Optical Transients from Fast Radio Bursts Heating Companion Stars in Close Binary Systems

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Abstract

Fast radio bursts (FRBs) are bright radio transients with short durations and extremely high brightness temperatures (e.g., Lorimer et al. 2007; Thornton et al. 2013), and their physical origins are still unknown (e.g., Cordes & Chatterjee 2019; Petroff et al. 2019; Zhang 2020b; Xiao et al. 2021). Up to the present, hundreds of FRB sources have been detected, and dozens of them showed repeating behaviors (e.g., The CHIME/FRB Collaboration et al. 2021). The recent discovery of Galactic FRB 200428, associated with SGR J1935+2154, suggests that at least some FRBs originate from magnetars born from the core collapses of massive stars (Bochenek et al. 2020; Mereghetti et al. 2020; CHIME/FRB Collaboration et al. 2020a; Li et al. 2021a; Ridnaï et al. 2021; Tavani et al. 2021), but the emission regions and the coherent mechanisms are still not confirmed (e.g., Katz 2016; Murase et al. 2016; Beloborodov 2017; Kumar et al. 2017; Yang & Zhang 2018; Metzger et al. 2019; Ioka 2020; Lu et al. 2020; Margalit et al. 2020; Wadiasingh et al. 2020; Lyubarsky 2021; Wang et al. 2021; Yang & Zhang 2021).

Very recently, a repeating FRB source, FRB 20200120E, was found to be associated with a globular cluster in the nearby M81 galaxy at 3.6 Mpc (Bhardwaj et al. 2021; Kirsten et al. 2021). Since globular clusters host old stellar populations, such an association challenges FRBs originating from active magnetars born from core-collapse supernovae, and implies that there might be more than one formation channel for magnetars as the central engines of FRBs (Kremer et al. 2021; Lu et al. 2021). Besides, a repeating source, FRB 180916B with a periodic activity of 16 days (CHIME/FRB Collaboration et al. 2020b), was localized to be ~250 pc offset from the brightest region of the nearest young stellar clump that was proposed to be its birth place (Tendulkar et al. 2021), which means that the source of FRB 180916B would take 0.8–7 Myr to traverse to the current observed position with the typical projected velocities (60–750 km s⁻¹) of neutron stars in binaries. Because the traversal timescale is inconsistent with the active times (<10 kyr) of a magnetar, this observation also challenges whether FRB 180916B was emitted from a magnetar born from a core-collapse supernova. Meanwhile, the observed 16 day periodic activity also implies that it may be in a binary system as proposed by some FRB models (Zhang 2017, 2020a, 2020c; Dai & Zhong 2020; Ioka & Zhang 2020; Lyutikov et al. 2020; Deng et al. 2021; Geng et al. 2021; Sridhar et al. 2021; Wada et al. 2021; Li et al. 2021c), which is consistent with the recent stellar population synthesis simulation (Zhang & Gao 2020).

The detection of associated multiwavelength or multimessenger counterparts is helpful to identify the physical origins and radiation mechanisms of FRBs. In general, there are four physical mechanisms that can give rise to an FRB-associated multiwavelength counterpart: (1) the extension of the radiation mechanism of FRB emission to higher frequencies (e.g., Yang et al. 2019), (2) the multicolorafterglow associated with an FRB explosion energy (Yi et al. 2014; Wang & Lai 2020), (3) the inverse Compton scattering processes associated with an FRB (Yang et al. 2019), and (4) the astrophysical events directly associated with FRBs (Beloborodov 2017; Metzger et al. 2019).

However, in addition to an X-ray burst associated FRB 200428 (Mereghetti et al. 2020; Li et al. 2021a; Ridnaï et al. 2021; Tavani et al. 2021), there is no confirmed multiwavelength or multimessenger transient being associated with other FRBs so far, which might be due to three main reasons (e.g., Wang et al. 2020b): (1) the typical fluxes of the multiwavelength counterparts of FRBs are lower than the sensitivities of current detectors, like faint FRB afterglows (Yi et al. 2014); (2) the durations of the multicolorafterglows are shorter than the time resolutions of detectors, like fast optical bursts (Tingay & Yang 2019; Yang et al. 2019); (3) the delay time between an FRB and its associated transient is much longer than the observation time, like the scenario of gamma-ray burst to FRB association (Wang et al. 2020a, 2020b).

Inspired by the recent discovery of FRB 20200120E associated with a globular cluster in the M81 galaxy, we are interested in the process where an FRB heats the companion star in a close binary system and the corresponding multiwavelength radiation. In a globular cluster similar to the host of FRB 20200120E, the...
number of main sequence stars, white dwarfs, and neutron stars might be about $10^7$, $10^5$, and $10^2$, respectively (Kremer et al. 2021). We assume that neutron stars are the engines of FRBs as proposed by most FRB models (e.g., Kumar & Bošnjak 2020; Lu et al. 2020; Yang & Zhang 2021), and they would likely have companions, such as low-mass main sequence stars in the globular cluster. Due to the large burst energy of an FRB, when a companion star stops an FRB, its surface would be heated and make a reemission, as shown in Figure 1. We discuss the radiation from the heated companion star in Section 2, including the cases of the companion star not filling/filling the Roche lobe in Sections 2.1 and 2.2, respectively. The results are summarized in Section 3 with some discussions. The convention $Q_\odot \equiv Q/10^5$ is adopted in cgs units, unless otherwise specified.

2. FRB Heating a Companion Star

Globular clusters are believed to host Population II stars with the mass–age relation satisfying (Lejeune & Schaerer 2001; Schaerer 2002; Cooray et al. 2012)

$$\log \left( \frac{\tau_{\text{age}}}{\text{yr}} \right) = 9.59 - 2.79 \log \left( \frac{M}{M_\odot} \right) + 0.63 \left[ \log \left( \frac{M}{M_\odot} \right) \right]^2.$$  (1)

For a globular cluster with age $\tau_{\text{age}} \approx 9.13$ Gyr (Kirsten et al. 2021), like the host of FRB 20200120E, the mass of Population II stars inside it are required to be

$$M \lesssim 0.75 M_\odot.$$  (2)

Thus, in the following discussion, we focus on a close binary system with an FRB source, i.e., neutron star, and a low-mass main sequence star. We assume that the neutron star has a mass of $M_{\text{NS}} \sim 1.4 M_\odot$ and the companion star has a possible mass range of $M = (0.1-1)M_\odot$.

According to Kepler’s third law, for a binary system with orbital period $P$ and total mass $M_{\text{tot}} = M + M_{\text{NS}}$, the semimajor axis of the binary is

$$a = \left( \frac{GM_{\text{tot}}}{\Omega^2} \right)^{1/3} = (3.4-3.9) \times 10^{11} \text{ cm} \ P_{2/3 \text{day}},$$  (3)

where $P_{\text{day}} = P/(1 \text{ day})$ and $M_{\text{tot}} = M + M_{\text{NS}} \sim (1.5-2.4)M_\odot$ is taken. We take $a \sim 3.65 \times 10^{11} \text{ cm} P_{2/3 \text{day}}^2$ as an approximation. If an FRB heats the companion star and makes reemission in the binary system, the intrinsic time delay between the FRB and the reemission is about

$$t_{\text{delay}} \sim \frac{a \cos i}{c} \approx 12 \text{ s} \ P_{2/3 \text{day}}^2 \cos i,$$  (4)

where $i$ is the orbital inclination of the binary system, and a detailed result is dependent on the relative position of both stars. If the companion star is very close to the FRB source, a large radiation pressure of FRB would act on the surface of the companion star:

$$P_{\text{FRB}} \approx \frac{E_{\text{FRB}}}{4\pi c^2 \Delta \Omega_{\text{FRB}}} = 2 \times 10^9 \text{ dyne cm}^{-2} E_{\text{FRB}} \Delta \Omega_{\text{FRB}}^{-1} P_{2/3 \text{day}}^{-4/3},$$  (5)

where $E_{\text{FRB}} = 10^{59} \text{ erg}$ is the isotropic energy in radio emission emitted by an FRB. Here, we take the typical value based on the luminosities of most extragalactic FRBs (e.g., Luo et al. 2020). Some physical scenarios proposed that the total radio fluence is only a small fraction of the total energy required by an FRB’s central engine (e.g., Metzger et al. 2019; Lu et al. 2020; Margalit et al. 2020), in which case the heating effect by total pressure (including radiation pressure at all bands and particle gas pressure) would be more significant than that given by Equation (5). However, a low radio emission fraction, e.g., $\xi \sim 10^{-5}$ for FRB 200428, implies that the active age of a repeating FRB source would be very short (Yang & Zhang 2021), which is inconsistent with the observation of FRB 121102 with a large burst event rate of many years (Li et al. 2021b). In the following discussion, at a conservative estimate, we mainly focus on the energy contribution by radio emission of an FRB. The radiation pressure given by Equation (5) is much greater that the radiation pressure at the companion star’s surface: $P \approx L/4\pi c R^2 = 2 \text{ dyne cm}^{-2} L_\odot R_\odot^{-2}$, where $L_\odot = L/L_\odot$, $R_\odot = R/R_\odot$. The intense radiation pressure of an FRB pulse would push an overdense target inwards, steepening the density profile, and a shock would be generated in this process, as shown in Figure 1.

The radiation-induced shock sweeps the surface medium of the companion star and is finally choked at the region where the stellar pressure is balanced with the radiation pressure, as shown in Figure 1. Before analyzing how much the surface medium is heated by the shock, we first make a discussion about the stellar structure. For a spherically symmetric star in hydrostatic equilibrium, the basic structure equations involve $\partial \rho/\partial m = 1/4\pi \rho^2$ and $\partial P/\partial m = -Gm/4\pi \rho^4$, where $P$ is the pressure, $\rho$ is the mass density, and $m$ is the mass inside $r$. For the uniform-density stellar model, $r = (3m/4\pi \rho)^{1/3}$. Eliminating $r$, one obtains $dP/dm = -(G/4\pi)(4\pi \rho/3)^{2/3}m^{-1/3}$ from the hydrostatic equation, which can be integrated to yield

$$P - P_0 = -\frac{3G}{8\pi} \left( \frac{4\pi \rho}{3} \right)^{4/3} m^{2/3}.$$  (6)

At the star surface, due to $m = M$ and $P \sim 0$, one obtains $P_0 = (3G/8\pi)(4\pi \rho/3)^{4/3}M^{2/3}$. Then the mass at the region

\footnote{Although we take a uniform-density stellar model as an approximation here, the following result of a pressure–mass relation is consistent with the standard solar model (e.g., Guenther et al. 1992).}
where the pressure is less than $P$ can be calculated by

$$m(<P) = M - \left[ M^{2/3} - \frac{8\pi P}{3G} \left( \frac{M}{R^3} \right)^{-4/3} \right]^{3/2}$$

$$\simeq \frac{4\pi PR^4}{GM}. \quad (7)$$

The radiation-induced shock is finally choked at the region where the pressure is balanced, $P = P_{\text{FRB}}$, and one has $m(<P_{\text{FRB}}) = (10^{-8} - 10^{-10})M_\odot$ for a companion star with a mass of $M = (0.1-1)M_\odot$. Notice that the mass given by Equation (7) is isotropic, and a detailed analysis will be discussed in Sections 2.1 and 2.2. Therefore, the larger the radiation pressure of an FRB, the larger the swept mass.

Since the FRB pulse cannot penetrate into the companion star, the absorbed energy is first transported mostly by energetic electrons, and further transfers to ions via collision processes. The energy of energetic electrons is of the order of the cycle-averaged oscillation energy in the electric field of the FRB in a vacuum (Macchi et al. 2013; Yang & Zhang 2020),

$$E_e = (\gamma - 1)mc^2 = mc^2\left( \sqrt{1 + e^2}\frac{a_{\text{str}}^2}{2} - 1 \right), \quad (8)$$

where the Lorentz factor $\gamma = (1 + a_{\text{str}}^2)^{1/2}$ of energetic electrons depends on the strength parameter as follows (e.g., Yang & Zhang 2020):

$$a_{\text{str}} = \frac{eE}{mc^\gamma} = \frac{eL_{\text{FRB}}^{1/2}}{2\pi mc^3/2aV}$$

$$= 44E_{\text{FRB},39}^{1/2}L_{\text{FRB},39}^{-1/2}V_{9}^{-1}P_{\text{day}}^{-2/3}. \quad (9)$$

where $E$ is the oscillating electric field of electromagnetic waves, and $L_{\text{FRB}}$ is the isotropic luminosity of the FRB. Therefore, the typical energy of energetic electrons is estimated by

$$\bar{E}_e \sim \frac{a_{\text{str}} mc^2}{\sqrt{2}}$$

$$= 16\text{ MeV} \ E_{\text{FRB},39}^{1/2}L_{\text{FRB},39}^{-1/2}V_{9}^{-1}P_{\text{day}}^{-2/3}. \quad (10)$$

At the depth with a density of $\rho \sim 10^{-5} \text{ cm}^{-3}$, where the pressure is balanced as discussed in the following (see Equation (18) and Equation (36)), the typical timescale of electron-electron collisions is (e.g., Somov 2012)

$$t_{ee} \sim \frac{m_e^2}{\bar{E}_e mc^2} \ \pi e^4 \rho/m_p$$

$$= 4 \text{ ms} \ E_{\text{FRB},39}^{3/4}L_{\text{FRB},39}^{-3/4}V_{9}^{-3/2}P_{\text{day}}^{1/2} \rho_5^{-1}. \quad (11)$$

For the same typical parameters, the timescale of proton–proton collisions is $t_{pp} \simeq 43t_{ee} \simeq 0.2 \text{ s}$, and the timescale of electron–proton collisions is $t_{pe} \simeq 950t_{ee} \simeq 3.7 \text{ s}$ (see Section 8.3 of Somov 2012). Therefore, at the pressure balance region, most particles would be heated and become thermal in a short time compared with the radiation timescale. In the following discussion, we will analyze the reemission process of an FRB heating a companion star, including two cases: (1) the companion star not filling its Roche lobe (Section 2.1) and (2) the companion filling its Roche lobe (Section 2.2).

### 2.1. Case A: Companion Star Not Filling the Roche Lobe

First, we consider that the companion star in the binary system does not fill its Roche lobe. For an isolated star with mass $M \lesssim 1M_\odot$, the mass–radius relation satisfies

$$R \simeq R_\odot \left( \frac{M}{M_\odot} \right)^{0.8} \text{ or } M \simeq M_\odot \left( \frac{R}{R_\odot} \right)^{1.25}, \quad (12)$$

where $M_\odot = 2 \times 10^{33} \text{ g}$ and $R_\odot = 7 \times 10^{10} \text{ cm}$ (e.g., Kippenhahn et al. 2012). Thus, the low-mass main sequence stars with mass $M \sim (0.1-1)M_\odot$ have a radius of $R \sim (0.16-1)R_\odot$. We assume that the FRB emission region is inside the binary orbit, as proposed by most close-in models (e.g., Kumar & Bosnjak 2020; Lu et al. 2020; Yang et al. 2020); then the solid angle of the companion star opened to the FRB source is

$$\Delta \Omega \sim \frac{\pi R^2}{a^2} = 0.12R_\odot^2P_{\text{day}}^{-4/3}. \quad (13)$$

for $R \ll a$. We assume that the radiation beaming of FRBs, $\Delta \Omega_{\text{FRB}}$, is larger than $\Delta \Omega$. It is not clear whether the FRB emission mechanism is beamed. If the FRB radiation is highly beamed, only a part of the companion surface would be heated; meanwhile, the radio burst would fail to interact with the companion star for a large fraction of the time. Recently, Connor et al. (2020) proposed that a selection effect by beamed emission causes the observed differences in burst durations of repeating FRBs and one-off FRBs. In this scenario, the relation between the beaming solid angle and duration is proposed to satisfy $\Delta \Omega_{\text{FRB}} \sim 0.2\Delta \Omega_{\text{FRB},-3}$, which is slightly larger than $\Delta \Omega$ given by Equation (13). According to Equations (3) and (13), the radiation energy of the FRB emission toward the companion star is

$$E_{\Delta \Omega} \sim \left( \frac{\Delta \Omega}{4\pi} \right)E_{\text{FRB}} \sim 10^{37} \text{ erg} \ E_{\text{FRB},39}R_{\odot,2}P_{\text{day}}^{-4/3}. \quad (14)$$

The large radiation pressure would sweep the surface medium of the companion star and finally be choked at the pressure balance region. According to Equations (5), (7), and (12), the mass of the shocked medium is given by

$$m(<P_{\text{FRB}}) = 2.3 \times 10^{-8}M_\odot E_{\text{FRB},39}\Delta \Omega_{\text{FRB},-3}P_{\text{day}}^{-4/3}R_{\odot,5}^{11/4}. \quad (15)$$

We notice that the above mass is isotropic, and only a fraction $f_b \sim (0.1-1)$ of the surface medium within $\Delta \Omega$ could be swept by the radiation pressure of the FRB, as shown in Figure 1. Therefore, the number of shocked particles is given by

$$N \simeq f_b m(<P_{\text{FRB}}) = 2.8 \times 10^{48}f_{b,-1}$$

$$\times E_{\text{FRB},39}\Delta \Omega_{\text{FRB},-3}P_{\text{day}}^{-4/3}R_{\odot,5}^{11/4}. \quad (16)$$

The radiation energy would transfer to thermal energy of the particles by the shock, and the temperature of the shocked medium is estimated by

$$kT \approx \frac{\eta E_{\Delta \Omega}}{N} = 2.2 \text{ eV}f_{b,-1}^{-1}\Delta \Omega_{\text{FRB},-3}P_{\text{day}}^{-4/3}R_{\odot,5}^{11/4}. \quad (17)$$
where $\eta$ is the absorption factor that is contributed by free–free absorption, synchrotron absorption, and plasma absorption (e.g., Yang et al. 2016; Kundu & Zhang 2021). It is noteworthy that the particle temperature is independent of the FRB burst energy. The reason is that the mass of the shocked medium is proportional to the burst energy according to Equations (5) and (7); this leads to the observation that the accelerated energy of each particle is independent of the effective temperature after thermalization. According to Equations (5) and (17), the mass density of the shocked medium is

$$\rho \approx \frac{m_p P_{\text{FRB}}}{k T_{\text{eff}}} = 10^{-5} \text{ g cm}^{-3} \eta^{-1} f_{b,-1}$$

and the thickness of the shocked medium is estimated by

$$l \approx \frac{f_{b} m(< P_{\text{FRB}})}{2\pi R^2 \rho} = 1.5 \times 10^7 \text{ cm } \eta \Delta T_{\text{FRB},-3}.$$ (19)

Based on Equations (18) and (19), the optical depth for Thomson scattering is

$$\tau \approx \kappa \rho l = 60 f_{b,-1} E_{\text{FRB},39} \Delta T_{\text{FRB},-3}^{-2} \rho_{3/4} R_{0,\odot}^{-3/4},$$ (20)

where $\kappa \sim 0.4 \text{ cm}^2 \text{ g}^{-1}$ is the opacity contributed by Thomson scattering of fully ionized hydrogen. One can see that the larger the separation, the smaller the optical depth. The reemission depends on the effective temperature of the blackbody radiation. According to the theory of radiative transfer, the effective temperature depends on the optical depth,

$$T_{\text{eff}} = T \left( \frac{1}{2} + \frac{3}{4} \tau \right)^{-1/4}.$$ (21)

In the following discussion, we will discuss both cases with $\tau \gg 1$ and $\tau \ll 1$. First, for the case with $\tau \gg 1 (P \ll 22$ days with the above typical parameters), the effective temperature is

$$k T_{\text{eff}} \approx k T \left( \frac{3}{4} \tau \right)^{-1/4} = 0.85 \text{ eV } \eta f_{b,-1}^{-5}$$

$$\times E_{\text{FRB},39}^{-1/4} \Delta T_{\text{FRB},-3}^{-5/4} \rho_{3/4}^{-15/16} R_{0,\odot}^{-15/16}.$$ (22)

Considering that only about half of the surface area could be heated, as shown in Figure 1, the luminosity of reemission by the FRB heating the companion is estimated as

$$L_{\text{re}} \approx 2 \pi R^2 \sigma T_{\text{eff}}^4 = 1.6 \times 10^{34} \text{ erg s}^{-1} \eta^{-3/4} f_{b,-1}^{-5}$$

$$\times E_{\text{FRB},39}^{-1/4} \Delta T_{\text{FRB},-3}^{-5/4} P_{\text{day}}^{1/3} R_{0,\odot}^{-7/4},$$ (23)

which is much larger than the solar luminosity. The typical timescale of reemission is about

$$t_{\text{re}} \approx \frac{\eta \Delta \Omega}{L_{\text{re}}} = 620 \text{ s } \eta^{-3} f_{b,-1}^{-5}$$

$$\times E_{\text{FRB},39}^{-1} \Delta T_{\text{FRB},-3}^{-5} P_{\text{day}}^{-8/3} R_{0,\odot}^{15/4}.$$ (24)

Equation (23) gives the luminosity of the reemission at all bands. For an optical band at $\lambda \sim 5000\text{Å}$, the observed flux at distance $d$ is estimated by

$$F_{\nu} = \pi B_{\nu} \left( \frac{R}{d} \right)^2 \approx \frac{2\pi k T_{\text{eff}}}{c^2} \left( \frac{R}{d} \right)^2 = 1.8 \times 10^{-3} \text{ Jy}$$

$$\times \eta f_{b,-1}^{-5/4} \epsilon_{\text{FRB},39}^{-1/4} \Delta T_{\text{FRB},-3}^{-5/4} P_{\text{day}}^{1/3} R_{0,\odot}^{17/16} \lambda_{\text{opt}}^{2} d_{1,\text{kpc}}^{-2},$$ (25)

where $d_{1,\text{kpc}} = d/(10 \text{ kpc})$ and $\lambda_{\text{opt}} = \lambda/5000 \text{ Å}$ are adopted. On the other hand, for the case with $\tau \ll 1 (P \gg 22$ days with the above typical parameters), the effective temperature is $T_{\text{eff}} = 2^{1/4} T$. The luminosity of reemission by the FRB heating the companion is

$$L_{\text{re}} \approx 2 \pi R^2 \sigma T_{\text{eff}}^4$$

$$= 1.5 \times 10^{36} \text{ erg s}^{-1} \eta^{-3} f_{b,-1}^{-4} \Delta T_{\text{FRB},-3}^{-4} P_{\text{day}}^{4/3} R_{0,\odot}^{-3},$$ (26)

which is about a few hundreds times greater than the solar luminosity. The typical timescale of reemission is about

$$t_{\text{re}} \approx \frac{\eta \Delta \Omega}{L_{\text{re}}}$$

$$= 6.7 \text{ s } \eta^{-3} f_{b,-1}^{-4} E_{\text{FRB},39} \Delta T_{\text{FRB},-3}^{-4} P_{\text{day}}^{4/3} R_{0,\odot}^{-3}.$$ (27)

The observed flux at the optical band is estimated by

$$F_{\nu} \approx 2\pi k T_{\text{eff}} \left( \frac{R}{d} \right)^2$$

$$= 5.4 \times 10^{-3} \text{ Jy s}^{-1} \eta^{-3} f_{b,-1}^{-4} \Delta T_{\text{FRB},-3}^{-4} P_{\text{day}}^{4/3} \lambda_{\text{opt}}^{-2} d_{1,\text{kpc}}^{-2}.$$ (28)

### 2.2. Case B: Companion Star Filling the Roche Lobe

Next, we consider that the companion star in the binary system has filled its Roche lobe, in which case the mass of the companion star would depend on the orbital semimajor or period. The Roche radius of the companion with mass $M$ is approximately given by (e.g., Frank et al. 2002)

$$R_{\text{Roche}} = 0.462 \left( \frac{M}{M_{\odot}} \right)^{1/3}$$ (29)

as an approximation for the range of $0.1 \lesssim M/M_{\text{NS}} \lesssim 0.8$. For a companion star filling the Roche lobe, its radius satisfies $R \approx R_{\text{Roche}}$. According to Equations (3) and (29), the radius–mass is approximately given by

$$M \approx M_{\odot} \left( \frac{R}{R_{\odot}} \right) = 2.7 M_{\odot} P_{\text{day}}.$$ (30)

One can see that the mass of the companion star filling its Roche lobe only depends on the orbital period. For the companion star with mass $M < 1 M_{\odot}$, the orbital period is required to satisfy $P < 0.4$ days, if its Roche lobe has been filling. In this case, the solid angle of the companion star opened to the FRB source is

$$\Delta \Omega \approx \frac{\pi R_{\text{Roche}}^2}{\alpha^2} = 0.84 P_{\text{day}}^{2/3}.$$ (31)

The following discussion is similar to the case of the companion star not filling the Roche lobe as discussed in Section 2.1. According to Equation (31), the FRB radiation
energy toward to the companion star is
\[ E_{\Delta \Omega} \simeq \left( \frac{\Delta \Omega}{4 \pi} \right) E_{\text{FRB}} \sim 7 \times 10^{37} \text{ erg} E_{\text{FRB,39}}^2 P_{\text{day}}^2, \tag{32} \]
Based on Equations (7) and (30), the mass of the shocked medium in this case is
\[ m(< P_{\text{FRB}}) = 4.5 \times 10^{-7} M_\odot E_{\text{FRB,39}} \Delta t_{\text{FRB,39}}^{-1} P_{\text{day}}^5, \tag{33} \]
and the number of shocked particles is given by
\[ N \simeq \frac{f_p m(< P_{\text{FRB}})}{m_p} = 5.4 \times 10^{49} f_{b,-1} \times E_{\text{FRB,39}} \Delta t_{\text{FRB,39}}^{-1} P_{\text{day}}^5. \tag{34} \]
The thermal energy assigned to each particle by the thermalization process is
\[ kT \simeq \frac{\eta E_{\Delta \Omega}}{N} \approx 0.8 \text{ eV} f_{b,-1} \Delta t_{\text{FRB,39}}^{-1} P_{\text{day}}^{-1}, \tag{35} \]
and the mass density of the shocked medium is
\[ \rho \simeq \frac{m_p E_{\text{FRB}}}{kT} = 2.6 \times 10^{-5} \text{ g cm}^{-3} \eta^{-1} f_{b,-1} \times E_{\text{FRB,39}} \Delta t_{\text{FRB,39}}^{-1} P_{\text{day}}^{-1/3}. \tag{36} \]
The thickness of the shocked medium is estimated by
\[ l \simeq \frac{f_p m(< P_{\text{FRB}})}{2 \pi R^2 \rho} = 1.5 \times 10^7 \text{ cm} \eta^{-1} f_{b,-1}. \tag{37} \]
Therefore, the optical depth for Thomson scattering is
\[ \tau \simeq \kappa \lambda l = 160 f_{b,-1} E_{\text{FRB,39}} \Delta t_{\text{FRB,39}}^{-1} P_{\text{day}}^{-1/3}. \tag{38} \]
For the orbital period of \( P < 0.4 \) days that we are interested in here, the optical depth always satisfies \( \tau \gg 1 \); then the effective temperature is
\[ kT_{\text{eff}} \simeq kT \left( \frac{3}{4} \right)^{-1/4} \approx 0.24 \text{ eV} \eta^{-5/4} f_{b,-1} \times E_{\text{FRB,39}} \Delta t_{\text{FRB,39}}^{-4/3} P_{\text{day}}^{11/12}. \tag{39} \]
The reemission luminosity from the companion star heated by an FRB is
\[ L_{\text{re}} \simeq 2 \pi R^2 \sigma T_{\text{eff}}^4 = 7.6 \times 10^{32} \text{ erg s}^{-1} \eta^{-4} f_{b,-1} \times E_{\text{FRB,39}} \Delta t_{\text{FRB,39}}^{-5} P_{\text{day}}^{5/3}, \tag{40} \]
and the corresponding typical timescale of the reemission is about
\[ t_{\text{re}} \simeq \frac{\eta E_{\Delta \Omega}}{L_{\text{re}}} = 9.2 \times 10^4 \text{ s} \eta^{-3} f_{b,-1} E_{\text{FRB,39}} \Delta t_{\text{FRB,39}}^{-5} P_{\text{day}}^{7/3}. \tag{41} \]
For an optical band at \( \lambda \approx 5000 \text{ Å} \), the observed flux is given by
\[ F_{\nu} \approx \frac{2 \pi \nu^2}{c^2} kT_{\text{eff}} \left( \frac{R}{a} \right)^2 = 3.6 \times 10^{-3} \text{ Jy} \eta f_{b,-1}^{-5/4} \times E_{\text{FRB,39}} \Delta t_{\text{FRB,39}}^{-5/4} P_{\text{day}}^{15/12} \lambda_{\text{opt}}^{-2} d_{1.5 \text{ kpc}}^{-2}. \tag{42} \]
Finally, we predict the spectrum and lightcurve of the reemission. Since the reemission is blackbody radiation, its spectrum satisfies Planck’s law. For a given optical band, the observed flux is proportional to the effective temperature, \( F_{\nu} \propto T_{\text{eff}} \). Thus, the lightcurve would mainly depend on the temperature evolution of the heated surface of the companion star. The internal energy stored by shocked particles is \( U \approx \eta E_{\Delta \Omega} \sim NKT \), and the temperature evolution is given by \( dU/dt \approx -2 \pi R^2 \sigma T_{\text{eff}}^4 \). Taking \( T_{\text{eff}} \sim T \) as an approximation, the cooling time from temperature \( T_0 \) to \( T \) could be approximately solved as
\[ t - t_0 = \frac{NkT_{\text{eff}}}{6 \pi R^2 \sigma} \left( \frac{1}{T^3} - \frac{1}{T_0^3} \right) \tag{43} \]
where \( T_0 \) is the initial temperature at time \( t_0 \). For \( T \ll T_0 \), the temperature evolves as \( T \propto (t/t_0)^{-1/3} \). Therefore, the optical flux decreases as \( F_{\nu} \propto T \propto (t/t_0)^{-1/3} \); then it tends to the constant optical flux of the companion star without a heating effect.

3. Results and Discussion

Based on Equations (25), (28), and (42), we plot the results as shown in Figure 2, taking the source distance as \( d \approx 10 \text{ kpc} \), the FRB isotropic energy as \( E_{\text{FRB}} = 10^{39} \text{ erg} \), and the FRB duration as \( \Delta t_{\text{FRB}} = 1 \text{ ms} \). The optical magnitude is calculated by \( m_{AB} = -2.5 \log(F_{\nu}/3631 \text{ Jy}) \). For the case of the companion star not filling its Roche lobe, the larger the companion star mass, the brighter the reemission, and the longer the duration. For a given companion star mass, the larger the separation, the shorter the duration and the brighter the reemission. The brightness of the reemissions tend to constant values as the separations increase. On the other hand, for the case of the companion star filling its Roche lobe, the brightness and duration only depend on the separation. The larger the separation, the brighter the reemission and the longer the duration. If the binary system at distance \( d \approx 10 \text{ kpc} \) has an orbital period of \( P \approx 1 \text{ day} \) and a companion star mass of \( M \approx M_\odot \), the optical magnitude of the reemission would be \( m_{AB} \approx 15.8 \text{ mag} \) (the absolute magnitude is \( m_{abs} \approx 0.8 \text{ mag} \)), and its duration is \( t_{re} \approx 10^3 \text{ s} \). For an optical telescope by typical optical transient survey with limiting magnitude \( m_{\text{lim}} = 20 \text{ mag} \), the threshold flux is \( F_{\nu, \text{th}} = 3.6 \times 10^{-7} \text{ Jy} \) and the threshold distance is about \( d \approx 71 \text{ kpc} \). For a large aperture telescope with limiting magnitude \( m_{\text{lim}} = 25 \text{ mag} \), the threshold flux is \( F_{\nu, \text{th}} = 3.6 \times 10^{-7} \text{ Jy} \) and the threshold distance is about \( d \approx 0.7 \text{ Mpc} \). FRB 20200120E was found in the M81 galaxy at \( d = 3.6 \text{ Mpc} \) (Bhardwaj et al. 2021; Kirsten et al. 2021). If a bright radio burst with energy compared to extragalactic FRBs was emitted from this source, the apparent magnitude from the heated companion star would be \( m_{AB} = 28.5 \text{ mag} \). Therefore, such an optical transient would be more likely detected from Galactic FRB sources. Recently, some optical follow-up observations were performed to detect the optical counterparts of FRBs (Tominaga et al. 2018; Tingay & Yang 2019; Marnoch et al. 2020; Tingay 2020; Kilpatrick et al. 2021; Nuñez et al. 2021; Xin et al. 2021).
Since the predicted duration in this scenario is less than a few times $10^4$ s, a short exposure time is preferred to search and identify the optical transient. For example, Kilpatrick et al. (2021) gave a limiting magnitude $m_{\text{lim}} = 24$ mag with exposure 30 s, Tingey & Yang (2019) gave a limiting magnitude $m_{\text{lim}} = 16$ mag with exposure 30 minutes, and Xin et al. (2021) gave a limiting magnitude $m_{\text{lim}} = 15.4$ mag with exposure 10 s. These capabilities are helpful to search for Galactic optical transients by FRBs heating companion stars. On the other hand, considering that such a transient might be in a globular cluster, one needs to judge whether it is resolvable. The number density of main-sequence stars in a globular cluster similar to the host of FRB 20200120E is $n \sim 10^5$ pc$^{-3}$ at its center. The separation is $l \sim n^{-1/3} \sim 0.02$ pc $n^{-1/3}_{5}$ pc$^{-3}$, and the angle separation is $l/d = 0.4 d_{10^{-1}} n^{-1/3}_{5}$ pc$^{-3}$, which is resolvable for an optical telescope with subarcsecond resolution.

In the above discussion, we assume that the FRB beaming solid angle is larger than the solid angle of the companion star opened to the FRB source, as shown in Figure 1. In this case, if the FRB beaming direction points to the observer, we can see both the FRB and its reemission. However, if the FRB beaming solid angle is smaller than the solid angle of the companion star opened to the FRB source, or the FRB hits the companion star but does not point to the observer, one may only observe an optical flare without an FRB. Therefore, we propose that some optical flares in Galactic globular clusters might be triggered by FRBs generated by active neutron stars, which might appear observable when repeating radio behaviors in the future. On the other hand, if the FRB misses the star, the heating effect would not be produced. We assume that the intrinsic beaming solid angle of an FRB is $\Delta\Omega_{\text{FRB}}$. For a binary system with an FRB source, the solid angle of the companion star opened to the FRB source is $\Delta\Omega$ given by Equations (13) and (31). If the radiation beam is random, the probability that the FRB will hit the companion star would be $p \sim (\Delta\Omega + \Delta\Omega_{\text{FRB}})/4\pi$. If $\Delta\Omega_{\text{FRB}} \ll \Delta\Omega$, one has $p \sim 0.01 R_{\odot}^2 P_{\text{day}}^{-2/3}$ for the companion star not filling its Roche lobe and $p \sim 0.07 P_{\text{day}}^{2/3}$ for the companion star filling its Roche lobe; if $\Delta\Omega_{\text{FRB}} \gg \Delta\Omega$, one has $p \sim \Delta\Omega_{\text{FRB}}/4\pi$.

The periodic activities of some FRB sources imply that they might be in binary systems. Besides FRB 180916B with 16.35 day periodic activity (CHIME/FRB Collaboration et al. 2020b), some FRB sources and Galactic magnetars also appear as possible periodic activities. The first repeating source, FRB 121102, showed a possible long period of $\sim 160$ days (Rajwade et al. 2020), and the X-ray bursts from Galactic magnetars SGR 1806-20 and SGR 1935+2154 were also found to have periodic activities of 398.2 days (Zhang et al. 2021) and 237 days (Zou et al. 2021), respectively. If these periodic activities are indeed due to the orbital periods of binary systems, FRB sources with bright radio bursts similar to the typical extragalactic FRBs might have the opportunity to produce the observable optical transients if they are in the Milky Way.

In this work, we mainly focus on the interaction process between the strong radio emission of an FRB and its companion star, and predict its multiwavelength observation. If the radio emission energy is only a small fraction of the total energy required by the FRB central engine, the heating effect would be more significant than what we estimate here. The above results would replace $E_{\text{FRB}}$ with $E_{\text{FRB}}/\xi$, where $\xi$ is the radio emission fraction. For example, the isotropic energy of Galactic FRB 200428 is $E_{\text{FRB}} \sim 10^{45}$ erg, and the energy of the associated X-ray burst is $E_{\text{XRB}} \sim 10^{46}$ erg. Thus, the radio emission fraction would be $\xi \sim 10^{-2}$. If such an FRB source with $E_{\text{FRB}} \sim 10^{45}$ erg and $\xi \sim 10^{-5}$ is in a close binary system in the Milky Way, the optical brightness by the heating effect would be comparable with or even larger than the typical value given by the above discussion. However, on the other hand, the large burst event rate of FRB 121102 suggested that its radio emission fraction cannot be as small as that of FRB 200428 (Yang & Zhang 2021). Based on the observed event rate of FRB 121102 (Li et al. 2021b), if its central engine is a magnetar with a magnetic field of $\sim 10^{15}$ G and the radio emission fraction the same as that of FRB 200428, its active age would be less than 1 yr, which is much less than the observed active age of $\gtrsim 8$ yr. This result implies that FRB 121102 has a much larger radio emission fraction than that of FRB 200428.

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