Rotating asymmetric solitons in competing nonlinear media

Liangwei Dong\textsuperscript{1,2,\textcopyright}, Dongshuai Liu\textsuperscript{1}, Zhijing Du\textsuperscript{1}, Kai Shi\textsuperscript{1} and Changming Huang\textsuperscript{3}

\textsuperscript{1} Department of Physics, Shaanxi University of Science & Technology, Xi'an, 710021, People's Republic of China
\textsuperscript{2} Institute of Theoretical Physics, Shaanxi University of Science & Technology, Xi'an 710021, People's Republic of China
\textsuperscript{3} Department of Electronic Information and Physics, Changzhi University, Changzhi, Shanxi 046011, People's Republic of China

\textsuperscript{\textcopyright} Author to whom any correspondence should be addressed.
E-mail: dlw_0@163.com

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Abstract

We predict a novel family of off-center localized nonlinear modes in a rotating optical system. The whispering-gallery-like solitons bifurcate out from the symmetric fundamental solitons through a symmetry breaking. They can appear as localized spots, nearly flat-top beams and crescent-like patterns extending over the entire range of polar angles. At critical rotation frequency, asymmetric solitons transform into vortex solitons, whose topological charge gradually increases with the growth of the propagation constant. Asymmetric solitons rotate around the origin persistently during propagation and preserve their shape over arbitrary distance in almost the whole existence domain. Thus, we put forward the first example of stable asymmetric intermediate states in optical systems that bridge the symmetric fundamental solitons and the symmetric vortex solitons with different topological charges, through the variance of the propagation constant and rotation frequency.

1. Introduction

The generation and propagation of nonlinear localized waves is a problem of continuously renewed interest in diverse areas of physics, for example, nonlinear optics \cite{1,2}, Bose–Einstein condensates (BECs) \cite{3}, and electron beams \cite{4}. A variety of mechanisms supporting the formation of soliton states are known \cite{1–8}. Optical solitons have found various applications, including optical trapping, diffraction control and quantum information, to name just a few \cite{1,8}.

In two-dimensional (2D) Kerr or saturable media, azimuthal instability usually breaks a soliton into several fragments. It was revealed that competing nonlinearity, such as the combination of \(\chi^{(2)}\) and \(\chi^{(3)}\) nonlinearity \cite{9} and the cubic-quintic \cite{10} nonlinearity, can suppress the azimuthal instability effectively. Similar competing nonlinearity occurs in binary BECs in the form of a cubic mean-field attraction and a quartic repulsion characterized by the \(s\)-wave scattering lengths with Lee–Huang–Yang corrections \cite{11,12}. This special type of nonlinearity were shown to support various types of ultradilute quantum droplets in cold–atom systems \cite{13–15}. The competing nonlinearity mentioned above is crucial for the stabilization of off-center asymmetric nonlinear modes.

In contrast to nonrotating systems, linear and nonlinear waves exhibit unique proprieties in rotating regimes \cite{16–21}. The Coriolis force induced by the rotation exerts substantial influences on the dynamics of linear and nonlinear localized modes. For example, new types of localized linear and nonlinear modes were demonstrated in truncated rotating square waveguide arrays \cite{16}. Surface solitons can rotate stably at the edge of several concentric rings \cite{17}. Vector and gap solitons were observed in a rotating waveguide array \cite{18}. Rotating vortex clusters were predicted in the media with inhomogeneous defocusing nonlinearity whose strength grows to the periphery at a fast rate \cite{19}. Robust ultrashort light bullets were studied in strongly twisted waveguide arrays \cite{20}. Chiral light was observed in optically induced helical waveguide arrays \cite{21}.

In BECs, matter-wave solitons were investigated in rotating optical lattices \cite{22,23}. Metastable rotating vortex clusters were revealed in the form of quantum droplets carrying multiple singly quantized vortices...
held in a parabolic potential [24]. Very recently, a novel type of 2D and 3D stable rotating quantum droplets were predicted in an anharmonic potential [25], modeled by the Gross–Pitaevskii equation augmented with Lee–Huang–Yang corrections.

Symmetry breaking is universal in nonlinear systems [26]. As is well known, the symmetry of all linear states is either the same as or the opposite to the symmetry of the external potential. Yet, when nonlinearity is involved, this symmetry law can be changed under appropriate conditions. For example, if a nonlinear system is modulated by a double-well potential (1D) or a potential with a minimum ring (2D), a noise can trigger an initially symmetric system to evolve toward one of its nonsymmetric states [27]. In other words, with the growth of energy, the symmetric state becomes an asymmetric one. This phenomenon is the so-called symmetry breaking. Due to its fundamentally physical interest, symmetry breaking has been paid special attention since its prediction [28]. In one-dimensional (1D) [29] or 2D [30] double-well potentials, two branches of asymmetric solitons respecting a mirror symmetry can bifurcate out from the symmetric ones belonging to the base branch. Self-trapping in one of two identical guiding channels was observed in diverse nonlinear optical settings [31, 32] and BECs [33]. Stationary crescent surface solitons pinged to a boundary were generated in a microstructured optical cavity [34].

Thus far, symmetry breaking in rotating optical systems is still an open problem. More importantly, the transition states, connecting the fundamental solitons at one end and the vortex solitons with different topological charges at the other end, have not yet been revealed in nonlinear optical systems. Though rich dynamics of rotating asymmetric quantum droplets were reported in binary BECs [25], the experimental realization of them is an uneasy work. Optics can provide a fertile ground where cold–atom related concepts can be realized and experimentally tested. Technically, the rigorous linear stability analysis has not been performed on solitons in any rotating systems. Exploration of the existence, stability and propagation dynamics of rotating asymmetric solitons is, therefore, an interesting problem. Our prediction will provide a helpful hint for the experimental creation of elusive self-sustained rotating asymmetric states in optical configurations.

2. Theoretical model

We consider the propagation of light beams along the z axis in media with a competing cubic-quintic nonlinearity. The evolution of the dimensionless field amplitude $\Psi$ is governed by the 2D nonlinear Schrödinger equation:

$$i \frac{\partial \Psi}{\partial z} = \left[ -\frac{1}{2} \nabla^2 + V_H + V_G - |\Psi|^2 - |\Psi|^4 \right] \Psi,$$

where the longitudinal $z$ and transverse $x, y$ coordinates are scaled to the diffraction length and the input beam width, respectively. The linear refractive index is modulated by an external harmonic trap in the form $V_H = \omega_z^2 r^2 / 2$, where $r = \sqrt{x^2 + y^2}$ is the radial coordinate and $\omega_z$ is the trapping frequency. To capture off-center solitons, we introduce a weak Gaussian potential $V_G = V_0 \exp(-r^2 / a^2)$ to create a shallow radial minimum of the refractive index, which usually favors the formation of whispering-gallery modes. Specifically, we set $\omega_z = 0.1$, $V_0 = 0.4$ and $a = 5$ in the following discussions. The harmonic potential and the Gaussian potential rotate around the $z$ axis with a rotation frequency (angular velocity) $\omega$.

To find stationary rotating asymmetric modes, we move to the rotating frame $x' = x \cos(\omega z) + y \sin(\omega z), \ y' = y \cos(\omega z) - x \sin(\omega z)$ in which the external potentials are independent of the propagation distance $z$. In the rotating frame, the nonlinear Schrödinger equation with omitted primes can then be written as:

$$i \frac{\partial \Psi}{\partial z} = \left[ -\frac{1}{2} \nabla^2 + V_H + V_G - |\Psi|^2 - |\Psi|^4 - \omega L_z \right] \Psi,$$

where $-\omega L_z \Psi = -\omega (xp_y - yp_x) \Psi = i\omega (x\partial \Psi / \partial y - y\partial \Psi / \partial x)$ is the Coriolis term induced by the rotation. Rotation frequency $\omega > 0$ corresponds to a counterclockwise rotation. Evolution of beams is characterized by the power (energy flow) $U = \int \int |\Psi(x, y)|^2 dxdy$ and the angular momentum $L = \int \int \Psi^* L_z \Psi dxdy$.

We search for stationary solution of equation (2) in rotating frame by assuming

$$\Psi(x, y, z) = \psi(x, y) \exp(ibrz) = [\psi_1 + iv_1]\exp(ibrz),$$

where $\psi, \psi_1$ and $v_1$ are the complex, real and imaginary parts of soliton profiles, respectively, and $b$ is a nonlinear propagation constant. The field modulus is defined as $|\psi_1| = \sqrt{\psi_1^2 + v_1^2}$ and the phase structure is characterized by the topological charge or winding number which is given by $m = \int \int \arctan(\psi_1 / v_1) dxdy / 2\pi$. Substitution of the light field $\Psi(x, y, z)$ into equation (2) yields an ordinary differential equation, from which stationary soliton solutions at $z = 0$ can be solved numerically using the relaxation iterative algorithm or the Newton-conjugate gradient method [35].
3. Numerical results and discussion

Typical profiles of the symmetric and asymmetric solitons solved from equation (2) are illustrated in figure 1. Due to the bulge in the Gaussian potential, the radially symmetric soliton at low power has a low-lying area around its center [figure 1(a1)]. As \( b \) increases, the symmetric soliton shrinks and its peak value grows. The dominate nonlinearity now is still a focusing one. With the growth of \( b \), the progressively increasing focusing nonlinearity becomes strong and overcomes the trapping capability of the shallow refractive-index modulation. As a result, the soliton concentrates in the central area of the harmonic potential [figure 1(a2)]. With the further increase of the soliton power, the dominate nonlinearity becomes a defocusing one, which results in the expansion of soliton [figure 1(a3)]. These fundamental solitons are characterized by their trivial phase distributions.

Asymmetric soliton resides on a ring corresponding to the shallow radial minimum of the external potentials [figures 1(b) and (c)]. The radial symmetry of the system makes the asymmetric states shown in figures 1(b) and (c) remain rotation-invariant. When \( \omega = 0 \), asymmetric soliton appears as a off-center light spot at lower \( b \) values and extends over the origin \((r = 0)\) for \( b \) near its upper cutoff \([b_{\text{cut}}(\omega = 0) = -0.1309]\). Nonrotating modes have a trivial phase and only one density maximum. As \( \omega \) grows, an increasing number of spatially separated phase dislocations come from transverse infinity and approach to the origin. There exists a maximum \( \omega_m \), at which, the power of a rotating soliton approaches
Figure 2. (a) Dependence of power $U$ on propagation constant $b$ for nonrotating symmetric (red) and asymmetric (blue) solitons. (b) Power $U$ versus $b$ for rotating symmetric (red) and asymmetric (blue) solitons at $\omega = 0.04$. (c) $U(\omega)$ curves for asymmetric solitons at different $b$ values. (d) Variation of $U$ versus $\omega$ for nearly flat-top asymmetric solitons ($b$ increases along the arrow from $-0.164$ to $-0.154$ in steps of $\delta b = 0.002$). All quantities are plotted in dimensionless units.

zero or an asymmetric soliton becomes a symmetric one with invariant power, which ceases the existence of solitons. For details of $\omega_m$, see the later discussions of figures 2(c) and (d) and 3(a). When $\omega$ reaches its maximum $\omega_m$, the number of phase singularities attains its maximum. All phase singularities merge together at the origin and the asymmetric mode is transformed as a symmetric vortex soliton eventually. For instance, a phase singularity with topological charge $m = 1$ emerges in figure 1(b2) at $\omega = 0.042$ and two singularities merge together in figure 1(b3) at $\omega_m = 0.059$. The soliton at $\omega_m = 0.059$ is a radially symmetric vortex soliton with $m = 2$ [figure 1(b3)]. Near $\omega_m$, the soliton appears as a crescentlike pattern extending over almost the entire range of polar angles. The rotation leads to the expansion of asymmetric soliton at higher $b$ values (e.g., $b = -0.154$) [figure 1(c)]. An asymmetric soliton is transformed into a fundamental soliton at $\omega = \omega_m = 0.059$. The nonrotating asymmetric solitons near the upper cutoff $b_{\text{upp}} = -0.1309$ and rotating asymmetric solitons near $\omega_m$ all look like nearly flat-top beams.

The dependence of the power of nonrotating solitons on the propagation constant is displayed in figure 2(a). Symmetric fundamental solitons bifurcate out from the linear mode corresponding to the largest discrete eigenvalue ($b = -0.3327$) of the linear system. The power of symmetric solitons at $\omega = 0$ increases monotonously with the growth of $b$ in the whole existence region. When $b = -0.1302$, the power curve becomes vertical. It implies that the solitons originating from the linear mode stop to exist for $b > -0.1302$. Interestingly, we find that another branch of solitons with opposite power slope ($\partial U / \partial b < 0$) exist at higher power. The upper-branch soliton merges with the lower-branch one at $b = -0.1302$. The competing nonlinearity and special potential with a radial minimum are responsible for the coexistence of two branches of symmetric fundamental solitons [see the description of figure 1(a)].

Due to the symmetry breaking, asymmetric solitons bifurcate out from the symmetric one when propagation constant or power is increased to a certain value [figure 2(a)]. Unlike the symmetry breaking occurs in 1D system [29] and 2D asymmetric system [36], the asymmetric patterns shown in figures 1(b) and (c) remain unchangeable by applying a rotation of arbitrarily angle. The power of asymmetric solitons is lower than that of symmetric ones for $b < -0.1721$ and greater than the power of asymmetric solitons when $b > -0.1721$. The power reaches a maximum at $b = 0.1455$ and decreases until it merges with the power curve of asymmetric solitons at $b = -0.1309$. The restoration of symmetric solitons from asymmetric soliton at high power cannot happen in Kerr media [29, 36]. The variation of the power of symmetric and asymmetric solitons in a rotating system ($\omega \neq 0$) is similar to that in the nonrotating system [figures 2(a)].
Figure 3. (a) Existence domain of the rotating solitons on the $(b,\omega)$ plane. With the increase of $b$, $m$ value at $\omega_m$ increases by 1 marked by the dots with alternate color. (b) Dependence of per-photon angular momentum on $\omega$ for solitons at $b = -0.22$. (c) Instability growth rate versus $\omega$ for solitons at $b = -0.14$. All quantities are plotted in dimensionless units.

and (b)]. However, the restoration of symmetry breaking does not occur when $\omega$ exceeds a certain value [figure 2(b)].

At low $b$ values, the power of asymmetric solitons decreases with the rotation frequency $\omega$ [figure 2(c)]. As $\omega$ reaches its allowed maximal value $\omega_m$, asymmetric soliton transforms into a symmetric vortex mode [figure 1(b3)]. In other words, asymmetric solitons originate from the vortex eigenmode of the rotating linear system with rotation frequency $\omega_m$. The marked circle at $\omega_m = 0.0579$ denotes the symmetry-breaking point at which the symmetric branch merges with the asymmetric branch. The symmetry-breaking point disappears for $b > -0.245$. The $\omega_m$ increases with $b$ firstly. When $b \geq -0.159$, the power becomes an increasing function of $\omega$ [figure 2(d)]. The corresponding asymmetric solitons occupy a region greater than the area of a semicircle [see, e.g., figure 1(c)]. The lower $\omega_m$ here also explains why the restoration of symmetry breaking cannot occur for a larger $\omega$ [figure 2(b)]. The symmetric power curves displayed in figure 2(d) show the equivalences between the counterclockwise rotation and clockwise rotation of asymmetric solitons.

We summarize the properties of rotating asymmetric solitons in figure 3(a) in the form of the dependence of $\omega_m$ on $b$. Rotating asymmetric solitons can be found in the region below $\omega_m$. At $\omega_m$, they transform into vortex solitons. The topological charge at $\omega_m$ increases gradually from $m = 0$ at $b = -0.3327$ up to $m = 8$ at $b = -0.1592$ in steps of $\delta m = 1$. When $b > -0.1592$, the existence domain shrinks suddenly and the maximal frequency $\omega_m$ drops down. Asymmetric solitons here feature nearly flat-top states and have trivial phase distributions [figure 1(c)]. The two gaps near the two ends of $b$ correspond to the radially symmetric fundamental solitons before the first symmetry-breaking point and after the second symmetry-breaking point, respectively.

To shed more light on rotating solitons, we show the dependence of angular momentum carried by per photon $L/U$ in figure 3(b). In contrast to the radially symmetric modes, the $L/U$ of the rotating asymmetric solitons increases monotonously with the growth of $\omega$. The per-photon angular momentum increases continuously from 0 at $\omega = 0$ up to 5 at $\omega_m$, which equals exactly the topological charge of the vortex eigenmode of the corresponding linear system with $\omega = \omega_m$. It manifests that the angular momentum induced by system rotation is transferred to the nonlinear states with a non-trivial phase.
Figure 4. (a)–(c) Stable propagation examples of rotating asymmetric solitons at $b = -0.22$. Rotating frequency $\omega = 0.038$ in (a), 0.07 in (b) and 0.085 in (c). There are 1, 3, and 5 phase singularities in (a)–(c), respectively. (d) Unstable propagation of nonrotating soliton at $b = -0.14$ and $\omega = 0$. The growth rate is shown in figure 3(c). In all the panels, $(x, y) \in [-15, 15]$. All quantities are plotted in dimensionless units.

The stability of rotating solitons can be analyzed by the linear stability analysis, i.e., solving the perturbed solutions of equation (2) in the form $\Psi(x, y, z) = [\psi(x, y) + u(x, y)\exp(\lambda z) + v^*(x, y)\exp(\lambda^*z)]\exp(ibz)$, where $u$ and $v \ll 1$ are infinitesimal perturbations and $\lambda$ is the instability growth rate. Linearization of equation (2) around $\psi$ yields an eigenvalue problem [37]:

$$
i \begin{bmatrix}
M_1 & M_2 \\
-M_2^* & -M_1^*
\end{bmatrix}
\begin{bmatrix}
u \\
v
\end{bmatrix}
= \lambda
\begin{bmatrix}
u \\
v
\end{bmatrix},$$

(3)

here, $M_1 = \nabla^2/2 - V_H - V_G - b + 2|\psi|^2 - 3|\psi|^4 - i\omega(x\partial/\partial y - y\partial/\partial x)$, $M_2 = \psi^2(1 - 2|\psi|^2)$, and $^*$ denotes complex conjugate. Equation (3) can be solved by Fourier collocation method [35]. Solitons are stable only when all positive real parts of eigenvalues $\lambda$ equal zero.

Linear stability analysis results reveal that asymmetric solitons are stable in almost their whole existence domain. A weak instability occurs for nearly flat-top solitons at larger $b$ values [figure 3(c)]. The narrow instability window of the propagation constant ($b \in [-0.1452, -0.1321]$) occupies about 7.2% of the whole existence domain of asymmetric solitons ($b \in [-0.3142, -0.1321]$). At fixed $b$, the instability growth rate decreases with $\omega$. Thus, the instability of solitons can be suppressed by the increase of rotation frequency completely.

To verify the stability analysis results, we perform direct numerical propagation simulations on rotating solitons using the split-step Fourier algorithm. The initial input is perturbed by a broadband noise in the form $\Psi(x, y, z = 0) = \psi(x, y)[1 + \rho(x, y)]$, where $\psi(x, y)$ is the profile of a soliton solution, and $\rho(x, y)$ is a
noise with variance $\sigma_{\text{noise}}^2 = 0.01$. Typical propagation examples are shown in figure 4. For illustration, we present the evolution of asymmetric solitons at a fixed $b$ but different $\omega$. The initial input at $z = 0$ is located on the left side of the $y$ axis, similar to the solitons shown in figures 1(b) and (c). The rotational periodicity estimated by the relation $T = 2\pi/\omega$ predicts the propagation dynamics of asymmetric solitons precisely (figures 4(a)–(c)). Solitons rotate persistently around the $z$ axis without any deformation. We stress again that, with the increase of $\omega$, some phase singularities come into the window from infinity merge together at the origin when $\omega \rightarrow \omega_{\text{ny}}$ as shown in figure 4(c4). The number of phase singularities in figures 4(a)–(c) is 1, 3, and 5, respectively. The unstable soliton with nearly flat-top distribution at $b = -0.14$ looks like an erratic breather.

4. Conclusions

Summarizing, we investigated the existence, stability and propagation dynamics of rotating asymmetric solitons in cubic-quintic media modulated by a radially symmetric potential. Through symmetry breaking, asymmetric soliton bifurcates out from symmetric solitons at a certain power and return to symmetric modes at another certain power. They exhibit various appearances at different parameters and rotate periodically around the $z$ axis during propagation. With the growth of the rotation frequency, some phase singularities come into the fields of asymmetric solitons. We performed the rigorous linear stability analysis on asymmetric solitons in rotating regimes for the first time and found they are stable in almost the whole existence region. Our results can be generalized to the rotating vector solitons in media with $\chi^{(2)} - \chi^{(3)}$ nonlinearity. The findings are also relevant for matter-wave solitons or superfluids trapped in a radial potential.

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ORCID iDs

Liangwei Dong https://orcid.org/0000-0002-7429-3200

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