Gravitational Dark Matter Decay and the ATIC/PPB-BETS Excess

Fuminobu Takahashi\textsuperscript{(a)} and Eiichiro Komatsu\textsuperscript{(a,b)}

\textsuperscript{a} Institute for the Physics and Mathematics of the Universe, University of Tokyo, Chiba 277-8568, Japan

\textsuperscript{b} Department of Astronomy, The University of Texas at Austin, Austin, TX 78712, USA

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Abstract

The hidden scalar field, which couples to the visible sector only through Planck-suppressed interactions, is a candidate for dark matter owing to its long lifetime. Decay of such a scalar field offers observational tests of this scenario. We show that decay of the hidden scalar field can explain the observed excess of high-energy positrons/electrons observed by ATIC/PPB-BETS, for a suitable choice of the mass and the vacuum expectation value of the field. We also show that the same choice of the parameters gives the observed dark matter abundance. Such a remarkable coincidence suggests that the Planck-suppressed interactions may be responsible for the observed excess in the cosmic-ray positrons/electrons.
The presence of dark matter has been established firmly by numerous observations, e.g., [1]. While we have not yet understood the nature of dark matter, recent experimental data from the cosmic-ray physics may be providing us with new insight into dark matter properties. The PAMELA data [2] showed that the positron fraction starts to deviate from a theoretically expected value for secondary positrons at around 10 GeV, and continues to increase up to about 100 GeV. The ATIC balloon-borne experiment collaboration [3] has recently released their data, showing a clear excess in the total flux of electrons plus positrons peaked at around 600 − 700 GeV, in agreement with the PPB-BETS observation [4]. The excess may be explained by astrophysical sources such as pulsars [5, 6], microquasars [7] and gamma-ray bursts [8]. An alternative explanation is the decay or annihilation of dark matter particles.

In this letter we focus on the decaying dark matter scenario as an explanation for the observed excess of high-energy positrons and electrons. In order to explain the ATIC/PPB-BETS excess, the dark matter particles must produce electrons and positrons with a hard energy spectrum, and satisfy the following properties:

\[ m \simeq (1-2) \text{ TeV}, \]

\[ \tau = O(10^{26}) \text{ sec}, \]

where \( m \) and \( \tau \) are the mass and the lifetime of the dark matter particles, respectively. The constraint on \( m \) comes from the observed energy spectrum of the positron/electron excess, which has been detected up to \( \sim \text{TeV} \) energy with a suggestive cut-off at \( \sim 600 \text{ GeV} \) #1, while that on \( \tau \) comes from the observed flux. We assume that the decaying dark matter accounts for most of the observed dark matter density throughout this letter.

We shall show that, if the dark matter is a hidden scalar field #2 whose decay through Planck-suppressed dimension 6 operators explains the ATIC/PPB-BETS excess, the observed dark matter abundance can also be explained naturally and simultaneously. This is a non-trivial coincidence; thus, we shall conclude that the dark matter decaying through gravitational (Planck-suppressed) interactions may be responsible for the observed excess in the cosmic-ray positron/electron flux.

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#1 Such an excess is indicative of a particle with \( m \sim 1.2 \text{ TeV} \) for the two-body decay, and \( m \sim 1.8 \text{ TeV} \) for the three-body decay.

#2 See, e.g., Refs. [9, 10, 11, 12, 13] for other explanations.
The longevity of dark matter is a puzzle, especially if its mass is as heavy as 1 TeV; the dark matter particle may sequester itself from the standard-model sector, or it may be protected by some discrete symmetry, or perhaps both.

Let us assume that the dark matter particle is a scalar field of mass given by (1), and has only Planck-suppressed interactions with the standard-model particles. If the scalar is a singlet under any symmetries, the lifetime will be much shorter than the present age of the universe, as the scalar field can certainly have dimension 5 operators. Let us therefore assume that the hidden scalar is charged under a symmetry (say, $Z_2$ symmetry). Then the hidden scalar field decays through Planck-suppressed dimension 6 operators, and the lifetime can be as long as (2) for an appropriate amount of the symmetry breaking.

For concreteness, we consider the following form of the Lagrangian density, the so-called $f(\phi)R$ gravity #3:

$$\mathcal{L} = \sqrt{-g} \left( \frac{1}{2} f(\phi) R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right) + \mathcal{L}_m, \quad (3)$$

where $V(\phi)$ is the scalar potential of $\phi$ and $\mathcal{L}_m$ is the matter Lagrangian density. Note that this particular choice of Lagrangian (i.e., $f(\phi)R$ gravity) is not essential. Our result applies to any models in which the decay rate is (approximately) given by (8). As we shall discuss later, our argument applies to other set-ups, e.g. supergravity theories, in a straightforward way. We use the following form of $f(\phi)$,

$$f(\phi) = M_P^2 + \xi (\phi^2 - v^2), \quad (4)$$

where $M_P \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass, $\xi$ is a numerical coefficient of order unity, and $v$ denotes the vacuum expectation value (vev) of $\phi$, i.e., $v \equiv \langle \phi \rangle$. The scalar potential, $V(\phi)$, is chosen such that the scalar field, $\phi$, acquires a vev, $v$ #4. The conventional Einstein gravity is restored in the low energy limit, where $\phi$ has settled into the vev, i.e., $f(v) = M_P^2$. This form of $f(\phi)$ is realized when we impose a $Z_2$ symmetry on $\phi$, which is spontaneously broken by $\langle \phi \rangle$ in the vacuum.

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#3 Here we adopt a convention that $\sqrt{-g}$ is included in the Lagrangian density, following Ref. [14].

#4 A potential production of domain walls can be made harmless by making the imposed $Z_2$ symmetry approximate rather than exact, e.g., by adding a small explicit breaking of the $Z_2$ symmetry [15]. It is even possible that the vev of $\phi$ is induced entirely by an explicit breaking of the $Z_2$ symmetry; domain walls are not produced in this case, and thus they would not affect our arguments.
Let us decompose $\phi$ into the classical part ($v$) and the quantum fluctuation ($\sigma$) as $\phi = v + \sigma$, and assume that $v \neq 0$. The previous work \cite{14, 16} has shown that $\sigma$ is generically coupled to any matter fields, even if $\phi$ does not have direct couplings with them in $\mathcal{L}_m$, as long as the matter fields are not conformally invariant.\footnote{The scalar field can also decay into gauge fields, which are conformally invariant at the tree level – the conformal invariance is broken at the one-loop level \cite{17}. The presence of such interactions does not change our arguments, as the amplitudes are one-loop suppressed.} The interaction vertices are induced by the mixing of $\sigma$ with gravity. It may be easier to understand how the interactions arise by performing the Weyl transformation to make the gravity canonically normalized (i.e., the Einstein gravity). Since the Weyl transformation depends on the scalar $\phi$, the interactions between $\sigma$ and the matter fields are induced in the Einstein frame. See \cite{14} for details.

As for the matter Lagrangian density, $\mathcal{L}_m$, let us consider another scalar field, $\chi$, with the following form for simplicity:

$$\mathcal{L}_m = -\frac{\sqrt{-g}}{2} \left( \partial_\mu \chi \partial^\mu \chi + m_\chi^2 \chi^2 \right).$$

(5)

The hidden scalar, $\sigma$, then decays into a pair of $\chi$'s through vertices $\sim v \sigma (\partial \chi)^2 / M_P^2$ and $vm_\sigma^2 \sigma^2 / M_P^2$. The decay rate has been calculated by Ref. \cite{14} for a general form of $f(\phi)$ and is given by

$$\Gamma = \frac{\hat{g}_\chi^2}{8\pi m_\sigma} \left( 1 - 4 \frac{m_\chi^2}{m_\sigma^2} \right)^{\frac{1}{2}},$$

(6)

where

$$\hat{g}_\chi \equiv \frac{f'(v)}{2M_P^2} \frac{m_\chi^2 + m_\sigma^2}{\sqrt{1 + 3 \left( \frac{f'(v)}{M_P^2} \right)^2}}.$$  

(7)

Here, $m_\sigma$ is the mass of $\sigma$, and $f'(v) \equiv \partial f / \partial \phi |_{\phi = v}$. If $m_\chi$ is much smaller than $m_\sigma$, the decay rate is approximately given by

$$\Gamma \approx \frac{\xi}{32\pi} \left( \frac{v}{M_P} \right)^2 \frac{m_\sigma^3}{M_P^2},$$

(8)

where we have used the form of $f(\phi)$ given by Eq. (4), and assumed $v \ll M_P$ (which gives $f'(v) \ll M_P$).

In our scenario the scalar $\phi$ is the dominant component of dark matter, which decays into the standard-model particles through the gravitational couplings, and the decay products are the source for the observed excess in the cosmic-ray positrons/electrons. This may be
realized if the scalar matter field, $\chi$, promptly decays into an electron-positron pair \[18\].

In order to meet the required lifetime $\tau$, the vev of $\phi$ must satisfy

$$ v \simeq 2 \times 10^8 \text{GeV} (\xi N)^{-\frac{1}{2}} \left( \frac{m_{\sigma}}{1 \text{ TeV}} \right)^{-\frac{3}{2}} \left( \frac{\tau_{\phi}}{10^{20} \text{ sec}} \right)^{-\frac{1}{2}}, $$

where we have assumed that $\sigma$ decays into different $N$ scalars, $\chi_i$ ($i = 1, \cdots, N$). Thus, in this model, the mass and vev of $\phi$ are fixed by the requirements (1) and (2). In Fig. 1 we show (1) and (9) on the $(m_{\sigma}, v)$-plane, where we have varied $\xi N$ from 0.1 to 10.

So far, we have merely shown that we can explain two observables (energy and flux of the high-energy cosmic ray positrons and electrons) by tuning two parameters, $m_{\sigma}$ and $v$, which may not be so remarkable. In the following we shall show that the same set of parameters can explain the cosmological abundance of $\phi$ simultaneously, i.e., two parameters can explain three observables.

We estimate the cosmological abundance of $\phi$ as follows. In general we expect that the initial position of $\phi$ set during inflation was different from the potential minimum in the low energy. For instance, if the $Z_2$ symmetry was respected during inflation, the $\phi$ field would sit at the origin, displaced from the low energy minimum by $v$. The $\phi$ field would start to oscillate about the potential minimum when the Hubble parameter became comparable to the mass, i.e., $H \simeq m_{\phi}$, with an amplitude around $v$, where $m_{\phi}$ is the mass at the initial position. Throughout this letter we shall assume $m_{\phi} \simeq m_{\sigma}$ only for simplicity: $m_{\phi}$ can be different from $m_{\sigma}$ in general, depending upon the shape of $V(\phi)$. If $m_{\sigma} \neq m_{\phi}$, it is $m_{\sigma}$ that must satisfy the mass constraint given by (1).

Assuming that reheating of the universe after inflation has been completed by the beginning of oscillations of $\phi$ (see below for the other case), we estimate the cosmological abundance of $\phi$ as

$$ \frac{\rho_{\phi}}{s} = \frac{4}{27} \left( \frac{m_{\phi}^2 v^2}{g_*} \right) \frac{3 m_{\phi}^2 M_{Pl}^2}{(3M_{Pl}^2)^{\frac{1}{2}}} \left( \frac{3m_{\phi} M_{Pl}^2}{2g_*} \right)^{\frac{3}{2}}, $$

$$ \simeq 6 \times 10^{-10} A \text{ GeV} \left( \frac{g_*}{100} \right)^{-\frac{1}{2}} \left( \frac{v}{10^9 \text{ GeV}} \right)^2 \left( \frac{m_{\sigma}}{1 \text{ TeV}} \right)^{\frac{1}{2}}, $$

where $\rho_{\phi}$ is the energy density of $\phi$, $s$ the entropy density, and $g_*$ the relativistic degrees of freedom at $H = m_{\phi}$. We have introduced a numerical coefficient $A$ to parametrize an $O(1)$

\#6 The energy spectrum of the electrons and positrons depends on the details of their production. It may be possible to distinguish different production processes by measuring the spectrum precisely in future observations \[19\].
uncertainty in the above estimate. Note that we have used $m_\phi \simeq m_\sigma$. One may also write this result in the following form:

$$\Omega_\phi h^2 = 0.2 A \left( \frac{g_\ast}{100} \right)^{-\frac{1}{2}} \left( \frac{v}{10^9 \text{GeV}} \right)^2 \left( \frac{m_\sigma}{1 \text{ TeV}} \right)^{\frac{1}{2}}.$$  \hspace{1cm} (11)

Here, $\Omega_\phi$ is the density parameter of $\phi$, and $h$ the present Hubble in units of 100 km/s/Mpc.

The predicted dark matter abundance, for $v$ and $m_\sigma$ that are required to explain the ATIC/PPB-BETS excess, is in remarkable agreement with the measured dark matter abundance, $\Omega_m h^2 \simeq 0.11$ \cite{1}. No tuning of parameters, apart from choosing the two parameters, $v$ \cite{9} and $m_\sigma$ \cite{1}, to explain the ATIC/PPB-BETS excess, was required. We have plotted the region of $(m_\sigma, v)$ where the dark matter abundance agrees with the observed value in Fig. 1. In the figure we have varied $A$ from 0.1 to 10.

The three conditions, the mass \cite{1}, the lifetime \cite{2}, and the cosmological abundance \cite{11} are a priori independent of one another. Nevertheless, if we assume that the hidden scalar dark matter is coupled to the visible sector only by the Planck-suppressed interactions, those three conditions meet at a single point on the $(m_\sigma, v)$ plane, i.e., $m = \mathcal{O}(1) \text{ TeV}$ and $v = \mathcal{O}(10^9) \text{ GeV}$. Did this happen merely by chance? Such a remarkable coincidence may suggest that the Planck-scale physics is playing an important role in the decaying dark matter scenario that accounts for the ATIC/PPB-BETS excess. For comparison, we also show in Fig. 1 a constraint from the lifetime if the cut-off scale is the grand unification theory (GUT) scale instead of the Planck-scale (the lower (gray) band). The other two lines (from mass and abundance) are the same. The three lines no longer meet at one point.

To obtain the abundance \cite{11} we have assumed that the reheating has been completed before the $\phi$ field began to oscillate. This assumption can be translated into the lower bound on the reheating temperature: $T_R \gtrsim \sqrt{m_\phi M_P} \sim 10^{10} \text{ GeV}$. On the other hand, if the reheating temperature was as high as $10^{14} \text{ GeV}$, the thermal production ($\chi \chi \rightarrow \phi \phi$) through the Planck-suppressed interaction, $\sim \phi^2 \chi^2 / M_P^2$ (see Sec. IV of Ref. \cite{20}), would give a significant contribution to the dark matter abundance, while the thermal production can be neglected for $T_R < 10^{14} \text{ GeV}$. Therefore the above estimate \cite{11} is valid for the reheating temperature between $10^{10} \text{ GeV}$ and $10^{14} \text{ GeV}$. What if the reheating was not completed when the $\phi$ began to oscillate, i.e., $T_R \lesssim 10^{10} \text{ GeV}$? The abundance would be diluted by the entropy production during reheating,
and $\Omega_\phi h^2$ would be given by

$$\Omega_\phi h^2 \approx 0.06 A \left( \frac{v}{10^9 \text{GeV}} \right)^2 \left( \frac{T_R}{10^9 \text{GeV}} \right),$$

(12)

which is necessarily smaller than the previous estimate (11), and therefore the agreement of three lines shown in Fig. 1 would not be as good.

Our discussion so far did not use supergravity; however, in supergravity there is a notorious gravitino problem [21, 22, 23] (see [24] and references therein for the recent constraint from Big Bang Nucleosynthesis), which requires care when the reheating temperature is higher than $10^{10}$ GeV or so. The reheating temperature higher than $10^{10}$ GeV is allowed for a gravitino mass in the following ranges: (i) $m_{3/2} \lesssim 10 \text{ eV}$ [25], (ii) $m_{3/2} \gtrsim 100 \text{ GeV}$ (if the gravitino is the lightest supersymmetric particle) and (iii) $m_{3/2} \gtrsim O(10) \text{ TeV}$. The presence of $R$-parity violation and/or a light $R$-parity odd field in a hidden sector may enlarge the allowed parameter space in some cases, but further discussion is beyond the scope of this letter.

As mentioned earlier, a coupling similar to (3) is generically present in the supergravity. In the conformal frame there is a term given by

$$\mathcal{L} = -\sqrt{-g} \frac{e^{-K/3}}{2} R + \cdots,$$

(13)

where $K$ is the Kähler potential. After performing the field-dependent Weyl transformation, we generically obtain quartic couplings such as $\sim \int d^4 \theta |\Phi|^2 |Q|^2 / M_P^2$ in the Einstein frame, where $\Phi$ and $Q$ denote the dark matter and a matter field, respectively. We assume that $\Phi$ is odd under a $Z_2$ symmetry, and the lowest component, $\phi$, is the hidden scalar dark matter. The decay into the fermionic partner and the gravitino is assumed to be kinematically forbidden. The scalar matter field that appeared in our discussion so far, $\chi$, may be identified with a slepton within the context of supersymmetry. The decay into a pair of sleptons is induced by a quartic coupling in the Kähler potential such as $\sim \int d^4 \theta |\Phi|^2 |e_i|^2 / M_P^2$, where $e_i$ denotes the right-handed lepton superfield in the $i$-th generation. The decay into a pair of sleptons through this coupling is suppressed by $(m_{\tilde{e}_{R,i}} / m_\phi)^4$ with respect to (8), if we redefine the vev as $v \equiv \langle |\phi| \rangle / \sqrt{2}$ #7. However, as the suppression is not so significant for the slepton mass of $O(100)$ GeV, our previous arguments are still valid without modification.

#7 Note that $\phi$ is a complex scalar here.
For instance, the suppression factor including the phase space is $\sim 0.02$ for $m_\phi = 1.4\text{TeV}$ and $m_{\tilde{e}_{R,1}} = 600\text{GeV}$. We have varied the decay rate by two orders of magnitudes in Fig. 1 which can account for this kind of possible uncertainty. In fact, the agreement of the three lines in Fig. 1 becomes even better in this case.

If the $Z_2$ symmetry is explicitly broken by a small amount, $\epsilon \ll M_P$, the vev of $\phi$ is expected to be of order of $\epsilon$. Also there may be a linear term in the Kähler potential: $\delta K \sim \epsilon \Phi + \text{h.c.}$. In the presence of such a linear term, the initial position of the $\phi$ field during inflation is naturally displaced from the potential minimum by $\mathcal{O}(\epsilon)$. Thus our estimate on the cosmological abundance of the $\phi$ is also valid in this case.

In this letter we have shown that a hidden scalar field dark matter, which couples to the standard-model sector only through the Planck-suppressed dimension 6 interactions, can explain the excess of cosmic-ray positrons/electrons observed by ATIC/PPB-BETS for a suitable choice of two parameters: the mass and the vacuum expectation value. We have also shown that the same parameters, without any further tuning or introduction of parameters, yield the correct dark matter abundance. Such a non-trivial coincidence suggests that the dark matter decaying through the Planck-suppressed interactions may be responsible for the ATIC/PPB-BETS excess. We have presented an explicit example using the so-called $f(\phi)R$ gravity, and also shown how it can be embedded in supergravity easily. We have seen that the vev of the hidden scalar field is necessarily $\mathcal{O}(10^{8-9})\text{GeV}$ in order to account for the ATIC/PPB-BETS excess. The origin of such an intermediate scale would require further explanation.

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FIG. 1: Constraints on the mass, $m_\sigma$, and the vacuum expectation value, $v$, of the hidden scalar field dark matter from the ATIC/PPB-BETS excess and the cosmological dark matter abundance. The energy and flux of cosmic-ray positrons/electrons detected by ATIC/PPB-BETS give $m_\sigma$ (vertical (red) band) and the lifetime (oblique (blue) band), respectively. The nearly horizontal (green) band shows the abundance constraint. The widths of the bands show the uncertainties in the details of the model: for the mass and lifetime we vary $\xi N$ from 0.1 to 10, and for the abundance we vary $A$ from 0.1 to 10. Three constraints meet at one point represented by a star. The lower (gray) band shows the constraint from the lifetime with the Planck-suppressed interaction \[8\] replaced by the GUT-scale-suppressed one, $M_{\text{GUT}} = 2 \times 10^{16}$ GeV.

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