Bayesian Network Inference in Binary Logistic Regression: A Case Study of *Salmonella sp* Bacterial Contamination on Vannamei Shrimp

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**Abstract:** Recently binary logistic regression has been used to identify four factors or predictor variables that supposedly influence the response variable, which is testing result of *Salmonella sp* bacterial contamination on vannamei shrimp. Binary logistic regression analysis results that there are two predictor variables which is significantly affect the testing result of *Salmonella sp* bacterial contamination on vannamei shrimp, those are the testing result of *Salmonella sp* bacterial contamination on farmers hand swab and the subdistrict of vannamei shrimp ponds. Those significant predictor variables selected have been modelled in binary logit model. This paper proposes to study the statistical associations between the two significant predictor variables and the contamination of *Salmonella sp* bacterial on vannamei shrimp and to build a numerical simulation of two significant predictor variables parameters using bayesian network inference. Directed Acyclic Graph (DAG) is applied for modelling binary logit model of significant factors in bayesian network inference.

**Keywords:** Binary Logistic Regression, Bayesian Network, *Salmonella sp* Bacterial Contamination, Vannamei Shrimp, Parameters

The logistic regression model is:

\[ g(x) = \beta_0 + \beta_1 x_1 + \ldots + \beta_p x_p \]  

Then the logit of that model could be written as:

\[ \pi(x) = \frac{\exp(g(x))}{1 + \exp(g(x))} \]  

If those \( p \) predictor variables are discrete or have nominal scale, the method of choice is to use dummy variables. If a nominal scaled variable has \( m \) possible values, then \( m-1 \) dummy variables will be needed. Suppose that the \( j^{th} \) predictor variable \( x_j \) has \( m_j \) levels. The \( m_j - 1 \) dummy variables will be indicated as \( D_{jk} \) and the coefficients for these dummy variables will be indicated as \( B_{jk} \), \( k = 1,2,\ldots,m_j-1 \). Then the logit of this case could be written as:

\[ g(x) = \beta_0 + \beta_1 x_1 + \ldots + \beta_p x_p + \sum_{a=1}^{m-1} \beta_a D_a + \beta_p x_p \]  

Binary logistic regression is a logistic regression where the response variable used is dichotomous (or it is qualitative data which has binary or two categories) and the predictor variables are polychotomous (it could be qualitative or quantitative data). Recently binary logistic regression has been used by the researchers to identify four factors or predictor variables \( (X_1, X_2, X_3, X_4) \) that supposedly influence the response variable \( (Y) \), which is the testing result of *Salmonella sp* bacterial contamination on vannamei shrimp. This response variable \( (Y) \) has two categories: \( 0= \) if testing result of *Salmonella sp* bacterial contamination on vannamei shrimp indicate that there is no *Salmonella sp* on vannamei shrimp; \( 1= \) if testing result of *Salmonella sp* bacterial contamination on vannamei shrimp indicate that there is *Salmonella sp* on vannamei shrimp. While there are four predictor variables used: \( X_1 \); The testing result of *Salmonella sp* bacterial contamination on farmers hand swab (nominal scaled variable), \( X_2 \); The subdistrict of vannamei shrimp ponds (nominal scaled variable), \( X_3 \); The fish processing unit that supplaid by (nominal scaled variable) and \( X_4 \); The pond area in hectare (ratio scaled variable).
Bayesian Network Inference

This paper proposes to study the statistical associations between two significant predictor variables and the contamination of Salmonella sp bacterial on vannamei shrimp and to build a numerical simulation of two significant predictor variables using Bayesian network inference. DAG is applied for modelling binary logit model of significant factors in Bayesian network inference.

Bayesian Network Inference

Neapolitan (1989) in Stephenson (2000) explains that Bayesian network is a specific type of graphical model, namely DAG. All of the edges in the graph are directed (the edges point in a particular direction) and there are no cycles (there is no way to start from any node and travel along a set of directed edges in the correct direction and arrive back at the starting node. The edges in Bayesian network explain the joint distribution of all variables. The joint probability indicated by one set of variables, i.e., $X_1$ and $X_2$. Those significant predictor variables have been modelled in binary logit model.

This paper proposes to study the statistical associations between two significant predictor variables and the contamination of Salmonella sp bacterial on vannamei shrimp and to build a numerical simulation of two significant predictor variables parameters using Bayesian network inference. DAG is applied for modelling binary logit model of significant factors in Bayesian network inference.

Table 1: Link function in DAG

| Link function | Formula |
|---------------|---------|
| Identity      | $x_i$   |
| Logit         | $\log pr(p_i) = \log \frac{p_i}{1 - p_i}$ |
| Probit        | $\text{Probit}(x_i)$ |
| Cloglog       | $\log(-\log(1 - p_i))$ |
| Log           | $\log(x_i)$ |

This method obtains that there are two significant predictor variables, i.e., $X_1$ and $X_2$. Those significant predictor variables have been modelled in binary logit model.

Analysis and Result

Binary logistic regression analysis results that there are two predictor variables which is significantly affect the testing result of Salmonella sp bacterial contamination on vannamei shrimp ($Y$), those are the testing result of Salmonella sp bacterial contamination on farmers hand swab ($X_1$) and the subdistrict of vannamei shrimp ponds ($X_2$). All of the research variables are shown in Table 2.

This Bayesian network analysis is performed by using WinBUGS software. The purpose of this analysis is to get all of the parameters estimation in binary logit model ($\hat{g}(X) = (\beta_0 + \beta_1 x_1 + \beta_2 x_2)$), those are $\beta_0$, $\beta_1$, and $\beta_2$, where the parameter $\beta_2$ is written as $\beta_2(1)$, $\beta_2(2)$, $\beta_2(3)$ and $\beta_2(4)$ (according to: $X_1(1)$ (Subdistrict $B$), $X_1(2)$ (Subdistrict $C$), $X_1(3)$ (Subdistrict $D$) and $X_1(4)$ (Subdistrict $E$)).

The parameters estimation that is obtained by Bayesian network is expected to show the statistical associations between $X_1$ and $X_2$ clearly. This Bayesian network of binary logistic regression is also use the first reference category as same as the previous binary logistic regression. The DAG of this Bayesian network is shown in Fig. 2. The model of DAG is denoted in Fig. 3.

In this Bayesian network analysis, three markov chains iteration is used in simulation process. There are two conditions to continue Bayesian analysis; those are the posterior distribution of parameters built should be stationary and the parameters should be convergence. Time series plot of history chains is used to check the stationary of posterior distribution. By looking Fig. 4, it obtains that the posterior distribution of parameters are stationary. Figure 5 is the Gelman Rubin statistics of parameters. It shows that the parameters are convergence. Therefore, Bayesian network analysis could be continued.

![Fig. 1: The icon specification in DAG (Source: Liu, 2012)](image-url)
Table 2: Research variables

| Variable | Definition | Category | Scale |
|----------|------------|----------|-------|
| $Y$      | The testing result of *Salmonella sp* bacterial contamination on vannamei shrimp | 0 : There is no *Salmonella sp* | Nominal |
|          |            | 1 : There is *Salmonella sp* |       |
| $X_1$    | The testing result of *Salmonella sp* bacterial contamination on farmers hand swab | 0 : There is no *Salmonella sp* | Nominal |
|          |            | 1 : There is *Salmonella sp* |       |
| $X_2$    | The subdistrict of vannamei shrimp ponds | 0 : Subdistrict A | Nominal |
|          |            | 1 : Subdistrict B |       |
|          |            | 2 : Subdistrict C |       |
|          |            | 3 : Subdistrict D |       |
|          |            | 4 : Subdistrict E |       |

![Fig. 2: The DAG for bayesian network of binary logistic regression](image)

```r
model;  
  for(i in 1:N) {
    logit(theta[i]) <- b0 + b1 * x1[i] + b2.1 * x2.1[i] + b2.2 * x2.2[i] + b2.3 * x2.3[i] + b2.4 * x2.4[i]
  }
  for(i in 1:N) {
    y[i] ~ dbern(theta[i])
  }
  b0 ~ dnorm(0.01,1.0E-4)
  b1 ~ dnorm(0.01,1.0E-4)
  b2.1 ~ dnorm(0.01,1.0E-4)
  b2.2 ~ dnorm(0.01,1.0E-4)
  b2.3 ~ dnorm(0.01,1.0E-4)
  b2.4 ~ dnorm(0.01,1.0E-4)
```

![Fig. 3: The model of DAG](image)

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The results of parameters estimation using Bayesian network are:

\[ \beta_0 = -1.428, \beta_1 = 1.261, \beta_2(1) = -0.514, \beta_2(2) = -1.374, \]
\[ \beta_2(3) = -0.464 \text{ and } \beta_2(4) = -0.855 \]

The binary logit model obtained is:

\[ \hat{g}(x) = -1.428 + 1.261x_1 - 0.514x_2(1) - 1.374x_2(2) \]
\[ -0.464x_2(3) - 0.855x_2(4) \]

The probability of presence or absence of *Salmonella sp* in testing result on vannamei shrimp according to the testing result of *Salmonella sp* contamination on farmers hand swab and the subdistrict of vannamei shrimp ponds with all of the possible combinations is shown in Table 3. Those probabilities are calculated by logit model of each combination, which are obtained by regress \( Y \) and \( X \) in all possibilities using DAG. The highest probability of the presence of *Salmonella sp* in testing result on vannamei shrimp (0.350) is obtained if the vannamei shrimp is farmed in Subdistrict A and the testing result of *Salmonella sp* contamination on farmer hand swab show that there is *Salmonella sp*; while the smallest probability of the presence of *Salmonella sp* in testing result on vannamei shrimp (0.021) is obtained if the vannamei shrimp is farmed in Subdistrict C and the testing result of *Salmonella sp* contamination on farmer hand swab show that there is no *Salmonella sp*.

Therefore, the highest probability of the absence of *Salmonella sp* in testing result on vannamei shrimp (0.979) is obtained if the vannamei shrimp is farmed in Subdistrict C and the testing result of *Salmonella sp* contamination on farmer hand swab show that there is no *Salmonella sp*; while the smallest probability of the absence of *Salmonella sp* in testing result on vannamei shrimp (0.650) is obtained if the vannamei shrimp is farmed in Subdistrict A and the testing result of *Salmonella sp* contamination on farmer hand swab show that there is *Salmonella sp*. 
between the significant predictor variables and the contamination of Salmonella sp in testing result on vannamei shrimp obtained if the vannamei shrimp is farmed in Subdistrict A and the testing result on vannamei shrimp obtained if the vannamei shrimp is farmed in Subdistrict B and the testing result on vannamei shrimp obtained if the vannamei shrimp is farmed in Subdistrict C and the testing result on vannamei shrimp obtained if the vannamei shrimp is farmed in Subdistrict D and the testing result on vannamei shrimp obtained if the vannamei shrimp is farmed in Subdistrict E and the testing result on vannamei shrimp obtained if the vannamei shrimp is farmed in Subdistrict F.

### Table 3: The probability of presence or absence of Salmonella sp in testing result of Salmonella sp contamination on vannamei shrimp

| The testing result of Salmonella sp contamination on farmer hand swab | The subdistrict of vannamei shrimp ponds | The probability of absence of Salmonella sp in testing result on vannamei shrimp | The probability of presence of Salmonella sp in testing result on vannamei shrimp |
|---|---|---|---|
| There is no Salmonella sp | Subdistrict A | 0.864 | 0.136 |
| | Subdistrict B | 0.874 | 0.126 |
| | Subdistrict C | 0.979 | 0.021 |
| | Subdistrict D | 0.874 | 0.126 |
| | Subdistrict E | 0.977 | 0.023 |
| There is Salmonella sp | Subdistrict A | 0.650 | 0.350 |
| | Subdistrict B | 0.671 | 0.329 |
| | Subdistrict C | 0.932 | 0.068 |
| | Subdistrict D | 0.671 | 0.329 |
| | Subdistrict E | 0.924 | 0.076 |

### Conclusion

The result of bayesian network analysis of binary logistic regression obtain the statistical associations between the significant predictor variables and the contamination of Salmonella sp bacterial on vannamei shrimp which is show in probability as following:

- The probability of the presence of Salmonella sp in testing result on vannamei shrimp obtained if the vannamei shrimp is farmed in Subdistrict A is 0.350
- The probability of the presence of Salmonella sp in testing result on vannamei shrimp obtained if the vannamei shrimp is farmed in Subdistrict B is 0.329
- The probability of the presence of Salmonella sp in testing result on vannamei shrimp obtained if the vannamei shrimp is farmed in Subdistrict C is 0.068
- The probability of the presence of Salmonella sp in testing result on vannamei shrimp obtained if the vannamei shrimp is farmed in Subdistrict D is 0.329
- The probability of the presence of Salmonella sp in testing result on vannamei shrimp obtained if the vannamei shrimp is farmed in Subdistrict E is 0.076

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### Author’s Contributions

**Pratnya Paramitha Oktaviana:** Participated in all experiments, coordinated the data-analysis and contributed to the writing of the manuscript.

**Kartika Fithriasari:** Guide the first author and contributed to the writing of the manuscript.
Ethics

The authors confirm that this paper is original and approved the manuscript. There is no ethical issues involved.

References

Chen, C., G. Zhang, R. Tarefder, J. Ma and H. Wei et al., 2015. A multinomial logit model-bayesian network hybrid approach for driver injury severity analyses in rear-end crashes. Elsevier: Accident Anal. Prevent., 80: 76-88. DOI: 10.1016/j.aap.2015.03.036

Hosmer, D.W. and D.S. Lemeshow, 2000. Applied Logistic Regression. 2nd Edn, John Wiley and Sons, Inc., New York, ISBN-10: 0471356328, pp: 373.

Liu, Y., 2012. Doodlebugs application. Department of Statistics, University of Missouri, Columbia.

Stephenson, T.A., 2000. An introduction to bayesian network theory and usage. Research Report, Idiap Research Institute, Martigny-Valais, Switzerland.