Stability and Control of the Convergence Point for Two-strand Yarn Spinning

Wenyuan Liu\textsuperscript{1}, Yanping Yu\textsuperscript{2}, Feng Li\textsuperscript{1} and Yajun Zhao\textsuperscript{1}

\textsuperscript{1}China Construction Science & Technology (Beijing) CO.TLD. Longhe Hi-tech Park Tiangao Road No.86, Anci District, Langfang City, Hebei Province, China
\textsuperscript{2}College of Textiles, Donghua University, Shanghai 201620, China
Email: w.y.liu@163.com

Abstract: In this paper, stability of the convergence point in Siro-spinning composite area and control of tension compensator (TC) to the unstable motion of the convergence point in Sirofil spinning were kinetically studied. The control effect was compared with corresponding experiments. It is concluded that the adoption of TC can greatly improve the stability of the system, apparently reduce the vibration of the convergence point, and simultaneously reduce the hairiness of Sirofil composite yarn and improve the property of abrasion-resistance enormously.

1. Introduction

Siro-spinning, one kind of ring spinning, is now widely employed in the textile industry. It is known that Sirospun yarns are conducted on a conventional ring frame by feeding two rovings, drafted simultaneously, into the apron zone at a predetermined separation. Emerging from the nip point of the front rollers, the two strands are twisted together to form the two-strand yarn. It is called Sirofil spinning if men substitute one filament yarn for one of the two rovings in the spinning process \cite{1} (also see Figure 1). Mechanics and mathematics approaches have been involved in solving the problems in such a spinning process. Quasistatic models by He et al. \cite{2}, Yu et al.\cite{5} and Liu \cite{6}, and dynamic models by He et al. \cite{3} \cite{4} for two-strand yarn spinning to analyze instability phenomenon of Siro-spinning have been established. We also studied the control of TC to the properties of Sirofil composite yarn from the static point of view \cite{7}, and now we study the control of TC to the unstable movement of the convergence point in the view of dynamics.

In Sirospun spinning, the component and modulus of the two rovings are the same. The twisting is stable and balanced, and the structure of resultant yarn is regular. But in Sirofil spinning, due to the modulus difference between roving and filament yarn, the two components are distributed asymmetrically over the Sirofil composite area, and the convergence point is thus unstable, leading to irregularity of yarn structure, i.e., staple yarn partially wrapping filament yarn. A tension compensator is introduced to control the irregular vibration of the convergence point, so as to improve the unevenness and the abrasion property of Sirofil composite yarn \cite{7}. In this paper, from the dynamical point of view, we study the movement of convergence point before and after the control of TC, and the effect of TC on the stability of Sirofil composite area is apparently shown. It is analytically and qualitatively seen that our control results are consistent with the experimental data shown by \cite{7}.
2. Stability of the Convergence Point in the Symmetric Two-strand Yarn Spinning

We first study the symmetric two-strand yarn spinning. The system is assumed initially in a stable condition. Due to some perturbations, the convergence point (equilibrium position $O$ in Figure 2) will move to an instantaneous position ($O'$). The distance $x$ and $y$ are measured from the equilibrium position.

For this case, let the ends of the two strands above convergence point be fixed a distance $2L$ apart, and let the equilibrium position be $H$ below. $F$ is the force in the two-strand yarn below the convergence point, and $F_1$, $F_2$ are the forces in the two strands above the convergence point. The angles $\alpha$ and $\beta$ are also defined in Figure 2. It is easy to see that the equations of motion in $x$- and $y$-directions are
Substitution of Equations (2)-(5) into Equations (1), approximately regarding that 

\[ F_1 = F_2 = f, \]

\[ F = 2f \cos \alpha, \quad \cos \alpha = H/\sqrt{L^2 + H^2}, \quad F = 2fH/\sqrt{L^2 + H^2}, \]

gives

\[ \ddot{x} + \omega^2_1 x - axy = 0 \]
\[ \ddot{y} + \omega^2_1 y - bx^2 - cy^2 = 0 \]

where

\[ \omega^2_1 = 2fH^2 / \left[ \frac{M (L^2 + H^2)^{3/2}}{M (L^2 + H^2)^{3/2}} \right], \]
\[ a = 2f \left[ -H (L^2 + H^2)^{-3/2} + 3L^2 H (L^2 + H^2)^{-3/2} \right] / M, \]
\[ b = f \left[ -H (L^2 + H^2)^{-3/2} + 3L^2 H (L^2 + H^2)^{-3/2} \right] / M, \]
\[ c = f \left[ -3H (L^2 + H^2)^{-3/2} + 3H^3 (L^2 + H^2)^{-3/2} \right] / M. \]

The equivalence autonomous nonlinear system like \( \dot{x} = X(x) \) of Equations (6) is

\[ \dot{x}_1 = x_2, \]
\[ \dot{x}_2 = -\omega^2_1 x_1 + ax_1 y, \]
\[ \dot{y}_1 = y_2, \]
\[ \dot{y}_2 = -\omega^2_1 y_1 + bx^2 + cy^2. \]

In terms of Taylor expansion, it approximates the following first-order linear system

\[ \dot{\hat{x}} = A \hat{x} \]
Where
\[ A = \frac{\partial X(x)}{\partial x} \mid _{x=0} = \begin{bmatrix} \frac{\partial X_1(x)}{\partial x_1} & \cdots & \frac{\partial X_n(x)}{\partial x_n} \\ \vdots & \cdots & \vdots \\ \frac{\partial X_n(x)}{\partial x_1} & \cdots & \frac{\partial X_n(x)}{\partial x_n} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega_x^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\omega_y^2 & 0 \end{bmatrix} \]

is the Jacobi matrix of the system, i.e., the first-order linear system is
\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\omega_x^2 x_1 \\
\dot{y}_1 &= y_2 \\
\dot{y}_2 &= -\omega_y^2 y_1
\end{align*}
\]

(9)

In the phase space of \((x_1, x_2) = (x_1, \dot{x}_1)\) and \((y_1, y_2) = (y_1, \dot{y}_1)\), the phase trajectories of Equations (9) are the integrals of the following expressions
\[
\frac{dx_1}{dx_1} = -\omega_x^2 \frac{x_1}{x_2}, \quad \frac{dy_1}{dy_1} = -\omega_y^2 \frac{y_1}{y_2}
\]
i.e.,
\[
\frac{1}{2} x_2^2 + \frac{1}{2} \omega_x^2 x_1^2 = H_x - \frac{1}{2} y_2^2 + \frac{1}{2} \omega_y^2 y_1^2 + l
\]

(10)

Where the integral constants \(H_x\) and \(H_y\) determined by the initial state, are respectively conservative, representing the horizontal and vertical energy of the system. \(x_2^2/2\) and \(y_2^2/2\) are the horizontal and vertical kinetic energy of the system, while \(\omega_x^2 x_1^2/2\) and \(\omega_y^2 y_1^2/2\) are the potential energy of the two directions. In the phase space, Equations (10) are respectively a set of closed elliptical curves around the center \((0,0)\) as shown ones in Figure 3. The motion of the convergence point is stable, though not asymptotically stable.

3. Stability and Control of the Convergence Point in the Asymmetric Two-strand Yarn Spinning

3.1 Stability of the Convergence Point in the Asymmetric Two-strand Yarn Spinning

For the asymmetric case, i.e. Sirowfil spinning, the dynamics of the system is quite different. The schematic diagram is shown as Figure 4.
The equations of motion are established as the symmetric ones, i.e. Equations (1). Also applying the binomial theorem to expand the square-root terms, substitution of the corresponding $\cos \alpha$, $\sin \alpha$, $\cos \beta$, $\sin \beta$ into Equations (1) gives

\[\begin{align*}
\dot{x} + \omega_x^2 x + dy + a_x x^2 + \frac{1}{2} by^2 + cxy &= 0 \\
\dot{y} + \omega_y^2 y + dx + \frac{1}{2} cx^2 + a_y y^2 + bxy &= 0
\end{align*}\]

(11)

where

\[\begin{align*}
\omega_x^2 &= \left( F_1 \cos^3 \alpha_1 + F_2 \cos^3 \alpha_2 \right)/MH \\
\omega_y^2 &= \left( F_1 \sin^2 \alpha_1 \cos \alpha_1 + F_2 \sin^2 \alpha_2 \cos \alpha_2 \right)/MH \\
d &= \left( F_2 \sin \alpha_2 \cos^2 \alpha_2 - F_1 \sin \alpha_1 \cos^2 \alpha_1 \right)/MH \\
a_1 &= \left( 3F_1 \sin \alpha_1 \cos^4 \alpha_1 - 3F_2 \sin \alpha_2 \cos^4 \alpha_2 \right)/2MH^2 \\
a_2 &= \left( 3F_1 \sin^2 \alpha_1 \cos^3 \alpha_1 + 3F_2 \sin^2 \alpha_2 \cos^3 \alpha_2 \right)/2MH^2 \\
b &= \left[ -F_1 \sin \alpha_1 \cos^2 \alpha_1 (3\cos^2 \alpha_1 - 1) + F_2 \sin \alpha_2 \cos^2 \alpha_2 (3\cos^2 \alpha_2 - 1) \right]/MH^2 \\
c &= \left[ -F_1 \cos^3 \alpha_1 (3\sin^2 \alpha_1 - 1) - F_2 \cos^3 \alpha_2 (3\sin^2 \alpha_2 - 1) \right]/MH^2 \\
\sin \alpha_1 &= L_x/\sqrt{L_x^2 + H^2} \\
\cos \alpha_1 &= H/\sqrt{L_x^2 + H^2} \\
\sin \alpha_2 &= L_y/\sqrt{L_y^2 + H^2} \\
\cos \alpha_2 &= H/\sqrt{L_y^2 + H^2}
\end{align*}\]

The equivalence autonomous nonlinear system like $\dot{x} = X(x)$ of Equations (11) is

\[\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\omega_x^2 x_1 - dy_1 - a_x x_1^2 - \frac{1}{2} by_1^2 - cxy_1 \\
\dot{y}_1 &= y_2 \\
\dot{y}_2 &= -\omega_y^2 y_1 - dx_1 - \frac{1}{2} cx_1^2 - a_y y_1^2 - bxy_1
\end{align*}\]

(12)

In terms of Taylor expansion, it approximates the following first-order linear system

\[\dot{x} = Ax\]

(13)

Where
\[ A = \frac{\partial X(x)}{\partial x} \bigg|_{x=0} = \begin{bmatrix} \frac{\partial X_1(x)}{\partial x_1} & \cdots & \frac{\partial X_1(x)}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial X_n(x)}{\partial x_1} & \cdots & \frac{\partial X_n(x)}{\partial x_n} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega_x^2 & 0 & -d & 0 \\ 0 & 0 & 0 & 1 \\ -d & 0 & -\omega_y^2 & 0 \end{bmatrix} \]

is the Jacobi matrix of the system, its characteristic equation is

\[ |A - \lambda E| = \begin{vmatrix} -\lambda & 1 & 0 & 0 \\ -\omega_x^2 - \lambda & -d & 0 \\ 0 & 0 & -\lambda & 1 \\ -d & 0 & -\omega_y^2 - \lambda \end{vmatrix} = \lambda^4 + (\omega_x^2 + \omega_y^2)\lambda^2 + \omega_x^2\omega_y^2 - d^2 = 0 \quad (14) \]

So the eigenvalues can be solved by

\[ \lambda^2 = -\frac{(\omega_x^2 + \omega_y^2) \pm \sqrt{(\omega_x^2 + \omega_y^2)^2 - 4(\omega_x^2\omega_y^2 - d^2)}}{2} \quad (15) \]

If \( \omega_x^2\omega_y^2 < d^2, \lambda_{1,2}^2 < 0, \lambda_{3,4}^2 > 0 \), one of the four eigenvalues \( \lambda \) is positive, thus the system is unstable; If \( \omega_x^2\omega_y^2 > d^2, \lambda_{1,2}^2 < 0, \lambda_{3,4}^2 < 0 \), the eigenvalues are two pairs of pure imaginary, and the equilibrium point is called center. The movement of the convergence point is stable, though not asymptotically stable. Thus, it is known that bifurcation occurs when \( \omega_x^2\omega_y^2 = d^2 \).

3.2 Control of the convergence point in the asymmetric two-strand yarn spinning

We then discuss the movement of the convergence point after tension compensator (TC) control. As shown in Figure 5, C is the convergence point, D is the point where the TC applies. Analyzing the undergoin force at points C and D respectively, the corresponding equations of motion of convergence point C are

\[ M\ddot{x} + F_x \cos \beta - F_1 \cos \alpha - F \tan \gamma = 0 \]
\[ M\ddot{y} + F - F_1 \sin \alpha - F_2 \sin \beta = 0 \quad (16) \]

Where \( \gamma \) is the ‘ber’ arranged in the experiments [7] as shown in Figure 5. Expanding the Equations (16), the equivalence autonomous nonlinear expressions for the controlled system in terms of Taylor expansion approximate the same as Equations (13), but with different values of \( \omega_x, \omega_y, a_1, a_2, b, c, d \) for the varying of \( F_1, F_2 \) when the tension compensator applied.

As we have known that the convergence point may occur large vibration and be unstable (i.e. \( \omega_x^2\omega_y^2 < d^2 \)) in the asymmetric two-strand yarn spinning, which do not appear in the symmetric case. Applying tension compensator (TC) to the system, and adjusting the \( \gamma \), i.e. correspondingly the \( F_1, F_2 \) to some value may make the system stable, i.e. \( \omega_x^2\omega_y^2 > d^2 \). So the system may turn out to be stable, and the effect of TC control can be apparently seen.

In the experiments [7], the properties of the hairiness and abrasion-resistance of Sirofil composite yarn with and without Tension Compensator control are shown in Table 1 and Fig.2-3, including hairiness quantity versus length or yarn configuration experiments with the same filament pre-tension and with different filament pre-tension, and shown in Table 2-5, including Two-sample average analysis, before and after yarn steaming of the abrasion-resistance experiments with and without TC. In general, the experiments concluded that the adoption of TC greatly reduce the hairiness of Sirofil.
composite yarn, and enormously improve the property of abrasion-resistance, which are consistent very well with the present theoretic derivation and analysis.

4. Conclusions
Following our work in the static aspect [7], we sequentially and kinetically disclosed the mechanism of TC control to Sirofil spinning. It is once more concluded that the TC control can increase the stability of the convergence point in Sirofil composite area, and correspondingly improve the Sirofil yarn properties.

5. References
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