Abstract We propose a simple modified gravity model without any initial matter fields in terms of several alternative non-Riemannian spacetime volume elements within the metric (second order) formalism. We show how the non-Riemannian volume-elements, when passing to the physical Einstein frame, create a canonical scalar field and produce dynamically a non-trivial inflationary-type potential for the latter with a large flat region and a stable low-lying minimum. We study the evolution of the cosmological solutions from the point of view of theory of dynamical systems. The theory predicts the spectral index $n_s \approx 0.96$ and the tensor-to-scalar ratio $r \approx 0.002$ for 60 e-folds, which is in accordance with the observational data. In the future Euclid and SPHEREx missions or the BICEP3 experiment are expected to provide experimental evidence to test those predictions.

Developments in cosmology have been influenced to a great extent by the idea of inflation [1]-[5], which provides an attractive solution of the fundamental puzzles for the standard Big Bang model, as the horizon and the flatness problems. In addition, providing a framework for sensible calculations of primordial density perturbations were discussed in [6]-[7]. However, it has been recognized that a successful implementation requires some very special restrictions on the dynamics that drives inflation. In particular, in New Inflation [4], a potential with a large flat region, which then drops to zero (or almost zero) in order to reproduce the vacuum with almost zero (in Planck units) cosmological constant of the present universe, is required.

In a parallel development, extended (modified) gravity theories as alternatives/generalizations of the standard Einstein General Relativity (for detailed accounts, see Refs. [8]-[11]) are being extensively studied in the last decade or so with the main motivation coming from cosmology (problems of dark energy and dark matter), quantum field theory in curved spacetime (renormalization in higher loops) and string theory (low-energy effective field theories).

One broad class of actively developed modified/extended gravitational theories is based on employing alternative non-Riemannian spacetime volume-forms (metric-independent generally covariant volume elements) in the pertinent Lagrangian actions instead of the canonical Riemannian one given by the square-root of the determinant of the Riemannian metric $\sqrt{-g} \equiv \sqrt{-\det|g_{\mu\nu}|}$ (originally proposed in [12]-[16]), for a concise geometric formulation, see [17,18]).

This formalism was used as a basis for constructing a series of extended gravity-matter models describing unified dark energy and dark matter scenario [19,20], quintessential cosmological models with gravity-assisted and inflaton-assisted dynamical suppression (in the “early” universe) or generation (in the post-inflationary universe) of electroweak spontaneous symmetry breaking and charge confinement [21-23], and a novel mechanism for the supersymmetric Brout-Englert-Higgs effect in supergravity [17].

In the present paper we propose a very simple gravity model without any initial matter fields involving several non-Riemannian volume-forms instead of the standard Riemannian volume element $\sqrt{-g}$. We show how the non-Riemannian volume-elements, when passing to the physical Einstein frame, generate a canonical scalar field $u$ and manage to create dynamically a non-trivial inflationary-type potential for $u$ with a large flat region for large positive $u$ and a stable low-lying minimum, i.e., $u$ will play the role of a dynamically created “inflaton”.

We study the evolution of the cosmological solutions from the point of view of the theory of dynamical systems and calculate the spectral index $n_s$ and the tensor-to-scalar ratio $r$ in our model whose values are in accordance with the observational data.
1 Non-Riemannian Volume-Forms Formalism

Let us recall that volume-forms in integrals over differentiable manifolds (not necessarily Riemannian one, so no metric is needed) are given by nonsingular maximal rank differential forms $\omega$

$$\int d^n \Omega(\omega) = \int d^n \Omega(\omega),$$

$$\omega = \frac{1}{D!} \epsilon_{\mu_1 \ldots \mu_D} dx^\mu_1 \ldots dx^\mu_D \Omega,$$

$$\omega_{\mu_1 \ldots \mu_D} = -\epsilon_{\mu_1 \ldots \mu_D} \Omega,$$

(our conventions for the alternating symbols $\epsilon^\mu_1 \ldots^\mu_D$ and $\epsilon_{\mu_1 \ldots \mu_D}$ are: $\epsilon^{\mu_1 \ldots \mu_D} = 1$ and $\epsilon_{\mu_1 \ldots \mu_D} = -1$). The volume element $\Omega$ transforms as scalar density under general coordinate reparametrizations.

In Riemannian $D$-dimensional spacetime manifolds a standard generally-covariant volume-form is defined through the “$D$-bein” (frame-bundle) canonical one-forms $e^A = e^A_\mu dx^\mu (A = 0, \ldots, D - 1)$:

$$\omega = e^{\mu_1} \ldots e^{D-1} = \det [e^A_\mu] dx^\mu_1 \ldots dx^{D-1} \Longrightarrow \Omega = \det [e^A_\mu] = \sqrt{-g} \det [g_{\mu\nu}].$$

To construct modified gravitational theories as alternatives to ordinary standard theories in Einstein’s general relativity, instead of $\sqrt{-g}$ we can employ one or more alternative non-Riemannian volume element(s) as in (1) given by non-singular exact $D$-forms $\omega = dA$ where:

$$A = \frac{1}{(D-1)!} \epsilon_{\mu_1 \ldots \mu_{D-1}} dx^\mu_1 \ldots dx^{D-1} \Longrightarrow \Omega \equiv \Phi(A) = \frac{1}{(D-1)!} e^{\mu_1 \ldots \mu_{D-1}} \partial_{\mu_1} A_{\mu_2 \ldots \mu_{D-1}}.$$

Thus, a non-Riemannian volume element is defined in terms of the (scalar density of the) dual field-strength of an auxiliary rank $D-1$ tensor gauge field $A_{\mu_1 \ldots \mu_{D-1}}$.

In general, modified gravity Lagrangian actions based on the non-Riemannian volume-form formalism have the following generic form (here and below we are using units with $16\pi G_{\text{Newton}} = 1$):

$$S = \int d^4 x \left\{ \Phi_1(A) \left[ R + L^{(1)} \right] + \Phi_2(B) \left[ L^{(2)} + \Phi_0(C) \sqrt{-g} \right] + \ldots \right\}.$$

Here $\Phi_1(A), \Phi_2(B), \Phi_0(C)$ are several different non-Riemannian volume elements of the form (4), i.e., defined by auxiliary rank 3 tensor gauge fields $A_{\mu\nu\lambda}, B_{\mu\nu\lambda}, C_{\mu\nu\lambda}$; $R$ denotes the scalar curvature in either first-order (Palatini) or second order (metric) formalism; $L^{(1)}$ and $L^{(2)}$ are some matter field Lagrangians; the dots indicate possible additional terms containing higher powers of the non-Riemannian volume elements, e.g., $(\Phi_1(A))^2 / \sqrt{-g}$. The specific forms of $L^{(1)}$ and $L^{(2)}$ can be uniquely fixed via the requirement for invariance of (4) under global Weyl-scale invariance (see (5) below).

A characteristic feature of the modified gravitational theories (4) is that when starting in the first-order (Palatini) formalism all non-Riemannian volume-forms are almost pure-gauge degrees of freedom, i.e. they do not introduce any additional propagating gravitational degrees of freedom when passing to the physical Einstein frame except for few discrete degrees of freedom with conserved canonical momenta appearing as arbitrary integration constants. This is explicitly shown within the canonical Hamiltonian treatment (see Appendices A in Refs. [18, 21]).

Unlike Palatini formalism, when we treat (4) in the second order (metric) formalism, while passing to the physical Einstein frame via conformal transformation:

$$g_{\mu\nu} \rightarrow g_{\mu\nu} = \chi g_{\mu\nu}, \quad \chi_1 \equiv \frac{\Phi_1(A)}{\sqrt{-g}},$$

the first non-Riemannian volume element $\Phi_1(A)$ in (4) is not any more a “pure gauge”, but creates a new dynamical canonical scalar field $u$ via $\chi_1 = \exp \frac{u}{\sqrt{-g}}$. In the following Section we will see how a non-trivial inflationary potential for $u$ is dynamically generated.

2 Einstein Frame - the Effective Scalar Potential

Let us now consider the simplest member in the class of modified gravitational models (4) with no original matter fields. i.e., $L^{(1)} = 0$ and $L^{(2)} = 0$, and where we only add a quadratic term w.r.t. non-Riemannian volume element $\Phi_1(A)$:

$$S = \int d^4 x \left\{ \Phi_1(A) \left[ R - 2\Lambda_0 \frac{\Phi_1(A)}{\sqrt{-g}} \right] + \Phi_2(B) \frac{\Phi_0(C)}{\sqrt{-g}} \right\},$$

Here $R$ is the scalar curvature in the second order (metric) formalism and:

$$\Phi_1(A) \equiv \frac{1}{3!} e^{\mu\nu\lambda \kappa} \partial_{\mu} A_{\nu\lambda \kappa}, \quad \Phi_2(B) \equiv \frac{1}{3!} e^{\mu\nu\lambda \kappa} \partial_{\mu} B_{\nu\lambda \kappa}, \quad \Phi_0(C) \equiv \frac{1}{3!} e^{\mu\nu\lambda \kappa} \partial_{\mu} C_{\nu\lambda \kappa}.$$

The specific form of the action (6) is dictated by the requirement about global Weyl-scale invariance under:

$$g_{\mu\nu} \rightarrow \lambda g_{\mu\nu} \lambda, \quad A_{\mu\nu\lambda} \rightarrow \lambda A_{\mu\nu\lambda}, \quad B_{\mu\nu\lambda} \rightarrow \lambda^2 B_{\mu\nu\lambda}, \quad C_{\mu\nu\lambda} \rightarrow C_{\mu\nu\lambda}.\ \ \ (7)$$

Scale invariance played an important role since the original papers on the non-canonical volume-form formalism [14]. Also let us note that spontaneously broken dilatation symmetry models constructed along these lines are free of the Fifth Force Problem [15].
The equations of motion resulting from (9) upon variation w.r.t. the auxiliary gauge fields $A_{\mu \nu \lambda}, B_{\mu \nu \lambda}, C_{\mu \nu \lambda}$ yield, respectively:

$$R - 4A_0 \frac{\Phi_0(A)}{\sqrt{-g}} = -M_1 \equiv \text{const},$$  \hspace{1cm} (9)

$$\Phi_0(C) = -M_2 \equiv \text{const}, \quad \frac{\Phi_2(B)}{\sqrt{-g}} = \chi_2 \equiv \text{const}. \hspace{1cm} (10)$$

Here $M_1, M_2$ and $\chi_2$ are (dimensionful and dimensionless, respectively) integration constants. The appearance of $M_1, M_2$ indicate spontaneous breaking of global Weyl symmetry (8).

The equations of motion w.r.t. the auxiliary scalar potential $S$ is (5) using (see e.g. Ref. [25]), bars indicate magnitudes in the $g_{\mu \nu}$-frame:

$$R_{\mu \nu} + \frac{1}{\chi_1} \left( g_{\mu \nu} \Box \chi_1 - \nabla_\mu \nabla_\nu \chi_1 \right) - \frac{\chi_2 M_2}{\chi_1} g_{\mu \nu} = 0,$$  \hspace{1cm} (11)

with $\chi_1$ as in (5). Following Ref.[24], upon taking into account relations (9)–(10) and performing the conformal transformation (5) using (see e.g. Ref.[25]), bars indicate magnitudes in the $\bar{g}_{\mu \nu}$-frame:

$$R_{\mu \nu}(\bar{g}) = R_{\mu \nu}(g) - \frac{3}{\chi_1} \bar{g}^{\kappa \lambda} \partial_\kappa \chi_1^{1/2} \partial_\lambda \chi_1^{1/2} + \chi_1^{-1/2} \left( \bar{g}_{\mu \nu} \chi_1^{1/2} + \bar{g}_{\mu \nu} \Box \chi_1^{1/2} \right),$$  \hspace{0.5cm} (12)

$$\Box \chi_1 = \chi_1 \left( \Box \chi_1 - 2 \chi_1^{1/2} \frac{\partial_\mu \chi_1^{1/2}}{\chi_1^{1/2}} \right),$$  \hspace{0.5cm} (13)

Eqs. (11) can be rewritten in the standard form of Einstein equations w.r.t. the new “Einstein-frame” metric $\bar{g}_{\mu \nu}$:

$$R_{\mu \nu}(\bar{g}) = \frac{1}{2} \bar{g}_{\mu \nu} R(\bar{g}) = \frac{1}{2} \left[ \partial_\mu u \partial_\nu u - \bar{g}_{\mu \nu} \left( \frac{1}{2} \chi_1^{1/2} \partial_\kappa \chi_1^{1/2} \partial_\lambda u + U_{\text{eff}}(u) \right) \right],$$  \hspace{0.5cm} (14)

where we have redefined:

$$\Phi_0(A)/\sqrt{-g} \equiv \chi_1 = \exp \left( u/\sqrt{3} \right)$$  \hspace{0.5cm} (15)

in order to obtain a canonically normalized kinetic term for the scalar field $u$, and where:

$$U_{\text{eff}}(u) = 2A_0 - M_1 \exp \left( -\frac{u}{\sqrt{3}} \right) + \chi_2 M_2 \exp \left( -\frac{2u}{\sqrt{3}} \right).$$  \hspace{0.5cm} (16)

Accordingly, the corresponding Einstein-frame action reads:

$$S_{\text{eff}} = \int d^4x \sqrt{-\bar{g}} \left[ R(\bar{g}) - \frac{1}{2} \bar{g}^{\mu \nu} \partial_\mu u \partial_\nu u - U_{\text{eff}}(u) \right].$$  \hspace{0.5cm} (17)

We now observe an important result – in [17] we have a dynamically created scalar field $u$ with a non-trivial effective scalar potential $U_{\text{eff}}(u)$ entirely dynamically generated by the initial non-Riemannian volume elements in (6) because of the appearance of the free integration constants $M_1, M_2, \chi_2$ in their respective equations of motion (9)–(10).

The qualitative shape of (16) is depicted in Fig.1. The effective potential $U_{\text{eff}}(u)$ has two main features relevant for cosmological applications. First, $U_{\text{eff}}(u)$ possesses a flat region for large positive $u$ and, second, it has a stable minimum for a small finite value $u = u_*$:

(i) $U_{\text{eff}}(u) \sim 2A_0$ for large $u$;

(ii) $\frac{\partial^2 U_{\text{eff}}}{\partial u^2} = 0$ for $u \equiv u_*$ where:

$$\exp \left( -\frac{u_*}{\sqrt{3}} \right) = \frac{M_1}{2\chi_2 M_2}, \quad \frac{\partial^2 U_{\text{eff}}}{\partial u^2} \bigg|_{u=u_*} = \frac{M_1^2}{6\chi_2 M_2} > 0.$$  \hspace{0.5cm} (18)

The flat region of $U_{\text{eff}}(u)$ for large positive $u$ correspond to “early” universe’ inflationary evolution with energy scale $2A_0$. On the other hand, the region around the stable minimum at $u = u_*$ correspond to “late” universe’ evolution where the minimum value of the potential:

$$U_{\text{eff}}(u_*) = 2A_0 - \frac{M_1^2}{4\chi_2 M_2} \equiv 2\Lambda_{\text{DE}}$$  \hspace{0.5cm} (19)

is the dark energy density value [26] [27]. Thus, to conform to the observational data we have to choose:

$$A_0 \sim M_1 \sim M_2 \sim 10^{-8} M_{Pl}^4 \quad , \quad \chi_2 \sim 1.$$  \hspace{0.5cm} (20)

where $M_{Pl}$ is the Planck mass.

Let us note that the effective potential $U_{\text{eff}}(u)$ generalizes the well-known Starobinsky inflationary potential [11] [16] reduces to Starobinsky potential upon taking the following special values for the parameters: $A_0 = 1/3 M_1 = 1/3 \chi_2 M_2$.

3 Evolution of the homogenous solution

We now consider reduction of the Einstein-frame action [17] to the Friedmann-Lemaître-Robertson-Walker (FLRW) setting with metric $ds^2 = -N^2 dt^2 + a(t)^2 dx^2$, and with $u = u(t)$. In order to study the evolution of the scalar field $u =
u(\tau) and the Friedmann scale factor \( a = a(\tau) \), it is useful to use the method of autonomous dynamical systems.

The FLRW action describes a minimally coupled canonical scalar field \( u \) with specific potential \( U_{\text{eff}}(u) \) \cite{16} (using again units with 16\(\pi G_{\text{Newton}} = 1 \)):

\[
S_{\text{FLRW}} = \int \! dt \left[ -\frac{a^2}{\mathcal{N}} + \mathcal{N} a^3 \left( \frac{1}{2} \dot{a}^2 + M_1 e^{-\eta/\sqrt{3}} - M_2 \chi_5^2 e^{-2\eta/\sqrt{3}} \right) \right].
\]  

(21)

Variations w.r.t. \( \mathcal{N}, a, u \) (and subsequently using the gauge \( \mathcal{N} = 1 \) for the lapse function) yield the pertinent Friedmann and field equations (\( H = \dot{a}/a \) being the Hubble parameter):

\[
H^2 = \frac{1}{6} \rho , \quad \rho = \frac{1}{2} u^2 + U_{\text{eff}}(u),
\]  

(22)

\[
H = -\frac{1}{4} (\rho + p) , \quad p = \frac{1}{2} u^2 - U_{\text{eff}}(u),
\]  

(23)

\[
\dot{u} + 3H u + \frac{\partial U_{\text{eff}}}{\partial u} = 0.
\]  

(24)

In the treatment of Eqs. (22)-(24) it is instructive to rewrite them in terms of a set of dimensionless parameters (following the approach in Ref. \cite{28}):

\[
x := \frac{\dot{u}}{\sqrt{2}H}, \quad y := \frac{\sqrt{U_{\text{eff}}(u) - 2\Lambda_{\text{DE}}}}{\sqrt{6}H}, \quad z := \frac{\sqrt{\Lambda_{\text{DE}}}}{\sqrt{3}H},
\]  

(25)

with \( \Lambda_{\text{DE}} \) as in \cite{19}. In these coordinates the system defines a closed orbit:

\[
x^2 + y^2 + z^2 = 1,
\]  

(26)

which is equivalent to the first Friedmann equation \cite{22}.

Employing the variables \((x, y, z)\) in Eqs. (22)-(24) and taking into account the constraint \cite{36} we obtain the autonomous dynamical system w.r.t. \((x, z)\):

\[
x' = \frac{\sqrt{3}}{2\Lambda_{\text{DE}}} z^2 \left[ -M_1 \xi(x, z) + 2M_2 \chi_5^2(x, z) \right] - 3x(1 - x^2),
\]  

\[
z' = 3z x^2,
\]  

(27)

where the primes denote derivative w.r.t. the parameter \( N = \log a \), and the function \( \xi(x, z) \) is defined as:

\[
\xi(x, z) = \frac{M_1}{2\chi_5^2 M_2} \left[ 1 - \frac{8\Lambda_0 M_2 \chi_5^2 (1 - x^2 - z^2)}{M_1^2} \right].
\]  

(28)

There are two critical points in the system. The stable point \( A (x = 0, z = 1) \) corresponds to the “late” universe de Sitter solution with the asymptotic cosmological constant \( \Lambda_{\text{DE}} \) \cite{19}.

The second point \( B (x = 0, z = \sqrt{\Lambda_{DE}/\Lambda_0}) \) is totally repulsive corresponding to the beginning of the universe’ evolution in the “early” universe at \( a \rightarrow \infty \).

Numerical solutions are demonstrated in Fig. 3. One can see that the Hubble parameter begins and ends with two different values. The first one is related to the inflationary epoch and the other related to the dark energy in the late universe. The scalar field \( u \) oscillates around the minimum point \( u_0 \) \cite{18} of \( U_{\text{eff}} \) \cite{16}, which corresponds to particle creation in the reheating epoch.

### 4 Perturbations

In order to check the viability of the model we investigate the perturbations of the above background evolution, in particular focusing on the inflationary observables such as the scalar spectral index \( n_s \) and the tensor-to-scalar ratio \( r \). As usual, we introduce the Hubble slow-roll parameters \cite{29,30}, which in our case using the potential \cite{16} read:

\[
\epsilon = \frac{1}{2} \left( \frac{U''_{\text{eff}}(u)}{U_{\text{eff}}(u)} \right)^2 = \frac{8(\xi^{\prime \prime} - 2)^2}{(\xi^{\prime \prime} + (\delta + 1) \xi^{\prime} - 4)^2} \quad \text{(29a)}
\]  

\[
\eta = \frac{U''_{\text{eff}}(u)}{U_{\text{eff}}(u)} = 16 - 4\xi^{\prime \prime} + \frac{16}{(\delta + 1) \xi^{\prime} - 4} \quad \text{(29b)}
\]

where \( \zeta \equiv M_1/\chi_2 M_2 \). Here we assumed that the fraction \( \delta \equiv \Lambda_{\text{DE}}/\Lambda_0 \ll 1 \) with \( \Lambda_{\text{DE}} \) as in \cite{19}.

Inflation ends when \( \epsilon(u_i) = 1 \):

\[
\xi^{\prime \prime} \{u_i\} = 2 \left( 1 + \sqrt{2} \right) \quad \text{(30)}
\]

Thus, if the initial point of the scalar field is \( u_i \) then the number of e-foldings reads:

\[
N = \int_{u_i}^{u_f} \frac{U_{\text{eff}}(u)}{U_{\text{eff}}(u)} du
\]

which gives:

\[
N = \frac{1}{4} (\xi^{\prime \prime} - 2u_i) - \frac{1}{4} (\xi^{\prime \prime} - 2u_i) + O(\delta) \quad \text{(31)}
\]
In order to simplify the analysis and to get solvable equations, we take the limit of a big scalar field $u$, and therefore $\exp u \gg u$:

$$N \approx \frac{1}{4} \left( \zeta \exp [u] - 2 \left( \sqrt{2} + 1 \right) \right)$$

(32)

Using the slow-roll parameters, one can calculate the values of the scalar spectral index and the tensor-to-scalar ratio respectively as [32, 33]:

$$r \approx 16 \varepsilon, \quad n_s \approx 1 - 6 \varepsilon + 2 \eta$$

(33)

From the connection we found between the initial scalar field value and the number of $e$-folds, one can get the observable values:

$$r = \frac{16}{(\sqrt{2}N + 1)^2}, \quad n_s = 1 - \frac{4}{(\sqrt{2}N + 1)^2} - \frac{2\sqrt{2}}{\sqrt{2}N + 1}.$$  

(34)

One viable example in our model is to take $N = 60$ $e$-folds. In such way, the observable are predicted to be:

$$n_s \approx 0.96, \quad r \approx 0.002,$$

(35)

which are well inside the PLANCK observed constraints [31]:

$$0.95 < n_s < 0.97, \quad r < 0.064$$

(36)

Fig. 3 demonstrates the relation between the number of $e$-folds and the dimensionless parameters. One can see that all those values fit the latest PLANCK collaboration constraints.

5 Conclusions

We propose a very simple gravity model without any initial matter fields in terms of several alternative non-Riemannian spacetime volume elements within the second order (metric) formalism. We show how the non-Riemannian volume-elements, when passing to the physical Einstein frame, create a canonical scalar field and produce dynamically a non-trivial inflationary-type potential for the latter with a large flat region and a stable low-lying minimum. We study the evolution of the cosmological solutions from the point of view of the theory of dynamical systems. Our model predicts scalar spectral index $n_s \approx 0.96$ and tensor-to-scalar ratio $r \approx 0.002$ for 60 $e$-folds, which is in accordance with the observational data.

A natural next step is to consider two-field inflation (Refs. [28, 34, 35], for a recent rigorous geometric treatment see Ref. [36], and references therein) by adding a new scalar field $\phi$ with non-trivial potentials in the starting modified
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