Robust Trust Region for Weakly Supervised Segmentation

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Abstract

Acquisition of training data for the standard semantic segmentation is expensive if requiring that each pixel is labeled. Yet, current methods significantly deteriorate in weakly supervised settings, e.g. where a fraction of pixels is labeled or when only image-level tags are available. It has been shown that regularized losses—originally developed for unsupervised low-level segmentation and representing geometric priors on pixel labels—can considerably improve the quality of weakly supervised training. However, many common priors require optimization stronger than gradient descent. Thus, such regularizers have limited applicability in deep learning. We propose a new robust trust region approach\footnote{https://github.com/dmitrii-marin/robust_trust_region} for regularized losses improving the state-of-the-art results. Our approach can be seen as a higher-order generalization of the classic chain rule. It allows neural network optimization to use strong low-level solvers for the corresponding regularizers, including discrete ones.

1. Introduction

Our paper proposes a higher-order optimization technique for neural network training. While focused on semantic image segmentation, our main algorithmic idea is simple and general - integrate the standard trust region principle into the context of backpropagation, i.e. the chain rule. We reinterpret the classic chain rule: instead of the chain of gradients/derivatives for a composition of functions, we formulate the corresponding chain of hidden optimization sub-problems. Then, inspired by the trust region principle, we can substitute a standard linear approximation solver (gradient descent) at any chain with a better higher-order solver. In short, we replace the classic differentiation chain rule by the trust region chain rule in the context of backpropagation.

Our work is motivated by the well-known challenges presented to the gradient descent by typical regularization losses or geometric priors/energies ubiquitous in the context of weakly-supervised or unsupervised segmentation.

To validate our approach, we present semantic segmentation results improving the state-of-the-art in the challenging setting where the training data has only a fraction of pixels labeled. The generality of our main principle (trust region chain rule) and our promising results for a difficult problem encourage further research. In fact, this work applies trust region principle only to the last “chain” in the network. We discuss several promising extensions for future work.

The rest of the introduction is organized as follows. To create a specific context for our general approach to network training, we review loss functions relevant for weakly-supervised or unsupervised segmentation. First, Sec. 1.1 discusses several standard geometric priors, regularization energies, clustering criteria, and their powerful solvers originally developed for low-level segmentation or general machine learning. Then, Sec. 1.2 outlines the use of such regularization objectives as losses for network training in the context of weakly supervised semantic (high-level) segmentation. We also review the standard trust region principle (Sec. 1.4) and highlight our main contributions (Sec. 1.5) based on the general idea of applying trust region (with powerful solvers) to network training.

1.1. Regularized energies in low-level segmentation

Assuming discrete segmentation \( s \in \{1, 2, \ldots, K\}^N \) where \( K \) is the number of categories and \( N \) is the number of image pixels, one common low-level segmentation energy can be represented as

\[
E(s) = -\sum_i \log P(I_i|s_i) + \sum_{\{i,j\} \in \mathcal{N}} w_{ij} [s_i \neq s_j] \tag{1}
\]

where \( I_i \) is a low-level feature (e.g. intensity, color, texture) at pixel \( i \) with distribution functions \( P(\cdot|k) \) for each category \( k \), neighborhood system \( \mathcal{N} \) describes any pairwise connectivity (typically 4-, 8-grid [5] or denser [35]), weights \( w_{ij} \) represent given pairwise affinities (typically Gaussian kernel for low-level features \( I_i \) and \( I_j \) [7, 5, 54, 35]), and \([\cdot]\) is the Iverson bracket operator returning 1 if the argument is true and 0 otherwise. The energy above combines the log-likelihoods term enforcing consistency with given (low-level) feature distributions and a pairwise reg
The neighborhood system regularization is closely related to the Normalized cut affinities

\[ w_{ij} \] difficult to optimize. In the formula above, vector \( \mathbf{s} \) combines self- and all pairwise (e.g. Gaussian) affinities \( w_{ij} \) between image pixels. Note that Potts regularization is closely related to the Normalized cut objective

\[ \sum_k (1-s_k^k)^\top W s_k^k \] for unsupervised segmentation [58].

It is common to combine energies like (1),(2),(3) with constraints based on user interactions (weak supervision). While there are different forms of such supervision, the most basic one is based on adding the seed loss [5] defined over pixels in subset \( \Omega_{\text{seeds}} \) with user-specified category labels \( y_i \). Assuming \( s_i \in \Delta_K \), it can be written as a partial cross entropy (PCE) for pixels \( i \in \Omega_{\text{seeds}} \)

\[ E_{\text{seeds}}(s) = - \sum_{i \in \Omega_{\text{seeds}}} \log s_i^{y_i} \] (4)

and, when restricted to one-hot \( s_i \) representing hard segmentation, it reduces to the hard constraints over seeds [5]. That is, for integer-valued \( s_i \in \{1, \ldots, K\} \) the seed loss is equivalent to \( \sum_{i \in \Omega_{\text{seeds}}} \lambda [s_i = y_i] \) for infinitely large \( \lambda \).

The log-likelihood loss, e.g. the first term in (1) or (3), is common in low-level segmentation and its importance cannot be underestimated. In basic formulations, the distributions of (low-level) features \( P(\cdot | k) \) can be assumed given for each category \( k \). However, if such distributions are not known a priori, their representation \( P(\cdot | \theta_k) \) can explicitly include unknown distribution parameters \( \theta_k \) for each category \( k \). Then, the overall loss \( E(s, \theta) \) adds \( \theta = \{\theta_k\} \) as an extra variable. Optimization of \( E(s, \theta) \) over both \( s \) and \( \theta \) corresponds to joint estimation of segmentation and maximum likelihood (ML) estimation of distribution parameters, as in well-known unsupervised low-level segmentation formulations by Zhu & Yuille [67] and Chan & Vese [14]. Similar ideas are also used in box-interaction methods [54].

1.2. Regularized losses in DNN segmentation

Unlike low-level segmentation methods based on readily available low-dimensional features (like color, texture, contrast edges), deep neural network (DNN) approaches to segmentation learn complex high-dimensional “deep” features that can discriminate semantic categories. Thus, one can refer to such methods as high-level segmentation, and to such learned features as high-level features.

The most standard way to train segmentation networks is based on full supervision requiring a large collection of images where all pixels are accurately labeled. Such training data is expensive to get. The training is based on minimizing the cross-entropy (CE) loss similar to the seed loss in low-level segmentation. For simplicity focusing on a single training image, CE loss is

\[ E_{\text{CE}}(s(\theta)) = - \sum_i \log s_i^{y_i}(\theta) \] (5)

where \( s(\theta) = f(\theta) \in \Delta_K^N \) is the (relaxed) segmentation output of the network \( f(\theta) \) with parameters \( \theta \). For brevity, here and later in this paper we omit the actual test image from the arguments of the network function \( f \).

The fundamental difference with low-level segmentation reviewed above is that instead of minimizing losses \( E \) directly over segmentation variable \( s \), now the optimization arguments are parameters \( \theta \) of the network producing such segmentation. Estimating parameters \( \theta \) can be interpreted as learning deep features. Note that this task is much more complex than ML estimation of distribution parameters for...
P(I|θ) in low-level segmentation with fixed low-level features I, as reviewed above. This explains why network optimization requires a large set of fully labeled training images, rather than a single image (unlabeled or partially-labeled), as in low-level segmentation.

The goal of weakly supervised segmentation is to train the network with as little supervision as possible. First of all, it is possible to train using only a subset of labeled pixels (seeds) in each image \([32, 61]\) in exact analogy with (4)

\[
E_{\text{PCE}}(s(θ)) = - ∑_{i∈Ω_{\text{seeds}}} \log s^i_\nu(θ) \quad (6)
\]

In particular, as shown in [61], this simple, but principled approach can outperform more complex heuristic-based techniques. To improve weakly-supervised training, it is also possible to use standard low-level regularizers, as in Sec. 1.1, that leverage a large number of unlabeled pixels \([66, 32, 61, 62, 42]\). For example, [62] achieves the state-of-the-art using bilinear relaxation of the Potts model in (3)

\[
E_{\text{Potts}}^\text{bl}(s(θ)) = ∑_k (1 - s_k(θ))^\top W s_k(θ) \quad (7)
\]

as an additional regularization loss over all (including unlabeled) pixels. For some \(ν > 0\), their continuous total loss

\[
E = E_{\text{PCE}} + ν E_{\text{Potts}}^\text{bl}. \quad (8)
\]

More generally, standard regularization losses from low-level segmentation are commonly used in the context of segmentation networks. Such losses and their solvers are ubiquitous in weak-supervision techniques using seeds or boxes to generate fully-labeled proposals \([29, 39]\). Optimization of low-level regularizers is also common for network’s output post-processing, typically improving performance during testing [16]. Also, the corresponding low-level solvers can be directly integrated as solution-improving layers [66].

1.3. Weakly supervised semantic segmentation

Weak supervision for deep neural network semantic segmentation comes in many different forms, e.g. image-level tags \([50, 49, 32]\), scribbles/clicks \([39, 61, 62, 42]\), and bounding boxes \([49, 29, 28]\). These works employ a large variety of strategies to compensate for the lack of labels. The concept of multiple instance learning (MIL) naturally fits the weakly supervised setting. Since generic MIL methods produce small unsatisfactory segments, more specialized methods are needed. For example, methods \([50, 28]\) impose constraints on the output of the neural network during learning. There are several segmentation-constraint constraints, such as size bias, constraints on present labels, tightness \([38]\), etc. \([32, 62, 42]\) incorporate edge alignment constraints. Proposal generation methods \([29, 39]\) aim to generate/complete the ground truth to use fully-supervised learning. However, DNNs are vulnerable to errors in proposals. More robust approaches use EM \([49]\) or ADMM \([42]\) to iteratively correct errors in “proposals”.

Some related prior work on weakly supervised DNN segmentation \([39]\) uses some specific non-robust version of the joint loss related to our approach. Similar losses (studied in segmentation since 1980s) do not imply similar algorithms. In particular, they iterate explicit low-level segmentation of super-pixels \([21]\) and pixel-level network training, where at each iteration the network is trained from scratch and to convergence. They motivate such integration by improved results only. They also argue that “when network gradually learns semantic content, the high-level information can help with the graph-based scribble propagation”, suggesting their main focus on improved “proposals”. As shown in \([61, 62]\), their method is outperformed by using only the partial cross entropy on seeds (6).
In these works, the “separation” of the gradient update into implicit layer-wise optimization problems is formulated as a gradient update of a certain energy function. Taylor et al. [63] use ADMM splitting approach to separate optimization over different layers in distributed fashion. These works focus on neural network parameter optimization replacing backpropagation altogether. In contrast to [9, 63, 23], we are primarily focused on optimization for complex loss functions in the context of the weakly supervised semantic segmentation, see Sec.1.2, while others focus on replacing the backpropagation in the intermediate layers. Also, unlike us, these methods use the squared Euclidean norm in their proximal formulations. Chen and Teboulle [15] generalize the proximal methods to Bregman divergences, a more general class of functions which includes both the Euclidean distance and KL-divergence. Nesterov in [45] uses the Euclidean norm with a higher power improving the convergence of the proximal method.

Our contribution are as follows:

• New trust region optimization for DNN segmentation integrating higher-order low-level solvers into training. Differentiability of the loss is not required as long as there is a good solver, discrete or continuous. The classic differentiation chain rule is replaced by the trust region chain rule in the context of backpropagation.

• The local optimization in trust region framework allows to use arbitrary metrics, instead of Euclidean distance implicit for the standard gradient ascent. We discuss different metrics for the space of segmentations and motivate a robust version of KL-divergence.

• We show benefits of our optimization for regularization losses in weakly supervised DNN segmentation, compared to the gradient descent. We set new state-of-the-art results for weakly supervised segmentation with scribbles achieving consistently the best performance at all levels of supervision, i.e. from point-clicks to full-length scribbles.

2. Trust region for loss optimization

Backpropagation is the dominant method for optimizing network losses during training. It represents the gradient descent with respect to model parameters $\theta$ where the gradient’s components are gradually accumulated using the classic chain rule while traversing the network layers starting from the output directly evaluated by the loss function.

Motivated by the use of hard-to-optimize regularization losses (Sec. 1.1) in the context of weakly-supervised segmentation (Sec. 1.2), we propose higher-order trust region approach to network training. While this general optimization approach can be developed for any steps of the backpropagation (i.e. chain rule) between internal layers, we focus on the very first step where the loss function is composed with the network output

$$\min_{\theta \in \mathbb{R}^m} E(f(\theta))$$

where some scalar loss function

$$E : \mathbb{R}^n \rightarrow \mathbb{R}^1$$

is defined over $n$-dimensional output of a network/model

$$f : \mathbb{R}^m \rightarrow \mathbb{R}^n.$$

Since during training the network’s input is limited to fixed examples, for simplicity we restrict the arguments of network function $f$ to its training parameters $\theta \in \mathbb{R}^m$. Also note that, as a convention, this paper reserves the boldface font for vector functions (e.g. network model $f$) and for matrix functions (e.g. model’s Jacobian $J_f$).

The main technical ideas of the trust region approach to network optimization (9) in this section are fairly general. However, to be specific and without any loss of generality, this and (particularly) later sections can refer to the output of the network as segmentation so that

$$\mathbb{R}^n = \mathbb{R}^{N \times K}$$

where $N$ is the number of image pixels and $K$ is the number of distinct semantic classes. This is not essential.

Our general trust region approach to (9) can be seen as a higher-order extension of the classic chain rule for the composition $E \circ f$ of the loss functions $E$ and model $f$. For the classic chain rule in the standard backpropagation procedure, it is critical that both $E$ and $f$ are differentiable. In this case, the classic chain rule for the objective in (9) gives the following gradient descent update for parameters $\theta$

$$\Delta \theta = -\alpha \nabla E^\top J_f$$

where $\Delta \theta \equiv \theta - \theta_t$ is an update of the model parameters from the current solution, $\alpha$ is the learning rate, $\nabla$ is the gradient operator, and $J_f$ is the model’s Jacobian

$$J_f := \begin{bmatrix} \frac{\partial f}{\partial \theta^1} \\ \vdots \\ \frac{\partial f}{\partial \theta^m} \end{bmatrix}.$$

We would like to rewrite the classic chain rule (10) in an equivalent form explicitly using a variable for segmentation $s \in \mathbb{R}^n$, which is an implicit (hidden) argument of the loss function $E$ in (9). Obviously, equation (10) is equivalent to two separate updates for the segmentation $\Delta s \equiv s - s_t$ and for the model parameters $\Delta \theta \equiv \theta - \theta_t$

$$\Delta s = -\alpha \nabla E^\top$$

$$\Delta \theta = \Delta s J_f$$

(11)
where the gradient $\nabla E$ is computed at the current segmentation $s_t := f(\theta_t)$. Note that $s \in \mathbb{R}^n$ represents points (e.g., segmentations) in the same space as the network output $f(\theta) \in \mathbb{R}^n$, the two should be clearly distinguished in the discourse. We will refer to $s$ as (explicit) segmentation variable, while $f(\theta)$ is referred to as segmentation output.

The updates in (11) and (12) correspond to two distinct optimization sub-problems. Clearly, (11) is the gradient descent step for the loss $E(s)$ locally optimizing its linear Taylor approximation $\tilde{E}_{\text{linear}}(s) = E(s_t) + \nabla E^s \Delta s$ over (explicit) segmentation variable $s \in B(s_t) \subset \mathbb{R}^n$ in a neighborhood (ball) around $s_t$

$$s_{t+1} = \arg \min_{s \in B(s_t)} \tilde{E}_{\text{linear}}(s). \tag{13}$$

While less obvious, it is easy to verify that $\theta$-update in (12) is exactly the gradient descent step

$$\Delta \theta = -\frac{1}{2} \nabla_\theta \|s_{t+1} - f(\theta)\|^2 \tag{14}$$

corresponding to optimization of the least-squares objective

$$\min_{\theta} \|s_{t+1} - f(\theta)\|^2 \tag{15}$$

based on the solution $s_{t+1} \equiv \Delta s + f(\theta_t)$ for problem (13).

Our trust region approach to network training (9) is motivated by the principled separation of the chain rule (10) into two sub-problems (13) and (15). Instead of the gradient descent, low-level optimization of the loss in (13) can leverage powerful higher-order solvers available for many popular loss functions, see Sec. 1.1. In particular, the majority of common robust loss functions for unsupervised or weakly-supervised computer vision problems are well-known to be problematic for the gradient descent. For example, their robustness (boundedness) leads to vanishing gradients and sensitivity to local minima. At the same time, the gradient descent can be left responsible for the least-squares optimization in (15). While it is still a hard problem due to size and non-convexity of the typical models $f(\theta)$, at least the extra difficulties introduced by complex losses $E$ can be removed into a different sub-problem.

Formally, our trust-region approach to training (9) generalizes our interpretation of the classic chain rule in sub-problems (13) and (15) as shown in iterative stages A, B:

**STAGE A** (low-level optimization)

$$s_{t+1} = \arg \min_s \tilde{E}(s) + \lambda d_\alpha(s, f(\theta_t)) \tag{16}$$

**STAGE B** (network parameters update)

$$\min_{\theta} d_\beta(s_{t+1}, f(\theta)) \tag{17}$$

$$\Delta \theta = -\gamma \nabla_\theta d_\beta(s_{t+1}, f(\theta)) \tag{18}$$

where $\tilde{E}$ is some loss approximation, $d_\alpha$ and $d_\beta$ are some distance/divergence measures. Instead of $\alpha$ in (11) and fixed weight $\frac{1}{2}$ in (14), the overall learning speed of our training procedure is controlled by two parameters: (A) scalar $\lambda$ indirectly determining the step size from the current solution $s_t = f(\theta_t)$ in (16), and (B) scalar $\gamma$ defining the step size for the gradient descent in (18). While both $\lambda$ and $\gamma$ are important for the learning speed, we mostly refer to $\lambda$ as a trust region parameter, while the term learning rate is reserved primarily for parameter $\gamma$ in (18), as customary for the gradient descent step size in network optimization. Note that similarly to the gradient descent (10), stages A/B are iterated until convergence. While it is sensible to make several B-steps (18) in a row, in general, it is not necessary to wait for convergence in sub-problem (17) before the next A-step.

Our formulation offers several significant generalizations of the classic chain rule. **First**, instead of the linear approximation (13) implied by the gradient descent (11), we target higher-order approximations of the loss $\tilde{E}$ in (16). In some cases, the exact loss $E$ could be used\(^2\). The corresponding powerful low-level solvers for (16) are readily available for many types of useful robust losses, see Sec. 1.1. Note that for exact solvers when $\tilde{E} = E$, one may argue for $\lambda = 0$ allowing the network to learn from the best solutions for regularized loss $E$ implying global optima in (9). However, such fixed proposals (Sec. 1.2) may result in overfitting to mistakes due to well-known biases/weaknesses in common regularizers. Constraining loss optimization (9) to the network output manifold in $\mathbb{R}^n$ motivates $\lambda > 0$ in (16). More discussion is in Sec. 5.1.

**Second**, besides continuous/differentiable losses required by the standard backpropagation (chain rule), our trust region approach (stages A/B) allows training based on losses defined over discrete domains. There are several reasons why this extension is significant. For example, besides continuous solvers, optimization in (16) now can use a significantly larger pool of solvers including many powerful discrete/combinatorial methods. Moreover, this approach enables training of models with discrete decision functions, e.g. step function instead of sigmoid, or hard-max instead of the soft-max. This is further discussed in Sec. 5.1.

**Third**, the standard gradient descent (10) is implicitly defined over Euclidean metric, that manifests itself in our equations (13) and (15) via the local neighborhood topology (Euclidean ball $B$) and the least-squares objective (squared Euclidean distance). In contrast, when replacing ball $B(s_t)$ in (13) by the trust region term in (16), we explicitly define the trust region “shape” using function $d_\alpha$. It could be any application-specific distance metric, quasi- or pseudo-

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\(^2\)Note that parameter $\lambda$ in (16) controls two properties: the size of the trust region for approximation $E$, as well as the network’s training speed. While using exact loss $E = \tilde{E}$ implies that the trust region for such “approximation” should be the whole domain (i.e. $\lambda = 0$), the competing interest of limiting the training speed in (17) may require $\lambda > 0$. 

metric, divergence, etc. Similarly, any appropriately motivated distance, distortion, or divergence function $d_a$ in (17) can replace the least squares objective in (15).

On the negative side, our trust region formulation could be more expensive due to the computational costs of the low-level solvers in stage A. In practice, it is possible to amortize stage A over multiple iterations of stage B.

### 3. Robust metric for trust region

The choice of metrics $d_a$ and $d_b$ defining the shape of the trust region above is application dependent. In the case of segmentation, the output of a neural network is typically obtained via the soft-max function. Hence, the space, in which the trust region operates, is the space of multiple categorical distributions over $K$ categories: $\Delta^K_N$.

Below, we generally discuss (robust) metrics over pairs of arbitrary probability distributions $p, q$ in $\Delta^K_N$. The goal of this section is to motivate our choice of metrics $d_a$ and $d_b$ in problems (16), (17) so that distribution $p$ can be associated with the segmentation variable $s$, and distribution $q$ can be associated with the network output $f(\theta)$. Besides this connection, the following discussion of metrics over probability distributions is independent of the context of networks.

Note, metrics $d_a$ or $d_b$ do not have to be proper distances for the purposes of trust region optimization. Instead, one may use any divergence measure defined on space $\Delta^K_N$. Let us consider the Kullback–Leibler divergence:

$$KL(p||q) = \sum_{i=1}^{N} \sum_{l=1}^{K} p_l^i \log \frac{p_l^i}{q_l^i} = -\sum_{i=1}^{N} \sum_{l=1}^{K} p_l^i \log q_l^i - H(p)$$

where $p, q \in \Delta^K_N$, and $p_l^i$ is the probability of pixel $i$ to have label $l$, and $H(p)$ is the entropy of distribution $p$.

A practically important case is when the distribution $p$ is degenerate or one-hot, i.e., for each pixel $i$ there exists label $y_i$ such that $p^l_i = 1$ and for any other label $l \neq y_i$ probability $p^l_i = 0$. In that case $H(p) = 0$ and

$$KL(p||q) = \sum_i - \log q_i^{y_i},$$

which is the cross-entropy or negative log-likelihood, a standard loss when $q$ is the probability estimate outputted by a neural network. In the following we assume (19).

During the trust region procedure, intermediate solutions generated by a solver in (16) may have a noticeable amount of misclassified pixels. It is known that many standard losses for neural networks, including cross-entropy (19), can result in training sensitive to idiosyncrasies in the datasets including mistakes in the ground truth [24, 40, 22]. Therefore, a robust distance measure may be needed. Our experiments show that robustness is critical. We propose a simple error model depicted in graphical model in Fig. 1.

Figure 1. The unknown true labeling $Z$ corresponds to observed image $I$. The observed labeling $Y$ is assumed to be generated from the true $Z$ by a simple corruption model (20).

Let random variable $Y_i$ be the observed noisy label of pixel $i$ and $Z_i$ be its hidden true label. We assume that the probability of observing label $l$ given true label $k$ is

$$\Pr(Y_i = l \mid Z_i = k) = \begin{cases} 1 - \varepsilon, & l = k, \\ \frac{\varepsilon}{K-1}, & l \neq k, \end{cases}$$

where $\varepsilon$ is called the outlier probability [37]. The probability of pixel $i$ having label $l$ given image $I$ is

$$\Pr(Y_i = l \mid I) = \sum_{z=1}^{K} \Pr(Y_i = l \mid Z_i = z) \Pr(Z_i = z \mid I) = a + b \Pr(Z_i = l \mid I)$$

where $a = \frac{\varepsilon}{K-1}$ and $b = 1 - K a$. The probability $\Pr(Z_i = z \mid I)$ is unknown and is replaced by probability estimate $q_i^l$ yielding a robust version of divergence (19):

$$\sum_i - \log \left( a + b q_i^{y_i} \right).$$

Figure 2 compares cross-entropy (19) with robust loss (22).

Our robust cross-entropy (22) is related to a more general approach for classification [51, 59]. In [51], the corresponding robust cross-entropy (forward correction) is

$$\sum_i - \log q_i^{y_i}$$

...
algorithms for (26). Alg. 1 with combinatorial solver for \( \tilde{E} \) known for sure. So,\( E \) where \( \Omega \) achieved at \( \varepsilon = 0 \). The best accuracy is achieved at \( \varepsilon = 0.4 \), which is close to the actual noise level.

where \( \tilde{q}_i = T^T q_i \), and \( q_i \) is the vector of probability estimates at pixel \( i \), and \( T = [T_{ik}] \) is the noise transition matrix: \( T_{ik} = \Pr(Z = i | Y = k) \). The effect of different \( \varepsilon \) is shown in example in Fig. 3.

In practice, different pixels require different values of \( \varepsilon \) in (20). For example, in the scribble-based weakly supervised segmentation, the labels of seed pixels \( \Omega \) are known for sure. So, \( \varepsilon = 0 \) for such pixels, and \( \varepsilon > 0 \) for other pixels. Thus, the robust “metric” is

\[
\text{KL}_{\varepsilon, \Omega_{\text{seeds}}} (p||q) = \sum_{i \in \Omega_{\text{seeds}}} - \log (a + b q_i^{y_i}) + \sum_{i \in \Omega_{\text{seeds}}} - \log q_i^{y_i}.
\]

(24)

In sum, we propose the following robust metrics for the trust region iterations (16) and (18):

\[
\begin{align*}
d_a(p, q) &= \text{KL}(p||q), \\
d_b(p, q) &= \text{KL}_{\varepsilon, \Omega_{\text{seeds}}}(p||q).
\end{align*}
\]

(25)

4. Results in weakly supervised segmentation

To validate our approach (16-18) we use standard efficient discrete solvers [7] for the loss

\[
\tilde{E} = E_{\text{PCE}} + E_{\text{Pam}}
\]

(26)

where \( E_{\text{Pam}}(s) = \sum_{(i,j) \in \mathcal{N}} w_{ij} [s_i \neq s_j] \) is the second (regularization) term in standard low-level energy (1). In this case, optimization in (16) is limited to the corners of the simplex where \( E_{\text{PCE}} \) reduces to the hard constraints over the seeds. In (16-18) we use robust metrics (25). The overall method is summarized in Alg. 1.

One natural baseline for Alg. 1 is a standard method based on stochastic gradient descent (SGD) for regularized loss (8) proposed in [62], see Sec. 1.2. Indeed, \( E_{\text{Pam}} \) is a relaxation of \( E_{\text{Pam}} \), as discussed in Sec. 1.1. Thus, (8) is a relaxation of (26). Alg. 1 with combinatorial solver for \( \tilde{E} \) in (26) can be seen as a discrete trust region approximation for (8). In general, our approach (16-18) allows other discrete or continuous solvers and/or other approximations \( \tilde{E} \).

First, PCE-GD baseline is the standard SGD optimizing partial cross-entropy (6). It has been shown in [62, 61] that such approach outperforms more complex proposal (fake ground truth) generation methods such as [39]. Second, Grid-GD is the SGD over regularized loss (8) where the CRF neighbourhood is the standard 8-grid. Third, Dense-GD is the approach of [62] that uses the common fully-connected (dense) Potts CRF of [35].

We use the ScribbleSup [39] annotations for Pascal VOC 2012 [20] dataset. ScribbleSup supplies scribbles, i.e. a small subset of image pixels (\( \approx 3\% \)) is labeled while the vast majority of pixels is left unlabeled.

4.1. Implementation details

In all our experiments we used DeeplabV3+ [17] with MobileNetV2 [55] as a backbone model.

Pretraining: We use the standard ImageNet [19] pretraining of the backbone models. In addition, before the optimization via Grid-GD (7) and Grid-TR (16-18) starts, the DeeplabV3+ models are pretrained by the PCE loss (6).

Meta-parameters: We train 60 epochs. We tuned the learning rates for all methods on the val set. Other meta-parameters for competitive methods were set as in the corresponding papers/code. The learning rate is polynomial with power 0.9, momentum is 0.9, batch size is 12.

Grid-TR Stage A (16): The low-level solver\(^4\) of the

\[\text{GCoV3.0: } \text{https://vision.cs.uwaterloo.ca/code/}\]

Algorithm 1: Robust Trust Region for Potts model

1. Initialize model \( f \) using ImageNet pretraining;
2. Tune parameters \( \theta \) of model \( f \) by optimizing PCE-GD loss (6);
3. Initialize \( \gamma \) with the base learning rate;
4. repeat
   for each image in dataset do
   compute segmentation variable \( s \) via (16) using metric \( d_a \) in (25) and loss (26);
   end
   for \( M \) epochs do
   for each image (batch) in dataset do
   update the network parameters \( \theta \) using stochastic gradient descent for loss (17) with robust metric \( d_b \) in (25);
   update rate \( \gamma \) in accord with schedule;
   end
   end
5. until required number of epochs is reached;
grid CRF is the α-expansion [8, 34, 6] with 8-grid neighbourhood system. The max number of α-expansion iterations is 5 achieving convergence in most cases. We restrict the set of labels to those present in the image. We amortize the STAGE A compute time by integrating it with data loading. The training is 1.3 times slower than Dense-GD.

Grid-TR STAGE B (18): To amortize the time consumed by the graph cuts, we perform $M = 5$ epochs of neural network weights updates (17) for each update of the segmentation variables (18). We use a global learning rate schedule spanning throughout iterations. See Alg. 1.

4.2. Segmentation quality

The quantitative results of the weakly supervised training for semantic segmentation are presented in Figure 5 and Tab. 1. The results are presented with different levels of supervision varying from the clicks (denoted as length 0) to the full-length scribbles (denoted as length 1). Decreasing supervision results in degraded performance for all methods. We are interested to compare how different approaches perform at different levels of supervision. Our Grid-TR outperforms all the competitors at each level of supervision.

The examples of images and results shown in Fig. 4 demonstrate the advantages of our method, particularly w.r.t. edge alignment. Quantitatively, we evaluate the accuracy of semantic boundaries using standard trimaps [31, 35, 16, 41]. A trimap corresponds to a narrow band around the ground truth segment boundaries of varying width. An accuracy measure, e.g. mIoU, is computed for pixels within each band. The results are shown in Fig. 6 where our approach demonstrates superior performance.

5. Discussion

5.1. On parameter $\lambda$ in (16)

As discussed below equations (16) - (18) in the paper, even for exact (global) solvers using $\tilde{E} = E$ in (16), the choice of $\lambda = 0$ could be sub-optimal, as demonstrated empirically here in Figure 7. As argued in the paper, while $\lambda = 0$ with an exact solver may seem like a good approach to training $\min_{\theta} E(\theta)$ suggesting globally optimal loss, empirically this leads to overfitting to mistakes or biases of the regularizer (e.g. the Potts model). One argument for $\lambda > 0$ discussed in the paper is that this corresponds to the constrained optimization of (9) over the network manifold in $\mathbb{R}^n$. Such formulation of the training could be preferred as constraining to neural networks can be seen as incorporation of the “deep priors”, e.g. [64]. One can also argue that local minima of $E$ inside the manifold of the network output in $\mathbb{R}^n$ may be preferable to the global optimum of $E$ due to limitations of the basic (but solvable) regularizers.

Empirically, $\lambda = 0$ in (16) leads to a fixed set of proposals generated in a single run of stage A completely independent of the network. In contrast, $\lambda > 0$ leads to multiple distinct iterations of stage A where the network is in the feedback loop. Vice versa, instead of fixed proposals, for $\lambda > 0$ the network is exposed to a substantially larger set of solutions in stage B reduces overfitting.

Moreover, the objective in (16) can be motivated on its own merits independently of the objective in (9). It can be seen as a low-level segmentation objective that integrates class likelihoods produced by the neural network, replacing the basic likelihoods using low-level features, e.g. colors, as discussed in Sec.1.1. Iterations A/B can be seen as joint segmentation and model estimation, as typical for well-known low-level segmentation methods like Zhu-Yuille [67], Chan-Vese [14], or GrabCut [54]. The main difference is that our stages A/B use “deep” models. In contrast to standard methods [67, 14, 54] estimating model parameters for some standard class of probability distributions (e.g. GMM) over fixed low-level features like colors, we estimate deep models with millions of parameters that can be interpreted as learning high-level (semantic) features.

5.2. On discrete losses and decisions/activations

Our approach can train networks using discrete decisions/activations and losses defined over discrete domains. For example, (16)-(18) do not require that $E$ is differentiable. In particular, (16) can be optimized over “hard” segmentations $s \in \{0, 1\}^{N \times K} \subset \Delta_N^K$ even if the network produces soft segmentations $f(\theta) \in \Delta_N^K$, as long as $d_s$ in (16) can measure a distance between discrete and continuous solutions, e.g. $KL(s, f)$ for one-hot and soft distributions. It is also possible to train the models with discrete decision functions $D(l)$ such that $f(\theta) = D(l(\theta))$ where $l$ are logits. Then, all arguments in (16) are discrete. Optimization in (17) can be formulated over real-valued logits using $d_{\text{logits}}$ measuring a distance to subset $\{l \mid D(l) = s_{t+1}\} \subset \mathbb{R}^{N \times K}$.

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| scribble length | 0 | 0.3 | 0.5 | 0.8 | 1 |
|-----------------|---|-----|-----|-----|---|
| full supervision |  |  |  |  | 0.7 |
| PCE-GD          | 0.50 | 0.57 | 0.59 | 0.61 | 0.61 |
| Dense-GD        | 0.55 | 0.61 | 0.62 | 0.63 | 0.64 |
| Grid-GD         | 0.54 | 0.60 | 0.62 | 0.64 | 0.64 |
| Grid-TR (our)   | 0.57 | 0.63 | 0.64 | 0.66 | 0.67 |

Table 1. Results for ScribbleSup, see description in Figure 5.
Figure 4. Examples of the full-scribble training results, see Tab. 1 and Figure 5. Note the better edge alignment of our Grid-TR.

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mean intersection over union (mIoU)

0.50
0.55
0.60
0.65
0.70
0.00 0.25 0.50 0.75 1.00
Full Supervision PCE-GD Dense-GD
Grid-GD Grid-TR

Figure 5. Segmentation performance on the val set of ScribbleSup [39, 20] using DeeplabV3+ [17] with MobileNetV2 [55] backbone. The supervision level varies horizontally, with 1 corresponding to the full scribbles. Our “Grid-TR” outperforms other competitors for all scribble lengths and provides a new state-of-the-art.

distance to the boundary (trimap width), px

Figure 6. The quality of segment boundary alignment. The networks were trained on the full-length scribbles.

value of $\lambda$ - Lagrange multiplier in (16)

Figure 7. Empirical evaluation of Lagrange multiplier $\lambda$ for the Trust Region term in (16): the plot shows how mobile-net training quality depends on $\lambda$. The context is the weakly-supervised semantic segmentation in Sec.4 with regularization loss $E$ (using 8-grid Potts and full scribbles) based on our Trust Region chain rule with robust metric $d_B$ in (18). For $\lambda = 0$ equation (16) generates fixed low-level segmentation proposals completely independent of the network. Then, the network overfits to mistakes in such proposals due to biases/weaknesses of the regularizer. As $\lambda \to \infty$, trust region becomes too small and our approach loses its advantages due to better (e.g. higher-order) approximation $\tilde{E}$ in (16). Conceptually speaking, it should get closer to the results of gradient descent, which uses basic first-order approximations.

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