A Scale-Consistent Approach for Recommender Systems

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Abstract

In this paper we propose and develop a relatively simple and efficient approach for estimating unknown elements of a user-rating matrix in the context of a recommender system (RS). The critical theoretical property of the method is its consistency with respect to arbitrary units implicitly adopted by different users to construct their quantitative ratings of products. It is argued that this property is needed for robust performance accuracy across a broad spectrum of RS application domains.

I. INTRODUCTION

Given a matrix (or higher tensor composition) in which the value associated with each element either represents a user’s score/ranking of a particular item or is unfilled, a recommender system (RS) is intended to use information from existing values to estimate a value for each unfilled element, i.e., to estimate how a user is likely to score a particular product that has not yet been evaluated, in order to determine whether the product should be recommended to the user ([1], [3], [5], [6]). Because the problem is inherently ill-defined, certain assumptions must be made based on a premise that users who give similar scores to a common set of products are likely to give similar scores to other products. Unfortunately, this premise is far too simplistic to capture the complex multivariate structures underpinning human interests and preferences.

To appreciate the RS challenge, consider a pair of users Alice and Bob of the same age and with identical backgrounds (e.g., geographical, cultural, socio-economic, etc.), and then attempt to explain the source of differences in their scores for a given set of products. A first potential source of difference arises from the fact that terms like score and rating are not rigorously defined, i.e., Alice and Bob are likely to adopt different “units of quality” when making their respective assessments. These implicit units introduce scale factors that affect their distribution of scores even when they are forced to convert to a fixed scale, e.g., 0 to 10 or 1 to 100. Moreover, they are likely to apply different units, or gradations of discrimination, to different product types, e.g., toothpaste versus jackets. Their relative scores are also likely to reflect distinct personal preferences, e.g., horror films versus romantic comedies. For example, Bob may enjoy horror films and apply a scale in which his favorite horror films receive high scores while other horror films receive low scores, whereas Alice may strongly dislike horror films and give the lowest possible score to every film in that genre. In fact, a user may even consciously recognize that he is applying different units of quality/preference when giving the same high score to, e.g., the Orson Welles film “Citizen Kane” and Bruce Lee’s “Enter the Dragon” – two films from completely different genres with almost entirely disjoint attributes of appeal.

In this paper we propose use of a scale-consistent matrix transformation to permit unrated elements of a matrix to be estimated in a manner that is robust to scale factors associated with the scores of different users and with respect to scale factors associated with user ratings of
distinct product types or genres. For example, if Alice tends to give a 20% higher score to products in a particular category relative to Bob, then estimates/predictions of their respective scores for a new product in that category should reflect that 20% factor.

II. DIAGONAL MATRIX SCALINGS

To achieve unit-scale robustness in recommender system applications it is necessary to estimate unrated values in a way that is as insensitive as possible to scale-factor differences in scores given by different users across all products and with respect to scale-factor differences in scores for different product types/genres as received from across all users. To achieve this it is necessary to reduce the given rating matrix to a unique scale-invariant form from which the estimates can then be computed. More specifically, for a given \( m \times n \) nonnegative matrix \( M \) a diagonal scaling is required that produces a unique matrix \( M' \) as

\[
M' = D \cdot M \cdot E
\]  

(1)

where \( D \) and \( E \) are nonnegative diagonal matrices. A wide variety of scaling methods have been investigated in the case of square \( M \) for the purpose of improving its condition number as a precursor (preconditioning step) to performing a numerical linear algebra operation that is highly sensitive to that condition number. Most methods scale only the rows or the columns, and the ones that do scale both are not generally unique, i.e., very different scalings (\( M' \)) may yield the same optimal condition number.

An approach that does yield a unique result for nonnegative \( M \) is the Sinkhorn scaling ([8], [9]). The Sinkhorn scaling is achieved by iteratively scaling the rows to have unit sum, then the columns, etc., to convergence. Sinkhorn showed that this algorithm does in fact always converge to a unique result with unit row and column sums. However, that result is highly sensitive to the distribution of zeros in the matrix, e.g., if applied to a triangular matrix the process will drive all nondiagonal values to zero with no ability to define finite row and column (\( D \) and \( E \)) scalings.

A less well known matrix scaling was defined by Rothblum and Zenios [7] and can be used to produce a scaled matrix with the property that the product of the nonzero elements in each row and column is 1. As pointed out in [10], the provable uniqueness of this scaling is analytically more important than the properties (e.g., condition number) of the result as it allows, e.g., the derivation of a scale-consistent generalized matrix inverse that is required in many practical engineering and robotics applications ([10], [11], [12]). As will be shown in the next section, it is also what is needed for RS robustness.

III. THE ALGORITHM

Using the Rothblum & Zenios (RZ) algorithm it is possible to obtain a unique scaled matrix \( S \) from nonnegative matrix \( M \) as

\[
S = D \cdot M \cdot E
\]  

(2)

where \( D \) and \( E \) are nonnegative diagonals and the product of the nonzero elements of each row and column of \( S \) is 1.

It should be noted that zero elements are invariant in the RZ decomposition, so if unfilled entries in \( M \) are taken to be zero then they will also be zero in \( S \). Maintaining the distinction between zeros representing scores versus zeros representing unfilled/unknown values, we replace the latter with 1s in \( S \) based on the rationale that 1 is the only nonzero value that can be used
that will preserve the unit-product property of $S$. Denoting the result as $S'$, a filled matrix $M'$ can be obtained by inverting the original scaling as

$$M' = D^{-1} \cdot S' \cdot E^{-1}$$  \ (3)

where each estimated value in $M'$ is scaled consistently with respect to the values in its associated row and column. For example, suppose that $M(i, j)$ is unknown. It can be verified that if row $i$ of $M$ is scaled by a factor $\alpha$, and column $j$ is scaled by a factor $\beta$, then the value estimated for element $(i, j)$ will be $\alpha \beta \cdot M'(i, j)$. In other words, estimated values are scaled consistently with respect to the implicit units (scale factors) associated with each row and column, i.e., as associated with different users and products.

With regard to computational complexity, the run-time complexity to determine the RZ scaling is $O(mn)$, which is optimal for dense $M$, and the algorithm can be further refined to exploit sparsity so that the scaling of $M$ with only $p$ nonzero elements can be evaluated in $O(p)$ time\(^1\). As for space, once the scaling is determined there is actually no need to explicitly produce $M'$ because all that needs to be stored are the diagonal elements of $D$ and $E$ because the value of each unfilled element $(i, j)$ is completely determined as $1/(D_{ii} \cdot E_{jj})$.

The natural question is: Does scale consistency produce estimated values that accurately predict user preferences? The answer of course can only be assessed by empirical evaluation, which will be the focus of future work. However, a related question can also be asked: Can a general method be expected to yield accurate predictions across a range of RS domains if it does not maintain scale consistency? In other words, scale-consistency may not be sufficient to produce acceptable results, but there is good reason to believe that it will need to be preserved – at least approximately – unless obviated by the availability of stronger assumptions about how users produce their scores/ratings.

More generally, any RS approach must assume or develop a parameterized model for how users determine their evaluation scores. The effectiveness with which that model actually captures the implicit evaluation formula of a given user can be assessed by comparing its predictions against ground truth scores from that user. Assuming there will be some users who are highly eccentric in the way they determine scores, i.e., very different from the implicit assumptions of the model, it makes sense to identify and remove them so that their inputs don’t degrade predictions for other users for whom the model appears effective\(^2\).

IV. SUMMARY

In this paper we have described a Recommendation System (RS) approach that provides consistency with respect to arbitrary units that are implicitly used by different evaluators when determining their product scores/ratings. We have provided a rationale for why this approach may be effective, and we have discussed how the accuracy of it (and other RS approaches) potentially may be improved by a simple regression scheme.

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\(^1\)Optimal complexity in the sparse limit can justify a simpler model in order to permit practical solutions to extremely large problems\(^4\) or in high data-rate applications\(^2\).

\(^2\)The predictions for deviant/outlier users can be retained as special cases from the initial calculation while those for the remaining users will be evaluated from the more refined scaling obtained from the regression process.
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