Experimental Demonstration of the Dependence of Temperature Decrease on the Hydraulic Radius of Regenerator in a Traveling-Wave Thermoacoustic Refrigerator

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Abstract. Thermoacoustic refrigerators are cooling devices that employ sound wave in heat removal process. They mostly use air or other inert gas as working fluid and hence they are environmentally benign. The cooling process takes place in a porous medium (regenerator or stack). This paper presents an experiment on a loudspeaker-driven traveling-wave thermoacoustic refrigerator to demonstrate the dependence of temperature decrease on the hydraulic radius ($r_h$) of regenerator. The refrigerator has a looped tube and a resonator tube, with the length of 260 cm and 150 cm, respectively. Six different regenerators are used, each of which is made of a tight stack of stainless-steel wire-mesh screens with different mesh sizes, providing $r_h$ from 0.07 mm until 0.28 mm. The regenerator length is 60 mm. Atmospheric air is used as the working gas, driven at operating frequency of 30 Hz, giving thermal penetration depth ($\delta_\kappa$) of 0.49 mm. The experimental results show that there is an optimum $r_h$ which gives the largest temperature decrease at the cooling point. It is found, in this case, that the optimum $r_h$ is 0.10 mm (that is, $r_h/\delta_\kappa = 0.20$) and the corresponding temperature decrease is 18.7 °C (i.e. from 28.0 °C down to 9.3 °C).

1. Introduction
Thermoacoustic refrigerators are cooling devices that employ sound wave to absorb heat from a cold region and release more heat to the environment. They mostly use air or other inert gas as working fluid and hence they are environmentally benign.[1, 2] The cooling process is induced by thermoacoustic effects which are thermal interactions between the oscillating gas within the flow channels of a porous medium and the solid walls of the channels. Detail description on the thermal interaction and thermodynamic cycle executed by the oscillating gas which interacts with the solid wall can be found elsewhere.[1, 3, 4]

The important parameter of the porous medium is the hydraulic radius ($r_h$) of the flow channels. In thermoacoustic devices, the hydraulic radius is usually compared to the thermal penetration depth ($\delta_\kappa$) of the working gas to specify the thermal contact condition between the gas and channel walls.[4, 5] If $r_h \gg \delta_\kappa$, there is no thermal contact between the gas and the walls. The gas parcels will expand and contract adiabatically and reversibly, no heat transfer will take place, and hence the thermoacoustic effects will not occur. On the other hand, if
If \( r_h < \delta_n \), a perfect thermal contact condition between the gas and the walls is obtained, and the porous medium is called a "regenerator", whereas if \( r_h \sim \delta_n \), the situation is referred to an imperfect thermal contact because there is a time delay between gas motion and heat transfer, and the porous medium is called a "stack". Standing wave thermoacoustic devices require the imperfect thermal contact condition to work, while the traveling wave thermoacoustic devices can operate under the perfect thermal contact circumstance.[3, 4] The former applies Brayton cycle on the working gas, consists of two reversible adiabatic steps and two irreversible isobaric steps, whereas the latter is based on Stirling cycle which is inherently reversible, and therefore their thermal efficiencies can easily surpass that of standing wave devices.[6]

Because the thermal performance (efficiency) of the traveling-wave refrigerators is higher than those of standing-wave refrigerators, and due to the importance of the role of regenerator hydraulic radius in thermoacoustic devices as described above, we conduct an experiment on a traveling-wave thermoacoustic refrigerator with various hydraulic radius of regenerator. The aim is to experimentally demonstrate the influence of the hydraulic radius on the maximum temperature decrease that can be achieved by the refrigerator. The detail of the experimental setup and method is described in the following section.

2. Experimental Setup and Method
The traveling-wave thermoacoustic refrigerator used in this experiment is driven by a loudspeaker and mainly consists of a looped tube, a resonator tube, a regenerator, and a loudspeaker, as shown schematically in Fig. 1. The looped tube and resonator are made of polyvinyl chloride (PVC) pipes with inner diameter of around 55 mm. The total length of the looped tube is 220 cm, whereas the resonator tube has the length of 150 cm. The working gas inside the looped-tube and resonator is free air at atmospheric pressure and room temperature (\( \approx 28^\circ C \)). A 8-inch loudspeaker is used to provide acoustic wave into the tubes. The sound frequency is controlled by using a digital function generator, while the electrical input power is adjusted by an audio amplifier and calculated from the electric current and voltage measured by a digital ampere-meter A and a digital volt-meter V, respectively. A regenerator, with length of 60 mm, is placed inside the looped tube at a distance of 25 cm from the top left corner of the looped-tube illustrated in Fig. 1 (measured to the center of regenerator). The temperatures at both ends of regenerator, denoted as \( T_h \) and \( T_c \), are measured by using LM35 sensors which are connected to a data logger and a computer.

![Figure 1. The schematic diagram of the experimental setup.](image-url)
Table 1.
Six regenerators with different mesh sizes \((n)\) and wire diameter \((D)\) used in this experiment. The estimated hydraulic radius of the regenerator is \(r_h\).

| Regenerator | \(n\) (inch\(^{-1}\)) | \(D\) (mm) | \(r_h\) (mm) |
|-------------|-----------------|---------|------------|
| 1           | 24              | 0.21    | 0.28       |
| 2           | 30              | 0.18    | 0.22       |
| 3           | 40              | 0.16    | 0.16       |
| 4           | 50              | 0.14    | 0.13       |
| 5           | 60              | 0.14    | 0.10       |
| 6           | 80              | 0.12    | 0.07       |

In order to demonstrate the influence of the hydraulic radius of regenerator on the temperature decrease, six regenerators with different hydraulic radius are used. They are made of stainless-steel wire-mesh screens which are stacked tightly inside the PVC tube. Various hydraulic radius of the regenerators are obtained by using different mesh sizes \((n)\) of the wire-mesh screen for each regenerator. (The mesh size represents the number of holes per unit length, usually expressed in inch\(^{-1}\).) Table 1 presents various mesh sizes which are used in this experiment, together with the wire diameter \(D\), and the hydraulic radius \((r_h)\) of the corresponding regenerators. The \(r_h\) is estimated by using equation [7]

\[
r_h = D \frac{\phi}{4(1 - \phi)},
\]

where \(\phi = 1 - \pi n D / 4\) is the porosity of regenerator. From Table 1 we see that the hydraulic radius of the regenerators vary from 0.07 mm until 0.28 mm.

The experimental procedure consists of two steps as followed. The first step is to determine the operating frequency of the refrigerator. The frequency of sound produced by the loudspeaker is varied by using the digital function generator. For each frequency variation, the temperatures \(T_h\) and \(T_c\) at both ends of regenerator are measured as the refrigerator operated, i.e. as functions of time, and the maximum temperature decrease \(\Delta T_{c,\text{max}}\) that can be achieved by the refrigerator for each frequency is recorded. The operating frequency is then determined as the frequency that gives the largest \(\Delta T_{c,\text{max}}\). The second step is to see the influence of different hydraulic radius \(r_h\) of regenerator on the \(\Delta T_{c,\text{max}}\). The hydraulic radius is varied by using six regenerators which have various hydraulic radius as listed in Table 1. For each \(r_h\) variation, the temperatures \(T_h\) and \(T_c\) are measured as functions of time and the \(\Delta T_{c,\text{max}}\) is noted. In this case, the refrigerator is driven at its operating frequency which has been found in the first step described above.

3. Results and Discussion
For a given regenerator (that is, for a given \(n\) or \(r_h\)), the sound frequency is varied from lowest value of 30 Hz, which is the lower limit of frequency range of the loudspeaker that we used, with the increment of 5 Hz. The typical measurement results of the temperatures \(T_h\) and \(T_c\) as functions of time are shown in Fig. 2 which presents the result for the regenerator 5 (i.e. with \(r_h = 0.10\) mm and \(n = 60\) inch\(^{-1}\)) and frequency of 30 Hz. It reveals, in this case, that at steady state (after operates for about 500 s) the temperature at the cooling point \((T_c)\) is 9.3 °C. It is the lowest temperature that can be achieved by the refrigerator which operates at the condition mentioned above. It means that the maximum temperature decrease \((\Delta T_{c,\text{max}})\) is 18.7 °C recalling that the initial temperature is 28.0 °C.
Figure 2. A typical temperatures history at the both ends of regenerator ($T_h$ and $T_c$). In this case, the regenerator 5 is used with sound frequency of 30 Hz.

Figure 3. The influence of the sound frequency $f$ on the maximum temperature decrease $\Delta T_{c,\text{max}}$. The solid line is a guide to the eye.

Next, Fig. 3 shows the influence of sound frequency on $\Delta T_{c,\text{max}}$ at the cooling point in the thermoacoustic refrigerator with regenerator 5. This figure shows the fact that $\Delta T_{c,\text{max}}$ is significantly depending on the driving frequency, and the largest $\Delta T_{c,\text{max}}$ is obtained by using 30 Hz sound frequency. Because 30 Hz is the lower limit of frequency range of the loudspeaker that we used, it is deduced that the operating frequency of the thermoacoustic refrigerator is 30 Hz. This frequency produces the largest $\Delta T_{c,\text{max}}$ probably because it is close to the resonance frequency of the refrigerator.

The thermal penetration depth $\delta_\kappa$ of gas is dependent on frequency according to the equation[7]

$$\delta_\kappa = \sqrt{\frac{2\kappa}{\omega}}$$

where $\kappa$ is the thermal diffusivity of gas and $\omega$ is the angular frequency of sound wave. Therefore, our thermoacoustic refrigerator with atmospheric air ($\kappa = 2.26 \times 10^{-5} \text{ m}^2/\text{s}$) as working gas and
driven at the operating frequency of 30 Hz will have $\delta_\kappa = 0.49$ mm. Furthermore, if we compare this thermal penetration depth with the hydraulic radius presented in Table 1, we find that the condition of $r_h \ll \delta_\kappa$ occurs in our traveling-wave thermoacoustic refrigerator.

Figure 4 depicts the dependence of $\Delta T_{c,\text{max}}$ on the hydraulic radius of regenerator which is normalized by the thermal penetration depth, $r_h/\delta_\kappa$. It can be seen that there is an optimum value of $r_h/\delta_\kappa$ ratio which gives the largest $\Delta T_{c,\text{max}}$. In this case, the optimum $r_h/\delta_\kappa$ is 0.20 (i.e. $r_h = 0.10$ mm) and the related temperature decrease $\Delta T_{c,\text{max}}$ is 18.7 $^\circ$C. This fact indicates that the use of regenerator with $r_h$ about $1/5\delta_\kappa$ yields the most effective heat pumping in this thermoacoustic refrigerator. When $r_h < 1/5\delta_\kappa$, the channel size is relatively too small ($r_h < 0.10$ mm), resulting in a large viscous dissipation in the regenerator and hence a less effective heat pumping process along the regenerator. On the other hand, when $r_h > 1/5\delta_\kappa$ (that is, for the right branch of the curve in Fig. 4), the channel is relatively wide ($r_h > 0.10$ mm), the distance of gas parcels to channel wall is too far, thus the heat exchange between the gas and channel wall does not effectively occur, and the cooling process is not optimal.

4. Conclusion and Future Work

The dependence of temperature decrease on the hydraulic radius of regenerator in a loudspeaker-driven traveling-wave thermoacoustic refrigerator has been experimentally demonstrated. The experimental results show that there is an optimum hydraulic radius which gives the largest temperature decrease. It is found that the optimum hydraulic radius is 0.20 times thermal penetration depth of the working gas. In this thermoacoustic refrigerator, in which the atmospheric air is used as the working gas, the optimum hydraulic radius is 0.10 mm, and the related temperature decrease is 18.7 $^\circ$C (i.e. from 28.0 $^\circ$C down to 9.3 $^\circ$C). This result implies that we need to design a regenerator with hydraulic radius of around 0.20 thermal penetration depth in order to get the lowest temperature decrease in a traveling wave thermoacoustic refrigerator.

Improving the performance of the thermoacoustic refrigerator is very important, and having an optimum hydraulic radius of the regenerator is one of the keys. The next effort that could be taken in order to upgrade the performance is by employing high pressure working gas. In this case, in designing a proper regenerator (with optimum hydraulic radius), we should be aware that the gas pressure will affect the operating (resonance) frequency and thermal penetration depth.
depth of the working gas [8].

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