Probe Branes Dynamics in Nonconstant Background Fields

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Abstract

We consider probe p-branes dynamics in string theory backgrounds of general type. We use an action, which interpolates between Nambu-Goto and Polyakov type actions. This allows us to give a unified description for the tensile and tensionless branes. Firstly, we perform our analysis in the frequently used static gauge. Then, we obtain exact brane solutions in more general gauges. The same approach is used to study the Dirichlet p-brane dynamics and exact brane solutions are also found. As an illustration, we apply our results to the brane world scenario in the framework of the mirage cosmology approach.

Keywords: string theory, p-branes, D-branes, mirage cosmology.

1 Introduction

The classical p-brane is a p-dimensional relativistic object, which evolving in the space-time, describes a (p + 1)-dimensional worldvolume. In this terminology, p = 0 corresponds to a point particle, p = 1 corresponds to a string, p = 2 corresponds to a membrane and so on.

Every p-brane characterizes by its tension $T_p$ with dimension of $(mass)^{p+1}$. When the tension $T_p = 0$, the p-brane is called tensionless one. This relationship between the tensionless branes and the tensile ones generalizes the correspondence between massless and massive particles for the case of extended objects. Thus, the tensionless branes may be viewed as a high-energy limit of the tensile ones.

The point particle (0-brane) couples to 1-form gauge field (the electro-magnetic potential). Generalizing to arbitrary p, one obtains that the p-branes couple to $(p + 1)$-form gauge potentials.

As is known, there exist five consistent superstring theories in ten dimensions: Type IIA with $N = 2$ non-chiral supersymmetry, type IIB with $N = 2$ chiral supersymmetry, type I with $N = 1$ supersymmetry and gauge symmetry $SO(32)$ and heterotic strings with $N = 1$ supersymmetry and $SO(32)$ or $E_8 \times E_8$ gauge symmetry.

The superstring dynamics unify all fundamental interactions between the elementary particles, including gravity, at super high energies. The p-branes arise naturally in the superstring theory, because there exist exact brane solutions of the superstring effective equations of motion. The 2-branes and the 5-branes are the fundamental dynamical objects in eleven dimensional M-theory, which is the strong coupling limit of the five superstring theories in ten dimensions, and which low energy field theory limit is the eleven dimensional supergravity. Particular type of 3-branes arise in the Randall-Sundrum brane world scenario.

In this talk, we consider the probe p-branes dynamics in string theory backgrounds of generic type. In the probe brane limit, one neglects the back-reaction of the brane on the background fields. We use an action, which interpolates between Nambu-Goto and Polyakov type actions. Thus, we give a unified description for the tensile and tensionless branes. Our aim is to obtain the conditions on the background metric and the $(p+1)$-form gauge field, under which the nonlinear brane equations of motion and constraints can be solved exactly. Firstly, we perform our analysis in the frequently used static gauge. Then, we obtain exact brane solutions in a more general gauge. The same approach is used to study the Dirichlet p-brane dynamics and exact solutions are also found. As an illustration, we apply our results to the brane world scenario in the framework of the mirage cosmology approach.
2 Probe $p$-Branes Dynamics

The Nambu-Goto type action for bosonic $p$-brane in a $D$-dimensional curved space-time with metric tensor $g_{MN}(x)$, interacting with a background $(p+1)$-form gauge field $B_{p+1}$ via Wess-Zumino term, can be written as

$$S_p^{NG} = -T_p \int d^{p+1} \xi \left\{ \sqrt{-\det[g_{MN}(X) \partial_0 X^M \partial_0 X^N g_{MN}(X)]} ight. $$

$$- \frac{\varepsilon^{m_1 \ldots m_{p+1}}}{(p+1)!} \partial_{m_1} X^{M_1} \ldots \partial_{m_{p+1}} X^{M_{p+1}} B_{M_1 \ldots M_{p+1}}(X) \left\{, x^M = X^M(\xi^m), \partial_m = \partial/\partial \xi^m, \right. $$

$$m, n = 0, 1, \ldots, p; \quad M, N = 0, 1, \ldots, D - 1. $$

The corresponding Polyakov type action is given by

$$S_p^P = -\frac{T_p}{2} \int d^{p+1} \xi \left\{ \sqrt{-\gamma} \left[ \gamma^{mn} \partial_m X^M \partial_n X^N g_{MN}(X) - (p-1) \right] \right. $$

$$- \frac{\varepsilon^{m_1 \ldots m_{p+1}}}{(p+1)!} \partial_{m_1} X^{M_1} \ldots \partial_{m_{p+1}} X^{M_{p+1}} B_{M_1 \ldots M_{p+1}}(X) \left\{, \gamma = \text{determinant of the auxiliary worldvolume metric } \gamma_{mn}, \text{ and } \gamma^{mn} \text{ is its inverse.} \right. $$

None of these actions is appropriate for description of the tensionless branes. To have the possibility for unified description of the tensile and tensionless branes, we will use the action

$$S_p = \int d^{p+1} \xi \left\{ \frac{1}{4\lambda^0} \left[ g_{MN}(X) \left( \partial_0 - \lambda^i \partial_i \right) X^M \left( \partial_0 - \lambda^j \partial_j \right) X^N \right] \right. $$

$$- (2\lambda^0 T_p)^2 \det[g_{MN}(X) \partial_i X^M \partial_j X^N] \right. $$

$$+ T_p B_{M_0 \ldots M_p}(X) \partial_0 X^{M_0} \ldots \partial_p X^{M_p} \right\}, \quad (i, j = 1, \ldots, p), $$

in which the limit $T_p \to 0$ may be taken. It can be shown that this action is classically equivalent to the above two actions. The Lagrange multipliers $\lambda^0$ and $\lambda^i$ are connected to the lapse function $N$ and the shift vector $N^i$ as follows:

$$\left(2\lambda^0 T_p\right)^2 = N^2, \quad \lambda^i = N^i. $$

Varying the action $S_p$ with respect to $\lambda^m$, one obtains the constraints

$$g_{MN} \left( \partial_0 - \lambda^i \partial_i \right) X^M \left( \partial_0 - \lambda^j \partial_j \right) X^N = 0, $$

$$g_{MN} \left( \partial_0 - \lambda^i \partial_i \right) X^M \partial_j X^N = 0. $$

We will work in the gauge $\lambda^m = \text{constants}$, in which the equations of motion take the form:

$$g_{LN} \left[ \left( \partial_0 - \lambda^i \partial_i \right) \left( \partial_0 - \lambda^j \partial_j \right) X^N - (2\lambda^0 T_p)^2 \partial_i \left( \mathcal{G}^{ij} \partial_j X^N \right) \right] $$

$$+ \Gamma_{L,MN} \left[ \left( \partial_0 - \lambda^i \partial_i \right) X^M \left( \partial_0 - \lambda^j \partial_j \right) X^N - (2\lambda^0 T_p)^2 \mathcal{G}^{ij} \partial_j X^M \partial_i X^N \right] $$

$$= 2\lambda^0 T_p H_{L,M_0 \ldots M_p} \partial_0 X^{M_0} \ldots \partial_p X^{M_p}, $$

where

$$\mathcal{G} = \det(G_{ij}) = \det\left( g_{MN} \partial_i X^M \partial_j X^N \right), \quad H_{p+2} = dB_{p+1}, $$

$$\Gamma_{L,MN} = \frac{1}{2} \left( \partial_M g_{NL} + \partial_N g_{ML} - \partial_L g_{MN} \right).$$
2.1 Static Gauge

In the commonly used static gauge, we have the following identification: $X^m(\xi) = \xi^m$. The other part of the coordinates $X^a$ are supposed to be functions only of $\xi^0 = \tau$. The typical string theory backgrounds do not depend on $x^m$, so that

$$\partial g_{MN}/\partial x^m = 0, \quad \partial B_{Ma...M_p}/\partial x^m = 0.$$ 

Under these conditions, the action $S_p$ reduces to

$$S_p^{SG} = \int d\tau L^{SG}(\tau), \quad V_p = \int d^p \xi,$$

$$L^{SG} = \frac{V_p}{4\lambda^0} \left[ g_{ab} \dot{X}^a \dot{X}^b + 2 (g_{0a} - \lambda^i g_{ia}) \dot{X}^a + g_{00} - 2\lambda^i g_{0i} + \lambda^i \lambda^j g_{ij} - (2\lambda^0 T_p)^2 \det(g_{ij}) + 4\lambda^0 T_p B_{01...p} \right].$$

To have finite action, we require the fraction $V_p/\lambda^0$ to be finite. In the string case ($p = 1$) and in conformal gauge, for example, this corresponds to have the fraction $V_1/\alpha' = 2\pi V_1 T_1$ finite.

Now, the constraints are:

$$g_{ab} \dot{X}^a \dot{X}^b + 2 (g_{0a} - \lambda^i g_{ia}) \dot{X}^a + g_{00} - 2\lambda^i g_{0i} + \lambda^i \lambda^j g_{ij} - (2\lambda^0 T_p)^2 \det(g_{ij}) + 4\lambda^0 T_p B_{01...p} = 0.$$

The Lagrangian $L^{SG}$ does not depend on $\tau$ explicitly, so the energy $E$ is conserved:

$$g_{ab} \dot{X}^a \dot{X}^b - g_{00} + 2\lambda^i g_{0i} - \lambda^i \lambda^j g_{ij} + (2\lambda^0 T_p)^2 \det(g_{ij}) - 4\lambda^0 T_p B_{01...p} = \frac{4\lambda^0 E}{V_p} = \text{constant}.$$

With the help of the constraints, we can replace this equality by the following one

$$g_{0a} \dot{X}^a + g_{00} - \lambda^i g_{0i} + 2\lambda^0 T_p B_{01...p} = \frac{2\lambda^0 E}{V_p}.$$

It can be shown that the equations of motion and all these constraints can be reduced to the equalities

$$g_{ab} \ddot{X}^b + \Gamma_{a, bc} \dot{X}^b \dot{X}^c + \frac{1}{2} \partial_a V^{SG} = 2\partial_{(a} A_{b)}^{SG} \dot{X}^b,$$

$$g_{ab} \dot{X}^a \dot{X}^b + V^{SG} = 0,$$

where

$$V^{SG} = (2\lambda^0 T_p)^2 \det(g_{ij}) - g_{00} + 2\lambda^i g_{0i} - \lambda^i \lambda^j g_{ij} - 4\lambda^0 (T_p B_{01...p} + E/V_p),$$

$$A_a^{SG} = g_{aa} - \lambda^i g_{ai} + 2\lambda^0 T_p B_{a1...p}.$$ 

It turns out that for background fields depending on only one coordinate $x^a$, we can always integrate these equations, and the solution is

$$\tau \left( X^a \right) = \tau_0 \pm \int_{\dot{X}_b^a}^{X^a} \left( -\frac{V^{SG}(g_{aa})}{\dot{X}^a} \right)^{-1/2} du.$$ 

Otherwise, supposing the metric $g_{ab}$ is a diagonal one, we can rewrite the equations of motion in the form

$$\frac{d}{d\tau} \left( g_{aa} \dot{X}^a \right)^2 + \dot{X}^a \partial_a \left( g_{aa} V^{SG} \right) + \dot{X}^a \sum_{b \neq a} \left[ \partial_a \left( \frac{g_{aa}}{g_{bb}} \right) (g_{bb} \dot{X}^b)^2 - 4\partial_{(a} A_{b)}^{SG} g_{aa} \dot{X}^b \right] = 0.$$ 

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To find solutions of the above equations without choosing particular background, we fix all coordinates $X^a$ except one. Then the exact probe brane solution of the equations of motion and constraints is given again by the same expression for $\tau (X^a)$.

To find solutions depending on more than one coordinate, we have to impose further conditions on the background fields. An example of such sufficient conditions, which allow us to reduce the order of the equations of motion by one, is given below (we split the index $a$ in such a way that $X^r$ is one of the coordinates $X^a$, and $X^\alpha$ are the others):

$$\partial_{\alpha} \left( \frac{g_{\alpha\alpha}}{g_{aa}} \right) = 0, \quad \partial_{\alpha} \left( g_{rr} \dot{X}^r \right)^2 = 0,$$

$$\partial_r \left( \frac{g_{\alpha\alpha} X^\alpha}{g_{aa}} \right)^2 = 0, \quad \mathcal{A}_{\alpha}^{\mathcal{SG}} = \partial_{\alpha} f.$$

The result of integrations is the following

$$\left( g_{\alpha\alpha} X^\alpha \right)^2 = D_\alpha \left( X^{a \neq \alpha} \right) + g_{\alpha\alpha} \left[ 2 \left( \mathcal{A}_r^{\mathcal{SG}} - \partial_r f \right) X^r - V^{SG} \right],$$

$$\left( g_{rr} \dot{X}^r \right)^2 = E_r \left( X^{r} \right),$$

$$\dot{Z}^r = \dot{X}^r + \frac{n_{\alpha}}{g_{rr}} \left( \mathcal{A}_r^{\mathcal{SG}} - \partial_r f \right),$$

where $D_\alpha$, $E_\alpha$ and $E_r$ are arbitrary functions of their arguments, and $n_{\alpha}$ is the number of the coordinates $X^\alpha$.

Further progress is possible, when working with particular background configurations.

### 2.2 Rotated Gauge

Now we will repeat our analysis of the probe p-brane dynamics in a more general gauge than the static one. Namely, our ansatz for the coordinates $X^m(\xi)$ is the following

$$X^m(\xi) = \Lambda^m_n \xi^n = \Lambda^m_r \tau + \Lambda^m_i \xi^i, \quad \Lambda^m_n = \text{constants}.$$

We call this ansatz rotated gauge, because $\Lambda^m_n \xi^n$ look like rotations in the space described by the coordinates $\xi^n$. However, there are no restrictions on the parameters $\Lambda^m_n$. They are arbitrary constants. For $\Lambda^m_n = \delta^m_n$, we come back to static gauge.

In the same way as before, one obtains the Lagrangian

$$L^R = \frac{V_p}{4\lambda^0} \left\{ g_{ab} \dot{X}^a \dot{X}^b + 2 \left[ (\Lambda^0_n - \lambda^i \Lambda^0_i) g_{na} + 2\lambda^0 T_p B_a \right] \dot{X}^a \right\}$$

$$+ \left( \Lambda^0_n - \lambda^i \Lambda^0_i \right) \left( \Lambda^0_m - \lambda^i \Lambda^0_j \right) g_{nm} - \left( 2\lambda^0 T_p \right)^2 \det(\Lambda^0_n \Lambda^0_m) g_{nm} + 4\lambda^0 T_p \Lambda^0_m B_m \right\}, \quad B_M \equiv B_{Mm_1 \ldots m_p} \Lambda^1_{m_1} \ldots \Lambda^p_{m_p},$$

the constraints

$$g_{ab} \dot{X}^a \dot{X}^b + \left( \Lambda^0_n - \lambda^i \Lambda^0_i \right) g_{na} \dot{X}^a + \left( \Lambda^0_n - \lambda^i \Lambda^0_i \right) \times \left( \Lambda^0_m - \lambda^j \Lambda^0_j \right) g_{nm} + \left( 2\lambda^0 T_p \right)^2 \det(\Lambda^0_n \Lambda^0_m) g_{nm} = 0,$$

$$\Lambda^0_n \left[ g_{na} \dot{X}^a + \left( \Lambda^0_m - \lambda^j \Lambda^0_j \right) g_{nm} \right] = 0,$$

the conserved energy

$$\Lambda^0_n \left[ g_{na} \dot{X}^a + \left( \Lambda^0_m - \lambda^j \Lambda^0_j \right) g_{nm} + 2\lambda^0 T_p B_a \right] = -\frac{2\lambda^0 E}{V_p},$$
and finally, the equations of motion and the effective constraint
\[
g_{ab} \ddot{X}^b + \Gamma_{a,bc} \dot{X}^b \dot{X}^c + \frac{1}{2} \partial_a V^R = 2 \partial_a A_{b}^R \dot{X}^b,
\]
\[
g_{ab} \dot{X}^a \dot{X}^b + V^R = 0.
\]

We see that these equalities have the same form as in static gauge, but with \( V^R \), \( A_{\alpha}^R \) replaced with \( V^S G \), \( A_{\alpha}^S G \) and finally, the equations of motion and the effective constraint
\[
S_D = \int d^{p+1} \xi e^{-\phi} 4\lambda^0 \left[ G_{00} - 2 \lambda^i G_{0i} + (\lambda^i \lambda^j - \kappa^i \kappa^j) G_{ij} - (2 \lambda^0 T_D)^2 \det(G_{ij}) + 2 \kappa^i (F_{0i} - \lambda^i F_{ji}) + 4 \lambda^0 T_D e^{\phi} C_{M_0 \ldots M_p} \partial_0 X^{M_0} \ldots \partial_p X^{M_p} \right],
\]
\[
F_{mn} = B_{mn} + 2 \pi \alpha' F_{mn}.
\]

Here additional Lagrange multipliers \( \kappa^i \) are introduced, in order to linearize the quadratic term
\[
(F_{0i} - \lambda^i F_{ki}) (G^{-1})^{ij} (F_{0j} - \lambda^j F_{lj})
\]
in the action.

For simplicity, we restrict our considerations to constant dilaton \( \Phi = \Phi_0 \) and constant electromagnetic field \( F_{mn} \) on the Dp-brane worldvolume.

3 Probe Dp-Branes Dynamics

The Dirac-Born-Infeld type action for the bosonic part of the super-Dp-brane in a 6-dimensional curved space-time with metric tensor \( g_{MN}(x) \), interacting with a background \((p+1)\)-form Ramond-Ramond gauge field \( C_{M_1 \ldots M_{p+1}} \) via Wess-Zumino term, can be written as
\[
S = -T_D \int d^{p+1} \xi e^{-a(p,D)\Phi} \sqrt{-\det(G_{mn} + B_{mn} + 2 \pi \alpha' F_{mn})}
- \frac{e^{m_1 \ldots m_{p+1}}}{(p+1)!} \partial_{m_1} X^{M_1} \ldots \partial_{m_{p+1}} X^{M_{p+1}} C_{M_1 \ldots M_{p+1}} \right].
\]

\( T_D = (2\pi)^{-(p-1)/2} g_s^{-1} T_p \) is the D-brane tension, \( g_s = \exp(\Phi) \) is the string coupling expressed by the dilaton vacuum expectation value (\( \Phi \)) and \( 2 \pi \alpha' \) is the inverse string tension. \( G_{mn} = \partial_m X^M \partial_n X^N g_{MN}(X) \), \( B_{mn} = \partial_m X^M \partial_n X^N b_{MN}(X) \) and \( \Phi(X) \) are the pullbacks of the background metric, antisymmetric tensor and dilaton to the Dp-brane worldvolume, while \( F_{mn}(\xi) \) is the field strength of the worldvolume \( U(1) \) gauge field \( A_m(\xi) \). The parameter \( a(p,D) \) depends on the brane and space-time dimensions \( p \) and \( D \) respectively.

To be able to take the limit \( T_D \to 0 \), we will work with the classically equivalent action
\[
S_D = \int d^{p+1} \xi e^{-\phi} 4\lambda^0 \left[ G_{00} - 2 \lambda^i G_{0i} + (\lambda^i \lambda^j - \kappa^i \kappa^j) G_{ij} - (2 \lambda^0 T_D)^2 \det(G_{ij}) + 2 \kappa^i (F_{0i} - \lambda^i F_{ji}) + 4 \lambda^0 T_D e^{\phi} C_{M_0 \ldots M_p} \partial_0 X^{M_0} \ldots \partial_p X^{M_p} \right],
\]
\[
F_{mn} = B_{mn} + 2 \pi \alpha' F_{mn}.
\]

Here additional Lagrange multipliers \( \kappa^i \) are introduced, in order to linearize the quadratic term
\[
(F_{0i} - \lambda^i F_{ki}) (G^{-1})^{ij} (F_{0j} - \lambda^j F_{lj})
\]
in the action.

For simplicity, we restrict our considerations to constant dilaton \( \Phi = \Phi_0 \) and constant electromagnetic field \( F_{mn} \) on the Dp-brane worldvolume.
The idea of the mirage cosmology approach \cite{1} - \cite{16} to the brane world model is the following \cite{2}.

### 4 Application to Mirage Cosmology

The motion of the probe brane in a curved higher dimensional bulk space induces cosmological evolution on the universe brane that is indistinguishable from a similar one induced by matter density on the brane. It can be shown that the motion of the probe brane in ambient space induces cosmological expansion or contraction on our universe simulating various kinds of "matter" or a cosmological constant (inflation).

There are two steps in the procedure:

1. Determine the probe brane motion by solving the worldvolume field equations.

2. Determine the induced metric on the brane which now becomes an implicit function of time. This gives a cosmological evolution in the induced brane metric. This cosmological evolution can be reinterpreted in terms of cosmological "mirage" energy densities on the brane via a Friedman-like equation for the scale factor \( A \):

### 3.1 Static Gauge

In static gauge, the reduced Lagrangian is given by

\[
L_D^{SG} = \frac{V_0 e^{-\alpha \phi_0}}{4 \lambda^0} \left[ g_{ab} \dot{X}^a \dot{X}^b + g_{00} - 2 \lambda^i g_{0i} \right. \\
\left. + 2 \left( \lambda^0 g_{0a} + 2 \lambda^0 T_D e^{m \phi_0} C_{a1...p} + \kappa^i b_{ai} \right) \dot{X}^a \right.
\]

where now

\[
V_D^{SG} = (2 \lambda^0 T_D)^2 \det (g_{ij}) - g_{00} + 2 \lambda^i g_{0i} - (\lambda^i \lambda^j - \kappa^i \kappa^j) g_{ij}
\]

The corresponding exact solution, depending on one of the coordinates \( X^a \), is

\[
\tau (X^a) = \tau_0 \pm \int_{X^a_0}^{X^a} \left( - \frac{V_D^{SG}}{g_{00}} \right)^{-1/2} du.
\]

### 3.2 Rotated Gauge

In the same way, one obtains the exact probe Dp-brane solution in this gauge. It is given by the above equality for \( \tau (X^a) \), where instead of \( V_D^{SG} \) we have to use the potential

\[
V_D^R = (2 \lambda^0 T_D)^2 \det (\Lambda^n_a \Lambda^n_m g_{nm}) - \left( \Lambda^n_0 - \lambda^i \Lambda^n_i \right) \left( \Lambda^m_0 - \lambda^j \Lambda^m_j \right)
\]

where

\[
C_m = C_{mm1...mp} \Lambda^m_1 \ldots \Lambda^m_p.
\]
\[
\left( \frac{1}{a} \frac{da}{d\eta} \right)^2 = \frac{8\pi}{3} \rho_{\text{eff}}.
\]

In addition, one defines the effective pressure \( p_{\text{eff}} \) through the equality
\[
\frac{1}{a^2} \frac{d^2 a}{d\eta^2} = -\frac{4\pi}{3} (\rho_{\text{eff}} + 3p_{\text{eff}}).
\]

We showed already how the brane equations of motion and constraints can be solved exactly. The induced line element on the brane is
\[
ds^2 = G_{mn} dx^m dx^n = G_{00}(dx^0)^2 + 2G_{0j} dx^0 dx^j + G_{ij} dx^i dx^j.
\]

Introducing the cosmic time \( \eta \) by the relation
\[
d\eta = \sqrt{-G_{00}} dx^0,
\]
one obtains
\[
ds^2 = -d\eta^2 + 2 \frac{G_{0j}}{\sqrt{-G_{00}}} d\eta dx^j + G_{ij} dx^i dx^j.
\]

Now, let us give the metrics induced on the brane in the different cases considered. In static gauge, we have the following \( p \)-brane metric
\[
G_{00} = - \left(2\lambda^0 T_p\right)^2 \det (g_{ij}) + \lambda^i \lambda^j g_{ij},
G_{0j} = \lambda^j g_{ij}, \quad G_{ij} = g_{ij}.
\]

The corresponding generalization for the \( Dp \)-brane metric is given by
\[
G_{00} = - \left(2\lambda^0 T_D\right)^2 \det (g_{ij}) + \left(\lambda^i \lambda^j - \kappa^i \kappa^j\right) g_{ij}
- 2\lambda^i \kappa^j \left(b_{ij} + 2\pi \alpha' F_{ij}\right),
G_{0j} = \lambda^j g_{ij} + \kappa^i \left(b_{ij} + 2\pi \alpha' F_{ij}\right), \quad G_{ij} = g_{ij}.
\]

In rotated gauge, the induced \( p \)-brane metric is
\[
G_{00} = - \left(2\lambda^0 T_p\right)^2 \det (\Lambda^m_i \Lambda^n_j g_{mn}) + \lambda^i \lambda^j \Lambda^m_i \Lambda^n_j g_{mn},
G_{0j} = \lambda^j \Lambda^m_i \Lambda^n_j g_{mn}, \quad G_{ij} = \Lambda^m_i \Lambda^n_j g_{mn}.
\]

In the same gauge, the metric induced on the \( Dp \)-brane is the following
\[
G_{00} = - \left(2\lambda^0 T_D\right)^2 \det (\Lambda^m_i \Lambda^n_j g_{mn}) + \left(\lambda^i \lambda^j - \kappa^i \kappa^j\right) \Lambda^m_i \Lambda^n_j g_{mn}
- 2\lambda^i \kappa^j \left(\Lambda^m_i \Lambda^n_j b_{mn} + 2\pi \alpha' F_{ij}\right),
G_{0j} = \lambda^j \Lambda^m_i \Lambda^n_j g_{mn} + \kappa^i \left(\Lambda^m_i \Lambda^n_j b_{mn} + 2\pi \alpha' F_{ij}\right),
G_{ij} = \Lambda^m_i \Lambda^n_j g_{mn}.
\]

We will not consider here any particular mirage cosmology. We just mention that the obtained results allow us to find exact probe branes solutions in more general background fields and in more general gauges, than known so far. Therefore, we can induce more general metrics on the probe brane, and investigate the corresponding mirage cosmology. For example, we can consider generalized Kasner type metrics, which appear in the superstring cosmology [17].

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