A Feasible Strategy for Building Distant Retrograde Orbits

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Abstract. Specific application of satellites require the building and maintenance of orbits with high altitude. In these cases, conventional orbital transfer methods may prove to be infeasible, especially with respect to fuel consumption. In this work we present an alternative strategy for an orbital transfer of a spacecraft from a low circular orbit to a high and retrograde orbit in the Earth-Moon System. Considering a four-body scenario, the technique consists in the use of appropriate perturbations applied around unstable periodic orbits predicted in the Restricted Three-Body Problem (R3BP), associated with the exploration of the sensitive dependence of the transition regions between capture and escape in the Earth-Moon System. In addition to small transfer times, the results obtained show that it is possible to obtain stable high and inclined final orbits with a low cost in terms of increment of velocity.

1. Introduction

Many of the applications of space missions, such as space telescopes, are designed to collect data from specific regions of the sky or to detect particular wavelengths, as in the case of X-rays. In these cases, in which the Earth’s atmosphere makes it impossible to observe from the ground, it is necessary to build and maintain satellites with specific orbital characteristics, mainly with regard to altitude and inclination. One way to avoid the near-Earth electromagnetic interferences is to make use of Distant Retrograde Orbits (DRO), which are essentially characterized by semimajor axis values larger than the Earth-$L_1$ distance [1]. However, the high altitude value is usually associated with a high energy/velocity increment required.

Several works in the literature, using impulsive or finite burn propulsion, are dedicated to optimized strategies of implementation of DROs [2, 3, 4]. Most of the traditional orbital transfer techniques to DRO require a high fuel consumption or even long transfer times to place a vehicle into an orbit with high altitude. A smart approach to overcome these issues is to explore the intrinsic features of the unstable orbits presents in R3BP. Due to its rich phase-portrait structure, arising from its manifolds crosses, the exploration of the unstable orbits allow to conjecture about several possible transfers strategies in astrodynamics applications [5, 6, 7, 8].

In this work we present a strategy for an orbital transfer that allows to take a vehicle, initially in a Low Earth Orbit (LEO), to a distant and retrograde orbit. The strategy is to obtain the
energy needed to reach the altitude of the DRO through the use of judiciously impulses applied in specific points of unstable periodic orbits combined with gravity assisted maneuvers.

This paper is organized as follow: Section 2 presents the fundamental concepts related to unstable orbits and escape trajectories making up the theoretical scope of this work. Section 3 brings the methodology adopted here to search the required impulses that promotes the final desired orbits. Some case studies carried out in this article are presented in section 4, along with their corresponding discussions. The final considerations and perspectives for extensions of this work are shown in section 5.

2. Periodic Orbits and Escape Trajectories
The Restricted Three Body Problem ([9, 10]) describes the motion of a particle under the action of the gravitational fields from other two massive bodies, called primaries, whose masses are much larger than the particle mass, such that their motions are not influenced by the particle. Moreover, the primaries move in circular and coplanar orbits about their common center of mass.

In a Cartesian reference frame, called synodic frame \((x, y, z)\), we consider that the primaries remain fixed over the \(x\)-axis, as shown in Figure 1. This frame can be normalized such that total mass of the system is \(\mu\), composed by Earth, Moon and particle, are taken as unity. The primaries masses are \(m_{\text{Earth}} = 5.97420 \times 10^{24}\) kg and \(m_{\text{Moon}} = 7.25901 \times 10^{22}\) kg. The distance between the primaries is constant and equal to the average distance between the Earth and the Moon, 384,400 km, and is taken as unit of length. The mean motion, \(n\), related to a inertial frame \((\xi, \eta, \zeta)\) called sideral frame, of the primaries is equal to 1.

Among the orbits showed in [9], a special group, called Familly G of periodic orbits, establish a way back and forth between the Earth and the Moon. An orbit that started in the vicinity of the Earth, after a few days passes near the lunar surface, returning to the terrestrial neighborhood after that. The Figures 2(a, b) show such orbit in the synodic and sideral frames, respectively.

Considering a spacecraft in circular LEO, whose altitude is equivalent to the perigee distance of the orbit G, [11] showed that it is possible to apply a proper impulse \(\Delta V_1\) in order to insert the vehicle in a path that makes a passage through the neighborhood of the Moon and promotes an alteration in a periodic orbit. An illustration of this strategy is shown in Figure 3. The Figure 4 shows a numerical relation obtained between the lowest value of \(\Delta V_1\) that promotes the lunar approach (and the consequent escape from Earth-Moon System), and the altitude \(h_0\) of the initial LEO.

![Figure 1: Restricted Three Body Problem in the Synodic \((x, y, z)\) and Sideral \((\xi, \eta, \zeta)\) frames.](image-url)
Figure 2: A periodic orbit of Family G viewed from Synodic frame in (a) and from Sideral frame in (b). The axes are scaled in units of Earth-Moon distances.

Figure 3: Scheme of the escape trajectory obtained through the perturbation in a periodic orbit of the Family G.

In a previous work, [12] showed that different magnitudes of the impulse $\Delta V_1$ promote lunar passages at different altitudes, hence, different types of final trajectories are obtained. In Figure 5 a diagram with the sequence of different types of final trajectories is shown as a function of the initial impulse applied at the departure from a LEO with altitude $h_0 = 200$ km.

The lowest magnitudes of $\Delta V_1$ promote trajectories with lowest apogee of escape trajectory. Following an increasing order, the first interval promotes passages on the side of the Moon that generates a reduction in vehicle speed. A small set of values promotes collision with the Moon. After these, there is a range of values that produces the closest lunar approaches on the opposite side of the Moon, with the highest gain of velocity, consequently, these trajectories are equivalent to the direct escape from the Earth-Moon system (green lines in Figure 6).

As we increase the magnitude of $\Delta V_1$, the vehicle’s path goes farther from the lunar surface, consequently, as predicted in [13] the energy gain in this swing-by is lower than in the
2.75
2.8
2.85
2.9
2.95
3
3.05
3.1
3.15

200  400  600  800  1000  1200  1400  1600  1800  2000
ΔV1 (km/s)
h0 (km)

Figure 4: Dependence between the lowest value of $\Delta V_1$ that promotes the escape and the altitude $h_0$.

| 0.054195 km/s |
|----------------|
| - Lunar Collision or Energy Reduction swing-by: $\Delta V_1 < 3.171353$ km/s |
| - Direct Escapes Trajectories, $3.171353 \leq \Delta V_1 < 3.171692$ km/s |
| - Escape via WSB: $3.171353 \leq \Delta V_1 \leq 3.171910$ km/s |
| - Geocentric Trajectories: $3.171910 \leq \Delta V_1 < 3.226105$ km/s |
| - Escape velocity in Two-body Problem $\Delta V_1 \geq 3.226105$ km/s |

Figure 5: Sequence of different types of final trajectories as a function of $\Delta V_1$ applied at the departure from a $h_0 = 200$ km LEO.

first cases, so that the vehicle does not escape directly from the Earth-Moon system, but travels to a region located around 1.5 and 2.0 million kilometers away from the Earth (the purple line in Figure 6), then returning to the vicinity of the system. From the perspective of phase space, this region corresponds to the position and velocity conditions in which the vehicle is at the transition between escapes and captures [14], thus constituting a state quite sensitive to slight variations in the initial conditions. Analogously to other known examples in systems with chaotic behavior, this feature can be exploited in order to guide the vehicle to the desired final state in a more optimized way.

In Figure 7(a) an example of this type of trajectory is presented. In this case, around 50 days after the swing-by with the Moon (Fig. 7(b)), the vehicle reaches the above-mentioned condition with velocity close to zero (Fig. 7(c)) and its energy relative to the Earth, as shown in Figure 7(d), approaches a value at which the vehicle-Earth system would no longer constitute a bound state.

3. Methodology
The approach proposed consists in exploring a new impulse $\Delta V_2$ of much smaller magnitude than $\Delta V_1$, applied when the vehicle’s velocity is near zero during its passage through the
Figure 6: Examples of direct and indirect escapes trajectories, along with the lunar orbit, viewed from geocentric frame.

Figure 7: In (a) the indirect escape trajectory viewed from geocentric reference (planar case). In (b) and (c) are shown, respectively, the temporal variation of the vehicle’s distance and velocity, relative to the Earth. In (d) is shown the variation of the vehicle’s two-body energy, also relative to the Earth.
sensitive dependence region, as illustrated in Figure 8. This impulse can guide the vehicle to a second swing-by, through the Earth or the Moon, allowing it to obtain different final escape orbits, with a range greater than the one obtained with only one single swing-by (direct escapes showed in Fig. 6).

In a geocentric cartesian frame \((X,Y,Z)\) in which those axes are respectively parallel to \((x,y,z)\) axis of the synodic frame, the cases in which \(\Delta V_2\) has components in the form \((\Delta V_{2X}, \Delta V_{2Y}, 0)\) are associated to new swing-bys with the Earth or the Moon that can generate escape trajectories or geocentric planar orbits with elevated semimajor axis. In this work, the impulse \(\Delta V_2\) has a nonzero Z-component of sufficient magnitude so that the final trajectory of the vehicle will be inclined relative to the Earth.

Starting from an initial circular LEO, with \(h_0 = 200 km\), considering the Moon's inclination relative to the Earth is zero, it was selected the cases related to indirect escape trajectories. At the moment of minimum velocity, it was tested several \(\Delta V_2\) impulses in order to find those that promote stable final geocentric orbits. The stability criterion adopted here is associated with orbits whose greater semimajor axis, eccentricity and inclination with low fluctuations over time. All cases were numerically integrated considering a four body scenario composed by Sun \((1.98911 \times 10^{30} \text{ kg})\), Earth, Moon and a vehicle with mass \(10^3 \text{ kg}\). The simulations were performed with a Gauss-Radau spacing integrator [15].

After identifying the values of \(\Delta V_2^*\) that gives stable inclined orbits, a refinement around the components \((\Delta V_{2X}^*, \Delta V_{2Y}^*, \Delta V_{2Z}^*)\) is performed in order to obtain a more stable final orbit. In the next section we present some results obtained for the transfers with inclined final orbits.

4. Results and Discussion

The departure LEO has an initial altitude of 200 km, which corresponds to a launch window at approximately every 88 minutes. The first impulse \(\Delta V_1 = (0.0, 3.1457, 0.0) \text{ km/s}\), responsible for inserting the vehicle in the path that will perform the swing-by with the Moon, is applied at the moment where the Earth, vehicle and Moon are necessarily aligned in that order, according the orientation shown in Figure 3. After leaving the lunar neighborhood, at the moment of the passage by the region of zero velocity, it is applied the second impulse \(\Delta V_2\). Table 1 brings the components of \(\Delta V_2\) and the total impulse needed for the orbital transfer.

Figures 9 and 10 show, for these impulse components, the complete trajectories from the parking LEO departure to the final retrograde orbit. In each one of these figures we have in (a) the indirect escape trajectory viewed from the geocentric reference (planar case). In (b) and (c) are shown, respectively, the temporal variation of the vehicle’s distance and velocity, relative to the Earth. In (d) is shown the variation of the vehicle’s two-body energy also relative to the Earth.
Table 1: Components of the second impulse, magnitude of $\Delta V_2$ and total magnitude $\Delta V_1 + \Delta V_2$ for each orbital maneuver, in km/s.

| Case | $\Delta V_{2X}$ | $\Delta V_{2Y}$ | $\Delta V_{2Z}$ | $\Delta V_2$ | $\Delta V_1 + \Delta V_2$ |
|------|-----------------|-----------------|-----------------|-------------|-----------------------------|
| 01   | -0.04142        | -0.09964        | -0.00124        | 0.10791     | 3.25500                     |
| 02   | -0.05993        | -0.07835        | -0.00291        | 0.09868     | 3.24577                     |

Figure 9: Results obtained for Case 01: $\Delta V_2 = (-0.04142, -0.09964, -0.00124)$ km/s

In both cases, the perturbation applied at the moment of the passage through the zero velocity region allowed the spacecraft to reach of a stable final retrograde orbit over a long interval of time. The exploration of a route derived from a periodic orbit combined with a gravity assisted maneuver with the Moon allows a reduction in the magnitude required to the first impulse $\Delta V_1$. In comparison with the Patched-Conic Approximation implemented in [16], our strategy to generate an escape trajectory requires an impulse 4% lower.

The strategy proposed here allows to achieve stable DROs even in a more realistic scenario where the influence of the Sun is considered. A total $\Delta V$ of 3.245 km/s and a time transfer around 145 days, which is compatible with the results showed in [1] where, also starting from a prograde parking LEO with 200 km altitude, under a three body perspective, demands a total $\Delta V$ of 3.315 km/s and a transfer time of 230 days to build a final DRO with semimajor axis of $1.5 \times 10^6$ km.
Figure 10: Results obtained for Case 02: $\Delta V_2 = (-0.05993, -0.07835, -0.00291)$ km/s

5. Final Remarks

In this work we presented a viable strategy for the construction of retrograde orbits. Through an application of appropriate impulses at specific points of unstable trajectories and a lunar swing-by maneuver, its possible to obtain stable DROs even in the four-body problem scenario.

In addition to applications such as space telescopes, the strategy proposed here also allows several others applications developments such as the possibility of transfers to the Lagrangian points of the Sun-Earth system. Our strategy has the potential to reduce the fuel consumption required to send a spacecraft to lunar DROs, which can be very convenient in asteroid capture missions, as the NASA’s Asteroid Redirect Mission, recently proposed. Other studies to reach final orbits that link Earth to Mars or Earth to Venus are also being conducted.

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