Error analysis in solving prism and pyramid problems

A N Hasanah¹, K Yulianti²

¹Mathematics Education Study Program, Post Graduate School, Universitas Pendidikan Indonesia, Jl. Dr. Setiabudhi No. 229, Bandung 40154, Indonesia
²Department of Mathematics Education, Universitas Pendidikan Indonesia, Jl. Dr. Setiabudhi No. 229, Bandung 40154, Indonesia

*corresponding author: annisanurhasanah@upi.edu

Abstract. This research is descriptive qualitative research. This study aims to analyze students' mistake in solving problems of surface area and volume on the prism and pyramid then to expose the causal factors. The subject of this research was 29 students in 9th class of one junior high school in Bandung, that was chosen by purposive sampling technique. Data was collected from students through validated essay tests and interviews. The data was analyzed by descriptive method. The results show that some of students (1) did not understand the problems, (2) could not represent the prism and pyramid in other form, (3) did some error in using formula of surface area and volume of prism and pyramid, (4) did some error in calculations and (5) had misconceptions. The causal factors were students were not accustomed to identify and write what known and asked when solving a mathematical problem, difficulty in reading information, did not understand concept of the prism and pyramid, carelessness in calculation, haste in answering questions, and lack of effort.

1. Introduction

Mathematics is one of the subject that students learn from the elementary level, due to its functions that are so important. In everyday life, one of its functions is used to solve various problems. Mathematical concepts are also often used in developing various other scientific fields, such as in natural sciences, social sciences, arts, technology and other fields. In addition, mathematics also can develop thinking ability.

Geometry is one of the branches in mathematics. Basically students have known geometry before they enter the formal education. It is because geometry is found in everyday life such as observing lines, plane and space. Relationship between points, lines, angles, and plane are studied in geometry. So, the study of geometry problems had an important place in our lives because it is relevant to our daily lives and very important to learn [1].

Geometry aspects that related to space and plane involves other branches of mathematics such as arithmetics, algebra and statistics. "Geometry is a unifying theme to the entire curriculum of algebraic arithmetical, and statistical concepts" [2]. Thus, indirectly the material about geometry of space became a bridge to understand other branches of mathematics.

Polya constructed theory to solve a problem more systematically, which is called heuristic Polya. The four steps in Heuristic Polya in solving problems are understanding questions, designing solutions, implementing plans, and reviewing answers [3]. By applying the four steps in solving problems, risk of students making mistakes in solving problems would be reduced.
There were several opinions regarding meanings and types of errors in learning. Errors and misconceptions are two things that have similar but have different meanings. Errors can be seen in the text or sentence arrangement, in this case the student's answer sheet, while the misconception is often hidden behind the correct answer. This correct answer is not the answer obtained through the right steps, but a coincidence [4].

Errors in learning can be found in the form of errors or inaccuracies in answering questions [5]. This type of error is then divided into two: systematic errors and non-systematic errors. A systematic error is an error that occurs repeatedly in the same section, while a non-systematic error is an error that does not occur repeatedly. Based on this explanation, a systematic error can be caused by failure to form a complete concept, which is called a misconception.

In solving mathematical problems, Newman's Error Analysis can be used in identifying and analyzing student errors [6]. Error Analysis according Newman is divided into five categories. First, reading error in which students cannot understand the meaning of the problem, it can occur because they do not understand one term or a sentence as a whole. Second, comprehension error in which students can read the words and questions but cannot interpret them as a whole so they cannot decide the problem solving steps. Third, transformation error in which students have understood the question and how to solve the problem that should have been done but cannot identify the operation. Fourth, mathematical processing error in which students can identify the operation of the problem solving steps but do not use them accurately. Fifth, encoding error in which students have found a solution to the problem but cannot express it in the right form [7].

Errors made by students in learning process can be used as a tool in diagnosing learning errors and determining remediation materials. It was revealed by Borasi, "In particular, it has been pointed out that errors can be a powerful learning tool for diagnosing difficulties and consequently direct remediation" [8]. In line with that opinion, revealing the diagnosis of learning difficulties can be done through Error Analysis [9].

If any of the factors that cause students to make mistakes associated with Polya heuristics, then type of errors can be divided as follows: (1) Errors in understanding problems, in which described as students who cannot answer the given problem so that students does not have any meaning or concept and students who does not answers the question, (2) errors in thinking of a plan, in which described as students who can already determine the problem by using some information but have not been able to solve the problem well, (3) errors in implementation of the plan, in which described as students who can already solve the problem and can finish but not solve the problem correctly, (4) errors in review, in which described as students who can answer the questions correctly, can think of a plan and write results but did not check the answers again and (5) misconception. Based on the description that has been explained, the researcher made an Error Analysis on the topic surface area and volume of prism and pyramid in SMP Negeri 40 Bandung. The objective of this research is to investigate whether there were still mistakes made by students at the school while working on the topic issues of surface area and volume of prism and pyramid. The researcher describes the types of errors that occur. In more detail, this study also analyzes the causes of the error. It is hoped that this research can become reference material as to be referred as actual information on mathematics learning, especially the topic of surface area and volume of prism and pyramid.

2. Methods
Selected subject of this research was 29 students who enrolled in the IX D class, SMP Negeri 40 Bandung. Sample was chosen by purposive sampling technique. Purposive sampling technique is a nonprobability sampling technique that researcher used to choose a sample of subjects or units from a population. In other word purposive sampling technique is a non-random selection of participants [10].

This research was done in the first semester of the academic year of 2017-2018. Data was collected from students through essay test in 60 minutes duration. Topic of the validated essay test was prism and pyramid surface and volume. Test was used to obtain data about students’ error in said mathematical problems. Collected data in this research was analyzed by using qualitative technique.
Qualitative technique was a technique analysis done by describing, explaining, and interpreting collected data [11].

The collection of data in this study is completed by using two main instruments, that were main and supporting instruments. The main instrument in this study is a set of questions that has been compiled based on problem solving indicators. The supporting instrument is an interview with some of the subject. Interview conducted i.e., face-to-face interview between researcher and the subject. The interview used in this research is natural interview, in which said interview was designed to bring up opinion the subject [12].

3. Result and Discussion
In this paper, the researcher discuss a sample question. The question can be seen in Table 1

| Original Assessment Material | Translated Assessment Material |
|-----------------------------|-------------------------------|
| Diketahui balok ABCD.EFGH, T adalah titik potong diagonal AH dan DE. Jika panjang rusuk AB = 10 cm, BC = 3cm dan GC = 4cm, carilah volume Limas T.ABCD! | Given cuboid ABCD. EFGH, T is the intersection point of diagonals AH and DE. If AB = 10 cm, BC = 3cm and GC = 4cm, find the volume of pyramid T.ABCD! |

Based on students’ answers, it was obtained that 13 of 29 students (44.8%) could not answer the question. Grouping the number of students who were wrong in completing prism and pyramid surface and volume problems can be seen in Table 2.

| Error                           | The number of students | Percentage |
|---------------------------------|------------------------|------------|
| errors in understanding the problem | 15                     | 51.7 %     |
| errors in thinking of a plan     | 6                      | 20.7 %     |
| errors in implementation of a plan | 2                     | 6.9 %      |
| errors in the review             | 8                      | 27.6 %     |
| misconceptions                   | 13                     | 44.8 %     |

Based on table 2 it was obtained that most students make mistakes due to errors in understanding the problem. And the second factor is misconceptions. It can be seen that the percentage in the table does not add up to 100%, because students can repeat mistakes more than once. For example, there are students who encountered an error in understanding problems and also due to misconceptions. Likewise with other mistakes, so a student can experience several type of errors at once. A sample of students’ errors in completing prism and pyramid surface and volume problems were described as follows:
3.1. Error in understanding the problem
The students do mistakes to count volume of prism. Percentage of students’ committing this error is 51.7% (15 of 29 students). The student can wrote what is known and asked. However, the student wrote the wrong formula, in which they wrote formula to measure the volume of the cupoid \( V = p \times l \times t \), instead of pyramid. It can be concluded this student did not understand the problem and committed error in using formula. This error can be interpreted that student do not understand the problem.

3.2. Error in thinking of a plan

![Figure 1](image)

**Figure 1.** Student’s error in thinking of a plan

The percentage of students doing this error was 20.7% (6 of 29 students). In Figure 1, it can be seen that student did not write what is known and asked and that they did not think of the plan properly. However, student wrote the correct formula for the volume of the pyramid \( V = \frac{1}{3} \times L \times \text{area} \times t \), but they could not identify which one is the height of the pyramid. In other word this student cannot interpret a pyramid when it is presented in other form. This student’ errors can be categorized as errors in thinking of a plan.

3.3. Error in implementation of the plan

![Figure 2](image)

**Figure 2.** Student’s error in implementation of the plan

The percentage of students doing this error is 6.9% (2 of 29 students). In Figure 2, it can be seen that student did not write correct answer. Student had plotted the answer well by writing down what is known and asked. Student also had written a plan of completion by writing the formula for the pyramid volume \( V = \frac{1}{3} \times L \times \text{area} \times t \). However, student made mistake in calculations. The correct answer should be 20 cm\(^3\) but student answered 80 cm\(^3\) This type of error could be categorized as errors in implementation of a plan.

3.4. Error in review
The percentage of students error when each side measure of prism known was 27.6% (8 of 29 students). Students experience errors in checking answers. In the answer above the student immediately
wrote the height of the pyramid is 2, without writing down where the height was obtained and did not write conclusion. Based on the results of interviews conducted, student admitted that they did not check the answers and that they did not write the answers completely. Errors in review occurred because students were not accustomed to write conclusions after each work on math problems and then did not crosscheck whether the answer is correct or not. Since the students only thought that the most important thing was that they got answers from the given mathematical problem [13, 14].

3.5. Misconceptions
The percentage of students doing this error was 44.8% (13 of 29 students). Students who had misconceptions did not write down the answers. Misconceptions are revealed from the results of interviews with students. At the time of the interview the students claimed they do not understand the form of pyramid in the given question. They only understand a form of pyramid in which its height is located in the middle of the pyramid. Therefore, it can be concluded that these students experience misconceptions.

Among the errors that have been identified before, the most errors that the students faced is error in understanding the problem. The results of this research are the same as the results of research conducted by Fitriani and Erna which showed that most students experienced errors in understanding problems [6,13]. The low level of thinking ability and creativity of students in solving mathematical problems and manipulating them into algebra as well as the use of procedures for solving unusual problems are the most influential factors in these errors [13]. Factors that cause errors in understanding the problem include students that were not accustomed to identifying and writing what was known and asked each time they solve a mathematical problem, because in their learning process such actions were never treated as errors [14]. Familiarizing students in working on the problem by writing down what is known and asked can reduce risk of errors occurring. Other causes of errors that occur are the students find difficulty in reading information, carelessness, errors in understanding questions, haste in answering questions, lack of effort made by some students in working on problems and understanding incomplete concepts and getting answers from others.

Based on these five type of errors, we could determine what are the causal factors of students’ errors. The factors can come from themselves or others student. Based on the result of test and interview, causal factors of student error was found to be some students were not accustomed to identify and write what was known and asked while solving a mathematical problem. And another factor was that student face difficulty in reading information, did not understand concept of the prism and pyramid, carelessness in calculation, haste in answering questions, and lack of effort.

4. Conclusion
In conclusion based on the results of the research, it was found that the errors made by students were quite diverse. Because this research pays attention to the problem solving steps of Polya, the mistakes made by students in this study are divided into four types of errors, namely errors in understanding the problem, errors in thinking of a plan, errors in implementation of the plan, errors in review. In addition, this study found errors which included misconceptions in students. Then the factors that cause student errors are that students were not accustomed to identify and write what was known and asked when solving a mathematical problem, difficulty in reading information, did not understand concept of the prism and pyramid, carelessness in calculation, haste in answering questions, and lack of effort.

5. References
[1] Aydoğdu MZ and Keşan, C 2014 A research on geometry problem solving strategies used by elementary mathematics teacher candidates Engineering Sciences & Technologies 4 1
[2] Idris N 2009 The impact of using Geometers’ Sketchpad on Malaysian students’ achievement and van Hiele geometric thinking Journal of mathematics Education 2 2 pp. 94-107.
[3] Polya G 1973. *How to Solve It, Second Edition* Standford University California: Princeton University Press

[4] Smith JP Disessa AA and Roschelle J 1994 Misconceptions reconceived: A constructivist analysis of knowledge in transition *The journal of the learning sciences* 3 2 pp.115-163.

[5] Luneta K 2015 Understanding students' misconceptions: an analysis of final Grade 12 examination questions in geometry *Pythagoras* 36 1 pp.1-11.

[6] Fitriani H N Turmudi T and Prabawanto S 2018 December. Analysis of students error in mathematical problem solving based on Newman’s error analysis In *International Conference on Mathematics and Science Education of Universitas Pendidikan Indonesia* 3 pp. 791-796

[7] Junaedi I Suyitno A Sugianto E and Eng C K 2015 Disclosure Causes of Students Error in Resolving Discrete Mathematics Problems Based on NEA as A Means of Enhancing Creativity *International Journal of Education* 7 4 pp.31-42.

[8] Borasi R 1987 Exploring mathematics through the analysis of errors *For the learning of Mathematics* 7 3 pp.2-8.

[9] Herholdt R and Sapire I 2014 An error analysis in the early grades mathematics-A learning opportunity? *South African Journal of Childhood Education* 4 1 pp.43-60.

[10] Etikan I Musa S A and Alkassim R S 2016 Comparison of convenience sampling and purposive sampling *American journal of theoretical and applied statistics* 5 1 pp.1-4.

[11] Williams C 2011 Research methods. *Journal of Business & Economics Research (JBER)* 5 3.

[12] Creswell J W and Creswell J D 2017 *Research design: Qualitative, quantitative, and mixed methods approaches* Sage publications

[13] Erna H W and Budi M 2016 *Kesalahan Siswa SMP dalam Menyelesaikan Soal Matematika Berbasis PISA Pada Konten Changer and Relationship* (Prosiding: Konferensi Nasional Penelitian Matematika dan Pembelajaran) p. 199-209

[14] Nurussafa’at F A Sujadi I and Riyadi R 2016 Analisis kesalahan siswa dalam menyelesaikan soal cerita pada materi volume prisma dengan fong’s schematic model for error analysis ditinjau dari gaya kognitif siswa *Jurnal Pembelajaran Matematika* 4 2.

Acknowledgments
Researcher sincerely thank and appreciate to teachers and students in SMP Negeri 40 Bandung for the participation.