Spatial Coherence of Tunneling in Double Wells.

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Abstract

Tunneling between two 2D electron gases in a weak magnetic field is of resonance character, and involves a long lifetime excitonic state of an electron and hole \textit{uniformly spread} over cyclotron orbits. The tunneling gap is linear in the field, in agreement with the experiment, and is anomalously sensitive to the electron density mismatch in the wells. The spatial coherence of tunneling along the orbit can be probed by magnetic field parallel to the plane, which produces an Aharonov-Bohm phase of the tunneling amplitude, and leads to an oscillatory field dependence of the current.

Electrons in GaAs quantum wells have very high mobility, and at low temperature form an almost ideal Fermi liquid. Strong Coulomb interaction and sensitivity to external magnetic fields make the physics of this system rich and interesting. Recently, several phenomena have been discovered in tunneling experiments in quantum wells, including: a tunneling gap induced by a magnetic field \cite{1}; resonance peaks of conductivity near zero bias \cite{2}; and excitonic effects \cite{3}. Typically, one has two wells containing 2D Fermi liquids with an electron density of the order of $10^{11}\text{cm}^{-2}$, separated by an oxide barrier of few tens nanometers thick. It is characteristic that the barrier thickness is very uniform, so that tunneling is coherent in lateral dimensions. This results in conservation of both momentum and energy. Basically, at a zero magnetic field, only tunneling between identical plane wave states can occur, which restricts the phase space of final states and leads to an unusual $I-V$ curve with a sharp peak near zero bias \cite{2}. Finite width of the peak is determined by elastic scattering. One
can probe spatial coherence by applying a magnetic field parallel to the barrier. Having practically no effect on the dynamics in the plane, the vector potential of the parallel field shifts electron momenta in one plane, which creates a mismatch of the Fermi surfaces of different planes. The effect on the tunneling rate can be easily accounted for by the free electron picture [2].

A magnetic field applied perpendicular to the plane makes the situation more interesting, especially in high fields, when the system is in the Quantum Hall state [1]. Compared to the $I-V$ curve at zero field, the current peak is shifted away from zero bias to some finite voltage. Also, the peak broadens as the field increases. The current is almost entirely suppressed below the lower edge of the peak, called a “tunneling gap.” Recently the tunneling gap has been intensively studied because it is believed that it can be used to probe the QH state. The gap depends on the field linearly at weak field ($\nu \gg 1$) [4], and saturates at higher field ($\nu \simeq 1$) [1]. To summarize, one can say that the $I-V$ curve can be interpreted in terms of a resonant tunneling mechanism, involving some intermediate state with the lifetime given by inverse peak width, and of the work needed to create this state corresponding to the gap.

The gap at high field is quite well understood [5–7]: its energy scale is of the order of $e^2/\epsilon a$, where $a = n^{-1/2}$ is the interparticle distance, and $\epsilon$ is the dielectric constant. Until recently, the low field gap has received less attention. The only available theory is by Aleiner, Baranger, and Glazman [8], who developed a hydrodynamical picture by treating the system as an ideal compressible conducting liquid. They wrote down classical electrodynamics equations in terms of charge and current densities, and derived the tunneling gap $\Delta = (\hbar \omega_c / \nu) \ln(\nu e^2 / e \hbar v_F)$. At constant electron density, they predict quadratic dependence of the gap on the magnetic field, which is different from the linear dependence found experimentally [1].

The reason for the disagreement, in our opinion, is that in a clean metal, such that $\omega_c \tau \gg 1$, one can use classical electrodynamics only on a scale much bigger than the cyclotron radius $R_c = v_F / \omega_c$. However, we will argue that the important scale of the problem is of the order of $R_c$, and thus one has to have a Fermi liquid in magnetic field. On this scale,
the state formed at tunneling has a non-trivial spatial structure, which makes the physics very different from that of the gap in high fields. Also, we will find that at weak field a large simplification occurs, because the problem is semiclassical, and one can use the classical Fermi liquid equation to describe the dynamics in terms of Fermi surface fluctuations.

Another point is that in two dimensions the energy and momentum conservation prescribes that all quantum numbers of the final and initial tunneling states coincide. In a magnetic field, this implies that the radius of an electron orbit as well as the guide center of the orbit are conserved at tunneling. In a weak field, this results in spatial coherence of tunneling over a large distance of the order of $R_c$, and leads to interesting effects.

**Summary of results.** Our goal is to explain the linear field dependence of the gap in a weak magnetic field, and to propose experiments that will reveal spatial structure of the intermediate tunneling state. Although at the end we are going to do a rigorous Fermi liquid calculation, it is instructive to begin with a semiclassical picture of electron states localized near classical cyclotron orbits and weakly interacting with each other. At tunneling, an electron hops from the orbit of radius $R_c$ in one layer to the identical orbit in the other layer, and leaves a hole on the first orbit. This creates an electrostatic configuration of two oppositely charged rings of radius $R_c$ separated by the barrier of width $d \ll R_c$. The energy of this charge distribution is

$$
\Delta = \frac{e^2}{\epsilon \pi R_c} \ln \frac{d}{l_B},
$$

where $\epsilon$ is the dielectric constant, and the magnetic length $l_B = \sqrt{\hbar c/eB}$ characterizes the ring's "thickness." Basically, we are saying that, in order to transfer an electron, one has to charge the "two ring capacitor," and its charging energy $e^2/2C$ constitutes the tunneling gap. The result (1) holds for $l_B \leq d$, i.e., fields which are not too low, and fall in the experimental range [4]. The gap $\Delta$ dependence on magnetic field is nearly linear, since the log term is roughly constant. The dependence on the barrier width is in agreement with the excitonic picture [3,7]. By the order of magnitude the gap (1) agrees with the experiment [4], which raises the question of why there is no gap suppression due to Coulomb screening. Basically,
the reason is that he charge distribution is localized in a very thin ring, of the thickness \( l_B \) comparable to the screening length, which makes the screening effects not too dramatic: they simply change the log in Eq. (1) by a constant of the order of one.

That the tunneling is coherent along the cyclotron orbit can be easily verified by applying a magnetic field parallel to the barrier, in addition to the perpendicular field. The parallel field flux “captured” between electron and hole trajectories will give an Aharonov-Bohm phase to the tunneling amplitude, and make it proportional to

\[
\int \int e^{\frac{\Phi}{\hbar} R_c dB_\parallel (\cos \theta_1 - \cos \theta_2)} d\theta_1 d\theta_2 .
\]

Thus the current will oscillate as a square of the Bessel function:

\[
I(V) = J_0^2 \left( \frac{p_F d}{\hbar B_\perp} B_\parallel \right) I(V)_{B_\parallel=0} .
\]

For an ideal two dimensional system, the current dependence on \( B_\parallel \) factors out because a parallel field does not affect the motion in plane. For real wells of finite width, the factorization (3) should still be a good approximation at small \( B_\perp \), when the cyclotron radius is big compared to the well width. However, since the parallel field will squeeze the states in the wells, and effectively increase the barrier width, the tunneling rate may acquire an additional non-oscillatory suppression factor.

Also, it is clear from what has been said, that the gap will be very sensitive to any asymmetry between the wells. For example, if there is a small mismatch \( \delta n \) of densities in the wells, the radii of the electron and hole orbits will become different, \( \delta R_c \simeq (R_c/2n)\delta n \), which will reduce mutual capacitance of the orbits, and thus raise the electrostatic energy. The gap will be enhanced by the order of its magnitude at \( \delta R_c \simeq l_B \), which corresponds to the density mismatch \( \delta n/n \simeq l_B/R_c \) anomalously small at weak field.

**Fermi liquid calculation.** To study the screening effects, we will use the Landau equation

\[
\left( \partial_t - D(1 + \hat{F}) \right) \delta n(p, r, t) = 0 ,
\]

where \( D = v_p \cdot \nabla + \omega_c m v_p \times \nabla_p \) contains the Lorentz force term. In Eq. (4) we have omitted a collision integral, which is legitimate in the semiclassical limit \( \hbar \omega_c \ll E_F \). We will find a
stationary solution that fully accounts for the screening of the electron and hole charges. The resulting distribution consists of two parts: singular and smooth. The singular distribution is localized near the electron and hole cyclotron orbits, within a short distance of the order of magnetic length. The smooth part is spread over a distance larger than the cyclotron radius. It is worthwhile to make a comparison with the standard picture of a particle in a Fermi liquid, which consists of a quasiparticle excitation combined with a background density fluctuation, and to draw a relation with the singular and smooth densities.

For simplicity, let us assume that \( \hat{F} \) corresponds to a pure density-density interaction:

\[
(\hat{F}\delta n)_\alpha(p, r) = \sum_{r', \nu', \beta} U_{\alpha\beta}(r - r') \delta n_\beta(p', r'),
\]

where \( \alpha, \beta = R, L \) label wells, and \( U_{\alpha\beta}(r) \) is the Coulomb potential. It will be straightforward to modify the calculation for a more general Fermi liquid interaction. One can write \( \delta n(p, r) \) corresponding to the electron and hole being uniformly spread over cyclotron orbits:

\[
\delta n^{(0)}_{R(L)}(p, r) = \pm \int_0^T dt \frac{dT}{T} \delta(p - p(t)) \delta(r - r(t)) ,
\]

where \( r(t), p(t) \) is the classical circular trajectory with \( |p| = p_F \), and with the period \( T = 2\pi/\omega_c \). Semiclassically, the density (6) corresponds to a state of the lowest Landau level available for tunneling. At tunneling, initially, the states (6) are created, and then, over the time scale \( \sim \omega_c^{-1} \), they relax to a stationary state, whose energy gives the tunneling gap.

In order to find the system response to the appearance of the electron and hole, we have to solve the equation:

\[
\left( \partial_t - D(1 + \hat{F}) \right) \delta n = \delta(t) \delta n^{(0)} .
\]

Formally, the solution of Eq.(7) can be written as a “ladder”:

\[
\delta n = \left( G + GD\hat{F}G + GD\hat{F}GD\hat{F}G + \ldots \right) \delta n^{(0)} ,
\]

where \( G = (\partial_t - D)^{-1} \) is readily evaluated in the Fourier representation:
\[ G(k, p, p') = \sum_n e^{\imath \theta p - \imath \theta p'} + \imath k \times (p - p')/m \omega_c, \] (9)

The density (8) satisfies the equation (4) with \( \hat{F} = 0 \). The first term of the ladder (8) gives \( \delta n(0)(p, r) \theta(t) \), the singular part of the density. The rest is the smooth “background” density.

In terms of \( \delta n \) the gap is given by

\[ \Delta = \sum_{p, r, \alpha, p', r', \beta} \delta n(0)_\alpha(p, r) \hat{F}_{\alpha \beta} \delta n(0)_\beta(p', r', t \to \infty), \] (10)

By summing up the ladder (8) in a standard Fermi liquid fashion, one gets

\[ \Delta = \sum_{k, p, p'} \delta n(0)_\alpha(p, r) \hat{F}_{\alpha \beta} \delta n(0)_\beta(p', r', t \to \infty), \] (11)

(1 + \( \hat{F}_k \)) \( \hat{F}_{\omega, k} = \hat{F}_k \left( 1 + \omega \sum_{n} \frac{J^2_n(k R_c)}{\omega - n \omega_c} \hat{F}_{\omega, k} \right) \), (12)

Here \( J_n \) are Bessel functions. At \( \omega \to 0 \) it turns into \( \hat{F}_k = \hat{F}_k \left( 1 + \hat{F}_k(1 - J^2_0(k R_c)) \right)^{-1} \). By plugging it in Eq. (11) together with the Fourier transform of density, \( \int d^2 p \delta n(0)_{k, R_L}(p) = \pm J_0(k R_c) \), we finally get

\[ \Delta = \int_0^\infty J^2_0(k R_c) e^{2 U_k} \frac{k dk}{2\pi}, \] (13)

where \( U_k = 2\pi(1 - e^{-kd})/\epsilon k \) is the 2D Fourier transform of the difference of the in-plane and interplane Coulomb interaction, and \( \nu = m/\pi \hbar^2 \) is the compressibility.

**Discussion of Eq. (13).** We can evaluate the contributions of two regions in the \( k \)-space, \( k R_c \gg 1 \) and \( k R_c \ll 1 \), by replacing \( J_0(x) \) with its asymptotic expressions:

\[ \begin{align*}
(i) \quad & \Delta_{k R_c \gg 1} = \frac{e^2}{\pi \epsilon \hbar u_F} \hbar \omega_c \ln \frac{d}{r_s}; \\
& \Delta_{k R_c \ll 1} = \frac{\epsilon \omega_c^2}{m u_F^2} \ln \frac{d}{r_s}.
\end{align*} \] (14)

According to the experiment [4], we now assume that \( l_B < r_s < d \ll R_c \), where \( l_B \) is magnetic length, and \( r_s \) is the Coulomb screening length. The contribution (14) coincides with the estimate (11), except for the log term suppressed by screening. When the result
(14) is compared to the experiment [1], one has to be careful because the screening length is probably set by the well width, rather than by the electrons' compressibility. Because this may alter the log term, we can claim only an agreement with the experimentally measured $\Delta$ by the order of magnitude.

It is interesting to note that the contribution (14) coincides with the gap found by hydrodynamics theory [8], the difference in the log term being due the effect of screening by another plane. The two terms (14),(15) account for contributions of the two parts of the charge density, the singular and the smooth. Not surprisingly, the singular density dominates in the energy, which one can understand by comparing charging energy of a two-ring capacitor with that of a parallel plate capacitor, both of the size $R_c$.

Bosonization calculation: a sketch. Now we proceed with an accurate derivation of tunneling current. In the weak field, $\hbar \omega_c \ll E_F$, the electron motion is semiclassical. Therefore, instead of doing full many-body theory, we can write down the action that corresponds to the classical Landau Fermi liquid equation, and use it to study dynamics in imaginary time. By that, we will determine optimal path in imaginary time (instanton) whose action will give the exponent of the tunneling rate.

The advantage of this approach, besides making a clear relation with classical theory, is the possibility of including effects of a parallel magnetic field and of a scattering by disorder in a natural fashion.

We use a bosonized Fermi liquid picture [9,10] to write electron operators in terms of Bose fields:

$$\psi_{R(L)}(r) = \frac{1}{\sqrt{2\pi v_F \tau_0}} \int e^{i p_F n r - i \phi_{R(L)}(n,r,t)} d\mathbf{n}.$$  \hfill (16)

Here the unit vector $\mathbf{n} = p/p_F$ labels points of the 2D Fermi-surface, and $\tau_0$ is a cutoff of the order of the inverse bandwidth that appears in the bosonization formalism. The integral $\int \ldots d\mathbf{n}$ means $\int \ldots d\theta/2\pi$, where $\theta$ is the polar angle in the $p-$plane.

The bosonized imaginary time action for $\phi(n,r,t)$ is

$$S = \frac{m}{4\pi} \int \phi D \left( i \partial_t - D \left( 1 + \hat{F} \right) \right) \phi d^2\mathbf{r} \, dt.$$  \hfill (17)
Here \( \phi = (\phi_R, \phi_L) \), and \( \hat{F} \) is the integral operator (3).

Note that \( u = D\phi(n, r, t) \) has a clear meaning of the displacement of the Fermi-surface at the point \( n \) in the normal direction, i.e., along \( n \). The saddle points \( \phi(n, r, t) \) of this action satisfy Eq. (4). According to Haldane \[9\], in the context of the Fermi liquid bosonization, the operator \( D \) has an interpretation of a covariant derivative.

To get the tunneling current, one has to find the amplitude

\[
K(r, \tau) = \langle T_\tau \psi_R^+(r, i\tau)\psi_R(0, 0)\psi_L(r, i\tau)\psi_L^+(0, 0) \rangle.
\]

(18)

Then, one continues \( K \) to the real time axes from the upper and the lower halfplane. (Following [11], we denote these functions as \( K^>(\tau) \) and \( K^<(\tau) \). By the standard formalism [12], one can express the tunneling current as

\[
I = e^{-|t_0|^2} \text{Im} \int d^2x dt \left( \Phi_{r, t} e^{-ieVt} + \Phi_{r, -t} e^{ieVt} \right),
\]

(19)

where \( \Phi_{r, t} = K^>(r, t) - K^<(r, t) \). To find \( K \), one has to evaluate the functional integral of the product of the exponentials [16] with the weight \( e^{-S} \). One notes that the integral is gaussian, and writes:

\[
K(r, t) = \frac{1}{(2\pi v_F t_0)^2} \int e^{-S_{n_0, n_1}} dn_0 dn_1.
\]

(20)

Here the action

\[
S_{n_0, n_1} = \frac{4\pi^2}{m} \langle JD^{-1}(i\partial_t - D + \hat{F})^{-1} J \rangle,
\]

(21)

where \( \langle ... \rangle = \int ... dn d^2R dt \), and

\[
J = \delta(r) \delta(t) \delta(n - n_0) - \delta(r - R) \delta(t - \tau) \delta(n - n_1).
\]

(22)

The source term \( J \) describes an electron injected at \( t = 0, r = 0 \) at the point \( n_0 \) of the Fermi surface, and then removed at \( t = \tau, r = R \) from a different point \( n_1 \).

Calculation of the action (21) involves inverting the operator \( i\partial_t - D + \hat{F} \), which has been done above (see (8), (9)), and then applying it to \( J \). The latter procedure requires
a lot of attention, because of the singular nature of the operators in (21). A complete
discussion of this operator, including its relation to classical trajectories, and regularization
technique will be presented elsewhere [14]. The result of this calculation, however, is very
simple: \( I(V) \approx \delta(eV - \Delta)\text{sign}(V) \), where \( \Delta \) is given by Eq.(13). In other words, the rigorous
calculation confirms the semiclassical result (13) for the gap, and predicts an infinitely sharp
resonance peak at \( eV = \Delta \).

To include scattering by disorder, we modify the action (17) by adding a “collision
integral” term:

\[
S = \frac{m}{4\pi} \int \phi_{-\omega}(n, r) D\hat{W}\phi_{\omega}(n, r) \text{sign}\frac{d\omega}{2\pi} d\omega d^2r. \tag{23}
\]

Here \( \hat{W} \) is the collision integral operator

\[
\hat{W}u(n) = -\frac{1}{\tau_s}(u(n) - \int u(n) \, d\nu), \tag{24}
\]

taken in the so called \( \tau \)–approximation. It is straightforward to verify that the new action
leads to the classical Fermi liquid kinetic equation with the scattering term. Let us note that
in the imaginary time representation the action (23) is non-local, because it is dissipative.

The Gaussian functional integral over \( \phi \) again gives (20), where now

\[
S_{n_0, n_1} = \frac{4\pi^2}{m} \langle JD^{-1} \left(i\partial_t - D(1 + \hat{F}) - \hat{W}\right)^{-1} J \rangle. \tag{25}
\]

In the absence of a magnetic field, by evaluating the action (25), then plugging it in
Eqs.(20),(19), and doing the integral [14], we get

\[
I_0(V) = \frac{e|t_0|^2}{2\pi v_F p_F \tau_s} \frac{eV}{eV^2 + \frac{1}{\tau_s}^2}. \tag{26}
\]

The peak of conductivity \( dI/dV \) is located at zero bias, and has finite width due to elastic
scattering, and Lorentzian tails, in agreement with the experiment [2,4].

A perpendicular magnetic field creates the tunneling gap. Calculation [14] shows that
the \( I - V \) curve is related to the zero-field curve (20) in a simple way:
\[
I(V) = \begin{cases} 
I_0(V - \Delta/e) & \text{at } V > \Delta; \\
0 & \text{at } -\Delta < V < \Delta; \\
I_0(V + \Delta/e) & \text{at } V < -\Delta
\end{cases}
\]  
\tag{27}

where \( \Delta \) is given by Eq.(1). So, at finite \( B_\perp \) the peaks of \( I_0(V) \) are simply shifted by \( \Delta/e \) away from zero bias.

This formula correctly describes the width of the gap and the peak shape, however it doesn’t hold deep inside the gap, near \( V = 0 \), because our simple approach to scattering breaks down at \( t \gg \tau_s \). In this limit the charge spreading regime changes to diffusive, and one has to deal with a classical electrodynamis problem \cite{13}.

To include a parallel field, we modify the amplitude \( K(r,t) \) by introducing in (20) the Aharonov-Bohm phase given by the parallel field flux:

\[
\tilde{K}(r,t) = e^{-i \frac{e}{\hbar c} B_\parallel dy \int K(r,t)} ,
\]  
\tag{28}

where \( y \) is the component of the radius vector \( r \) perpendicular to the field \( B_\parallel \). The expression (19) then produces the integral (2) equal to \( J_0^2 \left( \frac{p_\parallel d}{\hbar B_\perp} B_\parallel \right) \). This gives a prefactor to the tunneling current, which makes the current an oscillating function of \( B_\parallel \).

**Conclusion.** We interprete the \( I - V \) curve in terms of a resonance tunneling mechanism that involves an excitonic state, whose structure in the weak field limit can be studied by the Fermi liquid theory. We calculate the tunneling gap, and find linear dependence on the magnetic field, in agreement with the experiment. The structure of the excitonic state manifests itself in the gap high sensitivity to any asymmetry between the wells, like density mismatch. Also, it can be probed by a magnetic field parallel to the barrier, by observing oscillatory dependence of the current on the field.

We study the problem by means of the Fermi liquid bosonization theory, and obtain a complete \( I - V \) curve that includes the effects of the perpendicular and parallel fields, as well as of elastic scattering.
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