Three-body nonleptonic $B$ decays in perturbative QCD

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We develop perturbative QCD formalism for three-body nonleptonic $B$ meson decays. Leading contributions are identified by defining power counting rules for various topologies of amplitudes. The analysis is simplified into the one for two-body decays by introducing two-meson distribution amplitudes. This formalism predicts both nonresonant and resonant contributions, and can be generalized to baryonic decays.

The fundamental concept of perturbative QCD (PQCD) is to separate hard and soft dynamics in a QCD process. The former is calculable in perturbation theory, while the latter, though not calculable, is treated as a universal input. The separation can be performed in the framework of collinear factorization [1] or of $k_T$ factorization [2,3], in which an amplitude is expressed as a convolution of a hard kernel $H$ with a hadron distribution amplitude $\Phi(x)$ or with a hadron wave function $\Phi(x,k_T)$, $x$ and $k_T$ being a longitudinal momentum fraction and a transverse momentum, respectively. Collinear factorization works, if it does not develop an end-point singularity from $x \to 0$ in the above convolution. If it does, collinear factorization breaks down, and $k_T$ factorization is more appropriate.

It has been known that collinear factorization of charmed and charmless two-body $B$ meson decays suffers endpoint singularities. The PQCD formalism for these modes based on $k_T$ factorization theorem was then derived [4–6], which has been shown to be infrared-finite, gauge-invariant, and consistent with the factorization assumption in the heavy-quark limit [7,8]. If one still employs collinear factorization, an alternative approach, the so-called QCD-improved factorization [9], can be developed. In this approach the end-point singularities in the leading contributions are absorbed into $B$ meson transition form factors, and those appearing at the subleading level are regularized by arbitrary (nonuniversal) infrared cutoffs of momentum fractions $x$. Without the arbitrary cutoffs, PQCD has a predictive power, whose predictions for $B \to PP$, $VP$, and $VV$ modes are all in agreement with data [10].

Three-body nonleptonic $B$ meson decays have been observed recently [11,12]. Viewing the experimental progress, it is urgent to construct a corresponding framework. Motivated by its theoretical self-consistency and phenomenological success, we shall generalize PQCD to these modes. A direct evaluation of the hard kernels, which contain two virtual gluons at lowest order, is not practical due to the enormous number of diagrams. On the other hand, the region with the two gluons being hard simultaneously is power-suppressed and not important. Therefore, a new input is necessary in order to catch dominant contributions to three-body decays in a simple manner. The idea is to introduce two-meson distribution amplitudes [13], by means of which a factorization formula for a $B \to h_1h_2h_3$ decay amplitude is written, in general, as

$$\mathcal{M} = \Phi_B \otimes H \otimes \Phi_{h_1h_2} \otimes \Phi_{h_3}.$$  

(1)

It will be shown that both nonresonant contributions and resonant contributions through two-body channels can be included through the parametrization of the two-meson distribution amplitude $\Phi_{h_1h_2}$.

Three-body decay amplitudes are classified into four topologies, depending on number of light mesons emitted from the four-fermion vertices. Topologies I and III, shown in Figs. 1(a) and 1(c), are associated with one light meson emission and three light meson emission, respectively. The bubbles denote the distribution amplitudes, which absorb nonperturbative dynamics. The hard kernel $H$ contains only a single hard gluon exchange. The former involves transition of the $B$ meson into two light mesons. In the latter case a $B$ meson annihilates completely. For two light meson emission shown in Fig. 1(b), we assign IIs to the special amplitude corresponding to the scalar vertex, and II to the rest of the amplitudes. Both topologies II and IIs are expressed as a product of a heavy-to-light form factor and a time-like light-light form factor in the heavy-quark limit.

The dominant kinematic region for three-body $B$ meson decays is the one, where at least one pair of light mesons has the invariant mass of $O(\Lambda M_B)$ for nonresonant contributions and of $O(\Lambda^2)$ for resonant contributions, $\Lambda = M_B - m_b$ being the $B$ meson and $b$ quark mass difference. An example is the configuration, where all three mesons carry

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momenta of \( O(M_B) \), but two of them move almost parallelly. In the above dominant region collinear factorization theorem applies to topology I, since it is free of end-point singularities as shown below. With the pair of mesons emitted with a small invariant mass, the evaluation of topologies II and IIs is the same as of two-body decays. The contribution from the region, where all three pairs have the invariant mass of \( O(M_B^2) \), is power-suppressed. This contribution is the one, which can be calculated perturbatively in terms of the diagrams with two hard gluon exchanges.

We define the power counting rules for the various topologies in the dominant kinematic region, and identify the leading ones. Consider first nonresonant contributions. Topology I behaves like \( (\bar{\Lambda} M_B)^{-2} \), where one power of \( (\bar{\Lambda} M_B)^{-1} \) comes from the hard gluon in Fig. 1(a), which kicks the soft spectator in the \( B \) meson into a fast one in a light meson [7], and another power is attributed to the invariant mass of the light meson pair. The overall product of the meson pair constants is not shown explicitly. Topology II exhibits the same power behavior as topology I: the hard gluon in Fig. 1(b), i.e., the \( B \) meson transition form factor, gives a power of \( (\bar{\Lambda} M_B)^{-1} \), and the light-light form factor gives another power. The scalar vertex introduces an extra power \( m_\omega/M_B, m_\omega \) being the chiral symmetry breaking scale, to topology IIs. Topology III must involve large energy release for producing at least a pair of fast mesons with the invariant mass of \( O(M_B^2) \). That is, it behaves like \( (\bar{\Lambda} M_B)^{-1} M_B^{-2} \). Hence, we have the relative importance of the decay amplitudes,

\[
M_1 : M_{II} : M_{IIs} : M_{III} = 1 : 1 : \frac{m_\omega}{M_B} : \frac{\bar{\Lambda}}{M_B},
\]

indicating that topology III is negligible. For resonant contributions, we replace the power of \( (\bar{\Lambda} M_B)^{-1} \) associated with the light meson pair by \( \bar{\Lambda}^{-2} \). Therefore, Eq. (2) still holds.

Take topology I for the \( B^+ \to K^+\pi^+\pi^- \) mode as an example, in which the \( B \) meson transit into a pair of pions. The \( \pi^+ \) and \( \pi^- \) mesons carry the momenta \( P_1 \) and \( P_2 \), respectively. The \( B \) meson momentum \( P_B \), the total momentum of the two pions, \( P = P_1 + P_2 \), and the kaon momentum \( P_3 \) are chosen as

\[
P_B = \frac{M_B}{\sqrt{2}} (1, 1, 0_T), \quad P = \frac{M_B}{\sqrt{2}} (1, \eta, 0_T), \quad P_3 = \frac{M_B}{\sqrt{2}} (0, 1 - \eta, 0_T),
\]

with the variable \( \eta = w^2/M_B^2, w^2 = P^2 \) being the invariant mass of the two-pion system. The light-cone coordinates have been adopted here. Define \( \zeta = P_1^+ / P^+ \) as the \( \pi^+ \) meson momentum fraction, in terms of which, the other kinematic variables are expressed as

\[
P_1^+ = (1 - \zeta) P^+, \quad P_1^- = (1 - \zeta) \eta P^+, \quad P_2^- = \zeta P^+, \quad P_{1T}^2 = P_{2T}^2 = \zeta (1 - \zeta) w^2.
\]

The two pions from the \( B \) meson transition possess the invariant mass \( w^2 \sim O(\bar{\Lambda} M_B) \), implying the orders of magnitude \( P^+ \sim O(M_B), P^- \sim O(\bar{\Lambda}) \) and \( P_T \sim O(\sqrt{\bar{\Lambda} M_B}) \). In the heavy-quark limit, the hierarchy \( P^+ \gg P_{1T} \gg P^- \) corresponds to a collinear configuration. Therefore, we introduce the two-pion distribution amplitudes [13],

\[
\Phi_\nu(z, \zeta, w^2) = \frac{1}{2\sqrt{2N_c}} \int \frac{dy^-}{2\pi} e^{-izP^+y^-} \langle \pi^+(P_1)\pi^-(P_2) | \bar{\psi}(y^-) \gamma^\nu \gamma^5 | 0 \rangle |
\]

\[
\Phi_s(z, \zeta, w^2) = \frac{1}{2\sqrt{2N_c}} \int \frac{dy^-}{2\pi} e^{-izP^+y^-} \langle \pi^+(P_1)\pi^-(P_2) | \bar{\psi}(y^-) T \psi(0) | 0 \rangle |
\]

\[
\Phi_t(z, \zeta, w^2) = \frac{1}{2\sqrt{2N_c}} \int \frac{dy^-}{2\pi} e^{-izP^+y^-} \langle \pi^+(P_1)\pi^-(P_2) | \bar{\psi}(y^-) i\sigma_{\mu\nu} n_\mu P^\nu T \psi(0) | 0 \rangle |
\]

with \( \Phi_\nu \) being the twist-2 component, and \( \Phi_s \) and \( \Phi_t \) the twist-3 components. \( T = \tau^3/2 \) is for the isovector \( I = 1 \) state, \( \psi \) the \( u-d \) doublet, \( z \) the momentum fraction carried by the spectator \( u \) quark, and \( n_\mu = (0, 1, 0_T) \) a dimensionless vector. The constant \( f_{2\pi}^+ \) of dimension of mass is defined via the local matrix element [13],

\[
\lim_{w^2 \to 0} \langle \pi^+(P_1)\pi^-(P_2) | \bar{\psi}(0) i\sigma_{\mu\nu} n_\mu P^\nu T \psi(0) | 0 \rangle = \frac{w^2}{f_{2\pi}^+}(2\zeta - 1)P^+.
\]

The matrix element with the structure \( \gamma_5 \gamma^\nu \gamma^5 \) vanishes for topologies I and IIs, and contributes to topology II at twist 4. The one with the structure \( \gamma_5 \gamma^\nu \) vanishes. For topologies II and IIs, a kaon-pion distribution amplitude is introduced in a similar way. For other two-pion systems, the distribution amplitudes can be defined with the appropriate choice of the matrix \( T \). For instance, \( T = 1/2 \) is for the \( \pi^0 \pi^0 \) isoscalar \( (I = 0) \) state.
A two-pion distribution amplitude can be related to the pion distribution amplitude through the calculation of the process $\gamma \gamma^* \rightarrow \pi^+ \pi^-$ at large invariant mass $w^2$ [14]. The extraction of the two-pion distribution amplitudes from the $B \rightarrow \pi\pi\nu\bar{\nu}$ decay has been discussed in [15]. Here we pick up the leading term in the complete Gegenbauer expansion of $\Phi _1 (z, \zeta ,w^2 )$ [13]:

$$
\Phi _{\nu,t} (z, \zeta ,w^2 ) = \frac{3F_\nu (w^2 )}{\sqrt{2N_c}} z(1-z)(2\zeta -1), \quad \Phi _s(z, \zeta ,w^2 ) = \frac{3F_s(w^2 )}{\sqrt{2N_c}} z(1-z),
$$

where $F_{\nu,s,t}(w^2)$ are the time-like pion electromagnetic, scalar and tensor form factors with $F_{\nu,s,t}(0) = 1$. That is, the two-pion distribution amplitudes are normalized to the time-like form factors. For $\Phi _1$ in Eq. (9), we have adopted the parametrization,

$$
\langle \pi^+ (P_1) \pi^- (P_2) | \bar{\psi} (0) i\sigma_{\mu \nu} n^\mu P^\nu \Psi (0) | 0 \rangle = \frac{w^2}{f^2 \pi} F_1 (w^2 ) (2\zeta -1) P^+. \tag{10}
$$

Note that the asymptotic functional form for the $z$ dependence of $\Phi _s$ is an assumption.

For the $B$ meson distribution amplitude, we employ the model [5],

$$
\Phi _B (x) = N_B x^2 (1-x)^2 \exp \left[ -\frac{1}{2} \left( \frac{xM_B}{\omega_B} \right)^2 \right], \tag{11}
$$

with the shape parameter $\omega_B = 0.4$ GeV, and the normalization constant $N_B$ related to the decay constant $f_B = 190$ MeV (in the convention $f_\pi = 130$ MeV) via $\int_0^1 \Phi _B (x) dx = f_B / (2\sqrt{2N_c})$. The above $\Phi _B$ is identified as $\Phi _+$ in the definition of the two leading-twist $B$ meson distribution amplitudes $\Phi _\pm$ given in [16,17]. Equation (11), vanishing at $x \rightarrow 0$, is consistent with the behavior required by equations of motion [18]. Another distribution amplitude $\Phi _\delta$ in our definition, identified as $\Phi _\delta = (\Phi _B - \Phi _B^0 )/\sqrt{2}$ with a zero normalization, contributes at the next-to-leading power $\Lambda /M_B$ [7]. It has been verified numerically [19] that the contribution to the $B \rightarrow \pi$ form factor from $\Phi _B$ is much larger than from $\Phi _\delta$.

The total decay rate is written as

$$
\Gamma = \frac{G_F^2 M_B^5}{512\pi^4} \int_0^1 \! dx \int_0^1 \! d\eta |M|^2 , \quad M = M_1 + M_{II} + M_{II\delta} , \tag{12}
$$

with the amplitudes,

$$
M_1 = f_K \left( V_+^* \sum_{i=4,6} F^{P(u)}_{ei} - V_u^* F_{e2} \right) , \quad M_{II\delta} = V_t^* F_s (\omega^2 ) F^{P(d)}_{e6} ,
$$

$$
M_{II} = (2\zeta -1) F_\pi (\omega^2 ) \left[ V_t^* \left( \sum_{i=3}^5 F^{P(d)}_{ei} + \sum_{i=3,5} F^{P(u)}_{ei} \right) - V_u^* F_{e1} \right] . \tag{13}
$$

For a simpler presentation, we have assumed that the kaon-pion time-like form factor in topology $II\delta$ is equal to the pion time-like form factor multiplied by the ratio of the decay constants $f_K /f_\pi$. This assumption is in fact not necessary, and the property of the kaon-pion form factor will be discussed elsewhere. The superscript $P(q)$ stands for the amplitude from a penguin operator producing a pair of quarks $q$. Those without $P(q)$ arise from tree operators. The subscript $ei$ stands for the emission topology (in contrast to the annihilation topology $III$) from the effective four-fermion operator $O_4$ in the standard notation.

We calculate the hard kernels by contracting the structures, which follow Eqs. (5)-(7),

$$
\left( \frac{P_B + M_B}{\sqrt{2N_c}} \right) \Phi _B (x) , \quad \frac{1}{\sqrt{2N_c}} \left[ F_P \Phi _e (z, \zeta ,w^2 ) + w \Phi _s (z, \zeta ,w^2 ) + \frac{P_1 - P_2}{w(2\zeta -1)} \Phi _t (z, \zeta ,w^2 ) \right] , \tag{14}
$$

to Fig. 1. The factorization formulas for the $B \rightarrow 2\pi$ transition amplitudes are given by

$$
F_e^{P(u)} = 8\pi C_F M_B^2 (1-\eta) \int_0^1 \! dx_1 dz_1 \frac{\Phi _B (x_1 )}{x_1 z_1 M_B^2 + P_T^2} \times \left[ (1+z)\Phi _e (z, \zeta ,w^2 ) + \sqrt{\eta}(1-2z)\Phi _t (z, \zeta ,w^2 ) + \sqrt{\eta}(1-2z)\Phi _s (z, \zeta ,w^2 ) \right] \alpha_s (\mu^2 ) a_4 (u^2 ) a_1 (1) \frac{1}{z M_B^2 + P_T^2} \tag{14}
$$

3
\[ \mathcal{F}^{P(u)}_{6} = -16\pi C_F M_B^2 r_0 \int_{0}^{1} dx_1 dx_2 \frac{\Phi_B(x_1)}{x_1 z M_B^2 + P_T^2} \times \left\{ \left[ (1 + \eta - 2z\eta)\Phi_v(z, \zeta, w^2) - \sqrt{\eta z} \Phi_I(z, \zeta, w^2) + \sqrt{\eta (2 + z)} \Phi_s(z, \zeta, w^2) \right] \frac{a_s(t_e(1)) a_u(t_e(1))}{z M_B^2 + P_T^2} \right\}, \]

with \( r_0 = m_0/M_B \). \( \mathcal{F}_{6,2} \) is the same as \( \mathcal{F}^{P(u)}_{4} \) but with \( a_4^{(u)} \) replaced by \( a_2 \) (here \( a_2 \) is close to unity). The definitions of the Wilson coefficients \( a^{(q)}(t) \) are referred to [20]. The hard scales are defined by \( t_e(1) = \max[\sqrt{\mathcal{M}}B, P_T] \) and \( t_e(2) = \max[\sqrt{x}M_B, P_T] \). The above collinear factorization formulas are well-defined, since the invariant mass of the two-pion system, proportional to \( P_T \), smears the end-point singularities from \( z \to 0 \).

The \( B \) meson transition form factors involved in topologies II and IIs are

\[ F^{P(d)}_{4,6} = 8\pi C_F M_B^2 \int_{0}^{1} dx_1 dx_2 \int_{x_2}^{\infty} b_1 b_2 \Phi_B(x_1, b_1) \times \left\{ \left[ (1 - \eta)(1 + (1 - \eta)x_2)\Phi_K(x_2) + r_0(1 + \eta - 2(1 - \eta)x_2)\Phi_K(x_2) \right. \right. + \left. \left. r_0(1 - \eta)(1 - x_2)\Phi^\sigma(x_2) \right] E_4^{(d)}(t_e(1)) h_e(x_1, (1 - \eta)x_2, b_1, b_3) + 2r_0(1 - \eta)\Phi^\sigma_K(x_2) E_4^{(d)}(t_e(2)) h_e((1 - \eta)x_2, x_1, b_1, b_3) \right\}, \]

\[ F^{P(d)}_{6,6} = 16\pi C_F M_B^2 \sqrt{\eta} \int_{0}^{1} dx_1 dx_2 \int_{x_2}^{\infty} b_1 b_2 \Phi_B(x_1, b_1) \times \left\{ \left[ (1 - \eta)\Phi_K(x_2) + 2r_0\Phi^\sigma_K(x_2) + r_0(1 - \eta)x_2 \left( \Phi^K(x_2) - \Phi^\sigma_K(x_2) \right) \right] \times E_6^{(d)}(t_e(1)) h_e(x_1, (1 - \eta)x_2, b_1, b_3) + 2r_0(1 - \eta)\Phi^\sigma_K(x_2) E_6^{(d)}(t_e(2)) h_e((1 - \eta)x_2, x_1, b_1, b_3) \right\}. \]

The definitions of the evolution factors \( E_i^{(q)}(t) \), which contain the Wilson coefficients \( a_i^{(q)}(t) \), of the hard functions \( h_e \), and of the kaon distribution amplitudes \( \Phi_K, \Phi_{K'} \) and \( \Phi^\sigma \), are referred to [20]. \( F^{P(q)}_{6,4} \), \( F^{P(q)}_{6,5} \) and \( F^{P}(\eta) \) are obtained from \( F^{P(d)}_{6,4} \) by substituting \( a_3^{(q)} \) for \( a_1 \) and \( a_2 \) for \( a_3 \), respectively.

The PQCD evaluation of the form factors indicates the power behavior in the asymptotic region, \( F_{\pi}(w^2) \sim 1/w^2 \), and their relative importance: \( F_{s,t}(w^2)/F_{\pi}(w^2) \sim m_0/w \). Therefore, the twist-3 contributions in Eq. (15) are down by a power of \( \sqrt{m_0/w} = m_0/M_B \) compared to the twist-2 ones, which is the accuracy considered here. To calculate the nonresonant contribution, we propose the parametrization for the whole range of \( w^2 \),

\[ F^{(nr)}_{\pi}(w^2) = \frac{m^2}{w^2 + m^2}, \quad F^{(nr)}_{s,t}(w^2) = \frac{m_0 m^2}{w^3 + m_0 m^2}, \]

where the the parameter \( m = 1 \) GeV is determined by the fit to the experimental data \( M_{J/\psi}^2 |F_{\pi}(M_{J/\psi}^2)|^2 \sim 0.9 \) GeV\(^2 \) [21], \( M_{J/\psi} \) being the \( J/\psi \) meson mass. These form factors can carry strong phases, which are assumed to be not very different, i.e., overall and negligible here.

To calculate the resonant contribution, we parametrize it into the time-like form factors,

\[ F^{(r)}_{\pi,s,t}(w^2) = \frac{M_{\pi}^2}{\sqrt{(w^2 - M_{\rho}^2)^2 + \Gamma_{\pi}^2 w^2}} - \frac{M_{\rho}^2}{w^2 + M_{\rho}^2}, \]

with \( \Gamma_{\pi} \) being the width of the meson \( V \). The subtraction term renders Eq. (18) exhibit the features of resonant contributions: the normalization \( F^{(r)}_{\pi}(0) = 0 \) and the asymptotic behavior \( F^{(r)}_{\pi}(w^2) \sim 1/w^4 \), which decreases at large \( w \) faster than the nonresonant parametrization in Eq. (17). Equation (18) is motivated by the pion time-like form factor measured at the \( \rho \) resonance [22]. It is likely that all \( F^{(r)}_{\pi,s,t} \) contain the similar resonant contributions.
The relative phases among different resonances will be discussed elsewhere by employing the more sophisticated parametrization [23]. Here we assume the absence of the interference effect.

We adopt $m_0 = 1.4 (1.7)$ GeV for the pion (kaon) and the unitarity angle $\phi_3 = 90^\circ$ [5]. For the $B^+ \to \rho(770)K^+$ and $B^+ \to f_0(980)K^+$ channels, we choose $\Gamma_{\rho} = 150$ MeV and $\Gamma_{f_0} = 50$ MeV [24]. The nonresonant contribution $0.61 \times 10^{-6}$ to the $B^+ \to K^+\pi^+\pi^-$ branching ratio is obtained. Our results $1.8 \times 10^{-6}$ and $13.2 \times 10^{-6}$ are consistent with the measured three-body decay branching ratios through the $B^+ \to \rho(770)K^+$ and $B^+ \to f_0(980)K^+$ channels, $< 12 \times 10^{-6}$ and $(9.6^{+2.5+1.5+3.4}_{-2.3-1.3-0.8}) \times 10^{-6}$ [11], respectively. Since the $f_0$ width has a large uncertainty, we also consider $\Gamma_{f_0} = 60$ MeV, and the branching ratio reduces to $10.5 \times 10^{-6}$. The resonant contributions from the other channels can be analyzed in a similar way. For example, the $K^*(892)$ resonance can be included into the $K^-\pi$ form factors by choosing the width $\Gamma_{K^*} = 50$ MeV. The nonresonant and resonant contributions to the $B^+ \to K^+\pi^+\pi^-$ decay spectrum are displayed in Fig. 2.

In the above formalism nonfactorizable contributions arise from the diagrams, in which a hard gluon attaches the spectator quark and the meson emitted from the weak vertex (topology I) or the meson pair (topologies II and IIs). The nonfactorizable contributions, suppressed by $\ln^{-1}(M_B/\Lambda)$ [8], and topology III, being of $O(\Lambda/M_B)$, can be evaluated systematically by means of the two-meson distribution amplitudes. The framework presented here is not only applicable to the study of three-body mesonic B meson decays, but also to baryonic decays [25], such as $B \to p\bar p K$. One simply introduces two-proton distribution amplitudes, and the calculation of the corresponding hard kernel is similar.

In this letter we have proposed a promising formalism for three-body nonleptonic B meson decays. This formalism, though at its early stage, is general enough for evaluating both nonresonant and resonant contributions to various modes, and as simple as that for two-body decays. In the future we shall discuss more delicate issues, such as CP asymmetries [26], phase shifts from meson-meson scattering [27], and interference effects among different resonances [28].

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FIG. 1. Graphic definitions for topologies I, II(s), and III.

FIG. 2. (a) [(b)] Nonresonant (resonant) contribution to the $B^+ \to K^+\pi^+\pi^-$ decay spectrum with respect to the two-pion invariant mass $M(\pi^+\pi^-)$. The sharp peak corresponds to the $f_0$ resonance with the width 50 MeV.