Discrete optical Zeno effect for polarization of light

K O Sedykh\textsuperscript{1,2} and D V Sych\textsuperscript{1,2,3,4}

\textsuperscript{1} Moscow State Pedagogical University, Moscow 119992, Russia
\textsuperscript{2} National University of Science and Technology MISiS, Moscow 119049, Russia
\textsuperscript{3} Sirius University of Science and Technology, 1 Olympic Ave, 354340, Sochi, Russia
\textsuperscript{4} P. N. Lebedev Physical Institute, Moscow 119991, Russia

kseniaolegovna98@gmail.com

Abstract. Quantum Zeno effect concerns deterministic dynamics of a quantum system induced by a series of projective quantum measurements. Applying this effect in optics, one can achieve an arbitrary lossless transformation of linear polarization of light with help of linear polarizers. However, to demonstrate this effect in practice, we have to take into account unavoidable losses in each polarizer that limits probability of successful transformations. In this work, we theoretically study a realistic quantum Zeno effect with an optimal discrete set of polarizers and find the maximum success probability.

1. Introduction
There is no motion according to Zeno’s arrow paradox [1, 2]. Since a flying arrow is at rest at every moment of time in a certain point in space, this means that it is at rest all the time. In other words, it is motionless. The quantum Zeno effect is the inhibition of the evolution of the state of a quantum system by method of repeated measurements [3, 4]. The measurement result in quantum mechanics is intrinsically probabilistic. However, repeated measurement of a physical quantity after measurement should give the same result. Thus, the evolution of a quantum system is inhibited. If there is a particle which tends to abandon its initial quantum state and we try to observe how it changes its quantum state, the probability of the change tends to zero. With quite frequent observation rate, we can keep the state of the quantum system, and the probability for this tends to unity.

The relevance of the research topic is due to the development of the direction of quantum control and quantum interaction-free measurements research [5–7], and finds applications in quantum communication [8–16]. Quantum Zeno effect can be applied in the counterfactual communication in which information is transferred by the phase part of wave function [17].

In our work, we investigate a discrete version of the quantum Zeno effect and want to make an arbitrary deterministic transformation of the quantum polarization state of light with help of only projective measurements. Consider a vertically polarized photon directed through a set of linear polarizers, slightly tilted with respect to each other. Each of polarizers tilts the polarization plane to a certain angle depending on the number of polarizers. In this case, a vertically polarized photon can be transformed into horizontally polarized one at the end of the set of polarizers. This polarization rotation is an example of the evolution of the quantum system that we want to guide by quantum measurements. In another interpretation of quantum Zeno effect, the dynamics of a quantum system...
can be inhibited by frequent observations [18-23]. In this work, we provide theoretical investigation of
the discrete optical Zeno effect on the example of polarization of light and a set of lossy polarizers.

2. Ideal and real cases of Zeno effect

2.1. Ideal Zeno effect

Consider an optical system consisting of a set of linearly polarized light source, a set of polarizers and
power meter. If a horizontal polarizer is added to the system, then only horizontal component of light
remains. Consider a case, when we put horizontally and vertically oriented polarizers one after
another. Apparently, there is no light passes through such system. However, if we add a diagonally
oriented polarizer between them, we can observed a part of light that passes through the system. The
more polarizers we add to the system, the greater part of light passes through it, if we pay attention
how to put these devices to the optical scheme. There is a special relation between the number of
added polarizers and the correct angle of their relative polarization. There is a rotation by \( \frac{\pi}{2N} \), so the
initially polarized light is orthogonally polarized at the end.

The probability of light passing through the set of polarizers depends on the number of additional
polarizers:

\[
P_N(\theta) = \left( \cos^2 \frac{\pi}{2N} \right)^N
\]

(1)

The more polarizers are added to the optical system, the more light we can obtain (A1). In the limit
of infinitely many polarizers, we obtain 100% light transmission. This is the ideal case of the quantum
optical Zeno effect.

\[
\lim_{N \to \infty} \left( \cos^2 \frac{\pi}{2N} \right)^N = 1
\]

(2)

2.2. Real Zeno effect

Consider a realistic optical system linearly polarized light source, a set of lossy polarizers and a power
meter. For the real case of quantum Zeno effect, we need to take into account the non-unit
transmittance of the polarizers due to various losses, which affects the result. For different
transmittance of polarizers, we need to add different number of polarizers to achieve maximum light
transmittance. The probability of light passing through a set of polarizers depending on the
transmittance coefficient \( k \) is

\[
P_N(\theta) = \left( k \cdot \cos^2 \frac{\pi}{2N} \right)^N
\]

(3)

For example, if the transmittance \( k = 0.9 \), then the maximum light passing through the system can be
obtained using four additional polarizers (B1). In the limit of infinitely many polarizers, we obtain no
light, in contrast to the ideal quantum Zeno effect:

\[
\lim_{N \to \infty} \left( k \cdot \cos^2 \frac{\pi}{2N} \right)^N = 0
\]

(4)

In the real case of the Zeno effect, the probability of light passing depends on the transmittance
coefficient of the polarizers. Due to losses on each device, instead of 100% light transmission, only a
small part (~35%) remains for the case of 10% loss on each polarizer.
3. Conclusion
In this work, we consider a realistic version of the quantum optical Zeno effect on the example of changes of polarization of light with a set of polarizers. In the ideal case, the more polarizers we put to the system, the more light we can obtain at the end. In the real case, we have to take into account unavoidable losses in each polarizer, and the maximum amount of light can be obtained with a finite number of polarizers. This effect can be helpful for quantum state preparation via projective measurements, e.g. for quantum communication.

Acknowledgments
The reported study was funded by RFBR, Sirius University of Science and Technology, JSC Russian Railways and Educational Fund “Talent and success”, project number 20-32-51004.

Appendices
A1. Probability of light passing through the set of polarizers in ideal case.

| # of added polarizers | Angle (°) | \( P(\theta) \) |
|-----------------------|-----------|-----------------|
| 1                     | 45        | 0.25            |
| 2                     | 30        | 0.422           |
| 3                     | 22.5      | 0.531           |
| 4                     | 18        | 0.605           |
| 5                     | 15        | 0.659           |
| 6                     | 12.9      | 0.699           |
| 7                     | 11.25     | 0.733           |
| 8                     | 10        | 0.759           |
| 9                     | 9         | 0.781           |
B1. Probability of light passing through the set of polarizers in real case with transmittance coefficient $k=0.9$.

| # of added polarizers | Angle (°) | $P(\theta)$ |
|------------------------|-----------|-------------|
| 1                      | 45        | 0.206       |
| 2                      | 30        | 0.308       |
| 3                      | 22.5      | 0.348       |
| 4                      | 18        | 0.356       |
| 5                      | 15        | 0.351       |
| 6                      | 12.9      | 0.334       |
| 7                      | 11.25     | 0.316       |
| 8                      | 10        | 0.294       |
| 9                      | 9         | 0.272       |

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