Allometric Models to Predicate Single-Tree Biomass in the Eurasian Larix spp. Forest

Vladimir Andreevich Usoltsev¹,², Seyed Omid Reza Shobari¹*, Ivan Stepanovich Tsepordey², Walery Zukow³

¹Ural State Forest Engineering University, 620100 Yekaterinburg, Sibirskiy Trakt, 37, Russia
²Botanical Garden of Ural Branch of RAS, 620144 Yekaterinburg, str. 8 Marta, 202а, Russia
³Faculty of Earth Sciences and Spatial Management, Nicolaus Copernicus University, Lwowska 1 str., 87-100 Toruń, Poland

*corresponding author’s e-mail: Omidshobeyri214@gmail.com

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Abstract. Today, estimating of biological productivity or carbon-depositing ability of forests is going on the global level, and its increase is one of the major factors of climate stabilization. In recent years, two trends in the harmonization of allometric models of tree biomass have been developing. The first of them is related to ensuring the additivity of the biomass component composition, and the second one – to the search for the so-called generic model applicable to a wide range of environmental conditions. However, all "pseudo-generic" models give significant biases in their application in local conditions. In our modeling, we adhere to the principle of biomass additivity, split "generic" model into regional variants by introducing dummy variables, and build the model at the transcontinental level for the first time. When using the unique in terms of the volume of database of trees of the genus Larix Mill. In a number of 420 sample trees, the trans-Eurasian additive allometric models of biomass of trees for Eurasian larch forests are developed, and thereby the combined problem of model additivity and generality is solved. The additive model of tree biomass of Larix is harmonized in two ways: it eliminated the internal contradictions of the component and the total biomass equations, and in addition, it takes into account regional differences of trees of equal sizes on their biomass, i.e. it reflects the regional peculiarities of the component structure of tree biomass.

Key words: genus Larix, equations additivity, biosphere role of forests, biomass of single-trees, allometric models, sample plots, biological productivity, transcontinental tables of biomass.

1. Introduction

Study of biosphere role and climate conditionality of biological productivity of world forest is one of the most priority directions of ecology and biogeography sciences. It is known that changes in the vegetation cover of Eurasia occur both in the latitudinal direction due to changes in the solar radiation (Grigoriev & Budyko, 1956), and in the meridional direction due to changes in the continental climate (Komarov, 1921). Therefore, models of the biomass of trees and stands have been developed, including their mass-forming indices as independent variables, as well as indices of natural zoning and climate continentality (Usoltsev et al., 2018).
Due to current climate changes, priority is given to changes in the biomass of forest ecosystems under the influence of average temperatures and precipitation (Usoltsev et al., 2019a, 2019b; Hubau et al., 2020). Since these predictive models reflect long-term adaptive responses of stands to regional climate conditions, but do not take into account rapid trends of current changes, they are considered to be preliminary (Usoltsev et al., 2019b).

Along with biomass models that take into account both the natural zoning and continental climate, as well as temperature and precipitation, models and taxation standards for estimating biomass distributed by geographical regions are necessary for practical purposes. Generic allometric models only work well for aboveground biomass without component differentiating, mainly in tropical forests (Chave et al., 2001). However, when assessing the regional component composition of biomass, such models give biases (Usoltsev et al., 2017). Modern methods of modelling the biological productivity of single-trees and forest stands are developed in terms of biomass component additivity (Bi et al., 2010; Dong et al., 2015) and the transition from "pseudo-generic" allometric models to really generic, supposing regionalization of biomass model by introducing dummy variables (Fu et al., 2012). The database of single-tree biomass compiled for forest-forming species in Eurasia (Usoltsev, 2016) has enabled these modern methodologies to begin modelling additive tree biomass on transcontinental level.

In this article, an attempt to develop additive allometric models of single-tree biomass of one of the most common species in Eurasia - larch (Larix spp.). These models will provide the basis for the development of regional transcontinental standards for estimating tree and forest stand biomass and its temporal changes using the traditional forest mensuration and inventories.

2. Materials and methods

Of the mentioned database the materials in a number of 420 sample trees of four vicarage species of the genus Larix are taken, that are distributed in eight eco-regions and marked respectively by eight dummy variables from \( X_0 \) to \( X_7 \) (Table 1). The distribution of sample plots, on which sample trees are taken in different ecoregions of Eurasia, is shown in Figure 1.
Table 1. The scheme of regional coding actual biomass of 420 sample trees by dummy variables

| Region* | Species** of *Larix* | Dummy variables | Range of DBH, cm | Range of tree height, m | Number of measurements |
|---------|----------------------|----------------|------------------|------------------------|------------------------|
| WME     | *L. decidua* Mill.    | $X_1 = 0$ $X_2 = 0$ $X_3 = 0$ $X_4 = 0$ $X_5 = 0$ $X_6 = 0$ $X_7 = 0$ | $7.1\div47.8$ | $9.8\div34.0$ | 14 |
| ER      | *L. sibirica* Ledeb.  | $X_1 = 1$ $X_2 = 0$ $X_3 = 0$ $X_4 = 0$ $X_5 = 0$ $X_6 = 0$ | $1.0\div35.0$ | $2.3\div28.0$ | 25 |
| Tst     | *L. sibirica* Ledeb.  | $X_1 = 0$ $X_2 = 1$ $X_3 = 0$ $X_4 = 0$ $X_5 = 0$ | $6.2\div28.0$ | $7.9\div17.8$ | 28 |
| WSn     | *L. sibirica* Ledeb.; *L. gmelinii* (Rupr.) Kuzen. | $X_1 = 0$ $X_2 = 0$ $X_3 = 0$ $X_4 = 0$ | $2.1\div38.0$ | $2.9\div24.8$ | 116 |
| ESn     | *L. gmelinii* (Rupr.) Kuzen. | $X_1 = 0$ $X_2 = 0$ $X_3 = 1$ $X_4 = 0$ $X_5 = 0$ | $0.3\div22.7$ | $1.4\div14.8$ | 66 |
| FEn     | *L. gmelinii* (Rupr.) Kuzen. | $X_1 = 0$ $X_2 = 0$ $X_3 = 0$ $X_4 = 1$ $X_5 = 0$ | $3.9\div52.8$ | $2.9\div30.0$ | 43 |
| Ch      | *L. sibirica* Ledeb.; *L. gmelinii* Rupe. Kuzen. | $X_1 = 0$ $X_2 = 0$ $X_3 = 0$ $X_4 = 1$ $X_5 = 0$ | $0.5\div31.0$ | $1.5\div24.3$ | 50 |
| Jap     | *L. kaempferi* (Lamb.) Carrière | $X_1 = 0$ $X_2 = 0$ $X_3 = 0$ $X_4 = 0$ $X_5 = 1$ | $4.0\div35.9$ | $4.3\div26.7$ | 73 |

*Region designations: WME – West and Middle Europe; ER – European part of Russia, Central territory; Tst – Turgay steppe; WSn – Western Siberia, northern taiga; ESn – Eastern Siberia, Northern taiga; FEn – Far East, northern taiga; Ch – Northeast China; Jap – Japanese islands.

**According to World Flora Online, An Outline Flora of All Known Plants (WFO, 2020) *Larix sukaczewii* Dylis is synonym of *L. sibirica* Ledeb.; *L. cajanderi* Mayr is synonym of *L. gmelinii* (Rupr.) Kuzen.; and *L. leptolepis* (Siebold & Zucc.) Gordon is synonym of *L. kaempferi* (Lamb.) Carrière.
Figure 1. The distribution of sample plots, on which sample trees are taken in different ecoregions of Eurasia

According to the structure of disaggregating three-step additive model system (Tang et al., 2000; Dong et al., 2015), total biomass, estimated by the total equation, exploded into components according to the scheme presented in Figure 2. The coefficients of the regression models for all three steps are evaluated simultaneously, which ensures additivity of the all components: total, intermediate and initial ones (Dong et al., 2015).

Figure 2. The pattern of disaggregating three-step proportional weighting additive model. Designation: $P_t$, $P_r$, $P_a$, $P_c$, $P_s$, $P_f$, $P_w$ and $P_{bk}$ are tree biomass respectively: total, underground (roots), aboveground, crown (needles and branches), stems above bark (wood and bark), needles, branches, stem wood and stem bark correspondingly, kg.

### 3. Results and discussion

Initial allometric models are calculated:

\[
\ln P_i = a_i + b_i (\ln D) + c_i (\ln H) + d_{ij} (\ln D)(\ln H) + \sum g_{ij}X_j, \quad (1)
\]

where $P_i$ – biomass of $i$-th component, kg; $D$ – diameter on breast height, cm; $H$ – tree height, m; $i$ – index of biomass component: total ($t$), aboveground ($a$), roots ($r$), tree crown ($c$), stem above bark ($s$), foliage ($f$), branches ($b$), stem wood ($w$) and stem bark ($bk$); $j$ - index (code) of dummy variable, from 0 to 7 (see Table 1). $\sum g_{ij}X_j$ – block of dummy variables for $i$-th biomass component of $j$-th ecoregion. Model (1) after antilogarithmic procedure has the form:

\[
P_i = e^{a_i}D^{b_i}H^{c_i}D^{d_{ij}(\ln H)}e^{\sum g_{ij}X_j}. \quad (2)
\]

Rationale for the structure of the regression model (1) was made earlier (Usoltsev et al., 2017). Since calculation of regression coefficients in the model (1) is made in the transformed data, to eliminate biases caused by logarithmic modification of variables, in the equation the amendment proposed by G.L. Baskerville (1972) is introduced. Using the programme of common regression analysis the calculation of coefficients of equations (1) is performed and their characteristic is obtained, that is given in the Table 2 after correcting the logarithmic transformation by G.L. Baskerville (1972) and bringing it to the form (2). All the
regression coefficients for numerical variables of the equations (2) are significant at the level of probability of 0.95 or higher, and the equations are adequate to empirical data.

By substituting the regression coefficients of initial equations from Table 2 into the structure of the additive model, presented in Table 3, when using three-step scheme of proportional weighting, we got transcontinental additive model of component composition of larch tree biomass of double harmonization, the final appearance of which is given in Table 4. The model is valid in the range of actual data of height and diameter of the sample trees shown in the Table 1.

By tabulating the model obtained (Table 4) according to the given values of $D$ and $H$ as well as by the values of the dummy variables, localizing the general model for eco-regions, you can calculate regional transcontinental standards for Eurasia, intended for estimating tree and forest additive biomass components.

Because sometimes it is impossible to measure the height of trees in sample plots, for such cases when calculating the biomass per ha the auxiliary equation intended for using the proposed model (2) is calculated:

$$H = 2.387D^{0.7114}e^{-0.1649/D}e^{-0.1886X_1}e^{-0.1254X_2}e^{-0.1644X_3}e^{-0.4100X_4}e^{-0.2704X_5}e^{-0.2219X_6}e^{-0.2346X};$$

$$adjR^2 = 0.918.$$  \hspace{1cm} (3)

Variable $(1/D)$ introduced in the structure of the model (3) for correction of allometry, biased in small trees due to the shift of diameter $D$ in the upper part of tree crown. Tabulating of built additive models (2) in Excel format is fulfilled. Because the volume of tables obtained can exceed the format of journal article, we are limit ourselves to some regional characteristics analysis of the structure of biomass of trees of the same size when using the fragment of summary table for larch (Table 5). In their analysis, you can see that the maximum values for total biomass of equal trees occur in Western (82 kg) and Eastern (74-98 kg) parts of the studied area, that are under the influence of a humid climate of the Atlantic and Pacific oceans, respectively. From the Atlantic coast to the eastern direction, on the territory of European Russia and in Turgai Depression the total tree biomass is reduced to 55-62 kg. But in the subzone of northern taiga in Siberia the total tree biomass increases up to 82-110 kg due to the low density of standing trees in the permafrost. Approximately the same regional patterns are inherent and for aboveground biomass.

It was found (Cunia & Briggs, 1984; Reed & Green, 1985), that the correction of internal inconsistency of biomass equations by ensuring their additivity does not necessarily means improvements in the accuracy of biomass estimating, it is necessary to ascertain,
whether adequate the additive model obtained and how its adequacy characteristics are related to the same indices of independent (trivial) equations?
Table 2. The characteristic of independent allometric equations for larch trees

| Biomass component | Independent variables and the model regression coefficients | \(adjR^2\)* |
|-------------------|-----------------------------------------------------------|------------|
| \(P_t\)           | \(D^{1.5672} \quad H^{0.4054} \quad D^{0.0931 \ln H} \quad e^{-0.3963 X_1} \quad e^{-0.2728 X_2} \quad e^{0.0016 X_3} \quad e^{0.1681 X_4} \quad e^{0.5002 X_5} \quad e^{0.1786 X_6} \quad e^{-0.1001 X_7}\)| 0.987     |
| \(P_a\)           | \(D^{1.4212} \quad H^{0.5154} \quad D^{0.1841 \ln H} \quad e^{-0.1885 X_1} \quad e^{-0.0747 X_2} \quad e^{0.1839 X_3} \quad e^{0.1837 X_4} \quad e^{-0.0635 X_5} \quad e^{-0.0947 X_6} \quad e^{-0.1224 X_7}\)| 0.991     |
| \(P_r\)           | \(D^{1.9268} \quad H^{0.1825} \quad D^{0.0308 \ln H} \quad e^{-0.6254 X_1} \quad e^{-0.0191 X_2} \quad e^{-0.3614 X_3} \quad e^{0.4929 X_4} \quad e^{1.2546 X_5} \quad e^{1.1750 X_6} \quad e^{0.2900 X_7}\)| 0.954     |
| \(P_c\)           | \(D^{2.2312} \quad H^{-1.7550} \quad D^{0.2494 \ln H} \quad e^{-0.1750 X_1} \quad e^{-0.3042 X_2} \quad e^{-0.5668 X_3} \quad e^{-0.3290 X_4} \quad e^{-0.2613 X_5} \quad e^{-0.3483 X_6} \quad e^{-0.1243 X_7}\)| 0.904     |
| \(P_s\)           | \(D^{1.3238} \quad H^{0.6808} \quad D^{0.1700 \ln H} \quad e^{-0.2019 X_1} \quad e^{-0.0271 X_2} \quad e^{-0.0833 X_3} \quad e^{0.2779 X_4} \quad e^{-0.0286 X_5} \quad e^{-0.0654 X_6} \quad e^{-0.1733 X_7}\)| 0.992     |
| \(P_f\)           | \(D^{2.0986} \quad H^{-1.5555} \quad D^{0.1874 \ln H} \quad e^{-0.3966 X_1} \quad e^{0.1968 X_2} \quad e^{-0.1625 X_3} \quad e^{0.0886 X_4} \quad e^{-0.0193 X_5} \quad e^{0.0847 X_6} \quad e^{0.3110 X_7}\)| 0.855     |
| \(P_b\)           | \(D^{2.3314} \quad H^{-1.7586} \quad D^{0.2438 \ln H} \quad e^{-0.3327 X_1} \quad e^{0.4231 X_2} \quad e^{0.6662 X_3} \quad e^{-0.3923 X_4} \quad e^{-0.3125 X_5} \quad e^{-0.4403 X_6} \quad e^{-0.2260 X_7}\)| 0.908     |
| \(P_w\)           | \(D^{1.3125} \quad H^{0.7886} \quad D^{0.1730 \ln H} \quad e^{-0.1860 X_1} \quad e^{0.0654 X_2} \quad e^{-0.2185 X_3} \quad e^{0.3077 X_4} \quad e^{0.0552 X_5} \quad e^{-0.0828 X_6} \quad e^{-0.0020 X_7}\)| 0.993     |
| \(P_{bk}\)        | \(D^{1.3274} \quad H^{0.1312} \quad D^{0.2344 \ln H} \quad e^{-0.2909 X_1} \quad e^{0.1207 X_2} \quad e^{0.1761 X_3} \quad e^{0.6553 X_4} \quad e^{-0.2840 X_5} \quad e^{0.2626 X_6} \quad e^{0.2783 X_7}\)| 0.978     |

* \(adjR^2\) – adjusted coefficient of determination.
Table 3. The structure of three-step additive models obtained by proportional weighting. Symbols here and further see in equation (1)

| Step     | Formula                                                                                                                             |
|----------|-------------------------------------------------------------------------------------------------------------------------------------|
| Step 1   | $P_r = \frac{1}{1 + \frac{a_a D^{b_a} H^{c_a} D^{d_a}(\ln H) e^{\xi \varepsilon a_j X_j}}{a_r D^{b_r} H^{c_r} D^{d_r}(\ln H) e^{\xi \varepsilon r_j X_j}} \times P_t}$ |
| Step 2   | $P_a = \frac{1}{1 + \frac{a_r D^{b_r} H^{c_r} D^{d_r}(\ln H) e^{\xi \varepsilon r_j X_j}}{a_a D^{b_a} H^{c_a} D^{d_a}(\ln H) e^{\xi \varepsilon a_j X_j}} \times P_t}$ |
| Step 2   | $P_c = \frac{1}{1 + \frac{a_c D^{b_c} H^{c_c} D^{d_c}(\ln H) e^{\xi \varepsilon c_j X_j}}{a_c D^{b_c} H^{c_c} D^{d_c}(\ln H) e^{\xi \varepsilon c_j X_j}} \times P_a}$ |
| Step 3a  | $P_s = \frac{1}{1 + \frac{a_s D^{b_s} H^{c_s} D^{d_s}(\ln H) e^{\xi \varepsilon s_j X_j}}{a_s D^{b_s} H^{c_s} D^{d_s}(\ln H) e^{\xi \varepsilon s_j X_j}} \times P_a}$ |
| Step 3a  | $P_f = \frac{1}{1 + \frac{a_f D^{b_f} H^{c_f} D^{d_f}(\ln H) e^{\xi \varepsilon f_j X_j}}{a_f D^{b_f} H^{c_f} D^{d_f}(\ln H) e^{\xi \varepsilon f_j X_j}} \times P_c}$ |
| Step 3a  | $P_b = \frac{1}{1 + \frac{a_f D^{b_f} H^{c_f} D^{d_f}(\ln H) e^{\xi \varepsilon f_j X_j}}{a_f D^{b_f} H^{c_f} D^{d_f}(\ln H) e^{\xi \varepsilon f_j X_j}} \times P_c}$ |
| Step 3b  | $P_w = \frac{1}{1 + \frac{a_w D^{b_w} H^{c_w} D^{d_w}(\ln H) e^{\xi \varepsilon w_j X_j}}{a_w D^{b_w} H^{c_w} D^{d_w}(\ln H) e^{\xi \varepsilon w_j X_j}} \times P_s}$ |
| Step 3b  | $P_{bk} = \frac{1}{1 + \frac{a_{bk} D^{b_{bk}} H^{c_{bk}} D^{d_{bk}}(\ln H) e^{\xi \varepsilon bk_j X_j}}{a_{bk} D^{b_{bk}} H^{c_{bk}} D^{d_{bk}}(\ln H) e^{\xi \varepsilon bk_j X_j}} \times P_s}$ |
Table 4. Three-step additive model of component biomass composition for larch trees, obtained by proportional weighing

| Step 1 | Pa = \( \frac{1}{1 + 0.2093D^{0.5056}H^{-0.1309}D^{-0.1532} \ln H} \times P_t \) | Pr = \( \frac{1}{1 + 4.7775D^{0.5056}H^{0.1309}D^{0.1532} \ln H} \times P_t \) |
|--------|-----------------------------------------------------------------|-----------------------------------------------------------------|
| Step 2 | Pc = \( \frac{1}{1 + 0.1505D^{-0.9074}H^{2.4448}D^{0.0794} \ln H} \times Pa \) | Ps = \( \frac{1}{1 + 6.6460D^{-0.9074}H^{-2.4448}D^{0.0794} \ln H} \times Pa \) |
| Step 3a| Pf = \( \frac{1}{1 + 3.5479D^{0.2328}H^{-0.2033}D^{0.0565} \ln H} \times Pc \) | Pb = \( \frac{1}{1 + 0.2819D^{-0.2328}H^{0.2033}D^{-0.0565} \ln H} \times Pc \) |
| Step 3b| Pw = \( \frac{1}{1 + 0.6248D^{0.0150}H^{-0.6574}D^{0.0614} \ln H} \times Ps \) | PbK = \( \frac{1}{1 + 1.6605D^{0.0150}H^{0.6574}D^{-0.0614} \ln H} \times Ps \) |
Table 5. Fragments of the table of additive tree biomass for diameter 14 cm and tree height of 14 m according to the eco-regions and corresponding species of the genus *Larix*

| Biomass component | WME L. decidua | ER L. sibirica | Tst L. sibirica | WSn L. sibirica | ESn L. gmelinii | FEn L. gmelinii | Ch L. sibirica | L. gmelinii | Jap L. kaempferi |
|-------------------|----------------|----------------|-----------------|-----------------|-----------------|-----------------|----------------|--------------|-----------------|
| Total biomass     | 81.65          | 54.93          | 62.15           | 81.78           | 96.59           | 110.24          | 97.62          | 73.87       |
| Roots             | 13.24          | 6.10           | 10.90           | 11.21           | 20.12           | 46.25           | 39.82          | 16.70       |
| Aboveground       | 68.41          | 48.83          | 51.26           | 70.57           | 76.47           | 63.99           | 57.80          | 57.16       |
| Tree crown        | 11.39          | 8.32           | 6.75            | 7.74            | 7.51            | 8.75            | 7.56           | 9.91        |
| Foliage           | 1.70           | 2.22           | 1.66            | 1.75            | 1.64            | 1.67            | 1.73           | 2.29        |
| Branches          | 9.69           | 6.09           | 5.08            | 5.99            | 5.87            | 7.08            | 5.83           | 7.62        |
| Stem above bark   | 57.01          | 40.52          | 44.51           | 62.83           | 68.96           | 55.24           | 50.23          | 47.25       |
| Stem wood         | 48.49          | 34.98          | 37.42           | 51.74           | 55.22           | 48.97           | 40.67          | 38.38       |
| Stem bark         | 8.52           | 5.54           | 7.09            | 11.09           | 13.74           | 6.27            | 9.56           | 8.87        |
Table 6. The characteristics of "methodized" independent allometric equations for larch trees

| Biomass component | Components of regression models |
|-------------------|--------------------------------|
| $P_t$             | $D^{1.56/2}$ $H^{0.4054}$ $D^{0.0931(\ln H)}$ $e^{-0.3963 X_1}$ $e^{-0.2728 X_2}$ $e^{0.0016 X_3}$ $e^{0.1681 X_4}$ $e^{0.3002 X_5}$ $e^{0.1786 X_6}$ $e^{-0.1001 X_7}$ |
| $P_a$             | $D^{1.5126}$ $H^{0.4540}$ $D^{0.1251(\ln H)}$ $e^{-0.2248 X_1}$ $e^{-0.2433 X_2}$ $e^{-0.0219 X_3}$ $e^{0.1354 X_4}$ $e^{-0.0317 X_5}$ $e^{-0.1557 X_6}$ $e^{-0.1784 X_7}$ |
| $P_r$             | $D^{1.9950}$ $H^{-0.0153}$ $D^{0.0517(\ln H)}$ $e^{-0.6577 X_1}$ $e^{-0.6169 X_2}$ $e^{-0.4111 X_3}$ $e^{0.4411 X_4}$ $e^{1.1507 X_5}$ $e^{1.1536 X_6}$ $e^{0.2453 X_7}$ |
| $P_c$             | $D^{2.2573}$ $H^{-1.1987}$ $D^{0.2028(\ln H)}$ $e^{0.0476 X_1}$ $e^{-0.3003 X_2}$ $e^{-0.7037 X_3}$ $e^{-0.0857 X_4}$ $e^{-0.1963 X_5}$ $e^{-0.0858 X_6}$ $e^{-0.4087 X_7}$ |
| $P_s$             | $D^{1.4184}$ $H^{0.8823}$ $D^{0.0846(\ln H)}$ $e^{-0.2805 X_1}$ $e^{-0.1982 X_2}$ $e^{0.0849 X_3}$ $e^{0.2029 X_4}$ $e^{0.0009 X_5}$ $e^{-0.0675 X_6}$ $e^{-0.1183 X_7}$ |
| $P_f$             | $D^{2.0846}$ $H^{-0.8148}$ $D^{0.1199(\ln H)}$ $e^{0.8548 X_1}$ $e^{0.5336 X_2}$ $e^{-0.0424 X_3}$ $e^{0.5368 X_4}$ $e^{0.1914 X_5}$ $e^{0.4773 X_6}$ $e^{0.0693 X_7}$ |
| $P_b$             | $D^{2.3484}$ $H^{-1.2307}$ $D^{0.2026(\ln H)}$ $e^{-0.1894 X_1}$ $e^{-0.5605 X_2}$ $e^{-0.8354 X_3}$ $e{-0.2175 X_4}$ $e{-0.2737 X_5}$ $e{-0.7171 X_6}$ $e{-0.4807 X_7}$ |
| $P_w$             | $D^{1.3125}$ $H^{0.7886}$ $D^{0.1730(\ln H)}$ $e^{-0.1860 X_1}$ $e^{0.0454 X_2}$ $e{-0.0218 X_3}$ $e{0.3077 X_4}$ $e{0.0332 X_5}$ $e{-0.0282 X_6}$ $e{0.0050 X_7}$ |
| $P_{bk}$          | $D^{1.3274}$ $H^{0.1312}$ $D^{0.2344(\ln H)}$ $e{-0.2909 X_1}$ $e{0.1207 X_2}$ $e{0.1761 X_3}$ $e{0.6553 X_4}$ $e{-0.2840 X_5}$ $e{0.2626 X_6}$ $e{0.2783 X_7}$ |
To this purpose, the estimates of biomass obtained from independent and additive equations are compared with actual biomass values by calculating the coefficient of determination $R^2$ calculated by the formula:

$$R^2 = 1 - \frac{\sum_{i=1}^{N} (Y_i - \bar{Y})^2}{\sum_{i=1}^{N} (Y_i - \bar{Y})^2},$$  \hspace{1cm} (4)

where $Y_i$ - actual biomass values; $\bar{Y}_i$ - predicted biomass values; $\bar{Y}$ - the mean actual value of all ($N$) trees.

To properly compare the adequacy of independent and additive equations, we reproduce the original data in a comparable condition, i.e. independent equations for all components of biomass are calculated according to the same data that the additive ones and the equations for the total biomass. Description of such "methodized" equations is given in the Table 6. The results of the comparison (Table 7) indicate that while additive equations internally consistent, but compared to the independent equations they have better characteristics of adequacy not for all component biomass.

The ratio of actual values and derived ones by tabulating independent and additive tree biomass models (Fig. 3) shows the degree of correlativeness of the actual and calculated values and, in many cases, the absence of visible differences in the structure of residual variances obtained on two named models. More or less the value of $R^2$ of one or the other model is determined by the random position of actual values of biomass of largest trees in confidence range and uneven dispersion, namely accidental because of their small number and the greatest contribution to the residual variance (see Fig. 3).

Table 7. Comparison of the adequacy indices of the independent and additive equations for larch tree biomass

| Adequacy index | Pt | Pa | Pr | Ps | Pw | Pbk | Pc | Pb | Pf |
|----------------|----|----|----|----|----|-----|----|----|----|
| **Independent equations** |    |    |    |    |    |     |    |    |    |
| $R^2$           | 0.975 | 0.973 | **0.852** | **0.928** | **0.921** | 0.956 | 0.613 | 0.585 | 0.624 |
| **Additive equations** |    |    |    |    |    |     |    |    |    |
| $R^2$           | 0.975 | 0.967 | **0.891** | **0.985** | **0.965** | 0.907 | 0.749 | 0.737 | 0.721 |

*Designations see Figure 2. Bold components, for which $R^2$ values of the additive models higher than independent ones.
Figure 3. The ratio of observed values and the values derived by calculation of independent (a) and additive (b) models of tree biomass.
4. Conclusions

When using the unique in terms of the volumes of databases on the levels of a single-tree of the genus *Larix*, the trans-Eurasian additive allometric models of biomass of trees for Eurasian larch forests are developed for the first time, and thereby the combined problem of model additivity and generality is solved. The additive model of tree biomass of *Larix* is harmonized in two ways: it eliminated the internal contradictions of the component and the total biomass equations, and in addition, it takes into account regional differences of trees of equal sizes on their biomass, i.e. it reflects the regional peculiarities of the component structure of tree biomass. The proposed model and corresponding tables for estimating tree biomass makes them possible to calculate larch stand biomass (t/ha) on Eurasian forests when using measuring taxation.

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