Instability of Boundary Layer on Two-dimensional Corrugation with Various Wavelengths*

Mochamad Dady MA’MUN, Masahito ASAI, and Ayumu INASAWA

Department of Aerospace Engineering, Tokyo Metropolitan University, Tokyo 191–0065, Japan

The instability of the boundary layer on two-dimensional (2-D) surface corrugation was examined experimentally. The corrugation amplitude $A_w$ was one order of magnitude smaller than the displacement thickness of the boundary layer. Sinusoidal-geometry corrugations with various wavelengths were considered to see how the growth of Tollmien-Schlichting (T-S) waves was affected by the scale of corrugation. The corrugation wavelength $\lambda_c$ ranged from about one-tenth to the same order as the T-S wavelength. The destabilizing effect of the 2-D corrugation was observed for a wide range of corrugation wavelengths and was not strongly dependent on the wavelength. Even when a separation bubble was formed in each furrow for the smallest wavelength, the amplification of T-S waves was almost the same as in the case of longer wavelengths.

Key Words: Boundary Layer, Instability, Tollmien-Schlichting Waves, Roughness

1. Introduction

Our knowledge of the mechanism of boundary-layer transition has improved considerably over the last few decades. However, some fundamental problems have still remained. One of those is the influence of small-height distributed roughness on instability and transition, which is quite important in engineering applications such as laminar flow technologies. In order to better understand the influence of surface roughness on boundary-layer transition, we have been studying instability and transition on a corrugated wall with sinusoidal geometry in a zero-pressure-gradient boundary layer. First, we briefly reviewed related work regarding the roughness effect on instability and transition.

When roughness height is sufficiently large, an isolated roughness or roughness elements can lead to by-pass transition, skipping the growth stage of Tollmien-Schlichting (T-S) waves through strong inflectional instability directly caused by each roughness element. For instance, Mochizuki1] studied roughness-induced transition in a flat-plate boundary layer using a semi-sphere as an isolated roughness and showed that transition location moved upstream with increasing the roughness Reynolds number. Regarding such roughness-induced transition, Reshotko2] and Morkovin3] introduced a critical roughness Reynolds number $Re_k = U_k k / \nu$, where $k$ is the roughness height, $U_k$ is the Blasius flow velocity at the height $k$ and $\nu$ is the kinematic viscosity, and proposed a critical value of 25, above which roughness elements could cause boundary-layer transition.

On the other hand, when the roughness height is so small that the roughness only slightly modifies the boundary layer profiles, the role of roughness, especially the role of distributed roughness, in the transition process is not yet fully understood. Corke et al.4] compared the amplification of T-S waves in zero-pressure-gradient boundary layers on a smooth wall and on a rough wall under natural disturbance conditions experimentally. They used sand-paper roughness as the distributed roughness. They found that the transition occurred more rapidly on the rough wall than on the smooth wall. However, the destabilizing mechanism caused by distributed roughness was not clarified. The present study discusses the effects of surface corrugation (surface waviness) on the stability of a laminar boundary layer. The surface corrugation is regarded as a simplified model of distributed roughness. For such simple roughness geometry, it is easy to control the streamwise and spanwise scales of roughness as well as the height (i.e., the roughness Reynolds number), and thus this enables us to examine the roughness effect on boundary-layer instability in detail.

In regard to two-dimensional (2-D) corrugation, early theoretical and experimental work5–7] showed that 2-D surface waviness (with wavelength of the same order as T-S wavelength) modified the neutral stability curve and reduced the critical Reynolds number. A numerical simulation8] also demonstrated the destabilizing effect of 2-D surface waviness, although the wavelengths considered were much larger than those of the unstable T-S wave. Floryan9] analyzed the destabilizing effect of 2-D corrugation with various geometries (i.e., sinusoidal, triangular and rectangular), theoretically using a Floquet theory in a plane channel flow and showed that 2-D corrugations could really promote the growth of T-S waves, even for an extremely small corrugation amplitude. The theoretical prediction regarding the destabilizing effect of small corrugation amplitude was verified experimentally for the corrugation with sinusoidal geometry by Asai and Floryan.10] Ma’mun et al.11,12] examined the effects of 2-D and 3-D surface corrugations on the growth of T-S waves in a boundary layer, and revealed that the
3-D corrugation little enhanced the growth of T-S waves and thus the destabilizing effect was peculiar to 2-D corrugation. The experimental verification of the destabilizing effect of corrugation mentioned above only focused on corrugations with wavelengths of the same order as the T-S wavelengths. In the present paper, we focus on how the destabilizing effect of 2-D corrugation depends on the corrugation wavelength. When the corrugation wavelength is smaller than a certain value, the boundary layer can separate on the corrugated wall and a small separation bubble can form in the furrow of the corrugation. So, it is interesting to examine how the destabilizing effect is changed as the corrugation wavelength is decreased below a critical value to form a separation bubble.

2. Experimental Setup and Procedure

The experiment was conducted in an open-jet, low-turbulence wind tunnel, with a settling chamber having a cross-section of 1200 × 1200 mm². The area ratio of the contraction to the test section of 400 × 400 mm² was 9. The freestream turbulence was less than 0.1% over the velocity range of 2–12 m/s. Two sidewalls maintained the two-dimensionality of the main stream in the test section and the upper area was covered with 24-mesh wire-gauze. The facility was the same as in our previous study.¹¹,¹²

A boundary-layer plate of 1195 mm long was set parallel to the oncoming uniform flow in the test section. As illustrated in Fig. 1, the boundary-layer plate consisted of a 5-mm-thick and 300-mm-long aluminum plate, followed by a 20-mm-thick and 895-mm-long resin plate. The leading edge had an elliptical shape whose major axis was 20 times the minor axis. The resin plate could be replaced, having either a smooth wall. The velocity at the boundary-layer edge was the same as that of the upstream flat-plate part. The boundary-layer edge was a separation bubble, which started from the y-position of the corrugated surface and its average height was the same as that of the upstream flat-plate part. The corrugation amplitude used in the present experiment conditions, the largest corrugation wavelength (λₜ = 32 mm) was about two-thirds the wavelength of the amplified T-S waves, and the shortest wavelength (λₜ = 4 mm) was about one-tenth the T-S wavelength.

3. Base Flow and Excitation of T-S Waves

First, we examined the laminar boundary layer on the smooth wall. The velocity at the boundary-layer edge Uₑ quickly approached the value in the uniform flow U∞ due to the thin leading-edge (5 mm thick). Figure 3(a) shows the y-distributions of streamwise velocity U at x = 312, 440 and 728 mm, where Reₑ = Uₑδₑ/ν varied from 574 to 868; here, ν is the kinematic viscosity. Under the present experimental conditions, the largest corrugation wavelength (λₜ = 32 mm) was about two-thirds the wavelength of the amplified T-S waves, and the shortest wavelength (λₜ = 4 mm) was about one-tenth the T-S wavelength.

Fig. 1. Schematic of the boundary-layer plate (dimensions in mm).

Fig. 2. Photograph of 2-D surface corrugation (λₜ = 8 mm).
that boundary-layer growth started at a location 10 mm upstream of the leading-edge; that is, \( \delta^+ = 1.7208|x - x_0|/U_\infty^{1/2} \) with \( x_0 = -10 \text{ mm} \). The shape factor was slightly larger than the value of the Blasius flow \( (H = 2.59) \) over 50 mm < \( x < 250 \text{ mm} \) due to the elliptical leading-edge, suggesting that a weak adverse pressure gradient might exist there. Beyond \( x = 300 \text{ mm} \), the shape factor \( H \) exhibited a constant value of \( H = 2.60 \), and thus the zero-pressure-gradient laminar boundary layer (Blasius flow) developed there in the present experiment.

Then, the vibrating ribbon was driven at a single frequency \( f = f_1 \) to excite two-dimensional T-S waves in the boundary layer. The displacement-thickness Reynolds number was \( Re^* = 410 \) at the location of the vibrating ribbon \( (x = 150 \text{ mm}) \). Note that the critical Reynolds number for the linear instability of the Blasius flow is \( Re^* = 520 \) according to linear stability analysis with parallel flow assumption. It is about \( Re^* = 450 \) according to the direct numerical simulation (DNS) of the Navier-Stokes equations\(^{13} \) and the calculation based on the parabolized stability equation (PSE).\(^{14} \)

Figures 4(a) and (b) illustrate the \( \gamma \)-distributions of the root-mean-square (rms) amplitude \( u'_1 \) of the streamwise velocity component and phase of the excited wave at \( x = 600 \text{ mm} \) \( (Re^* = 780) \) being compared to the theoretical results calculated from the Orr-Sommerfeld stability equation. The forcing frequency component \( u_1 \) was singled out from the measured velocity by spectral analysis. Here, the rms amplitude was normalized by the \( \gamma \)-maximum \( u'_{1m} \) in Fig. 4(a).

The vibrating ribbon was driven at \( f_1 = 40 \text{ Hz} \) and its non-dimensional frequency \( F = 2\pi f_1/vU_\infty^2 \) was \( 1.5 \times 10^{-4} \). Good agreement was obtained between the theory and experiment. We also confirmed that the wavelength of the T-S wave, which was measured to be about 48 mm around \( x = 600 \text{ mm} \) for \( f_1 = 40 \text{ Hz} \), was in good agreement with that calculated from the Orr-Sommerfeld equation.

Figure 5 shows the streamwise growth of T-S waves with frequencies \( F = 1.5 \times 10^{-4} \) and \( F = 1.7 \times 10^{-4} \) compared to the linear stability theory (parallel flow theory) in terms of the \( N \)-factor defined as \( N = \ln[u'_{1m}/(u'_{1m})_0] \), where \( (u'_{1m})_0 \) denotes \( u'_{1m} \) at the lower branch (branch I) of the neutral stability curve. Here, the amplitudes of excited waves were controlled such that they were less than 0.3% in terms of the maximum rms amplitude \( u'_{1m}/U_\infty \), which was sufficiently small, staying below the threshold amplitude for the onset of secondary instability leading to 3-D wave growth.\(^{15} \) The development of T-S waves followed the disturbance growth predicted by the linear stability theory almost completely for both frequencies, which enabled us to make accurate comparisons of the stability nature between the smooth-wall and corrugated-wall cases in the next section. It should also be noted that the spanwise variation in the amplitude \( u'_{1m} \) was about 10% of the average in terms of the peak-to-peak value over \(-40 \text{ mm} < z < 40 \text{ mm} \) and the two-dimensionality of T-S waves was kept downstream.
Before comparing the dependence of two-dimensional corrugation wavelength on the growth of T-S waves, we first examined the effect of 2-D corrugation on the mean flow $U$. When the wavelength-to-depth ratio $\lambda_w/(2A_w)$ was decreased, there was a possibility that the flow separated at each crest of the corrugated wall forms a small separation bubble in each furrow. Figures 6(a) to (d) illustrate the streamwise ($x$) distributions of $U$ (over two corrugation wavelengths) at $y=0.45$, 0.65, 0.9, 1.1 and 1.3 mm, for the corrugation models with $\lambda_w = 32, 16, 8$ and 4 mm, respectively. Here, crest locations are $x = 632$ and 664 mm in Fig. 6(a), 604 and 620 mm in Fig. 6(b), 630 and 638 mm in Fig. 6(c), and 631, 635 and 639 mm in Fig. 6(d), respectively. We can see slightly-wavy variations of $U$ only for $y<1.1$ mm ($\gamma/\delta^*<0.5$) for $\lambda_w = 32, 16$ and 8 mm. The peak-to-peak value of the wavy variation was 2.2% of $U_\infty$ at $y=0.45$ mm for $\lambda_w = 32$ mm, and gradually decreased to 1.3% and 0.7% of $U_\infty$ for $\lambda_w = 16$ and 8 mm, respectively. The streamwise variations in $U$ did not coincide with those of the surface corrugation, but exhibited a phase shift of about one-eighth the wavelength. The feature of phase difference between the wavy surface and the streamwise variation of $U$ was also reported by Kachanov et al.\textsuperscript{16} For the smallest wavelength $\lambda_w = 4$ mm, on the other hand, the streamwise distribution of $U$ exhibited no waviness, even in the vicinity of the corrugated wall ($y=0.55$ mm), strongly suggesting that the near-wall flow was separated so that it did not flow along the corrugation surface. For the

---

**Fig. 5.** Streamwise growth of T-S wave, $N$ versus $Re$. (a) $F = 1.5 \times 10^{-4}$, (b) $F = 1.7 \times 10^{-4}$.

**Fig. 6.** The $x$-distribution of $U$ on the 2-D corrugation ($A_w = 0.21$ mm) at $U_\infty = 5$ m/s. $\circ$: $y = 0.45$ mm (0.55 mm), $\triangledown$: $y = 0.65$ mm (0.75 mm), $\bullet$: $y = 0.9$ mm (0.95 mm), $\blacktriangle$: $y = 1.1$ mm (1.15 mm), $\blacktriangledown$: $y = 1.3$ mm (1.35 mm). The values in the bracket are $y$-positions in (d). (a) $\lambda_w = 32$ mm, (b) $\lambda_w = 16$ mm, (c) $\lambda_w = 8$ mm, (d) $\lambda_w = 4$ mm. Crests locations are indicated by arrows.

©2015 JSASS
larger wavelengths ($\lambda_w = 32$ and $16$ mm), no evidence of separation on the corrugated wall was found.

As for the occurrence of separation bubble on the corrugated wall, the wavelength-to-depth ratio is important as well as the Reynolds number, as studied in the related work on the flow in wavy-walled channels. In the present experiment, this ratio is defined as $\lambda_w/(2A_w)$, and thus was 19 and 9.5 for $\lambda_w = 8$ and 4 mm, respectively, and separation occurred between these wavelengths. In order to clarify the change in flow close to the corrugated wall as the wavelength was reduced, Figs. 7(a) to (d) compare the y-distributions of streamwise velocity $U$ in the vicinity of the surface at the crest, trough and neutral positions of the corrugation with wavelengths $\lambda_w = 32$, 16, 8 and 4 mm, respectively. Here, $y = 0$ represented the average y-position of the corrugated wall for all the velocity profiles. In the cases of the larger wavelengths $\lambda_w = 32$, 16 and 8 mm, Figs. 7(a) to (c), the velocity profiles coincided with the Blasius profile for $y > 1.1$ mm ($y/\delta^* > 0.5$), but they were only slightly shifted towards positive $y$ (i.e., by 0.05 mm or less). The influence of surface corrugation on the base flow soon vanished for $y/\delta^* > 0.5$. In this case, we did not find any evidence of separation on the corrugated wall, although the velocity profile at the trough location became slightly inflectional in the vicinity of the wall. In the case of the shortest wavelength $\lambda_w = 4$ mm, Fig. 7(d), corresponding to the disappearance of waviness in the streamwise distribution of $U$ (see Fig. 6(d)), the velocity profile was shifted much more distinctly by a distance $\approx A_w$ ($= 0.21$ mm), as if the corrugated wall were replaced with a flat plate with a forward-facing step of height $A_w$: The velocity profiles still exhibited the Blasius flow profile away from the wall.

Figures 8(a) and (b) compare the streamwise development of T-S waves on the corrugated wall with four different wavelengths $\lambda_w = 32$, 16, 8 and 4 mm for $F = 1.5 \times 10^{-4}$ and $F = 1.7 \times 10^{-4}$, respectively. In the figure, the corresponding results in the smooth-wall case are also plotted for comparison. In each case, the rms amplitude $u'_{\text{rms}}$ is scaled with $(u'_{\text{rms}})_0$, the value at $Re^* \approx 600$, which corresponded to the lower branch (branch I) of the neutral stability location at $F = 1.5 \times 10^{-4}$ in the smooth-wall case. Therefore, $\ln(u'_{\text{rms}}/(u'_{\text{rms}})_0)$ represents the $N$-factor for the smooth-wall case (shown in Fig. 5). We see that the wall corrugation significantly enhanced the amplification of T-S waves compared to the smooth wall case for a wide range of corrugation wavelengths. A close comparison shows that, with decreasing $\lambda_w$, the maximum value of $N$ increased slightly to a maximum for $\lambda_w = 8$ mm and then decreased. However, the dependency of the destabilizing effect on the corrugation wavelength was very weak, at least for the range of $\lambda_w$ examined. Even when separation occurred in the furrow of corrugation for $\lambda_w = 4$ mm, the destabilizing effect was not enhanced. Similar results regarding the dependency on the corrugation wavelength were also obtained for a higher frequency $F = 2.0 \times 10^{-4}$.

The destabilizing mechanism enhancing the growth of the T-S wave is basically an inflectional instability, and the appearance of inflectional velocity profiles in the region very close to the wall affected the viscosity-conditioned instability, which is governed by the behavior of the viscous Stokes
layer having a thickness of the order of \((A_w = 0.21 \text{ mm})\) was sufficiently small compared to the boundary-layer displacement thickness (i.e., on the order of 10% of the displacement thickness of the boundary layer in the present experimental condition). The streamwise growth of T-S waves on the corrugated wall was compared to that in the Blasius boundary layer on the smooth surface. The results showed that 2-D corrugation significantly enhanced boundary-layer instability over a wide range of corrugation wavelengths. It is emphasized that, for the shortest-wavelength case with \(\lambda_w = 4 \text{ mm}\) where the wavelength-to-amplitude ratio \(\lambda_w/(2A_w)\) was about 9.5, the flow very close to the surface was separated and did not enter the furrow of the corrugation. Nevertheless, the growth rates of the T-S wave were found to be almost the same as those for the longer-wavelength corrugations without separation (\(A_w = 8, 16\) and 32 mm).

### Acknowledgments

This work was, in part, supported by JSPS KAKENHI (No. 21560820) and Grant for Scientific Research from Tokyo Metropolitan Government.

### References

1. Mochizuki, M.: Smoke Observation on Boundary Layer Transition Caused by a Spherical Roughness Element, J. Phys. Soc. Jpn., 16, 5 (1961), pp. 995–1008.
2. Reshotko, E.: Disturbances in a Laminar Boundary Layer Due to Distributed Surface Roughness, Turbulence and Chaotic Phenomena in Fluids, Tatsunami, T., ed., Elsevier, Amsterdam, 1984, pp. 39–46.
3. Morkovin, M. V.: On Roughness-induced Transition: Facts, Views and Speculations, Instability and Transition, Hussaini, M. Y. and Voigt, R. G., eds., Vol. 1, Springer-Verlag, New York, 1990, pp. 281–295.
4. Corke, T. C., Sever, A. Bar, and Morkovin, M. V.: Experiments on Transition Enhancements by Distributed Roughness, Phys. Fluids, 29 (1986), pp. 3199–3213.
5. Levchenko, V. Ya. and Solovlev, A. S.: Stability of Boundary Layer on Wavy Surface, Zh. Akad. Nauk, SSSR, Mekh. Zhidk. Gaza, 3 (1972), pp. 11–16.
6. Kachanov, Yu. S., Kozlov, V. V., and Levchenko, V. Ya.: Experimental Study of the Stability of Boundary Layer on a Wavy Surface, Zv. Sib. Otd. Akad. Nauk, SSSR, Ser. Tekh. Nauk, 3 (1974), pp. 3–6.
7. Lessen, M. and Gangwani, S. T.: Effect of Small Amplitude Wall Waviness upon the Stability of the Laminar Boundary Layer, Phys. Fluids, 19 (1976), pp. 510–513.
8. Wu, Y. S. and Malik, M. R.: Effect of Surface Waviness on Boundary-layer Transition in Two-dimensional Flows, Comput. Fluids, 27 (1998), pp. 157–181.
9. Floryan, J. M.: Two-dimensional Instability of Flow in a Rough Channel, Phys. Fluids, 17 (2005), 044101.
10. Asai, M. and Floryan, J. M.: Experiments on the Linear Instability of Flow in a Wavy Channel, Eur. J. Mech. B/Fluids, 25 (2006), pp. 971–986.
11. Ma’mun, M. D., Asai, M., and Inasawa, A.: Effects of Surface Corrugation on the Instability of Zero-pressure-gradient Boundary Layer, J. Fluid Mech., 741 (2014), pp. 228–251.
12. Ma’mun, M. D. and Asai, M.: Influences of Oblique Surface Corrugation on Boundary Layer Instability, J. Phys. Soc. Jpn., 83 (2014), 084402.
13. Fasel, H. and Konzelmann, U.: Non-parallel Stability of a Flat-plate Boundary Layer using the Complete Navier-Stokes Equations, J. Fluid Mech., 221 (1990), pp. 311–347.
14. Bertolotti, F. P., Herbert, Th., and Spalart, P. R.: Linear and Nonlinear Stability of the Blasius Boundary Layer, J. Fluid Mech., 242 (1992),
15) Herbert, Th.: Secondary Instability of Boundary Layers, *Annu. Rev. Fluid Mech.*, 20 (1988), pp. 487–526.

16) Kachanov, Yu. S., Kozlov, V. V., Kotjolkin, Ju. D., Levchenko, V. Ya., and Rudnitsky, A. L.: Laminar Boundary Layer on a Wavy Wall, *Acta Astronautica*, 2 (1975), pp. 557–559.

17) Nishimura, T., Nurakami, S., Arakawa, S., and Kawamura, Y.: Flow Observations and Mass Transfer Characteristics in Symmetrical Wavy-walled Channels at Moderate Reynolds Numbers for Steady Flow, *Int. J. Heat Mass Transfer*, 33, 5 (1990), pp. 835–845.

N. Tsuboi
Associate Editor