Investigation on unified model of constitutive relations for masonry

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Abstract. In order to establish a unified model of constitutive relations for masonry, explain the relationship between energy dissipation and damage energy in the process of uniaxial masonry compression. Based on the application of energy principle and the theory of damage mechanics, a constitutive relation of masonry with parameters is proposed by combining a damage function and the stress-strain curve’s characteristics which is under axial compression. The constitutive relation expressions of masonry for different materials can also be determined, only in combination with the peak stress and strain of the axial compression test. To verify this model, the constitutive relation of the perforated shale brick masonry was put forward according to it’s compression tests results and the constitutive relation fits well with the experimental data. Besides, not only in the small deflection, it’s curve shape is basically in line with these constitutive relations suggested by others, but it can also avoid the defects of these relations to reflect the curve features in the large deformation.

1 Introduction

In both seismic regions and non-seismic regions, the masonry structure has a wide range of applications in the multi-storey building and the high-rise buildings. So far, researchers at home and abroad have done a lot of studies on the basic mechanical properties of masonry⁴¹⁴⁵, but the constitutive relations researches are relatively few and all these constitutive relations are empirical formulas which have been put forward only based on the experimental results⁶⁻¹⁰. A perfectable constitutive relation model of masonry should have the following characteristics. Firstly, for the different materials of masonry, the constitutive relation can also be proposed by determining the parameters which are related to the masonry compression strength and the peak strain. Secondly, the expression must be terse in form and can show the relationship between stress and strain of masonry under the action of axial compression. Thirdly, the constitutive relation curve must be continuous and smooth in the peak point to ensure the calculation results of convergence and so that it can be applied in the finite element simulation software, and besides, there is only an inflection point in the
curve of the decline period. Apart from these features, the constitutive relation suggested not only based on the test results, but also have the theoretical inference.

2 Unified model of constitutive

Masonry is a two-phase hybrid material, and it’s blocks in the process of blocking and sintering forms some micro cracks and holes. From the macroscopic mechanical properties, the rise of the masonry stress-strain curve shows a stiffness degradation phenomenon, this suggests that loading process of masonry under the compression is essentially a material degradation process caused by its internal defect. Therefore, damage mechanics can be used to explain the process of crack extension until failure due to accumulation along masonry damage.

2.1 Masonry compression energy dissipation

The variation of the state or the motion state of objects is a thermodynamic process, and this process is not reversible while the variations accompany an energy dissipation. Due to the masonry damage process under static load changes slowly, so in any short period of time, the masonry can be considered as a state of balance.

Assume that masonry axial compression test is conducted in an adiabatic system with no heat loss, and thus the work done by compressive stress $\Delta W$ is converted into masonry’s strain energy $\delta K$ and internal energy $\delta U$ and kinetic energy $\delta E$ . Where the internal energy increased $\delta U$ is essentially the heat masonry accepted $\delta Q$. According to the first law of thermodynamics, it can be derived equation (1).

$$\Delta W = \delta K + \delta U + \delta E \quad (1)$$

In the loading process of masonry under axial compression, the kinetic energy $\delta E$ keeps very close to zero for the speed of lower loading. But in practice, the loading process is conducted in a thermostat system, for this reason, the internal energy of masonry is unchanged and masonry have $\delta Q$ of heat released, so in the process of masonry compression, the process of masonry internal damage the same as heat dissipation is an irreversible process. Therefore, the energy dissipation of damage owing to development of cracks can be considered as heat released in the processes of masonry compression. So get equation (2).

$$\Delta W = \delta K + \delta Q \quad (2)$$

Make a division on the opposite side of the equal sign by divide the volume $V$ of masonry, and get equation (3).

$$W(\varepsilon) = W_0(\varepsilon) + W_k(\varepsilon) \quad (3)$$

Where $W(\varepsilon)$ is strain energy density of the elastic system when the strain is $\varepsilon$, $W_0(\varepsilon)$ is the energy density released as the strain from 0 to $\varepsilon$, during the masonry compression damage, $W_k(\varepsilon)$ is strain energy density of the elastoplastic system.

On the basis of above study, a damage variable $D$ caused by external compressive strain can be defined.

$$D = \frac{W_0(\varepsilon)}{W(\varepsilon)} = 1 - \frac{2}{E_0 \varepsilon^2} \frac{\epsilon d\epsilon}{\varepsilon^3} \quad (4)$$

Where $E$ is the initial elastic modulus of masonry compression, $\sigma$ and $\varepsilon$ is stress and strain, respectively. Take the derivative of equation (3) and get (5).

$$\sigma = E\varepsilon(1 - D) + \frac{1}{2}E\varepsilon \frac{dD}{d\varepsilon} \quad (5)$$

Damage variable is a constant for the sick of the irreversible nature of the masonry compression in
the unloading process and so \( \frac{dD}{d\varepsilon} = 0 \). Then, get the masonry damage constitutive relationship expression (6).

\[
\sigma = E\varepsilon (1 - D)
\]

Expression (6) shows that the masonry Elastic Modulus is \( E(1 - D) \) result from stiffness deterioration cause by the damage development.

Due to the masonry damage is irreversible, and its degree increased along with the strain growth, so the damage function \( D \) is a monotone increasing function, when \( D = 0 \) is denote masonry without damage, and \( D = 1 \) is entirely damaged. According to the characteristics of damage function \( D \), various expressions of damage functions can be used, such as polynomial, piecewise function, as well as probability distribution functions like Weibull distribution function, logarithmic normal distribution, etc. Thus there are varieties of forms for constitutive relation expression can be constructed. In order to establish a simple expression so that it can be accepted by engineers, expression (7) is chosen as the damage function \( D \).

\[
D = 1 - \frac{a}{a + (1 - a) \cdot \left( \frac{\varepsilon}{\varepsilon_0} \right)^b}
\]

Where \( a \) and \( b \) are parameters determined by the compressive strength of masonry and the peak strain. Expression (8) can be derived by combining expression (6) and (7).

\[
\sigma = \frac{Ea}{a + (1 - a) \cdot \left( \frac{\varepsilon}{\varepsilon_0} \right)^b}
\]

Masonry compression stress-strain curve have the following characteristics: \( \bullet \) at the origin of stress-strain curve, where \( \varepsilon = 0 \), there are \( \sigma = 0 \) and \( d\sigma / d\varepsilon = E \). That is, the elastic modulus of the origin is \( E \). \( \bullet \) Where \( \varepsilon = \varepsilon_0 \), there are \( \sigma = f_m \) and \( d\sigma / d\varepsilon = 0 \). Obviously, expression (8) meets the feature \( \bullet \)

According to the feature \( \bullet \), the following results can be derived.

\[
a = \frac{f_m}{E\varepsilon_0}, \quad b = \frac{1}{1 - a}
\]

According to equation (8) and (9), the unified constitutive relation model with parameter for masonry-expression (10)-been put forward. Obviously, for different materials of brick masonry, the constitutive relation can be proposed by determine the parameters \( a \) and \( b \).

\[
y = \frac{x}{a + (1 - a)x^b}
\]

3 Application and validation

3.1 Experiment introduction

Adopting sintered shale porous brick with strength grade of MU10 and ordinary cement mortar with strength grade of M7.5, making 9 groups of specimens. The specimens size and constructing procedure and method of loading, enforcing strictly according to the standards specifications, namely Standard for Test Method of Basic Mechanics Properties of Masonry (GB/T 50129-50129). The height and length and width of each specimen are 710 mm×365 mm×240 mm. After 28-day standard curing at room temperature, the specimens are conducted compression tests using pressure testing machine with pressures value 200-T. Fig. 1 presents the test unit for specimens.
In the process of loading, the test data and the phenomena in the loading will be carefully recorded.

![Fig1. Test device for specimens compression](image)

3.2 Experimental results

According to the standards, Code for Design of Masonry Struction (GB50003-2011), expression(11) was recommended for calculating the average compressive strength for masonry.

\[
f_m = k_1 k_2 f_1^m (1 + 0.07 f_2) \tag{11}\]

Where \( f_m \) is the average compressive strength of masonry (MPa), \( k_1 \) is parameters related to the types of masonry, and for sintered porous brick \( k_1 = 0.78 \). \( k_2 \) is also parameters related the kind of mortar, usually \( k_2 = 1.0 \). \( \alpha \) is determined by the block materials, here \( \alpha = 0.5 \), \( f_1 \) and \( f_2 \) are the average compressive strength for bricks and mortar (MPa), respectively. The main test results are shown in table 1. where \( f \) is the measured compressive strength of the specimens in the axial compression test, \( \varepsilon_0 \) is the measured peak strain for masonry. According to the table, it was clear that the measured values were greater than the values of calculating suggested by expression(12).

| Number | Compressive Strength \( f \) /MPa | Average Compressive Strength \( f_m \) /MPa | Peak Strain \( \varepsilon_0 \times 10^{-3} \) | Average Strain \( \varepsilon_n \times 10^{-3} \) |
|--------|-------------------------------|-----------------------------------|---------------------------------|-------------------|
| KY--1  | 4.20                          | 2.19                              |                                |                   |
| KY--2  | 4.19                          | 2.26                              |                                |                   |
| KY--3  | 4.13                          | 2.18                              |                                |                   |
| KY--4  | 4.03                          | 2.55                              |                                |                   |
| KY--5  | 4.31                          | 3.78                              | 2.41                            | 2.27              |
| KY--6  | 4.14                          | 2.02                              |                                |                   |
| KA--7  | 4.05                          | 2.23                              |                                |                   |
| KA--8  | 4.11                          | 2.13                              |                                |                   |
| KA--9  | 4.05                          | 2.42                              |                                |                   |

3.3 Constitutive relation for sintere-shale porous brick

The elastic value of masonry is determined by formula (12)[9].

\[
E = 370 f_m \sqrt{f_m} \tag{12}
\]

The constitutive relation for sintere-shale porous brick-expression(13) - can be put forward as the parameters \( a \) and \( b \) was determined by combine with the test results of masonry and expression(10).

\[
y = \frac{x^{0.6 + 0.4 x^{2.5}}}{0.6 + 0.4 x^{2.5}} \tag{13}
\]
3.4 Validation and analysis
The test data contains only the data in the process of loading, on account of the insufficient stiffness of the testing machine. As is shown in Fig. 2, in the rising step these results fit well with the stress-strain’s curve shape, which is suggested in this paper. In order to further verify the curve of expression (16) in the downward section, a comparison between constitutive relations which were suggested by others and in this paper is shown in Fig. 3. In small deformation range of the decline period, the curve of the formula (16) coincides better with other formulas. In addition to this, the formula (16) also avoids the deficiency of the existing constitutive relations. So the compressive constitutive relations of masonry, formula (16), can be used to reflect the stress-strain relationship of masonry under compression. And thus, the unified constitutive relation model with parameters for masonry, expression (14) is also a rational model.

Table 2. Expressions of compressive constitutive relations of masonry

| Number | Formulas | References |
|--------|----------|------------|
| 14     | \[ y = \begin{cases} 
1.52x - 0.279x^2, & x \leq 1 \\
1 - 0.483x + 0.724x^2, & 3.4x - 1.13x^2, & x > 1 \\
1 + 1.4x - 0.13x^2 & \end{cases} \] | [13] |
| 15     | \[ y = 6.4x - 5.4x^{1.17} \] | [12] |
| 16     | \[ y = 2.3x - 1.555x^2 + 0.195x^3 + 0.075x^4 - 0.015x^5 \] | [14] |
| 27     | \[ y = xe^{1-x} \] | [15] |

4 Conclusions
According to the principle of conservation of energy, combining with the damage mechanics theory, established a constitutive relation model of masonry. This model is only related with three parameters-elastic modulus, peak stress and strain. All these parameters can be obtained from masonry axial compression test. According to the axial compression test results of sintered shale porous brick masonry structure, gets the constitutive relationship formula (16). The curve of the formula (16) is agree very well with test data. More importantly, this model avoids the deficiency of the existing constitutive relations.
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