Stability of the de Sitter spacetime in Horava-Lifshitz theory

Yongqing Huang \textsuperscript{a,b} \footnote{Electronic address: yongqing.huang@baylor.edu}, Anzhong Wang \textsuperscript{a} \footnote{Electronic address: anzhong.wang@baylor.edu} and Qiang Wu \textsuperscript{a} \footnote{Electronic address: qiang.wu@baylor.edu}

\textsuperscript{a} GCAP-CASPER, Physics Department, Baylor University, Waco, TX 76798-7316, USA
\textsuperscript{b} Department of Physics, Zhejiang University of Technology, Hangzhou 310032, China

(Dated: February 28, 2011)

The stability of de Sitter spacetime in Horava-Lifshitz theory of gravity with projectability but without detailed balance condition is studied. It is found that, in contrast to the case of the Minkowski background, the spin-0 graviton now is stable for any given $\xi$, and free of ghost for $\xi \leq 0$ in the infrared limit, where $\xi$ is the dynamical coupling constant.

PACS numbers: 04.60.-m; 98.80.Cq; 98.80.-k; 98.80.Bp

I. INTRODUCTION

Formulating a proper theory of quantum gravity has been one of the most challenging questions in physics over the past several decades \cite{1,2}. Despite of innumerable efforts, so far there are only two major candidates, the loop quantum gravity \cite{3} and string/M theory \cite{4,5}. While the latter seems to be the best bet for such a theory that unifies all the known forces, it is often too complicated to deal with. The former, on the other hand, has its own challenging problems.

Recently, Horava proposed a theory of quantum gravity \cite{6–8}, motivated by the Lifshitz theory in solid state physics \cite{9}. The Horava-Lifshitz (HL) theory is based on the perspective that Lorentz symmetry should appear as an emergent symmetry at long distances, but can be fundamentally absent at high energies \cite{10,11}. At low energies, the theory is expected to flow to GR, whereby the Lorentz invariance is “accidentally restored.” The theory is non-relativistic, ultra-violet (UV) complete, explicitly breaks Lorentz invariance at short distances, but is expected to recover GR in the infrared (IR) limit.

The effective speed of light in this theory diverges in the UV regime, which could potentially resolve the horizon problem without invoking inflation \cite{12}. The spatial curvature is enhanced by higher-order curvature terms \cite{13–16}, and this opens a new approach to the flatness problem and to a bouncing universe \cite{13–16,17,18}. In addition, in the super-horizon region scale-invariant curvature perturbations can be produced without inflation \cite{19,22}, and the perturbations become adiabatic during slow-roll inflation driven by a single scalar field and the comoving curvature perturbation is constant \cite{22}. Due to all these remarkable features, the theory has attracted lot of attention lately \cite{23–151}.\footnote{So far most of the work has abandoned the projectability condition but kept the detailed balance \cite{23,151}. However, with detailed balance a scalar field is not UV stable \cite{13,14}, and gravitational perturbations in the scalar section have ghosts \cite{6–8} and are not stable for any given value of the dynamical coupling constant $\xi (=1-\lambda)$ \cite{154}. In addition, detailed balance also requires a non-zero (negative) cosmological constant, breaks the parity in the purely gravitational sector \cite{157,158} and makes the perturbations not scale-invariant \cite{158}. Breaking the projectability condition, on the other hand, can cause strong couplings \cite{159,162} and gives rise to an inconsistency theory \cite{152,153}.}

To resolve these problems, various modifications have been proposed \cite{163–171}. In particular, Blas, Pujolas and Sibiryakov (BPS) \cite{172,173} showed that the strong coupling problem can be solved without projectability condition when terms constructed from the gradient of the lapse function are included. However, as shown in \cite{174}, strong coupling may still exist, unless the scale appearing in front of the higher order terms is much lower than the Planck scale \cite{172,173}. It is also not clear how the inconsistency problem found in \cite{152,153} is resolved in such a setup. Moreover, in the IR limit the theory is identical to a special case of the Einstein-aether theory, where the vector field is hypersurface-orthogonal \cite{173}. But, the latter seemingly conflicts with observations \cite{176}.

On the other hand, Sotiriou, Visser and Weinhardt (SVW) formulated the most general HL theory with projectability condition but without the detailed balance \cite{152,157}. Writing the 4-dimensional metric in the ADM form,

$$ ds^2 = -N^2 c^2 dt^2 + g_{ij} \left( dx^i + N^i dt \right) \left( dx^j + N^j dt \right), $$ \hspace{1cm} (1.1)
the projectability condition requires that
\[ N = N(t), \quad N^i = N^i(t, x), \quad g_{ij} = g_{ij}(t, x). \] (1.2)
Note that in [16, 22, 177], the constant \( c \) representing the speed of light was absorbed into \( N \). The ADM form (1.1) is preserved only by the types of coordinate transformations,
\[ t \to f(t), \quad x^i \to \zeta^i(t, x). \] (1.3)
Due to these restricted diffeomorphisms, one more degree of freedom appears in the gravitational sector - a spin-0 graviton. This is potentially dangerous, and needs to be highly suppressed in the IR regime, in order to be consistent with observations. Similar problems also raise in other modified theories, such as massive gravity [178].

Then, it can be shown that the most general action, which preserves the parity and is with projectability but without detailed balance condition, is given by [154, 157],
\[ S = \zeta^2 \int dt dx^3 \sqrt{g} \left( L_K - L_V + \zeta^{-2} L_M \right), \] (1.4)
where \( g = \det g_{ij}, \) \( L_M \) denotes the matter Lagrangian density, and
\[
\begin{align*}
L_K &= K_{ij} K^{ij} - (1 - \xi) K^2, \\
L_V &= 2\Lambda - R + \frac{1}{\zeta} (g_{ij} R^2 + g_i R_j R^{ij}) \\
&\quad + \frac{1}{\zeta^2} \left( g_{ij} R^2 + g_i R_j R^{ij} + g_k R_j R^j R^k \right) \\
&\quad + \frac{1}{\zeta^2} \left[ g_{ij} R^2 R + g_8 (\nabla_i R_{jk}) (\nabla^i R^j k) \right],
\end{align*}
\] (1.5)
where \( \zeta^2 = 1/16\pi G, \) and the covariant derivatives and Ricci and Riemann terms are all constructed from the three-metric \( g_{ij}, \) while \( K_{ij} \) is the extrinsic curvature,
\[ K_{ij} = \frac{1}{2N} (-\delta_{ij} + \nabla_i N_j + \nabla_j N_i), \] (1.6)
where \( N_i = g_{ij} N^j. \) The constants \( \xi, g_I (I = 2, \ldots, 8) \) are coupling constants, and \( \Lambda \) is the cosmological constant. In the IR limit, all the high order curvature terms (with coefficients \( g_I \)) drop out, and the total action reduces when \( \xi = 0 \) to the Einstein-Hilbert action.

It should be noted that, although the SVW generalization seems very promising, the gravitational scalar perturbations in such a setup either have ghosts (\( 0 \leq \xi \leq 2/3 \)) or are not stable (\( \xi < 0 \)) in [16, 179]. In order to avoid ghost instability, one needs to assume \( \xi \leq 0. \) Then, the sound speed \( c_s^2 = \xi/(2 - 3\xi) \) becomes imaginary, which leads to an IR instability. Izumi and Mukohyama showed that this type of instability does not show up if \( |c_s| \) is less than a critical value [180].

In this brief report, we show explicitly that this is no longer the case in the de Sitter background. The gravitational scalar perturbations are stable for any given \( \xi, \) and are free of ghosts for \( \xi \leq 0. \) In particular, in the next section we shall give a brief review of scalar perturbations in a flat FRW background, while in Section III we consider perturbations in a de Sitter background. The paper is ended with Section IV, where our main conclusions are presented.

It should be noted that gravitational scalar perturbations in the HL theory with detailed balance was studied in [181], while the stability of the Einstein static universe was considered in [182, 183].

II. SCALAR PERTURBATIONS IN FLAT FRW BACKGROUNDS

We give a very brief introduction to the scalar perturbations of flat FRW background in the HL gravity without detailed balance, but with the projectability condition. For detail, we refer readers to [16, 22]. The homogeneous and isotropic flat universe is described by the FRW metric, \( ds^2 = a^2(\eta) (-d\eta^2 + \delta_{ij} dx^i dx^j). \) For this metric, \( K_{ij} = -a\dot{H}\delta_{ij} \) and \( R_{ij} = 0, \) where \( \dot{H} = a'/a \) and an overbar denotes a background quantity. Then, the Hamiltonian constraint yields,
\[ \left( 1 - \frac{3}{2}\xi \right) \frac{\mathcal{H}^2}{a^2} = \frac{8\pi G}{3} \rho + \frac{\Lambda}{3}, \] (2.1)
while the dynamical equations give rise to
\[ \left( 1 - \frac{3}{2}\xi \right) \frac{\dot{\mathcal{H}}}{a^2} = -\frac{4\pi G}{3} (\dot{\rho} + 3\dot{\rho}) + \frac{1}{3} \Lambda, \] (2.2)
where \( \dot{\rho} \) and \( \ddot{\rho} \) denote the energy density and pressure of matter of the FRW background. Similarly to GR, the super-momentum constraint is then satisfied identically, while the conservation laws of energy and momentum yield,
\[ \ddot{\rho} + 3\mathcal{H} (\dot{\rho} + \ddot{\rho}) = 0. \] (2.3)
This can be also obtained from Eqs. (2.1) and (2.2).

Clearly, replacing \( G \) and \( \Lambda, \) respectively, by \( G/(1 - 3\xi/2) \) and \( \Lambda/(1 - 3\xi/2), \) Eqs. (2.1) and (2.2) reduce exactly to the ones given in GR.

Linear scalar perturbations of the metric are given by
\[
\begin{align*}
\delta g_{ij} &= a^2(\eta) (-2\psi\delta_{ij} + 2E_{ij}), \\
\delta N_i &= a^2(\eta) B_i, \\
\delta N &= a(\eta) \phi(\eta),
\end{align*}
\] (2.4)
Choosing the quasi-longitudinal gauge [16],
\[ \phi = 0 = E, \] (2.5)
we find that the two gauge-invariant quantities defined in [16] reduce to,
\[ \Phi = \mathcal{H} B + B', \quad \Psi = \psi - \mathcal{H} B, \] (2.6)
and that to second order the actions take the forms,

\[
S_{K}^{(2)} = \zeta^2 \int d\eta^3 x^a \left\{ (3\xi - 2) \left[ 3\psi'' + 6\mathcal{H}\psi' + 2\psi'\partial^2 B \\
+ \frac{9}{2} \mathcal{H}^2 \psi^2 \right] + \xi B\partial^2 B \right\},
\]

\[
S_{V}^{(2)} = \zeta^2 \int d\eta^3 x^a \left\{ 2 (\partial(3\xi - 2) - 3\Lambda a^2 \psi^2 - \frac{2\alpha_1}{a^2} (\partial^2 \psi)^2 \\
+ \frac{2\alpha_2}{a^2} \psi\partial^2 \psi \right\},
\]

where \(\alpha_1 \equiv (8g_2 + 3g_3)/\zeta^2\) and \(\alpha_2 \equiv (3g_8 - 8g_7)/\zeta^4\).

The matter perturbations are written as

\[
\delta J' = -2\delta \mu, \quad \delta J^i = \frac{1}{a} q^i
\]

\[
\delta \tau^{ij} = \frac{1}{a^2} \left[ (\delta \mathcal{P} + 2\bar{\rho}\psi) \gamma^{ij} + \Pi^{(ij)} \right].
\]

The angular brackets on indices define the trace-free part:

\[
f_{(ij)} \equiv f_{ij} - \frac{1}{3} \delta_{ij} f_{kk}.
\]

In GR, \(\delta \mu\) reduces to the density perturbation \(\delta \rho\), and \(q, \delta \mathcal{P}, \Pi\) to, respectively, \(-\alpha(\bar{\rho} + \bar{\bar{\rho}})(v + B)\), the pressure perturbation \(\delta p\), and the scalar part of the anisotropic pressure.

To first-order the Hamiltonian constraint is

\[
\int d^3 x \left[ \partial^2 \psi - \left( 1 - \frac{3\xi}{2} \right) \mathcal{H} (\partial^2 B + 3\psi') - 4\pi G a^2 \delta \mu \right] = 0.
\]

The integrand is a generalization of the Poisson equation in GR [184]. The supermomentum constraint, on the other hand, reads

\[
(2 - 3\xi) \psi' = \xi \partial^2 B + 8\pi G a^2 q, \tag{2.12}
\]

which generalizes the general relativity \(0i\) constraint [184]. The trace and trace-free parts of the perturbed dynamical equations yield, respectively,

\[
\psi'' + 2\mathcal{H}\psi' - \frac{\xi}{2 - 3\xi} \left( 1 + \frac{\alpha_1}{a^2} \partial^2 + \frac{\alpha_2}{a^2} \partial^4 \right) \partial^2 \psi
\]

\[
= \frac{8\pi G a^2}{3(2 - 3\xi)} \left[ 3\delta \mathcal{P} + (2 - 3\xi) \partial^2 \Pi \right], \tag{2.13}
\]

\[
(a^2 B)' \equiv \left( a^2 + \alpha_1 \partial^2 + \frac{\alpha_2}{a^2} \partial^4 \right) \psi - 8\pi G a^4 \Pi. \tag{2.14}
\]

The conservation laws give

\[
\int d^3 x \left[ \delta p' + 3\mathcal{H}(\delta \mathcal{P} + \delta \mu) - 3(\bar{\rho} + \bar{\bar{\rho}}) \psi' \right] = 0, \tag{2.15}
\]

\[
q' + 3\mathcal{H}q = a\delta \mathcal{P} + \frac{2}{3} a\partial^2 \Pi. \tag{2.16}
\]

### III. STABILITY OF THE DE SITTER SPACETIME IN THE IR LIMIT

To see how the ghost and instability problems of scalar perturbations are avoided in the de Sitter background, it is instructive first to recall how they raise in the Minkowski background [16, 172]. Since in this section we are mainly concerned with IR limit, the terms proportional to \(\alpha_1\) and \(\alpha_2\) are highly suppressed by the Planck scales \(M^2_{pl}\) and \(M^4_{pl}\), respectively. Then, in the following discussions it is quite safe to neglect all these terms.

In the Minkowski background, without matter perturbations, Eq. (2.12) in the momentum space gives,

\[
k^2 B_k = \frac{3\xi - 2}{\xi} \psi'_k, \tag{3.1}
\]

for \(\xi \neq 0\). Then, Eq. (2.13) becomes

\[
\frac{1}{c^2_{\psi}} \psi''_k + k^2 \psi_k = 0, \tag{3.2}
\]

where \(c^2_{\psi} \equiv \xi/(2 - 3\xi)\). Clearly, it is stable only when \(c^2_{\psi} \geq 0\), that is, \(0 < \xi \leq 2/3\). However, submitting Eq. (3.1) into Eqs. (2.7) and (2.8), we find that

\[
\mathcal{L} \equiv \mathcal{L}_K - \mathcal{L}_V = \left( \frac{1}{c^2_{\psi}} \psi'^2 - \left( \partial \psi \right)^2 \right). \tag{3.3}
\]

Therefore, unless \(c^2_{\psi} < 0\) (or \(\xi < 0\)), the spin-0 graviton is a ghost. But, when \(\xi < 0\) the scalar field becomes unstable. Note that the spin-0 graviton becomes stable when \(\xi = 0\) [16].

The de Sitter spacetime is given by \(a(\eta) = 1/(H\eta)\), where \(\eta \leq 0\). In particular, \(\eta = -\infty\) corresponds to the initial \((t = 0)\) of the universe, while \(\eta = 0\) to the future infinity \((t = \infty)\). When matter is not present, we have

\[
q = \delta \mathcal{P} = \delta \mu = \Pi = 0, \tag{3.4}
\]

and the momentum constraint (2.12) yields the same equation (3.1) for \(\xi \neq 0\). Then, from Eqs. (2.7) and (2.8), we find that

\[
\mathcal{L} = \alpha^2 \left\{ \frac{2(3\xi - 2)}{\xi} \psi'^2_k + \left[ \frac{9}{2}(2 - \xi)(3\xi - 2)\mathcal{H}^2 \\
- 2k^2 \right] \psi''_k \right\}, \tag{3.5}
\]

where

\[
\psi_k = \alpha \psi_k, \quad \alpha = \frac{\alpha_0}{a^{3\xi/2}}, \tag{3.6}
\]

with \(\alpha_0\) being an arbitrary constant. Thus, to have the kinetic part non-negative, we must assume either \(\xi \leq 0\) or \(\xi \geq 2/3\). However, the GR limit requires \(\xi = 0\). Therefore, one needs to restrict to the range \(\xi \leq 0\). But, in
the gauge-invariant quantities $\Phi$ and $\Psi$ are given by

\[
\chi'' + \left( \frac{\xi k^2}{2 - 3\xi} - \frac{2}{\eta^2} \right) \chi_k = 0, \tag{3.7}
\]

where $\chi_k = a\psi_k$. Depending on the values of $\xi$, the above equation has different solutions. In the following we consider them separately.

**Case 1)** $\xi/(2 - 3\xi) < 0$: In this case, Eq. (3.7) has the general solution,

\[
\chi_k = c_1 \left( 1 - \frac{1}{z} \right) e^{+z} + c_2 \left( 1 + \frac{1}{z} \right) e^{-z}, \tag{3.8}
\]

where $c_1$ and $c_2$ are two integration constant, and

\[
z = \left[ \frac{\xi k^2}{2 - 3\xi} \right]^{1/2} \eta = z_0 \eta. \tag{3.9}
\]

Then, $\psi_k$ and $B_k$ are given by

\[
\psi_k = \tilde{c}_1 (z - 1) e^{+z} + \tilde{c}_2 (z + 1) e^{-z},
\]

\[
B_k = \frac{(3\xi - 2)z}{\xi k^2} \left( \tilde{c}_1 e^{z} - \tilde{c}_2 e^{-z} \right), \tag{3.10}
\]

which are all finite as $k\eta \to 0^-$ or $t \to \infty$, where $\tilde{c}_1 \equiv -H c_1/z_0$. Inserting the above into Eq. (2.6), we find that the gauge-invariant quantities $\Phi$ and $\Psi$ are given by

\[
\Phi_k = \frac{z_0 z(3\xi - 2)}{\xi k^2} \left( \tilde{c}_1 e^{z} + \tilde{c}_2 e^{-z} \right),
\]

\[
\Psi_k = \tilde{c}_1 \left[ z + \frac{z_0 (3\xi - 2) - \xi k^2}{\xi k^2} \right] e^{z} + \tilde{c}_2 \left[ z - \frac{z_0 (3\xi - 2) - \xi k^2}{\xi k^2} \right] e^{-z}, \tag{3.11}
\]

which are also finite in the IR limit $k\eta \to 0^-$.

**Case 2)** $\xi/(2 - 3\xi) > 0$: In this case, Eq. (3.7) has the general solution,

\[
\chi_k = c_1 \sin \left( z + c_2 \right) + \frac{c_1}{z} \cos \left( z + c_2 \right), \tag{3.12}
\]

while $\psi_k$, $B_k$ and the gauge-invariant quantities $\Phi$ and $\Psi$ are given, respectively, by

\[
\psi_k = \tilde{c}_1 \left[ z \sin \left( z + c_2 \right) + \cos \left( z + c_2 \right) \right],
\]

\[
B_k = \frac{\tilde{c}_1 z_0 (3\xi - 2)}{\xi k^2} z \cos \left( z + c_2 \right),
\]

\[
\Phi_k = -\frac{\tilde{c}_1 z_0^2 (3\xi - 2)}{\xi k^2} z \sin \left( z + c_2 \right),
\]

\[
\Psi_k = \frac{\tilde{c}_1}{\xi k^2} \left[ \xi k^2 + z_0^2 (3\xi - 2) \right] \cos \left( z + c_2 \right) + \tilde{c}_1 z \sin \left( z + c_2 \right), \tag{3.13}
\]

which in the IR limit $k\eta \to 0^-$ are finite, too.

**Case 3)** $\xi = 0$: In this case, Eq. (2.12) yields $\psi_k = \psi_k^0$, where $\psi_k^0$ is a constant, while Eq. (2.11) gives

\[
B_k = \psi_k^0 \eta + c_0 H^2 \eta^2, \tag{3.14}
\]

where $c_0$ is another integration constant. Clearly, both of these two terms represent decaying modes $(k\eta \to 0^-)$. Then, the corresponding $\Phi$ and $\Psi$ are given by

\[
\Phi_k = \Psi_k = c_0 H^2 \eta \simeq 0, \tag{3.15}
\]

as $\eta \to 0^-$. Therefore, it is concluded that for any given $\xi$ the de Sitter spacetime is stable against the gravitational scalar perturbations.

### IV. CONCLUSIONS

In this brief report, we have studied the stability of the de Sitter spacetime in the framework of HL theory of quantum gravity with projectability but without detailed balance condition. In contrast to the Minkowski case [16, 179], the gravitational scalar perturbations are stable in the IR limit for any given coupling constant $\xi$. The model is free of ghost for $\xi \leq 0$. Thus, restricting $\xi$ to this range, we can see that both of the ghost and instability problems disappear here in de Sitter background.

It should be noted that the analysis given in Section III is valid only in the IR limit, as we dropped all terms proportional to $\alpha_1$ and $\alpha_2$. In particular, in the UV regime, these terms become dominant, and the gravitational scalar perturbations will be quite different from the ones given by Eqs. (3.11), (3.13) and (3.15). However, by properly choosing the coupling constants $g_2$, $g_1$, $g_7$ and $g_8$, it can be shown that the solutions are free of ghosts and stable for $\xi \leq 0$ in the UV regime, too.

Finally, we note that the initial motivation of Horava was to construct a UV complete theory of quantum gravity, while in the IR limit it reduces to GR [3, 8]. So, in this sense the HL theory can be considered as an ultra-violet-modified gravity. Recently, infrared-modified gravities have also attracted a lot of attention [178]. Simple examples are the massive gravity [185] and the DGP models [186]. Although the HL theory and these infrared-modified gravities represent modifications of GR in two opposite regimes, surprisingly they face similar problems: ghosts, tachyons, and strong couplings. For example, in massive gravity, the massless spin-0 graviton does not decouple in the massless limit in flat space, the well-known vDVZ discontinuity [187, 188]. Although such a discontinuity can disappear when $M/H \to 0$ in the de Sitter background, ghosts appear for $0 < M^2 < 2A/3$ [189–191]. Ghosts and strong couplings also happen in DGP models [192, 194]. Interestingly, the strong coupling in the DGP models actually helps to screen the spin-0 mode so that the models are consistent with solar system tests [193, 196]. In addition, the vDVZ discontinuity also disappears in the anti de Sitter background [191, 197].
Acknowledgements: We would like to thank Roy Maartens and Antonios Papazoglou for valuable discussions and comments. Part of the work was supported by NNSFC under Grant 10703005 and No. 10775119 (AZ & QW).
I. Adam, I.V. Melnikov and S. Theisen, JHEP, 0909, 005 (2009) [arXiv:0909.2219].

P. Wu and H. Yu, arXiv:0909.2821.

G. Leon, and E.N. Saridakis, arXiv:0909.3571.

C. G. Boehmer and F.S. N. Lobo, arXiv:0909.3986.

B. Chen, S. Pi, and J.-Z. Tang, arXiv:0910.0338.

Y.S. Myung, arXiv:0911.0724.

S. Dutta and E.N. Saridakis, arXiv:0911.1435.

Y.S. Myung, Y.-W. Kim, W.-S. Son, and Y.-J. Park, arXiv:0911.2525.

I. Bakas, F. Bourliot, D. Lust, and M. Petropoulos, arXiv:0911.2665.

X. Gao, Y. Wang, W. Xue, and R. Brandenberger, arXiv:0911.3196.

M. R. Setare, and M. Jamil, JCAP, 02, 010 (2010) [arXiv:1001.1251].

Q. J. Cao, Y.X. Chen, and K.N. Shao, arXiv:1001.2597.

Y.S. Myung, Y.W. Kim, W.S. Son, and Y.J. Park, arXiv:1001.3921.

S.W. Wei, Y.X. Liu, and Y.Q. Wang, arXiv:1001.5238.

J.O. Gong, S. Koh, and Sasaki, arXiv:1002.1429.

T. Moon, P. Oh, and J. Sohn, arXiv:1002.2549.

T. Kobayashi, Y. Urakawa, and M. Yamaguchi, arXiv:1002.3101.

M. Visser, arXiv:0902.0290.

M. Visser, arXiv:0912.4757.

L. Maccione, A.M. Taylor, D.M. Mattingly, and S. Liberati, arXiv:0902.1756.

P.R. Carvalho and M. Leite, arXiv:0902.1972.

A. Volovich and C. Wen, arXiv:0903.2455.

A. Jenkins, arXiv:0904.0453.

H. Nikolic, arXiv:0904.3412.

H. Nastase, arXiv:0904.3604.

R.G. Cai, Y. Liu, and Y.W. Sun, arXiv:0904.4104.

G.E. Volovik, arXiv:0904.4113.

B. Chen and Q.-G. Huang, arXiv:0904.4565.

D. Orlando and S. Reffert, arXiv:0905.0301.

A. Volovich and C. Wen, arXiv:0905.0310.

T. Nishioka, arXiv:0905.0473.

A. Ghodsi, arXiv:0905.0836.

J.B. Jimenez and A.L. Maroto, arXiv:0905.1245.

R.A. Konoplya, arXiv:0905.1523.

Y.-W. Kim, H.W. Lee, and Y.S. Myung, arXiv:0905.3423.

M. Sakamoto, arXiv:0905.4326.

M. Botta-Canciello, N. Grandi, and M. Sturla, arXiv:0906.0582.

Y. S. Myung, arXiv:0906.0848.

A. Ghodsi and E. Hatefi, arXiv:0906.1237.

T. Harko, . Koves, and F.S. N. Lobo, arXiv:0907.1449.

S. Mukohyama, JCAP, 0909, 005 (2009) [arXiv:0906.5069].

A. Kobakhidze, arXiv:0906.5401.

T. Harko, Z. Kovaecs, and F.S. N. Lobo, Phys. Rev. D80, 044021 (2009) [arXiv:0907.1449].

I. Adam, I.V. Melnikov and S. Theisen, JHEP, 0909, 130 (2009) [arXiv:0907.2156].

N. Afshordi, arXiv:0907.5201.

Y.S. Myung, arXiv:0907.5296.

T. Harko, Z. Koves, and F.S. N. Lobo, arXiv:0908.2874.

Y.S. Myung, arXiv:0909.2075.

I. Cho and G. Kang, arXiv:0909.3065.

T. Suyama, arXiv:0909.4833.

L. Iorio and M.L. Ruggiero, arXiv:0909.5355.
Seahra, arXiv:1001.1266.

[184] K.A. Malik and D. Wands, Phys. Reports 475, 1 (2009).
[185] M. Fierz and W. Pauli, Proc. Roy. Soc. 173, 211 (1939).
[186] G.R. Dvali, G. Gabadadze, and M. Porrati, Phys. Lett. B484, 112 (2000).
[187] H. van Dam, M. Veltman, Nucl. Phys. B22, 397 (1970).
[188] V.I. Zakharov, JETP Lett. 12, 312 (1970).
[189] A. Higuchi, Nucl. Phys. B282, 397 (1987).
[190] A. Higuchi, Nucl. Phys. B325, 745 (1989).
[191] I.I. Kogan, S. Pouslopoulos, and A. Papazoglou, Phys. Lett. B503, 173 (2001).

[192] M.A. Luty, M. Porrati, and R. Rattazzi, JHEP, 09, 029 (2003).
[193] R. Durrer and R. Maartens, arXiv:0811.4132.
[194] K. Koyama, Class. Quant. Grav. 24, R231 (2007) [arXiv:0709.2399].
[195] C. Deffayet, G. Dvali, G. Gabadadze, and A. Vainshtein, Phys. Rev. D65, 044026 (2002).
[196] E. Babichev, C. Deffayet, R. Ziour, Phys. Rev. Lett. 103, 201102 (2009) [arXiv:0907.4103].
[197] M. Porrati, Phys. Lett. B498, 92 (2001).