The analysis research on the movement of machine Tools for processing free-form surface by ball cutter

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Abstract. In order to resolve the function design issue of the machine tools for processing free-form surface, the workpiece is divided into n equal parts; each one consists of a generatix and a guideline. Using the generative design principle, the mathematical description matrix of the ball cutter and the machined surface are established, and thus determine the cutter’s positional and gesture matrixes and the possible movement scheme for processing free-form surface. The method is used for processing free-form surface by different tools; the results provide a good theoretical basis for the optimal design for machine tools and the design of combined machine tools.

1. Introduction

With the development of science and technology, the products with free-form surface character emerge in endlessly, and have been extensively used in the field such as Air, Aerospace, Mould, more attention has been focused on the processing of free-form surface. The design of moving functions in machine tools is the first task for the design of machine tools, many domestic scholars have made great efforts in the generative design of moving functions in machine tools [1][2], but the study on the generative design of moving functions in machine tools for processing free-form surface is rare. The analytic method is adopted in generative design of moving functions in machine tools for machining free-form surface, the possible movement schemes of the machine tools are presented, and a solid foundation is laid in optimization design for the machine tool structure and the design of combined machine tools.

2. The Analysis Principles of the Motion Schemes of Machine Tools

The processed surface of workpiece is formed by the relative motion between the tool and the work piece, the relative motion is realized by the movements of machine tools, the analysis design principle raised in this study on motion schemes of the machine tools is as follows

(1) The information description of tool, workpiece and the determination of the tool positional and gesture matrixes. For ball cutter, the contact point between the tool and the workpiece generate the cutting surface through turning motion. Mathemetic description of workpiece machining surface is given, the positional and gesture matrixes are determined by means of the contact between the cutting surface of tool and machining surfaces, the interference condition.

(2) The determination of the machine tools motion generation and its movement function. According to the positional and gesture matrixes of the cutter, the motions that generate the generatrix and the lead
have been carried out, then the motions are synthesized, and the impossible motion schemes of machine tools is resolved. Each of those has a generating motion expression, which is to be evaluated and chose to use when designing the solution of machine tool structure, and is provided a good theoretical basis for the optimal design combined machine tool.

3. The Information Descriptions of Cutting Tool and the Workpiece

As shown in Figure 1, the ball cutter is machining free-form surface.

![Figure 1. The sketch for processing free-form surface](image1)

Suppose the free-form surface is divided into n equal amounts, each one corresponds to a machining curve. The height of each amount is $\Delta h$, the ball cutter processes the curve (a height $h$) after moving up $\Delta h$, the cutter processes the curve (a height $h+\Delta h$), the curve is the generatrix, and the guideline is a $\Delta h$-length straight line in the direction of $h$. The smaller $\Delta h$ is, the higher the accuracy machining free-form surface is.

![Figure 2. The schematic diagram for processing processes the curve (a height h)](image2)

As shown in Figure 2, Ow is workpiece coordinate system, Op is the coordinates of the tool cutting surface, Oc is the coordinates of contact point of cutting surface, the radius of processed point is $R_i$ that changes with $\theta ZW$, OS is the coordinates of contact point on the cutting curve. The turning radius of cutter is $r$, the turning radius $r$ of the ball cutter changes with inclination angle $\theta$. the rotational motion of the cutter $V_p$ generates a cylindrical cutting surface. The mathematical description matrix during 2D processing in Op coordinates are as follows[3][4]
The mathematical description matrix of the machining surface are follows[5]

\[
[T_w] = \begin{bmatrix}
C V_p & -S V_p & 0 & -r C V_p \\
S V_p & C V_p & 0 & -r S V_p \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (1)

The mathematical description matrix of the machining surface are follows[5]

\[
[T_w] = \begin{bmatrix}
C_0 Z_w & -S_0 Z_w & 0 & -R_c C_0 Z_w \\
S_0 Z_w & C_0 Z_w & 0 & -R_s S_0 Z_w \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (2)

OC coordinates coincide with OS coordinates at cutting point, that a certain cutting conditions are satisfied, the cutter’s positional matrixe by the contact conditional expressions (1) (2), that the position and orientation matrix for Op coordinates relative to Ow is[6]

\[
[T_p] = \begin{bmatrix}
C(\theta Z_w - V_p) & -S(\theta Z_w - V_p) & 0 & (R_c + r_c)(\theta Z_w - V_p) \\
S(\theta Z_w - V_p) & C(\theta Z_w - V_p) & 0 & (R_s + r_s)(\theta Z_w - V_p) \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (3)

Because the radius of ball cutter is less than the smallest curvature radius of the machined surface, the tool make no interference with the machined surface, the formula 3 is the positional and gesture conditional expression that must be satisfied when the tool generate the machined surface(curve).

4. The Motion Generation of Machine Tools
As Figure 1 shows, the ball cutter first machine the curve (a height h), and then go up \( \Delta Z \), then the curve of the height of \( h + \Delta Z \) is machined, the curve is the generatrix, and guideline is a \( \Delta Z \)-length straight line in the direction of Z. For the generatix (curve), in order to satisfy position and posture matrix \([T_w]_p\) of the cutters, it is needed to determine the motion units required by machine tool from Op to Ow, set the motion scattering matrix as \([T_{wp}]\), and the matrix meet the following relationship[7]

\[
[T_{wp}] = [T_{wp}]
\] (4)

Equis.4 is solved by analytical method, if the setting motion unit is the functions that contains parametric variables, the motion unit is necessary, otherwise, the setting motion unit will be redundant. According to the characteristic of Equis.4, the setting motion scattering matrix is as follows[8]

\[
[T_{wp}] = \begin{bmatrix}
100 X & 100 a_x & [CV-SV 0 a_x] & 100 a_p \\
010 b_i & 010 Y & SV CV 0 b_i & 010 b_p \\
0010 & 0010 & 0 0 1 0 & 0010 \\
0001 & 0001 & 0 0 0 1 & 0001
\end{bmatrix}
\] (5)

Here, X, Y is linear motions, V is the rotational motion around Z axis, b1, a2, a3, ap, bP are constants, the following equation is gotten from formula(3)-formula(5)
\[
\begin{align*}
V &= \theta_{ZW} - V_p \\
X + a_1 + a_3 CV - b_2 SV &= (R_i + r) \theta_{ZW} \\
Y + b_3 + a_2 SV + b_3 CV &= (R_i + r) \theta_{ZW}
\end{align*}
\] (6)

X, Y, V, Vp are the four motion variable in the formula above, but there is only three equations, so they have multiple solutions, the following three solutions can be gotten

Solution 1: \( V=0 \), X, Y, Vp can be determined uniquely from the equations, they are functions that depend on \( \theta_{ZW} \), Ri, and so on.

Solution 2: \( X=0 \), V, Y, Vp can be determined uniquely from the equations, they are functions that depend on \( \theta_{ZW} \), Ri, and so on.

Solution 3: \( Y=0 \), V, X, Vp can be determined uniquely from the equations, they are functions that depend on \( \theta_{ZW} \), Ri, and so on.

Therefore, the movement schemes for machining generatix are as follows

\[
\begin{align*}
W/X, Y \cdot V_p/T \\
W/V, Y \cdot V_p/T \\
W/V, X \cdot V_p/T
\end{align*}
\] (7)

The Z direction of workpiece is in the same direction along the X direction of the machine tools, so the another three motion schemes for machining generatix are

\[
\begin{align*}
W/Z, Y \cdot V_p/T \\
W/Vx, Y \cdot V_p/T \\
W/Vx, Z \cdot V_p/T
\end{align*}
\] (8)

If the Z direction of workpiece is in the same direction along the Y direction of the machine tool, it can be omitted like the above scheme. Suppose the machine tool is a vertical machine in the above-mentioned facts, the spindle rotate about Z-axis and move along Z-axis.

For the guideline, if the ball cutter goes up \( \Delta Z \), the machine tool is needed to rotate above X, Y and move along Z, which is as follows

\[
W/VX, VY \cdot Z /T
\] (9)

If the Z-axis of workpiece is in the same direction along the X-axis of the machine tool, the machine tool is also needed to rotate around Z, Y-axis and move along X, namely

\[
W/VY, VZ \cdot X /T
\] (10)

If the Z-axis of workpiece is in the same direction along the Y-axis of the machine tool, the machine tool is also needed to rotate around Z, X-axis and move along Y, the scheme is of the same nature with the above scheme and omitted. So, the six different motion schemes are gotten as follows

**Table 1.** The different motion schemes processing free-form surface by ball cutter

| the direction of the workpiece | the schemes of the machine tool |
|-------------------------------|--------------------------------|
| Z-axis of the workpiece is in the same direction as the Z axis of machine tool | \( W/X, Y, Vx, Y \cdot V_p, Z/T \) \\
|                               | \( W/V, Y, Vx, Y \cdot V_p, Z/T \) \\
|                               | \( W/V, X, Vx, Y \cdot V_p, Z/T \) |
| Z-axis of the workpiece is in the same direction as the X axis of machine tool | \( W/Z, Y, Vx, Z \cdot V_p/X/T \) \\
|                               | \( W/Vx, Y, Vx \cdot V_p, X/T \) \\
|                               | \( W/Vx, Z, Vx, Z \cdot V_p/X/T \) |
5. Conclusion

Through the analyses above, the following conclusion are given:

1. The workpiece is divided into an equal parts, the mathematical description matrixes of the cutter and generatix and the cutter’s positional matrixes are established. Generative results of the motion schemes for processing the generatix are given, in order to ensure the slop angle between the ball cutter and the processed surface, the motion schemes required for processing the guideline are determined. Thus, the six motion schemes have been gotten for processing free-form surface.

2. The method illustrated the possible motion schemes for processing the free-form surface in theory; it has the versatility and applies to other types of cutter for processing the free-form surface as well.

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