STUDYING QUARK CONDENSATES
WITHIN MODELS OF FOUR-QUARK INTERACTIONS

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Analysing two models of four-quark interactions which are of intrinsic difference in the behaviours of their correlation lengths some issues of quark condensations are considered. It is demonstrated that the quark condensates substantially are not sensitive to the details of those interactions in the range of coupling constants interesting for applications.

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Studying the phase structure of strongly interacting matter is one of the most complicated tasks of up-to-date theoretical development and any theoretical achievement here is highly non-trivial being very indicative and guiding for planning and designing next round of physics experiments. It concerns not only the heavy ion experiments which are very instrumental to analyse the phase diagram of QCD matter at high temperatures and low baryon number densities but also the fundamental astrophysical research as being very informative to trace the phase diagram at low temperatures and very high baryon number densities. Moreover, nowadays it becomes clear these sources of experimental information on matter phase structure should take into account the strong magnetic fields that are generated in non-central collisions of relativistic heavy ions and are present in cores of neutron stars. Magnetic fields induce non-trivial effects in quark matter probing, first of all, the topological gluon configurations at high temperatures, and modifying color superconducting phase at high baryon number densities and low temperatures.

Obviously, the reliable information may be obtained only in these two extreme regions (asymptotically high temperatures or baryon number densities) of the QCD phase diagram where perturbative calculations are possible, and the studies at finite values of these parameters rely on the model considerations (or effective field theories). As known a dynamical content of relativistic quantum theories is ciphered in their correlation functions which are usually rather complicated functions of incoming and outgoing particle momenta. If one realizes the source of the most complicated behavior in a correlation function (for example, the standard propagators of the intermediate states) it becomes clear the contribution of heavy states to the correlation function of interest will be well approximated by the first terms of series expansion if the kinematics allows. Following such an idea in expanding the Lagrangian in powers of external momenta of the light fields leads to an effective field theory which allows to proceed further into a non-perturbative region with maintaining the symmetry constraints of the underlying theory.

The investigation of color superconductivity problems that followed such a strategy provided us with a new look at the QCD ground state (a quark condensate) in a region of high baryon number densities and shed a new light on the possible QCD phase structure. However, many interesting questions of this research field are still unanswered and we try to clarify those in current note. In particular, if a quark interaction is approximated as a point-like it is not clear what takes place beyond the momentum cut-off and what is a true condensate profile as a function of quark momentum. The nonlocal models with diverse ensembles of gluon vacuum fields are dealing with a zero-mode approximation but a pressing demand to get out of this approach has been discussed (see, for example, ). It remains still unclear in which extent a separable form of interaction, that is quite convenient for analytical study, distorts a genuine picture as well as a role of quark and anti-quark condensates in the different coupling (scalar-pseudoscalar or vector) channels. Novel moment of our analysis is based on the view of these questions from two akin but polar opposite channels. Novel moment of our analysis is based on the view of these questions from two akin but polar opposite channels. Novel moment of our analysis is based on the view of these questions from two akin but polar opposite channels.

Here Hamiltonian density which we are interested in has the form

\[ H = -\bar{q} (i\gamma \nabla + m) q - j_\mu^a \int d\mathbf{y} \langle A_\mu^a A_\nu^{a\prime} \rangle j_\nu^{a\prime} , \quad (1) \]

where \( j_\mu^a = \bar{q} \gamma_\mu q \) is the quark current, with operators of the quark fields \( q, \bar{q} \), taken in spatial point \( \mathbf{x} \) (the variables with prime corresponds to the \( \mathbf{y} \) point), \( m \) is the current quark mass, \( t^a = \lambda^a / 2 \) is the color gauge group.
SU(Nc) generators, μ, ν = 0, 1, 2, 3. The gluon field correlator \( \langle A^a_p A^{\dagger b}_v \rangle \) is taken in the simplest color singlet form with a time contact interaction (without retardation)

\[
\langle A^a_p A^{\dagger b}_v \rangle = G \delta^{ab} \delta_{\mu\nu} F(x - y),
\]

(we do not include corresponding delta-function on time in this formula). Generally speaking the terms spanned on the relative distance are permitted but for the sake of simplicity we neglect corresponding contribution. This simple correlation function is a fragment of corresponding ordered exponent and besides four-fermion interaction accompanied infinite number of multi-fermion vertices arise. But for our purposes it would be quite enough to restrict ourselves with this simple form. The mentioned above effective interactions appear in natural way by the coarse-grained description of the system with the corresponding averaging procedure having in mind that the vacuum gluon field changing stochastically (for example, in the form of instanton liquid, see \[7\]). A formfactor \( F(x) \) is interpreted in a simple manner as an interaction potential of point-like particles. The correlation function itself looks like formally a gauge non-invariant object, but it turns out that there exist an effective way to compensate this shortage by examining, in some sense, all potentials which would be of interest. For example, this set would be convincing enough if it is possible to compare two akin but opposite potentials, one behaving as the delta function formfactor in the coordinate space (one-flavour Nambu–Jona–Lasinio (NJL) model \[8\]) and another as the delta function formfactor in the momentum space, an analog of which is well known in condensed matter physics as the Keldysh model (KKB) \[9\] and corresponds to an infinite correlation length in the coordinate space. It is worth of remarking here that the only feature of this model essential for us further concerns the fact that the complicated integral equations become algebraic ones thanks to the formfactor form. Tuning the scale of coupling constant \( G \) that is interesting for applications should be done by attaching it to the corresponding PDG meson observables.

It is believed that at sufficiently large interaction the ground state of the system transforms from trivial vacuum \( |0\rangle \) (the vacuum of free Hamiltonian) into the mixed state (with quark–anti-quark pairs with the opposite momenta of vacuum quantum numbers) which is presented as the Bogolyubov trial function (in that way some separate reference frame is introduced, and chiral phase becomes fixed)

\[
|\sigma\rangle = T|0\rangle, \quad T = \prod_{p,s} \exp[\varphi_p (a^+_p s b^+_{-p,s} + a^-_{p,s} b^-_{p,s})].
\]

Here \( a^+, a, b^+, b \) are the quark creation and annihilation operators, \( a(0) = 0, b(0) = 0 \). The dressing transformation \( T \) transmutes the quark operators to the creation and annihilation operators of quasiparticles \( A = T a T^\dagger \), \( B^+ = T b^+ T^\dagger \).

The pairing angle can be found from the condition of mean energy minimum \( \langle \sigma | H | \sigma \rangle \). Investigating the Bogolyubov transformation as a function of formfactor demonstrates that the quark–anti-quark pairing angle (dynamical quark mass) does not show any essential dependence on the formfactor profile \[7\]. The most profitable coupling angles \( \theta = 2\varphi \) are presented for comparison in Fig.1 with the solid line for the NJL model and dashed one for the KKB model under normal conditions \( (T = 0, \mu = 0) \). For the delta-like potential in coordinate space (the NJL model) the mean energy diverges and to obtain the reasonable results the upper limit cutoff in the momentum integration \( \Lambda \) is introduced being one of the tuning model parameters along with the coupling constant \( G \) and current quark mass \( m \). Below we use one of the standard sets of the parameters for the NJL model \[10\]: \( \Lambda = 631 \text{ MeV}, G A^2/(2\pi^2) \approx 1.3, m = 5 \text{ MeV}, \) whereas the KKB model parameters are chosen in such a way that for the same quark current masses the dynamical quark ones in both NJL and KKB models coincide at vanishing quark momentum. It is interesting to notice that the momentum \( p_0 \) (parameter) corresponds to the maximal attraction between quark and anti-quark. The value of this parameter reversed determines a characteristic size of quasiparticle. For the models under consideration it is of order of \( p_0 \sim (mM_q)^{1/2} \), where \( M_q \) is a characteristic quark dynamical mass, i.e. the quasiparticle size is comparable with the size of \( \pi \)-meson (Goldstone particle). It is a remarkable fact that the quasiparticle, as it is seen from Fig.11 does not depend noticeably on the formfactor profile or, in other words, on the scale, but is rather dependent on the coupling constant. It is worthwhile to mention here that now we understand (with high clarity) the vacuum ensemble establishes a characteristic scale of the correlation length order. Then the KKB model limit corresponds just to the characteristic system size. The gluon correlation functions as measured in the lattice calculations show for the characteristic size of corresponding configurations the estimates 0.1–0.2 fm \[11\] (see also \[12\]). Besides, the lattice data for gluon propagator which could be interpreted as a gluon mass

![FIG. 1: The most stable equilibrium angles \( \theta \) (in degrees) as function of momentum \( p \) in MeV. The solid line shows the result for the NJL model, dashed one corresponds to the KKB model.](image-url)
Generation in a finite momentum interval support such an estimate [13].

Dynamical quark mass $M_q$ can be expressed via a pairing angle by the relation

$$\sin (\theta - \theta_m) = \frac{M_q}{P_0},$$

where $P_0 = [\mathbf{p}^2 + M_q^2(p)]^{1/2}$ is the energy of quark quasiparticle, below notation $E_p$ would also be applied. The auxiliary angle $\theta_m$ is determined by the relation $\sin \theta_m = m/p_0$, where $p_0 = [\mathbf{p}^2 + m^2]^{1/2}$. It can be shown that the dynamical quark mass can be determined through the equation

$$M_q(p) = m + 2G \int d\mathbf{q} \left(1 - n' - \bar{n}'\right) \frac{M_q'}{P_0} F(p + q),$$

where $n$, $\bar{n}$ is the quark antiquark distribution function under external conditions, $\beta = T^{-1}$, $T$ is the ensemble temperature, $\mu$ is the quark chemical potential

$$n = \left[e^{\beta(P_0 - \mu)} + 1\right]^{-1}, \quad \bar{n} = \left[e^{\beta(P_0 + \mu)} + 1\right]^{-1},$$

integration is performed over momentum $\bar{p} = p/(2\pi)^3$. The relation between coupling constants $\bar{g}$ and $G$ would be considered below. In particular at normal conditions ($T = 0$, $\mu = 0$) the dynamical quark mass in the NJL model is $M_q \sim 340$ MeV. Dynamical quark mass in the KKB model ($F(p) = \delta(p)$) is defined by the equation

$$M(p) = 2G \frac{M_q(p)}{P_0},$$

and the dynamical quark mass is related to the induced quark mass by the relation $M_q = m + M$. In practice, it turns out to be more convenient to use an inverse function $p(M_q)$. In particular, in the chiral limit $M_q = (4G^2 - \mathbf{p}^2)^{1/2}$, at $|\mathbf{p}| < 2G$, $M_q = 0$, at $|\mathbf{p}| > 2G$. Then, the quark states with momenta $|\mathbf{p}| < 2G$ are degenerate in energy $P_0 = 2G$. Fig. 2 demonstrates three branches of the equation solutions for dynamical quark mass. The dots show the imaginary part of solutions which are generated at the point where two real solution branches are getting merged.

Now focusing on the analysis of the KKB model in mean field approximation we present the Lagrangian density as follows:

$$\mathcal{L} = \bar{q} (i\gamma_\mu \partial_\mu + \mu - m) q + \bar{q} \gamma^a J^a_{\mu} \gamma^\mu q,$$

where additional summand with the quark chemical $\mu = \mu \gamma_\mu$ potential was introduced for a convenient work with the Green’s function. We do not show here an integration over $y$ coordinate with the corresponding formfactor to simplify our setup and recall only that the primed variables are associated with point $y$.

Making some transformations with color matrices arising at the Fierz transformation of the quark fields in mean field approximation and taking the well-known relations for color SU(3) group generators

$$\lambda^2_{ij}\lambda^3_k = 2 \delta^i_j \delta^k_l - \frac{2}{3} \delta^i_j \delta^k_l,$$

together with the identity

$$\varepsilon^{ijk}\varepsilon_{jkl} = \delta^i_j \delta^k_l - \delta^i_l \delta^k_j,$$

where $\varepsilon$ is entirely antisymmetric unit tensor) we receive the interaction term separating diquark channel [14] as

$$\lambda^2_{ij}\lambda^3_k = \left(4\alpha - \frac{2}{3}\right) \delta^i_j \delta^k_l + (2 - 4\alpha) \delta^i_j \delta^k_l + 4\alpha \varepsilon^{ijk} \varepsilon_{jkl},$$

where $\alpha$ is an arbitrary number. Now making use the Fierz identity for color SU(3) matrices

$$\delta^i_j \delta^k_l = \frac{1}{3} \delta^i_l \delta^k_j + \frac{1}{2} \lambda^2_{ij}\lambda^3_k,$$

we transform the interaction term to the following form:

$$g \lambda^2_{ij}\lambda^3_k = g_s \delta^i_l \delta^k_j + g_o \lambda^2_{ij}\lambda^3_k + g_d \varepsilon^{ijk} \varepsilon_ {jkl},$$

$g_s = \frac{8}{3} \left(\frac{2}{3} - \alpha\right) g$, $g_o = 2 \left(\alpha - \frac{1}{6}\right) g$, $g_d = 4\alpha g$, which contains the singlet, octet and diquark terms. Besides, the Fierz identities for the spin $\gamma$-matrices should be taken into account

$$\gamma^{\mu}_{\alpha\beta} \gamma^{\mu}_{\gamma\delta} = F^A \gamma^A_{\alpha\beta} \gamma^A_{\gamma\delta},$$

similarly for the diquark coupling we need

$$\gamma^{\mu}_{\alpha\beta} \gamma^{\mu}_{\sigma\delta} = - F^A \gamma^A_{\alpha\beta} C_{\sigma\gamma} \gamma^A_{\delta\beta},$$

where the index $A$ refers to the channels: 1, $\gamma^0$, $\gamma^\mu$, $\gamma^\mu\gamma^5$, $F^A = 1, -1, -1/2, -1/2$, correspondingly (we take here that a permutation of quark fields does not result in changing the sign in identity), $C = \gamma^A_\alpha \gamma^5_\alpha$ is the charge conjugation matrix, $\gamma^T$ denotes transposed matrix. As a
result the interaction term can be brought to the following form

\[
\mathcal{L}_{\text{int}} = g_s F^A \bar{q} \gamma^A q' \cdot q' \gamma^A q +
+ g_\rho F^A \bar{q} \gamma^A \lambda q' \cdot q' \gamma^A \lambda q +
+ g_d F^A \bar{q} \varepsilon^\rho \gamma^A \sigma_{\rho} \lambda \cdot q' \\varepsilon^\rho \gamma^A \sigma_{\rho} \lambda q',
\]

where the aggregated notations are used. It is convenient to introduce the bispinors associated with antiquarks \( \bar{q}_c \), \( q_c \) while dealing with a color superconductivity

\[
q_c = C q^T, \quad \bar{q}_c = q^T C.
\]

Remembering now the following identities

\[
\bar{q} q = \bar{q}_c q_c, \quad \bar{q} \varepsilon_\mu q = -\bar{q}_c \varepsilon_\mu q_c,
\]

the interaction term can be rewritten as

\[
\mathcal{L}_{\text{int}} = g_s F^A \bar{q} \gamma^A q' \cdot q' \gamma^A q +
+ g_\rho F^A \bar{q} \gamma^A \lambda q' \cdot q' \gamma^A \lambda q +
+ g_d F^A \bar{q} \varepsilon^\rho \gamma^A \sigma_{\rho} \lambda \cdot q' \\varepsilon^\rho \gamma^A \sigma_{\rho} \lambda q',
\]

It is of importance to mention the change of sign in the last summand occurs due to the identity valid for the charge conjugation matrix \( C^T = -C \). Now the singlet and octet components symmetric over \( q_c, \bar{q}_c \) fields may be presented as

\[
\mathcal{L}_{\text{int}} = \frac{g_s}{4} F^A [\bar{q} \gamma^A q' \cdot q' \gamma^A q'] : [\bar{q}^\prime \gamma^A q(\pm) A q' \gamma^A q] +
+ \frac{g_\rho}{4} F^A [\bar{q} \gamma^A \lambda q' \cdot q' \gamma^A \lambda q] : [\bar{q}^\prime \gamma^A \lambda q(\pm) A \gamma^A \lambda q] +
- g_d F^A \bar{q} \varepsilon^\rho \gamma^A \sigma_{\rho} q' \cdot q' \varepsilon^\rho \gamma^A \sigma_{\rho} q,
\]

sign \( (\pm)^A \) is determined by the interaction channel \( \gamma^A \).

Using the quark doublets \( Q = (\bar{q}, q_c) \), \( \bar{Q} = (\bar{q}_c, q) \) the Lagrangian \( \mathcal{L} \) can be written as

\[
\mathcal{L} = \frac{1}{2} \bar{Q} \left[ (i\gamma_\mu \partial_\mu - m) \sigma_0 + \mu \sigma_3 \right] Q +
+ \frac{g_s}{4} F^A \bar{Q} \gamma^A \sigma_A Q' \cdot Q' \gamma^A \sigma_A Q +
+ \frac{g_\rho}{4} F^A \bar{Q} \gamma^A \lambda \sigma_A Q' \cdot Q' \gamma^A \lambda \sigma_A Q +
- g_d F^A \bar{Q} \varepsilon^\rho \gamma^A \sigma_{\rho} Q' \cdot Q' \varepsilon^\rho \gamma^A \sigma_{\rho} Q,
\]

where \( \sigma_{\pm} = (\sigma_1 \pm i \sigma_2)/2 \). The matrices \( \sigma \) re acting in the space of variables \( Q, \bar{Q} \), and \( \sigma_0 \) is a unit matrix, \( \sigma_i \) are the Pauli matrices. The notations of direct products of matrices were omitted here and it was meant that either the matrix \( \sigma_0 \) or \( \sigma_3 \) appear in this expression in dependence on the channel \( A \), but for simplicity we indicate by this notation that a concrete form of the matrix \( \sigma_A \) should be specified.

The mean field approximation assumes an identification of non-trivial vacuum expectation values and formulation of respective self-consistency conditions. Now we consider several particular examples, first of all, for the normal conditions \( T = 0, \mu = 0 \) without resorting the approximations related to separating out the dominant interaction (as it is usually done), and try to take into account the contributions of all channels exactly. It becomes clear later this task is very hard to be completed accurately but it is possible to cover almost an entire spectrum of options by analysing the channels.

**Nontrivial average in scalar channel**

For consistency, we will first reproduce the already known result for the quark dynamical mass. Let the nontrivial vacuum expectation value is generated in a scalar channel

\[
\mathcal{L}_{\text{int}} \simeq 2 g_s \bar{q} q' \langle \text{Tr} q' q \rangle = -2 g_s \bar{q} q' \text{Tr} \{iS\}.
\]

Then using the Green function \( S = -i \langle q q' \rangle, S^{-1} = \hat{\rho} - M_q \), for the KKB model one can obtain

\[
M = 2 g_s \text{Tr} \left\{ \frac{1}{\hat{\rho} - M_q} \right\} = 2 G_s \frac{M_q}{E_p},
\]

\( E_p = (p^2 + M_q^2)^{1/2}, \) \( G_s = 2 N_c g_s \). Here, taking the trace means also integration in \( \int dp_0/(2\pi) \). The expression deduced coincides with Eq. \( \text{[9]} \).

We need also the energy density of the quark ensemble \( E = 2 N_c w \), which can be expressed through specific energy attributed to a one quark, see \( \text{[15]} \), in form

\[
w = \int d\vec{p} \, p_0 - \int d\vec{p} \, (1 - n - \tilde{n}) \, p_0 +
+ \frac{1}{4 G_s} \int d\vec{p} \, d\vec{q} \, F(p + q) \, \tilde{M}(p) \tilde{M}(q).
\]

In this expression a density of induced quark mass is used

\[
M_q(p) = m + M(p) = m + \int d\vec{q} \, F(p + q) \, \tilde{M}(q).
\]

The density of mean quark ensemble energy \( \text{[9]} \) is nothing more than the energy functional of the Landau’s Fermiliquid theory \( \text{[16]} \) (see also \( \text{[17]} \)), variation of which over density of induced quark mass gives equation for dynamical quark mass \( \text{[14]} \).

**Scalar and pseudoscalar channels**

Let us suppose non-trivial contributions occur in scalar and pseudoscalar channels:

\[
\mathcal{L}_{\text{int}} \simeq 2 g_s \bar{q} q' \langle \text{Tr} q' q \rangle + 2 g_s x_i \bar{q} i \gamma^5 q' \langle \text{Tr} q' i \gamma^5 q \rangle =
= -2 g_s \bar{q} q' \text{Tr} \{iS\} - 2 g_s \bar{q} i \gamma^5 q' \text{Tr} \{i\gamma^5 iS\}.
\]

Now the Green function is determined by the relation

\[
S^{-1} = \hat{\rho} - M_q - i \gamma^5 C_q.
\]
Then the selfconsistency relations take the form
\[ M = 2G_s \text{Tr} \left\{ \frac{1}{\bar{p} - M_q - i\gamma^5C_q} \right\} = 2G_s \frac{M_q}{E_p}, \]
\[ C = 2G_s \text{Tr} \left\{ \frac{i\gamma^5}{\bar{p} - M_q - i\gamma^5C_q} \right\} = 2G_s \frac{C_q}{E_p}, \tag{11} \]
where \( C_q = c + C \), \( E_p = (p^2 + M_q^2 + C_q^2)^{1/2} \). Here \( c \) is a bare ("current") "mass" in pseudoscalar channel. At \( c \equiv 0, C = 0 \), because in general case it is impossible to satisfy both conditions for dynamical masses \( M \). Let us stress that at \( c \equiv 0, m \equiv 0 \) it turns out impossible to determine the phase of dynamical mass.

Let us select the following averages in this configuration:
\[ \langle \bar{q}q \rangle = \sum_{\mu} \langle \bar{q}_\mu q^\mu \rangle, \]
\[ \langle \bar{q}q' \rangle = \sum_{\mu} \langle \bar{q}_\mu q'^\mu \rangle, \]
\[ \langle \bar{q}q^i \rangle = \sum_{\mu} \langle \bar{q}_\mu q^i \rangle, \]
\[ \langle \bar{q}q_{ij} \rangle = \sum_{\mu} \langle \bar{q}_\mu q_{ij} \rangle, \]

The Green function is found from the following relation
\[ \Delta^\rho = -2g_d \text{Tr} \langle \bar{q}q^i \rangle, \]
\[ \Delta^\delta = -2g_d \text{Tr} \langle \bar{q}q_{ij} \rangle, \]

Scalar and Isotriplet of the Octet Channel for SU(3) group

Let us select the following averages in this configuration:
\[ \mathcal{L}_{int} \approx 2g_s \bar{q}q' \left( \text{Tr}q\bar{q} + 2g_d \frac{\lambda}{2} q' \left( \text{Tr}q \right) \frac{\lambda}{2} \right) = -2g_s \bar{q}q' \text{Tr} \left\{ \lambda \right\} - 2g_d \bar{q} \lambda \left( \text{Tr} \lambda \right). \]

Diquark Condensation (Color Superconductivity),
\[ \mu \neq 0, T = 0 \]

We restrict ourselves here with considering the color superconductivity in pseudoscalar channel \( \gamma_5 \), where, as we demonstrate below, there exist an interesting solution with a real gap. The self-consistency relations in this case take the form
\[ M = -2g_s \text{Tr} \langle \bar{q}q' \rangle, \]
\[ \Delta^\rho = -2g_d \text{Tr} \langle \bar{q}q^\rho \rangle, \]
\[ \Delta^\delta = -2g_d \text{Tr} \langle \bar{q}q^\delta \rangle. \]

In order to simplify an analysis we select the third component \( \rho = 3 \) as nontrivial for color superconductivity. The matrix of inverse Green function has the form
\[ S^{-1} = \begin{pmatrix} (\hat{p}_+ - M_q) \delta_{ij} & \Delta^\rho \delta_{ij} \gamma_5 \\ \Delta^\rho \delta_{ij} \gamma_5 & (\hat{p}_- - M_q) \delta_{ij} \end{pmatrix}. \]

Such a form of the matrix is a direct consequence of the spinor \( \bar{q}_\mu, q^\mu \) forms. The Green’s function itself is obtained from the Frobenius formula for block matrices
\[ S^{-1} = \begin{pmatrix} A & I \\ J & B \end{pmatrix}, \quad S = \begin{pmatrix} A^{-1} - A^{-1}IL^{-1} & A^{-1}IL^{-1} \\ -L^{-1}JA^{-1} & L^{-1} \end{pmatrix}. \]

There exist two diagonal in color components \( A = (\hat{p}_+ - M_q) \delta_{ij}, B = (\hat{p}_- - M_q) \delta_{ij}, \) and two components including antisymmetric in indices tensor \( I = \Delta^3 \delta_{ij} \gamma_5, J = \Delta^3 \delta_{ij} \gamma_5. \) In order to proceed we need to consider the blocks of the Green function construction
\[ K = A - IB^{-1}J = K_\sigma \Sigma_{ij} + K_d D_{ij}, \]
\[ K_\sigma = \hat{p}_+ - M_q, \quad K_d = \left( \hat{p}_+ - M_q - \Delta^2 \frac{\hat{p}_+ - M_q}{(p_0 - \mu)^2 - E_p^2} \right). \]

To get this expression we use the fact that product of two antisymmetric tensors generates (doublet) projector upon the 1 and 2 components of the color space
\[ D_{ij} = \delta_{i1}\delta_{j1} + \delta_{i2}\delta_{j2} = -\epsilon^{3ij} \epsilon^{3kj}. \]

Then, we have introduced an additional singlet component
\[ \Sigma_{ij} = \delta_{i3}\delta_{j3}, \quad \delta_{ij} = \Sigma_{ij} + D_{ij}, \]
and decomposed the matrix block \( K \) into two parts—the singlet \( K_\sigma \) and the doublet \( K_d \). In principle, a similar
Utilizing these results we can expand the Green function

\[ L = B - JA^{-1}I = L_{\sigma} \Sigma_{ij} + L_{d} D_{ij}, \]

where the tensor components are

\[ L_{\sigma} = \hat{p}_{-} - M_{q}, \quad L_{d} = \hat{p}_{-} - M_{q} - \Delta^{2} \frac{\hat{p}_{+} - M_{q}}{(p_0 + \mu)^2 - E_{p}}, \]

Having found the inverse components \( K_{\sigma}^{-1}, L_{\sigma}^{-1} \), in particular,

\[ K_{\sigma}^{-1} = \left[[p_{-}^{2} - M_{q}^{2})\hat{p}_{+} + M_{q}] - \Delta^{2}(\hat{p}_{-} + M_{q})]/Z_{T}, \]
\[ L_{\sigma}^{-1} = \left[[p_{+}^{2} - M_{q}^{2})(\hat{p}_{-} + M_{q}) - \Delta^{2}(\hat{p}_{+} + M_{q})] / Z_{T}, \]
\[ Z_{d} = [(p_{0}^{2} - (E + \mu)^2 - \Delta^{2}][p_{0}^{2} - (E - \mu)^2 - \Delta^{2}], \]

where \( p^{2}_{\pm} = \hat{p}_{\pm}\hat{p}_{\pm} \), we are able to obtain the following expressions for the building elements of the Green function

\[ -A^{-1}IL^{-1} = \gamma_{5}\Delta^{3} [L_{d}(\hat{p}_{+} + M_{q})]^{-1} \varepsilon^{3ij}, \]
\[ -L^{-1}JA^{-1} = \gamma_{5}\Delta^{3} [(\hat{p}_{+} + M_{q})L_{d}]^{-1} \varepsilon^{3ij}. \]

Utilizing these results we can expand the Green function in the components of the matrix \( \sigma_{i} \) as

\[ S = \sum_{i=0,\ldots,3} S^{i} \sigma_{i}, \quad (12) \]

\[ S^{0} = s^{0}_{0} \Sigma_{ij} + s^{0}_{d} D_{ij}, \]
\[ S^{\sigma} = [(p_{0}^{2} + \mu^{2} - E_{p}^{2})(\hat{p} + M_{q}) - 2\mu p_{0} \hat{p}]/Z_{\sigma}, \]
\[ s^{d}_{d} = [(p_{0}^{2} + \mu^{2} - E_{p}^{2} - \Delta^{2})(\hat{p} + M_{q}) - 2\mu p_{0} \hat{p}]/Z_{d}, \]
\[ s^{1 + 2} = \gamma_{5}\Delta^{3} \varepsilon^{3ij}(p_{0}^{2} - \mu^{2} - E_{p}^{2} - \Delta_{q}^{2} + \hat{p} \hat{p} - \hat{p} \hat{p})/Z_{d}, \]
\[ s^{3} = s^{3}_{0} \Sigma_{ij} + s_{3} D_{ij}, \]
\[ s_{3}^{0} = [(p_{0}^{2} + \mu^{2} - E_{p}^{2}) \hat{p} - 2\mu p_{0} (\hat{p} + M_{q})]/Z_{3}, \]
\[ s_{3}^{d} = [(p_{0}^{2} + \mu^{2} - E_{p}^{2} + \Delta_{q}^{2}) \hat{p} - 2\mu p_{0} (\hat{p} + M_{q})]/Z_{d}, \]

and

\[ Z_{\sigma} = [(p_{0}^{2} + \mu^{2} - E_{p}^{2}[[p_{0}^{2} - \mu^{2} - E_{p}^{2}]]. \]

calculating the traces, which include the integration over the component \( p_{0} \), and allocating channels in the self-consistency equation (12), finally we obtain the following decomposed system of equations

\[ M = 4 \ g_{s} \ \frac{M_{q}}{E_{p}}, \]
\[ M = 4 \ g_{s} \ \frac{M_{q}}{E_{p}} \left[ \frac{E_{p} + \mu}{P_{+}} + \frac{E_{p} - \mu}{P_{-}} \right], \quad (13) \]
\[ 1 = 4 \ g_{d} \left[ \frac{1}{P_{+}} + \frac{1}{P_{-}} \right], \]

where \( P_{\pm} = [(E_{p} \pm \mu)^{2} + \Delta^{2}]^{1/2} \). The first line describes the third (singlet) component in the color space. Comparison it with the equation (10) we conclude that in general case the diquark channel has no any impact on the dynamical mass, only the coupling constant is getting three times smaller value. The second and third lines describe the color components of doublets 1 and 2. It is reasonable to mention here that we have not introduced the bare condensate \( \Delta_{0} \) and it is why the third equation does not contain a gap explicitly. It is also convenient to introduce the following designations of the coupling constants \( G_{s} = 2 \ N_c g_{s}, \ G_{d} = 2 \cdot 2 g_{d} \). One curious fact of this machinery is that if the scalar channel is used for color superconductivity instead of the pseudoscalar one the system of equations (10) becomes inconsistent (controversial) because in the lowest line a negative unity appears instead of positive unity. We do not introduce the additional separate notations for the masses of singlet and doublet channels in order not to overload the formulæ. A thorough analysis of this equation system shows the system has no the consistent real solutions, and they appear only when either the induced mass or the gap are getting zero values separately. The easiest way to observe this fact is to analyse the particular case when, for example, \( \mu = 0 \), i.e. in practice such a complex equation system is unnecessary, and we return to the equation (10) at \( \Delta = 0 \), and have the third line of the system (13) valid when the induced quark mass is zero \( (M = 0) \). In order to have a reasonable estimate of the gap characteristic values we consider the particular situation of normal conditions with the parameter \( \alpha = 1/6 \). The octet channel contribution disappears for such a configuration. Setting up the coupling constant \( g \) such a value that the dynamical quark mass at zero momentum for normal conditions \( (\mu = 0) \) at zero temperature in the chiral condensate phase equals to \( M_{q}(0) \approx 345 \) MeV we have from the third line of the system (13) for the gap energy \( P = 2G_{d} \approx 114 \) MeV. In the chiral limit \( m = 0 \), \( M_{q}(0) \approx 340 \) MeV then

\[ \Delta = (4G_{d}^{2} - P^{2})^{1/2}. \]

In Fig. (4) the solutions for the gap equation are displayed when the current quark mass is \( m = 5 \) MeV. The leftmost curve is obtained for normal conditions. The following right ones were received for the growing chemical potential with the step 100 MeV. It is clear that, although the color superconductivity can be developed for normal conditions, the gap value is essentially smaller (approximately three times less) than the chiral condensate. However, at the chemical potential growing the dynamical quark mass in the phase of non-zero chiral condensate falls quickly down and one could expect the color superconductivity phase more advantageous at large chemical potentials. In order to clarify this point we need to compare the energies and baryon charges of both phases. We are doing that here but in the chiral limit only. Then the quark energy in the phase of non-zero chiral quark condensate is degenerate as it follows
from equation (10), $E_p = 2G_s$. Therefore, to deal with the quark ensemble of changing a baryon/quark density we need to introduce the Fermi momentum $P_F$ that characterises the process of filling up the Fermi sphere by quark quasiparticles (the details can be found in [15]). Having determined the energy density and baryon charge density of an ensemble we may obtain an inter-relation between the Fermi momentum and the chemical potential. It is worth to remember here the dynamical quark mass has the zero value for momenta smaller than the Fermi momentum of $M_q = 0, p < P_F$ in the chiral limit. Beyond the Fermi momentum the dynamical quark mass is defined as $M_q = (4G_s^2 - p^2)^{1/2}$, $P_F < p < 2G_s$. At large momenta $2G_s < p$ the dynamical quark mass has the zero value. The ensemble energy density is defined as (we have omitted an unimportant normalization constant, i.e. the first term in Eq. (11))

$$H_{cc} = \int_0^{2G_s} d\tilde{p} \left( -2N_c P_0 + \frac{M^2}{4G_s} \right) = \int_0^{P_F} d\tilde{p} \left( -2N_c P_0 + \frac{M^2}{4G_s} \right)$$

$$H_{cc} = \frac{N_c}{2\pi^2} \frac{2G_s}{15} (2G_s) \left( \frac{2(2G_s)^5}{15} - (2G_s)^2 P_F^3 + \frac{P_F^5}{5} \right) , \quad P_F < 2G_s$$

$$H_{cc} = \frac{N_c}{4\pi^2} \left[ P_F^3 (2G_s)^4 \right] , \quad P_F > 2G_s .$$

The baryon number density is defined as

$$Q_{cc} = 2N_c \int_0^{P_F} d\tilde{p} = \frac{N_c}{3\pi^2} P_F^3 .$$

Now we are able to define the energy density and baryon number density of the superconducting state $H_{sc}$. By definition, the energy is given by

$$H_{sc} = \frac{1}{2} \int d\tilde{p} \int \frac{dp_0}{2\pi} \text{Tr} \{ \gamma^0 \sigma_3 \otimes [\Sigma + D] S e^{-ipc} \} + \int d\tilde{p} \frac{\Delta^2}{4g_d} .$$

(16)

Here, the direct products of the free Hamiltonian part pick out a spinor structure, we are interested in, $\gamma_0$, unit matrix $\sigma_3$ acting in the quark doublet space and, finally, an identity matrix of color space decomposed in the singlet $S$ and doublet $D$ components. The second term describes the interaction energy and may be obtained in a way similar to the interaction contribution in Eq. (9) (similar Eq. (13)) by calculating the average energy over the state which we are interested in. Making use the expansion of matrix $S$ (12) and calculating the corresponding integrals we obtain

$$H_{sc} = H_\sigma + H_d ,$$

where $H_\sigma$ is the contribution of singlet component (3) in color space and $H_d$ is the contribution of doublet 1, 2. The color component 3 being pure as to the color superconducting condensate can be easily calculated as

$$H_\sigma = 2 \int d\tilde{p} p \frac{1}{4\pi^2} P_F^3 .$$

(18)

It looks interesting to note this contribution is in factor $N_c$ weaker than the contribution of $H_{cc}$ because of the color singlet lost. Moreover, as it was mentioned, in the color superconducting phase the dynamical quark mass equals precisely to zero because the self-consistent solutions of the system (13) are absent, i.e. the quark energy contribution is $p$. The contribution of the quark doublet looks like

$$H_d = -2 \cdot 2 \int d\tilde{p} \frac{1}{2} (P_+ + P_-) + \int d\tilde{p} \frac{\Delta^2}{4g_d} .$$

(19)

Here in the free part (the first term) one factor 2 corresponds to the contribution of two color components and another factor 2 corresponds to the contribution of the two spin components. It is also interesting to note that these expressions are the energy functionals of the Landau theory of Fermi liquid, and and the equation system (13) can be calculated by taking their variations over dynamical mass and gap. Similarly, we have for the baryon charge

$$Q_{sc} = \frac{1}{2} \int d\tilde{p} \int \frac{dp_0}{2\pi} \text{Tr} \{ \gamma^0 \sigma_3 \otimes [\Sigma + D] S e^{-ipc} \} ,$$

$$Q_{sc} = Q_\sigma + Q_d ,$$

(20)

$$Q_\sigma = \frac{1}{3\pi^2} P_F^3 ,$$

$$Q_d = 2 \cdot 2 \int d\tilde{p} \frac{1}{2} \left( \frac{E_p + \mu}{P_+} - \frac{E_p - \mu}{P_-} \right) .$$
Comparing the bottom line to Eq. (19) we find the following identity

$$Q_d = -\frac{\partial H_d}{\partial \mu},$$

because only first summand of the type of free energy shows an obvious dependence on the chemical potential in Eq. (19). Now we find the inter-relation between the Fermi momentum and chemical potential $P_F(\mu)$. By definition, a chemical potential is an energy necessary to add (remove) a quasiparticle into a system

$$\mu = \frac{dH}{dQ} = \frac{\partial H}{\partial P_F} dP_F + \frac{\partial Q}{\partial P_F} dP_F + \frac{\partial Q}{\partial \mu} d\mu.$$

It allows us to obtain the differential equation for the function $P_F(\mu)$

$$\frac{dP_F}{d\mu} = -\frac{\mu}{\partial Q/\partial \mu} \frac{\partial Q}{\partial P_F} - \frac{\partial H}{\partial P_F}.$$

In the case of interest, the obvious dependence on the Fermi momentum is available in the contributions $H_\sigma$ and $Q_\sigma$. The free part of energy $H_d$ and baryon charge $Q_d$ are obviously depend on the chemical potential

$$\frac{\partial Q}{\partial P_F} = \frac{\partial Q_\sigma}{\partial P_F} = \frac{P_F^2}{\pi^2}, \quad \frac{\partial H}{\partial P_F} = \frac{\partial H_\sigma}{\partial P_F} = \frac{P_F^2}{\pi^2}.$$

$$\frac{\partial Q}{\partial \mu} = \frac{\partial Q_d}{\partial \mu} = 2 \cdot 2 \int d\tilde{p} \frac{1}{2} \Delta^2 \left( \frac{1}{P_F^+} + \frac{1}{P_F^-} \right).$$

Using these relations we obtain

$$\frac{P_F^2 (P_F - \mu)}{\pi^2} dP_F d\mu = \mu \frac{\partial Q_d}{\partial \mu} + Q_d = \frac{\partial Q_d}{\partial \mu}.$$

which results in the definition of the Fermi momentum as

$$P_F^4 - \mu \cdot P_F^3 - 3\pi^2 \mu Q_d = 0.$$

We can also consider Eq. (23) as a differential equation with the natural initial condition $P_F(0) = 0$. In particular, at $\mu \to 0$ we can obtain $Q(0) = 0$, $\partial Q/\partial \mu = 4(2G_d)^2/(15\pi^2)$. Then for the Fermi momentum we have approximately

$$P_F^4 \approx \frac{4}{5} (2G_d)^2 \mu^2.$$

At large $\mu$ the Fermi momentum is linearly increasing, see Fig. 4.

In order to find the density of the baryon charge $Q_d$ (and its derivative $\partial Q/\partial \mu$) we need to find out the boundary of momentum integration area with nontrivial superconducting color condensate. Taking $\Delta$ in the third equation of system (13) the zero value limit we obtain

$$p_{\text{min}} = 0, \quad p_{\text{min}} = \mu \left( 1 - \frac{4G_s \mu}{P_F^+} \right) \frac{1}{2}, \quad \mu > 2G_s,$$

$$p_{\text{max}} = 2G_s + \left( 4G_s^2 + \mu^2 \right)^{1/2}. $$

Fig. 5 demonstrates the energy (in an arbitrary unit) of the phase of non-zero chiral condensate (solid line) and color superconducting phase (dashed line) as a function of baryon number density which by definition is in factor three smaller than the density of quark baryon charge $Q_B = Q/3$. It is visible that at low densities a formation of chiral condensate is advantageous. At densities slightly higher than the density of normal nuclear matter $\sim 0.217/fm^3$ ($P_F^+ \approx 364$ MeV, $\mu^c \approx 330$ MeV and for comparison the dynamical quark mass is $M(0) = 2G_s = 340$ MeV) the color superconductivity state becomes profitable. It is interesting to emphasize an important role of the singlet component. The dashed
line is quickly going down without its contribution and the color superconductivity phase would be dominating already at unreasonably low baryon number density.

We obtained the estimate of the chemical potential (the density of the quark ensemble) in the chiral limit and expect that reaching this value a system could undergo the phase transition into a color superconducting phase. However, beyond the chiral limit the situation remains somewhat unclear. The energy of quark ensemble in a state with non-zero value of chiral condensate is given by the following expression similar to (14)

\[ E(N) = -\frac{g_N Q^2}{4}\int d^4x\left(\langle\bar{q}q\rangle + \langle\bar{q}q\rangle^2\right) + \left(\frac{1}{g_N}ight)\left(\frac{1}{2}\langle\bar{q}q\rangle^2 + \frac{1}{2}\langle\bar{q}q\rangle \langle\bar{q}q\rangle^2\right), \]

where \(p_0 = (p^2 + m^2)^{1/2}\), \(P_0 = (p^2 + M_q^2)^{1/2}\). The first term appears here because of a normalization reasons in order to keep zero energy of ensemble at switching of the interaction. Now turning to Eq. (6) we conclude that the asymptotic behavior of induced quark mass and its energy at large momenta \(p \to \infty\) are the following

\[ M \to 2G_s \frac{m}{p_0}, \quad P_0 \to p_0 + \frac{M_m}{p_0}. \]

Substituting the asymptotic expression in the (25), and bearing in mind the definition of coupling constant \(G_s = 2N_c g_s\), we conclude that the ensemble energy diverges linearly, and is going to the negative infinity but in the chiral limit the ensemble energy is finite. Discontinuity of the functional of average energy as a function of current mass has been found in [2] where it was mentioned that any formal conclusion about the color superconductor phase based on comparing the energy of two phases is unreliable. But it is worth of noting that the infinite energy comes from integration with asymptotically low value of chiral condensate, and then it looks quite possible to consider a mixed state at large momentum but an analysis of its formation dynamics is beyond the scope of this our study.

In this article we consider a special choice of the parameter \(\alpha\) responsible for the separation of singlet, octet and diquark channels of interaction in order to neutralize the octet channel. Obviously, it would be interesting to find out a way of fixing it grounded on the argument of the energy gain.

**NJL model**

Now to have a deeper insight we present results for the model with a gluon correlator behaving as a delta-function in coordinate space. The corresponding equation system the dynamical mass and gap looks like

\[ M = 4 g_s \int d^4p \frac{M_q}{E_p}, \]

\[ M = 4 g_s \int d^4p \frac{M_q}{E_p} \left[ \frac{E_p + \mu}{P_+} + \frac{E_p - \mu}{P_-} \right], \quad (26) \]

\[ \Delta = 4 g_d \Delta_q \int d^4p \left[ \frac{1}{P_+} + \frac{1}{P_-} \right], \]

and we suppose the quark mass and the superconductor gap do not depend on momentum. As in previous section solutions are searched either in the form of the chiral condensate \(M \neq 0, \Delta = 0\), or in the form of color superconductor \(M = 0, \Delta \neq 0\), or in the form of color superconductor \(M = 0, \Delta = 0\).

FIG. 6: Dynamical quark mass (solid line) and the gap (dashed line) as a functions of the baryon density in the NJL model.

It is essential to keep in mind that the coupling constant \(G_s\), and some others, which are present in this consideration have a different dimension comparing to the coupling constants handled in previous section. The constant \(G_s\) is chosen in such a way to have the induced quark mass equal to \(M = 340\) MeV. The parameter \(\alpha = 1/6\) fixes the inter-relation between the constant magnitudes in different channels, similarly to what we have for the KKB model. The quark energy at given momentum denotes with a corresponding index, for example \(E_A\). To make our analysis transparent we are dealing here with the chiral limit again putting the current quark mass equal to zero and further follow the same line as in the considered KKB model. Then the energy density of ensemble in the

\[ M = \frac{G_s}{2\pi^2} M_q \left( AE_A - P_F E_F - M_q^2 \ln \frac{\Lambda + E_A}{P_F + E_F} \right). \quad (27) \]
The baryon number density in the color superconducting phase looks like

\[ Q_d = 2 \int d\mathbf{p} \left( \frac{E_p + \mu}{P_+} - \frac{E_p - \mu}{P_-} \right) = \]

\[ = 2 \left\{ \frac{1}{6\pi^2} \left[ E_{A+\mu}^3 - E_{A-\mu}^3 \right] + \frac{\mu^2 - \Delta^2}{2\pi^2} \left[ E_{A+\mu} - E_{A-\mu} \right] - \frac{\mu}{2\pi^2} \left[ (A + \mu) E_{A+\mu} + (A - \mu) E_{A-\mu} - \Delta^2 \ln \left( \frac{\Lambda + \mu + E_{A+\mu}}{\Delta} \right) \right] \right\}. \]

Now we have the full setup to perform an analysis similar to that done for the KKB model. In particular, for example, the Fermi momentum dependence on chemical potential \( P_F(\mu) \) (see Fig. 3) occurs almost linear function but we do not present these data here. Fig. 6 demonstrates the dynamical quark mass (solid line) and the gap (dashed line) in the NJL model as the functions of baryon number density. In distinction with the KKB model at low density the gap is not appreciably developing, although, in fact, it is easy to see from the equation system (29) that at low values of chemical potential we are dealing with the equation of the same type as an equation for the dynamical quark mass. The reason why the gap does not reach an appreciable value is related to the smallness of coupling constant \( G_d \) comparing to the constant \( G_s \) (it is weaker approximately in factor three) and the corresponding integral can not provide a proper critical value. It should be noted, however, that at high baryon number densities the values obtained values are not reliable because the characteristic quark momenta available in the problem become quite comparable with the cutoff parameter \( \Lambda \). Fig. 7 demonstrates the energy (in some arbitrary units) of non-zero chiral condensate phase and the phase of color superconductor (dashed line) as a functions of baryon number density. Comparing this figure to the feature 5 we can see that the phase transition boundary in the color superconductor state in both models are roughly the same. It is also easy to guess how to rewrite the necessary formulae for the arbitrary form-factor \( F(p) \).
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