Analogs of time-dependent quantum phenomena in optical fibers

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Abstract. Review of the paraxial approximation to the problem of light beams propagating in optical fibers is presented. The analogy of time-dependent integrals of motion for the quantum system and space-dependent invariants for the radiation in inhomogeneous optical waveguides is elucidated. A specific tomographic probability distribution associated with the light-pulse profile is considered analogously to tomographic probabilities in quantum mechanics. The new inequality is presented for the tomographic probability.

1. Introduction

The Schrödinger stationary equation with a time-independent Hamiltonian describes quantization of energy of physical systems and provides the possibility to obtain their energy levels explicitly. The Schrödinger evolution equation describes the propagation of wave packets for both time-independent and time-dependent Hamiltonians. For the case of time-dependent Hamiltonians, the energy levels of a quantum system do not exist. The energy for such nonstationary systems does not preserve. Nevertheless, for nonstationary systems there exist integrals of motion [1], which in the limit of stationary systems coincide with the energies.

It is remarkable that there exist purely classical phenomena which can be studied, in view of an analogy with quantum phenomena. For the problem of electromagnetic-wave propagation in media with inhomogeneous refractive index, it was shown by Fock and Leontovich [2, 3] that the problem can be solved using the paraxial approximation of the Helmholtz equation. In this approximation, the Helmholtz equation is reduced to the Schrödinger-like evolution equation. For such equation, the longitudinal coordinate plays the role of time, the refractive index plays the role of the potential energy, and the Planck’s constant is replaced the electromagnetic-radiation wavelength. Such a quantumlike behaviour of classical processes is available for many other phenomena [4, 5].

The aim of this work is to focus on the properties of quantumlike phenomena taking place in optical fibers [6–8]. We review the properties of space-dependent analogs of time-dependent quantum integrals of motion for electromagnetic radiation in optical fibers and the properties of quantumlike packets propagating along inhomogeneous fibers [9–12]. The approach based on the quantumlike behaviour of light radiation in the fibers (or other waveguides) was suggested to be applied for studying some aspects of quantum computing and quantum-information processing in [13–16].
In quantum mechanics, there exists the probability representation where the states are described by tomographic probability densities or tomograms. We consider here one kind of such tomograms called Fresnel tomograms for classical light beams in optical fibers.

The paper is organized as follows. In section 2 we review the Fock–Leontovich approximation. In section 3 we study modes in inhomogeneous waveguides using the space analogs of time-dependent quantum integrals of motion for quantum systems with nonstationary Hamiltonians. In section 4 we consider the Fresnel tomogram for light beams in optical fibers. In section 5 we obtain the integral inequality for tomograms. The perspectives and conclusions are presented in section 6.

2. Fock–Leontovich approximation for paraxial beams of electromagnetic radiation

First we study light propagation in a planar waveguide. This problem can be considered in the Fock–Leontovich approximation and it is an analog of a quantum system with one degree of freedom. Given the Helmholz equation for the electric-field component of laser beam in a planar waveguide, which is obtained for a fixed frequency neglecting the media dispersion and influence of the polarization

\[ \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial z^2} + k^2 n^2(x, z) E = 0. \]  

In Eq. (1), \( \lambda = \frac{2\pi}{k} \) is the wavelength in vacuum and \( z \) is the longitudinal coordinate.

Let us introduce the complex function \( \psi(x, z) \), which is slowly varying amplitude of the electric field in the electromagnetic wave:

\[ E(x, z) = n_0^{-1/2}(z) \psi(x, z) \exp \left[ ik \int_0^z n_0(\xi) \, d\xi \right]. \]

Inserting (2) into (1) and neglecting the term with second derivative in \( z \) and cross-terms containing the first derivative of the refractive index on the axis, we obtain the Schrödinger-like equation from the Helmholz equation

\[ i\lambda \frac{\partial \psi(x, z)}{\partial z} = -\frac{1}{2n_0(z)} \frac{\partial^2 \psi(x, z)}{\partial x^2} + U(x, z) \psi(x, z), \]

with \( U(x, z) \) being an effective potential related to the refractive index of medium \( n(x, z) \)

\[ U(x, z) = \frac{1}{2n_0(z)} \left[ n_0^2(z) - n^2(x, z) \right], \]

where \( n_0(z) = n(0, z) \) is the medium refractive index at the beam axis. The condition

\[ \frac{\lambda}{n_0^2(z)} \left| \frac{dn_0(z)}{dz} \right| \ll 1 \]

is supposed to be fulfilled, which means that the refractive index has very small changes in the direction of \( z \)-axis on the distance equal to the wavelength \( \lambda \). Thus Eq. (3) is an analog of the Schrödinger equation for the quantum system with one degree of freedom and with time-dependent Hamiltonian.

The light beam propagating in the optical fiber is described by the equation

\[ i\lambda \frac{\partial \psi(x, y, z)}{\partial z} = -\frac{\lambda^2}{2n_0(z)} \left( \frac{\partial^2 \psi(x, y, z)}{\partial x^2} + \frac{\partial^2 \psi(x, y, z)}{\partial y^2} \right) \]

\[ + \frac{1}{2n_0(z)} \left[ n_0^2(z) - n^2(x, y, z) \right] \psi(x, y, z), \]
which is a Schrödinger-like equation for a wave-like function of a system with two degrees of freedom (coordinates $x$ and $y$). The longitudinal coordinate $z$ along the fiber axis plays the role of time in the Schrödinger equation. The $z$-evolution of the electromagnetic field is described by the evolution operator $\hat{U}(z)$ as follows:

$$\hat{U}(z)\psi(x, y, 0) = \psi(x, y, z),$$

which is defined by the Hamiltonian

$$\hat{H}(z) = \left(\frac{\hat{p}_x^2}{2} + \frac{\hat{p}_y^2}{2}\right) \frac{1}{n_0(z)} + U(x, y, z). \quad (5)$$

Here, the momentum-like operators read

$$\hat{p}_x = -i\lambda \frac{\partial}{\partial x}, \quad \hat{p}_y = -i\lambda \frac{\partial}{\partial y},$$

and the potential-like function has the form

$$U(x, y, z) = \frac{1}{2n_0(z)} \left[n_0^2(z) - n^2(x, y, z)\right].$$

The quadrature components $\hat{x}_0$, $\hat{y}_0$, $\hat{p}_x$, and $\hat{p}_y$ in the problem of light-beam propagating along the fiber have the following physical meaning. The light rays corresponding to the “classical mechanics” limit of the quantumlike picture of the light propagation have the characteristics — position coordinates $x$ and $y$ in the plane perpendicular to the fiber axis and direction of the ray. The ray direction is described by two angles $p_x$ and $p_y$ between the ray and the axes $x$ and $y$. Due to the quantumlike nature of these characteristics which reflects the phenomenon of light diffraction, the position of the ray and its direction cannot be determined simultaneously, and this phenomenon is an analog of the quantum uncertainty relations.

3. Modes in optical fibers

The electromagnetic radiation propagates along the optical-fiber axis (coordinate $z$) according to Eq. (4). In the case where the refractive-index profile has the form of inverse well [that provides the well form of potential $U(x, y, z)$ in Eq. (5)], the optical fiber traps the discrete modes described by a solution of Eq. (4) $\psi_{n_1n_2}(x, y, z)$ satisfying the normalization condition

$$\int |\psi_{n_1n_2}(x, y, z)|^2 \, dx \, dy = 1, \quad (6)$$

where $n_1$ and $n_2$ are some integers; we use dimensionless units.

The unitary evolution operator in the position (or quadrature) representation is the Green function of the Schrödinger-like equation (4)

$$G(x, y, z, x', y', z_0) = \langle x, y | \hat{U}(z, z_0) | x', y'\rangle. \quad (7)$$

In the optical fiber, there exist four $z$-dependent integrals of motion, which are the initial beam position-like quadratures

$$\hat{x}_0(z) = \hat{U}(z, z_0)\hat{x}\hat{U}^{-1}(z, z_0), \quad \hat{y}_0(z) = \hat{U}(z, z_0)\hat{y}\hat{U}^{-1}(z, z_0) \quad (8)$$
and the initial beam momentum-like quadratures

\[ \hat{p}_{x_0}(z) = \hat{U}(z, z_0) \hat{p}_x \hat{U}^{-1}(z, z_0), \quad \hat{p}_{y_0}(z) = \hat{U}(z, z_0) \hat{p}_y \hat{U}^{-1}(z, z_0). \]  

(9)

Since the unitary evolution preserves the commutation relations and hermiticity of quadrature operators, formulae (8) and (9) can be considered as canonical map of the initial quadratures onto quadratures at the distance \( z \) along the optical fiber. In the discrete basis, the evolution operator is described by the matrix

\[ U_{n_1, n_2, m_1, m_2}(z, z_0) = \langle n_1, n_2, z | \hat{U}(z, z_0) | m_1, m_2, z_0 \rangle. \]  

(10)

We define the modes \( | m_1, m_2, z_0 \rangle \) in Eq. (10) as eigenmodes of instant Hamiltonian \( \hat{H}(z_0) \) (5)

\[ \hat{H}(z_0) | m_1, m_2, z_0 \rangle = E_{m_1 m_2}(z_0) | m_1, m_2, z_0 \rangle \]  

(11)

and the modes \( | n_1, n_2, z \rangle \) as eigenmodes of the Hamiltonian \( \hat{H}(z) \)

\[ \hat{H}(z) | n_1, n_2, z \rangle = \tilde{E}_{n_1 n_2}(z) | n_1, n_2, z \rangle. \]  

(12)

In the case of instant change of the refractive index (the Born–Oppenheimer approximation), the energy of the initial mode \( | m_1, m_2, z_0 \rangle \) is distributed among the final modes with the distribution function given by the Frank–Condon factor

\[ P_{m_1 m_2}^{n_1 n_2} = \left| \int \psi^s_{n_1 n_2}(x, y, z) \psi_{m_1 m_2}(x, y, z_0) \, dx \, dy \right|^2. \]  

(13)

The probability distribution (13) for given initial mode \( | m_1, m_2, z_0 \rangle \) has a maximum as a function of \( n_1 \) and \( n_2 \) given by the Frank–Condon principle. It can be obviously formulated for the planar-waveguide picture [17]. The instant change of the refractive index at the “moment of time” \( z \) means in the quantum-like language that the parameters of the potential well instantly changed. Then it is clear that if, for energy level \( n \), a quantum particle was at the rest before instant change of the potential, it keeps to be at the rest. But the rest state in a new potential well corresponds to another level \( m \) in the new well with quantum numbers different from that it was in the initial well. Namely, the level \( m \) is maximally excited in the process described. The other levels are also excited but with smaller probability. The approach to calculate the Frank–Condon factors was developed for studying the polyatomic molecules (see, for example, [18, 19]).

In the case where the refractive index has the parabolic profile (the so-called selfoc), one has the following function for the quantumlike potential energy

\[ U(x, y, z) = a(z)x^2 + b(z)y^2 + d(z)xy + e(z)x + f(z)y + l(z), \]  

(14)

where the coefficients of the quadratic form can be \( z \)-dependent functions. In this case, the quantumlike problem can be solved in the explicit form. In fact, for selfoc the integrals of motion \( \hat{x}_0(z), \hat{y}_0(z), \hat{p}_x(z) \), and \( \hat{p}_y(z) \) can be found explicitly as the operator vector

\[ \hat{Q}_0(z) = \begin{pmatrix} \hat{p}_{x_0}(z) \\ \hat{p}_{y_0}(z) \\ \hat{x}_0(z) \\ \hat{y}_0(z) \end{pmatrix} = \Lambda(z) \begin{pmatrix} \hat{p}_x \\ \hat{p}_y \\ \hat{x} \\ \hat{y} \end{pmatrix} + \begin{pmatrix} \delta_1(z) \\ \delta_2(z) \\ \delta_3(z) \\ \delta_4(z) \end{pmatrix}, \]  

(15)

with the real symplectic matrix \( \Lambda(z) \) satisfying the equation of motion

\[ \ddot{\Lambda}(z) = -i B(z) \Sigma \Lambda(z); \quad \Lambda(z_0) = 1, \]  

(16)
where the $4 \times 4$-matrix $B(z)$ is determined by the parameters of the refractive index

$$
B(z) = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 2a(z) & d(z) \\
0 & 0 & d(z) & 2b(z)
\end{pmatrix}
$$

(17)

and the $4 \times 4$-matrix $\Sigma$ has the form

$$
\Sigma = \begin{pmatrix}
0 & 0 & -i & 0 \\
0 & 0 & 0 & -i \\
i & 0 & 0 & 0 \\
0 & i & 0 & 0
\end{pmatrix}.
$$

(18)

Now we introduce the vector $\vec{\sigma}(z)$, which has four components used in Eq. (15),

$$
\vec{\sigma}(z) = (\delta_1(z), \delta_2(z), \delta_3(z), \delta_4(z)).
$$

Then after introducing a 4-vector $C(z) = (0, 0, e(z), f(z))$, one can see that the 4-vector $\vec{\sigma}(z)$ satisfies the equation

$$
\dot{\vec{\sigma}}(z) = -i\Lambda(z)\Sigma C(z); \quad \vec{\sigma}(z_0) = 0.
$$

(19)

The presence of the vector term in Eq. (15) corresponds to linear in $x$ and $y$ terms in the refractive-index function. This reflects a curvature of the optical axis which can deviate from a straight line. These integrals of motion are analogs of time-dependent integrals of motion for two-dimensional parametric driven oscillator.

4. Fresnel tomogram of wave packets

For a quantum state with two degrees of freedom, one can construct the Fresnel tomogram which is the joint probability distribution of two positions $X_1$ and $X_2$ determining the quantum state. Here we follow the results of our study [20–26]. It is worthy noting that the analogous approach has been applied for determining the quantum states of helium atom in [27].

In view of the analogy with the quantum picture, we construct the Fresnel tomogram of the electromagnetic-field pulse propagating along an inhomogeneous waveguide. This tomogram reads

$$
w_F(X_1, X_2, \nu_1, \nu_2, z) = \frac{1}{(2\pi)^2|\nu_1||\nu_2|} \left| \psi(y_1, y_2, z) \exp \left[ \frac{i(X_1 - y_1)^2}{2\nu_1} + \frac{i(X_2 - y_2)^2}{2\nu_2} \right] dy_1 dy_2 \right|^2.
$$

(20)

The kernel in (20) is similar to the Green function of a free quantum particle. Parameters $\nu_1$ and $\nu_2$ are real and they are analogs of times for the free particle propagations.

For normalized pulse profile $\psi(y_1, y_2, z)$, the Fresnel tomogram can be considered as the probability density satisfying the normalization condition

$$
\int w_F(X_1, X_2, \nu_1, \nu_2, z) \, dX_1 dX_2 = 1.
$$

The Fresnel tomogram can be obtained from symplectic tomogram $w(X_1, \mu_1, \nu_1, X_2, \mu_2, \nu_2)$, which depends on positions $X_1$ and $X_2$, real parameters $\mu_1$, $\mu_2$ and $\nu_1$, $\nu_2$, and given, for example, in [28] as follows:

$$
w(X_1, \mu_1, \nu_1, X_2, \mu_2, \nu_2) = \langle \delta(X_1 - \mu_1\hat{q}_1 - \nu_1\hat{p}_1)\delta(X_2 - \mu_2\hat{q}_2 - \nu_2\hat{p}_2) \rangle
$$

(21)
by the replacement
\[ w_F(X_1, X_2, \nu_1, \nu_2, z) = w(X_1, 1, \nu_1, X_2, 1, \nu_2), \tag{22} \]
which means that
\[ w(X_1, \mu_1, \nu_1, X_2, \mu_2, \nu_2) = \frac{1}{|\mu_1|} \frac{1}{|\mu_2|} w_F \left( \frac{X_1}{\mu_1}, \frac{\nu_1}{\mu_1}, \frac{X_2}{\mu_2}, \frac{\nu_2}{\mu_2} \right). \tag{23} \]

For a quantumlike mean value, in the definition of symplectic tomogram in Eq. (21), we used the brackets notation \( \langle \hat{\kappa} \rangle \), where \( \hat{\kappa} \) is product of two delta-functions of operator arguments. Since symplectic tomogram provides the optical tomogram \([29, 30]\), which depends on two angles \( \theta_1 \) and \( \theta_2 \)
\[ w_{op}(X_1, \theta_1, X_2, \theta_2) = w(X_1, \cos \theta_1, \sin \theta_1, X_2, \cos \theta_2, \sin \theta_2), \]
the Fresnel tomogram can be expressed in terms of the optical tomogram as well
\[ w_{op}(X_1, \theta_1, X_2, \theta_2) = \frac{1}{\cos \theta_1 \cos \theta_2} w_F \left( \frac{X_1}{\cos \theta_1}, \frac{\sin \theta_1}{\cos \theta_1}, \frac{X_2}{\cos \theta_2}, \frac{\sin \theta_2}{\cos \theta_2} \right). \tag{24} \]
Thus, for an arbitrary profile of the light beam \( \psi(x, y, z) \) propagating along the optical fiber, one has the Fresnel tomogram along with the optical tomogram. These characteristics can be used in complete analogy with their use in quantum mechanics.

5. Inequalities for Fresnel tomogram

For the optical tomogram, the entropic uncertainty relation was found in \([31–35]\) and it looks as the following inequality:
\[
(q - 1) \ln \left\{ \int_{-\infty}^{\infty} dX_1 dX_2 \left| w(X_1, \theta_1, X_2, \theta_2) \right|^{1/q} \right\} + (q + 1) \ln \left\{ \int_{-\infty}^{\infty} dX_1 dX_2 \left| w(X_1, \theta_1 + \pi/2, X_2, \theta_2 + \pi/2) \right|^{1/q} \right\} \geq (q - 1) \ln \pi(1 - q) + (q + 1) \ln \pi(1 + q), \tag{25} \]
where \( 0 < q < 1 \).

Now we write the similar inequality for the Fresnel tomogram, i.e., in the above inequality we replace the optical tomogram, in view of expression (24), in terms of the Fresnel tomogram. One obtains
\[
(q - 1) \ln \left\{ \int_{-\infty}^{\infty} dX_1 dX_2 \left[ \frac{1}{\cos \theta_1 \cos \theta_2} w_F \left( \frac{X_1}{\cos \theta_1}, \frac{\tan \theta_1}{\cos \theta_1}, \frac{X_2}{\cos \theta_2}, \frac{\tan \theta_2}{\cos \theta_2} \right) \right]^{1/q} \right\} + (q + 1) \ln \left\{ \int_{-\infty}^{\infty} dX_1 dX_2 \left[ \frac{1}{\cos(\theta_1 + \pi/2) \cos(\theta_2 + \pi/2)} \right]^{1/q} \right\} \times w_F \left( \frac{X_1}{\cos(\theta_1 + \pi/2)}, \frac{\tan(\theta_1 + \pi/2)}{\cos(\theta_1 + \pi/2)}, \frac{X_2}{\cos(\theta_2 + \pi/2)}, \frac{\tan(\theta_2 + \pi/2)}{\cos(\theta_2 + \pi/2)} \right) \right\} \right\} \geq (q - 1) \ln \pi(1 - q) + (q + 1) \ln \pi(1 + q). \tag{26} \]
The inequality is valid for both different values of the angles and equal values \( \theta_1 = \theta_2 \) that corresponds to \( \nu_1 = \nu_2 \) in the Fresnel tomogram. The inequality can be used to study constraints onto the tomographic-probability profile of the electromagnetic-field radiation at the given optical fiber. In fiber optics, both inequalities (25) and (26) can be used to control the accuracy of measurements of the intensity and phase of light waves. If one measures the intensity only but for different distances, the phase of the wave can be also reconstructed. In fact, such measurement means that tomogram is measured. Fulfilling the entropic inequality (25) and (26) can be used as a method of controlling the accuracy of measuring the tomograms.
6. Conclusions

To conclude, we summarize the main results of our study.

We pointed out the analogy of the light-beam propagation in inhomogeneous optical fibers and the time evolution of a wave packet in quantum mechanics in the case of nonstationary Hamiltonian. The light beams in inhomogeneous optical fibers are analogs of nonstationary systems of quantum oscillators and they possess the space-dependent invariants, which are analogs of the time-dependent invariants of the systems of quantum oscillators.

We constructed the specific tomographic-probability distribution (Fresnel tomogram) for the light beam in inhomogeneous optical waveguide. The new inequality for the Fresnel tomogram was obtained and this inequality can be used to study the light-beam profile in optical waveguides.

It is worthy pointing out that the results obtained can be implemented for other quantumlike processes, for example, to study acoustic wave propagating in inhomogeneous media.

Acknowledgments

This study was supported by the Russian Foundation for Basic Research under Project No. 07-02-00598. The author thanks the Organizers of the 395th WE–Heraeus Seminar “Time-dependent Phenomena in Quantum Mechanics” (September 12-16, 2007, Blaubeuren, Germany) for invitation and the Wilhelm and Else Heraeus Foundation for support and kind hospitality.

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