Potential energy analysis based on piezoelectric cantilever beam structure

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Abstract. The piezoelectric vibration energy collection technology utilizes the positive piezoelectric effect of the piezoelectric material, and collects the energy by attaching the material to a clamping structure and deforming under the external excitation, thus accumulating the charge and generating the potential difference. In this paper, the cantilever beam is used as a clamping structure because of its large deflection and simple structure. In this paper, the raindrop impulsion is applied to the piezoelectric material, the piezoelectric vibration energy capture device is used to improve the energy collection efficiency, and the electromechanical coupling model is established to simplify the potential energy analysis by using the conservation of energy and Newton’s second law. In this paper, Runge-kutta method is used to simulate the differential equation, which reduces the computational cost of the system and has a certain precision for analysis and solution. Based on Al-‘Ula’s Bernoulli beam theory, the model was simulated by using the differential equation method and the ordinary differential equation group.

1. Introduction
At present, the research of piezoelectric energy harvesting technology has been relatively mature, and there are many simulation analyses of piezoelectric energy systems. At present, some studies have made corresponding improvements to the conventional cantilever piezoelectric energy flotation device [1], which improves the energy harvesting efficiency of the harvester, but ignores the electromechanical coupling effect of the energy harvester; there are also studies using Matlab software to The piezoelectric energy trap system was simulated [2], which verified the stability of the output voltage of the system, and intuitively reflected the output characteristics of the piezoelectric trap system. However, this method has a large computational cost and a complicated solution process; there are researches using COMSOL simulation to analyze the piezoelectric energy trap system [3], the influence of piezoelectric materials on the energy trap is obtained. Based on the existing related research and improving the shortcomings of the existing technology, this paper uses piezoelectric vibration energy harvester to improve the energy collection rate, and establishes an electromechanical coupling model [4] to simplify the potential energy analysis. At the same time, this paper establishes a corresponding mathematical model based on the system dynamics of the piezoelectric cantilever, simulates the influence of the precipitation effect on the potential energy and dynamic factors, and uses the Runge-Kutta numerical differential equation method to simulate the model accordingly. The corresponding system output response law curve is obtained, which more intuitively reflects the relationship between the raindrop diameter and the system output voltage.
2. System model

2.1. Formatting author names
In order to analyze and calculate the kinetic energy of raindrops falling on a piezoelectric plate, there are many uncertain scenarios and the process is relatively complicated. Therefore, the acceleration or deceleration of raindrops caused by wind when raindrops fall and the change of raindrop shape are neglected in this paper, dynamical system raindrops.

If a raindrop falls in a spherical structure, the vertical air drag $F$ and the gravity on the raindrop are expressed as follows:

\[ F_{\text{air}} = \frac{1}{2} \rho_a s \xi v^2 \]  
\[ F_G = \frac{4}{3} \pi \left( \frac{D}{2} \right)^3 \rho_w g \]

Among them, $\rho_a$ is air density, $S$ is cross-sectional area of raindrop, $C$ is air drag coefficient ($C=0.45$), $v$ is raindrop velocity, $r$ is raindrop diameter, $\rho_w$ is raindrop density, $g$ is gravitational acceleration.

To establish a differential equation for the motion of raindrops:

\[ \frac{dE}{\sqrt{2mE}} \frac{dt}{dt} = \frac{6(F_G - F_{\text{air}})}{\pi D^3 \rho_w} \]  
\[ v = \sqrt{\frac{\pi D^3 \rho_w g}{3 \rho_a s \xi}} \left(1 - \frac{\frac{2}{\exp\left(\frac{\pi D^3 \rho_w s \xi}{3} \right)} + 1}{\exp\left(\frac{\pi D^3 \rho_w s \xi}{3} \right)}\right) \]

The force of a raindrop can be expressed as:

\[ F = \frac{dmv}{dt} = \frac{1}{6} \rho_w \pi D^2 v^2 \]

3. Power Generation System Mode

3.1. Mechanical Model
Since the main motion form of the piezoelectric cantilever beam power generation system is first-order mode, in this paper, it is equivalent to a quality-damping-as-a-degree piezoelectric model, and carries on the related stress analysis and the modelling to it.

According to Newton’s second law and Kirchhoff’s circuit laws, the equations governing the system’s mechanical motion are derived:

\[ \tilde{m} \frac{d^2z}{dt^2} + \tilde{c} \frac{dz}{dt} + \tilde{k}z + \tilde{\alpha} V_R - F_{\text{z}} = -F(t) \]
\[ \tilde{\alpha} \frac{dz}{dt} - \frac{V_R}{R} = C_p \frac{dV_R}{dt} \]

Among them, $\tilde{m}$ is the equivalent mass, $\tilde{c}$ is the equivalent damping, $\tilde{\alpha}$ is the electromechanical coupling coefficient, $\tilde{k}$ is the equivalent stiffness, $F_{\text{z}}$ is the fractional repulsive force in the z direction between two magnets. $Z$ is the relative displacement of cantilever beam in equivalent model. $V_R$ is the voltage across the load resistance $R$. $C_p$ is the capacitance of piezoelectric film. Let the falling
process of raindrops be simplified as a uniform falling with time interval $\tau$. $F(t) = F_{G_x}(t), G_x(t)$ is the pulse sequence with period $\tau$.

It is assumed that the piezoelectric plate adheres tightly to the cantilever beam, and the motion of the cantilever beam accords with Al-Ula’s theory, and the cross section characteristics and geometric parameters of the cantilever beam do not change when the cantilever beam deforms, and when the piezoelectric cantilever beam deforms, its cross-section characteristics and geometric parameters do not change\(^5\).

### 3.2. Magnetic Models

A mechanical analysis of the repulsion between magnets is carried out according to the mechanical model of the magnetic dipole\(^6\) in Figure. 1:

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{magnetic_dipole_model.png}
\caption{magnetic dipole mechanics model}
\end{figure}

Find the magnetic induction $B_{BA}$ of Magnet B at Magnet A:

$$B_{BA} = -\frac{\mu_0}{4\pi} \nabla \cdot \frac{\vec{N}_B \cdot \vec{r}_{BA}}{|\vec{r}_{BA}|^3}$$ \hspace{1cm} (7)

Among them, $\mu_0$ is the vacuum permeability, $\nabla$ is the vector gradient, $\vec{r}_{BA}$ is the direction vector, $\vec{N}_B$ is the magnetic moment of Magnet B.

Find the magnetic potential energy between two magnets:

$$U(z) = \frac{\mu_0}{4\pi} \cdot \frac{M_AV_A M_B V_B z^2 d}{(d^2 + z^2)^{3/2}} \cdot \frac{V_A \cos \theta}{\sqrt{z^2 + l^2}}$$ \hspace{1cm} (8)

Among them, $M_B$ is the remanent magnetization of Magnet B, $V_B$ is the volume of Magnet B, $M_A = M_B$, $V_A = V_B$, $d$ is the horizontal distance between the centers of two magnets, $l$ is the distance from the center of Magnet A to the base of the cantilever beam, $Z$ is the displacement away from the center line when the upper end of the magnet A syncline is deflected at an angle $\theta$. The $Z$ direction repulsive force between the magnets is obtained.

$$F_z = \frac{\partial U(z)}{\partial z} = \frac{3\mu_0 M_A V_A M_B V_B}{2\pi d^4 l} \cdot \left[ z - \left( \frac{5}{d^2} + \frac{1}{l^2} \right) z^3 \right]$$ \hspace{1cm} (9)
4. Model results and analysis

Through MATLAB2020b to carry on the simulation test obtains the raindrop falling speed simulation:

As can be seen from Figure 2, the larger the diameter of the raindrop, the faster it falls, taking into account the drag of the air, in accordance with the actual situation, the final velocity of Raindrops falling on the ground is in the range of 7 ~ 17 m/s.

The frequency of raindrops hitting a cantilever beam is set here, and the frequency of raindrops falling is shown in Figure 3, taking into account the periodicity of the pulse sequence:

The potential kinetic model for solving the system can be reduced to the following second order differential equation:

\[
\begin{cases}
\ddot{z} + \frac{c}{m} \dot{z} + \frac{k}{m} z - F_R = F(t) \\
\dot{\alpha} \frac{dV_R}{dt} = C_p \frac{dV_R}{dt}
\end{cases}
\]  

(10)

Can be translated into:

\[
\begin{align*}
\ddot{z} + \frac{c}{m} \dot{z} + \frac{k}{m} z - F_R &= F(t) \\
\dot{\alpha} \frac{dV_R}{dt} &= C_p \frac{dV_R}{dt}
\end{align*}
\]
This can be solved using the Runge-Kutta differential equation method:

\[ \frac{dp}{dt} = \frac{F_x - F(y) - (\ddot{p} + \ddot{c} + \ddot{V})}{m} \]  
\[ \frac{dV}{dt} = \frac{\dot{p} - \frac{V}{R}}{C_p} \]  
\[ \frac{dz}{dt} = \dot{p} \]  
(11)

Hypothetically,

\[ y_{i+1} = y_i + c_1 K_i + c_2 K_2 + c_3 K_3 + c_4 K_4 \]  
(12)

K1, K2, K3, and K4 are the values of the function at four different points, each set to

\[ K_1 = hf(x_i, y_i) \]  
\[ K_2 = hf(x_i + a_2 h, y_i + b_{21} K_1) \]  
\[ K_3 = hf(x_i + a_3 h, y_i + b_{31} K_1 + b_{32} K_2) \]  
\[ K_4 = hf(x_i + a_4 h, y_i + b_{41} K_1 + b_{42} K_2 + b_{43} K_3) \]  
(13)

\[ c_1, c_2, c_3, c_4, a_1, a_2, a_3, a_4, b_{21}, b_{31}, b_{32}, b_{41}, b_{42}, b_{43} \] are all undetermined coefficients.

In this paper, the power series of h at Point xi of K2, K3 and K4 are respectively taken into the linear combination, and the obtained formula is compared with the Taylor series of y(xi+1) at Point xi, so that the right side of the formulas knows that the coefficients of h4 are equal, a group of special solutions about \( a_i, b_{ij} \) and \( c_j \) can be obtained by the complex process of solving equations. The function values at four different points K1, K2, K3 and K4 can be obtained by substitution.

The final simulation results are shown in figures 4, 5 and 6:

![Figure 4. V-T simulation curve](image_url)

![Figure 5. Z-T simulation curve](image_url)
As can be seen from Figure 4, the diameter of the raindrop is sensitive to the maximum potential energy of the system, but the time trend of the total potential energy and the extreme point of the system potential energy are almost the same; As can be seen from Figure 5, the change rate of the height difference of the magnet increases gradually in the later period, which indicates that the larger the diameter of raindrop, the richer the precipitation, and the larger the jitter amplitude and rate of the magnet; As can be seen from Figure 6, the larger the diameter of raindrops is, the greater the storage potential energy of the system is, and when the height difference reaches a certain threshold, the potential energy begins to reverse, the vibration amplitude of the piezoelectric cantilever is also larger and larger, which increases the potential energy.

5. Conclusion
1. Based on the mathematical model of the System dynamics cantilever, the effect of precipitation on the potential energy and the influence of the dynamic factors are simulated to study the impact of continuous raindrops on the cantilever, the output response curve of the system is obtained. The results show that the larger the diameter of the raindrop, the easier it is for the system to make large amplitude motion, and the number of large movements will increase, the output voltage is also greater.

2. In this paper, the kinetic energy of raindrop falling is used as the energy source of piezoelectric device with cantilever structure, and it is equivalent to the mass damping stiffness piezoelectric model. The second order ordinary differential equations are solved, and the numerical simulation is carried out by Runge Kutta. The results verify the reliability of raindrop power generation.

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