Mode stability in $\delta$ Scuti stars: linear analysis versus observations in open clusters

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ABSTRACT
A comparison between linear stability analysis and observations of pulsation modes in five $\delta$ Scuti stars, belonging to the same cluster, is presented. The study is based on the work by Michel et al. (1999), in which such a comparison was performed for a representative set of model solutions obtained independently for each individual star considered. In this paper we revisit the work by Michel et al. (1999) following, however, a new approach which consists in the search for a single, complete, and coherent solution for all the selected stars, in order to constrain and test the assumed physics describing these objects. To do so, refined descriptions for the effects of rotation on the determination of the global stellar parameters and on the adiabatic oscillation frequency computations are used. In addition, a crude attempt is made to study the role of rotation on the prediction of mode instabilities. The present results are found to be comparable with those reported by Michel et al. (1999). Within the temperature range $\log T_{\text{eff}} = 3.87-3.88$ agreement between observations and model computations of unstable modes is restricted to values for the mixing-length parameter $\alpha_{\text{NL}} \simeq 1.50$. This indicates that for these stars a smaller value for $\alpha_{\text{NL}}$ is required than suggested from a calibrated solar model. We stress the point that the linear stability analysis used in this work still assumes stellar models without rotation and that further developments are required for a proper description of the interaction between rotation and pulsation dynamics.

Key words: (Stars: variables:) $\delta$ Sct – Stars: rotation – Stars: oscillations – Stars: fundamental parameters – Stars: evolution – (Galaxy:) open clusters and associations: general

1 INTRODUCTION

With spectral types from A2 to F0, the $\delta$ Scuti stars are pulsating variables both in the main sequence and in the sub-giant evolution phase (hydrogen shell burning). They are considered as good candidates for asteroseismic studies. Such stars show modes excited over a wide frequency range, including mixed modes known to be sensitive to the structure of the deep interior. These characteristics, together with the absence of magnetic field or metallicity peculiarities make them suitable for the asteroseismic study of hydrodynamical processes occurring in stellar interiors, such as convective overshooting, mixing of chemical species, and redistribution of angular momentum (Zahn 1992). Recently, Suárez, Goupil & Morel (2006) reported that the effect of the distribution of angular momentum on adiabatic oscillation frequencies in $\delta$ Scuti stars is significant and that it can be detected by space missions such as CoRoT. Due to the complexity of the oscillation spectra in $\delta$ Scuti stars, their pulsating behaviour is not completely understood (see Cox 2002 for a complete review of unsolved problems in stellar pulsation physics). In particular, great efforts are made nowadays to solve the problem of mode identification. Ground based multisite campaigns are regularly organised within coordinated networks, e.g.: STEPHI (Michel et al. 2000) or DSN (Breger 2000; Handler 2000). Only a few tens of the low-degree modes have been detected from ground-based observations, with a maximum of around 75 modes for the $\delta$ Scuti star FG Vir (Breger et al. 2005). The recently launched space mission CoRoT ([Baglin et al. 2002]) represents a unique opportunity for investigating such stars considering its much lower detection threshold and for obtaining data from quasi-uninterrupted time series over 5 months.
Additional uncertainties arise from the effect of rapid rotation, both directly on the hydrostatic balance in the star and through mixing caused by circulation or instabilities induced by rotation. The δ Scti stars are commonly fast rotators (100 < V$_{\text{sin}i}$ < 200 km s$^{-1}$), and consequently the symmetry of the multiplets is broken by the rotation. In the framework of a perturbation analysis, the second order effects induce strong asymmetries in the splitting of multiplets (Said 1981; Dziembowski & Goode 1992) and frequency shifts must not be neglected even for radial modes (Soufi et al. 1995). In this context, Michel et al. (1999), hereafter M99, proposed a technique to estimate and correct the effect of fast rotation on the determination of fundamental parameters for stars in clusters. Applying this technique to a set of δ Scti stars belonging to the Praesepe cluster, the authors showed that it was possible to reach a reasonable agreement between ranges of observed oscillation modes and overstable radial modes predicted by a linear stability analysis. However, agreement was found only for certain values of $\alpha_{\text{NL}}$, the mixing-length parameter that was used in the non-local time-dependent convection treatment of the stability computations. This implies that models for convective heat transport in the outer stellar layers may be calibrated against observations.

In M99, different series of stellar model solutions were obtained, which assumed a wide range of physical parameters, e.g. overshooting, mixing-length parameter, metallicity, stellar age, etc. Considering these results as our reference domain of possible solutions, the present work revisits the work by M99 but adopts an improved approach. The main idea is to search for one particular solution that explains the whole set of observations, instead of sets of individual solutions for each star. To do so, refined techniques for modelling intermediate mass stars are required, in particular such techniques that take different effects of rotation into account. In concrete terms, in M99 the photometric parameters are corrected for the effect of rotation using the results of Maeder & Puetmann (1995) who applied the von Zeipel gravity-darkening law to estimate the emergent flux of a rotating star. In the present work we follow the method described by Pérez Hernández et al. (1999) (hereafter PH99) which improves such a calculation by means of, among other modelling aspects, updated models and consider the gravity-darkening law given by Claret et al. (1998), which, unlike the von Zeipel law, is also valid for non-fully radiative stars. Moreover, in M99 non-rotating equilibrium models were used whereas here we employ pseudo-rotating models. Such pseudo-models take first-order effects of rotation into account by means of an effective gravity (see Sect. 3.2). One of the major improvements with respect to M99’s work lies in the computation of adiabatic oscillation frequencies. In M99 the effects of rotation on the oscillation frequencies (up to the second order) are included in the manner of Perez Hernández et al. (1995), considering the perturbed frequency $\nu^\prime$ as

$$\nu^\prime = \nu_a + (A_n + B) \frac{\nu_{\text{rot}}^2}{\nu_a^2},$$

where $\nu_a$ is the unperturbed frequency, $\nu_{\text{rot}}$ is the rotation frequency, $B$ is a constant with an asymptotic value of 5/3, and $A_n$ corresponds to a coefficient depending on the eigenfunctions, and thus, on the stellar model. These coefficients are obtained by interpolating in the values computed by Said (1981) for an $n = 3$ polytrope. In the present work, following Dziembowski & Goode (1992) and Soufi et al. (1998) (see also Suárez, Goupil & Morel 2006) a complete treatment of second order effects (including near degeneracy) is used. Furthermore, this formalism takes the effect of the star deformation due to rotation into account.

With these improvements a similar methodology than that adopted in M99 is used here to study five δ Scti stars of the Praesepe cluster, four of which have already been included in the sample considered by M99. The fundamental parameters of these stars are determined from taking into account the effect of rotation on the photometric observables using the updated method by PH99. These fundamental parameters are then used to build representative asteroseismic models for each star, consisting of pseudo-rotating equilibrium models (instead of non-rotating equilibrium models used in M99) and their corresponding adiabatic oscillation spectra (properly corrected for the effect of rotation). These asteroseismic models are then used to determine the observed ranges radial orders, which, as it was done in M99, are then confronted with mode instability predictions obtained from a linear stability analysis. We note that, as in M99, the instability predictions are carried out using equivalent envelope models which do, however, not take the effect of rotation into account. This is so because, up to date, there are no reliable theories available which describe the effect of rotation on mode stability. Nevertheless, a crude estimate of this effect is addressed, which assumes that mode stability depends predominantly on the effective temperature of the model (Pamiatnykh 1975).

The paper is structured as follows: In Section 2 the main characteristics of the Praesepe cluster and the observational material are presented. Equilibrium models and isochrone computations are discussed in Section 3. Different aspects of the oscillation computations are described in Section 4. In Section 5 the consistency of our solutions with the results obtained by M99 is examined. Then, a detailed discussion of the present results is given. Finally, conclusions and perspectives are summarised in Section 6.
2 OBSERVATIONAL DATA

The selection of stars considered here contains five \( \delta \) Scuti stars belonging to the Praesepe cluster. Four of these stars: BW Cnc, BS Cnc, BU Cnc and BN Cnc (HD 73798, HD 73450, HD 73756, and HD 73763 respectively) were observed by several campaigns of the STEPHI network \cite{Michel1995}. The fifth star, BV Cnc (HD 73746), was observed by \cite{Frandsen2001}. The observed frequencies of these stars are listed in Table 1.

Regarding the cluster parameters, its distance modulus \((6.28 \pm 0.12 \text{ mag})\) was taken from \cite{Robichon1999} who derived it from mean cluster parallaxes computed from \textit{Hipparcos} intermediate data. We assume a metallicity value \( [M/H] \) of 0.170 which was derived from Geneva photometry of single stars with spectral types in the range F4-K3 \cite{Grenon2000,Robichon1999}. Apparent magnitudes and indices were taken from the Rufener’s catalogue \cite{Rufener1988}. In Table 2 the specific photometric data for the five selected \( \delta \) Scuti stars are listed. The projected velocity values \((V \sin i)\) were taken from \cite{Royer2002}.

| Star   | HD   | \( M_v \) | \( B_2 - V_1 \) | \( V \sin i \) |
|--------|------|-----------|----------------|-------------|
| BW Cnc | HD 73576 | 2.18 | 0.063 | 170 |
| BS Cnc | HD 74763 | 2.19 | 0.048 | 135 |
| BV Cnc | HD 73798 | 2.36 | 0.079 | 110 |
| BU Cnc | HD 73450 | 1.37 | -0.004 | 205 |
| BN Cnc | HD 73746 | 1.52 | 0.016 | 200 |

3 MODELLING

In the process of searching representative models for the five selected \( \delta \) Scuti stars, we take advantage of their cluster membership in several aspects: firstly, the chemical composition and age are assumed to be the same (to a relatively good extent) for all objects. In addition, metallicities and distances are commonly estimated better for stars in clusters than for field stars. Secondly, the photometric parameters of target stars are corrected for the effect of rotation following M99 and PH99. Such corrections also provide improved estimates for the stellar parameters required for the modelling.

3.1 Stellar models and isochrones

To characterise theoretically the observed stars within the cluster we first construct standard non-rotating models and isochrones that take into account the observed parameters presented in Section 2 and second, we also build models including uniform rotation (pseudo-rotating models) as described in Section 3.2. The evolutionary stellar models are computed with the CESAM code \cite{Morel1997}. Opacity tables are taken from the OPAL package \cite{Iglesias1994}, complemented at low temperatures \((T < 10^4 \text{ K})\) by the tables provided by \cite{Alexander1994}. The atmosphere is constructed from a grey Eddington \( T - \tau \) relation. The Praesepe’s metallicity value of \( Z/X \) is derived from the \([M/H] \) value given in Section 2, assuming \((Z/X)_\odot = 0.0245 \) \cite{Grevesse1993}. \( Y_p = 0.235 \), and \( Z_p = 0 \) for the helium and heavy element primordial mass abundances, and the value \( \Delta Y/\Delta Z = 2 \) for the enrichment ratio. This yields an initial helium abundance \( Y = 0.285 \) and a heavy element abundance \( Z = 0.025 \) for the cluster. Convection is treated with a local mixing-length model \cite{Bohm-Vitense1958} and we assume various values for the mixing-length parameter \( \alpha_{MLT} \). For the overshoot parameter \( d_{OV} = l/H_p \) (\( l/H_p \) being the penetration length of the convective elements) we assume values between 0.0 and 0.3.

For the construction of non-rotating isochrones, various sets of evolutionary sequences were computed for masses between 1 \( M_\odot \) and 5 \( M_\odot \), evolved from the zero-age main sequence (ZAMS) to the sub-giant branch. Isochrones were then obtained using the Geneva Isochrone Code. Three representative isochrones around the age of the Praesepe cluster are depicted in Fig. 1 after having previously transformed their effective temperatures and luminosities into the Geneva photometric system. This transformation is performed using the calibration proposed by \cite{Schmidt-Kaler1982} for \( M_v \), and by \cite{Kunzle1997} for \( (B_2 - V_1) \). Potential binarity and the effect of fast rotation, discussed in Section 3.2, are expected to induce systematic shifts of the models towards higher luminosities and lower effective temperatures when compared with single non-rotating stars. Therefore, in order to avoid both effects, the fit of isochrones has to be made by adjusting the isochrones to the bottom envelope of the cluster in the colour–magnitude diagrams (hereafter CM diagrams).

Considering the physical parametrisation described above, a range of ages between 650 Myr and 750 Myr is then found to be representative of the cluster (Fig. 1). This range is in agreement with values found earlier in the literature for the Praesepe cluster. The age will be adjusted more carefully in Section 3.2.
The modelling of the observed \( \delta \) Scuti stars requires the knowledge of certain basic stellar parameters including the values for mass, effective temperature and luminosity (and/or gravity). Two of these quantities can be determined from high resolution spectroscopy or, as in the present case, by means of photometric observations. However, as shown in M99, fast rotation must be taken into account when estimating fundamental parameters from photometric indices. In order to take this into account, the authors also considered three main effects:

Firstly, the shape distortion caused by rotation is such that a star seen pole-on has a larger projected area and hence looks brighter than if it were seen equator-on. Secondly, the flux (and hence the effective temperature) is larger at the poles than at the equator (gravity-darkening effect), and thirdly, rotation decreases the intrinsic luminosity and increases the mean radius of the star. As a result of these effects the location of a rotating star in a colour-magnitude diagram (and thereby in an HR diagram) depends not only on the angular velocity of the star but also on its angle of inclination \( i \). The basic idea of the method is to obtain, for a given rotating star and for a given \( V_{\text{sin}i} \) value, a segment covering the different potential positions in the colour-magnitude diagram (i.e. for the corresponding putative stars which are not rotating). The corrections so obtained for the selected \( \delta \) Scuti stars are displayed in Fig. 2 in which the positions of the non-rotating co-partners are varied with the angle of inclination \( i \), and with the rotation rate \( \omega_c \) defined as \( \omega = \Omega/\Omega_c \), where \( \Omega \) is the angular rotational velocity of the star, and \( \Omega_c \) is its break-up rotational velocity. Although the corrections are model-dependent, for a given isochrone, this dependence is weak and the uncertainty introduced smaller than that coming from the observational error in \( V_{\text{sin}i} \). An error on the \( V_{\text{sin}i} \) values of \( \pm 10 \% \) is assumed, which yields two segments per star.

After an iterative process (more details in PH99) it is possible, in most of the cases, to converge to an intersection between the isochrone and the segments. The non-rotating co-partners of our observed stars closest to the intersection between the isochrone and the segments give us an estimate of both \( i \) and \( \omega_c \). Additionally, the isochrone models at the intersection provide estimates of the required remaining stellar parameters.

As mentioned in PH99, the photometric correction for rotation depends on the selected isochrone. In the present case, the correction parameters \( \alpha_{\text{MLT}} \) and \( d_i \) were adjusted within the ranges given in the previous section; the best solution (given by this correction) was obtained for \( \alpha_{\text{MLT}} = 1.614 \) and \( d_i = 0.2 \), leading to an optimal age of 650 Myr for the cluster. This solution was selected under two main criteria: first, a standard fitting of isochrones, and second, the existence of suitable photometric corrections (see PH99 and Suárez et al. 2002 for more details) for the whole sample of stars. Both criteria make the solution converge rapidly to an age of 650 Myr (\( \pm 20 \) – 40 Myr). The age uncertainty of 20–40 Myr can be neglected in terms of global characteristics of the non-rotating co-partners (see the influence of age on photometric corrections for rotation in different open clusters in Suárez et al. 2002).

For this solution, the following range of masses are found: from \( M = 2.05 - 2.11 M_\odot \) (our high mass stars) to \( 1.64 - 1.65 M_\odot \) (our low mass stars), which is within the range of masses obtained in M99. In particular, the high mass models are found to be representative of BU Cnc & BN Cnc; the \( M = 1.70 - 1.73 M_\odot \) models are representative of BW Cnc & BS Cnc, and finally, the \( M = 1.64 - 1.65 M_\odot \) models are representative of BV Cnc. The characteristics of the different models are summarised in Table 3. We recall that the different ranges listed in Table 3 correspond to solutions in which \( V_{\text{sin}i} \) was varied by \( \pm 10 \% \) from its observed value. As explained in PH99 and Suárez et al. (2002), this uncertainty largely dominates the errors of the method.

We then compute stellar models including uniform rotation for the five considered stars, adopting the corrected values for their masses and ages obtained from the method described above. To take first order effects of rotation into account, the equilibrium equations are modified in the CESAM code in the manner described in Kippenhahn & Weigert (1990). In particular, the spherically averaged contribution of the centrifugal acceleration is included by means of an effective gravity \( g_{\text{eff}} = g - A_r(r) \), where \( g \) is the local gravity, \( r \) is the radius, and \( A_r(r) = \frac{1}{2} r \Omega^2(r) \) is the centrifugal acceleration of matter elements. This spherically averaged component of the centrifugal acceleration does not change the order of the hydrostatic equilibrium equations. From now on we shall refer to such models as ‘pseudo-rotating’ models. Although the non-spherical components of the centrifugal acceleration are not considered in the equilibrium models, they are included in the adiabatic oscillation computations by means of a linear perturbation analysis according to Soufi, Goupil & Dziembowski (1998) (see also Suárez, Goupil & Morel 2006).

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2 This corresponds to the rotational velocity that the star would have if the centrifugal force is balanced by the gravitational attraction at the equator.
Table 3. Characteristics of the non-rotating counterparts for the selected δ Scuti stars of the Praesepe cluster. These characteristics are obtained with the method for correcting the effect of rotation on photometric parameters (Section 3.2). The indicated ranges correspond to solutions, from left to right, for which $V_{\sin i}$ was varied by $+10\%$ and $-10\%$ from its observed value (see more details in Section 3.2). The best results with this method are obtained for an age of 650 Myr for the cluster. The different columns are: the star identification from the General Catalogue of Variable Stars (GCVS), the absolute magnitude $M_V$; the colour index $B_2 - V_1$; the mass $M$ (in units of $M_\odot$); the radius $R$ (in units of $R_\odot$); the effective temperature $T_{\text{eff}}$ (log, in K); the gravity $g$ (log, in cm s$^{-2}$); the rotation rate $\omega$ (in units of the break-up frequency $\Omega_c$); the cyclic rotation frequency $\nu_{\text{rot}}$ (in $\mu$Hz), and finally the inclination angle $i$ of the star.

| ID     | $M_V$ | $B_2 - V_1$ | $M$  | $R$   | $T_{\text{eff}}$ | $g$   | $\omega$ | $\nu_{\text{rot}}$ | $i$           |
|--------|-------|-------------|------|-------|-----------------|-------|---------|---------------------|--------------|
| BU Cnc | 1.29–1.36 | -0.070–(-0.057) | 2.09–2.11 | 2.52–2.56 | 3.91–3.91 | 3.95–3.94 | 0.95–0.95 | 19.05–14.70 | 56.27–51.17 |
| BN Cnc | 1.36–1.35 | -0.062–(-0.055) | 2.05–2.05 | 2.42–2.42 | 3.91–3.91 | 3.98–3.98 | 0.90–0.80 | 18.93–16.83 | 90–66.30    |
| BW Cnc | 2.19–2.23 | 0.011–0.014 | 1.73–1.70 | 1.87–1.84 | 3.88–3.88 | 4.13–4.14 | 0.90–0.90 | 24.14–24.71 | 42.63–48.25 |
| BS Cnc | 2.21–2.24 | 0.011–0.014 | 1.73–1.70 | 1.86–1.83 | 3.88–3.88 | 4.13–4.14 | 0.80–0.80 | 20.04–20.35 | 45.47–40.22 |
| BV Cnc | 2.41–2.42 | 0.043–0.044 | 1.65–1.64 | 1.75–1.74 | 3.87–3.87 | 4.16–4.17 | 0.80–0.80 | 24.58–24.72 | 34.70–31.42 |

Figure 3. Observed and predicted (using linear stability analysis) ranges of unstable radial modes for the selected δ Scuti stars ($n_1$ is the lowest value, and $n_2$ the largest value of the radial order of the unstable modes). In panel (a) we re-plot the results obtained by M99 for BU Cnc, BN Cnc, BS Cnc and BW Cnc, and in panel (b) the current results are displayed. Filled circles represent the observed ranges. Rhombus and squares correspond to predicted radial order ranges for $\alpha_{NL} = 1.89$ and $\alpha_{NL} = 1.50$, respectively. Each diagonal-dashed line represents the width (in radial orders) of the represented ranges.

4 DETERMINATION OF OBSERVED AND THEORETICAL RANGES OF UNSTABLE MODES

We compute synthetic adiabatic oscillation spectra corresponding to the pseudo-rotating evolutionary models described in Section 3.2. We restrict our computations to modes with degree $\ell \leq 2$ since modes with $\ell \geq 3$ are generally considered to be invisible in photometry (only visible in spectroscopy) for distant stars due to geometric cancellation. The ranges of observed radial and nonradial modes are similar to the predicted ranges of computed overstable radial modes. This is to be expected, because driving and damping in δ Scuti stars takes place predominantly in the HeII ionisation zone, which is rather close to the stellar surface, where the vertical scale is much less than the horizontal scale of the oscillations and when $\ell$ is low the modal inertia is quite insensitive to degree $\ell$. It is therefore plausible to use only radial modes in the computations the results of which are also applicable to modes of low degrees (e.g. Dziembowski et al. 2001; Daszyńska-Daszkiewicz et al. 2005). Then, proceeding as in M99,
for each star, the range of predicted unstable radial modes (according to linear growth rates) is compared with observed mode ranges.

4.1 Determination of observed ranges of radial modes

Theoretical adiabatic oscillation spectra are computed with the oscillation code *filou* ([Tran Minh & Lépine 1993; Suárez 2002]). As mentioned in Section 3, we are using an improved version of M99’s treatment for second-order effects of rotation on the adiabatic oscillation frequencies. In particular, we use the complete treatment of second-order effects of rotation by Suárez et al. (2006), which is based on the formalism by Dziembowski & Goode (1992) and Soufi et al. (1998). Furthermore, the theoretical adiabatic oscillation frequencies are computed from the pseudo-rotating equilibrium models (Section 3), whereas M99 considered only non-rotating equilibrium models. We recall that these pseudo-rotating equilibrium models, representative of each star, are computed from the rotationally corrected stellar parameters of Sect. 3.2.

For each star the range of radial orders of the observed modes is then determined in the manner of M99, i.e. the locations of the peaks in the observed power spectrum are compared with the locations of radial modes in the theoretical eigenspectrum of the corresponding pseudo-rotating models.

4.2 Predicted unstable modes

The stability computations are carried out in the manner of Balmforth (1992). We include the Lagrangian perturbations of the turbulent fluxes, i.e. the convective heat flux \( \delta F_c \) and turbulent pressure \( \delta p \), in the stability computations according to the non-local, time-dependent mixing-length model by Gough (1977a,b). The equilibrium envelope models are obtained by specifying mass, luminosity and effective temperature as provided by the evolutionary computations of Section 3, but they are not corrected for rotational effects. The radiation field is treated in the Eddington approximation, and the atmosphere is considered to be grey and plane-parallel. We adopt the nomenclature of M99, i.e. \( \alpha_{NL} = l/H_p \) is the mixing-length parameter of the non-local convection model used in the stability computations. The non-local mixing-length parameter \( \alpha_{NL} \) is calibrated to the same depth of the outer convection zone as suggested by the evolutionary computations (see Section 3) which use the standard mixing-length formulation by Bohm-Vitense (1958) and a local mixing-length parameter \( \alpha_{SUN} = 1.614 \). The so obtained calibrated value of \( \alpha_{NL} = 1.89 \). To study the effect of varying the mixing length on mode stability we computed a second series of stellar models with \( \alpha_{NL} = 1.50 \). Further details on the stability computations can be found in Houdek et al. (1999).

5 COMPARISON BETWEEN OBSERVED AND PREDICTED RANGES OF UNSTABLE RADIAL MODES

We proceed as in M99 and compare ranges of observed and predicted radial orders \( n \) of unstable modes for two values of \( \alpha_{NL} \): 1.89 and 1.50 (see Section 4.2). We recall that this comparison does not imply that all observed modes are identified as radial modes, but their frequency ranges are analysed in terms of unstable modes described within a range of radial modes (for details see M99).

The errors in the calibration from colour indices to effective temperatures can reach 150 K. It is therefore adequate to accept an uncertainty of \( \pm 1n \) in the determination of the range of radial orders. In addition, this error also accounts for (1) the difference of ranges estimated in the co-rotating and the observer’s frame, i.e., the rotational splitting; for modes up to \( \ell = 2 \), this represents a frequency range shift up to \( \nu_{rot} \ell \sim 40 \mu Hz \), with \( \nu_{rot} \) being the stellar rotational frequency (up to \( \sim 20 \mu Hz \); and (2), for the possible \( \ell = 1 \) and 2 modes with frequencies close to the boundaries of the radial order ranges.

5.1 Consistency with the M99 results

As a first step, we examine the consistency of the present results with the results by M99. We do so by comparing the ranges of radial orders (observed and predicted ranges) obtained here (Fig. 3b) with the ranges reported by M99 (Fig. 3a) for BU Cnc, BN Cnc, BW Cnc and BS Cnc. In both panels, the observed ranges of radial orders are represented by filled circles; those obtained from the stability analysis using \( \alpha_{NL} = 1.89 \) and 1.50 are illustrated by rhombus and squares, respectively. The coordinates \((n_1, n_2)\) correspond to ranges of radial orders between \( n_1 \) and \( n_2 \).

When performing this comparison, it has to be kept in mind that the present work includes various improvements in the stellar modelling over M99 and consequently represents a more specific and more precise solution, whereas the M99 results represent an extended and conservative set of solutions in terms of model parameters. In this view, we are interested to check if our new solutions do belong to the set of solutions by M99 (and if so, to what extent), or if they differ significantly.

Considering the observed ranges, Fig. 3 shows that the new solutions lie within those reported by M99. The only significant difference is observed for BN Cnc and which has to be attributed to the updated \( V_{Sini} \) value adopted in the present work.

As for the instability predictions, our results for the two considered values of \( \alpha_{NL} \) are in general similar, within \( \pm 1n \), to the results reported by M99.

5.2 Discussion of the present results

Focusing on the present results, we come to the comparison between observed and predicted ranges which is illustrated in Fig. 3a. Generally, the observed ranges are in good agreement with the theoretical predictions using \( \alpha_{NL} = 1.50 \), whereas with \( \alpha_{NL} = 1.89 \) the predicted ranges are in grave disagreement with the observations for the intermediate mass stars BW Cnc and BS Cnc. In M99 this agreement (and disagreement) was observed for the set of four stars. However, in M99 the comparison between observations and theory was performed for a representative set of model solutions which were obtained independently for each individual star. On one hand, M99 reported that the observed and theoretical results could not be made to agree with \( \alpha_{NL} = 1.89 \). On the other hand, their comparison was in reasonable agreement for models computed with \( \alpha_{NL} = 1.50 \) which, however, does not necessarily mean that they found one consistent model solution for all four objects (stars) all of which have the same age and metallicity. The results reported here, particularly the fact that we also do not find agreement between observations and theory for models obtained with
αNL = 1.89, support the conclusions reported by M99, but limited to only one consistent model solution for all considered stars, which is the main objective of the present work.

For the massive objects, BU Cnc and BN Cnc, the predicted unstable ranges are compatible with the observed results (±1 n), for both αNL = 1.50 and αNL = 1.89. The results for both objects are thus not sensitive to the value of αNL. This is to be expected because these more massive stars have shallower outer convection zones and thus their structures are less sensitive to the assumed value of the mixing-length parameter. In contrast, unstable ranges predicted for BW Cnc & BS Cnc show a strong dependence on this parameter. The discrimination between the two αNL values reported in M99 is then partially reproduced. Finally, for the less massive object, BV Cnc, the values for αNL cannot be distinguished. Nevertheless, the observed ranges agree with the theoretical predictions within ±1 n. Therefore, in general, these results constitute a consistent solution in terms of physics and cluster membership, and the observed and theoretical ranges of radial orders are in reasonable agreement for all the stars considered in this work.

At this point, we should keep in mind that, as in M99, the ranges of unstable modes were estimated from non-rotating models for the observed stars. Therefore, we may wonder how accurately the so obtained theoretical ranges of unstable modes represent the observations of the rotating stars. However, the lack of theories describing the effect of rotation on mode stability makes it difficult to overcome this problem. Nevertheless, assuming that mode stability depends predominantly on the effective temperature of the models (Pamiatnykh 1975), it is possible to make a rough estimate of the effect of varying the model’s effective temperature on the location of the range of unstable radial modes. The effective temperature varies if the photospheric parameters are corrected for the effect of rotation (see Section 5.2). Keeping the remaining model parameters (mass, luminosity and chemical composition) constant we re-calculate for each model the linear growth rates. The so obtained ranges of unstable radial modes are compared with the observations in Fig. 4.

In general, the results are similar to those in Fig. 3, but the ranges of unstable modes are shifted. For αNL = 1.50 the unstable ranges are shifted to lower values in the case of BU Cnc and BN Cnc, but remain almost unaltered for the remaining stars. On the other hand, for αNL = 1.89 the unstable ranges are shifted to higher values of n for the more massive objects, and to lower values for the rest of stars. In particular, the discrimination pattern of BW Cnc and BS Cnc in Fig. 3b, (obtained from the non-rotating equilibrium model parameters) is similar to the pattern of BU Cnc and BN Cnc in Fig. 4 (obtained from model parameters for which Teff is corrected for rotational effects), which are more massive and hotter stars (see Table 3). The effective temperatures of BW Cnc and BS Cnc in Fig. 4 are similar to the effective temperature of BV Cnc in Fig. 3 and, as expected, the stability results are comparable.

6 CONCLUSION

In this work we studied ranges of unstable modes predicted by a stability analysis and compared the results with observations for a selected sample of δ Sct stars in the Praesepe cluster. With improved descriptions for rotational effects in both the evolutionary and adiabatic oscillations computations, we searched for a single consistent solution in terms of physics and cluster membership in order to match reasonably the observed and theoretically predicted ranges of overstable low-degree modes for all considered stars.

Consistent solutions were found for models with an age of 650 Myr, and which were computed with a mixing-length parameter of αMLT = 1.614 and with a value for dν that is compatible with the grid of solutions provided by M99. In addition, for stellar models with log Teff = 3.87 – 3.88 a value of αNL ≈ 1.50 leads to a reasonable agreement between theoretical predictions and observations in ranges of unstable modes, indicating that for these stars a smaller mixing-length parameter αNL is required than suggested from a calibrated solar model. The need of a smaller value for αNL than that from a calibrated solar model was also reported by Daszyńska-Daszkiewicz et al. (2005) for the δ Sct star FG Vir.

Existing stability studies (including the present work) rely on calculations of non-adiabatic quantities without the inclusion of rotational effects; considering these effects in future investigations would lead to a substantial improvement of the stability analysis. In a first approximation we noticed an effect on mode stability through the change in effective temperature if it is corrected for rotational effects (see Section 5.2). The results presented here (Section 5.2) suggest, at least for some objects, a significant shift in the predicted ranges of unstable modes. Conclusions going beyond these findings, however, would require the development of a theory that describes the interaction between rotation and pulsation dynamics.
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