A Multi-scale Inversion Method Based on Convolutional Wavelet Transform Applied in Cross-Hole Resistivity Electrical Tomography

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Abstract. With the increasing demand for fine exploration, there is also a growing demand on the use of the cross-hole electrical resistivity tomography (ERT). However, the boundaries of geological bodies cannot be accurately described when traditional smooth constraint method was adopted. To solve this problem, we studied a multi-scale inversion method based on convolution wavelet transform. The finite element method is used in the modeling process. The imaging effects of numerical simulation are discussed, such as a single rectangular target, two rectangular targets, Irregular targets, and layered models. The effectiveness of multi-scale methods in cross-hole ERT has been revealed in this work.

1. Introduction
Cross-hole electrical resistivity tomography (ERT) is a great improvement of DC electrical exploration, and its imaging results are more accurate than the surface method for deep geological bodies. Therefore, this method has been applied in many fields requiring fine detection[1-3].

The imaging effect of the cross-hole ERT method is mainly affected by two factors. (1) For data collection, the complexity of the target (shape, size, and distance from the electrode), drilling spacing, electrode spacing, and electrode arrangement are important sub-factors to be considered. These sub-factors have been studied by many scholars through numerical simulations and experiments. Further, the data collection parameters are effectively obtained[1,4,5,6]. (2) For data processing, the inversion results are easily converged to the local optimum, due to incomplete and inconsistent data. The least square inversion with smoothness constraint is one of the most frequently-used solutions to solve the considerations. However, the size of the target in the imaging result using this method is often larger than the actual geological body, and the location of the boundary is inaccurate [7], which is affected by isotropic smoothness.

In addition, the global optimal solutions can be obtained by multi-scale inversion method. Multi-scale inversion methods based on wavelet transform have been widely used in the fields of electromagnetics and earthquakes[8-10]. However, the data of the DC electrical method cannot be directly processed by wavelet transform because they are usually potentials or apparent resistivity in a stable electric field. An effective solution is as follows: The model parameters are processed from the
spatial domain to the wavelet domain using the wavelet transform. Then, the inversion process is placed in the wavelet domain. Finally, the inversion results are processed from the wavelet domain to the space-time domain using the inverse wavelet transform. This solution has been applied to seismic [11],[12], gravity [13], magnetotellurics[14], airborne electromagnetics [18] and ERT[15].

In this paper, a multi-scale inversion method based on convolutional wavelet transform is applied to cross-hole ERT. Our procedure was improved drawing on the method proposed by Pang[15]. Multiple types of models were used to verify the validity of the multi-scale inversion method in numerical simulation.

2. Theory of inversion

2.1. Cross-hole ERT

The electrodes are placed in the hole when cross-hole ERT was used. For the 2D cross-hole ERT, data was commonly collected by current or potential electrodes on the survey lines in the two holes. Three typical electrode arrangements are widely used, including dipole-dipole array, bipole-bipole array and pole-tripole array[13,16,17]. The electrode arrangement is shown in Fig.1.

![Figure 1. Three typical electrode arrangements](image)

2.2. Multi-scale resistivity inversion

The inverse problem equation of the multi-scale resistivity inversion method can be expressed as [15]:

\[
\left( \mathbf{J}^* \mathbf{J} + \mu \mathbf{I} \right) \Delta \mathbf{m} = \mathbf{J}^* \left( \mathbf{d} - \mathbf{G}(\mathbf{m}) \right) - \frac{1}{2} \lambda
\]

where \( \Delta \mathbf{m} \) is a parameter in the wavelet domain. This parameter is obtained by a series of 2D discrete convolution transforms from the grid resistivity \( \mathbf{m} \) in the spatial domain. The four convolution kernels

\[
K_1 = \begin{pmatrix}
\frac{1}{2} & 0 \\
0 & \frac{1}{2} 
\end{pmatrix},
K_2 = \begin{pmatrix}
0 & \frac{1}{2} \\
\frac{1}{2} & 0 
\end{pmatrix},
K_3 = \begin{pmatrix}
\frac{1}{2} & 0 \\
0 & -\frac{1}{2} 
\end{pmatrix}
\]

are:

\[
K_4 = \begin{pmatrix}
0 & \frac{1}{2} \\
-\frac{1}{2} & 0 
\end{pmatrix}
\]

Approximation coefficients are obtained using \( K_1 \) and \( K_2 \). Detail coefficients are obtained using \( K_3 \) and \( K_4 \). The specific operations are as follows: (1) First, the approximate coefficients and detail coefficients on the first layer scale are obtained by using convolution transformation for all grid resistivities \( \mathbf{m} \). (2) The spatially adjacent approximate coefficients continue to be operated using convolution transformations.

\( \mathbf{J} \) represents the sensitivity matrix in the wavelet domain. Its expression is \( \mathbf{J} = \frac{\partial \mathbf{d}}{\partial \mathbf{m}} \), where \( \mathbf{d} \) is the potential data. \( \mu \mathbf{I} \) is the damping factor, which stabilizes the inversion process. \( \lambda \) is an extension vector of \( \lambda \) balancing the weights of model terms and data terms. In this work, the values of \( \mu \) and \( \lambda \) are \( 6 \times 10^{-5} \) and \( 5 \times 10^{-6} \).
2.3. Synthetic model test

In this section, many geoelectric models were used to test the multi-scale resistivity inversion method. The sizes of these geoelectric models were set to $8 \text{ m} \times 16 \text{ m}$ (length $\times$ depth), and discretized using a quadrilateral grid. The size of each grid was defined as $0.5 \text{ m} \times 0.5 \text{ m}$. Data was acquired via 2D cross-hole ERT, in which a total of 64 electrodes (spaced 0.5 m apart) were installed on both sides of the model. The measurement array we used including bipole-bipole, dipole-dipole, and pole-tripole, where the complete data set contained 4,256 independent data points. Results were compared with smooth constraint inversion methods in order to test the effectiveness of the multi-scale inversion method for boundary identification.

2.4. Simple models

A geoelectric model with a single rectangular target (low resistivity or high resistivity) was tested, as shown in Figure 2. The resistivity of the rectangular body was 20 Ohm·m or 500 Ohm·m, and the background resistivity is 100 Ohm·m. It can be seen that the imaging results of the multi-scale inversion method were similar to the true shape, while the imaging results of the smooth constrained method were enlarged.

As shown in Figure 3, a geoelectric model with two rectangular targets (both are low resistivity or both are high resistivity) was tested. The resistivity of the target was 20 Ohm·m or 1000 Ohm·m. A new situation can be seen in Figure 3c. Both targets were described using the new method, but the imaging of targets becomes blurred using traditional smoothing constraints. For this geoelectric model, situation of trapping in a local optimal solution may occurs while the traditional method was under used. And the size and location of the target geological body cannot be accurately detected.

Figure 2. Geoelectric model with a single rectangular target
2.5. Complex models

To further test the performance of the multi-scale inversion method, two complex models were designed ("L" shape or layered model). The specific target body and background resistivity values were shown in Figure 4 and Figure 5.

For the "L" shape models shown in Fig.4, it can be seen that the target position in the imaging results using traditional smooth constraint methods was accurate, but the scale was usually larger than the actual model. Inaccuracy of the large-size target body caused by the smoothness can be reduced because the smoothness of the new method was anisotropic. Nevertheless, the actual meaning of the characteristic parameter is the boundary in this work. The final feature parameters were obtained by the inversion method in the wavelet domain. The imaging effect was good because the boundary information becomes significant in the inversion. It was obvious that the new method was also subject to the sensitivity of the data. Areas close to the electrode had better imaging results due to high sensitivity, while artifacts were producing in the areas far from the electrode. In future research, this problem could be improved by drawing on ideas such as data weighting in Cross-hole ERT.

For layered model imaging results show in Fig.5, the effect of the layered imaging feature using the new method was worse than using that of the smooth constraint method. Because there were few large-scale feature data in the wavelet domain, and its proportion of imaging results was very small. One of the solutions could be weighting data at different scales or progressively inverting feature data at different scales.
3. Conclusions
In this study, we applied a multi-scale inversion method based on convolution wavelet transform to the cross-hole ERT. The feasibility of the method was verified by a large number of numerical simulations. The boundary of the geological body near the electrode point can be accurately imaged, which can be shown by the test results. This work provided a reference for the optimization of the multi-scale resistivity inversion method.

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