Cosmological perturbations of a perfect fluid and non-commuting variables

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We describe the linear cosmological perturbations of a perfect fluid at the level of an action, providing thus an alternative to the standard approach based only on the equations of motion. This action is suited not only to perfect fluids with a barotropic equation of state, but also to those for which the pressure depends on two thermodynamical variables. By quantizing the system we find that 1) some perturbation fields exhibit a non-commutativity quite analogous to the one observed for a charged particle moving in a strong magnetic field, 2) local curvature and pressure perturbations cannot be measured simultaneously, 3) ghosts appear if the null energy condition is violated.

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I. INTRODUCTION

The theory of cosmological perturbations (TCP) for a perfect fluid has always been an important issue in cosmology. It enables us to understand how small fluctuations seeded in the early universe eventually evolved into the present large scale structure. Also, TCP has been extremely useful to put constraints on various cosmological models.

TCP for a perfect fluid has been developed and studied at the level of the basic equations of motion, i.e., the Einstein equations of general relativity (GR) and the energy-momentum conservation law \([1, 2]\). Yet, TCP for a perfect fluid can be also studied at the level of the action. Although these two approaches are classically equivalent, the latter gives the following advantage. In TCP, one first has to perturb all the fields appearing in the equations of motion or in the action, such as the metric components and the energy density. However, as is well known, not all the perturbation fields are dynamical. Actually, GR with a perfect fluid has only one dynamical field for the scalar-type perturbation. But an identification of this field as well as a derivation of its closed evolution equation by means of the equations of motion alone are not straightforward.

The situation becomes worse when going to extended gravity models. For illustration, \(f(R, G)\) theories (\(R\) being the Ricci scalar, and \(G\) the Gauss-Bonnet term) with a perfect fluid involve two dynamical fields for the scalar-tensor perturbation. In such theories, the usual approach based on the equations of motion requires a rather strong intuition because the closed evolution equations for those dynamical fields have to be extracted from rather complicated coupled differential equations. On the other hand, the action approach advocated in this paper allows a straightforward identification of the auxiliary fields just by checking the absence of any kinetic terms in the second order action. Once the auxiliary fields are found, they can easily be eliminated through their trivial equations of motion. What is then left is an action containing only the dynamical fields, from which we can derive the closed evolution equations. In \([3, 4]\), we have explicitly checked that the action approach indeed works for \(f(R, G)\) gravity models with no matter and with a scalar field, respectively.

In this paper, we want to describe first-order TCP for GR with a perfect fluid at the level of the action, in a way consistent with the principles of thermodynamics. To this end, we use the action for a perfect fluid proposed by Schutz \([3]\) and do the quantization of the perturbations, which might also be of some interest beyond a pure academic point of view. Indeed, the quantization of the background universe with a perfect fluid has been discussed by many authors \([3, 13]\). But here, we prove that quantizing the perturbation fields leads to non-standard commutation relations and, consequently, to unexpected effects upon the physical properties of any perfect fluid in quantum cosmology.

A TCP action approach for fluids was first introduced in \([17]\) in the context of k-inflation. This approach was taken also in \([18]\) to study non-linear cosmological perturbations in the matter dominated universe. However, the fluid discussed there is the so-called scalar fluid whose energy-momentum tensor is completely written in terms of a scalar field and its derivative. By construction, the scalar fluid cannot have vector-type perturbations. Although there is an exact correspondence between a perfect fluid and the scalar fluid for the scalar-type perturbation at the linear order, it is no longer true for higher order perturbations because of the mixture of scalar and vector-type perturbations. On the other hand, the Schütz’s action we will use here is for a perfect fluid. Therefore the action exactly describes the dynamics of a perfect fluid at any order. As far as we know, this is the first time TCP is fully developed within the Schütz’s action. We believe our approach is suited for studying cosmology in extended gravity models.

Before introducing the action for a perfect fluid, let us briefly review the thermodynamics needed to describe it. In this paper, we consider a “single” fluid, that is, a fluid whose thermodynamical quantities are completely determined by only two variables, e.g. the chemical potential \(\mu\) and the entropy per particle \(s\) \([14]\). In this sense one first needs to give two equations of state, \(n = n(\mu, s)\) and...
$T = T(\mu, s)$, where $n$ is the number density and $T$ is the temperature of the fluid. Using then the first law of thermodynamics, $dp = n d\mu - n T ds$, one obtains the pressure as $p = p(\mu, s)$. Finally, the energy density is given by $\rho = \mu n - p$. This is enough to describe the system thermodynamically. Single fluids also satisfy particle number conservation, namely $N = n V$ is a constant. The second law of thermodynamics imposes $d(N s) = N d s$ $\geq 0$ such that $d s = 0$ at equilibrium.

A single perfect fluid is also defined through its stress-energy tensor $T_{\mu\nu}$ and is defined as follows

$$T_{\mu\nu} = (\rho + p) u_{\mu} u_{\nu} + \rho g_{\mu\nu}.$$  

In a Friedmann-Lemaître-Robertson-Walker (FLRW) background the conservation of energy-momentum, $T_{\mu\nu;\nu} = 0$, implies that $\rho + 3H(\rho + p) = 0$ or, equivalently, $d(\rho V) + p dV = 0$ since $V \propto a^3$ with $a$, the cosmological scale factor, and $H \equiv \dot{a}/a$, the Hubble parameter. This, in turn, implies that $d N = 0$ and $d s = 0$. In any FLRW universe we thus have $n a^3 = N = \text{constant}$ and $\dot{s} = 0$.

## II. ACTION

The action considered here has been introduced by Schutz [3] and is defined as follows

$$S = \int d^4 x \sqrt{-g} \left[ \frac{R}{16 \pi G} + p(\mu, s) \right].$$  

(1)

Alternative functionals have been proposed, all being physically equivalent as shown in [15]. We chose the version [11] as it was the most convenient for our purpose. The four-velocity of the perfect fluid is defined via potentials [3]:

$$u_{\nu} = \frac{1}{\mu} \left( \partial_{\nu} \ell + \theta \partial_{\nu} s + A \partial_{\nu} B \right),$$  

(2)

where $\ell$, $\theta$, $A$ and $B$ are all scalar fields. The normalization for the four-velocity, $u^{\nu} u_{\nu} = -1$, gives $\mu$ in terms of the other fields. The fundamental fields over which the action [11] will be varied are $g_{\mu\nu}$, $\ell$, $\theta$, $s$, $A$, and $B$.

Having chosen the Lagrangian for gravity to be the one of GR, we recover $G_{\mu\nu} = 8 \pi G T_{\mu\nu}$ by varying with respect to the metric field. Besides the conservation of particle number and entropy already discussed, the other equations of motion derived from Eq. [11] are [3]:

$$u^\alpha \partial_\alpha \theta = 0, \quad u^\alpha \partial_\alpha A = 0, \quad u^\alpha \partial_\alpha B = 0.$$  

(3)

In a FLRW universe, $u_i = 0$ and $u_0 = -1$ such that the solutions to Eq. [3] are simply

$$A = A(\vec{x}), \quad B = B(\vec{x}), \quad \theta = \int T(t') dt' + \tilde{\theta}(\vec{x}).$$  

(4)

There is a complete freedom for the functions $A$, $B$, and $\tilde{\theta}$ [21], any choice leading to the same physical background. We will take advantage of this freedom to simplify our study of the scalar and vector perturbations.

## III. PERTURBATIONS

Once and for all, we work within a spatially flat FLRW universe. At first order in perturbation theory we have $\delta u_0 = \frac{1}{2} \delta g_{00}$ and

$$\delta u_i = \partial_i \left( \frac{\delta \ell + \theta \delta s + A \delta B}{\mu} \right) + \frac{W_i}{\mu},$$  

(5)

with

$$W_i = B_i \delta A - A_i \delta B - \tilde{\theta}_i \delta s \equiv \partial_i u_s + \ddot{u}_i.$$  

(6)

Note that $W_i$ is gauge invariant since, following ref. [16], the perturbation fields transform respectively as

$$\delta \ell \rightarrow \delta \ell + \mu \xi^0 + A \partial_i B \xi^i,$$

$$\delta s \rightarrow \delta s,$$

$$\delta \theta \rightarrow \delta \theta - T \xi^0 - \partial_i \tilde{\theta} \xi^i,$$

$$\delta A \rightarrow \delta A - \partial_i B \xi^i,$$

$$\delta B \rightarrow \delta B - \partial_i B \xi^i,$$  

(7)

under the gauge transformation $x^0 \rightarrow x^0 + \xi^0$. In Eq. (6) we have decomposed $W_i$ into scalar ($u_s$) and divergence-less vector modes ($\ddot{u}_i$). So, in general $W_i$ will generate both scalar and vector perturbations. However, we can efficiently use the freedom of choosing the time-independent background quantities $A$, $B$ and $\theta$ given in Eq. (11) to disentangle them. Any such choice does not fix a gauge as no conditions are imposed on the perturbation fields themselves.

### A. Scalar type perturbations

Let us simply consider the choice

$$A = B = \tilde{\theta} = 0,$$  

(10)

to remove the vector perturbations arising from $W_i$. Regarding the metric, $\delta g_{00}$ and $\delta g_{i0}$ are auxiliary fields such that the only scalar component which will be dynamical is the curvature perturbation $\phi$ defined by $\delta g_{ij} = 2a^2 \phi \delta_{ij}$, with $\phi \rightarrow \phi - H \xi^0$ under a gauge transformation.

We introduce the new quantity $v = \delta \ell + \theta(t) \delta s$ such that $\delta u_i = \partial_i (v/\mu)$. Therefore, $v$ represents the velocity perturbation of a perfect fluid. We then define two gauge invariant fields, $\Phi = \phi + Hv/\mu$ and $\delta \theta = \partial_0 \theta + Tv/\mu$, to expand the action [11] at second order, in a gauge-independent way:

$$S_S = \int dt d^3 \vec{x} \left\{ \frac{a^3 Q s}{2} \left[ \dot{\Phi}^2 - \frac{c^2}{a^2} (\vec{\nabla} \Phi)^2 \right] + C \delta s \dot{\Phi} - \frac{D}{2} \dot{s}^2 - E (\delta \theta \delta s - \delta s \delta \theta + \delta A \delta B - \delta B \delta A) \right\}.$$  

(11)

The perturbation fields $\Phi$ and $\dot{\delta} \theta$ are related to the curvature and temperature, respectively. In the comoving gauge $v = 0$ where a perfect fluid remains static, $\Phi = \phi$
and \( \delta \theta = \theta \). The coefficients for the kinetic terms are given by

\[
Q_S = \frac{\rho + p}{c_s^2 H^2}, \quad c_s^2 \equiv \left( \frac{\partial p}{\partial \rho} \right)_s, \tag{12}
\]

whereas the remaining coefficients are

\[
C = \frac{n a^3}{H} \left[ \mu \left( \frac{\partial T}{\partial \mu} \right)_s - T \right], \quad E = \frac{n a^3}{2}, \tag{13}
\]

and

\[
D = na^3 \left[ T \left( \frac{\partial T}{\partial \mu} \right)_s + \left( \frac{\partial T}{\partial s} \right)_s \right]. \tag{14}
\]

The general solution for \( \delta s, \delta A, \) and \( \delta B \) is their initial values since Eq. (11) forces them to be time-independent. As a consequence, the non trivial equations of motion are

\[
\frac{1}{a^3 Q_S} \frac{d}{dt} (a^3 Q_S \Phi) - \frac{c_s^2}{a^2} \nabla^2 \Phi = -\frac{\dot{C}}{a^3 Q_S} \delta s, \tag{15}
\]

\[
a \frac{d}{dt} (b \delta \theta) - D \delta s + C \Phi = 0. \tag{16}
\]

These equations exactly coincide with those derived by perturbing the Einstein equations and the conservation law for the entropy, as it should be. In general, \( \Phi \) is sourced by \( \delta s \). For example, if the perfect fluid is an ideal non-relativistic gas characterized by \( T = \frac{\xi}{\mu} (\mu - m_0) \), \( m_0 \) being the mass of the particles, then \( C \neq 0 \) and we have to solve two coupled equations to know the time evolution of \( \Phi \) and \( \delta \theta \).

However, if \( T = f(s) \mu \), which is equivalent to having a barotropic equation of state \( p = p(\rho) \), then \( C = 0 \). (Note that both radiation and dust fulfill this condition, while a cosmological constant has vanishing \( Q_S \) so that no contribution for perturbations arises, as is well known). In this case, the sign of \( Q_S \) cannot be known from the usual approach based on the equations of motion alone. On the other hand, the action approach advertised here leads to an exact expression for \( Q_S \), which will be used to avoid ghost degrees of freedom when quantizing the perturbations.

We also conclude that the fields \( \Phi \) completely decouples from \( \delta s \) and propagates with a sound speed \( c_s \) if \( C = 0 \) and \( c_s^2 > 0 \).

\[ \]

### B. Vector type perturbations

To arrive at the desired action via the shortest path, let us first assume that all the perturbation variables propagate only in one direction, say the \( z \)-direction. This should be allowed, as we know that perturbations with different wavenumber vectors do not mix in a FLRW universe. Once we obtain the action for this particular mode, we can then easily infer the general action.

The vector contributions come only from the component \( \bar{u}_i \) of \( W_i \) defined in Eq. (9). It is not easy to extract \( \bar{u}_i \) from this equation since the functions \( A, B \) and \( \bar{\theta} \) depend in general on the spatial coordinates. Yet, taking again advantage of the freedom to select these background functions, we can make the simplest choice that contains all the information needed for the vector modes, namely

\[
A = \bar{\theta} = 0, \quad B_{,i} = b_i, \tag{17}
\]

where \( \bar{b} = (b, 0, 0) \) is a constant vector orthogonal to the \( z \)-direction. With this assumption, we have \( w_s = 0 \) and \( \bar{u}_i = b_i \delta A(t, z) \) for \( W_i \).

Regarding the vector perturbation of the metric, we follow again ref. [16] and denote \( \delta g_{0i} = a G_i, \) and \( \delta g_{ij} = a^2 (C_{i,j} + C_{j,i}), \) with transverse conditions \( G_{i,i} = C_{i,i} = 0, \) or, in our setup, \( G_z = C_z = 0. \) Then, we impose the gauge condition \( \delta B = 0. \) However, this condition alone does not completely fix the gauge, as only the component of \( \xi^i \) parallel to \( \bar{b} \) gets frozen by Eq. (9). Therefore we can still choose \( \xi^y \) such that \( C_y = 0, \) and \( \bar{C} = \bar{C}^y \) is parallel to \( \bar{b}. \) Finally, we find that the action for the vector perturbations is given by

\[
S_V = \int d^4x \left\{ \frac{a}{32 \pi G} \left[ (\partial_0 V_x)^2 + (\partial_0 V_y)^2 \right] + na^3 b \delta A \dot{\bar{C}} x \right. \\
+ na^2 b V_x \delta A + 2 \pi G b^2 n^2 a \delta A^2 / H \right\}, \tag{18}
\]

where \( V_i \equiv G_i - a \bar{C}_i \) is a gauge invariant field. This action can be immediately extended to the general case where the perturbation variables depend now on \( (x, y, z) \). In the gauge \( \delta B = 0, \) the result is given by

\[
S_V = \int d^4x \left[ \frac{a}{32 \pi G} (\partial_i V_j) (\partial_j V_i) + a^3 (\rho + p) \dot{\bar{C}}_i \delta u_i \right. \\
+ a^2 (\rho + p) V_i \delta u_i - \frac{1}{2} a (\rho + p) \delta u_i \delta u_j \right], \tag{19}
\]

where we substituted \( \delta u_i \) for \( b_i \delta A/\mu \). Variations with respect to \( V_i \) and \( C_i \) yield the following equations,

\[
\Delta V_i = 16 \pi G a (\rho + p) \delta u_i, \tag{20}
\]

\[
\frac{d}{dt} [(\rho + p)a^3 \delta u_i] = 0, \tag{21}
\]

respectively. Again, these equations exactly coincide with those derived by perturbing the Einstein equations and the energy-momentum conservation law [16]. This provides thus a cross-check that the calculations presented here are correct. In fact, the main novelty in our approach is to be found when we quantize the system.

To summarize section III, the known results on first-order TCP for a perfect fluid can be directly derived from variations of the classical action given in Eq. (11). Note that a similar action approach has been already performed in [17, 18]. Yet, the system studied there cannot represent a perfect fluid. Indeed, as already mentioned in the introduction, the action proposed in [17, 18] is made
of a real scalar field. So, this system, by construction, cannot have vector perturbations, as the only new perturbed field is the scalar one. Therefore the system studied there is not a perfect fluid, otherwise a perfect fluid would have no vector perturbation. It can be thought of as a scalar fluid, but, once more, not as a perfect fluid. It is simply a different physical system whose squared sound speed \(c_s^2\) is not equal to \(\bar{p}/\bar{\rho}\).

IV. QUANTIZATION

The most important advantage of the action approach proposed in this paper is, of course, that it allows us to quantize the system. Although the inhomogeneities of the present universe, such as the galaxy distribution, are clearly described by the classical theory, the quantization of a perfect fluid may have something to do with the early universe if the seeds for structure formation are provided by quantum fluctuations of fields generated during inflation. Yet, besides its practical utility, our action approach also opens new theoretical prospects, as discussed below. In the following, we will again treat the quantization for the scalar and vector type perturbations separately.

A. Scalar type perturbations

To quantize the scalar perturbations, let us first introduce the canonical field \(\psi \equiv \sqrt{a}\Psi\). To avoid the appearance of a ghost, we assume that \(Q_S\) is positive. According to Eq. (12), this means that \((\rho + p)/c_s^2 > 0\). Such a constraint, together with the stability of the perturbations, \(c_s^2 > 0\), lead to the null energy condition \(\rho + p > 0\). Using the new variable \(\psi\), the action (11) is rewritten as

\[
S_S = \int d^4x \left[ \frac{\dot{\psi}^2}{2} - \frac{c_s^2}{2a^2}(\nabla \psi)^2 + C_1 \delta s \dot{\psi} + C_2 \delta s \dot{\psi} - \frac{N}{2}(\delta \Theta \delta s - \delta s \delta \Theta) - \frac{D}{2} \delta s^2 \right],
\]

where we have neglected \(\delta A\) and \(\delta B\) as they do not contribute to the Hamiltonian. The field \(\psi\) has a canonical kinetic term, whereas the quadratic terms for \(\delta s\) and \(\delta \Theta\) are at most linear in their time derivatives. Yet, it is known (19) that a consistent quantization of such a singular Lagrangian can be done provided one introduces the following equal-time commutation conditions,

\[
[\hat{\psi}(t, \vec{x}), \hat{\pi}(t, \vec{y})] = i\delta(\vec{x} - \vec{y}),
\]

\[
[\hat{\delta s}(t, \vec{x}), \hat{\delta \Theta}(t, \vec{y})] = -\frac{i}{N}\delta(\vec{x} - \vec{y}).
\]

All the other commutators are zero and \(\pi\) is the canonical conjugate momentum of \(\psi\). The corresponding Hamiltonian is given by

\[
\hat{H} = \int d^3\vec{x} \left[ \frac{1}{2} \left( \hat{\pi} - C_1 \delta s \right)^2 + \frac{c_s^2}{2a^2}(\nabla \hat{\psi})^2 \right. \\
- \left. C_2 \delta s \hat{\psi} + \frac{D}{2} \delta s^2 \right].
\]

One can easily check that the Heisenberg equations, with the help of the commutation relations, yield the same equations of motion as the classical ones derived from the variation of Eq. (22).

The relation (24) shows that \(\delta s\) and \(\delta \Theta\) become non-commuting variables at the quantum level. In Quantum Field Theory, different fields (i.e., different particles) can be simultaneously observed at the same position. Here the perturbation fields related to the entropy and the temperature, to which we may individually attribute arbitrary numbers at the classical level, cannot be measured at the same space-time point. That this non-commutativity arises from the action of a perfect fluid is thus intriguing.

We should concede that consequences directly linked to present observations are missing. However, at this level it is quite interesting to compare the action (22) with the one of the Landau problem (19) of an archetype of non-commutative geometry. Regarding \(\delta s\) and \(\delta \Theta\), the action (24) is essentially the same as the one for a charged particle moving on a two-dimensional surface with a constant magnetic field background in the transverse direction:

\[
S = \int dt \left[ \frac{m}{2}(\dot{x}^2 + \dot{y}^2) - \frac{B}{2}(\dot{x}y - \dot{y}x) - V(x, y) \right].
\]

Within this analogy, the perturbation fields \((\delta s, \delta \Theta)\) correspond to the \((x, y)\) space coordinates for the particle, and the number of particles \(N = na^3\) plays the role of the constant magnetic field \(B\). Interestingly enough, while the quite heuristic non-commutative relation \([\hat{x}, \hat{y}] = -i/B\) in the Landau problem (19) holds only in the absence of the kinetic term in Eq. (26), which is valid in the large magnetic field limit, the non-commutative relation (24) of a perfect fluid is exact for any finite number of particles. So, perfect fluids provide a nice example of non-commutativity between different fields.

The other non-commutation relation (24) leads also to an interesting physical consequence. By using once more the Einstein equations and the energy-momentum conservation law, we find that the pressure perturbation in the comoving gauge \((v = 0)\) is given by \(\delta \rho = -\langle\rho + p\rangle / H\). Then, the commutator between \(\phi\) and \(\delta \rho\) becomes

\[
[\hat{\phi}(t, \vec{x}), \hat{\delta \rho}(t, \vec{y})] = -ic_s^2H\delta(\vec{x} - \vec{y})/a^3.
\]

Consequently, local curvature and pressure perturbations cannot be measured simultaneously.
B. Vector type perturbations

Time derivatives of $V_i$ and $\delta u_i$ do not appear in the action \[^{[19]}\]. Therefore, those are auxiliary fields which can be eliminated through their equations of motion. The action \[^{[19]}\] becomes then a functional which depends only on $C_i$. To make this action canonical, we introduce a new variable $F_i(k,t) = \sqrt{a^3Q_V(k,t)C_i^2}\hat{C}_i(k,t)$, where $C_i^2(\vec{x},t)$ is the Fourier transform of $C_i^2(\vec{x},t)$ and $Q_V$ is given by

$$Q_V(k,t) = \frac{a^2k^2(\rho + p)}{k^2 + 16\pi Ga^2(\rho + p)}. \quad (28)$$

To avoid the appearance of ghosts, $Q_V$ must be positive. So, as for the scalar modes we require $\rho + p > 0$, i.e. the null energy condition to hold. In terms of $F_i$, the canonical action in Fourier space is given by

$$S_V = \int dt d^3k \left( \frac{1}{2} \hat{F}_i^\dagger \dot{\hat{F}}_i - \frac{i}{2} m^2 k^2 \hat{F}_i^\dagger \hat{F}_i \right), \quad (29)$$

with

$$m^2 = -\frac{1}{2} \frac{d^2}{dt^2} \log(a^3Q_V) - \frac{1}{4} \left( \frac{d}{dt} \log a^3Q_V \right)^2. \quad (30)$$

Now the quantization is done by imposing the following canonical condition for $F_i$ and its conjugate momentum

$$[\hat{F}_i(t,\vec{k}), \pi_j^\dagger(t,\vec{k}')] = i\delta(\vec{k} - \vec{k}') \left( \delta_{ij} - \frac{k_i k_j}{k^2} \right). \quad (31)$$

The corresponding Hamiltonian is given by

$$\hat{H} = \int d^3k \left( \frac{i}{2} \pi^\dagger \pi_i + \frac{i}{2} m^2 k^2 \hat{F}_i^\dagger \hat{F}_i \right), \quad (32)$$

and the evolution of the operators is given by the Heisenberg equation with the help of the commutation relation \[^{[31]}\]. The quantum version of Eq. \[^{[20]}\] implies $[\hat{V}_i(t,\vec{x}), \delta u_j(t,\vec{y})] = 0$. Therefore, the gauge invariant metric perturbation and the vorticity of the perfect fluid can be measured at the same time, at the same position.

As for the tensor perturbations, they come only from the metric perturbation. The action for the tensor perturbations and its quantum aspects have been widely studied in the literature (e.g. \[^{[10]}\]), mainly in connection with the quantum generation during inflation. So, we do not discuss it any longer.

V. CONCLUSIONS

We have studied the theory of cosmological perturbations for a perfect fluid in GR at the action level. Starting from the action proposed by Schutz, we first reproduced the known results derived from the equations of motion alone. This enabled us to illustrate the advantage of our action approach at the classical level. Quantizing then the perturbation fields, we found that some of them do not commute, leading thus to a non-commutative field-geometry. In particular, we pointed out that a simultaneous measurement of local curvature perturbations and pressure inhomogeneities is not allowed at the quantum level. Finally, we proved that both the null energy condition and a positive squared sound speed have to hold at all times in order to avoid ghost degrees of freedom.

Another advantage of our action approach is that one can easily obtain the second order action depending only on the dynamical fields. Such an approach is thus suited to study cosmology in extended gravity models with more than one dynamical field. In particular, we expect that the approach presented here will be quite useful for the perturbation analysis of $f(R,G)$ gravity models \[^{[20]}\], or for the treatment of non-gaussianities for the entropy and vector perturbations on perfect fluids following \[^{[18]}\].

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[21] Since \(u_\nu = (-1, \vec{0})\), we also have that \(\ell = -\int t' \mu(t') dt' + \tilde{\ell}\), and \(\vec{\nabla} \tilde{\ell} = -A \vec{\nabla} B\), which implies that \(\vec{\nabla} A \times \vec{\nabla} B = 0\).
[22] For \(c_s^2\) we used the fact that \(\dot{\rho} = (\partial p/\partial \rho)_{\dot{s}} \dot{s} + (\partial p/\partial s)_{\rho} \dot{\rho}\).
[23] In this case, we obtain \((\partial \mu/\partial s)_{\rho} = T\) such that \((\partial p/\partial s)_{\rho} = n[(\partial \mu/\partial s)_{\rho} - T] = 0\).