Quantum Information Processing in Nanostructures

**Alexandra Olaya-Castro**  
Centre for Quantum Computation, Clarendon Laboratory  
Department of Physics, University of Oxford  
Parks Road, Oxford OX1 3PU, United Kingdom  
E-mail: a.olaya@physics.ox.ac.uk  
Phone: 44 1865 272308  
Fax: 44 1865 272400

**Neil F. Johnson**  
Centre for Quantum Computation, Clarendon Laboratory  
Department of Physics, University of Oxford  
Parks Road, Oxford OX1 3PU, United Kingdom  
E-mail: n.johnson@physics.ox.ac.uk  
Phone: 44 1865 272287  
Fax: 44 1865 272400

to appear in “Handbook of Theoretical and Computational Nanotechnology”
1 Introduction

What can be accomplished in a world governed by quantum correlations? This question has brought together researchers from a wide range of backgrounds – from fundamental physics through to information technology – in the common pursuit of understanding, designing and eventually building quantum information processing (QIP) systems.

The fundamental unit of quantum information is the quantum bit, or qubit, which is a quantum version of the classical binary bit (i.e. 0 and 1). Thanks to the enormous amount of work in the QIP field over the past decade, we now know that it should be possible to use qubits to realize fundamentally new and more powerful methods for computation and communication. In other words, one ought to be able to harness the ‘spookiness’ of quantum mechanics, in particular quantum correlations, to achieve a revolutionary form of information processing. It has been shown, for example, that algorithms for factorizing large integers can be realized faster in a suitably chosen QIP system than in a conventional computer[1]. The teleportation of a quantum state between spatially distinct locations, has been experimentally demonstrated[2, 3]. An application of increasing importance in this field, concerns the use of QIP to simulate other quantum systems[4, 5, 6]. Interesting by-products of this research include a new understanding of physical behaviour at a quantum phase transition[7, 8, 9]. The promise of further exciting applications and a deeper fundamental understanding of quantum mechanics, has sparked off a very active QIP research community spanning physicists, mathematicians, materials scientists, chemists and computer scientists.

1.1 The Challenge of Scalability

Any practical implementation of a QIP system needs to meet several stringent criteria in order to operate successfully[10]. First of all, the qubits (which can also be thought of as ‘quantum memories’) must be sufficiently isolated so that they can then be directly and conditionally manipulated in a controlled environment. They need to be initialized precisely and then efficiently measured. The effective interactions among qubits should also be carefully tuned, and a set of universal quantum operations should be made possible in order to perform any other required quantum gate. Most importantly, the system must be scalable to more than a few qubits. Consequently a large-scale QIP system is expected to include of the order of tens to hundreds of qubits arranged in a configurable way, depending on the quantum routine to be achieved. In order to maintain the quantum correlations, parallel addressing of spatially separated units is also required, with operation times smaller than both local and non-local decoherence times. Scalability and robustness are hence arguably two of the most demanding requirements.
facing the practical implementation of quantum information processing.

1.2 Nanostructures for scalable QIP

The clearest demonstrations of QIP involving massive particles, have come from the field of interacting atoms/ions and photons. Individual atoms or ions, which can be strongly or weakly coupled to the photon field, are manipulated in controlled environments with well-understood couplings and well-defined decay channels, thereby offering an ideal experimental set-up to systematically study the basic principles of quantum computation and communication. In fact, trapped ions in optical lattices have become an ideal test ground for the study of more complex QIP architectures. Despite these achievements, there is a growing feeling within the QIP community that the potential integration of basic science and technology which will be required for building large-scale quantum processors, will be best achieved in solid-state systems. After all, the transistor gave us classical computing, so why shouldn’t a quantum version give us quantum computing?

In the last decade, there have been a number of proposals of solid state systems in which quantum information processing might be achieved experimentally. All these proposals are conceptually similar in the sense that they require manipulation of the coherent dynamics of a quantum system which is embedded in a complex environment. Hence they all share the same challenge of having to resist highly non-trivial decoherence sources. Given that decoherence times in solid-state systems are short, the structures themselves must be sufficiently small that the quantum information can be manipulated and/or transferred in a time which is shorter than the decoherence time. Remarkably, such small structures, or nanostructures, are now beginning to be built. In fact, the incredible advances in solid-state nanofabrication mean that artificial and natural nanostructures, such as quantum dots, are becoming prime candidates for building a robust large-scale QIP system.

Semiconductor quantum dots (QDs) are structures with the dimensions of the order of few nanometers. Their electronic and optical properties can be tailored through the quantum confinement which results from their finite size. In particular, they exhibit physical features similar to individual atoms but are typically a few orders of magnitude larger and have an energy scale which is a few orders of magnitude smaller. Because of their atom-like properties but their key difference in size, they can be regarded as localized and addressable units for storing and manipulating quantum information. Quantum information can be encoded in QDs through a variety of effective two-level systems – for example, electron spin or charge excitation. Even more interesting is the possibility of building an artificial ‘molecule’ by coupling together two QDs. As we will discuss later, the nature of the interaction between QDs not only offers different perspec-
tives for QIP, but also establishes a direct link with biological and organic systems.

Although the earliest proposals for QIP with QDs envisioned spin interactions controlled by electrical gates\cite{14, 23}, it is now thought that schemes based on all-optical control of the quantum system and its interactions are possibly more desirable since they offer two principal advantages. First, the use of ultrafast laser technology\cite{24} means that quantum operations can be carried out within the coherence time. Second, the interaction between a QD (acting as a stationary quantum memory) and photons (acting as a channel for information transmission) make this setup attractive for both the manipulation and transfer of quantum information over relatively large distances. In this latter context, QDs can also be integrated with optical nano-cavities offering a means for exploiting the techniques of cavity quantum electrodynamics\cite{25, 26}.

With this backdrop, the aim of this Chapter is to survey the current state of theoretical proposals of QIP in semiconductor QDs, focusing on schemes which exploit optical control of qubit interactions. Readers interested in other solid state proposal are invited to consult Ref.\cite{27}. The Chapter is organized as follows. General theoretical aspects of the field of QIP are reviewed in Section II. Theoretical proposals for all-optical QIP using QDs are reviewed in Section III. Finally, future trends and developments are discussed in Section IV.
2 Theoretical Background

The coherent superposition and entanglement of quantum states are the features lying at the heart of any QIP protocol. They account for quantum interference effects within a single system (superposition) or between different quantum systems (entanglement). In any experimental situation, however, the unavoidable interaction of a quantum system with its environment leads to decoherence processes which eventually destroy superpositions and entanglement and yield statistical mixtures of states. Understanding such complex decoherence mechanisms and finding approaches to make quantum interference effects robust in the face of such decoherence, are still the major challenges facing many QIP proposals.

2.1 Basic QIP toolbox: Superposition and Entanglement

Quantum coherence represents the ability of a quantum system to be in a superposition of different states, and is the crucial element that leads to the definition of the fundamental unit of quantum information: the quantum bit (qubit). Qubits are quantum systems that can be represented to a good approximation as a two-state system. As a consequence, the state of an isolated qubit can be expressed as a coherent superposition of these two states:

$$|\Phi\rangle = \alpha|0\rangle + \beta|1\rangle$$  \hspace{1cm} (2.1)

where $|\alpha|^2$ and $|\beta|^2$ are the probabilities that the system be found in state $|0\rangle$ or $|1\rangle$, respectively. Such a coherent superposition implies that there is always a basis in which the value of the qubit is well-defined, as opposed to an incoherent mixture in which the qubit’s state becomes a statistical mixture irrespective of the basis used\[25\].

A single two-state quantum system can be represented as an effective spin-$\frac{1}{2}$ particle in a local magnetic field\[29\], whose Hamiltonian can be written as

$$H_0(t) = B(t) \cdot \sigma$$  \hspace{1cm} (2.2)

Here $\sigma_{x,y,z}$ are the Pauli matrices describing the qubit to be manipulated. Full control over the coherent dynamics of the system is possible if at least two components of the effective field $B(t)$ can be switched arbitrarily. This Hamiltonian corresponds to an ideal physical scheme for realizing single-qubit unitary operations described by the operator $U(t,0) = T \exp(\int_0^t H_0(t')dt')$ ($T$ denotes time-ordering). For instance, under the action of $U(t,0)$ it is possible to perform a Hadamard gate\[30\] which transforms the states $|0\rangle$ and $|1\rangle$ into coherent superpositions of the type described in Eq.\[2.1\] as $H|0\rangle \mapsto (1/\sqrt{2})(|0\rangle + |1\rangle)$ and $H|1\rangle \mapsto (1/\sqrt{2})(|0\rangle - |1\rangle)$.

However, any physical realization of a qubit – for example, an electron’s spin up or down, an optically excited QD with one exciton or no exciton, or
the flux in a SQUID – is embedded in a weakly interacting environment with a large number of degrees of freedom. This yields transient behavior with only partial coherence. Thus a density matrix formalism is more convenient to describe the qubit dynamics, where the reduced density operator in the basis \{\ket{0}, \ket{1}\} can be written as

\[
\hat{\rho}_q = \begin{pmatrix}
\rho_{00} & \rho_{01} \\
\rho_{10} & \rho_{11}
\end{pmatrix}
\]  

The diagonal elements \(\rho_{00}\) and \(\rho_{11}\) denote the populations of each level and the off-diagonal elements \(\rho_{10}\) and \(\rho_{01}\) define the coherences between states \(|0\rangle\) and \(|1\rangle\). The population of the excited level \(\rho_{11}\) is typically described as having a characteristic decay time known as \(T_1\), while the typical decay time of the coherences \(\rho_{10}\) and \(\rho_{01}\) is known as the dephasing or coherence time \(T_2\). If, however, the qubit dynamics are explored over timescales significantly smaller than \(T_2\) using ultrafast spectroscopy, the dephasing processes may not have sufficient time to be completed and hence the effects of memory in these processes must be accounted for. The Markov approximation in which memory effects are neglected yielding a characteristic exponential decay, may no longer be sufficiently accurate. Consequently \(T_2\) may depend on the temporal correlations between the qubit system and the environment and within the environment itself. In semiconductor nanostructures such non-Markovian effects are under active experimental and theoretical investigation\[31, 32, 33\].

The processing of quantum information requires qubits to be coherently manipulated on timescales shorter than \(T_2\). From the experimental point of view, a fundamental probe of coherent manipulation of a qubit is the identification of Rabi oscillations, which are produced in resonantly driven two-state systems\[34\] and have no classical analog. They correspond to a sinusoidal time-evolution of the population difference in a two-level system for timescales short as compared to the coherence time.

**Entanglement** occurs as a result of quantum interference among states of composite systems, giving rise to non-classical correlations between spatially separated quantum systems. An essential manifestation of these non-classical correlations is that a measurement performed in one of the subsystems determines the state of the other even if they are significatively far apart. For a long time this was considered a bizarre if not ‘spooky’ feature of quantum mechanics. However the modern view of entanglement is to see it as a fundamental nonlocal resource which can be used in any QIP protocol. The power of this resource has been demonstrated in quantum communication schemes performed with entangled photons, including state teleportation\[2, 3\], cryptographic key distribution\[35\] and quantum dense coding\[36\]. Entanglement of massive particles has been experimentally achieved in cavity QED\[37\], ion traps\[38\], and superconducting qubits\[39\].
The most general entangled state of \( N \) qubits can be expressed as a superposition of \( 2^N \) states with complex and time-dependent amplitudes \( a_i \) with \( i = 0, 1, \ldots , 2^N - 1 \):

\[
|\Psi\rangle = a_0(t)|010203\ldots 0_N\rangle + a_1(t)|110203\ldots 0_N\rangle + \cdots + a_{2^N-1}(t)|111213\ldots 1_N\rangle
\]  

As long as this state cannot be factorized into a product of single qubit states, then there will be non-classical correlations which can be exploited to store more information than a comparable set of classical counterparts, and which can be exploited to perform parallel operations on different qubits. Therein lies the extraordinary potential for quantum computation.

The quantification of entanglement has become a central problem in quantum information theory. In the case of bipartite quantum systems, several measures have been proposed (see Ref. [41]) including the entanglement of formation [42] which has the clear meaning of being the asymptotic number of Bell pairs required to prepare the state using only local unitary transformations and classical communication. Intimately related to this measure is the notion of concurrence [43]. While the bipartite case is well understood, a general formulation of multipartite entanglement remains an outstanding open problem.

### 2.2 Universal resources for QIP

One of the key issues that has been addressed in QIP science is to determine what subsets of physical resources are capable of achieving universal quantum computation and, more generally, how to quantify such resources for different QIP tasks [44]. The idea of universality is to identify a set of gates that are sufficient to perform any other quantum operation on arbitrary \( N \) qubits – for example, in order to transform any initial separable state of \( N \) qubits into an arbitrary entangled state of the type described in Eq. (2.4).

It is well known that certain two-qubit gates are universal for quantum computation when assisted with arbitrary one-qubit gates [45]. An example is the controlled-not (CNOT) operation which transforms as

\[
|m,n\rangle \rightarrow |m,m \oplus n\rangle \quad m \epsilon \{0,1\}
\]  

where \( \oplus \) denotes addition modulo 2. Recently it has become clearer what general class of two qubit gates are universal. It turns out that any interaction creating entanglement between any pair of qubits is universal for quantum computation [46, 47, 48]. In particular, it has been demonstrated that a fixed entangling two-qubit interaction can be used to perform universal quantum computation, when assisted by single-qubit transformations between applications of the entangling gate \( V \) [48]. This can be expressed as follows

\[
U = (A_1 \otimes B_1)e^{iV}(A_2 \otimes B_2)
\]

7
where $A_j, B_j$ are one qubit gates and $V = \sum_{X,Y,Z} \theta_\alpha \sigma_\alpha \otimes \sigma_\alpha$ with $-\pi/4 \leq \theta_\alpha \leq -\pi/4$. The key point in the demonstration is that by using single-qubit gates, any gate $U$ can be transformed into $W = e^{i\phi \sigma_2 \otimes \sigma_2}$ with $0 < |\phi| < \pi/2$, which is a natural gate for implementing the CNOT operation.

### a. Fundamental two-qubit interactions

The physical implementation of a two-qubit gate depends on both the system’s network structure and the nature of the effective interaction coupling together two qubits. The network structure refers to whether each qubit can be coupled to any other qubit, or whether each qubit can only be coupled to a few other qubits (e.g. nearest neighbors). For a given quantum task, this underlying structure is related to the number of operations required to achieve that task. This however also depends on the microscopic nature of the interaction between qubits. For instance, it has been discussed that a CNOT operation can be more efficiently realized if the available interaction is given by an Ising Hamiltonian rather than when it is described by an isotropic Heisenberg interaction\[49\]. The nature of the interaction also determines whether it is possible to use recoupling to encode qubits and hence reduce the number of single-qubit operations\[50\].

In a large number of proposals for QIP in solid state systems, the fundamental two-qubit gate is generated by an exchange interaction between qubits $i$ and $j$ which has the general form

$$\mathcal{H}_{\text{int}}(t) = \sum_{\alpha=X,Y,Z} J_\alpha(t) \sigma_\alpha^i \cdot \sigma_\alpha^j \quad (2.7)$$

This interaction may arise naturally in the system, or it might be effectively created by manipulating local variables like individual energy transitions or individual couplings to a common collective mode.

The isotropic Heisenberg model, which corresponds to $J_\alpha(t) \equiv J(t)$, describes the fundamental two-qubit interaction in proposals for the implementation of spin-based quantum computation\[14, 15, 23, 51\]. It has been shown that the Heisenberg interaction is sufficient to perform universal quantum gates when a logical qubit is encoded in two or more qubits\[52\]. These schemes require tunability of the interaction which may prove very difficult to achieve experimentally. As an alternative, schemes for quantum computation with an always-on Heisenberg interaction have been proposed\[53\]. By tuning individual energy transitions via local or global addressing, the effective interaction between qubits in a linear array becomes an effective Ising interaction: $\mathcal{H}_{\text{int}} \simeq J \sum \sigma_i^x \sigma_{i+1}^x$. The interesting point here is that up to single-qubit rotations, the CNOT operation is a natural two-qubit gate for the Ising Hamiltonian as discussed in Ref.\[48, 49\] and hence it is a resource for universal quantum computation.
The anisotropic $XXZ$ model, which corresponds to the case where $J^X(t) = \pm J^Y(t) \neq J^Z(t)$, is the natural interaction of electrons on a helium surface[18]. The scaling properties of entanglement close to the phase transition described by this model, have been discussed[9] as well as schemes for encoded QIP[50].

The $XY$ model, corresponding to the case $J^X(t) = J^Y(t), J^Z = 0$, has been intensively studied in the context of QIP[7, 9, 49, 54]. These studies include the generation of so-called $W$ states[55] in a one dimensional $XY$ model[54] as well as scaling properties of entanglement in the vicinity of a quantum phase transition described by the $XY$ interaction in a transverse magnetic field[7, 9]. Moreover it has been shown that the effective dipole-dipole interaction dominates the generation of entangled states in a wide range of systems, from excitons in quantum dots[16] through to both atomic systems[56] and quantum-dot spins in cavities[25]. In addition, it has been shown that a natural two-qubit gate for this interaction is a simultaneous CNOT and SWAP operation[49]. The $XY$ interaction therefore has an important potential role to play in QIP implementations.

b. Cavity QED as a resource for QIP

The nature of the photon-matter interaction in cavity quantum electrodynamics (QED) has very important implications for the processing of QIP[57] for several reasons. In the strong coupling regime, it is possible to use the cavity field as an intermediary or ‘bus’ to create entanglement between qubits. Several two-qubit entangling protocols using this scheme have been proposed[56, 58, 59] with some of them being experimentally realized[37]. Cavity QED thus provides a scenario for exploring more complex QIP protocols in ‘clean’ environments. In addition, qubit-cavity systems are arguably the most promising candidates for the distribution of quantum information in a quantum network[60]. For example, qubits driven by lasers and strongly coupled to spatially separated cavities can be entangled via photons travelling from one cavity to another[60]. Finally, cavity QED has well-defined decoherence channels. Subject to high-efficiency measurements, this provides an experimental set-up for studying the conditional evolution of open quantum systems[57].

In cavity-assisted entanglement, non-interacting qubits can be prepared in an entangled state through either dispersive[56] or resonant[59] interactions with a common quantized field. Entanglement is achieved in the transitory regime where the strong interaction between qubits and a vacuum cavity field is switched on and off on a time scale shorter than the cavity-field decay time and the dipole decay time. Such an interaction can be effectively created using a chain of single qubits, for example via a stream of flying qubits which enter and leave the cavity, as well as via a set of qubits which simultaneously interact with the field.
The most general situation is described in the interaction picture by the Hamiltonian ($\hbar = 1$)

$$H_c = \sum_{i=1,2} f_i(t_i, t, \tau_i) \{ e^{-i\delta t} a^\dagger \sigma_i^- + e^{i\delta t} \sigma_i^+ a \}$$ (2.8)

where $\sigma_i^+ = |1_i\rangle \langle 0_i|$, $\sigma_i^- = |0_i\rangle \langle 1_i|$ with $|1_i\rangle$ and $|0_i\rangle$ ($i = 1, 2$) being the excited and ground states of the $i$'th qubit. Here $a^\dagger$ and $a$ are respectively the creation and annihilation operators for the cavity photons and $\delta$ is the detuning between the qubit transition frequency and the cavity field frequency. The time-dependent coupling of the cavity field with the $i$'th qubit, which is injected at $t_i$ and interacts during a time $\tau_i$, is given by a time-window function,

$$f_i(t_i, t, \tau_i) = [\Theta(t - \tau_i) - \Theta(t - \tau_i - t_i)] \gamma_i(t)$$ (2.9)

where $\gamma_i(t)$ is the time-dependent qubit-field coupling strength. For the situation in which the second qubit interacts with the cavity just after the first did, i.e. $t_2 = t_1 + \tau_1$, the Hamiltonian corresponds to the Jaynes-Cummings interaction for each qubit separately. Under resonant conditions $\delta = 0$, the initial maximum entanglement between a first qubit and the field is converted into qubit-qubit entanglement during the interaction of the second qubit with the cavity mode. Under non-resonant conditions $\delta \gg \gamma_1 \gamma_2$, the common quantized field is only virtually excited. This gives rise to an effective interaction corresponding to the Heisenberg $XY$ model in which a coherent energy transfer takes place between pairs of qubits. This scheme has been experimentally realized with two Rydberg atoms crossing a nonresonant cavity.

For the case of two qubits which simultaneously interact with the same cavity field, we have $t_1 = t_2$ and $\tau_1 = \tau_2$. A far richer control space can be explored if we generalize the setup to different and time-dependent qubit-cavity couplings. This latter situation is described by a generalized two-qubit dynamical Dicke (DD) model. Under non-resonant conditions where $\delta \gg \gamma_1 \gamma_2$, the common quantized field is only virtually excited. This gives rise to an effective interaction corresponding to the Heisenberg $XY$ model in which a coherent energy transfer takes place between pairs of qubits. This scheme has been experimentally realized with two Rydberg atoms crossing a nonresonant cavity.

When both qubits are on resonance with the cavity mode and are simultaneously interacting, the generation of entanglement can be controlled using asymmetric qubit-cavity couplings. This scheme exploits a trapping vacuum condition in which the cavity-qubit state becomes separable, leaving the cavity photon number unchanged but the two-qubit subsystem in an entangled state. Such a trapping condition does not arise for identical couplings.
2.3 General aspects of decoherence

Decoherence is generally a fast process whose timescale depends primarily on the size and temperature of the system, but may also depend on other factors as well (e.g. sample preparation, errors in the prescribed unitary evolution, different sources of thermal and quantum noise). The original meaning of decoherence was specifically designated to describe the loss of coherence in the off-diagonal elements of the density operator in the energy eigenbasis [63]. More recently, with the additional goal of better understanding the effects of environmental interactions on QIP and other forms of quantum control, decoherence has been defined as non-unitary dynamics induced by system-environment couplings [64, 65].

This non-unitary dynamics is not limited to the known relaxation processes of dissipation and dephasing, traditionally associated with $T_1$ and $T_2$ processes. Non-unitary or ‘irreversible’ dynamics also includes processes in which the system is conditioned to follow a specific quantum path by a measurement at certain times, i.e. conditional quantum evolution or processes in which correlations in the bath properties may play an important role in the system’s dynamics (non-Markovian effects).

Irreversible dynamics is generally seen as an undesirable consequence of QIP hardware. However, recent studies have shown that decoherence channels can actually be exploited to favor quantum coherence (e.g. in order to prepare two-qubit entangled states [66, 67] or to perform high-fidelity quantum gate operations [68]). Here we briefly summarize three approaches that give insight into how to avoid decoherence sources, or how to use them in order to promote the persistence of quantum coherence. The first two approaches can be derived within the general framework of processes for which all memory effects in the environment can be discarded, i.e. Markovian dynamics which can be described by Lindblad-like equations [63]. The third type of approach, in contrast, is aimed at understanding and exploiting the bath’s memory effects.

a. Conditional quantum evolution

Conditional quantum evolution, also known as the quantum jump approach, has mainly been studied in connection with optical quantum systems [69]. In this approach, the environment or bath functions are subject to a continuous series of measurements which have the potential to interrupt the unitary evolution of the system at infinitesimal time intervals. Conditioned on a particular observation record for the decay channels, the system follows non-Hermitian dynamics in which the effective time-evolution does not preserve the norm of the state. This approach has been reviewed in Ref. [69].

The possibility of using decay channels to prepare entangled states have been discussed in the case of non-interacting qubits [66] as well as for inter-
acting qubits that are identically coupled to the quantized mode inside a leaky cavity. Subject to the condition that no photon is detected, they both demonstrate that a successful measurement performed on a sufficiently large timescale, generates an uncorrelated state for the qubits and cavity, leaving the two-qubit subsystem in its antisymmetric maximally entangled state. In the framework of this conditional evolution, it has also been shown that dissipation can be exploited to implement fast two-qubits gates with ground state qubits coupled to a common vibrational mode.

A common criticism of the above schemes is that the gate operation is probabilistic since there is always a non-zero probability of obtaining an undesirable outcome, i.e. a photon is detected and hence the gate operation has failed. Therefore in a broader perspective, a primary motivation for research on conditional quantum evolution is the prospect of developing general methods for real-time feedback control of open quantum systems.

b. **Decoherence-free subspaces and subsystems**

The central idea of decoherence-free subspaces is the identification of the dynamical symmetries in the system-environment interaction, in order to construct a basis of collective states that are robust to dissipation and dephasing. In this approach, all qubits are identically coupled to the same environment such that the qubit-permutation symmetry gives rise to collective decoherence which can be effectively overcome if the qubits are encoded in decoherence-free states. These states have been identified as singlets or one-dimensional irreducible representations of the algebra generating the dynamical symmetry.

Following these ideas there have been proposals for inducing subspaces which are effectively decoupled, via external time-dependent Hamiltonians. This is known as dynamical decoupling, and it appears to be a promising alternative for suppressing decoherence in solid-state systems which are subject to strong low-frequency noise. A further generalization of various schemes to avoid decoherence is provided by the concept of noiseless subsystems in which the information can be encoded in higher-dimensional representations. This generalization has become the basis of a full theory of universal and fault tolerant QIP on decoherence-free states. A recent review of the theory of decoherence-free subspaces and its extension to subsystems is given in Ref.

A particular motivation for understanding non-Markovian effects in the dynamics of an open quantum system, comes from experimental and theoretical studies in semiconductor nanostructures which have shown that the relaxation-time approximation may be questionable in the ultrafast optical
regime within which these systems are being explored for QIP. The reason is that these approximations overestimate decay effects on the short timescales within which the system is dynamically evolving [31, 32, 33].

These studies tie in to the more fundamental goal of advancing the theory of open quantum systems, in order to develop analytical methods in which correlations in the environment are incorporated [63]. A specific goal is to construct evolution equations for the reduced density matrix, that generalize the Markovian Lindblad master equation in order to include bath memory effects and yet which remains both numerically and analytically tractable [76, 77, 78]. Such non-Markovian extensions are required in order to preserve complete positivity, thereby ensuring that the system dynamics is compatible with the joint system-environment unitary evolution.

The importance of considering correlation effects within the bath, lies in the fact that such interference effects might actually reduce the effective decay-rate for coherence. For example, it has recently been shown that decoherence control is possible through a Parrondo-like effect [79] in which two correlated decoherence sources are made to effectively cancel.
3 All-optical QIP in semiconductor nanostructures

The possibility of controlling the coherent dynamics of an open quantum system, is a fascinating topic from both technological and fundamental physics perspectives. In semiconductor nanostructures, the availability of ultrafast lasers with their wide ranges of pulse widths, wavelengths, pulse energies, and pulse repetition rates, has made possible the coherent control of matter dynamics in the transient regime, i.e. before relaxation processes destroy the coherence created by an ultrafast optical excitation[24]. For optically-induced polarization of a quantum dot, the timescale of the transient regime has been found to range from tens of picoseconds[80] at low temperatures, down to several hundreds of femtoseconds at room temperature[81]. At low temperatures the mechanism of decoherence is mainly determined by radiative decay[80] while at higher temperatures it is dominated by pure dephasing processes which are themselves driven by interactions of the charge carriers with acoustic phonons[82].

Unprecedented levels of coherent control have been demonstrated on superpositions of exciton and biexciton states in a single quantum dot, using ultrafast optics[83, 84, 85, 86] (see figure 3.1). Earlier experiments showed coherent manipulation of the exciton wave function[80], coherent evolution between two different excitonic states with orthogonal polarizations[83], as well as Rabi oscillations between the vacuum state of excitons and a linearly polarized excitonic state[84]. Furthermore quantum superpositions have been reported between the ground state and the biexciton state[85] as well as exciton-to-biexciton Rabi oscillations[86]. These experimental achievements have been integrated in a set-up to perform an all-optical quantum gate in a single quantum dot[86]. Although this system is not scalable by simply adding arbitrary numbers of excitons into a dot, it demonstrates the potential for ultrafast optically-driven manipulation in scalable architectures based on multidot arrays.

Proposals for large-scale QIP in semiconductor QDs have been made using two different realizations of the qubit: the excess electron spin[14, 23] and an electron-hole pair excitation or exciton[16, 89, 17, 90, 91]. The former qubit benefits from a weak coupling to the environment, yielding longer coherence times that are of the order of microseconds[93]. However it lacks a fast and easy tunability of coupling between qubits. The latter candidate provides an extremely fast interaction between qubits, of the order of picoseconds for several proposals, but it suffers from short dephasing times. In order to achieve a high ratio between coherence and gate operation times, hybrid schemes which merge these various advantages have recently been proposed[94, 95, 96, 97]. In particular, these latter approaches use optical excitations to control the coupling between QD spins. Schemes based on spin-flip Raman transitions mediated by a high-Q semiconductor microcavity, have also been proposed[25, 87, 88].
The common goal of these approaches is to use ultrafast technology to achieve all-optical control of the qubit state and inter-qubit interactions in semiconductor QDs, and to then scale up this procedure to produce large-scale QIP. In this section we review proposals aimed at this end.

### 3.1 Ultrafast schemes with excitons as qubits

Optically-driven semiconductor QDs lie at the heart of many QIP proposals\cite{16, 89, 17, 90, 91}. These systems offer discrete energy levels which can be quickly and accurately addressed in space, and can be built using well-established semiconductor fabrication technology. The natural qubit in the QD system is given by the absence (\(|0\rangle\)) and presence (\(|1\rangle\)) of a ground-state electron-hole pair, the so-called exciton as illustrated in figure 3.3. Experimental evidence suggests that the decoherence time of this qubit is mainly limited by the radiative lifetime\cite{80}.

Several QIP schemes have proposed exploiting exchange and/or direct Coulomb interactions between spatially separated excitonic qubits in coupled QD structures\cite{16, 89, 17, 91}. The Coulomb exchange interaction in QD molecules gives rise to a non-radiative resonant energy transfer (i.e. Förster process) which corresponds to the exchange of a virtual photon, thereby destroying an exciton in a dot and then re-creating it in a close by dot (see left-hand side of figure 3.3). Quiroga and Johnson\cite{16} and Reina et al.\cite{89} have discussed how to exploit the Förster interaction to prepare both Bell and Greenberger-Horne-Zeilinger (GHZ) entangled states of excitons, using far-field light excitation to globally address two and three quantum dots in a spatially symmetric arrangement. The Hamiltonian describing the formation of single excitons within the individual QDs and its interdot Förster hopping is given by\cite{16}

\[
H(t) = H_0 + H_F + H_{ext}(t) . \tag{3.10}
\]

Here the single-exciton Hamiltonian is given by

\[
H_0 = \frac{\varepsilon}{2} \sum_{l=1}^{N} (e_l^\dagger e_l - h_l h_l^\dagger) , \tag{3.11}
\]

the interdot Förster interaction can be written as

\[
H_F = -\frac{V_F}{2} \sum_{l,l'}^{N} (e_l^\dagger h_{l'} e_l h_{l'}^\dagger + h_{l'}^\dagger e_l h_{l'} e_l) , \tag{3.12}
\]

the coupling of the carrier system with a classical laser of amplitude \(E(t)\) is described by

\[
H_{ext}(t) = E(t) \sum_{l=1}^{N} e_l^\dagger h_l^\dagger + E^*(t) \sum_{l=1}^{N} h_l e_l \tag{3.13}
\]
and all constant energy terms have been ignored. The electron (hole) creation operator in the $l$th QD is designated by $e^\dagger_l(h^\dagger_l)$, $\varepsilon$ is the QD band gap and $V_F$ denotes the strength of the Förster coupling between dots. The equidistant configuration of the multidot system favors the definition of global quasispin operators, which in turn enable an understanding of the dynamics of the multie exciton system in terms of the dynamics of its associated global quasispin\textsuperscript{16}. Starting with the initial condition of a vacuum of excitons, the preparation of a Bell state such as $|\Psi\rangle = \alpha|00\rangle + \beta|11\rangle$ where $|11\rangle$ denotes the simultaneous presence of two excitons in a double dot structure, follows from the application of a finite rectangular pulse of the form $E(t) = A\cos(\omega t)$ with central frequency $\omega$ and sub-picosecond duration.

When the multidot system is arranged in a linear configuration where exciton hopping takes place only between nearest neighbors, the Hamiltonian $\mathcal{H}(t)$ takes the form of the one-dimensional $XY$ Heisenberg model\textsuperscript{98}. The derivation of the effective Hamiltonian relies on introducing the following local $1/2$–pseudospin operators for electron-hole pairs in each dot:

$$\sigma^+_l = \frac{1}{2}(e^\dagger_l h^\dagger_l + h^\dagger_l e_l)$$
$$\sigma^y_l = -\frac{i}{2}(e^\dagger_l h^\dagger_l - h^\dagger_l e_l)$$
$$\sigma^z_l = \frac{1}{2}(e^\dagger_l e_l - h^\dagger_l h^\dagger_l).$$

Hence without the optical field term, the effective $XY$ Hamiltonian reads

$$\mathcal{H}_{eff}(t) = \varepsilon \sum_{l=1}^{N} \Delta \sigma^z_l - V_F \sum_{l=1}^{N-1} (\sigma^-_l \sigma^+_l + \sigma^+_l \sigma^-_{l+1})$$

(3.15)

where $\Delta$ is the detuning from the resonant excitation and $\sigma^\pm_l = \sigma^x_l \pm i \sigma^y_l$.

An interesting point which emerges from this analysis is the difference of timescales required to generate entanglement in the different configurations. In particular, it turns out that GHZ states require longer times in a symmetric multidot arrangement than in a linear configuration\textsuperscript{98}. It would be interesting to check whether such a statement also applies to a more general QIP protocol.

The direct electrostatic interaction between two excitons (see right-hand side of figure\textsuperscript{14}) changes the energy of the excitonic transition, and is known as a ‘bi-excitonic’ shift\textsuperscript{20}. In the presence of an in-plane electric field the excitons acquire a permanent electric dipole and the bi-excitonic shift becomes significant\textsuperscript{99}. This electrostatic dipole-dipole interaction has been proposed as a physical mechanism for implementing a controlled-NOT operation in a double dot structure, as discussed by Biolatti et al.\textsuperscript{17}. Assuming that the distance between the dots is large enough to prevent single-particle tunnelling but at the same time that they are sufficiently close to assure
a strong interdot Coulomb interaction, the effective Hamiltonian governing
the dynamical evolution of the system can be written as

$$H_{\text{eff}}(t) = H_0 + H_{xx} = \sum_{l=a,b} E_l \hat{n}_l + \frac{1}{2} \sum_{l \neq l'} \Delta E_{ll'} \hat{n}_l \hat{n}_{l'}$$  \hspace{1cm} (3.16)

where $\hat{n}_l$ denotes the excitonic occupation number operator with eigenvalues
0 and 1, with 0 and 1 denoting the absence and presence of an exciton
in the $l$–th QD respectively. Here $E_l$ denotes the ground-state exciton
energy; $\Delta E_{ll'}$ is the bi-excitonic shift in the presence of an electric field,
which only arises if the qubits in dots $l$ and $l'$ are in state $|1\rangle$ as illustrated
in right-hand side of figure 3.2. This state-dependent interaction can be
exploited to design conditional operations with properly adjusted two-color
laser pulses. For example in the case of two coupled dots $a$ and $b$, the
transition $|1_a 0_b\rangle \rightarrow |1_a 1_b\rangle$ could be achieved on the sub-picosecond timescale
by the following sequence: First apply a $\pi$ rotation of the state of qubit
$a$ ($|0\rangle \rightarrow |1\rangle$) and then apply a second pulse with frequency $E_b + \Delta E_{ab}$
to perform a $\pi$ rotation of qubit $b$. We note that this analysis neglects
the presence of the Förster interaction. In addition, one of the possible
difficulties with this scheme is the need for an external electric field, since
this would require the presence of electrical contacts which not only increase
the complexity of the set-up but would also imply an additional source of
decoherence due to electromagnetic fluctuations. In order to circumvent
this requirement, the same group have proposed the use of QD structures
with built-in electric fields, as observed in GaN dots.

The interplay between the resonant energy transfer ($V_F$) and the inter-
dot bi-exciton binding energy ($\Delta E_{ll'} = V_{xx}$) has been studied numerically by
Lovett et al. These authors have shown that by taking into account
both the Förster interaction and the bi-exciton binding energy, it is possible
to develop an energy-selective approach to prepare entangled states of excitons
and to perform the CNOT operation in QD molecules. They consider
two QDs $a$ and $b$ having different sizes, which implies that in the absence
of interdot interactions there is an energy difference between the excitonic
transitions in the dots denoted by $\Delta_0$. Hence two regimes, determined by
the ratio $V_F / \Delta_0$, can be explored. When the Förster coupling is dominant
($V_F / \Delta_0 \gg 1$) and the initial state is $|0_a 1_b\rangle$ which denotes a single exciton
in dot $a$ and no-exciton in dot $b$, the system can naturally evolve into a singlet which is maximally entangled state. When $V_F / \Delta_0 \ll 1$, the bi-exciton
binding energy $V_{xx}$ becomes dominant and a CNOT operation can be
implemented which is driven by pulses of different frequencies. In this case,
entangled states can also be prepared by a properly designed laser-pulse
sequence starting from the vacuum of excitons.

Inter-dot interactions in the presence of interband excitations, have
recently been described in the context of a multipolar Quantum Electrodynamics (QED) Hamiltonian. This treatment allows one to understand
the previously discussed interactions in terms of the exchange of transverse photons and electrostatic contributions, but also points out the physical mechanism to induce dipole-forbidden transitions that are mediated by an optical near-field [100].

All these schemes share the advantage of providing fast two-qubit operations within the sub-picosecond time scale, as a result of resonant energy transfer or the interdot-biexciton binding energy. However this timescale is still comparable with the dephasing time of the exciton dipole. A good question at this point is then: is it possible to take advantage of this ultrafast interaction, and hence ultrafast technology itself, to develop a combined scheme which integrates a long coherence qubit with fast two-qubit operations? This is precisely the question that recent proposals have been exploring [94, 95, 96, 97]. In what follows, we summarize proposals that share the idea of exploiting electron-hole excitations to control the coupling between QD spin qubits.

3.2 Exciton-assisted spin-based quantum computation

Single electron spins confined in quantum dots have been proposed as qubits [14, 23] because of their long relaxation times which are well into the microsecond timescale [93]. It has been suggested that the interaction between spins could be controlled by electronic gates [14]. However an all-optical approach is clearly more desirable since this would avoid the additional fabrication of gates and the unavoidable fluctuations in electromagnetic fields arising from these gates. As for the other all-optical approaches which have formed the focus of this article, it instead makes sense to try to exploit the major advances in ultrafast laser technology which continue to be developed [24]. Indeed, it has already been demonstrated experimentally that a single electron spin in a QD can be probed optically [101].

The idea behind spin-based QIP assisted by excitons [94, 95, 96, 97] is to exploit virtual or resonant interband excitations to optically induce and control interactions among spin-qubits which are localized in different QDs. In these schemes the logical qubit is defined by the spin-states of a single conduction-band electron confined in a QD: $|0\rangle = |m_z = -1/2\rangle$ and $|1\rangle = |m_z = 1/2\rangle$ as shown in figure 3.5.

The first scheme which used exciton states to induce indirect exchange interactions between electron spins localized on different QDs, was discussed by Piermarocchi et al. [94]. The excess electrons confined in spatially separated QDs, interact with a delocalized electron-hole pair which has been excited by an optical pulse which is itself detuned with respect to the continuum of exciton states in the host material. Keeping only the lowest-order contribution, exchange Coulomb interactions between the localized and the optically-excited conduction electrons lead to an effective spin-spin exchange coupling between QD spins of the type discussed in the previ-
ous section, i.e. $H_{\text{eff}} = -2J_{12}S_1S_2$ with $S_{i=a,b}$ being the electron spin in dot $i$. In this scheme the coupling parameter $J_{12}$ is always positive (ferromagnetic interaction) and depends on the detuning frequency between the laser and the excitonic transition in the continuum, and the interdot separation as illustrated in figure 3.6. This coupling mechanism is analogous to a Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction between two well separated magnetic impurities mediated by either conduction electrons or excitons\[102\], except that in the present case the intermediate electron-hole pair is produced by an external optical field.

The second scheme we discuss is based on conditional spin and exciton dynamics and employs a Pauli-blocking mechanism\[95, 96\]. As illustrated in figure 3.7 when the QD is excited with left-handed circularly polarized light the presence of an excess electron in its spin-down state ($|0\rangle$) inhibits the creation of an exciton with electron angular momentum $m_z = -1/2$. By contrast if the qubit is in its spin-up state ($|1\rangle$) then nothing prevents the creation of the electron-hole pair. This yields a state-selective coupling of the logical states $|0\rangle$ and $|1\rangle$ to the auxiliary state, defined as $|x^{-}\rangle$, which describes a QD with two electrons with opposite spins occupying the same energy level together with a single hole. This scheme also exploits the Pauli exclusion principle and electrostatic exciton-exciton interactions in order to implement a two-qubit phase gate. A rotation, given by an accumulated phase $\theta$, only occurs when both qubits are in their logical state $|1\rangle$:

$$|m,n\rangle \rightarrow e^{i\theta mn}|m,n\rangle \quad m \in \{0,1\}.$$

(3.17)

As discussed by Calarco et al\[96\], the key mechanism here is the energy shift due to the electrostatic dipole-dipole interaction between excitons when an external static electric field is applied (see figure 3.5). As mentioned earlier, this bi-excitonic shift occurs only in the case when both qubits are in their spin-up state. The effective Hamiltonian describing the situation for two adjacent QDs ($a$ and $b$) is given by\[96\]

$$H_{\text{eff}} = \sum_{\nu=a,b} \left( \frac{\Omega_{\nu}(t)}{2} |x^-\rangle_{\nu} \langle 1| + H.c. \right) - \Delta E_{ab} |x^-\rangle_a \langle x^-| \otimes |x^-\rangle_b \langle x^-| \quad (3.18)$$

Here $\Omega(t)$ is the effective Rabi frequency between the single-electron state $|1\rangle$ and the trion state $|x^-\rangle$. In this proposal, as in most of the spin-based schemes, realizing the optical rotation of a single spin represents a significant challenge. To overcome this difficulty, the authors suggest the implementation of Raman transitions via light-hole levels $m_h^z = \pm 1/2$ in situations where these states are the hole ground-states, a situation which does occur in II-VI semiconductors nanostructures.

A modification of the above scheme to achieve the spin-couplings via inter-dot resonant or Förster energy interaction ($V_F$) has been discussed in Ref.\[97\]. This modified scheme requires the excitation of a single ground-state exciton instead of interdot-biexciton states.
3.3 QIP schemes with microcavities and quantum dots

As discussed in the previous section, previous work on conditional quantum dynamics has demonstrated that cavity QED could play a key role in future quantum communication or computation schemes. More specifically, it has been shown that cavity-mediated interactions provide a means for preparing entangled states, in addition to providing a means for transmitting quantum information between distant nodes in a quantum network. Following this line of thought, Imamoglu et al. [25, 87] proposed a system which relies on the use of a quantized cavity mode and applied laser fields in order to mediate the interaction between spins of distant, doped QDs. The central idea in this scheme is to implement cavity assisted spin-flip Raman transitions and to couple pairs of qubits via virtual photons in the common vacuum cavity mode.

Assuming that a uniform magnetic field is applied along the $x$ direction, the QD qubit is defined by the spin states $|m_x = -1/2⟩ = |0⟩$ and $|m_x = 1/2⟩ = |1⟩$ of the single conduction-band electron. The scheme’s proposers consider QD spins interacting with a $x$–polarized vacuum cavity mode and a $y$–polarized laser field in order to implement cavity-assisted spin-flip Raman transitions between the two spin states, in close analogy with atomic cavity-QED schemes [103]. By adjusting the frequencies of the individual lasers in order to establish a near two-photon resonance condition, such that the cavity is virtually excited, the following effective Hamiltonian can be obtained:

$$H_{int} = \sum_{i \neq j} \frac{\tilde{g}_{ij}(t)}{2} [\sigma_i^{10} \sigma_j^{01} + \sigma_j^{10} \sigma_i^{01}] .$$

(3.19)

Here $\sigma_i^{10} = |1⟩⟨0|_i$ is the spin projection operator for the $i$th QD, and $\tilde{g}_{ij}(t)$ corresponds to the product of the two-photon coupling coefficients for the spins in QDs $i$ and $j$. It has been shown that the two-qubit coupling between any pair of QDs can be carried out in parallel and on sub-nanosecond timescales.

A modification of the above scheme so as to combine Pauli-blockade effects with the microcavity scheme has been discussed in Ref. [88]. In this modified scheme, the quantum information is defined by the states $|m_z = ±1/2⟩$ of the single conduction electron. The cavity mode, which is assumed to be right-hand polarized, induces an electron-hole pair excitation in a dot only if the excess spin electron is in the state $|m_z = -1/2⟩$. Individual lasers are assumed to be linearly polarized, and have their frequencies adjusted such that the Raman transition between the spin states can occur.
3.4 Decoherence control through optical pumping

Despite its atom-like properties, there are fundamental features that distinguish a quantum dot from an atom. One of these is the completely different role that hyperfine interactions play in the two quantum systems. In contrast to valence electrons of an atom, a single electron in a dot is confined on a lengthscale which extends over many lattice sites. Consequently, the electron spin interacts with a few thousand randomly-oriented nuclear spins. Interactions with this unpolarized bath present a decoherence mechanism for an electron spin in a quantum dot\cite{104}. Imamoglu et al.\cite{105} have discussed an optical scheme to suppress this decoherence mechanism. The main idea is to use hyperfine interactions to polarize the nuclear field. This is achieved by shifting the energy of the initial spin-up electronic state via the ac-Stark effect in order to create a resonant condition for the electron-nuclear spin-flip transition.

Interestingly, Shabaev et al.\cite{106} have shown that even in the presence of random hyperfine interactions with nuclear spins, a strong resonant optical excitation of the electron spin to an intermediate trion state provides readout capability of the spin state during the relaxation process of the trion state\cite{106}. Combined with a permanent transverse magnetic field, such an optical excitation also provides a way to initialize the spin into a state with a well-defined phase\cite{106}.

3.5 Concluding remarks

Optical properties of semiconductor QDs can be tailored by varying the size, shape and composition material of the QD, thereby offering a suitable scenario to implement all-optical approaches for the coherent control of qubits and their interactions. Semiconductor nanostructures integrated with ultrafast optics technology, are therefore an attractive solid-state alternative for constructing scalable and fault-tolerant architectures in order to implement quantum computation and communication, as well as quantum simulation protocols.

There is still a very long way to go before large-scale quantum processors can be made out of QD arrays. Indeed the state of the field is such that it would be a major scientific breakthrough if someone were to demonstrate quantum entanglement involving just a few QDs, let alone control or manipulate this entanglement. However there are many experimental groups trying to do exactly that – and eventually someone will manage. Along the way, there are many open problems which will need to be solved, and several new research themes will emerge. These open problems include a deeper understanding of decoherence mechanisms and readout for a single electron-spin, and experimental signatures of superposition and entanglement for excitonic and spin qubits and for the entanglement between a QD
system and photons. In the next section we discuss some of the trends we foresee for future developments.
4 Trends for future developments

The study of QIP in nanostructures has opened up many new questions. As a result of the presence of similar physical mechanisms of interactions in organic and semiconductor systems, one such question concerns the extent to which organic and biomolecular systems offer a viable alternative for QIP, e.g. ‘bio-QDs’ such as the LHI and LHII complexes used by Nature to harness the energy of a photon in photosynthesis. A second set of questions relates to how one can best exploit the interaction between non-classically correlated photons and nanostructures. We address each of these briefly.

4.1 QIP in organic and biomolecular nanostructures

Quantum dots embedded in organic structures offer novel physical properties. In particular they allow the formation of exciton states which exhibit large oscillator strengths and strong coupling to the light. These properties yield a large coherence length and high optical nonlinearities\(^\text{[108]}\) which may be exploited for QIP.

It has also been recognized that the coupling mechanisms occurring in QD molecules have the same physical origin as those present in existing natural systems, such as the light-harvesting complexes (LHI and LHII) for which excitonic interactions and energy transfer processes play a central role\(^\text{[109]}\). At nanosecond timescales, the fluorescence resonance energy transfer (FRET) which is associated with the Förster process in light-harvesting complexes and in FRET-coupled dye pairs, exhibits an incoherent dynamics. However, coherent FRET signals might possibly be found at pico- and femtosecond timescales, as discussed earlier for coupled QDs. Due to these basic similarities between inorganic QDs and biomolecular nanostructures, novel ideas for using these latter natural structures to process quantum information are beginning to be explored\(^\text{[91, 111]}\).

From our own perspective, we believe that hybrid bio-nano QIP systems will emerge as an important field of study in the future. This interdisciplinary field will need to combine novel ideas and understanding from the physical, chemical and biological sciences, as well as the biotechnology and nanofabrication industries. Whether fully quantum, mixed quantum-classical, or just classical devices can be built, remains to be seen. However all three prospects are exciting, whether it be classical information processing (IP) or full quantum information processing (QIP) systems which finally emerge.

4.2 Nanostructures and entangled photons

We have mentioned that one can take advantage of the Förster interaction between two quantum dots, which are globally addressed by a laser beam,
in order to generate Bell states of the type of $\alpha|00\rangle + \beta|11\rangle$, where the state $|11\rangle$ indicates the simultaneous presence of two excitons in the double-dot array [16]. This indicates that the formation of such entangled states can be achieved through a two-photon excitation. In fact, it has been experimentally shown that the coherent resonant dipole interaction between molecules can be probed via a two-photon transition in which both molecules are simultaneously excited [113]. An interesting question therefore arises as to what kind of quantum interference phenomena occur when the molecular (or QD) pair is excited by two photons which are entangled. Sources of entangled pairs of photons in the visible spectrum, are now available [107]. In fact sources of three and four entangled photons have very recently been reported [114, 115], which in turn allows one to extend the question beyond two molecules (or QDs) to many-particle systems. This is also an exciting area for future study, both theoretically and experimentally.

4.3 Photon statistics and dynamics of non-classical correlations

In quantum systems with optical outputs, it is expected that experimental signatures of entanglement take the form of non-classical statistical correlations of the emitted light [110]. In matter-light coupled systems, photon statistics has already proved to be a valuable tool for the identification of quantum signatures such as photon-antibunching in the resonance fluorescence of a two-level atom or a single quantum dot [112]. Indeed, second-order photon correlations are expected to exhibit signatures of coherent superpositions in single and double QD nanostructures [32, 33]. In a parallel development, Hanbury-Brown-Twiss experiments have been performed to measure intensity correlations in the nonlinear response of strongly and coherently coupled molecules [113], as well as in the FRET in coupled dye pairs [116]. These experimental achievements suggest that photon statistics could be used to characterize the interactions in coupled quantum dots. However further theoretical studies are required to investigate the precise details of such a characterization.

5 Acknowledgements

We are extremely grateful to Luis Quiroga for useful discussions and critical reading of this manuscript. We acknowledge funding from the Clarendon Fund and ORS (AOC), and the DTI-LINK project ‘Computing at the Quantum Edge’ (NFJ).
References

[1] P. Shor, SIAM J. Comp. 26, 1484 (1997)

[2] D. Bouwmeester, J.-W. Pan, K. Mattle, M. Eibl, H. Weinfurter, and A. Zeilinger, Nature 390, 575 (1997)

[3] D. Boschi, S. Branca, F. De Maritini, L. Hardy, and S. Popescu, Phys. Rev. Lett. 80, 1121 (1998)

[4] S. Somaroo, C.H. Tseng, T. Havel, R. Laflamme, and D.G. Cory, Physical Review Letters 82, 5381 (1999)

[5] Tseng, C.H., S.S. Somaroo, Y.S. Sharf, E. Knill, R. Laflamme, T.F. Havel, and D.G. Cory, Phys. Rev. A 61, 12302 (2000)

[6] G. Vidal, e-print quant-ph/0310089

[7] A. Osterloh, L. Amico, G. Falci, and R. Fazio, Science 416, 608 (2002)

[8] T.J. Osborne and M.A. Nielsen, Phys. Rev. A 66, 032110 (2002)

[9] G. Vidal, J.I. Latorre, E. Rico, and A.Y. Kitaev, Phys. Rev. Lett. 90, 227902 (2003)

[10] D.P. DiVincenzo, Fortschr. Phys. 48, 771 (2000)

[11] C. Monroe, Nature 416, 238 (2002)

[12] F. Schmidt-Kaler, H. Hffner, M. Riebe, S. Gulde, G. P. T. Lancaster, T. Deuschle, C. Becher, C. F. Roos, J. Eschner and R. Blatt, Nature 422, 408 (2003)

[13] S. Gulde, M. Riebe, G. P. T. Lancaster, C. Becher, J. Eschner, H. Hffner, F. Schmidt-Kaler, I. L. Chuang, and R. Blatt, Nature 421, 48 (2003)

[14] D. Loss and D.P. DiVincenzo, Phys. Rev. A 57, 120 (1998)

[15] B.E. Kane, Science 393, 133 (1998)

[16] L. Quiroga and N. F. Johnson, Phys. Rev. Lett. 83, 2270 (1999)

[17] E. Biolatti, R.C. Iotti, P. Zanardi, and F. Rossi, Phys. Rev. Lett. 85, 5647 (2000)

[18] P.M. Platzman and M.I. Dykman, Science 284, 1967 (1999)

[19] Y. Makhlin, G. Schön, and A. Shnirman, Rev. Mod. Phys. 73, 357 (2001)
[20] L. Jacak, P. Hawrylack, and A. Wojs, *Quantum Dots*, Springer, Berlin (1998)

[21] D.Bimberg, M. Grundmann, and N.N. Ledentosow, *Quantum Dot Heterostructures*, John Wiley & Sons, Chichester (1999)

[22] P.Harrison, *Quantum Wells, Wires and Dots*, Wiley, New York (2001)

[23] G. Bukard, D.Loss, and D.P. DiVincenzo, Phys. Rev B 59, 2070 (1999)

[24] J. Shah *Ultrafast spectroscopy of semiconductors and semiconductor structures*, Springer, Berlin (1998)

[25] A.Imamoglu, D.D.Awschalom, G.Bukard, D.P. DiVincenzo, D.Loss, M. Sherwin, and A. Small, Phys. Rev. Lett. 83, 4204 (1999)

[26] A. Kiraz, C. Reese, B. Gayral, L.Zhang, W. V. Schoenfeld, B. D. Gerardot, P. M. Petroff, E. L. Hu and A. Imamoglu, J. Opt. B: Quantum Semiclass. Opt. 5,129 (2003)

[27] See for instance the proceedings of the Discussion Meeting at The Royal Society in November 2002. Phil. Trans, R. Soc. Lond. A 361 July (2003)

[28] D. Bouwmeester and A. Zeilinger, in *The Physics of Quantum Information* edited D. Bouwmeester, A. Ekert and A. Zeilinger, Springer-Verlag, Berlin (2000), p.1.

[29] M.E.Rose *Elementary theory of Angular Momentum*, Jhon Wiley & sons, New York (1967)

[30] M.A. Nielsen and I.L. Chuang *Quantum computation and Quantum information*, Cambridge University Press, United Kingdom (2000).

[31] P. Borri, W. Langebin, S. Schenider, U. Woggon, R.L. Sellin, D. Ouyang, and D. Bimberg, Phys. Rev. Lett. 87, 157401 (2001)

[32] F.J. Rodríguez, L. Quiroga, and N.F. Johnson, Phys. Rev. B 66, 161302(R) (2001).

[33] L. Quiroga, F.J.Rodríguez, and N.F.Johnson, Microelectronics Journal 35, 95 (2004)

[34] L. Allen and J.H. Eberly, *Optical resonance and Two-Level atoms*, Dover, New York (1987)

[35] C. H. Bennett, Phys. Rev. Lett. 68, 3121 (1992)

[36] C. H. Bennett and S. Wiesner, Phys. Rev. Lett. 69, 2881 (1992)
[37] J. M. Raimond, M. Brune, and S. Haroche, Rev. Mod. Phys. 73, 565 (2001)

[38] C. A. Sackett, D. Kielpinski, B. E. King, C. Langer, V. Meyer, C. J. Myatt, M. Rowe, Q. A. Turchette, W. M. Itano, D. J. Wineland and C. Monroe, Nature 404, 256 (2000)

[39] A. J. Berkley, H. Xu, R. C. Ramos, M. A. Gubrud, F. W. Strauch, P. R. Johnson, J. R. Anderson, A. J. Dragt, C. J. Lobb, and F. C. Wellstood, Science 300, 1548 (2003)

[40] T. Yamamoto, Yu. A. Pashkin, O. Astafiev, Y. Nakamura, and J. S. Tsai, Nature 425, 941 (2003)

[41] D. Bruss, J. Math. Phys. 43, 4237 (2002)

[42] W. K Wootters, Quant. Inf. and Comp. 1, 27 (2001)

[43] W. K. Wootters, Phys. Rev. Lett. 80, 2245 (1998)

[44] M. A. Nielsen, C. M. Dawson, J. L. Dodd, A. Gilchrist, D. Mortimer, T. J. Osborne, M. J. Bremner, A. W. Harrow, and A. Hines, Phys. Rev. A 67, 052301 (2003)

[45] A. Barenco, C. Bennett, R. Cleve, D. DiVincenzo, N. Margolus, P. Shor, T. Sleator, J. Smolin, and H. Weinfruther, Phys. Rev. A 52, 346 (1995)

[46] W. Dür, G. Vidal, J. I. Cirac, N. Linden, and S. Popescu, Phys. Rev. Lett. 87, 137901 (2001)

[47] C. H. Bennett, J. I. Cirac, M. S. Leifer, D. W. Leung, N. Linden, S. Popescu, and G. Vidal, Phys. Rev. A. 66, 012305 (2002)

[48] M. J. Bremner, C. M. Dawson, J. L. Dodd, A. Gilchrist, A. W. Harrow, D. Mortimer, M. A. Nielsen, and T. J. Osborne, Phys. Rev. Lett. 89, 247902 (2002)

[49] N. Schuch and J. Siewert, Phys. Rev. A. 67, 032301 (2003)

[50] D. A. Lidar and L.-A. Wu, Phys. Rev. Lett. 88, 017905 (2002)

[51] R. Vrijen, E. Yablonovitch, K. Wang, H. W. Jiang, A. Balandin, V. Roychowdhury, T. Mor, and D. DiVincenzo, Phys. Rev. A 62, 012306 (2000).

[52] D. P. DiVincenzo, D. Bacon, J. Kempe, and K. B. Whaley, Nature 408, 339 (2000)

[53] S. C. Benjamin and S. Bose, Phys. Rev. Lett. 90, 247901 (2003)
[54] X. Wang, Phys. Rev. A. 64, 012313 (2001)

[55] W. Dürr, G. Vidal and J.I. Cirac, Phys. Rev. A. 62, 017901 (2000)

[56] Shi-Biao Zheng and Guang-Can Guo, Phys. Rev. Lett. 85, 2392 (2000)

[57] H. Mabuchi and A.C. Doherty, Science 298, 1372 (2002)

[58] X.X. Yi, X.H. Su, and L. You, Phy. Rev. Lett. 90, 097902 (2003)

[59] A. Olaya-Castro, N.F. Johnson and L. Quiroga, e-print quant-ph/0311181

[60] J.I. Cirac, P. Zoller, H.J. Kimble, and H. Mabuchi, Physical Review Letters 78, 3221 (1997)

[61] A. Olaya-Castro, N.F. Johnson and L. Quiroga, J. Opt. B: Quantum Semiclass. Opt. 6, S1 (2004)

[62] S. Osnaghi, P. Bertet, A. Auffeves, P. Maioli, M. Brune, J. M. Raimond, and S. Haroche, Phys. Rev. Lett. 87, 037902 (2001)

[63] H.-P. Breuer and F. Petruccione The theory of open quantum systems, Oxford University Press, Oxford, (2002)

[64] W.H. Zurek Rev. Mod. Phys. 75, 715 (2003)

[65] D.A. Lidar and K. B. Whaley, in Irreversible quantum dynamics edited F. Benatti and R. Floreanini, Spring Lecture Notes in Physics, Vol. 62, Berlin (2003), p.83.

[66] M. B. Plenio, S. F. Huelga, A. Beige, and P. L. Knight, Phys. Rev. A 59, 2468 (1999)

[67] S. Nicolosi, A. Napoli, A. Messina, and F. Petruccione e-print quant-ph/0402211

[68] A. Beige Phys. Rev. A 67, 020301(R) (2003)

[69] M.B. Plenio and P.L. Knight, Rev. Mod. Phys. 70, 101 (1998)

[70] L.-M Duan and G.-C. Guo.. Phys. Rev. A, 57, 737 (1998)

[71] D.A. Lidar, I.L. Chuang and K.B. Whaley. Phys. Rev. Lett., 81, 2594 (1998)

[72] L. Viola, Phys. Rev. A 66, 012307 (2002)

[73] K. Shiokawa, and D. A. Lidar, Phys. Rev. A 69, 030302(R) (2004)

[74] P. Zanardi, Phys. Rev. A 63, 012301 (2001)
[75] J. Kempe, D. Bacon, D.A. Lidar, and K.B. Whaley, Phys. Rev. A, 63, 042307 (2001)

[76] S. Daffer, K. Wodekiewicz, J.D. Cresser, and J.K. McIver, e-print quant-ph/0309081

[77] H.-P. Breuer e-print quant-ph/0403117

[78] A. Shabani and D.A. Lidar eprint quant-ph/0404077

[79] C.F. Lee, N.F. Jonson, F.J. Rodríguez, and L. Quiroga, special Issue of Fluctuation and Noise Letters 2, L305 (2002)

[80] N. H. Bonadeo, J. Erland, D. Gammon, D. Park, D. S. Katzer, and D. G. Steel, Science 282, 1473 (1998)

[81] P. Borri, W. Langbein, J.M. Mork, J.M. Hwam, F. Heinrichsdorf, M.-H. Mao, and D. Bimberg, Phys. Rev. B 60, 7784 (1999)

[82] B. Krummheuer, V.M. Axt, and T. Khun, Phys. Rev. B 65, 195313 (2002)

[83] G. Chen, N. H. Bonadeo, D. G. Steel, D. Gammon, D. S. Katzer, D. Park, and L. J. Sham, Science 289, 1906 (2000)

[84] T. H. Stievater, Xiaoqin Li, D. G. Steel, D. Gammon, D. S. Katzer, D. Park, C. Piermarocchi, and L. J. Sham, Phys. Rev. Lett. 87, 133603 (2001).

[85] G. Chen, T. H. Stievater, E. T. Batteh, Xiaoqin Li, D. G. Steel, D. Gammon, D. S. Katzer, D. Park, and L. J. Sham, Phys. Rev. Lett. 88, 117901 (2002)

[86] Xiaoqin Li, Yanwen Wu, D. Steel, D. Gammon, T. H. Stievater, D. S. Katzer, D. Park, C. Piermarocchi, and L. J. Sham, Science 301, 809 (2003)

[87] A. Imamoglu, Fortsch. Phys. 48, 987 (2000).

[88] M. Feng, I. D’Amico, P. Zanardi, and F. Rossi, Phys. Rev. A 67, 014306 (2003)

[89] J.H. Reina, L. Quiroga and N.F. Johnson, Phys. Rev. A. 62, 012305 (2000)

[90] S. De Rinaldis, I. D’Amico, E. Biolatti, R. Rinaldi, R. Cingolani, and F. Rossi, Phys. Rev. B 65, 081309(R) (2002).

[91] B. W. Lovett, J.H. Reina, A. Nazir, and G.A.D. Briggs, Phys. Rev. B 68, 205319 (2003)
[92] B. W. Lovett, J.H. Reina, A. Nazir, B. Kothari, and G.A.D. Briggs, Phys. Lett. A B. 315, 136 (2003)
[93] J.M. Kikkawa and D.D. Awschalom, Phys. Rev. Lett. 80 4313 (1998)
[94] C. Piermarocchi, P. Chen, and L.J. Sham, Phys. Rev. Lett. 89, 167402 (2002)
[95] E. Pazy, E. Biolatti, T. Calarco, I. D’Amico, P. Zanardi, F. Rossi, and P. Zoller, Europhys. Lett. 62, 175 (2003).
[96] T. Calarco, a. Datta, P. Fedichev, E. Pazy, and P. Zoller, Phys. Rev. A 68, 012310 (2003)
[97] A. Nazir, B.W. Lovett, S.D. Barrett, T.P. Spiller, and G.A.D. Briggs, e-print quant-ph/0403225
[98] P. Zhang, C.K. Chan, Q.-K. Xue, and X.-G. Zhao Phys.Rev. A. 67, 012312 (2003).
[99] E. Biolatti, I. D’Amico, P. Zanardi, and F. Rossi, Phys. REv. B 65 075306 (2002).
[100] S. Sangu, K. Kobayashi, A. Shojiguchi, and M. Ohtsu, Phys. Rev. B 69, 115334 (2004).
[101] J. A. Gupta, R. Knobel, N. Samarth, D. D. Awschalom, Science 292, 2458 (2001)
[102] C. A. R.S´ a de Melo, Phys. Rev. B. 51, 8922 (1995)
[103] T. Pellizari, S.A.Gardiner, J.I. Cirac, and P. Zoller, Phys. Rev. Lett 75, 3788 (1995)
[104] A. V. Khaetskii, D. Loss, and L. Glazman, Phys. Rev. Lett. 88, 186802 (2002)
[105] A. Imamoglu, E. Knill, L.Tian, P. Zoller, Phys. Rev. Lett. 91, 017402 (2003)
[106] A. Shabaev, Al. L. Efros, D.Gammon, and I.A. Merkulov, Phys.Rev. B. 68, 201305 (2004)
[107] J. Volz, C. Kurtsiefer, and H. Weinfurther, Appl. Phys. Lett. 79, 869 (2001)
[108] N.Q. Huong and J.L. Birman, Phys. Rev. B 61, 13131 (2000)
[109] X.Hu, T. Ritz, A. Damjanovic, F.Auterieth, and K. Shulten, Q. Rev. Biophys. 35, 1 (2002)
[110] A. Olaya-Castro, F. J. Rodriguez, L. Quiroga, and C. Tejedor, Phys. Rev. Lett. 87, 246403 (2001)

[111] S. K. Sekatskii, M. Chergui, and G. Dietler, Europhys. Lett. 63, 21 (2003)

[112] P. Michler, A. Imamoglu, M. D. Mason, P. J. Carson, G. F. Strouse, and S. L. Buratto, Nature 406, 968 (2000)

[113] C. Hettich, C. Schmitt, J. Zitzmann, S. Kühn, I. Gerhardt, V. Sandoghdar, Science 298, 385 (2002)

[114] P. Walther, J.-W. Pan, M. Aspelmeyer, R. Ursin, S. Gasparoni, A. Zeilinger, Nature 429, 158 (2004)

[115] M. W. Mitchell, J. S. Lundeen, and A. M. Steinberg, Nature 429, 161 (2004)

[116] A. J. Berglund, A. C. Doherty, and H. Mabuchi, Phys. Rev. Lett. 89, 068101 (2002)
Figure 3.1: Schematic diagram of the experimental setup used for ultrafast coherent control of an exciton in a dot in Ref. [80]. Wave interferometry is achieved by using two phase-locked optical pulses $\varepsilon_1$ and $\varepsilon_2$ delayed a time $\tau = t_f + t_c$. The pulses interact with the QD at times $t = 0$ and $t = \tau$ creating a coherent superposition of the dipole excited state.

Figure 3.2: Left panel: Shematic diagrams of quantum dots. An island formed due to fluctuations in a quantum well width (left top) and self-assembled quantum dots grown by the Stranski-Krastanow process (left bottom). The base diameter $d$ and the height $h$ are in the range of $20 - 40\, nm$ and $3 - 6\, nm$ respectively, depending on the growing conditions. These latter dots offer high potential for scalable architectures for QIP. Right panel: Relevant energy levels of a $III-V$ semiconductor quantum dot. Strong confinement is assumed to be in the $z$ direction. Here $\sigma^\pm$ indicates right(left)-hand circularly polarized light.
Figure 3.3: Logical states of an excitonic qubit. $|0\rangle$ denotes vacuum of excitations and $|1\rangle$ refers to the presence of a ground-state exciton made up of a hole with angular momentum $+3/2$ and an electron with angular momentum $1/2$.

$$|0\rangle = |\text{vac}\rangle \quad |1\rangle = e_{-1/2}^+ h_{+3/2}^+ |\text{vac}\rangle$$

Figure 3.4: Schematic illustration of interdot interactions. Resonant energy transfer (Förster) process thereby an exciton is destroyed in one quantum dot and re-created on the other quantum dot via exchange of a virtual photon (left side). When each dot contains an exciton, direct electrostatic Coulomb interaction between quantum dots takes place. This lead to individual energy shifts which in presence of a static electric field in the $xy$ plane become significant, and can be exploited to generate an energy selective two-qubit gate (right side).

$$|0\rangle = e_{-1/2}^+ |\text{vac}\rangle \quad |1\rangle = e_{+1/2}^+ |\text{vac}\rangle$$

Figure 3.5: Logical states associated to the spin of a single electron confined in a QD.
Figure 3.6: Schematic illustration of the effective optical RKKY exchange interaction between electrons in separated quantum dots via an intermediate continuum of exciton states produced by an off-resonance optical excitation. Here $\delta$ is the detuning between the laser frequency and the band gap frequency in the host material, $R$ is the distance between confined electrons.

Figure 3.7: Optically controlled Pauli-blocking mechanism. If the spin qubit is in $|0\rangle$ a creation of an exciton with electron angular momentum $-1/2$ is inhibited (left side). If the qubit is in $|1\rangle$, then nothing forbids the creation of such exciton. If the qubit is in a coherent superposition of its logical states, then it is transformed into a charge superposition: $|0\rangle + |1\rangle \leftrightarrow |0\rangle + |X^-\rangle$. 
Figure 3.8: Bi-excitonic shift. Interaction between QD spins is controlled by the electrostatic Coulomb interaction between optically created trion states in dots $|x_{a}^−\rangle$ and $|x_{b}^−\rangle$. In presence of a static electric field the trion states acquire permanent dipoles while the Coulomb interaction induces a significant energy shift $\Delta E_{ab}$ in exciton states. In combination with the Pauli-exclusion mechanism, this energy shift can be exploited to perform a quantum phase gate.

Figure 3.9: Scanning electron micrograph of the GaAs microdisk nanostructure reproduced from Ref.[26] with permission of the Institute of Physics Publishing ©2003. The diameter of the cavity is 4.5$\mu$m with a corresponding cavity-mode volume of $V_{cav} \simeq 200(\lambda/2n)^3$. The highest value of Q measured in this structure exceeded 1800. The cavity contains InAs QDs at locations fixed during the growth. The density of QDs is $2 \times 10^6 \text{cm}^{-2}$.  

35