$D_{s0}^{*}(2317)$ and $D_{s1}(2460)$ mesons in two-body $B$-meson decays

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We analyze the branching ratios of $B \to D^{(*)}D_{s0}^{*}(D_{s1})$ decays using the factorization hypothesis. The $B \to D^{(*)}$ transition form factors are taken from a model-independent analysis done by Caprini, Lellouch and Neubert based on heavy quark spin symmetry and dispersive constraints, including short-distance and power corrections. The leptonic decay constants $f_{D_{s0}^{*}}$ and $f_{D_{s1}}$ are calculated assuming a molecular structure for the $D_{s0}^{*}$ and $D_{s1}$ mesons. The calculated branching ratios of $B$-meson two-body decays are compared with experimental data and other theoretical results.

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I. INTRODUCTION

Presently there is strong interest to study a newly observed mesons and baryons in the context of a hadronic molecule interpretation (for overview see e.g. Ref. [1]). In the present work we focus on weak production properties of scalar $D_{s0}^*(2317)$ and axial $D_{s1}(2460)$ charm-strange mesons (for a review see e.g. [2]). The $D_{s0}^*(2317)$ meson was discovered just a few years ago by the BABAR Collaboration at SLAC in the inclusive $D^+_s\pi^0$ invariant mass distribution from $e^+e^-$ annihilation data [3]. The nearby state $D_{s1}(2460)$ decaying into $D_s^*\pi^+$ was observed by the CLEO Collaboration at CESR [4]. Both of these states have been confirmed by the Belle Collaboration at KEKB [5]. In the interpretation of these experiments it was suggested that the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ mesons are the $P$-wave charm-strange quark states with spin-parity quantum numbers $J^P = 0^+$ and $J^P = 1^+$, respectively. In the following the Belle [6] and BABAR [7] Collaborations observed the production of $D_{s0}^*(2317)$, $D_{s1}(2460)$ and their subsequent strong and radiative transitions in the nonleptonic two-body $B$ decays. The most recent data of BABAR and Belle on two-body $B$-meson decays into $D_{s0}^*(2317)$ and $D_{s1}(2460)$ states can be found in Refs. [8, 9]. It is worth noting that the existing experimental information on the properties of $D_{s0}^*(2317)$ and $D_{s1}(2460)$ mesons [10] leaves quite a significant uncertainty in their interpretation as $J^P = 0^+$ and $J^P = 1^+$ states.

Theoretical analysis of $B \rightarrow D^{(*)} D_{s0}^*(D_{s1})$ decays has been performed in different approaches [11]-[20] based on the factorization hypothesis, which essentially simplifies the calculation of the transition amplitude. The factorizable amplitude $B \rightarrow D^{(*)} D_{s0}^*(D_{s1})$ is given by the product of the corresponding form factors (or their combination) describing semileptonic $B \rightarrow D^{(*)}\ell\nu\ell$ transitions and the leptonic decay constant $f_{D_{s0}^*}(f_{D_{s1}})$. The leptonic decay constants $f_{D_{s0}^*}$ and $f_{D_{s1}}$ have been calculated directly or extracted from the analysis of $B \rightarrow D^{(*)} D_{s0}^*(D_{s1})$ decays in Refs. [11, 12, 14, 17, 18, 20-23]. The form factors of $B \rightarrow D^{(*)}\ell\nu\ell$ transitions have been analyzed and calculated in various theoretical approaches such as: heavy quark effective theory, QCD sum rules, lattice QCD, different types of quark and soliton models, approaches based on the solution of Bethe-Salpeter and Faddeev equations, etc.

In this paper we assume the $D_{s0}^*$ and $D_{s1}$ mesons to be hadronic molecules - bound states of $D, K$ and $D^*, K^*$ mesons, respectively. Using this molecular picture, in Refs. [26, 27] we calculated strong and radiative decays of $D_{s0}^*$ and $D_{s1}$ mesons. The obtained results are in agreement with other theoretical approaches, e.g. the strong decay widths $D_{s0}^* \rightarrow D_s^0\pi^0$ and $D_{s1} \rightarrow D_s^*\pi^0$ are of the order of $10^2$ KeV and the radiative decays $D_{s0}^* \rightarrow D_s^*\gamma$, $D_{s1} \rightarrow D_s^*\gamma$, etc. are of the order of a few KeV. Here, using the same approach, we calculate the leptonic decay constants $f_{D_{s0}^*}$ and $f_{D_{s1}}$. For the form factors governing the semileptonic $B \rightarrow D^{(*)}\ell\nu\ell$ transitions we use the model-independent results obtained by Caprini, Lellouch and Neubert (CLN) [28] on the basis of heavy quark spin symmetry and dispersive constraints, including short-distance and power corrections. Note, that in Ref. [18] the authors already used the CLN results in their analysis of two-body $B \rightarrow DD_{s0}^*(D_{s1})$ transitions restricting themselves to the heavy quark limit and modes with a pseudoscalar $D$ meson in the final state. Using the experimental lower limits for the $B \rightarrow DD_{s0}^*(D_{s1})$ branching ratios they derived lower limits for the products $|a_1|f_{D_{s0}^*}$ and $|a_1|f_{D_{s1}}$, where $a_1$ is a combination of the short-distance Wilson coefficients [29-31].

In the present paper we proceed as follows. First, in Section II, we discuss the basic notions of our approach. We indicate and evaluate the effective mesonic Lagrangian for the treatment of charmed mesons $D_{s0}^*(2317)$ and $D_{s1}(2460)$ as $D K$ and $D^* K$ bound states, respectively. Then in Section III we discuss the calculation of the leptonic decay constants $f_{D_{s0}^*}$ and $f_{D_{s1}}$. In Section IV we present a detailed analysis of two-body bottom meson decays $B \rightarrow D^{(*)}D_{s0}^*(D_{s1})$ applying the factorization hypothesis. As we already stressed before, in this analysis we use the model-independent CLN results [28] for the weak form factors defining the $B \rightarrow D^{(*)}\ell\nu\ell$ transitions. In Section V we give a short summary of our results.

II. MOLECULAR STRUCTURE OF $D_{s0}^{(*)}(2317)$ AND $D_{s1}^{(*)}(2460)$ MESONS

In this section we discuss the formalism for the study of the $D_{s0}^{(*)}(2317)$ and $D_{s1}^{(*)}(2460)$ mesons as hadronic molecules, represented by $D K$ and $D^* K$ bound states, respectively. We adopt that the isospin, spin and parity quantum numbers of $D_{s0}^{(*)}(2317)$ and $D_{s1}^{(*)}(2460)$ are: $I(J^P) = 0(0^+)$ and $I(J^P) = 0(1^-)$, while for their masses we take the values: $m_{D_{s0}^{(*)}} = 2.3173$ GeV and $m_{D_{s1}} = 2.4589$ GeV [10]. Our framework is based on effective interaction Lagrangians describing the couplings of $D_{s0}^*(2317)$ and $D_{s1}(2460)$ mesons to their constituents:

\begin{align}
\mathcal{L}_{D_{s0}^*}(x) &= g_{D_{s0}^*} D_{s0}^*(x) \int dy \Phi_{D_{s0}^*}(y^2) D(x + w_{KD} y) K(x - w_{DK} y) + \text{H.c.}, \\
\mathcal{L}_{D_{s1}}(x) &= g_{D_{s1}} D_{s1}(x) \int dy \Phi_{D_{s1}}(y^2) D_{s1}^*(x + w_{KD} y) K(x - w_{DK} y) + \text{H.c.},
\end{align}
where the doublets of $D^{(*)}$ and $K$ mesons are defined as

$$ D = \left( \begin{array}{c} D^0 \\ D^+ \end{array} \right), \quad D^* = \left( \begin{array}{c} D^{*0} \\ D^{*+} \end{array} \right), \quad K = \left( \begin{array}{c} K^+ \\ K^0 \end{array} \right). $$

The summation over isospin indices is understood. The molecular structure of the $D^*_{s0}$ and $D^*_{s1}$ states is:

$$ |D^*_{s0}^+\rangle = |D^+K^0\rangle + |D^0K^+\rangle, \quad |D^*_{s0}^-\rangle = |D^-\bar{K}^0\rangle + |D^0\bar{K}^-\rangle, \quad |D^*_{s1}^+\rangle = |D^{*+}K^0\rangle + |D^{*0}K^+\rangle, \quad |D^*_{s1}^-\rangle = |D^{*+}\bar{K}^0\rangle + |D^{*0}\bar{K}^-\rangle. $$

The correlation functions $\Phi_M$ with $M = D^*_{s0}$ or $D^*_{s1}$ characterize the finite size of the $D^*_{s0}(2317)$ and $D^*_{s1}(2460)$ mesons as $DK$ and $D^*K$ bound states and depend on the relative Jacobi coordinate $y$ with, in addition, $x$ being the center of mass (CM) coordinate. Note, that the local limit corresponds to the substitution of $\Phi_M$ by the Dirac delta-function: $\Phi_M(y^2) \rightarrow \delta^4(y)$. In Eqs. (1) and (2) we introduced the kinematical parameters $w_{ij}$:

$$ w_{ij} = \frac{m_i}{m_i + m_j}, $$

where $m_D$, $m_{D^*}$ and $m_K$ are the masses of $D$, $D^*$ and $K$ mesons. The Fourier transform of the correlation function reads

$$ \Phi_M(y^2) = \int \frac{d^4p}{(2\pi)^4} e^{-ip\xi} \Phi_M(-p^2), \quad M = D^*_{s0}, D^*_{s1}. $$

A basic requirement for the choice of an explicit form of the correlation function is that it falls down sufficiently fast in the ultraviolet region of Euclidean space to render the Feynman diagrams ultraviolet finite. We adopt the Gaussian form

$$ \Phi_M(p^2_E) \sim \exp(-p^2_E/\Lambda^2_M), $$

for the vertex function, where $p_E$ is the Euclidean Jacobi momentum. Here $\Lambda_{D^*}$ is a size parameter, which parametrizes the distribution of $D$ and $K$ mesons inside the $D^*_{s0}$ molecule, while $\Lambda_{D^*_{s1}}$ is the size parameter for the $D^*_{s1}$ molecule. For simplicity we will use a universal scale parameter $\Lambda_M = \Lambda_{D^*_{s0}} = \Lambda_{D^*_{s1}}$, i.e. the same size for $D^*_{s0}$ and $D^*_{s1}$ mesons.

The coupling constants $g_{D^*_{s0}}$ and $g_{D^*_{s1}}$ are determined by the compositeness condition [32, 33], which implies that the renormalization constant of the hadron wave function is set equal to zero:

$$ Z_{D^*_{s0}} = 1 - \Sigma'_{D^*_{s0}}(m^2_{D^*_{s0}}) = 0, \quad Z_{D^*_{s1}} = 1 - \Sigma'_{D^*_{s1}}(m^2_{D^*_{s1}}) = 0. $$

Here, $\Sigma'_{D^*_{s0}}(m^2_{D^*_{s0}}) = g^2_{D^*_{s0}} \Pi_{D^*_{s0}}(m^2_{D^*_{s0}})$ is the derivative of the $D^*_{s0}$ meson mass operator. In the case of the $D^*_{s1}$ meson we have $\Sigma'_{D^*_{s1}}(m^2_{D^*_{s1}}) = g^2_{D^*_{s1}} \Pi_{D^*_{s1}}(m^2_{D^*_{s1}})$, which is the derivative of the transverse part of its mass operator $\Sigma^\mu_{D^*_{s1}}$, conventionally split into transverse $\Sigma_{D^*_{s1}}$ and longitudinal $\Sigma_{D^*_{s1}}^L$ parts as:

$$ \Sigma^\mu_{D^*_{s1}}(p) = g^\mu_{D^*_{s1}} \Sigma_{D^*_{s1}}(p^2) + \frac{p^\mu p^\nu}{p^2} \Sigma^L_{D^*_{s1}}(p^2), $$

where

$$ g^\mu_{D^*_{s1}} = g^\mu - \frac{p^\mu p^\nu}{p^2}, \quad g^\mu_{D^*_{s1}} p^\mu = 0. $$

The mass operators of the $D^*_{s0}$ and $D^*_{s1}$ mesons are described by the diagram of Fig.1.

Following Eqs. (8) and (9) the coupling constants $g_{D^*_{s0}}$ and $g_{D^*_{s1}}$ can be expressed in the form:

$$ \frac{1}{g^2_{D^*_{s0}}} = \frac{2}{(4\pi\Lambda_M)^2} \int_0^1 dx \int_0^\infty \frac{d\alpha}{(1+\alpha)^3} \left[ \frac{d}{dz_0} \tilde{\Phi}_M(z_0) \right], $$

$$ \frac{1}{g^2_{D^*_{s1}}} = \frac{2}{(4\pi\Lambda_M)^2} \int_0^1 dx \int_0^\infty \frac{d\alpha}{(1+\alpha)^3} \left[ \frac{1}{2\mu^2_{D^*_{s1}}(1+\alpha)} - \frac{d}{dz_1} \tilde{\Phi}_M(z_1) \right] $$

where
where
\[ P_0(\alpha, x) = \alpha^2 x(1 - x) + w_{D^0}^2 \alpha x + w_{K^0}^2 \alpha(1 - x), \]
\[ P_1(\alpha, x) = \alpha^2 x(1 - x) + w_{D^{*0}}^2 \alpha x + w_{K^{*0}}^2 \alpha(1 - x), \]
\[ z_0 = \mu_D^2 \alpha x + \mu_K^2 \alpha(1 - x) - \frac{P_0(\alpha, x)}{1 + \alpha} \mu_{D^0}^2, \]
\[ z_1 = \mu_{D^*}^2 \alpha x + \mu_K^2 \alpha(1 - x) - \frac{P_1(\alpha, x)}{1 + \alpha} \mu_{D^{*0}}^2, \]
\[ \mu_M = \frac{m_M}{\Lambda_M}. \]

Above expressions are valid for any functional form of the correlation function $\bar{\Phi}_M$.

Let us note, that the compositeness condition of the type \( \Phi \), \( \Psi \) was originally applied to the study of the deuteron as a bound state of proton and neutron \( \Phi \), \( \Psi \). Then this condition was extensively used in low-energy hadron phenomenology as the master equation for the treatment of mesons and baryons as bound states of light and heavy constituent quarks \( \Phi \), \( \Psi \). In Refs. \( \Phi \), \( \Psi \), this condition was used in the application to glueballs as bound states of gluons and light and heavy scalar mesons as hadronic molecules.

### III. LEPTONIC DECAY CONSTANTS $f_{D^{*0}}$ AND $f_{D^{*1}}$

The decay constants of the scalar $D_{s0}^{*}$ and axial $D_{s1}$ mesons are defined by

\[ \langle 0 | \bar{s}O_{\mu}c| D_{s0}^{*+}(p) \rangle = p_{\mu} f_{D_{s0}^{*}}, \]
\[ \langle 0 | \bar{s}O_{\mu}c| D_{s1}^{+}(p, \epsilon) \rangle = \epsilon_{\mu}(p) m_{D_{s1}} f_{D_{s1}}, \]

where $O_{\mu} = \gamma_{\mu}(1 - \gamma_5)$. Note, parity symmetry implies that only the vector component of the weak $V - A$ current contributes to the transition $D_{s0}^{*+} \rightarrow W^+$, while for the transition $D_{s1}^{+} \rightarrow W^+$ only the axial component is present. The one-loop meson diagrams describing the matrix elements of Eqs. (15), (16) are given in Figs. 2a and 2b. In other words, adopting the molecular picture for the $D_{s0}$ and $D_{s1}$ mesons we need to evaluate the two-point $DK$ loop diagram describing the transition of the $D_{s0}$ meson to the vector current and the two-point $D^*K$ loop diagram corresponds to the transition of the $D_{s1}$ meson to the axial current. The coupling of the weak $c \rightarrow s$ flavor-changing vector current (FCVC) $V_\mu$ to the $DK$ pair and the weak $c \rightarrow s$ flavor-changing axial current (FCAC) $A_\mu$ to the $D^*K$ pair can be extracted from data. In particular, the couplings are extracted by deriving the effective Lagrangians from the matrix elements for the $D \rightarrow K^{(*)} \bar{\ell} \ell$ semileptonic transitions. First, the matrix element for the $D \rightarrow K^{(*)} \bar{\ell} \ell$ transition at zero momentum transfer $q = p - p' \rightarrow 0$ is approximated by

\[ \langle K(p')|V^\mu(0)|D(p)\rangle \simeq f_+^{DK}(0)(p + p')^\mu. \]

Therefore, the effective Lagrangian describing the coupling of $D$ and $K$ mesons with the weak vector current is given by

\[ L_{VDK}(x) = f_+^{DK}(0) V^\mu(x) D(x) i\partial_\mu K(x) + H.c. \]

where $A\partial_\mu B = (\partial_\mu A)B - A\partial_\mu B$.

The effective coupling of the weak axial current $A_\mu$ with $D^*$ and $K^*$ mesons can be related to the corresponding coupling with $D$ and $K^*$ mesons on the basis of SU(4) flavor symmetry arguments. Both matrix elements for the semileptonic transitions $D^* \rightarrow K \bar{\ell} \ell$ and $D \rightarrow K^* \bar{\ell} \ell$ semileptonic transitions at zero momentum transfer can be approximately written as:

\[ \langle K(p')|A^\mu(0)|D^*(p, \epsilon)\rangle \simeq \epsilon^\mu (m_D + m_K) A_1^{D^*K}(0), \]
\[ \langle K^*(p', \epsilon')|A^\mu(0)|D(p)\rangle \simeq \epsilon'^\mu (m_{D^*} + m_{K^*}) A_1^{D^*K*}(0). \]

At present no experimental information on the axial form factor $A_1^{D^*K}(0)$ is available, while the form factor $A_1^{D^*K*}(0)$ is known, although with significant uncertainties. SU(4) symmetry requires that the axial form factors $A_1^{D^*K}(0)$ and $A_1^{DK*}(0)$ satisfy the relation:

\[ (m_D + m_K) A_1^{D^*K}(0) = (m_{D^*} + m_{K^*}) A_1^{DK*}(0). \]
where the coupling constants \( g \) law is different from the 1

\[ A_{1}^{DK}(0) A_{\nu}(x) \left\{ D_{\nu}(x) K(x) + D(x) K_{\nu}(x) \right\} + \text{H.c.} \]  

(22)

In the following we calculate the leptonic decay constants \( f_{D*0} \) and \( f_{D*1} \) based on the effective interaction Lagrangian \( \mathcal{L}_{\text{eff}} \), which includes the couplings of \( D_{s0} \), \( D_{s1} \) mesons and the weak currents (vector \( V^\mu \) and axial \( A^\mu \)) with \( DK \) and \( D^*K \) meson pairs:

\[ \mathcal{L}_{\text{eff}}(x) = \mathcal{L}_{D*0}(x) + \mathcal{L}_{D*1}(x) + \mathcal{L}_{VDK}(x) + \mathcal{L}_{AD*K}(x). \]  

(23)

The corresponding diagrams are given in Fig.2. After a straightforward evaluation we obtain the following analytical expressions for the leptonic decay constants:

\[ f_{D*0} = \frac{g_{D*0}^{2}}{8\pi^{2}} f_{+}^{DK}(0) \int_{0}^{1} dx \int_{0}^{\infty} d\alpha \frac{\alpha}{(1+\alpha)^{2}} \tilde{\Phi}_{M}(z_{0}) \left[ 1 - \frac{2(w_{KD} + \alpha x)}{1 + \alpha} \right], \]  

(24)

\[ f_{D*1} = \frac{g_{D*1}^{2}}{8\pi^{2}} \frac{m_{D} + m_{K^{*}}}{m_{D_{s1}}} A_{1}^{DK^{*}}(0) \int_{0}^{1} dx \int_{0}^{\infty} d\alpha \frac{\alpha}{(1+\alpha)^{2}} \tilde{\Phi}_{M}(z_{1}) \left[ 1 + \frac{\alpha}{4\mu_{D}^{2}} \frac{d_{z_{1}}}{d\alpha} \right], \]  

(25)

where the coupling constants \( g_{D*0} \) and \( g_{D*1} \) are defined by Eqs. (12) and (13). We have only one model parameter in our calculations - the scale parameter \( \Lambda_{M} \) which was previously fixed to the value of \( \Lambda_{M} = 2 \text{ GeV} \) from strong and radiative decays of \( D_{s0} \) and \( D_{s1} \) mesons. Further quantities, entering in Eqs. (24) and (25), are chosen as follows. The masses of mesons we take from the PDG [10]

\[ m_{D*0} = 2.3173 \text{ GeV}, \quad m_{D_{s1}} = 2.4589 \text{ GeV}, \]  

\[ m_{K} \equiv m_{K^{*}} = 493.677 \text{ MeV}, \quad m_{K^{*}_{0}} \equiv m_{K^{*}_{0}} = 891.66 \text{ MeV}, \]  

\[ m_{D} \equiv m_{D^{*}} = 1.8693 \text{ GeV}, \quad m_{D_{s}} \equiv m_{D^{*}_{s}} = 2.010 \text{ GeV}. \]  

(26)

For the values of the \( D \to K^{(*)} \bar{\nu}_{\nu} \) semileptonic form factors at zero recoil, entering in our calculation, we use the world average data [39, 40, 41] of:

\[ f_{+}^{DK}(0) = 0.75 \pm 0.05, \quad A_{1}^{DK^{*}}(0) = 0.65 \pm 0.05. \]  

(27)

Using this input we get the following results for \( f_{D*0} \) and \( f_{D*1} \):

\[ f_{D*0} = 67.1 \pm 4.5 \text{ MeV}, \quad f_{D*1} = 144.5 \pm 11.1 \text{ MeV}. \]  

(28)

In Table 1 we summarize the present results for \( f_{D*0} \) and \( f_{D*1} \) obtained in different approaches (either on the basis of hadronic models or from the analysis of experimental data on two-body B-meson decays). Our results are in agreement with the predictions of Refs. [12, 17, 18, 20], especially with the lower limits derived from an analysis of the branching ratios of \( B \to D^{(*)} D_{s0}(D_{s1}) \) decays [18, 20].

At this point it is worth discussing the heavy quark limit (HQL) for the leptonic decay constants \( f_{D*0} \) and \( f_{D*1} \), where the masses of \( D, D^{*}, D_{s0} \) and \( D_{s1} \) mesons together with the charm quark mass \( m_{c} \) approach infinity. In the HQL the \( D^{(*)} \) mesons in the \( D_{s0}/D_{s1} \) hadronic molecules move to the center of mass and are surrounded by a light \( K \) meson in analogy with the heavy-light \( Qq \) mesons. It is known (see e.g. discussion in Refs. [17, 18]) that in the two-quark picture for the \( D_{s0}/D_{s1} \) mesons HQL leads to degenerate values for these couplings

\[ f_{D*0} \cong f_{D*1} \sim \frac{1}{\sqrt{m_{c}}}. \]  

(29)

In the present molecular approach we can also guarantee that the couplings are degenerate in the HQL, but the scaling law is different from the \( 1/\sqrt{m_{c}} \) behavior. In a first step we apply the HQL to the coupling constants \( g_{D*0} \) and \( g_{D*1} \), which are degenerate:

\[ \frac{1}{g_{D*0}^{2}} = \frac{1}{g_{D*1}^{2}} = \frac{I_{0}}{(4\pi m_{c})^{2}}, \]  

(30)

\[ I_{0} = \int_{0}^{\infty} \frac{d\alpha}{1 + \alpha} \Phi_{M}(\mu_{K}^{2} \alpha). \]
The structure integrals \( \int_0^1 \frac{d\alpha}{\alpha} \int 0^\infty \) entering in Eqs. (24) and (25) are also degenerate and equal
\[
I_1 = \frac{\Lambda_M}{m_c} \int_0^\infty \frac{d\alpha \sqrt{\alpha}}{1 + \alpha} \Phi_M^2(\mu^2 R, \alpha).
\]

Finally, the leptonic decay constants \( f_{D_{s0}} \) and \( f_{D_{s1}} \) in the HQL are given by:
\[
\begin{align*}
   f_{D_{s0}} &= \frac{\Lambda_M}{2\pi} g_{V_{DK}}^b h_{DK} \frac{I_1}{I_0}, \\
   f_{D_{s1}} &= \frac{\Lambda_M}{2\pi} g_{A_{DK}}^b h_{DK} \frac{I_1}{I_0},
\end{align*}
\]
where we introduced the effective \( V_{DK} \) and \( A_{DK} \) couplings in the HQL: \( g_{V_{DK}}^b \) and \( g_{A_{DK}}^b \). From Eqs. (32) and (33) we deduce that in the HQL \( f_{D_{s0}} \) and \( f_{D_{s1}} \) in the HQL limit do not depend on \( m_c \) at all, unlike as in Eq. (29), and are degenerate for \( g_{V_{DK}}^b = g_{A_{DK}}^b \). The latter condition can be eventually fulfilled, e.g. at finite masses (see Eqs. (24) and (25)) the ratio \( g_{A_{DK}}^b / g_{V_{DK}}^b \) is close to 1:
\[
\frac{g_{A_{DK}}^b}{g_{V_{DK}}^b} = \frac{m_B + m_K}{m_{D_{12}}} A_{DK}(0) / f_{DK}^b(0) \simeq 1.
\]

To give an estimate for the absolute values of the leptonic decay constants \( f_{D_{s0}} \) and \( f_{D_{s1}} \) in the HQL we vary the coupling constants \( g_{V_{DK}}^b = g_{A_{DK}}^b \) in the region \( 0.75 \pm 0.25 \). The corresponding result is:
\[
f_{D_{s0}} = f_{D_{s1}} = 205.2 \pm 68.4 \text{ MeV}.
\]

The following comment is in order. As was already stressed in Ref. \([18]\) the HQL does not give a reasonable approximation for the \( P \)-wave \( D_{s0} \) meson system. According to the data on the two-body decays \( B \to D^{(*)} D_{s0}^* (D_{s1}) \) the physical value of the decay constant \( f_{D_{s1}} \) should be about twice as large as \( f_{D_{s0}} \) \([18, 20]\).

### IV. Weak Two-Body Decays \( B \to D^{(*)} D_{s0}^* (D_{s1}) \)

In this section we give the predictions for the branching ratios of \( B \to D^{(*)} D_{s0}^* (D_{s1}) \) decays. For this purpose we use leptonic decay constants \( f_{D_{s0}} \), \( f_{D_{s1}} \) and, in addition, model-independent results for the form factors of \( B \to D^{(*)} \ell \bar{\nu}_\ell \) transitions obtained by Caprini, Lellouch and Neubert (CLN) \([28]\). Latter derivations are based on heavy quark spin symmetry, dispersive constraints, including short-distance and power corrections. Note, that in Ref. \([18]\) the authors already used the CLN results in their analysis of two-body \( B \to D D_{s0}^* (D_{s1}) \) transitions, restricting to modes containing the pseudoscalar meson \( D \) in the final state.

Working with the factorization approximation we first write down the factorizable amplitudes for the four decay modes \( B^0 \to D^+ D_{s0}^* \), \( D^+ D_{s1}^* \), \( D^{*+} D_{s0}^* \) and \( D^{*+} D_{s1}^* \) as expressed in terms of matrix elements of the semileptonic \( B \to D^{(*)} \ell \bar{\nu}_\ell \) and the leptonic \( D_{s0}^* \), \( D_{s1}^* \) transitions and (see details in Refs. \([17, 18, 20]\)):
\[
\begin{align*}
   M(B^0 \to D^+ D_{s0}^*) &= G_{eff}(D_{s0}^*- (q) \bar{s} O_{\mu} c (0) \langle D^+(p') | \bar{c} O^\mu p | B^0(p) \rangle), \\
   M(B^0 \to D^{*+} D_{s0}^*) &= G_{eff}(D_{s0}^*- (q) \bar{s} O_{\mu} c (0) \langle D^{*+}(p', \epsilon') | \bar{c} O^\mu p | B^0(p) \rangle), \\
   M(B^0 \to D^+ D_{s1}^*) &= G_{eff}(D_{s1}^* (q, \epsilon) \bar{s} O_{\mu} c (0) \langle D^+(p') | \bar{c} O^\mu p | B^0(p) \rangle), \\
   M(B^0 \to D^{*+} D_{s1}^*) &= G_{eff}(D_{s1}^* (q, \epsilon) \bar{s} O_{\mu} c (0) \langle D^{*+}(p', \epsilon') | \bar{c} O^\mu p | B^0(p) \rangle).
\end{align*}
\]

Here
\[
G_{eff} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* a_1
\]
and \( a_1 = c_2 + c_1/N_c \) is the combination of the short-distance Wilson coefficients \( c_1 \) and \( c_2 \) \([29, 31]\). Using the Wirbel-Stech-Bauer (WSB) decomposition \([33, 42]\) of the hadronic \( B \to D^{(*)} \) matrix elements
\[
\langle D^+(p') | V_{\nu}^\mu | B^0(p) \rangle = \left\{ (p + p')^\mu - \frac{m_B^2 - m_D^2}{q^2} q^\mu \right\} F_1(q^2) + \frac{m_B^2 - m_D^2}{q^2} q^\mu F_0(q^2),
\]

(41)
where $A_3(q^2) = \frac{m_B + m_{D^*}}{2m_{D^*}} A_1(q^2) - \frac{m_B - m_{D^*}}{2m_{D^*}} A_2(q^2)$.

we arrive at [17, 18, 20]:

$$M(B^0 \to D^+ D_s^{*-}) = G_{eff} f_{D_s^{*-}} (m_B^2 - m_{D_s^{*-}}^2) F_0(m_{D_s^{*-}}^2),$$

$$M(B^0 \to D^0 D_s^{*-}) = G_{eff} f_{D_s^{*-}} m_{D_s^{*-}} \epsilon^* (p + p') F_1(m_{D_s^{*-}}^2),$$

$$M(B^0 \to D^{*-} D_{s1}^{*-}) = 2G_{eff} f_{D_s^{*-}} m_{D_s^{*-}} \epsilon^* p_A (m_{D_s^{*-}}^2),$$

$$M(B^0 \to D^{*+} D_{s1}) = G_{eff} f_{D_{s1}} m_{D_{s1}} \left\{ \frac{2i\epsilon^{\mu \rho \alpha \beta}}{m_B + m_{D^*}} \epsilon^{\mu \nu}_{\rho} p_A \rho_B \epsilon^{\rho \sigma}_{\nu} V(m_{D_{s1}}^2) \right\} + \epsilon^* \epsilon^* (m_B + m_{D^*}) A_1(m_{D_{s1}}^2) - \frac{2 \epsilon^* p_A \rho_B}{m_B + m_{D^*}} A_2(m_{D_{s1}}^2).$$

In the following it is convenient to express the WSB set of form factors through a set of form factors $h_i(w)$ depending on the kinematical variable $w = \nu \cdot \nu'$, the scalar product of the four-velocities of the $B$ and $D^{(*)}$ mesons [28, 42, 43]:

$$\langle D(\nu')|V^\mu|B(\nu)\rangle = h_+(w) (v + v')^\mu + h_-(w) (v - v')^\mu,$$

$$\langle D^*(\nu', v')|V^\mu|B(\nu)\rangle = i h_1(w) \epsilon^{\mu \rho \alpha \beta} \epsilon_{\rho}^{\mu} v_{\alpha} v_{\beta},$$

$$\langle D^*(\nu', v')|A^\mu|B(\nu)\rangle = h_{A_1}(w) (w + 1) \epsilon^{\mu \nu} - \left[ h_{A_2}(w) v^\mu + h_{A_3}(w) v^\nu \right] \epsilon^* v,$$

where the meson states $|M(\nu)|$ obey the mass-independent normalization condition

$$\langle M(\nu')|M(\nu)\rangle = 2v^0 (2\pi)^3 \delta^3 (p - p'),$$

instead of the relativistic one used before:

$$\langle M(\nu')|p(\nu)\rangle = 2p^0 (2\pi)^3 \delta^3 (p - p').$$

The WSB form factors are expressed in terms of the $h_i(w)$ form factors as:

$$F_1(q^2) = \frac{1 + r^*}{2\sqrt{r^*}} V_1(w),$$

$$F_0(q^2) = \frac{2\sqrt{r^*}}{1 + r^*} \frac{w + 1}{2} V_1(w) R_1(w),$$

$$V(q^2) = \frac{1 + r^*}{2\sqrt{r^*}} h_{A_1}(w) R_1(w),$$

$$A_1(q^2) = \frac{2\sqrt{r^*}}{1 + r^*} \frac{w + 1}{2} h_{A_1}(w),$$

$$A_2(q^2) = \frac{1 + r^*}{2\sqrt{r^*}} h_{A_1}(w) R_2(w),$$

$$A_0(q^2) = \frac{1}{2\sqrt{r^*}} h_{A_1}(w) R_0(w),$$

where $r = m_D/m_B, r^* = m_{D^*}/m_B$ and

$$V_1(w) = G(w) = h_+(w) - \frac{1 - r}{1 + r} h_-(w).$$
Here we use the well-known form factor ratios $R_1(w)$ and $R_2(w)$ [28, 42]:

$$R_1(w) = \frac{h_V(w)}{h_{A_1}(w)}, \quad R_2(w) = \frac{h_{A_3}(w) + r^* h_{A_2}(w)}{h_{A_1}(w)}.$$  \hspace{1cm} (61)

In the literature the form factor $V_i(w)$ is also denoted as $G(w)$. In order to express all $h_i(w)$ form factors, completely defining the $B \to D^{(*)}$ transitions, through the two form factors $V_i(w)$ and $h_{A_1}(w)$ we introduce the additional ratios $R(w), R_0(w), R_2^*(w)$:

$$R(w) = \frac{h_+(w) - \frac{1 + r}{1 - r} h_-(w)}{h_+(w) - \frac{1 + r}{1 - r} h_-(w)}, \quad R_2^*(w) = \frac{h_{A_3}(w) - r^* h_{A_2}(w)}{h_{A_1}(w)}.$$ \hspace{1cm} (62)

and $R_0(w)$, which is just the combination of $R_2(w)$ and $R_2^*(w)$:

$$R_0(w) = (w + 1)[1 - R_2^*(w)] + \left(\frac{1 + r^*}{2r^*}\right)^2 \left[R_2^*(w) - \frac{1 - r^*}{1 + r^*} R_2(w)\right].$$ \hspace{1cm} (63)

For the functions $V_i(w) = G(w), h_{A_1}(w), R_0(w), R_1(w), R_2(w)$ and $R_2^*(w)$ we use the model-independent results derived by Caprini, Lellouch and Neubert (CLN) [28]:

$$\frac{G(w)}{G(1)} = 1 - 8 \rho_G^2 z + (51 \rho_G^2 - 10) z^2 - (252 \rho_G^2 - 84) z^3,$$ \hspace{1cm} (64)

$$\frac{h_{A_1}(w)}{h_{A_1}(1)} = 1 - 8 \rho_{h_{A_1}}^2 z + (53 \rho_{h_{A_1}}^2 - 15) z^2 - (231 \rho_{h_{A_1}}^2 - 91) z^3,$$ \hspace{1cm} (65)

$$R(w) = 1.004 - 0.007(w - 1) + 0.002(w - 1)^2,$$ \hspace{1cm} (66)

$$R_1(w) = 1.27 - 0.12(w - 1) + 0.05(w - 1)^2,$$ \hspace{1cm} (67)

$$R_2(w) = 0.80 + 0.11(w - 1) - 0.06(w - 1)^2,$$ \hspace{1cm} (68)

$$R_2^*(w) = 1.15 - 0.07(w - 1) - 0.11(w - 1)^2,$$ \hspace{1cm} (69)

where

$$z = \frac{\sqrt{w + 1} - \sqrt{2}}{\sqrt{w + 1} + \sqrt{2}}.$$ \hspace{1cm} (70)

In the derivation of the expressions for the ratios $R_0(w)$ and $R_2^*(w)$ we used the results of Ref. [28] (see Appendix). Note, that $R(w)$ is very close to 1 for $w$ in the interval $1 \leq w \leq w_{\text{max}} = (m_B^2 + m_D^2)/(2m_Bm_D)$.

The two-body decay widths $\Gamma(\bar{B} \to D^{(*)}D_{s0}^-)$ and $\Gamma(\bar{B} \to D^{(*)}D_{s1}^-)$, given in terms of the CLN form factors, are expressed by the formulas (see e.g. Ref. [18]):

$$\Gamma(\bar{B} \to D D_{s0}^-) = \frac{G_B^2}{8\pi m_B} f_{D_{s0}}^2 (m_B - m_D)^2 m_D^2 (w_1 + 1)^2 \frac{\sqrt{w_1^2 - 1}}{[G(w_1) R(w_1)]^2}$$ \hspace{1cm} (71)

$$\Gamma(\bar{B} \to D D_{s1}^-) = \frac{G_B^2}{8\pi m_B} f_{D_{s1}}^2 (m_B + m_D)^2 m_D^2 (w_2^2 - 1)^{3/2} \frac{G^2(\omega_2)}{2r^*}$$ \hspace{1cm} (72)

$$\Gamma(\bar{B} \to D^* D_{s0}^-) = \frac{G_B^2}{8\pi m_B} f_{D_{s0}}^2 m_B^2 m_{D^*}^2 (w_3^2 - 1)^{3/2} [h_{A_1}(w_3) R_0(w_3)]^2$$ \hspace{1cm} (73)

$$\Gamma(\bar{B} \to D^* D_{s1}^-) = \frac{G_B^2}{8\pi m_B} f_{D_{s1}}^2 (m_B - m_{D^*})^2 m_{D^*}^2 (w_4 + 1)^2 \frac{\sqrt{w_4^2 - 1}}{[h_{A_1}(w_4) \beta h_{A_1}(w_4)]^2},$$ \hspace{1cm} (74)

where the kinematical variables $w_i$ are defined as follows:

$$w_1 = \frac{m_B^2 + m_D^2 - m_{D_{s0}}^2}{2m_Bm_D}, \quad w_2 = \frac{m_B^2 + m_D^2 - m_{D_{s1}}^2}{2m_Bm_D},$$

$$w_3 = \frac{m_B^2 + m_{D^*}^2 - m_{D_{s0}}^2}{2m_Bm_{D^*}}, \quad w_4 = \frac{m_B^2 + m_{D^*}^2 - m_{D_{s1}}^2}{2m_Bm_{D^*}}.$$ \hspace{1cm} (75)
and
\[
\beta_{h_{A1}}(w) = \frac{1 - 2wr^* + r^{*2}}{(1 - r^*)^2} \left[ 1 + \frac{w - 1}{w + 1} R_1(w) \right] + \left[ 1 + \frac{w - 1}{1 - r^*} (1 - R_2(w)) \right]^2.
\] (76)

Note that the product \( h_{A1}^2 \beta_{h_{A1}} \) defines the well-known function \( F(w) \) [28, 42], which governs the semileptonic transition \( B \to D^* \ell \bar{\nu}_\ell \) (see e.g. Eq.(35) in Ref. [28]):
\[
h_{A1}^2 \beta_{h_{A1}}(w) \equiv F^2(w) \left[ 1 + \frac{4w}{w + 1} \left( 1 - 2wr^* + r^{*2} \right) \right].
\] (77)

In the numerical calculation we use the value for the Cabibbo-Kobayashi-Maskawa (CKM) matrix element \( V_{cb} \) [10, 41] and the 2006 averaged values from the Heavy Flavor Averaging Group (HFAG) [44]:
\[
|V_{cb}| F(1) = (36.2 \pm 0.8) \times 10^{-3}, \quad \rho_1^2 = 1.19 \pm 0.06, \quad \rho_2^2 = 1.17 \pm 0.18,
\] (78)
\[
|V_{cb}| G(1) = (42.4 \pm 4.4) \times 10^{-3}, \quad \rho_1^2 = 1.4 \pm 0.18,
\] (79)
where the normalization of \( F(w) \) and its slope \( \rho_1^2 \) at \( w = 1 \) are related to the characteristics of the \( h_{A1}(w) \) form factor as [28, 42]:
\[
F(1) \equiv h_{A1}(1), \quad \rho_1^2 \simeq \rho_{h_{A1}}^2 - 0.21.
\] (80)

For the parameter \( a_1 \) we use the value of 1.05 from Ref. [31]. A detailed discussion concerning the choice of \( a_1 \) can be found in Ref. [30].

In Table 2 we present our predictions for the branching ratios of two-body decays \( B \to D^*(s)D_{s0}(s_1) \). For the data we use the averaged lower limits from PDG [10] and in addition for the modes with \( D_{s1}(2460) \) meson in the final state the direct results of the BABAR Collaboration [9]. Our predictions are in good agreement with the experimental data except for the marginal situation in the case of \( B^0 \to D^{*-}D^s_0 \) decay, where our prediction is slightly lower than the experimental limit. In Table 3 we present the results for the ratios \( \Gamma(B \to D^*D_{s0}(s_1))/\Gamma(B \to DD_{s0}(s_1)) \) and \( \Gamma(B \to D^*D_{s1})/\Gamma(B \to DD_{s1}) \). We also use the compilation of experimental data and theoretical results within the covariant light-front (CLF) approach [17] summarized in Table 10 of Ref. [20]. Here, our predictions are in good agreement with the existing experimental data. In comparison with the CLF approach our predictions are lower, although not significantly.

V. CONCLUSION

We considered the new charmed-strange mesons \( D^*_0(2317) \) and \( D_{s1}(2460) \) as hadronic molecules, that is \( DK \) and \( D^*K \) bound states, respectively. Using an effective Lagrangian approach describing the coupling of \( D^*_0(2317) \) and \( D_{s1}(2460) \) states with their constituents we determined the leptonic decay constants \( f_{D^*_0} \) and \( f_{D_{s1}} \). Then we presented a detailed analysis of two-body bottom mesons decays \( B \to D^*(s)D_{s0}(s_1) \) using the factorization hypothesis and model-independent results for the form factors of the \( B \to D^*(s)\ell \bar{\nu}_\ell \) transitions obtained by Caprini, Lellouch and Neubert (CLN) [28]. The decay widths are derived in terms of the CLN form factors. Calculated branching ratios for \( B \to D^*(s)D_{s0}(s_1) \) decays are in good agreement with experimental results (or the lower limits), supporting a possible interpretation of \( D^*_0(2317) \) and \( D_{s1}(2460) \) as hadronic molecules.

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[1] J. L. Rosner, Phys. Rev. D 74, 076006 (2006) arXiv:hep-ph/0608102.
Table 1. Leptonic decay constants $f_{D^*_s}$ and $f_{D_{s1}}$.

| Approach | $f_{D^*_s}$ (MeV) | $f_{D_{s1}}$ (MeV) |
|----------|-----------------|-----------------|
| Ref. [23]| 225 ± 25        | 225 ± 25        |
| Ref. [24]| 206 ± 120       |                 |
| Ref. [25]| 200 ± 50        |                 |
| Ref. [21]| 170 ± 20        | 247 ± 37        |
| Ref. [14]| 138 ± 16        | 259 ± 13        |
| Ref. [22]| 110 ± 18        | 233 ± 31        |
| Ref. [18]| (74 ± 11)/|a_1| | (166 ± 20)/|a_1| |
| Ref. [17]| 71              | 117             |
| Ref. [17]| 60 ± 13         | 150 ± 40        |
| Ref. [20]| (58 - 86)/|a_1| | (90 - 228)/|a_1| |
| Ref. [12]| 67 ± 13         |                 |
| Ref. [11]| 44              | 41              |
| Our results | 67.1 ± 4.5       | 144.5 ± 11.1    |

Table 2. Branching ratios of $B \to D^{(*)} D^*_s (D_{s1})$ decays (in units of $10^{-3}$).

| Mode | Data (averaged) [10] | BABAR [9] | Our results |
|------|----------------------|-----------|-------------|
| $B^- \to D^*_s D^0$ | $> 0.74^{+0.23}_{-0.19}$ | 1.03 ± 0.14 | |
| $B^0 \to D^*_s D^+$ | $> 0.97^{+0.14}_{-0.34}$ | 0.96 ± 0.13 | |
| $B^- \to D^0 D^+$ | $> 1.4^{+0.5}_{-0.5}$ | 4.3 ± 1.6 ± 1.3 | 2.54 ± 0.39 |
| $B^0 \to D^0 D^+$ | $> 2.0^{+0.6}_{-0.5}$ | 2.6 ± 1.5 ± 0.74 | 2.36 ± 0.36 |
| $B^- \to D^0 D^{0*}$ | $> 0.9 ± 0.6^{+0.4}_{-0.3}$ | | 0.50 ± 0.07 |
| $B^0 \to D^* D^{0*}$ | $> 1.5 ± 0.4^{+0.3}_{-0.4}$ | | 0.47 ± 0.06 |
| $B^- \to D^0 D^{*+}$ | $> 5.5 ± 1.2^{+2.2}_{-1.6}$ | 11.2 ± 2.6 ± 2.0 | 7.33 ± 1.12 |
| $B^0 \to D^0 D^{*+}$ | $> 7.6 ± 1.7^{+3.2}_{-2.4}$ | 8.8 ± 2.0 ± 1.4 | 6.85 ± 1.05 |

Table 3. Ratios $MD^*/MD = \Gamma(B \to MD^*)/\Gamma(B \to MD)$ for $M = D^*_s, D_{s1}$.

| Mode | Data [20] | CLF [20] | Our results |
|------|-----------|----------|-------------|
| $D^*_s D^{0*}/D^*_s D^0$ | 0.91 ± 0.73 | 0.49 | 0.48 |
| $D^*_s D^{*+}/D^*_s D^+$ | 0.59 ± 0.26 | 0.49 | 0.48 |
| $D^0 D^{*0}/D^0 D^+$ | 3.4 ± 2.4 | 3.6 | 2.9 |
| $D^+ D^{*+}/D^+ D^+$ | 2.6 ± 1.5 | 3.6 | 2.9 |
FIG. 1: Mass operators of $D^*_0(2317)$ and $D_{s1}(2460)$ mesons.

FIG. 2: Diagrams related to the leptonic decay constants of $D^*_0(2317)$ and $D_{s1}(2460)$ mesons.