Perturbative QCD analysis of $B \to \phi K$ decays and power counting

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Abstract

We investigate exclusive nonleptonic $B$ meson decays $B \to \phi K$ in perturba-
tive QCD formalism. It is shown that the end-point (logarithmic and linear) singu-
larities in decay amplitudes do not exist, after $k_T$ and threshold resum-
mations are included. Power counting for emission and annihilation topologies of di-
agrams, including both factorizable and nonfactorizable ones, is discussed with Sudak-
off effects taken into account. Our predictions for the branching ratios $B(B \to \phi K) \sim 10 \times 10^{-6}$ are larger than those ($\sim 4 \times 10^{-6}$) from the factorization approach because of dynamical enhancement of penguin contributions. Whether this enhancement is essential for penguin-dominated modes can be justified by experimental data.

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I. INTRODUCTION

Perturbative QCD (PQCD) factorization theorem for the semileptonic decay $B \to \pi l\bar{\nu}$ has been proved in [1], which states that the soft divergences in the $B \to \pi$ form factor can be factorized into a light-cone $B$ meson distribution amplitude and the collinear divergences can be factorized into a pion distribution amplitude order by order. The remaining finite contribution is assigned into a hard amplitude, which is calculable in perturbation theory. A meson distribution amplitude, though not calculable, is universal, since it absorbs long-distance dynamics, which is insensitive to specific decays of the $b$ quark into light quarks with large energy release. The universality of nonperturbative distribution amplitudes is one of the important ingredients of PQCD factorization theorem. Because of this universality, one extracts distribution amplitudes from experimental data, and then employ them to make model-independent predictions for other processes. In this paper we shall assume that PQCD factorization theorem holds for two-body nonleptonic $B$ meson decays, to which there is no difficulty to generalize the proof in [1]. The one-loop proof for the PQCD factorization of two-body decays has been given in [2].

The PQCD formalism for the charmed decays $B \to D^{(*)}\pi(\rho)$ [2,3] is restricted to twist-2 (leading-twist) distribution amplitudes. For charmless decays such as $B \to K\pi, \pi\pi$ and $KK$ [4–8], contributions from two-parton twist-3 (next-to-leading-twist) distribution amplitudes are introduced via the penguin operators $O_{5-8}$ in the effective Hamiltonian for weak decays. It has been argued [9] that two-parton twist-3 contributions are in fact not suppressed by a power of $1/M_B$, $M_B$ being the $B$ meson mass. A complete leading-power PQCD analysis of the heavy-to-light $B \to \pi, \rho$ form factors, including both twist-2 and twist-3 contributions, has been performed in [10]. There exist many other higher-twist sources in $B$ meson decays, whose contributions are indeed down by a power of $1/M_B$. These sources include the $B$ meson and $b$ quark mass difference $\bar{\Lambda} = M_B - m_b$, the light quark masses $m_u, m_d$ and $m_s$, and the light pseudo-scalar meson masses $M_\pi$ and $M_K$. Those from three-parton distribution amplitudes are further suppressed by the coupling constant $\alpha_s$. All these
sub-leading contributions will be neglected in the current formalism.

In this work we shall perform a PQCD analysis of the $B \to \phi K$ decays up to corrections of $O(\bar{\Lambda}/M_B)$. These modes involve different topologies of diagrams, such as factorizable (also nonfactorizable) emission and annihilation. We predict the branching ratios and CP asymmetries of the modes

$$B_d^0 \to \phi K^0, \quad B^\pm \to \phi K^\pm.$$  \hspace{1cm} (1)

It will be found that two-parton twist-3 contributions are comparable to twist-2 ones as expected. Our predictions for the branching ratios

$$B(B^\pm \to \phi K^\pm) = (10.2^{+3.9}_{-2.1}) \times 10^{-6},$$

$$B(B_d^0 \to \phi K^0) = (9.6^{+3.7}_{-2.0}) \times 10^{-6},$$  \hspace{1cm} (2)

are larger than those from the factorization approach \[11\] and from the QCD factorization approach \[12\], which are located within $4.3^{+3.0}_{-1.4} \times 10^{-6}$ \[13,14\] with the uncertainty arising mainly from the inclusion of annihilation contributions \[14\]. The mechanism responsible for the larger branching ratios in the PQCD formalism is dynamical enhancement of penguin contributions \[4,5\]. Note that the current experimental data of $B(B^\pm \to \phi K^\pm)$,

CLEO [15] : \hspace{1cm} $(5.5^{+2.1}_{-1.8} \pm 0.6) \times 10^{-6},$

Belle [16] : \hspace{1cm} $(10.6^{+2.1}_{-1.9} \pm 2.2) \times 10^{-6},$

BaBar [17] : \hspace{1cm} $(7.7^{+1.6}_{-1.4} \pm 0.8) \times 10^{-6},$  \hspace{1cm} (3)

and those of $B(B^0 \to \phi K^0)$,

CLEO [15] : \hspace{1cm} $< 12.3 \times 10^{-6},$

Belle [16] : \hspace{1cm} $(8.7^{+3.8}_{-3.0} \pm 1.5) \times 10^{-6},$

BaBar [17] : \hspace{1cm} $(8.1^{+3.1}_{-2.5} \pm 0.8) \times 10^{-6},$  \hspace{1cm} (4)

are still not very consistent with each other.
The PQCD factorization formulas for $B \to \phi K$ decay amplitudes have been derived independently in [18]. Here we shall further discuss the power behavior of the factorizable emission and annihilation amplitudes, and the nonfactorizable amplitudes in $1/M_B$, whose relative importance is given by

$$\text{emission} : \text{annihilation} : \text{nonfactorizable} = 1 : \frac{2m_0}{M_B} : \frac{\bar{\Lambda}}{M_B},$$

(5)

with $m_0$ being the chiral symmetry breaking scale. In the heavy quark limit the annihilation and nonfactorizable amplitudes are indeed power-suppressed compared to the factorizable emission ones. Therefore, the PQCD formalism for two-body charmless nonleptonic $B$ meson decays coincides with the factorization approach as $M_B \to \infty$. We shall also explain why the annihilation amplitudes are mainly imaginary and investigate theoretical uncertainty in the PQCD approach.

Dynamical enhancement is a unique feature of the PQCD approach, which does not exist in the factorization or QCD factorization approach. We argue that the $B \to \phi K$ modes are more appropriate for testing this mechanism in penguin-dominated nonleptonic $B$ meson decays compared to the $B \to K\pi$ decays [9]. The large $B \to K\pi$ branching ratios may not be regarded as an evidence of dynamical enhancement: they can also be achieved by chiral enhancement (the kaon is a pseudo-scalar meson) and by choosing a large unitarity angle $\phi_3 \sim 120^\circ$ [19], which leads to constructive interference between penguin and emission contributions. The $B \to \phi K$ modes are not chirally enhanced, because $\phi$ is a vector meson, and insensitive to the variation of the angle $\phi_3$, because they are pure penguin processes. If the data of $B(B \to \phi K)$ are settled down at values around $10 \times 10^{-6}$ in the future, dynamical enhancement will gain a strong support.

On the other hand, precise measurement of the CP asymmetry in the $B \to \phi K$ decays is important for new physics search and for the determination of the unitarity angle $\phi_1$ with high degree of accuracy [20,21]. This measurement is experimentally accessible at the early stage of the asymmetric $B$ factories. The $B \to \phi K$ decays arise from penguin (loop) effects, while the $B \to J/\psi K$ decays involve only tree amplitudes. The search for
different CP asymmetries in the $B \to J/\psi K_s$ and $\phi K_s$ decays, with the common source from $B^0-\bar{B}^0$ mixing, provides a promising way to discover new physics \cite{22,23}: a difference of $|A_{CP}(J/\psi K_s) - A_{CP}(\phi K_s)| > 5\%$ would be an indication of new physics. This subject will be addressed elsewhere \cite{24}. Furthermore, the $\phi$ and $J/\psi$ mesons are bound states of the $s\bar{s}$ and $c\bar{c}$ quarks. It is also interesting to compare their branching ratios, which reflect the mass effect from charm quarks.

We demonstrate the importance of $k_T$ and threshold resummations by studying the $B \to \pi, K$ transition form factors in Sec. II. The power counting and the factorization formulas for various topologies of amplitudes are given in Sec. III. The numerical analysis is performed in Sec. IV. Section V is the conclusion. Twist-2 and two-parton twist-3 distribution amplitudes for the kaon and for the $\phi$ meson are defined in the Appendix.

II. SUDAKOV SUPPRESSION

In this section we briefly review the importance of $k_T$ and threshold resummations for an infrared finite PQCD calculation of heavy-to-light transition form factors \cite{10}. Consider the leading diagram shown in Fig. 1 for the $B \to \phi K$ decays in the kinematic region with a fast-recoil kaon. The $B$ meson momentum $P_1$, the $\phi$ meson momentum $P_2$ and the longitudinal polarization vector $\epsilon$, and the kaon momentum $P_3$ are chosen, in light-cone coordinates, as

$$P_1 = \frac{M_B}{\sqrt{2}}(1, 1, 0_T), \quad P_2 = \frac{M_B}{\sqrt{2}}(1, r_{\phi}^2, 0_T), \quad P_3 = \frac{M_B}{\sqrt{2}}(0, 1 - r_{\phi}^2, 0_T),$$

$$\epsilon = \frac{1}{\sqrt{2r_{\phi}}}(1, -r_{\phi}^2, 0_T), \quad (6)$$

with the ratio $r_{\phi} = M_\phi/M_B$, $M_\phi$ being the $\phi$ meson mass. At the end of the derivation of the factorization formulas, the terms $r_{\phi}^2 \sim 0.04$ in the above kinematic variables will be neglected. We treat the kaon as a massless particle, and define the ratio $r_K = m_0/M_B$ for the kaon, which will appear in the normalization of the twist-3 kaon distribution amplitudes. The $B$ meson is at rest under the above parametrization of momenta.
FIG. 1: Leading contribution to the $B \to \phi K$ decays, where $H$ denotes the hard amplitude and $k_i, i = 1, 2, 3,$ are the parton momenta.

A. $k_T$ and threshold resummations

It has been known that the lowest-order diagram in Fig. 2(a) for the $B \to \pi$ form factor $F^{B\pi}$ is proportional to $1/(x_1 x_3^2)$ without including parton transverse momenta $k_T$, where $x_1 = k_1^+/P_1^+$ and $x_3 = k_3^-/P_3^-$ is the momentum fraction associated with the spectator quark on the $B$ meson (pion) side. If the pion distribution amplitude vanishes like $x_3$ as $x_3 \to 0$ (in the twist-2 case), $F^{B\pi}$ is logarithmically divergent. If the pion distribution amplitude is a constant as $x_3 \to 0$ (in the twist-3 case), $F^{B\pi}$ even becomes linearly divergent. These end-point singularities have caused critiques on the perturbative evaluation of the $B \to \pi$ form factor. Several methods have been proposed to regulate the above singularities. An on-shell $b$ quark propagator has been subtracted from the hard amplitude as $x_3 \to 0$ in [25]. However, this subtraction renders the lepton energy spectrum of the semileptonic decay $B \to \pi l \bar{\nu}$ vanishes as the lepton energy is equal to half of its maximal value. Obviously, this vanishing is unphysical, indicating that the subtraction may not be an appropriate way to regulate the singularity. The subtraction also leads to a value of $F^{B\pi}$ at maximal recoil, which is much smaller than the expected one 0.3. A lower bound of $x_3$ of $O(\bar{\Lambda}/M_B)$ has been introduced in [20] to make the convolution integral finite. However, the outcomes depend on the cutoff sensitively, and PQCD loses its predictive power.
A self-consistent prescription has been proposed in [27], where parton transverse momenta \( k_T \) are retained in internal particle propagators involved in a hard amplitude. In the endpoint region the invariant mass of the exchanged gluon is only \( O(\bar{\Lambda}^2) \) without including \( k_T \). The inclusion of \( k_T \) brings in large double logarithms \( \alpha_s \ln^2(k_T/M_B) \) through radiative corrections, which should be resummed in order to improve perturbative expansion. \( k_T \) resummation \([28,29]\) then gives a distribution of \( k_T \) with the average \( \langle k_T^2 \rangle \sim O(\bar{\Lambda}M_B) \) for \( M_B \sim 5 \text{ GeV} \). The off-shellness of internal particles then remains of \( O(\bar{\Lambda}M_B) \) even at the end point, and the singularities are removed. Hence, it can not be self-consistent to treat \( k_T \) as a higher-twist effect as the endpoint region is important. The expansion parameter \( \alpha_s(\bar{\Lambda}M_B)/\pi \sim 0.13 \) is also small enough to justify PQCD evaluation of heavy-to-light form factors \([4,5]\). This result is so-called Sudakov suppression on the endpoint singularities in exclusive processes \([30]\).

The above discussion applies to the \( B \to K \) form factor and to the \( B \to \phi K \) decays. \( k_T \) resummation of large logarithmic corrections to the \( B, \phi \) and \( K \) meson distribution amplitudes lead to the exponentials \( S_B, S_\phi \) and \( S_K \), respectively:

\[
S_B(t) = \exp \left[ -s(x_1 P_1^+, b_1) - 2 \int_{1/b_1}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu}^2)) \right],
\]

\[
S_\phi(t) = \exp \left[ -s(x_2 P_2^+, b_2) - s((1-x_2)P_2^+, b_2) - 2 \int_{1/b_2}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu}^2)) \right],
\]

\[
S_K(t) = \exp \left[ -s(x_3 P_3^-, b_3) - s((1-x_3)P_3^-, b_3) - 2 \int_{1/b_3}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu}^2)) \right],
\]

with the quark anomalous dimension \( \gamma = -\alpha_s/\pi \). The variables \( b_1, b_2, \) and \( b_3 \), conjugate to the parton transverse momenta \( k_{1T}, k_{2T}, \) and \( k_{3T}, \) represent the transverse extents of the

FIG. 2: Lowest-order diagrams for the \( B \to \pi, K \) transition form factors.
$B$, $\phi$ and $K$ mesons, respectively. The expression for the exponent $s$ is referred to $^{28-30}$.

The above Sudakov exponentials decrease fast in the large $b$ region, such that the $B \rightarrow \phi K$ hard amplitudes remain sufficiently perturbative in the end-point region.

Recently, the importance of threshold resummation $^{31-33}$ has been observed in exclusive $B$ meson decays $^{34}$. As $x_3 \rightarrow 0$ (to be precise, $x_3 \sim O(\bar{\Lambda}/M_B)$) in Fig. 2(a), the internal $b$ quark, carrying the momentum $P_1 - k_3$, becomes almost on-shell, indicating that the end-point singularity is associated with the $b$ quark. Additional soft divergences then appear at higher orders, and the double logarithm $\alpha_s \ln^2 x_3$ is produced from the loop correction to the weak decay vertex. This double logarithm can be factored out of the hard amplitude systematically, and its resummation introduces a Sudakov factor $S_t(x_3)$ into PQCD factorization formulas $^{34}$. Similarly, another lowest-order diagram Fig. 2(b) with a hard gluon exchange between the $d(s)$ quark and the spectator quark gives an amplitude proportional to $1/(x_1^2 x_3)$. In the threshold region with $x_1 \rightarrow 0$ (to be precise, $x_1 \sim O(\bar{\Lambda}^2/M_B^2)$), additional collinear divergences are associated with the internal $d(s)$ quark carrying the momentum $P_3 - k_1$. The double logarithm $\alpha_s \ln^2 x_1$ is then produced from the loop correction to the weak decay vertex. Resummation of this type of double logarithms leads to the Sudakov factor $S_t(x_1)$.

The above formalism can be generalized to factorizable annihilation diagrams easily. For the lowest-order diagram with the internal quark carrying the momentum $P_2 + k_3$, the end-point region corresponds to $x_3 \rightarrow 0$. Hence, threshold resummation of the double logarithm gives the Sudakov factor $S_t(x_3)$. For the lowest-order diagram with the internal quark carrying the momentum $P_3 + k_2$, the end-point region corresponds to $x_2 \rightarrow 0$. Threshold resummation of the double logarithm then gives the Sudakov factor $S_t(x_2)$. The Sudakov factor from threshold resummation is universal, independent of flavors of internal quarks, twists and topologies of hard amplitudes, and decay modes. To simplify the analysis, we have proposed the parametrization $^{10}$,

$$S_t(x) = \frac{2^{1+2c} \Gamma(3/2 + c)}{\sqrt{\pi} \Gamma(1 + c)} [x(1-x)]^c.$$  (8)
with the parameter $c = 0.3$. This parametrization, symmetric under the interchange of $x$ and $1 - x$, is convenient for evaluation of factorizable annihilation amplitudes. It is obvious that threshold resummation modifies the end-point behavior of the meson distribution amplitudes, rendering them vanish faster at $x \to 0$.

Threshold resummation for nonfactorizable diagrams is weaker and negligible. The reason is understood as follows. Consider the diagram with a hard gluon exchange between the spectator quark and the $s$ quark in the $\phi$ meson, in which the internal $s$ quark carries the momentum $k_2 - k_1 + k_3$. To obtain additional infrared divergences, i.e., the double logarithms, the $s$ quark must be close to mass shell. We then have the threshold region defined, for example, by $k_1 \sim O(\bar{\Lambda}/M_B)$, $k_2 \sim O(\bar{\Lambda}/M_B)$, and $k_3 \sim O(M_B)$ simultaneously. That is, this region has more limited phase space compared to that for factorizable amplitudes. Furthermore, soft contribution to a pair of nonfactorizable diagrams cancels, such that the end-point region is not important. Based on the above observations, we shall not include threshold resummation for nonfactorizable amplitudes.

$k_T$ and threshold resummations arise from different subprocesses in PQCD factorization. They can be derived in perturbation theory, and are not free parameters. Their combined effect suppresses the end-point contributions, making PQCD evaluation of exclusive $B$ meson decays reliable. If excluding the resummation effects, the PQCD predictions for the $B \to K$ form factor are infrared divergent. If including only $k_T$ resummation, the PQCD predictions are finite. However, the two-parton twist-3 contributions are still huge, so that the $B \to K$ form factor has an unreasonably large value $F_{BK}^{BK} \sim 0.57$ at maximal recoil. The reason is that the double logarithms $\alpha_s \ln^2 x$ have not yet been organized. If including both resummations, we obtain the reasonable result $F_{BK}^{BK} \sim 0.35$. This study indicates the importance of resummations in PQCD analyses of $B$ meson decays. In conclusion, if the PQCD evaluation of the heavy-to-light form factors is performed self-consistently, there exist no end-point singularities, and both twist-2 and twist-3 contributions are well-behaved.

The mechanism of Sudakov suppression can be easily understood by regarding a meson as a color dipole. In the region with vanishing $k_T$ and $x$, the meson possesses a huge extent
in the transverse and longitudinal directions, respectively. That is, the meson carries a large color dipole. At fast recoil, this large color dipole, strongly scattered during $B$ meson decays, tends to emit infinitely many real gluons. However, these emissions are forbidden in an exclusive process with final-state particles specified. As a consequence, contributions to the $B \rightarrow \pi, K$ form factors from the kinematic region with vanishing $k_T$ and $x$ must be highly suppressed.

**B. Form Factors**

We demonstrate that $k_T$ and threshold resummations must be taken into account in order to obtain reasonable results for the $B \rightarrow K, \pi$ form factors. In the PQCD approach these form factors are derived from the diagrams with one hard gluon exchange as shown in Fig. 2. Soft contribution from the diagram without any hard gluon is Sudakov suppressed [10]. For a rigorous justification of this statement in QCD sum rules, refer to [33]. The two form factors $F_{+,0}^{BK}(q^2)$ are defined by

\[
\langle K(P_3)|\bar{b}(0)\gamma_\mu s(0)|B(P_1)\rangle = F_{+}^{BK}(q^2) \left[ (P_1 + P_3)_\mu - \frac{M_B^2 - M_K^2}{q^2} q_\mu \right] \\
+ F_{0}^{BK}(q^2) \frac{M_B^2 - M_K^2}{q^2} q_\mu ,
\]

where $q = P_1 - P_3$ is the outgoing lepton-pair momentum. $F_{+}^{BK}$ and $F_{0}^{BK}$ at maximal recoil are, quoted from Eq. (39) below, written as

\[
F_{+,0}^{BK}(0) = 8\pi C_F M_B^2 \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \Phi_B(x_1, b_1) \\
\times \left\{ \left[ (1 + x_3) \Phi_K(x_3) + (1 - 2x_3) r_K (\Phi_K^p(x_3) + \Phi_K^\sigma(x_3)) \right] \\
\times \alpha_s(t_e^{(1)}) S_B(t_e^{(1)}) S_K(t_e^{(1)}) h_e(x_1, x_3, b_1, b_3) \\
+ 2r_K \Phi_K^p(x_3) \alpha_s(t_e^{(2)}) S_B(t_e^{(2)}) S_K(t_e^{(2)}) h_e(x_1, x_3, b_1, b_3) \right\} ,
\]

with $C_F = 4/3$ being a color factor. The hard function $h_e(x_1, x_3, b_1, b_3)$, referred to Eq. (14), contains the threshold resummation factor $S_i(x_3)$. The hard scales $t_e$ are defined in Eq. (13). The expression for the $B \rightarrow \pi$ form factor $F^{B\pi}(q^2)$ is similar to Eq. (10).
In Eq. (10) we have included the complete two-parton twist-3 distribution amplitudes associated with the pseudo-scalar and pseudo-tensor structures of the kaon. We adopt the mass \( m_0 = 1.7 \text{ GeV} \) [4,5], the \( B \) meson distribution amplitude \( \Phi_B \) proposed in [4,5] (see Eq. (21)), and the kaon distribution amplitudes \( \Phi_K, \Phi_P^K, \Phi_\sigma^K \) derived from QCD sum rules [36] (see Eqs. (65)-(67)). Since \( \Phi_B \) is still not well determined, we consider the variation of its shape parameter within \( 0.36 \text{ GeV} < \omega_B < 0.44 \text{ GeV} \) [37] around its central value \( \omega_B = 0.4 \text{ GeV} \). Turning off \( k_T \) and threshold resummations and fixing \( \alpha_s \), the \( B \to \pi, K \) form factors are divergent and not calculable. With \( k_T \) resummation, the results become finite as shown in Table I, but still much larger than the expected ones 0.3-0.4. Further including the threshold resummation effect, we obtain the reasonable values \( F_{BK}(0) \sim 0.35 \pm 0.06 \) and \( F_{B\pi}(0) = 0.30 \pm 0.04 \). These ranges of the form factors have been usually adopted as model inputs in the literature, and are consistent with the results from lattice calculations [38–40] extrapolated to the small \( q^2 \) region and from light-cone QCD sum rules [41,42].

We also present our results of the time-like form factors \( F_{\phi K}^2 \), which govern the factorizable annihilation amplitudes. The factorization formulas are, quoted from Eqs. (41) and (42), given by

\[
F_{(V-A)}^{\phi K} = 8\pi C_F M_B^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \\
\times \left[ ((1-x_3) \Phi_\phi(x_2) \Phi^*_K(x_3) + 2r_K \Phi^*_\phi(x_2) ((2-x_3) \Phi_P^K(x_3) + x_3 \Phi^*_K(x_3))) \right] \\
\times \alpha_s(t_a^{(1)}) S_{\phi}(t_a^{(1)}) S_{K}(t_a^{(1)}) h_a(x_2, 1-x_3, b_2, b_3) \\
- \left[ x_2 \Phi_\phi(x_2) \Phi^*_K(x_3) + 2r_K \Phi^*_\phi(x_2) ((1-x_2) \Phi^*_\phi(x_2) - (1-x_2) \Phi_\phi(x_2)) \Phi^*_K(x_3) \right] \\
\times \alpha_s(t_a^{(2)}) S_{\phi}(t_a^{(2)}) S_{K}(t_a^{(2)}) h_a(1-x_3, x_2, b_2, b_3) \right].
\]  

(11)
\[
4r_K \Phi_\phi(x_2) \Phi_K^r(x_3) + 2x_2 r_\phi \left( \Phi_\phi^r(x_2) - \Phi_\phi^t(x_2) \right) \Phi_K(x_3)
\]
\[
\times \alpha_s \left( t_a^{(2)} \right) S_\phi \left( t_a^{(2)} \right) S_K \left( t_a^{(2)} \right) h_a \left( 1 - x_3, x_2, b_3, b_2 \right)
\]  
(12)

The hard function \( h_a(x_2, x_3, b_2, b_3) \), referred to Eq. (13), contains the threshold resummation factor \( S_t(x_3) \). The hard scales \( t_a \) and the \( \phi \) meson distribution amplitudes \( \Phi_\phi, \Phi_\phi^t \) and \( \Phi_\phi^r \) are defined in Eq. (17) and in Eqs. (62)-(64), respectively. It is obvious that \( F^{\phi K}_{(V-A)} \) contains both logarithmic and linear divergences, and \( F^{\phi K}_{(V+A)} \) contains logarithmic divergences if the Sudakov factors are excluded. To include annihilation contributions in the QCD factorization approach, several arbitrary complex infrared cutoffs must be introduced to regulate the above end-point singularities [44]. These cutoffs are process-dependent. Hence, the PQCD approach with the Sudakov effects has a better control on annihilation contributions.

We obtain \( F^{\phi K}_{(V-A)} = (1.78 + 0.63i) \times 10^{-2} \) and \( F^{\phi K}_{(V+A)} = (-3.48 + 13.52i) \times 10^{-2} \) for \( \omega_B = 0.4 \text{ GeV} \) and \( m_0 = 1.7 \text{ GeV} \). The smaller value of \( F^{\phi K}_{(V-A)} \) is due to mechanism similar to helicity suppression: note the partial cancellation between the two terms in Eq. (11). \( F^{\phi K}_{(V+A)} \) is mainly imaginary, whose reason will become clear after we explain the power behavior of the various decay amplitudes in the next section. The time-like form factors are independent of the shape parameter \( \omega_B \). The change of the \( B \to \pi, K \) form factors and of the time-like form factors with the chiral enhancement factor \( m_0 \) is displayed in Table II.

### III. POWER COUNTING AND DECAY AMPLITUDES

We discuss the power counting rules in the presence of the Sudakov effects and present the factorization formulas for the \( B \to \phi K \) decays. The effective Hamiltonian for the flavor-changing \( b \to s \) transition is given by

\[
H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} V_q \left[ C_1(\mu)O_1^{(q)}(\mu) + C_2(\mu)O_2^{(q)}(\mu) + \sum_{i=3}^{10} C_i(\mu)O_i(\mu) \right],
\]  

with the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements \( V_q = V_{qs}^* V_{qb} \) and the operators.

13
\[ O_1^{(q)} = (\bar{s}_i q_j)_{V-A}(\bar{q}_j b_i)_{V-A}, \quad O_2^{(q)} = (\bar{s}_i q_i)_{V-A}(\bar{q}_j b_j)_{V-A}, \]
\[ O_3 = (\bar{s}_i b_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V-A}, \quad O_4 = (\bar{s}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V-A}, \]
\[ O_5 = (\bar{s}_i b_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V+A}, \quad O_6 = (\bar{s}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V+A}, \]
\[ O_7 = \frac{3}{2}(\bar{s}_i b_i)_{V-A} \sum_q e_q(\bar{q}_j q_j)_{V+A}, \quad O_8 = \frac{3}{2}(\bar{s}_i b_j)_{V-A} \sum_q e_q(\bar{q}_j q_i)_{V+A}, \]
\[ O_9 = \frac{3}{2}(\bar{s}_i b_i)_{V-A} \sum_q e_q(\bar{q}_j q_j)_{V-A}, \quad O_{10} = \frac{3}{2}(\bar{s}_i b_j)_{V-A} \sum_q e_q(\bar{q}_j q_i)_{V-A}, \]
\[ i \text{ and } j \text{ being the color indices.} \]

Using the unitarity condition, the CKM matrix elements for the penguin operators \( O_3-O_{10} \) can also be expressed as \( V_u + V_c = -V_t \). The unitarity angle \( \phi_3 \) is defined via

\[ V_{ub} = |V_{ub}| \exp(-i\phi_3) \].

Here we adopt the Wolfenstein parametrization for the CKM matrix up to \( O(\lambda^3) \):

\[
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix} =
\begin{pmatrix}
1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \lambda^2/2 & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix}.
\]

A recent analysis of quark-mixing matrix yields \[15\]

\[ \lambda = 0.2196 \pm 0.0023 , \]
\[ A = 0.819 \pm 0.035 , \]
\[ R_b \equiv \sqrt{\rho^2 + \eta^2} = 0.41 \pm 0.07 . \]

The hard amplitudes contain factorizable diagrams, where hard gluons attach the valence quarks in the same meson, and nonfactorizable diagrams, where hard gluons attach the valence quarks in different mesons. The annihilation topology is also included, and classified into factorizable and nonfactorizable ones according to the above definitions. The \( B \to \phi K \) decay rates have the expressions

\[ \Gamma = \frac{G_F^2 M_B^3}{32\pi} |A|^2 . \]

14
The amplitudes for $B_d \to \phi K^0$, $B_d \to \phi K^0$, $B^+ \to \phi K^+$ and $B^- \to \phi K^-$ are written, respectively, as

\begin{align}
A &= f_\phi V_t^* F_e^{P(s)} + V_t^* M_e^{P(s)} + f_B V_t^* F_a^{P(d)} + V_t^* M_a^{P(d)}, \\
\bar{A} &= f_\phi V_t^* F_e^{P(s)} + V_t^* M_e^{P(s)} + f_B V_t^* F_a^{P(d)} + V_t^* M_a^{P(d)}, \\
A^+ &= f_\phi V_t^* F_e^{P(s)} + V_t^* M_e^{P(s)} + f_B V_t^* F_a^{P(u)} + V_t^* M_a^{P(u)} - f_B V_u^* F_a - V_u^* M_a, \\
A^- &= f_\phi V_t^* F_e^{P(s)} + V_t^* M_e^{P(s)} + f_B V_t^* F_a^{P(u)} + V_t^* M_a^{P(u)} - f_B V_u^* F_a - V_u^* M_a.
\end{align}

In the above expressions $F$ ($M$) denote factorizable (nonfactorizable) amplitudes, the subscripts $e$ ($a$) denote the emission (annihilation) diagrams, and the superscripts $P(q)$ denote amplitudes from the penguin operators involving the $q\bar{q}$ quark pair, and $f_B$ ($f_\phi$) is the $B$ ($\phi$) meson decay constant.

### A. Power Counting

Before presenting the factorization formulas of the above amplitudes, we discuss their power behavior in $1/M_B$. The spectator quark in the $B$ meson, forming a soft cloud around the heavy $b$ quark, carries momentum of $O(\bar{\Lambda})$. The spectator quark on the kaon side carries momentum of $O(M_B)$ in order to form the fast-moving kaon with the $s$ quark produced in the $b$ quark decay. These dramatic different orders of magnitude in momenta explain why a hard gluon is necessary. Based on this argument, the hard gluon is off-shell by order of $\bar{\Lambda}M_B$. This special scale, characterizing heavy-to-light decays, is essential for developing the PQCD formalism of exclusive $B$ meson decays. Below we shall explicitly show how to construct this power behavior, and argue that all the topologies of diagrams should be taken into account in the leading-power PQCD analysis.

We start with the twist-2 contribution to the factorizable emission amplitude, which contains the integrand (see Eq. (10)),

\begin{equation}
I^{e2} = \frac{x_3 M_B^2}{[x_1 x_3 M_B^2 + O(\bar{\Lambda}M_B)][x_3 M_B^2 + O(\bar{\Lambda}M_B)]},
\end{equation}

15
where the factor $x_3$ in the numerator comes from the twist-2 kaon distribution amplitude $\Phi_K(x_3)$, the first and second factors in the denominator come from the virtual gluon and quark propagators, respectively. The terms of $O(\bar{\Lambda}M_B)$ represent the order of magnitude of parton transverse momenta $k_T^2$ under Sudakov suppression. The momentum fraction $x_1$ is assumed to be of $O(\bar{\Lambda}/M_B)$. Here we concentrate on the important end-point region with $x_3 \sim O(\bar{\Lambda}/M_B)$. It is trivial to find that in the end-point region $I^{e2}$ behaves like

$$I^{e2} \sim \frac{1}{\bar{\Lambda}M_B},$$

as argued above.

We then consider the first twist-3 term in Eq. (10),

$$I^{e3} = \frac{r_K M_B^2}{[x_1 x_3 M_B^2 + O(\bar{\Lambda}M_B)][x_3 M_B^2 + O(\bar{\Lambda}M_B)]}. \tag{25}$$

For small $x_3 \sim O(\bar{\Lambda}/M_B)$, we have the power law,

$$I^{e3} \sim \frac{r_K}{\bar{\Lambda}^2} = \frac{m_0}{\bar{\Lambda}} \frac{1}{\bar{\Lambda} M_B}. \tag{26}$$

Hence, the twist-3 contribution is not power-suppressed compared to the twist-2 one in the $M_B \to \infty$ limit [10], though the two-parton twist-3 distribution amplitudes are proportional to the ratio $r_K = m_0/M_B$. The presence of the potential linear divergence in the twist-3 contribution modifies the naive power counting rules from twist expansion of meson distribution amplitudes. The power behavior of the other twist-3 term in Eq. (10) is the same.

A folklore for annihilation contributions is that they are suppressed by a power of the small ratio $f_B/M_B$, and negligible compared to emission contributions. The annihilation amplitudes from the operators $O_{1,2,3,4}$ with the structure $(V-A)(V-A)$ are small because of mechanism of helicity suppression. This argument applies exactly to the $B \to \pi\pi$ decays, and partially to the $B \to K\pi$ and $\phi K$ decays, since the kaon distribution amplitudes are not symmetric in the momentum fraction $x$, and the two final-state mesons are not identical. Those from the operators $O_{5,6}$ with the structure $(S-P)(S+P)$ survive under
helicity suppression [4,5]. Below we shall argue that the annihilation amplitudes from $O_{5,6}$ are proportional to $2r_K$, which is in fact of $O(1)$ for $M_B \sim 5$ GeV. That is, annihilation contributions vanish in the heavy quark limit $m_b \to \infty$, but are important for the physical mass $m_b$.

Referred to Eq. (11), the integrand for the twist-2 annihilation amplitudes from $O_{1,2,3,4}$ is written as,

$$I^{a2}_{(V-A)} = \frac{x_2 x_3^2 M_B^2}{[x_2 x_3 M_B^2 - O(\Lambda M_B) + i\varepsilon][x_3 M_B^2 - O(\Lambda M_B) + i\varepsilon]}, \quad (27)$$

where the factor $x_2$ in the numerator comes from $\Phi_{\phi}(x_2)$ and $x_3^2$ comes from $x_3 \Phi_K(x_3)$. Here we have interchanged $x_3$ and $1-x_3$ for the convenience of discussion. In the region with $x_3 \sim O(\bar{\Lambda}/M_B)$ and with arbitrary $x_2$, we obtain the power laws of the real and imaginary parts of Eq. (27),

$$Re(I^{a2}_{(V-A)}) \sim \frac{1}{M_B^2}, \quad Im(I^{a2}_{(V-A)}) \sim \frac{1}{M_B^2}.$$ \quad (28)

Hence, the twist-2 annihilation contributions from $O_{1,2,3,4}$ are negligible.

The integrand for the $O(r^2)$ terms in Eq. (11) is written as,

$$I^{a4}_{(V-A)} = \frac{2r_K r_{\phi} M_B^2}{[x_2 x_3 M_B^2 - O(\Lambda M_B) + i\varepsilon][x_3 M_B^2 - O(\Lambda M_B) + i\varepsilon]}.$$ \quad (29)

We express the quark propagator as

$$\frac{1}{x_3 M_B^2 - O(\Lambda M_B) + i\varepsilon} = \frac{P}{x_3 M_B^2 - O(\Lambda M_B)} - \frac{i\pi}{M_B^2} \delta \left( x_3 - O \left( \frac{\bar{\Lambda}}{M_B} \right) \right), \quad (30)$$

where $P$ denotes the principle-value prescription. The gluon propagator is expressed in a similar way. The real part then behaves like

$$Re(I^{a4}_{(V-A)}) = 2r_K r_{\phi} M_B^2 \left[ \frac{P}{[x_2 x_3 M_B^2 - O(\Lambda M_B)][x_3 M_B^2 - O(\Lambda M_B)]} \right]$$

$$\left[ \frac{P}{[x_2 x_3 - O(\bar{\Lambda}/M_B)]} \delta \left( x_3 - O \left( \frac{\bar{\Lambda}}{M_B} \right) \right) \right],$$

$$\sim 2r_K r_{\phi} \left( \frac{1}{\bar{\Lambda}^2} - \frac{\pi^2}{\Lambda M_B} \right) = 2r_K M_{\phi} \frac{1}{\bar{\Lambda}} \frac{1}{\Lambda M_B} \left( 1 - \frac{\pi^2 \bar{\Lambda}}{M_B} \right). \quad (31)$$
The imaginary part of Eq. (29) behaves like

$$\text{Im}(I^4_{(V-A)}) = -2\pi r_K r_\phi \left[ \frac{P}{x_2 x_3 M_B^2 - O(\Lambda M_B)} \delta \left( x_3 - O \left( \frac{\Lambda}{M_B} \right) \right) \right] + \frac{P}{x_2 x_3 M_B^2 - O(\Lambda M_B)} \delta \left( x_2 x_3 - O \left( \frac{\Lambda}{M_B} \right) \right),$$

$$\sim -2 r_K r_\phi \frac{2\pi}{\Lambda M_B} = -2 r_K \frac{M_\phi}{\Lambda} \frac{2\pi}{M_B^2}. \quad (32)$$

For the above estimates the end point $x_3 \sim O(\Lambda/M_B)$ with arbitrary $x_2$ corresponds to the important region. Note that the ratio $M_\phi/\Lambda$ is of $O(1)$. Compared to Eq. (24), the first term in Eq. (31), multiplied by a chiral factor $2r_K \sim O(1)$, is not down by the small ratio $f_B/M_B$. The second term in Eq. (31) and the imaginary part, though scaling like $1/M_B^2$, are enhanced by the factors $\pi^2$ and $2\pi$, respectively. However, the mechanism of helicity suppression renders these annihilation contributions turn out to be small ($\sim 1/M_B^2$) as shown in Table II.

Referred to Eq. (12), the general integrand for the twist-3 annihilation amplitudes from $O_{5,6}$ is given by,

$$I^{a3}_{(V+A)} = \frac{2r_K x_2 x_3 M_B^2}{[x_2 x_3 M_B^2 - O(\Lambda M_B) + i\epsilon][x_3 M_B^2 - O(\Lambda M_B) + i\epsilon]}, \quad (33)$$

By means of a similar argument, the real and imaginary parts behave, respectively, like

$$\text{Re}(I^{a3}_{(V+A)}) \sim 2r_K \frac{1}{\Lambda M_B} \left( 1 - \frac{\pi^2 \Lambda}{M_B} \right) \quad (34)$$

$$\text{Im}(I^{a3}_{(V+A)}) \sim -2 r_K \frac{2\pi}{M_B^2}. \quad (35)$$

It is observed that the twist-3 amplitudes in Eq. (12) possess the same power law as the $r^2$ terms in Eq. (11). The difference is only the $O(1)$ ratio $M_\phi/\Lambda$. As stated before, the potential linear divergence in the $r^2$ terms alters the naive power counting rules from twist expansion.

Because of $O(M_B/\Lambda) \sim \pi^2$ for $M_B \sim 5$ GeV, the two terms in Eq. (31) almost cancel each other. The imaginary part in Eq. (35) is enhanced by the factor $2\pi$. There is no helicity suppression in this case. It is then understood that the annihilation amplitudes from $O_{5,6}$ are
mainly imaginary, and their magnitude is few times smaller than the factorizable emission one as exhibited in Table II. If the mass $M_B$ changes, the relative importance of the real and imaginary parts will change. For example, as $M_B$ increases, the cancellation in Eq. (34) is not exact, such that the real part becomes larger. On the other hand, the imaginary part in Eq. (35) decreases. It has been checked that for $M_B = 10$ GeV, $Re(F^{\phi K}_{(V+A)})$ is of the same order as $Im(F^{\phi K}_{(V+A)})$. We conclude that the smallness of $Re(F^{\phi K}_{(V+A)})$ in Table II is due to the special value of the $B$ meson mass $M_B \sim 5$ GeV. For general $M_B$, it is more appropriate to state that annihilation amplitudes from $O_{5,6}$, without distinguishing their real and imaginary parts, scale like $2r_K/(\bar{\Lambda}M_B)$ according to Eq. (34).

The above reasoning is applicable to nonfactorizable amplitudes. It can be found, referred to Eqs. (19)-(51) and to the asymptotic behavior of the meson distribution amplitudes, that the twist-2 term of each nonfactorizable diagram scales like $1/(\bar{\Lambda}M_B)$. However, because of the soft cancellation between a pair of nonfactorizable diagrams in the end-point region of $x_3$, the sum of twist-2 terms turns out to scale like $1/M^2_B$. The twist-3 and $O(r^2)$ terms in each nonfactorizable diagram scale like

$$\frac{r_K}{\bar{\Lambda}M_B}, \quad \frac{r_Kr_\phi}{\bar{\Lambda}^2} = r_K \frac{M_\phi}{\bar{\Lambda}} \frac{1}{M_B},$$

respectively. The cancellation between a pair of nonfactorizable diagrams modifies the above power behaviors into

$$\frac{r_K}{M^2_B}, \quad \frac{r_Kr_\phi}{\bar{\Lambda}M_B} = r_K \frac{M_\phi}{\bar{\Lambda}} \frac{1}{M^2_B},$$

respectively. For the nonfactorizable annihilation amplitudes (see Eqs. (53) and (54)), the soft cancellation does not exist, since the $B$ meson is a heavy-light system. However, it can be shown that the twist-2, twist-3 and $O(r^2)$ terms in each nonfactorizable annihilation diagram possess the power behaviors,

$$\frac{1}{M^2_B}, \quad \frac{r_K}{M^2_B}, \quad \frac{r_K}{\bar{\Lambda}M^2_B},$$

respectively.
We emphasize that it is more appropriate to count the power of each individual diagram, instead of the power of sum of diagrams. In some cases, factorizable contributions are suppressed by a vanishing Wilson coefficient, so that nonfactorizable contributions become dominant. For example, factorizable internal-W emission contributions are strongly suppressed by the Wilson coefficient $a_2$ in the $B \to J/\psi K^{(*)}$ decays [3]. In some cases, such as the $B \to D \pi$ decays, there is no soft cancellation between a pair of nonfactorizable diagrams, and nonfactorizable contributions are significant [3]. In summary, we derive the relative importance of the various topologies of amplitudes given in Eq. (5). The annihilation and nonfactorizable amplitudes are indeed negligible compared to the factorizable emission ones in the heavy quark limit. However, for the physical mass $M_B \sim 5$ GeV, the annihilation contributions should be included.

**B. Factorization Formulas**

Below we calculate the hard amplitudes for the emission and annihilation topologies, which have been obtained independently in [18]. The factorizable penguin contribution is written as

$$F^P_e = -8\pi C_F M_B^2 \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \Phi_B(x_1, b_1)$$

$$\times \left\{ [(1 + x_3) \Phi_K(x_3) + r_K(1 - 2x_3)(\Phi^K_p(x_3) + \Phi^K_{s}(x_3))] \right. $$

$$\times \left. E^{(s)}_e \left( t^{(1)}_e \right) h_e(x_1, x_3, b_1, b_3) \right. $$

$$+ 2r_K \Phi^K_p(x_3) E^{(s)}_e \left( t^{(2)}_e \right) h_e(x_3, x_1, b_3, b_1) \right\}. \quad (39)$$

The factorizable annihilation contribution is given by

$$F^P_a = F^P_{a4} + F^P_{a6}, \quad (40)$$

with

$$F^P_{a4} = 8\pi C_F M_B^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3$$
the Wilson coefficient $q$ for the light quarks $a$

the Wilson coefficients $S$

The hard functions $h(t)$:

The factors $E(t)$ contain the evolution from the $W$ boson mass to the hard scales $t$ in the Wilson coefficients $a(t)$, and from $t$ to the factorization scale $1/b$ in the Sudakov factors $S(t)$:

$$E^{(q)}_c(t) = \alpha_s(t) a^{(q)}_c(t) S_B(t) S_K(t) ,$$

$$E^{(q)}_{ai}(t) = \alpha_s(t) a^{(q)}_i(t) S_B(t) S_K(t) .$$

The hard functions $h$'s are

$$h_c(x_1, x_3, b_1, b_3) = K_0(\sqrt{x_1 x_3} M_B b_1) S_t(x_3)$$

$$\times [\theta(b_1 - b_3) K_0(\sqrt{x_3} M_B b_1) I_0(\sqrt{x_3} M_B b_3)$$

$$+ \theta(b_3 - b_1) K_0(\sqrt{x_3} M_B b_3) I_0(\sqrt{x_3} M_B b_1)] ,$$

$$h_a(x_2, x_3, b_2, b_3) = \left(\frac{i\pi}{2}\right)^2 H^{(1)}_0(\sqrt{x_2 x_3} M_B b_2) S_t(x_3)$$
\[
\times \left[ \theta(b_2 - b_3) H_0^{(1)}(\sqrt{x_3 M_B b_2}) J_0(\sqrt{x_3 M_B b_3}) \\
+ \theta(b_3 - b_2) H_0^{(1)}(\sqrt{x_3 M_B b_3}) J_0(\sqrt{x_3 M_B b_2}) \right], \tag{45}
\]

where \( S_t(x) \) is the evolution function from threshold resummation discussed in Sec. II, and \( K_0, I_0, H_0 \) and \( J_0 \) are the Bessel functions.

The hard scales \( t \) are chosen as the maxima of the virtualities of internal particles involved in the hard amplitudes, including \( 1/b_i \):

\[
t^{(1)}_e = \max(\sqrt{x_3 M_B}, 1/b_1, 1/b_3),
\]
\[
t^{(2)}_e = \max(\sqrt{x_1 M_B}, 1/b_1, 1/b_3), \tag{46}
\]
\[
t^{(1)}_a = \max(\sqrt{1-x_3 M_B}, 1/b_2, 1/b_3),
\]
\[
t^{(2)}_a = \max(\sqrt{x_2 M_B}, 1/b_2, 1/b_3), \tag{47}
\]

which decrease higher-order corrections \[46\]. The Sudakov factor in Eq. (7) suppresses long-distance contributions from the large \( b \) (i.e., large \( \alpha_s(t) \)) region, and improves the applicability of PQCD to \( B \) meson decays. We emphasize that the special intermediate scales \( t \sim O(\sqrt{\Lambda M_B}) \) lead to predictions for penguin-dominated decay modes, such as \( B \to \phi K \), which are larger than those from the factorization and QCD factorization approaches. When PQCD analyses are extended to \( O(\alpha_s^2) \) \[46\], the hard scales can be determined more precisely and the scale independence of our predictions will be improved. The \( O(\alpha_s^2) \) corrections to two-body nonleptonic \( B \) meson decays have been computed in the generalized factorization approach \[44,48\], which indeed improve the scale independence of the predictions.

For the nonfactorizable amplitudes, the factorization formulas involve the kinematic variables of all the three mesons \[49\], and the Sudakov factor is given by \( S = S_B S_\phi S_K \). Their expressions are

\[
\mathcal{M}^{P(q)}_e = \mathcal{M}^{P(q)}_{e3} + \mathcal{M}^{P(q)}_{e4} + \mathcal{M}^{P(q)}_{e5} + \mathcal{M}^{P(q)}_{e6}, \tag{48}
\]

with

\[
\mathcal{M}^{P(q)}_{e4} = 16\pi C_F M_B^2 \sqrt{2N_c} \int_0^1 d[x] \int_0^\infty b_1 db_1 b_2 db_2 \Phi_B(x_1, b_1) \Phi_\phi(x_2)
\]

22
\[ M_{e_5}^{P(q)} = 16\pi C_F M_B^2 \sqrt{2N_c} \int_0^{\frac{1}{d}} d[x] \int_0^{\infty} b_1 db_1 b_2 db_2 \Phi_B(x, b_1) \Phi_{\phi}(x_2) \times \left\{(x_2 + x_3)\Phi_K(x_3) - r_K x_3 (\Phi^p_K(x_3) + \Phi^s_K(x_3)) \right\} \times E_{e_4}^{(q)'} \left(t_{d_1}^{(1)}\right) h_{d_1}^{(1)}(x_1, x_2, x_3, b_1, b_2) \]
\[- \left\{[(1 - x_2)\Phi_K(x_3) - r_K x_3 (\Phi^p_K(x_3) - \Phi^s_K(x_3))] \right\} \times E_{e_4}^{(q)'} \left(t_{d_2}^{(2)}\right) h_{d_2}^{(2)}(x_1, x_2, x_3, b_1, b_2) \right\} , \]

\[ M_{e_6}^{P(q)} = -16\pi C_F M_B^2 \sqrt{2N_c} \int_0^{\frac{1}{d}} d[x] \int_0^{\infty} b_1 db_1 b_2 db_2 \Phi_B(x, b_1) \times \left\{ \right\}
\times E_{e_6}^{(q)'} \left(t_{d_1}^{(1)}\right) h_{d_1}^{(1)}(x_1, x_2, x_3, b_1, b_2) \]
\[ + \left\{ \right\} \times E_{e_6}^{(q)'} \left(t_{d_2}^{(2)}\right) h_{d_2}^{(2)}(x_1, x_2, x_3, b_1, b_2) \right\} , \]

where \( N_c \) is the number of colors and \( d[x] \) denotes \( dx_1 dx_2 dx_3 \). The amplitude \( M_{e_5}^{P(q)} \) is the same as \( M_{e_4}^{P(q)} \) but with the Wilson coefficient \( a_{5}^{(q)'} \). The nonfactorizable annihilation amplitudes are given by

\[ M_{a}^{P(q)} = M_{e_3}^{P(q)} + M_{e_5}^{P(q)} , \]

with
The hard functions \( h \)

The amplitude \( M \)

The evolution factors are given by

\[
M = \left( P_a (P_a) = 3 \right) \times \left( P_3 (P_3) \right) - \left( \Phi_\phi (x_2) \Phi_\phi (x_3) - \Phi_\phi (x_2) \Phi_\phi (x_3) \right)
\]

\[
- r_K r_\phi (1 + x_2 - x_3) \left( \Phi_\phi (x_2) \Phi_\phi (x_3) - \Phi_\phi (x_2) \Phi_\phi (x_3) \right)
\]

\[
+ r_K r_\phi (1 + x_2 + x_3) \left( \Phi_\phi (x_2) \Phi_\phi (x_3) + (3 + x_2 - x_3) \Phi_\phi (x_2) \Phi_\phi (x_3) \right)
\]

\[
\times E_{a_3}^{(q)'} \left( t_f^{(1)} \right) h_f^{(1)} (x_1, x_2, x_3, b_1, b_2)
\]

\[
- [x_2 \Phi_\phi (x_2) \Phi_\phi (x_3)
\]

\[
- r_K r_\phi (1 + x_2 - x_3) \left( \Phi_\phi (x_2) \Phi_\phi (x_3) - \Phi_\phi (x_2) \Phi_\phi (x_3) \right)
\]

\[
+ r_K r_\phi (1 + x_2 + x_3) \left( \Phi_\phi (x_2) \Phi_\phi (x_3) + (3 + x_2 - x_3) \Phi_\phi (x_2) \Phi_\phi (x_3) \right)
\]

\[
\times E_{a_3}^{(q)'} \left( t_f^{(2)} \right) h_f^{(2)} (x_1, x_2, x_3, b_1, b_2)
\]

\[
(53)
\]

\[
M = -16 \pi C_F M_B^2 \sqrt{2 N_c} \int_0^1 d[x] \int_0^\infty b_1 db_1 b_2 db_2 \Phi_B (x_1, b_1)
\]

\[
\times \left\{ [r_\phi x_2 \left( \Phi_\phi (x_2) + \Phi_\phi (x_2) \right) \Phi_\phi (x_3)
\]

\[
- r_K (1 - x_3) \Phi_\phi (x_2) (\Phi_\phi (x_3) - \Phi_\phi (x_3))
\]

\[
\times E_{a_5}^{(q)'} \left( t_f^{(1)} \right) h_f^{(1)} (x_1, x_2, x_3, b_1, b_2)
\]

\[
+ [r_\phi (2 - x_2) \left( \Phi_\phi (x_2) + \Phi_\phi (x_2) \right) \Phi_\phi (x_3)
\]

\[
- r_K (1 + x_3) \Phi_\phi (x_2) (\Phi_\phi (x_3) - \Phi_\phi (x_3))
\]

\[
\times E_{a_5}^{(q)'} \left( t_f^{(2)} \right) h_f^{(2)} (x_1, x_2, x_3, b_1, b_2)
\]

\[
(54)
\]

The amplitude \( M_a \) is the same as \( M_{a_3}^{(u)} \) but with Wilson coefficient \( a'_i \).

The evolution factors are given by

\[
E_{c_1}^{(q)'} (t) = \alpha_s (t) a_s^{(q)'} (t) S (t) \big|_{b_3 = b_1} ,
\]

\[
E_{a_1}^{(q)'} (t) = \alpha_s (t) a_{a_1}^{(q)'} (t) S (t) \big|_{b_3 = b_2} .
\]

(55)

The hard functions \( h^{(j)} \), \( j = 1 \) and \( 2 \), are written as

\[
h_d^{(j)} = [\theta (b_1 - b_2) K_0 (D M_B b_1) I_0 (D M_B b_2)
\]

\[
+ \theta (b_2 - b_1) K_0 (D M_B b_2) I_0 (D M_B b_1)]
\]

24
\[ \times K_0(D_j M_B b_2) , \quad \text{for } D_j^2 \geq 0 \]
\[ \times \frac{i\pi}{2} H_0^{(1)} \left( \sqrt{|D_j^2|M_B b_2} \right) , \quad \text{for } D_j^2 \leq 0 , \quad (56) \]
\[ h_J^{(j)} = \frac{i\pi}{2} \left[ \theta(b_1 - b_2) H_0^{(1)} (FM_B b_1 J_0 (FM_B b_2) \right. \]
\[ + \left. \theta(b_2 - b_1) H_0^{(1)} (FM_B b_2 J_0 (FM_B b_1) \right] \]
\[ \times K_0(F_j M_B b_1) , \quad \text{for } F_j^2 \geq 0 \]
\[ \times \frac{i\pi}{2} H_0^{(1)} \left( \sqrt{|F_j^2|M_B b_1} \right) , \quad \text{for } F_j^2 \leq 0 , \quad (57) \]

with the variables

\[ D^2 = x_1 x_3 , \]
\[ D_1^2 = (x_1 - x_2) x_3 , \]
\[ D_2^2 = -(1 - x_1 - x_2) x_3 , \quad (58) \]
\[ F^2 = x_2 (1 - x_3) , \]
\[ F_1^2 = (x_1 - x_2) (1 - x_3) , \]
\[ F_2^2 = x_1 + x_2 + (1 - x_1 - x_2) (1 - x_3) . \quad (59) \]

The hard scales \( t^{(j)} \) are chosen as

\[ t_d^{(1)} = \max \left( DM_B, \sqrt{|D_1^2|M_B, 1/b_1, 1/b_2 \right) , \]
\[ t_d^{(2)} = \max \left( DM_B, \sqrt{|D_2^2|M_B, 1/b_1, 1/b_2 \right) , \]
\[ t_f^{(1)} = \max \left( FM_B, \sqrt{|F_1^2|M_B, 1/b_1, 1/b_2 \right) , \]
\[ t_f^{(2)} = \max \left( FM_B, \sqrt{|F_2^2|M_B, 1/b_1, 1/b_2 \right) . \quad (60) \]

In the above factorization formulas the Wilson coefficients are defined by

\[ a_1 = C_1 + \frac{C_2}{N_c} , \quad a'_1 = \frac{C_1}{N_c} , \]
\[ a_2 = C_2 + \frac{C_1}{N_c} , \quad a'_2 = \frac{C_2}{N_c} , \]
\[ a_3^{(q)} = C_3 + \frac{C_4}{N_c} + \frac{3}{2} e_q (C_9 + \frac{C_{10}}{N_c}) , \]
\[ a_3^{(q)'} = \frac{1}{N_c} \left( C_3 + \frac{3}{2} e_q C_9 \right) , \]
\[ a_4^{(q)} = C_4 + \frac{C_3}{N_c} + \frac{3}{2} e_q \left( C_{10} + \frac{C_9}{N_c} \right), \]
\[ a_4^{(q)'} = \frac{1}{N_c} \left( C_4 + \frac{3}{2} e_q C_{10} \right), \]
\[ a_5^{(q)} = C_5 + \frac{C_6}{N_c} + \frac{3}{2} e_q \left( C_7 + \frac{C_8}{N_c} \right), \]
\[ a_5^{(q)'} = \frac{1}{N_c} \left( C_5 + \frac{3}{2} e_q C_7 \right), \]
\[ a_6^{(q)} = C_6 + \frac{C_5}{N_c} + \frac{3}{2} e_q \left( C_8 + \frac{C_7}{N_c} \right), \]
\[ a_6^{(q)'} = \frac{1}{N_c} \left( C_6 + \frac{3}{2} e_q C_8 \right), \]
\[ a_{e}^{(q)} = a_3^{(q)} + a_4^{(q)} + a_5^{(q)}. \]

### IV. NUMERICAL ANALYSIS

For the \( B \) meson distribution amplitude, we adopt the model \[4,5\]
\[
\Phi_B(x, b) = N_B x^2 (1 - x)^2 \exp \left[ -\frac{1}{2} \left( \frac{x M_B}{\omega_B} \right)^2 - \frac{\omega_B^2 b^2}{2} \right],
\]
with the shape parameter \( \omega_B = 0.4 \text{ GeV} \). The normalization constant \( N_B = 91.784 \text{ GeV} \) is related to the decay constant \( f_B = 190 \text{ MeV} \) (in the convention \( f_\pi = 130 \text{ MeV} \)). As stated before, \( \Phi_B \) has a sharp peak at \( x \sim \bar{\Lambda}/M_B \sim 0.1 \). The \( \phi \) and \( K \) meson distribution amplitudes are derived from QCD sum rules \[36,50\],
\[
\Phi_\phi(x) = \frac{3 f_\phi}{\sqrt{2 N_c}} x (1 - x),
\]
\[
\Phi_\phi^i(x) = \frac{f_\phi^i}{2 \sqrt{2 N_c}} \left\{ 3 (1 - 2x)^2 + 0.21 \left[ 3 - 30 (1 - 2x)^2 + 35 (1 - 2x)^4 \right] 
+ 0.69 \left[ 1 + (1 - 2x) \ln \frac{x}{1 - x} \right] \right\},
\]
\[
\Phi_\phi^s(x) = \frac{f_\phi^s}{4 \sqrt{2 N_c}} \left[ 3 (1 - 2x)(4.5 - 11.2x + 11.2x^2) + 1.38 \ln \frac{x}{1 - x} \right],
\]
\[
\Phi_K(x) = \frac{3 f_K}{\sqrt{2 N_c}} x (1 - x) \left\{ 1 + 0.51(1 - 2x) + 0.3 [5(1 - 2x)^2 - 1] \right\},
\]
\[
\Phi_K^i(x) = \frac{f_K^i}{2 \sqrt{2 N_c}} \left[ 1 + 0.24 C_2^{1/2}(1 - 2x) - 0.11 C_4^{1/2}(1 - 2x) \right],
\]
\[
\Phi_K^s(x) = \frac{f_K^s}{2 \sqrt{2 N_c}} (1 - 2x) \left[ 1 + 0.35(10x^2 - 10x + 1) \right],
\]
\[
\Phi_K^p(x) = \frac{f_K^p}{2 \sqrt{2 N_c}} \left[ 1 + 0.24 C_2^{3/2}(1 - 2x) - 0.11 C_4^{3/2}(1 - 2x) \right].
\]
with the Gegenbauer polynomials

\[ C^{1/2}_2(\xi) = \frac{1}{2} [3\xi^2 - 1], \quad C^{1/2}_4(\xi) = \frac{1}{8} [35\xi^4 - 30\xi^2 + 3]. \]  

(68)

To derive the coefficients of the Gegenbauer polynomials, we have assumed \( M_K = 0.49 \) GeV and \( m_0 = 1.7 \) GeV. The terms \( 1 - 2x \), rendering the kaon distribution amplitudes a bit asymmetric, corresponds to the \( SU(3) \) symmetry breaking effect. We employ \( G_F = 1.16639 \times 10^{-5} \) GeV\(^{-2} \), the Wolfenstein parameters \( \lambda = 0.2196, A = 0.819 \), and \( R_b = 0.38 \), the unitarity angle \( \phi_3 = 90^\circ \), the masses \( M_B = 5.28 \) GeV and \( M_\phi = 1.02 \) GeV, the decay constants \( f_\phi = 237 \) MeV, \( f_\phi^T = 220 \) MeV and \( f_K = 160 \) MeV, and the \( B_d^0 (B^-) \) meson lifetime \( \tau_{B^0} = 1.55 \) ps \((\tau_{B^-} = 1.65 \) ps) [51]. Note that the \( B \to \phi K \) branching ratios are insensitive to the variation of \( \phi_3 \).

We present values of the factorizable and nonfactorizable amplitudes from the emission and annihilation topologies in Table III. Contributions from twist-2 and two-parton twist-3 distribution amplitudes are displayed separately. It is found that the latter, not power-suppressed, are in fact more important for \( f_\phi F_e^P \). According to the power counting in Sec. III, the twist-2 contributions to the annihilation amplitudes \( f_B F_a^P \) are negligible. This has been explicitly confirmed in Table III. As expected, the factorizable amplitudes \( f_\phi F_e^P \) dominate, and the annihilation amplitudes \( f_B F_a^P \) are almost imaginary and their magnitudes are only few times smaller than \( f_\phi F_e^P \). The nonfactorizable amplitudes \( M_e^P \) and \( M_a^P \) are down by a power of \( \Lambda/M_B \sim 0.1 \) compared to the factorizable ones \( f_\phi F_e^P \) and \( f_B F_a^P \), respectively. The cancellation between the twist-2 and twist-3 contributions makes them even smaller. \( M_a \) and \( f_B F_a \) from the operators \( O_{1,2} \) are of the same order because of the partial cancellation between the two terms in the factorization formula for \( F_a \) (helicity suppression).

We demonstrate the importance of penguin enhancement in Table IV. It has been known that the RG evolution of the Wilson coefficients \( C_{4,6}(t) \) dramatically increases as \( t < m_b/2 \), while that of \( C_{1,2}(t) \) almost remains constant [52]. With this penguin enhancement of about 40\%, the branching ratios of the \( B \to K\pi \) decays, dominated by penguin contributions, are about four times larger than those of the \( B \to \pi\pi \) decays, which are dominated by tree
contributions. This is the reason we can explain the observed $B \to K\pi$ and $\pi\pi$ branching ratios using a smaller unitarity angle $\phi_3 < 90^\circ$ [4,5]. In the factorization approach [11] and in the QCD factorization approach [12], it is assumed that factorizable contributions are not calculable. The leading contribution to a nonleptonic decay amplitude is then expressed as a convolution of a hard part with a form factor and a meson distribution amplitude. In both approaches the hard scale is $m_b$ and the intermediate scale $\Lambda M_B$ can not appear, so that the dynamical enhancement of penguin contributions does not exist. To accommodate the $B \to K\pi$ data in the factorization and QCD factorization approaches, one relies on the chiral enhancement by increasing the mass $m_0$ to a large value $m_0 \sim 3$ GeV, or on a large unitarity angle $\phi_3 \sim 120^\circ$ [19], which leads to constructive (destructive) interference between penguin and emission amplitudes for the $B \to K\pi$ ($B \to \pi\pi$) decays.

Whether dynamical enhancement or chiral enhancement is essential for the penguin-dominated decay modes can be tested by measuring the $B \to \phi K$ modes. In these modes penguin contributions dominate, and their branching ratios are almost independent of the angle $\phi_3$. Since $\phi$ is a vector meson, the mass $m_0$ is replaced by the $\phi$ meson mass $M_\phi \sim 1$ GeV, and chiral enhancement does not exist. Annihilation contributions can not enhance the $B \to \phi K$ branching ratios too much, because they are assumed to be a $1/m_b$ effect in the QCD factorization approach [12]. In the PQCD approach annihilation amplitudes reach 40%, which is reasonable according to our power counting. However, they, being mainly imaginary, are not responsible for the large $B \to \phi K$ branching ratios as shown in Table IV. If the $B \to \phi K$ branching ratios are around $4 \times 10^{-6}$ [13,14], the chiral enhancement may be essential. If the branching ratios are around $10 \times 10^{-6}$, the dynamical enhancement may be essential. Therefore, the $B \to \phi K$ decays are the appropriate modes to distinguish the QCD and PQCD factorization approaches. The branching ratios of $B^0_d \to \phi K^0$ and of $B^\pm \to \phi K^\pm$ are almost equal. We have also evaluated the CP asymmetries of the $B \to \phi K$ decays, and found that they are not significant: their maxima, appearing at $\phi_3 \sim 90^\circ$, are less than 2%.

We emphasize that $m_0(\mu)$, appearing along with the twist-3 kaon distribution amplitudes,
is defined at the factorization scale $1/b$ as low as 1 GeV in the PQCD formalism \cite{23}. Hence, its value should be located within $1.6 \pm 0.2$ GeV \cite{30}. Between the hard scale and the factorization scale, there is the Sudakov evolution. In the QCD factorization approach, $m_0(\mu)$ defined at $m_b$ is as large as 3 GeV, leading to chiral enhancement. It has been argued that $m_0(\mu)$ and $a_6(\mu)$ form a scale-independent product (that is, $m_0(\mu)$ increases, while $a_6(\mu)$ decreases with $\mu$), such that chiral and dynamical enhancements can not be distinguished \cite{53}. However, dynamical enhancement exists in both twist-2 and twist-3 contributions, but chiral enhancement exists only in twist-3 ones (see Eq. (39)). Therefore, they are indeed different mechanism.

At last, we examine the uncertainty from the variation of the hard scales $t$, which provides the information of higher-order corrections to the hard amplitudes. We notice that this is the major source of the theoretical uncertainty. The values of $\omega_B$ and $m_0$ have been fixed at around 0.4 GeV and 1.7 GeV, respectively, which are preferred by the $B \to K\pi$, $\pi\pi$ data. The light meson distribution amplitudes have been fixed more or less in QCD sum rules. The possible 30\% variation of the coefficients of the Gegenbauer polynomials lead to minor changes of our predictions. Since the analyses of the $B \to \pi$ and $B \to K$ form factors are the same, we constraint the ranges of the hard scales $t$, such that our predictions for the $B \to K\pi$ branching ratios are within the data uncertainties. The resultant approximate range of the hard scales $t_e$ is given by

$$\max(0.75\sqrt{x_3}M_B, 1/b_1, 1/b_3) < t_e^{(1)} < \max(1.25\sqrt{x_3}M_B, 1/b_1, 1/b_3),$$

$$\max(0.75\sqrt{x_1}M_B, 1/b_1, 1/b_3) < t_e^{(2)} < \max(1.25\sqrt{x_1}M_B, 1/b_1, 1/b_3).$$

(69)

Note that the coefficients of the factorization scales $1/b$, associated with the definition of the meson distribution amplitudes, do not change. The variation of the other hard scales $t$ is similar, but does not affect the results very much. The theoretical uncertainty for the $B \to \phi K$ branching ratios in Eq. (2) is then obtained.
V. CONCLUSION

In this paper we have shown that a leading-power PQCD formalism should contain contributions from both twist-2 and two-parton twist-3 distribution amplitudes. Threshold and $k_T$ resummations are essential for infrared finite PQCD analyses of $B$ meson decays. Without Sudakov suppression from these resummations, all topologies of decay amplitudes possess infrared (logarithmic or linear) divergences. We have explained the power counting rules for the factorizable (also nonfactorizable) emission and annihilation amplitudes under the sufficiently strong Sudakov effects. The annihilation and nonfactorizable amplitudes are suppressed by $2m_0/M_B$ and by $\bar{\Lambda}/M_B$ in the heavy quark limit, respectively, compared to the factorizable emission ones. For the physical mass $M_B \sim 5$ GeV, the former should be taken into account. In the PQCD formalism the annihilation amplitudes can be calculated in the same way as the emission ones without introducing any new free parameters. Hence, our formalism has more precise control on the annihilation effects than the QCD factorization approach. Annihilation contributions of $40\%$ at the amplitude level are reasonable according to our power counting. However, these amplitudes are not responsible for the large $B \to \phi K$ branching ratios, since they are mainly imaginary.

We have emphasized that exclusive heavy meson decays are characterized by a lower scale $\bar{\Lambda}M_B$, for which penguin contributions are dynamically enhanced. This enhancement renders penguin-dominated decay modes acquire branching ratios larger than those from the factorization and QCD factorization approaches, even when the final-state particle is a vector meson. We have proposed the $B \to \phi K$ decays as the ideal modes to test the importance of this mechanism. If their branching ratios are as large as $10 \times 10^{-6}$ (independent of the unitarity angle $\phi_3$), dynamical enhancement will gain a convincing support. The answer will become clear, when the consistency among the BaBar, Belle, and CLEO data is achieved. We have also found that the CP asymmetries in the $B \to \phi K$ modes are vanishingly small (less than $2\%$).
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APPENDIX A: TWO-PARTON DISTRIBUTION AMPLITUDES

1. $B$ Meson Distribution Amplitudes

The $B$ meson distribution amplitudes are written as [10,54]

$$\left( P_1 + M_B \right) \gamma_5 \frac{\Phi_B(x,b)}{\sqrt{2N_c}}, \quad \left( P_1 + M_B \right) \gamma_5 \frac{\hat{n}_+ - \hat{n}_-}{\sqrt{2}} \Phi_B(x,b). \quad (A1)$$

with the dimensionless vectors $n_+ = (1,0,0_T)$ and $n_- = (0,1,0_T)$. As shown in [10], the contribution from $\Phi_B$ is negligible, after taking into account the equation of motion between $\Phi_B$ and $\Phi_B$. Hence, we consider only a single $B$ meson distribution amplitude in the heavy quark limit in this work. As the transverse extent $b$ approaches zero, the $B$ meson distribution amplitude $\Phi_B(x,b)$ reduces to the standard parton model $\Phi_B(x) = \Phi_B(x,b = 0)$, which satisfies the normalization

$$\int_0^1 \Phi_B(x)dx = \frac{f_B}{2\sqrt{2N_c}}. \quad (A2)$$

2. $\phi$ Meson Distribution Amplitudes

To define the $\phi$ meson distribution amplitudes, we consider the following nonlocal matrix elements [54],

31
The spin structures associated with the $\phi$ meson can be derived from Eqs. (A3)-(A5).
3. Kaon Distribution Amplitudes

The general expressions of the relevant nonlocal matrix elements for a kaon are given by \[36\],

\[
\langle 0|\bar{s}(0)\gamma_5\gamma_\mu u(z)|K(P_3)\rangle = -i f_K P_{3\mu} \int_0^1 dx_3 e^{-ix_3 P_3 \cdot z} \phi_v(x_3)
- \frac{i}{2} f_K M_K^2 \frac{z_\mu}{P_3 \cdot z} \int_0^1 dx_3 e^{-ix_3 P_3 \cdot z} g_K(x_3),
\]

\[
(A8)
\]

\[
\langle 0|\bar{s}(0)\gamma_3 u(z)|K(P_3)\rangle = -i f_K m_0 \int_0^1 dx_3 e^{-ix_3 P_3 \cdot z} \phi_p(x_3),
\]

\[
(A9)
\]

\[
\langle 0|\bar{s}(0)\gamma_5\sigma_{\mu\nu} u(z)|K(P_3)\rangle = i \frac{f_K m_0}{2} \left(1 - \frac{M_K^2}{m_0^2}\right) (P_{3\mu} z_\nu - P_{3\nu} z_\mu)
\times \int_0^1 dx_3 e^{-ix_3 P_3 \cdot z} \phi_\sigma(x_3),
\]

\[
(A10)
\]

with the mass

\[
m_0 = \frac{M_K^2}{m_s + m_d}.
\]

\[
(A11)
\]

\(f_K\) is the kaon decay constant and \(x_3\) the momentum fraction associated with the \(u\) quark at the coordinate \(z \propto (1, 0, 0 T)\). The explicit expression of the wave functions \(\phi\) and \(g_K\) with unit normalization are referred to \[36\].

The contributions from the distribution amplitudes \(\phi_v, \phi_p, \phi_\sigma\) and \(g_K\) are twist-2, twist-3, twist-3, and twist-4, respectively. Note that \(g_K\) does not contribute to factorizable emission and annihilation amplitudes. Hence, the factorizable annihilation amplitude in Eq. (41) is complete in the \(r^2\) terms. We consider the following three kaon final-state distribution amplitudes

\[
\frac{\gamma_5 P_3}{\sqrt{2N_c}} \Phi_K, \quad m_0 \frac{\gamma_5}{\sqrt{2N_c}} \Phi_K^p, \quad \frac{m_0 \gamma_5 (\frac{1}{2} h_- - h_+ - 1)}{\sqrt{2N_c}} \Phi_K^\sigma,
\]

\[
(A12)
\]

with

\[
\Phi_K(x) = \frac{f_K}{2\sqrt{2N_c}} \phi_v(x), \quad \Phi_K^p(x) = \frac{f_K}{2\sqrt{2N_c}} \phi_p(x), \quad \Phi_K^\sigma(x) = \frac{f_K}{12\sqrt{2N_c}} \frac{d}{dx} \phi_\sigma(x),
\]

\[
(A13)
\]

where the term \(M_K^2/m_0^2\) in Eq. (A10) has been neglected. The spin structures associated with the kaon distribution amplitudes can be derived from Eqs. (A8)-(A10).
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Figure Captions

FIG. 1: Leading contribution to the $B \to \phi K$ decays, where $H$ denotes the hard amplitude and $k_i$, $i = 1, 2, 3$ are the parton momenta.

FIG. 2: Lowest-order diagrams for the $B \to \pi, K$ transition form factors.
TABLE I. $B \to \pi, K$ transition form factors without threshold resummation (column A) and with threshold resummation (column B).

| $\omega_B$ (GeV) | $F_{B\pi}(0)$   |               | $F_{BK}(0)$   |               |
|------------------|-----------------|---------------|-----------------|---------------|
|                  | A               | B             | A               | B             |
| 0.35             | 0.603           | 0.356         | 0.703           | 0.430         |
| 0.36             | 0.579           | 0.343         | 0.675           | 0.413         |
| 0.37             | 0.557           | 0.330         | 0.647           | 0.398         |
| 0.38             | 0.535           | 0.318         | 0.621           | 0.382         |
| 0.39             | 0.515           | 0.306         | 0.597           | 0.368         |
| 0.40             | 0.496           | 0.295         | 0.574           | 0.354         |
| 0.41             | 0.478           | 0.285         | 0.552           | 0.342         |
| 0.42             | 0.461           | 0.275         | 0.532           | 0.329         |
| 0.43             | 0.445           | 0.266         | 0.512           | 0.318         |
| 0.44             | 0.429           | 0.257         | 0.494           | 0.307         |
| 0.45             | 0.414           | 0.248         | 0.476           | 0.296         |
TABLE II. Time-like form factors $F^{\phi K}$

| $m_0$(GeV) | $F^{B\pi}(0)$ Re | $F^{BK}(0)$ Re | $F^{\phi K}_{(V-A)} \times 10^2$ Re | $F^{\phi K}_{(V+A)} \times 10^2$ Re |
|------------|-----------------|----------------|-------------------------------|-------------------------------|
| 1.4        | 0.295           | 0.312          | 1.49                         | -2.58                        |
| 1.5        | 0.308           | 0.326          | 1.61                         | -3.06                        |
| 1.6        | 0.320           | 0.340          | 1.71                         | -3.20                        |
| 1.7        | 0.333           | 0.354          | 1.78                         | -3.48                        |
| 1.8        | 0.345           | 0.368          | 1.92                         | -3.79                        |
| 1.9        | 0.357           | 0.383          | 2.02                         | -3.99                        |
| 2.0        | 0.370           | 0.396          | 2.10                         | -4.40                        |
TABLE III. Twist-2 and higher-twist contributions to the $B \to \phi K$ decay amplitudes.

| Decay | $B^\pm \to \phi K^\pm$ |
|-------|----------------------|
|       | $B^0 \to \phi K^0$   |

| Components | twist-2 ($10^{-4}$ GeV) | higher-twist ($10^{-4}$ GeV) | total ($10^{-4}$ GeV) |
|------------|------------------------|-------------------------------|-----------------------|
|           | Re         | Im   | Re         | Im   | Re    | Im   |
| $f_\phi F^P_e$ | 9.48       | —    | 27.71       | —    | 37.19  | —    |
| $M^P_e$     | -3.30      | 2.44 | 1.60        | -1.11| -1.70  | 1.33 |
| $f_B F^P_a$ | 0.07       | -0.01| -2.57       | -15.47| -2.50  | -15.48 |
| $M^P_a$     | 0.06       | 0.41 | 0.30        | -0.25| 0.36   | 0.16 |
| $f_B F_a$   | -1.21      | 0.47 | 41.00       | 13.52| 39.79  | 13.99 |
| $M_a$       | 0.88       | -12.74| -7.05       | 2.61 | -6.17  | -10.13 |

| Components | twist-2 ($10^{-4}$ GeV) | higher-twist ($10^{-4}$ GeV) | total ($10^{-4}$ GeV) |
|------------|------------------------|-------------------------------|-----------------------|
|           | Re         | Im   | Re         | Im   | Re    | Im   |
| $f_\phi F^P_e$ | 9.48       | —    | 27.71       | —    | 37.19  | —    |
| $M^P_e$     | -3.30      | 2.44 | 1.60        | -1.11| -1.70  | 1.33 |
| $f_B F^P_a$ | 0.07       | -0.01| -2.68       | -15.76| -2.61  | -15.77 |
| $M^P_a$     | -0.03      | 0.77 | 0.48        | -0.28| 0.45   | 0.49 |
TABLE IV. Enhancement effects in the $B^\pm \rightarrow \phi K^\pm$ decay amplitudes

| Scales   | $\mu = t$ | $\mu = 2.5$ GeV |
|----------|-----------|-----------------|
| Amplitudes | Re ($10^{-4}$ GeV) | Im ($10^{-4}$ GeV) | Re ($10^{-4}$ GeV) | Im ($10^{-4}$ GeV) |
| $f_\phi F_\phi^P$ | 37.19 | — | 23.14 | — |
| $M_\phi^P$ | -1.70 | 1.33 | -1.05 | 0.62 |
| $f_B F_a^P$ | -2.50 | -15.48 | 1.92 | -12.83 |
| $M_a^P$ | 0.36 | 0.16 | -0.05 | 0.19 |
| $f_B F_a$ | 39.79 | 13.99 | 37.73 | 13.14 |
| $M_a$ | -6.17 | -10.13 | -0.56 | -9.05 |
| Branching Ratio (without Ann.) | $9.8 \times 10^{-6}$ | $3.8 \times 10^{-6}$ |
| Branching Ratio (with Ann.) | $10.2 \times 10^{-6}$ | $5.6 \times 10^{-6}$ |