Low Fine Tuning in Yukawa-deflected
Gauge Mediation

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Abstract

We discuss a class of models with gauge-mediated supersymmetry breaking characterized by a non-unified messenger sector inducing non-standard gaugino mass ratios, as well as by additional contributions to the soft mass terms from a matter-messenger coupling. The well-known effect of this coupling is to generate A-terms at one-loop level, hence raising the Higgs mass without relying on super-heavy stops. At the same time, a hierarchy between Wino and gluino masses, as induced by the non-unified messenger fields, can greatly lower the radiative corrections to the Higgs soft mass term driven by the high-energy parameters, thus reducing the fine tuning. We search for models with low fine tuning within this scenario, and we discuss the spectrum, collider phenomenology, constraints, and prospects of the found solutions. We find that some setups are accessible or already excluded by searches at the Large Hadron Collider, and all our scenarios with a tuning better than about 2% can be tested at the International Linear Collider.

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1 Introduction

Despite the numerous dedicated searches, the first years of run of the Large Hadron Collider (LHC) have given no sign of new phenomena associated with supersymmetry (SUSY). The stringent limits set by ATLAS and CMS on the SUSY spectrum, as well as the measured Higgs boson mass, \( m_h \approx 125 \) GeV, certainly challenge the paradigm of low-energy SUSY as a solution of the electro-weak hierarchy problem, especially within the simplest realizations, such as minimal supergravity, minimal gauge mediation (mGMSB) etc. For this reason, we find it timely to study less minimal constructions in search for realizations that can better accommodate (i) the observed Higgs mass, and (ii) a spectrum heavy enough to evade the direct LHC searches, but (iii) without a large fine-tuning (FT) among the theory’s parameters.

Although any attempt to quantify the FT price of a model – which is obviously not a physical observable – is to some extent subjective, we still regard the FT measures proposed in the literature, starting from \([1]\), as useful tools to compare different models and SUSY spectra, and identify those that are most likely realized in nature.

In a previous work \([2]\), we employed the “electro-weak FT”, \( \Delta_{\text{EW}} \), introduced in \([3, 4]\), in order to show that models with gauge-mediated SUSY breaking \([5]\) with messengers in incomplete representations of a grand unified gauge group can lower the FT by more than one order of magnitude with respect to mGMSB, i.e. \( \Delta_{\text{EW}} \lesssim 50 \). In particular, solutions with a reduced FT are possible if such non-unified messenger sector gives raise to a Wino mass substantially larger than the gluino mass at the mediation scale. This in turn induces a compensation between gauge and Yukawa radiative corrections to the Higgs soft mass \( \tilde{m}_{H_u}^2 \), reducing its sensitivity to stop and gluino masses. This effect can be appreciated by inspecting the renormalization group equation (RGE) of \( \tilde{m}_{H_u}^2 \), whose one-loop expression is:

\[
16\pi^2 \frac{d}{dt} \tilde{m}_{H_u}^2 \approx 6y_t^2 [\tilde{m}_{H_u}^2 + \tilde{m}_{Q_3}^2 + \tilde{m}_{U_3}^2] + 6A_t^2 - 6g_2^2 M_2^2 \left( t \equiv \log \frac{\mu}{M} \right),
\]

where \( \mu \) is the renormalization scale and \( M \) a reference scale, and we omitted the hypercharge-dependent terms. The terms proportional to the top A-term \( A_t \) and the top Yukawa \( y_t \) (where the left-handed (LH) and right-handed (RH) stop masses \( \tilde{m}_{Q_3}^2 \) and \( \tilde{m}_{U_3}^2 \) appear) have opposite sign with respect to the SU(2) gauge term (where \( g_2 \) is the gauge coupling and \( M_2 \) the gaugino mass). As a consequence a compensation between the two terms, hence a reduced value of \( |\tilde{m}_{H_u}^2| \) at low energies, is possible provided that \( M_2 > M_3 \) (as the gluino mass \( M_3 \) induces large positive contributions to the stop masses in the running). For related works, see \([6-11]\).

In \([2]\), we found that the typical spectra of these low-tuned models with \( M_2 > M_3 \) tend to lie in the multi-TeV range (mainly because of \( m_h \)), and the only sub-TeV states are the Higgsino and possibly Bino and right-handed sleptons. The absence of signals at the LHC searches is therefore a natural consequence of the framework. However, this also makes it very difficult to test even in the long run. A similar conclusion is shared in the context of gravity mediation by grand unified theory (GUT) models with the non-universal gaugino masses induced by the breaking of the GUT group \([12-19]\).

The aim of the present work is to verify whether a deformation of models with non-unified messenger sectors can modify the above conclusion, and investigate possible handles
to test this class of low-tuned models at the LHC. For this purpose, we introduce in the
Lagrangian a Yukawa-like coupling between messenger and matter superfields, which provides
additional contributions to the soft terms besides those purely due to gauge interactions.
Models of this kind – sometimes labelled as “Yukawa-deflected gauge mediation” or “Extended
gauge mediation” – have been proposed long ago [20–22], and more recently received renewed
attention [23–46], especially after the discovery of the Higgs, since they can induce one-loop
contributions to the stop A-term \( A_t \) – in contrast to ordinary gauge mediation setups – such
that \( m_h \approx 125 \text{ GeV} \) can be accommodated without paying the price of multi-TeV stop masses.
Despite that, it has been shown that such models typically do not improve much the FT over
mGMSB [47,48], because the messenger-matter coupling does not only generate \( A_t \), but also
large negative contributions to \( \tilde{m}^2_{H_u} \) of the order \( \approx -|A_t|^2 \), so that large cancellations among
the parameters in the Higgs potential are still needed in order to achieve a correct electro-
weak symmetry breaking (EWSB). However, to the best of our knowledge, marrying “Yukawa
deflection” with a non-unified messenger sector has not been attempted yet. In this extended
setup, we expect that – similarly to the case of our previous work – negative contributions
to the Higgs soft masses can be compensated by a heavy Wino in the renormalization group
running, reducing the amount of cancellation required by EWSB, i.e. the fine tuning. At
the same time, the \( A_t \) induced by the matter-messenger coupling should allow to obtain \( m_h \approx 125
\text{ GeV} \) with a \( O(1) \) TeV stop, unlike in [2].

The rest of the paper is organized as follows. In section 2, we present the model setup
and the high-energy parameters that control the soft SUSY-breaking mass terms. In section
3 we show the results of a numerical scan over the space of models, highlighting the solutions
with reduced fine tuning and the typical features of their SUSY spectra. We discuss the
LHC phenomenology in section 4 of the different classes of low-tuned solutions we found, in
particular present collider constraints, and testability prospects. We draw our conclusions in
section 5.

2 Model setup and soft masses

As in [2], we work in the context of the minimal supersymmetric standard model (MSSM) and
we consider a Gauge Mediation setup where SUSY breaking is transmitted to the visible sector
trough loops involving heavy messengers in vectorlike representations of the Standard Model
(SM) gauge group, which however do not necessarily belong to complete multiplets of a grand
unified group. The resulting contributions to gaugino and sfermion masses then depend on
three independent parameters \( b_1^M, b_2^M, b_3^M \), denoting the shifts induced at the messenger scale
\( M \) to the 1-loop \( \beta \)-function coefficients of the SM gauge couplings \( g_1, g_2, g_3 \). As in minimal
Gauge Mediation, gaugino masses \( (M_a, a = 1, 2, 3) \) arise at 1-loop, scalar masses \( (\tilde{m}^2_X(M), \ X = Q, U, D, L, E, H_u H_d) \) at 2-loops. Their expressions for non-unified messengers read at

\[ 1 \] A discussion of the possible sets of messengers and the resulting \( b_a^M \) has been presented in [2].
the messenger scale \[49\]:

\[ M_a(M) = \frac{\alpha_a(M)}{4\pi} b_a^{M} \Lambda, \quad a = 1, 2, 3, \]  

\[ \tilde{m}_X^2(M) = 2 \sum_{a=1,3} \left( \frac{\alpha_a(M)}{4\pi} \right)^2 C_a^X b_a^M \Lambda^2, \]  

where \( \Lambda \equiv F/M \) is the ratio of a single SUSY-breaking F-term and the mediation scale \( M \), and \( C_a^X \) (\( a = 1, 2, 3 \)) is the quadratic Casimir of the representation of \( X \) under \( SU(3) \times SU(2) \times U(1) \). Being of a purely gauge origin, the sfermion masses are flavor universal.

In the spirit of ‘Yukawa-deflected’ gauge mediation, we also allow for matter-messenger couplings. Among several possibilities \[33\], we choose to introduce a single new coupling involving only the third generation quarks, of the form:

\[ W \supset \lambda_t Q_3 U_3 \Phi_u, \]  

where \( \Phi_u \) is a messenger superfield with the same quantum numbers as \( H_u \).

Unlike in mGMSB, squark A-terms are generated at 1-loop and read the messenger scale \[24,33\]:

\[ A_t(M) = -\frac{3\Lambda}{16\pi^2} \lambda_t^2 y_t, \quad A_b(M) = -\frac{\Lambda}{16\pi^2} \lambda_b^2 y_b, \]  

where \( y_t \) and \( y_b \) are the ordinary top and bottom Yukawas.

Additionally, negative 1-loop contribution to the stop masses are generated \[24,33\]:

\[ \Delta \tilde{m}^{(1)}_{Q_3} = -\frac{\Lambda^2}{96\pi^2} \lambda_t^2 g(x), \quad \Delta \tilde{m}^{(1)}_{U_3} = -\frac{\Lambda^2}{48\pi^2} \lambda_t^2 g(x), \]  

with

\[ g(x) = 3 \left( \frac{x - 2}{x^2} \log(1 - x) - \frac{2 + x}{x^2} \log(1 + x) \right) = x^2 + \frac{4}{5} x^4 + O(x^6), \quad x = \frac{\Lambda}{M}. \]  

These contributions become irrelevant for \( M \gg \Lambda \).

Finally, third generation squarks and the two Higgs soft masses are deflected by additional 2-loop contributions that do not vanish for large messenger scales \[24,33\]:

\[ \Delta \tilde{m}^{(2)}_{Q_3} = \frac{\Lambda^2}{256\pi^4} \left[ -\left( \frac{13}{15} g_1^2 + 3 g_2^2 + \frac{16}{3} g_3^2 \right) \lambda_t^2 + 6 \lambda_t^4 + 6 \lambda_t^2 y_t^2 \right], \]  

\[ \Delta \tilde{m}^{(2)}_{U_3} = \frac{\Lambda^2}{128\pi^4} \left[ -\left( \frac{13}{15} g_1^2 + 3 g_2^2 + \frac{16}{3} g_3^2 \right) \lambda_t^2 + 6 \lambda_t^4 + 6 \lambda_t^2 y_t^2 + \lambda_t^2 y_b^2 \right], \]  

\[ \Delta \tilde{m}^{(2)}_{D_3} = -\frac{\Lambda^2}{128\pi^4} \lambda_t^2 y_b^2, \]  

\[ \Delta \tilde{m}^{(2)}_{H_u} = -\frac{9\Lambda^2}{256\pi^4} \lambda_t^2 y_t^2, \]  

\[ \Delta \tilde{m}^{(2)}_{H_d} = -\frac{3\Lambda^2}{256\pi^4} \lambda_t^2 y_b^2. \]  

Unlike the standard contributions to the sfermion masses in Eq. \[3\], these additional terms induced by \( \lambda_t \) only concern the stops, thus they provide a departure from flavor universality,
hence from the Minimal Flavor Violation (MFV) [50] structure of low-energy squark mass matrices that is characteristic of pure GM frameworks. However, it has been shown that this does not induce unacceptably large flavor-changing neutral current processes, as long as the matter-messenger couplings feature a flavor hierarchical structure resembling that of the ordinary Yukawa interactions [23][34][38]. This is trivially the case in the present setup, as we introduced only a third-generation coupling $\lambda_t$ of $\mathcal{O}(1)$.

3 Models with low tuning and typical spectra

In order to explore the class of models defined in the previous section, we performed a numerical scan employing a version of the routine ISAJET 7.85 [51] that we modified implementing the Yukawa-deflection contributions to the soft terms given in Eqs. (5)-(12). As in [2], the messenger contributions to the $\beta$-function coefficients $b_1^M$, $b_2^M$, $b_3^M$ are integer numbers that we randomly varied within these intervals:

$$1 \leq (5 \times b_1^M) \leq 75, \quad 1 \leq b_2^M \leq 20, \quad 1 \leq b_3^M \leq 7.$$  \(13\)

For the other parameters we took the following ranges:

$$5 \times 10^4 \text{ GeV} \leq \Lambda \leq 10^6 \text{ GeV}, \quad 2 \times \Lambda \leq M \leq 10^{15} \text{ GeV},$$

$$5 \leq \tan \beta \leq 50, \quad 0 \leq \lambda_t \leq 1.5.$$  \(14\)

In the following, we base our naturalness considerations on the ‘electro-weak’ fine tuning, defined as [3][4]

$$\Delta_{\text{EW}} \equiv \frac{\text{max}_x |C_x|}{m_Z^2/2},$$  \(15\)

where $C_x$ are the terms in the right-hand side of the minimization condition of the Higgs potential:

$$\frac{m_Z^2}{2} = \frac{(\tilde{m}_{H_d}^2 + \Sigma_d) - (\tilde{m}_{H_u}^2 + \Sigma_u) \tan^2 \beta - \mu^2}{\tan^2 \beta - 1}.$$  \(16\)

The quantities $\Sigma_{u,d}$ express the 1-loop corrections to the tree-level potential [52].

We show in Fig. 1 the resulting $\Delta_{\text{EW}}$ for the points of our scan, as defined by Eqs. \(13\)\(14\). The orange points correspond to the observed Higgs mass, once a theoretical uncertainty of 3 GeV is taken into account: 122 GeV $\leq m_h \leq 128$ GeV. As we can see, this condition is satisfied by points with $\Delta_{\text{EW}}$ as low as $\approx 20$, which corresponds to a tuning of about 5%, cf. the top-left plot of Fig. 1. From the expression for $\Delta_{\text{EW}}$, Eq. \(15\), it is clear that models with reduced tuning will require in particular that $|\tilde{m}_{H_u}^2|$ and $|\Sigma_u|$ are not much larger than $m_Z^2$. The first condition is facilitated if the messenger sector is such that $b_2^M > b_3^M$, i.e. the Wino mass exceeds the gluino mass at the messenger scale, as shown in [2]. In fact, this can lead to a compensation of the terms $\propto y_t^2 (\tilde{m}_{Q_3}^2 + \tilde{m}_{L_3}^2)$ and $\propto g_2^2 M_2^2$ in the $\beta$-function of $\tilde{m}_{H_u}^2$, cf. Eq. \(1\), reducing the sensitivity of the low-energy value of $|\tilde{m}_{H_u}^2|$ on heavy gluinos and...
Figure 1: First line: \( \Delta_{\text{EW}} \) vs. \( m_h \) (left), the lightest stop mass (right). Second line \( \Delta_{\text{EW}} \) vs. the ratio \( b_2^M/b_3^M \) (left), \( \lambda_t \) (right). Orange points correspond to \( 122 \text{ GeV} \leq m_h \leq 128 \text{ GeV} \).

This mechanism is particularly efficient for \( b_2^M \approx 3 \times b_3^M \) as shown in the bottom-left panel of Fig. 1. A very heavy stop sector would however reintroduce a fine tuning problem inducing large finite radiative corrections encoded in \( \Sigma_u \). The additional contributions induced by the coupling \( \lambda_t \) allow for solutions with a light stop, which is a qualitatively different with respect to the models with \( \lambda_t = 0 \) considered in our previous work. This is shown in the top-right panel of Fig. 1, where we see that points with \( \Delta_{\text{EW}} \lesssim 50 \) can feature \( m_{\tilde{t}_1} \) down to 1 TeV. This is due to different effects: the new contributions to the stop soft masses in Eqs. (8, 9) are negative for \( \lambda_t \lesssim 0.7 \) \( ^{34} \); a sizable \( A_t \) (generated at 1-loop \( \propto \lambda_t^2 \)) allows for \( m_h \approx 125 \text{ GeV} \) with a lighter stop sector and, at the same time, lowers the mass of the lightest stop eigenstate through a large LR stop mixing. As a result, we find the lowest values of \( \Delta_{\text{EW}} \) for \( \lambda_t \approx 0.6 \div 0.8 \), as shown in the bottom-right plot of Fig. 1.

Characteristic features of the spectrum of solutions with low fine tuning and \( 122 \text{ GeV} \leq m_h \leq 128 \text{ GeV} \) can be seen in Figs. 2-4 where different ranges of \( \Delta_{\text{EW}} \) are plotted in

\footnote{In models with \( \lambda_t = 0 \), \( b_2^M/b_3^M \) can not be larger than about 2.5, a value that provides the solutions with best FT \(^2\), because of failure of EWSB for a too large positive contribution of \( M_2 \) to the running of \( \bar{m}_{H_u}^2 \). This effect is compensated for \( \lambda_t \neq 0 \) by the additional negative contribution in Eq. (11), and large ratios \( b_2^M/b_3^M \) are accessible.}
Figure 2: Gluino (left panel) and Higgsino (right panel) mass vs. lightest stop mass for different ranges of $\Delta_{EW}$.

Figure 3: Stop mass vs. the matter-messenger coupling $\lambda_t$ (left panel); Higgsino mass vs. Wino mass (right panel). Color code as in Fig. 2.

Different colors. From these plots we see that the spectrum of the solutions with lowest $\Delta_{EW}$ is characterized by the following mass ranges:

$$1 \text{ TeV} \lesssim m_{\tilde{t}_1} \lesssim 2.5 \ (4.5) \text{ TeV},$$

$$\Delta_{EW} \lesssim 50 \ (100) \Rightarrow 2 \ (1) \text{ TeV} \lesssim m_{\tilde{g}} \lesssim 4.5 \ (6.5) \text{ TeV},$$

$$100 \text{ GeV} \lesssim m_{\tilde{\chi}_1^0} \lesssim 450 \ (650) \text{ GeV}. \quad (17)$$

Furthermore, we see that solutions $\Delta_{EW} < 50$ (yellow points) have quite heavy Winos at low energy ($M_2 \gtrsim 3.5 \text{ TeV}$), which reflects the high-energy condition on gaugino masses, as explained above, and a sizable Yukawa deflection is preferred, $\lambda_t \gtrsim 0.4$, cf. Fig. 3.

We are particularly interested in the nature of the next to lightest SUSY particle (NLSP) (the lightest SUSY particle (LSP) is always a light gravitino, as typical of Gauge Mediation), as it determines most of the collider phenomenology of our models, as we will discuss in the next section. Information about the NLSP can be read in Fig. 4. Our typical low-tuned solutions, e.g. $\Delta_{EW} < 50$ (yellow points) and $\Delta_{EW} < 100$ (red points), feature as NLSP a
neutralino that is mostly Higgsino: indeed, the corresponding points typically lie in the top-right quadrant of the second plot. However, we also found some solutions with a Bino-like NLSP (top-left quadrant) and stau NSLP (bottom quadrants).

The above possibilities are exemplified by the benchmark models whose spectra are listed in Tab. 1. Models A1-A3 are examples of the typical low-tuned setup with a rather light Higgsino NLSP. In particular, model A1 corresponds to the solution with the lowest $\Delta_{EW}$ we found in the scan, and illustrates a typical spectrum with Higgsino NLSP and heavy spectrum, with the $SU(3)$ and $SU(2)$ singlets as the only other states possibly lighter than 1 TeV. Model A2 also features a Higgsino NSLP, but lighter stop and gluino masses (around 1.1 and 2.2 TeV respectively).

Besides $\Delta_{EW}$, we also computed the Barbieri-Giudice FT measure $\Delta_{BG}$ \cite{1} for the benchmark models:

$$\Delta_{BG} \equiv \max A \Delta_{BG}(A), \quad \Delta_{BG}(A) = \left| \frac{\partial \log m^2_Z}{\partial \log A} \right|,$$

where $A$ run over the fundamental high-energy parameters: $\Lambda$, $M$, $\mu^2$, and the matter-messenger coupling $\lambda_t$. In Tab. 1, we display $\Delta_{BG}$ both with and without taking into account $\Delta_{BG}(\lambda_t)$. As we can see, the resulting $\Delta_{BG}$ is of the same order as $\Delta_{EW}$, if we do not consider $\Delta_{BG}(\lambda_t)$, while this latter quantity can be considerably larger. This is not surprising given the large negative Yukawa-deflection contribution to $\tilde{m}_{H_u}^2$, cf. Eq. (11). Such a sensitivity on $\lambda_t$ is in fact larger for scenarios with higher values of $\lambda_t \times \Lambda$ (that we also show in the Table), since this is the quantity that actually controls the size of the Yukawa-deflected contributions to the soft masses, cf. Eqs. (8,12), in particular to $\tilde{m}_{H_u}^2$. We can argue that indeed $\Delta_{BG}(\lambda_t)$ should not be considered in computing the FT (similarly to $\Delta_{BG}(\mu_t)$), since $\lambda_t$ is a Yukawa coupling. In fact, assuming that the MSSM arise from string theories with suitable compactifications and moduli stabilizations, one can in principle calculate the corresponding gauge couplings and Yukawa couplings at the string scale, which should be required to be consistent with the low-energy experimental values via RGE running. In particular, gauge and Yukawa couplings are completely determined by the string compactifications and moduli stabilizations above the

*Figure 4: Lightest stau vs. lightest neutralino mass (left panel); stau-neutralino mass ratio vs. Bino-Higgsino mass ratio (right panel). Color code as in Fig. 2.*
Table 1: Spectrum and parameters of six representative models. Models A1, A2, and A3 belong to the most typical class of low-tuned solutions, those featuring an almost pure (long-lived) Higgsino NLSP. A1 and A2 correspond respectively to the model with lowest tuning and lightest $\tilde{t}_1$ we found in the scan. Model A3 is characterized by a reduced sensitivity on $\lambda_t$, due to a small value of $\lambda_t \times \Lambda$, see the text for details. Models B1, B2, and B3 illustrate particular corners of the parameter space. Model B1 is an example of the (long-lived) stau NLSP scenario. Model B2 features a Bino-like neutralino NLSP. Model B3 show a peculiar solution with a short-lived (promptly-decaying) Higgsino NLSP. Dimensionful quantities are in GeV unless otherwise indicated.

SUSY-breaking scale. Thus, they are not related to the naturalness of the MSSM, and then we will not consider their fine-tuning measures, in particular $\Delta_{BG}(\lambda_t)$, for our phenomenological considerations. Furthermore, we see that even considering $\Delta_{BG}(\lambda_t)$, we can find solutions with a reduced FT price taking smaller values of $\lambda_t \times \Lambda$. Model A3 is an example of reduced sensitivity to $\lambda_t$, featuring Higgsino NLSP.

Finally, models B1-B3 represent interesting corners of the parameter space. B1 is an example of a scenario with a stau NLSP, while a Bino-like neutralino is the NLSP for point B2. In both cases, the NLSP is long lived, given the large mediation scale $M$, as we will discuss in the next section. B3 instead is a model with low $M$, hence a fast-decaying (Higgsino) NLSP.
4 LHC phenomenology

As is any scenario with gauge-mediated SUSY breaking, the collider phenomenology of our models crucially depends on nature and properties of the NLSP, its life-time in particular. In fact, independently of the production mechanism, any cascade decay will end with the NLSP decaying into a light gravitino LSP, whose mass is in terms of our parameters

\[ m_{\tilde{G}} = \Lambda \times M / (\sqrt{3} M_{Pl}) , M_{Pl} = 2.4 \times 10^{18} \text{ GeV}. \]

As we have seen, the NLSP is the lightest neutralino in most of our parameter space. The decay modes crucially depend on the size of gaugino and Higgsino components of the lightest neutralino:

\[ \tilde{\chi}_1^0 = N_{11} \tilde{B} + N_{12} \tilde{W}^0 + N_{13} \tilde{H}_d^0 + N_{14} \tilde{H}_u^0 . \]

In terms of the entries \( N_{1k} \), the widths of the possible decay modes of a neutralino NLSP to gravitino read [53,54]:

\[
\Gamma(\tilde{\chi}_1^0 \to \tilde{G} Z) \simeq \frac{m_{\tilde{\chi}_1^0}}{48\pi m_{\tilde{G}}^2 M_{Pl}^2} \left( |N_{12} c_\theta - N_{11} s_\theta|^2 + \frac{1}{2} |N_{13} c_\beta - N_{14} s_\beta|^2 \right) \times \left( 1 - \frac{m_Z^2}{m_{\tilde{\chi}_1^0}^2} \right)^4 ,
\]

\[
\Gamma(\tilde{\chi}_1^0 \to \tilde{G} h) \simeq \frac{m_{\tilde{\chi}_1^0}}{96\pi m_{\tilde{G}}^2 M_{Pl}^2} |N_{13} c_\beta + N_{14} s_\beta|^2 \left( 1 - \frac{m_h^2}{m_{\tilde{\chi}_1^0}^2} \right)^4 ,
\]

\[
\Gamma(\tilde{\chi}_1^0 \to \tilde{G} \gamma) \simeq \frac{m_{\tilde{\chi}_1^0}}{48\pi m_{\tilde{G}}^2 M_{Pl}^2} |N_{11} c_\theta + N_{12} s_\theta|^2 ,
\]

where \( s_\theta \equiv \sin \theta_W, c_\theta \equiv \cos \theta_W \), and \( s_\beta \equiv \sin \beta, c_\beta \equiv \cos \beta \), and the dependence of the phase space on the gravitino mass has been neglected. As we can see from these equations, a mostly Higgsino NLSP will decay to \( Z \) and \( h \), while a sizable branching ratio to \( \gamma \) is only possible if \( \tilde{\chi}_1^0 \) is mostly gaugino, e.g. \( \tilde{B} \).

According to our scan, the only other possible NLSP is the lightest stau. In such a case, the decay width is:

\[
\Gamma(\tilde{\tau}_1 \to \tilde{G} \tau) \simeq \frac{m_{\tilde{\tau}_1}}{48\pi m_{\tilde{G}}^2 M_{Pl}^2} ,
\]

The expressions in Eqs. (19-22) show that the NLSP decay rate is always inversely proportional to the gravitino mass, hence on the gauge mediation scale, as \( m_{\tilde{G}} \propto M \). Therefore, higher messenger scales will correspond to more long-lived NLSP. We employ these formulae to compute the NLSP decay length \( c_{\tau_{NLSP}} = c/\Gamma_{NLSP}^\tau \), for the points of our scan. The result is shown in Fig. 5. The horizontal lines correspond to \( c_{\tau_{NLSP}} = 0.1 \text{ mm} \) and 10 m. Points below \( c_{\tau_{NLSP}} = 0.1 \text{ mm} \) feature a NLSP decay that will mostly appear at the LHC as occurring promptly at the \( pp \) collision point, while points above \( c_{\tau_{NLSP}} = 10 \text{ m} \) likely give raise to NLSP decays outside the detector. As we can see from the plot, solution with low fine tuning tend to have a long-lived NLSP. This is because a large mediation scale, thus a long running, more easily achieve a partial cancellation among the terms in Eq. (1). Nevertheless, we see that some points with \( \Delta_{EW} < 100 \) correspond to a promptly-decaying neutralino NLSP.

We are now ready to discuss phenomenology and constraints of our models at colliders, in particular at the LHC, according to the properties of the NLSP.
Long-lived Higgsino NLSP. As we have seen, our typical low $\Delta_{EW}$ scenarios feature a small Higgsino mass and a large mediation scale, hence almost degenerate $\tilde{\chi}_{1,2}^0$ and $\tilde{\chi}_1^\pm$, with $\tilde{\chi}_1^0$ as a long-lived NLSP. Models A1-A3 shown in Tab. 1 belong to this class. If Higgsinos are the only light states, while the other SUSY particles such as stops and gluinos lie in the multi-TeV range, there is little room to test this case at the LHC. The cross section of the direct EW production of $\tilde{\chi}_1^0$, $\tilde{\chi}_2^0$, and $\tilde{\chi}_1^\pm$ is rather large at the LHC with $\sqrt{s} = 13$ TeV ($\approx 34$ fb for $\mu = 500$ GeV, summing together all modes [55]) but $\tilde{\chi}_1^0$ leaves the detector unseen, and the decay products of $\tilde{\chi}_2^0$ and $\tilde{\chi}_1^\pm$ are too soft to be observable at the LHC, due to the small mass gaps. Searches for events with large missing energy and a single jet (from initial state radiation) do not improve the situation for Higgsinos heavier than 150–200 GeV [56–61]. However, this scenario is easily accessible at $e^+e^-$ colliders with $\sqrt{s} \gtrsim 2 \times m_{\tilde{\chi}_1^\pm}$, such as the International Linear Collider (ILC) [62]. From the scan, we found that $\tilde{\chi}_1^0 \lesssim 450$ (650) GeV for $\Delta_{EW} < 50$ (100), cf. Fig. 4 left. From this, we see that ILC operating at $\sqrt{s}$ up to about 1 TeV would test all our models with $\Delta_{EW} < 50$, i.e. tuning better than the 2% level. Another possibility relies on the production of heavier SUSY particles. As we have seen, the novel feature of models with matter-messenger couplings such as $\lambda_t$ (compared to our previous study [2]) is the possibility of a light stop (and relatively lighter gluino), as illustrated by Fig. 2 and the model A2 of Tab. 1. This case is in principle testable at the LHC (relying on stop/gluino production) through searches for events with b-jets and missing energy aiming at standard ‘natural SUSY’ scenarios. According to the recent study [63], a spectrum like that of model A2 already lies at the edge of the exclusion provided by the early 13 TeV LHC data, which approximately corresponds to $m_{\tilde{t}_1} \gtrsim 1$ TeV for $m_{\tilde{g}} = 2$ TeV. Therefore, we expect that the present LHC run will start testing at least the bottom-left corner of the left plot in Fig. 2. The possible presence of an intermediate Bino does not change the picture as the stop will always prefer to decay directly to Higgsinos, given the large $\sim y_t$ stop-Higgsino couplings.
Promptly-decaying Higgsino NLSP. As we mentioned above, certain solutions, shown in the right-bottom corner of Fig. 5, have a (mostly Higgsino) neutralino, with such a short lifetime that it would always appear to decay promptly at the collision point. An example is provided by the model B3 in Tab. 1. From Eqs. (19, 20), we see that an almost pure Higgsino mostly decays to $Z$ or $h$ and the gravitino, while the BR($\tilde{\chi}_1^0 \rightarrow \gamma \tilde{G}$) is suppressed by the negligible gaugino component in $\tilde{\chi}_1^0$. Furthermore, for moderate to large values of $\tan \beta$, the rates of the two modes $\tilde{\chi}_1^0 \rightarrow Z \tilde{G}$ and $\tilde{\chi}_1^0 \rightarrow h \tilde{G}$ only differ by the phase space, typically giving BR($\tilde{\chi}_1^0 \rightarrow Z \tilde{G}$) $\simeq 60 \pm 50\%$, BR($\tilde{\chi}_1^0 \rightarrow h \tilde{G}$) $\simeq 40 \pm 50\%$, for the range of $m_{\tilde{\chi}_1^0}$ we are interested in. A search for Higgsinos promptly decaying to a light gravitino employing the 8 TeV LHC data has been published in [64]. The limit is given as a function of BR($\tilde{\chi}_1^0 \rightarrow h \tilde{G}$) and reads $m_{\tilde{\chi}_1^0} \gtrsim 325$ (300) GeV for BR($\tilde{\chi}_1^0 \rightarrow h \tilde{G}$) = 40 (50)%. This constraint is too weak to exclude our models with $\Delta_{\text{EW}} < 100$, but we can expect a substantial improvement from the data collected at 13 TeV.

Long-lived stau NLSP. Models with a long-lived stau (hence charged) NLSP can be much more easily tested at the LHC through searches for charged tracks. The points in the top-left sector of Fig. 5 in particular the model B1 of Tab. 1 belong to this scenario. This model is already excluded by a CMS search with 8 TeV data for long-lived massive particles [65], according to which the bound on the stau mass for staus directly produced through the Drell-Yan mechanism is:

$$m_{\tilde{\tau}_1} > 339 \text{ GeV}. \quad (23)$$

This limit has not been improved yet by 13 TeV data [66]. This bound obviously becomes stronger if the production cross section of the heavier particles (e.g. charged and neutral Higgsinos in our case) is larger than the stau production. In fact, any SUSY event will eventually feature two charged tracks at the end of the cascade decay. Assuming that the acceptance of events from such cascade decays is the same as in the case of direct stau production, we can approximately recast the CMS bound by simply requiring that the production cross section of the heavier SUSY particles (that we computed by means of PROSPINO [67]) does not exceed the Drell-Yan cross section of a mostly RH stau with $m_{\tilde{\tau}_1} = 339$ GeV: $\sigma_{\text{prod}} \lesssim 0.32 \text{ fb}$. Such bound translates on the following limits on the relevant sparticle masses, valid for all models with a long-lived stau NLSP:

$$m_{\tilde{\ell}_R} \gtrsim 390 \text{ GeV}, \quad |\mu| \gtrsim 840 \text{ GeV},$$

$$m_{\tilde{t}_1} \gtrsim 1 \text{ TeV}, \quad m_{\tilde{g}} \gtrsim 1.5 \text{ TeV}, \quad (24)$$

where the bound on $m_{\tilde{\ell}_R}$ refers to mass-degenerate RH electron and smuon, and the bound on $|\mu|$ comes from all possible combinations of production involving neutral and charged Higgsinos. This latter constraint is particularly stringent, and excludes all our points with $\Delta_{\text{EW}} < 100$ featuring a long-lived stau NLSP. This can be seen comparing Fig. 6 where

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3This assumption is corroborated by the fact that the limits reported in [65] on the production cross section for both direct and indirect (cascade decay) production almost overlap.

4If instead only the bound from direct stau production, Eq. (23), is applied, only about one fourth of the solutions with $\Delta_{\text{EW}} < 100$ are excluded.
Figure 6: The same as Fig. 4 after imposing the bounds in Eqs. (23, 24) for the points with a long-lived stau NLSP.

the above bounds were applied, with the previous Fig. 4. Thus, searches for charged tracks already provide a stringent constraint on natural models belonging to this class, leaving as viable solutions only models with a tuning more severe than 1%.

**Long-lived Bino NLSP.** This case is represented by the points in the top-left quadrant of Fig. 4. In our case, models with a Bino-like neutralino NLSP resemble the ordinary searches for SUSY in gravity mediation, as the neutralino is long lived and decays outside the detector, appearing as missing energy. In our scenario several particles need to be light if Bino is light: those whose masses are controlled by $b_{1/2}^0$ in Eqs. (2, 3), i.e. the RH stau and sleptons, plus of course the Higgsinos in the case of low-tuned models. This is the case of our model B2. Therefore, there is room to test models of this kind by means of the EW production of these particles and the following cascade decays. The most promising channels rely on production of Higgsinos, that will decay dominantly to staus (given the hierarchy of the lepton Yukawas) or directly to $\tilde{\chi}_1^0$ in the low $\tan \beta$ regime, as well as production of first and second generation RH sleptons:

$$pp \to \tilde{\chi}_1^+ \tilde{\chi}_1^- \to \tilde{\tau}_1^+ \tilde{\tau}_1^- \nu_\tau \bar{\nu}_\tau \to \tau^+ \tau^- + \nu_\tau \bar{\nu}_\tau + 2 \tilde{\chi}_1^0$$

$$pp \to \tilde{\chi}_1^0 \tilde{\chi}_{2,3} \to \tilde{\tau}_1^+ \tilde{\tau}_1^- \tilde{\tau}_1^\pm \nu_\tau \to \tau^+ \tau^- + \nu_\tau + 2 \tilde{\chi}_1^0$$

$$pp \to \tilde{\chi}_2^0 \tilde{\chi}_3^0 \to \tilde{\tau}_1^+ \tilde{\tau}_1^- \tilde{\tau}_1^+ \tilde{\tau}_1^- \to \tau^+ \tau^- + \nu_\tau + 2 \tilde{\chi}_1^0$$

$$pp \to \tilde{\chi}_2^0 \tilde{\chi}_3^0 \to \tilde{\tau}_1^+ \tilde{\tau}_1^- \tilde{\tau}_1^+ \tilde{\tau}_1^- \to \tau^+ \tau^- + \nu_\tau + 2 \tilde{\chi}_1^0$$

$$pp \to \tilde{\tau}_1^+ \tilde{\tau}_1^- \to \ell^+ \ell^- + 2 \tilde{\chi}_1^0$$

All these modes have been intensively searched for by ATLAS and CMS with the 8 TeV data set. The limits on the Higgsino mass reach up to 450 GeV, but quickly drop for $m_{\chi_1^0}$ above 100–150 GeV or small mass splittings [68–72], hence have no impact on the parameter space of our models. Nevertheless, scenarios with much heavier neutralinos will be accessible at the high-luminosity runs of the LHC, as demonstrated by e.g. a prospect study for direct stau pair production [73].
NLSP decaying inside the detector. As shown in Fig. 5, we found some low-tuned models ($\Delta_{\text{EW}} < 100$) with $0.1 \text{ mm} < c\tau_{\text{NLSP}} < 10 \text{ m}$. In this regime the NLSP is likely to decay inside the detector after traveling a finite distance. Most of the solutions of this kind feature a Higgsino NLSP that, as we have seen above, decays with comparable probabilities to $\tilde{G}Z$ or $\tilde{G}h$, cf. Eqs. (19, 20). Possible strategies to test this kind of displaced neutralino decays at the LHC have been proposed in [74]. A search for this kind of topology in association with jets – thus sensitive to the production of colored superpartners – has been published by ATLAS employing the 8 TeV run data [75]. Assuming $\text{BR}(\tilde{\chi}_1^0 \to \tilde{g}h) = 100\%$, they set a limit on the production cross section up to 1 fb for $10 \text{ mm} \lesssim c\tau_{\tilde{\chi}_1^0} \lesssim 100 \text{ mm}$, corresponding to a gluino as heavy as $\sim 1.4 \text{ TeV}$. Our solutions with a Higgsino NLSP in this regime feature heavier gluinos $\gg 2 \text{ TeV}$, but this shows that searches for displaced vertices may become sensitive to this special class of models in the future.

From Fig. 5 we can also see that a few low-tuned models feature a stau NSLP with lifetime in this intermediate regime. Since the stau can only decay to $\tilde{G}_\tau$, the characteristic signature would be a charged track followed by a tau, i.e. by a lighter lepton or a jet. Although triggering on these daughter particles at the LHC seems to be unfeasible, searches for disappearing tracks [76, 77] are sensitive to such a scenario, as shown in [78], maximally for $c\tau_{\tilde{\tau}_1} \approx 50 \text{ cm}$. Furthermore, for $c\tau_{\tilde{\tau}_1} \gtrsim 2 \text{ m}$, a substantial fraction of the staus decay outside the detector such that the searches for stable charged particles discussed above become increasingly sensitive [78]. This seems to be the best way to test the few models of this kind we found, that have $m_{\tilde{\tau}_1} \approx 300 \text{ GeV}$ and $c\tau_{\tilde{\tau}_1} \gtrsim 3 \text{ m}$.

5 Summary and discussion

Within the MSSM with gauge-mediated SUSY breaking, we have discussed a class of models characterized by two deviations from minimal setups: (i) a non-unified messenger sector providing large freedom in the gaugino mass ratios, and (ii) a matter-messenger coupling $\lambda_t$ inducing 1-loop $A$-terms and additional contributions to the scalar masses. The first ingredient allows for a compensation in the running of the Higgs soft mass that reduces its sensitivity on the stop and gluino masses, thus reducing the fine tuning. The best solutions were found for $M_2/M_3 \approx 3$ at the mediation scale. The second ingredient generalizes our previous work [2] and gives the possibility of building models with a lighter stop (due to some negative contributions to its mass, as well as the impact on the Higgs mass of the $\lambda_t$-induced $A_t$) and FT as low as $\Delta_{\text{EW}} \approx 20$, i.e. around 5%, while in setups with $\lambda_t = 0$ no solutions with a tuning better than about 2% were found.

The typical spectra of the models with best FT that we found feature light Higgsinos and possibly light Bino and RH sleptons (in particular the lightest stau), since the masses of these particles receive no radiative contribution at one loop from the Wino mass that instead is required to be very heavy to trigger the compensation in the running that we discussed above. Other particles are typically heavy, with the possible exception of the lightest stop that can be as light as 1 TeV. Gluinos and RH squarks lie above 2 TeV, while LH squarks
and sleptons are multi-TeV again because of the large Wino mass. Therefore, this class of models is a remarkable example of SUSY scenarios with low fine tuning but spectra that can be easily evade current LHC searches\footnote{A notable exception is provided by models featuring a long-lived stau NLSP that, as we have seen in the previous section, are already excluded down to $\text{FT} \approx 1\%$.} in contrast to the classical ‘natural SUSY’ framework whose status has been recently studied in [63]. This feature is shared by other SUSY setups, for instance in the context of gravity mediation by ‘radiatively natural’ models (for recent discussions see [79, 80]). Our framework can be in principle distinguished by this latter one, because it features lighter gluinos, some lighter scalars, and have instead heavy Winos. Besides these differences, we also find that light higgsinos is a minimal condition for low tuning, while other SUSY particles are not necessarily light. Approximately we have 5% (2%) FT for 300 (450) GeV Higgsinos. This makes a strong case for the ILC, that could in fact test several classes of models with tuning better than 2%, if operating at the center of mass up to 1 TeV. This is an important conclusion that has been reached elsewhere in the literature, see e.g. [81], and we want to remark it here. Possible exceptions to the above conclusion could be given by models with a high degree of correlations among the high energy parameters, such as in ‘supernatural’ SUSY scenarios in the context of no-scale supergravity [82–85], or by models with non-minimal Higgs sectors, e.g. resembling N=2 SUSY [86]. In these cases low tuning can be achieved even for heavy Higgsinos.

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