Unruh effect for a Dvali-Gabadadze-Porrati Brane

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In braneworld cosmology the brane accelerates in the bulk, and hence it perceives Unruh radiations in the bulk. We discuss the Unruh effect for a Dvali-Gabadadze-Porrati (DGP) brane. We find that the Unruh temperature is proportional to the acceleration of the brane, but chemical potential appears in the distribution function for massless modes. The Unruh temperature does not vanish even at the limit $r_c \to \infty$, which means the gravitational effect of the 5th dimension vanishes. The Unruh temperature equals the geometric temperature when the the density of matter on the brane goes to zero for branch $\epsilon = 1$, no matter what the value of the cross radius $r_c$ and the spatial curvature of the brane take. And if the state equation of the matter on the brane satisfies $p = -\rho$, the Unruh temperature always equals the geometric temperature of the brane for both the two branches, which is also independent of the cross radius and the spatial curvature. The Unruh temperature is always higher than geometric temperature for a dust dominated brane.

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I. INTRODUCTION

One of the most remarkable prediction of the quantum field theory in curved space is that the concept of particle is observer dependent, which is called Unruh effect . In 1976, Unruh found that an accelerating particle detector with proper acceleration $A$ in Minkowski space perceived different excited modes exactly in Bose-Einstein distribution with a temperature $T = A/2\pi$ [1]. However this amazing effect generates some puzzles, sometimes even treated as paradoxes.

For example, Unruh’s original construction is not consistent because his quantization is not unitarily equivalent to the standard construction associated with Minkowski vacuum. Hence some authors used mathematically more rigorous methods to solve this problem [2].

The temperature of a particle Unruh detector, which is in thermal equilibrium with Unruh radiation it experienced in Minkowski vacuum, is higher than the temperature of the vacuum to an inertial observer. Hence, does it really emit radiation for an inertial observer, just like an

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accelerated charged particle? It was argued that there was no radiation flux from an Unruh observer in [3]. Unruh made an almost the same calculation as in [3], but he found the extra terms in the two point correlation function of the field which would contribute to the excitation of a detector [4]. It was pointed out the extra terms was missed in [3], which shapes a polarization cloud about an Unruh detector [5]. The above analysis and detailed analysis (including non-uniformly accelerated observers) in [6] showed that in a 2 dimensional toy model, there is no radiation flux from the detector. Recently it was found that there does exist a positive radiated power of quantum nature emitted by the detector in 4 dimensional space [7].

The response of an Unruh observer to the Minkowski vacuum is independent of its inherent structure: The distribution of the different excited modes perceived by the observer depends only the acceleration of the observer. This imply the Unruh temperature is a “geometric” effect, which may cause boundary puzzle in the Unruh effect [8]. About this problem and various other puzzles, see clarifications in [9].

The Unruh’s original result has been generalized to accelerating observer in de Sitter (dS) and Anti-de Sitter (AdS) [10], who perceives a thermal bath with temperature

$$T = \frac{(A^2 + \frac{\Lambda}{3})^{1/2}}{2\pi}. \quad (1)$$

There is a problem in this equation at first sight: If the acceleration of the observer is too small, i.e., $A^2 < -\frac{\Lambda}{3}$ in the AdS case, he will perceive an imaginary temperature. The resolution is that his motion becomes spacelike. A brane is one of the most important conception in high energy physics and cosmology in recent years. Braneworld cosmology has been extensively studied [11]. In the braneworld scenario a brane is moving in the bulk, and generally speaking, its proper acceleration does not vanish. One may expect that a brane, qua an observer in the bulk space, should perceive the Unruh radiation. The Unruh effect of RS II braneworld is investigated in [12]. It is found that (1) keeps valid. One only needs to replace $\Lambda$ by the effective cosmological constant on the brane. There is slight difference on the explanation from an ordinary particle observer in AdS space. $3A^2$ is unnecessarily larger than the absolute value of the effective cosmological constant on the brane. If $A^2 < |\Lambda|_{\text{eff}}/3$, the brane just perceives a vacuum state. However, there is a serious problem to deduce the Unruh effect in AdS space. It is found that there are several natural choices of vacua, corresponding to the fact that AdS space is not globally hyperbolic [13]. The specification of the vacuum state then depends on boundary conditions imposed at timelike infinity. A particle detector following a timelike worldline in an AdS space will in some cases detect a flux of particles with a thermal or modified thermal spectrum. Compared to the RS model, the
brane world model proposed by Dvali, Gabadadze and Porrati (DGP) is also very interesting. In the DGP model, the bulk is a flat Minkowski space, but a reduced gravity term appears on the brane without tension. The nature choice of the vacuum state of the Minkowski space is unique. The quantum fluctuations of DGP model is recently investigated in. We shall investigate the Unruh effect for DGP brane in this paper.

This paper is organized as follow: In the next section we give the basic construction of DGP model and present the Unruh temperature the perceived following the method in. In section III, we investigate the unruh temperature of a DGP brane in detail and find the thermal equilibrium condition for the Unruh radiation and the brane (evaluated by its geometric temperature). Our conclusion and discussion appear in section IV.

II. UNRUH EFFECT FOR A DGP BRANE

The basic construction of DGP model can be written as follow,\footnote{14}

\[
S = S_{\text{bulk}} + S_{\text{brane}},
\]

where

\[
S_{\text{bulk}} = \int d^5 x \sqrt{-(5)g} \left[ \frac{1}{2 \kappa^2} (5) R \right],
\]

and

\[
S_{\text{brane}} = \int d^4 x \sqrt{-g} \left[ \frac{1}{\kappa^2} K^\pm + L_{\text{brane}}(g_{\alpha\beta}, \psi) \right].
\]

Here \( \kappa^2 \) is the 5-dimensional gravitational constant, \((5) R \) is the 5-dimensional scalar curvature. \( x^\mu \) (\( \mu = 0, 1, 2, 3 \)) are the induced 4-dimensional coordinates on the brane, \( K^\pm \) is the trace of extrinsic curvature on either side of the brane and \( L_{\text{brane}}(g_{\alpha\beta}, \psi) \) is the effective 4-dimensional Lagrangian, which is given by a generic functional of the brane metric \( g_{\alpha\beta} \) and matter fields \( \psi \) on the brane. Consider the brane Lagrangian consisting of the following terms

\[
L_{\text{brane}} = \frac{\mu^2}{2} R - L_m,
\]

where \( \mu \) is the reduced 4 dimensional Planck mass, \( R \) denotes the scalar curvature on the brane, and \( L_m \) stands for the Lagrangian of other matters on the brane. Assuming an Friedmann-Robertson-Walker (FRW) metric on the brane, we derive the Friedmann equation on the brane\footnote{16},

\[
X = \frac{\rho}{3 \mu^2} + \frac{2}{r_c^2} + \frac{2 \epsilon}{r_c} \left( \frac{\rho}{3 \mu^2} + \frac{1}{r_c^2} \right)^{1/2},
\]
where $\rho$ denotes matter energy density on the brane, $r_c = \kappa^2 \mu^2$, denotes the cross radius of DGP brane, $\epsilon = \pm 1$, represents the two branches of DGP model, and $X$ is defined as

$$X = H^2 + \frac{k}{a^2}.$$  

(7)

Here $H$ is Hubble parameter, $k$ is spatial curvature, and $a$ is the scale factor of the brane. By using the junction condition across the brane

$$[K_{\mu\nu}]^\pm = -\kappa^2 (T_{\mu\nu} - \frac{1}{3} g_{\mu\nu} T),$$  

(8)

where $[K_{\mu\nu}]^\pm$ denotes the difference of the extrinsic curvatures of the two sides of the brane, $T_{\mu\nu}$ represents the energy-momentum confined to the brane, we derive the acceleration of brane [11],

$$A = \frac{\kappa^2}{6} (2\rho_e + 3p_e).$$  

(9)

Here $\rho_e$ and $p_e$ are the effective energy density and pressure of the brane, respectively. We see that, generally speaking, $A$ does not vanish. So the brane should perceive Unruh-type radiation in the bulk. Several different methods have been proposed to derive Unruh effect since Unruh’s original work [17].

From now on we consider an $n-1$ dimensional DGP brane embedded in an $n$ dimensional space. For simplicity we consider a detector coupling to a massless scalar field. Assume the brane, as a particle detector, moves along the worldline $x^i(\tau) (i = 0, ..., n - 1)$, where $\tau$ is proper time of the line. The Lagrangian $cm(\tau)\phi[x(\tau)]$ describes the interaction between detector and field, where $c$ is the coupling constant and $m$ denotes the momentum operator of the detector. Following the method in [17] that, for a small number $c$, the probability amplitude of the transition from the ground state $|E_0 >$ of a particle detector coupled to a scalar field in its vacuum state $|0_{DGP} >$, where $DGP$ stands for DGP brane world, reads

$$c^2 \sum_E | < E|m|0_{DGP}, E_0 > |^2 \mathcal{F}(E)$$  

(10)

where the detector response function $\mathcal{F}(E)$ are defined as

$$\mathcal{F}(E) = \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau' G^+(x(\tau), x(\tau')) e^{-i(E-E_0)(\tau-\tau')},$$  

(11)

and the Wightman Green function $G^+(x, x')$ is defined as

$$G^+ = | < 0|\phi(x)\phi(x')|0 > .$$  

(12)

It is clear that for a general trajectory the response function is not zero. For a stationary trajectory the Wightman Green function becomes a function of $\Delta \tau = \tau - \tau'$. Since the detector
will detect infinite particles in its whole history, the response function (11) is not well-defined. Under this situation it will make sense to consider the response function per unit time

$$\frac{\mathcal{F}(E)}{U} = \int_{-\infty}^{\infty} d\Delta \tau e^{-i(E-E_0)\Delta \tau} g^+(\Delta \tau).$$

(13)

The explicit form of Wightman Green function of a massless scalar field in n-dimensional Minkowski space reads,

$$G^+(x,x') = \frac{\Gamma(n/2 - 1)}{4\pi^{n/2} s^{n-2}} e^{-ir_c |y|/r_c},$$

(14)

where $s$, $r$ are defined as

$$s^2 = (t - t' - i\epsilon)^2 - (r - r')^2 - (y - y')^2,$$

(15)

and

$$r^2 = (t - t' - i\epsilon)^2 - (r - r')^2,$$

(16)

where $t$ is a time-like coordinate, $r$ denotes spatial position vector on the brane of the detector, and $y$ denotes the coordinate of the 5th dimension. For a trajectory with constant acceleration $A$, one can calculate the unit-time response function (13) directly,

$$\frac{\mathcal{F}(E)}{U} = \frac{\Gamma(n/2 - 1)E^{n-3}b_{n-3}\epsilon^{n}/r_c}{(4\pi)^{n/2} 1} e^{2\pi(E+r_c^{-1})/A - (-1)^n},$$

(17)

where the sequence $b_n$ reads

$$b_n = 4\pi \sum_{l=0}^{n-3} \frac{1}{l!},$$

(18)

for any dimension $n \geq 3$. It is easy to check that it goes to the standard result when $n = 4$ and $r_c \to \infty$. An interesting property of the distribution function (17) is that a term $r_c^{-1}$, which can be treated as chemical potential of the Unruh radiations, appears, though we only consider the massless modes. When $r_c \to \infty$ this chemical potential disappears. The other property is that when $n$ is an odd number, the Unruh radiation particles behaves as Fermion, though all the excited modes we integrated to derive the response function are Bosonic. It is not a phenomenon completely new. In 1986, Unruh pointed out that the Fermi-Dirac factor would appear in the response function for an accelerated monopole of a massless field in an odd number of space dimension, arising from integration over all modes for a scalar field (18). From (17) we obtain the familiar result, the temperature of the Unruh radiation

$$T = \frac{|A|}{2\pi}$$

(19)
Here, if $A$ is a negative number, which only means the direction of the acceleration is contrary to our reference direction. The corresponding Unruh temperature is the same as the case of the positive acceleration $-A$. So we just take the absolute value of $A$.

Now we come back to a 4 dimensional DGP brane in 5 dimensional bulk. Consider (20), we get the temperature of the brane,

$$T = \frac{\kappa^2}{12\pi} |2\rho_e + 3p_e|.$$  \hspace{1cm} (20)

III. A COMPARATION BETWEEN THE UNRUH TEMPERATURE AND \textit{GEOMETRIC} THE TEMPERATURE OF THE BRANE

We see that the temperature of Unruh radiation the brane perceived, displayed in (20), only relates to the energy density and pressure of the brane for a given DGP model. If there is no energy exchange between bulk and brane, a natural question emerges: Is the Unruh radiation hotter or colder than the brane? If there is energy exchange between bulk and brane, we also need the temperature of the brane to decide the direction of the energy flux. But there several different particles on the brane. They had gone out of thermal equilibrium long before. Hence we’d better to find the characteristic temperature of the braneworld independent of its detailed microscopic construction. As we have pointed out Unruh radiation is a geometric feature of the space, which is unrelated to the construction of the detector. So we require geometric temperature of the brane to make this comparison.

We know that the formulae of black hole entropy and temperature have a certain universality in the sense that the horizon area and surface gravity are purely geometric quantities determined by the space geometry, once Einstein equation determines the space geometry. But how does general relativity know that the horizon area of black hole is related to its entropy and the surface gravity to its temperature? In fact, it was found in [19] that one can obtain Einstein field equation from the proportionality of entropy to the horizon area together with the fundamental relation $\delta Q = TdS$, supposing the relation holds for all local Rindler causal horizons through each space point. More directly, applying the first law of thermodynamics to the apparent horizon of an FRW universe and assuming the geometric entropy given by a quarter of the apparent horizon area and the temperature given by the inverse of the apparent horizon, the Friedmann equation can be derived [20]. This celebrated result implies that the inverse number of the apparent horizon is the geometric temperature of the universe, which is independent of the various particles confined to the brane. The apparent horizon in the dynamical universe is a marginally trapped surface with
vanishing expansion. Naive calculation yields the radius of the apparent horizon

\[ R_A = X^{-1/2}, \]  

(21)

where \( X \) is defined in (14). And then the geometric temperature of the brane reads

\[ T' = \frac{R_A}{2\pi} = \frac{X^{1/2}}{2\pi} = \frac{1}{2\pi} \left\{ \frac{\rho}{3\mu^2} + \frac{2}{r_c^2} + \frac{2\epsilon}{r_c} \left( \frac{\rho}{3\mu^2} + \frac{1}{r_c^2} \right)^{1/2} \right\}. \]  

(22)

To obtain the explicit form of the Unruh temperature (20), we need the effective density \( \rho_e \) and pressure \( p_e \). By a variation of action (2) with respect to the 5 dimensional metric and by the junction condition across the brane, we get the effective 4 dimensional Einstein equation on the brane, which says [21],

\[ \rho_e = \rho - \mu^2 (3X), \]  

(23)

\[ p_e = p + \mu^2 \left( \frac{2a^2}{a} + X \right). \]  

(24)

As for standard cosmology, \( \frac{2a}{a} \) can be obtained from the Friedmann equation and the continuity equation \( \dot{\rho} + 3H(\rho + p) = 0 \). Here, similarly, from Friedmann equation (6) and the continuity equation, we derive

\[ \frac{\ddot{a}}{a} = X - \frac{1}{2\mu^2} \frac{\rho + p}{1 + \frac{\epsilon}{\sqrt{X/r_c}}}, \]  

(25)

where we have supposed there is no energy exchange between bulk and brane. Substitute (25) into (21) and then substitute (24) and (23) into (9), we arrive at

\[ T = \left| \frac{\kappa^2}{12\pi} (2\rho + 3p) + \frac{r_c}{4\pi} \left( X - \frac{1}{2\mu^2} \frac{\rho + p}{1 + \frac{\epsilon}{\sqrt{X/r_c}}} \right) \right|, \]  

(26)

where \( X \) is given by (6). Therefore we obtain the explicit form temperature of the brane, which is fully determined by its energy density and pressure. We see that both Unruh temperature (26) and geometric temperature (22) are unrelated to the spatial curvature of the brane.

To further investigate the evolution of the Unruh temperature and the geometric temperature during the history of the universe, we write them in dimensionless form,

\[ \frac{T'}{H_0} = \frac{X^{1/2}}{2\pi H_0} = \frac{1}{2\pi} \left[ x\Omega_m + 2\Omega_{r_c} + 2\epsilon \sqrt{\Omega_{r_c}} (x\Omega_m + \Omega_{r_c})^{1/2} \right]^{1/2}, \]  

(27)

\[ \frac{T}{H_0} = \frac{|A|}{2\pi H_0} = \left| \frac{x\Omega_m}{2\pi \sqrt{\Omega_{r_c}}} (1 + \frac{3w}{2}) + \frac{1}{4\pi \sqrt{\Omega_{r_c}}} \left[ \frac{X}{H_0^2} - 3\Omega_m x (1 + w) \left( \frac{1}{1 + \frac{\epsilon}{\sqrt{X/H_0}} \sqrt{\Omega_{r_c}}} \right) \right] \right|, \]  

(28)
where $H_0$ is the present value of the Hubble parameter, $w$ is the parameter of state equation of the matter confined to the brane, $x \triangleq \rho/\rho_0$, and $\Omega_m, \Omega_{rc}$ are defined as

$$\Omega_m = \frac{\rho_0}{3\mu^2 H_0^2},$$

$$\Omega_{rc} = \frac{1}{r_c^2 H_0^2}.$$  \hfill (29)

Here $\rho_0$ denotes the present density. First, we study various limits of the two types of the temperatures. When $r_c \rightarrow \infty$, $\Omega_{rc} \rightarrow 0$, we expect DGP brane theory to recover to standard 4 dimensional cosmology. It is just the case for geometric temperature of the brane,

$$\lim_{r_c \rightarrow \infty} T' = \frac{\sqrt{\rho}}{2\pi \sqrt{3\mu}}.$$  \hfill (31)

But for Unruh temperature, it is a different situation. Though all dynamical effect of the 5th dimension vanishes when $r_c \rightarrow \infty$, the Unruh temperature does not vanish. One can check

$$\lim_{r_c \rightarrow \infty} T = \frac{1}{2}(5 + 3w) \frac{\sqrt{\rho}}{2\pi \sqrt{3\mu}}.$$  \hfill (32)

This result implies that even the gravitational effect of the extra dimension vanishes, the quantum effect resides. The other important limit is the limit when the matter on the brane is infinitely diluted, i.e., $\rho \rightarrow 0$,

$$\lim_{\rho \rightarrow 0} T' = \frac{\sqrt{2} \sqrt{1 + \epsilon}}{2\pi r_c},$$

$$\lim_{\rho \rightarrow 0} T = \frac{1}{2\pi} \frac{1 + \epsilon}{r_c}.$$  \hfill (33)

One may conclude

$$\lim_{\rho \rightarrow 0} T' = \lim_{\rho \rightarrow 0} T,$$  \hfill (35)

for either $\epsilon = 1$ or $\epsilon = -1$. However

$$\lim_{\rho \rightarrow 0} \frac{T'}{T} = 1,$$  \hfill (36)

for $\epsilon = 1$,

$$\lim_{\rho \rightarrow 0} \frac{T'}{T} = \frac{1}{|2 + 3w|},$$  \hfill (37)

for $\epsilon = -1$, since the “speeds” are different when $T$ and $T'$ go to zero.

The other point deserved to note is $T'$ will be always equals $T$ if the parameter of state equation of the matter confined to brane $w = -1$. In this case,

$$T = T' = \frac{1}{2\pi} \frac{1}{r_c} + \epsilon \sqrt{\frac{1}{r_c^2} + \frac{\rho}{3\mu^2}}.$$  \hfill (38)
FIG. 1: $\eta \equiv T'/T$ as a function of $x$ and $\Omega_{rc}$. In this figure we consider the pure dust universe, i.e., $w = 0$ and we set $\Omega_m = 0.3$. (a) The branch $\epsilon = -1$. (b) The branch $\epsilon = +1$.

![Graph showing $\eta$ as a function of $x$ and $\Omega_{rc}$](image)

FIG. 2: $u \equiv 2\pi T/H_0$ as a function of $v \equiv 2\pi T'/H_0$. In this figure we also consider the pure dust universe, i.e., $w = 0$ and we set $\Omega_m = 0.3$. (a) The branch $\epsilon = 1$. (b) The branch $\epsilon = -1$.

![Graph showing $u$ as a function of $v$](image)

We see that the expressions of $T$ and $T'$ are rather complicated. Hence we plot two figures to give their visual profiles. Fig. 1 illustrates $T'/T$ as a function of $x$ and $\Omega_{rc}$. As we have pointed out, when $\rho \rightarrow 0$, $T'/T \rightarrow 1$ for the branch $\epsilon = 1$; while when $\rho \rightarrow 0$, $T'/T \rightarrow 1/|2 + 3w| = 1/2$ for the branch $\epsilon = -1$. Fig. 2 directly displays $T$ as a function of $T'$. It is clear that $T \rightarrow 0$ when $T' \rightarrow 0$, as shown by [33] and [34] for the branch $\epsilon = -1$. Our numerical result also shows an interesting property of the branch $\epsilon = 1$: $T$ is almost a linear function of $T'$. 

![Graph showing $u$ as a function of $v$](image)
IV. CONCLUSION AND DISCUSSION

How to choose ground state or vacuum is a very important issue in modern field theory and cosmology. Unruh effect is one of the most impressive effect in quantum field theory, which un-obscurely shows that there are different vacua even in Minkowski space. The vacuum state for an inertial observer becomes a thermal state for an accelerating observer. In braneworld theory, generally speaking the brane accelerates in the bulk. Hence it should also perceive Unruh radiation in the bulk. In this paper we show for a DGP brane in a Minkowski bulk it is just the case. We investigate the case that the brane, as a particle detector, coupled to a massless scalar filed in the bulk. As for a point detector, the temperature of the radiation perceived is proportional to the acceleration of the brane. But a new term, which can be regarded as chemical potential, appears in the distribution function.

We may be curious that whether the Unruh radiation is hotter or colder than the brane. The Unruh effect is a geometric effect in the sense that it is independent of the construction of the detector. We also find a characteristic geometric temperature of the brane, which is proportional to the inverse of the apparent horizon. By the precondition that there is no energy exchange between bulk and brane, we compare the two temperatures in various cases. We find Unruh temperature equals the geometric temperature when the the density of matter on the brane goes to zero for branch $\epsilon = 1$, no matter what the value of the cross radius $r_c$ and the spatial curvature of the brane. And if the state equation of the matter on the brane satisfies $p = -\rho$, the Unruh temperature always equals the geometric temperature of the brane for both the two branches, which is also independent of the cross radius and the spatial curvature.

A significant difference between particle Unruh observer and brane Unruh observer is that particle observer is always in thermal equilibrium with the Unruh radiation it perceived, while an brane observer can have a different temperature of the Unruh radiation, which has been shown in the text and in [12]. As we pointed out before, the validity of Unruh radiation have been confirmed in detail [7]. Therefore if energy exchange is allowed between bulk and matter brane, an energy flux between the bulk radiation and the matter confined to the brane will come forth, which may accelerate our universe. Clearly, in all cases $\rho > 0$, the Unruh temperature of the bulk radiation $T$ is higher than than the geometric temperature $T'$ for a dust dominated brane, which means an energy influx to the brane can appear. We shall study this possibility in [22].

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