COSMOLOGICAL CONSTRAINTS FROM A COMBINED ANALYSIS OF THE CLUSTER MASS FUNCTION AND MICROWAVE BACKGROUND ANISOTROPIES

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ABSTRACT

We present constraints on several cosmological parameters from a combined analysis of the most recent cosmic microwave background anisotropy data and the Sloan Digital Sky Survey cluster mass function. We find that the combination of the two data sets breaks several degeneracies among the parameters and provides the following constraints: $\sigma_8 = 0.76 \pm 0.09$, $\Omega_m = 0.26^{+0.06}_{-0.07}$, $h = 0.66^{+0.05}_{-0.06}$, $n = 0.96 \pm 0.05$, and $\tau = 0.07^{+0.07}_{-0.05}$.

Subject headings: cosmic microwave background — cosmological parameters — galaxies: clusters: general

On-line material: color figure

1. INTRODUCTION

The last few years have seen a spectacular increase in the amount and quality of available cosmological data. The new results on the cosmic microwave background (CMB) angular power spectrum (Netterfield et al. 2001; Halverson et al. 2002; Lee et al. 2001; Pearson et al. 2002; Scott et al. 2002; Benoit et al. 2003) have confirmed the theoretical prediction of acoustic oscillations in the primordial plasma and constrained theories of large-scale structure formation (see, e.g., Wang, Tegmark, & Zaldarriaga 2002). At the same time, early data from the Two-Degree Field Galaxy Redshift Survey (Percival et al. 2001) and the Sloan Digital Sky Survey (SDSS; York et al. 2000; Stoughton et al. 2002) have provided an unprecedented view of the large-scale structure of the universe as traced by galaxies.

Combined analysis of these independent CMB and galaxy data sets is placing strong constraints on some of the fundamental cosmological parameters (Bahcall et al. 1999; Efstathiou et al. 2002; Lahav et al. 2002; Melchiorri & Silk 2002). Together with the high-redshift supernovae results (Perlmutter et al. 1999; Filippenko & Riess 2000), a concordance model of the cosmological parameters of the $\Lambda$-CDM model has become the current paradigm.

The goal of these analyses is to determine the precise values of the cosmological parameters of the $\Lambda$-CDM model. Because of "cosmic degeneracy," the CMB data alone are unable to place tight constraints on several fundamental parameters, such as the rms amplitude of the mass fluctuations $\sigma_8$, the Hubble parameter $h$, and the optical depth of the universe $\tau$, even if one restricts the analysis to a flat universe.

In the present Letter, we combine the most recent CMB anisotropies data with the constraints obtained from the mass function of clusters of galaxies determined from early commissioning imaging data of the SDSS (Bahcall et al. 2003) to break the degeneracy among the cosmological models, allowing a determination of the best-fit values for individual parameters.

2. METHOD

2.1. CMB Data and Analysis

We consider a template of flat, adiabatic, $\Lambda$-CDM CMB spectra computed with CMBFAST (Seljak & Zaldarriaga 1996), sampling the various parameters as follows: the physical density in CDM $\Omega_{cdm} h^2 = \omega_{cdm} = 0.05, \ldots, 0.40$, in steps of 0.02; the physical density in baryons $\Omega_b h^2 = \omega_b = 0.009, \ldots, 0.030$, in steps of 0.003; and the cosmological constant $\Omega_{\Lambda} = 0.5, \ldots, 0.95$, in steps of 0.05. For each set of these parameters, the scalar spectral index $n$ is varied over the relevant inflationary values of $n = 0.8, \ldots, 1.2$, in steps of 0.02. The value of the Hubble constant is not an independent parameter, since $h = (\omega_{cdm} + \omega_b)/(1 - \Omega_{\Lambda})^{1/2}$; we include a top hat prior $h = 0.7 \pm 0.2$ (Freedman et al. 2001). Only models with age $t_0 > 11$ Gyr are considered.

We allow for a reionization of the intergalactic medium by varying the Compton optical depth parameter $\tau_e$ in the range $\tau_e = 0.0, \ldots, 0.45$ in steps of 0.05. High values of $\tau_e$ are in disagreement with recent estimates of the redshift of reionization $z_{re} \sim 6 \pm 1$ (see, e.g., Fan et al. 2002; Gnedin 2001), which point toward $\tau_e \sim 0.05$–0.10. However, since the mechanism of reionization is still not clear, we allow this parameter to vary freely within the above conservative range. As shown below, the combination of the CMB and cluster mass function (CMF) data provides an independent constraint on this parameter.

For the CMB data, we use the recent results from the BOOMERANG-98 (Netterfield et al. 2001), DASI (Halverson et al. 2002), MAXIMA-1 (Lee et al. 2001), CBI (Pearson et al. 2002), VSA (Scott et al. 2002), and Archeops (Benoit et al. 2003) experiments.

The power spectra from these experiments were estimated in 19, 9, 13, 14, 10, and 16 bins, respectively. For the CBI, we use the data from the Mosaic configuration (Pearson et al. 2002), spanning the range $2^\circ \leq l \leq 1500^\circ$. We also use the COBE data from the RADPACK compilation (Bond, Jaffe, & Knox 2000).

For the Archeops, CBI, DASI, MAXIMA-I, and VSA experiments, we use the publicly available correlation matrices and window functions. For the BOOMERANG experiment, we assign a flat interpolation for the spectrum in each bin $i(l + 1)C_l/2\pi = C_n$, and we approximate the signal $C_n$ inside the bin to be a Gaussian variable. The CMB likelihood for a given theoretical model is defined by $-2 \ln L^{CMB} = (C_n - C_n^{\text{obs}}) M_{\text{diag}}^{-1}(C_n^{\text{obs}} - C_n^{\text{diag}})$, where $M_{\text{diag}}$ is the Gaussian curvature of the likelihood matrix at the peak.

We consider 7%, 10%, 4%, 5%, 3.5%, and 5% Gaussian distributed calibration errors (in $\Delta t$) for the Archeops, BOOMERANG-98, DASI, MAXIMA-1, VSA, and CBI experiments, respectively, and we include the beam uncertainties by the analytical marginalization method presented in Bridle et al. (2002). Finally,
we rescale the spectrum by a prefactor $C_{10}^\nu$ assumed to be a free parameter, in units of $C_{10}^{cobi}$.

In order to constrain a parameter $x$, we marginalize over the values of the other parameters $y$. This yields the marginalized likelihood distribution

$$L(x) = \int L(x, y) dy.$$ (1)

The central values and 1 $\sigma$ limits are then found from the 16%, 50%, and 84% integrals of $L(x)$.

2.2. Cluster Mass Function Analysis

We use the CMF obtained from the early SDSS commissioning data (Bahcall et al. 2003). This CMF was derived from 294 clusters in the redshift range $z = 0.1-0.2$ selected by the hybrid matched filter (HMF) method. The HMF mass function was compared with large-scale cosmological simulations as well as with model predictions, as discussed in Bahcall et al. (2003). To compute a likelihood for a given set of cosmological parameters, the linear matter power spectrum is computed, which in turn is used to predict a CMF. Thus, for each differential mass bin, the predicted number of clusters in the survey volume can be compared with the observed number, using the error bars as given in Bahcall et al. (2003). The 68%, 95%, and 99% confidence contours (allowing $h$ and $n$ to vary as in the previous section) are shown by the dashed curves in Figure 1. If the Hubble constant and spectral index are kept constant at $h = 0.72$ and $n = 1$, then the best-fit relation between amplitude and density can be summarized as $\sigma_8\Omega_m^{1.6} = 0.33$. The CMF is not sensitive to the choice of the optical depth parameter $\tau_c$.

The shape of the CMF only partially breaks the degeneracy between $\Omega_m$ and $\sigma_8$ in the above relation: low values of $\sigma_8$ yield a steeper CMF shape at the high-mass end (i.e., fewer high-mass clusters) than do low $\Omega_m$ values (which produce a flatter CMF shape). The CMF prefers a low value for the mass density parameter and a relatively high value for the amplitude $\sigma_8$: the best-fit parameters for the HMF clusters are $\Omega_m = 0.18$ and $\sigma_8 = 0.92$; similar results are obtained for SDSS clusters selected by the maxBCG method (see Bahcall et al. 2003). The above $\sigma_8\Omega_m$ relation is consistent with recent results from the X-ray cluster temperature function (see Seljak 2001; Ikebe et al. 2002; Reiprich & Böhringer 2002; Pierpaoli et al. 2002) and from recent cosmic shear lensing observations (Hamana et al. 2002; Jarvis et al. 2003). The above relation is considerably lower than the "old" cluster normalization of $\sigma_8\Omega_m^{1.5} = 0.5 \pm 0.05$ obtained from previous observations of the cluster mass and temperature functions, as well as weak lensing analyses (see, e.g., Refregier, Rhodes, & Groth 2002 and references therein). However, most recent results that use larger samples and improved calibrations agree with the lower normalization relation; see comparisons and discussion in Bahcall et al. (2003).

The observed CMB spectrum of fluctuations suggests a lower amplitude value for $\sigma_8$ (which is, however, degenerate with the optical depth parameter for CMB) and a somewhat larger value for the mass density parameter ($\Omega_m \sim 0.3$ for $h = 0.72$) than given by the CMF above (see, e.g., Lahav et al. 2002; Melchiorri & Silk 2002; Wang et al. 2002 and references therein). However, the CMB and the CMF results are consistent with each other within 1 $\sigma$. Combining the CMB and CMF data will clearly result in intermediate values for the cosmological parameters, shifting the CMF constraints presented above toward a somewhat lower amplitude and higher mass density regime.

We combine the CMF results with those of the CMB by multiplying the two likelihoods $L(CMB) L(CMF)$, using the same range of parameters ($\Omega_m$, $\sigma_8$, $h$, and $n$), and marginalizing over the nuisance parameters as discussed in the previous section. Note that the CMF likelihood is independent of $\tau_c$.

3. RESULTS

The main result of our analysis is presented in Figure 1, in which we plot likelihood contours in the $\Omega_m$-$\sigma_8$ plane for the two data sets, separately and combined. It can be seen that both the CMB and the CMF data sets are affected by degeneracies between $\sigma_8$ and $\Omega_m$. In the case of the CMF, an increase of $\Omega_m$ results in a larger number of clusters, and $\sigma_8$ must be reduced to bring the predicted CMF back in line with observations. On the other hand, the CMB is only weakly sensitive to the trade-off between density and amplitude, so models with higher $\Omega_m$ and $\sigma_8$ can be in agreement with CMB data. The degeneracy in the CMB data set is therefore opposite to the one in the CMF data, and the two measurements complement each other. The combination of the two data sets provides the constraints $\Omega_m = 0.26^{+0.08}_{-0.07}$ and $\sigma_8 = 0.76 \pm 0.09$ at 1 $\sigma$ confidence level, as shown in Figure 1. The relatively low value of $\sigma_8$ obtained here is consistent with several recent combined analyses of CMB data with large-scale structure and/or supernovae observations (see, e.g., Melchiorri & Silk 2002; Lahav et al. 2002). However, interpretations of the recent small-scale anisotropy ($l > 2500^\circ$) excess from the CBI experiment as due to a Sunyaev-Zeldovich effect give $\sigma_8 = 1.05 \pm 0.15$ at 95% confidence level, which is inconsistent at the level of about 2 $\sigma$. A similar result of $\sigma_8 = 0.98 \pm 0.1$ is obtained from the evolution of cluster abundance (Bahcall & Bode...
Since this result is still compatible with the CMF analysis only if \( \omega_m \sim 0.15 \), it will be the duty of the next satellite and ground-based CMB multifrequency experiments to further scrutinize this discrepancy.

The CMB + CMF combination can also break additional degeneracies. The optical depth \( \tau_c \) and the Hubble parameter \( h \) are better determined after the inclusion of the CMF data. From the combined analysis, we obtain (at 1 \( \sigma \) confidence level)
\[
\begin{align*}
\Omega_m & = 0.26_{-0.05}^{+0.06}, \\
\Omega_b & = 0.05_{-0.05}^{+0.06}, \\
h & = 0.70_{-0.11}^{+0.09}, \\
\Omega_c & = 0.07, \\
\end{align*}
\]
These values can be compared with
\[
\begin{align*}
\Omega_m & = 0.11_{-0.09}^{+0.09}, \\
\Omega_b & = 0.06_{-0.06}^{+0.06}, \\
h & = 0.70_{-0.09}^{+0.09}, \\
\Omega_c & = 0.13_{-0.07}^{+0.07}, \\
\end{align*}
\]
from the CMB-only analysis.

4. CONCLUSIONS

We combine the CMB anisotropy and SDSS CMF data to produce constraints on several cosmological parameters. The complementary nature of the two data sets breaks existing degeneracies among cosmological parameters. The CMB data tend to indicate a higher \( \Omega_m \) and a lower amplitude \( \sigma_8 \) than suggested by the cluster data; thus, the combined result for \( \Omega_m \) is pulled to a lower value than suggested by the CMB alone, and the amplitude \( \sigma_8 \) is lower than suggested by the cluster data alone. We find that the combined data suggest a mass density \( \Omega_m = 0.26_{-0.05}^{+0.06} \), a normalization of the matter power spectrum \( \sigma_8 = 0.76 \pm 0.09 \), Hubble parameter \( h = 0.66_{-0.06}^{+0.06} \), a nearly scale-invariant spectrum of primordial fluctuations \( n = 0.96 \pm 0.05 \), and an optical depth of the universe \( \tau_c = 0.07_{-0.05}^{+0.05} \).

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