Comparing contact and dipolar interaction in a Bose-Einstein condensate

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We have measured the relative strength $\varepsilon_{dd}$ of the magnetic dipole-dipole interaction compared to the contact interaction in a chromium Bose-Einstein condensate. We analyze the asymptotic velocities of expansion of a dipolar chromium BEC with different orientations of the atomic magnetic dipole moments. By comparing them with numerical solutions of the hydrodynamic equations for dipolar condensates, we are able to determine $\varepsilon_{dd} = 0.159 \pm 0.034$ with high accuracy. Since the absolute strength of the dipole-dipole interaction is known exactly, the relative strength of the dipole-dipole interaction can be used to determine the s-wave scattering length $a = 5.08 \pm 1.06 \cdot 10^{-3} \mathrm{m} = 96 \pm 20 \, a_0$ of $^{52}\text{Cr}$. This is fully consistent with our previous measurements on the basis of Feshbach resonances.

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Gaseous Bose-Einstein condensates (BEC) with dipole-dipole interaction (DDI) have become a fast growing field of theoretical and experimental interest. Many new exciting phenomena are expected. Due to the anisotropic character of the DDI, most of them depend strongly on the symmetry of the trap. The expected phenomena range from modifications of the ground state wave function \cite{1, 2}, the expansion \cite{3, 4, 5}, the excitation spectrum \cite{6, 7, 8}, and stability criteria \cite{9, 10, 11} to the occurrence of new quantum phases in optical lattices \cite{11} and dramatic influence on the formation of vortices and vortex lattices \cite{12, 13}. Dipolar BECs are now also discussed in the context of spinor condensates \cite{14, 15}, where the combination of large spin and magnetic moment leads to new effects like the conversion of spin into angular momentum \cite{11, 16}. For all of these phenomena, the relative strength of the DDI compared to the contact interaction is a very important parameter.

In two recent publications, we have reported on the generation of a BEC of chromium atoms ($^{52}\text{Cr}$) \cite{17} and the observation of magnetic dipole-dipole interaction (MDDI) in the BEC \cite{18}. In the latter one, we have shown that depending on the orientation of the magnetic moments of the condensed chromium atoms with respect to the long axis of our optical dipole-trap, the expansion dynamics of the BEC is modified. The dynamic behavior of the condensate aspect ratio after release from an anisotropic trap was studied experimentally and compared to numerical calculations based on the description of the chromium BEC by superfluid hydrodynamic theory including dipole-dipole interaction \cite{3, 4, 10}. The observed behavior showed an excellent qualitative agreement with the theoretical prediction. In this paper, we discuss a method that allows to determine the strength of the MDDI compared to the contact interaction with an accuracy on the percent level.

Since the absolute strength of the MDDI is known, one can use a measurement of the relative strength of the MDDI to determine the s-wave interaction strength, which is proportional to the s-wave scattering length $a$. Many different techniques have been used to determine the s-wave scattering lengths of ultra-cold atoms but most of them come along with large error bars - often because the number of atoms enters the measurement. Examples are the $^{23}\text{Na}$ scattering length in $|F = 1, m_F = -1\rangle$ of $a_{\text{Na}} = 92 \pm 25$ (i.e. 27% error) determined from thermalization measurements \cite{20}, and $a_{\text{Na}} = 65 \pm 30$ (i.e. 46% error) from the measurement of the mean field energy of a BEC \cite{21}. The scattering length of metastable He$^+$ $a_{\text{He}^+} = 16 \pm 8 \mathrm{nm}$ was determined from the mean field energy by analyzing the size of the BEC \cite{22} where the error of 50% stems from an uncertainty of the number of atoms in the condensate. Our first determination of the chromium scattering length \cite{23} was based on cross-dimensional thermalization measurements \cite{24} and resulted in $a_{\text{Cr}} = 170 \pm 39 \mathrm{a}_0$. The error was mostly due to an uncertainty in the density and atom-number determination. We will show in this paper, that a measurement of the relative strength of the magnetic dipole-dipole interaction in a BEC can be used to obtain precise values for $a$ without such a strong dependence on the determination of the number of atoms.

The interaction energy of two magnetic dipoles separated by the distance $\vec{r}$ is given by

$$U_{dd}(\vec{r}) = \frac{\mu_0 \mu^2 m}{4 \pi r^3} \left(1 - \frac{3 (\vec{e}_\mu \vec{r})^2}{r^2}\right)$$

where the strength of the dipole-dipole interaction is measured by the pre-factor of $U_{dd}$ and the orientation of the dipoles $\vec{e}_\mu$ is parallel to an external magnetic field $\vec{B}$. This strength can be compared to the coupling constant $g$ of the s-wave interaction

$$U_{sw}(\vec{r}) = g \delta(\vec{r}) = \frac{4 \pi \hbar^2 a}{m} \delta(\vec{r})$$

and is measured by the dimensionless dipole-dipole interaction

$$\varepsilon_{dd} = \frac{U_{dd}(\vec{r})}{U_{sw}(\vec{r})} = \frac{4 \pi \hbar^2 a}{m} \frac{\delta(\vec{r})}{\frac{4 \pi \hbar^2 a}{m} \delta(\vec{r})} = \frac{\delta(\vec{r})}{\delta(\vec{r})} = 1$$

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strength parameter

\[ \varepsilon_{dd} = \frac{\mu_0 k_{\text{m}}^2 m}{12 \pi \hbar^2 a}. \]

It is chosen such that a homogeneous condensate is unstable if \( \varepsilon_{dd} > 1 \) in a static magnetic field \( 25 \). In contrast to the s-wave interaction which can be understood as a local, contact-like interaction \( 14 \), the dipole-dipole interaction is long-range and anisotropic. In a condensate with density distribution \( n(\vec{r}) = |\phi(\vec{r})|^2 \), it gives rise to the mean-field potential \( 18 \)

\[ \Phi_{dd}(\vec{r}) = \int U_{dd}(\vec{r} - \vec{r}') |\phi(\vec{r}')|^2 d^3 r'. \]

The integral in \( 4 \) reflects the non-local character of the interaction. If this interaction in addition to the contact interaction is taken into account, the well known Gross-Pitaevskii equation gets the form

\[ i\hbar \frac{\partial}{\partial t} \phi(\vec{r}, t) = \left( -\frac{\hbar^2}{2m} \nabla^2 + U_{\text{ext}}(\vec{r}) + g|\phi(\vec{r}, t)|^2 + \int U_{dd}(\vec{r} - \vec{r}') |\phi(\vec{r}', t)|^2 d^3 r' \right) \phi(\vec{r}, t). \]

O’Dell et al. have shown in \( 8 \), that even under the influence of the dipole-dipole mean field potential \( \Phi_{dd}(\vec{r}) \), the density distribution has the shape of an inverted parabola in the Thomas-Fermi limit. Like in the case of pure contact interaction, a wave function of the form

\[ |\phi(\vec{r})|^2 = n_{c,0} \left( 1 - \frac{x^2}{R_x^2} - \frac{y^2}{R_y^2} - \frac{z^2}{R_z^2} \right) \]

is a self consistent solution of the superfluid hydrodynamic equations \( 26 \) derived from the Gross-Pitaevskii equation \( 14 \), even in presence of dipole-dipole interaction \( 8, 19 \). \( R_x, R_y, \) and \( R_z \) are the Thomas-Fermi radii of the condensate. The anisotropy of the dipole-dipole interaction manifests itself in a modification of the aspect ratio of the trapped condensate \( 10, 25 \). This anisotropy also reveals during the expansion of a dipolar condensate \( 2, 18 \).

In the following we will determine the dipole-dipole strength parameter \( \varepsilon_{dd} \) by analyzing the dynamic behavior of the Thomas-Fermi radii \( R_i(t) \) of expanding dipolar condensates. The experimental apparatus and techniques that are used are described in detail in \( 3, 27 \). By applying a small homogeneous external field (~11.5 G), oriented either along the \( y \)- or \( z \)-axis, shortly (~7 ms) before releasing the condensate from the trap, we obtained two sets of measured radii of a ballistically expanding condensate with different alignment of the atomic magnetic moments. The trap from which the condensate was released was a crossed optical dipole trap that was elongated in \( z \)-direction with trap parameters of \( \omega_x = 2\pi 942 \text{ Hz}, \omega_y = 2\pi 712 \text{ Hz}, \) and \( \omega_z = 2\pi 128 \text{ Hz} \). In the asymptotic limit of long times of flight, which is governed by a collisionless and potential free (except for gravity) ballistic flight, the radii of the cloud can be parameterized as

\[ R_i(t) = R_i^* + v_i^* t, \]

where the index \( i = [x, y, z] \) indicates the direction of expansion that is considered. It is worthwhile to mention that the initial values \( R_i(0) = R_i^* \) are not the Thomas-Fermi radii \( R_i \). Note that the radii \( R_i(t) \) as well as the asymptotic velocities \( v_i^* \) of the expansion for long times \( (t \gg 1/\omega) \) depend on the direction in which the atoms are polarized. As shown in \( 2 \), they are thus proportional to \( (Na)^{1/5} \). In particular,

\[ v_i^* = C(Na)^{1/5} \]

with a constant of proportionality \( C \) that only depends on the known or measured quantities that determine the chemical potential, i.e. the trap parameters \( \omega_x, \omega_y, \omega_z \) the atomic mass \( m \) and a small contribution of \( \varepsilon_{dd} \). Using the hydrodynamic theory of an expanding dipolar condensate \( 3 \), the asymptotic velocity for a certain number of atoms and scattering length can be easily calculated numerically. Table I shows the expected asymptotic velocities \( v_i^* \) for \( C \cdot (30000 \cdot 103a_0)^{1/5} \) in \( y \)-direction and the corresponding values for \( C \), calculated for pure contact interaction \( (\varepsilon_{dd} = 0) \), \( y \)-polarization and \( z \)-polarization. The numbers are calculated for the measured trap parameters, 30000 atoms, and a scattering length of \( a = 103a_0 \) \( 25 \). The scattering length of \( 103a_0 \) corresponds to a dipole-dipole strength parameter of \( \varepsilon_{dd} = 0.148 \).

To determine the asymptotic velocity, we use the condensate radii \( R_i(t) \) measured in a time-of-flight series. Since the number of camera pixels that are covered by the

| polarization   | \( v_i^* [10^{-5} \text{m/s}] \) | \( C [\text{m}^{1/5}] \) |
|---------------|-------------------------------|-------------------|
| no dipoles    | 8.528                         | 0.0488            |
| \( y \)-polarization | 9.085                     | 0.0519            |
| \( z \)-polarization | 8.283                     | 0.0474            |

TABLE I: Asymptotic velocity in \( y \)-direction and corresponding proportionality constant \( C \) calculated numerically for the case of vanishing dipole-dipole interaction \( \varepsilon_{dd} = 0 \), and for \( \varepsilon_{dd} = 0.148 \) and polarization along \( y \) and \( z \). Velocities calculated for 30000 atoms, and \( a = 103a_0 \).
condensate is much larger in the direction of fast expansion (y−direction in our setup), one can expect the most accurate results when considering \( R_y(t) \). Figure 1 shows the dependence of the condensate radius \( R_y(t) \) with polarization along \( \hat{y} \) (left figure) and \( \hat{z} \) (right figure) on the time of flight for 67 different expansion times. Because the number of atoms is fluctuating during such a series of experiments, the radii fluctuate due to their dependence on \( N \). To get rid of these fluctuations, we divide each measured radius \( R_y \) by the fifth root of the number of condensed atoms \( N \) in the corresponding experiment and multiply them with the mean value of the fifth root of the atom-numbers \( < N^{1/5} > \) in all experiments.

\[
\overline{R}_y = \frac{R_y}{N^{1/5}} < N^{1/5} > .
\]

In this way we get a series of time dependent radii which are now independent of the atom-number. Open circles in Fig. 1 represent the measured \( R_y(t) \), crosses with error-bars mark the re-scaled \( \overline{R}(t) \) which show much less fluctuations. A linear fit to the re-scaled data for times larger than 3 ms to focus only on the asymptotic behavior, yields \( v_y^* = 9.56 \pm 0.24 \text{ m/s} \) for \( y- \)polarization. For \( z- \)polarization, we get \( v_y^* = 8.78 \pm 0.12 \text{ m/s} \).

By using the above re-scaling, the errors \( \Delta v_y^* = \pm 0.24 \text{ m/s} \) and \( \Delta v_y^{*5} = \pm 0.12 \text{ m/s} \) in the fitted slope \( v_y^* \) for \( y- \) and \( z- \)polarization, respectively, do not contain fluctuations of the atom-number anymore.

If we consider only the case of \( y- \)polarization, invert equation (8) and insert the fitted velocity \( v^*_y \) and the constant \( C_y \) from table 1, we get

\[
a = \frac{1}{< N >} \left( \frac{v^*_y}{C} \right)^5 = 138a_0 \tag{10}
\]

for the scattering length. The error on this measurement consists of two contributions: first the fitted asymptotic velocity comes with an error \( \Delta v^* \) due to such kind of noise on the data that is not correlated with the number of atoms. Since \( v^* \) appears in the fifth power in \( a \), \( \Delta v^* \) appears with a factor of 5 in \( \Delta a \). Second, the mean value of the number of atoms has an uncertainty, mainly due to an uncertainty in the detuning of the probe beam. Since a detuning from resonance can only lead to an underestimation of the number of atoms, the error in the scattering length \( a \) caused by this uncertainty is only towards smaller values of \( a \). We estimate a maximum detuning of \( \Delta \delta_{probe} = \pm 0.25 \text{ GHz} \) which leads to an estimated error in the number of atoms of \( \Delta N/N = -0.25\% \). The relative error in \( a \) is then

\[
\frac{\Delta a}{a} = \frac{\Delta < N >}{< N >} + 5 \frac{\Delta v^*}{v^*} = -0.25 \pm 0.075. \tag{11}
\]

Hence the scattering length of \( ^{52}\text{Cr} \) determined with this method is

\[
a_{\text{Cr}} = \left( 138^{+10}_{-45} \right) a_0. \tag{12}
\]

For \( z- \)polarization, we get a consistent value of \( a_{\text{Cr}} = \left( 133^{+10}_{-43} \right) a_0 \). Due to the relatively large systematic error in the number of atoms, this way to determine the scattering length yields only quite inaccurate values, typical for condensate expansion experiments. In the following, we will use the full set of data from both polarizations to determine the scattering length with much higher accuracy and independent of the number of condensed atoms. We use the two rescaled asymptotic velocities

\[
\overline{v}_y = \frac{v_y^*}{< N^{1/5} >} \tag{13}
\]

\( \overline{v}_y(\overline{B}_{||\hat{y}}) \) and \( \overline{v}_y(\overline{B}_{||\hat{z}}) \) for polarization along \( \hat{y} \) and \( \hat{z} \), respectively to determine \( \varepsilon_{dd} \) by analyzing their ratio. To first order in \( \varepsilon_{dd} \) (in the expected range of \( \varepsilon_{dd} \), higher orders are negligible), the ratio has the form

\[
\frac{\overline{v}_y(\overline{B}_{||\hat{y}})}{\overline{v}_y(\overline{B}_{||\hat{z}})} = 1 + D\varepsilon_{dd}. \tag{14}
\]

It depends only on the asymmetry introduced by the dipole-dipole interaction because the contribution of the s-wave scattering to the total energy is independent of the polarization. \( D \) is again a numerical constant. If we use the measured asymptotic velocities, we obtain

\[
\varepsilon_{dd} = 0.159 \pm 0.034 \tag{15}
\]

in very good agreement with the value of \( \varepsilon_{dd} = 0.148 \) that one would expect for \( a_{\text{Cr}} = 103a_0 \). In turn, since the dipole-dipole interaction strength can be exactly calculated from 25, this result can be used to determine the scattering length:

\[
a = \frac{\mu_0\mu_m^2 m}{12\hbar^2 \varepsilon_{dd}} = 96 \pm 20 \ a_0.
\]

This result is in excellent agreement with the value of \( 103 \pm 13\ a_0 \) that has been obtained by comparing the measured positions of Feshbach resonances in chromium collisions with multichannel calculations 26. Furthermore,
In conclusion, we have measured the relative strength of the magnetic dipole-dipole interaction in a chromium condensate with different polarization after release from an anisotropic trapping potential. This result was used to determine the s-wave scattering length of $^{52}\text{Cr}$ $a = 96 \pm 20 \ a_0$ in excellent agreement with the results of theoretical analysis of measured Feshbach resonances ($a_{52\text{Cr}} = 103 \pm 13 a_0$) \[28\]. In contrast to many other methods that are commonly used to determine the s-wave scattering length, this method does not depend on the accuracy of the atom-number determination. Furthermore, unlike other methods that deliver results with the same or even better accuracy like Feshbach resonance measurements or photoassociation spectroscopy \[29\] , \[30\] , \[31\] , it does not require knowledge of any details of the molecular potentials. We expect it to be well suited to determine the scattering length close to a Feshbach resonance with high accuracy, especially for small scattering lengths where the dipole-dipole interaction becomes as important as the contact interaction. The excellent agreement between experimental results and theory constitutes a confirmation of the theoretical approach that is used to describe the dipolar BEC.

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| method | year | scattering length $a_{52\text{Cr}}$ [$a_0$] | relative error |
|--------|------|------------------------------------------|----------------|
| cross-dimensional thermalization \[23\] | 2003 | $170 \pm 39$ | 23% |
| Feshbach resonances \[28\] | 2005 | $103 \pm 13$ | 13% |
| condensate expansion in one direction (this paper) | 2006 | $138 \pm 10 a_0$ and $133 \pm 10 a_0$ | 13% |
| dipolar expansion (this paper) | 2006 | $96 \pm 20$ | 13% |

TABLE II: Comparison of experimental values of the $^{52}\text{Cr}$ s-wave scattering length.
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