Kelvin’s ideal foam structure

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Abstract. Among his many extraordinary accomplishments, Kelvin was a pioneer of crystallography, elasticity and materials science. These interests came together to inspire his speculation on the nature of the ether in 1887. He conceived it to be an ordered liquid foam, of minimal surface area. Kelvin’s ideal structure of foam has been realised in the laboratory only recently. In the meantime it was surpassed (in terms of surface area minimisation) by the Weaire-Phelan foam, which is the basis for one of the main buildings of the Beijing Olympics.

1. Kelvin’s versatility
The span of Kelvin’s published work is enormous. Not only did he publish in many fields of physics, but he cross-fertilised them, often bringing the fundamental and the applied to bear on each other. It is an all-embracing style that few could even aspire to emulate today, even if they had his talent and energy.

His proposal of a structure for the ether of space [1] is a good example and it happens to be topical today. Here he was addressing one of the great fundamental questions of his age: what invisible material fills space, enabling light waves to travel through it? What is the nature of this ether? It was the “dark matter” of the 19th century.

It had been thought for half a century or more that the light waves must be elastic waves in a material ether, but ordinary materials do not have the right properties to account for light waves in this way.

Even as Kelvin dwelt upon this paradox, others began to consider it false or misguided. Instead of the elusive ether they accepted the more abstract field equations of Maxwell [2], and the doctrine of the so-called Maxwellians [3]. But Kelvin was not persuaded by that alternative view, and insisted that the ether was “a real thing”. He was familiar with many materials that had unusual properties: might one of these not be a candidate for the kind of stuff that made up the ether?

He continued to turn the problem over his mind until, lying in bed in his seaside mansion in 1887, he dreamed up his foam model of the ether. In framing and elaborating this he was to call on his knowledge of elasticity, wave theory and crystallography as well as his general background in materials science.

2. Kelvin and materials
No doubt pure curiosity played a role in Kelvin’s interest in materials, but it also derives from the applied research that made much of his reputation and most of his riches.

Beginning with a theoretical study of the propagation of telegraphic signals, he was gradually drawn into a concern for the properties of the very materials that constituted the telegraphic cable. On board the cable ships, struggling with practical realities, he saw the essential role of those materials. If they failed, then so did the entire enterprise.
The story of the gradual improvement and optimal exploitation of those materials is reminiscent of twentieth-century solid state physics. Just as Kelvin was led to demand better characterised materials, so in our own time did the semiconductor industry require improved control of silicon, and arcane research was seen to underpin industrial success. If we trace a thread of continuity between the two periods, Kelvin deserves to be called the first solid state physicist.

The telegraph cable was at that time (and for a long time to come) primarily made up of a copper core and a gutta-percha sheath. They played the role of conductor and insulator respectively, and those properties required to be standardised and optimised. Kelvin’s advocacy of proper industrial standards, based on this case, was to give Britain a great advantage over its industrial competitors in later years.

His dogged persistence in the campaign for proper units and standards is exemplified by a comical excerpt from a verbatim report on Electrical Standards Committee of the Board of Trade (1891), in Strutt’s biography of his father, Lord Rayleigh [4].

The Chairman: I am bound to tell the Committee that I did speak to Sir Michael Higgs Beach [President of the Board of Trade], about this particular definition, and that it certainly frightened him a great deal.

Sir William Thomson: A page from the theory of averages would frighten him still more.

Kelvin was not prepared to be obstructed by bureaucratic blockheads.

It is sad to reflect that even this huge economic benefit to the British economy did not, of itself, provide the motivation for his peerage, which was conferred only when he strayed into active politics.

3. Foam

By 1887, he was ideally qualified to design a material ether, and his musings in bed were quickly turned into a very strange model of an ether that consisted of a liquid foam without any enclosed gas.

To see that this should have the required mechanical properties was an extraordinary feat of imagination, and we have no idea where it came from. Maxwell wrote a poem about “empty bubbles” rising out of the unconscious during sleep, which is curiously apt to describe Kelvin’s revelation, but what more rational sources it may have had are obscure. It was a totally new and utterly brilliant idea, however out of tune with a time when the concept of ether was becoming obsolete.

The mathematical result that he conjectured has turned out to be incorrect. It was supposed to answer the question: what foam structure, made up of bubbles of equal size, has the lowest surface area, and hence energy? Although eventually surpassed, the Kelvin foam structure remains of interest today – truly a triumph of failure.

From the very outset his colleagues were not impressed [1] (“utterly frothy”, “a pure waste of time” etc.) [5]. They were particularly inspired by the experiment of Hertz, the youngest of the Maxwellians, and its Maxwellian interpretation by Fitzgerald. Many scathing comments were made (mostly in private), such as that of Rayleigh to Schuster [4]:

“Sir W. is full of a froth theory of the ether! This will lend itself to sarcasm even better than the jelly theory”.

The reference is to an earlier notion of Stokes, which Kelvin had also found interesting [6].
Even compendious histories of the ether pay little attention to Kelvin’s foam. In his long list of 650 or so papers, his proposal [1] of an ideal foam structure, intended for the ether, has been considered relatively insignificant by biographers. Nevertheless it would rank high in terms of the fashionable citation analysis of today. Mathematicians, physicists, materials scientists and biologists have all found it stimulating, when they consider foam or other cellular structures. So it is still worthy of a close reading, and accordingly a critique of it has recently been published [5].

Perhaps because the paper was composed in some haste and probably not subjected to review before publication (since Kelvin was the Editor), it is hardly a model of clarity. It recounts the author’s original thought processes rather than some reconstructed logic later formulated in tranquillity, in the interests of transparency. Just what we tell graduate students not to do?

That first paper, obviously intended to lay the foundation for much that was to come (but never did) was entitled:

*On the Division of Space with Minimum Partitional Area*

It was confined to the problem of the specification of the foam structure of equal bubbles that has minimal surface area: the ether is not mentioned! The question addressed, and encapsulated in the title, is today called the Kelvin Problem [6].

Kelvin’s instincts and logic led him to what he called the *tetrakaidecahedron*, as the shape of the cells packed together in the foam. Today’s physicists would relate it to the Wigner-Seitz cell of the bcc structure, although it is not precisely the same, in that some of its faces are subtly curved. Kelvin’s mathematical treatment of this difficulty is highly impressive, involving judicious approximation and numerical calculation, which he got right.

In general this kind of problem is impossibly difficult without the aid of computers. Only in the last decade or so has it become routinely possible to compute such surfaces accurately.

![Figure 1](image)
The Kelvin Cell, or tetrakaidecahedron, consists of six flat quadrilateral and eight hexagonal faces, which have a subtle curvature. It can be packed to fill all space, in a body-centred cubic arrangement.

4. Later developments

The unresolved Kelvin Problem [7] has lingered in the minds of mathematicians: *was his suggested structure really the best?*

If so, one might hope to see it in nature, but even individual Kelvin cells proved to be very rare. Was this because of the trapping of real macroscopic structures in haphazard disordered configurations, or the variation in cell sizes, or was it because Kelvin’s structure was not optimal after all? Opinion remained divided until the 1990’s.

In 1994, Weaire and Phelan [8] published the first counter-example to the conjecture that the Kelvin structure has lowest surface area.

Nevertheless, the Kelvin structure lives on. Only recently we have found out how to make it. Ironically, we cannot yet make the superior Weaire-Phelan foam in the laboratory. But it does loom large in a somewhat artificial manifestation in 2008.
5. Bubbles in Beijing
The Water Cube, the aquatic centre of the Olympic Games, consists of a gigantic steel framework, clad in plastic. Its beams represent the Plateau borders of the Weaire-Phelan structure. The designers started with the Kelvin structure, in a search for something emblematic of water, but eventually turned to the more irregular (but nevertheless periodic) WP structure, as conforming better to today’s aesthetic [9].

**Figure 2.** Part of the original design for the interior of the Water Cube, based on the Weaire-Phelan Structure. This is a more complex structure than that of Kelvin, with two kinds of cells. Pentagonal faces predominate.

It will be admired by many millions. For the physicist, it may be a monument to Kelvin’s restless curiosity, expressed in the Kelvin Problem, which as a problem in rigorous geometry still remains open for future generations of geometers.

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