A Theory of Heap for Constrained Horn Clauses
(Extended Technical Report)

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Abstract. Constrained Horn Clauses (CHCs) are an intermediate program representation that can be generated by several verification tools, and that can be processed and solved by a number of Horn solvers. One of the main challenges when using CHCs in verification is the encoding of heap-allocated data-structures: such data-structures are today either represented explicitly using the theory of arrays, or transformed away with the help of invariants or refinement types, defeating the purpose of CHCs as a representation that is language-independent as well as agnostic of the algorithm implemented by the Horn solver. This paper presents an SMT-LIB theory of heap tailored to CHCs, with the goal of enabling a standard interchange format for programs with heap data-structures. We introduce the syntax of the theory of heap, define its semantics in terms of axioms and using a reduction to SMT-LIB arrays and data-types, and discuss its properties and outline possible extensions and future work.

1 Introduction

Constrained Horn Clauses (CHCs) are a convenient intermediate verification language that can be generated by several verification tools in many settings, ranging from verification of smart contracts [16] to verification of computer programs in various languages [10,11,15,22,34]. The CHC interchange language provides a separation of concerns, allowing the designers of verification systems to focus on high-level aspects like the applied proof rules and verification methodology, while giving CHC solver developers a clean framework that can be instantiated using various model checking algorithms and specialised decision procedures. Solver performance is evaluated in the annually held CHC-COMP [33].

CHCs are usually expressed using the SMT-LIB standard, which itself is a common language and interface for SMT solvers [2]. Abstractly, both SMT solvers and CHC solvers are tools that determine if a first-order formula is satisfiable modulo background theories such as arithmetic, bit-vectors, or arrays.

One of the main challenges when using CHCs, and in verification in general, is the encoding of programs with mutable, heap-allocated data-structures. Since there is no native theory of heap in SMT-LIB, one approach to represent such data-structures is using the theory of arrays (e.g., [13]). This is a natural encoding since heap can be seen as an array of memory locations; however, as the encoding is byte-precise, in the context of CHCs it tends to be low-level and often yields clauses that are hard to solve.
An alternative approach is to transform away such data-structures with the help of invariants or refinement types (e.g., [31,25,15]). In contrast to approaches that use the theory of arrays, the resulting CHCs tend to be over-approximate (i.e., can lead to false positives), even with smart refinement strategies that aim at increasing precision. This is because every operation that reads, writes, or allocates a heap object is replaced with assertions and assumptions about local object invariants, so that global program invariants might not be expressible. In cases where local invariants are sufficient, however, they can enable efficient and modular verification even of challenging programs.

Both approaches leave little design choice with respect to handling of heap to CHC solvers. Dealing with heap at encoding level implies repeated effort when designing verifiers for different programming languages, makes it hard to compare different approaches to encode heap, and is time-consuming when a verifier wants to switch to another encoding. The benefits of CHCs are partly negated, since the discussed separation of concerns does not carry over to heap.

The vision of this paper is to extend CHCs to a standardised interchange format for programs with heap data-structures. To this end, we present a high-level theory of heap that does not restrict the way in which CHC solvers approach heap, while covering the main functionality of heap needed for program verification: (i) representation of the type system associated with heap data; (ii) reading and updating of data on the heap; (iii) handling of object allocation.

We use algebraic data types (ADTs), as already standardised by SMT-LIB v2.6, as a flexible way to handle (i). The theory offers operations akin to the theory of arrays to handle (ii) and (iii). The theory is deliberately kept simple, so that it is easy to add support to SMT and CHC solvers: a solver can, for instance, internally encode heap using the existing theory of arrays (we provide one such encoding in Section 6.2), or implement transformational approaches like [4,25]. Since we want to stay high-level, arithmetic operations on pointers are excluded in our theory, as are low-level tricks like extracting individual bytes from bigger pieces of data through pointer manipulation. Being language-agnostic, the theory of heap allows for common encodings across different applications, and is in the spirit of both CHCs and SMT-LIB.

Listing 1: The motivating example in Java

```java
abstract class IntList {
    protected int _sz;

    abstract int hd();
    abstract void setHd(int hd);
    abstract IntList tl();

    int sz() { return _sz; }
}

class Nil extends IntList {
    Nil() {_sz = 0;}

    int hd() {err();}
    void setHd(int hd) {err();}
    IntList tl() {err();}
}

class Motivation {
    void main() {
        IntList l = new Cons(42,
            new Nil());
        l.setHd(l.hd()+1);
        assert(l.hd() == 43);
    }
}
```
Contributions of the paper are (i) the definition of syntax and two possible formulations of semantics of the theory of heap (axiomatic and through an encoding into the theory of arrays); (ii) a discussion on how programs can be encoded using the heap theory; and (iii) properties of the theory.

Acknowledgements. This is the first full paper introducing the theory of heap. An earlier version of the theory was presented at the HCVS Workshop 2020 \[8\] and the SMT Workshop 2020. An invited paper at LOPSTR 2020 discusses preliminary work on decision and interpolation procedures \[9\]. We are grateful for the discussion and feedback provided by the different communities.

2 Motivating Example

We start with a high-level explanation how heap is handled by our theory. Listing 1 shows a simple Java program which constructs a singly-linked list, highlighting various heap interactions such as allocating objects on the heap (lines 26–27), as well as reading (lines 28–29) and modifying (line 28) heap data.

In order to encode this program we use Constrained Horn Clauses (CHCs), which we assume knowledge of (see Section 3 for a brief introduction). Although we present the theory in the context of CHCs, there is nothing CHC-specific in the theory itself; as discussed earlier support for the theory can easily be added to both SMT and CHC solvers since it is kept deliberately high level and simple. The encoding is given in Listing 2 in SMT-LIB v2.6 format.

Heap declaration To encode this program using the theory of heap, first a heap has to be declared that covers the program types as shown at lines 1–12 of Listing 2. Each heap comes with its own sorts for the heap itself and for heap locations (or addresses). Lines 2 and 3 are the names of declared heap and address sorts. We next need to define which data can be placed on the heap, which is done by choosing the sort of heap objects; this sort can be any of the sorts declared prior to or together with the heap declaration, excluding the heap sort itself. Line 4 specifies the object sort to be the ADT Object, declared later.

Line 5 defines the object assumed to be stored at unallocated heap locations. Since functions in SMT-LIB are total, semantics has to be defined also for reads from such unallocated addresses. The theory of heap leaves the choice of object produced by such reads to the user; the term specified at line 5 must have the object sort chosen at line 4. We call this the default object (or defObj), which in this case is created using the object constructor O_Empty.

The rest of the heap declaration at lines 6–12 corresponds to an SMT-LIB data-type declaration. In line 6, in addition to Object we declare data-types IntList, Cons, and Nil, encoding the classes of the program. The constructors at lines 7–9 specify the fields of each class, and in addition give Cons and Nil each a field containing the parent IntList object. In lines 10–12, the constructors of the Object sort are declared, which correspond to the classes Cons and Nil, as well as the default object O_Empty. The class IntList is abstract and does not occur directly on the heap, so that no constructor for this type is provided.
Since each heap theory has its own address sort, cases are immediately prevented in which multiple heaps share the same address sort, or in which some other interpreted sort (say, \texttt{Int}) is used to store addresses. This rules out accidental cases of pointer arithmetic, and leaves full flexibility to solvers on how to internally represent addresses (e.g., see [14]). This choice also makes it necessary to include the ADT declarations within \texttt{declare-heap}, since ADTs representing objects often have to refer to the address sort.

Within one heap, all pointers are represented using a single \texttt{Address} sort, and no distinction is made between pointers to objects from different constructors. This is close in semantics to languages like C, where casts between arbitrary pointer types are possible, and it has to be verified for each heap access that indeed an object of the right type is accessed. In languages like Java, the stronger type system will provide information about the objects a variable can refer to, but exceptions can be raised when performing casts. The theory of heap is flexible enough to cover those different settings.

Apart from the sorts mentioned, the heap declaration implicitly declares an ADT \texttt{ARHeap} (also called \texttt{AllocationResultHeap} later in the paper) that holds pairs \(<\texttt{Heap}, \texttt{Address}>\) returned as a result of allocations.

**Program encoding** Invariants representing program states are declared at lines 14–17. The first set of arguments in the parentheses list the sorts of the variables we want to keep track of at that point. E.g., for line 17, we want to have a global view of the heap, as well as all variables on the stack at that point. The only variable on the stack at this point is a temporary variable \(p\) that corresponds to the newly allocated \texttt{Nil} object’s address (line 27 in Listing 1).

Line 19 is the program entry point, where the heap is initially empty. The function \texttt{emptyHeap} returns an empty heap (i.e., unallocated at all locations) of the declared \texttt{Heap} sort specified at line 2. Lines 20–26 allocate, respectively, a \texttt{Nil} object and a \texttt{Cons} object on the heap. Allocation is done using the \texttt{allocate} function of the theory, which takes as arguments the old heap and the new object to be put on the heap, and returns an \texttt{ARHeap} pair with the new heap and the allocated address. Constructor calls are inlined and slightly simplified in the encoding. For example, line 25 shows the simplified encoding of the Java constructor for \texttt{Cons} at lines 20–23 of Listing 1. The updating of the \texttt{sz} field is simplified by directly assigning a value to it, which would actually require another clause with a read due to the statement at line 23 of Listing 1.

Lines 27–33 correspond to the statement at line 28 from Listing 1 which calls the methods \texttt{hd} and \texttt{setHd} corresponding to a read-modify-write operation on the list. We again inline these methods in the encoding; however, since both \texttt{Nil} and \texttt{Cons} define these methods, we add a clause for each (lines 29–31 encode \texttt{Cons.hd}() and \texttt{Cons.setHd}(), while lines 31–33 encode \texttt{Nil.hd}()). For brevity we do not show the clause for the inlined call to \texttt{Nil.setHd}(), which is similar to the encoding at lines 32–33. The assertion at lines 27–28 checks the validity of accesses in order to ensure memory safety.

Lines 27–33 illustrate the use of \texttt{read} and \texttt{write} functions. \texttt{read} reads from the provided heap at the given location, and \texttt{write} writes the provided object to the
Listing 2: SMT-LIB encoding of the motivating example from Listing 1. The symbols of some sorts and operations of the theory are abbreviated and the list of quantified variables are skipped in some cases for brevity.

```
(declare-heap
 Heap
 Addr
 Object
 0_Empty
 ((IntList 0) (Cons 0) (Nil 0) (Object 0)) ; ADTs
 ((IntList sz ...) Int)) ; Class constructors
 (Cons (parentCons IntList) (hd Int) (tl Addr))
 (Nil (parentNil IntList))
 (0_Cons (getCons Cons)) ; Object sort constructors
 (0_Nil (getNil Nil))
 (0_Empty)))
)

(assert (I1 emptyHeap))
(assert (forall ((h Heap) (hil Heap) (p1 Addr))
 (=> (and (I1 h) (* (AHeap hil p1) (allocate h (0_Nil (Nil (IntList 0))))))
  (I2 hil p1)))
(assert (forall (...) (=> (and (I2 h p)
  (* (AHeap hil p1) (allocate h (0_Cons (Cons (IntList 1) 42 p))))))
  (I3 hil p1)))
(assert (forall (...) (=> (and (I3 h l) (not (valid h l))) false))
(assert (forall (...) (=> (and (I3 h l) (not (valid h l))) false))
(assert (forall (...) (=> (and (I4 h l) (not (is-0_Cons (read h l)))) false)))
(assert (forall (...) (=> (and (I4 h l) (not (is-0_Cons (read h l)))) false)))
```

heap at the specified location. The dynamic dispatch needed when calling \texttt{hd} is implemented through pattern matching using the \texttt{0_Cons} and \texttt{0_Nil} constructors: in lines 29–31 the method call is successful, and the heap object is subsequently updated, while the clause at lines 32–33 models the error when executing \texttt{Nil.hd}. The same property can be expressed using the tester `is=0_Cons` in lines 37–38.

Lastly, lines 34–36 encode the assertion at line 29 from Listing 2.

### 3 Preliminaries

**Definition of a Theory** A *signature* (or vocabulary) \( \Sigma \) of a many-sorted logic is defined as the triple containing a set \( S \) of sorts, a set \( \Sigma_f \) of function symbols, and a set \( \Sigma_r \) of relation symbols. The arguments of functions and relations, and the values of functions are specified using sorts from \( S \). A \( \Sigma \)-formula uses only non-logical symbols from \( \Sigma \), in addition to logical symbols. A \( \Sigma \)-sentence is a \( \Sigma \)-formula that contains no free variables.
A $\Sigma$-theory $T$ is defined as a set of $\Sigma$-sentences closed under entailment. A $\Sigma$-formula $\phi$ is said to be $T$-satisfiable if a structure exists which satisfies both the sentences of $T$ and $\phi$. We call this structure a $T$-model of $\phi$.

**The Theory of Arrays** The idea of a (non-extensional) first order theory of arrays was first introduced by McCarthy [23]. It has the two functions select and store, whose semantics are given through the following read-over-write axioms:

\[
\forall a, i, j, e. (i = j \rightarrow \text{select}(\text{store}(a, i, e), j) = e) \quad \text{[array-row1]}
\]

\[
\forall a, i, j, e. (i \neq j \rightarrow \text{select}(\text{store}(a, i, e), j) = \text{select}(a, j)) \quad \text{[array-row2]}
\]

where $a$ is an array, $i$ and $j$ are indices, and $e$ is an element stored in the array.

Extensionality is introduced by an additional axiom, which allows reasoning about equality between two arrays:

\[
\forall a_1, a_2, i. (\text{select}(a_1, i) = \text{select}(a_2, i) \rightarrow a_1 = a_2) \quad \text{[array-ex]}
\]

The theory of arrays is one of the background theories defined by SMT-LIB, and as such, many solvers have specialised decision procedures for the decidable fragments of this theory.

**Algebraic Data-Types (ADTs)** Algebraic data-types (ADTs or data-types) provide a flexible way to represent types in many programming languages, and many SMT solvers provide native decision procedures to solve them efficiently [1,32,26]. They are supported in the SMT-LIB standard since version 2.6 through the declare-datatype and declare-datatypes commands. Non-recursive ADTs can be used to represent programming types such as enumerations, records and unions, while recursive ADTs can represent types such as arrays, lists and strings.

**Constrained Horn Clauses (CHCs)** A CHC is a sentence

\[
\forall x_1, x_2, \ldots, (C \land B_1 \land \ldots \land B_n \rightarrow H)
\]

where $H$ is either an application of a $k$-ary predicate $p(t_1, \ldots, t_k)$ to first-order terms or false, $B_i$ (for $i = 1 \ldots n$) is an application of an $m$-ary predicate $p_i(t_1, \ldots, t_m)$ to first-order terms, and $C$ is a constraint over some background theories (in this case including the proposed theory of heap). The universal quantification of first-order variables in a clause is usually not explicitly specified.

CHCs provide a natural way to encode programs: invariants represent program states, state transitions and assertions can be encoded through constraints and contradictions. A set of CHCs is solvable if no contradiction can be derived. We refer to other sources such as [3,10] for a more comprehensive explanation.

4 Vocabulary and Syntax of the Theory of Heap

4.1 SMT-LIB-style Declaration of Heaps

A theory of heap is declared as follows:

\[
\text{[declare-heap } c_h \ c_a \ c_o \ \tau_o \ ((\delta_1 k_1) \ldots (\delta_n k_n)) (d_1 \ldots d_n)]
\]
where \( c_h, c_a, c_o \) are symbols corresponding to the names of declared heap, declared address and chosen object respectively. \( \tau_o \) is a term of the chosen object which is returned on invalid accesses (i.e., the default object). The object sort can be chosen as any sort except \( c_h \). The rest of the declaration resembles the `declare-datatypes` declaration from the SMT-LIB standard v2.6 [2], with the exception that polymorphism is (currently) not supported in constructor declarations, and that there should be \( n \) (where \( n \geq 0 \)) instead of \( n + 1 \) ADT sort declarations (i.e., the object sort can also be declared before the heap declaration and specified using \( c_o \), if it does not use the address sort \( (c_a) \) in its declaration).

The concrete syntax for the heap declaration is given below, which extends \( \langle \text{command} \rangle \) in the concrete syntax of SMT-LIB v2.6.

\[
\langle \text{command} \rangle := \ldots
| ( \text{declare-heap} \langle \text{symbol} \rangle \langle \text{symbol} \rangle \langle \text{sort} \rangle \langle \text{term} \rangle
\quad (\langle \text{sort}_{\text{dec}} \rangle^n) \quad (\langle \text{heap}_{\text{datatype}_{\text{dec}}} \rangle^n) )
\]

\[
\langle \text{heap}_{\text{datatype}_{\text{dec}}} \rangle := (\langle \text{constructor}_{\text{dec}} \rangle^+)\]

The first two symbols and the following sort in the declaration correspond respectively to \( c_h, c_a \) and \( c_o \) from the abstract syntax. \( \langle \text{term} \rangle \) is the default object.

### 4.2 Sorts

Each heap declaration introduces several sorts. The names of these sorts are defined by the variables in the `declare-heap` command, which we assume in this paper to be `Heap` for \( c_h \) and `Address` for \( c_a \):

- a sort `Heap` of heaps,
- a sort `Address` of heap addresses,
- zero or more ADT sorts used to represent heap data,
- an additional ADT sort that holds the pair \( \langle \text{Heap}, \text{Address} \rangle \) which is the result of calling `allocate`. In order to make this ADT sort distinguishable, it is suffixed with associated heap sort `Heap` (e.g. `AllocationResultHeap`).

### 4.3 Operations

Below we describe each function of the theory; the semantics are given more formally through the axioms in Section 6.1. They are not listed below, but we also get access to all ADT operations as a side effect of heap declarations. Some operations contain the symbols `Heap` and `Address` in their signatures. This is done with the assumption that the declared heap and address sorts are named `Heap` and `Address` respectively. E.g. `nullAddress` would be `nullA` if the declared address sort was named \( A \), and it would return the sort \( A \). Including the sort name in some function and sort names makes it possible to determine their associated heap declarations without using the SMT-LIB command “as”. This is not required in sorts and functions where the associated heap sort is clear, such as in `read` (its first argument is of heap sort).

\[
\text{nullAddress} : () \rightarrow \text{Address}
\]
Function `nullAddress` returns an `Address` which is always unallocated/invalid.

```latex
\text{emptyHeap} : () \rightarrow \text{Heap}
```

`emptyHeap` returns the `Heap` that is unallocated everywhere.

```latex
\text{allocate} : \text{Heap} \times \text{Object} \rightarrow \text{Heap} \times \text{Address} \ (\text{AllocationResultHeap})
```

Function `allocate` takes a `Heap` and an `Object`, and returns `AllocationResultHeap`. `AllocationResultHeap` is a data-type representing the pair \( \langle \text{Heap}, \text{Address} \rangle \). The returned `Heap` at `Address` contains the passed `Object`, with all other locations unchanged. The pair ADT is required as the return sort since it is not possible in SMT-LIB to return the two sorts separately. In Section 8 we discuss other alternatives such as using multiple allocation functions.

```latex
\text{valid} : \text{Heap} \times \text{Address} \rightarrow \text{Bool}
```

The predicate `valid` checks if accesses to the given `Heap` at the given `Address` are valid. We say that an access is valid if and only if that location was allocated beforehand by using the function `allocate`.

```latex
\text{read} : \text{Heap} \times \text{Address} \rightarrow \text{Object}
\text{write} : \text{Heap} \times \text{Address} \times \text{Object} \rightarrow \text{Heap}
```

Functions `read` and `write` are similar to the array `select` and `store` operations described in Section 3; however, unlike an array, a heap also carries information about allocatedness. This means the `read` and `write` functions only behave as their array counterparts if the considered address is allocated. If the read address is unallocated, a default `Object` is returned to make the function total (as explained in Section 2 / Heap Declaration).

The function `write` normally returns a new `Heap` if the access is valid. If not, then the original `Heap` is returned without any changes. Validity of a `write` can be checked via memory-safety assertions as shown in lines 27–28 of Listing 2.

We propose a further short-hand notation `nthAddress`, which is useful when presenting satisfying assignments. It is used to concisely represent `Address` values which would be returned after `i` `allocate` calls, which is only possible with the deterministic allocation axiom \[ \text{alloc2} \] given in Section 6.1.

5 Encoding of Different Programming Languages

Java and Java-like Languages We have outlined in Section 2 how a Java class hierarchy can be encoded using the theory of heap, and how the different Java instructions can then be translated to CHCs. Every class is mapped to one
ADT, representing inheritance by adding a parent field to the sub-classes of a class, and defining an Object ADT as the union of the types that can occur on the heap. Java interfaces do not have to be considered explicitly, since in Java they are abstract and do not store data. Arrays and strings can in principle be handled using recursive ADTs, although it is probably more efficient to integrate the theory of arrays for this purpose (Section 8). Java also supports parametric polymorphism (generics), but implements it using type erasure, which means that type parameters do not explicitly occur on the heap and do not have to be stored. In languages with native polymorphism, for instance C#, types can be encoded using dedicated ADTs as part of a heap declaration, and type parameters of classes and methods can be represented using explicit fields/arguments.

Programs in C and C++ Our theory implements a relatively abstract view of the heap, and does not provide a byte-level heap model, which implies that not all C features can be handled directly. We believe that the theory represents a good trade-off, however, for analysing functional aspects of C programs that avoid undefined behaviour. Structs, enums, and unions in C can all be mapped to ADTs in a similar way as Java classes. C can in addition store native types like int on the heap, which can be encoded easily through further ADTs. Unsafe pointer conversions can be supported by verifying, using appropriate CHCs, that read/write accesses to objects only happen through the correct type; CHCs can also define certain byte-level conversions of objects. This way it is possible, among others, to give correct semantics to patterns like byte-level heap allocation using malloc or calloc.

Several other C features cannot be supported within the heap theory. The theory strictly rules out pointer arithmetic between objects at different addresses; it would be possible, however, to encode pointer arithmetic within an object already at the encoding level. Stack pointers are outside of the scope of the heap theory, but can also to some degree be handled during the CHC encoding. A further operation allowed by C, but not considered in the theory, is the deallocation of heap locations; this could be supported with the addition of a free function to the theory. Extensions to the theory are discussed in Section 8.

Heap in C++ can be modelled essentially by combining the techniques discussed for Java and C. Multiple inheritance of classes, which is possible in C++, can be encoded by adding multiple parent fields in the sub-classes. C++ templates, realising compile-time polymorphism, can be handled by adding separate ADTs for each template instance.

6 Semantics of the Theory

6.1 Axiomatic Semantics

We first propose and discuss a set of axioms defining the semantics of the heap theory. All variables occurring in the axioms are universally quantified with sorts \( h : \text{Heap} \), \( p : \text{Address} \), \( o : \text{Object} \) and \( ar : \text{AllocationResultHeap} \). Variables can also appear subscripted. \( \text{AllocationResultHeap} \) is the pair \((\text{Heap}, \text{Address})\), we use \( ar._1 \) and \( ar._2 \) to select the \( \text{Heap} \) and \( \text{Address} \) fields of \( ar \), respectively.
Array-like axioms

\[ \text{valid}(h, p) \rightarrow \text{read}(\text{write}(h, p, o), p) = o \]  \[\text{row1}\]

\[ p_1 \neq p_2 \rightarrow \text{read}(\text{write}(h, p_1, o), p_2) = \text{read}(h, p_2) \]  \[\text{row2}\]

\[ (\forall p : \text{Address}. (\text{valid}(h_1, p) \leftrightarrow \text{valid}(h_2, p)) \land \text{read}(h_1, p) = \text{read}(h_2, p)) \rightarrow h_1 = h_2 \]  \[\text{ext}\]

The extensionality axiom \[\text{ext}\] states that, given any \text{Address} \( p \), if two \text{Heaps} have the same allocation state at \( p \), and reads from \( p \) return the same \text{Object} in both, then the two \text{Heaps} must be the same. This axiom differs from the extensionality axiom of the theory of arrays \[\text{array-ex}\] only with the validity checks.

### Axioms about allocation

\[ \text{allocate}(h, o) = ar \rightarrow \text{read}(ar._1, ar.) = o \]  \[\text{roa1}\]

The axiom \[\text{roa1}\] states that reading from a \text{Heap}, using the \text{Address} returned from an allocation using \text{Heap} \( h \) and \text{Object} \( o \), returns \( o \).

\[ \text{allocate}(h, o) = ar \land p \neq ar._2 \rightarrow \text{read}(ar._1, p) = \text{read}(h, p) \]  \[\text{roa2}\]

The axiom \[\text{roa2}\] states that reading from a \text{Heap} using an \text{Address} \( p \) that is different than the \text{Address} returned from the allocation, which was done using \text{Heap} \( h \) and \text{Object} \( o \), is the same as directly reading \( p \) from \( h \).

\[ \text{allocate}(h, o) = ar \rightarrow \neg\text{valid}(h, ar._2) \land \text{valid}(ar._1, ar._2) \land (\forall p : \text{Address}. (ar._2 \neq p \rightarrow (\text{valid}(h, p) \leftrightarrow \text{valid}(ar._1, p)))) \]  \[\text{alloc1}\]

\[ (\forall p : \text{Address}. (\text{valid}(h_1, p) \leftrightarrow \text{valid}(h_2, p))) \rightarrow \text{allocate}(h_1, o_1) = \text{allocate}(h_2, o_2) \]  \[\text{alloc2}\]

The axiom \[\text{alloc2}\] ensures that the allocations are deterministic. If two \text{Heaps} are valid at the same \text{Addresses} (i.e., due to the same number of \text{allocate} calls), then allocating a new \text{Object} on either will return the same \text{Address}. 

Axioms about validity

\[ \neg \text{valid}(h, p) \rightarrow \text{write}(h, p, o) = h \]  \[\text{[ivwt]}\]

The axiom \[\text{[ivwt]}\] states that a write to an invalid Address of Heap \(h\) returns \(h\), in other words, the heap is unchanged by invalid writes, which eliminates the need for a validity check on the left-hand side of the implication in \[\text{row2}\].

\[ \neg \text{valid}(h, p) \rightarrow \text{read}(h, p) = \text{defObj} \]  \[\text{[ivrd]}\]

The axiom \[\text{[ivrd]}\] states that a read from an invalid Address of a Heap returns the default Object i.e., defObj (as explained in Section 2 / Heap Declaration).

\[ \neg \text{valid}(\text{emptyHeap}, p) \]  \[\text{[vld1]}\]

The axiom \[\text{[vld1]}\] states that emptyHeap is unallocated at every Address.

\[ \neg \text{valid}(h, \text{nullAddress}) \]  \[\text{[vld2]}\]

The axiom \[\text{[vld2]}\] states that nullAddress is unallocated in every Heap.

No-junk (or constructability) axiom

\[
\exists f : \text{Nat} \rightarrow \text{Heap}, g : \text{Nat} \rightarrow \text{Address}.
\begin{align*}
    f(0) &= \text{emptyHeap} \land g(0) = \text{nullAddress} \land \\
    \forall i : \text{Nat}. & (f(i + 1), g(i + 1)) = \text{allocate}(f(i), \text{defObj}) \land \\
    \forall p : \text{Address}. & \exists i : \text{Nat}. g(i) = p
\end{align*}
\]  \[\text{[cons]}\]

The axiom \[\text{[cons]}\] makes the Heap constructable by enumerating every Heap and Address. It is required in order to ensure that there are no heap terms in the models which cannot be generated. defObj is used in this axiom as a generic object since the allocated object in this case is not of importance.

6.2 Constructing a Model of the Axioms

We now discuss how a model of the axioms can be defined in terms of the theory of arrays. Such a reduction to arrays has multiple use cases: (i) it witnesses consistency of the axioms; (ii) in SMT solvers (but probably not in CHC solvers, as shown in our experiments below) it gives rise to a practical decision procedure; and (iii) it enables us to carry over complexity results for the theory of arrays.

The first attempt to define such a model is shown in Listing 3. The address sort Addr is represented using integers, and ADT declarations that were previously part of a heap declaration are turned into a datatype declaration. Each Heap term is associated with an array and an integer counter keeping track of the number of allocations (lines 6–7).
Each operation of the theory is then defined according to the axioms of the theory. valid becomes a simple check on heapSize and the integer value of the Address (lines 10–11). Line 12 declares an uninitialised array which is used to construct the emptyHeap on the next line. read and write operations become simple wrappers for array accesses, where the partial mapping is achieved using the if-then-else (ite) operator. allocate semantics are achieved by incrementing the heapSize after each allocation, and storing the allocated object at this location.

The encoding in Listing 3 approximates heaps, but still violates several of the heap axioms. Firstly, it does not establish extensionality (axiom [ext]), since array extensionality does not exactly correspond to heap extensionality; the latter only considers allocated addresses. This can be addressed by defining a heap-eq predicate replacing negative occurrences of the built-in equality = on heaps.

Secondly, the use of the sort Int for addresses and heap size is not consistent with the semantics stipulated by the axioms. Negative addresses would describe memory locations that are not reachable through allocate, violating [cons], and existence of heaps with negative heap size violate the axioms [roa1] and [alloc1]. Since addresses can also be stored in heap objects, fixing these issues by introducing additional well-formedness constraints on the SMT-LIB level is cumbersome. The problems go away when switching from Int to the natural numbers Nat, which is not possible in SMT-LIB but easily doable solver-internally.

7 Properties of the Theory of Heap

The reduction shown in Section 6.2 indicates that basic properties of the theory of arrays carry over to the heap theory, and in particular that satisfiability of quantifier-free heap formulas is NP-complete (provided that the theory chosen to represent heap objects is by itself in NP). Like for arrays, NP-completeness can be observed already for conjunctions of heap literals. Proofs are in the appendix.
Lemma 1. Consider an instance of the heap theory with uninterpreted object sort $O$. It is an NP-complete problem to check satisfiability of formulas $\phi_1 \land \cdots \land \phi_n$, in which each $\phi_i$ is (i) an equation between terms involving variables and the functions $\text{read}$, $\text{write}$, $\text{allocate}$, $\text{nullAddress}$, $\text{emptyHeap}$; or (ii) an atom $\text{valid}(h, p)$; or (iii) the negation of an atom as in (i) or (ii).

Lemma 2. The theory of heap does not admit quantifier-free Craig interpolation: there are unsatisfiable quantifier-free conjunctions $A \land B$ that do not have quantifier-free interpolants.

8 Alternative Definitions and Extensions

This section explains the rationale behind some of the design choices in the theory of heap, as well as some natural extensions. It is intended as a starting point for further discussions and a standardisation within SMT-LIB.

**AllocationResultHeap** Allocation on the heap needs to produce both a new heap and a fresh address. In our theory, the pair of new heap and new address is handled using the ADT $\text{AllocationResultHeap}$, which enables us to stick to just a single allocation function $\text{allocate}$. Alternatively, $\text{allocate}$ could be represented using a pair of functions, as in $\text{allocate}(h, o) = (\text{allocHeap}(h, o), \text{allocAddress}(h, o))$; this would be preferable from a solver implementation point of view, but not necessarily for users. Altogether this point is more of aesthetic concern.

**Deterministic allocation** In the current semantics of the heap theory, object allocation is deterministic: since $\text{allocate}$ is a function, it will always produce the same fresh address when applied to the same arguments. Moreover, $\text{alloc2}$ implies that the new address is determined entirely by the set of already allocated addresses on the heap. Determinism is required for constructability of heaps, and for presentation of counterexamples. It also simplifies the computation of program invariants, since it implies the existence of a linear order of the heap addresses, as witnessed by the array semantics in Section 6.2: an invariant can distinguish fresh and used addresses using a simple inequality. Determinism will in many practical cases not be observable in programs: the syntax of the heap theory prevents arithmetic on addresses, and normal program semantics does not allow $\text{allocate}$ to be called repeatedly on the same heap in any case.

In cases where it is needed, there is an elegant way to reintroduce non-determinism: the $\text{allocate}$ function can be given a third entropy argument, as in $\text{allocate}(h, o, e)$, and the axiom $\text{cons}$ be relativized to only hold for fixed values of $e$. The axiom $\text{alloc2}$ could be dropped. The translation of programs to CHCs can then choose a non-deterministic value for $e$ when encoding an allocation operation like $\text{new}$. A side effect of this change would be that decision procedures and correct encoding of heap using arrays become more complex, and for instance have to store the allocation status of each address using a bit-array.
Deallocation A natural extension of the theory is the addition of a function for deallocating objects, which would obviously be helpful to capture languages without garbage collector, like C/C++; for such languages deallocation otherwise has to be encoded using an explicit flag added to objects. The effect on the theory semantics would be similar as for non-deterministic allocation: decision procedures would need to maintain an additional bit-array to remember the allocation status of addresses.

Integration of arrays A second relevant extension is the integration of the theory of heap with McCarthy-style arrays. As defined here, the heap theory readily allows arrays with primitive index and value-type to be stored as objects on the heap. However, it is not possible to store arrays containing addresses, in the same way in which the command

\[\text{(declare-datatype A ((a (x (Array Int A)))))}\]

is not a well-formed declaration in SMT-LIB. Storing addresses in arrays on the heap probably does not pose any challenge for implementing decision procedures, but requires a suitable generalisation of the definitions in Section 4.

Polymorphic Heap Objects The declare-heap syntax is already prepared for including ADT constructors with sort parameters (as in declare-datatypes), and in some cases parametric polymorphism would help to represent class hierarchies of programs more succinctly. A full extension to polymorphic heap objects requires further research, however, and the overall value is not clear. Our experience is that the type systems in intermediate verification languages, even when they provide polymorphism [21], are often too weak to directly capture the type systems of real-world programming languages (with idiosyncratic sub-typing rules, native types vs. boxed types, etc.), so that still an encoding is necessary. Such an encoding can be done using ADTs in our theory already now.

9 Related Work

Separation Logic extends the assertions of Hoare’s logic [12] to succinctly express properties of heap and shared mutable data-structures [30]. Research has been done on specialised decision procedures for separation logic in SMT [29,27], and there is a proposal for encoding separation logic in SMT-LIB 2.5 [13].

The theory of heap and separation logic both provide mechanisms for reasoning about the heap; however, their approaches are quite orthogonal. Separation logic extends the assertion language with additional operators, while the theory of heap provides an interchange format for encoding programs with the goal of preserving as much information about the heap as possible. Both could be used in a complementary way to encode program assertions and the program itself.
Linear Maps provide a similar proof strategy to that of separation logic, while staying within the confines of classical logic [20]. The authors describe a two-way erasure transformation, transforming between imperative programs with a single unified heap and programs with multiple disjoint linear maps. Since the transformation is completely in classical logic, off-the-shelf SMT solvers and theorem provers can be used without a special decision procedure by making use of the existing theories such as the theory of arrays and the theory of sets.

Unlike the transformational approach of linear maps, the theory of heap aims to defer the handling of heap to the solvers. In fact, the linear maps strategy could also make use of the theory of heap in order to have access to more specialised decision procedures, and not be restricted to the theory of arrays.

Other related work The authors of [28] extend an SMT solver with a decision procedure to decide unbounded heap reachability with support for Boolean and integer data fields. [19] also describes a decision procedure for verification of heap-manipulating programs. Both papers are about verifying heap reachability, and both of them highlight the need for a standard theory of heap as that would have provided a framework for the research and ease the adoption of proposed decision procedures by different solvers.

10 Preliminary Experiments and Conclusions

We have proposed a theory of heap, along with its syntax and semantics, and discussed possible alternative definitions and extensions in Section 8. The intention is that the ideas presented here will initiate discussions, and eventually result in a common interchange language for programs with heap. As a long-term goal, we would like to include a heap track also at the CHC-COMP competition.

In order to highlight the feasibility of using the theory in a more concrete setting, we collected C benchmarks from SV-COMP’s ReachSafety and MemSafety categories[1] and extended TriCera, a model checker for C programs, in order to produce the CHCs in the theory of heap. To create a preliminary set of CHC benchmarks modulo heap, we filtered out programs that require heap, but none of the features not yet supported in our setting (e.g., stack pointers or arrays). In the end, 111 unique benchmarks remained.

To experiment with those benchmarks, the SMT solver PRINCESS [32] was extended to support the theory using the reasoning and interpolation procedures from [9], and the CHC solver ELDARICA [13] was extended to make use of the newly added theory in PRINCESS. We have made available the benchmarks and the version of ELDARICA used during the experiments[4]

1 https://github.com/sosy-lab/sv-benchmarks
2 https://github.com/uuverifiers/tricera/tree/heaptheory
3 https://github.com/uuverifiers/eldarica/releases/tag/v2.0.5-heap
The experiments were run on an AMD Opteron 2220 SE machine with 64-bit Linux. The results are given in Table 1. ELDARICA could solve 26 benchmarks, while others timed out (T/O) after 600 seconds or were unsolvable due to quantified interpolants (as stipulated by Lemma 2).

In order to show how the same benchmarks could be encoded using the theory of arrays, we also provide the array theory versions, which were translated using the encoding shown in Listing 3. At the time of writing this paper, none of the other current CHC solvers that we know of could solve this particular encoding of the benchmarks, mostly due to not supporting the theory combination of ADTs and arrays.

It has to be stressed that the experiments are early, and no conclusions should be drawn other than that real-world C programs can indeed be encoded and analysed using the proposed theory. The algorithms from [9] used in the experiments are direct and unrefined adaptations of procedures for the theory of arrays, and more work is needed to obtain, e.g., practical interpolation methods. However, now that the design choice is shifted to the solvers, alternative approaches can be employed to improve the results without changing the CHC representation of programs. In this context, two directions we are currently pursuing are improved decision and interpolation procedures for the heap theory, and the adaptation of the invariant-based heap encoding used in JayHorn [15].

| sat / unsat | t/o | other | total |
|-------------|-----|-------|-------|
| 8           |     | 40    | 111   |

Table 1: Results for ELDARICA 2.0.5-heap
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A Proof of Lemma 1

Membership in NP follows from the polynomial-time reduction of heap formulas to array formulas (Section 6.2), and NP-completeness of the theory of arrays combined with NP theories to represent indexes [5].

NP-hardness follows using a similar reduction of the Boolean SAT problem as in [7]. Suppose $C_1 \land \cdots \land C_k$ is a conjunction of clauses over Boolean variables $x_1, \ldots, x_m$. We introduce object variables $T, F : O$ to represent truth values, and for each Boolean variable $x_i$ two address variables $a_i, \bar{a}_i$. We then create a heap $h$ with exactly two valid addresses, and represent each clause as a chain of write operations. The resulting heap formula is equisatisfiable to $C_1 \land \cdots \land C_k$:

$$T \neq F \land h = \text{allocate(allocate(emptyHeap, F)}_1, F)_1$$

$$\land \bigwedge_{i=1}^m (\text{valid}(h, a_i) \land \text{valid}(h, \bar{a}_i) \land a_i \neq \bar{a}_i)$$

$$\land \bigwedge_{i=1}^k T = \text{read}(W_i, \text{nthAddress}_1)$$

where the term $W_i$ for a clause $C_i$ is defined by:

$$W_i = \text{write}(\cdots \text{write}(\text{write}(h, t^i_1, T), t^i_2, T), \cdots, t^i_m, T)$$

$$t^i_j = \begin{cases} \text{nthAddress}_1 & \text{if both } x_j \text{ and } \neg x_j \text{ occur in } C_i \\ a_j & \text{if } x_j \text{ occurs in } C_i \\ \bar{a}_j & \text{if } \neg x_j \text{ occurs in } C_i \\ \text{nullAddress} & \text{otherwise} \end{cases}$$

B Proof of Lemma 2

This result carries over from the result for arrays [17,24]. As an example, consider the following for $A$ and $B$:

$$A : h_2 = \text{write}(h_1, p_1, o_1) \land \text{valid}(h_1, p_1)$$

$$B : p_2 \neq p_3 \land \text{read}(h_2, p_2) \neq \text{read}(h_1, p_2) \land \text{read}(h_1, p_3) \neq \text{read}(h_2, p_3)$$

$$\land \text{valid}(h_1, p_2) \land \text{valid}(h_1, p_3)$$

Then the only possible interpolants are quantified ones such as:

$$\forall x, y. (x = y \lor \text{read}(h_1, x) = \text{read}(h_2, x) \lor \text{read}(h_1, y) = \text{read}(h_2, y) \lor \neg \text{valid}(h_1, x) \lor \neg \text{valid}(h_1, y))$$