Isospin equilibration in multi-fragmentation processes and dynamical correlations

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The asymptotic time derivative of the total dipole signal is proposed as an useful observable to study Isospin equilibration phenomenon in multi-fragmentation processes. The study proceeds through the investigation of the $^{40}\text{Cl} + ^{28}\text{Si}$ system at 40 MeV/nucleon by means of semiclassical microscopic many-body calculations based on the CoMD-II model. In particular, the study has been developed to describe charge/mass equilibration processes involving the gas and liquid ”phases” of the total system formed during the early stage of a collision. Through the investigation of dynamical many-body correlations, it is also shown how the proposed observable is rather sensitive to different parameterizations of the isospin dependent interaction.

I. INTRODUCTION

An interesting subject related to Heavy Ions Isospin physics \cite{1} is the process leading to the equilibration of the charge/mass ratio between the main partners of the reaction as well described in Ref.\cite{2}. The so called ”isospin diffusion” phenomenon is the relevant mechanism acting between the reaction partners in binary processes\cite{3} \cite{4} \cite{5} \cite{6}. In particular, in the collision of the 124 and 112 Tin isotopes at 50 MeV/nucleon \cite{3}, evidence of partial equilibrium in the charge/mass ratios of the quasi-projectile and quasi-target has been deduced through the study of the iso-scaling parameters related to the isotopic distributions. In this case dynamical calculations based on the Boltzmann-Uehling-Uhlenbeck model \cite{7} show that the degree of equilibration depends on the behavior of the symmetry potential $U^{*}$ as a function of the density. The analysis of the experimental data in this kind of studies is based on the linear relation between the iso-scaling parameter and the relative neutron excess $\beta$ of the emitting sources (typical in several statistical models). It is also assumed that both quantities weakly depend on the secondary statistical decay processes. In general, as discussed in ref.\cite{8}, this last condition can be affected by the fragment excitation energies, (or temperatures) and by the distinctive features of the models used to simulate the secondary decay stage. In this work we want to extend the study of the isospin equilibration processes, looking at the whole system, by using the following quantity $\vec{V}(t) = \sum_{i=1}^{Z_{\text{tot}}} \vec{v}_{i}$. The sum on the index $i$ is performed on all the $Z_{\text{tot}}$ protons of the system. $\vec{V}(t)$ corresponds, apart from the elementary charge $e$, to the time derivative of the total dipole of the system. The velocities $\vec{v}_{i}$ are computed in the center of mass (c.m.) reference frame. Several studies were based on this dynamical variable to describe pre-equilibrium $\gamma$-ray emission (see Refs.\cite{9} \cite{10} \cite{11} and references therein ). Various reasons suggest us to use the same variable to also describe isospin equilibration processes.

- (i) After the pre-equilibrium stage, starting from the time $t_{\text{pre}}$, when a second stage characterized by an average isotropic emission of the secondary sources (statistical equilibrium) takes place, the ensemble average of $\vec{V}(t)$ satisfies the following relation: $\vec{V}(t_{\text{pre}}) = \vec{V}(t > t_{\text{pre}}) \equiv \vec{V}$ \cite{12}. The average value of this dynamical variable at $t_{\text{pre}}$ is in fact invariant with respect to statistical processes and therefore the value of $\vec{V}$ is determined only by the complex dynamics which characterizes the early stage of the collision, when fast changes of the average nuclear density are expected. In particular, $\vec{V}$ can be expressed as a function of the charge $Z$, mass $A$, average multiplicity $m_{Z,A}$ and the mean momentum $C_{Z,A}$ of the detected particles having charge $Z$ and mass $A$ in the generic event:

\begin{equation}
\vec{V} = \sum_{Z,A} \frac{Z}{A} m_{Z,A} \langle \vec{P}_{Z,A} \rangle C_{Z,A}^{Z,A} \langle \vec{P} \rangle
\end{equation}

\begin{equation}
C_{Z,A}^{Z,A} \langle \vec{P} \rangle = \frac{m_{Z,A} \langle \vec{P}_{Z,A} \rangle}{\langle \vec{P}_{Z,A} \rangle m_{Z,A}}
\end{equation}

$C_{Z,A}^{Z,A}$ is the correlation function between the multiplicity and the mean momentum. This correlation function plays a key role for the invariance property and therefore requires for an event by event analysis in which many-body correlations can not be neglected.

For symmetry reasons, $\vec{V}$ lies on the reaction plane. It is directly linked with a weighted mean of the charge/mass ratio, as Eq.(1) suggests. It also takes into account the average isospin flow direction through the momenta $\langle \vec{P} \rangle_{Z,A}$.

- (ii) In the general case, we find attractive the following decomposition: $\vec{V} = \vec{V}_{G} + \vec{V}_{L} + \vec{V}_{GL}$ where $\vec{V}_{G}$ and $\vec{V}_{L}$ are the average dipolar signals associated to the gas ”phase” (light charged particles) and to the ”liquid” part \cite{12}, corresponding to the motion of the produced heavy fragments. The signal $\vec{V}_{GL}$ is instead associated to the relative motion of the two ”phases”. By supposing, for simplicity, that the gas ”phase” is formed by neutrons and protons, $\vec{V}$ can be further decomposed as:

\begin{equation}
\vec{V} = \frac{A_{G}(1-\beta_{G})}{4} \vec{v}_{r}^{NP} + \frac{\mu_{G,L}(\beta_{L}-\beta_{G})}{2} \vec{v}_{cm,LG} + \vec{v}_{r,L}
\end{equation}
In the above expression the first term represents the contribution related to the neutron-proton relative motion of the gas "phase" expressed through the relative velocity \( \vec{v}_{np} \); the second term concerns the relative velocity \( \vec{v}_{cm,LG} \) between the centers of mass of the "liquid" complex and the "gas"; the last term represents the contribution produced by the relative motion of the fragments. A similar expression can be obtained including others light particles in the gas "phase" as, for example, the Intermediate Mass Fragments (IMF). From this decomposition we can see how the isospin equilibration condition (\( \vec{V} = 0 \), for the total system, requires a very delicate balance which depends on the average neutron excess (\( \beta = \frac{N-Z}{4} \)) of the produced "liquid drops" \( \beta_L \), on the one associated to the gas "phase" \( \beta_G \), and on the relative velocities between the different parts. To enlighten the role played by some of the terms reported in Eq.(3), we can discuss the idealized decay of a charge/mass asymmetric source through neutron and proton emission (or the case in which the liquid drops are produced through a statistical mechanism \( \vec{V}_{r,L} = 0 \)). Moreover, for simplicity, we can consider uncorrelated fluctuations between the velocities, masses and neutron excesses. In absence of pre-equilibrium emission or for identical colliding nuclei the second term of Eq.(3) is zero (\( \vec{v}_{cm,LG} = 0 \)), and the isospin equilibration requires a neutron or proton gas "phase" or absence of relative neutron-proton motion. For non-identical colliding nuclei, if pre-equilibrium emission exists, then \( \vec{v}_{cm,LG} \neq 0 \). In this case, if \( \beta_G \neq \beta_L \), due, for example, to the isospin "distillation" phenomenon, the first term has to be necessarily different from zero and it will contribute to the neutron-proton differential flow (see also Sect.III). Therefore, according to our description, the understanding of the isospin equilibration process for the total system requires the gas "phase" contribution to be taken into account. This term can be regarded as a kind of "dissipation" with respect to the system formed by the liquid part. In this work, as an example, we will discuss the results obtained through the Constrained Molecular Dynamics-II approach (CoMD-II) \cite{13,14} applied to the charge/mass asymmetric system \(^{40}\text{Cl} + ^{28}\text{Si}\) at 40 MeV/nucleon. The study is performed by using different options for the symmetry potential term.

Before to show the results of our calculations, in the following section we briefly recall the way in which the isospin interaction is introduced in CoMD-II model.

II. SYMMETRY INTERACTION AND CORRELATIONS

According to the results obtained in Ref.\cite{15}, in the simple case of large and compact systems, the isospin dependent part of the interaction is expressed, in the so called Non-Local (N.L.), approximation as:

\[
U^\tau_{N.L.} = \frac{a_{sym}}{2S_{g.s.}} \rho A^2 F'[(1 + \frac{1}{2}(\alpha - \alpha'))\beta^2 - \frac{1}{2}\alpha] \quad (4)
\]

\[
\alpha' = \frac{1}{4} \frac{\partial^2 \alpha}{\partial \beta^2} \bigg|_{\beta=0} \quad (5)
\]

\( \beta^4 \) terms are neglected in the previous expression. \( \alpha \equiv \alpha(\rho) \) represents the correlation coefficient related to the difference in the dynamics of the n-p couples with respect to the n-n and p-p ones. It is evaluated at \( \beta = 0 \) and describes the main effect of the many-body correlations on the iso-vectorial interaction. \( \alpha \) depends on the average overlap integral per couple of nucleons \( \rho \) which reflects the degree of compression. The coefficient \( a_{sym} = 72 \text{MeV} \) determines the strength of the iso-vectorial interaction at the ground state density (g.s.). \( F' \) is a form factor which modulates the changes of the iso-vectorial interaction as a function of the average overlap integral \( S \). In particular for the Stiff I option we use \( F' = \frac{2S}{\sqrt{S^2 - 1}} \), for the Stiff II case \( F' = 1 \) and for the Soft I option \( F' = \bigg( \frac{S_0}{S} \bigg)^{1/2} \). From eq.(4) we note that the isospin forces if treated in a self-consistent many-body approach generates, beyond the \( \beta^2 \) dependent potential, also another iso-vectorial density dependent term, not proportional to \( \beta^2 \), but proportional to the degree of correlation \( \alpha \). As discussed in \cite{15,16}, at small asymmetries, this term determines the high sensitivity of the experimental observable to the different functional form of \( F' \). The limiting case corresponding to vanishing values for the correlation \( \alpha \) represents the so called Iso-vectorial Mean Field Approximation (I.M.F.A.). In this case the average overlap integrals per couple of nucleons related to neutron-neutron, proton-proton and neutron-proton interactions have the same values and the iso-vectorial interactions generate only the usual symmetry potential term which is proportional to \( \beta^2 \) (see eq. 4). A comparison between results obtained by full CoMD-II calculations and the one derived in the I.M.F.A. limit will be discussed in the next section.

III. CALCULATION RESULTS

Now we discuss the results concerning the isospin equilibration process for the \(^{40}\text{Cl} + ^{28}\text{Si}\) system at 40 MeV/nucleon. For this collision, as an example, in Fig. 1 we show the average total dipolar signals evaluated through CoMD-II calculations along the \( \hat{z} \) beam direction \( \vec{V} \) and along the impact parameter direction \( \hat{x} \), \( \vec{V}' \), respectively. The reference frame is the c.m. one. The impact parameter \( b \) is equal to 3 fm, in panels (a) and (c), and 1.5 fm in panels (b) and (d). In Fig.1(a) and Fig.1(c), the average dipolar signals are shown for the first 150 fm/c. Different lines refer to different iso-vectorial potentials. The isospin independent compressibility is equal to 220 MeV, according to Ref. \cite{13}. In the
first 150 fm/c we can see that in all the cases wide oscillations exist. They are responsible for the pre-equilibrium γ-rays emission [3, 10]. The damped oscillations converge towards smaller and constant values (within the uncertainty of the statistics associated to the ensemble average procedure). This can be seen in Fig. 1(b) and Fig. 1(d) in which the dynamical evolution is followed from 100 fm/c up to 300 fm/c. The time interval in which the stationary behavior is reached is related to the life time of the coherent dipolar collective mode and it is strictly linked to the average time for the formation of the main fragments and pre-equilibrium emission. For the asymptotic components, global results are shown in Fig. 2. They concern different impact parameters and different interaction options. In particular, in Fig. 3(a), we show for \( b = 3 \text{ fm} \) the charge distribution evaluated after 650 fm/c for the Stiff2 option. The shape of the distribution clearly show that at this impact parameter the mechanisms evolve essentially throughout a multifragmentation process. Under the same conditions, we show in Fig. 3(b) and in Fig. 3(c) the ratio \( R_{GL} = |\langle V_z \rangle|/|\langle V_x \rangle| \) between \( \hat{z} \) asymptotic components of the dipolar signal related to the light particles (first 2 terms of eq. (3)) and the one associated to the two biggest fragment (last term of eq. (3)). \( R_{GL} \) is plotted as a function of the reduced impact parameter \( b_r = \frac{b}{b_{max}} \) (\( b_{max} \approx 7.5 \text{ fm} \)). In Fig. 3(b) we can see that for central and mid-peripheral collisions the "gas" component (pre-equilibrium contribution) is comparable or larger than the one associated to the "liquid" one. Fig. 3(c) shows that the relevance of the "gas" light particle signal rapidly decreases with the increasing of the impact parameter.

As discussed in the introductory section when the collision partners are not identical nuclei the isospin equilibration process produce a neutron-proton differential flow contribution if the neutron and proton "gases" have different c.m. velocities. These pre-conditions are verified for the studied collision. For \( b = 3 \text{ fm} \) and for the Stiff1 and Soft options in Fig. 3(d) we show the neutron-proton differential flow \( F_{np} \) as a function of the particles rapidity, \( y \), normalized to the projectile one \( y_{beam} \). The rapidity values are evaluated in the c.m. reference frame. From the figure we can see that, on average, the neutron-proton transversal velocity has a negative value. This reflects the "bending" of the relative motion between the c.m. of the emitted neutrons and protons, through the half-plane opposite to the impact parameter direction. These results can be compared with the calculations displayed in Fig. 3(e) obtained by subtracting, event by event, the c.m. relative neutron-proton motion related to the "gas" phase. As can be seen, similarly to the case of identical nuclei, this correction restores (within the errors associated to the statistics of simulations) the almost specular of the \( F_{np} \) behavior with respect the rapidity axes. The correction acts also along the beam direction. The final results also show an enhanced sensitivity to the options related to the isospin potential.

In the following we want to discuss in some detail the sensitivity of the dipolar signal to different options concerning the iso-vectorial interaction and different approximation scheme. In Fig. 4(a) we show as a function of the reduced impact parameter \( b_r = \frac{b}{b_{max}} \) (\( b_{max} \approx 7.5 \text{ fm} \)) the asymptotic values of \( V_x \) and \( V_z \). The arrow indicates the direction of increasing impact parameters corresponding to the marked points. Different symbols represent different options. In the region of the bending of the lines, around \( b = 3 \text{ fm} \), we observe the greater sensitivity to the different options. This result is particularly evident by studying the ratios \( R = \langle V_x \rangle/\langle V_z \rangle \). In Fig. 4(b) we in fact show the relative change \( \Delta r = \frac{\Delta R}{R} \) between couples of different options. We can see that for \( b_r \) less than about 0.6 large changes are predicted according to the different form factor shapes. This impact parameter region is clearly dominated by large overlap between projectile and target nuclei which gives rise to processes changing from incomplete fusion reactions to IMF production. The region of intermediate impact parameters show the higher
FIG. 3: (a) charge $Z$ distribution of the reaction products obtained at 650 fm/c for the Stiff2 option and $b=3$ fm. In panels (b) and (c), the $R_{GL} = \frac{|V_z^G|}{|V_z^L|}$ ratio (see the text) is plotted as a function of the reduced impact parameter. Panel (d) displays neutron-proton differential flow $F_{np}$ as a function of the c.m. rapidity. Different symbols refer to different options describing the iso-vectorial interaction. (e) the same quantities reported in the panel (d) are plotted after the correction for the relative velocity associated to the c.m. of the neutron and protons "gases".

In particular, according to what previously observed (see for example point (ii)), we have evaluated the partial contributions $V_x$ and $V_z$ related to the two main fragments. As an example, for $b = 3$ fm and for the Stiff2 option, the "liquid" asymptotic values are $V_x = -0.120c$ and $V_z = 0.162c$ while the total contributions are $V_x = 0.044c$ and $V_z = -0.027c$. It results therefore that the contributions carried by the two main fragments only partially contribute to the isospin equilibration process. The remanent part ("gas"), which in this case we have associated to particles and to the IMF, generates a term with opposite sign and similar strength for both directions. It contributes in a decisive way to the global equilibration process. In particular, as an example, for $b = 3$ fm, in Fig 4(b) we show with the star symbol, the sensitivity parameter $r$ evaluated by changing the option from Stiff1 to Stiff2 and by only taking into account the two main fragments contributions. As we can see, the partial contribution shows a rather reduced sensitivity to the different options as compared to the case obtained by using the complete information of the system.

Finally, in the following, we show the role of the correlation coefficient $\alpha$, introduced in Sec.II, into determine the sensitivity of the investigated observable to the density dependence of the iso-vectorial interaction. For this aim, in Fig 4(c), we compare, for different impact parameters, the values of $r$ obtained for the Stiff2-Stiff1 options with the ones obtained in the I.M.F.A. limit. Fig. 4(c) clearly shows that in the I.M.F.A. case, at the investigated energies, the sensitivity of our investigated phenomenon to the behavior of the symmetry interaction is rather reduced. The I.M.F.A. also strongly affects the values of $V_z$. In particular, independently from the used options, it produces for $b_r=0.4$, values of $|V_z|$ about four times larger than the ones obtained with full CoMD-II calculations indicating a reduced capacity to obtain the isospin equilibration along the $\hat{z}$ direction.

IV. CONCLUSIVE REMARKS

In summary, in this work the isospin equilibration process for the asymmetric charge/mass system $^{40}$Cl+$^{28}$Si at 40 MeV/nucleon has been investigated by studying the ensemble average of the time derivative of the total dipole $\overrightarrow{V}$ evaluated through CoMD-II calculations. Some general properties of this quantity have been discussed. In particular, it allows to generalize the definition of isospin equilibration also in complex reactions evolving through multi-fragmentation processes. CoMD-II calculations show that the asymptotic values of $\overrightarrow{V}$ for these processes are quite sensitive to different symmetry po-
tential options; moreover, in central and mid-peripheral, the dipolar contribution associated to the pre-equilibrium emission of charged particles is relevant to determine the value of $\overline{V}$ and the related sensitivity to different density dependent form factors. CoMD-II calculations performed in the so called I.M.F.A. scheme also highlights the fundamental role played by the many-body correlations in the study of the isospin equilibration processes.

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