Development of Sand Volume Estimator for Under-Struck Excavator Bucket Using Single Camera

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Abstract
To support the intelligence of construction environment, it is important to measure the workload of excavator in real time. But, previous studies are expensive to implement or can not processed in real time. In this paper, an image-based method to estimate the workload of the excavator bucket especially for the state of under-struck is proposed by assuming the shape of bucket and the shape of sand inside the bucket as geometric models. By analyzing the relation between single camera image and actual bucket geometry, the volume of sand which is proportional to the excavator workload is estimated. The experimental results show 93.5% accuracy even though only some part of sand region is unseen.

Keywords: Sand volume estimator, Excavator bucket modeling, Single camera

1. Introduction
It is not easy to measure in real time the amount of sand which an excavator load in working environment [1, 2]. Previous method to measure this workload in real environment was to measure the charge of weight of dump truck. In order to make this weight change, it is needed to stop the truck in a certain area. However, this method is inefficient in terms of time because it decreases the working efficiency by stopping the truck. This paper proposes a new method which can estimate automatically the volume of sand in a bucket by capturing an image from a single camera in the excavator and analyzing the image specifically focused on the under-struck state.

2. Background
Depending on the amount of sand piled up in a bucket of excavator, states could be divided like Figure 1. It is called as struck state if the volume of bucket and that of sand are equal. It is called as under-struck state if the volume of sand is less than struck state. If the volume of sand is larger than struck state, it is called as heaped state. This paper focuses on under-struck state. The purpose of this paper is to estimate the amount of sand from the bucket image based on a single camera image processing [3, 4].

This paper assumes A1-A4 to estimate the volume of under-struck state.
A1: The shape of bucket is modeled as a combination of half cylinder and right triangular prism.
A2: The bucket diameter ‘2a’, bucket width ‘b’ and bucket teeth length ‘c’ are given by the excavator company.

A3: When the image is captured, the upper side of bucket is horizontal to the ground.

A4: In Figure 2, the pixel locations of points (P$_1$ to P$_8$) and the highest vertical line (U) can be calculated with image processing [3, 4].

3. Single Camera-based Sand Volume Estimation Algorithm for Under-Struck State

Under-struck state means that the highest line of sand U is below the line (Figure 3). This state could be divided into three cases in order to reduce the complexity of the formula. (Figure 2) The condition dividing the under-struck state is as follows:

L1: line segment $AB$
L2: parallel line segment with $AB$ passing through O
L3: Parallel line segment with $AB$ passing through D

If ($U < L3$) state $\leftarrow$ Under-Struck-Case1
Else if ($U < L2$) state $\leftarrow$ Under-Struck-Case2
Else if ($U < L1$) state $\leftarrow$ Under-Struck-Case3

Algorithm 3.1. Volume_Estimator

INPUT
P$_1$, ..., P$_6$: Six trapezoidal points of the bucket
U: The uppermost edge point of the sand region in photographed image
V$_1$: Center point of $P_1P_2$
V$_2$: Center point of $P_3P_6$
V$_3$: Center point of $P_4P_5$
C: Intersection point of line segment $V_1V_3$ and the center line of the image
D$_1$: Intersection of $P_1P_3$ and $P_2P_5$
D$_2$: Intersection of $P_1P_4$ and $P_2P_5$

OUTPUT
V: The volume of sand accumulated in the bucket

FUNCTION
Volume_Estimator(P$_1$, ..., P$_6$, U, V$_1$, V$_2$, V$_3$, C, D$_1$, D$_2$)

$\theta_C \leftarrow \cos^{-1} \left( \frac{f_{1(m+1)}^2 + (m+1)^2f^2 - b^2 - f_{1(m+1)}^2 + f^2(m+1)^2 + b^2f^2}{2bf_{1(m+1)}^2} \right)$
$h \leftarrow \frac{l_{1}^\prime m(m' - 1)b}{l_{1}^\prime l_{2(m'+1)}^\prime \sin \theta_y + l_{1}^\prime l_{2(m'+1)}^\prime m' \cos \theta_y}$
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In order to calculate the volume, metric unit such as mm is known by default such as l. Specific parameters are required to estimate the volume in case of under-struck. The unit of parameters l, w, h are mm, and so on. These basic parameters can be obtained by measuring each point (P1 ~ P8, U) through image processing.

In Figure 5,

\[ l_w = \sqrt{(\alpha_2 - \alpha_1)^2 + (\beta_2 - \beta_1)^2}, \]
\[ l_w'' = \sqrt{(\alpha_3 - \alpha_2)^2 + (\beta_3 - \beta_2)^2}, \]
\[ l_h''' = \sqrt{(\alpha_7 - \alpha_1)^2 + (\beta_7 - \beta_1)^2 - \frac{(l_w - l_w'')^2}{2}}, \]
\[ l_h'' = \sqrt{(\alpha_5 - \alpha_1)^2 + (\beta_5 - \beta_1)^2 - \frac{(l_w - l_w'')^2}{2}}. \]

The unit of parameters l_w, l_w'', l_h, l_h''' are number of pixels. In order to calculate the volume, metric unit such as mm is needed instead of number of pixels. This conversion is possible by multiplying the actual pixel size of image sensor.

### 3.2 Calculation of Angle between the Camera Centre Line ZF and the Bucket Line AB

We can not always take the picture at the same angle. That changes the parameter values of bucket. This paper use the expression of the angle between the camera centre line and the bucket to solve this problem. This is called \( \theta_C \). This section will describe the process of calculating \( \theta_C \).

In Figure 6, lets assume that

\[ r_2 = k r_1. \]

By the similarity between \(\triangle EDD'\) and \(\triangle EC'B\),

\[ 2a \cos \theta_c : r_1 = d : (r_1 + r_2) = d : (1 + k) r_1, \]
\[ k = \frac{d}{2a \cos \theta_c} - 1. \]

In Figure 6,

\[ L_1 = 2a \sin \theta_c - k r_1 = 2a \sin \theta_c - \left( \frac{d}{2a \cos \theta_c} - 1 \right) r_1. \]

By the similarity between \(\triangle FZY\) and \(\triangle FC'B\),

\[ d : f = L_1 : l_2, \]
\[ l_2 = \frac{f}{d} \left[ 2a \sin \theta_c - \left( \frac{d}{2a \cos \theta_c} - 1 \right) r_1 \right]. \]
In Figure 3, the ratio of \( l \) is fixed angle so it can be obtained by Eq. (25).

\[
\theta = \tan^{-1}\left( \frac{c}{2a} \right).
\]

By Figure 5 and Figure 6, \( l_2 = l_1 + l_2 = \frac{f}{d}(2a \sin \theta_c + r_1) \)

By Eq. (15) and Eq. (19),

\[
l_1 = \frac{dl''}{2a \cos \theta_c} - f \tan \theta_c,
\]

\[
l_2 = l'' + f \tan \theta_c - \frac{dl''}{2a \cos \theta_c}.
\]

\( l_1 \) and \( l_2 \) are decided based on the centerline of the camera image. Let's assume that the ratio of \( l_2 \) and \( l_1 \) is \( m \).

\[
l_2 = ml_1, \quad l''_m = (1 + m)l_1, \quad l_1 = \frac{l''_m}{1 + m}.
\]

\[
\theta_y \text{ can be obtained by using } \theta_C \text{ and } \theta_B. \quad \theta_y = \frac{\pi}{2} - \theta_C + \theta_B.
\]

In Figure 7, let's assume that

\[
r_2' = k'r_1'.
\]

The value \( k' \) can be obtained by using the similarity of \( \triangle EB'H' \) and \( \triangle EC'F \).

\[
h \cos \theta_y : r_1' = d' : (r_1' + r_2') = d' : (1 + k')r_1'.
\]

By the similarity between \( \triangle FEC' \) and \( \triangle FYZ \)

\[
d' = f = \frac{d'r_1'}{h \cos \theta_y} : l_2',
\]

\[
l_2' = \frac{f r_1'}{h \cos \theta_y}.
\]

By the similarity between \( \triangle FHC' \) and \( \triangle FXA \)

\[
d' = f = L_1' : l_1',
\]

\[
L_1' = k'r_1' - h \sin \theta_y = \left( \frac{d'}{h \cos \theta_y} - 1 \right) r_1' - h \sin \theta_y,
\]

\[
l_1' = \frac{f}{d'} \left[ \left( \frac{d'}{h \cos \theta_y} - 1 \right) r_1' - h \sin \theta_y \right].
\]
Figure 7. Camera geometry for calculating $h$.

In Figure 5,

$$l'_h = l'_2 - l'_1 = \frac{f}{d}(h \sin \theta_y + r'_1), \quad (35)$$

$$r'_1 = \frac{d' l'_h}{f} - h \sin \theta_y. \quad (36)$$

By Eq. (34) and Eq. (36),

$$l'_1 = \frac{d' l'_h}{h \cos \theta_y} - l'_h - f \tan \theta_y. \quad (37)$$

Let’s assume that

$$l'_2 = m' l'_1, \quad (38)$$

$$l'_h = (m' - 1) l'_1, \quad (39)$$

$$l'_1 = \frac{l'_h}{m' - 1}. \quad (40)$$

In Figure 5,

$$b : d' = l''_w : f, \quad (41)$$

$$d' = \frac{b f}{l''_w}. \quad (42)$$

By Eq. (37), Eq. (40), and Eq. (42),

$$l'_h = \frac{m''(m' - 1) f h \sin \theta_y}{(m' - 1) b f - m'' m' h \cos \theta_y}, \quad (43)$$

$$h = \frac{m''(m' - 1) f \sin \theta_y + l'_w l''_w m' \cos \theta_y}{l'_w (m' - 1) f}, \quad (44)$$

$$h_l = h - h_o, \quad (45)$$

$$h_o = a \sin \theta_B, \quad (46)$$

$$h_l = \frac{m''(m' - 1) f h \sin \theta_y + l'_w l''_w m' \cos \theta_y}{l'_w (m' - 1) f \sin \theta_y + l''_w m' \cos \theta_B} - a \sin \theta_B. \quad (47)$$

3.5 Estimation of Under-Struck State Volume

3.5.1 Estimation of sand volume for Under-struck-case 1 condition

The volume can be estimated by sector $OXY$ and $\triangle OYX$. Here, $V(x)$ means volume made by expanding the area $x$ along bucket width $b$.

$$V = V(\text{sector } OXY) - V(\triangle OYX), \quad (48)$$

$$\theta_z = \cos^{-1}\left(\frac{h_l}{a}\right), \quad (49)$$

By Eq. (49)

$$\text{sector } OXY = a^2 \cos^{-1}\left(\frac{h_l}{a}\right), \quad (50)$$

$$XY = 2\sqrt{a^2 - h_l^2}, \quad (51)$$

$$\triangle OYX = h_l \sqrt{a^2 - h_l^2}, \quad (52)$$

$$V = a^2 b \cos^{-1}\left(\frac{h_l}{a}\right) - bh_l \sqrt{a^2 - h_l^2}. \quad (53)$$

3.5.2 Estimation of sand volume for under-struck-case 2 condition

In under-struck-case 2, the formula for calculating the volume can be obtained by subtracting sector $BOY$ and $\triangle YOC$ from sector $BDO$ and adding $\triangle XDC$. See Figure 9.

$$V = V(\text{semicircle}) - V(\text{sector } BOY) - V(\triangle YOC) + V(\triangle XDC). \quad (54)$$

First, to obtain sector $BOY$, we need to find $u_1$ and $u_2$.

$$u_1 = \theta_B, \quad (55)$$
\[ u_2 = \frac{\pi}{2} - \cos^{-1}\left(\frac{h_l}{a}\right), \quad (56) \]

\[ \text{sector } BOY = \frac{a^2}{2} \left( \theta_B + \frac{\pi}{2} - \cos^{-1}\left(\frac{h_l}{a}\right) \right). \quad (57) \]

To find the area of \( \triangle XDC \), you need to find the base length and height. In Figure 10,

\[ \begin{align*}
2a \cos \theta_B : h_o + a \sin \theta_B &= v_2 : a \sin \theta_B - h_l, \\
2a : h_o + a \sin \theta_B &= v_3 : a \sin \theta_B - h_l.
\end{align*} \quad (58) \]

By Eq. (46),

\[ \begin{align*}
v_2 &= \frac{a \sin \theta_B - h_l}{\sin \theta_B}, \\
v_3 &= a - \frac{h_l}{\sin \theta_B}, \\
\triangle XDC &= \frac{1}{2} v_3 \sqrt{v_2^2 - v_3^2} = \frac{1}{2} \tan \theta_B \left( \frac{a \sin \theta_B - h_l}{\sin \theta_B} \right)^2. \quad (62) \]

In Figure 10,

\[ v_4 = a - v_3. \quad (63) \]

\[ \Delta YOC = \frac{1}{2} h_l \left( \sqrt{a^2 - h_l^2} + h_l \frac{\cos \theta_B}{\sin \theta_B} \right), \quad (64) \]

The total volume can be obtained by summing each areas.

\[ V = \frac{\pi a^2 b}{2} - \frac{a^2 b}{2} \left( \frac{\theta_B + \pi}{2} - \cos^{-1}\left(\frac{h_l}{a}\right) \right) \\
+ \frac{b h_l}{2} \left( \sqrt{a^2 - h_l^2} + h_l \frac{\cos \theta_B}{\sin \theta_B} \right) + \frac{b}{2} \tan \theta_B \left( \frac{a \sin \theta_B - h_l}{\sin \theta_B} \right)^2. \quad (65) \]

### 3.5.3 Estimation of sand volume for under-struck-case 3 condition

The equation for obtaining the volume in under-struck-case 3 can be obtained by adding \( \triangle Y'OY \) and \( \triangle XY'D \) in the semi-circle-subtracted sector \( BYO \). See Figure 12.

\[ V = V(\text{semicircle}) - V(\text{sector } BYO) \\
+ V(\triangle Y'OY) + V(\triangle XY'D). \quad (66) \]

In Figure 13,

\[ 2a : 2a \sin \theta_B = 2a - v_5 : 2a \sin \theta_B - h, \quad (67) \]

\[ \frac{2a}{\cos \theta_B} : 2a \sin \theta_B = v_6 : 2a \sin \theta_B - h, \quad (68) \]

\[ v_5 = \frac{h}{\sin \theta_B}, \quad (69) \]

\[ v_6 = \frac{2a \sin \theta_B - h}{\sin \theta_B \cos \theta_B}, \quad (70) \]

\[ \triangle XY'D = \frac{1}{2} \tan \theta_B \left( \frac{2a \sin \theta_B - h}{\sin \theta_B} \right)^2. \quad (71) \]

In Figure 14,

\[ v_8 = \sqrt{(a - v_5)^2 - (a \sin \theta_B - h)^2}, \quad (72) \]

\[ v_7 = \sqrt{a^2 - (a \sin \theta_B - h)^2} - v_8, \quad (73) \]
\[ v_7 + v_8 = \sqrt{a^2 - (a \sin \theta_B - h)^2}. \]  
\text{(74)}

In Figure 15,
\[ \theta_B - u_3 = \cos^{-1} \left( \frac{\sqrt{a^2 - (a \sin \theta_B - h)^2}}{a} \right), \]  
\text{(75)}
\[ u_3 = \theta_B - \cos^{-1} \left( \frac{\sqrt{a^2 - (a \sin \theta_B - h)^2}}{a} \right), \]  
\text{(76)}
\[ \text{sector } BYO = \frac{a^2}{2} \left( \theta_B - \cos^{-1} \left( \frac{\sqrt{a^2 - (a \sin \theta_B - h)^2}}{a} \right) \right). \]  
\text{(77)}

In Figure 16,
\[ \triangle Y'YO = \frac{1}{2} (a \sin \theta_B - h) \left( \sqrt{a^2 - (a \sin \theta_B - h)^2} - \frac{a \sin \theta_B - h}{\tan \theta_B} \right), \]  
\text{(78)}
\[ V = \frac{\pi a^2 b}{2} + \frac{b}{2} \tan \theta_B \left( \frac{2a \sin \theta_B - h}{\sin \theta_B} \right)^2 \]  
\[ - \frac{a^2 b}{2} \left( \theta_B - \cos^{-1} \left( \frac{\sqrt{a^2 - (a \sin \theta_B - h)^2}}{a} \right) \right) \]  
\[ + \frac{b}{2} (a \sin \theta_B - h) \left( \sqrt{a^2 - (a \sin \theta_B - h)^2} - \frac{a \sin \theta_B - h}{\tan \theta_B} \right). \]  
\text{(79)}

4. Experimental Results

Several experiments were performed to confirm the accuracy of the algorithm presented in this paper. The volume expression contains a parameter called focal length. This focal length parameter may or may not appear in the H/W specification of commercial camera. Here, we assumed that the focal length is not given and try to find by experiments using pre-known \( l_w \). The estimated focal length was 19.0011 mm as Table 1 when we use SPC-B900W camera. The size of image sensor in the camera was 21.12 mm \( \times \) 11.88 mm.

The experiment to find the focal length was done by taking a picture of an object with pre-known size at a pre-known distance. Then the focal length is found through the number of
Table 1. Focal length experiment

| Length between object and camera lens (mm) | Object length (mm) | Image pixel length (pixel) | Focal length (mm) |
|-------------------------------------------|-------------------|----------------------------|------------------|
| 450                                       | 113               | 441                        | 19.3181          |
| 400                                       | 113               | 499                        | 19.4301          |
| 350                                       | 113               | 559                        | 19.0456          |
| 300                                       | 113               | 652                        | 19.0407          |
| 250                                       | 113               | 783                        | 19.0553          |
| 200                                       | 113               | 963                        | 18.7487          |
| 150                                       | 113               | 1258                       | 18.3690          |
| Average focal length                      |                   |                            | 19.0011          |

Table 2. Density experiment of object

| Number of balls | Volume (mL) | Weight (g) | Density (g/mL) |
|-----------------|-------------|------------|----------------|
| 10              | 1.25        | 2.0        | 1.6            |
| 20              | 2.5         | 4.0        | 1.6            |
| 30              | 3.75        | 6.0        | 1.6            |
| 40              | 5           | 8.0        | 1.6            |
| Average density |             |            | 1.6            |

The experiment to find the density of a ball was done by finding the volume and weight of a fixed number of 6 mm balls. The density is determined by dividing the weight by the measured volume.

The sand used in the experiment is a type of ball with 6 mm diameter. The method to estimate the volume is as follows: The original weight is obtained through an electronic scale. Actual Volume is the volume measured when pouring water into the beaker and then pouring balls there. Estimated volume is obtained through the proposed algorithm. Estimated weight is obtained by multiplying the estimated volume by the average 1.6 g/ml density obtained in Table 2.

The experimental results are shown in Table 3. The error rate of case 1 was 10.6% and 6.1%, the error rate of case 2 was 9.8% and 0.5%, and the error rate of case 3 was 2.0% and 10.3%. Most of errors may be from the uneven surface of the sand region made by the fixed size of ball.

5. Conclusions

In this paper, we proposed a novel method to estimate the amount of sands in the excavator bucket based on a single camera by using the image processing technique and mathematical modeling of bucket. For each of three under-struck states, a closed form of mathematical solution to estimate the sand volume of excavator bucket was implemented. The experimental results show that the error rate is within 10.6% and the minimum error rate is 0.5% in a case of under-struck-case 2.
Conflict of Interest

No potential conflict of interest relevant to this article was reported.

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