Superprocesses as models for information dissemination in the Future Internet

Laura Sacerdote, Michele Garetto, Federico Polito, Matteo Sereno

Abstract

Future Internet will be composed by a tremendous number of potentially interconnected people and devices, offering a variety of services, applications and communication opportunities. In particular, short-range wireless communications, which are available on almost all portable devices, will enable the formation of the largest cloud of interconnected, smart computing devices mankind has ever dreamed about: the Proximate Internet. In this paper, we consider superprocesses, more specifically super Brownian motion, as a suitable mathematical model to analyse a basic problem of information dissemination arising in the context of Proximate Internet. The proposed model provides a promising analytical framework to both study theoretical properties related to the information dissemination process and to devise efficient and reliable simulation schemes for very large systems.

Laura Sacerdote (corresponding author)
Dipartimento di Matematica, Università di Torino, Italy. e-mail: laura.sacerdote@unito.it

Michele Garetto
Dipartimento di Informatica, Università di Torino, Italy. e-mail: michele.garetto@unito.it

Federico Polito
Dipartimento di Matematica, Università di Torino, Italy. e-mail: federico.polito@unito.it

Matteo Sereno
Dipartimento di Informatica, Università di Torino, Italy. e-mail: matteo@di.unito.it
1 Introduction

So many events had a significant impact on our life in these last 50 years that it can be difficult to narrow it down to a few. Flights have restricted our perception of distances, radio and television have made people aware of facts and news, dangerous illnesses have disappeared thanks to vaccines while new antibiotics have helped the recovery from many dangerous diseases, medicine has decreased childhood mortality in many countries. The list could continue but we focus here on one of the most sudden events of our last thirty years that impacts our daily life everywhere: the advent of the Internet. More than one billion people on Earth use the Internet nowadays.

Due to the importance of the Internet for our planet development, in Section 2 we briefly list some important facts about it and its diffusion. Then in the same section we present the past and the future role that wireless technology played and will play for the development of possible future services of the Internet.

Next, we move to the problem considered in this paper, i.e., the dissemination of information in large, disconnected mobile ad-hoc networks [Klein et al., 2010a]. Such networks will arise naturally in a possible Future Internet scenario, where an increasing number of users carrying existing and novel wireless devices (smartphones, tablets, laptops, smartwatches, smartglasses, etc.) will form the so-called Proximate Internet, i.e., the largest cloud of interconnected, smart computing devices mankind has ever dreamed about. In the Proximate Internet, users will be able to directly communicate among themselves exploiting short-range radio communications, enabling a variety of interesting applications, and making the search and distribution of information much more efficient (in terms of spectrum usage, energy and monetary costs for the users) than traditional (cellular) networks.

In this scenario, information spreads among the wireless nodes in an epidemic fashion, i.e., like an infection in a human population. Hence mathematical models developed for epidemiology can be applied, after properly adapting them to our specific context. Indeed, users carrying wireless devices are typically in motion over a certain area (e.g., a city), and they can communicate with other devices within a limited communication range. Hence the epidemic model must account for these fundamental features.

The availability of mathematical models is of paramount importance to understand and design future applications running in the Proximate Internet. For example, models can predict the information dissemination speed, and thus the delays incurred to reach far-away users, and the efficiency of the dissemination in terms of area coverage, as function of a variety of system parameters.

The rest of the paper is organized as follows. In Section 2 we start with a general introduction to the topic of mathematical modelling of the Internet. In Section 3 we provide a high-level description of the specific scenario of wireless content distribution that we consider in our work, before going
into technical details. In Section 4, we introduce some necessary background to the mathematical tools adopted in our model. In particular, we briefly recall how to relate the discrete space and time branching process, i.e. the Galton–Watson model, with its continuous time counterpart, i.e. the continuous space branching process. Then we add to each particle of the branching model a movement mechanism and we introduce the resulting super Brownian motion as a measure valued process suitable to describe the information dissemination in a wireless cloud. For each of these processes we recall those properties which are most significant for our modeling purposes.

In Section 5, we present our model as an instance of super Brownian motion, explaining why this representation provides a much powerful performance evaluation methodology than traditional detailed simulations. We conclude with directions of future research and a with a detailed reference section which can be a helpful tool for the interested reader.

2 Internet, Planet Earth and Mathematics

It is impressive to realize that the Internet has existed for a so short period considering how strongly it changed the life on our Planet and the speed of its penetration worldwide. In Fig. 1 and 2, we illustrate the growth and the current presence of the Internet in Europe. The Internet is also a formidable tool for the evolution of underdeveloped countries where it allows the diffusion of books and culture as well as to give long distance medical support to people living in unreachable areas of our planet. The list of the changes determined in everyday life is incredibly long and this short contribution is not the place where to discuss such important political and social events.

We prefer to just provide an example taken from our everyday experience as mathematical researchers, that clearly show the fundamental role played by the Internet. The oldest between us still have memory of the incredible waste of time related with any bibliographic research less than twenty years ago. Youngest have never spent hours in a department library consulting books of Mathematical Reviews, writing down the references and then moving to look for the suggested journals, climbing ladders and moving heavy volumes up and down. Now we simply Google the title or the subject of the paper eventually using Mathematical Reviews online. When we recognize the title of an interesting paper a further click miraculously let it appear on our screen. Going further, who has never checked the definition of some mathematical object on Wikipedia for a fast suggestion?

The evolution of the Internet is so fast that we get immediately used to any change and we finally disregard principia allowing the creation of these new tools. However the improvements of the Internet are often related with mathematical results since its first appearance. To cite a famous example we recall that the first network between computers was made possible by Kleinrock
Fig. 1 Percentage of households with broadband access with at least one member aged 16 to 74 (source: Eurostat, see also Fig. 2).

with his mathematical theory of packet networks, the technology underlying today’s Internet [Kleinrock, 1961]. A more recent example is the mathematical work of [Brin and Page, 1998] underlying the algorithms of Google search engine (many mathematicians are still working on their improvements).

The Internet is also a source of new questions of high mathematical interest. To give a couple of examples, let us think to the preferential attachment phenomenon. In the Internet, preferential attachment makes nodes (that can represent either routers, or web pages, or people in social networks) with higher degree to have stronger ability to grab links added to the network. It is well known that preferential attachment generates scale-free networks, i.e. networks whose degree distribution follows a power law, at least asymptotically [Barabási and Albert, 1999] [Gonzalez et al., 2008] [Candia et al., 2008]. Various analytical results are known for this model but, for example, its clustering coefficient is only numerically known and its analytical expression is still an open problem.

Many other examples of open mathematical problems could be cited, determined for example by congestion problems and transmission optimization [Srikant, 2004].

We can pursue different philosophiae to deal with fundamental problems related to the current and future Internet which translates into different needs
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Fig. 2 Percentage of households with broadband access with at least one member aged 16 to 74 (source: Eurostat, see also Fig. 1)

for mathematical models and analyses. Next generation Internet involves different topics that can be roughly classified under the primary networking function to which they are related: routing, management and control (centralized or scalable), security (additional overlap or part of the architecture), challenged network environments (continuously connected–intermittent connectivity; tools for connectivity), content delivery (robust and scalable methods) are a possible classification of the involved topics.

Furthermore, each of these subjects can be studied both under the so called clean-slate paradigm or as corrections and improvements of present situation. The clean-slate design is the philosophy adopted by a project started at Stanford University\(^1\) based on the belief that the current Internet has significant deficiencies that need to be solved before trying to improve future features of our global communication infrastructure. The basic idea is to explore what kind of Internet we would design if we were to start from scratch, with 20–30 years of hindsight. In this framework many interesting mathematical problems arise. A good example coming from the area of network security are the so called adaptive resilient hosts, based on the general idea that a system should adapt to attacks by changing process code, perhaps also limiting

\(^1\) Clean Slate Program, Stanford University, [http://cleanslate.stanford.edu](http://cleanslate.stanford.edu).
services temporarily. Other typical examples arise in the context of congestion control algorithms [Alizadeh et al., 2011]. On the other hand, many studies renounce the clean-slate approach, and just consider improvements of existing protocols and codes.

Almost invariably, while modelling the Internet, evident difficulties arise both technically and mathematically, when the focus moves from an isolated problem to the whole Internet, which is an incredibly complex system whose evolution is not simply driven by technological factors.

In our work we focus on a specific problem related to the transmission of files to large populations of users. Hence we recall some basic facts about this topic. Wireless and wireline digital infrastructure currently coexist to transmit digital and analog files of data, voice, video, text, image, fax and streaming media. This coexistence will grow up in the next future giving rise to new Internet services, day after day more user-friendly. Due to the growing importance of the wireless technology, we provide here a brief summary of its history. Wireless transmission has its origins around 1895 when Tesla and Marconi independently developed the wireless telegraphy but its mathematical foundations date back to 1864 when J.C. Maxwell mathematically predicted the existence of radio waves. Mathematical results were then experimentally verified by D.E. Hughes who first transmitted Morse code. After wireless telegraph in 1900 wireless technology was used to transmit voice over the first radio and following developments determined the diffusion of radios around the globe. Then wireless technology did not give rise to new epochal instruments until the digital age of the Internet in the latter part of the twentieth century, with the appearance of digital cellular phones, mobile networks, wireless network access and so on. However the beginning of the Internet was wireline based and only recently the power of wireless communications has been unleashed.

L. Kleinrock in his paper on the History of Internet and its Flexible Future [Kleinrock, 2008] recognizes the following five phases through which the Internet will evolve in the next few years: nomadic computing, smart spaces and smart networks, ubiquitous computing, platform convergence and intelligent agents. Indeed there is an increasing number of travelling users that requests trouble free Internet services from any device, any place and any time (nomadic computing). Meanwhile it is no more science fiction to think to a cyberspace where intelligent sensors give the alarm in the presence of sudden health problems of aged people or where we can use voice commands to interact with devices in a room (smart spaces and smart networks). Ubiquitous computing is related with the necessity to have Internet services available wherever, for example to consult maps or train timetables. As Kleinrock remarks it is ridiculous to travel with a number of electronic devices: cellular phones, notebook computers, clocks, microphones, cameras, batteries, chargers. All these devices should converge in one, equipped with all

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2 CRASH—Clean-slate Resilient Adaptive Secure Hosts, Cornell University, http://www.nuprl.org/crash/introextended.php.
necessary technologies. Finally, intelligent agents, i.e., autonomous software modules acting on data, observing trends, carrying out tasks or adapting to the environment, will generate an increasing amount of traffic that should use wireless channels.

Nowadays wireless technology is becoming absolutely fundamental both to access the Internet in the traditional sense and to communicate locally with nearby users and objects, like sensors and actuators (i.e., the Internet of Things). This evolution is made possible by the increasing popularity of portable devices, and the availability of a tremendous number of different services and applications enabled by these devices.

3 Wireless information dissemination: system overview

In our problem, we exploit device-to-device (wireless) communications to distribute information from one or more sources to a large number of mobile users belonging to the same cloud (the Proximate Internet). The exploitation of local, short-range communications (using for example WiFi or Bluetooth technology) permits to offload cellular networks (3G/4G), with significant advantages both for cellular operators and for the users, who can obtain almost for free the contents they are interested in directly from other nearby users.

More specifically, let us focus on a piece of information (an entire file or just a portion of it), hereinafter called the message, which is initially stored at one or more nodes (called the seeds). These source nodes might have retrieved the message from the Internet by other means (i.e., different wired or wireless technologies) or they can have generated it locally. The message can be transmitted to other nodes interested in it, but only within a limited communication range, depending on the specific wireless technology (in the order of a few tens of meters). By so doing, the message gets replicated in the wireless cloud in an epidemic fashion, reaching more and more nodes with the passing of time. At the same time, nodes who have already received the message may stop contributing to the dissemination process, due to several reasons (the application is closed, the wireless interface is switched off, the message is cancelled from the local memory to make room to other messages, etc). Furthermore, users can move while carrying their devices, and we assume that the movements are random and independent from user to user.

Our goal is to define a suitable mathematical framework to model the above message replication process in the case of a very large number of users, so that we can study the dynamics of wireless information dissemination over a large area (like a big city). The model should be able to estimate basic performance metrics such as:

1. the delay associated to the transfer of the information, as function of the distance from the source;
2. an index of the achieved city-level coverage, and in particular a measure of the possible zones that will never be reached by the information;
3. the probability that an uncovered zone will be covered again after a reasonable delay.

The availability of such a model would be very useful to plan and design applications exploiting the proximate Internet. For example, it could be used to optimize the number and locations of the initial seeds, trading off the various costs associated to bandwidth/energy/memory constraints, according to different objective functions.

4 Mathematical background: Super Brownian motion

In this section we present a brief description of the simplest superprocess that can be defined, arising as a weak limit of standard branching Brownian motions, the so-called super Brownian motion. In the most basic model of super Brownian motion we have critical behaviour, branching is quadratic, and jumps are not admitted. Plainly, more general models of super Brownian motions can be constructed, as we will see in the following. We start by analyzing the underlying branching structure and then we will add movement to the model.

Let us therefore first consider separately the branching structure. In practice the evolution of the number of nodes (i.e., the amount of messages disseminated in the network) is governed by continuous state branching processes (CSBPs), introduced by Jiřina [1958] and studied by various researchers during the past decades [see for example Lamperti, 1967a,b; Feller, 1951; Grey, 1974; Silverstein, 1969; Bingham, 1970; Le Gall, 1999; Li, 2006; Kale and Deshmukh, 1992; Li, 2009; Caballero et al., 2009; Li, 2012a; Kashikar and Deshmukh, 2012]. In order to understand what a CSBP is, we start by describing a related discrete time and discrete state space model: the so-called Galton–Watson process. In general a Galton–Watson process [Watson and Galton, 1875; Athreya and Ney, 1972; Athreya and Jagers, 1997; Haccou et al., 2005] models the evolution of a population composed by individuals acting independently and behaving homogeneously. Individuals live for only one generation (one time step) then they die producing a random number $Z$ of offsprings with a non negative discrete probability distribution $P(Z = z)$, $z \in \mathbb{N} \cup \{0\}$ and with mean value $E[Z] = \zeta < \infty$. Let us now write $N_m$, $m \in \mathbb{N} \cup \{0\}$ for the process counting the number of individuals in the population at time $m$. We put $N_0 = 1$ for simplicity (a single initial progenitor) and write that

$$N_{m+1} = \sum_{j=1}^{N_m} Z_j, \quad m \in \mathbb{N} \cup \{0\},$$

(1)
where \( Z_j \), are independent and identically distributed random variables representing the random number of offsprings generated by individual \( j \). By means of simple calculations it can be proven that the mean value \( \mathbb{E} N_m = \zeta^m \).

The Galton–Watson process is critical, subcritical or supercritical if \( \zeta = 1 \), \( \zeta \in (0, 1) \) or \( \zeta > 1 \), respectively. See Fig. 3 for an example of a realization of a Galton–Watson counting process together with the associated underlying tree coding the whole genealogy. For more information on Galton–Watson and related processes the reader can refer to the already cited classical or more modern references.

Consider now a Markov process \( Y^x_t \), \( t \geq 0 \), taking values in the positive real line, starting from point \( x \in \mathbb{R} \cup \{0\} \) and having right-continuous with left limits (c\`adl\`ag) paths. We refer to [Sharpe 1988] or [Blumenthal and Getoor 1968] for an introduction to the general theory of Markov processes. We say that \( Y^x_t \) satisfies the branching property if

\[
Q_t(x + z, \cdot) = Q_t(x, \cdot) * Q_t(z, \cdot),
\]

where * represents the convolution operator and \( (Q_t)_{t \geq 0} \) is the associated transition semigroup. This can be defined by its Laplace transform as

\[
\mathbb{E} \exp(-\mu Y^x_t) = \int_0^\infty \exp(-\mu y) Q_t(x, dy) = \exp(-x v_t(\mu)), \quad \mu > 0,
\]

where \( x \in \mathbb{R}^+ \cup \{0\} \) is the starting point (initial population density) and the mapping \( t \mapsto v_t(\mu) \) is the unique positive solution of the integral equation

\[
v_t(\mu) = \mu - \int_0^t \phi(v_s(\mu)) \, ds, \quad t \geq 0.
\]

The function \( \phi \) in the previous equation is called the branching mechanism of the continuous state branching process and in general it can be of the form [Silverstein 1969]
\[ \phi(z) = bz + cz^2 + \int_0^\infty (e^{-zy} - 1 + zy) \pi(dy), \quad z \geq 0, \]  
(5)

where \((y \wedge y^2)\pi(dy)\) is a finite measure concentrated on the positive real line and \(b \in \mathbb{R}, \ c \in \mathbb{R}^+ \cup \{0\}\). Note that the above representation is related to the Lévy–Khintchine representation for the characteristic function of a Lévy process \cite{Bertoin1998, Sato1999, Kyprianou2007}. The three different cases of \(b > 0, \ b < 0, \) and \(b = 0\) correspond respectively to the supercritical, subcritical, and critical cases. As we recalled above one of the simplest possible branching mechanisms is the quadratic (or binary) branching which corresponds to

\[ \phi(z) = cz^2, \quad c \in \mathbb{R}^+. \]  
(6)

Continuous state branching processes with quadratic branching are of the critical type and are known in literature as the Feller’s branching diffusions \cite{Feller1951, Pardoux2011}. From (3) it is possible to determine the mean behaviour

\[ \mathbb{E}Y_t^x = \int_0^\infty y Q_t(x, dy) = x \exp(-bt), \quad t \geq 0, \ x \in \mathbb{R}^+ \cup \{0\}, \]  
(7)

for the general case (5). When the branching mechanism reduces to (6) we obviously obtain that \(\mathbb{E}Y_t^x = x\) which characterizes a critical behaviour. In turn, when the branching is quadratic it is immediate to calculate an explicit form of the function \(v_t(\mu)\) (see for example \cite{LeGall1999}) as

\[ v_t(\mu) = \frac{\mu}{1 + ct\mu}. \]  
(8)

Continuous state branching processes are very well studied models of population evolution. For more in-depth results standard references include \cite{Jirina1958, Lamperti1967a, Lamperti1967b, Watanabe1969, Silverstein1969, Bingham1976, Li2011, LeGall1999, Kyprianou2007}, and many others. Furthermore, it is worthy of notice that CSBPs arise naturally as weak limits of a sequences of rescaled Galton–Watson processes when waiting times between splits become negligible (see for example \cite{Lamperti1967b, Haccou2005, LeGall1999}). This last consideration will prove very useful in the interpretation of super Brownian motion.

Consider therefore the \(d\)-dimensional Euclidean space \(\mathbb{R}^d\) and a random measure \cite{Kallenberg1976} for reference on it determining the location of particles undergoing branching (both births and deaths) at time \(t\), written as

\[ X_t = \frac{1}{\beta} \sum_j \delta_{B_t^j}, \quad t \geq 0. \]  
(9)
In \((9)\), for each alive particle \(j\), the process \(B_j^t, t \geq 0\), is a \(d\)-dimensional Brownian motion on \(\mathbb{R}^d\) with initial starting point \(x_j^0 = B_j^0\). Furthermore \(B_j^t\) is independent of \(B_i^t, i \neq j\), \(\delta_{B_j^t}\) is a Dirac delta measure on \(B_j^t\), and \(\beta\) is a normalizing factor related to the population size. The process \((9)\) is a measure valued process on \(\mathbb{R}^d\) representing simultaneous motion of non-interacting particles.

![Fig. 4](https://example.com/fig4.png) A sketch of a possible realization of the genealogy tree with Brownian movement. Death of particles is represented by black squares.

Beside this, each particle undergoes branching such that the total number of alive particles follows a Galton–Watson process:

\[
W_t = \frac{1}{\beta} \sum_{j \in [\mathbb{N}_t]} \delta_{B_j^t}, \quad t \geq 0.
\]  

\(10\)

Note that branching occurs at time 1, 2, 3, ..., that is the waiting times between branching events are deterministic and all equal to unity. See in Fig. 4 a sketch of a possible realization of the complete process where particles move following independent Brownian motions and branch at fixed times \(t_1 = 1, t_2 = 2\), and so forth.

We are in fact interested in the weak limit of the rescaled processes

\[
X_t^k = \frac{1}{\beta_k} \sum_{j} \delta_{B_j^{t,k}}, \quad t \geq 0, \quad k \geq 1,
\]  

\(11\)

with \(B_0^j = x_j^{0,k}\). Clearly, aside the sequence \((X_t^k)_{k \geq 1}\), we consider the related sequence of processes

\[
W_t^k = \frac{1}{\beta_k} \sum_{j \in [\mathbb{N}_t]} \delta_{x_j^{t,k}}, \quad t \geq 0, \quad k \geq 1,
\]  

\(12\)
which measures the total mass present (amount of messages in the network) and where the branching waiting times are all equal to $t/k$. The relation which links the two processes $W_k^t$ and $X_k^t$ is clearly that $\langle X_k^t, 1 \rangle = W_k^t$. Let us now consider the space $M(\mathbb{R}^d)$ of finite measures on $\mathbb{R}^d$. If the sequence of the initial measures converges towards the finite measure $\xi \in M(\mathbb{R}^d)$ we have that $(X_k^t, t \geq 0)_{k \geq 1}$ converges weakly towards a $M(\mathbb{R}^d)$-valued Markov process $X_t$, $t \geq 0$ (see Li [2012b], Theorem 2.1.9, Ethier and Kurtz [2009], Chapter 9). Correspondingly $(W_k^t, t \geq 0)_{k \geq 1}$ converges weakly to a continuous state branching process with a general branching mechanism (5). Since the underlying spatial motion is of Brownian type, the process obtained is called $\phi$-super Brownian motion. In the simplest case of a quadratic (or binary) branching it is simply called super Brownian motion. Relevant references for super Brownian motion and superprocesses in general are Dawson [1993], Li [2011], Le Gall [1999], Etheridge [2000], Perkins [2002], Le Gall and Perkins [1995], Fitzsimmons [1988], Kyprianou et al. [2012], Mörters [2001]. Moreover, it should be noted that also more general superprocess are already described in the literature [see e.g. Engländer and Kyprianou, 2004, Engländer and Fleischmann, 2000, Fleischmann and Klenke [2000], Dawson et al., 2001, Wang, 2002].

5 Wireless information dissemination: basic model and future directions

Now we can explain how the information dissemination scenario introduced in Section 3 can be represented in terms of a super Brownian motion of the simplest kind, as described in Section 4.

To do so, some simplifying hypotheses are necessary in order to obtain a tractable model of the considered system. We assume that each user independently moves over the infinite plane according to a simple random walk. We know that realistic models of human mobility suggest that individuals’ movements should be modeled through Lévy flights, i.e. as a random walk in which the step-lengths have a probability distribution that is heavy-tailed [Brockmann et al., 2006]. Alternatively truncated Lévy flights were proposed after analysing data [Gonzalez et al., 2008]. However, for a first simplified model we consider the movements of each user as a simple random walk, allowing us to adopt a standard Brownian motion to describe the macroscopic mobility of each node.

While moving, nodes come in contact with other nodes (i.e., other nodes fall in the communication range of the considered node), allowing the opportunistic, direct transfer of the message of interest. A standard computation, that takes jointly into account the node density, the transmission range, and the node speed, permits computing the rate $\lambda$ at which the message gets duplicated. We do not repeat the details of this computation, which can be
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found in [Klein et al., 2010b]. Hence, the birth rate $\lambda$ can be considered as a primitive system parameter.

We instead denote by $\mu$ the death rate of each node (i.e., the rate at which a node transits to a state in which it no longer contributes to the message dissemination), which is supposed to be given. At last, let $\sigma^2$ be the infinitesimal variance of the Brownian motion describing the macroscopic mobility of each node. Moreover, we assume that, at time $t = 0$, there is a given number $N_0$ of seeds, initially co-located at the origin of the plane.

Disregarding for now the nodes’ movements, these hypotheses characterize the previously presented Galton–Watson tree, i.e. the model that describes the evolution of a population that starts with a given size at time $t = 0$. Depending on the value of $\lambda$ with respect to $\mu$ the population becomes extinct with probability 1 (subcritical case $\lambda < \mu$ and critical case $\lambda = \mu$) or has a positive probability to survive forever (supercritical case: $\lambda > \mu$). When the population becomes sufficiently large its size can be modelled as a continuous state branching process, as we pointed out in the previous paragraph. Hence the number of nodes storing the message of interest can be described through these processes.

However, nodes also move according to a random walk, and we are especially interested in characterizing the distribution on the plane of the nodes who are currently storing the message, as function of time. In our brief review in Section 4 we pointed out that, when the size of the population is large and microscopic displacements of the nodes are frequent enough, one can rescale the Galton–Watson tree with moving particles obtaining a super Brownian motion.

For the super Brownian motion, results are available for the asymptotic speed of diffusion. These results can be directly used to study the delay incurred by the message delivery to far-away users.

In our case nodes move over $\mathbb{R}^2$ and the super Brownian motion is known to have singular distribution on $\mathbb{R}^2$. Furthermore this distribution is uniform on its random support. Then the study of the support properties of a super Brownian motion is a possible approach to deal with the city coverage problem: we assume that each device covers a region of radius $r$, equal to its transmission range. This leads to study the coverage of the support of the Super Brownian Motion through balls. The aim of this study should be to determine which percentage of the plane is covered by at least one ball. Possible zones that will never be reached by the information could also be investigated following this approach.

Beside the above theoretical results, our model also allows us to devise efficient simulation schemes for scenarios in which no analytical results are available, such as those in which the birth and/or death rates of the nodes, and/or the infinitesimal variance of the associated brownian motion, are a function of space and/or time. On this regard, we emphasize that a brute-force simulation approach in which each node is modelled in all details becomes infeasible for large number of nodes. In particular, in the super-critical
regime it is basically impossible to simulate the system after a very short time, due to the exponential growth of the number of nodes. Therefore, alternate simulation approaches are necessary.

Simulations represent the only methodology to study scenarios of particular interest, in which the parameters of the process are not homogeneous in space (presence of lakes, zones without inhabitants and so forth). The available techniques for these simulations do not consider the singular nature of the super Brownian motion. Hence reliable and efficient simulation methods will be the subject of future work, together with the study of more theoretical results of interest for our application scenario.

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References

M Alizadeh, A Javanmard, and B Prabhakar. Analysis of DCTCP: stability, convergence, and fairness. In Proceedings of the ACM SIGMETRICS joint international conference on Measurement and modeling of computer systems, pages 73–84, 2011.
KB Athreya and P Jagers. Classical and modern branching processes. Springer, 1997.
KB Athreya and PE Ney. Branching processes, volume 28. Springer-Verlag Berlin, 1972.
A-L Barabási and R Albert. Emergence of scaling in random networks. Science, 286(5439):509–512, 1999.
J Bertoin. Lévy processes, volume 121. Cambridge university press, 1998.
NH Bingham. Continuous branching processes and spectral positivity. Stochastic Processes and their Applications, 4(3):217–242, 1976.
RM Blumenthal and RK Getoor. Markov processes and potential theory. Courier Dover Publications, 1968.
S Brin and L Page. The anatomy of a large-scale hypertextual Web search engine. Computer networks and ISDN systems, 30(1):107–117, 1998.
D Brockmann, L Hufnagel, and T Geisel. The scaling laws of human travel. Nature, 439(7075):462–465, 2006.
ME Caballero, A Lambert, and GU Bravo. Proof(s) of the Lamperti representation of continuous-state branching processes. Probability Surveys, 6:62–89, 2009.
J Candia, MC González, P Wang, T Schoenharl, G Madey, and A-L Barabási. Uncovering individual and collective human dynamics from mobile phone records. *Journal of Physics A*, 41(22):224015, 2008.

DA Dawson. Measure-valued Markov processes. In *École d’été de probabilités de Saint-Flour XXI-1991*, pages 1–260. Springer, 1993.

DA Dawson, Z Li, and H Wang. Superprocesses with dependent spatial motion and general branching densities. *Electronic Journal of Probability*, 6:1–33, 2001.

J Engländér and K Fleischmann. Extinction properties of super-Brownian motions with additional spatially dependent mass production. *Stochastic processes and their applications*, 88(1):37–58, 2000.

J Engländér and AE Kyprianou. Local extinction versus local exponential growth for spatial branching processes. *The Annals of Probability*, 32(1A): 78–99, 2004.

A Etheridge. *An introduction to superprocesses*, volume 20. American Mathematical Society, 2000.

SN Ethier and TG Kurtz. *Markov processes: characterization and convergence*, volume 282. Wiley, 2009.

W Feller. Diffusion processes in genetics. In *Proc. Second Berkeley Symp. Math. Statist. Prob.*, volume 227, page 246, 1951.

PJ Fitzsimmons. Construction and regularity of measure-valued Markov branching processes. *Israel Journal of Mathematics*, 64(3):337–361, 1988.

K Fleischmann and A Klenke. The biodiversity of catalytic super-Brownian motion. *Annals of Applied Probability*, pages 1121–1136, 2000.

MC Gonzalez, CA Hidalgo, and A-L Barabasi. Understanding individual human mobility patterns. *Nature*, 453(7196):779–782, 2008.

DR Grey. Asymptotic behaviour of continuous time, continuous state-space branching processes. *Journal of Applied Probability*, pages 669–677, 1974.

P Haccou, P Jagers, and VA Vatutin. *Branching processes: variation, growth, and extinction of populations*, volume 5. Cambridge University Press, 2005.

M Jiřina. Stochastic branching processes with continuous state space. *Czechoslovak Mathematical Journal*, 8(2):292–313, 1958.

M Kale and SR Deshmukh. Estimation in a continuous state space branching process. *Biometrical journal*, 34(8):1001–1006, 1992.

O Kallenberg. *Random measures*. Akademie-Verlag Berlin, 1976.

AS Kashikar and SR Deshmukh. Second order branching process with continuous state space. *Statistics & Probability Letters*, 2012.

DJ Klein, J Hespanha, and U Madhow. A reaction-diffusion model for epidemic routing in sparsely connected MANETs. In *INFOCOM, 2010 Proceedings IEEE*, pages 1–9. IEEE, 2010a.

DJ Klein, J Hespanha, and U Madhow. A Reaction-diffusion Model For Epidemic Routing In Sparsely Connected MANETs. In *Proceedings of IEEE INFOCOM 2010*, pages 884–892, 2010b.

L Kleinrock. *Information Flow in Large Communication Nets*. Phd thesis proposal, Massachusetts Institute of Technology, May 1961.
L. Sacerdote et al.

L Kleinrock. History of the Internet and its flexible future. *Wireless Communications, IEEE*, 15(1):8–18, 2008.

AE Kyprianou. *Introductory lectures on fluctuations of Lévy processes with applications*. Springer, 2007.

AE Kyprianou, R-L Liu, A Murillo-Salas, and Y-X Ren. Supercritical super-Brownian motion with a general branching mechanism and travelling waves. *Annales de l’Institut Henri Poincaré-Probabilités et Statistiques*, 48(3):661–687, 2012.

J Lamperti. Continuous-state branching processes. *Bull. Amer. Math. Soc*, 73(3):382–386, 1967a.

J Lamperti. The limit of a sequence of branching processes. *Probability Theory and Related Fields*, 7(4):271–288, 1967b.

J-F Le Gall. *Spatial branching processes, random snakes and partial differential equations*. Birkhauser Basel, 1999.

J-F Le Gall and E Perkins. The Hausdorff measure of the support of two-dimensional super-Brownian motion. *The Annals of Probability*, pages 1719–1747, 1995.

Y Li. A weak limit theorem for generalized Jiřina processes. *Journal of Applied Probability*, pages 453–462, 2009.

Z Li. *Measure-valued branching Markov processes*. Springer, 2011.

Z Li. Path-valued branching processes and nonlocal branching superprocesses. *arXiv preprint arXiv:1203.6150*, 2012a.

Z Li. Continuous-state branching processes. *arXiv:1202.3223 [math.PR]*, 2012b.

Z-H Li. Branching processes with immigration and related topics. *Frontiers of Mathematics in China*, 1(1):73–97, 2006.

P Mörters. The average density of super-Brownian motion. *Annales de l’Institut Henri Poincaré (B) Probability and Statistics*, 37(1):71–100, 2001.

E Pardoux and A Wakolbinger. From Brownian motion with a local time drift to Feller’s branching diffusion with logistic growth. *Electronic Communications in Probability*, 16:720–731, 2011.

E Perkins. Dawson-watanabe superprocesses and measure-valued diffusions. *Lectures on probability theory and statistics*, pages 125–329, 2002.

K Sato. *Lévy processes and infinitely divisible distributions*. Cambridge university press, 1999.

M Sharpe. *General theory of Markov processes*, volume 133. Academic press, 1988.

ML Silverstein. Continuous state branching semigroups. *Zeitschrift für Wahrscheinlichkeitstheorie und Verwandte Gebiete*, 14(2):96–112, 1969.

R Srikanth. *The mathematics of Internet congestion control*. Birkhauser Boston, 2004.

H Wang. State classification for a class of interacting superprocesses with location dependent branching. *Elect. Commun. Probab*, 7(16):157–167, 2002.

S Watanabe. On two-dimensional Markov processes with branching property. *Transactions of the American Mathematical Society*, 136:447–466, 1969.
HW Watson and F Galton. On the probability of the extinction of families. *The Journal of the Anthropological Institute of Great Britain and Ireland*, 4:138–144, 1875.