ELECTROMAGNETIC RADIATION PRODUCES FRAME DRAGGING

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It is shown that for a generic electrovacuum spacetime, electromagnetic radiation produces vorticity of worldlines of observers in a Bondi–Sachs frame. Such an effect (and the ensuing gyroscope precession with respect to the lattice) which is a reminiscense of generation of vorticity by gravitational radiation, may be linked to the nonvanishing of components of the Poynting and the super–Poynting vectors on the planes orthogonal to the vorticity vector. The possible observational relevance of such an effect is commented.

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I. INTRODUCTION

Nowadays high–accuracy experiments in space can test relativistic gravity. These endeavours surely should lead to direct detection of gravitational waves \cite{1}. Particularly interesting are the frame dragging observations using cryogenic gyroscopes in Gravity Probe B searches \cite{2} and from data provided by LAGEOS satellites \cite{3, 4}. These successful experiments have fundamental implications for general relativity since frame dragging is one of the most conspicuous general relativistic effect \cite{5}. Furthermore, they are also very important in astrophysics because the frame dragging has been invoked as a mechanism to drive relativistic jets emanating from galactic nuclei \cite{6}. Dual jets are expected for inspiral supermassive binary black hole systems which produce prodigious quantities of gravitational waves and energetic electromagnetic events \cite{7}, which in some way have to be closely related. Less impressive outcomes but of utmost importance are the intense electromagnetic outbursts from hyperenergetic phenomena such as collapsing hypermassive neutron stars \cite{8} and Gamma Ray Bursts \cite{9}.

Now, it is a well established fact that gravitational radiation produces vorticity in the congruence of observers with respect to the compass of inertia \cite{10–14}. Since the vorticity vector describes the proper angular velocity of the compass of inertia (gyroscope) with respect to reference particles \cite{16}, it is clear that a frame dragging effect is associated with gravitational radiation.

In \cite{14} it was further shown that such a vorticity is closely related to the non–vanishing of super–Poynting vector components on the plane orthogonal to the vorticity vector. Later it was shown that the vorticity appearing in stationary vacuum spacetimes is also related to the existence of a flow of superenergy on the plane orthogonal to the vorticity vector \cite{16}.

The rationale to link vorticity and the super–Poynting vector comes from an idea put forward by Bonnor in order to explain the appearance of vorticity in the space–time generated by a charged magnetic dipole \cite{17}. Indeed, Bonnor observes that for such a system there exists a non–vanishing component of the Poynting vector, describing a flow of electromagnetic energy round in circles \cite{18}. He then suggests that such a circular flow of energy affects inertial frames by producing vorticity of congruences of particles, relative to the compass of inertia. Later, this conjecture was shown to be valid for a general axially symmetric stationary electrovacuum metric \cite{12}.

In \cite{13} it was suggested for the first time that a similar mechanism might be at the origin of vorticity in the gravitational case, i.e. a circular flow of gravitational energy would produce vorticity. However due to the well known problems associated to a local and invariant definition of gravitational energy it was unclear at that time what expression for the “gravitational” Poynting vector should be used. Following a suggestion by Roy Maartens to one of us (L.H.) we tried in \cite{14} with the super–Poynting vector based on the Bel–Robinson tensor \cite{20–23}. Doing so we were able to establish the link between gravitational radiation and vorticity, invoking a mechanism similar to that proposed by Bonnor for the charged magnetic dipole.

Our purpose with this work is to tie up an important loose end related to this problem, namely: Does electromagnetic radiation produces vorticity?, and if so can we explain such vorticity through a Bonnor–like mechanism?

We shall analyze a generic electrovacuum spacetime following the scheme developed by van der Burg \cite{24}, which is an extension of the Bondi formalism \cite{25–26} as to include electromagnetic fields. After a brief review of van der Burg paper we shall proceed to calculate the vorticity, the Poynting and the super–Poynting vectors. Based on the analysis of the obtained results we shall
then answer the two questions raised above.

II. ASYMPTOTIC EXPANSIONS FOR THE EINSTEIN–MAXWELL FIELD

In [23] van der Burg presents a generalization of the Bondi–Sachs formalism for the Einstein–Maxwell system. This has, among other things, the virtue of providing a clear and precise criterion for the existence of gravitational and electromagnetic radiation. Namely, if the news functions (gravitational and/or electromagnetic) are zero over a time interval, then there is no radiation (gravitational and/or electromagnetic) during that interval.

The formalism has as its main drawback [27] the fact that it is based on a series expansion which could not give closed solutions and which raises unanswered questions about convergence and appropriateness of the expansion. However since we shall consider regions of spacetime very far from the source, we shall use in our calculations only the leading terms in the expansion of metric functions. Furthermore, since the source is assumed to radiate during a finite interval, then no problem of convergence appears [28].

The general form of the metric in the Bondi–Sachs formalism can be written as [26]

$$ds^2 = \left(\frac{e^{2\beta} V}{r} - r^2 h_{AB} U^A U^B\right) du^2 + 2e^{2\beta} du dr + 2r^2 h_{AB} U^B du A^A dr A^B,$$

(1)

where all the metric components are functions of $x^0 = u$, $x^1 = r$, $x^2 = \theta$, $x^3 = \phi$. $\beta$ is the expansion of outward null rays, $V$ the Newtonian like potential. If we make a $2 + 1$ foliation doing constant $r$, $e^{2\beta} V/r$ is the lapse, $U^A$ the shift and $h_{AB}$ the 2–surface metric of constant $u$, satisfying $h^{AB} h_{BC} = \delta^A_C$. $u$ is a timelike coordinate, constant along outgoing radial null geodesics, while $r$ is a luminosity–distance parameter.

The electromagnetic field is characterized by a skew–symmetric tensor $F_{\mu\nu}$ giving rise the energy–momentum tensor $T_{\mu\nu}$ defined by

$$T_{\mu\nu} = \frac{1}{4} g_{\mu\nu} F_{\gamma\delta} F^{\gamma\delta} - g_{\gamma\delta} F_{\mu\gamma} F_{\nu\delta},$$

(2)

being $g_{\mu\nu}$ the metric tensor given by [1]. The Einstein–Maxwell field equations for the electro–vacuum spacetime outside the source are

$$R_{\mu\gamma} + T_{\mu\gamma} = 0,$$

(3)

$$F_{\mu\nu;\gamma} = 0,$$

(4)

$$F_{\nu\mu} = 0,$$

(5)

where $R_{\mu\gamma}$ is the Ricci tensor. Note that the units used are $16\pi G = c = 1$. Our purpose here is to extract general information from the radiative zone, at $J^+$ (future null–infinity), where the well known asymptotic expansion in power of $r^{-1}$ of Bondi seems to be the most convenient and precise.

The metric [11] can be written as follows [20]

$$ds^2 = (V r^{-1} e^{2\beta} - r^2 e^{2\gamma} U^2 cos\theta - r^2 e^{-2\gamma} W^2 \times$$

$$\cos\theta - 2r^2 (U W \sin\beta) du \times$$

$$+ (e^{2\gamma} U \cos\theta - 2r^2 (U W \sin\beta) du \phi + 2r^2 (e^{-2\gamma} W \cos\theta - 2r^2 \cos\theta du \phi + e^{-2\gamma} \cos\theta du \phi + 2 \sin\theta du \phi) \times$$

$$+ \cos\theta - 2r^2 (U W \sin\beta) du \phi + 2r^2 (e^{-2\gamma} W \cos\theta - 2r^2 \cos\theta du \phi + e^{-2\gamma} \cos\theta du \phi + 2 \sin\theta du \phi),$$

(6)

if we choose the gauge of Bondi, $det(h_{AB}) = det(q_{AB})$, where $q_{AB}$ is the unit 2–sphere metric; $\gamma$, $\delta$, $U$, $W$ are functions of $(u, r, \theta, \phi)$. In the present case the spacetime is asymptotically flat and necessarily Minkowskian by construction.

The procedure follows the script established by Bondi et al. [27]. Thus, four functions are assumed to be expanded as power series in negative powers of $r$, and prescribed on a hypersurface $u = u_0 =$ constant, namely (see [24] for details)

$$\gamma = cr^{-1} + \left[ C - \frac{1}{6} c^3 \right] r^{-3} + D r^{-4} + ...,$$

(7)

$$\delta = dr^{-1} + \left[ H + c^2 d/2 - d^3/6 \right] r^{-3} + K r^{-4} + ..., $$

(8)

$$F_{12} = er^{-2} + (2E + ec + fc) r^{-3} + ..., $$

(9)

$$F_{13} \csc \theta = fr^{-2} + (2F + cd - fc) r^{-3} + ..., $$

(10)

where all coefficients are functions of $u$, $r$ and $\theta, \phi$.

Next, from (7)–(10) and a subset of field equations (main equations), the following expressions are obtained for metric and electromagnetic variables:

$$\beta = -(c^2 + d^2) r^{-2}/4 + ...,$$

(11)

$$U = -(c_\theta + 2c \cot \theta + d_\phi \csc \theta) r^{-2}$$

$$+ [2N + 3(c_\theta + d_\phi)] + 4(c^2 + d^2) \cot \theta$$

$$- 2(c_\theta - cd_\phi \csc \theta) r^{-3}$$

$$+ \frac{1}{2} \{3C_\phi + 2C \cot \theta + H_\phi \csc \theta - 6(cN + dQ)$$

$$- 4(2c^2 c_\theta + c_\phi d_\phi + c_\phi d^2) - 8c(c^2 + d^2) \cot \theta - 4(c^2 d_\phi$$

$$+ c_\phi d + 2d^2 d_\phi) \csc \theta - (ee - \mu f) \r^{-4} + ...,$$

(12)

$$W = -(d_\theta + 2d \cot \theta - c_\phi \csc \theta) r^{-2}$$

$$+ [2Q + 2(c_\phi d -\phi)] + 3(\phi + d_\phi) c\theta) r^{-3}$$

$$+ \frac{1}{2} \{3(H_\theta + 2H \cot \theta - C_\phi \csc \theta) - 6(cQ - dN)$$

$$- 4(2d^2 d_\phi + c_\phi d_\phi + c_\phi d^2) - 8d(c^2 + d^2) \cot \theta + 4(c_\phi d^2$$

$$+ c_\phi d + 2c^2 c_\phi) \csc \theta + (\mu e + e f) r^{-4}) \r^{-4} + ...,$$

(13)
\[ V = r - 2M - [N_\theta + N \cot \theta + Q_\phi \csc \theta - \frac{1}{2}(c^2 + d^2)] - (c_\phi^2 + d_\phi^2) \cot \theta - 4(c_\phi d - c_\theta \csc \theta) \cot \theta + 2(c_\phi \theta - c_\theta \csc \phi) \cot \theta \] 
\[ + 2(c_\phi \theta - c_\theta \csc \phi) \csc \theta \left( - \frac{1}{2}(c^2 + d^2) \right) r^{-1} + \{ \cdots - \mu(f_\theta + f \cot \theta - e_\phi \csc \theta) \} r^{-2} \cdots, \] (14)

\[ F_{01} = -c \theta^{-2} + (e_\phi + e \cot \theta + f_\phi \csc \theta) \theta^{-3} + ..., \] (15)

\[ F_{02} = X + (e_\phi - e \cot \theta - f_\phi \csc \theta) \theta^{-1} + \{ [E + \frac{1}{2}(e \theta + f \phi)]_u \theta^{-2} + ..., \] (16)

\[ \csc \theta F_{03} = Y + (c_\phi \csc \theta - f_\phi) \theta^{-1} + \{ [F + \frac{1}{2}(e \phi - f \theta)]_u \theta^{-2} + ..., \] (17)

\[ \csc \theta F_{23} = -\mu - (f_\theta + f \cot \theta - e_\phi \csc \theta) \theta^{-1} + \cdots, \] (18)

where again, all coefficients are functions of \( u, \theta \) and \( \phi \), and subscript letters denote derivatives.

At this point we have nine functions of three variables which are undetermined on the initial hypersurface, namely \( M, N, Q, \epsilon, \mu, c_u, d_u, X, Y \). However using the remaining field equations (supplementary conditions), equations for the \( u \)-derivatives of \( M, N, Q, \epsilon, \mu \), in terms of the prescribed functions, can be obtained. Hence if \( \gamma, \delta, F_{12}, F_{13}, M, N, Q, \epsilon, \mu \) are prescribed on a given initial hypersurface \( u = \) constant, the evolution of the system is fully determined provided the four functions, referred to as news functions, \( c_u, d_u, X, Y \) are given for all \( u \). In other words, whatever happens at the source leading to changes in the field, it can only do so by affecting news functions and viceversa. In light of this comment the relationship between the news functions and the occurrence of radiation becomes clear.

Furthermore, the mass function \( m(u) \) which coincides with the Schwarzchild mass in the static case, is defined by

\[ m(u) = \int_0^{2\pi} \int_0^\pi M^* \sin \theta d\theta d\phi, \] (19)

with

\[ M^* = M + \frac{1}{2}i(\partial/\partial \theta + \cot \theta)(d \theta + 2d \cot \theta - c_\phi \csc \theta) - \csc \theta/\partial \phi(c_\phi + 2e \cot \theta + d_\phi \csc \theta). \] (20)

Then, introducing the intermediate complex quantities:

\[ e^* = e + id, \quad X^* = X + iY, \] (21)

it follows from one of the supplementary conditions (see [24] for details)

\[ m_u = -\int_0^{2\pi} \int_0^\pi \left( c_u^* c_u^* + \frac{1}{2} \bar{X}^* X^* \right) \sin \theta d\theta d\phi, \] (22)

clearly exhibiting the decreasing of the mass function in the presence of news (bar denotes complex conjugate).

Finally, the following equations derived from the supplementary conditions (Eqs.(14)-(20) in [21] will be used:

\[ M_u^* = -c_u^* c_u^* - \frac{1}{2} \bar{X}^* \bar{X}^* + \frac{1}{2} \bar{L}_\bar{L}_2 e_u^*, \] (23)

\[ N_u^* = \mathcal{L}_0 M^* + 2c^* \bar{L}_2 e_u^* + \bar{L}_2 (c_u^* c_u^*) - \bar{c}^* \bar{X}^*, \] (24)

\[ 4C_u^* = 2c^* \bar{c}^* + 2c^* \bar{M}^* + \mathcal{L}_1 N^* + e^* X^*, \] (25)

\[ 4D_u^* = -\bar{L}_2 (\mathcal{L}_2 c^* + 2c^* N^*) + \frac{1}{3} \bar{L}_1 e^* - \bar{c}^* \bar{c}^* + \frac{4}{3} E^* X^*, \] (27)

\[ 2c_u^* = -\mathcal{L}_0 \bar{c}^* - 2c^* \bar{X}^*, \] (28)

\[ 4E_u^* = -\bar{L}_2 (\mathcal{L}_1 e^* - 2c^* \bar{c}^*), \] (29)

where

\[ N^* = N + iQ, \quad C^* = C + iH \quad D^* = D + iK, \cdots \] (30)

\[ e^* = e + i\mu, \quad e^* = e + if \quad E^* = E + iF, \cdots \] (31)

and

\[ \mathcal{E}_p = -(\partial/\partial \theta - p \cot \theta + i \csc \theta \partial/\partial \phi). \] (32)

We shall next calculate the general expressions for the vorticity, Poynting (electromagnetic) and super-Poynting vectors for our system.
III. VORTICITY, POYNTING AND SUPER–POYNTING VECTORS

The vorticity vector is defined as usual by
\[ \omega^\alpha = \frac{1}{2\sqrt{-g}} \eta^{\alpha \gamma \lambda} u_\gamma u_\lambda, \]
where \( \eta_{\alpha \beta \gamma \delta} = +1 \) for \( \alpha, \beta, \gamma, \delta \) in even order, \(-1\) for \( \alpha, \beta, \gamma, \delta \) in odd order and 0 otherwise; \( u_\mu \) is the 4–velocity vector for an observer at rest in the considered frame. The absolute value of \( \omega^\alpha \) is denoted by \( \Omega \), i.e.
\[ \Omega = |\omega^\alpha \omega_\alpha|^{1/2}. \]

The electromagnetic Poynting vector is by definition
\[ S^\alpha = T^\alpha_\beta u_\beta, \]
whereas the super-Poynting vector based on the Bel–Robinson tensor, as defined in [21], is
\[ P_\alpha = \eta_{\alpha \beta \gamma \delta} E^\beta \eta^{\gamma \rho} u_\delta, \]
where \( E_{\mu \nu} \) and \( H_{\mu \nu} \), are the electric and magnetic parts of Weyl tensor, respectively, formed from Weyl tensor

\[ \omega^\alpha = -\frac{1}{2e^2 \sin \theta} \left( r^2 e^{-2\beta} (WU_r - UW_r) + 2r^2 \sinh^2 \gamma \cosh \delta - 2U \left( W^2 e^{2\gamma} + W^2 e^{-2\gamma} \right) + 4U W r^2 \cosh^2 \gamma \right) e^{-2\beta} \gamma_r \\
+ 2r^2 e^{-2\beta} (W^2 e^{-2\gamma} - U^2 e^{2\gamma}) \delta_r + e^{2\beta} [e^{-2\beta} (W \sinh \delta + e^{-2\gamma} W \cosh \delta)]_{\theta} \\
- e^{2\beta} [e^{-2\beta} (W \sinh 2\delta + e^{-2\gamma} U \cosh 2\delta)]_{\phi}, \]

\[ \omega^\theta = \frac{1}{e^{2\beta} \sin \theta} \left( 2e^2 A^{-2} \left[ (U^2 e^{2\gamma} + W^2 e^{-2\gamma}) \sinh 2\delta \cosh \delta + U W \cosh^2 2\delta \right] + (W^2 e^{-2\gamma} - U^2 e^{2\gamma}) \delta_u \\
+ \frac{1}{2} (W U - U W) \right] + A^2 \left[ A^{-2} (W e^{-2\gamma} \cosh 2\delta + U \sinh 2\delta) \right]_{\theta} - A^2 \left[ A^{-2} (W \sinh 2\delta + U e^{2\gamma} \cosh 2\delta) \right]_{\phi}, \]

\[ \omega^\phi = \frac{1}{2e^2 \sin \theta} \left( A^2 e^{-2\beta} [e^{2\beta} (W \sinh 2\delta + U e^{2\gamma} \cosh 2\delta) r] - e^{2\beta} A^{-2} [e^{-2\beta} r^2 (U \sinh 2\delta + e^{-2\gamma} W \cosh 2\delta)]_{\theta} + e^{2\beta} A^{-2} (e^{-2\beta} A^2)_{\phi}, \]

and

\[ \omega = \frac{1}{2e^2 \sin \theta} \left( A^2 e^{-2\beta} [e^{2\beta} (W \sinh 2\delta + U e^{2\gamma} \cosh 2\delta) r] - e^{2\beta} A^{-2} [e^{-2\beta} r^2 (U \sinh 2\delta + e^{-2\gamma} W \cosh 2\delta)]_{\theta} + A^{-2} e^{2\beta} (e^{-2\beta} A^2)_{\phi}. \]
Although algebraic manipulation by hand is feasible for the Bondi metric, for the Bondi–Sachs one is quite cumbersome. Therefore we write a Maple 15 script (available upon request) which uses intrinsic procedures to deal with tensors. We proceed in two steps. First, we calculate and save (37), (38) and (39), using the general form [1], that is, without using the metric function expansions [11]–[13]. Second, we expand all the relevant objects separately (metric, Weyl, electric and magnetic parts) up to the leading order. After these two simple steps we were able to write the output for the super–Poynting.

We use the shift vector \( U^A = (U, W/\sin \theta) \) and the 2–surfaces metric of constant \( u \)

\[
h_{\alpha \beta} = \begin{pmatrix} e^{2\gamma} \cosh 2\delta & \sinh 2\delta \sin \theta \\ \sinh 2\delta \sin \theta & e^{-2\gamma} \cosh 2\sin^2 \theta \end{pmatrix}.
\]  

(45)

In our calculations we keep these auxiliary variables, \( U^A \) and \( h_{\alpha \beta} \), as far as was possible.

Thus, for the absolute value of \( \omega^\mu \) we get

\[
\Omega = \Omega_G r^{-1} + \cdots + \Omega_{\mathcal{EM}} r^{-3} + \cdots,
\]

(46)

where subscripts \( G, \mathcal{EM} \) and \( E \mathcal{M} \) stand for gravitational, gravito–electromagnetic and electromagnetic. At this point a remark on the meaning of this notation is in order: “gravitational” terms refer to those terms containing exclusively functions \( M, N, Q, c_u, d_u \) and their derivatives. “Electromagnetic” terms are those containing exclusively functions \( \epsilon, \mu, X, Y \) and their derivatives, whereas “gravitomagnetic” terms refer to those containing functions of either kind and/or combination of both. It should be clear that all functions are related through field equations, and therefore the established division is rather formal, however the splitting as indicated is useful for the discussion below.

In order to get some insight let us first consider the axially and reflection symmetric case, i.e., \( \partial/\partial \phi = d = f = H = K = F = Y = Q = 0 \). Thus, we obtain

\[
\Omega_{\mathcal{EM}} = -\frac{1}{4}[4 c_u M \epsilon - 2 N \epsilon + 2c_u N + 3 \epsilon \c_n + N - 4(M c) - (\epsilon \c) + 2(4M - c)N \]

\[-4c c \c_n + 8 M \epsilon M - 2(1 + 2c_u)N \cot^2 \theta + (8c_M - 8M^2 - 2N - 3c^2) \c_n \theta \]

\[+2(3c_u c^2 - 2c_u N - 8c_M^2 + 4c_u c_M \]

\[-N \c_n + 4c M + 3c_u + N)] \cot \theta],
\]

(47)

where we observe the contribution \( \frac{1}{4}(\epsilon \c) \), which is purely electromagnetic. This latter term together with terms proportional to \( N \) and \( C_n \) may in turn be expressed through electromagnetic news \( (X^*) \) by means of (24)–(26), and (28). Doing so we clearly identify electromagnetic radiation as a voritcous source.

For the angular contravariant super–Poynting components (in the axially symmetric case) we identify many \( \mathcal{EM} \) contributions. One of them is explicit, e.g. \( c_{uu}(\epsilon \c)_u \) in \( P^\theta \) (the complete term is too long to display here; see the appendix) whereas other terms appear by replacing \( u \) derivatives of metric variables by means of (28)–(32). It is remarkable the following simplification

\[
P^\phi_{\mathcal{EM}} = -\csc \theta c_{uu} (\mu c)_u.
\]

(48)

Let us now get back to the most general three–dimensional case, where except for the leading term,

\[
\Omega_G = -\frac{1}{2} \left[(c_{uu} + 2c_u \cot \theta + d_{uu} \csc \theta)^2 \right. \]

\[+ (d_{uu} + 2d_u \cot \theta - c_{uu} \csc \theta)^2 \right]^{-1/2},
\]

(49)

the next leading graviito–electromagnetic term is too long to write here or anywhere (the expression is available upon request).

However, the important point to stress here is that \( \Omega_{\mathcal{EM}} \) contains electromagnetic news functions \( (X^*) \) by means of, for instance of terms

\[
3 \sin \theta c_{uu} d_{uu} \csc \theta, \quad 3c_{uu} d_{uu} f \mu_u,
\]

(50)

which are typical and selected contributing terms to \( \Omega_{\mathcal{EM}} \).

Next, we write the electromagnetic Poynting vector as

\[
S^u = S^u_{\mathcal{EM}} r^{-4} + S^u_{\mathcal{EM}} r^{-5} \cdots,
\]

(51)

\[
S^r = S^r_{\mathcal{EM}} r^{-4} + S^r_{\mathcal{EM}} r^{-5} \cdots,
\]

(52)

\[
S^\theta = S^\theta_{\mathcal{EM}} r^{-4} + S^\theta_{\mathcal{EM}} r^{-5} \cdots,
\]

(53)

\[
S^\phi = S^\phi_{\mathcal{EM}} r^{-4} + S^\phi_{\mathcal{EM}} r^{-5} \cdots,
\]

(54)

where

\[
S^u_{\mathcal{EM}} = \frac{1}{2}(\mu^2 + \epsilon^2),
\]

(55)

\[
S^u_{\mathcal{EM}} = (\mu f - \epsilon \c) \cot \theta - (\epsilon f + \mu f) \csc \theta
\]

\[+ \mu f + (\mu f + (\mu f + \epsilon f) \csc \theta,
\]

(56)

\[
S^r_{\mathcal{EM}} = X^2 + Y^2,
\]

(57)

\[
S^r_{\mathcal{EM}} = -2X c_u + M S^r_{\mathcal{EM}} + 2X c_u - 4d XY
\]

\[+ 2c(Y^2 - X^2) - 2Y f + 2c_y \csc \theta,
\]

(58)

\[
S^\theta_{\mathcal{EM}} = \epsilon X + \mu Y,
\]

(59)

\[
S^\theta_{\mathcal{EM}} = -(Y f \c + X f - \mu \c) \csc \theta - (X e - Y f) \cot \theta
\]

\[+ M S^\theta_{\mathcal{EM}} - 2c(Y X + X c) + \epsilon c - X c - \mu f + Y f - c c u,
\]

(60)
\[ S_{EM}^\phi = \csc \theta (\epsilon Y - \mu X), \]  
\[ S_{\theta EM}^\phi = -\csc \theta (Ye + Xf) \cot \theta - MS_{EM}^\phi \sin \theta \]
\[ - (X e_\phi - Ye f_\phi + e_\phi \epsilon) \csc \theta + (2f_\phi f_\theta + \epsilon)X \]
\[ + (\epsilon - 2c_\phi Y) + \epsilon f_u + (\epsilon - c_u) \mu]. \]  
(62)

It is worth noticing that terms explicitly containing \( X^* \) appear in \( \theta \) and \( \phi \) component of the Poynting vector.

Next, calculation of the super–Poynting gives the following result

\[ P_\mu = (0, P_r, P_\theta, P_\phi), \]  
(63)

where the explicit terms are too long to be written at this point.

The leading terms for each super–Poynting (contravariant) component are

\[ P^u = P^u_{\theta r} r^{-4} + \ldots, \]
\[ P^r = P^r_{\theta r} r^{-4} + \ldots, \]
\[ P^\theta = P^\theta_{\theta r} r^{-4} + \ldots + P^\theta_{\theta \phi} r^{-6} + \ldots, \]
\[ P^\phi = P^\phi_{\theta r} r^{-4} + \ldots + P^\phi_{\theta \phi} r^{-6} + \ldots, \]  
(64)

where

\[ P^u_{\theta r} = 2[2(d_{uu} d_u + c_{uu} c_u) \cot \theta + c_{uu} c_{\theta u} \]
\[ + (c_{uu} d_{\theta u} - d_{uu} c_{\theta u}) \csc \theta + d_{uu} d_{\theta u}], \]
\[ P^\theta_{\theta r} = 2 \csc \theta [2(c_{uu} d_u - d_{uu} c_u) \cot \theta + c_{uu} d_{\theta u} \]
\[ - d_{uu} c_{\theta u} - (c_{uu} c_{\theta u} + d_{uu} d_{\theta u}) \csc \theta]. \]  
(65)

Other terms are too long to display. However the important point is that, again, terms containing electromagnetic news appear in \( P_n^u \), for example

\[ \cot^4 \theta c_u u c_\epsilon \]
\[ \cot^4 \theta c_u u f \mu_u \]  
(67)

and

\[ 2 \csc^3 \theta \cot^2 \theta d_{uu} e \epsilon_u, \]
\[ \csc^5 \theta d_{uu} f \mu_u, \]  
(68)

which are typical and selected contributing terms to the super–Poynting components \( P^u_{\theta EM} \) and \( P^\phi_{\theta EM} \) respectively.

It is easy to check that the stationary case satisfy well known results, for instance, that of Bonnor or the Kerr–Newman. Now, contributions to each relevant object are superior for \( s \Omega \) and \( s P^\alpha \) with respect to the general (radiative) case, but it is not true for \( s S^\alpha \) which keeps the same leading terms:

\[ s \Omega = s \Omega_{EM} r^{-2} + \ldots + s \Omega_{\theta EM} r^{-4} + \ldots \]  
(69)
\[ s S^u = s S^u_{EM} r^{-4} + s S^u_{\theta EM} r^{-5} + \ldots \]  
(70)
\[ s S^r = s S^r_{EM} r^{-4} + \ldots \]  
(71)
\[ s S^\theta = s S^\theta_{EM} r^{-5} + \ldots \]  
(72)
\[ s S^\phi = s S^\phi_{EM} r^{-5} + \ldots \]  
(73)
\[ s P^u = s P^u_{EM} r^{-6} + \ldots, \]  
(74)
\[ s P^r = s P^r_{EM} r^{-6} + \ldots \]  
(75)
\[ s P^\theta = s P^\theta_{EM} r^{-7} + \ldots + s P^\theta_{\theta EM} r^{-9} + \ldots \]  
(76)
\[ s P^\phi = s P^\phi_{EM} r^{-7} + \ldots + s P^\phi_{\theta EM} r^{-9} + \ldots \]  
(77)

where

\[ s \Omega_{EM} = \frac{1}{2} \left[ \left( 4 (d_{\phi \phi} c_{\phi \theta} - 3 d_{\phi \theta} c_{\phi \theta}) \cos \theta \right) \right. \]
\[ + \left. 2 (c_{\phi \phi} c_{\phi \theta} - 3 c_{\phi \theta} d_{\phi \theta}) \right] \cos \theta + \left[ 4 (d_{\phi \phi} - d_{\phi \theta} + M_{\phi}^2 + M_{\theta}^2 + c_{\phi \theta}^2 + d^2) \right. \]
\[ + \left. (d_{\phi \phi} - d_{\phi \theta})^2 \right] \cos \theta + \left[ 2 (c_{\phi \phi} c_{\phi \theta} - 3 c_{\phi \theta} \cos \theta) (2d - d_{\phi \theta}) \right. \]
\[ + \left. 4 (d_{\phi \phi} d_{\phi \theta} - d_{\phi \theta}^2 + M_{\phi}^2) \right] \cos \theta \cot \theta \sin^2 \theta + \left[ 4 c_{\phi \phi} c_{\phi \theta} + (4 d_{\phi \phi} - d_{\phi \theta}^2 - M_{\phi}^2 - c_{\phi \theta}^2) \right. \]
\[ - \left. (c_{\phi \phi} - d_{\phi \theta} d_{\phi \theta}) \cot \theta \right] \}^{1/2} \]  
(78)

The expression for \( s \Omega_{\theta EM} \) is too long to write here or anywhere; we check that is manifestly gravito–electromagnetic

\[ s S_{EM}^u = S_{EM}^u \]  
(79)
\[ s S_{EM}^u = (\mu f - \epsilon) \cot \theta \]
\[ - (\epsilon f_{\theta} + \mu e_{\phi}) \csc \theta \]
\[ + \mu f_{\theta} - \epsilon e_{\phi} + MS_{EM}^u \]  
(80)
\[ s S_{EM}^\phi = \epsilon^2 + c_{\phi}^2 \csc^2 \theta \]  
(81)

(coefficients for \( s P^\alpha \) are too long to display anywhere).

The above expressions illustrate the Bonnor–like mechanism for any stationary electrovacuum solution (see 19 for further discussion).
IV. DISCUSSION

We have seen that electromagnetic radiation as described by electromagnetic news functions does produce vorticity. It is important to stress that this is so even in the case of minimum electromagnetic degrees of freedom (which is one), the reflection and axially symmetric case, when \( X \neq 0 \) and \( Y = 0 \), implying that the above mentioned effect is generic. Since vorticity unavoidably produces frame dragging, we have established the link between these two physical effects. Furthermore we have identify the presence of electromagnetic news both in the Poynting and the super-Poynting components orthogonal to the vorticity vector. Doing so we have proved that a Bonnor-like mechanism is at work in this case too. However it is important to emphasize that in the present situation vorticity is generated by the contributions of, both, the Poynting and the super-Poynting vectors, on the planes orthogonal to the vorticity vector. It must be stressed that the mechanism to explain how electromagnetic and gravitational radiation produce vorticity invokes the concept of superenergy (its flow), which is one of the most important concepts in general relativity involving the congruence of observers [30].

Before proceeding further with our discussion two comments are in order:

- We have clearly established the link between vorticity and electromagnetic radiation, which as mentioned before implies a link between electromagnetic radiation and frame dragging. This was the main goal of our work. However we have not calculated in detail the resulting precession rate of a falling gyroscope under such a circumstance, since it is out of the scope of this manuscript. It goes without saying that for an explicit experiment setup such a calculation should be provided.

- It should be clear that the magnitude of the mentioned effect is directly related to the intensity of the electromagnetic emission \( (Y^*) \).

Simulations from numerical relativity could shed some light to figure out how to measure the effect reported here. First, in the study of binary black holes dynamics near electromagnetic fields and plasmas [31], it was displayed how the system imprints characteristics on the two induced wavebands. In the present case, as an inverse problem, the electromagnetic and gravitational radiation produce precession (on test gyroscopes) which has to be imprinted by the waves. Second, almost in the same aforementioned context, it was possible to track the precession of compact binaries from gravitational wave signals [32], locating the frame from which the \( (l = 2, |m| = 2) \) modes are maximized. We suppose that in the same way as the “quadrupolar–aligned” frame is located, the “dipolar–aligned” frame could be find from electromagnetic modes. This simple method can be applied to the ensuing gyroscope precession, as reported in here.

The potential observational consequences of the reported effect should be seriously considered. Indeed, as we mentioned before, intense electromagnetic outbursts are expected from hyperenergetic phenomena such as collapsing hypermassive neutron stars and Gamma Ray Bursts (see [1], [8] and references therein). Although we are not able at the present to estimate the required sensitivity of the gyroscope to measure such an effect, the high intensity of radiation in the above mentioned scenarios leaves open the question about its detection with present technology. In fact, the direct experimental evidence of the existence of the Lense–Thirring effect [2] brings out the high degree of development achieved in the required technology. In the same direction point recent proposals to detect frame dragging by means of ring lasers [23–37].

Finally we would like to mention that frame dragging produced by the so called optical vortices has been recently described in the linear regime [38]. However it should be observed that the effect reported here stems from non–linear terms, as it can be seen from (50), (67) and (68), bringing out the relevance of nonlinearities in the general relativistic description of radiation.

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Appendix: Expression for the axially and reflection symmetric $P^0_{G\Sigma M}$

$$P^0_{G\Sigma M} = \frac{1}{4} [3c_\theta c_\theta a c_u - 4c_\theta a c_u c_\theta - 4c_u c_u e - 24c_u c_N u - 4c_u e c_u + 14c_\theta a c_u - 60c_\theta a c_u M^2 - 32M c_\theta a c_u$$

$$- 18c_u c_\theta c_\theta a c_u c_u - 12c_\theta a c_u c_u c_\theta - 4c_\theta a c_u c_\theta - 24M c_\theta c_\theta a c_u + 8c_u c_\theta c_\theta a c_u - 17c_\theta a c_\theta a c_u - 12c_\theta a c_\theta a c_u - 16c_u c_\theta a c_u$$

$$+ 36M c_\theta a c_u M + 30c_\theta a c_\theta c_\theta a c_u - 18c_u M c_\theta a c_u - 18c_\theta a c_\theta c_\theta a c_u - 24c_u c_\theta c_\theta a c_u + 8c_u a c_\theta a c_u + 12M c_\theta a c_u - 8c_\theta a c_\theta a c_u$$

$$- 4c_\theta a M c_\theta a - 16c_u a c_u - 12M c_\theta a c_u - 6c_\theta c_\theta a c_u - 8c_u c_\theta a c_u - 6c_\theta c_\theta a c_u - 6M c_\theta a c_u + (16 c_\theta a M c_\theta a - 24 c_u c_\theta a c_u$$

$$- 24M c_\theta a c_u - 16c_u a c_u - 48c_u a c_u c_\theta a c_u - 120c_u c_u M^2 c_\theta a - 2c_\theta a M_\theta - 8c_\theta a c_u - 25c_\theta a + 12a c_u M c_\theta a - 8c_\theta a c_\theta a c_u$$

$$+ 12M c_\theta a a c_\theta a - 8c_u a c_\theta a c_u + 24c_\theta a c_\theta a c_u - 16c_u N c_\theta a - 8c_\theta a c_\theta a c_u + 8c_u a c_\theta a c_u - 64 c_\theta a M - 41c_\theta a c_\theta a c_u$$

$$+ 16c_u a c_u - 160c_u a c_u c_\theta a - 24c_u a c_u c_\theta a - 24c_u a c_u M_c_\theta a] \cot \theta].$$

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