Nonlinear dynamics of non-equilibrium holes in p-type modulation-doped GaInNAs/GaAs quantum wells

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Abstract
Nonlinear charge transport parallel to the layers of p-modulation-doped GaInNAs/GaAs quantum wells (QWs) is studied both theoretically and experimentally. Experimental results show that at low temperature, T = 13 K, the presence of an applied electric field of about 6 kV/cm leads to the heating of the high mobility holes in the GaInNAs QWs, and their real-space transfer (RST) into the low-mobility GaAs barriers. This results in a negative differential mobility and self-generated oscillatory instabilities in the RST regime. We developed an analytical model based upon the coupled nonlinear dynamics of the real-space hole transfer and of the interface potential barrier controlled by space-charge in the doped GaAs layer. Our simulation results predict dc bias-dependent self-generated current oscillations with frequencies in the high microwave range.

Introduction
During the past decade, dilute nitrides, particularly the quaternary material system of GaInNAs/GaAs, have attracted a great deal of attention, both because of unusual physical properties and potential applications for a variety of optoelectronic devices. The addition of a small amount of nitrogen induces a strong perturbation in the conduction band of matrix semiconductors, while having a negligible effect on the valence band. As a result, the electron mobility is greatly lowered and the hole mobility can become higher than the electron mobility, in materials with relatively high nitrogen content. High hole mobility coupled with the low hole confinement energy (110 meV in our calculation for the samples investigated in this study) [1] in the GaInNAs/GaAs quantum well (QW) structure makes it possible for holes in the well to gain enough energy to overcome the small band discontinuity under an electric field applied parallel to the layer interface, and to transfer into the low-mobility p-doped GaAs layer. This leads to a negative differential mobility (NDM) caused by real-space hot hole transfer, as we previously observed [1]. Therefore, under dc conditions, a self-generated current oscillation in the real-space regime, as proposed by Schöll and co-authors [2-5], is expected in p-modulation-doped GaInNAs/GaAs heterostructures.

In this work, we study the nonlinear charge transport in a modulation-doped GaInNAs/GaAs semiconductor heterostructure where the GaAs barrier layer is intentionally p-doped. The charge transport processes perpendicular and parallel to the layers far from thermodynamic equilibrium are modeled by several coupled nonlinear dynamics equations. In this model, self-generated current oscillations can be described in the following way. Real-space transfer (RST) of holes out of the GaInNAs well layer leads to an increase of the hole density in the GaAs barrier, which diminishes the negative space charge that controls the band bending (Figure 1). Consequently, the potential barrier ΦB decreases, with some delay due to the finite dielectric relaxation time. This leads to an increased thermionic emission backward current J_b-w into the GaInNAs well, which decreases the hole density in the GaAs barrier. As a result, the space charge and ΦB are increased in the GaAs. This, in turn, decreases the thermionic emission backward current from the well into the barrier [6].

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Negative differential resistance instabilities in p-modulation-doped GaInNAs/GaAs QWs

The layer structure of the sample used in this study is given in Table 1. The sample, which was grown by molecular beam epitaxy (MBE) on semi-insulating GaAs substrate, consists of three 7 nm thick GaInNAs QWs, separated by 20 nm thick Be-doped GaAs barriers. These p-type-doped barriers are separated from the QWs by 5 nm undoped spacer layers to reduce the remote impurity scattering. The mole fraction of indium and nitrogen in the Ga1-xInxNyAs1-y QWs is x = 0.3 and y = 0.015, respectively. The sample was fabricated in the shape of a simple bar for I-V measurement. Fabrication details are given somewhere else [1].

The nonlinear transport processes depicted in Figure 1 are modeled by a set of dynamic equations relevant to current instabilities in semiconductors. We derive a set of nonlinear partial differential equations for the hole density in the well and in the barriers, the potential barrier in each of GaAs layers, and the dielectric relaxation of the applied parallel field (ξII).

The dynamics of the carrier density in the well and in the barrier are given by [5]

\[ \frac{\partial p_w}{\partial t} = \frac{1}{qL_w} \left( I_{w-b} - I_{b-w} \right) \]  

(1)

Table 1 Numerical parameters used in the simulation for the GaInNAs/GaAs sample [7]

| Material     | Thickness (Å) | Doping (m⁻³) |
|--------------|---------------|--------------|
| GaAs (cap)   | 500           | Be: 1 × 10²⁴ | x3 |
| GaAs (barrier) | 200           | Be: 1 × 10²⁴ | x3 |
| GaAs (spacier) | 50            | UD           | x3 |
| Ga1-xInxNyAs1-y, QW | 70            | UD           | x3 |
| GaAs (spacier) | 50            | UD           | x3 |
| GaAs (barrier) | 200           | Be: 1×10²⁴   | x3 |
| GaAs (buffer)  | 500           | UD           | x3 |

Semi-insulating GaAs substrate

where \( I_{w-b} \) and \( I_{b-w} \) are the thermionic currents flowing from the GaInNAs well layers to the GaAs barrier and from the barrier into the well, respectively, \( q \) is the positive electron charge, and \( L_w \) and \( L_b \) are the width of the GaInNAs QW and the GaAs barrier, respectively. The electric field parallel to the layer interface \( \xi_{II} \) can be derived from Poisson’s equation and is given by

\[ \varepsilon_0 \varepsilon_s \frac{\partial \xi_{II}}{\partial y} = \frac{q}{L_w + L_b} \left( \left( N_A - p_b \right) L_b + p_w L_w \right) \]  

(3)

where \( \varepsilon_0 \) and \( \varepsilon_s \) are the absolute and relative permittivity, respectively. Using Equations (1)-(3), the dielectric relaxation of the applied parallel field \( \xi_{II} \) can be written as

\[ \begin{align*}
\varepsilon_0 \varepsilon_s \frac{\partial \xi_{II}}{\partial y} &= \frac{q}{L_w + L_b} \left[ \frac{1}{qL_b} \left( I_{b-w} - I_{w-b} \right) + \mu_w \frac{\partial (p_w\xi_{II})}{\partial x} \right] L_b + \\
&= \frac{1}{qL_w} \left( I_{w-b} - I_{b-w} \right) + \mu_w \frac{\partial (p_w\xi_{II})}{\partial x} L_w
\end{align*} \]  

(4)

\[ \begin{align*}
\varepsilon_0 \varepsilon_s \frac{\partial \xi_{II}}{\partial t} - \frac{1}{L_w + L_b} \left[ \frac{\partial}{\partial x} \right] \left[ \frac{1}{qL_b} \left( \xi_{II} - q_p \mu_b \frac{\partial \xi_{II}}{\partial x} \right) + \frac{q_p \mu_b L_b}{\partial x} \right] \\
&= \sigma_L \left( \xi - \xi_{II} \right) - \frac{1}{L_w + L_b} \left[ \frac{\partial}{\partial x} \right] \left[ \frac{q_p \mu_b L_b \xi_{II}}{\partial x} \right] + \frac{q_p \mu_b L_b \xi_{II}}{\partial x}
\end{align*} \]  

(5)

Where \( \xi_0 = U_0/d \) is the applied field and \( U_0 \) is the applied voltage, \( \sigma_L = d/\hbar (L_w + L_b) q \mu_w R_{N_A} \) is connected to the load resistance \( R_L \), \( d \) is the sample length, \( \hbar \) is the width of the sample, \( \mu_w \) and \( \mu_b \) are the hole mobility in the QW and the GaAs barrier, respectively. By integrating both sides of Equation (5), we finally have the dielectric relation of the parallel electric field

\[ \begin{align*}
\frac{\partial \xi_{II}}{\partial t} &= J_{II} - \frac{1}{L_w + L_b} \left( q p_w \mu_w L_w + q p_b \mu_b L_b \right) \xi_{II}
\end{align*} \]  

(6)

where \( J_{II} \) is the external current density flowing through the external circuit at applied bias voltage \( U_b \). Here, we define the current density flowing through the sample as a function of applied parallel field, using

\[ J_{II} = \frac{1}{L_w + L_b} \left( q p_w \mu_w L_w + q p_b \mu_b L_b \right) \xi_{II} \]  

(7)
The time-dependent potential barrier in the GaAs layer is given by

\[
\frac{\partial \Phi_B}{\partial t} = \frac{q^2}{\varepsilon_0 \varepsilon_s} \left[ -\frac{\mu_B N_A}{q} \Phi_B + \frac{q \mu_B L_b^2}{2 \varepsilon_0 \varepsilon_s} (N_A - p_b)^2 \right]
\]

Equations (1) and (2) represent particle continuity, where the thermionic current densities \(J_{b-w}\) and \(J_{w-b}\) can be calculated using Bethe’s theory, by assuming that the width of the space charge is comparable to the mean free path \(L_m\) of the holes [4,5]

\[
J_{b-w} = -q \mu_w \left( \frac{K_B T_w}{2 \pi m_w^*} \right)^{1/2} \exp \left( -\frac{\Delta E_v}{K_B T_w} \right)
\]

\[
J_{w-b} = -q \mu_b \left( \frac{K_B T_b}{2 \pi m_b^*} \right)^{1/2} \exp \left( -\frac{\Phi_b}{K_B T_b} \right)
\]

where \(m_w^*\) and \(m_b^*\) are the hole effective mass in the GaInNAs QW and GaAs barrier, respectively, and the hole temperatures in the well and barrier are approximately given by

\[
T_w = T_L + \tau_{E_w} q \mu_w \xi_{II}^2 \quad T_b = T_L + \tau_{E_b} q \mu_b \xi_{II}^2
\]

Since the number of holes in the well and the barrier are related to each other, their total number is conserved [1]:

\[
\rho_0 L_w = \rho_u L_{w-b} + \rho_b L_{b-w}
\]

where \(\rho_0\) is the 3D hole density in the well at low field.

**Numerical results**

The steady-state can be evaluated by setting Equations (1), (2), (6), and (8) to zero, and using the parameters listed in Table 2. The resulting static current density characteristic as a function of the static electric field is shown in Figure 2. The measured \(I-V\) curve obtained with the same sample in our previous study is placed in the figure inset for comparison [1]. Simulation results predict that the RST of hot holes leads to an N-shaped characteristic with a regime of negative differential resistance [4,7,8]. The critical field for the onset of NDM is the order of 6 kV/cm, which agrees well with our experimental results.

The time-dependent nonlinear Equations (1), (2), (6), and (8) have been numerically resolved using Euler’s methods. The simulation reveals that the instability of the dynamic system is strongly dependent on the applied dc bias field, \(\xi_{II} = U_0/d\). We found that self-generated nonlinear oscillation appears in a range of applied dc electric fields where the load line lies in the NDM regime, as shown in Figure 3a. Figure 3b shows the corresponding current-density oscillations with frequency of 44 GHz, for \(\xi_{II} = 10.1 \text{ kV/cm}\) and \(N_A = 2.2 \times 10^{16} \text{ cm}^{-3}\).

It is interesting to find that the oscillation frequency is strongly dependent on the dopant concentration in the barrier and the barrier thickness, as shown in Figure 4. The oscillation frequency increases from 29 to 50 GHz as the dopant concentration in the barrier increases from \(1.9 \times 10^{16} \text{ cm}^{-3}\) to \(2.4 \times 10^{16} \text{ cm}^{-3}\), accompanied by gradually reduced oscillation amplitude. Finally, the periodic oscillation damps out when the dopant concentration is above \(2.4 \times 10^{16} \text{ cm}^{-3}\), as shown in Figure 5. The oscillation shows similar behavior as the barrier thickness increases. The fact that the self-generated oscillation frequency can be tuned by the doping concentration and the layer width can be explained by the nonlinear combination of the effective thermionic

| Table 2 Numerical parameters used in the simulation for the GaInNAs/GaAs sample [1] |
|-----------------|-------------------|
| \(L_w\) | 7 mm |
| \(L_b\) | 25 mm |
| \(\Delta E_v\) | 0.12 eV |
| \(m_w^*\) | 0.105 \(m_0\) |
| \(m_b^*\) | 0.62 \(m_0\) |
| \(d\) | 50 \(\mu m\) |
| \(h\) | 28 \(\mu m\) |
| \(\tau_{E_w}\) | 0.2 ps |
| \(\tau_{E_b}\) | 0.1 ps |
| \(T_L\) | 13 K |
| \(\mu_w\) | 0.3 m²/Vs |
| \(\mu_b\) | 0.021 m²/Vs |

Figure 2 Static current density-field characteristic as a function of the static electric field \(\xi_{II}\). The measured \(I-V\) characteristic of p-modulation-doped sample is shown in the inset.
Figure 3 (a) Static current density versus electric field curve. The load line (straight line) lies within the NDM area to determine the applied dc field. (b) Time-dependent current density curve, with $N_A = 2.2 \times 10^{16} \text{cm}^{-3}$.

Figure 4 Oscillation frequency as a function of (a) barrier thickness and (b) doping concentration in the GaAs barrier for $\zeta_0 = 24 \text{kV/cm}$.

Figure 5 Periodic oscillation damping with $N_A = 2.4 \times 10^{16} \text{cm}^{-3}$. 
emission time, \( \tau_0 = e^{3/2} L_w \sqrt{3 \pi m^*_w / \Delta E_v} \) and the dielectric relaxation time, \( \tau_r = \sqrt{\varepsilon_0 \mu_e N_d} \) as suggested by Döttling and Schöll [9]. The hysteretic switching transitions between the stable stationary state and the periodic oscillation in a uniform dynamic system depend on the ratio of the effective thermionic emission time and the dielectric relaxation time, \( \gamma \). In our case, \( \tau_0 = 0.21 \text{ps} \), the change in dopant concentration from \( 1.8 \times 10^{16} \text{ cm}^{-3} \) to \( 2.5 \times 10^{16} \text{ cm}^{-3} \) leads to \( \gamma \) increases from 0.076 to 0.12 resulting in phase transition in dynamic system.

**Conclusion**

In this work, we studied the transport processes parallel and perpendicular to the layers of \textit{p}-type modulation-doped GaInNAs/GaAs multi-QW structures far from the thermodynamic equilibrium. The simulation results of the steady-state predict an NDM induced by RST of hot holes in the QWS and the critical electric field of the onset of NDM to be the order of 6 kV/cm. This value agrees well with our previous experimental results. The numerically time-dependent simulations indicate that the self-generated oscillation caused by RST with the frequency in the range 20-50 GHz appears under the right applied electric field. The frequency of self-generated oscillation can be flexibly optimized to the range of considerable interest for applications as a simple way of generating high-frequency microwave power based on GaInNAs material system. According to our simulation, the predicted self-generated oscillation can be observed if the GaInNAs QW structure is optimized around 25 nm barrier and less than \( 2.4 \times 10^{16} \text{ cm}^{-3} \) doping concentration. The current oscillation measurements will be performed using optimized structures fabricated into two terminal devices, and shunted with a 50 \( \Omega \) resistor and high-speed circuit (high-speed oscilloscope and pulse generator). The experiment results are expected to be published in the near future.

**Abbreviations**

NDM: negative differential mobility; QWs: quantum wells; RST: real-space transfer.

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**Authors’ contributions**

HMK carried out the theoretical calculations, in collaboration with AA. MS grew the sample according to the specifications. YS fabricated the devices, carried out the experiments. HMK and YS wrote up the article. NB, is the supervisor of the project. All authors read and approved the final manuscript.

**Competing interests**

The authors declare that they have no competing interests.

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