Comment on the Nature of the $D_{s1}^*(2710)$ and $D_{sJ}^*(2860)$ Mesons

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Two charm-strange mesons, the $D_{s1}^*(2710)$ and the $D_{sJ}^*(2860)$, have recently been observed by several experiments. There has been speculation in the literature that the $D_{s1}^*(2710)$ is the $2^3S_1 (cs)$ state and the $D_{sJ}^*(2860)$ is the $1^3D_3 (cs)$ state. In this paper we explore this and other explanations in the context of the relativized quark model and the pseudoscalar emission decay model. We conclude that the $D_{s1}^*(2710)$ is most likely the $1^3D_1 (cs)$ state and the $D_{sJ}^*(2860)$ is most likely the $1^3D_3 (cs)$ state with the $1D_2$ resonances also contributing to the observed signals and explaining the observed ratios of branching ratios to $D^* K$ and $DK$ final states. We point out that measuring the $D_{sJ}^*(2860)$ spin can support or eliminate this explanation and that there are six excited $D_s$ states in this mass region; the $2^3S_1$, $2^3S_0$, $1^3D_1$, $1^3D_3$, $1^1P_1$ and two $1D_2$ states. Observing some of the missing states would help confirm the nature of the $D_{s1}^*(2710)$ and the $D_{sJ}^*(2860)$ states.

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I. INTRODUCTION

Heavy-light mesons provide a unique window into heavy quark dynamics and therefore provide an important test of our understanding of quantum chromodynamics in the non-perturbative regime [1–3]. In recent years three new excited charm-strange mesons have been observed for the first time which can test calculations and help improve our understanding of hadron spectroscopy. They are the $D_{s1}^*(2700)$, $D_{sJ}^*(2860)$, and $D_{sJ}^*(3040)$, [4–8]. We will focus on the first two states which have been observed by multiple experiments. The Particle Data Group averages for the masses, decay widths and ratios of branching fractions for the $D_{s1}^*(2700)^\pm$ and $D_{sJ}^*(2860)^\pm$ are [8]:

$$M(D_{s1}^*(2710)^\pm) = 2709 \pm 4 \text{ MeV} \quad (1)$$

$$\Gamma(D_{s1}^*(2710)^\pm) = 117 \pm 13 \text{ MeV} \quad (2)$$

$$\frac{\Gamma(D_{s1}^* \rightarrow D^* K)}{\Gamma(D_{s1}^* \rightarrow DK)} = 0.91 \pm 0.13(\text{stat}) \pm 0.12(\text{syst}) \quad (3)$$

and

$$M(D_{sJ}^*(2860)^\pm) = 2863^{+4.0}_{-2.6} \text{ MeV} \quad (4)$$

$$\Gamma(D_{sJ}^*(2860)^\pm) = 58 \pm 11 \text{ MeV} \quad (5)$$

$$\frac{\Gamma(D_{sJ}^* \rightarrow D^* K)}{\Gamma(D_{sJ}^* \rightarrow DK)} = 1.10 \pm 0.15(\text{stat}) \pm 0.19(\text{syst}) \quad (6)$$

Both states are observed decaying into both $DK$ and $D^*K$ so have natural parity $J^P = 1^-$, $2^+$, $3^-$, …… In this paper we compare the observed properties of these states to the mass predictions of the relativized quark model and decay predictions of a pseudoscalar emission model [2] to determine their spectroscopic assignments. This leads to further predictions that can test these assignments and help fill gaps in excited $D_s$ multiplets.

The $D_{s1}^*(2710)^\pm$ and $D_{sJ}^*(2860)^\pm$ mesons have been studied in the context of various models [10–27]. The $D_{s1}^*(2710)^\pm$ has been identified with the first radial excitation of the $D_{s1}^*(2112)^\pm$ or the $D_{s1}^*(1^3D_1)$ or some mixture of them [10–21] and the $D_{sJ}^*(2860)^\pm$ as the $D_s(2^3P_0)$ [10–23], the $D_{sJ}^*(1^3D_1)$ or the $D_{sJ}^*(1^3D_3)$ [11–18, 22]. The $D_{sJ}^*(2^3P_0)$ does not appear to be a viable explanation because the $D_{sJ}^*(2860)^\pm$ has been observed in both $DK$ and $D^*K$ final states, although van Beveren and Rupp [24] argue that the signal could be the result of two overlapping resonances. The theoretical predictions for these states are not completely consistent with their observed properties so it is useful to further test calculations against the experimental measurements. In the past we have found that the pseudoscalar emission model [2] for OZI allowed hadronic decays provides a useful consistency check for other models such as the $^3P_0$ strong decay model [28–30]. We therefore calculate the strong decay widths of excited charm-strange mesons using the pseudoscalar emission model in this spirit, that it is useful to compare the predictions of different models while acknowledging that some other models can be applied to a broader range of decays than the pseudoscalar emission model.

In this paper we study the decay widths of excited $D_s$ mesons and compare the predicted and measured properties of the $D_{s1}^*(2710)^\pm$ and $D_{sJ}^*(2860)^\pm$ states. In the following section we begin by giving the relativized quark model mass predictions for the charm-strange mesons [3]. We then give the partial decay widths for the $S$ and $1D$ charm-strange mesons calculated using the pseudoscalar emission model. In section III we use these results
FIG. 1: The mass spectrum. The mixed $L_{J^P}=L$ states are described by eqn. 7 with the primed states in the figure corresponding to the primed mixed state in eqn. 7 and the are described by eqn. 7 with the primed states in the figure.

In this chapter, we discuss the possible spectroscopic assignments of the $D_s^*(2710)^\pm$ and $D_{sJ}^*(2860)^{\pm}$ states. We summarize our conclusions in section IV.

II. 2S AND 1D $D_s$ PROPERTIES

A. Spectroscopy

We start with the mass predictions for the charm-strange mesons of the relativized quark model. The details of this model can be found in Ref. [3] and [31] and we do not repeat them here. This model has ingredients common to many quark potential models [20, 35–38]. Almost all such models are based on some variant of the Coulomb plus linear potential expected from QCD and relativistic effects are often included at some level. The relativized quark model has been reasonably successful in describing most known mesons. However in recent years, starting with the discovery of the $D_{sJ}(2317)$ [39] and $X(3872)$ states [42], an increasing number of states have been observed that do not fit into this picture [43, 44] pointing to the need to include physics which has hitherto been neglected such as coupled channel effects [47]. As a consequence of neglecting coupled channel effects and the crudeness of the relativization procedure we do not expect the mass predictions to be accurate to better than $\sim 10–20$ MeV. The mass predictions for this model are shown in Fig. 1. Lattice gauge theory predictions [48] for the excited $1^-$ and $3^-$ charm-strange mesons are given in Table 1 for comparison. See also Ref. [49] and [50].

For the case of a quark and antiquark of unequal mass, charge conjugation parity is no longer a good quantum number so that states with different total spins but with the same total angular momentum, such as $^3P_1$ and $^3S_1$ to $^3D_2$ and $^1D_2$ pairs, can mix via the spin orbit interaction or some other mechanism such as mixing via coupled channels. Consequently, the physical $J = 2$ $D$-wave states are linear combinations of $^3D_2$ and $^1D_2$ which we describe by:

$$
D_2 = ^3D_2 \cos \theta_{nD} + ^3D_2 \sin \theta_{nD}
$$

$$
D'_2 = ^1D_2 \sin \theta_{nD} + ^3D_2 \cos \theta_{nD}
$$

(7)

where $D \equiv L = 2$ designates the relative angular momentum of the $q\bar{q}$ pair and the subscript $J = 2$ is the total angular momentum of the $q\bar{q}$ pair which is equal to $L$ with analogous expressions for other values of $L$. We obtain $\theta_{nD} = \pm 38.5^\circ$ (for $c\bar{s}$). This notation implicitly implies $L-S$ coupling between the quark spins and the relative orbital angular momentum. In the heavy quark limit (HQL) in which the heavy quark mass $m_Q \to \infty$, the states can be described by the total angular momentum of the light quark, $j_q$, which couples to the spin of the heavy quark and corresponds to $j-j$ coupling. In this limit the state that is mainly spin singlet has $j_q = l + \frac{1}{2}$ while the state that is mainly spin triplet has $j_q = l - \frac{1}{2}$ and is labelled with a prime [51]. For $L = 2$ the HQL gives rise to two doublet states with $j_q = 3/2$ and $j_q = 5/2$ with $\theta_{nD} = -\tan^{-1}(\sqrt{2/3}) = -39.2^\circ$ where the minus sign arises from our $c\bar{s}$ convention [12, 14, 15, 21, 22, 24, 51, 52]. In some approaches it is more natural to use the $j-j$ basis but because we solve our Hamiltonian equations assuming $L-S$ eigenstates and then include the LS mixing we use the notation of eqn. 7. It is straightforward to transform between the $L-S$ basis and the $j-j$ basis [51]. We also note that the definition of the mixing angles is fraught with ambiguities and one should be extremely careful comparing predictions from different publications [54].

B. Strong Decays

We calculate decay widths using the pseudoscalar emission model (see Ref. [3] and references therein). There are a number of predictions for $D_s$ decay widths in the literature using the $3P_0$ model [10, 11, 16, 18, 53] and other models [13, 15, 21, 22, 52]. A weakness of the pseudoscalar emission model is that it can only calculate partial widths that include light (non-charm) pseudoscalar mesons in the final state. For a few states that we consider, decays to light vector mesons are kinematically allowed. We expect the partial widths to a charm plus light vector to be small based on limited phase space. While this is generally supported by other calculations, in a few cases the BR to these final states was found to be as large as $\sim 20\%$. The $3P_0$ model is more general and can therefore also be used in these cases. Nevertheless we have found that the pseudoscalar emission model provides a useful check of those results [28, 50]. In this spirit we calculate strong decays of excited $D_s$ mesons using the pseudoscalar emission model as another approach to understand the nature of the recently observed
In this model the decay is assumed to proceed through a single-quark transition. While the details of this model are given elsewhere, for completeness, we give the amplitude \[ A(q̅,q̅) \left( M^*(K̅,s) \rightarrow M(K̅',s)P^i(q̅') \right) = \pm i \frac{\sqrt{2c_2c_3s}}{(2\pi)^3/2} \langle M(s') \rangle \left\{ \left( g + \frac{h}{2} \frac{m_{q̅}(q̅)-m_q(q)}{m_{q̅}+m_q} \right) \tilde{a}_{q̅}(q̅') \cdot \tilde{q} \pm h \tilde{a}_{q̅}(q̅') \cdot \tilde{p} \right\} e^{\pm i \frac{m_{q̅}(q̅)+m_q}{m_{q̅}+m_q} \tilde{r} \cdot \tilde{r} X^i_q(q̅) M^*(s)} \right\} \right(8)\]

where \( \tilde{a}_{q̅}/2 \) and \( \tilde{r}_{q̅} \) are the spin and position of the quark (antiquark), \( \tilde{p}' = -i \tilde{\nabla} \) is the momentum operator acting on the final-state wave function and the upper (lower) sign refers to the \( q̅(q̅) \) case. The \( X^i_q(q̅) \) are flavour operators. The calculations are most readily performed by taking \( \tilde{q} = q̅ \bar{q} \) thereby calculating helicity amplitudes \( H_m \) where \( m = s' = s \) and then transforming to the usual partial-wave basis. The details of this approach are given in Ref. \[9\].

In our calculations we use harmonic oscillator wave functions with the oscillator parameter obtained by fitting the rms radius of the harmonic oscillator wavefunction to the rms radius of the full wavefunction. Because the harmonic oscillator parameter, \( \beta \), enters the decay expressions in combinations, we averaged the initial and final state \( \beta \)'s to obtain the effective \( \bar{\beta}_{c,\bar{c}} \) used in our numerical results. This allows us to calculate the amplitudes analytically, revealing relations between them that lets us classify the resulting amplitudes into two classes: “structure independent” amplitudes which have only momentum dependence dictated by angular momentum considerations along with the elastic form factor \( F(q^2) \) defined below, and “structure dependent” amplitudes which have additional polynomial momentum dependences which are sensitive to the structure of the states.

The resulting partial wave amplitudes are given in Table II with the following definitions for the amplitudes appearing in the table. The following form factor is factored out and needs to be included to obtain numerical values:

\[ F(q^2) = \left( \frac{q}{2\pi} \right)^{1/2} \exp \left[ -\frac{1}{4} \left( \frac{m_c}{m_c+m_s} \right) \frac{q^2}{\beta_{c,\bar{c}}^2} \right] \]

The amplitudes in Table II are the “structure independent” amplitude

\[ A_{c,\bar{c}} = \left( g + \frac{h}{2} \frac{m_c}{m_c+m_s} \right) \beta_{c,\bar{c}} \]

and the “structure dependent” amplitudes

\[ P_{c,\bar{c}} = \left( 3h - \frac{3}{2} A_{c,\bar{c}} \left( \frac{m_c}{m_c+m_s} \right) \frac{q^2}{\beta_{c,\bar{c}}^2} \right) \beta_{c,\bar{c}} \]

\[ D_{c,\bar{c}} = \left( 3h - \frac{3}{5} A_{c,\bar{c}} \left( \frac{m_c}{m_c+m_s} \right) \frac{q^2}{\beta_{c,\bar{c}}^2} \right) \beta_{c,\bar{c}} \]

In these expressions \( m_s = 0.5 \text{ GeV} \) and \( m_c = 1.7 \text{ GeV} \) are the relevant constituent quark masses used in the decay calculation, \( \beta = 0.4 \text{ GeV} \) and \( \beta_{c,\bar{c}} = 0.53 \) are harmonic oscillator wavefunction parameters used in obtaining these amplitudes. \( \beta \) is taken from the light meson decay analysis of Ref. \[8\] and \( \beta_{c,\bar{c}} \) was obtained as described above. Rather than calculate these various reduced amplitudes in terms of \( g \) and \( h \) we approximate the two amplitudes in terms of two parameters, \( A_{c,\bar{c}} \) and \( S_{c,\bar{c}} \) and use the numerical values \( S_{c,\bar{c}} = 3.27 \) and \( A_{c,\bar{c}} = 1.67 \) obtained from the light meson decay analysis of Ref. \[8\]. Further, following Ref. \[8\], we take \( P_{c,\bar{c}} = D_{c,\bar{c}} = S_{c,\bar{c}} = 3.27 \). We found that if we instead use fitted values for \( g \) and \( h \) we obtain very similar numerical results for the partial widths.

To simplify the notation in the table we define the amplitudes:

\[ \tilde{A}_{c,\bar{c}} = A_{c,\bar{c}} \left( \frac{m_c}{m_c+m_s} \right)^2 \frac{q^3}{\beta_{c,\bar{c}}^2} F(q^2) \]

\[ \tilde{P}_{c,\bar{c}} = P_{c,\bar{c}} \left( \frac{m_c}{m_c+m_s} \right) \frac{q}{\beta_{c,\bar{c}}} F(q^2) \]

\[ \tilde{D}_{c,\bar{c}} = D_{c,\bar{c}} \left( \frac{m_c}{m_c+m_s} \right) \frac{q}{\beta_{c,\bar{c}}} F(q^2) \]

The amplitudes and partial widths for the 2S and 1D multiplets are given in Table II. The widths in column 4 of Table II were calculated using the predicted masses for the initial states and the measured values from the PDG \[8\] for the final states. The widths shown in column 5 were obtained using the measured masses of the \( D_{c,\bar{c}}^* \) and \( D_{c,\bar{c}}^* \) states: \( M(2S_1) = M(D_{c,\bar{c}}^*) = 2709 \text{ MeV} \) and \( M(1^3D_1) = M(1^3D_3) = M(D_{c,\bar{c}}^*) = 2863 \text{ MeV} \). In Table II we compare our results to other calculations where the widths shown here used the observed \( D_{c,\bar{c}}^* \) masses. For the D-wave states we find that there are two narrow and two broad states corresponding to the \( j = 5/2 \) and \( 3/2 \) doublets of the \( mQ \rightarrow \infty \) limit which are composed of the \((3^-,2^-)\) and \((2^-,1^-)\) states respectively. The \( j = 5/2 \) states are narrower due to the higher angular momentum barrier. This pattern is similar to what is expected and observed for the P-waves states \[1, 28, 34\]. This may be an important piece of the puzzle which we will return to in the next section.
TABLE I: Partial widths for the 2S and 1D $c\bar{s}$ mesons calculated using the pseudoscalar emission model. The widths in column 4 were calculated using the predicted masses for the initial states and the PDG values $^8$ for the final states. The widths given in the last column were calculated using $M = 2710$ MeV for the $2^S_1$ initial state and $M = 2860$ MeV for the $1^D_1$ and $1^D_3$ initial states. The angles $\theta_{c\bar{s}} = -38.5^\circ$ and $\phi = \tan^{-1}(\sqrt{2}/3) \simeq 39.23^\circ$. Details of the calculations are given in the text.

| State | Decay | Amplitude | Width (MeV) | Width (MeV) |
|-------|-------|-----------|-------------|-------------|
| $D_{s}^*(2^S_1)(2732)$ | $D_s^*(2^S_1) \to DK$ | $-i\sqrt{\frac{2710}{2}} P_{c\bar{s}}$ | 13 | 12 |
| | $D_s^*(2^S_1) \to D^*K$ | $-i\sqrt{\frac{2710}{2}} P_{c\bar{s}}$ | 13 | 12 |
| | $D_s^*(2^S_1) \to D_s\eta$ | $+i\sqrt{\frac{2710}{2}} P_{c\bar{s}}$ | 1.7 | 1.4 |
| | $D_s^*(2^S_1) \to D_\eta$ | $+i\sqrt{\frac{2710}{2}} P_{c\bar{s}}$ | 0.7 | 0.4 |
| $D_s(2^S_0)(2673)$ | $D_s(2^S_0) \to D^*K$ | $+i\sqrt{\frac{2673}{2}} P_{c\bar{s}}$ | 13 | |
| | $D_s(2^S_0) \to D_s\eta$ | $-i\sqrt{\frac{2673}{2}} P_{c\bar{s}}$ | 0.1 | |
| $D_s^*(1^D_3)(2917)$ | $D_s^*(1^D_3) \to DK$ | $+i\sqrt{\frac{2917}{2}} A_{c\bar{s}}$ | 27 | 19 |
| | $D_s^*(1^D_3) \to D^*K$ | $+i\sqrt{\frac{2917}{2}} A_{c\bar{s}}$ | 13 | 8 |
| | $D_s^*(1^D_3) \to D_s\eta$ | $-i\sqrt{\frac{2917}{2}} A_{c\bar{s}}$ | 2.7 | 1.6 |
| | $D_s^*(1^D_3) \to D_\eta$ | $-i\sqrt{\frac{2917}{2}} A_{c\bar{s}}$ | 0.8 | 0.3 |
| $D_s(D^*_2)(2926)$ | $D_s^2 \to [D^*K]_{P}$ | $-i\sqrt{\frac{2926}{2}} D_{c\bar{s}} \cos(\theta + \phi)$ | 116 | |
| | $D_s^2 \to [D^*K]_{P}$ | $+i\sqrt{\frac{2926}{2}} A_{c\bar{s}} \sin(\theta + \phi)$ | $\sim 0$ | |
| | $D_s^2 \to [D^*\eta]_{P}$ | $+i\sqrt{\frac{2926}{2}} D_{c\bar{s}} \cos(\theta + \phi)$ | 17 | |
| | $D_s^2 \to [D^*\eta]_{P}$ | $-i\sqrt{\frac{2926}{2}} A_{c\bar{s}} \sin(\theta + \phi)$ | $\sim 0$ | |
| $D_s(D^*_2)(2900)$ | $D_s^2 \to [D^*K]_{P}$ | $-i\sqrt{\frac{2900}{2}} D_{c\bar{s}} \sin(\theta + \phi)$ | $\sim 0$ | |
| | $D_s^2 \to [D^*K]_{P}$ | $-i\sqrt{\frac{2900}{2}} A_{c\bar{s}} \cos(\theta + \phi)$ | 20 | |
| | $D_s^2 \to [D^*\eta]_{P}$ | $+i\sqrt{\frac{2900}{2}} D_{c\bar{s}} \sin(\theta + \phi)$ | $\sim 0$ | |
| | $D_s^2 \to [D^*\eta]_{P}$ | $+i\sqrt{\frac{2900}{2}} A_{c\bar{s}} \cos(\theta + \phi)$ | 1.0 | |

Before proceeding to the discussion we make a brief digression regarding kinematically allowed decays to $DK^*$ final states. As pointed out above, decays to light vector mesons are beyond the scope of the pseudoscalar emission model. However, we can use heavy quark symmetry to estimate their partial widths. In the heavy quark limit the light quark angular momentum is separately conserved from the heavy quark spin. For the $1^3D_3$ state $j_{q}^P = 5/2^-$ and for the $1^3D_1$ state $j_{q}^P = 3/2^-$. For the decay $1^3D_3 \to DK$ the final state must be in an $F$-wave to conserve angular momentum while for $1^3D_3 \to DK^*$ the final state can be in a $P$-wave. Assuming the underlying amplitudes are similar and including angular momentum factors, elastic form factors, and the momentum factors appropriate to $P$ and $F$ waves we find $\Gamma(1^3D_3 \to DK^*) \simeq 0.08 \times \Gamma(1^3D_3 \to DK)$ which is consistent with the various results given in Table I. We make a similar estimate for the decays $1^3D_1 \to DK$ and $1^3D_1 \to DK^*$ which both have final states in $P$-waves. For this case we estimate that the partial width to $DK^*$ is about 10% of that to the $DK$ final state. This is only consistent with one of the results listed in Table I with the other two calculations predicting a significantly larger partial width to $DK^*$. However those two cases have larger total widths so that in all cases the partial width to $DK^*$ will only modify the total width by at most $\sim 20\%$.

III. ON THE NATURE OF THE $D_s(2700)$ AND $D_s(2860)$ STATES

With the mass and width predictions for excited $D_s$ states in hand we examine whether the $D_{s1}^*(2710)^+$ state can be identified with the $2^S_1(c\bar{s})$ or $1^1D_1(c\bar{s})$ and the $D_{sJ}^{*+}(2860)^+$ with the $1^3D_1(c\bar{s})$ or $1^3D_3(c\bar{s})$ states. In what follows we will refer to the decay results summarized in Table I.

The predicted mass for the $2^S_1(c\bar{s})$ is 2732 MeV compared to the PDG $^8$ averaged mass for the $D_{s1}^*(2710)$ of 2709 $\pm$ 4 MeV and the predicted masses for the $1^3D_1(c\bar{s})$ and $1^3D_3(c\bar{s})$ states are 2899 and 2917 MeV respectively, which are compared to the measured $D_{sJ}^{*+}(2860)$ mass of 2863 $^{+4.0}_{-2.6}$ MeV. The measured $D_{s1}^*(2710)$ mass is consistent with the predicted $2^S_1$ mass within the accuracy of the model. The measured $D_{s1}^*(2860)$ mass is $\sim 36$ MeV lower than the predicted $1^3D_1(c\bar{s})$ mass and $\sim 54$ MeV...
lower than the predicted $1^3D_3(c\bar{s})$ mass. Depending on how well the predicted and measured decay widths agree one could accept the discrepancies in the mass predictions as being within the uncertainties of the model.

The predicted total width for the $2^3S_1(c\bar{s})$ is 26 MeV versus the $D_{s1}(2710)\pm$ measured width of $117 \pm 13$ MeV and the predicted value for the ratio of partial widths is $\Gamma(D_{s1}(2^3S_1) \to D^*K)/\Gamma(D_{s1}(2^3S_1) \to D^0K) = 0.99$ versus the observed value of $0.91 \pm 0.13(\text{stat}) \pm 0.12(\text{syst})$. While the $\Gamma(D^*K)/\Gamma(DK)$ ratio is in good agreement there is significant disagreement for the total widths even if we accept that the predicted widths could be off by a factor of two. Similarly, the measured $D_{s1}^*(2860)\pm$ width and $\Gamma(D^*K)/\Gamma(DK)$ ratio are $58 \pm 11$ MeV and $1.10 \pm 0.15(\text{stat}) \pm 0.19(\text{syst})$ respectively compared to the predicted values of 145 MeV and 0.34 for the $1^3D_1(c\bar{s})$ state. We conclude that the decay properties of the $D_{s1}^*(2710)\pm$ and the $D_{s1}^*(2860)\pm$ are not consistent with those of the $2^3S_1(c\bar{s})$ and $1^3D_1(c\bar{s})$ states. This conclusion is consistent with those of other calculations although it should be noted that Ref. [54] and [18] (unequal \(\bar{s}\) case) find the $2^3S_1(c\bar{s})$ total width to be consistent with the $D_{s1}^*(2710)\pm$ width although the $\Gamma(D^*K)/\Gamma(DK)$ ratio is in substantial disagreement.

To understand the $D_{s}^*$ states, other possibilities have been put forward. In one scenario the $D_{s1}^*(2710)\pm$ is a mixture of $2^3S_1(c\bar{s})$ and $1^3D_1(c\bar{s})$ [10, 13, 15, 19] and the

| State | Experiment [8] | Lattice [48] | PSEM | CS [54] | ZLDZ [11] | ZZ [13] | LM [17] | YCZ 1 [18] | YCZ 2 [18] |
|-------|----------------|-------------|------|---------|-----------|--------|--------|------------|------------|
| $M(2^3S_1)(2709)$ | $D_{s1}$ $2709 \pm 4$ | $2757 \pm 6$ | $2732$ | $2730$ | $2715$ | $2710$ | $2710$ | $2709$ | $2709$ |
| $\Gamma(\to DK)$ | $12$ | $17$ | $3.2$ | $11$ | $4.4$ | $9.4$ | $32$ |
| $\Gamma(\to D^*K)$ | $12$ | $81$ | $27.2$ | $18.1$ | $34.9$ | $41$ | $85$ |
| $\Gamma(\to D_s\eta)$ | $1.4$ | $2.6$ | $0.05$ | $1.7$ | $0.8$ | $2.0$ | $20$ |
| $\Gamma(\to D_s^*\eta)$ | $0.4$ | $4.1$ | $0.54$ | $0.7$ | $1.4$ | $2.0$ | $11$ |
| $\Gamma_{\text{Total}}$ | $117 \pm 13$ | $25$ | $105$ | $32$ | $31$ | $41.4$ | $55$ | $148$ |
| $\Gamma(\to D^*K)/\Gamma(\to DK)$ | $0.91 \pm 0.18$ | $0.99$ | $4.8$ | $8.5$ | $1.65$ | $7.93$ | $4.4$ | $2.67$ |

| $M(1^3D_1)(2863)$ | $D_{s1}$ $2863^{+4.0}_{-2.6}$ | $2888 \pm 5$ | $2899$ | $2900$ | $2860$ | $2862$ |
| $\Gamma(\to DK)$ | $93$ | $120$ | $84$ | $63$ |
| $\Gamma(\to D^*K)$ | $32$ | $74$ | $14$ | $38$ |
| $\Gamma(\to D_s\eta)$ | $16$ | $39$ | $24$ | $20$ |
| $\Gamma(\to D_s^*\eta)$ | $4.0$ | $17$ | $2$ | $8.5$ |
| $\Gamma(\to DK^*)$ | $-$ | $81$ | $7.8$ | $38$ |
| $\Gamma_{\text{Total}}$ | $58 \pm 11$ | $145$ | $331$ | $132$ | $168$ |
| $\Gamma(\to D^*K)/\Gamma(\to DK)$ | $1.10 \pm 0.24$ | $0.34$ | $0.62$ | $0.17$ | $0.60$ |

| $M(1^3D_3)(2863)$ | $D_{s1}$ $2863^{+4.0}_{-2.6}$ | $2942 \pm 6$ | $2917$ | $2920$ | $2860$ | $2862$ | $2862$ | $2862$ |
| $\Gamma(\to DK)$ | $19$ | $82$ | $22$ | $24$ | $36$ | $33$ | $42$ |
| $\Gamma(\to D^*K)$ | $8.1$ | $67$ | $13$ | $10$ | $27$ | $23$ | $24$ |
| $\Gamma(\to D_s\eta)$ | $1.6$ | $4.5$ | $1.2$ | $1.7$ | $1.6$ | $1.9$ | $6.2$ |
| $\Gamma(\to D_s^*\eta)$ | $0.3$ | $2.2$ | $0.3$ | $0.6$ | $0.7$ | $1.5$ |
| $\Gamma(\to DK^*)$ | $-$ | $14$ | $0.71$ | $0.2$ | $2.7$ | $2.1$ | $1.4$ |
| $\Gamma_{\text{Total}}$ | $58 \pm 11$ | $29$ | $222$ | $37$ | $36$ | $67$ | $60$ | $75$ |
| $\Gamma(\to D^*K)/\Gamma(\to DK)$ | $1.10 \pm 0.24$ | $0.43$ | $0.82$ | $0.59$ | $0.40$ | $0.75$ | $0.69$ | $0.56$ |
$D_{s1}^*(2860)$ is its orthogonal partner \[16\]–\[18\]:

\[
\begin{align*}
D_{s1}^* &= 2S_1 \cos \theta + 1^3 D_1 \sin \theta \\
D_{sJ}^* &= -2S_1 \sin \theta + 1^3 D_1 \cos \theta.
\end{align*}
\tag{16}
\]

The partial and total widths and the $\Gamma(D^*K)/\Gamma(DK)$ ratios for the $D_{s1}^*$ and $D_{sJ}^*$ mixed states defined by Eqn. 16 are plotted as a function of the mixing angle in Fig. 2. Eqn. 16 is a simplification as in general the mixing angle should depend on energy and a coupled channel analysis is required. However, this simplification is adequate for a preliminary study to determine whether mixing is a viable explanation for the observed decay properties and whether a more detailed analysis is warranted. The shaded bands in these plots represent the one standard deviation regions of the measured values for these quantities \[8\]. One can see that reasonable agreement is obtained with a $2^3 S_1 - 1^3 D_1$ mixing angle of $\sim 90^\circ$. Calculating the $\chi^2$ for the four observables finds the best fit for $\theta \approx 88^\circ$. In other words we obtain a reasonable fit for the decay widths assuming that the $D_{s1}^*(2710)^\pm$ is primarily the $1^3 D_1(c\bar{s})$ state and the $D_{sJ}^*(2860)$ is primarily the $2^3 S_1(c\bar{s})$ state. This, however, is inconsistent with the mass predictions and implies that the $2^3 S_1(c\bar{s})$ is more massive than the $1^3 D_1(c\bar{s})$. We are aware that in this scenario the predicted $D_{s1}^*(2710)^\pm$ width is smaller than the observed width but as stated previously, we consider the difference to be within the predictive power of the model.

A more serious discrepancy is that for the $1^3 D_1(c\bar{s})$ state the predicted $D_{s1}^*(2700)^\pm \Gamma(D^*K)/\Gamma(DK)$ ratio is about a factor of two smaller than what is observed. It is possible that various corrections could bring the predicted value for this ratio into closer agreement with experiment. For example, the amplitudes for the decays
The predicted total width and \( \Gamma(D_s^0) \) are the “structure dependent” type, Eqn. \[ \text{(12)} \] which we approximated with a constant. In previous calculations we found that this approximation gave reasonably good agreement with experiment and other models. However, if in Eqn. \[ \text{(12)} \] one takes into account the smaller phase space of the decay to \( D^*K \) compared to \( DK \) we expect an enhancement in the \( \Gamma(D^*K)/\Gamma(DK) \) ratio, although it is unlikely to be sufficient to bring it into agreement with experiment.

This brings us back to a point mentioned previously, that the physical \( J = 2 \) states are mixtures of \( 1^{3}D_2 \) and \( 1^{1}D_3 \) resulting in one narrow and one broad state approximately degenerate with the \( 1^{3}D_1 \) and \( 1^{3}D_3 \) states. Perhaps the peak seen at 2710 MeV has contributions from both the \( 1^{3}D_1 \) and the \( 1^{3}D_2 \) states and be enhanced relative to what would be predicted from the \( 1^{3}D_1 \) on its own, thereby explaining the observed \( \Gamma(D^*K)/\Gamma(DK) \) ratio. With sufficient statistics it should be possible to study the angular distributions of the final states and test this possibility. This explanation leaves the \( D_s^*(2860) \) unaccounted for.

A possibility put forward in the literature for the \( D_{sJ}^*(2860) \) is that it is the \( 1^{3}D_3(c\bar{s}) \) state \[ \text{(11),(15,22)} \]. The predicted total width and \( \Gamma(D^*K)/\Gamma(DK) \) ratio are 29 MeV and 0.42 respectively. As already stated, we can accept that our width predictions are off by a factor of two but we still need to understand the discrepancy in the ratio of BR’s. Unlike the \( 1^{3}D_1 \) decays, the amplitudes for the \( 1^{3}D_3 \to D^*K \) and \( DK \) are “structure independent” amplitudes so we do not expect the ratio of BR’s to change. A possible explanation is the same as we suggested for the \( D_{sJ}^*(2710)^\pm \) state, that the bump at 2860 MeV could be explained as two overlapping resonances, the \( 1^{3}D_3(c\bar{s}) \) state and a \( 1^{1}D_3 \) which would explain the discrepancy in the \( \Gamma(D^*K)/\Gamma(DK) \) ratio \[ 13 \].

Putting these pieces together we arrive at the possibility that the \( D_{sJ}^*(2710)^\pm \) is the \( 1^{3}D_1 \) state and the \( D_{sJ}^*(2860) \) is the \( 1^{3}D_3 \) state with overlapping \( 1^{1}D_2 \) states that modify the observed \( \Gamma(D^*K)/\Gamma(DK) \) ratios. However this does lead to a number of questions. First, with this explanation the measured \( 1^{3}D_1 - 1^{1}D_3 \) splitting, \( \sim 154 \) MeV, would be much much larger than the predicted splitting of 18 MeV. For comparison, in the strange meson system, the only light-heavy system for which there are candidate states for the \( 1^{3}D_1 \) and \( 1^{3}D_3 \), the splitting is \( 59 \pm 28 \) MeV. So although we might not want to rule out such a large splitting it is inconsistent with the predictions of this model. Next, if we identify the \( D_{sJ}^*(2710)^\pm \) with the \( 1^{3}D_1 \), where is the \( 2^{3}S_1 \) state?

Again, comparing to the strange mesons where the \( 2^{3}S_1 \) is predicted to lie 200 MeV below the \( 1^{3}D_1 \) state. This is roughly comparable to the \( 1^{3}D_1 - 2^{3}S_1 \) splitting of 168 MeV predicted for the charm-strange mesons. However, in the strange sector, the mesons identified with the \( 2^{3}S_1 \) and \( 1^{3}D_1 \) states have a measured mass difference of 303 MeV. So it is possible that the model gets this wrong and the \( 2^{3}S_1(c\bar{s}) \) state is also much lighter than predicted. We note that B. Zhang et al. \[ 11 \] also come to this conclusion by calculating decay widths using the \( ^3P_0 \) model and A. Zhang \[ 12 \] and Colangelo et al. \[ 22 \] come to the same conclusion treating the \( D_{sJ}^*(2710)^\pm \) and \( D_{sJ}^*(2860) \) as \( J^P = \frac{3}{2}^- \) and \( \frac{3}{2}^- \) states in the heavy quark limit and calculating the partial widths using an effective Lagrangian approach.

We have examined three possible interpretations of the \( D_{sJ}^*(2710)^\pm \) and \( D_{sJ}^*(2860) \) mesons based on the masses predicted by the relativized quark model and strong decay properties predicted by the pseudoscalar emission model (PSEM). None of these possibilities is without flaws and we conclude that the most likely explanation is that the \( D_{sJ}^*(2710)^\pm \) and \( D_{sJ}^*(2860) \) mesons are the \( 1^{3}D_1 \) and \( 1^{3}D_3 \) \( c\bar{s} \) states. However the predicted ratios of partial widths for these states to \( D^*K \) and \( DK \) final states are in sharp disagreement with the BaBar measurements \[ 6 \]. We suggest that this might be due to the \( 1^{3}D_2 \) states, which decay to \( D^*K \), overlapping with the \( 1^{3}D_1 \) and \( 1^{3}D_3 \) states thereby enhancing the \( D^*K \) signal. In Table \[ 11 \] we compare our predictions to a broad range of calculations and for the most part find similar results within the uncertainties of the models. The most noteworthy exceptions are the results of Close and Swanson \[ 54 \] who predict much larger total widths for the \( 2^{3}S_1 \), \( 1^{3}D_1 \) and \( 1^{3}D_3 \) states. In addition, all calculations other than ours find much larger values for the \( \Gamma(2^{3}S_1 \to D^*K)/\Gamma(2^{3}S_1 \to DK) \) ratio. As a consequence, further measurements will be needed to confirm or rule out the various quark model assignments of these states.

The most useful measurement is to determine the spin of the \( D_{sJ}^*(2860) \) which would settle whether it is the \( 1^{3}D_3(c\bar{s}) \) or one of the \( J^P = 1^- \) states. With sufficient statistics one could also determine whether there is more than one state contributing to both the \( D_{sJ}^*(2710)^\pm \) and \( D_{sJ}^*(2860) \) bumps. This would be able to rule out or confirm our suggestion of overlapping resonances.

Six states are predicted to lie in this mass region, the \( 2^{1}S_0 \), \( 2^{3}S_1 \) and the 4 D-wave states, but only two states have been observed. Observing the 4 missing states would go a long way to shed light on these states. We predict the \( D_s(2^{1}S_0) \) mass to be 2673 MeV and to be relatively narrow decaying predominantly to the \( D^*K \) final state. It should therefore be possible to observe it in \( D^*K \) by the experiments that have seen the \( D_{sJ}^*(2710)^\pm \) and \( D_{sJ}^*(2860) \) in this final state. If the \( D_{sJ}^*(2710)^\pm \) is in fact the \( 1^{3}D_1 \) \( c\bar{s} \) it should also be possible to observe the \( 2^{3}S_1 \) \( c\bar{s} \) in \( DK \) and \( D^*K \) final states. If it isn’t, as stated previously, it will be important to measure the \( D_{sJ}^*(2860) \) spin to distinguish between the \( 1^{3}D_1 \) and \( 1^{3}D_3 \). Finally, we expect the \( 1^{1}D_2 \) states to be close in mass to the \( 1^{3}D_1 \) and \( 1^{3}D_3 \) states so that high statistics will be needed to see them as separate resonances or to be able to per-
form an angular momentum analysis that can distinguish two overlapping states with different angular momentum. It is possible that the four missing states have not yet been observed because of insufficient statistics or that the invariant mass spectra are dominated by the tails of the $D_{s2}^*(2573)$ and $D_{s1}(2536)$ resonances in this mass region.

IV. SUMMARY

In this paper we attempted to understand the nature of the $D^*_{s1}(2710)^\pm$ and $D^*_{sJ}(2860)$ states as charm-strange mesons using the relativized quark model and the pseudoscalar emission model for their decays. We considered a number of possibilities; that the $D^*_{s1}(2710)^\pm$ is the $2^3S_1(\bar c s)$ state, that it is a linear combination of $2^3S_1$ and $1^3D_1(\bar c s)$ and the $D^*_{sJ}(2860)$ is its orthogonal partner. We came to the conclusion that they are best described as the $1^3D_1$ and $1^3D_3(\bar c s)$ states. However, the predicted ratios of partial widths for these states to $D^*K$ and $DK$ final states do not agree with the measured ratios. We suggest that this could be a result of overlapping $D_s$ states which enhances the $D^*K$ final state. A crucial measurement to distinguish these possibilities is the measurement of the $D^*_{sJ}(2860)$ spin and with sufficient statistics to determine whether the bumps at 2710 MeV and 2860 MeV contain two resonances with different spins. We also point out that 6 states are expected in this mass region so that 4 are still to be accounted for. Finding them would fill in gaps in the excited $D_s$ spectrum and confirm the nature of the $D^*_{s1}(2700)^\pm$ and $D^*_{sJ}(2860)$ states.

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