Approximate Analytical Conditions of a Panel RC Slab for Reproducing the Fatigue Behaviors of a Real Bridge RC Slab

Arslan Qayyum Khan1, Pengru Deng2* and Takashi Matsumoto3

Received 12 October 2019, accepted 17 December 2019 doi:10.3151/jact.18.1

Abstract

This study proposes approximate boundary conditions for a panel reinforced concrete (RC) slab to simulate the fatigue behaviors of a real bridge RC slab. To validate the approach, fatigue analysis of the panel RC slab is conducted using a finite element method (FEM) based numerical model with the bridging stress degradation concept. Both the proposed approximate boundary conditions, and those that have been typically employed in past studies, are used. The numerical results demonstrate that the panel RC slab with approximate boundary conditions experiences the same bending moment distribution and deformations around the loading locations as the load moves along the slab axis, similar to those of a real bridge RC slab. Moreover, the approximate boundary conditions reproduce the extensive grid crack pattern in the panel RC slab, which is similar to that generally observed in a real bridge RC slab. The results of this study establish that panel RC slab with approximate boundary conditions behaves in a similar manner to a real bridge RC slab, which results in a more realistic fatigue life estimation.

1. Introduction

RC slabs are one of the most critical members in a bridge and are susceptible to fatigue failure, as they directly experience repetitive moving wheel loads. Owing to insufficient slab thickness in previous designs, a brittle punching shear failure mode appeared as a key failure phenomenon for bridge RC slabs in service, especially with the increase in vehicle weight and traffic volume. Consequently, the fatigue strength of bridge RC slabs is significantly reduced, posing a great threat to the structure and human life (Sonoda and Horikawa 1982; Schlafli and Bruhwiler 1998). Therefore, various researches have been carried out to investigate the fatigue behaviors of bridge RC slabs (Perdikaris and Beim 1988; El-Ragaby et al. 2007; Maki et al. 2019; Cheng 2011). Due to cost, time, and space constraints, a panel of RC slab has generally been considered in the existing experimental studies in lieu of a real bridge RC slab (Maeda and Matsui 1984; Matsu 1987; Shakushiro et al. 2011; Mitamura et al. 2012). In these experimental studies, the boundary conditions comprising of simple supports along the longitudinal edges and steel I-beams along the transverse edges of the panel RC slab have been typically used to represent a real bridge RC slab with continuity between adjacent spans. These studies employed cyclic moving wheel loads, and the punching shear failure mode and a shorten fatigue life of bridge RC slabs were successfully reproduced. Moreover, fatigue analyses of panel RC slabs with typically used boundary conditions have been carried out based on various theories. In a numerical method, reported by the National Institute for Land and Infrastructure Management (NILIM), Japan, the stiffness of the damaged concrete elements was simply reduced by applying the damage accumulation theory (NILIM 2015). However, this numerical method failed to capture the dominant degradation mechanism in the panel RC slabs under fatigue loading, because only the two extreme states (entirely damaged and undamaged) were considered, rather than reducing the stiffness of the concrete element gradually.

Li and Matsumoto (1998) introduced the bridging stress degradation concept in the crack growth analysis of fiber reinforced concrete. Based on this concept, a numerical model for analyzing the fatigue behaviors of the panel RC slabs was developed, with which the fatigue life was predicted successfully (Suthiwarapirak and Matsumoto 2006). The bridging stress degradation concept was further extended to analyze the fatigue behaviors of the panel RC slabs reinforced with plain bars (Drar and Matsumoto 2016). These studies confirm the applicability of the numerical model based on the bridging stress degradation concept for the panel RC slabs subjected to moving wheel loads.

However, panel RC slabs with the typically used boundary conditions failed to reproduce the behaviors of real bridge RC slabs. For example, in a real bridge RC slab, the bending moment distribution and deformations around the loading locations remain almost unchanged as the load moves along the slab axis (Mabsout et al. 2004; Frederick 1997). On the other hand, a panel RC slab with typically used boundary conditions shows
remarkable variations of bending moment distribution and deformations as the wheel load moves along the slab axis. Moreover, the typically used boundary conditions lead to cracks originating from the loading point and propagating in a diagonal direction towards the corners of the panel RC slab. This is unlike the extensive grid crack pattern that is generally observed in real bridge RC slabs (Perdikaris and Beim 1988; Okada et al. 1982; Perdikaris et al. 1989). These differences in the behavior of a panel RC slab compared to a real bridge RC slab are attributed to the steel I-beams in the typically used boundary conditions, which are not truly representative of the continuity of the RC slab along the longitudinal direction of the real bridge. The steel I-beams in the typically used boundary conditions restrict the displacement of the panel RC slab at the steel I-beams, and this behavior is different from that of a real bridge RC slab. Furthermore, the steel I-beams reduce the displacements significantly when a load is applied at locations near the supports compared to those at the mid-span of the panel RC slab for the same applied load. As a result, the cracks propagate in a diagonal direction originating from the loading point towards the corners of the panel RC slab. Thus, it is essential to elucidate the effects of the boundary conditions and determine approximate ones for the panel RC slab, which means simulating real fatigue behaviors in terms of the extensive grid crack pattern and an accurate fatigue life estimation.

Therefore, this study mainly aims to propose the boundary conditions numerically for a panel RC slab, which can reproduce the realistic behaviors of a bridge RC slab. Using an FEM based numerical model with the bridging stress degradation concept, fatigue analysis is conducted for a panel RC slab with the approximate boundary conditions in addition to those typically used in the previous studies. Compared to the panel RC slab with the typically used boundary conditions, the analytical results of the panel RC slab with the approximate boundary conditions show deformation and cracking behaviors closer to those of a real bridge RC slab. Furthermore, the approximate boundary conditions permit the propagation of the cracked elements in the panel RC slab along the longitudinal direction to a greater extent compared to the typically used ones. As a result, the approximate boundary conditions reproduce the extensive grid crack pattern in the panel RC slab similar to that generally observed in a real bridge RC slab.

2. Method for the determination of approximate boundary conditions

To facilitate the investigation of fatigue behaviors of a real bridge RC slab, a panel RC slab equipped with boundary conditions comprising of simple supports along the longitudinal edges and steel I-beams along the transverse edges is generally employed in the previous studies (Maeda and Matsui 1984; Matsui 1987; Shashiro et al. 2011; Mitamura et al. 2012). The steel I-beams in the typically used boundary conditions restrict the panel RC slab from experiencing the same bending moment distribution and displacements around the loading locations as the wheel load moves along the slab axis. However, in a real bridge RC slab, the bending moment distribution and displacements around the loading locations remain almost unchanged as the wheel load moves along the slab axis. A real bridge RC slab and its corresponding panel RC slab are shown in Fig. 1, where the deformations of both slabs under the same load are shown schematically as well. Even though the displacements at the location of the load application are almost the same for both the real bridge RC slab and its corresponding panel RC slab, the displacements deviate from each other at locations away from the loading point. This difference is more pronounced near the steel I-beams in the panel RC slab, as can be seen in Figs. 1(c) and 1(d). This is because the vertically placed steel...
I-beams along the transverse edges restrict the displacement of the panel RC slab at the supports, which is contrary to the case of a real bridge RC slab. Furthermore, the vertically placed steel I-beams lead to a difference in the rotation around the steel I-beams in the panel RC slab compared to that in the real bridge RC slab, as shown in Figs. 1(c) and 1(d).

In real RC bridge slab, the displacement ($\delta_{\text{real}}$) and rotation ($\theta_{\text{real}}$) at the corresponding locations of the transverse edges of the panel RC slab are constrained from the adjacent parts of the RC slab rather than any vertical support as shown in Fig. 1(c). To simulate the deformation in the panel RC slab, the steel I-beams are horizontally placed along the transverse edges of the panel RC slab in this study. As a result, the real deformation of the bridge slab can be reproduced in the panel RC slab by adjusting the bending and rotational stiffness of the horizontally placed I-beams. The flowchart of the method used to determine the approximate boundary conditions for the panel RC slab is shown in Fig. 2, wherein the panel RC slab is modeled in a finite element analysis (FEA) software. To save time and simplify the calculation of the sectional moments in the panel RC slab, an elastic analysis method is considered in this section. Moreover, the panel RC slab without reinforcement bars is considered in the elastic analysis method. The elastic analysis method is available, and most design codes permit its use for the RC slabs (Muspratt 1978). The height of the flange, length of the web and web thickness of the steel I-beam are assumed to be constant parameters. However, the flange thickness of the steel I-beam is considered as a variable parameter, since the bending stiffness of the steel I-beam in this horizontal orientation is majorly depended upon the flange thickness. By varying the flange thickness, the bending stiffness of the steel I-beam is changed which changes the deformation behavior of the panel RC slab. Thus, with an appropriate flange thickness of the steel I-beam, the deformation behavior of the panel RC slab similar to that of a real bridge RC slab can be achieved.

A panel RC slab with loading locations is displayed in Fig. 3. A static load is applied at the center of the loading zone (loading location A), and the elastic analysis is carried out. The bending moments parallel to the x- and y-axes ($M_{xx}$ and $M_{yy}$, respectively) are calculated in the panel RC slab equipped with the selected boundary conditions. Similarly, $M_{xx}$ and $M_{yy}$ are calculated for the loading locations away from the center of the loading zone (i.e., at loading locations B, C, and D). To determine the cracking zones for the bending moments, the cracking bending moment ($M_{\text{crack}}$) is calculated as follows:

$$M_{\text{crack}} = \frac{\sigma_t I_c}{y}$$  

where $I_c$ is the second moment of area per unit width; $\sigma_t$ is the tensile strength of the concrete; and $y$ is equal to half of the slab thickness. The cracking zones for $M_{xx}$ and $M_{yy}$ are checked for loading locations A, B, C, and D. If the cracking zones are similar around all the loading locations for $M_{xx}$ as well as $M_{yy}$, the assumed di-
Dimensions of the steel I-beams would allow the panel RC slab to experience the same bending moment distribution around all the loading locations. Consequently, the displacement profile in the longitudinal and transverse directions would also be the same for all the loading locations. In such a case, the assumed configuration for the boundary conditions would be assigned to the approximate boundary conditions for the panel RC slab. If the bending moment distribution around all the loading locations is not similar, the flange thickness of the steel I-beams would be further adjusted. This process is repeated until the approximate boundary conditions are obtained for the panel RC slab.

3. Fatigue analysis method

3.1 Concrete model

An FEM based on the Newton-Raphson iteration technique is applied to solve the nonlinear behavior of the concrete. Nonlinear constitutive laws of the concrete in tension and compression are shown in Table 1 (Maekawa et al. 2003).

The bridging stress degradation concept is introduced as a degradation mechanism of concrete under repetitive loading in this fatigue analysis. A crack begins with length \(a\) and width \(w\) due to the first loading as displayed in Fig. 4(a). Under repetitive loading, the crack is subjected to a process of opening and closing, which leads to a reduction in the bridging stress between the crack surfaces. Due to the reduction of bridging stress, the existing crack propagates with the additional length \(da\) and additional width \(dw\) as shown in Fig. 4(b). The reduction in bridging stress is termed as bridging stress degradation (Li and Matsumoto 1998). The bridging stress degradation equation can be expressed as follows (Zhang et al. 2001):

\[
\frac{\sigma_N}{\sigma_i} = 1 - d \log(N)
\]

(2)

where \(\sigma_N\) and \(\sigma_i\) are bridging stress at the Nth and the first cycle, respectively; and \(d\) is the stress degradation factor which can be related with the maximum crack width \(W_{max}\) as (Zhang et al. 2001):

\[
d = d_0 + \gamma W_{max}
\]

(3)

where \(d_0\) is the stress degradation factor at minimum crack width corresponding to zero load; and \(\gamma\) is the slope of the linear relation of \(d\) and \(W_{max}\). Zhang (1998) analyzed a large number of experimental results of plain concrete under cyclic uniaxial tensile tests and found that the model predicted the test results reasonably when \(d_0 = 0.08\) and \(\gamma = 4\) mm\(^{-1}\). In order to introduce the bridging stress degradation relations to smeared crack elements, \(W_{max}\) is written in the form of maximum tensile strain \(\varepsilon_{max}\) by considering unit element size as \(W_{max} = \varepsilon_{max} \times 1\) (Suthiwarapirak and Matsumoto 2004). The stress-strain behavior of concrete in tension under cyclic loading is shown in Fig. 5.

Concrete cracks are allowed to initiate in three perpendicular directions in each crack element. The first crack is assumed to start perpendicular to the direction of the maximum principal strain in the concrete matrix when the tensile strain is larger than the cracking strain as shown in Fig. 6(a). After the first crack appears, the second crack can propagate perpendicular to the first crack when the tensile strain in that perpendicular direction exceeds the cracking strain. Similarly, the third

![Fig. 4 Crack propagation due to the bridging stress degradation.](image)

![Fig. 5 Tensile behavior of the concrete under cyclic loading.](image)
crack can start in the perpendicular direction to the existing two cracks when the tensile strain in that direction exceeds the cracking strain as shown in Fig. 6(b) (Drar and Matsumoto 2016).

### 3.2 Reinforcement bar model

The smeared model is adopted for the reinforcement bar embedded in concrete. A bilinear curve with yield stress ($f_y$) is used for the representation of the stress-strain behavior of the reinforcement bar. The Giuffré-Menegotto-Pinto model is utilized to represent the tensile behavior of the reinforcement bar under repetitive loading as follows (Menegotto and Pinto 1973):

$$\sigma = \frac{H}{R} \varepsilon + \left(1 - \frac{H}{R} \right) \frac{\varepsilon}{\varepsilon_y} \left(1 + \left(\frac{\varepsilon}{\varepsilon_y}\right)^{R-1}\right)$$

where $H$ is hardening parameter; $R_0$ and $R$ are transition parameters between elastic hardening for the first and the Nth cycle ($R_0 = 20$); $\varepsilon_{max}$ is the maximum excursion in the plastic range; $a_1$ and $a_2$ are the parameters for change of $R$ with repetitive load history, equal to 18.5 and 0.00015, respectively. The yield strength reduces as the number of cycles increases as shown in Fig. 7.

The bond between the deformed reinforcing bar and its surrounding concrete is assumed to be perfect. Moreover, the bond degradation due to a repetitive loading is not considered since the bond strength does not change much under a repetitive loading for the non-corroded steel bars, as reported in (Lin et al. 2017).

### 3.3 Analytical procedure

An FEA software MSC/MARC (MARC 2017) is used to model the slab, a 3D model using solid elements, as displayed in Fig. 8(a). Due to symmetry in loading and boundary conditions, only half of the slab is analyzed. Firstly, the moving load is applied to the central elements of the slab. Then, these elements are unloaded while the elements adjacent to the right side of the loaded elements are loaded simultaneously. In this technique, a load is made to move along the longitudinal direction back and forth. This loading leads to the propagation of cracked elements in the first cycle as shown in Fig. 8(b). According to the bridging stress degradation concept, the constitutive law for the cracked elements is modified. After the second cycle of moving load, new cracked elements appear as displayed in Fig. 8(c). The overall panel RC slab stiffness is decreased with the increase of cycles of moving load due to bridging stress degradation and crack propagation. The procedure is continued until the final cycle of moving wheel load, and the numerical results are recorded in each cycle of moving wheel load.

### 4. Studied panel RC slab

In this study, a panel RC slab is selected from the technical note of NILIM, Japan (NILIM 2015). The reason...
for selecting this slab is that the panel RC slab has been designed according to the recent design specifications for highway bridges (JRA 1996), incorporating improved fatigue durability of slabs and increased loading levels.

4.1 Dimensions and reinforcement details

The panel RC slab used in this study is an RC plate with (L × W × H) dimensions of (4500 × 2800 × 250) mm. In the tension zone, the slab is reinforced with D19@150 mm along the transverse direction and D16@125 mm along the longitudinal direction. Similarly, D19@300 mm along the transverse direction and D16@250 mm along longitudinal direction are provided in the compression zone. The slab dimensions, reinforcement details and moving wheel load zone are shown in Fig. 9. Material properties of the concrete and steel reinforcement of the panel RC slab are presented in Table 2.

4.2 Loading sequence

The panel RC slab is subjected to moving wheel load along the longitudinal direction with the dimensions of the loading zone of (3000 × 500) mm. The same stepwise loading sequence with the experimental study (NILIM 2015) has been used. In the stepwise loading sequence, the magnitude of moving wheel load is increased in intervals after a certain number of cycles to promote the deterioration at the early stages to save time. The stepwise loading sequence used in this study, which has an initial value of 157 kN and an increasing step of 19.6 kN load after every 40 000 cycles of moving wheel load, is shown in Fig. 10.

4.3 Boundary conditions

In this study, a total of five cases of boundary conditions are considered as shown in Fig. 11. In Case 1, the same boundary conditions as those of the experimental study (NILIM 2015) are considered. As discussed in detail in section 2, in real RC bridge slab, the displacement (\(\delta_{\text{real}}\)) and rotation (\(\theta_{\text{real}}\)) at the corresponding locations of the transverse edges of the panel RC slab are constrained from the adjacent parts of the RC slab rather than any vertical support. Therefore, for Cases 2, 3, 4 and 5, the steel I-beams are horizontally placed as supports along the transverse edges of the slab to simulate the deformation in the panel RC slab. Moreover, the flange thickness of these horizontally placed steel I-beams is progressively varied. The reason for varying the flange thickness is to obtain the appropriate bending stiffness of the steel I-beams, which allows the panel RC slab to experience the same bending moment distribution and displacements around the loading locations as the load moves along the slab axis. It is worth mentioning here that the primary objective of this study is to numerically propose the boundary conditions for a panel RC slab for simulating the behaviors of a real bridge RC slab.

In this study, after determining the approximate boundary conditions for the panel RC slab, fatigue analyses are conducted with the approximate and typically used boundary conditions. The following two cases of boundary conditions, Case 1 and Case 5, are considered for the fatigue analysis of the panel RC slab.

1. Typically used boundary conditions

In the case of the simulations with the typically used boundary conditions, the panel RC slab is supported by simple supports along the longitudinal edges and the steel I-beams are vertically placed along the transverse edges of the slab. The bottom flanges of the steel I-beams are restrained for vertical displacement. In this study, for the I-beams, the same dimensions as those adopted in the experimental study (NILIM 2015) are used.

| Material   | Property                      | Value (MPa) |
|------------|-------------------------------|-------------|
| Concrete   | Compression strength \(f'_c\) | 33.1        |
|            | Tensile strength \(f_t\)     | 1.9         |
|            | Elastic modulus \(E_c\)      | 27 900      |
| Steel rebar| Yield strength \(f_y\)       | 345         |
|            | Elastic modulus \(E_s\)      | 200 000     |

Table 2 Material properties.
(2) Approximate boundary conditions
In the case of the simulations with the approximate boundary conditions, the simple supports along the longitudinal edges of the panel RC slab are the same as those in the typically used boundary conditions case; however, the steel I-beams are horizontally placed as supports along the transverse edges to achieve a deformation behavior similar to that of a real bridge RC slab. The outer flange of the steel I-beams is restrained in the longitudinal direction of the panel RC slab. The dimensions of steel I-beams used for the approximate boundary conditions are shown in Fig. 11.

5. Results and discussion
5.1 Determination of approximate boundary conditions
Only one half of the panel RC slab is modeled in view of the symmetry in the loading and boundary conditions. A static load of 157 kN, which is the initial load of the stepwise loading sequence, is applied at the center of the panel RC slab (loading location A). To simplify the calculation of sectional moments in the panel RC slab, the elastic analysis is conducted for this specific purpose as most design codes permit the use of elastic analysis for the RC slabs (Muspratt 1978). Moreover, the panel RC slab without reinforcement bars is considered in this elastic analysis. The bending moments parallel to x- and y-axes (\( M_x \) and \( M_y \), respectively) are calculated in the slab for this loading location A. Similarly, the elastic analysis is conducted, and \( M_x \) and \( M_y \) are calculated for the loading locations B, C, and D as well. The loading locations in the panel RC slab are displayed in Fig. 12. Furthermore, the displacements in the longitudinal and transverse directions are obtained for the loading locations A, B, C, and D. Five cases of boundary conditions are considered in this study.

(1) Bending moment cracking zones
A comparison of the bending moment cracking zones
for $M_{xx}$ and $M_{yy}$ for the five cases of boundary conditions is shown in Fig. 13, corresponding to a static load of 157 kN applied at the loading locations A, B, C, and D successively.

For Case 1, the cracking zones for $M_{xx}$ due to the load at locations A, B, C, and D are similar to one another as shown in Fig. 13(a). However, the cracking zones for $M_{yy}$ are reduced significantly with an increase in the distance of the loading point from the center. This effect is more pronounced for the loading locations C and D. This is because the vertically placed steel I-beams result in a higher bending stiffness parallel to the y-axis. These strong steel I-beams restrict the panel RC slabs from possessing the same bending moment distribution and deformations around the loading locations as the wheel load moves along the slab axis. This phenomenon is contrary to that observed in a real bridge RC slab.

For Cases 2, 3 and 4, the cracking zones for $M_{xx}$ as well as the cracking zones for $M_{yy}$ are quite different from each other as shown in Figs. 13(b), (c) and (d).
However, the difference in bending moments cracking zones is reduced with an increase in the flange thickness of the steel I-beams. This is because the horizontally placed steel I-beams with appropriate stiffness in the transverse direction allow the panel slab to experience the same deformations around the loading locations as the load moves along the slab axis.

For Case 5, the cracking zones for $M_{yy}$ due to the load at locations A, B, C, and D are approximately the same as one another. In addition, a similar trend is observed for the cracking zones for $M_{xx}$ for all the loading locations [Fig. 13(e)].

(2) Longitudinal and transverse displacement distributions

The displacements in the longitudinal and transverse sections for the loading locations A, B, C, and D are compared for the five cases of boundary conditions and shown in Fig. 14.

In Fig. 14(a), the displacement for the load at location A is quite high compared to those for the load at locations B, C, and D for Case 1. This is because the steel I-beams play a significant role in reducing the displacements as the load moves along the slab axis. However, a real bridge RC slab experiences the same displacements around the loading locations as the load moves along the slab axis.

The displacement for the load at location D is quite high compared to that at the other loading locations for Case 2. It demonstrates that the horizontally placed steel I-beams have low bending stiffness. For Case 3, the bending stiffness of the steel I-beams is increased by increasing the flange thickness of the steel I-beams, which results in reducing the variation in the displacements around the loading locations as shown in Fig. 14(e). The flange thickness of the steel I-beams is further increased for Case 4, and consequently, the variation in the displacements around the loading locations is reduced even more significantly. Thus, the flange thickness of steel I-beams is increased in order to reduce the displacement variation. For Case 5, the bending stiffness of the steel I-beams becomes such that it allows the panel RC slab to experience the same displacements around all the loading locations (A, B, C, and D) as shown in Fig. 14(e).

A comparison of the displacement values for loading locations A, B, C, and D is made among all the five cases as presented in Table 3. The difference in the maximum displacement between loading locations A and D is 40.2% for Case 1, which shows that the vertically placed I-beams reduce the displacement significantly as the load moves along the slab axis. The corresponding difference in the maximum displacement is –22.1% for Case 2, implying that the bending stiffness of the steel I-beams is very low when placed horizontally. As the flange thickness of the steel I-beams is increased for Cases 3, 4 and 5, the bending stiffness is improved in these cases, resulting in the decrease of the displacement difference as can be seen from Table 3. Finally, the difference in the maximum displacement between the loading locations A and D is decreased to a reasonably small value of –3.1% for Case 5, which is determined as the approximate boundary conditions. This observation implies that the panel RC slab with the approximate boundary conditions experiences almost the same displacements around the loading locations as the load moves along the slab axis. As discussed earlier in detail in section 2, in real RC bridge slab, the displacements are not restricted by any vertical support at the corresponding locations of the transverse edges of the panel RC slab (see Fig. 1). As a result, displacements around the loading locations would remain almost similar as the load moves along the slab axis of the real bridge RC slab. Thus, it can be safely said that the panel RC slab with approximate boundary conditions can replicate the displacement behavior of a real bridge RC slab.

### Table 3 Comparison of maximum displacement.

| Cases of boundary conditions | Maximum displacement (mm) | Displacement difference (%) |
|-----------------------------|---------------------------|-----------------------------|
| Load at A                   | Load at B | Load at C | Load at D | (A–D)/A |
| Case 1                      | 0.403    | 0.378    | 0.331    | 0.241   | 40.2    |
| Case 2                      | 0.458    | 0.467    | 0.496    | 0.559   | –22.1   |
| Case 3                      | 0.454    | 0.459    | 0.482    | 0.531   | –16.0   |
| Case 4                      | 0.448    | 0.449    | 0.463    | 0.492   | –9.8    |
| Case 5                      | 0.444    | 0.446    | 0.448    | 0.458   | –3.1    |

5.2 Fatigue behaviors of panel RC slab

Fatigue analysis is conducted for the panel RC slab with the approximate boundary conditions as well with those typically used for the panel RC slab in the past studies.

(1) Propagation of cracked elements

In this numerical model, the constitutive law for the cracked elements is modified after each cycle according to the bridging stress degradation concept. The cracked elements propagate and degrade after moving wheel load cycles. Therefore, it is imperative to describe the propagation of cracked elements. The propagation of cracked elements of the panel RC slab with the typically used and approximate boundary conditions at different numbers of cycles is shown in Fig. 15. The uncracked elements are shown by white color, and the cracked elements caused by the different number of cycles of moving wheel load are displayed with different colors.

For the case of typically used boundary conditions, only a few elements on the bottom of the panel RC slab are cracked after the first cycle of moving wheel load as displayed in Fig. 15(a). With the increase in the number of cycles of moving wheel load, the cracked elements propagate in the longitudinal, transverse and vertical directions. This is because the bridging degradation of the concrete occurs due to the process of crack opening
and closing. The load capacity from the concrete bridging stress cannot reach the same level as in the 1st cycle with the already formed cracked state, and the cracked elements propagate in the other directions to reach the load level. At the 200 000th cycle, the cracked elements propagate in the diagonal direction and reach to the corner supports. At the 280 000th cycle, almost all the elements on the bottom surface of the panel RC slab are cracked, and the cracked elements propagate vertically up to 3/4th of the total thickness of the panel RC slab at the loading point and corner supports. At the last cycle of moving wheel load (i.e., 500 000th cycle), the corner supports are entirely cracked, and the cracked elements propagate in a diagonal direction on the upper layers from the corner supports. This is because the corner supports experience a negative bending and the top surface elements of the corner supports are cracked. These cracked elements at the top surface of the corner supports then propagate in the diagonal direction from the corner supports towards the loading point, whereas a real RC bridge slab does not experience this kind of crack propagation on the top surface.

On the other hand, for the approximate boundary conditions case, significant elements on the bottom of the panel RC slab are cracked after the application of the first loading cycle as shown in Fig. 15(b), as the horizontally placed I-beams allow the panel RC slab to experience the same cracking behavior as the wheel load moves along the slab axis. The cracked elements propagate in the longitudinal and transverse directions, as well as the vertical direction, with an increase in the number of cycles of moving load. At the 200 000th cycle, the cracked elements propagate in the longitudinal and transverse directions, in contrast to the typically used boundary conditions case. At the 280 000th cycle, the cracked elements reach the corner supports, start to propagate in the vertical direction, and reach up to 3/4th of the total thickness of the panel RC slab at the loading point. At the last cycle of moving load (500 000th cycle), the corner cracked elements are reached up to the 3/4th of the total thickness of the panel RC slab. The corner supports are not fully cracked, and no cracked elements propagate in the diagonal direction from the corner supports to the loading location on the top surface of the slab.

The volumes of the cracked elements for the typically used and approximate boundary conditions cases are compared and are plotted in Fig. 16. The cracked elements volume for the typically used boundary conditions case is smaller than that for the approximate boundary conditions case for the first 200 000 cycles. Subsequently, for the typically used boundary conditions case, the cracked elements propagate in the diagonal direction, reach the corner supports, and propagate in the vertical and diagonal directions. This phenomenon results in a larger cracked elements volume after the 200 000th cycle for the typically used boundary conditions case compared to the approximate boundary conditions case. The average degradation ratios for the typically used and approximate boundary conditions cases are 2487 mm³/cycle and 2017 mm³/cycle, respectively. The formation of the cracked elements in the diagonal

---

**Fig. 14 Displacement distributions in the longitudinal and transverse sections.**

![Displacement distributions](image-url)
direction triggers the degradation phenomenon in the panel RC slab with the typically used boundary conditions, which leads to the panel RC slab degradation earlier than with approximate boundary conditions.

The panel RC slab with the typically used boundary conditions experiences smaller cracked elements zone at the bottom surface of the slab after the first loading cycle as compared to the approximate boundary conditions case. However, the cracked elements propagate in the diagonal direction and reach the corner supports in fewer cycles, compared to the approximate boundary conditions case. This is because the vertically placed steel I-beams in the typically used boundary conditions case restrict the panel RC slab from deflecting in the longitudinal direction around these steel I-beams. As a result, the cracked elements propagate in the diagonal direction instead of the longitudinal and transverse directions, which is generally not observed in a real bridge RC slab. The corner cracked elements then propagate in the vertical and diagonal directions, resulting in a larger cracked elements zone in the typically used boundary conditions case. The larger cracked elements zone is undergone the bridging stress degradation mechanism [Eq. (2)], causing the deterioration of the panel RC slab to a greater extent. This additional deterioration due to the propagation of the cracked elements in the diagonal direction leads to a shorter fatigue life of the panel RC slab with the typically used boundary conditions compared to that of a real bridge RC slab.

(2) Maximum principal strain distribution at the bottom surface of the slab

Figure 17 shows a comparison of the maximum principal strain distribution on the bottom surface of the panel RC slab at different loading cycles for both cases of boundary conditions. In all contours, the maximum principal strain value is higher than the cracking strain value of the concrete except for the contour shown by dark gray color. After the first loading cycle, the maximum principal strain distribution is spread to a greater extent in the longitudinal direction in the typically used boundary conditions case compared to the typically used boundary conditions case. At the 200 000th cycle, the maximum principal strain distribution is in the diagonal direction starting from the loading point and reaching towards the slab corners for the typically used boundary conditions case. This is different from the maximum principal strain distribution along the longitudinal and transverse directions in the approximate boundary conditions case. This distribution confirms the crack propagation in the diagonal direction in the typically used boundary conditions case. At the 280 000th cycle, the maximum principal strain distribution is mainly along the diagonal direction with significantly increased strain value around the loading point and at the slab corners in the typically used boundary conditions case; however, the distribution is along the longitudinal and transverse directions for the approximate boundary conditions case. At the last cycle of moving wheel load (i.e., 500 000th cycle), the value of maximum principal strain is highest around the loading point, and it spreads in the diagonal direction in the typically used boundary conditions case while the spread is in the longitudinal and transverse directions in the approximate boundary conditions case.

After the first loading cycle, the panel RC slab with the approximate boundary conditions experiences the cracking strain at a larger portion of the bottom surface in the longitudinal direction, compared to the typically used boundary conditions case. With an increase in the cycles of moving wheel load, the maximum principal...
strain spreads in the longitudinal direction as well in the transverse direction of the panel RC slab for the approximate boundary conditions case. This is because of the fact that the horizontally placed steel I-beams in the approximate boundary conditions allow the maximum principal strain to spread along the longitudinal and transverse directions of the panel RC slab similar to that of a real bridge RC slab. However, the vertically placed steel I-beams in the typically used boundary conditions have high bending stiffness and these steel I-beams lead to the maximum principal strain distribution in the diagonal direction of the panel RC slab. Despite a very high maximum principal strain in the diagonal direction, there are uncracked zones in the longitudinal direction at the bottom surface of the panel RC slab with the typically used boundary conditions even at the final cycle of moving wheel load, which is not observed in a real bridge RC slab.

(3) Center displacement evolution
For the typically used boundary conditions case, the displacements at the center of the panel RC slab obtained during the fatigue analysis at different loading cycles in this study are compared with the experimental results from a previous study (NILIM 2015) as shown in Fig. 18.

In this study, the numerical model is based on the bridging stress degradation concept and the reduction of bridging stress in the concrete element is employed after each cycle of moving wheel load. The numerical model results in smaller center displacements at initial cycles compared to those of experimental, but it accurately predicts the experimental center displacements after the first few cycles of moving load. The reason for the accurate displacement prediction is that the numerical model is capable of capturing the dominant degradation mechanism. However, the degradation mechanism may not be pronounced during the initial cycles. Furthermore, the center displacement evolution by the numerical model for the typically used boundary conditions case is compared with that of the approximate boundary conditions case. The center displacement for the typically used boundary conditions case is lesser at initial cycles compared to the approximate boundary conditions case. However, it is comparatively more at the last cycles of moving load as shown in Fig. 18.

---

**Fig. 17 Maximum principal strain distribution on the bottom surface.**
Figure 19 shows a comparison of the displacement jump obtained during the fatigue analysis at different loading cycles in this study for the typically used and the approximate boundary conditions cases. The displacement jump indicates a sudden increase in the displacement due to an increase in the intensity of the moving wheel load in a stepwise loading sequence. For the typically used boundary conditions case, a significant high displacement jump is observed at the 200,000th cycle due to the formation of diagonal cracks, contrary to the approximate boundary conditions case. However, there is another significantly large displacement jump at the 280,000th cycle for both the boundary conditions cases. This is because almost all the elements on the bottom surface of the panel RC slab are cracked, and the cracked elements propagate up to 3/4th of the total thickness of the panel RC slab at the loading point leading to a decrease in the compression zone. For the typically used boundary conditions case, the displacement jump is larger after the 320,000th cycle as compared to the approximate boundary conditions case due to the propagation of the corner cracked elements in the vertical direction, in addition to their propagation in the diagonal direction. The propagation of the corner cracked elements in the vertical and diagonal directions leads to a larger displacement in the typically used boundary conditions case in contrast to a real RC bridge slab.

The panel RC slab with the typically used boundary conditions exhibits smaller displacement evolution at the initial cycles of the moving wheel load compared to the approximate boundary conditions case. The reason for this is that the vertically placed steel I-beams in the typically used boundary conditions case provide high resistance against bending and deflection as the wheel load moves along the slab axis. However, diagonal cracks are formed and the corner cracks propagate in the vertical direction, resulting in larger displacement in the typically used boundary conditions case, which is dissimilar to a real bridge RC slab.

(4) Crack pattern

The crack patterns on the bottom surface of the panel RC slab under moving wheel load at the last loading cycle for both the boundary conditions cases are compared and displayed in Fig. 20. For the typically used boundary conditions case, the main crack originates at the center, which is the first location of the moving wheel load; subsequently, it extends to the supporting corners. As the load begins to move, the diagonal cracks are formed between the locations of the moving wheel load in the longitudinal direction and the supporting corner, making the first crack set. At the same time, other cracks are formed perpendicular to the existing cracks as second and third crack sets, surrounding the moving wheel load zone. An increase in the number of cycles results in much more extensive cracking. However, the horizontal cracks are formed in the approximate boundary conditions case, different from the fore-
mation of the diagonal cracks in the typically used boundary conditions case. Moreover, the cracks propagate until the end of the moving wheel load zone in the longitudinal direction for the typically used boundary conditions case. This is in contrast to the cracks propagation that happened throughout the length of the RC slab, surrounding the moving wheel load zone in the longitudinal direction for the approximate boundary conditions case.

The crack angles at three different sections (I, II and III) are determined for both the boundary conditions cases and shown in Fig. 21. The crack is categorized into two types depending upon the angle of the crack in the 1st crack set obtained from the numerical model. If the angle of the crack with the x-axis is lesser than 22.5°, it is considered as a horizontal crack. Otherwise, it is considered as a diagonal crack (angle greater than 22.5°). The horizontal cracks form the grid crack pattern, and the diagonal cracks lead to the formation of the diagonal crack pattern.

The grid crack patterns for both the boundary conditions cases are compared in Fig. 22. For the typically used boundary conditions case, the grid crack pattern is reproduced only within the moving wheel load zone in the longitudinal and transverse directions, in contrast to a real bridge RC slab. However, the panel RC slab with approximate boundary conditions reproduces a grid crack pattern in the transverse direction that is almost twice as the grid crack pattern reproduced by the panel RC slab with typically used boundary conditions. Moreover, the approximate boundary conditions allow the propagation of the grid crack pattern in the panel RC slab along the longitudinal direction to a greater extent compared to the typically used boundary conditions case.

It is important to mention here that degradation and
crack patterns of real bridge slabs are primarily associated with the accumulated loads of daily traffics, i.e., moving wheel loads. Therefore, in this study, the moving wheel load is considered as the main cause of degradation and crack pattern.

However, the other factors, such as drying shrinkage, restriction of steel girder, less concrete cover due to poor construction and reinforcement bar arrangements, can also play a part in producing the crack pattern of a real bridge RC slab. These factors should also be considered together with the moving wheel load in the future study. The panel RC slab with the approximate boundary conditions considering the moving wheel load and other important factors would yield to a more realistic simulation of the real bridge RC slab.

6. Conclusions

In this study, approximate boundary conditions for a panel RC slab are proposed to reproduce the fatigue behaviors of a real bridge RC slab. First, a method for the determination of the approximate boundary conditions for the panel RC slab is presented, with which the approximate boundary conditions for the panel RC slabs of different dimensions can easily be determined. Second, fatigue analysis of a panel RC slab with the approximate boundary conditions is conducted in addition to the panel RC slab with the boundary conditions typically used in the past studies. An FEM based numerical model considering the bridging stress degradation is used in the fatigue analysis. Based on the numerical results, specific outcomes and conclusions can be drawn, which are summarized as follows:

1. The approximate boundary conditions reproduce the same bending moment distribution and displacements around the loading locations as the wheel load moves along the slab axis in the panel RC slab, similar to a real bridge RC slab.

2. The approximate boundary conditions do not result in a negative bending at the corners of the panel RC slab and the cracked elements do not propagate on the top surface from the corners to the loading point, which is similar to a real bridge RC slab. However, in the panel RC slab with the typically used boundary conditions, the negative bending at the corners and the propagation of the cracked elements on the top surface from the corners to the loading point produce an additional deterioration in the panel RC slab, which leads to a shorter fatigue life estimation of a real bridge RC slab.

3. The panel RC slab with the approximate boundary conditions reproduces the extensive grid crack pattern well in accordance with that of a real bridge RC slab. However, in contrast to a real bridge slab, the panel RC slab with the typically used boundary conditions reproduces the grid crack pattern only within the moving wheel load zone.

In conclusion, this study mainly focused on proposing the boundary conditions for a panel RC slab numerically, which can simulate the realistic behaviors of a bridge RC slab. The numerical results show that the panel RC slab with proposed approximate boundary conditions behaves in the same manner as a real bridge RC slab. Consequently, it is expected to yield a more realistic fatigue life estimation. In the future study, the experimental setup will be designed in such a way to ensure the proposed boundary conditions.

Acknowledgment

The authors would like to acknowledge Japan Bridge Engineering Center (JPEC) for supporting and conducting this study through “Grants for research and development related to bridge technology.”

References

Cheng, L., (2011). “Flexural fatigue analysis of a CFRP form reinforced concrete bridge deck.” Composite Structures, 93(11), 2895-2902.

Drar, A. A. M. and Matsumoto, T., (2016). “Fatigue analysis of RC slabs reinforced with plain bars based on the bridging stress degradation concept.” Journal of Advanced Concrete Technology, 14(1), 21-34.

El-Ragaby, A., El-Salakawy, E. and Bennokrane, B., (2007). “Fatigue life evaluation of concrete bridge deck slabs reinforced with glass FRP composite bars.” Journal of Composites for Construction, 11(3), 258-268.

Frederick, G. R., (1997). “Experimental and analytical investigation of load distribution in concrete slab bridges.” In: Proc. Spring Conference of Society for Experimental Mechanics, Bellevue, Washington 2-4 June 1997. Connecticut, USA: Society for Experimental Mechanics.

JRA, (1996). “Design specifications for highway bridges.” Tokyo: Japan Road Association. (in Japanese)

Li, V. C. and Matsumoto, T., (1998). “Fatigue crack growth analysis of fiber reinforced concrete with effect of interfacial bond degradation.” Cement and Concrete Composites, 20(5), 339-351.

Lin, H., Zhao, Y., Ozbolt, J. and Hans-Wolf, R., (2017). “The bond behavior between concrete and corroded steel bar under repeated loading.” Engineering Structures, 140, 390-405.

Mabsout, M., Tarhini, K., Jabahkanji, R. and Awwad, E., (2004). “Wheel load distribution in simply supported concrete slab bridges.” Journal of Bridge Engineering, 9(2), 147-155.

Maeda, Y. and Matsui, S., (1984). “Fatigue of reinforced concrete slabs under trucking wheel load.” Proceedings of Japan Concrete Institute, 6, 221-224. (in Japanese)

Maekawa, K., Pimanmas, A. and Okamura, H., (2003). “Nonlinear mechanics of reinforced concrete.” London, UK: Spon Press.

Maki, Y., Ha, T. M., Fukuda, S., Torii, K. and Ono, R., (2019). “Stiffness evaluation and current status of a degraded road bridge slab located in a mountainous area.” Journal of Advanced Concrete Technology,
Matsui, S., (1987). “Fatigue strength of RC-slabs of highway bridge by wheel running machine and influence of water on fatigue.” *Proceedings of Japan Concrete Institute*, 9(2), 627-632. (in Japanese)

Menegotto, M. and Pinto, P. E., (1973). “Method of analysis for cyclically loaded R.C. plane frames including changes in geometry and non-elastic behaviour of elements under combined normal force and bending.” In: *Proc. IABSE Symposium on Resistance and Ultimate Deformability of Structures Acted on by Well-Defined Repeated Loads*, Lisbon, Portugal September 1973. Zurich: International Association for Bridge and Structural Engineering, 15-22.

Mitamura, H., Syakushiro, K., Matsumoto, T. and Matsui, S., (2012). “Experimental study on fatigue durability of RC deck slabs with overlay retrofit.” *Journal of Structural Engineering*, 58(A), 1166-1177.

MARC, (2017). “Advanced nonlinear simulation solution programming software MARC v. 2017 [online]”. MSC Corporation, Newport Beach, California, USA. Available from: <https://www.mscsoftware.com/product/marc>.

Muspratt, M. A., (1978). “Elastic analysis of slabs.” *Building and Environment*, 13(1), 51-59.

NILIM, (2015). “Technical note no. 844: Research on fatigue durability evaluation for highway bridge concrete slabs.” Tokyo: National Institute for Land and Infrastructure Management. (in Japanese)

Okada, K., Okamura, H., Sonoda, K. and Shimada, I., (1982). “Cracking and fatigue behavior of bridge deck RC slabs.” *Proceedings of the Japan Society of Civil Engineers*, 321, 49-61. (in Japanese)

Perdikaris, P. C. and Beim, S., (1988). “RC bridge decks under pulsating and moving load.” *Journal of Structural Engineering*, 114(3), 591-607.

Perdikaris, P. C., Beim, S. R. and Bousias, S. N., (1989). “Slab continuity effect on ultimate and fatigue strength of reinforced concrete bridge deck models.” *ACI Structural Journal*, 86(4), 483-491.

Schlafl, M. and Bruhwiler, E., (1998). “Fatigue of existing reinforced concrete bridge deck slabs.” *Engineering Structures*, 20(11), 991-998.

Shakushiro, K., Matamuro, H., Watanabe, T. and Kishi, N., (2011). “Experimental study on fatigue durability of RC slabs reinforced with round steel bars.” *Journal of Structural Engineering*, 57(A), 1297-1304.

Sonoda, K. and Horikawa, T., (1982). “Fatigue strength of reinforced concrete slabs under moving load.” In: *Proc. IABSE Colloquium on Fatigue of Steel and Concrete Structures*, Lausanne, Switzerland 24-26 March 1982. Zurich: International Association for Bridge and Structural Engineering, 455-462.

Suthiarapirak, P. and Matsumoto, T., (2004). “3D fatigue analysis of RC bridge slabs and slab repairs by fiber cementitious materials.” In: V. C. Li, C. K. Y. Leung, K. J. William and S. L. Billington, Eds. *Proc. International Conference FraMCos-5*, Colorado, USA 12-16 April 2004. Illinois, USA: International Association for Fracture Mechanics of Concrete and Concrete Structures, Vol. 2, 677-684.

Suthiarapirak, P. and Matsumoto, T., (2006). “Fatigue analysis of RC slabs and repaired RC slabs based on crack bridging degradation concept.” *Journal of Structural Engineering*, 132(6), 939-948.

Zhang, J., (1998). “Fatigue fracture of fiber reinforced concrete – an experimental and theoretical study.” Thesis (PhD). Department of Structural Engineering, Technical University of Denmark.

Zhang, J., Stang, H. and Li, V. C., (2001). “Crack bridging model for fiber reinforced concrete under fatigue tension.” *International Journal of Fatigue*, 23(8), 655-670.