Lepton flavor violating $Z \rightarrow l_i l_j$ in flavor-universal topcolor-assisted technicolor

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In the context of flavor-universal topcolor-assisted technicolor (TC2) models, we calculate the lepton flavor violating (LFV) $Z \rightarrow l_i l_j$ decays. We find that the extra $U(1)$ gauge boson $Z'$ can give significant contributions to these LFV processes. With reasonable values of the parameters, the branching ratios of the processes $Z \rightarrow \tau \mu$ and $Z \rightarrow \tau e$ can approach the experimental upper limits. The indirect bound on the process $Z \rightarrow \mu e$ can give a severe constraint on the flavor-universal TC2 models.

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The high statistic results of the Superkamiokande (SK) atmospheric neutrino experiment [1] and the solar neutrino experiment [2] have made one to believe that neutrinos are massive and oscillate in flavor and are of interest in the lepton flavor violating (LFV) processes. Among them, the LFV Z decays, such as $Z \to \tau e$, $Z \to \tau \mu$, and $Z \to \mu e$ are important for the search of neutrinos and the physics beyond the standard model (SM). The Giga Z option of the TESLA linear collider project will work at the Z resonance and increase the production rate of Z boson at resonance [3]. This force one to study the LFV Z decays more precisely.

It is well known that the lepton numbers are automatically conserved and the tree level LFV Z decays are absent in the SM. Thus, one needs an extended theory to describe the LFV Z decays. The $\nu_SM$ [4], which takes neutrinos massive and permits the lepton mixing mechanism, is one of the possibility. Ref.[5] has studied the LFV Z decays in the context of $\nu_SM$. However, the branching ratios ($BR's$) are very small, i.e., $BR(Z \to e\mu) \sim BR(Z \to e\tau) \sim 10^{-54}$ and $BR(Z \to \mu\tau) < 4 \times 10^{-60}$. The $BR's$ of Z decays can be increased in the framework of the $\nu_SM$ with heavy neutrino [6]. Recently, the LFV Z decays are studied in the context of the Zee model [7] and the general 2HDM type III [8]. Ref.[8] has shown that, with the model parameters in the restriction region, the $BR's$ can be highly enhanced which can reach $7 \times 10^{-11}$ and $2.5 \times 10^{-9}$ for $BR(Z \to \mu e)$ and $BR(Z \to \tau e)$, respectively.

The aim of this letter is to point out that the $BR's$ of the LFV Z decays can be significantly enhanced in the flavor-universal topcolor assisted technicolor (TC2) models [9], which may approach the present experimental upper limits:

$$BR(Z \to \tau e) < 9.8 \times 10^{-6} [10,11],$$
$$BR(Z \to \tau \mu) < 1.2 \times 10^{-6} [10,12],$$
$$BR(Z \to \mu e) < 1.7 \times 10^{-6} [10].$$

and with the improved sensitivity at Giga-Z, these numbers could be pulled down to [13]:

$$BR(Z \to \tau e) < f \times 1.5 \times 10^{-8}, \quad BR(Z \to \tau \mu) < f \times 2.2 \times 10^{-8}, \quad BR(Z \to \mu e) < 2 \times 10^{-9}.$$  

with $f = 0.2 - 1.0$.

Given the large value of the top quark mass and the sizable splitting between the masses of the top and bottom quarks, it is natural to wonder whether $m_t$ has a different origin from the masses of the other quarks and leptons. There may be a common origin for electroweak symmetry breaking (EWSB) and top quark mass generation. Much theoretical work has been carried out in connection to the top quark and EWSB. The TC2 models [14] and the flavor-universal TC2 models [9] are two of such examples. All of these models suppose that top quark condensation contribute to some of EWSB. The strong $U(1)_b$ structure is required to tilt the chiral condensation in the $t\bar{t}$ direction and not form a $b\bar{b}$ condensation. Thus, this kind of models generally predict the existence of the extra $U(1)$ gauge boson $Z'$, which has flavor changing coupling vertices, such as $Z' tc, Z' \tau \mu, Z' \tau e$. The new gauge boson $Z'$ may have significant contributions to some flavor changing neutral current processes [15]. Thus, in this letter, we calculate the contributions of the gauge boson $Z'$ to the LFV Z decays.

In the flavor-universal TC2 models [9], the gauge group is the same as in the traditional TC2 models [14]:
\[G_{ETC} \times SU(3)_1 \times SU(3)_2 \times SU(2) \times U(1)_1 \times U(1)_2.\]

At an energy scale \(\Lambda\), the color sector \((SU(3)_1 \times SU(3)_2)\) breaks to its diagonal subgroup \(SU(3)_c\) and the hypercharge groups break in the pattern \(U(1)_1 \times U(2)_2 \rightarrow U(1)_y\). Thus these models also predict the existence of two additional gauge bosons: colorons and \(Z'\). However, the fermion charge assignments are significantly different from those in TC2 models. In quark sector, all quarks are \(SU(3)_c\) triplets and \(SU(3)_c\) singlets. In the hypercharge sector, the third generation of fermions transforms under the stronger \(U(1)_1\) and the others transform under the weaker \(U(1)_2\). All of the quarks and leptons have the same weak charge assignments as in the SM, which are showed in Table 1 of Ref.[9].

The flavor-diagonal couplings of \(Z'\) to leptons can be written as:

\[\mathcal{L}_{Z'}^{FD} = -\frac{1}{2} g_1 \cot \theta_y Z'_\mu (\bar{\tau}_L \gamma^\mu \tau_L + 2 \bar{\tau}_R \gamma^\mu \tau_R) + \frac{1}{2} g_1 \tan \theta_y Z'_\mu (\bar{\mu}_L \gamma^\mu \mu_L + 2 \bar{\mu}_R \gamma^\mu \mu_R + \bar{\epsilon}_L \gamma^\mu \epsilon_L + 2 \bar{\epsilon}_R \gamma^\mu \epsilon_R),\]

where \(g_1\) is the ordinary hypercharge gauge coupling constant, \(\theta_y\) is the mixing angle with \(\tan \theta_y = \frac{g_2}{\sqrt{\pi} R_1}\). To obtain the top quark condensation, there must be \(\tan \theta_y \ll 1\). The flavor changing couplings of \(Z'\) to leptons can be written as:

\[\mathcal{L}_{Z'}^{FC} = -\frac{1}{2} g_1 Z'_\mu \left[ K_{\tau\mu} (\bar{\tau}_L \gamma^\mu \tau_L + 2 \bar{\tau}_R \gamma^\mu \tau_R) + K_{\mu e} \tan^2 \theta_y (\bar{\mu}_L \gamma^\mu \mu_L + 2 \bar{\mu}_R \gamma^\mu \mu_R) \right],\]

where \(K_{ij}\) are the flavor mixing factors. In the following estimation, we will assume \(K_{\tau\mu} = K_{\tau e} = K_{\mu e} = K = \lambda [17]\), where \(\lambda = 0.22\) is the Wolfenstein parameter [17].

From Eq.(4) and Eq.(5), we can see that the extra gauge boson \(Z'\) exchange can indeed induce the LFV \(Z\) decays \(Z \rightarrow l_l l_j\). The relevant diagrams are depicted in Fig.1. Similar to Ref.[18], we can calculate these diagrams. In our calculation, we have taken \(m_\mu \approx 0, m_\tau \approx 0,\) and \(m_e \approx 0\). It is easy to see that, relative to the contributions of Fig.1(a), the contributions of Fig.1(b) and Fig.1(c) to \(Z \rightarrow \tau \mu\) decay are suppressed by the factors \(\tan^2 \theta_y\) and \(\tan^3 \theta_y\), respectively. The conclusions are also apply to \(Z \rightarrow \tau e\) and \(Z \rightarrow \mu e\) decays. Then, the LFV couplings of \(Z\) arising from the gauge boson \(Z'\) exchange can be written as:

\[\delta g_{L}^\tau = \delta g_{L}^e \simeq \frac{K_1 \tan \theta_y}{6\pi} \frac{1}{g_L K_2} \frac{m_\tau^2}{M_Z^2} \ln \frac{M_Z^2}{m_\tau^2},\]

\[\delta g_{R}^\tau = \delta g_{R}^e \simeq \frac{2K_1 \tan \theta_y}{3\pi} \frac{1}{g_R K_2} \frac{m_\mu^2}{M_Z^2} \ln \frac{M_Z^2}{m_\mu^2},\]

\[\delta g_{L}^\mu = \delta g_{L}^e \simeq \frac{K_1 \tan^2 \theta_y}{6\pi} \frac{1}{g_L K_2} \frac{m_\tau^2}{M_Z^2} \ln \frac{M_Z^2}{m_\tau^2},\]

\[\delta g_{R}^\mu = \delta g_{R}^e \simeq \frac{2K_1 \tan^2 \theta_y}{3\pi} \frac{1}{g_R K_2} \frac{m_\mu^2}{M_Z^2} \ln \frac{M_Z^2}{m_\mu^2},\]
with

\[ g_L^l = \frac{e}{s_W c_W} \left( \frac{1}{2} + s_W^2 \right), \quad g_R^l = \frac{e}{s_W c_W} (s_W^2), \tag{10} \]

where \( g_L^l(g_R^l) \) is the left(right)-handed \( Z - l - l \) coupling constant in the SM.

In general, the partial widths of the LFV \( Z \) decays can be written as:

\[ \Gamma(Z \rightarrow l_i l_j) = \frac{G_F m_Z^3}{3\sqrt{2}\pi} \left[ (\delta g_{ij}^L)^2 + (\delta g_{ij}^R)^2 \right] \tag{11} \]

To obtain numerical results, we take \( m_Z = 91.18 \text{GeV} \), \( G_F = 1.1664 \times 10^{-5} \text{GeV}^{-2} \), \( \Gamma_Z = 2.495 \text{GeV} \), \( s_W^2 = 0.2322 \) \cite{13}. To obtain proper vacuum tilting (the topcolor interactions only condense the top quark but not the bottom quark), the coupling constant \( K_1 \) should satisfy certain constraint, i.e. \( K_1 \leq 1 \) \cite{11}. Ref.\[20\] has given the lower bounds on the \( Z' \) mass from precision electroweak fits, which range from 500GeV to 2TeV depending on the value of \( K_1 \). Recently, Simmons \[21\] has shown that the B-meson mixing can give lower bound. The results are \( M_Z \geq 590 \text{GeV} \) if ETC does not contribute to \( \epsilon \) which is the CP-violation parameter and \( M_Z > 910 \text{GeV} \) if it does. In the following calculation, we will take the \( Z' \) mass \( M_Z \) and \( K_1 \) as free parameters.

In Fig.2, we plot the branching ratios of the LFV \( Z \) decay processes \( Z \rightarrow \tau \mu, Z \rightarrow \tau e \) as function of the \( Z' \) mass \( M_Z \) for three values of the parameter \( K_1 \). From Fig.2 we can see that the branching ratios decrease with \( M_Z \) increasing and increase with \( K_1 \) increasing. In most of the parameter space of the flavor-universal TC2 models, the branching ratios of the processes \( Z \rightarrow \tau \mu \) and \( Z \rightarrow \tau e \) are larger than \( 1 \times 10^{-10} \). For \( K_1 = 1 \) and \( M_Z = 500 \text{GeV} \), we have \( BR(Z \rightarrow \tau \mu, \tau e) = 7.0 \times 10^{-9} \), which can approach the future experimental upper limits: \( BR(Z \rightarrow \tau e) < f \times 1.5 \times 10^{-8} \) and \( BR(Z \rightarrow \tau \mu) < f \times 2.2 \times 10^{-8} \) with \( f = 0.2 - 1.0 \) \cite{13}. We plot the branching ratio \( BR(Z \rightarrow \mu e) \) versus \( M_Z \) in Fig. 3 for the three values of the parameter \( K_1 \). One can see from Fig. 3 that \( BR(Z \rightarrow \mu e) \) is smaller than \( BR(Z \rightarrow \tau \mu \) or \( \tau e) \). For \( K_1 = 0.6, BR(Z \rightarrow \mu e) \) varies between \( 3.8 \times 10^{-10} \) and \( 1.2 \times 10^{-11} \) for \( M_Z \) in the range of 500GeV - 1500GeV.

Using the current upper bound on the process \( \mu \rightarrow 3e \) and other data pertaining to the \( e^+e^- \) widths of the electroweak gauge boson \( Z \), Ref.\[22\] gives a set of bounds for the LFV \( Z \) decays:

\[ BR(Z \rightarrow \mu e) \leq 5 \times 10^{-13}, \quad BR(Z \rightarrow \tau l) \leq 3 \times 10^{-6}, \tag{12} \]

with \( l = e \) or \( \mu \). From Fig.2 and Fig.3, we can see that these bounds can not give any constraint on the flavor-universal TC2 models via the LFV processes \( Z \rightarrow \tau \mu \) and \( Z \rightarrow \tau e \). However, it is not this case for the process \( Z \rightarrow \mu e \). To see how the bound \( BR(Z \rightarrow \mu e) \leq 5 \times 10^{-13} \) constrains the flavor-universal TC2 models, we give the contour line of \( BR(Z \rightarrow \mu e) = 5 \times 10^{-13} \) in the \( (K_{\mu e}, M_Z) \) plane for \( 500 \text{GeV} \leq M_Z \leq 1500 \text{GeV} \) and three values of \( K_1 \) in Fig.4. We can see from Fig.4 that the constraints on the flavor mixing factor \( K_{\mu e} \) from the indirect bound \( BR(Z \rightarrow \mu e) \leq 5 \times 10^{-13} \) are very strong. If we take \( K_1 = 0.2, M_Z \leq 1500 \text{GeV} \), there must be \( K_{\mu e} \leq 0.17 \).

Similar to the flavor-universal TC2 models, TC2 models predict the existence of an extra \( U(1)_y \) gauge boson \( Z' \), which can also induce the tree-level flavor changing couplings. Thus,
the $Z'$ can also give contributions to the LFV $Z$ decays $Z \to l_\ell l_j$. However, Ref.[23] has shown that $B\bar{B}$ mixing provides lower bounds on the mass of the gauge boson $Z'$ predicted by TC2 models, i.e., it must be larger than 4TeV. So the contributions of the TC2 models to the LFV $Z$ decays are much smaller than those of the flavor-universal TC2 models. In most of the parameter space of the TC2 models, we have $BR(Z \to \tau \mu) = BR(Z \to \tau e) < 1 \times 10^{-11}$ and $BR(Z \to \mu e) < 1 \times 10^{-13}$.

To completely avoid the problems, such as triviality and unnaturalness arising from the elementary Higgs in the SM, various kinds of dynamical EWSB theories have been proposed and among which the strong top dynamical EWSB theories are very interesting. This type of models generally predict the existence of an extra $U(1)_y$ gauge boson $Z'$ which can induce the flavor changing coupling vertices, such as $Z'\tau \mu, Z'\tau e, Z'\mu e$. Thus, the gauge boson $Z'$ may have significant contributions to the LFV $Z$ decays $Z \to l_\ell l_j$. In this letter, we calculate the contributions of the $Z'$ predicted by the flavor-universal TC2 models to these decay processes. We find that the branching ratios of the LFV $Z$ decays indeed can be significantly enhanced by the $Z'$ exchange. With reasonable values of the parameters, the branching ratios of the processes $Z \to \tau \mu$ and $Z \to \tau e$ can reach $1 \times 10^{-8}$ which can approach the future experimental upper limits. We further consider whether the indirect bound of the process $Z \to \mu e$ given by Ref. [21] can give severe constraint on the flavor-universal TC2 models. Our results show that it is indeed this case. For $K_1 = 1$, there must be $M_Z \geq 5.2$TeV for $K_{\mu e} = 0.22$ and $K_{\mu e} \leq 0.038$ for $M_Z = 500$GeV.
Figure captions

Fig.1: The diagrams of $Z \rightarrow \tau\mu$ decay due to $Z'$ exchange in the flavor-universal TC2 models. The diagrams of $Z \rightarrow \tau e$ and $Z \rightarrow \mu e$ decays can be obtained by appropriate replacement of the internal fermion.

Fig.2: The branching ratios of the LFV $Z$ decays $Z \rightarrow \tau\mu$ and $Z \rightarrow \tau e$ as function of $M_Z$.

Fig.3: The branching ratios of the LFV $Z$ decay $Z \rightarrow \mu e$ as a function of $M_Z$.

Fig.4: The contour line of $BR(Z \rightarrow \mu e) = 5 \times 10^{-13}$ in the $(K_{\mu e}, M_Z)$ plane for $K_1 = 0.1$ (solid line), 0.5 (dashed line) and 1 (dotted line).
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Fig. 3
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