Exclusive $\rho^0$ electroproduction on the proton: GPDs or not GPDs?

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We discuss the interpretation of the $ep \rightarrow ep\rho^0$ process in terms of, on the one hand, Generalized Parton Distributions and, on the other hand, an effective hadronic model based on Regge theory.

Keywords: Nucleon structure; Generalized Parton Distributions; $\rho^0$ electroproduction

The $\gamma^{(*)}p \rightarrow p\rho^0$ process is, due to its relatively high cross section over a wide range of energy (from threshold at $W \approx 1.75$ GeV up to $W \approx 200$ GeV, where $W$ is the center of mass energy of the $(\gamma, p)$ system), one of the exclusive processes on the proton the most studied experimentally and, consequently, also theoretically.

In photoproduction above the nucleon resonance region the reaction is understood to proceed at low $W$ through the exchange of mesons ($\sigma, f_2,\ldots$) in the so-called $t$-channel (in the form of Regge trajectories) and at high $W$ through the exchange of the Pomeron trajectory, which carries the quantum numbers of the vacuum (see Fig. 1). These diagrams are based on hadronic degrees of freedom and the complex physics of QCD (Quantum Chromodynamics) is absorbed into effective meson-nucleon and photon-meson coupling constants plus potentially additional hadronic form factors.
Going to electroproduction, the increasing virtuality $Q^2$ of the initial photon allows us to probe shorter and shorter distances, and be sensitive to partonic (quarks and gluons) degrees of freedom. Then, at sufficiently high $Q^2$, the process should be understandable in terms of the so-called “handbag” diagrams illustrated in Fig. 2, where the incoming virtual photon scatters directly off a quark whose interaction can be calculable in the framework of perturbative QCD. The complex non-perturbative QCD structure of the nucleon is then represented by (quark and gluon) Generalized Parton Distributions (GPDs), which were shown to factorize from the QCD perturbative process in the handbag process for incoming longitudinal photons.\(^1\)

The GPDs might actually provide the most complete information accessible on the structure of the nucleon because they describe the (correlated) spatial and momentum distributions of the quarks in the nucleon (including the polarization aspects), its quark-antiquark content, a way to access the orbital momentum of the quarks, etc. We refer the reader to Refs.\(^2\)–\(^4\) for example, for very detailed and quasi-exhaustive reviews on the GPD formalism and the definitions of some of the variables that will be employed in the following.

The question then arises for what ranges in $Q^2$ are the hadronic and partonic descriptions valid and applicable. A first very simple image that one can form is that the quark handbag diagram (Fig. 2 left) might be associated with the meson Regge exchange mechanism (Fig. 1 left and center) while the gluon handbag diagram (Fig. 2 right) might correspond to
the exchange of the quantum numbers of the vacuum (as gluons form an isosinglet state) and thus to the Pomeron exchange mechanism. Indeed, “gluonic” GPDs, which obviously reflect the glue content of the nucleon, are expected to contribute at high $W$, i.e. in the low $x_B$ “sea” domain, while the “quark” GPDs should be predominant at low $W$, i.e. in the large $x_B$ “valence” domain, where $x_B$ can be related to $W$ by $x_B \approx \xi = \frac{Q^2}{W^2+Q^2-m^2}$. This correspondence is not fully exact in the sense that the quark handbag diagram can also carry the quantum numbers of the vacuum when sea quarks are exchanged and thus contribute to a Pomeron-type mechanism. Also, GPDs are not only $t$-channel mechanisms (which are just the $|x| < \xi$ part of the GPDs), there is also a part associated with standard “diagonal” Parton Distribution Functions (PDFs), which is not contained in $t$-channel Regge exchanges.

Experimentally, in the high $W$ domain, data for the exclusive electroproduction of the $\rho^0$ meson have been obtained at several $Q^2$ values by the H1 and ZEUS (at HERA), NMC (at CERN) and E665 (at Fermilab) experiments. It was shown by Frankfurt et al.\textsuperscript{5} that these high $W$ data can be well interpreted in terms of the 2-gluon exchange mechanism of Fig. 2 right. However, in order to achieve this, because the $Q^2$ domain where the data exist is relatively low, corrections have to be incorporated to the pure leading-twist calculation of the handbag diagrams. The straightforward leading twist calculation leads at low $Q^2$ values to an overestimate of the data by a large factor (up to 2 at $Q^2=10$ GeV$^2$ and higher at smaller
Also, the leading-twist calculation predicts that $\frac{d\sigma}{dt}$ should behave as $\frac{1}{Q^2}$, at fixed $x_B$ whereas a flatter $Q^2$ dependence is observed in the data.

The origin of the overestimate of the data lies in the presence of gluon exchange and the associated strong coupling to quarks $\alpha_s$ in the handbag diagrams of Fig. 2. Indeed, $\alpha_s$ is “running” and, in particular, is rising as $Q^2$, or more generally, the scale associated to the quark-gluon vertex (to which $Q^2$ is proportional), decreases. Also, the gluon exchange is associated with a propagator of the form $\frac{1}{zQ^2}$, where $z$ is the momentum fraction carried by the quark(s) interacting with the gluon. As $z$ runs between 0 and 1, at low $z$, this can lead to strong enhancements of the cross section.

The handbag diagrams deal mainly with longitudinal degrees of freedom $(x, \xi, \ldots)$. The idea brought by Frankfurt et al. to remedy the “overshooting” of the leading twist calculation was to also take into account the transverse momentum (“$k_\perp$”) dependence in the calculations of the handbag processes. This approach, also called the “modified parturbative approach”, was originally initiated in Ref. 6. The consequence is that there will always be a “minimum” momentum scale (the average transverse momentum) and therefore singularities of the type $z \to 0$ will be much avoided. With such a prescription, it was found a remarkable agreement for the $x_B$ (mainly driven by the momentum fraction distribution of the gluons) dependence and the $Q^2$ (asymptotic $\frac{1}{Q^2}$ modulated by the “$k_\perp$” correction factor) dependences of the $ep \to ep\rho^0$ cross section (as well as for the absolute normalization).

We have just discussed the high $W$ domain where, in summary, the $\rho^0$ electroproduction data look interpretable in terms of the 2-gluon exchange mechanism of Fig. 2 right (modulo “$k_\perp$” corrections). Now, in the low $W$ region, we ask whether, similarly, the quark handbag diagram of Fig. 2 left (modulo “$k_\perp$” corrections) is also at play? The few data that exist for the $ep \to ep\rho^0$ process at low $W$ include early data (1976) with a 7.2 GeV electron beam at DESY 7 and with a 11.5 GeV electron beam at Cornell 8 (1981) and more recently, with a 470 GeV muon beam at Fermilab, 9 a 27 GeV positron beam at HERMES, 10 and a 4.2 GeV electron beam at JLab 11 using the CLAS detector. Currently, JLab experiment E99-105 13 which recently took data with a 5.75 GeV electron beam and the CLAS detector is in the final stage of data analysis and we will have here a quick snapshot to some PRELIMINARY results.

The amplitude for the “quark” handbag diagram of Fig. 2 left (with “$k_\perp$” corrections) has been calculated, for instance, in Ref. 14. The main input to this calculation is the parametrization of the GPDs. As a first approximation, only $H(x, \xi, t)$ is considered in the following. The $(x, \xi)$ depen-
The only free parameters are $\alpha'$ which is fit, and strongly constrained, to reproduce the nucleon form factors and $b$ which governs the $(x, \xi)$ dependence but has, in effect, relatively little influence on the cross section.

As a second approximation, in the following, the quark and gluon handbags of Fig. 2 are summed at the cross section level. In Ref., this sum is treated at the amplitude level. It has been checked that, except in the intermediate $W$ region where interference is maximal, the two (independent) calculations are in remarkable agreement.

Fig. 3 shows the total longitudinal cross section for the exclusive $\rho^0$ electroproduction on the proton as a function of $W$ over a wide range for $Q^2 \approx 2.35 \text{ GeV}^2$ with the current world’s data. The cross sections exhibit clearly two different behaviors as a function of $W$: starting from low $W$, the cross section decreases with $W$. Then, at $W$ around $\approx 10 \text{ GeV}$, the $W$ slope changes and the cross section slowly rises. The handbag calculation (sum of the quark and gluon processes), just described above, shown by the dashed curve, gives a decent description of the high and intermediate $W$ region down to $\approx 6 \text{ GeV}$. This result was already observed by the HERMES collaboration. In particular, at high $W$, the rise of the cross section is due to the gluon and sea contributions.

Now, focusing on lower $W$ values, the GPD calculation clearly misses the data. This discrepancy can reach an order of magnitude at the lowest $W$ values. The trend of the GPD calculations is to decrease as $W$ decreases whereas the data increase. The trend of the GPD calculation is readily understandable: GPDs are approximately proportional to the forward quark densities $q(x)$ (the relation is not direct since, among other aspects, the quark densities are, in the Double Distributions, convoluted with a meson distribution amplitude but, still, the main trends remain; in the following
argument, we take \( x = x_B \). Therefore, as \( x \) increases (\( W \) decreases), they tend to 0 (\( q(x) \approx (1 - x)^3 \) for \( x \) close to 1). There might be a slight local increase around \( x \approx 0.3 \), due to the valence contribution (which is slightly apparent on Fig. 3) but it can never explain the increase of an order of magnitude, especially at low \( W \).

Fig. 3 also shows (dotted curve) the results of the calculation based on the “hadronic” \( t \)-channel diagrams of Fig. 1 which are “reggeized”. This is the co-called JML model\(^{19}\) and it reproduces fairly well the two general behaviors with \( W \) just mentioned. Here, the drop of the cross section at low \( W \) is due to the \( t \)-channel \( \sigma \) and \( f_2 \) meson exchange diagrams (the intercept \( \alpha(0) \) of the \( f_2 \) trajectory is \( \approx 0.5 \) and therefore the cross sections decrease with energy as \( \frac{1}{x^{0.5}} \)). The slow rise of the cross section which is observed above \( W \approx 10 \text{ GeV} \) is attributed to the Pomeron trajectory which has an intercept \( \alpha(0) \approx 1 + \epsilon \).

\[
2.20 < Q^2 (\text{GeV}^2) < 2.50
\]

Fig. 3. \( W \) dependence of the \( \gamma_L p \to p \rho^0 \) at \( Q^2 \approx 2.35 \text{ GeV}^2 \). Dashed curve : the “standard” handbag calculation (quark and handbag diagrams added incoherently) based on the Double Distribution ansatz for the quark GPDs. The thin solid curve is the result of the calculation including a (“renormalized”) D-term-like contribution. The dotted curve is the result of the Regge JML calculation.

Coming back to the GPD calculation, the conclusions that one can draw are two-fold:
• The handbag mechanism formalism is not at all the dominant mechanism in the low $W$ (valence) region and higher twists or (so far) uncontrollable non-perturbative effects suppress the handbag mechanism. In this case, one needs to explain why the handbag mechanism works at high/intermediate $W$ ($x_B$) domain and, quite abruptly, no more in the valence region. Higher twist can certainly depend on energy but such a strong variation with $W$ is certainly puzzling.
• An alternative explanation, based on and supported by the success of the handbag mechanism at high and intermediate $W$ values, is that the GPD formalism is indeed at work in the valence region but that a significant and fundamental contribution, besides the Double Distributions, is missing in our parametrization of the GPDs.

What could such a missing contribution in the low “$W$” regime be? In the framework of the JML model, the strong rise of the cross section as $W$ decreases is due to the $t$-channel $\sigma$ and $f_2$ meson exchange processes. It could then be tempting to associate the potentially missing piece in the GPDs with $t$-channel meson exchanges. The best example of such a contribution in GPDs is the so-called $D$-term which was originally introduced in Ref,\textsuperscript{20} where it was shown that it was required to introduce such a term in the most general parametrisation of GPDs in addition to Double Distributions, in order to satisfy the polynomiality rule. We recall that the $D$-term is non-zero only in the $-\xi < x < \xi$ domain, it is odd in $x$ and it is usually parametrized in terms of Gegenbauer polynomials of argument $\hat{x}$. The $-\xi < x < \xi$ region corresponds to the $q\bar{q}$ component of the GPDs and therefore the $D$-term can be thought of as representing the exchange of mesonic degrees of freedom in the $t$-channel. We see that a structure that exists only in the $-\xi < x < \xi$ domain would naturally provide a contribution that decreases with $W$, since $W \approx \frac{1}{\xi}$. In other words, as $W$ increases $\xi$ decreases, and therefore the support of such structure decreases and, as a consequence, its contribution diminishes up to 0 in the $\xi = 0$ limit.

The $D$-term is associated to a Lorentz scalar in the GPD definition and thus accounts only for scalar (isosinglet) meson exchange (as the $\sigma$ meson). Other meson contributions (for instance, tensor as the $f_2$ meson) are not included in the $D$-term. However, there is, in principle, in the GPD formalism, no reason to restrict these $q\bar{q}$ contributions to the scalar component as it is imposed in the $D$-term and there is freedom to add more general $q\bar{q}$ contributions, which can be an argument, in the following, to readjust its normalization. Because such contribution “exist” only in the $-\xi < x < \xi$
region (i.e. they vanish at \( \xi = 0 \)), they are not sensitive to the relation linking GPDs to quark densities at \( \xi=0 \). Furthermore, depending on their parity in \( x \), these contributions may or may not contribute to the sum rules linking the GPDs to form factors (FFs). For instance, the D-term is odd in \( x \) and doesn’t contribute to the form factor sum rule. One sees, in a very general way, that there can be contributions to the GPDs which completely escape any normalization or constraint.

The solid curve on Fig. 3 shows the results of a calculation with such a D-term which has been renormalized in order to fit the data. It is of course not very satisfying to add a term with unconstrained normalization to the GPDs parametrization. The purpose here is simply to illustrate that such a term provides just the right \( W \) dependence and is physically motivated in the sense that it is meant to parametrize \( q\bar{q} \) contributions, which are definitely part of the GPD concept. Now, the question of course remains whether one can justify such a strong contribution (the normalization of the D-term such as given in Ref\(^{20} \) barely changes the dashed curve on Fig. 3).

The CLAS experiment E99-105, which is in the final stage of analysis, will soon bring more than 25 new \((Q^2,x_B)\) data points to this low \( W \) domain, along with differential cross sections. For instance, as a snapshot, Fig. 4 shows the PRELIMINARY distribution \( dN/dt \) of events for the \( \gamma_L p \rightarrow p p^0 \) process. Because the analysis is still in progress, the normalization is arbitrary but the distributions are, for the most part, corrected for the acceptance of CLAS. One readily sees the large \((Q^2,x_B)\) domain spanned by these data. What is also obvious is that the slope of these \( t \) distributions vary with the kinematics and, in particular, increases with decreasing \( x_B \). In the framework of GPDs, this is a feature which is expected because the \( t \) dependence of the GPDs can be related,\(^{21-23} \) via a Fourier transform, to the transverse spatial distribution of the partons in the nucleon. A general “3-dimensional” image of the nucleon is then that “low \( x \)” sea quarks (which can be associated to the “pion cloud”) sit at the periphery of the nucleon (corresponding to large impact parameters and therefore large \( t \) slopes) while “large \( x \)” valence quarks sit in the core of the nucleon (corresponding to low impact parameters and therefore low \( t \) slopes). The evolution with \( x_B \) of the \( t \) slopes of Fig. 4 is in agreement with this image.

Finally, if a D-term like contribution is responsible for most of the cross section in this low \( W \) domain, it remains the “technical” problem to reconcile a varying \( t \) slope (with energy) with a D-term as the polynomiality rule imposes that the \( t \) dependence of the D-term be factorized (independent of
Fig. 4. PRELIMINARY distribution $dN/dt$ of events for the $\gamma_{LP} \rightarrow p\rho^0$ process from the CLAS E99-105 experiment. Units are arbitrary on the $y$ axis. All $x$ and $y$ axis have the same scales in each plot. The thick solid line is an exponential fit $Ae^{bt}$ to the data yielding the $b$ slope parameters. The thin solid line is the result of a GPD calculation including a (“renormalized”) D-term-like contribution. The dotted line is the result of the Regge JML calculation.

$x$ and $\xi$). We propose here a simple ansatz which generalizes the D-term:

$$GPD'(x,\xi,t) = \xi \int d\alpha d\beta \delta(x - \beta - \xi \alpha) DD'(\alpha,\beta,t)$$

(4)

with

$$DD'(\alpha,\beta,t) = ah(0,\alpha) \frac{b' | t |}{| \beta |^{b'+1}}$$

(5)

which has the nice feature of recovering the D-term in the forward limit since:

$$\lim_{|t| \to 0} \frac{b' | t |}{| \beta |^{b'+1}} = \delta(\beta)$$

(6)

The $\alpha$ factor allows us to respect the polynomiality rule.
The thin solid line in Fig. 4 shows the result of the calculation with such an ansatz and illustrates the variation of the slope with $x_B$ (the $t$ slope is, additionally, also non-constant in this calculation). The dashed line is the JML calculation (which uses “saturating” Regge trajectories) and the thick solid line is a simple exponential fit to the data, yielding the $b$ slope parameters which are on the figure.

Let us note the almost flat $t$ slopes at large $x_B$. Such flat $t$ slopes are basically impossible to reproduce with the standard double distributions contributions to the $H$ and $E$ GPDs. Indeed, their $t$-dependence is strongly constrained by the sum rule linking the GPDs to the FFs. If one is indeed sensitive to GPDs in this low $W$ regime, the very flat $t$ slopes observed can only arise from GPD contributions not constrained by the FF sum rule.

Let us finally mention that, at large $x_B$, the minimum value of $t$ that is kinematically accessible is large (i.e. $\approx 1.6 \text{ GeV}^{-2}$ at $< x_B > = 0.67$). A possible explanation of the disagreement between the “base” (i.e. without additional D-term-like $t$-channel meson exchange contribution) GPD model, could simply be that, at such large $t$ values, higher twists contributions can be extremely important.

Only additional and precise data for the $\gamma_L p \rightarrow pp^0$ process will allow us to distinguish between the two hypothesis raised above: GPDs or not GPDs?

With an important new set of data soon to be released, the E99-105 experiment using CLAS will already allow us to study in a more detailed manner the mechanisms at play in this reaction. In the longer term, the upgrade of JLab to 12 GeV, will permit a precise mapping of the $Q^2$, $x_B$ and $t$ dependences over a broad phase space. This, along with the measurement of new observables such as the transversely polarized target asymmetry, should bring a more definite answer as to the role or not of GPDs in the valence region for exclusive $\rho^0$, and more generally, other meson production.

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