Shear viscosity of a pion gas due to $\rho\pi\pi$ and $\sigma\pi\pi$ interactions

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We have calculated the shear viscosity of pion medium, where propagating pion has some finite thermal width due to interaction with the low mass resonances, $\sigma$ and $\rho$. With the help of standard thermal field theoretical technique, the thermal width of the pion has been calculated from the pion self-energy diagram for $\pi\sigma$ and $\pi\rho$ loops, where an effective Lagrangian density has been used for interaction part. We have found a very small value of shear viscosity by entropy density ratio ($\eta/s$), which is very close to the KSS bound.

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1. Introduction

The new state of matter, which is expected to be produced due to high energy nucleus-nucleus collisions at Relativistic Heavy Ion Collider (RHIC), is likely to have a very small ratio of shear viscosity to entropy density, $\eta/s$. This conclusion has been made by different hydrodynamical and transport simulation to explain the elliptic flow parameter, $v_2$, extracted from data collected at RHIC. Such a small $\eta/s$ is not really compatible with standard finite temperature calculation of Quantum Chromo Dynamics (QCD), which exhibits weakly interacting gas due to the asymptotic freedom of QCD at high temperature. Therefore, several investigations on microscopic and model dependent calculation of $\eta/s$ for quark matter as well as hadronic matter have been done in recent times. The latter one suddenly attract some extra importance when Niemi et al. [1] shows that the extracted transverse momentum $p_T$ dependence on elliptic flow parameter, $v_2(p_T)$, of RHIC data is highly sensitive to the temperature dependent $\eta/s$ in hadronic matter. In this context, we have calculated $\eta$ as well as $\eta/s$ of a pion gas using an effective Lagrangian for $\pi\pi\sigma$ and $\pi\pi\rho$ interactions and then we have also compared our results to others of the recent literature.

2. Formalism

Let us start with the standard expression of shear viscosity for pion gas, obtained from relaxation time approximation (RTA) [5, 6, 4]:

$$\eta = \frac{\beta}{10\pi^2} \int \frac{d^3k k^6}{\Gamma_{\pi}(k, T) \omega_k^2} n(\omega_k) [1 + n(\omega_k)], \quad (2.1)$$

where $n(\omega_k) = 1/\{e^{\beta \omega_k} - 1\}$ is Bose-Einstein (BE) distribution function for a temperature $T = 1/\beta$, with $\omega_k = (k^2 + m_\pi^2)^{1/2}$, and $\Gamma_{\pi}(k, T)$ is identified as the thermal width of $\pi$ mesons in the medium.

As $\sigma$ and $\rho$ resonances come into the picture of $\pi\pi$ scattering cross section, therefore we have adopted an effective interaction Lagrangian density,

$$\mathcal{L} = g_\rho \rho \mu \cdot \pi \times \partial^\mu \pi + \frac{g_\sigma}{2} m_\sigma \pi \cdot \pi \sigma, \quad (2.2)$$

as a dynamical guidance of pion interaction with mesonic medium. We have taken coupling constants $g_\rho = 6$ and $g_\sigma = 5.82$, which are fixed from experimental decay widths of $\rho$ and $\sigma$ respectively.

The pionic thermal width $\Gamma_{\pi}$ has been obtained from the imaginary part of pion self-energy $\Pi_{\pi(\pi R)}$ for $\pi R$ loops, where resonance $R$ stands for $\sigma$ and $\rho$ mesons. The self-energy diagram is shown in the left panel of Fig. [4]. Our required relation, after thermal field theoretical deduction, is [4]

$$\Gamma_{\pi}(k, T) = \sum_{R=\sigma, \rho} \Gamma_{\pi(\pi R)} = \sum_{R=\sigma, \rho} \frac{\text{Im} \Pi_{\pi(\pi R)}(k_0 = \omega_k, \vec{k}, T)}{m_\pi}$$

$$= \sum_{R=\sigma, \rho} \frac{1}{m_\pi} \int \frac{d^3l}{32\pi^2 \omega_k \omega_l} L(k, l)|_{l_0 = -\omega_k, k_0 = \omega_k} \left\{n(\omega_k) - n(\omega_l)\right\} \delta(\omega_k + \omega_l - \omega_n), \quad (2.3)$$
where \( n(\omega_l) \) and \( n(\omega_u) \) are BE distribution functions of intermediate \( \pi \) and \( R \) mesons for \( \omega_l = \{ \vec{l}^2 + m^2_\pi \}^{1/2} \) and \( \omega_u = \{ |\vec{k} - \vec{l}|^2 + m^2_R \}^{1/2} \) respectively. The vertex factors -

\[
L(k,l) = -\frac{g^2 m^2_\sigma}{4}, \quad \text{for } R = \sigma, \\
= -\frac{g^2}{m^2_\rho} \left[ k^2 \left( k^2 - m^2_\rho \right) + l^2 \left( l^2 - m^2_\rho \right) - 2 \{ (k \cdot l) m^2_\rho + k^2 l^2 \} \right], \quad \text{for } R = \rho
\]

(2.4)
can be obtained from the effective Lagrangian density, given in Eq. 2.2.

Next, to take into account the widths of the resonances, at first their masses \( m_R \) are taken as free parameter like invariant mass \( M \). Then the modified \( \Gamma_{\pi(R)}(\vec{k},T,M) \) of Eq. (2.3) are folded by vacuum spectral functions of corresponding resonances, whose general form is

\[
\rho_R(M) = \frac{1}{\pi} \text{Im} \left[ \frac{1}{M^2 - m^2_R + iM\Gamma_R(M)} \right].
\]

(2.5)

With the help of the Lagrangian density [22], the vacuum decay width \( \Gamma_R(M) \) of the \( R = \sigma, \rho \) mesons can be obtained as

\[
\Gamma_\sigma(M) = \frac{3 g^2 m^2_\sigma}{32 \pi M} \left( 1 - \frac{4m^2_\pi}{M^2} \right)^{1/2},
\]

(2.6)

\[
\Gamma_\rho(M) = \frac{g^2 M}{48 \pi} \left( 1 - \frac{4m^2_\pi}{M^2} \right)^{3/2}.
\]

(2.7)

The normalized relation for this folding technique, which we have taken, is

\[
\Gamma_{\pi(R)}(\vec{k},T,m_R) = \frac{\int dM^2 \rho_R(M) \Gamma_{\pi(R)}(\vec{k},T,M)}{\int dM^2 \rho_R(M)}.
\]

(2.8)
3. Results and Discussion

Using Eq. (2.3) and (2.8), without (dotted line) and with (solid line) folding results of thermal width as a function of momentum are respectively estimated and displayed in the right panel of Fig. 1 at $T = 0.170$ GeV (upper panel) and $T = 0.150$ GeV (lower panel). We see that due to folding effect, modification of $\Gamma_\pi(\vec{k})$ at low momentum becomes larger than that at high momentum and this modification increases with the increasing of temperature. This momentum distribution of $\Gamma_\pi$ will be inversely integrated out to obtain shear viscosity $\eta$ of pion gas, as expressed in Eq. (2.1).

The temperature dependence of $\eta$ is originated from two sources - the Bose-Einstein distribution of pion and its thermal width $\Gamma_\pi$. In the left panel of Fig. 2, the temperature dependence of $\eta$ is presented using without (upper panel) and with (lower panel) folding results of $\Gamma_\pi$. The shear viscosity due to $\pi \rho$ (dotted line) and $\pi \sigma$ (dashed line) loops become divergent in the higher ($T > 0.100$ GeV) and lower ($T < 0.100$ GeV) temperature regions respectively. This complementary features of these two loops indicates that consideration of both resonances in $\pi - \pi$ scattering is strictly necessary to obtain a smooth, non divergent $\eta$ for temperatures below the critical, $T_c \simeq 0.175$ GeV. Though without folding results of $\eta$ at very low temperatures ($T < 0.020$ GeV) tends to diverge, but after adopting folding this trend disappears.

![Figure 2: (Color online) Left : With (lower panel) and without (upper panel) folding results of $\eta$ vs $T$ due to $\pi \sigma$ (dashed lines), $\pi \rho$ (dotted lines) loops and their total (solid line). Right : Our estimation of $\eta(T)$ (upper panel) $\eta/s(T)$ (lower panel) have been compared to some other results. Horizontal red line indicates the KSS bound of $\eta/s$.](image)

Our results have been compared with the earlier results obtained by Fraile et al. [2], Lang et al. [3] and Mitra et al. [4], which are attached in the upper and right panel of Fig. (2). Shear viscosity of pion gas, obtained by Refs. [2, 3, 4] in the kinetic theory approach, has been found as a monotonically increasing function of temperature in the hadronic temperature range ($0.100$ GeV $< T < 0.175$ GeV) for vanishing baryon chemical potential ($\mu = 0$) whereas Lang et al. [3] in the Kubo approach has shown $\eta$ as a decreasing function of temperature. Similar decreasing nature of $\eta(T)$ are followed in the Kubo-approach calculation by Fraile et al. [2] before unitarization of pion thermal width but it turns to an increasing function after dynamically generating low mass resonances ($\rho$ and $\sigma$) via unitarization technique. This change in nature of $\eta$ vs $T$ may be because of the transformation from the scenario of pionic medium without resonances to one with resonances. Using the effective Lagrangian density, we have approximately mapped a similar kind of
scenario, where probabilities of pion scattering with the low mass resonances $\rho$ and $\sigma$ have been found from the finite temperature calculation of pion self-energy for $\pi\rho$ and $\pi\sigma$ loops. Similar to the results obtained by Fraile et al. [2] after unitarization, we have observed an increasing $\eta(T)$ after $T = 0.100$ GeV but lower in magnitude and increasing slope.

Using ideal expression of entropy density $s$ for pion, we have shown the variation of the ratio $\eta/s$ with respect to $T$ in the lower panel of Fig. (2), situated in the right side. Within the hadronic temperature domain, our results of $\eta/s$ respect the KSS bound $\frac{1}{4\pi}$ as shown by horizontal red line. Since Niemi et al. [1] have taken a similar range of $\eta/s(T)$, close to the KSS bound and found its important role to fit $v_2(P_T)$ of RHIC data, therefore our estimation has a good association with phenomenological side also.

4. Summary and Perspectives

By considering the interaction of pion with the low mass resonances $\sigma$ and $\rho$, we have calculated the shear viscosity coefficient of hot pion gas. In the framework of thermal field theoretical technique, the thermal width $\Gamma_\pi$ has been calculated from one-loop pion self-energy at finite temperature. For the interaction part, we have taken an effective Lagrangian density, by which pion self-energy for $\pi\sigma$ and $\pi\rho$ loops have been evaluated at finite temperature. To treat $\sigma$ and $\rho$ resonances as two-pion states, we have folded the self-energy of $\pi\sigma$ and $\pi\rho$ loops by the Breit-Wigner type spectral function of $\sigma$ and $\rho$ with their vacuum width in $\pi\pi$ channel respectively. We have seen a complementary role of $\pi\sigma$ and $\pi\rho$ loops to make $\eta$ be non-divergent in the lower ($T < 0.100$ GeV) and higher ($T > 0.100$ GeV) temperature regions respectively. From the investigations of Niemi et al. [1], we see that $v_2(P_T)$ of RHIC data prefers these small values of $\eta/s(T)$ for hadronic matter. This provides experimental justification to the microscopic calculation of shear viscosity due to $\sigma\pi\pi$ and $\rho\pi\pi$ interaction performed in this work.

References

[1] H. Niemi, G.S. Denicol, P. Huovinen, E. Molnár, D.H. Rischke, *Influence of the shear viscosity of the quark-gluon plasma on elliptic flow in ultra-relativistic heavy-ion collisions*, Phys. Rev. Lett. 106 (2011) 212302 [nucl-th/11012442].

[2] D. Fernandez-Fraile and A. Gomez Nicola, *Transport coefficients and resonances for a meson gas in chiral perturbation theory*, Eur. Phys. J. C 62 (2009) 37 [hep-ph/09024829].

[3] R. Lang, N. Kaiser, and W. Weise, *Shear viscosity of a hot pion gas*, Eur. Phys. J. A 48 (2012) 109 [hep-ph/12056648].

[4] S. Mitra, S. Ghosh, and S. Sarkar, *Effect of a spectral modification of the $\rho$ meson on the shear viscosity of a pion gas*, Phys. Rev. C 85 (2012) 064917 [nucl-th/12042388].

[5] S. Gavin, *transport coefficients in ultra-relativistic heavy ion collisions*, Nucl. Phys. A 435 (1985) 826.

[6] M. Prakash, M. Prakash, R. Venugopalan, and G. Welke, *Non-equilibrium properties of hadronic mixtures*, Phys. Rep. 227 (1993) 321.

[7] S. Ghosh, G. Krein, S. Sarkar, *Shear viscosity of a pion gas resulting from $\rho\pi\pi$ and $\sigma\pi\pi$ interactions*, Phys. Rev. C 89 (2014) 045201 [nucl-th/14015392].