Virtual source for Lommel–Gauss beams

Chen Yue¹, Ren Zhijun¹ ², Dong Yongsheng³ and Peng Baojin¹ ²

¹ Key Laboratory of Optical Information Detecting and Display Technology, Zhejiang Normal University, Jinhua, Zhejiang 321004, People’s Republic of China
² Institute of Information Optics, Zhejiang Normal University, Jinhua, 321004, People’s Republic of China
³ Department of Physics, Jining Normal University, Jining, Inner Mongolia, 012000, People’s Republic of China

E-mail: renzhijun@zjnu.cn

Received 16 May 2018, revised 9 July 2018
Accepted for publication 31 July 2018
Published 13 August 2018

Abstract
We introduce a group of virtual sources for generating Lommel–Gauss beams based on beam superposition to analyze nonparaxial propagation. We typically derive the paraxial approximation and integral representations of the nth-order Lommel–Gauss beams. The first three orders of the nonparaxial corrections for the on-axis field of the Lommel–Gauss beams are analytically obtained. The on-axis intensity distribution of the corresponding nonparaxial corrections is also provided.

Keywords: Lommel-Gauss beams, virtual sources

1. Introduction

Beam propagation is governed by the Helmholtz wave equation, which can be separately solved in orthogonal coordinate systems. Four special solutions of the wave equation, namely, the exponential, Bessel, Mathieu, and parabolic functions, can be obtained in the Cartesian, cylindrical, elliptical, and parabolic coordinates, respectively [1]. The function forms of the four special solutions are independent of the propagation distance. Accordingly, these forms correspond to four types of important nondiffracting beams: ideal plane (Cosh), Bessel [2], Mathieu [3], and parabolic [4] beams, respectively. Notably, none of the Cosh, Bessel, Mathieu, and parabolic functions are integrally square. Therefore, ideal nondiffracting beams with an infinite extent and energy are not physically achievable. In fact, the Cosh–Gauss [1, 5], Bessel–Gauss [1, 6], Mathieu–Gauss [1, 7], and parabolic–Gauss [1] beams, which have finite energy, can be achieved experimentally. These beams can be regarded as quasi nondiffracting beams because they can propagate over an extensive range without significant diffraction [1]. Particularly, Bessel beams are the first nondiffracting beam with practical value amongst the four types of beams, and are extensively applied in laser manipulation [8], medical imaging [9], and other fields. The classical Bessel beam is a centrally symmetric concentric circle. Recently, Kovalev and Zhao [10, 11] theoretically introduced and generated a new type of beam called the Lommel beam, which can be mathematically expressed in terms of the high-order Lommel functions with two variables. Lommel beams are a linear superposition of Bessel beams with identical axial distribution of the former has a reflection symmetry with respect to the Cartesian coordinate axes. Particularly, the intensity pattern across the beam’s section can be tuned continuously through a simple adjustment of the beam parameters [11]. Furthermore, another advantage of Lommel beams lies in the continuous change of its orbital angular momentum, whereas Bessel beams exhibit a discrete change. These distinct properties will provide immense potential for optical trapping, the rotation of microparticles [11], and multiplexing communication [12]. In order to use Lommel beams in experiments, the propagation properties of the Lommel beams should be analyzed. Similar to the Bessel–Gauss beams, only Lommel–Gauss beams [12, 13] with finite energy actually exist. Hence, this study focuses on the Lommel–Gauss beams.

By constructing virtual source points [5–7, 14–16], researchers have studied the nonparaxial propagation characteristics of several kinds of nondiffracting beams with important application value, such as the Bessel–Gauss [6], Hermite–Gauss [15], and Laguerre–Gauss [16] beams. To research the
propagation characteristics of the Lommel–Gauss beams, we introduce a group of virtual sources for generating the nth-order Lommel–Gauss beams, thereby establishing the inhomogeneous Helmholtz equation. Thereafter, we derive the paraxial approximation and nonparaxial integral representations of the Lommel–Gauss beams by using the Fourier–Bessel transform pair and Weber integral formula. Lastly, we obtain the first three orders of nonparaxial corrections for the on-axis field of the Lommel–Gauss beams. Since the research results provide a theoretical foundation for accurately revealing the propagation nature of Lommel–Gauss beams, they lay a theoretical foundation for the application of the Lommel–Gauss beams.

2. Theory

The Lommel beams can be described through a series of Bessel functions of orders \( n + 2p \) (\( p = 0, 1, 2, \ldots \)) with cylindrical coordinates as follows [10, 11, 13, 17]:

\[
E_n(\rho, \varphi, z) = \exp(i\sqrt{k^2 - \alpha^2}) \sum_{p=0}^{\infty} (-1)^p e^{2i\rho} \times \exp[i(n + 2p)\varphi] J_n + 2p(\beta_p),
\]

where \( \exp[i\sqrt{k^2 - \alpha^2}] \) is the propagation factor, \( k = 2\pi/\lambda \) is the wave number of the monochromatic light with wavelength \( \lambda \), \( \beta \) is the beam’s scaling factor, \( c \) is the asymmetry parameter, \( n \) is the integer parameter defining the order of the Lommel beams, and \( J_{n+2p}(\alpha) \) is the \((n + 2p)\)th order of the first type of Bessel function.

The reason why the Lommel–Gauss beams expressed in equation (1) can be expanded into the summation of the infinite terms of the Bessel functions of different order numbers in the cylindrical coordinate system can be understood as the principle of the independence and superposition of beam propagation. In the initial plane of \( z = 0 \), the Lommel–Gauss beams can be expressed as follows [13]:

\[
E_n(\rho, \varphi, z = 0) = \sum_{p=0}^{\infty} E_{n,2p}(\rho, \varphi, z = 0) = \sum_{p=0}^{\infty} (-1)^p e^{2i\rho} \exp[i(n + 2p)\varphi] J_n + 2p(\beta_p) \exp\left(-\frac{\rho^2}{\omega_0^2}\right),
\]

where \( \omega_0 \) is the beam waist width of the Gaussian envelope in \( z = 0 \).

In the physical space \( z > 0 \), constructing a scalar Lommel–Gauss beam that propagates along the \( z \)-axis is preferred. We can assume that \( E_{n,2p}(\rho, \varphi, z) \) is generated by the circular loops [1, 5–7, 14–16] of current strength \( S_{n,2p} \) and radius \( \rho = \rho_z \) situated in the plane \( z = z_{ex} \), with their axes oriented along the \( z \)-axis and having an azimuthal variation given by \( \exp[i(n + 2p)\varphi] \), where \((n + 2p)\) is the azimuthal mode number. The parameters \( S_{n,2p} \), \( \rho_{ex} \), and \( z_{ex} \) are subsequently determined to yield the desired beam. The field \( E_{n,2p}(\rho, \varphi, z) \) is assumed to have the following form:

\[
E_{n,2p}(\rho, \varphi, z) = U_{n,2p}(\rho, z) \times \exp[i(n + 2p)\varphi].
\]

The function \( U_{n,2p}(\rho, z) \) in equation (3) satisfies the inhomogeneous Helmholtz equation:

\[
\left[ \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} - \frac{(n + 2p)^2}{\rho^2} \right] U_{n,2p}(\rho, z) = -S_{n,2p}(n + 2p) \delta(\rho - \rho_{ex}) \delta(z - z_{ex}),
\]

where \( k \) is the wave number, and \( \delta(\cdot) \) is the Dirac delta function. A differential equation for the radial spectrum of \( U_{n,2p}(\rho, z) \) is determined from equation (4) using the Fourier–Bessel transform pair [6]:

\[
U_{n,2p}(\rho, z) = \int_{0}^{\infty} J_{n+2p}(\eta \rho) \tilde{U}_{n,2p}(\eta, z) \eta d\eta,
\]

\[
\tilde{U}_{n,2p}(\eta, z) = \int_{0}^{\infty} J_{n+2p}(\eta \rho) U_{n,2p}(\rho, z) \rho d\rho,
\]

where \( \tilde{U}_{n,2p}(\eta, z) \) is the radial spectrum of \( U_{n,2p}(\rho, z) \), and \( \eta \) is the radial component of the wave vector \( \vec{k} \). By substituting the solution of \( \tilde{U}_{n,2p}(\eta, z) \) into equation (5), we obtain the following expression:

\[
U_{n,2p}(\rho, z) = \int_{0}^{\infty} \eta d\eta J_{n+2p}(\eta \rho) \tilde{U}_{n,2p}(\eta, z) \frac{iS_{n,2p}(n + 2p)}{2\zeta} \exp[i\zeta(z - z_{ex})],
\]

where \( Re(z - z_{ex}) > 0 \) and \( \zeta = \sqrt{(k^2 - \eta^2)} \).

Equation (7) is a complex expression. Hence, we have to deal with it approximately. Under the restriction of \( \eta^2 \ll k^2 \), \( \zeta \) can be expressed as a series expansion of a small amount of \( \eta^2 \). That is, \( \zeta = k - \eta^2/2k + \ldots \). We retain the leading term for the amplitude factor and the first two terms for the phase factor in equation (7). In this approximation, equation (7) transforms as follows:

\[
U_{n,2p}(\rho, z) \approx \exp[ik(z - z_{ex})] iS_{n,2p}(n + 2p) \frac{2k}{2k} \times \int_{0}^{\infty} \eta d\eta J_{n+2p}(\eta \rho) \tilde{U}_{n,2p}(\eta \rho_{ex}) \exp\left[ -i\frac{\eta^2}{2k}(z - z_{ex}) \right].
\]

From equations (2) and (7), we know that the scale length of the variation of \( \rho \) is \( \omega_0 \), and the significant range of the variation of \( \eta \) is from 0 to a quantity of the order of \( 1/\omega_0 \). Given that the beam waist \( \omega_0 \) is generally larger than the wavelength \( k = 2\pi/\lambda \), the condition of \( \eta^2 \ll k^2 \) is especially easy to satisfy.

According to the Weber integral formula [18, 19],

\[
\int_{0}^{\infty} \eta \exp(-q^2 \eta^2) J_{n+2p}(\eta \rho) \tilde{U}_{n,2p}(\eta \rho_{ex}) d\eta = \frac{1}{2q^2} \int_{0}^{\infty} \exp\left[ -\frac{1}{2}(n + 2p) \pi \eta^2 \right] \exp\left( -\frac{\rho_{ex}^2 + \rho^2}{4q^2} \right) \times J_{n+2p}\left( \frac{\rho \rho_{ex}}{2q^2} \right),
\]
we substitute equation (9) into equation (8) to obtain the following expression:

\[
U_{n,2p}(\rho, z) = \exp\left[ -\frac{1}{2}(n + 2p)i \right] S_{ex}(n + 2p) z^{-z_{ex}} \times \exp[ik(z - z_{ex})] J_{n+2p} \left( \frac{kp_{ex}}{z - z_{ex}} \right).
\]

When \( z = 0 \), \( n \) takes even numbers, \( J_{n+2p}(\alpha) \) is an even function, and equation (10) transforms as follows:

\[
U_{n,2p}(\rho, z = 0) = \exp\left[ -\frac{1}{2}(n + 2p)i \right] S_{ex}(n + 2p) \times \exp(-ikz_{ex}) J_{n+2p} \left( \frac{kp_{ex}}{z_{ex}} \right).
\]

The parameters \( S_{ex}, \rho_{ex}, \) and \( z_{ex} \) can be determined by comparing equation (10) with equation (2). Thus, we can readily obtain the following expressions:

\[
z_{ex} = \frac{ik\omega_0^2}{2} \equiv ia,
\]

\[
\rho_{ex} = \frac{\beta}{k} z_{ex} = \frac{i\omega_0^2}{2} \equiv ib,
\]

\[
S_{ex} = -2ia(-1)^{p} e^{-2p} \exp(-ka) \exp\left[ \frac{1}{2}(n + 2p)i \right] \times \exp\left( -\frac{\beta^2\omega_0^4}{4} \right).
\]

By substituting equations (12) to (14) into equation (10), the paraxial approximation to \( U_{n,2p}(\rho, z) \) can be obtained. Therefore, the paraxial approximation to \( U_n(\rho, \varphi, z) \) of the Lommel–Gauss beams is written as follows:

\[
E_n(\rho = 0, \varphi, z) = \sum_{p=0}^{\infty} E_{n,2p}(\rho, \varphi, z) = \sum_{p=0}^{\infty} U_{n,2p}(\rho, z) \exp[i(n + 2p)\varphi]
\]

\[
= \sum_{p=0}^{\infty} (-1)^{p} e^{2p} \exp\left( -\frac{\beta^2\omega_0^4}{4} \right) \frac{\exp(ikz)}{1 + iz/a} \times \exp\left( -\frac{\rho^2}{\omega_0^2(1 + iz/a)} \right) \frac{\exp(ikz)}{1 + iz/a} \times J_{n+2p} \left( \frac{\beta \rho}{1 + iz/a} \right) \exp[i(n + 2p)\varphi].
\]

The additional subscript \( P \) represents the paraxial approximation.

By substituting equations (12) to (14) into equation (7), we also obtain the precise integral expression for \( U_{n,2p}(\rho, z) \) as follows:

\[
U_{n,2p}(\rho, z) = a(-1)^{p} e^{2p} \exp(-ka) \exp\left[ \frac{1}{2}(n + 2p)i \right] \times \exp\left( -\frac{\beta^2\omega_0^4}{4} \right) \frac{\exp(\eta b)}{1 + iz/a} \times \exp[i(\zeta(z - ia))].
\]

Thereafter, the on-axis field for the \( n \)-th order Lommel–Gauss beams is obtained using equations (3) and (16) as follows:

\[
E_n(\rho = 0, \varphi, z = 0) = \sum_{p=0}^{\infty} E_{n,2p}(\rho = 0, \varphi, z = 0) = \sum_{p=0}^{\infty} \exp(\eta b) \exp(\zeta(z - ia))
\]

\[
= \sum_{p=0}^{\infty} (-1)^{p} e^{2p} \exp(-ka) \exp\left[ \frac{1}{2}(n + 2p)i \right] \times \exp\left( -\frac{\beta^2\omega_0^4}{4} \right) \exp(i(n + 2p)\varphi)
\]

\[
	imes \int_{0}^{\infty} \eta b \eta J_{n+2p}(\eta b) \exp[i(\zeta(z - ia))].
\]

The distribution of the field of \( \rho = 0 \) should be analyzed because researchers in the majority of cases mainly focus on the on-axis intensity distribution when using various types of beams in scientific experimentation.

For equation (17), we perform the series expansion of \( 1/\zeta \) and \( \exp[i(\zeta(z - ia)) \] in the powers of \( \eta \). When \( \eta^2 \ll k^2 \), the product of both series’ terms up to the order \((k\omega_0)^{-2m}(m = 0, 1, 2, \ldots)\) is retained, and the \( m \)-th order nonparaxial corrections are obtained [7]. The product of both series terms up to the order \((k\omega_0)^{-6}\) is determined to obtain the first three nonparaxial corrections of the \( n \)-th order Lommel–Gauss beams on the axis for \( m = 3 \). Equation (17) can be presented as follows:

\[
E_n(\rho = 0, \varphi, z = 0) = \sum_{p=0}^{\infty} (-1)^{p} e^{2p} \exp\left[ \frac{1}{2}(n + 2p)i \right] \times \exp\left( -\frac{\beta^2\omega_0^4}{4} \right) \frac{\omega_0^2}{2} \exp(ikz + i(n + 2p)\varphi) \times \int_{0}^{\infty} \eta b \eta J_{n+2p}(\eta b) \exp\left[ \frac{i\eta^2}{2k}(z - ia) \right] \sum_{m=0}^{3} G^{(2m)}(\eta, z) \sum_{m=0}^{3} G^{(2m)}(\eta, z),
\]

where

\[
G^{(2m)}(\eta, z) = 1 + \frac{G^{(2)}(\eta, z)}{(k\omega_0)^{2m}} + \frac{G^{(4)}(\eta, z)}{(k\omega_0)^{2m}},
\]

\[
G^{(0)}(\eta, z) = 1,
\]

\[
G^{(2)}(\eta, z) = \frac{\omega_0^2 \eta^4}{2} 16\eta^2,
\]

\[
G^{(4)}(\eta, z) = \frac{3\omega_0^4 \eta^8}{8} - \frac{\omega_0^8 \eta^8}{16\eta^2} + \frac{\omega_0^8 \eta^8}{512\eta^4}.
\]
The function $M_z$ is a confluent hypergeometric function. The following relationship is used [18, 19]:

$$\int_0^\infty J_{n+2p}(\gamma b \rho) \exp(-q^2 \gamma^2) \eta^{\mu-1} d\eta, \quad (21)$$

where

$$M(\alpha, \gamma, z) = \sum_{n=0}^{\infty} \frac{\alpha^n}{n!} \gamma^n z^n = 1 + \frac{\alpha}{\gamma} z + \frac{\alpha(\alpha + 1)}{\gamma(\gamma + 1)} z^2 + \ldots, \quad (22a)$$

$$\Gamma(n + 1) = n!. \quad (22b)$$

The function $F_\alpha(\rho = 0, \varphi, z)$ is a confluent hypergeometric function, and the function $\Gamma$ is a gamma function [18]. Thus, we can derive the following equation from equations (18) to (22):

$$E_\alpha(\rho = 0, \varphi, z) = \sum_{p=0}^{\infty} (-1)^p e^{2p} \exp \left[ \frac{1}{2}(n + 2p)\pi i \right] \times \exp \left( -\frac{\beta^2 \omega_0^2}{4} \right) \times \frac{\omega_0^2}{4} \times \exp[i k z + i(n + 2p)\varphi] \times \frac{(ib)^{n+2p}}{\Gamma(n + 2p + 1)} \times \sum_{t=1}^{7} A(2t) g_{n+2p,t}(z), \quad (23)$$

where

$$g_{n+2p,t}(z) = \frac{\Gamma[(n + 2p + 2t)/2]}{q^2} \times M \left( \frac{n + 2p + 2t}{2}, n + 2p + 1, \frac{b^2}{4q^2} \right), \quad (24a)$$

$$q^2 = \frac{i(z - ia)}{2k} \equiv w. \quad (24b)$$

The corresponding coefficient $A(2t)$ is derived as follows:

$$t = 1, A(2t) = 1, g_{n+2p,t}(z) = \frac{\Gamma[(n + 2p + 2)/2]}{w} \times M \left( \frac{n + 2p + 2}{2}, n + 2p + 1, \frac{b^2}{4q^2} \right), \quad (24a)$$

$$t = 2, A(2t) = \frac{1}{2k} \Gamma[(n + 2p + 4)/2] \times M \left( \frac{n + 2p + 4}{2}, n + 2p + 1, \frac{b^2}{4q^2} \right), \quad (24a)$$

$$t = 3, A(2t) = \frac{3}{8k^3} \frac{\omega_0^2}{16k^2 h} \times g_{n+2p,t}(z) = \frac{\Gamma[(n + 2p + 6)/2]}{w^3} \times M \left( \frac{n + 2p + 6}{2}, n + 2p + 1, \frac{b^2}{4q^2} \right), \quad (24a)$$

$$t = 4, A(2t) = \frac{5}{16k^6} \frac{\omega_0^2}{16k^2 h} \times g_{n+2p,t}(z) = \frac{\Gamma[(n + 2p + 8)/2]}{w^4} \times M \left( \frac{n + 2p + 8}{2}, n + 2p + 1, \frac{b^2}{4q^2} \right), \quad (24a)$$

$$t = 5, A(2t) = \frac{\omega_0^2}{512k^3 h^2} \times g_{n+2p,t}(z) = \frac{\Gamma[(n + 2p + 10)/2]}{w^5} \times M \left( \frac{n + 2p + 10}{2}, n + 2p + 1, \frac{b^2}{4q^2} \right), \quad (24a)$$

$$t = 6, A(2t) = \frac{3\omega_0^2}{1024k^2 h^3} \times g_{n+2p,t}(z) = \frac{\Gamma[(n + 2p + 12)/2]}{w^6} \times M \left( \frac{n + 2p + 12}{2}, n + 2p + 1, \frac{b^2}{4q^2} \right), \quad (24a)$$

$$t = 7, A(2t) = \frac{6\omega_0^2}{4096k^3 h^3} \times g_{n+2p,t}(z) = \frac{\Gamma[(n + 2p + 14)/2]}{w^7} \times M \left( \frac{n + 2p + 14}{2}, n + 2p + 1, \frac{b^2}{4q^2} \right), \quad (24a)$$

The analytical expressions for the first-order and second-order nonparaxial corrections and the analytical expression of the zero-order paraxial approximation can be obtained using the same method.

As an example, we use the nonparaxial corrections for the on-axis field of the Lommel–Gauss beams using the typical parameters $\lambda = 632.8$ nm, $\beta = 80$ m$^{-1}$, and $\omega_0 = 60 \mu$m to calculate the on-axis intensity distribution of the Lommel–Gauss beams for $n = 0$. The calculation results are shown in figure 1.

In figure 1, the normalized intensity between the non-paraxial corrections and paraxial approximation are not
precisely the same when the zero-order Lommel–Gauss beams propagate in a short distance. This result indicates that the paraxial theory cannot be used to accurately calculate the near-field (nearly Rayleigh length) propagation characteristics of the Lommel–Gauss beam. Hence, the results obtained in this study should be important for using the Lommel–Gauss beams in near-field experimentation. However, the increase in the propagation distance makes the calculation results of the nonparaxial corrections get closer and closer to the calculation results of the paraxial approximation. Instead of making nonparaxial corrections, the relatively simple paraxial approximation theory can be conveniently used to accurately calculate the far-field (far beyond Rayleigh length) propagation. These conclusions are also consistent with classical optical theory. When \( n \) takes other integers in equation (23), the on-axis field of other-order Lommel–Gauss beams can also be accurately calculated. A special feature of nonparaxial corrections of beam propagation is also noted and described in and beyond the paraxial limit, accordingly.

In short, the in-depth study of nonparaxial propagation characteristics of Lommel–Gauss beams has uncovered some new and instructive information about the beam propagation. Since Lommel–Gauss beams have unique optical properties and new uses in some science fields [10–13], such analytical results will lay a foundation for the improved application of Lommel–Gauss beams in future scientific research, especially in near-field science.

3. Conclusion

In summary, we introduced a group of virtual sources for generating \( n \)-th-order Lommel–Gauss beams based on the principle of beam superposition to establish the corresponding inhomogeneous Helmholtz equation. We also rigorously derived the paraxial approximation and nonparaxial integral representations of the Lommel–Gauss beams based on the Fourier–Bessel transform pair and Weber integral formula. The first three orders of the nonparaxial corrections for the on-axis field of the Lommel–Gauss beams were analytically obtained, and the on-axis intensity distribution of the corresponding nonparaxial corrections was provided. The analytical expression about the Lommel–Gauss beam propagation in this study can be used to lay a theoretical foundation for improved application of Lommel–Gauss beams in scientific experimentation.

Acknowledgments

This work was supported by the National Natural Science Foundation of China under Grant No. 11674288.

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