1 Introduction

Along with the ordinary particle world, there may exist a hidden gauge sector in the form of its exact copy. Such a parallel sector, coined as mirror world \cite{1}, can have many interesting phenomenological and cosmological implications (for reviews, see \cite{2}). The Universe, besides the ordinary particles: electrons, nucleons, photons, etc., should also contain their invisible twins: mirror electrons, mirror nucleons, mirror photons, etc. having exactly the same mass spectrum and couplings constants. Mirror matter, being dark in terms of ordinary photons and interacting with ordinary matter via gravity, can be a viable candidate for dark matter \cite{3} (for earlier works, see also \cite{4}). The baryon asymmetry of the Universe can be generated via the out-of-equilibrium, $B-L$ and $CP$ violating processes between the ordinary and mirror particles \cite{5} which mechanism naturally explains the intriguing correspondence between the visible and dark matter fractions in the Universe \cite{6}. These processes can be mediated by some heavy gauge singlet particles as are the right handed neutrinos \cite{7}, or by extra heavy gauge bosons/gauginos interacting with both sectors \cite{8}.

These interactions can also induce the particle mixing and oscillation phenomena between the ordinary and mirror sectors. Any neutral particle, elementary or composite, can have a mixing with its mirror counterpart: e.g., photon with mirror photon \cite{8}, neutrinos with mirror neutrinos \cite{9}, pions with mirror pions \cite{2}, etc.

The mixing between the ordinary neutron $n$ and its mirror partner $n'$ via a small mass term $\varepsilon (\pi n' + \pi' n)$ was suggested in ref. \cite{10}. It was pointed out that the present experimental limits do not exclude a rapid $n - n'$ oscillation, with the timescale $\tau_{nn'} = \varepsilon^{-1}$ of order 1 s or at least much smaller than the neutron decay time $\tau_{\text{dec}} \simeq 10^3$ s. It is important that $n - n'$ mixing cannot destabilize nuclei and thus nuclear stability limits do not apply in this case.\cite{1} From the theoretical side, $n - n'$ mixing can be induced from the effective six-fermion operators like $\langle (1/M)^5 (u d d')(u' d' d'') \rangle$ between the ordinary quarks $u, d$ and their mirror twins $u', d'$, with $\varepsilon \sim (10 \text{ TeV}/M)^5 \times 10^{-15}$ eV, $M$ being the relevant cutoff scale. While the underlying TeV scale physics can be accessible at the LHC, the $n-n'$ oscillation itself can have interesting and testable astrophysical implications, e.g. for the propagation of ultrahigh energy cosmic rays \cite{11} or for the neutrons from solar flares \cite{12}. Moreover, it can be experimentally tested with slow neutron facilities \cite{10} and in particular with the ultra-cold neutron (UCN) storage chambers (for relevant discussions, see also \cite{13, 14}).

As far as mirror neutrons are sterile with respect to ordinary interactions, $n \rightarrow n'$ transition can only manifest as anomalous disappearance of the neutrons, in addition to the decay, absorption and other regular channels of their losses. However, in contrast to the latter, the neutron losses due to $n - n'$ transition depend on the magnetic field.

As far as the ordinary and mirror neutrons have equal masses and decay widths and also their gravitational po-

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1. Compare with neutron - antineutron ($n - \bar{n}$) oscillation \cite{11}: the direct limit from the neutron experiments in free flight gives $\tau_{n\bar{n}} > 10^8$ s \cite{12} while the nuclear stability tests yield even stronger bounds.

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More about neutron – mirror neutron oscillation

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Abstract. It was pointed out recently that oscillation of the neutron $n$ into mirror neutron $n'$, a sterile twin of the neutron with exactly the same mass, could be a very fast process with the the baryon number violation, even faster than the neutron decay itself. This process is sensitive to the magnetic fields and it could be observed by comparing the neutron lose rates in the UCN storage chambers for different magnetic backgrounds. We calculate the probability of $n - n'$ oscillation in the case when a mirror magnetic field $B'$ is non-zero and show that in this case it can be suppressed or resonantly enhanced by applying the ordinary magnetic field $B$, depending on its strength and on its orientation with respect to $B'$. The recent experimental data, under this hypothesis, still allow the $n - n'$ oscillation time order $1$ s or even smaller. Moreover, they indicate that the neutron losses are sensitive to the orientation of the magnetic field. If these hints will be confirmed in the future experiments, this would point to the presence of the mirror magnetic field on the Earth of the order of $0.1$ G, or some equivalent spin-dependent force of the other origin that makes a difference between the neutron and mirror neutron states.
tentials are universal, the non-relativistic Hamiltonian describing the $n - n'$ oscillation in the vacuum has the form

$$H_I = \left( \mu B \cdot \sigma \frac{\varepsilon}{\varepsilon - \mu B' \cdot \sigma} \right)$$

(1)

where $\mu = -6 \cdot 10^{-12}$ eV/G is the magnetic moment of the neutron, $B$ and $B'$ respectively are the ordinary and mirror magnetic fields, and $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli matrices.

It was naively assumed in [10] that there is no significant mirror magnetic field at the Earth, $B' = 0$. Therefore, the level splitting in the Hamiltonian [1] that corresponds to the energy (frequency) scales as

$$2\omega = |\mu B| = 6 \cdot 10^{-12} \left( \frac{B}{1 \text{ G}} \right) \text{ eV} \cdot 9 \cdot 10^3 \left( \frac{B}{1 \text{ G}} \right) \text{ s}^{-1}.$$  

(2)

Thus, for the angle of $n - n'$ mixing we have $|\tan 2\theta| = \varepsilon/\omega$ and the oscillation frequency is $\Omega = \sqrt{\omega^2 + \varepsilon^2}$. The probability of $n \rightarrow n'$ transition after the flight time $t$ reads

$$P_B(t) = \sin^2 2\theta \sin^2(\Omega t) = \frac{\varepsilon^2}{\omega^2 + \varepsilon^2} \sin^2 \left( \sqrt{\omega^2 + \varepsilon^2} t \right).$$

(3)

Clearly, $P_B$ depends on the magnetic field strength but does not depend on its orientation. Even if $\varepsilon t \ll 1$, the time-oscillating term can be averaged in strong magnetic field, when $\omega t \gg 1$, and the mean oscillation probability reads $P_B = \varepsilon^2/2\omega^2$.

On the other hand, once $B' = 0$, the Hamiltonian [1] becomes degenerate in the limit $B = 0$. Then $n - n'$ mixing is maximal ($\theta_0 = 45^\circ$), the oscillation frequency is $\Omega_0 = \varepsilon$ and the oscillation probability becomes $P_0(t) = \sin^2(\varepsilon t)$. If the neutron free flight is long enough, $t \gg \tau_{nn'}$, the $n - n'$ transition probability averaged over many oscillations becomes 1/2.

In the real experiments, the mean free flight time of the neutrons is limited by technical reasons. So far up to $t \approx 0.1$ s can be achieved for the cold neutrons propagated at distances $\sim 100$ m [12] or in the UCN traps of the dimensions $\sim 1$ m as in the experiments [16,17,18].

For $t \ll \tau_{nn'}$, the oscillation probability in zero magnetic field is $P_0(t) = (\varepsilon t)^2 \ll 1$. In the weak magnetic field, with $\omega \ll t^{-1}$, the oscillation probability remains essentially the same, $P_B(t) = P_0(t) = (\varepsilon t)^2$. Namely, for $t \approx 0.1$ s it would be the case for $B < 10^{-5} G$ [18]. However, if the magnetic field is strong, $\omega \gg t^{-1}$, it should suppress the $n - n'$ transition: $P_B(t) = (\varepsilon/\omega)^2 \sin^2(\omega t) \ll (\varepsilon t)^2$.

Therefore, experiments that compare the neutron losses in the conditions of zero (i.e. weak, $\omega \ll 1$) and non-zero (i.e. strong, $\omega \gg 1$) magnetic fields in otherwise similar conditions can directly trace the difference $P_B - P_0$ since the uncertainties related to the neutron decay and other regular channels can be canceled out.

Three experiments [16,17,18] of this kind were performed during the last years at the Institute of Laue-Langevin (ILL), Grenoble. The UCN losses were compared for different configurations of the magnetic fields and the limits

$$\begin{align*}
\tau_{nn'} &> 103 \text{ s} \quad (95\% \text{ CL}) \\
\tau_{nn'} &> 414 \text{ s} \quad (90\% \text{ CL}) \\
\tau_{nn'} &> 403 \text{ s} \quad (90\% \text{ CL})
\end{align*}$$

(4)

were reported in refs. [16,17,18] respectively, implying an upper bound $\varepsilon < 2 \cdot 10^{-18}$ eV or so. However, in these experiments the data were analyzed taking $B' = 0$ and hence assuming that (a) the probability of $n - n'$ oscillation $P_B$ does not depend on the magnetic field direction; (b) magnetic field can only suppress the oscillation, $P_B < P_0$, so that the UCN counts should be larger when the magnetic field is on than when it is off.

In the present paper we study the more general case when the mirror magnetic field $B'$ is non-zero. This makes the situation very different. Indeed, if at the Earth $B' \neq 0$ by some virtue, it cannot be screened in the experiments and contributes the mirror neutron energy levels in the Hamiltonian [1] as $2\omega' = |\mu B'|$. Hence, the $n - n'$ oscillation probability in the limit $B = 0$, if $\omega' t \gg 1$, can be averaged in time and it becomes $P_0 = 1/2(\varepsilon/\omega')^2$, instead of $P_0 = (\varepsilon t)^2$. Then the non-zero $B'$ can either suppress or resonantly enhance the $n - n'$ oscillation, depending on its strength as well as on its orientation with respect to $B'$.

Therefore experimental data on $n - n'$ oscillations should be interpreted more carefully. In particular, in the presence of mirror field $B' > 10^{-2}$ G or so, the experiments [16,17,18] cannot impose the limits [1], and as we see below, the $n - n'$ oscillation time $\tau_{nn'}$ can easily be order 1 s or even smaller.

The paper is organized as follows. In section 2 we study the Hamiltonian [1] in the general case when both ordinary and mirror magnetic fields $B$ and $B'$ are non-zero and arbitrarily oriented. We derive the exact formula for the $n - n'$ oscillation probability which essentially depends on two parameters: relative strength of the fields, $\eta = B/B'$, and the angle $\beta$ between the vectors $B$ and $B'$. In section 3 we discuss the implications of our results for the UCN storage experiments and re-visit the experimental data reported in refs. [16,17,18]. Namely, in the experiments [16,18] the UCN losses were measured for the vertical directions of the magnetic field, up ($B_1$) and down ($B_1$), respectively at $B = 0.06$ G [16] and $B = 0.2$ G [18].

Interestingly, these experimental data indicate a deviation from zero for the up-down asymmetry of the neutron losses, at about $3\sigma$ level. If these asymmetries are real, they may indicate the presence at the Earth of a mirror magnetic field $B'$ in the range around $0.1 - 1$ G, with a significant vertical component. The mechanisms that could generate mirror magnetic fields on the Earth, in solar system or in the Galaxy and their implications for the time variation of the signal are discussed in section 4. In addition, the implications of the possible matter effects or

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2 The DUSEL project [19] can significantly increase the neutron free flight time.

3 The experimental limit [12] on the deficit of cold neutrons propagating in a weak magnetic field $B \sim 10^{-4}$ G with a mean time $t \sim 0.1$ s implies $P_0(t) = (\varepsilon t)^2 < 10^{-2}$, and thus imposes the bound $\tau_{nn'} > 1$ s or so [10].
any other spin-independent effects that may lift the degeneracy between the $n$ and $n'$ states are also studied and the relevant formulas for the oscillation probabilities are given. Finally, we briefly discuss the possible strategies to search for the disappearance of the unpolarized or polarized neutrons due to the $n \to n'$ oscillation and for the neutron regeneration $n \to n'$, as well as for measurements of the neutron precession frequencies as a function of the magnetic field strength and its orientation.

2 $n-n'$ oscillation in the background of ordinary and mirror magnetic fields

Let us study the free neutron-mirror neutron oscillation in general case, when ordinary and mirror magnetic fields, $B$ and $B'$, are both non-zero and have arbitrary orientations. Denoting $\mu B = 2\omega$ and $\mu B' = 2\omega'$, the Hamiltonian (1) can be rewritten as

$$H_I = \begin{pmatrix} 2\omega & \sigma & \sigma & \sigma \\ 2\omega' & \sigma & \sigma & \sigma \\ -\sigma & 0 & 0 & 0 \\ -\sigma & 0 & 0 & 0 \end{pmatrix},$$

where we introduce the combinations $b = \omega' + \omega$ and $a = \omega' - \omega$. We can choose the basis of wavefunctions $(\psi_+, \psi_-, \psi'_+, \psi'_-)$ corresponding to $n$ and $n'$ states with the spins parallel (+) or antiparallel (−) to the direction of vector $b$ taken as $z$-axis: $b = (0, 0, b)$, while the vector $a$ is taken in the $xz$ plane: $a = (a_x, 0, a_z)$, so that $a_x b = a_x^2 - \omega^2$, $a_x b = |a x b| = 2|\omega x \omega'|$ and $b = |\omega' + \omega| = [\omega^2 + \omega'^2 + 2\omega\omega']^{1/2}$. Hence, in this basis $b\sigma = b\sigma_z$, $a\sigma = a_x\sigma_x + a_z\sigma_z$, and the Hamiltonian has the explicit form of the $4 \times 4$ matrix

$$H_I = \begin{pmatrix} b - a_x & -a_x & \epsilon & 0 \\ -a_x & b + a_x & \epsilon & 0 \\ \epsilon & 0 & a_x & b - a_x \\ 0 & \epsilon & b + a_x & a_x \end{pmatrix}. \tag{6}$$

It can be exactly diagonalized by the unitary transformation

$$H_I \rightarrow S^\dagger H_I S = \begin{pmatrix} 2\omega & 0 & 0 & 0 \\ 0 & -2\omega & 0 & 0 \\ 0 & 0 & 2\omega' & 0 \\ 0 & 0 & 0 & -2\omega' \end{pmatrix}, \tag{7}$$

using the mixing matrix of the form

$$S = \begin{pmatrix} \cos \theta & 0 & \sin \theta & 0 \\ 0 & \cos \theta & 0 & -\sin \theta \\ -\sin \theta & 0 & \cos \theta & 0 \\ 0 & \sin \theta & 0 & \cos \theta \end{pmatrix} \times \begin{pmatrix} \cos \phi & \sin \phi & 0 & 0 \\ -\sin \phi & \cos \phi & 0 & 0 \\ 0 & 0 & \cos \phi' & -\sin \phi' \\ 0 & 0 & -\sin \phi' & \cos \phi' \end{pmatrix}. \tag{8}$$

As for the eigenvalues, we obtain:

$$2\tilde{\omega} = \frac{b - \sqrt{a_x^2 + \epsilon^2}}{\cos 2\phi},$$

$$\sqrt{2(\omega^2 + \omega'^2) + 2(\omega^2 - \omega'^2)} \sqrt{1 + \tan^2 2\theta + \epsilon^2},$$

$$2\tilde{\omega}' = \frac{b + \sqrt{a_x^2 + \epsilon^2}}{\cos 2\phi'},$$

$$\sqrt{2(\omega^2 + \omega'^2) - 2(\omega^2 - \omega'^2)} \sqrt{1 + \tan^2 2\theta + \epsilon^2}. \tag{10}$$

Therefore, the probability of $n-n'$ transition after the flight time $t$ reads\footnote{Here the partial probabilities of the transitions $n \to n'$ and $n \to n'$ are summed up, and so $P_B(t)$ does not depend on the initial neutron polarization.}

$$P_B(t) = \sin^2 2\tilde{\omega}_0 \left[ \cos^2 (\phi - \phi') \sin^2 (\Omega^+ t) + \sin^2 (\phi - \phi') \sin^2 (\Omega^- t) \right], \tag{11}$$

where two characteristic frequencies are respectively $\Omega^\pm = |\omega' \pm \omega|$. The magnitude of $n-n'$ oscillation is essentially determined by the angle $\theta$ while the angles $\phi, \phi'$ merely describe the spin precessions: in fact, they do not enter in the averaged oscillation probability $P_B = \frac{1}{2} \sin^2 2\tilde{\omega}$. From (10) we obtain:

$$\tan^2 2\theta = \frac{\epsilon^2 (\omega + \omega')^2}{(\omega^2 - \omega'^2)^2} = \frac{1 + \eta^2 + 2\eta \cos \beta}{(1 - \eta^2)^2} \tan^2 2\theta_0, \tag{12}$$

where $\eta = \omega/\omega' = B/B'$, $\beta$ is the angle between the vectors $B$ and $B'$ and $\tan 2\theta_0 = \varepsilon/\omega'$ stands for $n-n'$ mixing angle in the limit $B = 0$. In this limit the oscillation frequency is $\Omega_0 = (\omega^2 + \epsilon^2)$ and the $n-n'$ oscillation probability becomes

$$P_B(t) = \sin^2 2\tilde{\omega}_0 \sin^2 (\Omega_0 t) = \frac{\epsilon^2}{\omega^2 + \epsilon^2} \sin^2 \left( \sqrt{\omega^2 + \epsilon^2} t \right). \tag{13}$$

Rewriting eq. (12) as

$$\sin^2 2\tilde{\omega} = \frac{\sin^2 2\theta_0}{\sin^2 2\theta_0 + \frac{(1 - \eta^2)^2}{1 + \eta^2 + 2\eta \cos \beta} \cos^2 2\theta_0}, \tag{14}$$

we see that $\sin^2 2\tilde{\omega}$ can be smaller or larger than $\sin^2 2\theta_0$, depending on the values $\eta$ and $\cos \beta$. If $\eta > 2$, we have $\sin^2 2\tilde{\omega} < \sin^2 2\theta_0$ for any value of $\cos \beta$. However, we get $\sin^2 2\tilde{\omega} > \sin^2 2\theta_0$ if $\eta < 2$ and $2 \cos \beta > \eta^3 - 3\eta$. (If $\eta < \sqrt{3}$ the latter condition can be satisfied also for negative $\cos \beta$, i.e. $\beta > \pi/2$.) The angle of $n-n'$ mixing can be resonantly amplified if $\eta$ is enough close to 1. Namely, if $\eta = 1$ ($B = B'$ exactly), we have maximal mixing, $\sin^2 2\tilde{\omega} = 1$, for any angle $\beta \neq \pi/2$. However, the width

$$\sin 2\tilde{\omega} = \frac{2\omega}{\sin 2\theta_0 + \frac{(1 - \eta^2)^2}{1 + \eta^2 + 2\eta \cos \beta} \cos^2 2\theta_0}.$$

This is a rather interesting feature of the system described by the Hamiltonian (1): the resonance (level-crossing) condition does not require the exact coincidence of the vectors $\omega$ and $\omega'$; it is sufficient that their modules are equal, $\omega = \omega'$, while their directions can be different.
of the resonance depends on $\beta$. Namely, inspection of eq. (14) gives that for arbitrary $\beta$, the resonance half-width at half-maximum is $\gamma_{\text{res}} = |\tan 2\theta_0 \cos(\beta/2)|$. It becomes maximal, $\gamma = |\tan 2\theta_0|$, when the vectors $B$ and $B'$ are parallel $(\beta = 0)$, gets smaller for non-zero $\beta$ and vanishes for $B$ and $B'$ being antiparallel $(\beta = \pi)$. Hence, for these limiting cases we have

$$
\sin^2\theta(\beta=0) = \frac{\tan^22\theta_0}{\tan^22\theta_0 + (1-\eta)^2},
$$

$$
\sin^2\theta(\beta=\pi) = \frac{\tan^22\theta_0}{\tan^22\theta_0 + (1+\eta)^2},
$$

(15)

while for the case of orthogonal $B$ and $B'$ $(\beta = \pi/2)$ we have

$$
\sin^2\theta(\beta=\pi/2) = \frac{\tan^22\theta_0}{\tan^22\theta_0 + (1-\eta^2)^2/(1+\eta^2)},
$$

(16)

which is larger than $\sin^2\theta_0$ if $\eta < \sqrt{3}$. The mirror magnetic field $B'$, if it exists at the Earth by some circumstances, cannot be screened in the experiments. Then the probability of $n \rightarrow n'$ transition should depend on the magnitude and orientation of the original magnetic field $B$, provided that the neutron free flight time $t$ is enough large.\(^6\) In particular, if $\omega' t > 1$, the oscillating term in the transition probability $P_0(t)$ \(^{13}\) can be averaged in time, and we obtain

$$
P_0 = \frac{1}{2} \sin^2\theta_0 = \frac{\varepsilon^2}{2\omega^2}.
$$

(17)

E.g., for $t \sim 0.1$ s we have $\omega' t > \pi$ if $2\omega' > 5 \times 10^{-14}$ eV, or $B' > 7$ mG. In addition, $P_0 < 1$ implies that $\omega' > \varepsilon$. In particular, the bound on neutron losses in the experiment \(^{12}\) yields the limit $P_0 < 10^{-2}$ or so.\(^{10}\) On the other hand, assuming that $B$ is far enough from the resonant value $B = B'$, so that $\theta \ll 1$ and $(1-\eta^2)^2 \gg P_0$, for the average oscillation probability \(^{11}\) in the magnetic field $B \neq 0$ we get

$$
P_B = \frac{1}{2} \sin^2\theta = P_0 \frac{1 + \eta^2 + 2\eta \cos \beta}{(1-\eta^2)^2} = P_{B\perp} + D_B \cos \beta,
$$

$$
P_{B\perp} = P_0 \frac{1 + \eta^2}{(1-\eta^2)^2},
$$

$$
D_B = P_0 \frac{2\eta}{(1-\eta^2)^2},
$$

(18)

where $P_{B\perp}$ corresponds to the case when $B$ and $B'$ are orthogonal, i.e. $\cos \beta = 0$. Hence, the experimentally measurable difference of the probabilities

$$
P_B - P_0 = P_{B\perp} - P_0 + D_B \cos \beta = P_0 \frac{\eta^2(3-\eta^2) + 2\eta \cos \beta}{(1-\eta^2)^2}
$$

(19)

depends on the orientation of the magnetic field $B$ (angle $\beta$). Changing the magnetic field direction to the opposite, $B \rightarrow -B$, while $B'$ remains fixed, i.e. $\beta \rightarrow \pi - \beta$, one sees that $P_{-B} \neq P_B$ unless $B$ and $B'$ are orthogonal. However, the average between $P_B$ and $P_{-B}$ does not depend on $\beta$ while their difference is proportional to $\cos \beta$:

$$
P_B + P_{-B} = 2P_{B\perp}, \quad P_B - P_{-B} = 2D_B \cos \beta,
$$

(20)

So, it is convenient to measure experimentally the latter difference as well as

$$
\Delta B = P_{B\perp} - P_0 = P_0 \frac{\eta^2(3-\eta^2)}{(1-\eta^2)^2}.
$$

(21)

The sign of $\Delta B$ depends on the strength of the magnetic field $B$. Namely, it is positive for weaker fields, when $B < \sqrt{3}B'$ and becomes negative for stronger fields, $B > \sqrt{3}B'$. For $B \ll B'$ the stronger effect should be observed by measuring $D_B$ which is nearly linear in $\eta$ while $\Delta B$ is quadratic. For the ratio of two effects we get

$$
\frac{\Delta B}{|D_B|} = \frac{1}{2} \eta(3-\eta^2).
$$

(22)

This ratio reaches its maximal value 1 at $\eta = 1$; vanishes at $\eta = \sqrt{3}$, turns to the value $-1$ at $\eta = 2$ and rapidly decreases further with increasing $\eta$. Hence, for $B \ll B'$ the dominant effect should be observed by measuring $\Delta B$.

The formulas above assume that $n \rightarrow n'$ oscillation is far from the resonance regime. At the resonance, when $\omega' \sim \varepsilon$, $\sin^2\theta = 1$ and for the oscillation probability we get

$$
P_B(t) = \frac{4\omega^2 - \varepsilon^2 \cos^2 \beta}{4\omega^2 - \varepsilon^2} \sin^2 \left( t \varepsilon \cos \frac{\beta}{2} \right) + \frac{\varepsilon^2 \sin^2 \frac{\beta}{2}}{4\omega^2 - \varepsilon^2} \sin^2 \left( t \sqrt{4\omega^2 + \varepsilon^2 \sin^2 \frac{\beta}{2}} \right).
$$

(23)

Hence, if $\omega' t \gg 1$ but $t \varepsilon \ll 1$, the dominant contribution comes from the first term which cannot be averaged in time and so $P_B(t) \approx \cos^2(\beta/2)(\varepsilon t)^2$ and $P_{-B}(t) \approx \sin^2(\beta/2)(\varepsilon t)^2$. Therefore, we get $D_B = P_{B\perp} = (t/\tau_{mn'})^2 \gg 2P_0 = (1/\omega'\tau_{mn})^2$, where $\tau_{mn'} = \varepsilon^{-1}$, and hence at the resonance $\Delta B/|D_B| = 1$, in agreement with (22). For example, for $\tau_{mn'} \sim 10$ s and $t = 0.1$ s, we would have $\Delta B \sim 10^{-4}$. Concluding this section, let us remark that non-$n'$ mixing in the background of the mirror magnetic field should affect also the neutron spin precession. For the Hamiltonian \(^{16}\) the probability of the neutron polarization change (transition from $\psi_+ \rightarrow \psi_-$ state) reads

$$
P_{\psi}(t) = \cos^4 t \sin^2 2\phi \sin^2 (2\omega t) + \frac{1}{2} \sin^2 2\theta \sin 2\phi \sin 2\phi' \sin(\omega t) \sin(\omega' t) + \sin^4 t \sin^2 \phi' \sin^2 (2\omega' t)
$$

(24)

In the limit $\theta = 0$ (no $n \rightarrow n'$ mixing), we have $P_{\psi}(t) = \sin^2 2\phi \sin^2 (2\omega t)$, the Larmor precession with normal frequency $2\omega = |\mu B|$. However, for non-zero $\theta$ the precession
frequency is affected. (In particular, the neutron gets a non-zero spin precession even if the magnetic field is exactly zero, \( B = 0 \): this is an evident consequence of the mixing with the neutron state while the spin of the latter precesses with respect the mirror magnetic field \( B' \).) Namely, for \( \theta \ll 1 \) eq. \((10)\) leads to

\[
\dot{\omega}_B = \omega \left[ 1 + \frac{P_0 + (\eta^2 - 1) P_B}{4 \eta^2} \right] = \omega + \frac{\eta + \cos \beta}{2(\eta^2 - 1)} P_0 \omega'.
\]

(25)

From the experimental point of view, the effect can be detected by comparing the results of the neutron magnetic moment measurements at different values and/or directions of the magnetic field. In particular, one can measure the difference between the precession frequencies \( \delta \omega_B = \omega_B - \omega_{-B} \) for the magnetic fields of the opposite direction \( B \) and \(-B\). In doing so, we expect

\[
\delta \omega_B = \frac{\eta^2 - 1}{4 \eta^2} \left[ P_B - P_{-B} \right] = \frac{\eta^2 - 1}{2 \eta^2} D_B \cos \beta = \frac{P_0 \cos \beta}{\eta (\eta^2 - 1)}.
\]

(26)

These shifts of the precession frequency can be measured in the experiments similar to the ones designed for a search of the neutron EDM in which the relative orientation of the applied electric and magnetic fields are tested. However, in these experiments usually the electric field direction is altered while the magnetic field is kept fixed. In our case no electric field is needed and only the magnetic field should be applied, altering its direction from \( B \) to \(-B\).

3 \( n - n' \) oscillation in the UCN storage chambers

In the UCN storage chamber \( n - n' \) oscillations are restricted by the free flight time \( t_f \) between the neutron collisions on the walls. During the free flight the initial neutron state gets a small admixture of mirror neutron state and thus per each wall collision it can escape from the trap with a mean probability \( P \) which is equal to the \( n - n' \) oscillation probability averaged over the neutron distribution. Therefore, given that the initial amount of the neutrons \( N_{in} \) is fixed, the amount of the neutrons that remain in the trap after the storage time \( t_s \) should be

\[
N(t_s) = N_{in} \exp \left[ - (\Gamma + R_s + P \nu) t_s \right], \quad \nu = 1/t_f
\]

is a mean collision frequency, \( \Gamma = \tau_{dec} \) is the neutron decay width and \( R_s \) accounts for the regular UCN losses due to absorption and upscattering during the wall collisions or in collisions with the residual gas in the chamber. The neutron losses due to \( n - n' \) oscillation can be smaller than the latter effects. However, if the \( n - n' \) oscillation probability depends on the magnetic field, then by comparing the neutron counts \( N_1(t_s) \) and \( N_2(t_s) \) measured for two configurations \( B_1 \) and \( B_2 \) with different strength and/or orientation of the magnetic field, the dependence on \( \Gamma \) and \( R_s \) cancels out and the ratio \( N_1(t_s)/N_2(t_s) = \exp \left[ (P_2 - P_1) \nu t_s \right] \) directly traces the difference between the oscillation probabilities \( P_1 = P(B_1) \) and \( P_2 = P(B_2) \).

In particular, one can measure the neutron counts \( N_B \) for an applied magnetic field \( B \) after a holding time \( t_s \), then change the direction of the magnetic field and measure the neutron counts \( N_{-B} \) in the same conditions. The expected directional asymmetry of the neutron counts can be simply calculated and we get

\[
A(t_s) = \frac{N_B(t_s) - N_{-B}(t_s)}{N_B(t_s) + N_{-B}(t_s)} = \frac{e^{-n_s P_B} - e^{-n_s P_{-B}}}{e^{-n_s P_B} + e^{-n_s P_{-B}}} = -\tanh(n_s D_B \cos \beta)
\]

(27)

where \( n_s = \nu t_s \) is the mean amount of wall collisions per neutron during the time \( t_s \), and \( P_B - P_{-B} = 2 D_B \cos \beta \) (see eq. \((26)\)). On the other hand, one can compare the neutron counts for zero magnetic field, \( N_0 \), with the averaged counts between the opposite directions of the non-zero magnetic field, \( N_B = \frac{1}{2}(N_B + N_{-B}) \). Then we have

\[
1 + E(t_s) = \frac{N_0(t_s)}{N_B(t_s)} = \frac{2 e^{-n_s P_0}}{e^{-n_s P_B} + e^{-n_s P_{-B}}} = \exp(n_s \Delta B \cos \beta) \]

(28)

where \( \Delta B = \frac{1}{2}(P_B + P_{-B}) - P_0 \) (see eq. \((21)\)). For small oscillation probabilities, when \( D_B, \Delta B \ll n_s \), we have approximately \( A(t_s) = -n_s D_B \cos \beta \) and \( E(t_s) = n_s \Delta B \), and hence the ratio \( E(t_s)/|A(t_s)| = \Delta B/|\Delta B| \) should not depend on the holding time \( t_s \).

The experiment \(16\) used the UCN chamber where the mean free flight time between the wall collisions was \( t_f \approx 0.05 \) s and hence \( \nu = t_f^{-1} \approx 20 \) s\(^{-1}\). The measurements were performed for the storage times \( t_s = 50 \) s, 100 s and 175 s and the neutron counts were compared in the conditions of “zero” (\( B_0 < 0.5 \) mG) and “non-zero” (\( B = 0.06 \) G) magnetic fields. The direction of the latter was vertical and altered from up (\( B_1 \)) to down (\( B_1 \)) repeating the configuration sequence: \( B_0, B_1, B_1, B_0, B_0, B_1, B_0, B_1 \). Another sequence \( B_1, B_1, B_0, B_0 \) was also used for \( t_s = 50 \) s. Taking into account that the neutrons can oscillate also during the filling and emptying of the chamber, the effective holding time was estimated as \( t_s = t_s + 23 \) s.

Results of the measurements \(16\) are reported in Table 1. They indicate that the neutron counts in the non-zero magnetic field depend on the direction of the latter. We observe that \( N_{B1} \) regularly exceeds \( N_{B1} \) for all 4 cycles of data taking. As far as under the naive assumption \( B' = 0 \) this feature was not expected, since in this case the probability of \( n - n' \) oscillation is independent of the magnetic field direction, in ref. \(16\) this effect was neglected, the counts \( N_{B1} \) and \( N_{B1} \) were averaged and compared to \( N_0 \). As a result, the first limit in \(4\) was imposed. However, in the presence of mirror magnetic field \( B' \) the difference \( N_{B1} - N_{B1} \) must depend on the angle \( \beta' \) between \( B' \) and \( B_1 \). Thereby, let us analyze the data of ref. \(16\) allowing the oscillation probabilities \( P_{B1} \) and \( P_{B1} \) to be different. Then, fitting \( E(t_s) \) and \( A(t_s) \) given in Table 1 we obtain within 1σ error-bars:

\[
\Delta_{B=0.06G} = (2.9 \pm 4.4) \times 10^{-7}, \; \chi^2_{d.o.f.} = 6.9/3 \quad (29)
\]

and

\[
D_{B=0.06G} \cos \beta_V = (6.2 \pm 2.0) \times 10^{-7}, \; \chi^2_{d.o.f.} = 0.52/3 \quad (30)
\]
As we see, \( D_B \) has a 3\( \sigma \) deviation from zero while there is no pronounced effect for \( \Delta_B \).

A more recent experiment \cite{18} has collected much bigger statistics. The “up-down” asymmetry of the neutron counts \( N_H \) and \( N_B \) was measured for a vertical applied magnetic field \( B \approx 0.2 \) G, repeating the configuration sequence: \( B_1, B_1, B_1, B_1, B_1, B_1, B_1 \). Such a sequence of measurements gives an important advantage since it allows to remove a linear drift and also an eventual quadratic drift in the neutron flux, vacuum conditions, etc.. In addition, the neutron monitor was during the filling of the trap. After the UCN holding time \( t_s = 300 \) s, the neutrons were counted using two independent detectors.

As a consequence, for an effective exposition time estimated as \( t_s = 370 \) s, the following result was obtained:

\[
\Delta_V(t_s) = (3.8 \pm 1.2) \times 10^{-4}
\]

If this 3\( \sigma \) deviation is not related to statistical fluctuations or some unknown systematic effects, then it may point to the \( n - n' \) oscillation in the background of a mirror magnetic field \( B' \) with a significant vertical component: \( \cos \beta_V \sim 1 \). The control measurements performed in the UCN flow mode in order to check whether this deviation was induced by the influence of the current switching on the electronic systems have shown no systematic effects of such type at the accuracy level \( 10^{-4} \). Then, using eq. \text{(27)} and the effective amount of wall collisions per neutron estimated as \( n_s = \nu t_s \approx 4 \times 10^4 \), this translates to

\[
D_{(B=0.2\text{G})} \cos \beta_V = (9.5 \pm 3.0) \times 10^{-8}.
\]

In the same experiment \cite{18} the asymmetry of the UCN counts between the configurations \( B_H \) to \( -B_H \) was measured also for the horizontal magnetic fields \( (B = 0.2 \) G), directed roughly towards North-East, with the following result:

\[
A_H(t_s) = (0.3 \pm 5.1) \times 10^{-5},
\]

which translates as

\[
D_{(B=0.2\text{G})} \cos \beta_H = (0.1 \pm 1.3) \times 10^{-8},
\]

with \( \beta_H \) being the angle between \( B_H \) and \( B' \). At the same time, for the difference between the UCN counts measured in \( "\text{small}" \) \( (B < 0.012 \) G) and \( "\text{large}" \) \( (B = 0.2 \) G) magnetic fields, that according to \text{(28)} should not depend essentially on the magnetic field orientation, the following result was obtained:

\[
\Delta_{B=0.2\text{G}} = -(3.5 \pm 2.5) \times 10^{-8},
\]

In the above considerations it was implicitly assumed that the angles \( \beta_V \) and \( \beta_H \) are constant in time. This would occur e.g. if the mirror magnetic field \( B' \) is related to the Earth, rotates with the Earth at the same angular velocity, and so its orientation with respect to the experimental site does not change in time.\footnote{The analysis would be more complicated if \( \beta_V \) and \( \beta_H \) vary with time, e.g. if the background field \( B' \) has a fixed direction in the solar system while the direction of experimental field \( B \) changes with the Earth rotation, as discussed in Section 4. In the experiment \cite{18} the time interval between the configurations \( B_1 \) and \( B_2 \) was typically few hours, and hence at the corresponding time moments the angles \( \beta_V \) and \( \beta_H \) could be different. (In stable experimental conditions the neutron counts at zero magnetic field, \( N_0 \), should anyway remain time independent.) Therefore, instead of eqs. \text{(20)} and \text{(21)} we would get \( D_B = 2P_0 \eta \cos \beta \cos \alpha / (1 - \eta^2) \) and \( \Delta_B = P_0 (3 \eta^2 - \eta^4 - 2 \eta \sin \beta \sin \alpha) / (1 - \eta^2)^2 \), where \( \beta = (\beta_V + \beta_H) / 2 \) and \( \alpha = (\beta_V - \beta_H) / 2 \). Notice that this could imitate negative \( \Delta_B \) for small \( \eta \)’s while the truly equal time measurement of \( N_B \) and \( N_{B1} \) should always give positive \( \Delta_B \) if \( \eta < \sqrt{3} \) (c.f. eq. \text{(21)})}.
oscillation times $\tau_{nn'} = \varepsilon^{-1}$ are allowed? From eqs. (20) and (17), we obtain $P_0 = D_B(1 - \eta^2)^2/(2\eta)$ and $\varepsilon = \omega \sqrt{\Delta B / (1 - \eta^2)^2 / \eta^2}$; $B = 0.2$ G means $\omega = 6 \times 10^{-13}$ eV.

For example, let us take a central value $D_B = 0.2$ G = $9.5 \times 10^{-7}$ and $B' = 0.11$ G ($\eta > 1.8$). Then we get $P_0 = 1.4 \times 10^{-7}$, $\tau_{nn'} = \varepsilon^{-1} = 3.8$ s, and also $D_B = 0.6$ G = $3 \times 10^{-7}$, which curiously is consistent with the fit (20) following from the experiment [15]. Along the same lines for e.g. $B' = 0.15$ GeV we get $P_0 = 2.1 \times 10^{-8}$ and $\tau_{nn'} = 5.5$ s. As for the small $\eta$ branch, $B' > 1$ GeV implies $P_0 > 2 \times 10^{-7}$ and $\tau_{nn'} < 0.3$ s.

As for another $3\sigma$ deviation [20] indicated by the data of ref. [16], it probably should not be taken very seriously. Let us discuss, nevertheless, what size of mirror magnetic fields it requires. Then the lower bound can be settled from the value of $\Delta B$ [20]. As one can see from eq. (21), $\Delta B$ could be positive if $B'$ is large enough, namely if $\eta = B / B' < \sqrt{3}$. On the other hand, according to eq. (22), for $\eta > 2$ one would expect $\Delta B$ to be negative and larger than $D_B$, which seems incompatible with the indications [20] and [24]. Thus, one can set a conservative lower bound $B' > 0.3$ G or so. This in turn implies that for the neutron free flight time being $t_f \approx 0.5$ s in the experiment [16], we have $\omega t_f < 13$ and hence the approximation of the time-averaged oscillation probabilities is valid. The upper limit on $B'$ can be imposed from the following consideration. As follows from eq. (24), for $\eta < 1$ we get $P_0 \approx D_B(2\eta \cos \beta)' = D_B(B'/0.12)$ G. Then, assuming $D_B > 4 \times 10^{-7}$, $B' > 3$ G would imply $P_0 > 10^{-5}$.

On the other hand, for $B' > 3$ G the oscillation probability $P_B$ in the Earth magnetic field ($B \approx 0.5$ G in Grenoble), would be in fact larger than $P_0$ and thus larger than $10^{-5}$, which is excluded by the upper bound on the UCN losses in the Earth magnetic field [20]. Therefore, rather conservatively, one can impose an upper bound $B' < 3$ G, which by strict analysis can be improved by a factor of 2 or so.

The experiment [17] used the horizontally directed magnetic field $B_H$ with $B = 0.02$ G, without altering its direction. Unfortunately, since in this experiment the magnetic field direction was not altered, the values of $D_B$ and $\Delta B$ cannot be evaluated. Its result reads $P_{B_H} - P_0 = \Delta B + D_B \cos \beta_H = -(1.7 \pm 3.6) \times 10^{-8}$. The angle $\beta_H$ between $B'$ and $B_H$ can vary between $\pi / 2 \pm \beta_V$, $\beta_V$ being the angle between $B'$ and $B$. $\tau_{nn'}$ is by chance parallel to the Earth rotation axis. Namely, we would have

$$\cos \beta_V = \cos \gamma \sin \varphi + \sin \gamma \cos \varphi \cos \left(2\pi \frac{t - t_0}{t_{sd}}\right),$$

where $\gamma$ is an angle that vector $B'$ makes with the Earth rotation axis, $\varphi$ is a latitude of the experimental site (≈ 45° for Grenoble), and $t_{sd} = 23.9345$ h is a sidereal day. As for the case of horizontal magnetic fields, directed e.g. to North or to East, we respectively get

$$\cos \beta_N = \cos \gamma \cos \varphi - \sin \gamma \sin \varphi \cos \left(2\pi \frac{t - t_0}{t_{sd}}\right),$$

$$\cos \beta_E = -\sin \gamma \sin \left(2\pi \frac{t - t_0}{t_{sd}}\right).$$

The experimental data [17,18] can be carefully analyzed in order to see whether there are the traces of such variations in $P_B - P_{-B} = 2D_B \cos \beta$ and certainly new dedicated experiments in which the strength and orientation of the applied magnetic field can be varied are desirable. Let us recall that no variations should be expected for $\Delta B$ since $P_B + P_{-B}$ does not depend on the angle $\beta$.

Another possibility is that the mirror matter is captured by the solar system and correspondingly in the solar neighborhood the substantial mirror magnetic field is present. One can also expect that in this case $B'$ is not homogeneous in the solar system and it varies around the Earth orbit (both the strength and the direction). In this case short time measurements should indicate a day-night oscillations in $P_B - P_{-B}$ with a fixed $P_B + P_{-B}$, while long

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9. As for the experiment [16], all data were taken during the daytime and thus the possibility of day-night variation instead of the constant fit [20] cannot be a priori excluded.
time experiments must show certain annual modulations that would depend on the pattern of the mirror magnetic field lines that cross the Earth orbit.

According to common sense, there should be no significant amount of the mirror matter in the Earth. The gravitational potential of the Earth is not efficient to capture a large amount of cosmic mirror particles. However, the situation could change if there are some stronger interactions between the ordinary and mirror matters, e.g. due to photon - mirror photon kinetic mixing as discussed e.g. in refs. [22]. Such interactions could give also a consistent explanation to the DAMA/Libra results on dark matter search [23]. On the other hand, if the neutron - mirror neutron mixing is possible, it would look pretty natural that also neutral mesons of the ordinary and mirror sectors have a reasonable mixing: \( \pi - \pi', \eta - \eta', \rho - \rho' \) etc. that would mediate enough strong “nuclear” forces that could efficiently capture the mirror nuclei in the Earth, with cross-sections up to few pb. Interestingly, the geo-physical constraints on the amount of the mirror matter within the Earth appear to be rather flexible, allowing for up to 0.4 per cent of the Earth mass constituted by mirror particles [24]. If so, then the existence of the mirror magnetic field \( B' \) of order 1 G or even larger should not be a surprise, if one takes into account that the Earth’s rotation itself can give rise to the asymmetric capture of the mirror matter that can give rise to circular currents, as well as the possibility of very efficient dynamo mechanism. In the view of the latter, the mirror magnetic field could have time variations much faster than the terrestrial magnetic field: the latter changes its polarity in every few million years. Depending on the interaction strength between the ordinary and mirror particles as well as on the chemical composition of the latter, the two following situations can be envisaged: first, when the mirror matter forms a puffy cloud around the Earth, with a size much larger than the Earth radius, that can have a differential rotation. In this case the measurements of \( P_B - P_{-B} \) can exhibit a quasi-periodic pattern originated by superposition of the Earth rotation with the slower rotation of the mirror cloud; and second, when captured mirror particles form a compact body inside the Earth that rotates together with the latter with the same angular velocity. In this case no time variation of the signal should be expected.

Another possibility is that the Earth itself is the origin of some pseudo-magnetic field acting on the \( n' \) state. Imagine, for example, a light axion-like boson \( \chi \) that has a pseudoscalar coupling with mirror neutrons \( ig \chi \bar{n}^\gamma n = g(\partial_a \chi/m)n^\gamma n \), but also has scalar couplings with normal matter components. Such a hybrid boson would mediate the long range Yukawa type “fifth forces” violating the weak equivalence principle, and also induce the CP-violating monopole-dipole interactions discussed in ref. [26]. This could occur, if e.g. the mirror axion having the Yukawa interactions with mirror baryons [25], is mixed in some way with a dilaton like scalar coupled to the trace of the energy-momentum tensor of the normal matter. Interestingly, in the mirror gravity scenario with Lorentz-violation [27], even the massive graviton could mix with the axion like scalar). Then, if \( \chi \) is very light, with a Compton length comparable to the Earth radius, the Earth itself acts as a source for a spin-dependent static potential \( (g \nabla \chi/m) \pi \Sigma n' \) where \( m \) is the neutron mass, and \( \Sigma = \text{diag}(\sigma, \sigma) \) is the spin matrix, and \( \nabla \chi \) acts as a vertically directed pseudo-magnetic field, i.e. \( \beta \nu = 90^\circ \) and \( \beta_H = 0 \).

The following remark is in order. In the case of the Earth bounded mirror matter, a non-zero density of mirror gas can induce a significant spin-independent contribution in the Hamiltonian of \( n' \), state. (as for the ordinary gas, in the UCN chambers it is pumped out in order not to affect significantly the neutron propagation). In this case the effective Hamiltonian describing the \( n \rightarrow n' \) oscillations becomes

\[
H_I = \left( \begin{array}{cc} v + 2 \omega \sigma & \varepsilon \\ \varepsilon & v' + 2 \omega' \sigma \end{array} \right),
\]

where \( 2\omega = \mu B, 2\omega' = \mu B' \), and \( v \) and \( v' \) are the matter induced spin-independent potentials respectively for ordinary and mirror neutrons. In the basis of the wave-functions \( (\psi_+, \psi_-, \psi'_+, \psi'_-) \) where \( \psi_\pm \) correspond to the neutron states with the spins parallel/antiparallel to \( B \), and \( \psi'_\pm \) to the mirror neutron states with the spins parallel/antiparallel with respect to \( B' \), the Hamiltonian has the form

\[
H_I = \left( \begin{array}{cccc} 2(\rho + \omega) & 0 & \varepsilon \cos \frac{\beta}{2} - \varepsilon \sin \frac{\beta}{2} & 0 \\
0 & 2(\rho - \omega) & \varepsilon \sin \frac{\beta}{2} & \varepsilon \cos \frac{\beta}{2} \\ \varepsilon \cos \frac{\beta}{2} & \varepsilon \sin \frac{\beta}{2} & 2\omega' & 0 \\ -\varepsilon \sin \frac{\beta}{2} & -\varepsilon \cos \frac{\beta}{2} & 0 & -2\omega' \end{array} \right),
\]

where \( 2\rho = v - v' \) and \( \beta \) is the angle between the vectors \( \omega \) and \( \omega' \). Hence, the neutron states with the ( + ) and ( - ) polarizations have different mixings with the mirror neutron states. Assuming that the mixing angles are small \((\lesssim 1)\), for the time-averaged oscillation probabilities \( n_+ \rightarrow n' \) and \( n_- \rightarrow n' \) respectively we obtain\[^{11}\]

\[
P_{+}(B) = \frac{\varepsilon^2 \cos^2 \beta}{2(\omega' - \omega - \rho)^2} + \frac{\varepsilon^2 \sin^2 \beta}{2(\omega' + \omega + \rho)^2},
\]

\[
P_{-}(B) = \frac{\varepsilon^2 \sin^2 \beta}{2(\omega' + \omega - \rho)^2} + \frac{\varepsilon^2 \cos^2 \beta}{2(\omega' - \omega + \rho)^2},
\]

\[^{10}\]More generally, there can be other reasons that may provide different spin-independent potentials between \( n \) and \( n' \) states, e.g. if the gravitation forces are not quite universal between the ordinary and mirror matters [27]. In fact, the Hamiltonian describes also a situation when the ordinary and mirror neutrons are not quite degenerate and their masses \( m \) and \( m' \) have a small splitting.

\[^{11}\]Here the sum is taken over the polarizations of the final \( n' \) states. Clearly, in the limit \( \rho = 0 \) we have \( P_{+}(B) = P_{-}(B) = P(B) \), the latter given by eq. (18).
where \( y = \rho/\omega' \) and \( \eta = \omega/\omega' \). Obviously, here we assumed that neither \( P_{\rho}(B) \) nor \( P_{-\rho}(B) \) are at the resonance. The difference between \( P_{\rho}(B) \) and \( P_{-\rho}(B) \) can be measured in the experiments with the polarized neutrons. For unpolarized neutrons, as in the case of experiments [16, 17, 18], two probabilities (38) can be averaged. Thus we obtain:

\[
P_B = \frac{1}{2} \left[ P_{\rho}(B) + P_{-\rho}(B) \right] = P_{B\perp} + D_B \cos \beta \quad (39)
\]

where

\[
P_{B\perp} = \frac{P_B + P_{-B}}{2} = \frac{\varepsilon^2}{4\omega'^2} \left[ 1 + \eta_+^2 \left( \frac{1}{(1 - \eta_+^2)^2} + \frac{1 + \eta_-^2}{(1 - \eta_-^2)^2} \right) \right],
\]

\[
D_B = \frac{P_B - P_{-B}}{2 \cos \beta} = \frac{\varepsilon^2}{4\omega'^2} \left[ \frac{2\eta_+}{(1 - \eta_+^2)^2} + \frac{2\eta_-}{(1 - \eta_-^2)^2} \right] \quad (40)
\]

\( (\eta_\pm = \eta \pm y) \), while the averaged oscillation probability in the limit of zero magnetic field, \( B = 0 \), is

\[
P_0 = \frac{1}{2} \left( P_0^+ + P_0^- \right) = \frac{1}{2} \left[ \frac{\varepsilon^2}{2(\rho + \omega'^2)} + \frac{\varepsilon^2}{2(\rho - \omega'^2)} \right] = \frac{\varepsilon^2}{2\omega'^2} \left( 1 + \frac{y_+^2}{1 - y_+^2} \right) \quad (41)
\]

where we assumed that \( y^2 \neq 1 \), i.e. \( P_0 \) has no resonance at \( \omega = 0 \). Thus, the values \( D_B \) and

\[
\Delta_B = P_{B\perp} - P_0 = \frac{\varepsilon^2}{2\omega'^2} \left[ \eta_+^2 \left( 3 - \eta_+^2 \right) + \eta_-^2 \left( 3 - \eta_-^2 \right) \right] + \frac{\varepsilon^2}{2(1 - \eta_+^2)^2} \frac{y_+^2(3 - y_+^2)}{1 - y_+^2} \quad (42)
\]

where \( y = (\eta_+ - \eta_-)/2 \), can be tested experimentally with magnetic fields of varying strength and direction.

New interesting features emerge with respect to the case \( \rho = 0 \), when the \( n - n' \) oscillation probability had only one resonance at \( \omega = \omega' \). Now we have two resonance values of \( \omega \). Namely, if \( 0 < y < 1 \), \( P_{\rho}(B) \) has a resonance at \( \omega = \omega' - \rho \) while \( P_{-\rho}(B) \) has a resonance at \( \omega = \omega' + \rho \). For \( \omega' \) and \( \rho \) fixed, the sign of \( \Delta_B \) depends on \( \omega \), while the sign of \( D_B \) depends on \( \beta \) but does not depend on \( \omega \). On the other hand, for \( y > 1 \), \( P_{\rho}(B) \) has two resonances, at \( \omega = \rho \pm \omega' \) while the sign of \( D_B \) changes with increasing \( \omega \). Notice also, that due to the difference between \( P_{\rho}(B) \) and \( P_{-\rho}(B) \), the UCN with + and – polarizations should disappear with the different rates and thus surviving neutrons should have a preferred polarization even if the neutrons initially were unpolarized. These effects can be tested experimentally also if one varies the value of \( \rho = (v - v')/2 \) by changing the residual gas pressure in the UCN traps.

5 Discussion and outlook

Summarizing, if a reasonably large mirror magnetic field, say \( B' > 0.01 \) G, exists on the Earth or its environments, it cannot be screened in the terrestrial experiments and can strongly affect the neutron to mirror neutron oscillation features. In particular, the oscillation probability becomes dependent on the strength and the direction of the applied magnetic field \( B \). Therefore, the experimental data [16, 17, 18] on the \( n - n' \) oscillation should be analyzed with more care. In particular, \( \tau_{nn'} \) cannot be anymore restricted by the limits of about 400 s [4], and in fact it easily could be of order 1 s or even smaller.

The issue of a fast \( n - n' \) oscillation can have interesting links. Namely, the questions whether the \( n - n' \) oscillation is related to anomalous neutron losses observed for different material surfaces [20], or whether it is relevant for understanding the 6.5σ discrepancy between the last precise measurements of the neutron lifetime, \( \tau_{dec} = (885.4 \pm 0.9,_{stat} \pm 0.4,_{syst}) \) s [28] and \( \tau_{dec} = (878.5 \pm 0.7,_{stat} \pm 0.3,_{syst}) \) s [29], remain still open: in fact, the external magnetic fields were neither screened nor controlled in these experiments. The fact that the baryon number violating process can be so fast, much faster the neutron decay, is interesting per se and certainly constitutes a strong challenge. Such a fast oscillation, with \( P(t) = \sin^2(t/\tau_{nn'}) \) can occur in a deep cosmos where both ordinary and mirror magnetic fields are expected to be rather small, and in any case it would have consequences for the propagation of the ultra-high energy cosmic rays [10].

The effect of the \( n - n' \) oscillation can be experimentally tested by comparing the neutron loss rates for opposite directions of the applied magnetic field. By varying the strength of the magnetic field in these experiments the resonance is achieved when \( B = B' \). (recall however that resonance conditions change if \( n \) and \( n' \) states have also different spin-independent potentials.) It may be convenient to use in the experiments inhomogeneous magnetic fields with smooth profile for achieving the MSW-like resonant transitions between \( n - n' \).

If the resonant amplification of neutron losses will be really observed, this would point to the \( n - n' \) oscillation, but would also allow to provide the crucial test by observing the neutron regeneration \( n \to n' \to n \): the neutrons disappear from the UCN traps but they can reappear in the neighboring trap with the same magnetic conditions with a measurable probability. In addition, as far as \( n - n' \) mixing changes the neutron precession as well, the effect can be observed in the neutron precession and depolarization experiments similar to those that are used for the search of the neutron EDM.

If the mirror magnetic field has a galactic origin or it is related to other extended structures like the mirror molecular clouds or mirror matter in the solar system, \( n - n' \) oscillations should exhibit specific day-night and perhaps also other seasonal variations.

The non-zero up-down asymmetries (30) and (32), observed in the experiments [16 and 18] for the applied magnetic fields of \( B = 0.06 \) G and \( B = 0.2 \) G respectively, could be a signal for the \( n - n' \) oscillation in the background of a mirror magnetic field. However, new dedicated experiments are needed to verify if at least one of these \( 3\sigma \) deviations can be real.
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