GRAVITATIONAL WAVES FROM VACUUM FIRST-ORDER PHASE TRANSITIONS

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Daniel Cutting
University of Sussex

Phys. Rev. D 97, 123513 [DC, Mark Hindmarsh, David Weir]
• First order phase transitions proceed through bubble nucleation and merger.
• Collision of bubble walls can source gravitational waves.
• In a vacuum transition bubble walls accelerate until collision.
  ➢ i.e Fluid effects on wall minimal, behaves as if in a vacuum.
• Previous studies mostly use envelope approximation:
  ➢ Stress-energy concentrated in infinitesimally thin shell at bubble wall.
  ➢ Neglect overlap regions once bubbles have collided.

[Kosowsky et al, 1992] [Huber and Konstandin, 2008][Weir, 2016] [Konstandin, 2017]
• LISA will be sensitive to gravitational waves from a first-order transition around the electroweak scale.

• Extensions to the Standard Model can generate a first order phase transition around the electroweak scale.

• Can probe BSM physics if we can characterise the GW signal.
TOY MODEL

- Single real scalar field $\phi(x, t)$ with potential as follows:
  \[ V(\phi) = \frac{1}{2} M^2 \phi^2 + \frac{1}{3} \delta \phi^3 + \frac{1}{4} \lambda \phi^4. \]
- Vary parameters $M^2, \delta, \lambda \rightarrow$ critical bubble radius $R_c$, and wall width $l_0$.
- The scalar field evolves according to:
  \[ \Box \phi - V'(\phi) = 0. \]
- Then the energy momentum tensor is given by:
  \[ T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - \eta_{\mu\nu} \left( \frac{1}{2} (\partial \phi)^2 + V(\phi) \right). \]
Transverse traceless metric perturbation evolves as

$$\Box h^{TT}_{ij} = 16\pi G T^{TT}_{ij}. $$

$P_\dot{h}(k, t)$ is the spectral density of the time derivative of $h^{TT}_{ij}$:

$$\langle \dot{h}^{TT}_{ij}(k, t) \dot{h}^{TT}_{ij}(k', t) \rangle = P_\dot{h}(k, t)(2\pi)^3 \delta(k + k').$$

Then the gravitational wave density parameter power spectrum is

$$\frac{d\Omega_{GW}}{d\ln(k)} = \frac{d\rho_{GW}}{d\ln(k)} \frac{1}{\rho_c} = \frac{1}{32\pi G \rho_c} \frac{k^3}{2\pi^2} P_\dot{h}(k, t).$$
INITIAL CONDITIONS

• Different nucleation scenarios:
  - simultaneous
  - constant nucleation rate
  \[ p(t) = p_c \]
  - exponential nucleation rate
  \[ p(t) = p_0 \exp(\beta t) \]

• Mean bubble separation:
  \[ R_\ast = \left( \frac{V}{N_b} \right)^{1/3} \]

• Lorentz factor of bubble wall at collision given by
  \[ \gamma_\ast = \frac{1}{2} \frac{R_\ast}{R_c} \]
\( \gamma_* \approx 2, N_b = 512 \)

- Dual peak structure.

- IR peak scales with bubble radius (dotted line).

- UV peak scale set by wall width (coloured dashed lines).

- Most models have \( R_* \gg l_0 \) and so UV peak suppressed.
The figure shows the evolution of the primordial gravitational wave spectrum, with the peak at IR frequencies. The spectrum is plotted on a logarithmic scale, with the wave number $k$ on the x-axis and the dimensionless quantity $d\Omega_{gw}/d\ln(k)$ on the y-axis. The spectrum is approximately given by $k^3$ for small $k$, $k^{-1}$ in the intermediate region, and $k^{-1.5}$ for large $k$.

For the $N_b$ values of 8, 64, 512, and 4096, the spectrum is shown in different colors. The parameters $\beta/M$ and $p_c/M^4$ are also indicated in the legend. The figure highlights the dependence of the spectrum on these parameters.

The label $\gamma_* \approx 4$ is placed in the figure, indicating a specific value or behavior in the context of the graph.
CONCLUSIONS

• Peak gravitational wave power has approximate agreement with envelope approximation fit.
• Peak frequency agrees well with the envelope approximation.
• Power law steeper than envelope for large $\gamma_*$, with $k^{-1.5}$ instead of $k^{-1}$.
• Second peak from scalar field oscillations found in UV.
• UV peak will have negligible contribution for most models.
ENVELOPE APPROXIMATION

• Assumptions:
  Ø Stress-energy concentrated in infinitesimal thin shell.
  Ø Neglect any region where bubbles overlap.

• Broken power law:
  Ø Rises as $k^3$ in IR.
  Ø Falls like $k^{-1}$ towards UV.
  Ø Peak location and amplitude given by $R_*$.

[Huber and Konstandin, 2008] [Konstandin, 2017]
UV PEAK GROWTH

- Linear growth of UV peak contribution to spectrum $\Omega_{gw}^{osc}$,
  \[
  \frac{d\Omega_{gw}^{osc}}{dt} \sim 10^{-1} \frac{(H_* l_0 \Omega_{vac})^2}{R_*}.
  \]

- Contribution from the bubble collisions is
  \[
  \Omega_{gw}^{coll} \sim 10^{-3} (H_* R_* \Omega_{vac})^2.
  \]

- Ratio of contributions:
  \[
  \frac{\Omega_{gw}^{osc}}{\Omega_{gw}^{coll}} \lesssim 10^2 \frac{n_b}{H^3} \left( \frac{M_b}{m_{Pl}} \right)^2.
  \]
$\gamma_* \simeq 4, N_b = 512$

- Can turn off evolution of metric perturbations until bubbles finish colliding.
- Ringing in the IR for scales above $R_*$. 
- UV peak continues to grow until late times.
- Growing plateau extending from $R_*$ up until the UV peak.
**GAMMA DEPENDENCE**

- IR peak dependence on $\gamma_*$ only via $R_*$

- Power law towards UV becomes steeper than $k^{-1}$ for $\gamma_* > 2$. 

- $\sim k^{-1}$

\[
\frac{1}{(H_* R_*)^2 \Omega_{\text{GW}}} \sim k^{-1}
\]
\[ \gamma_* \approx 4, N_b = 64 \]
Simultaneous nucleation

- Black dots gives spectrum from envelope simulation.
- Coloured lines show spectrum from lattice simulation.
• 3+1 dimensional classical lattice simulation.
• Built using LATfield2, an open source massively parallel lattice code. [Daverio, Hindmarsh and Bevis, 2015]
• Periodic boundary conditions.
• The leapfrog algorithm to evolve fields.
• Resolve the bubble wall:
\[ dx \ll l_* = l_0/\gamma_* \]
COSMOLOGICAL FIRST-ORDER PHASE TRANSITIONS

• In cosmological phase transitions (PTs) an (effective) scalar field transitions from false vacuum to true vacuum.
• In a first-order PT phases separated by potential barrier.
• Proceed through the nucleation and merger of bubbles.
• Extensions to the Standard Model can generate a first-order PT at the electroweak scale.
THERMAL VS VACUUM PHASE TRANSITIONS

• Thermal PTs:
  - Thermally fluctuate over barrier.
  - Fluid shell around bubble wall, exerts friction.
  - Terminal wall velocity.
  - Free energy difference mostly shared between bulk motion of fluid and thermal radiation.

• Vacuum PTs:
  - Quantum tunnel through barrier.
  - Fluid effects negligible.
  - Bubble wall accelerates until collision.
  - Free energy difference deposited into motion of the bubble wall.
  - Limit of highly super cooled PTs or a PT in a hidden sector.
Colliding bubbles break spherical symmetry → radiate gravitational waves (GWs).

In thermal PTs the dominant GW signal is from acoustic oscillations in fluid.

In vacuum PTs the GW signal expected to be from shear stress in scalar field at bubble wall.

Scalar field contribution to GW signal previously studied using envelope approximation.

E.g [Kosowsky et al, 1992] [Huber and Konstandin, 2008] [Weir, 2016] [Konstandin, 2017]
LASER INTERFEROMETER SPACE ANTENNA (LISA)

• Space based gravitational wave observatory using laser interferometry.
• Planned launch date of 2034.
• Three arms with length of 2.5 million km.
• Among goals is direct detection of a stochastic GW background of cosmological origin. [arXiv:1702.00786]
• Sensitive to GWs with frequencies of $10^{-4} Hz$ to $10^{-1} Hz$. [arXiv:1702.00786] (Electroweak scale)
CRITICAL BUBBLE PROFILE

\[ V(\phi) / \Delta V \]

\[ \phi / \phi_b \]

\[ \phi / \phi_b \]

\[ rM \]
BUBBLE EXPANSION

Thin, $t = 0.1$

Thick, $t = 0.1$

Movie credit: Elva Granados Escartin