Optimal Pricing and Assembly Rate for Prefabricated Construction Assembler with Subsidy Policy

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Abstract. With the development of the construction industry, assembly construction has received an increasing number of attention. The government introduced subsidy policy to promote prefabricated construction. We have considered subsidy into the traditional prefabricated construction, in which case decision on retail pricing and construction assembly rate will be complicated. We discussed the impact of subsidy policy on assembler pricing and assembly rate. The paper finds that subsidy policy can lower the retail price of prefabricated construction and increase the assembly rate, the assembler must control the cost factor for increasing assembly rate within a certain range to obtain higher profit.

1. Introduction

In the construction industry, prefabricated construction is the mainstream trend for future development. It is not only an important measure for national energy saving and emission reduction, but also one of the important ways to promote the development of the construction industry. Prefabricated construction has many significant sustainable advantages: lower labors and material costs, higher speed of construction, improved waste reduction, enhanced construction quality, along with a cleaner working environment [1]. Vigorously developing prefabricated construction is an inevitable requirement for green, recycling and low-carbon development.

China has actively explored the development of prefabricated construction in recent years, its construction ratio and scale are still lower. The cost is identified as one of the most critical barriers [2]. Because of its high cost of component production and transportation, the overall cost of fabricated construction is higher than traditional construction. In order to expand the industrial scale of the prefabricated construction, the provinces have actively introduced corresponding subsidy policy for prefabricated construction. In prefabricated construction, we consider government subsidy can reduce costs, but it also complicates assembler decisions. First, assembler need to determine construction assembly rate and retail pricing based on subsidy policy and supplier wholesale price. Second, assembly rate and pricing will affect demand. Therefore, it is of great theoretical and practical value to study the assembler optimization and coordination under the government subsidy policy.

Two research streams are closely related to this paper and will be reviewed to highlight our contributions. The first stream related to our work is the literature on subsidy policies for construction companies. Lena et al. (2010) used a multiple regression of two models to research explore the extent that interest subsidies have impacted on the total production of Swedish single and multifamily houses [3]. The results showed that a targeted subsidy may change the balance between different types of housings since lower construction costs due to the subsidy could favor the development of certain profitable housing types. Ma et al. (2015) examined the impact of consumer subsidies and manufacturer
subsidies on the closed-loop supply chain \cite{4}. They showed that after introducing government subsidy program in the closed-loop supply chain, both the manufacturer and the retailer have benefited from the government-subsidy. By comparison, the paper find that the subsidies to consumer can bring more benefits for the manufacturer and the retailer. The literature above mentioned didn’t consider the impact of government subsidies on prefabricated construction. The second stream related to our work is the literature on the prefabricated building supply chain management strategy. Hong et al. (2018) established a cost–benefit analysis framework to explore the basic cost composition of prefabrication and examined the effect of adopting prefabrication on the total cost of real building projects \cite{5}. The results showed that in order to fully gain the economic benefits from the precast construction, the future focus should lie in providing financial support for promoting the development of prefabrication technology, optimizing the structure integrity of prefabricated construction, and improving the maturity of the precast market. Li et al. adopted a data collection and processing method to systematically summarize the research progress of the prefabricated component management discipline \cite{6}. The analysis revealed that prefabrication is becoming increasingly important to the entire construction industry. Chen et al. (2016) analyzed the interest between government and construction industry by using evolutionary game theory and build a game model of government-construction enterprises \cite{7}. The government will achieve its purpose of promoting prefabricated construction through subsidies and fines, and builders through technological innovation to achieve the purpose of increasing construction efficiency. Jiang and Wu (2019) studied proposed an effective incentive mechanism for the contractor through game theory approach \cite{8}. The results show that incentive mechanism can successfully lower total cost for the developer and increases the profit for the contractor. The literature above mentioned laid the foundation for our study but did not consider the application of government subsidy policies to the assembler decision-making research.

In order to fill the gap presented by the literature review, the paper researches the pricing of prefabricated construction assembler and assembly rate under subsidy policies.

The rest of this paper is organized as follows. In Section 2, the model descriptions and assumptions are presented. In Sections 3 and 4, the paper formulate models for no government subsidy and government subsidy assembler, we derive the optimal decisions for the assembler, and discuss the impact of cost factor. The paper conclude our key findings in Section 5 and highlight possible future works.

2. Model Descriptions and Assumptions

We consider an assembler to place orders with a supplier, which is then assembled by the assembler and sold to the customer. The assembler faces prefabricated construction subsidy policy, which means that they need to increase the assembly rate to get more subsidies, which can reduce costs. In addition, we define some parameters and variables as summarized in Table 1.

| Notations | Descriptions |
|-----------|--------------|
| \( \omega \) | Supplier’s unit wholesale price. |
| \( p \) | Assembler’s unit retail price. |
| \( c \) | Assembler’s unit cost. |
| \( \delta \) | Construction assembly rate, \( 0 < \delta < 1 \). |
| \( \delta_0 \) | Assembly rate subsidy floor imposed by the government, \( 0 < \delta_0 < 1 \). |
| \( \theta \) | Unit subsidy ratio imposed by the government, \( 0 < \theta < 1 \). |
| \( \epsilon \) | Cost factor for increasing assembly rate. |

We assume that the demand function depends on the price. That is \( D(p) = \alpha - \beta p \) (\( \alpha \) and \( \beta > 0 \)) , where \( \alpha \) is the initial market and \( \beta \) is self-price sensitivity. Government subsidy is related to the assembly rate. When \( \delta \geq \delta_0 \), that is \( I(\delta) = \theta \delta(c + \omega)(\alpha - \beta p) \), \( \theta \) and \( \delta_0 \) are imposed by the government. The assembler’s cost of assembly rate investment is a function with respect to \( \delta \), that is \( \mu(\epsilon_\delta) = \epsilon \delta^2 \) represents the cost factory of assembly rate investment of the assembler. The large \( \epsilon \) means the lower assembly rate. The supplier’s unit wholesale price (\( \omega \)) is fixed.
3. Base model
In this section, we propose a base model without subsidy policy, examine the assembler optimal pricing and assembly rate. The supplier and the assembler make their decisions in the following sequence. First, the supplier determines the wholesale price. Then, the assembler determines the pricing and assembly rate according to the wholesale price.

According to assumptions abovementioned, the assembler’s profit \( \pi_0(p, \delta) \), is:
\[
\pi_0(p, \delta) = (p - c - \omega)D(p) - \mu(e_i)
\]  
(1)

First term is the profit from product wholesaling. The second term indicates the cost of increasing the assembly rate of the construction.

We have \( D(p) = \alpha - \beta p \) and \( \mu(e_i) = \epsilon \delta^2 \). Substitute them into \( \pi_0(p, \delta) \), the paper can rewrite \( \pi_0(p, \delta) \) as:
\[
\pi_0(p, \delta) = (p - c - \omega)(\alpha - \beta p) - \epsilon \delta^2
\]  
(2)

Proposition 1 The optimal pricing of the assembler \( (p_1) \) and the construction assembly rate \( (\delta_1) \), listed as follows:
\[
p_1 = \frac{\alpha + \beta(c + \omega)}{2\beta}
\]
\[
\delta_1 = 0
\]

Proof. We can obtain the first order condition of \( \pi_0(p, \delta) \):
\[
\frac{\partial \pi_0(p, \delta)}{\partial p} = \alpha - 2\beta p + \beta(c + \omega)
\]
\[
\frac{\partial \pi_0(p, \delta)}{\partial \delta} = -2\epsilon \delta
\]

In order to make the Hessian Matrix in the basic model be negatively definite, we assume \( \epsilon > 0 \).

We have \( \frac{\partial^2 \pi_0(p, \delta)}{\partial p^2} = -2\beta < 0 \), \( \frac{\partial^2 \pi_0(p, \delta)}{\partial \delta^2} = -2\epsilon < 0 \) and \( \frac{\partial^2 \pi_0(p, \delta)}{\partial p \partial \delta} = \frac{\partial^2 \pi_0(p, \delta)}{\partial \delta p} = 0 \). So the paper can obtain the determinant of this problem, \( H = \begin{vmatrix} -2\beta & 0 \\ 0 & -2\epsilon \end{vmatrix} = 4\beta \epsilon > 0 \). Then we have that \( \pi_0 \) is joint concave in \( p_1 \) and \( \delta_1 \). This completes the proof.

Proposition 1 shows that assembler’s optimal retail price and construction assembly rate uniquely exist on basic model. When there is no subsidy policy, the assembly rate is lower because of the higher cost of prefabricated construction.

4. Subsidy model
4.1. The optimal pricing and assembly rate
In this section, we propose a model with subsidy policy, research the assembler optimal pricing and the assembly rate. Subsidy can be obtained when the construction assembly rate is higher than the minimum assembly rate set by the government. That means that the Assembler must decide assembly rate with the constraint \( \delta \geq \delta_0 \).

According to assumptions abovementioned, the assembler’s profit \( \pi_2(p, \delta) \), is:
\[
\pi_2(p, \delta) = (p - c - \omega)D(p) + I(\delta) - \mu(e_i)
\]  
(3)

The first term is the profit from product wholesaling. The second term indicates the government subsidy. The third term represents the cost of increasing the construction assembly rate. Then \( \pi_2(p, \delta) = (p - c - \omega)(\alpha - \beta p) + \theta \delta(c + \omega)(\alpha - \beta p) - \epsilon \delta^2 \)  
(4)

Proposition 2 The optimal pricing of the assembler \( (p_2) \) and the construction assembly rate \( (\delta_2) \), listed as follows:
\[
p_2 = \frac{2\epsilon[\alpha + \beta(c + \omega)] - \alpha \beta \theta^2(c + \omega)^2}{4\epsilon \beta - \beta^2 \theta^2(c + \omega)^2}
\]
\[
\delta_2 = \frac{(c + \omega)\theta[\alpha - \beta(c + \omega)]}{4\epsilon - \beta\theta^2(c + \omega)^2}
\]

**Proof.** We can obtain the first order condition of \( \pi_a^2(p, \delta) \):

\[
\frac{\partial \pi_a^2(p, \delta)}{\partial p} = \alpha - 2\beta p + \beta(c + \omega) - \theta\delta\beta(c + \omega)
\]

\[
\frac{\partial \pi_a^2(p, \delta)}{\partial \delta} = \theta(c + \omega)(\alpha - \beta p) - 2\epsilon\delta
\]

In order to make the Hessian Matrix in the basic model be negatively definite, we assume \( 4\epsilon > \beta\theta^2(c + \omega)^2 \).

We have \( \frac{\partial^2 \pi_a^2}{\partial p^2} = -2\beta < 0, \frac{\partial^2 \pi_a^2}{\partial \delta^2} = -2\epsilon < 0 \) and \( \frac{\partial^2 \pi_a^2}{\partial p\partial \delta} = \frac{\partial^2 \pi_a^2}{\partial \delta p} = -\beta\theta(c + \omega) \). So the paper can obtain the determinant of this problem, \( H = \begin{bmatrix} -2\beta & -\beta\theta(c + \omega) \\ -\beta\theta(c + \omega) & -2\epsilon \end{bmatrix} = \beta [4\epsilon - \beta\theta^2(c + \omega)^2] > 0 \). So the Hessian Matrix of this problem is negative definite. Then the paper have that \( \pi_a^2 \) is joint concave in \( p_2 \) and \( \delta_2 \). This completes the proof.

Proposition 3 shows that assembler’s optimal retail price and construction assembly rate uniquely exist on basic model. Assembler can adjust pricing and assembly rate based on assembly rate cost factory and subsidy policy to optimize operational decisions.

**Proposition 3** We find out that the retail price is positively related to the cost coefficient. The construction assembly rate is negatively related to the cost factor.

\[
\frac{\partial p_2}{\partial \epsilon} = \frac{2\beta^2\theta^2(c + \omega)^2[\alpha - \beta(c + \omega)]}{4\epsilon\beta - \beta^2\theta^2(c + \omega)^2} > 0
\]

\[
\frac{\partial \delta_2}{\partial \epsilon} = -\frac{(4\epsilon - \beta\theta^2(c + \omega)^2)^2}{4\epsilon\beta - \beta^2\theta^2(c + \omega)^2} < 0
\]

**Proof.** In model, we have \( p_2 = \frac{2\epsilon[\alpha - \beta(c + \omega)]}{4\epsilon\beta - \beta^2\theta^2(c + \omega)^2} \) and \( \delta_2 = \frac{\theta(c + \omega)[\alpha - \beta(c + \omega)]}{4\epsilon\beta - \beta^2\theta^2(c + \omega)^2} \). Now we take the derivative of \( p_2 \) and \( \delta_2 \) respectively. We have \( (c + \omega) < p \), then \( \alpha > \beta(c + \omega) \). This completes the proof.

Proposition 3 shows that when the government subsidizes the assembler, the assembler can increase the order quantity and negotiate with the supplier to obtain a lower wholesale price to reduce the cost factor of the assembly rate. This can increases assembler profit and building assembly rates.

When \( \delta < \delta_2 \), the assembler cannot get the subsidy, the assembler pricing and construction assembly rate will be the same as the model without subsidy policy.

### 4.2. Discussion

In this section, we discuss the impact of government subsidy on construction assembly rate, assembler pricing and assembler profit.

**Proposition 4:** (1) \( p_1 > p_2 \), (2) \( \delta_1 < \delta_2 \).

**Proof.** (1) \( p_2 - p_1 = -\frac{\beta\theta^2(c + \omega)^2[\alpha - \beta(c + \omega)]}{2[4\epsilon\beta - \beta^2\theta^2(c + \omega)^2]} < 0 \), the paper have \( \alpha > \beta(c + \omega) \) and \( 4\epsilon > \beta\theta^2(c + \omega)^2 \), we can obtain \( p_1 > p_2 \). (2) \( \delta_2 - \delta_1 = \frac{\theta(c + \omega)[\alpha - \beta(c + \omega)]}{4\epsilon\beta - \beta^2\theta^2(c + \omega)^2} > 0 \), the paper have \( \alpha > \beta(c + \omega) \) and \( 4\epsilon > \beta\theta^2(c + \omega)^2 \), we can obtain \( \delta_1 < \delta_2 \). This completes the proof.

Proposition 4 shows that when prefabricated construction consider government subsidy, the construction assembly rate will increase and will lower the retail price.

**Proposition 5:** When \( \frac{\beta\delta(c + \omega)^2\theta^2 - \sqrt{\beta\delta(c + \omega)^2\theta^2}[2\alpha + \beta(c + \omega)(\delta\theta - 2)]}{8\delta} < \epsilon < \frac{\beta\delta(c + \omega)^2\theta^2 + \sqrt{\beta\delta(c + \omega)^2\theta^2}[2\alpha + \beta(c + \omega)(\delta\theta - 2)]}{8\delta} \), \( \pi_a^2 > \pi_a^1 \).

**Proof.** \( \pi_a^1 - \pi_a^2 = \frac{\theta(c + \omega)[\alpha - \beta(c + \omega)]}{4[\beta(c + \omega)^2\theta^2 - 4\epsilon\theta^2]} \), we have
\[ \alpha > \beta(c + \omega) \]. Whether the increase in profit is related to the cost coefficient, when
\[
\frac{\beta \delta}{(c+\omega)^2 \theta^2 - \sqrt{\beta \delta \theta (c+\omega)^3 \theta}} < \varepsilon < \frac{\beta \delta}{(c+\omega)^2 \theta^2 + \sqrt{\beta \delta \theta (c+\omega)^3 \theta}}
\]
we have \( \pi^2_{\text{max}} > \pi^1_{\text{max}} \). When
\[
\varepsilon > \frac{\beta \delta}{(c+\omega)^2 \theta^2 + \sqrt{\beta \delta \theta (c+\omega)^3 \theta}}\delta
\]
we have \( \pi^2_{\text{max}} < \pi^1_{\text{max}} \). This completes the proof.

Proposition 5 shows that when prefabricated construction consider government subsidy, the assembler profit correlates with cost coefficient. The cost factory is within the above range, and profit will increase.

5. Conclusion and Future Research

In order to promote prefabricated construction and reduce costs, the assembly company introduced a subsidy policy, but after considering the subsidy policy, it will complicate the decision of the assembler. Response to the challenge, we introduce subsidy policy into the traditional prefabricated buildings model. First, we establish models without government subsidy and models with government subsidy, we obtain the optimal pricing and assembly rate for assembler. The effects of subsidy on assembler pricing and assembly rates are then discussed. We find that subsidy policy can lower the retail price of prefabricated construction and increase the assembly rate, the assembler must control the cost factor for increasing assembly rate within a certain range to obtain higher profit.

Our paper researches that the impact in assembler pricing and assembly rate when the government subsidy assembler. In fact, the government can subsidize supplier, which will also cause impact in pricing and assembly rate. Therefore, the other direction of this paper is to research the effect on pricing and assembly rate of supplier with and without government subsidy.

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