A Statistical Method for Non-Linear Quantization in Lossy JPEG2000 Compression

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Abstract—The paper presents a non-linear quantization method for detail components in the JPEG2000 standard. The quantization step sizes are determined by actual statistics of the wavelet coefficients. Mean and standard deviation are the two statistical parameters used to obtain the step sizes. Moreover, weighted mean of the coefficients lying within the step size is chosen as quantized value - contrary to the uniform quantizer. The empirical results are compared using mean squared error of uniformly quantized and non-linearly quantized wavelet coefficients w.r.t. original wavelet coefficients. Through empirical results, it has been concluded that for low bit rate compression, the quantization error introduced by uniform quantizer is higher than that of non-linear quantizer, and thus suggesting the use of non-linear quantizer for low bit rate lossy compression.

Index Terms—Mean and standard deviation, quantization, discrete wavelet transform, image compression, image processing, JPEG2000 standard.

I. INTRODUCTION

JPEG2000 standard (hereinafter: standard) is a state-of-art compression for diverse applications and platforms embedded into a single system and a single compressed bit stream [1]–[3]. Apart from providing higher compression efficiency and improved quality of an image compared to baseline JPEG, additional features like multiresolution representation, embedded coding, signal to noise ratio scalability, region of interest coding and error robustness are included in the standard [2]–[4].

One major reason for better performance of standard over baseline JPEG and other image compression formats is the introduction of discrete wavelet transform (DWT) into the standard. The perfect reconstruction of DWT has enabled lossy and lossless compression into a unified system. Moreover, DWT possesses multiresolution capability which naturally allows this property in the standard. Further, DWT provides high energy compaction resulting in better compression ratio. Also, DWT removes blocking affects due to higher decorrelation of the image because it is applied to a complete image [4].

Another important factor that results in improved performance is the embedded coding of DWT coefficients, which is accomplished at present by an uniform quantizer [4]–[6]. Fig. 1 displays the fundamental block diagram of the standard [4], [5], [7]. introduced perceptual criteria before quantizing the coefficients in the uniform quantizer to incorporate the human visual system.

However, there are some major drawbacks of using the uniform quantizer. First, except for the case where coefficient values lie nearby zero, the uniform quantizer treats the information provided by each fixed step size in the image as equally important. This assumption is flawed because the detail components represent the high frequency coefficients in the horizontal, vertical and diagonal directions of the image. This implies that the coefficients lying farther from the origin are more important. These high frequency coefficients capture the edge information in the images, hence depict the overall structure of the image and if quantized using the uniform quantizer then poses the problem of enhanced structural deformation. Thus, it is important to minimize quantization error as quantization step size approaches the end of the histogram of the detail component. This can be achieved by having variable step sizes for quantization. Second, the uniform quantizer has a deadzone region where all the coefficient values near zero, are assigned the quantized value zero. Although these coefficients are the least important, carrying minimal information, they do provide some information about the majority of coefficients, which needs to be represented in an adequate manner. The adequate manner can be used to assign a quantized value within the range of quantized step sizes, if the step sizes are of varying length. In addition to the above mentioned points, higher compression at the same desired quality can be obtained through a better bit representation of quantized values (discussed in section V).

This paper describes a non-linear quantizer to overcome the problems posed by the uniform quantizer. The quantizer uses variable step sizes. The range of the step sizes reduces as they approach the end of the histogram of each detail component, as they approach the end of the histogram of each detail.
component. Moreover, the quantized values assigned are the weighted mean of all the coefficients present in the respective step size. This would result in minimal error based on the real statistics of the detail component for a given image. Furthermore, it reduces the number of quantization step sizes. The proposed non-linear quantization method in the detail components for lossy compression in the standard is the advancement and combination of three papers. The non-linear quantizer described in this paper was used in [8] to segment an image in space domain for object separation. Subsequently, [9] compared the efficacy of different wavelets to achieve effective segmentation in the wavelet domain using the same quantizer. Finally, [10] proposed the concept of non-linear quantizer in the standard by showing the high Peak Signal to Noise Ratio (PSNR) of the reconstructed images. However, the scope of [10] was limited to the concept of non-linear quantization in the standard. It lacked the implementation of the algorithm in the standard and its subsequent comparison with the uniform quantizer applied in the detail components. This work builds on [10] and shows that the proposed non-linear quantizer is better suited for low bit rate lossy image compression applications. The novelty of this work is the comprehensive implementation of non-linear quantizer in the detail components of the standard based on the actual statistics of the coefficients. In addition, the paper also proposes the usage of a table to encode quantized values to further increase the compression.

The paper is organized as follows. Section II describes the uniform quantizer currently used in the standard, followed by section III which illustrates the non-linear quantizer used in the paper. In section IV, the experimental results show the comparison of the uniform and non-linear quantizer. Subsequently, the discussion of experimental results is carried out in section V. Lastly, section VI concludes the paper with future work.

II. QUANTIZATION IN THE STANDARD

The part I of the standard uses uniform scalar quantizer having deadzone around the origin. The quantization step size is same throughout a subband, but step sizes vary among the subbands. The step size reduces towards the subbands that are representing higher decomposition levels [5]. Quantized values for the wavelet coefficients are calculated as,

\[ q_b(i, j) = \text{sign}(y_b(i, j)) \left\lfloor \frac{y_b(i, j)}{\Delta_b} \right\rfloor \]

where \( q_b(i, j) \) is the quantized value, \( y_b(i, j) \) is the input DWT coefficient, and \( \Delta \) is the quantization step size. The subscript \( b \) represents the subband. For deadzone region, the quantization size is \( 2\Delta_b \).

In Part II of the standard [5], the deadzone region is devised to be flexible i.e., the deadzone region can be of variable length. The rest of the interval have fixed width \( \Delta \). Similar to Part I, the formula for calculating quantized values is given by,

\[ q_b(i, j) = \text{sign}(y_b(i, j)) \left\lfloor \frac{y_b(i, j) + k\Delta}{\Delta_b} \right\rfloor \]

where \( k \) is the parameter which varies the deadzone step size. The quantization step size of the deadzone is \( 2(1-k)\Delta \).

In scalar uniform quantizer with fixed deadzone width as well as variable deadzone width, the number of bits required to represent all the quantized step sizes is \( B = \max \{ \log_2 q(i, j) \} \). For detailed information about quantization in the standard, refer [5].

III. NON-LINEAR QUANTIZER

As mentioned in the introduction, the non-linear quantizer decides the varying step sizes with the help of mean and standard deviation. The formulae for obtaining the thresholds or boundaries of step sizes is given as,

\[ T_L = \mu_L - \kappa_1 \sigma_L \]

\[ T_R = \mu_R + \kappa_2 \sigma_R \]

where \( T \) is the boundary of the step sizes, \( \mu \) is the weighted mean of the considered part of the histogram, \( \sigma \) is the standard deviation of the considered part of the histogram, \( \kappa \) is the skewness parameter. Here, \( \kappa \) is similar to the \( \hat{k} \) of the uniform quantizer with the variable deadzone. It allows the non-linear quantizer to further vary the step size. In general, \( \kappa_1 \) and \( \kappa_2 \) can have different values, but it is suggested that \( \kappa_1 = \kappa_2 = 1 \) as it serves the purpose in most cases. The subscripts \( L \) and \( R \) represents the left and the right part of the histogram, respectively. The step sizes boundaries are chosen in the following way. Initially, the complete histogram is considered, and \( T_L \) and \( T_R \) is calculated using \( \mu \) and \( \sigma \) of the complete histogram, where \( \mu = \mu_R = \mu_L \) and \( \sigma = \sigma_R = \sigma_L \), because the complete histogram is considered. From the next steps, the
Finally, the result of the uniform quantizer and the non-linear quantizer are compared using mean square error (MSE) on uncompressed high resolution gray scale images. The images are taken from www.imagecompression.info/test_images. MSE can be calculated as \( \text{MSE} = \frac{1}{N} \sum |X(i) - \hat{X}(i)|^2 \), where \( N \) is the total number of detail coefficients, \( X \) is the uncompressed image detail coefficient, and \( \hat{X} \) is the compressed image detail coefficient, both at location \( i \). Experimental setup is as follows. First, each test image is compressed with the uniform quantizer using compression ratios (CRs) from 1 to 10 (The compression ratio is the ratio of desired output size and the input size of the image. In other words, it represents bits per pixel for the compressed image). After that, DWT of the compressed image for all CRs is carried out using coiflet wavelet family (In this paper, only results of coiflets are displayed. However, similar results were achieved using other wavelets. Coiflets are better suited for the analysis of the images with well defined global structures and textures. These basis sets satisfy the so called mini-max condition, which ensures that, the maximum error in capturing structured features are minimized). Concurrently, at this step, non-linear quantizer is applied to the discrete wavelet transformed uncompressed test image. Here, two points have to be noted:(1) uniform and non-linear quantizer is applied to each horizontal, vertical and diagonal component, and (2) there are three sets of detail components available at this stage i.e., uniformly quantized detail components at different CRs, non-linearly quantized detail components at different number of step sizes, and detail components of original uncompressed image. Finally, the MSE of uniformly quantized detail components at CRs 1 to 10, and the MSE of non-linearly quantized detail components at the number of step sizes 2 to 10 are found w.r.t. the original detail components. At this point, it needs to be clarified that...
single MSE is calculated for horizontal, vertical and diagonal component, while quantization is carried out separately.

Fig. 3 shows the MSE plots to evaluate the performance of both uniform and non-linear quantizer of the images shown in Fig. 4. Due to space constraints, only results utilizing “coiflet3” and “coiflet4” wavelets at decomposition level 1 are shown. It can be observed that MSE reduces drastically with increased number of step sizes in non-linear quantizer. Also, MSE increases substantially with the increase of CR. Interestingly, after threshold 5 (i.e., step sizes 6), for all images and both coiflets, the MSE decreases with decreasing rate. Hence, it can be interpreted that 6 is the optimal number of step sizes for non-linear quantizer. Moreover, performance of non-linear quantizer is better than uniform quantizer from and above CR5, at non-linear step segments 6. This implies that uniform quantizer give superior quality at higher bit rate, while non-linear quantizer is best suited for low bit rate applications.

It needs to be emphasized that this paper is focussed on the overall performance of uniform quantizer and non-linear quantizer in terms of the MSE and is independent of the number of quantization step sizes. The number of quantization step sizes in both these quantizers are only used to calculate the overall MSE and are not compared directly. There is no one to one mapping. With reference to Fig. 3, the different CRs are directly correlated with the number of quantized step sizes used in the uniform quantizer and can therefore be meaningfully compared with the MSE of the non-linear quantizer. This paper compares the MSEs because the bits required to represent the quantized values in both quantizers could be same as followed in the standard (see section 5 for better representation of quantized values).

V. DISCUSSION

Through this paper, there are two advantages that can be deduced of non-linear quantizer over uniform quantizer. One that at low bit rate, the performance of the non-linear quantizer is superior, and other that the number of bits required to represent the quantized values is less than uniform quantizer. To illustrate, B number of bits are required to represent quantized values in the uniform quantizer (see section II). On the other hand, \( \lceil \log_2(\text{number of segments}) \rceil \) bits are required for non-linear quantizer. A small table can be created to map the bit representation of quantized values. However, this concept can also be extended to uniform quantizer. There is one more important observation. Instead of applying equation (1) or (2) to obtain quantized values for uniform quantizer, the weighted mean of all the values lying in that step size should be quantized value. Current method does not incorporate the actual statistics of the coefficients. This would yield better performance within the uniform quantizer.

VI. FUTURE WORK

Although the results of non-linear quantizer described in this paper are encouraging, it needs development of a comprehensive and robust theory for more quantitative analysis. One of the key problems to solve would be finding the theoretical upper bound for non-linear quantization noise.

Another important theoretical challenge would be to find the relationship among quality factor, compression ratio, number of step sizes required, and impact of varying \( k \).

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