Radiative symmetry breaking, cosmic strings and observable gravity waves in \( U(1)_R \) symmetric \( SU(5) \times U(1)_{\chi} \)

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Abstract. We implement shifted hybrid inflation in the framework of supersymmetric \( SU(5) \times U(1)_{\chi} \) GUT model which provides a natural solution to the monopole problem appearing in the spontaneous symmetry breaking of \( SU(5) \). The \( U(1)_{\chi} \) symmetry is radiatively broken after the end of inflation at an intermediate scale, yielding topologically stable cosmic strings. The Planck’s bound on the gravitational interaction strength of these strings, characterized by \( G_N \mu_s \), are easily satisfied with the \( U(1)_{\chi} \) symmetry breaking scale which depends on the initial boundary conditions at the GUT scale. The dimension-5 proton lifetime for the decay \( p \to K^+ \bar{\nu} \), mediated by color-triplet Higgsinos is found to satisfy current Super-Kamiokande bounds for SUSY breaking scale \( M_{\text{SUSY}} \gtrsim 12.5 \text{ TeV} \). We show that with minimal Kähler potential, the soft supersymmetry breaking terms play a vital role in bringing the scalar spectral index \( n_s \) within the Planck’s latest bounds, although with small tensor modes \( r \lesssim 2.5 \times 10^{-6} \) and \( SU(5) \) gauge symmetry breaking scale in the range \( (2 \times 10^{15} \lesssim M_{\alpha} \lesssim 2 \times 10^{16}) \text{ GeV} \). By employing non-minimal terms in the Kähler potential, the tensor-to-scalar ratio approaches observable values \( (r \lesssim 10^{-3}) \) with the \( SU(5) \) symmetry breaking scale \( M_{\alpha} \approx 2 \times 10^{16} \text{ GeV} \).

Keywords: inflation, supersymmetry and cosmology, Cosmic strings, domain walls, monopoles

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1 Introduction

SUSY hybrid inflation [1, 2, 4, 5] provides fascinating framework to realize inflation within the grand unified theories (GUTs) of particle physics. Several GUT models such as, SU(5) [6], Flipped SU(5) [7–9] and the Pati-Salam symmetry SU(4)_C × SU(2)_L × SU(2)_R [10–12], have been employed successfully to realize standard, shifted and smooth variants of hybrid inflation [13–18]. The SU(5) × U(1)_χ gauge symmetry is another suitable choice as a GUT model due to its various attractive features [19, 20]. The whole gauge group of the model is embedded in SU(5) × U(1)_χ ⊂ SO(10), owing to special U(1)_χ charge assignment [19]. In contrast to the SU(5) [6] model, a discrete Z_2 symmetry that avoids rapid proton decay, naturally arises after the breaking of U(1)_χ factor. This Z_2 symmetry not only serves as the Minimal Supersymmetric Standard Model (MSSM) matter parity, but also ensures the existence of a stable lightest supersymmetric particle (LSP) which can be a viable cold dark matter candidate. Furthermore, the right-handed neutrino mass is naturally generated by the breaking of U(1)_χ symmetry after one of the fields carrying U(1)_χ charge acquires a Vacuum Expectation Value (VEV) at some intermediate scale. The well-known advantages of U(1)_χ symmetry include seesaw physics to explain neutrino oscillations, and baryogenesis via leptogenesis [21, 22].

The breaking of SU(5) part of the gauge symmetry leads to copious production of magnetic monopoles [23] in conflict with the cosmological observations whereas, the breaking of U(1)_χ factor yields topologically stable cosmic strings [19, 24, 25]. The cosmic strings can be made to survive if U(1)_χ breaks after the end of inflation. In order to avoid the undesired monopoles, the shifted or smooth variant of hybrid inflation [16] can be employed, where the gauge symmetry is broken during inflation and disastrous monopoles are inflated away. In the simplest SUSY hybrid inflationary scenario the potential along the inflationary track is completely flat at tree level. The inclusion of radiative corrections (RC) to the scalar potential provide necessary slope needed to drive inflaton towards the SUSY vacuum and in doing so the gauge symmetry G breaks spontaneously to its subgroup H.
In this paper, we implement shifted hybrid inflation scenario in the SU(5) × U(1)χ GUT model [20] where the SU(5) symmetry is broken during inflation and the U(1)χ symmetry radiatively breaks to its Z2 subgroup at some intermediate scale. The scalar spectral index ns lies in the observed range of Planck’s results [26] provided the inflationary potential incorporates either the soft supersymmetry (SUSY) breaking terms [27–32, 32–34], or higher-order terms in the Kähler potential [17, 35]. Without these terms, the scalar spectral index ns lies close to 0.98 which is acceptable only if the effective number of light neutrino species are slightly greater than 3 [36]. We show that, by taking soft SUSY contribution into account along with the supergravity (SUGRA) corrections in a minimal Kähler potential setup, the predictions of the model are consistent with the Planck’s latest bounds on scalar spectral index ns [36], although the values of tensor to scalar ratio remain small. By employing non-minimal Kähler potential, large tensor modes are easily obtained, approaching observable values potentially measurable by near-future experiments such as, PRISM [37], LiteBird [38], CORE [39], PIXIE [40], CMB-S4 [41], CMB-HD [42] and PICO [43]. Moreover, the U(1)χ symmetry radiatively breaks after the end of inflation at an intermediate scale, yielding topologically stable cosmic strings. The Planck’s bound [44, 45] on the strength of gravitational interaction of the strings, GNµs are easily satisfied with the U(1)χ symmetry breaking scale obtained in the model, which depends on the initial boundary conditions at the GUT scale. Furthermore, the Super-Kamiokande bounds [46] on dimension-5 proton decay lifetime are easily satisfied for SUSY breaking scale MSUSY ≳ 12.5 TeV.

The rest of the paper is organised as follows. Section 2 provides the description of the SU(5) × U(1)χ model. The implementation of shifted hybrid inflation including the mass spectrum of the model, gauge coupling unification and dimension-5 proton decay is discussed in section 3. The results and inflationary predictions of the model with minimal Kähler potential are presented in section 4 and with non-minimal Kähler potential in section 5. The radiative breaking of U(1)χ symmetry and cosmic strings is discussed in section 6. Finally we summarize our results in section 7.

2 The U(1)R symmetric SU(5) × U(1)χ model

The 10, 5 and 1 dimensional representations of the group SU(5) constitute the 16 (spinorial) representation of SO(10) and contains the MSSM matter superfields. Their decomposition with respect to the MSSM gauge symmetry is

\[ F_i \equiv (10, -1) = Q(3, 2, 1/6) + u^c(\bar{3}, 1, -2/3) + e^c(1, 1, 1), \]
\[ \tilde{f}_i \equiv (\bar{5}, +3) = u^c(\bar{3}, 1, 1/3) + \ell(1, 2, -1/2), \]
\[ \nu^c_i \equiv (1, -5) = \nu^c(1, 1, 0), \]

where \( i = 1, 2, 3 \) denotes the generation index. The scalar sector of SU(5) × U(1)χ consists of the following superfields: a pair of Higgs fiveplets, \( h \equiv (5, 2), \bar{h} \equiv (\bar{5}, -2) \), containing the electroweak Higgs doublets \( (h_d, h_u) \) and color Higgs triplets \( (D_h, \bar{D}_{\bar{h}}) \); a Higgs superfield \( \Phi \) that belongs to the adjoint representation \( \Phi \equiv 24_0 \) and responsible for breaking SU(5) gauge symmetry to MSSM gauge group; a pair of superfields \( (\chi, \bar{\chi}) \) which trigger the breaking of U(1)χ into a Z2 symmetry which is realized as the MSSM matter parity; and finally, a gauge singlet superfield \( S \) whose scalar component acts as an inflaton. The decomposition of
the above SU(5) representations under the MSSM gauge group is
\[
\Phi \equiv (24, 0) = \Phi_{24}(1, 1, 0) + W_H(1, 3, 0) + G_H(8, 1, 0)
\]
\[
+ Q_H(3, 2, -5/6) + Q_H(3, 2, 5/6),
\]
\[
h \equiv (5, 2) = D_h(3, 1, -1/3) + h_u(1, 2, 1/2),
\]
\[
\bar{h} \equiv (5, -2) = D_{\bar{h}}(3, 1, 1/3) + h_d(1, 2, -1/2),
\]
\[
\chi \equiv (1, 10), \quad \bar{\chi} \equiv (1, -10), \quad S \equiv (1, 0),
\]
where the singlets (\(\chi, \bar{\chi}\)) originate from the decomposition of 126 representation of SO(10)
\[
126 = (1, -10) + (5, -2) + (10, -6) + (15, 6) + (45, 2) + (50, -2).
\]

Following [16], the U(1)_R charge assignment of the superfields is given in table 1 along with their transformation properties. The SU(5) \times U(1)_{\chi} and U(1)_R, symmetric superpotential of the model with the leading-order non-renormalizable terms is given by
\[
W = S \left[ \kappa M^2 - \kappa \text{Tr}(\Phi^2) - \frac{\beta}{m_p} \text{Tr}(\Phi^3) + \sigma \chi \bar{\chi} \right] + \gamma \bar{h} \Phi h + \delta \bar{h} h
\]
\[
+ y_{ij}^{(u)} F_i F_j h + y_{ij}^{(d,e)} F_i f_j \bar{h} + y_{ij}^{(\nu)} \nu_i^c \bar{\nu}_j h + \lambda_{ij} \chi \nu_i^c \nu_j^c,
\]
(2.4)
where \(M\) is a superheavy mass and \(m_P = 2.43 \times 10^{18}\) GeV is the reduced Planck mass. The terms in square bracket in the first line are relevant for shifted hybrid inflation while, the last two terms are involved in the solution of doublet-triplet splitting problem, as discussed in section 3.2. The Yukawa couplings \(y_{ij}^{(u)}, y_{ij}^{(d,e)}, y_{ij}^{(\nu)}\) in the second line of (2.4) generate Dirac masses for quarks and leptons after the electroweak symmetry breaking, whereas \(m_{\nu_{ij}} = \lambda_{ij} \langle \chi \rangle\) is the right-handed neutrino mass matrix, generated after \(\chi\) acquires a VEV through radiative breaking of U(1)_\chi symmetry, as discussed in section 6.

The superpotential \(W\) exhibits a number of interesting features as a consequence of global U(1)_R symmetry. First, it allows only linear terms in \(S\) in the superpotential, omitting the higher order ones, such as \(S^2\) which could generate an inflaton mass of Hubble size, invalidating the inflationary scenario. Second, the U(1)_R symmetry naturally avoids the so called \(\eta\) problem [3], that appears when SUGRA corrections are included. Finally, several dangerous dimension-5 proton decay operators are highly suppressed.

3 Shifted hybrid SU(5) × U(1)_\chi inflation

In this section, the effective scalar potential is computed considering contributions from the F- and D-term sectors. The superpotential terms relevant for shifted hybrid inflation are
\[
W \supset S \left[ \kappa M^2 - \kappa \text{Tr}(\Phi^2) - \frac{\beta}{m_p} \text{Tr}(\Phi^3) \right] + \gamma \bar{h} \Phi h + \delta \bar{h} h
\]
\[
+ \sigma \chi S \chi \bar{\chi} + \lambda_{ij} \chi \nu_i^c \nu_j^c.
\]
(3.1)
In component form, the above superpotential is expanded as follows,
\[
W \supset S \left[ \kappa M^2 - \kappa \frac{1}{2} \sum_i \phi_i^2 - \frac{\beta}{4m_P} d_{ijk} \phi_i \phi_j \phi_k \right] + \delta \bar{h}_a h_a + \gamma T_{ab} \phi^b \bar{h}_a h_b
\]
\[
+ \sigma \chi S \chi \bar{\chi} + \lambda_{ij} \chi \nu_i^c \nu_j^c,
\]
(3.2)
Superfields & Representations under SU(5) $\times$ U(1)$_{\chi}$ & Global U(1)$_{R}$ \\
\hline
Matter sector & & \\
\hline
$F_i$ & (10, $-1$) & 3/10 \\
$\bar{f}_i$ & (5, 3) & 1/10 \\
$\nu_i^c$ & (1, $-5$) & 1/2 \\
\hline
Scalar sector & & \\
\hline
$\Phi$ & (24, 0) & 0 \\
$h$ & (5, 2) & 2/5 \\
$\bar{h}$ & (5, $-2$) & 3/5 \\
$\chi$ & (1, 10) & 0 \\
$\bar{\chi}$ & (1, $-10$) & 0 \\
$S$ & (1, 0) & 1 \\
\hline
\end{tabular}

Table 1. The representations of matter and scalar superfields under SU(5) $\times$ U(1)$_{\chi}$ gauge symmetry and global U(1)$_{R}$ symmetry in shifted hybrid inflation model.

where $\Phi = \phi_i T^i$ with $\text{Tr}[T_i T_j] = \frac{1}{2}\delta_{ij}$ and $d_{ijk} = 2\text{Tr}[T_i \{T_j, T_k\}]$ in the SU(5) adjoint basis. The $F$-term scalar potential obtained from the above superpotential is given by

\[ V_F = \left| \kappa M^2 - \frac{1}{2}\sum_i \phi_i^2 - \frac{\beta}{4m_P} d_{ijk} \phi_i \phi_j \phi_k + \sigma_\chi \chi \bar{\chi} \right|^2 \\
+ \sum_i \left| \kappa S \phi_i + \frac{3\beta}{4m_P} d_{ijk} S \phi_j \phi_k - \gamma T_{ab}^i \bar{h}_a h_b \right|^2 \\
+ \sum_b \left| \gamma T_{ab}^i \phi^a \bar{h}_a + \delta h_b \right|^2 + \sum_b \left| \gamma T_{ab}^i \phi^a \bar{h}_a + \delta h_b \right|^2 \\
+ \left| \sigma_\chi S \bar{\chi} + \lambda_{ij} \nu_i^c \nu_j^c \right|^2 + \left| \sigma_\chi S \chi \right|^2 + \left| 2\lambda_{ij} \chi \nu_i^c \right|^2, \]  

(3.3)

where the scalar components of the superfields are denoted by the same symbols as the corresponding superfields. The VEV’s of the fields at the global SUSY minimum of the above potential are given by,

\[ S^0 = h_a^0 = \bar{h}_a^0 = \nu_i^0 = 0, \quad \chi^0 = \bar{\chi}^0 = 0 \]  

(3.4)

with $\phi_i^0$ satisfying the following equation:

\[ \sum_{i=1}^{24} (\phi_i^0)^2 + \frac{\beta}{2\kappa m_P} d_{ijk} \phi_i^0 \phi_j^0 \phi_k^0 = 2M^2. \]  

(3.5)

The superscript ‘0’ denotes the field value at its global minimum. The superfield pair ($\chi, \bar{\chi}$) break U(1)$_{\chi}$ to $Z_2$, the matter parity. This symmetry ensures the existence of a lightest supersymmetric particle (LSP) which could play the role of cold dark matter. Further, as discussed in [20], this $Z_2$ symmetry yields topologically stable cosmic strings.
Using SU(5) symmetry transformation the VEV matrix $\Phi^0 = \phi_i^0 T^i$ can be aligned in the 24-direction,

$$\Phi^0_{24} = \frac{\phi_{24}^0}{\sqrt{10}} (1, 1, 1, -3/2, -3/2).$$  \hspace{1cm} (3.6)

Thus the SU(5) gauge symmetry is broken down to Standard Model gauge group $G_{SM}$ by the non-vanishing VEV of $\phi_{24}^0$ which is a singlet under $G_{SM}$.

The $D$-term scalar potential,

$$V_D = \frac{g_2^2}{2} \sum_i \left( f^{ijk} \phi_j \phi_k^\dagger + T^i \left( |h_{a1}|^2 - |\tilde{h}_{a1}|^2 \right) \right)^2 + \frac{g_2^2}{2} \left( q_x |\chi|^2 + q_{\bar{x}} |\bar{\chi}|^2 + (q_{\bar{x}} + q_x) \varsigma \right)^2,$$

also vanishes for this choice of the VEV (since $f^{t,24,24} = 0$) and for $|\tilde{h}_{a1}| = |h_{a1}|, |\bar{\chi}| = |\chi|$. The scalar potential in eq. (3.3) can be written in terms of the dimensionless variables

$$z = \frac{|S|}{M}, \quad y = \frac{\phi_{24}^0}{M \sqrt{2}}.$$  \hspace{1cm} (3.8)

as follows,

$$\hat{V} = \frac{V}{\kappa^2 M^4} = \left( 1 - y^2 + \alpha y^3 \right)^2 + 2 z^2 y^2 \left( 1 - 3 \alpha y/2 \right)^2,$$

where $\alpha = \beta M/\sqrt{30} \kappa m_p$. This dimensionless potential exhibits the following three extrema

$$y_1 = 0,$$

$$y_2 = \frac{2}{3\alpha},$$

$$y_3 = \frac{1}{3\alpha} + \frac{1}{3 \sqrt{2} \alpha} \left( \sqrt{-2 + 27\alpha^2 + \sqrt{(2 - 27\alpha^2)^2 + 4 (9\alpha^2 z^2 - 1)^3}} - \sqrt{-2 + 27\alpha^2 + \sqrt{(2 - 27\alpha^2)^2 + 4 (9\alpha^2 z^2 - 1)^3}} \right).$$  \hspace{1cm} (3.12)

for a constant value of $z$ and is displayed in figure 1 for different values of $\alpha$. The first extremum $y_1$ with $\alpha = 0$ corresponds to the standard hybrid inflation for which $y = 0, z > 1$ is the only inflationary trajectory that evolves at $z = 0$ into the global SUSY minimum at $y = \pm 1$ (figure 1(a)). For $\alpha \neq 0$, a shifted trajectory appears at $y = y_2$, in addition to the standard trajectory at $y = y_1 = 0$, which is a local maximum (minimum) for $z < \sqrt{\frac{4}{27}\alpha^2 - 1}$ ($z > \sqrt{\frac{4}{27}\alpha^2 - 1}$). For $\alpha < \sqrt{\frac{2}{27}} \simeq 0.27$, this shifted trajectory lies higher than the standard trajectory (figure 1(b)). In order to have suitable initial conditions for realizing inflation along the shifted track, we assume $\alpha > \sqrt{\frac{2}{27}}$, for which the shifted trajectory lies lower than the standard trajectory (figure 1(c)). Moreover, to ensure that the shifted inflationary trajectory at $y_2$ can be realized before $z$ reaches zero, we require $\alpha < \sqrt{\frac{4}{27}} \simeq 0.38$. Thus, for $0.27 < \alpha < 0.38$, while the inflationary dynamics along the shifted track remain the same as for the standard track, the SU(5) gauge symmetry is broken during inflation, hence alleviating the magnetic monopole problem. As the inflaton slowly rolls down the inflationary valley and enters waterfall regime at $z = \sqrt{\frac{4}{27}\alpha^2 - 1}$, its fast
Figure 1. The tree-level, global dimensionless scalar potential \( \tilde{V} = V/\kappa^2 M^4 \) versus \( y \) and \( z \) for: \( \alpha = 0 \) (a), \( \alpha = 0.25 \) (b) and \( \alpha = 0.3 \) (c). The standard track corresponds to \( \alpha = 0 \) whereas, \( \alpha \neq 0 \) corresponds to two shifted trajectories. The inflationary track feasible for realizing successful inflation is shown in panel (b) for \( \alpha = 0.3 \).

rolling ends inflation, and the system starts oscillating about the vacuum at \( z = 0 \) and \( y = y_3 \). In order to calculate one-loop radiative correction along \( y_2 \), we need to compute the mass spectrum of the model along this track where both SU(5) gauge symmetry and SUSY are broken.

During inflation, the field \( \Phi \) acquires a VEV in the \( \phi_{24} \) direction which breaks the SU(5) gauge symmetry down to SM gauge group \( G_{SM} \) while, the \( U(1)_Y \) symmetry remains unbroken. The potential in eq. (3.3) generates the following masses for: 2 real scalars

\[
m_{24\pm}^2 = \pm \kappa^2 M^2_{\alpha} + \kappa^2 |S|^2,
\]

22 real scalars

\[
m_{i\pm}^2 = \pm 5\kappa^2 M^2_{\alpha} + 25\kappa^2 |S|^2, \quad i = 1, \ldots, 8, 21, 22, 23,
\]

and 4 real and pseudo scalars

\[
m_{(\chi, \bar{\chi})\pm}^2 = \pm \kappa \sigma_{\chi} M^2_{\alpha} + \sigma_{\chi}^2 |S|^2,
\]

where \( M^2_{\alpha} = M^2 \left( \frac{1}{27\alpha^2} - 1 \right) \). The superpotential (3.2) generates: a Majorana fermion with mass squared,

\[
m_{24}^2 = \kappa^2 |S|^2,
\]
where $z$ is observable sector field, $W_i = \frac{\partial V}{\partial z_i}$, $m_{3/2}$ is the gravitino mass and $A$ is the complex coefficient of the trilinear soft-SUSY-breaking terms. The effective contributions of
soft SUSY breaking terms during inflation can be written as,

$$V_{\text{Soft}} = am_{3/2}kM_3^2x + M_S^2M_3^2x^2 + \frac{8M_\phi^2M_3^2}{9(4/27 - \alpha^2)},$$  \hspace{1cm} (3.23)

with

$$a = 2|A - 2|\cos(\arg S + \arg |A - 2|),$$  \hspace{1cm} (3.24)

where \(a\) and \(M_S\) are the coefficients of soft SUSY breaking linear and mass terms for \(S\), respectively, \(M_\phi\) is the soft mass term for \(\phi_{24}\) and \(m_{3/2}\) is the gravitino mass.

### 3.1 Gauge coupling unification

After the breaking of the SU(5) symmetry, the octet \(G_H\) and triplet \(W_H\) from the adjoint Higgs field remain massless, as shown in [48, 49]. The presence of these flat directions is a generic feature of simple groups like the SU(5) with a U(1)\(_R\) symmetry, as discussed in [48, 49]. These fields, however, acquire relatively light masses \(\mathcal{O}(\sim \text{TeV})\) from the soft SUSY-breaking terms in our model, which spoils the unification of the gauge couplings. In order to preserve gauge-coupling unification we add the following combination of vectorlike particles

$$5 + \bar{5} + E + \bar{E} = (D + \bar{D}, L + \bar{L}) + E + \bar{E},$$  \hspace{1cm} (3.25)

with the \(R\)-charge, \(R(5, \bar{5}, E, \bar{E}) = (1, 1)\) and allow mass-splitting within a multiplet with some fine tuning. The superpotential of these vectorlike fermions is given by [50],

$$W \supset A_{ij}(E, \bar{E}) \frac{\text{Tr}(\Phi^2)E_i\bar{E}_j}{m_P} + B_{ij}(E, \bar{E}) \frac{\text{Tr}(E_i\Phi^2\bar{E}_j)}{m_P} \quad \text{+}\quad \frac{A_{ij}(5, \bar{5})}{m_P} \text{Tr}(\Phi^2) \text{Tr}(5_i\bar{5}_j) + \frac{B_{ij}(5, \bar{5})}{m_P} \text{Tr}(5_i\Phi^2\bar{5}_j),$$

$$\supset M_EEE + M_D\bar{D}D + M_LL\bar{L}. \quad (3.26)$$

Assuming \(A_{ij} = \delta_{ij}A\) and \(B_{ij} = \delta_{ij}B\) for convenience, we obtain the following masses of the MSSM field components of vectorlike particles,

$$M_E = \frac{40A(E, \bar{E}) + 12B(E, \bar{E})}{45\alpha^2} \left(\frac{M^2}{m_P}\right),$$  \hspace{1cm} (3.27)

$$M_D = \frac{60A(5, \bar{5}) + 8B(5, \bar{5})}{135\alpha^2} \left(\frac{M^2}{m_P}\right),$$  \hspace{1cm} (3.28)

$$M_L = \frac{20A(5, \bar{5}) + 6B(5, \bar{5})}{45\alpha^2} \left(\frac{M^2}{m_P}\right).$$  \hspace{1cm} (3.29)

The masses for \(E + \bar{E}\) and \(L + \bar{L}\) can be made light with fine tuning on the parameters such that

$$40A(E, \bar{E}) + 12B(E, \bar{E}) \sim 0, \quad 20A(5, \bar{5}) + 6B(5, \bar{5}) \sim 0.$$

The mass of \(D + \bar{D}\) component is then given as

$$M_D \sim \frac{21A(5, \bar{5})}{81\alpha^2} \left(\frac{M^2}{m_P}\right).$$  \hspace{1cm} (3.30)
Figure 2. The evolution of the inverse gauge couplings $\alpha_i^{-1}$ with the energy scale $\Lambda$ in $U(1)_R$ symmetric SU(5) $\times$ U(1)$_\chi$ model, with SUSY breaking scale $M_{\text{SUSY}} = 12.5$ TeV (left) and $M_{\text{SUSY}} = 25$ TeV (right). Unification is achieved with three generations of vectorlike fermions and GUT scale at $M_{\text{GUT}} \sim 2 \times 10^{16}$ GeV in both cases.

| SUSY breaking scale $M_{\text{SUSY}}$ | Vectorlike Fermion masses (GeV) $M_D$ | $M_L$ | $M_E$ | Unification scale $M_{\text{GUT}} \sim 2 \times 10^{16}$ GeV |
|--------------------------------------|-------------------------------------|-------|-------|----------------------------------|
| 12.5 TeV                            | $6.5 \times 10^{13}$                | $2.14 \times 10^9$ | $1.0 \times 10^8$ | $M_{\text{GUT}}$ |
| 25 TeV                              | $6.5 \times 10^{13}$                | $3.16 \times 10^9$ | $2.5 \times 10^8$ | $M_{\text{GUT}}$ |

Table 3. The effective SUSY breaking scales, $M_{\text{SUSY}}$ and corresponding mass splitting patterns of vectorlike families. Unification occurs at the same scale for both cases.

Figure 2 shows successful gauge-coupling unification with three generations of the vectorlike families and different mass-splittings, as listed in table 3, for two SUSY-breaking scales, $M_{\text{SUSY}} = (12.5, 25)$ TeV. Here, we assume the masses of the octet and the triplet to be near the SUSY-breaking scale, $M_{\text{SUSY}} \simeq M_{\text{GUT}}$ = $M_{\text{GUT}}$. In both cases, the unification is achieved at $M_{\text{GUT}} = (5\sqrt{2}/9\alpha)g_5M \sim 2 \times 10^{16}$ GeV, where $g_5$, the gauge coupling of SU(5) gauge group, is unified with $g_{\chi}$, the gauge coupling of U(1)$_\chi$ group.

3.2 Dimension-5 proton decay

In this section, the implementation of the douplet-triplet solution to the well known issue of the color triplets $D_h, \bar{D}_h$ embedded in the same representations 5 and $\bar{5}$ with the MSSM Higgs fields is briefly discussed. The relevant superpotential terms are

$$W \supset \gamma h \Phi h + \delta \bar{h} h.$$  \hspace{1cm} (3.32)

After the SU(5) symmetry breaking, these can be written in terms of the MSSM fields as follows

$$W \supset \left( \delta - \frac{3\gamma \phi_0^2}{2\sqrt{15}} \right) h_u h_d + \left( \gamma \frac{\phi_0^2}{\sqrt{15}} \right) \bar{D}_h D_h \supset \mu_H h_u h_d + M_{D_h} \bar{D}_h D_h.$$  \hspace{1cm} (3.33)

We observe that the doublet-triplet splitting problem is resolved by requiring fine-tuning of the involved parameters, such that

$$\delta \sim \frac{3\gamma \phi_0^2}{2\sqrt{15}}.$$
Here $\mu_H$ is identified with the MSSM $\mu$ parameter taken to be of the order of TeV scale while, $M_{D_h}$ is the color triplet Higgs mass parameter given by,

$$M_{D_h} \sim \frac{5\gamma \phi_4^{14}}{2\sqrt{15}} = \frac{5\gamma M_{\alpha} y_2}{\sqrt{30 \left( \frac{4}{27\alpha^2} - 1 \right)}}.$$ (3.34)

The dominant contribution to dimension-5 proton decay amplitude comes from color-triplet Higgsinos and typically dominates the decay rate from gauge boson mediated dimension-6 operators. The proton lifetime for the decay $p \rightarrow K^+\bar{\nu}$ mediated by color-triplet Higgsinos is approximated by [51]:

$$\tau_p \simeq 4 \times 10^{35} \times \sin^4 2\beta \left( \frac{M_{\text{SUSY}}}{10^2 \text{TeV}} \right)^2 \left( \frac{M_{D_h}}{10^{16} \text{GeV}} \right)^2 \text{years},$$ (3.35)

which depends on Higgino mass as well as the SUSY breaking scale $M_{\text{SUSY}}$. The Super-Kamiokande experiment places a lower bound on proton lifetime of $\tau_p = 5.9 \times 10^{33}$ years at 90% confidence level for the channel $p \rightarrow K^+\bar{\nu}$. With $M_{\alpha} \simeq 2 \times 10^{16}$ GeV, this translates into a lower bound on $M_{\text{SUSY}}$,

$$M_{\text{SUSY}} \gtrsim 12.5 \text{TeV}.$$ (3.36)

This can also be seen in figure 3 where SU(5) gauge symmetry breaking scale $M_{\alpha}$ is plotted against the SUSY breaking scale $M_{\text{SUSY}}$. The curves represent different values of $\tan \beta$ and drawn for proton lifetime fixed at Super-Kamiokande bounds [46].

4 Minimal Kähler potential

The minimal canonical Kähler potential is given as,

$$K = |S|^2 + Tr|\Phi|^2 + |h|^2 + |\bar{h}|^2 + |\chi|^2 + |\bar{\chi}|^2 + |\nu|^2.$$ (4.1)
The F-term SUGRA scalar potential is given by
\[ V_{\text{SUGRA}} = e^{K/m^2} \left( K^{-1} D_{z_i} W D_{z_j} W^* - 3m_P^2 |W|^2 \right), \] (4.2)
with \( z_i \) being the bosonic components of the superfields \( z_i \in \{ S, \Phi, h, \chi, \bar{\chi}, \cdots \} \), and we have defined
\[ D_{z_i} W \equiv \frac{\partial W}{\partial z_i} + m_P^2 \frac{\partial K}{\partial z_i} W, \quad K_{ij} \equiv \frac{\partial^2 K}{\partial z_i \partial z_j^*}, \] (4.3)
and \( D_{z_i} W^* = (D_{z_i} W)^* \). The SUGRA scalar potential during inflation becomes
\[ V_{\text{SUGRA}} = \kappa^2 M_\alpha^4 \left[ 1 + \left( \frac{4}{9(4/27 - \alpha^2)} \right) \left( \frac{M_\alpha}{m_P} \right)^2 ight. \\
\left. + \left( \frac{4(2 + 9x^2 (4/27 - \alpha^2))}{81(4/27 - \alpha^2)^2} + \frac{1}{2} x^4 \right) \left( \frac{M_\alpha}{m_P} \right)^4 + \cdots \right]. \] (4.4)
Putting all these corrections together, we obtain the following form of inflationary potential,
\[ V \simeq V_{\text{SUGRA}} + V_{1\text{-loop}} + V_{\text{Soft}} \]
\[ \simeq \kappa^2 M_\alpha^4 \left[ 1 + \left( \frac{4}{9(4/27 - \alpha^2)} \right) \left( \frac{M_\alpha}{m_P} \right)^2 ight. \\
\left. + \left( \frac{4(2 + 9x^2 (4/27 - \alpha^2))}{81(4/27 - \alpha^2)^2} + \frac{1}{2} x^4 \right) \left( \frac{M_\alpha}{m_P} \right)^4 ight. \\
\left. + \frac{\kappa^2}{16\pi^2} \left[ F(M_\alpha^2, x^2) + 11 \times 25 F(5M_\alpha^2, 5x^2) \right] + \frac{\sigma^2}{8\pi^2} F(M_\alpha^2, y^2) \right] \\
\left. + \frac{am_{3/2} x}{\kappa M_\alpha} + \frac{M_\phi^2 x^2}{\kappa^2 M_\alpha^2} + \frac{8M_\sigma^2}{9\kappa^2 M_\alpha^2 (4/27 - \alpha^2)} \right]. \] (4.5)
The inflationary slow roll parameters are given by,
\[ \epsilon = \frac{1}{4} \left( \frac{m_P}{M_\alpha} \right)^2 \left( \frac{V'}{V} \right)^2, \quad \eta = \frac{1}{2} \left( \frac{m_P}{M_\alpha} \right)^2 \left( \frac{V''}{V} \right), \quad \alpha^2 = \frac{1}{4} \left( \frac{m_P}{M_\alpha} \right)^4 \left( \frac{V'V''}{V^2} \right). \] (4.6)
Here, the derivatives are with respect to \( x = |S|/M_\alpha \), whereas the canonically normalized field \( \sigma \equiv \sqrt{2}|S| \). In the slow-roll (leading order) approximation, the tensor-to-scalar ratio \( r \), the scalar spectral index \( n_s \), and the running of the scalar spectral index \( dn_s/d\ln k \) are given by
\[ r \simeq 16 \epsilon, \] (4.7)
\[ n_s \simeq 1 + 2\eta - 6\epsilon, \] (4.8)
\[ \frac{dn_s}{d\ln k} \simeq 16 \epsilon \eta - 24 \epsilon^2 - 2\xi^2. \] (4.9)
The last \( N_0 \) number of e-folds before the end of inflation is,
\[ N_0 = 2 \left( \frac{M_\alpha}{m_P} \right)^2 \int_{x_e}^{x_0} \frac{V}{V'} dx, \] (4.10)
where \(x_0\) is the field value at the pivot scale \(k_0\), and \(x_e\) is the field value at the end of inflation, defined by \(|\eta(x_e)| = 1\). Assuming a standard thermal history, \(N_0\) is related to \(T_r\) as \[N_0 = 54 + \frac{1}{3} \ln \left( \frac{T_r}{10^9 \text{GeV}} \right) + \frac{2}{3} \ln \left( \frac{V(x)^{1/4}}{10^{15} \text{GeV}} \right), \tag{4.11}\]

where \(T_r\) is the reheat temperature and in numerical calculation we set \(T_r = 10^9 \text{GeV}\). This could easily be reduced to lower values if the gravitino problem is regarded to be an issue.\(^1\) The non-perturbative effects, i.e. preheating, are generally suppressed in supersymmetric hybrid inflation \([33, 54, 55]\) and are only efficient if both inflaton and waterfall field are coupled to some other scalar field and if inflaton is relatively strongly coupled to the scalar field. In our case, \(\chi\) and \(h\) in the D-flat direction represent such scalar fields and efficient preheating requires \(\sigma_\chi \gg \kappa\) or \(\gamma \gg \kappa\). In our analysis, \(\gamma \simeq \sigma_\chi \simeq \kappa\), therefore the non-perturbative effects via preheating are suppressed. Furthermore, due to Pauli blocking, the preheating of fermions is generically expected to be subdominant.

The amplitude of the curvature perturbation is given by \([56]\):

\[
A_s(k_0) = \frac{1}{24 \pi^2} \left( \frac{V/m_P^4}{\epsilon} \right) \bigg|_{x=x_0}, \tag{4.12}
\]

where \(A_s = 2.137 \times 10^{-9}\) is the Planck normalization at \(k_0 = 0.05 \text{Mpc}^{-1}\). In our numerical calculations, we have taken \(M_S = M_S^0\) and have set the dimensionless couplings equal, \(\kappa = \sigma_\chi\), such that \(\zeta = 1\). Figure 4, shows our results without soft SUSY mass terms where various parameters are plotted against the scalar spectral index \(n_s\). It can be seen that without the soft mass terms, the scalar spectral index \(n_s\) cannot be achieved within Planck 2-\(\sigma\) bounds. With the inclusion of soft mass terms, the scalar spectral index \(n_s\) can easily be obtained within Planck’s 2-\(\sigma\) bounds.

The soft mass terms, seem to play an important role in inflationary predictions. Figure 5 shows our numerical results with soft mass terms where the behavior of SU(5) gauge symmetry breaking scale \(M_\alpha\) (upper left panel), \(\kappa\) (lower left panel) and \(S_0/m_P\) (lower right panel) is depicted as a function of soft mass parameter \(|M_S|\) for different values of the gravitino mass \(m_{3/2}\). The behavior of SU(5) gauge symmetry breaking scale \(M_\alpha\) with respect to the tensor to scalar ratio \(r\) is shown in the upper right panel. In obtaining these results, we have fixed the scalar spectral index \(n_s\) at the central value (0.9655) of Planck’s latest bounds. The soft mass squared parameter \(M_S^2\) and the combination \(am_{3/2}\) can be either positive or negative. We consider the following possible cases in our numerical calculations,

\[
\begin{align*}
am_{3/2} & > 0 & M_S^2 & > 0, \\
am_{3/2} & < 0 & M_S^2 & < 0, \\
am_{3/2} & < 0 & M_S^2 & > 0, \\
am_{3/2} & > 0 & M_S^2 & < 0. \tag{4.13}
\end{align*}
\]

The first case with \(M_S^2 > 0\) and \(am_{3/2} > 0\) \((a = +1)\) is inconsistent with Planck’s results. A red tilted scalar spectral index \((n_s < 1)\) is compatible with Planck’s latest bounds is obtained for the rest of the cases. The yellow curves are drawn for \(am_{3/2} < 0\) \((a = -1)\) where the solid lines correspond to the case when \(M_S^2 < 0\) and dashed lines correspond to \(M_S^2 > 0\). For lower values

\(^1\)For a recent discussion on the gravitino overproduction problem in hybrid inflation see ref. [53].
of $|M_S| \sim (1 - 10^4)$ TeV, the radiative corrections provide dominant contribution, whereas both SUGRA corrections and soft mass squared terms are suppressed. The suppression of supergravity corrections in this region is supported by small values of $S_0/m_P \sim 10^{-3}$, as shown in lower right panel of figure 5. For $|M_S| \gtrsim 10^4$ TeV, the soft mass squared term begins to take over, which drives the curve upward for $M_S^2 < 0$, and downward for $M_S^2 > 0$. For $M_S^2 < 0$ and $|M_S| \gtrsim 10^4$ TeV, $\kappa$ takes on large values, $M_\alpha$ approaches $\sim 1.5 \times 10^{16}$ GeV and supergravity corrections become important.

It is useful to analytically examine some approximate equations to understand the behavior depicted in figure 5. In the slow-roll approximation, the amplitude of the power spectrum of scalar curvature perturbation $A_s$ and the scalar spectral index $n_s$ is given by,

$$A_s(k_0) \simeq \frac{\kappa^2}{6 \pi^2} \left( \frac{M_\alpha}{m_p} \right)^6 \left( 2x_0^3 \left( \frac{M_\alpha}{m_p} \right)^4 - \frac{2M_S^2x_0}{\kappa^2M_\alpha^2} + \frac{am_{3/2}}{\kappa M_\alpha} + \frac{278\kappa^2}{16 \pi^2} F''(5x_0) \right)^{-2}, \quad (4.14)$$

$$n_s \simeq 1 + \left( \frac{m_p}{M_\alpha} \right)^2 \left( 6x_0^2 \left( \frac{M_\alpha}{m_p} \right)^4 - \frac{2M_S^2}{\kappa^2M_\alpha^2} + \frac{278\kappa^2}{16 \pi^2} F''(5x_0) \right). \quad (4.15)$$

Taking the contributions of the soft linear mass term comparable to mass squared term, we
Figure 5. The scalar spectral index $n_s$ vs. the SU(5) symmetry breaking scale $M_\alpha$, the tensor-to-scalar ratio $\kappa$ and $S_0/m_P$ for minimal Kähler potential without the soft mass terms.

obtain the following analytical expressions for $M_\alpha$, $\kappa$ and $|M_S|$:}

\begin{align}
\kappa &\simeq \left( \frac{8\pi^3}{139} \right)^3 \frac{(1 - n_s)}{\left| F'(5x_0)^2 F''(5x_0) \right|} \left( \frac{m_{3/2}}{m_P} \right)^{1/4}, \\
M_\alpha &\simeq \left( \frac{139 |F''(5x_0)|^3}{8\pi^2 (1 - n_s)^3 |F'(5x_0)|^2} \right)^{1/8} \left( m_{3/2} m_P^3 \right)^{1/4}, \\
|M_S| &\simeq \kappa^2 M_\alpha \sqrt{\frac{139}{32\pi^2}} F'(5x_0).
\end{align}

It can be checked that, for $m_{3/2} \simeq 10$ TeV, $n_s = 0.9655$, $x_0 \sim 1$, we obtain $\kappa \simeq 1.3 \times 10^{-4}$, $M_\alpha \simeq 2 \times 10^{15}$ GeV and $|M_S| \simeq 2 \times 10^4$ TeV. Also, for $m_{3/2} \simeq 1000$ TeV, $n_s = 0.9655$, $x_0 \sim 1$, we obtain $\kappa \simeq 4.5 \times 10^{-4}$, $M_\alpha \simeq 5.5 \times 10^{15}$ GeV and $|M_S| \simeq 7 \times 10^5$ TeV. These estimates are in excellent agreement with the numerical results shown in figures 5. Therefore, for $m_{3/2} \lesssim 1000$ TeV and $M_S \lesssim 10^4$ TeV, only radiative corrections and linear soft mass term dominate, whereas the SUGRA corrections and soft mass-squared term are suppressed. It should be noted that larger values of $m_{3/2}$ shift the contribution of the soft mass-squared term towards larger values of $M_S$. For example, with $m_{3/2} \simeq 10$ TeV, the soft mass-squared term begins to take over for $M_S \gtrsim 10^4$ TeV, whereas with $m_{3/2} \simeq 1000$ TeV, the soft mass-squared term becomes important for $M_S \gtrsim 5 \times 10^5$ TeV. Furthermore, for larger values of $m_{3/2}$ ($\gtrsim 1000$ TeV), the SUGRA corrections become important and large values of $M_\alpha$ can
be obtained, independent of $M_S$. The curves exhibit similar behavior for $|M_S| \gtrsim 10^6$ TeV for all three cases but decouple for value of $|M_S|$ below $10^6$ TeV.

The fourth case with $am_{3/2} > 0$ ($a = +1$) and $M_2^S < 0$, generates large $M_a$ that easily approaches $M_{\text{GUT}}$. For $|M_S| \lesssim 10^6$ TeV, $M_a$ takes on large values, whereas the radiative corrections become suppressed owing to small values of $\kappa$. This is in contrast to the other two cases where the contribution of soft mass-squared term becomes negligible below $|M_S| \lesssim 10^6$ TeV. With radiative corrections suppressed, eqs. (4.14) and (4.15) simplify to the following form,

$$A_s(k_0) \simeq \frac{\kappa^2}{6 \pi^2} \left( \frac{M_a}{m_p} \right)^6 \left( 2x_0^3 \left( \frac{M_a}{m_p} \right)^4 - \frac{2M_x^2 x_0^4}{\kappa^2 M_a^2} + \frac{am_{3/2}}{\kappa M_a} \right)^{-2},$$  \hspace{1cm} (4.19)

$$n_s \simeq 1 + \left( \frac{m_p}{M_a} \right)^2 \left( 6x_0^2 \left( \frac{M_a}{m_p} \right)^4 - \frac{2M_x^2}{\kappa^2 M_a^2} \right).$$  \hspace{1cm} (4.20)

Taking the soft mass-squared term to be comparable to linear soft SUSY-breaking term, we obtain the following analytical expressions for $\kappa$ and $M$ in terms of $m_{3/2}$ and $M_S$:

$$\kappa \simeq 2 \left( 2 \left( 1 - n_s \right) \right)^{1/2} \frac{|M_S|^3}{m_{3/2} m_p},$$  \hspace{1cm} (4.21)

$$M \simeq \left( \frac{1}{2(1-n_s)} \right)^{1/2} \left( \frac{m_{3/2} m_p}{|M_S|} \right).$$  \hspace{1cm} (4.22)

Using $n_s \simeq 0.9655$, $m_{3/2} \simeq 10$ TeV and $|M_S| \simeq 4.6 \times 10^3$ TeV, we obtain $\kappa \simeq 2 \times 10^{-7}$ and $M_a \sim 2 \times 10^{15}$ GeV. Similarly for $n_s \simeq 0.9655$, $m_{3/2} \simeq 1000$ TeV and $|M_S| \simeq 4.5 \times 10^5$ TeV, we obtain $\kappa \sim 2 \times 10^{-5}$ and $M_a \sim 2 \times 10^{16}$ GeV. These estimates are in good agreement with our numerical results displayed in figure 5.

The behavior of $M_a$ with respect to the tensor to scalar ratio $r$ is shown in the upper right panel of figure 5 and can be understood from the following approximate relation between $r$, $M_a$ and $\kappa$, obtained by using the Planck’s normalization constraint on $A_s$,

$$r \simeq \left( \frac{2\kappa^2}{3\pi^2 A_s(k_0)} \right) \left( \frac{M_a}{m_p} \right)^4.$$  \hspace{1cm} (4.23)

This shows that $r$ is proportional to both $M_a$ and $\kappa$ and large values of $r$ are obtained for large $M_a$ and $\kappa$. It can readily be checked that for $M_a \simeq 2.4 \times 10^{15}$ GeV and $\kappa \simeq 1.3 \times 10^{-4}$, the above equation gives $r \simeq 4.7 \times 10^{-7}$. Similarly, for $M_a \simeq 1.3 \times 10^{16}$ GeV and $\kappa \simeq 0.01$, we obtain $r \simeq 2.5 \times 10^{-6}$. These approximate values are very close to the actual values obtained in our numerical calculations. The above equation therefore gives a valid approximation of our numerical results. The tensor to scalar ratio $r$ varies in the range $4.7 \times 10^{-13} \lesssim r \lesssim 2.5 \times 10^{-6}$ and is beyond the current measuring limits of various upcoming experiments.

## 5 Non-minimal Kähler potential

In this section we employ a non-minimal Kähler potential including non-renormalizable terms up to sixth order;

$$K = |S|^2 + \text{Tr} |\Phi|^2 + |\tilde{h}|^2 + |\tilde{\chi}|^2 + |\chi|^2$$

$$+ \kappa_{\Phi} \frac{|S|^2 \text{Tr} |\Phi|^2}{m_p^2} + \kappa_{S\tilde{h}} \frac{|S|^2 |\tilde{h}|^2}{m_p^2} + \kappa_{SH} \frac{|S|^2 |H|^2}{m_p^2} + \kappa_{S\chi} \frac{|S|^2 |\chi|^2}{m_p^2} + \kappa_{S\tilde{\chi}} \frac{|S|^2 |\tilde{\chi}|^2}{m_p^2}.$$
Including the one loop radiative corrections and soft SUSY mass terms, the full scalar potential during inflation then reads as,

\[
V \simeq V_{\text{SUGRA}} + V_{1\text{-loop}} + V_{\text{Soft}} \\
\simeq \kappa^2 M_\alpha^4 \left[ 1 + \left( \frac{4(1 - \kappa S \phi)}{9(4/27 - \alpha^2)} - \kappa S x^2 \right) \left( \frac{M_\alpha}{m_p} \right)^2 \\
+ \left( \frac{4(1 - 2\kappa S \phi)^2 + 1 + \kappa \phi}{81(4/27 - \alpha^2)^2} \\
+ \frac{4((1 - 2\kappa S \phi)^2 - \kappa S(1 - 2\kappa S \phi))x^2 + \gamma S x^4}{9(4/27 - \alpha^2)} \right) \left( \frac{M_\alpha}{m_p} \right)^4 \\
+ \frac{\kappa^2}{16\pi^2} \left[ F(M_\alpha^2, x^2) + 11 \times 25 F(5M_\alpha^2, 5x^2) \right] + \frac{\sigma \chi}{8\pi^2} F(M_\alpha^2, y^2) \\
+ \frac{a m_{3/2} x}{\kappa M_\alpha} + \frac{M_\chi^2 x^2}{\kappa^2 M_\alpha^2} + \frac{8M_\phi^2}{9 \kappa^2 M_\alpha^2 (4/27 - \alpha^2)} \right],
\]

(5.2)

where \( \gamma_S = 1 - \frac{7\kappa S}{\kappa S} + 2\kappa S^3 - 3\kappa \Sigma S \). The results of our numerical calculations with a non-minimal Kähler potential are displayed in figures 6–8. In obtaining these results, we have used up to second order approximation on the slow-roll parameters and the SU(5) gauge symmetry breaking scale \( M_\alpha \) is fixed at \( M_{\text{GUT}} \simeq 2 \times 10^{16} \text{GeV} \). We have also fixed the soft SUSY masses at \( m_{3/2} \simeq M_S \simeq 10 \text{TeV} \), with \( a = 1 \) and \( M_S^2 > 0 \).

As compared to the minimal case, the non-minimal Kähler potential increases the parametric space and with the addition of new parameters, we now expect to obtain \( n_s \) within the latest Planck bounds with large values of tensor-to-scalar ratio \( r \). The radiative corrections and SUGRA corrections parameterized by \( \kappa_S \) and \( \kappa_{\Sigma S} \), dominate the global SUSY potential while the soft mass terms with \( m_{3/2} \simeq M_S \simeq 10 \text{TeV} \) are adequately suppressed. To keep the SUGRA expansion under control we impose \( S_0 \leq m_p \). We also restrict the non-minimal couplings \( |\kappa_S| \leq 1 \) and \( |\kappa_{\Sigma S}| \leq 1 \). These two constraints are shown in figures 6–8 by the red \( (S_0 = m_p) \) and blue \( (\kappa_{\Sigma S} = 1) \) curves. The lighter (darker) yellow region represents the Planck 2-\( \sigma \) (1-\( \sigma \)) bounds on scalar spectral index \( n_s \). By employing non-minimal Kähler potential, there is a significant increase in the tensor-to-scalar ratio \( r \) and both \( \kappa_S \) and \( \gamma_S \) play vital role to bring the scalar spectral index \( n_s \) within Planck 2-\( \sigma \) data bounds, with a large value of tensor to scalar ratio \( r \simeq 10^{-3} \).
Figure 6. Behavior of $\kappa$ (right) and tensor-to-scalar ratio $r$ (left) with respect to scalar spectral index $n_s$ for SU(5) breaking scale $M_\alpha \simeq M_{\text{GUT}} = 2 \times 10^{16} \text{GeV}$. The lighter (darker) shaded region represents the Planck 2-$\sigma$ (1-$\sigma$) bounds, whereas the red and blue curves correspond to the $S_0 = m_P$ and $\kappa_{SS} = 1$ constraints, respectively.

Figure 7. Behavior of tensor-to-scalar ratio $r$ with respect to the non-minimal coupling $\kappa_S$ (left) and quartic coupling $\gamma_S$ (right) for SU(5) breaking scale $M_\alpha \simeq M_{\text{GUT}} = 2 \times 10^{16} \text{GeV}$. The lighter (darker) shaded region represents the Planck 2-$\sigma$ (1-$\sigma$) bounds, whereas the red and blue curves correspond to the $S_0 = m_P$ and $\kappa_{SS} = 1$ constraints, respectively.

Figure 8. Behavior of non-minimal coupling $\kappa_{SS}$ (left) and quartic coupling $\gamma_S$ (right) with respect to the non-minimal coupling $\kappa_S$ for SU(5) breaking scale $M_\alpha \simeq M_{\text{GUT}} = 2 \times 10^{16} \text{GeV}$. The lighter (darker) shaded region represents the Planck 2-$\sigma$ (1-$\sigma$) bounds, whereas the red and blue curves correspond to the $S_0 = m_P$ and $\kappa_{SS} = 1$ constraints, respectively.
The behavior of tensor-to-scalar ratio $r$ and $\kappa$, as displayed in figure 6, can be understood from the explicit relation (4.23) between $r$, $\kappa$ and $M_\alpha$ which shows that larger values of $r$ are expected when $\kappa$ or $M_\alpha$ is large. Since $M_\alpha$ is fixed, larger $r$ values should be obtained for large $\kappa$. For fixed $M_\alpha \approx 2 \times 10^{10}$ GeV, the largest value of $r$ ($\sim 1.5 \times 10^{-3}$) obtained in our numerical results occurs for $\kappa \approx 0.1$. The behavior of tensor to scalar ratio $r$ with respect to $\kappa_S$ and $\gamma_S$ is presented in figure 7, while figure 8 depicts the behavior of $\kappa_{SS}$ and $\gamma_S$ with respect to $\kappa_S$. It can be seen that the large $r$ values are obtained with non-minimal couplings $\kappa_S < 0$, $\kappa_{SS} > 0$ and the quartic coupling $\gamma_S < 0$. Moreover, in the large $r$ limit, both $\kappa_S$ and $\kappa_{SS}$ are tuned to make $\gamma_S$ very small ($\sim -0.003$). Note that large tensor modes can be obtained for any value of scalar spectral index $n_s$ within Planck $2\sigma$ bounds. Finally, smaller $r$ values ($\sim 10^{-6}$) are obtained for $S_0 \lesssim 0.05 m_P$ and $\kappa_{SS} \approx 1$ for which $\gamma_S$ is negative and fairly large ($\sim -2$).

The spectral index $n_s$ and tensor to scalar ratio $r$ in the leading order slow-roll approximation are given by

$$n_s \approx 1 - 2\kappa_S + \left(6 \gamma_S x_0^2 + \frac{8 (1 - \kappa_S)}{9 (4/27 - \alpha^2)} \right) \frac{M_\alpha}{m_P} + \frac{278 \kappa^2 F''(5x_0)}{16\pi^2} \frac{m_P}{M_\alpha}^2,$$

$$r \approx 4 \left(\frac{m_P}{M_\alpha}\right)^2 \left(-2\kappa_S x_0 \left(\frac{M_\alpha}{m_P}\right)^2 + \left(2\gamma_S x_0^3 + \frac{8 (1 - \kappa_S)}{9 (4/27 - \alpha^2)} \right) \left(\frac{M_\alpha}{m_P}\right)^4 + \frac{278 \kappa^2 F''(5x_0)}{16\pi^2}\right)^2.$$

Solving these two equations simultaneously for $S_0 \approx m_P$, $r \approx 10^{-3}$, and $n_s \approx 0.9655$ we obtain $\kappa_S \approx -0.006$ and $\gamma_S \approx -0.005$. Similarly in the small $r$ region for $S_0 \approx (0.05)m_P$, $r \approx 3 \times 10^{-6}$, and $n_s \approx 0.9655$ we obtain $\kappa_S \approx -0.005$ and $\gamma_S \approx -2$. These approximate values are very close to the actual values obtained in the numerical calculations. The above analytical equations therefore gives a valid approximation of our numerical results displayed in figures 6–8. For non-minimal couplings ($-0.011 \lesssim \kappa_S \lesssim -0.00063$) and $(0.34 \lesssim \kappa_{SS} \lesssim 1)$, we obtain the scalar spectral index $n_s$ within the Planck 2-$\sigma$ bounds and tensor to scalar ratio $r$ in the range $(9.7 \times 10^{-7} \lesssim r \lesssim 1.5 \times 10^{-3})$.

6 Radiative breaking of $U(1)_\chi$ symmetry

After the end of inflation, the effective unbroken gauge symmetry is $\text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y \times \text{U}(1)_\chi$. The charge assignments of the fields under this symmetry are displayed in table 4. The superpotential terms relevant for $U(1)_\chi$ symmetry breaking are given by

$$W = W_{\text{MSSM}} + W_\chi$$

$$W_\chi = \sigma_\chi S \bar{\chi} + \lambda_{ij} \nu_i^c \nu_j^c.$$  

(6.1)

From the above equation, we obtain

$$F_{\bar{\chi}} = \frac{\partial W}{\partial S} = \sigma_\chi \bar{\chi},$$

$$F_\chi = \frac{\partial W}{\partial \chi} = \sigma_\chi S \chi = 0,$$

$$F_{\bar{\chi}} = \frac{\partial W}{\partial \bar{\chi}} = \sigma_\chi S \bar{\chi} + \lambda_{ij} \nu_i^c \nu_j^c = 0,$$

$$F_{\nu^c} = \frac{\partial W}{\partial \nu^c} = 2 \lambda_{ij} \nu_i^c \nu_j^c = 0.$$  

(6.2)
Superfields | Representations under SU(3)_{C} \times SU(2)_{L} \times U(1)_{Y} \times U(1)_{\chi}
---|---
Matter sector | |
\( Q \) & (3, 2, 1/6, −1) \\
\( u^{c} \) & (3, 1, −2/3, −1) \\
\( d^{c} \) & (3, 1, 1/3, 3) \\
\( \ell \) & (1, 2, −1/2, 3) \\
\( e^{c} \) & (1, 1, 1, −1) \\
\( \nu^{c} \) & (1, 1, 0, −5) \\
Scalar sector | |
\( h_{u} \) & (1, 2, 1/2, 2) \\
\( h_{d} \) & (1, 2, −1/2, −2) \\
\( \chi \) & (1, 1, 0, 10) \\
\( \bar{\chi} \) & (1, 1, 0, −10) \\
\( S \) & (1, 1, 0, 0)

Table 4. Superfields and their representations under the effective unbroken gauge symmetry SU(3)_{C} × SU(2)_{L} × U(1)_{Y} × U(1)_{\chi} after the end of inflation.

This leads to the following vacua;

\[
\langle \chi \rangle = \langle \bar{\chi} \rangle = 0, \quad \langle \nu_{i}^{c} \rangle = 0, \quad \langle S \rangle = \text{Arbitrary.} \tag{6.3}
\]

In order to break the U(1)_{\chi} symmetry, a non-zero VEV of the field \( \chi \) is desired; \( \langle \chi \rangle = \langle \bar{\chi} \rangle \neq 0 \). Including the soft SUSY breaking mass terms,

\[
V_{\text{Soft}} = m_{S}^{2}|S|^{2} + m_{\chi}^{2}|\chi|^{2} + m_{\bar{\chi}}^{2}|\bar{\chi}|^{2} + m_{\nu_{i}^{c}}^{2}|\nu_{i}^{c}|^{2} + A_{\nu} \lambda_{ij} \nu_{i}^{c} \nu_{j}^{c} + A_{\bar{\nu}} y_{ij} \nu_{i}^{c} \nu_{j}^{c} + A_{\chi} \sigma_{\chi} S \chi \bar{\chi} + \frac{1}{2} M_{\chi} Z_{\chi} Z_{\chi}, \tag{6.4}
\]

where \( A_{\nu} \) and \( A_{\chi} \) are coefficients of linear soft mass terms, \( Z_{\chi} \) is the U(1)_{\chi} gaugino and \( M_{\chi} \) is the gaugino mass. The full scalar potential is then given by,

\[
V = V_{F} + V_{D} + V_{\text{Soft}} = \sigma_{\chi}^{2} |\chi|^{2} + \lambda_{ij} \nu_{i}^{c} \nu_{j}^{c} |^{2} + |\sigma_{\chi} S \bar{\chi} + |\sigma_{\chi} S \chi|^{2} + |2 \lambda_{ij} \nu_{i}^{c} |^{2} + 50 \frac{g_{\chi}^{2}}{2} \left(|\chi|^{2} - |\bar{\chi}|^{2}\right)^{2} + m_{S}^{2}|S|^{2} + m_{\chi}^{2}|\chi|^{2} + m_{\bar{\chi}}^{2}|\bar{\chi}|^{2} + m_{\nu_{i}^{c}}^{2}|\nu_{i}^{c}|^{2} + A_{\nu} \lambda_{ij} \nu_{i}^{c} \nu_{j}^{c} + A_{\chi} \sigma_{\chi} S \chi \bar{\chi} + \frac{1}{2} M_{\chi} Z_{\chi} Z_{\chi}. \tag{6.5}
\]


The potential minima can be obtained as follows:

\[
\frac{\partial V}{\partial S^\dagger} = \sigma^2_S S \left( |\chi|^2 + |\bar{\chi}|^2 \right) + \sigma_\chi \lambda_{ij} \nu_i \nu_j^c \bar{\chi}^\dagger + m_S^2 S = 0,
\]

\[
\frac{\partial V}{\partial \chi} = \sigma^2_\chi \left( \chi |\chi|^2 + \bar{\chi} |S|^2 \right) - 100 g_{\chi S}^2 \chi \left( |\chi|^2 - |\bar{\chi}|^2 \right) + \sigma_\chi \lambda_{ij} \nu_i \nu_j^c S^\dagger + m_\chi^2 \bar{\chi} = 0,
\]

\[
\frac{\partial V}{\partial \nu_i^c} = \sigma^2_i \left( \chi |\chi|^2 + \bar{\chi} |\nu|^2 \right) + 100 g_{\chi \nu_i}^2 \chi \left( |\chi|^2 - |\bar{\chi}|^2 \right) + 4 \lambda_{ij} |\nu_i^c|^2 \chi + m_{\nu_i^c}^2 \nu_i^c = 0. \tag{6.6}
\]

Conservation of R-parity requires, \( \langle \nu_i^c \rangle = 0 \). The VEV of the fields \( \chi, \bar{\chi} \) is found to be,

\[
\langle |\bar{\chi}| \rangle = \langle |\chi| \rangle = \sqrt{\frac{m_S^2}{2 \sigma_\chi}}. \tag{6.7}
\]

The negative mass squared, \( m_S^2 < 0 \) should be satisfied at an intermediate scale \( M_i \) below the GUT scale to realize the correct \( U(1)_\chi \) symmetry breaking. A negative mass squared can be achieved through the RG running from the GUT scale to an intermediate scale with a large enough Yukawa coupling even if the mass squared is positive at the GUT scale. We consider the \( U(1)_\chi \) renormalization group equations and analyze the running of the scalar masses \( m_\chi^2, m_{\nu_i^c}^2, m_{S_i}^2 \) and \( m_S^2 \). A negative mass-squared \( m_S^2 \) will trigger the radiative breaking of \( U(1)_\chi \) symmetry. We show that the mass-squared of the fields \( \chi, \bar{\chi}, \nu_i^c \) and \( S \) evolve in such a way that \( m_S^2 \) becomes negative whereas \( m_{\nu_i^c}^2, m_\chi^2 \) and \( m_\bar{\chi}^2 \) remain positive. The renormalization group equations are given by

\[
16\pi^2 \frac{dg_\chi}{dt} = \frac{57}{5} g_\chi^3,
\]

\[
16\pi^2 \frac{dM_\chi}{dt} = \frac{114}{5} g_\chi^2 M_\chi,
\]

\[
16\pi^2 \frac{d\lambda_i}{dt} = \lambda_i \left( 8 \lambda_i^2 + 2 \text{Tr} \lambda^2 + \sigma_\chi^2 - \frac{15}{2} g_\chi^2 \right),
\]

\[
16\pi^2 \frac{d\sigma_\chi}{dt} = \sigma_\chi \left( 3 \sigma_\chi^2 + 2 \text{Tr} \lambda^2 - 10 g_\chi^2 \right),
\]

\[
16\pi^2 \frac{dm_\chi^2}{dt} = 2 \sigma_\chi \left( m_\chi^2 + m_\bar{\chi}^2 + m_S^2 \right) + 4 m_\chi^2 \text{Tr} \lambda^2 + 8 \text{Tr} \left( m_{\nu_i^c}^2 \lambda^2 \right) + 4 T_{\sigma_\chi}^2 - 20 g_\chi^2 M_\chi^2,
\]

\[
16\pi^2 \frac{dm_\bar{\chi}^2}{dt} = 2 \sigma_\bar{\chi} \left( m_\bar{\chi}^2 + m_\chi^2 + m_S^2 \right) + 2 T_{\sigma_\bar{\chi}}^2 - 20 g_{\bar{\chi}}^2 M_\chi^2,
\]

\[
16\pi^2 \frac{dm_{\nu_i^c}^2}{dt} = 8 \lambda_i^2 \left( m_{\nu_i^c}^2 + 2 m_\chi^2 \right) + 8 T_{\nu_i^c}^2 - 5 g_{\nu_i^c}^2 M_\chi^2,
\]

\[
16\pi^2 \frac{dm_S^2}{dt} = 2 \sigma_S^2 \left( m_S^2 + m_\chi^2 + m_\bar{\chi}^2 \right) + 2 T_{\sigma_S}^2,
\]

\[
16\pi^2 \frac{dm_{\nu_i^c}}{dt} = 2 \sigma_{\nu_i}^2 \left( m_{\nu_i^c}^2 + 2 m_{\chi}^2 \right) + 2 T_{\sigma_{\nu_i}}^2,
\]

\[
16\pi^2 \frac{dm_{\tau_i}}{dt} = 2 \sigma_{\tau_i}^2 \left( m_{\tau_i}^2 + 2 m_{\chi}^2 \right) + 2 T_{\sigma_{\tau_i}}^2.
\]
Figure 9. The evolution of scalar squared masses $m^2_{\chi}$, $m^2_{\nu_i}$, $m^2_{\tilde{\chi}}$ and $m^2_S$ from the GUT to TeV scale.

\[ 16\pi^2 \frac{dT_{\chi}}{dt} = T_{\chi} \left( 9g^2_{\chi} + 2 \text{Tr} \lambda^2 \right) , \quad (6.16) \]

\[ 16\pi^2 \frac{dT_{\nu_i}}{dt} = T_{\nu_i} \left( \sigma^2_{\chi} + 2 \text{Tr} \lambda^2 + 24\lambda_i^2 - \frac{15}{2} g^2_{\chi} \right) \]

\[ + 2\lambda_i \left( 2 \text{Tr} \lambda^2 + 15g^2_{\chi} M_{\chi} + T_{\sigma_{\chi}} \sigma_{\chi} + 2 \text{Tr} (\lambda_i T_{\nu_i}) \right) , \quad (6.17) \]

where $\lambda_{ij} = \text{diag}(\lambda_1, \lambda_2, \lambda_3)$. The evolution of these parameters depends on the boundary conditions at GUT scale, $M_{\text{GUT}} = 2 \times 10^{16}$ GeV. We assume universal soft SUSY breaking at this scale,

\[ m^2_{\chi} = m^2_{\tilde{\chi}} = m^2_S = m^2_0, \quad M_{\chi} = 16 \text{ TeV}, \]

\[ g^2_{\chi} = 1.04, \quad \lambda_3 = 0.4, \quad \sigma_{\chi} = 0.3, \quad m_0 = 3.2 \text{ TeV}, \quad (6.18) \]

Here we let the tri-linear couplings ($A_\nu = A_\chi = 0$) vanish as they have negligible effect on the overall running of the parameters. Also, for simplicity, we neglect the couplings of the first two right handed neutrino (RHNs) generations ($\lambda_1 = \lambda_2 = 0$). Figure 9 shows the running of scalar masses from GUT scale. It can be seen that the mass-squared $m^2_S$ turns negative at scale $\sim 8 \times 10^{12}$ GeV, whereas $m^2_{\tilde{\chi}}$ rapidly increase and approaches $\sim 10^9$ GeV² around TeV scale. Note that, although the running of mass squared $m^2_{\nu_i}$ and $m^2_{\chi}$ decreases from $M_{\text{GUT}}$, they always remain positive.

The breaking of $U(1)_\chi$ at the end of inflation yields topologically stable cosmic strings on which, the observational bounds are given in terms of the dimensionless quantity $G_N\mu_s$, which characterizes the strength of the gravitational interaction of the strings. Where $G_N$ is the Newton’s constant, and $\mu_s \simeq 2\pi \langle \chi \rangle^2$ denotes the mass per unit length of the string. The Planck bound on $G_N\mu_s$ derived from constraints on the string contribution to the CMB power spectrum is given by [44, 45]

\[ G_N\mu_s \lesssim 2.4 \times 10^{-7}. \quad (6.19) \]

This then translates to the following upper bound on $U(1)_\chi$ breaking scale

\[ \langle \chi \rangle \lesssim 2.35 \times 10^{15} \text{ GeV}, \quad (6.20) \]
which is easily satisfied as can be seen from figure 9 and depends on the initial boundary conditions at GUT scale. After the U(1)$_\chi$ symmetry breaking, the RHNs and the U(1)$_\chi$ gauge boson ($Z'$) acquire the following masses:

$$M_{Z'}^2 \simeq g_\chi^2 \langle \chi \rangle^2, \quad m_{\nu_3}^2 \simeq \lambda_3^2 \langle \chi \rangle^2,$$

which for the particular boundary condition in (6.18) yields, $M_{Z'} \simeq 4.89$ TeV and $m_{\nu_3} \simeq 2.44$ TeV. The bound on $M_{Z'}$ is constantly being updated by comprehensive analyses. The severest bound on $M_{Z'}$ comes from negative results of LEP data, $M_{Z'}/g_\chi \gtrsim 6$ TeV [83].

Even though considering its decay modes can lower the mass bound on $Z'$ [84–87], setting $M_{Z'} \gtrsim 4$ TeV [88] guarantees avoiding possible exclusions limit due to the light $Z'$ mass.

The mass of RHNs predicted by the above model is of the order of TeV scale. Since they are singlet under the SM gauge group, a mixing between the RHNs and the SM neutrinos is generated through the Dirac Yukawa coupling in the seesaw mechanism. As a result, the RHN mass eigenstates couple to the weak gauge bosons is generated through the Dirac Yukawa coupling in the seesaw mechanism. As a result, the RHN mass eigenstates couple to the weak gauge bosons through this mixing. Although in general, this mixing can be made sizable even for TeV-scale RHNs, contrary to the naive seesaw expectations, under special textures of the Dirac and RHN Majorana mass matrices [57–67], it has been shown [68] that this mixing has an upper bound of $O(0.01)$ to satisfy various experimental constraints, such as the neutrino oscillation data, the electroweak precision measurements, neutrinoless double beta decay and the charged lepton flavor violating (LFV) processes. Hence, the canonical production cross section of TeV-scale RHNs through either the weak gauge bosons [69–77] or the Higgs boson [78–82] at the LHC is expected to be very small within the minimal seesaw.

Leptogenesis can still occur with light RHNs masses. In particular, resonant leptogenesis [64, 89, 90] can occur with heavy Majorana neutrinos even as light as $\sim 1$ TeV. In this scenario the mass degeneracy of RHNs enhances the CP violating effects and their decay leads to the resonant production of lepton asymmetry. Adequate baryon asymmetry then can be generated with RHNs of masses $\sim 1$ TeV using their flavor oscillations [91]. In the above model, all SM fermions as well as the RHNs have non-zero U(1)$_\chi$ charges, and therefore, the RHNs can be efficiently produced at colliders, in particular, through the resonant production of $Z'$ boson, if kinematically allowed, and its subsequent decay into a pair of RHNs.

7 Summary

We have explored shifted hybrid inflation in the framework of supersymmetric SU(5) × U(1)$_\chi$ model where SU(5) gauge symmetry is spontaneously broken during inflation, inflating the disastrous magnetic monopoles away. The U(1)$_\chi$ symmetry is radiatively broken after the end of inflation at an intermediate scale, yielding topologically stable cosmic strings. The symmetry breaking scale of U(1)$_\chi$ depends on the initial boundary conditions at the GUT scale and easily satisfies Planck’s bound on $G_N\mu_s$. The $d = 5$ proton lifetime for the decay $p \rightarrow K^+\bar{\nu}$, mediated by color-triplet Higgsinos is found to satisfy Super-Kamiokande experimental bounds for SUSY breaking scale $M_{\text{SUSY}} \gtrsim 10$ TeV. We have shown that with minimal Kähler potential, the soft supersymmetry breaking terms play a vital role in bringing the scalar spectral index $n_s$ within the Planck’s latest bounds and the SU(5) gauge symmetry breaking scale is obtained in the range $(2 \times 10^{15} \lesssim M_\alpha \lesssim 2 \times 10^{16})$ GeV with small values of tensor-to-scalar ratio $r \lesssim 2.5 \times 10^{-6}$. In a non-minimal Kähler potential setup, large values of tensor to scalar ratio are obtained ($r \lesssim 10^{-3}$) with non-minimal couplings ($-0.011 \lesssim \kappa_S \lesssim -0.00063$) and $(0.34 \lesssim \kappa_{SS} \lesssim 1)$ and symmetry breaking scale $M_\alpha \simeq M_{\text{GUT}} = 2 \times 10^{16}$ GeV.
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