Understanding Nuclei in the upper $sd$ - shell

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Nuclei in the upper-$sd$ shell usually exhibit characteristics of spherical single particle excitations. In the recent years, employment of sophisticated techniques of gamma spectroscopy has led to observation of high spin states of several nuclei near $A \approx 40$. In a few of them multiparticle, multihole rotational states coexist with states of single particle nature. We have studied a few nuclei in this mass region experimentally, using various campaigns of the Indian National Gamma Array setup. We have compared and combined our empirical observations with the large-scale shell model results to interpret the structure of these nuclei. Indication of population of states of large deformation has been found in our data. This gives us an opportunity to investigate the interplay of single particle and collective degrees of freedom in this mass region.

I. INTRODUCTION

Nuclei in the $sd$ shell [1] has been studied since 1960s. In these earlier experiments, mostly low spin states were studied. Observation of spherical as well as deformed states were reported. However, study of the high-spin state was scarce due to experimental limitations in the sensitivity and efficiency of the detection system. Similarly, on the theoretical front, shell model calculations were also limited by severe truncation. In the recent years, sophisticated techniques of gamma spectroscopy permitted observation of high spin states of several nuclei in this mass region. States of pure shell model characteristics have been found to coexist with those having features of permanent deformation. Superdeformation (SD) in $^{40}Ca$ [2] and $^{36}Ar$ [3] have been reported. Strong violation of isospin symmetry observed in mirror pairs has been an issue of serious discussion [4]. The nuclei in the neighbourhood of doubly closed $^{40}Ca$ are usually suitable for applications of spherical shell model calculations. Theoretical interpretation of SD bands in upper-$sd$ shell [3] provides an ideal opportunity to extend the microscopic description of collective rotation in nuclei where cross-shell correlations play an important role. Indications of breaking of a shell closure (near $N = 20$) far from stability have been observed in $^{31}Na, ^{32}Mg$ -in the "island of inversion. Large basis cross-shell calculations have indicated the need for change of the $sd – fp$ energy gap for reliable reproduction of negative parity and high spin positive parity states in nuclei near stability also.

The gamma spectroscopic studies of these light mass nuclei are different from those of the heavier ones. These low $Z$ nuclei have lower Coulomb barriers. In fusion - evaporation reaction, the number of competing channels with evaporation of charge particles becomes very large with increasing excitation energy of the compound nucleus. At low excitation energies their level density is also low, the energies of the gamma rays connecting these states are usually very high ($\approx 2-3$ MeV) where the efficiencies of the normal HPGe detectors fall off sharply. Moreover, the correlated gammas emitted have low multiplicity, as the structure of these nuclei are dominated by shell model states. The maximum angular momentum of the single particle orbit in this mass region is $7/2$ in $1f_{7/2}$. So the angular momentum of the compound system is also restricted. As the spin increases, the energies of the transitions become higher implying lower detection efficiency and poorer resolution. In this respect the Clover detectors in their addback mode show excellent improvement over the normal detectors, so the Indian National Gamma Array (INGA) is an ideal setup for studying these nuclei.

In the present contribution, the gamma spectroscopic studies of fusion evaporation reaction residues with $A < 40$ done at various Indian accelerator facilities using the INGA [6] array will be discussed. It will encompass a few observations and related queries which were experienced while studying these nuclei lying on or close to the line of stability.
II. EXPERIMENTS

So far using different campaigns of INGA, we have been studying several upper sd shell nuclei near \( A=40 \). The primary motivations for these studies are:

- To understand the issue of the variation of \( sd - fp \) shell gap on the excitation spectra of nuclei on the stability line, viz., \(^{35}Cl\), \(^{30}P\), \(^{34}Cl\), \(^{36}Cl\), \(^{37}Ar\), etc.

- In search of a 2 particle \( n-p \) state in \(^{30}P\),

- The issue of isospin symmetry breaking and conservation in mirror nuclei \(^{35}Cl\) and \(^{37}Ar\)

- Collectivity and deformation in sd shell nuclei like, \(^{35}Cl\), \(^{34}Cl\), \(^{38}Ar\) etc.

The specifications of experiments done at different centres to study these issues are listed in Table I.

A. Preludes

As mentioned above, the gamma rays depopulating the excited states in the nuclei in this mass region usually have very high energies (\( \simeq 2 \) MeV or more). Since suitable radioactive sources having gamma rays of energies greater than 1.5 MeV are not easily available, the energy and efficiency calibration of the Clover detectors were done using \(^{66}Ga\) (\( T_{1/2} = 9.41 \) h) along with standard sources like \(^{60}Co\), \(^{133}Ba\) and \(^{152}Eu\). The radioactive \(^{66}Ga\) nuclei emit several gamma-rays with energies ranging from 833 to 4806 keV. This source was prepared by \(^{52}Cr(\alpha,pn)^{66}Ga\) reaction at 55 MeV \[9\]. Gamma ray spectra of two \((p,\gamma)\) resonances \( (^{13}C(p,\gamma)^{14}N, ^{27}Al(p,\gamma)^{28}Si) \) have been utilised for the characterisation of the Clover detector at energies beyond 5 MeV \[10\]. Apart from the efficiency and the resolution of the detector, the shapes of the full energy peaks as well as the nature of the escape peaks which are also very crucial at higher energies have been analysed with special attention.

In all the INGA experiments, the Clover detectors are used as polarimeters for measuring polarization asymmetry of gamma-rays to determine the electric or magnetic character of transitions. The sensitivity of different implementations of INGA setup are compared with earlier measurements of Palit et al. \[11\] (Fig. 1). The sensitivity for an ideal point scatterer is also shown in the figure for comparison.

### TABLE I: The list of experiments done at various INGA campaigns.

| Reaction | Beam energy (MeV) | Centre | No of clovers |
|----------|-------------------|--------|---------------|
| \(^{16}O\) \(^{16}O\) | 40 | TIFR | 8 |
| \(^{14}N\) \(^{27}Al\) | 66 | TIFR | 7 |
| \(^{16}O\) \(^{27}Al\) | 115 | VECC | 8 |
| \(^{28}Si\) \(^{12}C\) | 70, 88, 110 | TIFR, IUAC, IUAC | 8, 8, 13 |
| \(^{12}C\) \(^{27}Al\) | 40 | TIFR | 15 |
B. Measurements and analysis

In most of our on-line experiments at different acceler-ator centres, we have used reactions in inverse kinemat-
ics. In inverse kinematics, the recoils have large veloc-
ities ($\beta = v/c = 5-6\%$) and they move within a narrow
cone (half angle $13^\circ$) in the forward direction facilitating
observation of large lineshape of emitted gammas. The
spread in the recoil velocities is also within $7\%$ in the in-
verse reaction compared to $17\%$ in forward reaction. This
reduction in the spread decreases the uncertainty in the
lineshape fitting. However, in some cases, the large shifts
and broadening pose additional problems in analysing the
lineshape data.

In these experiments, substantial number of reaction
channels are populated which result in the presence of
many overlapping gamma - rays, This makes determin-
ation of intensities from a singles spectrum extremely
difficult, if not impossible, in many cases. Therefore,
relative intensities have been extracted from symmet-
ric matrices having data from all the detectors on both
the axes. The multipolarities of transitions have been
obtained through a measurement of directional correla-
tion (DCO) of gamma -rays deexciting oriented states.
For assignment of spins and gamma -ray multipole mix-
ing ratios, the experimental DCO ratios were compared
with the theoretical ones using a standard program. We
have performed integrated polarization asymmetry mea-
surements (IPDCO). For this purpose two asymmetric
IPDCO matrices named parallel and perpendicular were
constructed from the data [7, 8]. Lifetime analysis using
Doppler Shift Attenuation Method (DSAM) was done us-
ing asymmetric coincidence matrices having on one axis
events from the detector at a particular angle and on
the second axis the coincident gamma rays registered in

FIG. 5: The results of crude shell model calculations for $7^+$
($\pi_1f_{7/2} - \nu_1f_{7/2}$) and $5^-$ ($\pi_1d_{3/2} - \nu_1f_{7/2}$) states in odd-odd
nuclei.

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FIG. 4: Experimental and theoretical spectra for $^{34}$Cl. The new transitions are marked by star (*)
any other detector. Level lifetimes were extracted using both the centroid shift method and the lineshape analysis. The details about our analysis may be found in any one of our previous work \[7, 8\].

**III. RESULTS AND DISCUSSION**

A. The effect of the sd – fp shell gap

\(^{35}\)Cl is a stable nucleus with 75.77\% abundance in natural Chlorine. With \(^{16}\)O core, it has nine and ten valence protons and neutrons, respectively. In its low excitation spectra, the first positive parity state at 1219 keV and the most intense gamma is emitted from the lowest negative parity state at 3163 keV. Therefore, inclusion of a negative parity orbital in the valence space is essential to explain the low energy spectrum. In the shell model calculations, in particular for the negative parity states and for the positive parity states of relatively higher spins, a nuclear Hamiltonian over the sd – fp valence space is needed. The Hamiltonian thus consists of three parts, viz., sd and fp shell interactions and the cross-shell ones. Large basis shell model calculations have been done using the code OXBASH \[12\]. The valence space consists of \(1d_{5/2}, 1d_{3/2}, 2s_{1/2}, 1f_{7/2}, 1f_{5/2}, 2p_{3/2}\) and \(2p_{1/2}\)-orbitals for both protons and neutrons above the \(^{16}\)O inert core. The sd\(p_{3/2}\) interaction used is taken from WBMB \(sdfp\) shell Hamiltonian \[13\]. Our observations from these studies are:

- To reproduce the low-lying positive parity states \[7, 8\], \(0\hbar\omega\) excitation has been considered, i.e., only the full sd-shell has been used as the valence space. Low energy positive parity spectra are reproduced quite accurately with this truncation.

- However, the energies of higher spin positive parity states beyond \(9/2^+\) are predicted substantially higher than the experimental ones. This indicates that at relatively higher spins, the valence space considered become inadequate for a proper description of the state. Nucleon excitations to the neighbouring fp shell are therefore essential at these spins.

- The simplest way to get negative parity states is to consider \(1\hbar\omega\) excitation, i.e. only \(1p - 1h\), sd \(\rightarrow\) fp excitations are allowed.

- For these excitations, the calculated energies of the negative parity levels are consistently higher compared to the experimental values. This feature was also observed by previous workers, viz., \[7, 14\] where the predictions for negative parity states with accuracy better than 500-600 keV were found to be difficult. This was attributed to the overestimation of the sd – fp shell gap in the corresponding interaction.

Later we included a few modifications in the Hamiltonian, which led to remarkable improvement of results. The observations made consequently are:

- Excitations to fp shell are essential to reproduce even the positive parity higher spin states.

- The sd – fp shell gap has to be decreased to reproduce both positive and negative parity levels. To improve the agreement for the negative parity and high spin positive parities states, we have depressed the single particle energies (SPES) of \(1f_{7/2}\) and \(2p_{3/2}\) so that the first negative parity state is exactly reproduced.

We have found that similar modifications in the sd – fp shell gap are needed to reproduce our experimental data for negative parity as well as high spin positive parity states in \(^{30}\)P \[8\], \(^{36}\)Cl (Fig. 2), \(^{37}\)Ar (Fig. 3) \[15\]. We have studied \(^{34}\)Cl (Fig. 4), an odd-odd nucleus in the sd shell, which is of interest for its importance in astrophysical scenario. Spherical shell model calculations have been done to interpret the experimental data (Fig 4). Several options of calculations with different values of mass normalisation for the sd shell interaction as well as variation...
in the monopole shift in the fp interaction (Theo-I, II and III) have been adopted to study the effect of the variation of sd – fp shell gap in this cross-shell calculations.

In a few recent work, Bender et al. [17] and Steppenbeck et al. [18] used the new WBP-a Hamiltonian to calculate the negative-parity states by allowing 1p1h excitations within a model space incorporating the sd shell with the 1f7/2 and 1p3/2 orbitals. In this modified interaction, the energies of the two fp-shell orbits were reduced by 1.8 and 0.5 MeV to better reproduce level energies. On the other hand, Ionescu-Bujor et al. [19] used two different interactions for this mass regions. They indicated that the shell gap between the sd and fp shells produced by the first interaction (sdfp) is somewhat underestimated showing a need for increasing the sd – fp gap, while this is overestimated for SDPF – M interaction indicating a need for decreasing the sd – fp gap.

To understand the origin of this difference, we carefully investigated the choices of single-particle energies in the interactions used in this mass region. There are primarily two different sets used for these calculations. In the Set A, the single-particle energies (SPEs) are determined so as to reproduce the neutron separation energies and the one particle spectra of 17O (sd shell) and 41Ca (fp shell). SDPF-M, sdfp,mw Hamiltonians use this Set A.

On the other hand, the sdfp effective interaction takes 28Si as a core, and the single-particle energies (SET B) are chosen in order to reproduce the single-particle states in 29Si. The calculated energies of the negative parity states are systematically smaller than the experimental ones indicating that the shell gap between the sd and fp shells produced by the sdfp interaction is somewhat underestimated. So the SET B is modified by increasing the 1f7/2 and 1p3/2 single-particle energies. If one analyses the experimental energies of the first negative parity states observed in these upper-sd shell nuclei, it may appear that the sd – fp shell gap evolves as a function of proton number. We are continuing this study for more conclusive results. This study emphasizes that the issues like erosion of shell gap and observation of "island of inversion" thought to be relevant for nuclei away from stability, should also be studied over a broader region of nuclear territory. With the latest developments in the experimental facilities, the nuclei near the stability line should be re-investigated for finding possible correlations with those away from stability.
B. In search of the $1\pi - 1\nu$ $7^+$ state in $^{30}P$

We have studied $^{30}P$ in one of our earlier experiments using the Indian National Gamma (Clover) Array (INGA) up to moderate spins ($I = 5$). To understand the underlying structure of the levels and transition mechanisms, experimental data have been compared with the results from large basis cross-shell calculations. The results for the negative parity states are especially important in this respect. Positive parity states indicate an onset of collectivity manifested through large configuration mixing, whereas the negative parity states are members of neutron-proton multiplets. Later in continuation to this work, we have used the DCO and polarisation measurement data to determine the multipolarity and character of the 2858 keV transition de-exciting the 7202 keV state in $^{30}P$. Unambiguous assignment of spin could not be made after analysing these data. Untruncated full $sd$ shell calculations favour assignment of a $6^+$ option to the state. However, a truncated $sd$ - $fp$ shell calculation and a comparison of the measured and calculated half-lives of the level from two calculations do not totally rule out a $7^+$ option for it. So we have done a crude shell model calculation (Fig. 5) to identify the $(1f_{7/2})^2$ state by comparison with neighbouring nuclei. In this model the $(1f_{7/2})^2$ states in an odd-odd nucleus $(2Z+1, 2N+1)$ have been obtained by exciting one proton and one neutron in $1f_{7/2}$ state. The excitation energy of that state is obtained by simply adding their single particle energies ($spe$). The simplest and the best way to calculate the $1f_{7/2}$ proton single particle energy is from the experimental $7/2^-$ state of the isotopic $(2Z+1,2N)$ odd-A nucleus and for neutrons from that in the isotonic $(2Z,2N+1)$ odd-A nucleus. The energy of the $(1f_{7/2})^2$ state in $^{30}P$ is indicated in the Fig. 6. This study favours the assignment of $1\pi - 1\nu 7^+$ to the state at 7202 keV from which the 2858 keV gamma ray in $^{30}P$ is emitted.

C. Studies on Mirror Energy differences (MED)

Studies on MED has been pursued to test the charge symmetry of nuclear force. It is well known that a complementary way to test isospin symmetry is based on investigation of the electromagnetic decay properties in mirror pairs. Anomalous MED in $sd$ shell nuclei, $^{35}Cl$ and $^{35}Ar$ has been observed [4]. In the present effort, we have done measurements of the lifetimes of excited higher spin levels of $^{35}Cl$, mixing ratios of a few gamma transitions in $^{35}Ar$ and determined polarisation asymmetry of some gamma rays (Fig. 6) of both the nuclei. We are analysing these data further to compare the electromagnetic decay properties of the mirror partners.

D. Collectivity and deformation in $sd$-shell nuclei

The nuclei in the neighborhood of doubly closed $^{40}Ca$ usually exhibit characteristics of single particle excitation. The spectroscopy of several nuclei in the mass region revealed deformed states (even Superdeformation) at low excitation energies, indicating that the nuclei near the closed shell with $Z=20$ and $N=20$ can easily lose spherical shape.

1. Indication of deformation in $^{35}Cl$

In our earlier work discussed in Ref.[7], we have discussed about the six shifted peaks from around 900 keV to 3000 keV. We concluded that the lifetimes of the corresponding states must be shorter than the characteristic stopping time of $^{35}Cl$ recoils in gold (Au) backing. This indicates that the corresponding transition probabilities
FIG. 13: The shifted gammas in $^{34}\text{Cl}$ gamma spectrum.

must be large implying large deformation. So we investigated whether these gammas belong a single deformed band. DCO values have been extracted to determine their multipolarities. The exact energies of these shifted peaks have been determined by plotting the centroid energies as function of $\cos(\theta)$ have been plotted (Fig. 7). The fractional Doppler shift ($F(\tau)$) for these gammas are shown in Fig 8.

To facilitate determination of correlation between these shifted gammas a $90^0 \rightarrow 90^0$ matrix has been generated. It has been found that only four of these gammas are in a sequence. All these four gammas have E2 multipolarity. The lifetimes of these four states have been determined using Doppler shift attenuation method to get the quadrupole moment of the band. The quadrupole moment extracted from preliminary analysis using centroid shift ($F(\tau)$) method is around 0.7 eb.

As an initial attempt, we have done theoretical calculations using a version of the Particle-Rotor Model (PRM), where experimental energies of the core can be directly given as inputs \cite{21}. Detailed large scale shell model calculations have been done to understand this band. We have done two sets of calculation to distribute 7 nucleons in the $2s_{1/2} - 1d_{5/2} - fp$ orbitals. In the first set, they are distributed with $(sd)^3 - (fp)^4$ - as shown in Fig. 9. Although the calculated states are strongly configuration mixed - they do not seem to form a collective band. The gamma energies connecting these states when plotted as a function of angular momentum show erratic distribution for both the signatures, instead of showing regularity. On the other hand, with $(sd)^4 - (fp)^3$ - a linear dependence (Fig. 10) is found with a kink at 19/2$^{-}$. The experimental data also agree reasonably well with calculations. A comparison between the wavefunction structures of 7/2$^{-}$ generated from $(sd)^6 - (fp)^3$ and $(sd)^4 - (fp)^3$ (Fig. 11), clearly show a strongly collective structure for $fp - 3$ band. The calculated B(E2) values are also large (Fig. 12) with gradually decreasing trend for the higher spins while approaching the termination of this band. The experimental angular correlation, polarisation and lifetime data are being analysed in detail to reach to a firm conclusion.

For other nuclei in this mass regions, viz., $^{34}\text{Cl}$ (Fig. 13) and $^{38}\text{Ar}$ gamma spectra show similar shifted peaks indicative of states with large deformation. In future we shall try to understand the evolution of collectivity with increasing neutron number as well as its interplay with the single particle modes in these light-mass nuclei.

IV. CONCLUSION

The intruder orbitals from the $fp$ shell play an important role in the structure of nuclei around the line of stability in the upper $sd$ shell. Experimental and theoretical studies of these nuclei near stability with higher precision have deeper implication for understanding the issues relevant for exotic nuclei in the island of inversion. Interplay between collectivity and single particle behaviour is also important for these nuclei. Shell model calculations for explaining these experimental features give us an opportunity to investigate this interplay microscopically.

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