Scaffolding in mathematical problem-solving

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Abstract. This research is a descriptive study that aims to describe the scaffolding conducted by the teacher in helping students and students' strategies in solving problems. The subjects of this research were 20 students grade IX of junior high school number 3 Palembang. The instruments used in this study were field notes, observation sheets, student worksheets and interviews. Based on the research results, scaffolding can help solve students' problem solving. Scaffolding in form a written questions and direct guidance with step by step are as a bridge in solving problems. Scaffolding which starts with sketching can be teacher feedback in knowing the concept of student images. Differences in students solution and student strategies are influenced by prior knowledge, background experience, intellectual abilities, and writing abilities in mathematical language.

1. Introduction
Problem solving is the most important ability [1-4]. When solving problems, students not only utilize existing mathematical knowledge but students must involve all high-level thinking skills [2, 5-9]. The importance of problem solving is to increase perseverance when solving problems, improve self-concept, increase awareness, encourage students to believe in their own abilities [10-13].

The Problem solving is explained as a complex process [2, 14], that requires a lot of skills and many steps to find the relationship between experiences or past schemes and the problems they are currently facing [15, 16]. Problem solving starts with interpreting problems that aim to be able to understand the problem, choosing the information needed, turning information obtained into mathematical symbols aimed at planning alternative solutions through the use of various techniques, methods, and problem solving strategies, and producing alternative solutions as the ultimate goal of problem solving [14, 17, 18].

However, there are still many difficulties faced by students when solving problems [19]. Students difficulty on problems that cannot be solved by one routine procedure, requires critical analysis [20, 21], and requires an understanding of a concept and application of mathematical skills, and have difficulty with problems that the solution is no single and there are variations in answers [1, 19-21].

One of strategies that used for these problems is scaffolding. Previous studies explained that scaffolding is very useful in mathematics education, especially when there is dialogue between teachers and students [22]. One of dialogue between teachers and students is scaffolding. Scaffolding is a teaching strategy that provides support to both groups and individuals [23-25]. The provision of scaffolding in learning mathematics can improve students' understanding of concepts [26], as well as contribute to helping students solve problems. Scaffolding can improve student learning outcomes and independence [27]. Scaffolding is very dependent on the knowledge and abilities of teachers [28].
Scaffolding can also influence students' future cognitive development [29]. Scaffolding is given to students who are unable to complete their assignments independently [30, 31]. Scaffolding can be done in two ways namely oral and written through the use of worksheets and structured assignments [32]. Scaffolding can be obtained from teachers and friends [33]. Other individuals (friends) who are more knowledgeable provide support to facilitate student development. Scaffolding can be in the form of modeling desired behavior, offering explanations, inviting students to participate, verifying and clarifying student understanding, and inviting students to contribute instructions [34]. Scaffolding can be used to reduce mistakes made by students when solving problems and further facilitate students in constructing knowledge [35].

In relation to problem solving, this research also provides scaffolding through the provision of student worksheets that are equipped with probing and prompting questions. Problems presented in worksheets are in the form of contextual problems. The provision of scaffolding begins when students are asked to understand the contextual problem. The provision of scaffolding aims so that students understand the intent of the problem and present it in a simple form. Provision of scaffolding is then done when students are asked to connect related concepts that exist in the problem. Students are asked to think reflectively related knowledge that can be used in problem solving. Provision of scaffolding by emphasizing the conceptual relationship contained in the problem in order to take steps to solve. Based on the description above, this study aims to describe the scaffolding by the teacher in helping students solve students' mathematical problems.

2. Method
This research is a descriptive study that aims to describe the scaffolding conducted by the teacher in helping students solve mathematical problems. The subjects of this study were grade IX students of junior high school number 3 in Palembang. Data collection techniques using non-test. The instruments used were student worksheets, documentation and field notes. During the learning process, students are given student worksheets that contain problems. All teacher and student activities are recorded and recorded. all data collected is then summarized and described. The use of worksheets not only aims to facilitate the provision of scaffolding but also to find out the strategies used by students in solving problems. The following is one of the problems presented in the worksheet.

A beam-shaped tub is above the mayor's building. The tub has a capacity of 1,200 cubic meters with a base area of 250 square meters. At first, the reservoir is full of water, and after 5 hours flowing into the surrounding area, the height of the remaining water in the tub is 3 meters. If the water starts to flow at 6:00 in the morning and the water flows at a constant speed. At what time does the water no longer flow?

Figure 1. mayor’s building
3. Result and Discussion
In this study there are three scaffolding phases, namely understanding the problem, planning the solution, and the problem solving process. Scaffolding begins when students are asked to understand contextual problems. In the phase of understanding the problem, initially students are asked to read the problem on the worksheet and tell it in their own language. Next, students are guided and led to write whatever is known of a problem and write it in mathematical symbols. Students who have difficulty understanding the problem are guided by the teacher through instructions and additional questions that are investigative. The following form of scaffolding in the phase of understanding the problem.

**Teacher** : Try to read each sentence carefully. Do you know and understand all the words in the problem. is there a foreign term so that it obscures your understanding.

**Student** : Yes sir. I can understand all the words.

**Teacher** : Can you tell the purpose of this problem.

**Student** : There is a tub above the mayor's building, the volume of the tub is 1200 cubic meters and the base area is 250 square meters. in five hours there is water flowing from the tub. water starts flowing at 6:00 in the morning.

**Teacher** : Try to illustrate the situation with the problem.

**Student** : Yes sir. (while illustrating the problem in sketch form)

Based on the conversation above, it appears that the scaffolding given by the teacher. The teacher gives scaffolding in the form of an invitation to read the problem slowly and then asks students to identify whether there are any unknown terms. To see students' understanding, the teacher asks students to tell the problem being solved, and the next form of scaffolding is to lead students to pour the known information in the form of sketches. The following are some sketches made by students as a result of scaffolding activities.

![Figure 2](image)

**Figure 2.** the problem in sketch form

Figure 2 above, it appears that the results of students' sketches are different. This is influenced by students' understanding of the problem. On the problem it is written that there is "a tub above the mayor's building", so students sketch a tub at the height. Based on the results of the sketch, there was a student sketching a clock that showed 6:00 a.m. This is the impact of the words "water starts flowing at 06.00 am".

Next, the teacher continues scaffolding to state the problem in mathematical symbols. The following form of scaffolding.
Teacher : Okay, good. Furthermore, what should we look for in this matter?
Student : What time does the water not flow, sir?
Teacher : Can you write down all the information you have told me in the form of mathematical symbols. Both the information that is known and the information that you want to find on this problem.
Student : Yes, sir.

Based on the conversation above, it appears that the teacher gives scaffolding in the form of an invitation to read the problem slowly and then asks students to write in mathematical language and mathematical symbols. The following are a few examples of student answers as a result of scaffolding activities.

![Figure 3. The information in the form of mathematical symbols](image)

Figure 3 above, it appears that the results of writing in mathematical symbols are almost the same. This is influenced by information and numbers that are clearly seen in the problem. For example "the volume of a tub of 1200 cubic meters, the area of the base of a tub of 250 square meters, and water has been flowing for 5 hours." This information makes it easy for students to write in the form of mathematical symbols.

The second phase of scaffolding is planning for completion. Scaffolding is given to students who have difficulty connecting related concepts in the problem. Students are asked to think reflectively and think of prior knowledge that can be used in problem solving. The following is one form of scaffolding given by the teacher when guiding students to plan problem solving.

Teacher : Do you think that with the available information (known data), the solution can be immediately sought.
Student : No, sir, we don't know how tall the tub is.
Teacher : How to find the height of the tub.
Student : We can use the beam volume formula and we can find it by sharing the volume and area alone, sir
Teacher : Next, what is the height?
Student : From that height we can know how much water comes out after 5 hours.
Teacher : In your opinion, what related material is used to solve the problem?
Students : Many sir. Materials related to this problem are volume, area and time.
Based on the conversation above, it appears that the teacher provides scaffolding in the form of questions that are exploring prior knowledge and knowledge that has been learned before. The teacher asks about material that is relevant to the problem and what concepts can be used to solve the problem. The following are a few examples of student answers as a result of scaffolding activities.

![Figure 4](image)

**Figure 4.** Related material is used to solve the problem

Figure 4 above, it appears that students easily write material related to the problem. There is information that is the key word to the problem, for example there is the word "volume, area and time". From the students' answers, it appears that almost all students wrote the same answers.

The third phase is solving the problem. In this phase students are guided and led to solve problems using strategies that have been selected and are deemed to be in accordance with students' own abilities. Scaffolding is given in the form of emphasizing the conceptual relationship contained in the problem to take steps to solve it. The teacher guides students so that they can decide what methods and strategies are used to solve the problem. The following is one form of scaffolding given by the teacher when guiding students in solving problems.

Table: Scaffolding in Problem Solving

| Teacher | We have found height. then what should we look for again. |
|---------|----------------------------------------------------------|
| Student | We don't know what time the water will run out, sir.      |
| Teacher | Your imagination, how much water has been flowed and how much water is left. |
| Student | Wait a minute; I'm looking for first (while counting). Yes sir I already know that water that has been flowing for 5 hours is equivalent to 1.8 meters high. |
| Teacher | So, how high is the water that hasn't been drained? |
| Student | There are still 3 meters left. |
| Teacher | Your imagination, if 1.8 meters is equivalent to 5 hours. Then 3 meters will be equivalent to how many hours. |
| Student | Oh yes sir. I know how to solve it. |

Based on the conversation above, it appears that the teacher provides scaffolding in the form of questions that are leading to the completion step. Students already know height, but students don't know what to look for next. The teacher leads students to determine how much water has come out in the first 5 hours, and then by using the same concepts and thinking lines, students are asked to calculate how much time is needed to spend the entire contents of the tub. Figure 5 shows some student strategies and answers as a result of scaffolding activities.

It appears that almost all students think to find how much water is flowed every hour (water discharge every hour). For students this is a basic key in solving this problem. From the students' answers there are three different strategies. The first student (S1) uses the strategy by calculating how long it takes to spend the remaining water in the tub. Because the remaining water is 3 meters and the speed of water is 0.36 meters / hour. S1 gets the time needed to spend the rest of the water is 8 hours.
20 minutes, then in determining when the water does not flow, S1 writes in the form "5 hours + 8 hours 20 minutes = 13 hours 20 minutes", and the problem is that water begins to flow from 06.00 the water does not flow at 7:20 p.m.

The second student (S2) uses a strategy similar to S1, but the difference is S2 directly calculates how long it takes to spend the entire contents of the body (assuming: the tub is in a full state). Based on the calculation results, S2 gets the time needed to spend the entire contents of the tub is "13 hours 20 minutes", so that the water does not flow at 19:20 at night.

![Figure 5. students' answers](image)

Third student (S3) uses a different strategy from S1 and S2. S3 knows that there are 1.8 meters of water flowing in 5 hours. This means that there is 3.6 water flowing in 10 hours. This also implies that there are 1.2 meters of water that has not been drained, and the time needed to drain the water is less than 5 hours. From the calculation results it is obtained that the time needed to drain 1.2 meters of water is 3 hours 20 minutes. So the time to spend the entire contents of the tub is "5 hours + 5 hours + 3 hours 20 minutes = 13 hours 20 minutes".

From the scaffolding activities and the analysis results of the students’ answers it was found that the scaffolding provided had a positive role in the students' problem solving abilities [36]. Based on scaffolding activities, it is found that students who have moderate and high cognitive levels tend to be guided more quickly than students who have low cognitive levels [37]. Based on the results of students' answers, there are several representations and different ways of solving that students do. In terms of making sketches, differences in student sketches are influenced by what is imagined by students and how the concept of images that exist in students' brains [38]. Mathematical symbols made by students are influenced by previous habits and are influenced by prior student knowledge [39]. From the aspect of strategy in solving problems, differences in student strategy are influenced by prior knowledge, background experience, intellectual abilities, and the ability to write in mathematical language [40].

4. Conclusion
Scaffolding can help solve students' problem solving. Written questions and direct guidance step by step act as a bridge in solving problems. Scaffolding which starts with sketching can be teacher
feedback in knowing the concept of student images. Different forms of resolution and student strategies are influenced by prior knowledge, background experience, intellectual abilities, and writing abilities in mathematical language.

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6. References
[1] Arifin S, Zulkardi, Putri R I I, Hartono Y and Susanti E 2017 J. Phys. Conf. Ser. 943 012038
[2] Saygili S 2017 E-International Journal of Educational Research 8 91
[3] Chaudhry N G and Rasool G 2012 World Appl. Sci. J. 20 34
[4] Yu, KC Fan, SC and Lim, KY 2014 Int. J. Sci. Math. Educ. 13 1377
[5] Firdaus, Kailani I, Nor and Bak M 2015 EduLearn 9 226
[6] Carson J 2007 Math. Educ. 17 7
[7] Wyndham J and Saljö R 1997 Learn. Instr. 7 361
[8] Krulik S and Rudnick J A 1996 The New source book for teaching reasoning and problem solving in junior and senior high schools (Boston, MA: Allyn and Bacon)
[9] Cai J and Lester F 2010 Why is Teaching with Problem Solving Important to Student Learning? (Reston, VA: National Council of Teachers of Mathematics)
[10] Paolini A 2015 The Journal of Effective Teaching 15 20
[11] Surya E, Putri F A and Mukhtar 2017 IndoMS-JME 8 85
[12] Cornoldi C, Carretti B, Drusi S and Tencati C 2015 Br. J. Educ. Psychol. 85 424
[13] Rustam E, Simamora, Saragih S and Hasratuddin 2019 Int. Elect. J. Math. Ed. 14 61
[14] Polya G 1957 How to solve it: A new aspect of mathematical method. 2nd ed. (New York: Double Day and Co)
[15] Mayer R E, Sims V and Tajika H 1995 Am. Educ. Res. J. 35 443
[16] Kirkley J 2003 Principles for teaching problem solving: Technical (Indiana University: Plato Learning)
[17] Artzt A and ve Armour-Thomas E 1992 Cognition Instruct. 9 137
[18] Hall A 2000 Math Forum: Learning and Mathematics: Common–Sense Questions–Polya online: http://mathforum.org/~sarah/discussion/Sessions/Polya.html
[19] Angateeh K S 2017 International Journal of Learning and Teaching 3 46
[20] Novriana M R and Surya E 2017 International Journal of Sciences: Basic and Applied Research 33 63
[21] Toharudin U 2017 International Journal of Science and Research 6 2004
[22] Bakker A, Smit J and Wegerif R 2015 Mathematics Education 47 1047
[23] Stuyf R R V D 2002 Adolescent Learning and Development 52 5
[24] Panselinas G and Komis V 2009 Think. Skills Creativity 4 86
[25] Chang K, Chen I and Sung Y 2002 J. Exp. Educ. 71 5
[26] Van Breukelen D, Smeetsa M and de Vries M 2015 Journal of Research in STEM Education 1 87
[27] Anghileri J 2006 J. Math. Teach. Educ. 9 33
[28] Holton D and Clarke D 2006 Int. J. Math. Educ. Sci. Technol. 37 127
[29] Landry S H, Miller-Loncar C L, Smith K E and Swank P R 2010 Dev. Neuropsychol. 21 15
[30] Alibali M W and Nathan M J 2007 Video Research in the Science ed R Goldman, R Pea, B Barron and S J Derry (Mahwah, NJ: Erlbaum) p 349
[31] Anwar, Yuwono I, Irawan E B and As’ari A R 2016 IndoMS-JME 8 65
[32] Ge X and Land S M 2004 Educ. Tech. Res. Dev. 51 21
[33] Frederick M L, Courtney S and Caniglia J 2014 Investigations in Mathematics Learning 7 21
[34] Roehler, L., and Cantlon, D 1997 Scaffolding Student Learning: Instructional Approaches and
Issues ed K Hogan and M Pressley (Cambridge, MA: Brookline) pp 27-37

[35] Sari Y M and Valentino E 2016 J. Res. Adv. Math. Educ. 1 90
[36] Ismail N, Ismail K and Aun N S M 2015 Int. Proc. of Economics Development and Research IPEDR vol 85 (Singapore: IACSIT Press) p 154
[37] van de Pol J, Volman M, Oort F and Beishuizen J 2015 Instr. Sci. 43 615
[38] Nathan M J, Wolfgram M, Srisurichan R, Walkington C and Alibali M W 2017 J. Educ. Res. 110 272
[39] Cuoco A, Goldenberg E P and Mark J 1997 Journal of Mathematical Behavior 15 375
[40] Hohensee C 2016 J. Res. Math. Educ. 47 17