Study on radiative decays of $D_{sJ}^*(2860)$ and $D_{s1}^*(2710)$ into $D_s$ by means of LFQM

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Abstract

The observed resonance peak around 2.86 GeV has been carefully reexamined by the LHCb collaboration and it is found that under the peak there reside two states $D_{s1}^*(2860)$ and $D_{s3}^*(2860)$ which are considered as $1^3D_1(c\bar{s})$ and $1^3D_3(c\bar{s})$ with slightly different masses and total widths. Thus, the earlier assumption that the resonance $D_{s1}^*(2710)$ was a $1D$ state should not be right. We suggest to measure the partial widths of radiative decays of $D_{sJ}^*(2860)$ and $D_{s1}^*(2710)$ to confirm their quantum numbers. We would consider $D_{s1}^*(2710)$ as $2^3S_1$ or a pure $1^3D_1$ state, or their mixture and respectively calculate the corresponding branching ratios as well as those of $D_{sJ}^*(2860)$. The future precise measurement would provide us information to help identifying the structures of those resonances.

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I. INTRODUCTION

Resonance $D_s^*(2860)$ was experimentally observed [1–4], but its quantum number is still to be eventually identified because the ratio $\Gamma(D_s^*(2860) \to D^*K)/\Gamma(D_s^*(2860) \to DK)$ is not well understood [5, 6]. A careful reexamination on the spectrum peak around 2.86 GeV recently has been carried out by the LHCb collaboration and it is found that a spin-1 state and a spin-3 state overlap under the peak. They are $D_{s1}^*(2860)$ with mass and width as $M(D_{s1}^*(2860)) = (2859 \pm 12 \pm 6 \pm 23)$ MeV, $\Gamma(D_{s1}^*(2860)) = (159 \pm 23 \pm 27 \pm 72)$ MeV [7] and $D_{s3}^*(2860)$ with mass and width as $M(D_{s3}^*(2860)) = (2860.5 \pm 2.6 \pm 2.5 \pm 6.0)$ MeV, $\Gamma(D_{s3}^*(2860)) = (53 \pm 7 \pm 4 \pm 6)$ MeV [8]. Based on the new data Godfrey and Moats suggest that $D_{s1}^*(2860)$ and $D_{s3}^*(2860)$ should be identified as $1^3D_1(c\bar{s})$ and $1^3D_3(c\bar{s})$. Previously $D_{s1}^*(2710)$ [2] was measured and its mass and width are $M(D_{s1}^*(2710)) = (2709 \pm 4)$ MeV, $\Gamma(D_{s1}^*(2710)) = (117 \pm 13)$ MeV. It was assigned to be $1^3D_1$ or $2^3S_1$ or their mixture [3, 4, 8, 9]. Obviously the $1^3D_1$ assignment of $D_{s1}^*(2710)$ conflicts with the LHCb’s new observation, because the $1^3D_1$ state of $c\bar{s}$ is occupied by $D_{s1}^*(2860)$, so there is no room to accommodate $D_{s1}^*(2710)$. Therefore one can conjecture that as long as $D_{s1}^*(2860)$ is in the $1^3D_1$ state, $D_{s1}^*(2710)$ should be regarded as a $2^3S_1$ state or others [3, 4, 8, 9]. Since all resonances $D_{s1}^*(2860)$ and $D_{s3}^*(2860)$ and $D_{s1}^*(2710)$ have been undoubtedly reconstructed in the hadronic processes under investigation, the best channels to determine their quantum identities are their respective strong decays [3, 12, 13] which are in fact the dominant ones. However, on other aspect, one still has a chance to observe the resonances in their electromagnetic decays where excited states transit into ground states by emitting a photon. Especially the calculation on the electromagnetic decays is more reliable. In Ref. [14], the authors study the radiative decays of $D_s^*(1^3D_1)$ and $D_s^*(3^3D_1)$ into a P-wave $c\bar{s}$ meson. In this paper we will study the radiative decay of a D-wave meson into an S-wave $c\bar{s}$ meson. The results may help us to determine the quantum number of these particles in addition to the studies via strong processes.

In this work, we employ the light-front quark model (LFQM) to estimate the branching ratios. This relativistic model has been thoroughly discussed in literatures [15, 16] and applied to study hadronic transition processes [17, 19]. The results obtained in this framework qualitatively agree with the data for all the concerned processes.

In conventional LFQM the radiative decay of a $1^{--}$ (S-wave) meson into a $0^{-+}$ meson was evaluated [20] and the same formula can also be generalized to the covariant LFQM [21]. In our earlier papers [22, 24] we studied radiative decays of some mesons in covariant LFQM and now we will concentrate our attention to the radiative decays of $1^{--}$ (D-wave) mesons to $0^{-+}$ mesons. The results would be useful for confirming the identities of the aforementioned mesons. Since the Lorentz structure of the vertex functions of D-wave is the same as that of S-wave [25], the formulas for decays of the $1^{--}$ D-wave mesons can be simply obtained by replacing several functions which were used for the decays of the $1^{--}$ S-wave mesons.

This paper is organized as following: after this introduction, we derive the theoretical formulas in next section where we also present relevant formulas given in literatures, and then in Sec. III, we present our numerical results along with all inputs which are needed for
the numerical computations. In the last section we draw our conclusion and make a brief discussion.

II. THE FORMULAS FOR THE RADIATIVE DECAY OF $1^{-+}$ MESON IN LFQM

![Feynman diagrams depicting the radiative decay.](image)

**FIG. 1:** Feynman diagrams depicting the radiative decay.

In the light front quark model, the transition matrix elements for the decay of $1^{-+}(V) \rightarrow 0^{-+}(P)\gamma$ were examined (Fig.1) and the form factor $F_{V \rightarrow P}(q^2)$ can be expressed as \cite{20}:

$$F_{V \rightarrow P}(q^2) = e_1 I(m_1, m_2, q^2) + e_2 I(m_2, m_1, q^2),$$  \hspace{1cm} (1)

where $e_1$ and $e_2$ are the electrical charges of charm and strange quarks, $m_1 = m_c$, $m_2 = m_s$ and

$$I(m_1, m_2, q^2) = \int_0^1 \frac{dx}{8\pi^3} \int d^2 p_\perp \frac{\phi \phi'}{x_1 M_0 M'_0} \left\{ A + \frac{2}{w_{3S_1}} \frac{[p^2_\perp - (p_\perp q_\perp)^2]}{q^2_\perp} \right\},$$  \hspace{1cm} (2)

where $h_{3S_1} h'_P = (M^2 - M_0^2) \sqrt{\frac{2 + x}{N_c \sqrt{2} M_0}} \phi$, $w_{3S_1} = M_0 + m_1 + m_2$, $A = x_2 m_1 + x_1 m_2$ and $x = x_1$. It is noted that the $1^{-+}$ meson in Ref.\cite{20, 21} just refers to $3S_1$ state. The other variables in Eq. (2) are presented in the Appendix.

Obviously, a $1^{-+}$ meson may be in a $3D_1$ state or a $3S_1$ state or their mixture.

In Ref.\cite{25} the vertex function for $3D_1$ states was deduced and its Lorentz structure is the same as that of $3S_1$ state, so Eq. (2) is also valid for the radiative decay of $3D_1$ through replacing the functions $h_{3S_1}$ and $w_{3S_1}$ by

$$h_{(3D_1)} = -(M^2 - M_0^2) \sqrt{\frac{x_1 x_2}{N_c \sqrt{2} M_0}} \frac{1}{12 \sqrt{5} M_0^2 \beta^2} [M_0^2 - (m_1 - m_2)^2][M_0^2 - (m_1 + m_2)^2] \phi,$$

$$w_{(3D_1)} = \frac{(m_1 + m_2)^2 - M_0^2}{2 M_0 + m_1 + m_2}.$$
The decay width is
\[ \Gamma(V \to P + \gamma) = \frac{\alpha}{3} \left( \frac{m_V^2 - m_P^2}{2m_V} \right)^3 \mathcal{F}_{V \to P}^2(0). \] (3)

III. NUMERICAL RESULTS

Before we carry out our numerical computations for evaluating the branching ratios of the D-wave mesons, we need to determine a nonperturbative parameter \( \beta \) which exists in the wave function, in a proper way. In Ref. [16] the authors suggested that via calculating the decay constant of the ground state one can determine \( \beta \). Alternatively, we also can get the value of \( \beta \) by fitting the spectra of the relevant mesons as done in Ref. [20]. In this work we follow the first scheme. With the averaged decay branching ratio of \( D_s \to \mu \nu \) (5.56 \pm 0.25) \times 10^{-3} [27] one obtains its decay constant as \( f_{D_s} = (247 \pm 6) \) MeV. Then using the Eq.(6) in Ref.[21] \( \beta \) is fixed as (0.534 \pm 0.015) GeV\(^{-1}\) when we set \( m_c = 1.4 \) GeV, \( m_s = 0.37 \) GeV [16] and \( m_{D_s} = 1.9685 \) GeV [27].

A. The radiative decays of \( D_{s1}^*(2860) \) and \( D_{s3}^*(2860) \)

In our numerical computations we adopt the assumption that \( D_{s1}^*(2860) \) and \( D_{s3}^*(2860) \) are \( 1^3D_1(c\bar{s}) \) and \( 1^3D_3(c\bar{s}) \) respectively.

Using the parameters we calculate the form factor \( F(0) \) for \( D_{s1}^*(2860) \to D_s \gamma \) which is (0.0168 \pm 0.0002) GeV\(^{-1}\). The decay width \( \Gamma(D_{s1}^*(2860) \to D_s \gamma) \) is (0.291 \pm 0.006) keV. Comparing with the total width the value is rather small, namely the branching ratio is small, but one still has a chance to measure it in more accurate experiments. To explore its dependence on the parameter \( \beta \) we vary \( \beta \) from 0.35 GeV\(^{-1}\) to 0.6 GeV\(^{-1}\). The results are depicted in Fig. 2. One can notice that the result is not sensitive to the value of \( \beta \) after all.

Since the vertex function of the \( 3^3D_3 \) state is more complicated we are not going to directly deduce the transition matrix elements for the radiative decays in this framework. Instead, we would take an approximate but reasonable scheme to estimate the radiative decay width of \( 3^3D_3 \). Namely, one obtains the rate of \( 3^3D_3 \) radiative decay in terms of that of the \( 3^3D_1 \) radiative decay. Under the nonrelativistic approximation the authors of Ref. [26] presented a formula to calculate the widths for the \( M1 \) transition as
\[ \Gamma(i \to f \gamma) = \frac{\alpha}{3} \left( \frac{e_c}{m_c} - \frac{e_s}{m_s} \right)^2 E_{\gamma}^3(2J_f + 1) \langle f | j_0(kr/2) | i \rangle^2. \] (4)

If we ignore the spin-orbit coupling term in the potential which results in the fine-structure of spectra, the wave functions of \( D_{s1}^*(2860) \) and \( D_{s3}^*(2860) \) obtained by solving the Schrödinger equation would be the same because they have the same orbital angular momentum and intrinsic spin, thus we would naturally get \( \langle D_s | j_0(kr/2) | D_s(3^3D_3) \rangle = \langle D_s | j_0(kr/2) | D_s(3^3D_1) \rangle \).
FIG. 2: $\Gamma(D_{s1}^*(2860) \rightarrow D_s\gamma)$ dependence on $\beta$.

TABLE I: The form factor for $D_{s1}^*(2710) \rightarrow D_s$.

|                | D-wave | S-wave(1) | S-wave(2) |
|----------------|--------|-----------|-----------|
| $\mathcal{F}(0)$ (GeV$^{-1}$) | $-0.0168 \pm 0.0002$ | $0.099 \pm 0.001$ | $0.112 \pm 0.001$ |
| $\Gamma$ (keV) | $0.179 \pm 0.004$ | $6.18 \pm 0.07$ | $8.00 \pm 0.02$ |

Since the mass of $D_{s1}^*(2860)$ is close to that of $D_{s1}^*(2860)$, it hints that the contributions of the spin-orbit coupling term to spectra and wave function are less important. By including all factors, it is straightforward to estimate $\Gamma(D_s(3D_3) \rightarrow D_s\gamma) \approx \Gamma(D_s(3D_1) \rightarrow D_s\gamma)$.

B. The radiative decay of $D_{s1}^*(2710)$

After $D_{s1}^*(2710)$ was found, a lot of work has been done to investigate its identity. In Ref. [11] the authors suggested that $D_{s1}^*(2710)$ should be a $2^3S_1$ state, rather than a $1^3D_1$ state. To be more open, here let us assume $D_{s1}^*(2710)$ to be respectively a $2^3S_3$ state or a $1^3D_1$ state and under the different assumptions, we calculate its radiative decay width. The results are listed in Table I. For the S-wave state ($2^3S_3$) we employ the conventional wave function (S-wave(1)) and modified wave function (S-wave(2)) which was discussed in Ref. [22]. Then we continue to calculate the rate of radiative decay of the D-wave state in the aforementioned approximation.

One would notice that there exists a huge gap between the S-wave and D-wave cases. If we assume that $D_{s1}^*(2710)$ is the mixture of $2^3S_3$ and $1^3D_1$ i.e. $|D_{s1}^*(2710)\rangle = \cos \theta |2^3S_3\rangle - \sin \theta |1^3D_1\rangle$ [14], using the values of $\mathcal{F}(0)$ given in Table I, the corresponding radiative decay width is re-calculated. In Fig 3 the dependence of the decay width on the mixing angle $\theta$ is depicted where the modified wavefunction is used for the 2S state.

In Ref. [14] the authors studied $\Gamma(D_s(2710) \rightarrow D_{s2}(2573)\gamma)$, $\Gamma(D_s(2710) \rightarrow D_{s0}(2317)\gamma)$,
\[ \Gamma(D_s(2710) \rightarrow D_{s1}(2460)\gamma) \] and \[ \Gamma(D_s(2710) \rightarrow D_{s1}(2536)\gamma) \] which are 0.09 \( \sim \) 0.12keV, 7.80 \( \sim \) 7.97 keV, 1.47 \( \sim \) 1.56 keV and 0.27 \( \sim \) 0.29 keV respectively. The above cited estimates are about radiative decays of \( D_s(2710) \) into a P-wave meson plus a photon, as we noted that for finally identifying the quantum numbers of \( D_s(2710) \), the decay mode under investigation: \( D_s(2710) \rightarrow D_s(1963)\gamma \) which is \( D_s(2710) \) decaying into a S-wave meson plus a photon, is not less important.

\[ \text{FIG. 3: dependence of } \Gamma(D_{s1}^{*}(2710) \rightarrow D_{s}\gamma) \text{ on the mixing angle } \theta. \]

IV. SUMMARY

In this work we study the radiative decay of \( D_{s1}^{*}(2860) \), \( D_{s3}^{*}(2860) \) and \( D_{s1}^{*}(2710) \) respectively in terms of LFQM. Assuming \( D_{s1}^{*}(2860) \), \( D_{s3}^{*}(2860) \) to be \( 1^3D_1 \) and \( 1^3D_3 \) states, we obtain their partial widths. Our estimates on \( \Gamma(D_{s1}^{*}(2860) \rightarrow D_{s}\gamma) \) and \( \Gamma(D_{s1}^{*}(2860) \rightarrow D_{s}\gamma) \) are approximately 0.291 keV. The estimated branching ratios of the radiative decays of \( D_{s1}^{*}(2860) \) and \( D_{s3}^{*}(2860) \) are about 1.9\( \times \)10\(^{-6} \) and 5.8 \( \times \)10\(^{-6} \). By the achieved integrated luminosity at LHCb (3.0 fb\(^{-1}\)), the LHCb Collaboration\[8\] collected 12450 \( B_s^0 \rightarrow \bar{D}^0 K^- \pi^+ \) samples where only a part of the events concern \( D_{s1}^{*}(2860) \) and \( D_{s3}^{*}(2860) \). Their radiative decays have not been observed yet due to the small database for \( D_{s1}^{*}(2860) \). Indeed we need longer time and higher luminosity to observe the radiative decays \( \Gamma(D_{s1}^{*}(2860) \rightarrow D_{s}\gamma) \) and \( \Gamma(D_{s1}^{*}(2860) \rightarrow D_{s}\gamma) \).

Though the fractions of the radiative decays are small, they have clear signal to be observed from the background, therefore the advantage of detecting those modes is obvious. Thus we expect our experimental colleagues to carry out accurate experiments to measure them.

Concerning \( D_{s1}^{*}(2710) \), as discussed in the introduction, if \( D_{s1}^{*}(2860) \) and \( D_{s3}^{*}(2860) \) are confirmed to be the D-wave \( D_s \) meson, \( D_{s1}^{*}(2710) \) cannot be a pure 1D-wave \( c\bar{s} \) system, we
calculate its radiative decay rate by assuming two possible assignments: $2^3S_3$ or $1^3D_1$ respectively. Our numerical results show that if it is a $2^3S_3$ state the corresponding branching ratio is about $5.2 \times 10^{-5} \sim 6.7 \times 10^{-5}$, instead while it is $1^3D_1$, the corresponding rate is around $1.5 \times 10^{-6}$. There is an obvious gap between the estimated rates for the two assignments.

Because the LFQM is a relativistic model and its validity is widely recognized due to its success for explaining the available data for hadronic decays of heavy mesons, we may believe that the numerical results obtained in this framework is trustworthy, at most they could only decline from the real values by a small factor less than 2 which was confirmed by other phenomenological studies in terms of the same model. The possible uncertainties are incurred by the inputs. Even so, the results could help identifying the quantum numbers since in the two cases the resultant ratios of $\Gamma(D^*_s(2710) \rightarrow D_s \gamma)$ are apparently apart.

No doubt, the final decision will be made by the future precise measurements. Our work only indicates the importance of studying the radiative decays because of their obvious advantage and strongly suggest to search such decay modes at the coming super-BELLE or next run of LHCb, even the expected ILC.

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Appendix A: Notations

Here we list some variables appearing in the context. The incoming meson in Fig. 1 has the momentum $P = p_1 + p_2$ where $p_1$ and $p_2$ are the momenta of the off-shell quark and antiquark and

\[
p^+_1 = x_1 P^+, \quad p^+_2 = x_2 P^+,
\]

\[
p_1 \perp = x_1 P \perp + p_\perp, \quad p_2 \perp = x_2 P \perp - p_\perp,
\]

with $x_i$ and $p_\perp$ are internal variables and $x_1 + x_2 = 1$.

The variables $M_0$ and $\tilde{M}_0$ are defined as

\[
M_0^2 = \frac{p_\perp^2 + m_1^2}{x_1} + \frac{p_\perp^2 + m_2^2}{x_2},
\]

\[
\tilde{M}_0 = \sqrt{M_0^2 - (m_1 - m_2)^2},
\]

\[
\phi(1S) = 4\left(\frac{\pi}{\beta^2}\right)^{3/4} \sqrt{-\frac{\partial p_z}{\partial x_2}} \exp\left(-\frac{p_z^2 + p_\perp^2}{2\beta^2}\right),
\]

\[
\phi(2S) = 4\left(\frac{\pi}{\beta^2}\right)^{3/4} \sqrt{-\frac{\partial p_z}{\partial x}} \exp\left(-\frac{1}{2} \frac{p_z^2 + p_\perp^2}{\beta^2}\right) \left(3 - 2\frac{p_z^2 + p_\perp^2}{\beta^2}\right)^{1/2},
\]

\[
\phi_M(2S) = 4\left(\frac{\pi}{\beta^2}\right)^{3/4} \sqrt{-\frac{\partial p_z}{\partial x_2}} \exp\left(-\frac{2a_2 - b_2 - 2a_2 - b_2}{\beta^2}\right). \quad (A2)
\]
with $p_z = \frac{x_2 M_0}{2} - \frac{m_2^2 + p^2_{\perp}}{2x_2 M_0}$, $\delta = 1/1.82$, $a_2 = 1.88684$ and $b_2 = 1.54943$. 

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