FAILURE OF THE LADDER APPROXIMATION TO QCD

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The proof of the failure of the ladder approximation to QCD is given in manifestly gauge-invariant way. This proof is valid for the full gluon propagator and for all types of quarks. The summation of the ladder diagrams within the Schwinger-Dyson integral equation for the quark-gluon vertex, on account of the corresponding Slavnov-Taylor identity, provides an additional constraint on the quark Schwinger-Dyson equation itself in the ladder approximation. It requires that there is neither running nor current quark masses in the ladder approximation. Thus, all the results based on the nontrivial (analytical or numerical) solutions to the quark Schwinger-Dyson equation in the ladder approximation should be reconsidered, and its use in the whole energy/momentum range should be abandoned.

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I. INTRODUCTION

There are only three different types of interactions in Nature: gravitational, electroweak and strong. However, all these interactions are described by the gauge theories. They differ from each other by the strength of the coupling constant, the gauge group structure and the fundamental constituents involved. In the two first theories the coupling constant is very weak and rather weak, respectively. The gauge theory of strong interactions – Quantum Chromodynamics (QCD) – can be considered in both the strong and the weak coupling regimes (asymptotic freedom) \[^1\]. It is well known that the full dynamical information on any gauge theory such as QCD and Quantum Electrodynamics (QED) is contained in the corresponding quantum equations of motion, the so-called Schwinger-Dyson (SD) equations for propagators (lower Green’s functions) and vertices (higher Green’s functions) \[^1\]. The Bethe-Salpeter (BS) type integral equations for higher Green’s functions and bound-state amplitudes \[^2\] should also be included into this system. The Green’s functions, bound-state amplitudes and scattering kernels are the most important objects in any gauge theory. This system should be complemented by the corresponding Word-Takahashi (WT) identity in QED and Slavnov-Taylor (ST) identities in QCD \[^1\], which, in general, relate lower and higher Green’s functions to each other. These identities are consequences of the exact gauge invariance and therefore “are exact constraints on any solution to QED/QCD” \[^3\]. To solve this system means to solve QED/QCD itself and vice versa.

It is important to understand however, that the above mentioned system is an infinite chain of strongly coupled highly nonlinear integral equations, so there is no hope for an exact solution(s). Let us also remind that this system is, in fact, an infinite chain of relations between different propagators, vertices, kernels. Thus, truncations/approximations are necessary for sure in order to formulate a closed system of equations in this or that sector of the theory. QED being a gauge theory with simple gauge group $U(1)$, contains only two different sectors: electron/positron-photon and BS in the sense that its system of the dynamical equation of motion contains the electron/positron and photon SD equations and the corresponding integral equation for the electron/positron-photon vertex only. At the same time, QCD being a gauge theory with rather complicated gauge group $SU(3)$ color, contains many sectors of different nature, such as quark, ghost, Yang-Mills (YM), Nambu-Goldstone (NG), BS, etc. Making use of some truncation/approximation scheme in one sector, it is necessary to be sure that nothing is going wrong in other ones, since the SD system of equations may remain coupled even after truncation/approximation made. As noted above, the main tool to maintain the self-consistent treatment of different sectors in QED/QCD is the use of the above-mentioned identities.

In theories with weak coupling constant the general method of their investigation is the perturbation theory (PT). It allows one to find solutions term by term in powers of the coupling constant. In order to sum up the PT series just the above-mentioned system of dynamical equations of motion is to be used. Within these equations such kind of the summation is known as the ladder approximation (LA) truncation scheme to QED and QCD (for its general description see subsection below). Being thus a generalization of PT, the main problem of the LA is, of course, its self-consistency. There already exists one serious problem with the LA to QCD, namely this truncation scheme is

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strongly gauge dependent (and why this is not acceptable in QCD is explained in Ref. [3]). In the LA one should omit ghost and non-Abelian corrections to the vertex and the corresponding identity. They may appear in the full gluon propagator (see below). By omitting ghosts, however, one cannot already cancel the unphysical (longitudinal) degrees of freedom of the gauge bosons. So, all results based on the LA will be, in principle, plagued by the unphysical singularities (the unitarity of the $S$-matrix is violated) in this case. This is the reason why the LA is usually used in the Landau gauge only, in which the longitudinal component of the full gluon propagator vanishes. On the other hand, this means that any solution in the Landau gauge obtained within the LA is a gauge artefact, and therefore cannot be used for the calculation of any physical observable, which is, by definition, manifestly gauge-invariant.

In Ref. [4], we have investigated the validity of the LA to QCD, keeping the ghost degrees of freedom in the corresponding ST identity for the quark-gluon vertex. It has been exactly proven that even in this case the quark propagator is almost trivial one, i.e., there is no "dressed" quark propagator in the LA to QCD. This proof was given in manifestly gauge-invariant way and for any types of quarks. We did not also use some specific solution for the full gluon propagator, i.e., it remained completely arbitrary. In this way, we have formulated an exact criterion to prove (in QED) or disprove (in QCD) the LA in gauge theories. Nevertheless, there are a lot of recent papers [2, 3, 4] (and references therein) in which the LA is being continuously used for the calculation of physical observables in QCD.

The main purpose of this Letter is to explicitly show that the LA to QCD is not only almost trivial one (which was proven in our previous publication [1]), but it does not take into account the response of the true QCD vacuum at all (proven here), which is only one modifies the quark self-energy (to make it "dressed"). The second point, which was also missed in the previous publication [3], is to emphasize a possible connection between the validity of the LA and the gauge group structure of the corresponding theories (QED and QCD). Contrary to the previous publication [3], the proof of the general failure of the LA to QCD is given in such details which can be understood and checked in all its steps by scientists who are not experts in this field. Here we precisely describe how to formulate the above-mentioned criterion (constraint) in the most transparent way. We hope that all this will make it possible to prevent from the future exploitation of the LA in QCD due to its internal inconsistency proven in this Letter.

A. General description of the LA

Let us now describe what the LA is about in QED and QCD. In general, it consists of approximating the full quark/electron-gluon/photon vertices by their point-like ("bare") counterparts in the corresponding kernels of the above-mentioned SD integral equations for the quark/electron propagator and the quark/electron-gluon/photon vertex (see Eq. (2.1) and Eq. (4.1) below, respectively). Approximating in addition the full gluon/photon propagator by its free expression in the BS-type integral equation for the vertex, one gets the so-called quenched LA (QLA) scheme, which became known as the "rainbow" approximation for the quark/electron SD equation. However, very soon it has been realized that QLA was too crude for QCD and should be improved in order to incorporate the QCD renormalization group results for the running coupling constant (asymptotic freedom) into the system of the SD equations. The so-called improved LA (ILA) was proposed [2, 3]. It makes it possible to take into account the self-interacting gluon modes, including the 3-gluon coupling, at least (non-Abelian character of QCD), at the level of the full gluon propagator only. At the same time, the quark-gluon vertices in the quark and in the kernel of the vertex SD equations remain intact, i.e., they remain point-like ones (see below). Thus, the exact definition of the LA excludes the non-Abelian corrections to the quark-gluon vertices, while retaining them in the full gluon propagator. The LA scheme with the full gluon propagator is known as the generalized, or simply LA. For more detail description of the LA in the quark and BS sectors see sections 2 and 3 below, respectively.

II. QUARK SD EQUATION

In what follows we will investigate the validity of the LA by comparing QED and QCD, and using precisely the corresponding equations of motion. We begin with the SD equation for the quark propagator. In the LA and in the momentum space it is

$$S^{-1}(p) = S_0^{-1}(p) + i\Sigma(p) = S_0^{-1}(p) - g_F^2 \int \frac{d^4l}{(2\pi)^4} \gamma_\alpha S(l)\gamma_\beta D_{\alpha\beta}(q),$$

(2.1)

where $i\Sigma(p)$ is the quark self-energy and $q = p - l$. We assign the factor $-ig\gamma$ to the point-like vertices with the corresponding Dirac indices. Here $g_F^2 = g^2C_F$ and $C_F$ is the eigenvalue of the quadratic Casimir operator in the
fundamental representation (for SU(N), in general, \(C_F = (N^2 - 1)/2N = 4/3, \ N = 3 \) for QCD). The free quark propagator is

\[
S_0^{-1}(p) = -i(\hat{p} - m_0)
\]  

(2.2)

with \(m_0\) being the current ("bare") mass of a single quark. Here and everywhere below \(D_{\alpha\beta}(q)\) is the full gluon propagator in an arbitrary covariant gauge

\[
D_{\alpha\beta}(q) = -i \left\{ \left[ g_{\alpha\beta} - \frac{q_{\alpha\beta} q_{\beta\alpha}}{q^2} \right] d(q^2; \xi) + \xi \frac{q_{\alpha\beta} q_{\beta\alpha}}{q^2} \right\} \frac{1}{q^2},
\]  

(2.3)

and \(\xi\) is the gauge fixing parameter (for example, \(\xi = 0 - \) Landau gauge). The free gluon propagator \(D_{\alpha\beta}^0(q)\) is given by Eq. (2.3) with \(d(q^2; \xi) = 1\), by definition. The formal iteration solution in powers of the coupling constant squared \(g^2\) of the quark SD equation (2.1) with \(D_{\alpha\beta}(q) = D_{\alpha\beta}^0(q)\) is known as the "rainbow" approximation, while with the full gluon propagator as the generalized "rainbow" approximation. Just the latter one includes all the non-Abelian gluon modes (the 3- and 4-gluon couplings as well as the ghost-gluon vertex) at the level of the full gluon propagator only (for the structure of the SD equation for the full gluon propagator see Ref. \(\mathbb{R}\) and our paper \(\mathbb{S}\)). As emphasized above, the non-Abelian corrections to the quark-gluon vertices should be excluded, by definition, otherwise the approximation becomes not "rainbow" at all. Let us note that if in the full gluon propagator the special dependence on the gluon momentum is chosen \(\mathbb{S}\), then the corresponding iteration solution is known as the improved "rainbow" approximation, mentioned above.

Differentiating Eq. (2.1) with respect to \(p_\mu\), one obtains

\[
\partial_\mu S^{-1}(p) = -i \gamma_\mu + \partial_\mu \Sigma(p) = -i \gamma_\mu - \partial_\mu g_F^2 \int \frac{d^4l}{(2\pi)^4} \gamma_\alpha S(l) \gamma_\beta D_{\alpha\beta}(q)
\]

\[
= -i \gamma_\mu - g_F^2 \int \frac{d^4l}{(2\pi)^4} \gamma_\alpha \partial_\mu S(l) \gamma_\beta D_{\alpha\beta}(q),
\]  

(2.4)

up to an unimportant total derivative (as usual), which is assumed to vanish at the ends of integration. This is the differential form of the quark SD equation (2.1) relevant for further discussion. Here and below \([\partial_\mu S(l)]\) denotes the differentiation with respect to \(l_\mu\). Let us remind that in QED the electron SD equation (2.1) and hence its differential form (2.4) is the same, apart from unimportant replacement \(g_F^2 \rightarrow g^2\) for the case of QED.

Concluding, let us make a few remarks. In order not to complicate notations, here and below we use the system of the SD equations for the unrenormalized Green’s functions. However, all the integrals are divergent (this is the general feature of quantum field theory, in particular QED and QCD). In order to assign a mathematical meaning to them and formal (mainly algebraic) operations with them, we will implicitly assume that they are regularized at the upper limit, and the ultra-violet cut-off will go to infinity at the final stage. Evidently, none of the exact operations here and below will be affected by such assumed regularization (for more remarks on the issue of renormalization see below in section 7).

### III. BS SECTOR

It is well known that, in general, the SD equation for the quark-gluon vertex contains four unknown scattering kernels \(\mathbb{H}\), from which only one is the BS scattering kernel, known from QED. It is the sum of an infinite number of the corresponding skeleton diagrams. The three others are rather complicated objects, containing among them the ghost-quark scattering kernel. The two others scattering kernels are to be combined with the 3- and 4-gluon vertices. However, in order to proceed to the LA for the quark-gluon vertex it is necessary to omit all the above-mentioned three scattering kernels and retain only one, namely the BS scattering kernel. That is why we call the below-displayed integral equation as the BS type or simply BS integral equation. In turn, only the first skeleton diagram should be retained from this kernel. Moreover, the quark-gluon vertices in this term should be replaced by their point-like counterparts. Just in this way one gets the BS type integral equation in the LA for the quark-gluon vertex \(\Gamma^\alpha_\mu(p, k)\), which analytically can be written down as follows:

\[
\Gamma^\alpha_\mu(p, k) = -i \gamma_\mu T^a \gamma_\alpha T^b S(l) \Gamma^a_\mu(l, k) S(l - k) \gamma_\beta T^b D_{\alpha\beta}(q),
\]  

(3.1)
where \( k = p - p' \) is the momentum transfer and again \( q = p - l \). In QED the same integral equation takes place, apart from the \( SU(3) \) color group generators, \( T^a, T^b \). Just this group structure makes the principal difference between QED and QCD (see below). The coupling constant \( g \) is already cancelled from both sides of this equation. The formal iteration solution of this equation with \( D_{\alpha\beta}(q) = D_{\alpha\beta}^0(q) \) is known as the QLA, mentioned above, while with the full gluon propagator as the generalized LA. Let us emphasize once more that just the latter one includes all the non-Abelian gluon modes (the 3- and 4-gluon couplings as well as the ghost-gluon vertex) at the level of the full gluon propagator only. As underlined above, the non-Abelian corrections to the quark-gluon vertices should be excluded, by definition, otherwise the approximation becomes not "ladder" at all. If again in the full gluon propagator the special dependence on the gluon momentum is chosen \( S(l, k) \), then the corresponding iteration solution is known as the ILA, also mentioned above.

For our purposes, it is sufficient to exactly decompose the vertex and the quark propagator as follows:

\[
\Gamma^\alpha_{\mu}(p, k) = \Gamma^\alpha_{\mu}(p, k) + \Gamma^\alpha_{\mu}(p, 0) - \Gamma^\alpha_{\mu}(p, 0) = \Gamma^\alpha_{\mu}(p, 0) + O^\alpha_{\mu}(p, k),
\]

\[
S(l - k) = S(l - k) + S(l) - S(l) = S(l) + O(l, k),
\]

where, obviously, the terms \( O^\alpha_{\mu}(p, k) \) and \( O(l, k) \) depend on the momentum transfer \( k \) linearly and higher, i.e., they are terms, at least, of the order \( (k^2) \). Here a few short remarks are in order. Due to the correspondence between the point-like quark-gluon vertex and the proper one, when all its momenta goes to zero, the analyticity of the vertex at zero momentum transfer in QCD is implicitly assumed. Non-analytical dependence (if it makes sense at all) is completely different story, and is beyond the scope of this paper. At the same time, how to remove unphysical (kinematical) singularities from the vertex is well-known procedure \[10\].

Substituting now these exact decompositions into the previous BS equation, one obtains

\[
\Gamma^\alpha_{\mu}(p, 0) + O^\alpha_{\mu}(p, k) = -i\gamma_{\mu} T^a - g^2 \int \frac{d^4l}{(2\pi)^4} \gamma_{\alpha} T^b S(l) [\Gamma^\alpha_{\mu}(l, 0) S(l) + L^\alpha_{\mu}(l, k) \gamma_{\beta} T^b D_{\alpha\beta}(q)],
\]

where

\[
L^\alpha_{\mu}(l, k) = \Gamma^\alpha_{\mu}(l, 0) O(l, k) + O^\alpha_{\mu}(l, k) S(l) + O^\alpha_{\mu}(l, 0) O(l, k).
\]

The composition \( L^\alpha_{\mu}(l, k) \) depends on \( k \) linearly, at least. Equating now the terms at zero order in powers of the momentum transfer \( k \), the previous BS equation becomes equivalent to the system of two equations, namely

\[
\Gamma^\alpha_{\mu}(p, 0) = -i\gamma_{\mu} T^a - g^2 \int \frac{d^4l}{(2\pi)^4} \gamma_{\alpha} T^b S(l) [\Gamma^\alpha_{\mu}(l, 0) S(l) \gamma_{\beta} T^b D_{\alpha\beta}(q)],
\]

and

\[
O^\alpha_{\mu}(p, k) = -g^2 \int \frac{d^4l}{(2\pi)^4} \gamma_{\alpha} T^b S(l) L^\alpha_{\mu}(l, k) \gamma_{\beta} T^b D_{\alpha\beta}(q).
\]

The BS equation (3.5) will be the main subject of our consideration. The BS equation (3.6) is, in fact, an infinite chain of equations, obtained by equating the terms at every order in powers of the momentum transfer \( k \) (starting at the order \( k \)). Each of these equations, however, should be solved on account of the solution to the zero order equation (3.5), which should be agreed with the solution of the WT identity (see below).

**IV. WT IDENTITY**

The ST identity in QCD contains the above-mentioned ghost-quark scattering kernel \[11\]. In order to get the SD equation in the LA for the quark-gluon vertex it should be omitted (see discussion above). So, omitting it here for the sake of self-consistency this identity becomes the WT identity of QED and has the form

\[
k_{\mu} \Gamma^\alpha_{\mu}(p, k) = T^a S^{-1}(p) - S^{-1}(p - k) T^a.
\]
It provides an exact solution for the zero momentum transfer $k = p - p' = 0$ as $(\Gamma_\mu^a(p, p) \equiv \Gamma_\mu^a(p, 0))$

$$\Gamma_\mu^a(p, 0) = T^a \partial_\mu S^{-1}(p).$$  \hfill (4.2)

In what follows we will use the relation

$$\partial_\mu S^{-1}(p) = -S^{-1}(p) [\partial_\mu S(p)] S^{-1}(p),$$  \hfill (4.3)

which stems from the obvious identity $S^{-1}(p) S(p) = S(p) S^{-1}(p) = 1.$

\section{Triviality of the Quark Propagator in the LA}

On account of the WT identity (4.2) and the relation (4.3), the BS equation (3.5) becomes

$$\partial_\mu S^{-1}(p) T^a = -i \gamma_\mu T^a + g^2 T^b T^a T^b \int \frac{d^4l}{(2\pi)^4} \gamma_\alpha [\partial_\mu S(l)] \gamma_\beta D_{\alpha\beta}(q).$$  \hfill (5.1)

Let us now use the commutation relation between color group generators $[T^a, T^b] = i f_{abc} T^c$, where $f_{abc}$ are the antisymmetric $SU(3)$ structure constants with non-zero values, given, for example in Ref. [11]. After doing some group algebra, one obtains

$$T^b T^a T^b = [C_F - \frac{1}{2} C_A] T^a,$$  \hfill (5.2)

where $C_F$ is the above mentioned eigenvalue of the quadratic Casimir operator in the fundamental representation while $C_A$ is the same but in the adjoint representation (for $SU(N)$, in general, $C_A = N = 3$ for QCD). So, from this relation and because of the notation $g^2 C_F = \tilde{g}_F^2$, one arrives at

$$\partial_\mu S^{-1}(p) = -i \gamma_\mu + [g_F^2 - \frac{1}{2} g^2 C_A] \int \frac{d^4l}{(2\pi)^4} \gamma_\alpha [\partial_\mu S(l)] \gamma_\beta D_{\alpha\beta}(q),$$  \hfill (5.3)

where we have already cancelled the color group generator $T^a$ from both sides of this equation (In QED the factor $[g_F^2 - (1/2)g^2 C_A]$ is to be simply replaced by $g^2$). Comparing Eq. (5.3) with that of Eq. (2.4) (the second line), one immediately concludes that

$$-\frac{1}{2} g^2 C_A \int \frac{d^4l}{(2\pi)^4} \gamma_\alpha [\partial_\mu S(l)] \gamma_\beta D_{\alpha\beta}(q) = 0.$$  \hfill (5.4)

However, comparing the first and second lines in Eq. (2.4), and taking into account the last relation, one obtains that it can be reduced to

$$\partial_\mu i \Sigma(p) = 0.$$  \hfill (5.5)

This constraint has only a trivial solution, namely

$$\Sigma(p) = m_c,$$  \hfill (5.6)

where $m_c$ is the constant of the dimensions of mass (constant of integration). From the quark SD equation (2.1), taking into account Eq. (2.2), and on account of the trivial solution to the quark self-energy (5.6) obtained above, it follows that

$$S(p) = \frac{i}{p - (m_0 + m_c)}.$$  \hfill (5.7)

Thus, the solution of the constraint equation (5.6) in the LA to covariant gauge QCD requires the quark propagator to be a free one, apart from the redefinition of the quark mass. In other words, there should be neither analytical nor numerical solution to the quark SD equation in the LA. So, there is no "running/dressed" quark mass in the LA to QCD. However, the real situation is even more catastrophic.
VI. INTERNAL INCONSISTENCY OF THE LA TO QCD

Substituting the obtained solution for the quark propagator (5.7) back to the quark SD equation (2.1), and taking the trace from its both sides and omitting some rather simple algebra, one obtains

\[ m_c = (m_0 + m_c) f(p^2) \]  

(6.1)

with

\[ f(p^2) = ig_F^2 \int \frac{d^4l}{(2\pi)^4} \frac{[3d(q^2;\xi) + \xi]}{(l^2 - (m_0 + m_c)^2)q^2}, \]

(6.2)

where let us remind \( q = p - l \) and the dependence on \( \tilde{m} = m_0 + m_c \) and \( \xi \) is not shown, for simplicity. Since we are seeking the solution of the relation (6.1) explicitly, the regularization of the divergent integral (6.2) with the help of the assumed ultra-violet cut-off is not appropriate. Obviously, the solution in this case may depend on it which is not acceptable. In this case, the regularization with the help of the corresponding subtraction is much more appropriate. So, let us define as usual \( f_R(p^2) = f(p^2) - f(0) \), and the relation (6.1) should be then replaced as

\[ m_c = (m_0 + m_c) f_R(p^2). \]  

(6.3)

It does not depend on the ultra-violet cut-off and the finite function enters it. Thus, its only solution is

\[ m_c = m_0 = 0. \]  

(6.4)

From Eq. (5.7) then it follows that the quark propagator in the LA to QCD finally becomes

\[ S(p) = i/\hat{p}. \]  

(6.5)

This solution is not acceptable, since even the current masses of light quarks are not, in general, zero. So, the LA to QCD cannot be used, it is simply wrong. Let us underline that we do not specify the full gluon form factor \( d(q^2;\xi) \) and hence the full gluon propagator itself.

Due to the expression (5.7), the vertex at zero momentum transfer is always trivial one anyway, i.e., it is

\[ \Gamma_\mu(p, 0) = -i\gamma_\mu, \]  

(6.6)

while the BS integral equation (3.6) for the non-zero momentum transfer is not trivial.

VII. CONCLUSIONS AND DISCUSSION

1. The proof of the general failure of the LA to QCD has been given in manifestly gauge invariant way (without specifying the gauge fixing parameter in the full gluon propagator), and for any gluon propagator (the full, QLA, ILA or something else, it does not matter), and for all types of quarks (light or heavy).

2. The failure of the generalized LA itself, stems from the color charge interactions in QCD (nontrivial gauge group structure).

3. In theories without colors, for example in QED (simplistic gauge group structure), there is no constraint. Formally, in this case we can put \( C_A = N = 0 \) in Eq. (5.4), while omitting the dependence on \( N \) in the quark SD equation, making it thus the electron SD equation. So, the constraint equation (5.4) will identically vanish. In its turn this means that the electron SD equation obtained from the BS equation, on account of the WT identity as described in this Letter, completely coincides with the electron SD equation itself (more precisely with its differential form). This justifies the use of the LA in QED.

4. In QED an expansion in powers of the external momentum makes not much sense, since there are no stable bound states (the positronium is unstable).

5. In QCD the same expansion is relevant because of the existence of the NG sector. Many important physical quantities such as scattering length, pion charge radius, etc. are defined as coefficients of the pion form factor expansion.
in powers of the external momentum (the chiral perturbation theory \[12\] and its counterpart at the fundamental quark level \[13\]). In QCD the zero external momentum transfer has a physical meaning, as relating directly to the various physical observables.

6. The self-consistency of any other truncation schemes, for example such as planar, \(1/N_c\) limit, etc. ought to be investigated in the same or other way. As emphasized in our paper, this is important in theories with complicated gauge group such as QCD.

We have investigated the self-consistency of the LA to QCD and QED. The wide-spread opinion that different sectors in QCD in this approximation can be decoupled from each other is not justified. They remain connected even in the LA (nontrivial gauge group structure). This clearly shows the general failure of the LA to QCD. All the nontrivial (analytical or numerical) results, obtained by the solution of the quark SD equation only in the LA, should be abandoned. The use of the LA to QCD in the whole energy/momentum range is forbidden, while to use it in the high energy/momentum region only (i.e., as a part of some another approximation) is possible. Within the LA the non-Abelian degrees of freedom (for example, the 3- and 4-gluon couplings) can be only taken into account in the full gluon propagator, and such kind of corrections to the quark-gluon vertices are forbidden, by definition. So, there is no room for improvement in order to formulate more sensible LA, i.e., it is too rigid truncation scheme. In other words, it is either LA or it is completely different approximation if one includes some other corrections to the vertices, which investigation is completely beyond the scope of this Letter, indeed.

Apparently, the smallness of the coupling constant in the gauge theories is not sufficient to use PT (and the LA as its generalization). This could be only first necessary condition. The second sufficient condition is the structure of the corresponding gauge group. It should be simple enough to validate the use of PT. Thus, the use of PT in the theory of gravitation becomes doubtful, since its gauge group is not so simple. One can change the regime of the consideration in QCD (weak or strong couplings), but impossible to change its gauge group structure. So, a possible correspondence between gauge group structure and the complexity of the corresponding vacuum structure should be also mentioned. Simplistic gauge group structure corresponds to the almost simple vacuum structure like in QED. Complicated gauge group structure corresponds to the highly nontrivial structure of the true QCD vacuum. In this connection let us remind that just its response, which is only one which modifies the quark self-energy. In the LA to QCD it cannot modify it, that is why it is simply wrong. Thus, due to its internal inconsistency, we propose to completely abandon it from the use in QCD.

Renormalization, of course, cannot change our main result, namely the proof of the internal inconsistency of the LA. That is why we started, for simplicity, from the unrenormalized Green’s functions. The issue of renormalization has been discussed in all details (i.e., explicitly and carefully) in our previous publication \[4\]. It causes only technical complications, which make no any sense to repeat here again. Evidently, the internal inconsistency of the LA does not depend on the issue of multiplicative renormalizability of QCD.

Ghost degrees of freedom cannot change this result either. In principle, one may keep them in the quark-gluon ST identity, making it "exact" in the LA to the above-mentioned ghost-quark scattering kernel, while necessarily omitting them in the vertex SD equation. This technical complication has been also investigated in our previous publication \[4\]. Since we do not specify the full gluon propagator, the quark propagator in the LA will remain an almost trivial one in any noncovariant gauge as well.

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