Measurement of the $Q^2$-evolution of the Bjorken integral and extraction of an effective strong coupling constant at low $Q^2$.

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We report on the measurement of the Bjorken sum in the range $0.16 < Q^2 < 1.1$ GeV$^2$. The extraction of an effective strong coupling constant is then discussed.

1. Bjorken Sum Rule

The Bjorken sum rule [1] has been of central importance for studying the spin structure of the nucleon. Accounting for finite $Q^2$ corrections to the sum rule, it reads:

$$\int_0^1 (g_1^p - g_1^n) dx = \frac{g_a}{6} \left[1 - \frac{\alpha_s}{\pi} - 3.58 \left(\frac{\alpha_s}{\pi}\right)^2 - 20.21 \left(\frac{\alpha_s}{\pi}\right)^3 + \ldots \right] + \sum_{i=1}^{\infty} \frac{\mu_{2i+2}^{p-n}}{Q^{2i}},$$

where the $\mu_{2i+2}^{p-n}$ are higher twist terms. The sum rule has been checked experimentally at $Q^2=5$ GeV$^2$ to better than 10%. As recently realized, the Bjorken sum rule is related to a more general sum rule, the generalized Gerasimov-Drell-Hearn (GDH) sum rule [2,3]:

$$\int_0^1 (g_1^p - g_1^n) dx = \frac{Q^2}{16\pi^2\alpha_s} \left(\text{GDH}_p(Q^2) - \text{GDH}_n(Q^2)\right).$$

Since the generalized GDH sum is, in principle, calculable at any $Q^2$, it can be used to study the transition from the partonic to hadronic degrees of freedom of the strong force. However, the validity domains for chiral perturbation theory ($\chi$PT) at low $Q^2$ and pQCD calculations at higher $Q^2$ used to calculate the GDH sum do not overlap. The Bjorken sum is the flavor non-singlet part of the GDH sum. This leads to simplifications that may help in linking the ($\chi$PT) validity domain to the pQCD validity domain [4]. Hence the Bjorken sum appears as a key quantity to study the hadron-parton transition.

We used data from the Thomas Jefferson National Accelerator Facility (JLab) [5,6,7] to extract the Bjorken sum from $Q^2 = 0.16$ to 1.1 GeV$^2$ [8] (Fig. 1 panel A). At low $Q^2$, we can compare to $\chi$PT calculations [9,10]. The data agree well with models [11,12] and also with Eq. 1 calculated to third order in $\alpha_s$ and to leading twist. The agreement between the data and the leading twist calculation down to quite low $Q^2$ indicates that overall higher twist effects [8] are small.

2. The Effective Strong Coupling Constant, $\alpha_s^{\text{eff}}$

$\alpha_s$ can be extracted from Eq. 1 if higher twists are known or negligible. This is not the case here. This difficulty disappears when considering effective coupling constants [13].
In that case, $\alpha_s^{\text{eff}}$ contains higher twists and QCD radiation effects. As a consequence, $\alpha_s^{\text{eff}}$ is analytical at any $Q^2$ and renormalization scheme independent. However, $\alpha_s^{\text{eff}}$ becomes process dependent which is not a problem since these different coupling constants are related by “commensurate scale relations” that connect observables without scheme or scale ambiguity [14,15]. Following this procedure, $\alpha_s^{\text{eff}}$ is extracted using the equation:

$$\Gamma_1^{p-n} = \frac{1}{6}g_a[1 - \frac{\alpha_s^{\text{eff}}}{\pi}].$$

Such $\alpha_s^{\text{eff}}$ is shown in Fig. 1B, together with $\alpha_s^{\text{eff}}$ extracted using $\Gamma_1^{p-n}$ from Eq. 1 computed to third order in $\alpha_s$ and to leading twist. Also shown are $\alpha_s$ calculated to order $\beta_0$, $\alpha_s^{\text{eff}}$ calculated with the model [12], and $\alpha_s^{\text{eff}}$ extracted from world data on the Bjorken sum. $\alpha_s^{\text{eff}}$ merges with $\alpha_s$ at large $Q^2$ since their difference is due to higher twists and gluon bremsstrahlung. At $Q^2 = 0$, the GDH sum rule gives the slope of $\alpha_s^{\text{eff}}$. The data, together with the constraint at $Q^2 \simeq 0$, hint that $\alpha_s^{\text{eff}}$ has no $Q^2$ scale dependence at low $Q^2$.

In QED or QCD, only loops on the exchanged photon or gluon are responsible for the running of the coupling constant because of the Ward identities. Consequently, theoretical calculations of the running coupling constant deal only with dressed propagators. We can assume that, in order to compare to non-perturbative calculations of $\alpha_s^{\text{eff}}$, we do not need to include in $\alpha_s^{\text{eff}}$ the gluon bremsstrahlung and vertex corrections. This amounts to not folding the QCD radiative corrections into $\alpha_s^{\text{eff}}$ and redefining it using the equation:

$$\Gamma_1^{p-n} = \frac{g_a}{6}[1 - \frac{\alpha_s^{\text{eff}}'}{\pi} - 3.58 \left(\frac{\alpha_s^{\text{eff}}'}{\pi}\right)^2 - 20.21 \left(\frac{\alpha_s^{\text{eff}}'}{\pi}\right)^3 - 130 \left(\frac{\alpha_s^{\text{eff}}'}{\pi}\right)^4 - 893 \left(\frac{\alpha_s^{\text{eff}}'}{\pi}\right)^5].$$

The error from the series truncation is estimated by taking the difference between the fourth and fifth orders. With this redefinition, $\alpha_s^{\text{eff}}'$ becomes scheme-dependent (we work in the MS scheme). The result is shown in the panel C of Fig. 1 along with world data, the running of $\alpha_s$ from pQCD and estimates of the phenomenological running constant. In ref. [16] a solution to the Dyson-Schwinger equations regularizes the infrared behavior of $\alpha_s$ by generating an effective gluon mass that is found to be $m_g = 500 \pm 200$ MeV. For us, $m_g$ is constrained by the GDH sum rule which determines the derivative of the Bjorken integral at $Q^2 = 0$. This imposes $\alpha_s^{\text{eff}}'(Q^2 = 0) = 0.629 \pm 0.086$ which in turn constrains the gluon mass at the photon point to be $407 \pm 51$ MeV. Mattingly and Stevenson [17] used $e^+e^-\rightarrow e^+e^-$ annihilation to extract an effective $\alpha_s$. The curve from Godfrey and Isgur [18] shows the coupling constant needed in their quark model to reproduce hadron spectroscopy.

Lattice QCD results present more often the gluon propagator rather than the coupling constant. Since the behavior of $\alpha_s^{\text{eff}}'$ at low $Q^2$ may be accounted for by a dynamical gluon mass, which modifies the gluon propagator, we can extract from our result an “effective gluon propagator” and compare it to Lattice calculations. The Dyson-Schwinger equations provide the non-perturbative approach needed for studying the gluon propagator. However, the necessity of approximations to solve the equations results in some ambiguity. We use the results of Cornwall [16] which provide good comparison with results from various studies. Results on the gluon propagator multiplied by $Q^2$ are shown on the panel D of Fig. 1 along with quenched and unquenched Lattice QCD results [19]. More Lattice results exist but they are mostly quenched and agree with Ref. [19].
3. Summary and Conclusion

We have presented the Bjorken sum in the $Q^2$ range of 0.16-1.1 GeV$^2$. The gap between the parton to hadron descriptions of the strong interaction, if smaller, is not bridged yet. With these data, we extracted an effective coupling for the strong interaction. We hypothesize that $\alpha_{s}^{\text{eff}}$ defined when QCD radiations are not folded in can be compared to the various effective couplings available from theories. These physical couplings, obtained within very different areas of strong interaction (hadron spectroscopy, non-perturbative calculations, lattice QCD and moments of structure functions) agree with our data. $\alpha_{s}^{\text{eff}}$ can be used to parametrize the strong force at any $Q^2$. Our data and the $Q^2 = 0$ constraint hint that $\alpha_{s}^{\text{eff}}$ loses its scale dependence at very low $Q^2$. This will be checked by upcoming experimental results at very low $Q^2$ \[20,21\].

4. Acknowledgments

This work was supported by the U.S. Department of Energy (DOE) and the U.S. National Science Foundation. The Southeastern Universities Research Association operates the Thomas Jefferson National Accelerator Facility for the DOE under contract DE-AC05-84ER40150.

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Figure 1. Left top (Panel A): $Q^2$-evolution of the Bjorken sum. The dark (light) horizontal band is the experimental systematic error corresponding to the neutron extracted using $^3$He (D). Right top (Panel B): Effective strong coupling constant as defined by Eq. 3. Left bottom (Panel C): Extracted $\alpha_s^{\text{eff}}(Q)$ together with experimental $\alpha_s(Q)$, running of pQCD and phenomenological $\alpha_s(Q)$. The vertical band represents $\Lambda_{QCD}$ and its uncertainty. The dark band gives the uncertainty on the Cornwall calculation due to $\Lambda_{QCD}$. The lighter band is the uncertainty on the Burkert-Ioffe model due to the truncation of the leading twist series to 5th order. Right bottom (Panel D): The gluon transverse propagator multiplied by $Q^2$, extracted from our result together with quenched and unquenched Lattice QCD calculations [19].