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Exponential behavior of neutral impulsive stochastic integro-differential equations driven by Poisson jumps and Rosenblatt process

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Abstract: In this article, we are concerned with the neutral impulsive stochastic integro-differential equations driven by Poisson jumps and Rosenblatt process. By using resolvent operator and some analysis techniques, we ensure existence and uniqueness of solutions. Further, we investigate exponential stability of mild solutions. We have also given an example to illustrate our theoretical results.

Keywords: Existence results, Stability, Poisson jumps, Rosenblatt process, Resolvent operator

MSC: 60H15, 35R60

1 Introduction

In the past decades, the theory of nonlinear functional differential or integro-differential equations with resolvent operators has become an active research field due to their applications in many physical phenomena. The resolvent operator is comparable to the semigroup operator for abstract differential equations in Banach spaces. However, the resolvent operator does not satisfy semigroup properties. The study of deterministic neutral functional differential equations was initiated by Hale and Mayer [17]. For more details on the theory and their applications, we also refer the readers to Hale and Lunel [18], Kolmanovskii and Nosov [20] and so on. Deterministic and stochastic differential equations have gained great popularity in the last few years due to their use in many areas, such as physics, electronics, control theory, engineering and economics. Several authors have considered the existence, uniqueness and asymptotic behavior of mild solutions, and many important theory and applications findings have been obtained. For more details we refer to the papers by Ali et al. [4], El-Borai et al. [11], Gorec and Sathananthan [12], Gupta and Dabas [15], Gupta and al. [16], Laksmikantha [22], Ahmed [2], Ahmed et al. [3], Arthi and Balachandran [5], Gupta et al. [14], Levin et al. [24].

On the other side, fractional Brownian motion was intensively explored due to their applications in various domain. We point out that, a fractional Brownian motion (fBm) of Hurst index $H \in (0, 1)$ (see [21, 26] for...
more details) is a Gaussian process $B^H = \{B^H(t), t \geq 0\}$, centered, with the covariance function

$$R_H(t, s) = \mathbb{E}\left( B^H(t) B^H(s) \right) = \frac{1}{2} \left( t^{2H} + s^{2H} - |t - s|^{2H} \right).$$

We also mention that fBm is not a semimartingale and when $H = 1/2$, the fBm becomes standard Brownian motion. Further, if $H > 1/2$, the fBm $B^H$ have a long-memory and this property makes it an ideal process to modeling in biology, mathematics / finance \cite{7, 8} etc.

Moreover, the fBm belongs to Hermite family processes, it’s self-similar which being defined as limits that appear in the so-called Non-Central Theorem. For $d \geq 1$, they have the following representation

$$Y_{t}^{H,d} = c(H_0) \int_{\mathbb{R}} \ldots \int_{\mathbb{R}} \left( \int_{0}^{t} \prod_{j=1}^{d} (s - x_j)^{H_0 - 1} \, ds \right) dB_{x_1} \ldots dB_{x_d}, \quad \forall t > 0,$$

where $\{B_{x_i} : x_i \in \mathbb{R}, i = 1, \ldots, d\}$ are some two-sided Brownian motions, $c(H_0)$ is a normalizing constant such that

$$\mathbb{E}|Y_{t}^{H,d}|^2 = 1, \quad H_0 = \frac{1}{2} + \frac{H - 1}{d}, \quad H \in (\frac{1}{2}, 1).$$

When $d = 1$ the process become a fractional Brownian motion, thus Taqqu \cite{32} have named the Rosenblatt process when $d = 2$, which is not Gaussian process but they have stationary increments and long-range-dependency. Recently, the Rosenblatt process have attracted attention of many authors due to their properties. For example, Meajima and Tudor \cite{29}, Veillette and Taqqu \cite{36} have given many important properties of distributions, Bernet and Tudor \cite{6}, Viens and Tudor \cite{35} established the construction of estimator for the self-similarity parameter $H$.

Based on the above works, we investigate the following neutral stochastic functional integro-differential equations with delay and impulses effects

$$
\begin{align*}
\begin{cases}
\frac{d}{dt} \left[ u(t) - q(t, u_t) \right] &= \left[ A \left( u(t) - q(t, u_t) \right) + \int_{0}^{t} Y(t - s) \left[ u(s) - q(s, u_s) \right] \, ds + f(t, u_t) \right] \\
&\quad + \int_{0}^{t} g(t, s, u_s) \, ds \right] dt + \int_{\theta}^{t} h(t, u_t, v) N(dt, dv) + \sigma(t) dZ^H(t), \quad t \in [0, T], \; t \neq t_k, \\
\Delta u(t) &= I_k(u(t_k)) \quad t = t_k, \quad k = 1, 2, \ldots , \\
u_{0}(t) &= \phi(t) \in \mathcal{P}C \left( [-r, 0], \mathbb{H} \right), \quad -r \leq t \leq 0,
\end{cases}
\end{align*}
$$

where $A : D(A) \subset \mathbb{H} \to \mathbb{H}$ is the infinitesimal generator of a $C_0$-semigroup $(T(t))_{t \geq 0}$ of bounded linear operators in a Hilbert space $\mathbb{H}$; for $t \geq 0$, $Y(t)$ a closed linear operator on $\mathbb{H}$, with $D(A) \subset D(Y)$. The jump moments $t_k$ satisfy the condition $0 < t_1 < t_2 < \cdots < t_k < \cdots$, $\lim_{k \to \infty} t_k = \infty$, $I_k : \mathbb{H} \to \mathbb{H}$, $\Delta u(t_k) = u(t_k^+) - u(t_k^-)$, $u(t_k^-)$ and $u(t_k^+)$ are the right and left limits of $u(t)$ at $t_k$, respectively which is the jump size of the state $u$ at $t_k$. For $\phi \in \mathcal{P}C$, $||\phi|| = \sup_{t \in [-r, 0]} ||\phi(t)|| < \infty$, where $\mathcal{P}C = \{ \phi : [-r, 0] \to \mathbb{H}, \phi(t) \}$ is continuous everywhere except a finite number of points $i$ at which $\phi(t^-_i), \phi(t^+_i)$ exist and $\phi(t_i^-) = \phi(t_i^+)\}$. For any $t \in [0, T]$ and any continuous function $u$, the element of $\mathcal{P}C$ is defined by $u_i(\theta) = u(t + \theta)$, $-r \leq \theta \leq 0$. The functions $q, f : [0, +\infty) \times \mathcal{P}C \to \mathbb{H}$, $g : [0, +\infty) \times [0, +\infty) \times \mathcal{P}C \to \mathbb{H}$, $h : [0, +\infty) \times \mathbb{H} \times \theta \to \mathbb{H}$ and $\sigma : [0, +\infty) \to L_0^2(Y, \mathbb{H})$ are appropriate functions, and $Z^H_0$ is assumed to be a Rosenblatt process. The particular case $Y(t) = 0$ and $h = 0$ of Eq. (1) has been considered by Ma et al \cite{25}, where the authors used an impulsive integral inequality to prove their result. It should be mentioned that there is no work yet reported on the exponential stability of neutral impulsive stochastic integro-differential equations driven by Poisson jumps and Rosenblatt process. Motivated by this facts, our main objective is to study the exponential stability for a class of neutral impulsive stochastic integro-differential equations\cite{1}. In this paper, we derive existence and exponential results for the system (1) with the help of resolvent operator and fixed point techniques. In the first result, we obtain the sufficient conditions proving existence and uniqueness of the mild solution.
of (1) by utilizing Banach fixed point theorem under Lipschitz conditions on nonlinear terms. While in the second result, we have proved the exponential stability of mild solution via an integral inequality. Our article, expands the usefulness of stochastic integro-differential equations, since the literature shows results for existence and exponential stability for such equations in the case of semigroup only (see [2, 3, 11, 14, 16] and the references therein). The results obtained improve, extend and complete many other important ones in the literature.

The following is the organization of this paper. We recall some preliminary definitions and outcomes in Section 2. Section 3 is devoted to investigate existence and uniqueness of mild solution. The exponential stability for the mild solution is also discussed. We give an example in the fourth section to illustrate the results. The last section is dedicated to conclude this paper.

2 Preliminaries

In this section, we provided some basic results about Poisson process, Resolvent operator and Rosenblatt process.

2.1 Poisson jumps process

Let \( \mathcal{B}(\mathcal{H}) \) the Borel \( \sigma \)-algebra of \( \mathcal{H} \). Let \( (p(t)), (t \geq 0) \) be an \( \mathcal{H} \)-valued, \( \sigma \)-finite stationary \( \mathcal{F}_t \)-adapted Poisson point process on \( (\Omega, \mathcal{F}, \mathbb{P}) \). The counting random measure \( N \) defined by

\[
N((t_1, t_2] \times \theta)(w) = \sum_{t_1 < s \leq t_2} 1_{l}(p(s)(w)),
\]

for any \( \mathcal{B} \in \mathcal{B}(\mathcal{H} - \{0\}) \), where \( 0 \notin \mathcal{F} \) is called the Poisson random measure associated to Poisson point process \( p \). The following notation is used

\[
N(t, \theta) = N((0, t] \times \theta).
\]

Then it is known that there exists a \( \sigma \)-finite measure \( \theta \) such that

\[
\mathbb{E}(N(t, \theta)) = \nu(\theta)t,
\]

\[
\mathbb{P}(N(t, \theta) = k) = \frac{\exp(-t\nu(\theta))(tv(\theta))^k}{k!}.
\]

This measure \( \nu \) is said Levy measure. Then the measure \( \tilde{N} \) is defined by

\[
\tilde{N}((0, t] \times \theta) = N((0, t] \times \theta) - t\nu(\theta).
\]

This measure \( \tilde{N}(dt, du) \) is called the compensated Poisson random measure, and \( d\nu(\theta) \) is called the compensator (see [30]).

**Definition 2.1.** Let \( \theta \in \mathcal{B}(\mathcal{H} - \{0\}) \), \( P^2([0, T] \times \theta; \mathcal{H}) \) is the space of all predictable mappings \( h : [0, T] \times \theta \times \Omega \rightarrow \mathcal{H} \) for which

\[
\int_0^T \int_\theta \mathbb{E}\|h(t, \nu)\|^2 dt d\nu < \infty.
\]

We may then define the \( \mathcal{H} \)-valued stochastic integral \( \int_0^T \int_\theta L(t, \nu)\tilde{N}(dt, d\nu) \), which is a centered square-integral martingale [30].
2.2 Rosenblatt process

In this subsection, we recall some basic concepts on the Rosenblatt process as well as the Wiener integral with respect to it. Consider \((\xi_n)_{n \in \mathbb{Z}}\) a stationary Gaussian sequence with mean zero and variance 1 such that its correlation function satisfies that \(R(n) := \mathbb{E}(\xi_0 \xi_n) = n^{-\frac{2H}{1+2H}} L(n)\), with \(H \in \left(\frac{1}{2}, 1\right)\) and \(L\) is a slowly varying function at infinity. Let \(g\) be a function of Hermite rank \(k\), that is, if \(g\) admits the following expansion in Hermite polynomials

\[
g(x) = \sum_{j=0}^{k} c_j H_j(x), \quad c_j = \frac{1}{\pi} \mathbb{E}(g(\xi_0 H_j(\xi_0))),
\]

then \(k = \min\{j \mid c_j \neq 0\} \geq 1\), where \(H_j(x)\) is the Hermite polynomial of degree \(j\) given by \(H_j(x) = (-1)^j e^{x^2} \frac{d^j}{dx^j} e^{-x^2}\). Then, the Non-Central Limit Theorem (see, for example, Dobrushin & Major [10]) says \(\frac{1}{\sqrt{n}} \sum_{j=1}^{[nt]} g(\xi_j)\) converges as \(n \to \infty\), in the sense of finite dimensional distributions, to the process

\[
Z^k_H(t) = c(H, k) \int_0^t \left( \prod_{j=1}^k (s - y_j)^{\frac{1}{2} - \frac{1+4H}{2}} \right) ds d B(y_1) \cdots d B(y_k),
\]

where the above integral is a Wiener-Itô multiple integral of order \(k\) with respect to the standard Brownian motion \((B(y))_{y \in \mathbb{R}}\) and \(c(H, k)\) is a positive normalization constant depending only on \(H\) and \(k\). The process \((Z^k_H(t))_{t \geq 0}\) is called as the Hermite process and it is \(H\) self-similar in the sense that for any \(c > 0\), \((Z^k_H(ct)) \overset{d}{=} (c^H Z^k_H(t))\) and it has stationary increments.

The fractional Brownian motion (which is obtained from (2) when \(k = 1\)) is the most used Hermite process to study evolution equations due to its large range of applications. When \(k = 2\) in (2), Taqqu [32] named the process as the Rosenblatt process. The stationarity of increments, self-similarity and long range dependence (see Tindel, Tudor and Viens [33]) were made that the Rosenblatt process is very important in practical applications. However, it is noted that Rosenblatt process is not Gaussian. In fact, due to their properties (long range dependence, self-similarity), the fractional Brownian motion process has large utilization in practical models, for instance in telecommunications and hydrology. So, many researchers prefer to use fractional Brownian motion than other processes because it is Gaussian and it facilitate calculations. However in concrete situations when the Gaussianity is not plausible for the model, one can use the Rosenblatt process.

In recent years, there exists many works that investigated on diverse theoretical aspects of the Rosenblatt process. For example, Leonenko and Ahn [23] gave the rate of convergence to the Rosenblatt process in the Non-Central Limit Theorem and the wavelet-type expansion has been presented by Abry and Pipiras [1]. Tudor [34] established, the representation as a Wiener-Itô multiple integral with respect to the Brownian motion on a finite interval and developed the stochastic calculus with respect to it by using both pathwise type calculus and Malliavin calculus (see also Maejima and Tudor [27]). For more details on Rosenblatt process, we refer the reader to Maejima and Tudor [28, 29], Pipiras and Taqqu [31] and the references therein.

Consider a time interval \([0, T]\) with arbitrary fixed horizon \(T\) and let \(\{Z_H(t), t \in [0, T]\}\) be a one-dimensional Rosenblatt process with parameter \(H \in \left(\frac{1}{2}, 1\right)\). According to the work of Tudor [34], the Rosenblatt process with parameter \(H > \frac{1}{2}\) can be written as

\[
Z_H(t) = d(H) \int_0^t \int_0^t \left\{ \int_{y_1 \leq y_2} \frac{\partial K^H}{\partial u}(u, y_1) - \frac{\partial K^H}{\partial u}(u, y_2) du \right\} dB(y_1)dB(y_2),
\]

where \(K^H(t, s)\) is given by

\[
K^H(t, s) = c_H s^{\frac{1}{2} - H} \int_s^t (u-s)^{H-3/2} u^{H-1/2} du \text{ for } t > s,
\]

with

\[
c_H = \sqrt{\frac{H(2H-1)}{\beta(2-2H, H - \frac{1}{2})}},
\]
The covariance structure of the Rosenblatt process allows to construct Wiener integral with respect to it. We refer to Maejima and Tudor [27] for the definition of Wiener integral with respect to general Hermite processes and to Kruk, Russo, and Tudor [19] for a more general context (see also Tudor [34]).

Note that

\[
\beta(\ldots) \text{ denotes the Beta function, } K^H(t, s) = 0 \text{ when } t \leq s, \quad (B(t), t \in [0, T]) \text{ is a Brownian motion, } H' = \frac{H+1}{2} \quad \text{and} \quad d(H) = \frac{1}{\Gamma(T)} \sqrt{\frac{H}{\pi^{H-1}}} \text{ is a normalizing constant. The covariance of the Rosenblatt process } \{Z_H(t), t \in [0, T]\} \text{ satisfy}
\]

\[
E(Z_H(t)Z_H(s)) = \frac{1}{2} \left(s^{2H} + t^{2H} - |s-t|^{2H}\right).
\]

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Let

\[
Z_H(t) = \int_{0}^{T} \int_{0}^{T} I(1_{[0,1]})(y_1, y_2)dB(y_1)dB(y_2),
\]

where the operator \(I\) is defined on the set of functions \(f : [0, T] \to \mathbb{R}\), which takes its values in the set of functions \(g : [0, T]^2 \to \mathbb{R}^2\) and is given by

\[
I(f)(y_1, y_2) = d(H) \int_{y_1 \vee y_2}^{T} f(u) \frac{\partial K^H}{\partial u}(u, y_1) \frac{\partial K^H}{\partial y_2}(u, y_2) du.
\]

Let \(f\) be an element of the set \(E\) of step functions on \([0, T]\) of the form

\[
f = \sum_{i=0}^{n-1} a_i 1_{(t_i, t_{i+1})}, \quad t_i \in [0, T].
\]

Then, it is natural to define its Wiener integral with respect to \(Z_H\) as

\[
\int_{0}^{T} f(u)dZ_H(u) := \sum_{i=0}^{n-1} a_i (Z_H(t_{i+1}) - Z_H(t_i)) = \int_{0}^{T} I(f)(y_1, y_2)dB(y_1)dB(y_2).
\]

Let \(H\) be the set of functions \(f\) such that

\[
||f||^2_{H} := 2 \int_{0}^{T} \int_{0}^{T} (I(f)(y_1, y_2))^2 dy_1 dy_2 < \infty.
\]

It follows that (see Tudor[34])

\[
||f||^2_{H} = H(2H-1) \int_{0}^{T} \int_{0}^{T} f(u)f(v)|u-v|^{2H-2}dudv.
\]

It has been proved in Maejima and Tudor [27] that the mapping

\[
f \to \int_{0}^{T} f(u)dZ_H(u)
\]

defines an isometry from \(E\) to \(L^2(\Omega)\) and it can be extended continuously to an isometry from \(H\) to \(L^2(\Omega)\) because \(E\) is dense in \(H\). We call this extension as the Wiener integral of \(f \in H\) with respect to \(Z_H\). It is noted that the space \(H\) contains not only functions but its elements could be also distributions. Therefore it is suitable to know subspaces \(|H|\) of \(H\) : \(|H| = \left\{ f : [0, T] \to \mathbb{R} \mid \int_{0}^{T} \int_{0}^{T} f(u)||f(v)||u-v|^{2H-2}dudv < \infty \right\} \). The space \(|H|\) is not complete with respect to the norm \(\|\cdot\|_{|H|}\) but it is a Banach space with respect to the norm

\[
||f||^2_{|H|} = H(2H-1) \int_{0}^{T} \int_{0}^{T} f(u)||f(v)||u-v|^{2H-2}dudv.
\]
As a consequence, we have 
\[ L^2([0, T]) \subset L^{1/H}([0, T]) \subset \mathcal{H} \subset \mathcal{K}. \]
For any \( f \in L^2([0, T]) \), we have 
\[ \|f\|^2_{2^H} \leq 2HT^{2H-1} \int_0^T |f(s)|^2 \, ds \]
and 
\[ \|f\|^2_{\mathcal{K}} \leq C(H)\|f\|^2_{L^{1/H}([0, T])}, \]
for some constant \( C(H) > 0 \). Let \( C(H) > 0 \) stands for a positive constant depending only on \( H \) and its value may be different in different appearances.

Define the linear operator \( K_H^\ast \) from \( \mathcal{E} \) to \( L^2([0, T]) \) by 
\[ (K_H^\ast f)(y_1, y_2) = \int_{y_1 \vee y_2}^T f(t) \frac{\partial \mathcal{K}}{\partial t}(t, y_1, y_2) \, dt, \]
where \( \mathcal{K} \) is the kernel of Rosenblatt process in representation (3)
\[ \mathcal{K}(t, y_1, y_2) = 1_{[0,1]}(y_1)1_{[0,1]}(y_2) \int_{y_1 \vee y_2}^t \frac{\partial K_H^\ast}{\partial u}(u, y_1)\frac{\partial K_H^\ast}{\partial u}(u, y_2) \, du. \]
Note that \( (K_H^\ast 1_{[0,1]})(y_1, y_2) = \mathcal{K}(t, y_1, y_2)1_{[0,1]}(y_1)1_{[0,1]}(y_2) \). The operator \( K_H^\ast \) is an isometry between \( \mathcal{E} \) to \( L^2([0, T]) \), which can be extended to the Hilbert space \( \mathcal{K} \). In fact, for any \( s, t \in [0, T] \) we have
\[ \left\langle K_H^\ast 1_{[0,1]}, K_H^\ast 1_{[0,s]} \right\rangle_{L^2([0,T])} = \left\langle \mathcal{K}(t, \ldots), 1_{[0,1]}(y_1)1_{[0,1]}(y_2) \right\rangle_{L^2([0,T])} \]
\[ = \int_{t \land s}^{t \land s} \int_0^t \mathcal{K}(t, y_1, y_2) \mathcal{K}(s, y_1, y_2) \, dy_1 \, dy_2 \]
\[ = H(2H-1) \int_0^t \int_0^s |u-v|^{2H-2} \, du \, dv \]
\[ = \left\langle 1_{[0,1]}, 1_{[0,s]} \right\rangle_{\mathcal{K}}. \]
Moreover, for \( f \in \mathcal{K} \), we have
\[ Z_H(f) = \int_0^T \int_0^T (K_H^\ast f)(y_1, y_2) \, dB(y_1) \, dB(y_2). \]
Let \( \{Z_n(t)\}_{n \in \mathbb{N}} \) be a sequence of two-sided one dimensional Rosenblatt process mutually independent on \((\Omega, \mathcal{F}, \mathbb{P})\). We consider a \( K \)-valued stochastic process \( Z_Q(t) \) given by the following series:
\[ Z_Q(t) = \sum_{n=1}^{\infty} Z_{Q(n)}(t)Q^{1/2}e_n, \quad t \geq 0. \]
Moreover, if \( Q \) is a non-negative self-adjoint trace class operator, then this series converges in the space \( K \), that is, it holds that \( Z_Q(t) \in L^2(\Omega, K) \). Then, we say that the above \( Z_Q(t) \) is a \( K \)-valued \( Q \)-Rosenblatt process with covariance operator \( Q \). For instance, if \( \{\sigma_n\}_{n \in \mathbb{N}} \) is a bounded sequence of non-negative real numbers such that \( Qe_n = \sigma_n e_n \), by assuming that \( Q \) is a nuclear operator in \( K \), then the stochastic process
\[ Z_Q(t) = \sum_{n=1}^{\infty} Z_{Q(n)}(t)Q^{1/2}e_n = \sum_{n=1}^{\infty} \sqrt{\sigma_n}Z_{Q(n)}(t)\sigma_n, \quad t \geq 0, \]
is well-defined as a \( K \)-valued \( Q \)-Rosenblatt process.
Definition 2.2. (Tudor[34]). Let \( \varphi : [0, T] \to L_0^2(K, \mathbb{H}) \) such that \( \sum_{n=1}^{\infty} \| K_T^r(\varphi Q^{1/2} e_n) \|_{L^2([0, t]; \mathbb{H})} \) \( < \infty \). Then, its stochastic integral with respect to the Rosenblatt process \( Z(t) \) is defined, for \( t \geq 0 \), as follows:

\[
\int_0^t \varphi(s) dZ(s) := \sum_{n=1}^{\infty} \int_0^t \varphi(s) Q^{1/2} e_n dz_n(s) = \sum_{n=1}^{\infty} \int_0^t (K_T^r(\varphi Q^{1/2} e_n))(y_1, y_2) dB(y_1) dB(y_2). \tag{6}
\]

Lemma 2.1. For \( \varphi : [0, T] \to L_0^2(K, \mathbb{H}) \) such that \( \sum_{n=1}^{\infty} \| \varphi Q^{1/2} e_n \|_{L^2([0, t]; \mathbb{H})} \) \( < \infty \) holds, and for any \( a, b \in [0, T] \) with \( b > a \), we have

\[
E \left\| \int_a^b \varphi(s) dZ(s) \right\|^2 \leq c(H)(b-a)^{2H-1} \sum_{n=1}^{\infty} \int_a^b \| \varphi(s) Q^{1/2} e_n \|^2 ds.
\]

If, in addition,

\[
\sum_{n=1}^{\infty} \| \varphi(t) Q^{1/2} e_n \| \text{ is uniformly convergent for } t \in [0, T],
\]

then, it holds that

\[
E \left\| \int_a^b \varphi(s) dZ(s) \right\|^2 \leq C(H)(b-a)^{2H-1} \int_a^b \| \varphi(s) Q^{1/2} e_n \|^2 ds.
\]

Proof. Let \{e_n\}_n be the complete orthonormal basis of \( K \) introduced above. Applying (6) and Hölder inequality, we have

\[
E \left\| \int_a^b \varphi(s) dZ(s) \right\|^2 = E \left\| \sum_{n=1}^{\infty} \int_a^b \varphi(s) Q^{1/2} e_n dz_n(s) \right\|^2
\]

\[
= \sum_{n=1}^{\infty} E \left\| \int_a^b \varphi(s) Q^{1/2} e_n dz_n(s) \right\|^2
\]

\[
= \sum_{n=1}^{\infty} H(2H-1) \int_a^b \int_a^b \| \varphi(s) Q^{1/2} e_n \| \| \varphi(t) Q^{1/2} e_n \| |t-s|^{2H-2} ds dt
\]

\[
\leq C(H) \sum_{n=1}^{\infty} \left( \int_a^b \| \varphi(s) Q^{1/2} e_n \|^{1/2} ds \right)^{2H}
\]

\[
\leq C(H)(b-a)^{2H-1} \sum_{n=1}^{\infty} \int_a^b \| \varphi(s) Q^{1/2} e_n \|^2 ds.
\]

Lemma 2.2. [37] Let a function \( f : \mathbb{R} \to (0, \infty) \) be such that there exist positive constants \( \omega > 0, \alpha_j (j = 1, 2, 3) \), and \( \beta_i (i = 1, 2, \cdots) \) such that

\[
\Gamma(t) \leq \begin{cases} 
\alpha_1 e^{-\omega t} & \text{for } t \in [-r, 0], \\
\alpha_1 e^{-\omega t} + \alpha_2 \sup_{\theta \in [-r, 0]} \Gamma(t + \theta) + \alpha_3 \int_0^t e^{-\omega(t-s)} \sup_{\theta \in [-r, 0]} \Gamma(t + \theta) ds \\
+ \sum_{i \leq t} \beta_i e^{-\omega(t-i)} \Gamma(i) & \text{for } t \geq 0.
\end{cases}
\]
If \( \alpha_2 + \frac{a_3}{(\omega - \gamma)} + \sum_{i=1}^{\infty} \beta_i < 1 \), the \( \Gamma(t) \leq Ne^{-\gamma t} \) for \( t \geq -r \), where \( \Gamma > 0 \) is the unique solution to the equation

\[
\alpha_2 + \frac{a_3}{(\omega - \gamma)} e^{\gamma t} + \sum_{i=1}^{\infty} \beta_i = 1 \quad \text{and} \quad N = \max\{a_1, \frac{a_1(\omega - \gamma)}{a_1 e^{\gamma t}}\} > 0.
\]

### 2.3 Partial integro-differential equations in Banach spaces

In this section, we recall some fundamental results needed to establish our main results. For the theory of resolvent operators we refer the reader to [13]. Throughout this paper, \( \mathbb{H} \) is a Banach space, \( A \) and \( Y(t) \) are closed linear operators on \( \mathbb{H} \). \( Y \) represents the Banach space \( D(A) \) equipped with the graph norm defined by

\[
|x|_Y := |Ay| + |x| \quad \text{for} \quad y \in Y.
\]

The notations \( C([0, +\infty); Y), \mathcal{B}(Y, \mathbb{H}) \) stand for the space of all continuous functions from \([0, +\infty)\) into \( Y \), the set of all bounded linear operators from \( Y \) into \( \mathbb{H} \), respectively. We consider the following Cauchy problem

\[
\begin{align*}
\rho'(t) &= A\rho(t) + \int_0^t Y(t-s)\rho(s)ds, \quad \text{for} \quad t \geq 0, \\
\rho(0) &= \nu_0 \in \mathbb{H}.
\end{align*}
\]

**Definition 2.3.** ([13]). A resolvent operator for Eq. (7) is a bounded linear operator valued function \( \Pi(t) \in \mathcal{L}(\mathbb{H}) \) for \( t \geq 0 \), satisfying the following properties:

(i) \( \Pi(0) = I \) and \( |\Pi(t)| \leq Me^{\beta t} \) for some constants \( M \) and \( \beta \).

(ii) For each \( x \in \mathbb{H} \), \( \Pi(t)x \) is strongly continuous for \( t \geq 0 \).

(iii) \( \Pi(t) \in \mathcal{L}(Y) \) for \( t \geq 0 \). For \( x \in Y \), \( \Pi(\cdot)x \in C^1([0, +\infty); \mathbb{H}) \cap C([0, +\infty); Y) \) and

\[
\Pi'(t)x = A\Pi(t)x + \int_0^t Y(s)(t-s)\Pi(s)xds
\]

\[
= \Pi(t)Ax + \int_0^t \Pi(t-s)Y(s)xds \quad \text{for} \quad t \geq 0.
\]

**Remark 2.1.** There exist a constant \( \hat{M} > 0 \) such that \( ||\Pi(t)|| \leq \hat{M} \), for \( t \in [0, T] \).

The following assumptions are imposed on the system under consideration:

**(H1)** \( A \) is the infinitesimal generator of a strongly \( C_0 \)-semigroup \( \{S(t)\}_{t \geq 0} \) on \( \mathbb{H} \).

**(H2)** For all \( t \geq 0 \), \( Y(t) \) is a closed linear operator from \( D(A) \) to \( \mathbb{H} \), and \( Y(t) \in \mathcal{B}(Y, \mathbb{H}) \). For any \( y \in Y \), the map \( t \to Y(t)y \) is bounded, differentiable and the derivative \( t \to Y(t)y \) is bounded and uniformly continuous on \( \mathbb{R}^+ \).

**Theorem 2.3.** ([13, Theorem 3.7]) Assume that **(H1)**-**(H2)** hold. Then there exists a unique resolvent operator for the Cauchy problem (7).
3 Main Results

3.1 Existence of mild solution

In this section, we present and prove the existence and uniqueness of mild solutions of Eq. (1) by means of the theory of resolvent operator and contraction mapping principle. First of all, we begin with the definition of mild solution for Eq. (1)

**Definition 3.1.** An $H$-valued stochastic process $u(t), t \in [−\tau, T]$, is called a mild solution of Eq. (1) if

1. $u(\cdot) \in \mathcal{PC}([-\tau, T], L^2(\Omega, H))$,
2. $u(t) = \phi(t), t \in [−\tau, 0]$,
3. for $t \in [0, T], u(t)$ satisfies the following integral equation:

$$
u(t) = \Pi(t) [\phi(0) - q(0, \phi)] + q(t, u_t)
+ \int_{0}^{t} \Pi(t - s) f(s, u_s) ds + \int_{0}^{t} \Pi(t - s) \int_{t}^{s} g(s, s, u_t) d\eta ds
+ \sum_{0 < t_k < t} \Pi(t - t_k) I_k(u(t_k)) + \int_{0}^{t} \Pi(t - s) h(s, u_s) N(ds, dv)
+ \int_{0}^{t} \Pi(t - s) \sigma(s) d\mu(s), t \in [0, T]$$

(8)

In order to attain the result, we impose the following assumptions:

(H3) For all $t \in [0, T]$, there exist constants $0 < K_1 < 1$ such that, for $\psi_j \in \mathcal{PC}, j = 1, 2$, the $H$-valued function $q : [0, +\infty) \times \mathcal{PC} \to H$ satisfies the condition

$$||q(t, \psi_1) - q(t, \psi_2)|| \leq K_1 ||\psi_1 - \psi_2||.$$  

Also, $\bar{K}_1 = \sup_{t \in [0, T]} ||q(t, 0)||$.

(ii) The function $q$ is continuous in the quadratic mean sense. $\forall \phi \in \mathcal{PC},$

$$\lim_{t \to s} \mathbb{E}||q(t, \phi) - q(s, \phi)||^2 = 0.$$  

(H4) There exists a constant $K_2 > 0$ such that, for $\psi_j \in \mathcal{PC}, j = 1, 2$, the mapping $f : [0, +\infty) \times \mathcal{PC} \to H$ satisfies the following Lipschitz condition for all $t \in [0, T]:$

$$||f(t, \psi_1) - f(t, \psi_2)|| \leq K_2 ||\psi_1 - \psi_2||.$$  

Here $\bar{K}_2 = \sup_{t \in [0, T]} ||f(t, 0)||$.

(H5) The mapping $g : [0, +\infty) \times [0, +\infty) \times \mathcal{PC} \to H$ satisfies the following Lipschitz condition. For $t \in [0, T]$,

$$\int_{0}^{t} ||g(t, s, \psi_1) - g(t, s, \psi_2)|| ds \leq K_3 ||\psi_1 - \psi_2||.$$  

Here $\bar{K}_3 = T \sup_{0 < s < T} ||g(t, s, 0)||.$
We have the following two conditions for the complete orthonormal basis $\{a_n\}_{n \in \mathbb{N}}$ in $Y$.

\[(H6)\] The mapping $h : [0, +\infty) \times \theta \times \mathcal{P}C \to \mathbb{H}$ satisfies the following Lipschitz condition. For $t \in [0, T]$, there exists a constant $K_4 > 0$ such that, for $\psi_1, \psi_2 \in \mathcal{P}C$, $j = 1, 2$,

$$\int \frac{||h(t, \psi_1, v) - f(t, \psi_2, v)||^2 \lambda(dv)}{2} \leq K_4 ||\psi_1 - \psi_2||.$$ 

\[(H7)\] The impulsive function $I_k : \mathbb{H} \to \mathbb{H}$ is continuous. There exists constants $d_k > 0$ ($k = 1, 2, \ldots$) satisfying \[\sum_{k=1}^{\infty} d_k < \infty,\] such that

$$||I_k(\psi_1) - I_k(\psi_2)|| \leq d_k ||\psi_1 - \psi_2||, \quad ||I_k(0)|| = 0 \quad \text{for all } \psi_1, \psi_2 \in \mathcal{P}C.$$ 

\[(H8)\] The function $\sigma : [0, +\infty) \to L^0_Q(Y, \mathbb{H})$ satisfies

$$\int_0^t ||\sigma(s)||_{L_0}^2 ds < \infty, \quad \forall \ t \in [0, T].$$

We now establish the existence and uniqueness results for Eq.(1).

**Theorem 3.1.** Assume that hypotheses (H1)–(H7) are satisfied for all $\phi \in \mathcal{P}C$, $T > 0$, and

$$K_1 + \frac{4M^2}{(1 - K_1)} \left( \sum_{i=1}^{\infty} d_i \right)^2 < 1. \quad (9)$$

Then, the Eq.(1) has a unique mild solution on $[-\tau, T]$.

**Proof.** To begin, we introduce $\Lambda_T := \mathcal{P}C([-\tau, T], L^2(\Omega, \mathbb{H}))$ the Banach space of all continuous functions from $[-\tau, T]$ into $L^2(\Omega, \mathbb{H})$ equipped with the supremum norm $||\xi||_{\Lambda_T} = \sup_{s \in [-\tau, T]} (\mathbb{E}||\xi(s)||^2)$. Now consider the closed subset $\hat{\Lambda}_T = \{ u \in \Lambda_T : u(\tau) = \phi(\tau) \text{ for } \tau \in [-\tau, 0] \}$ of $\Lambda_T$ endowed with the norm $||\cdot||_{\hat{\Lambda}_T}$. The problem (1) is transformed into a fixed point problem. We define the operator $\Psi : \hat{\Lambda}_T \to \hat{\Lambda}_T$ by

\[(\Psi u)(t) = \begin{cases} 
\phi(t) - g(0, \phi) + g(t, u_t) 
+ \int_0^t \Pi(t - s)f(s, u_s)ds + \int_0^t \Pi(t - s)h(s, \eta, u_\eta)d\eta ds \\
+ \sum_{0 < t_k < t} \Pi(t - t_k)I_k(u(t_k)) + \int_0^t \Pi(t - s)\sigma(s)dZ^0(s) \quad P - a.s. 
\end{cases} \]
Now, to prove the existence result of mild solution of Eq. (1), it is sufficient to show that $Ψ$ has a fixed point. To this end we subdivide the proof into two steps.

**Step 1:** First, we show that the map $t \to (Ψx)(t)$ is continuous on the interval $[0, T]$. Let $|t|$ be sufficiently small, for $u \in \mathcal{A}_T$ and $0 < t < T$. We get

\[
E|||\{(Ψu)(t + \tilde{t}) - (Ψu)(t)\}||^2 \leq 12E \left\| \left( \Pi(t + \tilde{t}) - \Pi(t) \right) \left( \phi(0) - q(0, \phi) \right) \right\|^2 + 12E \left\| q(t + \tilde{t}, u_{t+\tilde{t}}) - q(t, u_t) \right\|^2
\]

\[
+ 12E \left\| \int_0^t \left( \Pi(t + \tilde{t} - s) - \Pi(t - s) \right) f(s, u_s) ds \right\|^2 + 12E \left\| \int_0^t \Pi(t + \tilde{t} - s)f(s, u_s) ds \right\|^2
\]

\[
+ 12E \left\| \int_0^t \left( \Pi(t + \tilde{t} - s) - \Pi(t - s) \right) g(s, \eta, u_\eta) d\eta ds \right\|^2
\]

\[
+ 12E \left\| \int_0^t \left( \Pi(t + \tilde{t} - s) - \Pi(t - s) \right) h(s, u_s, \nu) \bar{N}(ds, du) \right\|^2
\]

\[
+ 12E \left\| \int_0^t \left( \Pi(t + \tilde{t} - s) - \Pi(t - s) \right) k(s, \phi) \bar{N}(ds, dv) \right\|^2
\]

\[
+ 12E \left\| \Pi(t + \tilde{t}) - \Pi(t) \right\|^2
\]

\[
+ 12E \left\| \int_0^t \Pi(t + \tilde{t} - s) \sigma(s) dZ^H_0(s) \right\|^2 + 12E \left\| \int_0^t \Pi(t + \tilde{t} - s) \sigma(s) dZ^H_0(s) \right\|^2
\]

\[
\leq 12 \sum_{j=1}^{12} P_j.
\]

By Definition 2.3-(ii), we have

\[
\lim_{\tilde{t} \to 0} P_1 = 0.
\]

By using (ii) of (H3) we obtain that

\[
\lim_{\tilde{t} \to 0} P_2 = 0.
\]

By using Hölder inequality we have:

\[
P_3 \leq tE \int_0^t \left\| \left( \Pi(t + \tilde{t} - s) - \Pi(t - s) \right) f(s, u_s) \right\|^2 ds.
\]

By Definition 2.3-(ii), we have

\[
\lim_{\tilde{t} \to 0} \left( \Pi(t + \tilde{t} - s) - \Pi(t - s) \right) f(s, u_s) = 0
\]

and by combining the assumption (H4) and Definition 2.3-(ii), we obtain

\[
\left\| \left( \Pi(t + \tilde{t} - s) - \Pi(t - s) \right) f(s, u_s) \right\|^2 \leq 2M^2(e^{2(t+\tilde{t})\beta} + e^{2t\beta})(K_2||u_t||^2 + K_2),
\]

\[
E|||\{(Ψu)(t + \tilde{t}) - (Ψu)(t)\}||^2 \leq 12E \left\| \left( \Pi(t + \tilde{t}) - \Pi(t) \right) \left( \phi(0) - q(0, \phi) \right) \right\|^2 + 12E \left\| q(t + \tilde{t}, u_{t+\tilde{t}}) - q(t, u_t) \right\|^2
\]

\[
+ 12E \left\| \int_0^t \left( \Pi(t + \tilde{t} - s) - \Pi(t - s) \right) f(s, u_s) ds \right\|^2 + 12E \left\| \int_0^t \Pi(t + \tilde{t} - s)f(s, u_s) ds \right\|^2
\]

\[
+ 12E \left\| \int_0^t \left( \Pi(t + \tilde{t} - s) - \Pi(t - s) \right) g(s, \eta, u_\eta) d\eta ds \right\|^2
\]

\[
+ 12E \left\| \int_0^t \left( \Pi(t + \tilde{t} - s) - \Pi(t - s) \right) h(s, u_s, \nu) \bar{N}(ds, du) \right\|^2
\]

\[
+ 12E \left\| \int_0^t \left( \Pi(t + \tilde{t} - s) - \Pi(t - s) \right) k(s, \phi) \bar{N}(ds, dv) \right\|^2
\]

\[
+ 12E \left\| \Pi(t + \tilde{t}) - \Pi(t) \right\|^2
\]

\[
+ 12E \left\| \int_0^t \Pi(t + \tilde{t} - s) \sigma(s) dZ^H_0(s) \right\|^2 + 12E \left\| \int_0^t \Pi(t + \tilde{t} - s) \sigma(s) dZ^H_0(s) \right\|^2
\]

\[
\leq 12 \sum_{j=1}^{12} P_j.
\]
The Lebesgue dominated theorem, implies that 
\[ \lim_{l \to 0} P_3 = 0. \]

Using Hölder inequality, we have
\[
P_4 \leq E \left( \int_{t}^{t+1} \left\| \Pi(t + \tilde{l} - s)f(s, u_s) \right\| ds \right)^2
\leq \int_{t}^{t+1} \left\| \Pi(t + \tilde{l} - s) \right\|^2 ds E \int_{t}^{t+1} \left\| f(s, u_s) \right\|^2 ds
\leq \tilde{M}^2 \int_{t}^{t+1} E \left\| f(s, u_s) \right\|^2 ds.
\]

Thus, we have 
\[ \lim_{l \to 0} P_4 = 0. \]

By using similar arguments to \( P_2 \) and combining assumption \((H5)\) we obtain
\[ \lim_{l \to 0} P_5 = 0. \]

Also, with the same argument to \( P_3 \) and using assumption \((H5)\) we obtain that 
\[ \lim_{l \to 0} P_6 = 0. \]

Similarly, with the same argument to \( P_3 \) and using assumption \((H5)\) we obtain that
\[ \lim_{l \to 0} P_{11} = 0, \quad \lim_{l \to 0} P_{12} = 0. \]

For \( P_7 \), application of Lemma 2.1 gives
\[
P_7 \leq 2cH(2H - 1)\tau^{2H-1} \int_{0}^{t} \left\| \left( \Pi(t + \tilde{l} - s) - \Pi(t - s) \right) \sigma(s) \right\|_{L^2_0}^2 ds.
\]

By Definition 2.3-(ii) we have
\[ \left( \Pi(t + \tilde{l} - s) - \Pi(t - s) \right) \sigma(s) \to 0, \text{ as } \tilde{l} \to 0 \]
and by Definition 2.3-(i) we have the inequality:
\[ \left\| \Pi(t + \tilde{l} - s) - \Pi(t - s) \sigma(s) \right\|_{L^2_0}^2 \leq 2M^2 (e^{2(\tilde{l} - \tilde{t})} + e^{2\tilde{t}\beta}) \left\| \sigma(s) \right\|_{L^2_0}^2, \]
and by the Lebesgue dominated convergence theorem, we get 
\[ \lim_{l \to 0} P_7 = 0. \]

By Lemma 2.1 we have
\[
P_8 \leq 2cH(2H - 1)\tau^{2H-1} \int_{t}^{t+1} \left\| \Pi(t - s) \sigma(s) \right\|_{L^2_0}^2 ds.
\]
since $s \in [0, T]$, we have
\[ P_8 \leq 2cH(2H - 1)T^{2H-1}M^2 \int_t^{t+1} ||\sigma(s)||^2_{L_q} \; ds, \]
and regarding to (H7), we get
\[ \lim_{l \to 0} P_8 = 0. \]
By using Definition 2.3-(i) we have
\[ \left\| \left( \Pi(t + \frac{l}{2} - t_k) - \Pi(t - t_k) \right) I_k(u(t_k)) \right\|^2 \leq 2M^2(e^{2(t-t_k)} + e^{2(t+\frac{l}{2})}) ||u(t_k)||^2. \]
By combining assumption (H6) and Definition 2.3-(i), we have
\[ \lim_{l \to 0} P_9 = 0. \]
By using assumption (H6) and Definition 2.3-(i), we get
\[ \lim_{l \to 0} P_{10} = 0. \]
Therefore, we can conclude
\[ \lim_{l \to 0} E||(\Psi x)(t + \tau) - (\Psi x)(t)||^2 = 0. \]
Hence, the above arguments imply that function $t \to (\Psi x)(t)$ is continuous on the interval $[0, T]$.

**Step 2:** In this part of the proof, we will verify that $\Psi$ is contraction mapping in $\hat{A}_{T_1}$ with some $T_1 \leq T$ to be specified later. Let $u, v \in \hat{A}_{T_1}$ and $t \in [0, T]$. By virtue of elementary inequality we obtain
\[
\left\| (\Psi u)(t) - (\Psi v)(t) \right\|^2 \leq \frac{1}{M} \left\| q(t, t_k) - q(t, t_k) \right\|^2 + \frac{1}{t - \theta} \left\{ \left\| \int_0^t \Pi(t - s) \left[ f(s, u_s) - f(s, v_s) \right] \; ds \right\|^2 \\
+ \left\| \int_0^t \int_0^s g(s, u, u) \; du - \int_0^t \int_0^s g(s, u, v) \; du \; ds \right\|^2 \\
+ \left\| \int_0^t \int_0^s \left[ h(s, u, v) - h(s, v, v) \right] N(ds, dv) \; ds \right\|^2 \\
+ \left\| \sum_{0 < t_k < t} \left[ I_k(x(t_k)) - I_k(y(t_k)) \right] \right\|^2 \right\}.
\]
By using assumptions (H3)-(H6), Definition 2.3 together with Hölder’s inequality, we get
\[
E||(\Psi u)(t) - (\Psi v)(t)||^2 \leq K_1 E||u(t) - v(t)||^2 + \frac{4M^2K^2_t}{1 - K_1} \int_0^t E||u_s - v_s||^2 \; ds + \frac{4M^2K^2_t}{1 - K_1} \int_0^t E||u_s - v_s||^2 \; ds + 4M^2 \left( \sum_{i=1}^{\infty} d_i \right)^2 \E||u(t_k) - v(t_k)||^2.
\]
Hence, we have
\[ \sup_{s \in [-r, t]} E||(\Psi u)(s) - (\Psi v)(s)||^2 \leq \theta(t) \sup_{s \in [-r, t]} E||u(s) - v(s)||^2, \]
where
\[ \theta(t) = K_1 + 4M^2 \left( \frac{K^2_t + K^2_t + K^2_t}{1 - K_1} \right) t + 4M^2 \left( \sum_{i=1}^{\infty} d_i \right)^2. \]
By inequality (9), we have

$$\theta(0) = K_1 + \frac{4M^2}{(1 - K_1)} \left( \sum_{k=1}^{+\infty} d_k \right)^2 < 1.$$ 

Then there exists $0 < T_1 \leq T$ such that $0 < \theta(T_1) < 1$ and the operator $\Psi$ is a contraction on $\hat{A}_{T_1}$ and hence it has a unique fixed point on $[-\tau, T_1]$, which is a mild solution of Eq.(1) on the interval $[-\tau, T_1]$. By repeating a similar process the solution can be extended to the entire interval $[-\tau, T]$.

**Remark 3.1.** Notice that we can extend the solution for $t \geq T$. Indeed, if we assume that the constants $K_1, K_2, K_3, K_4$ which appear in assumptions (H3)-(H6) are independent of $T > 0$, then the mild solution is defined for all $t \in [-\tau, T]$, for each $T > 0$. This will play a crucial role in our analysis of stability. Therefore, in the next section we will assume that the solutions are defined globally in time (for instance, under the previous assumptions).

### 3.2 Exponential stability

In this subsection, it is established the exponential stability in the mean square moment of the mild solution for Eq.(1), we need to state the following additional assumptions.

(H9) The corresponding resolvent operator $(II(t))_t$ of Eq. (7) verifies the following: There exist $\gamma > 0$ and $M > 0$ such that $\|II(t)\| \leq Me^{-\gamma t}$, for all $t \geq 0$.

(H10) There exist nonnegative real numbers $R_i > 0$ and continuous functions $\xi_i : [0, +\infty) \to \mathbb{R}_+$, $\xi_i(t) \leq a_i e^{-\gamma t}$, $i = 1, 2, 3, 4$, $a_i > 0$ such that for all $t \geq 0$ and $\psi_i \in \mathcal{Y}$

$$\|q(t, \psi_1)\|^2 \leq R_1 \|\psi_1\|^2 + \xi_1(t),$$

$$\|f(t, \psi_2)\|^2 \leq R_2 \|\psi_2\|^2 + \xi_2(t),$$

$$\left\| \int_{0}^{t} h(t, s, \psi_3) ds \right\|^2 \leq R_3 \|\psi_3\|^2 + \xi_3(t),$$

$$\int_{0}^{\theta} \left( \int_{\theta}^{t} \left\| h(t, \psi_4, v) \right\|^2 \lambda(dv) \right)^{1/2} \leq R_4 \|\psi_4\|^2 + \xi_4(t).$$

(H11) The function $\sigma : [0, +\infty) \to L_0^2(Y, \mathbb{H})$ satisfies the following condition in addition to assumptions (C.1) and (C.2):

$$\int_{0}^{+\infty} e^{\gamma s} \|\sigma(s)\|^2 ds < \infty.$$

**Theorem 3.2.** Assume that the conditions of Theorem 3.1 are satisfied and (H9)–(H11) are fulfilled. Then the mild solution of Eq.(1) is exponentially stable in mean square moment provided

$$\frac{5 \left[ M^2 (R_2 + R_3 + R_4) / \gamma^2 \right] + M^2 \left( \sum_{k=1}^{+\infty} d_k \right)^2}{(1 - k)^2} < 1,$$  

where $k := \sqrt{Q_1}$.

**Proof.** From (10), it is possible to find a suitable number $\bar{\gamma} > 0$ small enough such that

$$k + \frac{5M^2(R_2 + R_3 + R_4)}{\gamma(\gamma - \bar{\gamma})(1 - k)} + \frac{5M^2(\sum_{k=1}^{+\infty} d_k)^2}{(1 - k)} < 1.$$
Let assume that \( \mu = \gamma - \bar{t} \) and \( u(t) \) be a mild solution of Eq. (1). Then, from (8) we have

\[
E||u(t)||^2 \leq \frac{1}{k} E||q(t, u_t)||^2 + \frac{6}{1-k} E\left\{ ||II(t)[\phi(0) - q(0, \phi)]||^2 \right. \\
+ \left. \left| \left| \int_0^t II(t-s)f(s, u_s)ds \right| \right|^2 + \left| \left| \int_0^t II(t-s)\sigma(s)dz_Q(s) \right| \right|^2 \\
+ \left| \left| \int_0^t II(t-s)\int g(s, \eta, u_\eta)d\eta ds \right| \right|^2 + \left| \left| \int_0^t II(t-s)h(s, u_s, v)N(ds, dv) \right| \right|^2 \\
+ \left| \left| \sum_{0 \leq \tau \leq t} II(t-t_k)I_k(u(t_k)) \right| \right|^2 \right\} \\
\leq \sum_{j=1}^7 J_j(t),
\]

By assumption \((H10)\) we get

\[
J_1(t) = \frac{1}{k} E||q(t, u_t)||^2 \\
\leq \frac{1}{k} \{ R_1 E||u_t||^2 + \xi_1(t) \} \\
\leq k E||u_t||^2 + \lambda_1 e^{-\gamma t},
\]

where \( \lambda_1 = \frac{\alpha_1}{k} \).

According to \((H9)\) and \((H10)\) we obtain

\[
J_2 \leq \frac{10}{1-k} E||II(t)\phi(0)||^2 + \frac{10}{1-k} E||II(t)q(0, \phi)||^2 \\
\leq \frac{10M^2}{1-k} e^{-2\gamma t} E||\phi(0)||^2 + \frac{10M^2}{1-k} e^{-2\gamma t} \{ R_1 E||\phi||^2 + \xi_1(t) \} \\
\leq \lambda_2 e^{-\mu t},
\]

where \( \lambda_2 = \frac{10M^2}{1-k} [E||\phi(0)||^2 + \{ R_1 E||\phi||^2 + \alpha_1 \}] \).

Employing \((H9)\), \((H10)\) and Hölder’s inequality, we get

\[
J_3(t) = \frac{5}{1-k} E\left| \left| \int_0^t II(t-s)f(s, u_s)ds \right| \right|^2 \\
\leq \frac{5}{1-k} E\left( \int_0^t e^{-\gamma(t-s)} ||f(s, u_s)||ds \right)^2 \\
\leq \frac{5M^2 R_2}{\gamma(1-k)} e^{-\gamma(t-s)} E||u_s||^2 ds + \lambda_3 e^{-\mu t}
\]

where \( \lambda_3 = \frac{5M^2 R_3}{\gamma(1-k)} \frac{\alpha_3}{\gamma - \mu} \).
Also with the same arguments, we have

\[
J_4(t) \leq \frac{5}{1-k} \mathbb{E} \left( \int_0^t Ne^{-\gamma(t-s)} \left\| \int_0^t g(s, \eta, x_0) d\eta \right\| ds \right)^2
\]

\[
\leq \frac{5M^2 R_4}{\gamma(1-k)} \int_0^t e^{-\gamma(t-s)} E \left\| u_s \right\|^2 ds + \lambda_4 e^{-\mu t},
\]

where \(\lambda_4 = \frac{5M^2 R_4}{\gamma(1-k)} \frac{a_4}{\gamma - \mu} \).

By Lemma 2.1 and (H7) we have

\[
J_5 \leq \frac{5}{1-k} M^2 c(H) t^{2H-1} \int_0^t e^{-\gamma(t-s)} \left\| \sigma(s) \right\|^2_{L^2} ds
\]

\[
\leq e^{-\mu t} \frac{5M^2}{1-k} c(H) t^{2H-1} e^{-\gamma t} \int_0^t e^{\gamma s} \left\| \sigma(s) \right\|^2_{L^2} ds.
\]

According to assumption (H11), there exist a constant \(\gamma_5 > 0\) such that, for all \(t \geq 0\),

\[
\frac{5M^2}{1-k} c(H) t^{2H-1} \int_0^t e^{-\gamma(t-s)} ds \leq \lambda_5.
\]

Thus

\[
J_5(t) \leq \lambda_5 e^{-\mu t}.
\]

and

\[
J_6(t) \leq \frac{6}{1-k} \mathbb{E} \left( \left\| \int_0^t \int_0^t h(s, u_s, v) \tilde{N}(ds, dv) \right\| \right)^2
\]

\[
\leq \frac{6M^2}{\gamma(1-k)} R^4 \int_0^t e^{-\gamma(t-s)} E \left\| u_s \right\|^2 ds + \lambda_6 e^{-\mu t},
\]

where \(\lambda_6 = \frac{6M^2}{\gamma(1-k)} \frac{a_6}{\gamma - \mu} \).

By using (H7) and (H9) we have

\[
J_7(t) \leq \frac{5M^2}{1-k} \left( \sum_{i=1}^{+\infty} d_k \right)^2 e^{-\gamma(t-t_i)} \mathbb{E} \left\| u(t_k) \right\|^2
\]

\[
\leq \frac{5M^2}{1-k} \left( \sum_{i=1}^{+\infty} d_k \right) \left( \sum_{i=1}^{+\infty} d_i \right) e^{-\gamma(t-t_i)} \mathbb{E} \left\| u(t_k) \right\|^2.
\]

The above inequalities (11)-(17) together with Lemma 2.2, imply that

\[
\mathbb{E} \left\| u(t) \right\|^2 \leq ve^{-\mu t} \text{ for } t \in [-\tau, 0]
\]

and

\[
\mathbb{E} \left\| u(t) \right\|^2 \leq ve^{-\mu t} + k \sup_{-\tau \leq \theta \leq 0} \mathbb{E} \left\| u(t + \theta) \right\|^2 + \kappa \int_0^t e^{-\mu(t-s)} \sup_{-\tau \leq \theta \leq 0} \mathbb{E} \left\| u(t + \theta) \right\|^2 ds
\]

\[
+ \sum_{i=1}^{+\infty} \omega_k e^{-\mu(t-t_i)} \mathbb{E} \left\| u(t_k) \right\|^2, \quad t \geq 0
\]
where
\[ \hat{k} = \frac{5M^2(R_2 + R_3 + R_4)}{\gamma(1 - k)} \] and \( v = \max \left( \sum_{i=1}^{6} \lambda_i, \sup_{-\tau \leq t \leq 0} \mathbb{E}[|\phi(t)|^2] \right). \]

And we observe that \( k + \frac{\hat{k}}{\mu} + \sum_{k=1}^{\infty} \omega_k < 1. \)
Thus, the mild solution of Eq.(1) is exponentially stable in mean square moment, since \( k + \frac{\hat{k}}{\mu} + \sum_{k=1}^{\infty} \omega_k < 1 \) and by Lemma 2.2 we have the existence of two positive constants \( C \) and \( r \) such that \( \mathbb{E}[|u(t)|]^2 \leq Ce^{-rt} \) for any \( t \geq -\tau \), where \( \theta > 0 \) is the unique solution to the equation \( k + \frac{\hat{k}}{\mu} + \sum_{k=1}^{\infty} \omega_k = 1 \) and \( C = \max\{v, \frac{\nu}{\mu - r}\} > 0. \)
This completes the proof of the theorem.

\[ \square \]

4 Illustration

This part consist to make an application of the theory studied above. Consider the impulsive neutral stochastic partial integrodifferential equations of the form

\[
\begin{align*}
\{ & d[w(t, \xi) - \beta_1(t, w(t - r, \xi))] = \left[ \frac{\partial^2}{\partial x^2}[w(t, \xi) - \beta_1(t, w(t - r, \xi))] \\
& + \int_0^t b(t - s) \frac{\partial^2}{\partial x^2}[w(s, \xi) - \beta_1(s, w(s - r, \xi))]ds \\
& + \beta_2(t, w(t - r, \xi)) + \int_0^t \beta_3(t, s, w(s - r, \xi))ds \right] dt \\
& + \int_0^\theta \frac{\partial}{\partial \theta}[\beta_4(t, x(t - s), z)]N(ds, dz) + \sigma(t)dzH(t), \ 0 \leq \xi \leq \pi, \ t \neq t_k, \ t \in [0, T], \\
& w(t, 0) = w(t, \pi) = 0, \ 0 \leq t \leq T, \\
& \Delta w(t_k, \cdot)(\xi) = \beta_5 w(t_k, \xi), \ t = t_k, \ k = 1, 2, \ldots, \\
& w(t, \xi) = \phi(t, \xi) \in \mathcal{P}C([-r, 0], L^2[0, \pi]), \ -r \leq t \leq 0,
\end{align*}
\]

where \( Z^H(t) \) is Rosenblatt process define on probability space \( (Q, \mathcal{F}, \mathbb{P}) \), \( \beta_1, \beta_2 : \mathbb{R}^+ \times \mathbb{R} \rightarrow \mathbb{R} \), \( \beta_3, \beta_4 : \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R} \rightarrow \mathbb{R} \), \( b : \mathbb{R}^+ \rightarrow \mathbb{R} \) are continuous functions, \( \beta_5 \geq 0 \) constant and \( \sigma : [0, \infty) \rightarrow L^0_0(L^2([0, \pi]), L^2([0, \pi])) \).

Let \( H = Y = L^2[0, \pi] \) with the norm \( \| \cdot \| \) and \( e_n(x) = \sqrt{\frac{2}{\pi}} \sin nx, \ n = 1, 2, \ldots \). Then \( (e_n)_{n \in \mathbb{N}} \) is a complete orthogonal basis in \( Y \). In order to define the operator \( Q : Y \rightarrow Y \), we chose a sequence \( (\sigma_n)_{n=1} \subset \mathbb{R}^+ \) and set \( Qe_n = \sigma_ne_n \), and assume that \( \sum_{n=1}^{\infty} \sqrt{\sigma_n} < \infty \). Define the process \( Z^H(t) \) by

\[
Z^H(t) = \sum_{n=1}^{\infty} \sqrt{\sigma_n}e^H_n e_n,
\]

where \( H \in (\frac{1}{2}, 1) \) and \( \left\{ z^H_n \right\}_{n \in \mathbb{N}} \) is a sequence of two-side one-dimensional Rosenblatt process mutually independent.

Define the operator \( A : D(A) \subset H \rightarrow H \) by \( A = \frac{\partial}{\partial t} \) with domain \( D(A) = H^1_0(0, \pi) \cap H^2(0, \pi) \).

It is well-known that \( A \) is the infinitesimal generator of a strong continuous semigroup \( \{T(t)\}_{t \geq 0} \) on \( H \), which is given by

\[
T(t)x = \sum_{n=1}^{\infty} e^{-n^2t}(x, e_n)e_n \text{ and } Ax = -\sum_{n=1}^{\infty} n^2(x, e_n)e_n, \ x \in D(A).
\]

We suppose that:
(i) There exist a positive constant $l_1$, $0 < \pi l_1^2 < 1$ such that
\[
|\beta_1(t, y_1) - \beta_1(t, y_2)| \leq l_1 |y_1 - y_2|, \quad t \geq 0, \ y_1, y_2 \in \mathbb{R}.
\]

(ii) There exist a positive constant, $l_2 > 0$ such that
\[
|\beta_2(t, y_1) - \beta_2(t, y_2)| \leq l_2 |y_1 - y_2|, \quad t \geq 0, \ y_1, y_2 \in \mathbb{R}.
\]

(iii) There exist a positive constant, $l_3 > 0$ such that
\[
\left| \int_0^t [\beta_3(t, y_1) - \beta_3(t, y_2)] \, ds \right| \leq l_3 |y_1 - y_2|, \quad t \geq 0, \ y_1, y_2 \in \mathbb{R}.
\]

(iv) There exist a positive constant, $l_4 > 0$ such that
\[
\left| \int_\theta^\infty [\beta_4(t, y_1, v) - \beta_4(t, y_2, v)] \tilde{N}(ds, dv) \right| \leq l_4 |y_1 - y_2|, \quad t \geq 0, \ y_1, y_2 \in \mathbb{R}.
\]

(v) There exist nonegative real numbers $Q_1, Q_2, Q_3, Q_4 > 0$ and functions $\xi_1, \xi_2, \xi_3, \xi_4 : [0, \infty) \mapsto \mathbb{R}^+$ with $\xi_i(t) \leq P_i e^{-\lambda t} (i = 1, 2, 3, 4)$, $P_1 > 0$, such that
\[
\begin{align*}
|\beta_1(t, y)|^2 &\leq Q_1 |y|^2 + \xi_1(t), \\
|\beta_2(t, y)|^2 &\leq Q_2 |y|^2 + \xi_2(t), \\
\left| \int_0^t \beta_3(t, s, y) \, ds \right|^2 &\leq Q_3 |y|^2 + \xi_3(t), \\
\left| \int_\theta^\infty \beta_4(t, v, y) \tilde{N}(ds, dv) \right|^2 &\leq Q_4 |y|^2 + \xi_4(t).
\end{align*}
\]

(vi) The function $\sigma : [0, \infty) \mapsto L^0_\sigma \left( L^2([0, \pi]), L^2([0, \pi]) \right)$ satisfies
\[
\int_0^t \| \sigma(s) \|_{L^0_\sigma}^2 \, ds < \infty, \quad t \in [0, T] \quad \text{and} \quad \int_0^\infty e^{ts} \| \sigma(s) \|_{L^0_\sigma}^2 \, ds < \infty.
\]

Let $Y : D(A) \subset \mathbb{H} \rightarrow \mathbb{H}$ the operator define by $Y(t)z = b(t)Az$ for $t \geq 0$, $z \in D(A)$. For $\xi \in [0, \pi]$, we define the operators $g, f : [0, \infty) \times \mathbb{H} \mapsto \mathbb{H}, g : [0, \infty) \times [0, \infty) \times \mathbb{H} \mapsto \mathbb{H}, h : [0, \infty) \times \theta \times \mathbb{H} \mapsto \mathbb{H}$ and $I_k : \mathbb{H} \mapsto \mathbb{H}$ by
\[
\begin{align*}
G(t, \phi)(\xi) &= \beta_1(t, \phi(\xi)), \\
F(t, \phi)(\xi) &= \beta_2(t, \phi(\xi)), \\
\int_0^t \tilde{H}(t, s, \phi)(\xi) \, ds &= \int_0^t \beta_3(t, s, \phi(\xi)) \, ds, \\
\int_\theta^\infty h(t, \phi)(\xi) &= \beta_4(t, \phi(\xi)), \\
I_k(\phi)(\xi) &= \beta_k^2 \phi(\xi) \quad (k = 1, 2, \ldots).
\end{align*}
\]

If we put
\[
\begin{cases}
\begin{aligned}
u(t, \xi) &= w(t, \xi), &\text{for } t \geq 0, \ \text{and} \ \xi \in [0, \pi], \\
\phi(\theta)(\xi) &= u_0(\theta, \xi), &\text{for } -\tau \leq \theta \leq 0, \ \text{and} \ \xi \in [0, \pi],
\end{aligned}
\end{cases}
\]

...
then, Eq.(18) takes the following abstract form

\[
\begin{align*}
\frac{d}{dt} [u(t) - q(t, u_t)] &= A(u(t) - q(t, u_t)) + \int_0^t Y(t - s) [u(s) - q(s, u_s)] \, ds + f(t, u_t) \\
&\quad + \int_0^t g(t, s, u_s) \, ds dt + \int_0^t h(t, u_t, v) N(dt, dv) + \sigma(t) dz(t), \quad t \in [0, T], \ t \neq t_k, \\
\Delta u(t) &= I_k(u(t_k)), \quad t = t_k, k = 1, 2, \ldots, \\
u_0(t) &= \phi(t) \in \mathcal{D}_{[\mathcal{L}, 0], \mathbb{H}}, \quad -\tau \leq t \leq 0,
\end{align*}
\]

(19)

Moreover, if \( b \) is bounded and \( C^1 \) function such that \( b' \) is bounded and uniformly continuous, then \((H1)\) and \((H2)\) are satisfied, and hence, by Theorem 2.3, Eq.(7) has a resolvent operator \((R(t))_{t \geq 0}\) on \( \mathbb{H} \). Using Lemma 5.2 [9], let \( \mu > \delta > 1 \) and \( b(t) < \exp(-\beta t) \), for all \( t \geq 0 \). Then the above resolvent operator decays exponentially to zero. Specifically \( \| R(t) \| \leq \exp(-at) \) where \( a = 1 - 1/\delta \).

For \((t, s, \phi_j) \in \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{H}, j = 1, 2, \) respectively by assumptions (i), (ii), (iii) and (iv), we get

\[
\begin{align*}
&\| q(t, \phi_1) - q(t, \phi_2) \| \leq l_1 \| \phi_1 - \phi_2 \|, \\
&\| f(t, \phi_1) - f(t, \phi_2) \| \leq l_3 \| \phi_1 - \phi_2 \|, \\
&\left\| \int_0^t [g(t, s, \phi_1) - g(t, s, \phi_2)] \, ds \right\| \leq l_2 \| \phi_1 - \phi_2 \|, \\
&\left\| \int_2^t h(t, \phi_1, v) - h(t, \phi_2, v) N(dt, dv) \right\| \leq l_4 \| \phi_1 - \phi_2 \|, \\
&\| q(t, \phi_1) \|^2 \leq Q_1 \| \phi_1 \|^2 + \xi_1(t), \\
&\| f(t, \phi_1) \|^2 \leq Q_2 \| \phi_1 \|^2 + \xi_2(t), \\
&\left\| \int_0^t g(t, s, \phi_1) \, ds \right\|^2 \leq Q_3 \| \phi_1 \|^2 + \xi_3(t), \\
&\left\| \int_0^t h(t, \phi_1, v) \, dv \right\|^2 \leq Q_4 \| \phi_1 \|^2 + \xi_4(t).
\end{align*}
\]

We have also that

\[
\left\| I_k(\phi_1) - I_k(\phi_2) \right\| \leq d_k \| \phi_1 - \phi_2 \|,
\]

where \( d_k = \frac{\beta_k}{\tau}, k = 1, 2, \ldots \) and \( \sum_{k=1}^{\infty} d_k = \sum_{k=1}^{\infty} \frac{\beta_k}{k_1} < \infty \).

Thus, all assumptions of Theorem 3.1 are fulfilled. Therefore, the existence of a mild solution of Eq.(18) follows. In addition, by Theorem 3.2, we easily see that the mild solution of Eq.(18) is exponentially stable in the 2nd moment.

5 Conclusion

In this article, we showed existence and unicity of mild solution to Eq.(1) by using Banach fixed point theorem. Further, we investigated exponential stability of mild solutions. The last part of this paper is devoted to an example to illustrate our results.
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