Finite strain transient creep of D16T alloy: identification and validation employing heterogeneous tests

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Abstract. A cyclic creep damage model, previously proposed by the authors, is modified for a better description of the transient creep of D16T alloy observed in the finite strain range under rapidly changing stresses. The new model encompasses the concept of kinematic hardening, which allows us to account for the creep-induced anisotropy. The model kinematics is based on the nested multiplicative split of the deformation gradient, proposed by Lion. The damage evolution is accounted for by the classical Kachanov-Rabotnov approach. The material parameters are identified using experimental data on cyclic torsion of thick-walled samples with different holding times between load reversals. For the validation of the proposed material model, an additional experiment is analyzed. Although this additional test is not involved in the identification procedure, the proposed cyclic creep damage model describes it accurately.

1. Introduction
Numerous experimental observations carried out on metals subjected to elevated temperatures indicate that creep is a loading history dependent anisotropic phenomenon [1, 2, 16]. An accurate modelling of the creep behaviour is important for the analysis of various metal forming applications and for the safety analysis of high-temperature structural components operating under cyclic loading [6]. A creep-induced anisotropy has to be taken into account in the analysis of creep under rapidly changing loadings. One of the most popular approaches to the description of transient creep occurring shortly after load changes is based on backstresses (see, among others, [14, 11, 25, 26]). Backstresses are stress-like quantities which represent internal stress fields, acting on the microstructural level. In metals, backstresses may represent a resistance to dislocation motion caused by dislocation pile-ups [25] (atomistic scale), while other backstresses are caused by plastic strain incompatibilities between grains (mesoscopic scale) [4]. Within the phenomenological approach, the evolution of backstresses is typically assumed in a hardening/recovery format. The hardening is governed by the creep strain rate. The recovery can be strain-controlled, also known as dynamic recovery (cf. [15]). Alternatively, it can be temperature-controlled, which is also known as static recovery (cf. [14, 11, 25]). In the material model used here, both types of recovery are combined (see [23] for the justification of this combined approach).

There is only a small number of publications dealing with finite strain creep under rapidly changing loadings. Although different phenomenological modelling approaches to the kinematic
hardening can be used, we employ here the multiplicative split of the deformation gradient; it has several advantages, as discussed in [20]. More specifically, we employ a nested multiplicative split proposed in [13]. The seminal idea of the nested split was successfully used for metal plasticity with nonlinear kinematic hardening [19, 3, 12] as well as its extensions to shape memory alloys [8], thermo-plasticity [18], ductile damage [21], evolution of microstructure [24] and other physically nonlinear phenomena. One of the goals of the current study is to promote this approach by presenting practical identification and validation procedures. Toward that end, experiments on deformation of D16T aluminium alloy (similar to AlCuMg2 and 24ST4) are considered. Here we focus on heterogeneous experiments, since they can be performed under more general conditions than homogeneous tests. In the particular case of torsion tests, experiments on thick-walled tubes allow one to induce larger strains than in experiments on thin-walled samples. However, an improved flexibility of the heterogeneous testing comes at the cost of increased computational efforts during the identification of parameters. In the current study, the torsion tests are simulated numerically using the nonlinear finite element method (direct problem). The material parameters are identified by the minimization of a least square error functional, reflecting the deviation of the FEM simulation results from the experimental data (inverse problem). After the parameters are identified, they are validated by simulating an additional torsion test. A good correspondence between experimental and theoretical results is observed both in identification and validation.

2. Material model of cyclic creep in the finite strain range

The material model used here is a slight modification of the model proposed in [23]. Here, for simplicity, we consider isothermal processes only.\(^1\) Just as in [23], we employ the Rabotnov damage variable \(\omega \in [0, 1]\). Here, \(\omega = 0\) and \(\omega = 1\) characterize respectively the virgin state and the fully destroyed state [9, 17]. A rheological motivation of the model is shown in Figure 1a. This motivation represents an idealized device which is build of two elastic springs, two viscous dashpots and one rate-independent dashpot. The model kinematics is based on the nested multiplicative split of the deformation gradient, originally proposed by Lion in [13] for metal plasticity. The deformation gradient \(\mathbf{F}\) is a product of a creep-related part \(\mathbf{F}_{\text{cr}}\) and an elastic part \(\mathbf{F}_{\text{e}}\); the creep part itself is decomposed multiplicatively into a certain dissipative part \(\mathbf{F}_{\text{ii}}\) and a conservative part \(\mathbf{F}_{\text{ie}}\)

\[
\mathbf{F} = \hat{\mathbf{F}}_{\text{e}} \mathbf{F}_{\text{cr}}, \quad \mathbf{F}_{\text{cr}} = \hat{\mathbf{F}}_{\text{ie}} \mathbf{F}_{\text{ii}}. \tag{1}
\]

The multiplicative decompositions are summarized in a commutative diagram shown in Figure 1b.

In order to formulate the constitutive equations on the reference configuration, we introduce the right Cauchy-Green tensor \(\mathbf{C}\), the right Cauchy-Green tensor of creep \(\mathbf{C}_{\text{cr}}\), and the inelastic right Cauchy-Green tensor of substructure \(\mathbf{C}_{\text{ii}}\)

\[
\mathbf{C} := \mathbf{F}^T \mathbf{F}, \quad \mathbf{C}_{\text{cr}} := \mathbf{F}_{\text{cr}}^T \mathbf{F}_{\text{cr}}, \quad \mathbf{C}_{\text{ii}} := \mathbf{F}_{\text{ii}}^T \mathbf{F}_{\text{ii}}. \tag{2}
\]

Using these quantities, the second Piola-Kirchhoff stress \(\mathbf{T}\) is computed through (cf. [23])

\[
\mathbf{T} = (1 - \omega)^{n_e} \left[ \frac{k}{10} \left( (\det \mathbf{C})^{5/2} - (\det \mathbf{C})^{-5/2} \right) \mathbf{C}^{-1} + \mu \mathbf{C}^{-1} (\mathbf{C} \mathbf{C}_{\text{cr}}^{-1} \mathbf{D}) \right], \tag{3}
\]

where \(k > 0\) and \(\mu > 0\) are the elasticity parameters of the virgin material, \(n_e > 0\) is a material parameter governing the deterioration of elastic properties;\(^2\) the symbol \((\cdot)^{\text{th}}\) stands for the

\(^1\) Thus, we assume that the temperature is constant in space and time. The model can be generalized to the thermo-mechanical case using the procedure discussed in [18].

\(^2\) In the classical Kachanov approach, \(n_e = 1\).
deviatoric part of a tensor, by the overline (·) we denote the unimodular part of a tensor. The
used in (3) ansatz for the hydrostatic component of the stress complies with basic plausibility
restrictions (cf. [7]); if necessary, it can be replaced by any alternative. Next, a backstress
operating on the reference configuration is computed by
\[ \tilde{X} = (1 - \omega)^n c \frac{C_{cr}^{-1} (C_{cr} C_{ii}^{-1})^D}{2}, \]
where \( c \geq 0 \) is a constant pertaining to the intact material. Further, an effective stress tensor,
which essentially governs the creep, is defined by the equation
\[ \tilde{\Sigma} := C_T - C_{cr} \tilde{X}. \]
Using it, we define the magnitude of the driving force:
\[ \tilde{\mathcal{F}} := \mathcal{N}(\tilde{\Sigma}^D), \quad \text{where} \quad \mathcal{N}(A) := \sqrt{\text{tr}(A^2)}. \]
A Mandel-like backstress on the reference configuration is introduced through
\[ \tilde{\Xi} := C_{cr} \tilde{X}. \]
The evolution of \( C_{cr} \) is governed by the flow rule of \( J_2 \) type
\[ \dot{C}_{cr} = 2 \frac{\lambda}{\delta} \tilde{\Sigma}^D C_{cr}, \]
where \( \lambda \geq 0 \) is the creep strain rate. The material rate \( \dot{C}_{ii} \) is given by (cf. [23])
\[ \dot{C}_{ii} = (\kappa_{\text{dynam}} \mathcal{N}(C_{cr}^{-1} \dot{C}_{cr}) + 2 \kappa_{\text{stat}}) \tilde{\Xi}^D C_{ii}, \]
where \( \kappa_{\text{dynam}} \) and \( \kappa_{\text{stat}} \) are respectively the parameters of static and dynamic recovery. Note
that these parameters do not depend on the damage \( \omega \). Although this assumption is very simple,
it allows one to achieve a good accuracy in some applications (cf. the next section). Probably,
one of the most simple assumptions governing the creep rate \( \lambda \) is provided by the Norton creep
claw combined with Kachanov’s approach to creep damage
\[ \lambda = A (1 - \omega)^{-m} (\tilde{\mathcal{F}} / \sigma_0)^n, \]
where \( A \geq 0, m \geq 0, \) and \( n \geq 0 \) are material parameters; \( \sigma_0 := 1 \leq 1 \) MPa is not a material constant.
The evolution of the damage variable is described by the following modification of the classical
Kachanov-Rabotnov relation (cf. [10, 2])
\[ \dot{\omega} = B (1 - \omega)^{-l} \left( \mathcal{N}((C_T)^D) / \sigma_0 \right)^k, \]
where $B \geq 0$, $l \geq 0$, and $k \geq 0$ are material constants. Here, in contrast to the damage evolution rule used in [23], the damage growth is governed by the norm of the deviatoric Kirchhoff stress, which is equal to $\|\mathbf{CT}^D\|$. As will be seen from the next section, this assumption allows us to obtain reasonable results in a wide range of cyclic creep scenarios. In order to close the system of constitutive equations, the following initial conditions need to be specified:

$$C_{cr}|_{t=0} = C_{cr}^0, \quad C_{ii}|_{t=0} = C_{ii}^0, \quad \omega|_{t=0} = \omega^0.$$  \hspace{1cm} (12)

Here, $\omega^0 \in [0, 1)$ is the initial damage. Note that an appropriate choice of the initial values $C_{cr}^0$ and $C_{ii}^0$ allows one to change the reference configuration and to introduce initial anisotropy.

For $n_e = 1$, the introduced model is a special case of the creep damage model presented in [23] with exception of the damage evolution rule (11). Similar to the proof presented in [23], it is possible to show that this model is thermodynamically consistent and w-invariant. Obviously, this model does not account for the damage-induced dilatation of the material due to void nucleation and growth. This drawback can be overcome using the modelling steps described in [21].

The numerical implementation of this model is essentially based on the efficient time stepping scheme presented in [22]. The algorithm exactly preserves the incompressibility condition, and it is first order accurate. In the damage-free case, it is unconditionally stable. The model is implemented into MSC.MARC employing the user-defined material subroutine via the Hypela2 interface.

### 3. Parameter identification and validation

In this section we use a series of torsion tests carried out in [5] on the D16T aluminium alloy (similar to AlCuMg2 and 24ST4) at 250°C. The experiments are conducted on thick-walled tubular samples. The advantage of this experimental set-up is that it allows one to achieve large strains without loss of stability and without the undesired influence of friction. The sample dimensions in the gage area are as follows: sample length $L = 70$ mm, inner radius $r_i = 5$ mm, outer radius $r_0 = 10$ mm. Since the stress distribution within the sample is heterogeneous, the analysis of the experiment requires a solution of a boundary value problem. The corresponding FEM mesh and boundary conditions are the same as discussed in [23].

The initial state of the material is assumed to be stress-free, undeformed, and isotropic. Therefore we set the following initial conditions

$$C_{cr}|_{t=0} = C_{cr}^0 = \mathbf{1}, \quad C_{ii}|_{t=0} = C_{ii}^0 = \mathbf{1},$$  \hspace{1cm} (13)

where $\mathbf{1}$ is the identity tensor. The material parameters are identified from two different torsion experiments. In both tests, a piecewise constant torque is applied with $|M| = 67.75$ N·m. In the first test, the applied torque is reversed every 24 hours, in the second every 96 hours. The experimentally measured twist angle of the sample is plotted in Figure 2 for different holding times between load reversals. Some of the parameters are determined by general considerations (cf. [23]). The remaining parameters are identified by the minimization of the least square error functional, reflecting the deviation of the simulated twist angle from the corresponding experimental result. The identification results are summarized in Table 1. There, parameters which appeared in the minimization problem are marked by the asterisk *. As can be seen from Figure 2, the obtained set of material parameters provides a good accuracy.

Now we proceed to the validation of the parameters from Table 1. An additional cyclic creep torsion test with $|M| = 67.75$ N·m and holding times of 48 hours between load reversals is simulated using these parameters. The corresponding experimental data and simulation results are shown in Figure 3. Although these data are ignored during parameter identification, a very good correspondence between simulation and experiment is observed.

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Table 1. Material parameters of the D16T alloy.

| parameter | value            | brief explanation                          | equation |
|-----------|------------------|--------------------------------------------|----------|
| $k$       | 73500 MPa        | bulk modulus of intact material            | (3)      |
| $\mu$    | 28200 MPa        | shear modulus of intact material           | (3)      |
| $c$       | 20000 MPa        | shear modulus of intact substructure       | (4)      |
| $n_e$     | 35.24 [-]        | impact of damage on the elastic properties | (3), (4) |
| $A$       | $3.574 \cdot 10^{-13}$ h$^{-1}$ | parameter of Norton’s law | (10)     |
| $n$       | 5 [-]            | parameter of Norton’s law (exponent)       | (10)     |
| $m$       | 68.49 [-]        | impact of damage on the creep rate         | (10)     |
| $\gamma_{\text{dyam}}$ | 0.0586 MPa$^{-1}$ | dynamic recovery coefficient               | (9)      |
| $\gamma_{\text{stat}}$ | 0.0 MPa$^{-1}$h$^{-1}$ | static recovery coefficient               | (9)      |
| $B$       | $3.428 \cdot 10^{-13}$ h$^{-1}$ | damage evolution parameter | (11)     |
| $l$       | 5 [-]            | damage evolution parameter                 | (11)     |
| $k$       | 5 [-]            | damage evolution parameter                 | (11)     |
| $\omega_0$ | 0.01 [-]       | initial damage                             | (12)     |

Figure 2. Cyclic torsion of thick-walled tubular specimens with different holding times: experimental data from [5] and corresponding simulation results visualizing the identified material parameters.

4. Discussion and conclusions

Intuitively, the torsion test with holding times of 48 hours is located “between” the tests with holding times of 24 and 96 hours. In other words, we have checked the ability of the
Figure 3. Validation of the material model using additional experiment on cyclic torsion of a thick-walled tubular specimen: experimental data from [5] and simulation results using the previously identified material parameters.

In the current study a practical example of parameter identification and validation is presented, basing on the evaluation of heterogeneous test data. It is shown that a rather complex finite strain cyclic creep damage model can be calibrated using relatively simple tests.

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