Thermophoresis of an Antiferromagnetic Soliton

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We study dynamics of an antiferromagnetic soliton under a temperature gradient. To this end, we start by phenomenologically constructing the stochastic Landau-Lifshitz-Gilbert equation for an antiferromagnet with the aid of the fluctuation-dissipation theorem. We then derive the Langevin equation for the soliton’s center of mass by the collective coordinate approach. An antiferromagnetic soliton behaves as a classical massive particle immersed in a viscous medium. By considering a thermodynamic ensemble of solitons, we obtain the Fokker-Planck equation, from which we extract the average drift velocity of a soliton. The diffusion coefficient is inversely proportional to a small damping constant $\alpha$, which can yield a drift velocity of tens of m/s under a temperature gradient of $1\, \text{K/mm}$ for a domain wall in an easy-axis antiferromagnetic wire with $\alpha \sim 10^{-4}$.

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Introduction.—Ordered magnetic materials exhibit solitons and defects that are stable for topological reasons. Well-known examples are a domain wall (DW) in an easy-axis magnet or a vortex in a thin film. Their dynamics have been extensively studied because of fundamental interest as well as practical considerations such as the racetrack memory [1]. A ferromagnetic (FM) soliton can be driven by various means, e.g., an external magnetic field [2] or a spin-polarized electric current [3]. Recently, the motion of an FM soliton under a temperature gradient has attracted a lot of attention owing to its applicability in an FM insulator [4–7]. A temperature gradient of $20\, \text{K/mm}$ has been demonstrated to drive a DW at a velocity of $200\, \mu\text{m/s}$ in a yttrium iron garnet film [8].

An antiferromagnet (AFM) is of great current interest in the field of spintronics [9–11] due to a few advantages over an FM. First, the characteristic frequency of an AFM is several orders higher than that of a typical FM, e.g., a timescale of optical magnetization switching is an order of ps for AFM NiO [12] and ns for FM CrO$_2$ [13], which can be exploited to develop faster spintronic devices. Second, absence of net magnetization renders the interaction between AFM particles weak, and, thus, leads us to prospect for high-density AFM-based devices. Dynamics of an AFM soliton can be induced by an electric current or a magnon current [14,15].

A particle immersed in a viscous medium exhibits a Brownian motion due to a random force that is required to exist to comply with the fluctuation-dissipation theorem (FDT) [17]. An externally applied temperature gradient can also be a driving force, engendering a phenomenon known as thermophoresis [18]. Dynamics of an FM and an AFM includes spin damping, and, thus, involves thermal fluctuations at a finite temperature [19]. The effect of thermal stochastic field has been studied for the dynamics of an FM soliton [7,20], but not for an AFM one.

In this Letter, we study the Brownian motion of a soliton in an AFM under a temperature gradient. We derive the stochastic Landau-Lifshitz-Gilbert (LLG) equation for an AFM with the aid of the FDT, which relates the fluctuation of the staggered and net magnetization to spin damping. We then derive the Langevin equation for the soliton’s center of mass by employing the collective coordinate approach [15,21]. We develop the Hamiltonian mechanics for collective coordinates and conjugate momenta of a soliton, which sheds light on stochastic dynamics of an AFM soliton; it can be considered as a classical massive particle moving in a viscous medium. By considering a thermodynamic ensemble of solitons, we obtain the Fokker-Planck equation, from which we extract the average drift velocity of a soliton. As a case study, we compute the drift velocity of a DW in a quasi one-dimensional easy-axis AFM.

Thermophoresis of a Brownian particle is a multi-faceted phenomenon, which involves several competing mechanisms. As a result, a motion of a particle depends on properties of its environment such as the medium or a temperature $T$. For example, in a general aqueous medium, a particle moves to a colder region for $T > 4^\circ\text{C}$ and otherwise to a hotter region [18]. Thermophoresis of
an AFM soliton would be at least as complex as that of a Brownian particle. We have focused on one aspect of it in this Letter; the effect of thermal stochastic force on dynamics of the soliton. We discuss two other possible mechanisms, the effects of a thermal magnon current and an entropic force \[1\], later in the Letter.

**Main results.**—Before pursuing details of derivations, we first outline our three main results. Let us consider a bipartite AFM with two sublattices that can be transformed into each other by a symmetry transformation of the crystal. Its low-energy dynamics can be developed in terms of two fields: the unit staggered spin field \( \mathbf{n} \equiv (\mathbf{m}_1 - \mathbf{m}_2)/2 \) and the net spin field \( \mathbf{m} \equiv (\mathbf{m}_1 + \mathbf{m}_2)/2 \) perpendicular to \( \mathbf{n} \). Here, \( \mathbf{m}_1 \) and \( \mathbf{m}_2 \) are unit vectors along the directions of spin angular momentum in the sublattices. Our first main result is the stochastic LLG equation \([1]\),

\[
\begin{align*}
\dot{\mathbf{n}} + \beta \mathbf{n} \times \mathbf{m} &= \mathbf{n} \times (\mathbf{h} + \mathbf{h}^\text{th}), \\
\dot{\mathbf{m}} + \beta \mathbf{m} \times \mathbf{m} + \alpha \mathbf{n} \times \dot{\mathbf{n}} &= \mathbf{n} \times (\mathbf{g} + \mathbf{g}^\text{th}) \\
&+ \mathbf{m} \times (\mathbf{h} + \mathbf{h}^\text{th}),
\end{align*}
\]

in conjunction with the correlators of the thermal stochastic fields \( \mathbf{g}^\text{th} \) and \( \mathbf{h}^\text{th} \),

\[
\begin{align*}
\langle g^\text{th}_i(r, t)g^\text{th}_j(r', t') \rangle &= 2k_BT\alpha s\delta_{ij}\delta(\mathbf{r} - \mathbf{r}')\delta(t - t'), \\
\langle h^\text{th}_i(r, t)h^\text{th}_j(r', t') \rangle &= 2k_BT\beta s\delta_{ij}\delta(\mathbf{r} - \mathbf{r}')\delta(t - t'),
\end{align*}
\]

which are independent of each other. Here \( \alpha \) and \( \beta \) are the damping constants associated with \( \mathbf{n} \) and \( \mathbf{m} \), \( \mathbf{g} \equiv -\delta U/\delta \mathbf{n} \) and \( \mathbf{h} \equiv -\delta U/\delta \mathbf{m} \) are the effective fields conjugate to \( \mathbf{n} \) and \( \mathbf{m} \), \( U[\mathbf{n}, \mathbf{m}] = U[\mathbf{n}] + \int d\mathbf{V}|\mathbf{m}|^2/(2\chi) \)

is the potential energy (\( \chi \) represents the magnetic susceptibility), and \( s = \hbar S/\mathcal{V} \) is the spin angular momentum density (\( \mathcal{V} \) is the volume per spin) per each sublattice.

Slow dynamics of stable magnetic solitons can often be expressed in terms of a few collective coordinates \( \mathbf{q} = \{q_1, q_2, \cdots \} \) parametrizing slow modes of the system. The center of mass \( \mathbf{R} \) represents the proper slow modes of a rigid soliton when the translational symmetry is weakly broken. Translation of the stochastic LLG equation \([1]\) into the language of the collective coordinates results in our second main result, a Langevin equation for the soliton’s center of mass \( \mathbf{R} \):

\[
M\ddot{\mathbf{R}} + \Gamma \dot{\mathbf{R}} = -\partial U/\partial \mathbf{R} + \mathbf{F}^\text{th},
\]

which adds the stochastic force \( \mathbf{F}^\text{th} \) to Eq. (5) of Tveten et al. \[16\]. The mass and dissipation tensors are symmetric and proportional to each other: \( M_{ij} = \rho \int d\mathbf{V}(\partial_j \mathbf{n} \cdot \partial_i \mathbf{n}) \) and \( \Gamma_{ij} = M_{ij}/\tau \), where \( \tau = \rho/(\alpha s) \) is the relaxation time, \( \rho \equiv \chi s^2 \) is the inertia of the staggered spin field \( \mathbf{n} \). The correlator of the stochastic field \( \mathbf{F}^\text{th} \) obeys the Einstein relation

\[
\langle F^\text{th}_i(t)F^\text{th}_j(t') \rangle = 2k_BT\gamma\delta_{ij}\delta(t - t').
\]

Temperature gradient \( T = T(x) \) along a certain axis \( \hat{x} \) causes a translational motion of the AFM soliton. In the absence of a deterministic force, the average drift velocity is proportional to a temperature gradient \( V \propto k_B

\[
\text{in the linear response regime. The form of the proportionality constant can be obtained by a dimensional analysis. Let us suppose that the mass and dissipation tensors are isotropic. The Langevin equation \([1]\) is, then, characterized by three scalar quantities: the mass \( M \), the viscous coefficient \( \Gamma \), and the temperature \( T \), which define the unique set of natural scales of time \( \tau = M/\Gamma \), length \( l = \sqrt{k_B TM/\Gamma^2} \), and energy \( \epsilon = k_BT \). Using these scales to match the dimension of a velocity yields \( V = -c\mu(k_BT) \), where \( \mu = \Gamma^{-1} \) is the mobility of an AFM soliton and \( c \) is a numerical constant. The explicit solution of the Fokker-Planck equation, indeed, shows \( c = 1 \). This simple case illustrates our last main result; a drift velocity of an AFM soliton under a temperature gradient in the presence of a deterministic force \( F \) is given by

\[
V = \mu F - \mu(k_BT).
\]

For a DW in an easy-axis one-dimensional AFM, the viscosity coefficient is \( \Gamma = \lambda/(2\alpha s) \), where \( \lambda \) is the width of the wall. For a numerical estimate, let us take an angular momentum density \( s = 2\hbar \text{nm}^{-1} \), a width \( \lambda = 100 \text{ nm} \), and a damping constant \( \alpha = 10^{-4} \) following the previous studies \[16,22\]. For these parameters, the AFM DW moves at a velocity \( V = 32 \text{ m/s} \) for the temperature gradient \( \nabla T = 1 \text{ K/mm} \).

**Stochastic LLG equation.**—Long-wave dynamics of an AFM at zero temperature can described by the Lagrangian

\[
L = s \int d\mathbf{V} \mathbf{m} \cdot (\mathbf{n} \times \dot{\mathbf{n}}) - U[\mathbf{n}, \mathbf{m}].
\]

We use the potential energy \( U[\mathbf{n}, \mathbf{m}] = \int d\mathbf{V}|\mathbf{m}|^2/(2\chi) + U[\mathbf{n}] \) throughout the paper. Minimization of the action subject to nonlinear constraints \( |\mathbf{n}| = 1 \) and \( \mathbf{n} \cdot \mathbf{m} = 0 \) yields the equations of motion for the fields \( \mathbf{n} \) and \( \mathbf{m} \). Damping terms that break the time reversal symmetry can be added to the equations of motion to the lowest order, which are first order in time derivative and zeroth order in spatial derivative. The resultant phenomenological LLG equations are given by

\[
\begin{align*}
\dot{\mathbf{n}} + \beta \mathbf{n} \times \mathbf{m} &= \mathbf{n} \times \mathbf{h}, \\
\dot{\mathbf{m}} + \beta \mathbf{m} \times \mathbf{m} + \alpha \mathbf{n} \times \dot{\mathbf{n}} &= \mathbf{n} \times \mathbf{g} + \mathbf{m} \times \mathbf{h}
\end{align*}
\]

\[22\]. The damping terms can be derived from the Rayleigh dissipation function

\[
R = \int d\mathbf{V}(\alpha s|\dot{\mathbf{n}}|^2 + \beta s|\dot{\mathbf{m}}|^2)/2,
\]

which is related to the energy dissipation rate by \( -\dot{U} = 2R \). The microscopic origin of damping terms does not
concern us here but it could be, e.g., caused by thermal phonons that deform the exchange and anisotropy interaction.

At a finite temperature, thermal agitation causes fluctuations of the staggered spin field $\mathbf{n}$ and net spin field $\mathbf{m}$. These thermal fluctuations can be considered to be caused by the stochastic fields $\mathbf{g}^{\text{th}}$ and $\mathbf{h}^{\text{th}}$ with zero mean, which are conjugate to $\mathbf{n}$ and $\mathbf{m}$, respectively; their noise correlators are then related to the dissipative coefficients by the FDT. The standard procedure to construct the noise sources yields the stochastic LLG equation $[1]$. The correlator of the stochastic fields are obtained in the following way $[17, 22]$. Casting the linearized LLG equation $[7]$ into the form $\{\mathbf{h}, \mathbf{g}\} = \gamma \otimes \{\mathbf{h}, \mathbf{m}\}$ provides the kinetic coefficients $\gamma$. Symmetrizing the kinetic coefficients $\gamma$ produces the correlators $[2]$ of the stochastic fields consistent with the FDT.

Langevin equation.—For slow dynamics of an AFM, the energy is mostly dissipated through the temporal variation of the staggered spin field $\mathbf{n}$ due to $|\mathbf{m}|^2 \times |\mathbf{n}|^2$ (from Eq. $[7a]$), which allows us to set $\beta = 0$ to study long-term dynamics of the magnetic soliton $[16]$. At this point, we switch to the Hamiltonian formalism of an AFM $[23]$, which sheds light on the stochastic dynamics of a soliton. The canonical momentum field $\pi$ conjugate to the staggered spin field $\mathbf{n}$ is

$$\pi \equiv \delta L/\delta \dot{\mathbf{n}} = s \mathbf{m} \times \mathbf{n}. \quad (9)$$

The canonical momentum represents essentially the net spin field $\mathbf{m}$. The stochastic LLG equations $[7]$ can be interpreted as Hamilton’s equations,

$$\dot{\mathbf{n}} = \delta H/\delta \pi = \pi/\rho, \quad \dot{\pi} = -\delta H/\delta \mathbf{n} - \delta R/\delta \dot{\mathbf{n}} + \mathbf{g}^{\text{th}}, \quad (10)$$

with the Hamiltonian

$$H \equiv \int dV \pi \cdot \dot{\mathbf{n}} - L = \int dV \frac{\pi^2}{2\rho} + U[\mathbf{n}], \quad (11)$$

where $\rho \equiv \chi s^2$ parametrizes the inertia of the staggered spin field.

Long time dynamics of magnetic texture can often be captured by focusing on a small subset of slow modes, which are parametrized by the collective coordinates $\mathbf{q} = \{q_1, q_2, \cdots \}$. A classical example is a DW in a one-dimensional easy-axis magnet described by the position of the wall $X$ and the azimuthal angle $\Phi$ $[2]$. Translation from the field language into that of collective coordinates can be done as follows. If the staggered spin field $\mathbf{n}$ is encoded by coordinates $\mathbf{q}$ as $\mathbf{n}(\mathbf{x}, t) = \mathbf{n}(\mathbf{x}; \mathbf{q}(t))$, time dependence of $\mathbf{n}$ reflects evolution of the coordinates: $\dot{n}_i = \dot{q}_i \delta \mathbf{n}/\delta q_i$. With the canonical momenta $\mathbf{p}$ defined by

$$p_i \equiv \frac{\partial L}{\partial \dot{q}_i} = \int dV \frac{\partial \mathbf{n}}{\partial \dot{q}_i} \cdot \pi, \quad (12)$$

Hamilton’s equations $[10]$ translate into

$$M \ddot{\mathbf{q}} = \mathbf{p}, \quad \dot{\mathbf{p}} + \Gamma \dot{\mathbf{q}} = \mathbf{F} + \mathbf{F}^{\text{th}}, \quad (13)$$

where $\mathbf{F} \equiv -\partial U/\partial \mathbf{q}$ is the deterministic force and $F_i^{\text{th}} \equiv \int dV \partial q_i \mathbf{n} \cdot \mathbf{g}^{\text{th}}$ is the stochastic force. Hamilton’s equations $[13]$ can be derived from the Hamiltonian in the collective coordinates and conjugate momenta,

$$H \equiv \mathbf{p}^T M^{-1} \mathbf{p}/2 + U(\mathbf{q}), \quad (14)$$

with the Poisson brackets $\{q_i, p_j\} = \delta_{ij}$, $\{q_i, q_j\} = \{p_i, p_j\} = 0$. An AFM soliton, thus, behaves as a classical particle moving in a viscous medium.

We focus on a translational motion of a rigid AFM soliton by choosing its center of mass as the collective coordinates $\mathbf{q} = \mathbf{R}$. Eliminating momenta from Hamilton’s equations $[13]$ and multiplying both sides by $M^{-1}$ yield the Langevin equation for the soliton’s center of mass:

$$\ddot{\mathbf{R}} + \dot{\mathbf{R}} = \mu \mathbf{F} + \eta. \quad (15)$$

where $\eta \equiv \mu \mathbf{F}^{\text{th}}$ is the stochastic velocity. Here the mobility tensor of the soliton $\mu \equiv T^{-1}$ relates a deterministic force to a drift velocity $\langle \mathbf{R} \rangle = \mu \mathbf{F}$ at a constant temperature. The mobility is inversely proportional to a damping constant, which can be a small number for an AFM, e.g., $\alpha \sim 10^{-4}$ for NiO $[25]$. The correlator of the stochastic velocity is given by

$$\langle \eta_i(t) \eta_j(t') \rangle = 2k_B T \delta_{ij} \delta(t - t') \equiv 2D_{ij} \delta(t - t'). \quad (16)$$

The Langevin equation $[15]$ with the correlator of the stochastic velocity $[16]$ determines the slow dynamics of the magnetic soliton at a finite temperature.

From Eq. $[16]$, we see that diffusion of the soliton caused by thermal fluctuation and energy dissipation through spin damping respect the Einstein relation: $D = \mu k_B T$, which can be naturally understood in the following way. The Hamiltonian $[14]$ is composed of both the kinetic and potential energy. A system of an ensemble of magnetic solitons at thermal equilibrium is described by the partition function $Z \equiv \int \Pi[dp_i dx_i / (2\pi \hbar)] \exp(-H/k_b T)$, which provides the autocorrelation of the velocity, $\langle \dot{x}_i \dot{x}_j \rangle/2 = M^{-1}_{ij} k_B T/2$ (essentially the equipartition theorem). In the absence of an external force, multiplying $\tau \ddot{x}_i + \dot{x}_i = \eta_i$ $[15]$ by $x_j$ and symmetrizing it with respect to indices $i$ and $j$ give the equation, $\tau d^2 \langle x_i x_j \rangle / dt^2 + d/\langle x_i x_j \rangle / dt = 2\tau \langle \ddot{x}_i x_j \rangle$, where the first term can be neglected for long-term dynamics $t \gg \tau$. This equation in conjunction with the autocorrelation of the velocity allows us to obtain the diffusion coefficient $D_{ij}$ in Eq. $[16]$, $\langle x_i x_j \rangle = 2k_B T \tau M^{-1}_{ij} t = 2D_{ij} t$, without prior knowledge about the correlator $[2]$ of the stochastic fields.

Average dynamics.—An AFM soliton exhibits Brownian motion at a finite temperature. The following Fokker-Planck equation for an ensemble of solitons in an inhomogeneous medium describes the evolution of the density
\[ \rho(R, t) \text{ at time } t \gg \tau : \]
\[ \frac{\partial \rho}{\partial t} + \nabla \cdot j = 0, \quad \text{with } j = \mu F \rho - D \nabla \rho - D_T(k_B \nabla T), \]
where the thermophoretic mobility \( D_T \equiv \mu \) is usually dubbed “thermal diffusion coefficient” [18] [20]. The stochastic part of the soliton current \( j \) is composed of a Brownian diffusive term \( -D \nabla \rho \) and a thermal diffusive term \( -D_T(k_B \nabla T) \). The average drift velocity is determined by the thermal diffusive term [27]:
\[ V = \mu F - \mu(k_B \nabla T). \tag{18} \]

**Example**—Let us take an example of a DW in a quasi one-dimensional easy-axis AFM with the energy
\[ U[n] = \int dV(A[\partial_x n]^2 - K n_x^2)/2. \]
A DW in the equilibrium is \( n^{(0)} = (\sin \theta \cos \Phi, \sin \theta \sin \Phi, \cos \theta) \) with \( \cos \theta = \tan h[(x - X)/\lambda] \), where \( \lambda \equiv \sqrt{A/K} \) is the width of the wall. There are two zero-energy modes of the system: the position \( X \) and the azimuthal angle \( \Phi \) of the DW, which are engendered by the translational and spin-rotational symmetry of the energy. Their dynamics are decoupled, \( \Gamma_X = 0 \), which allows us to study the dynamics of \( X \) separately from \( \Phi \). The mobility of the DW is \( \mu = \lambda/(2\alpha\sigma) \), where \( \sigma \) is the cross-sectional area of the AFM perpendicular to the \( x \)-axis. The average drift velocity [18] is given by
\[ V = \frac{1}{2\alpha} \frac{k_B}{\sigma} \lambda. \tag{19} \]

For a numerical estimate, let us use an angular momentum density \( \sigma \alpha = 2h \text{nm}^{-1} \), a DW width \( \lambda = 100 \text{nm} \), and a damping constant \( \alpha = 10^{-4} \) following the previous studies [16] [22]. With these parameters, the AFM DW moves at a velocity \( V = 32 \text{m/s} \) for the temperature gradient \( \nabla T = 1 \text{K/mm} \).

**Discussion**—We have showed that an AFM soliton at a finite temperature can be treated as a classical particle moving in a viscous medium. Thermal fluctuations of magnetic moments in an AFM translate into Brownian motion of a soliton; a small damping constant of an AFM translate into a large diffusion coefficient. We have studied an example of a DW in a narrow AFM wire with a small damping constant, which can be driven efficiently by applying a temperature gradient.

The deterministic force \( F \) on an AFM soliton can be extended to include the effect of an electric current, an external field, and a magnon current [14] [10]. It depends on details of interaction between the soliton and the external degrees of freedom. Quantitative understanding of the deterministic drift velocity \( \mu F \), therefore, demands the accurate knowledge on the interactions. The drift velocity \( V \) [15] of the soliton under a temperature gradient, however, is completely determined by local property of the soliton.

We have focused on the thermal stochastic force as a trigger of thermophoresis of an AFM soliton in this Letter. There are two other possible ingredients of thermophoresis of a soliton. One is a thermal magnon current, scattering with which could exert a force on a soliton. The effect is, however, negligible at temperature lower than the spin wave gap (\( \sim 40 \text{K} \) for NiO [25]). For our example—a DW in a one-dimensional easy-axis AFM—thermal magnons pass through the DW without any backscattering [10] [23], which leads us to expect their effect on other solitons to be small as well. The other is an entropic force, which originates from thermally-induced decreasing of a spin length [3]. For an FM, the effect of the entropic force on a soliton could be studied in the framework of the Landau-Lifshitz-Bloch equation [29]. An analogous framework for an AFM has not been developed yet. A naive estimate for a drift velocity caused by the entropic force can be guessed from the result for an FM DW [1]: \( V^{\text{en}} \sim k_B \lambda / \alpha \); the thermal stochastic force dominates the entropic force for dynamics of a wide DW in a thin wire. A DW in a wire with a large crosssection \( \alpha \gg a^2 \) forms a membrane at its center, whose fluctuations foment additional soft modes of the dynamics; understanding of a motion of such a DW requires the study of equations of motions [13] involving the soft modes.

We have studied dynamics of an AFM soliton in the Hamiltonian formalism. Hamiltonian’s equations [13] for the collective coordinates and conjugate momenta can be derived from the Hamiltonian [14] with the conventional Poisson bracket structure. In particular, the Hamiltonian allows us to describe a thermodynamic ensemble of magnetic solitons, which are subject to thermal fluctuations. By replacing the Poisson brackets with the commutators, the collective coordinates and conjugate momenta can be promoted to quantum operators. This may provide a one route to study the effect of quantum fluctuations on dynamics of an AFM soliton [30].

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