Switching between deterministic and accidental Dirac degeneracy by rotating scatterers and the multi-channel topological transport of sound

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Abstract

In this study, a Dirac cone is yielded at the corner of the Brillouin zone of a two-dimensional solid phononic crystal, which consists of a triangular array of hexagonal steel with a trilobal hole drilled in the center. With the rotation of the trilobal hole, the Dirac degeneracy switches between the deterministic form and accidental form. Every Dirac point is the critical state of a topological phase transition process, which occurs six times within an angular period of $120^\circ$. This structure offers a flexible way to achieve topological transitions and provides multiple routes to construct an interface that supports valley-dependent edge states between two topologically distinct PCs. The associated backscattering suppressed wave propagation along the multiple curved interface channels is also demonstrated. In addition, our design shows the robust propagation of topological interface states against the perturbation of the structure. This study could potentially be significant in the design of acoustic devices for practical applications, such as acoustic signal lossless transport and tunable multi-channel sound transmission.

1. Introduction

The Dirac point, a unique linear dispersion relation at the center or corner of the Brillouin zone (BZ), was first proposed for relativistic particles based on the Dirac equation \cite{1, 2}. It is a nodal point degeneracy connected with linear dispersion in momentum space and has attracted significant attention because of its many intriguing wave transport properties. This special kind of conic dispersion is not limited to atomic crystals. It has also been widely observed and studied in classical wave systems, including photonic and phononic crystals (PCs) \cite{3–12}. Interesting physical phenomena accompanied by the appearance of the Dirac point, such as pseudo-diffusion, object cloaking, and classical analogs of the Zitterbewegung, have been reported \cite{13–15}. Dirac cones can be classified into two kinds. One is generated deterministically and the other is generated accidentally. Generally, the deterministic Dirac cone is formed and protected by the high symmetry of crystals. While the accidental Dirac cone emerges because of appropriately designed sample structures. Commonly used methods to generate an accidental cone mainly include the adjustment of the filling ratio or elastic parameters of constituents.

Recently, with the advent of topology mechanisms, the exploration of the topological states in a classical wave system has been an active research area \cite{16–27}. One of the necessary conditions for the topological transition is the band inversion process of the opening, closing, and reopening of the band gap. The point where two bands touch is the Dirac point, which is the critical state in the topological transition process. In the field of artificial crystals (ACs), the spatial symmetry of the crystal and the scattering strength can be precisely
customized, and the topological properties of the bands for the AC can be easily controlled. ACs are suitable platforms to study topologically protected edge wave propagation because topological edge states occur when crystals of different topological properties are spliced together. As a kind of classical AC, PCs have been extensively studied and designed to produce topological states. For example, Lu and colleagues extended the concept of valley states from valleytronics to acoustics. The vortex nature of valley states has been revealed and the experimental observation of the topological valley transport of sound in sonic crystals has been reported \[19, 20\].

In this work, a solid two-dimensional PC that features Dirac cones at the corner of the BZ is designed. By means of the rotating-scatterer mechanism, we realize the switch between deterministic and accidental Dirac degeneracies. Through the analysis of the eigen-field distributions for the PCs, the band inversion and corresponding topological transition are demonstrated. In addition, the dispersion relation of topological interface states is calculated using the supercell method. The topologically protected edge wave propagation along the multiple interface channels and the tunable propagation with negligible backscattering along the right-angled channel are realized. Finally, the robust propagation of topological interface states against the perturbation of the structure is presented. In this study, all full-wave simulations are performed by the commercial software COMSOL Multiphysics.

2. Unit cell, Dirac degeneracy and band inversion

The PC consists of a triangular array of hexagonal steel with a trilobal hole drilled in its center. As shown in figure 1(a), the structural geometry of the unit cell is described by the lattice constant $a = 10$ mm, blade width $l = 4$ mm, modulation parameter $ht$ characterizing the size of the hole, and angle $\alpha$ indicating the orientation of the hole with respect to the horizontal axis. The material parameters of the steel are chosen as follows: longitudinal wave velocity $c_p = 6010$ m s$^{-1}$, shear wave velocity $c_s = 3320$ m s$^{-1}$, and mass density $\rho = 7760$ kg m$^{-3}$. Air is in the trilobal hole, and because of the large mismatch of impedance between steel and air, rigid boundary conditions are applied to the interface between the steel and trilobal hole. In figure 1(b), the blue solid lines denote the band structure for the PC with rotation angle $\alpha = 0^\circ$, while the red solid lines indicate the band structure for the PC with rotation angle $\alpha = 3^\circ$, both with specific modulation parameter $ht = 4/\sqrt{3}$ mm. When $\alpha = 0^\circ$, because of the $C_{3v}$ symmetry of the lattice, a deterministic Dirac degeneracy emerges near the frequency of 0.332 MHz at the K point in the BZ. However, when $\alpha = 3^\circ$, because of the breaking of mirror symmetry and the lattice becoming $C_3$ symmetry, the Dirac point opens to become a band gap, whose upper and lower edges are known as the acoustic valley states \[19, 20\]. It should be noted that the considered bands are the in-plane elastic wave modes. They are hybridized bands stem from the coupling of P mode and SV mode of elastic wave.

In figure 2, the evolution of the band-edge frequencies versus the rotation angle $\alpha$ and the eigen-field distributions of the valley states are calculated. As is shown, within an angular period of 120°, Dirac degeneracy...
occurs six times when $\alpha$ is equal to $-60.00^\circ$, $-41.07^\circ$, $-18.93^\circ$, $0.00^\circ$, $18.93^\circ$ and $41.07^\circ$. The eigen-field distributions of four valley states when the rotation angles are $\alpha = -13^\circ$ and $13^\circ$ are indicated. When $\alpha = -13^\circ$, the energy flow for the upper valley state appears clockwise, while that for the lower state appears anticlockwise. However, when $\alpha = 13^\circ$, the rotation direction of energy flow is exactly the opposite. It is easy to reach the conclusion that the band inversion process occurs around the Dirac degeneracy at $\alpha = 0^\circ$ [20]. Similarly, the same band inversion process occurs around all the other five Dirac points. Curves of two different colors in figure 2 indicate two kinds of topological states. All the states that feature a clockwise energy flow can be defined as pseudospin-down states and are presented as red curve. The rest of the states that feature an anticlockwise energy flow are defined as pseudospin-up states and presented as blue curve. Two different color blocks (yellow and cyan) between the two bands indicate two kinds of distinct topological bandgaps. They appear alternately and the intersection between each of the two connected color blocks is the Dirac point.

To investigate the properties of the Dirac points, the shape and size of the trilobal holes are continuously adjusted. By adjusting the modulation parameter $ht$ and the blade width $l$, the evolution of four Dirac points between $0^\circ$ and $60^\circ$ (the evolution of the Dirac points between $-60^\circ$ and $0^\circ$ is identical) is shown in figure 3. In figure 3(a), with the increase of $ht$, the Dirac points with rotation angles at $0^\circ$ and $60^\circ$ remain constant, while the rotation angles for the other two Dirac points continuously vary. It should be noted that when $ht = 2\sqrt{3} \approx 3.464$ mm, the hole turns into a regular hexagon, which leads to the Dirac points emerging at any rotation angles. It originates from its $C_{6v}$ lattice symmetry [11]. Maintaining the ratio $ht/l = 1/\sqrt{3}$, the size of the trilobal hole can be changed by varying the magnitude of the blade width $l$, which is the equivalent of scaling the trilobal hole. In figure 3(b), the evolution of the rotation angles for the Dirac points versus the blade width $l$ is presented. Similarly, the Dirac points with rotation angles at $0^\circ$ and $60^\circ$ remain constant, while the rests float in the range of $\alpha = 20^\circ$–$40^\circ$.

It can be concluded from figure 3 that the Dirac degeneracies occur at $\alpha = 0^\circ$ and $60^\circ$ with the lattice symmetry of $C_{3v}$, are deterministic, while the rest of the Dirac degeneracies with the lattice symmetry of $C_{3}$ are accidental. The physical mechanism for the generation of the accidental Dirac degeneracy is unambiguous. As can be seen from figure 1, when the lattice symmetry changes from $C_{3v}$ to $C_{3}$ which means that the mirror symmetry is broken, the deterministic Dirac point opens to become a bandgap [11]. Meanwhile, as the rotation angle of the anisotropic hole changes, the interaction among the scatterers is gradually altered, which results in the closure of the bandgap, i.e. appearance of an accidental Dirac degeneracy at some specific orientations. Clearly, the accidental Dirac degeneracy is sensitive to the shape of the scatterer. However, unlike the accidental double Dirac degeneracy found by Li et al which is very sensitive to the filling ratio of the core–shell structure [7], the accidental degeneracy in our work is much more sensitive to the shape of hole, rather than the filling ratio, which is clearly demonstrated in figure 3.
3. Topological edge states and their robustness

In order to verify the topological phase transition aforementioned in figure 2, two different supercells are designed to calculate the dispersion relation of interface states: one is built by two PCs with $\alpha = 10^\circ$ and $50^\circ$, and the other is built by two PCs with $\alpha = 10^\circ$ and $30^\circ$. Each PC block consists of 10 unit cells that are arranged periodically, and two different PC blocks are spliced together to form a ribbon that has one unit cell in one direction and 20 unit cells along the other direction. In figure 4, the upper panel presents the band structure of the former supercell and the lower panel presents that of the latter one. For the former one, the edge spectrum is completely gapped as the two PCs share the same type of topological bandgaps (same color block in figure 2).

However, for the band structure of the latter one, which consists of PCs with different kinds of topological bandgaps (different color block in figure 2), a pair of gapless topological edge states naturally appears in the bandgap.
It should be noted that the band inversion process occurs six times during the rotation within an angular period of 120°. This property provides multiple routes to construct an interface that can support valley-dependent edge states between two topologically distinct PCs. To confirm this, two hexagonal crystal structures with multiple interfaces are constructed, as shown in figures 5(a) and (b). In figure 5(a), six PC subblocks with \( \alpha = -50°, -30°, -10°, 10°, 30° \) and 50° are placed in turn, and there are correspondingly six different interfaces. All the interfaces are spaced by two topologically distinct PC subblocks. A 0.35 MHz point source is placed at the center of the sample and the magnitude of the displacement field distribution is plotted. As can be seen from figure 5(a), sound waves can be transmitted smoothly along all six channels. In figure 5(a), the positions of the PCs with \( \alpha = 30° \) and 50° are exchanged, while the other PC subblocks are maintained in the same location. The number of channels in this hexagonal crystal structure is still six; however, the upper-right and lower-right interfaces are spaced by two topologically identical PC subblocks. Certainly, only four channels can support topological interface states, which is clearly shown by the field distribution in figure 5(b). In fact, each PC subblock with different \( \alpha \) can be considered as a piece of the whole device. By designing the permutation and combination of each piece, like building a Lego structure, the direction and number of available channels that can support topological interface states can be easily achieved according to requirement. This provides great flexibility in manufacturing diversified devices.

The propagation of topological edge states is immune to defects or disorders in the crystal. To confirm this, the negligible backscattering of the valley-protected edge states along the curved channels, a T-shaped crystal model is built. As shown in figures 5(c) and (d), the sample consists of three parts; two fixed crystal blocks (PC_1 and PC_2) are placed at the bottom and one tunable block (PC_3) is placed at the top, which can adjust the rotation angle of the trilobal hole. The rotation angles of PC_1 and PC_2 are fixed to \( \alpha_1 = -30° \) and \( \alpha_2 = -50° \), respectively. They have different kinds of topological bandgaps; thus, the topological edge states always exist on the interface between them. A point source is placed at the bottom center, and the wave first travels along the interface between PC_3 and PC_2 from the bottom to top. Then, it meets a fork that leads to different paths forward. In fact, the direction in which the wave continues to travel depends on the topological properties of the bandgap for PC_3, which is always identical to one PC and different from the other one at the bottom. As shown in figure 5(c), when the topological properties of the bandgap for PC_3 is different from that for PC_1, for example, \( \alpha_3 = -10° \), the interface wave travels along the left channel. However, when \( \alpha_3 = 10° \), the bandgap for PC_3 is topologically different from that for PC_2 and then the wave travels to the right channel, as shown in figure 5(d). Supposing the rotation angle of PC_3 can be continuously adjusted, the propagation of the wave will gradually
and dynamically shift from one channel to the other channel (the dynamic evolution can be found in the supplementary material, available online at stacks.iop.org/NJP/21/073047/mmedia). Meanwhile, the wave propagating along the constructed sharply curved edge shows negligible backscattering. According to figure 3, the deterministic Dirac point is not related to the structural parameters. When a device that can support topological interface states is designed, only the rotation angle $\alpha$ should be carefully chosen, while the other parameters can suffer somewhat perturbation. This anti-interference property of our structure is presented in figure 6, in which all three horizontal interface channels are constructed using two PCs with $\alpha = 10^\circ$ and $-10^\circ$. A 0.35 MHz point source is placed in each model. Figure 6(a) presents the wave propagation along the interface, where all trilobal holes are identical but only have different rotation angles in the upper and bottom PCs. Compared to figure 6(a), 10% random perturbation of $ht$ and 10% perturbation of $l$ are added in all trilobal holes in figures 6(b) and (c), respectively. As represented by the displacement field distribution, we can clearly see that the wave can still propagate along the interface channel with very little energy loss. Figure 6(d) shows the transmission spectra in these three cases. The black line, blue line and red line represent the transmission spectra of (a) with the lattice having identical trilobal holes, (b) with perturbation in $ht$ of trilobal holes and (c) with perturbation in $l$ of trilobal holes. As can be seen from the transmission spectra, in the frequency range of bulk gap, the most energy can still be transmitted even when the perturbation is added, that indicates the survival of topological edge states. However, the transmissivity becomes smaller when the perturbation is introduced, that is because some energy has been reflected as the result of the broken of lattice-
periodicity. Similar anti-interference property also exists for the interface states in the gap when accidental degeneracy is opened, but the rotation angle of hole should be chosen more carefully as that for the accidental Dirac points are more sensitive to structure.

4. Conclusions

In summary, a new design of a solid PC that features a Dirac cone at the corner of the BZ has been proposed. By rotating the trilobal hole in scatterers, the switch between the deterministic Dirac degeneracy and accidental Dirac degeneracy is achieved. Within an angular period of 120°, the Dirac degeneracy appears six times and the band inversion process occurs when switching between each of those six Dirac degeneracies. By controlling the shape and the size of the hole, it is found that the rotation angle at which the deterministic Dirac degeneracy emerges always remains constant, while that for the accidental Dirac degeneracy is sensitive to the structural parameters of the trilobal hole. The topological transitions are further confirmed by the gapless edge states along the interfaces separating two topologically distinct PCs. Subsequently, the topologically protected edge wave propagation along the multiple interface channels and the associated valley-protected backscattering suppression around T-shaped curved interface channels are demonstrated. Finally, our structure shows great robustness against the perturbation of trilobal holes in the lattice. Our solid structure is easy to fabricate and assemble, which is very significant for device manufacturing.

This work presents a new development in the study of Dirac degeneracy and is very instructive for the research of solid acoustic devices. Compared to the studies about implemented topological acoustic devices in the fluid background, those have been conducted on the solid medium are usually more difficult [25–27]. However, in practice, most high-frequency and even radio-frequency devices are made from solids, such as acoustic surface wave filters and bulk acoustic wave filters in mobile phones. For example, in the realm of the ultra-long acoustic delay line, the delay time depends on the delay distance and the speed of sound in the device. For a certain sound speed, longer delay times require longer delay distances, which increase the space occupied by the device. If we apply the backscattering suppression characteristics of sound propagation in our structure, a long propagation length can be compressed into a zigzag path within a smaller space. It will greatly reduce the size of the device. In addition, with the increase in the operating frequency of the device, the feature size will decrease and the requirements for machining accuracy will increase. This will lead to inevitable machining errors in the process. Fortunately, our structure can tolerate a certain degree of machining errors in the device. Because of these outstanding properties, small solid acoustic devices can potentially be manufactured and applied, such as a more compact high-frequency ultra-long acoustic delay line device.

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