Non-universal gauge bosons $Z'$ and lepton flavor-violation tau decays

Chongxing Yue$^{(a,b)}$, Yanming Zhang$^b$, Lanjun Liu$^b$

a: CCAST (World Laboratory) P.O. BOX 8730. B.J. 100080 P.R. China

b: College of Physics and Information Engineering, Henan Normal University, Xinxiang 453002. P.R.China *

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Abstract

There are many models beyond the standard model predicting the existence of non-universal gauge bosons $Z'$, which can give rise to very rich phenomena. We calculate the contributions of the non-universal gauge bosons $Z'$, predicted by topcolor-assisted technicolor (TC2) models and flavor-universal TC2 models, to the lepton flavor-violation tau decays $\tau \rightarrow l_l l_j l_k$ and $\tau \rightarrow l_l \gamma$. We find that the branching ratio $B_r(\tau \rightarrow l_l l_j l_k)$ is larger than that of the process $\tau \rightarrow l_l \gamma$ in all of the parameter space. Over a sizable region of the parameter space, we have $B_r(\tau \rightarrow l_l l_j l_k) \sim 10^{-8}$, which may be detected in the future experiments.

*E-mail:cxyue@public.xxptt.ha.cn*
Although the standard model (SM) has been successful in describing the physics of electroweak interactions, it is quite possible that the SM is only an effective theory valid below some high energy scale. Extra gauge bosons $Z'$ are the best motivated extensions of the SM. If discovered they would represent irrefutable proof of new physics, most likely that the SM gauge groups must be extended [1]. If these extensions are associated with flavor symmetry breaking, the gauge interactions will not be flavor-universal [2], which predict the existence of non-universal gauge bosons $Z'$. Furthermore, the possible anomalies in the $Z$ pole $b\bar{b}$ asymmetries may suggest a non-universal $Z'$ [3].

After the mass diagonalization from the flavor eigenbasis into the mass eigenbasis, the non-universal gauge interactions result in the tree level flavor-changing neutral currents (FCNC’s) couplings. Thus, the non-universal gauge bosons $Z'$ may have significant contributions to some FCNC processes. For example, the non-universal gauge bosons $Z'$ predicted by strong top dynamical models, such as topcolor-assisted technicolor (TC2) models [4] and flavor-universal TC2 models [5], can give significant contributions to some FCNC processes [6, 7, 8]. If these effects are indeed detected at LHC, LC or other experiments, it will be helpful to identify the gauge bosons $Z'$, and hence unravel underlying theory [9].

The high statistic results of the SuperKamiokande (SK) atmospheric neutrino experiment [10] and the solar neutrino experiment [11] have made one believe that neutrinos are massive and oscillate in flavor and are of interest in the lepton flavor violation (LFV) processes, such as $l_i \rightarrow l_j \gamma$ and $l_i \rightarrow l_j \gamma l_i$. These LFV processes are practically suppressed to zero in the SM, due to the unitarity of the leptonic analog of CKM mixing matrix and the near masslessness of the three neutrinos. Thus, the observation of any rate for one of these processes would be a signal of new physics. The improvement of their experimental measurements forces one to make more elaborate theoretical calculation in the framework of some specific models beyond the SM and see whether the LFV effects can be tested in the future experiments. For instance, the LFV tau decays, such as $\tau \rightarrow \mu \gamma$, $\tau \rightarrow e \gamma$, $\tau \rightarrow 3\mu$ and $\tau \rightarrow 3e$, have been studied in a model independent way in Ref.[12], in the SM with extended right-handed and left-handed neutrino sectors [13], in supersymmetric
models\cite{14}, in the two Higgs doublets model type III \cite{15} and in Zee model\cite{16}.

In this letter, we focus our attention on the LFV decays of tau lepton. Now the bounds on the LFV tau decay modes are being improved by the Belle experiment at KEK. The new bounds are \cite{17}:

\[
\begin{align*}
B_r(\tau \rightarrow \mu \gamma) &< 5 \times 10^{-7}, \\
B_r(\tau \rightarrow 3\mu) &< 8.7 \times 10^{-7}, \\
B_r(\tau \rightarrow 2\mu e) &< 7.7 \times 10^{-7}, \\
B_r(\tau \rightarrow 2e\mu) &< 3.4 \times 10^{-7}, \\
B_r(\tau \rightarrow 3e) &< 7.8 \times 10^{-7}.
\end{align*}
\]

We will calculate the contributions of the non-universal gauge bosons $Z'$, predicted by TC2 models and flavor-universal TC2 models, to the branching ratios $B_r(\tau \rightarrow l_i \gamma)$ and $B_r(\tau \rightarrow l_i l_j l_k)$. It will be shown that the branching ratios of the processes $\tau \rightarrow l_i l_j l_k$, which occur at the tree level are much larger than those of the processes $\tau \rightarrow l_i \gamma$, which arise from $\gamma$-penguins with the photon on the mass shell. With reasonable values of the parameters, the values of $B_r(\tau \rightarrow l_i l_j l_k)$ can reach $4.1 \times 10^{-8}$, which may be tested by the near future experiments. For the branching ratio $B_r(\tau \rightarrow l_i \gamma)$, it is possible to reach $2.6 \times 10^{-9}$.

To completely avoid the problems, such as triviality and unnaturalness arising from the elementary Higgs in the SM, various kinds of dynamical models are proposed and among which TC2 models are very interesting because they explain the large top quark mass and provide possible dynamics of electroweak symmetry breaking (EWSB)\cite{1}. A common feature of these models is that the SM gauge groups are extended at energy well above the weak scale. Breaking of the extended gauge groups to their diagonal subgroups produces non-universal massive gauge bosons. For instance, TC2 models and flavor-universal TC2 models all predict the existence of the non-universal gauge bosons $Z'$. These new particles treat the third generation fermions differently from those in the first and second generations. Thus, they can lead to FC couplings.
The flavor-diagonal couplings of $Z'$ to leptons can be written as [4, 6]:

\[ L^F D_{Z'} = - \frac{1}{2} g_1 \cot \theta' Z'_{\mu} (\bar{\tau}_L \gamma^\mu \tau_L + 2 \bar{\tau}_R \gamma^\mu \tau_R) \]

\[ - \frac{1}{2} g_1 \tan \theta' Z'_{\mu} (\bar{\mu}_L \gamma^\mu \mu_L + 2 \bar{\mu}_R \gamma^\mu \mu_R + \bar{\epsilon}_L \gamma^\mu \epsilon_L + 2 \bar{\epsilon}_R \gamma^\mu \epsilon_R), \quad (1) \]

where $g_1$ is the ordinary hypercharge gauge coupling constant, $\theta'$ is the mixing angle with $\tan \theta' = \frac{g_1}{\sqrt{4\pi k_1}}$. To obtain the top quark condensation and not form a $b\bar{b}$ condensation, there must be $\tan \theta' << 1$ [4, 5]. The flavor changing couplings of $Z'$ to leptons can be written as:

\[ L^F C_{Z'} = - \frac{1}{2} g_1 Z'_{\mu} [k_{\tau\mu}(\bar{\tau}_L \gamma^\mu \mu_L + 2 \bar{\tau}_R \gamma^\mu \mu_R) + k_{\tau e}(\bar{\tau}_L \gamma^\mu \epsilon_L + 2 \bar{\tau}_R \gamma^\mu \epsilon_R) + k_{\mu e} \tan^2 \theta(\bar{\mu}_L \gamma^\mu \epsilon_L + 2 \bar{\mu}_R \gamma^\mu \epsilon_R)], \quad (2) \]

where $k_{ij}$ are the flavor mixing factors. In the following estimation, we will take $k_{\tau\mu} = k_{\tau e} = k_{\mu e} = k = \lambda [3]$, where $\lambda = 0.22$ is the Wolfenstein parameter [18].

From Eq.(1) and Eq.(2), one can see that the LFV tau decays can be generated via exchange of $Z'$ in the TC2 models and flavor-universal TC2 models. We first consider the LFV decay processes $\tau \rightarrow \mu \gamma$ and $\tau \rightarrow e \gamma$. These processes are generated by the on-shell photon penguin diagrams Fig.1(a). The internal fermion line may be $\tau$, $\mu$ or $e$. However, the two internal $\tau$ propagators provide a term proportional to $m_\tau^2$ in the numerator, which is not cancelled by the $m_\tau^2$ in the denominator since the heavy $Z'$ boson mass dominate the denominator. Thus, we ignore the contributions of the LFV couplings $Z'\mu e$, $Z'\mu \mu$ and $Z'ee$ and only consider those of the LEV couplings $Z'\tau \tau$, $Z'\tau \mu$ and $Z'\tau e$. After straightforward calculation, we can derive the widths of $\tau \rightarrow \mu \gamma$ and $\tau \rightarrow e \gamma$, which are [19]:

\[ \Gamma(\tau \rightarrow \mu \gamma) = \Gamma(\tau \rightarrow e \gamma) = \frac{\alpha^2 k_1}{1152\pi^2 C_W^2 M_Z^2} \frac{m_\tau^5}{k^2}, \quad (3) \]

where $C_W^2 = \cos^2 \theta_W$, $\theta_W$ is the Weinberg angle and $M_Z$ is the mass of the non-universal gauge bosons $Z'$. The non-universal gauge bosons $Z'$ can contribute to the processes $\tau \rightarrow l_i l_j l_k$ via the Feynman diagrams as depicted in Fig1.(b) and Fig1.(c). The contributions of the off-shell photon penguin and $Z$ penguin to these processes are much smaller than
those of the $Z'$ exchange at tree level. Thus we ignore the contributions of Fig.2(b) to

$\tau \rightarrow l_il_jl_k$ in our calculation. The partial widths can be written as [20]:

$$\Gamma(\tau \rightarrow 3\mu) = \Gamma(\tau \rightarrow 3e) = \Gamma(\tau \rightarrow ee\mu) = \Gamma(\tau \rightarrow 3\mu) = 25\frac{\alpha^3}{384\pi k_1 C_W^6 M_Z^4} k^2$$

The width of $\tau \rightarrow \mu\gamma$ is equal to that of $\tau \rightarrow e\gamma$ and the widths of the process $\tau \rightarrow l_il_jl_k$ are equal each other. This is because the non-universal gauge bosons $Z'$ only treat the fermions in the third generation differently from those in the first and second generation and treat the fermions in the first generation same as those in the second generation. The $e\bar{\nu}_e\nu_\tau$ is one of the dominant decay modes of the lepton $\tau$. The branching ratio $B_r(\tau \rightarrow e\bar{\nu}_e\nu_\tau)$ has been precisely measured, i.e. $B_r(\tau \rightarrow e\bar{\nu}_e\nu_\tau) = (17.83 \pm 0.06)\%$ [21]. Thus, we can use $B_r(\tau \rightarrow e\bar{\nu}_e\nu_\tau)$ to represent the branching ratios of the LFV tau decays:

$$B_r(\tau \rightarrow l_i\gamma) = B_r(\tau \rightarrow e\bar{\nu}_e\nu_\tau) \frac{\Gamma(\tau \rightarrow l_i\gamma)}{\Gamma(\tau \rightarrow e\bar{\nu}_e\nu_\tau)},$$

$$B_r(\tau \rightarrow l_il_jl_k) = B_r(\tau \rightarrow e\bar{\nu}_e\nu_\tau) \frac{\Gamma(\tau \rightarrow l_il_jl_k)}{\Gamma(\tau \rightarrow e\bar{\nu}_e\nu_\tau)},$$

with

$$\Gamma(\tau \rightarrow e\bar{\nu}_e\nu_\tau) = \frac{m^5 G_F^2}{192\pi^3}.$$  

Here the Fermi coupling constant $G_F = 1.16639 \times 10^{-5} GeV^{-2}$ [21].

It has been shown that vacuum tilting and the constraints from $Z$-pole physics and

$U(1)$ triviality require $k_1 \leq 1$ [3]. The limits on the mass of $Z'$ can be obtained via studying

its effects on various experimental observables [3]. For example, Ref. [2] has shown that
to fit the electroweak measurement data, the $Z'$ mass must be larger than 1 TeV. As
numerical estimation, we take the $Z'$ mass $M_Z$ and $k_1$ as free parameters.

In Fig.2 and Fig.3 we plot the branching ratios $B_r(\tau \rightarrow l_i\gamma)$ and $B_r(\tau \rightarrow l_il_jl_k)$ as functions of $M_Z$ for three values of the parameter $k_1$: $k_1 = 0.2, 0.5$ and 1, respectively.

From Fig.2 and Fig.3 we can see that the branching ratio $B_r(\tau \rightarrow l_il_jl_k)$ is larger than

that of the process $\tau \rightarrow l_i\gamma$ in all of the parameter space. The values of $B_r(\tau \rightarrow l_i\gamma)$

increase with $k_1$ increasing, while the values of $B_r(\tau \rightarrow l_il_jl_k)$ decrease with $k_1$ increasing.
For $k_1 = 1$, $M_Z = 1 TeV$, the value of $B_r(\tau \rightarrow l_1 l_j l_k)$ can reach maximum value i.e. $B_r(\tau \rightarrow l_1 l_j l_k) = 4.1 \times 10^{-8}$. For $B_r(\tau \rightarrow l_i \gamma)$, the maximum value is $2.6 \times 10^{-9}$.

The existence of non-universal gauge bosons $Z'$ is a key feature of all TC2 models. The new gauge bosons can lead to the flavor-changing couplings, and hence may have significant contributions to some FCNC processes. In this letter, we calculated the contributions of non-universal gauge bosons $Z'$ predicted by TC2 models and flavor-universal TC2 models to the LFV $\tau$ decays. We found that, in most of parameter space, the branching ratio $B_r(\tau \rightarrow l_i \gamma)$ is smaller than $2.6 \times 10^{-9}$, which is very difficult to be tested. However, the branching ratio $B_r(\tau \rightarrow l_j l_l l_k)$ can be larger over a wide range of parameter space, $B_r(\tau \rightarrow l_j l_l l_k) \sim 10^{-8}$. Thus, the effects of non-universal gauge bosons $Z'$ on the tau decays $\tau \rightarrow l_j l_l l_k$ may be detected in the near future experiments.

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Figure captions

Fig.1: Feynman diagrams for lepton flavor-violation $\tau$ decays $\tau \to l_i \gamma$ and $\tau \to l_i l_j l_k$ induced by non-universal gauge bosons $Z'$ exchange.

Fig.2: Branching ratios for $\tau \to l_i \gamma$ as functions of the gauge bosons $Z'$ mass $M_Z$ for $k_1 = 0.2$ (solid line), 0.5 (dashed line) and 1 (dotted line).

Fig.3: Branching ratios of $\tau \to l_i l_j l_k$ as functions of the gauge bosons $Z'$ mass $M_Z$ for $k_1 = 0.2$ (solid line), 0.5 (dashed line) and 1 (dotted line).
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Fig. 1
