Query-based Adversarial Attacks on Graph with Fake Nodes

Zhengyi Wang 1 Zhongkai Hao 1 Hang Su 1 Jun Zhu 1

Abstract

While deep neural networks have achieved great success on the graph analysis, recent works have shown that they are also vulnerable to adversarial attacks where fraudulent users can fool the model with a limited number of queries. Compared with adversarial attacks on image classification, performing adversarial attack on graphs is challenging because of the discrete and non-differential nature of a graph. To address these issues, we proposed Cluster Attack, a novel adversarial attack by introducing a set of fake nodes to the original graph which can mislead the classification on certain victim nodes. Specifically, we query the victim model for each victim node to acquire their most adversarial feature, which is related to how the fake node’s feature will affect the victim nodes. We further cluster the victim nodes into several subgroups according to their most adversarial features such that we can reduce the searching space. Moreover, our attack is performed in a practical and unnoticeable manner: (1) We protect the predicted labels of nodes which we are not aimed for from being changed during attack. (2) We attack by introducing fake nodes into the original graph without changing existing links and features. (3) We attack with only partial information about the attacked graph, i.e., by leveraging the information of victim nodes along with their neighbors within k-hop instead of the whole graph. (4) We perform attack with a limited number of queries about the predicted scores of the model in a black-box manner, i.e., without model architecture and parameters. Extensive experiments demonstrate the effectiveness of our method in terms of the success rate of attack.

1. Introduction

Graphs play important roles in many domains by representing the relationships between real-world identities. Recent research in Graph Neural Networks (GNNs) has shown a promising performance on various applications to graph data including the recommendation systems (Ying et al., 2018), social networks (Qiu et al., 2018), electronic commerce (Chen et al., 2019), etc. Just like other types of deep learning models, recent studies have shown that GNNs are vulnerable to adversarial attack (Dai et al., 2018; Zügner et al., 2018). The performance of a well-trained GNN can be significantly degenerated by adversarial manipulations, which are carefully crafted inputs with small perturbations added. For example, adversarial attacks on graph may ruins the credit prediction application if one fraudulent user can disguise himself by adding illegal friendship connections with others, which may cause severe consequences.

Compared with the adversarial attacks on image classification (Madry et al., 2018; Athalye et al., 2018), the study of adversarial attacks on graph data is at its infancy age. As the graph data have much more complex structures, researchers have proposed different adversarial attack setups includes different assumptions of the attackers’ knowledge (white-box or black box), perturbation strategies (node injection or feature modification), and the constraints on adversarial attacks (norm of feature modification or number of edge modifications). Representative works includes greedy-method (Zügner et al., 2018), RL (reinforcement learning) etc.
learning)-based method (Dai et al., 2018; Sun et al., 2020) and gradient-based method (Wu et al., 2019b; Xu et al., 2019a). Despite numerous efforts, there still exists significant gap between most of the existing attack setups and the practice. The unreasonable assumptions include that some attackers can alter a large proportion of nodes, or the attacker may have full knowledge about the underlying GNN model for the victims. Some works only evaluate the performance on the victim nodes without considering the side effects on other graph node, which will make attacks more detectable.

In this work, we consider a more practical scenario in adversarial attack on graph data, which aim to mislead the predicted labels of certain victim nodes without sacrificing the prediction on other nodes significantly by adding mild perturbation to the graph. In our setting, we consider to protect the neighbors of victim nodes within $k$-hop from being misclassified. Moreover, we choose to introduce extra fake nodes into the original graph as the perturbation allowed for us to manipulate the graph instead of directly modifying the original graph, since it is more practical to add fake nodes in real-world scenario rather than modifying the existing nodes and edges. For example, it’s often prohibitive for the attacker to have others’ password in a financial network and it is much more practical to register fake accounts and make connection to the attacked users.

It is also noted that in the practical scenarios, we can only access part of the graph since it is usually impossible to observe the whole graph especially when attacking a large network. In our setting, we can only access partial information of the graph data by observe the nodes being attacked with their neighbor nodes within $k$-hop. The other part of the graph leaves unknown to us, which means that we cannot utilize the whole graph structure when performing attack. As a more challenging setup, we have no access to the model parameters, which means a black-box attack. We can only have a limited number of queries on the victim model about the predicted scores of certain nodes when performing adversarial attack. For example, in a financial network we may query the credit of attacked users for several times and adjust the attributes and the connections of the fake accounts accordingly to perform our attack.

To summarize, we study a more restricted and more practical adversarial attacks for node classification on graph data, which considers the scenarios that the adversary has only partial information of the graph data and black-box knowledge of the model parameters. Moreover, our setup also differs from existing works with a novel constraints on the side effects of the adversarial attack that the attacker also aims to preserve the predictions for other nodes in the graph, which will reduce the probability to be detected. The challenges of our attacks also lie in the discrete and non-differential nature of the graph which makes it hard to directly apply the existing query-based adversarial attack method based on gradient methods. Directly applying greedy search requires a large number of queries to the model due to the large searching space which is not affordable. It’s even more challenging since we have to decide both the attributes and the connections of the fake nodes at the same time.

To tackle the discrete optimization problem, we propose our Cluster Attack, which attacks by divide the victim nodes into several clusters according to their most adversarial features, which can be approximately computed within limited queries. The most adversarial feature proposed by us is the feature of the fake node connected to the victim node which minimizes our adversarial loss. It represents the vulnerability of each victim node. The problem becomes several sub-problems in each cluster which are easy to optimize. We connect each fake node with each cluster and use the average of most adversarial feature of each victim node in the cluster to initialize the corresponding fake node’s feature. After that a greedy method is applied to enhance the fake node’s feature. Thus our method prevents the time-consuming and query-consuming searching of the large searching space.

To attack certain victim nodes without affecting others, we formulate our adversarial loss as a trade-off formulation between victim nodes and protected nodes.

Our contribution can be summarized as follows:

- We propose a practical threat model on graph adversarial attack. We perform query-based adversarial attack on graph with partial information about the graph with non-targeted nodes protected.
- We propose Cluster Attack, an effective and efficient adversarial attack on graph structured data which attacks by divide victim nodes into several smaller clusters. Our method decides both the attributes and the connections of the fake nodes at the same time with a limited budget of queries.

2. Background

In this section, we first describe our setting of node classification along with related work and then summarize related works on general adversarial attack and adversarial attack on graph specifically.

2.1. Node Classification on Graph

Node classification on graph has drawn increasing attention in recent years. Many methods have been proposed including (Kipf & Welling, 2016; Hamilton et al., 2017; Veličković et al., 2017; Wu et al., 2019a; Xu et al., 2019b). They make classification by aggregating the information from neighboring nodes.
Recent works witness node classification using one of the most representative methods, GCN (Graph Convolutional Network) (Kipf & Welling, 2016). Given a graph $G = (A, X)$, where $A \in \{0, 1\}^{N \times N}$ represents the adjacency matrix and $X \in \{0, 1\}^{N \times D}$ represents the feature matrix, where $N$ is the number of nodes of graph $G$ and $D$ is the dimension of node’s feature. Given a subset of labeled nodes in the graph denotes as $\Phi$, the goal of GCN is to predict the labels of the remaining unlabeled nodes in the graph. The GCN (Kipf & Welling, 2016) makes the prediction as

$$f(G) = \text{softmax}(\hat{A} \sigma(AXW^{(0)})W^{(1)}),$$

where $\hat{A} = \hat{D}^{-\frac{1}{2}}(A + I)\hat{D}^{-\frac{1}{2}}$ is normalized adjacency matrix with $\hat{D}$ being the diagonal degree matrix as $\hat{D}_{ii} = \sum_j(A + I)_{ij}$; $W^{(0)} \in \mathbb{R}^{D \times DH}$ and $W^{(1)} \in \mathbb{R}^{DH \times DL}$ are parameter matrices where $D_H$ and $D_L$ denote the dimension of hidden layer and the number of the categories of the labels, respectively; $\sigma$ is the activation function $\sigma(x) = \text{ReLU}(x) = \max(x, 0)$; and $f(G) \in \mathbb{R}^{N \times DL}$ is the prediction corresponding to the probability of each node to be each label.

### 2.2. Adversarial Attacks

Ever since the seminal work of (Szegedy et al., 2013), adversarial attack on deep neural networks has drawn much attention. FGSM (Goodfellow et al., 2014), along with I-FGSM (Kurakin et al., 2016) and MI-FGSM (Dong et al., 2017), uses gradient-based method to perform white-box attack. (Moosavi-Dezfooli et al., 2015) proposed DeepFool, an algorithm treating the victim model as a hyper-plane classification model. (Carlini & Wagner, 2016) proposed C&W Attack, a powerful method based on projected gradient descent. One-pixel Attack (Su et al., 2019) performs adversarial attack on images by only modifying several pixels. ZOO (Chen et al., 2017) uses zeroth order optimization method to perform black-box attack. (Ilyas et al., 2018) proposed black-box adversarial attacks with limited queries and information.

In recent years, many methods were proposed to perform adversarial attack on graph. For greedy methods, NET-TACK (Zügner et al., 2018) proposed to attack the graph by greedily modifying the edges and features of the graph. (Wang et al., 2018) proposed to attack by adding fake nodes and then greedily modifying the edges and features. For methods based on reinforcement learning, (Dai et al., 2018) proposed RL-S2V to attack by changing existing edges in the graph, while (Sun et al., 2020) proposed NIPA to attack by adding fake nodes. (Xu et al., 2019a; Wu et al., 2019b; Chen et al., 2020b;a) proposed to attack using gradient information. (Zügner & Günnemann, 2019) uses meta-learning to attack. (Ma et al., 2019) attacks by Rewiring, a special operation on the graph. (Chang et al., 2019) proposed GFA, a restricted black-box adversarial framework. (Ma et al., 2020) proposed a black-box attack strategy manipulating the original graph. (Xu et al., 2020) works contemporarily with us which performs white-box attack by adding fake nodes. The above methods are either not adoptable in our setting or having poor performance in our setting.

### 3. Methodology

In this section, we describe the problem formulation of our attack along with the attack algorithm. We provide theoretical analysis about the convergence of Cluster Attack.

#### 3.1. Problem Formulation

Given a set of victim nodes $\Phi_\mathcal{A} \subseteq \Phi$ in the graph, our goal is to perform mild perturbations on the graph $G = (A, X)$, leading to $G^+ = (A^+, X^+)$, such that the predicted labels of as many nodes as possible in $\Phi_\mathcal{A}$ change. $A^+ = \begin{bmatrix} A & B \end{bmatrix}$ and $X^+ = \begin{bmatrix} X \\ X_{\text{fake}} \end{bmatrix}$, where B represents the connections between original nodes and fake nodes. $\Phi^+ = \Phi \cup \Phi_{\text{fake}}$ denotes the node set of $G^+$. Starting from $A_{\text{fake}} = 0, B = 0$, we manipulate $A_{\text{fake}}, B$ and $X_{\text{fake}}$, leading to as low classification accuracy on $\Phi_\mathcal{A}$ as possible. We perform evasion attack, which means that our model does not get retrained after perturbations.

Our thread model is defined as follows.

**Adversarial Budget** To ensure our perturbation is unnoticeable, we limit the number of new connections between fake nodes and original nodes by $\Delta_{\text{edge}}$. The connections between fake nodes, i.e., $A_{\text{fake}}$, are free for us to modify while the connections between original nodes, i.e., $A$, are not allowed for us to modify. We can decide $X_{\text{fake}}$ at will but cannot change $X$, the features of original nodes. We limit the number of fake node by the number of rows of $A_{\text{fake}}$.

**Protected Nodes** Owing to the non-i.i.d nature of graph data, attacking victim nodes may have side effects on their neighboring nodes which we are not aimed for. While attacking victim nodes, we aim to keep the labels of other nodes which are not targeted unchanged at the same time to make our perturbation unnoticeable. In our setting, we consider to protect $\mathcal{V}_k(\Phi_\mathcal{A})$, neighbors of victim nodes within $k$-hop from being misclassified.

**Partial Information** The attacker is only accessible to partial graph. We can only observe the victim nodes with their neighbors within $k$-hop and fake nodes and we can only make connections between victim nodes and fake nodes. Also, we can only query the classification results of the victim nodes with their neighbors within $k$-hop and fake nodes. The main graph structure leaves unknown to us.
**Query-based Adversarial Attacks on Graph with Fake Nodes**

### Limited Queries

It’s more practical in real-world scenario that we have a limited number of queries to victim model than that we have full outputs of arbitrarily many chosen inputs. In our setting, we can totally query $K$ times for the predicted scores of all victim nodes with their neighbors within $k$-hop. The architecture and parameters about victim model are unknown by the attacker.

We aim to make the classifier misclassify as many nodes in $\Phi_A$ as possible. We formulate our problem as an optimization problem. Directly optimizing the number of misclassified nodes is difficult as the objective is discrete. Thus we try to optimize a substitute loss function as

$$\min_{G^+} \mathbb{L}(G^+; \Phi_A) \triangleq \sum_{v \in \Phi_A} \ell(G^+, v) + \lambda \sum_{v \in N_k(\Phi_A)} \ell_{\mathcal{N}}(G^+, v),$$

subject to $N_r(G^+) \leq N_{fake}$, $N_c(G^+) \leq \Delta_{edge}$.

where $G^+ = ([A \ B] \ [T \ X])$, $N_r(G^+)$ denotes the number of rows of matrix $A_{fake}$ and is no more than $N_{fake}$, which means that we at most introduce $N_{fake}$ fake nodes into the original graph. $N_c(G^+)$ represents the number of non-zero elements of $B$ and is no more than $\Delta_{edge}$, which means that we can at most add $\Delta_{edge}$ extra links. $\ell(G^+, v)$ and $\ell_{\mathcal{N}}(G^+, v)$ represents loss function for every victim node and every protected node, respectively. Smaller $\ell(G^+, v)$ means node $v$ is more likely to be misclassified by victim model $f$ and smaller $\ell_{\mathcal{N}}(G^+, v)$ means the predicted label of node $v$ is less likely to be changed during our attack. We perform targeted attack, which means the labels of victim nodes have to be misclassified as the ones which we specify.

Here we choose the C&W loss (Carlini & Wagner, 2016)

$$\ell(G^+, v) = \max_{y_i \neq y_i} \max \{ [f(G^+)]_{v, y_i} - [f(G^+)]_{v, y_i}, 0 \},$$

for our attack, where $y_i$ stands for the target label of node $v$ and the attacker succeeds only when node $v$ is misclassified as $y_i$. $[f(G^+)]_{v, y_i}$ denotes the output value of node $v$ having label $y_i$. For protected nodes, we have

$$\ell_{\mathcal{N}}(G^+, v) = \max_{y_i \neq y_i} \max \{ [f(G^+)]_{v, y_i} - [f(G^+)]_{v, y_i}, 0 \},$$

where $y_i$ stands for the ground-truth label of node $v$ provided by victim model. Overall loss $\mathbb{L}(G^+; \Phi_A)$ is summed over the loss of each victim node along with the loss of each protected neighboring node with a trade-off parameter $\Delta$.

Optimization of Eq. (2) is challenging due to the discrete nature of $G^+$ and large space of possible choices of $G^+$. To tackle the optimization problem, we propose our Cluster Attack.

### 3.2. Cluster Attack

For our problem, the main concern is to find a query-efficient method to determine the connections between the original nodes and the fake nodes and determine the corresponding features of the fake nodes.

In an adversarial scenario, it is often the case that the number of fake nodes we are allowed to add to the graph is much smaller than the number of victim nodes. To make use of each fake node better, we have to connect every fake node to several victim nodes. However, due to the structural complexity of the graph, different victim nodes may have very different local structures and the corresponding feature information, especially when our victim nodes are sparsely scattered in the whole graph. Consequently, a fake node with certain feature may change the predicted label of one victim node after connecting to it, but may not change another victim node’s.

Building on the above perspective, we have the insight that, if we connect a fake node to several victim nodes which share a similarity that their predicted labels are all easily changed after they are connected to fake nodes with similar features, then we have more chance of changing the predicted labels of those victim nodes. As a result, we can divide the victim nodes into several clusters according to this similarity and for each cluster we assign one fake node to attack.

**Most Adversarial Feature**

In our attack, we use the most adversarial feature, which represents the vulnerability of each victim node, to denote the above similarity of victim nodes. It’s the feature of the fake node connected to the victim node which minimizes our loss.

Formally, for victim node $v$, consider that we only connect one fake node to and only to $v$. (To achieve this, we may temporarily remove all the extra edges in $G^+$ and temporarily add one edge connecting this victim node and one fake node $v_f$. After the computation, the edges in $G^+$ become the same before the computation.) $x_{vf}$ is the feature of the newly added fake node $v_f$. The MAF of $v$ is defined as followed

$$\text{MAF}(v) = \arg\min_{x_{vf}} \mathbb{L}(G^+; \Phi_A),$$

where $\mathbb{L}(G^+; \Phi_A)$ is defined in Eq. (2). Although the computation of MAF depends on an extra fake node, the MAF is computed for every victim node and it’s related to the victim node’s gradient towards adversarial examples.

Direct computation of MAF for every victim node may be extremely time-consuming and intractable with only a limited number of queries. Here we propose an greedy algorithm to approximate $\text{MAF}(v)$ with $K_i$ queries for each...
victim node which is summarized in Algorithm 1. For conve-
ience and consistency, we use the same notation MAF($v$) to denote approximated MAF of node $v$ computed using
Algorithm 1 in the following sections.

Algorithm 1 Fast Approximation of MAF with a Fixed
Number of Queries

\begin{algorithm}
\textbf{Input:} Graph $G^+ = (A^+, X^+)$. Victim node $v$. Number of
queries $K$.
\textbf{Output:} Approximated most adversarial feature $\text{MAF}(v)$ for $v$.
\begin{enumerate}
\item \textbf{initialize} Choose one fake node $v_f$ and connect it to and
only to $v$, randomly initialize the fake node’s feature $x_{v_f}$. Keep other fake nodes isolated.
\item Randomly sample a sequence $I_t$ from $\{1, 2, ..., D\}$ with
length $K_t$. $D$ is the dimension of nodes’ feature.
\item \textbf{for} $i \in I_t$ \textbf{do}
\item \hspace{0.5cm} if $x_{v_f}[i] \leftarrow 1 - x_{v_f}[i]$ makes $L(G^+; \Phi_A)$ smaller
\hspace{0.5cm} then
\item \hspace{1cm} $x_{v_f}[i] \leftarrow 1 - x_{v_f}[i]$
\item \hspace{0.5cm} end if
\item \textbf{end for}
\item \textbf{return} $\text{MAF}(v) \leftarrow x_{v_f}$
\end{enumerate}
\end{algorithm}

Clustering the Victim Nodes After the computation of most adversarial features, we aim to divide the victim nodes $\Phi_A$ into $N_{fake}$ clusters $C = \{C_1, C_2, ..., C_{N_{fake}}\}$ according to their MAFs to minimize the object function of clustering

$$
\min_C \sum_{C_i \in C} \sum_{v \in C_i} \|\text{MAF}(v) - c_i\|_2^2, 
$$

s.t. \hspace{1cm} \sum_{C_i \in C} |C_i| \leq \Delta_{edge},

where $\| \cdot \|_2$ denotes $l^2$-norm. We have $\sum_{C_i \in C} |C_i| \leq \Delta_{edge}$ since it can be verified that $\sum_{C_i \in C} |C_i|$ equals to $\mathbb{N}_e(G^+)$ using Eq. (8) and it has to be not larger than $\Delta_{edge}$.

And

$$
c_i = \frac{1}{|C_i|} \sum_{v \in C_i} \text{MAF}(v),
$$

denotes the cluster center of each cluster $C_i$. The optimization of Eq. (6) can be easily solved by any cluster algorithm. After the computation of clusters $C$, we connect the $i$th fake node $v_{N+i}$ to nodes in cluster $C_i$ accordingly, i.e.,

$$
B_{ij} = \begin{cases} 1, & \text{if } v_j \in C_i \\ 0, & \text{otherwise} \end{cases}
$$

Feature of Fake Nodes It’s intuitive that the feature of every fake node in our attack derives from the MAFs of the victim nodes that it’s connected to. For our attack, we use the cluster center of the MAFs of the victim nodes as the fake node’s feature,

$$
x_{v_{N+i}} = c_i,
$$

where $x_{v_{N+i}}$ is the feature of the $i$th fake node $v_{N+i}$. In practice, we use greedy search to enhance the fake node feature $x_{v_{N+i}}$ computed by Eq. (9). We make extra $K_f$ queries for every fake node to enhance their features.

Overall Algorithm In all, our method is summarized in
Algorithm 2. Figure 2 is an overview of our attack algorithm. We only consider the case that $\Delta_{edge} > |\Phi_A|$, otherwise some victim nodes do not have the chance to connect directly to fake nodes. For now, we keep $A_{fake} = 0$ during our whole algorithm. We leave utilizing $A_{fake}$, the connections between fake nodes as our future work.

Our method solves the optimization problem in Eq. (2) by minimize the object function Eq. (6), which can be easily solved by any cluster algorithm. Our method prevents the time-consuming search of the large space of $A_{fake}, B, X_{fake}$ and is thus an efficient method.

3.3. Theoretical Analysis

To show the convergence of Cluster Attack, we study the gradient of the classifier inspired by (Andriushchenko et al.,
Algorithm 2 Cluster Attack

Input: Graph $G^+ = (A^+, X^+)$. Victim node set $\Phi_A$. Number of fake nodes $N_{fake}$. Number of query $K_i$ for each victim node and $K_f$ for each fake node.

Output: Manipulated graph $G^+ = (A^+, X^+)$. 

1: Initialize $A^+ = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}$ and randomly initialize $X_{fake}$.
2: for all $v \in \Phi_A$ do
3: \hspace{10pt} Compute $MAF(v)$ using Algorithm 1 with $K_i$ queries.
4: end for
5: Divide nodes in $\Phi_A$ into $N_{fake}$ clusters according to their $MAF$ values minimizing Eq. (6), using cluster algorithm.
6: Decide $B$ using Eq. (8).
7: Initialize $X_{fake}$ using Eq. (9) with rounding.
8: for $i = 1, 2, \ldots, N_{fake}$ do
9: \hspace{10pt} Randomly sample a sequence $I_f$ from $\{1, 2, \ldots, D\}$ with length $K_f$. $D$ is the dimension of nodes’ feature.
10: for $j \in I_f$ do
11: \hspace{10pt} if $x_{vN+},[j] = 1 - x_{vN+},[j]$ makes $L(G^+; \Phi_A)$ smaller then
12: \hspace{20pt} $x_{vN+},[j] = 1 - x_{vN+},[j]$
13: \hspace{10pt} end if
14: end for
15: end for
16: return $G^+ = (A^+, X^+)$

Proof 1 The key step of the proof is to use the Lipschitz inequality to bound the gradient for every node and then sum them together.

We write the Lipschitz inequality for $L$-smooth function:
\[
\|\nabla g(x_u) - \nabla g(x_v)\|_2 \leq L \|x_u - x_v\|_2,
\]
where $u$ and $v$ are 2 arbitrarily chosen nodes. In $i$-th iteration, the update is $\delta_i = x_u^{(i+1)} - x_v^{(i)}$, we have
\[
g(x_u^{(i+1)}) \leq g(x_v^{(i)}) + \langle \nabla g(x_v^{(i)}), \delta_i \rangle + \frac{L}{2} \|\delta_i\|_2^2.
\]
Then we have
\[
g(x_u^{(i+1)}) \leq g(x_v^{(i)}) + \min \left\{ 0, \langle \nabla g(x_v^{(i)}), \delta_i \rangle + \frac{L}{2} \|\delta_i\|_2^2 \right\}.
\]

Notice that $\min \{a, b\} = \frac{a+\min\{|a-b|,0\}}{2}$, we have
\[
g(x_u^{(i+1)}) \leq g(x_v^{(i)}) + \frac{L}{2} \|\delta_i\|_2^2.
\]

By taking expectation, we have
\[
\mathbb{E} g(x_u^{(i+1)}) \leq \mathbb{E} g(x_v^{(i)}) - \frac{1}{2} \mathbb{E} \|\nabla g(x_v^{(i)}), \delta_i\|_2^2.
\]
Since the $\delta_i$ satisfies $\|\delta_i\|_2^2 \leq 1$, we have
\[
\mathbb{E} g(x_u^{(i+1)}) \leq \mathbb{E} g(x_v^{(i)}) - m \mathbb{E} \|\nabla g(x_v^{(i)})\|_2 + \frac{L}{2}.
\]
where $m > 0$ is a constant. Then
\[
\mathbb{E} \|\nabla g(x_v^{(i)})\|_2 \leq \frac{2}{m} \left( \mathbb{E} g(x_v^{(i)}) - \mathbb{E} g(x_v^{(i+1)}) + \frac{L}{2} \right).
\]
sum these steps from 0 to $|I|$,
\[
\min_{0 \leq i \leq |I|} \mathbb{E} \|\nabla g(x_v^{(i)})\|_2 \leq \frac{1}{|I|} \sum_{0 \leq i \leq |I|} \mathbb{E} \|\nabla g(x_v^{(i)})\|_2 \leq \frac{2}{|I|m} \left( g(x_v^{(0)}) - \mathbb{E} g(x_v^{(|I|+1)}) \right) + \frac{L}{m}
\]
At last we sum for all victim nodes in the graph,
\[
\min_{0 \leq i \leq |I|} \sum_{v \in \Phi_A} \mathbb{E} \|\nabla g(x_v^{(i)})\|_2 \leq \sum_{v \in \Phi_A} \left( \frac{2}{|I|m} (g(x_v^{(0)}) - \mathbb{E} g(x_v^{(|I|+1)})) + \frac{L}{m} \right).
\]
This proposition shows that the gradient of fake node’s feature after $|I_f|$ steps of update converges at a level of $\Theta(L)$ which suggests that the gradient can be bounded by a constant that depends on the Lipschitz constant. Since the features in our problem are discrete values, the gradient might not be able to converge to 0.

The following proposition demonstrates that the optimization of fake nodes’ features will stop at a local minima after certain iterations.

**Proposition 2** The optimization of fake nodes’ features will stop at a local minima after finite iterations.

**Proof 2** For a given node $v$, $D$ is the dimension of its feature. There are at most $2^D$ cases for its feature and the corresponding loss value $\mathcal{L}(G^+, \Phi_A)$. In each iteration our algorithm optimize the feature to make the loss smaller. Since there are more no than $2^D$ different loss values, after $2^D$ steps of optimizations the feature of fake node $v$ will converge to a local minima.

The above propositions support the convergence of our method to a certain degree.

### 4. Experiments

#### 4.1. Experimental Setup

We do our experiments on Cora and Citeseer (Sen et al., 2008), two benchmark citation networks. Nodes in the datasets are papers and the edges represent the relationship of citation. In this case, fake nodes and extra edges may be papers of low quality without peer review process and citations which may be a potential threat when analyzing academic data using machine learning models. The statistics of the datasets are shown in Table 1.

For each experimental setting, we run 100 times of experiments and report the average results. Each round we randomly sample $|\Phi_A|$ nodes in the graph as victim nodes. To reduce the variance in the training process of victim model, we retrain the victim GCN model every 5 rounds of attack. Number of queries $K$ is set to $K = |\Phi_A| \cdot K_t + N_{fake} \cdot K_f$, where we set $K_t = D$ and $K_f = D$. We set $k = 1$ in $N_k(\Phi_h)$, which means we can only observe 1-hop neighbors of victim nodes and we aim to protect those 1-hop neighbors. By default we set trade-off parameter $\lambda = 0$ without specification. We currently choose $K$-Means as our cluster algorithm and leave the exploration of more adaptive options of cluster algorithms as future work.

Although there are lots of proposed method to perform adversarial attack on graph, many of them cannot be easily adapted in our query-based setting without a great loss of performance. We include the following baselines:

**Random Attack** Randomly decide the fake nodes’ features and randomly choose the connections between fake nodes and original nodes.

**NETTACK** We adapt NETTACK (Zügner et al., 2018), one of the most effective attack, in our setting. We add several nodes at once and then greedily add edges between the fake nodes and original nodes and decide the features of the fake nodes.

**NETTACK - Sequential** We adapt NETTACK (Zügner et al., 2018) to a greedy method by sequentially adding fake nodes. Greedily make connections between the newly added fake node and original nodes and greedily decide the feature of the fake node every time a fake node is added.

**Fake Node Attack** Fake Node Attack (Wang et al., 2018) attacks by adding fake nodes. It decides the links between fake nodes and victim nodes and the feature of fake nodes using simple greedy method.

**KDD Cup 1st Attack** We adapt the method proposed by (kdd, 2020). It won 1st place in KDD Cup competition on graph adversarial attack. Since the feature in our experiments are discrete compared to its original concrete setting, we use our greedy method to attack instead of the original gradient descent.

We don’t include NIPA (Sun et al., 2020) as our baseline. It is based on reinforcement learning and it’s too time-consuming and impractical to train an RL-agent for every round of our attack. For baselines, we don’t limit the number of queries.

#### 4.2. Quantitative Evaluation

##### 4.2.1. PERFORMANCE WITH DIFFERENT NUMBER OF VICTIM NODES.

We first evaluate the performance of Cluster Attack along with other baselines with different number of victim nodes. Without loss of generality, we uniformly set $N_{fake} = 4$, $\Delta_{edge} = |\Phi_A|$ and let the number of victim nodes varies to see the performance under different $N_{fake} : |\Phi_A|$. We compare Cluster Attack with other baselines. The results are shown in Table 2. Our algorithm outperforms all baselines in terms of success rates. The success rate of Cluster Attack gets higher when the number of victim nodes gets smaller with a fixed number of fake nodes. We conjecture that this

| Table 1. Statistics of the datasets |
|-----------------------------------|
| Name    | Nodes | Edges | Features | Classes |
|---------|-------|-------|----------|---------|
| Cora    | 2708  | 5429  | 1433     | 7       |
| Citeseer| 3327  | 4732  | 3702     | 6       |
Table 2. Success rates of Cluster Attack along with other baselines. \( T \) denotes number of victim nodes. For \( T = 3 \), we only add 3 fake nodes in our Cluster Attack.

| Method                  | Cora  |        |        |        | Citeseer |        |        |        |
|-------------------------|-------|--------|--------|--------|----------|--------|--------|--------|
|                         | \( T = 3 \) | \( T = 5 \) | \( T = 7 \) | \( T = 10 \) | \( T = 3 \) | \( T = 5 \) | \( T = 7 \) | \( T = 10 \) |
| Random                  | 0.07  | 0.08   | 0.04   | 0.05   | 0.04     | 0.02   | 0.03   | 0.03   |
| NETTACK                 | 0.61  | 0.57   | 0.55   | 0.53   | 0.75     | 0.71   | 0.66   | 0.61   |
| NETTACK - Sequential    | 0.68  | 0.73   | 0.72   | 0.70   | 0.76     | 0.74   | 0.72   | 0.67   |
| Fake Node Attack        | 0.61  | 0.58   | 0.54   | 0.52   | 0.76     | 0.68   | 0.62   | 0.60   |
| KDD Cup 1st Attack      | 0.61  | 0.55   | 0.51   | 0.42   | 0.55     | 0.56   | 0.51   | 0.45   |
| Cluster Attack          | 0.99  | 0.93   | 0.84   | 0.72   | 1.00     | 0.89   | 0.80   | 0.70   |

is because the number of victim nodes in each cluster get smaller and thus the influence of the fake node on each victim node get relatively larger.

4.2.2. PERFORMANCE WITH DIFFERENT NUMBER OF FAKE NODES.

In this section, we evaluate the performance of Cluster Attack along with other baselines with different number of fake nodes. We fix the number of victim nodes at 10 and vary the number of fake nodes to examine the success rates. We set \( \Delta_{\text{edge}} = |\Phi_A| = 10 \). The success rates are shown in Figure 3. We only list the results of NETTACK - Sequential in Figure 3 without NETTACK since we found that NETTACK - Sequential performs better than NETTACK. We see that the success rate is higher when there are more fake nodes. For Cluster Attack, we conjecture that this is because the number of clusters get larger when there are more fake nodes. Thus the MAFs of the victim nodes in each cluster are able to be closer to each other and they are easier to be attacked by the same fake node. Among all methods, our Cluster Attack achieves the highest success rate.

![Figure 3. Success Rates of Cluster Attack with Different Number of Fake Nodes.](image)

4.2.3. PERFORMANCE WITH DIFFERENT TRADE-OFF PARAMETER \( \lambda \).

In this section, we examine the performance of Cluster Attack with different trade-off parameter \( \lambda \) between fake nodes and protected nodes. We examine the performance under different \( \lambda \) in Cora dataset. We uniformly set \( N_{\text{fake}} = 4 \), \( \Delta_{\text{edge}} = |\Phi_A| = 10 \). We choose 2 most competitive baselines, NETTACK - Sequential and Fake Node Attack and adapt their loss function to our trade-off format. The results are shown in Figure 4. It can be seen from Figure 4 that our algorithm performs the best among all baselines in terms of attack success rates. When \( \lambda \) goes up, which means that we pay more attention to the protected nodes, the percentage of protected nodes whose labels remain unchanged during attack goes up while the success rates of attack drops. When \( \lambda \) gets large enough (\( \lambda \geq 10 \)), nearly all protected nodes are successfully protected, which demonstrates the effectiveness of our trade-off formulation in our loss function Eq. (2). It shows that to protect the labels of not-targeted nodes from being changed during attack, we can simply set a large \( \lambda \). Also, our trade-off formulation between victim nodes and protected nodes in Eq. (2) not only applies to our Cluster Attack, but also applies to other baselines.

![Figure 4. Cluster Attack along with Other Baselines in Cora with Different \( \lambda \).](image)

4.2.4. PERFORMANCE WITH DIFFERENT NUMBER OF QUERIES.

In this section, we examine the performance of Cluster Attack with different number of queries. We set \( K_t = K_f = \alpha \cdot D \) and examine the performance under different \( \alpha \).
in Cora and Citeseer dataset. We uniformly set \( N_{\text{fake}} = 4 \), \( \Delta_{\text{edge}} = |\Phi_A| = 10 \). The results are shown in Figure 5. The success rate of Cluster Attack drops as the number of queries drops. Our algorithm still performs well when the number of queries drops not very much, especially when \( \alpha \geq 0.4 \). It demonstrates that the most adversarial feature can be approximated and the fake nodes’ features can be optimized with a smaller number of queries without a great decrease in the success rate. It shows that our Cluster Attack can work in a query-efficient manner.

**Figure 5.** Success Rates of Cluster Attack with Different Number of Queries in Cora and Citeseer.

### 4.2.5. Analysis of Cluster Attack on Victim Nodes with Different Degrees.

In this section, we evaluate the performance of Cluster Attack on victim nodes with different degrees. We uniformly set \( N_{\text{fake}} = 4 \), \( \Delta_{\text{edge}} = |\Phi_A| = 10 \). The success rates of Cluster Attack on victim nodes with different degrees are shown in Figure 6 along with the proportion of sampled victim nodes with each degree. Victim nodes with degrees larger than or equal to 7 are counted together since they only account for a small proportion.

**Figure 6.** Success Rates of Cluster Attack on Victim Nodes with Different Degrees.

It can be seen from Figure 6 that victim nodes with higher degrees are more robust to our attack in general. We conjecture that when a victim node has a relatively large number of neighbors, adding one fake node as its neighbor has less impact on it and thus is less likely to change its predicted label.

### 4.2.6. Ablation Study.

In this section, we examine the effectiveness of our most adversarial feature (MAF). We uniformly set \( N_{\text{fake}} = 4 \), \( \Delta_{\text{edge}} = |\Phi_A| = 10 \). We compare the success rate of Cluster Attack without MAF, i.e., the victim nodes’ MAFs are randomly set. The results are shown in Table 3. Cluster Attack without MAF performs worse than normal Cluster Attack with MAF, which demonstrates the effectiveness of our MAF. MAF is related to the vulnerability of victim nodes. Nodes with similar MAFs in a cluster are easier to be affected together by one fake node. Thus the success rate of Cluster Attack with MAF is better than without MAF.

**Table 3.** Success Rates of Cluster Attack with and without MAF in Cora.

| Method                  | Success Rate |
|-------------------------|--------------|
| Cluster Attack - without MAF | 0.62         |
| Cluster Attack           | 0.72         |

### 5. Conclusion

In this paper, we propose Cluster Attack, an algorithm of adversarial attack on graph structured data. We perform query-based black-box adversarial attack on graph by adding fake nodes with partial information about the graph. To make our adversarial perturbation unnoticeable, we further consider to protect the predicted labels of neighboring nodes of victim nodes from being changed. We propose to attack by clustering the victim nodes according to the similarity in their most adversarial features, which can be approximated by a limited number of queries. Experimental results demonstrate our method has strong performance in terms of success rate of attacking.

### References

Kdd cup, 2020. URL https://www.biendata.xyz/competition/kddcup_2020/.

Andriushchenko, M., Croce, F., Flammarion, N., and Hein, M. Square attack: A query-efficient black-box adversarial attack via random search. In Vedaldi, A., Bischof, H., Brox, T., and Frahm, J. (eds.), Computer Vision - ECCV 2020 - 16th European Conference, Glasgow, UK, August 23-28, 2020, Proceedings, Part XXIII, volume 12368 of Lecture Notes in Computer Science, pp. 484–501. Springer, 2020. doi: 10.1007/978-3-030-58592-1_29. URL https://doi.org/10.1007/978-3-030-58592-1_29.

Athalye, A., Carlini, N., and Wagner, D. Obfuscated gradients give a false sense of security: Circumventing de-
Query-based Adversarial Attacks on Graph with Fake Nodes

fenses to adversarial examples. In ICML, pp. 274–283, 2018.

Carlini, N. and Wagner, D. Towards evaluating the robustness of neural networks, 2016.

Chang, H., Rong, Y., Xu, T., Huang, W., Zhang, H., Cui, P., Zhu, W., and Huang, J. A restricted black-box adversarial framework towards attacking graph embedding models, 2019.

Chen, J., Chen, Y., Zheng, H., Shen, S., Yu, S., Zhang, D., and Xuan, Q. MGA: momentum gradient attack on network. CoRR, abs/2002.11320, 2020a. URL https://arxiv.org/abs/2002.11320.

Chen, J., Chen, Y., Zheng, H., Shen, S., Yu, S., Zhang, D., and Xuan, Q. Mga: Momentum gradient attack on network, 2020b.

Chen, P.-Y., Zhang, H., Sharma, Y., Yi, J., and Hsieh, C.-J. Zoo: Zeroth order optimization based black-box attacks to deep neural networks without training substitute models. In Proceedings of the 10th ACM Workshop on Artificial Intelligence and Security. AISec ’17, pp. 15–26, New York, NY, USA, 2017. Association for Computing Machinery. ISBN 9781450352024.

Chen, W., Gu, Y., Ren, Z., He, X., Xie, H., Guo, T., Yin, D., and Zhang, Y. Semi-supervised user profiling with heterogeneous graph attention networks. In IJCAI, volume 19, pp. 2116–2122, 2019.

Dai, H., Li, H., Tian, T., Huang, X., Wang, L., Zhu, J., and Song, L. Adversarial attack on graph structured data, 2018.

Dong, Y., Liao, F., Pang, T., Su, H., Zhu, J., Hu, X., and Li, J. Boosting adversarial attacks with momentum, 2017.

Goodfellow, I. J., Shlens, J., and Szegedy, C. Explaining and harnessing adversarial examples, 2014.

Hamilton, W. L., Ying, R., and Leskovec, J. Inductive representation learning on large graphs, 2017.

Ilyas, A., Engstrom, L., Athalye, A., and Lin, J. Black-box adversarial attacks with limited queries and information. In Proceedings of the 35th International Conference on Machine Learning. ICML 2018, July 2018. URL https://arxiv.org/abs/1804.08598.

Kipf, T. N. and Welling, M. Semi-supervised classification with graph convolutional networks, 2016.

Kurakin, A., Goodfellow, I., and Bengio, S. Adversarial examples in the physical world, 2016.

Ma, J., Ding, S., and Mei, Q. Towards more practical adversarial attacks on graph neural networks. arXiv: Learning, 2020.

Ma, Y., Wang, S., Derr, T., Wu, L., and Tang, J. Attacking graph convolutional networks via rewiring, 2019.

Madry, A., Makelov, A., Schmidt, L., Tsipras, D., and Vladu, A. Towards deep learning models resistant to adversarial attacks. In ICLR, 2018.

Moosavi-Dezfooli, S., Fawzi, A., and Frossard, P. Deepfool: a simple and accurate method to fool deep neural networks. CoRR, abs/1511.04599, 2015.

Qiu, J., Tang, J., Ma, H., Dong, Y., Wang, K., and Tang, J. Deepinf: Social influence prediction with deep learning. In Proceedings of the 24th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining, pp. 2110–2119, 2018.

Sen, P., Namata, G., Bilgic, M., Getoor, L., Gallagher, B., and Eliassi-Rad, T. Collective classification in network data. AI Magazine, 29(3):93, Sep. 2008. doi: 10.1609/aimag.v29i3.2157. URL https://www.aaai.org/ojs/index.php/aimagazine/article/view/2157.

Su, J., Vargas, D. V., and Sakurai, K. One pixel attack for fooling deep neural networks. IEEE Transactions on Evolutionary Computation, 23(5):828–841, Oct 2019. ISSN 1941-0026.

Sun, Y., Wang, S., Tang, X., Hsieh, T.-Y., and Honavar, V. Adversarial attacks on graph neural networks via node injections: A hierarchical reinforcement learning approach. In Proceedings of The Web Conference 2020, pp. 673–683, 2020.

Szegedy, C., Zaremba, W., Sutskever, I., Bruna, J., Erhan, D., Goodfellow, I., and Fergus, R. Intriguing properties of neural networks, 2013.

Velicković, P., Cucurull, G., Casanova, A., Romero, A., Liò, P., and Bengio, Y. Graph attention networks, 2017.

Wang, X., Cheng, M., Eaton, J., Hsieh, C.-J., and Wu, F. Attack graph convolutional networks by adding fake nodes, 2018.

Wu, F., Zhang, T., de Souza Jr., A. H., Fifty, C., Yu, T., and Weinberger, K. Q. Simplifying graph convolutional networks, 2019a.

Wu, H., Wang, C., Tyshetskiy, Y., Docherty, A., Lu, K., and Zhu, L. Adversarial examples on graph data: Deep insights into attack and defense, 2019b.
Xu, K., Chen, H., Liu, S., Chen, P.-Y., Weng, T.-W., Hong, M., and Lin, X. Topology attack and defense for graph neural networks: An optimization perspective, 2019a.

Xu, K., Hu, W., Leskovec, J., and Jegelka, S. How powerful are graph neural networks? In 7th International Conference on Learning Representations, ICLR 2019, New Orleans, LA, USA, May 6-9, 2019. OpenReview.net, 2019b. URL https://openreview.net/forum?id=ryGs6iA5Km.

Xu, X., Du, X., and Zeng, Q. Attacking graph-based classification without changing existing connections. In Annual Computer Security Applications Conference, ACSAC ’20, pp. 951–962, New York, NY, USA, 2020. Association for Computing Machinery. ISBN 9781450388580. doi: 10.1145/3427228.3427245. URL https://doi.org/10.1145/3427228.3427245.

Ying, R., He, R., Chen, K., Eksombatchai, P., Hamilton, W. L., and Leskovec, J. Graph convolutional neural networks for web-scale recommender systems. In Proceedings of the 24th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining, pp. 974–983, 2018.

Zügner, D. and Günnemann, S. Adversarial attacks on graph neural networks via meta learning. arXiv preprint arXiv:1902.08412, 2019.

Zügner, D., Akbarnejad, A., and Günnemann, S. Adversarial attacks on neural networks for graph data. Proceedings of the 24th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining - KDD ’18, 2018.