We report the recent progress in the studies of D-wave heavy quarkonia production from gluon fragmentation at the Tevatron and in \( Z^0 \) decays, and the D-wave charmonia production in \( B \) decays. We show that the color-octet contributions are of crucial importance to the production of D-wave quarkonia. Signals, as strong as that for \( J/\psi \) and \( \psi' \), should be seen for the D-wave charmonium in the processes. These will provide a test of the color-octet production mechanism.

1 Introduction

Studies of heavy quarkonium production in high energy collisions provide important information on both perturbative and nonperturbative QCD. In recent years, a new framework for treating quarkonium production and decays has been advocated by Bodwin, Braaten and Lepage in the context of nonrelativistic quantum chromodynamics (NRQCD). In this approach, the production process is factorized into short and long distance parts, while the latter is associated with the nonperturbative matrix elements of four-fermion operators. This factorization formalism gives rise to a new production mechanism called the color-octet mechanism, in which the heavy-quark and antiquark pair is produced at short distance in a color-octet configuration and subsequently evolves nonperturbatively into physical quarkonium state. The color-octet term in the gluon fragmentation to \( J/\psi(\psi') \) has been considered to explain the \( J/\psi(\psi') \) surplus problems discovered by CDF. Taking the nonperturbative \( \langle O_{J/\psi}(3S_1) \rangle \) and \( \langle O_{\psi'}(3S_1) \rangle \) as input parameters, the CDF surplus problem for \( J/\psi \) and \( \psi' \) can be explained as the contributions of color-octet terms due to gluon fragmentation. In the past few years, applications of the NRQCD factorization formalism to \( J/\psi(\psi') \) production at various experimental facilities have been studied.

Even though the color-octet mechanism has gained some successes in describing the production and decays of heavy quark bound systems, it still needs more effort to go before finally setting its position and role in heavy quarkonium physics. (For instance, the photoproduction data from HERA put a question about the universality of the color-octet matrix elements, in which the fitted values of the matrix elements \( \langle O_{J/\psi}(1S_0) \rangle \) and \( \langle O_{\psi'}(3P_J) \rangle \) are one order of magnitude smaller than those determined from the Tevatron data).

In this report, we discuss the production of D-wave heavy quarkonia under the framework of NRQCD factorization formalism, including gluon fragmentation at the hadron col-
lider in the $Z^0$ decays, and the $D$-wave charmonia production in $B$ decays. All these results show that the color-octet mechanism is crucially important to $D$-wave charmonium production, and the color-octet contributions are found to be over two orders of magnitude larger than the color-singlet contributions. This because, in essence, the above processes are associated with the color-octet $^3S_1$ intermediate configuration, which is the same as that in $J/\psi$ or $\psi'$ production at the Tevatron via gluon fragmentation. Therefore, the production of $D$-wave quarkonia would provide a crucial test of the color-octet production mechanism. On the other hand, even if the color-singlet model predicts the $D$-wave production rates too small to be visible, they could now be detected after and only after including the color-octet production mechanism. It is the color-octet mechanism that could make it possible to search for the $D$-wave heavy quarkonium states, and then complete the studies of charmonium and bottomonium families.

2 Gluon Fragmentation to $^3D_J$ Heavy Quarkonia at the Tevatron

We choose a special process to study the gluon fragmentation to color-singlet $^3D_J$ quarkonium,

$$Q^* \rightarrow Qg^*; \quad g^* \rightarrow ^3D_J gg.$$

The decay widths of a virtual quark $Q^*$ to color-singlet quarkonium state $^3D_J$ by gluon fragmentation can be evaluated via

$$\Gamma(Q^* \rightarrow Qg^*; g^* \rightarrow ^3D_J gg) = \int_{\mu^2_{\text{min}}}^{s} d\mu^2 \Gamma(Q^* \rightarrow Qg^*(\mu)) \cdot P(g^* \rightarrow ^3D_J gg),$$

where $s$ is the invariant mass squared of $Q^*$; $\mu$ is the virtuality of the gluon, and its minimum value squared $\mu^2_{\text{min}} = 12m_Q^2$ corresponding to the infrared cutoff as discussed below; $P$ is the decay distribution defined as

$$P(g^* \rightarrow AX) \equiv \frac{1}{\pi \mu^3} \Gamma(g^* \rightarrow AX).$$

The calculations of $\Gamma(g^* \rightarrow AX)$ are lengthy but straightforward. So, the fragmentation functions can be calculated

$$D_{g^* \rightarrow ^3D_J}(z, 2m_Q, s) = \frac{d\Gamma(Q^* \rightarrow ^3D_J gg Q)/dz}{\Gamma(Q^* \rightarrow Qg)},$$

where $z \equiv \frac{2P \cdot k}{\mu^2} = 2 - x_1 - x_2$. At high energy limit, the interaction energy $s$ goes up to infinity, then the definition of $z$ here is identical with that in Ref.[2] multiplied by a factor of two and the fragmentation functions decouple from any specific gluon splitting processes, which just reflects the universal spirit of fragmentation. The fragmentation function of Eq.(4) is evaluated at the renormalization scale $2m_Q$, which corresponds to the minimum value of the invariant mass of the virtual gluon.
The calculation of color-octet fragmentation functions in \( g^* \to 3D_J(3S_1,8) \) processes is trivial. They may be obtained directly from color-octet \( g^* \to J/\psi(3S_1,8) \) process:

\[
D_{g^*\to 3D_J}(z,2m_Q) = \frac{\pi\alpha_s(2m_Q)}{24m_Q^2}\delta(1-z)(\langle O_8^{3D_J}(3S_1) \rangle).
\] (5)

For the numerical calculations, we choose

\[
m_c = 1.5 \text{ GeV}, \quad m_b = 4.9 \text{ GeV}, \quad \alpha_s(2m_c) = 0.26, \quad \alpha_s(2m_b) = 0.19,
\]

\[
|\langle (cc) \rangle(0)|^2 = 0.015 \text{ GeV}^7, \quad |\langle (bb) \rangle(0)|^2 = 0.637 \text{ GeV}^7.
\] (6)

And then, for color-singlet gluon fragmentation we obtain

\[
D_{g^*\to 3D_1(cc)}^{(1)} = 5.6 \times 10^{-8}, \quad D_{g^*\to 3D_2(cc)}^{(1)} = 3.1 \times 10^{-7},
\]
\[
D_{g^*\to 3D_3(cc)}^{(1)} = 2.2 \times 10^{-7}, \quad D_{g^*\to 3D_1(bb)}^{(1)} = 2.5 \times 10^{-10},
\]
\[
D_{g^*\to 3D_2(bb)}^{(1)} = 1.4 \times 10^{-9}, \quad D_{g^*\to 3D_3(bb)}^{(1)} = 9.9 \times 10^{-10}.
\] (7)

For color-octet gluon fragmentation, the fragmentation probabilities are proportional to the nonperturbative matrix elements \( \langle O_8^{3D_J}(3S_1) \rangle \) which have not been extracted out from experimental data, nor from the Lattice QCD calculations. Based upon the NRQCD velocity scaling rules, here we tentatively assume

\[
\langle O_8^{3D_2(cc)}(3S_1) \rangle \approx \langle O_8^{3D_3(cc)}(3S_1) \rangle = 4.6 \times 10^{-3} \text{ GeV}^3
\] (8)

and further extend this relation to the \( bb \) system

\[
\langle O_8^{3D_2(bb)}(3S_1) \rangle \approx \langle O_8^{3D_3(bb)}(3S_1) \rangle = 4.1 \times 10^{-3} \text{ GeV}^3.
\] (9)

Note that the NRQCD scaling rules tell us that the two matrix elements in (8) (or (9)) are of the same order, but not necessarily equal, therefore the assumed relations (8) and (9) certainly possess uncertainties to some extent. However, from the calculated results below we are confident that it will not destroy our major conclusion. From the approximate heavy quark spin symmetry relation, we have

\[
\langle O_8^{3D_1(3S_1)} \rangle \approx \frac{3}{5}\langle O_8^{3D_2(3S_1)} \rangle \approx \frac{5}{1}\langle O_8^{3D_3(3S_1)} \rangle
\] (10)

for both \( bb \) and \( cc \) systems.

Using Eqs.(5), (8), and (9)-(10), we readily have

\[
D_{g^*\to 3D_1(cc)}^{(8)} = 4.2 \times 10^{-5}, \quad D_{g^*\to 3D_2(cc)}^{(8)} = 7.0 \times 10^{-5},
\]
\[
D_{g^*\to 3D_3(cc)}^{(8)} = 9.7 \times 10^{-5}, \quad D_{g^*\to 3D_1(bb)}^{(8)} = 2.5 \times 10^{-6},
\]
\[
D_{g^*\to 3D_2(bb)}^{(8)} = 4.2 \times 10^{-6}, \quad D_{g^*\to 3D_3(bb)}^{(8)} = 5.9 \times 10^{-6}.
\] (11)
Comparing (11) with (7), we come to an anticipated conclusion that at the Tevatron the gluon fragmentation probabilities through color-octet intermediates to spin-triplet D-wave charmonium and bottomonium states are over $2 \sim 4$ orders of magnitude larger than that of color-singlet processes. As a result, the production rates of $^3D_J$ charmonium states are about the same amount as $\psi'$ production rates. Compared with the $\psi'$ production at the Tevatron, the gluon fragmentation color-octet process plays an even more important role in the $^3D_J$ quarkonium production, and it also gives production probabilities larger than the quark fragmentation process.

3 In $Z^0$ Decays

For the $D$-wave heavy quarkonium production in $Z^0$ decays, the dominant process is $Z^0 \to ^3D_J q\bar{q}$. This is similar to $Z^0 \to \psi q\bar{q}$ discussed in Ref.[11]. Here $q$ represents $u, d, s, c$ or $b$ quarks. In the limit $m_q = 0$, we readily have

$$\Gamma(Z \to ^3D_J q\bar{q}) = \Gamma(Z \to q\bar{q}) \frac{\alpha_s^2(2m_c)(O_8^{3D_J}(3S_1))}{36} \{5(1 - \xi^2) - 2\xi \ln \xi + [2Li_2(1 + \xi) - 2Li_2(1 + \xi)] - 2\ln(1 + \xi) \ln \xi + 3\ln \xi + \ln^2 \xi(1 + \xi)^2\},$$

(12)

where $Li_2(x) = -\int_0^x dt \ln(1 - t)/t$ is the Spence function. The calculation with physical masses, say, e.g. $m_b = 5$ GeV, has also been performed by us, which does not show much difference from the case of $m_q = 0$.

From Eq.(12) (with slight modification due to nonzero $m_q$), we can get the branching ratios of $Z^0 \to ^3D_J q\bar{q}$. Summing over all the quark flavors ($q = u, d, s, c, b$) with their physical masses, we obtain the fraction ratios

$$\frac{\Gamma(Z^0 \to ^3D_1 q\bar{q})}{\Gamma(Z^0 \to q\bar{q})} = 2.0 \times 10^{-4}, \frac{\Gamma(Z^0 \to ^3D_2 q\bar{q})}{\Gamma(Z^0 \to q\bar{q})} = 3.4 \times 10^{-4}, \frac{\Gamma(Z^0 \to ^3D_3 q\bar{q})}{\Gamma(Z^0 \to q\bar{q})} = 4.8 \times 10^{-4}.$$

(13)

The color-singlet processes include the quark fragmentation contributions and the gluon fragmentation contributions. For quark fragmentation, the branching ratios of $^3D_J$ production in color-singlet processes are $2.3 \times 10^{-6}$, $3.6 \times 10^{-6}$, $1.7 \times 10^{-6}$ for $J = 1, 2, 3$, respectively. The gluon fragmentation processes are more complicated. For in the most important kinematic region the virtual gluon is nearly on its massshell, $^3D_J$ production in the gluon fragmentation color-singlet process may be separated into $Z^0 \to q\bar{q}^*$ and $g^* \to ^3D_J gg$. The decay widths of $Z^0$ to color-singlet charmonium state $^3D_J$ by gluon fragmentation can be evaluated via

$$\Gamma(Z^0 \to q\bar{q}^*: g^* \to ^3D_J gg) = \int_{\mu_{\text{min}}^2}^{\mu_i^2} d\mu^2 \Gamma(Z^0 \to q\bar{q}^*)P(g^* \to ^3D_J gg),$$

(14)
where the cutoff $\Lambda = m_c$ is transformed into a lower limit on $\mu_{\text{min}}^2 = 12m_c^2$. Summing over all the flavors $q (q = u, d, s, c, b)$, we obtain the fraction ratios for the production of $^3D_J$ quarkonium are $4.3 \times 10^{-7}$, $2.1 \times 10^{-6}$, $1.2 \times 10^{-6}$ for $J = 1, 2, 3$, respectively.

4 D-wave Charmonia Production in $B$ Decays

We first calculate the color-singlet contribution to $^3D_1$ charmonium production in $b$ decays. The matrix elements $\langle 0| (c\bar{c})_{V-A}|^3D_1 \rangle$ can be calculated as

$$\langle 0| (c\bar{c})_{V-A}|^3D_1 \rangle = \frac{20\sqrt{3}}{\sqrt{2\pi}} \epsilon_{\mu} \frac{R_{D}^f(0)}{M^3 \sqrt{M}}.$$  \hspace{1cm} (15)

So the partial width for color-singlet $\delta_{1}^c$ production in $b$ decays will be

$$\Gamma(b \to c\bar{c}(^3D_1^{(1)}) + s, d \to \delta_{1}^c + X) = \frac{5}{9} \frac{\langle O^{c_{10}}(3D_1) \rangle}{M_b^6} (2C_+ - C_-)^2 (1 + \frac{8M_b^2}{M_c^2}) \hat{\Gamma}_0.$$

4.1 $\delta_{1}^c$ Production

The color-octet contributions to $\delta_{J}^c$ production are similar to that to the $J/\psi$ production

$$\Gamma(b \to (c\bar{c})_S + s, d \to \delta_{J}^c + X) = \left( \frac{\langle O^{d_{10}}(3S_1) \rangle}{2M_c^2} + \frac{\langle O^{d_{30}}(3P_1) \rangle}{M_c^4} \right) \times (C_+ + C_-)^2 (1 + \frac{8M_b^2}{M_c^2}) \hat{\Gamma}_0.$$  \hspace{1cm} (17)

The predicted branching fractions for $D$-wave charmonium states are

$$B(B \to \delta_{1}^c + X) = 0.28\%, \quad B(B \to \delta_{2}^c + X) = 0.46\%, \quad B(B \to \delta_{3}^c + X) = 0.65\%.$$  \hspace{1cm} (19)

5 Conclusions

In conclusion, we have shown that in the production of $D$-wave heavy quarkonia, the color-octet processes are always dominant. Their contributions are orders of magnitude larger than the color-singlet contributions. According to the NRQCD velocity scaling rules, the production rate of $D$-wave quarkonium is the same order as that of $\psi'$. Among the three triplet states of $D$-wave charmonium, $\delta_{2}^c$ is the most prominent candidate to be discovered.

$a$Here, we denote the physical $D$-wave charmonium state as $\delta_{J}^c$, and $^3D_J$ represents the leading part of the Fock state components of $\delta_{J}^c$.  

5
firstly. It is a narrow resonance, and the branching fraction of the decay mode $J/\psi \pi^+\pi^-$ is estimated to be

$$B(\delta^+_2 \rightarrow J/\psi \pi^+\pi^-) \approx 0.12,$$

which is smaller than that of $B(\psi' \rightarrow J/\psi \pi^+\pi^-)$ by only a factor of 3. Therefore, $2^{-} D$-wave charmonium should be observable at the Tevatron via color-octet gluon fragmentation, and at the CLEO via $B$ decays. This would provide a crucial test for the color-octet production mechanism. We expect the experimental data will be soon obtained.

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China, the State Education Commission of China, and the State Commission of Science and Technology of China. One of us (K.T.C.) would like to thank Prof. S.K. Kim-Ewha Womans University, Prof. C. Lee-Seoul National University, and Prof. K. Kang-Brown University for their outstanding organization and hospitality during this conference.

References

1. G. T. Bodwin, E. Braaten and G. P. Lepage, Phys. Rev. D 51, 1125 (1995).
2. E. Braaten and S. Fleming, Phys. Rev. Lett. 74, 3327 (1995); M. Cacciari, M. Greco, M.L. Mangano and A. Petrelli, Phys. Lett. B 356, 553 (1995).
3. P. Cho and K. Leibovich, Phys. Rev. D 53, 150 (1996); ibid, Phys. Rev. D 53, 6203 (1996).
4. For a recent review see E. Braaten, S. Fleming, and T. C. Yuan, Annu. Rev. Nucl. Part. Sci. 46, 197 (1996).
5. H1 Collab., S. Aid et al, Nucl. Phys. B 472, 3 (1996); ZEUS Collab., M. Derrick et al, DESY-97-147.
6. M. Cacciari and M. Krämer, Phys. Rev. Lett. 76, 4128 (1996).
7. C.F. Qiao, F. Yuan, and K.T. Chao, Phys. Rev. D 55, 5437 (1997).
8. C.F. Qiao, F. Yuan, and K.T. Chao, Phys. Rev. D 55, 4001 (1997).
9. F. Yuan, C.F. Qiao, and K.T. Chao, Phys. Rev. D 56, 329 (1997).
10. K. Cheung and T.C. Yuan, Phys. Rev. D 53, 3591 (1996).
11. K. Cheung, W.-Y. Keung, and T. C. Yuan, Phys. Rev. Lett. 76, 877 (1996).
12. G.T. Bodwin, E. Braaten, T.C. Yuan and G.P. Lepage, Phys. Rev. D 46, R3703 (1992); P. Ko, J. Lee, and H.S. Song, Phys. Rev. D 53, 1409 (1996).
13. P. Ko, J. Lee, and H.S. Song, Phys. Lett. B 395, 107 (1997).