Activity Induced Synchronization: From Mutual Flocking to Chiral Self-Sorting

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Synchronization, the temporal coordination of coupled oscillators, allows fireflies to flash in unison, neurons to fire collectively and human crowds to fall in step on the London millenium bridge. Here, we interpret active (or self-propelled) chiral microswimmers with a distribution of intrinsic frequencies as motile oscillators and show that they can synchronize over very large distances, even for local coupling in 2D. This opposes to canonical non-active oscillators on static or time-dependent networks, leading to synchronized domains only. A consequence of this activity-induced synchronization is the emergence of a “mutual flocking phase”, where particles of opposite chirality cooperate to form superimposed flocks moving at a relative angle to each other, providing a chiral active matter analogue to the celebrated Toner-Tu phase. The underlying mechanism employs a positive feedback loop involving the two-way coupling between oscillators’ phase and self-propulsion, and could be exploited as a design principle for synthetic active materials and chiral self-sorting techniques.

Populations of motile entities, from bacteria to synthetic microswimmers, can spontaneously self-organize into phases which are unattainable in passive matter. Examples range from the spectacular murmuration of starlings and bacterial swarming [1–4], to collective behavior of synthetic self-propelled grains [5, 6], emulsion droplets [7] or assemblies of robots [8]. Active colloids [9] in particular, spontaneously form living clusters in low density suspensions which, unlike equilibrium clusters, continuously break up and reform [9–15]. Remarkably, when active particles align with their neighbors, they can self-organize into a polarly ordered phase, featuring long-range order in 2d [16–19]. Thus, motile entities, like wildebeests or sheep, can order at long-distances and all run in the same direction, whereas immotile entities like spins in a 2d magnet cannot display (true) long-range order when locally coupled [20, 21]. Likewise, oscillators which are localized in space, like metronomes or neurons [22, 23], can synchronize only locally in 2d or 3d when coupled to their neighbors [24, 25].

Here, we consider ensembles of chiral active particles with a tendency to swim in circles and interpret them as ‘active oscillators’, to link generic synchronization theories with active matter physics. Despite the common occurrence of such active oscillators both in nature, e.g. microorganisms [28–32] or cell-components [33–35], and in the world of synthetic microswimmers [36–43], their generic large-scale synchronization behavior remains surprisingly unclear: (i) In active matter, previous stud-

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FIG. 1: (a) Phase diagram of active oscillators comprising the mutual flocking phase induced by activity-induced synchronization: Lines and symbols show analytical predictions and simulation for the phase boundaries (red symbols for two species, black ones for a normal frequency distribution) using two different densities ($\rho_0 = 10, 20$ shown in black crosses and squares). The weak deviation between simulations and predictions probably originates from a reduction of the growth rate of unstable modes as $\Omega$ increases (see SM [27]). Insets show snapshots in the chiral phase for two species (b) and a Gaussian distribution (c) both for $\text{Var}(\Omega) = 2.0$; $g\rho_0 = 2.8$; colors show intrinsic frequencies.

ies have focused on pattern formation in chiral active particles [43–45], but leave the synchronization behavior of large ensembles essentially open. (ii) In the field of synchronization, in turn, there is an impressive body of knowledge on the large-scale synchronization of oscillators which are localized in space [22, 23], or move in a way that is unaffected by their phases [49, 52] – but not for the above active-oscillator examples, which show a two-
way coupling between phase and displacement through space.

Here, we establish a framework to explore the large-scale synchronization of active oscillators showing that the two-way coupling, naturally present in active oscillators, creates a novel synchronization behavior allowing for synchronization over very large distances. This not only transcends a fundamental limitation of synchronization in locally coupled oscillators (which can synchronize only locally in 2d or 3d \[23, 26\]), but also creates an undiscovered active matter phase, which we call the *mutual flocking phase*. Here, active oscillators with opposite chirality cooperate to move coherently at a relative angle, forming non-rotating flocks, akin to the celebrated Toner-Tu phase in linear active matter \[17–19\] (see Fig. 1, orange domain). This suggests that the universality of polar active matter might also apply to active oscillators, yet in a remarkably nontrivial way, involving synchronization across non-identical particles (i.e. a distribution of frequencies).

In addition, for comparatively broad frequency distributions, we find that active oscillators spatially segregate according to their chirality and form rotating macroscopic clusters (see Fig. 4 green domain). These feature internal synchronization (see snapshots Fig. 1 b,c) and grow linearly with system size. This *chiral phase* constitutes a second example where activity leads to synchronization on the macroscale. Notice, that the chiral self-sorting underlying this phase allows to segregate active particles by chirality, without requiring chemical reactions \[53\], external flows or environmental chirality \[59\].

To demonstrate the impact of activity on the large-scale synchronization of coupled oscillators, we consider \(N\) overdamped particles with 2d positions \(r_α\) and alignment interactions of strength \(K\) in a square box of length \(L\), which self-propel with a constant speed \(v\) along direction \(n_α = (\cos θ_α, \sin θ_α)\), as described by:

\[
\dot{r}_α = v n_α, \quad \dot{θ}_α = \omega_α + \frac{K}{π R^2} \sum_{ν ∈ 0,} \sin(θ_ν − θ_α) + \sqrt{2D r} \eta_α
\]

Here, \(η\) represents Gaussian white noise with unit variance and zero mean. The intrinsic frequencies \(ω_α\) are randomly drawn from a distribution \(\Delta(ω)\). Note that, conversely to more complicated models, involving e.g. hydrodynamic interactions, excluded volume effects or sophisticated couplings, the present model focuses on the essential ingredients required to demonstrate the generic synchronization scenario shown by active oscillators. Moreover, direct experimental realizations of the present model are also possible, e.g. using 3D-printed granular particles on vibrated plates \[6, 54\]. For example, disk-shaped granulates featuring asymmetric legs \[6\], both swim in circles and locally align with each other \[6, 54\]. Refs. \[54, 55, 57\] provide other very recent setups containing the key-ingredients of the present model. Since we want to understand the synchronization behavior arising from the competition between the distribution of natural frequencies and the coupling, we consider only distributions with zero mean. Indeed, here we establish the role played by the frequency dispersion (through its variance \(\text{Var}(ω)\)) on the collective behavior of active oscillators, a control parameter that has no equivalent in previous works on chiral active matter. The sum eq. \(\sum\) runs over all the neighbors of particle \(α\), defined by the cutoff distance \(R\). For \(v = 0\) our model reduces to the noisy Kuramoto model of locally coupled oscillators, which is known to show only local synchronization in 2d \[23, 26\]. For \(ω_α = 0\), in turn, the model is equivalent to (a smooth variant of) the Viscek model for which it is known that self-propulsion induces long-range order in 2d. The crucial feature of the present model is that it identifies the oscillators phase with their direction of self-propulsion, giving rise to circular motion at the individual level and the emergence of a novel synchronization scenario at the collective level. In the spirit of the Kuramoto model \[58\] and simple models of self-propelled particles (like the Vicsek and smooth variants of it \[59–61\] or the Active Brownian Particle model \[62–64\]), our model provides a minimal framework to study the generic behavior emerging in systems of non-identical active oscillators or circle swimmers.

However, most previous studies on synchronization in dynamic networks have focused on identical oscillators whose phase does not affect their motion in space \[49, 52\], showing that, the absence of global synchronization (and other scaling properties) for Kuramoto oscillators in 2d is generically preserved, even if carried by active particles. Note that, despite this recent boost of activity on synchronization of motile oscillators, the dispersion of natural frequencies, a main feature in synchronization problems, has not been considered yet. Therefore, in order to better understand the fundamental synchronization mechanisms involved in active systems, we have carried extensive simulations of our model in the absence of the back-coupling between the phase of the oscillators and their direction of motion and show that global phase synchronization cannot be achieved (see SM \[27\] for further details). Models considering agents whose phase, or a different internal state, affect the way they move in space appeared recently \[65, 68\]. However, none of the previous studies consider particles self-propelled in the direction of an internal Kuramoto phase (with a distribution of natural frequencies and local coupling). Then, as expected, such models cannot account for the generic synchronization scenario of active oscillators we establish here, in particular, the possibility to sustain global phase synchronization in 2d (and 3d) active matter and provide a route to chiral self-sorting. To understand the key control parameters of the present system, we express times and lengths in units of \(1/D_r\) and \(R\), respectively, leading to the dimensionless quantities: (i) \(g = K/(π R^2 D_r)\), the reduced coupling; (ii) \(\text{Pe} = v/(D_r R)\), the rotational Péclet number; (iii) \(Ω_1 = ω_1/D_r\), the reduced rotation frequency, and (iv) \(ρ_0 = NR^2/L^2\), the average number density. We denote by \(ρ^{(i)} = N^{(i)} R^2/L^2\) the den-
sity of particles with natural frequency \( \Omega_i \) (species \( i \)), where \( N^{(i)} \) counts particles sharing the same natural frequency. For simplicity, we exemplify our key results for two species with equal overall density and opposite chirality. As we will show, the synchronization behavior is controlled by \( \text{Var}(\Omega) \), such that this distribution is largely representative to cases of several species and continuous frequency distributions.

Field Theory We first derive a set of field equations describing the collective dynamics of chiral active particles. Here, we assume that the system contains \( M \) different species with frequencies \( \Omega_i \) (\( i = 1..M \)) and describe the dynamics of species-\( i \)-particles using the density field \( \rho \equiv \rho^{(i)}(x,t) \) and the polarization density \( \mathbf{w} \equiv \mathbf{w}^{(i)}(x,t) \), where \( \mathbf{w}/|\mathbf{w}| \) is the average self-propulsion direction of particles of species \( i \). The resulting equations read (see SM for details [27]):

\[
\dot{\rho} = - \rho \nabla \cdot \mathbf{w} \quad (3)
\]

\[
\dot{\mathbf{w}} = - \mathbf{w} + \sum_{i=1}^{M} \frac{g_{\rho}^{(i)}}{2} \mathbf{w}^{(i)} + \Omega \mathbf{w}_\perp - \frac{\rho}{2} \nabla \rho
\]

\[
+ \frac{\rho^2}{2b} \nabla^2 \mathbf{w} + \frac{\rho^2 \Omega}{4b} \nabla^2 \mathbf{w}_\perp
\]

\[
- \frac{g^2}{b} \left( \mathbf{w} + \frac{\Omega \mathbf{w}_\perp}{2} \right) \left( \sum_{i=1}^{M} \mathbf{w}^{(i)} \right)^2 + \mathcal{O} (\nabla \mathbf{w}^2) \quad (4)
\]

Here, \( b = 2(4 + \Omega^2) \), \( \mathbf{w}_\perp = (-w_y, w_x) \) and \( \mathcal{O} (\nabla \mathbf{w}^2) \) represents terms which are both nonlinear in \( \mathbf{w} \) and involve gradients, and are not of interest for our following purposes. Eq. (3) simply reflects that particles on average self-propel in the direction of \( \mathbf{w} \). The first term in Eq. (4) represents a decay of the polarization due to rotational diffusion of the particles, which happens in competition with the second term creating alignment among all species; the third term represents a rotation of the average (local) polarization direction with a species-specific frequency while the forth term expresses a statistical tendency for self-propulsion away from high-density regions; remaining terms ‘smear out’ regions of high and low polarization and lead to saturation. For weak interactions, the system is in a disordered uniform phase, given by \( (\rho, \mathbf{w}) = (\rho_0, \mathbf{0}) \), which is a solution of Eqs. [3] and represented by the blue domain in the phase diagram Fig. 1.

Mutual Flocking Phase To understand the onset of synchronization we perform a linear stability analysis for a bimodal frequency distribution \( \Omega_1 = -\Omega_2 = \Omega \) of the uniform disordered phase \( (\rho^{(1)}, \rho^{(2)}) = \rho_0/2, \mathbf{w}_1 = \mathbf{w}_2 = 0 \) [27]. It is instructive to first focus on the zero wavenumber limit \( (q = 0) \). Here, we find that the uniform phase looses stability if \( \rho_0 > 2(1 + \Omega^2) \), suggesting the existence of a nontrivial ordered uniform phase. We indeed find an exact solution of our field equations [3] representing such a phase: a state of uniform density \( \rho^{(1)} = \rho^{(2)} = \rho_0/2 \), but with a finite polarization, spontaneously breaking the symmetry of the disordered phase. In this state, circle swimmers of both species cooperate and form two individual and superimposed flocks moving linearly and at a relative angle \( \Delta \Phi \) to each other. We thus call it mutual flocking phase. Defining the overall polarization \( \mathbf{P} = (\mathbf{w}_1 + \mathbf{w}_2)/\rho_0 \), where \( |\mathbf{w}_1| = |\mathbf{w}_2| = \rho_0 P/\sqrt{2(1 + \cos \Delta \Phi)} \), the exact expressions representing the mutual flocking phase read

\[
P = \frac{\sqrt{2}}{\rho_0} \sqrt{\sqrt{(\rho_0)^2 + 6(\rho_0 - 6)} + \rho_0 + 2\Omega^2 - 4} \quad (5)
\]

\[
\Delta \Phi = -i \ln \left[ \frac{2\Omega (6 - \frac{\rho_0}{2}) + 4i \sqrt{\frac{(\rho_0)^2}{4} + 3\Omega^2 (\frac{\rho_0}{2} - 3)}}{\rho_0 (\Omega + 2i)} \right]
\]

At low frequency \( \Omega < 1/\sqrt{2} \), this solution exists (is real and positive) if \( \rho_0 > 2(1 + \Omega^2) \), i.e. precisely when the field equations show a linear instability at \( q = 0 \).

Brownian dynamics simulations at high coupling and high density confirm the existence of the mutual flocking phase. As shown in Fig. 1(a), we find the mutual flocking phase essentially in the whole parameter regime where it exists according to our field theory. Fig. 2 shows in turn a close quantitative agreement between theoretical predictions (Eqs. 5) and our simulations, both for the angle between the two flocks and for the overall polarization (see Movie 1 [27]). (In our simulations, we measure the partial and overall polarization, \( \mathbf{P} = |\mathbf{P}_{1,2}| \), where \( \mathbf{P}_i = \frac{2}{N} \sum \mathbf{n}_j \delta_{\omega_j, \omega_i} \), and \( \mathbf{P} = \frac{1}{N} \sum \mathbf{n}_j \), respectively.) In Fig. 2(c) we show the orientational self-correlation function \( C(r) = \langle \mathbf{n}_i \cdot \mathbf{n}_j \rangle \), strongly suggesting the emergence of global synchronization for active oscillators in contrast to immobile oscillators \( (v = 0 \text{ in eq. 1}) \).

Activity-induced synchronization To generally understand when the disordered phase looses stability, we now explore its linear stability at finite wavenumber \( q \neq 0 \).
This is equivalent to accounting for the impact of motility on the onset of synchronization, as the coefficients of all gradient terms in \( \mathbf{g}^2 \) are non-vanishing only if \( \text{Pe} \neq 0 \). While passive oscillators need a stronger coupling to synchronize as the frequency dispersion increases \([23, 58]\), our linear stability calculation shows that self-propulsion generally induces an instability for any \( g_{\rho_0} > 2 \) \([27]\), independently of \( \text{Var}(\Omega) \). That is, self-propulsion induces phase ordering even in parameter regimes where a corresponding non-motile system (\( \text{Pe} = 0 \)) would simply show asynchronous oscillations of the individual particles. As the present instability emerges at finite \( q \), i.e. for localized perturbations, we do not expect a uniform phase, but rather the emergence of localized synchronous structures. Our particle based simulations confirm this. Above the critical value \( g_{\rho_0} \gtrsim 2 \) [Fig. 3(a)], we find large rotating clusters of opposite chirality featuring internal phase-synchronization, as illustrated by the snapshots Fig. 3 (b-c) (and Movie 2 [27]). From \( C(r) \) we extract a characteristic length \( \xi \), which, as shown in Fig. 3(a), grows linearly with \( \sqrt{N} \) (at fixed \( \rho_0 \)), meaning that clusters are macroscopic in this regime and tend to phase separate. Thus, the chiral phase represents a second example where self-propulsion induces (long-range) phase-synchronization among each species. As shown in Fig. 3(b), above a critical coupling strength, \( \tilde{P} \) starts to increase, allowing us to locate the phase boundaries Fig. 1(a) (Particle segregation, quantified by the probability \( \Psi \) to find an excess of particles of one chirality, offers an alternative route to identify the emergence of the chiral phase, see [27]).

**Continuous frequency distributions** To demonstrate the generality of our results, we now simulate active oscillators with Gaussian distribution of natural frequencies (with zero mean and variance \( \text{Var}(\Omega) \)). As for two species, we predict the stability threshold of the disordered phase at \( g_{\rho_0} > 2 \) for 3 and 4 species using linear stability analysis (see SM [27]) and confirm with simulations that the stability threshold of the disordered state does not change as compared to the 2-species case, up to numerical accuracy (Fig. 1). At \( g_{\rho_0} \gtrsim 2 \), we find the same two phases as for two species: a mutual flocking phase, further illustrated in the SM [27], and a phase comprising synchronized macroclusters scaling linearly with system size (see Fig. 1(c) and movie 3 [27]). Now, the macroclusters involve a continuous range of frequencies and feature frequency synchronization across species.

**Physical Mechanism** What is the physical mechanism allowing motility to qualitatively change the synchronization of oscillators? The absence of global synchronization is well-known for static structures of dimension \( d < d_c = 4 \) (where \( d_c \) denotes its lower critical dimension) \([24, 28]\), and particle motion itself does not affect such scenario if the oscillators motion is independent of their phase \([27, 52]\). Hence, compared to oscillators on dynamic networks, the crucial new ingredient here is that the phase of an oscillator determines its direction of motion. It follows that two circle swimmers sharing the same phase are aligned and move together, enhancing their interaction time, which in turn enhances their alignment and fosters synchronization. This can be viewed as a positive feedback between alignment (local synchronization) and interaction-time, mutually supporting each other, which is absent for oscillators on dynamic networks but plays a fundamental role for synchronization of active oscillators. The impact of such feedback on the emergence of order is even more dramatic in the present case than in linear active matter. Unlike the static XY-model for immotile spins, featuring quasi-long-range order in \( d = d_c = 2 \), the Vicsek model (and its continuous variants) shows long-range order in the form of a flocking (Toner-Tu) phase. Although for phase-oscillators (Kuramoto model) \( d_c = 4 \), global phase synchronization occurs in \( d = 2 \) for active oscillators, well below the critical dimension of its passive counterpart.

**Conclusions** Chiral active particles, here interpreted as 2D active oscillators, can synchronize over very long distances, even for a purely local coupling. This contrasts the synchronization behavior of the huge class of non-active oscillators in 2d or 3d structures \([23]\), which can only synchronize within domains. A consequence of global synchronization in active oscillators, is the emergence of the mutual flocking phase as a new active matter phase, akin to the celebrated Toner-Tu phase in linear active matter. Our results transcend a knowledge boundary at the interface of active matter physics and synchronization and could be useful e.g. to sort active enantiomers.

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