GRAVITATIONAL WAVE SIGNAL FROM ASSEMBLING THE LIGHTEST SUPERMASSIVE BLACK HOLES

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ABSTRACT

We calculate the gravitational wave signal from the growth of $10^{7} M_{\odot}$ supermassive black holes (SMBHs) from the remnants of Population III stars. The assembly of these lower mass black holes (BHs) is particularly important because observing SMBHs in this mass range is one of the primary science goals for the Laser Interferometer Space Antenna (LISA), a planned NASA/ESA mission to detect gravitational waves. We use high-resolution cosmological N-body simulations to track the merger history of the host dark matter halos, and model the growth of the SMBHs with a semianalytic approach that combines dynamical friction, gas accretion, and feedback. We find that the most common source in the LISA band from our volume consists of mergers between intermediate-mass BHs and SMBHs at redshifts less than 2. This type of high mass ratio merger has not been widely considered in the gravitational wave community; detection and characterization of this signal will likely require a different technique than is used for SMBH mergers or extreme mass ratio inspirals. We find that the event rate of this new LISA source depends on prescriptions for gas accretion onto the BH as well as an accurate model of the dynamics on a galaxy scale; our best estimate yields ~40 sources with a signal-to-noise ratio greater than 30 occurring within a volume like the Local Group during SMBH assembly—extrapolated over the volume of the universe yields ~500 observed events over 10 years, although the accuracy of this rate is affected by cosmic variance.

Key words: black hole physics – galaxies: evolution – gravitational waves – methods: numerical

1. INTRODUCTION

Any mass configuration that creates a time-dependent quadrupole of the stress-energy tensor will radiate away energy in gravitational waves. One of the best examples of this is the binary pulsar system J1915+1606, whose measured orbital decay of $76 \mu s/yr$ is extremely well described by gravitational wave emission (Hulse & Taylor 1975). However, even for very massive and compact astrophysical objects, the strain amplitude of this fluctuation in spacetime, $h$, is much less than $10^{-20}$. For this reason, gravitational radiation has yet to be directly detected by current ground-based gravitational wave observatories such as the LIGO, VIRGO, and TAMA.

The strongest expected gravitational wave sources occur from the inspiral and merger of binary supermassive black holes (SMBHs). At the current epoch, nearly every galaxy is thought to host an SMBH with a mass of $10^6$–$10^{10} M_{\odot}$ (e.g., Kormendy & Richstone 1995). Observational evidence suggests that the black holes (BHs) are embedded in galactic nuclei even at high redshift and grow more massive as the galaxy grows (e.g., Soltan 1982; David et al. 1987; Silk & Rees 1998; Kauffmann & Haehnelt 2000; Monaco et al. 2000; Schneider et al. 2002a; Granato et al. 2001; Merloni 2004). Since galaxies are thought to assemble by merging, each galaxy merger is expected to spawn a binary SMBH (Bogdanović et al. 2009; Mayer et al. 2007; Escala et al. 2006; Hopkins et al. 2006; Wyithe & Loeb 2005; Komossa et al. 2003; Menou et al. 2001; Adams et al. 2001; Haehnelt & Kauffmann 2002). In general, the inspiral and coalescence of low mass SMBHs are expected to spawn the largest amplitude gravitational wave signal (e.g., Haehnelt 1998). The lightest SMBH binaries, with masses $10^5$–$10^7 M_{\odot}$, have coalescence frequencies between $\sim 10^{-4}$ and 1 Hz—squarely within the frequency range of the NASA/ESA Laser Interferometer Space Antenna (LISA), which has been proposed to launch in the next decade. LISA observations will complement ground-based gravitational wave observatories by broadening the observable gravitational wave spectrum to include low frequencies. In particular, gravitational wave detections made by LISA will be able to directly map the assembly of these lightest SMBHs (Haehnelt 1994; Menou et al. 2001; Enoki et al. 2004).

In the current picture of SMBH assembly, the BH begins life as a low mass “seed” BH at high redshift. It is not clear, though, when exactly these BH seeds emerge or what mass they have at birth. SMBH seeds may have been spawned from the accretion of low angular momentum gas in a dark matter halo (Koushiappas et al. 2004; Bromm & Loeb 2004), the coalescence of many seed BHs within a halo (Begelman & Rees 1978; Islam et al. 2004), or from an IMBH formed, perhaps, by runaway stellar collisions (Portegies Zwart et al. 2004; Miller & Colbert 2004; van der Marel 2004). However, the most likely candidates for SMBH seeds are the remnants that form from the first generation of stars sitting deep within dark matter halos (Madau & Rees 2001; Heger et al. 2003; Madau & Rees 2001; Volonteri et al. 2003a; Islam et al. 2003; Wise & Abel 2005)—so called Population III stars. With masses $< 10^5 M_{\odot}$, these relic seeds are predicted to lie near the centers of dark matter halos between $z \sim 12$ and 20 (Bromm et al. 1999; Abel et al. 2000, 2002). Structure formation dictates that dark matter halos form in the early universe and hierarchically merge into larger bound objects, so naturally as dark matter halos merge, seed BHs sink to the center through dynamical friction and eventually coalesce. Dark matter halo mergers become synonymous, then, with BH mergers at these masses and redshifts. This means that although the seed formation stops at $z \sim 12$ as Population III supernovae rates drop to zero (Wise & Abel 2005), SMBH growth continues as dark matter halo mergers proceed to low redshifts.
Gas accretion is thought to play a critical role in fueling the early stages of BH growth (David et al. 1987; Kauffmann & Haehnelt 2000; Merloni 2004), and this may explain the tightness of the $M_{BH} - \sigma$ relation (Burkert & Silk 2001; Haehnelt & Kauffmann 2000; Di Matteo et al. 2005; Kazantzidis et al. 2005; Robertson et al. 2006). Since high-redshift galaxies are thought to be especially gas rich, each merger brings a fresh supply of gas to the center of the galaxy, and new fuel to the growing SMBH (Mihos & Hernquist 1994; Di Matteo et al. 2003). From a combination of gas accretion and binary BH coalescence, it is thought that these Pop III generated seeds may form the SMBHs we observe today (Soltan 1982; Schneider et al. 2002b).

During a galaxy merger, each BH sinks to the center of the new galaxy potential due to dynamical friction and eventually becomes bound as a binary (Kazantzidis et al. 2005; Escala et al. 2005). Dynamical friction then continues to shrink the orbit until the binary is hard (i.e., the separation between each BH, $d_{BBH}$, is such that the system tends to lose energy during stellar encounters; Heggie et al. 2007). Thereafter, further decay is mediated by three-body scattering with the ambient stellar background until the binary becomes so close that the orbit can lose energy via gravitational radiation. In studies of static, spherical potentials, it may be difficult for stellar encounters alone to cause the binary to transition between the three-body scattering phase and the gravitational radiation regime (Milosavljevic & Merritt 2003). However, in gas-rich or non-spherical systems, the binary rapidly hardens and coalesces into one BH, emitting copious gravitational radiation in the process (Mayer et al. 2007; Kazantzidis et al. 2005; Berczik et al. 2006; Sigurdsson 2003; Holley-Bockelmann & Sigurdsson 2006).

In our previous work, we calculated the cosmological merger rate for BHs between 200 and $3 \times 10^6 M_\odot$ from redshift 49-0 (Micic et al. 2007, 2008). Our approach combined high-resolution, small-volume cosmological N-body simulations with analytic prescriptions for the dynamics of merging BHs below our resolution limit; this allowed us to explore different BH growth mechanisms and seed formation scenarios while also accurately simulating the rich and varied merger history of the host dark matter halos.

In this paper, we calculate the gravitational wave signal from the BH mergers involved in assembling an SMBH at the center of the Milky Way analogue our simulation volume. The volume is designed to provide one possible evolutionary path for a region like our Local Group, and as such, it should contain SMBHs on the light end of the SMBH mass spectrum. We review the details of our simulation in Section 2, and in Section 3 we describe how to calculate the gravitational wave signal from two merging BHs. We discuss our results and implications in Section 4.

2. TRACKING THE MERGER HISTORY OF SMBHs

In Micic et al. (2008), we performed a high-resolution “zoom-in” cosmological N-body simulation of a comoving section of a ΛCDM universe ($\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$, $\sigma_8 = 0.9$, and $h = 0.7$) from $z = 49$ to $z = 0$. The high-resolution region was a box $10 h^{-1}$ Mpc on each side, for a total comoving volume of $1000 h^{-3}$ Mpc$^3$. The volume is designed to provide one possible evolutionary path for a region like our Local Group, and as such, it should contain SMBHs on the light end of the SMBH mass spectrum. Our mass resolution is $8.85 \times 10^5 M_\odot$, and our spatial resolution is 2 kpc. After the simulation is complete, we identified halos with at least 32 particles using P-Groupfinder (Springel et al. 2001) and seed BHs those halos in the appropriate mass and redshift range to host Pop III stars. Note that we are using WMAP5 (Spergel et al. 2007) cosmological parameters in this study to compare with our previous work; however, at the time of this paper’s submission, “zoom-in” simulations of several small volumes are underway with WMAP5 parameters to better explore cosmic variance. This will allow us to better pin down the rate of these new gravitational wave sources.

In our hybrid method, we combine the dark matter halo merger trees obtained in numerical simulations with an analytical treatment of the physical processes that arise in the dynamics of galaxy and BH mergers. Since some of the processes are ill constrained, we probe the effect of different BH growth recipes on the final BH mass function. In general, we assume that each dark matter halo is described by a Navarro–Frenk–White profile (Navarro et al. 1997), and include the effects of dynamical friction and merger-induced gas accretion onto the SMBH, as well as the SMBH merger itself. This hybrid approach generates, for each recipe, a census of the number and mass ratio of the mergers in our volume at each redshift.

One of the surprises from this method is that the BH mass function is quite sensitive to the dynamics of the host dark matter halo on large scales. Previous semianalytic work on SMBH growth all assume that the host galaxies assembled from binary mergers whose timescale was dictated by Chandrasekhar dynamical friction (Chandrasekhar 1943). Numerical simulations, however, indicate that galaxies can often assemble from multiple near-simultaneous mergers, and that the merger...
timescale is longer than what the Chandrasekhar dynamical friction timescale suggests. Simulations indicate that the merger timescale is only smaller by a factor 3 for minor mergers at $z = 0$ (Boylan-Kolchin et al. 2008), so given the myriad uncertainties in the dynamics acting in the other stages of the merger, this factor of a few was neglected. However, a longer dynamical friction timescale reduces the total number of BH mergers in a volume of the universe by pushing some mergers to the future. For example, we found that the Boylan-Kolchin treatment for dynamical friction resulted in 1056 SMBH mergers in our volume, while Chandrasekhar dynamical friction yielded 1245 mergers. In addition, most of the current work on SMBH evolution indicates that gas accretion dominates the BH’s growth and that this gas is funneled onto the SMBH by the merger process itself (e.g., Robertson et al. 2006; Hopkins et al. 2005; Di Matteo et al. 2003; Volonteri et al. 2003b); if the gas is driven to the center over a longer timescale, then each merger may excite a longer gas-fueled growth spurt. In the context of gravitational wave source prediction, these two effects may combine to make the SMBH mergers louder and at lower frequency than what has been predicted with a Chandrasekhar dynamical friction prediction. We will return to this in Section 4.

In this paper, we select a sample of the BH growth recipes from Micic et al. (2008) and calculate the gravitational wave signal from each of the BH mergers in the volume. Briefly, the recipes span two choices for the dynamical friction timescale and three choices for the mass ratio of the halos that excite merger-driven gas accretion onto the BH (4:1, 10:1, and $\infty$ – a.k.a. “dynamical friction”).

The next section outlines this BH growth prescription.

### 2.1. SMBH Growth Prescription

The SMBH in our model grows through a combination of BH mergers and gas accretion. To better separate the effects of gas accretion on the BH, we include a dry growth scenario, where the BH grows through mergers only. At high redshift, this galaxy merger-driven approach is likely a good assumption, though note that at low redshift when mergers are infrequent, secular evolution, such as bar instabilities, may dominate the gas (and therefore BH) accretion. Integrated over the whole of a BH lifetime, though, this major merger-driven accretion is likely to be the dominant source of gas inflow. Since the BH growth is so strongly dependent on what fuel is driven to the center during galaxy mergers, it is important to characterize this merger-driven gas inflow, including the critical gas physics that may inhibit or strengthen this nuclear supply. We are motivated by a recent suite of numerical simulations that include radiative gas cooling, star formation, and stellar feedback to study the starburst efficiency for unequal mass ratio galaxy mergers (Cox et al. 2008), which finds that the gas inflow depends strongly on the mass ratio of the galaxy (see also, e.g., Hernquist 1989; Mihos & Hernquist 1994). This study parameterizes the efficiency of nuclear star formation (i.e., gas supply and inflow), $\alpha$, as a function of galaxy mass ratio:

$$\alpha = \left(\frac{M_i}{M_p} - \alpha_0\right)^{0.5},$$

where $\alpha_0$ defines the mass ratio below which there is no enhancement of nuclear star formation (i.e., gas inflow). Here, the gas accretion efficiency has a maximum of 0.56 for 1:1 halo mergers and falls to zero at $\alpha_0$. This parameterization is insensitive to the stellar feedback prescription. We use $\alpha$ to define how efficiently the merger funnels the galaxy’s gas to the BH accretion disk. Our equation for $\alpha$ differs only slightly from the Cox et al. (2008) work in that we adjust $\alpha_0$ to trigger gas inflow, while their $\alpha_0 = 0.09$.

In particular, we contrasted the BH growth that would result from only major mergers with the growth that would occur if minor mergers were included as well. Our more conservative criterion, for example, allows BHs to accrete gas only if the mass ratio of the host dark matter halo is less than 4:1 (this is synonymous with setting $\alpha_0 = 0.25$). This cuts off the gas inflow earlier than predicted by the galaxy merger simulations, but is useful as a conservative estimate and to compare to our previous work.

We can set an upper constraint on the final BH mass by allowing gas accretion as long as the merging dark matter halos have a mass ratio less than 10:1 (consistent with $\alpha_0 = 0.1$). Note that $\alpha_0 = 0.09$ would imply that mass ratios less than roughly 11:1 would drive gas inflow (Cox et al. 2008), rather than 10:1. Adopting $\alpha_0 = 0.09$ would increase the BH mass only slightly, and since our main goal is to compare to our earlier work, we retain $\alpha_0 = 0.1$.

We used a semianalytic formalism to calculate the dynamical friction decay time and subsequent merger timescale of each BH once it enters a dark matter halo. Now that we have a realistic description of the merger time for each BH within a halo, we can allow the BHs to grow for a physically motivated accretion timescale. The accretion of gas onto both incoming and the central BH starts when the two BHs are still widely separated, at the moment of the first pericenter passage, and continues until the BHs merge (c.f. Di Matteo et al. 2005; Colpi et al. 2007). This sets the accretion timescale, $t_{\text{acc}}$, as follows: $t_{\text{acc}} = t_{\text{dyn}}(r = R_{\text{vir}}) - t_{\text{dyn}}(r = R_{\text{vir}})$, where $t_{\text{dyn}}(r = R_{\text{vir}})$ is the merger timescale including dynamical friction; and $t_{\text{dyn}}(r = R_{\text{vir}})$ is dynamical time at virial radius $R_{\text{vir}}$, which marks the first pericenter pass of the BH. By stopping the accretion as the BHs merge, we roughly model the effect of BH feedback in stopping further accretion.

Putting these pieces together, the mass accreted by a BH during $t_{\text{acc}}(r = R_{\text{vir}})$ is

$$M_{\text{acc}} = M_{\text{BH,0}}(e^{\frac{t_{\text{acc}}}{t_{\text{sal}}}} - 1),$$

where $M_{\text{BH,0}}$ is initial BH mass, and $\alpha$ is starburst efficiency (Cox et al. 2008), and $t_{\text{sal}} \equiv c M_{\text{BH,0}} c^2 / [(1 - \epsilon) L]$, where $c$ is the speed of light, with the SMBH at the center and a new SMBH is formed after having accreted gas for $t_{\text{acc}}$. The accretion time and efficiency both implicitly encode the large-scale dynamics of the merger and the bulk gas accretion into the nuclear region, while $t_{\text{sal}}$ describes the accretion disk physics. As before, we set $t_{\text{sal}}$ to describe sustained Eddington-limited accretion with an efficiency of $\epsilon = 0.1$ (Shakura & Sunyaev 1973).

In Micic et al. (2008), we found that a gas accretion triggered by major mergers (4:1) and a Boylan-Kolchin dynamical friction prescription produced a 4 × $10^7 M_{\odot}$ SMBH from a 200 $M_{\odot}$ seed in place by $z = 5$. We consider this our preferred model as it produces an SMBH consistent with the Sgr A*. Note, however, that the minor merger (10:1) Boylan-Kolchin prescription also generates a 1.3 × $10^7 M_{\odot}$ BH from the same seed, and can be considered a viable model for an M31-like SMBH.

### 3. Calculating the Gravitational Wave Signal

In order to compare our results with other published results, we follow the approach of Sesana et al. (2004) to calculate the gravitational wave signal from the BH mergers in our simulation.
The orbital motion of the binary system constitutes an important aspect of gravitational radiation. The gravitational wave strain, \( h \), measures the strength of the propagating wave. The amplitude depends on the comoving distance to the binary, \( d \), the rest-frame gravitational wave frequency, \( f_r \), and the chirp mass of the binary, \( M \). The rest-frame gravitational wave frequency is related to the orbital period, \( P \), of the binary, \( f_r = 2/P \), and the chirp mass depends on the mass of each binary component as follows: \( M = (m_1 m_2)^{3/5}/(m_1 + m_2)^{1/5} \). When averaged over the sky position, polarization, and period, the strain can be written as:

\[
h = \frac{8\pi^{2/3} G^{5/3} M^{5/3}}{10^{1/2}} \frac{c^4 d}{f_r^{2/3}}. \tag{3}
\]

This gravitational radiation will cause the binary components to inspiral on ever tighter and faster orbits. When massive BH binaries are widely separated and orbiting at low frequencies, the gravitational radiation emitted is relatively weak, and therefore the frequency shift per orbit is minuscule. Most of the evolution is spent in this phase, where a frequency shift of order unity takes many orbits to achieve.

As the binary separation slowly shrinks, the gravitational radiation emitted increases dramatically until the binary coalesces. Close to coalescence, the binary rapidly sweeps through many frequencies in one orbit, and at the innermost stable circular orbit (ISCO), the change in frequency per orbit is of order unity.

We set the minimum observed frequency to \( f_{\text{ISCO}} \) for a test particle orbiting around a single Schwarzschild BH. In the BH mergers we consider, the mass ratios are always well outside this test particle limit; we adopt the conventional definition for \( f_{\text{ISCO}} \) nonetheless:

\[
f_{\text{ISCO}} = \frac{c^3}{6^{3/2} \pi G (m_1 + m_2)} (1 + z)^{-1}, \tag{4}
\]

where \( c \) and \( G \) are the speed of light and gravitational constant, respectively, \( m_1 \) and \( m_2 \) are the BH masses, and \( z \) is the redshift.

The frequency shift rate in the rest frame is, to first order,

\[
f_r = \frac{96\pi^{8/3} G^{5/3}}{5c^5} f_r^{11/3} M^{5/3} \tag{5}
\]

The orbital periods of million solar mass BH binaries in the gravitational radiation regime are on the order of hours, which means that multi-year observations could accumulate many cycles of an inspiral at a particular frequency. LISA, for example, is expected to observe the sky for at least a year, and projections place the LISA lifespan at roughly a decade. Million solar mass BHs orbiting in the LISA frequency band of \(~10^{-2}–1\) Hz are years or less from coalescence, so LISA observations will be able to track the inspiral and coalescence phases of SMBH binaries.

The number of cycles accumulated during the inspiral in an observation of duration \( \tau \) depends on the observed gravitational wave frequency, \( n = f \tau \), where \( f = f_r/(1 + z) \). Note that there are two regimes: \( f > n/\tau \), where the binary sweeps through many frequencies in an observation, and \( f < n/\tau \), where the signal builds from several orbits at one frequency.

The observed characteristic strain, \( h_c \), is the strain accumulated in a single observation:

\[
h_c = h\sqrt{n} \sim \frac{1}{\sqrt{3\pi^{2/3} c^3 d}} f_r^{1/6} \quad n < f \tau, \tag{6}
\]

\[
h_c = h\sqrt{f \tau} \sim \frac{8\pi^{2/3} G^{5/3} M^{5/3}}{10^{1/2}} f_r^{1/6} \quad n > f \tau.
\]

Since we have the component mass and redshift of each BH merger in our simulation volume, we can calculate \( h \) and \( h_c \) over an observation span \( \tau \) before merger. Most BHs in our volume will merge within a year of reaching the LISA band, so a three-year observation window should catch these mergers in the act—if this volume is a representative slice of the universe. We distinguish between the mergers directly found in our volume and the mergers extrapolated throughout the universe in Table 2.

4. RESULTS

4.1. A New Gravitational Wave Source

Overall, most of the mergers in our volume are between seed and intermediate-mass BHs (\( O(10^4) M_\odot \)) at redshifts greater than 5 (Figure 1). Though these mergers were critical in assembling the eventual SMBH in our Milky Way analogue, the generated LISA signal-to-noise ratio (S/N) of most of these low mass mergers at high redshift was much less than 1.0. Figures 2 and 3 demonstrates that the overwhelming majority of BH mergers will fall below LISA’s detection limit. Although there are a few mergers that are resolvable by LISA at \( z > 6 \), the number of resolvable sources increases once the SMBH grows to a mass within the LISA band at \( z < 5 \), as can be seen in Figures 4 and 5. This broadly agrees with Sesana et al. (2005), which showed that \~10\% of the precursors to \( 10^9 M_\odot \) SMBHs are expected to be observable with LISA out to redshift 10.

However, Figure 1 also reveals a substantial class of BH mergers with very high mass ratios; these arise from the accretion of smaller satellite halos at low redshift. Since these mergers are nearby, they are easily detectable with LISA. This is in stark contrast to the predictions for assembling the massive end of the SMBH mass spectrum. Lippai et al. (2009) concentrate on merger rates for mass ratios larger than 10; indeed Sesana et al. (2007) showed that the detectable mass ratios are equally distributed in the range of 1–10. In fact, Figure 6 indicates that the most commonly observed BH mergers in our volume have mass ratios of \~1000 or more (depending on the BH growth prescription). Mergers of more equal mass dark matter halos (and subsequently, the coalescence of more equal mass BHs) occurred in this volume at \( z > 8 \), and at this epoch, most of our BHs were too small to be observed by LISA. Figures 1 and 5 bear this out by plotting mapping the number of resolvable sources as a function of mass ratio and redshift for one BH growth prescription over a 10-year observation.

The signals from an equal mass inspiraling system and those with mass ratios of above 10,000 can be quite different. From an analytical standpoint, the difference in signals can be understood by investigating the post-Newtonian expansion of the gravitational wave strain, which uses the orbital velocity as an expansion parameter. Each term in the expansion results in harmonics of the orbital frequency Blanchet et al. (1996). Starting at the first full order, certain select frequencies are scaled by \( \eta \equiv m_1 m_2/(m_1 + m_2)^2 \), which suppresses those frequencies for systems with large mass ratios. Figure 7 compares the power...
Figure 1. Map of the number of BH mergers in our 1000 Mpc$^3$ volume as a function of the log of the BH mass ratio and the redshift of coalescence. The colors represent the number of BH mergers, with a scaling as denoted by the color bar to the left of the map. Here, the dynamical friction is treated with a Boylan-Kolchin formula and gas accretion is triggered for dark matter halo mass mergers with mass ratios smaller than 4:1—major mergers. Note the bimodal distribution and the fact that most mergers are low mass and large mass ratio.

Figure 2. Map of the log of the S/N as a function of the log of the BH mass ratio and the redshift of coalescence. The color bar to the left translates the color scale to the log of the S/N—an S/N of 5 would be dark blue. Here, the dynamical friction is treated with a Boylan-Kolchin formula and gas accretion is triggered for dark matter halo mergers with mass ratios smaller than 4:1—major mergers. Over this 10-year LISA observation, we can expect to detect no equal mass mergers, but that the loud sources will all have mass ratios larger than 100. We re-emphasize that this map tracks the sources only from the formation channel of the lightest SMBHs.

Spectral densities for two SMBH systems: one with two $10^7 M_\odot$ SMBHs and the other with $m_1 = 10^7 M_\odot$ and $m_2 = 10^4 M_\odot$. These figures demonstrate how the equal mass system sweeps through all frequencies while the large mass ratio system shows much more structure, with many of the frequencies being suppressed.

The LISA data streams present unique challenges for data mining. Unlike electromagnetic observatories, LISA is simultaneously sensitive to gravitational wave sources located all throughout the sky. LISA will also be sensitive to a wide variety of astrophysical sources—from SMBH binaries, to millions of close white dwarf binaries within our Galaxy, to EMRIs. The data analysis objective is to conclusively detect a signal and to extract descriptive parameter values. In preparation for the immense data analysis challenge, simulated data have been produced and distributed to the LISA community as part of the Mock LISA Data Challenge (Vallisneri 2009). So far the challenges have focused on order unity mass ratios for the simulated SMBH inspirals. While for most analysis techniques an assumption about the mass ratio is not hardwired into the routine, the
Figure 3. Histogram of the S/N of the BH mergers in our volume (scaled to cosmological volumes) for an assumed three-year *LISA* observation window. See Figure 6 for a description of the panels and colors. Since our cosmological volume is small, we are unable to resolve the growth of the most massive BHs. Most of our mergers are at high redshift between seed and intermediate-mass BHs involved in assembling the light end of the SMBH spectrum; these high redshift, low mass mergers yield low S/N sources.

Table 1

| $M_{bh1}$ | $M_{bh2}$ | Redshift in Volume | Max Redshift |
|-----------|-----------|--------------------|--------------|
| 19398     | 16654     | 8.881              | 9.1          |
| 83201     | 20548     | 7.958              | 17.5         |
| 2723      | 929       | 0.726              | 0.8          |
| 46124     | 9093      | 1.169              | 9.8          |
| 13054368  | 435       | 0.017              | 0.5          |
| 13055368  | 200       | 0.006              | 0.4          |
| 13055168  | 600       | 0.006              | 0.5          |

Notes. Column 1: primary BH mass. Column 2: secondary BH mass. Column 3: redshift at which the merger occurred within the simulation volume. Column 4: the redshift to which the merger can be detected at an S/N of 5 for a three-year *LISA* observation.

routines have not been tested at the more extreme ratios of $10^5$ as suggested here. It is possible that the suppressed harmonics will cause false positives or negatives in some routines because the system will be missed or misidentified.

4.2. The *LISA* Signal from Assembling a Milky Way SMBH

Figure 8 presents the total characteristic strain for seven bright BH mergers in our volume, assuming a 10:1 BH growth recipe and a Boylan-Kolchin dynamical friction treatment. The differences in the two classes of source are clear—the brightest sources are high mass ratio mergers which coalesce in the *LISA* band at low redshift, while the other bright class probes more equal mass mergers of low mass BHs at high redshift. Note that the BHs in the bottommost track actually merge outside the *LISA* band and will be considered an inspiral; this was the only detectable inspiral in our volume.

In Table 1, we determined the redshift at which these seven mergers would become undetectable in a three-year observation.

Understandably, the highly unequal mass mergers can only be detected out to a luminosity distance of roughly 3 Gpc, which is similar to the distance probed by EMRIs (Gair et al. 2004). *LISA* observations of this class of high mass ratio merger, then, may be useful to probe the fraction of BH mass if accreted at late times for this low mass SMBH range.

Figure 9 presents the total characteristic strain for the *LISA* detector from all the BH mergers in our 1000 $h^{-3}$ Mpc$^3$ volume over a three-year observation span. Each curve has a low frequency rise that drops steeply off before hitting a shallow plateau; the rise and drop-off is the signature of the accumulated high mass ratio mergers, while the shallow segment marks the sum of the relatively equal mass mergers. We caution that this implicitly assumes that all of these mergers would take place during this three-year period, or alternatively that this volume is representative of the universe. Issues of cosmic variance aside, we can see that the BH growth prescription strongly influences the total strain in the *LISA* band; this is best seen when dynamical friction is described by Boylan-Kolchin et al. (2008) where the drop-off changes by a decade in frequency in response to a change in the typical mass of the local SMBHs.

4.3. Extrapolating to Universal Black Hole Merger Rates

Although this is plagued by cosmic variance, it is instructive to estimate the number of mergers observable by *LISA* in the universe expected from assembling these lightest SMBHs. Table 2 presents the extrapolated universal *LISA* merger rates. One surprising point is that the *LISA* merger rates depend so strongly on the adopted form of the dynamical friction force. All previous *LISA* merger rate estimates have used a semianalytic technique that employs Chandrasekhar dynamical friction to merge the dark matter halos and to usher the BHs to the inner kiloparsec. However, both perturbation theory (Colpi et al. 1999) and numerical simulations (Weinberg 1989; Holley-Bockelmann & Richstone 1999; Boylan-Kolchin et al. 2008) have shown that the drop-off changes by a decade in frequency in response to a change in the typical mass of the local SMBHs.
2008) have shown that Chandrasekhar dynamical friction approximates the merger time to within a factor of 2, at best. When we employ a dynamical friction formalism that is based on fits to merger timescales from numerical simulations, we find that the BH mergers are delayed to lower redshift. Naively, this would simply make every merger louder. However, in our SMBH growth prescription, there is an additional effect: since the incoming SMBH drives gas inflow to the primary SMBH over a longer timespan, resulting SMBH ultimately grows more massive than with Chandrasekhar dynamical friction. This can place the SMBHs in our volume just out of LISA’s “sweet spot” which will decrease the $S/N$ of the more massive BHs. Paradoxically, then, the LISA event rate for more correct merger times drops by as much as an order of magnitude. For our most realistic model to assemble the lightest SMBHs—gas accretion triggered by major mergers, with a Boylan-Kolchin dynamical friction merger timescale—the merger rate drops by a factor of 2. Over a three-year LISA observation, we should be able to detect at least 70 SMBH mergers with an $S/N$ greater than 30 that are involved in assembling the light end of the SMBH mass spectrum in the universe. If we consider longer LISA observation windows, the number of observed sources increases, naturally, and for a 10-year observation, LISA should detect nearly 500 mergers from the assembly of the lightest SMBHs alone.

5. DISCUSSION

By concentrating on a small cosmological volume, we have been able to model the gravitational wave sources that result from the assembly of a $\sim 10^6$–$10^7 M_\odot$ BH in a Milky Way mass halo. We have calculated the gravitational wave strain for each of the BH mergers in our volume and have determined which of those will be detectable with LISA. Of the 1500 mergers in our volume, we uncovered approximately 300–1200 mergers detectable with an $S/N$ greater than 5 over a LISA observation span of three years.

We found that the most common class of observable BH merger in our volume is between an SMBH and IMBH (of 200–2000 $M_\odot$) at $z < 0.05$. These IMBHs originally resided in
Figure 7. Plus polarization of the gravitational wave amplitude as a function of frequency for two representative BH mergers at redshift 3. Sources are placed at the same random sky position and orientation. Left: merger with BH masses $M_1 = 10^7 M_\odot$ and $M_2 = 10^4 M_\odot$. Right: merger with equal BH masses of $10^7 M_\odot$. Notice the broader bandwidth and richer structure of the high mass ratio merger.

Table 2

| Gas Accretion Trigger | Dynamical Friction Type | $S/N$ | Observation Time | Number of Mergers |
|-----------------------|-------------------------|-------|-----------------|------------------|
| Salpeter 10:1         | Chandra                 | 30    | 10              | 1033             |
| Salpeter 4:1          | Chandra                 | 30    | 10              | 741              |
| Dry mergers           | Chandra                 | 30    | 10              | 608              |
| Salpeter 10:1         | Boylan-Kolchin          | 30    | 10              | 474              |
| Salpeter 4:1          | Boylan-Kolchin          | 30    | 10              | 851              |
| Dry mergers           | Boylan-Kolchin          | 30    | 10              | 425              |
| Salpeter 10:1         | Chandra                 | 5     | 10              | 3768             |
| Salpeter 4:1          | Chandra                 | 5     | 10              | 3367             |
| Dry mergers           | Chandra                 | 5     | 10              | 2892             |
| Salpeter 10:1         | Boylan-Kolchin          | 5     | 10              | 1775             |
| Salpeter 4:1          | Boylan-Kolchin          | 5     | 10              | 3476             |
| Dry mergers           | Boylan-Kolchin          | 5     | 10              | 2516             |
| Salpeter 10:1         | Chandra                 | 30    | 5               | 644              |
| Salpeter 4:1          | Chandra                 | 30    | 5               | 511              |
| Dry mergers           | Chandra                 | 30    | 5               | 400              |
| Salpeter 10:1         | Boylan-Kolchin          | 30    | 5               | 172              |
| Salpeter 4:1          | Boylan-Kolchin          | 30    | 5               | 510              |
| Dry mergers           | Boylan-Kolchin          | 30    | 5               | 280              |
| Salpeter 10:1         | Chandra                 | 5     | 5               | 2893             |
| Salpeter 4:1          | Chandra                 | 5     | 5               | 2674             |
| Dry mergers           | Chandra                 | 5     | 5               | 2261             |
| Salpeter 10:1         | Boylan-Kolchin          | 5     | 5               | 729              |
| Salpeter 4:1          | Boylan-Kolchin          | 5     | 5               | 2176             |
| Dry mergers           | Boylan-Kolchin          | 5     | 5               | 1726             |
| Salpeter 10:1         | Chandra                 | 30    | 3               | 438              |
| Salpeter 4:1          | Chandra                 | 30    | 3               | 401              |
| Dry mergers           | Chandra                 | 30    | 3               | 316              |
| Salpeter 10:1         | Boylan-Kolchin          | 30    | 3               | 73               |
| Salpeter 4:1          | Boylan-Kolchin          | 30    | 3               | 316              |
| Dry mergers           | Boylan-Kolchin          | 30    | 3               | 194              |
| Salpeter 10:1         | Chandra                 | 5     | 3               | 2200             |
| Salpeter 4:1          | Chandra                 | 5     | 3               | 2323             |
| Dry mergers           | Chandra                 | 5     | 3               | 1898             |
| Salpeter 10:1         | Boylan-Kolchin          | 5     | 3               | 328              |
| Salpeter 4:1          | Boylan-Kolchin          | 5     | 3               | 1288             |
| Dry mergers           | Boylan-Kolchin          | 5     | 3               | 1896             |

Notes. Columns 1 and 2 describe the BH growth model; Column 1 indicates which halo mass ratio would trigger gas accretion onto the primary BH; Column 2 indicates the dynamical friction formalism that dictates the gas accretion timescale. Column 3: $S/N$ for detection. Column 4: observation duration. Column 5: predicted number of LISA events. Note that Column 5 is not the entire LISA event rate; it is the conjectured rate from the class of sources involved in assembling the lightest SMBHs. Also note that although the predicted event rate varies here by a factor of 50, even the most pessimistic rate estimate still will provide ample new sources for LISA to observe.
small dark matter halos that merged with the massive primary halo at high redshift and had very long dynamical friction timescales. Before the merger occurs, the incoming IMBH may be observed with the next generation of X-ray telescopes as an Ultra Luminous X-ray source (ULX) with a rate of about $\sim 3–7 \text{ yr}^{-1}$ for $1 < z < 5$. Because of their potential tie to observable ULXs, and because this class of source has a different waveform character than other well known gravitational wave sources, such as equal mass mergers, intermediate mass ratio inspirals (IMRIs), or EMRIs, we have nominally dubbed this class of source as ultra large inspirals (ULIs). The other class was IMBH–IMBH mergers at $z = 2–8$.

Note that we could not resolve the growth of the most massive dark matter halos with our technique and miss the mergers that arise from the assembly of the most massive SMBHs. Therefore, we consider our approach to be complementary to studies like Sesana et al. (2005; 2007)—we believe that the high mass ratio mergers identified here would add to the results found in previous studies that focus on BH growth in present-day halos larger than $\sim 10^{11} M_\odot$. However, the high mass ratio channel that we have identified may well dominate the \textit{LISA} BH merger events. In fact, even our most pessimistic estimate ($\sim 70$ events) yields a comparable number of \textit{LISA} sources as Sesana et al. (2005; $\sim 90$ events).

The S/N listed in Table 1 assumes simple data analysis techniques. If we were to employ similar sophisticated tools as are being developed for the EMRIs, such as matched filtering, it is possible that this class of source may probe slightly larger distances than EMRIs. In any event, current data analysis techniques to extract SMBH signals from \textit{LISA} data streams all assume that the binaries have mass ratios close to unity. As we have shown here, the more complex waveform structure of these ULIs may require a different data analysis strategy.

In our volume, the SMBH was in place at nearly its final mass at redshift 5, and it was built by mergers of hundreds of $O(10^2) M_\odot$ BHs at $z > 5$—these mergers were too weak to be detectable by \textit{LISA}. In fact, the first trace of the growing SMBH occurs at about redshift 7 for our most aggressive gas accretion prescription. This may have implications for how well \textit{LISA} observations can constrain the early growth of these lightest SMBHs.

In this paper, we have neglected gravitational wave recoil, a potentially important mechanism that may inhibit BH growth. Binary BHs strongly radiate linear momentum in the form of gravitational waves during the plunge phase of the inspiral—resulting in a “kick” to the new BH. This, in itself, has long been predicted as a consequence of an asymmetry in the binary orbit or spin configuration. Previous kick velocity estimates, though, were either highly uncertain or suggested that the resulting gravitational wave recoil velocity was relatively small, astrophysically speaking. Now, recent results indicate the recoil can drive a gravitational wave kick velocity as fast as $\sim 4000 \text{ km s}^{-1}$ (e.g., Herrmann et al. 2007; González et al. 2007a, 2007b; Baker et al. 2006; Koppitz et al. 2007; Campanelli et al. 2007; Schnittman & Buonanno 2007). In reality, much smaller values than this maximum may be expected in gas-rich galaxies due to the alignment of the orbital angular momentum and spins of both BHs (Bogdanović et al. 2007). However, even typical kick velocities ($\sim 200 \text{ km s}^{-1}$) are interestingly large when compared to the escape velocity of typical astronomical systems—low mass galaxies, as an example, have an escape velocity of $\sim 200 \text{ km s}^{-1}$ (e.g., Holley-Bockelmann et al. 2008). The effect of large kicks combined with low escape velocity from the centers of small dark matter halos at high redshift plays a major role in suppressing the growth of BH seeds into SMBH. Even the most massive dark matter halo at $z \geq 11$ cannot retain a BH that receives $\geq 150 \text{ km s}^{-1}$ kick (Merritt et al. 2004; Micic et al. 2006). We have submitted a companion paper that incorporates the effect of recoil velocity on the expected merger rates and the growth

Figure 8. Characteristic strain over a three-year observation for selected classes of resolvable BH mergers in the simulation as a function of observed frequency. Here, the dynamical friction is treated with a Boylan-Kolchin formula and gas accretion is triggered for dark matter halo mergers with mass ratios smaller than 10:1. The merger redshift and pre-merger masses of the BH binary are labeled; our loudest sources are either caused by mergers between two intermediate-mass BHs, or between a very light seed BH and the most massive SMBH in our volume.

Figure 9. Total characteristic strain as a function of observed frequency for all the massive BH mergers in the cosmological volume for a three-year observation. The solid curve is the sensitivity curve for \textit{LISA} for a single-arm Michelson configuration.
of a Milky Way mass SMBH, and find that if there is no spin alignment mechanism, then a Pop III seed BH can reach $10^6 M_\odot$ only 20% of the time through merger-driven gas accretion. We are exploring the effect of recoil on the gravitational wave signal in a forthcoming paper.

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