Seiberg-Witten maps and anomalies in noncommutative Yang-Mills theories

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Summary. A BRST-cohomological analysis of Seiberg-Witten maps and results on gauge anomalies in noncommutative Yang-Mills theories with general gauge groups are reviewed.

1 Introduction

We shall discuss two aspects of noncommutative Yang-Mills theories of the type introduced in [1] (see sect. 2 for a brief review). The first aspect concerns the construction of these theories which is based on so-called Seiberg-Witten mappings (SW maps, for short). These mappings express ‘noncommutative’ fields and gauge transformations in terms of the standard (‘commutative’) fields and gauge transformations. The mappings have been named after Seiberg and Witten because they were established first in [2] for the particular case of $U(N)$-theories. However, it should be kept in mind that in the present context they are not limited to $U(N)$-theories but extended to other gauge groups. This raises the questions whether and why SW maps exist for general gauge groups, how they can be constructed efficiently and to which extend they are unique resp. ambiguous. These questions are the topic of sect. 3 which reviews work in collaboration with G. Barnich and M. Grigoriev [3, 4, 5].

Sect. 4 reports on work in collaboration with C.P. Martín and F. Ruiz Ruiz [6]. It addresses the question whether the gauge symmetries of noncommutative Yang-Mills theories can be anomalous when one applies the standard perturbative approach to (effective) quantum field theories. It is not to be discussed here whether or not such an approach makes sense; currently there is hardly an alternative perspective on these theories in the general case (i.e., for a general gauge group) since the theories are constructed only by means of SW maps and no formulation in terms of ‘noncommutative’ variables is known. Hence, at present we have to content ourselves with a formulation of the ‘effective type’ that is not renormalizable by power counting, i.e., a Lagrangian containing field monomials of arbitrarily high mass dimension. As a
consequence, there is no simple argument which can rule out from the outset the possible occurrence of gauge anomalies with mass dimensions larger than 4. This complicates the anomaly discussion as compared to renormalizable Yang-Mills theories whenever the gauge group contains at least one abelian factor since in that case there is an infinite number of candidate gauge anomalies in addition to the well-known chiral gauge anomalies.

2 Brief review of noncommutative Yang-Mills theories

The noncommutative Yang-Mills theories under consideration involve a \(*\)-product given by the Weyl-Moyal product,

\[ f_1 \star f_2 = f_1 \exp(-\frac{i}{2} \tau \theta^{\mu\nu} \frac{\partial}{\partial \nu}) f_2, \quad \theta^{\mu\nu} = -\theta^{\nu\mu} = \text{constant}. \]

\(\tau\) is a constant deformation parameter that has been introduced for the sake of convenience. The ‘noncommutative’ generalization of the Yang-Mills action reads

\[ \hat{I}[\hat{A}] = -\frac{1}{4} \int d^n x \text{Tr} (\hat{F}_{\mu\nu} \star \hat{F}^{\mu\nu}), \quad \hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu + \hat{A}_\mu \star \hat{A}_\nu - \hat{A}_\nu \star \hat{A}_\mu \]  

(1)

where \(\hat{A}_\mu\) is constructed from ‘commutative’ gauge potentials \(A_\mu\) by means of a SW map. \(A_\mu\) lives in the Lie algebra of the gauge group and has the standard Yang-Mills gauge transformations,

\[ \delta_\lambda A_\mu = \partial_\mu \lambda + [A_\mu, \lambda] \equiv D_\mu \lambda, \]

(2)

where \(\lambda\) denotes Lie algebra valued gauge parameters. SW maps, by definition, express the noncommutative gauge potentials \(\hat{A}_\mu\) and gauge parameters \(\hat{\lambda}\) in terms of \(A_\mu\) and \(\lambda\) such that (2) induces the noncommutative version of Yang-Mills gauge transformations given by

\[ \hat{\delta}_\lambda \hat{A}_\mu = \partial_\mu \hat{\lambda} + \hat{A}_\mu \star \hat{\lambda} - \hat{\lambda} \star \hat{A}_\mu \equiv \hat{D}_\mu \hat{\lambda}. \]

(3)

Furthermore we require that \(\hat{A}_\mu\) and \(\hat{\lambda}\) coincide with \(A_\mu\) and \(\lambda\) at \(\tau = 0\),

\[ \hat{A}_\mu = \hat{A}_\mu(A, \tau) = A_\mu + O(\tau), \quad \hat{\lambda} = \hat{\lambda}(\lambda, A, \tau) = \lambda + O(\tau). \]

Hence, SW maps are required to fulfill

\[ \delta_\lambda \hat{A}_\mu(A, \tau) = (\hat{\delta}_\lambda \hat{A}_\mu)(A, \tau). \]

For the inclusion of fermions see, e.g., [1, 4].
3 Analysis of SW maps

Noncommutative Yang-Mills theories can be regarded as consistent deformations of corresponding commutative Yang-Mills theories. This allows one to apply BRST-cohomological tools to analyse SW maps along the lines of [7].

In the following, we first review briefly the BRST-cohomological approach to consistent deformations and then the results on SW maps.

3.1 Consistent deformations

Consider an action $I^{(0)}[\varphi]$ with gauge invariance $\delta^{(0)}_\lambda I^{(0)}[\varphi] = 0$.

Consistent deformations of $I^{(0)}[\varphi]$ and $\delta^{(0)}_\lambda$ are power series $I[\varphi, \tau]$ and $\delta_\lambda$ in a deformation parameter $\tau$, such that the deformed action is invariant under the (possibly) deformed gauge transformation,

$$I[\varphi, \tau] = I^{(0)}[\varphi] + \sum_{k \geq 1} \tau^k I^{(k)}[\varphi], \quad \delta_\lambda = \delta^{(0)}_\lambda + \sum_{k \geq 1} \tau^k \delta^{(k)}_\lambda, \quad \delta_\lambda I[\varphi, \tau] = 0.$$

Two such deformations are called equivalent ($\sim$) if they are related by mere field redefinitions $\hat{\varphi}(\varphi, \tau)$, $\hat{\lambda}(\lambda, \varphi, \tau)$:

$$\hat{I}[\hat{\varphi}(\varphi, \tau), \tau] = I[\varphi, \tau], \quad (\hat{\delta}_\lambda \hat{\varphi})(\varphi, \lambda, \tau) \approx \delta_\lambda \hat{\varphi}(\varphi, \tau),$$

where $\approx$ is ‘equality on-shell’ (equality for all solutions to the field equations).

Accordingly, a deformation is called trivial if the deformed action and gauge transformations are equivalent to the original action and gauge transformations, i.e., if $I \sim I^{(0)}$ and $\delta \sim \delta^{(0)}$.

We may distinguish two types of nontrivial deformations:

Type I: $I \not\sim I^{(0)}$, $\delta^{(k)}_\lambda \sim \delta^{(k)}_\lambda$, i.e., the deformation of the action is nontrivial whereas the deformation of the gauge transformations is trivial.

Type II: $I \not\sim I^{(0)}$, $\delta^{(k)}_\lambda \not\sim \delta^{(k)}_\lambda$, i.e., the deformations of both the action and the gauge transformations are nontrivial.

Notice that in this terminology noncommutative Yang-Mills theories as described in sect. 2 are type I deformations of Yang-Mills theories because SW maps are field redefinitions that bring the noncommutative gauge transformations back to the standard (commutative) form, i.e., the deformation of the gauge transformations is trivial.

3.2 BRST-cohomological approach to consistent deformations

The BRST-cohomological approach to consistent deformations is most conveniently formulated in the so-called field-antifield formalism. The ‘fields’ $\phi^a$ of that formalism are the fields $\varphi^i$ occurring in the action $I[\varphi]$, ghost fields...
\( C^\alpha \) corresponding to the nontrivial gauge symmetries of the action, as well as ghost fields of higher order (‘ghosts for ghosts’) if the gauge transformations are reducible. Each field is accompanied by an antifield \( \phi^*_a \) according to definite rules which are not reviewed here. In particular this allows one to define the so-called antibracket \( (\ , \ ) \) of functions or functionals of the fields and antifields according to

\[
(F, G) = \int d^n x \ F \left( \frac{\delta}{\delta \phi^a(x)} \frac{\delta}{\delta \phi^*_a(x)} - \frac{\delta}{\delta \phi^*_a(x)} \frac{\delta}{\delta \phi^a(x)} \right) G.
\]

A central object of the formalism is the master action \( S \). Its importance originates from the fact that it contains both the action \( I[\varphi] \) and all information about its gauge symmetries, such as the gauge transformations, their commutator algebra, reducibility relations etc. In particular the gauge transformations occur in \( S \) via terms \( \varphi^i \delta C^i \) where \( \delta C^i \) is a gauge transformation of \( \varphi^i \) with ghost fields \( C \) in place of gauge parameters \( \lambda \). The information about the gauge symmetry is encoded in the master equation \( (S, S) = 0 \),

\[
S[\phi, \phi^*] = I[\varphi] + \int d^n x \, \varphi^*_i \delta C^i \varphi^i + \ldots \text{ such that } (S, S) = 0.
\]

In particular \( S \) defines the BRST differential \( s \) via the antibracket with \( S \). The master equation \( (S, S) = 0 \) implies that \( s \) squares to zero \( (s^2 = 0) \),

\[
s = (S, \cdot) \quad (\Rightarrow \ s^2 = 0).
\]

These properties of \( S \) make it so useful in the context of consistent deformations. Indeed, the fact that \( S \) contains both the action and the gauge transformations allows one to analyse consistent deformations in terms of the single object \( S \) that has to satisfy the master equation,

\[
S = S^{(0)} + \sum_{k \geq 1} \tau^k S^{(k)}, \quad (S, S) = 0.
\]

The first relation to BRST-cohomology can be established by differentiation of the master equation with respect to the deformation parameter:

\[
(S, S) = 0 \quad \Rightarrow \quad \left( S, \frac{\partial S}{\partial \tau} \right) = 0 \quad \Leftrightarrow \quad s \frac{\partial S}{\partial \tau} = 0.
\]

This shows that \( \partial S/\partial \tau \) is a cocycle of \( s \). The second relation to BRST-cohomology derives from the fact that field redefinitions (of \( \varphi \) and/or the gauge parameters) translate into anticanonical transformations \( \hat{\delta}(\phi, \phi^*, \tau), \hat{\delta}^*(\phi, \phi^*, \tau) \) (these are transformations generated via the antibracket by some functional \( \Xi \)). This implies:
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\[
\frac{d\hat{\phi}}{d\tau} = (\Xi, \hat{\phi}), \quad \frac{d\hat{\phi}^*}{d\tau} = (\Xi, \hat{\phi}^*) \Rightarrow \frac{dS(\hat{\phi}, \hat{\phi}^*, \tau)}{d\tau} = \frac{\partial S}{\partial \tau} - (S, \Xi) = \frac{\partial S}{\partial \tau} - s \Xi.
\]

As a consequence, master actions of equivalent deformations are related as follows:

\[
S \sim S' \Rightarrow \frac{\partial S}{\partial \tau} - \frac{\partial S'}{\partial \tau} = s \Xi.
\]

This shows that consistent deformations are determined by the BRST-cohomology \( H(s) \) in ghost number 0 since \( \partial S/\partial \tau \) (i) has to be a BRST-cocycle, (ii) is defined only up to a BRST-coboundary, and (iii) has ghost number 0 (\( S \) has ghost number 0 according to the standard ghost number assignments).

### 3.3 BRST-cohomological analysis of SW maps

To describe SW maps in the field-antifield formalism we denote the ‘noncommutative fields’ by \( \hat{\phi} \) and the ‘commutative’ fields by \( \phi \). Actually we enlarge the setup here as compared to sect. 2: all the fields \( \hat{\phi} \) and \( \phi \) take values in the enveloping algebra of the Lie algebra of the gauge group, resp. some representation \( \{T_A\} \) thereof. The superfluous fields \( \phi \) (those that do not belong to the Lie algebra of the gauge group) are set to zero at the end of the construction, see [4] for details. Dropping again the fermions, we have

\[
\{\hat{\phi}^a\} = \{\hat{A}^A, \hat{C}^A\}, \quad \{\phi^a\} = \{A_A^A, C^A\}.
\]

The ‘noncommutative’ master action reads

\[
S[\hat{\phi}, \hat{\phi}^*, \tau] = \int d^n x \left[ -\frac{1}{4} \text{Tr} (\hat{F}_{\mu\nu} \star \hat{F}^{\mu\nu}) + \hat{A}^*_A \star (\hat{D}_\mu \hat{C})^A + \hat{C}^*_A \star (\hat{C} \star \hat{C})^A \right].
\]

The existence of a SW map means that the gauge transformations can be brought to the standard Yang-Mills form, which particularly does not depend on \( \tau \). In terms of the master action this means that there is an anticanonical transformation \( \hat{\phi}(\phi, \phi^*, \tau), \hat{\phi}^*(\phi, \phi^*, \tau) \) which casts \( S[\hat{\phi}, \hat{\phi}^*, \tau] \) in the form of an effective type Yang-Mills action \( I_{\text{eff}}[A, \tau] \) plus a piece that involves the antifields and encodes gauge transformations of Yang-Mills type (for the enveloping algebra),

\[
S[\hat{\phi}(\phi, \phi^*, \tau), \hat{\phi}^*(\phi, \phi^*, \tau), \tau] = I_{\text{eff}}[A, \tau] + \int d^n x \left[ A^{*\mu} D_\mu C + C^* C \right].
\]

where indices have been dropped (\( A^{*\mu} D_\mu C \) means \( A_A^{*\mu} (D_\mu C)^A \) etc). Differentiating with respect to \( \tau \) und using the properties of anticanonical transformations (see above), we obtain

\[
\frac{\partial S}{\partial \tau} - s \Xi = \frac{\partial I_{\text{eff}}[A, \tau]}{\partial \tau}, \quad \frac{d\hat{\phi}}{d\tau} = (\Xi, \hat{\phi}), \quad \frac{d\hat{\phi}^*}{d\tau} = (\Xi, \hat{\phi})^*.
\]
Hence, in order to find and analyse SW maps one may analyse whether \( \partial S/\partial \tau \) can be written as a BRST-variation \( s \Xi \) up to terms that do not involve antifields. Notice that \( \Xi \) gives the SW map. For \( \partial S/\partial \tau \) one obtains

\[
\frac{\partial S}{\partial \tau} = \frac{i\theta^{\alpha\beta}}{2} \int d^n x \left[ \text{Tr} \left( -\hat{F}^{\mu\nu} \star \partial_\alpha \hat{A}_\mu \star \partial_\beta \hat{A}_\nu \right) \right. \\
+ \hat{A}^{\alpha\mu} \star \{ \partial_\alpha \hat{A}_\mu \star \partial_\beta \hat{C} \} + \hat{C}^{\alpha} \star \partial_\alpha \hat{C} \star \partial_\beta \hat{C} \],
\]

where \( \{ \, \star , \, \} \) denotes the \( \star \)-anticommutator,

\[
\{ X \star Y \} = X \star Y + Y \star X.
\]

This expression for \( \partial S/\partial \tau \) is indeed BRST-exact up to terms that do not contain antifields. One can infer this by means of so-called contracting homotopies for derivatives of the ghost fields used already in [10, 11]. We shall not review the construction of these homotopies here since this is a somewhat technical matter. Rather, we shall only present the result. It is actually ambiguous as we shall discuss below. In particular it depends on the specific contracting homotopy one uses (there are various options). A particularly nice version of the result is

\[
\Xi = \frac{i}{4} \theta^{\alpha\beta} \int d^n x \left( -\hat{A}^{\alpha\mu} \{ \hat{F}_{\alpha\mu} + \partial_\alpha \hat{A}_\mu \star \hat{A}_\beta \} + \hat{C}^{\alpha} \{ \hat{A}_\alpha \star \partial_\beta \hat{C} \} \right),
\]

\[
\frac{d\hat{A}_\mu}{d\tau} = (\Xi, \hat{A}_\mu) = \frac{i}{4} \theta^{\alpha\beta} \{ \hat{F}_{\alpha\mu} + \partial_\alpha \hat{A}_\mu \star \hat{A}_\beta \},
\]

\[
\frac{d\hat{C}}{d\tau} = (\Xi, \hat{C}) = \frac{i}{4} \theta^{\alpha\beta} \{ \hat{A}_\alpha \star \partial_\beta \hat{C} \},
\]

\[
\frac{dI_{\text{eff}}[\hat{A}(A, \tau), \tau]}{d\tau} = i\theta^{\alpha\beta} \int d^n x \text{Tr} \left( \frac{1}{8} \hat{F}_{\alpha\beta} \hat{F}_{\mu\nu} \hat{F}_{\mu\nu} - \frac{1}{2} \hat{F}_{\alpha\mu} \hat{F}_{\beta\nu} \hat{F}_{\nu\mu} \right).
\]

The expressions for \( d\hat{A}_\mu/d\tau \) and \( d\hat{C}/d\tau \) are differential equations for SW maps of the same form as derived in [2] for \( U(N) \)-theories.

The ambiguities of the result can be described in terms of \( \Xi \) as shifts \( \Xi + \Delta \Xi \) of \( \Xi \) which satisfy

\[
0 = s(\Delta \Xi) + \text{terms without antifields},
\]

where the terms without antifields yield the shift \( d(\Delta I_{\text{eff}})/d\tau \) corresponding to \( \Delta \Xi \). This is again an equation that can be analysed by cohomological means which are not reviewed here, and we only present the result: the general SW map \( \hat{A}_\mu(A, \tau), \hat{\lambda}(\lambda, A, \tau) \) for the gauge fields and gauge parameters can be written as

\[
\hat{A}_\mu(A, \tau) = \left[ A^{-1} \star \hat{A}_\mu^{sp} \star A + A^{-1} \star \partial_\mu A \right]_{A_\mu \rightarrow \hat{A}_\mu^{sp}(A, \tau)},
\]

\[
\hat{\lambda}(\lambda, A, \tau) = \left[ A^{-1} \star \hat{\lambda}^{sp} \star A + A^{-1} \star \delta_\lambda A \right]_{\lambda_\mu \rightarrow \hat{\lambda}^{sp}(\lambda, A, \tau)}.
\]
where

\[ A(A, \tau) = \exp(\int B(A, \tau) T_B) \text{ with arbitrary } f_B(A, \tau), \]

\[ \hat{A}^{sp}_\mu(A, \tau), \hat{\lambda}^{sp}(\lambda, A, \tau) \text{ is a particular SW map}, \]

\[ A^{\mu B}_\nu(A, \tau) = [A_\mu + W_\mu(A, \tau)]^C R^B_C(\tau) \text{ where:} \]

\[ \delta_\lambda W_\mu(A, \tau) = [W_\mu(A, \tau), \lambda] \text{ (i.e., } W_\mu \text{ is gauge covariant)}, \]

\[ T_B \rightarrow R^B_C(\tau) T_C \text{ is an (outer) Lie algebra automorphism}. \]

Recall that \( \{T_A\} \) is (a representation of) the enveloping algebra of the Lie algebra of the gauge group. Hence, the Lie algebra automorphisms \( T_B \rightarrow R^B_C(\tau) T_C \) that enter here refer to the Lie algebra of \( \{T_A\} \) rather than to the Lie algebra of the gauge group. Without loss of generality one may restrict these automorphisms to outer automorphisms since inner ones are already covered by the \( A \)-terms. Note that the latter are (field dependent) noncommutative gauge transformations of a special SW map \( \hat{A}_\mu^{sp} \).

Hence, SW maps are determined only up to (compositions of) noncommutative gauge transformations of \( \hat{A}_\mu \), gauge covariant shifts of enveloping algebra valued gauge fields \( A_\mu \), and outer automorphisms of the Lie algebra of the enveloping algebra.

### 4 Gauge anomalies

A 1-loop computation, performed with dimensional regularization, yields the following expression for gauge anomalies in four-dimensional noncommutative Yang-Mills theories with chiral fermions [6]:

\[ \mathcal{A}[\hat{C}, \hat{A}, \tau] = \int \text{Tr}[\hat{C} \star d(\hat{A} \star d\hat{A} + \frac{1}{2} \hat{A} \star \hat{A} \star \hat{A})], \tag{4} \]

where we used differential form notation (\( d = dx^\mu \partial_\mu, \hat{A} = dx^\mu \hat{A}_\mu \)). This expression is reminiscent of anomalies in ordinary (commutative) Yang-Mills theories since it arises from the latter by replacing commutative fields \( C \) and \( A_\mu \) with their noncommutative counterparts and ordinary products with \( \star \)-products. However, the presence of \( \star \)-products poses an apparent puzzle: \( \mathcal{A} = 0 \) does not only impose the usual anomaly cancellation conditions \( \text{Tr}(T_{(a} T_{b} T_{c)}) = 0 \) but additional conditions at higher orders in \( \theta \), such as \( \text{Tr}(T_{(a} T_{b} T_{c}) = 0 \). On the other hand all candidate gauge anomalies of noncommutative Yang-Mills theories of the type considered here are known because these theories can be considered Yang-Mills theories of the effective type whose anomalies were exhaustively classified (see [12] for a review). These known results state in particular that the chiral (Bardeen) anomalies exhaust
all candidate gauge anomalies when the gauge group is semisimple. According to this result \( \theta \)-dependent terms in \( \mathcal{A} \) are BRST-exact when the gauge group is semisimple.

The situation is more involved when the gauge group contains an abelian factor. In this case there are additional, and in fact infinitely many, candidate anomalies, and it is not obvious from the outset whether or not some of them occur in \( \mathcal{A} \). The question is thus: is \( \mathcal{A} \) always cohomologically equivalent to a standard chiral anomaly, even when the gauge group contains abelian factors? The answer is affirmative, as was shown in [6]. Again, we shall only briefly sketch how this result was obtained and drop all details.

The idea is to differentiate \( \mathcal{A} \) with respect to \( \tau \) and to show that the resultant expression is BRST-exact. The reason for dealing with \( \frac{d\mathcal{A}}{d\tau} \) rather than with \( \mathcal{A} \) itself is that, as it turns out, \( \frac{d\mathcal{A}}{d\tau} \) is the BRST-variation of an expression that can be compactly written as an integrated \( \star \)-polynomial of the noncommutative variables \( \hat{A}_\mu \):

\[
\frac{d\mathcal{A}}{d\tau} = s\mathcal{B}_* ,
\]

\[
\mathcal{B}_* = \frac{i\theta^\alpha\beta}{2} \int \text{Tr} (\hat{A}_\alpha \star \partial_\beta d\hat{A} \star d\hat{A} - \frac{1}{2} d\hat{A}_\alpha \star \hat{A}_\beta \star d\hat{A} \star \hat{A} \\
+ \frac{3}{2} d\hat{A} \star d\hat{A}_\alpha \star \hat{A} \star \hat{A}_\beta - \frac{1}{2} d\hat{A}_\alpha \star \hat{A}_\beta \star d\hat{A} \star \hat{A} \\
+ \partial_\alpha \hat{A}_\beta \star d\hat{A} \star \hat{A} \star \hat{A} + \text{terms with 5 or 6 } \hat{A}'s) .
\]

We remark that \( \mathcal{B}_* \) is not unique (it is determined only up to BRST-cocycles with ghost number 0) and can be written in various ways. Hence, the expression given above is just one particular choice. The desired result for \( \mathcal{A} \) is now obtained using \( \mathcal{A}(\tau) = \mathcal{A}(0) + \int_0^\tau d\tau' \frac{d\mathcal{A}}{d\tau'} \). This gives

\[
\mathcal{A} = \int \text{Tr}[Cd(AdA + \frac{1}{2} A^3)] + s\mathcal{B}[A, \tau], \quad \mathcal{B}[A, \tau] = \int_0^\tau d\tau' \mathcal{B}_*[\hat{A}(A, \tau'), \tau'] .
\]

Notice that \( \mathcal{B} \), in contrast to \( \mathcal{B}_* \), can not be naturally written as an integrated \( \star \)-polynomial of the noncommutative variables \( \hat{A}_\mu \) because of the dependence of \( \hat{A}(A, \tau') \) on \( \tau' \). \( \mathcal{B}_* \) shows that \( \mathcal{A} \) is indeed given by the standard chiral gauge anomaly \( \int \text{Tr}[Cd(AdA + \frac{1}{2} A^3)] \) up to a BRST-exact piece \( s\mathcal{B} \). Hence, at least at the 1-loop level, noncommutative Yang-Mills theories do not possess additional gauge anomalies or anomaly cancellation conditions as compared to the corresponding commutative theories, even when the gauge group contains abelian factors (the above results apply to all gauge groups). Notice that \( -\mathcal{B} \) is the counterterm that cancels the \( \theta \)-dependent terms in \( \mathcal{A} \).
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