Improved Quenched QCD on Large Lattices – First Results

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Continuing our investigations of quenched QCD with improved fermions we have started simulations for lattice size $32^3 \times 64$ at $\beta = 6.2$. We present first results for light hadron masses at $\kappa = 0.13520, 0.13540$, and $0.13555$. Moreover we compare our initial experiences on the T3E with those for APE/Quadrics systems.

1. INTRODUCTION

High computer costs turned out to be a major problem when performing quenched QCD simulations at smaller lattice spacing $a$. Improving the action in order to reduce cut-off effects therefore became an important goal.

While standard gluonic action has discretization errors of $O(a^2)$, those for Wilson fermions are of $O(a)$. Sheikholeslami and Wohlert\textsuperscript{[1]} proposed a modified fermion action:

$$S_F = S_F^{(0)} - \frac{i}{2} \kappa c_{SW}(g^2) a^3 \sum_x \bar{\psi}(x) \sigma_{\mu\nu} F_{\mu\nu}(x) \psi(x)$$

(1)

where $S_F^{(0)}$ is the standard Wilson action and

$$F_{\mu\nu}(x) = \frac{1}{8ig^2} \sum_{\mu,\nu=\pm} (U(x)_{\mu\nu} - U(x)_{\nu\mu}^\dagger).$$

(2)

If the coefficient $c_{SW}$ of the so-called clover term is chosen appropriately, this action removes all $O(a)$ errors from on-shell quantities like hadron masses. A non-perturbative calculation of $c_{SW}$ as a function of $g^2$ was done by the Alpha collaboration\textsuperscript{[2]}.

Until now the QCDSF collaboration has presented results for the light hadron spectrum using the improved action for two values of the coupling, $\beta = 6.0$ and 6.2 (see\textsuperscript{[3,4]}). These calculations have been carried out on APE Quadrics computers on lattice sizes up to $24^4 \times 48$. In order to allow a more reliable estimate of the chiral limit for $\beta = 6.2$ we started calculations with a hopping parameter $\kappa$ closer to the critical value, $\kappa_c$, on a lattice of size $32^3 \times 64$.

At present we have evaluated $O(75)$ configurations. We hope for higher statistics, and so the following results should be regarded as preliminary.

2. SIMULATION DETAILS

We perform the quenched QCD simulations at $\beta = 6/g^2 = 6.2$. To generate a new gauge configuration we use 100 cycles consisting of a single 3-hit Metropolis sweep followed by 16 over-relaxation sweeps using the $SU(3)$ algorithm suggested by Creutz\textsuperscript{[5]}.
We use Jacobi smearing \cite{6} for source and sink. We chose the number of smearing steps to be \(N_s = 100\) and for the smearing hopping parameter we took \(\kappa_s = 0.21\), for which the radius of the smeared source \(r_a\) is about 3.5\(a\) which roughly corresponds to 0.4\(fm\). Although we have calculated the propagator for both smeared and unsmeared sink, we will only use the results for smeared sink here.

The simulations are performed for three different hopping parameters, \(\kappa = 0.13520, 0.13540, 0.13555\), with clover coefficient \(c_{SW} = 1.614\) chosen according to \cite{2}. For the matrix inversion we mainly use BiCGstab \cite{7,8}. The minimal residue algorithm is used in case BiCGstab does not converge. As convergence criterion we chose \(r \leq 10^{-15}\), where \(r = |M\chi - \phi|/|\chi|\).

We found up to 4 configurations per \(\kappa\) which show an exceptional pattern (see Fig. 1). They have been excluded from the evaluation.

3. RESULTS

Until now we have looked at the \(\pi, \rho\) and nucleon masses. We find good plateaus when plotting the effective mass \(m(t) = \ln[c(t)/c(t+1)]\), as shown in Fig. 3 for the \(\pi\).

Plots of the dimensionless ratio \(m_X/m_\rho\) as a function of \((m_\pi/m_\rho)^2\) (so-called APE plot) were found to be rather different for improved and Wilson fermions at \(\beta = 6.0\) \cite{3}. As can be seen from Fig. 3, the results at \(\beta = 6.2\) seem to confirm that the improved results come closer to the physical value than the Wilson results.

To see how masses scale as \(\beta\) is changed, it has been suggested \cite{3} that \(m_\rho\) should be plotted in units of the square-root of the string tension \(K\) which has cut-off errors of \(O(a^2)\) only. In Fig. 3 the ratio \(m_\rho/\sqrt{K}\) is shown as a function of \(a/\sqrt{K}\) for fixed physical \(\pi\) masses with \(m_\pi^2 = 0, 2K\) and \(4K\). To obtain the \(\rho\) mass for these values of \(m_\pi\) we extrapolated, or interpolated, \(m_\rho\) using the phenomenological ansatz \cite{3}

\[m_X^2 = b_0 + b_2m_\pi^2 + b_3m_\pi^3, \quad X = \rho, N\]  

4. T3E PERFORMANCE

On the \(32^3 \times 64\) lattice our current program needs 13.1s per BiCGstab iteration step on a T3E with 128 DEC Alpha 21164 5/375MHz microprocessors. Simulations done on a 256 node Quadrics QH2 need 6.3s for the same operations on a \(24^3 \times 48\) lattice. Comparing the peak perf-
Figure 3. APE plot at $\beta = 6.2$ for improved (□ [3]) and Wilson fermions (○ [10,11]). The new improved results on larger lattices are marked by ×. This data can be compared with the mass ratio (∗) at the physical quark mass and in the heavy quark limit. The solid line comes from a fit using the phenomenological ansatz, eq. 3, with the new preliminary data included.

Figure 4. The ratio $m_\rho/\sqrt{K}$ as a function of the lattice spacing for Wilson fermions (○ [10,11]) and improved (□ [3] and ×, this work). This is compared with the experimental value (∗) using $\sqrt{K} = 427\text{MeV}$.

REFERENCES

1. B. Sheikholeslami and R. Wohlert, Nucl. Phys. B259 (1985) 572.
2. M. Lüscher et al., Nucl. Phys. B491 (1997) 323, hep-lat/9609035, ibid B491 (1997) 344, hep-lat/9611015.
3. M. Göckeler, R. Horsley, H. Perlt, P. Rakow, G. Schierholz, A. Schiller and P. Stephenson, DESY preprint DESY 97-125 (1997), hep-lat/9707021.
4. P. Stephenson et al., this conference.
5. M. Creutz, Phys. Rev. D36 (1987) 516.
6. C. R. Allton et. al., Phys. Rev. D47 (1993), 5128.
7. H. van der Vorst, SIAM J. Sc. Stat. Comp. 12 (1992) 631.
8. A. Frommer, V. Hannemann, B. Nockel, T. Lippert and K. Schilling, Int. J. Mod. Phys. C5 (1994) 1073.
9. R. Sommer, Phys. Rep. 275 (1996) 1.
10. C. R. Allton V. Gimenez, L. Giusti, and F. Rapuano, Nucl. Phys. B480 (1997) 427.
11. S. Collins, Nucl. Phys. B (Proc. Suppl.) 30 (1993) 393.