Self consistent and covariant propagation of pions, nucleon and isobar resonances in cold nuclear matter

C.L. Korpa\textsuperscript{a} and M.F.M. Lutz\textsuperscript{b,c}

\textsuperscript{a}Department of Theoretical Physics, University of Pecs, Ifjusag u. 6, 7624 Pecs, Hungary
\textsuperscript{b}Gesellschaft für Schwerionenforschung (GSI), Planck Str. 1, 64291 Darmstadt, Germany
\textsuperscript{c}Institut für Kernphysik, TU Darmstadt D-64289 Darmstadt, Germany

Abstract

We evaluate the in-medium spectral functions for pions, nucleon and isobar resonances in a self consistent and covariant manner. The calculations are based on a recently developed formulation which leads to predictions in terms of the pion-nucleon scattering phase shifts and a set of Migdal parameters describing important short range correlation effects. We do not observe significant softening of pion modes if we insist on reasonable isobar resonance properties but predict a considerable broadening of the $N(1440)$ and $N(1520)$ resonances in nuclear matter. Contrasted results are obtained for the s-wave $N(1535)$ and $N(1650)$ resonances which are affected by a nuclear environment very little. The properties of slowly moving isobar’s in nuclear matter are found to depend very sensitively on a soft form factor in the $\pi NN$ vertex, which is not controlled by the $\pi N$ scattering data.

1 Introduction

The study of pion propagation in dense nuclear matter is of central importance when addressing the in-medium modifications of nucleon and delta resonances \cite{1,2,3,4,5,6,7,8,9,10,11,12,13,14}. There are strong hints from the empirical photon nucleus absorption cross section as well as from pion-nucleus scattering data that the low-lying nucleon and delta resonances do change their properties in nuclear matter substantially already at nuclear saturation density \cite{15,16,17,18,19,20}. Recent data on electroproduction of isobars off helium three \cite{21} are interpreted in terms of a repulsive in-medium mass change...
and an increased decay width of the isobar-resonance [21]. Since most nucleon and isobar resonances have a substantial decay fraction into one nucleon and one pion, the in-medium resonance structure reflects to a large extent the medium modified propagation properties of pions. The pion self energy in nuclear matter is quantitatively constrained by pionic atom data [6,22,23]. Most exciting are the recently established states where a negatively charged pion is bound by a heavy nucleus in an s-wave or p-wave state [24]. It is still an open problem to find a quantitative and microscopic derivation in particular for the large absorptive part in the nuclear optical potential needed in the phenomenological description of pionic atom data [6,22,23]. The problem requires a non-perturbative many-body approach, since the low-density expansion ceases to converge rapidly at the relevant nuclear densities even when chiral correction terms are considered [25,26,27].

In this work we generalize the covariant framework of [28], which was recently proposed for the self consistent propagation of antikaons and hyperon-resonances in nuclear matter, to the problem of pion, nucleon- and isobar-resonance propagation incorporating short range correlation effects. The merit of our scheme is that it is formulated entirely in terms of the two-body scattering phase-shifts properly extrapolated to subthreshold energies. Given the empirical \( \pi N \) phase shifts and values for the Migdal parameters [6,11,12,13] describing important short range correlations the scheme is parameter free. We expect self consistency to lead to a broadening of the pion spectral function which may help to establish a microscopic understanding of pionic atom data and also offer an improved understanding of the expected broadening of nucleon and isobar resonances in nuclear matter. Our present approach constitutes a significant progress as compared to previous self consistent calculations [8,9,10] which were based on p-wave pion-nucleon dynamics only. Moreover this work is the first attempt to consider the effects of the in-medium mixing of partial wave amplitudes. In particular the feedback effect of the in-medium modified s-wave \( N(1520) \) and \( N(1650) \), the p-wave \( N(1440) \) and the d-wave \( N(1535) \) resonances to the propagation properties of pions will be addressed in this work. Similarly the isobar resonances s-wave \( \Delta(1620) \), p-wave \( \Delta(1232) \) and \( \Delta(1600) \) and the d-wave \( \Delta(1700) \) resonances are considered.

2 Self consistent nuclear pion dynamics

We briefly recall the self consistent and covariant many-body framework introduced in [28] appropriately adjusted to pion propagation in nuclear matter. First we recall the vacuum on-shell pion-nucleon scattering amplitude

\[
\langle \pi^j(\vec{q}) N(\vec{p}) | T | \pi^i(q) N(p) \rangle = (2\pi)^4 \delta^4(q + p - \vec{q} - \vec{p})
\]
where $\delta^4(\ldots)$ guarantees energy-momentum conservation and $u(p)$ is the nucleon isospin-doublet spinor. The vacuum scattering amplitude is decomposed into its isospin channels. Applying standard notation for the Pauli matrices $\tau_i$ we write

$$T_{\pi N \to \pi N}(\bar{q}, \bar{p}; q, p) = T^{(1/2)}(\bar{k}, k; w) P^{ij}_{(I=1/2)} + T^{(3/2)}(\bar{k}, k; w) P^{ij}_{(I=3/2)},$$

$$P^{ij}_{(I=1/2)} = \frac{1}{3} \tau^i \tau^j, \quad P^{ij}_{(I=3/2)} = \delta^{ij} 1 - \frac{1}{3} \tau^i \tau^j,$$

where $q, p, \bar{q}, \bar{p}$ are the initial and final pion and nucleon 4-momenta and

$$w = p + q = \bar{p} + \bar{q}, \quad k = \frac{1}{2} (p - q), \quad \bar{k} = \frac{1}{2} (\bar{p} - \bar{q}).$$

In quantum field theory the scattering amplitudes follow as the solution of the Bethe-Salpeter matrix equation

$$T(\bar{k}, k; w) = K(\bar{k}, k; w) + \int \frac{d^4 l}{(2\pi)^4} K(\bar{k}, l; w) G(l; w) T(l, k; w),$$

$$G(l; w) = -i S(\frac{1}{2} w + l) D(\frac{1}{2} w - l),$$

in terms of the Bethe-Salpeter kernel $K(\bar{k}, k; w)$, the free space nucleon propagator $S(p) = 1/(p - m_N + i \epsilon)$ and pion propagator $D(q) = 1/(q^2 - m^2_{\pi} + i \epsilon)$. Following [29] we neglect self energy corrections in the nucleon and pion propagators. In a chiral scheme such effects are of subleading order. The Bethe-Salpeter equation (4) properly implements Lorentz invariance and unitarity for the two-body scattering process. The generalization of (4) to a coupled-channel system is straightforward.

The pion-nucleon scattering process is readily generalized from the vacuum to the nuclear matter case. In compact notation we write

$$\mathcal{T} = \kappa + \kappa \cdot \mathcal{G} \cdot \mathcal{T}, \quad \mathcal{T} = \mathcal{T}(\bar{k}, k; w, u), \quad \mathcal{G} = \mathcal{G}(l; w, u),$$

where the in-medium scattering amplitude $\mathcal{T}(\bar{k}, k; w, u)$ and the two-particle propagator $\mathcal{G}(l; w, u)$ depend now on the 4-velocity $u_\mu$ characterizing the nuclear matter frame. For nuclear matter moving with a velocity $\vec{v}/c$ one has

$$u_\mu = \left( \frac{1}{\sqrt{1 - \vec{v}^2/c^2}}, \frac{\vec{v}/c}{\sqrt{1 - \vec{v}^2/c^2}} \right), \quad u^2 = 1.$$
We emphasize that (5) is properly defined from a Feynman diagrammatic point of view even in the case where the in-medium scattering process is no longer well defined due to a broad pion spectral function. We consider the effect of an in-medium modified two-particle propagator $\mathcal{G}$

$$
\Delta S(p, u) = 2\pi i \Theta[p \cdot u] \delta(p^2 - m_N^2) (\not{p} + m_N) \Theta(k_F^2 + m_N^2 - (u \cdot p)^2),
$$

$$
S(p, u) = S(p) + \Delta S(p, u), \quad \mathcal{D}(q, u) = \frac{1}{q^2 - m_\pi^2 - \Pi(q, u)},
$$

$$
\mathcal{G}(l; w, u) = -i S(\frac{1}{2} w + l, u) \mathcal{D}(\frac{1}{2} w - l, u),
$$

where the Fermi momentum $k_F$ parameterizes the nuclear density $\rho$ with

$$
\rho = -2 \text{tr} \gamma_0 \int \frac{d^4 p}{(2\pi)^4} i \Delta S(p, u) = \frac{2 k_F^3}{3 \pi^2 \sqrt{1 - \bar{v}^2/c^2}}. \quad (8)
$$

In the rest frame of the bulk with $u_\mu = (1, \vec{0})$ one recovers with (8) the standard result $\rho = 2 k_F^3 / (3 \pi^2)$. In a more complete approach the effect of nucleonic correlation and binding on $\Delta S$ should be considered. This is beyond the scope of this work. The pion self energy $\Pi(q, u)$ is evaluated self consistently in terms of the in-medium scattering amplitudes

$$
\Pi(q, u) = 2 \text{tr} \int \frac{d^4 p}{(2\pi)^4} i \Delta S(p, u) \hat{T}(\frac{1}{2} (p - q), \frac{1}{2} (p - q); p + q, u)
$$

$$
+ \Delta \Pi(q, u), \quad \hat{T} = \frac{1}{3} \hat{T}^{(I=1/2)} + \frac{2}{3} \hat{T}^{(I=3/2)}, \quad (9)
$$

where the in-medium amplitudes, $\hat{T}^{(I)}(\vec{k}, k; w, u)$, are defined with respect to the free-space interaction kernel, $K$,

$$
\hat{T} = K + K \cdot \mathcal{G} \cdot \hat{T} = T + T \cdot \Delta \mathcal{G} \cdot \hat{T}, \quad \Delta \mathcal{G} = \mathcal{G} - G. \quad (10)
$$

Additional contributions induced by the in-medium modification of the interaction kernel $K = K + \Delta K$ are cast into the term, $\Delta \Pi(q, u)$, of (9). It is crucial to consider the latter term since it will introduce in particular the important short range correlation effects described by the Migdal parameters. This will be discussed in more detail below.

With (10) the self consistent set of equations (5,7,9) is rewritten in a way that one may start with a set of tabulated free-space scattering amplitudes $T^{(I)}$ in terms of which self consistency is achieved. Coupled channel effects, which are known to be important for $\pi N$ scattering at higher energies, are included by assigning $\hat{T}$, $\mathcal{G}$ and $K$ the appropriate matrix structures. The effect of
all inelastic channels can be accounted for by a renormalization leading to a complex scattering kernel $K$ of the $\pi N$ channel. Therefore we may assume that the amplitudes $T^{(l)}$ in (10) already include the dynamics of all inelastic coupled channels. In this work we study the consequence of the in-medium modified $\pi N$ channel exclusively. Thus it is not necessary to make the coupled channel structure of the $\pi N$ amplitudes explicit [28]. Ultimately it would be desirable to also evaluate the in-medium modification of the inelastic processes described for instance by the $\pi \Delta$, $\rho N$ and $\omega N$ channels.

The scattering amplitudes can be systematically decomposed into covariant projectors $Y_n(\pm) (\bar{q}, q; w)$ with good angular momentum and parity $J^P = (n + \frac{1}{2})^\pm$. We write

\[
T^{(l)}(\bar{k}, k; w) = \sum_{n=0}^{\infty} Y_n(+) (\bar{q}, q; w) M_I^+(\sqrt{s}, n) + \sum_{n=0}^{\infty} Y_n(-) (\bar{q}, q; w) M_I^-(\sqrt{s}, n),
\]

where $w^2 = s$ and $k = \frac{1}{2} (p - q)$ and $\bar{k} = \frac{1}{2} (\bar{p} - \bar{q})$. The representation (11) implies a particular off-shell behavior of the scattering amplitude, which was obtained in the course of constructing a systematic on-shell reduction of the covariant Bethe-Salpeter equation that does not depend on the choice of meson and baryon interpolation fields [29]. In this work we focus on the leading $J = \frac{1}{2}$ and $J = \frac{3}{2}$ channels with $Y_0^+(s\text{-wave}), Y_0^-(p\text{-wave})$ and $Y_1^+(p\text{-wave}), Y_1^-(d\text{-wave})$. For details on the construction and the properties of these projectors we refer to [29,28].

For energies above threshold $\sqrt{s} > m_N + m_\pi$ the invariant amplitudes are uniquely determined by the scattering phase shifts $\delta_j^{(l)}(\sqrt{s})$ and inelasticity parameters $\eta_j^{(l)}(\sqrt{s})$,

\[
M^{(\pm)}(\sqrt{s}, J - \frac{1}{2}) = -i \frac{8 \pi s \eta_j^{(l)}(\sqrt{s})}{p_{\pi N}^{2J}} \left( \epsilon^{2i \delta_j^{(l)}(\sqrt{s})} - 1 \right),
\]

\[
p_{\pi N}^2 s = \frac{1}{4} \left( 1 - \frac{(m_N + m_\pi)^2}{s} \right) \left( 1 - \frac{(m_N - m_\pi)^2}{s} \right).
\]

We use $m_N = 939$ MeV and $m_\pi = 139$ MeV throughout this work. At sub-threshold energies $\sqrt{s} < m_N - m_\pi$ the on-shell amplitudes are again determined by the scattering phase shifts once crossing symmetry is invoked. In the region $m_N - m_\pi < \sqrt{s} < m_N + m_\pi$ an analytic continuation is required as for instance implied by the dispersion-integral representation,
\[ \Re M_l^{(\pm)}(\sqrt{s}, n) = D_l^{(\pm)}(\sqrt{s}, n) + \mathcal{P} \int_{m_N + m_\pi}^{\infty} \frac{dw}{\pi} \Im M_l^{(\pm)}(w, n), \]  

(13)

where the functions \( D_l^{(\pm)}(\sqrt{s}, n) \) represent all left-hand cut contributions [30]. Since the cut structure in \( D_l^{(\pm)}(\sqrt{s}, n) \) is largely dominated by the s- and u-channel nucleon-exchange contributions, it is straightforward to analytically continue the on-shell amplitudes into the region \( m_N - m_\pi < \sqrt{s} < m_N + m_\pi \) as to arrive at approximate partial-wave amplitudes specified for all energies. As a consequence the amplitudes would receive large and strongly energy-dependent contributions in the interval \( m_N - m_\pi < \sqrt{s} < m_N + m_\pi \).

We point out, however, that the formulation as presented above would lead to unphysical and misleading results for the pion self energy unless the dependence of the partial-wave amplitudes on \( q^2 \neq m_\pi^2 \) at subthreshold energies is taken into account in some way. This is important, since the u-channel contribution to the partial wave amplitudes shows an extremely strong dependence on \( q^2 \). Typically for given \( \sqrt{s} \) with \( m_N - m_\pi < \sqrt{s} < m_N + m_\pi \) and \( q^2 < 0 \) the contribution is small and negligible but at the on-shell point \( q^2 = m_\pi^2 \) it is large. Since the subthreshold amplitudes are probed by the pion self energy only for \( q^2 < m_\pi^2 \), it is clear that the \( q^2 \)-dependence must be considered. Rather than extending our scheme for the most general off-shell structure in the scattering amplitude we propose a simple modification of the scheme that successfully circumvents the artifacts of a pure on-shell scheme. It would be unclear in any case what to use for the off-shell dependence, since such a dependence is highly scheme-dependent.

The idea the proposed scheme is based on, exploits crossing symmetry as a tool that determines the troublesome off-shell part of the scattering amplitude. We seek a representation of the free-space scattering amplitude of the form

\[ T_s^{(\pm)}(\bar{q}, q; w) \]  

that manifests the desired crossing symmetry of the amplitudes explicitly but masks the constraints set by unitarity. The decomposition (14) is not unique per se but can be tailored to our application. In particular we insist that \( T_s \) contains only s-channel unitarity cuts but shows no cuts in the u-channel. Similarly we insist that \( T_u \) is real for \( \sqrt{s} > m_N + m_\pi \). In general there will be contributions present in \( T \) which lead to both an s-channel and u-channel cut. Such contributions must be put into a remainder term not displayed in (14), if we wish to keep perfect analytic properties of the amplitudes \( T_s \) and \( T_u \). We point out however, that such contributions are zero identically, if one
considered elastic scattering processes only. Therefore we expect the degree of analyticity violation, when demanding $T_s$ to represent the complete strength of the s-channel unitarity cut, to be reasonably small if energies are not excessively large. Within a given application window there is still the freedom how to distribute a polynomial background contribution. Here it is advantageous to insist on $T_s = T$ beyond a certain scale $\sqrt{s} > \Lambda$. Consistency then requires that for $\sqrt{s} < (m_N^2 - m_p^2)/\Lambda$ one should obtain $T_s = 0$ or equivalently $T = T_u$. The scattering amplitude $T_s$ is decomposed into partial wave amplitudes $\bar{M}_I^{(\pm)}(\sqrt{s}, n)$. In practise we use the representation (13) with the function $D_I^{(\pm)}(\sqrt{s}, n)$ replaced by a second order polynomial in $\sqrt{s}$. The coefficients of the latter are determined to guarantee first, that the real parts of $\bar{M}_I^{(\pm)}(\sqrt{s}, n)$ vanish identically at $\sqrt{s} = (m_N^2 - m_p^2)/\Lambda$ and second, is such that a smooth matching to the real part of the partial wave amplitudes, $M_I^{(\pm)}(\sqrt{s}, n)$, at $\sqrt{s} = \Lambda$ as given in (12) is obtained. Of course in the $P_{11}$ channel the nucleon pole contribution is incorporated in addition. The pion-nucleon coupling constant, $g_{\pi NN} = 13.34$, of [31] is used. With $\Lambda = 1600$ MeV a remarkably smooth matching is achieved for all considered channels as described above. The deviation of $\bar{M}_I^{(\pm)}(\sqrt{s}, n)$ from $M_I^{(\pm)}(\sqrt{s}, n)$, both shown in Fig. 2, is a measure for the importance of u-channel cuts in a given partial wave.

The idea is to use the amplitudes $T_s$ in (14) properly decomposed into partial waves via (11). The effect of $T_u$ is incorporated into the pion self energy by adding to (9) the term induced by $T_s$ but with $q \to -q$ replaced. As we apply the decomposition (14) for a many-body evaluation of the pion self energy, the parameter $\Lambda$ plays the role of a scale that determines at what energy the manifestly crossing-symmetric scheme recovers two-body unitarity exactly. For energies close to the pion-nucleon threshold a unitarization is not really required for a quantitative description of the scattering amplitude. The amplitudes are largely dominated by the s- and u-channel baryon exchange contributions. In contrast, at higher energies the unitarity constraint becomes more and more important rendering any constraint from crossing symmetry rather implicit.

The solution of the in-medium Bethe-Salpeter equation is considerably complicated by the presence of further tensor structures $P_{[ij]}$ and $Q_{[ij]}$ not required in the vacuum. For a complete collection of the in-medium projectors together with their induced in-medium loop functions, $\Delta J_{[ij]}^{(p,q)}(w, u)$, representing the object $\Delta G$ in (10), see [28]. In this work the loop functions are evaluated by their dispersion-integral representation in terms of their imaginary parts. A subtraction constant is determined by insisting that the scattering amplitudes do not renormalize the nucleon mass

$$\Delta J_{[ij]}^{(p,q)}(w, u)|_{w^2 = m_N^2} = 0 \to \quad T^{(I)}_s(\tilde{q}, q; w, u)|_{w^2 = m_N^2} = T^{(I)}_s(\tilde{q}, q, w), \quad (15)$$
Fig. 1. Partial wave πN scattering amplitudes at subthreshold energies as reconstructed from the SM01 phase shifts [32]. The solid and dashed lines give the real and imaginary parts of the $M^\pm_I(\sqrt{s}, J - 1/2)$ amplitudes of (12). The additional solid lines extending down to subthreshold energies represent the amplitudes $\bar{M}^\pm_I(\sqrt{s}, J - 1/2)$ that represent $T^{(f)}_s(\bar{q}, q; w)$ of (14) and are free of u-channel cuts. For $w \cdot u > 0$ such a condition is well justified in the present scheme since neither correlations nor binding effects are incorporated. That is outside the scope of this work. Furthermore we drop a small contribution of intermediate hole states, not consistently treated here in any case, but part of the relativistic scattering equation (5). This leads to an in-medium scattering amplitude, whose imaginary part,

$$
\Im T_s(\bar{q}, q; w, u) = 0 \quad \text{if} \quad w \cdot u < \sqrt{m_N^2 + k_F^2}, \tag{16}
$$

vanishes below the chemical potential as expected from the Pauli principle. To be precise the property (16) requires also the consideration of the Pauli blocking effect in the s-channel nucleon exchange contribution, i.e. $K \not= K$.

We turn to the effects of short range correlation [6]. Here we follow the recent work [14] and apply covariant expressions parameterized in terms of the Migdal parameters $g'_{11}, g'_{12}$ and $g'_{22}$. The delta-hole term is folded by an isobar spectral function obtained from the appropriate in-medium scattering amplitude...
assuming the averaged value $\bar{w} = 200$ MeV. We then approximate the isobar spectral function to be a function of $w_0^2 - \bar{w}^2$. Thus, the value for $g'_{22}$ used in this work is introduced with respect to the in-medium pion-isobar coupling constant. The latter will show a sizeable in-medium reduction in our scheme. The expression for the pion self energy as given in [14] properly subtracted by its leading order contribution, defined in the limit $g'_{ij} \to 0$, is identified with $\Delta \Pi$ in (9). The real part of this contribution is evaluated in terms of a dispersion-integral representation with a subtraction constant fixed at $\omega^2 - \bar{q}^2 = m^2_\pi$ matched to the real part of the original $\Delta \Pi$ contribution. Since the nucleon-hole contribution as derived in terms of the in-medium scattering amplitude includes an appreciable momentum dependent renormalization of the pion-nucleon coupling constant, a corresponding renormalization is applied in $\Delta \Pi$. This effect reduces the strength of the nucleon-hole contribution. The values we quote here for Migdal’s $g'_{11}$ and $g'_{12}$ parameters are defined with respect to the free-space pion-nucleon but in-medium pion-isobar coupling constants [14].

We reconstruct the real part of the pion self energy, $\Re \Pi(q, u)$, in terms of its imaginary part, $\Im \Pi(q, u)$. From now on we work in the rest frame of nuclear matter with $u_\mu = (1, 0)$ for convenience and write $\Pi(q, u) = \Pi(\omega, \vec{q})$. A subtracted dispersion-integral representation is imposed

$$\Pi(\omega, \vec{q}) = c(\vec{q}) + \int_0^\lambda d\bar{\omega} \frac{\Im \Pi(\bar{\omega}, \vec{q})}{\bar{\omega}^2 - \omega^2 - i\epsilon},$$

(17)
cut off by $\lambda = 1.2$ GeV. The reflection property $\Pi(\omega, \vec{q}) = \Pi(-\omega, \vec{q})$ holds for isospin symmetric nuclear matter as a direct consequence of our manifestly crossing-symmetric scheme. The subtraction constant $c(\vec{q})$ is fixed such that the real part of the self energy reproduces the corresponding value for the contribution of the amplitude defined by (9) at the point

$$\omega = \Max[\sqrt{\Lambda^2 + (|\vec{q}| + k_F)^2} - \sqrt{m_N^2 + k_F^2}, \sqrt{\Lambda^2 + |\vec{q}|^2 - m_N}].$$

This condition guarantees that the real part of the in-medium amplitude is probed only for $w_0^2 - \bar{w}^2 \geq \Lambda^2$, where the effective amplitude $T^{(I)}(q, q; w)$ represents the exact scattering amplitude (see (def-decom)). We assure that the sum rule

$$\int_{-\infty}^{+\infty} \frac{d\omega}{\pi} |\omega| \Im D(\omega, \vec{q}) = -1,$$

(18)
holds accurately within 1% in our scheme.
The self consistent set of equations (9,5) is solved numerically by iteration where we start with the evaluation of the in-medium self energy. Convergence is typically found after 5 to 6 iterations. The energies and the three-momenta are restricted by $|\omega| < 1.2 \text{ GeV}$, $|\vec{q}| < 0.8 \text{ GeV}$ and $w_0 < 2 \text{ GeV}$, $|\vec{u}| < 1.2 \text{ GeV}$. The free-space partial wave amplitudes, $\bar{M}_I^{(\pm)}(\sqrt{s}, n)$, are put to zero for $\sqrt{s} < 2 \text{ GeV}$ and $\sqrt{s} > 0.54 \text{ GeV}$.

3 Results

We give a presentation and discussion of our results for the in-medium modification of the pion and the $J = \frac{1}{2}, \frac{3}{2}$ nucleon and isobar resonance properties. The resonance propagator can be identified with the appropriate pion-nucleon scattering amplitude of a given partial wave. In the self consistent scheme of section 2 the in-medium scattering process is intimately related to the pion spectral function. According to (9) once the self consistent in-medium scattering process is established the pion self energy follows by averaging the in-medium scattering amplitudes (9) over the Fermi distribution. Therefore the pertinent structures in the in-medium amplitudes already tell the characteristic features expected in the pion spectral function.

Since the present day literature does not offer a unique set of Migdal parameters $g'_{11}, g'_{22}$ and $g'_{12}$ we studied first the sensitivity of our approach to different choices thereof. We use two sets of parameters in this work. Set I) is assuming $g'_{11} = g'_{12} = g'_{22} = 0.8$. Set II) is given by the values $g'_{11} = 0.585, g'_{12} = 0.15$ together with $g'_{22} = 0.60$. We remind the reader that we use here a convention in which $g'_{11}$ is defined with respect to the free-space pion-nucleon coupling constant whereas $g'_{22}$ is defined with respect to the in-medium pion-nucleon-isobar coupling constant.

With the nuclear density set to $\rho = 0.17 \text{ fm}^{-3}$ Fig. 2 shows the resulting pion self energy together with the real and imaginary parts of the in-medium $\pi N$ scattering amplitude in the $P_{33}$ channel. As illustrated by the figure the effect of different choices for the Migdal parameters affects the pion self energy mostly at not too large energies and intermediate momenta. As illustrated by the right hand panels the effects of such differences are quite important for the properties of the $\Delta(1232)$-isobar in nuclear matter. Only set I) leads to acceptable properties of the isobar though we may still overestimate the broadening of its width in nuclear matter for high velocities. Note that the depletion of the isobar peak in Fig. 2 is a combined effect of reduced pion-isobar coupling constant and increased decay width. Thus the $g'_{22}$ parameter as defined more conventionally with respect to the free-space pion-nucleon-isobar coupling constant is in fact smaller than 0.8 about 0.6 only. For isobar propagation with non-zero momenta set II) leads to a substantial broaden-
Fig. 2. In-medium pion self energy (left hand panels) and $\pi N$ scattering amplitude in the $P_{33}$ channel (right hand panels) at nuclear saturation density. The solid lines show the results obtained with set I) of Migdal parameters, the dashed lines those for set II) as specified in the text. The upper two (lower two) right hand panels describe isobar propagation in nuclear matter with total isobar three momentum $|\vec{w}| = 0.0$ GeV ($|\vec{w}| = 0.4$ GeV), compared to free space results.

...unlikely to be consistent with photo-absorption data of the nucleus [16]. We confirm the results of [3] that the splitting of transverse (Q-space) and longitudinal (P-space) modes is small. In our formulation such effects reflect the in-medium mixing of partial wave amplitudes. In none of the channels we observed significant effects thereof. The attractive mass shift for the isobar of about 80 MeV is in qualitative agreement with previous microscopic calculations [3]. Referring to the argument put forward in [1,3] the apparent repulsive mass shift as extracted from photo absorption data [4] is a combined effect of an attractive isobar self energy and short range correlation effects.

The results differ from conclusions of previous works [8,9,10] that also incorporated self consistency but did not observe such a significant broadening of the isobar. This difference is a direct consequence of the soft form factor in the pion-isobar vertex used in [8,9,10]. In our scheme there is not much place for such phenomenology, since the applied phase shifts entail already form factor effects that one may want to use modeling the pion-nucleon interaction. We emphasize our goal to develop a microscopic understanding of pion propagation in nuclear matter. First we continue to present and discuss results solely...
based on the $\pi N$ phase shifts and a set of Migdal parameters only. Nevertheless, we will return to this issue and also present results that follow upon incorporating a phenomenological form factor into our scheme.

3.1 Propagation without form factors

In the form factor-free scheme the isobar width can be protected against excessive increase to some extent by an appropriate choice of Migdal parameters. Large $g'_{12}$ and $g'_{22}$ parameters together with a reasonably large $g'_{11} \sim 0.6 - 0.8$ lead to a suppression of fast soft modes that are responsible for the broadening of the isobar, i.e. the low-energy tail of the pion spectral function at momenta $|\vec{q}| \sim 400 - 500$ MeV is suppressed. We are aware that this conclusion is possibly inconsistent with the conclusion of the recent work [33]. This issue certainly deserves more detailed investigations. Vertex correction diagrams that mask the pion spectral function as probed in the isobar self energy but not considered in the present scheme yet may lead to a similar suppression of fast soft modes. Thus we would not exclude a small value of $g'_{12}$ once such effects are incorporated.

A further interesting question is the amount of softening found in the pion self energy. Typically the traditional nucleon and isobar-hole model leads to a minimum of the function

$$S(\vec{q}) = \vec{q}^2 + m^2_{\pi} + \Pi(0, \vec{q}), \quad (19)$$

at some intermediate momentum. Such a minimum helps to explain for instance unnatural parity states of finite nuclei [6]. To be specific the covariant nucleon and delta-hole model proposed in [14] shows a minimum at $|\vec{q}| \simeq 365$ MeV and nuclear saturation density using the parameter set II). In the present self consistent scheme we do not find a minimum of $S(\vec{q})$ for either of the two choices of Migdal parameters studied here. The size of the function at a given momentum, say $|\vec{q}| = 300$ MeV, strongly depends on the choice of Migdal parameters, with $S \simeq 3.4 m^2_{\pi}$ and $S \simeq 2.4 m^2_{\pi}$ for set I) and II) respectively. By lowering $g'_{11}$ and $g'_{12}$ down to unrealistic values it is possible to produce a minimum of $S(\vec{q})$ in our present scheme, however at the prize that the isobar resonance is dissolved almost completely.

At small pion momenta $|\vec{q}| < 40$ MeV the pion self energy may be approximated by

$$\Pi(\omega, \vec{q}) \bigg|_{\omega^2 = \Re[\alpha m^2_{\pi} + \beta \vec{q}^2]} \simeq (\alpha - 1) m^2_{\pi} + (\beta - 1) \vec{q}^2, \quad (20)$$
Fig. 3. Real (left hand panel) and imaginary (right hand panel) part of the pion self energy at $\rho = 0.5 \rho_0$, $\rho_0$ and $1.5 \rho_0$ with $\rho_0 = 0.17$ fm$^{-3}$. The results were obtained with parameter set I).

the values of the parameters known to some extent from pionic atom data [5,7]. For set I) and II) we obtain $(\alpha, \beta) \simeq (1.09 - i 0.05, 0.56 - i 0.25)$ and $(\alpha, \beta) \simeq (1.06 - i 0.07, 0.74 - i 0.44)$. We do not find any significant effect from a possible wave function renormalization. Comparing (20) with the results of [7] the s-wave absorption strength parameterized by $\Im \alpha$ comes out about a factor two too small. This reflects the fact that not all absorption channels are included in our work. The value for $\Re \alpha$ is reasonably close to phenomenological values. The p-wave absorption strength related to $\Im \beta$ is overestimated somewhat [5]. This may be linked to a possibly too large in-medium isobar width obtained in our present scheme. The parameter $\Re \beta$ of set I) is however close to the empirical value [5]. Given the fact that our scheme is parameter free except for the choices of the Migdal parameters, these results are encouraging. However, it should be emphasized that one expects some non-linear dependence of $\alpha$ and $\beta$ on the density. For instance at $0.5 \rho_0$ we obtain $(\alpha, \beta) \simeq (1.09 - i 0.02, 0.46 - i 0.11)$ for set I). To further illustrate the amount of non-linear behavior in the pion self energy we include Fig. 3 which displays the results for the pion self energy for the nuclear densities $0.5 \rho_0$, $1.0 \rho_0$ and $1.5 \rho_0$. A striking non-linear behavior is typically seen at small pion energies.

In Fig. 4 our results for the pion spectral function and the $J = \frac{3}{2}$ isobar
Fig. 4. Pion spectral function (left hand panels) and $J = \frac{3}{2}$ nucleon resonance propagators (right hand panels) at $\rho_0 = 0.17$ fm$^{-3}$ (solid lines) and $2 \rho_0$ (dashed lines) as functions of the pion, $\omega$, and resonance, $w_0$, energy respectively. The vertical lines in the left hand panels show the energy of a pion in free space at given momentum $|\vec{q}|$. The upper (lower) two right panels show the in-medium $\pi N$ scattering amplitude in the $P_{33}$ ($D_{13}$) channel. Results are shown for a resonance at rest with $|\vec{w}| = 0$ (thick lines) and for three-momentum $|\vec{w}| = 0.4$ GeV (thin lines) with longitudinal polarization.

$\Delta(1232)$ and $N(1520)$ resonances are shown for set I) of Migdal parameters at two different nuclear densities, $\rho_0 = 0.17$ fm$^{-3}$ and $2 \rho_0$. Of course the results at $2 \rho_0$ should be considered cautiously because nuclear binding and correlation effects were not yet fully included in the present scheme. The pion spectral function clearly exhibits the three well known modes, zero-sound, isobar-hole and pion branch with weighting factors strongly dependent on the pion momentum. We wish to make two points here. First, in contrast to the standard delta-hole model for the pion [1], density we do not observe a significant strength of soft pion modes at nuclear saturation in our work. This is due to self consistency. Using somewhat smaller values $g^{'ij} = 0.6$ does not change our conclusion qualitatively. The resulting spectral function is quite similar to the one shown in Fig. 4 only that for instance at $|\vec{q}| = 400$ MeV about 25% of the strength sitting in the pion branch is moved up to the delta-hole branch and to a lesser degree into the zero-sound branch. Our finding confirms the results of previous self consistent approaches to pions in nuclear matter [9,8,10] qualitatively and should have important consequences in various applications of the pion spectral function to hadron properties in nuclear matter. Second,
the significant broadening and repulsive shift of the main mode at large momenta is obviously a result of the inclusion of s-, p- and d-waves in our scheme. Such effects are lacking in a model that incorporates nucleon- and delta-hole terms only.

We turn to the resonance properties. Fig. 4 shows the narrowing of the isobar resonance with increasing density due to the growing importance of Pauli blocking. A similar but much less pronounced effect is observed for the $N(1520)$ resonance. The significant broadening of the latter resonance at nuclear saturation density is in qualitative agreement with previous phenomenological works [17,18,19,20] and may help to arrive at a microscopic understanding of photo absorption data in the second resonance region expected to be dominated by the contribution of the $N(1520)$ resonance [34,35,36]. The fact that at nuclear saturation density we do not obtain any large in-medium effect from the $\pi N$ channel on the remaining resonances, $N(1535)$, $N(1650)$, $\Delta(1620)$, $\Delta(1600)$ and $\Delta(1700)$ except for the $N(1440)$, for which we find a significantly increased width, is interesting and deserves further studies. The $D_{13}$ and $P_{11}$ resonances $N(1520)$ and $N(1440)$ appear particularly sensitive to the in-medium dressing of the $\pi N$ channel due to their d-wave and p-wave phase space behavior. Clearly the effect of possible in-medium modifications of inelastic channels like $\pi\Delta$, $\rho N$, $\omega N$ on the resonance properties asks for further detailed studies [37].

3.2 Propagation with form factors

Until now we did not include any phenomenological form factors into the computation, motivated by the fact that most of such effects are already taken care of by using the physical scattering amplitudes. However, a certain form of pion-nucleon-nucleon and pion-nucleon-isobar form factors commonly used in calculations of pion, nucleon and isobar self energies based on the relevant 3-point functions can also be introduced in the present scheme. The condition is that the solution of the Bethe-Salpeter equation in vacuum is not affected for the on-shell scattering amplitudes. Indeed, the use of scattering phase shifts incorporates the form factors necessary to describe the pion-nucleon interaction, but only for on-shell particles. Off-shell-pion effects may allow the inclusion of phenomenological form factors of the form,

$$F_{\pi NX}(q^2) = \exp \left[ - (q^2 - m_{\pi}^2)^2 / \Lambda_{\pi NX}^4 \right], \quad (21)$$

which suppress off-shell-pion contributions from relevant channels (with $X$ being $N$ or $\Delta$) when calculating the pion self energy. It should be emphasized, however, that the phenomenological need for such a form factor may stem from medium modifications of the interaction vertices rather than a strong off-shell dependence of the vertices in free-space. We do not modify the loop-
Fig. 5. In-medium $\pi N$ scattering amplitude in the $P_{33}$ channel at nuclear saturation density, compared to the vacuum amplitude (solid line). The values of Migdal parameters are: $g'_{11} = g'_{12} = g'_{22} = 0.8$. The dashed line corresponds to $\Lambda_{\pi NN} = 0.5$ GeV and $\Lambda_{\pi N\Delta} \to \infty$, the dash-dot line to $\Lambda_{\pi NN} = \Lambda_{\pi N\Delta} = 0.5$ GeV, the dash-dot-dot line to the calculation without form factors. The left two (right two) panels describe isobar propagation in nuclear matter with total isobar three momentum $|\vec{w}| = 0$ GeV ($|\vec{w}| = 0.4$ GeV).

In Fig. 5 we show the effect of the form factors on the $\pi N$ scattering amplitude in the $P_{33}$ channel at nuclear saturation density (other channels are affected marginally). Whereas the form-factor influence is quite pronounced at momentum $|\vec{w}| = 0$ it is of minor importance at $|\vec{w}| = 0.4$ GeV. As expected, the $\pi NN$ form factor reduces the strength of the pion spectral function at low energy (less than $m_{\pi}$), thus suppressing the isobar decay amplitude and decreasing its width. The influence of the $\pi N\Delta$ form factor is less pronounced, since the delta-hole branch of the pion spectral function is not so far off-shell. In the present approach it somewhat broadens the isobar and also makes its mass shift less pronounced.

The pion spectral function is not dramatically changed by the form factors, whose effect is illustrated in Fig. 6. The figure shows the effect of the $\pi NN$ form factor since it produces the most pronounced effect. In general we get slightly more softening of the main pion mode and a suppression of the very-low-energy part. The pion-nucleon-isobar form factor does not bring in no-
In Fig. 6 we show the pion self energy at nuclear saturation density, for momenta $|\vec{q}| = 0.2$ GeV and $|\vec{q}| = 0.4$ GeV. The most prominent feature of the self energy is the strong suppression of the delta-hole contribution for momenta around 0.2 GeV, as well as the contributions of higher-mass resonances which leave the imaginary part non-vanishing even at energies up to 1 GeV.

These results show that further investigations of the effects of vertex modifications in the medium are required, which respect the features intrinsic to our present scheme.
Fig. 7. In-medium pion self energy at nuclear saturation density, for $|\vec{q}| = 0.2$ GeV and $|\vec{q}| = 0.4$ GeV pion momentum. The full line corresponds to a calculation without form factors with $g'_{11} = g'_{12} = g'_{22} = 0.8$. The dash-dot line shows the results for $\Lambda_{\pi NN} = \Lambda_{\pi N\Delta} = 0.5$ GeV and unchanged $g'$ values. For comparison we show the results (dashed line) of a non-relativistic computation from ref. [9].

4 Summary

In this work we evaluated the pion self energy in nuclear matter in a self consistent and covariant manner. A novel framework based on the $\pi N$ phase shifts, as measured in free space, together with a set of Migdal parameters was developed. Important constraints of crossing symmetry and unitarity were incorporated approximatively. Using reasonable values for the Migdal parameters we found that the nucleon resonances $N(1535)$ and $N(1650)$ are basically unaffected by the nuclear environment. Contrasted results were obtained for the p-wave $N(1440)$ and d-wave $N(1520)$ resonances for which we predict considerable broadening already at nuclear saturation density.

Our result for the isobar resonance are not satisfactory at this stage, due to a significant overestimate of its in-medium decay width. Improved results were obtained by incorporating a soft phenomenological form factor into the $\pi NN$ vertex. Whereas the properties of slow isobars in nuclear matter are changed significantly, a soft form factor has rather moderate effects on the pion spectral function. These results show that further detailed investigations of the effects of vertex modifications in the medium are required to arrive at a fully microscopic understanding of the properties of isobars in nuclear matter.

Acknowledgments

This research was supported in part by the Hungarian Research Foundation (OTKA) grant T030855. M.F.M. L. acknowledges useful discussions with
E.E. Kolomeitsev, E.E. Saperstein and D. Voskresensky. C.L.K would like to thank the NWO (Netherlands) for providing a visitors stipend and the K.V.I. (Groningen) for the kind hospitality.

References

[1] E. Oset, H. Toki and W. Weise, Phys. Rep. 83 (1982) 281.

[2] W.H. Dickhoff et al., Phys. Rev. C 23 (1981) 1154.

[3] E. Oset and L.L. Salcedo, Nucl. Phys. A 468 (1987) 631; R.C. Carrasco and E. Oset, Nucl. Phys. A 536 (1992) 445.

[4] Y. Horikawa, M. Thies and F. Lenz, Nucl. Phys. A 345 (1980) 386.

[5] T.E.O. Ericson and W. Weise, Pions and Nuclei (Clarendon Press, Oxford, 1988).

[6] A.B. Migdal et al., Phys. Rep. 192 (1990) 181.

[7] J. Nieves, E. Oset and C. Garcia-Recio, Nucl. Phys. A 554 (1993) 554.

[8] L. Xia, P.J. Siemens and M. Soyeur, Nucl. Phys. A 578 (1994) 493.

[9] C.L. Korpa and R. Malfliet, Phys. Rev. C 52 (1995) 2756.

[10] F. Riek and J.Knoll, nucl-th/0402090.

[11] V.F. Dmitriev and T. Suzuki, Nucl. Phys. A 438 (1985) 697; T. Herbert, K. Wehrberger and F. Beck, Nucl. Phys. A 541 (1992) 699; M. Nakano et al., Int. J. Mod. Phys. E 10 (2001) 459.

[12] M. Hjorth-Jensen, H. Müther and A. Polls, Phys. Rev. C 50 (1994) 501.

[13] H. Kim, S. Schramm and S.H. Lee, Phys. Rev. C 56 (1997) 1582.

[14] M.F.M. Lutz, Phys. Lett. B 552 (2003) 159; Erratum ibd B 566 (2003) 277.

[15] J.H. Koch, E.J. Moniz and N. Ohtsuka, Ann. Phys. 154 (1984) 99.

[16] J. Ahrens et al., Phys. Lett. B 146 (1984) 303; N. Bianchi et al., Phys. Lett. B 299 (1993) 219; Th. Frommhold et al., Zeit. Phys. A 350 (1994) 249; N. Bianchi et al., Phys. Rev. C 54 (1996) 1688.

[17] L.A. Kondratyuk et al., Nucl. Phys. A 579 (1994) 453.

[18] W.M. Alberico, G. Gervino and A. Lavango, Phys. Lett. B 321 (1994) 177.

[19] C.M. Chen et al., Phys.Rev. C 52 (1995) 485.

[20] S. Boffi et al., Phys. Atom. Nucl. 60 (1997) 1193.

[21] M. Kohl et al., Phys. Lett. B 530 (2002) 67.
[22] C.J. Batty, E. Fridman and A. Gal, Phys. Rep. 287 (1997) 385.
[23] E.E. Kolomeitsev, N. Kaiser and W. Weise, Phys. Rev. Lett. 90 (2003) 092501.
[24] H. Gilg et al., Phys. Rev. C 62 (2000) 025201; K. Itahasi et al., Phys. Rev. C 62 (2000) 025202.
[25] N. Kaiser and W. Weise, Phys. Lett. B 512 (2001) 283.
[26] T.-S. Park, H. Jung and D.-P. Min, J. Korean Phys. Soc. 41 (2002) 195.
[27] U.G.- Meißner, J.A. Oller and A. Wirzba, Ann. Phys. 297 (2002) 27.
[28] M.F.M. Lutz and C.L. Korpa, Nucl. Phys. A 700 (2002) 309.
[29] M.F.M. Lutz and E.M. Kolomeitsev, Nucl. Phys. A 700 (2002) 193.
[30] G. Höhler, in: H. Schopper (Ed), Landoldt-Börnstein, Vol. I/9b2, Springer, Berlin, 1983.
[31] T.E.O. Ericson, B. Loiseau, A.W. Thomas, hep-ph/0009312
[32] SAID on-line programm, http://gwdac.phys.gwu.edu/
[33] T. Suzuki, H. Sakei and T. Tatsumi, Proc. RCNP Int. Sympo. on "Nuclear Responses and Medium Effects”, Univ. Academy Press, 7-83 (1999).
[34] B. Krusche et al., Phys. Rev. Lett. 86 (2001) 4764.
[35] M. Effenberger, A. Hombach, S. Teis and U. Mosel, Nucl. Phys. A 613 (1997) 353.
[36] J. Lehr and U. Mosel, Phys. Rev. C 64 (2001) 042202.
[37] W. Peters et al., Nucl. Phys. A 632 (1998) 109.