Non-abelian vector backgrounds with restored Lorentz invariance

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Abstract

The influence of vector backgrounds with restored Lorentz invariance on non-abelian gauge field theories is studied. Lorentz invariance is ensured by taking the average over a Lorentz invariant ensemble of background vectors. Like in the abelian case [1], the propagation of fermions is suppressed over long distances. Contrary to the fermionic sector, pure gauge configurations of the background suppress the long-distance propagation of the bosons only partially, i.e. not beyond the leading contribution for a large number of colours.

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I. INTRODUCTION

Apart from appearances and applications in many other fields of physics, vector backgrounds can be used in order to incorporate mass dimension two vector condensates into gauge field theories in a Lorentz invariant way [1, 2, 3] or, in a different context, in theories breaking Lorentz invariance explicitly [4]. The latter approach is motivated from string [5] and non-commutative field theory [6]. Of late, various aspects of mass dimension two condensates have attracted attention (see e.g. [7]). In order to incorporate the condensates’ influence, the gauge field is shifted by a background vector Φ. In general it is taken to be constant [1, 2, 3], thereby corresponding to the long wave-length limit of a more general approach. A homogeneous background alone does not break translational invariance. In case translational invariance should be broken one would have to average over all translations in order to restore it. Be this as it may, in any single background Lorentz invariance is violated. It has to be reinstated by taking the mean over a Lorentz invariant ensemble of background vectors. A Lorentz invariant ensemble is a set of vectors which is mapped onto itself under any Lorentz transformation, while, of course, almost every single element changes. In [1, 2, 3] it has been demonstrated that the elementary fermions and/or bosons are removed from the spectrum of asymptotically freely propagating particles. Usually the ensembles of backgrounds used in the literature [2, 3] contain gauge field configurations leading to zero as well as to non-zero field tensors. However, the work presented in [1] shows that the propagation of fermions is already stopped in pure gauge configurations of the background in Euclidean as well as in Minkowski space. The present article goes beyond the abelian limit and investigates the influence of non-abelian pure gauge backgrounds on the propagation of quarks and gluons. Here also the restoration of gauge invariance is an issue.

Section III discusses the Lorentz invariant weight functions characterising the ensembles of background vectors for non-abelian gauge groups and the preservation of gauge invariance. Section IIIA investigates the influence of the backgrounds on quarks and gluons for a SU(3) gauge group. The main ingredients of the theory, i.e. the generating functional for the Green functions (section IIIA) and the two-point functions (section IIIB) are treated. Section IV summarises the results.

II. WEIGHT CLASSIFICATION

At the beginning let us remember some facts about the abelian case [1] also needed for the non-abelian theory. The Lorentz invariant ensemble of backgrounds is characterised by a weight function $W(\Phi)$ which appears in the averaging prescription $(\langle O \rangle_W := \int d^4\Phi)$$:

$$\langle O \rangle_W = \int_\Phi W(\Phi)O,$$

which does not change under Lorentz transformations and is normalised in such a way that:

$$\int_\Phi W(\Phi) = 1.$$

Except for when $\Phi = 0$, which leads to the unmodified theory, all other Lorentz invariant quantities depending on the vector $\Phi$ must be functions of $\Phi^2$. Thus every allowed weight function $W$ can be cast into the form:

$$W(\Phi) = c\delta^{(4)}(\Phi^2) + w(\Phi^2),$$

where $w$ is a normalisable function of $\Phi^2$.

In euclidean space $\Phi^2 = 0$ also means $\Phi = 0$, whereby that case could be encoded within $w_E(\Phi^2)$, where the subscript $E$ marks the euclidean space. However, here, the possible contribution from $\Phi = 0$, from the unmodified theory is to remain marked clearly and the delta term is kept explicitly. Thereby the normalisation condition (2) becomes:

$$\pi^2 \int_0^\infty v \, dv \, w_E(v) = 1 - c,$$
with \( v := \Phi^2 \). The Lorentz invariant weight functions can be decomposed into elementary delta weights

\[
w^v_\lambda(\Phi^2) := (4\pi\lambda)^{-1}\delta(\Phi^2 - \lambda), \tag{5}\]

given by the sum of three terms \([1]\):

\[
w^v(\Phi^2) = \int 4\pi\lambda \, d\lambda \, w^v_\lambda(\Phi^2) \, w_E(\lambda). \tag{6}\]

In Minkowski space the time ordered formalism is to be pursued the basis should be chosen differently. From

\[
2\pi i \delta(\Phi^2 - \lambda) = S^-_\lambda(\Phi) - S^+_\lambda(\Phi), \tag{7}\]

with the time ordered (+) and anti time-ordered (−) scalar propagators

\[
S^\pm_\lambda(\Phi) = (\Phi^2 - \lambda \pm i\epsilon)^{-1}, \tag{8}\]

follows that the elementary weight function of choice is constructed from the scalar Feynman propagator \( S^+_\lambda(\Phi) \).

Yet in Minkowski space the hyperboloid pair defined by \( \Phi^2 = \text{const} \) has infinite content. Thus the minimal normalisable weight is given by the sum of three terms \([1]\):

\[
w_M(\Phi^2) = \sum_{j=1}^3 a_j S^+_\lambda_j(\Phi), \tag{9}\]

satisfying: \( \sum_{j=1}^3 a_j = 0 \), \( \sum_{j=1}^3 a_j \lambda_j = 0 \), and the normalisation condition: \( (4\pi^2/4) \sum_{j=1}^3 a_j \lambda_j \ln \lambda_j = 1 - c \).

The subscript \( M \) stands for "Minkowski". On top of that, the case \( \Phi = 0 \) cannot be described by a function \( w_M = w_M(\Phi^2) \) and must be added in form of a delta term where required.

1. **Non-abelian**

In the present article, already at this point the background vector is identified with a component of the gauge field and the approach is generalised to pure gauge configurations in non-abelian gauge groups. In the non-abelian case the gauge field carries colour. Nevertheless, the different Lorentz components of a homogeneous pure gauge background commute. The background is to transform homogeneously under gauge transformations \( U \): \( \Phi^a \to U^\dagger \Phi^a U \). Therefore one can decompose the background (for the fundamental representation) according to:

\[
\Phi_\mu = \sum_{a=1}^{N_c} U^\dagger \Theta^a U \Phi^a_\mu, \tag{10}\]

where the \( \Theta^a \) are \( N_c \) projectors. Only summations on colour indices are carried out, which are marked explicitly. The projectors satisfy \( \Theta^a \Theta^b = \delta^{ab} \Theta^a \), where \( \delta^{ab} \) is the Kronecker symbol. Thus a function of the background vector \( \Phi \) can be deconstructed in the basis of the projectors \( \Theta^a \):

\[
f(\Phi) = \sum_{a=1}^{N_c} U^\dagger \Theta^a U f(\Phi^a). \tag{11}\]

The gauge dependence resides entirely in the transformations \( U \) of the projectors \( \Theta^a \). The \( \Phi^a \) being gauge invariant quantities one can study each addend separately. Averaging each of them over its proper weight yields:

\[
\langle f(\Phi) \rangle_{(\Lambda^a)} = \sum_{a=1}^{N_c} \Theta^a_U \int_{\Phi^a} W_{\Lambda^a}(\Phi^a) f(\Phi^a), \tag{12}\]

where \( \Theta^a_U := U^\dagger \Theta^a U \). This is the same as introducing the product weight function:

\[
W_{(\Lambda^a)}(\Phi) := \prod_{a=1}^{N_c} W_{\Lambda^a}(\Phi^a) \tag{13}\]

and integrating over the \((\mathbb{R}^+)^{N_c}\), because every single weight \( W_{\Lambda^a}(\Phi^a) \) is already normalised.

In situations where the gauge invariance of the resulting quantity is required, it can be ensured by averaging over all gauge transformations \( U \) of the background \( \Phi \):

\[
u(U^\dagger \Theta^a U) := \int [dU] U^\dagger \Theta^a U = \nu N_c^{-1}, \tag{14}\]

where \( \nu \) stands for the volume of the gauge group. The result is proportional to unity in the corresponding space, because all non-singlet contributions drop out. That result finally leads to:

\[
\langle \langle f(\Phi) \rangle_{(\Lambda^a)} \rangle_U = N_c^{-1} \sum_{a=1}^{N_c} \int_{\Phi^a} W_{\Lambda^a}(\Phi^a) f(\Phi^a). \tag{15}\]

If all \( \Lambda^a \) are the same the abelian case is recovered. Gauge invariance can already be achieved by taking the average with the weight \( W \) if it, in itself, is gauge invariant. In the event where \( W \) is gauge invariant calculating the mean over the gauge group does not change the result anymore.

For the fundamental representation of a \( U(N_c) \) gauge group all ingredients for the averaging procedure have been presented above. In the case of a \( SU(N_c) \) gauge group an additional constraint arises from the tracelessness of the generators:

\[
\sum_{a=1}^{N_c} \Phi^a = 0, \tag{16}\]

Its incorporation leads to a coupling of the channels and thereby to a modification of Eq. (12) which can be expressed by the weight function:

\[
W_{SU}^{\Lambda^a}(\Phi) := \delta^{(4)} \left( \sum_{a=1}^{N_c} \Phi^a \right) \times \prod_{a=1}^{N_c-1} W_{\Lambda^a}(\Phi^a). \tag{16}\]
2. Adjoint representation

The background for bosonic correlators transforms under the adjoint representation of the gauge group. The adjoint representation of the SU(Nc) can be embedded in the fundamental of the SU(Nc^2 - 1) or the U(Nc^2 - 1). At the end the relevant result is extracted by imposing additional constraints. In general, members of the adjoint representation of SU(Nc) are hermitian and antisymmetric. Therefore the eigenvalues either vanish or come in pairs with opposite sign [10].

For the adjoint representation of SU(2) embedded in U(3) this means that the combination of eigenvalues \( \phi^3 = -\phi^2 \) and \( \phi^5 = 0 \) always exists simultaneously. For the adjoint representation of SU(3) embedded in U(8) one has \( \phi^1 = -\phi^2, \phi^3 = -\phi^4, \phi^5 = -\phi^6 \), and \( \phi^7 = 0 = \phi^8 \) [10]. This induces the following form for the average:

\[
\langle f(\phi) \rangle^{SU(3)}_{\{\lambda^a\}} = \sum_{\alpha \in \mathcal{M}} \int_{\phi^a} W_\lambda^\ast (\phi^a)(\theta_U^\ast + \theta_U^{\ast +1}) f(\phi^a) + (\theta_U^\ast + \theta_U^{\ast +1}) f(0),
\]

with \( \theta^a \) the U(Nc^2 - 1) projectors, \( \mathcal{M} := \{ 1; 3; 5 \} \), and where use has been made of the fact that \( W \) is an even function of \( \phi^a \). In the case of the adjoint representation taking the average over the gauge group yields a prefactor \( (N_c^2 - 1)^{-1} \) instead of \( N_c^{-1} \).

III. QCD WITH THE BACKGROUND

A. Generating functional

The generating functional for the time-ordered Green functions of QCD reads:

\[
Z = Z_{int} Z_A Z_X Z_\psi
\]

with the interaction part \( Z_{int} \), whose non-fermionic part depends on the chosen gauge and will not be specified here. The other (free) factors are the bosonic

\[
Z_A = \exp\left\{ i \frac{1}{2} \int_{x,y} J(x) \cdot \Gamma_0(x-y) \cdot J(y) \right\},
\]

the ghost

\[
Z_X = \exp\left\{ -i \int_{x,y} \xi^\dagger(x) \Gamma_0(x-y) \xi(y) \right\},
\]

and the fermionic part

\[
Z_\psi = \exp\left\{ -i \int_{x,y} \bar{\eta}(x) \Gamma_0(x-y) \eta(y) \right\},
\]

with the free boson propagator \( \Gamma_0^{\mu\nu}(x-y) \), the free ghost propagator \( \Gamma_0(x-y) \), the free fermion propagator \( G_0(x-y) \), as well as the currents \( J, \xi, \xi^\dagger, \eta, \) and \( \bar{\eta} \).

The background is included in the theory by translating the gauge field \( A \) by the background \( \Phi - A_\mu \rightarrow A_\mu + \Phi_\mu \) —and afterwards taking the mean over the ensemble of these vectors and restoring gauge invariance with respect to gauge transformations of the background. Doing so leads to the modified generating functional:

\[
Z = \langle \langle Z_{int}^\Phi Z_A^\Phi Z_X^\Phi Z_\psi^\Phi \rangle_W \rangle_U,
\]

with the different parts evaluated in a single background \( \Phi \). The non-interacting factors are obtained by replacing the free two-point functions by those in a single background \( \Phi \). They shall be defined in the next section. \( Z_{int}^\Phi \) contains all terms of third and fourth order in the dynamic fields. Its \( \Phi \) dependence originates from the four-gluon vertex with three dynamical gluon legs and one coupling to the background. The propagators studied in the following section are obtained by taking the functional derivatives of the above generating functional with respect to the corresponding currents.

In Eq. (22) a common mean over the sectors of the generating functional transforming under the fundamental and the adjoint representation of the gauge group, respectively, is taken. For this reason a connection has to be made between the averages calculated with the weights [11] and Eq. (17), respectively. Especially this fact is important if correlators are to be calculated that involve bosonic and fermionic fields simultaneously. In SU(3) one has the connection [11]:

\[
2(\phi^1_\mu)^2 = \Phi^2_\mu[1 - \cos(\theta_\mu)],
\]

\[
2(\phi^3_\mu)^2 = \Phi^2_\mu[1 + \cos(\theta_\mu - \pi/3)],
\]

\[
2(\phi^5_\mu)^2 = \Phi^2_\mu[1 + \cos(\theta_\mu + \pi/3)],
\]

(23)

with

\[
1 + \cos^3(\theta_\mu) = 18(\Phi^2_{\mu})^{-3} \prod_{a=1}^{3} (\Phi^a_{\mu})^2.
\]

This relationship holds separately for each Lorentz component, which is why one does not sum over the index \( \mu \) anywhere in the previous equations. These relations allow to construct the corresponding weight for one representation from the weight function for the other.

B. Two-point functions

A principal ingredient of the modified generating functional and thereby of the modified theory are the two-point correlators in the background. They also serve to establish the link to the results presented in [1].

With the usually made assumption that all other condensates be absent [2,3] the fermionic propagator in the background obeys the equation of motion

\[
[i \not\partial(x) + \Phi - m]G_\Phi(x-y) = \delta^{(4)}(x-y).
\]

(25)
Its solution in a pure gauge background is given by:
\[ G_\Phi(z) = e^{i\Phi\cdot z}G_0(z), \]
where \( G_0(z) \) is the solution of the free equation.

The bosonic and ghost propagators depend on the chosen gauge. The gauge-fixing contribution to the lagrangian density in background-field Feynman gauge is given by (this time summing over all colour indices):
\[ \mathcal{L}_{GF} = -\frac{1}{2}\left\{ \left[ \partial_\mu - i\Phi_\mu \right] a_{\nu} \partial^a A^\nu \right\} \{ \left[ \partial_\nu - i\Phi_\nu \right] a^c A^{\mu \nu} \}, \]
which is Lorentz covariant. Without further spontaneous symmetry breaking, i.e. with \( \langle A \rangle = 0 \), the equation of motion for the gluon propagator reads \[ \delta^{(4)}(x-y)g^{\mu\nu}(28) \]
where matrix multiplication of the colour matrices is understood implicitly and the \( \tilde{\cdot} \) indicates that the adjoint representation has to be used. In the same gauge the ghost propagator obeys the equation of motion \[ \tilde{\delta}^{(4)}(x-y). \]

Therefore one has in background field Feynman gauge:
\[ \Gamma_\Phi^{\mu\nu}(x-y) = g^{\mu\nu}\Gamma_\Phi(x-y) \]
and it is sufficient to study one of the two propagators, e.g. the ghost propagator \( \Gamma(x-y) \). In a pure gauge background one has then:
\[ \Gamma_\Phi(z) = e^{i\Phi\cdot z}G_0(z), \]
with \( G_0(z) \) the solution of the background-free equation.

As had already been seen in the abelian case \[ \tilde{\Phi} \] the Fourier phases of the propagators in the background \( \Phi \) lead to the averaging procedure being identical to a Fourier transformation of the weight function. For this reason, in coordinate space the n-point functions of the modified theory are the n-point functions of the theory without background multiplied by a n-point function which is the Fourier transformed weight. The latter is a genuine n-point function. That means that even if in the original theory the higher correlators factorise into lower ones, they will not in the modified theory. Thereby especially if the original theory should display gaussianity on a certain level, the modified theory will not.

There are other situations where such field configurations play a rôlé. The zero components of aforesaid Fourier phases resemble chemical potentials \[ \tilde{\Omega} \]. The spatial components are similar to what one encounters for twisted boundary conditions on compact spaces \[ \tilde{\Omega} \]. In a colour superconductor a field configuration corresponding to a zero field tensor serves to restore its colour neutrality \[ \tilde{\Omega} \].

The theory in the pure gauge background can be seen as one with a modified vacuum structure without background energy density. One could express this by writing:
\[ \langle 0|T\psi^n\psi^n A^\nu|0 \rangle = \langle 0|T\psi^n\psi^n A^\nu|\Omega \rangle, \]
where \( |\Omega \rangle \) represents the non-trivial vacuum. The vacuum expectation values of the modified theory \( \langle 0|O|\Omega \rangle \) are the averaged vacuum expectation values \( \langle 0|O|\Omega \rangle_W \) each in a single background.

1. Euclidean space

Computing the average of the fermionic propagator \[ 20 \] according to Eq. \[ 15 \] \( U_N(\tilde{\Lambda}) \) with the elementary weights \[ \tilde{\Omega} \], choosing \( c = 0 \), and averaging over the gauge group leads to:
\[ \langle \langle G_\Phi(z) \rangle \rangle_{U_N(\tilde{\Lambda})} \]
In a \( SU(N_c) \) gauge group the result reads [see Eq. \[ 16 \]]:
\[ \langle \langle G_\Phi(z) \rangle \rangle_{U_N(\tilde{\Lambda})} = N_c^{-1} \sum_{a=1}^{N_c} \sin \sqrt{\alpha^2 z^2} G_0(z) + N_c^{-1} \prod_{a=1}^{N_c-1} \sin \sqrt{\alpha^2 z^2} \]

For large distances \( \sqrt{z^2} \), the propagator is damped with respect to the free one. At short distances \( \sqrt{z^2} \) the free propagator is recovered. In momentum space this manifests itself in the on-shell pole being removed and a pole proportional to \( 1/\sqrt{\mathcal{L}} \) being introduced \[ \tilde{\Phi} \]. The elementary fermions are no longer part of the spectrum of freely propagating particles.

This result resembles the one for the tree-level propagator \[ 2 \, 3 \]. However, there, as opposed to here configurations of the gauge field were taken into account that lead to a non-vanishing field tensor. The weight used there \( W_{\text{HP}}(\Phi) \sim \exp(-\Phi^2/\mathcal{L}^2) \) but constrained to zero field tensors in coordinate space gives \( U_N(\tilde{\Lambda}) \):
\[ \langle \langle G_\Phi(z) \rangle \rangle_{W_{\text{HP}}} = \exp[-z^2 \mathcal{L}^2/4] G_0(z), \]

or in a \( SU(N_c) \) gauge group:
\[ \langle \langle G_\Phi(z) \rangle \rangle_{W_{\text{HP}}} = \left\{ (1 - N_c^{-1}) \exp\left[-z^2 \mathcal{L}^2/4\right] + N_c^{-1} \exp\left[-z^2 \mathcal{L}^2(N_c - 1)/4\right] \right\} G_0(z), \]

which for a large number of colours \( N_c \) reduces to the \( U_N(\tilde{\Lambda}) \) result. Again the propagation over large distances \( \sqrt{z^2} \) is suppressed while the free propagator is obtained for \( \sqrt{z^2} \to 0 \).

As Eq. \[ 20 \] is satisfied by every fermion propagator which is a singlet of the colour group, the previous result holds for those correlators, too. If a fermionic two-point function \( g(x-y) \) should not be colour neutral, in general, the projectors \( \Theta^a \) do not commute with the correlator. This leads to a breaking of translational invariance:
\[ g_\Phi(x-y) = e^{i\Phi\cdot x} g(x-y) e^{-i\Phi\cdot y} = \sum_{a,b=1}^{N_c} e^{i(\Phi^a\cdot x - \Phi^b\cdot y)} \Theta^a g(x-y) \Theta^b \]
As mentioned above it usually would have to be restored by
averaging over all translations. However here it is
already required to take the average over the gauge group
anyhow. This is equivalent—up to a factor—to taking
the trace over the colour matrices:

\[ N_{c}(\Theta^{a}g(z)\Theta^{b})_{U} = \text{tr}\{\Theta^{a}g(z)\Theta^{b}\} = \text{tr}\{\Theta^{a}g(z)\}\delta^{ab} \]

which is non-zero only for \( a = b \) due to the projection
properties of the \( \Theta^{a} \). Therefore translational invariance
is restored. Enforcing Lorentz invariance, for example,
in the case of a \( U(N_{c}) \) gauge group leads to Eq. (33)
but where the free propagators \( G_{0}(z) \) are replaced by
\( \text{tr}\{\Theta^{a}g(z)\} \). That means that also here the propagation
is stopped at large distances and almost free at short
distances.

In background field Feynman gauge for the present in-
vestigation it is sufficient to study either the gluon or
the ghost propagator. Let us take the latter. Calculat-
ing only partially. In momentum space the suppressed
channels behave like the fermionic propagators in the
adjoint representation equals \( N_{c} - 1 \) and the multiplicity of zero
eigenvalues is \( N_{c} - 1 \) they are of subleading importance
for large values of \( N_{c} \) and go unnoticed in an analysis
based on the leading order of an expansion in \( N_{c}^{-1} \).

2. Minkowski space

Taking the average of the fermionic propagator according to
Eq. (16) [\( U(N_{c}) \)] with the elementary weights \( \Theta \), choosing \( c = 0 \), and averaging over the gauge group
yields:

\[ \langle G_{\phi}(z) \rangle_{M,SU}^{A_{3}}(z) U = N_{c}^{-1} \sum_{a=1}^{N_{c}} \sum_{j=1}^{3} a^{a}_{j}s^{+}_{j}(z)G_{0}(z), \tag{41} \]

with \( s^{+}_{j}(z) := 4\pi^{2}\sqrt{N_{c}}(\sqrt{\lambda_{j}z^{2}} + \lambda_{j})/\sqrt{z^{2} + \lambda_{j}} \). In a \( SU(N_{c}) \) gauge group the result becomes [see Eq. (16)]:

\[ \langle G_{\phi}(z) \rangle_{M,SU}^{A_{3}}(z) U = N_{c}^{-1} \sum_{a=1}^{N_{c}-1} \sum_{j=1}^{3} a^{a}_{j}s^{+}_{j}(z)G_{0}(z) + \]

\[ +N_{c}^{-1} \prod_{a=1}^{N_{c}-1} \sum_{j=1}^{3} a^{a}_{j}s^{+}_{j}(z) G_{0}(z). \tag{42} \]

For \( z^{2} = 0 \) the previous expressions reduce to the free
propagator. For large absolute values of \( z^{2} \) the prop-
agator is suppressed at least proportionally to \( |z^{2}|^{-3/4} \)
with respect to the free propagator. Therefore the
fermions cannot propagate over arbitrarily large
distances.

Like in the abelian case all terms for the \( U(N_{c}) \) gauge
group correspond to contributions of scalars to the self
energy of the fermion without external legs, thereby indi-
cating that no freely propagating particles are described.
For the \( SU(N_{c}) \) gauge group the first terms have the
same interpretation. Only the last one corresponds to a
sum over multiple \( (N_{c} - 1) \)-loop contributions of scalars
to the self energy of the fermion once more without ex-
ternal legs. Thus what was said concerning the absence
of freely propagating particles remains valid.

Computing the average of the ghost propagator according to
Eq. (17) with the elementary weights \( \Theta \), choosing \( c = 0 \), and averaging over the gauge group
yields:

\[ \langle G_{\phi}(z) \rangle_{M,SU}^{A_{3}}(z) U = \frac{N_{c}}{4} \sum_{a=1}^{N_{c}} \sum_{j=1}^{3} a^{a}_{j}s^{+}_{j}(z) \tag{43} \]

For small absolute values of \( z^{2} \) the free propagator is re-
obtained. At large absolute values of \( z^{2} \) two \( (N_{c} - 1) \) of
the eight \((N_c^2 - 1)\) channels remain unaltered and correspond to freely propagating gluons.

The terms for which the propagation over long distances is suppressed again do resemble the contribution of scalars to the self energy of the considered particle—whence now is the gluon/ghost—without external legs.

The observations for ghosts and gluons remain the same in all background field Lorentz gauges. For those the ghost propagator \((\Gamma_0^g)\) stays the same. In the gluon propagator only the free part \(\Gamma_0^g(z)\) changes, which leaves the envelope function \(\tilde{W}(z)\) unchanged.

Also in Minkowski space the above findings are valid for every propagator that is a colour singlet. For non-singlet propagators, translational invariance is recovered after the average over the gauge group has been taken. The consequences remain the same as in euclidean space.

IV. SUMMARY

Non-abelian gauge field theories have been studied whose correlators are defined with an additional average over a Lorentz invariant ensemble of homogeneous vector backgrounds. The ensembles are constrained to pure scalars to the self energy of the particle without external legs.

The feature that the propagation over short distances remains essentially unchanged while it is modified over long distances is shared by non-commutative field theories preserving Lorentz invariance \(^\text{13}\). The gauge fields as

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