Radiation-induced resistance oscillations in a 2D hole gas: a demonstration of a universal effect

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Abstract

We report on a theoretical study about the microwave-induced resistance oscillations and zero resistance states when dealing with p-type semiconductors and holes instead of electrons. We consider a high-mobility two-dimensional hole gas hosted in a pure Ge/SiGe quantum well. Similarly to electrons we obtain radiation-induced resistance oscillations and zero resistance states. We analytically deduce a universal expression for the irradiated magnetoresistance, explaining the origin of the minima positions and their 1/4 cycle phase shift. The outcome is that these phenomena are universal and only depend on radiation and cyclotron frequencies. We also study the possibility of having simultaneously two different carriers driven by radiation: light and heavy holes. As a result the calculated magnetoresistance reveals an interference profile due to the different effective masses of the two types of carriers.

Keywords: microwaves, magnetoresistance, holes

(Some figures may appear in colour only in the online journal)

1. Introduction

High-mobility two-dimensional electron systems (2DES) are fantastic platforms for studying transport and coupling with different potentials, static or time-dependent (radiation) in nano-systems. In the last decade two of the most striking effects involving radiation-matter coupling in 2DES were discovered: microwave-induced resistance oscillations (MIRO) and zero resistance states (ZRS) [1, 2]. They are indeed remarkable effects that surprised the condensed matter community when they were discovered. Mainly because they involve simultaneously radiation-matter interaction [3] and transport excited by radiation in a nanoscopic system. On the other hand, their discovery was also considered very important, especially in the case of zero resistance states, because they were obtained without quantisation in the Hall resistance. The interest in both effects is focused not only on the basic explanation of a physical effect but also on their potential applications. They are obtained when 2DES, in high mobility samples at low temperature (∼1 K), are subjected to a perpendicular magnetic field (B) and radiation (microwave (MW) band) simultaneously. In these experiments, for an increasing radiation power (P), one first obtains longitudinal magnetoresistance (Rxx) oscillations which turn into zero resistance states (ZRS) at high enough P.

Many experiments have been carried out [4–17] and theoretical explanations [18–24] have been given to try to explain their physical origin, but to date it still remains controversial. Yet, we can cite interesting experimental results about radiation-induced magnetotransport oscillations on a different non-degenerate 2D system such as electrons on a liquid helium surface [13, 14]; they may share a similar physical origin to MIRO. In this way we wonder if MIRO and ZRS are universal effects and if, as a result, they can be observed in different platforms. For instance, different materials and carriers such as holes working with valence bands in p-type materials [25]. In contrast to 2DES, a two-dimensional hole gas (2DHG) presents more non-linearities and a more interesting dynamics when it comes to coupling with radiation.
In this article, we demonstrate that these effects are universal phenomena and that they can also be obtained in a 2DHG.

Based on a previous theoretical model [18, 26–28], we obtain a universal expression for irradiated $R_{\alpha \gamma}$ and according to it, MIRO only depends on radiation and cyclotron frequencies and not on the type of semiconductor material. We are able to explain the experimentally obtained MIRO extrema and node positions and the 1/4 cycle phase shift of MIRO minima. We have applied the results to the case of holes obtaining MIRO and ZRS in a high-mobility 2DHG hosted in a pure Ge/SiGe quantum well. According to this theory, when a Hall bar is illuminated, the orbit centers of the Landau states perform a classical trajectory consists in a harmonic motion along the direction of the current. Thus, the 2D carriers move in phase and harmonically at the radiation frequency altering dramatically the scattering conditions and giving rise eventually to MIRO and, at higher $P$, ZRS.

Working with the valence band gives us a new scenario, as does the possibility of having two different carriers sustaining the current and being coupled simultaneously with radiation. We expect that this situation will profoundly change the MIRO profile. They would be light and heavy holes being driven by MW and give rise to a clear interference regime being evidenced in the calculated $R_{\alpha \gamma}$. In the same way, the interplay of lower temperatures ($T$) and higher $P$ can reveal two different resonance peaks at different $B$ in $R_{\alpha \gamma}$, each one corresponding to heavy and light holes. Finally, we have studied the hole-based MIRO dependence on $T$ and $P$ obtaining similar results as with electrons. For instance, the calculated dependence on $P$ follows a sublinear power law that was already obtained in previous experiments [16] with electrons and was theoretically confirmed [29, 30]. As expected, the corresponding exponent of the power law is around 0.5.

### 2. Theoretical model

The **radiation-driven electron orbits model**, was developed to explain the magnetoresistance response to the radiation of a 2DEG at low $B$ [18, 26–28]. We first obtain an exact expression of the electronic wave vector for a 2DES in a perpendicular $B$ and radiation. Thus, the total Hamiltonian $H$ can be written as:

$$
H = \frac{p^2}{2m^*} + \frac{1}{2} m^* \omega^2 (x - X)^2 - e E_{dc} X \\
+ \frac{1}{2} m^* \frac{E^2}{B^2} - e E_0 \cos wt (x - X) \\
- e E_0 \cos wt X
$$

(1)

$x$ is the centre of the orbit for the electron spiral motion and is dependent on $B$ and $E_{dc}$, which is the dc electric field in the $x$ direction. $E_0$ is the intensity for the MW field and $H_1$ is the Hamiltonian corresponding to a forced harmonic oscillator whose orbit is centered at $X$. $H_1$ can be solved exactly [27, 28] making possible an exact solution for the total wave function of $H$ [18, 26–29]:

$$
\Psi_b(x, t) \propto \phi_b(x - X - x_\alpha(t), t)
$$

(2)

where $\phi_b$ is the solution for the Schrödinger equation of the unforced quantum harmonic oscillator. $x_\alpha(t)$ is the classical solution of a forced and damped harmonic oscillator:

$$
x_\alpha(t) = \frac{e E_0}{m^* \sqrt{(\omega^2 - \omega_0^2)^2 + \gamma^2}} \cos \omega t = A \cos \omega t
$$

(3)

where $\gamma$ is a phenomenologically-introduced damping factor for the interaction of carriers with acoustic phonons. $w$ is the MW angular frequency, where $w = 2\pi f$, $f$ being the frequency. Since this model can be applied equally either to electrons or holes, we will refer to them as carriers for the rest of the paper.

Then, the obtained wave function is the same as the standard harmonic oscillator, where the centre is displaced by $x_\alpha(t)$. Thus, the carriers orbit centres are not fixed, but they oscillate harmonically at the MW frequency. This radiation – driven behaviour will dramatically affect the charged impurity scattering and eventually the conductivity. Next, we introduce the scattering suffered by the carriers due to charged impurities. If the scattering is weak, we can apply time-dependent first-order perturbation theory. First, we calculate the impurity scattering rate, $W_i = 1/\tau_i$, between two oscillating Landau states, the initial $\Psi_i(t)$ and the final state $\Psi_o(t)$ [18, 26, 31], being $\tau_i$ the scattering time.

Secondly, and in order to calculate the drift velocity, we find the average effective distance advanced by the carrier in every scattering jump [18, 26, 31], $\Delta X^{MW}$. Without radiation, one carrier in an initial Landau state $\Psi_i$ in an orbit centre position $X_{cl}^i$ undergoes a scattering process and jumps to a final Landau state $\Psi_o$ with an orbit centre position $X_{cl}^o$.

On average the carrier orbit centre moves in the $x$ direction a distance given by the difference of the two orbits’ centre positions, $\Delta X^0 = X_{cl}^o - X_{cl}^i$ (see figure 1(a)). With radiation, the carriers orbit centre position changes in the $x$ direction harmonically with time and is given according to our model by $X^{MW}(t) = X^0 + A \cos wt - \theta$. $\theta$ is a phase constant being calculated applying the initial conditions, i.e. for $t = 0$, $X^{MW}(0) = X^0$, then $\theta = \pi/2 \Rightarrow X^{MW}(t) = X^0 + A \sin wt$. In other words, due to the MW field all the carrier orbit centres oscillate in phase back and forth in the $x$ direction with $A \sin wt$. On the other hand, after the MW is on, in a given time the carrier will undergo a scattering event. If this happens when the orbits, driven by MW, move backwards we have the situation depicted in figure 1(b). This corresponds to an increase in the average distance advanced by the carriers giving rise to a MIRO peak. If the orbits move forwards, we will have the opposite situation, a drop in the average advanced distance, producing a MIRO valley.

If at the moment of scattering the carrier is in the oscillating Landau state $\Psi_i(t)$ at the position $X_i(t) = X_{cl}^i + A \sin wt$, after a time $\tau$, what we call flight time, it will reach a final oscillating Landau state $\Psi_o(t + \tau)$ located in the position given by $X_o(t + \tau) = X_{cl}^o + A \sin w(t + \tau)$. In general, this position is no longer occupied by $\Psi_o$ since its position, $X_{cl}^o(t)$, has been
Then, the advanced distance under MW is:

\[ \Delta X^0 \]

with respect to the average advanced distance shift \( \Delta = \Delta^0 - A \sin w \tau \). To have an advanced distance, the term \( \pi \tau \) is going to be the term responsible of the MW driven \( R_{\alpha} \) oscillations or MIRO. Therefore, according to that expression if \( X_f^0 > X_2^0 \) we will have a larger advanced distance in the x direction producing a peak in the conductivity and in turn in \( R_{\alpha} \). In the opposite situation if \( X_f^0 < X_2^0 \) we would obtain a valley in \( R_{\alpha} \) with respect to the dark scenario. When \( X_f^0 = X_2^0 \) we would obtain a node, where \( R_{\alpha} \) with radiation is equal to the dark \( R_{\alpha} \), and finally the most striking situation happens when \( X_f^0 < X_2^0 \) where ZRS occur.

To calculate the distance shift we have to take into account, as we said above, that the position occupied by the orbit \( \Psi_2 \) at a given time \( t \) will be taken by the orbit \( \Psi_2 \) after a time \( t + \tau \), \( X(t) = X(t + \tau) \). On the other hand, mid-positions of the orbits are obtained when \( wt = 2\pi n \), \( n \) being a positive integer, and substituting this condition in the last equation:

\[ X_f^0 = X_0^0 + A \sin w \tau \]

Thus, the two key variables to observe MIRO and ZRS are \( w \) and \( \tau \).

The longitudinal conductivity \( \sigma_{xx} \) is given by

\[ \sigma_{xx} \propto \int dE E^2 \frac{\langle \Delta X^{MW} \rangle}{\tau_s} \]

being \( E \) the energy. To obtain \( R_{\alpha} \) we use the relation

\[ R_{\alpha} = \frac{\sigma_{uu} + \sigma_{yy}}{\sigma_{xy}} \approx \frac{\sigma_{uu}}{\sigma_{xy}} \]

where \( \sigma_{uu} \approx \sigma_{xy} \), \( p_i \) being the holes density, and \( \sigma_{xx} \ll \sigma_{xy} \). Thus,

\[ R_{\alpha} \propto -A \sin w \tau \]

From this expression the MIRO minima positions fulfill the condition:

\[ w \tau = \frac{\pi}{2} + 2\pi n \Rightarrow w = \frac{2\pi}{\tau} \left( \frac{1}{4} + n \right) \]

\( n \) being a positive integer. And for the MIRO maxima:

\[ w \tau = \frac{3\pi}{2} + 2\pi n \Rightarrow w = \frac{2\pi}{\tau} \left( \frac{3}{4} + n \right) \]

We can also obtain expressions for the MIRO nodes, or the points where the radiation curve crosses the dark curve. If we take as a reference any MIRO peak, the right node fulfills the condition.

\[ \Delta \tau \cdot \Delta E \approx \hbar \]

We obtained the energy we would obtain either \( E_a \) or \( E_m \). If we measured the energy we would obtain either \( E_a \) or \( E_m \), then the uncertainty of the energy is: \( \Delta E = |E_a - E_m| = \hbar \omega \). For the...
The semiclassical assessment of \( \tau \) would consist in the following: during the scattering jump from one orbit to another, in a time \( \tau \), the carriers in their orbits complete one full loop, which implies that \( \tau = T_c \). Therefore, the carrier involved in the scattering ends up in the same relative position inside the final orbit as the one it started from in the initial one. The reason for this is that the dynamics of the orbits (Landau states and the elastic scattering between them) is governed on average by the position of the centre of the orbit irrespective of the carrier position inside the orbit when the scattering takes place. Then, on average, both the initial and final semiclassical positions are identical in their respective orbits. Then, during the flight time, the carriers in their orbits complete one full loop, in a time \( \tau \), which implies that \( \tau = T_c \). If next, we substitute the result \( \tau = \frac{2\pi}{w_c} \) in the MIRO extrema expressions, we obtain:

\[
\frac{w_w}{w_c} = \left( \frac{1}{4} + n \right)
\]

for the minima and,

\[
\frac{w_w}{w_c} = \left( \frac{3}{4} + n \right)
\]

for the maxima. The above expressions are exactly the same as the ones experimentally obtained previously by Mani et al.\cite{4, 7}. Therefore, we can conclude, based on our theory, that the physical origin of the 1/4-cycle phase shift in MIRO has to do with the harmonically swinging nature of the irradiated Landau states and the elastic scattering between them. Radiation frequency and carrier flight time are the key variables ruling the process.

Nevertheless, the most important result turns out to be the final expression of irradiated magnetoresistance, where the part responsible for MIRO can be written as:

\[
R_{xx} \propto -A \sin \left( 2\pi \frac{w}{w_c} \right)
\]

This result can be described as universal since it depends only on external variables such as radiation and magnetic field and it is totally independent of the type of the sample semiconductor material\cite{34-35}.

Finally, in a scenario where we had two different type of carriers simultaneously coupled to MW as light and heavy holes, \( R_{xx} \) would be written as:

\[
R_{xx} \propto \left[ A_{lh} \sin \left( 2\pi \frac{w}{w_{c, lh}} \right) + A_{hh} \sin \left( 2\pi \frac{w}{w_{c, hh}} \right) \right]
\]
In the inset we present a schematic diagram explaining the physical origin of ZRS: if we increase further the MW power, we will eventually reach the situation where the orbits are moving forwards but their amplitude is larger than the scattering jump. In this case the hole jump is blocked because the final state is occupied.

In the hypothetical case of having a 2DHG in unstrained Ge, we would have the heavy and light hole valence bands degenerate at the $\Gamma$ point. As a result, both type of holes would be available to participate in the transport and couple to MW. The theoretical outcome, according to equation (17), would be an interference scenario that would be evidenced in $R_{xx}$. This is presented in figure 4, where we plot the calculated $R_{xx}$ versus $B$ for unstrained Ge, MW of frequency 100 GHz and $T = 1$ K. We have considered the bulk effective masses for Ge: $m^*_{hh} = 0.28m_0$ and $m^*_{lh} = 0.044m_0$.

0.10 and 0.12 T. In the inset we present a schematic diagram explaining the physical origin of ZRS: if we increase further the MW power, we will eventually reach a situation where the orbits are moving forwards but their amplitude is larger than the scattering jump. In this case the hole jump is blocked because the final state is occupied.

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As expected, we obtain a very clear interference profile for MIRO. For a more realistic scenario we have considered the case of a 100 Å GaAs/GaAlAs quantum well [37]. For this platform it is possible by applying a uniaxial stress to shift downwards the highest heavy hole band, making it degenerate with a light hole band at the $\Gamma$ point [37]. The corresponding calculated effective masses turn out to be: $m^*_{hh} = 0.38m_0$ and $m^*_{lh} = 0.09m_0$ [37]. In figure 5 we present the calculated $R_{xx}$ versus $B$ for this case. The MW frequency is 100 GHz and $T = 1$ K. We obtain an interference profile for MIRO.

In figure 6 we present a similar case as in figure 5, but now the MW frequency is 50 GHz and $T = 0.2$ K. We have considered the bulk effective masses for GaAs: $m^*_{hh} = 0.28m_0$ and $m^*_{lh} = 0.044m_0$. As expected, we obtain a very clear interference profile for MIRO. For a more realistic scenario we have considered the case of a 100 Å GaAs/GaAlAs quantum well [37]. For this platform it is possible by applying a uniaxial compressive stress to shift downwards the highest heavy hole band, making it degenerate with a light hole band at the $\Gamma$ point [37]. The corresponding calculated effective masses turn out to be: $m^*_{hh} = 0.38m_0$ and $m^*_{lh} = 0.09m_0$ [37]. In figure 5 we present the calculated $R_{xx}$ versus $B$ for this case. The MW frequency is 100 GHz and $T = 1$ K. We obtain an interference profile for MIRO.
lowered the temperature in order to weaken the damping $\gamma$, (see the expression of $A$) and obtain the corresponding resonance peaks of light and heavy holes. The former is observed at $B_0 \approx 0.2$ T and the latter at $B_0 \approx 0.7$ T. Interestingly, from the $B$-position of these peaks we could obtain simultaneously the effective masses of carriers involved in the magnetotransport. The heavier the carrier the more displaced the peak to higher $B$.

In figure 7(a) we present the $P$-dependence of irradiated $R_{xx}$ versus $B$ for $f = 40$ GHz and $T = 0.6$ K for a 2DGH in a Ge/SiGe quantum well. We observe that the radiation-induced oscillation increases giving rise to larger peaks and deeper valleys. b) Calculated amplitude $\Delta R_{xx} = R_{xx}^{\text{rad}} - R_{xx}^{0}$ versus $P$, for data coming from the main peak. $R_{xx}^{0}$ is the magnetoresistance for dark and $R_{xx}^{\text{rad}}$ when the radiation field is on. We fit the data obtaining a sublinear $P$-dependence, where the exponent is close to 0.5.

Figure 7. a) Calculated $P$-dependence of $R_{xx}$ versus $B$ for $f = 40$ GHz and $T = 0.6$ K for a 2DGH in a Ge/SiGe quantum well. We sweep the radiation power $P$ from dark to $P = 10.7$ mW. We observe that the radiation-induced oscillation increases giving rise to larger peaks and deeper valleys. b) Calculated amplitude $\Delta R_{xx} = R_{xx}^{\text{rad}} - R_{xx}^{0}$ versus $P$, for data coming from the main peak. $R_{xx}^{0}$ is the magnetoresistance for dark and $R_{xx}^{\text{rad}}$ when the radiation field is on. We fit the data obtaining a sublinear $P$-dependence, where the exponent is close to 0.5.

$R_{xx}^{\text{rad}}$ with the radiation field is on. We fit the data obtaining a sublinear $P$-dependence,

$\Delta R_{xx} \propto P^{0.58}$

and in agreement with previous experimental [16] and theoretical [30] results obtained for electrons.

In figure 8(a), we present the $T$-dependence of irradiated $R_{xx}$ versus $B$ for $f = 100$ GHz of a 2DGH in a Ge/SiGe quantum well. We sweep the temperature from $T = 1$ K to $T = 5$ K. We observe a clear decrease in the oscillation for increasing $T$, eventually reaching a $R_{xx}$ response similar to dark, but without the Shubnikov-de Haas oscillations that are much affected by increasing $T$. b) Absolute values of $R_{xx}$ amplitudes, $\Delta R_{xx}$, of the labelled peak $+2$ and the labelled valley $-1$ versus $T^2$. The two curves are vertically shifted for clarity.

Figure 8. a) Calculated $T$-dependence of $R_{xx}$ versus $B$ for $f = 100$ GHz of a 2DGH in a Ge/SiGe quantum well. We sweep the temperature from $T = 1$ K to $T = 5$ K. We observe a clear decrease in the oscillation for increasing $T$. Eventually a $R_{xx}$ response similar to dark is reached, but without the Shubnikov-de Haas oscillations that are much affected by increasing $T$ making them vanish. The $T$-dependence, according to the model, is explained with the damping parameter $\gamma$, which represents the interaction of the carriers with the lattice ions giving rise to the emission of acoustic phonons. This interaction can be calculated in terms of the scattering rate of the carriers with longitudinal acoustic phonons through Fermi’s golden rule [38, 39]. Thus, $\gamma$ turns out to be linear with $T$, and then an increasing $T$ means an increasing $\gamma$ and smaller $R_{xx}$ oscillations. When the damping is strong enough (higher $T$) the $R_{xx}$ oscillations collapse. In figure 8(b), we present $\Delta R_{xx}$, of the peak labelled in the upper panel with $+2$ and the valley labelled with $-1$
We observe that for high $T$, $\Delta R_{xx}$ is approximately linear with $T^2$ and for low $T$, $\Delta R_{xx}$ falls below the linear dependence approaching a constant value, i.e. independent of $T^2$. We can find an explanation considering the expressions obtained from the model:

$$\Delta R_{xx} \propto \frac{eE_w}{m^* \sqrt{(w_c^2 - w^2)^2 + \gamma^4}}$$  \hspace{1cm} (20)$$

and $\gamma \propto T$. Accordingly, with high $T$, $\gamma^4 > (w_c^2 - w^2)^2$ and then we can approximate $\Delta R_{xx} \propto T^2$ giving a linear dependence. Yet, for low $T$, $\gamma^4 < (w_c^2 - w^2)^2$ making $\Delta R_{xx}$ independent of $T$ and approaching a horizontal line, as plotted in figure 8(b). To confirm this last result we have calculated $R_{xx}$ for much lower $T$ reaching 50 mK. We obtain that $\Delta R_{xx}$ clearly tends to a constant value. We present these results in figure 9, where we plot $\Delta R_{xx}$ versus $T$. Here we sweep $T$ from 50 mK to 5 K. According to our theory, when $T$ tends to 0, $\Delta R_{xx}$ tends to a constant value, (when $w_c$ is far from resonance), as can be obtained from equation (20). This is a prediction from our theoretical model that can also be applied to electrons. Experiments on MIRO studying $T$-dependence have not reached such low temperatures to date. Therefore, the real MIRO behaviour at such very-low-temperature could serve to discriminate between the existing theories and give credibility to the ones predicting similar results as the experiments.

4. Conclusions

In summary, we have presented a theoretical study on MIRO in a 2DGH hosted in a Ge/SiGe quantum well in order to demonstrate that MIRO and zero resistance states are universal effects. We obtain the calculated $R_{xx}$ revealing MIRO and ZRS. We have analytically deduced a universal expression for the irradiated magnetoresistance, explaining the origin of the minima positions and their 1/4 cycle phase shift. The outcome is that these phenomena are universal and only depend on radiation and cyclotron frequencies. On the other hand, they turn out to be independent of the type of semiconductor material. Interestingly, we study the possibility of having simultaneously two different carriers driven by radiation: light and heavy holes. As a result the calculated magnetoresistance reveals an interference regime due to the different effective masses of the two types of carriers. In the same way, we obtain two different resonance peaks at a low enough temperature, corresponding to the two carriers. Finally, we study the dependence on microwave power and temperature obtaining a similar behaviour to electrons. In the power dependence we obtain a sublinear law, which relates the amplitude of the resistance oscillations and the applied power, being the exponent approximately equal to 0.5. For the temperature dependence we obtain, as expected, a vanishing effect on the radiation-induced resistance oscillations for increasing temperature. Interestingly, we also obtain that the amplitude of MIRO tends to a constant values as the temperature tends to 0.

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References

[1] Mani R G, Smet J H, von Klitzing K, Narayanamurti V, Johnson W B and Umansky V 2002 Nature 420 646
[2] Mani R G, Narayanamurti V, von Klitzing K, Smet J H, Johnson W B and Umansky V 2004 Phys. Rev. B 69 161306
[3] Mani R G, Narayanamurti V, von Klitzing K, Smet J H, Johnson W B and Umansky V 2004 Phys. Rev. B 70 155310
[4] Zudov M A, Du R R, Pfeiffer L N and West K W 2003 Phys. Rev. Lett. 90 046807
[5] Iñarrea J, Platero G and Tejedor C 1994 Semicond. Sci. Tech. 9 515
[6] Iñarrea J and Platero G 1995 Phys. Rev. B 51 5244
[7] Iñarrea J and Platero G 1996 Europhys. Lett. 34 43
[8] Iñarrea J and Platero G 1996 Europhys. Lett. 33 477
[9] Iñarrea J, Aguado R and Platero G 1997 Europhys. Lett. 40 417
[10] Mani R G, Smet J H, von Klitzing K, Narayanamurti V, Johnson W B and Umansky V 2004 Phys. Rev. Lett. 92 146801
[11] Mani R G, Smet J H, von Klitzing K, Narayanamurti V, Johnson W B and Umansky V 2004 Phys. Rev. B 69 193304
[12] Willett R L, Pfeiffer L N and West K W 2004 Phys. Rev. Lett. 93 026804
[13] Mani R G 2004 Physica E 22 1
[14] Smet J H, Gorshunov B, Jiang C, Pfeiffer L, West K, Umansky V, Dressel M, Meisels R, Kuchar F and von Klitzing K 2005 Phys. Rev. Lett. 95 116804
[15] Yuan Z Q, Yang C L, Du R R, Pfeiffer L N and West K W 2006 Phys. Rev. B 74 075313
[16] Mani R G, Johnson W B, Umansky V, Narayanamurti V and Ploog K 2009 Phys. Rev. B 79 205320
