The use of the wxMaxima linear algebra module on Gauss elimination lesson for mathematics education students

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Abstract. Students’ difficulties in solving problems in Linear Algebra course could be assisted by the use of wxMaxima Linear Algebra module. This paper focuses on the learning on Gauss elimination topic using the wxMaxima Linear Algebra module. The wxMaxima Linear Algebra module has been developed using the ADDIE development model. The participants were 18 mathematics education students of UIN Ar-Raniry Banda Aceh. This study used the lecturer’s self-reflection sheet to see the relevance of the lesson plan and the learning implementation. The learning process implemented direct instruction model and the time allocated for the class session was 150 minutes. The learning activities began with the explanation about elementary row operations (ERO) and Gauss elimination (GE) by the lecturer and followed by solving problems manually solving using the paper-pencil method. Furthermore, the solutions obtained manually were checked using the wxMaxima software. Learning activities were then continued to the concept of Gauss Jordan elimination method. Although the students spent much time to solve the Gauss elimination problems manually, overall the learning process went well as the time allocated in the lesson plan. The findings suggest that the wxMaxima Linear Algebra module could be used for other linear algebra topics.

1. Introduction
Mathematics education department of UIN Ar-Raniry utilizes Indonesian National Qualification Framework-based curriculum. One of the objectives is to educate students as prospective teachers in mastering technology that will be implemented in secondary schools. Students are also required to master mathematics concept during their study in university. One of the courses that must be undertaken by students in mathematics education department is linear algebra.

One of the topics required in a linear algebra class is Gauss elimination (GE). By learning GE, the students are expected to be able to change the enlarged matrix into row echelon form. After that, the students were expected to understand Gauss-Jordan elimination to determine the solution of the linear equation system. However, some students still made mistakes in solving problems related to Gauss-Jordan elimination. Their mistakes generally involved calculation. This is possibly caused by the lack of accuracy in calculating elementary row operations and the limited time to understand the lesson [1]. Some students also had difficulties in solving problems with very large linear equation system since it requires much time to solve. Therefore, it needs a media to facilitate students in solving such problems. Calculation of very large and complex systems require computers to make it easier [2]. This is relevant with the learning objectives expected by the university. That is, the learning should give students the opportunity to learn and use information and communication technology (ICT).
Specifically, the learning should focus students’ attention on how to use technology in mathematics appropriately, and provide opportunities for students to apply the technology in mathematics learning [3].

Using software in mathematics learning can improve students' understanding and performance in mathematics [4,5]. One of software that can be used for linear algebra is wxMaxima software. wxMaxima is a computer algebra system (CAS) that combines graphics, symbols, and numerical capabilities. Professor Schelter was an expert on wxMaxima software [6]. The wxMaxima is developed by the MACSYMA system in the Massachusetts Institute of Technology. There is other CAS software such as Mathcad, Mathematica, Maple, and Smath Studio are also useful for linear algebra. However, wxMaxima is free and easily accessible for students; thus, wxMaxima is utilized in this study.

The use of wxMaxima to solve linear algebra problems can help students build their own knowledge and develop the desired competencies [7]. Therefore, this article aims to explain the learning process using Linear Algebra module with wxMaxima software on Gauss elimination topic.

2. Method

Linear Algebra module with wxMaxima software has been developed using ADDIE development model. The ADDIE model consists of five development stages: analysis, design, development, implementation/execution, and evaluation [8]. This study is part of the implementation phase of the first meeting on GE in a large group to describe the learning process on GE. The participants were 18 mathematics education students. The research data is taken from the lecturer's self-reflection sheet to see the relevance between the lesson plan and the implementation. Data were analyzed descriptively about the learning process on GE. The learning activities began with the explanation about elementary row operations (ERO) and GE by the lecturer and followed by solving the problem manually using the paper-pencil method. Then, the solutions obtained manually were checked using the wxMaxima software.

3. Results and discussion

The purpose of this learning process was that students could solve problems related to elementary row operations (ERO), solutions of homogeneous linear equation system, and the concept of Gauss Jordan elimination method. GE lesson using wxMaxima Linear Algebra module began with reviewing the previous lesson about ERO, then followed by explaining the GE steps to students and completing problems related to GE.

Students were assigned several problems on the wxMaxima Linear Algebra module to be solved manually. The assigned problems on wxMaxima Linear Algebra module were adopted from Linear Algebra Elementary book on page 16 [2]. One of the problems assigned to students is presented in Figure 1.

| Complete the matrix of the following system of equations to be a row echelon matrix using GE. |
| 3x1 + 2x2 - 2x3 + 2x5 = 0 |
| 2x1 + 6x2 - 5x3 - 2x4 + 4x5 - 2x6 = -1 |
| 5x3 + 10x4 + 15x6 = 5 |
| 2x1 + 6x2 + 8x4 + 4x5 + 18x6 = 6 |

**Figure 1.** One of the problems assigned to students.

The students’ answers to the problem were varied. Some students answered correctly with the final answer as shown in Figure 2.
The answer in Figure 2 indicates that the students had understood the concepts and procedures of the GE. The first procedure is that if a row does not consist entirely of zeros, then the first non zero number in the row is a 1, called a leading 1. The second procedure is that if there are any rows that consist entirely of zeros, then the rows are grouped together at the bottom of the matrix. The third procedure is that in any two successive rows that do not consist entirely of zeros, the leading 1 in the lower row occurs farther to the right than the leading 1 in the higher row [2]. However, some students answered the problem incorrectly as shown in Figure 3.

Figure 3. The students’ incorrect answer in term of miscalculation.
Figure 3 shows that the students answered incorrectly in determining the multiplication result of the matrix element. The students repeated the mistake in the next step. As a result, the final solution obtained was also wrong. Based on the students’ answer in Figure 3, it shows that the students made a mistake out of the GE procedure. The error was in determining the result of GE operation caused by lack of accuracy in calculation. The students were not aware of the mistake they made because the solution is lengthy. The same mistake was also experienced by other students, as shown in Figure 4.

Figure 4. Another students’ wrong answer in term of miscalculation.
In addition to the errors in the calculation of the matrix elements, there were also some students who had a mistake in the concept when solving the given problem. The answers given by the students were not based on the GE rules. The students' answer is shown in Figure 5.

Figure 5. The student's wrong answer in term of misconception.

Figure 5 shows that the student experienced an error concept in the second GE procedure, namely: If there are any rows that consist entirely of zeros, then they are grouped together at the bottom of the matrix. The students should add the next step of exchanging line III with line IV.

When asked to determine the correct answer, all those students could not find it because they assumed that their answers were correct already. Therefore, the students needed to check their answers that had been obtained manually by using the wxMaxima software. Checking the calculation steps by using wxMaxima software was provided in the module as well. Utilizing the software, the students got a correct answer as shown in Figure 6.
Figure 6. The answer to the problem displayed on the wxMaxima software.

Students who answered differently from the answer displayed on the wxMaxima software were required to double check their initial answer. At this stage of learning, the students could find out the mistakes they made in their calculation. The same learning procedures were carried out to Gauss Jordan elimination lesson. The Gauss Jordan elimination is a further step of the GE to obtain a reduced row echelon matrix. Therefore, it is essential for students to check the GE steps to improve the answers obtained before they proceed to the next step.

The learning process in the Linear Algebra course especially on GE lesson run well. The students did the exercises and seemed enthusiastic to check their solution using the wxMaxima software. Learning steps in Linear Algebra course especially GE lesson using wxMaxima software could be implemented in accordance with the activities on the lesson plan. The time allocated was also well executed in the learning process.

Based on the learning implementation, it shows that the use of wxMaxima software could help the students recognize their wrong calculation and misconception. This finding is aligned with the results of a previous study indicating that the use of mathematical software has created a major influence on students' understanding of mathematics and their subsequent performance in learning [9].

During the learning process, the students looked enthusiastic in completing the given task and double-checking their answers using the wxMaxima software. This finding is supported by previous research that found that students were active in learning with the use of the software. The use of wxMaxima Linear Algebra module could improve student engagement and provide positive effects to students during the learning process [10].

The module also provided assistance for students to follow the learning process. Students could understand the lesson from the module and check their solution based on the solutions presented in the module without waiting for the assistance from the lecturer. Also, the use of the module in learning could attract students' interests, made students more active in learning, and helped students improve their learning achievement [11].

4. Conclusion

Learning linear algebra especially GE topic by using wxMaxima software could be implemented in the classroom. The learning activities run well as the activities and the allocated time in the lesson plan. The use of wxMaxima software in the learning process also could help students find and notice the wrong calculations and concepts. This is valuable for the students to learn from their mistakes.
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