Effective delivering capacity in traffic dynamics based on scale-free network

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Abstract

We investigate the percentage of delivering capacities that are actually consumed in a typical traffic dynamics where the capacities are uniformly assigned over a scale-free network. Theoretical analysis, as well as simulations, reveal that there are a large number of idle nodes under both free and weak congested state of the network. It is worth noting that there is a critical value of effective betweenness to classify nodes in the congested state, below which the node has a constant queue size but above which the queue size increases with time. We also show that the consumption ratio of delivering capacities can be boosted to nearly 100% by adopting a proper distribution of the capacities, which at the same time enhances the network efficiency to the maximum for the current routing strategy.

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I. INTRODUCTION

The interplay between the dynamics and the topology of network can be used to model the evolutions and structures of a wide variety of complex systems in nature and human society. A branch of dynamics research that focuses on the data traffic in the network has drawn a lot of interests. The studies in this field are mainly about capturing the attributes of the data transportation and providing hints on how to make the physical communication network more efficient. Previous structure analysis based on real data spanning several disciplines revealed that many networks, including the Internet, exhibit scale-free characteristics [1]. The degree distribution of these networks follows a power law. Having explored the topological properties [2, 3], researchers habitually choose the scale-free network as the underlying structure when studying the traffic dynamics in order to observe phenomena that are close to those in the real world [4, 5, 6].

The traffic models proposed by recent works generally consist of four components: the underlying network topology, the information packet generation, the routing strategy and the packet delivering capacity of individual nodes [7, 8, 9, 10, 11]. They all have counterparts in the reality. Take the Internet as an example. The four components correspond to the physical links between computers, the average data flow, the algorithm implemented on routers and the number of parallel packet processors per site, respectively. They also have other interpretations in the field of path navigation [12] and organization design [13, 14]. In these models, a continuous phase transition of network from free state to congested state is observed when the packet generation rate exceeds a critical value [10]. The core purpose of researches on these models is the optimization that enhances the network capacity which is measured by the critical value of packet generation rate. Some of the previous efforts concentrate on finding a most suitable network topology [7, 8, 9], while the others are mainly about deciding an optimal routing strategy [5, 6, 9, 15]. However, in most of the models, constant delivering capacity is assigned to nodes. It is not economical as the loads on different nodes vary. In fact, the delivering capacities are processor resources that they should also be taken into consideration during optimization attempts.

With the hope of discovering potential possibilities of improvements on the communication network, we analyze the percentage of delivering capacities that are truly active in our traffic model where the capacities are uniformly distributed over a scale free network.
It is concerned that congestion is sometimes inevitable in real lives, for example, everyone has the experience of traffic jam in rush hours, so the performance of delivering processors under congested state of network is paid special attentions. We find that some of the nodes in a weak jammed network always have constant number of packets waiting to be delivered which is the same situation as in the free state of the network, whereas the others have increasing number of packets. These two types can be distinguished by referring to the effective betweenness which is an attribute of node defined in Ref. [8]. Our theoretical estimates and simulations reveal that delivering capabilities of nodes are not well utilized even under the congested state of network, though optimal routing strategy based on local degree information discussed in Ref. [11] is adopted. To alleviate the waste of processor resources, we apply a non-uniform distribution of the delivering capacities which adjusts the network to the best performance under the given conditions.

This paper is organized as follows. The traffic model is described in Sec. II. In Sec. III, theoretical analysis and simulations of the model are provided in both free and congested state. The conclusion is given in Sec. IV.

II. TRAFFIC MODEL

We use the famous model proposed by Barabási and Albert [16] to build the underlying scale-free network. BA model, which features growth and preferential attachment, uncovers for the first time the mechanism controlling the emergence of power-law degree distribution observed in real networks. We set the model parameters $m_0 = m = 5$ and network size $N = 1000$. In the traffic dynamics, network nodes are thought to be sources that produce information packets, routers that deliver packets and first-in-first-out queues with unlimited size that store to-be-processed packets. At each time step, for a node $i (i = 1, \ldots, N)$, the following procedures are done. A packet is generated with probability $\rho_i$. The new packet whose destination is randomly selected among the other $N - 1$ nodes is placed at the back of queue $i$. Meanwhile, packets are picked from the front of queue $i$. If the picked packet has destination node $i$, it is removed; otherwise, it is delivered to node $j$, one of the neighbors of node $i$, with preferential weight

$$\Pi_j = \frac{k_j^{-1}}{\sum_i k_i^{-1}},$$

(1)
where sum runs over the neighbors of node \( i \) and \( k_i \) is the degree. In Eq. (1), we have used the optimal routing strategy based on the knowledge of only neighbor’s degree [11]. At most \( C_i \) packets can be processed by node \( i \) if there are adequate ones in queue \( i \). \( C_i \) is the delivering capacity of node \( i \). We set \( \rho_i = \rho \), \( C_i = C = 10 \) following the parameters used in Ref. [11] in order to meet the conditions for the optimal routing strategy. These procedures are carried out for different \( i \) in parallel. What we concern is the ratio between the number of packets actually processed by the network during a unit time and the total capacities assigned, which is defined as the effective delivering capacity.

III. EFFECTIVE DELIVERING CAPACITY

Depending on \( \rho \), there is a continuous phase transition from free state to congested state [12]. When \( \rho \) is small, the packets flow freely in the network as the nodes always have available processing abilities to send them to the next positions. The average travel time \( \tau \) of the packets keeps constant and the total number \( N_p(t) \) of packets floating in the network at time step \( t \) fluctuates slightly around \( \rho N \tau \). However, when \( \rho \) is large, packets beyond the delivering capacities accumulate continuously in the queues, causing \( \tau \) to diverge. Since the number of packets the network can manage and the generation rate \( \rho \) are both constants, the amount of accumulation at every time step is also constant. As a result, \( N_p(t) \) increases linearly with time. To watch the transition accurately, we use the order parameter \( \xi \) introduced in Ref. [17],

\[
\xi = \lim_{t \to \infty} \frac{1}{\rho N} \frac{\langle N_p(t + \Delta t) - N_p(t) \rangle}{\Delta t},
\]  

(2)

where \( \langle \ldots \rangle \) indicates an average over time windows of width \( \Delta t \). From the properties of \( N_p(t) \), we know that \( \xi \) is zero in free state but non-zero in congested state. \( \rho_c \), the critical value of \( \rho \), which is also the maximum generation rate keeping the system in free state, measures the network capacity.

To characterize the situation in free state, we use the method introduced in Ref. [8] with our modifications. Let us focus on a packet at node \( i \) whose destination is node \( k \). The probability for this packet to go to node \( j \) in one time step is denoted as \( p_{ij}^k \). The precise form of \( p_{ij}^k \) depends on the routing strategy. For the scheme used in the model,

\[
p_{ij}^k = (1 - \delta_{ik}) \frac{A_{ij} k_j^{-1}}{\sum_l A_{il} k_l^{-1}},
\]  

(3)
where $A_{ij}$ is the element of the adjacent matrix. The strategy is Markovian that each packet is delivered independently. As the network is in free state, packets pass through nodes without any wait. The probability for the above packet to go to node $j$ in $n$ time steps is given by

$$P_{ij}^k(n) = \sum_{l_1,l_2,\ldots,l_{n-1}} p_{ii}^k p_{i1}^k \cdots p_{l_{n-1}j}^k.$$  \hspace{1cm} (4)

Treating $p_{ij}^k$ and $P_{ij}^k(n)$ as elements of matrices $p^k$ and $P^k(n)$ respectively, we have

$$P^k(n) = (p^k)^n.$$ \hspace{1cm} (5)

Let us switch the focus to the centrality of node. In the traffic model, packets coming from neighbors at a specific time step are a fraction of packets generated before. Supposing that one packet with target node $k$ is generated at node $i$ each time step, we calculate $b_{ij}^k(t)$, the average number of such packets moving to node $j$ at time step $t$,

$$b^k(t) = \sum_{n=1}^{t} (p^k)^n.$$ \hspace{1cm} (6)

As $N_p(t)$ is stable, when $t \to \infty$,

$$b^k = \sum_{n=1}^{\infty} (p^k)^n = (I - p^k)^{-1} p^k.$$ \hspace{1cm} (7)

Summing over all the possible sources and targets of packets yields the effective betweenness of node $j$, $B_j$,

$$B_j = \sum_{i,k} b_{ij}^k.$$ \hspace{1cm} (8)

Note that $B_j$ depends on both the routing strategy and the network topology. As the routing algorithm can vary, highly connected nodes do not always have large effective betweenness. The load $L_j$, which is the average number of packets to be delivered by node $j$ every time step, is written as,

$$L_j = \rho + \frac{\rho B_j}{N-1},$$ \hspace{1cm} (9)

where the first term on the right side corresponds to packets generated by $j$ itself while the second term corresponds to packets coming from neighbors. Any node $j$ with $L_j > C$ will cause the network to be congested. Thus we estimate $\rho_c$ by

$$\rho_c = \frac{C}{B^*} + 1 \approx \frac{C(N-1)}{B^*},$$ \hspace{1cm} (10)
where $B^*$ is the maximum effective betweenness. In our model, we had $\rho_c = 0.0045$. When $\rho < \rho_c$, the number of packets processed at node $j$ per time step is exactly its load $L_j$. Therefore, effective delivering capacity $\eta$ under free state is

$$\eta = \frac{\sum_j L_j}{CN},$$

(11)

which is proportional to $\rho$.

For $\rho$ larger than $\rho_c$, Eq. (11) is invalid because the system is not expected to be in free state when $t \to \infty$ in Eq. (7). Back to time step $t = 1$, the average load in the network is equal to the generation rate $\rho$. Here we limit our discussion to a weak congested scenario where $\rho$ is not supposed to be extremely large so that the nodes are still able to process all the packets in the queues for the first few time steps. During these time steps, we can obtain the instantaneous load $L_j(t)$ by replacing Eq. (7) with Eq. (6) in the previous calculation for free state. As $L_j(t)$ is a monotonically increasing function of time, it is at a later time step $t_c$ that the largest $L_j(t)$ starts to exceed the delivering capacity $C$. While the packets beyond the capacity are hindered at the corresponding node, they do not flow in the network. Comparing with an imaginary case where no constraints on the delivering capacities are applied, the actual instantaneous load increases with time more slowly. Though $L_j(t)$ for $t > t_c$ can not be directly worked out by analytical calculation, we can predict that with more nodes suffering from packet overloading, $L_j(t)$ increases even more slowly. After a transient time, the instantaneous load over the whole network stops increasing and remains stationary. Thereafter, all the nodes in the network maintain their states. Those with load higher than delivering capacity have growing queues, while the others keep constant queues. Since the effective betweenness $B_j$ determines free state load $L_j$ by Eq. (9), it is plausible to suppose that nodes with greater effective betweenness become overloaded more easily. We expect a critical effective betweenness $B_c$ for current $\rho$, which can be used as the criterion to classify the two types of node. Once $B_c$ is found, effective delivering capacity can be considered by separate examinations on different types of node.

To find $B_c$, we look at the congested state of system when $t \to \infty$. The increasing queues are already considerably long. A newly hindered packet at a specific node will wait in the queue for so long a time that it seems to be disappeared from the views of the other nodes in the network. We assume that a node with an increasing queue is a “target” for information transportation where packets disappear. To make it more like a “target”, the $C$ packets it
lets out at every time step are shifted to the neighbors as equivalent generation rates. By this imagination, the original congested network turns out to be a smaller one in free state, and method for \( \rho < \rho_c \) can be exploited. We use the following algorithm:

1. All the nodes in the network are sorted by effective betweenness \( B_j \) in descending order. The first node in the sequence is added to an initially empty set \( S \).

2. The new generation rate \( \rho'_j \) for node \( j \) is

\[
\rho'_j = \rho + \sum_{i \in S} Cp_{ij},
\]

where sum runs over all the elements in \( S \) and \( p_{ij} \) represents the routing strategy,

\[
p_{ij} = \frac{A_{ij}k^{-1}_j}{\sum_l A_{il}k^{-1}_l}.
\]

3. In order to suppress packets arriving at “targets”, row \( i \) in the original matrix \( p^k \) is set to 0 for each \( i \in S \).

4. Employing \( \rho'_i \) and the modified \( p^k \), the new load \( L'_j \) is evaluated by

\[
L'_j = \begin{cases} 
0 & j \in S \\
\rho + \frac{1}{N-1} \sum_{i,k} \rho'_i b^k_{ij} & \text{otherwise}
\end{cases}
\]

5. The validity of

\[
\text{max}(L'_j) < C
\]

must be ensured, which corresponds to constant queue size for nodes not in \( S \). If there exists load \( L'_j \) larger than \( C \), the next node in the sequence descending ordered by \( B_j \) is added to \( S \).

6. Step 2 to 5 are repeated until Eq. (15) is fulfilled. \( B_c \) for current \( \rho \) is just the critical \( B_j \) separating nodes within \( S \) from those outside.

To describe the changes of queues in the simulation, we define \( \kappa_j \) for node \( j \),

\[
\kappa_j = \lim_{t \to \infty} \frac{\langle Q_j(t + \Delta t) - Q_j(t) \rangle}{\Delta t},
\]

where \( Q_j(t) \) is the queue size of node \( j \) at time step \( t \) and \( \langle \ldots \rangle \) indicates an average over time windows of width \( \Delta t \). \( \kappa_j \) is zero if node \( j \) has a constant queue or positive if node \( j \) has
FIG. 1: Simulation results of $\kappa$ versus $B$. The critical point $B_c$ is found by calculation. In the inset, we plot $\mu_j = (Q_j(2t) - Q_j(t))/Q_j(2t)$ when $t \to \infty$ for nodes with $B_j > B_c$, which shows that the queues increase linearly with time.

an increasing queue. Taking $\rho = 0.02$ as an example, we illustrate $\kappa_j$ versus $B_j$ in Fig. 1, where $B_c$ is obtained by calculation. It is shown that below $B_c$ nodes have constant queues but above $B_c$ the queues increase linearly with time. Besides, $\kappa_j \sim B_j - B_c$ for $B_j > B_c$.

We are ready to estimate the effective delivering capacity under congested state. For the nodes with constant queues, the packets delivered are equal to the load $L'_j$. For the nodes with increasing queues, they work at the capacity $C$. Note that during the calculation, packets processed by these nodes are shifted to the neighbors, which indicates that the packets pass through nodes twice. The effective delivering capacity for congested state, $\eta'$, is then given as

$$\eta' = \frac{\sum_j L'_j + 2CN_s}{CN},$$

where $N_s$ is the number of elements in $S$.

As shown in Fig. 2, we calculated the effective delivering capacity for different $\rho$ ranging from 0.001 to 0.03 and compared the results with simulation where packets processed in a unit time are directly counted. In the free state, $\eta$ increases with $\rho$ according to Eq. (11). In the congested state, though the increment of $\rho$ shows a trend to enlarge $L'_j$, there is a cutoff imposed by the delivering capacity $C$. With the overloaded nodes ignored, the sum of $L'_j$ does not vary too much. $\eta'$ increases only due to the increment of $N_s$ which is small when $\rho$ changes hardly.
FIG. 2: Effective delivering capacity $\eta$ and order parameter $\xi$ for $C = 10$. $\rho_c$ follows Eq. (10).

It is undesirable that up to $\rho = 0.03 \approx 6.7\rho_c$ the effective delivering capacity is no more than 75%, despite the use of optimal local routing algorithm. In other words, while the network is not able to handle all the information packets, 25% processors are idle. An optimization problem arises naturally: given the fixed number of processors, how to exploit them all? The key is to avoid resource wastes on less loaded nodes. We introduce a new delivering capacity distribution,
\[ C'_i = \frac{B_i}{\sum_j B_j} CN, \]  
where the total capacity $CN$ is conserved. With respect to Eq. (9), when $\rho$ is approaching the critical value, the delivering capacity just meets the load, and no waste happens. Above the critical value of $\rho$, cutoff imposed by delivering capacity does not reduce the load on the nodes with constant queues, and the utilization ratio keeps 100%. In particular, the network capacity is enhanced to the maximum since all the processors are active. Similar to Eq. (10), for an arbitrary distribution of delivering capacity, the critical point $\rho_c$ is
\[ \rho_c = \min \left\{ \frac{C_j(N - 1)}{B_j}, j = 1, \ldots, N \right\}, \]  
with constraint $\sum_j C_j = CN$. The optimal value of $\rho_c$ is obtained by neutralizing the differences among $C_j(N - 1)/B_j$ under the constraint. It can be accomplished by employing Eq. (18). The corresponding critical point $\rho_{c\text{ opt}}$ is
\[ \rho_{c\text{ opt}} = \frac{CN(N - 1)}{\sum_j B_j}. \]
Calculation yields $\rho_{c\ opt} = 0.0074$. In the simulation, delivering capacity given by Eq. (18) is converted to integer where errors are involved. Figure 3 illustrates simulation results of the new distribution. It is clear that the effective delivering capacity is very close to 100% for $\rho > \rho_{c\ opt}$ and $\rho_{c\ opt}$ is consistent with theoretical estimates.

IV. CONCLUSION

In this paper, we study the effective delivering capacity for a typical traffic model under both free and weak congested state. It is shown that with equal number of packet processors assigned to nodes, a large percentage of the processors are idle even under the congested state. By redistributing the processors according to effective betweenness, all of them are activated, which corresponds to maximum network capacity for the routing strategy adopted. Considering the difficulties to change the topologies of real networks, previous works [9, 11] on optimization mainly focus on finding the routing algorithm best fits the underlying structure. We rise the importance of choosing a proper distribution of the delivering capacity which is an essential supplement to the optimized routing strategies. In addition, we successfully separate nodes with constant queues from those with increasing queues in a weak congested network by referring to a critical value of effective betweenness, which may provide some hints for further studies on the congested network.
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