Parameter-Free Deterministic Reduction of the Estimation Bias in Continuous Control

Baturay Saglam, Enes Duran, Dogan C. Cicek, Furkan B. Mutlu, and Suleyman S. Kozat, Senior Member, IEEE

Abstract—Approximation of the value functions in value-based deep reinforcement learning systems induces overestimation bias, resulting in suboptimal policies. We show that when the reinforcement signals received by the agents have a high variance, deep actor-critic approaches that overcome the overestimation bias lead to a substantial underestimation bias. We introduce a parameter-free, novel deep Q-learning variant to reduce this underestimation bias for continuous control. By obtaining fixed weights in computing the critic objective as a linear combination of the approximate critic functions, our Q-value update rule integrates the concepts of Clipped Double Q-learning and Maxmin Q-learning. We test the performance of our improvement on a set of MuJoCo and Box2D continuous control tasks and find that it improves the state-of-the-art and outperforms the baseline algorithms in the majority of the environments.

Index Terms—Deep reinforcement learning, continuous actor-critic methods, action value estimation, estimation bias, underestimation bias

I. INTRODUCTION

A. Preliminaries

Deep approaches to the policy optimization in reinforcement learning (RL) have achieved notable successes in a wide range of tasks such as playing complex video games [1]–[4], board games [5], [6], neural network optimization [7]–[10] and autonomous driving [11], [12]. However, there are problems in approximating the functions for deep reinforcement learning setting [13]. The systematic estimation bias that prevents the learning agents from attaining maximum performance and applicability of the deep techniques to diverse real-world problems are one of the difficulties originating from the function approximation [14], [15], [13]. For discrete action spaces, the estimation bias on the value estimates has been widely investigated for the value-based RL algorithms [16]–[26]. In addition, similar work is done in the continuous action domain for actor-critic techniques for a subtype of the estimation bias, namely, overestimation bias [13]. However, [27] and [28] demonstrate that actor-critic methods that overcome the overestimation bias and accumulated high variance induce an underestimation bias on the action value estimates. Our study employs a probabilistic approach to this problem and improves current state-of-the-art performance on several continuous reinforcement learning tasks.

In value-based deep reinforcement learning, the estimation bias on the action value estimates is generally divided into underestimation and overestimation [13]. Overestimation bias, caused by maximizing the noisy estimates in traditional Q-learning, results in a cumulative inaccuracy on the value estimates throughout the learning stage [29]. Since deep neural networks represent the action and value functions in the function approximation setting, such an estimation noise is inevitable [13]. Due to the temporal difference (TD) learning [14], this inaccuracy in the value estimation is further amplified [15]. Underestimation bias, on the other hand, is a result of Q-learning variants that focuses on eliminating the accumulated overestimation bias along with the excessive variance on the value estimates [13], [18], [27], [28], [30]. Although the recent objective function proposals [13], [18] are shown to eliminate the overestimation bias and amplified variance, they can nevertheless decrease an RL agent’s performance by assigning low values to optimal state-action pairs and thus, result in suboptimal policies, and divergent behaviors [30].

We begin by demonstrating that in continuous action domains, this underestimation phenomenon arises in a delayed policy gradient which employs the minimum of a pair of critics; specifically, Clipped Double Q-learning [13], a variant of the Q-learning [1]. In the Clipped Double Q-learning [13], two critics with the same structure but random weights are initialized before the learning process [13]. The minimum of these critics’ estimations is utilized to create the objective Q-value during learning. Despite the decoupled actor and critics, usage of the minimum Q-value in learning the targets results in persistent underestimation of the state-action values [27], [28], [30].

To address this issue, we first show that if the vari-
ance of the encountered reinforcement signals increases, the underestimation bias on the Q-values increases. To overcome the underestimation bias and improve the performance of continuous control reinforcement learning algorithms, we then present a clipped double Q-learning [13] variant, Fixed Weighted Twin Critic Update, that combines the notions behind clipped double Q-learning [13] and Maxmin Q-learning [18]. Our approach derives a linear combination of the functions of two approximate critics with fixed weights that does not require further parameter tuning.

We build our deep Q-learning modification on the state-of-the-art deterministic continuous control algorithm, Twin Delayed Deep Deterministic Policy Gradient (TD3) [13], and introduce the Fixed Weighted Twin Delayed Deep Deterministic Policy Gradient (FWTD) algorithm. Our deterministic continuous control algorithm substantially reduces the underestimation bias to a negligible margin and simultaneously inhibits the overestimation bias and high accumulated variance. We evaluate the performance of our approach on 12 continuous control tasks from MuJoCo [31] and Box2D [32] physics engines interfaced by OpenAI Gym [33], where we show that Fixed Weighted Twin Critic Update substantially improves the performance of TD3 [13] and outperforms the current baselines that aim to reduce the underestimation bias. We run our experiments across a large set of seeds for a fair evaluation procedure and reproducibility.

B. Related Work

Prior studies on the function approximation error in reinforcement learning have been done by [34] and [35] in terms of the estimation bias and resulting high variance build-up. This paper focuses on one of the function approximation error outcomes: underestimating the state-action values.

1) Discrete Action Spaces: For deep Q-learning [1], many techniques are proposed to mitigate the impacts of overestimation bias caused by the function approximation and policy optimization. [17] addresses the function approximation error for discrete action spaces in their work, Double Q-learning [16], one of the successor works to the deep Q-learning [1], by employing two independent and identically structured Q-value approximators to obtain unbiased state-action values. [18] modifies the traditional deep Q-learning [1] through the utilization of multiple action value estimators. Their approach, Maxmin Q-learning [18], uses multiple action value estimates selected through partial maximum operators, and the minimum of which constructs the deep Q-learning target [1] for discrete action domains. Although [18] primarily aims to eliminate the overestimation, they show that their method may yield in underestimation [18]. Additionally, methods that employ multi-step returns are shown to overcome the estimation bias [19], [24], [36] and prove to be effective through distributed approaches [20], [21], approximate upper and lower bounds [22], [23], weighted Q-learning [36], and importance sampling for off-policy correction [19], [21], [23]–[25]. However, these methods introduce a trade-off between the biased action value estimates and accumulated variance as shown by [24]. Furthermore, these approaches use impractical longer horizons than one-step solutions to the bias and variance build-up trade-off [24]. [37] proposes a one-step improvement for the reduction of the contribution of each erroneous estimate by reducing the discount factor in a structured manner. Moreover, the variance introduced by Q-value estimates is overcome by [26] through the average of multiple value estimates as a one-step, deterministic solution.

2) Continuous Action Spaces: A direct and one-step solution to the overestimation and variance accumulation has been proposed by [13] and is shown to be effective in eliminating the function approximation error for the deep setting of continuous actor-critic methods. Their research demonstrates that the deep function approximation of Q-values causes overestimation bias and cumulative variance in continuous action domains. An extension of temporal difference learning [19] in deep deterministic policy gradients [38], Twin Delayed Deep Deterministic Policy Gradient (TD3) [13], on which we build our algorithm, presents a direct remedy to the overestimation problem by employing two identically structured Q-networks, delayed actor updates, and target policy regularization through additive policy noise. TD3 algorithm [13] is shown to exhibit state-of-the-art performance by a large margin in a sufficient number of training iterations on a variety of continuous control tasks. TD3 [13] overcomes the overestimation build-up by performing the target Q-value computation through the minimum of two approximate critics. Their introduced update rule, clipped double Q-learning [13], is used in many state-of-the-art continuous control algorithms such as in both deterministic and stochastic variants of the Soft Actor-Critic algorithm [39].

Even though the improvements of clipped double Q-learning [13] can eliminate cumulative estimation error, the use of the minimum of two critics causes an underestimation bias in the Q-value estimations [13], [27], [28], [30]. Several techniques have been proposed to address the underestimation problem, including the use of a linear combination of the Q-value estimates by approximate critics to compute the objective for Q-network update [15], [27], [28]. The minimum and maximum action value estimates are combined in a weighted linear combination by [15]. The Weighted Deep Deterministic Policy Gradient (WD3) method [27]
C. Contributions

Our main contributions are as follows:

- We theoretically show that if the rewards that the agent receives vary on a large scale, the underestimation of the action value estimates detrimentally increases.
- We empirically verify our claims by comparing the actual and estimated Q-values and demonstrating that the underestimated state-action values can still degrade the agents’ performance in terms of the highest evaluation returns and convergence rates.
- We introduce a Double Q-learning variant as a linear combination of two approximate critics that upper and lower bounds the value estimates without introducing any hyper-parameter or network. In this way, we reduce the underestimation to a negligible level and prevent the overestimation and high-variance build-up.
- Our algorithm has the same linear time complexity as in the state-of-the-art continuous control algorithm TD3 [13]. Thus, with the same run time, our method can achieve accurate Q-value estimates.
- Our introduced temporal difference model can readily be adapted to any continuous control, actor-critic method that employs temporal difference learning [19].
- Through an extensive set of experiments, we show that our method obtains more accurate Q-value estimates, improves the performance of the state-of-the-art, and outperforms the current baselines by attaining higher evaluation returns in fewer time steps on several challenging OpenAI Gym [33] continuous control tasks.

II. BACKGROUND

Reinforcement learning paradigm considers an agent that interacts with its environment to learn the optimal, reward-maximizing behavior. The standard reinforcement learning is represented by a partially or fully observable Markov Decision Process (MDP) defined by the tuple \((S, A, p, \gamma)\) where \(S\) and \(A\) denote the state and action spaces, respectively, \(p\) is the transition dynamics and \(\gamma\) is the constant discount factor. At each discrete time step \(t\), the agent observes its state \(s \in S\) and chooses an action \(a \in A\) according to its policy \(\pi_\phi\), stochastic or deterministic, parameterized by \(\phi\). Then, based on its action decision given the observed state, the agent receives the reward \(r\) from a reward function corresponding to its environment, and observes a next state \(s’\) such that \(s’, r \sim p(s, a)\). The objective of the agent is to maximize the cumulative reward defined as the discounted sum of future rewards \(R_t = \sum_{i=t}^{T} \gamma^i r(s_i, a_i)\) where the discount factor \(\gamma \in [0, 1)\) downscales the long-term rewards to prioritize the short-term rewards more.

The agent learns the optimal policy \(\pi_\phi^*\) that maximizes the expected return \(J(\phi) = \mathbb{E}_{s, \sim p_s, a, \sim \pi}[R_t]\). In actor-critic settings, parameterized policies represented by deep neural networks are optimized by computing the gradient of the expected return \(\nabla_{\phi} J(\phi)\) through the deterministic policy gradient (DPG) algorithm [40]:

\[
\nabla_{\phi} J(\phi) = \mathbb{E}_{s, \sim p_s} [\nabla_a Q^\pi(s, a) | a = \pi(s)] \nabla_{\phi} \pi_\phi(s). \tag{1}
\]

The expected return after taking the action \(a\) given the observed state \(s\) under the current policy \(\pi\) is computed by the critic or action-value functions \(Q^\pi(s, a) = \mathbb{E}_{s’, \sim p_{s’}, a’, \sim \pi}[R_t | s, a]\) which values the quality of the action decision given the observed state. The critic evaluates and improves the agent’s policy to obtain higher quality action choices.

In standard Q-learning, if the transitions dynamics of the environment is accessible, the action-value function \(Q^\pi\) is estimated through recursive Bellman optimization [41] given the transition tuple \((s, a, r, s’)\):

\[
Q^\pi(s, a) = r + \gamma \mathbb{E}_{s’, a’}[Q^\pi(s’, a’), a’ \sim \pi(s’)]. \tag{2}
\]

For large state and action spaces, the action-value function is usually estimated by function approximators \(Q_\theta(s, a)\) parameterized by \(\theta\), also known as the Q-networks. In the deep setting of Q-learning [11], the Q-network is updated through the temporal difference learning [19] by a secondary frozen target network \(Q_\theta^*(s, a)\) to construct the objective for behavioral Q-network:

\[
y = r + \gamma Q_\theta^*(s’, a’); \quad a’ \sim \pi_\theta(s’), \tag{3}
\]

where the next actions given the observed next state are obtained from a separate target actor network \(\pi_\theta\) for actor-critic settings in continuous control. The target networks are either updated by a small proportion \(\tau\) at
each time step, i.e., $θ' ← τθ + (1−τ)θ'$, called soft-update, or periodically to exactly match the behavioral networks called hard-update.

III. The Underestimation Bias in Continuous Control

Overestimation of action values due to analytical maximization is a well-known and well-studied artifact for discrete action domains [16]–[26]. The presence and effects of overestimation in actor-critic settings are highlighted in [13] through gradient ascent policy updates. [13] computes the target action value at each iteration using the minimum value by two critics, as well as delayed actor updates and smoothed target policy. However, using the minimum operator to compensate for the overestimation of Q-values may result in an underestimated action-value estimates [13], [27], [28]. We begin by proving through basic assumptions and claims that the underestimation phenomenon exists in continuous control, actor-critic methods for environments with varying reinforcement signals. Then, we introduce our modified target Q-value update rule for the actor-critic setting to minimize the underestimation bias while remaining in the “safe zone” of the function approximation error.

In the TD3 algorithm [13], the policy is updated using the minimum value estimate by two approximate critics, $Q_{θ_1}$ and $Q_{θ_2}$, parameterized by $θ_1$ and $θ_2$, respectively. Without loss of generality, we assume that the both critics overestimate the action values, and the policy is updated with respect to the first approximate critic, $Q_{θ_1}(s, a)$, through the deterministic policy gradient algorithm [40]. We show that this update rule for Q-networks introduces an underestimation bias in environments with large-scale rewards.

Let $φ_{approx}$ define the parameters from the actor update by the maximization of the first approximate critic $Q_{θ_1}(s, a)$:

$$φ_{approx} = φ + \frac{η}{Z_1}E_{a ∼ p_a}\left[\nabla_φ πφ(s)\nabla_θ Q_{θ_1}(s, a)\right]_{a = πφ(s)},$$

where $Z_1$ is the gradient normalizing term such that $Z_1||E[.]|| = 1$. As the actor is optimized with respect to $Q_{θ_1}(s, a)$ and the gradient direction is a local maximizer, there exists $ζ$ sufficiently small such that if $η < ζ$, then the approximate value of the policy, $π_{approx}$, by the first critic will be bounded below by the approximate value of the policy by the second critic:

$$E[Q_{θ_1}(s, π_{approx}(s))] ≥ E[Q_{θ_2}(s, π_{approx}(s))].$$

Then, we can treat the function approximation error for both critics as distinct Gaussian random variables:

$$Q_{θ_1}(s, a) − Q^*(s, a) = N_1 ∼ N(μ_1, σ_1),$$
$$Q_{θ_2}(s, a) − Q^*(s, a) = N_2 ∼ N(μ_2, σ_2).$$  (6)

Following (5), we have $μ_1 ≥ μ_2 ≥ 0$. As the same experience replay buffer [42] and opposite critics are used in learning the target Q-values, critics and therefore, error Gaussian’s denoted by [6] are not entirely independent [13]. Through the first moment of the minimum of two correlated Gaussian random variables [43], the expected estimation error for the clipped Double Q-Learning algorithm [13] becomes:

$$E[\min_{i=1,2}\{N_i\}] = μ_2 + (μ_1 − μ_2)Φ((μ_1 − μ_2)/θ) − θψ((μ_1 − μ_2)/θ),$$

(7)

where $θ := \sqrt{σ_1^2 + σ_2^2 − 2ρσ_1σ_2}$, $ρ$ is the correlation coefficient between $N_1$ and $N_2$, and $Φ(·)$ and $ψ(·)$ are the cumulative distribution function (CDF) and probability density function (PDF) of the standard normal distribution, respectively. Due the presence of the delayed actor updates, the mean function approximation errors by both critics are not very distant due to the decoupled actor and first critic, that is, $μ_1 ≈ μ_2$. Using this, (7) reduces to:

$$E[\min_{i=1,2}\{N_i\}] = μ_1 − θ/\sqrt{2π},$$

(8)

since $Φ(0) = 1/2$, $ψ(0) = 1/\sqrt{2π}$. Hence, if $σ_1, σ_2 > √π/(1−ρ)μ_1$, then the action value estimate will be underestimated:

$$E[\min_{i=1,2}\{Q_θ(s, a)\} − Q^*(s, a)] < 0.$$  (9)

From $σ_1, σ_2 > √π/(1−ρ)μ_1$ condition, if the pair of critics are highly correlated, underestimation does not exist. However, there exists a moderate correlation between the pair of critics due to the delayed policy updates which increases the underestimation possibility [13].

Although the improvements by [13] aim to reduce the estimation error growth, the variance of the Q-values cannot be eliminated as they are adhered to the variance of the future value estimates and rewards [13]. Furthermore, the Bellman equation in function approximation settings cannot be exactly satisfied [13], which results in erroneous Q-value estimates as a function of the actual TD error expressed by (6). Then, we can show that the variance of the value estimates increases as the agent receives reward signals that vary on a large scale due to the exploration [44]. As shown in [13], the Q-value can be expressed in terms of the expected sum of discounted future rewards:

$$Q_{θ_1}(s, a) = E_{a ∼ p_a, a_i ∼ π_i} [ ∑_{i=t}^{T} γ^{t−i} r_i ] + μ_1 ∑_{i=t}^{T} γ^{t−i}.$$  (10)

If the expected estimation errors by both critics are constant, varying reinforcement signals increase the variance of the Q-value estimates resulting in an increasing underestimation bias. Since an extensive exploration is
a mandatory requirement for continuous action spaces\cite{15}, the variance of the reinforcement signals usually increases throughout the learning phase. Therefore, the underestimation bias on the value estimates becomes unavoidable. Moreover, in the underestimation case, the estimation error is not accumulated due to the temporal difference learning\cite{14},\cite{19} and thus, the underestimation bias is far more preferable to the overestimated Q-values in the actor-critic setting\cite{13},\cite{15}. Nevertheless, underestimated action values may discourage agents from choosing good state-action pairs for a long period and reinforce agents to value suboptimal state-action pairs more frequently\cite{30}.

We can show the existence of the underestimation bias in practice by comparing the true and estimated Q-values while an agent under the TD3 algorithm\cite{13} is learning on a set of OpenAI Gym\cite{33} continuous control tasks over a training duration of 1 million time steps. The simulation results is reported by Fig. 1. We randomly select 1000 state-action pairs at every step and obtain the estimated Q-values by the first Q-network. The true Q-values are obtained at every 100,000 time steps by computing the discounted sum of rewards starting from a randomly sampled 1000 states following the current policy. The Monte-Carlo simulation\cite{45} is used over the randomly selected states and state-action pairs to obtain the average true and estimated Q-values.

From Fig. 1, we observe an apparent underestimation bias throughout the learning phase such that the estimated Q-values are smaller than the true ones except for a small proportion of the initial time steps. The underestimation bias arises depending on the environment and either grows or settles to a fixed level. These simulation results verify our claims; the approximate critics overestimate the actual Q-values at initial time steps. However, when the agent starts exploring the environment and encounters varying rewards, the variance of the value estimates increases, and the underestimation bias starts growing. For the majority of the environments, underestimation bias becomes fixed after a duration. This is due to the span of the reward space. If the agent encounters a sufficiently large subspace, the underestimation bias cannot become larger. However, if the agent does not receive a significantly large subspace, the underestimation bias keeps growing even with the delayed target and actor updates as in Ant and HumanoidStandup environments. Although the continuous, multi-dimensional, and large state-action spaces contribute to the growth of error, the scale of the OpenAI Gym benchmarks\cite{33} is still very small compared to the real-world tasks\cite{15}. Hence, the underestimation bias will be a more detrimental and inevitable problem when larger-scaled tasks are introduced.

To overcome the shown underestimation bias, we introduce our novel, hyper-parameter-free modification on the target Q-value update as a linear combination of two approximate critics, Fixed Weighted Twin Critic Update, that significantly reduces the underestimation bias.

IV. FIXED WEIGHTED TWIN CRITIC UPDATE

Several approaches have been proposed to eliminate the underestimation bias introduced by the minimum of two critics\cite{27},\cite{28}. These methods approach the estimation bias by different treatments of the function approximation error as random variables. Albeit these approaches can reduce the underestimation bias through modified Q-value updates, they introduce additional hyper-parameters or networks to be tuned and optimized. In this section, we present our parameter-free, novel modification on the Q-network update for continuous control, a variant of Double Q-learning\cite{16}.

We highlight that the bias introduced by two approximate critics is a function of the expected function approximation error and variance of the value estimates in the Clipped Double Q-learning\cite{13}. Since the expected function approximation error is assumed to be constant for both critics, the function approximation error can be either an underestimation or overestimation, depending on the variance of the value estimates, proportional to the received reinforcement signals. However, in practice, comprehensive exploration induces the variance of the Q-value estimates to be greater than the expected function approximation error. Thus, the underestimation is present for continuous control due to the minimum operator in target Q-value computation such as in \cite{13} and
Consequently, the good state-action pairs may be less valued. We overcome this problem, approached by a different perspective [27, 28, 30], by constructing a linear combination of two critics in computing the target Q-value to simply upper- and lower-bound the Q-value estimates, without introducing any hyper-parameter or network to be tuned and optimized.

Consider an additional third critic \( Q_{\theta_3} \) with corresponding estimation error distribution \( N_3 \sim N(\mu_3, \sigma_3) \). As the first critic is used to optimize the policy and due to the randomness in transition sampling, the same probability distribution can represent the errors corresponding to the second and third critics, i.e., \( N_3 \sim N(\mu_2, \sigma_2) \).

Then, the following update rule can upper- and lower-bound the Q-value estimates by taking the minimum of the maximum of the first two critics and the third critic:

\[
y = r + \gamma \min \left( \max_{i=1,2} Q_{\theta_i}'(s', \pi_{\phi'}(s')), Q_{\theta_3}'(s', \pi_{\phi'}(s')) \right).
\]  

(11)

The expected function approximation error induced by the Q-value target expressed by (11) is given as:

\[
E[\min(\max(N_1, N_2), N_3)] = (E[\min \{ N_i \} ] + \mu_2) / 2,
\]  

(12)

as shown in Appendix A. This expected estimation bias is slightly less than the average of the underestimation in TD3 [13] and overestimation in DDPG [38]. As the variance of the value estimates by two correlated critics are greater than the expected function approximation error, (12) is still an underestimation. We can further reduce this underestimation by replacing \( \mu_2 \) with \( \mu_1 \) in (12) as \( \mu_1 \geq \mu_2 \geq 0 \). Such an estimation error corresponds to the following update rule, Fixed Weighted Twin Critic Update:

\[
y = r + \gamma \left( \min_{i=1,2} Q_{\theta_i}'(s', \pi_{\phi'}(s')) + Q_{\theta_3}'(s', \pi_{\phi'}(s')) \right). 
\]  

(13)

Observe that this update rule balances the overestimation and underestimation, introduced by a single approximate critic and minimum of two approximate critics, respectively. This is done by choosing either average or minimum of the Q-value estimates by both critics, depending on the overestimation of the first critic. If the function approximation error is large, then this update rule results in a slight overestimation, and if reward signals vary on a large scale, then there is a slight underestimation. Regardless, such small estimation errors can be tolerated by agents [44], in contrast to TD3 [13] and DDPG [38] algorithms.

As we highlight the large variance of the value estimates, the Fixed Weighted Twin Critic Update usually underestimates the Q-values. Nonetheless, the induced estimation error is slightly larger than half of the bias in the Clipped Double Q-learning algorithm [13] yielding a greatly reduced underestimation bias and more accurate Q-value estimates without an introduction of additional parameters or networks. Moreover, since the underestimation is more preferable to the overestimation due to the non-accumulating nature [15], the Fixed Weighted Twin Critic Update remains in the “safe zone” of the function approximation error in general. Furthermore, the Fixed Weighted Twin Critic Update offers more accurate value estimates than TD3 [13] for two extreme ends where the variance of the value estimates is very large or very small.

**Algorithm 1 FixedWeightedTwinCriticUpdate**

- **Input** \( Q_{\theta_1}', Q_{\theta_2}', s', \tilde{a} \)
- \( y \leftarrow r + \frac{\gamma}{2} \left( \min_{i=1,2} (Q_{\theta_i}'(s', \pi_{\phi'}(s')) + Q_{\theta_3}'(s', \pi_{\phi'}(s'))) \right) \)
- **return** \( y \)

**Algorithm 2 FWTD**

- Initialize critic networks \( Q_{\theta_1}, Q_{\theta_2} \), and actor network \( \pi_{\phi} \) with randomly initialized parameters \( \theta_1, \theta_2, \phi \)
- Initialize target networks \( \phi' \leftarrow \phi, \theta_1' \leftarrow \theta_1, \theta_2' \leftarrow \theta_2 \)
- Initialize replay buffer \( B \)

for \( t = 1 \) to \( T \) do

Select action with exploration noise \( a \sim \pi_{\phi}(s) + \epsilon, \epsilon \sim \mathcal{N}(0, \sigma) \) and observe reward \( r \) and new state \( s' \)

Store transition tuple \((s, a, r, s')\) in \( B \)

Sample mini-batch of \( K \) transitions \((s, a, r, s')\) from \( B \)

\( \tilde{a} \leftarrow \pi_{\phi'}(s') + \epsilon; \epsilon \sim \text{clip}(\mathcal{N}(0, \tilde{\sigma}), -c, c) \)

\( y \leftarrow \text{FixedWeightedTwinCriticUpdate}(Q_{\theta_1}', Q_{\theta_2}', s', \tilde{a}) \)

Update critics \( \theta_i \leftarrow \text{argmin}_{\theta_i} \sum (y - Q_{\theta_i}(s, a))^2 / K \)

if \( t \text{ mod } d \) then

Update \( \phi \) by the deterministic policy gradient:

\( \nabla_{\phi} J(\phi) = \frac{1}{K} \sum \nabla_{\phi} Q_{\theta_1}(s, a)_{a=\pi_{\phi}(s)} \nabla_{\phi} \pi_{\phi}(s) \)

Update target networks:

\( \theta_i' \leftarrow \tau \theta_i + (1 - \tau) \theta_i' \)

\( \phi' \leftarrow \tau \phi + (1 - \tau) \phi' \)

end if

end for

For the stability concerns, the actor should always be optimized with respect to the first critic, and the Q-network order in the Fixed Weighted Twin Critic Update must remain constant. As a result, our modification offers accurate Q-value estimates without introducing hyper-parameters and networks or requiring more training iterations.

We summarize our introduced approach built on the Clipped Double Q-learning algorithm [13] in Algorithm 1 and the resulting algorithm, Fixed Weighted Twin Delayed Deep Deterministic Policy Gradient (FWTD), in Algorithm 2. In the following section, we present the empirical results for our method in terms of the Q-value comparisons and learning evaluations on several OpenAI Gym [33] continuous control tasks.
Fig. 2. Measuring estimation bias produced WD3 and TADD versus FWTD while learning on MuJoCo and Box2D environments over 1 million time steps. Estimated and true Q-values are computed through Monte Carlo simulation for 1000 samples.

V. Experiments

We evaluate the performance of our estimation bias correction approach by first demonstrating the estimated and actual Q-values for FWTD versus the state-of-the-art continuous control method TD3 [13] and two baseline algorithms that aim to reduce the underestimation: WD3 [27] and TADD [28]. Then, we evaluate the learning performances of RL agents under FWTD, TD3 [13], WD3 [27] and TADD [28] algorithms on 12 OpenAI Gym [33] continuous control tasks powered by MuJoCo [31] and Box2D [32] physics engines. For the sake of reproducibility and a fair evaluation procedure, we directly follow the same set of tasks from [31] and [32] with no modifications on the environment dynamics.

A. Implementation Details & Experimental Setup

To implement the TD3 algorithm [13], we use the author’s GitHub repository (https://github.com/sfujim/TD3) [46]. The implementation of TD3 [13] is the fine-tuned version of the algorithm updated by the author as of September 2021. This version of TD3 [13] differs from the one introduced in [13] such that the number of hidden units in all networks is reduced to 256, the batch size is increased from 100 to 256, learning rates for the behavioral actor and critic Adam optimizers [47] are decreased from $10^{-3}$ to $3 \times 10^{-4}$, and 25000 time steps of pure exploratory policy is employed on all environments. We built our modification on this implementation such that the target Q-value computation is replaced by (13). To implement the baseline algorithms, WD3 [27] and TADD [28], we use the same repository [46] from [13]. We follow the same parameter, network, and Q-value update structures in [27] and [28] such that we replace the target Q-value computation and initialize an additional Q-network if required. For the weight parameter ($\beta$) that controls the underestimation bias in WD3 [27] and TADD [28] algorithms, we use the values for the environments presented in the respective papers. We manually fine-tune the $\beta$ value over a training duration of 1 million time steps for 10 random seeds for the rest of the environments. The values with the average highest evaluation returns are chosen to train WD3 [27] and TADD [28] algorithms. The used $\beta$ values are given in Appendix B.

Each task in the Q-value comparisons is run for 1 million time steps, and curves are derived through the same procedure explained in section III. We perform evaluations on every task by running the algorithms over 1 million time steps and evaluating the agent’s performance on a distinct evaluation environment without exploration noise at every 5000 time steps. Each evaluation report is an average of 10 episode rewards. The results are reported over 10 random seeds of the Gym [33] simulator, network initialization, and code dependencies.

B. Discussion

1) Q-value Comparisons: True and estimated Q-value comparisons for our approach versus TD3 [13], and WD3 [27] and TADD [28] over 6 OpenAI Gym [33] continuous control tasks are reported in Fig. 1 and 2 respectively. FWTD obtains more accurate Q-value estimations than TD3 [13] and the baseline algorithms, WD3 [27] and TADD [28], on all environments. Our empirical findings indicate two cases. First, our method estimates the Q-values with a small margin of error. Second, even though FWTD considerably reduces estimation bias, an underestimation error still occurs, which is slightly larger
than half of the underestimation in TD3 \cite{13}. These results verify our argument that an increasing variance of rewards increases the underestimation, and the expected function approximation error can be reduced further, as shown in this study.

2) Evaluation: Table I and Fig. 3 report the evaluation results. Our method either matches or outperforms the performance of TD3 \cite{13} and baseline algorithms in terms of the learning speed and highest evaluation return. We observe that estimation bias, either underestimation or overestimation, prevents agents from reaching higher possible evaluation returns. Our approach obtains higher and more stable evaluation returns through a fixed linear combination of two approximate critics by substantially reducing the underestimation bias by balancing the over and underestimation.

VI. CONCLUSION

We show in this paper that receiving different reward signals that vary on a large scale increases the underestimation of the action value estimates. Then, we introduce a novel deep Q-learning variant that dramatically reduces the underestimation bias to a negligible level. Having
our claims and empirical results combined, this improvement establishes our efficient, parameter-free update rule, Fixed Weighted Twin Critic Update, which significantly improves both the learning speed and performance of the state-of-the-art continuous control algorithm TD3 [13] in several challenging continuous control tasks. By obtaining more accurate Q-value estimates, our method, Fixed Weighted Twin Delayed Deep Deterministic Policy Gradient (FWTD), outperforms the state-of-the-art and baseline algorithms on all environments tested. Our modification is orthogonal and can be easily adapted to any continuous control actor-critic method.

\section*{Appendix A}
\textbf{Proof of (12)}

First, expand $\min(\max(N_1, N_2), N_3)$ in terms of the maximum of error Gaussian’s:

$$\min(\max(N_1, N_2), N_3) = \frac{1}{2} \max(N_1, N_2) + \frac{1}{2} N_3 - \frac{1}{2} \max(N_1, N_2) - N_3. \tag{14}$$

It is not trivial to compute the expectation of the latter term in the right-hand side of (14). However, we can rewrite (14) in terms of the maximum of three correlated Gaussian’s and use the derivation for its expectation for equal means case from [48]. For this purpose, let $N_{\text{max}} = \max(\max(N_1, N_2), N_3) = \max(N_1, N_2, N_3)$. Then, the expected value of (14) can be expressed as:

$$\mathbb{E}[\min(\max(N_1, N_2), N_3)] = \mathbb{E}[\max(N_1, N_2)] - \frac{\mathbb{E}[N_3] - \mathbb{E}[N_{\text{max}}]}{2}. \tag{15}$$

Under the assumption made in section III that $\mu_1 \approx \mu_2 = \mu_3$, let us define $\mu := \mu_1 = \mu_2 = \mu_3$. Now, we can directly import the special case for the expectation of maximum of correlated Gaussian’s from [48]. The equal means case states that if $N_i \sim \mathcal{N}(\mu, \sigma_i)$, then the expected value of maximum of 3 Gaussian’s can be expressed as:

$$\mathbb{E}[\max(N_1, N_2, N_3)] = \mu + \frac{1}{2\sqrt{2\pi}}(\theta_{1,2} + \theta_{1,3} + \theta_{2,3}), \tag{17}$$

where $\theta_{i,j} := \sqrt{\sigma_i^2 + \sigma_j^2 - 2\rho_{i,j}\sigma_i\sigma_j}$. Due to the same experience replay [42] used in updating the Q-networks and decoupled actor and the first critic, without loss of generality, we can further assume that $\theta := \theta_{1,2} = \theta_{1,3} = \theta_{2,3}$. Then, (17) reduces to:

$$\mathbb{E}[N_{\text{max}}] = \mathbb{E}[\max(N_1, N_2, N_3)] = \mu + \theta/\sqrt{2\pi}. \tag{18}$$

Furthermore, using the exact distribution of $\mathbb{E}[\max(N_1, N_2)]$ from [43], similar to (7), we have:

$$\mathbb{E}[\max\{N_i\}] = \mu_2 + (\mu_1 - \mu_2)\Phi((\mu_1 - \mu_2)/\theta) + \theta\psi((\mu_1 - \mu_2)/\theta). \tag{19}$$

Using the assumptions made, we can simplify (19) into:

$$\mathbb{E}[\max(N_1, N_2)] = \mu + \theta/\sqrt{2\pi}. \tag{20}$$

Inserting (18), (20) and $\mathbb{E}[N_3] = \mu$ into (16), we derive:

$$\mathbb{E}[\min(\max(N_1, N_2), N_3)] = \mu - \theta/\sqrt{2\pi}. \tag{21}$$

Replacing $\mu$ with $\mu_2$, we can express expected function approximation error for $\min(\max(Q_1, Q_2, Q_3))$ in terms of the expected error for the clipped Double Q-learning [13] denoted by (7) as:

$$\mathbb{E}[\min(\max(N_1, N_2), N_3)] = (\mathbb{E}[\min\{N_i\}] + \mu_2)/2. \tag{22}$$

\section*{Appendix B}
\textbf{Used weight parameter values in baseline algorithms training}

Table II presents the used environment specific weight parameter ($\beta$) values for WD3 [27] and TADD [28] algorithms. Values that we fine-tune and presented in [27] and [28] are marked.

\begin{table}[h]
\centering
\begin{tabular}{lccc}
\hline
Environment & WD3 & TADD & \multicolumn{1}{c}{Values in Baseline Algorithms Training} \\
\hline
Ant-v2 & 0.75 & 0.95 & \checkmark \\
BipedalWalker-v3 & 0.45 & 0.80 & \checkmark \\
HalfCheetah-v2 & 0.45 & 0.95 & \checkmark \\
Hopper-v2 & 0.50 & 0.95 & \checkmark \\
Humanoid-v2 & 0.60 & 0.95 & \checkmark \\
HumanoidStandup-v2 & 0.30 & 0.95 & \checkmark \\
InvertedDoublePendulum-v2 & 0.75 & 0.95 & \checkmark \\
InvertedPendulum-v2 & 0.75 & 0.95 & \checkmark \\
LunarLanderContinuous-v2 & 0.75 & 0.80 & \checkmark \\
Reacher-v2 & 0.15 & 0.95 & \checkmark \\
Swimmer-v2 & 0.75 & 0.20 & \checkmark \\
Walker2d-v2 & 0.45 & 0.95 & \checkmark \\
\hline
\end{tabular}
\caption{WD3 environment specific weight values}
\end{table}

$^1$ As given in the paper

$^2$ Fine-tuned

\section*{References}

[1] V. Mnih, K. Kavukcuoglu, D. Silver, A. Graves, I. Antonoglou, D. Wierstra, and M. A. Riedmiller, “Playing atari with deep reinforcement learning,” \textit{CoRR}, vol. abs/1312.5602, 2013. [Online]. Available: http://arxiv.org/abs/1312.5602.

[2] OpenAI, “Openai five,” https://blog.openai.com/openai-five/2018.

[3] O. Vinyals, I. Babuschkin, W. Czarnecki, M. Mathieu, A. Dudzik, J. Chung, D. Choi, R. Powell, T. Ewalds, P. Georgiev, J. Oh, D. Horgan, M. Kroiss, I. Danihelka, A. Huang, L. Sifre, T. Cai, J. Agapiou, M. Jaderberg, and D. Silver, “Grandmaster level in starcraft ii using multi-agent reinforcement learning,” \textit{Nature}, vol. 575, 11 2019.

[4] J. Schrittwieser, I. Antonoglou, T. Hubert, K. Simonyan, L. Sifre, S. Schmitt, A. Guez, E. Lockhart, D. Hassabis, T. Graepel, and et al., “Mastering atari, go, chess and shogi by planning with a learned model,” \textit{Nature}, \textbf{vol. 550}, no. 7666, pp. 588–604, Dec 2020. [Online]. Available: http://dx.doi.org/10.1038/s41586-020-30501-4.

[5] D. Silver, J. Schrittwieser, K. Simonyan, I. Antonoglou, A. Huang, A. Guez, T. Hubert, L. Baker, M. Lai, A. Bolton, Y. Chen, T. Lillicrap, F. Hui, L. Sifre, G. Driessche, T. Graepel, and D. Hassabis, “Mastering the game of go without human knowledge,” \textit{Nature}, \textbf{vol. 550}, pp. 354–359, 10 2017.
