Analytic Two-loop Higgs Amplitudes in Effective Field Theory and Maximal Transcendentality Principle

Qingjun Jin\textsuperscript{a} and Gang Yang\textsuperscript{a}

\textsuperscript{a}CAS Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China

We obtain for the first time the two-loop amplitudes for Higgs plus three gluons in Higgs effective field theory including dimension-seven operators. This provides the S-matrix elements for the top mass corrections for Higgs plus a jet production at LHC. The computation is based on the on-shell unitarity method combined with integration by parts reduction. We work in conventional dimensional regularisation and obtain analytic expressions renormalized in the MS scheme. The two-loop anomalous dimensions present operator mixing behavior. The infrared divergences agree with that predicted by Catani and the finite remainders take remarkably simple forms, where the maximally transcendenttal parts are identical to the corresponding results in $\mathcal{N} = 4$ super-Yang-Mills theory. The parts of lower transcendentality turn out to be also largely determined by the $\mathcal{N} = 4$ results.

Introduction.—The discovery of a Standard-Model-like Higgs boson at Large Hadron Collider (LHC) set a milestone in particle physics. A major goal of the present and future collider experiments is to make precise measurements of the Higgs properties, which is crucial to understand the precise mechanism of electroweak symmetry breaking and to probe potential new physics beyond the Standard Model. Given the substantial increase in statistical precision of the experiments, a precise theoretical prescription of the scattering processes is thus mandatory.

The dominant Higgs production mechanism in LHC is the gluon fusion through a top quark loop \cite{1, 2}. In the approximation that the top mass $m_t$ is much larger than Higgs mass $m_H$, the computation can be greatly simplified using an effective field theory (EFT) where the top quark is integrated out \cite{3–7}. The leading term of the effective Lagrangian, corresponding to taking $m_t$ to be infinity, is given by a unique dimension-5 operator, $H \text{Tr}(G_{\mu\nu}G^{\mu\nu})$, where $H$ is the Higgs field and $G_{\mu\nu}$ is the gauge field strength. The leading term contribution provides an accurate approximation for the inclusive Higgs production and has been computed to N$^3$LO QCD accuracy \cite{8}. Differential results for Higgs plus a jet production were also computed at N$^3$LO \cite{9, 10} in the large $m_t$ limit. However, for large Higgs transverse momentum which is near (or larger than) the top mass threshold, the large $m_t$ limit is no longer a good approximation, and the contribution of the higher dimension operators in the EFT can have significant effect. For Higgs plus a jet production, at NLO order, progress of including finite top mass effect were made recently in \cite{11, 12}. The planar master integrals (which are two-loop integrals with a massive quark loop) for the NLO virtual QCD corrections were known analytically in \cite{13}.

In this paper, we compute for the first time the analytic two-loop amplitudes of Higgs plus three gluons with dimension-7 operators in the EFT. This provides the building blocks for the correction of top mass effect for Higgs plus a jet production at N$^3$LO order. We obtain the two-loop anomalous dimensions of the dimension-7 operators which exhibit the operator mixing effect.

Analytic results are crucial for uncovering hidden structures of the amplitudes and are thus important to improve our understanding of high loop structure of perturbative QCD. In this respect, the analytic results obtained in this paper allow to test the “maximal transcendentality principle” which conjectures an intriguing correspondence between QCD and the maximally supersymmetric Yang-Mills ($\mathcal{N} = 4$ SYM) theory. It was first observed in \cite{20, 21} that, for the anomalous dimensions of twist-two operators, the $\mathcal{N} = 4$ SYM theory results can be obtained from the leading transcendental part of the QCD results \cite{22}. A further surprising observation in \cite{23} is that the two-loop form factor of stress-tensor multiplet in $\mathcal{N} = 4$ SYM coincides with the maximally transcendental part of the QCD Higgs plus three-gluon amplitudes in the heavy top quark limit \cite{24}. The same maximally transcendental function was also found in the $\mathcal{N} = 4$ two-loop Konishi form factor \cite{25}. Further evidence of this correspondence were also found for certain configurations of Wilson lines \cite{26, 27}. This principle has also been used to extract from known QCD data the analytic planar four-loop collinear anomalous dimension in $\mathcal{N} = 4$ SYM \cite{28}. While counter examples of this principle are known such as one-loop amplitudes (see also the study of high energy limit of amplitudes \cite{29}), it is interesting to explore to which extent this principle is valid.

The analytic results in QCD obtained in this paper provide concrete new examples to test the maximal transcendentality principle. The two-loop three-gluon form factor of $\text{Tr}(G^3)$ operator in $\mathcal{N} = 4$ SYM was obtained in \cite{30}, in which it was also argued that the maximally transcendentnal part should be equal between the $\mathcal{N} = 4$ result and the corresponding Higgs amplitude in QCD. Our explicit QCD result confirms this argument. More intriguingly, it turns out that the sub-leading transcendental parts in QCD are also closely related to the $\mathcal{N} = 4$ re-
sult. In particular, except for the transcendental degree zero part, all terms having rational kinematics coefficients are identical between two theories.

The computation of two-loop amplitudes in QCD, as is well known, is a challenging problem. While the two-loop four-gluon amplitudes are known analytically long time ago [31–33], the planar two-loop five-gluon amplitudes are still in progress [34–43]. The computation of Higgs amplitudes has extra complications. The inclusion of higher dimension operators introduces new complex interaction vertices and also increases the powers of loop momenta in the integral numerators. Furthermore, since the Higgs boson is a color singlet, one encounters non-planar integrals even for planar Higgs amplitudes, which makes the reconstruction of full integrand via on-shell unitarity method highly non-trivial.

In this paper, we develop an efficient approach to compute Higgs amplitudes by combining the unitarity method and the integration by parts (IBP) reduction [42, 49–53] in an ‘unconventional’ way. In particular, we apply the IBP reduction directly for the cut integrands, which computes the final coefficients of master integrals, thus avoiding reconstructing the full integrand. Besides, the IBP reduction, which is often the most time consuming part of the calculation, can be simplified using the on-shell condition. Similar strategy of combining unitarity cut and IBP reduction has also been used in [43], see also [42, 49, 53].

Setup.—Higgs production from gluon fusion can be computed using an effective Lagrangian

\[ \mathcal{L}_{\text{eff}} = \hat{C}_1 O_0 + \frac{1}{m^2} \sum_{i=1}^{4} \hat{C}_i O_i + \mathcal{O} \left( \frac{1}{m^2} \right), \]  

where \( O_0 = H \text{Tr}(G^2) \) is the leading term, and the sub-leading terms contain dimension-7 operators [54–58]

\[ O_1 = H \text{Tr}(G_{a \mu}^G G_{b \rho}^G G_{c \nu}^G), \]
\[ O_2 = H \text{Tr}(D_{\rho} G_{a \mu} D_{\rho} G_{b \mu} G_{c \nu}^G), \]
\[ O_3 = H \text{Tr}(D^\rho G_{a \mu} D_{\rho} G_{b \mu} G_{c \nu}^G), \]
\[ O_4 = H \text{Tr}(G_{a \mu} D_{\rho} D_{\sigma} G_{b \mu} G_{c \nu}^G). \]

The explicit form of the coefficients \( \hat{C}_i \) is not important for this study, although their renormalization is determined by the renormalization constant mentioned later. In this paper, we will focus on the pure gluon sector. The last two operators have zero contribution in the sector and can contribute when there are internal quark lines, see e.g. [58].

An amplitude with a Higgs boson and \( n \) gluons is equivalent to a form factor with the operator \( O_i \)

\[ \mathcal{F}_{O_i,n} = \int d^4 x \, e^{-i q \cdot x} \langle p_1, \ldots, p_n | O_i(x) | 0 \rangle, \]

where the operator \( O_i \) corresponds to a Higgs-gluon interaction term \( O_i \) in the EFT with the Higgs field stripped off, i.e. \( O_i = H O_i \). \( q \) is the total momentum flowing through the \( O_0 \) operator, satisfying \( q^2 = m_H^2 \).

In the following, we will refer to Higgs amplitudes as form factors.

Using Bianchi identity one can decompose the operator \( O_2 \) as (see e.g. [55])

\[ O_2 = \frac{1}{2} q^2 O_0 - 4 g_{\text{YM}} O_1 + 2 O_4. \]  

In the pure gluon sector, since the form factor of \( O_4 \) is zero, we have the relation for the form factors

\[ F_{O_2} = \frac{1}{2} q^2 F_{O_0} - 4 g_{\text{YM}} F_{O_1}, \]  

where the partial derivatives reduce to \( q^2 \). This will serve as self-consistency check for the result.

A further simplification of the computation is that for the form factors with three gluons in the pure YM sector, the color factorizes out up to two loops as

\[ \mathcal{F}^{(l)}(1_{a_1}, 2_{a_2}, 3_{a_3}) = f^{a_1 a_2 a_3} N_c F^{(l)}(1, 2, 3) \]  

for \( l \leq 2 \), where \( f^{a_1 a_2 a_3} \) is the structure constant of the gauge group. This can be easily seen by examining the color factors of various two-loop topologies. This implies that the form factor has only planar contribution. Below we consider only the color stripped form factor \( F^{(l)}(1, 2, 3) \), and the full color dependence can be easily reproduced using [40].

Computation.—Unitarity method is a powerful tool to construct loop amplitudes or form factors from their discontinuities, i.e. by applying cuts. On the cut, the loop integrand factorizes into a product of tree-level or lower-loop amplitudes and form factors. The commonly used strategy of unitarity method is to reconstruct the full integrand from the cuts. We will use a different strategy where the IBP reduction is applied directly for the cut integrand. In this way, there is no need to construct the full integrand, and one obtains directly the final coefficients \( c_i \) of IBP master integrals:

\[ F^{(l)}|_{\text{cut}} = \sum_{\text{helicities}} F^{\text{tree}} \prod_{j} A^{\text{tree}}_{j} = \sum_{i} c_i M_i|_{\text{cut}}, \]

where \( M_i \) are IBP master integrals. Note that a coefficient \( c_i \) computed in a single cut channel must be the final answer. This is because the master integrals are ‘irreducible’, and the coefficients are loop momenta independent without ambiguity. We would like to stress that the on-shell cut conditions greatly simplify the IBP reduction. First, the cut integrand is much simpler than the full integrand. Furthermore, the integrals without the cut propagators are dropped off during the reduction. Below we describe our strategy in more details.

We apply \( D \)-dimensional planar unitarity method. Tree amplitudes and form factors valid in \( D \) dimensions.
can be computed using planar Feynman diagrams, or recursive techniques such as Berends-Giele method. Although one could also carry out non-planar unitarity cut, in which the building blocks are the complete amplitudes or form factors with color factors, the non-planar contribution vanishes up to two loops as explained before, so these two methods are equivalent.

We need to sum over all helicity states for the cut legs, for which we contract the internal gluon polarization vectors using the rule:

$$\sum_{\text{helicities}} \varepsilon_i^{\mu} \varepsilon_i^{\nu} = \eta^{\mu \nu} - \frac{q^{\mu} p_i^{\nu} + q^{\nu} p_i^{\mu}}{q \cdot p_i} ,$$  \hspace{1cm} (11)

where $q^{\mu}$ is an arbitrary null momentum. This corresponds to using conventional dimensional regularization (CDR) scheme.

Since the cut-integrand is gauge invariant, we can further expand it using a set of gauge invariant basis $B_\alpha$ (see e.g. [24], and also [13, 60] for recent general discussion)

$$F_n(\varepsilon_i, p_1, l_0)|_{\text{cut}} = \sum_\alpha f_n^\alpha(p_1, l_0) B_\alpha .$$  \hspace{1cm} (12)

After projection, all polarization vectors are contained in the basis $B_\alpha$, and $f_n^\alpha$ contain only scalar product of loop and external momenta, which can be reduced directly by IBP, using e.g. public codes [61, 62].

As an example, consider the triple-cut of $F_{O_2}^{(2)}(p_1, p_2)$ as shown in Figure 1. Using the procedure described above, this cut allows us to fix the coefficients of two masters integrals, the sunrise and the cross-ladder integrals, as shown in Figure 1. To determine the coefficients of all master integrals, there are four other cuts to consider, as shown in Figure 2.

Let us comment on an important new feature of form factors comparing to amplitudes computation. Since the operator (or the Higgs particle) is a color singlet, the leg carrying momentum $q$ can appear in the ‘internal’ of the graph, even for the color planar contribution. This explains the appearance of the cuts (c) and (e) in Figure 2. These two cuts determine the coefficients of master integrals (3) and (5) in Figure 3. Although mathematically these two master integrals are the same as integrals (2) and (4) of Figure 3 respectively, they have different physical origin in the planar contribution and should be considered separately. The full form factor $F_{O_2}^{(2)}$ can be given as

$$F_{O_2}^{(2)}(p_1, p_2) = \left( \sum_{i=1}^{4} c_i M_i + \sum_{i=5,6} c_i^2 M_i \right) + \text{perms}(p_1, p_2),$$  \hspace{1cm} (13)

where $M_i$ correspond to the integrals with label (i) in Figure 3. Note the factor $\frac{1}{2}$ is necessary for integrals (5) and (6), since the permutation does not alter the diagram.

For the three-point two-loop form factors, all the cuts needed are given in Figure 4. The master integrals are shown in Figure 5. While all cuts are needed for the form factor of length-2 operator $O_0$ and $O_2$, only the first four cuts contribute to $O_1$, since the tree form factors of $O_1$ contain at least three gluons. Correspondingly, only the first seven master integrals in Figure 5 appear in the form factor of $O_1$. With the coefficients of masters integrals, the full form factor is obtained by adding all the master integrals and taking into account the symmetry factors properly, similar to the two-point case in (13).

We compute all two-loop form factors of $O_i$, $i = 0, 1, 2$ with two and three external gluons. The linear relation (8) means that these three form factors are not independent. This will serve as a non-trivial check of the computation as well as mentioned later. We would like to emphasize that the computation of form factor for $O_2$ is more involved than the known result of $O_0$ due to extra derivatives in the operator.

Let us mention that the above strategy can be also applied to $\mathcal{N} = 4$ SYM. One can use four dimensional helicity tree amplitudes and form factors in the cuts, which corresponds to the use of four-dimensional helicity (FDH) scheme. The computation as well as the final results are usually much simpler than the corresponding QCD quantities. In this way, we obtain the $\mathcal{N} = 4$ form factors of...
The super extension of $\mathcal{O}_0$ and $\mathcal{O}_1$, which were previously computed in $^{23}$ and $^{30}$, respectively.

For the form factors considered in this paper, all master integrals have been obtained in terms of harmonic polylogarithms $^{65,66}$. Thus we obtain the bare form factors in explicit transcendental functions.

**Divergence subtraction and checks.**—The bare form factors contain both ultraviolet (UV) and infrared (IR) divergences. Our QCD results are regularized in the CDR scheme, and we use MS renormalization scheme $^{67}$. To remove the UV divergences in the form factors, both the gauge coupling and the operator require renormalization. For the IR divergences, we apply the subtraction formula by Catani $^{68}$.

At two-loop, all poles in $1/\epsilon^m, m = 4, 3, 2$ are totally fixed by the universal IR structure and the one-loop data, which provides non-trivial consistency check of the results. From the $1/\epsilon$ UV pole one can extract the two-loop anomalous dimension of the operator, which is related to the renormalization constant of the operator by

$$\gamma = \mu \frac{\partial}{\partial \mu} \log Z. \quad (14)$$

Our computations reproduce all known results, including the non-trivial two-loop QCD amplitudes of Higgs plus three gluons with the operator $\mathcal{O}_0$ $^{24}$ (see also $^{69}$). For the latter, we match not only the divergences but also the finite remainders exactly, which provides a non-trivial check for our computation. The $N = 4$ computations also reproduce those in $^{23}$ and $^{30}$.

As a further consistency check of the new results of dimension-7 operators, we find the form factor results satisfy exactly the linear relation $^{8}$. This is true already for the expressions in terms of IBP master integrals.

**Operator mixing at two loops.**—At two-loop the operator mixing appears. Let us first consider $\mathcal{O}_2$. Based on $^{8}$, we can define a new operator

$$\tilde{\mathcal{O}}_2 = -\frac{3}{2} (\mathcal{O}_2 + 8 g_{\text{YM}} \mathcal{O}_1) = -\frac{3}{4} q^2 \mathcal{O}_0. \quad (15)$$

The new operator $\tilde{\mathcal{O}}_2$ has no mixing with others. The anomalous dimension of $\tilde{\mathcal{O}}_2$ is identical to that of $\mathcal{O}_0$, and the form factor of $\tilde{\mathcal{O}}_2$ is proportional to that of $\mathcal{O}_0$ as

$$F_{\tilde{\mathcal{O}}_2} = -\frac{3}{4} q^2 F_{\mathcal{O}_0}. \quad (16)$$

Below we only focus on the results for the operator $\mathcal{O}_1$.

The normalization constant $-\frac{3}{4}$ is introduced such that $F_{\mathcal{O}_2}(1^-, 2^-, 3^-)/F_{\mathcal{O}_1}(1^-, 2^-, 3^-) = 1/(uvw)$, where

$$u = \frac{s_{12}}{q^2}, \quad v = \frac{s_{23}}{q^2}, \quad w = \frac{s_{13}}{q^2}, \quad q^2 = s_{123}. \quad (17)$$

To study the operator mixing effect for $\mathcal{O}_1$, we first consider the form factor with two external gluons $F_{\mathcal{O}_1}^{(1)}(1^-, 2^-)$. The tree and one-loop results are zero, while at two-loop we obtain

$$F_{\mathcal{O}_1}^{(2)}(1^-, 2^-) = F_{\mathcal{O}_2}^{(0)}(1^-, 2^-) \left( -\frac{1}{\epsilon} + 2 \log s_{12} - \frac{487}{72} \right) + \mathcal{O}(1^). \quad (18)$$

This is completely an operator mixing effect between $\mathcal{O}_1$ and $\tilde{\mathcal{O}}_2$. Furthermore, for the three-point form factor $F_{\mathcal{O}_1}^{(2)}(1^-, 2^-, 3^-)$, its $Z^{(2)}$ part is given as

$$F_{\mathcal{O}_1}^{(2)}(1^-, 2^-, 3^-)|_{Z^{(2)}\text{-part}} = F_{\mathcal{O}_1}^{(0)}(1^-, 2^-, 3^-) \left( -\frac{19}{24\epsilon^2} + \frac{25}{12\epsilon} - \frac{1}{uvw} \frac{1}{\epsilon} \right) = \left( \frac{19}{24\epsilon^2} + \frac{25}{12\epsilon} \right) F_{\mathcal{O}_1}^{(0)}(1^-, 2^-, 3^-) - \frac{1}{uvw} \frac{1}{\epsilon}. \quad (19)$$

The term $\frac{1}{uvw}$ is precisely due to the operator mixing, and its divergence is consistent with $^{18}$.

Similar to $^{14}$, we can define a new operator which avoids the operator mixing as

$$\tilde{\mathcal{O}}_1 = \mathcal{O}_1 + \frac{1}{\epsilon} g_{\text{YM}} \left( \frac{\alpha_s}{4\pi} \right)^2 \tilde{\mathcal{O}}_2, \quad (20)$$

and we have

$$Z_{\tilde{\mathcal{O}}_1}^{(2)} = -\frac{19}{24\epsilon^2} + \frac{25}{12\epsilon}, \quad \gamma_{\tilde{\mathcal{O}}_1}^{(2)} = \frac{25}{3}, \quad (21)$$

in which the two-loop anomalous dimensions is computed using $^{14}$. We emphasize that it is an important consistency check that the $1/\epsilon^2$ term in the two-loop renormalization constant cancel exactly by the one-loop data.

**Two-loop finite remainder.**—After renormalization and subtracting the IR divergences, the two-loop finite remainder of $F_{\mathcal{O}_1}^{(2)}(1^-, 2^-, 3^-)$ is given in terms of harmonic polylogarithms, which can be simplified using the
symbology technique for transcendental functions \[70\]. The final expression takes a remarkable simple form. It can be decomposed as:

\[
F_{R_{\mathcal{O}_1}}^{(2), \text{fin}} = F_{\mathcal{O}_1}^{(0)} \sum_{i=0}^{4} \Omega_{\mathcal{O}_1;i}^{(2)},
\]

where \(\Omega_{\mathcal{O}_1;i}^{(2)}\) has uniform transcendental weight \(i\).

The corresponding \(N = 4\) result was first computed in \[30\] which we have reproduced using our method. To properly compare the two results, we recompute the \(N = 4\) result in the Catani subtraction scheme \[68\], denoted by \(\Omega_{\mathcal{O}_1;i}^{(2),N=4}\). This is different from the result given in \[30\] which is based on BDS subtraction scheme \[71\].

Below we give the explicit QCD results according the transcendental weight, and we comment on their relation to the corresponding \(N = 4\) counterparts. As we will see, not only the maximally transcendental parts are identical, the lower transcendental parts are also closely related to each other.

Weight 4: The maximally transcendental part is give by:

\[
\Omega_{\mathcal{O}_1;4}^{(2)} = -\frac{3}{2} \text{Li}_4(u) + \frac{3}{4} \text{Li}_4 \left(\frac{u}{w}\right) - \frac{3}{2} \log(w) \text{Li}_3 \left(\frac{-u}{v}\right) + \frac{\log^2(u)}{32} \left[\log^2(u) + \log^2(v) + \log^2(w) - 4 \log(v) \log(w)\right] + \frac{\zeta_2}{8} \left[5 \log^2(u) - 2 \log(v) \log(w)\right] - \frac{1}{4} \zeta_4 - \frac{1}{2} \zeta_3 \log(-q^2) + \text{perms}(u,v,w).
\]

We find a precise match between QCD and \(N = 4\) results:

\[
\Omega_{\mathcal{O}_1;4}^{(2)} = \Omega_{\mathcal{O}_1;4}^{(2),N=4},
\]

which confirms the argument made in \[30\]. Note that the expression is slightly different from the result of \[30\], which also appears in other form factors in \(N = 4\) SYM.\[72\][73]; this is purely due to the change of scheme between Catani and BDS subtraction.

Weight 3: The transcendentality-3 part is give by:

\[
\Omega_{\mathcal{O}_1;3}^{(2)} = \left(1 + \frac{u}{w}\right) T_3 + \frac{143}{72} \zeta_3 - \frac{11}{24} \zeta_3 \log(-u q^2) + \text{perms}(u,v,w),
\]

where

\[
T_3 := \left[- \text{Li}_3 \left(\frac{-u}{w}\right) + \log(u) \text{Li}_2 \left(\frac{v}{1-u}\right) - \frac{1}{2} \log(1-u) \log(u) \log \left(\frac{u^2}{1-u}\right) + \frac{1}{2} \text{Li}_3 \left(\frac{-u}{w}\right) + \frac{1}{2} \log(u) \log(v) \log(w) + \frac{1}{12} \log^3(w) + (u \leftrightarrow v)\right] + \text{Li}_3(1-v) - \text{Li}_3(u) + \frac{1}{2} \log^2(v) \log \left(\frac{1-u}{w}\right) - \zeta_2 \log \left(\frac{uv}{w}\right).
\]

Very interestingly, the corresponding \(N = 4\) SYM result is given by

\[
\Omega_{\mathcal{O}_1;3}^{(2),N=4} = \left(1 + \frac{u}{w}\right) T_3 + \text{perms}(u,v,w).
\]

The function \(T_3\) was also given as the building block of the corresponding \(N = 4\) result \[30\] and also appeared in the form factor in the SU(2) sector in \(N = 4\) \[73\].
Weight 1 and 0: The weight-1 part is given by:

\[
\Omega_{O^{(2)}}^{(2)} = \left( \frac{119}{18} + \frac{v}{w} + \frac{u^2}{2uw} \right) \log(u) \\
+ \left( \frac{119}{18} - \frac{1}{3uw} \right) \log(-q^2) + \text{perms}(u,v,w),
\]

where the terms with coefficients that are rational functions of \(\{u,v,w\}\) are identical to the \(\mathcal{N} = 4\) SYM result. Finally, the weight-0 part is given by:

\[
\Omega_{O^{(2)}}^{(2)} = \frac{487}{72} \frac{1}{uvw} - \frac{14075}{216},
\]

in which we note that the coefficient of \(\frac{1}{uvw}\) equals the finite rational number in (18).

Summary and discussion.—In this paper, we compute for the first time the two-loop Higgs amplitudes with dimension-7 operators in the Higgs EFT. This provides the virtual amplitudes for the top mass corrections for Higgs plus a jet production at NLO order. Our computation is based on an efficient application of the modern on-shell unitarity method coupled with IBP reduction.

The final results are given in complete analytic form which reveal direct connections between QCD and \(\mathcal{N} = 4\) SYM. The maximally transcendental part of the form factor of \(\text{Tr}(G^3)\) turns out to be equivalent to that of \(\mathcal{N} = 4\) SYM, as was argued in [30] (see also [77] for the study of \(\mathcal{N} < 4\) supersymmetric theories). More intriguingly, we find the parts of lower transcendentalities are also similar to the \(\mathcal{N} = 4\) SYM blocks. In particular, except the transcendental weight zero terms, all terms having coefficients of rational functions of \(\{u,v,w\}\) are identical between two theories. Furthermore, because of the linear relation [8], this implies that the maximal transcendentalty principle applies also to the \(\text{Tr}(DGDG)\) operator. The simplicity of the result and the connection to \(\mathcal{N} = 4\) SYM indicate there may be alternative path to understand or derive the results in a more direct way, such as using bootstrap (see e.g. [78, 80]).

An obvious next step is to compute the full QCD results including massless fundamental quarks in the internal or as external on-shell states. These results as well as computational details will be presented in [81].

Given the effectiveness of the method, a further interesting goal is to consider dimension-9 operators and higher order terms in the EFT using on the method of this paper. It is reasonable to expect similar simplicity will show up there. Analytic results in certain limit often provide crucial constraints which may allow to reconstruct full results. Given the top mass corrections to certain order in EFT, a deeper understanding may be possible for the finite top mass dependence at NLO order.

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