Possible Spontaneous Breaking of Lorentz and CPT Symmetry

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One possible ramification of unified theories of nature such as string theory that may underlie the conventional standard model is the possible spontaneous breakdown of Lorentz and CPT symmetry. In this talk, the formalism for inclusion of such effects into a low-energy effective field theory is presented. An extension of the standard model that includes Lorentz- and CPT-breaking terms is developed. The restriction of the standard model extension to the QED sector is then discussed.

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I. Introduction and Motivation

Virtually all modern particle physics theories are constructed using Lorentz invariance as a basic axiom. Local, point-particle field theories, coupled with this assumed Lorentz invariance along with some mild technical assumptions leads one to conclude that CPT must also be preserved.[1] The standard model, its supersymmetric extensions and grand unified models are all of this type.

However, if the fundamental theory underlying the standard model is constructed using nonlocal objects such as strings, Lorentz symmetry may be spontaneously broken in the low-energy limit of the full theory. An explicit mechanism of this type has been proposed in the context of string theory.[2, 3] The Lorentz- and CPT-violating terms are generated when tensor fields gain vacuum expectation values through spontaneous symmetry breaking.
The approach adopted here is to use the mechanism of spontaneous symmetry breaking to generate a list of possible Lorentz violating interactions between standard model fields. The standard model extension is then constructed by selecting those terms satisfying $\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$ gauge invariance and power-counting renormalizability.\cite{4} By only using the property of spontaneous symmetry breaking and not referring to explicit details of the underlying theory, we are able to construct a general model of Lorentz breaking in the context of the conventional standard model.

Many experimental tests of Lorentz and CPT invariance have been performed, so it is useful to have a general theory with explicit parameters that can be used to relate the various experiments as well as motivate new ones. For example, high precision measurements involving atomic systems, \cite{3,5} clock comparisons, \cite{6} and neutral meson oscillations \cite{8,9} provide stringent tests of Lorentz and CPT symmetry. The implications of CPT-violating terms on baryogenesis have also been investigated.\cite{10}

To describe spontaneous Lorentz and CPT breaking, it is convenient to first review the Higgs mechanism in the standard model. Conventional spontaneous symmetry breaking occurs in the Higgs sector of the standard model where the Higgs field obtains an expectation value, thereby partially breaking $\text{SU}(2) \times \text{U}(1)$ gauge invariance. This happens because an assumed potential for the Higgs field is minimized at some nonzero value of the field.

As an example, consider a simple Lagrangian describing a single fermion field $\psi$ and a single scalar field $\phi$ of the form

$$L = L_0 - L', \quad (1)$$

where

$$L' \supset \lambda \phi \bar{\psi}\psi + \text{h.c.} - (\phi^\dagger \phi - \alpha^2)^2. \quad (2)$$

A nonzero vacuum expectation value $\langle \phi \rangle$ for the scalar field will minimize the energy, hence generating a mass for the fermion of $m_f = \lambda \langle \phi \rangle$. This expectation value of the scalar field breaks $\text{SU}(2) \times \text{U}(1)$ gauge invariance because $\langle \phi \rangle$ no longer transforms in a nontrivial way under this gauge group. Lorentz symmetry is maintained in this case because $\langle \phi \rangle$ and $\phi$ are both scalars under the action of the Lorentz group.
Suppose instead that a tensor \( T \) gains a nonzero vacuum expectation value, \( \langle T \rangle \). Lorentz invariance is spontaneously broken in this case. To see how this form of symmetry breaking might occur, consider a Lagrangian describing a fermion \( \psi \) and a tensor \( T \) of the form

\[
\mathcal{L} = \mathcal{L}_0 - \mathcal{L}' ,
\]

where

\[
\mathcal{L}' \supset \frac{\lambda}{M^k} T \cdot \overline{\psi} \Gamma (i \partial)^k \psi + \text{h.c.} + V(T) .
\]

In this expression, \( \lambda \) is a dimensionless coupling, \( M \) is a heavy mass scale of the underlying theory, \( \Gamma \) denotes a general gamma matrix structure in the Dirac algebra, and \( V(T) \) is a potential for the tensor field. (indices are suppressed for notational simplicity) Terms contributing to \( V(T) \) are precluded from conventional renormalizable four-dimensional field theories, but may arise in the low-energy limit of a more general theory such as string theory. [2]

If the potential \( V(T) \) is such that it has a nontrivial minimum, a vacuum expectation value \( \langle T \rangle \) will be generated for the tensor field. There will then be a term of the form

\[
\mathcal{L}' \supset \frac{\lambda}{M^k} \langle T \rangle \cdot \overline{\psi} \Gamma (i \partial)^k \chi + \text{h.c.} ,
\]

present in the Lagrangian after spontaneous symmetry breaking occurs. These terms can break Lorentz invariance and various discrete symmetries C, P, T, CP, and CPT.

II. Relativistic Quantum Mechanics and Field Theory

To develop theoretical techniques for treating generic terms of the type given in Eq. (5), we first study a specific example. The example presented here involves a lagrangian for a single fermion field containing Lorentz-violating terms with no derivative couplings \( (k = 0) \) that also violate CPT.

We proceed by listing the possible gamma-matrix structures that could arise within such a term:

\[
\Gamma \sim \{1, \gamma^\mu, \gamma^5 \gamma^\mu, \sigma^{\mu\nu}, \gamma^5\} .
\]

The condition that a fermion bilinear with no derivative couplings violates CPT is
equivalent to the requirement that $\Gamma$ be chosen such that $\{\Gamma, \gamma^5\} = 0$. Half the matrices in Eq. (4) satisfy this condition: $\Gamma \sim \gamma^\mu$ and $\Gamma \sim \gamma^5 \gamma^\mu$. The contribution to the lagrangian from these terms can be written as

$$L'_a \equiv a_\mu \bar{\psi} \gamma^\mu \psi, \quad L'_b \equiv b_\mu \bar{\psi} \gamma^5 \gamma^\mu \psi,$$

where $a_\mu$ and $b_\mu$ are constant coupling coefficients that parameterize the tensor expectation values and relevant coupling constants arising in Eq. (5). These parameters are assumed suppressed with respect to other physically relevant energy scales in the low-energy effective theory in order to be in agreement with current experimental bounds.

Including these contributions from the spontaneous symmetry breaking mechanism into a theory containing a free Dirac fermion yields a model lagrangian of

$$L = \frac{i}{2} \bar{\psi} \gamma^\mu \stackrel{\leftrightarrow}{\partial}_\mu \psi - a_\mu \bar{\psi} \gamma^\mu \psi - b_\mu \bar{\psi} \gamma^5 \gamma^\mu \psi - m \bar{\psi} \psi.$$  

Several features of this modified theory are immediately apparent upon inspection. The first feature is that the lagrangian is hermitian, thereby leading to a theory obeying conventional quantum mechanics, conservation of probability and unitarity. The second feature is that translational invariance implies the existence of a conserved energy and momentum. This conserved four-momentum is explicitly constructed as

$$P_\mu = \int d^3 x \Theta^0_\mu = \int d^3 x \frac{1}{2} i \bar{\psi} \gamma^0 \stackrel{\leftrightarrow}{\partial}_\mu \psi,$$

just as in the conventional case. The third feature is that the Dirac equation resulting from Eq. (8) is linear in the fermion field allowing an exact solution to the free theory. Finally, a global U(1) invariance of the model lagrangian implies the existence of a conserved current $j_\mu = \bar{\psi} \gamma^\mu \psi$.

The Dirac equation obtained by variation of Eq. (8) with respect to the fermion field is

$$(i \gamma^\mu \partial_\mu - a_\mu \gamma^\mu - b_\mu \gamma^5 \gamma^\mu - m) \psi = 0.$$  

Due to the linearity of the equation, plane-wave solutions

$$\psi(x) = e^{\pm i p_\mu x^\mu} w(\vec{p})$$.
are used to solve the equation exactly. Substitution of the plane-wave solution into the modified Dirac equation yields

\[(\pm p_\mu \gamma^\mu - a_\mu \gamma^\mu + b_\mu \gamma^5 \gamma^\mu - m)w(\vec{p}) \equiv M_\pm w(\vec{p}) = 0.\]

(12)

A nontrivial solution exists only if \(DetM_\pm = 0\). This imposes a condition on \(p^0(\vec{p}) \equiv E(\vec{p})\), hence generating a dispersion relation for the fermion.

The general solution involves finding the roots of a fourth-order polynomial equation. The solutions can be found algorithmically, but the resulting solution is complex and not very illuminating. For simplicity we consider only the special case of \(\vec{b} = 0\) here. The exact dispersion relations for this case are

\[E_+(\vec{p}) = \left[m^2 + (|\vec{p} - \vec{a}| \pm b_0)^2\right]^{1/2} + a_0,\]

(13)

\[E_-(\vec{p}) = \left[m^2 + (|\vec{p} + \vec{a}| \mp b_0)^2\right]^{1/2} - a_0.\]

(14)

Examination of the above energies reveals several qualitative effects of the CPT-violating terms. The usual four-fold energy degeneracy of spin-\(1/2\) particles and antiparticles is removed by the \(a_\mu\) and \(b_0\) terms. The particle-antiparticle energy degeneracy is broken by \(a_\mu\) and the helicity degeneracy is split by \(b_0\). The corresponding spinor solutions \(w(\vec{p})\) have been explicitly calculated, forming an orthogonal basis of states as expected.

An interesting feature of these solutions is the unconventional relationship that exists between momentum and velocity. A wave packet of positive helicity particles with four momentum \(p^\mu = (E, \vec{p})\) has an expectation value of the velocity operator \(\vec{v} = i[H, \vec{x}] = \gamma^0 \vec{\gamma}\) of

\[\langle \vec{v} \rangle = \left\langle \frac{(|\vec{p} - \vec{a}| - b^0)(\vec{p} - \vec{a})}{(E - a^0)|\vec{p} - \vec{a}|} \right\rangle.\]

(15)

Examination of the above velocity using a general dispersion relation reveals that \(|v_j| < 1\) for arbitrary \(b_\mu\), and that the limiting velocity as \(\vec{p} \to \infty\) is 1. This implies that the effects of the CPT violating terms are mild enough to preserve causality in the theory. This will be verified independently using the perspective of field theory that will now be developed.
To quantize the theory, the general expansion for $\psi$ in terms of its spinor components given by

$$\psi(x) = \int \frac{d^3 p}{(2\pi)^3} \sum_{\alpha=1}^{2} \left[ \frac{m}{E_u^{(\alpha)}} b_{(\alpha)}(\vec{p}) e^{-i p_{(\alpha)} \cdot x} u_{(\alpha)}(\vec{p}) + \frac{m}{E_v^{(\alpha)}} d_{(\alpha)}(\vec{p}) e^{i p_{(\alpha)} \cdot x} v_{(\alpha)}(\vec{p}) \right],$$

is promoted to an operator acting on a Hilbert space of basis states. The energy is calculated from Eq. (9) using conventional normal ordering. The result is a positive definite quantity (for $|a^0| < m$) provided the following nonvanishing anticommutation relations are imposed on the creation and annihilation operators:

$$\{ b_{(\alpha)}(\vec{p}), b_{(\alpha')}^\dagger(\vec{p'}) \} = (2\pi)^3 \frac{E_u^{(\alpha)}}{m} \delta_{\alpha \alpha'} \delta^3(\vec{p} - \vec{p'}) ,$$

$$\{ d_{(\alpha)}(\vec{p}), d_{(\alpha')}^\dagger(\vec{p'}) \} = (2\pi)^3 \frac{E_v^{(\alpha)}}{m} \delta_{\alpha \alpha'} \delta^3(\vec{p} - \vec{p'}) .$$

The resulting equal-time anticommutators for the fields are

$$\{ \psi_{(\alpha)}(t, \vec{x}), \psi_{(\beta)}^\dagger(t, \vec{x'}) \} = \delta_{\alpha \beta} \delta^3(\vec{x} - \vec{x'}) ,$$

$$\{ \psi_{(\alpha)}(t, \vec{x}), \psi_{(\beta)}(t, \vec{x'}) \} = 0 ,$$

$$\{ \psi_{(\alpha)}^\dagger(t, \vec{x}), \psi_{(\beta)}^\dagger(t, \vec{x'}) \} = 0 .$$

These relations show that conventional Fermi statistics remain unaltered in the presence of Lorentz- and CPT-violating terms.

The conserved charge $Q$ and four-momentum $P^\mu$ are computed as

$$Q = \int \frac{d^3 p}{(2\pi)^3} \sum_{\alpha=1}^{2} \left[ \frac{m}{E_u^{(\alpha)}} b_{(\alpha)}^\dagger(\vec{p}) b_{(\alpha)}(\vec{p}) - \frac{m}{E_v^{(\alpha)}} d_{(\alpha)}^\dagger(\vec{p}) d_{(\alpha)}(\vec{p}) \right] ,$$

$$P_{\mu} = \int \frac{d^3 p}{(2\pi)^3} \sum_{\alpha=1}^{2} \left[ \frac{m}{E_u^{(\alpha)}} p_{\alpha \mu}^{(\alpha)} b_{(\alpha)}^\dagger(\vec{p}) b_{(\alpha)}(\vec{p}) + \frac{m}{E_v^{(\alpha)}} p_{\alpha \mu}^{(\alpha)} d_{(\alpha)}^\dagger(\vec{p}) d_{(\alpha)}(\vec{p}) \right] .$$

From these expressions we see that the charge of the fermion is unperturbed and the energy and momentum satisfy the same relations that are found using relativistic quantum mechanics.
Causality is governed by the anticommutation relations of the fermion fields at unequal times. Explicit integration in the special case $\vec{b} = 0$ proves that

$$\{\psi_\alpha(x), \overline{\psi}_\beta(x')\} = 0,$$

for spacelike separations $(x - x')^2 < 0$. The above result shows that physical observables separated by spacelike intervals will in fact commute (for case $\vec{b} = 0$). This agrees with our previous results obtained by examination of the velocity using relativistic quantum mechanics.

Next, the problem of extending the free field theory to interacting theory is addressed. Much of the conventional formalism developed for perturbative calculations in the interacting theory carries over directly to the present case. The main reason that these techniques work is that the Lorentz violating modifications which are introduced are linear in the fermion fields. The main result is that the usual Feynman rules apply provided the Feynman propagator is modified to

$$S_F(p) = \frac{i}{p_\mu \gamma^\mu - a_\mu \gamma^\mu - b_\mu \gamma_5 \gamma^\mu - m},$$

and the exact spinor solutions of the modified free fermion theory are used on the external legs of the diagrams.

III. Extension of The Standard Model

In this section the question of how to apply spontaneous symmetry breaking to generate Lorentz-violating terms using standard model fields is addressed. Our approach involves consideration of all possible terms that can arise from spontaneous symmetry breaking that satisfy power-counting renormalizability and preserve the $\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$ gauge invariance of the standard model.\[4\] Even with these constraints, terms are found to contribute to all sectors of the standard model. In listing the terms here, the Lorentz violating terms are classified according to their properties under the CPT transformation.

In the lepton sector the left- and right-handed multiplets are defined as

$$L_A = \left( \begin{array}{c} \nu_A \\ l_A \end{array} \right)_{L}, \quad R_A = (l_A)_R,$$
where $A = 1, 2, 3$ labels the flavor:

$$\begin{align*}
l_A &\equiv (e, \mu, \tau) , \\
\nu_A &\equiv (\nu_e, \nu_\mu, \nu_\tau)
\end{align*} \quad (24)$$

The Lorentz-violating terms that satisfy the required properties are

$$\begin{align*}
\mathcal{L}_{\text{lepton}}^{\text{CPT-even}} &= \frac{1}{2} i (c_L)_{\mu \nu AB} \bar{T}_A \gamma^\mu \hat{D}^\nu L_B \\
&\quad + \frac{1}{2} i (c_R)_{\mu \nu AB} \bar{R}_A \gamma^\mu \hat{D}^\nu R_B ,
\end{align*} \quad (25)$$

$$\begin{align*}
\mathcal{L}_{\text{lepton}}^{\text{CPT-odd}} &= -(a_L)_{\mu AB} \bar{T}_A \gamma^\mu L_B \\
&\quad - (a_R)_{\mu AB} \bar{R}_A \gamma^\mu R_B .
\end{align*} \quad (26)$$

In the above expression $c_{\mu \nu}$ and $a_\mu$ are constant coupling coefficients related to the background expectation values of the relevant tensor fields, and $D^\mu$ is the conventional covariant derivative.

The final form of the standard model terms is different because the SU(2) × U(1) symmetry is broken by the Higgs mechanism. Once this breaking occurs, the fields in Eq. (26) are rewritten in terms of the physical Dirac spinors corresponding to the observed leptons and neutrinos. As an example, the CPT-odd lepton terms become

$$\begin{align*}
\mathcal{L}_{\text{lepton}}^{\text{CPT-odd}} &= -(a_L)_{\mu AB} \bar{T}_A \frac{1}{2} \gamma^\mu \gamma^5 \nu_B \\
&\quad - (a_l)_{\mu AB} \bar{T}_A \gamma^\mu l_B \\
&\quad - (b_l)_{\mu AB} \bar{T}_A \gamma^5 \gamma^\mu l_B .
\end{align*} \quad (27)$$

Note that $b_\mu$ coupling coefficients arise in the process of combining the right- and left-handed fields into Dirac spinors.

If we now examine the first generation electron contribution corresponding to $A = B = 1$, we find the terms

$$\mathcal{L}_{\text{lepton}}^{\text{CPT-odd}} \supset -(a_l)_{\mu 11} \bar{e} \gamma^\mu e - (b_l)_{\mu 11} \bar{e} \gamma^5 \gamma^\mu e . \quad (28)$$

These terms are exactly the form as the contributions to the model lagrangian of Eq. (8) that were analyzed in the previous section. The relativistic quantum mechanics and field theoretic techniques that were developed to handle these terms are
therefore directly applicable to electrons. Terms in Eq. (27) of the form $A \neq B$ contribute small lepton flavor-changing amplitudes.

The construction of the standard model extension in the quark sector is similar to that in the lepton sector. The main difference is that corresponding right-handed quark fields are present for each left-handed field unlike the case in the lepton sector. The left- and right-handed quark multiplets are denoted

\[ Q_A = \begin{pmatrix} u_A \\ d_A \end{pmatrix}_L, \quad U_A = (u_A)_R, \quad D_A = (d_A)_R, \quad (29) \]

where $A = 1, 2, 3$ labels quark flavor

\[ u_A \equiv (u, c, t), \quad d_A \equiv (d, s, b). \quad (30) \]

The Lorentz-violating terms in the quark sector are of the same form as in the lepton sector. The diagonal $A = B$ terms are again of the same form as Eq. (7). The quark $a_\mu$ terms are particularly interesting because they can lead to observable CPT-violating effects in neutral meson systems.\[11\]

In the Higgs sector, there are contributions involving two Higgs fields, and generalized Yukawa coupling terms involving a single Higgs and two fermion fields. The Lorentz-violating terms that are quadratic in the Higgs fields are

\[
L_{\text{Higgs}}^{\text{CPT-even}} = \frac{1}{2} (k_\phi^{\mu\nu}) D_\mu \phi \phi D_\nu \phi + \text{h.c.} \\
- \frac{1}{2} (k_{\phi B})^{\mu\nu} \phi_{\dot{\phi}} B_{\mu\nu} \\
- \frac{1}{2} (k_{\phi W})^{\mu\nu} \phi \phi W_{\mu\nu} + \text{h.c.}, \quad (31)
\]

\[
L_{\text{Higgs}}^{\text{CPT-odd}} = i (k_\phi) \phi \phi D_\mu \phi + \text{h.c.}, \quad (32)
\]

where $W_{\mu\nu}$ and $B_{\mu\nu}$ are the field strengths for the SU(2) and U(1) gauge fields and the various $k$ parameters are coupling constants related to tensor expectation values.

The Yukawa type terms involving one Higgs field are

\[
L_{\text{Yukawa}}^{\text{CPT-even}} = -\frac{1}{2} \left[ (H_L)_{\mu\nu AB} \overline{T}_A \phi \sigma^{\mu\nu} R_B \\
+ (H_U)_{\mu\nu AB} \overline{Q}_A \phi_\sigma^{\mu\nu} U_B \\
+ (H_D)_{\mu\nu AB} \overline{Q}_A \phi \sigma^{\mu\nu} D_B \right] + \text{h.c.}, \quad (33)
\]
where the \( H \) parameters are related to tensor expectation values.

One interesting result of including these terms into the standard model is a modification of the conventional SU(2)\( \times \)U(1) breaking. When the full static potential is minimized, the \( Z^0 \) boson gains an expectation value of

\[
\langle Z^0_\mu \rangle = \frac{1}{q} \sin 2\theta_W (\text{Re} \hat{k}_\phi)_{\mu\nu}^{-1} k_\phi^{\nu} ,
\]

(34)

where \( \hat{k}_{\phi\phi} = \eta_{\mu\nu} + k_{\phi\phi}^{\mu\nu} \), \( q \) is the electric charge, and \( \theta_W \) is the weak mixing angle. If the CPT-odd term \( k_\phi \) vanishes then \( \langle Z^0_\mu \rangle = 0 \). This is reasonable since a nonzero value of \( \langle Z^0_\mu \rangle \) violates CPT symmetry.

The gauge sector is the final sector to be examined. The various Lorentz-breaking terms satisfying the relevant criteria are

\[
L_{\text{gauge}}^{\text{CPT-even}} = -\frac{1}{2} (k_G)_{\kappa\lambda\mu\nu} \text{Tr}(G^{\kappa\lambda} G^{\mu\nu}) \\
- \frac{1}{2} (k_W)_{\kappa\lambda\mu\nu} \text{Tr}(W^{\kappa\lambda} W^{\mu\nu}) \\
- \frac{1}{4} (k_B)_{\kappa\lambda\mu\nu} B^{\kappa\lambda} B^{\mu\nu} ,
\]

(35)

\[
L_{\text{gauge}}^{\text{CPT-odd}} = k_{3e} e^{\kappa\lambda\mu\nu} \text{Tr}(G^\kappa G^\mu G^\lambda G^\nu) \\
+ k_{2e} e^{\kappa\lambda\mu\nu} \text{Tr}(W^\kappa W^\mu W^\lambda W^\nu) \\
+ k_{1e} e^{\kappa\lambda\mu\nu} B^\kappa B^\mu B^\lambda B^\nu .
\]

(36)

In these expressions, the \( k^\prime \) terms are constant coupling constants and the \( G^{\mu\nu} \), \( W^{\mu\nu} \), and \( B^{\mu\nu} \) are the field strengths for the SU(3), SU(2), and U(1) gauge fields respectively.

The CPT-odd terms can generate negative contributions to the conserved energy\cite{12}, hence creating an instability in the theory. It is therefore desirable to set these coefficients to zero, provided they remain zero at the quantum level. This procedure has been carried out to the one-loop level by utilizing an anomaly cancellation mechanism that must be inherited from any consistent theory underlying the standard model\cite{14}. This point is discussed further in the following section.

IV. QED Restriction

We now restrict our attention to the theory of electrons and photons that results
from the above extension of the standard model. The conventional QED Lagrangian is

$$\mathcal{L}^\text{QED}_{\text{electron}} = \frac{1}{2} \bar{\psi} \gamma^\mu \not{D}_\mu \psi - m_e \bar{\psi} \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \ ,$$  

(37)

where $\psi$ is the electron field, $m_e$ is its mass, and $F^{\mu\nu}$ is the photon field strength tensor.

The CPT-even electron terms that violate Lorentz symmetry in the full standard model extension are

$$\mathcal{L}^\text{CPT-even}_{\text{electron}} = \cdots + \frac{1}{2} i c_{\mu\nu} \bar{\psi} \gamma^\mu \not{D}^\nu \psi \cdots \ ,$$  

(38)

where $H$, $c$, and $d$ are constant coupling coefficients. The CPT-odd electron terms are

$$\mathcal{L}^\text{CPT-odd}_{\text{electron}} = \cdots + \frac{1}{2} i d_{\mu\nu} \bar{\psi} \gamma_5 \gamma^\mu \not{D}^\nu \psi \cdots \ ,$$  

(39)

where $a$ and $b$ are parameters analogous to those in Eq. (7) applied to electrons.

Experiments involving conventional QED tests can be used to place stringent bounds on the above violation parameters. For example, Penning traps may be used to compare energy levels of $e^-$ and $e^+$ or $p$ and $\bar{p}$ orbits to constrain various combinations of parameters to few parts in $10^{20}$. In addition, tests involving comparison of hydrogen and antihydrogen $1S - 2S$ and hyperfine transitions can place comparable bounds on other combinations of parameters.

The corrections to the photon from the gauge sector are given by

$$\mathcal{L}^\text{CPT-even}_{\text{photon}} = \cdots - \frac{1}{4} (k_F)_{\kappa\lambda\mu\nu} F^{\kappa\lambda} F^{\mu\nu} \ ,$$  

(40)

and

$$\mathcal{L}^\text{CPT-odd}_{\text{photon}} = + \frac{1}{2} (k_{AF})^\kappa \epsilon_{\kappa\lambda\mu\nu} A^{\lambda} F^{\mu\nu} \ ,$$  

(41)

where the parameters $k_F$ and $k_{AF}$ are the appropriate linear combinations of parameters in Eqs. (35) and (36) that result when the photon is defined as the unbroken U(1) electric force mediator.
A stringent limit of \((k_{AF})^\mu < 10^{-42}\text{GeV}\) has been placed on the CPT-odd term using cosmological birefringence tests. Coupled with the theoretical difficulties involving negative contributions to the energy, this experimental bound indicates that this coefficient should be set identically to zero in the theory. At first sight, radiative corrections appear to induce a nonzero term at the quantum level. However, such corrections must cancel provided the underlying theory is anomaly free.

The only QED correction term with matching C, P, and T symmetry properties that contributes to \((k_{AF})^\mu\) is \(b^\mu\). The one-loop diagram produces an ambiguous, finite, and regularization dependent correction of \((k_{AF})^\mu = \zeta b^\mu\), where \(\zeta\) is an arbitrary constant. When this correction is summed over all fermion species, the contributions must cancel provided there is no anomaly in the full underlying theory. A zero result to lowest order in \(b^\mu\) has also been argued as a consistent choice using arguments based on the gauge invariance of the lagrangian. Several other recent works have shown similar results in various regularization schemes.

More recently, a calculation to all orders in \(b^\mu\) using the exact modified propagator has been carried out. Remarkably, the full result is the same as the correction generated by the linear term. This means that the anomaly cancellation mechanism applies to all orders in \(b^\mu\) and the coefficient \((k_{AF})^\mu\) remains zero at the quantum level.

The CPT-even terms are more interesting for several reasons. First, the total canonical energy is positive provided the couplings are reasonably suppressed. Secondly, the contribution to cosmological birefringence is suppressed relative to the CPT-odd term. Constraints of a few parts in \(10^{23}\) have been obtained on the rotationally invariant term using cosmic-ray tests. More general terms can be bounded to \(k_F \leq 10^{-28}\) using cosmological birefringence measurements.

V. Summary

A framework has been presented that incorporates Lorentz- and CPT-violating effects into the context of conventional quantum field theory. Using a generic spontaneous symmetry breaking mechanism as the source for these terms, an extension
of the standard model that includes Lorentz and CPT breaking was developed. This extension preserves power-counting renormalizability and SU(3)×SU(2)×U(1) gauge invariance. The parameters that have been introduced can be used to establish quantitative bounds on CPT- and Lorentz-breaking effects in nature. Implications for electron and photon propagation in the QED sector were discussed.

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