Fractional plateaus of the Coulomb blockade of coupled quantum dots

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Ground-state properties of a double-large-dot sample connected to a reservoir via a single-mode point contact are investigated. When the interdot transmission is perfect and the dots controlled by the same dimensionless gate voltage, we find that for any finite backscattering from the barrier between the lead and the left dot, the average dot charge exhibits a Coulomb-staircase behavior with steps of size $e/2$ and the capacitance peak period is halved. The interdot electrostatic coupling here is weak. For strong tunneling between the left dot and the lead, we report a conspicuous intermediate phase in which the fractional plateaus get substantially altered by an increasing slope.

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I. INTRODUCTION

At low temperature, the charge on an isolated metallic grain (micronmetric dot) is known to be quantized in units of the electron charge $e$. Even when the grain is weakly-coupled to a bulk lead, so that electrons can occasionally hop from the lead to the dot and back, the grain charge remains to a large extent quantized. This is commonly referred to as the Coulomb blockade. In the opposite limit of perfect transmission between the reservoir and the dot, the average dot charge now depends (in a continuous manner) linearly on the applied gate voltage and the Coulomb blockade disappears. However, Matveev has shown that a crossover from the linear charge-voltage dependence to a Coulomb-staircase function occurs for any finite backscattering from the quantum point contact (QPC) between the grain and the lead. The physics remains qualitatively unchanged by increasing the reflection amplitude at the QPC.

Furthermore, close to the steps, the charge exhibits a nonanalytic logarithmic dependence on the voltage due to the presence of two spin channels entering the dot, resulting in an underlying two-channel Kondo model.

Note also that the Coulomb blockade can be smeared out by applying an in-plane magnetic field.

A direct measurement of the average grain charge, has been made possible using a single-electron transistor (SET) which has a sensitivity well below a single charge as well as a small input capacitance. In particular, some of the predictions above have been checked experimentally and its superiority to conductance measurements of charge fluctuations demonstrated.

Here, we investigate exotic Coulomb staircases with fractional plateaus.

The simplest system we consider comprises two large symmetric dots, which can be viewed as an artificial molecule, connected to a single reservoir via a single-mode QPC (Fig. 1). For a recent review on artificial molecules built up with two dots, see Ref. [1]. Here, each dot is coupled with the same capacitance $C_{gd}$ to a side-gate. The term “large dot” implies that the spacing $\Delta \sim L^{-2}$ of the energy levels on each dot vanishes compared to the dot’s charging energy $E_c = e^2/(2C_{\Sigma}) \sim L^{-1}$.

$\frac{1}{2}$

\begin{align*}
H_c[N] &= \frac{E_c}{2} (Q - 2N)^2 + 2E_c (Q_1 - \frac{Q}{2})^2 - 2E_c N^2. 
\end{align*}

The interdot capacitive coupling is weak in order to maximize the interdot charge fluctuations. Moreover, the symmetric dots are controlled by the same gate voltage $V_G$ and $N = V_G C_{gd}/e$. From the electrostatic Hamiltonian, it can be easily inferred that the double dot behaves as a single composite conductor of quantized charge $\epsilon Q = 2eQ_1$ determined by the total gate voltage $2N$. When an electron tunnels into the left dot, i.e., $Q = 1$, this implies that a charge $\epsilon$ is fluctuating back and forth between the dots and clearly $Q_{1,2} [N]$ exhibits steps of size $1/2$. Moreover, close to a point $2N = (2n + 1)/2$ ($n \in N$), the charge states with $Q = n$ ($Q_1 = n/2$)
and $Q = n + 1$ ($\bar{Q}_1 = n/2 + 1/2$) are degenerate resulting in (sharp) peaks in the single dot capacitance
$C_1 \propto \partial \bar{Q}_1 /\partial N$ (Fig. 2). Similar to the conductance peaks for two large dots tunnel-coupled to leads $^{10,11}$, we then observe that strong interdot charge fluctuations produce the {	extit{halving}} of the capacitance peak period. For an experimental proof, see e.g. Ref. $^{[11]}$.  

Based on two-impurity two-channel Kondo models ($2\text{CKMs}$) (small dots coupled to leads are described by a two-impurity $1\text{CKM}$ $^{15}$), below we thoroughly analyze the evolution of the fractional steps as a function of the hopping parameters $t_1$ and $t_2$ (Fig. 1). Some aspects of the problem will join up with previous works on the conductance through a double (large) dot structure $^{16,17}$. 

From here on, we assume that a single orbital channel with two spin polarizations $\alpha = \uparrow, \downarrow$ enters the double dot. Again, we assume that the level spacing on each dot (almost) vanishes which means that we consider a continuous spectrum in each dot and we neglect the mesoscopic corrections to the capacitance $C_{gd}$; the size of a dot can thus exceed the effective Bohr radius ($\sim \mu m$ in Refs. $^{[1,11]}$). Temperature will be taken to be zero ($T = 0$).

II. WEAK COUPLING WITH LEAD

Weak tunneling ($t_1 \ll 1$) between the lead and the composite dot produces corrections to the Coulomb staircase behavior found above.

More precisely, for perfect interdot transmission ($t_2 \to 1$), we can describe the composite dot in the vicinity of the two QPCs by the same field operator $\Psi_{co}(x)$. Additionally, close to a degeneracy point $N^* = (2n + 1)/4$, only the states with $Q = n$ and $Q = n + 1$ are allowed and, thus, following Ref. $^{[11]}$, the tunneling Hamiltonian for this truncated system takes the form:

$$H_t = \sum_\alpha \left(t_1 \Psi_{co}^\dagger(0) \Psi_{ra}(0) S^+ + h.c.\right).$$  

(2)

$\Psi_{ra}$ stands for the electron operator in the lead, and the spin operator $S^+$ guarantees that when an electron tunnels into the double dot, the total charge $Q$ only changes from $n$ to $n + 1$; we then have the equalities $^{15}$

$$\bar{Q} = 2\bar{Q}_1 = (n + 1/2) + \bar{S}_z.$$  

(3)

Following the route of the single-dot problem $^{15,16}$, now we can identify $s_{\alpha} = \Psi_{co}^\dagger(0) \Psi_{ra}(0)$ as an electron pseudo-spin operator acting on the orbital indices $\alpha = r, c$ and finally recover a $2\text{CKM}$. $^{16,17}$ The two channels are the two spin states of an electron. In particular, Eq. (3) can be viewed as a local magnetic field $h \bar{S}_z$ with $h \propto (2n + 1 - 4N)$. This results in

$$\bar{Q}_1 - \frac{2n + 1}{4} \propto (2n + 1 - 4N) \ln \left(|N - \frac{2n + 1}{4}|\right).$$  

(4)

To sum it up, we recover a standard logarithmic form

$$\delta C_1 = C_1 - C_{gd} \propto -\ln \left(|N - \frac{2n + 1}{4}|\right),$$  

(5)

for the capacitance peaks.

We now discuss the situation in which the interdot tunneling is strongly decreased, ($t_1; t_2 \ll 1$). Each dot is described by its own operator $\Psi_{ia}$ and the Coulomb term should be written in a more common way as $^{11,12}$

$$H^0_t[n] + H^0_c[n] = E_c \sum_{i=1,2} \left(Q_i - N\right)^2 - 2E_cN^2. $$  

(6)

When $t_2 \to 0$, we converge to a \textit{single-dot} problem $^{16}$. Each dot is quantized and for symmetric dots this guarantees $Q_1 = n/2$ until $t_2 \to 0$ (Eqs. (1), (2)).

FIG. 2: Charging energies ($+\delta E$) of the \textit{“composite”} dot as a function of $N$ given in units of $E_c$; $t_1$ is small. Each eigenstate with $Q = n$ gives rise to a parabola. The solid lines correspond to $t_2 = 0$ and dashed lines to increasing $t_2$ couplings. For $r_1 = 1 - t_1 \to 1$, $Q$ is quantized and for symmetric dots this guarantees $Q_1 = n/2$ until $t_2 \to 1$ (Eqs. (1), (2)).

As soon as $t_2$ is finite ($t_2 \sim t_1$) and $N \approx 1/2$, we propose to modify the tunneling Hamiltonian as:

$$H_t = \sum_\alpha \left(t_1 \Psi_{1a}^\dagger(0) \Psi_{ra}(0) + t_2 \Psi_{2a}^\dagger(L) \Psi_{1a}(L) + h.c.\right).$$  

(7)

where $S_{\alpha}^\dagger (S_{\alpha}^-)$ emphasizes that the charge on dot 1(2) only changes from 0 to 1; For more details, see note $^{[13]}$

Again, the index $j = 1, 2$ - which designates the location of an electron in the setup- in the $\Psi_{ja}$ operator can mimic an internal \textit{“orbital”} degree of freedom. It is then straightforward to define two spin operators at $x = 0, L$ acting on the orbital space, similar as in Ref. $^{[11]}$.

$$s_{\alpha} = \Psi_{1a}^\dagger(0) \Psi_{ra}(0)$$

(8)

$$s^-_{\alpha} = \Psi_{2a}^\dagger(L) \Psi_{1a}(L).$$

This two-impurity (two-channel) Kondo model is particularly convenient to revisit the behavior of charge fluctuations close to the degeneracy points $N^*_1$; the crucial
point being that a finite bare coupling \( t_2 \) (like \( t_1 \)) will be strongly renormalized at low temperature.\(^{20}\)

At the fixed point \((T = 0)\) and, e.g., close to the degeneracy point \( N_s^* = 1/2 \), the two dots will merge into one and therefore by analogy to Eq. (3) we must correctly reidentify

\[
\hat{Q} = (j + 1/2) \hat{S}_z = 2\hat{Q}_1,
\]

where \( j = (0;1) \).\(^{20}\) Moreover, the Coulomb term in the fixed-point basis takes the form \( hS_z \) where \( h \approx (1 - 2N + 2\kappa T_2) \) for \( j = (0;1) \); \( T_2 = (t_2)^2 \) and \( \kappa > 0 \). Away from the point \( N_s^* = 1/2 \), second order perturbation theory in \( t_2 \) is accurate, and we have taken into account the relative energy shift between even and odd \( Q \)-states.\(^{10,11}\)

\[
\delta E \propto -4T_2 \ln 2.
\]

Similar to Eq. (4), we are thus led to (for \( j = 0, 1 \) respectively)

\[
\hat{Q}_1 = \begin{cases} 
\frac{1}{4} - b(\frac{N}{2} + \kappa T_2) \ln \left| \left( N - \frac{1}{2} + \kappa T_2 \right) \right| , \\
\frac{1}{4} - b(\frac{N}{2} - \kappa T_2) \ln \left| \left( N - \frac{1}{2} - \kappa T_2 \right) \right| .
\end{cases}
\]

\[
(11)
\]

\( b > 0 \) is a parameter which is inversely proportional to the Kondo energy scale. By continuity, a tiny step appears at \( \hat{Q}_1 = 1/2 \), and the single-dot capacitance peaks are already split by \( \sim 2\kappa T_2 \) (Fig. 2).

The progressive pairing of the capacitance sub-peaks close to \( t_2 = 1 \) will be studied later (Eq. (24)).

III. STRONG COUPLING WITH LEAD

Now, we mainly consider the case where all the junctions have conductances close to \( 2e^2/h \), i.e., reflection amplitudes are small \((r_1; r_2) \ll 1\).

In this case, the whole system can be viewed as a single conductor and, for convenience, we will use the unique field operator \( \Psi_{ra}(x) \). We can write \( \Psi_{ra}(x) = \exp(ik_Fx)\Psi_{a}(x) + \exp(-ik_Fx)\Psi_{-a}(x) \). \( \Psi_{+a} \) and \( \Psi_{-a} \) describe right- and left-moving fermions respectively.

The kinetic energy obeys

\[
H_k = iv_k \int_{-\infty}^{+2L} dx \left( \Psi_{\downarrow a}^\dagger \partial_x \Psi_{\uparrow a} - \Psi_{\downarrow a}^\dagger \partial_x \Psi_{\downarrow a} \right),
\]

\[
(12)
\]

\( v_F \) being the Fermi velocity. The backscattering term(s) takes the standard form:

\[
H_b = v_F \sum_{a} \left( r_{12} \Psi_{\downarrow a}^\dagger (0,L) \Psi_{\downarrow a} (0,L) + h.c. \right),
\]

\[
(13)
\]

and interactions in a grain are embodied via the general Coulomb Hamiltonians \( H_c[N_1] \) and \( H_c[N_2] \) in Eq. (4).

At low energy, we proceed with this model by bosonization of the one-dimensional Fermi fields. In those variables, the kinetic energy yields a separation of the spin and charge and the resulting Hamiltonians have plasmon-like excitations. Here, \( \partial_x \phi_j \) with \( j = (c, s) \) measures fluctuations of charge/spin density and \( \Pi_j = \partial_x \theta_j \) being its conjugate momentum. The Coulomb Hamiltonians take the forms (We could also employ Eq. (1)):

\[
H_c^1[N_1] = \frac{2E_c}{\pi} \left( \phi_c(0) - \phi_c(L) - \sqrt{\frac{\pi}{2}} N_1 \right)^2 - E_cN_1^2
\]

\[
(14)
\]

\[
H_c^2[N_2] = \frac{2E_c}{\pi} \left( \phi_c(L) - \phi_c(2L) - \sqrt{\frac{\pi}{2}} N_2 \right)^2 - E_cN_2^2.
\]

To minimize the Coulomb energies when the transmissions at the two QPCs are both perfect, we easily recover that the dot’s charges evolve continuously (linearly) as a function of the gate voltages: \(^{34}\)

\[
\hat{Q}_1 = \frac{\sqrt{2}}{\pi} \left( \phi_c(0) - \phi_c(L) \right) = N_1
\]

\[
(15)
\]

\[
\hat{Q}_2 = \frac{\sqrt{2}}{\pi} \left( \phi_c(L) - \phi_c(2L) \right) = N_2.
\]

Remember that for \( r_1 = r_2 = 0 \), the Coulomb blockade physics is totally suppressed. In our geometry there are no charge fluctuations at \( x = 2L \) and then \( \phi_c(2L) = \text{const} \).

Furthermore, following the traditional route of the single-dot problem for this regime, the backscattering term may be rewritten as

\[
H_b = \sqrt{\frac{\gamma a E_c v_F}{\pi a}} 4r_1 \cos(\pi(N_1 + N_2)) \cos(\sqrt{2\pi} \phi_s(0)) \mathcal{T}_{1x}
\]

\[
+ \sqrt{\frac{\gamma a E_c v_F}{\pi a}} 4r_2 \cos(\pi(N_2)) \cos(\sqrt{2\pi} \phi_s(L)) \mathcal{T}_{2x}.
\]

(16)

Since the charge fluctuations on each dot cannot depend on the precise size of a dot, we must equate \( \phi_c(2L) = 2k_F L/\sqrt{2\pi} \) and rescale \( \phi_s(0) \rightarrow \phi_s(0) + 2k_F L/\sqrt{2\pi} \) here \( \gamma \) obeys \( \gamma = e^C \) where \( C \approx 0.5772 \ldots \) is the Euler-Mascheroni constant and \( a \) is a short-distance cutoff. We have introduced two commuting impurity spins \( \mathcal{T}_1 \) and \( \mathcal{T}_2 \) (which here are not related to the charge on each dot).

Clearly, the \( \mathcal{T}_{1x} \) and \( \mathcal{T}_{2x} \) spin-operators both commute with the Hamiltonian and must be simply identified as numbers, i.e., \( \mathcal{T}_{1x} = 1/2 \) (or \(-1/2\)) and similarly for \( \mathcal{T}_{2x} \). Eq. (10) must be viewed as an extension of the 2CKM at the Emery-Kivelson limit.\(^{22}\)

To compute the correction to the average dot charge(s), here we must “de-bosonize” the problem as

\[
H_b \approx \frac{iJ_{1x}}{\sqrt{4\pi a}} \left( \psi(0) + \psi^\dagger(0) \right) \zeta_1
\]

\[
+ \frac{iJ_{2x}}{\sqrt{4\pi a}} \left( \psi(L) + \psi^\dagger(L) \right) \zeta_2.
\]

\( \zeta_1 \) and \( \zeta_2 \) are two Majorana fermions, and the Kondo exchanges above lead \( J_{1x} = 4r_1 \sqrt{\gamma a E_c v_F} \cos(\pi(N_1 + N_2)) \) and \( J_{2x} = 4r_2 \sqrt{\gamma a E_c v_F} \cos(\pi N_2) \). In the absence of an applied magnetic field, there is no net magnetization and no spin current on the whole region \([-L; L]\) and, thus, we have approximated, \( \psi(L) \approx \exp(i\sqrt{2\pi} \phi_s(L)) \) (For more
explanation, see Ref. 6). The fermionic model here generates two Kondo resonances

\[ \Gamma_1 = \frac{J_1^2}{4\pi u_F} = \frac{E_c \gamma}{\pi} (2r_1)^2 \cos^2(\pi(N_1 + N_2)) \]

and all the quantities of interest will be now inferred from the quantum correction of the ground state energy

\[ \delta E = -\frac{\Gamma_1}{\pi} \ln(E_c/\Gamma_1) - \frac{\Gamma_2}{\pi} \ln(E_c/\Gamma_2), \]

which implies that impurities are independently screened. Let us discuss the case of symmetric dots: \( N_1 = N_2 = N \).

The correction to the average charge on each dot \( \delta \bar{Q}_i \) and the dot’s differential capacitance \( \delta C_i \) obey: \( \delta \bar{Q}_i = \bar{Q}_i - N \propto -\delta E/(E_c \partial N) \) and \( \delta C_i \propto \partial \delta \bar{Q}_i/\partial N \). For clarity’s sake, results have been summarized in Figs. 3, 4.

A. \( r_2 \to 0 \)

For a double dot connected by a reflectionless constriction \( 0 \to r_2 \ll r_1 \ll 1 \), using the formulas above, we easily recover fractional charge plateaus with steps 1/2 and capacitance peaks with halved period. Again, the double dot behaves as a single composite conductor of quantized charge \( Q \approx 2\bar{Q}_1 \). In particular, we predict that the logarithmic singularity \( \delta C_i \propto -\ln(|N - \frac{1}{2}|) \) should be observed at any value of \( t_1 \neq 1 \) (as nicely illustrated in Figs. 3 and 4).

B. \( r_2 \to 1 \)

In the opposite limit \( r_1 \ll r_2 \to 1 \), the interdot constriction considerably impedes the charge spreading between the dots. \( Q_2 \) becomes an integer-valued operator describing electrons which tunnel into the dot 2 (Eqs. 6), and charge fluctuations in dot 1 closely resemble the ones of a single dot which is strongly-coupled to one lead:

\[ H_b \propto r_1 (-1)^{Q_2} \cos(\pi N) \cos\left(\sqrt{2\pi}\phi_s(0)\right) T_{1x}. \]

The Kondo energy scale

\[ \Gamma_1 = \frac{E_c \gamma}{\pi} (2r_1)^2 \cos^2(\pi N), \]

is identical to the one of the single-dot problem, and assuming \( r_1 \neq 0 \), \( Q_1 \) becomes also quantized.

The small term \( t_2 \psi_{1a}(L)\psi_{1a}(L)S_z^2 + h.c. \) here mostly produces slight charge fluctuations in the dot 2, and \( \delta C_2 \propto -\ln(|N - 1/2|) \).

C. \( r_1 \approx r_2 \ll 1 \)

For \( r_1 \approx r_2 \), a strong opposition between the single-dot (\( Q_1 \) is quantized for \( r_2 \gg r_1 \)) and the composite-dot ground-state (\( Q_1 = Q/2 \) for \( r_1 \gg r_2 \)) arises giving a fascinating “hybrid” regime where the fractional plateaus become gradually destroyed by acquiring a positive slope (Fig. 4); Close to \( N = 1/2 \), exploiting Eqs. 6 and 10, we can approximate \( (f(R_1) = \ln(R_1 + \text{const.}) \)

\[ \delta \bar{Q}_1 \propto (N - 1/2) \left(R_1 f(R_1) - R_2 \ln(|N - 1/2|)\right), \]

then inducing an exotic “3-peak” capacitance profile; \( R_1 = (r_1)^2 \) (inset in Fig. 4). The central peak becomes more pronounced by slightly increasing \( r_2 \), whereas the external peaks only depends on \( r_1 \) (as long as \( r_2 \ll 1 \)).
D. \( r_1 \to 1 \) and \( r_2 \ll 1 \)

It is worthwhile to compare with the case \( r_1 \to 1 \) and \( r_2 \ll 1 \). Here, \( Q = \sqrt{2/\pi \phi_c(0)} = n \) must be an integer-valued operator which guarantees \( Q_1 = n/2 \). The fractional plateaus remain by decreasing the inter-dot coupling and only their widths progressively reduce: \( N^*(n = 1) - N^*(n = 0) = 1/2 - 2nR_2 \ln(1/R_2); \) \( \eta > 0 \) is a constant parameter. More precisely, for \( N_1 = N_2 = N \), it is easy to rewrite the backscattering term as:

\[
H_b = \frac{\sqrt{\gamma} aE_0 v_F}{\pi a} 4r_2 \cos \left( \frac{n\pi}{2} \right) \cos \left( \sqrt{2\pi \phi_c(L)} \right) T_{2z},
\]

(23)

to Eqs. \( \text{[4], [1]} \), which then produces a Kondo energy scale \( \Gamma_2 = E_0 \gamma(2r_2)^2 \cos^2 \left( \frac{n\pi}{2} \right) / \pi \), and then a relative energy shift \( \delta E \propto R_2 \ln(1/R_2) \) between even and odd states.\(^{10,11}\)

This engenders that the positions of the capacitance (sub-)peaks (furnished by Eq. 1) are shifted as:

\[
N^* = (2n + 1)/4 + (-1)^n \eta R_2 \ln(1/R_2).
\]

(24)

The capacitance (sub-)peaks are not equally spaced anymore and progressively pair around the points \( N^*_n = (2n + 1)/2 \) (Fig. 2). Finally, we have checked that integer plateaus become more prominent: \( N^*(n = 2) - N^*(n = 1) = 1/2 + 2nR_2 \ln(1/R_2) \).

IV. CONCLUSION

In closing, based on two-impurity two-channel Kondo models, we have presented a detailed discussion on the evolution of the fractional plateaus as a function of the hopping parameters \( t_1 \) and \( t_2 \) for a double-dot coupled via a single-mode QPC to a reservoir.

Again, for perfect interdot transmission, Coulomb steps of size 1/2 occur for any finite backscattering between the lead and the left dot. When an electron enters the artificial molecule, a charge 1 is fluctuating back and forth between the two dots. We are hopeful that this can be observed via capacitance measurements.\(^{14,16}\)

Substantially decreasing the interdot coupling inevitably restores the single-dot Coulomb blockade and the capacitance peak period doubles.

For strong coupling between the lead and the left dot \( (r_2 \ll 1) \), we find a striking intermediate range \( (r_2 \approx r_1) \) where the fractional steps become progressively unstable, i.e., show an increasing positive slope; this happens due to the strong competition between a single-dot and a composite-dot ground state. On the contrary, when \( r_1 \to 1 \), \( Q \) must be quantized and \( Q_1 = Q/2 \); the fractional steps persist.

For asymmetric dots, e.g., with different gate-dot capacitances, we report that the Coulomb staircase with halved steps is gradually altered.

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2 spin operators) $t_2 \Psi_{2\alpha}^\dagger(L)\Psi_{1\alpha}(L)S_2^+S_1^-$; but, this is not “renormalized” at low energy and we ignore it. Again, here the coupling $S_{1z}S_{2z}$ ($Q_1Q_2$) is negligible.

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