Research on Seismic Data Interpolation and Reconstruction

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Abstract. The seismic record samples are frequently larger than appropriate and the record data is often lost in survey, so we need to perform interpolation and reconstruct of seismic data. In this thesis, generalized F-K seismic trace interpolation method is applied to resolve the problem of the serious insufficiency of spatial sampling. $\tau$-$p$ transformation is applied to reconstruct seismic data technique to resume lost seismic data. Finally, models of theoretic data and practical seismic data can verify that the interpolation and reconstruction methods described in this thesis own high applicability and efficiency.

1. Introduction
In actual seismic exploration, excessive trajectory spacing will cause spatial false frequencies. Missing trajectories will lose some important underground information and affect the quality of seismic data processing. At present, there are many studies about the theory and method of seismic data interpolation at home and abroad \cite{1,7}. But these are interpolation methods based on Fourier transform and least squares inversion theory. The calculation process is complicated and the amount of calculation is large. In this paper, the generalized F-K domain seismic data interpolation method is used to perform uniform intra-channel interpolation, and the data reconstruction method based on $\tau$-$p$ transformation is used to perform non-uniform data reconstruction. The theoretical and practical data verification has achieved ideal results.

2. Interpolation of Generalized F-K Seismic Data

2.1. Principle
The basic principle of the generalized F-K domain seismic data interpolation method is to insert a new gathering between the original collecting data. The sampling interval of the original gathering is L times the sampling interval of the new gathering after interpolation (L can be any integer, L is Interpolation coefficient).

Assuming that the seismic profile has N data, there is only one linear in-phase axis, the wavelet spectrum is $W (f)$, and the time sampling interval is $t_n$, then the Fourier transform of the nth channel is:

$$S_n (f) = W (f) e^{j2\pi fn}\quad n=0, 1, 2..., N-1$$

(1)
If the Fourier transform in the K direction is performed on equation (1), the seismic data changes from T-X domain to F-K domain:

\[
A(k, f) = \sum_{n=0}^{N-1} W(f) e^{i2\pi \Delta t n} e^{-i2\pi \Delta k n / N} \quad k=0, 1, 2..., N-1
\]  

(2)

Summing equation (2) by equal series, then:

\[
A(k, f) = W(f) \frac{1 - e^{i2\pi \Delta t N}}{1 - e^{i2\pi \Delta t} e^{-i2\pi \Delta k / N}} \quad k=0, 1, 2..., N-1
\]  

(3)

After the original data is interpolated with L-channel zero-track data, the track pitch will become 1 / L times the track pitch of the original data, and the maximum wave number in the frequency wavenumber domain will be expanded to the original L times, but the distance between the first channel and the last one has not changed, so the sampling interval of wave number remains unchanged. The F-K spectrum of the data after interpolating L channels and zero channels from the original data is:

\[
B(k, f) = W(f) \frac{1 - e^{i2\pi \Delta t N}}{1 - e^{i2\pi \Delta t} e^{-i2\pi \Delta k / N}} \quad k=0, 1, 2..., L\times(N-1)
\]  

(4)

Expand the original data by L times in the time and space direction, and zero-fill the expanded part. Its F-K spectrum is:

\[
C(k, f) = W(f) \frac{1 - e^{i2\pi \Delta t N}}{1 - e^{i4\pi \Delta t} e^{-i2\pi \Delta k} / N} \quad k=0, 1, 2..., N-1
\]  

(5)

After zero-filling the even-numbered channels of the original data, then expand it by L times in the time and space direction, and expand some data to zero-fill, its F-K spectrum is:

\[
D(k, f) = W(f) \frac{1 - e^{i2\pi \Delta t N}}{1 - e^{i4\pi \Delta t} e^{-i2\pi \Delta k} / N} \quad k=0, 1, 2..., N-1
\]  

(6)

After interpolation being inserted correctly, the track spacing of the interpolated seismic data is 1 / L times the track spacing of the original data, and the maximum wave number in the frequency wavenumber domain is L times the maximum wavenumber in the original data frequency wavenumber domain. Select C(k,2f) and D(k,2f) for the first half of the frequency, the F-K spectrum of the interpolated seismic data is:

\[
E(k, f) = \frac{C(k/2, f/2)}{D(k/2, f/2)} B(k, f)
\]  

(7)

Therefore, a filter factor can be obtained by equation (7), and then the original data is interpolated with L-channel zero-channel data, that is, the F-K spectrum of the interpolated seismic data is obtained, and the generalized F-K is completed by inverse F-K transformation Seismic data interpolation. This method is also applicable to the interpolation of seismic data with multiple cross-phase axes.

2.2. Model Verification

The data is expected to be sampled at 1 ms, and there are two linear in-phase axes with inclination angles of 4 ms/channel and -8 ms/channel. Take the even-numbered tracks of the desired data to form the original data, as shown in figure 1a. The inclination angle of the original data on the same phase axis is 8 ms/channel and -16 ms/channel respectively. Figure 1c is the F-K spectrum of the original data. At this time, the -16ms/channel in-phase axis generates spatial aliasing after 250 Hz.

Through generalized F-K domain seismic data interpolation, the F-K spectrum of the interpolated data is obtained, and the inverse transform is to obtain the interpolated data, as shown in figures 1b and 1d. After interpolation, the waveform of the data is natural, and no spatial aliasing is generated.
2.3. Interpolation of Actual Data

Figure 2 is an actual data application. Take the even-numbered channels of actual data as the data to be interpolated, and use the generalized F-K domain seismic data interpolation method to perform interpolation processing to obtain the interpolation data. It can be seen that the difference between the interpolated data and the actual seismic data is very small.

3. Reconstruction of Seismic Data Based on $\tau - p$ Transform
### 3.1. Principle
For the seismic data sampled and observed in the spatial direction, the basic formula of $\tau - p$ positive transformation is:

$$\psi(\tau, p) = \sum \varphi(\tau + px, x)$$  \hspace{1cm} (8)

The basic formula of $\tau - p$ inverse transformation is:

$$\varphi(t, x) = \sum \psi(t - px, p)$$  \hspace{1cm} (9)

The implementation steps of transform-based seismic data reconstruction method are as follow:

1. Fill zeros in the original data to be reconstructed;
2. Select the appropriate $\Delta p$, $p_{\text{max}}$, $p_{\text{min}}$, and after the transformation, a non-zero track is generated at the position to be reconstructed;
3. Replace the zero-value track with the reconstructed data to obtain the reconstructed data. Through the above processing, seismic data reconstruction is achieved. If the reconstruction effect is not good, iterative processing can be performed to improve the reconstruction effect.

### 3.2. Model Validation
It is expected that there are 60 data in total, the length of the channel is 1500 ms, the distance between the channels is 40 m, the time sampling interval is 1ms, and there are four reflective layers. Zero-fill the 11 and 12 seismic data of the expected data to obtain the data to be reconstructed, as shown in figure 3a. Figure 3b shows the reconstructed data based on transformation. The reconstruction effect of seismic data is very good, the amplitude difference is small, and all coherent and in-phase axes are correctly reconstructed.

![Figure 3](image.png)

**Figure 3.** Reconstruction processing of large offset.

### 3.3. Reconstruction of Actual Data
Figure 4 is the actual single shot unilateral seismic record, in which 17, 18 seismic traces are missing. After reconstruction, the missing seismic traces are basically restored, and the recovered in-phase axis maintains the characteristics of the original section.
4. Conclusion
The generalized F-K domain seismic data interpolation method has the characteristics of being simple principle, fast operating speed and high efficiency. It can not only remove spatial false frequencies, but also interpolate any number of seismic data uniformly between seismic traces. However, the calculation effect on uneven interpolation is poor. Transform-based seismic data reconstruction can perform non-uniform seismic data reconstruction. The reconstruction effect is ideal. It can accurately recover all in-phase axes without generating false in-phase axes. It is suitable for reconstruction of seismic records with linear or medium complexity. In actual data processing, the F-K domain method can be used to perform regular interpolation on the data to complete most of the interpolation of the missing data, and then perform the transformation-based interpolation calculation on the uneven missing data. To achieve the best calculation efficiency and effect.

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