Mathematical modelling of pipelines, including equipment, levelling sharp changes in fluid pressure

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Abstract. The article deals with the issues related to the modelling of unsteady fluid flow in short and long pipelines, including various equipment: pumping stations (equipped with centrifugal or piston pumps), valves of different operating principle, pressure stabilizers, etc. Pressure stabilizers (especially in short pipelines) are able to protect the pipeline from sudden pressure surges. This is important not only in terms of the strength of the pipeline system, but also in short pipeline circuits for a uniform flow of liquid fuel supply. As a mathematical tool for numerical solution of differential equations of quasi-hyperbolic type, the method of characteristics is applied, which allows to consider the problems that develop over time. The program is written in C++ with graphical interpretation in Maple.

1. Introduction

Sudden unexpected increases (or decreases) in the pressure in the hydraulic system caused by various reasons can cause damage to the equipment, as well as lead to the failure of individual parts of the pipeline (pipeline rupture and the occurrence of plastic deformations [11]). Especially unpleasant are the effects of water hammer on mechanical equipment, such as pumps. A sharp increase in pressure can also cause the pipes to break (especially in places prone to corrosion processes). In the case of an extended breakage and other unusual situations, human casualties are possible. Therefore, the creation of devices that can protect short and long pipelines is currently relevant.

Analysis unsteady fluid motion problems occupies the minds of mankind since there were structures associated with hydraulics: bridges, pipelines, etc. By the beginning of the twentieth century, the theory of unsteady fluid motion was mainly built by M.V. Lomonosov, L. Euler, N.E. Zhukovsky, etc. With the development of computer technology, numerical algorithms were required such that would not be too complex and would guarantee good accuracy of the solution results. Among modern scientists should be noted I.A. Charny [1], who introduced the hypothesis of quasistatic fluid motion, thus making it possible to apply a solution algorithm that takes into consideration the instantaneous velocity of the fluid in the pipe. I would also like to note the great role of D.A. Fox [2], who developed...
graphical algorithms, which were later successfully used to compile numerical algorithms for unsteady fluid motion. Such scientists as B.F. Glickman [4], K.S. Kolesnikov [5], M.S. Natanzon [6] worked in the field of short pipelines used, for example, for supplying liquid fuel to rocket engines. Significant theoretical and practical contribution to the issues of pressure stabilization was made by scientists R.F. Ganiev, H.N. Nizamov, E.I. Derbukov [3]. In the works of Mass E.I., Alyshev V.M. and others were presented algorithms for numerical calculations of pipelines for multi-phase fluid [7].

2. Main part
The equations of fluid motion and continuity of fluid flow are:

\[
\frac{\partial}{\partial x} \left( \rho g z + p + \alpha \rho \frac{v^2}{2} \right) + \beta \rho \frac{\partial v}{\partial t} + \frac{\rho A}{2D} v | v | = 0, \left[ N/m^3 \right],
\]

\[
\rho \frac{\partial p}{\partial x} + \rho \frac{\partial p}{\partial t} + \Gamma v \frac{\partial v}{\partial x} = 0, \left[ N/(m^2 \cdot \text{sec}) \right] \]

These equations are quasi-hyperbolic partial differential equations. Unknown functions are the pressure \( p = p(x, t) \) and the fluid flow rate in the pipeline \( v = v(x,t) \), depending on the longitudinal coordinate of the pipeline \( x \) and time \( t \). The calculation is carried out by the numerical method of characteristics with the implementation of one boundary condition \( v \) or \( p \) or their dependence. The method is considered in detail in [8]. In [8], it is also shown that for a small flow velocity \( v \) compared to the wave propagation velocity of the high-pressure wave \( c \) is convenient to use a regular grid of characteristics (figure 1). The error of the numerical solution is insignificant.

![Figure 1. Regular characteristics grid.](image1)

![Figure 2. Valve in the characteristics grid.](image2)

![Figure 3. Boundary conditions.](image3)
As a boundary condition, the pressure constancy condition is used (that is, a reservoir with a constant water level at \( H_0 \) is modeled. If a constant discharge to the node or outflow from the node of liquid is set, then a constant discharge of liquid in the boundary pipe is set as the boundary condition. At the intersection of branches of the pipeline, the equality pressure \( p \) condition at the point of intersection of the pipes is used to construct a numerical algorithm (figure 3).

The pressure loss in the Valve (figure 2) installed in the \( i \)-th cross section of the structural section of the pipeline is determined by the Weisbach formula:

\[
h_v = \frac{\xi_v v^2}{2g},
\]

where \( \xi_v \) is the coefficient of hydraulic losses of the valve. The valve is modeled under various conditions: instantaneous closing, closing according to a certain law depending on the flow rate and other parameters. To determine the hydraulic resistance, the authors used the reference book of I.E. Idelchik [9].

The issues of quenching the kinetic energy of the flow were considered in [10]-[16].

In the node of hydraulic system, a pressure stabilizer is installed in the \( i \)-th cross section of the structural section of the pipeline (figure 5). In this paper, we consider a device without perforation, which additionally extinguishes the flow energy in the hydraulic system. When the pressure increases at a point \( C \), part of the liquid with the flow rate \( Q_c \) enters the pressure stabilizer, the membrane moves up from the section \( C-C \). Thus, the pressure is stabilized at point \( C \). When the pressure decreases, the membrane bends down. The stabilization effect depends on the useful volume of the device and its location in the hydraulic circuit.

For the polytropic law of expansion-compression of air is as follows:
\[ p_{C,j+1} w_{j+1}^x = p_{C,j} w_j^x , \]

where \( p_C \) - the pressure in the section C-C, \( w \) - the useful volume of air in the pressure stabilizer, \( \chi \) - the indicator of the polytropy; \( i-1, i, i+1 \)-cross sections of the pipe along the \( x \) axis at a distance \( \Delta x \) from each other; \( j, j+1 \)-time along the \( t \) axis with step \( \Delta t \), \( p_C = p_w \).

![Figure 6. Perforation in the tube.](image)

![Figure 7. Apertures of perforation.](image)

It is obvious that \( w_{j+1} = w_j - Q_c \Delta t \). According to (4), we have:

\[ p_{C,j+1} = p_{C,j} \left( \frac{w_j}{w_{j+1}} \right)^x . \]

When liquid moves through a punched hole (figure 6), losses of a pressure are calculated on the following formula:

\[ p_{pun} = \xi_{pun} \frac{\rho v_{pun}^2}{2} , \]

where \( v_{pun} \) - the average speed of movement of liquid through a hole, \( \xi_{pun} \) - coefficient of losses of a pressure. In case of the long pipeline when time of movement of liquid in one direction is estimated in tens of seconds, then \( \xi_{pun} \) can be accepted as a constant.

In I.E. Idelchik [9] \( \xi_{pun} \) is given for various holes and traffic patterns of liquid. We will consider one of schemes represented in figure 7. For an opening in a thin wall (at a pipe thickness \( \delta < 3d \)), Reynolds numbers \( \text{Re} = \frac{v_{pun} d}{\mu} \geq 10^4 \) and \( \frac{v}{v_{pun}} < 0.5 \), \( \xi_{pun} \) is accepted equal to 2.7. Parameter \( \eta \) is called punching percent, i.e. the total ratio of the area of holes to the interior area of the pipe increased by 100% is used.

The dependence \( p=p(Q) \) of a Centrifugal Pump in steady motion can be described fairly accurately by the parabola equation if three points of the curve \( p = A + BQ - CQ^2 \), where \( Q \) is water discharge.

In the case of long pipelines, the pressure stabilizer may be economically inefficient. In this case it is sometimes advisable to use a Valve-Pull-Down Device (VPD) if liquid discharge into the atmosphere is possible (figure 4). The rigid membrane inside the devise is configured in such a way that at pressures that do not exceed the pressure in stationary mode \( p_{ST} \), the liquid is not discharged from the pipeline system, if pressure is greater than \( p_{ST} \), the membrane moves and the liquid flows out.
3. Numerical analyses and the main results.

**Problem 1**

Initial data: \( L = 3500\text{m}, \ L_1 = 2670\text{m}, \) diameter of the pipe \( D = 200\text{mm}, \) \( H_0 = 74.4\text{m}, \) \( h = 42.4\text{m} \) - friction losses, the velocity of a fluid in stationary motion \( v = 1.44\text{m/s} \) (flow rate \( Q = 0.044\text{m}^3/\text{sec} \)), hydraulic friction coefficient \( \lambda = 0.0239 \), the propagation velocity of the pressure wave \( c = 1000\text{m/sec} \), \( \xi_{pan} \) is accepted 2.7

Consider the pipeline diagram shown in figure 8. The numbers are: 1-tank with constant pressure \( H_0 \); 2-pipeline; 3-valve; 4-pressure stabilizer (PS); 5-PS connection node with the pipeline.

![Diagram of a pipeline with a pressure stabilizer.](image)

**Figure 8. Diagram of a pipeline with a pressure stabilizer.**

Figure 9 shows graphs of pressure changes at a point \( C \) depending on time: curve 1 – pressure fluctuations without a stabilizer; curve 2 – pressure fluctuations with PS of volume \( w_0 = 0.4\text{m}^3 \); curve 3 – pressure fluctuations with a pressure stabilizer of \( w_0 = 0.8\text{m}^3 \). The valve closes instantly at the moment \( t = 50\text{sec} \). The calculations without PS are in good agreement with the theoretical results.

![Line pressure.](image)

**Figure 9. Line pressure.**

Figure 10 shows graphs of pressure change in a point \( C \) depending on time: curve 1 – fluctuations in the pressure stabilizer, having an air volume \( w_0 = 0.4\text{m}^3 \) without perforation; curve 2 – the pressure fluctuations with the stabilizer, having a volume of air \( w_0 = 0.4\text{m}^3 \) and the percentage of perforation \( \eta = 14% \); curve 3 – pressure fluctuations with a cap, having a volume of air \( w_0 = 0.4\text{m}^3 \).
and the percentage of perforation $\eta = 9\%$. The valve closes in time $t = 50\text{ sec}$. Figure 11 shows a graph of the maximum pressure depending on percentage of perforation for stabilizer, having a volume of air $w_0 = 0.4\text{ m}^3$.

Summary for Problem 1:

1. Numerical experiments show that a relatively small pressure stabilizer with a useful volume of $w_0 = 0.8\text{ m}^3$ is required to reduce the pressure jump by half.
2. When the volume of the PS increases, the period of pressure fluctuations increases significantly.
3. Decrease of punched holes square area less than 10% should be well checked analytically and experimentally at the stage of system design.
4. Increase of punched holes square area more than 25% does not give a visible effect of reducing of pressure in the hydraulic system.
5. Proper selection of punched holes square area may decrease up to 30% water hammer when a predetermined volume of hydraulic stabilizer is established. This leads to a significant economic effect.
Problem 2

The authors continue to search for equipment and hydraulic circuits that can protect certain sections of the pipeline from sudden changes in pressure. One of the latter is the idea of creating oscillating circuits that can stabilize the pressure in certain parts of the hydraulic circuit.

Consider the hydraulic circuit shown in figure 12. The main valve is instantly triggered at \( t=1000 \) sec. When the water head in node 15 rises above 85 meters, the valves in nodes 9 and 13 are closed instantly. Between these nodes there is an oscillating circuit. The diameter of the pipeline is 1 m, the pressure and flow rate of the system at steady motion and after the operation of the valve in the node 16 are shown in figures 13 and 14. The length of the oscillating circuit between us 9 and 13 is 2600 m. The length of the pipeline from node 1 to node 9 is 17,000 m. The Hydraulic resistances are selected so that the flow rate in considered segment is 1 m / sec. Also are given graphs of pressure and water discharge in node 10 (figures 13, 14).

Summary for Problem 2:

1. The oscillating circuit due to fluctuations in the flow rate (and water speed) with a higher frequency provides the best conditions for quenching the kinetic energy of the flow.
2. Between nodes 9 and 13, the flow rate fluctuates with a greater frequency, which allows the use of pressure stabilizers of a smaller volume.
3. Additional energy quenching occurs due to the meeting of the rarefaction wave and the increased pressure wave in the node 11.

4. Conclusions
The relevance of the article lies in the fact that with a sharp increase in pressure in the hydraulic system, there is a danger of damage to the pipeline with unpredictable consequences. The main goal of the authors is to protect the pipeline from dangerous sudden increases (decreases) in pressure at the working section of the pipeline. Thanks to the installed equipment in the considered sections of the pipeline, the energy of the moving fluid is converted into kinetic energy of the oscillatory process with its successful damping due to the friction of the fluid against the walls of the pipeline. In the future, the authors plan to consider the installation of pressure stabilizers, which will smooth out the amplitude of oscillations.

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