Magnetizing a complex plasma without a magnetic field

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We propose and demonstrate a concept that mimics the magnetization of the heavy dust particles in a complex plasma while leaving the properties of the light species practically unaffected. It makes use of the frictional coupling between a complex plasma and the neutral gas, which allows to transfer angular momentum from a rotating gas column to a well-controlled rotation of the dust cloud. This induces a Coriolis force that acts exactly as the Lorentz force in a magnetic field. Experimental normal mode measurements for a small dust cluster with four particles show excellent agreement with theoretical predictions for a magnetized plasma.

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FIG. 1. (Color online) Estimates of $\omega_c/\omega_0$ in various strongly coupled plasmas: white dwarf stars $^7$ ($\rho_m \gtrsim 10^3$ kg m$^{-3}$, $B \lesssim 10^5$ T), neutrino stars $^{14}$ ($\rho_m \sim 10^7 \ldots 10^{12}$ kg m$^{-3}$, $B \lesssim 10^{12}$ T), ions in Penning traps $^{15,16}$ ($n \sim 10^2 \ldots 10^7$ cm$^{-3}$, fields of several Tesla), and complex plasmas $^8,10,11$. The method presented in this paper (rotating complex plasmas) covers a wide range of magnetizations at room temperature.

Strongly coupled Coulomb systems have very unusual properties including spontaneous spatial ordering and the formation of liquids or even crystals. Coulomb crystals and liquids, originally predicted by Wigner $^1$, were eventually observed on the surface of helium droplets $^2$, in ion traps $^3$, and in complex plasmas $^4,5$. They are also believed to occur in semiconductor quantum dots and quantum wells $^6$, as well as in white dwarf and neutron stars $^7$. Of particular current interest is their behavior in a magnetic field, where strongly modified oscillation spectra $^8,11$ or anomalous diffusion properties $^{12}$ have been predicted, and even applications to verify enhanced nuclear reaction rates have been demonstrated $^{13}$. Especially in neutron stars $^{14}$, giant magnetic fields are present that considerably modify the properties of the liquid and crystal states in the outer layers and alter the whole evolution of the star.

In a strongly coupled plasma (SCP), the Coulomb interaction energy of two particles, $Q^2/(4\pi\epsilon_0 a)$ [charge $Q$, typical inter-particle distance $a$], is much larger than their thermal energy, $k_BT$. In a magnetized SCP the particles are, in addition to the electrostatic interactions, subject to the Lorentz force. The degree of magnetization can be measured by comparing the relevant time scales associated with these two forces $^{10}$: While the Coulomb interactions lead to a characteristic vibration frequency $^8 \omega_0 = \sqrt{Q^2/(4\pi\epsilon_0 ma^3)}$, the cyclotron frequency $\omega_c = QB/m$ is the relevant parameter for the Lorentz force. For a given magnetic field $B$ and mass density of the plasma $\rho_m = mn$ (particle mass $m$, number density $n$), the ratio of the two becomes $\omega_c/\omega_0 = B\sqrt{3\epsilon_0/m}$. Estimates for various strongly coupled astrophysical $^7,14$ and laboratory plasmas $^{13,15}$ are presented in Fig. 1.

Complex (dusty) plasmas $^{17}$ have, in recent years, become a prototypical system to study strong correlation effects in unmagnetized Coulomb systems. They contain highly charged, micrometer-sized particles embedded in a partially ionized electron-ion plasma. Complex plasmas are found in numerous space environments including interstellar clouds, cometary tails, or planetary rings $^{18}$. In laboratory experiments, the large particle size and mass make it possible to follow individual particle trajectories with unprecedented spatial and temporal resolution $^{19,21}$ providing valuable insight into strong coupling phenomena. However, until now, it has not been possible to extend this analysis to magnetized Coulomb systems. The large particle mass (via large $\rho_m$) limits $\omega_c/\omega_0$ to values below 0.1…0.5, even if superconducting magnets and particles with sub-micron diameter are

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Here, \( F \) is the rotating frame reads
\[
\langle \sigma_{ij} \rangle = \frac{1}{2} \int d^3V \rho \langle \sigma_{ij}(r) \rangle \,
\]
where \( \sigma_{ij} \) is the matrix for rotations around the \( z \)-direction, \( \rho \) is the rotation frequency. Particle confinement is provided by a potential \( V(\rho, z) = \frac{m}{2} \left( \omega_\perp^2 \rho^2 + \omega_z^2 z^2 \right) \). The effective confinement potential is now given by
\[
\tilde{V}(\tilde{\rho}, \tilde{z}) = \frac{m}{2} \left( \omega_\perp^2 \tilde{\rho}^2 + \omega_z^2 \tilde{z}^2 \right).
\]

The first observation is that, in the rotating frame, the effective confinement frequency \( \tilde{\omega}_\perp = \sqrt{\omega_\perp^2 - \Omega^2} \) in the direction perpendicular to the rotation axis is being reduced as a consequence of the centrifugal force. Second, the rotation induces the Coriolis force
\[
\mathbf{F}_{\text{Cor}}(\tilde{r}) = m \tilde{r} \times (2\Omega \epsilon_z),
\]
which has the same form as the Lorentz force for a homogeneous magnetic field \( \mathbf{B}_{\text{eff}} = (2m\Omega/Q \epsilon_z) \). The doubled rotation frequency can be identified with the cyclotron frequency, \( \tilde{\omega}_z = 2\Omega \). The neutral gas flow velocity \( \mathbf{u}(r) \) no longer appears explicitly because the coordinate frame rotates at the same angular velocity as the gas. While the form of the inter-particle forces remains unaffected, \( F_{\text{int}}^{\text{int}} = -\nabla_i \phi(\rho_{ij}, \tilde{z}_{ij}) \), the components of the random force occur in a mixed form, \( \tilde{f}_i(t) = R^T(t) f_i(t) \). However, its statistical properties are the same as in the laboratory frame, i.e., a Gaussian white noise is retained, \( \langle \tilde{f}_i(t) \rangle = 0 \), and \( \langle f_i(t) \rangle \tilde{f}_j(t') \rangle = 2m \nu k_B \delta_{ij} \delta(t - t') \).

Let us now estimate the effective “magnetic fields” \( |\mathbf{B}_{\text{eff}}| \approx 2m\Omega/Q \) that can be reached. With rotation frequencies of \( \Omega \approx 10 \text{ Hz} \), a charge of \( |Q| \approx 10^4 e \), and a mass of \( m \approx 10^{-12} \text{ kg} \), one easily generates effective magnetic fields exceeding \( 10^4 \text{T} \), which is far beyond the capabilities of superconducting magnets. At very high rotation speeds, the centrifugal force weakens the horizontal confinement \( \tilde{\omega}_z \), which may give rise to a deformed plasma shape and a lower dust density. However, especially in two-dimensional systems \( \tilde{\omega}_z \), the shape of the dust monolayer will remain unaffected, and a lower density could even be beneficial as it decreases the dust monolayer will remain unaffected, and a lower density could even be beneficial as it decreases the dust monolayer will remain unaffected, and a lower density could even be beneficial as it decreases the dust monolayer will remain unaffected, and a lower density could even be beneficial as it decreases the dust monolayer will remain unaffected, and a lower density could even be beneficial as it decreases the dust monolayer will remain unaffected, and a lower density could even be beneficial as it decreases the dust monolayer will remain unaffected, and a lower density could even be beneficial as it decreases the dust monolayer will remain unaffected, and a lower density could even be beneficial as it decreases
parameter variation

\[
\frac{2\Omega}{\omega_\perp(\Omega)} = \frac{2(\Omega/\omega_\perp)}{(1 - \Omega^2/\omega_\perp^2)^{1/2}}.
\]

It is obvious that, already for a relatively slow rotation with \(\Omega = \omega_\perp/2\), the plasma is strongly “magnetized” \((2\Omega/\omega_\perp = 1.15)\), see the left panel of Fig. 2. The associated changes (compared to the non-rotating system) of the coupling and screening parameter are small. These parameters scale as \(\Gamma(\Omega)/\Gamma(0) = (1 - \Omega^2/\omega_\perp^2)^{1/3}\) and \(\kappa(\Omega)/\kappa(0) = (1 - \Omega^2/\omega_\perp^2)^{-1/3}\), respectively, and start to change substantially when the centrifugal force noticeably increases the inter-particle distance \((\Omega/\omega_\perp \gtrsim 0.7)\), see the left panel of Fig. 2. In this regime, the decrease of \(\omega_\perp\) is largely responsible for the dramatic increase of the magnetization parameter. We also show the dimensionless damping rate, which scales as \(\gamma(\Omega)/\gamma(0) = (1 - \Omega^2/\omega_\perp^2)^{-1/2}\). This means that, in experiments, the gas pressure should be sufficiently low allowing for a small neutral gas friction coefficient \(\nu\) before start of the rotation. The right panel of Fig. 2 shows \(\Gamma\) and \(\kappa\) as a function of the effective magnetization in a parameter regime that should be easily accessible in dusty plasma experiments, see below [32].

In the following, we present a proof-of-principle experiment to verify the efficiency of the proposed concept. A sketch of the experimental setup is shown in Fig. 3. The experiments were performed in a 13.56 MHz capacitively coupled radio-frequency discharge at a gas pressure of \(p = 0.4\) Pa (Argon). Spherical particles with a diameter of \(d = 21.8\) \(\mu\)m and a mass of \(m = 6.46 \cdot 10^{-12}\) kg are injected into the plasma, where they form two-dimensional clusters. The upper electrode can be set into rotation with frequencies up to 30 Hz, which causes a vertically sheared rotational motion of the neutral gas column [28].

We concentrate on the dynamics of a small ensemble of \(N = 4\) particles and analyze their normal modes. The normal modes of small 2D clusters have already been studied experimentally [33], but for magnetized dusty plasmas only theoretical predictions exist [34]. We first consider the center-of-mass (sloshing) mode. Since the effective confinement in the rotating frame is harmonic, the center-of-mass coordinate \(\bar{r}_{cm}(t)\) is independent of the interaction (Kohn theorem) and obeys the same equation of motion as a single particle [35]. In the absence of rotation, the two center-of-mass modes are degenerate with \(\omega_{cm} = \omega_\perp\). For \(\Omega > 0\), however, this degeneracy is lifted, and the frequencies read \(\omega_{cm}^\pm = \sqrt{(\omega_\perp/2)^2 + \omega_\perp^2} \pm \omega_\perp/2 = \omega_\perp \pm \Omega\). Here, \(\omega_\perp = \sqrt{\omega_\perp^2 - \Omega^2}\) is the effective trap frequency in the rotating frame. The experimental results obtained from the spectrum of \(\bar{r}_{cm}(t)\) are depicted in Fig. 4 and show remarkable agreement with the theoretical prediction.

It is crucial for our scheme to further verify the accuracy of the remaining modes, which are sensitive to both the magnetic field and inter-particle correlations. By lin-
earizing the equations of motion in the rotating frame, we determine the eigenfrequencies $\omega$ and eigenvectors $\mathbf{v}_i$ from:

$$\left(\omega^2 \delta_{ij} \delta^{\alpha\beta} - \hat{H}^{\alpha\beta}_{ij} / m - 2i \omega \Omega \delta_{ij} \epsilon^{\alpha\beta\gamma} \right) v^\beta_i = 0,$$

where $\alpha, \beta \in \{x, y\}$ and $i, j \in \{1, \ldots, N\}$. Further, $\hat{H}^{\alpha\beta}_{ij}$ denotes the Hessian of the total potential energy in the rotating frame, $\delta_{ij}$ ($\delta^{\alpha\beta}$) the Kronecker delta, and $\epsilon^{\alpha\beta\gamma}$ the Levi-Civita symbol. The cluster configuration for $N = 4$ is a square with particles located a distance $R$ from the trap center, which is calculated from Eq. (6) in Ref. [36]. Even though a finite dust-neutral friction parameter is essential to put the plasma into rotation, it is sufficiently low to be negligible for the calculation of the eigenfrequencies ($\nu/\omega_\perp \approx 1/50$). To determine the mode frequencies experimentally, we calculated the projection $P(t) = \sum_{i=1}^N \tilde{r}_i(t) \cdot \mathbf{v}_i$ of the particle trajectories $\tilde{r}_i(t)$ on the eigenvectors $\mathbf{v}_i$ of the unmagnetized system, see Fig. 4. The breathing mode corresponds to a radial expansion and contraction of the cluster. The spectrum of $P(t)$ shows a peak at the associated mode frequency, which can be tracked as the rotation frequency is varied.

The measurements are compared with the theoretical results in Fig. 4. As for the case of the sloshing modes, we observe excellent agreement with the normal modes of a magnetized plasma. Effective magnetizations $2\Omega/\bar{\omega}_\perp \gtrsim 3$ allow us to clearly verify the predicted splitting of the normal modes into the upper and lower bracket [34].

To summarize, we have presented a simple approach to “magnetize” a complex plasma. The idea is based on the correspondence between charged particles in a magnetic field and particles in a rotating gas column. The possibility to put the plasma into rotation takes advantage of the dissipative nature of complex plasmas in which the neutral gas acts as a highly effective transmission agent of angular momentum. We demonstrated that, with very modest rotation frequencies applied to only one electrode, strongly correlated particles in a rotating dusty plasma reproduce, to high accuracy, the normal modes of a magnetized system. Evidently, our concept can also be realized by other means and opens new unique possibilities for highly accurate studies of strongly correlated and strongly magnetized plasmas. The advantage compared to cryogenic ions is the broad range of accessible plasma parameters that can be varied independently (coupling strength, screening, dissipation) and the availability of single-particle resolution at room temperature conditions. It should be possible to create SCP states with extreme magnetizations potentially even comparable to those exotic ones in the outer layers of neutron stars. While our technique does not require any (superconducting) magnet at all, use of the latter in combination with plasma rotation allows to create novel types of plasmas. In such plasmas, there would be effectively two “magnetic fields” that can be controlled independently—one of which (the rotation) affects only the heavy particles whereas the second (“real”) field influences the electrons and ions, allowing for an effective control of the interaction between the dust particles. The combination of a rotating flow with a magnetic field also provides a promising avenue to access magnetorotational instabilities on the particle scale, which are relevant for the understanding of accretion disks [37].

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![Diagram](image-url)
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