1. Introduction

The FMEA (Failure mode and effect analysis) is extensively used as a powerful tool for system safety and reliability analysis of products and processes in a wide range of industries, including the aerospace, nuclear, automotive, electronics and medical industries. In conventional FMEA, the risk of a failure mode or cause is evaluated with RPN (Risk priority number), which is the mathematical product of its occurrence, severity and detection. Each value of the three components is usually determined as to the guidelines of FMEA handbook or similar documents. Occurrence and detection basically include the probability concepts in their nature, but severity is merely the relative rank associated with the most serious effect of a failure within the scope of the individual FMEA.

In traditional FMEA, the 1 to 10 scale for the three risk components depends largely on the past experience and intuition of the FMEA team. And its application has a number of limitations to be more carefully addressed for further improvement. Some important drawbacks are; (i) the subjective and inconsistent process of evaluating RPN, (ii) unreasonable allocation of the same weight on its three risk components, (iii) totally different contexts of risk possible with the same value of RPN, (iv) and dependence on intuition and experience rather than scientific method for estimating three risk components. Many authors have attempted to improve the RPN of FMEA. Abdelgawad and
Fayek4) used a combination of fuzzy FMEA and fuzzy AHP (Analytic hierarchy process) for risk management in the construction industries. Kumru and Kumru5) directly evaluate the linguistic assessment of factors based on fuzzy theory to obtain the RPN value. Liu et al.6) employed an intuitionistic fuzzy hybrid weighted Euclidean distance operator to overcome the limitations of traditional FMEA. Liu et al.7) proposed a risk priority model based on fuzzy set theory, treating the risk factors and their weights as fuzzy variables. And Liu et al.8) proposed a risk priority model based on the grey relational projection method.

There are few, if any, studies that consider the role of time in FMEA. Kwon et al.9) constructed a time dependent model with a quadratic loss function for the unfulfilled mission period, presenting an optimal monitoring policy. Kwon et al.3) proposed a time dependent expected loss model, assuming a homogeneous Poisson process for occurrence of failures and causes. Rhee and Ishii10) introduced a life cost-based FMEA for analyzing design alternatives of a particular system with Monte Carlo simulation.

In this paper, we consider the time sequence of failure occurrence, i.e. cause - failure - effect. It is quite natural to assume that a failure occurs after at least one of its causes has occurred. And also, it may take time to detect the failure after its occurrence. We consider a situation where the severity of a failure effect depends on the length of undetected time duration of failure. Generally, the detection of a failure cause prior to the actual failure will only require a constant cost of fixing the cause. If the failure is detected after its actual occurrence, then the loss may become significantly larger as the undetected time duration of the failure gets longer. We consider the linear and quadratic severity functions of undetected time duration. Assuming exponential probability law for failure occurrence and detection times, we suggest a risk evaluation model for improving the conventional FMEA.

2. Failure Risk Components

2.1 Time delay between occurrence and detection

In the conventional FMEA, RPN is obtained by

\[
RPN = O \times S \times D
\]

(1)

where O, S and D are the numbers corresponding to occurrence of a failure cause, severity of the failure effect and detection of the failure or its cause, respectively. In general, the three components are scored by experts using a numeric scale from 1 to 10, which is based on commonly agreed evaluation criteria1). A failure mode with a larger RPN carries a greater risk, and hence, special consideration should be made for such mode. Since the evaluation of the three components depends on subjectivity of the FMEA team, the RPN is quantitative but hardly objective. This is especially true of severity, as it is naturally linked with the size of a failure effect, unlike the probability-oriented components of occurrence and detection.

In FMEA, severity has quite a different meaning in the estimation of risk than those of the other two components. The numbers for ranking occurrence and detection are closely related with probability, whereas the number for ranking severity is far from probability concept11). Severity is merely a relative rank, being determined by individual FMEA teams. FMEA teams reach a consensus on severity ratings using the severity rating table2). This means that severity ratings might differ, depending on the team to carry out the assessment in the conventional FMEA. To complement these shortcomings, the process of failure occurrence and detection is first examined.

Suppose that an item is exposed to random events leading to a failure and any failure occurs after occurrence of its corresponding cause. Then the occurrence process of failure mode \( F_i \), \( i = 1, 2, \ldots, m \) can be described by the elapsed time \( T_{ij} \), \( j = 1, 2, \ldots, n_i \) from the occurrence of the \( j^{th} \) cause to the actual failure occurrence. There may also be a time delay \( D_{ij} \) from the occurrence of the \( j^{th} \) cause of failure mode \( F_i \) until its detection. This situation is depicted in Fig. 1. Thus, when an item is exposed to random events leading
to failure, a stochastic model will be appropriate to describe
the failure occurrence, detection and its severity.

2.2 Occurrence and detection of a failure cause
Suppose that the events leading to failure occur as to a
HPP (homogeneous Poisson process) with a constant rate.
Then the occurrence rate of the \( j \)th cause of failure mode
\( F_i \) can be described by \( \tau_{ij} \).

With most systems, occurrence of failure cause is not
usually detected instantaneously. There usually be a time
gap between occurrence and detection of a failure cause.
Let \( \tau_{ij} \) be the elapsed time from the occurrence of the
\( j \)th cause of failure mode \( \alpha_i \) to its detection. Under HPP
assumption with a constant detection rate \( \mu_{ij} \) over time,
the probability density function of \( \tau_{ij} \) will be

\[
f_{\tau_{ij}}(d) = \mu_{ij} e^{-\mu_{ij}d}, \quad d > 0. \tag{2}
\]

2.3 Occurrence of a failure
Let \( T_{ij} \) be the time elapsed from the occurrence of the
\( j \)th cause to the occurrence of failure mode \( F_i \). Under
HPP assumption with constant occurrence rate \( \lambda_{ij} \), \( T_{ij} \)
will be exponentially distributed with mean \( 1/\lambda_{ij} \) and its
probability density function will be

\[
f_{T_{ij}}(t) = \lambda_{ij} e^{-\lambda_{ij}t}, \quad t > 0. \tag{3}
\]

2.4 Severity
Note that the failure cause may be detected before or
after its corresponding failure actually occurs, as
described by case A and B of Fig. 1. Under case A
where the a failure cause is detected prior to the actual
failure occurrence, the severity of its effect will be small.
The only cost incurred in this case will be attributable to corrective action for the failure cause and will not be so
much dependent on the elapsed time to the cause detection. Under case B where a failure is detected after
its occurrence, the losses due to the failure may be accumulated until the corrective action is taken.

Assuming that an immediate corrective action is taken
on detection of failure cause, the severity of the failure effect will naturally depend on the elapsed time to
detection of the corresponding failure cause. Thus, the
severity of each failure mode may reasonably be estimated based on the length of undetected time duration
after its occurrence.

Considering two possible situations in Fig. 1, it is
reasonable to assume that the severity of a failure effect is constant when a cause is detected before its corresponding failure occurs. Let \( a_i \) be the constant severity coefficient
due to the \( j \)th cause of failure mode \( F_i \).

When a failure is detected after its occurrence, its
severity will depend on the time difference of \( |D_{ij} - T_{ij}| \).
Let \( S_{ij} \) denote the severity for the effect of failure mode
\( F_i \) in relation with its \( j \)th cause. \( S_{ij} \) will reasonably be a
non-decreasing function of \( |D_{ij} - T_{ij}| \). We consider the linear and quadratic functions of \( |D_{ij} - T_{ij}| \) for \( S_{ij} \).

For linear type, \( S_{ij} \) will be proportional to \( |D_{ij} - T_{ij}| \) with
constant severity coefficient \( b_j \). Thus, \( S_{ij} \) is defined by

\[
S_{ij} = \begin{cases} a_i, & 0 < D_{ij} \leq T_{ij} \\ a_i + b_j(D_{ij} - T_{ij}), & T_{ij} < D_{ij} \end{cases}
\]

(4)

When the risk increases dramatically as time goes by
with a failure left uncorrected, a quadratic \( S_{ij} \) will be
more appropriate. For quadratic type, \( S_{ij} \) is defined by

\[
S_{ij} = \begin{cases} a_j, & 0 < D_{ij} \leq T_{ij} \\ a_i + b_j(D_{ij} - T_{ij})^2, & T_{ij} < D_{ij} \end{cases}
\]

(5)

3. Risk evaluation
3.1 The expected failure severity
For linear \( S_{ij} \), its expected value was obtained by
using Equation (2), (3) and (4) as follows:

\[
E[S_{ij}] = a_i \int_0^{D_{ij}} \int_0^{T_{ij}} f_{D_{ij}}(d) \, dt \, dd + a_j \int_0^{T_{ij}} \int_0^{T_{ij}} f_{D_{ij}}(d) \, dt \, dd + b_j \int_0^{T_{ij}} \int_0^{T_{ij}} (D_{ij} - t_j) f_{D_{ij}}(d) \, dt \, dd
\]

\[
= a_i + b_j \left( \frac{\lambda_{ij}}{\mu_{ij} + \lambda_{ij}} \right).
\]

(6)

See Appendix I for detailed derivation.
For quadratic \( S_{ij} \), its expected value was obtained by
using Equation (2), (3) and (5) as follows:

\[ E[S_{ij}] = a_i \int_0^\infty \int_0^d f_{F_i}(t) f_{D_j}(d) \, dt \, dd \]
\[ + b_i \int_0^\infty \int_0^d (d_{ij} - t_i)^j f_{F_i}(t) f_{D_j}(d) \, dt \, dd \]
\[ = a_i + b_i \left( \frac{2\lambda_{ij}}{\mu^2_{ij}(\mu_{ij} + \lambda_{ij})} \right). \]  

(7)

See Appendix II for detailed derivation.

3.2 The risk evaluation metric

Since FMEA is a prevention-oriented technique, the risk linked with each failure cause is necessary to be evaluated. We incorporate the occurrence rate of each failure cause with the expected severity of the corresponding failure into one risk metric, i.e., the REM (Risk evaluation metric). The REM of to the \( j^{th} \) cause of failure mode \( F_i \) is simply defined by the mathematical product of the failure cause occurrence rate \( \tau_{ij} \) and \( E[S_{ij}] \).

\[ REM_{ij} = \tau_{ij} E[S_{ij}]. \]  

(8)

Thus, \( REM_{ij} \) is obtained by

\[ REM_{ij} = \tau_{ij} \left[ a_i + b_i \left( \frac{\lambda_{ij}}{\mu_{ij}(\mu_{ij} + \lambda_{ij})} \right) \right] \]  

for linear \( S_{ij} \),

\[ REM_{ij} = \tau_{ij} \left[ a_i + b_i \left( \frac{2\lambda_{ij}}{\mu^2_{ij}(\mu_{ij} + \lambda_{ij})} \right) \right] \]  

for quadratic \( S_{ij} \).  

(9)

(10)

Note that \( a_{ij} \) reflects the cost of correcting the \( j^{th} \) failure cause of failure mode \( F_i \), while \( b_i \) is a constant coefficient reflecting the failure effect including the cost for repairing failure mode \( F_i \).

4. Numerical example

4.1 An example

An example of a dual clutch transmission is excerpted from Zhang et al.\(^{12}\) to demonstrate application of the proposed FMEA model. Some numerical figures are supplemented or slightly changed as necessary to fit our models. Table 2 shows a part of conventional FMEA sheet.

The failure mode ‘clutch disengaging’ may result in ‘shifting failure’ or ‘loss of acceleration’, i.e., reduced level of performance. Thus, severity 7 is assigned to this failure mode according to the conventional FMEA guide. Since the failure mode ‘incorrect gear position’ may result in ‘damage of gear box’ or ‘possible collision’ which affects safe vehicle operation, its severity is ranked as 9. Other numbers for occurrence and detection are given in Table 2.

Table 2 uses RPN for evaluating the risk linked with each failure cause according to the conventional FMEA. Based on this table, the modified FMEA sheet is constructed to demonstrate a practical application of REM. Numerical figures are assigned for \( \lambda_{ij}, b_i, \lambda_{ij}, \mu_{ij} \) and \( \tau_{ij} \) considering those numbers given in Table 2.

Table 2. An illustrative example

| Failure Mode | Potential Effects | S | Failure Causes | O | D | RPN | Rank |
|--------------|------------------|---|----------------|---|---|-----|------|
| clutch dis-engageing | shifting failure \ loss of acceleration | 7 | dual clutch mechatronic assembly fault | 4 | 3 | 84 | 5 |
| incorrect gear position | damage of gearbox \ possible collision | 9 | sensor fault | 3 | 5 | 135 | 2 |
| | ECU fault | 2 | 5 | 90 | 4 |
| | actuator fault | 4 | 3 | 108 | 3 |

The severity of failure modes, 7 and 9 were used as the severity coefficient \( b_i \). The occurrence ranking of each failure cause is converted into the occurrence rate \( \tau_{ij} \) with reference to the FMEA handbook. For example,
Table 3. The corresponding numerical figures for the proposed model

| Failure mode       | Failure effect | Failure cause                  | Occurrence rate ($\lambda_{ij}$) | Severity coefficient ($a_{ij}$) | Failure rate ($\mu_{ij}$) | Occurrence rate ($\tau_{ij}$) |
|--------------------|----------------|-------------------------------|----------------------------------|--------------------------------|--------------------------|-----------------------------|
| clutch disengaging | 7             | dual clutch mechatronic assembly fault | 0.01                             | 1                              | 0.02                       | 0.001                        |
| incorrect gear position | 9         | sensor fault                   | 0.003                            | 1                              | 0.005                      | 0.0005                       |
|                    | 9             | ECU fault                      | 0.001                            | 1                              | 0.005                      | 0.0001                       |
|                    | 9             | actuator fault                 | 0.0001                           | 1                              | 0.02                       | 0.001                        |

4.2 Risk evaluation

To illustrate the effectiveness of the proposed FMEA approach, we used the above example to analyze and compare with the conventional FMEA. Table 4 provides the REM value for each failure cause. Comparing it with Table 2, we can see that the failure cause “shift mechatronic assembly fault” is ranked as 1 consistently with both the conventional FMEA and the proposed model. But this is not always the case. For example, the failure cause “dual clutch mechatronic assembly fault” is ranked as 3 for both linear and quadratic severity functions in Table 4, while ranked as 5 in Table 2. Under such an inconsistent situation, REM should be considered to be more important. In fact, the RPN value of the conventional FMEA has no practical meaning except priority order, which may also be doubtful. For example, the RPN difference between the two failure causes ‘dual clutch mechatronic assembly fault’ and ‘ECU fault’ is 90 - 84 = 6, while the difference between ‘ECU fault’ and ‘actuator fault’ is 108 - 90 = 18. But it does not mean the difference 18 is three times as large as 6 from the aspects of risk. On the other hand, with quadratic severity function, the corresponding REM differences 11.67 - 1.10 = 10.57 and 1.10 - 0.02 = 1.08 have practical meaning, that is, the risk difference 10.57 is about ten times as large as 1.08. Thus, the FMEA team will have more specific and realistic risk information with REM than the traditional method. The proposed FMEA model may also lead the FMEA team to anticipate the severity of the risk if sufficient data is available. In addition, the proposed model enables the FMEA team to analyze various situations by assigning different values to the cost or distribution parameters.

4.3 The sensitivity of REM to the distribution parameters

REM of every failure cause may be examined on sensitivity. The failure cause ‘dual clutch mechatronic assembly fault’ will be studied here since the patterns of variation in REM of the others will be similar. Sensitivity of REM may be analyzed for any of $a_{ij}$, $b_i$, $\tau_{ij}$, $\lambda_{ij}$ and $\mu_{ij}$. The effect of variation in $a_{ij}$ or $b_i$ or $\tau_{ij}$ can easily be predicted if the other parameters are fixed. Sensitivity analysis of REM is carried out only for $\lambda_{ij}$ and $\mu_{ij}$, which are the distribution parameters mostly affecting the severity of the failure cause. The sensitivity of REM is analyzed for each linear and quadratic severity function, assuming that the severity coefficients $a_{ij}$ and $b_i$ are fixed as 1 and 7, respectively, and that the occurrence rate $\tau_{ij}$ of failure cause was fixed as 0.1.

The REM values for linear and quadratic severity functions with different values of $\lambda_{ij}$ and $\mu_{ij}$ are shown in Fig. 2 and Fig. 3, respectively. The results show that; (i) REM with the quadratic severity function is more sensitive than REM with the linear severity function, (ii) smaller values of $\mu_{ij}$ make REM more sensitive, and (iii) if $\lambda_{ij}$ takes extremely small or large values, REM is relatively insensitive to $\lambda_{ij}$.

Fig. 2 and Fig. 3 show that REM has its maximum value when $\mu_{ij}$ and $\lambda_{ij}$ are 0.005 and 0.5 respectively. It simply means that the risk is maximized with a potential failure
cause which is hardly detectable but highly possible to incur a failure. There are big differences of REM values with $\lambda_j = 0.001, 0.01$, and $0.1$, while showing small differences with $\lambda_j = 0.1$ and $0.5$ or $\lambda_j = 0.0001$ and $0.001$. This implies that REM does not show much difference when the possibility of a failure is very high or very low. It means that (i) every cause that incurs a failure with a high possibility should be taken care of and (ii) on the other hand, a cause that hardly incurs a failure need not to be addressed so much. For the failure causes with medium possibility of incurring actual failures, the priority of corrective actions should be carefully determined.

5. Conclusion

A new risk metric REM is developed to compensate the shortcomings of RPN of the conventional FMEA. Assuming a homogeneous Poisson process for occurrence and detection of failure and its cause, linear and quadratic functions of the undetected failure time duration are used for evaluating severity. The REM of each failure cause is obtained by multiplying its occurrence rate with the expected severity of its corresponding failure mode.

Risk evaluation based on the REM provides a more meaningful metric than the conventional RPN. REM represents the size of risk induced by a failure cause, while RPN only represents the risk priority. That is, REM provides a meaningful quantitative measure, while RPN provides only comparative priority information with rarely practical meaning with its numerical value. Thus, REM not only provides priority information of potential failure cause to be corrected but also enables to evaluate effectiveness of corrective actions quantitatively.

Numerical analyses show that (i) REM provides the risk priority information that may be different from that of the conventional RPN, (ii) the difference in values of REM reflects the difference in size of practical risks, (iii) smaller detection rate makes REM more sensitive to failure occurrence rate, (iv) the priority of corrective actions should be rightly determined on the cause with medium possibility of incurring actual failures, while causes rarely incurring failure need not to be so carefully addressed. These results are consistent with our intuition.

This study assumes exponential probability models and independency of occurrence and detection times of a failure and its cause for analytic simplicity. The real situation, however, may be more complicated. Further studies are expected for the cases where (i) the time variable is not exponentially distributed, (ii) occurrence of a failure affects detection of its cause, (iii) some causes are interrelated and dependent on each other, and (iv) there exists a hierarchical relationship among some failure causes. In fact, these are only some examples of possible situations and there may be many others to be covered in the future study.

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Appendix

I. Proof of Equation (6) related with $b_i$

$$
\int_0^{\infty} \int_0^{\infty} \left( (d-t) f_{\tilde{D}_u}(d) f_{\tilde{F}_v}(t) \right) dt \, dd \\
= \int_0^{\infty} \left[ \int_0^{\infty} (d-t) \mu_i e^{-\mu \rho \lambda} e^{-\lambda d} dt \right] \, dd \\
= \int_0^{\infty} \left[ \mu_i \int_0^{\infty} e^{-\mu \rho \lambda} e^{-\lambda d} dt - \mu_i \int_0^{\infty} e^{-\mu \rho \lambda} e^{-\lambda d} dt \right] \, dd \\
= \mu_i \int_0^{\infty} e^{-\mu \rho \lambda} dt + \frac{\mu_i}{\lambda_j} \int_0^{\infty} e^{-(\mu_j + \lambda_j) d} \, dd - \frac{\mu_i}{\lambda_j} \int_0^{\infty} e^{-\rho \lambda \lambda_j} dt \\
= \frac{\mu_i}{\mu_i + \lambda_j}
$$

II. Proof of Equation (7) related with $b_i$

$$
\int_0^{\infty} \int_0^{\infty} \left( (d-t) f_{\tilde{D}_u}(d) f_{\tilde{F}_v}(t) \right) dt \, dd \\
= \int_0^{\infty} \left[ \int_0^{\infty} (d-t) \mu_i e^{-\mu \rho \lambda} e^{-\lambda d} dt \right] \, dd \\
= \int_0^{\infty} \left[ \left( \mu_i \int_0^{\infty} e^{-\mu \rho \lambda} e^{-\lambda d} dt \right) - 2 \mu_i \int_0^{\infty} e^{-\mu \rho \lambda} e^{-\lambda d} dt \right] \, dd \\
= \mu_i \int_0^{\infty} e^{-\mu \rho \lambda} dt + \frac{\mu_i}{\lambda_j} \int_0^{\infty} e^{-(\mu_j + \lambda_j) d} \, dd - \frac{2 \mu_i}{\lambda_j} \int_0^{\infty} e^{-\rho \lambda \lambda_j} dt \\
= \frac{2 \mu_i}{\lambda_j} \int_0^{\infty} e^{-\mu \rho \lambda} dt - \frac{2 \mu_i}{\lambda_j} \int_0^{\infty} e^{-(\mu_j + \lambda_j) d} \, dd + \frac{2 \mu_i}{\lambda_j} \int_0^{\infty} e^{-\rho \lambda \lambda_j} dt \\
= \frac{2 \mu_i}{\lambda_j} \int_0^{\infty} e^{-\mu \rho \lambda} dt
$$

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