Linear response in light deformed nuclei

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Abstract. We develop of a fully consistent Quasiparticle Random Phase Approximation (QRP A) method that employs the canonical harmonic oscillator (HO) Hartree–Fock–Bogoliubov (HFB) basis. Skyrme energy density functionals and density–dependent pairing functionals are used for both the HFB mean field and the QRP A approaches. We accurately describe multipole strength functions in axially–symmetric deformed even–even nuclei. Isoscalar and isovector responses in the deformed $^{24-26}$Mg and $^{34}$Mg isotopes are presented investigating the consequences of neglecting the spin–orbit and Coulomb residual interactions in QRP A.

1. Introduction
Correlations among particles composing many–body systems may be analyzed in terms of small amplitude oscillations such as RPA and Quasiparticle–RPA (QRPA) in superfluid systems where quasiparticle excitations play a role.

Several self-consistent RPA calculations based on the Skyrme–Hartree–Fock (SHF) method \cite{1, 2} and QRPA including pairing correlations \cite{3, 4, 5, 6, 7, 8} have been performed in spherical nuclei. The QRPA approach has been recently applied to light deformed nuclei (see e.g. \cite{9, 10} and references therein) and heavy deformed nuclei \cite{11, 12, 13}. At the same time new iterative methods have been developed to calculate QRPA strength functions for both spherical \cite{14, 15} and deformed \cite{16, 17, 18} nuclear systems.

Within this context we present a self–consistent HFB+QRPA approach for light deformed nuclei already presented in Ref. \cite{9}, discussing the role played by the residual interaction in the response of these nuclear systems to an external electric field.

2. Method
We start by solving HFB equations in a finite canonical harmonic oscillator (HO) basis with $N_{sh} = 15$ shells. See Ref. \cite{9} for details on the adopted method. We take the Skyrme SkM\textsuperscript{*} force together with a density–dependent pairing contact interaction \cite{19}

\begin{equation}
V_{\text{pair}}(r, r') = \frac{1 - P_\sigma}{2} \left[ V_0 + \frac{V_1}{6} \rho_0(r) \right] \delta (r - r'),
\end{equation}

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(\rho_0(r) \) is the associated isoscalar density and \( P_r \) the spin exchange operator) with quasiparticle energy cutoff \( E_{\text{cut}} = 50 \mev \) and parameters \( V_0 = -280 \mev \cdot \text{fm}^{-3} \), \( V_1 = 18.75V_0 \), \( \gamma = 1 \) such that a mixed surfaced–volume type of pairing potential is reproduced. The cutoff parameters adopted in the QRPA calculations on the single–particle energies and on the occupation probabilities are \( \varepsilon_{\text{crit}} = 200 \mev \) and \( v_{\text{crit}} = 10^{-2} \) respectively.

The strength functions

\[
S_j^\tau(E) = \sum_\lambda \sum_\Omega \frac{\Gamma/2}{\pi} \frac{\left|\langle \lambda | \hat{F}_{j\Omega}^\tau |0 \rangle \right|^2}{(E - E_\lambda)^2 + \Gamma^2/4},
\]

are calculated for \( \Gamma = 1 \mev \) with transition operator

\[
\hat{F}_{2\Omega}^{1S} = \frac{eZ}{A} \sum_{i=1}^{A} r_i^2 Y_{2\Omega}(\hat{r}_i),
\]
\[
\hat{F}_{1\Omega}^{IV} = \frac{eN}{A} \sum_{i=1}^{Z} r_i Y_{1\Omega}(\hat{r}_i) - \frac{eZ}{A} \sum_{i=1}^{N} r_i Y_{1\Omega}(\hat{r}_i),
\]

in the IS quadrupole and IV dipole modes, respectively for \(^{24,26}\text{Mg} \) and \(^{34}\text{Mg} \).

3. Results

Figure 1. (Color online) HFB potential energy curves \( E_{\text{HFB}} \) (upper panels), neutron and proton pairing energies \( E_{\text{pair}} \) (lower panels) as functions of axial deformation parameter \( \beta \) in \(^{24}\text{Mg} \) (left–hand panels) and \(^{26}\text{Mg} \) (right–hand panels).
3.1. \( ^{24,26}\text{Mg} \)

In figure 1 the potential energy curves for \(^{24}\text{Mg}\) and \(^{26}\text{Mg}\) are plotted as function of the deformation parameter

\[
\beta = \sqrt{\frac{3}{5} \langle Q \rangle_n + \langle Q \rangle_p},
\]

\(<Q>_n\) being the average value of the quadrupole–moment operator \(Q = 2z^2 - r^2_\perp\) for protons \((q = p)\) and neutrons \((q = n)\). The QRPA calculations are obtained using the HFB solutions with minimum energy as mean field basis.

In the upper panels of figure 1, \(^{24}\text{Mg}\) is found to be prolately deformed with deformation \(\beta = 0.39\), while \(^{26}\text{Mg}\) presents an oblate deformation at \(\beta = -0.18\). The lower panels of figure 1 show that for \(^{24}\text{Mg}\) the total pairing energy is zero at \(\beta = 0.39\), so QRPA calculations reduce to RPA ones. A full QRPA treatment is needed for \(^{26}\text{Mg}\) with HFB ground state having proton correlations.

The upper panels of figure 2 display the fraction EWSR for the IS \(2^+\) mode in \(^{24}\text{Mg}\) and \(^{26}\text{Mg}\). They present a giant resonance in the energy region 15–25 MeV. In \(^{24}\text{Mg}\), the three–peaks structure corresponds to the \(\Omega\)–splitting, with \(\Omega^+ = 0^+\) component along the longest axis at the lowest energy. The \(\Omega\)–splitting is not pronounced in \(^{26}\text{Mg}\) due to the smaller value of the deformation parameter [20]. Because of the oblate shape of \(^{26}\text{Mg}\) the peak at the lowest energy belongs to the \(\Omega^+ = 2^+\) component. The quadrupole response in \(^{24}\text{Mg}\) is compared with the experimental results of Youngblood et al. [21]. The shape of the experimental curve is quite well reproduced, however in our calculations the central peak at around 20 MeV is too pronounced and too little strength is found at lower energies. The sharp experimental peak at 15 MeV is of notice and it may be connected to the \(\Omega^+ = 0^+\) vibration which is calculated at 17.5 MeV. These differences are apparent from the values of the mean energies

\[
E(\text{GR}) = \frac{m_1 [E_{\text{min}}, E_{\text{max}}]}{m_0 [E_{\text{min}}, E_{\text{max}}]},
\]

with moments

\[
m_\lambda [E_{\text{min}}, E_{\text{max}}] = \sum_k \sum_\Omega E_k^\lambda |\langle k| \hat{F}_J \Omega |0\rangle|^2,
\]

shown in table 1 calculated in the energy interval \([E_{\text{min}}, E_{\text{max}}] = [9, 41]\) MeV. The theoretical centroid of 19.4 MeV overshoots the experimental mean energy by about 2 MeV. The calculated mean value of the giant resonance is also compared with that obtained by S. Pérus et al. [22] with the use of a Gogny force. The difference of about 1 MeV may be addressed to the lower effective mass \(m^*/m = 0.7\) for the Gogny force, having the values of \(m^*/m = 0.8\) for the SkM* force. A calculation performed with a SLy4 force \((m^*/m = 0.7)\) shows an increasing of the centroid energy of about 0.6 MeV, indicating that half of the difference between the two theoretical results is due to the effective mass.

The IV dipole fraction of EWSR of \(^{24}\text{Mg}\) and \(^{26}\text{Mg}\) are shown in the lower panels of figure 2 where they are compared to the experimental results of Irgashev et al. [23] and Fultz et al. [24] respectively. For \(^{24}\text{Mg}\), both curves display the same two–peaked structure, but the calculated peaks at 16 MeV and 21–22 MeV appear too low in energy of about 3 MeV. This difference is found also in the mean energy (see table 1) extracted from the experimental data in the energy range \([E_{\text{min}}, E_{\text{max}}] = [10, 29]\) MeV and equal to 22.1 MeV, to be compared to the theoretical values of 19.6 MeV. The centroid energy of 23.0 MeV obtained by the Gogny–force calculation [22] approaches more to the observed one. In \(^{26}\text{Mg}\), there is agreement between the calculated curve and the experimental one [24], both of them presenting two maxima at around 18 MeV
and 21–22 MeV separated by a minimum at about 20 MeV. Table 1 shows that our calculations differ from experimental centroid energies by 300 keV. Our calculated mean energies differ from those obtained by S. Pérusat al. [22] by about 2.5 MeV.

In both $^{24}\text{Mg}$ and $^{26}\text{Mg}$, the effects of the spin–orbit and Coulomb parts of the residual interaction are stronger for the IS $2^+$ mode than for the IV dipole response. The quadrupole calculations performed without these terms shift the strength function down in energy by about 0.9 MeV. The IS $2^+$ peaks calculated in $^{24}\text{Mg}$ by K. Yoshida et al. [25], who neglected these terms, are further lowered in energy of about 0.9 MeV with respect to those obtained by our calculations without spin–orbit and Coulomb terms. This may be due to the different basis used in the two calculations and to the renormalization of the interaction performed in Ref. [25].

Figure 2. (Color online) Fractions of IS $2^+$ (upper panels) and IV $1^-$ (lower panels) EWSR in $^{24}\text{Mg}$ (left–hand panels) and $^{26}\text{Mg}$ (right–hand panels) without the residual spin–orbit (so) and Coulomb (C) interaction (thin solid curve) and with all the terms included (dashed curve). These are compared with experimental data by Youngblood et al., Irgashev et al. and Fultz et al. for IS $2^+$, IV $1^-$ in $^{24}\text{Mg}$ and IV $1^-$ in $^{26}\text{Mg}$ respectively (thick solid curve).

3.2. $^{34}\text{Mg}$

In the upper panel of figure 3 $^{34}\text{Mg}$ is found to be prolately deformed with $\beta = 0.36$. Full QRPA calculations are performed for this nuclear system, with HFB ground state having neutron pairing correlations as shown in the lower panel of figure 3.

The upper panel of figure 4 shows the IS quadrupole response for the $\Omega^\pi = 0^+, 1^+, 2^+$ components. A low–lying peak and a giant resonance are given at 2–3 MeV and 15–22 MeV, respectively. The main contribution to the first low–lying peak belonging to the $\Omega^\pi = 0^+$ excitation, is given by the neutron particle–particle (pp) transitions $[N, n_z, \Lambda, \pm \Omega^\pi]_2^2 \equiv [2, 0, 2, \pm 3^+]_2^2, [3, 2, 1, \pm 3^+]_2^2, [3, 3, 0, \pm 1^+]_2^2$, the first of them coming from $(1d_{3/2})^2$ the latter
Table 1. Theoretical mean energy values (in MeV) in $^{24,26}$Mg obtained with the Gogny force by S. Péré et al., with the SkM* Skyrme force (present work) and experimental energy values by Youngblood et al. and Irgashev et al. of the IS giant quadrupole resonances (calculated in the energy interval $[E_{\text{min}}, E_{\text{max}}] = [9, 41]$ MeV), and of the IV giant dipole resonance (calculated in the energy interval $[E_{\text{min}}, E_{\text{max}}] = [10, 29]$ MeV) in $^{24}$Mg. For IV $1^−$ resonance in $^{26}$Mg the experimental mean energy value by Fultz et al. is also given.

| $E(\text{GR})$ | $^{24}$Mg $J^π = 2^+$ | $^{24}$Mg $J^π = 1^−$ | $^{26}$Mg $J^π = 2^+$ | $^{26}$Mg $J^π = 1^−$ |
|----------------|---------------------|---------------------|---------------------|---------------------|
| Interval       | [9,41]              | [10,29]             | [9,41]              | [10,29]             |
| Theor. SkM*    | 19.4                | 19.6                | 20.3                | 20.3                |
| Theor. SLy4    | 20.0                | 19.8                |                     |                     |
| Theor. D1S (Péré et al.) | 20.5              | 23.0                | 21.0                | 22.9                |
| Exp. (Youngblood et al.) | 16.9 ± 0.6       |                     |                     |                     |
| Exp. (Irgashev et al.) | 22.1              |                     |                     |                     |
| Exp. (Fultz et al.)  |                     |                     |                     | 20.6                |

two from $\left(1f_7/2\right)^2$. The data are treated in table 2 in detail and agree with those calculated by K. Yoshida et al. (see table I of Ref. [26]). The value of the two–quasiparticle (2qp) transition energies $E_{qpK} + E_{qpK'}$, obtained diagonalizing the HFB Hamiltonian in the canonical basis (cf. Eqs. (4.14b), (4.20) of Ref. [27]) is higher than the energy of the low–lying peak at 2–3 MeV. Thus, the residual interaction shifts down in energy the IS $2^+$ strength distribution in the pp channel.

The IV dipole response shown in the lower panel of figure 4 displays a fragmented behavior both in the lower energy peaks of the IV giant dipole resonance at (12–17) MeV belonging to the $\Omega^π=0^−$ projection and in the upper ones at (18–25) MeV, belonging to $\Omega^π=1^−$. The general behavior of the resonance displays anyhow the typical two–peak structure due to the $\Omega^π=0^−, 1^−$–splitting. The last two columns of table 2 show that the major components contributing to the peak at 15.57 MeV belonging to the $\Omega^π=0^−$ component, are the proton particle-hole (ph) and neutron hole-hole (hh) $\left[2, 2, 0, \pm \frac{1}{2}^+\right] \rightarrow \left[3, 3, 0, \pm \frac{1}{2}^-\right]$ transitions, coming from the $1d_{5/2} \rightarrow 1f_{7/2}$ transitions. It is interesting to see that the energy $E_{qpK} + E_{qpK'}$ of the unperturbed 2qp transitions is lower than the energy of the QRPA peak for the proton transitions, viceversa for the neutron transitions. One may conclude that the main contribution of the residual interaction in the ph and hh channels is to shift up and down in energy respectively the IV dipole response.

As seen in Section 3.1 for $^{24,26}$Mg, a significant contribution of the spin–orbit and Coulomb terms to the residual interaction is given in the $2^+$ modes. In figure 5 the total IS quadrupole strength distribution is displayed. The omission of the spin–orbit and Coulomb residual interaction shifts up in energy of about 0.2 MeV the low–lying peak belonging to the $\Omega^π=0^+, 2^+$ components and shifts down in energy the giant resonance by about 0.6 MeV. The general behavior of the response without spin–orbit and Coulomb terms is quite close to the corresponding calculations by K. Yoshida et al. [25]. Our giant resonance is shifted up in energy by about 1.3 MeV respect to that in Ref. [25], similarly to the difference discussed in Section 3.1 between the present calculations and those in [25] for $^{24}$Mg.
4. Conclusions
A fully consistent QRPA method is adopted to calculate the linear response of axially–
symmetric–deformed nuclei in the canonical HO HFB basis. We analyzed the IS 2+ and IV 1−
modes of 24−26Mg and 34Mg.
For 24−26Mg we focused on the role of deformation in the energy region of giant resonances.
It manifests itself in the splitting between the different projections which is more pronounced for
the strongly deformed 24Mg. An other feature of the giant modes is the fragmented behavior,
especially of their highest energy projections. In this respect, our calculations confirm earlier
theoretical results [22, 25] and are in quite good agreement with experimental data [21, 23, 24].
The inclusion of the spin-orbit and Coulomb parts of the residual interaction only introduces
minor changes in the giant resonances in the IS quadrupole modes, shifting the centroid by
about 1 MeV and leaving the widths rather unaffected.
In 34Mg we concentrated on the microscopic features of the low-lying states of the IS
quadrupole Ωπ = 0+ excitation and of the Ωπ = 0− component of the IV dipole giant resonance
and we found an overall agreement with the theoretical results of K. Yoshida et al. [25, 26].
In a general perspective, the present work represents the first, unavoidable step for a
consistent and more systematic study of collective modes in nuclei, in particular light exotic
nuclei, taking properly into account also medium polarization effects.

References
[1] Hamamoto I, Sagawa H and Zhang X Z 1996 Phys. Rev. C 53 765
[2] Shlomo S and Agrawal B K 2003 Nucl. Phys. A 722 98c
[3] Matsuo M 2001 Nucl. Phys. A 696 371
[4] Hagino K and Sagawa H 2001 Nucl. Phys. A 695 82
[5] Bender M, Dobaczewski J, Engel J and Nazarewicz W 2002 Phys. Rev. C 65 054322
Figure 4. (Color online) IS $2^+$ strength functions (upper panel) for the $\Omega^\pi = 0^+$ (solid curve), $1^+$ (dashed curve) and $2^+$ (dashed–dotted curve) excitations in $^{34}$Mg. IV $1^-$ strength functions (lower panel) for the $\Omega^\pi = 0^-$ (solid curve), $1^-$ (dashed curve) excitations.

Figure 5. (Color online) IS $2^+$ strength functions in $^{34}$Mg without the residual spin–orbit (so) and Coulomb (C) interaction (thin solid curve) and with all the terms included (dashed curve).

[6] Khan E, Sandulescu N, Grasso M and Van Giai N 2002 Phys. Rev. C 66 024309
[7] Yamagami M and Van Giai N 2004 Phys. Rev. C 69 034301
[8] Terasaki J, Engel J, Bender M, Dobaczewski J, Nazarewicz W and Stoitsov M 2005 Phys. Rev. C 71 034310
[9] Losa C, Pastore A, Dossing T, Vigezzi E, Broglia R A 2010 Phys. Rev. C 81 064307
[10] Martini M, Péru S and Dupuis M 2011 arXiv:1103.1553v1
[11] Péru S, Gosselin G, Martini M, Dupuis M, Hilarie S and Devaux J -C 2011 Phys. Rev. C 83 014314
[12] Terasaki J and Engel J 2011 arXiv:1105.3817v1
[13] Yoshida K. 2010 Phys. Rev. C 82 034324
[14] Toivanen J, Carlsson B G, Dobaczewski J, Mizuyama K, Rodríguez-Guzmán R R, Toivanen P, Veselý P 2010 Phys. Rev. C 81 034312
Table 2. QRPA amplitudes $[N,n_z,L,\pm\Omega^\pi]_K \rightarrow [N,n_z,L,\pm\Omega^\pi]_{K'}$ with isospin $q$ ($q = n$ for neutrons, $q = p$ for protons) of the IS quadrupole $\Omega^\pi = 0^+$ mode at 2.64 MeV (2nd–4th columns) and IV dipole $\Omega^\pi = 0^-$ mode at 15.57 MeV (5th and 6th columns) in $^{34}$Mg. The quasi particle occupations $v^2_{K,K'}$, the single particle energies $\varepsilon_K$ and the 2qp excitation energies $E_{qpK} + E_{qpK'}$, are given. Only components with $X^2_{KK'} - Y^2_{KK'} > 0.01$ are listed.

| $K$ | $[2,0,2,\pm\frac{3}{2}]$ | $[3,2,1,\pm\frac{3}{2}]$ | $[3,3,0,\pm\frac{1}{2}]$ | $[2,2,0,\pm\frac{1}{2}]$ | $[2,2,0,\pm\frac{1}{2}]$ |
|-----|------------------|-----------------|-----------------|-----------------|-----------------|
| $K'$ | $[2,0,2,\pm\frac{3}{2}]$ | $[3,2,1,\pm\frac{3}{2}]$ | $[3,3,0,\pm\frac{1}{2}]$ | $[3,3,0,\pm\frac{1}{2}]$ | $[3,3,0,\pm\frac{1}{2}]$ |
| $J^\pi, \Omega^\pi$ | $2^+, 0^+$ | $2^+, 0^+$ | $2^+, 0^+$ | $1^-, 0^-$ | $1^-, 0^-$ |
| $E_\lambda$ | 2.64 MeV | 2.64 MeV | 2.64 MeV | 15.57 MeV | 15.57 MeV |

$\varepsilon_K$ | $-3.20$ | $-4.91$ | $-6.84$ | $-23.13$ | $-17.13$ |
$\varepsilon_K'$ | $-3.20$ | $-4.91$ | $-6.84$ | $-11.51$ | $-6.84$ |
$X_{KK'}$ | $-0.64$ | $0.67$ | $0.27$ | $0.27$ | $0.22$ |
$Y_{KK'}$ | $-0.09$ | $0.03$ | $0.04$ | $-0.03$ | $-0.01$ |
$E_{qpK} + E_{qpK'}$ | $3.58$ | $3.49$ | $5.72$ | $10.95$ | $16.05$ |

[15] Avogadro P and Nakatsukasa T 2011 Phys. Rev. C 84 014314
[16] Nakatsukasa T, Inakura T and Yabana K 2007 Phys. Rev. C 76 024318
[17] Inakura T, Nakatsukasa T and Yabana K 2009 Phys. Rev. C 80 044301
[18] Stoitsov M, Kortelainen M, Nakatsukasa T, Losa C and Nazarewicz W 2011 arXiv: 1107.3530v1
[19] Chasman R R 1976 Phys. Rev. C 14 1935
[20] Bortignon P F, Bracco A and Broglia R A 1998 Giant Resonances (Amsterdam: Harwood Academic Press)
[21] Youngblood D H, Lui Y -W and Clark H L 1999 Phys. Rev. C 76 044304
[22] Pérus S and Goutte H 2008 Phys. Rev. C 77 044313
[23] Irgashev K M, Ishkhanov B S and Kapitonov I M 1966 Phys. Rev. C 145 771
[24] Fultz S C, Alvarez R A, Berman B L, Kelly M A, Lasher D R, Phillips T W and McElhinney J 1971 Phys. Rev. C 4 149
[25] Yoshida K and Van Giai N 2008 Phys. Rev. Phys. Rev. C 78 064316
[26] Yoshida K and Yamagami M 2008 Phys. Rev. C 77 044312
[27] Dobaczewski J, Nazarewicz W, Werner T R, Berger J F, Chinn C R and Dechargé J 1996 Phys. Rev. C 53 2809