Investigating the Effect of Family Non-universal $Z'$ Boson in $B \rightarrow \phi\phi$ Decay

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Within the perturbative QCD approach, we re-calculate the branching ratio and polarization fractions of the pure annihilation decay $B \rightarrow \phi\phi$ in both the standard model (SM) and the family non-universal $Z'$ model. We find that this decay is dominated by the longitudinal part, while the transverse parts are negligibly due to the absence of the $(S - P)(S + P)$-type operator. In SM, the branching ratio is predicted as $(4.4^{+0.8+0.3}_{-0.6-0.5}) \times 10^{-8}$, which is larger than the previous predictions. With an additional $Z'$ boson, the branching ratio can be enhanced by a factor of 2, or reduced one half in the allowed parameters space. These results will be tested by the ongoing LHCb experiment and forthcoming Super-B experiments. Moreover, if the $Z'$ boson could be directly detected at hadron collider, this decay can be used to constrain its mass and the couplings in turn.

I. INTRODUCTION

Despite the fact that the standard model of particle physics has various predictions that are in accordance with experimental data, it is generally viewed as an effective realization of an underlying theory to be discovered yet. Interestingly to understand the hierarchy problem of the Higgs mass, neutrino masses, and the CP asymmetry, one is often allured to resort to the new physics (NP) beyond SM. If existing, the NP degree of freedom may manifest itself either directly at the hadron collider or indirectly at low energy via its effects to observables that have been precisely constrained. Over the past years, processes induced by flavor-changing-neutral-current (FCNC) have been under sharper scrutiny, as these processes are forbidden at the tree level and thus arise only at the loop level within SM. Many NP models have different patterns with SM and enhance the FCNC transition at the tree or loop level, which are likely to affect some physical observables sizably compared to SM.

The rare decay $B \rightarrow \phi\phi$ is of this type and proceeds via a FCNC process $b \rightarrow d\bar{s}s$. Moreover,

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since all quarks in the final states are different from those in the initial $B$ meson, this decay involves only the pure annihilation topology. As a consequence of the power counting rules derived from the heavy quark effective theory, its branching ratio is expected to be very tiny. Meanwhile, on the experimental side, the signature of this decay is very clean. Due to these advantages, the $B \to \phi\phi$ has thus received considerable attentions in both theoretical [1–3] and experimental sides [4, 5] in the past few years.

To the best of our knowledge, an annihilation amplitude involving two light mesons suffers from the endpoint divergence, and many approaches have been advocated for dealing with it. In [1], by introducing the effective gluon mass $m_g = 500 \pm 200$ MeV, the authors predicted $Br(B^0 \to \phi\phi) = (2.1^{+1.6}_{-1.3}) \times 10^{-9}$ in SM. While in the R-parity violating supersymmetric model, the branching fraction of this decay could be enhanced to $10^{-7}$. In the QCD factorization approach, the endpoint singularity has been usually parameterized by two free parameters $\rho_A$ and $\phi_A$ in a phenomenology way, which are mode-dependent and cannot be calculated directly. As a result, only the upper limit of this decay $10^{-8}$ has been presented in [3]. By keeping the intrinsic transverse momenta $k_T$ of the valence quarks in the perturbative QCD (PQCD) approach, the annihilation topologies could be calculated directly, as the divergence can be eliminated by the Sudakov form factor and the threshold resummation. Within the PQCD approach, its branching ratio has been predicted to be $(1.89^{+0.61}_{-0.21}) \times 10^{-8}$ in [2], in which the longitudinal polarization fraction was estimated to be about 65%. However, as discussion the decay modes $B \to \phi K^*$ [6–8], it has been known that the longitudinal polarization fraction about 48% was measured in experiments. In the PQCD framework, the annihilation contribution from the $(S - P)(S + P)$ operators enhances the amplitudes remarkably due to the helicity flip, so the so-called ”polarization anomaly” could be well understood. However, because the $(S - P)(S + P)$ operator vanishes in this mode, it is hard for us to understand the large transverse polarization 35% predicted in [2]. Therefore, it is necessary to re-analyse this decay in SM within the PQCD approach.

As stated above, in SM, the decay $B \to \phi\phi$ is expected to have a small branching ratio, which allows us to search for possible NP effects. Hence, another purpose of this work is to explore the effects of an extra $Z'$ boson on this decay, which is allowed in a few well motivated extensions of SM due to an additional $U(1)'$ gauge symmetry. Among many $Z'$ models, the most general one is the family non-universal $Z'$ model, which can be realized in various grand unified theories, string-inspired models, dynamical symmetry breaking models, and the little Higgs models, just to name a few [9]. The $Z'$ boson in different representative models has been directly searched at
colliders as well as indirectly probed via a variety of precision data \cite{10}, which put limits on its
gauge coupling and/or mass. In such a model, the nonuniversal \(Z'\) couplings to fermions could
lead to FCNC at the tree level, which may enhance the branching ratios of some rare \(B\) decays
dominated by penguin operators. In recent years, the effects of the \(Z'\) boson have been studied
extensively in the low energy flavor physics phenomenology, such as in \(B\) physics, top physics,
and lepton decays \cite{11}.

In this work, we will first reanalyze \(B \rightarrow \phi \phi\) in SM within the PQCD approach in Sec.II, and
find that the results for branching ratios are larger than the predictions in \cite{2}. We then in Sec.III
consider the contribution of the non-universal \(Z'\) boson, which could change the branching ratio
in the suitable parameters space. At last, we summarize this work in Sec. IV.

II. CALCULATION IN SM

In SM, the relevant effective weak Hamiltonian related to \(B \rightarrow \phi \phi\) is given by:

\[
H^{SM}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{tb}^* V_{td} \sum_{i=3}^{10} C_i O_i.
\]

(1)

\(O_i\) are the four-quark operators and \(C_i\) are the corresponding Wilson coefficients, whose explicit
expressions are refereed to \cite{2}. \(V_{tb}\) and \(V_{td}\) are the Cabibbo-Kabayashi-Maskawa (CKM) matrix
elements. Then, the decay width for this decay is written as

\[
\Gamma = \frac{p_c}{8 S \pi m_B^2} \sum_{\sigma=L,T} \mathcal{M}^\sigma \mathcal{M}^{\sigma\dagger},
\]

(2)

where \(p_c\) is the momentum of the outgoing mesons and \(S = 2\) comes from the identical final
state particles. The decay amplitude \(\mathcal{M}^\sigma\) will be calculated later, where the subscript \(\sigma\) denotes
the helicity states of the two vector mesons with \(L(T)\) standing for the longitudinal (transverse)
component. Furthermore, the amplitude \(\mathcal{M}^\sigma\) can be decomposed into:

\[
\mathcal{M}^\sigma = m_B^2 \mathcal{M}_L + m_B^2 \mathcal{M}_N \epsilon_{2}^*(\sigma = T) \cdot \epsilon_3^*(\sigma = T) + i \mathcal{M}_T \epsilon_{\mu \nu \rho \sigma} \epsilon_2^{\mu^*} \epsilon_3^{\nu^*} P_2 P_3^\sigma,
\]

(3)

where \(\epsilon_{2(3)}\) and \(P_{2(3)}\) are the polarization vector and the four-momentum of the final state vector
meson, respectively. Conventionally, the longitudinal \(H_{00}\) helicity amplitudes and the transverse
helicity amplitudes \(H_{\pm \pm}\) are defined by

\[
H_{00} = m_B^2 \mathcal{M}_L
\]

(4)

\[
H_{\pm \pm} = m_B^2 \mathcal{M}_N \mp m_\phi^2 \sqrt{\kappa^2 - 1} \mathcal{M}_T,
\]

(5)
with the helicity summation,

$$\sum_{\sigma=L,T} \mathcal{M}^{\sigma\dagger} \mathcal{M}^{\sigma} = |H_{00}|^2 + |H_{++}|^2 + |H_{--}|^2,$$

(6)

and \( \kappa = (P_2 \cdot P_3)/m_{\phi}^2 \). Another equivalent set of definitions of helicity amplitudes is also used,

$$A_0 = -\zeta m_B^2 \mathcal{M}_L,$$

$$A_\parallel = \zeta \sqrt{2} m_B^2 \mathcal{M}_N,$$

$$A_\perp = \zeta m_\phi^2 \sqrt{\kappa^2 - 1} \mathcal{M}_T,$$

(7)

with the normalization factor \( \zeta \) satisfying

$$|A_0|^2 + |A_\parallel|^2 + |A_\perp|^2 = 1,$$

(8)

where the notations \( A_0, A_\parallel, A_\perp \) denote the longitudinal (\( f_L \)), parallel (\( f_\parallel \)), and perpendicular (\( f_\perp \)) polarization fractions, respectively.

![Feynman diagrams](image)

**FIG. 1:** Feynman diagrams for the \( B^0 \rightarrow \phi \phi \) decay in the PQCD approach.

Now, we will evaluate the hadronic matrix elements \( \mathcal{M}_L, \mathcal{M}_N \) and \( \mathcal{M}_T \) using the PQCD approach. According to the effective Hamiltonian in Eq. (1), we draw the lowest order diagrams of \( B \rightarrow \phi \phi \) as shown in Fig.1. In PQCD, the decay amplitude is factorized into the soft part \( \Phi \), the hard part \( H \), and the harder one \( C_i \) characterized by different scales. It is conceptually written as,

$$\mathcal{M} \sim \int dx_1 dx_2 dx_3 b_1 db_1 b_2 db_2 b_3 db_3 Tr[C(t)\Phi_B(x_1, b_1) \Phi_\phi(x_2, b_2)\Phi_\phi(x_3, b_3)H(x_i, b_i, t)S_i(x_i)e^{-S(t)}],$$

(9)

where \( x_i \) denotes the momentum fraction of a light quark in each meson, and \( b_i \) is the conjugate space coordinate of the transverse momentum. \( Tr \) means the trace over Dirac and color indices.
and $C(t)$ is the Wilson coefficient evaluated at scale $t$. The universal wave function $\Phi_M(M = B, \phi)$ describes hadronization of a quark and an anti-quark into the meson $M$, whose structure can be found in $[2, 6, 12]$. $H$ is the six-quark hard scattering kernel, which consists of the effective four quark operators and a hard gluon attaching to the spectator quark in the decay, so it can be perturbatively calculated. The function $S_t(x_i)$ describes the threshold resummation which smears the end-point singularities. The last term $e^{-S(t)}$, coming from the resummation of the double logarithm $\ln^2 k_T$, is the Sudakov form factor which suppresses soft dynamics effectively.

As shown in Fig. 1, there are four kinds of Feynman diagrams contributing to the $B \rightarrow \phi\phi$ decay at leading order. They involve two types: factorizable diagrams $(a)$ and $(b)$, and non-factorizable diagrams $(c)$ and $(d)$. After calculating these diagrams, we can get the amplitudes as follows:

$$M_{i=L,N,T} = \frac{2G_F}{\sqrt{2}}V_{tb}V_{td}^* \left\{ f_B F_{ann}^{LL,i} \left[ C_3 + \frac{1}{3} C_4 - \frac{1}{2} C_9 - \frac{1}{6} C_{10} \right] + M_{ann}^{LL,i} \left[ C_4 - \frac{1}{2} C_{10} \right] 
 + f_B F_{ann}^{LR,i} \left[ C_5 + \frac{1}{3} C_6 - \frac{1}{2} C_7 - \frac{1}{6} C_8 \right] + M_{ann}^{SP,i} \left[ C_6 - \frac{1}{2} C_8 \right] \right\} ,$$

where $i = L, N, T$ stands for the longitudinal polarization and the two transverse polarizations. $f_B F_{ann}^{LL(LR)}$ comes from the contribution of the factorizable diagrams with the operators $(V - A)(V - A)$ or $(V - A)(V + A)$, and $f_B$ is the decay constant of the $B$ meson. $M_{ann}^{LL(SP)}$ is the non-factorizable amplitude with the operator $(V - A)(V - A)$ or $(S - P)(S + P)$, and the latter operator is from the Fierz transformation of the operator $(V - A)(V + A)$. In $[6, 12]$, the authors had listed all formulae of $f_B F_{ann}^{LL(LR),i}$ and $M_{ann}^{LL(SP),i}$ at leading order in detail, thus it is not necessary to duplicate them in the current work.

Due to the current conservation, for the longitudinal and parallel polarization parts, the contributions from the factorizable diagrams $(a)$ and $(b)$ are canceled exactly by each other, leading to $f_B F_{ann}^{LL(LR),L(N)} = 0$. Therefore, there is only $f_B F_{ann}^{LL(LR),T}$ left for the factorizable diagrams, but it is suppressed by $(m_\phi/m_B)^2$. For the non-factorizable diagrams $(c)$ and $(d)$, the longitudinal parts give the leading and dominant contribution, and other terms are suppressed by $(m_\phi/m_B)^2$. That is to say, the contribution of the longitudinal and parallel polarization is only from the non-factorizable diagrams, but the latter one is suppressed by 4%. Although the perpendicular part receives another effect from the diagrams $(a)$ and $(b)$, but their contribution is negligible. Thus, the transverse parts can be dropped safely in SM.

In the numerical calculation, we must input the $B$ and $\phi$ meson distribution amplitudes, which
are nonperturbative parameters. For the $B$ meson, we employ the function

$$
\phi_B(x, b) = N_B x^2 (1-x)^2 \exp \left[ -\frac{1}{2} \left( \frac{xm_B}{\omega_B} \right)^2 - \frac{\omega_B^2 b^2}{2} \right],
$$

where the shape parameter $\omega_B = 0.4$ GeV has been adopted in all previous analysis of exclusive $B$ meson decays. The normalization constant $N_B = 91.784$ GeV is related to the decay constant $f_B = 190$ MeV. Since the $\phi$ meson is a vector particle, there are six distribution amplitudes up to twist 3, and all of them have been calculated in QCD sum rules [13]. The formulae have been also given explicitly in [6, 13].

Honestly speaking, there are many theoretical uncertainties in our calculation. For the penguin-dominated decays, one of the important uncertainties is from the hard scales $t$, which are defined as the invariant masses of internal particles and are required to be higher than the factorization scale $1/b$, $b$ being the transverse extents of the mesons. Another large uncertainty comes from the distribution amplitude of $B$ meson, since it cannot be calculated directly from the first principle. Varying the hard scales $t$ between 0.75 – 1.25 times the center values and the shape parameter $\omega_B = 0.40 \pm 0.05$, we then obtain the $B \to \phi\phi$ branching ratio

$$
Br(B^0 \to \phi\phi) = (4.4^{+0.8+0.3}_{-0.6-0.5}) \times 10^{-8}.
$$

The uncertainties from the $\phi$ meson distribution amplitudes are less than 20%, so we will not discuss them here. The above branching ratio can be measured at the Large Hadron Collider beauty (LHCb) experiments or the Super-$B$ factory in future, which helps test SM. For the longitudinal polarization fraction, it is given by

$$
f_L \approx 1.
$$

Compared with the results of [2], our branching ratio is about twice larger than theirs. For the polarization, it does not agree with theirs either. Furthermore, the large longitudinal polarization fraction in the annihilation decay mode $B^0 \to K^{*+}K^{*-}$ has been also confirmed in [14]. With the formulae and parameters given in [2], we get the branching ratio $3.9 \times 10^{-8}$ and $f_L \approx 1$, which agree with present results considering the difference of Wilson coefficients and other parameters.
III. EFFECT OF Z' BOSON

Now we are in position to analyze this process with an extra gauge boson $Z'$. In the gauge basis, ignoring the mixing between $Z$ and $Z'$, we write the $Z'$ term of the neutral-current Lagrangian as

$$\mathcal{L}^{Z'} = -g'Z'^{\mu} \sum_{i,j} \bar{\psi}_i^{\prime} \gamma_{\mu} [(\epsilon_{\psi_L})_{ij} P_L + (\epsilon_{\psi_R})_{ij} P_R] \psi_j^{\prime},$$

(14)

where $i$ is the family index and labels the fermions. $g'$ is the gauge coupling constant at the electro-weak scale $M_W$, and $P_{L,R} = (1 \mp \gamma_5)/2$. The superscript $I$ refers to the gauge interaction eigenstate, and $\epsilon_{\psi_L}$ ($\epsilon_{\psi_R}$) denotes the left-handed (right-handed) chiral coupling. After rotating to the physical basis, the fermion Yukawa coupling matrices $Y_{\psi}$ in the weak basis can be diagonalized as

$$Y_{\psi}^{\prime} = V_{\psi R} Y_{\psi} V_{\psi L}^\dagger$$

(15)

using the unitary matrices $V_{\psi L,R}$ in $\psi_{L,R} = V_{\psi L,R} \psi_{L,R}^{\prime}$, where $\psi_{L,R}^{\prime} \equiv P_{L,R} \psi_{L,R}$ and $\psi_{L,R}$ are the mass eigenstate fields. The CKM matrix is usually given by

$$V_{\text{CKM}} = V_u L V_d L^\dagger.$$

(16)

So, the chiral $Z'$ coupling matrices in the mass basis of down-type quarks could be written as

$$B_{dL} \equiv V_{dL} \epsilon_{dL} V_{dL}^{\dagger},$$

$$B_{dR} \equiv V_{dR} \epsilon_{dR} V_{dR}^{\dagger}.$$  

(17)

(18)

If $\epsilon_{dL,R}$ are not proportional to the identity matrix, $B_{dL,R}$ will have nonzero off-diagonal elements that induce FCNC interactions. In the current work, we will assume that the right-handed couplings are flavor-diagonal for simplicity.

With nonzero flavor-diagonal matrix elements, the $Z'$ boson contributes to FCNC at the tree level, and its contribution will interfere with SM contributions. In particular, the flavor-changing couplings of the $Z'$ boson with the left-handed fermions will contribute to the $O_9$ and $O_7$ operators for the left (right)-handed couplings at the flavor-conserving vertex, i.e., $C_{9,7}(M_W)$ receive contributions from the new $Z'$ boson. Then, the $Z'$ part of the effective Hamiltonian for $b \to d\bar{s}s$ transitions has the form

$$\mathcal{H}^{Z'}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} \left( \frac{g' M_Z}{g_Y M_{Z'}} \right)^2 B_{db}^{L} (B_{ss}^{L} O_9 + B_{ss}^{R} O_7) + \text{h.c.},$$

(19)
where \( g_Y = e / (\sin \theta_W \cos \theta_W) \) and \( M_{Z'} \) is the mass of the new gauge boson. \( O_{7,9} \) are the effective operators in SM. Due to the hermiticity of the effective Hamiltonian, we always assume that the diagonal elements of the effective coupling matrices \( B_{qq}^{L,R} \) are real. However, there is still a new weak phase \( \phi \) in the off-diagonal one of \( B_{bd}^L \). Compared with Eq.(1), the resultant \( Z' \) contributions to the Wilson coefficients are

\[
\Delta C_{9,7} = 4 \frac{|V_{tb}V_{td}^*|}{|V_{tb}V_{td}^*|} \xi^{L,R} e^{-i\phi},
\]

with

\[
\xi^{L,R} = \left( \frac{g' M_{Z'}}{g_Y M_{Z}} \right)^2 \frac{|B_{db}^L B_{ss}^{L,R}|}{|V_{tb}V_{td}^*|}.
\]

Since the heavy degrees of freedom in the theory have already been integrated out at the scale \( M_W \), the RG evolution of the Wilson coefficients after including the new contributions from \( Z' \) is exactly the same as in SM [15].

Generally, we always suppose \( g' \approx g_Y \) if both the \( U(1) \) and \( U'(1) \) gauge groups have the same origin from some grand unified theories. Though the \( Z' \) boson has not been detected in Large Hadron Collider (LHC) experiments, we always expect the mass \( M_{Z'} \) to be at the TeV scale, which would lead to \( M_Z / M_{Z'} \approx 0.1 \). In order to explain the mass differences of \( B_q - \overline{B}_q (q = d, s) \) and the \( CP \) asymmetry anomalies in \( B \to \phi K, \pi K, |B_{qq}^{L,R}| \) should be of \( O(1) \). More about constraints on these parameters are refereed to [16–18]. To quantify the effects of the \( Z' \) boson, we consider \( \xi^{L,R} \in [0.001, 0.02] \) in the following discussion. Moreover, for the new weak phase \( \phi \), we treat it as a free parameter.

Our analyses are divided into the following three scenarios with different simplifications, namely,

- **S1**: Ignoring the right-hand couplings, i.e., \( \xi^R = 0 \).
- **S2**: Supposing that the left-hand couplings share the same values as the right-hand values, i.e., \( \xi^L = \xi^R \).
- **S3**: Allowing arbitrary values for \( \xi^{L,R} \) without any simplifications.

With the possible parameter space, we evaluate the \( B \to \phi \phi \) branching ratios under the different
scenarios together with the SM contribution as

\[
\text{Br}(B \rightarrow \phi\phi) = \begin{cases} 
(3.6^{+0.5+0.3+2.8}_{-0.5-0.4-0.8}) \times 10^{-8}, & \text{S1;} \\
(5.1^{+0.9+0.5+0.8}_{-0.7-0.5-2.0}) \times 10^{-8}, & \text{S2;} \\
(5.1^{+0.9+0.5+2.9}_{-0.7-0.5-3.2}) \times 10^{-8}, & \text{S3;} \\
(4.4^{+0.8+0.3}_{-0.6-0.5}) \times 10^{-8}, & \text{SM,}
\end{cases}
\]

(22)

where the first two errors are from uncertainties of PQCD, i.e. the shape parameter \(\omega_B\) and the hard scale \(t\). For the \(Z'\) contribution, we scan all possible parameter space (\(\xi^{L,R}\) and the new weak phase \(\phi\)), and get the third uncertainties. As for the center values, we take \(\xi^{L,R} = 0.01\) and \(\phi = 0\). Under S1, it is clear that the \(Z'\) boson plays a destructive role for the branching ratio, while the branching ratio will be enhanced after adding the contribution from the right-hand couplings under S2 and S3. Since only one strong phase exists, there is no \(CP\) asymmetry in this decay. The polarizations are almost unchanged, though the new \(Z'\) particle could change the transverse parts of the amplitudes.

![Graph showing variation of branching ratio with new weak phase](image)

**FIG. 2:** Variation of the branching ratio with the new weak phase \(\phi\) under S1 (left panel) and S2 (right panel), where the dashed (red), solid (black) and dot-dashed (blue) lines correspond to \(\xi = 0.001, 0.01\) and \(0.02\), respectively. The range with horizontal lines shows the prediction in SM after adding the two errors in quadrature.

To study the effect of the \(Z'\) boson clearly, we plot the variation of the branching ratio as a function of the new weak phase \(\phi\) with different values of \(\xi = 0.001, 0.01, 0.02\) under S1 (left panel) and S2 (right panel), as shown in Fig. 2. According to these plots, we note that if \(\xi \leq 0.001\), i.e. a heavy \(Z'\) boson, the new physics effect is too small to be detected. Even for \(\xi \approx 0.01\) under both scenarios, its effect is also hard to measure in experiments, because it will be buried by the uncertainties of PQCD in SM. While \(\xi \approx 0.02\), the \(Z'\) boson will change the branching
FIG. 3: Variation of the branching ratio with the new weak phase $\phi$ under S3, where the solid (red) and dot-dashed (blue) curves correspond to maximal and minimal values, respectively. The range with horizontal lines shows the prediction in SM after adding the two errors in quadrature.

ratio remarkably, but the trends are different for different scenarios. Under S1, the branching ratio becomes larger and exceeds the predicted range in SM with a large phase, while it becomes smaller with a small weak phase, which can be seen from the left panel of Fig. 2. For S2, as seen from the right panel, it has an opposite situation. As for S3, by varying $\xi^L$ and $\xi^R$ independently, we present the maximal and minimal curves of the branching ratio as functions of $\phi$ in Fig. 3. It is found that the range of the branching ratio is much larger than the SM predictions, which is also shown in Eq. (22). When $\phi = 0$, the maximal value is about $10^{-8}$, which is about twice of prediction of SM. On the contrary, by setting $\phi = \pm 180^\circ$, it will be decreased to half of the center value of the SM prediction. All the above results can be tested in the current LHCb experiments or at the Super-B factory in future. Moreover, if the $Z'$ boson would be detected in future, the observation of this mode will in turn help us constrain the $Z'$ mass and its couplings to fermions.

IV. SUMMARY

In this work, we have re-calculated the branching ratio and the polarization fractions of the pure annihilation decay $B \to \phi\phi$ within the perturbative QCD approach in both SM and the non-universal $Z'$ model. We found that this mode is longitudinal part dominated and its longitudinal polarization fraction is about 1 because of absence of contributions from the operator $(S - P)(S + P)$. The branching ratio is estimated to be $(4.4^{+0.8+0.3}_{-0.6-0.5}) \times 10^{-8}$, which may be measured in the ongoing LHCb experiments or at the Super-$B$ factory in future. Considering the effect of an additional $Z'$ boson, we found that the branching ratio may be enhanced by a factor of 2, or
reduced to half in the allowed parameter space, as shown in Fig. 3. Thus, if this mode could be measured in the LHCb experiments and/or at the Super-\(B\) factory, it will provide a test of SM and the non-universal \(Z'\) model. Furthermore, if the \(Z'\) boson could be detected, these results can be used to constrain its mass and couplings in turn.

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