper proposes an optimization model that aims to improve the monitoring redundancy of sag
sensitive areas and meets the sag observability and configuration economy. It solves the problem that
the traditional optimal configuration model is difficult to uniquely determine the solution that meets the
engineering needs due to multiple solutions. On this basis, it is proposed to jointly optimize the
configuration of PMU and sag monitor to ensure that the number of sag monitors is as small as possible,
and the monitoring capability of the sag sensitive area in the system is the largest, which is of great
significance.

Acknowledgements
This work is supported by the science and technology project of State Grid Shandong electric power
company. The project name is "Research on panoramic voltage sag monitoring technology of
distribution network under the condition of power Internet of things", and the project number is
"520615200004".

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