Non-Markovian dynamics of entanglement for multipartite systems

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Abstract
Entanglement dynamics for a couple of two-level atoms interacting with independent structured reservoirs is studied using a non-perturbative approach. It is shown that the revival of atom entanglement is not necessarily accompanied by the sudden death of reservoir entanglement, and vice versa. In fact, atom entanglement can revive before, simultaneously or even after the disentanglement of reservoirs. Using a novel method based on the population analysis for the excited atomic state, we present the quantitative criteria for the revival and death phenomena. To give a more physically intuitive insight, the quasimode Hamiltonian method is applied. Our quantitative analysis is helpful for the practical engineering of entanglement.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Entanglement is highly relevant to the fundamental issues of quantum mechanics and plays a central role in the application of quantum information [1]. In recent years, it has been observed that two qubits interacting with independent reservoirs experience disentanglement in a finite time in spite of the asymptotical decoherence. The corresponding investigations for this phenomenon, called entanglement sudden death (ESD), give notable results both theoretically [2–7] and experimentally [8, 9]. Nevertheless, most previous works concentrate on the dynamical evolution of a bipartite system, while the understanding of how the information is transferred is to be explored in depth, especially for non-Markovian environment.

Recently, people begin to analyse the dynamics of an extended system which incorporates two qubits and their respective reservoirs and successfully explain the process of information transmission in a Markovian regime [10]. Thanks to these significant improvements and using the non-perturbative method [11–13], we extend the study of this composite system into a non-Markovian regime where the reservoir presents some internal structure.

First of all, we study the entanglement dynamics of two atoms and two reservoirs. As expected, when the atom entanglement is depleted permanently, it is completely transferred to the reservoir entanglement irrespective of the non-Markovian memory effect. The reservoir entanglement can not only exhibit entanglement sudden birth (ESB) but also ESD. Surprisingly, the revival of atom entanglement [14] is not always accompanied by the disentanglement of reservoirs, and vice versa. Entanglement of atom pairs can revive before, simultaneously or even after the ESD of reservoirs and these phenomena are independent of the relative strength of atom–reservoir interaction.

In addition, we analyse the effective bipartite entanglement and multipartite entanglement within the system and thus give a comprehensive interpretation of the information transmission in the non-Markovian regime. Applying the population analysis for the excited atomic state, we present quantitative criteria, which have been left obscure for quite a long time, of the occurrence and exact numbers for revivals of atom entanglement and death of reservoir entanglement. Our analytic and numerical analysis enables...
one to precisely manipulate the entanglement based on the atoms or/and quantum dots in high Q cavities [26–29] and thus have potential importance in application.

Finally, by transforming the true-mode Hamiltonian [12, 19] to the quasimode form [12] which explicitly separates the memory effect and the damping effect of the reservoir, we give a more physically intuitive insight into the non-Markovian phenomena.

2. Theoretical framework

Under the rotating-wave approximation, the true-mode Hamiltonian of the single-body system (atom and its reservoir) is \((\hbar = 1)\) [20, 21]

\[
H = H_0 + H_{\text{int}},
\]

where

\[
H_0 = \sigma_0 \sigma_+ + \int_{-\infty}^{\infty} \text{d} \omega_k \omega_k b^\dagger (\omega_k) b(\omega_k),
\]

\[
H_{\text{int}} = \int_{-\infty}^{\infty} \text{d} \omega_k g(\omega_k) \sigma_+ b(\omega_k) + \text{H.c.}
\]

Here, \(\sigma_\pm\) and \(\omega_0\) are the annihilation operators and transition frequency of the atom, and \(b(\omega_k), b^\dagger (\omega_k)\) are the annihilation and creation operators of the field mode of the reservoir with the eigenfrequency \(\omega_0\).

For an initial state of the form \(|e\rangle \otimes |0\rangle\), with \(|0\rangle = \prod_k |0_k\rangle\), the time evolution of the single-body system is

\[
|\psi(t)\rangle = c_0(t) e^{-i\omega_0 t}\langle e|0\rangle + \int_{-\infty}^{\infty} \text{d} \omega_k c_{\omega_k}(t) e^{-i\omega_0 t}\langle \omega|1_k\rangle,
\]

(3)

where \(|1_k\rangle\) is the state of the reservoir with only one exciton in the \(k\)th mode. According to the Schrödinger equation, the equations for the probability amplitudes take the forms

\[
\iota c_0(t) = \int_{-\infty}^{\infty} \omega_k g(\omega_k) e^{i(\omega_0-\omega_0) t} \text{d} \omega_k,
\]

(4)

\[
\iota c_{\omega_k}(t) = g^\dagger (\omega_k) e^{i(\omega_0-\omega_0) t} c_0(t).
\]

(5)

Eliminating the coefficients \(c_{\omega_k}(t)\) by integrating (5) and substituting the result into (4), one yields

\[
\dot{c}_0(t) = -\int_{0}^{t} c_0(t_1) f(t-t_1) \text{d} t_1,
\]

(6)

where the correlation function takes the form

\[
f(t-t_1) = \int_{-\infty}^{\infty} \text{d} \omega_k J(\omega_k) e^{-i(\omega_0-\omega_0)(t-t_1)}.
\]

(7)

Suppose the atom interacting resonantly with the reservoir with Lorentzian spectral density

\[
J(\omega_k) = |g(\omega_k)|^2 = W^2 \lambda / \pi [(\omega_k - \omega_0)^2 + \lambda^2],
\]

(8)

by employing Fourier transform and residue theorem, we get the explicit form \(f(t-t_1) = W^2 \exp(-\lambda |t-t_1|)\), where \(W\) is the transition strength and the quantity \(1/\lambda\) is the reservoir correlation time.

To construct a concrete physical insight, we adopt a non-perturbative method called a pseudomode approach [11–13] in the following analysis. Equation (6) can be restated as

\[
\dot{c}_0(t) = -iWb(t),
\]

(9)

\[
\dot{b}(t) = -\lambda b(t) - iWc_0(t),
\]

(10)

where

\[
b(t) = -iW \int_{0}^{t} c_0(t_1) e^{-\lambda(t-t_1)} \text{d} t_1
\]

is the pseudomode amplitude. Typically, there are two regimes [12]: the weak-coupling regime (\(\lambda < 2W\)), where the behaviour of the single-body system is Markovian and irreversible decay occurs, and the strong-coupling regime (\(\lambda > 2W\)), where non-Markovian dynamics occurs accompanied by an oscillatory reversible decay and a structured rather than a flat reservoir situation applies. We will limit our considerations to the latter case.

Define the normalized collective excited state of the reservoir as

\[
|1\rangle = \int_{-\infty}^{\infty} c_{\omega_k}(t)/c_2(t) e^{-i\omega_0 t}|1_k\rangle \text{d} \omega_k,
\]

(11)

one can rewrite (3) as

\[
|\psi(t)\rangle = c_1(t)|e\rangle|0\rangle_r + c_2(t)|g\rangle|1\rangle_r,
\]

(12)

where

\[
c_1(t) = e^{-i(\omega_0 t)/2}\left[\cos\left(\frac{d}{2}\right) + \frac{\lambda}{d} \sin\left(\frac{d}{2}\right)\right],
\]

with \(d = (4W^2 - \lambda^2)^{1/2}\). We note that \(c_1(t) = c_0(t) \exp(-i\omega_0 t), c_2(t) = (1 - |c_1(t)|^2)^{1/2}\) which can be calculated directly from the definition (11) and

\[
b(t) = -2iW \frac{\lambda}{d} e^{-\lambda t/2} \sin\left(\frac{d}{2}\right).
\]

The atom and its reservoir now evolve as an effective two-qubit system.

Now, we study the joint evolution of two identical single-body systems initially in the global state

\[
|\phi_0\rangle = \alpha |g\rangle_1|g\rangle_2 + \beta |e\rangle_1|e\rangle_2|0\rangle_1|0\rangle_2,
\]

(13)

where the real non-negative parameters \(\alpha\) and \(\beta\) satisfy \(\alpha^2 + \beta^2 = 1\) and \(i (i = 1, 2)\) denotes the \(i\)th single-body system. The evolution of the composite system reads

\[
|\phi(t)\rangle = \alpha |g\rangle_1|g\rangle_2|0\rangle_1|0\rangle_2 + \beta |\psi(t)\rangle_2|\psi(t)\rangle_2,
\]

(14)

where \(|\psi(t)\rangle (i = 1, 2)\) represents the single-body evolution and can be determined by (12). Employing the density matrix \(\rho(t) = |\phi(t)\rangle \langle \phi(t)|\) and the definition of concurrence [22], we can write down the concurrence of entanglement for different partitions: \(a_1 \otimes a_2, r_1 \otimes r_2\) and \(a_1 \otimes r_1\), respectively,

\[
C_{a_1a_2}(t) = \max\{0, 2\beta|c_1|^2(\alpha - |c_1|^2)\},
\]

(15a)

\[
C_{r_1r_2}(t) = \max\{0, 2\beta|c_2|^2(\alpha - |c_1|^2)\},
\]

(15b)

\[
C_{a_1r_2}(t) = 2\beta^2|c_2||c_1^2|.
\]

(15c)
α for the cases of atom entanglement inevitably vanishes due to the atom–reservoir interaction for each subsystem, entanglement. Suppose, initially, only two atoms are reservoirs, irrespective of the non-Markovian memory effect. Initially stored in the atoms is completely transferred to the reservoirs, (solid curve) as a function of the dimensionless quantity between two atoms (solid curve) and two reservoirs (dashed curve) in a non-Markovian regime, with the initial state being (13), for the cases of $\lambda/W = 0.2$ (up plots), (a) $\alpha = 1/\sqrt{2}$, (b) $\alpha = \sqrt{10}/5$, (c) $\alpha = 1/4$ and $\lambda/W = 0.1$ (bottom plots), (d) $\alpha = 2\sqrt{2}/5$, (e) $\alpha = 1/\sqrt{3}$, (f) $\alpha = \sqrt{3}/5$.

3. Dynamic analysis

We apply the asymptotic analysis to atom and reservoir entanglement. Suppose, initially, only two atoms are entangled: $C_{a_1a_2}(0) = 2\alpha \beta$ and $C_{r_1r_2}(0) = 0$. As $t \to \infty$, due to the atom–reservoir interaction for each subsystem, atom entanglement inevitably vanishes $C_{a_1a_2}(t \to \infty) = 0$ and reservoir entanglement necessarily tends to a final value $C_{r_1r_2}(t \to \infty) = 2\alpha \beta$. This indicates that the information initially stored in the atoms is completely transferred to the reservoirs, irrespective of the non-Markovian memory effect.

In figure 1, we show the time evolution of concurrence between two atoms (solid curve) and two reservoirs (dashed curve) as a function of the dimensionless quantity $\tau = \lambda t$ for six typical values of the parameter $\alpha$, namely, $\alpha = 1/\sqrt{2}, \sqrt{10}/5, 1/4$ ($\lambda/W = 0.2$) and $2/5, 1/\sqrt{3}, \sqrt{3}/5$ ($\lambda/W = 0.1$).

In figure 1(a), reservoir entanglement performs huge damped oscillations before it reaches its final value with no ESD or ESB phenomena and this holds when $\alpha > \beta$. In (b), atom entanglement can revive from its last death [14] and reservoirs can suddenly be entangled in a period of time. In (c), similar to the Markovian case [10], atom entanglement vanishes permanently after a finite time, while reservoir entanglement presents sudden birth and death even without the revival of atom entanglement, which is totally different from the Markovian case where reservoir entanglement just increases monotonically up to a stationary value. According to (b) and (c), the revival of atom entanglement does not necessarily indicate the sudden death of reservoir entanglement, and vice versa.

Besides these distinctive non-Markovian behaviours, we point out that entanglement of atom pairs can revive before, simultaneously or even after the disentanglement of reservoirs, as in figures 1(d)–(f). According to the expressions of $C_{a_1a_2}$ and $C_{r_1r_2}$, (15a) and (15b) respectively, the coincidence of revival and death means $2\beta |c_1|^2 (\alpha - \beta |c_2|^2) \geq 0$ and $2\beta |c_2|^2 (\alpha - \beta |c_1|^2) \leq 0$ where the two equalities hold at the same time and $2\alpha = \beta$. We can easily prove that revival ahead of (after) death requires $2\alpha > \beta$ ($2\alpha < \beta$). These counterintuitive phenomena do not rely on the explicit expressions of $|c_1|^2$ or $|c_2|^2$ and thus are independent of the relative strength of atom–reservoir interaction, that is, the ratio $\lambda/W$.

Taking the symmetry of the composite system into account, we analyse the bipartite entanglement of the partitions: $a_1 \otimes a_2, r_1 \otimes r_2, a_1 \otimes r_1$ and $a_1 \otimes r_2$ in strong non-Markovian limit ($\lambda/W = 0.1$), as shown in figure 2. In the initial period of time, due to the atom–reservoir couplings, $C_{a_1a_2}$ diminishes and $C_{a_1r_1}$ rises. According to (15e), $C_{a_1r_1}$ has a maximal value $\beta^2$ with $|c_1(t)| = 1/\sqrt{2}$ and if $2\alpha < \beta$, $a_1 \otimes a_2$ and $a_1 \otimes r_2$ would already have been disentangled at that time. With the successive decay of atoms, the reservoir entanglement emerges and evolves to its maximal value $2\alpha \beta$ with the two-reservoir state being $a|0\rangle_{r_1}|0\rangle_{r_2} + \beta |1\rangle_{r_1}|1\rangle_{r_2}$. In the Markovian case, the transmission of information comes to an end.

However, in the non-Markovian case, the information transferred to the reservoirs is fed back to the atoms due to the memory effect, which means the excited atomic state will rise again. Assuming $\partial |c_1(t)|^2/\partial t = 0$, we get $\sin(\theta - 2\pi = 0$ (valley) or $\tan(\theta = \tan^{-1}(d/\lambda)$. For the peaks, $|c_1(t_n)|^2 = \exp(-2n\pi \lambda/d)$ ($n \in N$) and $t_n = 2n\pi/d$. We analyse the behaviour of expression $2\beta |c_1|^2 (\alpha - \beta |c_2|^2)$ between the peak and valley of $|c_1(t)|^2$
Similar to the atom entanglement, the occurrence of ESD as in figures 1(a) and (b). The number of ESD is
\[ \alpha \]
and find that, the revival of atom entanglement is governed by the inequality
\[ \alpha > \frac{1 - \exp(-2\pi \lambda/d)}{\sqrt{1 + \vert 1 - \exp(-2\pi \lambda/d) \vert^2}}. \] (16)
For a given atom-reservoir interaction $\lambda/W$, the occurrence of revival means
\[ \alpha > \frac{1 - \exp(-2\pi \lambda/d)}{\sqrt{1 + \vert 1 - \exp(-2\pi \lambda/d) \vert^2}}. \] (17)
If $\alpha$ violates the criterion, $C_{a,0,2}$ fails to exhibit revival as in figure 1(c). Furthermore, with a given initial condition $\alpha$, the exact number of revivals is
\[ n_a = \lfloor \frac{d}{2\pi \lambda} \ln(\beta/(\beta - \alpha)) \rfloor, \] (18)
where $\lfloor \cdot \rfloor$ is the Gaussian function which represents the maximal integer smaller than or equal to $x$. The interval of revivals can be approximately estimated by
\[ t_i \approx t_{i+1} - t_i = 2\pi f/d, \] (19)
which is independent of the initial condition $\alpha$ and largely determined by the transition strength $W$. Therefore, if $\alpha$ satisfies (17), the period of revivals is the same. Here, we note that when $\alpha \geq 1/\sqrt{2}$, the atom entanglement performs damped oscillations and $n$ can be infinite as in figure 1(a).

In addition, it is not difficult to derive that the sudden death of reservoir entanglement is regulated by
\[ \alpha < \frac{\exp(-2\pi \lambda_2/d)}{\sqrt{1 + \exp(-4\pi \lambda_2/d)}}, \] (20)
Similar to the atom entanglement, the occurrence of ESD means
\[ \alpha < \frac{\exp(-2\pi \lambda/d)}{\sqrt{1 + \exp(-4\pi \lambda_2/d)}}, \] (21)
If $\alpha$ is big enough, there is no ESD between the two reservoirs as in figures 1(a) and (b). The number of ESD is
\[ n_r = \lfloor \frac{d}{2\pi \lambda} \ln(\beta/\alpha) \rfloor, \] (22)
where $\lfloor \cdot \rfloor$ is the Gaussian function. Based on (18) and (22), we can see that when $\alpha < \beta < 2\alpha$, revivals of atom entanglement are more than deaths of reservoir entanglement and $2\alpha = \beta$ equal, $2\alpha < \beta$ less. Though this is visible from the equations, it is not so obvious from our physical intuition.

Back to our analysis of bipartite entanglement, if the memory effect is strong enough, that is, the ratio $\lambda/W$ is very small, $|c_1(t_1)| = \exp(-\pi \lambda/d)$ is bigger than $1/\sqrt{2}$ and $C_{a,0,2}$ will rise again to its maximal value. If $\alpha$ satisfies (17) and (21), the reservoir entanglement will be disentangled and the atom reservoir entanglement will revive, as in figure 2. This means the information is transferred from two reservoirs to each subsystem and then to two atoms. This depletion-feedback process continues for some time with a damping of amplitudes because the memory effect is finite and the atoms will decay inevitably. Entanglement for other partitions is shown in figure 3. Curve (IV) depicts the multipartite entanglement between the four effective qubits, which is defined by multipartite concurrence $C_N$ [23]. Other bipartite entanglement is obtained through $t$-concurrence [24], as curves (I)-(III), (V) and (VI), which coincides with the concurrence in the pure two-qubit case.

Our simulation conditions $\lambda/W = 0.2$ and 0.1 can be well realized within the current experimental level [25]. A more intuitive interpretation of these results will be given in detail in the following section.

4. Explanations via quasimode Hamiltonian

Memory effect plays an important role in our analysis of non-Markovian dynamics. To separate it from the damping effect, we convert the true mode Hamiltonian with Lorentzian spectral density into the quasimode form. Applying the method in [12], the quasimode Hamiltonian of the single-body system can be given by
\[ H = H_0 + H_{\text{memory}} + H_{\text{damping}}, \] (23)
with
\[ H_0 = \omega_0 \sigma_x \sigma_+ + \omega_0 a^\dagger a + \int_{-\infty}^{\infty} \Delta c^\dagger(\Delta)c(\Delta) d\Delta, \] (24a)
\[ H_{\text{memory}} = W(\sigma_x \sigma_+ - \sigma_- \sigma_-), \] (24b)
\[ H_{\text{damping}} = (\lambda/\pi)^{1/2} \int_{-\infty}^{\infty} (\alpha^\dagger c(\Delta) + ac(\Delta)) d\Delta, \] (24c)
where $c^\dagger(\Delta), c(\Delta)$ are the creation and annihilation operators of the continuum quasimode with frequency $\Delta$ and other
parameters are the same as before. The particular conversion relations are given in the appendix in [21]. Analogous to the procedure in section 2, the evolution of the single-body system is

\[
|\psi(t)\rangle = c_a(t)|e\rangle|0\rangle_m|0\rangle_r + c_c(t)|g\rangle|1\rangle_m|0\rangle_r + c_r(t)|g\rangle|0\rangle_m|1\rangle_r, 
\]

with \(c_a(t) = c_1(t)\), \(c_m(t) = b(t) \exp(-ioDt)\) and \(c_r(t) = (1 - |c_a(t)|^2 - |c_m(t)|^2)^{1/2}\). This means that the pseudomode in section II is just the discrete quasimode and the quasimodes (both discrete and continuum) are our previous reservoir. We plot the time evolution of the population in figure 4 and explain how the memory effect and damping effect induce \(|c_1(t)|^2\) to perform damped oscillations.

According to (23)–(24e), the atom only interacts with the discrete mode and their coupling coefficient is just the transition strength \(W\), whereas the discrete mode interacts with a set of continuum modes and their coupling strength contains only the constant width of Lorentzian spectral density \(\lambda\). If we let the atom and the discrete mode be a new system and the continuum modes be the external environment, the behaviour of the new system will be exactly Markovian. Therefore, transmission of the exciton from atom to reservoir is a two-step process: first, a photon is created in a discrete (cavity) mode via the atom–discrete mode interaction; second, this photon is annihilated and a photon is created in a continuum (external) mode via the discrete–continuum mode coupling. The memory effect stems from the finite life span of the photon in the cavity [12, 13]. Thus, the reabsorbing phenomenon that causes the oscillatory entanglement, as shown in figures 1 and 2, only exists in the first step. In figure 4, the alternative peaks and valleys of \(|c_m(t)|^2\) and \(|c_1(t)|^2\) demonstrate that the energy of exciton exchanges between the two states \(|e\rangle|0\rangle_m|0\rangle_r\) and \(|g\rangle|1\rangle_m|0\rangle_r\) periodically.

Assuming \(\partial|c_m(t)|^2/\partial t = 0\), we get \(\sin(dt/2) = 0\) (the valley) or \(\tan(dt/2) = d/\lambda\) (the peak). The valley of \(|c_m(t)|^2\) coincides with the peak of \(|c_a(t)|^2\) at \(t = 2n\pi/d\) when \(\partial|c_r(t)|^2/\partial t = 0\), which makes \(|c_1(t)|^2\) act like a staircase curve. This means that if the exciton only exists in the atom, the damping process does not happen. The damping effect happens only when the photon escapes into the continuum modes and never comes back. In our case, there is no mechanism, such as the dipole–dipole interaction [21], to protect the exciton from escape. So the damping effect is unavoidable and the memory effect is just a finite-time phenomenon, which indicates that atom entanglement can never be completely reconstructed and the information will be totally transferred to the reservoirs, which fits the results in figures 1 and 3.

5. Conclusions

We study the dynamical evolution of a couple of two-level atoms interacting with independent structured reservoirs. We find that reservoir entanglement exhibits sudden birth, death and revival phenomena. The revival of atom entanglement does not necessarily indicate the disentanglement of reservoirs and vice versa. Applying the quantitative analysis, we derive the criteria for the revival (death) phenomena and prove that the atom entanglement can revive before, simultaneously or after the sudden death of reservoir entanglement, which is independent of the relative strength of atom–reservoir interaction. Besides, by studying the bipartite and
multipartite entanglement for different partitions, we present a comprehensive interpretation of the information transmission within this composite system in a non-Markovian regime. Our results and conclusions are desirable in the implementation of various optical schemes [26–29] for the preparation and manipulation of entanglement.

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