Decay rates of an interstitial two-level impurity in a ferromagnetic spin lattice

Yamen Hamdouni

Department of physics, Faculty of Exact Sciences, Mentouri University, Constantine, Algeria

Abstract

The decay rates of a two-level interstitial impurity, weakly coupled to an anisotropic ferromagnetic spin lattice are derived. Using the spin-wave theory and Bogoliubov transformations, the lattice Hamiltonian is diagonalized leading to the identification of the critical point. The elimination of the lattice spin degrees of freedom in the derivation of the master equation is restricted by the periodicity of the lattice which fixes the form of the spectral density in the continuum limit. This fact has a great impact on the form and the values of the decay rate and the Lamb shift. It is found that the time dependent decay rates exhibit strong non-Markovian features as they assume negative values. The exact analytical form of the decay rate and the Lamb shift are derived for particular situations. The analytical and the numerical calculations reveal that in addition to the lattice criticality, there exist other critical resonance-like points about which the behavior of the decay rate, and hence the reduced density matrix, changes drastically.

* hamdouniyamen@gmail.com
I. INTRODUCTION

The full description of the dynamics of quantum systems should take into account the influence of the surrounding environment on the different features of their evolution. This represents the basic concept behind the foundation of the theory of open quantum systems [1]. As a matter of fact, many interesting phenomena cannot be explained in a plausible way without the inclusion of the effect of the surrounding, as is the case for the decoherence, dephasing, and dissipation phenomena, to name a few [2–4]. This indicates that the study of the interaction of small quantum systems with their environment is of great importance, either theoretically and experimentally. On the theoretical side, this becomes evident because any attempt to solve the dynamics of an open system inevitably requires some how the elimination of the irrelevant degrees of freedom of the surrounding environment.

Very often, the properties of the environment, which a priori, is characterized by a large number of degrees of freedom, make it very difficult, if not impossible, to solve in an exact manner the evolution equations. This of course does not prevent the finding of some special case for which exact solutions do exist. In particular, the Jaynes-Cummings model [5] represents one of the most popular and important paradigms that enabled the investigation of the dynamics of open quantum systems. It has been widely used in many contexts and it is of great usefulness to test the validity of the numerical techniques used to solve complicated problems. Depending on whether the degrees of freedom are of bosonic or of spin nature, many techniques have been proposed in order to fulfill this task [6–17]. It should be emphasized that the two key points that make some models so interesting reside in the possibility to integrate out the bosonic degrees of freedom through the introduction of a spectral density (usually of Lorentzian form), and in the use of the so-called Born approximations and Markovian. The latter is widely used in, e.g., quantum optics, and is based on the assumptions that the characteristic time scale of the environment is much smaller that that associated with the central system. This leads to a loss of memory of the central system, which is characteristic for the Markov processes. As a consequence, the reduced system density matrix is found to satisfy a master equation which is in the Lindblad form. The latter is characterized by decay rates which are essentially positive and usually time-independent.

However, the validity of the Markovian approximation is not justified in many problems
that display features indicating strong non-Markovian behavior. This is for example the case when the decay rates become negative implying that information flows back from the environment to the system; consequently, the memory effects should be taken into account in this case. Actually, the non-Markovian dynamics of quantum system became over the last years one of the most interesting subjects in the theory of open quantum systems [18–20]. This is mainly due to the lack of an exact general non-Markovian master equation, in contrast to the known Lindblad form of the Markovian case.

In this manuscript, we focus on the study of the non-Markovian dynamics of a central spin impurity that is coupled to a ferromagnetic spin lattice. The latter presents periodic properties [21, 22] that fix in a unique manner the spectral density, and hence lead to drastically different outcomes as compared with the spin boson model. It should be stressed that the spin degrees of freedom are the most suitable candidates towards the the implementation of new quantum technologies [23, 24]. One can, for instance, profit from their properties to implement the proposed quantum algorithms [25–30]. In this work, We shall be mainly interested in the decay rates, whose properties determine the way the reduced density matrix behaves in time.

The manuscript is organized as follows. In section II we introduce the total Hamiltonian of the composite system. Then, through the Holstein-Primakoff transformation, we use the spin-wave theory to diagonalize the lattice Hamiltonian and to identify the critical magnetic field. Section III deals with the derivation of the time local master equation that describes the evolution of the impurity. Section IV is devoted to the study of the decay rate at zero temperature, where we use the exact spectral features of the lattice to explicitly integrate out its degrees of freedom. There we discuss the dependence of the evolution of the decay rate and the Lamb shift on the various model parameters. We end the the paper a brief conclusion.

II. MODEL

A. System Hamiltonian

The system we intend to study is a two-level impurity that is immersed in a ferromagnetic spin lattice in $d$ dimensions. The impurity is dealt with as a central system, which
couples to its surrounding environment, which plays the role of the spin bath. The total model Hamiltonian is given by the sum of the free Hamiltonian of central system $H_S$, the Hamiltonian of the lattice $H_B$, and the interaction Hamiltonian $H_{SB}$, which describes the coupling of the impurity to the spins of the lattice. Explicitly, we have:

$$H = H_S + H_B + H_{SB}$$  \hspace{1cm} (1)

The free Hamiltonian of the central two level system may expressed in terms of the usual Pauli matrices as:

$$H_S = \omega_0 \sigma_+ \sigma_-,$$  \hspace{1cm} (2)

where $\omega_0$ is the energy gap between the ground state and the excited state of the impurity. Note that the formalism we shall use applies to the general case of a qubit where the free Hamiltonian is written as $H_S = \omega_0 \sigma_z$, $\omega_0$ being the strength of the magnetic field applied to the qubit.

The Hamiltonian describing a ferromagnetic lattice in $d$ dimensions may expressed as

$$H_B = -\sum_{\langle i,j \rangle} (J_{i,j}^x S_i^x S_j^x + J_{i,j}^y S_i^y S_j^y + J_{i,j}^z S_i^z S_j^z) - h \sum_j S_j^z,$$  \hspace{1cm} (3)

where in the above equation $S_i^x$, $S_i^y$ and $S_i^z$ represent the components of the spin operator of the spin at site $i$, and the summation is performed with respect to all pairs of spins. The parameters $J_{i,j}^x$, $J_{i,j}^y$ and $J_{i,j}^z$ denote the coupling constants which are all positive in the ferromagnetic case; the values of the latter quantities are assumed different to account for the anisotropy of the lattice. The magnetic field applied along the $z$-direction is taken to be constant, the strength of which is $h$. In what follows we shall be dealing with the situation in which each spin of the lattice only interacts with its neighbor spins; this means that next-to-nearest neighbor interactions are neglected.

Under the above assumptions, the lattice Hamiltonian can be written as

$$H_B = -\sum_{j} (J\delta S_j^x + J\gamma S_j^y + J\gamma S_j^z) - h \sum_j S_j^z,$$  \hspace{1cm} (4)

where the summation is with respect to the $d$-dimensional vector $\delta$ joining each spin at site $j$ to its nearest neighbor spins. The parameters $J\delta$, $J\gamma$ and $J\gamma$ designate the coupling constants restricted to nearest-neighbor spins. For ease of notation, we simplify the expression of the lattice Hamiltonian, and rewrite it as

$$H_B = -J \sum_{j} (S_j^x + \gamma S_j^y + \gamma S_j^z) - h \sum_j S_j^z.$$  \hspace{1cm} (5)

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Here $J$ is now the main coupling constant of the model, and the dimensionless quantities $\gamma_y$ and $\gamma_z$ are the anisotropy parameters which, without loss of generality, are assumed to satisfy the condition $0 \leq \gamma_y \leq 1$ and $0 \leq \gamma_z < 1$. At this stage, it is useful to introduce the raising and lowering operators $S_j^\pm = S_j^x \pm iS_j^y$, which enables us to rewrite the lattice Hamiltonian in the form:

$$H_B = -\frac{J}{4} \sum_{j,\delta} \left[ (1 - \gamma_y)(S_j^+ S_{j+\delta}^+ + S_j^- S_{j+\delta}^-) + (1 + \gamma_y) \right. \\
\left. \times (S_j^+ S_{j+\delta}^- + S_j^- S_{j+\delta}^+) + 4\gamma_z S_j^z S_{j+\delta}^z \right] - \hbar \sum_j S_j^z. \quad (6)$$

We assume that the coupling between the two-level impurity and the lattice is of Heisenberg type, whose Hamiltonian is given explicitly by the formula:

$$H_{SB} = \sum_j (g_j \sigma_- S_j^+ + g_j^* \sigma_+ S_j^-) \quad (7)$$

where the parameter $g_j$ denotes the coupling constant of the central system to the spin located at site $j$; for the seek of generality, we assume it complex-valued.

**B. Spin-wave formulation**

The properties of ferromagnets at low temperatures can be investigated by means of the spin-wave theory, where the concept of the magnon naturally arises as the analogue of the photon in electromagnetic radiations, and of the phonon for the lattice vibrations. Generally speaking, magnons are ground state excitations that propagate through the spin lattice, as a result of thermal or quantum perturbations. The standard method in spin-wave theory consists in using suitable transformations that map the spin operators to bosonic operators. In this work we use the Holstein-Primakoff transformation which proved to be very convenient in solving such problems. Recall that the prescription employed in the use of the Holstein-Primakoff transformation resides in the following identities [31]:

$$S_j^- = \sqrt{2S} b_j^\dagger b_j, \quad S_j^+ = \sqrt{2S} b_j^\dagger b_j^\dagger, \quad S_j^z = S - b_j^\dagger b_j \quad (8)$$

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where $a_j$ are bosonic operators that satisfy $[b_j, b_{j'}^\dagger] = \delta_{jj'}$.

Before we apply the Holstein-Primakoff transformations we should first of all identify the classical spin configuration of the lattice that minimizes the classical energy. This task may be fulfilled by replacing every spin operator $S_j$ by its classical counterpart, which is simply a vector whose components are

$$S^x_j = S \sin(\theta_j) \cos(\varphi_j) \quad (10)$$

$$S^y_j = S \sin(\theta_j) \sin(\varphi_j) \quad (11)$$

$$S^z_j = S \cos(\theta_j) \quad (12)$$

where $\theta_j$ and $\varphi_j$ are the spherical angles that determine the orientation in space of the spin vector. Afterwards, the classical energy can be cast in the form:

$$E_{\text{class}} = -JS^2 \sum_{j, \delta} \left[ \sin(\theta_j) \sin(\theta_{j+\delta}) \sin(\varphi_j) \sin(\varphi_{j+\delta}) + \gamma_y \cos(\varphi_j) \cos(\varphi_{j+\delta}) + \gamma_z \cos(\theta_j) \cos(\theta_{j+\delta}) \right] - hSN \sum_j \cos(\theta_j). \quad (14)$$

The applied magnetic field forces the spins to point in the same direction, which corresponds to the classical ground state of the ferromagnetic lattice. Hence we exclude from our discussion the eventual Helical spin configuration, and we assume that all the angles $\theta_j$ are the same and are equal to theta; the same argument applies to the angles $\varphi_j$, with the common value being equal to $\varphi$. As a result, we can write:

$$E_{\text{class}} = -JS^2 \eta N \left[ \sin^2(\theta) \left( \cos^2(\phi) + \gamma_y \sin^2(\phi) \right) + \gamma_z \cos^2(\theta) \right] - hSN \cos(\theta), \quad (15)$$

where $\eta$ is the number of nearest neighbors characterizing the lattice (e.g. $\eta = 2$ for $d = 1$). Minimizing $E(\theta, \phi)$ we find that two situations arise: on the one hand when the condition

$$h \geq 2JS\eta(1 - \gamma_z) =: h_{\text{cri}} \quad (16)$$

is satisfied, the energy is minimum for $\theta = 0$ regardless of the value of the anisotropy parameter $0 \leq \gamma_y \leq 1$. This implies that all the spins are oriented along the $z$ axis, which coincides with the direction of the magnetic field. On the other hand when

$$0 \leq h < 2JS\eta(1 - \gamma_z), \quad (17)$$
it turns out that the energy is minimized for
\[ \theta = \arccos \left( \frac{h}{2JS\eta(1-\gamma)} \right) \]
and \( \varphi = 0, \pi \) provided that \( \gamma_y < 1 \). The case of \( \gamma_y = 1 \) bears a special character in the sense that the minimum energy is independent of the angle \( \varphi \).

1. The case of \( h > h_{\text{cri}} \)

When the strength of the magnetic field exceeds the critical value of Eq. (16) we can directly apply the Holstein-Primakoff transformation to the Hamiltonian (6) because in the ground state, all the spins of the bath are parallel to the magnetic field. Keeping only quadratic terms in the Hamiltonian, we obtain:

\[
H = -\frac{JS}{2} \sum_{j} \left[ (1 - \gamma_y)(b^\dagger_j b^\dagger_{j+\delta} + b_j b_{j+\delta}) + (1 + \gamma_y) \right. \\
\times \left. (b^\dagger_j b_{j+\delta} + b_j b^\dagger_{j+\delta}) + 4\gamma_z (b^\dagger_j b_j + b^\dagger_{j+\delta} b_{j+\delta}) \right] \\
+ \hbar \sum_j b^\dagger_j b_j - \hbar NS - J\eta\gamma_z NS^2
\] (18)

Next we Fourier transform the bosonic operators \( b_j \) as follows:

\[
b_j = \frac{1}{\sqrt{N}} \sum_k e^{ik\vec{r}_j} a_k
\] (19)

\[
a_k = \frac{1}{\sqrt{N}} \sum_j e^{-ik\vec{r}_j} b_j
\] (20)

where \( \vec{r}_j \) designates the \( d \)-dimensional real-space vector that determines the position of the spin at site \( j \) of the lattice. It can easily be verified that the operators \( a_k \) satisfy \( [a_k, a^\dagger_{k'\delta}] = \delta_{kk'} \). By virtue of Eqs. (19) and (20), the Hamiltonian becomes

\[
H = \sum_k \left[ (h + 2J\gamma_z \eta S - J\eta S(1 + \gamma_y)\tau_k) a^\dagger_k a_k \\
- \frac{1}{2} J\eta S(1 - \gamma_y)\tau_k (a^\dagger_k a^\dagger_{-k} + a_k a_{-k}) \right] - \hbar NS \\
- J\eta S/2 \sum_k \tau_k - J\eta\gamma_z NS^2
\] (21)

Here we have introduced the parameter:

\[
\tau_k = \frac{1}{\eta} \sum_\delta e^{ik\delta}
\] (22)
called the structure factor of the lattice, which will prove to be of great importance in the subsequent discussion.

To diagonalize the above quadratic Hamiltonian we invoke the following Bogoliubov transformation:

$$a_k = \cosh(\psi_k) \alpha_k - \sinh(\psi_k) \alpha_{-k}^\dagger$$  \hspace{1cm} (23)

$$a_{-k}^\dagger = \cosh(\psi_k) \alpha_{-k}^\dagger - \sinh(\psi_k) \alpha_{k}$$ \hspace{1cm} (24)

and choose the parameter \(\psi_k\) so that the non-diagonal term of Eq. (21), namely, \(a_{-k}^\dagger a_{-k} + a_{k} a_{k}\), disappears. It is worthwhile mentioning that, thanks to the property \(\cosh^2(\psi_k) - \sinh^2(\psi_k) = 1\), the new operators \(\alpha_k\) are also bosonic operators satisfying \([\alpha_k, \alpha_{k'}^\dagger] = \delta_{kk'}\). Inserting Eqs. (23) and (24) into Eq. (21), we find that in order to diagonalize the Hamiltonian \(H\) it is sufficient to choose

$$-\omega_k \cosh(\psi_k) \sinh(\psi_k) = \Lambda_k(\cosh^2(\psi_k) + \sinh^2(\psi_k))$$ \hspace{1cm} (25)

where we have introduced the quantities:

$$\omega_k = h + 2J\gamma z \eta S - J\eta S(1 + \gamma y)\tau_k$$ \hspace{1cm} (26)

$$\Lambda_k = -\frac{1}{2} J\eta S(1 - \gamma y)\tau_k.$$ \hspace{1cm} (27)

It follows that the sufficient condition to diagonalize the Hamiltonian reads:

$$\tanh(2\psi_k) = \frac{2\Lambda_k}{\omega_k}.$$ \hspace{1cm} (28)

By direct substitution in the expression of \(H\) we end up with the following diagonalized bosonic Hamiltonian:

$$H = \sum_k \left[ \sqrt{\omega_k^2 - 4\Lambda_k^2} \alpha_k \alpha_k^\dagger + \frac{1}{2}(\sqrt{\omega_k^2 - 4\Lambda_k^2} - \omega_k) 
- J\eta S/2\tau_k \right] - hNS - J\eta \gamma_z NS^2$$ \hspace{1cm} (29)

This implies that the dispersion relation of the ferromagnetic lattice reads

$$\Omega_k = \sqrt{\omega_k^2 - 4\Lambda_k^2}$$ \hspace{1cm} (30)

or more explicitly:

$$\Omega_k = \sqrt{[h - 2J\eta S\tau_k(1 - \gamma z)][h - 2J\eta S\tau_k(\gamma y - \gamma z)]}$$

$$= \sqrt{\left(h - h_{\text{crit}}\tau_k\right)\left(h - h_{\text{crit}}\frac{\gamma y - \gamma z}{1 - \gamma z}\right)}$$ \hspace{1cm} (31)
Notice that because of the conditions $|r_k| \leq 1$ and $|\gamma_y - \gamma_z| \leq 1$ when $0 \leq \gamma_y \leq 1$, $0 \leq \gamma_z < 1$, we ascertain that $\omega_k^2 - 4A_k^2 \geq 0$ provided that $h > h_{\text{crit}}$, as it should be.

Now, we can express the interaction Hamiltonian in terms of the bosonic operators $\alpha_k$ as:

$$H_{SB} = \sum_k [\cosh(\psi_k)(g_k\sigma_-\alpha_k^+ + g_k^*\sigma_+\alpha_k) - \sinh(\psi_k)(g_k\sigma_-\alpha_k + g_k^*\sigma_+\alpha_k^\dagger)]$$

(32)

where the new coupling constant $g_k$ is defined through the expression:

$$g_k = \sqrt{\frac{2S}{N}} \sum_j g_j e^{i\vec{k}\vec{r}_j}.$$  

(33)

At this stage, we perform the rotating-wave approximation to simplify the Hamiltonian of the composite system; this yields (we omit the constant terms):

$$H = \omega_0\sigma_+\sigma_- + \sum_k \Omega_k\alpha_k^\dagger\alpha_k + \sum_k \cosh(\psi_k)(g_k\sigma_-\alpha_k^+ + g_k^*\sigma_+\alpha_k),$$

(34)

which is the form we shall use in the subsequent discussion. Let us remark that for certain initial states, one can go beyond the rotating-wave approximation by applying the unitary transformation

$$\tilde{H} = \exp\{S\} H \exp\{-S\},$$

(35)

where

$$S = \sum_k \frac{\sinh \psi_k g_k}{\omega_0 + \Omega_k} \sigma_-\alpha_k + \frac{\sinh \psi_k g_k^*}{\omega_0 + \Omega_k} \sigma_+\alpha_k^\dagger.$$  

(36)

The resulting Hamiltonian takes the form

$$\tilde{H} = \left(\omega_0 + \sum_k \frac{\sinh^2 \psi_k |g_k|^2}{\omega_0 + \Omega_k}\right) \sigma_+\sigma_- + \sum_k \cosh(\psi_k)(g_k\sigma_-\alpha_k^+ + g_k^*\sigma_+\alpha_k)$$

$$+ \sum_k \Omega_k\alpha_k^\dagger\alpha_k + \text{higher order terms}$$

(37)

We see that in the weak-coupling approximation, after neglecting the higher-order terms, we obtain a Hamiltonian that resembles the rotating-wave one but with a renormalized energy of the two-level impurity. We stress here that the latter procedure requires that the initial state be invariant under the unitary transformation.
The case of $\gamma_y = 1$, which corresponds to the XXZ Hamiltonian is of particular importance; in this instance the Hamiltonian is diagonalized without having recourse to the Bogoliubov transformation. The dispersion relation simplifies to:

$$\Omega_k = h - 2J\eta S(\tau_k - \gamma_z)$$

while $H_{SB}$ reduces to:

$$H_{SB} = \sum_k (g_k \sigma_- \alpha_k^+ + g_k^* \sigma_+ \alpha_k).$$

2. The case of $0 \leq h < h_{crit}$

As we have already mentioned above, the ground state of the lattice when $0 \leq h < h_{crit}$ corresponds to the state in which all the spins point in the direction determined by the angle $\theta = \arccos \{h/(2J\eta(1-\gamma_z))\}$, around which the low energy excitations occur. This implies that the bosonization should be carried out on the components of the spin vectors with respect to a frame for which the $z$ axis coincides with the aforementioned direction. The latter task may be fulfilled through a rotation by the angle $\theta$ within the $x$-$z$ plane, so that:

$$S^x_j = \cos(\theta)S^{x'}_j + \sin(\theta)S^{z'}_j$$

$$S^y_j = S^{y'}_j$$

$$S^z_j = -\sin(\theta)S^{x'}_j + \cos(\theta)S^{z'}_j$$

where $S^{x'}_j$, $S^{y'}_j$, and $S^{z'}_j$ are the components of the spin operator at site $j$ with respect to the new frame. In terms of the latter components, the lattice Hamiltonian reads:

$$H = -J \sum_{j,\delta} \left[ (\cos^2(\theta) + \gamma_z \sin^2(\theta)) S^{x'}_j S^{x'}_{j+\delta} + \gamma_y S^{y'}_j S^{y'}_{j+\delta} + (\gamma_z \cos^2(\theta) + \sin^2(\theta)) S^{z'}_j S^{z'}_{j+\delta} \right] - h \cos(\theta) \sum_j S^{z'}_j$$

where all cross terms cancel. Following the same procedure we find that (apart from a trivial constant)

$$H = \sum_k \Omega_k \alpha_k^+ \alpha_k + C^{st}$$
where, in this case, the dispersion relation reads:

$$\Omega_k = \sqrt{(1 - \gamma_y \tau_k)[4J^2S^2y^2 - (h + 2J\eta S\gamma_z)^2\tau_k]} \quad (45)$$

The rotating-wave approximation interaction Hamiltonian reads now as:

$$H_{SB} = \frac{1}{2} \sum_k \left[ \cosh(\psi_k)(\cos \theta + 1) - \sinh(\psi_k)(\cos \theta - 1) \right] (g_k \sigma_+ \alpha_k^+ + g_k^* \sigma_- \alpha_k), \quad (46)$$

where:

$$\tanh(2\psi_k) = \frac{2\Lambda_k}{\omega_k}, \quad (47)$$

with:

$$\omega_k = h \cos \theta + 2J(\gamma_z \cos^2 \theta + \sin^2 \theta)\eta S - J\eta S(\gamma_y + \cos^2 \theta + \gamma_z \sin^2 \theta)\tau_k \quad (48)$$

$$\Lambda_k = -\frac{1}{2} J\eta S(\cos^2 \theta + \gamma_z \sin^2 \theta - \gamma_y)\tau_k. \quad (49)$$

### III. MASTER EQUATION

In this section we shall derive the master equation governing the evolution of the reduced density matrix of the central impurity in the weak coupling regime. This means that the strength of the coupling of the two level system is sufficiently weak to allow for a perturbative expansion with respect to the coupling constants $g_j$. The starting point is the Liouville-von Neumann equation

$$\frac{d\rho_{\text{tot}}}{dt} = \mathcal{L}\rho_{\text{tot}} \quad (50)$$

where $\rho_{\text{tot}}$ denotes the density matrix of the whole composite system (i.e, the impurity and the lattice), and where $\mathcal{L}$ is a superoperator whose action is defined by

$$\mathcal{L}A = -i[H, A] \quad (51)$$

In the interaction picture, the Liouville-von Neumann equation becomes

$$\frac{d\rho_{\text{tot}}^I(t)}{dt} = -i[H_I(t), \rho_{\text{tot}}^I(t)] \quad (52)$$

where

$$\rho_{\text{tot}}^I(t) = e^{-i(H_S + H_B)t}\rho_{\text{tot}}e^{i(H_S + H_B)t} \quad (53)$$
and

\[ H_I(t) = e^{i(H_S + H_B)t} H_{SB} e^{-i(H_S + H_B)t}. \] (54)

In what follows, in order to lighten the mathematical expressions, the calculations will be performed for the case of \( h > h_{cr} \); the same results hold when \( h < h_{cr} \), but with the substitution

\[ \cosh \psi_k \rightarrow \frac{1}{2}[\cosh \psi_k (\cos \theta + 1) – \sinh \psi_k (\cos \theta - 1)] \] (55)

Using the commutation relations satisfied by the operators \( \sigma_{\pm} \) and \( \alpha_k \), we find that

\[ H_I(t) = \sum_k \cosh(\psi_k)(g_k e^{-i(\omega_0 – \Omega_k)t} \sigma_- \alpha_k^+ + g_k^* e^{i(\omega_0 – \Omega_k)t} \sigma_+ \alpha_k) \] (56)

The solution of Eq.(52) can be written as:

\[ \rho_{\text{tot}}^I(t) = \rho_{\text{tot}}^I(0) – i \int_0^t dt_1 [H_I(t_1), \rho_{\text{tot}}^I(t_1)] \] (57)

Inserting the latter equation twice into Eq.(52) we obtain:

\[ \frac{d\rho_{\text{tot}}^I(t)}{dt} = -i[H_I(t), \rho_{\text{tot}}^I(0)] – \int_0^t dt_1[H_I(t), [H_I(t_1), \rho_{\text{tot}}^I(t_1)]] \\
+ i \int_0^t dt_1 \int_{t_1}^t dt_2[H_I(t), [H_I(t_1), [H_I(t_2), \rho_{\text{tot}}^I(t_2)]]] \] (58)

The last term in the above equation is of third order with respect to coupling constants, and hence can be omitted in the weak coupling regime. Moreover, the form of the interaction Hamiltonian \( H_{SB} \) ensures that

\[ [H_I(t), \rho_{\text{tot}}^I(0)] = 0 \] (59)

We thus end up with the time-local master equation

\[ \frac{d\rho_{\text{tot}}^I(t)}{dt} = - \int_0^t dt'[H_I(t'), [H_I(t'), \rho_{\text{tot}}^I(t)]] \] (60)

Inserting equation (56) into (60) yields the master equation

\[ \frac{d\rho_S(t)}{dt} = - \int_0^t dt'[\rho_S(t) \sigma_+ \sigma_- - \sigma_- \rho_S(t) \sigma_+] \Psi(t' – t) - \int_0^t dt' [\sigma_- \rho_S(t) - \sigma_+ \rho_S(t) \sigma_-] \Phi(t’ – t) \\
- \int_0^t dt'[\sigma_+ \sigma_- \rho_S(t) - \sigma_- \rho_S(t) \sigma_+] \Psi(t – t’) - \int_0^t dt' [\rho_S(t) \sigma_+ \sigma_- - \sigma_+ \rho_S(t) \sigma_-] \Phi(t’ – t)] \] (61)
where the bath’s correlation functions $\Psi(t)$ and $\Phi(t)$ are defined by

$$\Psi(t) = \sum_k \cosh^2 \psi_k |g_k|^2 e^{i(\omega_0 - \Omega_k)t} [n(\Omega_k) + 1],$$

$$\Phi(t) = \sum_k \cosh^2 \psi_k |g_k|^2 e^{-i(\omega_0 - \Omega_k)t} n(\Omega_k).$$  \hspace{1cm} (62)

In the above equations, $n(\Omega_k)$ is the mean number of magnons in mode $k$ at temperature $T$, namely:

$$n(\Omega_k) = \frac{1}{e^{\Omega_k/k_B T} + 1}. $$ \hspace{1cm} (63)

### IV. DYNAMICS AT ZERO TEMPERATURE

The state of the lattice at zero temperature is given by the fully polarized state. This means that the mean number of magnons is identically equal to zero, which implies that at $T = 0$

$$\Psi(t) = \sum_k \cosh^2 \psi_k |g_k|^2 e^{i(\omega_0 - \Omega_k)t},$$

$$\Phi(t) = 0.$$ \hspace{1cm} (64)

The master equation reduces to

$$\frac{d\rho_S(t)}{dt} = -\int_0^t dt' [\rho_S(t)\sigma_+\sigma_- - \sigma_-\rho_S(t)\sigma_+]\Psi(t' - t) - \int_0^t dt' [\sigma_+\sigma_-\rho_S(t) - \sigma_-\rho_S(t)\sigma_+]\Psi(t - t')$$ \hspace{1cm} (65)

Let us define the time-dependent parameters:

$$\kappa(t) = 2\text{Re} \int_0^t dt' \Psi(t - t'),$$

$$\xi(t) = 2\text{Im} \int_0^t dt' \Psi(t - t').$$ \hspace{1cm} (66)

Then we can rewrite the master equation in the interaction picture in the form:

$$\frac{d\rho_S(t)}{dt} = -i \left[ \frac{1}{2} \{\xi(t)\sigma_+\sigma_-, \rho_S(t)\} + \kappa(t) \left( \sigma_-\rho_S(t)\sigma_+ - \frac{1}{2} \{\sigma_+\sigma_-, \rho_S(t)\} \right) \right],$$ \hspace{1cm} (67)

where $\{A, B\}$ denotes the anticommutator of $A$ and $B$. Going back to the Schrödinger picture, the master equation becomes:

$$\frac{d\rho_S(t)}{dt} = -i [(\omega_0 + \xi(t)/2)\sigma_+\sigma_-, \rho_S(t)] + \kappa(t) \left( \sigma_-\rho_S(t)\sigma_+ - \frac{1}{2} \{\sigma_+\sigma_-, \rho_S(t)\} \right).$$ \hspace{1cm} (68)
Physically speaking the parameter $\kappa(t)$ represents the decay rate of the two-level impurity, while the renormalization parameter $\xi(t)$ plays the role of the Lamb shift due to the coupling to lattice. Our main task here is to calculate these quantities as functions of the model parameters.

**A. One-dimensional lattice**

We assume that the impurity lies in the middle between two lattice atoms, and that it interacts only with these two neighbors, with coupling constant $g$. We can easily show that

$$g_k = 2g\sqrt{\frac{2S}{N}}\cos\left(\frac{\delta k}{2}\right), \quad -\pi/\delta \leq k \leq \pi/\delta. \quad (69)$$

The structure factor for the one-dimensional lattice reads ($\eta = 2$):

$$\tau_k = \cos(\delta k). \quad (70)$$

Observing that:

$$\cosh^2 \psi_k = \frac{1}{2} \left( 1 + \frac{1}{\sqrt{1 - \frac{4\Lambda^2}{\omega_k^2}}} \right), \quad (71)$$

the correlation function takes the form

$$\Psi(t) = \frac{1}{N} \sum_k 4Sg^2 \left( 1 + \frac{1}{\sqrt{1 - \frac{4\Lambda^2}{\omega_k^2}}} \right) \cos^2 \left( \frac{\delta k}{2} \right) e^{i(\omega_0 - \Omega_k)t}. \quad (72)$$

In the continuum limit, we get:

$$\Psi(t) = \frac{1}{\pi} \int_{-\pi}^{\pi} 2Sg^2 \left( 1 + \frac{1}{\sqrt{1 - \frac{4\Lambda^2}{\omega(k)^2}}} \right) \cos^2 \left( \frac{k}{2} \right) e^{i(\omega_0 - \Omega(k))t} dk. \quad (73)$$

where the functions $\omega(k)$ and $\Lambda(k)$ are obtained by setting $\delta = 1$ in the expressions of $\omega_k$ and $\Lambda_k$, respectively.

Let us begin with the XXZ case for which $\gamma_y = 1, h > h_{cri}$, which yields $\Lambda(k) = 0$, $\cosh \psi_k = 1$. Under these conditions the correlation function simplifies to

$$\Psi(t) = 2Sg^2 e^{i(\omega_0 - h - 4JS\gamma_z) t} \int_{-\pi}^{\pi} \frac{1 + \xi}{\pi \sqrt{1 - \xi^2}} e^{i4JS\xi} d\xi \quad (74)$$
This integral can be evaluated exactly using the Bessel functions of the first kind:

$$J_n(z) = \frac{1}{\pi} \int_0^\pi \cos(z \sin \theta - n\theta) d\theta$$  \hspace{1cm} (75)

yielding

$$\Psi(t) = 2Sg^2 e^{i(\omega_0 - h - 4JS\gamma_z)t} [J_0(4JS t) + iJ_1(4JS t)]$$  \hspace{1cm} (76)

Consequently, the rate constant $\kappa$ can be expressed as:

$$\kappa(t) = 4g^2S \int_0^t dt' \left[ \cos \left[ (\omega_0 - h - 4JS\gamma_z)(t - t') \right] J_0(4JS(t - t')) 
- \sin \left[ (\omega_0 - h - 4JS\gamma_z)(t - t') \right] J_1(4JS(t - t')) \right]$$  \hspace{1cm} (77)

whereas the Lamb-shift $\xi$ takes the form

$$\xi(t) = 4g^2S \int_0^t dt' \left[ \sin \left[ (\omega_0 - h - 4JS\gamma_z)(t - t') \right] J_0(4JS(t - t')) 
+ \cos \left[ (\omega_0 - h - 4JS\gamma_z)(t - t') \right] J_1(4JS(t - t')) \right]$$  \hspace{1cm} (78)

The time dependence of the decay rate $\kappa$ and the Lamb shift $\xi$ is displayed in Figs. 1-4 for some particular values of the model parameters. There the above quantities are given in units of $g^2/J$, and for convenience, the time and the magnetic field $h$ as well as $\omega_0$ are given in units of $J$. It is found that as the strength of the magnetic field increases, the decay rate increases until some value of $h$ above of which the decay rate begins to decrease in magnitude. The same behavior applies as well to the Lamb shift. This indicates the presence of a critical behavior with respect to the variation of the strength of the applied magnetic field. To see that, let us investigate the asymptotic values of the decay rate and the Lamb shift. These can be derived by evaluating the long-time integral of the correlation function; we show that:

$$\kappa_\infty = \begin{cases} 
0 & \text{for } 4JS < |\omega_0 - h - 4JS\gamma_z| \\
\frac{g^2}{J} \sqrt{\frac{4JS - \omega_0 + h + 4JS\gamma_z}{\omega_0 - h - 4JS\gamma_z + 4JS}} & \text{for } 4JS > |\omega_0 - h - 4JS\gamma_z| 
\end{cases}$$  \hspace{1cm} (79)

and that

$$\xi_\infty = \begin{cases} 
\frac{g^2}{J} \left( 1 - \frac{\omega_0 - h + 4JS\gamma_z}{\sqrt{(\omega_0 - h - 4JS\gamma_z)^2 - (4JS)^2}} \right) & \text{for } 4JS < |\omega_0 - h - 4JS\gamma_z| \\
\frac{g^2}{J} & \text{for } 4JS > |\omega_0 - h - 4JS\gamma_z| 
\end{cases}$$  \hspace{1cm} (80)

In Figs. 5 and 6 we represent the variation of $\kappa$ and $\xi$ as functions of the time for different
FIG. 1. The decay rate $\kappa$ as a function of the time for different values of the strength of the magnetic field: $h = 1.2J$ (black curve), $h = 6J$ (Blue curve), $h = 8J$ (Red curve); other parameters are: $\omega_0 = 8J$, $S = 1$, $\gamma_y = 1$, $\gamma_z = 0.7$. For convenience, the decay rate is given in units of $2g^2/J$.

values of $\omega_0$ with $h$ fixed. It is quite clear that the decay rate and the Lamb shift exhibit the same dependence as that corresponding to varying the strength of the magnetic field with the parameter $\omega_0$ being fixed. The above results lead us to the study of the dynamics when $\omega_0$ coincides with $\Omega_k$ at the edges of the first Brillouin zone, that is for $k = \pm \pi$.

FIG. 2. The Lamb shift $\xi$ as a function of the time for different values of the strength of the magnetic field: $h = 1.2J$ (black curve), $h = 6J$ (Blue curve), $h = 8J$ (Red curve); other parameters are: $\omega_0 = 8J$, $S = 1$, $\gamma_y = 1$, $\gamma_z = 0.7$. For convenience, the Lamb shift is given in units of $2g^2/J$. 

FIG. 3. The decay rate $\kappa$ as a function of the time for different values of the strength of the magnetic field: $h = 6J$ (black curve), $h = 10J$ (Blue curve), $h = 12J$ (Red curve); other parameters are: $\omega_0 = 8J$, $S = 1$, $\gamma_y = 1$, $\gamma_z = 0.7$. For convenience, the decay rate is given in units of $2g^2/J$.

FIG. 4. The Lamb shift $\xi$ as a function of the time for different values of the strength of the magnetic field: $h = 6J$ (black curve), $h = 10J$ (Blue curve), $h = 12J$ (Red curve); other parameters are: $\omega_0 = 8J$, $S = 1$, $\gamma_y = 1$, $\gamma_z = 0.7$. For convenience, the Lamb shift is given in units of $2g^2/J$.

**Resonance-like behavior**

In the resonance-like case $\omega_0 - h_{\text{res}} - 4JS\gamma_z = \pm 4JS$, we can distinguish two possible situations:

* $\omega_0 < h_{\text{res}}$.

This yields the condition $h_{\text{res}} - \omega_0 = h_{\text{cri}} := 4JS(1 - \gamma_z)$. The decay rate turns out to be

$$\kappa(t) = 4g^2St\left\{[\cos(4JS)t]J_0(4JS) + \sin(4JS)t]J_1(4JS)\right\}$$

$$+ \frac{8}{3}(SJt)^2 {}_2F_3\left(\frac{5}{4}, \frac{7}{4}; \frac{2}{2}, \frac{5}{2}; -16J^2S^2t^2\right),$$

(81)
FIG. 5. The decay rate $\kappa$ as a function of the time for different values of $\omega_0$: $\omega_0 = 2J$ (black curve), $\omega_0 = 5J$ (Blue curve), $\omega_0 = 10J$ (Red curve); other parameters are: $h = 5J$, $S = 1$, $\gamma_y = 1$, $\gamma_z = 0.7$. For convenience, the decay rate is given in units of $2g^2/J$.

FIG. 6. The Lamb shift $\xi$ as a function of the time for different values of $\omega_0$: $\omega_0 = 2J$ (black curve), $\omega_0 = 5J$ (Blue curve), $\omega_0 = 10J$ (Red curve); other parameters are: $h = 5J$, $S = 1$, $\gamma_y = 1$, $\gamma_z = 0.7$. For convenience, the Lamb shift is given in units of $2g^2/J$.

where ${}_pF_q$ denotes the generalized hypergeometric function. The Lamb shift, on the other hand, is given by

$$
\xi(t) = 4g^2St\{[\cos(4JSt)J_1(4JSt) - \sin(4JSt)J_0(4JSt)] + \frac{1 - 2F_3\left(-\frac{1}{4}, \frac{1}{4}; -\frac{1}{2}, \frac{1}{2}, 1; -16J^2S^2t^2\right)}{4JSt}\} \quad (82)
$$

The particular feature of the above quantities stems in the fact that they grow relatively fast as the time increases; in particular, we find that $\lim_{t \to \infty} \kappa(t) = \lim_{t \to \infty} \xi(t) = \infty$, indicating a resonance-like behavior as is displayed in Figs. 7 and 8.

- $\omega_0 > h_{\text{res}}$. 

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When the above condition is satisfied, we find that the strength of the magnetic field reads:

\[ \omega_0 - h_{\text{res}} = 4JS(1 + \gamma_z) \]

provided that \( \omega_0 \geq 8JS \). A straightforward calculation leads to the decay rate

\[ \kappa(t) = \frac{g^2}{J} \sin(4JSt)J_0(4JSt) \] (83)

Similarly, we find that the Lamb shift takes the form:

\[ \xi(t) = \frac{g^2}{J}[1 - \cos(4JSt)J_0(4JSt)] \] (84)

In this case, we obtained a rather reduced decay rate, and in particular, it turns out that \( \kappa(t), \xi(t) \to 0 \) as \( t \to \infty \).

Before we proceed with the discussion of the variation of the above quantities when \( \gamma_y \neq 1 \), let us explore an other special case that leads to close analytical expressions. This instance corresponds to the XX Hamiltonian where \( \gamma_z = 0 \). When the strength of the magnetic field satisfies the condition \( h = \omega_0 \), we obtain that

\[ \kappa(t) = 4g^2St_1 F_2 \left( \frac{1}{2}, 1, \frac{3}{2}, -4J^2S^2t^2 \right) \],

\[ \xi(t) = \left( \frac{g^2}{J} \right)[1 - J_0(4JSt)] \] (86)

\[\text{FIG. 7. Resonance behavior of the decay rate } \kappa. \text{ The figure shows the time variation for different values of the strength of the magnetic field: } h = 6J \text{ (black curve), } h = 9.2J \text{ (Blue curve), } h = 10J \text{ (Red curve); other parameters are: } \omega_0 = 2J, S = 1, \gamma_y = 1, \gamma_z = 0.7. \text{ The critical value corresponds to } h = 9.2J. \text{ For convenience, the decay rate is given in units of } 2g^2/J. \]
FIG. 8. Resonance behavior of the Lamb shift $\xi$. The figure shows the time variation for different values of the strength of the magnetic field: $h = 6J$ (black curve), $h = 9.2J$ (Blue curve), $h = 10J$ (Red curve); other parameters are: $\omega_0 = 2J$, $S = 1$, $\gamma_y = 1$, $\gamma_z = 0.7$. The critical value corresponds to $h = 9.2J$. For convenience, the Lamb shift is given in units of $2g^2/J$.

case of $\gamma_y \neq 1$

When the anisotropy parameter $\gamma_y$ is different from unity, with $h > h_{\text{cri}}$, we find that for $h$ close to the critical value, the decay rate decreases as $\gamma_y$ increases; but for large values of the magnetic field, it can be noticed the behavior is reversed, in the sense that the decay rate assumes larger values, se Figs [9][11]. This result can be explained as earlier through the emergence of a critical resonance with respect to the dependence on $h$. To see this we can for instance evaluate the asymptotic value of the decay rate which turns out to be:

$$\kappa_\infty = 4Sg^2 \int_{-\pi}^{\pi} \left( \frac{\omega_0 + \omega(k)}{\omega(k)} \right) \cos^2 \left( \frac{k}{2} \right) \delta(\omega_0 - \Omega(k)) dk.\quad (87)$$

It follows that the resonance occurs at the zeros of the function

$$G(k) = \omega_0 - \Omega(k).\quad (88)$$

To close the discussion with the one-dimensional lattice at zero temperature, we give some results for $h < h_{\text{cri}}$. An example of the variation of the decay rate is displayed in Figs. We notice that as the magnetic field approaches its critical value from below, the decay rate increases. The identification of the resonance in this case is quite complicated and will not be shown here.
FIG. 9. The decay rate $\kappa$ as a function of time for different values of the anisotropy parameter: $\gamma_y = 0.8$ (black curve), $\gamma_y = 0.6$ (Blue curve); other parameters are: $h = 1.2J$, $\omega_0 = 8J$, $S = 1$, $\gamma_z = 0.7$. For convenience, the decay rate is given in units of $2g^2/J$.

FIG. 10. The decay rate $\kappa$ as a function of time for different values of the anisotropy parameter: $\gamma_y = 0.8$ (black curve), $\gamma_y = 0.6$ (Blue curve); other parameters are: $h = 7J$, $\omega_0 = 8J$, $S = 1$, $\gamma_z = 0.7$. For convenience, the decay rate is given in units of $2g^2/J$.

**B. Two-dimensional lattice**

We assume that the impurity lies in the center of a unit cell in the lattice so that the distance from it to any neighboring lattice spin is equal to $\delta/\sqrt{2}$. The coupling constant of the impurity to the lattice spins is denoted here also by $g$. Thus the squared modulus of the coupling constant $g_k$ is given by

$$|g_k|^2 = \frac{2Sg^2}{N} \left| 1 + e^{i k_x \delta / \sqrt{2}} (1 + e^{i k_y \delta / \sqrt{2}}) + e^{i k_x \delta / \sqrt{2}} \right|^2 \nonumber$$

$$= \frac{32gS}{N} \cos^2 \left( \frac{\sqrt{2}}{2} k_x \delta \right) \cos^2 \left( \frac{\sqrt{2}}{2} k_y \delta \right). \tag{89}$$
FIG. 11. The decay rate $\kappa$ as a function of time for different values of the anisotropy parameter: $\gamma_y = 0.8$ (black curve), $\gamma_y = 0.6$ (Blue curve); other parameters are: $h = 12J$, $\omega_0 = 8J$, $S = 1$, $\gamma_z = 0.7$. For convenience, the decay rate is given in units of $2g^2/J$.

FIG. 12. The decay rate $\kappa$ as a function of time for different values of the anisotropy parameter: $\gamma_y = 0.8$ (black curve), $\gamma_y = 0.6$ (Blue curve); other parameters are: $h = 7J$, $\omega_0 = 8J$, $S = 1$, $\gamma_z = 0.7$. For convenience, the decay rate is given in units of $2g^2/J$.

Moreover, the lattice structure factor reads now as:

$$\tau_k = \frac{1}{2}(\cos(k_x \delta) + \cos(k_x \delta))$$  \hspace{1cm} (90)$$

By direct substitution, we find that for $h > h_{\text{cri}}$ the correlation function in the continuum limit takes the form

$$\Psi(t) = \frac{4g^2}{\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \left(1 + \frac{1}{\sqrt{1 - \frac{4\Lambda(k)^2}{\omega(k)^2}}} \right) \left(\cos^2\left(\frac{\sqrt{2}k_x}{2}\right) \cos^2(\frac{\sqrt{2}k_y}{2})\right) e^{i(\omega_0 - \Omega(k))t} dk_x dk_y. \hspace{1cm} (91)$$
FIG. 13. The decay rate $\kappa$ as a function of time for different values of the anisotropy parameter: $\gamma_y = 0.8$ (black curve), $\gamma_y = 0.6$ (Blue curve); other parameters are: $h = 7J$, $\omega_0 = 8J$, $S = 1$, $\gamma_z = 0.7$. For convenience, the decay rate is given in units of $2g^2/J$.

The detailed study of the time dependence of the decay rate in this case, reveals essentially the same features obtained for the one-dimensional case, and hence we competent ourselves by giving an example of the evolution of the decay rate for as special case as displayed in figures 14 and 15.

FIG. 14. The decay rate $\kappa$ in two dimensions as a function of time for different values of the magnetic field: $h = 2.5J$ (black curve), $h = 7J$ (Blue curve), $h = 10J$ (Blue curve); other parameters are: $\gamma_y = 1$, $\omega_0 = 4J$, $S = 1$, $\gamma_z = 0.7$. For convenience, the decay rate is given in units of $2g^2/J$. 

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FIG. 15. The decay rate \( \kappa \) in two dimensions as a function of time for different values of the magnetic field: \( h = 2.5J \) (black curve), \( h = 7J \) (Blue curve), \( h = 10J \) (Blue curve); other parameters are: \( \gamma_y = 0.5, \omega_0 = 4J, S = 1, \gamma_z = 0.7 \). For convenience, the decay rate is given in units of \( 2g^2/J \).

V. CONCLUSION

In conclusion, we have derived the master equation of a two-level impurity coupled to an anisotropic ferromagnetic spin lattice. We have diagonalized the lattice Hamiltonian by using the Holstein-Primakoff transform that is essentially valid at sufficiently low temperatures. This corresponds to the low excitation domain of the lattice. We found that there exists a critical point of the magnetic field around which the orientation of the spins in the ground state changes. The bosonization is carried out above and below the critical point. Using the bosonic operators, we expressed the interaction Hamiltonian in the two cases. Using the Liouville equation, we derived the master equation for the reduced density matrix of the purity. The elimination of the lattice degrees of freedom is carried out by taking into account the spectral properties of the lattice which is uniquely fixed by its periodicity. We have discussed the variation of the decay rates with respect to the model parameters. It is found that for \( h \) exceeding the critical value the decay rate increases, with the increase of the magnetic field, until some point where in blows up as the time increases, which is identified as a resonance behavior; as the magnetic increases, the decay rate begins to decrease, and assumes lower asymptotic values. For \( h \) less than the critical value the decay rate assumes essentially lower values. These results turn out to be valid regardless of the dimension of the lattice. The next step would be to include the effect of the temperature on the decay.
rates and to investigate the consequences on the dynamics of the two-level system.

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