Phase Difference Function in Coherent Temporal-spatial Region and Unified Equations of Steady, Non-steady Interference

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(Dated: March 31, 2022)

Phase difference function is established by means of phase transfer function between time domains of source and interference point. The function reveals a necessary interrelation between outcome of two-beam interference, source’s frequency and measured subject’s kinematic information. As inference unified equations on steady and non-steady interference are derived. Meanwhile relevant property and application are discussed.

I. INTRODUCTION

On two beam interference, the explicit interrelationship among source’s frequency parameter, time functions of the beams between time domains of source and interference point and instantaneous outcome of the interference constructs a necessary foundation for any two-beam interferometry’s design and interpretation of measured data. By means of phase transfer function between time domains of source and observer, the accumulated phase or phase function for either of the wave beams at any spatial point in the coherent temporal spatial region can be determined by source’s parameter and corresponding instant. Upon the beam’s phase function in the coherent temporal-spatial region, the phase difference function, two variables function in interference temporal-spatial region, is established. Is the phase difference function a general interrelationship itself. From it, the unified equations on steady and non-steady interference are inferred directly under the cases respectively. For steady interference, conventional rule on the interference spatial distribution is a particular example of the unified equation as the two beam’s wavelengths are same. In addition Michelson-Morley experiment result is reinterpret with the equation. For non-steady interference two sets of the equations are derived for different interferometry outcomes: beat frequency and fringe’s instantaneous displacing velocity; moreover on some of typical dynamical measurement: history of distance, velocity and acceleration as well as source frequency property, the principle formulas are presented for application illustration.

II. DERIVATION OF PHASE DIFFERENCE FUNCTION IN INTERFEROMETRY TEMPORAL-SPATIAL REGION

In beam’s transfer time-space $t' \in \tau'_i(r)$, $r \in V_i$, there is phase transfer functions (Ref. [2]):

$$\varphi_i(r, t') = \int_{t'_i}^{t'} \omega'_i(r, \dot{t}) \, d\dot{t} + \varphi_i(r, t'_i)$$

$$= \int_{t'_i-T_i(r, t')}^{t'-T_i(r, t')} \omega_i(t) \, dt + \varphi_i(t_i)$$

$$= \varphi_i[t' - T_i(r, t')]$$ (1a)

here $\varphi_i(r, t'_i) = \varphi_i[t'_i - T_i(r, t'_i)] = \varphi_i(t_i) = \varphi[t'_i - T_i(r, t'_i)]$, and

$$\frac{\partial \varphi_i(r, t')}{\partial t'} = \omega'_i(r, t') = \omega[t' - T_i(r, t')] \frac{\partial t_i(r, t')}{\partial t'}$$

$$= \omega[t' - T_i(r, t')] \left[1 + \frac{\partial T_i(r, t')}{\partial t'}\right]$$ (1b)

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\[
\text{grad } \varphi_i(r, t') = -\omega [t' - T_i(r, t')] \text{grad } T_i(r, t') \\
= -\omega [t' - T_i(r, t')] \frac{1}{v_{ioi}(r, t')} \frac{\partial T_i(r, t')}{\partial t'} \vec{n} \text{grad } T_i \\
= -\omega [t' - T_i(r, t')] \frac{1}{v_{ioi}(r, t')} \left[ 1 - \frac{\partial T_i(r, t')}{\partial t'} \right] \vec{n} \text{grad } T_i
\] (1c)

\[
\frac{\partial \varphi_i(r, t')}{\partial t'} = \rho_c(r, t') v_{ioi}(r, t') = |\text{grad } \varphi_i(r, t')| v_{ioi}(r, t')
\] (1d)

In coherent temporal-spatial region \( r \in V_{i12} \in V_1 \cap V_2, \ t' \in [t'_0(r), t'_0(r) + \tau'_{c12}(r)] \in \tau'_1(r) \cap \tau'_2(r) \); where \( V_{i12} \) — coherent spatial volume of wave beam 1 and 2, \( \tau'_{c12} \) — coherent time interval of beam 1 and beam 2; two beams are split by same source but have different main trajectories to reach interference point.

Now define the phase difference function in this coherent temporal-spatial region,

\[
\psi(r, t') = \varphi_2(r, t') - \varphi_1(r, t')
\]

\[
\psi(r, t') = \int_{t_0}^{t'} \omega_2(r, t) dt + \varphi_2(r, t'_0) - \int_{t'_0}^{t'} \omega_1(r, t) dt - \varphi_1(r, t'_0) \\
= \varphi[t' - T_2(r, t')] - \varphi[t' - T_1(r, t')] \\
= \int_{t' - T_1(r, t')}^{t' - T_2(r, t')} \omega(t) dt
\] (2)

Thus there are the derivative property of the function:

\[
\frac{\partial \psi(r, t')}{\partial t'} = \omega_2(r, t') - \omega_1(r, t') \\
= \omega[t' - T_1(r, t')] \frac{\partial T_1(r, t')}{\partial t'} - \omega[t' - T_2(r, t')] \frac{\partial T_2(r, t')}{\partial t'} + \omega[t' - T_2(r, t')] - \omega[t' - T_1(r, t')] \\
= \frac{\partial \psi(r, t')}{\partial t'}
\] (3)

\[
\text{grad } \psi(r, t') = \text{grad } \varphi[t' - T_2(r, t')] - \text{grad } \varphi[t' - T_1(r, t')] \\
= \text{grad } \int_{t' - T_1(r, t')}^{t' - T_2(r, t')} \omega(t) dt \\
= \left\{ -\omega[t' - T_2(r, t')] \frac{\partial T_2(r, t')}{\partial t'} + \omega[t' - T_1(r, t')] \frac{\partial T_1(r, t')}{\partial t'} \right\} \vec{n} \text{grad } \psi \\
= \left\{ -\omega[t' - T_2(r, t')] \text{grad } T_2(r, t') + \omega[t' - T_1(r, t')] \text{grad } T_1(r, t') \right\} \cdot \vec{n} \text{grad } \psi \\
= \left\{ -\omega[t' - T_2(r, t')] \frac{\vec{n} \text{grad } T_2}{v_{io2}(r, t')} \left[ 1 - \frac{\partial T_2(r, t')}{\partial t'} \right] \\
+ \omega[t' - T_1(r, t')] \frac{\vec{n} \text{grad } T_1}{v_{io1}(r, t')} \left[ 1 - \frac{\partial T_1(r, t')}{\partial t'} \right] \right\} \cdot \vec{n} \text{grad } \psi
\] (4)

where (See Appendix):

\[
|\text{grad } T(r, t')| = \frac{1}{v_{io}(r, t')} \frac{dt}{dt'} = \frac{1}{v_{io}(r, t')} \left[ 1 - \frac{\partial T(r, t')}{\partial t'} \right]
\]

\[
\frac{\partial \psi(r, t')}{\partial t'} = \rho_c(r, t') v_{ioi}(r, t') \\
= |\text{grad } \psi(r, t')| v_{ioi}(r, t')
\] (5)
III. UNIFIED EQUATIONS OF STEADY AND NON-STEADY INTERFERENCE

A. When \( \frac{\partial \psi(r,t')}{\partial t'} \equiv 0 \) or \( v_{\text{fringe}}(r, t') \equiv 0 \) and \( \omega[t' - T_i(r, t')] = constant \), \( \frac{\partial T_i(r, t')}{\partial t'} = \frac{\partial T_i(r)}{\partial t} = 0 \) there steady or static spatial distribution of interference

\[
\frac{2\pi}{r^*} [T_1(r) - T_2(r)] = \begin{cases} 
2k\pi & \text{max.} \\
(2k + 1)\pi & \text{min.} 
\end{cases} \quad k = 0, \pm 1, \ldots 
\]  

or

\[
\int_{t'}^{t'+\tau} |\nabla \psi(r, t')| \, dr = 2\pi 
\]

\( d \) – short distance between two fringes

\[
T_1(r) - T_2(r) = \begin{cases} 
k \tau^+ & \text{max.} \\
(2k + 1) \frac{\tau^+}{2} & \text{min.} 
\end{cases} 
\]

When \( v_{\text{fringe}}(r) = v_{\text{light}}(r) = constant \), there is conventional rule on static spatial distribution of steady interference

\[
L_1(r) - L_2(r) = \begin{cases} 
k \lambda & \text{max.} \\
(2k + 1) \frac{\lambda}{2} & \text{min.} 
\end{cases} 
\]

B. For dynamical interferometry there exist general beat equation and fringe's instantaneous displacement velocity equation as following

\[
\int_{t'}^{t'+\tau_b} \omega_b(r, t) \, dt = \int_{t'}^{t'+\tau_b} [\omega_2'(r, t) - \omega_1'(r, t)] \, dt = \pm 2\pi 
\]  

(7a)

or

\[
\int_{t'}^{t'+\tau_b} \left\{ \omega[t - T_2(r, t')] - \omega[t - T_1(r, t')] + \omega[t - T_1(r, t')] \frac{\partial T_1(r, t)}{\partial t} - \omega[t - T_2(r, t')] \frac{\partial T_2(r, t)}{\partial t} \right\} \, dt = \pm 2\pi 
\]  

(7b)

\[
\int_{t'}^{t'+\tau_b} \left\{ \omega[t_2(r, t')] \frac{\partial T_2(r, t)}{\partial t} - \omega[t_1(r, t')] \frac{\partial T_1(r, t)}{\partial t} \right\} \, dt = \pm 2\pi 
\]  

(7c)

\[
v_{\text{fringe}}(r, t') = \left\{ \frac{\partial \psi(r, t')}{\partial t'} \right\} \n \bar{n}(|\nabla \psi|) / |\nabla \psi(r, t')| 
\]  

(8)

C. The principle formulas for a series of typically dynamical parameter measurement

(i) Ref. Fig. 1(a) \( T_2(r, t') = T_2(r, t) \), \( \omega(t) = \frac{2\pi}{r^*} = constant \), from Eq. (7b), there \( \frac{2\pi}{r^*} \left. \frac{\partial T_1(r, t')}{\partial t} \right|_{t'} = \pm 2\pi \) or \( \left. \frac{\partial T_1(r, t')}{\partial t} \right|_{t'} = \frac{1}{r^*} \nu_b \), the \( v(t) \) can be resolved from \( \frac{\partial T_1(r, t')}{\partial t} \), here \( T_1(t') = t' - t_1 + t_1 - t \).

(ii) To measure \( R(t') \) history by two-beam interferometry with modulated frequency source, Ref. Fig. 1(a) \( \frac{d\omega(t)}{dt} = constant \), \( T_2(t') = T_2 \equiv 0 \), \( T_1(t') - T_2(t') = \frac{2R(t')}{c} \), from Eq. (7c)

\[
\int_{t'}^{t'+\tau_b} \left\{ \int_{t(t)}^{t'} \frac{d\omega(t)}{dt} \, dt + \omega \left[ t(t) - \frac{2R(t)}{c} \right] \frac{2}{c} \frac{dR(t)}{dt} \right\} \, dt = 2\pi 
\]
A. Phase difference function, through phase transfer function and involved time function, reflects the theoretical interrelation between outcome of two-beam interference, frequency characteristics of two autonomic wave sources, for example, as beat equation

\[
\int_{t' - \tau}^{t' + \tau} \left\{ \omega_2 [t_2 (r, t')] \frac{\partial x_2 (r, t')}{\partial t} - \omega_1 [t_1 (r, t')] \frac{\partial x_1 (r, t')}{\partial t} \right\} \, dt = \pm 2\pi
\]

and subject’s kinematic information involved in corresponding time function. The time function applied in phase transfer function must be reversible, in most cases it is positive reversible, that is, \( \frac{dt'}{dt} = 1 + \frac{dT_2 (t')}{dt} > 0 \); all inference and property of the phase difference function can be applied in case of two autonomic wave sources, although in most cases two coherent wave sources are derived from one by splitting. Since phase difference function is consisted of or can be expressed by spatial frame independent physical quantities, the theoretical relation and consequent inferences do not depend upon spatial-frame’s selection.

B. Explanation of zero fringe displacing result of Michelson-Morley experiment.

According to steady interference equation \( 5a \) \( 5b \), the only existing explanation for the experimental result is phase difference at observing interference point remains same or invariable in two cases, that is, light speed with respect to interferometer remains same in both cases.

V. CONCLUSION

Phase difference function reveals the necessary interrelation between outcome of two-beam interference, frequency parameter of two autonomic wave sources, and concrete time function’s information affected by subject kinematic movement. Unified equation on steady and non-steady two-beam interference can be derived from the phase difference function. Phase difference function and related inference are independent to spatial-frame’s selection or remain invariable under frame transformation \( 6 \).
APPENDIX: DERIVATION OF $|\text{grad } T(r, t')|$ 

$$
|\text{grad } T(r, t')| = \lim_{\Delta r \to 0} \frac{T(r + \Delta r, t') - T(r, t')}{\Delta r} \\
= \lim_{\Delta r \to 0} \frac{t' - T(r, t') - [t' - T(r + \Delta r, t')]}{\Delta r} \\
= \lim_{\Delta r \to 0} \frac{t(r, t') - t(r + \Delta r, t')}{\Delta r} = \lim_{\Delta r \to 0} \frac{1}{\Delta r} \frac{\Delta t}{\Delta t'} \\
= \frac{1}{v_{io}(r, t')} \frac{dt}{dt'} = \frac{1}{v_{io}(r, t')} \left[ 1 - \frac{\partial T(r, t')}{\partial t'} \right]
$$

or from

$$
|\text{grad } T(r, t')| = |\text{grad } T[r, t' - T(r, t')]| = |\text{grad } T[r, t(r, t')]| \\
= \frac{\partial T(r, t')}{\partial r} = \frac{dT[r, t(r, t')]}{dr}
$$

and

$$
\frac{\partial T(r, t)}{\partial r} = \lim_{\Delta r \to 0} \frac{T(r + \Delta r, t) - T(r, t)}{\Delta r} \\
= \lim_{\Delta r \to 0} \frac{t + T(r + \Delta r, t) - [t + T(r, t)]}{\Delta r} \\
= \lim_{\Delta r \to 0} \frac{\Delta t'}{\Delta r} = \frac{1}{v_{io}(r, t)}
$$

there (Ref. [1])

$$
|\text{grad } T(r, t')| = \frac{\partial T(r, t')}{\partial r} = \frac{\partial T(r, t)}{\partial r} \left[ 1 + \frac{\partial T(r, t)}{\partial t} \right] \\
= \frac{1}{v_{io}(r, t)} \left[ 1 - \frac{\partial T(r, t')}{\partial t'} \right]
$$

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