Three Dimensional Analysis of Raft Foundation Rest on Homogenous Soil Using Finite Element Method

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Abstract. The study includes analysis of Al – Samaraie Hospital raft foundation by the finite element method (FEM) using program package Fear 4.0. The analysis conducted on the foundation is resting on infinite, homogeneous, isotropic soil. In this study, effect of raft thickness, soil properties, modulus of elasticity and Poisson's ratio of soil are investigated. The most dominant parameters such as thickness of raft foundation, modulus of elasticity and Poisson's ratio of soil are graphically plotted against maximum displacements, maximum positive and negative bending moments ($M_x$, $M_y$ and $M_{xy}$). The results showed that raft thickness and soil properties play a vital role in changing the values of maximum positive and negative moments ($M_x$, $M_y$) and maximum displacements, while their effect on maximum positive and negative moment ($M_{xy}$) can be considered as insignificant. Also, empirical equations have been developed to calculate maximum positive and negative bending moment ($M_x$, $M_y$) and maximum displacements beneath the raft. This is performed by the aid of statistical program (Statistic). The accuracy of the proposed equations was ranging between 92.7% and 99.9%.

1. Introduction

The mat footing is a common base that covers the entire area below the structure and supports all applied load. These mats are usually located directly on the soil or rocks, but can also be supported on piles as well [1].

Mat footing are usually used when the base soil has low bearing capacity and/or concentrated loads are too large to cover more than fifty percent of the area with conventional diffusion layers. It is common to use mat bases for both deep basements to spread column loads to a more uniform pressure distribution and to provide a floorboard down [2].

The rational approach in analyzing the soil-centered foundations should take into account both the soil deformation properties and the elasticity of the foundation [3].

The aim of this study to evaluating the effect of raft thickness, elastic modulus and Poisson ratio of soil on maximum bending moment and maximum displacement values, also formulating a reasonable mathematical model in the form of empirical equation to find maximum displacements and maximum negative and bending moments along x- and y-axes. This will help in signing these design criteria swiftly without establishing a prolonged solution depending on the reached accuracy.

2. Finite Element Method

This method possesses high accuracy in representing the foundation and the soil beneath foundation, however, the foundation and the soil are modelled triangular and/or tetrahedral elements, considering fine mesh at locations of severe changes in displacements. These elements are interconnected by number of nodes the numbering of which must submitted to induce minimum bandwidth.
2.1. Modulus of Subgrade Reaction
To calculate moment, shear and deflection in a mat foundation, subjected to given loads, one has to determine the modulus of subgrade reaction (also called soil spring constant or low reaction coefficient). This is defined as the ratio of the intensity of soil pressure to soil deflection at the point of contact surface. They are expressed as [4].

\[ k_s = \frac{q}{w} \]  

Where

- \( k_s \): modulus of subgrade reaction in force/lengths.
- \( q \): soil pressure in force/length²
- \( w \): deflection in soil in length units.

2.2. Soil Structure Interaction
The foundations received loads from the superstructure through the columns, or both, and transport these loads to the soil. This reaction may continue for a long time until a balance is created between the overlapping loads and the supporting soil reactions. Moments, shear and settlement can only be calculated if these soil reactions can be determined, American concrete Institute [5] and [6].

3. Analytical program (FEAR4)
The FEAR4 program (Finite Elements Analysis of Rafts - Version 4.0) is prepared by the University of Sydney, Geotechnical Research Center. Fear4.0 is a DOS-based program that allows analysis of rafts built on foundations consisting of several horizontal soil layers. The soil is treated as a flexible continuous chain (although each layer may have different elastic properties) which is an advantage over the use of spring models (Winkler springs) that do not allow spring-to-spring interaction. The interface between the raft and the soil should be smooth.

Rafts can be analysed that have rectangular uniform loaded area, point loads or point moments in the plan. Bending moments, rotation, fluctuation and displacement can be calculated in a large range. The distribution of contact stress between the raft and the soil can also be calculated.

Contour cutting can be performed either as a raft view or in an isometric form of a raft. Isometric pieces of distorted raft can also be created, and for easier viewing, the pieces can be rotated so that the distorted raft can be seen from different angles [7].

4. Case Study
The raft foundation of Kamal Al-Samaraie Hospital building is chosen as a case study because have all needed information which necessary input to program. The raft is holding 21 columns, the column loads and spacing is shown in figure 1. Finite element computer program (FEAR4.0) is used to analyse the problem.

![Figure 1. The basic problem geometry, after [8].](image)
4.1. Finite Element Mesh layout
Many finite element mesh sizes have been chosen in order to model the raft foundation, these are fine mesh (192 elements and 225 nodes), medium mesh (48 elements and 21 nodes), and coarse mesh (12 elements and 21 nodes).

The results of the medium mesh shown in figure 2 is considered to exhibit the results, in fact, the finer mesh, the more accurate is the results, but because of the large storage memory required for it and of no serious difference between the medium and fine meshes in comparison with results, the medium size mesh is considered which also greatly simplifies out the required results.

![Finite element mesh on raft foundation](image)

Figure 2. Finite element mesh on raft foundation.

4.2. Material Properties
The materials that will enter as a data to the program can be divided into foundation materials and soil materials, the properties of each material are shown in table 1, obtained from site investigation analysis.

Table 1. Soil and foundation properties used in the analysis.

| Properties of foundation | Properties of soil |
|--------------------------|-------------------|
| $E_s = 21538100.0$      | $q_a = 33 \text{kNm}^2$ |
| $\nu = 0.15$           | $\Theta = 0 - 20^\circ$ |
| $\gamma = 24 \text{kN/m}^3$ | $C_v = 9.14 - 47.87 \text{mm}^2/\text{min}$ |
|                          | $\gamma = 19 \text{kNm}^3$ |

5. Results of Analysis
Results of raft foundation analysis obtained by studying various factors affecting bending moments, vertical displacement and contact pressure induced in the raft foundation are presented.

Factors, which have been chosen concern soil and foundation materials properties. The analysis was performed using FEAR4 program. the factors include foundation thickness, Elastic modulus and Poisson ratio of the homogenous soil. figure 3 shows the profile of soil beneath Kamal Al-Samaraie Hospital foundation. The analysis requires estimation of Elastic modulus and Poisson ratio of the soil. Values of elastic modulus and Poisson ratio for most soil types can be found from [9] the soil encountered (sandy clayey silt) was not found in the tables indicated, hence the problem was solved by assigning initial the value of modulus of subgrade reaction ($K_s$) for the soil according to [9], in order to obtain the approximate type of soil from which the value of the two parameters ($E_s$ and $\nu$) can be predicted.

Hence, from [10], the value of ($K_s = 6000\text{kN/m}^3$) the soil type is loose sand. Depending on the approximate type of soil loose sand and from (Bowles,1982), $E_s = (10-24) = (10000 - 24000) \text{kPa}$ and $\nu = 0.2 - 0.3$.

Three values for modulus of elasticity are taken: $E_s = 10000 \text{kpa}$ (minimum value), $E_s = 17000 \text{kpa}$ (medium value) and $E_s = 24000 \text{kpa}$ (maximum value), and three values for Poisson's ratio are taken: $\nu = 0.2$ (minimum value), $\nu = 0.275$ (medium value) and $\nu = 0.35$ (maximum value).
The analysis is also requiring preliminary estimation of raft thickness. Since the study concern with relation between raft thickness and soil properties, three values of raft thicknesses were chosen: $t = 0.6$ m, $t = 0.8$ m and $t = 1.0$ m. In order to accommodate all the parameters entering the analysis, are the factors that affect this analysis.

5.1. Effect of foundation thickness
The effect of foundation thickness has been studied with the two other parameters, Elastic modulus and Poisson ratio of soil. Two cases were adopted: the first case, the modulus of elasticity is constant and Poisson's ratio is variable, where in the second, the elastic modulus is taken as variable and Poisson's ratio is kept constant.

Figure 4 shows the relation between maximum displacement beneath the raft and modulus of elasticity of soil. It is obvious from Fig. (4) that increasing the raft thickness will considerably reduce the maximum displacement beneath the raft, this behaviour is hold valid for other values of Poisson's ratio ($\nu = 0.275$ and $\nu = 0.35$).

Figure 5 and figure 6 clarify that the thickness reduction will result in reducing maximum negative and positive bending moments about x-axis for all values of Poisson's ratio.

Figure 7 reflects the effect of thickness upon maximum negative bending moments about the y-axis. Any reduction of thickness will reduce the moment ($M_y$) for all Poisson's ratio values.

Figure 8 indicates that increase of thickness will result in reducing maximum positive bending moment about the y-axis for all values of Poisson's ratio.

Figure 9 identify the effect of thickness on the negative bending ($M_{xy}$), the reduction of thickness will reduce the value of moments ($M_{xy}$), the reduction of thickness will reduce the value of moment for all values of Poisson's ratio.

Figure 10 represent the effect of thickness on positive bending moment ($M_{xy}$). The effect of raft thickness is minimal upon positive and negative bending moment ($M_{xy}$). The effect of thickness, modulus of elasticity and Poisson's ratio on the results of maximum negative and positive bending moments $M_{xy}$ is very small, therefore neglected the figures including the data of maximum negative and positive bending moments $M_{xy}$ with another cases.

5.2. Effect of thickness with Poisson's ratio
Figure 11 represent the effect of raft thickness on the maximum displacements beneath the foundation, the displacement will be reduced as the thickness increased for modulus of elasticity equal to 10000 kPa, the same behaviour for all modulus of elasticity, when the thickness of raft increases in order of 1.667 times itself, increase 1.75 for Poisson's ratio, and 2.4 times modulus of elasticity, the value of maximum displacement decreases 37.78%.

Figure 12 and figure 13 explain the effect of raft thickness on positive and negative bending moments about x-axis when the thickness reduction will result in reducing negative and positive bending moments.

Figure 14 shows that when the thickness reduced will be reduced the maximum negative bending moment about y-axis.

![Soil profile under the raft foundation, after [8].](image-url)
Figure 15 shows effect of thickness on maximum positive bending moments about y-axis, any increase in thickness reduction will result in bending moments reduction.

**Figure 4.** Effect of foundation thickness on the relation between maximum displacement and modulus of elasticity of soil.

**Figure 5.** Effect of foundation thickness on the relation between maximum negative bending moments and modulus of elasticity of soil.
Figure 6. Effect of foundation thickness on the relation between maximum positive bending moments $M_x$ and modulus of elasticity of soil.

Figure 7. Effect of foundation thickness on the relation between maximum negative bending moments $M_y$ and modulus of elasticity of soil.

Figure 8. Effect of foundation thickness on the relation between maximum positive bending moments $M_y$ and modulus of elasticity of soil.
Maximum negative bending moments $M_{x,y}$ (kN.m)

$1268 \quad 1272 \quad 1276 \quad 1280$

$v = 0.2$

$t = 0.6 \text{ m}$

$t = 0.8 \text{ m}$

$t = 1.0 \text{ m}$

Figure 9. Effect of foundation thickness on the relation between maximum negative bending moments $M_{x,y}$ and modulus of elasticity of soil.

Maximum positive bending moments $M_{x,y}$ (kN.m)

$0.29 \quad 0.3 \quad 0.31 \quad 0.32 \quad 0.33 \quad 0.34$

$E_s = 10000 \text{ kPa}$

$t = 0.6 \text{ m}$

$t = 0.8 \text{ m}$

$t = 1.0 \text{ m}$

Figure 10. Effect of foundation thickness on the relation between maximum positive bending moments $M_{x,y}$ and modulus of elasticity of soil.

Maximum displacements (m)

$0.29 \quad 0.3 \quad 0.31 \quad 0.32 \quad 0.33 \quad 0.34 \quad 0.36$

$E_s = 10000 \text{ kPa}$

$t = 0.6 \text{ m}$

$t = 0.8 \text{ m}$

$t = 1.0 \text{ m}$

Figure 11. Effect of foundation thickness on the relation between maximum displacement and Poisson's ratio of soil.
**Figure 12.** Effect of foundation thickness on the relation between maximum negative $M_x$ and Poisson's ratio of soil.

**Figure 13.** Effect of foundation thickness on the relation between maximum positive $M_x$ and Poisson's ratio of soil.

**Figure 14.** Effect of foundation thickness on the relation between maximum negative $M_y$ and Poisson's ratio of soil.
5.3. Effect of modulus of elasticity

Figure 16 represent the effect of modulus of elasticity upon maximum displacement beneath the raft, the increase in modulus of elasticity will result in reduction displacement for all Poisson's ratio values.

Figure 17 clarifies the effect of modulus of elasticity on maximum negative bending moments about x-axis (\(M_x\)) in which the increasing of modulus of elasticity will reduce moment values, when modulus of elasticity increases in the order of 2.4 times itself, increase 1.75 for Poisson's ratio, and decreases 0.6 times thickness of raft, the maximum value of negative bending moments \(M_x\) decreases 89.82%.

Figure 18 present effect of modulus of elasticity on maximum positive bending moments about x-axis (\(M_x\)), the same effects upon negative bending moments, when the modulus of elasticity increases in the order of 2.4 times itself, increase 1.75 for Poisson's ratio, and decreases times thickness of the raft, the maximum value of positive bending moments \(M_x\) decreases 81.73%.

Figure 19 and figure 20 present effect of modulus of elasticity upon positive and negative bending moments about y-axis (\(M_y\)).

Figure 21 through figure 25 represent the effect of modulus of elasticity upon maximum displacement and negative and positive bending moments (\(M_x\) and \(M_y\), the effect is obvious by reviewing the figures.
Figure 17. Effect of modulus of elasticity of soil on the relation between maximum negative bending moments $M_y$ and thickness of raft.

Figure 18. Effect of modulus of elasticity of soil on the relation between maximum positive bending moments $M_x$ and thickness of raft.

Figure 19. Effect of modulus of elasticity of soil on the relation between maximum negative bending moments $M_y$ and thickness of raft.
Figure (20): Effect of modulus of elasticity of soil on the relation between maximum positive bending moments $M_y$ and thickness of raft.

Figure 21. Effect of modulus of elasticity of soil on the relation between maximum displacement and Poisson's ratio of soil.

Figure 22. Effect of modulus of elasticity of soil on the relation between maximum negative bending moments $M_x$ and Poisson's ratio of soil.
Figure 23. Effect of modulus of elasticity of soil on the relation between maximum positive bending moments $M_x$ and Poisson's ratio of soil.

Figure 24. Effect of modulus of elasticity of soil on the relation between maximum negative bending moments $M_y$ and Poisson's ratio of soil.

Figure 25. Effect of modulus of elasticity of soil on the relation between maximum positive bending moments $M_y$ and Poisson's ratio of soil.
5.4. Effect of Poisson's ratio

Figure 26 clarifies the effect of Poisson's ratio on maximum displacements for the constant thickness equal to 0.6m. The graph shows that increasing Poisson's ratio will reduce maximum displacement for all values of thickness of the raft. Figs. (27) and (28) indicate that increasing Poisson's ratio will result in reducing maximum negative and positive bending moments about the x-axis.

Figure 29 indicates that increasing Poisson's ratio will reduce negative bending moments ($M_x$), when the modulus of elasticity increases in order of 2.4 times itself, increase 1.75 for Poisson's ratio, and decreases 0.6 times thicker of raft, the maximum value of positive bending moments $M_x$ decreases 91.54%.

Figure 30 indicates that increasing Poisson's ratio will increase positive bending moments ($M_y$), when decreases Poisson's ratio in the order of 0.57 times itself, increase 1.67 for raft thickness, and decrease 0.42 times modulus of elasticity, the maximum value of positive bending moments $M_y$ decrease 59.42%.

Figure 31 through figure 34 indicate the effect of Poisson's ratio upon maximum negative and positive bending moments ($M_x$ and $M_y$) and maximum displacements.

![Figure 26](image)

**Figure 26.** Effect of Poisson's ratio of soil on the relation between maximum displacement and modulus of elasticity of soil.

![Figure 27](image)

**Figure 27.** Effect of Poisson's ratio of soil on the relation between maximum negative bending moments $M_x$ and modulus of elasticity of soil.
Figure 28. Effect of Poisson's ratio of soil on the relation between maximum positive bending moments $M_x$ and modulus of elasticity of soil.

Figure 29. Effect of Poisson's ratio of soil on the relation between maximum negative bending moments $M_y$ and modulus of elasticity of soil.

Figure 30. Effect of Poisson's ratio of soil on the relation between maximum positive bending moments $M_x$ and modulus of elasticity of soil.
Figure 31. Effect of Poisson's ratio of soil on the relation between maximum displacement and thickness of raft.

Figure 32. Effect of Poisson's ratio of soil on the relation between maximum negative bending moments $M_x$ and thickness of raft.

Figure 33. Effect of Poisson's ratio of soil on the relation between maximum negative bending moments $M_y$ and thickness of raft.
6. Mathematical Models

Mathematical equations are derived that enable to predict the maximum displacements and maximum negative and positive bending moments \( M_x \) and \( M_y \) utilizing the statistical computer program (STATISTICA). In this case the modulus of elasticity of the soil, Poisson's ratio and raft thickness are related.

The following equations are obtained depending upon the results of (FEAR4.0) program and statistical Program (STATISTICA).

Maximum displacement \( w \) = \( c_1 t c_2 E_s c_3 \nu c_4 \)  
Maximum negative \( M_x \) = \( c_1 t c_2 E_s c_3 \nu c_4 \)  
Maximum positive \( M_x \) = \( c_1 t c_2 E_s c_3 \nu c_4 \)  
Maximum negative \( M_y \) = \( c_1 t c_2 E_s c_3 \nu c_4 \)  
Maximum positive \( M_y \) = \( c_1 t c_2 E_s c_3 \nu c_4 \)  

Where
- \( t \) : thickness of raft foundation (m).
- \( E_s \) : modulus of elasticity of soil (kPa).
- \( \nu \) : Poisson's ratio of soil.

The values of \( c_1 \), \( c_2 \), \( c_3 \), and \( c_4 \) are listed for each equation in table 2.

The accuracy of equations is ranging between accuracy \( R^2 = 99.9 \) and \( R^2 = 92.12 \) which can be regard as a very high.

| Equation number | \( c_1 \)     | \( c_2 \)     | \( c_3 \)     | \( c_4 \)     |
|-----------------|---------------|---------------|---------------|---------------|
| 2               | 1025.8        | -0.16479      | -0.90491      | -0.14356      |
| 3               | 2876.5        | 0.12617       | -0.04375      | -0.00697      |
| 4               | 2751.8        | 0.25327       | -0.07809      | -0.01202      |
| 5               | 2717.6        | 0.10131       | -0.03635      | -0.00587      |

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**Figure 34.** Effect of Poisson's ratio of soil on the relation between maximum positive bending moments \( M_y \) and thickness of raft.
7. Conclusions
The following conclusions can be drawn from the present study:

1. The maximum displacements induced beneath raft foundation are reduced when raft thickness, Elastic modulus and Poisson ratio of the soil layer are increased.

2. If the thickness of raft increases in the order of 1.667 times itself, 1.75 increases for Poisson's ratio, and 2.4 times modulus of elasticity, the displacement value decreases 37.78%.

3. The maximum positive and negative bending moments (M_x, M_y and M_{xy}) are reduced when raft thickness decreases and Elastic modulus and Poisson ratio of soil increases. The maximum positive bending moment (M_y) is excluded from this rule, i.e. it reduces as raft thickness increases and Elastic modulus and Poisson ratio of soil are decreases.

4. If the thickness of raft decreases in the order of 0.6 times itself, 1.75 increases for Poisson's ratio, and 2.4 times modulus of elasticity, the maximum negative bending moments M_x values decreases 89.82%, the maximum positive bending moments M_x values decreases 81.73%, and the maximum negative bending moments M_y values decreases 91.54%.

5. If the thickness of raft increases in the order of 1.667 times itself, 0.5714 decreases for Poisson's ratio, and 0.4167 times modulus of elasticity, the maximum positive bending moments M_y value decreases 59.42%.

8. Recommendations
1. Studying the raft foundation resting on nonhomogeneous layers.
2. Studying circular foundation resting on homogenous and nonhomogeneous layers.
3. Analysis the raft foundation carrying uniform and non-uniform loads.
4. Comparison the results with other programs.

9. References
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