Uncovering Locally Discriminative Structure for Feature Analysis

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Supplementary Material

Lemma 1. For any matrix $A$, $A(A^T A + \theta I)^{-1} = (AA^T + \theta I)^{-1} A$ holds.

Proof. For any matrix $A$, it is obvious that we have $(AA^T + \theta I)A = A(A^T A + \theta I)$. Then,

$$A(A^T A + \theta I)^{-1}$$

$$= (AA^T + \theta I)^{-1}(AA^T + \theta I)A(A^T A + \theta I)^{-1}$$

$$= (AA^T + \theta I)^{-1}A(A^T A + \theta I)(A^T A + \theta I)^{-1}$$

$$= (AA^T + \theta I)^{-1} A$$


Theorem 1. The objective function in (14) is equivalent to the following objective function:

$$\arg \min_{G_i} \text{Tr} \left( G_i^T L_i G_i \right),$$

where $L_i = H(\bar{X}_i^T \bar{X}_i + \theta I)^{-1} H$

Proof. According to Lemma 1 and $\bar{X}_i H_k = \bar{X}_i$, we have

$$G_i^T \left( \bar{X}_i^T (\bar{X}_i \bar{X}_i^T + \theta I)^{-1} \bar{X}_i \right) G_i$$

$$= G_i^T \left( H \bar{X}_i^T (\bar{X}_i \bar{X}_i^T + \theta I)^{-1} \bar{X}_i H \right) G_i$$

$$= G_i^T \left( H (\bar{X}_i \bar{X}_i^T + \theta I)^{-1} \bar{X}_i \bar{X}_i^T \bar{X}_i H \right) G_i$$

$$= G_i^T \left( H (\bar{X}_i \bar{X}_i^T + \theta I)^{-1} (\bar{X}_i \bar{X}_i^T \bar{X}_i + \theta I - \theta I) H \right) G_i$$

$$= G_i^T \left( H - \theta H (\bar{X}_i \bar{X}_i^T + \theta I)^{-1} H \right) G_i$$

Thus, the optimization problem in Eq. (14) is equivalent to the problem (15).