RESOLVING CLOSE ENCOUNTERS: STABILITY IN THE HD 5319 AND HD 7924 PLANETARY SYSTEMS

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ABSTRACT

Radial velocity searches for exoplanets have detected many multi-planet systems around nearby bright stars. An advantage of this technique is that it generally samples the orbit outside of the inferior/superior conjunction, potentially allowing the Keplerian elements of eccentricity and argument of periastron to be well characterized. The orbital architectures for some of these systems show signs of close planetary encounters that may render the systems unstable as described. We provide an in-depth analysis of two such systems: HD 5319 and HD 7924, for which the scenario of coplanar orbits results in their rapid destabilization. The poorly constrained periastron arguments of the outer planets in these systems further emphasizes the need for detailed investigations. An exhaustive scan of parameter space via dynamical simulations reveals specific mutual inclinations between the two outer planets in each system that allow for stable configurations over long timescales. We compare these configurations with those presented by mean-motion resonance as possible stability sources. Finally, we discuss the relevance to interpretation of multi-planet Keplerian orbits and suggest additional observations that will help to resolve the system stabilities.

Key words: astrobiology – planetary systems – techniques: radial velocities

1. INTRODUCTION

Detection of multi-planet systems via the radial velocity (RV) method are becoming increasing common as both the duration and sensitivity of RV surveys increase. A crucial step in examining these multi-planet systems is the analysis of the orbital stability of the planets over long timescales (Smith & Lissauer 2009). Since the RV technique can sample the orbit during any position of the orbital phase, it is particularly well suited to providing information on the eccentricity and argument of periastron for each planet. These Keplerian orbital elements can result in orbits that imply close encounters between the planets in the system. Mean-motion orbital resonances (MMR), such as the Pluto 2:3 resonance with Neptune, can prevent close encounters and result in orbital stability (for example, see Barnes & Greenberg 2006a, 2007 and references therein). Dynamical simulations of planetary systems with an additional planet inserted at an arbitrary semimajor axis are often used to numerically determine the location of the islands of stability at the MMRs (Kane 2015).

Two planetary systems were recently moved from single-planet to multi-planet status through additional observations. The star HD 5319 was found by Robinson et al. (2007) to harbor a planet, and then by Giguere et al. (2015) an additional planet. The first planet in the HD 7924 was discovered by Howard et al. (2009), after which the system was expanded by Fulton et al. (2015) with two more planets. Based upon the published orbital parameters, both of these systems exhibit evidence of orbital instability due to potential close encounters of the two outer planets. The inclination of the planetary orbits to the plane of the sky for these systems is unknown, and so a dynamical solution to avoiding close encounters may include mutual inclinations between the orbits in addition to MMRs that may be present.

In this paper, we present dynamical simulations of the HD 5319 and HD 7924 systems that help to resolve possible close encounters of the outer planets via mutual inclinations of the planetary orbits. Section 2 presents a description of the problem being addressed, including the relevant system parameters and quantifying the proximity of the planetary orbits to each other. Section 3 outlines the methodological approach and parameters used in the numerical simulations. The results for the HD 5319 and HD 7924 simulations are presented in Sections 4 and 5 respectively. Section 6 investigates the possibility of MMRs as additional sources of stability. Section 7 investigates the effect of periastron argument on system stability and details a strategy for further observations that could help to resolve the orbits of the planetary systems. We provide concluding remarks in Section 8.

2. SYSTEM PARAMETERS

The orbital and physical characteristics of the HD 5319 and HD 7924 planets were extracted from the publications by Giguere et al. (2015) and Fulton et al. (2015) respectively. In the case of HD 5319, Giguere et al. (2015) used the Keplerian Fitting Made Easy package (Giguere et al. 2012) to produce the final orbital solution, then validated the solution with a Differential-Evolution Markov Chain Monte Carlo (DE-MCMC) approach that included a 100 year dynamical stability constraint. Giguere et al. (2015) also performed stability simulations that showed the majority of the DE-MCMC results were unstable over 10^7 years with the exception of a subset of the results at the 4:3 MMR. For HD 7924, Fulton et al. (2015) used the RVLIN package (Wright & Howard 2009) to estimate the initial orbital solution and then used DE-MCMC to produce the final solution. The median fit parameters from the DE-MCMC analysis were used for a single stability simulation that proved to be stable for 10^5 years. The planetary parameters derived from these methods that are relevant to our analysis are shown in Table 1, including the mass of the host star (M_∗), orbital period (P), time of periastron passage (T_p), orbital eccentricity (e), argument of periastron (ω), and semimajor axis (a). The orbits of the planets in the HD 5319 and HD 7924 systems are depicted in the left and right panels of Figure 1,
respectively. Also shown in the panels are solid lines from the host star that represent the periastron arguments for the orbits.

An additional quantity we calculated for each planet was the Hill radius, given by

$$R_H = r \left( \frac{M_p}{3M_*} \right)^{1/3},$$

where $r$ is the time-dependent (for a non-circular orbit) star–planet separation. This separation is given by

$$r = a \left( 1 - e^2 \right) \frac{1}{1 + e \cos f},$$

where $f$ is the true anomaly. We include the mean Hill radius (where $r = a$) for each planet in Table 1.

We calculated star–planet separations through the entire orbit for each planet and determined the location of closest proximity for the outer planet orbits, assuming that the orbits are coplanar. This location is indicated by a dashed line in each of the panels of Figure 1. For the HD 5319 system, the closest proximity of the planetary orbits occurs at a star–planet separation and true anomaly of $r = 1.701\, \text{au}$ and $f = 158.3^\circ$ for planet b, and $r = 1.761\, \text{au}$ and $f = 23.3^\circ$ for planet c. The separation of the orbits at this location is 0.061 au, equivalent to 0.495 $R_H$ for planet b and 0.571 $R_H$ for planet c, where the Hill radii were calculated using Equation (1) at the location of closest approach. Similarly for the HD 7924 system, the values are $r = 0.118\, \text{au}$ and $f = 120.4^\circ$ for planet c, and $r = 0.125\, \text{au}$ and $f = 285.2^\circ$ for planet d. In this case the minimum separation between the outer planet orbits is 0.007 au, corresponding to 2.749 $R_H$ for planet c and 2.933 $R_H$ for planet d. Note that the HD 7924 system is smaller in scale than the HD 5319 system, both in terms of planetary masses and semimajor axes. The proximity of the orbits in each case emphasizes the need for detailed dynamical simulations to resolve potential close planetary encounters.

3. METHODOLOGY

To assess the orbital stability of the planetary systems, we made use of the Mercury Integrator Package, as described by Chambers (1999). The code performs N-body integrations based upon user-specified input parameters and starting conditions for the system. Our dynamical simulations use the hybrid symplectic/Bulirsch–Stoer integrator with a Jacobi coordinate system, which generally provides more accurate results for multi-planet systems (Wisdom & Holman 1991; Wisdom 2006).

For each system, we set up initial conditions using the parameters shown in Table 1. Each of the integrations was performed for a simulation duration of 10$^7$ years commencing at the present epoch. The time resolution for the simulations was chosen to meet the minimum timestep recommendation of Duncan et al. (1998): 1/20 of the shortest orbital period in the system. To meet this requirement, we used timesteps of 5 and 0.25 days for the HD 5319 and HD 7924 systems respectively. Results from each integration were output in steps of 100 years.

We conducted a single simulation for each system assuming that the orbits are approximately coplanar ($i = 90^\circ$), verifying that the systems are indeed unstable as described. We then extended our analysis by running an exhaustive set of simulations that slowly changed the orbital inclination of the outer planet, from $i = 90^\circ$ to $i = 60^\circ$ in steps of 0.1°. This introduces a mutual inclination between the two outer planets that allows a search for islands of stability that may resolve the close encounter dilemma.

The orbital parameters described in Section 2 have associated uncertainties that may also account for system stability. To investigate this, we conducted additional simulations that vary the argument of periastroid to locate islands of stability for the coplanar scenario. These results are presented in the context of refining the orbital parameters in Section 7.

4. The HD 5319 System

The two planets of the HD 5319 system have their closest approach where $\omega + f \sim 255^\circ$ (see Section 2). Our dynamical stability simulation for the assumption of coplanar orbits shows that the planetary system can only survive for ~330,000 years based on the system parameters from Section 2. The results of this simulation are shown in the top panel of Figure 2, where the dashed and dotted lines represent the orbital eccentricities of planets b and c respectively. Planet c remains in the system with an eccentricity of $e \sim 0.35$ after planet b is ejected. The results of the simulation that add a mutual inclination between planets b and c (see details in Section 3) are shown in the middle panel of Figure 2. The solid line indicates the changing mass of planet c as the inclination is gradually decreased from $i_c = 90^\circ$. Even with the increased mass of planet c, planet b remains the dominant mass during close encounters and thus remains in the system during the majority of inclination cases. We located three inclinations in the range $90^\circ > i_c > 60^\circ$ for which both planets survived the complete simulation duration of 10$^7$ years: 85°, 81°, and 80°.5.
of eccentricities for both planets where planet c has an inclination for planet d. The variation in orbital eccentricity for planets c and d are shown in the bottom panel of Figure 3, with the zoomed inset panel showing the angular momentum exchange of the two outer planets. These planets maintain stability with eccentricities staying below ~0.3. It should be noted that there are large uncertainties associated with the periastron argument for planet d. Thus there could be a more suitable value for \( \omega \) that would allow more stable configurations for the system.

6. MEAN-MOTION RESONANCES

A further consideration are MMRs as potential sources of dynamical stability. MMRs have been considered in detail by various authors (Varadi 1999; Petrovich et al. 2013; Antoniadou & Voyatzis 2014), including the effect of mutual inclinations (Barnes et al. 2015). Examples of stable MMRs are 3:2 (1.5), 4:3 (1.33), 5:4 (1.25), 5:3 (1.67), and 8:5 (1.6). From Table 1, the ratios of the orbital periods for the two outer planets are \( 1.38 \pm 0.01 \) and \( 1.598 \pm 0.001 \) for the HD 5319 and HD 7924 systems respectively.

For the HD 5319 system, the closest MMR is the 4:3 resonance, although it does not match to that resonance within the period uncertainties. It was shown by Barnes & Greenberg (2006b) that the long-term apsidal behavior of multi-planet orbital elements may be distinguished between libration and circulation, where the boundary between them is the secular separatrix. Stability simulations conducted by Giguere et al. (2015) using coplanar orbits found that several realizations maintained stability through a mean orbital period ratio close to the 4:3 resonance with a librating apsidal trajectory. For our stable configuration with \( i_c = 85^\circ \) (see Section 4) we computed the apsidal trajectory for the HD 5319 system using the eccentricity of the inner and outer planets (\( e_b \) and \( e_c \) respectively) and the difference in periastron arguments (\( \Delta \omega \)). These are represented graphically with polar coordinates.

The lower panel of Figure 2 shows the variation in orbital eccentricities for both planets where planet c has an inclination of \( i_c = 85^\circ \). The system is able to acquire a stable configuration whereby angular momentum is transferred between the two planets, oscillating the eccentricities over long timescales, as described by Kane & Raymond (2014). The zoomed inset in the lower panel of Figure 2 shows the eccentricity oscillations over a 40,000 year segment of the simulation.

5. THE HD 7924 SYSTEM

The orbits of the two outer planets (planet c and d) for the HD 7924 system have their closest approach where \( \omega + f \sim 147^\circ \) (see Section 2). The planets of this system are significantly less massive than those of the HD 5319 system and are several Hill radii apart at the closest approach. Even so, the dynamical stability of the system for the coplanar scenario is disrupted relatively early and planet d is lost after only 3700 years, as shown in the top panel of Figure 3. After this event, planet c oscillates in orbital eccentricity as it exchanges angular momentum with planet b. Note that, even though Fulton et al. (2015) found the system to be stable for \( 10^5 \) years, the stability duration is sensitive to the timestep used. As described in Section 3, we use a timestep of 0.25 days but there are a small fraction of timesteps in the range 0.05–0.5 days that produce stable outcomes that last slightly more than \( 10^5 \) years for the same initial conditions.

Similar to the procedure described in Section 4, we performed simulations that vary the inclination of the outer planet in the range \( 90^\circ > i_d > 60^\circ \). The results of these simulations are shown in the middle panel of Figure 3. Within the searched inclination range, our simulations revealed only one stable configuration, located at an inclination of \( i_d = 78^\circ \) for planet d. The variation in orbital eccentricity for planets c and d are shown in the bottom panel of Figure 3, with the zoomed inset panel showing the angular momentum exchange of the two outer planets. These planets maintain stability with eccentricities staying below ~0.3. It should be noted that there are large uncertainties associated with the periastron argument for planet d. Thus there could be a more suitable value for \( \omega \) that would allow more stable configurations for the system.

![Figure 1](image1.png)  
**Figure 1.** Top-down views of the HD 5319 (left) and HD 7924 (right) planetary systems. The orbits have been plotted using the system parameters shown in Table 1. The “scale” refers to the scale of the plot along a side, and “closest” refers to the closest approach of the two outer planet orbits assuming they are coplanar. The solid lines joining the host star to the orbits shows the periastron location for each orbit. The dashed line joining the host star to the orbits indicates the location of closest approach.
in the left panel of Figure 4. Clearly the apsidal trajectory evolution of this system is complex due to both secular and resonant dynamics. The system begins with librating apsidal modes (the points clustered near the origin) but moves to circulating apsidal modes since the polar trajectories encompass the origin. However, the system cannot be readily classified in terms of libration or circulation.

In the case of the HD 7924 system, the orbital period ratio for the outer planets is close to the 8:5 MMR. A coplanar stability simulation by Fulton et al. (2015) was only run for 10^5 years and did not encounter stability issues, nor explore resonances. As for the HD 5319 system, we calculated apsidal trajectories over the 10^7 year simulation for the stable configuration with \( i_p = 78^\circ \). The resulting polar coordinates using eccentricities of the outer planets \( e_c \) and \( e_d \) respectively are shown in the right panel of Figure 4. The apsidal modes of the system appear to be circulating although, as noted by Barnes & Greenberg (2006b), interpretation of systems with three or more planets is challenging since the secular interactions of all the planets result in a superposition of the various oscillation amplitudes. The effect of this is the homogeneous spreading of data points around the origin, as seen in the right panel of Figure 4.

An important point to note is that period ratios that diverge slightly from resonance, such as those described here, do not imply that the planets concerned do not lie in those resonances. There are in fact quite few planet pairs that have period ratios occurring exactly at resonance. It was shown by Goldreich & Schlichting (2014) that accounting for ongoing orbital dynamics explains the distribution of Kepler planets found to be near MMR. Additionally, it has been demonstrated by Goldreich & Schlichting (2014) that overstable librations, such as that seen for HD 5319, can result in eccentricity damping and a subsequent divergence from the expected value of MMR. The combination of these factors can lead to an orbital evolution that places the period ratio preferentially slightly above the expected MMR, as seen in the dynamical simulations performed by Giguere et al. (2015).
7. FURTHER OBSERVATIONS

A critical aspect for understanding the stability of a particular system is the accuracy of the orbital elements. Further RV observations of known exoplanet systems at calculated optimal times can provide a dramatic improvement to the planetary orbital parameters (Ford 2008; Kane et al. 2009). Such improvement is especially necessary for the eccentricity and argument of periastron, parameters that are often poorly constrained. For example, the argument of periastron uncertainties for HD 7924d (see Table 1) are $+210^\circ$ and $-97^\circ$.

The effect of varying the periastron argument of the outer planet on the proximity of the outermost planetary orbits are visualized as the solid line shown in Figure 5 for HD 5319 (top) and HD 7924 (bottom). As described in Section 2, the minimum separation of the orbits for the HD 7924 c and d planets is 0.007 au when using the measured orbital parameters. Moving the periastron argument in the negative direction increases the minimum separation of the orbits, whereas a positive shift decreases the minimum separation. The region where the periastron argument causes the orbits of the planets to cross has been shaded gray and the minimum separation fixed at zero. For mutual inclinations close to zero, such periastron arguments are usually untenable. The exceptions to this are cases where an MMR can prevent close encounters even with overlapping orbits, as described by Marzari et al. (2006). Since the variation of the argument of periastron can have such a significant effect on the orbital architecture of the system, further observations of these systems are highly encouraged to help resolve the close encounter issue of the outer planets.

To test for system stability as a function of periastron argument, we conducted additional simulations for each system using the methodology described in Section 3. These simulations assume orbital coplanarity and vary the periastron argument of the outer planet between $-100^\circ$ and $+100^\circ$ in steps of $1^\circ$ relative to the measured values from Table 1, shown as vertical dashed lines in each panel of Figure 5. Our...
These correspond to the stable configurations found in Sections 4 and 5 where \( i_p = 85^\circ \) for HD 5319 and \( i_p = 78^\circ \) for HD 7924. The apsidal modes for the HD 5319 planets move from libration to circulation due to the resonance dynamics. For the HD 7924 outer planets, the apsidal modes appear to be circulating but include oscillation modes due to secular interactions with the inner planet (planet b).

Simulations do not find any region of stability for the HD 5319 system in the range of periastron arguments explored for the coplanar case. An additional parameter to explore would be the variance of eccentricity, and possibly explain the stable region found by Giguere et al. (2015) in the simulation of the same system. For HD 7924, we find several isolated locations of stability including in the region where the orbits cross. As described above, such orbital architectures are possible when MMR is achieved ensuring that the planets do not experience close encounters over long timescales. The possibility of system stability at other periastron arguments emphasizes the need for further observations to refine the orbital parameters of the HD 7924 system.

Planetary multiplicity within a system can be used to constrain orbital inclination, such as for the HD 10180 system (Lovis et al. 2011; Kane & Gelino 2014). Such constraints are normally determined via stability simulations that assume coplanarity for the system. Furthermore, assuming orbital inclinations different from edge-on (as we have simulated in this work) can result in a modified dynamical interaction that may be inconsistent with the RV Keplerian solution. Laughlin & Chambers (2001) and Rivera & Lithauer (2001) investigated this effect for the GJ 876 system and found that significant divergence with the RV solution occurs for \( \sin i < 0.8 \). For our simulations, we explore the parameter space consisting of \( i > 0.866 \), with stable inclinations corresponding to \( \sin i = 0.996 \) and \( \sin i = 0.978 \) for HD 5219 and HD 7924 respectively. Thus it is not expected that the change in masses will cause significant divergence from the best-fit Keplerian model in our cases.

Ultimately, astrometric data will provide the information required to understand the true inclinations of the planetary orbits, both with respect to each other and the plane of the sky. The Gaia mission is an astrometry mission that was successfully launched by the European Space Agency in 2013. As well as determining a vast number of stellar parallaxes, the mission will provide astrometry for known exoplanetary systems in addition to discovering new systems (Perryman et al. 2014). The full capabilities of the Gaia mission are described in detail by de Bruijne (2012) and Bailer-Jones et al. (2013).

8. CONCLUSIONS

RV multi-planet systems provide excellent opportunities to study the orbital dynamics of Keplerian orbits. In some cases, such studies reveal complex problems with coplanar assumptions regarding the system, particularly if such an assumption causes the system to be unstable. A first-order analysis of the system is to determine the proximity of the planetary orbits in relation to the Hill radii of the planets. However, a thorough investigation via \( N \)-body numerical simulations is needed to fully resolve such complex cases.

Our analysis of the HD 5319 and HD 7924 systems show that they both suffer from a fundamental instability based upon the precise published orbital parameters, but long-term stable coplanar solutions may still be found within the published one-sigma uncertainties for both systems. Through an exhaustive suite of stability simulations that varied the mutual inclinations of the outer planets, we have located inclinations that satisfy system stability over a period of \( 10^7 \) years. Depending upon the system, there may be several such islands of stability in the parameter-space of inclination, keeping in mind that lowering the inclination also increases the mass of the planet in question. Our further analysis of the apsidal trajectory evolution for stable mutual inclinations of the systems shows that they can generally be described as having circulating apsidal modes due to orbits of the
outer planets that are near MMR. The complexity involved in achieving a full understanding of system stabilities is compounded by the uncertainty in the Keplerian orbital elements. Further RV and astrometric data for these and other similar systems will aid enormously in resolving the close encounters evident in the system architectures.

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Figure 5. The stability (dotted line) and minimum separation of the outer two planetary orbits (solid line) for the HD 5319 (top panel) and HD 7924 (bottom panel) systems. These are represented as a function of varying the argument of periastron of the outer planet from the measured position, indicated by the vertical dashed line. The gray shaded region in the bottom panel represents the periastron arguments for which the outer planet orbits cross.