Fits of the Electroweak Standard Model and Beyond using Gfitter

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The global fit of the Standard Model to electroweak precision data, routinely performed by the LEP electroweak working groups and others, has been revisited in view of (i) the development of the new generic fitting package, Gfitter, (ii) the insertion of constraints from direct Higgs searches at LEP and Tevatron, and (iii) a more thorough statistical interpretation of the results. This paper describes the Gfitter project, and presents state-of-the-art results for the global electroweak fit in the Standard Model, and for a model with an extended Higgs sector. Example results are an estimation of the mass of the Higgs boson ($M_H = 116.4^{+18.3}_{-13.3}$ GeV) and a forth-order result for the strong coupling strength ($\alpha_S(M_Z^2) = 0.1193^{+0.0028}_{-0.0027}(\text{exp}) \pm 0.0001(\text{theo})$). Using toy Monte Carlo techniques the $p$-value of the SM has been determined ($p = 0.22$). As an example of a New Physics model constraints are derived for the Two Higgs Doublet Model of Type-II using observables from the $B$ and $K$ physics sectors.

1. INTRODUCTION

Precision measurements allow us to probe physics at much higher energy scales than the masses of the particles directly involved in experimental reactions by exploiting contributions from quantum loops. A prominent example is the global fit of the Standard Model (SM) to electroweak precision data, routinely performed by the LEP electroweak working group and others (for latest results see [1]), which demonstrated impressively the predictive power of electroweak unification and quantum loop corrections. Several theoretical libraries within and beyond the SM have been developed in the past, which allowed to constrain the unbound parameters of the SM and models of New Physics. However, most of these programs are relatively old, were implemented in outdated programming languages, and are difficult to maintain in line with the theoretical and experimental progress expected during the forthcoming era of the LHC. These considerations led to the development of the generic fitting package Gfitter [2, 3], designed to provide a modular framework for complex fitting tasks in high-energy physics, like model testing and parameter estimation problems. Gfitter is implemented in C++ and relies on ROOT functionality. It consists of a core package providing the tools for data handling, fitting and statistical analyses and allows a consistent treatment of statistical, systematic and theoretical errors, possible correlations, and inter-parameter dependencies. Tools provided for statistical analyses include e.g. parameter scans, contours, Monte Carlo (MC) toy analyses and goodness-of-fit $p$-value evaluation. More details on the framework can be found at [2]. Gfitter performs the minimisation of a $\chi^2$ test statistics quantifying the deviation of the experimental data from the predictions in a certain physics model. The theoretical calculations are implemented via plug-in libraries for the Gfitter framework. In this paper we report results which are obtained using the first libraries implemented in the Gfitter package: SM predictions of the electroweak precision observables and predictions of $B$ and $K$ physics observables in a model with two Higgs doublets (2HDM).

2. THE GLOBAL ELECTROWEAK FIT

In the global electroweak fit with Gfitter state-of-the-art calculations are compared with the most recent experimental data to constraint the free parameters of the fit and to test the goodness-of-fit. The SM parameters relevant for the global electroweak analysis are the coupling constants of the electromagnetic, weak and strong interactions, and the masses of the elementary bosons and fermions. Electroweak unification and simplifications arising from fixing parameters with insignificant uncertainties compared to the sensitivity of the fit allow to reduce the number of free

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fit parameters. The remaining free parameters are the coupling parameters $\Delta c_{\text{had}}^{(5)}(M_Z^2)$ and $\alpha_S(M_Z^2)$, the masses $M_Z$, $m_c$, $m_b$, $m_t$ and $M_H$. In addition, four free parameters enter to include the theoretical uncertainties of $M_W$, $\sin^2 \theta^\text{eff}$ and the electroweak form factors $\rho_{W}$ and $\kappa_{Z}$.

For the prediction of the electroweak precision observables as measured by the LEP, SLC and Tevatron experiments the most up-to-date calculations are implemented in the Gfitter SM library using the OMS scheme. Wherever possible the results have been cross-checked against the ZFITTER package [4]. The full two-loop and leading beyond-two-loop correction are available for the computation of $M_W$ and $\sin^2 \theta^\text{eff}$ [6]. The partial and total widths of the $Z$ are known to leading order, while for the second order only the leading $m_t^2$ corrections are available. Among the new developments included is the NNNLO perturbative calculation of the massless QCD Adler function [7], contributing to the vector and axial-vector radiator functions in the prediction of the $Z$ hadronic width (and other observables). It allows to fit the strong coupling constant with unique theoretical accuracy. More details on the theoretical computations in Gfitter can be found in [8].

The following experimental measurements are used: The mass and width of the $Z$ boson, the hadronic pole cross section $\sigma_{\text{had}}^0$, the partial widths ratio $R_0$, and the forward-backward asymmetries for leptons $A_{FB}^0,\ell$, have been determined by fits to the $Z$ line-shape measured precisely at LEP (see [8] and references therein). Measurements of the $\tau$ polarisation at LEP [8] and the left-right asymmetry at SLC [8] have been used to determine the lepton asymmetry parameter $A_\ell$. The corresponding $c$ and $b$-quark asymmetry parameters $A_{c(b)}$, the forward-backward asymmetries $A_{FB}^{0,c(b)}$, and the widths ratios $R_c^0$ and $R_b^0$, have been measured at LEP and SLC [8]. In addition, the forward-backward charge asymmetry measurement in inclusive hadronic events at LEP was used to directly determine $\sin^2 \theta^\text{eff}$ [8]. For the running quark masses $m_c$ and $m_b$, the world average values are used. For $\Delta c_{\text{had}}^{(5)}(M_Z^2)$ we use the most recent phenomenological result [8]. Results presented in this paper are obtained using the combined LEP and Tevatron results on the mass and the width of the $W$ boson [8], $M_W = (80.399 \pm 0.025) \text{ GeV}$, $\Gamma_W = (2.098 \pm 0.048) \text{ GeV}$, and the latest combined result on the top mass [11], $m_t = (172.4 \pm 1.2) \text{ GeV}$, presented at this conference.

The direct searches for the SM Higgs Boson at LEP [12] and the most recent results from the Tevatron [13, 14], leading to a 95% confidence level (CL) exclusion for $M_H < 114.4 \text{ GeV}$ and at $M_H = 170 \text{ GeV}$ respectively, are included using a Gaussian approach that quantifies the difference between the observed test statistics (the log-likelihood ratios) and the expected values for the s+b hypothesis using the published values of the respective confidence level (CL$_{s+b}$). A contribution to the $\chi^2$ estimator of the fit is derived for each Higgs mass. We perform global fits in two versions: the standard (“blue-band”) fit makes use of all the available information except for the direct Higgs searches; the

![Figure 1](https://example.com/image1.png)
complete fit uses also the constraints from the direct Higgs searches.

Due to the restricted space available only example results of the electroweak fit are reported in the following. More fit results and thorough studies of its statistical properties are given in \[3\] where also the perspectives for the LHC, ILC and GigaZ data are discussed. The standard (complete) fit converges at the global minimum value $\chi^2_{\text{min}} = 16.4$ ($\chi^2_{\text{min}} = 18.0$) for 13 (14) degrees of freedom. The estimation for $M_H$ from the standard fit without the direct Higgs searches is $M_H = 80^{+30}_{-23}$ GeV and the 2σ and 3σ intervals are respectively $[39, 156]$ GeV and $[26, 209]$ GeV. The complete fit represents the most accurate estimation of $M_H$ considering all available data. We find $M_H = 116.4^{+18.3}_{-1.3}$ GeV. The resulting $\Delta \chi^2$ curve versus $M_H$ is shown in Fig. 1 (left). The shaded band indicates the influence of theoretical uncertainties. The one, two and three standard deviations from the minimum are indicated by the crossings with the corresponding horizontal lines. The 2σ and 3σ allowed regions of $M_H$, including all errors, are $[114, 145]$ GeV and $[[113, 168]$ and $[180, 225]]$ GeV, respectively. The inclusion of the direct Higgs search results from LEP leads to a strong rise of the $\Delta \chi^2$ curve below $M_H = 115$ GeV. The data points from the searches at the Tevatron, available in the range $110 \text{ GeV} < M_H < 200$ GeV increases the $\Delta \chi^2$ estimator for Higgs masses above 140 GeV beyond that obtained from the standard fit.

The strong coupling at the Z-mass scale is determined by the complete fit with $\alpha_S(M_Z^2) = 0.1193^{+0.0028}_{-0.0027} \pm 0.0001$ where the first error is experimental and the second due to the truncation of the perturbative QCD series.

Figure 1(right) compares the direct measurements of $M_W$ and $m_t$, shown by the shaded/green 1σ bands, with the 68%, 95% and 99% CL obtained for three sets of fits: the largest/blue (narrower/yellow) allowed regions are derived from the standard fit (complete fit) excluding the measured values in the fits. The inclusion of the LEP and Tevatron Higgs searches significantly impacts the constraints obtained. Good agreement between direct measurements and indirect fit results is observed. The third set of fits (narrowest/green) results from the complete fit including the measured values. Hence it uses all available information and leads to the narrowest allowed region.

The $p$-value of the global SM fit, quantifying the probability of wrongly rejecting the SM hypothesis, has been evaluated by means of toy MC experiments. For each MC experiment, the complete fit is performed yielding the $\chi^2_{\text{min}}$ distribution shown by the light shaded histogram in Fig. 2 (left). The monotonously decreasing curves give the $p$-value of the SM fit as a function of $\chi^2_{\text{min}}$ obtained by integrating the distribution between $\chi^2_{\text{min}}$ and infinity. The value of the global SM fit is given by $p$-value (data/S) $= 0.22 \pm 0.01 \pm 0.02$, where the first error is statistical and the second accounts for the shift resulting from theoretical uncertainties.

3. CONSTRAINTS IN THE 2HDM

As an example for a study beyond the SM we investigate models with an extended Higgs sector of two doublets. In the Type-II 2HDM, we constrain the mass of the charged Higgs and the ratio of the vacuum expectation values of the two Higgs doublets using current measurements of observables from the $B$ and $K$ physics sectors and their most recent theoretical 2HDM predictions, namely $R_b^0$ \[8\,\[15\], the branching ratio (BR) of $B \to X_s \gamma$ \[16\,\[17\], the BR of leptonic decays of charged pseudoscalar mesons ($B \to \tau \nu$ \[18\,\[19\], $B \to \mu \nu$ \[19\,\[20\] and $K \to \mu \nu$ \[21\]) and the BR of the semileptonic decay $B \to D \tau \nu$ \[22\,\[23\].

For each observable, individual constraints have been derived in the $(\tan \beta, M_{H^\pm})$ plane. Figure 2(right) displays the resulting 95% excluded regions derived assuming Gaussian behaviour of the test statistics, and one degree of freedom. The figure shows that $R_b^0$ is mainly sensitive to $\tan \beta$ excluding small values. BR($B \to X_s \gamma$) is only sensitive to $\tan \beta$ for values below $\approx 1$. For larger values it provides an almost constant exclusion of a charged Higgs lighter than $\approx 260$ GeV. For all leptonic observables the 2HDM contribution can be either positive or negative since signed terms enter the prediction of the BRs resulting in a two-fold ambiguity in the $(\tan \beta, m_{H^\pm})$ space.

In addition, we have performed a global fit combining the information from all observables. For the CL calculation in the 2-dim plane we performed toy MC tests in each scan point which allows to avoid the problem of ambiguities in the effective number of degrees of freedom. The 95% CL excluded region obtained are indicated in Fig. 2(right) by the area below the single solid line. We can exclude a charged Higgs mass below 240 GeV independently of $\tan \beta$. This limit increases towards larger $\tan \beta$, e.g., $M_{H^\pm} < 780$ GeV are excluded for $\tan \beta = 70$. 

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Figure 2: Left: Result of the MC toy analysis of the complete fit of the electroweak SM. Shown are the $\chi^2_{\text{min}}$ distribution of a toy MC simulation (open histogram), the corresponding distribution for a complete fit ignoring theoretical uncertainties (shaded/green histogram), an ideal $\chi^2$ distribution assuming a Gaussian case with $n_{\text{dof}} = 14$ (black line) and the $p$-value as a function of the $\chi^2_{\text{min}}$ of the global fit; Right: 95% CL exclusion regions in the ($\tan\beta, M_{H^\pm}$) plane from individual 2HDM constraints and the toy-MC-based result (hatched) from the combined fit overlaid.

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