Constraining parameter space in type-II two-Higgs doublet model in light of a 125 GeV Higgs boson

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Abstract

We explore the implications of a 125 GeV Higgs boson indicated by the recent LHC results for two-Higgs doublet model (2HDM). Identifying the 125 GeV Higgs boson as either the lighter or heavier of CP even neutral Higgs bosons in 2HDM, we examine how the masses of Higgs fields and mixing parameters can be constrained by the theoretical conditions and experimental constraints. The theoretical conditions taken into account are the vacuum stability and perturbativity required to be satisfied up to a cut-off scale. We also show how bounds on the masses of Higgs bosons and mixing parameters depend on the cut-off scale. In addition, we determine the regions of parameter space accommodating the enhanced di-photon signals and then predict the signal strengths for other accessible channels of the Higgs decay for the regions.

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I. INTRODUCTION

Both the ATLAS and CMS experiments have discovered a new particle consistent with the Higgs boson \[1\] with a mass of around 125 GeV at about 5\(\sigma\) significance \[2\]. A common belief among particle physicists that the SM is not the ultimate theory of fundamental interactions calls for new physics beyond the SM, such as supersymmetry (SUSY) and extra dimension models. Many new physics beyond the SM contain more than one Higgs doublet of the SM \[3\]. In this regards, it must deserve to examine whether signals detected at the LHC imply the existence of more Higgs sectors or not.

The purpose of this work is to examine the implications of a 125 GeV Higgs boson indicated by the recent LHC results for two-Higgs doublet model (2HDM). We will focus on how severe the theoretical conditions and experimental results on the Higgs sectors can constrain the masses of Higgs fields and mixing parameters in 2HDM in the light of a 125 GeV Higgs boson. The theoretical conditions taken into account are the vacuum stability and perturbativity which are required to be satisfied up to a cut-off scale. Then one can obtain constraints on the couplings of the Higgs potential in 2HDM, which in turn lead to bounds on the masses of Higgs bosons as well as mixing parameters. Although there are a few works on the estimation of bounds on the masses of Higgs fields in 2HDM by applying the vacuum stability and perturbativity \[4, 5\], our new points are to show how the parameter spaces in 2HDM are constrained by those theoretical conditions applied up to a cut-off scale by identifying the 125 GeV Higgs boson as either lighter or heavier of CP even neutral Higgs bosons, and to see how bounds on the masses of Higgs bosons depend on the cut-off scale. In addition, we will examine how experimental constraints on the parameters of Higgs bosons from the LEP can constrain the parameter spaces further. Finally, we will determine the regions of parameter spaces accommodating the enhanced di-photon signals observed at the LHC and then predict the signal strengths for other accessible channels of the Higgs decay for the regions.
II. HIGGS SECTOR IN 2HDM, THEORETICAL AND EXPERIMENTAL CONSTRAINTS

The model we consider is so-called type-II 2HDM with a $Z_2$ discrete symmetry which prevents dangerous flavor changing neutral currents (FCNC) mediated by neutral scalar boson at the tree level. The scalar potential of the model we consider is given by [6]

$$V = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - (m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.})$$
$$+ \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2)$$
$$+ \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \left\{ \frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \text{h.c.} \right\}.$$  \hspace{1cm} (1)

where $\Phi_1$ and $\Phi_2$ are two complex $SU(2)_L$ Higgs doublet scalar fields with $Y = 1$, and $m_{12}^2$ is taken to be zero due to $Z_2$ symmetry. We require that the scalar potential conserves the CP symmetry, so that all the parameters in Eq.(1) must be real. After spontaneous symmetry breaking, the Higgs doublets have the vacuum expectation values as follows,

$$< \Phi_1 > = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad < \Phi_2 > = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix},$$  \hspace{1cm} (2)

where $v^2 \equiv v_1^2 + v_2^2 = (246 \text{ GeV})^2$ and $v_2/v_1 = \tan \beta$. We take $v_1$ and $v_2$ to be positive, so that $0 \leq \beta \leq \pi/2$ is allowed.

There are five physical Higgs particles in 2HDMs: two CP-even Higgs $h$ and $H$ ($m_h \leq m_H$), a CP-odd Higgs $A$ and a charged Higgs pair ($H^\pm$). Following [6], the squared masses for the CP-odd and charged Higgs states are given by

$$m_A^2 = -\lambda_5 v^2, \quad m_{H^\pm}^2 = m_A^2 + \frac{1}{2} v^2 (\lambda_5 - \lambda_4),$$  \hspace{1cm} (3)

and the squared masses for neutral Higgs ($m_H \geq m_h$) are given by

$$m_{H,h}^2 = \frac{1}{2} \left[ A + B \pm \sqrt{(A - B)^2 + 4C^2} \right],$$  \hspace{1cm} (4)

where $A = \lambda_1 v_1^2, B = \lambda_2 v_2^2$ and $C = (\lambda_3 + \lambda_4 + \lambda_4)v_1 v_2$. and $s_\beta = \sin \beta, \ c_\beta = \cos \beta$.

The stable vacuum guaranteed when the scalar potential [1] is bounded from below can be obtained only if the following conditions are satisfied [5][8]

$$\lambda_{1,2} > 0, \quad \lambda_3 > -\sqrt{\lambda_1 \lambda_2}, \quad \lambda_3 + \lambda_4 - |\lambda_5| > -\sqrt{\lambda_1 \lambda_2}. \hspace{1cm} (5)$$
Since radiative corrections give rise to the modification of the couplings in the scalar potential, we need to require that the stability conditions (5) are valid for all energy scales up to a cut-off scale \( \Lambda \). As is known, the stability conditions (5) can lead us to lower bounds on the couplings \( \lambda_i \), which in turn give rise to bounds on the masses of the Higgs fields. In addition, we require the perturbativity for the quartic couplings \( \lambda_i < 4\pi \) in the scalar potential at all scales up to the cut-off scale \( \Lambda \). It is worthwhile to notice that those theoretical conditions can constrain not only Higgs masses but also mixing parameters \( \tan \beta \) and \( \alpha \) via the renormalization group (RG) evolutions. In our numerical analysis, we used RG equations for the parameters \( m^2_{ii}, \lambda_i \), gauge couplings \( g_i \) and Yukawa couplings presented in ref.[9]. In particular, we take the top quark pole mass and QCD coupling constant at \( Z \) boson mass scale \( (\alpha_s(M_Z)) \) to be 172 GeV and 0.1185, respectively.

On the other hand, experimental results from the LEP give rise to constraints on the masses of Higgs bosons and the mixing parameters in the case that the masses of light neutral Higgs bosons lie between 10 GeV and 150 GeV [10, 11]. For the charged Higgs bosons, the experimental lower bound on their masses is 79.3 GeV [12]. The invisible decay width of \( Z \)-boson from LEP indicates that only the Higgs masses satisfied with \( M_h + M_A > M_Z \) are kinematically allowed. In addition, when \( M_h \lesssim 115 \) GeV, non-observation of the Higgsstrahlung process \( e^+e^- \rightarrow hZ \) at LEP constrains the parameter space of \( \sin^2(\beta - \alpha) \) and \( M_h \) [19]. We also consider the Higgs pair production processes, \( e^+e^- \rightarrow hA(HA) \rightarrow b\bar{b}b\bar{b} \) and \( e^+e^- \rightarrow hA(HA) \rightarrow AAA \rightarrow b\bar{b}b\bar{b}b\bar{b}b\bar{b} \), if they are kinematically allowed. Non-observations of those Higgs pair productions can lead to constraints on light neutral Higgs masses and mixing parameters as shown in [11]. Combining the theoretical constraints with the experimental ones, we may obtain the allowed regions of the masses of Higgs bosons and mixing parameters.

In addition, we take into account the new physics contribution to the parameter \( \rho_0 \) defined by [13],

\[
\rho_0 \equiv \frac{M_W^2}{\rho M_Z^2 \cos^2 \theta_w} = 1 + \Delta \rho_0^{\text{new}},
\]

where \( \Delta \rho_0^{\text{new}} = \Delta \rho_{2\text{HDM}} - \Delta \rho_{\text{SM}} \) and the formulae for \( \Delta \rho_{2\text{HDM}} \) as well as \( \Delta \rho_{\text{SM}} \) are given in [14, 16]. From the global fit [13], the constraint of new physics becomes

\[
0.0001 \leq \Delta \rho_0^{\text{new}} \leq 0.0025.
\]
III. ALLOWED REGIONS OF PARAMETER SPACES

Let us study the implication of the 125 GeV Higgs boson indicated by the recent LHC results by identifying it as the lighter or heavier of the CP even neutral Higgs bosons.

FIG. 1: Allowed regions (uncolored) in the plains $(M_h, M_A)$ (left panels) and $(M_A, M_{H^\pm})$ (right panels). Upper, middle and lower panels correspond to $\Lambda \simeq 2 \times 10^{16}$ GeV, $2 \times 10^6$ GeV and 1 TeV, respectively. Shaded regions in yellow, green, brown and blue are excluded by the theoretical conditions, non-observations of the Higgsstrahlung and Higgs pair productions, and constraint on $\rho$ parameter, respectively. The shaded regions in red (or below the red line) and in magenta (or below the magenta line) are excluded by the direct searches for the charged Higgs and invisible $Z$-decay, respectively.
A. Case for $M_H = 125$ GeV

Assuming that the mass of the heavier neutral CP even Higgs is 125 GeV, let us examine how the parameter space of Higgs masses and mixing parameters can be constrained by theoretical conditions and experimental constraints explained in sec. II. Also, we show how those constraints depend on the cut-off scale.

Fig. 1 presents how the regions of parameter spaces in the plains $(M_h, M_A)$ and $(M_A, M_{H^+})$ can be constrained by the theoretical conditions and experimental constraints. The upper, middle and lower panels correspond to $\Lambda \simeq 2 \times 10^{16}$ GeV (GUT scale), $2 \times 10^6$ GeV and 1 TeV, respectively. The shaded regions in yellow, green and brown are excluded by the theoretical conditions, the direct searches for Higgs fields from the Higgsstrahlung, and Higgs pair productions, respectively. Imposing the experimental constraint on $\rho$ parameter, we can further exclude the regions in blue. The shaded regions in red (or below the red line) and magenta (or below the magenta line) are excluded by the direct searches for charged Higgs at ALEPH and invisible Z-decay, respectively. Finally, the uncolored regions are the over-all surviving regions of the parameter space. We see that the allowed regions strongly depend on the cut-off scale. The allowed regions get wider as the cut-off scale gets lower. For the GUT scale, the surviving regions are very tiny. Note that the theoretically allowed regions presented in Fig. 1(b) are split by two parts because of our convention, $\sin(\beta - \alpha) > 0$. It is worthwhile to notice that the mixing parameter $\alpha$ can be constrained by not only the LEP experiments but also the theoretical conditions.

In Fig. 2(a), we present how the allowed regions of $M_h$ from both theoretical conditions and experimental constraints can change by varying $\tan \beta$ for $\Lambda \simeq 2 \times 10^{16}$ GeV (magenta), $2 \times 10^6$ GeV (green), 10 TeV (blue) and 1 TeV (red). The shaded region in cyan is disfavored by non-observation of SM-like Higgs signal for those ranges at the LHC. The lines indicate the upper bounds on $M_h$ for given $\tan \beta$ and $\Lambda$. We see that the bound on $M_h$ increases as $\Lambda$ gets lower for a given $\tan \beta$. For a given cut-off scale, the bound on $M_h$ increases as $\tan \beta$ gets lower down to around 2 except for the case of GUT scale. From the analysis, we see that only $0.6 \lesssim \tan \beta \lesssim 45$ are allowed for $\Lambda \gtrsim 1$ TeV.

The relevant channels to probe the SM-like Higgs boson are $H \rightarrow \gamma\gamma$, $H \rightarrow WW^* \rightarrow l\nu l\nu$, $H \rightarrow ZZ^* \rightarrow 4l$ channels. The fit to the signal strength ($\sigma_{\gamma\gamma} = \sigma(h)_{\text{prod}} \times B(h \rightarrow \gamma\gamma)$) for
FIG. 2: (a) Bounds on $M_h$ as a function of tan $\beta$ for $\Lambda \simeq 2 \times 10^{16}$ GeV (magenta), $2 \times 10^6$ GeV (green), 10 TeV (blue) and 1 TeV (red). The shaded region in brown can accommodate the enhanced di-photon signal observed at the LHC. (b) Allowed region in the $\alpha - \tan \beta$ plain corresponding to the shaded region in brown in (a).

the di-photon channel measured at CMS is slightly deviated from the SM prediction \cite{2},

$$1.1 \lesssim \sigma^{\gamma\gamma}/\sigma_{SM}^{\gamma\gamma} \lesssim 2.1 \quad \text{at 1}\sigma \text{ C.L.} \quad (8)$$

The shaded region in brown in Fig. 2(a) can accommodate the enhanced di-photon signal giving rise to Eq.(8) and is compatible with $M_h \lesssim 110$ GeV which are uncovered at the LHC. We see that this region favors $2.2 \lesssim \tan \beta \lesssim 5.2$. Fig. 2(b) presents the allowed region in the $(\alpha, \tan \beta)$ plain which is converted from the parameter space corresponding to the shaded region in brown in Fig. 2(a).

For the allowed region accommodating Eq.(8), we estimate the signal strength defined by $\sigma^{XX} = \sigma(H)_{prod} \times B(H \rightarrow XX)$ for several final states $XX$. In Fig. 3 we present the predictions of the ratio of $\sigma^{XX}$ to the SM signal strength $\sigma_{SM}^{XX}$ as a function of $\alpha$ for $XX =$up quark pair (red), down quark (lepton) pair (green), $WW^*, ZZ^*$ (cyan) and $Z\gamma$ (black). Here, we take ($\tan \beta, M_{H^\pm}$) to be (a) (3, 200 GeV), (b) (3, 300 GeV), (c) (5, 200 GeV) and (d) (5, 300 GeV). In each panel, we present the regions of $M_A$ which are compatible with the measurement of $\sigma^{\gamma\gamma}/\sigma_{SM}^{\gamma\gamma}$ given in Eq.(8). Notice that the predictions of $\sigma^{XX}/\sigma_{SM}^{XX}$ are presented only for the region of $\alpha$ constrained as in Fig. 2(b). The predictions for $\sigma^{WW^*(ZZ^*)}/\sigma_{SM}^{WW^*(ZZ^*)}$ turn out to be larger than one, which is incompatible with the current results of LHC, but, when $\tan \beta = 5$, the right end points of the cyan lines are almost compatible with the recent LHC result for $H \rightarrow ZZ^*$. In addition, $\sigma^{Z\gamma}/\sigma_{SM}^{Z\gamma}$ is predicted to be less than 0.3, which would test this scenario from future measurement.
FIG. 3: Predictions of $\frac{\sigma^{XX}}{\sigma_{SM}^{XX}}$ as a function of $\alpha$ for $XX=$up quark pair (red), down quark (lepton) pair (green), $WW^*$, $ZZ^*$ (cyan) and $Z\gamma$ (black). Here we take $(\tan\beta, M_{H^\pm})$ to be (a) $(3, 200 \text{ GeV})$, (b) $(3, 300 \text{ GeV})$, (c) $(5, 200 \text{ GeV})$ and (d) $(5, 300 \text{ GeV})$.

B. Case for $M_h = 125$ GeV

Assuming the lighter CP-even neutral Higgs mass ($M_h$) is $125$ GeV, let us examine how the parameter space of Higgs masses and mixing parameters can be constrained, and how the allowed regions of the parameter spaces depend on the cut-off scale. In this case, the experimental results coming from the direct search for the Higgs bosons except for the charged Higgs at LEP do not further constrain the parameter space surviving the theoretical conditions.

In Fig. 4, we present how the parameter space in the plains $(M_H, M_A)$ (left panels) and $(M_A, M_{H^\pm})$ (right panels) can be constrained by the theoretical conditions and experimental constraints. The shaded regions in yellow, blue and red are excluded by the theoretical conditions, experimental constraint on $\rho$ parameter, and experimental lower bound on the charged Higgs mass $M_{H^\pm} > 79.3$ GeV [12]. The over-all surviving regions are uncolored. The upper, middle and lower panels correspond to $\Lambda \simeq 3 \times 10^8$ GeV, $2 \times 10^6$ GeV and 1
FIG. 4: The same as in Fig. 1 except that the $x$–axis in left panels is $M_H$, and upper, middle and lower panels correspond to $\Lambda \simeq 3 \times 10^8$ GeV, $2 \times 10^6$ and 1 TeV, respectively. Similar to the case for $M_H = 125$ GeV, the allowed regions get wider as the cut-off scale gets lower. We see from Fig. 4 that the bound on $M_H$ is 450 GeV for $\Lambda \simeq 1$ TeV, which may be incompatible with the LHC results for the search of Higgs boson. Taking the cut-off scale below 1 TeV, the allowed regions of the parameter spaces get enlarged.

In Fig. 5-(a), the region enclosed by the green curve represents how the allowed region of $M_H$ by the theoretical conditions can change by varying the cut-off scale. The region enclosed by the dashed red curve is allowed by the experimental constraint on the $\rho$ parameter. We see from Fig. 5- (a) that there is no allowed regions of parameter space above $\Lambda \simeq 2 \times 10^9$ GeV. In Fig. 5-(b), the green curve indicates how the bound on $\Lambda$ surviving the theoretical
conditions and experimental constraints can change by varying $\tan \beta$. In both panels of Fig. 5, we plot the shaded region where the experimental results for $\frac{\sigma^{\gamma\gamma}}{\sigma_{SM}^{\gamma\gamma}}$ given in Eq. (8) are accommodated for $M_{H^\pm} = 300$ GeV, in which the enhancement of di-photon channel constrains $\tan \beta$ to be $0.5 \lesssim \tan \beta \lesssim 3.4$. Fig. (a) presents the allowed region

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig5}
\caption{(a) Allowed regions in the plains (a) $M_h-\ln(\Lambda^2/M_Z^2)$ and (b) $\Lambda - \tan \beta$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig6}
\caption{(a) The same as in Fig. 2 (b) but for $M_h = 125$ GeV. Predictions of $\frac{\sigma^{X X}}{\sigma_{SM}^{X X}}$ as a function of $\alpha$ for $X X =$up quark pair (red), down quark (lepton) pair (green), $WW^*$, $ZZ^*$ (cyan) and $Z\gamma$ (black). Here we take $(\tan \beta, M_{H^\pm})$ to be (b) (1, 300 GeV), (c) (2, 300 GeV), (d) (3, 300 GeV).}
\end{figure}
in the $\alpha - \tan \beta$ plain which is compatible with the enhanced di-photon signal giving rise to Eq.(8) for the restricted regions of $\tan \beta$ presented in Fig. 5-(b). For the allowed region accommodating Eq.(8), we estimate the signal strength for several final states $XX$. In Fig. 6-(b-d), we present the predictions of $\sigma^{XX}/\sigma_{SM}^{XX}$ as a function of $\alpha$ for $XX =$ up quark pair (red), down quark (lepton) pair (green), $WW^*, ZZ^*$ (cyan) and $Z\gamma$ (black). Here we take ($\tan \beta, M_{H^\pm}$) to be (b) (1, 300 GeV), (c) (2, 300 GeV), and (d) (3,300 GeV). Similar to the case for $M_{H^\pm} =$ 125 GeV, the predictions of $\sigma^{WW^*(ZZ^*)}/\sigma_{SM}^{WW^*(ZZ^*)}$ turn out to be larger than one. Notice that the predictions of $\sigma^{XX}/\sigma_{SM}^{XX}$ are presented only for the region of $\alpha$ constrained as in Fig. 6-(a). We see that $\alpha \sim 0$ is disfavored for $\tan \beta > 1$. We see that the left end points of the cyan curves are compatible with the recent LHC results for $H \rightarrow ZZ^*$ channels. The prediction of $\sigma^{Z\gamma}/\sigma_{SM}^{Z\gamma}$ turns out to be even less than 0.3, which is characteristic feature of this scenario for the allowed regions of parameter spaces. In fact, the authors in [17] have obtained the parameter region explaining the enhanced di-photon signal in this case of 2HDM, but we have examined the same problem by taking into account the experimental constraints from the LEP experiments and theoretical conditions valid for all renormalization scales up to given cut-off scale. So we can obtain even stronger constraint on $\tan \beta$ compared with that obtained in [17].

In this work, we do not consider the constraint coming from the $b \rightarrow s\gamma$ decay, which may constrain the charged Higgs mass ($M_{H^\pm} \geq 295$ GeV) [18]. If we seriously take into account the constraint in the scenarios we consider, some part of the allowed regions are excluded. As can be seen from our plots, it is likely that most regions surviving the constraints from $b \rightarrow s\gamma$ in the case of $M_H = 125$ GeV prefer to the values of $M_A$ which lie in the excluded mass region obtained from the recent LHC results [2]. However, in the case of $M_h = 125$ GeV, one can find the parameter regions surviving both the constraint from $b \rightarrow s\gamma$ and the LHC results for search of Higgs boson by lowering the cut-off scale below 1 TeV.

In conclusion, we have examined the implications of a 125 GeV Higgs boson indicated by the recent LHC results for 2HDM with a $Z_2$ symmetry. Identifying the 125 GeV Higgs as either the lighter or heavier of the CP even neutral Higgs, we have obtained the allowed ranges of Higgs masses and mixing parameters by imposing the theoretical conditions and experimental results on the Higgs sectors. The theoretical conditions taken into account are the vacuum stability and perturbativity required to be satisfied up to a cut-off scale. So, the allowed regions are turned out to be strongly dependent of the cut-off scale. We have shown
how the experimental constraints on the parameters of Higgs bosons from the LEP as well as constraint on $\rho$–parameter can constrain the parameter spaces further. Finally, we have determined the regions of parameter spaces accommodating the enhanced di-photon signals observed at the LHC and then predicted the signal strengths for other accessible channels of the Higgs decay for the regions.

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