THE RR LYRAE STAR U COMAE AS A TEST FOR NONLINEAR PULSATION MODELS

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ABSTRACT

We use high-precision multiband photometric data of the first-overtone RR Lyrae star U Comae to investigate the predictive capability of full-amplitude, nonlinear, convective hydrodynamical models. The main outcome of this investigation is that theoretical predictions properly account for the luminosity variations along a full pulsation cycle. Moreover, we find that this approach, because of the strong dependence of this observable and of the pulsation period on stellar parameters, supplies tight constraints on stellar mass, effective temperature, and distance modulus. Pulsational estimates of these parameters appear in good agreement with empirical ones. Finally, a well-defined bump just before the luminosity maximum gave the unique opportunity to calibrate the turbulent convection model adopted for handling the coupling between pulsation and convection.

Subject headings: stars: distances — stars: evolution — stars: horizontal-branch — stars: individual (U Comae) — stars: oscillations — stars: variables: other

1. INTRODUCTION

Variable stars play a key role in many astrophysical problems, since their pulsation properties do depend on stellar parameters, and therefore they can supply valuable and independent constraints on a large number of current evolutionary predictions. In particular, the empirical evidence found a long time ago in Magellanic Cepheids of the correlation between period and luminosity was the initial step for a paramount theoretical and observational effort aimed at using variable stars as standard candles to estimate cosmic distances. The current literature is still hosting a vivid debate on the intrinsic accuracy of the Cepheid distance scale (Bono, Marconi, & Stellingwerf 1999; Laney 2000) and on the use of RR Lyrae stars to evaluate the distance—and the age—of Galactic globulars (Caputo 1998; Gratton 1998).

Theoretical insights into the problem of radial stellar pulsations came from the linearization of local conservation equations governing the dynamical instability of stellar envelopes. Linear, nonadiabatic models typically supply accurate pulsation periods and plausible estimates (necessary conditions) on the modal stability of the lowest radial modes. However, a proper treatment of radial pulsations does require the solution of the full system of hydrodynamic equations, including a nonlocal and time-dependent treatment of turbulent convection (TC) to account for the coupling between radial and convective motions (J. I. Castor 1968, unpublished; Stellingwerf 1982).

Nonlinear, convective hydrocodes (Stellingwerf 1982; Gehmeyr 1992; Bono & Stellingwerf 1994, hereafter BS94; Wuchterl & Feuchtinger 1998) gave the opportunity to provide plausible predictions on the properties of radial variables, and in particular on the topology of the instability strip as well as on the time behavior of both light and radial velocity curves. This new theoretical scenario allowed the investigation, for the first time, of the dependence of pulsation amplitudes and Fourier parameters on stellar mass, luminosity, and effective temperature (see, e.g., Kovacs & Kanbur 1998; Brocato, Castellani, & Ripepi 1996; Feuchtinger 1999, hereafter F99). However, all these investigations dealt with parameters related to the light curve, whereas nonlinear computations supply much more information, as given by the detailed predictions of the light variation along a full pulsation cycle. Therefore, the direct comparison between observed and predicted light curves appears as a key test only partially exploited in the current literature (Wood, Arnold, & Sebo 1997).

In order to perform a detailed test of the predictive capability of our nonlinear, convective models, we focused our attention on the photometric data collected by Heiser (1996, hereafter H96) for the field, first-overtone—RR,—variable U Comae. The reason for this choice relies on the detailed coverage of the $U$, $B$, and $V$ light curves as well as on the characteristic shape of the light curve, with a well-defined bump close to the luminosity maximum. This secondary feature provides a tight observational constraint to be nailed down by theory. Since the period of the variable strongly depends on the structural parameters (mass, luminosity, and radius) of the pulsator, the problem arises whether or not nonlinear pulsation models account for the occurrence of similar pulsators, and if affirmative, how precisely the observed light curves can be reproduced by theoretical predictions.

In § 2 we present the comparison between theory and observations, while in § 3 we discuss the calibration of the TC model. Finally, in § 4 we briefly outline the observables that can further validate this theoretical scenario.

2. COMPARISON BETWEEN THEORY AND OBSERVATIONS

On the basis of spectroscopic measurements, Fernley & Barnes (1997, hereafter FB97) estimated for U Com a metallicity $[\text{Fe/H}] = -1.25 \pm 0.20$, while Fernley et al. (1998a) found a negligible interstellar extinction $E(B-V) = 0.015 \pm 0.015$. According to this empirical evidence and to a well-established evolutionary scenario, we expect for a metal-poor RR Lyrae star a stellar mass of the order of $0.6 M_\odot$ and a luminosity ranging from $\log L/L_\odot = 1.6$ to $1.7$. At the same time, pulsation predictions on double-mode pulsators suggest similar mass values (Cox 1991; Bono et al. 1996a). As a consequence, we computed a sequence of nonlinear models at fixed chemical composition ($Y = 0.24$, $Z = 0.001$) and pulsation period $P = 0.29$ days. Along such an isoperiod sequence, the individual models were constructed at fixed mass value
(M/M_⊙ = 0.60), while both the luminosity and the effective temperature were changed according to the pulsation relation given by Bono et al. (1997, hereafter BCCM). Both linear and nonlinear models were computed by adopting the input physics and physical assumptions already discussed in BS94, BCCM, and Bono et al. (1999). According to Stellingwerf (1982), current models were computed by assuming a vanishing efficiency of turbulent overshooting in the region in which the superadiabatic gradient attains negative values. This means that the convective flux can only attain positive or vanishing values (F ≥ 0). In the next section, we show that the assumption adopted in our previous investigations—i.e., F can attain both positive and negative values—marginally affects the topology of the instability strip, but the predicted light curves are somewhat at variance with empirical ones.

The top panels of Figure 1 show that at fixed stellar mass (M/M_⊙ = 0.60), the double-peaked feature appears in models characterized by luminosities approximately equal to log L/L_⊙ ≈ 1.61 and effective temperatures ranging from 6950 to 7150 K. These models also present B amplitudes in reasonable agreement with empirical estimates (A_e = 0.64 mag). The bottom panels of Figure 1 display the light curves of models along the isoperiod sequence constructed by adopting a fixed effective temperature (T_e = 7100 K) but different assumptions on stellar mass and luminosity. A glance at these curves shows that the luminosity amplitude is mainly governed by the stellar mass, whereas the shape of the light curve is only marginally dependent on this parameter.

We find that the best fit to the observed B light curve is obtained for M/M_⊙ = 0.6, log L/L_⊙ = 1.607, T_e = 7100 K, and P = 0.290 days, together with a distance modulus (m_B - M_B) = 11.01 mag. The fit—although not perfect—appears rather satisfactory, thus supplying substantial support for the predictive impact of the adopted theoretical scenario. On the basis of this finding, we are now interested in testing the accuracy of theoretical predictions in different photometric bands. Figure 2 shows from left to right the comparison between predicted light curves (lines) and empirical data (circles) in the U, B, V, and K bands, respectively. The comparison was performed by adopting the same distance modulus, i.e., by neglecting the interstellar extinction, and the agreement between theory and observations seems even better than for the B light curve. This suggests that nonlinear models account for luminosity amplitudes which are a long-standing problem of pulsation theory.

Not surprisingly, we also find that the time-averaged colors predicted by our model appear, within current uncertainty on both reddening and photometry, in very good agreement with empirical estimates (see Table 1). This result supports the evidence that nonlinear models, at least in this case, can constrain stellar colors by best fitting the light curve in a single photometric band. At the same time, this agreement suggests that the pulsational constraints on the temperature of the pulsator, as derived by the B light curve, are consistent with the theoretical light curves in the other photometric bands.

However, we note that on the basis of both the period and the shape of the B light curve, we predicted the effective temperature, the intrinsic luminosity, and in turn the distance modulus of this object. The plausibility of the theoretical constraints can be further tested by comparing them with independent evaluations available in the literature. We find that the effective temperature predicted by nonlinear models (T_e = 7100 ± 50 K) is in remarkable agreement with the empirical temperature (T_e = 7100 ± 150 K) derived by adopting the true intensity mean color (⟨V⟩ - ⟨K⟩)_0 = 0.77 ± 0.07 provided by FB97 and the color-temperature relation by Fernley (1989). The same outcome applies by assuming E(B - V) = 0, and indeed ⟨⟨V⟩ - ⟨K⟩⟩ = 0.81 ± 0.07 → T_e = 7050 ± 150 K, while the semiempirical estimate provided by H96 suggests T_e = 7250 ± 150 K. We also note that the effective gravity of the best-fit model (log g ≈ 3.0) is also in very good agreement with both the photometric estimate obtained by H96 (log g = 3.1 ± 0.2) and the spectroscopic measurements for field RR Lyrae stars provided by Clementini et al. (1995) and by Lambert et al. (1996).

As far as the distance modulus is concerned, Figure 3 shows the comparison of our pulsational estimates (filled circles) with empirical and theoretical U Com absolute magnitudes obtained by adopting different methods. The top and the bottom panel

![Figure 1](image1.png)

**Figure 1.** Top: Blue light curves of isoperiod (P = 0.29 days) RR Lyrae models constructed by adopting the same chemical composition (Y = 0.24, Z = 0.001) and stellar mass (M/M_⊙ = 0.60), but different assumptions on effective temperatures and luminosities (see labeled values). Bolometric curves where transformed into B magnitudes by adopting bolometric corrections and color-temperature relations by Castelli, Gratton, & Kurucz (1997). Bottom: Similar to top panels, but for models constructed at fixed effective temperature (T_e = 7100 K) and different assumptions on stellar masses and luminosities (see labeled values).

**Figure 2.** Comparison between theory (lines) and observations (circles). From left to right the panels refer to photometric data in U, B, V (H96), and K (Fernley, Skillen & Burki 1993). Empirical data were plotted by assuming E(B - V) = 0. Photometric errors in optical bands are equal to the symbol size.

![Figure 2](image2.png)

**Table 1.** U Com: Theoretical and Empirical Colors

| Color | Theory | E(B - V) = 0 | E(B - V) = 0.015 |
|-------|--------|-------------|------------------|
| U     | 0.06 ± 0.01 | 0.11 ± 0.02 | 0.09 ± 0.02 |
| B     | 0.23 ± 0.01 | 0.21 ± 0.02 | 0.19 ± 0.02 |
| V     | 0.77 ± 0.01 | 0.81 ± 0.07 | 0.77 ± 0.07 |

**Note.** Values are given in magnitudes.

*Empirical estimates are based on photometric data collected by H96 and FB97. Theoretical colors refer to the best-fit model, and the errors were estimated by assuming an uncertainty of 50 K in the temperature of this model.*
A disagreement between observational and pulsation predictions concerning the luminosities of RR Lyrae stars was brought out by Caputo et al. (1999) and more recently by Castellani et al. (2000), who found that up-to-date He-burning models seem too bright when compared with the current pulsation determination. A disagreement between observational and pulsation predictions concerning the luminosities of RR Lyrae stars was brought out by Caputo et al. (1999) and more recently by Castellani et al. (2000), who found that up-to-date He-burning models seem too bright when compared with the current pulsation determination.

3. CALIBRATION OF THE TC MODEL

The first set of models we constructed for fitting the empirical light curves was characterized by an unpleasant feature: the peak of the bump was, in contrast with empirical evidence, brighter than the “true” luminosity maximum. According to Bono & Stellingwerf (1993), the bump along the rising branch presents a strong dependence on the free parameters adopted in the TC model. However, in the calibration of the TC model suggested by BS94, both the eddy viscosity and the diffusion scale lengths (see their eqs. [4] and [9]) were scaled to the value of the mixing length parameter. We performed several numerical experiments by changing along each sequence only one of the three free parameters. As a result, we find that full amplitude models constructed by adopting plausible changes of the free parameters do not simultaneously account for the pulsation amplitude, the shape of the light curve, and the temperature width of the first-overtone instability region.

Because of the lack of a self-consistent theory of time-dependent, nonlocal, convective transport, current investigations were mainly aimed at calibrating the free parameters adopted for treating the coupling between pulsation and convection (Yecko, Kollath, & Buchler 1998; F99). This is not a trivial effort, since the observables and the comparison between theory and observations are affected by the thorny problem of the transformation into the observational plane and/or by systematic deceptive errors such as reddening and distance estimates. In order to overcome some of these difficulties, F99 calibrated the TC model by performing a detailed comparison between theoretical and observed luminosity amplitudes of field RR Lyrae stars. On the basis of the fine tuning of both mixing length and turbulent viscosity length, F99 found that the Fourier parameters of fundamental light curves agree with observational data. The same outcome did not apply to RR variables, and indeed predicted values appear, at fixed period, smaller than the empirical ones. A detailed comparison with the convective structure of RR Lyrae models constructed by F99 is not possible because he adopted a convective flux limiter in the turbulent source function and in the convective flux enthalpy and neglected both the turbulent pressure and the turbulent overshooting. As a consequence, we decided to test the dependence of full-amplitude models on the last two ingredients.

Interestingly enough, we find that the models constructed by assuming a vanishing overshooting efficiency satisfy empirical constraints, i.e., the bump is dimmer than the luminosity maximum, the luminosity amplitudes attain values similar to the empirical ones. A detailed comparison with the convective structure of RR Lyrae models constructed by F99 is not possible because he adopted a convective flux limiter in the turbulent source function and in the convective flux enthalpy and neglected both the turbulent pressure and the turbulent overshooting. As a consequence, we decided to test the dependence of full-amplitude models on the last two ingredients.

The comparison between theory and observations, namely the period and the shape of the $B$ light curve of U Com, allowed...
Comparison between theory and observations shows that both the structural parameters and the distance are in very good agreement with estimates available in the literature. No evidence for a systematic discrepancy was found in pulsation estimates, thus supporting the evidence that the individual fit to light curves can supply independent and firm constraints on the actual parameters and distances of variable stars. This finding confirms the results of a similar analysis on a LMC bump Cepheid by Wood et al. (1997).

Accurate radial velocity data for U Com are not available in the literature, and therefore we could not constrain the accuracy of the velocity variation along the pulsation cycle. The three radial velocity points collected by Fernley et al. (1998b) agree quite well with the predicted curve. However, the radial velocity curve is a key observable for constraining the consistency of the adopted TC model (F99), and therefore new spectroscopic measurements of U Com would be of great relevance for assessing the predictive impact of nonlinear, convective models. Theoretical observables of the best-fit model discussed in this Letter, as well as both radius and radial velocity variations, are available upon request to the authors.

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Fig. 4.—$B$ light curves of RR Lyrae models vs. phase. Solid lines refer to models constructed by adopting the calibration of the TC model suggested by BS94, while dashed lines refer to models constructed by assuming that the convective flux is vanishing in the regions in which the superadiabatic gradient is negative (see eq. [7] and § 3 in BS94).