Periodic stability assessment of a flexible hub connection for load reduction on two-bladed wind turbines

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Abstract. The periodic stability of an innovative load reduction system for two-bladed wind turbines is investigated using Floquet theory. The load reduction system introduces an additional cardanic degree of freedom between the hub mount and the nacelle carrier flange. A reduced rotor model is considered to analyze the time-variant behavior as a function of rotor shapes and design parameters. Aerodynamic excitation of the rotor is neglected in this study. The linearized system equations of motion of this system indicate a dependance of the rotor shape and the periodic system stability. Due to the time-variance of the asymmetric rotor the Floquet multipliers are derived to determine system stability and the dominant periodic coefficients. It can be shown that the rotor shape has a significant impact on the stability of gimbal-hinged rotors. Gyroscopic effects are the reason for a stabilization of disc-like rotors, whereas a cylindrical, two-bladed rotor assembly is in general unstable. The introduction of an additional spring-damper coupling is a possibility to stabilize such configurations, but results show that resonance phenomena have to be accounted for.

1. Introduction
Rapid offshore wind energy development opens the opportunity for unusual, yet cost-effective wind turbine concepts. Among others the two-bladed, horizontal axis wind turbine is a potential solution to reduce operational and initial expenditure by both reducing installation and maintenance cost. However, dynamic loading remains a challenge due to time-variant system properties and inhomogeneous aerodynamic loading. The flexible hub connection is a possible concept for load alleviation in two-bladed wind turbines. Conceived for the Skywind 3.4 MW prototype, this system consists of an elastic interface between the nacelle carrier and the hub mount, which carries the drive train, the generator and the hub (see Fig. 1). Slight excursions of the hub mount mitigate asymmetric rotor loading induced by non-uniform inflow. Previous numerical studies have indicated a significant load reduction potential in a transient inflow environment [1], which is achieved by a combination of reduced structural transmittance and an alleviation of aerodynamic imbalances. Yet, by introducing an additional degree of freedom into the structure the stability of the system is no longer guaranteed throughout the operational range. According to [2] wind turbine instabilities can arise from resonance phenomena, aeroelastic blade interaction (e.g. flutter), tower-rotor coupling (e.g. whirling) and combinations of the latter. Due to the time-variant system properties of a two-bladed rotor, explicit Floquet analysis can be applied to determine the stability of simple linear, time-periodic systems. To
handle more complex models implicit Floquet analysis can be used to identify the dominant eigenvalues of the periodic system at a reduced computational cost [3].

Wind turbine concepts including additional degrees of freedom, like teeter, flap and lead-lag hinges, were investigated using the Floquet approach [4, 5, 6, 7]. Simplified models and limited physical detail were generally used. For these concepts centrifugal forces play a major role in stabilizing the uncoupled system. As the kinematic behavior of the flexible hub connection differs from those concepts, the contribution of circulatory forces to the rotor dynamics will be distinguished by the derivation of the homogeneous equations of motion. It is found that an increasing rotor asymmetry implies reduced periodic stability, which needs to be accounted for by the introduction of stabilizing spring-damper elements.

This study serves to identify potential periodic instabilities of the time-variant system by using simplified rotor models in combination with Floquet theory. The focus is to determine the stability zones associated to both symmetrical and asymmetrical rotor shapes in order to derive design recommendations for the coupled system.

2. Floquet Theory

The theory of the Floquet approach is briefly outlined here. Its formulation and application to rotor problems was performed in and [7] and [8]. Considering a linear, time-variant system with periodic coefficients an N degrees of freedom, the first order state-space formulation reads as

\[ \dot{x} = A(t)x \]  

where \( x = [x^T \dot{x}^T]^T \) is the state vector of length \( 2N \) and \( A \) is the linear, time-varying system matrix with periodic coefficients and period \( T \), such that \( A(t) = A(t + T) \). The solution of the system can be expressed as a function of the initial conditions by introducing the state-transition matrix \( \Phi(t, 0) \).

\[ x(t) = \Phi(t, 0)x(0) \]  

Floquet theory assumes that the transition matrix \( \Phi(t, 0) \) can be decomposed into a periodic matrix \( P(t) = P(t + T) \) and the matrix exponential of a constant matrix \( C \)

\[ \Phi(t, 0) = P(t)e^{Ct} \]  

The constant matrix \( C \) is called the monodromy matrix. It describes the logarithmic decrement of the state transition matrix per period.

\[ C = \frac{1}{T} \ln(\Phi(T, 0)) \]
The stability of the periodic system is determined by the eigenvalues of the monodromy matrix $\lambda_i$, which are called the characteristic exponents. To calculate the eigenvectors the state transition matrix after one period $\Phi(T,0)$ will be derived by integrating eq. 5 with a a set of $2N$ linearly independent initial conditions $\Phi(0,0) = I$.

$$\dot{\Phi}(t,0) = A(t)\Phi(t,0)$$

The eigenvalues of $\Phi(T,0)$ are called the characteristic multipliers $\sigma_i$. They yield the frequency and damping of the periodic modes associated with the characteristic exponents $\lambda_i = \zeta_i + j\omega_i$. The eigenvalue problem is formulated as

$$\Phi(T,0) = V\Sigma V^{-1}$$

where the diagonal matrix $\Sigma$ contains the eigenvalues $\sigma_i$ and $V$ the associated periodic eigenvectors respectively.

The damping $\zeta_i$ and frequency $\omega_i$ can be calculated from $\sigma_i$.

$$\zeta_i = \frac{1}{T}ln|\sigma_i|$$

$$\omega_i = \frac{1}{T}(\text{Arg}\{\sigma_i\} + 2\pi k)$$

As the argument $\text{Arg}\{\sigma_i\}$ is multivalued, the factor $k$ appears in the frequency solution. It can be identified by system response analysis. The periodic system is stable if $\lambda_i < 0$ for all modes, which implies $|\sigma_i| < 1$.

The approach requires a number of $2N$ different time integrations of one period. The calculation cost therefore becomes increasingly prohibitive for an increasing number of degrees of freedom. Reduced models are therefore recommended to capture the principal effects of time periodic systems.

### 3. Equations of motion

In this section the equations of motion of a simplified gimbal-supported rotor will be derived based on the Euler’s equation describing the rotation of a rigid body. The resulting equations allow one to estimate the impact of gyroscopic forces and precession due to external loading on the dynamics of the rotor mounting. Differences between a beam-like rotor and a disc-like rotor, can thereby be explained. The equations have been developed to study whirling of unsymmetrical rotors for propeller applications by Bergmann [5] and Brosens and Crandall [9], using a different convention for the definition of the Cardan angles.

In the presented approach the velocity and acceleration components of a mass point on a gimbal-mounted body serve to formulate the rotational momentum balance. By integrating over the body volume the homogenous equations of motion are obtained. Linearization yields the time-dependent mass matrix and the gyroscopic matrix. In all derivations the complexity of the hub mount assembly is simplified to a single body, which avoids a more complicated multi-body formulation.

#### 3.1. Kinematics

For the derivation of equations of motion a coordinate system according to the Germanischer Lloyd standard is used. The main rotational axis is pointing in the $x^{(1)}$-direction. A body frame (1) is defined with three rotational degrees of freedom relative to the inertial frame (0), Fig. 2. The rotor, represented by an arbitrary symmetric body, is connected to the hub mount, which tumbles around a hinge point in the origin of the inertia frame (0). The hub mount is
considered as a large mass with low rotational inertia. The three degrees of freedom are denoted \( \alpha^{(0)} = (\Omega t, \beta_0, \gamma_0) \) and describe a succession of Cardan angle rotations. The rotor speed \( \Omega \) is constant. The rotational sequence is defined as \( y^{(0)} - z^{(\beta)} - x^{(1)} \), where the intermediate frames \( (\beta) \) and \( (\gamma) \) are introduced to describe the tilting and yawing motion of the hub mount.

\[ \begin{align*}
\mathbf{u}^{(0)} &= T \mathbf{u}^{(1)} = T_{x^{(1)}} T_{z^{(\beta)}} T_{y^{(0)}} \mathbf{u}^{(1)} \\
&= T_{x^{(1)}} T_{z^{(\beta)}} T_{y^{(0)}} \mathbf{v}^{(0)} \\
&= \frac{\partial}{\partial t} \left( T \mathbf{u}^{(1)} \right) \\
&= \mathbf{T} \frac{\partial}{\partial t} \mathbf{u}^{(1)} \quad \text{(8)}
\end{align*} \]

\[ \begin{align*}
T_{x^{(1)}} &= \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \Omega t & -\sin \Omega t \\
0 & \sin \Omega t & \cos \Omega t
\end{bmatrix} \\
T_{z^{(\beta)}} &= \begin{bmatrix}
\cos \beta_0 & -\sin \gamma_0 & 0 \\
\sin \gamma_0 & \cos \gamma_0 & 0 \\
0 & 0 & 1
\end{bmatrix} \\
T_{y^{(0)}} &= \begin{bmatrix}
\cos \beta_0 & 0 & -\sin \beta_0 \\
0 & 1 & 0 \\
\sin \beta_0 & 0 & \cos \beta_0
\end{bmatrix}
\end{align*} \quad \text{(9)}

By applying this particular rotational sequence the rotor axis \( x^{(1)} \) is enabled to tumble about the hinge point in the rotational degrees of freedom \( \beta_0 \) and \( \gamma_0 \) independently from the actual rotor azimuth position \( \Omega t \).

The time derivatives of the position vector are transformed into the body fixed frame:

\[ \begin{align*}
\mathbf{v}^{(1)} &= T^T \mathbf{v}^{(0)} = T_{x^{(1)}} T_{z^{(\beta)}} T_{y^{(0)}} \mathbf{v}^{(1)} \\
&= \frac{\partial}{\partial t} \left( T \mathbf{u}^{(1)} \right) \\
&= \mathbf{T} \mathbf{v}^{(1)} \\
&= \omega^{(1)} \mathbf{u}^{(1)} \quad \text{(10)}
\end{align*} \]

and

\[ \begin{align*}
\mathbf{a}^{(1)} &= T^T \mathbf{a}^{(0)} = T_{x^{(1)}} T_{z^{(\beta)}} T_{y^{(0)}} \mathbf{a}^{(1)} \\
&= \frac{\partial^2}{\partial t^2} \left( T \mathbf{u}^{(1)} \right) \\
&= \mathbf{T} \mathbf{a}^{(1)} \\
&= \omega^{(1)} \mathbf{u}^{(1)} + \mathbf{\omega}^{(1)} \mathbf{u}^{(1)} \quad \text{(11)}
\end{align*} \]
The matrix $\omega^{(1)}$ is the asymmetrically filled cross product operator of the rotational speed vector $\omega^{(1)}$. It is obtained from the matrix multiplication $T^T T$

$$\omega^{(1)} = T^T T = \begin{bmatrix} 0 & -\omega^{(1)}_x & \omega^{(1)}_y \\ -\omega^{(1)}_x & 0 & -\omega^{(1)}_z \\ -\omega^{(1)}_y & \omega^{(1)}_z & 0 \end{bmatrix}$$  \hspace{1cm} (12)

3.2. Momentum equations

By introducing the differential external force vector $d\mathbf{p}_e^{(1)}$ acting on the mass element $dm$ the rotational momentum balance can be formulated in the body fixed frame.

$$0 = \int_V \mathbf{u}^{(1)} \left( d\mathbf{p}_e^{(1)} - \mathbf{a}^{(1)} dm \right)$$  \hspace{1cm} (13)

The expression $\mathbf{u}^{(1)}$ describes the cross product operator of the mass point position vector $\mathbf{u}^{(1)}$.

The rotational equations of motion in the body fixed frame, eq. 14, and in the stationary frame, eq. 15, are obtained by defining the inertia tensor with respect to the hinge point $\Theta_0^{(1)} = \int_V \mathbf{u}^{(1)} \mathbf{u}^{(1)} dm$.

$$\mathbf{p}_M^{(1)} = \Theta_0^{(1)} \mathbf{a}^{(1)} + \omega^{(1)} \Theta_0^{(1)} \dot{\omega}^{(1)}$$  \hspace{1cm} (14)

$$\mathbf{p}_M^{(0)} = \Theta_0^{(0)} \mathbf{a}^{(0)} + \omega^{(0)} \Theta_0^{(0)} \dot{\omega}^{(0)}$$  \hspace{1cm} (15)

where $\mathbf{p}_M$ describes the external moments with respect to the hinge point and $\Theta_0^{(0)}(t) = T(t) \Theta_0^{(1)} T^T(t)$ is the time-variant inertia tensor with respect to frame (0). Due to geometrical symmetry the time-invariant inertia tensor $\Theta_0^{(1)}$ is assumed to be diagonal.

These equations can be written as a function of the set of inertial degrees of freedom $\alpha^{(0)} = (\Omega t, \beta_0, \gamma_0)$. The resulting expressions are lengthy. Assuming small cardanic joint angles $[\beta_0, \gamma_0]$ and constant rotor speed $\Omega$, a linearization leads to a matrix formulation in the two tumbling degrees of freedom, eq. 16. The resulting systems of equations can be separated in an acceleration proportional mass matrix and a velocity proportional gyroscopic matrix. No position proportional terms appear in the equations of motion.

$$\mathbf{p}_M^{(0)} = M_0(t) \ddot{\alpha}^{(0)} + G_0(t) \dot{\alpha}^{(0)}$$

$$= \frac{1}{2} \begin{bmatrix} \Theta_y + \Theta_z + (\Theta_y - \Theta_z) \cos 2\Omega t \\ (\Theta_y - \Theta_z) \sin 2\Omega t \\ \Theta_y + \Theta_z - (\Theta_y - \Theta_z) \cos 2\Omega t \end{bmatrix} \begin{bmatrix} \gamma_0 \\ \beta_0 \\ \beta_0 \end{bmatrix}$$

$$+ \Omega \begin{bmatrix} -\Theta_y + (\Theta_y - \Theta_z) \cos 2\Omega t \\ \Theta_x + (\Theta_y - \Theta_z) \cos 2\Omega t \\ -\Theta_x + (\Theta_y - \Theta_z) \cos 2\Omega t \end{bmatrix} \begin{bmatrix} \gamma_0 \\ \beta_0 \\ \beta_0 \end{bmatrix}$$  \hspace{1cm} (16)

3.3. Time variance

Due to the constantly changing alignment of the beam-like rotor, two-bladed wind turbines are known to possess time-variant system properties. The mass matrices and the gyroscopic matrices in the inertial frame in eq. 16 contain periodic time-variant elements in the form of linear sine
and cosine functions, which are each multiplied by the inertia difference $\Theta_y - \Theta_z$. Consequently, if this inertia difference disappears, the matrices become time-invariant. This is the case for disc-like rotors hinged in the rotor center ($\Theta_x \approx 2\Theta_y = 2\Theta_z$), but also for three-bladed rotors. Two-bladed rotors resemble a beam-like, cylindrical structure. The inertia about the cylinder axis is very small relative to the other two inertias ($\Theta_z \ll \Theta_x = \Theta_y$), so the time-dependance of the system matrices is maximal.

3.4. Gyroscopic precession

If the rotational axis of a rotating body is deflected due to an external torque, the momentum change causes a reaction moment perpendicular to the cause. The body experiences subsequently an angular acceleration perpendicular to the direction of the initial deflection, resulting in a tumbling motion of the body. If the inertia distribution is symmetric about the spin axis and the external torque direction constant in the body fixed coordinates, the trajectory of the spin axis will describe a cone. Such systems are periodically stable due to the perpendicular reaction to an external excitation.

A phase shift of $90^\circ$ is present between the time-variant mass matrix and the gyroscopic matrix, visible in the oppositional distribution of cosines and sines in the matrices. Once excited, a rotating, tumbling and undamped system will therefore not return to a steady state, because the reaction of the system is always perpendicular to the cause and no external moments act on the body. Although proportional to the velocity, the gyroscopic matrix does not introduce damping to the system. The kinetic energy of the homogeneous system does not decrease. Still, gyroscopic precession can cause an apparent stiffening of the rotor, which is known on fast spinning disc rotors. For disc-like rotors ($\Theta_y = \Theta_z$) the time-variance of the matrices disappears. In this case the system behaves independently from the current azimuth position. For inhomogeneous rotor shapes ($\Theta_y \neq \Theta_z$) the stability of the system is determined by the entries of the inertia tensor. Due to the periodic coefficient in the system matrices, Floquet theory will be applied to assess the periodic stability of the system.

3.5. Scaling

The rotor speed appears as a factor of the gyroscopic matrix, so the corresponding accelerations are scaled, too. As the period reduces with increasing rotor speed, the velocities observed at a given rotor azimuth angle are in fact the same independently from the rotor speed. Analogously, the angular displacements are inversely scaled with the rotor speed. In consequence an increased rotor speed leads to reduced angular displacements of the rotor mount, despite the absence of centrifugal forces. The formal stability of the system is unaffected by this effect.

4. Numerical study

4.1. Modeling

Floquet theory is only valid for linearized, time-variant systems with periodic coefficients. Therefore, the linearized equations of motion need to be considered, which can be solved by implicit or explicit time-marching integrators provided for example by Matlab. The variable step continuous explicit solver ode45 is used in this study. The chosen model only takes into account the two degrees of freedom considered in eq. 16. Aerodynamic and aero-elastic effects were neglected to assess the sole impact of rotor shape and hub mount coupling.

The different rotor shapes are characterized by the normalized inertia distributions. As mentioned earlier the homogeneous equations of motion can be scaled by the inertia tensor without changing the principal dynamic properties at the same rotor speed. The scaled inertia tensor of the reference case of the Skywind 3.4 MW prototype are given in eq. 17. In both expressions of the off-axis inertias $\Theta_y$ and $\Theta_z$ the additional mass of the hub mount increases the values slightly with respect to the main inertia $\Theta_x$.  

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In the following sections the behavior of the freely tumbling system will be investigated for general rotor shapes, followed by considerations of different coupling parameters. For this matter the equations of motions in the stationary frame are expanded by the homogeneous stiffness and damping matrices. The stiffness is $C_0 = c_0 I$ and the damping matrix is $D_0 = d_0 I$, where $c_0$ and $d_0$ are the uniform, rotational stiffness and damping connecting the hub mount and the nacelle carrier respectively.

$$\Theta_0^{(1)} = \begin{bmatrix}
\Theta_x \\
\Theta_y \\
\Theta_z
\end{bmatrix} = \begin{bmatrix}
\Theta_x \\
1.05 \cdot \Theta_x \\
0.07 \cdot \Theta_x
\end{bmatrix}$$

(17)

The rotational spring and damper parameters $c_0$ and $d_0$ will be represented by equivalent system eigenfrequencies $\omega_C$ and $\omega_D$ with respect to the main rotor inertia $\Theta_x$:

$$\omega_C^2 = \frac{c_0}{\Theta_x} \quad \text{and} \quad \omega_D^2 = \frac{d_0}{\Theta_x}$$

(19)

4.2. Homogeneous system

First the uncoupled system will be considered to analyze the impact of the individual entries of the inertia tensor in conjunction. Different relative distributions of the inertias represent possible distinguishable rotor shapes. In Fig. 3 the stability map of the homogeneous system for different combinations of $\Theta_y/\Theta_x$ and $\Theta_z/\Theta_x$ is shown. Light shaded areas represent an unstable operational regime. It has to be noted that not every inertia combination is representable by actual bodies, but are still included in this study to demonstrate the methodology. Due to the unidirectional joint the stability maps are symmetric. The unstable region, bordered by the dashed line, comprises the inertia combinations $\Theta_y < \Theta_x < \Theta_z$ and $\Theta_y > \Theta_x > \Theta_z$ respectively. In reverse this implies that all those configurations are stable for which the off-axis inertias $\Theta_y$ and $\Theta_z$ are either both higher, or both lower than the main rotor inertia $\Theta_x$. With a cylinder-like two-bladed rotor ($\Theta_y/\Theta_x = 1, \Theta_z \ll \Theta_y$) the system is unstable. The same applies for the rotor configuration of the Skywind 3.4MW prototype. In contrast to a three-bladed rotor and to two-bladed teetering rotors, the circulatory forces acting on the rotor do not stabilize the system. To provide for stable operation the flexible hub connection requires an increased structural coupling by introducing both or either stiffness and damping elements at the hinge point.

4.3. Rotational damping

In Figs. 4a and 4b the stability maps for a coupled system including pure rotational damping are shown. The reference inertia ratios correspond to the Skywind 3.4MW properties. The rotor speed is set to $\Omega = 1$ rps. It can be observed that an increased damping leads to a stabilization of the system. For a very low reference inertia ratio, as shown in Fig. 4a, the unstable areas appear for corresponding ratios $\Theta_y/\Theta_x > 1$ due to high time-variance and strong gyroscopic impact on the relative accelerations. A stabilization effect is only observed for equivalent damping beyond $\omega_D^2 > 10/s$, unless combined with corresponding $\Theta_y/\Theta_x$ values close to 1. The corresponding comparison case is displayed in Fig. 4b. Again it is shown that the highest damping values are required for the highest rotor asymmetry.

The introduction of a pure damper element is a promising feature to provide for stable dynamic properties of the flexible hub connection. However, the absence of a position proportional term in the equations of motion requires the application of a spring element to restore the neutral joint position and to avoid large angular displacements.
4.4. Rotational stiffness
While the introduction of a connector stiffness is required to reduce the potential hub mount excursions, the dynamic coupling yields the risk of resonance phenomena in certain operational points. Consequently, the study has to be performed for different rotor speeds. In Figs. 5a and 5b the impact of a pure stiffness at a rotor speed of $\Omega = 1$ rps is shown. Considering the small reference inertia case a similar stability map is obtained as for the purely damped system. For inertias $\Theta_y/\Theta_x < 1$ the coupled system is generally stable. An increased stiffness leads eventually to a stabilization of the rotor shape configurations with $\Theta_y/\Theta_x > 1$.

Resonance phenomena are observed in the stability map the high reference inertia case ($\Theta_z/\Theta_x = 1.05$, Fig. 5b). The previously unstable inertia combinations $\Theta_y/\Theta_x < 1$ are effectively stabilized by an increased stiffness. Considering the range of larger inertias $\Theta_y/\Theta_x > 1$ however, a region of unstable configurations appears as a function of the corresponding stiffness. The appearance of this region can be explained by a resonance of the cyclic excitation induced by the rotor speed and the periodic system eigenfrequencies. This self-resonance is inherent for coupled, non-damped and rotating systems. The stability is consequently dependent upon the rotor speed, as can be seen in the Figs. 6a and 6b for the same reference inertia ratios.

![Figure 3: Stability map of uncoupled flexible hub connection. The dashed line indicates the stability limit where $|\sigma_{i,max}| = 1$.](image)

![Figure 4: Periodic stability maps of different rotor shapes as a function of the connector damping.](image)
maximum absolute Floquet multipliers are plotted as a function of the rotor speed. In Fig. 6a the specific inertia combination of the Skywind 3.4 MW turbine is displayed. For low equivalent system frequencies, corresponding to low stiffness values, the system is unstable for wide rotor speed ranges. If the stiffness is increased the system is stable for low speed operation. A critical rotor speed is observed that distinguishes stable and unstable operational regimes. This critical rotor speed increases for a higher connector stiffness.

A principally different behavior can be observed for a rotor shape where both off-axis inertias are greater than the main rotor inertia ($\Theta_y/\Theta_x > 1$ and $\Theta_z/\Theta_x > 1$). Displayed in Fig. 6b is a configuration of ($\Theta_y/\Theta_x = 1.05, \Theta_z/\Theta_x = 1.5$). Such an inertia combination can be realistic if the rotational inertia associated to the mass of the hub mount is significantly increased relative to the main rotor inertia $\Theta_x$. A resonance range can be observed where the system shows unstable dynamic properties. By increasing the stiffness the resonance induced periodic instability appears at higher rotor speeds. In contrast to the previously considered inertia combination the resonance range is bound, which means that the system stabilizes again for very high rotor speeds. Considering the stability criterion $|\sigma_i|_{\text{max}}$ the resonance yields significantly smaller Floquet multipliers.

Figure 5: Periodic stability maps of different rotor shapes as a function of the connector stiffness.

Figure 6: Stability criterion $|\sigma_i|_{\text{max}}$ as a function of rotor speed for the stiffened hub connection.
4.5. Combined stiffness and damping

It was shown in [1] that the hub connector stiffness and damping on the actual turbine affect the aero-elastic interaction of the rotor kinematics and the inflow. The choice of the connector parametrization has to take into account the stability criterion developed here, but also optimal and risk-free load reduction. The consideration of the combined impact of stiffness and damping on the stability is performed for the Skywind 3.4 MW inertia configuration. Results are shown in Fig. 7. For a low damped system an increased stiffness stabilizes this inertia combination for a wider rotor speed range. With higher damping ratios a stable operation can be obtained for lower stiffness values. A softer coupling is desirable as it determines the transmittance of structural loads and the load reduction potential.

![Image](image_url)

Figure 7: Periodic stability maps of the Skywind 3.4 MW inertia configuration as a function of the connector stiffness and damping.

5. Conclusion

In this study the periodic stability of the flexible hub connection is investigated. By introducing two additional degrees of freedom the flexible hub connection decouples rotor loads from the support and is thus a potential load reduction system for two- and more-bladed wind turbines. For design purposes the stability of the system and potential effects leading to instability need to be assessed. The linear, time-periodic equations of motion were developed for arbitrary rotor shapes. Floquet theory was applied to a simplified rotor model, which allows to study the stability limits as a function of the rotor shape and the interfaced spring-damper properties. Aerodynamic and aeroelastic effects were excluded for simplification.

It was shown by application of the Floquet approach to the simplified rotor model that a spinning disc-rotor exhibits stable properties. As no position-proportional matrix appears in the equations of motion, the stability is solely attributed to gyroscopic effects. Asymmetric rotor shapes, like two-bladed rotors, lead to instabilities that are independent from the rotor speed. For the application on two-bladed rotors structural coupling by spring-damper elements is hence required. It was shown that both stiffening and damping of the joint coupling leads to a stabilization of the rotor connection. However, the introduction of a rotational spring element yields a dependance of the rotor speed. Resonance phenomena are observed for certain rotor shapes that previously possessed stable properties in the unactuated case. For a given rotor shape and rotor speed only a limited parameter range of stiffness and damping is acceptable to guarantee stable operation. By providing general stability maps this study can be used as a starting point for design studies of a flexible hub connection. Both two and three-bladed rotors are covered by this methodology.

As periodic instability is also known to appear as a result of rotor-tower and rotor-aero interaction, it is intended to investigate higher physical and modeling detail in a future study.
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