Amplification of gravitational waves during inflation in Brans-Dicke Theory (Revised Version)

Marcelo S.Berman\textsuperscript{(1)}

and Luis A.Trevisan\textsuperscript{(2)}

\textsuperscript{(1)} Tecpar-Grupo de Projetos Especiais. R.Prof Algacir M. Mader 3775-CEP 81350-010 Curitiba-PR-Brazil
Email: marsambe@tecpar.br

\textsuperscript{(2)} Universidade Estadual de Ponta Grossa, Demat, CEP 84010-330, Ponta Grossa,Pr, Brazil email: latrevis@uepg.br

November 12, 2018

Abstract

Due to a previous result \cite{3}, there is a possibility of exponential inflationary phase in Brans-Dicke theory, even for an equation of state $p = \alpha \rho$ with $\alpha > 0$. In this case, we show that the perturbation in the metric would lead to an enormous amplification of gravitational waves that could detectable, making it possible to discard General Relativity theory in favour of Brans-Dicke theory.

PACS 98.80 Hw
AMPLIFICATION OF GRAVITATIONAL WAVES DURING INFLATION IN BRANS-DICKE THEORY. (Revised Version) MARCELO S. BERMAN and LUIS A. TREVISAN.

1 Introduction

One of the possibilities for inflationary models is the exponential scale-factor [1][2][4]. In such case, Einstein’s theory of General Relativity yields the equation of state:

\[ p = -\rho \]

where \( p \), \( \rho \) stand for pressure and energy density. Berman and Som [9] showed that under this equation of state, all the conformally flat space-times render a constant expansion rate in G.R., but the literature on Brans-Dicke [11] cosmology shows the possibility for other equations of state[3], like \( p = \alpha \rho \) with \( \alpha > 0 \). Being a viable alternative, B.D. gravitational theory must be taken into account when one delves into constant expansion rates.

“Extended ” inflation, put forward by La and Steinhardt,[10], with support of a paper by Barrow and Maeda [12], is one of such attempts. It seems to us that the current experimental data are not capable of outruling B.D. theory ( or its generalization involving a variable coupling “constant ”).

On the other hand, the study of gravitational waves production in the expanding Universe has been a subject of major research projects, either on theoretical or in the experimental fields. We shall show that, in contrast with G.R., we would have an enormous amplification of gravitational waves in the inflationary phase with \( p = \alpha \rho \) and \( \alpha > 0 \), in Brans-Dicke theory.

2 Derivation of Amplification for G.W ’s.

Barrow et al. [1] obtained the gravitational waves equation for a flat Robertson-Walker’s metric, in scalar tensor theories of gravity, as considered by Barrow [2]. Some particular cases were solved [1], in the conformal time \( \eta \) system, defined by

\[ dt = ad\eta \]  \hspace{1cm} (1)
where \( a = a(t) \) stands for the scale-factor in the metric:

\[
ds^2 = dt^2 - a^2(t) \left[ dx^2 + dy^2 + dz^2 \right]
\]

(2)

For an account of Brans-Dicke cosmologies, we refer to Weinberg [8].

The amplification in Brans-Dicke theory was derived by Barrow et al. [11], and is essentially given by the \( Y_k(t) \), with the equation:

\[
\ddot{Y}_k + \left[ \frac{\dot{\phi}}{\phi} - \frac{\dot{a}}{a} \right] \dot{Y}_k + \left[ \frac{k^2}{a^2} - \frac{2\dot{a}}{a} - 2\frac{\dot{a}\dot{\phi}}{a\phi} \right] Y_k = 0
\]

(3)

where \( k \equiv |\vec{k}| \), the comoving wave vector,

\[
k = \frac{2\pi a}{\lambda}
\]

(4)

and \( Y_k \) is the amplitude. The perturbation in the metric is given by:

\[
h_k = \frac{Y_k}{a^2}
\]

(5)

For a B.D inflationary phase of the exponential type, given by:

\[
a = a_0 e^{Ht}
\]

(6)

\((a_0, H \text{ constants})\), we find,

\[
\phi = \phi_0 e^{\beta t}
\]

(7)

\((\beta = \text{const})\).

\[
\rho = \rho_0 e^{\gamma t}
\]

(8)

\((\gamma, \rho_0 \text{ constants})\), and,

\[
p = \alpha \rho
\]

(9)

\((\alpha = \text{const})\).

Relation (9) makes for a perfect gas equation of state, while the relations between the constants of the theory were found by Berman and Som [3]. In particular,
\[ \gamma = -3H(1 + \alpha) = \beta \]  

(10)

When we plug (6), (7), (8) and (9) into (3), we find an exponentially increasing solution for \( h_k \), so that there is indeed amplification of gravitational waves, in Brans-Dicke theory, for such a phase, but only for the case \( \alpha > 0 \). This can be seen from the equation for \( Y_k \), which is,

\[ \ddot{Y}_k - H [4 + 3\alpha] \dot{Y}_k + \left\{ 2H^2 [2 + 3\alpha] + \frac{k^2}{a_0^2} e^{-2Ht} \right\} Y_k = 0. \]  

(11)

When we neglect the term in \( e^{-2Ht} \), we find,

\[ Y_k = Be^{\delta t} \]  

(12)

where \( B, C \) and \( \delta \) are constants, as a solution where \( \delta = \delta_1 = 2H \) or \( \delta = \delta_2 = H(2 + 3\alpha) \). The perturbation in the metric is given by a constant \( h_k \) for the first solution. In the second case we find

\[ h_k = Ce^{3\alpha Ht} \]  

(13)

so that, for \( \alpha > 0 \) an extraordinary amount of amplification is possible. The condition for \( \alpha \) results from requirement that \( h_k \) increase with time in expression (13).

Incidentally, Berman and Som[3] found the condition to be obeyed by \( H \) :

\[ H^2 = \frac{8\pi \rho_0}{3\phi_0 \left[ -2 - 3\alpha - \frac{3}{2}(1 + \alpha)^2 \right]} > 0 \]  

(14)

In reference [3], Berman and Som also found

\[ \omega = \frac{4}{(1 + \alpha)} \left[ \frac{9\alpha^2 + 12\alpha + 1}{9\alpha^2 - 30\alpha - 3} \right] \]  

(15)

with \( \alpha \neq -1 \).

The case \( \alpha = -1 \) recovers G.R, because \( \omega \to \infty \). The relation between \( G \) (Newton’s gravitational constant) and \( \phi \), is [8]:

\[ G = \left[ \frac{2\omega + 4}{2\omega + 3} \right] \phi^{-1} > 0 \]  

(16)
3 Conclusion

In conclusion, we have shown, by considering the inflationary (exponential) phase, in B.D theory, that amplification of gravitational waves is possible, with an enormous output in the $\alpha > 0$ case. This was a result obtained during the inflationary phase, and it does not need any phase transition consideration. In contrast, the corresponding equation in G.R, obtained by Grischuk (5),(6),(7), presents no such large amplification, for exponential scale-factor, when we focus only on such phase, because it is found $h_k = \text{const}$. It turns out that the gravitational waves originated in such period can be a decisive test for B.D. theory in particular, and to the existence of an inflationary period in the past history of the Universe. It also may be a test for the the homogeneous and isotropic metric that represents, according to our present model, the Universe, provided that we try to measure the same properties (homogeneity and isotropy) in the spectrum of such gravitational waves. The detection of such g.w’s could allow us to outrule General Relativity theory in favour of other scalar-tensor theory.

Acknowledgments

Both authors thank support by Prof Ramiro Wahrhaftig, Secretary of Science, Technology and Higher Education of the State of Paraná, and by our Institutions, especially to Jorge L.Valgas, Roberto Merhy, Mauro K. Nagashima, Carlos Fior, C.R. Kloss, J.L.Buso, and Roberto Almeida. The report of an anonymous referee saved us from a ”faux pas”, so that to him we extend our thanks.

References

[1] Barrow, J.D; Mimoso,J.P; Maia, M.R.G–Phys.Rev.D 48,3630, (1993).
[2] Barrow,J.D, –Phys.Rev.D 47, 5329, (1993)
[3] Berman,M.S;Som,M.M–Phys.Letters A 136, 206, (1989)
[4] For the literature on inflation, see, for instance, Linde, A. “Particle Physics and Inflationary Cosmology”, Harwood academic publ, N.Y, (1990).
[5] Grischuk, L.P–Sov. Phys. JETP 40, 409 (1975)
[6] Grischuk, L.P–Lett. Nuovo Cimento 12, 60, 9 (1975)
[7] Grischuk, L.P–Ann. N.Y Acad. Sci 302, 439, (1977).
[8] Weinberg, S. “Gravitation and Cosmology”, Wiley, N.Y, (1972)
[9] Berman, M.S; Som, M.M–Progress Theor. Phys, 81, 823, (1989).
[10] La, D; Steinhardt, P.J–Phys. Rev. Lett 62, 376, (1989).
[11] Brans, C.; Dicke, R.H.–Phys. Rev. 124, 925 (1961)
[12] Barrow, J.D.; Maeda, K.–Nuclear Physics B 341, 294 (1990).