Analytical research of heat and mass transfer in bulks of food raw materials in the presence of self-heating hearths

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Abstract. The article provides a more accurate analysis of the heat and mass transfer mode in conditions close to the actual storage of most types of agricultural raw materials. The issues of improving metrological support are most acute in industries that traditionally use fuzzy logic approaches. Organoleptic methods that are common in the food industry, in particular when analyzing the quality of agricultural raw materials after storage, often lead to significant losses and affect the overall performance of agricultural industry in total. Thus, the subject of the article is of interest to both developers of the corresponding measuring equipment and employees of food industries. The article considers the problem of storing bulk food raw material in cylindrical silos with moisture and heat insulated side and moisture insulated end surfaces. The analytical solution of the appropriate boundary value problem is obtained by the Laplace integral transformation method. The obtained analytical solutions are convenient for engineering calculations and can be used to determine the temperature at each moment of the storage process at any point of the bulk more accurately, thus allowing timely measures to be taken by the means of automation.

1. Introduction
The analytical research of the temperature and moisture content distribution in food raw materials bulks in the presence of high-temperature hearths in the form of a heating layer, which is a source of intensity of heat emission, is important.

When storing food raw materials in cylindrical silos with moisture and heat insulated side and moisture insulated end surfaces, the temperature changes following the change of the atmospheric air temperature at the borders between bulks of food raw material and the environment (as in [1,2-4]).

From a mathematical point of view, a food raw materials bulk in a silo-type storage can be taken as an unlimited plate (a limited rod with a heat-and-mass-insulated surface).

2. Problem statement
The boundary value problem of joint heat and mass transfer can be formulated as follows:

it is required to solve a system of differential equations of the thermal conductivity type [5] for a three-layer homogeneous and isotropic body of the basic geometric shape (a limited final rod) with the necessary initial and boundary conditions that reflect the real interaction of the food raw material bulk with the environment:

I — for the lower layer (0 < x < h, τ > 0)
\[
\begin{aligned}
&\frac{\partial t_1(x, \tau)}{\partial \tau} = \alpha_q \frac{\partial^2 t_1(x, \tau)}{\partial x^2} + \epsilon \rho \frac{\partial u_1(x, \tau)}{\partial \tau}, \\
&\frac{\partial u_1(x, \tau)}{\partial \tau} = \alpha_m \frac{\partial^2 u_1(x, \tau)}{\partial x^2} + \alpha_m \delta \frac{\partial t_1^2(x, \tau)}{\partial x^2}, \\
t_1(x, 0) = t_{10} = \text{const}, \\
u_1(x, 0) = u_{10} = \text{const}, \\
t_1(0, \tau) = \bar{\tau}_0 + t_m \cos(\omega(\tau + \tau_0)),
\end{aligned}
\]

(1)

\[
\begin{aligned}
&\frac{\partial t_1(x, \tau)}{\partial \tau} = \alpha_q \frac{\partial^2 t_1(x, \tau)}{\partial x^2} + \epsilon \rho \frac{\partial u_1(x, \tau)}{\partial \tau}, \\
&\frac{\partial u_1(x, \tau)}{\partial \tau} = \alpha_m \frac{\partial^2 u_1(x, \tau)}{\partial x^2} + \alpha_m \delta \frac{\partial t_1^2(x, \tau)}{\partial x^2}, \\
t_1(x, 0) = t_{10} = \text{const}, \\
u_1(x, 0) = u_{10} = \text{const}, \\
t_1(0, \tau) = \bar{\tau}_0 + t_m \cos(\omega(\tau + \tau_0)),
\end{aligned}
\]

(2)

(3)

(4)

(5)

(6)

(7)

(8)

(9)

(10)

II — for a layer with a variable heat source (h<x<l, \tau > 0)

\[
\begin{aligned}
&\frac{\partial t_2(x, \tau)}{\partial \tau} = \alpha_q \frac{\partial^2 t_2(x, \tau)}{\partial x^2} + \epsilon \rho \frac{\partial u_2(x, \tau)}{\partial \tau} + q(\tau), \\
&\frac{\partial u_2(x, \tau)}{\partial \tau} = \alpha_m \frac{\partial^2 u_2(x, \tau)}{\partial x^2} + \alpha_m \delta \frac{\partial t_2^2(x, \tau)}{\partial x^2}, \\
t_2(x, 0) = t_{20} = \text{const}, \\
u_2(x, 0) = u_{20} = \text{const}, \\
t_2(l, \tau) = t_3(l, \tau), \\
\frac{\partial t_2(l, \tau)}{\partial x} = \frac{\partial t_3(l, \tau)}{\partial x}, \\
u_2(l, \tau) = u_3(l, \tau), \\
\frac{\partial u_2(l, \tau)}{\partial x} = \frac{\partial u_3(l, \tau)}{\partial x}
\end{aligned}
\]

(11)

(12)

(13)

(14)

(15)

(16)

(17)

(18)

III — for the top layer (l < x < H, \tau > 0)

\[
\begin{aligned}
&\frac{\partial t_3(x, \tau)}{\partial \tau} = \alpha_q \frac{\partial^2 t_3(x, \tau)}{\partial x^2} + \epsilon \rho \frac{\partial u_3(x, \tau)}{\partial \tau}, \\
&\frac{\partial u_3(x, \tau)}{\partial \tau} = \alpha_m \frac{\partial^2 u_3(x, \tau)}{\partial x^2} + \alpha_m \delta \frac{\partial t_3^2(x, \tau)}{\partial x^2}, \\
t_3(x, 0) = t_{30} = \text{const}, \\
u_3(x, 0) = u_{30} = \text{const}, \\
\frac{\partial u_3(l, \tau)}{\partial x} + \delta \frac{\partial t_3(l, \tau)}{\partial x} = 0, \\
t_3(H, \tau) = \tilde{\tau}_0 + t_m \cos(\varphi \omega(\tau + \tau_0)),
\end{aligned}
\]

(19)

(20)

(21)

(22)

(23)

(24)
Here \( t_i(x, \tau) \) is the layer temperature \( i \) (i= 1,2,3); \( t_{0i} \) is the initial temperature of the corresponding layer, take further \( t_{0i}=t_0 \);
\( u_i(x, \tau) \) is the moisture content of the \( i \)-layer of the bulk of food raw materials, \( u_{0i}=u_0 \) is the initial moisture content;
\( x \) is the coordinate; \( \tau \) is the time;
\( \alpha_q \) is the thermal diffusivity;
\( \alpha_m \) is the coefficient of moisture conductivity;
\( \varepsilon \) is the coefficient of phase transformation;
\( \rho \) is the specific heat of the phase transformation;
\( c_q \) is the specific heat of bulk food raw materials;
\( \delta \) is the thermogradient coefficient;
\( t_{\bar{\theta}} \) is average temperature of atmospheric air;
\( t_m \) is the amplitude of the temperature fluctuation of the air;
\( \omega = \frac{2\pi}{p} \); \( p \) — the period of atmospheric air temperature fluctuation;
\( \tau_0 \) is the time shift of the moment putting bulk food raw materials in the storage from the moment of the summer maximum of atmospheric air temperature;
\( q(\tau) \) is the specific heat of respiration of bulk food raw materials.

Equations (1), (11) and (19) are the heat transfer equations and (2), (12) and (20) are the moisture transfer equations.

Equalities (3) - (4), (13) - (14), (21) - (22) - initial conditions.
(7) - (10), (15) - (18) are boundary conditions of the fourth kind, which determine the equality of temperatures, moisture contents and their flows at the boundaries of the corresponding layers.
(5) and (24) are boundary conditions of the first kind that describe the interaction of the bulk with the environment by setting the temperature on the layer surface [6-8].
(6) and (23) are moisture insulation conditions.

The summand \( \frac{q(\tau)}{c_q} \) is of the equation (11) is the heat source.

In this study, we take \( q(\tau) = q_1 + q_2 \exp(-k\tau) \), where \( k = const > 0 \) [1.9]. The expressions for the temperature and moisture fields of the boundary value problem (1) - (24) are heavy, so, we present a solution to the corresponding heat transfer problem, which is a solution of the heat conduction equation for all bulks of food raw materials with the necessary initial and boundary conditions:

for the lower layer \((0 < x < h, \tau > 0)\)
\[
\frac{\partial t_1(x, \tau)}{\partial \tau} = \alpha \frac{\partial^2 t_1(x, \tau)}{\partial x^2},
\]
(25)
\[
t_1(x, 0) = t_0,
\]
(26)
\[
t_1(h, \tau) = t_2(h, \tau),
\]
(27)
\[
\frac{\partial t_1(h, \tau)}{\partial x} = \frac{\partial t_2(h, \tau)}{\partial x},
\]
(28)
\[
t_1(0, \tau) = t_{\bar{\theta}} + t_m \cos(\omega(\tau + \tau_0)),
\]
(29)

for the layer \((h< x < l, \tau > 0)\)
\[
\frac{\partial t_2(x, \tau)}{\partial \tau} = \alpha \frac{\partial^2 t_2(x, \tau)}{\partial x^2} + \frac{q(\tau)}{c_q},
\]
(30)
\[
t_2(x, 0) = t_0,
\]
(31)
\[
t_2(l, \tau) = t_3(l, \tau),
\]
(32)
\[
\frac{\partial t_2(l, \tau)}{\partial x} = \frac{\partial t_3(l, \tau)}{\partial x},
\]
(33)

for the top layer \((1 < x < H, \tau > 0)\)
\[
\frac{\partial t_3(x,t)}{\partial t} = \alpha \frac{\partial^2 t_3(x,t)}{\partial x^2}, \quad (34)
\]

\[
t_3(x, 0) = t_0, \quad (35)
\]

\[
t_3(H, t) = t_0 + t_m \cos(\omega(t + \tau_0)), \quad (36)
\]

3. Discussion of results

The analytical solution of the boundary value problem (25) is obtained by the Laplace integral transformation method.

The temperature fields for the layers of bulk food raw materials are presented in the following form:

\[
T_1(X, Fo) = 1 + Po_1 k_\Delta \left(1 - \frac{k_\Delta}{2} + k_h\right) X + \frac{2Po_2}{Pd_k} \times 
\]

\[
\times \frac{\sin(\sqrt{Pd_k} k_h)}{\sin(\sqrt{Pd_k})} \left[\frac{\sin(\sqrt{Pd_k} k_h)}{\sin(\sqrt{Pd_k})} \right] \exp(-Pd_k Fo) + \cdots + \sum_{n=1}^{\infty} \frac{2}{n\pi} \cdot \Phi_n \sin(n\pi x) \exp(-(n\pi)^2 Fo) + \Phi_{1\omega}; \quad (37)
\]

\[
T_2(X, Fo) = 1 - \frac{Po_1}{2} \left[k_h^2 (1 - X) + k_1 (k_1 - 2) X + X^2\right] + \frac{2Po_2}{Pd_k} \times 
\]

\[
\times \left[\cos(\sqrt{Pd_k} k_h) \sin(\sqrt{Pd_k} (1 - X)) + \cos(\sqrt{Pd_k} (1 - k_1) \sin(\sqrt{Pd_k} X)) - 1\right] \sin(\sqrt{Pd_k}) \exp(-Pd_k Fo) + \cdots + \sum_{n=1}^{\infty} \frac{2}{n\pi} \cdot \Phi_n \sin(n\pi x) \exp(-(n\pi)^2 Fo) + \Phi_{2\omega}; \quad (38)
\]

\[
T_3(X, Fo) = 1 + Po_1 k_\Delta \left(k_h + \frac{k_\Delta}{2}\right) (1 - X) + \frac{2Po_2}{Pd_k} \times 
\]

\[
\times \frac{\sin(\sqrt{Pd_k} k_h + k_\Delta)}{\sin(\sqrt{Pd_k})} \left[\frac{\sin(\sqrt{Pd_k} (1 - X))}{\sin(\sqrt{Pd_k})} \right] \exp(-Pd_k Fo) + \cdots + \sum_{n=1}^{\infty} \frac{2}{n\pi} \cdot \Phi_n \sin(n\pi x) \exp(-(n\pi)^2 Fo) + \Phi_{3\omega}; \quad (39)
\]

where

\[
T_i(X, Fo) = \frac{t_i(x,t) - t_0}{t_0 - t_0} \text{ is dimensionless temperature (i=1.2.3)};
\]

\[
\Phi_n = \frac{(-1)^n - 1}{1 + \frac{2}{Pd_k} \left[\frac{Po_0}{Pd_k} - \frac{Po_1}{Pd_k}\right] \sin\left\{\frac{n\pi k_\Delta}{2}\right\}}, \quad (40)
\]

\[
+ \frac{2}{1 - \left(\frac{Pd_\omega}{Pd_k}\right)^2 \left(\frac{Po_0}{Pd_k} - \frac{Po_1}{Pd_k}\right) \sin\left\{\frac{n\pi k_\Delta}{2}\right\}}, \quad (41)
\]

\[
\Phi_{1\omega} = \Phi_{3\omega} = \frac{T_m}{(\text{chacosa})^2 + (\text{shasina})^2} \times
\]

\[
\times \left[\cos(Pd_\omega (Fo + F_{o0})) (\text{chacosa}_x \cos x + \text{shasina}_x \sin x) - \sin(Pd_\omega (Fo + F_{o0})) (\text{chacosa}_x \sin x - \text{shasina}_x \cos x)\right];
\]
\[ \Phi_{23} = \frac{T_m}{(\text{chasin})^2 + (\text{shacosa})^2} \times \]
\[ \times \left[ \cos(Pd_\omega(Fo + F_0)) (\text{chasin}_{\alpha x} \sin_{\alpha x} + \text{shacosa}_{\alpha x} \cos_{\alpha x}) - \right. \]
\[ \left. - \sin(Pd_\omega(Fo + F_0)) (\text{shacosa}_{\alpha x} \sin_{\alpha x} - \text{chasin}_{\alpha x} \cos_{\alpha x}) \right]; \]
\[ Fo = \frac{\alpha \tau}{H^2}, F_0 = \frac{\alpha \tau_0}{H^2} \] are Fourier numbers;
\[ P_{0l} = \frac{q_l H^2}{\alpha c \Delta \ell} \] is Pomerantsev number \((i = 1, 2, \ldots)\),
\[ P_{00} = P_{01} + P_{02}; \]
\[ P_{0k} = \frac{kH^2}{\alpha}, P_{d_\omega} = \frac{\omega H^2}{\alpha} \] are Predvoditelev numbers;
\[ X = \frac{x}{H}; k_l = \frac{l}{H}; k_h = \frac{h}{H}; k_\Delta = k_l - k_h; \]
\[ T_m = \frac{t_m}{\Delta t}, \Delta t = t_0 - t_0; \]
\[ \varphi = \arctg \frac{P_{d_\omega}}{(\pi \nu)^2}; \alpha = \frac{\sqrt{P_{d_\omega}}}{8}; \alpha_x = \frac{\sqrt{P_{d_\omega}}}{8} (1 - 2X). \]

When the storage time is long enough \((i.e., \tau \to \infty)\) solutions are simplified, and the temperature of bulk food raw materials outside the layer with the heat source will be equal to

\[ t_1(x, \tau) = t_0 + \frac{q_1 \Delta h}{\alpha c} \left( 1 - \frac{l + h}{H} \right) x; \] \(40\)
\[ t_3(x, \tau) = t_0 + \frac{q_1 \Delta h l + h}{2} \left( 1 - \frac{x}{H} \right); \] \(41\)

In this case, the temperature distributions considered depend on the thermophysical properties of bulk food raw materials, the height of the layer with a self-heating source \(\Delta h = l - h\), the average temperature of atmospheric air, a component of the heat source \(q_1\), and the regions in bulk food raw materials at where a self-heating hearths occur.

We consider a simplified problem \((25)-(36)\). We assume that

\[ t_1(0, \tau) = t_{1\Pi} = \text{const}; \] \(42\)
\[ t_3(H, \tau) = t_{2\Pi} = \text{const}; \] \(43\)

i.e. the surface temperature of bulk food raw material bordering the base of the silo ("flooring") and the temperature of the top surface of the silo remain constant throughout the storage process.

In this case, the temperature distributions will be as follows:

\[ t_1(x, \tau) = t_{1\Pi} + \left[ t_{2\Pi} - t_{1\Pi} + \frac{q_1 \Delta h}{\alpha c} \left( H - \left( h + \frac{\Delta h}{2} \right) \right) \right] \frac{x}{H} + \]
\[ + \frac{2q_2}{ck} \cdot \frac{\sin \left( \frac{kx}{\sqrt{\alpha}} \right)}{\sin \left( \frac{k}{\sqrt{\alpha H}} \right)} \cdot \sin \left( \frac{k}{\sqrt{\alpha}} \left( H - \left( h + \frac{\Delta h}{2} \right) \right) \right) \sin \left( \frac{k}{\sqrt{\alpha}} \frac{\Delta h}{2} \right) \exp(-k\tau) - \]
overheating can be prevented.

In connection with the remarks to the formulas (40) - (41) in (47) - (48), the average temperature of the atmospheric air is the temperature of the base of the silo and the temperature difference between the top and lower surfaces of bulk the food raw material. They can be controlled and thereby unauthorized overheating can be prevented.

\[
-t_1(x, \tau) = t_{11} + \left[ t_{21} - t_{11} + \frac{q_1}{ac} \left( h(H - \Delta h) + \Delta h \left( H - \frac{\Delta h}{2} \right) \right) \right] \frac{x}{H}; \quad (0 < x < h); \quad (47)
\]

\[
t_2(x, \tau) = t_{11} + \left[ t_{21} - t_{11} + \frac{q_1}{ac} \left( h(H - \Delta h) + \Delta h \left( H - \frac{\Delta h}{2} \right) \right) \right] \frac{x}{H}; \quad (h < x < l); \quad (48)
\]

\[
t_3(x, \tau) = t_{11} + \frac{q_1}{ac} \left( h + \frac{\Delta h}{2} \right) \left[ t_{21} - t_{11} - \frac{q_1}{ac} \left( h + \frac{\Delta h}{2} \right) \right] \frac{x}{H}; \quad (h < x < H); \quad (49)
\]
4. Conclusion
Using the formulas (44) - (46), the temperature distribution along the height of the bulk of the food raw material was calculated. The sizes of the silo type elevator are taken from the work of P. P. Ilyin [10]: H=9 m.

The design parameters and thermophysical characteristics are as follows:

\[ h = 0.4 N \quad 1) \Delta h = 1 \text{ m}; \quad 2) \Delta h = 0.5 \text{ m}; \]

\[ t_0 = t_{1\Pi} = t_{2\Pi} = 16^\circ \text{C}; \quad q_1 = 0.01 \frac{W}{kg}; \quad q_2 = 0.02 \frac{W}{kg}; \]

\[ \alpha = 10^{-7} \frac{m^2}{s}; \quad c = 1,6 \frac{J}{kg \cdot K}; \quad k = 2 \cdot 10^{-6} \frac{1}{s}; \quad \tau = 80 \text{ days}. \]

The calculation results are presented in figure. We can conclude that the storage temperature increases to a critical temperature when the width of the layer with a self-heating hearth approach 0.5 m under the chosen storage conditions (curve 2).

![Figure 1. Temperature distribution by the height of the food raw material layer: 1—Δh=1 m; 2—Δh = 0.5 m.](image)

Formulas (47) - (48) give higher values of temperatures that real ones, which allows earlier warning of autoignition of bulks of food raw materials.

The obtained analytical solutions (37) - (49) are convenient for engineering calculations and can be used to determine temperature at any time of the storage process at any region of the bulk of the food raw material.

Thus, using the developed mathematical model, it is possible to predict safe storage conditions for bulk food raw materials, the rate of temperature change, to prevent autoignition, thereby keeping the high quality of food raw materials in bulks.

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