A Candidate for a Supergravity Anomaly

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Abstract

Using elementary BRS cohomology theory, this paper describes a supergravity anomaly analogous to, but very different from, the well known gauge and gravitational anomalies. The form of the anomaly is simply the ghost charge one expression $A_{\text{Supergravity}}^1 = \int d^4x \varepsilon^{\lambda\mu\nu\sigma} \left\{ c_1 C_\alpha \partial_\lambda \Psi_\mu^\alpha \partial_\sigma V_\nu + c_2 \partial_\lambda \Psi_\mu^\alpha \partial_\sigma \Psi_\nu^\omega \right\}$. Here $\Psi_\mu^\alpha$ and $V_\mu$ are the gravitino and a gauge boson, and $C_\alpha$ and $\omega$ are their Faddeev Popov ghost fields. There is one boundary generator here, namely the ghost charge zero expression $A_{\text{Counterterm}}^0 = \int d^4x \varepsilon^{\lambda\mu\nu\sigma} \left\{ \Psi_\lambda^\alpha \partial_\mu \Psi_\nu^\alpha V_\sigma \right\}$, but there are two cycles in $A_{\text{Supergravity}}^1$, so the anomaly is present if the coefficients $c_1, c_2$ are not in the ratio that can arise from the boundary generator. A model that is likely to generate this supergravity anomaly is described. The coefficient of this anomaly, in perturbation theory with unbroken supergravity, appears to be zero, because no relevant diagrams are linearly divergent. However, when, and only when, there is spontaneously broken supergravity, there are counterterms needed in the action, and these counterterms are just right to contribute to linearly divergent diagrams that can contribute to the anomaly. We conjecture that the coefficient of the anomaly, for a general gauge group, is of the form $\kappa^3 M_G \langle D^a \rangle$, where $M_G$ is the mass of the gravitino, $\kappa$ is the Planck length, and $\langle D^a \rangle$ is the VEV of the gauge auxiliary field. This paper also discusses gauge parameters for the gravitino and the massive vector boson. It is possible that this anomaly might provide significant constraints to characterize viable theories that are based on supergravity or superstring theory.

1. Supergravity and Superstring Theory: These theories appeared likely to explain how we could generate a Grand Unified Theory, including supergravity, and then explain why the Standard Model looks so special and peculiar [1,2,3,4,5,6]. But the initial promise of these theories has yielded disappointment, because there does not seem to be any way to understand why one set of particles and groups is better than another. At the same time supersymmetry [7,8,9,10,11] has also been a disappointment, because no plausible superpartners have been observed in the many experiments that have looked for them. Anomalies in higher dimensional gravity theories generated much of the interest in superstring theory [12,13]. Gauge anomalies play a very important role in the Standard Model [14].

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It could be argued that adding supergravity has not really helped in the search for a Grand Unified Theory. Adding supergravity is very complicated. Incorporating it might seem worthwhile, if only it explained something. But, up to now, it has been generally believed that there are no further anomalies that are special to supergravity. However, this paper claims that there are such anomalies.

2. The main result of this paper is in paragraph 9, and it is very simple to describe, as follows: There is at least one expression, with ghost charge one, in the BRS cohomology of supergravity, which is not simply the supersymmetric version of any known gauge or gravitational anomaly, and it is in equation (14) below. It is also true that the methods needed for the calculation of the coefficient of this anomaly are very different from those for any other known anomaly, as will be shown below, particularly in paragraphs 14 and 16.

3. General Remarks about the BRS Cohomology of Supergravity: The questions regarding supersymmetry and BRS symmetry \([15,16,17,18,19,20,21]\) are popular topics. The subject, in general, is immense and confusing. A particularly important issue is that there are various claims that the BRS cohomology of supergravity and related theories have been examined, and that no ghost charge one objects, special to supersymmetry, exist. It seems to have been generally accepted that supergravity does not have any anomaly that is not simply the supersymmetric version of a well known non-supersymmetric anomaly. This paper shows that this viewpoint is not correct. At the end of this paper, there are some tables which summarize some of the notation and results in this paper.

4. BRS and ZJ and the SU(3) and U(1) Anomalies: The power of the Becchi Rouet Stora (BRS) and Zinn-Justin (ZJ) Master Equation technique is that it summarizes all the symmetries of an action, and its transformations, in a compact and powerful way. Originally BRS used the mathematical fact that if one takes

\[
\delta = \omega^a T^a - \frac{1}{2} f^{abc} \omega^b \omega^c \frac{\partial}{\partial \omega^a}
\] (1)
where $T^a$ are the antihermitian generators of the SU(3) group on some field, satisfying

$$[T^a, T^b] = f^{abc} T^c$$

then this generates cohomology because $\delta^2 = 0$. They noticed that the anomaly of the SU(3) Yang Mills theory in four dimensions arises from the totally antisymmetric tensor with five ghosts (here $d^{abc}$ is the symmetric tensor with three indices in the real adjoint representation of SU(3)):

$$d^{abc} f^{bde} f^{cpq} \omega^a \omega^d \omega^e \omega^p \omega^q$$

$$\rightarrow A_{\text{SU}(3) \text{ Anomaly}}^1 = \int d^4x d^{abc} \{ \varepsilon^{\mu\nu\lambda\sigma} \partial_\mu V_\nu^a \partial_\lambda V_\sigma^b \omega^c + \cdots \}$$

The $+ \cdots$ in the above are higher order terms in the coupling constant and we can ignore them when computing the anomaly. The notation $A_{\text{SU}(3) \text{ Anomaly}}^1$ is to remind us that this expression looks like a piece of the action, except that it has ghost charge one.

5. **Similarly the usual U(1) anomaly arises from the expression:**

$$A_{\text{U}(1) \text{ Anomaly}}^1 = \int d^4x \{ \varepsilon^{\mu\nu\lambda\sigma} \partial_\mu V_\nu \partial_\lambda V_\sigma \omega \}$$

The Grassmann odd ghost fields $\omega^a, \omega$ are the Faddeev Popov (FP) Ghosts. Early on, ZJ introduced a way of incorporating the nilpotent $\delta$ of BRS into the path integral technique. Later on, Batalin and Vilkovisky (BV) named these results the ‘Master Equation’ [22,23].

6. **Gauge Transformations:** These anomalies arise from the following nilpotent gauge transformations:

$$\delta V_\mu^a = \partial_\mu \omega^a + \cdots$$

$$\delta V_\mu = \partial_\mu \omega + \cdots$$

These simple terms are nilpotent because they are Grassmann odd. When the higher order terms are included, it gets more complicated.

7. **Why is the expression an anomaly?** Here is the reason. The expressions like (5) can be obtained, in perturbation theory, by taking the
BRS variation of some Feynman diagram one-particle-irreducible vertex, as follows, for example (see (19) below for an example of the calculation):

\[ A^1_{U(1) \text{ Anomaly}} = \delta G^0_{\text{Feynman}} \] (8)

This vertex \( G^0_{\text{Feynman}} \) is a complicated non-local expression and the calculation is an interesting and subtle process, which necessarily uses the linear divergence of the amplitude \( G^0_{\text{Feynman}} \). See [15] for a simple discussion of this process. Clearly one can try to remove the above expression (5) as follows:

\[ A^1_{U(1)} = \delta A^0_{U(1) \text{ Counterterm}} \] (9)

where one chooses a local counterterm in the action, of the form:

\[ A^0_{U(1) \text{ Counterterm}} = \frac{1}{2} \int d^4x \left\{ \varepsilon^{\mu\nu\lambda\sigma} V_\mu V_\nu \partial_\lambda V_\sigma \right\} \] (10)

The problem is that this particular expression \( A^0_{U(1) \text{ Counterterm}} = 0 \). We are antisymmetrizing a symmetric expression \( V_\mu V_\nu \). In fact there is no appropriate local expression that can satisfy equation (9). Exactly the same problem arises for expression (4). Our example of a supergravity anomaly (sugra-nomaly) below in (14) has exactly the same features, but it is trickier because it involves the gravitino.

8. BRS realized that anomalies can be identified as objects in the local BRS ghost charge one cohomology: The reason for this is simple. We observe that for example, it is true that for the relevant nilpotent BRS operators \( \delta \), the objects in equations (5) and (4) satisfy the equations

\[ \delta A^1_{SU(3) \text{ Anomaly}} = \delta A^1_{U(1) \text{ Anomaly}} = 0 \] (11)

but there do not exist any local polynomial objects such that

\[ A^1_{SU(3) \text{ Anomaly}} = \delta A^0_{SU(3) \text{ Counterterm}} \] (12)

\[ A^1_{U(1) \text{ Anomaly}} = \delta A^0_{U(1) \text{ Counterterm}} \] (13)

In practice, the expressions \( A^1_{SU(3) \text{ Anomaly}} \) do arise in perturbation theory, by taking the variation of some Green’s function. If the anomaly is in the local BRS cohomology space, it cannot be removed by any terms added to the
Lagrangian, since the latter is local and the 1PI vertices are not local. This is true at the lowest level, and it continues to be true at all levels in the relevant coupling constants.

9. **Supergravity Anomaly:** This paper will argue that supergravity has an anomaly of its own, of the form

\[ A_{\text{Supergravity}} = \int d^4x \varepsilon^{\lambda\mu\nu\sigma} \left\{ c_1 C_\alpha \partial_\lambda \Psi_\mu^\alpha \partial_\nu V_\sigma + c_2 \partial_\lambda \Psi_\mu^\alpha \partial_\nu \Psi_\sigma^\alpha \omega + \cdots \right\} \]  

(14)

where the numerical coefficients \( c_i, i = 1, 2 \) are discussed below. In the above formula, \( \Psi_{\mu\alpha} \) is the Grassmann odd spin \( \frac{3}{2} \) gravitino of supergravity (in the complex two-component formulation), and \( V_\mu \) is a (real) vector boson in a gauge multiplet coupled to chiral multiplets and to supergravity. The field \( C_\alpha \) is the Grassmann even FP ghost for the supersymmetry and these two fields have the nilpotent transformations:

\[ \delta \Psi_{\mu\alpha} = M_P \partial_\mu C_\alpha + \cdots, \quad \delta V_\mu = \partial_\mu \omega + \cdots \]  

(15)

where \( M_P \) is the Planck mass, and \( \omega \) is again the Grassmann odd scalar FP ghost, as in the above examples. For the purposes of calculating the coefficient of the anomaly the higher order terms + \( \cdots \) in equation (14) and equation (15) can be ignored, and that is very important for a complicated calculation like this.

10. **The counterterm situation for supergravity is more complicated than for the gauge theories:** The relevant possible counterterm like (10) would be

\[ A_{\text{Supergravity Counterterm 1}}^0 = \int d^4x \left\{ \varepsilon^{\mu\nu\lambda\sigma} \Psi_\mu^\alpha \Psi_\nu^\alpha \partial_\lambda V_\sigma \right\} = 0 \]  

(16)

and again this is zero, which is just like the situation for (10), except that here there is a triple antisymmetrization which makes this zero, whereas in (10) it is a single antisymmetrization. However, there is another possible counterterm in this supergravity case. This takes the form

\[ A_{\text{Supergravity Counterterm 2}}^0 = \int d^4x \left\{ \varepsilon^{\mu\nu\lambda\sigma} \partial_\lambda \Psi_\mu^\alpha \Psi_\nu^\alpha V_\sigma \right\} \neq 0 \]  

(17)
An effort to show that this is zero results in a quadruple antisymmetrization (including integration by parts), and so it does not succeed in showing this is zero. This expression generates objects that look exactly like (14):

\[ \delta A_{\text{Supergravity Counterterm}}^0 = \int d^4 x \varepsilon^{\lambda \mu \nu \sigma} \{ c'_1 C_\alpha \partial_\lambda \Psi^\alpha_\mu \partial_\nu \Psi_\sigma + c'_2 \partial_\lambda \Psi^\alpha_\mu \partial_\nu \Psi_\sigma \omega \} \]  

where we have integrated by parts. The values are \( M_P c'_1 = -c'_2 = 1 \). Note that this combination of these two factors occurs with a definite and unchangeable linear combination with definite numbers \( c'_1, c'_2 \) in the above expression (18). An effort to show that the second term on the right hand side in (18), is zero, results in a double antisymmetrization, and so the term is non-zero. The term in (17) can generate one particular linear combination of these numbers, but not two. Both of the expressions in \( A_{\text{Supergravity}}^0 \) in (14) separately yield zero when acted upon by \( \delta \). So there is no way, in general, to obtain expression (14) from the BRS variation of a local object with ghost charge zero in the theory. If it arises in perturbation theory with coefficients that do not have the same ratio as in the above expression (18), it will spoil the SUSY type gauge symmetry of supergravity in an incurable way. In quantum field theory, we expect that this will happen in general—the rule there is that what can happen, usually does happen, because the perturbation expansion tends to explore all the corners.

11. Equations of motion of the gravitino and the vector boson: We will not worry here about the issues that relate to the higher levels in the coupling. However, it is crucial to observe that a genuine anomaly must not vanish using the equations of motion. Sometimes this is called invariance under deformations of the transformation rules [24]. This requirement is equivalent to ensuring that the anomaly is in the cohomology space (a cocycle, but not a coboundary) of the entire \( \delta \) derived in paragraph 34 here. The equation of motion for the gravitino is discussed below in paragraph 32. The expression (14) does not vanish using any of the forms of the equations of motion of either the gravitino or the vector boson.

12. The supergravity anomaly requires the spontaneous breaking of supersymmetry in order to appear in the theory: Later in this
paper, starting with paragraph 23, we will write down the detailed components of a theory which we believe will generate this anomaly. Based on this theory, we present a diagram for the anomaly in (19). The surprising thing here is that this anomaly of supergravity does not make its appearance in perturbation theory in the simple way familiar from the two anomalies of gauge theory above. In fact, a preliminary examination of the diagrams indicates that there are no linearly divergent diagrams for the relevant amplitude, which excludes any possibility of generating it with a non-zero coefficient. I have tried to find a linearly divergent diagram for the relevant amplitude in the theory here, before introducing the terms described in paragraph 16, and it does not seem possible, but I have no proof that it does not happen.

13. **Conjecture for the supergravity anomaly:** My conjecture is, that for this anomaly to appear, SUSY and gauge symmetry must both be spontaneously broken. This means that the gravitino mass, and the vector boson mass, must both be present, and must arise from the spontaneous breaking of local supersymmetry and local gauge symmetry, using the well known Higgs mechanisms [25]. The reasoning behind this conjecture follows from the diagram (22) in paragraph 16 below. At the present time, this claim remains a conjecture.

14. **A diagram for the sugranomaly:** Here is an example of a diagram that appears likely to generate this anomaly in supergravity. It is a linearly divergent triangle diagram, which is a necessary feature for a diagram to generate an anomaly, by analogy with the known anomalies discussed above. However, there is a very serious problem in this diagram. It needs a propagator from $\overline{\lambda^i}$ to $\chi_1^{\delta_1}$ in the top left of the triangle (we are using a circle, actually), but no such propagator exists in the theory! This problem appears, at first sight, to be an insuperable difficulty for generating the anomaly. As we shall see below, this problem is actually a hint about how the anomaly can be generated.
The vertices for the above diagram can be found in the chiral action in paragraph 35, and in the gauge action in paragraph 37. However the new mixed kinetic term $c_{\text{new}}\lambda^\alpha \sigma^\mu \partial^\mu \overline{\chi}_1$ arises from the new counterterm (23) in the action, discussed in paragraph 16. The massive $VV$ propagator at the bottom comes from the terms in paragraph 29. Its mass could be dropped here, since only the linear divergence contributes to the anomaly.

Note that there is no problem in getting the propagator $M_{1,\text{mix}}\overline{\chi}_1 \chi_1$ here, since it arises immediately from the term $ig\overline{\chi}_1 \chi_1 \langle A_1 \rangle$ which is in the last line (91) of the chiral action below. It brings in a power of mass from the VEV. It is, of course, unusual to find a power of mass in an anomaly, but this diagram clearly has a power of $gM_{\text{OV}} = M_{1,\text{mix}}$ from one propagator.

15. **Calculation of the sugranomaly:** We now recall that the sugranomaly is present in the theory if the following equation holds true in perturbation theory:

\[
\delta G[\Psi, \Psi, V] = \int d^4x \varepsilon^{\mu\nu\lambda\sigma} \left\{ c_1 \partial_\lambda \Psi^\alpha_\mu C_\alpha \partial_\nu V_\sigma + c_2 \partial_\lambda \Psi^\alpha_\mu \partial_\sigma \Psi_\nu \right\} \]

(19)
where the vertex is the sum of all relevant diagrams, as usual:

\[ \mathcal{G} [\Psi, \Psi, V] = \sum_n \mathcal{G}_n [\Psi, \Psi, V] \]  

(21)

Here \( c_1 \) and \( c_2 \) are coefficients, calculated from perturbation theory. In order for this to be an anomaly, it is necessary that these coefficients not be in a ratio such that they can arise from equation (18). It is quite remarkable that this requires us to have a propagator resulting from a counterterm that arises from diagrams like (22) below. Note that by the usual interpretation of quantum field theory, this term appears with a new arbitrary coefficient \( c_{\text{new}} \), as further explained below in paragraph 41.

16. The Counterterms for the theory with spontaneously broken SUSY: As we mentioned above, the problem with the diagram in paragraph 14 is that there are no propagators that take \( \lambda^{\gamma_1} \) to \( \bar{\chi}^{\dot{\gamma}_1} \) in the theory. But consider the following diagram:

\[ \begin{align*}
\mathcal{G}_1 [\lambda, \bar{\chi}] &= \int d^4 p \lambda^\alpha (p) \bar{\chi}^{\dot{\gamma}_1} (-p) \frac{1}{(2\pi)^2} \int d^4 l \\
&\quad \kappa^2 (igM_O v) (M_G) \left( \frac{\xi^{\dot{\gamma}_1} \eta^{\mu} + \cdots}{l^2 + M_G^2} \right) \\
&\quad [\sigma_{\nu_1 \alpha_1} (l + p)_\nu - \sigma_{\nu \alpha} (l + p)_\nu] \left( \frac{\eta^{\nu_1 \nu_2} + \cdots}{(l + p)^2 + M_G^2} \right) (\bar{\sigma}_{\mu_2} \dot{\epsilon})^{\dot{\beta}}
\end{align*} \]

Diagram 1 for \( \mathcal{G} [\lambda, \bar{\chi}] \)  

(22)

The vertex at the left comes from the action \( A_{\text{Gauge}} \) in paragraph 37. Similarly, the vertex at the right comes from the action \( A_{A_1} \) in paragraph 35 below.
after the VEV \(< A_1 >= M_0 \nu\) described in paragraph 27 appears. There is a similar term and diagram from \(\mathcal{A}_{A_2}\). The propagator at the top comes from the action in paragraph 32. There is still some work needed to convert that to the massive case. The propagator at the bottom comes from the terms in paragraph 29. This diagram clearly vanishes if the Gravitino mass \(M_G\) vanishes.

17. **The Diagram (22) can exist only if the gravitino is massive:** Otherwise the propagator \(\nabla^\nu \nabla^\nu \hat{\psi}^1 \hat{\psi}^2\) does not exist. This diagram is linearly divergent, and we can expect that it will need a counterterm in our action. But this counterterm will be needed only if the SUSY symmetry is spontaneously broken. We expect that the calculations in our theory will satisfy the Master Equation (see below in equation (77)), which reflects the complicated symmetry of this action. What that means is that if there are counterterms needed in the theory, like the following

\[
\mathcal{A}_{A,1 \text{ new counterterm needed}} = c_{\text{new}} \int d^4x \left\{ \lambda^\alpha \sigma^{\mu \alpha \beta} \partial_\mu \chi^\beta_1 \right\}
\]

(23)

where \(c_{\text{new}}\) is the divergent coefficient calculated from diagrams like (22), then there will also be other counterterms needed with the same coefficient. These can all be generated together as a boundary, as will be discussed below in paragraph 41, after we have discussed more of the details of the Master Equation and actions here.

Note that \(c_{\text{new}}\) can indeed be dimensionless, because the counterterm with a divergence in the diagram above in (22) for \(\mathcal{G}_1 \left[ \lambda, \chi \right]\) is proportional to

\[
\int d^4x \kappa^2 M_O M_G \lambda^{\alpha} \sigma^{\mu}_{\alpha \beta} \partial_\mu \chi^\beta_1 \times \text{a dimensionless (but divergent) constant, so that the whole term is of dimension zero.}
\]

On the other hand, we also note that the counterterm with a divergence in the diagram for \(\mathcal{G}_1 \left[ \Psi, \Psi, V \right]\) has dimension \(\int d^4x \kappa^2 M_{1, \text{mix}} \Psi \partial \Psi V\), which is also of dimension zero. This means that we can expect to calculate the sugranomaly coefficients in the form

\[
c_1 = \kappa M_{1, \text{mix}} N_1, c_2 = \kappa^2 M_{1, \text{mix}} N_2, \]

where \(N_i\) are finite, but mass-dependent, numbers. See paragraph 21 for more comments on these issues.

18. **The counterterms like (23), calculated from diagrams like (22), which fall into groups as discussed in paragraph 41, must be added**
to the action when the gravitino is massive: They then allow the diagrams like (19) to appear in the theory, and so they allow the sugranomaly to arise. That is why we said in paragraph 14 that the strange propagators needed for that diagram are a hint as to how it arises.

19. In a more detailed analysis of this situation, one would want to diagonalize the various terms, so that one is dealing with propagators that are diagonal in the masses and in the kinetic terms. Here there is all sorts of mixing going on, and it takes a lot of effort to sort it out. Our claim is that we can still find a lot of information from the action without doing all that work. But that work now needs to be done to determine how these anomalies restrict the set of viable supergravity theories, particularly in the context of unification [29,30,31].

20. Is this a real supergravity anomaly? The reader might wonder whether this is just an example of one of the usual anomalies, but made supersymmetric. It seems clear that this is actually quite different from the known anomalies, although it certainly has a family resemblance, as noted in this paper. We can expect that eliminating it from theories will give rise to interesting new restrictions on viable theories. However, getting its general form, performing the Feynman calculations accurately, and figuring out what those restrictions are, are complicated and demanding tasks, given the complicated nature of the action and transformations.

21. Conjecture for the Coefficient of the Anomaly in General:

For the general case where we have an arbitrary gauge group and arbitrary representations for matter and Higgs fields coupled to that group and to supergravity, here is a conjecture about the form and coefficient of this anomaly.

\[
\mathcal{A}_{\text{Supergravity}}^{1} = \int d^4x \varepsilon^{\lambda\mu\sigma\rho} \left\{ c_1 C_\alpha \partial_{\lambda} \Psi_{\mu}^a \partial_{\nu} V^a + \kappa c_2 \partial_{\lambda} \Psi_{\mu}^a \partial_{\nu} \Psi_{\sigma a} \omega^a \right\} g^3 \kappa^3 M_G \langle \langle D^a \rangle \rangle 
\]

Here we have written the coefficients \(c_1, c_2\) so that they are dimensionless. The reasoning behind this conjecture resides simply in the above diagrams (19) and (22), as follows:

If we add group matrices to this paper rather than leave it in the Abelian
notation, then we get the following group matrix in (19):
\[ \kappa^3 g^3 M_G M^2 v^i (T^a T^b T^a)_{i j} = \left( \frac{1}{2} C_2(G) + C_1(G) \right) \kappa^3 g^3 M_G M^2 \langle D^a \rangle > \quad (25) \]

where \( T^a \) are the representation matrices for a Lie group, and
\[ \langle D^b > = M^2 v^i T^b_{i j} \langle A^i \rangle = M v^i, \quad (26) \]

and we are taking into account what we get for the mixed propagator from (22). We note that the vector propagator is diagonal in group space, if we drop the mass terms, and we can do that because we are interested only in the linearly divergent part of the diagram (19). But then we note that
\[ T^a T^b T^a = ([T^a, T^b] + T^b T^a) T^a = f^{abc} T^c T^a + T^b T^a T^a \quad (27) \]
\[ f^{abc} = \frac{1}{2} f^{cad} T^d + T^b T^a T^a = \left( \frac{1}{2} C_2(G) + C_1(G) \right) T^b \quad (28) \]

and so we get the form (24), where we absorb this constant \( \left( \frac{1}{2} C_2(G) + C_1(G) \right) \), made from Casimir coefficients of the group, into the coefficients \( c_i \). In [2,8] there are discussions about the question whether the auxiliary field \( D^a \) can have a VEV, and [32] is also useful.

22. Calculating the Diagrams: It would be nice to calculate the diagrams like (22) and (19). The diagram (22) needs to be non-zero in order for the diagram (19) to even exist. But that requires us to have a proper version for both massive propagators, with gauge parameters, in (22). That is why some effort has been made to start the discussion of the terms in (73) and (74) for the gravitino propagator in paragraph 32 below.

23. The Action for the model: We take supergravity coupled to a U(1) super gauge theory, coupled to two chiral multiplets \( \hat{A}_i = A_i + \theta^\alpha \chi_{i \alpha} + \frac{1}{2} (\theta^2) F_i, i = 1, 2 \), with opposite charges, and two uncharged chiral multiplets \( \hat{B}_i = B_i + \theta^\alpha \phi_{i \alpha} + \frac{1}{2} (\theta^2) G_i, i = 1, 2 \). We choose a simple superpotential of the O’Raiffertaigh [1] form, so that the theory will have spontaneous breaking of supergravity. This part can be done as if we were using merely rigid SUSY, and we use \( M_O \) for a mass parameter here:

\[ \mathcal{A} \text{Scalars from; Chiral Superpotential} \quad (29) \]
\[
\int d^4 x d^2 \beta \left\{ g_1 B_1 \hat{A}_1 \hat{A}_2 + \hat{B}_2 \left( g_2 \hat{A}_1 \hat{A}_2 - M_O^2 \right) \right\} + * \quad (30)
\]
\[
\int d^4 x \left\{ g_1 G_1 A_1 A_2 + g_1 B_1 A_1 F_2 + g_1 B_1 F_1 A_2 
+ g_1 B_1 \chi_1^\alpha \chi_2^\alpha + g_1 \phi_1^\alpha \chi_1 A_2 + g_1 \phi_1^\alpha A_1 \chi_2^\alpha
+ g_2 G_2 A_1 A_2 + g_2 B_2 A_1 F_2 + g_2 B_2 F_1 A_2 
+ g_2 B_2 \chi_1^\alpha \chi_2^\alpha + g_2 \phi_2^\alpha \chi_1 A_2 + g_2 \phi_2^\alpha A_1 \chi_2^\alpha - G_2 M_O^2 \right\} \quad (34)
\]

and we add to this the purely scalar terms relating to the auxiliary fields and scalars in the other parts of the actions:

\[
\mathcal{A}_{\text{Scalars from Action}} = \int d^4 s \left\{ \sum_{n=1}^{2} \left\{ F_n \overline{F}_n + G_n \overline{G}_n \right\} + D^2 + D g A_1 \overline{A}_1 - D g A_2 \overline{A}_2 \right\} \quad (35)
\]

We put these together into

\[
\mathcal{A}_{\text{SP}} = \mathcal{A}_{\text{Scalars from Action}}
+ \mathcal{A}_{\text{Scalars from:Chiral Superpotential}} + \overline{\mathcal{A}}_{\text{Scalars from:Chiral Superpotential}} \quad (37)
\]

24. The bosonic parts of the various auxiliary fields here are obtained from the bosonic parts of their equations of motion:

\[
\frac{\delta \mathcal{A}_{\text{SP}}}{\delta G_1} = \overline{G}_1 + g_1 A_1 A_2 = 0; \quad \frac{\delta \mathcal{A}_{\text{SP}}}{\delta G_2} = \overline{G}_2 + g_2 A_1 A_2 - M_O^2 = 0 \quad (38)
\]
\[
\frac{\delta \mathcal{A}_{\text{SP}}}{\delta F_1} = \overline{F}_1 + g_1 B_1 A_2 + g_2 B_2 A_2 = 0; \quad \frac{\delta \mathcal{A}_{\text{SP}}}{\delta F_2} = \overline{F}_2 + g_1 B_1 A_1 + g_2 B_2 A_1 = 0 \quad (39)
\]
\[
\frac{\delta \mathcal{A}_{\text{SP}}}{\delta D} = D + g \left( A_1 \overline{A}_1 - A_2 \overline{A}_2 \right) = 0 \quad (40)
\]

25. We integrate the auxiliary fields in the path integral, which replaces them with the negative of their values from their equations of motion, and this yields the following potential in the usual way:

\[
V = -\mathcal{L}_{\text{Scalar Potential}} = \sum_{n=1}^{2} \left\{ F_n \overline{F}_n + G_n \overline{G}_n \right\} + D D \quad (41)
\]

which is, using the equations in paragraph 24:

\[
V = |g_1 A_1 A_2|^2 + |g_2 A_1 A_2 - M_O^2|^2 + |g_1 B_1 A_2 + g_2 B_2 A_2|^2 \quad (42)
\]
\[ + |g_1 B_1 A_1 + g_2 B_2 A_1|^2 + |g (A_1 \overrightarrow{A}_1 - \overrightarrow{A}_2 A_2)|^2 \]  

**26. The minimum value of this expression occurs for Vacuum Expectation Values (VEVs) as follows:**

\[ \langle B_1 \rangle = \langle B_2 \rangle = \langle \overrightarrow{F}_1 \rangle = \langle \overrightarrow{F}_2 \rangle = 0 \]  

These zero results all follow from the fact that setting them to zero just contributes zero to the VEV of the expression (43). The VEVs of the auxiliary fields \( G_1, G_2, D \) may be non-zero, which means that the gravitino gains a mass, and that the supersymmetry here is spontaneously broken.

**27. Values of the VEVs:** These VEVs need to satisfy the equations obtained by taking the derivatives of the expression (43) with respect to the four scalar fields \( A_1, A_2, B_1, B_2 \) and setting the result to zero, so as to find an extremum. Then one chooses the minimum value for \( V \) among the extrema. We can choose real values for the VEVs here. The solution is unique for present purposes, and is

\[ \langle A_1 \rangle = \langle A_2 \rangle = M_O v = M_O \sqrt{\frac{g^2}{g^2_1 + g^2_2}} \]  

\[ \Rightarrow \langle G_1 \rangle = -M^2_O \frac{g_1 g_2}{g^2_1 + g^2_2}; \, \langle G_2 \rangle = +M^2_O \frac{g^2_1}{g^2_1 + g^2_2}; \, \langle D \rangle = 0 \]  

and it yields a minimum value for \( V \), with those VEVs, of:

\[ V = V \left[ A_1 \rightarrow \langle A_1 \rangle, A_2 \rightarrow \langle A_2 \rangle, B_1 \rightarrow 0, B_2 \rightarrow 0 \right] = M^4_O \frac{g^2_1}{(g^2_1 + g^2_2)} \]  

**28. We expect to need a non-zero cosmological constant counterterm in any gravitational theory:** In supergravity theories this is of the anti-de-Sitter kind, negative in value, whereas the value (47) is positive. We can and do ‘fine tune’ the above value to be equal to the cosmological constant, and so the result has zero cosmological constant corresponding to asymptotically flat spacetime. That is the origin of the mass of the gravitino.

**29. This is a bit different from the well known way that the mass of the vector boson appears in such theories:** In this case that mass
occurs in the form
\[ -D_\mu A_1 D_\nu A_1 - D_\mu A_2 D_\nu A_2 \rightarrow \] (48)
\[ -igM_O v \partial_\mu V^\nu A_1 + igM_O v \partial_\mu V^\nu A_1 \] (49)
\[ +igM_O v \partial_\mu V^\nu A_2 - igM_O v \partial_\mu V^\nu A_2 \] (50)
\[ -2g^2 M_O^2 v^2 V_\mu V^\mu \] (51)

30. **Cosmological term and gravitino mass**: In this model we need to suppose we start with a cosmological term. To preserve the BRS invariance of supergravity we also need to add a gravitino mass term and a term that changes the gravitino transformation.

\[
A_{\text{Cosmological}} = \int d^4x \left\{ eM_{cc}^4 - e \left( M_G \Psi \Psi - \frac{1}{e} M_{\text{New}}^2 \tilde{\Psi}^\mu \sigma_\mu \bar{C} + * \right) \right\} \] (52)
\[
= \int d^4x \left( 1 + \frac{\kappa}{6} h^\sigma + \cdots \right) \left\{ M_{cc}^4 - \left( M_G \Psi \Psi - \frac{1}{e} M_{\text{New}}^2 \tilde{\Psi}^\mu \sigma_\mu \bar{C} + * \right) \right\} \] (53)

and in order to have invariance we need
\[
M_G M_{\text{New}}^2 = \frac{\kappa}{6} M_{cc}^4 \] (54)

Now suppose that we have, using (47)
\[
M_O^4 \frac{g_1^2}{(g_1^2 + g_2^2)} = M_{cc}^4 \] (55)

so that the cosmological term \( \int d^4x M_{cc}^4 e \) is cancelled by the VEV of V. These terms have the right signs to accomplish this because the (uncancelled) cosmological term in supergravity corresponds to an anti de Sitter universe [2]. We still have the gravitino mass term and we still have the new term in the transformation of the gravitino, but we no longer have the cosmological term in terms of \( h \). How does the supergravity symmetry survive?

Strictly speaking we really should integrate the auxiliary terms out at this point. But for present purposes we can proceed as follows. We get several new kinds of terms from the VEV of the auxiliary terms at the same time. We get
the mixing terms from the lines that correspond to (89) in $A_{\text{Chiral Kinetic B 1}} + A_{\text{Chiral Kinetic B 2}}$. It is

$$A_{\text{Goldstino mixing}} = \int d^4 x \{ \bar{\phi}_1 \sigma^\mu \kappa < G_1 > + \bar{\phi}_2 \sigma^\mu \kappa < G_2 > \} \Psi_\mu$$ (56)

$$= \int d^4 x M_{\text{New,3}} \bar{\phi}_{\text{Goldstino}} \sigma^\mu \Psi_\mu$$ (57)

Also we get the terms in the BRS transformations of the $\phi$ fields in $A_{\text{ZJ Chiral;B 1}} + A_{\text{ZJ Chiral;B 2}}$ from the lines that correspond to (119). They are:

$$\int d^4 x \left\{ \bar{\phi}_1^\alpha \kappa < G_1 > C_\alpha + \bar{\phi}_2^\alpha \kappa < G_2 > C_\alpha + * \right\}$$ (58)

$$= \int d^4 x M_{\text{New,4}} \left\{ \bar{\phi}_{\text{Goldstino}}^\alpha C_\alpha \right\} + *$$ (59)

So now we have no cosmological term but we have the following special terms in the action

$$\int d^4 x \left\{ M_G \Psi \Psi + M_{\text{New,3}} \bar{\phi}_{\text{Goldstino}} \sigma^\mu \Psi_\mu \right\} + *$$ (60)

and special terms in the ZJ action too:

$$\int d^4 x \left\{ -M_{\text{New}}^2 \tilde{\Psi}^\mu \sigma_\mu C + M_{\text{New,4}} \tilde{\phi}_{\text{Goldstino}}^\alpha C_\alpha \right\} + *$$ (61)

$$+ \tilde{G}_1 \beta^\beta \sigma^{\mu\nu} \kappa < G_1 > \Psi_\mu \sigma_\nu + \tilde{G}_2 \beta^\alpha \sigma^{\mu\nu} \kappa < G_2 > \Psi_\mu \sigma_\nu \right\} + *$$ (62)

There would also be a term $-i\kappa \lambda^\alpha_\beta \langle D \rangle \Psi_\mu + *$ from line (95) in the gauge action $A_{\text{Gauge}}$ except that in this model, from paragraph 27, we have $\langle D \rangle = 0$

The Goldstino then is

$$\phi_{\text{Goldstino}} = \frac{\kappa}{2M_G} \left( \langle \overline{G}_1 \rangle \phi_1 + \langle \overline{G}_2 \rangle \phi_2 \right) = \frac{\kappa M_G^2}{g_1 + g_2} g_2 (g_2 \phi_1 + g_1 \phi_2)$$ (63)

This needs more attention to see exactly how the invariance remains, but we will not do that here for now.

31. **We can eliminate the Goldstino mixing term $A_{\text{Goldstino mixing}}$ in (57) by choosing a special form of the Ghost and Gauge Fixing Action:** This technique was invented by ’t Hooft for the case of the spontaneous
breaking of the massive gauge vector boson, and later it was updated using
the BRS transformations [15]. This also allows us to keep a free (and useful)
gauge parameter. We choose the following Ghost and Gauge Fixing (GGF)
action for the gravitino:

\[ A_{\text{Gravitino GGF}} = \int d^4x \delta \left\{ E^\alpha \left( \Lambda_\alpha + \left( \sqrt{m} \sigma^\mu_{\alpha \beta} \Psi^\beta_\mu + \sqrt{\frac{M^2_G}{m}} \phi_\alpha;\text{Goldstino} \right) \right) + * \right\} \] (64)

where \( E^\alpha \) is a Grassmann even antighost for the \( C_\alpha \) SUSY ghost, and \( \Lambda^\alpha \) is a
Grassmann odd ‘ghost auxiliary field’, and these are taken to have the BRS
variations:

\[ \delta E^\alpha = \Lambda^\alpha, \ \delta \Lambda^\alpha = 0 \] (65)

After the auxiliary \( \Lambda_\alpha \) and its complex conjugate are integrated out of the
path integral this becomes:

\[ A_{\text{Gravitino GGF}} \Rightarrow \int d^4x \left\{ - \left( \sqrt{m} \sigma^\mu_{\alpha \beta} \Psi^\beta_\mu + \sqrt{\frac{M^2_G}{m}} \chi_\alpha;\text{Goldstino} \right) \right\} \] (66)

\[ \epsilon^{\alpha \gamma} \left( \sqrt{m} \sigma^\mu_{\gamma \delta} \Psi^\delta_\mu + \sqrt{\frac{M^2_G}{m}} \chi_\gamma;\text{Goldstino} \right) + E^\alpha \delta \left( \sqrt{m} \sigma^\mu_{\alpha \beta} \Psi^\beta_\mu + \sqrt{\frac{M^2_G}{m}} \chi_\alpha;\text{Goldstino} \right) + * \] (67)

The cross term in line (67) cancels the mixing term (57), yielding a gauge
parameter dependent mass term for the Goldstino, and a gauge fixing term
for the Gravitino, along with a ghost action that involves the BRS variations
of the Gravitino and the Goldstino. The gauge parameter is \( m \) and the
Gravitino mass is \( M_G \). Both have the dimension of mass.

All of this is exactly like what happens for the massive vector boson and its
Goldstone boson and its gauge parameter, except that the gauge parameter
of the Gravitino has to have the dimension of mass. The old style of gauge
fixing the gravitino [34] did not accomodate any gauge parameter.

32. **The Gravitino Propagator with a gauge parameter** Calculating
the form of this with a gauge parameter is not simple. There are ten ‘spinor-
tensor-derivative’ operators that must be considered. Here is the kinetic
action which results after the mixing term (57) has been removed using the
methods in paragraph 31:
\[ A_{\text{Kinetic Gravitino}}^0 = \int d^4x \gamma^0 K^{\mu\nu} \Psi_{\nu} \]

\[ = \int d^4x \gamma^0 (\varepsilon^{\mu\nu\rho\kappa} \gamma_\rho \gamma_5 \partial_\kappa - M_G \eta^{\mu\nu} - m \gamma^\mu \gamma^\nu) \Psi_{\nu} \]

Here we have switched to real gamma matrices as used in [33]. This term is a Lorentz covariant tensor in the Lorentz indices \( \mu\nu \cdots \), and a Lorentz covariant matrix in the implicit spinor indices that go with \( \gamma_\mu \cdots \). and we want to find the propagator, which is the inverse:

\[ P^{\mu\nu} K_{\nu\lambda} = \delta^\mu_\lambda \]

To find this inverse we need to write down the general Lorentz covariant form of this tensor-matrix. This turns out to be:

\[ P^{\mu\nu} = \left\{ \left( a_1 \varepsilon^{\mu
u}_{\alpha\beta} \partial^\alpha \gamma_5 + a_2 \partial^\mu \gamma^\nu + a_3 \partial \eta^{\mu\nu} + a_4 \gamma^\mu \partial^\nu + \frac{c_1}{m^2} \partial^\mu \partial \phi^\nu \right) \\
+ \left( \frac{b_1}{m} \partial^\mu \partial^\nu + \frac{b_2}{m} \Box \eta^{\mu\nu} + \frac{b_3}{m} \Box \gamma^\mu \gamma^\nu + \frac{b_4}{m} \gamma^\mu \partial \phi^\nu + \frac{b_5}{m} \partial \phi^\nu \partial \gamma^\nu \right) \right\} \]

In (71) we need to assume that the ten coefficients \( a_1, \cdots c_1, b_1 \cdots \) are functions of the ratio

\[ R = \frac{\Box}{m^2} \]

which has mass dimension zero.

So far, this work has only been done for the case where \( M_G = 0 \). For that case, the solution is:

\[ P^{\mu\nu} = \frac{1}{2\Box} \left\{ Q^{\mu\nu} + \partial^\mu \partial^\nu \frac{2}{m} \right\} \]

where

\[ Q^{\mu\nu} = \varepsilon^{\mu
u}_{\alpha\beta} \partial^\alpha \gamma_5 + \partial^\mu \gamma^\nu - \partial \eta^{\mu\nu} + \gamma^\mu \partial^\nu - \partial \partial^\nu \frac{4\partial \phi^\nu}{\Box} \]

Note that the propagator has a simple form:

- When \( m \rightarrow 0 \) this becomes singular. There is no inverse when \( m = 0 \).
- There are no \( \frac{1}{\Box + m^2} \) poles. The only place that \( m \) appears in the propagator is in the term \( \frac{1}{m} \partial^\mu \partial^\nu \) which looks and acts like a gauge parameter term.
• The part $Q^\mu{}^\nu$ satisfies $\gamma^\mu Q^\mu{}^\nu = 0$
• The part $\varepsilon^{\mu\nu\rho\kappa} \gamma_5 \partial_\kappa$ satisfies

$$\varepsilon^{\mu\nu\rho\kappa} \gamma_5 \partial_\kappa \partial_\nu = 0 \quad (75)$$

For the case where $M_G \neq 0$ this still needs to be completed. Obviously, this propagator needs to be used so that one can calculate diagrams like (22)–the significant physics of that diagram needs to be gauge invariant under two different gauge parameters. The physical consequences, including the anomaly coefficient, of the diagram (19) also should be gauge invariant under the gauge parameter for the massive vector boson.

33. The Action and the BRS transformations for the model used in this paper: At this point we need to write down the complicated and long action and transformations for the theory used in this paper. These are needed to understand the remarks in 16 to see what is going on. The invariance of the action and the nilpotence of the BRS transformations are all summarized in the Master Equation, found in the next section.

34. The Master Equation: This summarizes all the consequences of the BRS symmetry of the theory. For the total action $\mathcal{A}$, which is the sum of all the actions and all the ZJ actions, it takes the form:

$$\mathcal{M}_{\text{Total}} = \mathcal{M}_{\text{Sugra}} + \mathcal{M}_{\text{Gauge}} + \mathcal{M}_{\text{A Chiral}; 1} + \mathcal{M}_{\text{A Chiral}; 2} + \mathcal{M}_{\text{B Chiral}; 1} + \mathcal{M}_{\text{B Chiral}; 2} = 0 \quad (77)$$

where

$$\mathcal{M}_{\text{Sugra}} = \int d^4x \left\{ \frac{\delta \mathcal{A}}{\delta h^\mu{^a}} \frac{\delta \mathcal{A}}{\delta h^\mu{^a}} + \frac{\delta \mathcal{A}}{\delta \bar{\Psi}^{\mu{^\alpha}}} \frac{\delta \mathcal{A}}{\delta \bar{\Psi}^{\mu{^\alpha}}} \right\} + \frac{\delta \mathcal{A}}{\delta V^\mu{^\mu}} \frac{\delta \mathcal{A}}{\delta V^\mu{^\mu}} + \frac{\delta \mathcal{A}}{\delta \rho_{ab}} \frac{\delta \mathcal{A}}{\delta \rho_{ab}} + \frac{\delta \mathcal{A}}{\delta \xi^\mu} \frac{\delta \mathcal{A}}{\delta \xi^\mu} + \frac{\delta \mathcal{A}}{\delta \bar{C}^{\alpha}} \frac{\delta \mathcal{A}}{\delta \bar{C}^{\alpha}} + \frac{\delta \mathcal{A}}{\delta \bar{A}} \frac{\delta \mathcal{A}}{\delta \bar{A}} \right\} \quad (78)$$

$$\mathcal{M}_{\text{Gauge}} = \int d^4x \left\{ \frac{\delta \mathcal{A}}{\delta V^\mu{^\mu}} \frac{\delta \mathcal{A}}{\delta V^\mu{^\mu}} + \frac{\delta \mathcal{A}}{\delta \lambda^\alpha} \frac{\delta \mathcal{A}}{\delta \lambda^\alpha} + \frac{\delta \mathcal{A}}{\delta \bar{A}} \frac{\delta \mathcal{A}}{\delta \bar{A}} + \frac{\delta \mathcal{A}}{\delta D} \frac{\delta \mathcal{A}}{\delta D} + \frac{\delta \mathcal{A}}{\delta \bar{\omega}} \frac{\delta \mathcal{A}}{\delta \bar{\omega}} \right\} \quad (80)$$

$$\mathcal{M}_{\text{A Chiral}; n=1,2} = \int d^4x \left\{ \frac{\delta \mathcal{A}}{\delta \bar{A}_n} \frac{\delta \mathcal{A}}{\delta \bar{A}_n} + \frac{\delta \mathcal{A}}{\delta \bar{\omega}_n} \frac{\delta \mathcal{A}}{\delta \bar{\omega}_n} \right\} \quad (81)$$
The Master Equation actually applies to the 1PI vertex functional $\mathcal{G} = \mathcal{A} + \hbar \mathcal{G}_1 + O(\hbar^2)$, and the action $\mathcal{A}$ is just the local part of that. The next sections will contain the various actions which sum to the total action $\mathcal{A}$. There are also some simple adjustments needed for complications that relate to the gauge fixing terms and ghost actions. We will ignore those complications for now.

The BRS ZJ $\delta$ operator is the ‘square root’ of the above, by which we mean

$$\delta = \int d^4x \left\{ \frac{\delta \mathcal{A}}{\delta h^{\mu a}} \frac{\delta}{\delta h_{\mu a}} + \frac{\delta \mathcal{A}}{\delta \Phi^{\mu a}} \frac{\delta}{\delta \Phi_{\mu a}} + \frac{\delta \mathcal{A}}{\delta G_{\mu a}} \frac{\delta}{\delta G_{\mu a}} + \cdots \right\}$$

(85)

Note that the nilpotence of $\delta$ and the vanishing of $\mathcal{M}_{\text{Total}}$ imply each other:

$$\mathcal{M}_{\text{Total}} = 0 \iff \delta^2 = 0$$

(86)

It should also be remembered that $\mathcal{M}$ is rather like a Poisson Bracket, and so it is invariant under transformations that are like canonical transformations [26, 27]. That is what is happening with the generation of the new counterterms in paragraph 41, as was explained in [28].

35. The Action and the BRS transformations for the Four Chiral Kinetic Multiplets, Coupled to Gauge Theory, and to Supergravity:

Here is the action for the theory used in this paper:

$$\mathcal{A}_{\text{Chiral Kinetic A1}} = \mathcal{A}_{A1} =$$

$$\int d^4x \left\{ (\partial_{\mu} A_1 - ig V_{\mu} A_1 + \kappa \Psi_{\mu} \cdot \chi_1) \left( \partial^{\mu} \bar{A}_1 + ig V^{\mu} \bar{A}_1 + \kappa \bar{\Psi}^{\mu} \cdot \bar{\chi}_1 \right) \right.$$  

$$+ \bar{\chi}_1 \sigma^{\mu} \left[ \partial_{\mu} \chi_{1\alpha} - ig V_{\mu} \chi_{1\alpha} + w_{\mu ab} (\sigma^{ab})_{\alpha\beta} \chi_1^{\beta} + \kappa F_1 \Psi_{\mu \alpha} \right]$$

(87)
\[ + \kappa \sigma^\tau_{\alpha \beta} \bar{\Psi}^\beta_\mu (\partial_\tau A_1 - igV_\tau A_1 + \kappa \Psi \cdot \chi_1) \] (90)

\[ + g \left( DA_1 A_1 + \lambda^\alpha \chi_1 A_1 + \bar{\lambda}^\alpha \bar{\chi}_1 A_1 \right) \] (91)

The action for \( A_{A_2} \) is obtained, from 1 above, by simply changing \( 1 \to 2 \) and also changing the sign of \( g \) throughout. The actions for \( A_{B_1} \) and \( A_{B_2} \) are the same except for the change of notation (the superfields are discussed above, and indicate the notation) and setting \( g \to 0 \), since they are neutral under the gauge group. In the above we are using ‘supercovariant derivatives’, as discussed in [1]. We have written these out in full in the first line (88) as \( \hat{D}_\mu A_1 \hat{D}_\mu A_1 \), for example.

36. **The Action for Supergravity**: Here is the Action for supergravity as used here:

\[ \mathcal{L}_{\text{Supergravity}} = M_5^2 e R + \varepsilon_{\mu \nu \lambda \sigma} \Psi^{\mu \alpha} \sigma^\nu_{\alpha \beta} \partial^\lambda \bar{\Psi}^\sigma \bar{\Psi}^\beta + \varepsilon_{\mu \nu \lambda \sigma} \Psi^{\mu \nu} w^{\lambda cd} \sigma_{cd} \bar{\Psi}^\sigma \] (92)

We use the formulation by Zumino and Deser in [35]. There is an important feature here that relates to the spectral sequence and to the quantization of these theories which we discuss in paragraph 43.

37. **The Action for the U(1) Gauge Theory, Coupled to Supergravity**:

\[ \mathcal{A}_{\text{Gauge}} = \int d^4 x \left\{ - \frac{1}{4} \left( F_{\mu \nu} + \kappa \Psi^\gamma_{[\mu} \sigma^\nu_{\gamma \delta] \bar{\lambda}^\delta + \kappa \bar{\Psi}^\gamma_{[\nu} \sigma^\mu_{\gamma \delta] \lambda^\delta} \right) \right. \] (93)

\[ \left( F^{\mu \nu} + \kappa \Psi^\gamma_{[\mu} \sigma^\nu_{\gamma \xi] \lambda^\xi + \kappa \bar{\Psi}^\gamma_{[\nu} \sigma^\mu_{\gamma \xi] \bar{\lambda}^\xi} \right) \} \] (94)

\[ + \int d^4 x \left\{ \delta^\beta_\mu \sigma^\mu_\beta \alpha \left[ \partial_\lambda \lambda^\alpha + w^{ab}_{\mu} (\sigma_{ab})^{\alpha \beta \lambda \beta} + \kappa (\sigma_{\sigma \tau})^{\alpha \beta} F^{\sigma \tau} \Psi_{\mu \beta} - iD \kappa \Psi_{\mu \alpha} \right] \right. \] (95)

\[ \left. + (\ast \text{ of previous term}) - D^2 \right\} \] (96)

where we define

\[ F_{\mu \nu} = \partial_\mu V_\nu - \partial_\nu V_\mu \] (97)

38. **The ZJ part of the Supergravity Action**: This is also needed for the theory. It describes the nilpotent BRS transformations that leave the
action invariant.

\[ \mathcal{L}_{ZJ, \text{Supergravity}} = \tilde{h}^{\mu a} \left\{ -M_P (\partial_\mu \xi_a) + M_P (\rho_{\mu a}) + \rho_a \hat{\partial} h_{\mu a} - h_{\mu a} \partial^\sigma \xi_a \right\} \quad (98) \]

\[ + \xi^\sigma \partial_{\sigma} h_{\mu a} + C\sigma_{a} \bar{\Psi}_\mu + C\sigma_{a} \bar{\Psi}_\mu + k \left( \Psi_{\mu} \sigma^\rho \mathcal{C} + C\sigma^\rho \bar{\Psi}_\mu \right) h_{\nu a} \right\} + \tilde{\omega}^{\mu ab} \{ \hat{\partial}_{\mu} \rho_{ab} \right\} \quad (99) \]

\[ + [\rho, w_{\mu}]_{ab} + \xi^\sigma \partial_{\sigma} w_{\mu ab} + k B_{\mu ab} + k B_{\mu ab} - k^2 \left( B_{\mu ab} - B_{\mu ab} \right) \quad (100) \]

\[ + \kappa^2 \left( B_{\mu ab} h_{\mu a} - B_{\mu ab} h_{\mu a} \right) - k \left( \Psi_{\mu} \sigma^\rho \mathcal{C} + C\sigma^\rho \bar{\Psi}_\mu \right) w_{\nu ab} \right\} \quad (101) \]

\[ + \tilde{\omega}^{\mu} \left\{ M_P (\partial_{\mu} C + w_{\mu ab} \sigma_{ab} C) + \rho_{ab} \sigma_{ab} \Psi_{\mu} \right\} \quad (102) \]

\[ + h_{\mu a} \partial^\nu C + \xi^\nu \partial_{\nu} \Psi_{\mu} - k \left( \Psi_{\mu} \sigma^\nu \mathcal{C} + C\sigma^\nu \bar{\Psi}_\mu \right) \Psi_{\nu} \right\} \quad (103) \]

where following Zumino and Deser in [35] we put

\[ B_{a}^{\lambda \mu} = \varepsilon^{\lambda \mu \nu \rho} \mathcal{C} \sigma_{a} D_{\nu} \psi_{\rho} \quad (108) \]

39. The ZJ part of the Gauge Action: This describes the nilpotent BRS transformations of the gauge ZJ sources, that leave the action invariant.

\[ A_{ZJ, \text{Gauge}} = \int d^4 x \left\{ \tilde{V}^{\mu} \left( \partial_{\mu} \omega + C^{\alpha}_{\beta} \sigma_{\alpha \beta} \lambda^\beta \right) \right\} \quad (109) \]

\[ + \tilde{C}^{\alpha}_{\beta} \sigma_{\alpha \beta} \lambda^\beta + \xi^\alpha \partial_{\alpha} V_{\mu} \right\} \quad (110) \]

\[ + \tilde{\lambda}^{\alpha} \left( \sigma^{\mu \nu}_{\alpha \beta} F_{\mu \nu} C^{\beta} + \rho_{ab} \sigma_{ab} \lambda^\beta + i D C_{\alpha} + \xi^\mu \partial_{\mu} \lambda^\alpha \right) \quad (111) \]

\[ + \tilde{\lambda}^{\alpha} \left( \sigma^{\mu \nu}_{\alpha \beta} F_{\mu \nu} \bar{C}^{\beta} + \rho_{ab} \sigma_{ab} \lambda^\beta - i D \bar{C}_{\alpha} + \xi^\mu \partial_{\mu} \lambda_{\alpha} \right) \quad (112) \]

\[ + \tilde{D} \left( i (C^{\alpha}_{\beta} \sigma^{\mu}_{\alpha \beta} \tilde{D}_{\mu} \lambda^\beta - \tilde{C}^{\alpha}_{\beta} \sigma^{\mu}_{\alpha \beta} \tilde{D}_{\mu} \lambda^\beta) + \xi^\mu \partial_{\mu} D \right) \quad (113) \]

\[ + \tilde{\omega} \left( C^{\alpha}_{\beta} \sigma^{\mu}_{\alpha \beta} \bar{C}^{\beta} V_{\mu} + \xi^\mu \partial_{\mu} \omega \right) \right\} \quad (114) \]

In the above we are using ‘supercovariant derivatives’, indicated with the notation \( \tilde{D}_{\mu} \lambda^\beta \), for example, as discussed in [1] and as used above in paragraph 37.
40. **The ZJ part of the Four Chiral Actions:** This describes the nilpotent BRS transformations of the four kinds of chiral ZJ sources, that leave the action invariant.

\[ \mathcal{L}_{ZJ} \text{ Chiral A } 1 = \]
\[ \tilde{\mathcal{A}}_1 [-ig\omega A_1 + C \cdot \chi_1 + \xi^\mu \partial_\mu A_1] \quad (116) \]
\[ + \tilde{\mathcal{A}}_1 [+ig\omega A_1 + \overline{C} \cdot \overline{\chi}_1 + \xi^\mu \partial_\mu \overline{A}_1] \quad (117) \]
\[ + \tilde{\chi}^\alpha_1 \left[ -ig\omega \chi_{1\alpha} + \rho^{\alpha\beta}(\sigma_{ab})_{\alpha\beta} \chi_1^\beta + F_1 C_\alpha + \xi^\mu \partial_\mu \chi_{1\alpha} \right] \quad (118) \]
\[ + \sigma^\tau_{\alpha\beta} \overline{C}^\beta_\tau (\partial_\tau A_1 - igV_\tau A_1 + \kappa \Psi^\beta_\tau \chi_{1\beta}) \right] + * \quad (119) \]
\[ + \tilde{F}_1 [-ig\omega F_1 + \xi^\mu \partial_\mu F_1] + * \quad (120) \]
\[ + \tilde{F}_1 \overline{C}_\beta \sigma^{\mu \alpha \beta} [\partial_\mu \chi_{1\alpha} - igV_\mu \chi_{1\alpha} + \kappa F_1 \Psi_{\mu \alpha}] + * \quad (121) \]
\[ + \tilde{F}_1 \overline{C}_\beta \sigma^{\mu \alpha \beta} \kappa \sigma^\tau_{\alpha\gamma} \Psi^\gamma_\mu [\partial_\tau A_1 - igV_\tau A_1 + \kappa \Psi^\beta_\tau \chi_{1\beta}] + * \quad (122) \]

The expressions for \( \mathcal{L}_{ZJ} \text{ Chiral A } 2 \) and \( \mathcal{L}_{ZJ} \text{ Chiral B } n, n = 1, 2 \) are obtained from the above in the same way that was described above for the actions.

41. **The Diagram (22) can exist only if the gravitino is massive:**

All these counterterms will be expected to be obtained from an expression like this (the meaning of the notation is given below). Note that the ghost charge of this expression is \( N_{\text{Ghost}} = -1 \): it is linear in ZJ sources:

\[ \mathcal{A}^{-1}_{\text{Generator}} = \int d^4x \left\{ if_1 \tilde{D} \left( F_1 - \overline{F}_1 \right) + if_2 \tilde{D} \left( F_2 - \overline{F}_2 \right) \right\} \quad (123) \]
\[ + if_3 \left( \tilde{\chi}^\alpha_1 \lambda_\alpha + \tilde{\chi}^\dot{\alpha}_1 \overline{\lambda}_\dot{\alpha} \right) + if_4 \left( \tilde{\chi}^\alpha_2 \lambda_\alpha + \tilde{\chi}^\dot{\alpha}_2 \overline{\lambda}_\dot{\alpha} \right) \quad (124) \]
\[ + if_5 \left( \tilde{F}_1 - \overline{F}_1 \right) D + if_6 \left( \tilde{F}_2 - \overline{F}_2 \right) D + \quad (125) \]
\[ + if_7 \left( \tilde{\chi}^\alpha_1 \chi_{1\alpha} + \tilde{\chi}^\dot{\alpha}_1 \overline{\chi}_{1\dot{\alpha}} \right) + if_8 \left( \tilde{\chi}^\alpha_2 \chi_{2\alpha} + \tilde{\chi}^\dot{\alpha}_2 \overline{\chi}_{2\dot{\alpha}} \right) + \cdots \right\} \quad (126) \]

To get the counterterms one takes

\[ \delta \mathcal{A}^{-1}_{\text{Generator}} = \mathcal{A}^0_{\text{Counterterms}} \quad (127) \]
where the operator $\delta$ comes from the ‘square root’ of the Master equation, found in equation (77). It is easy to verify that there are plenty of diagrams that appear when the gravitino is massive, and we expect them all to be related in this way. It is interesting to note that these counterterms change the scalar interaction in paragraph 23 in a simple way. That does not appear to spoil the general argument or the VEVs here, however.


42. Errors: Anyone who works with SUSY makes plenty of these, and the author is no exception. There are so many minus signs to take care of that it is exhausting, and short cuts must be taken. If the reader thinks there is a missing minus sign or factor or term, he may well be right. Some features are crucial of course, but they are usually simple. The crucial and simple issues in this paper are in paragraphs 9 and 10, and in the diagrams (19) and (22). The author apologizes for not yet having the energy to carefully and accurately check all the terms in the actions and transformations. I believe that these are pretty close to correct, but there are hundreds of terms to check, and the task is not easy. There are two stages in finding the action and transformations here. The first stage is to make sure that there are enough terms of the right kinds, with indices in the right places, to satisfy each identity. The second stage is to make sure that the signs and factors are correct. I believe that I have accomplished the first stage, but I have currently not yet had the energy to perform the second stage. In the case of $\delta w_{\mu ab}$, I am still working on the first stage. Much of the necessary material there is already in the paper [35]. However the current results reported here are, I think, sufficient to justify the conjecture that spontaneous breaking is needed to generate these sugranomalies. What difference does this make to this paper? I think that the fundamental result is not affected at all. It involves only the simple terms discussed above. The calculation of the anomaly coefficient and the question whether it requires spontaneous breaking of supersymmetry are crucially important, of course. To calculate the anomaly coefficient we need the exact action and BRS transformations. We need to be sure that the Master equation is correct and that $\delta^2 = 0$ is correct, and that no mistakes are made in the calculation. That is a large undertaking.
43. **First Order and Second Order Formalism:** Most work in supergravity has been done using the second order formalism, where the spin connection $w_{\mu ab}$ is replaced by its expression in terms of the vierbein $e_{\mu a} = \eta_{\mu a} + \kappa h_{\mu a}$. We do not do that here for two reasons. The first is that it seems easier to formulate the BRS formalism in terms of the first order formalism. The second is that it seems very strange to try to quantize the theory when $w_{\mu ab}$ is no longer present. It is the gauge field of the ghost $\rho^{ab}$. This transformation is the usual sort of gauge transformation: $\delta w_{\mu ab} = \partial_{\mu} \rho^{ab} + \cdots$. The absence of $w_{\mu}^{ab}$ means that in some sense, in the second order formalism, one needs to have two different antighosts for the gauge fixing term for the field $h_{\mu a}$, one for the general coordinate ghost $\xi_{\mu}$ and the other for the Lorentz ghost $\rho^{ab}$. From the spectral sequence point of view, this is particularly disturbing and it looks wrong. We also prefer the Weyl two component complex formalism because the Fierz transformations are so much simpler there. It seems clear that supersymmetry is more natural, and simpler, in that formalism. For example the anomaly (14) discussed here is very chiral–its complex conjugate

$$\overline{A}_{\text{Supergravity}}^{1} = \int d^{4}x \varepsilon^{\mu \nu \sigma} \left\{ \overline{c_{1}} \overline{C}_{\dot{\alpha}} \partial_{\lambda} \overline{\Psi}_{\mu}^{i} \partial_{\nu} V^{\sigma} + \overline{c_{2}} \partial_{\lambda} \overline{\Psi}_{\mu}^{i} \partial_{\nu} \overline{\Psi}_{\sigma \dot{\alpha}} \omega + \cdots \right\} \quad (128)$$

is also an anomaly form.

44. **Auxiliary Fields:** This paper is composed using the first order formalism and in terms of Weyl two component complex spinors. The reader might wonder whether we need additional auxiliary fields to close the algebra of supergravity, for example in the expressions in paragraph 38. Auxiliary fields are useful for generating a calculus where multiplets can be multiplied to make other multiplets and invariants. However they are not needed to create the nilpotent BRS transformations. The reason is that one can imagine having the auxiliary fields and integrating them out using the path integral [38]. That changes the terms in the Master Equation, but the derived $\delta$ is still nilpotent, because it must be nilpotent, given that it is the ‘square root’ of the Master Equation. As is well known, auxiliary fields are used to close transformations that do not close, except using the field equations. In the
Master Equation approach, exactly the same function is served by the Zinn sources. It appears to me that there is no need in the first order formalism for any auxiliary fields in the present paper—they would show up as a need for terms with doubled Zinn sources. But it seems likely that they would start to appear at higher orders, in particular because of the terms in paragraph 41.

45. **Spectral Sequence:** The results in this paper were obtained by the author using spectral sequences [36,37,39,40,41]. These methods provided the insight needed to deduce that there had to be an anomaly in supergravity. When a suitable spectral sequence can be used, the space $E_\infty$ represents all the possible invariants of the theory—but their explicit construction is still needed in that case. In the present situation, a spectral sequence indicated that there ought to be an anomaly, but its construction from that took a very long time. Unfortunately, those methods are quite difficult and confusing, though powerful. There are still lots of unsolved issues there when those methods are applied to supergravity, and indeed other theories too.

46. **Future Steps:** The action and the transformations, and their closure, need to be completed and rechecked. The work here on the massless gravitino propagator with a massive gauge parameter needs to be extended to the case where the gravitino is also massive. The supergravity anomalies need to be calculated, in many theories. The values of their coefficients should be independent of all gauge parameters. The significance of these supergravity anomalies needs to be understood. Is the conjecture about their coefficients in equation (24) correct? If it is correct, does that mean that the viable theories must be those which do have some $\langle F^i \rangle \neq 0$, but which also have the sugranomaly coefficient, or perhaps $\langle D^a \rangle$ itself, arranged to vanish, somehow? Is that easy? What kinds of theories are those? The spectral sequence work also needs to be advanced.

47. **Supergravitational fields in this model:**
Note that we define:

\[ e_{\mu a} = \eta_{\mu a} + \kappa h_{\mu a}; g_{\mu \nu} = e_{\mu a} \eta^{ab} e_{\nu b} \]  

(129)

which means that the inverse of \( e \) is an infinite series in \( h \), and I am ignoring that for now.

48. Gauge Particles for the Abelian Gauge Theory:

| Symbols | Ghost Number | Reality and Grassman | Dim \( m^d \) | Name |
|---------|--------------|----------------------|----------------|------|
| \( V_\mu \) | 0 | Real, Even | 1 | Gauge Vector Boson |
| \( \omega \) | 1 | Real, Odd | 0 | Gauge Ghost Field |
| \( \eta \) | -1 | Real, Odd | 2 | Gauge AntiGhost Field |
| \( H \) | 0 | Real, Odd | 2 | Gauge Ghost Auxiliary Field |
| \( \lambda^\alpha, \lambda^{\dot{\alpha}} \) | 0 | Complex, Odd | \( \frac{3}{2} \) | Gauge Fermion Field |
| \( D \) | 0 | Real, Even | 2 | Gauge Auxiliary Field |

49. Higgs Representations in this Model

| Symbols | Reality and Grassman | Dim \( m^d \) | Name |
|---------|----------------------|----------------|------|
| \( A_n, \tilde{A}_n \) | Complex, Even | 1 | Charged Chiral Scalar Pair: \( n=1,2 \) |
| \( \chi^\alpha, \chi^{\dot{\alpha}} \) | Complex, Odd | \( \frac{3}{2} \) | Charged Chiral Spinor Pair: \( n=1,2 \) |
| \( F_n, \tilde{F}_n \) | Complex, Even | 2 | Charged Chiral Auxiliary Pair: \( n=1,2 \) |
| \( B_n, \tilde{B}_n \) | Complex, Even | 1 | Singlet Chiral Scalars: \( n=1,2 \) |
| \( \phi^\alpha, \phi^{\dot{\alpha}} \) | Complex, Odd | \( \frac{3}{2} \) | Singlet Chiral Spinors: \( n=1,2 \) |
| \( G_n, \tilde{G}_n \) | Complex, Even | 2 | Singlet Auxiliary Scalars: \( n=1,2 \) |

50. Some Constants in this Model
51. Some Important Concepts in the paper

| Symbol      | Paragraph | Dim $m^d$ | Name and Comment                  |
|-------------|-----------|-----------|-----------------------------------|
| $\kappa$    | 34        | $-1$      | Planck Length                     |
| $M_p = \frac{1}{\kappa}$ |           | $+1$      | Planck Mass                       |
| $M_O$       | 23        | $+1$      | $0'$Raiffartagaiff Mass           |
| $M_G$       | 30,32     | $+1$      | Gravitino Mass                    |
| $m$         | 32        | $+1$      | Gravitino Gauge Parameter         |
| $M_{cc}$    | 30        | $+4$      | Cosmo Mass                        |
| $g_1$       | 23        | $0$       | Chiral Coupling                   |
| $g_2$       | 23        | $0$       | Chiral Coupling                   |
| $g$         | 27        | $0$       | VEV parameter                     |
| $M_{1,mix} = g < A_1 > = gvM_O$ |       | 35,14     | Mass term from VEV                |
| $\langle A_1 \rangle = M_O v$ | 27 | 1 | VEV of $A_1$ |
| $\langle A_2 \rangle = M_O v$ | 27 | 1 | VEV of $A_2$ |
| $\langle G_1 \rangle = -M_O^2 \frac{g_1 g_2}{g_1^2 + g_2^2}$ | 27 | 2 | VEV of $G_1$ |
| $\langle G_2 \rangle = M_O^2 \frac{g_2^2}{g_1^2 + g_2^2}$ | 27 | 2 | VEV of $G_2$ |

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