Movement Characteristics Analysis and Dynamic Simulation of Collaborative Measuring Robot

MA guoqing, LIU li, YU zhenglin, CAO guohua and ZHENG yanbin
School of Mechatronical Engineering, Changchun University of Science and Technology, Changchun 130022

Abstract. Human-machine collaboration is becoming increasingly more necessary, and so collaborative robot applications are also in high demand. We selected a UR10 robot as our research subject for this study. First, we applied D-H coordinate transformation of the robot to establish a link system, and we then used inverse transformation to solve the robot’s inverse kinematics and find all the joints. Use Lagrange method to analysis UR robot dynamics; use ADAMS multibody dynamics simulation software to dynamic simulation; verifying the correctness of the derived kinetic models.

1. Introduction
Since human-machine collaborations are currently in high demand, collaborative robot applications are being used with increasing frequency. UR robots have a force feedback function, meaning that when they collide, they automatically stop working. Therefore, in most applications a collaboration robot model does not require a security fence[1].

We selected a UR10 robot as our research subject for this study. First, we applied D-H coordinate transformation of the robot to establish a link system, and we then used inverse transformation to solve the robot’s inverse kinematics and find all the joints. Using Lagrange method to analysis UR robot dynamics; using ADAMS multibody dynamics simulation software to dynamic simulation; verifying the correctness of the derived kinetic models.

2. Kinematics Solution
The traditional process to solve kinematics involves studying movement relationships, but does not consider the interaction force between each link[2]. To solve a forward kinematics problem we must solve the position and orientation of the end-effector in the base coordinate system, which is in turn based on the joint angle and dimensions of the known links. Once this has been done, the essential components are in place and it is possible to solve kinematic equations. With inverse kinematics, we strike each joint’s angles when the end-effector reaches a location based on its known position and orientation the in base coordinate system.

2.1. Establishing the connecting coordinate system
UR10 is a series robot with six revolute joints. In this study, we used the D-H coordinate transformation method to solve the position and orientation relationship between each link of the robot. The steps are as follows[3]:
(1) Establish the coordinate system \( x_0, y_0, z_0 \) and confirm initial position and orientation;
(2) Establish a second joint coordinate system based on the DH coordinate principle;
(3) Establish a robot tool coordinate system $x_6, y_6, z_6$.

Fig. 1 below illustrates the DH coordinate system of a UR10 robot.

**Figure 1.** DH coordinate system of UR10

### 2.2. Forward kinematics solution

DH parameters can be determined based on the conversion relationship between adjacent joints and the links’ coordinate system. All the joints of UR10 are revolute joints. This means that only the joint angle $\theta$ is variable, while joints with a twist angle $\alpha$, link length $a$, and link distance $d$ are constant.

Table 1 shows the DH parameters for a UR10.

| joint | $\alpha/^\circ$ | $a/(\text{mm})$ | $d/(\text{mm})$ | $\theta/^\circ$ | range $/^\circ$ |
|-------|----------------|-----------------|-----------------|----------------|----------------|
| 1     | $-90^\circ$    | 0               | 0               | $\theta_1$    | $-360$ to $360$ |
| 2     | $0^\circ$      | $a_2$           | $d_2$           | $\theta_2$    | $-360$ to $360$ |
| 3     | $0^\circ$      | $a_3$           | $d_3$           | $\theta_3$    | $-360$ to $360$ |
| 4     | $90^\circ$     | 0               | 0               | $\theta_4$    | $-360$ to $360$ |
| 5     | $90^\circ$     | 0               | $d_5$           | $\theta_5$    | $-360$ to $360$ |
| 6     | $0^\circ$      | 0               | 0               | $\theta_6$    | $-360$ to $360$ |

The transformation matrix between the robot’s adjacent joint coordinates are shown in Formula (1).

$$ T_i^j = \text{Rot}(z, \alpha_i) \times \text{Trans}(0,0,d_i) \times \text{Trans}(a_i,0,0) \times \text{Rot}(x, \alpha_j) $$

$$ = \begin{bmatrix}
    c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\
    s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\
    0 & s\alpha_i & c\alpha_i & d_i \\
    0 & 0 & 0 & 1
\end{bmatrix} $$

In order to obtain the transformation matrix between the adjacent coordinate system, we must enter the DH parameters of Table 1 into Formula (1). Subsequently, we multiply the resulting transformation matrix, which provides positive solutions as shown in Formula (2).

$$ T_e = T_1^6 T_2^7 T_3^8 T_4^9 T_5^10 $$

$$ = \begin{bmatrix}
    n_1 & n_2 & n_3 & n_4 & n_5 & n_6 & n_7 & n_8 & n_9 & n_{10} \\
    a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 & a_{10} \\
    p_1 & p_2 & p_3 & p_4 & p_5 & p_6 & p_7 & p_8 & p_9 & p_{10} \\
    0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0
\end{bmatrix} $$

In which
Therefore, inverse kinematics for the UR robot exist. This meets the condition that three adjacent joint axes are parallel or intersecting at one point.

2.3. Formatting author affiliations

According to the configuration characteristics of UR10, the second, third, and fourth joint axes are parallel. Therefore, inverse kinematics for the UR robot exist.

1) Solve \( \theta_1 \)

Put \( \psi_n = T_1^r T_2^r T_3^r T_4^r T_5^r \) left and right side multiplied by \( (T_1^r)^{-1} \), then

\[
(T_1^r)^{-1} T_2 = \begin{bmatrix}
    -\frac{2}{p_2} + x_1, & c_1, & c_1, & c_1, & c_1,
    -\frac{2}{p_1} + x_1, & c_1, & c_1, & c_1, & c_1,
      -\frac{2}{p_1} + x_1, & c_1, & c_1, & c_1, & c_1,
    0, & 0, & 0, & 0, & 1
\end{bmatrix}\]

make: \( T_2^r T_3^r T_4^r T_5^r T_6^r = \frac{A_1}{A_2} A_3 A_4 \)

The third row and the fourth column of Formula (3) and Formula (4) correspond with:

\[
-s_3 p_r + c_3 p_r = d_z + d_3
\]

Perform the following triangle substitution for Formula (6)

\[
\begin{aligned}
    r &= r \cos \varphi \\
    p_y &= r \sin \varphi
\end{aligned}
\]

\[
\varphi = \arctan \left( \frac{P_x}{P_y} \right) \quad \theta_1 = \arctan \left( \frac{P_x}{P_y} \right) - \arctan \left( \frac{d_z + d_3}{r} \right)
\]

2) Solve \( \theta_1 \)

The third row and third column of Formula (3) and Formula (4) correspond with:

\[
-c_3 = c_3 a_3 - s_3 a_3
\]

So

\[
\theta_1 = \arccos(s_3 a_3 - c_3 a_3)
\]

3) Solve \( \theta_2 \)

The third row and the first column of Formula (3) and Formula (4) correspond with:

\[
c_3 s_5 = c_3 p_r - s_3 p_r
\]

So

\[
\theta_2 = \arccos(c_3 p_r - s_3 p_r)
\]

4) Solve \( \theta_2 \)

Put \( \psi_n = T_1^r T_2^r T_3^r T_4^r T_5^r \) left and right side multiplied by \( (T_1^r)^{-1} \), then

\[
(T_1^r)^{-1} T_2 = \begin{bmatrix}
    c_3 c_5 s_2^2 d_3, & s_3 c_5 s_2^2 d_3, & c_3 c_5 s_2^2 d_3, & c_3 c_5 s_2^2 d_3, & c_3 c_5 s_2^2 d_3,
    s_3 c_5 s_2^2 d_3, & s_3 c_5 s_2^2 d_3, & c_3 c_5 s_2^2 d_3, & c_3 c_5 s_2^2 d_3, & c_3 c_5 s_2^2 d_3,
    s_3 c_5 s_2^2 d_3, & c_3 c_5 s_2^2 d_3, & c_3 c_5 s_2^2 d_3, & c_3 c_5 s_2^2 d_3, & c_3 c_5 s_2^2 d_3,
    0, & 0, & 0, & 0, & 1
\end{bmatrix}
\]

make:

\[
(T_1^r T_2^r) = \begin{bmatrix}
    B_1, & B_2, & B_3, & B_4,
    B_1, & B_2, & B_3, & B_4,
    B_1, & B_2, & B_3, & B_4,
    B_1, & B_2, & B_3, & B_4
\end{bmatrix}
\]

The second column, third column and fourth elements (respectively):
\[
\begin{bmatrix}
B_{12} \\
B_{22} \\
B_{32} \\
B_{42}
\end{bmatrix} = \begin{bmatrix}
c_1c_2o_x + s_1c_2o_y - s_2o_z \\
-c_2o_x - s_1s_2o_y - c_2o_z \\
c_1o_y - o_zs_1 \\
0
\end{bmatrix}
\]

(11)

\[
\begin{bmatrix}
B_{13} \\
B_{23} \\
B_{33} \\
B_{43}
\end{bmatrix} = \begin{bmatrix}
c_1c_2o_x + s_1c_2o_y - s_2o_z \\
-c_2o_x - s_1s_2o_y - c_2o_z \\
c_1o_y - o_zs_1 \\
0
\end{bmatrix}
\]

(12)

\[
\begin{bmatrix}
B_{14} \\
B_{24} \\
B_{34} \\
B_{44}
\end{bmatrix} = \begin{bmatrix}
c_2p_x + s_1c_2p_y - s_2p_z - a_z \\
-c_2p_x - s_1s_2p_y - c_2p_z \\
p_y - p_z - a_1 \\
1
\end{bmatrix}
\]

(13)

The first row and second column of Formula (9) and Formula (10) correspond with:

\[
c_1c_2o_x + s_1c_2o_y - s_2o_z = s_yc_x - c_yc_z
\]

So

\[
\theta_i = \arctan \left( \frac{c_1o_y + s_1c_2 + s_2a_z}{s_2c_x + c_1c_z} \right)
\]

5) Solve \(\theta_j\) The second row and third column, and the second row and fourth column of Formula (9) and Formula (10) correspond with:

\[
c_2s_x = -c_1s_xa_y - s_2s_ya_z - c_2a_z
\]

\[
s_2a_x - d_1c_2 = c_1s_xp_y - s_2s_yp_z - c_2p_z
\]

So

\[
\theta_j = \arcsin \left( \frac{-c_1s_xp_y - s_2s_yp_z - c_2p_z + d_2c_1a_y + s_2c_2a_z}{s_2c_x + c_1c_z} \right)
\]

6) Solve \(\theta_4\) Based the above, joints \(\theta_{23}, \theta_3, \theta_4\) are known, so \(\theta_4 = \theta_{23} - \theta_3\).

\[
\theta_4 = \arcsin \left( -\frac{\theta_3}{s_2} \right) - \arcsin \left( \frac{c_1o_y + s_1c_2 + s_2a_z}{s_2c_x + c_1c_z} \right)
\]

3. Dynamics Equation of Measuring Robot

There are many methods to study robot dynamics, mainly Newton-Euler method (Newton-Euler)[4], Lagrangian (Lan-grange) method[5], Gauss (Gauss) law[6], Kane (Kane) method and Robertson-Weiden Fort (Robson-Wittenburg) method[7]. In this paper, creating dynamic equations of the robot used Lagrange way.Lagrange equation is:

\[
\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = Q_i \quad (i = 1,2,\ldots,6)
\]

(17)

Type, \(q_i\), \(\dot{q}_i\) is the generalized coordinates and generalized velocity; \(Q_i\) is the generalized force for the generalized coordinates.Setting the translational velocity vector is \(v_i\), the angular velocity vector is \(\omega_i\), for the quality is \(m_i\), relative to the center of mass of inertia tensor is \(I_i\) in the base coordinate system, the kinetic energy of \(i\) th joints is:

\[
T_i = \frac{1}{2} m_i v_i^T v_i + \frac{1}{2} \omega_i^T I_i \omega_i
\]

(18)
Among them, the first item is the kinetic energy of the object in the translational motion, the second is the kinetic energy of the object rotating around the center of mass. The \(i\)th joint velocity in basic coordinate system and the joints and before each joint can be expressed as the relationship between the speed

\[
\begin{bmatrix}
    v_i \\
    w_i
\end{bmatrix} = \begin{bmatrix}
    J_{ii} \\
    J_{ii}
\end{bmatrix} \begin{bmatrix}
    q_i \\
    \dot{q}_i
\end{bmatrix}
\]

Because joints for the rotational joint, so the total kinetic energy for the robot is:

\[
T = \sum_{i=1}^{n} T_i = \frac{1}{2} \sum_{i=1}^{n} M_i \dot{q}_i^2
\]

\[
M_{ij} \text{ is the n square matrix of M element}
\]

\[
M = \sum_{j=1}^{n} \left( m_i J_{ij}^{(ii)} + J_{ij}^{(ir)} J_{ij}^{(ri)} \right)
\]

The robot's potential energy \(U = \sum m_i g_i O_{ci}.\) Among them, \(g\) is acceleration of gravity vector on the basis of coordinates system, \(O_{ci}\) is the vector of origin of coordinates to the center of mass of the vector on the basis of the origin of coordinates. Generalized force is:

\[
Q = \tau + J^T F
\]

Amoong them, \(\tau\) and \(F\) respectively joint force vector and the force vector of end of actuator contact with the external environment.Put the robot's kinetic energy, potential energy and generalized force generation into the Lagrange equation, available the robot dynamics equation.

\[
\sum_{j=1}^{n} M_{ij} \ddot{q}_j + \sum_{j=1}^{n} \left( \frac{\partial M_{ij}}{\partial q_i} \dot{q}_j + \frac{1}{2} \frac{\partial M_{ij}}{\partial \dot{q}_j} \dot{q}_i \right) + \sum_{j=1}^{n} m_i g_j J_{ij}^{(ii)} = Q_i
\]

As can be seen by the above formula is derived to calculate the theoretical advantage of solving the dynamic equations of the robot more complex, so the general multi-body dynamics simulation method to accomplish each robot joint force or torque solving.

### 4. Dynamics Simulation of Measuring Robot

We us ADAMS to simulate the dynamics of the measuring robot, the simulation time is set to 5s, step number is set to 500. add drivers in each of the joints as follow setting.

1) The first joint rotation control function
   STEP(time,0,0,0,5,72d)+STEP(time,0,5,0,5,5,0)+STEP(time,5,5,0,6,-72d),
2) The second joint rotation control function
   STEP(time,0,0,1,40d)+STEP(time,1,0,9,0)+STEP(time,9,0,10,-40d)
3) The third joint rotation control function
   STEP(time,0,0,1,40d)+STEP(time,1,0,9,0)+STEP(time,9,0,10,-40d)
4) The forth joint rotation control function
   STEP(time,0,0,1,50d)+STEP(time,1,0,7,0)+STEP(time,7,0,8,-50d)
5) The fifth joint rotation control function
   STEP(time,0,0,1,60d)+STEP(time,1,0,6,0)+STEP(time,6,0,7,-60d)
6) The sixth joint rotation control function
   STEP(time,0,0,1,80d)+STEP(time,1,0,5,0)+STEP(time,5,0,6,-80d)

#### 4.1. Angular Velocity of each joint

Each joint angular velocity curve shown in figure 2, which play a very important role in the dynamics of the robot's overall design, therefore needs to measure each joint angular velocity changes during the simulation.
As we can see from the joint angular velocity graph, during the period of the robot dynamics simulation, the angular velocity of each joint in line with the changes of setted, uniformly accelerated start - uniform run - even retarding the working conditions, and in the acceleration phase and deceleration phase transition is relatively stable, there is no mutation phenomenon, each robot joint operation is in accordance with the actual situation, will not affect the overall performance of the robot.

4.2. Force and Moment of each joint
The force and moment of each joint in the robot dynamics simulation are an important parameter. By measuring the torque of each joint suffered, can verify whether the drive motor selected can meet the requirements, but also can verify whether the reduction ratio and torque harmonic reducer chosed can meet the design requirements, so dynamics simulation has play a very important role in the design of the robot. Joint forces and torques suffered is hown in fig 3 each joint of UR10 robot can withstand the torque value is shown in table 2.

![Figure 3](image)

**Table 2** withstand torque value of UR robot

| Joint | Joint | Joint | Joint | Joint | Joint |
|-------|-------|-------|-------|-------|-------|
| torq  | 330N  | 330N  | 150N  | 56N   | 56N   |
| m     | m     | m     | m     | m     | m     |

We can see from figure 3, the force of each joint and moment of force during robot movement are withstand the torque range of UR10 robot can suffer as shown in table 2 withstand the torque range, meets the needs of the movement, moment of force.

5. Conclusions
In this study, we selected a UR10 robot as our research subject. We first used D-H coordinate transformation of the robot to establish a link system. Then, we used inverse transformation to solve inverse kinematics for the robot and locate all the joints. Use Lagrange method to analysis the UR robot dynamics, use ADAMS multibody dynamics simulation software to realize dynamic simulation.
The results show that the measuring robot UR10 has a good static balance of performance, stability and dynamic response of the motion.

6. References
[1] LIU Kun, LI Shi-zhong, WANG Bao-xiang. Research of the Direct Teaching System Based on Universal Robot[J]. Science Technology and Engineering, 2015,10(28):22-26.
[2] Gao Yi, Ma Guoqing, Yu Zhenglin, et al. Kinematics Analysis of an 6-DOF Industrial Robot and Its Visualization Simulation[J]. China Mechanical Engineering, 2016, 27(13):1726-1731.
[3] Huang Xiaochen, Zhang Minglu, Zhang Xiaojun, et al. Improved DH Method to Build Robot Coordinate System[J]. Transactions of the Chinese Society for Agricultural Machinery, 2014, 45(10):313-318.
[4] ZHAO Tieshi, GENG Mingchao, CHEN Yuhang, et al. Kinematics and Dynamics Hessian Matrices of Manipulators Based on Screw Theory[J]. Chinese Journal of Mechanical Engineering, 2015, 02:226-235.
[5] Zhao Junwei, Li Xuefeng, Chen Guoqiang. The Dynamics Equation of A 3- P R S Parallel Manipulator Based on Lagrange Method[J]. Machine Design and Research, 2015, 31(2):1-5.
[6] LIU Yingjie, ZHAO Youqun, XU Jianxiong, et al. Vehicle Handling Inverse Dynamics Based on Gauss Pseudospectral Method while Encountering Emergency Collision Avoidance[J]. Journal of Mechanical Engineering, 2012, 48 (22):127-132.
[7] YU Changjuan, ZHANG Minglu, ZHANG Jianhua, et al. Dynamics modeling and simulation of multi-motion mode robot[J]. Journal of Machine Design. 2015, 3(3):68-71.