SUSY contributions to the charge asymmetry in $K^\pm \rightarrow \pi^\pm \ell^+\ell^-$ decays

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Abstract

We analyse the contributions to the charge asymmetry in $K^\pm \rightarrow \pi^\pm \ell^+\ell^-$ decays induced by gluino-exchange diagrams in the context of supersymmetric models with generic flavour couplings. We show that sizeable deviations with respect to the Standard Model are possible only under special circumstances. Within this scenario we set an upper limit of about $10^{-3}$ for the relative charge asymmetry – integrated for $M_{\ell^+\ell^-} > 2M_\pi$ – of both muon and electron channels. We also show that this limit is close to saturating a model-independent upper bound on the charge asymmetry derived from the present constraints on $\Gamma(K_L \rightarrow \pi^0 \ell^+\ell^-)$.
I. CP violation is one of the most interesting and least known aspects of particle physics. The recent results of KTeV [1] and NA48 [2] about $\varepsilon'/\varepsilon$ have unambiguously shown the existence of CP violation in $|\Delta S| = 1$ transitions (the so-called direct CP violation) and ruled out superweak scenarios. The experimental measurements of $\varepsilon'/\varepsilon$ are generally compatible with the theoretical expectations within the Standard Model (SM) [3]. Nevertheless, the latter are affected by sizeable theoretical uncertainties, and it is difficult to constrain non-standard effects. At the moment the theoretical uncertainties on $\varepsilon'/\varepsilon$ are so large that we cannot even exclude that this observable is completely dominated by new physics (NP) contributions. Given this situation, it is highly desirable to obtain new independent information about CP violation in $|\Delta S| = 1$ transitions.

Among direct-CP-violating observables, particularly interesting are those accessible in rare decays. In these processes the smallness of the SM contribution leads to identifying more easily a possible large NP effect, whereas the simplicity of the hadronic structure helps us to keep under control the theoretical uncertainties [4]. An observable that satisfies these requirements is the charge asymmetry in $K^\pm \to \pi^\pm \ell^+ \ell^-$, defined as

$$\Delta_\ell(s_0) = \frac{\int_{q^2 > s_0} \frac{d\Gamma}{dq^2}(K^+ \to \pi^+ \ell^+ \ell^-) - \frac{d\Gamma}{dq^2}(K^- \to \pi^- \ell^+ \ell^-)}{\int_{q^2 > s_0} \frac{d\Gamma}{dq^2}(K^+ \to \pi^+ \ell^+ \ell^-) + \frac{d\Gamma}{dq^2}(K^- \to \pi^- \ell^+ \ell^-)},$$

where $q^2 = M_{\ell^+ \ell^-}^2$ is the dilepton invariant mass and $\ell = e$ or $\mu$.

This asymmetry is a pure direct-CP-violating observable. A non-zero $\Delta_\ell$ is generated by the interference between the absorptive contribution of the long-distance amplitude and a CP-violating phase of short-distance origin [5]. As we shall discuss below, the kinematical cut on the dilepton invariant mass ($q^2 > s_0$) is a useful tool to maximize the CP-violating effect; indeed, the SM expectations of this asymmetry are given by

$$|\Delta_e(4m_e^2)|_{\text{SM}} \lesssim 10^{-5} \quad \text{and} \quad |\Delta_{e,\mu}(4m_\pi^2)|_{\text{SM}} \lesssim 10^{-4}.$$

The smallness of these figures leads to considering this observable as a good probe of possible NP effects. One of the most promising scenarios where sizeable non-standard CP-violating contributions could be generated is the supersymmetric (SUSY) extension of the SM with generic flavour couplings and minimal particle content. In particular, it has been recognized that in this context gluino-mediated penguin diagrams could naturally account for the observed value of $\varepsilon'/\varepsilon$ [6]. The purpose of this paper is to analyse the consequences that this scenario could have on $\Delta_\ell$. We shall therefore assume that the CP-violating phase of the $s \to d\ell^+\ell^-$ amplitude is entirely dominated by gluino-mediated penguin diagrams or, to be more specific, by the contributions of the dimension-5 chromomagnetic and electromagnetic dipole operators (CMO and EMO). The couplings of these operators, determined by the mismatch between quarks and squarks mass matrices, appear also in $\varepsilon$ and $\varepsilon'$. We will therefore extract the allowed range of these couplings from the measured values of $\varepsilon$ and $\varepsilon'$ in order to analyse the possible effects on $\Delta_\ell$.

Our conclusions are that, in the general case, SUSY effects are at most as large as the SM results in (2) and thus not particularly interesting. Only assuming a cancellation...
between two independent SUSY contributions to $\varepsilon'$, or in a fine-tuned scenario, is it possible to reach higher values. In any case, within the minimal SUSY extension of SM considered here, $\Delta_f$ cannot exceed the $10^{-3}$ level (independently of the $q^2$ cut). We will also show that this bound is very close to a model-independent limit on the charge asymmetry which was extracted, by means of isospin symmetry, from the present constraints on $\Gamma(K_L \rightarrow \pi^0 \ell^+ \ell^-)$.

This paper is organized as follows: we shall first discuss the amplitude decomposition of $K^\pm \rightarrow \pi^\pm \ell^+ \ell^-$ decays and the generic expression of the charge asymmetry; then we will introduce the effective Hamiltonian describing the SUSY short-distance contributions; finally we shall discuss the phenomenological constraints on the SUSY phases and the corresponding bounds for the charge asymmetry.

II. The charge asymmetry is produced by interference between the CP-conserving strong phase and the CP-violating weak phase of the decay amplitude. The latter is generated by the exchange of heavy particles (short-distance), the former is due to the $K \rightarrow 3\pi$ intermediate state and thus belongs to the long-distance part of the amplitude ($K^\pm \rightarrow 3\pi \rightarrow \pi^\pm \ell^+ \ell^-$). As shown in [6], the decay width of $K^\pm \rightarrow \pi^\pm \ell^+ \ell^-$ is largely dominated by long-distance effects and, as long as CP violation is not considered, short-distance contributions can be neglected.

In the long-distance part of the $K^\pm \rightarrow \pi^\pm \ell^+ \ell^-$ amplitude the two leptons are always produced by a virtual photon ($K^\pm \rightarrow \pi \gamma^* \rightarrow \pi^\pm \ell^+ \ell^-$). Since we are interested in the interference between short- and long-distance terms, all short-distance contributions where the two leptons are not in a vector state can be neglected. We can therefore parametrize the decay amplitude in terms of a single vector form factor, $W(z)$, defined as

$$\mathcal{A} \left( K^\pm(k) \rightarrow \pi^\pm(p)\ell^+(l^+)\ell^-(l^-) \right) = -\frac{e^2 G_F}{(4\pi)^2} W(z)(k + p)^{\mu}\bar{u}(l^+)\gamma_{\mu}v(l^-),$$

where $q = k - p$ and $z = q^2/M_K^2$. Integrating this amplitude over the phase space leads to

$$\frac{d\Gamma}{dz} = \alpha^2 G_F^2 M_K^2 \frac{\rho(z)|W(z)|^2}{12\pi(4\pi)^4},$$

where $4r_\ell^2 \leq z \leq (1 - r_\pi)^2$, $r_i = m_i/M_K$, and $\rho(z) = \sqrt{1 - 4r_\ell^2/z(1 + 2r_\ell^2/z)}\lambda^{3/2}(1, z, r_\pi^2)$.

Referring to the discussion in Ref. [3], we decompose the form factor as

$$W(z) = W^{pol}(z) + \frac{1}{G_F M_K^2} W^{\pi\pi}(z).$$

Here $W^{\pi\pi}(z)$ denotes a non-analytic contribution, largely dominated by the dipion intermediate state, as a consequence of the small $q^2$ of the lepton pair, which exhibits a branch cut along the real axis starting at $z = 4r_\ell^2$. The polynomial term $W^{pol}(z)$ includes both long- and short-distance contributions. The latter are completely dominant in the real part of $W^{pol}(z)$, which can be fitted from experiments. Parametrizing the polynomial term as $W^{pol}(z) = G_F M_K^2 (a + bz)$, consistently with a chiral expansion at the next-to-leading order [6], the values of $a$ and $b$ fitted by the BNL E865 Collaboration are reported in Table [4]. On the other hand, the imaginary part of $W^{pol}$ can be produced only by the direct CP-violating weak phase of short-distance origin.
\[
\begin{array}{|c|c|c|c|}
\hline
\text{Re}(a) & \text{Re}(b) & \alpha & \beta \\
\hline
(-0.587 \pm 0.010) & (-0.655 \pm 0.044) & (-20.6 \pm 0.5) \times 10^{-8} & (-2.4 \pm 1.2) \times 10^{-8} \\
\hline
\end{array}
\]

Table 1: Experimental values of the parameters determining \( W_{\text{pol}} \) and \( W_{\pi\pi}(z) \).

The function \( W_{\pi\pi}(z) \) has already been computed in the literature and we report it here for completeness:

\[
W_{\pi\pi}(z) = \frac{1}{r_\pi^2} \left[ \alpha + \beta \frac{z - z_0}{r_\pi^2} \right] \Phi(z) \chi(z),
\]

where the values of \( \alpha \) and \( \beta \) (fitted from \( K \to 3\pi \) decays) are shown in Table I, \( z_0 = 1/3 + r_\pi^2 \), \( \Phi(z) = 1 + z/r_\pi^2 \) \( (r_\pi^2 \simeq 2.5) \) and \( \chi(z) \) is defined by

\[
\chi(z) = \frac{4}{9} - \frac{4}{3} \frac{r_\pi^2}{z} - \frac{1}{3} \left( 1 - \frac{4r_\pi^2}{z} \right) G(z/r_\pi^2),
\]

\[
G(z/r_\pi^2) = \begin{cases} 
\frac{\sqrt{4r_\pi^2/z}}{2} - 1 \arcsin(\sqrt{z/4r_\pi^2}) & z \leq 4r_\pi^2, \\
-\frac{1}{2} \sqrt{1 - 4r_\pi^2/z} \left( \log \frac{1 - \sqrt{1 - 4r_\pi^2/z}}{1 + \sqrt{1 - 4r_\pi^2/z}} + i\pi \right) & z \geq 4r_\pi^2.
\end{cases}
\]

The interference between the CP-violating phase in \( W_{\text{pol}} \) and the absorptive contribution to \( W(z)_{\pi\pi} \) leads to the following difference between the widths of the charge-conjugated modes

\[
\Gamma_+ - \Gamma_- = \frac{\alpha^2 G_F^2 M_K^5}{12\pi(4\pi)^4} \int_{4m_\pi^2}^{(1-\rho)^2} 4\rho(z) \left[ \text{Im}(W(z)_{\text{pol}}) \cdot \text{Im}(W(z)_{\pi\pi}) \right] dz.
\]

Obviously, in order to obtain an adimensional asymmetry we should normalize the width difference to the sum of the widths. However, we stress here the strong dependence of this observable from the dilepton invariant mass: owing to the kinematical threshold in the absorptive part, only when the two-pion intermediate state can be on shell is the asymmetry different from zero. This observation suggests the construction of integrated asymmetry setting a cut on \( q^2 \) above \( 4m_\pi^2 \), i.e. setting \( s_0 = 4m_\pi^2 \) in Eq. (I). Since electron and muon channels have a very similar phase space for \( q^2 \geq 4m_\pi^2 \), with such a cut the normalized asymmetry turns out to be very similar in the two modes. On the other hand, since most of the available phase space for the electron channel is below this cut, from the experimental point of view the muon channel appears a better candidate for the study of this asymmetry.

**III.** As discussed in the previous section, we look for short-distance contributions to the \( s \to d\ell^+\ell^- \) amplitude, with the lepton pair in a vector state and possibly with a sizeable new weak phase. As shown in Ref. [10], in the presence of non-minimal flavour mixing,
the largest SUSY contributions to this type of amplitude are produced by the dimension-
5 CMO and EMO. All the other contributions are in fact naturally suppressed once the
bounds from other processes are taken into account (barring accidental cancellations). The
structure of the effective Hamiltonian necessary to describe these contributions reads
\[ H_{\text{eff}} = C_+^\gamma Q_+^\gamma + C_-^\gamma Q_-^\gamma + C_+^g Q_+^g + C_-^g Q_-^g + h.c. , \]
with the operators expressed in the following basis
\[ Q_+^\gamma = \frac{e Q_+ g}{16 \pi^2} (\bar{s}_L \sigma_{\mu\nu} F_{\mu\nu} d_R \pm \bar{s}_R \sigma_{\mu\nu} F_{\mu\nu} d_L) , \]
\[ Q_+^g = \frac{g}{16 \pi^2} (\bar{s}_L \sigma_{\mu\nu} t^a G_{\mu\nu} d_R \pm \bar{s}_R \sigma_{\mu\nu} t^a G_{\mu\nu} d_L) . \]
Note that in this basis the operators have well defined properties under parity transfor-
mations: \( Q_+^\gamma \) induce parity-conserving transitions and \( Q_-^\gamma \) parity-violating ones.

The Wilson coefficients of these operators induced by gluino exchange are given by [11, 10]
\[ C_+^\gamma (m_{\tilde{\gamma}}) = F(x) \frac{\pi \alpha_s (m_{\tilde{\gamma}})}{m_{\tilde{\gamma}}} \left[ (\delta_{LR})_{21} \pm (\delta_{LR})_{12} \right] , \]
\[ C_+^g (m_{\tilde{\gamma}}) = G(x) \frac{\pi \alpha_s (m_{\tilde{\gamma}})}{m_{\tilde{\gamma}}} \left[ (\delta_{LR})_{21} \pm (\delta_{LR})_{12} \right] . \]
Here \( (\delta_{LR})_{ij} = (M^2)_{ij,LR}/m_{\tilde{g}}^2 \) denotes the off-diagonal entries of the down-type matrix in
the super-CKM basis, and \( x = m_{\tilde{\gamma}}^2/m_{\tilde{g}}^2 \) the ratio of gluino and (average) squark masses
squared. The explicit expression of the loop functions \( F(x) \), \( G(x) \) can be found in [10].
We have also considered the CMO, even if it does not participate at tree level, because of
the large mixing between CMO and EMO induced by QCD interactions at the one-loop
level. Taking into account the 2 \( \times \) 2 anomalous-dimension matrix of these operators [10],
computed at lowest order, the Wilson coefficients evolved down to charm scales reads
\[ C_+^\gamma (m_c) = \eta^2 [C_+^\gamma (m_{\tilde{\gamma}}) + 8(1 - \eta^{-1}) C_+^g (m_{\tilde{\gamma}}) ] , \]
\[ C_+^g (m_c) = C_+^g (m_{\tilde{\gamma}}) , \]
where
\[ \eta = \left( \frac{\alpha_s (m_{\tilde{\gamma}})}{\alpha_s (m_t)} \right)^{\frac{1}{33}} \left( \frac{\alpha_s (m_t)}{\alpha_s (m_b)} \right)^{\frac{1}{33}} \left( \frac{\alpha_s (m_b)}{\alpha_s (m_c)} \right)^{\frac{1}{33}} = 0.89 \left( \frac{\alpha_s (m_{\tilde{\gamma}})}{\alpha_s (500 \text{ GeV})} \right)^{\frac{1}{33}} . \]
Starting from Eq. (15), we find it convenient to rewrite the Wilson coefficient of the EMO as
\[ C_+^g (m_c) = \frac{\pi \alpha_s (m_{\tilde{\gamma}})}{m_{\tilde{\gamma}}} Y (x) \delta^\pm , \]
where
\[ Y(x) = G(x) \eta^2 \left[ \frac{F(x)}{G(x)} + 8(1 - \eta^{-1}) \right] , \]
\[ \delta^\pm = (\delta_{LR})_{sd} \pm (\delta_{LR})_{ds}^* . \]  

In order to identify the SUSY contribution to the polynomial form factor, we need to evaluate the EMO matrix element between \( K^+ \) and \( \pi^+ \) external states:

\[ \langle \pi^+|Q_\pi^+|K^+\rangle = 2i \frac{eQ_d}{16\pi^2 M_K} p_\mu (\pi^+) p_\nu (K^+) F^{\mu\nu}. \]  

As is usually done in the literature, we have expressed the hadronic matrix element in terms of a suitable \( B_T \) parameter \([10, 12]\), expected to be \( \mathcal{O}(1) \), which encodes the non-perturbative dynamics. This parameter has recently been computed on the lattice \([14]\), confirming the estimate \( B_T \approx 1 \) made in Ref. \([12]\). We shall leave it as a parameter through all the paper, but for completeness we report here the recent lattice result

\[ B_T (\mu = 2 \text{ GeV}) = 1.21 \pm 0.09 \pm 0.04^{+0.07}_{-0.00} \]  

Now using Eqs. \((18)-(21)\) we can write the full matrix element of the Hamiltonian \((10)\) relevant to \( K^\pm \to \pi^+ \ell^+ \ell^- \) decays

\[ \langle \pi^\pm \ell^+ \ell^- | \mathcal{H}_{\text{eff}} | K^\pm \rangle = \alpha \alpha_s \frac{Q_d}{4} \left[ \frac{B_T}{M_K m_\tilde{g}} Y(x) \delta^+ \right] \times (k + p)^\mu \bar{u}(l^+) \gamma_\mu v(l^-). \]  

Then, according to the definition of \( W(z) \) in Eq. \((3)\), we obtain:

\[ \text{Im}(W_{\text{SUSY}}^{\text{pol}}) = \pi \alpha_s(m_\tilde{g}) \frac{Q_d}{G_F M_K} \frac{B_T}{m} \frac{Y(x)}{m} \text{Im}(\delta^+) , \]  

\[ = 15.2 \times \left( \frac{Y(x)}{Y_0(1)} \right) \left( \frac{500 \text{ GeV}}{m} \right) \left( \frac{\alpha_s(m_\tilde{g})}{\alpha_s(500 \text{ GeV})} \right) B_T \text{Im}(\delta^+) , \]  

where \( Y_0(1) = Y(x = 1; m_\tilde{g} = 500 \text{ GeV}) = 0.39 \). Inserting this result in Eq. \((3)\) we can finally write:

\[ |\Delta_s(4m_\pi^2)| = (1.0 \pm 0.1) \left[ \left( \frac{Y(x)}{Y_0(1)} \right) \left( \frac{500 \text{ GeV}}{m} \right) \left( \frac{\alpha_s(m_\tilde{g})}{\alpha_s(500 \text{ GeV})} \right) \right] \times B_T |\text{Im}(\delta^+)| , \]  

where the error includes the uncertainty in the experimental parameters.

**IV.** Before starting a numerical analysis of the SUSY contribution to the charge asymmetry, we discuss here a more general upper bound on \( |\text{Im}(W^{\text{pol}})| \) (and thus on the charge asymmetry), which can be extracted from the experimental upper bound on \( \Gamma(K_L \to \pi^\pm \ell^+ \ell^-) \).

Isospin symmetry relates in a model-independent way the short-distance components of \( K^\pm \to \pi^\pm \ell^+ \ell^- \) and \( K_L \to \pi^\pm \ell^+ \ell^- \) amplitudes. Indeed, in both cases, the hadronic current is necessarily a \( \Delta I = 1/2 \) operator of the type \( s\bar{d} \) (or \( d\bar{s} \)). In the \( K_L \) case, the approximate CP-odd combination of \( K^0 \) and \( \bar{K}^0 \) states select the imaginary part of this amplitude, which can therefore be constrained by using the experimental bounds on \( \Gamma(K_L \to \pi^\pm \ell^+ \ell^-) \). Assuming that long-distance contributions to \( K_L \to \pi^\pm \ell^+ \ell^- \) are
In general there are two classes of bounds: those coming from \(|\varepsilon|\) and \(\text{BR}(K_L \to \pi^0 e^+ e^-)\) (direct bounds) and those extracted from \(\text{Re}(\varepsilon'/\varepsilon)\) (indirect bounds). In the first case \(\text{Im}(\delta^+)\) is directly involved, whereas in the second case – dealing with parity-violating transitions – only \(\text{Im}(\delta^-)\) is directly involved. As clearly shown by Eq. (28), \(\delta^+\) and \(\delta^-\) are naturally related: if we assume these two quantities to be of the same order of magnitude, then the measurement of \(\text{Re}(\varepsilon'/\varepsilon)\) leads to a bound on \(|\text{Im}(\delta^+)|\) of

\[
|\Delta_e(4m^2_e)| \leq 43 \times \sqrt{\text{BR}(K_L \to \pi^0 e^+ e^-)} \leq 1.0 \times 10^{-3},
\]

where \(\varepsilon^\prime\) is the mean lifetime of the \(K_L\) and the integral on \(\rho(z)\) extends to the whole phase space. Using this relation we obtain the following model-independent upper limits on the charge asymmetries of electron and muon modes:

\[
\begin{align*}
|\Delta_e(4m^2_e)| & \leq 43 \times \sqrt{\text{BR}(K_L \to \pi^0 e^+ e^-)} \leq 1.0 \times 10^{-3}, \\
|\Delta_\mu(4m^2_\mu)| & \leq 82 \times \sqrt{\text{BR}(K_L \to \pi^0 \mu^+ \mu^-)} \leq 1.6 \times 10^{-3},
\end{align*}
\]

where the numerical values have been obtained using the experimental bounds \(\text{BR}(K_L \to \pi^0 e^+ e^-) < 5.8 \times 10^{-10}\) and \(\text{BR}(K_L \to \pi^0 \mu^+ \mu^-) < 3.8 \times 10^{-10}\) obtained by the KTeV Collaboration.

In Table 2 we compare the model-independent upper bounds for the charge asymmetries with SM and MSSM expectations, expressed in terms of the imaginary parts of the CKM factor \(\lambda_i = V_{ts}^* V_{td}\) and the SUSY parameter \(\delta^+\), respectively. Since \(\text{Im}(\lambda_i) \approx 10^{-4}\), it is clear that the SM could only account for a few per cent of the general bounds. From this perspective it is particularly interesting to understand how large the SUSY contribution could be.

The bounds on \(\text{Im}(\delta^+)\) [and more in general on the \(\text{Im}(\delta_{LR})\)’s] have been widely discussed in the recent literature (for extensive discussions, see Ref. [15] and references therein). In general there are two classes of bounds: those coming from \(|\varepsilon|\) and \(\text{BR}(K_L \to \pi^0 e^+ e^-)\) (direct bounds) and those extracted from \(\text{Re}(\varepsilon'/\varepsilon)\) (indirect bounds). In the first case \(\text{Im}(\delta^+)\) is directly involved, whereas in the second case – dealing with parity-violating transitions – only \(\text{Im}(\delta^-)\) is directly involved. As clearly shown by Eq. (28), \(\delta^+\) and \(\delta^-\) are naturally related: if we assume these two quantities to be of the same order of magnitude, then the measurement of \(\text{Re}(\varepsilon'/\varepsilon)\) leads to a bound on \(|\text{Im}(\delta^+)|\) of

\[
\begin{array}{|c|c|c|}
\hline
\text{ } & \text{SM} & \text{SUSY} \\
\hline
|\Delta_e(4m^2_e)| & 0.07 \times |\text{Im}(\lambda_e)| & 0.13 \times |\text{Im}(\delta^+)| \\
|\Delta_\mu(4m^2_\mu)| & 0.53 \times |\text{Im}(\lambda_\mu)| & 1.0 \times |\text{Im}(\delta^+)| \\
\hline
\end{array}
\]

Table 2: Summary of the results for both the electron and muon channel asymmetry, evaluated in the SM [2] and in the MSSM, with and without the kinematical cut on the dilepton square mass.
This observable could be particularly useful in an experimental set up where it is difficult to obtain a precise flux normalization and thus precise width measurements. In Figure 1 we plot $\delta \Delta_{\ell}(z)$, normalized to the SUSY phase $\text{Im}(\delta^+)$, as a function of $z$.

Before concluding, we point out that an observable particularly useful from the experimental point is the differential asymmetry, defined by:

$$
\delta \Delta_{\ell}(z) = \frac{d\Gamma_{\ell}^+(z)/dz - d\Gamma_{\ell}^-(z)/dz}{d\Gamma_{\ell}^+(z)/dz + d\Gamma_{\ell}^-(z)/dz}.
$$

This observable could be particularly useful in an experimental set up where it is difficult to obtain a precise flux normalization and thus precise width measurements. In Figure 1 we plot $\delta \Delta_{\ell}(z)$, normalized to the SUSY phase $\text{Im}(\delta^+)$, as a function of $z$.

V. In this letter we have analysed the charge asymmetry in $K^\pm \rightarrow \pi^\pm \ell^+ \ell^-$ decays in the framework of supersymmetric models with minimal particle content and generic flavour couplings. We have shown that in general the expectations are of the same order of magnitude as in the SM case. Nevertheless, under special circumstances it is possible to relax the indirect constraints on the SUSY CP-violating phases and obtain results in the
range \(10^{-4} < \Delta_\ell(4m_\pi^2) \lesssim 10^{-3}\), or above the SM expectation. We have also shown that under general assumptions \(\Delta_\ell(4m_\pi^2)\) cannot exceed the \(10^{-3}\) level. This model-independent bound is derived, using isospin invariance, from the experimental constraints on \(\Gamma(K_L \to \pi^0 \ell^+\ell^-)\).

The first measurement of the charge asymmetry in the muon mode has recently been announced by the HyperCP collaboration \([16]\):

\[
\Delta_\mu(4m^2_\mu) = -0.02 \pm 0.11 \text{ (stat)} \pm 0.04 \text{ (syst)}. \tag{31}
\]

Unfortunately at the moment the sensitivity is very far from the interesting region, still well above the model-independent bound. In view of possible improvements of this measure, we can summarize as follows the possible outcome of a non-vanishing result:

- \(\Delta_\ell(4m_\pi^2) \leq 10^{-4}\): we could conclude that the charged kaon system is well described by the SM. Moreover, we could put a new strong constraint on the SUSY phase \(\text{Im}(\delta^+)\).

- \(10^{-4} < \Delta_\ell(4m_\pi^2) \lesssim 10^{-3}\): we would have a clear signal of new physics, compatible with the results obtained within the Minimal Supersymmetric Model with generic flavour couplings.

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