Nuclear Shadowing in Neutrino-Nucleus Deeply Inelastic Scattering

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In the framework of the collinear factorized pQCD approach we calculate the small-$x_B$ process-dependent nuclear modification to the structure functions measured in neutrino-nucleus deeply inelastic scattering. We include both heavy quark mass corrections $(M^2/Q^2)$ and resummed nuclear-enhanced dynamical power corrections in the quantity $(\xi^2/Q^2)(A^{1/3}−1)$ with $\xi^2$ evaluated to leading order in $\alpha_s$. Our formalism predicts a measurable difference in the shadowing pattern of the structure functions $F_2^A(x_B,Q^2)$ and $F_3^A(x_B,Q^2)$ and a significant low- and moderate-$Q^2$ modification of the QCD sum rules. We also comment on the relevance of our results to the NuTeV extraction of $\sin^2\theta_W$.

PACS numbers: 12.38.Cy; 12.39.St; 24.85.+p; 25.30.-c

I. INTRODUCTION

Recent surprising results on $\sin^2\theta_W$, reported by the NuTeV collaboration and based on a comparison of charged and neutral current neutrino interactions with an iron rich target, renewed our quest for understanding the nuclear dependence in neutrino-nucleus deeply inelastic scattering (DIS). A possibility that process-dependent nuclear shadowing might affect the NuTeV extraction of the Weinberg angle $\theta_W$ was raised by Miller and Thomas. Although such scenario was considered unlikely by the collaboration, a systematic study and a clear understanding of the process-dependent nuclear effects in neutrino-nucleus shadowing will strengthen the importance of the NuTeV result.

Like all nuclear dependences in the physical cross sections, the small-$x_B$ shadowing in lepton-nucleus DIS has both process-dependent and process-independent contributions. While its universal part can be factorized in the leading twist nuclear parton distribution functions (nPDFs), the DIS-specific modifications arise from the higher twist (or power) corrections to the structure functions. In this letter, we present a calculation of the process-dependent shadowing in neutrino-nucleus deeply inelastic scattering by resumming heavy quark mass corrections, $M^2/(2p \cdot q) = x_B M^2/Q^2$, and nuclear size enhanced dynamical power corrections, $(\xi^2/Q^2)(A^{1/3}−1)$ with $\xi^2 \propto \langle F^+ F^- \rangle$, the gluon density in a large nucleus. The numerical value for the characteristic scale of higher twist $\xi^2$, extracted from DIS data on $\mu$-A interactions, is much less than $Q^2$ in the region which is perturbatively accessible. Therefore, we only evaluate $\xi^2$ to the leading order in $\alpha_s$, while resumming the power corrections to all orders in $(\xi^2/Q^2)(A^{1/3}−1)$. Using $\xi^2 = 0.09 − 0.12 \text{GeV}^2$, our results provide a good description of the deviation between the Gross-Llewellyn Smith QCD sum rule and the existing data. At small Bjorken $x_B$, the high twist components to the calculated structure functions $F_2^A(x_B,Q^2)$ and $F_3^A(x_B,Q^2)$ in neutrino-iron DIS qualitatively describe the low-$x_B$ and low-$Q^2$ suppression trend in the preliminary data, recently reported by the NuTeV collaboration at DIS 2003.

II. DIS KINEMATICS AND COHERENCE AT SMALL $x_B$

The charged current DIS cross section of a neutrino (or antineutrino) beam $(k)$ off a nuclear target $(P_A)$, as illustrated in Fig. (a), probes three independent structure functions, $F_i^A(x_B,Q^2)$ with $i = 1, 2, 3$:

$$\frac{d\sigma^{\nu(\bar{\nu})A}}{dx_B dy} = \frac{\pi \alpha^2_{em} m_N E}{2 \sin^2(\theta_W/(Q^2 + M_W^2))}$$

$$\times \left[ \frac{y^2}{2} x_B F_1^{\nu(\bar{\nu})A} + \left( 1 - y - \frac{m_N x_B}{2E} \right) F_2^{\nu(\bar{\nu})A} \right]$$

$$+ (-) \left( y - \frac{y^2}{2} \right) x_B F_3^{\nu(\bar{\nu})A},$$

(1)

where the Bjorken variable $x_B = Q^2/(2p \cdot q)$ with $p = P_A/A$, the exchanged W-boson momentum $q$ and its virtuality $Q^2 = -q^2$, and $y = p \cdot q/(p \cdot k)$. In Eq. (1), the “($-\cdot$)” represents the sign for an antineutrino beam, $m_N = M_A/A$ with nuclear mass $M_A$, $M_W$ is the W-boson mass, and $E$ is the beam energy. The
FIG. 1: (a) Illustration of the characteristic scales, $A_\perp = 1/Q^2$ and $\Delta z^{(-)} = 1/(x_B p^+)$ in the boosted frame, probed by the virtual meson in DIS. (b) Multiple final state interactions of the struck quark with the partons from the nucleus at a fixed impact parameter $b$.

often-referred longitudinal structure function, $F_L^{\nu(v)A} = F_2^{\nu(v)A}/(2x_B) - F_1^{\nu(v)A}$ if $4x_B^2m_N^2 \ll Q^2$. Here, we are predominantly interested in the small-$x_B$ region and neglect the target mass (rescaling) corrections [12].

The DIS cross section with an exchange of a $W$- or $Z$-boson of virtuality $Q^2$ and energy $\nu = Q^2/(2m_Nx_B)$ has an effective resolution in transverse area $A_\perp = 1/Q^2$, which is much less than the nucleon size, and an uncertainty in longitudinal direction $\Delta z^{(-)} = 1/(x_B p^+)$ with boosted nucleon momentum $p^+$. If $\Delta z^{(-)} = 1/(x_B p^+) \geq 2r_0(m_N/p^+)$ or $x_B \leq x_N = 1/(2m_Nr_0) \sim 0.1$, the neutrino will coherently interact with more than one nucleon inside the nucleus, and probe the nuclear dependence at a perturbative scale $Q^2 [14 15 16].$

III. CALCULATING MASS AND DYNAMICAL POWER CORRECTIONS

Electroweak charged and neutral current processes necessitate a discussion of final state charm mass effects in neutrino-nucleus DIS even if the leading twist charm quark parton distribution is neglected, $\phi_c(x, Q^2) = \phi_c(x, Q^2) = 0$ [13]. It is, therefore, critical to develop a systematic approach to the interplay of a heavy quark final state and the resumed nuclear enhanced power corrections discussed in [12]. We define the boost invariant mass fraction

$$x_M = \frac{M^2}{2p^+q} = \frac{x_B M^2}{Q^2}$$

and choose a frame such that $p^\mu = p^+n^\mu$ and $q^\mu = -x_B p^+n^\mu + Q^2/(2x_B p^+)$, where $n^\mu = [0, 1, 0, 1]$ and $\bar{n}^\mu = [0, 1, 0, 1]$ specify the "+" and "−" lightcone directions, respectively. With a non-vanishing quark mass $M$, the Feynman rule for the final state cut line of quark momentum $x_i p + q$ in Fig. 2(a) is

$$Cut = 2\pi \left( \frac{x_B}{Q^2} \right) (\gamma \cdot \bar{p} + M) \delta(x_i - x_B - x_M),$$

where

$$\bar{p}^\mu = x_M p^+ \bar{n}^\mu + \frac{Q^2}{2x_B p^+} n^\mu,$$

with $p^2 = M^2$. For $M \to 0$ we recover the known massless case, in which the scattered quark is moving along the "−" lightcone direction. A direct consequence of this Feynman rule is a tree-level coupling for longitudinally polarized vector mesons $\propto M^2/Q^2$. Contracting $e_{\mu\nu}^A$ with the charged current hadronic tensor $W_{\mu\nu}$ yields:

$$\frac{1}{A} F_L^{\nu A}(x_B, Q^2) = \sum_{D,U} |V_{UD}|^2 \frac{M_D^2}{Q^2} \phi_D^A(x_B + x_{MD}, Q^2)$$

$$+ \sum_{U,D} |V_{UD}|^2 \frac{M_D^2}{Q^2} \phi_D^A(x_B + x_{MD}, Q^2),$$

$$\frac{1}{A} F_L^{\nu A}(x_B, Q^2) = \sum_{D,U} |V_{UD}|^2 \frac{M_D^2}{Q^2} \phi_D^A(x_B + x_{MD}, Q^2)$$

$$+ \sum_{D,U} |V_{UD}|^2 \frac{M_D^2}{Q^2} \phi_D^A(x_B + x_{MD}, Q^2),$$

where the CKM matrix elements $V_{ij}$ parametrize the electroweak and mass eigenstate mixing with up-type quark $U = (u, c, t)$ and down-type quark $D = (d, s, b)$, and $\phi_i^A$ represent the flavor-$i$ universal twist-2 parton distribution functions (PDFs) of a nucleon $(A = 1)$ or a nucleus $B$. Eqs. (5) and (6) give a novel leading order $(a_0^2)$ power suppressed $(M^2/Q^2)$ quark mass contribution to the ratio of longitudinal and transverse structure functions $R(x_B, Q^2) = F_L^{\nu A}(x_B, Q^2)/F_T^{\nu A}(x_B, Q^2)$ for both nucleons and nuclei.

We calculate the nuclear enhanced dynamical power corrections in the lightcone $A^+=0$ gauge. In this gauge, other than the initial-state contact-term contributions, all leading order nuclear enhanced power corrections are from final-state multiple gluon interactions of the scattered quark in a large nucleus shown in Fig. 2(b). To resum all order nuclear enhanced power corrections with a non-vanishing (anti)quark mass, we examine its propagator structure [10]. For a quark momentum $x_i p + q$

$$Propagator = \pm i \left( \frac{x_B}{Q^2} \right) \frac{\gamma \cdot p}{x_i - (x_B + x_M) + i\epsilon},$$

where $\pm i, \pm i\epsilon$ correspond to propagators to the left or right of the $t = \infty$ cut. In the Fourier space conjugate to $x_i p^+$ the first term, free of $x_i$ pole, is $\propto \delta(y_i)$. The operators in the hadronic matrix element that this contact term ($\leftrightarrow$) separates can be evaluated in the same
nucleon state $h_0$. In contrast, the Fourier transform of the second term is $\propto \theta(y^-)$. Therefore, this pole term ($\rightarrow$) is the source of the $A^{1/3}$ nuclear size enhancement to the power corrections. The operators that it connects in the multi-field multi-local hadronic matrix element can be long-distance separated and thus approximately evaluated in different nucleon states $h_0$. Alternative operator decompositions as well as other terms that arise from a formal operator product expansion (OPE) are suppressed by powers of the nuclear size.

The case of massive final state quarks could be much more involved than the $M \to 0$ limit. The complexity of the calculation stems from the potentially dangerous exponential growth of the number of terms coming from products of propagators, see Fig. 2(b). In our calculation we first observe that products of propagators, see Fig. 2(b), in our calculation, the complexity of the calculation stems from the potentially dangerous exponential growth of the number of terms coming from products of propagators, see Fig. 2(b).

For the diagrams in Fig. 2(b) only one alternating sequence of short and long distance parts of the propagators Eq. (7), initiated by the $t = \infty$ cut, survives. Therefore, there must be an even number of gluon interactions between the cut line and any surviving pole term of a propagator in Fig. 2(b). We also note that

\[ \cdots \gamma \cdot \vec{p} (\gamma \cdot \vec{p} \mp M) \gamma \cdot \vec{p} \cdots \propto \gamma^- p^+ \left( \frac{Q^2}{2 M \vec{p} \gamma^+} \right) \gamma^- p^+ \]

leaves no mass dependence in the spinor trace of the diagrams. The basic unit for two-gluon exchange with a net momentum fraction flow $x_i - x_{i-1}$, and two-quark propagators (one contact plus one pole) [8], see Fig. 2(b), now reads:

\[
\text{Unit} = x_B \left( \frac{4 \pi^2 \alpha_s}{3 Q^2} \right) \int \frac{d\lambda_i}{2 \pi} e^{i \lambda_i (x_i - x_{i-1}) - i \epsilon} \times \left\{ \begin{array}{ll}
\gamma^- \gamma^+ & x_{i-1} - (x_B + x_M) - i \epsilon, \quad \text{left} \\
\gamma^- \gamma^+ & x_{i-1} - (x_B + x_M) + i \epsilon, \quad \text{right} \end{array} \right.
\]

In Eq. 8 the boost invariant $\lambda_i = p^+ y^- i$, the two cases correspond to a vertex to the left or right of the final state cut and $\tilde{F}^2(\lambda_i)$ is given by the intra-nucleon two-gluon field strength correlator defined in 8:

\[
\tilde{F}^2(\lambda_i) \equiv \int \frac{d\lambda_i}{2 \pi} \left( \frac{1}{(p^+)^2} \right)^2 F^{\alpha-\gamma} (\lambda_i) F_{\alpha-\gamma} (\tilde{\lambda}_i) \theta (\lambda_i - \tilde{\lambda}_i).
\]

We conclude that the dynamical nuclear-enhanced all twist contributions from the leading order in $\alpha_s$ Feynman diagrams with a massive quark final state are identical to the massless case up to the substitution $x_B \rightarrow x_B + x_M$ (rescaling) in the $\delta$-function in the cut, Eq. 8, and the propagator poles, Eq. 8. Effectively, we have shown that the mass and nuclear enhanced power corrections “commute” and $x_i = x_B + x_M$ for all “i”. This allows us to take all possible final state interaction diagrams and all possible cuts to explicitly carry out the resummation of coherent high-twist contributions to neutrino-nucleus DIS structure functions.

\[
\frac{1}{A} F_{1,3}^{\alpha A} (x_B, Q^2) \approx 2 \sum_{D,U} |V_{DU}|^2 \phi_D^A (x_B + x_{HT} + x_{M_D}, Q^2) \pm \sum_{D,U} |V_{UD}|^2 \phi_U^A (x_B + x_{HT} + x_{M_D}, Q^2).
\]

In Eqs. 10 and 11 the “±” signs refer to $F_1$ (parity conserving) and $F_3$ (parity violating) transverse struc-
ture functions, respectively. The factor \( \{2\} \) gives the standard normalization for \( F_3 \) only \( \xi \) and the isospin average in the PDFs over the protons and neutrons in the nucleus is implicit. In Eqs. (10) and (11) \( x_{HT} \) is the momentum fraction shift (rescaling) induced by nuclear enhanced dynamical power corrections and derived in Ref. [6]:

\[
x_{HT} = x_B \frac{\xi^2}{Q^2} (A^{1/3} - 1) f(x_B),
\]

where \( \xi^2 \) represents the effective scale for the dynamical power corrections. To the leading order in \( \alpha_s \) it is given by

\[
\xi^2 = \frac{3\pi\alpha_s(Q^2)}{8r_0^6} \langle p|\hat{F}^2|p \rangle,
\]

where \( \langle p|\hat{F}^2|p \rangle \) depends on the small-\( x \) limit of the gluon distribution in the nucleon/nucleus \( \xi \). While \( x_N = 0.1 \) is the limiting value for the onset of coherence, at \( x_A = 1/(2m_Nr_0A^{1/3}) < x_N \) the exchange vector meson already probes the full nuclear size, see Fig. 11. To first approximation, the function

\[
f(x_B) = \begin{cases} 
0, & x_B > x_N \\
\frac{x_B - x_N}{x_A - x_N}, & x_A \leq x_B \leq x_N \\
1, & x_B < x_A 
\end{cases}
\]

in Eq. (12) represents the interpolation between the two regimes based on the uncertainty principle [20]. The nuclear enhancement factor \( (A^{1/3} - 1) \) in Eq. (12) comes from the integration \( \int d\lambda_i \) in Eq. (8), the lower limit of which was chosen such that the effect vanishes for the proton \( (A = 1) \) case.

Including the dynamical power corrections, the longitudinal structure functions in Eqs. (5) and (6) become:

\[
\frac{1}{A} F_L^{A}(x_B, Q^2) \approx F_L^{(LT)}(x_B, Q^2) + \sum_{D,U} |V_{UD}|^2 \left[ \frac{M_D^2}{Q^2} + \frac{\xi^2}{Q^2} \left( 2 - \frac{M_D^2}{Q^2 + M_D^2} \right) \right] \phi_D (x_B + x_{HT} + x_{M_D}, Q^2)
\]

\[
+ \sum_{U,D} |V_{UD}|^2 \left[ \frac{M_D^2}{Q^2} + \frac{\xi^2}{Q^2} \left( 2 - \frac{M_D^2}{Q^2 + M_D^2} \right) \right] \phi_D (x_B + x_{HT} + x_{M_D}, Q^2),
\]

\[
= F_L^{(LT)}(x_B, Q^2) + \sum_{U,D} |V_{UD}|^2 \left[ \frac{M_D^2}{Q^2} + \frac{\xi^2}{Q^2} \left( 2 - \frac{M_D^2}{Q^2 + M_D^2} \right) \right] \phi_D (x_B + x_{HT} + x_{M_D}, Q^2)
\]

\[
+ \sum_{D,U} |V_{UD}|^2 \left[ \frac{M_D^2}{Q^2} + \frac{\xi^2}{Q^2} \left( 2 - \frac{M_D^2}{Q^2 + M_D^2} \right) \right] \phi_D (x_B + x_{HT} + x_{M_D}, Q^2).
\]

In Eqs. (15) and (16) we include the \( \mathcal{O}(\alpha_s) \) leading twist longitudinal structure functions \( F_L^{(LT)}(x, Q^2) \) since they are of the same order as the leading \( \xi^2 \) power. For numerical evaluation in the next Section we consider two quark generations, \( U = (u,c) \) and \( D = (d,s) \), and use \( |V_{ud}|^2 = |V_{cs}|^2 = \cos^2 \theta_c = 0.95 \), \( |V_{us}|^2 = |V_{cd}|^2 = \sin^2 \theta_c = 0.05 \) with Cabibbo angle \( \theta_c \). The u, d and s quarks are treated as massless and the charm quark mass is set to \( M_c = 1.35 \text{ GeV} \).

\section*{IV. HIGH TWISTS, SHADOWING AND QCD SUM RULE}

We first quantify analytically the differences in the “shadowing” pattern induced by valence and sea quarks, neglecting the charm mass effects that are shown to be small below. For isoscalar-corrected \( (Z = N = A/2) \) target nuclei we average over neutrino- and antineutrino-initiated charged current interactions, \( F_1^A(x_B, Q^2) = (F_{1u}^A(x_B, Q^2) + F_{1d}^A(x_B, Q^2))/2 \). In the leading-order and leading twist parton model \( F_1^A(x_B, Q^2) \) measures the valence quark number density with \( \phi_{val}(x) \propto x^{-\alpha_{val}} \) at small \( x \). \( F_2^A(x_B, Q^2) \), a singlet distribution, is proportional to the momentum density of all interacting quark constituents and for \( x_B \ll 0.1 \) is dominated by the sea contribution, \( \phi_{sea}(x) \propto x^{-\alpha_{sea}} \). Therefore, the \( x_B \)-dependent shift from dynamical nuclear enhanced power corrections, \( x_{HT} \) in Eq. (12), generates different modification to \( F_2^A(x_B, Q^2) \) and \( F_3^A(x_B, Q^2) \). Let \( R_{A/A'}^{sea/val}(x_B, Q^2) \) be the shadowing ratio determined from nuclei A and A’ (for example \( ^{56}\text{Fe} \) to \( ^2\text{D} \)) in \( F_2 \) and \( F_3 \). If the scale of high twist corrections \( \xi^2 \ll Q^2 \) and \( x_B \lesssim \min(x_A, x_A') \), we expand the PDFs to first order
in $x_{HT}$ to obtain:

$$R_{sea/val}^{A/A'}(x_B, Q^2) = \frac{F_3^A(x_B, Q^2)}{F_3^{A'}(x_B, Q^2)} / \frac{F_2^A(x_B, Q^2)}{F_2^{A'}(x_B, Q^2)} = 1 - (\alpha_{sea} - \alpha_{val.})(A^{1/3} - A'^{1/3})\xi^2/Q^2 + \cdots$$

(17)

Since $\alpha_{val.} \approx 0.5$ and $\alpha_{sea} \approx 1$ vary slowly with $Q^2$, Eq. (17) predicts a measurable difference in the nuclear shadowing for the structure function $F_2$ ($F_1$) in comparison to $F_3$.

Figure 3 shows the modification to the DIS structure functions from Eqs. (10), (16) for large nuclei ($^{12}C$, $^{56}Fe$ and $^{208}Pb$) relative to the deuteron, calculated with the CTEQ6L parton distribution functions (21). The bands correspond to a scale for power corrections $\xi^2 = 0.09 - 0.12$ GeV$^2$. The latest global QCD fits include $\nu(\bar{\nu})$-A DIS measurements on nuclear targets (24), these data lack the necessary atomic weight systematics to identify small-$x_B$ nuclear shadowing. We do, however, note that our results provide a consistent explanation of the observed small- and moderate-$Q^2$ power law deviantation at small-$x_B$ of the preliminary NuTeV data (11) on $F_2^A(x_B, Q^2)$ and $x_BF_3^A(x_B, Q^2)$ from the next-to-leading order leading twist QCD predictions using MRST parton distribution functions (20).

The latest global QCD fits include $\nu(\bar{\nu})$-A DIS data without nuclear correction is other than isospin (21). Such analysis would tend to artificially eliminate most of the higher twist contributions discussed here due to a trade off between the power corrections in a limited range of $Q^2$ and the shape of the fitted input distributions at $Q^2$, especially within the error bars of current data. An effective way to verify the importance of the nuclear enhanced power corrections for neutrino-nucleus deeply inelastic scattering is via the QCD sum rules, in particular, the Gross-Llewellyn Smith (GLS) sum rule (8)

$$S_{GLS} = \int_0^1 dx_B \frac{1}{2x_B} (x_B F_3^{sA} + x_B F_3^{SA}) .$$

(18)

At tree level Eq. (18) counts the number of valence quarks in a nucleon, $S_{GLS} = 3$. Since valence quark number conservation is enforced in the extraction of twist-2 nucleon/nucleus PDFs, the adjustments of input parton distributions can alter their shape but not the numerical contributions to the GLS sum rule.

The effect of scaling violations can modify $S_{GLS}$, and at $O(\alpha_s) (8)$

$$\Delta_{GLS} \equiv \frac{1}{3} (3 - S_{GLS}) = \frac{\alpha_s(Q^2)}{\pi} + \frac{G}{Q^2} + O(Q^{-4}).$$

(19)

Loop contributions to the GLS sum rule are known to $O(\alpha_s^2)$ (27). Although power corrections can also modify the shape of $nucleon$ structure functions, recent precision DIS data on both hydrogen and deuteron targets from JLab (28) indicate that effects from higher twist to the lower moments of structure functions are very small at $Q^2$ as low as 0.5 GeV$^2$, which confirms the Bloom-Gilman duality (29). A recent phenomenological study (30) also suggests that power corrections to the proton $F_2(x_B, Q^2)$ have different sign in the small- and large-$x_B$ regions and largely cancel in the QCD sum rule.

On the other hand, the coherence between different nucleons inside a large nucleus is only relevant for $x_B \leq x_N$. The suppression of structure functions at small Bjorken $x_B$ in Fig. 3 caused by the nuclear enhanced dynamical power corrections, cannot be canceled in the moments and further reduces the numerical value of $S_{GLS}$. Figure 4 shows a calculation of $\Delta_{GLS}$ from Eqs. (10) and (11) for $^{56}Fe$. While the effect of charm mass is seen to be small relative to $\alpha_s/\pi$, for $Q^2 \sim 1$ GeV$^2$ nuclear enhanced higher twists may contribute as much as $\sim 10\%$
data and other global data sets used in QCD parton structure analysis can all be consistent within the SM.

Because a heavy target was used, several nuclear effects can enter the cross sections to influence the extraction of $\sin^2 \theta_W$ \cite{22}. Since nuclear enhanced power corrections were not included in NuTeV’s analysis, Miller and Thomas pointed out that nuclear shadowing from a vector meson dominance (VMD) model could affect the charged and neutral current neutrino scattering differently, and therefore change the predictions for the ratios of neutral current (NC) over charged current (CC) cross sections, $R^{\nu(\bar{\nu})} = \sigma^{\nu(\bar{\nu})}/\sigma^{\nu CC(\nu(\bar{\nu}))}$, and the extraction of $\sin^2 \theta_W$ \cite{2}. The NuTeV collaboration argued \cite{3, 22} that such possibility was considered unlikely because $R^-$ has little sensitivity to process-dependent nuclear effects.

In this letter we calculated the process-dependent nuclear effects in the neutrino-nucleus differential cross sections, Eq. (11), in the perturbatively accessible DIS region. Our predictions on the nuclear modification to the $\nu(\bar{\nu})$-A structure functions in Fig. 4 should be relevant for $Q^2$ between 1 and 10 GeV$^2$. While for the mean $\langle Q^2 \rangle = 25.6$ GeV$^2$ and $\langle Q^2 \rangle = 15.4$ GeV$^2$ the effect of dynamical power corrections is small, a large fraction of the final data sample cover the $x_B < 0.1, Q^2 < 10$ GeV$^2$ range where shadowing can be as large as $\sim 20\%$. We note that the NuTeV measurement constitutes a $\sim 2\%$ increase in the value of $\sin^2 \theta_W$ relative to the SM, or equivalently $\sim 4\%$ reduction of the expected total neutrino-nucleus cross section. Including shadowing into the expected total cross sections will certainly reduce the discrepancy of $\sin^2 \theta_W$. However, without knowing the nuclear enhanced power corrections to the structure functions at $Q^2 < 1$ GeV$^2$, and the detailed Monte Carlo simulation of event distributions, it is difficult to estimate the precise corrections to the extraction of $\sin^2 \theta_W$. We, nevertheless, note that at small Bjorken $x_B$, the calculated nuclear structure functions $F_2^B(x_B, Q^2)$ and $F_4^A(x_B, Q^2)$ in neutrino-iron DIS qualitatively describe the low-$x_B$ and low-$Q^2$ suppression trend in the preliminary data, presented by the NuTeV collaboration at DIS 2003 \cite{11}.

VI. CONCLUSIONS

In the framework of the perturbative QCD collinear factorization approach \cite{33, 34}, we computed and resumed the tree level perturbative expansion of nuclear enhanced power corrections to the structure functions measured in inclusive (anti)neutrino-nucleus deeply inelastic scattering. We demonstrated that these corrections commute with the final state heavy quark effects and identified the new contributions to the longitudinal structure function $F_2^N(x_B, Q^2)$. Our calculated $Q^2$-dependent modification to the Gross-Llewellyn Smith sum rule agrees well with the existing measurements on an iron target \cite{11}. Our approach predicts a non-negligible difference in the small-$x_B$ shadowing of
the structure functions $F_2(x_B, Q^2)$ ($F_1^A(x_B, Q^2)$) and $F_3^A(x_B, Q^2)$, which is consistent with the trend in the preliminary NuTeV data\[11\]. Although our results, valid in the perturbative region, are unlikely to have an immediate impact on the NuTeV’s extraction of $\sin^2 \theta_W$, the predicted $x_B$-, $Q^2$-, and $A$-dependence of the structure functions in the shadowing region can be tested at the future Fermilab NuMI facility \[24\].

Acknowledgments

This work is supported in part by the US Department of Energy under Grant No. DE-FG02-87ER40371. We thank G. Zeller, J. Morfin, G. Sterman and E. Shuryak for useful discussion.

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