"Impress of rotation and an inclined MHD on waveform motion of the non-Newtonian fluid through porous canal"

T Sh Alshareef

"Department of mathematics, College of education for pure science \Ibn AlHaitham\, University of Baghdad, Iraq"

"tamara.shihab8282@gmail.com"

Abstract: Waveform flow of non-Newtonian fluid through a porous medium of the non-symmetric sloping canal under the effect of rotation and magnetic force, which has applied by the inclined way, have studied analytically and computed numerically. Slip boundary conditions on velocity distribution and stream function are used. We have taken the influence of heat and mass transfer in the consideration in our study. We carried out the mathematical model by using the presumption of low Reynolds number and small wave number. The resulting equations of motion, which are representing by the velocity profile and stream function distribution, solved by using the method of a domain decomposition analysis and we obtained the exact solutions of velocity, temperature, and concentration. The expressions of velocity, temperature, and concentration of the particles of the fluid have obtained and examined graphically by utilizing the soft wave of the Mathematica program. The efforts of various variables on mathematical modeling of motion and energy are discussed in detail. We found that.

Keywords: rotation effect, non-Newtonian fluid, porous medium, magnetic force, waveform transport.

1- Introduction

Motion through the porous area takes place in the filtering of fluids and leakage of water in the beds of rivers. The moment under the ground, oils and water are some important examples of flows through a porous medium. An oil barrage mostly includes the formation of sediments such as sandstone and limestone in which the oil is entrapped. Another example of motion through a porous medium is the leakage under a dam, which is very important. There are examples of nature's porous medium such as rye bread and beach sand. The transport through porous media discussed by (Sceidgger, 1963). The waveform motion of Newtonian fluid in a vertical asymmetric porous channel is studied by (Srinivas S and Gayathri R, 2009). The peristaltic transport of Jeffrey fluid under the effort of a magnetic field in an asymmetric porous canal is studied by (kothandapani & Srinivas, 2008). The impact of porous medium and magnetic force on the waveform flow of Jeffrey fluid is studied by (Mahmood, Afifi, & Al. Isede, 2011). The influence of the thickness of the porous material on the waveform pumping of Jeffrey fluid when the tube wall is provided with non-erodible lining is made by (Rathod & Channakote, 2011).

The MHD flow of a fluid in a channel with elastic, rhythmically contraction walls is of interest in connection with a certain problem of the movement of conductive physiological fluids, e.g., the blood and with the need for theoretical research on the operation of a peristaltic MHD Compressor. The effect of a moving magnetic field on blood flow was studied by (Stud V K, Sephone G S and Mishra R K G, 1977). And they observed that the effects of a suitable moving magnetic field accelerate the speed of blood. The blood as an electrically conducting fluid that constitutes a suspension of red cells in the plasma is considered by (Srivastava L M and Srivastava V P, 1984). The MHD flow of a conducting couple stress fluid in a slit channel with rhythmically contracting walls is...
analyzed by (Mekheimer Kh S, 2008). The MHD peristaltic motion of a Sisko fluid in an asymmetric channel is studied by (Wang Y, Hayat T, Ali N and Oberlack M, 2008). The peristaltic transport of a Jeffrey fluid under the effect of the magnetic field in an under the effect of a magnetic field in a symmetric channel with flexible rigid walls are examined by (Kothandapani M and Srinivas S, 2008). The effects of an endoscope and magnetic field on the peristalsis involving Jeffrey fluid has investigated by (Hayat T, Ahmed N and Ali N, 2008). Given these facts, it will be interesting to study the peristaltic flow of conducting Jeffrey fluid flow in a channel bounded by permeable walls. Waveform transport with heat and mass transfer has many applications in biomedical sciences and industry such as conduction in tissues, heat convection due to blood flow from the pores of tissues and radiation between environment and its surface, food processing and vasodilation. The processes of oxygenation and hemodialysis have also visualized by considering peristaltic flows with heat transfer. There is a certain role of mass transfer in all these processes. The mass transfer also occurs in many industrial processes like membrane separation process, reverse osmosis, distillation process, combustion process and diffusion of chemical impurities. The effect of heat transfer on the peristaltic flow of an electrically conducting fluid in a porous space is studied by (Hayat T, Qurashi M U and Hussain Q, 2009). The influence of heat transfer and slip on peristaltic transport is analyzed by (Hayat T, Hina S and Hendi A A, 2012). Heat transfer analysis of peristaltic flow in a curved channel is analyzed by (Ali N, Sajid M, Javed T and Abbas Z, 2010).

It is also of interest to remember that non-slip boundary conditions are unsuitable for most non-Newtonian fluids because they display microscopically the slip condition of the walls. The fluids that displaying the boundary slip condition give applications in technology such that the polishing of artificial heart, there are many studies, which are, using this condition, see (Abdulhadi A M and Al-Hadad A H, 2015), (Chaubey M K, Pandey S K and Tripathi D, 2010) & (Ali N, Wang Y, Hayat T and Oberlack M, 2009). Recently, magnetic field and rotation effects on the peristaltic transport of Jeffrey fluid in an asymmetric channel studied by (Abd-Alla, A M. and Abo-Dahab, S M, 2015). The effect of the rotation on wave motion through the cylindrical bore in a micropolar porous medium is discussed by (Mahmoud S R, Abd-alla A M and El-Sheikh M A, 2011). The effects of rotation and MHD on the nonlinear peristaltic flow of Jeffrey fluid in an asymmetric channel through a porous medium has discussed by (Abdulhadi A M and Al-Hadad A H, 2016).

Now in this paper, we discuss the waveform motion of the non-Newtonian fluid through a porous medium of non-symmetric sloping canal under the effect of rotation parameter of the channel and combined influence of inclined magnetic field as well as heat/mass transfer. We suppose that there is infinite number of waves, which are transporting with speed $c_1$ along the non-regular walls. We have chosen a system of rectangular coordinates for this channel with $X_1$ along the direction of wave's propagation and parallel to the cort line and the axis $Y_1$ is transverse to it. The mathematical model for the channel's walls can be described by:

2- Problem's Mathematical Pattern

Through our work, we have considered the waveform flow of non-Newtonian fluid through a porous medium of two-dimensional with non-symmetric and non-uniform inclined channel under the effect of rotation parameter of the channel and combined influence of inclined magnetic field as well as heat/mass transfer. We suppose that there is infinite number of waves, which are transporting with speed $c_1$ along the non-regular walls. We have chosen a system of rectangular coordinates for this channel with $X_1$ along the direction of wave's propagation and parallel to the cort line and the axis $Y_1$ is transverse to it. The mathematical model for the channel's walls can be described by:
\[ \begin{align*}
G_{11} &= -A - B_1, \text{left wall,} \ldots \\
G_{12} &= -A + B_2, \text{right wall,} \ldots 
\end{align*} \]...
(2.1)

\[ \begin{align*}
\overline{A} &= \varepsilon + m_1 X_1 \\
\overline{B}_1 &= \varepsilon_1 \sin \left[ \frac{2\pi}{\zeta}(X_1 - c_1 t) + \phi_1 \right] \\
\overline{B}_2 &= \varepsilon_2 \sin \left[ \frac{2\pi}{\zeta}(X_1 - c_1 t) \right] 
\end{align*} \]...
(2.2)

Such that \( \zeta \) is the wave’s length, \((2e)\) is the width of the channel at the inlet \((m_1 << 1)\) which is the non-uniform parameter, \((\varepsilon_1, \varepsilon_2)\) are the wave’s amplitudes, \(\phi_1\) is the phase difference of the waves which changes in the rate about \(\phi_1 \in [0, \pi]\) in which if \(\phi_1 = 0\) correspond to symmetric channel and the waves are out the phase and if \(\phi_1 = \pi\) represent to the waves in the phase. Moreover the parameters \(\varepsilon_1, \varepsilon_2, e\) and \(\phi_1\) achieved the following condition:

\[ e_1^2 + e_2^2 + 2e_1 e_2 \cos \phi_1 \leq (2e)^2 \]...
(2.3)

Also, it is worth noting through our study, we suppose the magnetic Reynolds number is small and hence the induced magnetic field is cancel.

### 3- Basic Equation

The system that governing the equations of motion and energy can give in the following formula:

\[ \begin{align*}
\frac{\partial W_1}{\partial X_1} + \frac{\partial W_2}{\partial X_2} &= 0 \\
\rho_i \left( \frac{\partial W_1}{\partial t} + W_1 \frac{\partial W_1}{\partial X_1} + W_2 \frac{\partial W_1}{\partial X_2} \right) - \rho_i \Omega (\Omega W_1) + 2 \frac{\partial W_2}{\partial Y_1} &= \frac{\partial P}{\partial X_1} + \frac{\partial}{\partial Y_1} (\bar{\tau}_{X_1 X_1}) + \frac{\partial}{\partial Y_1} (\bar{\tau}_{X_1 Y_1}) \\
-\sigma B_1^2 \cos \beta \bar{W}_1 \cos \beta - W_2 \sin \beta \right) - \frac{N_{\chi}}{k_1} W_1 + \rho_i g \sin \alpha_i. \\
\rho_i \left( \frac{\partial W_2}{\partial t} + W_1 \frac{\partial W_2}{\partial X_1} + W_2 \frac{\partial W_2}{\partial X_2} \right) - \rho_i \Omega (\Omega W_2) - 2 \frac{\partial W_2}{\partial Y_1} &= \frac{\partial P}{\partial X_1} + \frac{\partial}{\partial Y_1} (\bar{\tau}_{X_1 X_1}) + \frac{\partial}{\partial Y_1} (\bar{\tau}_{X_1 Y_1}) \\
+\sigma B_1^2 \sin \beta \bar{W}_1 \cos \beta - W_2 \sin \beta \right) - \frac{N_{\chi}}{k_1} W_2 - \rho_i g \cos \alpha_i. \\
\rho_i \zeta_1 \left( \frac{\partial \bar{F}}{\partial X_1} + \frac{\partial \bar{F}}{\partial Y_1} \right) = k_2 \left( \frac{\partial^2 \bar{F}}{\partial X_1^2} + \frac{\partial^2 \bar{F}}{\partial Y_1^2} \right) + N_{\delta} \left(2 \frac{\partial W_1}{\partial Y_1} \right)^2 + 2(\frac{\partial W_1}{\partial Y_1} \frac{\partial W_2}{\partial X_1})^2 + 2(\frac{\partial W_1}{\partial Y_1} \frac{\partial W_2}{\partial X_1})^2 + \frac{g \nu k_f}{k_1} \frac{\partial^2 \bar{F}}{\partial X_1^2} \\
+ \frac{\partial^2 \bar{F}}{\partial Y_1^2} \right) \\
\frac{\partial \bar{W}_1}{\partial X_1} + \frac{\partial \bar{W}_2}{\partial X_2} &= \frac{g \nu k_f}{k_1} \left( \frac{\partial^2 \bar{F}}{\partial X_1^2} + \frac{\partial^2 \bar{F}}{\partial Y_1^2} \right) \]...
(3.4)
(3.5)

Where \((\rho_i)\) is the fluid’s density, \(\Omega = \Omega \kappa\), \(\kappa\) is the unit vector parallel to \(z\)-axis, \(\Omega\) is the rotation parameter, \(\bar{W} = [\bar{W}_1(X_1, Y_1), \bar{W}_2(X_1, Y_1), 0]\) is the vector of velocity in two-dimensional coordinates.
\((\vec{X}_1,\vec{Y}_1)\), \(\rho_1\) is the fluid's pressure, \(\vec{\tau}\) is the flow's fluid time, \(\sigma\) is the fluid's electrical conductivity, \(B_0\) is the strength of the applied magnetic force. The absence of an electrical field characterized by the Lorentz force \((\vec{J} \times \vec{B})\), which takes the following formula:

\[
\vec{J} \times \vec{B} = -\sigma B_0^2 \cos \beta (\vec{W}_I \cos \beta - \vec{W}_2 \sin \beta) e_i + +\sigma B_0^2 \sin \beta (\vec{W}_I \cos \beta - \vec{W}_2 \sin \beta) e_i
\]

...(3.6)

Where \((e_i,e_j)\) are the unit vectors, \(\vec{J}\) is the induced current density. We observed that the effect of the magnetic field appears on the flow of \(\vec{X}_1\) direction due to the inclination angle \(\beta\) of magnetic field. Also, we have \(\alpha_i\), referred to inclination angle of the channel, \(g\) is the acceleration due to gravity, \(\text{No}\) is the fluid's viscosity, \(k_1\) is the porosity parameter of the canal, \(\zeta_1\) is the specific heat at constant pressure, \(\vec{F}\) is the fluid's temperature, \(\vec{f}\) is the fluid's concentration, \(k_2\) is the fluid's thermal conductivity, \(g_m\) is the coefficient of mass diffusivity, \(k_s\) is the concentration susceptibility, \(k_{F}\) is the thermal diffusion ratio and \(\vec{F}_m\) is the fluid's mean temperature.

The constituent equations for non-Newtonian incompressible fluid which characterized by rate type fluid can be shown as the form:

\[
\vec{S} = -\rho \vec{I} + \vec{\tau}
\]

...(3.7)

Where \(\vec{S}\) is the Cauchy stress tensor, \(\vec{I}\) is the identity tensor and \(\vec{\tau}\) is the extra stress for the fluid which is formed as [18]:

\[
\vec{\tau} = \frac{N_0}{1 + \lambda_1} (\vec{r} + \zeta_2 \vec{r}')
\]

...(3.8)

Where the ratio of repose to obstruction times is \(\lambda_1\), \(\zeta_2\) is the obstruction time, \(\vec{r}\) is the rate of shear, such that:

\[
\vec{r} = (\nabla \vec{W}) + (\nabla \vec{W})^T
\]

...(3.9)

\[
\vec{r}' = [(\frac{\partial}{\partial t} + \vec{W}_1 \frac{\partial}{\partial X_1} + \vec{W}_2 \frac{\partial}{\partial Y_1}) \vec{r}]
\]

...(3.10)

Now, if we substitute (3.10) into (3.8), we obtain:

\[
\vec{\tau} = \frac{N_0}{1 + \lambda_1} ((\frac{\partial}{\partial t} + \vec{W}_1 \frac{\partial}{\partial X_1} + \vec{W}_2 \frac{\partial}{\partial Y_1})) \vec{r}
\]

...(3.11)

Then the components of stress have given by:

\[
\tau_{\vec{X}_1,\vec{X}_1} = \frac{N_0}{1 + \lambda_1} (\vec{r}_{\vec{X}_1,\vec{X}_1} + \zeta_2 \vec{r}'_{\vec{X}_1,\vec{X}_1})
\]

\[
= \frac{2N_0}{1 + \lambda_1} [\vec{W}_1 + \zeta_2 (\frac{\partial^2 \vec{W}_1}{\partial \vec{X}_1 \partial \vec{X}_1} + \vec{W}_2 \frac{\partial^2 \vec{W}_2}{\partial \vec{Y}_1 \partial \vec{X}_1})]
\]

...(3.12)

\[
\tau_{\vec{X}_1,\vec{Y}_1} = \frac{N_0}{1 + \lambda_1} (\vec{r}_{\vec{X}_1,\vec{Y}_1} + \zeta_2 \vec{r}'_{\vec{X}_1,\vec{Y}_1})
\]
\[-\frac{N_0}{1+\lambda_1}(\frac{\partial \tilde{W}_1}{\partial Y_1} + \frac{\partial \tilde{W}_2}{\partial X_1}) + \zeta_1(\frac{\partial^2 \tilde{W}_1}{\partial t^2} + \frac{\partial^2 \tilde{W}_3}{\partial t \partial X_1}) + \tilde{W}_1(\frac{\partial^2 \tilde{W}_1}{\partial X_1^2} + \frac{\partial^2 \tilde{W}_3}{\partial X_1 \partial Y_1}) + \tilde{W}_2(\frac{\partial^2 \tilde{W}_1}{\partial Y_1^2} + \frac{\partial^2 \tilde{W}_3}{\partial Y_1 \partial X_1})]\] ...(3.13)

\[-\frac{\tau \partial \tilde{v}}{1+\lambda_1} + \frac{N_0}{1+\lambda_1}(r_{v,v} + \zeta_2 r_{v,v})\]

\[= \frac{2N_0}{1+\lambda_1}(\frac{\partial \tilde{W}_1}{\partial Y_1} + \zeta_2(\frac{\partial^2 \tilde{W}_2}{\partial t \partial Y_1} + \frac{\partial^2 \tilde{W}_3}{\partial X_1 \partial Y_1}) + \tilde{W}_1(\frac{\partial^2 \tilde{W}_2}{\partial X_1^2} + \frac{\partial^2 \tilde{W}_3}{\partial X_1 \partial Y_1}) + \tilde{W}_2(\frac{\partial^2 \tilde{W}_2}{\partial Y_1^2} + \frac{\partial^2 \tilde{W}_3}{\partial Y_1 \partial X_1})]\] ... (3.14)

Now, if we introduce the following non-dimensional parameters into Eq. (1-14) we obtain:

\[x = \frac{X_1}{\zeta}, \quad y = \frac{Y_1}{e}, \quad t = \frac{c_1 t}{\zeta}, \quad u = \frac{W_1}{c_1 \sigma_1}, \quad v = \frac{W_2}{c_1 \sigma_1}, \quad \delta_1 = \frac{\epsilon}{\zeta}, \quad g_1 = \frac{G_1}{\epsilon}, \quad g_2 = \frac{G_2}{\epsilon}, \quad \theta = \frac{\overline{F} - \overline{F}_0}{N_0 f_1 - f_0}, \quad \varphi = \frac{\overline{f} - f_0}{N_0 f_1 - f_0}, \quad \rho = \frac{c_1^2}{e} \]

\[\text{Re}_n = \frac{\rho e^2}{N_0}, \quad \text{Pr}_n = \frac{N_0}{k_2}, \quad M = \frac{\sigma}{N_0} \epsilon B_0, \quad a = \frac{e_1}{e}, \quad b = \frac{e_2}{e}, \quad \text{Sci} = \frac{N_0}{\rho_1 g_m}, \quad \text{Sor} = \frac{\rho_1 g_m k_f (F_1 - F_0)}{F_0 N_0 (f_1 - f_0)} \]

\[D_a = \frac{k_1}{e^2}, \quad \text{Ec} = \frac{c_1^2}{\zeta_1 (F_1 - F_0)^2}, \quad D_f = \frac{g_0 k_f (f_1 - f_0)}{\zeta_1 N_2 k_x (F_1 - F_0)}, \quad \tau = \frac{e^2}{N_0 \epsilon}, \quad P_1 = \frac{e^2 \overline{P}}{c_1^2 \epsilon N_0}, \quad \text{Fr} = \frac{c_1^2}{g e} \]

\[B_m = \text{Pr}_n \text{Ec}, \quad A = \frac{\overline{A}}{e}, \quad B_1 = \frac{\overline{B}_1}{e}, \quad B_2 = \frac{\overline{B}_2}{e}, \quad m_1 = \frac{m_1}{e} \]

... (3.15)

where \(\delta_1\) is the wavenumber, \(\text{Pr}_n\) is the Prandtl number, \(\text{Ec}\) is the Eckert number, \(\text{Re}_n\) is the Reynolds number, \(B_m\) is the Brinkman number, \(M\) is Hartmann number, \(a\) & \(b\) are the amplitudes of the wave, \(\text{Sci}\) is the Schmidt number, \(\text{Sor}\) is the sorption number, \(D_a\) is the porous medium parameter, \(\theta\) & \(\varphi\) are the non-dimensional of temperature and concentration respectively, \(\overline{P}\) is the pressure of the fluid, \(F_0\) & \(F_1\) are the temperature of the fluid at upper and lower side of the walls, \(c_0\) & \(c_1\) are the concentration of the fluid at upper and lower part of the walls \(g_1\) & \(g_2\) are non-dimensional of upper and lowers walls of the channel, \(D_f\) is the Dufour number, \(\text{Fr}\) is the Froude number.

So, Equations (3.1)-(3.14) will become:

\[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \text{(3.16)}\]

\[\text{Re}_n \delta_1 \left( \frac{\partial^2 u}{\partial t^2} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial P}{\partial x} + \delta_1 \frac{\partial}{\partial x} \tau_{xx} + \frac{\partial}{\partial y} \tau_{xy} + \frac{\rho_1 \overline{\Omega}^2 e^2}{N_0} u + 2 \delta_1^2 \text{Re}_n \frac{\partial v}{\partial t} \quad \text{(3.17)}\]

\[-N_1^2 \frac{\partial u}{\partial t} + \frac{\text{Re}_n}{\text{Fr}} \sin \alpha_i \]

\[\text{Re}_n \delta_1 \left( \frac{\partial^2 v}{\partial t^2} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial P}{\partial y} + \delta_1^2 \frac{\partial}{\partial y} \tau_{yy} + \delta_1 \frac{\partial}{\partial x} \tau_{xy} + \frac{\rho_1 \overline{\Omega}^2 e^2}{N_0} v - 2 \delta_1^2 \text{Re}_n \frac{\partial u}{\partial t} + \delta_1 M^2 \sin \beta \cos \beta u - \delta_1^2 N_1^2 \frac{\partial v}{\partial x} - \delta_1 \frac{\text{Re}_n}{\text{Fr}} \cos \alpha_i \quad \text{(3.18)}\]
\[
\text{Re}_n \delta_i \text{Pr} (\frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y}) = \delta_i^2 \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + Bm[2\delta_i^2 (\frac{\partial u}{\partial x})^2 + 2\delta_i^2 (\frac{\partial v}{\partial y})^2 + (\frac{\partial u}{\partial y} + \delta_i (\frac{\partial v}{\partial x}))^2] \\
+ \text{Pr} Df(\delta_i^2 \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}) 
\]

...(3.19)

\[
\text{Re}_n \delta_i \text{Sci} (u_0 \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y}) = (\delta_i^2 \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}) + \text{Sci Sor} (\frac{\partial \theta}{\partial y} + \delta_i \frac{\partial \theta}{\partial x}) 
\]

...(3.20)

In which \( N_1 = \sqrt{M^2 \cos^2 \beta + \frac{1}{Da}} \), \( N_2 = \sqrt{M^2 \sin^2 \beta + \frac{1}{Da}} \) and the components of shear stress are:

\[
\tau_{xx} = \frac{2\delta_i}{1 + \lambda_i} [1 + \frac{\zeta}{e} \frac{\partial}{\partial x} (\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y})] \frac{\partial u}{\partial x} 
\]

...(3.21)

\[
\tau_{xy} = \frac{1}{1 + \lambda_i} [1 + \frac{\zeta}{e} \frac{\partial}{\partial x} (\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y})] (\frac{\partial u}{\partial y} + \delta_i \frac{\partial v}{\partial x}) 
\]

...(3.22)

\[
\tau_{yy} = \frac{-2\delta_i}{1 + \lambda_i} [1 + \frac{\zeta}{e} \frac{\partial}{\partial x} (\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y})] \frac{\partial v}{\partial y} 
\]

...(3.23)

Now, by using the approximations of small wavenumber \( \delta_i \) and it's orders and the low value of Reynolds number (Re), Eq. (3.16)-3.22 will be in the following form:

\[
\frac{\partial P}{\partial x} = \frac{\partial}{\partial y} \tau_{xy} + \frac{\rho e^2}{N_0} u - N_1^2 u + \text{Re}_n \text{si} 
\]

...(3.24)

\[
\frac{\partial P}{\partial y} = 0 
\]

...(3.25)

\[
0 = \frac{\partial^2 \theta}{\partial y^2} + Bm(\frac{\partial u}{\partial y})^2 + \text{Pr} Df (\frac{\partial^2 \phi}{\partial y^2}) 
\]

...(3.26)

\[
0 = \frac{\partial^2 \phi}{\partial y^2} + \text{Sci Sor} \frac{\partial^2 \theta}{\partial y^2} 
\]

...(3.27)

\[
\tau_{xx} = 0, \tau_{xy} = \frac{1}{1 + \lambda_i} \frac{\partial u}{\partial y}, \tau_{yy} = 0 
\]

...(3.28)

Now, if we introduce the stream function \( \phi(x,y) \) in Eq. (3.24) by taking the formula \( u = \frac{\partial \psi}{\partial y} \), \( v = -\frac{\partial \psi}{\partial x} \), we get

\[
\frac{\partial P}{\partial x} = \frac{1}{1 + \lambda_i} \frac{\partial^2 \psi}{\partial y^2} - (N_1^2 - \frac{\rho e^2}{N_0}) \frac{\partial \psi}{\partial y} + \text{Re}_n \sin \alpha_i 
\]

...(3.29)

From Eq. (3.25) we deduce that the pressure \( \rho_1 \) of the fluid doesn’t depend on y, so if we derive Eq. (3.30) with respect to y we obtain:
0 = \frac{1}{1+\lambda_i} \frac{\partial^2 \psi}{\partial y^2} + \left( \frac{\rho_i \overline{\omega} e^2}{N_0} - N_i \right) \frac{\partial^2 \psi}{\partial y^2} \quad \text{(3.31)}

The boundary conditions, which are used through this study, can represent in the following:

\[ \psi = \frac{1}{2} \frac{\partial \psi}{\partial y} + \beta_i \frac{\partial^2 \psi}{\partial y^2} = -1 \quad \text{at} \quad y = g_2 \]

\[ \psi = -\frac{1}{2} \frac{\partial \psi}{\partial y} - \beta_i \frac{\partial^2 \psi}{\partial y^2} = -1 \quad \text{at} \quad y = g_1 \quad \text{(3.32)} \]

\[ \theta = 0, \varphi = 0 \quad \text{at} \quad y = g_2 \]

\[ \theta = 1, \varphi = 1 \quad \text{at} \quad y = g_1 \quad \text{(3.33)} \]

In which, \( g_2 = A + B_2, g_1 = A - B_1, A = 1 + m_i x, B_i = a \sin[2\pi (x - t) + \phi_i], B_2 = b \sin[2\pi (x - t)] \) \quad \text{(3-34)}

4. Problem’s solution

By using the method of "A domain decomposition", the Eq. (3.31) can be written as:

\[ \frac{\partial^2 \tilde{\psi}}{\partial y^2} = S^2 \frac{\partial^2 \psi}{\partial y^2} \quad \text{(4.1)} \]

In which \( S^2 = (1+\lambda_i)(N_i - \frac{\rho_i \overline{\omega} e^2}{N_0}) \), an operator \( (\tilde{\psi}) \) can write Eq. (4.1) as:

\[ (\tilde{\psi}) \quad \text{(4.2)} \]

In which \( \tilde{\psi} = 1 \frac{\partial^4}{\partial y^4} \) is a fourth-order difference operators \( (\tilde{\psi})^{-1} \) is a fourth-fold integration operator defined by:

\[ (\tilde{\psi})^{-1} = \int \int \int \int (\psi) dy dy dy dy \quad \text{(4.3)} \]

if we are operating with \( (\tilde{\psi})^{-1} \) on Eq. (4.2), we have:

\[ \psi = c_{11} + c_{12} y + c_{13} \frac{y^2}{2!} + c_{14} \frac{y^3}{3!} + S^2 (\tilde{\psi})^{-1} (\psi_m)_{yy} \quad \text{(4.4)} \]

In which the function \( c_{ij} (i = 1, j = 1,2,3,4) \) can be obtained by using the boundary conditions, Eq. (3.32)

The standard of A domain decomposition method, are get:

\[ \psi = \sum_{m=0}^{\infty} \psi_m \quad \text{(4.5)} \]

Where the components \( (\psi_m), m \geq 0 \), will be located frequently. The following repeated relation is got from Eq. (4.4):

\[ \psi_0 = c_{11} + c_{12} y + c_{13} \frac{y^2}{2!} + c_{14} \frac{y^3}{3!} + \ldots \quad \text{(4.6)} \]
\[ \psi_{m+1} = S^2 \left( \frac{1}{2} \right)^{(m+1)} \psi_m, \ m \geq 0 \]

Hence, we have:

\[ \psi_1 = \frac{1}{S^2 c_{13}} \frac{(Sy)^4}{4!} + \frac{1}{S^2 c_{14}} \frac{(Sy)^5}{5!} + \ldots \]  

\[ \psi_2 = \frac{1}{S^2 c_{13}} \frac{(Sy)^6}{6!} + \frac{1}{S^2 c_{14}} \frac{(Sy)^7}{7!} + \ldots \]  

\[ \vdots \]

\[ \psi_m = \frac{1}{S^2 c_{13}} \frac{(Sy)^{2m+2}}{(2m+2)!} + \frac{1}{S^2 c_{14}} \frac{(Sy)^{2m+3}}{(2m+3)!}, \ m \geq 0 \]  

Thus from Eq. (4.5), the formula for \( \psi \) is given as:

\[ \psi = c_{11} + c_{12}y + \frac{1}{S^2 c_{13}} (cosh Sy - 1) + \frac{1}{S^2 c_{14}} (sinh Sy - Sy) \]  

\[ u = (-c_s + c_s^2 + c_sCosh[sy] + c_sSinh[Sy]) / s^2 \]  

The expression of temperature and concentration distributions as follow:

\[ \theta = \frac{Br \epsilon C_y y^2}{4(-1+W1)} - \frac{Br \epsilon C_y y^2}{4s^2(-1+W1)} + a_1 + a_2 + \frac{Br \epsilon C_y Cosh[2sy]}{8s^2(-1+W1)} + \frac{Br \epsilon C_y Cosh[2sy]}{8s^2(-1+W1)} + \frac{Br \epsilon C_y Sinh[2sy]}{8s^2(-1+W1)} \]  

\[ \varphi = -((Br \epsilon C_y S_c S_y^2) / (4(-1+W1))) + (Br \epsilon C_y S_c S_y^2) / (4s^2(-1+W1)) + b_1 + b_2 - (Br \epsilon C_y S_c S_y Cosh[2sy]) / (4s^2(-1+W1)) - (Br \epsilon C_y S_c S_y Cosh[2sy]) / (4s^2(-1+W1)) - (Br \epsilon C_y S_c S_y Sinh[2sy]) / (4s^2(-1+W1)) \]

**5 – Discussion of the problem's results**

**5.1 Velocity's distribution**

By Equation (4-11), we can realize that velocity's profile is a function of \( y \).

In this section, we have displayed the results of the problem and have discussed for different physical parameters of interest. Figure (1) have used to show the distribution of axial velocity for various of the channel \( m_1 \), the phase difference of the channel \( \phi_1 \), the amplitudes of the channel's waves \( a \& b \), Hartmann number \( M \), the fluid's material parameter \( \lambda_1 \), fluid's density \( \rho_1 \), fluid's viscosity \( \nu_0 \), rotation parameter \( \Xi \), channel's width \( 2e \), inclination angle of magnetic field \( \beta \), volume flow rate \( \Theta_1 \) and the slip parameter \( \beta_1 \). In figure (1-a), we observed that an increase in \( m_1 \) leads to an increase in flow of fluid on the walls of the channel and decrease in the cott of channel at \( y \in (-0.7,0.9) \). Figure (1-b), shows the impact of \( \phi_1 \) on the velocity profile, it have found that the magnitude of velocity reduces at all the channel and especially at the lower wall of the channel. Figure (1-c,d,e) displays the effects of parameters \( a,b \& \theta \) on the velocity, it have noted that their behavior on velocity is opposite to phase difference's behavior on it. The efforts of \( M, \lambda_1,N_0 \) and \( \beta_1 \) on the velocity distribution have sketched on the figures(1-f,g,h,i) and we noticed that the rising values of the...
last parameters results an increase to amount of flow on the sides of the channel and decreasing in the center.
Fig(1-a,b,…,n): Effect of parameters on velocity profile

$m_1 = 1.5, t = 0.01, \phi_1 = \pi / 6, a = 0.2, b = 0.3, M = 1, \lambda_1 = 1.5, \rho_1 = 0.1, N_o = 0.2, \Omega = 1, e = 1, \beta \rightarrow \pi / 6, Da = 1, \theta_1 = 0.5, \beta_1 = 0.5, x = 0.3$

5.2 Temperature's characteristics

By Eq. (4-12), we can see that the distribution of fluid's temperature is a function of $y$. figure (2) have designed to show the changes of temperature distribution for various values of $m_1, \phi_1, a, b, M, \lambda_1, \rho_1, N_o, \Omega, e, \beta, Pr_n, Ec, D_x, Sci, D_x, Sor, \theta_1$ and $\beta_1$ .figure (2-a) have drawn to explain the effect of non-uniform parameter of channel $m_1$ on fluid's temperature, we have seen that the temperature increase on the walls of channel but it decreases in the part of center of channel when $y \in (-1.2, 0.2)$. The impact of phase difference $\phi_1$ on the distribution of temperature, it observed that an increase in this parameter leads to reduce in the heat of fluid on the lower part of channel but there is a slight increase in the cort of channel when $y \in (-0.8, 0.02)$ of wave's amplid we can see that in figure (2-b), opposite effectiveness can see in figure (2-c) for the influence of wave's amplitude $a$ on the heat of fluids and we see that it's temperature is low in the area when $y \in (-1.2, 0.3)$. Figure (2-d,e,f,g,h,i) are displayed
the efforts of wave's amplitude \( b \), fluid's density \( \rho \), fluid's viscosity \( N_0 \), rotation parameter \( \Omega \), inclination angle of magnetic force \( \beta \) and Darcy number \( D_a \) on heats distribution, we have noted that the temperature of fluid increases in the all parts of channel with an increase of these parameters. adverse effective can notice in figure (2-j,k) for the actions of Hartmann number \( M \) and fluid's material parameter \( \lambda \). Figure(2-l,m,n,o,p,q,r) is sketched to show the impress of Prandtl number \( Pr_n \), Eckert number \( Ec \), Dufour number \( D_f \), Schmidt number \( Sci \), half width \( e \), soret number \( Sor \) and volume flow rate \( \theta \), on the fluid's heat, we viewed that with an excess values of previous variables, the temperature of the fluid will raise at the middle of the channel but it goes down little at the endings of walls. In figure (2-s), the effectuation of slip parameter \( \beta \), on the fluid's temperature is offered, we have seen that this parameter show up cross attitude for the prandtl number's manner \( Pr_n \) on the heat of fluid.
Figures 2(a,b,...,s): Effect of parameters on temperature profile

\[
t = 0.01, \phi = \pi / 6, a = 0.2, b = 0.3, M = 1, \\
\lambda = 1.5, \rho = 0.1, N_o = 0.2, \Omega = 1, e = 1, \beta = 1.5, \\
\beta = \pi / 6, Pr = 2.5, Ec = 0.5, D_f = 0.5, Sci = 0.5 \\
D_a = 1, Sor = 0.5, \theta = 0.5, m_i = 1.5, x = 0.3
\]

5.3 Concentration's Profile

By equation (4-13), we can notice that the fluid's concentration is a function of y. Figure (3) have drawn to show the variation of concentration distribution for sundry value of \( m_1, \phi_1, a, b, M, \lambda, \rho_1, \\
N_o, \Omega, e, \beta, Pr, n, Ec, D_f, Sci, D_a, Sor, \theta_1 \) and \( \beta_1 \). The concentration's profile is opposite of temperature profile and the variables have behaved inverse action on concentration than a fluid's heat. Figure (3-a) have depicted to show the effect of non-regularity parameter of channel \( m_1 \) on fluid's concentration, we have seen that the concentration decreases at the walls of the channel but it have started to increase by a slightly way at the upper wall when \( y \in (0.2,1.5) \). Opposite conduct on the impact of (b) which have shown in figure (3-b). figure (3-c,d,e,f) are displayed the effects of \( M, \lambda, N_o \) and \( \phi_1 \), we have noted that the concentration is an increasing function of these parameters. Figure (3-g,h,l,j,k,l) are sketched to clarify the actions of \( \rho_1, N_o, \Omega, e, \beta, D_a \) and \( a \), we have observed that the concentration is an decreasing function of these parameters. the activity of Pr on the fluid's concentration have formalized in figure (3-m), we have perceived that the concentration is less in the center of channel but it is more at the walls of the channel. Similar effectiveness for the influence of \( Ec, D_f, Sci, Sor \) and \( \theta_1 \), and their efficacy have shown in figure (3-n, o, p, q), but we can see the inverse impress for the parameter \( \beta_1 \) and have seen it's influence in figure (3-s).
Fig (3-a,b,…,s): effect of parameter on concentration profile.

- $t = 0.1, \phi_t = \pi / 6, a = 0.2, b = 0.3, M = 1$
- $\lambda_i = 1.5, \rho_i = 0.1, N_o = 0.2, \Omega = 1, e = 1$
- $\beta = \pi / 6, Pr n = 3, Ec = 0.8, D_f = 0.3, SCI = 0.7$
- $D_a = 2, Sor = 0.7, \theta_1 = 0.5, m_1 = 1.5, x = 0.3$
5.4 Phenomenon of fluid's waves stream

The phenomenon of fluid's trapping is an motivating them in wave's transporting of fluids. The formulation of an inwardly revolving bolus of fluid through enclosed stream lines is known by trapping and this trapping bolus is derived a head a long with the contracted waves. The impacts of various parameters like $m_1, \phi_1, \sigma, \beta, \lambda, \rho, N_o, \overline{\Omega}, \epsilon, \beta_i, \Pr, \Ec, D_f, \Sci, D_n, \Sor, \theta_i$ and $\beta_i$ on trapping have seen through the figures (4-17). Figures (4-a,b)-(7-a,b) show that the number and size of trapping bolus increase with an increase of $m_1, \sigma, \rho$, and $\beta_i$. Inverse situation can noticed in the figures (8-a,b)-(12-a,b) for the actions of $\phi_1, \lambda, N_o, \overline{\Omega}$. The effect of $\beta$ is sketched in figure (13-a,b), at the beginning, we have noted that there is a connected wave but it have taken to separated different waves which is increasing in volume and number. The influence of $\epsilon$ have illustrated in figure (14-a,b), it have observed there is an increasing in volume and number of bolus in the right side of channel when $0.8 < x < 1.5$ and there is a decreasing in the size and number of bolus in the left part of channel when $0 < x < 0.6$. Similar effect for the activity of $\beta$ and $D_n$ on the waves of fluid and their effect have represented in figure (15-a,b)-(16-a,b) respectively, and we have noticed that there is clear boost in number of bolus in the right wall of channel when $0.8 < x < 1.5$. Where as in figure (17-a,b), we have viewed the contrary demeanor for the work of $\theta_1$ on the fluid's waves, we have recognized that the bolus of fluid have gone down in number for both sides of channel but they have enhanced in the size.
Fig (4-c) \( \beta = 0.2 \quad \beta = 0.3 \)

Fig (4-d) \( \beta = 0.5 \quad \beta = 1.5 \)

Fig (4-e) \( \phi = \pi/4 \quad \phi = \pi/3 \)

Fig (4-f) \( M = 1 \quad M = 3 \)
Fig (4-a,b,…,n): Effect of parameters on streamline

\[ m_i = 1.5, t = 0.01, \phi_i \rightarrow \pi / 6, a = 0.2, b = 0.3, M = 3, \lambda_i = 1.5, \]
\[ \rho_i = 0.1, N = 0.2, \Omega = 1, e = 1, Da = 1, \theta_i = 0.5, \beta_i = 0.5 \]
In the present study, we deal with the waveform transport of non-Newtonian fluid under the combined influence of inclined magnetic field and heat/mass transfer in the porous medium of non-symmetric inclined channel by using the effect of rotation parameter of the channel. Thus through our study we have conclude the following observations:

1. On the velocity's distribution, there is an enhancement on it's profile with an increase values of non-uniform parameter \( m_1 \) of the channel, amplitudes of channel \( (a \& b) \), Hartmann number \( M \), fluid's material parameter \( \lambda \), fluid's viscosity \( N_0 \), volume flow rate of fluid \( \theta_1 \) and slip parameter \( \beta_1 \). Opposite case is satisfied with an increase values of phase of fluid's wave \( \phi_1 \), fluid's density \( \rho_1 \). Rotation parameter of the channel \( \Omega \), half-width of channel \( c \) and slopping angle of magnetic field \( \beta \).

2. On temperature's distribution; there is an ascending on it's profile with an rising magnitude of left amplitude of wave \( b \), fluid's density \( \rho_1 \), darcy number \( D_\alpha \), rotation parameter \( \Omega \), half-width of channel \( c \) and slopping angle of magnetic field \( \beta \), inverse status is achieved with an increase of Hartmann number \( M \), fluid's material parameter \( \lambda \) and fluid's viscosity \( N_0 \).

3. With an increase of right amplitude of wave \( a \). There is clear increasing on fluid's heat on left wall of the channel and there is slight reducing in the middle part of the channel. We can see the opposite behavior for the influence of wave's phase.

4. With an increase of non-uniform parameter of channel and slip parameter \( \beta_1 \). There is clear increasing on fluid's temperature on the walls of the channel and it decreases at the center of channel. The contrary case can be seen with an increase of prandtl number \( Pr_n \), Eckert number \( Ec \), Dufour number \( D_f \), Schmidt number \( Sci \), soret number \( Sor \) and volume flow rate of the wave \( \theta_1 \).

5. There is a seriousness relationship between the distribution of velocity of fluid and it's temperature.

6. There is discrepant relationship between the distribution of fluid's temperature and it's concentration. So, we have noticed that the fluid's concentration is an ascending function of the parameters \( M, \lambda, N_0 \) and it is decreasing function of the parameters \( \rho_1, \Omega, E, \beta, D, a \).

7. With an increase of the following parameters \( Pr_n, Ec, D, Sci, Sor, \theta_1 \), the fluid's concentration have increased at the walls of the channel and have decreased at the center of the channel. We can observe the inverse case with an increase of \( (m_1, b, \beta_1) \).

8. The number and size of the trapping bolus have increased with an increase of \( m_1, (a, b), \rho_1 \) and \( \beta_1 \). Opposite p light with an increase of \( \phi_1, M, \lambda, N_0 \) and \( \Omega \).

9. With increase values of parameters, \( (e, \beta, D) \), there is clear increasing in size and number of bolus in the right side of channel and clear decreasing in it in the left side of channel.

10. The influence of volume flow rate \( \theta_1 \) on the trapping bolus of fluid's waves have promoted basically the size of these bolus, but it have negative effect on their number on both sides of channels walls.
References

[1] Sceidgger, A. E. (1963). The physics of flow through a porous media. University of Toronto press.

[2] Srinivas S and Gayathri R. (2009). Peristaltic transport of a Newtonian fluid in a vertical asymmetric channel with heat transfer and porous medium. Applied Mathematics and Computation, 215, pp:185-196.

[3] kothandapani, M., & Srinivas, S. (2008). Peristaltic transport of a Jeffery fluid under the effect of magnetic field in an asymmetric channel. Physics Letter A., 372, pp:4586-4591.

[4] Mahmood, S. R., Afifi, N. A., & Al. Isede, H. M. (2011). Effect of porous medium and magnetic field on the peristaltic transport of a Jeffery fluid. Journal of Math Analysis, 5, pp:1025-1034.

[5] Rathod, V. P., & Channakote, M. M. (2011). A study of ureteral peristalsis in cylindrical tube through porous medium. Advance in Applied Science Research, 134-140.

[6] Stud V K, Sephone G S and Mishra R K G. (1977). MHD peristaltic flow of a jeffrey fluid in an asymmetric channel with partial slip. Bull. Bial, 39, PP: 358-390.

[7] Srivastava L M and Srivastava V P. (1984). Peristaltic transport of blood: casson model-II. J. Biomech, 17, 821-829.

[8] Mekheimer Kh S. (2008). Effect of the induced magnetic field on peristaltic flow of a couple stress fluid. Phys. Lett. A, 372(23), PP: 4271-4278.

[9] Wang Y, Hayat T, Ali N and Oberlack M. (2008). Magnetohydrodynamic peristaltic motion of a sisko fluid in a symmetric channel. Physica A. Statistical Mechanics and its Applications, 387(2-3), PP: 347-362.

[10] Kothandapani M and Srinivas S. (2008). Peristaltic transport of a jeffrey fluid under the effect of magnetic field in an asymmetric channel. Int. J Non-linear Mech, 43, PP: 915-924.

[11] Hayat T, Ahmed N and Ali N. (2008). Effects of an endoscope and magnetic field on the peristalsis involving Jeffrey fluid. Communications in Nonlinear Science and Numerical Simulation, 13, PP: 1581–1591.

[12] Hayat T, Qurashi M U and Hussain Q. (2009). Effect of heat transfer on the peristaltic flow of an electrically conducting fluid in a porous space. Comm. Non linear Sci. Number, 33(4), PP: 1862-1873.

[13] Hayat T, Hina S and Hendi A A. (2012). Slip Effect on peristaltic transport of a maxwell fluid with heat and mass transfer. Journal of Mechanics in medicine and Biology, 12(1), PP: 1-22.

[14] Ali N, Sajid M, Javed T and Abbas Z. (2010). Heat transfer analysis of peristaltic flow in a curved channel. Int. J. Heat Mass transfer, 53(15-16), PP: 3319-3325.
[15] Ali N, Wang Y, Hayat T and Oberlack M. (2009). Slip effects on the peristaltic flow of a third grade fluid in a circular cylindrical tube. *J. Appl. Mech*(76), PP: 011006-011015.

[16] Chaube M K, Pandey S K and Tripathi D. (2010). Slip effects on peristaltic transport of a micropolar fluid. *Appl. Math. Sci*(4), PP: 2105-2117.

[17] Abdulhadi A M and Al-Hadad A H. (2015). Slip Effect on the Peristaltic Transport of MHD Fluid through a Porous Medium with Variable Viscosity. *Iraqi Journal of Science*, 56(3B), pp: 2346-2363.

[18] Abd-Alla, A M. and Abo-Dahab, S M. (2015). Magnetic field and rotation effects on the peristaltic transport of a Jeffery fluid in an asymmetric channel. *Journal of Magnetism and Magnetic Materials*, 374, PP: 680-689.

[19] Mahmoud S R, Abd-alla A M and El-Sheikh M A. (2011). Effect of the rotation on wave motion through cylindrical bore in a micro-polar porous medium. *International Modern Physics B*(25), PP: 2712-2728.

[20] Abdulhadi A M and Al-Hadad A H. (2016). Effects of rotation and MHD on the Nonlinear Peristaltic Flow of a Jeffery Fluid in an Asymmetric Channel through a Porous Medium. *Iraqi Journal of Science*, 57(10), PP: 223-240.