Mechanics of stability's loss in the skiing turning

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Abstract. The article discusses the mechanics of the loss of an athlete transverse steady state when performing a ski turn. The analysis of the loss of stable position of the rod model of the mechanical system "skier-skis" in the lateral sliding of skis and falling in the frontal plane was performed. The conditions and possible ways of preventing the skier from falling and restoring a stable position during turn are quantified.

1. Introduction
The exit skier from equilibrium, loss of sustainability in the best case leads to loss of speed movement athlete on the race course, and in the worst case to fall. The present work is devoted to the analysis of the General nature of changes in the state of the system "skier–skis” when it deviates from a stable position in the process of performing a ski turn.

Figure 1a. Figure 1b.
2. The analysis of stable motion
As was considered in [1], the line of action of the vector of resultant gravity and inertial forces $F$ is the orientation direction along which the athlete positions his body, keeping his position stable when performing a ski turn (figure 1a). Consider such a stable position, turning the figure so that the line of action of the force $F$ and the line of action of the resultant support reactions $R$ are directed vertically along the line $OO'$ (figure 1b).

In this case, the resultant of the support reactions $R$, having normal ($N$) and tangent ($F_{fr}$) components, is directed against the force $F$. And the angle between the surface of slope and the perpendicular to $OO'$ corresponds to the angle between the reference line $OO'$ and the normal to the slope, i.e. the angle of inclination $\delta$ of the reference line, as shown in the force diagram for the model of the mechanical system (figure 2).

Since the skier rides on two skis (feet), changing the angle of the reference line when moving along the trajectory of turn does not immediately lead him out of sustainable movement. When the reference line rotates, the support reactions are redistributed between the skis (feet), compensating for the varying rotational moment of the resultant force $F$. These redistributions are fixed by muscle receptors and the vestibular apparatus of the skier, the skier turns his body in a transverse plane to the movement, moving his center of mass to a new position of the support line and changing the skis edging. In this way, the skier reflexively adapts to the changing conditions of rotation − speed and turning radius.

A necessary condition for the stability of the position of the skier is the passage of the line of action of the vector $F$ through the support area, limited by the width of the statement of skis. The loss of stability occurs when the point of intersection of the slope and the line of action of the vector $F$ goes beyond the reference site. In this case, for the possibility of restoring stability, it is necessary that the process of falling (loss of stability) lasts no shorter than the characteristic time of human reaction to external influence, which is $0.15\pm0.2$ sec, only after the expiration of which the skier can adequately respond to the deviation of the position and restore the stability of the movement.

3. Deviation from a stable position
We estimate the time of the fall of the skier at a deviation from a stable position. To do this, consider the case of falling skier in the absence of resistance to loss of stability. Consider the simplest model of the mechanical system "skier−skis" in the form of a straight rod OA. It is obvious that such a mechanical model does not allow to accurately assess the parameters of the process of loss of stability due to inaccurate correspondence of the inertial characteristics of the human body and a homogeneous rod, however, its use makes it possible to analyze the general nature of the mechanics of changing the state of a system when it deviates from its stable position, which, in fact, is the subject of this study.

Figure 3 shows the scheme of the rotation of the OA rod around its butt-end supported on a support O under the action of a force $F$ applied to the center of mass C of the bar. Let $OO'$ be the line of stable position of the bar axis, it is also the line of action of the reference reaction in a stable position, as well as the line of action of the resultant $F$ gravity and inertia forces. Then $\varphi$ − the angle of rod's deviation
from a stable position, OA = L, OC = ℓ. The magnitude of the force vector $F$ expressed in terms of gravity $mg$ as $F = n \cdot mg$, where $n$ is the overload experienced by the skier in the turn.

Then the equation of the dynamics of rotational motion for such a system will be:

$$J_\alpha \varepsilon = n mg \sin \varphi$$

Here $\varepsilon = \dot{\varphi}$ is the angular acceleration, or the second time derivative of the rotation's angle. For a homogeneous rod $L = 2\ell$, then its moment of inertia relative to the end face is equal to:

$$J_\alpha = \frac{1}{3} mL^2 = \frac{4}{3} m\ell^2$$

After substitution and reduction by $m$ equation (1) is transformed into a nonlinear differential equation:

$$\dot{\varphi} - n \frac{3g}{4\ell} \sin \varphi = 0$$

Considering the process of deviation from a stable position, it is necessary to estimate the time for which the angle of deviation reaches such a value at which the fall becomes inevitable already with any further actions of the skier. Such a critical angle $\varphi_{cr}$ is approximately $30^\circ$, or 0.5 radian. As we will see from further consideration, it is convenient to set this angle in the following form:

$$\varphi_{cr} = \frac{\pi}{2e} \approx 0.578 \text{ rad} \approx 33.1^\circ.$$  

Given that we are interested in relatively small deflection angles, we linearize equation (3) to achieve the possibility of obtaining an analytical estimate of its solutions:

$$\dot{\varphi} - n \frac{3g}{4\ell} \varphi = 0$$

We will look for a solution in the form $\varphi = Ae^\eta$. Then, after substitution in (5), we obtain a characteristic equation of the form $\eta^2 - n \frac{3g}{4\ell} \eta = 0$, through the roots of which $\pm \frac{1}{2} \left( n \frac{3g}{4\ell} \right)^{\frac{1}{2}}$ the general solution of equation (5) is expressed:

$$\varphi = A_1 e^{\frac{1}{2} \left( n \frac{3g}{4\ell} \right)^{\frac{1}{2}}} t + A_2 e^{-\frac{1}{2} \left( n \frac{3g}{4\ell} \right)^{\frac{1}{2}}} t$$

4. Scenarios of falling

a) The deviation starts at some initial angle with zero initial angular velocity: $t_0 = 0; \quad \omega_0 = \varphi_0 = 0; \quad \dot{\varphi}_0 \neq 0$. Such initial conditions may be associated with a change in the relief of the slope, and hence the steepness (angle) of the slope when the skier moves on the race course. Using these initial conditions allows you to determine the unknown parameters $A_1$ and $A_2$ and get a solution in the form:

$$\varphi = \frac{1}{2} \varphi_0 \left( e^{\frac{1}{2} \left( n \frac{3g}{4\ell} \right)^{\frac{1}{2}}} t + e^{-\frac{1}{2} \left( n \frac{3g}{4\ell} \right)^{\frac{1}{2}}} t \right),$$
and the angular velocity when falling is described by the expression:

$$\omega = \frac{1}{4} \varphi_0 \left( n \frac{3g}{\ell} \right)^{1/2} \left( e^{\frac{1}{2} \left( n \frac{3g}{\ell} \right)^{1/2} t} - e^{-\frac{1}{2} \left( n \frac{3g}{\ell} \right)^{1/2} t} \right).$$

(8)

The solution of equation (7) for the angle $\varphi_a$ gives the value of time $\tau$ to reach the critical angle of deflection:

$$\tau = \frac{2}{\sqrt{n}} \left( \frac{\ell}{3g} \right)^{1/2} \ln \left( \frac{\pi}{2e \varphi_0} + \sqrt{\frac{\pi^2}{4e^2 \varphi_0^2} - 1} \right)$$

(9)

If for convenience of calculations the initial angle is set in the form $\varphi_0 = \frac{\pi}{2e}$, then at $k = 2$ the initial angle has a value of $\varphi_0 \approx 12.2^\circ$, at $k = 3$ its value is $\varphi_0 \approx 4.5^\circ$, and the formula (9) becomes simpler:

$$\tau = \frac{2}{\sqrt{n}} \left( \frac{\ell}{3g} \right)^{1/2} \ln \left( e^{k-1} + \sqrt{e^{2k-2} - 1} \right)$$

(10)

b) The deviation begins with a zero initial angle and with some initial angular velocity: $t_0 = 0$; $\omega_0 = \dot{\varphi}_0 \neq 0$; $\varphi_0 = 0$. This variant of the initial conditions can be formed during the passage of the skier, for example, uneven terrain – pit or knoll, overcoming which can cause transverse rotation of the body of the skier in the frontal plane. Applying these initial conditions gives a solution in the form:

$$\varphi = \frac{\omega_0}{4} \sqrt{\frac{3g}{\ell}} \ln \left( e^{\frac{1}{2} \left( n \frac{3g}{\ell} \right)^{1/2} \tau} - e^{-\frac{1}{2} \left( n \frac{3g}{\ell} \right)^{1/2} \tau} \right)$$

(11)

Let us estimate the minimum initial angular velocity of rotation, at which, during the standard human response delay time ($\tau_r = 0.15$ sec) the deflection angle reaches the critical value (4). From (11) follows:

$$\omega_0 = \frac{1}{\sqrt{n}} \frac{2\pi}{e} \sqrt{\frac{\ell}{3g}} \ln \left( e^{\frac{1}{2} \left( n \frac{3g}{\ell} \right)^{1/2} \tau} - e^{-\frac{1}{2} \left( n \frac{3g}{\ell} \right)^{1/2} \tau} \right)^{-1}$$

(12)

We draw up the results of calculations on points a) and b), placing them in Table 1, taking numerically: $g = 9.81 \text{ m/sec}^2$, $\ell = 0.8 \text{ m}$, $\varphi_{cr} \approx 33.1^\circ$, $\tau_r = 0.15$ sec.

Table 1. Deviation time $\tau$ (sec) to a critical angle $\varphi_{cr}$

| Initial deviation angle $\varphi_0$, degree | Initial angular velocity $\omega_0$, degree/sec | Overload $n$ |
|--------------------------------------------|-----------------------------------------------|--------------|
| 4.5                                        | 0                                             | 1 2 3        |
| 12.2                                       | 0                                             | 0.65 0.46 0.38 |
| 0                                          | 23.2                                          | 0.27 0.19 0.16 |
| 0                                          | 11.2                                          | – 0.15 –     |
| 0                                          | 7.2                                           | – 0.15 0.15  |


5. Loss of stability during side slip
The force scheme changes if the tangent component of the support reaction or the friction force \( F_{fr} \) that prevents lateral skidding disappears (partially or completely) (figure 4). This may occur due to the departure of the skier on the ice section of the slope, insufficient sharpening of the edges of the skis, changing the load due to the irregularities of the terrain, etc. As a result of the loss of grip, the skier will begin to skid down (or outward of the turn), while the action of a normal reaction results in the turn of the body the body to turn around the center of mass \( C \) (angle \( \varphi \)).

For the case of complete loss of friction it is given below a system of equations of dynamics of the center of mass \( C \) along the \( x \) and \( y \) axes, as well as rotation around the point \( C \), where \( OC = \ell \), \( OA = L \), and the value \( F \) through the overload \( n \) can be expressed as \( F = n \cdot mg \):

\[
\begin{cases}
ma_{cx} = F \sin \delta \\
ma_{cy} = -F \cos \delta + N \\
J_c \ddot{\varphi} = N \ell \sin \varphi
\end{cases}
\]  

(13)

From the first equation of system (13) it follows that the center of mass will move along the \( x \) axis with acceleration:

\[ a_{cx} = ng \sin \delta , \]

(14)

with speed:

\[ v_{cx} = ng \sin \delta \cdot t , \]

(15)

over a distance:

\[ x_c = \frac{1}{2}ng \sin \delta \cdot t^2 . \]

(16)

From the second equation of the system (13), the normal reaction of the support is expressed:

\[ N = ma_{cy} + n \cdot mg \cos \delta \]

(17)

If we take the homogeneous straight rod as the simplest model of a mechanical system, then \( L = 2 \ell \), and the central moment of inertia \( J_c = \frac{1}{12} mL^2 = \frac{1}{3} m\ell^2 \).

The distance \( y_c \) of the point \( C \) from the surface of the slope is related to the angle of deviation of the rod from the normal to the surface: \( y_c = \ell \cdot \cos \varphi \). The normal components of the speed \( v_{cy} \) and acceleration \( a_{cy} \) are determined through the first and, respectively, the second time derivatives of \( y_c \):

\[ v_{cy} = \dot{y}_c = -\ell \phi \sin \varphi , \]

(18)

\[ a_{cy} = \ddot{y}_c = \ddot{y}_c = -\ell (\ddot{\phi} \sin \varphi + \dot{\phi}^2 \cos \varphi) . \]

(19)

After substituting (19) into (17) and further into the third equation of system (13), we get:

\[ \frac{1}{3}m\ell^2 \ddot{\varphi} = n \cdot mg \ell \cos \delta \sin \varphi - m\ell^2 \sin \varphi (\ddot{\phi} \sin \varphi + \dot{\phi}^2 \cos \varphi) \]

(20)

After reduction by \( m \):
\[ \phi - n \cdot \frac{3}{\ell} g \cos \delta \sin \varphi + 3 \phi \sin^2 \varphi + 3 \phi^2 \sin \varphi \cos \varphi = 0 \] (21)

Equation (21) is a nonlinear differential equation that can be integrated numerically. Given that we are interested in relatively small angles, angular velocities, and angular accelerations, we linearize equation (21) by replacing the sine with the angle itself and discarding terms with a higher order than two:

\[ \phi - n \cdot \frac{3g}{\ell} \cos \delta \cdot \varphi = 0 \] (22)

As you can see, the equation (22) corresponds to the equation (5), which allows to obtain a general analytical solution in the form of:

\[ \varphi = A_1 e^{\left(\frac{2n^2 g \cos \delta}{\ell} t\right)} + A_2 e^{\left(-\frac{2n^2 g \cos \delta}{\ell} t\right)} \] (23)

Applying the initial conditions \((t_0 = 0; \omega_0 = \varphi_0 = 0; \varphi_0 = \delta)\) gives a solution in the form:

\[ \varphi = \frac{1}{2} \delta \left( e^{\left(\frac{2n^2 g \cos \delta}{\ell} t\right)} + e^{\left(-\frac{2n^2 g \cos \delta}{\ell} t\right)} \right), \] (24)

while the magnitude of the deviation \(\Delta \varphi\) from a stable position is equal to:

\[ \Delta \varphi = \frac{1}{2} \delta \left( e^{\left(\frac{2n^2 g \cos \delta}{\ell} t\right)} + e^{\left(-\frac{2n^2 g \cos \delta}{\ell} t\right)} \right) - \delta , \] (25)

and angular velocity:

\[ \omega = \frac{1}{2} \delta \left( n \frac{3 g \cos \delta}{\ell} \right)^{1/2} \left( e^{\left(\frac{2n^2 g \cos \delta}{\ell} t\right)} - e^{\left(-\frac{2n^2 g \cos \delta}{\ell} t\right)} \right). \] (26)

**Table 2.** The angle of deviation \(\Delta \varphi\) from a stable position and the angular velocity \(\omega\) of the fall at a time \(t\)

| Angle of the reference line \(\delta\), deg | Overload \(n\) | 1 | 2 | 3 | 1 | 2 | 3 |
|------------------------------------------|----------------|---|---|---|---|---|---|
| Fall time \(t\), sec                     | angle of deviation \(\Delta \varphi\), deg | 0.15 | 0.20 | 0.15 | 0.20 | 0.15 | 0.20 | 0.15 | 0.20 | 0.15 | 0.20 |
| 4.5                                      | 2.0            | 3.7 | 4.2 | 8.3 | 6.8 | 14 | 28 | 41.5 | 64 | 103 | 108 | 189 |
| 12.2                                     | 5.3            | 9.9 | 11 | 22 | 18 | 37 | 75 | 110 | 170 | 272 | 287 | 497 |
| 16.5                                     | 7              | 13 | 15 | 29 | 24 | 49 | 100 | 146 | 225 | 360 | 380 | 655 |
| 33.1                                     | 12             | 23 | 26 | 50 | 41 | 82 | 171 | 249 | 382 | 598 | 636 | 1066 |
| 48.6                                     | 14             | 26 | 29 | 55 | 46 | 90 | 194 | 276 | 421 | 638 | 687 | 1099 |
| 66.2                                     | 11             | 21 | 23 | 43 | 36 | 68 | 156 | 217 | 329 | 475 | 520 | 780 |
From expressions (25) and (26), it follows that the values of the parameters associated with each other ($\Delta \varphi$, $\omega$, $t$) practically will not differ by more than 10% from each other when the angle $\delta$ changes within $0^\circ \div 35^\circ$ (because $\sqrt{\cos 35^\circ} \approx 0.9$). Comparing (8) and (26), it should be noted that all other things being equal, the rate of change in the position of the mechanical system, or the rate of fall in the latter case, is almost twice as high.

Table 2 shows the results of calculations of the angle of rotation and the angular velocity of the fall for different values of the delay time range of the natural human reaction ($\tau_r = 0.15 \div 0.20$ sec).

As can be seen from the results, deviations from the stable position in the fall due to lateral skidding increase rapidly as the angle of inclination of the reference line increases, but at large angles – decrease due to a decrease in the value of the normal component of the reference reaction.

6. Restoring a stable position
Let's consider different scenarios of restoring stability by a skier.

1) Restoration with the help of a quick change feet's position in the frontal plane, for example, by means of a jump or by means of shifting (moving) the skis on a slope in the frontal plane. In practice, the recovery of stability by a jump is rarely realized for two reasons. Firstly, it requires considerable time for the sequential inclusion in the work of muscles - antagonists (flexors – extensors). Secondly, in the absence of friction, the skier loses the necessary support to a certain extent.

In the above cases 4A) and 4b) the skier can restore stability by rearranging the skis, or as a result of cross-displacement of the skis and increasing the distance between the legs (skis).

2) Restoring stability by changing the position of the skier's body.

The skier is in a constant process of maintaining stability while driving, controlling this process by changing the angulation [3,4]. In the event of an exit from a steady position, the skier reflexively changes the position of the body, disrupting his angulating position to restore stability.

The most radical and extreme way to restore stability is to change the position of the limbs (hands). In this case, the termination of the transverse rotation of the body and the restoration of a stable position is achieved by the reflex accelerated rotation of the hands (often, and the upper body) in the frontal plane in the direction of the transverse rotation of the body. In the process of restoring stability, flapping (rotation) of the hands is the most effective and dynamic way to restore stability. Let us estimate quantitatively the effectiveness of such a method.

For a rough estimate, let us consider the simplest models of the elements of a mechanical system - the skier and his hands in the form of uniform straight rods 1, 2 and 2a (figure 5). Let the lengths of these rods be $L = 2\ell = 1.6$ m and $\ell = 0.6$ m, with the mass of one human arm $m_2$ being [2] about 6% of the total body weight $m = m_1 + 2m_2$, or: $m_2 = 0.06m$, $m_1 = 0.88m$. Let rod 1 rotate with angular velocity $\omega_1$, and rods 2-2a with angular velocity $\omega_2$, moreover $\omega_2 >> \omega_1$. Then the kinetic moment of the entire system relative to the center C consists of the kinetic moment $K_{c1}$ of the rod 1 relative to the center of mass of the system and the kinetic momentum $K_{c2}$ of the system of rods 2-2a fixed on the rod 1 at the point M, which includes the angular momentum of the center of mass of the system of rods 2-2a relative to point C:
Here \( \omega_1 = \omega_{1}\cdot CM \) – the speed of rotation of point \( M \) about point \( C \).

The moment of inertia of the rod 1 is considered relative to the center of mass of system \( C \), and the moment of inertia of rods’ system \( 2-2a \) through the point \( M \) of its fixing on the rod 1:

\[
J_{1c} = \frac{1}{12} m_1 l^2; \quad J_{2-2a} = \frac{2}{3} m_2 l'^2.
\]  

(28)

Let the reference reaction be oriented with respect to the rod 1 at an angle \( \beta \), then in accordance with the theorem on the change of the kinetic moment:

\[
\frac{dK_{1c}}{dt} = M_c(\bar{R}), \quad \text{or}
\]

\[
\frac{d\left((J_{1c} + 2m_2CM^2)\omega_1 + J_{2-2a}\omega_2\right)}{dt} = R\ell \sin \beta
\]  

(29)

If we neglect the moment of reaction force, the kinetic moment of the system will not change:

\[
(J_{1c} + 2m_2CM^2)\omega_1 + J_{2-2a}\omega_2 = (J_{1c} + 2m_2CM^2 + J_{2-2a})\omega
\]  

(30)

In the latter equality for the initial angular velocity \( \omega \) of rotation of the system is taken as the angular velocity of the fall from the table. 2. Then, in the absence of a reference reaction to stop the rotation of the body 1, it is necessary that the value of the rotation speed of the rods 2-2a satisfies the equality (30) when \( \omega_1 = 0 \). Hence:

\[
\omega_2 = \frac{J_{1c} + 2m_2CM^2 + J_{2-2a}}{J_{2-2a}} \omega.
\]  

(31)

By the average value of the rotation speed, we determine the time of rotation of the system 2-2a by 180°, which will be equal to:

\[
\tau = \frac{180^\circ}{\langle \omega_2 \rangle} = \frac{2 \cdot 180^\circ}{J_{1c} + 2m_2CM^2 + J_{2-2a}} \cdot \frac{360^\circ}{\omega}.
\]  

(32)

When \( m_1 = 0.88m \), \( m_2 = 0.06m \), \( \ell = 0.8 \) meters, \( \ell_2 = 0.6 \) meters, and, for example, \( CM = 0.4 \) meters, \( \omega = 28^\circ/c \), we obtain \( \tau = 0.24 \) seconds. If it is considered roughly that the effect of rotating two symmetrically arranged rods 2 and 2a is twice as large as that of rotating one, the required time for stopping the rotation of rod 1 by the accelerated rotation of one rod 2 is halved: \( \tau = 0.12 \) sec.

Thus, if a full swing of the arm can be performed in 0.1 sec, then the skier can stop the rotation of the body at even a small initial angle of inclination of the support line 4.5° only when overloaded \( n=1 \), and at large angles of inclination of the support line, a fall from loss of stability with lateral skid becomes inevitable.

If you provide a change in the angular velocity of rotation to a greater extent than that defined in (31) by including not only both hands in the accelerated rotation, but also the upper part of the skier’s body, in accordance with (29) there will be a noticeable support reaction that allows the skier to gain short-term support and perform the restoration of a steady state, for example, by the methods described in section 1 of this article.
7. Conclusion
The paper analyzes the various mechanisms of loss of stability of the skier during his movement in the ski turn. It is shown that when the initial conditions change within a wide range, the time of skier’s body deviation to the critical angle is higher than the time of natural human reaction. This leaves him with the ability to take action to prevent a fall and restore stability up to a triple overload in a turn experienced by the skier.

It is defined that the rate of fall of the skier in a side skidding of the skis is almost twice that of no-skid. In this regard, the possibility of restoring the stable position of the skier is limited to very small angle of the reference line in the absence of a significant overload, and with an increase in these angles, the fall becomes inevitable.

In practice, the most typical case of loss of stability of an athlete – skier is lateral skidding. Therefore, to assess the possibility of restoring the stability of the athlete, it is necessary to use the quantitative parameters obtained in the version with lateral skidding of skis.

The human musculoskeletal system has immeasurably more degrees of freedom than the simplest models of the mechanical system considered in the article. In this regard, the quantitative estimates of the process of loss of stability obtained in this paper are very approximate. Nevertheless, this approach allows us to establish the general nature of the change in the state of the "skier–skis" system in the process of a skier’s ski turn, both when it deviates from a stable position and when it is restored.

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