DEVELOPMENT OF A METHOD FOR THE OBSERVATION OF LIGHTNING IN PROTOPLANETARY DISKS USING ION LINES

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Received 2014 November 10; accepted 2015 November 2; published 2015 December 11

ABSTRACT

In this paper, we propose observational methods for detecting lightning in protoplanetary disks. We do so by calculating the critical electric field strength in the lightning matrix gas (LMG), the parts of the disk where the electric field is strong enough to cause lightning. That electric field accelerates multiple positive ion species to characteristic terminal velocities. In this paper, we present three distinct discharge models with corresponding critical electric fields. We simulate the position–velocity diagrams and the integrated emission maps for the models. We calculate the measure-of-sensitivity values for detection of the models and for distinguishing between the models. At the distance of TW Hya (54 pc), LMG that occupies 2π in azimuth and has 25 AU < r < 50 AU is detectable at 1200σ to 4000σ. The lower limits of the radii of 5σ-detectable LMG clumps are between 1.6 AU and 5.3 AU, depending on the models.

Key words: instabilities – magnetohydrodynamics (MHD) – methods: observational – planets and satellites: formation – protoplanetary disks – submillimeter: planetary systems

1. INTRODUCTION

Lightning in protoplanetary disks is an important topic in protoplanetary disk physics. The existence of lightning is still an open question, and if it exists, it serves as an elementary electromagnetic process, as an observational clue to measure the electromagnetic states of the disk, and as a candidate mechanism for chondrule heating. The amount of observational data available today is huge, and open access to the observational results from the most advanced telescopes is available. However, the methods of observation of protoplanetary lightning using advanced telescopes have not been seriously studied.

There has been a controversial debate about the existence and the mechanism of protoplanetary disk lightning. Gibbard & Morfill (1997) argued that plasma conductivity is too large for lightning to take place. Pilipp et al. (1998) argued that an unknown, efficient grain–grain charging process is required to produce lightning. Despite these barriers, mechanisms that lead to lightning have been proposed: dust–dust collisional charging (Desch & Cuzzi 2000; Muranushi 2010), mutual positive feedback of thermal ionization and Joule heating (Hubbard et al. 2012; McNally et al. 2013), and an electric field generated by magnetorotational instability (MRI) (Inutsuka & Sano 2005; Muranushi et al. 2012). Magnetized chondrules found in meteorites carry evidence of a magnetic field of 500–1000 G during their formation, suggesting that they are struck by lightning (Wasilewski & Dickinson 2000).

Meanwhile, understanding of the lightning ignition mechanism has progressed in the last 20 years. Attempts have been made to explain the mechanism that causes discharge at a point well below the nominal dielectric strength of air (Nasser 1968; Phelps 1971). As result, new lightning models such as runaway breakdown (Gurevich et al. 1992; Gurevich & Zybin 2001) have been proposed. We adopt such progress in understanding terrestrial lightning, and propose a new method for the observational discrimination of protoplanetary disk lightning.

Observational studies searching for disk lightning will contribute to our understanding of the electromagnetic processes in protoplanetary disk, and the source of chondrule heating.

Lightning, or electrical breakdown, is a result of a large electric field $E$. The electric field $E$ in a protoplanetary disk can be generated by MRI or by the collective motion of charged dust. The breakdown model sets an upper limit $E \leq E_{\text{crit}}$ to the electric field amplitude. At the point $E = E_{\text{crit}}$ electric discharge takes place, which increases the degree of ionization of the medium and prevents further growth of the electric field amplitude. Thus, the electric field amplitude is kept below the upper limit ($E \leq E_{\text{crit}}$).

This electric field is a common feature of the large volume of gas surrounding lightning bolts. In this article, we study this electric-field feature as the possible predominant emitter of observational signals.

Lightning bolts themselves are difficult to observe because lightning is a transient events, and the typical radius of a lightning bolt is $5 \times 10^3$ times the mean free path (Pilipp et al. 1992). This radius is much smaller than the scale height of the disk. Hence even if the critical condition is met, most of the time most of the protoplanetary disk gas is in the region outside the lightning bolts. We call this lightning matrix gas (LMG). The properties of LMG are no different from those of disk gas without lightning except that LMG is subject to a critical electric field $E \lesssim E_{\text{crit}}$. In this paper, we explore the possible observational features of LMG.

This paper is organized as follows. In Section 2, we introduce the discharge models, taking the Earth’s atmosphere as an example (Sections 2.1, 2.2); introduce the protoplanetary disk model (Section 2.3); and apply the discharge model to the disk gas (Section 2.4). In Section 3 we establish our observation model by

1. calculating the terminal velocity of the ion molecules (Section 3.1),
2. MODEL

2.1. Dielectric Strength of Air

We begin by estimating the dielectric strength of the Earth’s atmosphere, in order to introduce the reader to the discharge models we later apply to the protoplanetary disk gas.

The dielectric strength of an insulating material is the maximum amplitude of the electric field that does not cause electric breakdown in the material. It is a physical property of central importance to discharge physics. Lightning on Earth is a discharge phenomenon in the air. However, it has been a longstanding mystery that lightning takes place under an electric field amplitude well below the dielectric strength of air measured in the laboratory.

The dielectric strength of the air at normal temperature and pressure (NTP; 20 °C and 1 atm) are well established from laboratory experiments (Rigden 1996):

\[ E_0 = 30 \text{ kV cm}^{-1}. \]

The long-distance limit of Paschen’s law states that the dielectric strength of gas depends linearly on the gas number density (Raizer 1997). However, in the case of the Earth’s atmosphere the dependence of the dielectric strength on the number density is known to be steeper than linear. This is explained by the effects of electron loss via three-body interactions and also collisions with water vapor molecules. Empirical formulae are known (Phelps & Griffiths 1976; Takahashi et al. 2009):

\[ E = E_0 \left( \frac{P}{P_0} \right)^{1.5-1.65}, \]

where \( E_0 \) and \( P_0 \) are the dielectric strength and the pressure of the air at ground level, respectively. The formula predicts the dielectric strength of air to be 17 kV cm\(^{-1}\) at an altitude of 3 km and 10 kV cm\(^{-1}\) at 6 km.

On the other hand, intracloud lightning is observed with an electric field amplitude of 140 V cm\(^{-1}\) (French et al. 1996) to 150 V cm\(^{-1}\) (Dye et al. 1986). Cloud-to-ground lightning is observed with an electric field amplitude of around 1 kV cm\(^{-1}\) (Takahashi 1983) to 2 kV cm\(^{-1}\) (Takahashi et al. 1999).

2.2. Breakdown Models on Earth

In this section we introduce three breakdown models that we are going to compare in this paper. Note that, in our formulation, the density functions of electrons and ions under a given electric field are well understood (see, e.g., Golant et al. 1980), and thus we use the same density function for all three breakdown models. What we do not understand well is the dielectric strength—the amplitude of the electric field at which breakdown takes place. The only difference among the three models is the assumed value of the dielectric strength.

– In the Townsend breakdown model (T) the critical electric field is such that an electron accelerated by the electric field over its mean free path gains kinetic energy large enough to ionize a neutral gas molecule. It has been widely used in the meteorological context, and also adopted into an astrophysical context, e.g., by Desch & Cuzzi (2000) and Muranushi (2010). This model explains laboratory gas discharge experiments (Equation (1)) well.

– Druyvesteyn & Penning (1940) derived the formulae for an equilibrium distribution of electrons under a constant electric field, neglecting the effects of inelastic collisions with atoms. When the electric field is weak, so that the work done by the electric field per mean free path \( eE_{\text{mfp}} \) is much smaller than the electron kinetic energy, the equilibrium distribution is nearly isotropic. The distribution is expressed as the sum of the isotropic equilibrium and its first-order perturbation. The average energy and the average velocity of the mean motion, \( \langle e \rangle \) and \( \langle v_e \rangle \), satisfy \( \langle e \rangle = 0.43 eE_{\text{mfp}} \sqrt{M/m_e} \) and \( \langle v_e \rangle = 0.9 \sqrt{eE_{\text{mfp}} (m_e M)^{1/4}} \), respectively. Here, \( m_e \) and \( M \) are the masses of the electron and its collision partner, respectively.

The Druyvesteyn–Penning breakdown model (DP) assumes that breakdown takes place when \( \langle e \rangle \) exceeds the ionization energy. Since the factor \( \sqrt{M/m_e} \) makes the average energy \( \langle e \rangle \) in the DP breakdown model nearly 100 times larger than that in the Townsend breakdown model, the former model allows for breakdown under an electric field amplitude about 10\(^{-2}\) times that of the latter. The model is introduced as a protoplanetary disk lightning model by Inutsuka & Sano (2005).

– Gurevich et al. (1992) have proposed the runaway breakdown model (R) and Gurevich & Zybin (2001) provided a detailed review of it. In this model, the equilibrium of the electrons with relativistic (~1 MeV) kinetic energy, much larger than the average of the Maxwellian energy distribution, plays an important role. Because the ionization losses for electrons are inversely proportional to the kinetic energy in the non-relativistic regime, the mean free path for such fast electrons is much longer than that for thermal electrons. In the runaway breakdown model, the number of relativistic electrons grows exponentially, once the electric field is large enough to balance the ionization losses for a certain energy range of the relativistic electrons. (We define that the acceleration criterion is met for those electrons.) The ionization processes generate a spectrum of fast and slow electrons. Fast electrons that meet the acceleration criterion contribute to the exponential growth, while slow electrons are large in number and increase the degree of ionization of the matter, ultimately leading to its electric breakdown. Thus, runaway breakdown can take place in an electric field much weaker than that of a Townsend breakdown. The runaway breakdown model better explains observations of lightning in the Earth’s atmosphere and is used as the discharge model in thunderstorm simulation studies, e.g., by Mansell et al. (2002).

In order to estimate the dielectric strength of gas, we need to compute the energy distribution of electrons. Since the interactions of electrons with even the simplest atoms and molecules have profound details (Itikawa 2000, 2001; Martienssen 2003), this requires difficult numerical computations.
The Astrophysical Journal, 815:84 (10pp), 2015 December 20

Table 1
Collsional Cross Sections of the Molecules for 15 eV Electrons

| Species | \(\sigma_{\text{inel}}\) (cm\(^{-2}\)) | \(\sigma_{\text{el}}\) (cm\(^{-2}\)) |
|---------|-----------------------------------|-----------------|
| \(\text{H}_2\) | \(1.6 \times 10^{-16}\) | \(6.6 \times 10^{-16}\) |
| \(\text{He}\) | 0.0 | \(3.6 \times 10^{-16}\) |
| \(\text{CO}\) | \(5.1 \times 10^{-18}\) | \(1.1 \times 10^{-15}\) |
| \(\text{O}_2\) | \(1.8 \times 10^{-16}\) | \(8.9 \times 10^{-16}\) |

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Table 2
The Terminal Velocities of the Molecular Ions for Cross-section Models XL, XM and XS

| Cross Section Model XL | \(\text{HCO}^+\) | \(\text{DCO}^+\) | \(\text{N}_2^+\) |
|------------------------|----------------|----------------|----------------|
| T                      | \(8.6 \times 10^4\) cm s\(^{-1}\) | \(8.6 \times 10^4\) cm s\(^{-1}\) | \(8.6 \times 10^4\) cm s\(^{-1}\) |
| DP                     | \(2.5 \times 10^4\) cm s\(^{-1}\) | \(2.5 \times 10^4\) cm s\(^{-1}\) | \(2.5 \times 10^4\) cm s\(^{-1}\) |
| R                      | \(2.2 \times 10^3\) cm s\(^{-1}\) | \(2.2 \times 10^3\) cm s\(^{-1}\) | \(2.2 \times 10^3\) cm s\(^{-1}\) |

| Cross Section Model XM | \(\text{HCO}^+\) | \(\text{DCO}^+\) | \(\text{N}_2^+\) |
|------------------------|----------------|----------------|----------------|
| T                      | \(4.2 \times 10^4\) cm s\(^{-1}\) | \(4.2 \times 10^4\) cm s\(^{-1}\) | \(4.2 \times 10^4\) cm s\(^{-1}\) |
| DP                     | \(1.2 \times 10^4\) cm s\(^{-1}\) | \(1.2 \times 10^4\) cm s\(^{-1}\) | \(1.2 \times 10^4\) cm s\(^{-1}\) |
| R                      | \(1.1 \times 10^4\) cm s\(^{-1}\) | \(1.1 \times 10^4\) cm s\(^{-1}\) | \(1.1 \times 10^4\) cm s\(^{-1}\) |

| Cross Section Model XS | \(\text{HCO}^+\) | \(\text{DCO}^+\) | \(\text{N}_2^+\) |
|------------------------|----------------|----------------|----------------|
| T                      | \(2.0 \times 10^4\) cm s\(^{-1}\) | \(2.0 \times 10^4\) cm s\(^{-1}\) | \(2.0 \times 10^4\) cm s\(^{-1}\) |
| DP                     | \(5.9 \times 10^3\) cm s\(^{-1}\) | \(5.9 \times 10^3\) cm s\(^{-1}\) | \(5.9 \times 10^3\) cm s\(^{-1}\) |
| R                      | \(5.2 \times 10^3\) cm s\(^{-1}\) | \(5.2 \times 10^3\) cm s\(^{-1}\) | \(5.2 \times 10^3\) cm s\(^{-1}\) |

Note. The terminal velocities are the characteristic features we use for observational measurement of the dielectric strength.

Our Seven Disk Models, Their Respective Discharge Models, and LMG Distribution Models

| Disk Model Name | Discharge | LMG Region |
|-----------------|-----------|------------|
| N               | no discharge |          |
| T25             | Townsend discharge | 25 AU < \(r\) < 50 AU |
| T50             | Townsend discharge | 50 AU < \(r\) < 100 AU |
| DP25            | Druyversteyn–Penning discharge | 25 AU < \(r\) < 50 AU |
| DP50            | Druyversteyn–Penning discharge | 50 AU < \(r\) < 100 AU |
| R25             | runaway discharge | 25 AU < \(r\) < 50 AU |
| R50             | runaway discharge | 50 AU < \(r\) < 100 AU |

(Imutsuka & Sano 2005)

\[
\langle \epsilon \rangle = 0.43 e I_{\text{mf}} \sqrt{\frac{M}{m_e}},
\]

where \(M\) is the mass of the collision partner, and the dielectric strength \(E_{\text{crit}}\) is the solution of \(\langle \epsilon \rangle = \Delta W\). In the case of air at NTP, since the mean molecular weight of air is 28.96 g mol\(^{-1}\), \(M = 4.81 \times 10^{-23}\) g. Note that \(I_{\text{mf}}\) in the DP model means the elastic mean free path \(I_{\text{mf}} = (\langle n, \sigma_{\text{el}} \rangle)^{-1} = 3.59 \times 10^{-3}\) cm. \(I_{\text{mf}}\) is calculated from the elastic cross sections of the elemental molecules at 12 eV (\(\sigma_{\text{el}} = 1.16 \times 10^{-15}\) cm\(^2\), 9.00 \(\times 10^{-16}\) cm\(^2\), 1.74 \(\times 10^{-15}\) cm\(^2\), respectively, for \(N_2, O_2, Ar\); see Itikawa 2000; Martienssen 2003.) Therefore, \(E_{\text{crit}} = 3.38\) kV cm\(^{-1}\).

Finally, according to the runaway breakdown model the dielectric strength \(E_{\text{crit}}\) is the electric field amplitude where acceleration by the electric field balances the ionization loss for minimum ionizing electrons. Minimum ionizing electrons are electrons with kinetic energy \(\varepsilon\) such that for them the ionization loss is the smallest. The kinetic energy of the minimum ionizing electrons is about 1 MeV, where ionization loss is the dominant energy sink for the electrons (Gurevich & Zybin 2001). The ionization loss of an electron per unit time as a function of \(\varepsilon\) was formalized by Bethe (1930, 1932) and Bloch (1933). We use the following form of the Bethe formula from Longair (2010, chap. 5.5):

\[
\frac{d\varepsilon}{dx} = \frac{e^2 \tilde{Z} n_a}{8\pi \varepsilon_0 m_e e^2} a(\gamma),
\]

where

\[
a(\gamma) = \left(\frac{\gamma}{\gamma - 1}\right)^2 \ln \left\{ \frac{\gamma^3 m_e^2 v^4}{2(1 + \gamma)^2} - \left(\frac{2}{\gamma} - 1 - \frac{1}{\gamma^2}\right) \right\}.
\]

Here, \(\gamma = (1 - v^2/c^2)^{-1/2}\) is the Lorentz factor of the electron, \(\varepsilon = (\gamma - 1)m_e c^2\) is the electron kinetic energy, and \(\tilde{Z} n_a\) is the number density of ambient electrons of the matter. \(I\) is the mean excitation energy, a parameter to be fitted to laboratory experimental data. We use the value \(I_{\text{mf}} = 86.3\) eV from the ESTAR database (Berger et al. 2009).

For the case of air \(a(\gamma)\) takes its minimum \(a_{\text{min}} = 20.2\) at \(\gamma = 3.89\) or \(\varepsilon = 1.48\) MeV. The dielectric strength \(E_{\text{crit}}\) is the...
solution of the following work-balance equation

$$eE - \frac{de}{dx} = 0,$$

which is

$$E_{\text{crit}} = \frac{e^2 Z n_e}{8 \pi \epsilon_0 m_e c^2} a_{\text{min}},$$

$$= 1.9 \text{ kV cm}^{-1}. \quad (8)$$

Here we summarize the three models. The dielectric strength of the gas is proportional to its number density. It is this proportional relation that leads to the constant ion velocity we present in this paper.

$$E_{c,T} = \frac{\Delta W}{e} \sigma_{\text{mol}} n_e = 30.1 \text{ kV cm}^{-1} \cdot \left( \frac{n_e}{n_{0,\text{air}}} \right)^1,$$

$$E_{c,\text{DP}} = \frac{\Delta W}{0.43 \sqrt{M}} \sigma_{\text{qf}} n_e = 3.4 \text{ kV cm}^{-1} \cdot \left( \frac{n_e}{n_{0,\text{air}}} \right)^1,$$

$$E_{c,R} = \frac{e^2 a_{\text{min}} Z}{8 \pi \epsilon_0 m_e c^2} n_e = 1.9 \text{ kV cm}^{-1} \cdot \left( \frac{n_e}{n_{0,\text{air}}} \right)^1. \quad (9)$$

2.3. The Disk Model

The minimum-mass solar nebula (MMSN) model (Hayashi 1981) has been widely used in studies of the protoplanetary disk, with fruitful results. Recent observations have contributed to sophistication of the disk models and have also reported qualitative values for the inner and outer edge radii of the disks (Kitamura et al. 2002; Andrews et al. 2009, 2010; Williams & Cieza 2011). However, such observational values for the geometry and mass of specific objects still contain uncertainty factors of 2–3 and are subjects of debate. See, e.g., Menu et al. (2014).

Some of the features common to the recent models are that the power-law index of the surface density distribution is close to 1 rather than 1.5, and that there is an exponential cut-off at the outer edge of the disk. Therefore, we use a simple model proposed by Akiyama et al. (2013) that captures these common features, and adopt the values of TW Hya reported by Calvet et al. (2002). Our disk model is as follows:

$$\Sigma(r) = 6.4 \times 10^2 \left( \frac{r}{1 \text{ AU}} \right)^{-1} \times \exp \left( -\frac{3r}{r_{\text{out}}} \right) \text{ g cm}^{-2} \text{ for } r > r_{\text{in}},$$

$$\Sigma(r) = 0 \text{ otherwise}. \quad (10)$$

Figure 1. Line profiles for HCO$^+$, DCO$^+$, and N$_2$H$^+$, assuming that the lightning takes place at 50 AU < r < 100 AU of the disk. The labels N, T, DP, and R for the curves corresponds to no lightning, the Townsend breakdown model, the Druyvesteyn–Penning breakdown model, and the runaway breakdown model, respectively.
Here, \( r_{\text{out}} = 150 \text{ AU} \) is the outer radius of our model disk. We also introduce an inner cut-off at \( r_{\text{in}} = 3.5 \text{ AU} \). The assumption of hydrostatic equilibrium leads to the vertical distribution of the gas:

\[
\rho(r, z) = \rho_0(r) \exp\left(-\frac{z^2}{2h^2}\right) = 5.08 \times 10^{-10} \left(\frac{r}{1 \text{ AU}}\right)^{-\frac{3}{2}} \exp\left(-\frac{z^2}{2h^2}\right) \text{ g cm}^{-3},
\]

(12)

where \( h(r) = \frac{c_s}{\Omega} \)

\[
h(r) = 3.29 \times 10^{-2} \left(\frac{r}{1 \text{ AU}}\right)^{\frac{5}{2}} \text{ AU},
\]

(13)

\[
c_s(r) = \sqrt{\frac{k_B T(r)}{\mu m_p}},
\]

(14)

\[
\Omega_K(r) = \sqrt{\frac{GM_\odot}{r^3}}.
\]

(15)

Here \( \mu m_p \) is the mean molecular mass of the gas. Therefore the number density of \( \text{H}_2 \) is

\[
n_{\text{H}_2}(r, z) = 1.52 \times 10^{14} \exp\left(-\frac{z^2}{2h^2}\right) \frac{\Sigma(r)}{\Sigma(1 \text{ AU})} \text{ cm}^{-3}.
\]

(17)

2.4. Breakdown Models on Protoplanetary Disks

Here we estimate the dielectric strength of the protoplanetary disk gas. In our disk model the gas density at the equatorial plane, \( r = 1 \text{ AU} \) is \( n_{\text{ppd}} = 1.52 \times 10^{14} \text{ cm}^{-3} \). We assume that protoplanetary disk gas consists of \( \text{H}_2, \text{He}, \text{CO, O}_2 \) and their volume fractions are 0.92, 7.8 \( \times 10^{-2}, 2.3 \times 10^{-4}, 1.3 \times 10^{-4} \) respectively (Lodders et al. 2009, chap. 3.4.6).

We used cross-section data for 15 eV electrons tabulated in Itikawa (2000) and Martienssen (2003; see Table 1), since \( \Delta W_{\text{B}} = 15.43 \text{ eV} \) and 15 eV is the closest table index that is found in the database.
Calculations similar to those in the previous section lead to the following values:

\[ E_{\text{c,T}} = \frac{\Delta W}{\varepsilon} (\sigma_{\text{inel}}) n_n = 3.0 \times 10^{-1} \left( \frac{n_n}{n_{0,\text{ppd}}} \right) \text{V cm}^{-1}, \]

\[ E_{\text{c,DP}} = \frac{\Delta W}{0.43 \sqrt{M}} \varepsilon_\eta n_n = 5.0 \times 10^{-2} \left( \frac{n_n}{n_{0,\text{ppd}}} \right) \text{V cm}^{-1}, \]

\[ E_{\text{c,R}} = \frac{\varepsilon \alpha_{\text{min}} Z}{8\pi\varepsilon_0 m e^2} n_n = 1.4 \times 10^{-3} \left( \frac{n_n}{n_{0,\text{ppd}}} \right) \text{V cm}^{-1}. \]

(18)

### Table 4

| Species | T25 | DP25 | R25 |
|---------|-----|------|-----|
| HCO\(^+\) | 3729.7 | 3021.6 | 1277.1 |
| T25 | 1508.4 | 3294.1 |
| DP25 | 2608.7 |
| DCO\(^+\) | 2488.4 | 2199.2 | 1277.4 |
| T25 | 1418.7 | 2316.5 |
| DP25 | 2125.5 |
| N\(_2\)H\(^+\) | 122.3 | 104.9 | 46.3 |
| T25 | 44.7 | 115.3 |
| DP25 | 99.3 |

### Table 5

| Species | T25 | DP25 | R25 |
|---------|-----|------|-----|
| N\(_2\)H\(^+\) | 5.8 | 5.2 | 2.3 |
| T25 | 2.0 | 5.8 |
| DP25 | 5.1 |

**Note:** The measure-of-sensitivity was estimated among N, T25, DP25, and R25 models, and among N, T50, DP50, and R50 models, using either one or all three of our lines HCO\(^+\) 3–2, DCO\(^+\) 3–2, and N\(_2\)H\(^+\) 3–2.

3. OBSERVATION

### 3.1. Calculation of the Terminal Velocity of the Ions

The goal of this section is to calculate the Doppler broadening of the molecular ion lines in the disk, which reflects the electric field strength in the protoplanetary disk. In order to establish the observational procedure, we calculate the collisional cross sections and the terminal velocities of the molecules. Then, we can estimate the optical depths and the spectral irradiances of the specific lines. We simulate the observational images using the calculated spectral irradiances. Finally, we establish a model discrimination procedure based on matched filtering.

We choose three ion species: HCO\(^+\), DCO\(^+\), and N\(_2\)H\(^+\) lines, whose observations have been performed (Öberg et al. 2010, 2011). Such charged chemical species are accelerated to their respective terminal velocities by the electric field of the LMG. Let \(\varepsilon_I\) be the kinetic energy of a particle of such an ion species \(I\). We can calculate the value of \(\varepsilon_I\) at the equilibrium by...
Table 6. Upper Limits to the Sizes of LMG Clumps that Exist on 25 AU < r < 50 AU and 50 AU < r < 100 AU Orbits

| 3 Species | T25 | DP25 | R25 |
|-----------|-----|------|-----|
| N         | 1.8 AU | 2.5 AU | 2.8 AU |
| T25       | 2.2 AU | 1.8 AU |             |
| DP25      | 2.5 AU |             |             |

| 3 Species | T50 | DP50 | R50 |
|-----------|-----|------|-----|
| N         | 4.0 AU | 4.9 AU | 5.3 AU |
| T50       | 4.9 AU | 3.9 AU |             |
| DP50      | 4.8 AU |             |             |

| 3 Species | T50 | DP50 | R50 |
|-----------|-----|------|-----|
| N         | 3.8 AU | 4.0 AU | 5.3 AU |
| T50       | 5.1 AU | 3.9 AU |             |
| DP50      | 4.1 AU |             |             |

Table 2. In the Appendix we describe the detail of the cross-section model we have used in order to estimate the above cross sections.

3.2. Estimation of the Observational Signals

The column density corresponding to optical depth $\tau_{e0} = 1$ is estimated as follows (see the appendix of Scoville et al. 1986):

$$N_{\tau_{e0}=1} = \frac{3\epsilon_0 k_B T_{ex} \Delta v_{gas}}{2\pi^2 B \mu^2 \cos \theta (J + 1)} \times \exp \left( \frac{h \nu_0}{k_B T_{ex}} \right) \left( 1 - \exp \left( -\frac{h \nu_0}{k_B T_{ex}} \right) \right),$$

(20)

where $T_{ex}$ is the excitation temperature, $\Delta v_{gas}$ is the Doppler broadening of the target molecule, $B$ is the rotational constant of the molecule, $\mu$ is its electric dipole matrix element, $\theta$ is the angle between the disk axis and the line of sight, $h \nu_0$ is the energy difference between the two levels, and $J$ is the rotational quantum number of the lower state.

The optical depth of the disk with column density $N$ is

$$\tau_{e0}(N) = N / N_{\tau_{e0}=1},$$

(21)

so that the intensity is

$$I(\nu_0) = B(\nu_0, T) \left( 1 - \exp \left( -\tau_{e0}(N) \right) \right).$$

(22)

Here $B(\nu_0, T)$ is the vacuum brightness of a blackbody at frequency $\nu_0$ (see Lang 2006, chap. 2.7).

The spectral irradiance of the disk $E(\nu)$ as a function of $\nu$ is

$$E(\nu) = \frac{1}{D^2} \int \int I(\nu_0, r) \times \exp \left( -\frac{m c^2 d(\nu, \nu_0, r)^2}{2 k_B T_{ex} \nu_0^2} \right) r \, dr \, d\varphi,$$

(23)

where $d(\nu, \nu_0, r) = \nu - \nu_0 - \frac{v_K(r)}{c} \cos \varphi \sin \theta$.

The integral is done in cylindrical coordinates $(r, \varphi)$ and $\theta$ is the inclination angle of the disk.

We consider HCO$^+$ 3–2, DCO$^+$ 3–2, and N$_2$H$^+$ 3–2 lines. Their frequencies are 267.56 GHz, 216.12 GHz, and 279.52 GHz, respectively. At 100 AU of the model disk $T = 27$ K.

For simplicity we assume that fractional abundances of the ion species are uniform within the disk. The population of the ionized species can be drastically increased or decreased as a result of lightning, but to estimate such a population is beyond the scope of this paper. We adopt the XR+UV-new chemical process model of Walsh et al. (2012), and use the abundance values at $r = 100$ AU, $z = 3h$, since the electric field is most likely to reach the critical amplitude at higher altitudes of the disk (Muranushi et al. 2012). Thus, we assume that the fractional abundance (relative to H$_2$) of N$_2$H$^+$ is $5.3 \times 10^{-10}$, We assume the fractional abundances of HCO$^+$ and DCO$^+$ to be $9.0 \times 10^{-9}$ and $3.0 \times 10^{-9}$, respectively, based on a paper by Matthews et al. (2013) that reports observation of enhancement in DCO abundance. Therefore, the column
densities of HCO\(^+\), DCO\(^+\), and N\(_2\)H\(^+\) are 1.8 \times 10^{15} \text{ cm}^{-2}, 6.0 \times 10^{14} \text{ cm}^{-2}, and 1.1 \times 10^{14} \text{ cm}^{-2}, respectively.

Assuming that there is no lightning and that the molecules are at their thermal velocities, \(N_{10t}=n\) for the three lines are 7.78 \times 10^9 \text{ cm}^{-2}, 6.74 \times 10^9 \text{ cm}^{-2}, and 1.39 \times 10^{10} \text{ cm}^{-2}, respectively. On the other hand, \(N_{10t}=n\) for the three lines are 6.15 \times 10^{11} \text{ cm}^{-2}, 5.42 \times 10^{11} \text{ cm}^{-2}, and 1.10 \times 10^{12} \text{ cm}^{-2}, respectively, if the molecules are accelerated by the lightning electric field.

We can see that the disk is optically thick for all of the lines at 100 AU. However, all the three lines become two orders of magnitude more transparent under the effect of the critical electric field. This is a result of the molecular speed becoming faster. Consequently, observed line profiles are broadened. This Doppler broadening of the lines of the charged molecular species is the key observational feature to observe the characteristic speed of the molecules, and therefore the electric field strength in the protoplanetary disk.

3.3. Calculations of the Line Profiles and Integral Maps by Radiative Transfer

We introduce seven disk models, as in Table 3. We calculate the line profiles for the three ion species with these seven disk models in order to study the ability to distinguish the lightning model from the line observations (Figure 1). The line profiles are obtained by performing the spectral irradiance integral (Equation (23)). We assume an isotropic distribution for the ion velocities, assuming that the electric field is turbulent. We simulate the channel maps using the spectral line radiation transfer code LIME by Brinch & Hogerheijde (2010). In Figure 2, we present the simulated channel maps of the HCO\(^+\) line for disks N, T25, and T50. We assumed that our model disk is located in the same way as TW Hya. That is, our model disk is at a distance of 54 pc and at an inclination angle of 7° (van Leeuwen 2007). Although we limit the disk parameters to this specific distance and inclination throughout this paper, our programs can be easily applied to other disk parameters.

3.4. Matched Filtering

We apply the matched filtering method (North 1963) in order to distinguish the lightning model by ALMA. Matched filtering is the optimal method for discriminating models from noisy observations; it has been well studied and has wide applications not only in radio astronomy (Ellingson & Hampson 2003) but also in extrasolar planet astronomy (Jenkins & Doyle 1996; Doyle et al. 2000), in gravitational wave astronomy (Owen & Sathyaprakash 1999; Vaishnav et al. 2007; Hotokezaka et al. 2013), and even in ocean tomography (Munk & Wunsch 1979). We follow the treatment by Creighton & Anderson (2011).

Given that the noise levels for HCO\(^+\), DCO\(^+\), and N\(_2\)H\(^+\) are 1.130 \times 10^{-2} \text{ Jy}, 1.330 \times 10^{-2} \text{ Jy}, and 1.800 \times 10^{-2} \text{ Jy}, respectively, their noise spectrum power densities \(S_b\) per square arcsecond are 5.843 \times 10^{-5} \text{ Jy}^2 \text{ km s}^{-1}, 8.094 \times 10^{-5} \text{ Jy}^2 \text{ km s}^{-1}, and 1.483 \times 10^{-4} \text{ Jy}^2 \text{ km s}^{-1}, respectively.
The measure-of-sensitivity $\sigma_{\text{mos}}$ of the matched filter between two images $h_1(x, y, v)$ and $h_2(x, y, v)$ is

$$
\sigma_{\text{mos}} = 4 \int_{v_{\text{min}}}^{v_{\text{max}}} \int_{(x,y) \in \text{image}} \left[ h_1(x, y, v) - h_2(x, y, v) \right]^2 \frac{dx \, dy \, dv}{S_h}.
$$

(24)

Here, $x$ and $y$ are image coordinates in arcseconds and $v$ is the velocity coordinate.

The measure-of-sensitivity values among the models using different lines are summarized in Table 4. The measure-of-sensitivity for any two different models is larger than 100, and the largest measure-of-sensitivity is greater than 1000. This is true even if we change the cross-section models (Table 5). Therefore an image like Figure 2 is not difficult to detect. However, no observation of a protoplanetary disk has been reported. Therefore, we can reject such forms of lightning models from observations. There are multiple alternative scenarios that observations suggest: (1) Protoplanetary disk lightning does not exist at all. (2) The probability of a protoplanetary disk with LMG is low, so that we have not observed one yet. (3) Protoplanetary disk LMG exists in the form of LMG clumps (protoplanetary “cumulonimbus clouds”) much smaller than the size of the protoplanetary disks. (4) Protoplanetary disk LMG is scattered in many smaller clumps in the protoplanetary disk with a certain volume-filling factor, so that their total cross section covers a fraction of the disk image. This case is reduced to case (3) by considering the total cross section of the clumps.

We can put an upper limit on the size of such LMG clumps by thresholding the measure-of-sensitivity. For example, if the radii of LMG clumps are smaller than the values in Table 6, they are not detectable at $5.0 \sigma$. The matched filter studies show that the Townsend breakdown model is the easiest model to detect, the DP model being next, and runaway breakdown being most difficult. This tendency is explained as the wider the Doppler broadening is, the easier the detection.

4. CONCLUSIONS AND DISCUSSIONS

Discharge phenomena take place in regions with the critical electric field (LMG), and we have established observable features for detecting LMGs from line observations of the accelerated molecular ions. The dielectric strength of the disk gas, being one of the crucial elementary properties, will open up our understanding of MRI in protoplanetary disks. Understanding of the MRI in weakly ionized accretion disks will contribute to the study of the dynamics of protoplanetary disks as well as circumplanetary disks (Keith & Wardle 2014).

We have presented three dielectric strength models for protoplanetary disks. They are the Townsend breakdown model, DP breakdown model, and runaway breakdown model. We have proposed a method to distinguish these three models observationally. The models are distinguishable with the sensitivity of advanced telescopes such as ALMA. It is now possible to reject some of the lightning models based on ground observations. Upper limits on the size of LMG clouds are given by the observations. Our lightning models treated here are quite simple. Further studies aim to apply this work to more realistic disk models as well as more detailed discharge models (Okuzumi & Inutsuka 2015).

We thank Yasuo Fukui, Hitoshi Miura, Munetake Momose, and Takayuki Muto for helpful discussions; Shinichi Enami for his advice on ice surface charge chemistry; Edward Kmett, Simon Peyton Jones, and Richard Eisenberg for their comments on our Haskell programs; Tooru Eguchi for instruction of Japanese Virtual Observatory; Motomichi Tashiro for discussion on the empirical formula of ion–molecule collision cross section. We thank Steven Rieder and Keigo Nitadori for helping us correct grammatical errors. We are grateful to the Institute of Low Temperature Science, Hokkaido University, for hosting the workshop “Recent Development in Studies of Protoplanetary Disks with ALMA” where the authors learned ALMA image analysis. This work is based on the landmark review of terrestrial lightning by Takahashi (2009). The parallel computation techniques used in this work are based on Marlow (2013). We used the units library (Muranushi & Eisenberg 2014) to thoroughly check for the consistency of physical dimensions and units in this paper. This work is supported by Grants-in-Aid for Scientific Research (#23103005, #24103506, #25887023, #26400224) from MEXT. This research used computational resources of the K computer provided by the RIKEN Advanced Institute for Computational Science (AICS). We thank RIKEN AICS for the support in conducting this research. We thank the referee for patient and constructive interaction, with which we have made a substantial improvement to this paper.

APPENDIX

CROSS-SECTION MODEL OF ION–NEUTRAL MOLECULAR COLLISION

We establish the model of ion–neutral collisional cross sections as functions of collision energy, in collaboration with Motomichi Tashiro.

In order to compute the equilibrium speed in an electric field for ion species $\text{HCO}^+$, $\text{DCO}^+$, and $\text{N}_2^+$, we need to know the energy-dependent cross section of collisions between $\text{H}_2$ and the ion species. However, no experimental values for such collisions exist. In general, collisional cross-section data for molecular ions and molecules are scarce, due to the difficulty of setting up such collision experiments. On the other hand, quantum-mechanical simulation of such collision events would require up to one month per collision (M. Tashiro 2015, private communication) and it requires many collision simulations with different collision parameters to establish a cross-section value for one collision energy. The computational cost prohibits the estimation of the collisional cross section by simulation.

Therefore, we construct and use a simple empirical model of the collisional cross section of molecular ions and molecules, following Tashiro’s advice.

There are collision cross-section data (Phelps 1990, 1991) for the following six pairs of molecular ions and molecules: $\text{H}^-–\text{H}_2$, $\text{H}_2^+–\text{H}_2$, $\text{H}_2^+–\text{H}_3$, $\text{N}_2^+–\text{N}_2$, $\text{N}_2^+–\text{N}_3$, and $\text{Ar}^2–\text{Ar}$. We use the following model of total collisional cross section $\sigma_{\text{I}}(\varepsilon)$ between molecular ion species $\text{I}$ and ion species $\text{I}^+$:

$$
\sigma_{\text{I},\text{I}^+}(\varepsilon) = p_0(\varepsilon) \mu(I, I^+) p_1(\varepsilon),
$$

(25)

where $\varepsilon$ is the collision energy and $\mu(I, I^+)$ is the residual mass of species $I^+$ and I. $p_0(\varepsilon)$ and $p_1(\varepsilon)$ are model parameters universal across all species.
We perform the fitting of the model so that the cost function

$$ C = \sum_{i, l, f, d} \left( \sigma_{i, l, f, d}(\varepsilon) - \sigma_{i, l, f, d, \exp}(\varepsilon) \right)^2 $$

(26)

is minimized. Here, $\sigma_{i, l, f, d, \exp}(\varepsilon)$ are experimentally known cross-section values for some fixed values of $\varepsilon$ found in Phelps (1990, 1991). The experimental data and the best-fit models are presented in Figure 3. Our cross-section model predicts almost identical cross sections for the three collision pairs HCO$^+$−H$_2$, DCO$^+$−H$_2$, and N$_2$H$^+$−H$_2$. This is because our model depends only on the residual mass of species $\mu(I^+, I)$, and all the three pairs HCO$^+$−H$_2$, DCO$^+$−H$_2$, and N$_2$H$^+$−H$_2$ have similar residual masses.

Available experimental data on ion–neutral collisional cross sections are scarce and do not justify models more complex than Equation (25). Instead of making better-fitting models, we study how the uncertainty in the model affects our results. In addition to the model Equation (25) with the fitted parameters (Figure 3), we study two alternative models, where

$$ \sigma_{i, l, f, d}^L(\varepsilon) = 10 \sigma_{i, l, f, d}(\varepsilon), $$

(27)

$$ \sigma_{i, l, f, d}^S(\varepsilon) = \frac{1}{10} \sigma_{i, l, f, d}(\varepsilon), $$

(28)

for $I^+ - I = $ HCO$^+$−H$_2$, DCO$^+$−H$_2$, and N$_2$H$^+$−H$_2$. The cross-section models of Equations (25), (27), and (28), are labeled XM, XL, and XS, respectively.

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