Simulation of Alfvén Wave Propagation in the Magnetic Chromosphere with Radiative Loss: Effects of Nonlinear Mode Coupling on Chromospheric Heating

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Abstract

We perform magnetohydrodynamic simulations to investigate the propagation of Alfvén waves in the magnetic chromosphere. We use the 1.5D expanding flux tube geometry setting and transverse perturbation at the bottom to generate the Alfvén wave. Compared with previous studies, our expansion is that we include the radiative loss term introduced by Carlsson & Leenaarts. We find that when an observation-based transverse wave generator is applied, the spatial distribution of the time-averaged radiative loss profile in our simulation is consistent with that in the classic atmospheric model. In addition, the energy flux in the corona is larger than the required value for coronal heating in the quiet region. Our study shows that the Alfvén wave-driven model has the potential to simultaneously explain chromospheric heating and how energy is transported to the corona.

Unified Astronomy Thesaurus concepts: Solar chromosphere (1479); Solar chromospheric heating (1978)

1. Introduction

How the solar chromosphere and the corona maintain their temperature at $10^6$ K is still under debate. On average, energy fluxes of $3 \times 10^7$ erg s$^{-1}$ cm$^{-2}$ and $4 \times 10^6$ erg s$^{-1}$ cm$^{-2}$ in the quiet region are required for coronal and chromospheric heating, respectively (Withbroe & Noyes 1977). The chromosphere is divided into two regions: a high-beta non-magnetic region below the magnetic canopy (Gabriel 1976), and a low-beta magnetic region in flux tubes, as well as in higher positions above the equipartition layer ($\beta = 1$ layer). The height of the equipartition layer in the quiet region is 0.8–1.6 Mm (Wiegelmann et al. 2014).

The dissipation of acoustic shock is considered a candidate for chromospheric heating in the non-magnetic chromosphere (Schwarzschild 1948; Anderson and Athay 1989; Jordan 1993). The dynamics of acoustic wave propagation in the non-magnetic chromosphere have been studied by hydrodynamic simulations with non-local thermodynamic equilibrium radiative transfer (Carlsson & Stein 1995, 1997), which shows that the synthesized emerging Ca II K line spectra are consistent with observations.

However, it is difficult for acoustic waves to supply energy in the high chromosphere (Ulmschneider & Stein 1982; Jordan 1993), as they dissipate energy fast at lower positions. At heights above the equipartition layer, low-beta magnetic regions occupy all the space where Alfvén waves (Alfvén 1942) are considered an important energy transporter (e.g., Alfvén 1947; Mathioudakis et al. 2013; Soler et al. 2019). Numerical studies suggest that the continuous input of transverse perturbations at the photosphere, which behave as Alfvén waves, could contribute to coronal heating (Kudoh & Shibata 1999; hereinafter KS99; Antolin & Shibata 2010; Matsumoto & Shibata 2010; hereinafter MS10). At the same time, as the nonlinearity increases with expansion of the flux tube, the nonlinear mode coupling (Hollweg et al. 1982; Ulmschneider et al. 1991; McAteer et al. 2003) generates acoustic (slow mode) waves, which steepen to produce shocks and dissipate to provide energy for chromospheric heating (Matsumoto & Suzuki 2012; Arber et al. 2016; Brady & Arber 2016). As a result, a scenario in which Alfvén waves carry energy to the higher chromosphere and the corona while the chromospheric heating is powered by the shock dissipation of longitudinal waves, which are initialized by the mode coupling from these Alfvén waves, has been promoted.

Previous studies of Alfvén wave propagation in the magnetic chromosphere have usually ignored or crudely treated the radiative loss in the chromosphere, which is the most significant source of energy loss (Withbroe & Noyes 1977). MS10 and Matsumoto & Suzuki (2012) include radiative loss while applying the approximation in Anderson and Athay (1989), where the radiative loss is only determined by the local density, which means that the chromospheric plasma has a constant cooling time. Brady & Arber (2016) also include radiative loss, where the radiative loss rate at a certain position is determined by the time average of the viscous heating during the previous 160 s. However, as pointed out by Hueneth & Ulmschneider (1995), radiative loss is much more narrowly concentrated in the hot region behind shocks, which cannot be correctly reflected by the treatments used in these studies. On the other hand, models with advanced 3D radiative MHD simulations (e.g., Gudiksen et al. 2011; Iijima & Yokoyama 2017), as well as synthesized observations, are widely used in diagnostics of spectral lines formed in the chromosphere and the transition region (e.g., Leenaarts et al. 2013; Rathore et al. 2015). However, their complexity makes understanding the underlying physical process difficult.

To investigate the applicability of previous Alfvén wave driving model to chromospheric heating, we conduct MHD simulations with an improved treatment of radiative loss introduced by Carlsson & Leenaarts (2012; hereinafter CL12). We ignore the longitudinal wave input at the photosphere to avoid mixture of mode coupling-initiated waves and the input longitudinal waves in the chromosphere. In this paper, we consider a similar geometry setting following KS99 and MS10. We study chromospheric heating by comparing the spatial distributions of the radiative loss profile in our simulation and the classic model VALC (Vernazza et al. 1981). The setting of our simulation is introduced in Section 2. The results are shown in Section 3. Discussions and a comparison with previous studies are included in Section 4. Finally, we summarize our results in Section 5.
2. Numerical Setting

We solve 1.5D ideal compressible MHD equations on an expanding flux tube whose cross-section area \( A \) is a function of the height \( z \), which does not change with time. The expression “1.5D” indicates that we have a one-dimensional geometry setting, while the velocity and magnetic field has two components, namely the \( s \) direction and the \( \phi \) direction. The \( s \) direction is curved along the flux tube, while the \( \phi \) direction is the azimuthal direction. The basic equations are

\[
\frac{\partial}{\partial t} \left( \rho A \right) + \frac{\partial}{\partial s} \left( \rho V_s A \right) = 0, \tag{1}
\]

\[
\frac{\partial}{\partial t} \left( \rho V_s A \right) + \frac{\partial}{\partial s} \left[ \left( \rho V_s^2 + P + \frac{B_s^2}{8\pi} \right) A \right] = \frac{f_s}{A} \rho \frac{dA}{ds}, \tag{2}
\]

\[
\frac{\partial}{\partial t} \left( \rho V_A z^2 \right) + \frac{\partial}{\partial s} \left[ \left( \rho V_A z^2 - \frac{B_A^2}{4\pi} \right) A z^2 \right] = A \rho L_{\text{iq}}, \tag{3}
\]

\[
\frac{\partial}{\partial t} \left( \sqrt{A} B_s \right) + \frac{\partial}{\partial s} \left[ (B_s V_s - B_V V_s) \sqrt{A} \right] = 0, \tag{4}
\]

\[
\frac{\partial}{\partial t} \left[ \left( \frac{\rho V_s^2}{2} + \frac{P}{\gamma - 1} + \frac{B_s^2}{8\pi} \right) A \right] + \frac{\partial}{\partial s} \left[ \left( \frac{\rho V_s^2}{2} + \frac{\gamma P}{\gamma - 1} + \frac{B_s^2}{4\pi} \right) V_s - \frac{B_s B_s V_s}{4\pi} \right] \right] = -L_{\text{rad}} A - \rho V_s g_0 \frac{dA}{ds} + \rho V_s \sqrt{A} L_{\text{iq}} + S_{\text{art}} A, \tag{5}
\]

and the ideal gas equation of state, which is given by

\[
P = \frac{k_B}{m} \rho T, \tag{6}
\]

where \( \rho \) is the density; \( A \) is the cross-section area; \( t \) is the time; \( s \) is the distance along the field line; \( V_s \) is the velocity along the \( s \) direction; \( P \) is the gas pressure; \( V_\phi \) is the velocity along the \( \phi \) direction; \( g_0 \) is the gravity; \( z \) is the height; \( B_s \) is the magnetic field along the \( s \) direction; \( B_\phi \) is the magnetic field along the \( \phi \) direction; \( L_{\text{rad}} \) is the radiative loss; \( \gamma \) is the ratio of specific heats, \( \gamma = \frac{5}{3} \); \( T \) is the temperature; \( m \) is the mass per particle, assuming \( m = m_{\text{H}} = 1.67 \times 10^{-24} \text{ g} \); \( k_B \) is the Boltzmann constant, \( k_B = 1.38 \times 10^{-16} \text{ erg K}^{-1} \); and \( S_{\text{art}} \) is an artificial heating term that is used to prevent the temperature from dropping too low. For the derivation of 1.5D MHD equations in curvilinear coordinates, one could refer to Shoda & Yokoyama (2018).

We set the expanding flux tube geometry according to KS99 by setting the radius of the flux tube \( r \) as a function of \( z \). The radius \( r \) and cross-section area \( A \) have the relation \( A = \pi r^2 \). The radius is given by

\[
r = \int \cos \alpha ds, \tag{7}
\]

\[
z = \int \sin \alpha ds, \tag{8}
\]

where

\[
\alpha = \alpha_t + (\alpha_r - \alpha_t) f_n, \tag{9}
\]

\[
\alpha_r = -\arctan \left( \frac{-4H_0}{r} \right), \tag{10}
\]

\[
\alpha_t = \arctan [k \sin (z/z_d)^2]. \tag{11}
\]

\[
f_n = -\frac{1}{2} \left\{ \tanh \left[ (z - 0.2z_d)/(0.1z_d) \right] \right\}^{-1}. \tag{12}
\]

We set \( z_d = 2250 \text{ km} \) and \( H_0 = 150 \text{ km} \) following KS99. Also, \( r \) as a function of \( z \) is obtained by numerically solving the ordinary differential equation:

\[
\frac{dr}{dz} = \cot \alpha. \tag{13}
\]

The degree of expansion can be varied by adjusting \( k \) in Equation (11). We set \( k = 1.2 \) for a typical case, while \( k \) is also adjusted for a parameter survey of the flux tubes, with the expansion factor changing within observational range. In addition, we keep the radius of the flux tube \( r \) constant when \( z < 0.1 \text{ Mm} \) and apply a Gaussian kernel smoothing with a width of 0.04 Mm to connect the low constant radius part and the expanding part of the flux tube. This setting mimics a flux tube expanding from the network region. The longitudinal section of the flux tube is shown in Figure 1. The radius of the flux tube \( r \) at the lower boundary \( z = 0 \) is set to 150 km, which is approximately the length of the pressure gradient height. The starting point \( z = 0 \) is at the same position where \( z = 0 \). The expansion factor \( f \) describes the degree of expansion of the flux tube, which is defined as

\[
f = A_{\text{low}} / A_{\text{top}}, \tag{14}
\]

where \( A_{\text{low}} \) and \( A_{\text{top}} \) are the cross-section areas of the flux tube at the top boundary and lower boundary, respectively.
There are three extra terms besides an ideal MHD model. The first one is gravity. In our model, gravity \( g_0 \) is calculated by
\[
g_0 = \frac{GM_\odot}{(z + R_\odot)^2},
\]
where \( G \) is the gravitational constant, \( M_\odot \) is the mass of the Sun, and \( R_\odot \) is the radius of the Sun.

The second term is the transverse torque \( L_{\text{eq}} \). The Alfvén wave is initialized by this transverse torque, which mimics the convection motion at the photosphere. Following KS99 and MS10, \( L_{\text{eq}} \) is modeled to have the following form:
\[
L_{\text{eq}}(t, z) = r W_0(t) \left( \frac{1}{\tanh \left( \frac{z - 0.75H_0}{0.075H_0} \right) - 1} \right),
\]
where \( W_0(t) \) determines the amplitude and time evolution of the artificial torque. We adjust the form and amplitude of the artificial torque by adjusting \( W_0(t) \), which is derived from the velocity spectra. The transverse velocity at the bottom has the form
\[
V_\psi(t, z = 0) = \sum_i C_i \sin(2\pi \nu_i t + \psi_i),
\]
where \( C_i \) determines the power of the transverse velocity at frequency \( \nu_i \) using the velocity spectra. Frequency \( \nu_i \) is chosen to be 100 points averagely distributed between the chosen minimum frequency \( f_{\text{min}} = 2 \times 10^{-4} \text{s}^{-1} \) and the maximum frequency \( f_{\text{max}} = 5 \times 10^{-2} \text{s}^{-1} \). \( C_i \) is obtained from the spectra of the observed transverse velocity of the photosphere shown in Figure 2 (modified from Figure 2 in MS10). The phases \( \psi_i \) are random numbers between 0 and \( 2\pi \) for each \( i \). To obtain this velocity distribution, we set the intensity of torque to be the acceleration that has the form
\[
W_0(t) = \frac{dV_\psi}{dt} = \sum_i 2\pi \nu_i C_i \cos(2\pi \nu_i t + \psi_i).
\]

We apply a multiplier to \( W_0(t) \) in order to adjust the root mean square of the transverse velocity at the lower boundary to be around 1 km s\(^{-1}\).

The third term is radiative loss \( L_{\text{rad}} \). Following CL12, we have
\[
L_{\text{rad}} = -\sum_{X_m} L_{X_m}(T) E_{X_m} \frac{N_{X_m}}{N_X} (T) A_X N_{H}\rho_{\text{ce}}.
\]

Here, the subscript \( X_m \) represents a component of element \( X \) in the ionization state \( m \). \( X_m \) used in this approximation method include neutral hydrogen (H), singly ionized calcium (Ca II), and singly ionized magnesium (Mg II), since they are the most important components for chromospheric radiative loss (Vernazza et al. 1981). \( A_X \) is the abundance of element \( X \). \( L_{X_m}(T) \) represents the optically thin radiative loss for different elements that are functions of \( T \). \( \frac{N_{X_m}}{N_X} (T) \) represents the fractions of specific ions or neutral atoms in the ionization state \( m \) of element \( X \), which are functions of \( T \). \( E_{X_m} \) is the escape probability. Escape probability are tabulated functions of column mass for Mg II and Ca II and neutral hydrogen column density for H. One could refer to Section 4.2 in CL12 for further explanation. The column mass is calculated by \( \int \rho dz \).

The neutral hydrogen column density is calculated by \( \int \rho / m_p N_H(T) \). All these functions are obtained by fitting with a detailed radiative transfer calculation. \( N_{H} \) is the number density of hydrogen element and \( n_e \) is the number density of electrons; \( N_{H} \) is determined by substituting temperature into the function of the fraction of neutral hydrogen. We assume \( n_e = \rho / m_H - N_{H} \). The aim of this approximation approach is to obtain a simple form of \( L_{X_m}(T) \), \( E_{X_m} \), and \( \frac{N_{X_m}}{N_X} \) as a function of some physical parameters, so that we can calculate the radiative loss rate by putting proper values into these functions without carrying out complete radiative transfer calculations.

Heat conduction is not included in the simulation, since the timescale for heat conduction in the chromosphere is much longer than the wave transition time. In addition, we also ignore the radiative loss in the corona, since we mainly focus on the chromosphere and we have a very crude grid size in the corona. As we also ignore heat conduction, we cannot treat the energy balance in the corona carefully.

For the initial condition, we assume a hydrostatic stratified atmosphere in which
\[
\frac{dP}{dz} = -\rho g_0.
\]

The initial temperature distribution is a combination of the classic VALC temperature model and a hyperbolic tangent distribution that is described below:
\[
T = \begin{cases} 
T_{\text{valc}}(z) & z \leq 1 \text{ Mm} \\
T_{\text{pho}} + \frac{1}{2}(T_{\text{cor}} - T_{\text{pho}}) \left( \tanh \left( \frac{z - z_r}{w_r} \right) + 1 \right) & z > 1 \text{ Mm}
\end{cases}
\]

where \( T_{\text{valc}}(z) \) is the temperature distribution as a function of height in the VALC model; \( T_{\text{cor}} \) is the temperature of the corona, which is set to be \( 10^6 \text{K} \); \( T_{\text{pho}} \) is the temperature of the photosphere, which is set to be 6000 K; \( z_r \) is the height of the transition region, which is set to be 2.25 Mm; \( w_r \) relates with the width of the transition region, which is set to be 0.05 Mm. The density at the lower boundary is set to be \( 2.53 \times 10^{-7} \text{ g cm}^{-3} \). After the temperature is determined, the pressure and density are
calculated as functions of height using Equation (20) and the equation of state of ideal gas (Equation (6)). The distributions of temperature, gas pressure, and density are shown in Figure 3. The background Alfvén speed, sound speed, plasma beta, and nonlinearity of the Alfvén wave are shown in Figure 4. The nonlinearity of the Alfvén wave is estimated by $\psi_{WKB}/C_A$, where $C_A$ is the background Alfvén speed and $\psi_{WKB}$ is the amplitude of the wave in the azimuthal direction estimated by WKB approximation. $B_s$ at the photosphere is determined by the gas pressure required to maintain the plasma beta around unity. As a result, the magnetic field at the bottom is 1812 G. The pressure at the bottom is $1.26 \times 10^3$ dyn cm$^{-2}$.

The MHD equations are solved using the upwind scheme with the Modified Harten-Lax-van Lee approximate Riemann solver (Miyoshi & Kusano 2005). We set the scheme to have second-order accuracy in terms of space and time by applying the Monotonic Upwind Scheme for Conservation Law (MUSCL) reconstruction (van Leer 1979), with a minmod slope limiter (Roe 1986) and the second-order Total Variation Diminishing Runge–Kutta scheme (Shu & Osher 1988) for time evolution. At the lower boundary, the density and pressure of the point at the outer boundary increase according to the hydrostatic stratification. For the momentum perpendicular to the boundary and $B_{\parallel}$, it has the same absolute value but opposite directions. The other physical parameters parallel to the boundary are symmetric. The top boundary is a free boundary. There is reflection of waves at the top boundary; it is more ideal if we can have an open boundary for waves propagating freely across the top boundary. However, since Alfvén waves are highly reflected at the transition region (Cranmer & van Ballegooijen 2005), the energy flux of the Alfvén wave in the corona is too small to affect the chromosphere. As a result, we can ignore the reflected wave from the top boundary. We simulate up to 9 Mm with an evenly distributed grid having a size of around 5 km. Above 9 Mm, the length of each grid increases gradually. The value of $z$ at the top of the simulation region is 200 Mm.

### 3. Results

The rms of the velocity and transverse magnetic field over time as well as the time-averaged temperature in the chromosphere for a typical case are shown in Figure 5. The waves in the chromosphere are shown by the nonlinearity of the time-averaged velocity, which is defined by the rms of the transverse (longitudinal) velocity divided by the time-averaged Alfvén (sound) speed (Figure 6). An increase in nonlinearity with height indicates steepening of waves as they propagate upward, especially for longitudinal waves.

An ideal way to compare with observations is to synthesize the emerging spectra and compare them with the observation.
However, it is difficult to perform synthesis in the chromosphere due to the NLTE condition in the chromosphere and the limitations of 1.5D geometry (Sukhorukov & Leenaarts 2017). Instead, we compare the radiative loss profile and temperature distribution in our simulation with those of the classic model. Since the timescale of radiative loss is around 200 s in the chromosphere and our calculation lasts 5000 s, which is around several tens of times the radiative cooling time, we expect that statistically, the energy balance between heating and radiative loss in the chromosphere has already been reached, and the time-averaged cooling rate is identical to the time-averaged heating rate. We further estimate the heating rate in Section 4.

The time-averaged effective radiative loss (ERL) profile for the typical case is shown by the thick black line in Figure 7. The ERL is defined as

\[ L_{\text{ERL}} = L_{\text{rad}} A / A_c, \]

where \( A \) is the cross-section area at that height, \( A_c \) is the cross-section area at the corona (defined at \( z = 8 \) Mm), and \( L_{\text{ERL}} \) is the radiative loss rate, with compensation for the expanding effect. Also, instead of being applicable just inside the flux tube, the ERL represents the averaged value across an entire slice of the cylinder, which has a constant cross-section \( A_c \). We define the ERL, since we are only focusing on the flux tube region and we want to emphasize that only the heating inside the flux tube could provide required heating for the chromosphere. We plot the profile as a function of the height instead of the mass to prevent the influence from height variation of the transition region caused by formation of spicules. In addition, the radiative loss in the classic atmospheric model VALC is overplotted by the thick dashed line. The dotted lines represent the results of simulations with adjustments in the background magnetic field. The VALC radiative loss profile is plotted by the thick dashed line and the blue region represents the temperature profile from the VALA to the VALF model (Vernazza et al. 1981). The energy flux in the corona (defined at \( z = 8 \) Mm) is \( 7.3 \times 10^6 \) erg cm\(^{-2}\)s\(^{-1}\). The energy flux required for coronal heating in the quiet region is \( 3.0 \times 10^5 \) erg cm\(^{-2}\)s\(^{-1}\) (Withbroe & Noyes 1977). In the calculations, with adjustment in the magnetic field as described above, the largest and smallest fluxes are \( 2.21 \times 10^6 \) erg cm\(^{-2}\)s\(^{-1}\) and \( 3.5 \times 10^6 \) erg cm\(^{-2}\)s\(^{-1}\), respectively. We conclude that in these simulations, enough energy, which could meet the requirement of coronal heating in the quiet region, is transported to the corona. This result is consistent with KS99 and MS10. We also note that the energy mass is based on Table 10 and Table 15 in Vernazza et al. (1981).

Our simulation results suggest that despite the change in the background magnetic field, the radiative loss profile in the simulation agrees quantitatively with the classic solar atmospheric model.

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flux in the typical case is much larger than that in MS10; we will discuss this in Section 4.

The time-averaged temperature as a function of height is shown in Figure 8, where the thick solid black line represents the time-averaged temperature profile and the thick dashed line represents the VALC temperature profile. Despite the result that the radiative loss profile is consistent with the classic atmospheric model, the time-averaged temperature profile is apparently lower than that in the classic model. In Figure 9, from the upper panel to the lower panel, the time-integrated ERL, ERL, and temperature at a certain height $z = 1.5$ Mm are shown as thick black solid lines. The slopes of the dashed–dotted lines in the upper panel represent the corresponding radiative loss rate at this height in the VALC model. We note that when the shock front propagates across this height, as shown by the high temperature in the lower panel, a sudden increase in radiative loss occurs, as shown in the comparison between the middle panel and lower panel. Also, in the upper panel, we note that there are corresponding jumps, which indicate strong radiative loss. As a result, a continuous shock wave could support enough radiative loss. However, the low-temperature region between the shocks dominates most of the time, which leads to a lower time-averaged temperature. A low temperature without a temperature increase in the chromosphere is also obtained in other dynamic chromospheric models (Carlsson & Stein 1994; Wedemeyer et al. 2004). Carlsson & Stein (1994) suggest that the averaged gas temperature in the dynamic model is lower than that in the hydrostatic equilibrium model despite that both models having similar emerging intensities. This is because high-temperature shocks make a significant contribution to intensity in the dynamic model. We need to point out that Carlsson & Stein (1994) focuses on the non-magnetic region, which is different from our simulation, but the effect of the shocks that cause the difference between the averaged gas temperature in the dynamic model and the hydrostatic equilibrium model is similar.

4. Discussion

The time slice of the density distribution is shown in Figure 10. The rise and fall of the transition region reflects the formation of spicules.

Our calculation is an extension of MS10, where they only apply crude treatment of radiative loss. In their study, by including only observation-based transverse wave drivers, energy is transported to the corona while achieving spicule formation at the same time. In our simulation, in addition to the energy flux in the corona and spicule formation, we emphasize that the time-averaged radiative loss profile is also consistent with the classic atmosphere model. Our result is consistent with Brady & Arber (2016) in that we obtain a radiative loss profile that is consistent with the classic model. However, our treatment of chromospheric radiative loss is different. In addition, we also perform a parameter survey for making changes to the magnetic field intensity and expansion factor, which confirms the robustness of this result.

In our simulation, the energy flux in the corona is $7.3 \times 10^5$ erg cm$^{-2}$ s$^{-1}$, which is around 2 times that in MS10; we conclude that this is mainly caused by the difference in stratification. In our simulation, the temperature in the photosphere is 6000 K compared with 5000 K in MS10, therefore we have a longer scale height in the photosphere, which leads to a higher density below the transition region. As a result, the Alfvén wave has a smaller phase velocity in our calculation, and hence a shorter wavelength. Although a shorter wavelength will increase the dissipation rate of an Alfvén wave in the chromosphere, it also makes the transmittance at the transition region become higher in our simulation. As a result, higher transmittance increases the energy flux in the corona.

We note that the height of spicules is shorter than that in KS99 and MS10. For KS99, there is no radiative loss in the chromosphere and the internal energy in the chromosphere increases constantly. As a result, the height of spicules increases with time. For comparison with MS10, due to the
difference in stratification, our simulation has a higher density below the transition region, which leads to the result that the spicule height in our simulation appears lower.

We also estimate the heating rate and compare the time-averaged heating rate with radiative loss. Since our simulation does not contain explicit dissipation, we estimate the heating rate at shock fronts from physical parameters at both upstream and downstream region. The positions of shock fronts are finally calculated following Cranmer et al. (2007) and specially averaged within the whole shock region:

$$\frac{\partial V_s}{\partial s} \leq -\frac{1}{t_c}$$

(23)

where $t_c$ is a parameter showing the threshold for identification of shock waves. We chose $t_c$ changes between 10 s and 30 s. The selection of $t_c$ will be discussed in detail in the Appendix. After identification of a shock front, we choose a local minimum or maximum of $\frac{\partial V_s}{\partial s}$ near the shock front as the position to pick up upstream and downstream physical parameters. The heating rate is finally calculated following Cranmer et al. (2007) and specially averaged within the whole shock region:

$$Q_{\text{heat}} = c_v u_t \rho_1 (T_2 - T_1 (\rho_2/\rho_1)^{\gamma})/w_{\text{shock}}.$$  

(24)

where $Q_{\text{heat}}$ is heating rate per unit volume; $c_v$ is specific heat capacity at constant volume per unit mass; $T_1$ and $\rho_1$ are temperature and density at the upstream region; $T_2$ and $\rho_2$ are temperature and density at the downstream region; $u_t = v_1 - u$ is the velocity of the upstream region in the shock rest frame, where $u$ is the propagating speed of the shock front and $v_1$ is velocity at the upstream region; $u$ is calculated using the jump condition of conservation of mass: $u = (\rho_1 v_1 - \rho_2 v_2)/(\rho_1 - \rho_2)$; $w_{\text{shock}}$ is the width of the shock wave, which is set to be 35 km. $w_{\text{shock}}$ dose not affect the total amount of heating rate (see the Appendix). The result is shown in Figure 11. We conclude that in the selected range of $t_c$, the estimated heating rate is found to be consistent with the radiative cooling rate. This result justifies our usage of cooling rate as an approximated value for heating rate.

For comparison with previous studies, we also perform the simulation with only simplified radiative loss $L_{\text{rad}} = 4.9 \times 10^9$ erg cm$^{-2}$ s$^{-1}$, which is included in MS10. The result of temperature and radiative loss at $z = 1.5$ Mm is shown in Figure 9. From the upper to the lower panel, the time-integrated ERL, ERL, and temperature for the simulation with simplified radiative loss are shown by thin gray solid lines. This simplification method assumes a constant cooling time across the entire chromosphere. In our calculation, we set a lower limit $T = 6000$ K to switch on the simplified radiative loss. This lower limit is compulsory, since at the lower-temperature region between shocks, the cooling timescale is much shorter than the acoustic wave transition timescale. If the simplified radiative loss is included without a switch, it will cause the temperature in regions between shock fronts to decrease to 0 when propagating upward (or if artificial heating is included, fixed at the temperature below which artificial heating will take effect). We note that the radiative loss at shock fronts in the simulation with simplified radiative loss is significantly smaller than that with CL12 radiative loss. According to the Rankine–Hugoniot condition, the compression ratio is smaller than 4, which results in a maximum of 4 times an increase in the radiative loss at shock fronts compared with their surroundings. However, in detailed radiative transfer calculations, the radiative loss at shock fronts could be a few orders larger than that in the surrounding area (e.g., Figure 14 in CL12, Figures 2 and 3 in Huenerth & Ulmschneider 1995). This leads to an underestimation of the radiative loss at shock fronts. We also estimate the robustness of the simplified radiative loss term. We apply the simplified radiative loss term with different low-temperature limits (6000 and 5000 K). We find that the low-temperature limit will directly affect the stratification in the chromosphere, which further leads to differences in height of the spicules and the energy flux of waves due to the reason that we have discussed above. This result suggests that there is a risk of loss of self-consistency when applying simplified radiative loss. Therefore, we consider that although $L_{\text{rad}} = 4.9 \times 10^9$ erg cm$^{-2}$ s$^{-1}$ is a good approximation for time-
averaged chromospheric radiative loss rate, one should be careful when applying this method to simulations studying chromospheric dynamics.

Our model is limited by the 1.5D geometry of a fixed flux tube. Additionally, as we only focus on transverse waves, mode conversion (Cally & Goossens 2008) from acoustic to Alfvén waves is ignored, although it is considered important for the generation of high-frequency Alfvén waves (Shoda & Yokoyama 2018). In this simulation, we also ignore the longitudinal acoustic wave input at the photosphere to avoid mixture of mode coupling-initiated waves and input acoustic waves in the chromosphere. However, a comprehensive understanding of the role of waves in heating the magnetic chromosphere requires identification of different wave modes in the chromosphere and a thorough consideration of other heating mechanisms. A comparison between shock heating, turbulence heating (van Ballegooijen et al. 2011), ambipolar diffusion (Leake et al. 2005; Khomenko & Collados 2012; Khomenko et al. 2018), and other heating mechanisms is further desired.

5. Conclusion

We solve 1.5D ideal MHD equations with CL12 approximated radiative loss model. We found that if observation-based transverse perturbation is involved, the Alfvén wave-driven model could reproduce the time-averaged radiative loss profile in the magnetic chromosphere. The time-averaged radiative loss profile is consistent with that in the classic atmospheric models. In addition, the energy transported to the corona could also meet the requirement of coronal heating in the quiet region, which is consistent with previous studies. However, the temperature in the magnetic chromosphere is apparently lower than that in the classic atmospheric model. Comparison with previous studies indicates that one needs to be careful when applying the simplified radiative loss term when studying chromospheric dynamics. For example, when quantifying spicule height and coronal energy flux, simplified radiative loss will involve new artificial parameters that affect stratification in the chromosphere and further lead to changes in spicule heights and the energy flux of waves.

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Appendix

Selection of Parameters in Estimation of Heating Rate

For estimation of heating rate in our simulation, we need to identify each shock by divergence of velocity. We use $\frac{\partial V_s}{\partial s} \leq -\frac{1}{t_c}$ to select shock regions (Equation (23)) and heating rate is calculated using physical parameters in upstream and downstream regions of the shock wave (Equation (24)). $t_c$ in Equation (23) is required to be large enough for including weaker shocks while small enough to exclude compression from linear propagation of waves. For this purpose, we plot the occurrence frequency distribution of $\partial V_s/\partial s$ in Figure 12. For linear propagating waves, it is expected to have a symmetric distribution with respect to $\partial V_s/\partial s = 0$. The actual distribution is asymmetric, which has a larger frequency for negative $\partial V_s/\partial s$ due to the formation of shocks. For distribution in $\partial V_s/\partial s > 0$, we plot its symmetric part with respect to $\partial V_s/\partial s = 0$ as the dotted line. The actual frequency distribution is much larger than the symmetric part for the threshold $t_c = 10$ s (red dotted line). We conclude that $t_c = 10$ s is small enough to exclude compression from the linear propagation of waves. We also apply $t_c = 20$ s and 30 s for comparison. A larger threshold will include weaker shocks as well as the possibility of overestimation of heating rate because compression in linear propagating waves may be included. Figure 11 shows that the heating rate above 1 Mm is similar for the three different thresholds, which indicates that the threshold $t_c = 10$ s gives good estimation of heating rate and we do not need to concern about an underestimated due to weak shocks that are excluded by this threshold.

In our calculation of heating rate, $w_{\text{shock}}$ is arbitrary and only affects the local spatial distribution of heating rate. For a single shock wave, as we set $Q_{\text{heat}} = 0$ outside the shock region and the heating rate is constant in the shock region, the spatial integration of $Q_{\text{heat}}$ does not depend on $w_{\text{shock}}$.

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Figure 12. The thick black line is the normalized occurrence frequency distribution of $\partial V_s/\partial s$. The dotted black line is normalized occurrence frequency vs. $-\partial V_s/\partial s$ for $\partial V_s/\partial s > 0$ (the dotted black line and the right part of the thick black line with $\partial V_s/\partial s > 0$ are symmetric with respect to $\partial V_s/\partial s = 0$). The vertical dashed red, green, and blue lines are $\partial V_s/\partial s = 1/10$ s$^{-1}$, 1/20 s$^{-1}$, and 1/30 s$^{-1}$, respectively.

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