Optical solitons in Bragg gratings fibers for the nonlinear (2+1)-dimensional Kundu–Mukherjee–Naskar equation using two integration schemes

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Abstract
The current work handles for the first time, dispersive optical solitons in fiber Bragg gratings for the nonlinear (2+1)-dimensional Kundu–Mukherjee–Naskar equation. Two integration schemes, namely, the modified Kudryashov’s approach and the addendum to Kudryashov’s methodology are applied. Dark and bright soliton solutions as long as explicit solutions are obtained. Also, combo bright-singular solutions are introduced.

Keywords Two integration technologies · Optical soliton solutions · Fiber Bragg gratings · Kundu–Mukherjee–Naskar equation

1 Introduction

Fiber Bragg gratings (FBGs) are of tremendous interests to physicists and engineers during the past couple of decades or more because of its applications. FBGs are applied in all optical communication systems and thus have been widely studied with several methods (Biswas et al. 2019, 2019a, b, c, d; Biswas 2018; Yıldırım et al. 2020; Arshed et al. 2020; Yıldırım 2019; Ekici et al. 2019; Yıldırım 2019; Kudryashov et al. 2019; Kudryashov 2020a, b; Yıldırım and Mirzazadeh 2020; Trik et al. 2021; Yıldırım 2019b; Chen et al. 2020; Xia et al. 2020; Chen et al. 2021; Lü and Chen 2021; Lü and Ma 2016; He et al. 2021; Yin et al. 2020; Lü et al. 2021; Chen et al. 2021; Xu et al. 2020; Lü et al. 2016). Optical nonlinearities, such as Kerr law, quadratic–cubic law, log-law, cubic–quintic–septic law, triple-power law, power-law, parabolic-law, anti-cubic law, dual-power law and parabolic-nonlocal law have been investigated by FBGs using integration technologies. Also, optical solitons in FBGs have been gained with governing models that describe the dynamics of soliton propagation, such as the Biswas–Milovic equation, Lakshmanan–Porsezian–Daniel model, Fokas–Lenells equation, Kundu–Mukherjee–Naskar model, Biswas–Arshed equation, complex Ginzburg–Landau equation, Kudryashov’s model, Chen–Lee–Liu equation, Sasa–Satsuma equation, Kaup–Newell
equation, Gerdjikov–Ivanov equation, Kundu–Eckhaus equation, Triki–Biswas model and Manakov model. The Kundu–Mukherjee–Naskar model corresponds to a case of NLSE with the different form nonlinearity and dispersion term (Yıldırım and Mirzazadeh 2020; Trik et al. 2021; Yıldırım 2019a, b). Actually, there are some new analytical solutions have been derived for PDEs (Chen et al. 2020; Xia et al. 2020; Chen et al. 2021; Lü and Chen 2021). It is worth mentioning here that there are also other studies on an interesting kind of exact solutions, such as, lump solution (Lü and Ma 2016), Bäcklund transformation, Pfaffian, Wronskian and Grammian solutions (He et al. 2021), localized characteristics of lump and interaction solutions (Yin et al. 2020), Painleve analysis, soliton solutions, Bäcklund transformation, Laxpair and infinitely many conservation laws (Lü et al. 2021) and other studies (Chen et al. 2021; Xu et al. 2020; Lü et al. 2016).

The current paper handles solitons in FBGs with the nonlinear (2+1)-dimensional Kundu–Mukherjee–Naskar equation that has solved by aid of two integration schemes. The governing model in FBGs is formulated in this the paper for the first time. Optical solitons of the model equation are revealed by the modified Kudryashov’s approach (Zayed et al. 2021a, b, c) and the addendum to Kudryashov’s methodology (Kudryashov 2020a; Zayed et al. 2021a, b, c). These solitons are reported in this paper for the first time. The details are sketched through, after a thorough introduction to the governing model.

1.1 Governing model

The dimensionless form of the nonlinear (2+1)-dimensional Kundu–Mukherjee–Naskar model in polarization-preserving fibers is written as (Yıldırım 2019; Ekici et al. 2019):

\[
i q_t + a q_{xy} + i b (q q_x^* - q^* q_x) q = 0,
\]

where \(q(x, y, t)\) is a complex-valued function representing the wave profile, while \(a\) and \(b\) are real-valued constants. The first term is the linear temporal evolution, the second term represents the dispersion term, while the third term represents the nonlinearity term and \(i = \sqrt{-1}\).

In Bragg gratings fibers, Eq. (1) can be written, for the first time as:

\[
i u_t + a_1 u_{xy} + i \left[ (b_1 u^2 + c_1 v^2) u_x^* - (d_1 |u|^2 + e_1 |v|^2) u_x \right] + i \alpha_1 u_x + \beta_1 v + \sigma_1 u^* v^2 = 0,
\]

and

\[
i v_t + a_2 v_{xy} + i \left[ (b_2 v^2 + c_2 u^2) v_x^* - (d_2 |v|^2 + e_2 |u|^2) v_x \right] + i \alpha_2 v_x + \beta_2 u + \sigma_2 v^* u^2 = 0,
\]

where \(u(x, y, t)\) and \(v(x, y, t)\) are complex-valued functions that represent the wave profiles, while \(a_j, b_j, c_j, d_j, e_j, \alpha_j, \beta_j, \sigma_j (j = 1, 2)\) are real-valued constants. Here, \(a_j\) are the coefficients of dispersion terms. The parameters \(b_j, c_j, d_j, e_j (j = 1, 2)\) are the coefficients of nonlinearity. Next, \(\alpha_j, \beta_j\) and \(\sigma_j\) give the inter-modal dispersions (IMD), the detuning parameters and the four wave mixing (4WM) parameters, respectively. Yıldırım (Yıldırım 2019) has discussed the birefringent fibers of Eq. (1) using the modified simple equation approach.

The main objective of this article is to apply the modified Kudryashov’s approach and the addendum to Kudryashov’s method to find the dark, bright and singular soliton solutions of Eqs. (2) and (3).

The organization of this article can be written as: The mathematical preliminaries are introduced in Sect. 2. In Sects. 3 and 4, we give the solutions of the system (2) and (3).
Sect. 5, the numerical simulations for some solutions are designed. In Sect. 6, conclusions are illustrated.

2 Mathematical preliminaries

In this section, we suppose that Eqs. (2) and (3) have the solutions:

\[ u(x, y, t) = P_1(\xi) \exp \left[ i(-\kappa_1 x - \kappa_2 y + \omega t + \theta_0) \right], \]

\[ v(x, y, t) = P_2(\xi) \exp \left[ i(-\kappa_1 x - \kappa_2 y + \omega t + \theta_0) \right], \]

and

\[ \xi = B_1 x + B_2 y - pt, \]  

where \( B_1, B_2, \rho, \kappa_1, \kappa_2, \omega \) and \( \theta_0 \) are all non-zero real constants to be determined. The parameters \( B_1 \) and \( B_2 \) represent direct cosines and are related to the inverse widths of the soliton in the \( x \)- and \( y \)-directions, respectively, while \( \rho \) is the soliton velocity. From the phase component, \( \kappa_1 \) and \( \kappa_2 \) give the frequencies of the solitons along the \( x \)- and \( y \)-directions, respectively, while \( \omega \) is the wave number and \( \theta_0 \) is the phase constant. Here, \( P_1(\xi) \) and \( P_2(\xi) \) are real valued functions which stand for the pulse shapes. If we substitute (4) and (5) into Eqs. (2) and (3) and separate the real and imaginary parts, we deduce that

\[ a_1 B_1 B_2 P''_2 + (B_1 - a_1 \kappa_1 \kappa_2) P_2 - \omega P_1 + \kappa_1 (b_1 - d_1) P_1^3 + \left[ \kappa_1 (c_1 - e_1) + \sigma_1 \right] P_1 P_2^2 = 0, \]

\[ a_2 B_1 B_2 P''_1 + (B_2 - a_2 \kappa_1 \kappa_2) P_1 - \omega P_2 + \kappa_2 (b_2 - d_2) P_2^3 + \left[ \kappa_2 (c_2 - e_2) + \sigma_2 \right] P_2 P_1^2 = 0, \]

and

\[ (a_1 B_1 - \rho) P'_1 - a_1 (\kappa_2 B_1 + B_2 \kappa_1) P'_2 + B_1 (b_1 - d_1) P_1^2 P'_1 + B_1 (c_1 - e_1) P_2^2 P'_1 = 0, \]

\[ (a_2 B_2 - \rho) P'_2 - a_2 (\kappa_1 B_2 + B_1 \kappa_2) P'_1 + B_2 (b_2 - d_2) P_2^2 P'_2 + B_2 (c_2 - e_2) P_1^2 P'_2 = 0. \]

Set

\[ P_2(\xi) = AP_1(\xi), \]

where \( A \) is a non zero constant, such that \( A \neq 1 \). Now, Eqs. (6)–(9) become

\[ a_1 B_1 B_2 A P''_1 + \left[ A(B_1 - a_1 \kappa_1 \kappa_2) - \omega \right] P_1 + \left[ \kappa_1 (b_1 - d_1) + A^2 \kappa_1 (c_1 - e_1) + A^2 \sigma_1 \right] P_1^3 = 0, \]

\[ a_2 B_1 B_2 P''_1 + (B_2 - a_2 \kappa_1 \kappa_2 - A\omega) P_1 + A \left[ \kappa_2 (b_2 - d_2) + \kappa_2 (c_2 - e_2) + \sigma_2 \right] P_1^3 = 0, \]

and

\[ [a_1 B_1 - \rho - a_1 A (\kappa_2 B_1 + B_2 \kappa_1)] P'_1 + [B_1 (b_1 - d_1) + A^2 B_1 (c_1 - e_1)] P_1^2 P'_1 = 0. \]
\[ [A(\alpha_2B_2 - \rho) - a_2(\kappa_1B_2 + B_1\kappa_2)]P'_1 + AB_2[A^2(b_2 - d_2) + c_2 - e_2]P''_1P'_1 = 0. \] (14)

Integrating Eqs. (13) and (14) with zero-integration constants, one gets
\[ [a_1B_1 - \rho - a_1A(\kappa_2B_1 + B_2\kappa_1)]P_1 + \frac{1}{3}B_1[b_1 - d_1 + A^2(c_1 - e_1)]P''_1 = 0, \] (15)
\[ [A(\alpha_2B_2 - \rho) - a_2(\kappa_1B_2 + B_1\kappa_2)]P_1 + \frac{1}{3}AB_2[A^2(b_2 - d_2) + c_2 - e_2]P''_1 = 0. \] (16)

Setting the coefficients of the linearly independent functions of Eqs. (15) and (16) to zero, yields
\[ \rho = a_1B_1 - a_1A(\kappa_2B_1 + B_2\kappa_1), \] (17)
\[ \rho = \frac{Aa_2B_2 - a_2(\kappa_1B_2 + B_1\kappa_2)}{A}, \] (18)

and the constraints conditions
\[ b_1 - d_1 + A^2(c_1 - e_1) = 0, \] (19)
\[ A^2(b_2 - d_2) + c_2 - e_2 = 0. \] (20)

Equations (11) and (12) are equivalent under the constraint conditions:
\[ a_1A = a_2, \] (21)
\[ A(B_1 - a_1\kappa_1\kappa_2) - \omega = B_2 - a_2\kappa_1\kappa_2 - A\omega, \] (22)
\[ \kappa_1(b_1 - d_1) + A^2\kappa_1(c_1 - e_1) + A^2\sigma_1 = A[A^2\kappa_2(b_2 - d_2) + \kappa_2(c_2 - e_2) + \sigma_2]. \] (23)

From (21) to (22), we have the wave number of the soliton:
\[ \omega = \frac{AB_1 - B_2}{(1 - A)}. \] (24)

Equation (11) can be rewritten in the form:
\[ P''_1(\xi) + L_1P'_1(\xi) + L_2P''_1(\xi) = 0, \] (25)
where
\[ L_1 = \frac{A(B_1 - a_1\kappa_1\kappa_2) - \omega}{a_1B_1B_2A}, \quad L_2 = \frac{\kappa_1(b_1 - d_1) + A^2\kappa_1(c_1 - e_1) + A^2\sigma_1}{a_1B_1B_2A}, \] (26)

provided \( a_1B_1B_2A \neq 0 \). In the next two sections, we solve Eq. (25) using the following two methods:
3 The modified Kudryashov’s method

According to this method, we balance $P''_1(\xi)$ with $P_1^3(\xi)$ in Eq. (25), we get:

$$M + 2p = 3M \implies M = p.$$  \hfill (27)

Now, the following cases can be considered.

**Case-1** Setting $p = 1$, then $M = 1$. Thus, Eq. (25) has the formal solution:

$$P_1(\xi) = A_0 + A_1Q(\xi),$$ \hfill (28)

where $A_0$ and $A_1$ are parameters, provided $A_1 \neq 0$. The function $Q(\xi)$ satisfies the differential equation:

$$Q'(\xi) = Q(\xi)[Q(\xi) - 1] \ln K, \quad 0 < K \neq 1.$$ \hfill (29)

Substituting (28) along with (29) into (25), setting all the coefficients of $Q(\xi)^i (i = 0, 1, 2, 3)$ to zero, we have the results

$$A_0 = -\sqrt{-\frac{\ln^2 K}{2L_2}}, \quad A_1 = 2\sqrt{-\frac{\ln^2 K}{2L_2}}, \quad L_1 = \frac{1}{2} \ln^2 K, \quad \text{provided } L_2 < 0.$$

Substituting (30) along with the well-known solutions of Eq. (29) obtained in Zayed et al. (2021a, 2021b, 2021c) into (28), we get the solutions:

$$u(x, y, t) = -A \sqrt{-\frac{\ln^2 K}{2L_2}} \left[ 1 - \frac{2}{1 + e^{K(\xi + \xi_0)}} \right] e^{(-\kappa_1 x - \kappa_2 y + \omega t + \theta_0)},$$ \hfill (31)

and

$$v(x, y, t) = -A \sqrt{-\frac{\ln^2 K}{2L_2}} \left[ 1 - \frac{2}{1 + e^{K(\xi + \xi_0)}} \right] e^{(-\kappa_1 x - \kappa_2 y + \omega t + \theta_0)}.$$ \hfill (32)

The solutions (31), (32) can be rewritten in the form

$$u(x, y, t) = -A \sqrt{-\frac{\ln^2 K}{2L_2}} \left[ 1 - \frac{2}{1 + e \left\{ \cosh [((\xi + \xi_0) \ln k] + \sinh [(\xi + \xi_0) \ln k] \right\}} \right] e^{(-\kappa_1 x - \kappa_2 y + \omega t + \theta_0)},$$ \hfill (33)

and

$$v(x, y, t) = -A \sqrt{-\frac{\ln^2 K}{2L_2}} \left[ 1 - \frac{2}{1 + e \left\{ \cosh [((\xi + \xi_0) \ln k] + \sinh [(\xi + \xi_0) \ln k] \right\}} \right] e^{(-\kappa_1 x - \kappa_2 y + \omega t + \theta_0)},$$ \hfill (34)

which represent the combo bright-singular solutions. In particular, if $e = 1$, we have the dark soliton solution:
\[ u(x, y, t) = \sqrt{-\frac{\ln^2 K}{2L_2}} \tanh \left( \frac{\xi + \xi_0}{2} \right) L_2 e^{i(-\kappa_1 x - \kappa_2 y + \omega t + \theta_0)}, \]  
(35)

and

\[ v(x, y, t) = -A \sqrt{-\frac{\ln^2 K}{2L_2}} \tanh \left( \frac{\xi + \xi_0}{2} \right) L_2 e^{i(-\kappa_1 x - \kappa_2 y + \omega t + \theta_0)}, \]  
(36)

while if \( e = -1 \), we have the singular solution:

\[ u(x, y, t) = \sqrt{-\frac{\ln^2 K}{2L_2}} \coth \left( \frac{\xi + \xi_0}{2} \right) L_2 e^{i(-\kappa_1 x - \kappa_2 y + \omega t + \theta_0)}, \]  
(37)

and

\[ v(x, y, t) = -A \sqrt{-\frac{\ln^2 K}{2L_2}} \coth \left( \frac{\xi + \xi_0}{2} \right) L_2 e^{i(-\kappa_1 x - \kappa_2 y + \omega t + \theta_0)}. \]  
(38)

**Case-2** Setting \( p = 2 \), then \( M = 2 \). Thus, Eq. (25) has the formal solution:

\[ P_1(\xi) = A_0 + A_1 Q(\xi) + A_2 Q^2(\xi), \]  
(39)

where \( A_0, A_1, \) and \( A_2 \) are parameters, provided \( A_2 \neq 0 \). The function \( Q(\xi) \) satisfies the differential equation:

\[ Q'(\xi) = Q(\xi) \left[ Q^2(\xi) - 1 \right] \ln K, 0 < K \neq 1. \]  
(40)

Substituting (39) along with (40) into (25), setting all the coefficients of \( Q(\xi)^l (l = 0, \ldots, 6) \) to zero, we have the results

\[ A_0 = -\frac{1}{2} \sqrt{-\frac{8 \ln^2 K}{L_2}}, \quad A_1 = 0, \quad A_2 = \sqrt{-\frac{8 \ln^2 K}{L_2}}, \quad L_1 = 2 \ln^2 K, \]  
(41)

provided \( L_2 < 0 \). Substituting (41) along with the well-known solutions of Eq. (40) obtained in Zayed et al. (2021a, 2021b, 2021c) into (39), we have the solutions:

\[ u(x, y, t) = \sqrt{-\frac{8 \ln^2 K}{L_2}} \left[ 1 - \frac{2}{1 + e^{k(2\xi + \xi_0)}} \right] e^{i(-\kappa_1 x - \kappa_2 y + \omega t + \theta_0)}, \]  
(42)

and

\[ v(x, y, t) = \sqrt{-\frac{8 \ln^2 K}{L_2}} \left[ 1 - \frac{2}{1 + e^{k(2\xi + \xi_0)}} \right] e^{i(-\kappa_1 x - \kappa_2 y + \omega t + \theta_0)}. \]  
(43)

The solutions (42), (43) can be rewritten in the following form of the combo bright-singulart solutions:
\[ u(x, y, t) = -\frac{1}{2} \sqrt{-\frac{8 \ln^2 K}{L_2}} \left[ 1 - \frac{2}{1 + e^{\left(\frac{(2\xi + \xi_0)}{2} \ln K\right) \sinh\left(\frac{(2\xi + \xi_0)}{2} \ln K\right)}} \right] e^{(-\kappa_1 x - \kappa_2 y + \omega t + \theta_0)}, \]  

(44)

and

\[ v(x, y, t) = -\frac{1}{2} A \sqrt{-\frac{8 \ln^2 K}{L_2}} \tan \left[ \frac{(2\xi + \xi_0)}{2} \ln K \right] e^{(\kappa_1 x - \kappa_2 y + \omega t + \theta_0)}. \]  

(45)

In particular, when \( \epsilon = 1 \), we have the dark soliton solution:

\[ u(x, y, t) = -\frac{1}{2} \sqrt{-\frac{8 \ln^2 K}{L_2}} \tanh \left[ \frac{(2\xi + \xi_0)}{2} \ln K \right] e^{(-\kappa_1 x - \kappa_2 y + \omega t + \theta_0)}, \]  

(46)

and

\[ v(x, y, t) = -\frac{1}{2} A \sqrt{-\frac{8 \ln^2 K}{L_2}} \tanh \left[ \frac{(2\xi + \xi_0)}{2} \ln K \right] e^{(-\kappa_1 x - \kappa_2 y + \omega t + \theta_0)}, \]  

(47)

while if \( \epsilon = -1 \), we have the singular solution:

\[ u(x, y, t) = -\frac{1}{2} \sqrt{-\frac{8 \ln^2 K}{L_2}} \coth \left[ \frac{(2\xi + \xi_0)}{2} \ln K \right] e^{(-\kappa_1 x - \kappa_2 y + \omega t + \theta_0)}, \]  

(48)

and

\[ v(x, y, t) = -\frac{1}{2} A \sqrt{-\frac{8 \ln^2 K}{L_2}} \coth \left[ \frac{(2\xi + \xi_0)}{2} \ln K \right] e^{(-\kappa_1 x - \kappa_2 y + \omega t + \theta_0)}. \]  

(49)

Similarly, we can find many other solutions by choosing other values for \( p \) and \( M \).

### 4 The addendum to Kudryashov’s method

According to this method, we balance \( P'_1(\xi) \) with \( P_1^3(\xi) \) in Eq. (25), we get the relation (27). Let us now discuss the following cases:

**Case-1** Choose \( p = 1 \), then \( M = 1 \). Now, we have the formal solution

\[ P_1(\xi) = \chi_0 + \chi_1 R(\xi), \]  

(50)

where \( \chi_0 \) and \( \chi_1 \) are parameters, provided \( \chi_1 \neq 0 \). Here \( R(\xi) \) satisfies the differential equation:

\[ R''(\xi) = R^2(\xi) \left[ 1 - \chi R^2(\xi) \right] \ln^2 K, \quad 0 < K \neq 1, \]  

(51)

where \( \chi \) is a constant. Substituting (50), (51) in (25) and setting all the coefficients of \([R(\xi)]^{q_1}[R'(\xi)]^{q_2}, (q_1 = 0, 1, 2, 3, q_2 = 0, 1)\), to zero, we have the results
\[ \chi_0 = 0, \quad \chi_1 = \sqrt{\frac{2\chi \ln^2 K}{L_2}}, \quad L_1 = -\ln^2 K, \]  

provided \( \chi L_2 > 0 \) and \( L_1 < 0 \). Substituting (52) along with the well-known solutions of Eq. (51) obtained in Zayed et al. (2021a, 2021b, 2021c) in (50), we have the solutions:

\[ u(x, y, t) = \sqrt{\frac{2\chi \ln^2 K}{L_2}} \left( \frac{4S}{4S^2 K \frac{\chi}{\xi} + \chi K^{-\frac{\chi}{\xi}}} \right) e^{i(-\kappa_1 x - \kappa_2 y + \omega t + \theta_0)}, \]  

and

\[ v(x, y, t) = A \sqrt{\frac{2\chi \ln^2 K}{L_2}} \left( \frac{4S}{4S^2 K \frac{\chi}{\xi} + \chi K^{-\frac{\chi}{\xi}}} \right) e^{i(-\kappa_1 x - \kappa_2 y + \omega t + \theta_0)}. \]

The solutions (53), (54) can be rewritten in the form:

\[ u(x, y, t) = \sqrt{\frac{2\chi \ln^2 K}{L_2}} \left[ \frac{4S}{(4S^2 + \chi) \cosh (\xi \ln K) + (4S^2 - \chi) \sinh (\xi \ln K)} \right] e^{i(-\kappa_1 x - \kappa_2 y + \omega t + \theta_0)}, \]  

and

\[ v(x, y, t) = A \sqrt{\frac{2\chi \ln^2 K}{L_2}} \left[ \frac{4S}{(4S^2 + \chi) \cosh (\xi \ln K) + (4S^2 - \chi) \sinh (\xi \ln K)} \right] e^{i(-\kappa_1 x - \kappa_2 y + \omega t + \theta_0)}, \]

which represent the combo bright-singular solution. In particular, when \( \chi = 4S^2 \), we have the bright soliton solution:

\[ u(x, y, t) = \sqrt{\frac{2 \ln^2 K}{L_2}} \sech (\xi \ln K) e^{i(-\kappa_1 x - \kappa_2 y + \omega t + \theta_0)}, \]  

and

\[ v(x, y, t) = A \sqrt{\frac{2 \ln^2 K}{L_2}} \sech (\xi \ln K) e^{i(-\kappa_1 x - \kappa_2 y + \omega t + \theta_0)}, \]

provided \( L_2 > 0 \), while if \( \chi = -4S^2 \), we have the singular solutions:

\[ u(x, y, t) = \sqrt{-\frac{2 \ln^2 K}{L_2}} \csch (\xi \ln K) e^{i(-\kappa_1 x - \kappa_2 y + \omega t + \theta_0)}, \]  

and

\[ v(x, y, t) = A \sqrt{-\frac{2 \ln^2 K}{L_2}} \csch (\xi \ln K) e^{i(-\kappa_1 x - \kappa_2 y + \omega t + \theta_0)}, \]
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\[ P_1(\xi) = \chi_0 + \chi_1 R(\xi) + \chi_2 R^2(\xi), \]  

(61)

where \( \chi_0, \chi_1 \) and \( \chi_2 \) are parameters, provided \( \chi_2 \neq 0 \). Here \( R(\xi) \) satisfies the differential equation:

\[ R''(\xi) = R^2(\xi) \left[ 1 - \chi R^4(\xi) \right] \ln^2 K, \quad 0 < K \neq 1, \]  

(62)

where \( \chi \) is a constant. Substituting (61), (62) in (25) and setting all the coefficients of \( [R(\xi)]^{q_1} [R'(\xi)]^{q_2} \), \( (q_1 = 0, \ldots, 6, q_2 = 0, 1) \) to zero, we have the results

\[ \chi_0 = 0, \chi_1 = 0, \chi_2 = \sqrt{\frac{8\chi \ln^2 K}{L_2}}, L_1 = -4\ln^2 K, \]  

(63)

provided \( \chi L_2 > 0 \) and \( L_1 < 0 \). Substituting (63) along with the well-known solutions of Eq. (62) obtained in Zayed et al. (2021a, 2021b, 2021c) into (61), we have the solutions:

\[ u(x, t) = \sqrt{\frac{8\chi \ln^2 K}{L_2}} \left( \frac{4S}{4S^2 K^2 + \chi K^{-2\xi}} \right) e^{i(-\kappa_1 x - \kappa_2 y + \omega t + \theta_0)}, \]  

(64)

and

\[ v(x, t) = A \sqrt{\frac{8\chi \ln^2 K}{L_2}} \left( \frac{4S}{4S^2 K^2 + \chi K^{-2\xi}} \right) e^{i(-\kappa_1 x - \kappa_2 y + \omega t + \theta_0)}. \]  

(65)

The solutions (64), (65) can be rewritten in the following form of the combo bright-singular solutions:

\[ u(x, t) = \sqrt{\frac{8\chi \ln^2 K}{L_2}} \left[ \frac{4S}{(4S^2 + \chi) \cosh(2\xi \ln K) + (4S^2 - \chi) \sinh(2\xi \ln K)} \right] e^{i(-\kappa_1 x - \kappa_2 y + \omega t + \theta_0)}, \]  

(66)

and

\[ v(x, t) = A \sqrt{\frac{8\chi \ln^2 K}{L_2}} \left[ \frac{4S}{(4S^2 + \chi) \cosh(2\xi \ln K) + (4S^2 - \chi) \sinh(2\xi \ln K)} \right] e^{i(-\kappa_1 x - \kappa_2 y + \omega t + \theta_0)}. \]  

(67)

In particular, when \( \chi = 4S^2 \), we have the bright soliton solution:

\[ u(x, y, t) = \sqrt{\frac{8\ln^2 K}{L_2}} \sech(2\xi \ln K) e^{i(-\kappa_1 x - \kappa_2 y + \omega t + \theta_0)}, \]  

(68)

and
provided $L_2 > 0$, while if $\chi = -4S^2$, we have the singular solution:

\[
u(x, y, t) = A\sqrt{\frac{8 \ln^2 K}{L_2}} \text{sech} \left(2\xi \ln K \right) e^{i(-\kappa_1 x - \kappa_2 y + \omega t + \theta_0)}, \quad (69)
\]

and

\[
u(x, y, t) = A\sqrt{\frac{8 \ln^2 K}{L_2}} \text{csch} \left(2\xi \ln K \right) e^{i(-\kappa_1 x - \kappa_2 y + \omega t + \theta_0)}, \quad (70)
\]

provided $L_2 < 0$.

Similarly, we can find many other solutions by choosing other values for $p$ and $M$.

5 Numerical simulations

In this section, we introduce the graphs of the absolute values of some solutions for Eqs. (35) and (36), Eqs. (48) and (49) and Eqs. (57) and (58). Let us now examine Figs. 1, 2 and 3, as it illustrates some of our solutions obtained in this paper. To this purpose, we choose some special values of the obtained parameters.

**Figure 1** shows the profile of the absolute values of the dark soliton solutions (35) and (36).

**Figure 2** shows the profile of the absolute values of the singular solutions (48) and (49).

**Figure 3** shows the profile of the absolute values of the bright soliton solutions (57) and (58).

**Fig. 1** The numerical simulations of the absolute values of the solutions (35) and (36) with the parameter values $a_1 = 0.02, a_2 = 0.03, \kappa_1 = 0.2, \kappa_2 = 0.1, b_1 = 0.1, d_1 = 0.2, c_1 = 0.01, e_1 = 0.03, \sigma_1 = -0.05, B_1 = 0.2, B_2 = 0.3, y = 0, a_3 = 0.2, K = 3, \xi_0 = 0.1, -5 \leq x, t \leq 5.
From the above Figures, one can see that the obtained solutions possess the dark soliton, the singular and the bright soliton solutions. Also, these Figures express the behavior of these solutions which give some perspective readers how the behavior solutions are produced.

6 Conclusions

This paper studies the nonlinear (2+1)-dimensional Kundu–Mukherjee–Naskar equation in fiber Bragg gratings by the aid of two mathematical approaches which are the modified Kudryashov’s approach and the addendum to Kudryashov’s technique. Optical solitons...
to the coupled system (2) and (3) are regained. These optical solitons are emerged from constraint conditions. The coupled system (2) and (3) is studied in this paper for the first time. The results of the current paper have not been determined before. Also, the graphs of Eqs. (35), (36), (48), (49), (57) and (58) are given by choosing special values of constants. Profiles of dark soliton, singular and bright soliton solutions respectively are represented in Figs. 1, 2 and 3 which express the behavior of optical solitons in fiber Bragg gratings.

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