The Spherical Relativistic Detonation of Scalaron Stars

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(6 September 2000)

Abstract

Now the hypothesis of existence of scalar fields of a various nature and energy density in the modern Universe is intensively explored. It can explain a nature of the dark (non-baryon) matter in the Universe and an existence of positive $\Lambda$-term [1]. One of component of such field has a cluster nature and organizes in the closed gravitational configurations from galactic scales up to relativistic microscopic stars. In the authors paper [2] the hypothesis of detonation of such fields was considered. As a result of phase transition behind the wavefront a relativistic plasma of high energy density can appear. This process is similar to a relativistic detonation and it can create macroscopic fireballs sufficient for an explanation of the phenomenon of gamma-ray bursts [3]. In Ref. [2] it was supposed that the front of such ”detonation” wave is entered by the flow of scalar fields with constant energy density. If the size of the formed plasma configuration is commensurable with the size of scalaron cluster, this hypothesis is not correct. It is necessary to take into account a modification of the energy density of the scalar field from centre to a periphery. It is changes the dynamics of the fireball on principle. The indicated problem in framework of special relativity is considered in this paper.

1 Introduction

Until recently the problems of a relativistic detonation when a velocities both of wave and gas (plasma) achieve of near-light velocities had only methodical interest. In connection with the hypothesis of a detonation regime of ”burning” of the cosmic scalar fields [4] this problem takes on physical interest. As well as in Ref. [2] we suppose that there is a scalaron star in which the pressure of the field is equilibrated by the weak gravitational forces (Newtonian approximation). Then it is possible to describe the specified process of detonation in framework of special relativity. Other field of application can be, in principle, the processes of laboratory scale [4].

At description of this phenomenon at the front of ”detonation” wave according to conservation laws both of relativistic momenta density and momenta density flow $T_0^0$(field) = $T_0^0$(plasma) and $T_1^1$(field) = $T_1^1$(plasma) for the scalar field in the rest, on which the wavefront with velocity $D$ is passing, we have:

$$D = \frac{2v_p}{1 + v_p^2}, \quad \varepsilon_p = \frac{2}{1 - \omega^2} \varepsilon_f .$$

(1)

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Here, $\varepsilon_f$ is the energy density of the scalar field in scalaron regime (it forms from density both of potential and kinetic energy of the field), $\varepsilon_p$ is the energy density of plasma behind the wavefront. Thus the plasma removes from the wavefront with the velocity of sound $\omega$ that corresponds to a condition of normal detonation. As well as in Ref. [2] we choose the scalar field of the simplest form

$$T^k_i = \varphi_i \varphi^k - \delta^k_i \left( \frac{1}{2} \varphi,_{\mu} \varphi^\mu - V(\varphi) \right), \quad V(\varphi) = m^2 \varphi^2 / 2, \quad (2)$$

which has rather small spatial derivatives. In Eqn. (1) $v_p$ is a velocity of high-temperature plasma relative to the wavefront $D$.

## 2 The Homogeneous Detonation

In this report we are interesting in dynamics of the detonation wave and the current behind one in the case when $\varepsilon_f$ depends on radial coordinate $r$. For considering of this problem it is expediently to repeat briefly the results of the homogeneous case. The set of equations of relativistic gas-dynamics in special relativity for spherical frame with use of usual radial velocity $v$ is possible to write as:

$$\frac{1}{\vartheta^2} \left( \frac{\partial v}{\partial \tau} + v \frac{\partial v}{\partial r} \right) + \frac{1}{w} \left( \frac{\partial p}{\partial r} + v \frac{\partial p}{\partial \tau} \right) = 0,$$

$$\frac{1}{w} \left[ \frac{\partial \varepsilon}{\partial \tau} + v \frac{\partial \varepsilon}{\partial r} \right] + \frac{1}{\vartheta^2} \left( \frac{\partial v}{\partial r} + v \frac{\partial v}{\partial \tau} \right) + \frac{2v}{r} = 0,$$

$$\frac{\partial \sigma}{\partial \tau} + v \frac{\partial \sigma}{\partial r} + \sigma \left[ \frac{1}{\vartheta^2} \left( \frac{\partial v}{\partial r} + v \frac{\partial v}{\partial \tau} \right) + \frac{2v}{r} \right] = 0. \quad (3)$$

Here, $\sigma$ is the entropy density, $\vartheta^2 = 1 - v^2$, $w = \varepsilon + p$ and $c = 1$. According to the classical theory of a spherical detonation considered by Zel’ dovich [3], the pattern of current is self-similar and depending from the unique variable

$$\xi = r/\tau. \quad (4)$$

For the relativistic problem we maintain the same self-similarity. The mentioned above velocity $v_p$ relative to the wavefront should be equal to the velocity of sound of relativistic plasma $\omega = 1/\sqrt{3}$. Thus the velocity of the detonation wave $D = \sqrt{3}/2$ according to Eqn. (1). The set of Eqns. (3) supposes the deriving of one equation relative to velocity $v$:

$$\frac{dv}{d\xi} \left[ \frac{1}{\omega^2} \left( \frac{v - \xi}{1 - v \xi} \right)^2 - 1 \right] = \frac{2v}{\xi} \frac{\theta^2}{1 - v \xi}. \quad (5)$$

The analysis of this equation is carried out similarly to Ref. [3] and the diagrams of solutions both for $v$ and energy density of plasma $\varepsilon_p$ are shown in Fig. 1. At the wavefront the derivative of the velocity tends to infinity. At $\xi = \omega$ the solution $v(\xi)$ is matched with $v = 0$ with maintenance of a continuity of first derivative. This point corresponds to a contact discontinuity. Thus the solution for $\varepsilon_p(\xi)$ is obtained from the second equation of system (3). It is easy to show that for photon gas with $p = \varepsilon/3$ and entropy density $\sigma \sim \varepsilon_p^{3/4}$ the third equation from (3) is satisfy identically. It is necessary to note that the nonrelativistic analog of this problem, considered for the first time by Zel’dovich, corresponds to isentropic flow of classical gas-dynamics. As it was specified above, it is not valid for a relativistic problem where the flow is adiabatic. In paper [3] the statement about isentropy in the relativistic problem is incorrect, though all obtained results are valid by virtue of the mentioned above note about identical realization of the equation for the entropy from (3).
3 The Inhomogeneous Detonation

As the scalar field $\varepsilon_f$ which is "burnt" in the detonation wave represents a gravitational cluster (scalaron "star"), its energy depends on coordinate $r$. In the example of solution considered in Ref. [2] for such cluster $\varepsilon_f \sim r^{-2}$ except for central area. In this connection it is interesting to find a self-similar solution for the relativistic detonation at $\varepsilon_f(r)$. Note that the velocity of the detonation wave according to Eqn. (1) does not depend on energy density $\varepsilon_f, \varepsilon_p$ and remains constant. On this reason the mentioned above dependence is possible to present as:

$$\varepsilon_f = \left( \frac{\tau_0}{\tau} \right)^n E(\xi),$$

where $\xi$ is still determined by expression (4) and $E(\xi)$ is a representative of the function $\varepsilon_f$. In this case also is possible to obtain one equation on $v(r)$ which is possible to write as

$$\frac{dv}{d\xi} \left[ \frac{1}{\omega^2} \left( \frac{v - \xi}{1 - v\xi} \right)^2 - 1 \right] = \frac{2v}{\xi} \frac{\theta^2}{1 - v\xi} - \frac{n\theta^4}{(1 + \omega^2)(1 - v\xi)^2}.$$  

(The equation for the entropy is still satisfy identically.) At $n = 0$ the last equation passes in Eqn. (5). The key difference of this equation from Eqn. (5) is the absence of the velocity $v = 0$ that changes qualitatively the form of solution shown in Fig. 1. On the other hand the small values of $n$ can not in essence change the form of the solution near to front. It is necessary to expect that there is a critical value $n_\ast$ since which this region of solution will be essential change also. It can be found from the following reasons: infinite derivative $dv/d\xi$ at the wavefront, as well as in Eqn. (5) (see Fig. 1), is achieved at tendency of the factor in brackets on the l.h.s. in Eqn. (7) to zero from above and positive determinancy of expression on the r.h.s. At realization of these requirements the flow behind the front will coincide qualitatively with the case considered already [2]. The differences are appeared that on the side of small values of $n$...
Figure 2: The dependence of the representative of energy density $E$ and velocity $v$ on the self-similar variable $\xi$ in the case of $n=0.5$. The singular point (node) of Eqn. (7) is in the point $\xi = 0.66$.

In central region the solution $v(\xi)$ will grow linearly from centre and the complete solution is obtained by matching of the specified two solutions. Thus the value of velocity remains continuous and the jump is experienced only by derivatives (Fig. 2). Note that the point of matching is not casual. According to the qualitative theory of the differential equations it corresponds to a singular point - node. The branches of solutions $v(\xi)$ going out from the point $v, \xi = 0$ and from the wavefront at $\xi = D$ and $v = \omega$ with inevitability fall into this point. Note that the centre ($\xi = 0$) is a singular point - saddle and the solution $v(\xi)$ in a neighbourhood of centre which is interesting for us is a separatrix going from a saddle to a node that is according to the qualitative theory of the differential equations. In accordance with increasing of $n$ the coordinate of this node on the axis $\xi$ is monotonically moved from the minimum value $n = 0, \xi = \omega$ up to $\xi = D$ at $n_* = 1 + \omega^2 = 4/3$. It is a limiting value of inhomogeneity of the energy density of the scalar field at which the solution $v(\xi)$ remains continuous from centre up to the wavefront. Since $n > n_*$ the condition of “over-compressed” detonation is realized [5] when the velocity of the detonation wave becomes larger then the specified value $D$ from Eqn. (1). Behind the wavefront the supersonic current takes place which terminates in a relativistic shock wave (SW). Behind the SW the solution is matched with continuous current of $v(\xi)$ going up to centre. The coordinate of the SW and the parameters of plasma are determined from conservation laws on the SW [4]. This variant is interesting because of the velocities both of the front $D$ and plasma behind it can achieve ultra relativistic values ($D, v \to 1$).

The case when the exponent $n < 0$ has the special interest. Last means that the energy density of the scalar field grows from centre to periphery. (Note that for field stars, when the effective negative pressure is realized, such variants are possible.) The analysis of the Eqn. (4) shows that as against considered before cases the velocity of the plasma in a neighbourhood of centre becomes negative (the motion to centre). It easy to understand qualitatively, as the increasing density of the field energy enters the wavefront. The adiabatic compression of gas
Figure 3: The dependence of the representative of energy density $E$ and velocity $v$ on the self-similar variable $\xi$ in the case of $n=-0.5$. The singular point (node) of Eqn. (7) is in the point $\xi = 0.51$.

in central region brings to increase of the energy density at centre (Fig. 3).

It is important to note that for laboratory problems the similar control of the energy density by laser radiation can bring to increase of temperature at centre. It can be important for processes of laser detonation in a spherical regime [4].

For relativistic scalar stars the considered processes should be described, certainly, in framework of general relativity.

References

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