Validity of strong lensing statistics for constraints on the galaxy evolution model

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2 February 2008

ABSTRACT
We examine the usefulness of the strong lensing statistics to constrain the evolution of the number density of lensing galaxies by adopting the values of the cosmological parameters determined by recent WMAP observation. For this purpose, we employ the lens-redshift test proposed by Kochanek (1992) and constrain the parameters in two evolution models, simple power-law model characterized by the power law indexes $\nu_n$ and $\nu_v$ and the evolution model by Mitchell et al. (2005) based on CDM structure formation scenario. We use the well-defined lens sample from the Sloan Digital Sky Survey (SDSS) and this is similarly sized samples used in the previous studies. Furthermore, we adopt the velocity dispersion function of early-type galaxies based on SDSS DR1 and DR5. It turns out that the indexes of power-law model are consistent with the previous studies, thus our results indicate the mild evolution in the number and velocity dispersion of early-type galaxies out to $z = 1$. However we found that the values for p and q used by Mitchell et al. are inconsistent with the presently available observational data. More complete sample is necessary to withdraw more realistic determination on these parameters.

Key words: gravitational lensing, dark matter;

1 INTRODUCTION
Since the pioneering work by Turner, Ostriker and Gott (Turner, Ostriker & Gott 1984), the strong lensing statistics is known as a powerful tool to constrain the cosmological parameters, especially the cosmological constant (Fukugita, Futamase, Kasai & Turner 1992), as well as to investigate the evolution of number density of lensing galaxies (Mao 1991, Mao & Kochanek 1994, Ofek, Rix & Maoz 2003, Mitchell et al. 2005, Chae, Mao & Kang 2006, and Capelo & Natarajan 2007). However this method is not competitive with other approaches such as Cosmic Microwave Background (CMB). In fact recent observation by WMAP (e.g. Spergel et al. 2007) determined the cosmological parameters within the accuracy of a few percent. The reason is mainly because of the shortage of observed lens systems as well as the partial knowledge of the redshift information of the sources and lensing galaxies.

Specifically, the current largest complete lens sample comes from the Cosmic Lens All Sky Survey (CLASS; Myers et al. 2003, Browne et al. 2003), an extension of the earlier Jodrell Bank/Very Large Array Astrometric Survey (JVAS; Patnaik et al. 1993, Pantaik et al. 1992 and Augusto et al. 2001), but the number of lensing events are only 13 lensed radio sources. On the other hand, the SDSS Quasar Lens Search (QLS) has already discovered approximately 20 new strongly lensed quasars, so this sample is expected to become the largest statistical sample of strongly lensed quasars.

On the other hand, there have been no well motivated and established theoretical models for the evolution of the number density of the lensing galaxies, and there seems no definite observational conclusion if the number density evolves or not.

Thus it will be timely to investigates the evolution of the number density of the lensing galaxies by adopting the values for the cosmological parameters determined by WMAP and using a new sample based on QLS.

In this paper, we focus on two typical redshift evolution models: One model is described by the power-law evolution for the number density and the characteristic velocity dispersion as $(1 + z)^{\nu_n}$ and $(1 + z)^{\nu_v}$, respectively. (see Chae & Mao (2003) in detail). The other is the model based on CDM structure formation scenario used by Mitchell et al. (2005) which we call Mitchell’s model. We use ‘lens-redshift test’ proposed by Kochanek (see Kochanek (1992)) to calculate the likelihood as a function of the parameters in the
models. This method has advantage not to use the total lensing probability and thus not to require the knowledge of the magnification bias in the sample. We also use the SLOS data whose separation is under 2 arcsec as the lens sample in order to select only the events caused by a single galaxy which is known to be well modeled by Singular Isothermal Sphere(SIS) or Singular Isothermal Ellipsoid(SIE). We have used the parameters in the velocity dispersion function given by SDSS DR1 and DR5. Thus we avoid ambiguities of the lens model as possible as we can.

The organization of this paper is as follows. In section 2, we explain some basic of lens statistics and show the lens models and the evolution models we adopt. In section 3, we describe the data that we use. We calculate maximum likelihood value and confidence level of the parameters of each evolution model using the differential lensing probability. In section 4, we present the results. Finally some discussion is given in section 5. Throughout this paper we adopt a dominant totally flat cosmology with the matter density Ω_m = 0.27.

2 BASIC OF STRONG LENSING STATISTICS

2.1 The optical depth

Following to Turner, Ostriker & Gott (1984), we consider the light ray propagating from source (QSO) to observer. The differential probability dr of the ray encountering a lens in traveling the path of the redshift interval dz_L is given by

\[ dr = n_L(z_L) \sigma_L c dt dz_L B \]

where n_L(z_L) is number density of the lens (see 2.3), σ_L is the lensing cross section (see (22) and B is the magnification bias (see Turner, Ostriker & Gott 1984). The quantity \( \frac{cdt}{dz_L} \)

is calculated in the FLRW geometry to be

\[ \frac{cdt}{dz_L} = \frac{R_0}{1 + z_L \sqrt{\Omega_m(1 + z_L)^3 + \Omega_\Lambda}} \]

where R_0 = 1/H_0 is Hubble distance, Ω_m is total mass density of the universe, Ω_Λ ≡ Λc^2/3H_0^2 is the normalized cosmological constant, and z_L is lens redshift.

Using (1), the differential optical depth in traversing dz_L with angular separation between Δθ and Δθ + dΔθ is calculated as d^2τ/dz_L dΔθ.

2.2 The lens models

We use a singular isothermal sphere (SIS) (Asada et al. 2003) and a singular isothermal ellipsoid (SIE) (Asada et al. 2003) as our lens model. The SIS model is widely used, so we don’t explain its property. (see, for example, Turner, Ostriker & Gott 1984 and Maoz & Rix 1993). The SIE model can be solved analytically (Asada et al. 2003). We refer the readers to Asada et al. (2003) for specific calculation. We only show the cross sections of this model for 4images (σ_L4im) and 2images (σ_L2im) here.

\[ \sigma_{L4im} = \sigma_{L2im} = \pi \sqrt{1 - e^2} \theta_E^2 - \sigma_L4im \]

where e represents ellipticity and this ellipticity is constrained as 0 ≤ e < 1/5. The SIS model is recovered in the e = 0 for σ_L4im case and this upper constrain e < 1/5 comes from the fact that the density contours must be convex, which is reasonable for an isolated relaxed system. And θ_E is Einstein radius for SIS lens and can be written as

\[ \theta_E = 4\pi (\frac{\sigma}{c})^2 \frac{D_{LS}}{D_{OS}} \]

where σ is one-dimensional velocity dispersion and D_{LS} and D_{OS} are the angular diameter distance between the lens and the source, the observer and the source, respectively.

2.3 Number density of galaxy lens

The number density of galaxies with velocity dispersion (we call this function velocity dispersion after this) lying between σ and σ + dσ is assumed to be described by modified Schechter function (Sheth et al. 2003, Mitchell et al. 2003) of the form,

\[ \phi(\sigma)d\sigma = \phi_* \left( \frac{\sigma}{\sigma_*} \right) ^\alpha \exp \left[ -\left( \frac{\sigma}{\sigma_*} \right) ^\beta \right] \frac{\beta}{\Gamma(\alpha/\beta)} \frac{d\sigma}{\sigma} \]

where \( \phi_* \) is the integrated number density of galaxies, \( \sigma_* \) is the characteristic velocity dispersion, \( \alpha \) is the low-velocity power-law index and \( \beta \) is the high-velocity exponential cutoff index. These parameters in the velocity function, \( \phi\), \( \alpha \), \( \beta \) and \( \sigma_* \), are determined by observations. \( \Gamma(x) \) represents the gamma function. The present day comoving number density of galaxies can be calculated by integrating (9).

For these parameters, we use the measured velocity dispersion function from SDSS DR1 (Sheth et al. 2003, Mitchell et al. 2003) and DR5 (Choi & Vogels 2007) for early-type galaxies. The measured values that Choi (2007) outlined are as follows,

\[ \phi(\sigma, \alpha, \beta)_DR1 = \left[ (4.1 \pm 0.3) \times 10^{-3} h^3 \text{Mpc}^{-3}, 88.8 \pm 17.7 \text{km s}^{-1}, 6.5 \pm 1.0, 1.93 \pm 0.22 \right] \]

\[ \phi(\sigma, \alpha, \beta)_DR5 = \left[ (8.0 \times 10^{-3} h^3 \text{Mpc}^{-3}, 161 \pm 5 \text{km s}^{-1}, 2.32 \pm 0.10, 2.67 \pm 0.07 \right] \]

where h is the Hubble constant in units of 100 km/s Mpc^{-3}.

2.4 The lens redshift distribution

In this subsection we show two evolution models of number density of galaxies for (9).

2.4.1 The power-law evolution model

The first evolution model is a power-law type as most authors adopted (for example, Chae & Mao 2003). In this model, we allow the number density of galaxies \( \phi \) and the velocity dispersion \( \sigma \) in (9) to vary with redshift as

\[ \phi(z) = \phi_0 (1 + z)^{\nu_\phi} \]

\[ \sigma(z) = \sigma_0 (1 + z)^{\nu_\sigma} \]
The no evolution model corresponds to \((\nu_s = 0, \nu_v = 0)\). Note that the case \((\nu_s > 0, \nu_v < 0)\) is consistent with the hierarchical model of galaxy formation with bottom-up assembly of structure.

2.4.2 Mitchell’s evolution model

The next evolution model is proposed by Mitchell et al. (2005). We briefly explain this model. They have taken into consideration the number density evolutions by combining a halo mass function with a velocity dispersion as a function of mass and redshift. They use the prediction by the extended Press-Schechter model of structure formation, which is calibrated by N-body simulations (Sheth & Tormen 1999) as the halo mass function. This halo mass function at epoch \(z\) is introduced by Sheth & Tormen (1999),

\[
\frac{d\ln n(M,z)}{d\ln M} = \bar{\rho}_M \sigma_v M^{1/3} \exp\left(-\frac{\bar{\rho}_M \sigma_v^2 M^{1/3}}{2}\right) \left(1 + \frac{\bar{\rho}_M \sigma_v^2 M^{1/3}}{2}\right)^{3/2}
\]

where \(\bar{\rho}_M\) is the mean density, \(\sigma_v^2 = \sigma_{\text{top}}^2 / \sigma_{\text{top}}^2(M, z)\), and \(\sigma_{\text{top}}^2(M, z)\) is the extrapolated linear overdensity of a spherical top-hat perturbation at the time it collapse, and \(\sigma_{\text{top}}^2(M, z)\) is the variance of the density field at epoch \(z\) in linear perturbation theory, smoothed with a top hat filter of radius \(R(M) = (3M/4\pi \bar{\rho})^{1/3}\). The parameters, \((p, q, A(p))\), are given by N-body simulations in cold dark matter models as \(p = 0.3, q = 0.75, A(p) = 0.3222\) (see Sheth & Tormen 1999).

Next, we show the velocity dispersion

\[
\sigma(M, z) = 92.3 \Delta_{\text{vir}}(z_f)^{1/6} E(z_f)^{1/3} \left(\frac{M}{10^{14} h^{-1} M_\odot}\right)^{1/3} \text{km s}^{-1}
\]

where \(\Delta_{\text{vir}}(z) = 18\pi^2 + 82(\Omega(z) - 1) - 39(\Omega(z) - 1)^2\),

\[
\Omega(z) = \frac{(1 + z)^3}{E^2(z)}
\]

3 DATA AND METHOD

3.1 The lens galaxy sample

Our lens sample is primarily drawn from CASTLES (Muñoz et al. 1998) data base. While this total sample size of CASTLES is 100 systems, a statistical analysis requires a sample from a survey that is complete and has well-characterized, homogeneous selection criteria. From CASTLES data base, we choose 56 samples discovered (re-discovered) by SDSS from CASTLES data. To eliminate biases in terms of the redshift of the lensing galaxy, we have selected systems with \(0.6 < z_L < 2.2\). This source redshift range comes from SDSS selection criterion of the statistical sample of strongly lensed quasars (Inada et al. 2007b).

Furthermore we exclude the systems whose lensing galaxies are the spiral galaxies because the lens model of spiral galaxies is not well established, and the fraction of spiral galaxies in the lensing galaxies has not been decided yet. Furthermore the contribution of the spiral galaxies on the lensing probability is expected to be small. We also exclude the systems whose image separation are large (\(> 2.0''\)) or the systems that contain multiple lensing galaxies.

Using above procedure, our sample contains the lensing systems whose lens redshift is unknown. We exclude these systems because we need the lens redshift and the source redshift information in using this lens-redshift test. Lenses at a low redshift are more likely to have a spectroscopically confirmed redshift than lenses at high redshift so this may bias the sample to systems where the lens is at an artificially low redshift. But our aim is to investigate the usefulness of the lens-redshift test in using the lensing sample from SDSS only. For this reason, we simply neglect these event at this time. In the end our lensing sample consists only of the systems with single early-type galaxy as lensing object and this contains 13 lensing systems (see Table 1).

3.2 The maximum likelihood method

We use a maximum likelihood estimator in our statistical analysis of lens redshift distributions. In this paper, we follow the method proposed by Kochanek (1992) and use the differential optical depth to lensing with respect to the angular separation \(\Delta\theta\) as the probability density. For each systems, we calculate the probability density \(P(z_L|z_S, \Delta\theta, X)\) which is normalized unity, where \(X\) is the set of galaxy evolution model parameters. For power-law evolution model, this set is \(X = (\nu_s, \nu_v)\), and for Mitchell’s evolution model, \(X = (p, q)\). The probability distributions of lens redshifts \(z_L\) with a given angular separation are calculated as follows,

\[
P(z_L|z_S, \Delta\theta, X) = \frac{d^2 P}{d\theta^2} (z_L, z_S, \Delta\theta, X)
\]

The likelihood as a function of \(X\) is given by,

\[
L(X) = \prod_{i=1}^{N} P_i(z_L|z_S, \Delta\theta, X)
\]

4 RESULTS

We investigate two evolution models by estimating the likelihood function (10).

First, we obtain constrains on the power-law evolution model parameters, \((\nu_s, \nu_v)\). We calculate the \(\nu_s-\nu_v\) contours using the SIS lens and the SIE lens for SDSS DR1 and DR5 data. The results are shown in Fig. 1. The maximum L value

\[1\] http://cfa-www.harvard.edu/castles/index.html
Table 1. The columns are: (1) lens number; (2) lens name; (3) source redshift, \( z_s \); (4) lens redshift, \( z_l \); (5) lens separation, \( \Delta \theta \), in arcseconds; (6) grade for the likelihood that the object is a lens: A=I’d bet my life, B=I’d bet your life, and C=I’d bet your life and you should worry (CASTLES); (7) Number of images corresponding to each source component, E means extended and R means there is an Einstein ring (CASTLES); (8) references.

| Number | Name (1) | \( z_s \) | \( z_l \) | \( \Delta \theta \) | Grade | \( N_{im} \) | References |
|--------|----------|----------|----------|----------|-------|---------|------------|
| 1      | SDSS0806+2006 | 1.540 | 0.573 | 1.40 | A | 2 | 1 |
| 2      | SBS0909+523 | 1.38 | 0.83 | 1.17 | A | 2 | 2 |
| 3      | SDSS0924+0219 | 1.52 | 0.39 | 1.75 | A | 4 | 3, 4 |
| 4      | FBQ0951+2635 | 1.24 | 0.20 | 1.11 | A | 2 | 5, 6 |
| 5      | B1030+0747 | 1.54 | 0.60 | 1.65 | A | 2 | 7 |
| 6      | B1152+2000 | 1.02 | 0.44 | 1.59 | A | 2 | 8 |
| 7      | SDSS1226-0006 | 1.12 | 0.52 | 1.26 | A | 2 | 9 |
| 8      | SDSS1332+0347 | 1.445 | 0.191 | 1.14 | A | 2 | 10 |
| 9      | SDSS1335+1138 | 1.629 | 0.3 | 1.41 | A | 2 | 11 |
| 10     | SDSS1406+6126 | 2.13 | 0.27 | 1.98 | A | 4 | 12 |
| 11     | SBS1520+530 | 1.86 | 0.72 | 1.59 | A | 2 | 13 |
| 12     | HST1543+5352 | 2.09 | 0.50 | 1.18 | A | 2R | 14 |
| 13     | MG1549+3047 | 1.17 | 0.11 | 1.7 | A | R | 15, 16 |

and the 1σ limits are \( \nu_v = -2.0^{+1.0}_{-1.0}, \nu_v = 0.15^{+0.40}_{-0.40} \) and \( \nu_v = -1.0^{+2.5}_{-2.5}, \nu_v = 0.08^{+0.50}_{-0.50} \) using the SIS lens for SDSS DR1(top left panel in Fig. 1) data and DR5 data(top right panel in Fig. 1), respectively. In case of SIE lens model, we have \( \nu_v = 1.5^{+3.0}_{-3.0}, \nu_v = -0.15^{+0.20}_{-0.10} \) and \( \nu_v = 2.0^{+4.0}_{-4.0} \), \nu_v = -0.25^{+0.20}_{-0.20} \) for SDSS DR1(bottom left panel in Fig. 1) data and DR5 data(bottom right panel in Fig. 1), respectively.

We compare the estimation of parameters used in the power-law evolution model with previous results. Ofek, Rix & Maoz (2003) give the 1σ limit d log \( \nu_v / dz = -0.1^{+0.06}_{-0.06} \) for the evolution of the characteristic velocity dispersion. The 1σ limit of the characteristic velocity dispersion of the lenses at \( z = 1 \) must be within 69%-91% of the present value. Meanwhile, Chae & Mao (2003) obtain limit \( \nu_v = 0.2^{+0.6}_{-1.0} \) (1σ) without using the lensing rate. The 1σ limit of the characteristic velocity dispersion of the lenses at \( z = 1 \) must be within 57%-74% of the present value. On the other hand, our results are 84%-104% (the SIE lens for DR1) and 73%-97% (the SIE lens for DR5). Thus we obtain the constrains on the evolution of the characteristic velocity dispersion which are consistent with these two previous studies.

Ofek, Rix & Maoz (2003) also give the 1σ limit \( d log \phi_v / dz = +0.7^{+1.2}_{-1.2} \) for the number density. The 1σ limit of number density of the lenses at \( z = 1 \) must be 30% of the present value. Our results are 35% (the SIE lens for DR1) and 50% (the SIE lens for DR5) and thus are consistent with their result.

Next, we calculate the \( p-q \) contours using the SIS lens and the SIE lens for SDSS DR1 and DR5 data. We limit our investigation to \( p < 5 \) and \( q < 3 \) because the value of \( \phi(\sigma; z) \) become unphysical beyond this range. The reason is as follows. We consider redshift evolution of the number density of lensing galaxies by multiplying current velocity function, \( \phi(\sigma; z) / \phi(\sigma; z = 0) \) (see (2.4.2)) just like Mitchell et al. (2005). If we try to calculate the ratio in the case of \( p > 5 \), the resulting ratios become negative. Moreover the parameter \( \alpha \) in the velocity function becomes negative in the case of \( q > 3 \).

Our results of the likelihood functions of the parameters, \( p, q \), are in Fig. 2. In Fig. 2, we obtain maximum values in L at \( (p = 5.0, q = 0.1) \) for the SIS lens and \( (p = 4.5, q = 0.1) \) for the SIE lens which are largely different from the estimation from the simulation (\( p = 0.3, q = 0.75 \) the white-dots in Fig. 2).

The maximum value, \( p \) and \( q \) are at the very corner of the parameter space so we expect that our best fit values of \( p \) and \( q \) are the maximum of the likelihood within our physically meaningful limits and it is probably not a true maximum of the likelihood. But we think it still can be said that the values for \( p \) and \( q \) used by Mitchell et al. are inconsistent with the presently available observational data.

Furthermore we compare the power-law evolution model and the model given by Mitchell et al. \( (p = 0.3, q = 0.75) \). We use our best fit parameter value of the power-law evolution model, \( \nu_v \), in the case of SIE lens, \( \nu_v = -0.15 \) for DR1 and \( \nu_v = -0.25 \) for DR5 and calculate the ratio of the velocity function at \( z = 0.3 \) and \( z = 1.0 \) to that at \( z = 0.0 \). The error of the parameter \( \nu_v \) is large, so we use only estimated \( \nu_v \) and set \( \nu_v = 0.0 \). Similarly, we use the Mitchell et al.’s evolution model\( (p = 0.3, q = 0.75) \) and calculate the ratio of the velocity function. This result is shown in Fig. 3. As a result, we find that the evolution model by the Mitchell et al. has possibility to underestimate the number density of galaxy whose velocity dispersion is large.

5 CONCLUSIONS AND DISCUSSION

We employed the ‘lens-redshift test’ proposed by Kochanek (1992) and constrain the parameters of two evolution models, the power-law evolution model and the evolution model
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Figure 1. The likelihood functions of the number density evolution index $\nu_n$ and the velocity dispersion evolution index $\nu_v$. The functions are normalized the corresponding maximum ($L_{\text{max}}$). The cross represent the maximum likelihood values. Top left panel: We use the SIS lens and the DR1 data as the parameters of the velocity function. The maximum value of $L$ is $\nu_n = -2.0$, $\nu_v = 0.15$. Top right panel: We use the SIS lens and the DR5 data as the parameters of the velocity function. The maximum value of $L$ is $\nu_n = -1.0$, $\nu_v = 0.0$. Bottom left panel: We use the SIE lens and the DR1 data as the parameters of the velocity function. The maximum value of $L$ is $\nu_n = 1.5$, $\nu_v = -0.15$. Bottom right panel: We use the SIE lens and the DR5 data as the parameters of the velocity function. The maximum value of $L$ is $\nu_n = 2.0$, $\nu_v = -0.25$.

proposed by Mitchell et al. (2005). We use the SDSS lens sample that is well-defined. It turns out that one of the parameters of the power-law evolution model, $\nu_v$ is consistent with that given by Ofek, Rix & Maoz (2003) and Chae & Mao (2003) and the other parameter $\nu_n$ is consistent with the result by Ofek, Rix & Maoz (2003).

On the other hand, the most likely values for the parameters used in the evolution model by Mitchell et al. (2005) are inconsistent with the predicted values from the presently available observational data. The inconsistency may be mainly caused by ignoring the total lensing rate in the present analysis. Although Chae & Mao (2003) have shown that the evolution model can be restricted more strictly by including the information of lensing rate, we have not included the effect because the total rate depends on the magnification bias which can not be calculated at the moment with very good accuracy in a consistent way with our SDSS sample. Furthermore we find that there is a possibility to underestimate the number density of galaxy which have large velocity dispersion by using the parameters in Mitchell et al.’s evolution model.

The SQLS is still ongoing and the number of the strong lensing events is steadily increasing. Thus if we are able to calculate accurate magnification bias based on SDSS sample, our method will be very useful to constrain the evolution models of the number density of the lensing galaxies and thus the CDM based structure formation scenario.

ACKNOWLEDGEMENTS

We thank Masamune Oguri for helpful discussions and the anonymous referees for helpful comments that improved our conclusion.

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Figure 2. The likelihood functions of the parameters, \((p, q)\) used in Mitchell’s evolution model. The functions are normalized the corresponding maximum \((L_{\text{max}})\). Top left panel: We use the SIS lens and the DR1 data as the parameters of the velocity function. The maximum value of L is \(P = 5.0, Q = 0.1\). Top right panel: We use the SIS lens and the DR5 data as the parameters of the velocity function. The maximum value of L is \(P = 5.0, Q = 0.1\). Bottom left panel: We use the SIE lens and the DR1 data as the parameters of the velocity function. The maximum value of L is \(P = 4.5, Q = 0.1\). Bottom right panel: We use the SIE lens and the DR5 data as the parameters of the velocity function. The maximum value of L is \(P = 4.5, Q = 0.1\). The white points represent estimation from simulation.

Figure 3. Ratio of the velocity function at \(z = 0.3\) and \(z = 1.0\) to that at \(z = 0\). The set, \((p = 0.3, q = 0.75)\) is estimated by numerical simulated values. And DR1 and DR5 in the figure show the ratio of velocity function when we use the best fit values estimated by using SIE lens model, \((\nu_n = 0.0, \nu_v = -0.15)\) and \((\nu_n = 0.0, \nu_v = -0.25)\), respectively. (see figure 1 and section 4)
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