Tests for Coherence in Neutral B Meson Decays

Michael Gronau
Department of Physics, Technion-Israel Institute of Technology
Technion City, 32000 Haifa, Israel

and

Jonathan L. Rosner
Enrico Fermi Institute and Department of Physics
University of Chicago, Chicago, Illinois 60637

ABSTRACT

A density-matrix method for the study of tagged states of neutral $B$ mesons with arbitrary coherence properties is applied to several examples, including $e^+e^-$ production both at and above the $\Upsilon(4S)$ resonance, and hadronic production. In the absence of coherence the only term modulating the exponential decay of a neutral $B$ meson behaves as $\cos \Delta mt$, while a $\sin \Delta mt$ modulation is a signal of partial or full coherence. Decays to CP eigenstates are needed to fully specify the density matrix. We relate these results to more familiar expressions for the cases of the $\Upsilon(4S)$ and incoherent production.

I Introduction

Neutral $B$ mesons undergo time-dependent oscillations with their antiparticles. This feature, first demonstrated for neutral kaons nearly half a century ago \cite{1}, has been crucial in extracting fundamental information on the mechanism of CP violation from the decays of neutral $B$’s. Moreover, the oscillations themselves have provided crucial information on the magnitude of electroweak couplings, and were one of the first pieces of evidence for a very heavy top quark \cite{2}.

The oscillations are characterized by splittings $\Delta m$ between mass eigenstates. For the $B_d = \bar{b}d$, the most recent world average \cite{3} is $\Delta m_d = 0.487 \pm 0.014 \, \text{ps}^{-1}$. For the $B_s = \bar{b}s$, only a lower limit \cite{3,4} $\Delta m_s > 15 \, \text{ps}^{-1}$ exists at present.

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In the study of CP violation in decays of a neutral $B$ meson, one frequently needs to know its flavor at the time of production. Was it a $B^0$ or a $B^0$? Was it a $B_s$ or a $B_s$? The dynamics of $B$ meson production affords several methods for identifying this flavor. “Same-side” tagging methods [5, 6, 7, 8] utilize the correlation of the flavor of a neutral $B$ with the charge of a kaon or pion which is produced near it in phase space. “Opposite-side” methods utilize the associated production of $b\bar{b}$ in electromagnetic or strong interactions to tag a neutral $B$ using the fragmentation products of the quark produced in association with it. The tagging methods are useful not only for the study of CP asymmetries, but also in the study of the oscillations themselves. For example, it is important to understand the systematic errors of tagging methods if a reliable estimate of $\Delta m_s$ is to be achieved.

The threshold for electromagnetic or strong production of a pair of nonstrange $B$ mesons [$M(B) = 5.28 \text{ GeV}/c^2$] is just below the $\Upsilon(4S)$ resonance, which lies at a center-of-mass energy of 10.58 GeV. At the $\Upsilon(4S)$, the reaction $e^+e^- \to BB$ produces the two mesons in an eigenstate of the charge conjugation operator $C$, with eigenvalue $\eta_C = -1$. The flavor oscillations of neutral $B$’s then manifest themselves as functions of the time difference $t - \bar{t}$ between their decays. Consequently, asymmetric $e^+e^-$ collisions have been adopted as a means of time-dilating the decays to enable the separation of their vertices [9, 10].

Another means of $B$ production is through the decay $Z \to b\bar{b}$, with subsequent fragmentation of the $b$ or $\bar{b}$ to the neutral meson of interest. Here, the observed $B$ and the tagging hadron (which could be any meson or baryon containing a $b$ or $\bar{b}$) are likely to be uncorrelated in their charge-conjugation properties, as has been assumed in several analyses (e.g., [11]).

The question of coherence between the tagging hadron and the detected neutral $B$ meson may not be so clear-cut in several cases intermediate between $\Upsilon(4S) \to B^0\bar{B}^0$ (full coherence) and $Z \to b\bar{b}$ (little or no coherence). For example, just above the threshold for $e^+e^- \to B^0\bar{B}^{*0}$ or $B^0\bar{B}^{*0}$, if the photon in $B^{*0} \to B^0\gamma$ or $B^{*0} \to B^0\gamma$ is detected, the $B^0\bar{B}^0$ pair will be in a state of $\eta_C = +1$. If the photon is not detected, however, there may be an additional contribution from $e^+e^- \to B^0\bar{B}^0$, in which the $B^0\bar{B}^0$ pair has $\eta_C = -1$. The relative probabilities $P_\pm$ of $\eta_C = \pm 1$ states are in any case unlikely to be equal.

In hadronic $b\bar{b}$ production the subprocesses $q\bar{q} \to b\bar{b}$ and $gg \to b\bar{b}$ generate a $b\bar{b}$ pair whose mass spectrum peaks at a scale of several times $m_b$. Additional $b\bar{b}$ pairs with an even sharper $M(b\bar{b})$ peak near threshold arise from splitting of a virtual gluon: $g^* \to b\bar{b}$. While incoherence has been assumed (e.g., [12]) in analyses of flavor oscillations in hadronic $B$ production, there may be effects of coherence if a $B^0$ and a $\bar{B}^0$ are produced in a state of low enough effective mass. This is particularly likely in the case of forward geometries (e.g., the HERA-B experiment at DESY [13], the BTeV experiment at Fermilab [14], and the LHCb experiment at CERN [15]), in which the $B$ and $\bar{B}$ are highly kinematically correlated. It is less likely to be the case for central $B$ production (as in the CDF and D0 experiments at Fermilab [16, 17] and the ATLAS and CMS experiments at CERN [18, 19]).

The practical effects of coherence cannot be ignored, since they lead to a char-
acteristic term proportional to $e^{-\Gamma t} \sin \Delta m t$ in the dependence on proper time $t$ of flavor oscillations. Such a term signals unequal probabilities for $B^0\overline{B}^0$ production in eigenstates of positive and negative charge-conjugation eigenvalue. In the absence of coherence, the only terms present are proportional to $e^{-\Gamma t}$ and $e^{-\Gamma t} \cos \Delta m t$. We thus advocate the inclusion of $e^{-\Gamma t} \sin \Delta m t$ terms in any analyses in which the presence of coherence is suspected. The present paper is devoted to the study of such effects. Although we shall generally speak of the $(B^0, \overline{B}^0)$ system, many of our results apply as well to neutral strange $B$ mesons.

In Section II we redervie a density-matrix formalism first introduced in [7, 8], using a more standard phase convention and correcting a sign error in the original references. This formalism is then applied to several cases, including mixed states with dilution of tagging efficiency (Section III), full coherence (Section IV), and intermediate cases (Section V). The means of fully specifying the density matrix is discussed in Section VI. The degree to which current and planned experiments can be expected to display coherence is given in Section VII, while Section VIII concludes.

II Density-matrix description

The density matrix is the appropriate means with which to discuss states with arbitrary coherence properties. We work in a two-component “quasi-spin” space [20, 21] with initial basis states

$$|B^0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad |\overline{B}^0\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$  

In this basis the most general density matrix $\rho$ satisfying $\rho = \rho^\dagger$, $\text{Tr}(\rho) = 1$ can be written

$$\rho = \frac{1}{2} [1 + \mathbf{Q} \cdot \mathbf{\sigma}],$$  

where $\mathbf{Q}$ describes polarization in quasispin space, $\mathbf{Q}^2 \leq 1$, and $\sigma_i$ ($i = 1, 2, 3$) are the Pauli matrices. A pure state can be described by a density matrix with $|\mathbf{Q}| = 1$, while a completely incoherent combination of $B^0$ and $\overline{B}^0$ with relative probabilities $P_{B^0}$ and $P_{\overline{B}^0} = 1 - P_{B^0}$ (a “mixed state”) corresponds to a diagonal density matrix with $Q_1 = Q_2 = 0$, $Q_3 = 2P_{B^0} - 1$. One describes the density matrices for initial $B^0$ and $\overline{B}^0$ by diag(1,0) and diag(0,1), respectively.

The probability for a transition from an initial state denoted by the density matrix $\rho_i$ to a final state denoted by $\rho_f$ is then

$$I(f) = \text{Tr} (\rho_i T^\dagger \rho_f T),$$  

where $T$ is the operator which time-evolves the state from $i$ to $f$. Here $f$ will denote an arbitrary coherent superposition of $B^0$ and $\overline{B}^0$ at time $t$, so that we shall be able to discuss decays to both flavor eigenstates (such as $J/\psi K^{*0} \rightarrow J/\psi K^+\pi^-$) and CP eigenstates (such as $J/\psi K_{S,L}$). The density matrix $\rho_f$ will take the appropriate form for each such final state.
It is most convenient to transform to the mass eigenstate basis

$$|B_L\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad |B_H\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

where “L” denotes “light” and “H” denotes “heavy,” with the relation between mass and flavor eigenstates given by

$$|B_L\rangle = p|B^0\rangle + q|\overline{B}^0\rangle, \quad |B_H\rangle = p|B^0\rangle - q|\overline{B}^0\rangle, \quad |p|^2 + |q|^2 = 1 .$$

In a standard convention \[22\] one has \( q/p = e^{-2i\beta} \), where \( \beta = \text{Arg}(-V_{cb}^*V_{cd}/V_{tb}^*V_{td}) \), and \( V_{ij} \) are elements of the Cabibbo-Kobayashi-Maskawa matrix specifying the charge-changing weak couplings of quarks. We then choose \( p = e^{i\beta}/\sqrt{2}, \quad q = e^{-i\beta}/\sqrt{2} \).

We shall neglect width differences in the following discussion. They are expected to be extremely small for nonstrange neutral \( B \)'s, although they may be as large as 10\% for \( B_s \) \[23, 24\]. In the mass eigenstate basis (4) the time evolution operator is

\[
\langle M'|T|M\rangle = e^{-iM_{D\tau}} + e^{-i\Gamma/2\text{diag}(e^{-i\Delta m_L}, e^{-i\Delta m_H})} = e^{-i\Gamma/2}\left.e^{i\sigma_3\Delta m/2}\right|,
\]

where \( \vec{m} \equiv (m_H + m_L)/2 \) and \( \Delta m \equiv m_H - m_L \). Transforming to the flavor basis (1), we find

\[
\langle F'|T|F\rangle = e^{-i\Gamma/2\text{diag}(\cos\Delta m/2, \sin\Delta m/2)}\left.i\text{e}^{2i\beta\sin\Delta m/2}\right| .
\]

It is most convenient to express the density matrix in the mass basis as well:

\[
\langle M'|\rho|M\rangle = \sum_{F,F'} \langle M'|F'\rangle \langle F'|\rho|F\rangle \langle F|M\rangle .
\]

We shall denote the density matrix in the mass basis by

\[
\rho' = \frac{1}{2}[1 + Q' \cdot \sigma];
\]

the vector \( Q' \) in the mass basis is related to the vector \( Q \) in the flavor basis by

\[
Q'_1 = Q_3, \quad Q'_2 = -(Q_1 \sin 2\beta + Q_2 \cos 2\beta), \quad Q'_3 = Q_1 \cos 2\beta - Q_2 \sin 2\beta.
\]

Since states which are pure \( B^0 \) or pure \( \overline{B}^0 \) correspond to \( Q_3 = \pm 1, \quad Q_1 = Q_2 = 0 \), their transformed density matrices are \( \rho'_f = (1/2)(1 \pm \sigma_1) \), respectively, since then \( Q'_1 = \pm 1, \quad Q'_2 = Q'_3 = 0 \). More generally, an incoherent state with \( Q_3 = 2P_{B^0} - 1, \quad Q_1 = Q_2 = 0 \) corresponds to \( \rho'_f = (1/2)[1 + (2P_{B^0} - 1)\sigma_1] \).

The transition probability can now be written in terms of traces as

\[
I(f) = \text{Tr}(\rho'_f e^{iM_{D\tau}}\rho'_f e^{-iM_{D\tau}}).
\]

For flavor eigenstates \( f \) corresponding to \( B^0 \) or \( \overline{B}^0 \), let us take as examples \( J/\psi K^{*0} \rightarrow J/\psi K^+\pi^- \) and \( J/\psi K^{*0} \rightarrow J/\psi K^-\pi^+ \), respectively. In the present convention both
Table I: Fractions \( f(H) \) of hadrons produced in \( b \) quark fragmentation.

| Hadron     | CDF (a)      | LEP          |
|------------|--------------|--------------|
| \( B^0 \)  | 0.375 ± 0.023 | 0.40 ± 0.01  |
| \( B^- \)  | 0.375 ± 0.023 | 0.40 ± 0.01  |
| \( B_s \)  | 0.160 ± 0.044 | 0.097 ± 0.012|
| Baryons    | 0.090 ± 0.029 | 0.104 ± 0.017|

(a) Assuming equal fractions of charged and neutral nonstrange \( B \) mesons.

decay amplitudes are equal to the same constant \( A \), since they involve the quark subprocesses \( \bar{b} \to \bar{c}c\bar{s} \) and \( b \to c\bar{c}s \), respectively. Then we find

\[
I \left( \frac{B^0}{\overline{B^0}} \right) \propto e^{-\Gamma t} [1 \pm (Q'_1 \cos \Delta m t + Q'_2 \sin \Delta m t)] \quad .
\] (14)

The sign in front of the \( Q'_2 \) term was incorrectly stated in Refs. [7] and [8].

As noted in Refs. [7] and [8], the component \( Q'_3 \) does not appear in these expressions. We shall return to the question of its determination in Section VI.

III Mixed state

A mixed state of \( B^0 \) and \( \overline{B^0} \) is one in which there are no amplitude correlations between the \( B^0 \) and \( \overline{B^0} \). Such a state will arise, in general, when a \( b\bar{b} \) pair is produced with high enough effective mass that the \( b \) and \( \bar{b} \) fragment independently. In this case we can consider a tagging method to indicate with probability \( P_r \) the right-sign neutral \( B \) and with probability \( P_w = 1 - P_r \) the wrong-sign neutral \( B \). The dilution factor \( D \) is

\[ D = P_r - P_w = 2P_r - 1. \]

Dilution can occur in various ways, depending on the tagging method. In opposite-side tagging at high \( M(b\bar{b}) \), the opposite-side quark may fragment into a charged or neutral nonstrange \( B \) meson, a strange \( B \) meson, or a beauty baryon. These fractions have been measured at CDF [25] and LEP [3] and are summarized in Table I.

The probability that a \( B^0 \) is detected as a \( B^0 \) is \( x_d^2/[2(1 + x_d^2)] \approx 0.18 \), where \( x_d \equiv \Delta m_d/\Gamma_d = (0.487 \pm 0.014 \text{ ps}^{-1})(1.56 \pm 0.04 \text{ ps}) = 0.76 \pm 0.03 \) [3, 26]. The corresponding probability of mis-detecting the flavor of a \( B_s \) is very close to 1/2. Assuming that the other flavors are detected with unit probability, the CDF results imply \( P_r \approx 0.85 \) and \( D = 0.70 \) while the LEP results imply \( P_r \approx 0.88 \) and \( D = 0.76 \). In practice many other factors of course contribute to the dilution of a tagging method.

Given a tag which should indicate the presence of a \( B^0 \) at time of production with tagging probability \( P_{B^0} \), the corresponding density matrix elements in the mass basis are \( Q'_1 = 2P_{B^0} - 1 \), \( Q'_2 = Q'_3 = 0 \). The time-dependence of the flavor-specific final state \( f \) arising from either a \( B^0 \) or \( \overline{B^0} \) decay is then given by

\[
I \left( \frac{B^0}{\overline{B^0}} \right) \propto e^{-\Gamma t} [1 \pm (2P_{B^0} - 1) \cos \Delta m t] \quad ,
\] (15)
without any sin $\Delta m t$ term. The quantity $P_{\ell 0}$ is usually determined empirically in a fit which also yields $\Delta m$.

**IV  Full coherence**

We now consider the case of fully coherent states of $B^0$ and $\bar{B}^0$ produced in states of definite charge conjugation eigenvalue. We denote a $C$ eigenstate of $B^0$ and $\bar{B}^0$ by

$$\Psi_C = \frac{1}{\sqrt{2}} \left[ B^0(\hat{p})\bar{B}^0(-\hat{p}) + \eta_C B^0(\hat{p})\bar{B}^0(-\hat{p}) \right] ,$$

where $\eta_C = \pm 1$ is the eigenvalue of the $C$ operator, and $\hat{p}$ and $-\hat{p}$ are unit vectors denoting the direction of the particles in their center-of-mass. The case of $\Upsilon(4S) \rightarrow B^0\bar{B}^0$ corresponds to $\eta_C = -1$.

The states $\Psi_C$ can be written in terms of mass eigenstates as

$$\Psi_C(\eta_C = -1) = \frac{1}{\sqrt{2}} \left[ B_H(\hat{p})B_L(-\hat{p}) - B_L(\hat{p})B_H(-\hat{p}) \right] ,$$

$$\Psi_C(\eta_C = +1) = \frac{1}{\sqrt{2}} \left[ B_L(\hat{p})B_L(-\hat{p}) - B_H(\hat{p})B_H(-\hat{p}) \right] .$$

These expressions allow us to write the elements $Q'_i$ of the density matrix in the mass-eigenstate representation and thereby to calculate the correlations between particles traveling along $\hat{p}$ (decaying at proper time $t$) and those traveling along $-\hat{p}$ (decaying at proper time $\bar{t}$). We shall derive the results using both the one-particle formalism given in Section II and a two-particle formalism more suitable for joint distributions.

**A. One-particle description**

We consider for definiteness the case in which a flavor tag ($B^0$ or $\bar{B}^0$) is applied at a time $t$. Recalling that

$$\langle B^0|B_L(-\hat{p}) \rangle = \langle B^0|B_H(-\hat{p}) \rangle = \frac{1}{\sqrt{2}} e^{i\beta} , \quad \langle \bar{B}^0|B_L(-\hat{p}) \rangle = -\langle \bar{B}^0|B_H(-\hat{p}) \rangle = \frac{1}{\sqrt{2}} e^{-i\beta} ,$$

we can calculate the dependence on the tagging particle’s decay time $\bar{t}$ of the production amplitudes of the states $|B_{L,H}(\hat{p})\rangle$. For example, the state $|B_L(\hat{p})\rangle$ appears in (17) with coefficient $-|B_H(-\hat{p})|/\sqrt{2}$, so it depends on $\bar{t}$ with coefficient $-(1/2)e^{i\beta}e^{-\Gamma\bar{t}/2}e^{-imH\bar{t}}$ for a $B^0$ tag and $(1/2)e^{-i\beta}e^{-\Gamma\bar{t}/2}e^{-imH\bar{t}}$ for a $\bar{B}^0$ tag. For states with both values of $\eta_C$ we then find

$$\eta_C = -1, B^0 \text{ tag : } \begin{bmatrix} |B_L(\hat{p})\rangle \\ |B_H(\hat{p})\rangle \end{bmatrix} = \frac{1}{2} e^{i\beta} e^{-\Gamma\bar{t}/2} \begin{bmatrix} -e^{-imH\bar{t}} \\ e^{-imH\bar{t}} \end{bmatrix} ,$$

$$\eta_C = -1, \bar{B}^0 \text{ tag : } \begin{bmatrix} |B_L(\hat{p})\rangle \\ |B_H(\hat{p})\rangle \end{bmatrix} = \frac{1}{2} e^{-i\beta} e^{-\Gamma\bar{t}/2} \begin{bmatrix} e^{-imH\bar{t}} \\ e^{-imH\bar{t}} \end{bmatrix} .$$
Table II: Density matrix elements corresponding to correlated $B^0\bar{B}^0$ production in states of definite charge-conjugation eigenvalue $\eta_C$.

| $\eta_C$ | Tag ($\hat{p}$) | $Q_1'$ | $Q_2'$ |
|----------|-----------------|--------|--------|
| $-1$     | $B^0$           | $-\cos \Delta m t$ | $-\sin \Delta m t$ |
| $\bar{B}^0$ | $\cos \Delta \bar{m} t$ | $\sin \Delta \bar{m} t$ |
| $+1$     | $B^0$           | $-\cos \Delta m \bar{t}$ | $\sin \Delta m \bar{t}$ |
| $\bar{B}^0$ | $\cos \Delta \bar{m} \bar{t}$ | $-\sin \Delta m \bar{t}$ |

\[
\eta_C = +1, B^0 \text{ tag : } \begin{bmatrix} |B_L(\hat{p})\rangle \\ |B_H(\hat{p})\rangle \end{bmatrix} = \frac{1}{2} e^{i\beta} e^{-\Gamma \bar{t}/2} \begin{bmatrix} e^{-im_L \bar{t}} \\ -e^{-im_H \bar{t}} \end{bmatrix}, \tag{22}
\]

\[
\eta_C = +1, \bar{B}^0 \text{ tag : } \begin{bmatrix} |B_L(\hat{p})\rangle \\ |B_H(\bar{p})\rangle \end{bmatrix} = \frac{1}{2} e^{-i\beta} e^{-\Gamma \bar{t}/2} \begin{bmatrix} e^{-im_L \bar{t}} \\ e^{-im_H \bar{t}} \end{bmatrix}. \tag{23}
\]

Translating these pure states into normalized density matrices with unit trace, we find the results summarized in Table II. The component $Q_3'$ is zero.

The density matrix elements in Table II can now be combined with the expression (14) to give joint rates for production of states with direction $\hat{p}$ decaying at time $t$ and direction $-\hat{p}$ decaying at time $\bar{t}$ [27]. We find:

\[
I[B^0(t), B^0(\bar{t})] = I[\bar{B}^0(t), \bar{B}^0(\bar{t})] = e^{-\Gamma(t+\bar{t})}[1 - \cos \Delta m(t + \eta_C \bar{t})], \tag{24}
\]

\[
I[B^0(t), \bar{B}^0(\bar{t})] = I[\bar{B}^0(t), B^0(\bar{t})] = e^{-\Gamma(t+\bar{t})}[1 + \cos \Delta m(t + \eta_C \bar{t})]. \tag{25}
\]

The above expressions are consistent with those in the literature (e.g., [28]) and make physical sense. Their dependence on $t + \eta_C \bar{t}$ is mandated by Bose statistics. When $\eta_C = -1$ and $t = \bar{t}$, one never sees the decay products of neutral $B$ mesons of the same flavor. This is also the case for $t = \bar{t} = 0$ when $\eta_C = +1$. When tags of each flavor are combined, the oscillatory terms cancel one another and one is left with a pure exponential $\sim e^{-\Gamma(t+\bar{t})}$.

**B. Two-particle description**

For entangled states such as described by $\Psi_C$ in Eqs. (17) and (18), one can use a direct-product notation [29]. Our convention will be such that the first state in the direct product refers to the particle with direction $\hat{p}$, while the second refers to that with $-\hat{p}$. Typical direct products are then

\[
\begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}. \tag{26}
\]

A spin-singlet state of two spin-1/2 particles is then represented by the four-component
Writing the states and their time-evolution as in the previous subsection, we then

\[ |S = 0\rangle = \frac{1}{\sqrt{2}} (\uparrow \downarrow - \downarrow \uparrow) = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} \]  

(27)

and by the density matrix

\[
\rho(S = 0) = \frac{1}{2} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} .
\]  

(28)

In terms of the direct product representation, this can be written as

\[
\rho(S = 0) = \frac{1}{4} I \otimes I - \frac{1}{4} \sigma_1 \otimes \sigma_1 - \frac{1}{4} \sigma_2 \otimes \sigma_2 - \frac{1}{4} \sigma_3 \otimes \sigma_3 ,
\]  

(29)

where \( I \) is the \( 2 \times 2 \) identity matrix. We have used the identities

\[
\sigma_1 \otimes \sigma_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} , \quad \sigma_2 \otimes \sigma_2 = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} ,
\]

\[
\sigma_3 \otimes \sigma_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} , \quad I \otimes I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} .
\]  

(30)

Other useful identities are

\[
\sigma_1 \otimes \sigma_2 = \begin{bmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{bmatrix} , \quad \sigma_2 \otimes \sigma_1 = \begin{bmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{bmatrix} .
\]  

(31)

Writing the states and their time-evolution as in the previous subsection, we then find the corresponding density matrices

\[
\rho(\eta_C = -1) = \frac{1}{2} e^{-\Gamma(t+i)} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -e^{i\Delta m(t-i)} & 0 \\ 0 & -e^{i\Delta m(t-i)} & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \frac{1}{4} e^{-\Gamma(t+i)} [I \otimes I - \sigma_3 \otimes \sigma_3 - (\sigma_1 \otimes \sigma_1 + \sigma_2 \otimes \sigma_2) \cos \Delta m(t - i) - (\sigma_1 \otimes \sigma_2 - \sigma_2 \otimes \sigma_1) \sin \Delta m(t - i)] ,
\]  

(32)

\[
\rho(\eta_C = +1) = \frac{1}{2} e^{-\Gamma(t+i)} \begin{bmatrix} 1 & 0 & 0 & -e^{i\Delta m(t+i)} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -e^{-i\Delta m(t+i)} & 0 & 0 & 1 \end{bmatrix} = \frac{1}{4} e^{-\Gamma(t+i)} [I \otimes I + \sigma_3 \otimes \sigma_3 - (\sigma_1 \otimes \sigma_1 - \sigma_2 \otimes \sigma_2) \cos \Delta m(t + i) + (\sigma_1 \otimes \sigma_2 + \sigma_2 \otimes \sigma_1) \sin \Delta m(t + i)] .
\]  

(33)

These results may be used to derive such expressions as (24) and (25) in an alternate way.
V Intermediate cases

The density matrices for \( \eta_C = +1 \) and \( \eta_C = -1 \) can be added to one another. If the probability of states with \( \eta_C = \pm 1 \) is denoted by \( P_\pm \) with \( P_+ + P_- = 1 \), the resulting elements for \( B^0(\bar{p}) \) production with a \( B^0(\bar{\bar{p}}) \) tag are

\[
B^0(\bar{p}), B^0(\bar{\bar{p}}): \quad Q'_1 = -\cos \Delta m \bar{t} , \quad Q'_2 = (P_+ - P_-) \sin \Delta m \bar{t} , \quad (34)
\]

with the signs of \( Q'_{1,2} \) changed for a \( B^0(\bar{\bar{p}}) \) tag. The joint probabilities for production of (opposite,same) flavors of neutral \( B \) mesons at times \( t \) and \( \bar{t} \) are then

\[
I \begin{pmatrix} \text{Opp} \\ \text{Same} \end{pmatrix} (t, \bar{t}) = e^{-\Gamma(t+\bar{t})} \begin{bmatrix} 1 & \pm 1 + \frac{1}{1 + x^2} \cos \Delta m t & \pm (P_- - P_+) \frac{x}{1 + x^2} \sin \Delta m t \end{bmatrix} , \quad (35)
\]

Equation (35) can be integrated with respect to time, with the result

\[
\int_0^\infty dt I \begin{pmatrix} \text{Opp} \\ \text{Same} \end{pmatrix} (t, \bar{t}) = \frac{1}{\Gamma} e^{-\Gamma t} \begin{bmatrix} 1 & \pm \frac{1}{1 + x^2} \cos \Delta m \bar{t} & \pm (P_- - P_+) \frac{x}{1 + x^2} \sin \Delta m \bar{t} \end{bmatrix} , \quad (36)
\]

where \( x \equiv \Delta m / \Gamma \). As long as \( P_- \neq P_+ \), the \( \sin \Delta m t \) term will be present.

The two-particle description can also be applied to intermediate cases. For a state which is a mixture of \( \eta_C = +1 \) with probability \( P_+ \) and \( \eta_C = -1 \) with probability \( P_- = 1 - P_+ \), the density matrix is

\[
\rho = \frac{1}{2} e^{-\Gamma(t+\bar{t})} \begin{bmatrix} P_+ & 0 & 0 & -P_+ e^{i\Delta m (t+\bar{t})} \\ 0 & P_- & -P_- e^{-i\Delta m (t-\bar{t})} & 0 \\ -P_+ e^{-i\Delta m (t+\bar{t})} & 0 & P_- & 0 \\ 0 & -P_- e^{i\Delta m (t-\bar{t})} & 0 & P_+ \end{bmatrix}
\]

\[
= \frac{1}{4} e^{-\Gamma(t+\bar{t})} \left\{ I \otimes I + (P_+ - P_-) \sigma_3 \otimes \sigma_3 - \begin{bmatrix} 0 & e^{i\Delta m t} \\ e^{-i\Delta m t} & 0 \end{bmatrix} \otimes \begin{bmatrix} 0 & e^{i\Delta m \bar{t}} \\ e^{-i\Delta m \bar{t}} & 0 \end{bmatrix} \right\} , \quad (37)
\]

This last equation says, in particular, that in order to specify a one-particle state in which the matrix element \( Q'_3 \) is non-zero, one must not only have \( P_+ \neq P_- \), but the “tagging” particle (with decay time \( \bar{t} \)) must also correspond to non-zero \( Q'_3 \).

These expressions hold for both nonstrange and strange neutral \( B \) mesons. They must be modified to take account of dilution effects such as those discussed in Section III. However, such effects should reduce the coefficients of the \( \cos \Delta m t \) and \( \sin \Delta m t \) terms by a common factor.

Since \( x \) is very large for \( B_s \) mesons, the presence of the \( \sin \Delta m t \) term may be difficult to demonstrate for them, unless one resolves the dependence on the “tagging” time \( \bar{t} \) and does not integrate with respect to it.
VI Full specification of the density matrix

As pointed out in Refs. [7, 8], it is necessary to observe decays to CP eigenstates and not just to flavor eigenstates in order to fully specify the density matrix, since the element $Q'_3$ does not appear in any of the previous expressions for rates. We consider decays to $J/\psi K_S$ and $J/\psi K_L$.

Taking account of the negative CP of $J/\psi K_S$ and positive CP of $J/\psi K_L$, the decay amplitudes of interest are

$$
\langle J/\psi K_S | B^0 \rangle = -\langle J/\psi K_S | \bar{B}^0 \rangle = \langle J/\psi K_L | B^0 \rangle = \langle J/\psi K_L | \bar{B}^0 \rangle = A'/\sqrt{2} .
$$

Then the density matrices for each final state in the flavor basis are

$$
\rho_{J/\psi K_S} = \frac{1}{2} |A'|^2 \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} , \quad \rho_{J/\psi K_L} = \frac{1}{2} |A'|^2 \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} ,
$$

while in the mass-eigenstate basis they are

$$
\rho'_{J/\psi K_S} = \frac{1}{2} |A'|^2 \begin{pmatrix} 1 - \cos 2\beta & -i \sin 2\beta \\ i \sin 2\beta & 1 + \cos 2\beta \end{pmatrix} ,
$$

$$
\rho'_{J/\psi K_L} = \frac{1}{2} |A'|^2 \begin{pmatrix} 1 + \cos 2\beta & i \sin 2\beta \\ -i \sin 2\beta & 1 - \cos 2\beta \end{pmatrix} .
$$

We then recover the results of Refs. [7, 8], aside from a sign in the $Q'_2$ term which we correct here:

$$
I \left( J/\psi K_L \atop K_S \right) \sim e^{-\Gamma t} \left\{ 1 \pm [Q'_3 \cos 2\beta + (Q'_1 \sin \Delta m t - Q'_2 \cos \Delta m t) \sin 2\beta] \right\} ,
$$

$$
\bar{I} \left( J/\psi K_L \atop K_S \right) \sim e^{-\Gamma \bar{t}} \left\{ 1 \pm [Q'_3 \cos 2\beta - (Q'_1 \sin \Delta m t - Q'_2 \cos \Delta m t) \sin 2\beta] \right\} ,
$$

where $I$ refers to a rate tagged with an opposite-side $B$, while $\bar{I}$ refers to a rate tagged with an opposite-side $\bar{B}$.

The determinations of $Q'_3$ and $\cos 2\beta$ are interrelated. Information on the sign of $\cos 2\beta$ would be useful in resolving the discrete ambiguity associated with extracting the value of $\beta$ from that of $\sin 2\beta$ [30]. However, in eigenstates of $C$ with $\eta_C = \pm 1$, the two-particle density matrix results indicate that the contributions from $\sigma_3$ for the particles decaying at times $t$ and $\bar{t}$ are correlated. Thus, in order to prepare a state with $Q'_3 \neq 0$ decaying at time $t$ it appears that one must tag with a CP eigenstate decaying at time $\bar{t}$. For example, in $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B^0 \bar{B}^0 \rightarrow (J/\psi K_{S,L})(\bar{p})(J/\psi K_{S,L})(-\bar{p})$, the effects of $Q'_3 \neq 0$ will always involve the term $\cos^2 2\beta$, so information on the sign of $\cos 2\beta$ is lost.

An explicit calculation with the two-particle density matrix leads to the following time-dependent rates for a mixture of $C$ eigenstates with probabilities $P_+$ and $P_- = 1 - P_+$:

$$
\frac{d^2 \Gamma}{dt \, d\bar{t}} [J/\psi K_{S,L}(t) J/\psi K_{S,L}(\bar{t})] \propto \left\{ 1 + (P_+ - P_-) \cos^2 2\beta \right\} .
$$
\[-\sin^2 2\beta [\sin \Delta m t \sin \Delta m \bar{t} + (P_- - P_+) \cos \Delta m t \Delta m \bar{t}] \] , \hspace{1cm} (42)

\[
\frac{d^2 \Gamma}{dt \, d\bar{t}} [(J/\psi K_{S,L}(t) J/\psi K_{L,S}(\bar{t})] \propto \{1 - (P_+ - P_-) \cos^2 2\beta \\
+ \sin^2 2\beta [\sin \Delta m t \sin \Delta m \bar{t} + (P_- - P_+) \cos \Delta m t \Delta m \bar{t}] \} . \hspace{1cm} (43)
\]

As noted, $\beta$ appears only through $\sin^2 2\beta$ and $\cos^2 2\beta = 1 - \sin^2 2\beta$.

In principle the reaction $e^+ e^- \rightarrow \Upsilon(4S) \rightarrow B^0 \bar{B}^0 \rightarrow (J/\psi K_{S,L})(\hat{p})(\pi^+ \pi^-)(-\hat{p})$ can provide additional information. The (normalized) density matrices for $\pi^+ \pi^-$ production are

\[
\rho_{\pi^+ \pi^-} = \frac{1}{2} \left[ \begin{array}{cc} 1 & e^{-i\gamma} \\ e^{i\gamma} & 1 \end{array} \right] \hspace{1cm} (44)
\]

in the flavor basis and

\[
\rho'_{\pi^+ \pi^-} = \frac{1}{2} \left[ \begin{array}{cc} 1 + \cos 2\alpha & -i \sin 2\alpha \\ i \sin 2\alpha & 1 - \cos 2\alpha \end{array} \right] \hspace{1cm} (45)
\]

in the mass-eigenstate basis. (We have neglected the effect of penguin amplitudes here.) The use of Eq. (37) then leads to expressions involving the combinations $\cos 2\beta \cos 2\alpha$ and $\sin 2\beta \sin 2\alpha$.

In a state of definite charge-conjugation eigenvalue we find, by substituting the values in Table II and noting that $Q_3' = 0$ for such a state, that

\[
I \left( \begin{array}{c} J/\psi \\ K_L \\ K_S \end{array} \right) \propto e^{-\Gamma(t+\bar{t})} \left[ 1 \mp \sin 2\beta \sin \Delta m (t + \eta_C \bar{t}) \right] \hspace{1cm} (46)
\]

\[
\bar{I} \left( \begin{array}{c} J/\psi \\ K_L \\ K_S \end{array} \right) \propto e^{-\Gamma(t+\bar{t})} \left[ 1 \pm \sin 2\beta \sin \Delta m (t + \eta_C \bar{t}) \right] \hspace{1cm} (47)
\]

The results for $\eta_C = -1$ agree with those in Ref. [28], while for $\eta_C = +1$ the sign of the $\sin 2\beta \sin \Delta m (t + \bar{t})$ term is reversed. Again, although we have used the one-particle expressions based on the vector $Q'$, these results can also be derived using the two-particle density matrices (32) and (33).

**VII Coherence expected in present and planned experiments**

The specific cases we have discussed so far range from fully coherent $B^0 \bar{B}^0$ production at the $\Upsilon(4S)$ to uncorrelated production at high effective $b\bar{b}$ masses (as in $Z^0 \rightarrow b\bar{b}$). A qualitative estimate of the degree of coherence expected between $B^0$ and $\bar{B}^0$ may be obtained by examining their relative orbital angular momenta. Since for a meson-antimeson pair with relative orbital angular momentum $L$ the charge-conjugation eigenvalue is $\eta_C = (-1)^L$, the degree of coherence is expected to decrease as the accessible values of $L$ increase.

Suppose, for example, that the population of orbital angular momentum levels of $B^0 \bar{B}^0$ is dictated by their statistical weights $2L + 1$ up to a maximum $L = L_{\text{max}}$. If
$L_{\text{max}} = 0$ the probability $P_+$ of $\eta_C = 1$ is 1, while the probability $P_-$ of $\eta_C = -1$ is 0. If $L_{\text{max}} = 1$ then $P_+ = 1/4$, $P_- = 3/4$. The general expressions are

\[
L_{\text{max}} \ \text{even} : \quad P_+ = \frac{L_{\text{max}} + 2}{2(L_{\text{max}} + 1)}, \quad P_- = \frac{L_{\text{max}}}{2(L_{\text{max}} + 1)},
\]

\[
L_{\text{max}} \ \text{odd} : \quad P_+ = \frac{L_{\text{max}}}{2(L_{\text{max}} + 1)}, \quad P_- = \frac{L_{\text{max}} + 2}{2(L_{\text{max}} + 1)}.
\]

Then one finds $P_+ - P_- = (-1)^{L_{\text{max}}}/(L_{\text{max}} + 1)$, and the magnitude of the coefficient of the sin $\Delta m t$ term in the time-dependence of a flavor eigenstate with a flavor tag decreases as $1/L_{\text{max}}$.

A semiclassical argument can be used to estimate $L_{\text{max}}$. Imagine a $b\bar{b}$ pair with squared c.m. energy $s$ to fragment into a pair of $B$ mesons. The fragmentation process is limited to impact parameters $b_0 \leq 1 \text{ fm} \simeq 5 \text{ GeV}^{-1}$. Thus

$$L_{\text{max}} = k b_0, \quad k \equiv \sqrt{\left(s/4\right) - m_b^2}.$$  \hspace{1cm} (50)

We now discuss the specific experimental cases mentioned in the Introduction, in decreasing order of likelihood of $B^0\bar{B}^0$ coherence.

1. **Production at the $\Upsilon(4S)$** leads to a $B^0\bar{B}^0$ pair in a state with $L = 1$, $\eta_C = -1$. The full-coherence arguments of Section III apply.

2. **Production through the process $e^+e^- \to B^0\bar{B}^{\prime 0}$ c.c.** gives rise to a $B^0\bar{B}^0$ pair with $\eta_C = +1$, since $B^{\prime 0}$ decays entirely to $\gamma B^0$.

3. **Contamination of $e^+e^- \to B^0\bar{B}^{\prime 0}$ c.c.** by $e^+e^- \to B^0\bar{B}^0$ is likely. At the threshold for (vector + pseudoscalar) meson production, pseudoscalar meson pair production is also kinematically allowed, leading to some admixture of the $\eta_C = -1$ state. The relative fractions of the two processes are not well known but can be measured in principle. One can ensure against the $\eta_C = -1$ state by detecting the 46 MeV photon in $B^{\prime 0}$ decay.

4. **Forward hadronic production of $B^0\bar{B}^0$** will lead to a pair with effective mass typically no more that a few times the threshold mass of $2m_b$. The subprocesses $q\bar{q} \to b\bar{b}$ and $gg \to b\bar{b}$ both favor low effective $b\bar{b}$ masses, while the gluon-splitting process $g^* \to b\bar{b}$ will favor even lower effective masses. The corresponding value of $k$ will then be of order $m_b$, leading to $L_{\text{max}} = \mathcal{O}[m_b \cdot (1 \text{ fm})] \simeq 25$. Thus one might expect magnitudes of $P_+ - P_-$ of at most a few percent.

5. **Central hadronic production** may lead to somewhat higher effective $b\bar{b}$ masses, especially if “opposite-side” tagging utilizes $B$’s produced in the opposite hemisphere of the detector. The probability of $B^0\bar{B}^0$ coherence is thus likely to be less than in forward geometries.

6. **Production in $Z \to b\bar{b}$** is expected to lead to very little $B^0\bar{B}^0$ coherence, since $k \simeq M_Z/2$ and $L_{\text{max}}$ consequently exceeds 200.

The best prospect for studying the coherence effects we have mentioned here thus seems to be $e^+e^-$ collisions not far above the $\Upsilon(4S)$, where the $\eta_C = +1$ and $\eta_C = -1$ states are not necessarily equally populated. Ultimately, however, the question is an experimental one, and such effects can be studied at any energy and in any configuration by searching for the sin $\Delta m t$ term.
VIII Conclusions

We have discussed the possibility of coherence of neutral $B$ meson pairs, using a density-matrix approach which describes situations ranging from fully correlated pairs to mixed (uncorrelated) states. The density matrix is parametrized by a “polarization” vector $Q'$ describing a direction of “quasi-spin.” Usual experiments determine only one component, $Q'_1$, of this vector, relating it to the dilution factor in flavor tagging. It gives rise to a characteristic modulation of exponential decay by a $\cos \Delta mt$ term. In general there can appear a term proportional to $\sin \Delta mt$ as well, which has not been taken into account in previous studies. This term arises from the component $Q'_2$ of the quasi-spin polarization vector, and is one signal of coherence. The component $Q'_3$ affects decays to CP eigenstates, and can be searched for by studying such final states as $J/\psi K_S$ and $J/\psi K_L$. However, its investigation probably involves correlations between decays to pairs of CP eigenstates, and thus may require the production of a considerable number of $B$ mesons.

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