An important advantage of fractional derivatives is that we can formulate models describing much better systems with memory effects. Fractional operators with different memory are related to the different type of relaxation process of the nonlocal dynamical systems. Therefore, we investigate the COVID-19 model with the fractional derivatives in this paper. We apply very effective numerical methods to obtain the numerical results. We also use the Sumudu transform to get the solutions of the models. The Sumudu transform is able to keep the unit of the function, the parity of the function, and has many other properties that are more valuable. We present scientific results in the paper and also prove these results by effective numerical techniques which will be helpful to understand the outbreak of COVID-19.

Key words
COVID-19, Mittag–Leffler kernel, numerical methods, Sumudu transform

JEL classification
37C75; 93B05; 93B07; 65L07

1 | INTRODUCTION

Epidemiological study presents a crucial role to see the impact of infectious disease in a community. In mathematical modeling, we check models, estimation of parameters, and measure sensitivity through different parameters and compute their numerical simulations through building models. The control parameters and ratio spread of disease can be understood through this type of research.1 These types of diseased models are often called infectious diseases (i.e., the disease which transferred from one person to another person). Measles, rubella, chicken pox, mumps, aids, and gonorrrhea syphilis are the examples of infectious disease.2

Severe acute respiratory syndrome (SARS) is caused by a coronavirus and plays important role for its investigation.3 According to the group of investigators that has been working since 30 years on the coronavirus family investigated that SARS and coronavirus have many similar features like biology and pathogenesis.4 RNA enveloped viruses known as coronavirus are spread among humans, mammals, and birds. Many respiratory, enteric, hepatic, and neurological diseases are caused by coronavirus.5 Human disease is caused by six types of coronavirus.6 In 2019, China faced a major outbreak of coronavirus disease 2019 (COVID-19), and this outbreak has the potential to become a worldwide pandemic. Interventions and real-time data are needed for the control on this outbreak of coronavirus.7 In previous studies, the transfer of the virus from one person to another person, its severity and
history of the pathogen in the first week of the outbreak, has been explained with the help of real-time analyses. In December 2019, a group of people in Wuhan admitted to the hospital that all were suffering from pneumonia, and the cause of pneumonia was idiopathic. Most of the people linked cause of pneumonia with the eating of wet markets meet and seafood. Investigation on etiology and epidemiology of disease was conducted on December 31, 2019, by Chinese Center for Disease Control and Prevention (China CDC) with the help of Wuhan city health authorities. Epidemically changing was measured by time-delay distributions including date of admission to hospital and death. According to the clinical study on the COVID-19, symptoms of coronavirus appear after 7 days of onset of illness. The time from hospital admission to death is also critical to the avoidance of underestimation when calculating case fatality risk.

The fractional order techniques are very helpful to better understand the explanation of real-world problems other than ordinary derivative. The idea of fractional derivative has been introduced by Riemann–Liouville, which is based on power law. The new fractional derivative which is utilizing the exponential kernel is prosed in Atangana and Alkahtani. A new fractional derivative with a nonsingular kernel involving exponential and trigonometric functions is proposed in previous studies, and some related new approaches for epidemic models have been illustrated here. Important results related to this new operator have been established, and some examples have been provided in Khan et al. The equation of one-dimensional finite element can be established generalized three-dimensional motion by using the Lagrange's equations. The problem is important in technical applications of the last decades, characterized by high velocities and high applied loads. A constant Thomson coefficient, instead of traditionally a constant Seebeck coefficient, is assumed. The charge density of the induced electric current is taken as a function of time. A normal mode method is proposed to analyze the problem and to obtain numerical solutions.

We organize our work as follows: We present the main definitions of fractional calculus in Section 2. We give the fractional order COVID-19 model in Section 3. Sumudu transform (ST) is applied in this section, also present some scientific theorems. We discuss Adams–Moulton method with the Mittag–Leffler kernel in Section 4. The new numerical scheme is developed in Section 5. Conclusion is provided in the last section.

2 | SOME BASIC CONCEPTS OF FRACTIONAL CALCULUS

We give some basic definitions related to fractional calculus and ST in this section.

Definition 2.1. For any function \( \phi(t) \) over a set, the Sumudu transform

\[
A = \left\{ \phi(t) : \text{there exist } \Lambda, \tau_1, \tau_2 > 0, |\phi(t)| < \Lambda \exp \left( \frac{t}{\tau_1} \right), \text{if } t \in (-1)^j \times [0, \infty) \right\}
\]

is defined by

\[
F(u) = ST[\phi(t)] = \int_0^\infty \exp(-u)\phi(ut)dt, \ u \in (-\tau_1, \tau_2).
\] (1)

Definition 2.2. We define the fractional order derivative of Atangana–Baleanu in Caputo sense (ABC) as

\[
^{ABC}_{a}D^\alpha_t \phi(t) = \frac{AB(\alpha)}{n-\alpha} \int_a^t \frac{d^n}{dw^n} f(w)E_\alpha \left\{ -\alpha \frac{(t-w)^\alpha}{n-\alpha} \right\}dw, \ n-1 < \alpha < n,
\] (2)

where \( E_\alpha \) is the Mittag–Leffler function and \( AB(\alpha) \) is normalization function and \( AB(0) = AB(1) = 1 \). The Laplace transform of Equation 2 is presented as

\[
L^{[ABC]}_{a}D^\alpha_t \phi(t) (s) = \frac{AB(\alpha) s^\alpha L[\phi(t)](s) - s^{\alpha-1} \phi(0)}{s^{\alpha} + \frac{a}{1-a}}.
\] (3)

By using ST for (2), we obtain
Definition 2.3. We have the Atangana–Baleanu fractional integral of order $\alpha$ of a function $\phi(t)$ as

$$A^{ABC}_t \mathcal{I}^\alpha \phi(t) = \frac{\Gamma(\alpha)}{B(\alpha)\Gamma(\alpha)} \int_0^t \phi(s)(t-s)^{\alpha-1} \, ds.$$  \hspace{1cm} (5)  

3 | FRACTIONAL ORDER COVID-19 MODEL

The model of COVID-19 with quarantine and isolation has eight sub-compartments which are $S(t)$ Susceptible individual, $E(t)$ Exposed individuals, $I(t)$ Infected individuals, $A(t)$ Asymptomatically infected, $Q(t)$ Quarantined, $H(t)$ Hospitalized, $R(t)$ recovered Individuals, $M(t)$ environmental generating function. The model parameters are the Birth rate is presented with $\Lambda$ in the model. The natural morality rate of the human population is described with $\mu$. The healthy individuals require infection after contacting with infected and asymptomatic infected individuals by a rate $\eta_1$, while $\psi$ denotes the transmissibility factor. The asymptomatic infection is generated by the parameter $\theta$. The incubation periods are shown by $\omega$ and $\rho$. The parameters $\tau_1$, $\tau_2$, $\phi_1$, and $\phi_2$ define, respectively, the recovery of infected, asymptomatically infected, quarantined, and hospitalized individuals. The hospitalization rates of infected and quarantined individuals are demonstrated, respectively, by $\gamma$ and $\delta_2$. The disease death rates of infected and hospitalized individuals are shown by $\xi_1$ and $\xi_2$. The variable $\delta_1$ shows the quarantine rate of exposed individuals. Individuals who are visiting the seafood market and catch the infection are increasing with rate $\eta_2$. The infection generated in the seafood market due to infected and asymptotically infected is presented by the parameters $q_1$ and $q_2$, respectively, while the removal of infection from the market is shown by $q_3$. The system of governing equations for the model is given as

$$\frac{dS}{dt} = \Lambda - \mu S(t) - \lambda(t)S(t),$$  
$$\frac{dE}{dt} = \lambda(t)S(t) - ((1-\theta)\omega + \theta\rho + \mu + \delta_1)E(t),$$  
$$\frac{dI}{dt} = (1-\theta)\omega E(t) - (\tau_1 + \mu + \xi_1 + \gamma)I(t),$$  
$$\frac{dA}{dt} = \theta\rho E(t) - (\tau_2 + \mu)A(t),$$  
$$\frac{dQ}{dt} = \delta_1 E(t) - (\mu + \phi_1 + \delta_2)Q(t),$$  
$$\frac{dH}{dt} = \gamma I(t) + \delta_2 Q(t) - (\mu + \phi_2 + \xi_2)H(t),$$  
$$\frac{dR}{dt} = \tau_1 I(t) + \tau_2 A(t) + \phi_1 Q(t) + \phi_2 H(t) - \mu R(t),$$  
$$\frac{dM}{dt} = q_1 I(t) + q_2 A(t) - q_3 M(t),$$  \hspace{1cm} (6)  

where

$$\lambda(t) = \frac{\eta_1 (I + \psi A)}{N} + \eta_2 M.$$  

We replace the classical derivative with the ABC derivative and obtain:
\[ ABC \frac{D^o}{0}S(t) = \Lambda - \mu S(t) - \lambda(t)S(t), \]
\[ ABC \frac{D^o}{0}E(t) = \lambda(t)S(t) - ((1 - \theta)\omega + \theta + \phi_1 + \phi_2)E(t), \]
\[ ABC \frac{D^o}{0}I(t) = (1 - \theta)\omega E(t) - (\tau_1 + \mu + \delta_1 + \gamma)I(t), \]
\[ ABC \frac{D^o}{0}A(t) = \theta \rho E(t) - (\tau_2 + \mu)A(t), \]
\[ ABC \frac{D^o}{0}Q(t) = \delta_1 E(t) - (\mu + \phi_1 + \phi_2)Q(t), \]
\[ ABC \frac{D^o}{0}H(t) = \gamma I(t) + \delta_2 Q(t) - (\mu + \phi_2 + \xi_2)H(t), \]
\[ ABC \frac{D^o}{0}R(t) = \tau_1 I(t) + \tau_2 A(t) + \phi_1 Q(t) + \phi_2 H(t) - \mu R(t), \]
\[ ABC \frac{D^o}{0}M(t) = q_1 I(t) + q_2 A(t) - q_3 M(t). \]

We apply the ST and get

\[ B(\alpha)\alpha \Gamma(\alpha + 1)E_{\alpha} \left( -\frac{1}{1 - \alpha} \omega \right) ST[S(t) - S(0)] = ST[\Lambda - \mu S(t) - \lambda(t)S(t)], \]
\[ B(\alpha)\alpha \Gamma(\alpha + 1)E_{\alpha} \left( -\frac{1}{1 - \alpha} \omega \right) ST[E(t) - E(0)] = ST[\lambda(t)S(t) - ((1 - \theta)\omega + \theta + \phi_1 + \phi_2)E(t)], \]
\[ B(\alpha)\alpha \Gamma(\alpha + 1)E_{\alpha} \left( -\frac{1}{1 - \alpha} \omega \right) ST[I(t) - I(0)] = ST[(1 - \theta)\omega E(t) - (\tau_1 + \mu + \xi_1 + \gamma)I(t)], \]
\[ B(\alpha)\alpha \Gamma(\alpha + 1)E_{\alpha} \left( -\frac{1}{1 - \alpha} \omega \right) ST[A(t) - A(0)] = ST[\theta \rho E(t) - (\tau_2 + \mu)A(t)], \]
\[ B(\alpha)\alpha \Gamma(\alpha + 1)E_{\alpha} \left( -\frac{1}{1 - \alpha} \omega \right) ST[Q(t) - Q(0)] = ST[\delta_1 E(t) - (\mu + \phi_1 + \phi_2)Q(t)], \]
\[ B(\alpha)\alpha \Gamma(\alpha + 1)E_{\alpha} \left( -\frac{1}{1 - \alpha} \omega \right) ST[H(t) - H(0)] = ST[\gamma I(t) + \delta_2 Q(t) - (\mu + \phi_2 + \xi_2)H(t)], \]
\[ B(\alpha)\alpha \Gamma(\alpha + 1)E_{\alpha} \left( -\frac{1}{1 - \alpha} \omega \right) ST[R(t) - R(0)] = ST[\tau_1 I(t) + \tau_2 A(t) + \phi_1 Q(t) + \phi_2 H(t) - \mu R(t)], \]
\[ B(\alpha)\alpha \Gamma(\alpha + 1)E_{\alpha} \left( -\frac{1}{1 - \alpha} \omega \right) ST[M(t) - M(0)] = ST[q_1 I(t) + q_2 A(t) - q_3 M(t)]. \]

Reorganizing system 8, we have

\[ ST[S(t)] = S(0) + \frac{1 - \alpha}{B(\alpha)\alpha \Gamma(\alpha + 1)E_{\alpha} \left( -\frac{1}{1 - \alpha} \omega \right)} ST[\Lambda - \mu S(t) - \lambda(t)S(t)], \]
\[ ST[E(t)] = E(0) + \frac{1 - \alpha}{B(\alpha)\alpha \Gamma(\alpha + 1)E_{\alpha} \left( -\frac{1}{1 - \alpha} \omega \right)} x ST[\lambda(t)S(t) - ((1 - \theta)\omega + \theta + \phi_1 + \phi_2)E(t)], \]
\[ ST[I(t)] = I(0) + \frac{1 - \alpha}{B(\alpha)\alpha \Gamma(\alpha + 1)E_{\alpha} \left( -\frac{1}{1 - \alpha} \omega \right)} ST[(1 - \theta)\omega E(t) - (\tau_1 + \mu + \xi_1 + \gamma)I(t)], \]
\[ ST[A(t)] = A(0) + \frac{1 - \alpha}{B(\alpha)\alpha \Gamma(\alpha + 1)E_{\alpha} \left( -\frac{1}{1 - \alpha} \omega \right)} ST[\theta \rho E(t) - (\tau_2 + \mu)A(t)], \]
\[ ST[Q(t)] = Q(0) + \frac{1 - \alpha}{B(\alpha)\alpha \Gamma(\alpha + 1)E_{\alpha} \left( -\frac{1}{1 - \alpha} \omega \right)} ST[\delta_1 E(t) - (\mu + \phi_1 + \phi_2)Q(t)], \]
\[ ST[H(t)] = H(0) + \frac{1 - \alpha}{B(\alpha)\alpha \Gamma(\alpha + 1)E_{\alpha} \left( -\frac{1}{1 - \alpha} \omega \right)} ST[\gamma I(t) + \delta_2 Q(t) - (\mu + \phi_2 + \xi_2)H(t)], \]
\[ ST[R(t)] = R(0) + \frac{1 - \alpha}{B(\alpha)\alpha \Gamma(\alpha + 1)E_{\alpha} \left( -\frac{1}{1 - \alpha} \omega \right)} ST[\tau_1 I(t) + \tau_2 A(t) + \phi_1 Q(t) + \phi_2 H(t) - \mu R(t)], \]
\[ ST[M(t)] = M(0) + \frac{1 - \alpha}{B(\alpha)\alpha \Gamma(\alpha + 1)E_{\alpha} \left( -\frac{1}{1 - \alpha} \omega \right)} ST[q_1 I(t) + q_2 A(t) - q_3 M(t)]. \]

Then, we apply the inverse ST and obtain
\[ S(t) = S(0) + ST^{-1} \left[ \frac{1-\alpha}{B(\alpha)\Gamma(\alpha+1)E_\alpha(-\frac{1}{\Gamma(\alpha+1)})} \right] \cdot ST\{ \Lambda - \mu S(t) - \lambda(t)S(t) \}, \]

\[ E(t) = E(0) + ST^{-1} \left[ \frac{1-\alpha}{B(\alpha)\Gamma(\alpha+1)E_\alpha(-\frac{1}{\Gamma(\alpha+1)})} \right] \cdot ST\{ \lambda(t)S(t) - ((1-\theta)\omega + \theta\rho + \mu + \delta_1)E(t) \}, \]

\[ I(t) = I(0) + ST^{-1} \left[ \frac{1-\alpha}{B(\alpha)\Gamma(\alpha+1)E_\alpha(-\frac{1}{\Gamma(\alpha+1)})} \right] \cdot ST\{ (1-\theta)\omega E(t) - (\tau_1 + \mu + \xi_1 + \gamma)I(t) \}, \]

\[ A(t) = A(0) + ST^{-1} \left[ \frac{1-\alpha}{B(\alpha)\Gamma(\alpha+1)E_\alpha(-\frac{1}{\Gamma(\alpha+1)})} \right] \cdot ST\{ \theta\rho E(t) - (\tau_2 + \mu)A(t) \}, \]

\[ Q(t) = Q(0) + ST^{-1} \left[ \frac{1-\alpha}{B(\alpha)\Gamma(\alpha+1)E_\alpha(-\frac{1}{\Gamma(\alpha+1)})} \right] \cdot ST\{ \delta_1 E(t) - (\mu + \phi_1 + \delta_2)Q(t) \}, \]

\[ H(t) = H(0) + ST^{-1} \left[ \frac{1-\alpha}{B(\alpha)\Gamma(\alpha+1)E_\alpha(-\frac{1}{\Gamma(\alpha+1)})} \right] \cdot ST\{ \gamma I(t) + \delta_2 Q(t) - (\mu + \phi_2 + \xi_2)H(t) \}, \]

\[ R(t) = R(0) + ST^{-1} \left[ \frac{1-\alpha}{B(\alpha)\Gamma(\alpha+1)E_\alpha(-\frac{1}{\Gamma(\alpha+1)})} \right] \cdot ST\{ \tau_1 I(t) + \tau_2 A(t) + \phi_1 Q(t) + \phi_2 H(t) - \mu R(t) \}, \]

\[ M(t) = M(0) + ST^{-1} \left[ \frac{1-\alpha}{B(\alpha)\Gamma(\alpha+1)E_\alpha(-\frac{1}{\Gamma(\alpha+1)})} \right] \cdot ST\{ q_1 I(t) + q_2 A(t) - q_3 M(t) \}. \]

Since

\[ \lambda(t) = \frac{\eta_1(I + \psi A)}{N} + \eta_2 M, \]

and \( \lambda(t) = \frac{\eta_1}{N} (I(t) + \psi A(t)) + \eta_2 M. \)

Therefore, the following is obtained:

\[ S_{(n+1)}(t) = S_n(t) + ST^{-1} \left[ \frac{1-\alpha}{B(\alpha)\Gamma(\alpha+1)E_\alpha(-\frac{1}{\Gamma(\alpha+1)})} \right] \cdot ST\left\{ \Lambda - \mu S_n(t) - \left( \frac{\eta_1(I(t) + \psi A(t))}{N} + \eta_2 M_n(t) \right) S_n(t) \right\}, \]

\[ E_{(n+1)}(t) = E_n(t) + ST^{-1} \left[ \frac{1-\alpha}{B(\alpha)\Gamma(\alpha+1)E_\alpha(-\frac{1}{\Gamma(\alpha+1)})} \right] \cdot ST\left\{ \left( \frac{\eta_1(I(t) + \psi A(t))}{N} + \eta_2 M_n(t) \right) S_n(t) - \left( (1-\theta)\omega + \theta\rho + \mu + \delta_1 \right) E_n(t) \right\}, \]

\[ I_{(n+1)}(t) = I_n(t) + ST^{-1} \left[ \frac{1-\alpha}{B(\alpha)\Gamma(\alpha+1)E_\alpha(-\frac{1}{\Gamma(\alpha+1)})} \right] \cdot ST\left\{ (1-\theta)\omega E_n(t) - (\tau_1 + \mu + \xi_1 + \gamma)I_n(t) \right\}, \]

\[ A_{(n+1)}(t) = A_n(t) + ST^{-1} \left[ \frac{1-\alpha}{B(\alpha)\Gamma(\alpha+1)E_\alpha(-\frac{1}{\Gamma(\alpha+1)})} \right] \cdot ST\left\{ \theta\rho E_n(t) - (\tau_2 + \mu)A_n(t) \right\}, \]

\[ Q_{(n+1)}(t) = Q_n(t) + ST^{-1} \left[ \frac{1-\alpha}{B(\alpha)\Gamma(\alpha+1)E_\alpha(-\frac{1}{\Gamma(\alpha+1)})} \right] \cdot ST\left\{ \delta_1 E_n(t) - (\mu + \phi_1 + \delta_2)Q_n(t) \right\}. \]

\[ H_{(n+1)}(t) = H_n(t) + ST^{-1} \left[ \frac{1-\alpha}{B(\alpha)\Gamma(\alpha+1)E_\alpha(-\frac{1}{\Gamma(\alpha+1)})} \right] \cdot ST\left\{ \gamma I_n(t) + \delta_2 Q_n(t) - (\mu + \phi_2 + \xi_2)H_n(t) \right\}, \]

\[ R_{(n+1)}(t) = R_n(t) + ST^{-1} \left[ \frac{1-\alpha}{B(\alpha)\Gamma(\alpha+1)E_\alpha(-\frac{1}{\Gamma(\alpha+1)})} \right] \cdot ST\left\{ \tau_1 I_n(t) + \tau_2 A_n(t) + \phi_1 Q_n(t) + \phi_2 H_n(t) - \mu R_n(t) \right\}, \]

\[ M_{(n+1)}(t) = M_n(t) + ST^{-1} \left[ \frac{1-\alpha}{B(\alpha)\Gamma(\alpha+1)E_\alpha(-\frac{1}{\Gamma(\alpha+1)})} \right] \cdot ST\left\{ q_1 I_n(t) + q_2 A_n(t) - q_3 M_n(t) \right\}. \]
And obtained solution of 10 is presented as

\[
S(t) = \lim_{n \to \infty} S_n(t); \quad E(t) = \lim_{n \to \infty} E_n(t); \quad I(t) = \lim_{n \to \infty} I_n(t); \quad A(t) = \lim_{n \to \infty} A_n(t);
\]

\[
Q(t) = \lim_{n \to \infty} Q_n(t); \quad H(t) = \lim_{n \to \infty} H_n(t); \quad R(t) = \lim_{n \to \infty} R_n(t); \quad M(t) = \lim_{n \to \infty} M_n(t).
\]

Assume that \((X, |\cdot|)\) is a Banach space and \(H\) is a self-map of \(X\). Suppose that \(r_{n+1} = g(Hr_n)\) is a specific recursive procedure. The following conditions must be satisfied for \(r_{n+1} = Hr_n\).

1. The fixed point set of \(H\) has at least one element.
2. \(r_n\) converges to a point \(P \in F(H)\).
3. \(\lim_{n \to \infty} S_n(t) = P\).

**Theorem 3.1.** Assume that \((X, |\cdot|)\) is a Banach space and \(H\) is a self-map of \(X\) fulfilling

\[
\|H_x - H_r\| \leq \theta \|X - H_x\| + \theta \|x - r\|. \tag{11}
\]

for all \(x, r \in X\) where \(0 \leq \theta < 1\). Let \(H\) be a picard \(H\)-stable.

We take into consideration Equation 5 and get

\[
\frac{1 - \alpha}{B(a)\alpha\Gamma(a + 1)E_a\left(\frac{1}{1 - \alpha}w^a\right)}. \tag{12}
\]

**Theorem 3.2.** We describe \(K\) as a self-map by

\[
K[S_{n+1}(t)] = S_{n+1}(t) = S_n(t) + ST^{-1}\left[\frac{1 - \alpha}{B(a)\alpha\Gamma(a + 1)E_a\left(\frac{1}{1 - \alpha}w^a\right)} \times ST\left\{\lambda - \mu S_n(t) - \left(\eta_1 I_n(t) + \eta_2 A_n(t)\right)N + \eta_3 M_n(t)\right\}S_n(t)\right],
\]

\[
K[E_{n+1}(t)] = E_{n+1}(t) = E_n(t) + ST^{-1}\left[\frac{1 - \alpha}{B(a)\alpha\Gamma(a + 1)E_a\left(\frac{1}{1 - \alpha}w^a\right)} \times ST\left\{\eta_1 I_n(t) + \eta_2 A_n(t)N + \eta_3 M_n(t)\right\}S_n(t) - (1 - \theta)w + \theta \|x - r\|E_n(t)\right\},
\]

\[
K[I_{n+1}(t)] = I_{n+1}(t) = I_n(t) + ST^{-1}\left[\frac{1 - \alpha}{B(a)\alpha\Gamma(a + 1)E_a\left(\frac{1}{1 - \alpha}w^a\right)} \times ST\left\{(1 - \theta)wE_n(t) - (\tau_1 + \mu + \xi_1 + \gamma)I_n(t)\right\},
\]

\[
K[A_{n+1}(t)] = A_{n+1}(t) = A_n(t) + ST^{-1}\left[\frac{1 - \alpha}{B(a)\alpha\Gamma(a + 1)E_a\left(\frac{1}{1 - \alpha}w^a\right)} \times ST\left\{\theta_1 E_n(t) - (\tau_2 + \mu)A_n(t)\right\},
\]

\[
K[Q_{n+1}(t)] = AQ_{n+1}(t) = AQ_n(t) + ST^{-1}\left[\frac{1 - \alpha}{B(a)\alpha\Gamma(a + 1)E_a\left(\frac{1}{1 - \alpha}w^a\right)} \times ST\left\{\delta_1 E_n(t) - (\mu + \phi_1 + \phi_2)Q_n(t)\right\},
\]

\[
K[H_{n+1}(t)] = H_{n+1}(t) = H_n(t) + ST^{-1}\left[\frac{1 - \alpha}{B(a)\alpha\Gamma(a + 1)E_a\left(\frac{1}{1 - \alpha}w^a\right)} \times ST\left\{\gamma I_n(t) + \delta_2 Q_n(t) - (\mu + \phi_2 + \xi_2)H_n(t)\right\},
\]

\[
K[R_{n+1}(t)] = R_{n+1}(t) = R_n(t) + ST^{-1}\left[\frac{1 - \alpha}{B(a)\alpha\Gamma(a + 1)E_a\left(\frac{1}{1 - \alpha}w^a\right)} \times ST\left\{\tau_1 I_n(t) + \tau_2 A_n(t) + \phi_1 Q_n(t) + \phi_2 H_n(t) - \mu R_n(t)\right\},
\]

\[
K[M_{n+1}(t)] = M_{n+1}(t) = M_n(t) + ST^{-1}\left[\frac{1 - \alpha}{B(a)\alpha\Gamma(a + 1)E_a\left(\frac{1}{1 - \alpha}w^a\right)} \times ST\left\{q_1 I_n(t) + q_2 A_n(t) - q_3 M_n(t)\right\}.
\]
Then, we reach

\[
\|K[S_n(t)] - K[S_m(t)]\| \leq \|S_n(t) - S_m(t)\| + ST^{-1}
\]

\[
\left[ \frac{1 - \alpha}{B(\alpha + 1)} \right] \times ST \left\{ (1 - \theta) \omega \| (E_n(t) - E_m(t)) \| - (\tau_1 + \mu + \xi_1 + \gamma) \| (I_n(t) - I_m(t)) \| \right\}
\]

\[
\|K[E_n(t)] - K[E_m(t)]\| \leq \|E_n(t) - E_m(t)\| + ST^{-1}
\]

\[
\left[ \frac{1 - \alpha}{B(\alpha + 1)} \right] \times ST \left\{ (1 - \theta) \omega \| (E_n(t) - E_m(t)) \| - (\tau_1 + \mu + \xi_1 + \gamma) \| (I_n(t) - I_m(t)) \| \right\}
\]

\[
\|K[I_n(t)] - K[I_m(t)]\| \leq \|I_n(t) - I_m(t)\| + ST^{-1}
\]

\[
\left[ \frac{1 - \alpha}{B(\alpha + 1)} \right] \times ST \left\{ (1 - \theta) \omega \| (E_n(t) - E_m(t)) \| - (\tau_1 + \mu + \xi_1) \| (A_n(t) - A_m(t)) \| \right\}
\]

\[
\|K[A_n(t)] - K[A_m(t)]\| \leq \|A_n(t) - A_m(t)\| + ST^{-1}
\]

\[
\left[ \frac{1 - \alpha}{B(\alpha + 1)} \right] \times ST \left\{ (1 - \theta) \omega \| (E_n(t) - E_m(t)) \| - (\tau_1 + \mu + \xi_1 + \gamma) \| (Q_n(t) - Q_m(t)) \| \right\}
\]

\[
\|K[Q_n(t)] - K[Q_m(t)]\| \leq \|Q_n(t) - Q_m(t)\| + ST^{-1}
\]

\[
\left[ \frac{1 - \alpha}{B(\alpha + 1)} \right] \times ST \left\{ (1 - \theta) \omega \| (E_n(t) - E_m(t)) \| - (\tau_1 + \mu + \xi_1 + \gamma) \| (Q_n(t) - Q_m(t)) \| \right\}
\]

\[
\|K[H_n(t)] - K[H_m(t)]\| \leq \|H_n(t) - H_m(t)\| + ST^{-1}
\]

\[
\left[ \frac{1 - \alpha}{B(\alpha + 1)} \right] \times ST \left\{ (1 - \theta) \omega \| (E_n(t) - E_m(t)) \| - (\tau_1 + \mu + \xi_1 + \gamma) \| (Q_n(t) - Q_m(t)) \| \right\}
\]

\[
\|K[R_n(t)] - K[R_m(t)]\| \leq \|R_n(t) - R_m(t)\| + ST^{-1}
\]

\[
\left[ \frac{1 - \alpha}{B(\alpha + 1)} \right] \times ST \left\{ (1 - \theta) \omega \| (E_n(t) - E_m(t)) \| - (\tau_1 + \mu + \xi_1 + \gamma) \| (Q_n(t) - Q_m(t)) \| \right\}
\]

\[
\|K[M_n(t)] - K[M_m(t)]\| \leq \|M_n(t) - M_m(t)\| + ST^{-1}
\]

\[
\left[ \frac{1 - \alpha}{B(\alpha + 1)} \right] \times ST \left\{ (1 - \theta) \omega \| (E_n(t) - E_m(t)) \| - (\tau_1 + \mu + \xi_1 + \gamma) \| (Q_n(t) - Q_m(t)) \| \right\}
\]

K satisfies the condition associated with the Theorem 3.1 if
The special solution of system 6 using the iteration method is unique singular solution.

**Theorem 3.3.** The special solution of system 6 using the iteration method is unique singular solution.

**Proof.** We consider the following Hilbert space $H = L^2((p,q) \times (0,T))$ which can be defined as

$$h : (p,q) \times (0,T) \to \mathbb{R}, \int ghgdh < \infty.$$  

Then, we take into consideration:

$$\theta(0,0,0,0,0,0,0,0), \theta = \left\{ \begin{array}{l} (\Lambda - \mu S(t) - \lambda(t)S(t), \\
\lambda(t)S(t) - ((1 - \theta)\omega + \theta \rho + \mu + \delta_1)E(t), \\
(1 - \theta)\omega E(t) - (\tau_1 + \mu + \xi_1 + \gamma)I(t), \\
\theta \rho E(t) - (\tau_2 + \mu)A(t), \\
\delta_1 E(t) - (\mu + \phi_1 + \delta_2)Q(t), \\
\gamma I(t) + \delta_2 Q(t) - (\mu + \phi_2 + \xi_2)H(t), \\
\tau_1 I(t) + \tau_2 A(t) + \phi_1 Q(t) + \phi_2 H(t) - \mu R(t), \\
q_1 I(t) + q_2 A(t) - q_3 M(t). \end{array} \right. \tag{13}$$

We establish that the inner product of

$$T((S_{11}(t) - S_{21}(t), E_{21}(t) - E_{22}(t), I_{31}(t) - I_{32}(t), A_{41}(t) - A_{42}(t), Q_{51}(t) - Q_{52}(t), H_{61}(t) - H_{62}(t), R_{71}(t) - R_{72}(t), M_{81}(t) - M_{82}(t), V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8)),\)$$

where

$$\{S_{11}(t) - S_{21}(t), E_{21}(t) - E_{22}(t), I_{31}(t) - I_{32}(t), A_{41}(t) - A_{42}(t), Q_{51}(t) - Q_{52}(t), H_{61}(t) - H_{62}(t), R_{71}(t) - R_{72}(t), M_{81}(t) - M_{82}(t)\)$$

are the special solutions of the system. Then, we have

$$\{\Lambda - \mu (S_{11}(t) - S_{21}(t)) - \lambda(t)(S_{11}(t) - S_{21}(t)), V_1\} \leq \Lambda ||V_1|| + \mu ||(S_{11}(t) - S_{21}(t))|| ||V_1|| + \lambda(t)||S_{11}(t) - S_{21}(t)|| ||V_1||, $$
\begin{align*}
\{\lambda(t)(S_{11}(t)-S_{21}(t)) - ((1-\theta)\omega + \theta p + \mu + \delta_1)(E_{21}(t)-E_{22}(t)), V_2 \} & \leq \lambda(t)\|S_{11}(t)-S_{21}(t)\|\|V_2\| \\
+ ((1-\theta)\omega + \theta p + \mu + \delta_1)\|(E_{21}(t)-E_{22}(t))\|\|V_2\|, \\
\{(1-\theta)\omega(E_{21}(t)-E_{22}(t)) - (\tau_1 + \mu + \xi_1 + \gamma)(I_{31}(t)-I_{32}(t)), V_3 \} & \leq (1-\theta)\omega\|(E_{21}(t)-E_{22}(t))\|\|V_3\| \\
+ (\tau_1 + \mu + \xi_1 + \gamma)\|(I_{31}(t)-I_{32}(t))\|\|V_3\| \\
\{\theta p(E_{21}(t)-E_{22}(t)) - (\tau_2 + \mu)(A_{41}(t)-A_{42}(t)), V_4 \} & \leq \theta p\|(E_{21}(t)-E_{22}(t))\|\|V_4\| + (\tau_2 + \mu)\|(A_{41}(t)-A_{42}(t))\|\|V_4\|, \\
\{\delta_1(E_{21}(t)-E_{22}(t)) - (\mu + \phi_1 + \delta_2)(Q_{51}(t)-Q_{52}(t)), V_5 \} & \leq \delta_1\|(E_{21}(t)-E_{22}(t))\|\|V_5\| \\
+ (\mu + \phi_1 + \delta_2)\|(Q_{51}(t)-Q_{52}(t))\|\|V_5\|, \\
\{\gamma(I_{31}(t)-I_{32}(t)) + \delta_2(Q_{51}(t)-Q_{52}(t)) - (\mu + \phi_2 + \xi_2)(H_{61}(t)-H_{62}(t)), V_6 \} & \leq \gamma\|(I_{31}(t)-I_{32}(t))\|\|V_6\| \\
+ \delta_2\|(Q_{51}(t)-Q_{52}(t))\|\|V_6\| + (\mu + \phi_2 + \xi_2)\|(H_{61}(t)-H_{62}(t))\|\|V_6\|, \\
\{\tau_1(I_{31}(t)-I_{32}(t)) + \tau_2(A_{41}(t)-A_{42}(t)) + \phi_1(Q_{51}(t)-Q_{52}(t)) + \phi_2(H_{61}(t)-H_{62}(t)) - \mu(R_{71}(t)-R_{72}(t)), V_7 \} & \leq \tau_1\|(I_{31}(t)-I_{32}(t))\|\|V_7\| \\
+ \tau_2\|(A_{41}(t)-A_{42}(t))\|\|V_7\| + \phi_1\|(Q_{51}(t)-Q_{52}(t))\|\|V_7\| + \phi_2\|(H_{61}(t)-H_{62}(t))\|\|V_7\| + \mu\|(R_{71}(t)-R_{72}(t))\|\|V_7\|, \\
\{q_1(I_{31}(t)-I_{32}(t)) + q_2(A_{41}(t)-A_{42}(t)) - q_3(M_{81}(t)-M_{82}(t)), V_8 \} & \leq q_1\|(I_{31}(t)-I_{32}(t))\|\|V_8\| + q_2\|(A_{41}(t)-A_{42}(t))\|\|V_8\| + q_3\|(M_{81}(t)-M_{82}(t))\|\|V_8\|. \\
\end{align*}

In the case for large number \(e_1, e_2, e_3, e_4, e_5, e_6, e_7, \) and \(e_8, \) both solutions happen to be converged to the exact solution. Applying the topology concept, we can get eight positive very small variables \((x_{e_1}, x_{e_2}, x_{e_3}, x_{e_4}, x_{e_5}, x_{e_6}, x_{e_7}, \) and \(x_{e_8}.\))

\begin{align*}
||S-S_{11}||, ||S-S_{12}|| & \leq \frac{X_{e_1}}{\sigma}, ||E-E_{21}||, ||E-E_{22}|| \leq \frac{X_{e_2}}{\xi}, ||I-I_{31}||, ||I-I_{32}|| \leq \frac{X_{e_3}}{\delta}, \\
||A-A_{41}||, ||A-A_{42}|| & \leq \frac{X_{e_4}}{\kappa}, ||Q-Q_{51}||, ||Q-Q_{52}|| \leq \frac{X_{e_5}}{\theta}, ||H-H_{61}||, ||H-H_{62}|| \leq \frac{X_{e_6}}{\gamma}, \\
||R-R_{71}||, ||R-R_{72}|| \leq \frac{X_{e_7}}{\nu} \text{ and } ||M-M_{81}||, ||M-M_{82}|| \leq \frac{X_{e_8}}{\epsilon},
\end{align*}

where

\begin{align*}
\sigma &= 8(\lambda(t))\|(S_{11}(t)-S_{21}(t))\| + \lambda(t)\|S_{11}(t)-S_{21}(t)\|\|V_1\| \\
\xi &= 8(\lambda(t))\|(S_{11}(t)-S_{21}(t))\| + ((1-\theta)\omega + \theta p + \mu + \delta_1)\|(E_{21}(t)-E_{22}(t))\|\|V_2\| \\
\theta &= 8((1-\theta)\omega)\|(E_{21}(t)-E_{22}(t))\| + (\tau_1 + \mu + \xi_1 + \gamma)\|(I_{31}(t)-I_{32}(t))\|\|V_3\| \\
\phi &= 8(\theta p)\|(E_{21}(t)-E_{22}(t))\| + (\tau_2 + \mu)\|(A_{41}(t)-A_{42}(t))\|\|V_4\| \\
\phi &= 8(\delta_1)\|(E_{21}(t)-E_{22}(t))\| + (\mu + \phi_1 + \delta_2)\|(Q_{51}(t)-Q_{52}(t))\|\|V_5\| \\
\phi &= 8(\gamma)(I_{31}(t)-I_{32}(t))\| + \delta_2\|(Q_{51}(t)-Q_{52}(t))\| + (\mu + \phi_2 + \xi_2)\|(H_{61}(t)-H_{62}(t))\|\|V_6\| \\
\phi &= 8(\tau_1)(I_{31}(t)-I_{32}(t))\| + \gamma\|(I_{31}(t)-I_{32}(t))\|\|V_7\| \\
\phi &= 8(\tau_2)(A_{41}(t)-A_{42}(t))\| + \phi_1\|(Q_{51}(t)-Q_{52}(t))\| + \phi_2\|(H_{61}(t)-H_{62}(t))\| + \mu\|(R_{71}(t)-R_{72}(t))\|\|V_7\| \\
\phi &= 8(q_1)(I_{31}(t)-I_{32}(t))\| + q_2\|(A_{41}(t)-A_{42}(t))\| + q_3\|(M_{81}(t)-M_{82}(t))\|\|V_8\|. \\
\end{align*}
But, it is obvious that

\[ (A + \mu \|S_{11}(t) - S_{21}(t)\| + \lambda(t)\|S_{11}(t) - S_{21}(t)\|) \neq 0 \]

\[ (\lambda(t)\|S_{11}(t) - S_{21}(t)\| + ((1-\theta)\alpha + \theta \rho + \mu + \delta_1)\|(E_{21}(t) - E_{22}(t))\|) \neq 0 \]

\[ ((1-\theta)\alpha\|(E_{21}(t) - E_{22}(t))\| + (\tau_1 + \mu + \xi_1 + \gamma)\|(I_{31}(t) - I_{32}(t))\|) \neq 0 \]

\[ (\theta \rho\|(E_{21}(t) - E_{22}(t))\| + (\tau_2 + \mu)\|(A_{41}(t) - A_{42}(t))\|) \neq 0 \]

\[ (\delta_1\|(E_{21}(t) - E_{22}(t))\| + (\mu + \phi_1 + \delta_2)\|(Q_{51}(t) - Q_{52}(t))\|) \neq 0 \]

\[ (\gamma\|(I_{31}(t) - I_{32}(t))\| + \delta_2\|(Q_{51}(t) - Q_{52}(t))\| + (\mu + \phi_2 + \xi_2)\|(H_{61}(t) - H_{62}(t))\|) \neq 0 \]

\[ (\tau_1\|(I_{31}(t) - I_{32}(t))\| + \tau_2\|(A_{41}(t) - A_{42}(t))\| + \phi_1\|(Q_{51}(t) - Q_{52}(t))\| + \phi_2\|(H_{61}(t) - H_{62}(t))\| + \mu\|(R_{71}(t) - R_{72}(t))\|) \neq 0 \]

\[ (\delta_1\|(E_{21}(t) - E_{22}(t))\| + (\mu + \phi_1 + \delta_2)\|(Q_{51}(t) - Q_{52}(t))\|) \neq 0, \]

where \( \|V_1\|, \|V_2\|, \|V_3\|, \|V_4\|, \|V_5\|, \|V_6\|, \|V_7\|, \|V_8\| \neq 0 \).

Therefore, we have

\[ S_{11} - S_{12} = 0, \quad E_{21} - E_{22} = 0, \quad I_{31} - I_{32} = 0, \quad A_{41} - A_{42} = 0, \]

\[ Q_{51} - Q_{52} = 0, \quad H_{61} - H_{62} = 0, \quad R_{71} - R_{72} = 0, \quad M_{81} - M_{82} = 0. \]

which yields that

\[ S_{11} = S_{12}, \quad E_{21} = E_{22}, \quad I_{31} = I_{32}, \quad A_{41} = A_{42}, \quad Q_{51} = Q_{52}, \quad H_{61} = H_{62}, \quad R_{71} = R_{72}, \quad M_{81} = M_{82}. \]

This completes the proof of uniqueness.

4 | ADAMS–MOULTON METHOD FOR ATANGANA–BALEANU FRACTIONAL DERIVATIVE

We define the numerical scheme of the Atangana–Baleanu fractional integral by using Adams–Moulton rule as\(^{20}\)

\[
{\begin{split}
\frac{AB}{0} I^\alpha_t f(t_{n+1}) &= \frac{1-\alpha}{\beta(\alpha)} \left[ f(t_{n+1}) - f(t_n) \right] + \frac{\alpha}{\Gamma(\alpha)} \sum_{k=0}^{\infty} \left[ f(t_{n+1}) - f(t_n) \right] d_k^n, \\
\end{split}}
\]

where

\[ d_k^n = (k+1)^{1-\alpha} - (k)^{1-\alpha}. \]

We obtain the following for system 6:

\[
S_{(n+1)}(t) - S_n(t) = S_0^n(t) + \left[ \frac{1-\alpha}{\beta(\alpha)} \left( A - \mu \left( \frac{S(t_{n+1}) - S(t_n)}{2} \right) - \left( \frac{\lambda(t_{n+1}) - \lambda(t_n)}{2} \right) \left( \frac{S(t_{n+1}) - S(t_n)}{2} \right) \right) \right] + \frac{\alpha}{\Gamma(\alpha)} \sum_{k=0}^{\infty} d_k^n \left[ A - \mu \left( \frac{S(t_{n+1}) - S(t_n)}{2} \right) - \left( \frac{\lambda(t_{n+1}) - \lambda(t_n)}{2} \right) \left( \frac{S(t_{n+1}) - S(t_n)}{2} \right) \right]
\]
$E_{(n+1)}(t) - E_{(n)}(t) = E_0^a(t) + \frac{1-\alpha}{\beta(\alpha)} \left\{ \frac{\lambda(t_{n+1})-\lambda(t_n)}{2} \left( \frac{S(t_{n+1})-S(t_n)}{2} \right) - \left( (1-\theta)\omega + \theta\rho + \mu + \delta_1 \right) \left( \frac{E(t_{n+1})-E(t_n)}{2} \right) \right\} + \frac{\alpha}{\Gamma(\alpha)} \sum_{k=0}^{\infty} d_k^a \left[ \frac{\lambda(t_{n+1})-\lambda(t_n)}{2} \left( \frac{S(t_{n+1})-S(t_n)}{2} \right) - \left( (1-\theta)\omega + \theta\rho + \mu + \delta_1 \right) \left( \frac{E(t_{n+1})-E(t_n)}{2} \right) \right].$ 

$I_{(n+1)}(t) - I_{(n)}(t) = I_0^a(t) + \frac{1-\alpha}{\beta(\alpha)} \left\{ (1-\theta)\omega \left( \frac{E(t_{n+1})-E(t_n)}{2} \right) - (\tau_1 + \mu + \xi_1 + \gamma) \left( \frac{I(t_{n+1}) - I(t_n)}{2} \right) \right\} + \frac{\alpha}{\Gamma(\alpha)} \sum_{k=0}^{\infty} d_k^a \left[ (1-\theta)\omega \left( \frac{E(t_{n+1})-E(t_n)}{2} \right) - (\tau_1 + \mu + \xi_1 + \gamma) \left( \frac{I(t_{n+1}) - I(t_n)}{2} \right) \right].$ 

$A_{(n+1)}(t) - A_{(n)}(t) = A_0^a(t) + \frac{1-\alpha}{\beta(\alpha)} \left\{ \theta\rho \left( \frac{E(t_{n+1})-E(t_n)}{2} \right) - (\tau_2 + \mu) \left( \frac{A(t_{n+1}) - A(t_n)}{2} \right) \right\} + \frac{\alpha}{\Gamma(\alpha)} \sum_{k=0}^{\infty} d_k^a \left[ \theta\rho \left( \frac{E(t_{n+1})-E(t_n)}{2} \right) - (\tau_2 + \mu) \left( \frac{A(t_{n+1}) - A(t_n)}{2} \right) \right].$ 

$Q_{(n+1)}(t) - Q_{(n)}(t) = Q_0^a(t) + \frac{1-\alpha}{\beta(\alpha)} \left\{ \delta_1 \left( \frac{E(t_{n+1})-E(t_n)}{2} \right) - (\mu + \phi_1 + \delta_2) \left( \frac{Q(t_{n+1}) - Q(t_n)}{2} \right) \right\} + \frac{\alpha}{\Gamma(\alpha)} \sum_{k=0}^{\infty} d_k^a \left[ \delta_1 \left( \frac{E(t_{n+1})-E(t_n)}{2} \right) - (\mu + \phi_1 + \delta_2) \left( \frac{Q(t_{n+1}) - Q(t_n)}{2} \right) \right].$ 

$H_{(n+1)}(t) - H_{(n)}(t) = H_0^a(t) + \frac{1-\alpha}{\beta(\alpha)} \left\{ \gamma \left( \frac{I(t_{n+1}) - I(t_n)}{2} \right) + \delta_2 \left( \frac{Q(t_{n+1}) - Q(t_n)}{2} \right) - (\mu + \phi_2 + \xi_2) \left( \frac{H(t_{n+1}) - H(t_n)}{2} \right) \right\} + \frac{\alpha}{\Gamma(\alpha)} \sum_{k=0}^{\infty} d_k^a \left[ \gamma \left( \frac{I(t_{n+1}) - I(t_n)}{2} \right) + \delta_2 \left( \frac{Q(t_{n+1}) - Q(t_n)}{2} \right) - (\mu + \phi_2 + \xi_2) \left( \frac{H(t_{n+1}) - H(t_n)}{2} \right) \right].$ 

$R_{(n+1)}(t) - R_{(n)}(t) = R_0^a(t) + \frac{1-\alpha}{\beta(\alpha)} \left\{ \epsilon_1 \left( \frac{I(t_{n+1}) - I(t_n)}{2} \right) + \epsilon_2 \left( \frac{A(t_{n+1}) - A(t_n)}{2} \right) + \phi_1 \left( \frac{Q(t_{n+1}) - Q(t_n)}{2} \right) + \phi_2 \left( \frac{H(t_{n+1}) - H(t_n)}{2} \right) - \mu \left( \frac{R(t_{n+1}) - R(t_n)}{2} \right) \right\} + \frac{\alpha}{\Gamma(\alpha)} \sum_{k=0}^{\infty} d_k^a \left[ \epsilon_1 \left( \frac{I(t_{n+1}) - I(t_n)}{2} \right) + \epsilon_2 \left( \frac{A(t_{n+1}) - A(t_n)}{2} \right) + \phi_1 \left( \frac{Q(t_{n+1}) - Q(t_n)}{2} \right) + \phi_2 \left( \frac{H(t_{n+1}) - H(t_n)}{2} \right) - \mu \left( \frac{R(t_{n+1}) - R(t_n)}{2} \right) \right].$ 

$M_{(n+1)}(t) - M_{(n)}(t) = M_0^a(t) + \frac{1-\alpha}{\beta(\alpha)} \left\{ q_1 \left( \frac{I(t_{n+1}) - I(t_n)}{2} \right) + q_2 \left( \frac{A(t_{n+1}) - A(t_n)}{2} \right) - q_3 \left( \frac{M(t_{n+1}) - M(t_n)}{2} \right) \right\} + \frac{\alpha}{\Gamma(\alpha)} \sum_{k=0}^{\infty} d_k^a \left[ q_1 \left( \frac{I(t_{n+1}) - I(t_n)}{2} \right) + q_2 \left( \frac{A(t_{n+1}) - A(t_n)}{2} \right) - q_3 \left( \frac{M(t_{n+1}) - M(t_n)}{2} \right) \right].$

5 NEW NUMERICAL SCHEME

In this section, we construct a new numerical scheme for nonlinear fractional differential equations with fractional derivative with nonlocal and nonsingular kernel. To do this, we consider the following nonlinear fractional ordinary equation.24
\[
\begin{aligned}
ABC\frac{Dy(t)}{dt} &= f(t, y(t)), \\
y(0) &= y_0.
\end{aligned}
\] (15)

The above equation can be converted to a fractional integral equation by using the fundamental theorem of fractional calculus.

\[
y(t) - y(0) = \frac{(1-\alpha)}{ABC(\alpha)} f(t, y(t)) + \frac{\alpha}{\Gamma(\alpha) \times ABC(\alpha)} \int_0^t f(\tau, y(\tau))(t-\tau)^{\alpha-1} d\tau.
\] (16)

At a given point \( t_{n+1}, n = 0, 1, 2, 3, \ldots \), the above equation is reformulated as follows:

\[
y(t_{n+1}) - y(0) = \frac{(1-\alpha)}{ABC(\alpha)} f(t_n, y(t_n)) + \frac{\alpha}{\Gamma(\alpha) \times ABC(\alpha)} \int_0^{t_{n+1}} f(\tau, y(\tau))(t_{n+1}-\tau)^{\alpha-1} d\tau
\]

\[= \frac{(1-\alpha)}{ABC(\alpha)} f(t_n, y(t_n)) + \frac{\alpha}{\Gamma(\alpha) \times ABC(\alpha)} \sum_{k=0}^n f(t_k, y(t_k)) (t_{n+1}-t_k)^{\alpha-1} d\tau
\] (17)

Within the interval \([t_k, t_{k+1}]\), the function \( f(\tau, y(\tau)) \), using the two-step Lagrange polynomial interpolation, can be approximate as follows:

\[
P_k(\tau) = \frac{\tau-t_{k-1}}{t_k-t_{k-1}} f(t_k, y(t_k)) - \frac{\tau-t_k}{t_{k-1}-t_k} f(t_{k-1}, y(t_{k-1}))
\]

\[= \frac{f(t_k, y(t_k))}{h} (\tau-t_{k-1}) - \frac{f(t_{k-1}, y(t_{k-1}))}{h} (\tau-t_k)
\]

\[= \frac{f(t_k, y_k)}{h} (\tau-t_{k-1}) - \frac{f(t_{k-1}, y_{k-1})}{h} (\tau-t_k).
\] (18)

The above approximation can therefore be included in Equation 17 to produce

\[
y_{n+1} = y_0 + \frac{(1-\alpha)}{ABC(\alpha)} f(t_n, y(t_n))
\]

\[+ \frac{\alpha}{\Gamma(\alpha) \times ABC(\alpha)} \sum_{k=0}^n \left( \frac{f(t_k, y_k)}{h} \int_{t_k}^{t_{k+1}} (\tau-t_{k-1})(t_{n+1}-\tau)^{\alpha-1} d\tau - \frac{f(t_{k-1}, y_{k-1})}{h} \int_{t_k}^{t_{k+1}} (\tau-t_k)(t_{n+1}-\tau)^{\alpha-1} d\tau \right).
\] (19)

For simplicity, we let

\[
A_{a,1,k} = \int_{t_k}^{t_{k+1}} (\tau-t_{k-1})(t_{n+1}-\tau)^{\alpha-1} d\tau
\] (20)

and also

\[
A_{a,2,k} = \int_{t_k}^{t_{k+1}} (\tau-t_k)(t_{n+1}-\tau)^{\alpha-1} d\tau
\]

\[= h^\alpha + 1 (n+1-k)^{\alpha}(n-k+2+\alpha) - (n-k)^{\alpha}(n-k+2+2\alpha)
\]

\[= \alpha(\alpha+1)
\]

\[
A_{a,3,k} = h^\alpha + 1 (n+1-k)^{\alpha+1} - (n-k)^{\alpha}(n-k+1+\alpha)
\]

\[= \alpha(\alpha+1).
\] (21)
Thus, integrating Equations 20 and 21 and replacing them in Equation 19, we obtain

\[
y_{n+1} = y_0 + \left(\frac{1-a}{ABC(a)}\int f(t_n, y(t_n)) + \frac{\alpha}{ABC(\alpha)} \sum_{k=0}^{n} \right) \left(\frac{h^a f(t_k, y_k)}{\Gamma(a+2)}(n+1-k)^{a}(n-k+2+a)-(n-k)^{a}(n-k+2+2a)\right) - \frac{h^a f(t_{k-1}, y_{k-1})}{\Gamma(a+2)}((n+1-k)^{a+1}-(n-k)^{a}(n-k+1+a))\right).
\]

We obtain the following for system 6:

\[
S(t) - S(0) = \frac{(1-a)}{ABC(a)} \left\{ \Lambda - \mu S(t) - \lambda(t)S(t) \right\} + \frac{\alpha}{\Gamma(\alpha) \times ABC(\alpha)} \int_{0}^{t} \left\{ \Lambda - \mu S(r) - \lambda(r)S(r) \right\} (t-r)^{\alpha-1} dr,
\]

\[
E(t) - E(0) = \frac{(1-a)}{ABC(a)} \left\{ \lambda(t)S(t) - ((1-\theta)\omega + \theta \rho + \mu + \delta_1)E(t) \right\} + \frac{\alpha}{\Gamma(\alpha) \times ABC(\alpha)} \int_{0}^{t} \left\{ \lambda(r)S(r) - ((1-\theta)\omega + \theta \rho + \mu + \delta_1)E(r) \right\} (t-r)^{\alpha-1} dr,
\]

\[
I(t) - I(0) = \frac{(1-a)}{ABC(a)} \left\{ (1-\theta)\omega E(t) - (\tau_1 + \mu + \delta_2)I(t) \right\} + \frac{\alpha}{\Gamma(\alpha) \times ABC(\alpha)} \int_{0}^{t} \left\{ (1-\theta)\omega E(r) - (\tau_1 + \mu + \delta_2)I(r) \right\} (t-r)^{\alpha-1} dr,
\]

\[
A(t) - A(0) = \frac{(1-a)}{ABC(a)} \left\{ \theta \rho E(t) - (\tau_2 + \mu)A(t) \right\} + \frac{\alpha}{\Gamma(\alpha) \times ABC(\alpha)} \int_{0}^{t} \left\{ \theta \rho E(r) - (\tau_2 + \mu)A(r) \right\} (t-r)^{\alpha-1} dr,
\]

\[
Q(t) - Q(0) = \frac{(1-a)}{ABC(a)} \left\{ \delta_1 E(t) - (\mu + \phi_1 + \delta_2)Q(t) \right\} + \frac{\alpha}{\Gamma(\alpha) \times ABC(\alpha)} \int_{0}^{t} \left\{ \delta_1 E(r) - (\mu + \phi_1 + \delta_2)Q(r) \right\} (t-r)^{\alpha-1} dr,
\]

\[
H(t) - H(0) = \frac{(1-a)}{ABC(a)} \left\{ \gamma I(t) + \delta_2 Q(t) - (\mu + \phi_2 + \delta_2)H(t) \right\} + \frac{\alpha}{\Gamma(\alpha) \times ABC(\alpha)} \int_{0}^{t} \left\{ \gamma I(r) + \delta_2 Q(r) - (\mu + \phi_2 + \delta_2)H(r) \right\} (t-r)^{\alpha-1} dr.
\]

\[
R(t) - R(0) = \frac{(1-a)}{ABC(a)} \left\{ \tau_1 I(t) + \tau_2 A(t) + \phi_1 Q(t) + \phi_2 H(t) - \mu R(t) \right\} + \frac{\alpha}{\Gamma(\alpha) \times ABC(\alpha)} \int_{0}^{t} \left\{ \tau_1 I(r) + \tau_2 A(r) + \phi_1 Q(r) + \phi_2 H(r) - \mu R(r) \right\} (t-r)^{\alpha-1} dr.
\]

\[
M(t) - M(0) = \frac{(1-a)}{ABC(a)} \left\{ q_1 I(t) + q_2 A(t) - q_3 M(t) \right\} + \frac{\alpha}{\Gamma(\alpha) \times ABC(\alpha)} \int_{0}^{t} \left\{ q_1 I(r) + q_2 A(r) - q_3 M(r) \right\} (t-r)^{\alpha-1} dr.
\]
At a given point $t_{n+1}, n = 0, 1, 2, \ldots$, the above equation is reformulated as

\[
S(t_{n+1}) - S(0) = \frac{(1-\alpha)}{ABC(\alpha)} \left\{ \Lambda - \mu S(t_n) - \lambda(t_n) S(t_n) \right\} \\
+ \frac{\alpha}{\Gamma(\alpha) \times ABC(\alpha)} \sum_{k=0}^{n} \int_{t_k}^{t_{k+1}} \{ \Lambda - \mu S(r) - \lambda(r) S(r) \} (t_{n+1} - r)^{\alpha-1} dr,
\]

\[
E(t_{n+1}) - E(0) = \frac{(1-\alpha)}{ABC(\alpha)} \left\{ \lambda(t_n) S(t_n) - ((1-\theta) \omega + \theta \rho + \mu + \delta_1) E(t_n) \right\} \\
+ \frac{\alpha}{\Gamma(\alpha) \times ABC(\alpha)} \sum_{k=0}^{n} \int_{t_k}^{t_{k+1}} \{ \lambda(r) S(r) - ((1-\theta) \omega + \theta \rho + \mu + \delta_1) E(r) \} (t_{n+1} - r)^{\alpha-1} dr,
\]

\[
I(t_{n+1}) - I(0) = \frac{(1-\alpha)}{ABC(\alpha)} \left\{ (1-\theta) \omega E(t_n) - (\tau_1 + \mu + \xi_1 + \gamma) I(t_n) \right\} \\
+ \frac{\alpha}{\Gamma(\alpha) \times ABC(\alpha)} \sum_{k=0}^{n} \int_{t_k}^{t_{k+1}} \{ (1-\theta) \omega E(r) - (\tau_1 + \mu + \xi_1 + \gamma) I(r) \} (t_{n+1} - r)^{\alpha-1} dr,
\]

\[
A(t_{n+1}) - A(0) = \frac{(1-\alpha)}{ABC(\alpha)} \left\{ \theta \rho E(t_n) - (\tau_2 + \mu) A(t_n) \right\} \\
+ \frac{\alpha}{\Gamma(\alpha) \times ABC(\alpha)} \sum_{k=0}^{n} \int_{t_k}^{t_{k+1}} \{ \theta \rho E(r) - (\tau_2 + \mu) A(r) \} (t_{n+1} - r)^{\alpha-1} dr,
\]

\[
Q(t_{n+1}) - Q(0) = \frac{(1-\alpha)}{ABC(\alpha)} \left\{ \delta_1 E(t_n) - (\mu + \phi_1 + \delta_2) Q(t_n) \right\} \\
+ \frac{\alpha}{\Gamma(\alpha) \times ABC(\alpha)} \sum_{k=0}^{n} \int_{t_k}^{t_{k+1}} \{ \delta_1 E(r) - (\mu + \phi_1 + \delta_2) Q(r) \} (t_{n+1} - r)^{\alpha-1} dr,
\]

\[
H(t_{n+1}) - H(0) = \frac{(1-\alpha)}{ABC(\alpha)} \left\{ \gamma I(t_n) + \delta_2 Q(t_n) - (\mu + \phi_2 + \xi_2) H(t_n) \right\} \\
+ \frac{\alpha}{\Gamma(\alpha) \times ABC(\alpha)} \sum_{k=0}^{n} \int_{t_k}^{t_{k+1}} \{ \gamma I(r) + \delta_2 Q(r) - (\mu + \phi_2 + \xi_2) H(r) \} (t_{n+1} - r)^{\alpha-1} dr,
\]

\[
R(t_{n+1}) - R(0) = \frac{(1-\alpha)}{ABC(\alpha)} \left\{ \tau_1 I(t_n) + \tau_2 A(t_n) + \phi_1 Q(t_n) + \phi_2 H(t_n) - \mu R(t_n) \right\} \\
+ \frac{\alpha}{\Gamma(\alpha) \times ABC(\alpha)} \sum_{k=0}^{n} \int_{t_k}^{t_{k+1}} \{ \tau_1 I(r) + \tau_2 A(r) + \phi_1 Q(r) + \phi_2 H(r) - \mu R(r) \} (t_{n+1} - r)^{\alpha-1} dr,
\]

\[
M(t_{n+1}) - M(0) = \frac{(1-\alpha)}{ABC(\alpha)} \left\{ q_1 I(t_n) + q_2 A(t_n) - q_3 M(t_n) \right\} \\
+ \frac{\alpha}{\Gamma(\alpha) \times ABC(\alpha)} \sum_{k=0}^{n} \int_{t_k}^{t_{k+1}} \{ q_1 I(r) + q_2 A(r) - q_3 M(r) \} (t_{n+1} - r)^{\alpha-1} dr.
\]

By using Equation 18, we have
\[ S_{n+1} = S_0 + \frac{(1-\alpha)}{ABC(\alpha)} \{ \Lambda - \mu S(t_n) - \lambda(t_n)S(t_n) \} + \frac{\alpha}{\Gamma(\alpha) \times ABC(\alpha)} \sum_{k=0}^{n} \left( \frac{\Lambda - \mu S_k - \lambda_k S_k}{h} \right) B_1 - \frac{\Lambda - \mu S_{k-1} - \lambda_{k-1} S_{k-1}}{h} A_{a,k,2}, \]

\[ E_{n+1} = E_0 + \frac{(1-\alpha)}{ABC(\alpha)} \{ \lambda(t_n)S(t_n) - ((1-\theta)\omega + \theta \rho + \mu + \delta_1)E(t_n) \} + \frac{\alpha}{\Gamma(\alpha) \times ABC(\alpha)} \sum_{k=0}^{n} \left( \frac{\lambda_k S_k - ((1-\theta)\omega + \theta \rho + \mu + \delta_1)E_k}{h} \right) B_1 - \frac{\lambda_{k-1} S_{k-1} - ((1-\theta)\omega + \theta \rho + \mu + \delta_1)E_{k-1}}{h} A_{a,k,2}, \]

\[ I_{n+1} = I_0 + \frac{(1-\alpha)}{ABC(\alpha)} \{ (1-\theta)\omega E(t_n) - (\tau_1 + \mu + \xi_1 + \gamma)I(t_n) \} + \frac{\alpha}{\Gamma(\alpha) \times ABC(\alpha)} \sum_{k=0}^{n} \left( \frac{(1-\theta)\omega E_k - (\tau_1 + \mu + \xi_1 + \gamma)I_k}{h} \right) B_1 - \frac{(1-\theta)\omega E_{k-1} - (\tau_1 + \mu + \xi_1 + \gamma)I_{k-1}}{h} A_{a,k,2}, \]

\[ A_{n+1} = A_0 + \frac{(1-\alpha)}{ABC(\alpha)} \{ \theta \rho E(t_n) - (\tau_2 + \mu)A(t_n) \} + \frac{\alpha}{\Gamma(\alpha) \times ABC(\alpha)} \sum_{k=0}^{n} \left( \frac{\theta \rho E_k - (\tau_2 + \mu)A_k}{h} \right) B_1 - \frac{\theta \rho E_{k-1} - (\tau_2 + \mu)A_{k-1}}{h} A_{a,k,2}, \]

\[ Q_{n+1} = Q_0 + \frac{(1-\alpha)}{ABC(\alpha)} \{ \delta_1 E(t_n) - (\mu + \phi_1 + \delta_2)Q(t_n) \} + \frac{\alpha}{\Gamma(\alpha) \times ABC(\alpha)} \sum_{k=0}^{n} \left( \frac{\delta_1 E_k - (\mu + \phi_1 + \delta_2)Q_k}{h} \right) B_1 - \frac{\delta_1 E_{k-1} - (\mu + \phi_1 + \delta_2)Q_{k-1}}{h} A_{a,k,2}, \]

\[ H_{n+1} = H_0 + \frac{1-(\alpha)}{ABC(\alpha)} \{ \gamma I(t_n) + \delta_2 Q(t_n) - (\mu + \phi_2 + \xi_2)H(t_n) \} + \frac{\alpha}{\Gamma(\alpha) \times ABC(\alpha)} \sum_{k=0}^{n} \left( \frac{\gamma I_k + \delta_2 Q_k - (\mu + \phi_2 + \xi_2)H_k}{h} \right) B_1 - \frac{\gamma I_{k-1} + \delta_2 Q_{k-1} - (\mu + \phi_2 + \xi_2)H_{k-1}}{h} A_{a,k,2}, \]

\[ R_{n+1} = R_0 + \frac{(1-\alpha)}{ABC(\alpha)} \{ \tau_1 I(t_n) + \tau_2 A(t_n) + \phi_1 Q(t_n) + \phi_2 H(t_n) - \mu R(t_n) \} + \frac{\alpha}{\Gamma(\alpha) \times ABC(\alpha)} \sum_{k=0}^{n} \left( \frac{\tau_1 I_k + \tau_2 A_k + \phi_1 Q_k + \phi_2 H_k - \mu R_k}{h} \right) B_1 - \frac{\tau_1 I_{k-1} + \tau_2 A_{k-1} + \phi_1 Q_{k-1} + \phi_2 H_{k-1} - \mu R_{k-1}}{h} A_{a,k,2}, \]

\[ M_{n+1} = M_0 + \frac{(1-\alpha)}{ABC(\alpha)} \{ q_1 I(t_n) + q_2 A(t_n) - q_3 M(t_n) \} + \frac{\alpha}{\Gamma(\alpha) \times ABC(\alpha)} \sum_{k=0}^{n} \left( \frac{q_1 I_k + q_2 A_k - q_3 M_k}{h} \right) B_1 - \frac{q_1 I_{k-1} + q_2 A_{k-1} - q_3 M_{k-1}}{h} A_{a,k,2}, \]

where \( A_{a,k,2} = \int_{l_0}^{l_{k+1}} (\tau - t_k)(t_{n+1} - \tau)\alpha^{-1} d\tau \) and \( B_1 = \int_{l_0}^{l_{k+1}} (\tau - t_{k-1})(t_{n+1} - \tau)\alpha^{-1} d\tau. \)

Thus, integrating Equations 20 and 21 and replacing them in equations of system 25, we get
In this manuscript, we investigated the COVID-19 model with the help of ST and some effective numerical methods. Developed for the model to prove the efficiency of the developed techniques. Results will be very helpful for analysis of COVID-19 outbreak and to check the actual behavior of this pandemic disease, also helpful for further study on fractional derivatives.

\[ S_{n+1} = S_0 + \sum_{k=0}^{n} \left( h^\alpha \{ \Lambda - \mu S_n - \lambda(t_n)S(t_n) \} + \frac{\alpha}{ABC(\alpha)} \sum_{k=0}^{n} \left( h^\alpha \{ \Lambda - \mu S_{k-1} - \lambda_{k-1}S_{k-1} \} \right) \right) / \Gamma(\alpha + 2) \],

\[ E_{n+1} = E_0 + \sum_{k=0}^{n} \left( h^\alpha \{ (1-\theta)\omega + \theta \rho + \mu + \delta_1 \} E_k \right) / \Gamma(\alpha + 2) \],

\[ I_{n+1} = I_0 + \sum_{k=0}^{n} \left( h^\alpha \{ (1-\theta)\omega I_k + \rho A_k \} \right) / \Gamma(\alpha + 2) \],

\[ A_{n+1} = A_0 + \sum_{k=0}^{n} \left( h^\alpha \{ \theta \rho E_k - (\tau_2 + \mu)A_k \} \right) / \Gamma(\alpha + 2) \],

\[ Q_{n+1} = Q_0 + \sum_{k=0}^{n} \left( h^\alpha \{ (1-\theta)\omega E_k - (\tau_1 + \mu + \xi_1 + \gamma)I_k \} \right) / \Gamma(\alpha + 2) \],

\[ H_{n+1} = H_0 + \sum_{k=0}^{n} \left( h^\alpha \{ \delta_1 E_k - (\mu + \phi_1 + \delta_2)Q_k \} \right) / \Gamma(\alpha + 2) \],

\[ R_{n+1} = R_0 + \sum_{k=0}^{n} \left( h^\alpha \{ (1-\theta)\omega R_k - (\tau_1 + \mu + \xi_2)H_k \} \right) / \Gamma(\alpha + 2) \],

\[ M_{n+1} = M_0 + \sum_{k=0}^{n} \left( h^\alpha \{ q_1 I_k + q_2 A_k - q_3 M_k \} \right) / \Gamma(\alpha + 2) \],

where \( A_1 = [(n+1-k)^{\alpha+1} - (n-k)^{\alpha}(n-k+1+a)] \) and \( A_2 = [(n+1-k)^{\alpha}(n-k+2+a) - (n-k)^{\alpha}(n-k+2+a)]. \)

**6 CONCLUSION**

In this manuscript, we investigated the COVID-19 model with the help of ST and some effective numerical methods. The Mittag–Leffler kernel is used to obtain very effective results for the proposed model. Some theoretical results are developed for the model to prove the efficiency of the developed techniques. Results will be very helpful for analysis of COVID-19 outbreak and to check the actual behavior of this pandemic disease, also helpful for further study on fractional derivatives.

**ACKNOWLEDGEMENT**

There are no funders to report for this submission.

**CONFLICT OF INTEREST**

This work does not have any conflicts of interest.
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How to cite this article: Aslam M, Farman M, Akgül A, Ahmad A, Sun M. Generalized form of fractional order COVID-19 model with Mittag–Leffler kernel. *Math Meth Appl Sci*. 2021;44:8598–8614. https://doi.org/10.1002/mma.7286