Modeling of liquid-solid flow erosion in curved pipes of gradually varying cross section

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Abstract. Liquid-solid flow erosion in curved pipes of gradually varying cross section is investigated. The model consists of three components integrated: Fluid Transport to describe the flowing fluid via Reynolds-Averaged Navier-Stokes equations with standard k-ε turbulence model, Particle Transport to describe the trajectories of the solid particles using Discrete Particle Modeling (DPM) and Particle Erosion to describe the erosion rate using Oka erosion model. The model is validated against experimental data for erosion in curved pipe of constant cross section. This study focuses at the geometrical effect of curved pipes, i.e. converging, constant cross section and diverging, on erosion. Both maximum erosion and locations where it occurs are identified. Results show that curved converging pipes suffer from high erosion rate concentrated especially in the outer wall of the curved section. Curved diverging pipes on the other hand has much lower erosion of erosion rate at least one order of magnitude lower compared against that of diverging pipes and concentrated on the side wall of the curved section. This very different erosion behavior is highlighted in the study.

1. Introduction

Solid particles are occasionally carried by fluids in engineering flows, e.g. sand particles carried by water flowing in pipes, oil flowing in production wells and gas flowing in ducts. These solid particles are normally sharp and hard. Impacts of these particles on conduit surfaces degrade and remove materials mechanically resulting in erosion. Severe erosion is normally localized in parts of the conduits with gradual or even worse sudden geometrical changes, e.g. contraction and bend, where complex changes in flow characteristics are induced. Such a localized erosion affects negatively operational reliability and is particularly critical from a safety perspective when the flowing fluid is corrosive, flammable or toxic. To reduce or prevent localized erosion in these flow configurations, a good understanding of the flow coupled erosion process is needed.

In this study, particle erosion in liquid-solid flow in curved pipes of varying cross section is investigated numerically. Curved pipes of varying cross section are relatively common in engineering flow systems to change the flow direction and at the same time accelerate (converging) and decelerate (diverging) the flow. The flow characteristics is more complex if compared to that in regular straight or curved pipes, characterized by the presence of strong adverse pressure gradient near the inner wall and intricate secondary flow structures. Particles carried by flow in curved pipes of varying cross section have more random trajectories, inducing very different erosion behaviors. As a result, accurate prediction of erosion in curved pipes of varying cross section is more challenging.
There are numerous investigations on erosion in curved pipe of constant cross section (including elbow and bend) for air-solid [1]-[8], liquid-solid [9]-[12] and gas-liquid-solid [13] flows. To the best knowledge of the authors, erosion in curved pipe of varying cross section, e.g. converging or diverging has not been considered in the existing studies. Therefore, in this study, erosion induced by liquid laden with solid particles flowing in curved pipes of gradually varying cross section either of the converging or the diverging types is investigated numerically. The numerical model for the prediction consists of three integrated components: Fluid Transport to describe the flowing fluid via Reynolds-Averaged Navier-Stokes equations with standard k-ε turbulence model, Particle Transport to describe the trajectories of the solid particles using Discrete Particle Modeling (DPM) and Particle Erosion to describe actual material removal on surface upon impacted by particles quantified in term of erosion rate using Oka erosion model.

The remaining of the article is divided into five sections. The problem of interest will be description in Section 2. In Section 3, the governing equations for Fluid Transport, Particle Transport and Particle Erosion for the current problem are presented followed by a brief description of the numerical solution procedure in Section 4. Then, results investigating erosion in various converging and diverging pipes are presented and discussed in Section 5. Finally, a few concluding remarks are given in Section 6.

2. Problem Description

Figure 1 shows a converging and a diverging curved pipes. Each curved pipe consists of three sections, i.e. a horizontal, a curved and a vertical sections. The diameter of the curved pipe varies along the pipe and is given by

\[
D = \begin{cases} 
D_i, & \theta < 0^\circ \\
D_i \left[ 1 + (\lambda - 1) \frac{\theta}{90^\circ} \right], & 0^\circ \leq \theta \leq 90^\circ \\
D_o, & \theta > 90^\circ
\end{cases} \tag{1a}
\]

where \( D_i \) and \( D_o \) are respectively inlet and outlet diameters. The convergent-divergent ratio \( \lambda \) is defined as

\[
\lambda = \frac{D_o}{D_i} \tag{1b}
\]

Note that \( \lambda = 1 \) for curved pipe of constant cross section. Liquid laden with solid particles flows into the curved pipe. At the inlet, the flow is steady and fully-developed and the solid particles are uniformly distributed. While flowing in the pipe, solid particles can impact on the wall, chip off material from the surface and cause erosion.
3. Mathematical Formulation
The model consists of three components, i.e. Fluid Transport, Particle Transport and Particle Erosion. Fluid Transport models the liquid flow to predict velocity and pressure governed by the Navier-Stokes equations with $k - \varepsilon$ turbulence closure.

$$
\frac{\partial}{\partial x_j}\left(\rho u_j\right) = 0
$$

$$
\frac{\partial}{\partial x_j}\left(\rho u_j u_i\right) = -\frac{\partial p}{\partial x_j} + \frac{\partial}{\partial x_j}\left[\mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right] + \rho g_i + \frac{\partial}{\partial x_j}\left(2\rho C\frac{k^2}{\varepsilon} S_j - \frac{2}{3} \rho k \delta_{ij}\right)
$$

where $\rho$, $\mu$, $g$, $u$ and $u'$ are respectively liquid density, liquid viscosity, gravitational acceleration, mean velocity, and velocity fluctuation. In last term of Eq. (3), $S_j$ is the mean rate of deformation. The turbulence kinematic energy $k$ and the dissipation rate $\varepsilon$ are governed by

$$
\frac{\partial}{\partial x_j}\left(\rho u_j k\right) = \frac{\partial}{\partial x_j}\left[\mu_t \left(\frac{\partial k}{\partial x_j}\right)\right] + 2\mu_t S_j \cdot S_j - \rho \varepsilon
$$

$$
\frac{\partial}{\partial x_j}\left(\rho u_j \varepsilon\right) = \frac{\partial}{\partial x_j}\left[\frac{\mu_t}{\varepsilon} \left(\frac{\partial \varepsilon}{\partial x_j}\right)\right] + C_{1\varepsilon} \frac{\varepsilon}{k} 2\mu_t S_j \cdot S_j - C_{2\varepsilon} \rho \varepsilon^2
$$

The five constants are set to $C_{\mu} = 0.09$, $C_{1\varepsilon} = 1.44$, $C_{2\varepsilon} = 1.92$, $\sigma_k = 1.0$ and $\sigma_\varepsilon = 1.3$ [14].

Within a discrete phase model (DPM), Particle Transport tracks the trajectories of solid particles by integrating Newton’s second law as

$$
\frac{d\vec{u}_p}{dt} = \vec{F}_D + \vec{F}_p + \vec{F}_{VM} + \vec{F}_B
$$

where $\vec{u}_p$ is the solid particle velocity. The forces are expressed in term of per unit particle mass: $\vec{F}_D$, $\vec{F}_p$, $\vec{F}_{VM}$ and $\vec{F}_B$ are respectively drag force, pressure gradient force, added mass force and buoyancy force [15].

Particle Erosion model the actual removal of material from surface quantified in term of erosion rate $ER$ defined as the mass of removed material from surface per unit area per unit of time (i.e. kg/m²s)

$$
ER = \frac{1}{A_f} \sum_{p=1}^{N} \dot{m}_pe_r
$$

where $A_f$ and $\dot{m}_p$ are respectively surface area and mass flowrate of each individual particle. The erosion ratio $e_r$ is the mass of removed material per mass of impinging particles modeled using Oka erosion model [16] as

$$
e_r = e_{90} f(\alpha) \frac{u_p}{u_{ref}}^{k_1} \left(\frac{d_p}{d_{ref}}\right)^{k_2}
$$

$$
f(\alpha) = (\sin \alpha)^{n_1} \left[1 + H_v(1 - \sin \alpha)\right]^{n_2}
$$

where $d_p$ is the particle diameter. In particular, $e_r$ at 90° impact angle $e_{90} = 0.0006154$, reference velocity $u_{ref} = 104$, reference particle diameter $d_{ref} = 0.326$, Vickers hardness $H_v = 1.5$ GPa, $k_2 = 2.35$, $k_3 = 0.19$, $n_1 = 0.8$ and $n_2 = 1.3$. 


4. Solution Procedure

The governing equations are solved employing a finite volume method implemented in ANSYS Fluent. While the convective terms in the momentum equations are treated with a second-order upwind scheme, those in Eqs. (4) and (5) for \( k \) and \( \varepsilon \) are discretized using a first-order upwind scheme. The velocity-pressure coupling is handled using a SIMPLE algorithm. For Particle Transport, automated tracking approach in ANSYS Fluent capable of switching from low order Euler implicit to higher order trapezoidal scheme is employed. One-way coupling between Fluid Transport and Particle Transport is adopted. Finally, erosion rate \( ER \) is determined.

5. Results and Discussions

For validation purpose, solution is obtained for flow in curved pipe of constant cross section with geometry and operating conditions identical to those of [17]. \( ER \) at the outer wall of the curved pipe is determined and plotted in Fig. 2 with the experimental measurements of [17] superimposed. Note that either particle-wall rebound model of Forder [18] or Grant [19] can be used in combination with Oka erosion model (referred respectively as Oka-Forder Oka-Grant). The present solution agrees reasonably well with that of [17].

![Figure 2: Validation against experimental data of [17].](image)

In the following investigation, water with density \( \rho = 1000 \text{ kg/m}^3 \) and viscosity \( \mu = 0.001 \text{ Pa.s} \) at an average velocity of \( u_o = 4 \text{ m/s} \) at the inlet is used. For the solid particles, particle diameter \( d_p = 150 \mu \text{m} \), particle density \( \rho_p = 2650 \text{ kg/m}^3 \) and particle mass flowrate \( m = 100 \text{ g/s} \). Mesh independent study was conducted. Mesh independent solutions for convergent-divergent ratio varied from \( \lambda = 0.6, 0.8, 1.0, 1.2 \) to 1.4 are obtained. \( ER \) at various locations of the curved pipes are shown in Fig. 3. The results will be discussed in detail for the curved section, i.e. \( 0^\circ \leq \theta \leq 90^\circ \).

Quantification in term of average \( ER \) in the curved pipe is less useful. The location and magnitude of maximum \( ER \) is more important from an engineering application point of view. If failure were to occur due to erosion, it must start from the location with maximum \( ER \). For converging pipes (\( \lambda = 0.6 \) and 0.8), location of maximum erosion occurs at the outer wall of curved section at around \( \theta = 89^\circ \) as shown in Figs. 3a and 3b. In particular for pipe of \( \lambda = 0.6 \), erosion is highly concentrated at this location with much lower \( ER \) elsewhere in the curved section. Physically, the solid particles are carried by water. However, because of inertia, these particles do not necessarily follow the flow
streamlines. In fact as they negotiate the curved section, many of their trajectories actually deviate from the flow streamlines ended up impacting and then eroding the outer wall of the curved section, see Fig. 4. Within the curved section, very few particles impact on the inner wall. Therefore, minimal erosion occurs at the inner wall of the curved section. The more diverging the pipe is (larger $\lambda$), the lower the curvature effect and deviation from streamline is smaller. As such, $ER$ generally is lower and location of maximum erosion moves gradually to the side wall of the curved section, see comparison between Figs. 3a, 3b and 3c. Note that the order of magnitude for maximum $ER$ varies from order of $10^{-5}$ to $10^{-7}$ kg/m²s upon increasing the convergent-divergent ratio from $\lambda = 0.6$ to 1.4.
Figure 3: ER in pipes of various $\lambda$.

Figure 4: Schematic of solid particles trajectories.

For pipe of $\lambda = 0.6$, region with maximum $ER$ is located at the outer wall around $\theta = 89^\circ$. For pipe of $\lambda = 0.8$, this region of maximum $ER$, although in term of magnitude smaller than that of $\lambda = 0.6$, expands circumferentially around the pipe. Therefore, it is essential to look at the circumferential distribution of $ER$ around the pipe at streamwise location with maximum $ER$. For this purpose, circumferential angle $\varphi$ measured starting from the side wall, with inner wall at $\varphi = 90^\circ$ and outer wall at $\varphi = 270^\circ$, is introduced in Fig. 5. With this, $ER$ around the pipe at streamwise location of maximum $ER$ can be plotted as in Fig. 6. Because the different order of magnitude for $ER$, the same plot is enlarged in Fig. 6b. Note that for pipe of $\lambda = 1.0$ or larger, the streamwise location of maximum $ER$ is no longer located within the curved section but downstream of it. For example, for pipe of $\lambda = 1.0$, streamwise location of maximum $ER$ occurs at $0.1d$ downstream of the curved section. For $\lambda = 1.2$, it is $0.76d$ downstream of the curved section. For pipes of larger $\lambda$, it
region of maximum $ER$ moves gradually to the inner wall (although the streamwise location is different). From an engineering context, the inner wall downstream of the curved section is increasingly affected by erosion for more diverging curved pipe.

![Figure 5: Definition of circumferential angle $\varphi$.](image)

![Figure 6: Circumferential distribution of $ER$ at streamwise location of maximum $ER$.](image)

6. Concluding Remarks

In the present work, liquid-solid flow erosion in curved pipes of gradually varying cross section is investigated. The model is validated against experimental data for erosion in curved pipe of constant cross section. Curved converging ($\lambda = 0.6, 0.8$), constant cross section ($\lambda = 1.0$) and divergent ($\lambda = 1.2, 1.4$) pipes are considered. Upon increasing the convergent-divergent ratio from $\lambda = 0.6$ to 1.4, maximum $ER$ can decrease one order of magnitude. Therefore, erosion is severe for curved converging pipes with region of high erosion located at the outer wall of the curved section. For more diverging pipes, erosion is less with region of high erosion located at the side walls of the curved section. For better erosion resistant, with locations of high erosion identified, curved pipes can be designed to have a locally thicker pipe wall or a local erosion resisting coating. From a maintenance point of view, the locations of curved pipes should be inspected more carefully for possible erosion cause failure. The present study focuses at the geometrical effect of curved pipes, i.e. converging or diverging. Of course, other parameters including type of liquid, liquid velocity at the inlet, particle type and particle mass flowrate are equally important. These are reserved for a separate study.
Nomenclature

| Symbol | Description                                      |
|--------|--------------------------------------------------|
| $A_f$  | surface area of individual particle              |
| $d_p$  | particle diameter                                |
| $D_i$  | inlet diameter                                  |
| $D_o$  | outlet diameter                                  |
| $e_r$  | erosion ratio                                   |
| $ER$   | erosion rate                                     |
| $F_B$  | buoyancy force                                   |
| $F_D$  | drag force                                       |
| $F_P$  | pressure gradient force                          |
| $F_{VM}$ | added mass force                                 |
| $g_1$  | gravitational acceleration                       |
| $k$    | turbulence kinematic energy                      |
| $m_p$  | mass flowrate of individual particle             |
| $S_{ij}$ | mean deformation rate                           |
| $t$    | time                                            |
| $u_i$  | mean velocity                                   |
| $u_i'$ | velocity fluctuation                            |
| $u_p$  | particle velocity                                |
| $\epsilon$ | dissipation rate                                |
| $\lambda$ | convergent-divergent ratio                      |
| $\mu$  | liquid viscosity                                 |
| $\rho$  | liquid density                                   |
| $\phi$ | circumferential angle                            |

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