Research on accelerated test method for fatigue life of cylindrical helical compression

Tao Fu¹, Decheng Wang¹,² and Peng Cheng¹,²

¹ China Academy of Machinery Science and Technology, Beijing 100044, China
² School of Mechanical Engineering, University of Science and Technology, Beijing 100083, China
³ E-mail: wangdc@cam.com.cn

Abstract. In order to accelerate fatigue test, this paper present a technique to transform fatigue lives obtained from specimen tests under high stress levels to the non-tested one under low stress level. This technique is presumably based on the principle of the probability percentiles correspondence of specimen fatigue lives under different cyclic stress levels. The life distribution under low stress level can be derived from the ones under high stress levels. The corresponding conversion process is given, and an example analysis is carried out. The SPSS software is used to calculate the difference of the mean and variance between the equivalent life span and the real life span. The consistency of the mean and variance between the equivalent life span and the real life span is obtained and acceptable. Therefore, spring fatigue tests can be accelerated, that is, low stress life distribution can be derived from the fatigue tests under high stress levels, rather than test it under low stress level.

1. Introduction
Spring is an important bearing component, and widely used in Industrial Engineering. With the demand improvement of the product reliability, spring fatigue life of is required to meet certain requirements. In order to ensure spring fatigue life, for every batch of spring fatigue tests are required because different batch performs differently. Due to the improvement of steel smelting technology, the mechanical properties of spring wire have been greatly improved, and then the spring fatigue life is improved. So the original guiding significance of fatigue data is declining. At present, the service life of the spring belongs to the high cycle fatigue life, and the fatigue test takes long time [1]. Therefore, a new fatigue test method is needed to obtain the latest fatigue data.

Fatigue life curve describes the fatigue life of the specimen under various stress levels, but the change of life mean does not take the stress condition into account [2]. So fatigue life conversion under different stress conditions can’t be performed by using fatigue curves. In this paper, we will present a technique to derive low stress life (high cycle fatigue) from high stress life (low cycle fatigue). Conversion process of fatigue life under different stress conditions is given. Only if data distribution in the low life zone is obtained, the fatigue life in the high life zone can be deduced, and the experimental effect of fatigue acceleration is achieved.

2. Materials and methods
The material used in this research is SWOSC-V spring steel. Table 1 shows its Chemical composition. Dimension of designed specimen springs is listed in table 2.
Fatigue failure will occur during the working process of the spring specimen, and fatigue life depends mainly on specimen properties and the service condition [3]. Properties of the specimens involve properties of the raw material and its processing quality. Under certain service conditions, lifetime distribution of a set of test specimens is determined by the properties of the test specimens.

| Table 1. Chemical composition of the wire. (wt%) |
| C   | S   | Si  | Mn  | P   | Cr  | Cu  |
|-----|-----|-----|-----|-----|-----|-----|
| 0.57| 0.0050| 1.47| 0.73| 0.018| 0.70| 0.010|

| Table 2. Dimension of designed specimen springs. |
| shape parameter | outer diameter of coil (mm) | spring index | Free height (mm) | Number of active coils | Total number of coils | End shape |
|-----|----------------|----------------|-----------------|----------------------|---------------------|------------|
| numerical value | 20.8 | 6.43 | 50.4 | 4.0 | 6.0 | Closed end, Grinding |

Dispersion of specimen life spans also depends on the test specimen performance. Under a specific cyclic load, fatigue life distribution in a set of identical specimens is obviously dispersed. Difference of specimen lifetime is mainly due to the difference of the specimen performance. Formation of fatigue life can be divided into two stages: "sample extraction" and "life formation". Samples are selected during the "sample extraction" stage. In "life forming" phase, life value of each selected sample specimen under specified working load condition is formed. Fatigue performance of a sample is random and can be represented as a random variable X, whose distribution is of probability density, \( f(x) \). "Sample extraction" is a random process to extract a sub-sample \( x_i \) in a matrix whose probability density function is \( f(x) \). "Life formation" is a process to change the fatigue life of the samples under different cyclic loading conditions. For the entire sample matrix, it is equivalent to transform the probability density function \( f(x_i) \) into the life distribution function \( g(n_i) \). If the performance of sample observations are \( x(1), x(2)…x(n) \), and the corresponding life under cyclic loading \( S_i \) are \( n_i(1), n_i(2)…n_i(n) \), which satisfies \( n_i(1)<n_i(2)<…<n_i(n) \) and \( P(x<x_i)=P(N_i<n_i) \), since it is impossible to directly get fatigue life of a single specimen at different stress levels, under the premise of ensuring the same fatigue failure mechanism of different stress levels, it can be predicted that whether stress level is high or not, fatigue life of the same specimen at different stress levels will correspond to the same probability distribution of the life matrix, determined by their respective stress levels[4].

The fatigue and wear life of a specimen is subject to the normal distribution model, which is usually applied in reliability analysis [5]. A relationship of normal distribution corresponding to log-normal distribution exists. Below is the expression of probability density function for normal distribution:

\[
 f(t) = \frac{1}{\sqrt{2\pi} \sigma} \exp \left[ -\frac{1}{2} \frac{(t-\mu)^2}{\sigma^2} \right], \quad -\infty < t < \infty \tag{1}
\]

Where \( \mu \) is the average value of life distribution, \( \sigma \) is the standard deviation of life distribution. The corresponding cumulative distribution function is as follows:

\[
 F(t) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi} \sigma} \exp \left[ -\frac{1}{2} \frac{(t'-\mu)^2}{\sigma^2} \right] dt' \tag{2}
\]

\[
 z = \frac{T - \mu}{\sigma} \tag{3}
\]

Then \( Z \) obeys the normal distribution, and there are:
\[
F(t) = \Pr\{ t \geq T \} = \Pr\left\{ \frac{t - \mu}{\sigma} \geq \frac{T - \mu}{\sigma} \right\} = \Pr\left\{ \frac{t - \mu}{\sigma} \geq z \right\} = \phi\left( \frac{t - \mu}{\sigma} \right) \quad (4)
\]

According to above fatigue probability distribution, there is a correspondence between the life sample points at different stress levels. Therefore, if we know the mean and variance of the fatigue life under different stress levels, the life transformation can be carried out. That is:

\[
\frac{n_i - \mu_j}{\sigma_j} = \frac{n_k - \mu_k}{\sigma_k}
\quad (5)
\]

Where \( n_i \) is the logarithmic life of sample \( x_i \) at the stress level of \( S_i \), \( \mu_j \) is the mean logarithmic life under the stress level of \( S_j \), \( \sigma_j \) is the standard deviation of logarithmic life under the stress level of \( S_j \) (The meaning is the same when the subscript \( j \) is changed to \( k \)). According to the literature, we can see that there is an approximate correspondence among the mean fatigue logarithmic life, the standard deviation and the stress level \( S \). Therefore, the fatigue curve can be extended to predict the high life distribution under low stress condition by the life distribution under high stress condition. The conversion process is as follows:

1. Whether the test of fatigue data is subordinate to the log-normal distribution;
2. Calculate of logarithmic life mean and logarithmic life standard deviation under different stress conditions
3. Find the linear relationship between the mean and standard deviation of logarithmic life and the stress level \( S \)
4. Calculate of logarithmic life mean and logarithmic life standard deviation under low stress condition by linear relation
5. Speculate the life value of low stress condition from the formula (5) under high stress condition

3. Results

Compression fatigue tests are performed on cylindrical helical compression springs, and the fatigue logarithmic mean life and logarithmic life standard deviation are obtained under three different stress conditions. The test site is shown in figure 1. Statistic of the fatigue data are shown in table 3.

![Fatigue test site photo](image)

**Figure 1.** Fatigue test site photo.

**Table 3.** Fatigue data statistics.

| Maximum stress level S (Mpa) | Mean value of logarithmic life \( \mu \) | Std difference of logarithmic life \( \sigma \) |
|-----------------------------|----------------------------------------|-----------------------------------------------|
| 1230                        | 12.505                                 | 0.401                                         |
| 1172                        | 13.084                                 | 0.429                                         |
| 1070                        | 14.111                                 | 0.463                                         |
Linear relationships between the mean logarithm life, the standard deviation of logarithmic life with stress levels are fitted in least square estimation. The corresponding relations are as follows respectively:

\[
\mu_i = -0.01 S_i + 24.854 \\
\sigma_i = -0.0004 S_i + 0.8723
\]  

(6)

(7)

Using above obtained formula, the logarithmic fatigue life mean value and logarithmic life span standard deviation with a stress level of 1020Mpa can be predicted, and the logarithmic life expectancy and logarithmic life span are 14.654 and 0.4643, respectively.

Fatigue test of the same batch of springs under the stress level of 1020Mpa is carried out, and the fatigue life of the specimen at 1020Mpa is obtained. The logarithmic life expectancy and logarithmic life span are calculated to be 14.66 and 0.486 respectively. Compared with the equivalent conversion life, the relative errors are 0.041% and 4.47%, respectively, and quite small. In order to accurately determine the difference between true life and the equivalent life, the SPSS software will be applied to analyze [6].

Figure 2 shows the fatigue fracture of the spring under two stress conditions, including fatigue source, expansion zone and short break zone. All are typical fatigue fractures and consistent with the fracture mechanism.
4. Discussions
SPSS software is now applied to do a T-test on the equivalent life sample and the real life sample to determine the difference between the logarithmic life mean value and the logarithmic life standard deviation of the two samples. The test results are shown in table 5 and table 6.

| Table 5. Comparison between equivalent life and real life. |
|-----------------------------------------------|
| N | Mean   | Std. Deviation | Std. Error Mean |
|---|--------|----------------|-----------------|
| Real life | 29 | 14.6609 | 0.23589 | 0.04380 |
| Equivalent life | 11 | 14.6540 | 0.18678 | 0.05632 |

| Table 6. T test result. |
|-------------------------|
| Levene's Test for Equality of Variances  | t-test for Equality of Variances  |
|------------------------------------------|-----------------------------------|
| Equal variances assumed | F | Sig. | t | df | Sig. (2-tailed) | Mean Difference | Std. Error Difference |
| 1.118 | 0.297 | 0.087 | 38 | 0.931 | 0.0688 | 0.07932 |
| Equal variances not assumed | 0.096 | 22.782 | 0.924 | 0.0688 | 0.07135 |

From table 6 it can be seen that the standard deviation F statistic P value is 0.297, greater than the significance level of 0.05. Accept the null hypothesis that the standard deviation of the two samples is equal, and consider that there is no difference in the overall variance of the two samples. The variances of the two samples are equal, so refer to the T-test result of the first row. The corresponding probability P value in the first row is 0.931, far greater than the significance level of 0.05. Therefore, accepting the null hypothesis that there is no significant difference in the average logarithmic life span of the two samples, that is, the true logarithmic life is equivalent to the equivalent logarithmic lifespan. Equivalent life can be used instead of the real life.

5. Conclusions
In this study, spring fatigue tests are performed and life conversion under different stress conditions is carried out. Following is a summary:

According to the principle of consistency of fatigue life probability points, a process method for deriving low stress life from cylindrical helical compression spring high stress life is constructed.

(1) Spring fatigue test under different stress conditions is performed, and spring fatigue data are obtained. Life distribution under low stress conditions is derived and compared with the real life. SPSS software is applied to verify the result, and it shows that there is no significant difference between the real life and the equivalent life.

(2) This method derives the fatigue life distribution under low stress conditions from the life distribution under high stress conditions, which achieves the transition from high stress to low stress conditions, and it can improve the efficiency of the fatigue test.

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