Non-linear vibrations in rotor systems with floating ring bearings induced by fluid-structure interactions

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Although the use of combustion engines will decrease in passenger cars due to ecological reasons, turbochargers (TC) as typical applications with floating ring bearings will furthermore be applied in other applications like combined heat and power plants, ships or emergency backup generators for data processing service center and similar systems. Thus, the development of TC concerning rotor- and hydrodynamics is an important task, which leads due to the non-linear behaviour to the necessity of advanced simulations for improving the performance.

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1 Introduction

For a sustainable analysis of the rotor dynamic behaviour of hydrodynamically supported systems the non-linear effects resulting from the journal bearings have to be considered. This can take place in various manners always with regard to the application and the task of the analysis. For transient loads and operational conditions analytical solutions of the governing Reynolds equation with usual simplifying assumptions show restricted accuracy concerning the non-linear force laws, which depend on position and velocity of the bearing partners. Thus, numerical approaches have to be used. At this point the interaction between the hydrodynamics and rotor dynamics can be enlarged by additional influencing field problems like thermodynamics (energy equation and thermal behaviour of the surrounding structure) or structural dynamics (local and/or global elastic deformations due to acting forces or even thermal loads). As a result, for specific systems like TC, crank drives or gas turbines the coupled effects have to be analysed concerning their influences on each other. Hence, adapted simulation methods have to be used. Furthermore, the necessary level of detail for the description of the non-linearity to assure appropriate simulation results for the vibration in an adequate time is rather unclear. Especially effects resulting from cavitation are often neglected in transient analyses. In this contribution, a TC rotor supported in full floating ring bearings is examined, cf. Fig. 1. For this sake, an extended and time efficient approach for the numerical consideration of cavitation in floating ring bearings is presented and studied for different conditions like varying bearing clearances. The results are validated against measurements and the model is compared concerning its advantages and disadvantages w.r.t. common approaches.

2 Fluid-structure interaction

Full floating ring bearings consist of two serially arranged journal bearings, leading to an outer and inner fluid film. While the outer film is directly supplied with oil, the inner film is connected to the outer one by a set of drill holes, cf. Fig. 2, leading to coupling effects concerning the pressure distributions in both fluid films. Furthermore, the resulting drag torque in the inner film accelerates the floating ring, while the drag torque in the outer film acts decelerating. Summing up the torques including inertia effects, the current acceleration and thus speed of the floating ring results depending on bearing clearances, temperatures of oil and bearing parts as well as on bearing kinematics caused by outer loads. As a result, the ring speed usually is in the range of \( \nu_{\text{ring}} = 0.2 \ldots 0.45 \times \nu_{\text{shaft}} \).

Due to the rotation of shaft and floating ring, the oil circulates in both fluid films, which causes the well known oil-whirl excitation [1]. The excitation frequency is determined by the surface velocities of the adjacent bearing partners, which can be expressed as \( f_{\text{exc},i} \approx (f_{\text{shaft}} + f_{\text{ring}})/2 \) and \( f_{\text{exc},o} \approx f_{\text{ring}}/2 \), respectively. Both excitation frequencies interact with the elastic properties of the shaft. Hence, resonance-like states can occur, if e.g. the excitation frequency \( f_{\text{exc},i} \) coincides with the eigenfrequency of the first forward whirl mode, cf. Fig. 3. In contrast to the usual resonance due to unbalance excitation, these subharmonic vibrations are perpetuated, even if the resonance condition is left by increasing the rotor speed. This is caused by the fluid films, since the phase angle between excitation and answer is limited to 90° here [2, 3]. Subsequently, the rotor is locked into the corresponding eigenfrequency, known as oil-whip, until a further subharmonic resonance with the next eigenfrequency occurs. The whole procedure repeats here with locking into the second forward whirl mode until at a certain rotor speed the outer excitation \( f_{\text{exc},o} \) coincides with the first forward whirl mode.

Finally, a series of subharmonic response vibrations results, which are usually named Sub1, Sub2 and Sub3 in the order of occurrence w.r.t. rotor speed. The resulting shaft motion in turn influences the non-linear bearing properties like stiffness, damping and the excitation frequency, which completes the fluid-structure interaction loop.

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3 Numerical model

In addition to these preliminary theoretical considerations, in real TC rotors two floating ring bearings with different loads and possibly different geometry occur leading to a more complex excitation scheme. Furthermore, gyroscopic effects of the rotor result in splitting of eigenfrequencies (EF) in forward (increasing EF) and backward (decreasing EF) whirl mode. Hence, a numerical time integration procedure is used to predict the system’s behaviour during a run-up. Thus, transient as well as non-linear effects due to the hydrodynamical bearings can be handled in detail. For this sake, the equations of motion for the rigid floating rings

\[ M_{FR} \ddot{x}_{FR} + h \omega_{FR} = h_{ex,FR} \]  

(1)

as well as for the elastic shaft, which is modelled using finite beam elements (Timoshenko theory),

\[ M_{FE} \ddot{x}_{FE} + (D_{FE} + \Omega G_{FE}) \dot{x}_{FE} + K_{FE} x_{FE} = h_{ex,FE} \]  

(2)

including mass properties of turbine and compressor wheel are used. From these equations, the accelerations are calculated and transferred in state space description in order to be integrated using an appropriate ODE solver [3].

Within Eq. 1 and Eq. 2 the external loads \( h_{ex} \) contain gravity forces and unbalance of the wheels. Beside that, here also the bearing forces are considered entailing a link between the motion of both floating rings and the shaft as well as between

\[ \gamma_i = \lambda_i (\Omega_{shaft} + \Omega_{ring}) \]

\[ \gamma_o = \lambda_o \Omega_{ring} \]

\[ f_1 \]

\[ f_1 + f_1 - f_2 \]

\[ f_2 + f_2 - f_2 \]

Fig. 3: Typical Campbell diagram of a turbocharger rotor with excitations \( \gamma_i \) and \( \gamma_o \) due to oil-whirl in case of full floating ring bearings and the resulting subharmonic response (bold line) [6].

Fig. 4: Switch function of Sigmoid type \( g(\Pi) = \frac{1}{2} \arctan\left(\frac{\Pi}{\Pi^*}\right) + \frac{1}{2} \) for different regularisation parameters \( \Pi \) [7].

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the body motion and the hydrodynamics. In order to calculated the bearing forces, the Reynolds PDE is solved numerically in each time step for the pressure distributions $p_i$ and $p_o$. This procedure provides the possibility to account for tilting motion including restoring torques, for cavitation effects and for coupling between inner and outer fluid film due to the drill holes. The resulting simulation loop for the fluid-structure interaction is depicted in Fig. 5.

An additional important point is the consideration of cavitation in the fluid films. Several approaches are used in that context with different modelling depth (e.g. non-mass conserving: Gümbel or mass conserving: JFO-theory). In this contribution the Elrod model is used, which on the one hand belongs to the mass conserving group and on the other hand yields a good compromise between accuracy and effort. Therefore, the film content $\vartheta$ is introduced as unknown in the cavitation zone, whereas the pressure is assumed to be known there as $p_{cav}$. Usually, a unique variable $\Pi$ is introduced, combining the complementary unknowns $p$ and $\vartheta$

\[
p = g \Pi \quad \text{and} \quad \vartheta = (1 - g) \Pi + 1 \tag{3}
\]

via a switch function $g = f(\Pi)$. Hence, the pressure is mapped to positive values of $\Pi$, whereas the film content is mapped into $\Pi = -1 \ldots 0$. The Reynolds PDE in dimensionless form according to [7] then reads

\[
\left[ \frac{\partial}{\partial X} \left( \frac{H^3}{12} \frac{\partial \Pi}{\partial X} \right) + \frac{\partial}{\partial Y} \left( \frac{H^3}{12} \frac{\partial \Pi}{\partial Y} \right) - \frac{\partial H}{\partial X} \frac{\partial H}{\partial T} \right] g = 0 \quad \text{(pressure zone)}
\]

\[
+ \left( \frac{\partial ((\Pi+1)H)}{\partial X} + \frac{\partial ((\Pi+1)H)}{\partial T} \right) (g-1) = 0 \quad \text{(cavitation zone)}, \tag{4}
\]

which is afterwards discretised with FVM yielding a set of $n$ equations

\[
A_p \varrho = r \quad \text{with} \quad \varrho = [\Pi_1 \ldots \Pi_n] \quad \text{and} \quad A = A_p g(\varrho) + A_{cav} (g(\varrho) - 1). \tag{5}
\]

Since the cavitation boundary and therefore the distribution of $g$ is initially unknown, Eq. 5 becomes non-linear. Depending on the choice of $g$, it can be solved either using fix-point iteration ($g$ is Heaviside function) or Newton-Raphson method ($g$ is continuously differentiable e.g. of Sigmoid type), cf. Fig. 4. The latter shows very similar results with a coarser mesh and additionally manages a better convergence rate leading to a significant reduction in calculation time. Finally, from the solution $\varrho$ the bearing forces and the restoring as well as the drag torques are calculated, which enter in Eq. 1 and Eq. 2.

### 4 Results

**Ring speed measurement** In order to verify the simulation, an experiment documented in [8] with measurement of ring speed ratio (RSR) on a single floating ring bearing under static load was modelled. The RSR is an integral quantity depending on several factors, e.g. drag torques, temperatures, hot clearances, viscosity and pressure gradient. Hence, all of them have to be mapped satisfactorily in order to match the measured RSR. Finally, the comparison with the simulated RSR for different values of bearing clearance, cf. Fig. 6, showed good agreement in a wide range of load (inverse Sommerfeld number).

**TC run-up** The next step of validation is done on a real TC driven by hot gas from a combustion engine. Fig. 7 shows a comparison of compressor end vibrations in form of a spectrogram, which visualises the harmonic as well as subharmonic frequency parts during the run-up. Again the agreement of simulation with the measurement is good, especially concerning the jump frequencies ($f_{\text{Shaft}}$ for Sub1 $\Rightarrow$ Sub2 and Sub2 $\Rightarrow$ Sub3). Further, the response frequencies of the Subs as well as their amplitudes correlate well.

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**Fig. 6:** Ring speed ratio vs. Sommerfeld number - comparison of simulation with measurements of Kettlebrough for different clearance ratios. [6].

**Fig. 7:** Measurement (left) vs. simulation (right) full-floating TC with standard supply pressure: Spectrogram of compressor side shaft end displacement [7]. Within simulation, inner and outer oil excitation as well as their interactions to EFs are visible.
Fig. 8: Simulation with different models: Elrod-Online (left), Gumbel-Online (mid) and Elrod-look-up table (right) with varied inner bearing clearance (c) and outer bearing clearance (f) according to [7].

Model depth Since the calculation time for a run-up is comparatively high ($\approx 20h$), the question of simplification arises. A change to Gumbel cavitation algorithm reduces Eq. 5 to a linear form. Another possibility is to pre-calculate the bearing behaviour to look-up tables under the assumption of parallel gaps. The latter is fast (< $1h$), but misses $Sub_1$ and $Sub_2$ completely here. Since Gumbel approach was used in an online form, it doesn’t save much time, yielding similar results but leads to a slight shift of jump frequencies, cf. Fig. 8.

5 Summary
A method for calculating the run-up of TC rotors with consideration of the bearings including cavitation effects was presented. The model was validated against measurements and showed good agreement. Finally, the model depth was varied.

Acknowledgements The results were generated in the framework of the project WO 2085/2, which is supported by the DFG (Deutsche Forschungsgemeinschaft / German Research Foundation). This support is gratefully acknowledged.

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