Lagrangian formulation of free arbitrary $N$-extended massless higher spin supermultiplets in 4D, AdS space

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Abstract

We derive the component Lagrangian for the free $N$-extended on-shell massless higher spin supermultiplets in four-dimensional anti-de Sitter space. The construction is based on frame-like description of massless integer and half-integer higher spin fields. The massless supermultiplets are formulated for $N \leq 4k$, where $k$ is a maximal integer or half-integer spin in the multiplet. The supertransformations that leave the Lagrangian invariant are found and it is shown that their algebra is closed on-shell.
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1 Introduction

The study of various aspects of higher spin fields is currently one of the actively developing areas of modern theoretical and mathematical physics (for review see e.g. [1–6]). Recently, there has been a surge of interest in constructing the supersymmetric higher spin models [7–20] and investigating the properties of such models. In this paper, we study a general problem of Lagrangian construction for arbitrary \(N\)-extended massless free on-shell supermultiplets in four dimensional \(AdS\) space and derive the Lagrangians describing the dynamics of such supermultiplets.

It is well known that in four dimensions the \(N\)-extended supermultiplets with maximal spin \(k = 1\) are restricted by the condition \(N \leq 4\) and the multiplets with maximal spin \(k = 2\) are restricted by the condition \(N \leq 8\). The supermultiplets with \(N > 8\) must contain the higher spins \(k > 2\). To be more precise, there is a specific relationship between the parameter \(N\) and the highest spin \(k\) in the supermultiplet, \(N \leq 4k\) (see e.g. [21]). Of course, if one does not restrict the maximal spin in the multiplet by the quantities \(k = 1, 2\), then for any \(N\) there exist the supermultiplets with arbitrary higher spins.

For the case of simple \(N = 1\) supersymmetry, the component Lagrangian formulation of on-shell higher spin supermultiplets in Minkowski space has been known for a long time [22], [23] and further the component approach has been generalized and studied in the works [24]–[28]. In particular, supertransformations were found that leave invariant the sum of Lagrangians for free massless fields with spins \(k\) and \(k + 1/2\). Completely off-shell Lagrangian formulation for such theories was constructed within the framework of the superfield approach [29, 30]. Off-shell formulation of the \(N=1\) higher superspin free Lagrangian theory in 4D, \(AdS\) space has been first developed in the work [32] from the very beginning in superfield formalism and its component form was derived from superfield theory. Quantization of this theory has been given in [33]. The \(N = 2\) supersymmetric higher spin models both in the Minkowski and \(AdS\) spaces were discussed in [34], the universal higher spin superfield approach in 4D, \(N = 1\) \(AdS\) superspace has been developed in [35].

Recently the on-shell superfield Lagrangian realization was constructed for the extended \(N = 1\) massless supermultiplets in the framework of light-cone gauge formalism [19]. Extension of this approach for on-shell \(N\)-extended superfield Lagrangian formulation has been given in [20] under the condition \(N = 4n\), where \(n\) is a natural number. In this paper we generalize the results of the work [20] and give explicit component Lagrangian construction of arbitrary \(N\)-extended massless higher spin on-shell supermultiplets in the four-dimensional anti-de Sitter space without the restriction of [20].

Our construction is based on the frame-like approach for higher spin fields which generalize the tetrad formulation of gravity. The Lagrangian description for free higher spin bosonic and fermionic fields was developed in [36]. We derive the supertransformations for \(N\)-extended massless higher spin supermultiplets that leave the corresponding system of their free Lagrangians invariant and show that the algebra of the supertransformations is closed on-shell.

The paper is organized as follows. In Section [2] we describe the basic elements of the frame-like Lagrangian formulation for free massless higher spin fields in 4D \(AdS\) space and

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\(^1\)Higher spin models are sometimes called in the literature the higher spin (super)gravities.

\(^2\)See also the later paper [31] on the same subject.
the 4D multispinor technique. In Section 3 we present the minimal massless $N = 1$ supermultiplets [37] which will be used as the building blocks to construct the $N$-extended supermultiplets. Section 4 is devoted to constructing the arbitrary $N$-extended massless supermultiplets in 4D AdS. For each case, we formulate the field contents and introduce the corresponding field variables. Then we derive the supertransformations for these supermultiplets and define the Lagrangian as a sum of Lagrangians for all integer and half-integer spin fields of the given supermultiplet. We prove that such a Lagrangian is invariant under the above transformations. Finally, we show that the constructed supertransformations form the closed $N$-extended 4D AdS superalgebras.

2 Free higher spin fields

In this section, we briefly consider the frame-like Lagrangian formulation of the free massless higher spin fields in the 4D AdS space and the corresponding 4D multispinor formalism.

In the frame-like approach, the massless fields with integer spin $k \geq 2$ are described by the dynamical 1-form $f^a(k-1)\dot{a}(k-1)$ and the auxiliary 1-form $\Omega^{a(k)\dot{a}(k-2)}$, $\Omega^{a(k-2)\dot{a}(k)}$ (see all notations in the appendix). These fields are totally symmetric with respect to the dotted and undotted indices and generalize the tetrad field and Lorentz connection in the frame formulation of gravity. We choose them to be real valued that is they satisfy the following rules of hermitian conjugation

$$ (f^{\alpha(k-1)\dot{a}(k-1)})^\dagger = f^{\alpha(k-1)\dot{a}(k-1)}, $$
$$ (\Omega^{a(k)\dot{a}(k-2)})^\dagger = \Omega^{a(k-2)\dot{a}(k)}. $$

The Lagrangian being the differential 4-form in 4D AdS space looks like

$$ \frac{(-1)^k}{k!} L_k = k\Omega^{a(k-1)\dot{a}(k-2)} E_\gamma \Omega_{a(k-1)\gamma k(k-2)} - (k-2)\Omega^{a(k)\dot{a}(k-3)\dot{b}} E_\gamma \Omega_{a(k)\dot{a}(k-3)\gamma k} $$
$$ + 2\Omega^{a(k-1)\dot{a}(k-2)} e_\beta \Omega_{\dot{b} k(k-2)\beta} $$
$$ + 2k\lambda^2 f^{a(k-2)\dot{a}(k-1)} E_\gamma \Omega_{a(k-2)\dot{a}(k-1) + h.c.}. $$

Here 1-form $e^{a\dot{a}}$ is the AdS background tetrad, $D$ is the AdS covariant derivative $De^{a\dot{a}} = 0$, $E_{\alpha\dot{b}}$ and $E^{a\dot{b}}$ are a double product of $e^{a\dot{a}}$ (see appendix for details). The form of Lagrangian (2.1) is determined by the invariance under the gauge transformations

$$ \delta f^{a(k-1)\dot{a}(k-1)} = D\xi^{a(k-1)\dot{a}(k-1)} + e_\beta \Omega_{\dot{b} k(k-2)\beta} + e_\beta \Omega_{a(k)\dot{a}(k-2)\gamma k}, $$
$$ \delta \Omega^{a(k),\dot{a}(k-2)} = D\eta^{a(k),\dot{a}(k-2)} + \lambda^2 e_\beta \xi^{a(k-1)\dot{a}(k-2)\beta}, $$
$$ \delta \Omega^{a(k-2),\dot{a}(k)} = D\eta^{a(k-2),\dot{a}(k)} + \lambda^2 e_\beta \xi^{a(k-2)\dot{a}(k-1)}. $$

The remarkable property of frame-like formulation is possibility to construct the gauge invariant objects which generalize the torsion and curvature in gravity

$$ T^{a(k-1)\dot{a}(k-1)} = Df^{a(k-1)\dot{a}(k-1)} + e_\beta \Omega_{\dot{b} k(k-2)\beta} + e_\beta \Omega_{a(k)\dot{a}(k-2)\gamma k}, $$
$$ R^{a(k),\dot{a}(k-2)} = D\Omega^{a(k),\dot{a}(k-2)} + \lambda^2 e_\beta f^{a(k-1)\dot{a}(k-2)\beta}, $$
$$ R^{a(k-2),\dot{a}(k)} = D\Omega^{a(k-2),\dot{a}(k)} + \lambda^2 e_\beta f^{a(k-2)\dot{a}(k-1)}. $$
To simplify the construction of the supermultiplets we do not introduce any supertransformations for the auxiliary fields $\Omega$. Instead, all calculations are done up to the terms proportional to the auxiliary field equations of motion. It is equivalent to the following "zero torsion conditions":

$$\mathcal{T}^{(k-1)} \approx 0 \Rightarrow e_\beta \dot{\alpha} \mathcal{R}^{(k-1)\beta\dot{\alpha}(k-2)} + e^\alpha_\beta \mathcal{R}^{(k-2)\dot{\alpha}(k-1)\dot{\beta}} \approx 0. \quad (2.2)$$

As for the supertransformations for the dynamical fields $f$, the corresponding variation of the Lagrangian can be compactly written as follows

$$(-1)^k \delta \mathcal{L}_k = -i2\mathcal{R}^{(k-1)\beta\dot{\alpha}(k-2)} e_\beta \dot{\alpha} \mathcal{R}^{(k-1)\dot{\alpha}(k-2)\dot{\beta}} + h.c.$$

Now let us turn to massless fields with half-integer spin $k+1/2 \geq 3/2$ which are described by 1-form $\Phi^{(k)\dot{\alpha}(k)}$, $\Phi^{(k-1)\dot{\alpha}(k)}$. To be Majorana fields they must satisfy the reality condition

$$(\Phi^{(k)\dot{\alpha}(k-1)})^+ = \Phi^{(k-1)\dot{\alpha}(k)}.$$

The corresponding Lagrangian has form

$$(-1)^k \mathcal{L}_{k+\frac{1}{2}} = \Phi^{(k)\dot{\alpha}(k)}\dot{\alpha} \mathcal{R}^{(k-1)\beta\dot{\alpha}(k-2)} + e_\beta \dot{\alpha} \mathcal{R}^{(k-1)\dot{\alpha}(k-1)\dot{\beta}}$$

$$+ \epsilon_{k+\frac{1}{2}} \epsilon_{k+\frac{1}{2}} \left[ (k+1)\Phi^{(k)\dot{\alpha}(k-1)} \mathcal{R}^{(k-1)\dot{\alpha}(k-1)} \mathcal{R}^{(k-1)\dot{\alpha}(k-1)} \right]$$

$$- (k-1)\Phi^{(k)\dot{\alpha}(k-2)} \mathcal{R}^{(k-2)\dot{\alpha}(k-2)} \mathcal{R}^{(k-2)\dot{\alpha}(k-2)} + h.c.]$$

The Lagrangian is invariant under gauge transformations

$$\delta \Phi^{(k)\dot{\alpha}(k-1)} = D_\xi \epsilon^{(k)\dot{\alpha}(k-1)} + e_\beta \dot{\alpha} \eta^{(k)\dot{\alpha}(k-2)} + \epsilon_{k+\frac{1}{2}} \lambda e^{(k)\dot{\alpha}(k-1)\dot{\beta}}$$

$$\delta \Phi^{(k-1)\dot{\alpha}(k)} = D_\xi \epsilon^{(k-1)\dot{\alpha}(k)} + e_\beta \dot{\alpha} \eta^{(k-2)\dot{\alpha}(k)} + \epsilon_{k+\frac{1}{2}} \lambda e^{(k-1)\dot{\alpha}(k-1)\dot{\beta}}$$

where $\epsilon_{k+\frac{1}{2}} = \pm 1$. Note that the above consideration does not fix a sign of $\epsilon_{k+\frac{1}{2}}$. As in integer spin case we can construct the gauge invariant curvatures

$$\mathcal{F}^{(k)\dot{\alpha}(k-1)} = D_\Phi \Phi^{(k)\dot{\alpha}(k-1)} + \epsilon_{k+\frac{1}{2}} \lambda e^{(k)\dot{\alpha}(k-1)\dot{\beta}}$$

$$\mathcal{F}^{(k-1)\dot{\alpha}(k)} = D_\Phi \Phi^{(k-1)\dot{\alpha}(k)} + \epsilon_{k+\frac{1}{2}} \lambda e^{(k-1)\dot{\alpha}(k-1)\dot{\beta}}$$

Then Lagrangian variation can be compactly written as follows

$$(-1)^k \delta \mathcal{L}_{k+\frac{1}{2}} = -\mathcal{F}^{(k)\dot{\alpha}(k-1)} \dot{\alpha} \mathcal{R}^{(k-1)\beta\dot{\alpha}(k-2)} + h.c.$$

Both in bosonic and in fermionic cases the variation of the higher spin Lagrangians is completely expressed in geometric terms.

### 3 Minimal $N = 1$ supermultiplets

In this section, we present the minimal massless $N = 1$ supermultiplets in $4D$ AdS. In the next sections, they will play the role of building blocks for construct the extended supermultiplets.
3.1 Higher superspins

Supermultiplet \((k+1/2, k)\) contains two massless fields with spin \(k\) and spin \(k+1/2\). They are described by the fields
\[
f^{\alpha(k-1)\dot{\alpha}(k-1)}, \quad \Omega^{\alpha(k)\dot{a}(k-2)}, \quad \Omega^{\alpha(k-2)\dot{a}(k)}
\]
and
\[
\Phi^{\alpha(k)\dot{a}(k-1)}, \quad \Phi^{\alpha(k-1)\dot{a}(k)}
\]
respectively. The corresponding supertransformations are written in the form
\[
\begin{align*}
\delta f^{\alpha(k-1)\dot{a}(k-1)} &= \alpha \Phi^{\alpha(k-1)\dot{a}(k-1)} \zeta_\beta - \bar{\alpha} \Phi^{\alpha(k-1)\dot{a}(k-1)} \bar{\zeta}_\beta \\
\delta \Phi^{\alpha(k)\dot{a}(k-1)} &= \beta \Omega^{\alpha(k)\dot{a}(k-2)} \dot{\zeta}^{\dot{a}} + \gamma f^{\alpha(k-1)\dot{a}(k-1)} \zeta^a \\
\delta \Phi^{\alpha(k-1)\dot{a}(k)} &= \bar{\beta} \Omega^{\alpha(k-2)\dot{a}(k)} \zeta^{a} + \bar{\gamma} f^{\alpha(k-1)\dot{a}(k-1)} \bar{\zeta}^{\dot{a}},
\end{align*}
\]
were \(\alpha, \beta, \gamma\) are the complex parameters. Parameters of the \(N = 1\) supertransformations \(\zeta^a, \bar{\zeta}^{\dot{a}}\) satisfy the relation
\[
D \zeta^a = -\lambda e^a_{\alpha} \zeta^\beta, \quad D \bar{\zeta}^{\dot{a}} = -\lambda e_{\dot{a}}^\beta \bar{\zeta}_\alpha. \tag{3.1}
\]
Note that here and further we do not introduce any supertransformation for auxiliary field \(\Omega\) since the calculations are done up to equations of motion for it \((2.2)\). Invariance of the Lagrangian, \(\delta (L_k + L_{k+\frac{1}{2}}) = 0\) requires the restrictions on the coefficients
\[
\alpha = i \frac{(k-1)}{4} \beta, \quad \gamma = \lambda \beta, \quad \beta = \epsilon_{k+\frac{1}{2}} \bar{\beta}, \quad \epsilon_{k+\frac{1}{2}} = \pm 1.
\]
The free complex parameter \(\beta\) can be taken pure real or pure imaginary. In AdS space it relates the sign of mass-like term for fermionic field and parity of bosonic field. Two cases \(\epsilon_{k+\frac{1}{2}} = \pm 1\) correspond to different \(N = 1\) massless supermultiplets with parity-even/odd boson. To fix the parameter \(\beta\) one calculates the commutator of two supertransformations on bosonic field
\[
\frac{1}{\rho} [\delta_1, \delta_2] f^{\alpha(k-1)\dot{a}(k-1)} = \Omega^{\alpha(k-1)\dot{a}(k-2)} \zeta^\beta \dot{\zeta}^{\dot{a}} + \bar{\Omega}^{\alpha(k-1)\dot{a}(k-1)} \bar{\zeta}_\beta \zeta^{a} + \lambda (f^{(k-2)\dot{a}(k-1)} \eta^\alpha_{\beta} + f^{(k-1)\dot{a}(k-2)} \eta^{\dot{a}}_{\dot{\beta}}), \tag{3.2}
\]
where
\[
\zeta^\beta \dot{\zeta}^{\dot{a}} = i (\zeta^{\beta} \zeta_{2\beta} - \zeta^{\dot{a}} \zeta_{1\beta}), \quad \eta^\alpha_{\beta} = i (\eta^\alpha_{\beta} \zeta_{2\beta} - \eta^{\dot{a}}_{\dot{\beta}} \zeta_{1\beta}) \tag{3.3}
\]
and
\[
\rho = \frac{(k-1)}{4} \bar{\beta} \beta.
\]
We see that the commutator of these supertransformations is combination of translation with parameter \(\xi^{\alpha\dot{a}}\) and Lorentz rotation with parameter \(\eta^\alpha_{\beta}, \eta^{\dot{a}}_{\dot{\beta}}\). It means that two corresponding supercharges \(Q_\alpha, Q_{\dot{\alpha}}\) satisfy the commutation relations of \(N = 1\), AdS superalgebra
\[
\{Q_\alpha, Q_{\beta}\} \sim P_{\alpha\dot{\beta}}, \quad \{Q_\alpha, Q_{\dot{\beta}}\} \sim \lambda M_{\alpha\beta}, \quad \{Q_{\dot{\alpha}}, Q_{\beta}\} \sim \lambda M_{\dot{\alpha}\dot{\beta}},
\]
where $P_{\dot{a}a}$, $M_{\alpha(2)}$, $M_{\dot{\alpha}(2)}$ are the AdS generators.

Supermultiplet $(k, k-1/2)$ contains massless integer spin $k$ and half-integer spin $k-1/2$. Corresponding fields are

$$f^{\alpha(k-1)\dot{a}(k-1)}, \quad \Omega^{\alpha(k)\dot{a}(k-2)}, \quad \Omega^{\alpha(k-2)\dot{a}(k)}$$

and

$$\Phi^{\alpha(k-1)\dot{a}(k-2)}, \quad \Phi^{\alpha(k-2)\dot{a}(k-1)}.$$ 

Supertransformations under the equations of motion for auxiliary field $\Omega$ (2.2) can be written

$$\delta f^{\alpha(k-1)\dot{a}(k-1)} = \alpha' \Phi^{\alpha(k-1)\dot{a}(k-2)} \zeta^{\dot{a}} - \bar{\alpha}' \Phi^{\alpha(k-2)\dot{a}(k-1)} \zeta^{\dot{a}},$$

$$\delta \Psi^{\alpha(k-1)\dot{a}(k-2)} = \beta' \Omega^{\alpha(k-1)\dot{b}(k-2)} \zeta^{\dot{b}} + \gamma' f^{\alpha(k-1)\dot{a}(k-2)} \zeta^{\dot{b}},$$

$$\delta \Psi^{\alpha(k-2)\dot{a}(k-1)} = \bar{\beta}' \Omega^{\alpha(k-2)\dot{b}(k-1)} \zeta^{\dot{b}} + \bar{\gamma}' f^{\alpha(k-2)\dot{b}(k-1)} \zeta^{\dot{b}}.$$ 

Lagrangian invariance $\delta (\mathcal{L}_k + \mathcal{L}_{k-\frac{1}{2}}) = 0$ gives

$$\alpha' = \frac{i}{4(k-1)} \bar{\beta}', \quad \gamma' = \lambda \beta', \quad \beta' = \epsilon_{k-\frac{1}{2}} \bar{\beta}', \quad \epsilon_{k-\frac{1}{2}} = \pm 1.$$ 

Again free parameter $\beta'$ can be pure real/imaginary. It corresponds to two different $N = 1$ massless supermultiplets with parity even/odd boson. Calculating commutator of two supertransformations which is equal (3.2) we fix $\beta'$

$$\rho = \frac{1}{4(k-1)} \bar{\beta}' \beta'.$$

### 3.2 Low superspins

Supermultiplet $(1, 3/2)$ contains the massless field with spin $3/2$ which is described by 1-forms $\Phi^\alpha$, $\Phi^{\dot{a}}$ with the Lagrangian (2.3) at $k = 1$

$$\mathcal{L}_{\frac{3}{2}} = -\Psi^\beta e^{\beta}_{\dot{\gamma}} D \Psi^{\dot{\gamma}} - \epsilon_{\frac{3}{2}} \lambda \Psi^\beta E^{\beta}_{\gamma} \Psi^\gamma + h.c.]$$

Massless spin 1 is described by dynamical 1-form $f$ and auxiliary 0-forms $W^{\alpha(2)}$, $W^{\dot{\alpha}(2)}$. The corresponding Lagrangian looks like

$$\frac{1}{k} \mathcal{L}_1 = 2E W^{\alpha(2)} + E^{\alpha(2)} D f + h.c.$$ 

It is evident that the Lagrangian is invariant under the gauge transformations

$$\delta f = D \xi, \quad \delta W^{\alpha(2)} = 0.$$ 

We do not introduce any supertransformation for auxiliary field $W^{\alpha(2)}$ since the calculations are done up to equations of motion which are equivalent to condition

$$\mathcal{T} = D f + 2(E^{\alpha(2)} W^{\alpha(2)} + E_{\alpha(2)} W_{\dot{\alpha}(2)}) \approx 0. \quad (3.4)$$
As an consequence of the above condition we have the relation
\[ E_{\alpha(2)} D W^{\alpha(2)} + E_{\dot{\alpha}(2)} D W^{\dot{\alpha}(2)} \approx 0. \]

Then the supertransformations can be rewritten in the form
\[
\begin{align*}
\delta f &= \alpha \Phi^{\alpha} \zeta - \bar{\alpha} \Phi^{\dot{\alpha}} \bar{\zeta}, \\
\delta \Phi^{\alpha} &= \beta e_{\beta} W^{\alpha\beta} \zeta + \gamma f \zeta \alpha, \\
\delta \Phi^{\dot{\alpha}} &= \bar{\beta} e_{\beta} W^{\alpha\dot{\beta}} \zeta + \bar{\gamma} f \zeta \dot{\alpha}.
\end{align*}
\]

Condition of the Lagrangian invariance \( \delta (L_1 + L_2) = 0 \) under the equation \( T \approx 0 \) yields
\[ \alpha = -i \frac{\bar{\beta}}{2}, \quad \gamma = -\frac{\lambda}{2} \beta, \quad \beta = \epsilon \frac{3}{2} \bar{\beta}, \quad \epsilon \frac{2}{2} = \pm 1. \]

Commutator of two supertransformations has the form
\[
\frac{1}{\rho} [\delta_1, \delta_2] f = -2 e_{\beta \bar{\beta}} (W^{\alpha\beta} \zeta^{\alpha \beta} + W^{\dot{\alpha} \dot{\beta}} \zeta^{\dot{\alpha} \dot{\beta}}), \quad (3.5)
\]
where \( \zeta^{\alpha \dot{\alpha}} \) is the same as in (3.3) and \( \rho = \frac{\bar{\beta} \beta'}{4} \).

**Supermultiplet (1, 1/2)** contains the massless spin 1 described in the same way as in the previous case and the massless spin 1/2 described by 0-forms \( Y^{\alpha}, Y^{\dot{\alpha}} \). The Lagrangian for spin 1/2 field has the form
\[ L = -Y_{\alpha} E^{\alpha}_{\ d} D Y^{\dot{\alpha}}. \]

Note that unlike the higher spin fermionic fields, there is no mass-like term in the above Lagrangian. Supertransformations, up to the equations of motion for auxiliary field \( W^{\alpha\beta} \), are written as follows
\[
\begin{align*}
\delta f &= \alpha' e_{\alpha \dot{\alpha}} Y^{\alpha} \zeta^{\dot{\alpha}} - \alphadot e_{\alpha \dot{\alpha}} Y^{\dot{\alpha}} \zeta^{\alpha}, \\
\delta Y^{\alpha} &= \beta' W^{\alpha\beta} \zeta^{\beta}, \\
\delta Y^{\dot{\alpha}} &= \bar{\beta}' W^{\dot{\alpha} \dot{\beta}} \zeta^{\dot{\beta}}.
\end{align*}
\]

Lagrangian invariance \( \delta (L_1 + L_2) = 0 \) under the equations \( T \approx 0 \) (3.4) yields
\[ \alpha' = -i \frac{\bar{\beta}'}{4}. \]

Calculating the commutator of the supertransformations leads to relations (3.5) and allows to fix the parameter \( \rho = \frac{\bar{\beta} \beta'}{8} \).

**Supermultiplet (0, 1/2)** contains the massless spin 1/2 and one massless spin 0 which is described by the dynamical 0-form \( W \) and the auxiliary 0-form \( W^{\alpha \dot{\alpha}} \). The Lagrangian for spin 0 has the form
\[
\frac{1}{i} L = -\frac{1}{2} E W_{\alpha \dot{\alpha}} W^{\alpha \dot{\alpha}} - E_{\alpha \dot{\alpha}} W^{\alpha \dot{\alpha}} D W + 2 \lambda^2 E W^2.
\]
Using the equation of motion for auxiliary field $W^{\alpha\dot{\alpha}}$

$$W = DW + e_{\alpha\dot{\alpha}}W^{\alpha\dot{\alpha}} \approx 0$$

one gets the relation

$$e_{\alpha\dot{\alpha}}DW^{\alpha\dot{\alpha}} \approx 0 \implies E^\alpha_{\dot{\alpha}}DW^{\beta\dot{\beta}} \approx \frac{1}{2} \varepsilon^{\alpha\beta\dot{\gamma}}E_{\dot{\gamma}\dot{\gamma}}DW^{\gamma\dot{\gamma}}.$$ 

We use the following anzac for supertransformations

$$\delta W = \alpha_0 Y^\alpha \zeta_\alpha - \bar{\alpha}_0 \bar{Y}^{\dot{\alpha}} \bar{\zeta}_{\dot{\alpha}},$$
$$\delta Y^\alpha = \beta_0 W^{\alpha\dot{\alpha}} \zeta_\alpha + \gamma_0 W^\alpha \zeta^\alpha,$$
$$\delta \bar{Y}^{\dot{\alpha}} = \bar{\beta}_0 \bar{W}^{\alpha\dot{\alpha}} \bar{\zeta}_\alpha + \bar{\gamma}_0 \bar{W}^\alpha \bar{\zeta}^{\dot{\alpha}}$$

with a set of arbitrary complex parameters $\alpha_0, \beta_0, \gamma_0$. Invariance of the Lagrangian $\delta(\mathcal{L}_0 + \mathcal{L}_\pm) = 0$ under the equation $W \approx 0$ gives restrictions on the parameters

$$\alpha_0 = \frac{i}{2} \bar{\beta}_0, \quad \gamma_0 = \lambda \beta_0.$$ 

Commutator of two supertransformations for spin 0 field has the form

$$\frac{1}{\rho} [\delta_1, \delta_2] W = W^{\alpha\dot{\alpha}} \xi_{\alpha\dot{\alpha}},$$

where $\xi_{\alpha\dot{\alpha}}$ is the parameter of translation (3.3) and $\rho = \bar{\beta}_0 \beta_0$. Parameter $\beta$ can be taken purely real/imaginary depending on supermultiplet with parity even/odd spin 0 field.

**Chiral supermultiplet** $(0_+, 0_-, 1/2)$ contains one massless spin 1/2 and two massless parity even/odd spins $0_+/0_-$. In this case the spin 0 is described by the complex scalar field $W$. The corresponding supertransformations have form

$$\delta W = 2\alpha_0 Y^\alpha \zeta_\alpha, \quad \delta Y^\alpha = \beta_0 W^{\alpha\dot{\alpha}} \zeta_\alpha + \gamma_0 W^\alpha \zeta^\alpha,$$
$$\delta \bar{W} = -2\bar{\alpha}_0 \bar{Y}^{\dot{\alpha}} \bar{\zeta}_{\dot{\alpha}}, \quad \delta \bar{Y}^{\dot{\alpha}} = \bar{\beta}_0 \bar{W}^{\alpha\dot{\alpha}} \bar{\zeta}_\alpha + \bar{\gamma}_0 \bar{W}^\alpha \bar{\zeta}^{\dot{\alpha}},$$

where

$$\alpha_0 = \frac{i}{2} \bar{\beta}_0, \quad \gamma_0 = \lambda \beta_0.$$ 

The commutators of the supertransformations are written as follows

$$\frac{1}{\rho} [\delta_1, \delta_2] W = W^{\alpha\dot{\alpha}} \zeta_{\alpha\dot{\alpha}}, \quad \frac{1}{\rho} [\delta_1, \delta_2] \bar{W} = \bar{W}^{\alpha\dot{\alpha}} \zeta_{\alpha\dot{\alpha}},$$

where $\rho = \bar{\beta}_0 \beta_0$. If to redenote the complex field in the form

$$W = W_+ + iW_- \quad W^{\alpha\dot{\alpha}} = W^{\alpha\dot{\alpha}}_+ + iW^{\alpha\dot{\alpha}}_-,$$
$$\bar{W} = W_+ - iW_- \quad \bar{W}^{\alpha\dot{\alpha}} = W^{\alpha\dot{\alpha}}_+ - iW^{\alpha\dot{\alpha}}_-,$$

then at real $\beta_0$ the field $W_+$ is parity even spin 0 field while $W_-$ is parity odd one. At imaginary $\beta_0$ the $W_+$ is parity odd field and $W_-$ is parity even one.
4 $N$-extended supermultiplets

In this section, we consider the massless $N$-extended higher spin supermultiplets in 4D AdS. As we pointed out in the Introduction, for given maximal integer or half-integer spin $k$ in the supermultiplet, the parameter $N$ satisfies the relation $N \leq 4k$. For each spin $k$ we describe the field contents and the corresponding field variables. Then, we derive the supertransformations and show that the specially defined free Lagrangians are invariant under these transformations. Finally, we prove that the constructed supertransformations form the on-shell closed $N$-extended 4D AdS superalgebras.

4.1 $N \leq 2k - 3$

In this case, the massless supermultiplets contain the massless fields with spins

$$k, k - \frac{1}{2}, k - 1, \ldots, k - \frac{N - 1}{2}, k - \frac{N}{2},$$

where $k$ is an arbitrary integer or half-integer. We will write it compactly as

$$k - \frac{m}{2}, \quad m = 0, 1, \ldots, N.$$

The number of massless fields with the given spin $k - \frac{m}{2}$ is equal to $\frac{N!}{m!(N-m)!}$. One can see that the minimal spin equals to $\frac{3}{2}$ in the boundary case $N = 2k - 3$. So all massless fields entering extended supermultiplets are uniformly described in the Section 1.

Let us introduce the bosonic field variables

$$f_{k - \frac{m}{2}, i[m]} \alpha^{(k - \frac{m+2}{2})\dot{\alpha}^{(k - \frac{m+2}{2})}}, \quad \Omega_{k - \frac{m}{2}, i[m]} \alpha^{(k - \frac{m}{2})\dot{\alpha}^{(k - \frac{m+4}{2})}}$$

and fermionic ones

$$\Phi_{k - \frac{m}{2}, i[m]} \alpha^{(k - \frac{m+1}{2})\dot{\alpha}^{(k - \frac{m+1}{2})}}.$$

Here the first lower index denotes the spin of the field, the compact index $i[m] = [i_1 i_2 \ldots i_m]$ denotes the antisymmetric combination of indices $i = 1, 2, \ldots, N$ and corresponds to antisymmetric representation of the internal symmetry group $SO(N)$. If the maximal spin $k$ is integer then $m$ takes even values $0, 2, \ldots, 2\left[\frac{N}{2}\right]$ for bosonic fields and odd values $1, 3, \ldots, 2\left[\frac{N-1}{2}\right] + 1$ for fermionic ones. In the case maximal half-integer spin $k$ the parameter $m$ takes the even values for fermions and odd ones for bosons.

The generic anzac for the linear supertransformations is chosen in the following form
with a set of arbitrary complex coefficients $\alpha_m, \alpha'_m, \beta_m, \beta'_m, \gamma_m, \gamma'_m$

$$
\delta f_{k-\frac{m}{2}, i[m]} \alpha(k-\frac{m+2}{2}) \dot{\alpha}(k-\frac{m+2}{2}) = \alpha'_m \Phi_{k-\frac{m+1}{2}, i[m]} \alpha(k-\frac{m+2}{2}) \dot{\alpha} \zeta \dot{\alpha} - \alpha'_m \Phi_{k-\frac{m+1}{2}, i[m]} \alpha(k-\frac{m+4}{2}) \dot{\alpha}(k-\frac{m+4}{2}) \zeta \dot{\alpha} + \alpha_m \Phi_{k-\frac{m+1}{2}, i[m]} \alpha(k-\frac{m+2}{2}) \beta \dot{\alpha}(k-\frac{m+2}{2}) \zeta_i \dot{\beta} - \alpha_m \Phi_{k-\frac{m+1}{2}, i[m]} \alpha(k-\frac{m+4}{2}) \dot{\alpha}(k-\frac{m+4}{2}) \zeta_i \dot{\beta} + \alpha_m \Phi_{k-\frac{m+1}{2}, i[m]} \alpha(k-\frac{m+2}{2}) \alpha(k-\frac{m+4}{2}) \dot{\alpha}(k-\frac{m+4}{2}) \zeta_i \dot{\beta} \tag{4.1}
$$

$$
\delta \Phi_{k-\frac{m}{2}, i[m]} \alpha(k-\frac{m+1}{2}) \dot{\alpha}(k-\frac{m+3}{2}) = \beta_m \Omega_{k-\frac{m+1}{2}, i[m]} \alpha(k-\frac{m+1}{2}) \dot{\alpha} \zeta \dot{\alpha} + \beta'_m \Omega_{k-\frac{m+1}{2}, i[m]} \alpha(k-\frac{m+1}{2}) \beta \dot{\alpha}(k-\frac{m+1}{2}) \zeta_i \dot{\beta} + \gamma_m \dot{f}_{k-\frac{m+1}{2}, i[m]} \alpha(k-\frac{m+1}{2}) \alpha(k-\frac{m+3}{2}) \dot{\alpha}(k-\frac{m+3}{2}) \zeta_i \dot{\beta} + \gamma'_m \dot{f}_{k-\frac{m+1}{2}, i[m]} \alpha(k-\frac{m+1}{2}) \alpha(k-\frac{m+3}{2}) \dot{\alpha}(k-\frac{m+3}{2}) \zeta_i \dot{\beta}. \tag{4.2}
$$

Here $\xi^{\alpha}_i, \xi^{\dot{\alpha}}_i$ are parameters of extended supertransformations satisfying the conditions (3.11). Lagrangian is defined as $L = \sum_m L_{k-\frac{m}{2}}$, where $L_{k-\frac{m}{2}}$ is the Lagrangian for free field with spin $k-\frac{m}{2}$. Invariance of the Lagrangian under these supertransformations leads to restrictions on the arbitrary parameters

$$
\alpha_m = \frac{i(k-\frac{m+2}{2})}{4} \beta_{m-1} \quad \gamma_m = \lambda \beta_m, \quad \beta_m = \epsilon_k \beta_m
$$

$$
\alpha'_m = \frac{i}{4(k-\frac{m+2}{2})} \beta_{m+1} \quad \gamma'_m = \lambda \beta'_m, \quad \beta'_m = \epsilon_k \beta'_m.
$$

In these relations $\epsilon_k = +1$ or $\epsilon_k = -1$ for any $m$ depending on parity of the corresponding field. It means that there are two families of parameters $\beta_m$ and $\beta'_m$. In order to relate them with each other we require the closer of the supertransformations algebra (4.1), (4.2). It yields the condition

$$
\alpha_m \beta_m - \alpha'_m \beta'_{m+1} = 0. \tag{4.3}
$$

Calculation of the commutator for the two supertransformations (4.1) allows to obtain the result

$$
\frac{1}{\rho} [\delta_1, \delta_2] \dot{f}_{k-\frac{m}{2}, i[m]} \alpha(k-\frac{m+2}{2}) \dot{\alpha}(k-\frac{m+2}{2}) = \Omega_{k-\frac{m}{2}, i[m]} \alpha(k-\frac{m+2}{2}) \beta \dot{\alpha}(k-\frac{m+2}{2}) \zeta \dot{\beta} + \Omega_{k-\frac{m}{2}, i[m]} \alpha(k-\frac{m+4}{2}) \dot{\alpha}(k-\frac{m+4}{2}) \zeta \dot{\beta} + \lambda \dot{f}_{k-\frac{m}{2}, i[m]} \alpha(k-\frac{m+2}{2}) \beta \dot{\alpha} \zeta_i \dot{\beta} \tag{4.4}
$$

where

$$
\xi^\beta_\dot{\alpha} = i(\zeta_{1j, \beta} \xi^j_\dot{\alpha} - \zeta_{2j, \beta} \xi^j_\dot{\alpha}), \quad \eta^\beta_\dot{\alpha} = i(\zeta_{1j, \dot{\alpha}} \xi^j_\beta - \zeta_{2j, \dot{\alpha}} \xi^j_\beta), \quad \zeta_i^j = i(\zeta_1^{i\beta} \zeta_2^{j\dot{\alpha}} - \zeta_2^{i\beta} \zeta_1^{j\dot{\alpha}}).
$$

$$
\zeta^j = -z^j, \quad \zeta^{ij} = -z^{ij}. \tag{4.6}
$$
One can see that commutator (4.4) is equal to combinations of translations, Lorentz rotations and internal \(\text{SO}(N)\) transformations with parameters \(\xi^{\alpha\dot{\alpha}}, \eta^{\alpha\beta}\) and \(z^{ij}\) respectively. From (4.3), (4.4) we have restriction for the parameters

\[
\beta_{m-1} \gamma_{m-1} = \frac{4\rho}{(k - \frac{m+2}{2})}, \quad \beta'_{m+1} \gamma'_{m+1} = 4\rho(k - \frac{m+2}{2}).
\]

The form of the above commutator shows to prove that the supercharges \(Q_{\alpha}^{i}, Q_{\dot{\alpha}}^{\dot{i}}\) corresponding to the supertransformations (4.1), (4.2) satisfy the commutation relations of the extended AdS superalgebra.

The supercharges \(Q_{\alpha}^{i}, Q_{\dot{\alpha}}^{\dot{i}}\) satisfy the commutation relations of the extended AdS superalgebra

\[
\begin{align*}
\{Q_{\alpha}^{i}, Q_{\dot{\alpha}}^{\dot{i}}\} &\sim \delta^{ij} P_{\alpha\dot{\beta}}, \\
\{Q_{\alpha}^{i}, Q_{\alpha}^{j}\} &\sim \lambda(\delta^{ij} M_{\alpha\beta} + \frac{1}{2} z_{\alpha\beta} T^{ij}), \\
\{Q_{\dot{\alpha}}^{\dot{i}}, Q_{\dot{\beta}}^{\dot{j}}\} &\sim \lambda(\delta^{ij} M_{\dot{\alpha}\dot{\beta}} + \frac{1}{2} \dot{z}_{\alpha\beta} T^{ij}),
\end{align*}
\]

where \(P_{\alpha\dot{\beta}}, M_{\alpha(2)}, M_{\dot{\alpha}(2)}\) are the AdS generators and \(T^{ij} = -T^{ji}\) are the generators of internal \(\text{SO}(N)\) group symmetry.

### 4.2 \(2k - 3 < N \leq 2k\)

In order to go beyond \(N > 2k - 3\) we should include the massless fields with lower spins to the supermultiplets. In the case \(N = 2k - 2\) it is sufficient to add the massless spin 1 and the corresponding fields

\[
f_{1,i[2k-2]}, \quad W_{i[2k-2]}^\alpha, \quad Y_{i[2k-1]}^\alpha, \quad Y_{i[2k-1]}^{\dot{\alpha}}.
\]

Analogically, in the cases \(N = 2k - 1\) and \(N = 2k\) we also should add the massless fields with spins \(\frac{1}{2}\)

\[
Y_{i[2k-1]}^\alpha, \quad Y_{i[2k-1]}^{\dot{\alpha}}
\]

and set of complex fields for spins 0

\[
W_{i[2k]}, \quad W_{i[2k]}^{\alpha\dot{\alpha}}, \quad \bar{W}_{i[2k]}, \quad \bar{W}_{i[2k]}^{\alpha\dot{\alpha}}.
\]

In this case the anzac for supertransformations with a set of arbitrary parameters looks like

\[
\begin{align*}
\delta \Phi_{\frac{1}{2}, i[2k-3]}^{\alpha} &\sim \beta'_{2k-3} \Omega_{2,i[2k-4]}^{\alpha\beta} \zeta_{i,\beta} + \gamma'_{2k-3} f_{2,i[2k-4]}^{\alpha\beta} \zeta_{i,\beta} + \beta'_{2k-3} \epsilon_{\alpha\beta} W_{i[2k-3]}^{\alpha\beta} \zeta_{i,\beta} + \gamma'_{2k-3} f_{1,i[2k-3]}^{\alpha\beta} \zeta_{i,\beta}, \\
\delta f_{1,i[2k-2]} &\sim \alpha_{2k-2} \Phi_{\frac{1}{2}, i[2k-3]}^{\alpha} \zeta_{i,\alpha} + \alpha'_{2k-2} \epsilon_{\alpha\beta} Y_{i[2k-2]}^{\alpha\beta} \zeta_{i,\beta},
\end{align*}
\]

\[
\begin{align*}
\delta Y_{i[2k-1]}^{\alpha} &\sim \beta'_{2k-1} W_{i[2k-2]}^{\alpha\beta} \zeta_{i,\beta} + \beta'_{2k-1} \epsilon_{\alpha\beta} \zeta_{i,\beta} + \gamma'_{2k-1} W_{i[2k-1]}^{\alpha\beta} \zeta_{i,\beta}, \\
\delta W_{i[2k]}^{\alpha} &\sim 2 \alpha_{2k} Y_{i[2k-1]}^{\alpha} \zeta_{i,\alpha} + \beta_{2k} \epsilon_{\alpha\beta} Y_{i[2k-1]}^{\alpha} \zeta_{i,\beta}.
\end{align*}
\]

\(\text{We have calculated such a commutator only for bosonic fields. For fermionic fields the result will be the same, however the computations became to be more complicated and tedious.}\)
Lagrangian is defined as a sum of Lagrangians for all fields in the supermultiplet. Invariance of the Lagrangian yields restrictions for the arbitrary parameters

\[ \alpha_{2k-2} = -i \frac{\bar{\beta}_{2k-3}}{2}, \quad \gamma_{2k-3} = -\frac{\lambda}{2} \beta_{2k-3}, \quad \beta_{2k-3} = \epsilon \bar{\beta}_{2k-3}, \quad \epsilon = \pm 1, \]

\[ \alpha'_{2k-2} = -i \frac{\bar{\beta}_{2k-1}}{4}, \quad \alpha_{2k} = i \bar{\beta}_{2k-1}, \quad \gamma_{2k-1} = \lambda \beta_{2k-1}. \]

As a result, there are three arbitrary complex parameters \( \beta_{2k-3}, \beta'_{2k-1} \) and \( \beta_{2k-1} \) which can be either purely real or purely imaginary depending on even or odd parity of bosonic fields entering supermultiplets. We fix them by the requirement that commutators for spin 1 and spin 0 are closed. It gives the condition

\[ \alpha_{2k-2} \beta_{2k-3} - \alpha'_{2k-2} \beta'_{2k-1} = 0. \] (4.12)

Then the commutators for the spin 1 field has the following form up to the gauge transformations

\[ \frac{1}{\rho} [\delta_1, \delta_2] f_i[2k-2] = -2e_{\alpha \dot{\alpha}} (W_i[2k-2]^{\alpha \beta} \xi^\beta \dot{\alpha} + W_i[2k-2]^{\dot{\alpha} \dot{\beta}} \xi^\alpha \dot{\beta}) + \lambda f_i[2k-3] j^i z^j, \] (4.13)

\[ \frac{1}{\rho} [\delta_1, \delta_2] W_i[2k] = W_i[2k]^{\alpha \dot{\alpha}} \xi^\alpha \dot{\alpha}, \] (4.14)

where \( \xi^{\alpha \dot{\alpha}} \) and \( z^{ij} \) are parameters of translations and internal \( SO(N) \) symmetry defined by (4.5) and (4.6). The relations (4.12), (4.13), (4.14) lead to the conditions for the parameters

\[ \bar{\beta}_{2k-3} \beta_{2k-3} = 4 \rho, \quad \bar{\beta}'_{2k-1} \beta'_{2k-1} = 8 \rho, \quad \bar{\beta}_{2k-1} \beta_{2k-1} = \rho. \]

Using the commutators (4.13), (4.14) we can show that the corresponding supercharges satisfy the relations (4.7).

4.3 \( 2k < N < 4k \)

To extend supersymmetry further, i.e. to consider the case \( N > 2k \) we should include into supermultiplets more massless fields with spins

\[ \frac{1}{2}, 1, \frac{1}{2} - k, \quad m = 2k + 3, 2k + 4, ..., N - 1, N. \]

We introduce the additional field variables for spin \( \frac{1}{2} \)

\[ Y_i[2k+1]^{\alpha}, \quad Y_i[2k+1]^{\dot{\alpha}}, \]

for spin \( 1 \)

\[ f_{1,i}[2k+2], \quad W_i[2k+2]^{\alpha(2)}, \]

and for higher spins

\[ f_{\frac{m}{2}-k,i}[m]^{\alpha\left(\frac{m-2}{2}-k\right)\dot{\alpha}\left(\frac{m-2}{2}-k\right)}, \quad \Omega_{\frac{m}{2}-k,i}[m]^{\alpha\left(\frac{m-4}{2}-k\right)\dot{\alpha}\left(\frac{m-4}{2}-k\right)}, \quad m = 2k + 4, ..., 2k + 4, \]

\[ \left\lfloor \frac{N}{2} \right\rfloor. \]
\[ \Phi_{m-k,i[m]} \alpha^{(m-1-k)} \beta^{(m-k)} = m = 2k + 3, \ldots, 2\left[ \frac{N-1}{2} \right] + 1 \]

Additional anzac for the supertransformations is chosen in the following form with a set of arbitrary parameters

\[
\delta f_{m-k,i[m]}^{(m-1-k)} = \alpha_{m} \Phi_{m-k,i[m]}^{(m-1-k)} \beta_{m}^{(m-k)} + \alpha_{m} \Phi_{m-k,i[m]}^{(m-1-k)} \beta_{m}^{(m-k)} \xi_{i}^{j} \beta_{i}^{j} + h.c. \quad m \geq 2k + 4,
\]

\[
\delta \Phi_{m-k,i[m]}^{(m-1-k)} = \beta_{m} \Omega_{m-k,i[m]}^{(m-1-k)} \alpha^{(m-1-k)} \beta^{(m-k)} \xi_{i}^{j} \beta_{i}^{j} + \beta_{m} \Omega_{m-k,i[m]}^{(m-1-k)} \alpha^{(m-1-k)} \beta^{(m-k)} \xi_{i}^{j} \beta_{i}^{j} + \gamma_{m} f_{m-k,i[m]}^{(m-1-k)} \alpha^{(m-1-k)} \beta^{(m-k)} \xi_{i}^{j} \beta_{i}^{j} + \gamma_{m} f_{m-k,i[m]}^{(m-1-k)} \alpha^{(m-1-k)} \beta^{(m-k)} \xi_{i}^{j} \beta_{i}^{j} \quad m \geq 2k + 5,
\]

Lagrangian is defined as a sum of the Lagrangians for all fields in the supermultiplet. Condition of invariance of the Lagrangian invariant yields restrictions on the arbitrary parameters

\[
\alpha_{m} = \frac{i}{4(m-2-k)} \beta_{m-1}, \quad \gamma_{m} = \lambda \beta_{m}, \quad \beta_{m} = \epsilon_{m-k} \beta_{m},
\]

\[
\alpha'_{m} = \frac{i(m-2-k)}{4} \beta'_{m-1}, \quad \gamma'_{m} = \lambda \beta'_{m}, \quad \beta'_{m} = \epsilon_{m-k} \beta'_{m},
\]

\[
\alpha''_{2k+1} = -i \frac{\beta''_{2k+3}}{2}, \quad \gamma''_{2k+3} = -\lambda \frac{\beta''_{2k+3}}{2}, \quad \beta''_{2k+3} = \epsilon \frac{\beta''_{2k+3}}{2}, \quad \epsilon_{2} = \pm 1,
\]

\[
\alpha''_{2k+2} = -i \frac{\beta''_{2k+1}}{4}, \quad \alpha''_{2k+1} = \frac{i}{2} \beta''_{2k+1}, \quad \gamma''_{2k+1} = \lambda \beta''_{2k+1}.
\]

To close algebra of the supertransformations we impose the conditions

\[
\alpha_{m} \beta_{m-1} - \alpha'_{m} \beta'_{m+1} = 0, \quad \alpha_{2k+2} \beta_{2k+1} - \alpha'_{2k+2} \beta'_{2k+3} = 0, \quad 2\alpha_{2k} \beta_{2k-1} + 2\alpha'_{2k} \beta'_{2k+1} = 0 \quad (4.15)
\]
As a result, the commutators of supertransformations have the form
\[ \frac{1}{\rho}[\delta_1, \delta_2] f_{m-k,i,j} = \Omega_{m-k,i,j} \alpha(k-\frac{m+4}{2}) \hat{\alpha}(k-\frac{m+2}{2}) \xi_\beta \hat{\xi}_\beta + \Omega_{m-k,i,j} \alpha(k-\frac{m+2}{2}) \hat{\alpha}(k-\frac{m+4}{2}) \eta_\beta \hat{\eta}_\beta + \lambda(f_{m-k,i,j} \alpha(k-\frac{m+2}{2}) \hat{\alpha}(k-\frac{m+4}{2}) \eta_\beta \hat{\eta}_\beta + f_{m-k,i,j} \alpha(k-\frac{m+4}{2}) \hat{\alpha}(k-\frac{m+2}{2}) \eta_\beta \hat{\eta}_\beta) \]
\[ (4.16) \]
\[ \frac{1}{\rho}[\delta_1, \delta_2] f_{1,i[2k+2]} = -2e_{\alpha \hat{\alpha}} (W_{i[2k+2]} \alpha \beta \hat{\xi}_\beta \hat{\xi}_\beta + W_{i[2k+2]} \alpha \beta \hat{\xi}_\beta \hat{\xi}_\beta) + \lambda f_{i[2k-3]} \alpha \beta \hat{\xi}_\beta \hat{\xi}_\beta, \]
\[ \frac{1}{\rho}[\delta_1, \delta_2] W_{i[2k]} = W_{i[2k]} \alpha \beta \hat{\xi}_\beta \hat{\xi}_\beta + \lambda W_{i[2k-1]} \alpha \beta \hat{\xi}_\beta \hat{\xi}_\beta, \]
where \( \xi_{\alpha \hat{\alpha}}, \eta_{\alpha \beta} \) and \( z^{ij} \) are defined by (4.5) and (4.6). Form of the above commutators and relation (4.15) allows to find the additional restrictions for the arbitrary parameters
\[ \bar{\beta}_{m-1} \beta_{m-1} = 4\rho \left( \frac{m-2}{2} - k \right), \quad \bar{\beta}'_{m+1} \beta'_{m+1} = \frac{4\rho}{\left( \frac{m-2}{2} - k \right)}, \]
\[ \bar{\beta}'_{2k+3} \beta'_{2k+3} = 4\rho, \quad \bar{\beta}_{2k+1} \beta_{2k+1} = 8\rho, \quad \bar{\beta}'_{2k+1} \beta'_{2k+1} = \rho. \]
The commutators (4.16) allows to calculate the algebra of supercharges which will have the form (4.7). As a result we obtain the on-shell \( N \)-extended component free Lagrangian formulations for the supermultiplets with \( 2k < N < 4k \).

### 4.4 \( N = 4k \)

Now we consider a special case of maximal \( N \)-extended supersymmetry with the highest spin \( k \) in the supermultiplet. Such a supermultiplet contains the massless fields with all spins from \( 0 \) to \( k \). Field variables are the same as in the \( N = 2k \) case but now \( i = 1, 2, ..., 4k \)
\[ f_{k-i,j} \alpha(k-\frac{m+2}{2}) \hat{\alpha}(k-\frac{m+4}{2}) \]
\[ \Omega_{k-i,j} \alpha(k-\frac{m+4}{2}) \hat{\alpha}(k-\frac{m+2}{2}) \]
for bosonic higher spin fields and
\[ \Phi_{k-i,j} \alpha(k-\frac{m+1}{2}) \hat{\alpha}(k-\frac{m+3}{2}) \]
for fermionic higher spin fields. For spin 1 we introduce the fields
\[ f_{1,i[2k-2]}, \quad W_{i[2k-2]} \alpha(2) \]
for spin 1/2 we introduce the fields
\[ Y_{i[2k-1]} \alpha, \quad Y_{i[2k-1]} \hat{\alpha}. \]
Besides, we introduce a set of complex fields for spin 0
\[ W_{i[2k]}, \quad W_{i[2k]} \alpha, \quad W_{i[2k]} \hat{\alpha}, \quad W_{i[2k]} \hat{\alpha}. \]
which are subject to the condition
\[ W_{i[2k]} = \frac{1}{(2k)!} \mathcal{E}_{i[2k]}^j[2k] \bar{W}_{j[2k]}, \]  
(4.17)

where \( k \) is an arbitrary integer. As before all field variables are totally antisymmetric over the indices \( i = 1, 2, \ldots, N \). Totally antisymmetric invariant tensors \( \mathcal{E}_{i[4k]} = \mathcal{E}_{i_1i_2\ldots i_{4k}} \) are normalized as follows \( \mathcal{E}_{12\ldots4k} = 1 \).

The supertransformations in the case under consideration are the same as in the case \( N = 2k \; (4.1), (4.2), (4.1), (4.8), (4.9), (4.10) \). While for the spin 0 components, the supertransformations compatible with (4.17) look like
\[
\delta W_{i[2k]} = 2\alpha_{2k} Y_{i[2k-1]}^\alpha \bar{\zeta}_{i\dot{\alpha}} - \frac{2\bar{\alpha}_{2k}}{(2k - 1)!} \mathcal{E}_{i[2k]}^j[2k] Y_{j[2k-1]}^\dot{\alpha} \bar{\zeta}_{i\dot{\alpha}}, \\
\delta \bar{W}_{i[2k]} = -2\bar{\alpha}_{2k} Y_{i[2k-1]}^{\dot{\alpha}} \bar{\zeta}_{i\alpha} + \frac{2\alpha_{2k}}{(2k - 1)!} \mathcal{E}_{i[2k]}^j[2k] Y_{j[2k-1]}^{\dot{\alpha}} \bar{\zeta}_{i\dot{\alpha}}.
\]  
(4.18)

In this case, again the algebra of supercharges has the form (4.7) and the Lagrangian is invariant under the transformations (4.1), (4.2), (4.1), (4.8), (4.9), (4.10), (4.18).

To conclude this subsection one notes that we have studied here only the case of maximal integer spin \( k \). In the case of maximal half-integer spin the above consideration is not applicable since the relation (4.17) is inconsistent with the half-integer \( k \). This case requires a special analysis. The matter is that the consistent usage of the generic scheme described the subsection 4.3 leads to double of all the fields in the supermultiplet. In the case of maximal integer spin we can avoid this doubling if one imposes, in particular, the condition (4.17). In the case of maximal half-integer spin such a condition is impossible.\(^4\)

5 Summary and Prospects

In this paper we have studied the field realization of arbitrary \( N \)-extended massless supermultiplets in 4D AdS space. For arbitrary highest integer or half-integer spin \( k \) fields entering the supermultiplets we realized the on-shell supersymmetric component Lagrangian formulations under the condition \( N < 4k \) and defined the higher superspin field Lagrangians as the sums of free Lagrangians for all the fields in the supermultiplets. We have constructed the supertransformations which form the closed on-shell algebras and leave invariant the Lagrangians. It is shown that the commutators of two such supertransformations form the \( N \)-extended AdS superalgebra, i.e. they are the combinations of the translations, Lorentz rotations and internal \( SO(N) \)-transformations. Also, we have realized the maximally extended supermultiplets where \( N = 4k \) in the case of the highest integer spin \( k \) and constructed the corresponding supertransformations.

We hope that our results can be helpful for the construction of the extended massive higher spin and massless infinite spin supermultiplets and their Lagrangian formulations extending the results of the \( N = 1 \) case [37, 39]. Besides, we point out some other open problems in the free supersymmetric higher spin field theory. First of all, this is a problem of the

\(^4\)We are grateful to Yu.M. Zinoviev for discussion of this question.
superfield Lagrangian formulation of 4D, \( N = 1 \) supersymmetric massive higher superspin fields. The corresponding massless theories have been constructed in the works [29, 30, 32]. As to massive theories, there are only partial examples of the higher superspin massive \( N = 1 \) superfield models [41]. Problem of formulating the extended supersymmetric higher spin theories in terms of unconstrained superfields is completely open. At present, the only case where such a possibility can in principle be realized is the 4D, \( N = 2 \) supersymmetry, where harmonic superfield approach [42] allows to construct the field models in terms of \( N = 2 \) unconstrained superfields. We hope to study some of these open problems in the forthcoming papers.

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A Notations and conventions

In the paper, we adopt the "condensed notation" for the indices. Namely, if some expression contains \( n \) consecutive indices, denoted by the same letter with different numbers (e.g. \( \alpha_1, \alpha_2, \ldots, \alpha_n \)) and is symmetric on them, we simply write the letter, with the number \( n \) in parentheses if \( n > 1 \) (e.g. \( \alpha(n) \)). For example:

\[
\Phi^{\alpha_1, \alpha_2, \alpha_3} = \Phi^{\alpha(3)}, \quad \zeta^{\alpha_1 \Omega^{\alpha_2 \alpha_3}} = \zeta^{\alpha \Omega^{(2)}}
\]

(A.1)

We define symmetrization over indices as the sum of the minimal number of terms necessary without normalization multiplier.

We use the multispinor formalism in four dimensions (see e.g. [40]). Every vector index is transformed into a pair of spinor indices: \( V^\mu \sim V^{\alpha \dot{\alpha}} \), where \( \alpha, \dot{\alpha} = 1, 2 \). Dotted and undotted indices are transformed into one another under the hermitian conjugation:

\[
(\Omega^{\alpha \dot{\alpha}(2)})^\dagger = \Omega^{\alpha(2) \dot{\alpha}}
\]

(A.2)

The spin-tensors, i.e. fields with odd number of indices, are Grassmannian. For example,

\[
A^{(2) \dot{\alpha} \eta^\alpha} = -\eta^\alpha A^{\alpha(2) \dot{\alpha}}
\]

(A.3)

Under the hermitian conjugation, the order of fields is reversed:

\[
(A^{\alpha(2) \dot{\alpha} \eta^\alpha})^\dagger = \eta^\alpha A^{\alpha(2) \dot{\alpha}} = -A^{(2) \dot{\alpha} \eta^\alpha}
\]

(A.4)

The metrics for the spinor indices is an antisymmetric bispinor \( \epsilon_{\alpha \beta} \) and inverse \( \epsilon^{\alpha \beta} \):

\[
\epsilon_{\alpha \beta} \xi^\beta = -\xi^\alpha, \quad \epsilon^{\alpha \beta} \xi_\beta = \xi^\alpha
\]

(A.5)
similarly for dotted indices.

In the frame-like formalism, two bases, namely the world one and the local one are used. We denote the local basis vectors as $e^{\alpha\dot{\alpha}}$; the world indices are omitted, and all the fields are assumed to be the differential forms. Similarly, all the products of the forms are exterior with respect to the world indices. In the paper, we use the basis forms, i.e. antisymmetrized products of basis vectors $e^{\alpha\dot{\alpha}}$. The forms are the 2-form $E^{\alpha(2)} + h.c.$, the 3-form $E^{\alpha\dot{\alpha}}$ and the 4-form $E$. The transformation law of the forms under the hermitian conjugation is:

\[
(e^{\alpha\dot{\alpha}})^\dagger = e^{\alpha\dot{\alpha}}, \quad (E^{\alpha(2)})^\dagger = E^{\dot{\alpha}(2)}, \quad (E^{\alpha\dot{\alpha}})^\dagger = -E^{\alpha\dot{\alpha}}, \quad (E)^\dagger = -E
\] (A.6)

The covariant AdS derivative satisfies the following normalization conditions:

\[
D \wedge D \Omega^{\alpha(m)\dot{\alpha}(n)} = -2\lambda^2(E^\alpha_\beta \Omega^{\alpha(m-1)\beta\dot{\alpha}(n)} + E^{\dot{\alpha}}_\beta \Omega^{\alpha(m)\dot{\alpha}(n-1)\beta})
\] (A.7)

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