Doping and energy evolution of spin dynamics in the electron-doped cuprate superconductor \( \text{Pr}_{0.88}\text{LaCe}_{0.12}\text{CuO}_{4-\delta} \)

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The doping and energy evolution of the magnetic excitations of the electron-doped cuprate superconductor \( \text{Pr}_{0.88}\text{LaCe}_{0.12}\text{CuO}_{4-\delta} \) in the superconducting state is studied based on the kinetic energy driven superconducting mechanism. It is shown that there is a broad commensurate scattering peak at low energy, then the resonance energy is located among this low energy commensurate scattering range. This low energy commensurate scattering disperses outward into a continuous ring-like incommensurate scattering at high energy. The theory also predicts a dome shaped doping dependent resonance energy.

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I. INTRODUCTION

The parent compounds of cuprate superconductors are believed to belong to a class of materials known as Mott insulators with an antiferromagnetic (AF) long-range order, then superconductivity emerges when charge carriers, holes or electrons, are doped into these Mott insulators. It has been found that only an approximate symmetry in the phase diagram exists about the zero doping line between the hole-doped and electron-doped cuprate superconductors, and the significantly different behavior of the hole-doped and electron-doped cases is observed, reflecting the electron-hole asymmetry.

Experimentally, by virtue of systematic studies using the nuclear magnetic resonance, and muon spin rotation techniques, particularly the inelastic neutron scattering (INS), the dynamical spin response in the hole-doped and electron-doped cuprate superconductors in the superconducting (SC) state has been well established now, where an important issue is whether the behavior of the magnetic excitations determined by the dynamical spin structure factor (DSSF) is universal or not. The early INS measurements on the hole-doped cuprate superconductors showed that the low energy spin fluctuations form a quartet of the incommensurate (IC) magnetic scattering peaks at wave vectors away from the AF wave vector \([\pi,\pi]\) (in units of inverse lattice constant). With increasing energy these IC magnetic scattering peaks are converged on the commensurate \([\pi,\pi]\) resonance peak at intermediate energy. Well above this resonance energy, the continuum of the spin wave like IC magnetic excitations are observed. Very recently, the INS measurements on the electron-doped cuprate superconductor \( \text{Pr}_{0.88}\text{LaCe}_{0.12}\text{CuO}_{4-\delta} \) showed that the IC magnetic scattering and inward dispersion toward a resonance peak with increasing energy appeared in the hole-doped cuprate superconductors are not observed in the electron-doped side. Instead, the magnetic scattering in the electron-doped cuprate superconductor \( \text{Pr}_{0.88}\text{LaCe}_{0.12}\text{CuO}_{4-\delta} \) has a broad commensurate peak centered at \([\pi,\pi]\) at low energy (≤ 50meV). In particular, the magnetic resonance is located among this low energy broad commensurate scattering range. In analogy to the hole-doped cuprate superconductors, the commensurate resonance with the resonance energy (≈ 10meV) in the electron-doped side scales with the SC transition temperature forming a universal plot for all cuprate superconductors irrespective of the hole-doped and electron-doped cases. Furthermore, the low energy broad commensurate magnetic scattering disperses outward into a continuous ring-like IC magnetic scattering at high energy (50meV < \( \omega < 300\text{meV} \)), this is the same as the hole-doped case. Therefore, the hour-glass shaped dispersion in the magnetic scattering of the hole-doped superconductors may not be a universal and intrinsic feature of all cuprate superconductors. Instead, the commensurate resonance itself appears to be a universal property of cuprate superconductors. At present, it is not clear how theoretical models based on a microscopic SC theory can reconcile the difference of the dynamical spin response in the hole-doped and electron-doped cuprate superconductors. No explicit predictions on the doping dependence of the resonance energy in the electron-doped cuprate superconductors has been made so far.

Within the framework of the kinetic energy driven SC mechanism, the dynamical spin response of the hole-doped cuprate superconductors has been discussed, and the results are in qualitative agreement with the INS experimental data. In this paper, we study the doping and energy dependence of the spin dynamics in the electron-doped cuprate superconductor \( \text{Pr}_{0.88}\text{LaCe}_{0.12}\text{CuO}_{4-\delta} \) along with this line. We calculate explicitly the dynamical spin structure factor (DSSF) of the electron-doped cuprate superconductor \( \text{Pr}_{0.88}\text{LaCe}_{0.12}\text{CuO}_{4-\delta} \), and reproduce qualitatively all main features of the INS experiments on the electron-doped cuprate superconductor \( \text{Pr}_{0.88}\text{LaCe}_{0.12}\text{CuO}_{4-\delta} \), including the energy dependence of the commensurate magnetic scattering and resonance at low energy and IC magnetic scattering at high energy. Our results also show that the difference of the low energy dynamical spin response between the hole-doped and electron-doped cuprate superconductors is mainly caused by the SC gap function in the electron-doped case deviated from the monotonic d-wave function.

The rest of this paper is organized as follows. The basic formalism is presented in Sec. II, where we generalize the calculation of the DSSF from the previous hole-doped case to the present electron-doped case. Within this theoretical framework, we discuss the dynamical spin response of the electron-doped cuprate superconductor.
Pr$_{0.88}$LaCe$_{0.12}$CuO$_{4-\delta}$ in the SC state in Sec. III, where we predict a dome shaped doping dependent resonance energy. Finally, we give a summary and discussions in Sec. IV.

II. THEORETICAL FRAMEWORK

In both hole-doped and electron-doped cuprate superconductors, the characteristic feature is the presence of the two-dimensional CuO$_2$ plane, and it seems evident that the unusual behaviors of cuprate superconductors are dominated by this plane. From the angle-resolved-photoemission spectroscopy (ARPES) experiments, it has been shown that the essential physics of the CuO$_2$ plane in the electron-doped cuprate superconductors is contained in the $t$-$t'$-$J$ model on a square lattice,

$$ H = t \sum_{\langle ij \rangle \langle \sigma \eta \rangle} PC_{ij \sigma}^\dagger C_{i+\pi \eta \sigma} P^\dagger - t' \sum_{\langle ij \rangle \langle \sigma \eta \rangle} PC_{ij \sigma}^\dagger C_{i+\pi \eta \sigma} P^\dagger - \mu \sum_{\langle ij \rangle} S_i \cdot S_{i+\pi}, $$

where $t < 0$, $t' < 0$, $\eta = \pm \hat{x}, \pm \hat{y}$, $\pi = \pm \hat{x} \pm \hat{y}$, $C_{ij \sigma}^\dagger$ ($C_{ij \sigma}$) is the electron creation (annihilation) operator, $S_i = C_{ij \pi}^\dagger \sigma C_{ij \sigma}/2$ is spin operator with $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ as Pauli matrices, $\mu$ is the chemical potential, and the projection operator $P$ removes zero occupancy, i.e., $\sum_{\sigma} C_{ij \sigma}^\dagger C_{ij \sigma} \geq 1$. In this case, an important question is the relation between the hole-doped and electron-doped cases. The $t$-$J$ model with nearest neighbor hopping $t$ has a particle-hole symmetry because the sign of $t$ can be absorbed by changing the sign of the orbital on one sublattice. However, the particle-hole asymmetry can be described by including the next neighbor hopping $t'$, which has been tested extensively in Ref. 14, where they use $ab$ initio local density functional theory to generate input parameters for the three-band Hubbard model and then solve the spectra exactly on finite clusters, and the results are compared with the low energy spectra of the one-band Hubbard model and the $t$-$t'$-$J$ model. They found an excellent overlap of the low lying wavefunctions for both one-band Hubbard model and the $t$-$t'$-$J$ model, and were able to extract the effective parameters as $J \approx 0.1 \sim 0.13$ eV, $t/J = 2.5 \sim 3$ for the hole doping and $t'/J = -2.5 \sim -3$ for the electron doping, and $t'/t$ is of order 0.2 $\sim$ 0.3, and is believed to vary somewhat from compound to compound. Although there is a similar strength of the magnetic interaction $J$ for both hole-doped and electron-doped cuprate superconductors, the interplay of $t'$ with $t$ and $J$ causes a further weakening of the AF spin correlation for the hole doping, and enhancing the AF spin correlation for the electron doping, therefore the AF spin correlations in the electron-doped case is stronger than those in the hole-doped side. In particular, it has been shown from the ARPES experiments that the lowest energy states in the hole-doped cuprate superconductors in the normal state are located at $k = [\pi/2, \pi/2]$ point, while they appear at $k = [\pi, 0]$ point in the electron-doped case. This asymmetry seen by the ARPES observation on the hole-doped and electron-doped cuprates is actually consistent with calculations performed within the $t$-$t'$-$J$ model based on the exact diagonalization studies, where all of the hopping terms have opposite signs for the electron and hole doping, and the sign of $t'$ is of crucial importance for the coupling of the charge motion to the spin background. Furthermore, the low energy electronic structures of the hole-doped and electron-doped cuprates have been well reproduced by the mean-field (MF) solutions within the $t$-$t'$-$J$ model.

For the hole-doped case, the charge-spin separation (CSS) fermion-spin theory has been developed to incorporate the single occupancy constraint. In particular, it has been shown that under the decoupling scheme, this CSS fermion-spin representation is a natural representation of the constrained electron defined in a restricted Hilbert space without double electron occupancy. To apply this theory in the electron-doped case, the $t$-$t'$-$J$ model (1) can be rewritten in terms of a particle-hole transformation $C_{ij \sigma} \rightarrow f_{i \sigma}$ as,

$$ H = -t \sum_{\langle ij \rangle} f_{i \sigma}^\dagger f_{i+\pi \sigma} + t' \sum_{\langle ij \rangle} f_{i \sigma}^\dagger f_{i+\pi \sigma} - \mu \sum_{\langle i \rangle} f_{i \sigma}^\dagger f_{i \sigma} + J \sum_{\langle ij \rangle} S_i \cdot S_{i+\pi}, $$

supplemented by a local constraint $\sum_{\sigma} f_{i \sigma}^\dagger f_{i \sigma} \leq 1$ to remove double occupancy, where $f_{i \sigma}^\dagger$ ($f_{i \sigma}$) is the hole creation (annihilation) operator, while $S_i = f_{i \uparrow}^\dagger \sigma f_{i \downarrow}/2$ is the spin operator in the hole representation. Now we follow the CSS fermion-spin theory, and decouple the hole operators as $f_{i \sigma} = a_{i \uparrow}^\dagger S_i^-$ and $f_{i \sigma} = a_{i \downarrow}^\dagger S_i^+$, with the spinful fermion operator $a_{i \sigma} = e^{-i \phi_{i \sigma}^\dagger} a_{i \sigma}$, describes the charge degree of freedom together with some effects of spin configuration rearrangements due to the presence of the doped electron itself (dressed charge carrier), while the spin operator $S_i$ describes the spin degree of freedom, then the single occupancy local constraint is satisfied. In this CSS fermion-spin representation, the $t$-$t'$-$J$ model (2) can be expressed as,

$$ H = -t \sum_{\langle ij \rangle} (a_{i \uparrow} S_i^+ a_{i+\pi \uparrow}^\dagger S_{i+\pi \uparrow}^\dagger + a_{i \downarrow} S_i^- a_{i+\pi \downarrow}^\dagger S_{i+\pi \downarrow}) $$

$$ + t' \sum_{\langle ij \rangle} (a_{i \uparrow} S_i^+ a_{i+\pi \uparrow}^\dagger S_{i+\pi \uparrow}^\dagger + a_{i \downarrow} S_i^- a_{i+\pi \downarrow}^\dagger S_{i+\pi \downarrow}) $$

$$ - \mu \sum_{\langle i \rangle} a_{i \uparrow} a_{i \downarrow} + J_{eff} \sum_{\langle ij \rangle} S_i \cdot S_{i+\pi}, $$

with $J_{eff} = (1 - x)^2 J$, and $x = \langle a_{i \sigma} a_{i \sigma} \rangle / \langle a_{i \sigma}^\dagger a_{i \sigma} \rangle$ is the electron doping concentration. As in the hole-doped case, the SC order parameter for the electron Cooper pair in the electron-doped case also can be defined as,

$$ \Delta = \langle C_{ij \uparrow}^\dagger C_{j \downarrow}^\dagger - C_{ij \downarrow} C_{j \uparrow} \rangle = \langle a_{i \uparrow} a_{j \downarrow} S_i^+ S_{j-\pi \downarrow}^\dagger - a_{i \downarrow} a_{j \uparrow} S_i^- S_{j-\pi \uparrow} \rangle $$

$$ = -(S_i^+ S_{j-\pi \downarrow}^\dagger) \Delta_n, $$

with the charge carrier pairing order parameter $\Delta_n = \langle a_{j \uparrow} a_{i \downarrow}^\dagger - a_{j \downarrow}^\dagger a_{i \uparrow} \rangle$. It has been shown from the ARPES experiments that the hot spots are located close to $[\pm \pi, 0]$.
most strongly couples to electrons, suggesting a spin-energy term in the underdoped regime, and reaches a maximum in the optimum temperature increases with increasing doping in the undoped regime. In particular, this SC-state is controlled by both SC gap function and quasiparticle coherence, which leads to the fact that the SC transition temperature increases with increasing doping in the underdoped regime, and reaches a maximum in the optimal doping, then decreases in the overdoped regime. Furthermore, superconductivity in the electron-doped cuprate superconductors has been also discussed under this kinetic energy driven SC mechanism, and the results show that superconductivity appears over a narrow range of doping, around the optimal electron doping $x = 0.15$. Within the kinetic energy driven SC mechanism, the DSSF of the hole-doped $t$-$t'$-$J$ model in the SC state with a monotonically d-wave gap function has been calculated in terms of the collective mode in the dressed charge carrier-particle-particle channel, and the results are in qualitative agreement with the INS experimental data on the hole-doped cuprate superconductors in the SC state. Following their discussions, we can obtain the DSSF of the electron-doped $t$-$t'$-$J$ model (3) in the SC state with the nonmonotonic d-wave gap function (5) as,

$$S(k, \omega) = -2[1 + n_B(\omega)] \text{Im} D(k, \omega),$$  

with the full spin Green’s function in the SC state,

$$D(k, \omega) = \frac{B_k}{\omega^2 - \omega^2_{\text{cm}} - B_k \Sigma^{(s)}(k, \omega)},$$

with $B_k = 2\lambda_1(\gamma_k - A_2) - \lambda_2(2\chi_2'\gamma_k' - \chi_2), \lambda_1 = 2ZJ_{\text{eff}}, \lambda_2 = 4Z\phi t', A_1 = \chi_1' + \chi_1/2, A_2 = \chi_1' + \chi_1/2, \epsilon = 1 + 2t\phi_1/J_{\text{eff}}, \gamma_k = (1/Z) \sum_{\eta} e^{i\lambda_\eta}, \gamma_k' = (1/Z) \sum_{\tau, \tau'} e^{i(k - p)\tau}, Z$ is the number of the nearest-neighbor or next-nearest-neighbor sites, the dressed charge carrier-particle-hole parameters $\phi_1 = \langle a_{\tau, 1}^\dagger a_{\tau + \eta} \rangle$, and the spin correlation functions $\chi_1 = \langle S_1^+ S_{\eta}^z \rangle$, $\chi_2 = \langle S_1^+ S_{\eta}^{-z} \rangle$, $\chi_1' = \langle S_1^+ S_{\eta}^{\dagger z} \rangle$ and $\chi_2' = \langle S_1^+ S_{\tau}^{\dagger z} \rangle$, and the MF spin excitation spectrum,

$$\omega_k^2 = \lambda_1^2 [A_3 - \alpha \gamma_k (1 - \epsilon \gamma_k) + \frac{1}{2}(A_3 - \frac{1}{2}(A_3 - \frac{1}{2}A_3))] + \lambda_1 \lambda_2 [\alpha \gamma_k (1 - \epsilon \gamma_k) + \frac{1}{2}(A_3 - C_3)] + \frac{3}{2} \epsilon \chi_2 \gamma_k'$$

with $A_3 = \alpha C_1 + (1 - \alpha)/(2Z), A_4 = \alpha C_1$ and $A_5 = \alpha C_2 + (1 - \alpha)/(2Z)$, and the spin correlation functions $C_1 = (1/Z^2) \sum_{\eta, \eta'} \langle S_{\eta}^+ S_{\eta'}^- \rangle$, $C_2 = (1/Z^2) \sum_{\tau, \tau'} \langle S_{\eta}^+ S_{\eta'}^- \rangle$, $C_3 = (1/Z^2) \sum_{\tau, \tau'} \langle S_{\eta}^+ S_{\eta'}^- \rangle$, and $C_4 = (1/Z) \sum_{\eta} \langle S_{\eta}^+ S_{\eta}^z \rangle$. In order to satisfy the sum rule of the correlation function $\langle S_{\eta}^+ S_{\eta}^- \rangle = 1/2$ in the case without AF long-range order, the decoupling parameter $\alpha$ has been introduced, which can be regarded as the vertex correction, while the spin self-energy function $\Sigma^{(s)}(k, \omega)$ in Eq. (7) is obtained from the dressed charge carrier bubble in the dressed charge carrier-particle-particle channel as,

$$\Sigma^{(s)}(k, \omega) = \frac{1}{N^2} \sum_{p, q} A(q, p, k) \frac{B_{q+k}}{\omega_{q+k}} \frac{Z_{p+q}}{4} Z_{p} Z_{p+q} \frac{\Delta_{p+q}}{E_{p+q+q}} \left( \frac{F^{(1)}_{s}(k, p, q)}{\omega^2 - (E_{p} - E_{p+q} + \omega_{q+k})^2} + \frac{F^{(2)}_{s}(k, p, q)}{\omega^2 - (E_{p+q} - E_{p} + \omega_{q+k})^2} + \frac{F^{(3)}_{s}(k, p, q)}{\omega^2 - (E_{p} + E_{p+q} + \omega_{q+k})^2} + \frac{F^{(4)}_{s}(k, p, q)}{\omega^2 - (E_{p+q} + E_{p} - \omega_{q+k})^2} \right),$$

where $A(q, p, k) = [(Zt\gamma_{q-p} - Zt\gamma_{q-p})^2 + (Zt\gamma_{q+p-k} - Zt\gamma_{q+p-k})^2]$, $N$ is the number of sites,
\[ F^{(1)}(k, p, q) = (E_p - E_{p+q} + \omega_{k+q})\{n_B(\omega_{q+k})n_F(E_p) - n_F(E_{p+q})\} - n_F(E_{p+q})n_F(-E_p), \]
\[ F^{(2)}(k, p, q) = (E_{p+q} - E_p + \omega_{k+q})\{n_B(\omega_{q+k})n_F(E_{p+q}) - n_F(E_p)\} - n_F(E_{p+q})n_F(-E_p), \]
\[ F^{(3)}(k, p, q) = (E_p + E_{p+q} + \omega_{k+q})\{n_B(\omega_{q+k})n_F(E_p) - n_F(E_{p+q})\} - n_F(E_{p+q})n_F(E_p), \]
\[ \Delta_{\alpha}(k) = \Delta_{\alpha}(\gamma_{k}^{(d)} - B\gamma_{k}^{(2d)}), \]
the dressed charge carrier quasiparticle spectrum \( E_k = \sqrt{\epsilon_k^2 + |\Delta_{\alpha}(k)|^2} \), and \( \epsilon_k = Z_k F \epsilon_k \), the MF dressed charge carrier excitation spectrum \( \epsilon_k = Z_k \chi_1 \gamma_k - Z_k \gamma_{2k} - \mu \), while the dressed charge carrier quasiparticle coherent weight \( Z_{\alpha} F \) and effective dressed charge carrier gap parameters \( \Delta_{\alpha} \) and \( B \) are determined by the following three equations:\[ \begin{align*}
1 &= \frac{1}{N^2} \sum_{k, q, p} \Gamma_{k+q}^{(d)} \left[ \gamma_{k}^{(d)} - B\gamma_{k}^{(2d)} \right] \frac{Z_{\alpha}^2 B_q B_p}{E_k \omega_p \omega_q} \left( \frac{F^{(1)}_1(k, q, p)}{(\omega_p - \omega_q)^2 - E_k^2} - \frac{F^{(2)}_1(k, q, p)}{(\omega_p + \omega_q)^2 - E_k^2} \right), \\
B &= -\frac{1}{N^2} \sum_{k, q, p} \Gamma_{k+q}^{(d)} \left[ \gamma_{k}^{(d)} - B\gamma_{k}^{(2d)} \right] \frac{Z_{\alpha}^2 B_q B_p}{E_k \omega_p \omega_q} \left( \frac{F^{(1)}_2(k, q, p)}{(\omega_p - \omega_q)^2 - E_k^2} - \frac{F^{(2)}_2(k, q, p)}{(\omega_p + \omega_q)^2 - E_k^2} \right), \\
\frac{1}{Z_{\alpha} F} &= 1 + \frac{1}{N^2} \sum_{q, p} \Gamma_{k+q}^{(d)} \frac{Z_{\alpha} F B_q B_p}{4\omega_p \omega_q} \left( \frac{F^{(1)}_2(q, p)}{(\omega_p - \omega_q - E_{p-q+k})^2} + \frac{F^{(2)}_2(q, p)}{(\omega_p - \omega_q + E_{p-q+k})^2} \right). 
\end{align*} \]
\[ \Delta_{\alpha}(k) = \Delta_{\alpha}(\gamma_{k}^{(d)} - B\gamma_{k}^{(2d)}), \]
the dressed charge carrier quasiparticle spectrum \( E_k = \sqrt{\epsilon_k^2 + |\Delta_{\alpha}(k)|^2} \), the dressed charge carrier quasiparticle spectrum \( E_k = \sqrt{\epsilon_k^2 + |\Delta_{\alpha}(k)|^2} \), and \( \epsilon_k = Z_k F \epsilon_k \), the MF dressed charge carrier excitation spectrum \( \epsilon_k = Z_k \chi_1 \gamma_k - Z_k \gamma_{2k} - \mu \), while the dressed charge carrier quasiparticle coherent weight \( Z_{\alpha} F \) and effective dressed charge carrier gap parameters \( \Delta_{\alpha} \) and \( B \) are determined by the following three equations:\[ \begin{align*}
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B &= -\frac{1}{N^2} \sum_{k, q, p} \Gamma_{k+q}^{(d)} \left[ \gamma_{k}^{(d)} - B\gamma_{k}^{(2d)} \right] \frac{Z_{\alpha}^2 B_q B_p}{E_k \omega_p \omega_q} \left( \frac{F^{(1)}_2(k, q, p)}{(\omega_p - \omega_q)^2 - E_k^2} - \frac{F^{(2)}_2(k, q, p)}{(\omega_p + \omega_q)^2 - E_k^2} \right), \\
\frac{1}{Z_{\alpha} F} &= 1 + \frac{1}{N^2} \sum_{q, p} \Gamma_{k+q}^{(d)} \frac{Z_{\alpha} F B_q B_p}{4\omega_p \omega_q} \left( \frac{F^{(1)}_2(q, p)}{(\omega_p - \omega_q - E_{p-q+k})^2} + \frac{F^{(2)}_2(q, p)}{(\omega_p - \omega_q + E_{p-q+k})^2} \right), 
\end{align*} \]

where \( \Gamma_{k+q} = Z_k \chi \gamma_k + Z_k \gamma_{2k} \), and \( \gamma_k^{(d)} \) and \( \gamma_k^{(2d)} \). and \( F^{(1)}(k, q, p) = (\omega_p - \omega_q)\{n_B(\omega_q) - n_B(\omega_p)\}[1 - 2n_F(E_k)] + E_k\{n_B(\omega_p)n_B(\omega_q) + n_B(\omega_q)n_B(\omega_p)\}, \quad F^{(2)}(k, q, p) = (\omega_p + \omega_q)\{n_B(\omega_p) - n_B(\omega_q)\}[1 - 2n_F(E_k)] + E_k\{n_B(\omega_p)n_B(\omega_q) + n_B(\omega_q)n_B(\omega_p)\}, \quad F^{(3)}(k, q, p) = n_F(E_{p+q+k})\{n_B(\omega_p) + n_B(\omega_q)\} - n_B(\omega_p)n_B(\omega_q) - n_B(\omega_q)n_B(\omega_p) - n_B(\omega_p)n_B(\omega_q) \]

III. DOPING AND ENERGY DEPENDENT INCOMMENSURATE MAGNETIC SCATTERING AND COMMENSURATE RESONANCE

We are now ready to discuss the doping and energy dependence of the dynamical spin response in the electron-doped cuprate superconductors in the SC state. In Fig.
FIG. 1: The dynamical spin structure factor $S(k, \omega)$ in the $(k_x, k_y)$ plane at $x = 0.15$ with $T = 0.002J$ for $t/J = -2.5$ and $t'/t = 0.3$ at (a) $\omega = 0.07J$, (b) $\omega = 0.12J$, and (c) $\omega = 0.36J$.

For determining the commensurate magnetic resonance energy in the SC state, we have made a series of calculations for the intensities of the DSSF in the SC state with the nonmonotonic d-wave gap function and normal state, and the differences between the SC state and normal state intensities at different energies in the low energy commensurate scattering range, and the results of the intensities of the DSSF in the (a) SC state, (b) normal state, and (c) the differences between the SC state and normal state intensities as a function of energy at $x = 0.15$ with $T = 0.002J$ for $t/J = -2.5$ and $t'/t = 0.3$ are plotted in Fig. 3. For comparison, the corresponding experimental data\cite{10} of the differences between the SC state and normal state intensities for Pr$_{0.88}$LaCe$_{0.12}$CuO$_{4-\delta}$ is also shown in Fig. 3c (inset). Obviously, the corresponding intensity of the DSSF in the normal state is much smaller than this in the SC state, then the commensurate resonance is essentially determined by the intensities of the DSSF in the SC state. In this case, a commensurate resonance peak centered at $\omega_r = 0.07J$ is obtained from Fig. 3c. In particular, this magnetic resonance energy is located among the low energy commensurate scattering range. Using a reasonably estimative value of $J \sim 150$ meV in the electron-doped cuprate superconductors\cite{8}, the present result of the resonance energy $\omega_r = 0.07J \approx 10.5$ meV is in quantitative agreement with the resonance energy $\approx 11$ meV observed\cite{10} in Pr$_{0.88}$LaCe$_{0.12}$CuO$_{4-\delta}$. Furthermore, we also find that the value of the resonance energy $\omega_r$ is dependent on the next neighbor hopping $t'$, i.e., with increasing $t'$, the value of the resonance energy $\omega_r$ increases. Since the value of $t'$ is believed to vary somewhat from compound to compound, therefore there are different values of the resonance energy $\omega_r$ for different families of the electron-doped cuprate superconductors. However, there is a substantial difference between theory and experiment, namely, the differences between SC state and normal state intensities in the DSSF show a flat behavior for Pr$_{0.88}$LaCe$_{0.12}$CuO$_{4-\delta}$ at low energies below 5meV\cite{10},

In this case, a commensurate resonance peak centered at $\omega_r = 0.07J$ is obtained from Fig. 3c. In particular, this magnetic resonance energy is located among the low energy commensurate scattering range. Using a reasonably estimative value of $J \sim 150$ meV in the electron-doped cuprate superconductors\cite{8}, the present result of the resonance energy $\omega_r = 0.07J \approx 10.5$ meV is in quantitative agreement with the resonance energy $\approx 11$ meV observed\cite{10} in Pr$_{0.88}$LaCe$_{0.12}$CuO$_{4-\delta}$. Furthermore, we also find that the value of the resonance energy $\omega_r$ is dependent on the next neighbor hopping $t'$, i.e., with increasing $t'$, the value of the resonance energy $\omega_r$ increases. Since the value of $t'$ is believed to vary somewhat from compound to compound, therefore there are different values of the resonance energy $\omega_r$ for different families of the electron-doped cuprate superconductors. However, there is a substantial difference between theory and experiment, namely, the differences between SC state and normal state intensities in the DSSF show a flat behavior for Pr$_{0.88}$LaCe$_{0.12}$CuO$_{4-\delta}$ at low energies below 5meV\cite{10},

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while the calculation anticipates the differences of the SC state and normal state intensities linearly increase from the zero energy towards to the resonance peak. However, upon a closer examination one sees immediately that the main difference is due to that the difference of the SC state and normal state intensities linearly increase at too low energies in the theoretical consideration. The actual range of rapid growth of the differences of the SC state and normal state intensities with energy (around 5meV $\sim 16$meV) is very similar in theory and experiments. We emphasize that although the simple $t$-$t'$-$J$ model (1) cannot be regarded as a comprehensive model for a quantitative comparison with the electron-doped cuprate superconductor Pr$_{0.88}$LaCe$_{0.12}$CuO$_{4-\delta}$, our present results for the SC state are in qualitative agreement with the major experimental observations on the electron-doped cuprate superconductors modulates the renormalized spin excitation spectrum in the electron-doped case and reaches a maximum in the optimal doping, then decreases in the overdoped regime. In comparison with the previous results for the hole-doped case, our present results also show that the commensurate magnetic resonance is a common feature for cuprate superconductors irrespective of the hole doping or electron doping, while the commensurate magnetic scattering at low energy in the present electron-doped case indicates that the intimate connection between the IC magnetic scattering and resonance in the hole-doped side at low energy is not a universal feature. In other words, the resonance energy itself is intimately related to superconductivity, other details such as the incommensurability and hour-glass dispersion found in different cuprate superconductors may not be fundamental to superconductivity.

The essential physics of the doping and energy dependence of the dynamical spin response in the electron-doped cuprate superconductor Pr$_{0.88}$LaCe$_{0.12}$CuO$_{4-\delta}$ in the SC state is the same as in the hole-doped case except the nonmonotonic d-wave gap function form (5). Although the momentum dependence of the SC gap function (5) is basically consistent with the d-wave symmetry, it obviously deviates from the monotonic d-wave SC gap function. This is different from the hole-doped case, where the momentum dependence of the monotonic d-wave SC gap function is observed. As seen from Fig. 2, the higher harmonic term in Eq. (5) mainly effects the low energy behavior of the dynamical spin response, i.e., the nonmonotonic d-wave SC gap function (5) in the electron-doped cuprate superconductors modulates the renormalized spin excitation spectrum in the electron-doped cuprate superconductors, and therefore leads to the difference of the low energy dynamical spin response between the hole-doped and electron-doped cuprate su-
perconductors in the SC state.

IV. SUMMARY AND DISCUSSIONS

In summary, we have shown very clearly in this paper that if the nonmonotonic d-wave SC gap function is set into account in the framework of the kinetic energy driven SC mechanism, the DSSF of the t-t’-J model can correctly reproduce all main features found in the INS measurements on the electron-doped cuprate superconductor Pr0.88LaCe0.12CuO4−δ, including the energy dependence of the commensurate magnetic scattering and resonance at low energy and IC magnetic scattering at high energy, without using adjustable parameters. We believe that the commensurate magnetic resonance is a universal feature of cuprate superconductors, as shown by the INS experiments on the hole-doped cuprate superconductors YBa2Cu3O7−δ, La2−xSrxCuO4−δ, Tl2Ba2CuO6+δ, Bi2Sr2CaCu2O8+δ, and electron-doped cuprate superconductors Pr1−xLaCeCuO4−δ and Nd2−xCeCuO4−δ. The theory also predicts a dome shaped doping dependent magnetic resonance energy, which should be verified by further experiments.

Within the framework of the kinetic energy driven SC mechanism, we have studied the electronic structure of the electron-doped cuprate superconductors in the SC state. It is shown that although there is an electron-hole asymmetry in the phase diagram, the electronic structure of the electron-doped cuprates in the SC state is similar to that in the hole-doped case. In particular, it is also shown that the higher harmonic term in Eq. (5) mainly affects the low energy spectral weight, i.e., the low energy spectral weight increases when the higher harmonic term is considered, while the position of the SC quasiparticle peak is slightly shifted away from the Fermi energy. This is consistent with the present result of the spin dynamics, and both studies indicate that the higher harmonic term in Eq. (5) mainly affects the low energy behavior of the systems.

Finally, we have noted that the differences of the intensities between SC and normal states become negative for the electron-doped cuprate superconductor Nd1.85Ce0.15CuO4−δ at low energies below 5meV, which shows that the DSSF intensity at energies below 5meV is suppressed in the SC state. Although it has been argued that this unusual behavior of the dynamical spin response of Nd1.85Ce0.15CuO4−δ may be related to the spin gap, the physical reason for this unusual behavior is still not clear. These and the related issues are under investigation now.

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