ABSTRACT

This study suggests Bonus-Malus System (BMS) to minimize the occupational accidents in Turkey. Having a 12-class structure with a specific premium ratio for each class considering the number of accidents, the suggested system depends on the principle that the premium ratios should vary according to the number of accidents occurring in a given business. To enter the system, all businesses start from class 6, with a premium ratio of 2%. For the calculation of premium ratios for each class, the expected value principle is used. It is observed by using Markov chains that the system reaches the steady state in 45 years’ time and the changes in the premium income of Social Security Institution (SSI) according to the distribution of businesses according to accident in the 5th, 10th, 20th, and 45th years are investigated. As a result, it is seen that the premium income of SSI will, in the course of time, decrease with BMS. In return, it is predicted that business are likely to try to escape punishment and turn to reward and increase measures against occupational accidents with a decrease in the number of accidents.

Keywords: Bonus-Malus System, Occupational accidents, Premium income, Bayesian estimation
TÜRKİYE’DE İŞ KAZALARININ AZALTILMASINA YöNELİK BİR SİSTEM: BONUS-MALUS SİSTEMİ

ÖZ

Bu çalışma Türkiye’de meslek kazalarının azaltılması için Bonus-Malus Sistemi’ni (BMS) önermektedir. Kaza sayısına göre her bir basamak için özel bir prim oranına sahip 12 basamaklı bir yapı olan bu sistem, prim oranlarının işletmede meydana gelen kaza sayısına göre değişmesi gerektiği ilkesine dayanmaktadır. Tüm işletmeler 6. basamaktan %2 prim oranıyla sisteme giriş yapacaklardır. Her bir basamak için prim oranlarının hesaplanmasına beklenen değer ilkesi kullanılmaktadır. Markov zincirlerinin kullanılmasıyla yapılan analizde, sistem 45 yıllık bir zaman diliminde istikrarlı duruma erişmektedir. 5., 10., 20. ve 45. yıllarda işletmelerin kaza dağılımına göre Sosyal Sigortalar Kurumunun (SSK) prim gelirlerindeki değişiklikler analiz edilmiştir. 5. ve 10. yıllarında SSK’nın prim gelirlerinde düşüş olacağını ifade etmiştir. 20. ve 45. yıllarında ise SSK’nın prima katılan işletmelerin cezadan kaçma eğiliminde olsa ve ödüle yönelerek meslek kazalarını önleme yönündeki önlemleri artıracığı ve böylece kaza sayısının azalacağı tahmin edilmiştir.

Anahtar Kelimeler: Bonus-Malus Sistemi, İş Kazaları, Prim Geliri, Bayesyen Tahmin

INTRODUCTION

Defined as any unplanned occurrence caused by unsafe conditions and careless act resulting in fatal, major or minor occupational injury and/or damage to the machines and tools in the workplace, occupational accidents create social and economic burdens for many countries. Some of these are losing workforce, compensation, slowdown in production, data loss, and health expenditure [1-3]. For a more detailed classification of occupational accidents, see Khanzode et al. [4].

Turkey ranks the third after El Salvador and Algeria in fatal accidents in the workplace in the world and is at the top of the list of occupational accidents in Europe. The rate of the fatal workplace accidents is 20.5 employees per 100 000 population in Turkey, but in such countries as Norway, Sweden, and Denmark, this number is two employees per 100 000 population [5]. According to the Ministry of Labor and Social Security, 172 daily workplace accidents occur in Turkey, with four deaths and six workers becoming incapable of work [6]. According to the Council of Workers’ Health and Workplace Safety, 1270 workers lost their lives in the first eight...
months of 2014, exceeding the 1235 deaths in 2013. In the last decade, 6428 workers have lost their lives due to workplace accidents. The number of the workers having lost their lives at work between 2000 and 2014 is over 15000 [7]. Furthermore, the increase in the number of fatal occupational accidents in mines and shipyards in recent years has made it necessary for the government to create incentives and impose sanctions, according to a statement made by the Turkish Prime Minister [8-10]. Consequently, it is seen that Turkey is in need of measures to reduce the number of occupational accidents. To this end, it seems necessary that the structure of SSI be changed so that it can serve a preventive function along with the compensative one. Within the framework of the social security system, determining the premium ratios according to the number of occupational accidents is one of the most effective systems to encourage the businesses to take the necessary precautions against occupational hazards.

Turkey has a social security system where the premiums paid over the wages of employees according to their insurance status are collected in a joint pool and the benefits are provided based on the paid premiums only when old-age pension is entitled. The amount of the benefits to be granted to the insurance holders in cases of retirement, accident and sickness varies by the income they previously had. The main actors in this system are employees, employers and representatives in the public sector. The sole responsible body for social security work is SSI in Turkey [11]. Work Health and Safety Act was introduced in 2012 to reduce the risk of occupational accidents in the workplace and to improve the current working conditions [12]. This act stipulates that businesses appoint from their human resources workplace safety experts, workplace doctors, and other medical staff to provide workplace safety service. If businesses do not have such staff, they have to receive all or some of such a service from common units for health and safety. One year after the act’s coming into effect, the premium ratios of short-term insurance branches were determined as 2% of the insurable earnings of insureds.

Premium ratio in Turkish social security system can be defined as the ratio of the amount of money to be paid by the employer to the SSI by deducting the amount equal to the rate of premium collected over total earnings from workers’ wages and adding the amount of premium to be paid by the employers depending on the law in return for the benefits the employee enjoys” [13]. This ratio is the same for all businesses and does not vary by the number of occupational accidents, risk category, and the areas of work, which we argue could not help decreasing the number of workplace accidents.

A more efficient alternative suggested in this study is the BMS. It is a method in which workplace statistics and measures taken to prevent occupational accidents and occupational diseases are monitored and the premium ratios are determined.
according to the success businesses can achieve in monitoring the statistics and the measures mentioned. In BMS, the premiums businesses should pay vary dynamically according to their history of occupational accidents yearly or in certain time intervals.

This system is used to determine the insurance premium ratios for occupational accidents and occupational diseases in many countries, such as Germany, France, Belgium, and Italy [14]. In Turkey, however, the premium ratios were determined according to the risk category the businesses belonged to before 2013, but today the premium ratio is 2% of the insurable earnings of insureds for all the businesses, irrespective of their status. To explain it more clearly, the premium ratio is not determined by occupational accident, occupational disease, pregnancy, and the risk of sickness, but a single ratio (2% of the insurable earnings of insureds) is used instead. This system disregards such risk factors likely to have an effect on the occurrence of occupational accidents as the quality of work and the success of the business in taking the measures against occupational accidents. Worse than that, this system disregards even the number of the occupational accidents. However, the number of occupational accidents represents most of the risk factors, and that’s why, occupational accidents should be used as a criterion in determining the premium ratios.

Being applied research, this study suggests that a BMS which exclusively takes the number of accidents into account should replace the current system, which applies 2% for all types of businesses irrespective of their status. In this way, the effectiveness of the social security system in Turkey could be improved. In BMS, as the number of accidents increases, the premium ratios go up; otherwise, the premium ratios go down. Thus, it is believed that such a system would, when in practice in Turkey, urge the businesses to take measures to prevent occupational accidents, which will directly or indirectly contribute substantially to the country’s economy.

What follows is the second section of the paper that explains the Bonus-Malus System briefly. The third section elaborates the methodology of the components used in calculating the premium ratios. The fourth section deals with the application of the methodology. The final section reports the conclusions.

1. The Bonus-Malus System

In the context of occupational accidents, BMS refers to a practice whereby the premium ratios are set by taking a given business’ statistical data into account. Calculations using this system may result in variations in the premium ratios in the same occupations, and even in the same occupational risk category.

It was suggested in the mid-1950s that the premium ratios should be determined considering the claims history of policyholders. The system came under
such names as merit-rating, experience-rating, no claim discount, and bonus-malus. The first area to which BMS was applied was automobile insurance [15]. Later uses of BMS included reducing the number of occupational accidents in many countries [16]. However, the method of collecting premiums for the insurance of occupational accidents varies from country to country. To illustrate, in Bulgaria, Germany, France, Ireland, Portugal, and Finland, BMS is used; in Austria, Sweden, Greece, and Turkey, fixed premium ratio is used; in Belgium, Denmark, Spain, and Luxemburg, the ratios of premiums vary according to occupational risk categories.

In BMS, the premiums businesses must pay go down as a reward if they reduce the number of accidents; otherwise, the premiums increase as a punishment. BMS naturally causes the businesses to take measures in order to minimize occupational accidents. On the one hand, if a business reduces the number of accidents, then this system means, to this business, a tool for financial incentives. The fact that the businesses make an effort to lower the number of accidents to benefit from such incentives helps reduce the costs which institutions financing the occupational risks must cover. It is known that occupational accidents tend to have a negative impact on the country’s economy [17]. On the other hand, with this system, more accurate forecasts could be produced from the existing data, which may facilitate policy-making by central institutions like SSI.

2. Statistical Analysis

This section gives the methodology of the components used in calculating the premium ratios. Therefore, it covers the methods of how BMS is formed. These are determining the distribution of the number of accidents, calculating the expected premiums based on the distribution of the number of accidents, and forming the Markov chains for the transitions from one state to another.

1.1. Poisson Distribution

The number of occupational accidents in a given time interval is Poisson distributed if they satisfy the following assumptions:

i. The accidents in disjoint time intervals are independent of each other.

ii. \( \Delta \) indicating a short time interval, the probability of an accident in \( \Delta \) time interval equals to \( \Delta \lambda \). Here \( \lambda \) is the Poisson parameter representing the mean.

iii. \( \Delta \) is defined to be sufficiently small such that the probability of more than one accident is in \( \Delta \) is negligible.

iv. The process is stationary and as \( \Delta \) increases, the probability of an accident occurring in \( \Delta \) increases, independent of the start of the interval.
Let \( k \) be the number of accidents in a given time interval. Then \( X \) is a Poisson random variable with parameter \( \lambda \). The probability function of the Poisson is written as:

\[
 f(k \mid \lambda) = P(X = k \mid \lambda) = \frac{e^{-\lambda} \lambda^k}{k!}, \quad k = 0, 1, 2, \ldots
\]

(1)

The mean and the variance of the Poisson are both equal to \( \lambda \) and \( \lambda \) is regarded to be a constant over time. However, in actuarial practice, \( \lambda \), average number of accidents varies by business and over time, so \( \lambda \) is assumed to have a distribution, which requires using Bayesian estimators. Therefore, Bayes is used to obtain the probability function of the number of accidents (marginal distribution) and to calculate the expected premium ratios. What follows are the steps involved in Bayesian estimation method.

1.2. Bayesian Estimation

Given a random sample of \( X_1, X_2, \ldots, X_n \), Bayesian estimation begins with selecting a prior distribution for the \( \lambda \) parameter of the distribution. Then the posterior distribution is obtained using Bayesian method. The expected value of the posterior distribution is the Bayesian estimator of the parameter [18]. The Bayesian estimator of \( \lambda \) is \( \hat{\lambda} = E(\lambda \mid k_1, k_2, \ldots, k_n) \). The steps of Bayesian estimation are given in Appendix A. As the steps in Appendix A show, in Bayesian method, information about the prior distribution along with the information obtained from the sample are used to reduce the uncertainty about the parameter [19].

1.3. Marginal Distribution

This section covers the marginal distribution of the number of accidents obtained through Bayesian method. This distribution is required to calculate the probability of occurrence of accidents in businesses. To do this, a prior distribution is selected. The support set of the parameter plays an important role in the selection of the prior distribution. The parameter of the Poisson distribution is defined as \( \lambda > 0 \), called the support set. In the prior distribution selected accordingly, the random variable must be defined as greater than 0. This must be taken into account in the selection of the Gamma distribution. The gamma distribution is the conjugate prior distribution for Poisson likelihood functions. Then the marginal distribution is obtained as
\[ m(k) = \binom{k+a-1}{k} \left( \frac{\tau}{1+\tau} \right)^a \left( \frac{1}{1+\tau} \right)^k \]  

(2)  

(See Appendix B).

Given \( \frac{\tau}{1+\tau} = p \) and \( \frac{1}{1+\tau} = q \), the expected value and variance of \( m(k) \) are respectively \( \frac{a}{\tau} \) and \( \frac{a}{\tau} \left( 1 + \frac{1}{\tau} \right) \), clearly a negative Binomial distribution.

This marginal distribution yields the probability of \( k \) number of accidents a business can have in a given time interval. The process of obtaining the marginal distribution of the random variable considering the distribution of \( \lambda \) is called Mixed Poisson Process [20]. The estimators of \( \tau \) and \( a \) parameters can be obtained by moments method as:

\[ \hat{\tau} = \frac{\bar{X}}{S^2 - \bar{X}} \quad \text{and} \quad \hat{a} = \frac{\bar{X}^2}{S^2 - \bar{X}} \]

These values of estimation are used to calculate the ratios of premiums for the businesses.

1.4. Premium Calculation

When the premium ratios per business are determined, it is important to classify the businesses into homogenous groups in insurance sector, especially the one with systems like BMS. Homogeneity of the groups is important for ease of modeling. In the case of homogenous groups, the average number of accidents for all the businesses can be assumed to be constant \( \lambda \). The average age and experience of the employee, the unit he works in, and the accident history of the business are considered to be key variables for homogeneity. However, there are other variables that cannot be measured, such as job satisfaction, stress etc. To incorporate the heterogeneity caused by such variables into the model, Poisson Mixed Models are used [21].

The distribution of the total number of accidents occurring in year \( t \) must be taken into account for premium calculation. Let the total number of accidents occurring
This function is known as the function of likelihood and, given \( \lambda \), gives the probability of occurrence of \( k_i \)'s. The posterior distribution is, using Bayesian method, written as (Appendix C):

\[
u(\lambda / k_1, \ldots, k_t) = \frac{(\tau + t)^{a+k} \lambda^{k+a-1} e^{-(t+\tau)\lambda}}{\Gamma(a+k)}
\]

where safety loading is denoted by \( (1 + \alpha) \):

\[
P(k_1, \ldots, k_t) / \lambda = P(k_1 / \lambda) \ldots P(k_t / \lambda)
\]
in \( t \) year(s) be \( k = \sum k_i \). Then

\[
\frac{\lambda^t e^{-t\lambda}}{\prod_{i=1}^{t} (k_i !)}
\]

\[
E[(\lambda / k_1, \ldots, k_t)] = \frac{a + k}{\tau + t}
\]

There are a number of different methods for premium calculation [22]. In literature, there are a number of methods used to calculate risks, such as net premium, the expected value principle and variance. These methods have been proposed as alternatives and can give similar results. In this study, the expected value principle, as it is the most common method, has been used to calculate the premium ratios. The simplest one is net premium plus a safety loading proportional to the net premium. This is called The Expected Value Principle [23]. The premium a business with an accident history \( (k_1, \ldots, k_t) \) must pay is calculated by using the following formula,
In actuarial science, the net premium is multiplied by a coefficient called safety loading so as to prevent other expenditure and additional costs to be created by unforeseeable risks [24]. In the present study, the net premium is weighted by Initial Premium / (a / τ) for premium calculations [25]. In this way, in case of time $t + 1$ and k number of accidents, the premium is calculated by:

$$P_{t+1}(k_1,\ldots,k_t) = (1 + \alpha)E[(\lambda/k_1,\ldots,k_t)]$$

$$= (1 + \alpha)\frac{a + k}{\tau + t}$$

(6)

In BMS, the businesses are categorized into classes by the premium ratios they must pay. As the class number (e.g., in a 12-class system, where the premium ratio increases from 1 to 12, 12 represents the highest level, and 1 the lowest) for a given business increases, the premium ratio it must pay increases, or vice versa. The transitions between classes depend on the number of accidents. Markov chains are used to illustrate probability of transitions between classes and to monitor the long-term behavior of the system.

1.5. Markov Chains

The stochastic process satisfying the following equation is called a Markov Chain:

$$P\{Y_{t+1} = j\mid Y_0,\ldots,Y_t\} = P\{Y_{t+1} = j\mid Y_t\}$$

(8)

A Markov chain is a sequence of random variables $Y_1, Y_2, Y_3,\ldots$ with the Markov property, that is, given the present state, the future and past states are independent [26]. Here $Y_t = j$ denotes the process in j case in time $t$.

1.5.1. Transition Probability and Transition Matrix

The probability of going through a single step from i to j is

$$P\{Y_{n+1} = j\mid Y_n = i\} = P(i,j)$$

(9)
Here the probability of transition is independent of $t$. If the $P(i, j)$ probabilities satisfy the following two conditions, transition matrix $P$ is called Markov matrix.

1. $0 \leq P(i, j) \leq 1$ for all $i, j$
2. $\sum_j P(i, j) = 1$ for all $I$

1.5.2. The n-step Transition Probabilities and Stationary Points

Let Markov chain $Y$ have transition matrix $P$ and state space $E$. Given $i, j, k \in E$

$$P\{Y_{i+n} = j \mid Y_i = i\} = P^n(i, j)$$ (10)

Therefore, the probability of going from state $i$ to state $j$ in $n$ time steps is in transition matrix $P$’s $n^{th}$ power $i^{th}$ row and $j^{th}$ column.

If $P$ is a steady-state matrix, namely given $n > 1$, all the elements of matrix $P^n$ are positive, then a steady-state probability vector can be obtained by

$$vP = v$$ (11)

This vector shows the probability of $Y$’s being in state $i$ after a long time passing.

1.5.3. Markov Chains in BMS

In a Markov chain, going from one state to another does not depend on the previous states of the system but depends on the state of the system that is one step before. Thus, information about the previous states of the system is irrelevant for Markov chains. In terms of Markov chains, the future class of any given business is determined by using the number of accidents occurring in the present year and the information about the present class of the business [27].

In BMS, it is necessary to define the rules for transition in order to calculate the transition probabilities. To put it another way, in what way the present class of a given business could change according to the number of accidents must be identified. Then possible transition probabilities can be calculated by using the distribution of the number of accidents and a Markov chain is formed.
2. Application

This section elaborates the suggested BMS for Turkey considering the occupational accidents.

2.1. Data and Variables

The data are obtained from the database of SSI in Turkey. Collected in the year 2011, the data is about the businesses where there occurred some occupational accidents and contains the information about the scale and the number of the accidents occurring in those businesses.

The businesses with fewer employees than 10 were excluded from the study, so the data includes 210,138 businesses with 10 or more employees. The number of businesses where one or more accidents took place was 33,122.

Since the number of accidents in a given business is proportional to the scale of the business, levels ($k$) based on frequency rather than number is used. To calculate the frequency of the accidents, the following formula is used:

$$\text{Frequency of Accident (FA)} = \frac{\text{Number of accidents}}{\text{Number of insureds}} \times 100 \quad (12)$$

[28].

Then the businesses are classified into groups according to the frequency of accidents, which yields the accident levels of the businesses. The coding for accident levels of the businesses is illustrated in Table 1.

| Accident level ($k$) | Interval       |
|----------------------|---------------|
| 0                    | No accidents  |
| 1                    | $0 < FA \leq 10$ |
| 2                    | $10 < FA \leq 20$ |
| 3                    | $20 < FA \leq 30$ |
| 4                    | $30 < FA \leq 40$ |
| 5                    | $40 < FA \leq 50$ |
| 6                    | $50 < FA$      |
2.2. Distribution of Accident Levels and BMS Premium Ratios

To calculate new premium ratios, the distribution of the accident levels of the businesses should be known. The distribution of the businesses by accident levels in 2011 is shown in Table 2.

**Table 2: Distribution of businesses by accident level**

| k | Number of businesses |
|---|----------------------|
| 0 | 177,016              |
| 1 | 30,867               |
| 2 | 1,868                |
| 3 | 279                  |
| 4 | 83                   |
| 5 | 17                   |
| 6 | 8                    |

The null hypothesis that “the data is drawn from the Poisson distribution” for the data in Table 2 is tested by *Kolmogorov–Smirnov test*. The test shows that the accident levels are drawn from Poisson distribution with \( \lambda = 0.1709 \) (p>0.05). The mean and variance of the data in Table 2 are \( \bar{X} = 0.1709 \) and \( S^2 = 0.1749 \), respectively. Thus, \( \hat{\tau} \) and \( \hat{a} \) are calculated as:

\[
\hat{\tau} = \frac{0.1709}{0.1749 - 0.1709} = 43.725
\]

and

\[
\hat{a} = \frac{(0.1709)^2}{0.1749 - 0.1709} = 7.473
\]

To calculate premium ratios, a premium ratio for the initial class should be determined. All the businesses pay the same amount of premium (2%) according to the present system. Accordingly, in the study, the premium ratio for the initial class is taken as 2%. Table 3 shows the BMS premium ratios obtained through net premium formula.
**Table 3: Premium ratios (%) in BMS**

| Accident levels (k) | 0     | 1     | 2     | 3     | 4     | 5     | 6     |
|---------------------|-------|-------|-------|-------|-------|-------|-------|
| Year (t)            | 0     | 1     | 2     | 3     | 4     | 5     | 6     |
| 0                   | 2.00  |       |       |       |       |       |       |
| 1                   | 1.96  | 2.22  | 2.48  | 2.74  | 3.00  | 3.26  | 3.53  |
| 2                   | 1.91  | 2.17  | 2.42  | 2.68  | 2.94  | 3.19  | 3.45  |
| 3                   | 1.87  | 2.12  | 2.37  | 2.62  | 2.87  | 3.12  | 3.37  |
| 4                   | 1.83  | 2.08  | 2.32  | 2.57  | 2.81  | 3.06  | 3.30  |
| 5                   | 1.79  | 2.03  | 2.28  | 2.52  | 2.76  | 3.00  | 3.24  |

For instance, if the accident level of a given business starting from the initial class is 0, then the premium ratio it should pay in the second year is calculated as:

\[
P'_2(0) = (1 + \alpha) \times \frac{\tau(a + k)}{a(\tau + t)} = 2 \times \frac{43.725}{7.473} \times \frac{7.473 + 0}{43.725 + 1} = 1.96
\]

### 2.3. The Number of Classes and Premium Ratios for the Classes

In BMS, businesses start from a certain initial class and their classes vary according to the number of accidents in a given year. In BMS, as the number of accidents increases, the premium ratios go up; otherwise, they go down.

The present study considers the number of classes as 12, which is the same number as the one used by SSI before 2013, when the businesses were categorized according to risk categories.

New premium ratios according to the classes are determined using Table 3 as shown below in Table 4:

**Table 4: New premium ratios (%) by class**

| Class | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  | 12  |
|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Premium ratio | 1.79 | 1.83 | 1.87 | 1.91 | 1.96 | 2.00 | 2.22 | 2.48 | 2.74 | 3.00 | 3.26 | 3.53 |

As is seen in Table 4, businesses start from the 6th class and continue to pay by the current premium ratio (2%). In the coming years, the class of a given business will be determined according to its accident level and the business will pay the new premium at its determined class.
2.4. The Rule for Transitions Between Classes

The next step after determining the number of classes in BMS is to decide how transitions between classes should be. The rule for transitions between classes is defined in this study as:

i. If the accident level of a given business is 0, then its class decreases by one level.

ii. If the accident level of a given business increases, then its class increases accordingly.

For example, if the accident level of a business at class 6 is 2, then its new class is 8 (6+2). On the other hand, if the accident level of a business at class 6 is 0, then its new class is 5 (6-1). However, as there are 12 classes totally, the highest class is 12.

Table 5 shows the rule for transitions between classes.

Table 5: Matrix of transitions between classes

| Current Class | Accident levels |
|---------------|-----------------|
|               | 0   | 1   | 2   | 3   | 4   | 5   | 6   |
| 1             | 1   | 2   | 3   | 4   | 5   | 6   | 7   |
| 2             | 1   | 3   | 4   | 5   | 6   | 7   | 8   |
| 3             | 2   | 4   | 5   | 6   | 7   | 8   | 9   |
| 4             | 3   | 5   | 6   | 7   | 8   | 9   | 10  |
| 5             | 4   | 6   | 7   | 8   | 9   | 10  | 11  |
| 6             | 5   | 7   | 8   | 9   | 10  | 11  | 12  |
| 7             | 6   | 8   | 9   | 10  | 11  | 12  | 12  |
| 8             | 7   | 9   | 10  | 11  | 12  | 12  | 12  |
| 9             | 8   | 10  | 11  | 12  | 12  | 12  | 12  |
| 10            | 9   | 11  | 12  | 12  | 12  | 12  | 12  |
| 11            | 10  | 12  | 12  | 12  | 12  | 12  | 12  |
| 12            | 11  | 12  | 12  | 12  | 12  | 12  | 12  |

*The numbers in Table 5 show at which class the business will be in the coming year.*
2.5. The Transition Matrix

If the distribution of the accident levels is Poisson, then transition matrix is calculated as:

\[
P = \begin{pmatrix}
0.84291 & 0.14405 & 0.01231 & 0.00070 & 0.00003 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.84291 & 0 & 0.14405 & 0.01231 & 0.00070 & 0.00003 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.84291 & 0.14405 & 0.01231 & 0.0007 & 0.00003 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.84291 & 0.14405 & 0.01231 & 0.0007 & 0.00003 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.84291 & 0.14405 & 0.01231 & 0.0007 & 0.00003 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.84291 & 0.14405 & 0.01231 & 0.0007 & 0.00003 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.84291 & 0.14405 & 0.01231 & 0.0007 & 0.00003 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.84291 & 0.14405 & 0.01231 & 0.00073 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.84291 & 0 & 0.14405 & 0.01304 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.84291 & 0.15709 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.84291 & 0.15709
\end{pmatrix}
\]

Here let \( t = 0 \) time be 2011. The value 0.84291 at the intersection of the 1st row and 1st column means the probability of staying in the same class for a given business during transition from 2011 to 2012, and it is calculated as:

\[
P(Y_1 = 1 | Y_0 = 1) = f(0 | \lambda) = P(X = 0 | \lambda) = \frac{e^{-\lambda} \lambda^0}{0!} = 0.84291
\]

Likewise, the value 0.14405 at the intersection of the 1st row and 2nd column means the probability of going from 1st class to 2nd class for a given business during transition from 2011 to 2012. With the matrix, the probabilities of transitions by year between classes are calculated, by means of which estimation of the SSI’s premium income until the system reaches steady state can be made. It is assumed that there occurs no inclusion and exclusion in the system. The system reaches the steady state in 45 years’ time. That is, the distribution of the number of businesses will probably be fixed in approximately 45 years. For the calculation of the steady state probabilities, the Markov module of \textit{WINQSB} has been used [29]. The steady state probabilities for all the classes are given in Table 6.
Table 6: Distribution of businesses by classes when system is in steady state

| Class | \( v \) |
|-------|---------|
| 1     | 0.7973  |
| 2     | 0.1486  |
| 3     | 0.0400  |
| 4     | 0.0104  |
| 5     | 0.0027  |
| 6     | 0.0007  |
| 7     | 0.0002  |
| 8     | 0.0000  |
| 9     | 0.0000  |
| 10    | 0.0000  |
| 11    | 0.0000  |
| 12    | 0.0000  |

Table 6 illustrates that about 80% of the businesses is at 1st, 15% 2nd, 4% 3rd, and 1% 4th classes, and the rest is at the other classes.

2.6. Distribution of Businesses by Classes and Premium Incomes for Prospective Years

2011 assumed as the first year, estimations as to distribution of businesses by class and premium incomes are made for the prospective 5th, 10th, 30th and 45th years (Table 7). To do this, it is necessary to calculate the distribution of businesses by class for these years. For example, the vector showing initial distribution of businesses by class should be multiplied by the transition matrix of the 5th year so as to calculate distribution of businesses by class in 2016.

Since all the businesses will be included in the system from class 6 at the outset, the vector showing initial distribution of businesses by class is:

\[
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 210138 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

The numbers of businesses by class obtained by multiplying \( P^6 \) by this vector are shown in column 2016 in Table 7, which also presents the numbers as to the distribution for the prospective 10th, 30th and 45th years.
Table 7: Distribution of businesses by class and premium income for prospective years

| Class | 2016  | 2021  | 2041  | 2056  |
|-------|-------|-------|-------|-------|
| 1     | 75,367| 156,816| 167,512| 167,531|
| 2     | 77,281| 40,219| 31,226| 31,223|
| 3     | 6,604 | 10,086| 8,417 | 8,411 |
| 4     | 33,394| 8,408 | 2,201 | 2,196 |
| 5     | 5,659 | 2,546 | 576   | 573   |
| 6     | 8,087 | 1,857 | 151   | 150   |
| 7     | 1,970 | 658   | 40    | 39    |
| 8     | 1,241 | 342   | 11    | 10    |
| 9     | 358   | 125   | 3     | 3     |
| 10    | 130   | 50    | 1     | 1     |
| 11    | 37    | 17    | 0     | 0     |
| 12    | 10    | 6     | 0     | 0     |

| Premium income (TL billion) | 65.5 | 63.7 | 63.2 | 63.2 |

As is seen in Table 7, as time goes by, the businesses are expected to be at lower classes, which is only possible if the number of occupational accidents in the businesses go down.

Another important point to consider is in what way the premiums SSI in Turkey will collect will be affected. Calculations for the prospective premium income SSI will receive can be made as such: for 2011 average premium earning per day is TL 46.41, and the average premium earning per year is TL 16,707.6. If you multiply this by the premium ratios given in Table 4, the premium incomes per business are obtained. Then the premium incomes per business are multiplied by the number of businesses in Table 7 to calculate the expected premium income of SSI. The related values are shown in the final row in Table 7. For example, this value is expected to be TL 65.5 billion in 2016. The current system, with a fixed premium ratio of 2%, would yield TL 70.2 billion. With years passing by, the premium incomes of SSI will decrease. However, if BMS is used, the number of occupational accidents is likely to go down because the businesses are expected to take precautions to prevent accidents so that they will pay lower premiums, which in return will help reduce the SSI’s expenses.

CONCLUSION

BMS is a system that determines the premium ratios by taking the number of accidents in businesses into account. Although the businesses pay a fixed ratio of 2% in the current system in Turkey, the need for a kind of reward and punishment system
has been voiced by some members of the government recently. The fatal occupational accidents in recent months occurring in the mines because of the inadequate safety measures have paved the way for a debate over the need for a premium system based on reward and punishment.

In this study, BMS is applied to Turkish case. Therefore, the suggested version of BMS yields premium ratios ranging from 1.79% to 3.53%, and all the businesses start from class 6, where the premium ratio is 2%. It has been demonstrated that the businesses in this system are likely to pay lower premiums in the course of time, going from higher classes to lower classes. It is apparent that 80% of the businesses will be at the lowest class in time (class 1), which is one of the expected outcomes of the study. It is predicted in this study that business are likely to try to avoid punishment and turn to reward and thus SSI’s premium income will decrease. This appears, at first glance, to be negative; however, it is of great importance for the country’s economy, for the reason for such a decrease in premium income is reduced number of occupational accidents. Fewer occupational accidents mean fewer incapacity benefits to be paid and less healthcare expenditure by class, which contributes substantially to the country’s economy.

It is important to collect data correctly to improve the practical efficiency of BMS. It is possible that businesses do not report the accurate number of the accidents to avoid punishment. Therefore, an effective supervision mechanism should be established and operated by SSI in cooperation with the Ministry of Labor and Social Security. Moreover, SSI can impose additional punishments on businesses by considering temporary and permanent incapacity benefits caused by occupational accidents, and survivors’ pensions caused by fatal accidents in order to minimize the severity of occupational accidents and to quicken the pace of going back to work.

As is seen in Section 4.Application, it takes about 45 years for the businesses to reach the steady state. To reduce this duration, the premium ratios could be differentiated so that the variance between classes could be widened. In such a case, the premium ratios would have a wider range than the current 1.79% - 3.53%.

This study does not differentiate between the types of accidents, such as near miss, without injury, with injury, and fatal accidents. However, a BMS could easily be adapted, considering the types of accidents for the calculation of the premium ratios.

In this study, no categorization by sector is made because it is assumed that the initial class premium is the same for all sectors. Thus, the suggested system requires assessing the business by its own risks, not by the sector it is classified into. For example, a given business in an apparently safe sector could pay a higher premium based on its accident level than another business in an apparently more risky sector.
Appendix A. The steps involved in obtaining the Bayesian Estimator

The following are the steps involved in obtaining the Bayesian estimator.

i. Prior distribution, \( u(\lambda) \), is selected.

ii. Given \( \lambda \), the sample from probability function is assumed as \( f(k / \lambda) \).

iii. The joint probability function of \( X \) and \( \lambda \) is obtained as

\[
 f(k, \lambda) = f(k / \lambda)u(\lambda)
\]

iv. From \( f(k, \lambda) \), the marginal function of \( X, m(k) \), is obtained.

v. Given \( k \), the conditional probability function of \( \lambda \) yields posterior distribution as follows:

\[
 u(\lambda / k) = \frac{f(k, \lambda)}{m(k)}
\]

vi. The Bayesian estimator of \( \lambda \) is

\[
 \hat{\lambda} = E(\lambda / k_1, k_2, ..., k_n)
\]

Hence, for this estimator, the likelihood function denoted as \( f(k_1, k_2, ..., k_n / \lambda) \) is used.

Appendix B. Obtaining the Marginal Distribution

The Gamma distribution with parameters \( a \) and \( \tau \) is written as:

\[
 1 = \frac{\tau^a e^{-\tau\lambda} \lambda^{a-1}}{\Gamma(a)}
\]

where the \( \Gamma(a) \) function is defined as

\[
 \Gamma(a) = \int_0^\infty t^{a-1}e^{-t}dt
\]

and has the property of \( \Gamma(a + 1) = a\Gamma(a) \). If \( a \) is an integer, \( \Gamma(a + 1) = a! \). The joint probability function of \( X \) and \( \hat{\lambda} \) is obtained as:
\[ f(k, \lambda) = f(k / \lambda)u(\lambda) = \frac{e^{-\lambda} \lambda^k}{k!} \frac{\tau^a e^{-\tau \lambda}}{\Gamma(a)} \]

Using this, the probability function of the number of accidents (marginal distribution), \( m(k) \), is

\[
m(k) = \int_0^\infty \frac{e^{-\lambda} \lambda^k}{k!} \frac{\tau^a e^{-\tau \lambda}}{\Gamma(a)} d\lambda
\]

\[
= \frac{\Gamma(k+a)}{\Gamma(k+1)\Gamma(a)} \frac{\tau^a}{(1+\tau)^{k+a}}
\]

\[
= \binom{k+a-1}{k} \left( \frac{\tau}{1+\tau} \right)^a \left( \frac{1}{1+\tau} \right)^k
\]

**Appendix C. Obtaining the Posterior Distribution**

\[
u(\lambda / k_1, \ldots, k_t) = \frac{P(k_1, \ldots, k_t / \lambda)u(\lambda)}{\bar{P}(k_1, \ldots, k_t)}
\]

\[
= \frac{\lambda^k e^{-\lambda} \tau^a e^{-\tau \lambda}}{\prod_{i=1}^t (k_i)! \Gamma a}
\]

\[
= \int_0^\infty \frac{\lambda^k e^{-\lambda} \tau^a e^{-\tau \lambda}}{\prod_{i=1}^t (k_i)! \Gamma a} d\lambda
\]

\[
= \frac{\lambda^{k+a-1} e^{-(\tau+t)\lambda}}{\int_0^\infty \lambda^{k+a-1} e^{-(\tau+t)\lambda} d\lambda}
\]

\[
= \frac{(\tau+t)^{a+k} \lambda^{k+a-1} e^{-(\tau+t)\lambda}}{\Gamma(a+k)}
\]
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