A random-signal approach to robust radar detection

Angelo Coluccia, Senior Member, IEEE and Giuseppe Ricci, Senior Member, IEEE
Dipartimento di Ingegneria dell’Innovazione
Università del Salento, Via Monteroni, 73100 Lecce, Italy.
E-Mail: name.surname@unisalento.it.

Abstract—The paper presents a novel family of robust radar receivers for the problem of detecting the possible presence of a coherent return from a given cell under test. The idea is to introduce a random signal in addition to the nominal steering vector, with a given structure, but whose entity is estimated from the data (hence can also be zero). While in the classical formulation the target signature is present only in the mean of the likelihood function, and recent work investigated tests based on the sole covariance matrix, the proposed approach splits the target signature between mean and covariance. In particular, the complex amplitude is modeled as an unknown deterministic parameter while the random part of the signature is included in the covariance to promote robustness. Preliminary results for different choices of the design matrix (that is, of the structure of the covariance matrix of the random signal) show that a palette of behaviors can be obtained, with different trade-offs between loss under matched conditions and robustness to mismatches.

Index Terms—adaptive radar detection, generalized likelihood ratio test (GLRT), robust receivers, array processing

I. INTRODUCTION

The well-known problem of detecting the possible presence of a coherent return from a given cell under test (CUT) in range, doppler, and azimuth, is classically formulated as the following hypothesis testing problem:

\[
\begin{cases}
H_0 : & z = n \\
H_1 : & z = \alpha v + n
\end{cases}
\]

where \( z \in \mathbb{C}^{N \times 1} \), \( n \in \mathbb{C}^{N \times 1} \), and \( v \in \mathbb{C}^{N \times 1} \) denote the received vector, the corresponding noise term, and the known space-time steering vector of the useful target echo. The noise term is commonly modeled according to the complex normal distribution with zero mean and unknown Hermitian positive definite matrix \( C \), denoted by \( \mathcal{CN}_N(0, C) \). Modeling \( \alpha \) has an unknown deterministic parameter returns a complex normal distribution for \( z \) under both hypotheses; the non-zero mean of the received vector under \( H_1 \) makes it possible to discriminate between the two hypotheses resorting to the GLRT for the test

\[
\begin{cases}
H_0 : & z \sim \mathcal{CN}_N(0, C) \\
H_1 : & z \sim \mathcal{CN}_N(\alpha v, C)
\end{cases}
\]  

(1)

In the pioneering paper by Kelly [1], the generalized likelihood ratio test (GLRT) is derived for (1), assuming a set of independent and identically distributed training data \( z_1, \ldots, z_K \), independent also of \( z \), free of signal components, and sharing with the CUT the statistical characteristics of the noise, is available.

In [2] the performance of such a detector is assessed when the actual steering vector is not aligned with the nominal one. Later, many works have addressed the problem of enhancing either the selectivity or the robustness of GLRT-based detectors to mismatches. In particular, the adaptive matched filter (AMF) [3], which solves (1) following a two-step approach, is a prominent example of robust detector, while the adaptive coherence estimator (ACE, also known as adaptive normalized matched filter) [4], [5], [6] and Kelly’s detector are selective receivers, i.e., they have excellent rejection capabilities of signals arriving from directions different from the nominal one [7]. Other detectors try to explicitly take into account rejection capabilities at the design stage, namely based on the adaptive beamformer orthogonal rejection test (ABORT) [8] or related ideas [9], [10], subspace detection [11], [12], [13], [14], [15], and cone-shaped constraints [16], [17], [18]. In the ABORT approach the idea is to modify the null hypothesis, by introducing (in addition to the noise term) a fictitious signal which is somehow orthogonal to the assumed target’s signature. This is meant to make the detector less inclined to declare a detection, as the null hypothesis will be more plausible than in the case of noise-only. In [19] randomness is introduced in the fictitious signal to obtain a tunable receiver.

On the other hand, modeling \( \alpha \) as a complex normal random variable with zero mean and variance \( \|\alpha\|^2 = E[|\alpha|^2] \), returns a zero-mean complex normal distribution for \( z \) under both hypotheses \( H_0 \) and \( H_1 \); more precisely, \( E[z z^H | H_i] = C + \delta_{i1} |\alpha|^2 vv^H \), \( i = 0, 1 \), where \( \delta_{i1} \) denotes the Kronecker delta, hence

\[
\begin{cases}
H_0 : & z \sim \mathcal{CN}_N(0, C) \\
H_1 : & z \sim \mathcal{CN}_N(0, C + |\alpha|^2 vv^H)
\end{cases}
\]

(2)

This is a “second-order” approach to target modeling: the presence of a useful signal (\( H_1 \) hypothesis) is modeled in terms of a rank one modification of the noise covariance matrix. Interesting properties in terms of either robustness or selectivity can be obtained by considering this random (instead of deterministic) model for the target signal, depending on the way problem (2) is solved and possibly on the presence of a fictitious signal under the null hypothesis [20], [21], [22].

Motivated by the above results, we investigate the potential of a detector that solves the hypothesis testing problem

\[
\begin{cases}
H_0 : & z = n \\
H_1 : & z = \alpha v + \theta + n
\end{cases}
\]

(3)

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where \( \alpha \) is an unknown deterministic parameter and \( \theta \sim \mathcal{CN}_N(0, \nu \Sigma) \) represents the random signal component. The resulting hypothesis testing problems turns out to be

\[
\begin{align*}
H_0 : & \quad z \sim \mathcal{CN}_N(0, C) \\
H_1 : & \quad z \sim \mathcal{CN}_N(\alpha v, C + \nu \Sigma)
\end{align*}
\]

where the structure term \( \Sigma \) in the covariance matrix is known, while the power term \( \nu \) is a non-negative unknown real. The idea is that a mismatch is allowed around the nominal steering vector \( v \), with a given structure, but whose type is estimated from the data (hence can also be zero)\(^1\).

For this problem, different detection strategies can be conceived, in particular by considering the one-step and two-step GLRT approaches under different choices of the design matrix \( \Sigma \). In this paper we present the general derivation of the two-step GLRT. To this end, we first derive the GLRT for known covariance matrix \( C \) and then we obtain an adaptive detector by replacing the unknown matrix and the threshold with proper estimates based upon the secondary data. The performance assessment, carried out by Monte Carlo simulation, shows that the proposed solution is very robust, with somewhat varying performance according to the choice of \( \Sigma \), while keeping a reduced loss under matched conditions.

It is important to stress the difference between the proposed approach and the ACE detector. More precisely, considering the latter as the GLRT to detect a coherent signal in partially-homogeneous noise [5], we highlight that therein the parameter \( \nu \) represents the possible mismatch between the power of the noise in the data under test and that of secondary data. Herein, instead, \( \nu \) enters the characterization of a possible random mismatch between the actual and the nominal useful signal. For this reason, we have a different structure of the covariance matrix and, in addition, \( \nu \) is not present under the \( H_0 \) hypothesis. This difference in the formulation of the hypotheses testing problem makes our detector more robust than Kelly’s detector [1] in presence of mismatched signals while it is well-known that the ACE is more selective than the latter (under the same operating conditions).

II. DERIVATION OF AN ADAPTIVE DETECTOR RESORTING TO THE TWO-STEP GLRT DESIGN PROCEDURE

To obtain the two-step GLRT, we first derive the GLRT under the assumption that the matrix \( C \) is known. To this end, consider the binary hypothesis testing problem (4) where \( \nu \geq 0 \) and \( \alpha \in \mathbb{C} \) are unknown quantities while the vector \( \nu \in \mathbb{C}^{N\times1} \), the design matrix \( \Sigma \), and the Hermitian positive definite matrix \( C \) are known. The corresponding GLRT is thus

\[
\Lambda(z, C) = \max_{\nu \geq 0} \max_{\alpha \in \mathbb{C}} \frac{f(z|\nu, \alpha, H_1)}{f(z|H_0)} \quad \frac{H_1}{H_0} > \eta
\]

where

\[
f(z|\nu, \alpha, H_1) = \frac{1}{\pi^N \det(C + \nu \Sigma)} e^{-(z - \alpha v)^H(C + \nu \Sigma)^{-1}(z - \alpha v)}
\]

and

\[
f(z|H_0) = \frac{1}{\pi^N \det(C)} e^{-z^HC^{-1}z}
\]

denote the pdf of \( z \) under \( H_1 \) and \( H_0 \), respectively. As to \( \eta \), it is the detection threshold to be set according to the desired probability of false alarm (\( P_{fa} \)). Maximization with respect to \( \alpha \) leads to [23]

\[
\Lambda(z, C) = \max_{\nu \geq 0} \max_{\alpha \in \mathbb{C}} \frac{e^{-(z - \alpha v)^H(C + \nu \Sigma)^{-1}(z - \alpha v)}}{\det(C + \nu \Sigma)^{1/2}} e^{-\frac{1}{2} \| z_{\nu}^H P_{\nu v} z_{\nu} \|^2}
\]

\[
\max_{\nu \geq 0} \frac{1}{\det(C)} e^{-\frac{1}{2} \| z_{\nu}^H P_{\nu v} z_{\nu} \|^2}
\]

where \( z_{\nu} = (C + \nu \Sigma)^{-1/2} z \), \( v_{\nu} = (C + \nu \Sigma)^{-1/2} v \), and

\[
P_{\nu v} = I_N - (C + \nu \Sigma)^{-1/2} \frac{v v^H}{v^H(C + \nu \Sigma)^{-1} v} (C + \nu \Sigma)^{-1/2}.
\]

Notice also that

\[
\| P_{\nu v} z_{\nu} \|^2 = z^H (C + \nu \Sigma)^{-1} z - \frac{| z^H (C + \nu \Sigma)^{-1} v |^2}{v^H(C + \nu \Sigma)^{-1} v}.
\]

A closed-form for the maximum in (8) is not available and, hence, its maximum has to be computed numerically. To this end, the numerator of (8) can be re-written in a different form. In particular, we can write

\[
C + \nu \Sigma = C^{1/2} \left( I_N + \nu \Sigma^{-1/2} \Sigma C^{-1/2} \right) C^{1/2}
\]

\[
= Q (I_N + \nu \Phi) Q^H
\]

with \( Q = C^{1/2} W \), where \( W \) is unitary and \( \Phi \) diagonal and have been obtained from the eigendecomposition

\[
C^{-1/2} \Sigma C^{-1/2} = W \Phi W^H.
\]

Thus, it follows that

\[
\det(C + \nu \Sigma) = \prod_{i=1}^{N} (1 + \nu \phi_i)
\]

where \( \phi_i \) are the (diagonal) elements of \( \Phi \). Moreover

\[
(C + \nu \Sigma)^{-1} = Q^{-H} (I_N + \nu \Phi)^{-1} Q^{-1}
\]

which has the great advantage that, clearly, \( (I_N + \nu \Phi)^{-1} \) is a diagonal matrix with elements \( 1/(1 + \nu \phi_i) \). With these expressions for the determinant and inverse, the maximization over \( \nu \) requires much less computations than computing directly

\[
\det(C + \nu \Sigma) \text{ and } (C + \nu \Sigma)^{-1}
\]

each time.

\(^1\)Notice that for deterministic \( \alpha \) and \( \theta \sim \mathcal{CN}_N(0, \sigma^2 \Sigma) \), the GLRT for problem (3) is equivalent to the case in which the \( H_1 \) hypothesis is instead \( z = \alpha(v + \theta) + n \), due to the invariance of the domain over which the maximization is performed. Interestingly, in this case we have

\[
\begin{align*}
H_0 : & \quad z \sim \mathcal{CN}_N(0, C) \\
H_1 : & \quad z \sim \mathcal{CN}_N(\alpha^2 \sigma^2 \Sigma)
\end{align*}
\]

i.e., the signal “signature” is present both in the first-order (mean) and second-order (covariance) statistics. Thus, the proposed model (3) represents a possible generalization of (1) and (2).
We consider data $z$ with the sample covariance matrix based on the secondary ratio (SNR), defined as

$$\nu = \frac{I_N + \hat{\nu}\Sigma^{-1}}{\nu} \left[ \frac{z^H(C + \nu\Sigma)^{-1}z + \frac{1}{\nu}H(C + \nu\Sigma)^{-1}v^2}{v^H(C + \nu\Sigma)^{-1}v} \right]^{1/2}$$

and $\hat{\nu}$ is the estimate of $\nu \geq 0$. It can be also re-written as

$$\Lambda(z, C) = \frac{1}{\det(C + \hat{\nu}\Sigma)} e^{-z^H(C + \hat{\nu}\Sigma)^{-1}z - \frac{1}{\nu}H(C + \nu\Sigma)^{-1}v^2}$$

Such a detector can be made fully adaptive by replacing $C$ with the sample covariance matrix based on the secondary data $z_1, \ldots, z_K$ (independent of $z$), that is

$$\hat{C} = \frac{1}{K} S$$

where $S$ is the scatter matrix, i.e.,

$$S = \sum_{k=1}^{K} z_kz_k^H.$$  \hfill (10)

Although the present general derivation of the two-step GLRT contains a final maximization that is not in closed-form, it only requires a simple one-dimensional search. It is also remarkable that, for specific choices of $\Sigma$, one-step and two-step GLRT-based solutions can be expressed in closed-form; such detectors are part of our ongoing work. As an example, setting $\Sigma = C$ yields the equivalent closed-form statistics

$$\Lambda^*(z, S) = \left\{ \begin{array}{ll} \frac{C^{(1/2)}}{\sqrt{\pi}} e^{\frac{v^H S^{-1}v}{2}} z^H S^{-1} z - \frac{|z^H S^{-1}v|^2}{v^H S^{-1} v} > \frac{N}{\nu} \\
|z^H S^{-1}v|^2 \end{array} \right.$$

whose derivation is sketched in Appendix. In the following, we perform a preliminary analysis for different choices of the design matrix $\Sigma$, so as to understand the main trade-off at play in the proposed random-signal approach.

### III. Performance Analysis

For the performance analysis, we resort to Monte Carlo simulation of a radar scenario with $N = 16$ pulses. The target signature is $v = [1 \ e^{2\pi f_d} \ldots \ e^{2\pi(N-1)f_d}]^T$ with normalized Doppler frequency $f_d = 0.08$, a value such that the target competes with noise. The target amplitude $\alpha$ is generated deterministically according to the signal-to-noise ratio (SNR), defined as

$$\text{SNR} = |\alpha|^2 v^H C^{-1} v.$$  \hfill (9)

We consider $K = 32$ secondary data and generate $z$, under $H_0$, and the $z_i$s, under both hypotheses, as (independent) random vectors ruled by a zero-mean, complex Gaussian distribution. As to the covariance matrix, we adopt as $C$ the sum of Gaussian-shaped clutter covariance matrix and a white (thermal) noise 10 dB weaker, i.e., $C = R_c + \sigma_n^2 I_N$ with $[R_c]_{m_1, m_2} \propto \exp(-2\pi^2 \sigma_f^2 (m_1 - m_2)^2)$ and $\sigma_f^2 = 0.01$.

We resort to 100/$P_{fa}$ independent trials to evaluate the thresholds necessary to ensure a preassigned value of $P_{fa}$ and to 10$^3$ independent trials to compute the probabilities to decide for $H_1$ when a useful signal is present ($P_d$).

A comparison is performed against the natural competitors for the problem at hand, which are designed based on the assumption that $\alpha$ is a deterministic parameter; obvious references for robust receivers are the AMF [3]

$$t_{\text{AMF}}(z, S) = \frac{|z^H S^{-1}v|^2}{v^H S^{-1}v}$$

and the (adaptive) energy detector (ED)

$$t_{\text{ED}}(z, S) = z^H S^{-1} z.$$  \hfill (11)

As benchmark for the performance under matched conditions, the most natural references are the Kelly’s detector [1]

$$t_{\text{Kelly}}(z, S) = \frac{|z^H S^{-1}v|^2}{v^H S^{-1}v (1 + z^H S^{-1}z)}$$

and the ACE [5]

$$t_{\text{ACE}}(z, S) = \frac{|z^H S^{-1}v|^2}{v^H S^{-1}v z^H S^{-1}z}.$$  \hfill (10)

To assess the robustness of the proposed detectors, we simulated a target with a mismatched steering vector having normalized Doppler frequency $f_d(1 + \delta/N)$, for $\delta \in \{0, 0.1, 0.2, 0.3, 0.4, 0.5\}$. Four different choices of $\Sigma$ are analyzed, two full-rank and two rank-one:

- $\Sigma = I_N$, a "neutral" choice (in the original signal space);
- $\Sigma = \hat{C} = S/K$, which can be interpreted as a "neutral" choice in the (quasi-)whitened signal space;
are anyway better than the ED.

The proposed detectors, being all based on a two-step GLRT approach, are less powerful than the Kelly’s detector.

Fig. 1. The proposed detectors, being all based on a two-step GLRT approach, are less powerful than the Kelly’s detector.

SNR [dB]
10 12 14 16 18 20 22
Pd
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
KELLY
ACE
AMF
proposed with Σ = IN
proposed with Σ = ˆC
Energy Detector
proposed with Σ = vvH
proposed with Σ = vmvH
m

Fig. 2. $P_d$ vs SNR in case of mismatched steering vector, for $\delta = 0.1$.

$\Sigma = vv^H$, which enforces the target signature in both the mean and covariance, hence it can be regarded as a combination of the two modeling approaches (1) and (2);

$\Sigma = \psi m^H \psi^H$ where $\psi m$ is a mismatched steering vector, aimed at making the $H_1$ hypothesis more plausible, hence the detector more robust (a mismatch of $0.2/N$ has been set in the simulations).

Results under matched conditions ($\delta = 0$) are reported in Fig. 1. The proposed detectors, being all based on a two-step GLRT approach, are less powerful than the Kelly’s detector, as one might expect; however, the two detectors with rank-one $\Sigma$ are more powerful than the ACE for moderate to high SNRs, and very close to the Kelly’s detector in that region. The two detectors with full-rank $\Sigma$ exhibit an higher loss, but are anyway better than the ED.

Fig. 2 reports the results for a slight mismatch of the steering vector ($\delta = 0.1$). The performance are still similar to the matched case, but the gap with the ACE starts to widen and becomes more evident in Fig. 3, which reports the case $\delta = 0.2$. Interestingly, already at this moderate mismatch the detectors with rank-one $\Sigma$ outperform the competitors, while the detectors with full-rank $\Sigma$ are in between the AMF and the ED, depending on the choice of the matrix.

As mismatch increases, the selective receivers (Kelly and ACE) start to miss the target: in Fig. 4, with $\delta = 0.3$, the different behaviors of robust and selective receivers become very marked. Interestingly, the proposed detector with $\Sigma = \psi m^H \psi_m^H$ exhibits the strongest robustness, while the detector with $\Sigma = vv^H$ for higher mismatches apparently performs similarly to the AMF. This becomes more evident for $\delta = 0.4$ and $\delta = 0.5$, as shown in Figs. 5-6, as well as the fact that the two proposed detectors with full-rank $\Sigma$ show an intermediate
behavior between AMF and ED.

From the preliminary analysis above, it emerges that the proposed approach yields a family of robust receivers, whose performance in terms of loss under matched conditions and robustness to mismatches can be tuned by acting on the design matrix $\Sigma$. The four choices analyzed above provide a palette of behaviors, which opens to different possible applications.

IV. CONCLUSIONS

In this paper, we have proposed a novel family of robust radar receivers for the problem of detecting the possible presence of a coherent return from a given cell under test. To this end, we introduced a random signal in addition to the nominal steering vector, with a given structure, but whose entity is estimated from the data (hence can also be zero).

The test is only loosely reminiscent of the one that leads to the ACE detector. In fact, in accordance with the discussion in Sec. I about the difference between the proposed approach and the ACE detector, the performance assessment has shown that the different meaning of the parameter $\nu$ — in the ACE representing the possible mismatch between the power of the noise in the data under test and that of secondary data, while in the proposed approach characterizing the entity of a possible random mismatch between the actual and the nominal useful signal (and $\nu$ is not present under the $H_0$ hypothesis) — leads to detectors with opposite performance. By acting on the design matrix $\Sigma$, the degree of robustness and loss under matched conditions can be traded-off.

While the general expression of the two-step GLRT given here includes a one-dimensional numerical maximization, one-step and two-step GLRT-based solutions can be expressed in closed-form for specific choices of $\Sigma$. This is our ongoing work; as an instance, the case $\Sigma = C$ has been reported here.

APPENDIX

Let us consider the proposed hypothesis testing problem for the case $\Sigma = C$, i.e.

$$
\begin{align*}
H_0 & : \quad z \sim \mathcal{CN}_N(0, C), \\
H_1 & : \quad z \sim \mathcal{CN}_N(\alpha v, (1 + \nu)C),
\end{align*}
$$

The corresponding GLRT is

$$
\Lambda(z, C) = \max_{\nu \geq 0} \max_{\alpha \in \mathbb{C}} \frac{f(z|\nu, \alpha, H_1)}{f(z|H_0)} \quad \frac{H_1}{H_0} < \eta \quad (12)
$$

where

$$
f(z|\nu, \alpha, H_1) = \frac{1}{\pi N} e^{-\frac{1}{2} \langle z - \alpha v \rangle^H C^{-1} (z - \alpha v)}
$$

while $f(z|H_0)$ is as in (7). Maximization over $\alpha$ leads to

$$
\Lambda(z, C) = \max_{\nu \geq 0} \frac{1}{(1 + \nu)^N} e^{-\frac{1}{2} \|P_{\tilde{z}} z\|^2} e^{-\nu^H C^{-1} \nu}
$$

where $\tilde{z} = C^{-1/2} z$, $\tilde{v} = C^{-1/2} v$, and

$$
P_{\tilde{z}} = I_N - C^{-1/2} v v^H C^{-1/2} v^H C^{-1} v.
$$

Finally, maximization of the numerator with respect to $\nu$ can be accomplished using the following lemma.

**Lemma 1:** The function

$$
f(\nu) = \frac{1}{(1 + \nu)^N} e^{-\frac{1}{1 + \nu^2}}, \quad a > 0,
$$

admits a unique maximum over $[0, +\infty)$ given by

$$
\max_{\nu \geq 0} f(\nu) = \left\{ \begin{array}{ll}
\left( \frac{\nu}{a} \right)^N e^{-\nu}, & a > N, \\
\nu^{-a}, & \text{otherwise}.
\end{array} \right.
$$

It follows that (12) is given by

$$
\Lambda(z, C) = \max_{\nu \geq 0} \frac{\left( \frac{\nu}{a} \right)^N e^{-\nu}}{\|P_{\tilde{z}} z\|^2} - 1 > 0,
$$

and, finally, its adaptive version can be rewritten as (11).

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