Sensitivity analysis and instability study of mechanical parameters of deterioration layer of RCC dam

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Abstract. Aiming at the characteristics of multi-layer structure of roller compacted concrete (RCC) dam and the complicated combination of layers, the three-dimensional calculation of the structure of RCC dam adopts the multi-layer elastoplastic constitutive model. The influence of various parameters of RCC dam deterioration on the strength of the dam body was analyzed in detail. The Latin hypercube algorithm was used to randomly sample each material parameter. The sensitivity of the degraded interlayer parameters was studied based on the Spearman rank correlation coefficient. To solve the simulation The non-stationary randomness of the degradation process, the introduction of the impact factor attenuation function to smooth the degradation, the establishment of a time-varying model of material parameter degradation, and on the basis of the strength reduction coefficient method, the degradation driving level of the RCC dam is calculated. Through the research results, it is possible to focus on monitoring the relevant aspects of the dam during operation, take relevant measures in real time, and feedback on the design and construction plan, which is of great significance for the research on the stability of the RCC dam against sliding.

1. Introduction

Due to the advantages of rapid construction, saving cement, reducing construction cost and shortening construction period, China’s roller compacted concrete dam has become an important dam type after more than 30 years of development. In addition to the various dynamic and static loads, the RCC dam also bears the effects of various sudden disasters and the impact of the natural harsh environment. The deterioration of the RCC dam is inevitable [1]. Whether the RCC dam can last long-term service is not only affected by the evolution of the properties of the RCC material, but also related to many factors such as the construction quality, safety monitoring, inspection and diagnosis and human activities of the RCC dam [2]. The performance degradation of the damming material and the weak zone of the dam is an internal factor that reduces the operational effectiveness of the RCC dam during service. Analyzing the driving mechanism of its structural performance evolution is also the basis for realizing the safety
monitoring of the performance of the RCC dam. At present, the research on the behavior of the layer is mainly through experiments, focusing on the analysis of the effect of the layer on the shear resistance, strength characteristics, seepage characteristics and deformation properties of the RCC. The test results show that the physical and mechanical properties of roller compacted concrete are affected by the amount of cement and fly ash cement, the intermittent time between the two layers, the treatment method of the layer, etc., and depend to a large extent on the density of vibration compaction. With the development of 3D modeling technology, many scholars have explored constructing a constitutive model of RCC dam to better simulate the characteristics of RCC dam. For example, Fang Haiting aimed at the gradual change of the structure of the impact zone of the RCC dam, and applied the mechanics of composite materials and the idea of series-parallel connection to establish the rheological characteristics of the analysis model of the gradual change of the zone impact zone [3]. Gu Chongshi et al. conducted a deeper study on the problem that the RCC dam construction rolling layer has a significant impact on the displacement of the dam body, analyzed the changing rules and characteristics of the rolling layer, and established the thickness and non-thickness construction layer. The analysis model can reflect the elastic deformation and attenuation creep of the layer [4]. Using the entropy theory and catastrophe theory, Li Bo et al. established the ultimate bearing capacity and failure criterion of RCC and determined the most dangerous sliding surface of RCC failure [5].

It can be seen from the above research that there are many studies on the physical properties, mechanical parameters and constitutive models of the layers of the RCC dam, and the research on the evolution process of the interlayer degradation of the RCC dam and the influence of its parameters on the stability of the dam body is not deep enough. Due to the relatively short history of RCC dam construction [6], the performance of damming materials is only 20 to 30 years of actual measured operating data. The characteristics of RCC materials for longer ages (such as 50 years and 100 years) are also unknown, the researchers are still unclear about the characteristics and evolution mechanism of the RCC dam material and the weak surface between the layers. Because each mechanical layer of RCC involves more mechanical parameters, such as elastic modulus, bulk density, friction coefficient, Poisson's ratio coefficient, cohesion. In order to analyze the degree of factors that influence the parameters of each layer on the stability of the dam body, Latin Hypercube Random Sampling (LHS) is used to randomly sample the parameters of each layer according to the uniform distribution. Latin hypercube random sampling is a method for generating a reasonable parameter value set distribution from a multidimensional distribution. It can better sample the influence factor set with many elements and has been widely used in many research fields [7]. In order to determine the sensitivity of each factor quantitatively, the stress of the dam body is calculated by the elastoplastic constitutive of the multilayer structure of the RCC dam, and the Spearman rank correlation coefficient [8] is used to influence the stability factors of the RCC dam layer. Sensitivity analysis can determine the level of related mechanical parameters and the impact of different parts on the safety and stability of the dam. Starting from the influencing factors of degradation, a dynamic model of material parameter random factor is introduced to characterize the evolution of the material performance of the RCC dam, a time-varying model of material degradation is established, and on this basis, the weak driving level of the degradation of the RCC dam is determined. The results are of great significance for the safe operation of the dam.

2. Sensitivity and time-varying function of mechanical parameters at the degradation level

2.1. Elastoplastic constitutive model of multilayer structure of RCC dam

Due to the characteristics of layered roller compaction and layered structure of the RCC dam, the dam body calculation model uses an incremental elastoplastic model to construct the elastoplastic constitutive relationship of the multilayer structure of the RCC dam, and schematic diagram is shown in Figure 1. The total strain increment of RCC includes elastic strain increment and plastic strain increment [9]:

\[ d\varepsilon = d\varepsilon^e + d\varepsilon^p \]  

(1)
Where \(d\varepsilon\) is the total strain increment of RCC, \(d\varepsilon^e\) is the elastic strain increment of RCC; \(d\varepsilon^p\) is the plastic strain increment of RCC.

The plastic strain increment of RCC consists of the plastic strain increments of the body and the layer, then:

\[
d\varepsilon^p = \sum_{i=1}^{n} d\varepsilon^p_{ai} + \sum_{j=1}^{n} d\varepsilon^p_{ij}
\]

The overall equivalent elastoplastic matrix of RCC is:

\[
D_{ep} = D_{eo}(I - \sum_{i=1}^{n} f_{ai} \frac{\partial F}{\partial \sigma} - \sum_{j=1}^{n} f_{aj} \frac{\partial F}{\partial \sigma})
\]

2.2. Parameter sensitivity analysis based on Spearman rank correlation coefficient

In order to be able to qualitatively and quantitatively study the sensitivity of different mechanical parameters between degraded layers, a random sampling should be conducted within the change interval of the mechanical parameters. Since Latin Hypercube Sampling (LHS) is an effective uniform sampling, the number of sampling is less, which can effectively avoid a lot of repeated sampling work of Monte Carlo method. Latin hypercube sampling is mainly sampling from a known variable \(x_i\) distribution, and each sample evaluates \(y\) once, so that the mean and mean square deviation of \(y\) can be obtained. The random pairing of the input variables is effectively screened, and an appropriate interval pairing is selected.

Because the damage of the layer of the RCC dam is formed under the combined action of various factors, the degree of influence of various mechanical parameters (elastic modulus, internal friction coefficient, cohesion, etc.) on the stability of the layer of the RCC dam determines the sliding Mechanism and failure mode, by establishing the elastoplastic constitutive relationship of the multilayer structure of the RCC dam and the Spearman rank correlation coefficient, the sensitivity analysis of the influencing factors of the stability of the layer of the RCC dam can determine the relevant mechanical parameters of the layer and the different parts. The degree of impact on dam safety and stability.

In nonparametric statistics, Spearman rank correlation coefficient is used to measure the correlation between two random variables. Assuming two random variables \(X\) and \(Y\), the sample values are \(X_1, X_2, X_3, \cdots, X_n\) and \(Y_1, Y_2, Y_3, \cdots, Y_n\), and \(R_1\) and \(Q\) represent \(X_i\) and \(Y_i\) in \((X_1, X_2, X_3, \cdots, X_n)\) and \((Y_1, Y_2, Y_3, \cdots, Y_n)\), the test statistic \(R_s\) is the Spearman rank correlation coefficient:
In actual calculations, if there is no same rank, you can use a simplified method of calculating $R_s$:

$$ R_s = \sqrt{\sum_{i=1}^{n} \frac{(R_i - \bar{R})(Q_i - \bar{Q})}{\sum_{i=1}^{n} (R_i - \bar{R})^2 \sum_{i=1}^{n} (Q_i - \bar{Q})^2}} $$

(4)

In actual calculations, if there is no same rank, you can use a simplified method of calculating $R_s$:

$$ R_s = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n(n^2 - 1)} $$

Where $d_i = R_i - Q_i$.

### 2.3. Time-varying function of RCC dam surface deterioration

In order to study the instability law at the driving level of the deterioration of roller compacted concrete during service, we must first understand the law of material performance evolution. In order to solve this problem, starting from the factors influencing the deterioration, a dynamic model of material parameter random factor is introduced to express the evolution law of material performance of RCC dam[10]. In actual engineering, the random factors that affect the deterioration of RCC dams change dynamically with time. The change process is often a one-dimensional or multi-dimensional non-stationary random process. It is necessary to stabilize the non-stationary random process $x(t)$ [11]. Since the statistical parameters such as the expectation and standard deviation of the stationary random process are independent of the starting point $t_0$ of the time calculation, if the stationary random process $x_0(t)$ at time $t_0$ is $x(0)$, it is expressed as:

$$ x(t) = x(0)\psi(t) $$

(6)

Where $\psi(t)$ the time-varying function is determined for the stabilization of the random factor, $x_0(t)$ is the stationary stochastic process; $x(0)$ is the stationary random process $x_0(t)$ at time $t_0$, $t$ is the time.

Over time, assuming that no engineering measures such as reinforcement are taken into consideration, factors such as gradual carbonization, dissolution, and aging of the anti-seepage curtain of the body and surface of the RCC dam will cause the bulk density of the material, The elastic modulus, tensile strength, internal friction coefficient, cohesion, etc. may show a decreasing trend or some parts may show an increasing trend [12]. The commonly used time-varying models of random influence factors include Weibull distribution and exponential distribution. The exponential distribution model is used here, and considering the possibility of unconventional loads such as floods and earthquakes, the following random influence factor dynamic model expressions are obtained:

$$ x(t) = x(0)\exp \left( \lambda(t + \xi_0) + \sum_{i=1}^{N_U} \xi_i \right) $$

(7)

Where $\lambda$ represents the rate of change, $\lambda$ positive represents the rate of increase, and $\lambda$ negative represents the rate of decay, $\xi_0$ represents the performance at the starting time, which needs to be determined according to the measured information at the starting time, $N_U$ is the number of unconventional loads experienced before time $t$, $\xi_i$ represents the $i$ The effect of secondary unconventional loads.

Combined with the dynamic model of random influence factors, the strength reduction coefficient method is used to determine the anti-slide stability of the dam body and the layer. That is, under the given convergence criterion [13], the strength reserve coefficient method is used for finite element calculation to draw the key points of the dam body. The relationship curve between the displacement and the strength reserve coefficient, at the same time, based on the convergence of the finite element calculation and the range of the plastic zone where the model appears, the strength reserve coefficient is comprehensively obtained.
3. Case study

3.1. Project overview and finite element model

The maximum height of a RCC gravity dam is 72.40m, the height of the crest is 634.40m, the length of the crest is 206m, the width of the crest is 7.50m, and the slope of the downstream face of the dam section is 1.0:72. The typical dam section is analyzed as an example. Figure 2 is a schematic diagram of the main divisions of the non-overflow dam section (6# dam section). The finite element computation ontology plus level model analysis model is divided into 39514 units and 41723 nodes, as shown in Figure 3. In the analysis and calculation, the upstream and downstream water pressure, lifting pressure and dam body weight are considered.

![Figure 2. The main partitions of the RCC dam](image1)

![Figure 3. The finite element of the 6# dam section](image2)

3.2. Strength reserve coefficient calculation and sensitivity analysis

According to the indoor and outdoor test data of the project, combined with the design data, according to the material partition, the value range is shown in Table 1. Among them, $E_1$, $f_1$ and $c_1$ are the parameters of the lower layer of R1, $E_2$, $f_2$ and $c_2$ are the parameters of the lower layer of R2, $E_3$, $f_3$ and $c_3$ are the parameters of the lower layer of R3, and $E_4$, $f_4$ and $c_4$ are the parameters of the base layer.

| parameter name | $E_1$ (Gpa) | $f_1$ | $c_1$ (Mpa) | $E_2$ (Gpa) | $f_2$ | $c_2$ (Mpa) | $E_3$ (Gpa) | $f_3$ | $c_3$ (Mpa) | $E_4$ (Gpa) | $f_4$ | $c_4$ (Mpa) |
|----------------|-------------|-------|-------------|-------------|-------|-------------|-------------|-------|-------------|-------------|-------|-------------|
| Parameter range| 12.41-25.5  | 0.93-1.26 | 1.12-2.04   | 10.76-22.1  | 0.83-1.12 | 0.99-1.8    | 12.41-25.5  | 0.98-1.32 | 0.79-1.44   |

According to the sample values involved in the previous section, there should be 12 random variables, that is, the minimum number of samples that can be obtained is 16. Sampling results are obtained by LHS sampling through the Lhsdesign function, and ranking is performed, as shown in Table 2.

| No. | $E_1$ | $f_1$ | $c_1$ | $E_2$ | $f_2$ | $c_2$ | $E_3$ | $f_3$ | $c_3$ | $E_4$ | $f_4$ | $c_4$ |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1   | 3     | 1     | 2     | 6     | 15    | 5     | 5     | 7     | 8     | 14    | 14    | 9     |
| 2   | 6     | 9     | 8     | 7     | 9     | 13    | 10    | 13    | 13    | 7     | 13    |
The above parameters are based on the LHS sampling results, and the remaining calculation parameters are determined based on the design data and indoor and outdoor test data. After 16 sample calculations, 16 sets of strength reserve coefficients are obtained and ranked. The results are shown in Table 3.

| No. | K Rank | No. | K Rank |
|-----|--------|-----|--------|
| 1   | 2.56   | 9   | 2.55   |
| 2   | 2.78   | 10  | 2.71   |
| 3   | 2.88   | 12  | 2.84   |
| 4   | 2.68   | 14  | 2.70   |
| 5   | 2.80   | 16  | 2.65   |
| 6   | 2.77   | 18  | 2.74   |
| 7   | 2.83   | 20  | 2.60   |

According to the Spearman rank correlation coefficient calculation formula, the solution results are shown in Table 4.

| Rs  | 0.0798 | 0.4385 | 0.4091 | -0.2877 | 0.2915 | 0.4855 | -0.1466 | 0.4590 | 0.408 | 0.2532 | 0.2901 | 0.8795 |

Find the absolute value of $Rs$ and sort it, we can get the sorting result of sensitivity of each parameter: $c_4 > c_2 > c_5 > f_4 > f_3 > f_2 > f_1 > c_1 > c_3 > E_2 > E_4 > E_1 > E_3$, as shown in Figure 4. It can be seen from Figure 4 that the influence of each mechanical parameter on the strength reserve coefficient is large. Among them, the strength parameter has a greater influence on the strength reserve coefficient, while the elastic modulus has a smaller influence on it. The coefficient has the greatest influence, and in addition, the influence at the R2 level is second.
3.3. Instability calculation of roller compacted concrete dam based on layer degradation attenuation function

Firstly, the specific time-varying function of material parameter deterioration is determined. According to the results of sensitivity analysis in 3.2, the cohesion of the dam base surface and the cohesion of the R2 plane have the greatest impact on the anti-sliding stability of the plane. Therefore, the time-varying process of this parameter degradation must ensure accuracy. In order to make the calculated value safer, it is assumed that the dam is constantly deteriorating, that is, the parameters of the material are in a decay trend, and the strength of the long-term hydration of the RCC dam body is not considered, but floods and earthquakes are not considered. Unusual load conditions that may occur. In terms of the time-varying rate of material degradation, the effect of high seepage pressure is considered: the degradation rate of the dam base and the lower part of the dam is large, and the degradation rate of the upper part of the hydraulic pressure is small; the degradation rate of the layer is much greater than that of the body. According to the indoor and outdoor test data of the engineering example, the initial values of the material parameters of each zone are shown in Table 5.

| Material partition | Elastic modulus (GPa) | Elastic modulus (kN/m³) | f  | C    |
|--------------------|-----------------------|--------------------------|----|------|
| Normal CC          | 24                    | 0.167                    | 24 | 1.26 | 2.625 |
| Body R1            | 24.38                 | 0.163                    | 24 | 1.2285 | 2.268 |
| Body R2            | 23.44                 | 0.163                    | 24 | 1.1235 | 2.205 |
| Body R3            | 22.5                  | 0.163                    | 24 | 1.05   | 2.0685 |
| Layer R1           | 19.215                | 0.3                      | 24 | 1.26 | 2.04  |
| Layer R2           | 18.06                 | 0.3                      | 24 | 1.12   | 1.8    |
| Layer R3           | 16.275                | 0.3                      | 24 | 1.08   | 1.14  |
| Construction base  | 19.215                | 0.3                      | 24 | 1.32 | 1.44  |
| basis              | 15                    | 0.27                     | /  | /     |

Each parameter is calculated from the deterioration time-varying function of 2.3. With time, the material parameter value in year $t$ is the initial value multiplied by the decay function $e^\lambda t$. The corresponding $\lambda$ of each parameter are shown in Table 6.

| Material partition | Elastic modulus | Elastic modulus | f    | C    |
|--------------------|-----------------|-----------------|------|------|
| Normal CC          | -0.005          | 0               | -0.0005 | -0.007 | -0.012 |
| Body R1            | -0.001          | 0               | -0.0005 | -0.0034 | -0.005 |
| Body R2            | -0.0009         | 0               | -0.0005 | -0.00335 | -0.00045 |
| Body R3            | -0.0008         | 0               | -0.0005 | -0.004 | -0.0065 |
| Layer R1           | -0.005          | 0               | -0.0005 | -0.007 | -0.012 |

Figure 4. Sensitivity distribution of various mechanical parameters to the overall stability
After the material degradation coefficients of each zone are determined, the strength reserve coefficients corresponding to different parameters can be calculated. The results of the initial value of the material parameter strength reserve coefficient are shown in Figure 5. After adjusting the material parameters according to the time-varying function in other years, the calculation is performed in the same way. The results are shown in Table 7. The course curve of the level of anti-skid stability strength reserve coefficient with service life is shown in Figure 6.

| Layer | Parameter | Strength Reserve Coefficient |
|-------|-----------|------------------------------|
| R2    | 0         | -0.0048                      |
| R3    | 0         | -0.0046                      |
| Base  | 0         | -0.0052                      |
|       | 0         | -0.0005                      |
|       | 0         | -0.0068                      |
|       | /         | -0.01                         |

![Figure 5](image)

Figure 5. The initial value of the material parameter strength reserve coefficient

It can be seen from Figure 6 that under normal conditions, the strength reserve factor gradually decreases as the material deteriorates without considering the impact of various engineering measures such as later reinforcement. From theoretical calculations, the strength reserve coefficient approaches 1 at the 80-year service period, indicating that the dam is basically unstable at 80 years, but this situation is too idealized and does not consider possible unconventional phenomena such as overload, floods, and earthquakes. Load conditions, therefore, it is unreasonable to use the strength reserve factor of 1 as an
indicator of instability. With reference to Chang Xiaolin et al. [14] on the calculation of the critical criterion formula of RCC dam stability and the design safety factor, the design safety factor under basic combined load is 1.4. In this paper, 1.4 is also used as the instability judgment index, which is about 56. The possibility of annual instability is greater. Therefore, safety monitoring should be strengthened and engineering measures should be taken in advance to reinforce or non-engineering measures to intervene to ensure the safety of the dam.

| Years of service (years) | Strength reserve factor |
|------------------------|------------------------|
| 0                      | 3.53                   |
| 10                     | 3.25                   |
| 20                     | 2.88                   |
| 30                     | 2.41                   |
| 40                     | 2.03                   |
| 50                     | 1.55                   |
| 60                     | 1.33                   |
| 70                     | 1.22                   |
| 80                     | 1.03                   |

**Figure 6. Process line of strength reserve coefficient with service life**

4. **Conclusion**

In this paper, the sensitivity analysis method of level mechanical parameters to stability and the law of level instability driven by the deterioration of roller compacted dam are studied. The main conclusions are as follows:

1. A new multi-factor sensitivity analysis method is established by combining Latin hypercube sampling method and Spearman rank correlation coefficient method. The analysis result of this method is consistent with the characteristics of RCC dam, which shows the feasibility of this method. Based on the time-varying attenuation model of material degradation and the strength reserve coefficient analysis method of anti-sliding stability, the results are consistent with the operation law of the RCC dam. This method can reveal the instability law of the driving level of the deterioration of the RCC dam.

2. According to the sensitivity analysis results, during the operation and maintenance stage of the dam, the monitoring and control of the dam base surface and the rolling surface under the high seepage pressure near the dam base should be focused on to ensure the dam safety. Moreover, in the design and construction, it provides a reference for the design and construction of similar projects of the RCC dam. It is necessary to focus on the quality of the roller compacted layer under the high seepage pressure near the dam foundation and especially to ensure its cohesion and internal friction coefficient. Meet the requirements. It can be seen from the situation that the strength reserve coefficient changes with the service life, and after 50 years of service, the dam should be comprehensively inspected and reinforced.

3. In order to simplify the model, this paper does not consider the strength increase of the long-term hydration of the RCC dam body, nor does it consider the unconventional load conditions that may occur such as floods and earthquakes. The researchers can further study these unconventional conditions.

**Acknowledgments**

This research has been partially supported by the National Key R&D Program of China (2018YFC1508603), National Natural Science Foundation of China (Grant Nos. 51739003, 51579086).
References

[1] Wang Chao, Song Ran, Wang Gaohui, et al. (2020) Modifications of the HJC (Holmquist-Johnson-Cook) Model for an Improved Numerical Simulation of Roller Compacted Concrete (RCC) Structures Subjected to Impact Loadings. Pubmed, 13(6):1361.

[2] Ashrafian Ali, Gandomi Amir H., Rezaie-Balf Mohammad, et al. (2020) An evolutionary approach to formulate the compressive strength of roller compacted concrete pavement. Measurement, 152(C).

[3] Fang Haiting. (2006) Study on Layer Mechanical Behavior and Properties of Roller Compacted Concrete Dams. Hohai University.

[4] Chongshi Gu, Guangming Huang, Daoping Lai. (2007) Analytic Model of Deformation of Construction Interfaces of RCCD. Applied Mathematics and Mechanics, 01:70-76.

[5] Li Bo, Chongshi Gu, Jinkun Wu. (2013) Gradual change law of elastic mechanical parameters of roller compacted concrete dam. Journal of Hydraulic Engineering, 44(12):1488-1497.

[6] Lanting Zhou, Chongshi Gu. (2009) Reckoning Equivalent Deformation Parameters of RCCD Construction Interfaces. Water Resources and Power, 27(4):77-80.

[7] Weifeng Zhang, Yanbo Che, Yangsheng Liu. (2015) Improved Latin Hypercube Sampling Method for Reliability Evaluation of Power Systems. Automation of Electric Power Systems, 4:52-57.

[8] Yongjiang Zhou, Yibin Yao, Yongliang Xiong. (2020) Study of Correlation between PWV and PM2.5 Based on Spearman Rank Correlation Coefficient. Journal of Geodesy and Geodynamics, 40(03):236-241.

[9] Huang Guangming, Li Yun, Gu Chongshi, et al. (2006) Retrieval of transversely isotropic viscoelastic parameters for RCC dams. Hydro-Science and Engineering, 4:15-20.

[10] Gu Chongshi, Li Bo, Yu Hong, et al. (2010) Back analysis of mechanical parameters of roller compacted concrete dam. Science China Technological Sciences, 6:651-656.

[11] Futian Jiang. (2008) Layer surface of RCC dam and its influence. Water Resources and Hydropower Engineering, 39(2):19-21.

[12] Yuxi Liu. (2015) Principle and Application of Construction Information Model for RCC Dam. Tianjin University.

[13] Zheng Wan, Yangping Yao, Da Meng. (2016) An elastoplastic constitutive model of concrete under complicated load. Chinese Journal of Theoretical and Applied Mechanics, 48(5): 1159-1171.

[14] Xiaolin Chang, Shuyuan Lu, Guowei Lai. (1998) Critical stability criterion formula and design safety factors of RCC gravity dams. Journal of Hydraulic Engineering, 29(5):0060-0065.