A Fuzzy Semantic for BDI Logic

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\textbf{ABSTRACT}

The BDI logic is an important and widely used theoretical apparatus to represent and reason about rational agents. However, the BDI logics are incomplete regarding the intention reconsideration, override of intention, the deliberation process, and belief revision. These are essential processes of the BDI model. Also, some rational agents, especially human being, have not the approximate reasoning well represented by the BDI logics. So, in this paper, we define fuzzy semantics for a BDI language that is capable to eliminate those limitations. Additionally, we show how this paper is related to current works about BDI agents and discuss how those limitations can be fixed through the extension of the BDI logic to a fuzzy BDI logic.

\textbf{KEYWORDS}

Agents; fuzzy logic; BDI logic; LoRA

1. Introduction

An agent is an entity which interacts with the resources from the environment or with other agents. The literature of multiagent systems usually classify the agents as reactive, rational (also known as cognitive, or intelligent), or hybrid agents. In this paper we focus on the rational one. The rational agents are able to acts in a deliberate manner (practical reasoning). How it can do this is a question, whose a widely used response is Belief-Desire-Intention (BDI for short) model (see [1,2]).

According to the BDI model, the practical reasoning is given by two steps: the deliberation process where the agent commits itself with an intention, from its own beliefs, desires and a previous intention; and the means-ends reasoning where the agent build a plan or recipe, i.e. a set of states to be achieved sequentially, to read the chosen intention.

Second [3], three points contribute to the success of the BDI model: it has a philosophical foundation [1]; some BDI logics formalisations were defined [4–7]\textsuperscript{1}; and some implementations use the BDI approach – as examples of implementations or computational tools which have conceptual roots in the BDI model, see [9–13].

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\textsuperscript{1} [4] is based on [8] and [1]. While [5–7] are based only on [1].

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The formalisation is the best way to understand and detail the processes of the BDI model. Moreover, the formalisation assures that an implementation of a BDI reasoning agree with the philosophy behind the model. The best-known BDI logics (based only on Bratman’s Theory) are BDI Logic by Rao and Georgeff – firstly described in [5] and revisited in [6] – and LoRA (Logic of Rational Agents) by Wooldridge [7]. Those logics are incomplete regarding the deliberation and the reconsideration of beliefs and intentions (see [6, section 7]). One way to bypass such limitations is to consider the uncertainties of the agents at the moment they express their beliefs, desires and intentions. So they can deliberate or reconsider based on the uncertainty degree of their beliefs, desires and intentions, by choosing what they are more confident of.

On the other hand, the classical formal semantics is founded under the principles of bivalence, i.e. sentences are either true or false, and truth functionality which means that the truth value of logically complex sentences are given in function of the truth values of their subsentences [14]. Historically, problems that arise from these two notions when dealing with some incomparable information have motivated to discard the principle of bivalence and consider three values or, more generically, many truth values [15]. The set of these truth values can be finite as in [16] or infinite as proposed by Lotfi Zadeh when introduced the Fuzzy logic in [17]. Fuzzy logics model the uncertainty, vagueness, and ambiguity present in the real world by mapping sentences to truth degrees of the real interval [0; 1] and by the basis for the approximate reasoning, i.e. methods and methodologies to reasoning with imprecise inputs to obtain meaningful outputs. Inference in approximate reasoning has strong and important differences with the inference in classical logic. Indeed, in the former, the consequences of a set of fuzzy propositions depend on the underlying meaning of such fuzzy propositions. Thus, inference in approximate reasoning is a computation with the possible fuzzy sets which give meaning to the set of fuzzy propositions [18]. Thereby, one way to overcome the limitations of BDI is to fuzzify the BDI logic, i.e. consider a fuzzy semantics for it. Also contributed to the employment of the Approximate Reasoning in the BDI model. The approximate reasoning becomes the BDI model more faithful to the human being reasoning representation and maintains the model capable of reasoning about simpler rational agency.

According to this new fuzzy BDI model, the agent has an initial intention and some initial beliefs – with possibly different degrees of truth indicating that there are some agent’s beliefs stronger than others. Based on them, the agent selects some states to reach (called states of desire) – each desire has a degree of will. From the degrees of will added to its beliefs and its previous intention, the agent decides for, and commits itself with, one of the states of desire (such committed state becomes the new agent’s intention). In the sequence, the agent realises the means-ends reasoning.

1.1. BDI Logic

The BDI logics are first-order multi-modal temporal action many-sorted logics. They are extensions of the classical first-order logic with three types of modalities: Bel, Des and Int. Beyond those modalities, there are temporalities (Table 1) and each world is defined as

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2 A fuzzy inference system can be used in this step.
3 Depending of the calculation used in this step, intentions with different truth degrees could be inferred, but just one of them (that one which has the major degree, for example) is adopted as the agent’s intention.
Table 1. Temporal operators.

| Formula | Interpretation                      |
|---------|-------------------------------------|
| ◻ϕ      | ϕ is true in the next time-point    |
| ◻̄ϕ     | ϕ is eventually true                |
| □ϕ      | ϕ is always true                    |
| ϕ ⋁ ψ   | ϕ is true until ψ becomes true      |
| ϕ ⋀ ψ   | ϕ is true unless ψ is true          |

Figure 1. Example of a world according to BDI logics.

Note: Note in the Figure 1 that one time-point – t0 – is adopted to represent the time ‘now’.

Table 2. [7,table 3.6] Constructions for action expressions.

| Formula | Interpretation            |
|---------|---------------------------|
| α; α'   | α followed by α'          |
| α|α'  | either α or α'            |
| α*     | α repeated more than once |
| ϕ?     | ϕ is satisfied            |

a time-tree (see Figure 1). Each node (or state) of this time-tree is a time-point which are related by an arc where the actions are presented. The time-tree is discrete, bounded in the past (there is an initial time), linear in the past (there is just one past history), unbounded in the future (there is no end-time) and branching in the future (the course of future events is yet to be determined). There are constructors of actions (Table 2) and operators over actions (Table 3). The BDI logics are said to be many-sorted since it permits the quantification over agents, (sequence of) actions, groups of agents and other individuals in the world.
The state of an agent is defined by its beliefs, desires and intentions and their semantics are given using the Kripke semantics [19].

The Rao and Georgeff’s BDI Logic has a similar definition for Model. The relevant difference between the semantics of the BDI logics is that BDI Logic assumes a specific relation between the mental attitudes called strong realism. Another important feature of the BDI Logic, which is not contemplated in LoRA, is that BDI Logic distinguishes a non-occurred event from a failed or successful event.

Another contribution given by the authors of [5,6] is the formal definition of the widely used term sub-world. We say that $w$ is a sub-world of $w'$, denoted by $w \sqsubseteq w'$, iff every paths and valuation of formulae contained in $w$ is also in $w'$.

In this paper, we intend to eliminate the mentioned limitations and to put the BDI model closer to the human reasoning representation by allowing agent’s beliefs, desires and intentions to be quantified into the interval $[0,1]$. Therefore, we concern to define a fuzzy BDI logic.

In this section we show a generic informal semantics found in the BDI logics [5,7] and based on this informal semantics, in Section 2 we define a semantic approach of a fuzzy BDI Logic using the classic-like fuzzy semantics defined in [20] and the first family of many-valued modal logic described by Fitting in [21]. In Section 3 are shown related work the BDI agents and applications. Further, in Section 4 we discuss about other related works and ratify the importance of the fuzzy semantics in the BDI model. For more details of the BDI logics presented in this section, see [5–7].

### 2. Fuzzy BDI Logic

Others fuzzy (or many-valued) modal logics have already been defined (see for example [21–27]). In the approach proposed in [23,28], the propositional connectives are interpreted as t-norms, t-conorms, fuzzy implications, fuzzy negations are considered in such a way that the classical tautologies are preserved as in [20,29] and the modal connectives have a semantic similar to the proposed by Caicedo and Rodríguez [30]. In particular, the authors seek a characterisation of fuzzy connective which maintains the usual theorems of modal logic into fuzzy setting.

Fitting [21], in particular, investigates two families of many-valued modal logics and the first one determines the truth value of a modal formula based on its sub-formulae values in each accessible world. This notion is extended in this paper to a fuzzy BDI semantics where the notions of t-norms, t-conorms, fuzzy negations, implication and bi-implication [18,20,31–33] to give semantic, respectively, to the propositional connectives and, $\lor$, $\neg$, $\rightarrow$, and $\leftrightarrow$.

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4 Read ‘iff’ as ‘if, and only if’.
Definition 2.1: Let $T, S, J, E : [0; 1]^2 \to [0; 1]$ and $N : [0; 1] \to [0; 1]$ be functions. Then

1. $T$ is a triangular norm (t-norm) if it satisfies commutativity, associativity, monotonicity and 1 acts like an identity element (1-identity).
2. $S$ is a triangular conorm (t-conorm) if it satisfies commutativity, associativity, monotonicity and 0 acts like an identity element (0-identity).
3. $N$ is a fuzzy negation if $N(0) = 1, N(1) = 0$ and $N$ is decreasing.
4. $J$ is a fuzzy implication if $J$ is decreasing with respect to the first variable, increasing with respect to the second variable and if it satisfies boundary conditions, i.e.: $J(0, 0) = J(0, 1) = J(1, 1) = 1$ and $J(1, 0) = 0$.
5. $E$ is a fuzzy bi-implications if it satisfies the following properties (1), (2) and (3).

$E(x, y) = E(y, x); E(0, 1) = 0; \text{ and } E(x, x) = 1$ (1)

If $x \leq y \leq z$, then $E(x, z) \leq E(x, y)$ (2)

If $x \leq y \leq z$, then $E(x, z) \leq E(y, z)$ (3)

2.1. BDI Syntax

The BDI language is defined in three steps. In the first one we define a language for events: $\mathcal{L}^E = (\Sigma^E, g^E)$, where $\Sigma^E = \Sigma_{Ac} \cup \Sigma_{\text{Construct}}$ is the alphabet of events and $g^E = \{E_1, E_2, E_3, E_4\}$ is the set of grammatical rules, such that:

$\Sigma_{Ac} = \{ \alpha_1, \alpha_2, \alpha_3, ..., \beta_1, \beta_2, \beta_3, ... \}$ is an enumerable set of actions;
$\Sigma_{\text{Construct}} = \{, /, * \}$ is the set of constructors of actions expressions; and

$$E_1 \xrightarrow{\alpha} E_2 \xrightarrow{\alpha/\beta} E_3 \xrightarrow{\alpha/\beta/\gamma} E_4 \xrightarrow{\alpha/\beta/\gamma}$$ (4)

Remark 2.1: The elements of such language are called events, and they are denoted by $e$. The interpretation of constructors of action expressions is the same viewed in Table 2.

In the second step, we define the language of terms $\mathcal{L}^T$ such that $\mathcal{L}^T = (\Sigma^T, g^T)$ with $\Sigma = X \cup \Sigma_C \cup \Sigma_F \cup \{ (, ) \}$ and $g^T = \{ T_1, T_2, T_3 \}$, where

1. $X = \{ x_1, x_2, x_3, ..., y_1, y_2, y_3, ..., z_1, z_2, z_3, ... \}$ is an enumerable set of symbols of variables;
2. $\Sigma_C = \{ a_1, a_2, a_3, ..., b_1, b_2, b_3, ..., c_1, c_2, c_3, ... \}$ is an enumerable set of symbols of constants;
3. $\Sigma_F = \{ f_1, f_2, f_3, ... \}$ is an enumerable set of symbols of functions; and
4. $T_1 \xrightarrow{a} x \in X; T_2 \xrightarrow{a} T_3 \xrightarrow{a \epsilon \Sigma_F}; a \in \Sigma_C; T_3 \xrightarrow{f_1, f_2, ..., f_n}; f \in \Sigma_F$ and $\text{arity}(f) = n$.

Finally, in the third step we define the BDI language $\mathcal{L}^{bd} = (\Sigma^{bd}, g^{bd})$, let $\Sigma^{bd}$ be the alphabet and $g^{bd}$ the grammatical rules of the BDI language such that $\Sigma^{bd} = \mathcal{L}^E \cup \mathcal{L}^T \cup \Sigma_R \cup \Sigma_P \cup L^{Gr} \cup \Sigma_L^{bd}$, where:

1. $L^{Gr} = \Sigma_{Ag} \cup \Sigma_{Gr}$ is a finite set composed by agents and groups of agents – an element of $L^{Gr}$ is denoted by $\iota$. 
(a) $\Sigma \text{Ag} = \{ \text{ag}_1, \text{ag}_2, \text{ag}_3, \ldots \}$ is an enumerable set of agents;
(b) $\Sigma \text{Gr} = \{ \text{gr}_1, \text{gr}_2, \text{gr}_3, \ldots \}$ is an enumerable set of groups of agents;
(2) $\Sigma_R = \{ P_1, P_2, P_3, \ldots \}$ is an enumerable set of symbols (or first-order predicates);
(3) $\Sigma_p = \{ (, ,, \} \}$ is an enumerable set of symbols of punctuation marks;
(4) $\Sigma_{\text{bdi}} = \{ \text{and}, \lor, \neg, \iff, \exists, \diamond, \Box, \forall, ?, \text{Inev}, \text{Opt}, \text{Bel}, \text{Des}, \text{Int}, \text{Succeeds}, \text{Fail}, \text{Does}, \text{Succeeded}, \text{Failed}, \text{Done}, \text{Agts}, \text{Achvs} \}$ is a set of logical symbols.

$\mathcal{G}_{\text{bdi}} = \{ F_1, \ldots, F_{30} \}$ such that:

$F_1$ $\frac{\varphi, \varphi, \ldots, \varphi}{\varphi \lor \varphi}$

$F_2$ $\frac{\varphi, \varphi, \ldots, \varphi}{\varphi \land \varphi}$

$F_3$ $\frac{\varphi}{\varphi \rightarrow \varphi}$

$F_4$ $\frac{\varphi, \varphi, \ldots, \varphi}{\varphi \leftrightarrow \varphi}$

$F_5$ $\frac{\varphi}{\forall \varphi}$

$F_6$ $\frac{\varphi, \forall \varphi}{\forall \forall \varphi}$

$F_7$ $\frac{\varphi}{\exists \varphi}$

$L_{\text{bdi}}$ is the language generated by the formal language $L_{\text{bdi}}$. The brackets in the grammatical rules $F_{23}$–$F_{28}$ indicate that the $\varphi$ is optional.\(^5\)

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\(^5\) The $\varphi$ will be added in the formula iff $\varphi$ is satisfiable after the event.
2.2. Fuzzy BDI Semantics

A BDI language supports several semantics for $L^{bdi}$. All these semantics are crisp, now will be presented a fuzzy semantics for the BDI language. Definition 2.2 shows a fuzzy semantic for a propositional language. Definition 2.6 presents an evaluation and Definition 2.7 presents a model $M$ of the fuzzy BDI Logic.

**Definition 2.2:** ([20]) A fuzzy semantic for the propositional connectives, or just a fuzzy semantics, is a tuple $T = \langle N, T, S, J, E \rangle$, where $N$ is a fuzzy negation, $T$ is a t-norm; $S$ is a t-conorm; $J$ is a fuzzy implication and $E$ is a fuzzy bi-implication.

**Definition 2.3:** Let $T = \langle N, T, S, J, E \rangle$ be a fuzzy semantic and $k \in (0, 1]$. We say that $T$ is $k$-crisp\(^6\) if for each $x, y \in [0, 1]$ we have that

1. $N(x) \geq k$ iff $x < k$;
2. $T(x, y) \geq k$ iff $x \geq k$ and $y \geq k$;
3. $S(x, y) \geq k$ iff $x \geq k$ or $y \geq k$;
4. $J(x, y) \geq k$ iff $x < k$ or $y \geq k$; and
5. $E(x, y) \geq k$ iff $(x \geq k$ and $y \geq k)$ or $(x < k$ and $y < k)$.

**Definition 2.4:** ([7]) A model for $LoRA$ is a structure

$$M_L = \langle T_P, R, W, D, \text{Act}, \text{Agt}, B, D, I, C, \Phi \rangle$$

where $T_P$ is a set of all time points, $R \subseteq T_P \times T_P$ is a total backwards-linear branching time relation over $T_P$, $W$ is a set of worlds over $T_P$, $D = \langle D_{Ag}, D_{Ac}, D_{Gr}, D_U \rangle$ is a domain composed by a non-empty set of agents\(^7\) ($D_{Ag}$), a non-empty set of actions ($D_{Ac}$), a non-empty set of group of agents ($D_{Gr}$) and a non-empty set of other individuals ($D_U$). $\text{Act}: R \rightarrow D_{Ac}$ associates an action with every arc in $R$; $\text{Agt}: D_{Ac} \rightarrow D_{Ag}$ associates an agent with every action. $B, D$ and $I$ are accessibility relations where $B: D_{Ag} \rightarrow \wp(W \times T_P \times W)$; $D \subseteq D_{Ag} \rightarrow \wp(W \times T_P \times W)$; and $I: D_{Ag} \rightarrow \wp(W \times T_P \times W)$. $C: \text{Const} \times T_P \rightarrow D (\bar{D} = D_{Ag} \cup D_{Ac} \cup D_{Gr} \cup D_U)$ is a function that interprets constants. $\Phi: \text{Pred} \times T_P \rightarrow \wp(\bigcup_{u \in \bar{N}} \bar{D}^u)$ is a function that interprets predicates.

The semantics of the modalities $\text{Bel}$, $\text{Des}$ and $\text{Int}$ are defined through those accessibility relations $B$, $D$ and $I$ as follows:

\[
\begin{align*}
M_L, V, w, t & \models \text{Bel}_i(\varphi) \iff \forall w' \in W, (w' \in B^w_i (i) \rightarrow M_L, V, w', t \models \varphi) \quad (5) \\
M_L, V, w, t & \models \text{Des}_i(\varphi) \iff \forall w' \in W, (w' \in D^w_i (i) \rightarrow M_L, V, w', t \models \varphi) \quad (6) \\
M_L, V, w, t & \models \text{Int}_i(\varphi) \iff \forall w' \in W, (w' \in I^w_i (i) \rightarrow M_L, V, w', t \models \varphi) \quad (7)
\end{align*}
\]

where $M_L$ is a model for $LoRA$, $V$ is a variable assignment, $w$ is a world in $M_L$ and $t \in T_P$ is a time point in $w$.

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\(^6\) A complete definition for $k$-crisp fuzzy semantics could be given using partitions as in [20,23].

\(^7\) The variable that denotes agent is $'i'$.
Thus, the semantics of $\text{Bel}$ relates a world $w$ at a time $t$ – called situation\(^8\) and denoted by $w_t$ or by the pair $(w, t)$ – to a belief-accessible world\(^9\) (belief-world for short) where the agent’s belief is true. $D$ and $I$ semantics are defined analogously.

$\text{D}_{w_t}$ is a subset of $\tilde{D}$ for the situation $w_t$. The truth values of free variables are assigned in each situation by a function $\rho_{w_t} : L^T \rightarrow \text{D}_{w_t}$, such that:

1. $\forall f \in \Sigma_F, w_t(f) : \text{D}_{w_t}^{\text{arity}(f)} \rightarrow \text{D}_{w_t}$;
2. $\forall P \in \Sigma_R, w_t(P) \subseteq \text{D}_{w_t}^{\text{arity}(P)}$;
3. $\forall a \in \Sigma_C, w_t(a) \in \text{D}_{w_t}$;
4. $\rho_{w_t}(a) = w_t(a)$ for each $a \in \Sigma_C$;
5. $\rho_{w_t}(x) = w_t(x)$ for each $x \in X$;
6. $\rho_{w_t}(f(\tau_1, \ldots, \tau_n)) = w_t(f)\rho_{w_t}(f(\tau_1), \ldots, \rho_{w_t}(\tau_n))$.

The relation symbols are interpreted as fuzzy relations (from the fuzzy theory), the propositional, modal and temporal symbols have a fuzzy semantics, but $\text{Succeeded}$, $\text{Failed}$, $\text{Done}$, $\text{Agt}$s and $\text{Ach}$s have not – since $\text{M}_l$ has not degrees over accessibility relations. Those cited connectives are interpreted similarly to the same symbols in the BDI logics\([5,7]\).

**Definition 2.5:** Each world is a tuple \(\langle T_{pw}, R_w, \text{Suc}_w, \text{Fal}_w \rangle\) where $T_{pw} \subseteq T_p$ is the set of time-points in the world $w$, $R_w \subseteq T_{pw} \times T_{pw}$ is a total backwards-linear branching time relation over $T_{pw}$, and $\text{Suc}_w, \text{Fal}_w : L^\text{Gr} \times R_w \rightarrow L^E$ are functions that map agents and adjacent time-points to events such that $\text{Suc}_w$ maps to time points that represent the successful (failure) occurrence of the event.

The definition of sub-world is similar to \([5,6,\text{definition 6}]\).

**Definition 2.6:** Let $T = \langle N, T, S, J, E \rangle$ be a fuzzy semantic. $V : S \times L^{\text{bdI}} \rightarrow [0, 1]$ is an evaluation function, with respect to $T$, if maps well-formed formulae (wff for short) of $L^{\text{bdI}}$, in a situation $(w, t)$ to a value into the interval $[0, 1]$ such that for each truth values assigned for the terms of free variables $\rho_{w_t}$ we have that\(^{10}\)

\[
V^w_t(R(\tau_1, \ldots, \tau_n)) = \mu_R(\rho_{w_t}(\tau_1), \ldots, \rho_{w_t}(\tau_n))
\]

\[
V^w_t(\neg \varphi) = N(V^w_t(\varphi))
\]

\[
V^w_t(\varphi \land \phi) = T(V^w_t(\varphi), V^w_t(\phi))
\]

\[
V^w_t(\varphi \lor \phi) = S(V^w_t(\varphi), V^w_t(\phi))
\]

\[
V^w_t(\varphi \rightarrow \phi) = J(V^w_t(\varphi), V^w_t(\phi))
\]

\(^{8}\) The set of all situations of a world $w$ is denoted by $S_w$ and $S = W \times T_p$ is the set of all situations of the model.

\(^{9}\) $B^{\text{bdI}}(i)$ is the set of all belief-accessible world from the situation $w_t$ by the agent $i$. The set of desire- and intention-accessible worlds have analogous denotation.

\(^{10}\) ‘path(1)’ indicates the next adjacent time-point (from the current time-point). ‘paths(w1)’ is the set of all paths of the world $w$ from the time-point $t$. The path that originates the time-point $t$ is denoted by $p(t)$. 
\[ V_t^w(\varphi \leftrightarrow \phi) = E(V_t^w(\varphi), V_t^w(\phi)) \]

\[ V_t^w(\forall x \cdot (\phi)) = \inf\{V_t^w(\phi)|V_t^w(y) = V_t^w(y) \forall y \cdot (y \neq x)\} \]

\[ V_t^w(\exists x \cdot (\phi)) = \sup\{V_t^w(\phi)|V_t^w(y) = V_t^w(y) \forall y \cdot (y \neq x)\} \]

\[ V_t^w(\Diamond \varphi) = \sup\{V_t^w(\varphi)|tRt'\} \]

\[ V_t^w(\Box \varphi) = \inf\{V_t^w(\varphi)|tRt'\} \]

\[ V_t^w(\bigcirc \varphi) = \inf\{V_t^w(\varphi)|tRt' \text{ and } t' = \text{path}(1)\} \]

\[ V_t^w(\varphi \land \psi) = \inf\{\{T(V_t^w(\varphi), V_t^w(\psi))\} \cup \{V_t^w(\varphi)\} \cup \{V_t^w(\psi)\}\} \forall j, i \cdot (t < j \leq t', t' \leq i \text{ and } j, t', i \in \text{paths}({w_i})) \]

\[ V_t^w(\varphi \lor \psi) = \min(\inf\{|T(V_t^w(\varphi), V_t^w(\psi))\} \forall j, i \cdot (t < j \leq t' \text{ and } j, t' \in \text{paths}({w_i}))\}, V_t^w(\varphi)) \]

\[ V_t^w(Bel_{\text{ag}}(\varphi)) = \inf\{V_t^w(\varphi)|B(w_i, w')\} \]

\[ V_t^w(Des_{\text{ag}}(\varphi)) = \inf\{V_t^w(\varphi)|D(w_i, w')\} \]

\[ V_t^w(Int_{\text{ag}}(\varphi)) = \inf\{V_t^w(\varphi)|I(w_i, w')\} \]

\[ V_t^w(Inev(\varphi)) = \inf\{V_t^w(\varphi)|t' \in \text{paths}(w_i)\} \]

\[ V_t^w(Opt(\varphi)) = \sup\{V_t^w(\varphi)|t' \in \text{paths}(w_i)\} \]

\[ V_{t_0}^w(Succeeds_{\text{ag}}(e)) = \begin{cases} 1, \text{if } Suc_w(\text{ag}, t_i, t_i + 1) = e \\ 0, \text{otherwise.} \end{cases} \]

\[ V_{t_0}^w(Fails_{\text{ag}}(e)) = \begin{cases} 1, \text{if } Fal_w(\text{ag}, t_i, t_i + 1) = e \\ 0, \text{otherwise.} \end{cases} \]

\[ V_{t_i}^w(Does_{\text{ag}}(e)) = \begin{cases} 1, \text{if } Suc_w(\text{ag}, t_i, t_i + 1) = e \text{ or } Fal_w(\text{ag}, t_i, t_i + 1) = e \\ 0, \text{otherwise.} \end{cases} \]

\[ V_{p(t_j)}^w(Succeeded_{\text{ag}}(e)) = \begin{cases} 1, \text{if } \exists t_i \text{ s.t. } t_i \in p(t_j) \text{ and } Suc_w(\text{ag}, t_i - 1, t_i) = e \\ 0, \text{otherwise.} \end{cases} \]

\[ V_{p(t_j)}^w(Failed_{\text{ag}}(e)) = \begin{cases} 1, \text{if } \exists t_i \text{ s.t. } t_i \in p(t_j) \text{ and } Fal_w(\text{ag}, t_i - 1, t_i) = e \\ 0, \text{otherwise.} \end{cases} \]

\[ V_{p(t_j)}^w(Done_{\text{ag}}(e)) = \begin{cases} 1, \text{if } \exists t_i \text{ s.t. } t_i \in p(t_j) \text{ and } Suc_w(\text{ag}, t_i - 1, t_i) = e \\ 0, \text{otherwise.} \end{cases} \]
\[ V_p^w\left(\text{Agts}_g\left(\text{a}, \text{e}\right)\right) = \begin{cases} 
1, & \text{if there exists } t \in p \text{ s.t. } V_t^w\left(\text{Succeeds}_g\left(e\right)\right) = 1 \\
0, & \text{otherwise.} 
\end{cases} \]

\[ V_p^w\left(\text{Achieve}_g\left(e, \text{p}\right)\right) = \begin{cases} 
V_{t+1}^w\left(\text{p}\right), & \text{if } V_t^w\left(\text{Succeeds}_g\left(e\right)\right) = 1 \\
0, & \text{otherwise.} 
\end{cases} \]

**Definition 2.7:** A model of the fuzzy BDI Logic is a tuple \( M = \langle T, M_L, V \rangle \) where \( T \) is a fuzzy semantic, \( M_L \) is a model for LoRA and \( V \) is an evaluation function with respect to \( T \). When \( T \) is \( k \)-crisp for some \( k \) we say \( M \) of \( k \)-model.

**Definition 2.8:** Let \( V: S \times L_{BDI} \rightarrow [0, 1] \) be an evaluation function and \( k \in [0, 1] \) a truth constant. A formula \( \varphi \in L_{BDI} \) is \( k \)-true in a model \( M \), denoted by \( M, V \models_{BDI}^k \varphi \) iff \( V_t^w(\varphi) \geq k \) for each situation \((w, t)\). Hence, when \( V_t^w(\varphi) \geq k \), \((M, V)\) is said to be a model of \( \varphi \).

**Definition 2.9:** Let \( \varphi \) be a formula of \( L_{BDI} \) then:

1. \( \varphi \) is \( k \)-true in a situation \( w_t \) of a model \( M \) and evaluation \( V \), denoted by \( M, V, w_t \models_{BDI}^k \varphi \), if \( V_t^w(\varphi) \geq k \).
2. \( \varphi \) is \( k \)-satisfiable in a model \( M \) and evaluation \( V \), denoted by \( M, V \models_{BDI}^k \varphi \), if there exists a situation \( w_t \) such that \( M, V, w_t \models_{BDI}^k \varphi \).
3. \( \varphi \) is \( k \)-unsatisfiable in a world \( w \) of a model \( M \) and evaluation \( V \), denoted by \( M, V \models_{BDI}^k \varphi \), if \( V_t^w(\varphi) < k \) for any \( w_t \in S_w \).
4. \( \varphi \) is \( k \)-true in \( M \), denoted by \( M \models_{BDI}^k \varphi \), if for all evaluation \( V \) and possible world \( w \) there is at least one situation \( w_t \in S_w \) such that \( V_t^w(\varphi) \geq k \), i.e. if \( \varphi \) is \( k \)-satisfiable in every world \( w \) of \( M \) and evaluation \( V \).
5. \( \varphi \) is \( k \)-false in \( M \) if it is \( k \)-unsatisfiable in every world \( w \) of \( M \) and evaluation \( V \).
6. \( \varphi \) is a \( k \)-contigent formula in \( M \) if for all evaluation \( V \) there are worlds \( w \) and \( w' \) in \( M \) such that \( V_t^w(\varphi) \geq k \) and \( V_t^w(\varphi) < k \).
7. \( \varphi \) is \( k \)-universally valid, denoted by \( \models_{BDI}^k \varphi \), if for all model \( M \), evaluation \( V \) and situation \( w_t, M, V, w_t \models_{BDI}^k \varphi \).

**Definition 2.10:** Let \( L_{BDI} \) be the BDI language, \( \Gamma \subseteq L_{BDI} \) a set of formulae of this language, and \( k \in (0, 1] \). \( \varphi \in L_{BDI} \) is a \( k \)-semantic consequence of \( \Gamma \), denoted by \( \Gamma \models_{BDI}^k \varphi \), iff for all \((w, t)\) and \( k \)-model \( M \), if \( M, V, w_t \models_{BDI}^k \Gamma \) then \( M, V, w_t \models_{BDI}^k \varphi \), that is, in every situation in which \( \Gamma \) is satisfiable, so is \( \varphi \).

**Definition 2.11:** Let \( L_{BDI} \) be the BDI language, then the fuzzy BDI Logic is

\[ \text{Log}^{BDI} = L_{BDI}, \models_{BDI}^k \]

To complete the presentation of the fuzzy BDI Logic, we demonstrate now that it satisfies the Deduction Theorem.

**Theorem 1:** Let \( M \) be a \( k \)-model of the fuzzy BDI logic. Let \( \varphi \) and \( \phi \) be a wff of the \( L_{BDI} \). So

\[ \varphi \models_{BDI}^k \phi \iff \models_{BDI}^k \phi \rightarrow \phi \]
Proof: Let $k$ be the truth constant and $\phi, \varphi \in L^{\text{bdi}}$.

$\Rightarrow$: Suppose that $\varphi \models^{\text{bdi}}_k \phi$. So, by Definition 2.10, for every situation $(w, t)$ of the model, if $V^w_t(\varphi) \geq k$, then $V^w_t(\phi) \geq k$. By Definition 2.6, $V^w_t(\phi) = J(V^w_t(\varphi), V^w_t(\phi))$ and if $V^w_t(\varphi) \geq k$ then $V^w_t(\phi \rightarrow \varphi) \geq k$, because $V^w_t(\phi) \geq k$ and $T$ is $k$-crisp. Therefore $\varphi \rightarrow \phi$.

$\Leftarrow$: Suppose that $\models^{\text{bdi}}_k \varphi \rightarrow \phi$. So by Definition 2.9, for all model $M$ and evaluation $V$ and situation $w$, we have that $V^w_t(\phi) \geq k$ and by Definition 2.6, $V^w_t(\phi \rightarrow \varphi) = J(V^w_t(\varphi), V^w_t(\phi))$. So, since $T$ is $k$-crisp, either $V^w_t(\phi) < k$ or $V^w_t(\phi) \geq k$. Thus $V^w_t(\phi) \geq k$ whenever $V^w_t(\phi) \geq k$. Therefore $\varphi \models^{\text{bdi}}_k \phi$. $\blacksquare$

Corollary 2.1: Let $\varphi_1, \ldots, \varphi_n, \varphi$ be wff of the $L^{\text{bdi}}$. So

$$\varphi_1, \ldots, \varphi_n \models^{\text{bdi}}_k \varphi \iff \varphi_1, \ldots, \varphi_n-1 \models^{\text{bdi}}_k \varphi_n \rightarrow \varphi$$

Proof: Let $k$ be the truth constant and $\varphi_1, \ldots, \varphi_n, \varphi \in L^{\text{bdi}}$.

$\Rightarrow$: Suppose that $\varphi_1, \ldots, \varphi_n \models^{\text{bdi}}_k \varphi$. So, by Definition 2.10, for every situation $(w, t)$ of the model, if $V^w_t(\varphi_i) \geq k$ for each $i = 1, \ldots, n$, then $V^w_t(\varphi) \geq k$. On the other hand, for some situation $(w, t)$ of a model, occurs that $V^w_t(\varphi) \geq k$ for each $i = 1, \ldots, n-1$. Thus, case (a) $V^w_t(\varphi_n) \geq k$, then $V^w_t(\varphi) \geq k$, by the hypothesis, and so $V^w_t(\varphi_n \rightarrow \varphi) = J(V^w_t(\varphi_n), V^w_t(\varphi)) \geq k$. Case (b) $V^w_t(\varphi_n) < k$, then, because $T$ is $k$-crisp, $V^w_t(\varphi_n \rightarrow \varphi) = J(V^w_t(\varphi_n), V^w_t(\varphi)) < k$. Hence $\varphi_1, \ldots, \varphi_n-1 \models^{\text{bdi}}_k \varphi_n \rightarrow \varphi$.

$\Leftarrow$: Suppose that $\varphi_1, \ldots, \varphi_n \models^{\text{bdi}}_k \varphi_n \rightarrow \varphi$. So by Definitions 2.10 and 2.5 and the properties showed in Definition 2.1, for every $(w, t)$ such that $V^w_t(\varphi_1, \ldots, \varphi_n-1) \geq k$, $V^w_t(\varphi) \geq k$ (for any value of $V^w_t(\varphi_n)$), or $V^w_t(\varphi) < k$ (for any value of $V^w_t(\varphi_n)$). In both cases, for every $(w, t)$ such that $V^w_t(\varphi_1, \ldots, \varphi_n) \geq k$, $V^w_t(\varphi) \geq k$. Hence $\varphi_1, \ldots, \varphi_n \models^{\text{bdi}}_k \varphi$. $\blacksquare$

3. Related Works

According to Santos [34] a fuzzy approach is important to personality-based social exchanges in multiagent systems. In that paper was used the fuzzy logic to deal with the interaction processes between agents with different types of personalities, evaluating an exchange process using pertinence degrees. Interactions between agents are based on Piaget’s theory of social exchanges. Since social exchange values are a qualitative nature, so they could be modelled by fuzzy BDI logic.

In order to structure the regulator agent decision process and the exchange strategy learning process of open multiagent systems, in [35] was proposed a combination of a partially observable Markov decision process (POMDP) and a hidden Markov model (HMM). That paper shows interactions between agents when cooperating/competing to achieve their individual, collective objectives. Can be interesting analyse the possibility of using an adaptation of fuzzy BDI logic for interaction process.

Casali, Godo and Sierra in [36,37] propose the g-BDI (graded BDI) architecture to represent and reason through gradual notions of desires and intentions, including a complete logical formalisation. The works show a framework for studying and developing this type of agents.

The graded BDI considers the following characteristics: (i) beliefs degrees; (ii) degrees of positive or negative desires; (iii) intention degrees. An agent with different types of behaviour can be modelled with g-BDI, based on the representation and interaction of
these characteristics. Moreover, the approach modal fuzzy is used to represent and reason about different degrees of mental attitudes and a multi-context system (MCS) was considered to integrate processes between g-BDI agents defining mental contexts, functional contexts and a set of bridge rules. A MCS for a g-BDI agent is defined as a tuple with (i) the mental contexts represent: beliefs context (BC), desires context (DC) and intentions context (IC); (ii) two functional contexts are used for planning and communication; (iii) a suitable set of rules encode to a particular pattern of interaction between beliefs, desires and intentions. Soundness and completeness was proved for desires context, the axiomatization is correct with respect to the defined semantics and it is complete as well for finite theories of modal formulae.

In [37] was developed a graded BDI agent framework and a case of study was developed using this framework to a recommendation system applied to tourism, showing the flexibility and performance of a g-BDI model.

A BDI software applications in the simulation area of human behaviour is explored in [38]. This work develops a study, exploring the differences in an approach based on classical rules and fuzzy rules. A fuzzy approach is made using BDI agents. It developed a game that showing the efficiency and realistic approach using BDI fuzzy systems, most adapting to human behaviour. The work does not explore mathematical concepts and an appropriate formalisation.

In [39], the authors strengthen the use of a fuzzy BDI semantic. They state that intelligent agents’ models which do not admit degrees of mental attitudes results systems incapable to take account of much of the useful information which helps to guide the human reasoning about the world. Moreover, rational agents (especially human being) believe or desire some states more than others, similarly, to intend something (regarding the common sense of the intention notion). In the real world we often must deal with different levels of confidence in our beliefs, desires, and intentions. For example, it is natural for a human to believe that he is 30 and hungry, at the same time, consequently he will desire to drink and eat something with different degrees of will. These degrees of will are determined considering what he believes to be his greater necessity at the moment and he will commit with one of the mental states: either no more 30 or no more hungry. We often lead with this type of decision-making and the fuzzy BDI semantics, defined in this paper, can perfectly model it.

Thus, the idea of extending the BDI model to a fuzzy BDI model or at least to admit a degree to reason about rational agency, is not new. Some works are related to these purposes – see [39–44] as examples. Although relevant works consider degrees in multiagent systems and they solve specific problems.

4. Final Remarks

In the initial of 1990s, Rao and Georgeff, in [6], claim that their BDI Logic was incomplete with respect to deliberation process and reconsideration and override of intention.

The authors also affirm that such problems could be solved adding to the BDI logic, a decision-theoretic technique. Such decision technique could be to map a utility for each course of action, or to add a fuzzy degree of truth for each sentence in the system, or even to employ Baysean methods. To Add a fuzzy degree in the semantics of a BDI logic is an interesting solution once that we can solve the cited limitations in the same level in which they appear.
With a fuzzy BDI logic, it is possible to make the revision of beliefs, desires and intentions by discarding the mental attitudes whose truth degree are less than a pre-defined $\alpha$ number ($\alpha$-cut concept). The deliberation process, the intention reconsideration and the override of intention could be easily implemented by searching for the greatest agent’s desire to become the new intention. Moreover, all those decision process could be improved by a fuzzy Inference System.

This paper tries to establish the concepts related to BDI Syntax where a language about an alphabet and a set of grammatical rules was defined. As far as Fuzzy BDI Semantics is concerned a fuzzy semantic for a propositional language, an evaluation function $V$, and a $M$ of fuzzy BDI Logic model are described. This paper does not address the proofs of soundness and completeness.

In the future we intend to expand this study by addressing some formal theories for BDI syntax. It is necessary to show some results of soundness and completeness for some fuzzy semantics, tried to characterise these results according to the fuzzy semantics of this paper. In addition, it also must extend the fuzzy BDI logic for a lattice-valued (fuzzy) BDI logic, i.e. a BDI logic where the truth degree took value in an arbitrary lattice.

Beyond of the contribution to the computational logic area – we propose a new fuzzy modal logic, and we define a new BDI logic with the limits between syntax and semantics clear and formally defined –, the fuzzy BDI semantics, defined in this paper, can be employed to solve a wide range of AI representation problems to model, through a logical apparatus: the deliberation process, the belief and desire revision, the reconsideration and override of intention. To make the BDI model more faithful to a representation of the human being reasoning was employed the approximate reasoning in the BDI model.

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