Quasi-two-body decays $B_{(s)} \to D(\rho(1450), \rho(1700)) \to D\pi\pi$ in the perturbative QCD factorization approach

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By employing a framework for the quasi-two-body decays in the perturbative QCD (PQCD) factorization approach, we calculate the branching ratios of the decays $B_{(s)} \to D(\rho(1450), \rho(1700)) \to D\pi\pi$ with $D = (D_{(s)}, D_{(s)})$. The pion vector form factor $F_\pi$, acquired from a BABAR Collaboration analysis of $e^+e^- \to \pi^+\pi^-(\gamma)$ data, is involved in the two-pion distribution amplitudes $\Phi_{\pi\pi}^{1}$. The PQCD predictions for the branching ratios of the considered quasi-two-body decays are in the range of $10^{-10} \sim 10^{-4}$. The PQCD predictions for $B(B^0 \to D^*(\rho(1450), \rho(1700)) \to D^0\pi^+\pi^-)$ agree well with the measured values as reported by LHCb if one takes still large theoretical errors into account. Unlike the traditional way of the PQCD approach, one can extract the decay rates for the two-body decays $B_{(s)} \to D(\rho(1450), \rho(1700))$ from the results of the corresponding quasi-two-body decays. The PQCD predictions for $B(B_{(s)} \to D\rho(1450))$ and $B(B_{(s)} \to D\rho(770))$ are similar in magnitude: an interesting relation to be tested by future experimental measurements.

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I. INTRODUCTION

Up to now, many hadronic charmed three-body $B_{(s)}$ meson decays have been measured by experiments, such as the Belle, BABAR, D0, CDF, and LHCb Collaborations [1–6]. The study for the $B_{(s)} \to Dh'h'$ decays with $D = (D_{(s)}, D_{(s)})$ and $h'^{(s)} = (\pi, K)$, for example, has played an important role in the precise determination of the Cabibbo-Kobayashi-Maskawa (CKM) angle $\gamma$ [7] and the study of the rich resonant structure [5]. Recently, a study of the $\pi^+\pi^-$ system by the LHCb Collaboration [5] was performed through Dalitz plot analysis [8] of $B^0 \to D^0\pi^+\pi^-$ decays. The phase-space range was broad, from 0.28 ($\approx 2m_\pi$) to 3.4 GeV ($\approx m_B - m_D$), and the first observation of the decay $B^0 \to D^0\rho(1450)$ was reported [5]. When a decay rate of $\rho(1450) \to \pi^+\pi^-$ determined by employing the Isobar model [9–11] or the $K$-matrix formalism [12] was used, respectively, the LHCb Collaboration reported their measurements for the branching fraction of the quasi-two-body decay $B^0 \to D^0\rho(1450)$ [5],

$$B(B^0 \to D^0\rho(1450) \to D^{0}\pi^+\pi^-) = \begin{cases} 1.36 \pm 0.28 \pm 0.08 \pm 0.19 \pm 0.06 \times 10^{-5} \text{ (Isobar)}, \\ 1.91 \pm 0.37 \pm 0.73 \pm 0.19 \pm 0.09 \times 10^{-5} \text{ (K - matrix)} \end{cases} \tag{1}$$

Meanwhile, the branching fraction of the quasi-two-body decay $B^0 \to D^0\rho(1700)$ with a given $\rho(1700) \to \pi^+\pi^-$ was also reported in Ref. [5],

$$B(B^0 \to D^0\rho(1700) \to D^0\pi^+\pi^-) = \begin{cases} 0.33 \pm 0.11 \pm 0.06 \pm 0.05 \pm 0.02 \times 10^{-5} \text{ (Isobar)}, \\ 0.73 \pm 0.18 \pm 0.53 \pm 0.10 \pm 0.03 \times 10^{-5} \text{ (K - matrix)} \end{cases} \tag{2}$$

Similar quasi-two-body decays like $B^0 \to K^+\rho^-(1450)$ and $B^- \to \pi^-\rho(1450)$ have been observed by the BABAR Collaboration [13, 14] with the cascade decay $\rho(1450) \to \pi\pi$. For $\rho(1450)$ and $\rho(1700)$ 1, there is a strong interference near 1.6 GeV.

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1 In the following sections, we generally use the abbreviation $\rho = \rho(770), \rho' = \rho(1450), \rho'' = \rho(1700), \rho''' = \rho(2254), \text{and } D = (D_{(s)}, D_{(s)})$. 

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High-statistics study of the $\tau^- \to \pi^-\pi^0\nu_\tau$ decay by Belle [15] reported the first observation of both $\rho'$ and $\rho''$. In the study of $e^+e^- \to \pi^+\pi^-(\gamma)$ by BABAR [16], a clear picture of the two $\pi^+\pi^-$ resonances interfering with the $\rho$ was reported. The basic properties of $\rho'$ and $\rho''$ mesons from PDG2016 [17] are listed in Table I.

On the theory side, the three-body $B_s(\bar{s})$ decays have been investigated by employing the QCD factorization approach [18–31], the perturbative QCD (PQCD) factorization approach [32–42], and the framework of the symmetry principles [43–51]. In three-body $B$ decays, there are two distinct final state interaction mechanisms: (a) the interactions between the meson pair in the resonant region associated with various intermediate states, and (b) the rescattering between the third particle and the pair of mesons usually ignored in the quasi-two-body approximation. In the real data analysis, most of the quasi-two-body decays are extracted from the Dalitz-plot analysis of the three-body ones; the study of quasi-two-body $B$ decays could be a starting point in the studies of the three-body decays.

The two-body $B \to D\rho$ decays have been studied intensively by using various theoretical methods or approaches [52–56]. But the three-body $B$ decays involving the radially excited $\rho$ mesons ($\rho', \rho''...$) have not attracted much attention in the literature. Recently, a study of the $\pi^+\pi^-\pi^-$ system was performed through Dalitz-plot analysis of $B_s^0 \to D^0\pi^+\pi^-$ decays by the LHCb Collaboration [5], while the quasi-two-body decays $B \to K(\rho, \rho') \to K\pi\pi$ were investigated by using the PQCD approach [37]. The resonant and nonresonant contributions between the $\pi\pi$ pair were parametrized into the timelike pion form factors involved in the two-pion distribution amplitudes [32, 33, 57–60]. Besides $\rho$, the contribution from the $\rho'$ intermediate state could also be singled out from the given timelike form factor $F_\pi$. By using the Gegenbauer moments of the $P$-wave two-pion distribution amplitudes, we can make the predictions for the branching ratios and the direct $CP$ asymmetries of the $B \to K\rho' \to K\pi\pi$ decays. Following Ref. [37], we have studied the $B_s(\bar{s}) \to P\rho(\rho', \rho'') \to P\pi\pi$ [39, 42] and $B_s(\bar{s}) \to D\rho \to D\pi\pi$ decays [38] where $P$ denotes the light pseudoscalar mesons $\pi, K, \eta, \eta'$ and $D$ stands for the charmed $D$ meson.

Based on our previous works in Refs. [37–39, 42], we here study all $B_s(\bar{s}) \to D(\rho', \rho'') \to D\pi\pi$ decay modes and present the PQCD predictions for their branching ratios. The typical Feynman diagrams that may contribute to the considered decay modes are the same ones as those illustrated in Figs. 1 and 2 of Ref. [38]. Since only tree operators are involved here, the direct $CP$-violating asymmetries for the considered decays are absent naturally. Without the information about the distribution amplitudes for $\rho'$ and $\rho''$, the PQCD approach does not work in calculating the branching ratios of the two-body decays $B_s(\bar{s}) \to D(\rho', \rho'')$ in a traditional way. Unlike the observed $B(\rho' \to \pi\pi) \sim 100\%$, the $\rho'$ also has other decay channels like $A\pi, K\bar{K}, \omega\pi$, etc. [17]. For $\rho''$, we also know that $\rho'' \to \rho\pi\pi$ is dominant [17]. In the quasi-two-body framework, fortunately, we can extract the branching ratios for the two-body decays $B_s(\bar{s}) \to D(\rho', \rho'')$ from the results of $B_s(\bar{s}) \to D(\rho', \rho'') \to D\pi\pi$ after making a reliable estimation for the branching fraction $B((\rho', \rho'') \to \pi\pi)$. This paper is organized as follows. In Sec. II, we give a brief introduction for the theoretical framework. The numerical values, some discussions and the conclusions are given in last two sections.

### II. THE THEORETICAL FRAMEWORK

For the considered $B_s(\bar{s}) \to D(\rho', \rho'') \to D\pi\pi$ decays, the effective Hamiltonian is of the form

\[
H_{\text{eff}} = \begin{cases} 
\frac{\mathcal{G}_A}{\sqrt{2}}V_{ub}^*V_{cq} [C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu)], \quad & \text{for } B_s(\bar{s}) \to D(\rho', \rho'') \to D\pi\pi \text{ decays,} \\
\frac{\mathcal{G}_A}{\sqrt{2}}V_{ub}^*V_{cq} [C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu)], \quad & \text{for } B_s(\bar{s}) \to D(\rho', \rho'') \to D\pi\pi \text{ decays,} 
\end{cases}
\]

where $O_{1,2}(\mu)$ represent the tree operators, $C_{1,2}(\mu)$ are the Wilson coefficients, $q = (d, s)$, and $V_{ij}$ are the CKM matrix elements.

In the framework of the PQCD approach for the quasi-two-body decays, the nonperturbative dynamics associated with the pair of the pion mesons are factorized into two-meson distribution amplitudes [32, 33, 57–60] due to two reasons [32, 33]. First, it is not practical to make a direct evaluation for the hard $b$-quark decay kernels containing two virtual gluon propagators at leading order, while the possible contribution in such a kinematic region is also power suppressed and not important. Secondly, the dominant contribution most possibly comes from the region where the involved two energetic mesons are almost collimating to each other and having an invariant mass below $O(\Lambda m_B)$ ($\Lambda = m_B - m_b$).

Analogous to the two-body $B$ decays, the decay amplitude $A$ for the quasi-two-body decays $B_s(\bar{s}) \to D(\rho', \rho'') \to D\pi\pi$ in

| Mesons | $I^G$ | $\bar{y}^{PC}$ | Mass (MeV) | Width (MeV) |
|--------|--------|---------------|------------|-------------|
| $\rho'$ | $1^+$ | $1^-$ | $1465 \pm 25$ | $400 \pm 60$ |
| $\rho''$ | $1^+$ | $1^-$ | $1720 \pm 20$ | $250 \pm 100$ |
the PQCD approach can be written conceptually as the convolution \([32, 33]\)

\[
A = \Phi_B \otimes H \otimes \Phi_D \otimes \Phi_{\pi\pi}^{I=1},
\]

(4)

where the hard kernel \(H\) describes the dynamics of the strong and electroweak interactions in the decays, \(\Phi_B, \Phi_D\) and \(\Phi_{\pi\pi}\) denote the distribution amplitudes for the \(B_1(s)\) meson, the final state \(D = (D(s), D(s))\) meson and the final state \(\pi\pi\) pair. In this work, the widely used wave functions for \(B_1(s)\) meson and \(D\) mesons as used for example in Refs. \([38, 39]\) are adopted.

For the \((\rho', \rho'')\) mesons, their longitudinal distribution amplitudes are defined in the same way as in Ref. \([37]\).

\[
\Phi_{\pi\pi}^{I=1} = \frac{1}{\sqrt{2N_c}} \left[ \frac{\bar{p}_1 \phi^0(z, \zeta, w^2) + w \phi^t(z, \zeta, w^2) + \bar{p}_2 \bar{p}_3 + p_3}{w(2\zeta - 1)} \phi^t(z, \zeta, w^2) \right],
\]

(5)

with the functions

\[
\phi^0(z, \zeta, w^2) = \frac{3F_2(s)}{2\sqrt{2N_c}} \left[ 1 - z \right] \left[ 1 + a_2^0 C_2^{3/2}(t) \right] P_1(2\zeta - 1),
\]

\[
\phi^t(z, \zeta, w^2) = \frac{3F_2(s)}{2\sqrt{2N_c}} \left[ 1 - 2z \right] \left[ 1 + a_2^t (1 - 10z + 10z^2) \right] P_1(2\zeta - 1),
\]

\[
\phi^t(z, \zeta, w^2) = \frac{3F_2(s)}{2\sqrt{2N_c}} \left[ 1 - 2z \right]^2 \left[ 1 + a_2^t C_2^{3/2}(t) \right] P_1(2\zeta - 1),
\]

(6)

where \(p_1\) and \(p_2\) denote the momentum of the pion pair, and \(p = p_1 + p_2\) is the momentum of the \(\rho\) or \(\rho''\) meson. The parameter \(z\) is the momentum fraction of the pion pair and \(\zeta\) denotes the momentum fraction for one pion among the pion pair, while \(s = w^2 = p^2\) denotes the invariant mass squared of the pion pair. The Gegenbauer polynomial \(C_2^{3/2}(t) = \frac{1}{2}(5t^2 - 1)\) and \(t = 2z - 1\), and the Legendre polynomial \(P_1(2\zeta - 1) = 2\zeta - 1\).

Based on the \(BABAR\) Collaboration analysis of \(e^+e^- \rightarrow \pi^+\pi^-(\gamma)\) data, the form factor \(F_\pi\) has been chosen as the form of \([16]\)

\[
F_\pi(s) = \frac{1}{1 + \sum_i c_i}, \quad \left\{ GS_{\rho}(s, m_\rho, \Gamma) + c_\omega BW_{\omega}(s, m_\omega, \Gamma) \right\} + \sum_i c_i GS_i(s, m_i, \Gamma_i) \right\},
\]

(7)

with

\[
BW_{\omega}(s, m, \Gamma) = \frac{m^2 - s - im\Gamma}{m^2 - s + f(s, m, \Gamma) - im\Gamma(s, m, \Gamma)},
\]

\[
GS_{\rho,i}(s, m, \Gamma) = \frac{m^2 [1 + d(m) \Gamma/m]}{m^2 - s + f(s, m, \Gamma) - im\Gamma(s, m, \Gamma)}
\]

(8)

In the above formulas, \(BW_{\omega}(s, m, \Gamma)\) is the Breit-Wigner (BW) function \([61]\) for the \(\omega\) meson, while \(GS_{\rho,i}(s, m, \Gamma)\) are the functions for the \(\rho\) meson and its excited states \(i = (\rho', \rho'', \rho''')\) as described by the Gounaris-Sakurai(GS) model based on the BW model \([61, 62]\). The explicit expressions of the functions and relevant parameters in Eqs. (7) and (8) can be found in Ref. \([16]\). In this work, we single out the component for \(\rho'\) and \(\rho''\) from the form factors as defined in Eq. (7). We here choose the Gegenbauer moments

\[
a_2^0 = 0.30 \pm 0.05, \quad a_2^t = 0.70 \pm 0.20, \quad a_2^t = -0.40 \pm 0.10
\]

(9)

by fitting the available experimental data for the decays \(B \rightarrow P\rho \rightarrow P\pi\pi\) \([39]\) where \(P\) represents the light pseudoscalar mesons \(\pi, K, \eta, \eta'\), which are slightly different from those determined from the decay \(B \rightarrow K\rho \rightarrow K\pi\pi\) \([37]\).

For the decays \(B(s) \rightarrow D(\rho', \rho'') \rightarrow D\pi\pi\), the differential decay rate can be written as

\[
\frac{d\mathcal{B}}{dw^2} = \tau_B \left| \frac{\bar{p}_\pi \bar{p}_D}{32\pi^3 m_B^4} |A|^2 \right|,\]

(10)

where \(\tau_B\) is the mean lifetime of the \(B\) meson, and \(|\bar{p}_\pi|\) and \(|\bar{p}_D|\) denote the magnitudes of the \(\pi\) and \(D\) momenta in the center-of-mass frame of the pion pair.

\[
|\bar{p}_\pi| = \frac{1}{2} \sqrt{w^2 - 4m_\pi^2},
\]

\[
|\bar{p}_D| = \frac{1}{2} \sqrt{[(m_B^2 - m_D^2)^2 - 2(m_B^2 + m_D^2)w^2 + w^4]/w^2}.
\]

(11)

The analytic formulas for the corresponding decay amplitudes and relevant functions for the considered decays \(B \rightarrow D(\rho', \rho'') \rightarrow D\pi\pi\) are the same in form as those given in Ref. \([38]\) for the cases of \(B(s) \rightarrow D\rho \rightarrow D\pi\pi\) decays.
III. NUMERICAL RESULTS

Besides those Gegenbauer moments in Eq. (9), the following input parameters [17] (the masses, decay constants and QCD scale are in units of GeV) are used in the numerical calculations:

\[
\begin{align*}
M_{MS}^{(\bar{c}q)} &= 0.25, \\
M_B &= 5.280, \\
M_{B_s} &= 5.367, \\
M_{D_z} &= 1.870, \\
M_{ar{B}^s} &= 1.865, \\
M_{D^*} &= 1.986, \\
m_{\pi^0} &= 0.140, \\
m_{\pi^0} &= 0.135, \\
m_B &= 4.8, \\
m_c &= 1.27, \\
f_{B_D} &= 0.19, \\
f_{B_{sD}} &= 0.236, \\
f_{D} &= 0.2119, \\
f_{D^*} &= 0.249, \\
\tau_{B^0} &= 1.520 \text{ ps}, \\
\tau_{B^+} &= 1.638 \text{ ps}, \\
\tau_{B_s} &= 1.510 \text{ ps},
\end{align*}
\]

and the Wolfenstein parameters \(\Lambda = 0.811 \pm 0.026, \lambda = 0.22506 \pm 0.00050, \bar{\rho} = 0.124^{+0.019}_{-0.018}, \bar{\eta} = 0.356 \pm 0.011.\)

TABLE II: The PQCD predictions for the branching ratios of the quasi-two-body decays \(B_{(s)} \to D\rho' \to D\pi\pi\) and the two-body decays \(B_{(s)} \to D\rho'.\)

| Decay modes | Quasi-two-body decays | Two-body decays |
|-------------|-----------------------|-----------------|
| \(B_{(s)} \to D_{(s)}\rho' \to D_{(s)}\pi\pi\) | \(B\) | \(B\) |
| \(B^+ \to D^0\rho^+ \to D^\pi^0\pi^0\) | \(8.68 \pm 1.14\times 10^{-4}\) | \(2.68 \pm 1.14\times 10^{-4}\) |
| \(B^0 \to D^+\rho^- \to D^-\pi^0\pi^0\) | \(6.80 \pm 1.14\times 10^{-4}\) | \(6.80 \pm 1.14\times 10^{-4}\) |
| \(B^0 \to D^0\rho^0 \to D^0\pi^0\pi^0\) | \(9.04 \pm 1.14\times 10^{-4}\) | \(9.04 \pm 1.14\times 10^{-4}\) |
| \(B^0 \to D^0\rho^0 \to D^+\pi^-\pi^0\) | \(2.41 \pm 1.14\times 10^{-4}\) | \(2.41 \pm 1.14\times 10^{-4}\) |
| \(B^0 \to D^0\rho^0 \to D^-\pi^0\pi^0\) | \(1.88 \pm 1.14\times 10^{-4}\) | \(1.88 \pm 1.14\times 10^{-4}\) |
| \(B^0 \to D^0\rho^0 \to D_\pm^\pi^+\pi^-\) | \(5.33 \pm 1.14\times 10^{-4}\) | \(5.33 \pm 1.14\times 10^{-4}\) |

In the second columns of Tables II and III, we present the PQCD predictions for the branching ratios of the quasi-two-body decays \(B_{(s)} \to D(\rho', \rho'') \to D\pi\pi\). The main errors come from the uncertainties of the input parameters in the wave functions of the \(B_{(s)}\) meson and the final state mesons: \(\omega_B = 0.40 \pm 0.04\) and \(\omega_{B_s} = 0.50 \pm 0.05, a_2^B = -0.40 \pm 0.10, a_2^B = 0.30 \pm 0.05\) and \(a_2^{B_s} = 0.70 \pm 0.20, C_D = 0.5 \pm 0.1\) and \(C_{D_s} = 0.4 \pm 0.1\), respectively.

As a special feature of our PQCD framework, we can extract the branching ratios for the two-body decays \(B_{(s)} \to D(\rho', \rho'')\) from the corresponding quasi-two-body decays if one knows the decay rates of \((\rho', \rho'') \to \pi\pi\) transitions reliably. In Ref. [37], the authors found

\[
B(\rho' \to \pi\pi) = \Gamma_{\rho' \to \pi\pi} / \Gamma_{\rho'} = (10.04 \pm 2.25) \times 10^{-2},
\]

by using the formula

\[
\Gamma_{\rho' \to \pi\pi} = \frac{g^{2}_{\rho' \pi \pi}}{6\pi} \left( \frac{m_{\rho'}^{2}}{m_{\pi}^{2}} \right)^{3}
\]

and the measured value of \(\Gamma_{\rho'} = 0.311 \pm 0.062\) GeV [63]. The value of \(B(\rho' \to \pi\pi) \approx 10\%\) [37] is also consistent with the range \([4.56\%, 10.0\%]\) as predicted in Refs. [63, 64]. By using the same method, we find \(f_{\rho''} = 0.103_{-0.012}^{+0.017}\) GeV [42] when \(\Gamma_{\rho'' \to \pi\pi} = 0.69 \pm 0.15\) keV [63] is adopted. Again we find [42]

\[
B(\rho'' \to \pi\pi) = (8.11_{-1.47}^{+2.22}) \times 10^{-2}.
\]

Of course, we know that the resonance parameters for \(\rho''\) are still not well determined [65]; more theoretical studies and experimental measurements are indeed required to improve the estimation for those parameters.
TABLE III: The PQCD predictions for the branching ratios of the quasi-two-body decays $B_{(s)} \to D \rho' \to D \pi \pi$ and the two-body decays $B_{(s)} \to D \rho$. 

| Decay modes | Quasi-two-body decays $B$ | Two-body decays $B$ |
|-------------|--------------------------|---------------------|
| $B^+ \to \bar{D}^0 \rho'^{++} \to \bar{D}^0 \pi^+ \pi^0$ | $(5.88 \pm 2.62) \times 10^{-3}$ | $(5.65 \pm 2.26) \times 10^{-3}$ |
| $B^0 \to D^- \rho'' \to D^- \pi^+ \pi^- \to D^0 \pi^+ \pi^0$ | $(3.30 \pm 2.09) \times 10^{-3}$ | $(4.07 \pm 2.60) \times 10^{-3}$ |
| $B^0 \to D^0 \rho'' \to D^0 \pi^+ \pi^- \to D^- \pi^+ \pi^- \to D^0 \pi^+ \pi^0$ | $(5.68 \pm 3.14) \times 10^{-3}$ | $(7.60 \pm 4.11) \times 10^{-4}$ |
| $B^0 \to D^\ast_\rho \rho'' \to D^\ast_\rho \pi^+ \pi^- \to D^\ast_\rho \pi^+ \pi^- \to D^0 \pi^+ \pi^0$ | $(2.08 \pm 0.43) \times 10^{-3}$ | $(2.56 \pm 0.97) \times 10^{-4}$ |
| $B^0 \to D^\ast_\rho \rho'' \to D^\ast_\rho \pi^+ \pi^- \to D^- \pi^+ \pi^- \to D^0 \pi^+ \pi^0$ | $(1.04 \pm 0.21) \times 10^{-3}$ | $(1.28 \pm 0.51) \times 10^{-4}$ |
| $B^0 \to D^\ast_\rho \rho'' \to D^\ast_\rho \pi^+ \pi^- \to D^- \pi^+ \pi^- \to D^0 \pi^+ \pi^0$ | $(2.57 \pm 0.86) \times 10^{-3}$ | $(3.17 \pm 1.82) \times 10^{-5}$ |

FIG. 1: (a) The differential branching ratios for the $B(B^0 \to \bar{D}^0 (\rho^0, \rho'') \to \bar{D}^0 \pi^+ \pi^-)$ decays. (b) The branching ratio of $B(B^0 \to D^0 (\rho^0, \rho'') \to D^0 \pi^+ \pi^-)$ decays with $a_2 = [-0.8, -0.3]$. 

By using the simple relation between the decay rate of the quasi-two-body decay and the corresponding two-body ones

$$B(B_{(s)} \to D(\rho', \rho'') \to D \pi \pi) = B(B_{(s)} \to D(\rho', \rho'') \to \pi \pi),$$

one can extract the branching ratios $B(B_{(s)} \to D \rho')$ and $B(B_{(s)} \to D \rho'')$ from the PQCD predictions for the branching ratios of those quasi-two-body decays $B_{(s)} \to D(\rho', \rho'') \to D \pi \pi$, if we take $B(\rho' \to \pi \pi)$ and $B(\rho'' \to \pi \pi)$ as given in Eqs. (13) and (15) as input. In the last column of Tables II and III, we listed the PQCD predictions for $B(B_{(s)} \to D \rho')$ and $B(B_{(s)} \to D \rho'')$, where the individual errors have been added in quadrature.

From our studies and the PQCD predictions as listed in above tables, we have the following observations:

(1) Unlike the fixed kinematics of the two-body $B_{(s)}$ meson decays, the decay amplitudes of the quasi-two-body $B_{(s)}$ meson decays considered in this paper do have a strong dependence on the $\pi^+ \pi^-$ invariant mass $s = w^2$. In Fig. 1(a), we plot the $w$-dependence of the differential decay rates for $B^0 \to \bar{D}^0 \rho^0 \to \bar{D}^0 \pi^+ \pi^-(\text{the red dots curve})$ and $B^0 \to \bar{D}^0 \rho'' \to \bar{D}^0 \pi^+ \pi^-(\text{the blue solid curve}).$ As discussed in Refs. [37, 38], the main contribution to the branching ratios lies in the region around the pole mass of the resonance $m_{\rho'} = 1.45 \text{ GeV}$ and $m_{\rho''} = 1.7 \text{ GeV}$. Numerically,
\(B(B^0 \to D^0 \rho^0 \to D^0 \pi^+ \pi^-)\) is a little larger than \(B(B^0 \to D^0 \rho^0 \to D^0 \pi^+ \pi^-)\), since the relevant parameters such as \(c_{\rho'}\) and \(c_{\rho''}\) are a little different for the decay involving \(\rho'\) or \(\rho''\). Such kinds of differences also exist for other decay channels; one can easily find them from the values as listed in Tables II and III.

(2) Our prediction for the central value of \(B(B^0 \to D^0 \rho^0 \to D^0 \pi^+ \pi^-) \approx 0.90 \times 10^{-5}\) is less than the experimental result reported by LHCb [5]: 1.36 (1.91) \(\times 10^{-5}\) in the Isobar model (K-matrix model). For \(B^0 \to D^0 \rho^0 \to D^0 \pi^+ \pi^-\), furthermore, our prediction is \(B(B^0 \to D^0 \rho^0 \to D^0 \pi^+ \pi^-) \approx 0.57 \times 10^{-5}\), while the measured value from LHCb was 0.33 (0.73) \(\times 10^{-5}\) in the Isobar model (K-matrix model) [5]. If we take the still large theoretical errors into account, our PQCD predictions as listed in Tables II and III do agree well with those currently available data.

(3) The dominant theoretical error comes from the uncertainty of \(\omega_{B^0} = 0.40 \pm 0.04\) and \(\omega_{B_s} = 0.50 \pm 0.05\): about 20\% – 50\% of the central values. The PQCD predictions also have a strong dependence on the magnitude of the Gegenbauer coefficients, specifically on the value of \(a_2^t\). In Fig. 1(b), we plot the PQCD predictions for the branching ratios of the decay \(B(B^0 \to D^0 \rho^0 \to D^0 \pi^+ \pi^-)\) (the red curve) and \(B(B^0 \to D^0 \rho^0 \to D^0 \pi^+ \pi^-)\) (the blue curve) in the range of \(a_2^t = [-0.8, -0.3]\) (although in this paper, we assume \(a_2^t = -0.4 \pm 0.1\)) while other parameters take their central values. For some decay modes, we observe similar strong \(a_2^t\)-dependence, as listed in Table II.

\[
\begin{align*}
B(B_s^0 \to D^- \rho'^+ & \to D^- \pi^+ \pi^0) = 4.21^{+1.10}_{-0.81}(a_2^t) \times 10^{-7}, \\
B(B_s^0 \to D^0 \rho^0 & \to D^0 \pi^+ \pi^-) = 1.88^{+0.57}_{-0.34}(a_2^t) \times 10^{-7}, \\
B(B^+ \to D^+ \rho^0 & \to D^+ \pi^+ \pi^-) = 5.88^{+1.46}_{-1.17}(a_2^t) \times 10^{-8}, \\
B(B^0 \to D^0 \rho^0 & \to D^0 \pi^+ \pi^-) = 9.75^{+4.05}_{-2.86}(a_2^t) \times 10^{-10}, \\
B(B^+ \to D_s^+ \rho^0 & \to D_s^+ \pi^+ \pi^-) = 1.38^{+0.42}_{-0.34}(a_2^t) \times 10^{-6}, \\
B(B^0 \to D_s^0 \rho^0 & \to D_s^0 \pi^- \pi^0) = 2.56^{+0.79}_{-0.60}(a_2^t) \times 10^{-6}. \\
\end{align*}
\]

It is easy to see that the theoretical error due to \(a_2^t = -0.4 \pm 0.1\) amounts to 20\% – 40\% to the central values for the decays in Eq. (17). For other remaining decays, the corresponding error due to \(a_2^t\) is only about 10\%. The same situation appears for the considered \(B(s) \to D \rho' \to D \pi \pi\) decays.

(4) We find a new way to estimate the decay rates of the two-body decays \(B(s) \to D(\rho', \rho'')\). The PQCD predictions for \(B(B(s) \to D(\rho', \rho''))\) are listed in the third column of Tables II and III. When compared with the numerical results for \(B(B(s) \to D \rho)\) decays as listed in Tables I and II of Ref. [38], we find that the PQCD predictions for the branching ratios of the similar decay modes but involving different \(\rho\) or \(\rho'\) as one of the two final state mesons are similar in magnitude: for example, \(B(B^+ \to D^0 \rho^+) \approx 0.87 \times 10^{-2}\) vs \(B(B^+ \to D^0 \rho^+) = 1.15 \times 10^{-2}\), and \(B(B^+ \to D^+ \rho^0) = 5.86 \times 10^{-7}\) vs \(B(B^+ \to D^+ \rho^0) = 5.33 \times 10^{-7}\).

IV. SUMMARY

In this paper, we calculated the branching ratios of the quasi-two-body \(B(s) \to D(\rho', \rho'')\) decays by employing the PQCD factorization approach. The contributions from the \(\rho'\) and \(\rho''\) resonant states were singled out from the given timelike form factor \(F_\pi\) in the \(P\)-wave two-pion distribution amplitudes \(\Phi_\pi\). With the estimated branching fraction for \(\rho' \to \pi \pi\) and \(\rho'' \to \pi \pi\), we have also extracted the theoretical predictions for the branching ratios for the two-body decays \(B(s) \to D \rho'\) and \(B(s) \to D \rho''\). From the analytical and numerical calculations, we found the following points:

(1) The PQCD predictions for the branching ratios of the considered quasi-two-body decays \(B(s) \to D(\rho', \rho'')\) are in the range of \(10^{-10} \sim 10^{-4}\). Those decay channels with large decay rate, say \(\geq 10^{-6}\), could be measured and tested at the future LHCb and Belle-II experiments.

(2) The PQCD predictions for \(B(B^0 \to D^0(\rho^0, \rho'\rho'') \to D^0 \pi^+ \pi^-)\) agree well with the measured values as reported by LHCb if one takes still large theoretical errors into account.

(3) One can extract the decay rates for the two-body decays \(B(s) \to D(\rho', \rho'')\) from the PQCD predictions for the branching ratios of the corresponding quasi-two-body decays \(B(s) \to D(\rho', \rho'')\) to \(D \pi \pi\).

(4) The PQCD predictions for \(B(B(s) \to D \rho')\) and \(B(B(s) \to D \rho)\) are similar in magnitude. It is an interesting relation to be tested by the future experimental measurements.
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