STATE OF THE UNIFICATION ADDRESS∗

M. J. DUFF†

Michigan Center for Theoretical Physics
Randall Laboratory, Department of Physics, University of Michigan
Ann Arbor, MI 48109–1120, USA

After reviewing how M-theory subsumes string theory, I report broadly on some new and interesting developments, focusing on the “brane-world”: circumventing no-go theorems for supersymmetric brane-worlds, complementarity of the Maldacena and Randall-Sundrum pictures; self-tuning of the cosmological constant. I conclude with the top ten unsolved problems.

1. Introduction

Mr. Chairman, members of DPF, honored guests, my fellow physicists: Today, I have the honor of reporting on the State of the Unification. By which I mean current developments in string theory and M-theory. Back in the Fall of 99 when my colleagues and I at the University of Michigan began planning for the organization of the Strings 2000 conference held in July, we were concerned, as conference organizers often are, that perhaps too little would have happened in the subject since Strings 99. In fact, we need not have worried. Although there was nothing to rival the magnitude of 1984 String Revolution or the 1995 M-theory revolution, the field is nevertheless very healthy with many new and unexpected areas of progress. These include topics which go by the names of: (1) non-commutative field theory, (2) open-membrane (OM) theory, (3) K-theory, (4) tachyon condensation, (5) strongly coupled gravity, (6) supersymmetric brane-worlds and how no-go theorems are circumvented, (7) complementarity of the Maldacena and Randall-Sundrum pictures (8) self-tuning of the cosmological constant. My job is to summarize these new developments, but in view of the time constraints and in view of the recent excellent reviews on topics (1)-(5), I shall focus mainly on topics (6)-(8) which have the common theme of the “brane-world.” Bearing in mind the audience at this DPF meeting, these are also the areas which seem closest to phenomenology. First, however, it is necessary to recall where we are at the moment.

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† mduff@umich.edu
a http://feynman.physics.lsa.umich.edu/strings2000/
b “Not close enough,” I hear some of you say. I share your pain.
2. The story so far

2.1. M-theory and dualities

Not so long ago it was widely believed that there were five different superstring theories each competing for the title of “Theory of everything,” that all-embracing theory that describes all physical phenomena. See Table 1.

Moreover, on the \((d, D)\) “branescan” of supersymmetric extended objects with \(d\) worldvolume dimensions moving in a spacetime of \(D\) dimensions, all these theories occupied the same \((d = 2, D = 10)\) slot. See table 2. The orthodox wisdom was that while \((d = 2, D = 10)\) was the Theory of Everything, the other branes on the scan were Theories of Nothing. All that has now changed. We now know that there are not five different theories at all but, together with \(D = 11\) supergravity, they form merely six different corners of a deeper, unique and more profound theory called “M-theory” where \(M\) stand for Magic, Mystery or Membrane. M-theory involves all of the other branes on the branescan, in particular the eleven-dimensional membrane \((d = 3, D = 11)\) and eleven-dimensional fivebrane \((d = 6, D = 11)\), thus resolving the mystery of why strings stop at ten dimensions while supersymmetry allows eleven.

Although we can glimpse various corners of M-theory, the big picture still eludes us. Uncompactified M-theory has no dimensionless parameters, which is good from the uniqueness point of view but makes ordinary perturbation theory impossible since there are no small coupling constants to provide the expansion parameters. A low energy, \(E\), expansion is possible in powers of \(E/M_P\), with \(M_P\) the Planck mass, and leads to the familiar \(D = 11\) supergravity plus corrections of higher powers in the curvature. Figuring out what governs these corrections would go a long way in pinning down what M-theory really is.

Why, therefore, do we place so much trust in a theory we cannot even define? First we know that its equations (though not in general its vacua) have the maximal number of 32 supersymmetry charges. This is a powerful constraint and provides many “What else can it be?” arguments in guessing what the theory looks like when compactified to \(D < 11\) dimensions. For example, when M-theory is compactified on a circle \(S^1\) of radius \(R_{11}\), it leads to the Type IIA string, with string coupling
constant $g_s$ given by

$$g_s = R_{11}^{3/2}$$  \hspace{1cm} (1)$$

We recover the weak coupling regime only when $R_{11} \to 0$, which explains the earlier illusion that the theory is defined in $D = 10$. Similarly, if we compactify on a line segment (known technically as $S^1/Z_2$) we recover the $E_8 \times E_8$ heterotic string. Moreover, although the corners of M-theory we understand best correspond to the weakly coupled, perturbative, regimes where the theory can be approximated by a string theory, they are related to one another by a web of dualities, some of which are rigorously established and some of which are still conjectural but eminently plausible. For example, if we further compactify Type IIA string on a circle of radius $R$, we can show rigorously that it is equivalent to the Type IIB string compactified on a circle or radius $1/R$. If we do the same thing for the $E_8 \times E_8$ heterotic string we recover the $SO(32)$ heterotic string. These well-established relationships which remain within the weak coupling regimes are called $T$-dualities. The name $S$-dualities refers to the less well-established strong/weak coupling relationships. For example, the $SO(32)$ heterotic string is believed to be $S$-dual to the $SO(32)$ Type I string, and the Type IIB string to be self-$S$-dual. If we compactify more dimensions, other dualities can appear. For example, the heterotic string compactified on a six-dimensional torus $T^6$ is also believed to be self-$S$-dual. There is also the phenomenon of duality of dualities by which the $T$-duality of one theory is the $S$-duality of another. When M-theory is compactified on $T^n$, these $S$ and $T$ dualities are combined into what are termed $U$-dualities. All the consistency checks we have been able to think of (and after 5 years there dozens of them) have worked and convinced us that all these dualities are in fact valid. Of course we can compactify M-theory on more complicated manifolds such as the four-dimensional $K3$ or the six-dimensional Calabi-Yau spaces and these lead to a bewildering array of other dualities. For example: the heterotic string on $T^3$ is dual to the Type I string on $K3$; the heterotic string on $T^6$ is dual to the the Type I string on Calabi-Yau; the Type IIA string on a Calabi-Yau manifold is dual to the Type IIB string on the mirror Calabi-Yau manifold. These more complicated compactifications lead to many more parameters in the theory, known to the mathematicians as moduli, but in physical uncompactified spacetime have the interpretation as expectation values of scalar fields. Within string perturbation theory, these scalar fields have flat potentials and their expectation values are arbitrary. So deciding which topology Nature actually chooses and the values of the moduli within that topology is known as the vacuum degeneracy problem.

2.2. Branes

In the previous section we outlined how M-theory makes contact with and relates the previously known superstring theories, but as its name suggest, M-theory also relies heavily on membranes or more generally $p$-branes, extended objects with $p = d - 1$ spatial dimensions (so a particle is a 0-brane, a string is a 1-brane, a membrane is a 2-brane and so on). In $D = 4$, a charged 0-brane couples naturally
Table 2: The branescan, where \( S \), \( V \) and \( T \) denote scalar, vector and antisymmetric tensor multiplets.

\[
\begin{array}{cccccccccc}
11 & 1 & S & T \\
10 & V & S/V & V & V & V & S/V & V & V & V & V \\
9 & S & & & & & & & & & \\
8 & S & & & & & & & & & \\
7 & S & & & & & & & & & \\
6 & V & S/V & V & S/V & V & V & V & V & V & V \\
5 & S & & & & & & & & & \\
4 & V & S/V & S/V & V & V & V & V & V & V & V \\
3 & S/V & S/V & V & V & V & V & V & V & V & V \\
2 & S & & & & & & & & & \\
1 & & & & & & & & & & \\
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & d & \rightarrow
\end{array}
\]

to an Maxwell vector potential \( A_\mu \), with field strength \( F_{\mu\nu} \) and carries an electric charge

\[
Q \sim \int_{S^2} \ast F_2
\]  
and magnetic charge

\[
P \sim \int_{S^2} F_2
\]  
where \( F_2 \) is the Maxwell 2-form field strength, \( \ast F_2 \) is its 2-form dual and \( S^2 \) is a 2-sphere surrounding the charge. This idea may be generalized to \( p \)-branes in \( D \) dimensions. A \( p \)-brane couples to \( (p+1) \)-form potential \( A_{\mu_1\mu_2...\mu_{p+1}} \) with \( (p+2) \)-form field strength \( F_{\mu_1\mu_2...\mu_{p+2}} \) and carries an “electric” charge per unit \( p \)-volume

\[
Q \sim \int_{S^{D-p-2}} \ast F_{D-p-2}
\]  
and “magnetic” charge per unit \( p \)-volume

\[
P \sim \int_{S^{p+2}} F_{p+2}
\]  
where \( F_{p+2} \) is the \( (p+2) \)-form field strength, \( \ast F_{D-p-2} \) its \( (D-p-2) \)-form dual and \( S^n \) is an \( n \)-sphere surrounding the brane. A special role is played in \( M \)-theory by the BPS (Bogomolnyi-Prasad-Sommerfield) branes whose mass per unit \( p \)-volume, or tension \( T \), is equal to the charge per unit \( p \)-volume

\[
T \sim Q
\]  
This formula may be generalized to the cases where the branes carry several electric and magnetic charges. The supersymmetric branes shown on the branescan are always BPS, although the converse is not true. \( M \)-theory also makes use of non-BPS
and non-supersymmetric branes not shown on the branescan, but the supersymmetric ones do play a special role because they are guaranteed to be stable.

The letters S, V, and T on the branescan refer to scalar, vector and antisymmetric tensor supermultiplets of fields that propagate on the worldvolume of the brane. Historically, these points on the branescan were discovered in three different ways. The S branes were classified by writing down spacetime supersymmetric worldvolume actions that generalize the Green-Schwarz actions on the superstring worldsheet. By contrast, the V and T branes were shown to arise as soliton solutions of the underlying supergravity theories. However, the solitonic V branes found this way were bound by $p \leq 7$. The 8-brane and 9-brane slots were included on the scan only because they were allowed by supersymmetry. Subsequently, all the V-branes were given a new interpretation as Dirichlet p-branes, called D-branes, surfaces of dimension p on which open strings can end and which carry R-R (Ramond-Ramond) charge. The IIA theory has D-branes with $p = 0, 2, 4, 6, 8$ and the IIB theory has D-branes with $p = 1, 3, 5, 7, 9$. They are related to one another by T-duality. In terms of how their tensions depend on the string coupling $g_s$, the D-branes are mid-way between the fundamental (F) strings and the solitonic (S) fivebranes:

$$T_F \sim m_s^2, \quad T_{Dp} \sim \frac{m_s^{p+1}}{g_s}, \quad T_{S5} \sim \frac{m_s^6}{g_s^2}$$

(7)

Since they are BPS, there is a no-force condition between the branes that allows us to have many branes of the same charge parallel to one another. The gauge group on a single D-brane is an abelian $U(1)$. If we stack $N$ such branes on top of one another, the gauge group is the non-abelian $U(N)$. As we separate them this decomposes into its subgroups, so in fact there is a Higgs mechanism at work whereby the vacuum expectation values of the Higgs fields are related to the separation of the branes. For example the theory that lives on a stack of $N$ Type IIB D3 branes is a four-dimensional $U(N) \times U(N)$ super Yang-Mills theory. In the limit of large $N$ the geometry of this configuration tends to the product of five dimensional anti-de Sitter space and a five dimensional sphere, $AdS_5 \times S^5$.

In $D = 11$, M-theory has two BPS branes, an electric 2-brane and its magnetic dual which is a 5-brane. Their tensions are related to each other and the Planck mass by

$$T_2 \sim T_5 \sim M_P^6$$

(8)

if we stack $N$ such branes on top of one another, the M2-brane geometry tends in the large $N$ limit to $AdS_4 \times S^7$ and the M5-brane geometry to $AdS_7 \times S^4$. In addition there are two other objects in $D = 11$, the plane wave and the Kaluza-Klein monopole, which though not branes are still BPS. When spacetime is compactified a p-brane may remain a p-brane or else become a $(p - k)$-brane if it wraps around $k$ of the compactified directions. For example, the Type IIA fundamental string

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5The 3-brane soliton of Type IIB supergravity was an early candidate for a ‘brane-world’, first because of its dimensionality and secondly because gauge fields propagate on its worldvolume. See section 4.2.
emerges by wrapping the M2-brane around $S^1$ and shrinking its radius to zero, and the Type IIA 4-brane emerges in a similar way from the M5-brane.

3. Spin-offs of M-theory

What do we now know with M-theory that we did not know with old-fashioned string theory? Here are a few examples, references to which may be found in Ref. 2.

1) Electric-magnetic (strong/weak coupling) duality in $D = 4$ is a consequence of string/string duality in $D = 6$ which in turn is a consequence of membrane/fivebrane duality in $D = 11$.

2) Exact electric-magnetic duality, first proposed for the maximally supersymmetric conformally invariant $n = 4$ super Yang-Mills theory, has been extended to effective duality by Seiberg and Witten to non-conformal $n = 2$ theories: the so-called Seiberg-Witten theory. This has been very successful in providing the first proofs of quark confinement (albeit in the as-yet-unphysical super QCD) and in generating new pure mathematics on the topology of four-manifolds. Seiberg-Witten theory and other $n = 1$ dualities of Seiberg may, in their turn, be derived from M-theory.

3) Indeed, it seems likely that all supersymmetric quantum field theories with any gauge group, and their spontaneous symmetry breaking, admit a geometrical interpretation within M-theory as the worldvolume fields that propagate on the common intersection of stacks of p-branes wrapped around various cycles of the compactified dimensions, with the Higgs expectation values given by the brane separations.

4) In perturbative string theory, the vacuum degeneracy problems arises because there are billions of Calabi-Yau vacua which are distinct according to classical topology. Like higher-dimensional Swiss cheeses, each can have different number of p-dimensional holes. This results in many different kinds of four-dimensional gauge theories with different gauge groups, numbers of families and different choices of quark and lepton representations. Moreover, M-theory introduces new non-perturbative effects which allow many more possibilities, making the degeneracy problem apparently even worse. However, most (if not all) of these manifolds are in fact smoothly connected in M-theory by shrinking the p-branes that can wrap around the $p$-dimensional holes in the manifold and which appear as black holes in spacetime. As the wrapped-brane volume shrinks to zero, the black holes become massless and effect a smooth transition from one Calabi-Yau manifold to another. Although this does not yet cure the vacuum degeneracy problem, it puts it in a different light. The question is no longer why we live in one topology rather than another but why we live in one particular corner of the unique topology. This may well have a dynamical explanation.

5) Ever since the 1970’s, when Hawking used macroscopic arguments to predict that black holes have an entropy equal to one quarter the area of their event horizon, a microscopic explanation has been lacking. But treating black holes as wrapped p-branes, together with the realization that Type II branes have a dual interpretation
as Dirichlet branes, allows the first microscopic prediction in complete agreement with Hawking. The fact that M-theory is clearing up some long standing problems in quantum gravity gives us confidence that we are on the right track.

6) It is known that the strengths of the four forces change with energy. In supersymmetric extensions of the standard model, one finds that the fine structure constants $\alpha_3, \alpha_2, \alpha_1$ associated with the $SU(3) \times SU(2) \times U(1)$ all meet at about $10^{16}$ GeV, entirely consistent with the idea of grand unification. The strength of the dimensionless number $\alpha_G = GE^2$, where $G$ is Newton’s constant and $E$ is the energy, also almost meets the other three, but not quite. This near miss has been a source of great interest, but also frustration. However, in a universe of the kind envisioned by Witten, spacetime is approximately a narrow five dimensional layer bounded by four-dimensional walls. The particles of the standard model live on the walls but gravity lives in the five-dimensional bulk. As a result, it is possible to choose the size of this fifth dimension so that all four forces meet at this common scale. Note that this is much less than the Planck scale of $10^{19}$ GeV, so gravitational effects may be much closer in energy than we previously thought; a result that would have all kinds of cosmological consequences.

So what is M-theory?

There is still no definitive answer to this question, although several different proposals have been made. By far the most popular is M(atrix) theory. The matrix models of M-theory are $U(N)$ supersymmetric gauge quantum mechanical models with 16 supersymmetries. Such models are also interpretable as the effective action of $N$ coincident Dirichlet 0-branes.

The theory begins by compactifying the eleventh dimension on a circle of radius $R$, so that the longitudinal momentum is quantized in units of $1/R$ with total $P_L N/R$ with $N \to \infty$. The theory is holographic in that it contains only degrees of freedom which carry the smallest unit of longitudinal momentum, other states being composites of these fundamental states. This is, of course entirely consistent with their identification with the Kaluza-Klein modes. It is convenient to describe these $N$ degrees of freedom as $N \times N$ matrices. When these matrices commute, their simultaneous eigenvalues are the positions of the 0-branes in the conventional sense. That they will in general be non-commuting, however, suggests that to properly understand M-theory, we must entertain the idea of a fuzzy spacetime in which spacetime coordinates are described by non-commuting matrices. In any event, this matrix approach has had success in reproducing many of the expected properties of M-theory such as $D = 11$ Lorentz covariance, $D = 11$ supergravity as the low-energy limit, and the existence of membranes and fivebranes.

It was further proposed that when compactified on $T^{d-1}$, the quantum mechanical model should be replaced by an $d$-dimensional $U(N)$ Yang-Mills field theory defined on the dual torus $\tilde{T}^{d-1}$. Another test of this M(atrix) approach, then, is that it should explain the $U$-dualities. For $d = 4$, for example, this group is $SL(3,Z) \times SL(2,Z)$. The $SL(3,Z)$ just comes from the modular group of $T^3$ whereas the $SL(2,Z)$ is the electric/magnetic duality group of four-dimensional
$n = 4$ Yang-Mills. For $d > 4$, however, this picture looks suspicious because the corresponding gauge theory becomes non-renormalizable and the full $U$-duality group has still escaped explanation. There have been speculations on what compactified $M$-theory might be, including a revival of the old proposal that it is really $M$-embranetheory. In other words, perhaps $D = 11$ supergravity together with its BPS configurations: plane wave, membrane, fivebrane, KK monopole and the $D = 11$ embedding of the Type $IIA$ eightbrane, are all there is to $M$-theory and that we need look no further for new degrees of freedom, but only for a new non-perturbative quantization scheme. At the time of writing this is still being hotly debated.

What seems certain, however, is that $M$-theory is not a string theory. It can be approximated by a string theory only in certain peculiar corners of parameter space. So “string phenomenology” will become an oxymoron unless, for some as yet unknown reason, our universe happens to occupy one of these corners.

3.1. $AdS/CFT$ and the brane-world

The year 1998 marked a renaissance in anti de-Sitter space ($AdS$) brought about by Maldacena’s conjectured duality between physics in the bulk of $AdS$ and a conformal field theory on its boundary. For example, $M$-theory on $AdS_4 \times S^7$ is dual to a non-abelian ($n = 8, d = 3$) superconformal theory. Type $IIB$ string theory on $AdS_5 \times S^5$ is dual to a ($n = 4, d = 4$) $U(N)$ super Yang-Mills theory and $M$-theory on $AdS_7 \times S^4$ is dual to a non-abelian ($n_+, n_- = (2, 0), d = 6$) conformal theory. In particular, as has been spelled out most clearly in the $d = 4$ $U(N)$ Yang-Mills case, there is seen to be a correspondence between the Kaluza-Klein mass spectrum in the bulk and the conformal dimension of operators on the boundary. We note that, by choosing Poincaré coordinates on $AdS_5$, the metric may be written as

$$ds^2 = e^{-2y/L}(dx^\mu)^2 + dy^2,$$

where $x^\mu$, ($\mu = 0, 1, 2, 3$), are the four-dimensional brane coordinates. In this case the superconformal Yang-Mills theory is taken to reside at the boundary $y \to -\infty$. The AdS length scale $L$ is given by

$$L^4 = 4\pi \alpha'^2 (g_{YM}^2 N)$$

The string coupling $g_s$ and the Yang-Mills coupling $g_{YM}$ are related by

$$g_s = g_{YM}^2$$

The full quantum string theory on this spacetime is difficult to deal with, but we can approximate it by classical Type IIB supergravity provided

$$L^2 >> \alpha'$$

so that stringy correction to supergravity are small, and that $g_s << 1$ or

$$N \to \infty$$
so that loop corrections can be neglected. There is now overwhelming evidence in favor of this correspondence and it allows us to calculate previously uncalculable strong coupling effects in the gauge theory starting from classical supergravity. Models of this kind, where a bulk theory with gravity is equivalent to a boundary theory without gravity, have also been advocated by ’t Hooft and by Susskind who call them holographic theories. Many theorists are understandably excited about the AdS/CFT correspondence because of what it can teach us about non-perturbative QCD. In my opinion, however, this is, in a sense, a diversion from the really fundamental question: What is $M$-theory? So my hope is that this will be a two-way process and that superconformal field theories will also teach us more about $M$-theory.

The Randall-Sundrum mechanism also involves AdS but was originally motivated, not via the decoupling of gravity from D3-branes, but rather as a possible mechanism for evading Kaluza-Klein compactification by localizing gravity in the presence of an uncompactified extra dimension. This was accomplished by inserting a positive tension 3-brane (representing our spacetime) into AdS$_5$. In terms of the Poincaré patch of AdS$_5$ given above, this corresponds to removing the region $y < 0$, and either joining on a second partial copy of AdS$_5$, or leaving the brane at the end of a single patch of AdS$_5$. In either case the resulting Randall-Sundrum metric is given by

$$ds^2 = e^{-2|y|/L}(dx^\mu)^2 + dy^2,$$

where $y \in (-\infty, \infty)$ or $y \in [0, \infty)$ for a ‘two-sided’ or ‘one-sided’ Randall-Sundrum brane respectively.

The similarity of these two scenarios led to the notion that they are in fact closely tied together. To make this connection clear, consider the one-sided Randall-Sundrum brane. By introducing a boundary in AdS$_5$ at $y = 0$, this model is conjectured to be dual to a cutoff CFT coupled to gravity, with $y = 0$, the location of the Randall-Sundrum brane, providing the UV cutoff. This extended version of the Maldacena conjecture then reduces to the standard AdS/CFT duality as the boundary is pushed off to $y \to -\infty$, whereupon the cutoff is removed and gravity becomes completely decoupled. Note in particular that this connection involves a single CFT at the boundary of a single patch of AdS$_5$. For the case of a brane sitting between two patches of AdS$_5$, one would instead require two copies of the CFT, one for each of the patches.

A third development in the brane-world has been the idea that the extra dimensions are compact but much larger than the conventional Planck sized dimensions in traditional Kaluza-Klein theories. This is possible if the standard model fields are confined to the $d = 4$ brane with only gravity propagating in the $d > 4$ bulk. We shall return to this possibility in section 4.4.
4. Developments on the Brane-World

4.1. No-go theorems for supersymmetry

If we are to give a “top-down” justification of the Randall-Sundrum brane-world by embedding it in string theory or M-theory, it is desirable that the R-S picture be consistent with supersymmetry. Indeed, such a supersymmetric brane-world is necessary if the Maldacena and Randall-Sundrum (R-S) pictures are to stand any chance of being complementary. At first, however, this seemed to be problematical and several papers appeared in the literature suggesting that R-S could not be supersymmetric. Some of these no-go theorems listed below are exactly as they appeared; with others I have taken the liberty of setting up the straw man so as more effectively to knock him down.

1) R-S branes cannot be SUSY because massless supergravity scalars give kink-up and not kink-down potentials which do not bind gravity to the brane.

2) R-S branes cannot be SUSY because their tension is not that of a BPS brane.

3) R-S branes cannot be SUSY because $\delta$-functions are incompatible with susy transformation rules.

4) R-S branes cannot be SUSY because the photon superpartners of the graviton cannot be bound to the brane.

4.2. Yes-go theorems for Supersymmetry of the brane-world

In fact, the domain-wall solution of Bremer et al provides a supersymmetric Type IIB Randall-Sundrum realization. See also Refs. and . So it is instructive to see how the no-go theorems are circumvented:

1) The required supergravity scalar is massive, being the breathing mode of the $S^5$ compactification. So the negative conclusions about massless scalars in Refs., while correct, are not relevant.

2) The tension comes from two sources: the BPS D3-branes and the kink. So the observation of Ref. that the D3 brane tension is only 2/3 of the R-S tension, while correct, is not relevant.

3) The sign flip of the coupling constant across the brane removes the $\delta$-functions in the supersymmetry transformation rules. So the problems raised by Ref., while correct, are not relevant.

4) Photons can be bound to the brane but their bulk origin is not Maxwell’s equations. So the result of Ref. that photons obeying Maxwell’s equations in the bulk cannot be bound to the brane, while correct, is not relevant.

An entirely different question is whether a smooth domain wall can provide

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$^d$Actually, these authors showed that, treated as test particles, Maxwell photons could be bound to the brane but their charge would be screened. However, the combined bulk Einstein-Maxwell equations rule out photons on the brane altogether.
a supersymmetric Randall-Sundrum realization, and here ordinary supergravity seems to fail, requiring some kind of higher derivative and presumably stringy corrections.\cite{23, 24}

4.3. Complementarity of the Maldacena and Randall-Sundrum pictures

In his 1972 PhD thesis under Abdus Salam, the author showed that, when one-loop quantum corrections to the graviton propagator are taken into account, the inverse square $r^{-2}$ behavior of Newton’s gravity force law receives an $r^{-4}$ correction whose coefficient depends on the number and spins of the elementary particles.\cite{35, 36} Specifically, the potential looks like

$$V(r) = \frac{GM}{r} \left( 1 + \frac{\alpha G}{r^2} \right),$$

where $G$ is the four-dimensional Newton’s constant, $\hbar = c = 1$ and $\alpha$ is a purely numerical coefficient given, in the case of spins $s \leq 1$, by

$$\alpha = \frac{45 \pi \alpha}{12 N_1 + 3 N_{1/2} + N_0},$$

where $N_s$ are the numbers of particle species of spin $s$ going around the loop.

Now fast-forward to 1999 when Randall and Sundrum proposed that our four-dimensional world is a 3-brane embedded in an infinite five-dimensional universe. Gravity reaches out into the five-dimensional bulk but the other forces are confined to the four-dimensional brane. Contrary to expectation, they showed that an inverse square $r^{-2}$ law for gravity is still possible but with an $r^{-4}$ correction coming from the massive Kaluza-Klein modes whose coefficient depends on the bulk cosmological constant. Their potential looks like

$$V(r) = \frac{GM}{r} \left( 1 + \frac{2 L^2}{3 r^2} \right),$$

where $L$ is the radius of AdS$_5$. Since (15) was the result of a four-dimensional quantum calculation and (16) the result of a five-dimensional classical calculation, they seem superficially completely unrelated. However, Ref.\cite{37} invokes the AdS/CFT correspondence of Maldacena to demonstrate that the two are in fact completely equivalent ways of describing the same physics. From (15), we see that the contribution of a single $n = 4 U(N)$ Yang-Mills CFT, with $(N_1, N_{1/2}, N_0) = (N^2, 4N^2, 6N^2)$, is

$$V(r) = \frac{GM}{r} \left( 1 + \frac{2 N^2 G}{3 \pi r^2} \right).$$

Using the AdS/CFT relation $N^2 = \pi L^3 / 2 G_5$ and the one-sided brane-world relation $G = 2 G_5 / L$, where $G_5$ is the five-dimensional Newton’s constant, we obtain exactly (16).

As discussed in the August 2000 edition of Scientific American,\cite{18} experimental tests of deviations from Newton’s inverse square law are currently under way.
4.4. Self-tuning of the cosmological constant

Imagining that our universe is a brane in a higher dimensional spacetime changes the nature of the cosmological constant problem. We must now explain why the four-dimensional cosmological constant $\Lambda_4$ is small, without unacceptable fine-tuning, as opposed to the bulk cosmological constant $\Lambda_5$. An interesting recent development is the following “self-tuning” idea.

Once again, we start with a five-dimensional bulk theory with a scalar field $\phi$ coupled to a four-dimensional brane source via an action:

$$\int d^4x \sqrt{-g} V e^{b\phi}$$

where $V$ and $b$ are constants. Let us allow Poincare supersymmetry in the bulk while the theory on the brane breaks supersymmetry. Hence, to first approximation, the bulk has vanishing cosmological constant (so this scenario is different from that of sections 4.1, 4.2 and 4.3). This has the important consequence that the bulk interactions are invariant under a shift in $\phi$:

$$\phi \rightarrow \phi + \text{constant}$$

The authors of Refs. 38 and 39 were able to find 3-brane solutions of the combined bulk-brane equations for which the four-dimensional cosmological constant vanishes. As usual, the question now is why it should remain zero after supersymmetry breaking and radiative corrections of the “standard model” theory on the brane. Normally we would expect huge corrections to $\Lambda_4$ of order $M_S^4$ where $M_S$ is the supersymmetry-breaking scale.

The crucial observation is that, as a result of (19), given a brane solution with one value of $V$, there exist another for any value, since shifts in $V$ can be absorbed by shifts in $\phi$, owing to the combination $V e^{b\phi}$. Hence one can effectively absorb any cosmological constant generated by standard model physics, and keep $\Lambda_4 = 0$ to lowest order.

Of course, the interactions between bulk and brane fields will result in supersymmetry breaking on the brane eventually feeding back to the bulk and spoiling the $\Lambda_5 = 0$ starting point. However, the resulting corrections to $\Lambda_4$ appear as a power series in $M_S/M_5$, which will be about $10^{-5}$, if we take $M_S \sim TeV$ and $M_5 \sim 10^{19}GeV$. This is still much bigger than the experimental bound and so this is by no means the final answer to the cosmological constant problem. There are other possible objections to these models that might be raised. However, the main culprit $M_S^4$ has at least been eliminated, and this may point the way to a new solution.

4.5. Five versus eleven

As we have seen M-theory requires eleven dimensions, whereas if the brane-world picture is correct, we really need only five with the other six going along for the
ride. Why should Nature behave like this? The only good answer to this question I could find is in Mother Goose's Nursery Rhymes:

\begin{quote}
Nature requires five,
Custom allows seven,
Idleness takes Nine,
And Wickedness Eleven.
\end{quote}

5. Top Ten Unsolved Problems

In 1900 the world-renowned mathematician David Hilbert presented twenty-three problems at the International Congress of Mathematicians in Paris. These problems have inspired mathematicians throughout the last century. As a piece of millennial madness, all participants of the Strings 2000 Conference were invited to help formulate the ten most important unsolved problems in fundamental physics. Each participant was allowed to submit one candidate problem for consideration. To qualify, the problem must not only have been important but also well-defined and stated in a clear way. The best 10 problems were selected at the end of the conference by a selection panel consisting of David Gross, Edward Witten and myself. The results in no particular order are:

(1) *Are all the (measurable) dimensionless parameters that characterize the physical universe calculable in principle or are some merely determined by historical or quantum mechanical accident and uncalculable?*

David Gross, Institute for Theoretical Physics, University of California, Santa Barbara.

(2) *How can quantum gravity help explain the origin of the universe?*

Edward Witten, California Institute of Technology and Institute for Advanced Study, Princeton.

(3) *What is the lifetime of the proton and how do we understand it?*

Steve Gubser, Princeton University and California Institute of Technology.

(4) *Is Nature supersymmetric, and if so, how is supersymmetry broken?*

Sergio Ferrara, CERN European Laboratory of Particle Physics; Gordon Kane, University of Michigan.

(5) *Why does the universe appear to have one time and three space dimensions?*

Shamit Kachru, University of California, Berkeley; Sunil Mukhi, Tata Institute of Fundamental Research; Hiroshi Ooguri, California Institute of Technology.

(6) *Why does the cosmological constant have the value that it has, is it zero and is it really constant?*
Andrew Chamblin, Massachusetts Institute of Technology; Renata Kallosh, Stanford University.

(7) What are the fundamental degrees of freedom of M-theory (the theory whose low-energy limit is eleven-dimensional supergravity and which subsumes the five consistent superstring theories) and does the theory describe Nature?
Louise Dolan, University of North Carolina, Chapel Hill; Annamaria Sinkovics, Spinoza Institute; Billy & Linda Rose, San Antonio College.

(8) What is the resolution of the black hole information paradox?
Tibra Ali, Department of Applied Mathematics and Theoretical Physics, Cambridge; Samir Mathur, Ohio State University.

(9) What physics explains the enormous disparity between the gravitational scale and the typical mass scale of the elementary particles?
Matt Strassler, Institute for Advanced Study, Princeton.

(10) Can we quantitatively understand quark and gluon confinement in Quantum Chromodynamics and the existence of a mass gap?
Igor Klebanov, Princeton University; Oyvind Tafjord, McGill University.

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