Numerical Simulation of 2D Flood Waves Using Shallow Water Equations

Wei Wenli      Zhang Pei       Gao Sheng

Institute of Water Conservancy and Hydraulic Engineering,
Xi’an University of Technology
Xi’an, China, 710048
weiwenli@xaut.edu.cn

Abstract—This paper is concerned with a mathematical model for numerical simulation of 2D flood waves due to complete or partial dam-break. The governing water equations are solved by the MacCormack’s predictor-corrector technique. The mathematical model is used to numerically predict 2D flood waves due to partial instantaneous dam-break in a rectangular open channel with a rectangular cylinder barrier downstream, and the reliability of numerical results is analyzed. The comparison and the analysis show that the proposed method is accurate, reliable and effective in simulation of dam-break flood waves.

Keywords—dam-break flood waves; numerical simulation; predictor-corrector scheme.

I. INTRODUCTION

The analysis of Dam-Break Flow (DBF) is a part of dam design and safety analysis. The importance of DBF analysis can not be overemphasized, as dams are potential sources of hazard to life and property. Therefore, considerable efforts have been made in past years to obtain satisfactory solutions for this problem.

Mathematically, the DBF is commonly described by the shallow water equations (also named the Saint Venant equations for the 1D case). One feature of hyperbolic equations of this type is the formation of bores (i.e., the rapidly varying discontinuous flow). It is an important basis for validating the numerical method whether the scheme can capture the dam-break bore waves accurately or not. This gives rise to an increasing interest in solving such a problem.

Form 1980 to 2000 several finite-difference schemes that handle discontinuities effectively were used to compute open-channel flows, such as the approximate Riemann solver [1-4]. Based on the above research results, the goal of the current work is to develop a mathematical model capable of dealing with hydraulic discontinuities such as steep fronts, hydraulic jump and drop, etc. The water governing equations has been solved by the MacCormack’s predictor-corrector technique. The comparisons with theoretical results as well as other numerical solutions show that the proposed method is comparatively accurate, fast, and reliable.

II. GOVERNING FLOW EQUATIONS

The equation governing the two-dimensional unsteady flow is the famous shallow water equations. If wind and Coriolis forces are neglected, they are written in matrix form as

\[ W_x + E_x + F_y + D = 0 \]  (1)

in which

\[ W = \begin{bmatrix} h \\ uh \\ vh \end{bmatrix} \quad E = \begin{bmatrix} u \\ u^2 + gh^2/2 \\ uh \end{bmatrix} \]

\[ F = \begin{bmatrix} vh \\ uvh \\ v^2h + gh^2/2 \end{bmatrix} \quad D = \begin{bmatrix} 0 \\ -gh(S_{ox} - S_{fx}) \\ -gh(S_{oy} - S_{fy}) \end{bmatrix} \]  (2)

where \( h \) is flow depth; \( u \) and \( v \) are depth-averaged velocity in the \( x \)-and \( y \)-directions; \( g \) is the gravity acceleration; \( S_{ox} \) and \( S_{oy} \) are the channel bottom slope in the \( x \)-and \( y \)-directions and they are defined as

\[ S_{ox} = \frac{\partial Z_0}{\partial x}, \quad S_{oy} = \frac{\partial Z_0}{\partial y} \]  (3)

where \( Z_0 \) is the bottom elevation; and \( S_{fx} \) and \( S_{fy} \) are the friction slopes in the \( x \)-and \( y \)-directions respectively, computed using the steady state friction formulas

\[ S_{fx} = \frac{n^2v\sqrt{u^2 + v^2}}{h^{4/3}}, \quad S_{fy} = \frac{n^2v\sqrt{u^2 + v^2}}{h^{4/3}} \]  (4)

in which \( n \) is the Manning’s roughness coefficient.

The system of Eqs.(1) is of conservative from, capable of dealing also with hydraulic discontinuities such as steep fronts, hydraulic jumps, etc [5-6].

III. NUMERICAL PROCEDURE

A. Discretization scheme

For the numerical solution of Eqs. (1), the MacCormack scheme has been widely used in computational fluid
dynamics. This scheme consists of a two-step predictor-corrector sequence. Flow variables are known at k time level and their values are to be determined at k+1 time level. Then for grid points i and j, the following finite difference equations may be written for Eqs. (1).

Predictor step

$$\hat{W}_{i,j} = W_{i,j}^{k+1} - \frac{\Delta t}{\Delta x} \nabla \cdot E_{i,j} - \frac{\Delta t}{\Delta y} \nabla_j F_{i,j} - \Delta t \Delta_b_{i,j}$$

(Ca)

Corrector step

$$\hat{W}_{i,j} = \hat{W}_{i,j} - \frac{\Delta t}{\Delta x} \Delta_x E_{i,j} - \frac{\Delta t}{\Delta y} \Delta_y F_{i,j} - \Delta t \Delta_b_{i,j}$$

(Ca)

in which \(\hat{W}\) and \(\hat{W}\) are the intermediate values for \(W\). The new values of \(W\) are then obtained from

$$W_{i,j}^{k+1} = \frac{1}{2}(\hat{W}_{i,j} + \hat{W}_{i,j})$$

(6)

The grid points are defined by subscripts i, j and k. The scheme first uses forward space differences (\(\nabla\) and \(\Delta\)) to predict an intermediate solution from known information at the k time level. Backward space differences (\(\Delta\) and \(\Delta\)) are then used in the second step to correct the predicted values. The forward and backward difference operators (\(\nabla\) and \(\Delta\)) are defined by

$$\nabla_x E_{i,j} = E_{i+1,j} - E_{i,j}, \Delta_x E_{i,j} = E_{i,j} - E_{i-1,j}$$

(7)

where the subscript indicates the direction of difference. The values of primitive variables are determined from the computed value of \(W\) at each step as follows

$$h^{k+1} = h^{k+1}, u^{k+1} = (uh)^{k+1} / h^{k+1}, v^{k+1} = (vh)^{k+1} / h^{k+1}$$

(8)

where \(k+1\) refers to an intermediate value obtained during a current predictor or corrector sequence.

B. BOUNDARY CONDITIONS

The inclusion of the boundary is very important in the successful application of any numerical technique. Hyperbolic equations are particularly sensitive because errors introduced at the boundaries are propagated and reflected throughout the grid. This, in many cases, may result in instability. In the present application, two types of boundaries are encountered: solid boundaries and water boundaries. For the solid boundaries, the governing equations do not include the turbulent viscosity, but the bottom friction, free-slip conditions may be considered, and the normal discharge to the wall is set to zero in order to represent no flux through the solid boundaries. The water boundaries, in particular, need to be treated. The local value of the Froude number, or whether the flow is subcritical or supercritical, is the basis for determining the number of boundary conditions. For the 2D subcritical flow, two external conditions are specified at the inflow boundary, and one is specified at the outflow boundary. For the supercritical flow, three boundary conditions at the inflow boundary and none at the outflow boundary have to be specified.

C. STABILITY CONDITION

The above-described numerical scheme is a time-marching method in which \(\Delta t\) must be satisfied with Courant-Friedrichs-Levy Condition. For every point i, j of the computational domain the \(\Delta t\) time step is expresses by

$$\Delta t = \min(DT_1, DT_2)$$

where

$$DT_1 = \frac{\Delta x}{|v| + \sqrt{gh}}, DT_2 = \frac{\Delta y}{|v| + \sqrt{gh}}.$$  

D. 2D Partial Dam-Break

This problem models a partial dam-break or the rapid opening of a sluice gate with a symmetrical breach. The discontinuous initial conditions impose severe difficulties, and most of the presently used non-Godunov-type numerical schemes create enormous errors or fail under such conditions. Here, the computational domain is defined by a channel 200m long, 200m wide, with a horizontal friction bottom. The symmetric breach is 40m wide and the thickness of dam is 5m [Fig.1]. At downstream in front of the breach there is a rectangular cylinder barrier that occupies an area of 15*15m, the distance among which is shown in Fig.1. Initially, the upstream and downstream water depths are set at 10 m and 5m, respectively. The dam wall is then breached, and instantaneously water discharges from the higher to the lower level as a downstream-directed bore while a depression wave propagates upstream. Herein, for comparison purposes, the water body is discretized by a uniform 5×5m square mesh that exactly fits the domain. The time-step \(\Delta t\) is 0.1s. Fig.1 shows computational velocity vectors and Fig.2 shows a 3D view of the water surface elevation 12s after dam failure when the waves have not yet reached all the boundaries. At both ends of the breach, the water depths are smaller than that at the centre of the breach. Flow separates from the truncated dam walls just downstream of the breach and forms counter rotating eddies. One can clearly see that the structure, shape, and location of vortices, as well as the reflection (bore front of the barrier), and the discontinuities change as time increases.
IV. CONCLUSION

The MacCormack two-step explicit scheme with second order accuracy can be employed for the solution of two-dimensional flow equations written in conservative form. The MacCormack scheme can effectively simulate the rapidly varying discontinuous water waves. The proposed mathematical model can effectively simulate the 2D flood waves due to complete or partial dam-break. When utilize the technique of boundary treatment such as a body-fitted coordinate system, the proposed model can also effectively simulate the flows with complex boundaries.

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