Approximation of Dielectric Properties using Linear Regression Model in Microwave Tomography

L. Mohamed\(^1\), Y. Kuwahara\(^2\)

\(^1\) Faculty of Electrical Technology Engineering, UniMAP, Arau, Perlis, Malaysia.
\(^2\) Department of Information Science and Technology, GSST, Shizuoka University, Japan.

Abstract. The Finite Difference Time Domain (FDTD) is the most commonly used method of numerical simulation to model the electromagnetic waves propagation in biological tissue. An MRI-derived FDTD breast model has been developed and characterized to its corresponding dielectric properties. The relative permittivity and conductivity are correlated using least-square fitting criteria to obtain a linear regression model, which provides the approximation of dielectric properties. This technique reduces the number of unknowns to be reconstructed to one parameter in the analysis region, hence decrease the complexity and computational cost of the microwave imaging problem, without sacrificing the imaging resolution.

1. Introduction

Microwave imaging (MWI) studies for early breast cancer detection have gained considerable interest [1]. In microwave tomography, for precise image reconstruction of the dielectric properties of the imaging region, the amount of diverse observation data needs to be increased. The multi-polarization approach was proposed as an efficient technique for collecting a variety of data from observations and its efficiency in image reconstruction was verified in [2][3].

The Finite Difference Time Domain (FDTD) approach is the commonly applied method of numerical simulation to represent the propagation of electromagnetic waves in biological tissue [4]. FDTD can model the propagation and accurately estimate the scattering field in the forward problem. In addition, through the use of the FDTD method, greater resolution and sensitivity of image restoration can be achieved. The breast’s physiological geometry, the actual heterogeneity and the nature dispersive of ordinary breast tissue must be taken into account by an accurate FDTD breast model. This approach differs from the method suggested in [3], which models the randomly distributed variance of dielectric properties.

Microwave-based detection of breast cancer relies on broad difference in electromagnetic properties between normal and malignant (cancer) tissues. Assume the inhomogeneous medium with a finite volume \(S\) is enclosed in uniform background relative permittivity \(\varepsilon^b\) and background conductivity \(\sigma^b\). At a point \(r(x,y,z)\), the complex permittivity \(\varepsilon^c(r)\) is denoted by Equation (1).

\[
\varepsilon^c(r) = \sigma(r) + j\omega\varepsilon_0\varepsilon_i(r) \tag{1}
\]

In quantitative microwave imaging, considering the biological tissues as dielectrics, the dielectric properties are reconstructed regarding the differences in the complex permittivity. Therefore, at a point \(r\), the contrast function, \(\chi\) is determined by Equation (2).
\[
\chi = (\sigma - \sigma^b) + j\omega\varepsilon_0(\varepsilon_r - \varepsilon^b)
\]

(2)

Where \(\varepsilon_0\) is the permittivity of free space, and \(\omega\) is the angular frequency. The two unknown parameters to be measured are the relative permittivity, \(\varepsilon_r\), and conductivity, \(\sigma\), of the imaging object. In order to obtain an accurate image reconstruction with high resolution using the FDTD method, the two parameters could lead to a large number of unknowns. As proposed in [5], a linear relationship between two parameters can be introduced to build an analytical model. This approach decreases the number of unknowns in the computation to one parameter per cell.

In this paper, a small sized imaging sensor with the multi-polarization configuration as proposed in [6] is considered. The Distorted Born Iterative Method (DBIM) described in [7] is used to reconstruct the image. The effectiveness of employing the analytical method in image reconstruction of an FDTD-based breast model is investigated.

2. Numerical Simulation Model

2.1. FDTD breast model

Figure 1(a) shows a numerical breast model constructed on the basis of the real MRI images acquired from Hamamatsu University School of Medicine, Shizuoka Japan. The 1.0 mm grid model of the breast consists of chest wall (muscle), cancer, skin, fibro-glandular tissue and adipose tissue [6]. In FDTD numerical simulation, the breast model's grid dimension is reduced to 3.0 mm.

For simplicity of the analysis region as demonstrated in Figure 1(b), the adipose tissue, fibro-glandular tissue and malignant tissue were considered and the background, muscle and skin were omitted. The model of the breast is defined by its corresponding dielectric properties, as tabulated in Table 1. In breast cancer detection, the high contrast between adipose and malignant tissue contributes an important factor for the benefits of microwave imaging. 6727 cells are the number of unknowns to be reconstructed within the analysis region.

![Figure 1](image)

(a) MRI-derived numerical model of breast

![Figure 1](image)

(b) MRI-derived FDTD breast model (analysis region)

Figure 1. MRI-derived breast model.
2.2. Antenna Configuration
The dimensions of the enclosed antenna array are 144 mm wide x 144 mm long x 60 mm high. The vertical polarization is taken to be the z-axis and the horizontal polarization is x-axis or y-axis. As a further simplification, a linear dipole with 1.8 GHz of a single frequency is used to simulate the transmit pulse. Figure 2 shows the 32 element antenna array with multi-polarization configuration. The antenna configurations for each side are same, and either the xz- or yz- plane is parallel to the four side panels.

![Figure 2. Antenna array for multi-polarization.](image)

3. Polynomial Fit for a Linear Model
In order to achieve a linear regression model (polynomial fit) to mend their discrepancy, the dielectric properties of conductivity, $\sigma$, and relative permittivity, $\varepsilon_r$, are correlated. A linear regression model is defined as an equation that can be depicted by a first-degree polynomial and is linear in the coefficients. Based on the Least Square (LS) method in [5], the polynomial fit is utilized to align the data with a linear model to data and yield the estimation of dielectric properties.

As the fibro-glandular tissue, malignant tissue and adipose tissue are the unknown parameters in the analysis region, the linear regression model in Figure 3 with Equation (3) is selected as the suitable linear model to estimate the parameters involved.

![Figure 3. Linear model based on Figure 1(b).](image)

$$\sigma = 0.0233\varepsilon_r + 0.0531 \quad (3)$$

Equation (3) is adapted to the contrast function, $\chi$, to substitute the conductivity, $\sigma$, in the reconstruction algorithm. Therefore, the unknown parameter for the linear regression model in the contrast function is limited to only one parameter, i.e. the relative permittivity, $\varepsilon_r$. This approach can reduce the number of unknown parameters in the reconstruction algorithm to estimate the parameters or contrast perturbations.
4. Numerical Simulation Results

The results for the non-linear and linear model after 10 iterations are presented. The Distorted Born Iterative Method (DBIM) described in [7] is used to reconstruct the image. The initial approximation is presented by \( \{ \varepsilon_r, \sigma \} = \{ 6.5, 0.20 \text{ [S/m]} \} \), which is equal to the adipose tissue. A clear bound restriction is added to the contrast perturbations to minimise the range of the optimal solution that contributes to an underdetermined and ill-posed system.

Figure 4 shows the actual and reconstruction images in axial view, where each image is divided to the relative permittivity (left) and conductivity (right). In the true model of Figure 4(a), the adipose tissue, fibro-glandular tissue, and malignant tissue (cancer) are demonstrated in blue, cyan and yellow (arrow), respectively, for both parameters.

The fibro-glandular tissue was gradually reconstructed for the non-linear model in Figure 4(b), but the presence of cancer is unrecognized. In comparison, when using the linear model, the location and shape of the cancer (arrow) are clearly noticeable, as shown in Figure 4(c). After ten iterations, the linear regression model shows good image reconstruction compared to the non-linear model. The result shows that the reduction of the unknown parameters by using the linear regression model will lessen the ill-posedness of the inverse problem.

**Figure 4.** Image reconstruction in axial view (xz- axis)
Figure 5 shows the analysis region with the distribution of dielectric properties. Each images is divided to relative permittivity (left) and conductivity (right). The x-axis shows the number of voxel, whilst the y-axis shows the true and approximated value for either conductivity or relative permittivity. The true and approximated values are denoted by red solid line and blue asterisk, respectively. After ten iterations, the approximated values in Figure 5(a) are low approximated, whilst the values are highly estimated in Figure 5(b). The regression linear model in Figure 5(b) demonstrates the approximation values of the non-linear model, hence yield an adequate image reconstruction.

5. Conclusion
The effectiveness of employing the analytical model in image reconstruction of an FDTD-based breast model is investigated. The analytical model is derived based on the polynomial fit to obtain a linear model, which reduces the number of unknowns to be reconstructed to one parameter in the analysis region. From results above, the linear regression model intensify the performance of non-linear model to provide better reconstruction, even in a small iteration number. Hence, the complexity and computational cost of the microwave imaging problem can be reduced, without sacrificing the imaging resolution.
6. References

[1] N. K. Nikolova, “Microwave Imaging for Breast Cancer,” IEEE Microwave Magazine, Vol. 12, no. 7, pp. 78–94, 2011.
[2] R. Owen Mays, N. Behdad, and S. C. Hagness, “A TSVD Analysis of the Impact of Polarization on Microwave Breast Imaging Using an Enclosed Array of Miniaturized Patch Antennas,” IEEE Antennas Wireless Propagation Letter, Vol. 14, pp. 418–421, 2015.
[3] L. Mohamed, Y. Ono, T. Kamiya, and N. Ozawa, “Study of Multi-Polarization in Microwave Tomography for Breast Cancer Detection,” IEICE (in Japanese), Vol. J99–C, no. 8, pp. 1–9, 2016.
[4] M. O’Halloran, R. Conceicao, D. Byrne, M. Glavin, and E. Jones, “FDTD Modeling Of The Breast: A Review,” Prog. Electromagnetic Res. B, Vol. 18, pp. 1–24, 2009.
[5] Ikram-e-Khuda, S. Khatun, K. J. Reza, M. M. Rahman, and M. M. Fakir, “Improved Debye Model for Experimental Approximation of Human Breast Tissue Properties at 6 GHz Ultra-Wideband Centre Frequency,” Int. J. Engineering Technology, Vol. 5, no. 6, pp. 4708–4717, 2014.
[6] Y. Ono, L. Mohamed, and Y. Kuwahara, “Optimization of Configuration on Imaging Sensor for Microwave Tomography,” in Proceedings of the 2016 IEICE Society Conference, 2016, Vol. C–2–79, no. ISSN 1349–144X (in Japanese).
[7] J. D. Shea, P. Kosmas, B. D. Van Veen, and S. C. Hagness, “Contrast-Enhanced Microwave Imaging Of Breast Tumors: A Computational Study Using 3D Realistic Numerical Phantoms,” Inverse Problem, Vol. 26, no. 074009, pp. 1–22, 2010.

Acknowledgments

The author would like to acknowledge Faculty of Electrical Engineering Technology (FTKE), Universiti Malaysia Perlis (UNIMAP) for the funding of this paper.