Quintessential Axions from a New Confining Force

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Almost massless mesons created at the condensation scale of extra nonabelian gauge group can be candidates of dark energy source. This can be another motivation for introducing an additional confining force.

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I. INTRODUCTION

The origin of dark energy (DE) in the Universe is most mysterious in the current cosmology. At present, the DE density (0.003 eV)\(^4\) is synonymous to the cosmological constant (CC). In the multiverse distinguished by CC, the anthropic choice allows some universes to evolve to the ones similar to our currently nearly flat and empty Universe to have creatures to quest about it now [1]. On the other hand, if the current DE is a transient one, parametrized by a varying \(\theta\), the classically evolving scalar field \(\theta\) is interpreted as the source for the current domination of the energy density of the Universe [2,3]. The name 'quintessential axion' [4–6] is by identifying \(\theta\) as the pseudo-Goldstone boson of spontaneously broken global symmetry \(U(1)_{DE}\).

Light pseudoscalar particles are the lampposts to the road leading to physics scales much above their masses. The well-known example is the pion triplet looked at low energy strong interaction scale, which guided toward the \(SU(2)\) symmetries such that the breaking scale is sufficiently small. If the pseudoscalar mass \(m\) is less than 1 MeV, it can rarely decay to two photons and two neutrinos. Using the above relation, we get an idea on the lifetime of the pseudoscalar. For it to be still present in our Universe, therefore we require \(\Gamma^{-1} > 4.3 \times 10^{17}\) s = \(1/(1.53 \times 10^{-33}\text{ eV})\). Using the above \(f - m\) relation, we get an idea on \(m\) from the condition that it survives until now, using the QCD scale \(\Lambda\),

\[
\Gamma^{-1} \approx \frac{64\pi^3 \alpha^4_{\text{em}} \Lambda_{\text{QCD}}}{a^4 m^4} \frac{1}{m} > \frac{1}{1.53 \times 10^{-33}\text{ eV}}
\]

or \(m < 65\text{ eV}\) for \(\Lambda_{\text{QCD}} = 380\text{ MeV}\).\(^1\) If we take \(\Lambda^4\) as the current energy density of the Universe, then pseudoscalars with mass less than \(3.3 \times 10^{-4}\text{ eV}\) survives until now.\(^2\)

\(^1\) For the very light axion, the specific coupling reduces it further to 24 eV [6].
\(^2\) Ultra-light axions (ULAs) [13,14] was suggested for the galactic scale structures.
We will be using two words for describing breaking the global symmetry. One is the spontaneous breaking scale \( f \) mentioned above. This is the vacuum expectation value (VEV) of the scalar field which carries the non-vanishing quantum number of the global symmetry and is determined from the interaction terms respecting the global symmetry. The other is the explicit breaking terms which are interaction terms explicitly breaking the global symmetry in question and provide the curvature in the direction of \( a \).

We will consider the QCD axion also because it is present in most models. Not only providing a solution of the strong CP problem, the QCD axion can work as a source of dark matter (DM) in the Universe, which depends on the confining scale \( \Lambda_{\text{QCD}} \) and the axion decay constant \( f_a \). So, let us start with building a field theoretic model housing both the QCD and quintessential axions. Working out an explicit example, we employ supersymmetry because supersymmetry reduces the number of couplings and hence the discussion for generating symmetry breaking terms is much simpler.

Let the gauge group for a new confining force be \( G \) and the flavor symmetry of ex-quarks under \( G \) be \( U(N) \times U(N) \) which reduces to \( SU(N) \times SU(N) \) by removing the heavy singlet meson obtaining mass by the anomaly contribution. The light mesons \( \Pi^i_j(i = 1, \cdots, N; j = 1, \cdots, N) \) with \( \text{Tr} \Pi = 0 \), belonging to the adjoint representation \((N^2 - 1)\) of \( SU(N)_A \subset SU(N) \times SU(N) \), obtain mass by explicit breaking of the flavor symmetry of ex-quarks.

One obvious scale breaking the global symmetry is the condensation scale \( \Lambda \) of the ex-quarks,

\[
\langle Q^a_jT^a_jQ_L \rangle = \Lambda^3 e^{\text{Im} \sigma_f/f}
\]

where \( \bar{Q}_LT^a_jQ_L \) is interpreted as the matrix for adjoint representation \( T^a \) sandwiched between the relevant fermion fields \( \psi_R^a \bar{T}^a \psi_L \). To construct a model based on discrete symmetry, supersymmetric extension is simpler and we will discuss supersymmetric extension below.

In the \( N = 1 \) supersymmetric extension, we include scalar partners of ex-quarks also. These scalar quarks are defined to have the chirality of the corresponding ex-quarks denoted by tilded fields. The \( SU(N)_L \times SU(N)_R \) representation of scalar ex-quarks are

\[
\begin{pmatrix}
(a_1) \\
(a_2) \\
\vdots \\
(a_N)
\end{pmatrix}_L, \\
\begin{pmatrix}
\bar{b}_1 \\
\bar{b}_2 \\
\vdots \\
\bar{b}_N
\end{pmatrix}_L
\]

(3)

Usually the same notations \( Q_L \) and \( Q_R \) are used for the corresponding superfields also, \( Q_L + \sqrt{2}Q_L \theta + F_{Q_L} \theta^2 \), etc., and interaction is given by \( \int d^4x d^2 \theta W \). In supersymmetric extension, the R-charges of chiral superfields determine possible terms in the superpotential \( W \). For dynamical SUSY breaking via the new confining gauge group \( G \), the gaugino condensation scale \( \Lambda \) is expected around \( 10^{13} \) GeV. With \( U(1)_{\text{DE}} \), we also consider the breaking scale of \( U(1)_{\text{PQ}} \) for the QCD axion. For simplicity, let us assume the VEV of the singlet scalar field \( \langle \sigma \rangle = V \) is the QCD axion scale \( f_a \approx 10^{10} \) GeV. In addition to singlet fields, below we will consider Higgs fields also.

In supersymmetric models, condensation is possible for bilinears of fermions and also bilinears of scalar ex-quarks (bi-scalar). Condensation of bi-scalars does not break supersymmetry. This is a merit introducing quintessential axion in the phases of bi-scalar condensation. So, \( f \) for the bi-scalar condensation can be different from the gaugino condensation scale \( \Lambda \).

Condensation of bi-scalars is expected when the gauge coupling of the new non-abelian force becomes strong at \( \Lambda \). Let the bi-scalar singlet be \( X \),

\[
\bar{Q}_LQ_L \equiv X.
\]

As mentioned above, nonzero \( X \) does not break supersymmetry. If we consider a superpotential in terms of \( X \),

\[
W = \Lambda X - \frac{1}{2M}X^2 + \cdots
\]

(5)

where \( \Lambda \) and \( 1/M \) can result from the VEVs of some singlet fields and \( M \) in the following is used for the Planck mass \( M_P = 2.43 \times 10^{18} \) GeV. So, the first minimum of the potential appears from \( V = \left( \Lambda - \frac{1}{M_P}X \right)^2 \) and the next minimum can be far above the Planck scale. Then, the decay constant \( f = \sqrt{X} \) is expected at a median of \( \Lambda \) and \( M_P \).

Now let us consider the explicit breaking terms. Anomalies of non-abelian gauge groups are inevitable breaking terms, and the QCD anomaly is used for the explicit breaking term of the \( U(1)_{\text{PQ}} \) global symmetry. If we consider a
FIG. 1: Potential generated by Yukawa terms breaking U(1)$_{\text{DE}}$. At the intersection of the blue curve and the $f_u = 1$ line, $v_d$ is 25.6 GeV.

phase of the *SM singlet scalar field*, the weak gauge group in the SM may work for an explicit breaking term of the U(1)$_{\text{DE}}$ global symmetry [20]. But our quintessence fields are mesons in the phase of bi-scalars, or scalar ex-quarks, and hence the SU(2)$_W$ anomaly cannot work for our quintessence fields. The only method is from the superpotential made of scalar fields. To have a sufficiently small numerical value for the vacuum energy, we consider the VEV scale of the SM Higgs fields. A possible superpotential is

$$\Delta W = \frac{1}{M^{n+3}} Q_L Q_L (H_u H_d)^2 \sigma^n, \quad (6)$$

where we started from $(H_u H_d)^2$. If we started with one power of $H_u H_d$, too large a value of $n$ would be required. Considering condensation of the hidden sector quark $Q$ in Eq. (6), the vacuum energy density is

$$\frac{4}{M^{n+3}} A^3 (v_u v_d)^2 V^n = \frac{4}{M^{n+3}} A^3 \frac{v_d^4}{\cos \beta} V^n \simeq (0.003 \text{eV})^4 \quad (7)$$

where $\tan \beta = v_u/v_d$.

The top quark mass $m_t$ is $f_u v_u/\sqrt{2}$ and the bottom quark mass $m_b$ is $f_d v_d/\sqrt{2}$. For the perturbative calculation to be possible, we limit the study in the region $f_u \leq 1$ and $f_d \leq 1$. Equation (7) has one parameter $\tan \beta$. For a given $\tan \beta$, relative values of $f_u$ and $f_d$ are given, as shown in Fig. 2. The blue curve in Fig. 2 is interpreted as the current DE, $(0.003 \text{eV})^4$, for $f_d = 1$.

As an example, taking $v_d = 20$ GeV and $m_t = 173$ GeV, Eq. (7) is estimated as,

$$\frac{4}{M^{n+3}} A^3 \frac{v_d^4}{\cos \beta} V^n \simeq \tan^4 \beta \left( \frac{V}{M} \right)^n \left( \frac{20}{2.43 \times 10^{18}} \right)^4 A^3 \cdot 0.97 \times 10^{28} \text{eV} = 10^{-10} \text{eV}^4 \quad (8)$$

leading to

$$\Lambda = \begin{cases} 2.7 \times 10^{14} \text{GeV}, \text{ for } n = 1 \\ 1.7 \times 10^{17} \text{GeV}, \text{ for } n = 2 \end{cases} \quad (9)$$

This example is shown as the red bullet in Fig. 2.

In Table I an example for the $Z_{6R}$ discrete symmetry is shown, from which $n = 1$ is chosen in Eq. (9). The $Z_{6R}$ is a discrete subgroup of R-symmetry U(1)$_{R}$. Because the antisymmetric parameter $\vartheta$ carries $-1$ unit of U(1)$_{R}$ charge, superpotential $W$ with +2 units of U(1)$_{R}$ charge survives in the integration $\int d^2 \vartheta W$. To assign tree level Yukawa couplings of the SM quarks, we have the following quantum numbers

$$q_L : 0, \quad u^c_L : -1, \quad d^c_L : 0, \quad (10)$$
such that both $q_L u_L^3 H_u$ and $q_L d_L^3 H_d$ carry two units (modulo 6) of $U(1)_R$ charge. To obtain a TeV scale $\mu$ term \[28\], one can introduce the following superpotential

$$ W_\mu = \frac{(10^{10} \text{ GeV})^2}{M} H_u H_d, \quad (11) $$

but should forbid the dimensions 2 and 3 superpotential terms $H_u H_d, H_u H_d \sigma$ and $H_u H_d S$. Equation \[11\] can be satisfied with $(S) \approx (\sigma) \approx 10^{10}$ GeV. Then, singlets $S$ and $\sigma$ lead to the following superpotential

$$ W = -\alpha \sigma S^2 + \frac{\varepsilon}{M^2} S^4 - \frac{x}{M^2} \sigma S Q_L \bar{Q}_L + \cdots, \quad (12) $$

where the neglected terms $\cdots$ denote other terms including $Q_L \bar{Q}_L$. The relations between the VEVs of $\sigma$ and $S$ are

$$ \frac{\partial W}{\partial \sigma} \to Q_L \bar{Q}_L = -\frac{\alpha M^2}{x}, \quad (13) $$

$$ \frac{\partial W}{\partial S} \to (x Q_L \bar{Q}_L + \alpha)\sigma = \frac{2\varepsilon}{M^2} S^2. $$

Relations in \[13\] give exact SUSY. But SUSY is broken dynamically by the condensation of ex-quarks at $\Lambda \simeq 10^{13}$ GeV. This SUSY breaking effect is added in the first SUSY relation,

$$ -\alpha S^2 - \frac{x}{M^2} S^2 Q_L \bar{Q}_L + \delta_1 \Lambda^2 = 0, $$

$$ \frac{2\varepsilon S^3}{M} - \alpha S - \frac{x}{M^2} SQ_L \bar{Q}_L \sigma + \frac{\delta_1 \Lambda^2}{2} = 0, $$

$$ 2 \frac{\varepsilon}{M} S^4 + (\delta_2 S - 2\delta_1 \sigma) \Lambda^2 = 0. \quad (14) $$

Solutions of $\sigma$ and $S$ satisfying Eq. \[14\] are shown in Fig. 2. So, the needed $\mu$ term is around the point $(1, 1)$ in Fig. 2.

The above $U(1)_R$ symmetry is a kind of the PQ symmetry because the $U(1)_R$ quantum number of $H_u H_d$ is non-zero. At the scale $V_{PQ} \equiv \sqrt{\sigma^2 + S^2}$, therefore, the PQ symmetry is spontaneously broken and there results the QCD axion discussed in the literature [6]. Because the quantum numbers of $\sigma$ and $S$ are relatively prime, all the axionic vacua are connected and the domain wall number is 1 [10] [29]. This QCD axion is responsible for the DM in the Universe.

Gravity effects of breaking global and discrete symmetries can be given by the terms explicitly breaking them. Since supergravity with superpotential $W$ has a fine-tuning problem of adjusting the cosmological constant [30], we lack any understanding on the magnitude of the cosmological constant in supergravity. So, no attempt is made to introduce explicit DE terms. However, we can discuss the gravity effects to the parameters in the gauge part, especially on the $\hat{\theta}$ of QCD, as done in [31]. Here, we consider only the potential $V$ satisfying the $Z_{6R}$ symmetry, but not the superpotential $W$. Possible $Z_{6R}$ invariant terms are

$$ \sigma^3, \sigma^*^3, S^2 \sigma^*, S^6, S^*^6, (\sigma \sigma^*)^n \text{ for } n = 1, 2, \cdots, (SS^*)^n \text{ for } n = 1, 2, \cdots, \text{ etc.} \quad (15) $$

Note that the $(\sigma \sigma^*)^n$ and $(SS^*)^n$ terms respect the PQ symmetry and do not shift $\hat{\theta}$. The leading terms in shifting $\hat{\theta}$ in Eq. \[15\] are the cubic terms. But these cubic terms do not arise from the superpotential and hence appear with

| Representation under $G \equiv SU(N)$ | SU(2)$_W \times U(1)_Y$ | $Z_{6R}$ |
|--------------------------------------|------------------------|---------|
| $Q_L$                                | $\mathcal{N}$          | 1       | +1      |
| $\bar{Q}_L$                          | $\overline{\mathcal{N}}$ | 1       | -1      |
| $H_u$                                | 1                      | $2_{+1/2}$ | +3      |
| $H_d$                                | 1                      | $2_{-1/2}$ | +2      |
| $\sigma$                             | 1                      | 1        | +4      |
| $S$                                  | 1                      | 1        | +5      |

**TABLE I: $Z_{6R}$ quantum numbers of relevant chiral superfields appearing in Eq. \[6\].**
supersymmetry breaking coefficients, i.e. the gaugino condensation \( \langle \mathcal{G} \mathcal{G} \rangle \) of the new confining force. Thus the \( \bar{\theta} \) shift is estimated from

\[
\frac{1}{M^2} \langle \mathcal{G} \mathcal{G} \rangle \sigma^3, \ldots
\]  

which can be compared to the axion (or Pontryagin number) shift by \( \langle G_{\mu\nu} \tilde{G}^{\mu\nu} \rangle \approx \Lambda^4 \),

\[
\Delta \bar{\theta} \approx \frac{\sigma^3}{M^2 \Lambda} \approx 1.7 \times 10^{-20},
\]

for \( \Lambda = 10^{13} \text{ GeV} \) and \( \sigma = 10^{10} \text{ GeV} \). So, the \( \bar{\theta} \) shift by gravity effects falls in the allowed region, \( |\bar{\theta}| \lesssim 10^{-11} \) [6]. In addition, the higher order terms will give a potential of the form given in Fig. 3. Restricting the vacuum for \( a \) below \( M_P \) as shown in the left-hand side of the dash line, the nonzero VEV of \( a \) is near the Planck scale. In addition, note that the effects of pseudo-Goldstone bosons may induce terms expected in general relativity [32]. We note, however, that all such effects with supersymmetry are included in Eq. (12).

In conclusion, we proposed a mechanism creating quintessential axions that are almost massless mesons created at the condensation scale of a new nonabelian gauge group. Since an extra nonabelian gauge group has been proposed for dynamical supersymmetry breaking, our mechanism is better suited in supersymmetric models.
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