Long-term Optical Polarization Variability and Multiwavelength Analysis of Blazar Mrk 421

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Abstract

The results of 8 yr R-band photopolarimetric data of blazar Mrk 421 collected from 2008 February to 2016 May are presented, along with extensive multiwavelength observations covering radio to TeV γ-rays around the flares observed in 2008 May, 2010 March, and 2013 April. The most important results are found in 2013, when the source displayed in the R band a very high brightness state of 11.29 ± 0.03 mag (93.60 ± 1.53 mJy) on April 10 and a polarization degree of 11.00% ± 0.44% on May 13. The analysis of the optical data shows that the polarization variability is due to the superposition of two polarized components that might be produced in two distinct emitting regions. An intranight photopolarimetric variability study carried out over seven nights after the 2013 April maximum found flux and polarization variations on the nights of April 14, 15, 16, and 19. In addition, the flux shows a minimum variability timescale of $\Delta t = 2.34 \pm 0.12$ hr, and the polarization degree presents variations of $\sim 1\%-2\%$ on a timescale of $\Delta t \sim$ minutes. Also, a detailed analysis of the intranight data shows a coherence length of the large-scale magnetic field of $l_B \approx 0.3$ pc, which is the same order of magnitude as the distance traveled by the relativistic shocks. This result suggests that there is a connection between the intranight polarimetric variations and spatial changes of the magnetic field. Analysis of the complete R-band data along with the historical optical light curve found for this object shows that Mrk 421 varies with a period of $16.26 \pm 1.78$ yr.

Key words: acceleration of particles – BL Lacertae objects: individual (Markarian 421) – galaxies: photometry – gamma rays: general – polarization – radiation mechanisms: non-thermal

Supporting material: machine-readable table

1. Introduction

Blazars, a special subclass of active galactic nuclei (AGNs), are characterized by having a relativistic jet closely aligned with the observer’s line of sight (estimated viewing angles $\theta \approx 10^\circ$). Blazars exhibit high variability in all frequencies of the electromagnetic spectrum. Several variability studies in blazars have been carried out with the aim of finding short and long timescale variabilities (Ulrich et al. 1997; Falomo et al. 2014). For instance, optical bands have been well studied to search for variations down to minutes (the so-called intraday variability; see, e.g., Wagner & Witzel 1995) and periodic or quasi-periodic variations on timescales of years (see, e.g., Sillanpää et al. 1988). Strong and variable polarization degrees higher than 3% have been observed in blazars in the radio and optical bands (Angel & Stockman 1980; Impey & Tapia 1990). Polarization variability studies can provide valuable information about the structure and strength of the magnetic field associated with the physical processes in the emitting region (see, e.g., Sorcia et al. 2013, 2014; Corvina et al. 2015).

Analysis of the equivalent width of the emission lines revealed that blazars can be divided in BL Lac objects and flat-spectrum radio quasars (FSRQs; Marcha et al. 1996; Beckmann & Shrader 2012). An additional analysis of the spectral features carried out by Abdo et al. (2009) showed a fairly different separation between FSRQs and BL Lac objects, with FSRQs having considerably softer spectra. They found that the spectral power-law index $\sim 2.2$ corresponds to the boundary between BL Lac objects and FSRQs. Later, Ghisellini et al. (2009) proposed that this division can be explained in terms of the different accretion regimes and the distinct radiation cooling mechanisms of electrons in these objects. It is worth noting that according to the location of the synchrotron peak in the spectral energy distribution (SED), BL Lac objects are classified into low-energy peaked BL Lac (LBL), intermediate-energy peaked BL Lac (IBL), and high-energy peaked BL Lac (HBL; Padovani & Giommi 1995).

At a distance of 134.1 Mpc, the BL Lac object Mrk 421 ($z = 0.03$; Sbarufatti et al. 2005) is one of the closest and most comprehensively studied sources of the HBL class. Mrk 421 was the first extragalactic object observed in the very-high-energy (VHE) γ-ray band (Punch et al. 1992). By analyzing the historical light curve comprising data for almost 100 yr in the optical B band, Liu et al. (1997) found two kinds of variability behaviors in Mrk 421. The first one consisted of nonperiodical rapid variations on timescales from hours to days. The second one consisted of periodic variations with possible periods of $23.1 \pm 1.1$ and $15.3 \pm 0.7$ yr, although the former period was not very significant.

Considered an excellent candidate to study the physical processes within blazar jets, Mrk 421 has been a frequent target of multiwavelength campaigns in order to study correlations among distinct energy bands. Correlations between TeV γ-ray and X-ray bands have been found several times during high-activity states or flares (Macomb et al. 1995; Fossati et al. 2004; Albert...
et al. 2007) and during quiescent states (Aleksić et al. 2015a; Baloković et al. 2016; Abdo et al. 2014). In only a few cases, TeV γ-rays without an X-ray emission counterpart have been reported (the so-called orphan flares; see, e.g., Blaževski et al. 2005; Acciari et al. 2009). Correlations found between optical bands and TeV γ-rays/X-rays are still controversial. For instance, optical and TeV γ-ray/X-ray correlations and anticorrelations have been found with different time lags in some studies (Horan et al. 2009; Aleksić et al. 2015a; Sandrinelli et al. 2017), and, in other ones, no correlations have been reported (Macomb et al. 1995; Aleksić et al. 2015a; Albert et al. 2007). Similarly, the radio bands are found not to be correlated with γ-rays (Acciari et al. 2011).

The broadband SED of Mrk 421 presents a double-humped shape; the lower energy hump has a peak located at a few keV, and the second hump has one at hundreds of GeV. Abdo et al. (2011) found that both leptonic and hadronic models are able to fit the SED of this object reasonably well, implying comparable jet powers but with very different characteristics for the emitting region. In the leptonic scenario, a one-zone synchrotron self-Compton (SSC) model with three accelerated electron power-law functions (through diffusive relativistic shocks with a randomly oriented magnetic field) has been used (Abdo et al. 2011). In the hadronic scenario (Abdo et al. 2011; Fraija & Marinelli 2015), the first peak is explained by electron synchrotron radiation. For the second peak that extends from low-energy γ-rays to VHE γ-rays, the synchrotron proton blazar (SPB) model is used (Mücke & Protheroe 2001; Mücke et al. 2003). It is worth noting that the strength of the magnetic field required to fit the SED with a hadronic model was 50 G. In contrast, using the shortest variability timescale of 1 hr, a magnetic field density of $8.2 \times 10^{-2}$ G was obtained when a leptonic model was considered.

The polarization observations of Mrk 421 have been widely studied with different optical bands. Hagen-Torn et al. (1983) studied the polarization and photometric behavior in the B and V bands during 1974–1982. They reported a variability timescale from days to years and polarization degrees less than 6% with a preferential position angle of 173°. Successively, many photopolarimetric observations were carried out from the ultraviolet to near-infrared regions (Mead et al. 1990; Takalo 1991; Takalo et al. 1992; Takalo & Sillanpää 1993), reporting a moderate polarization degree.

From 1994 October to 1997 June, polarimetric observations were performed in the radio and optical bands by Tosti et al. (1998). In particular, a large optical outburst was detected in the winter of 1996–1997, followed by a radio outburst with a delay of 30–60 days. At the same time, Mrk 421 increased its polarization level, reaching a value of ~12% in the V band. Ikejiri et al. (2011) reported correlations between the flux, color ($K$, $J$, and $K_s$ bands), and polarization variations on timescales from days to months. Recently, using the data collected with KVA and RINGO2 from 2008 to 2011, Jermak et al. (2016) reported a strong increase in the polarization degree and a 360° rotation of the position angle before the unprecedentedly large γ-ray flare occurred after 2012 June.

In this work, we report the results of a long-term R-band photopolarimetric observations of blazar Mrk 421 carried out from 2008 February to 2016 May. The paper is arranged as follows. In Section 2, the R-band optical polarimetric observations and data reduction are presented. In Section 3, the polarimetric variability analysis is shown. Section 4 describes the multiwavelength data used in this work. Section 5 shows the results of the analysis done on the multiwavelength data. Section 6 presents general results, and Section 7 presents the conclusions.

### 2. Optical Polarimetric Observations and Data Reduction

The optical R-band observations were performed with the 0.84 m f/15 Ritchey–Chrétien telescope at the Observatorio Astronómico Nacional de San Pedro Mártir (OAN-SPM) in Baja California, Mexico, and with the instrument POLIMA, a single-beam polarimeter.\(^8\) The exposure time was 60 s per frame of Mrk 421. For these observations, three different CCD cameras were used with a pixel size of 13.5–24 μm, 2 × 2 binning, and a plate scale of 0.′′22–0.′′39.\(^9\) Photometry was performed with an aperture radius of ~3″ after regular subtraction of bias, dark-current, and flat-field correction for each angular position of the polarizer. Images had an average FWHM of ~3″. Photometric calibrations were done using nearby standard stars from Fiorucci & Tosti (1996). Polarimetric calibrations were done using the polarized and unpolarized standard stars from Schmidt et al. (1992). A correction for polarization bias was done using the estimator of the polarization amplitude from normally distributed Stokes parameters (Plaszczynski et al. 2014). The R-band magnitudes were converted to apparent flux using $F_R = K_0 \times 10^{-0.4 \mu m}$, where $K_0 = 3.08 \times 10^6 \text{mJy for the effective wavelength of } \lambda = 6400$. The host galaxy contribution was removed from the total and polarized fluxes after fitting its surface brightness profile given by Sérsic (1968) and Caon et al. (1993):

$$I(r) = I(r_e)\exp\left(-b_n\left(\frac{r}{r_e}\right)^n - 1\right),$$

where $b_n = 0.868n - 0.142$ and $I(r_e) = f_k / [K_n r_e^2 (1 - \epsilon)]$, with $f_k$ the flux corresponding to a total magnitude and $K_0 = 0.03 [\log(1/n)]^2 + 0.441 \log(1/n) + 1.079$. Using a Sérsic index of $n = 4$, an ellipticity of $\epsilon = 0.21$, and a diameter of aperture 7″, we found a value of 14.48 ± 0.18 for the magnitude of the host galaxy in the R-band with an effective wavelength of $\lambda = 6400$ Å (see Nilsson et al. 1999; Soria et al. 2013). It is worth noting that the correction of the flux from the host galaxy depends strongly on the aperture radius used in the photometry, but the FWHM from seeing has only a minor effect (Nilsson et al. 2007).

The position angle was corrected by an ambiguity of 180° in such a way that the variations observed between the position angles of temporal consecutive data should be less than 90°. This difference can be written as

$$|\Delta \theta_m| = |\theta_{m+1} - \theta_m| - \sqrt{\sigma(\theta_{m+1})^2 + \sigma(\theta_m)^2},$$

where $\theta_m$ and $\theta_{m+1}$ are the mth and $m + 1$ position angles, respectively, and $\sigma(\theta_m)$ and $\sigma(\theta_{m+1})$ are their errors. If $|\Delta \theta_m| < -90°$, 180° are added to $\theta_{m+1}$; if $|\Delta \theta_m| > 90°$, 180° are added to $\theta_{m+1}$. If $|\Delta \theta_m| \leq 90°$, no correction is needed.

The R-band photopolarimetric observations of the BL Lac Mrk 421 were carried out from 2008 February to 2016 May (data available online; see Table 1). The total data collected during 8 yr were 385 points obtained in 30 observing runs.

Figure 1 shows the R-band photopolarimetric light curves of Mrk 421 obtained from 2008 February to 2016 May. The upper two panels show flux and R-band magnitude variation, and the

\(^7\) A detailed description of our photopolarimetric monitoring program on blazars can be found at http://www.astrossp.unam.mx/blazars.

\(^8\) Technical information about the POLIMA instrument can be found at http://karo.astrossp.unam.mx/blazars/Instrument/Instrument.html.
lower two panels show the polarization degree and the electric vector position angle (EVPA) variations. Hereafter, instead of EVPA, this parameter will be denoted as position angle. Table 2 shows the maximum and minimum brightness states of polarization degree and position angle with their respective dates. The polarization degree and position angle panels in Figure 1 exhibit random and significant variations around the average values.

3. Polarimetric Variability Analysis

Variability at long and short timescales is related to the radiative processes that occur in the emitting region and the strength of the
magnetic field. In the framework of synchrotron radiation, the Fermi-accelerated electrons described by a power law with spectral index $\alpha_e$ are injected into the emitting region. This electron population confined by a randomly oriented magnetic field radiates a photon distribution with a spectral index $(\alpha_e - 1)/2$. The size of the emitting region can be constrained using the variability timescale $\tau_v$ through the relation

$$r_d = \frac{\delta_D}{(1 + z)} \tau_{v, \text{min}} \leq \frac{\delta_D}{(1 + z)} \tau_v,$$

and the magnetic field can be estimated through the synchrotron cooling timescale

$$t_{\text{syn}} = \frac{6\pi}{\sigma_T} \sqrt{\frac{m_e q_e (1 + z)}{\delta_D \epsilon_R B^3}},$$

with $\tau_v \approx t_{\text{syn}}$, and then it can be written as

$$B = \left(36\pi^2 m_e q_e (1 + z)^{1/3}\right)^{1/3} \delta_D^{-1/3}. \tag{5}$$

Here, $z$ is the redshift, $m_e$ is the electron mass, $\sigma_T$ is the Thompson cross section, $q_e$ is the electric charge, $\epsilon_D$ is the Doppler factor, and $\epsilon_R$ is the energy of the $R$ band associated with the effective wavelength of $\lambda = 6400 \text{ Å}$ (see Nilsson et al. 1999; Sorcia et al. 2013). It is worth noting that the evolution of the magnetic field structure can be found following Korchakov & Syrovatskii (1962) and Bjornsson (1985). Bjornsson considered that one or more components could be present, and then each of these could dominate the emission at different epochs. Variations in the polarization degree $P$ are directly related to the evolution of the magnetic field structure in the emitting region, where the resulting magnetic field is produced by an ordered $(B_0)$ produced by shocks) and a chaotic $(B_i)$ immersed in the emitting region) magnetic field. Considering the spectral power index of the photon distribution radiated in this process, at the limit of $\beta = B_0/B_i \ll 1$, the polarization degree and electron power index are related by

$$P = \frac{(\alpha_e + 3)(\alpha_e + 5)}{32} \Pi_0 \beta^2,$$  \tag{6}

where $\Pi_0 = (\alpha_e + 1)/(\alpha_e + 7/3)$ is the polarization degree of a perfectly uniform magnetic field.

In particular, the timescale for flux variability can be estimated through the relation (Burbidge et al. 1974)

$$\tau_v = \left| \frac{d \ln F}{dt} \right|^{-1}, \tag{7}$$

where $dt$ is the time interval between two consecutive fluxes $F_i$ and $F_j$. For a data set with $i, j = 1, \ldots, N - 1$, where $N$ is the number of observations, the minimum timescale can be found by

$$\tau_{v, \text{min}} = \min \{\tau_v\}. \tag{8}$$

The minimum timescale equation is used in Section 5.5. For our statistical analysis, it is important to define the amplitude of the variations $Y(%)$, fluctuation index $\mu$, and fractional variability index $F$, which are (Heidt & Wagner 1996)

$$Y(%) = \frac{100}{hS} \sqrt{(S_{\text{max}} - S_{\text{min}})^2 - 2\sigma_e^2}, \tag{9}$$

$$\mu = \frac{100\sigma_e}{(S)} \%, \tag{10}$$

and

$$F = \frac{S_{\text{max}} - S_{\text{min}}}{S_{\text{max}} + S_{\text{min}}}, \tag{11}$$

respectively. Here, $S_i$ for $i = \text{max}$ and $\text{min}$ are the optical flux, polarization degree, and position angle; $(S)$ is the average value; and $\sigma_e^2 = \sigma_{\text{max}}^2 + \sigma_{\text{min}}^2$.

The long-term $R$-band photopolarimetric data correspond to 8 yr of observations of Mrk 421. During the monitoring period, the blazar displayed several flares; in particular, a set of maxima values was observed. In brightness, a value of $R = 11.29 \pm 0.03$ mag $(93.60 \pm 1.53 \text{ mJy})$ was detected on 2013 April 10; this is our maximum brightness detection. A month later, a maximum for the polarization degree of $P = 11.00\% \pm 0.18\%$ occurred on 2013 May 13. Finally, a maximum for the position angle $\theta = 285.4^\circ \pm 8.1^\circ$ was detected on 2014 March 27 (see Figure 1). Considering all of the data, Mrk 421 exhibited an average polarization degree of $\approx 3.6\%$ with a preferred position angle direction of $\sim 169^\circ$. The effective wavelength of the polarization degree and the preferred position angle direction are explained in terms of two different emitting regions where the dominated optical flux comes from a region permeated by a stable magnetic field (Hagen-Torn et al. 1983; Jermak et al. 2016).

In order to look for correlations per year between the optical flux, polarization degree, position angle, and normalized Stokes parameters $q$ and $u$, a statistical analysis and the Pearson’s correlation coefficients were done and the results are reported in Tables 3 and 4, respectively. Among the most outstanding results are the following. (i) In 2012, the object presented the lowest average values in polarization degree and position angle: $2.60\% \pm 0.05\%$ and $65.6^\circ \pm 0.5^\circ$, respectively. (ii) In 2013, the maximum variations in the $R$-band optical flux, $u$ at $u = U/F_R = P \cos \theta$ and $u = U/F_R = P \sin \theta$. To determine the correlation of the data, we use the Pearson’s correlation coefficients which are the covariance of the two variables divided by the product of their standard deviation.
In column 2, \( F_r \) is the \( R \)-band flux, \( P \) (%) is the polarization percentage, and \( \theta \) (deg) is the position angle. Columns (3)–(5) show the maximum, minimum, and average values of the parameters. Columns (6)–(8) are the amplitude of the variations, the fluctuation index, and the fractional variability index.

| Year (1) | Parameter | \( F_r \) (mJy) | \( P \) (%) | \( \theta \) (deg) | \( F_r \) (mJy) | \( P \) (%) | \( \theta \) (deg) | \( F_r \) (mJy) | \( P \) (%) | \( \theta \) (deg) | \( F_r \) (mJy) | \( P \) (%) | \( \theta \) (deg) |
|---------|-----------|----------------|----------|----------------|----------------|----------|----------------|----------------|----------|----------------|----------------|----------|----------------|
| 2008    | \( F_r \) (mJy) | 38.94 ± 1.62 | 14.86 ± 0.92 | 23.39 ± 0.09 | 102.80         | 0.40      | 0.45           |
| 2009    | \( P \) (%)  | 7.97 ± 0.28  | 1.28 ± 0.29  | 3.29 ± 0.05  | 102.41         | 1.39      | 0.72           |
| 2010    | \( \theta \) (deg) | 34.19 ± 1.4  | 10.22 ± 1.2  | 176.67 ± 0.45 | 62.02         | 0.26      | 0.35           |
| 2011    | \( F_r \) (mJy) | 28.21 ± 0.56 | 18.26 ± 0.44 | 24.58 ± 0.17 | 40.28         | 0.70      | 0.21           |
| 2012    | \( P \) (%)  | 7.43 ± 0.28  | 2.01 ± 0.27  | 4.06 ± 0.11  | 132.89         | 2.57      | 0.57           |
| 2013    | \( \theta \) (deg) | 316.3 ± 1.5  | 118.1 ± 1.4  | 137.33 ± 0.65 | 27.74         | 0.48      | 0.14           |
| 2014    | \( F_r \) (mJy) | 41.01 ± 0.73 | 19.16 ± 0.45 | 22.74 ± 0.17 | 95.92         | 0.73      | 0.36           |
| 2015    | \( P \) (%)  | 5.73 ± 0.30  | 1.25 ± 0.29  | 3.77 ± 0.09  | 166.09         | 2.51      | 0.72           |
| 2016    | \( \theta \) (deg) | 180.3 ± 2.8  | 114.3 ± 4.0  | 131.84 ± 0.83 | 49.78         | 0.63      | 0.22           |

Table 3
Pearson’s Correlation Coefficients per Year

Note. In column (2), \( F_r \) is the \( R \)-band flux, \( P \) (%) is the polarization percentage, and \( \theta \) (deg) is the position angle. Columns (3)–(5) show the maximum, minimum, and average values of the parameters. Columns (6)–(8) are the amplitude of the variations, the fluctuation index, and the fractional variability index.

| Year (1) | \( F_r \)–\( P \) | \( F_r \)–\( \theta \) | \( P \)–\( \theta \) | \( F_r \)–\( \mu \) | \( F_r \)–\( q \) | \( \mu \)–\( q \) |
|---------|----------------|----------------|----------------|----------------|----------------|----------------|
| 2008    | 0.37 (0.11)    | −0.19 (0.44)   | −0.51 (0.03)   | 0.12 (0.65)   | −0.26 (0.30)   | −0.44 (0.05)   |
| 2009    | −0.03 (0.94)   | −0.30 (0.43)   | 0.35 (0.36)    | 0.01 (0.98)   | −0.29 (0.47)   | −0.40 (0.27)   |
| 2010    | −0.04 (0.92)   | −0.18 (0.64)   | −0.24 (0.53)   | −0.41 (0.29)  | −0.41 (0.29)   | 0.88 (2.1 × 10^{-3}) |
| 2011    | −0.40 (0.13)   | 0.04 (0.88)    | −0.52 (0.04)   | −0.38 (0.17)  | −0.45 (0.06)   | 0.64 (0.01)    |
| 2012    | 0.24 (0.22)    | 0.05 (0.80)    | −0.45 (0.02)   | 0.44 (0.01)   | 0.36 (0.11)    | 0.33 (0.06)    |
| 2013    | −0.15 (0.02)   | 0.20 (1.2 × 10^{-3}) | 0.32 (1.4 × 10^{-3}) | 0.08 (0.19)   | −0.09 (0.15)   | 0.64 (5.6 × 10^{-3}) |
| 2014    | 0.34 (0.31)    | −0.21 (0.54)   | −0.38 (0.25)   | 0.51 (0.12)   | −0.45 (0.18)   | −0.62 (0.05)   |
| 2015    | −0.69 (0.01)   | −0.16 (0.62)   | −0.09 (0.78)   | −0.63 (0.04)  | 0.41 (0.17)    | −0.51 (0.08)   |
| 2016    | 0.40 (0.38)    | −0.53 (0.22)   | 0.49 (0.26)    | 0.35 (0.47)   | 0.37 (0.39)    | 0.71 (0.06)    |

Table 4
Pearson’s Correlation Coefficients in Flaring Events

Note. Column (2): optical flux and polarization degree. Column (3): optical flux and position angle. Column (4): polarization and position angle. Columns (5) and (6): optical flux and normalized Stokes parameters \( \mu \) and \( q \). Column (7): normalized Stokes parameters \( q \) and \( \mu \). In columns (2)–(7), numbers in parentheses are the corresponding \( r \) values.

polarization degree, and position angle were found. The optical flux reached a value of 65.90 mJy in a timescale of \( \Delta t = 2 \) months, the polarization degree went up to 10.65% in \( \Delta t = 2 \) months, and the position angle increased up to 186.8° in \( \Delta t = 4 \) months. (iii) In 2015, the object exhibited the lowest average flux of 17.70 ± 0.12 mJy with the smallest variations of 4.96 ± 0.61 mJy. It is worth noting that during this year, the highest average polarization degree, 6.52% ± 0.09% was found.

Also, it is found that the Stokes parameters are strongly correlated in 2010 and 2016, moderately correlated in 2011 and 2013 (anticorrelated in 2014 and 2015), and weakly correlated in 2012 (anticorrelated in 2008 and 2009).

4. Multiwavelength Data

To analyze the \( R \)-band data in a multiwavelength context, quasi-simultaneous observations from radio to TeV \( \gamma \)-ray bands were used. The complete data set used covers the outstanding flaring events of Mrk 421 that occurred in 2008 May, 2010 March, 2012 July/September, and 2013 April.
**TeV and GeV observations.** VERITAS observed Mrk 421 in 2007–2008 and collected data during 47.3 hr. Details of the data reduction can be consulted in Acciari et al. (2011). The Whipple 10 m telescope observed this source with the ON/OFF and TRK (tracking) modes. The data set in 2010 accounts for a total of 36 hr. Details about the analysis performed and normalization of the photon fluxes can be found in Aleksic et al. (2015). The MAGIC telescope system performed 11 observations during the flaring state of 2010 March. MAGIC collected data during 4.7 hr of this flare. Details on the weather conditions, technical problems, and analysis can be found in Aleksic et al. (2015). With a total exposure of 49 hr, this object was observed for 10 consecutive nights (from 2013 April 10 to 19) during the flare of 2013 April.11 The ARGO-YBJ data used in this work are reported in Bartoli et al. (2016). The GeV -ray fluxes were obtained in the energy range 0.1–300 GeV using the public database of Fermi-LAT.12 The reduction of these data was obtained following Cabrera et al. (2013).

**X-ray observations.** The Swift-BAT/XRT data used in this work are publicly available.13 The XRTE-PCA (ASM) data were obtained from the public MIT archive.14 The MAXI data are publicly available.15

**Additional optical observations.** Several campaigns were performed with the GLAST-AGILE Support Program (GASP) within the Whole Earth Blazar Telescope (WEBT)16 and the New Mexico Skies, Rovor, Blandford, and Perkins telescopes. Optical polarization measurements are shown from the Crimean and St. Petersburg observatories. The Swift-UVOT observations are given in the three bands, UVW1, UVM2, and UVW2, that are available at the HEASARC data archive.17

**Radio observations.** The Metsähovi radio telescope measurements were made at 36.8 GHz. The description of the data reduction and analysis is given in Teräsranta et al. (1998). UMRAO provided data at 4.8, 8, and 14.5 GHz between 2006 June and 2008 May. The calibration and reduction procedures are described in Aller et al. (1985).18 OVRO (Richards et al. 2011) data are publicly available.19 Carma, as a part of the MARMOT20 blazar monitoring project, has performed observations since 2013 February using the eight 3.5 m telescopes of the array with a central frequency of 95 GHz.

### 5. Multiwavelength Data Analysis

In order to analyze all of the multiwavelength data collected for Mrk 421, they have been divided by year. In the following subsections, the most important results of the analysis of the variability behavior around the observed flares are presented.

### 5.1. Flaring Event in 2008 May

The R-band optical light curve, together with the polarization degree and position angle variability presented in Figure 2, shows the variability behavior of Mrk 421 from 2008 April 8 (MJD 54564) to 2008 June 8 (MJD 54620). This figure also shows the TeV -ray, hard/soft X-ray, and radio observed variations. During the first period from 2008 May 4 (MJD 54590) to 8, a bright TeV -ray flux was observed (reaching a level of 10 Crabs) that was not detected in soft/hard X-ray activity, suggesting a possible “orphan flare” (Acciari et al. 2011; Fraija 2015). The TeV flare coincides with a significant drop of 60% in the position angle and a polarization degree of 2%. Subsequently, the TeV high-flux level decreased until it reached the quiescent level, whereas the optical flux continued to increase gradually. The Pearson’s correlation values are reported in Table 5 (column (2)). These values were estimated considering a maximum allowed time difference between data of t ≤ 0.45 days (Acciari et al. 2011). The analysis performed during the period in which the TeV -ray flux began to decline revealed that optical flux is anticorrelated with TeV -rays, hard/soft X-rays, polarization degree, and position angle. Similarly, this analysis showed that the polarization degree and position angle are strongly correlated. Figure 3 shows the correlation between polarization degree and position angle (right panel) and the anticorrelations of the optical flux with the polarization degree and position angle (left panels). The previous result indicates that there is a TeV -ray-emitting region with a high ordered magnetic field that is different from the zone where the optical flux is released (Marscher et al. 2008).

Recently, Ahnen et al. (2016) reported a TeV -ray flare during the second period (from 2008 June 3 to 8). With our data, we found that this TeV flare is strongly correlated with the polarization degree percentage (with a Pearson’s coefficient of 0.88 and p value of 0.02) and without any significant change of the position angle. Assuming this hypothesis is valid, a possible mechanism to explain this variability behavior is by means of perpendicular shocks moving along a uniform–axisymmetric jet axis. In these cases, the jet plasma is compressed, and the degree of polarization varies without significantly altering the value of the position angle (Marscher et al. 2008; Abdo et al. 2010). In addition, using Equations (7) and (8), the optical flux varies on timescales of 15.4 days, and the minimum variability timescale is ~9.6 hr. Both results are similar to those found in Acciari et al. (2011).

### 5.2. Flaring Event in 2010 March

Figure 4 shows the R-band photopolarimetric observations along with TeV -ray, hard/soft X-ray, optical, and radio data. Although there are only four photopolarimetric data points—two obtained on 2010 March 12 (MJD 55267) and two on 2010 March 14 (MJD 55269)—these points (black open triangles) are useful because, between 2010 March 10 (MJD 55265) and 22 (MJD 55277), Mrk 421 exhibited a high activity level in all bands. Considering R-band photopolarimetric data and the maximum allowed time difference t ≤ 0.45 days, the values of the Pearson’s correlation coefficients and p values were computed and reported in Table 5 (column (3)). But it is worthwhile to remember that these correlations are based on only four points. Based on this analysis, the optical flux is found to be correlated with the TeV -rays, hard/soft X-rays,
Figure 2. Mrk 421 light curves, including polarization and position angle data between 2008 April 22 (MJD 54578) and June 8 (MJD 54625), collected with Veritas, Swift, RXTE, OAN-SPM, GASP-WEBT, Metsähovi, and UMRAO. From top to bottom: TeV γ-rays, hard/soft X-rays, optical flux, polarization degree, position angle, and radio flux variations. Dashed vertical lines indicate the two analyzed periods.

Table 5
Pearson’s Correlation Coefficients

| Observables          | \( r_{10} \) (2) | \( r_{12} \) (3) | \( r_{13,\text{ep}1} \) (4) | \( r_{13,\text{ep}2} \) (5) | \( r_{13,\text{ep}3} \) (6) | \( r_{13,\text{ep}4} \) (7) |
|----------------------|-------------------|-------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| \( F_{\text{K}}-F_{\text{TeV}} \) | -0.88 (0.02)      | 0.98 (0.02)       | -0.82 (0.39)                 | 0.09 (0.69)                 | 0.54 (1.6 \times 10^{-5})  | \cdots                     |
| \( F_{\text{K}}-F_{\text{K, soft}} \) | -0.98 (5.9 \times 10^{-4}) | 0.98 (0.02)       | -0.93 (0.24)                 | -0.84 (1.0 \times 10^{-6}) | 0.48 (3.2 \times 10^{-5})  | 1.0 (1.9 \times 10^{-5})   |
| \( F_{\text{K}}-F_{\text{K, hard}} \) | -0.99 (1.5 \times 10^{-4}) | 0.98 (0.02)       | -0.98 (0.13)                 | 0.99 (1.3 \times 10^{-8})  | 0.30 (7.2 \times 10^{-8})  | -0.386 (0.39)              |
| \( F_{\text{K}}-F_{\text{K, 95 GHz}} \) | \cdots            | 0.94 (0.22)       | 0.99 (3.4 \times 10^{-9})    | \cdots                      | 1.00 (1.9 \times 10^{-5})  | \cdots                     |
| \( F_{\text{K}}-P \) | -0.91 (0.01)      | 0.95 (0.05)       | -0.18 (0.88)                 | -0.97 (1.2 \times 10^{-8}) | -0.41 (4.1 \times 10^{-8}) | 0.40 (0.38)                |
| \( F_{\text{K}}-\theta \) | -0.82 (8.9 \times 10^{-4}) | -0.97 (0.03)     | -0.65 (0.96)                 | 0.98 (5.9 \times 10^{-9})  | -0.33 (7.0 \times 10^{-7}) | -0.23 (0.62)               |
| \( \theta-P \) | 0.93 (5.1 \times 10^{-3}) | -0.92 (0.08)     | 0.99 (0.05)                  | -0.96 (6.7 \times 10^{-8}) | 0.47 (5.3 \times 10^{-8})  | -0.51 (0.24)               |
| \( F_{\text{K}}-u \) | -0.37 (0.46)      | -0.96 (0.02)      | -0.03 (0.98)                 | 0.97 (6.1 \times 10^{-8})  | -0.27 (1.5 \times 10^{-8}) | 0.56 (0.16)                |
| \( F_{\text{K}}-q \) | -0.84 (0.04)      | -0.93 (0.06)      | -0.73 (0.46)                 | -0.62 (1.3 \times 10^{-3}) | -0.36 (2.0 \times 10^{-8}) | 0.11 (0.83)                |
| \( u-q \) | 0.11 (0.85)       | 0.92 (0.07)       | -0.64 (0.57)                 | -0.73 (1.6 \times 10^{-4}) | 0.76 (3.3 \times 10^{-5})  | -0.62 (0.13)               |

Note. Here \( F_{\text{TeV}}, F_{\text{K, soft}}, F_{\text{K, hard}}, \) and \( F_{\text{K}} \) are the TeV γ-rays, soft X-ray, hard X-ray, and radio fluxes, respectively. Column (2): 2008 May. Column (3): 2010 March. Columns (4)–(7): 2013 April for periods 1, 2, 3, and 4. In columns (2)–(7), numbers in parentheses are the corresponding \( p \) values.
and polarization degree and anticorrelated with the position angle and Stokes parameters. During this flare, the polarization degree and position angle exhibited variations: 4% in the polarization degree and 10° in the position angle.

5.3. Flaring Event in 2012

The $R$-band photopolarimetric observations complemented with TeV $\gamma$-ray, hard/soft X-ray, and radio light curves are shown in Figure 5. The light curves include the period from 2012 February 18 (MJD 55975) and 2012 May 23 (MJD 56070). During this period, Mrk 421 shows moderate activity in some wavebands (e.g., the optical flux varied from 40.96 to 11.81 mJy in $\Delta t = 9$ days when the polarization degree and position angle showed variations of $\sim 0.3\%$ and $\sim 137°$, respectively). Afterward, this source went into a long-lasting outburst phase starting on 2012 July 9 (MJD 56117) and ending on 2012 September 17 (MJD 56187), when this object was not visible with optical telescopes. Only some instruments, including ARGO-YBJ (Bartoli et al. 2012), Fermi (D’Ammando & Orienti 2012), BAT, NuSTAR (Baloković et al. 2016), and OVRO (Hovatta et al. 2012; Hovatta et al. 2015) could observe the Mrk 421 outburst. Therefore, the Fermi collaboration reported the highest flux observed by this source since the beginning of the Fermi mission (D’Ammando & Orienti 2012), and ARGO-YBJ also detected a high flux level (Bartoli et al. 2012). Although this flare was intensively followed by Fermi-LAT and ARGO-YBJ, the MAXI-GSC and Swift-BAT instruments did not report the same high activity in the X-ray bands.

5.4. Flaring Event in 2013 April

The multiwavelength light curves including polarization degree and position angle variations from 2013 January 28 (MJD 56320) to 2013 June 17 (MJD 56460) are presented in Figure 6. This strong flare was widely monitored in several energy bands and was observed from 2013 April 9 (MJD 56391) to 19 in the TeV (Cortina & Holder 2013), GeV (Paneque et al. 2013), hard/soft X-ray (Balokovic et al. 2013; Negoro et al. 2013; Pian et al. 2014), and optical (Semkov et al. 2013; Fraija et al. 2015) bands. A blow-up of the light curves around the flare is shown in Figure 7. The $R$-band optical light curve, together with the polarization degree and position angle shown in this figure, includes 235 observations that were obtained during 10 consecutive nights. The photopolarimetric data were divided into three periods in accordance with the activity state of the source. These periods were defined by considering the TeV and X-ray data. The lowest activity state was defined by taking into account the fluxes observed on April 18 and 19 in both bands. This is shown by the red dashed lines in Figure 7. These lines indicate the average fluxes ($0.11 \pm 0.04 \times 10^{-9} \text{cm}^{-2} \text{s}^{-1}$ and $37.26 \pm 0.04 \text{counts cm}^{-2} \text{s}^{-1}$) for the TeV and X-ray bands, respectively. Therefore, high-activity states are defined when data are $3\sigma$ above the red dashed lines: the first period marks the high activity level, the second one marks the moderate activity level, and the third period shows the decreasing activity level.

First period: From 2013 April 10 (MJD 56392) to 13. The strongest flare was observed. The TeV $\gamma$-ray and optical fluxes were the highest ever recorded for this object (Cortina & Holder 2013). In the optical $R$-band, the flux reached an unprecedented value of $93.60 \pm 1.53 \text{mJy}$ ($11.29 \pm 0.03 \text{mag}$), which corresponds to an optical luminosity of $L_R \approx 9.3 \times 10^{44} \text{erg s}^{-1}$. The results of the statistical analysis of these data are presented in Table 8. It can be seen that while a difference in flux level of $\Delta F = 39.78 \text{mJy}$ was registered, no significant variations were found in the polarization degree and position angle; i.e., these parameters varied $\sim 1\%$ and $\sim 10^\circ$ over their average values, respectively.

Column (4) in Table 5 shows the Pearson’s correlation coefficients among TeV $\gamma$-ray, hard/soft X-ray, optical and radio fluxes, polarization degree, position angle, and Stokes parameters by considering the maximum allowed time difference $\Delta t \leq 0.45 \text{days}$. The analysis performed indicates that the optical $R$ band is strongly anticorrelated with the TeV $\gamma$-rays, hard/soft X-ray bands, and Stokes parameter $q$ and correlated with the radio wavelength, although high activity was not detected in this band. A strong correlation between the polarization degree and position angle was found, and a moderate correlation was found between the Stokes parameters.

Second period: From 2013 April 13 (MJD 56395) to 15. During this period, the object showed a moderate activity level. The statistical analysis is also reported in Table 8. Whereas no significant variations were found in the polarization degree, since it varied only $\sim 1\%$ over its average value, the position angle displays a long rotation of $\sim 100^\circ$. Column (5) in Table 5 shows the Pearson’s correlation coefficients among TeV $\gamma$-ray,
hard/soft X-ray, optical and radio fluxes, polarization degree, position angle, and Stokes parameters for a maximum allowed time difference of $\Delta t \leq 0.45$ days. The analysis indicates that the optical $R$ band is strongly anticorrelated with the soft X-ray band and correlated with the hard X-ray band, as well as the radio waveband (GHz). In addition, the optical flux is correlated with the position angle and Stokes parameter $U$, and the polarization degree is anticorrelated with the optical flux and position angle. Their respective Pearson’s coefficient values are reported in Table 5 (column (4)).

**Third period: From April 15 (MJD 56397) to 19.** From the multiwavelength light curves (Figure 7), it can be seen that TeV $\gamma$-ray, hard/soft X-ray, and optical fluxes show a tendency toward a decreasing activity level. No correlation was found among fluxes, polarization degree, position angle, and Stokes parameters (see column (6) in Table 5). Figure 7 shows that...
while the value of the position angle had very small variations around $\sim 185^\circ$, the polarization degree increased substantially, reaching a value of 8.33% around 2013 April 18 (MJD 56400). After this value, it started decreasing again.

5.5. R-band Intranight Polarimetric Variability Observations

Due to the high activity level observed during 2013 April in Mrk 421 (first period), a photopolarimetric monitoring campaign dedicated to detecting intranight variations started 3 days after the maximum flux observed in the R-band data. This monitoring has a cadence of $\sim 1$ day. It was performed during seven consecutive nights with a duration that ranged from 1 to 5 hr. The intranight variability observations were done from 2013 April 13 to 19 at the OAN-SPM and include 14, 14, 53, 35, 28, 33, and 58 observational points per night, respectively. In Figure 8, the results of the R-band differential photometry $\Delta m_R$ are shown. In Figure 9, the corresponding polarimetric intranight variations are shown. In these figures, there are clear hints of possible intranight variability in some nights. To confirm this, a statistical analysis of the data was done using the $F$ and ANOVA tests and following de Diego (2010, 2014). On the one hand, for the $F$-test, a star with a magnitude that is only 0.08 mag brighter than Mrk 421 was used; i.e., both objects have virtually the same photometric errors. On the other hand, ANOVA compares the dispersion between different data groups drawn out from the light curve of a single object, rather than comparing between different light curves of different objects. The detection limit of intranight variability was set at a level of significance $\alpha = 10^{-3}$, i.e., a probability of 0.1% that the light curve is obtained from nonvariable sources. However, a significance $\alpha = 10^{-2}$ is considered as a hint of possible variations. The values obtained and derived from the variability statistical analysis are reported in Table 6, where the $p$ value can be found in column (9). Hence, using the $F$ and ANOVA tests, intranight variability was found in the R-band differential magnitudes and in polarimetry (degree and position angle) during four nights (2013 April 14, 15, 16, and 19). Intranight variations are a valuable tool for estimating the minimum variability timescale per night using the photometric data. In column (10), the minimum variability timescale per night is given, and the minimum value was found on April 15, where it has a value of $2.34 \pm 0.12$ hr.

In order to search for correlations between variations in flux, polarization degree, and position angle, the Stokes parameters...
were plotted, as shown in Figure 10. The values of the slopes obtained from the fitting and the Pearson’s correlation coefficients are reported in Table 7.

Figure 11 displays the intranight variations of the electron power index as a function of time (Δt ~ minutes) and the different fractions of ordered and chaotic magnetic fields. The electron power indexes were derived from the polarization degree (see Equation (6)) during the flaring activity presented between 2013 April 13 (MJD 56395) and 19. Table 7 shows the average spectral index per night for three values of the magnetic field ratio (β = 0.1, 0.15, and 0.20). A description per day of the Stokes parameter correlations with the intranight variations of optical flux, polarization degree, and position angle is given in the following paragraphs. Additionally, the maximum changes in the values of the spectral index and timescales are reported in Table 7.

2013 April 13. With less than 2 hr of observations during this night, the optical flux, polarization degree, and position angle display small variations of 0.4 mJy, 0.2%, and 10°, respectively. The Stokes parameters were moderately anticorrelated. Very fast spectral index variations were detected during this night. The maximum changes in the spectral index were 0.85, 0.54, and 0.38 for β = 0.1, 0.15, and 0.20, respectively, on a timescale of ~11 minutes.

2013 April 14. Variations of the optical flux were found during this night. It varied by 57.34 mJy, whereas the polarization degree changed by only 1.10%. No significant correlation was found between the Stokes parameters. The maximum changes in the spectral index were 0.87, 0.55, and 0.37 for β = 0.1, 0.15, and 0.20, respectively, on a timescale of ~39 minutes.

2013 April 15. During this night, the optical flux, polarization degree, and position angle showed equal variations. No significant correlation was found between the Stokes parameters. The maximum changes in the spectral index were 2.62, 1.68, and 1.19 for β = 0.1, 0.15, and 0.20, respectively, on a timescale of ~4.6 hr.

2013 April 16. With less than 5 hr of observations during this night, the levels of the optical flux and position angle increased, whereas the polarization degree decreased. A strong anticorrelation was found between the Stokes parameters. The maximum changes in the spectral index were 2.14, 1.38, and
0.9 for $\beta = 0.1$, 0.15, and 0.20, respectively, on a timescale of $\sim 2.5$ hr.

2013 April 17. With almost 3 hr of observations during this night, the levels of the optical flux and position degree increased slowly, whereas the position angle had small changes around its average. A moderate correlation was found between the Stokes parameters. The maximum changes in the spectral index were 1.90, 1.24, and 0.89 for $\beta = 0.1$, 0.15, and 0.20, respectively, on a timescale of $\sim 3.2$ hr.

2013 April 18. The highest mean polarization value of 6.86% $\pm$ 0.03% was observed, whereas the position angle had small variations. No significant correlation was found between the Stokes parameters. The maximum changes in the spectral index were 2.26, 1.49, and 1.10 for $\beta = 0.1$, 0.15, and 0.20, respectively, on a timescale of $\sim 4.6$ hr.

2013 April 19. During this night, the polarization degree and position angle decreased. A strong correlation was found between the Stokes parameters. The maximum changes in the spectral index were 3.37, 2.20, and 1.59 for $\beta = 0.1$, 0.15, and 0.20, respectively, on a timescale of $\sim 3.5$ hr.

Subsequently, from 2013 May 10 (MJD 56422) to 17 (MJD 56429), a new flare was observed only in the $R$ band and radio wavebands, as shown in Figure 6. Mrk 421 showed the second brightness state of $64.49 \pm 1.08$ mJy ($11.70 \pm 0.04$ mag), which corresponds to an optical luminosity of $L_R \approx 6.4 \times 10^{44}$ erg s$^{-1}$. Moreover, the highest polarization degree value of 11.00% $\pm$ 0.18% in the $R$ band was detected. In Table 5 (column (7)), the Pearson’s correlation coefficients among fluxes, polarization degree, position angle, and Stokes parameters are shown for a maximum allowed time difference of $\Delta t \leq 0.45$ days. The values of these coefficients indicate that the soft X-ray, optical, and radio fluxes are correlated. In the high-energy bands, no significant correlations were found. It is worth noting that although the soft X-ray flux was correlated with the optical and radio wavebands, no high activity was detected. The minimum variability timescale found in this period was $\sim 7$ days.

6. Analysis and General Results

6.1. Two Emitting Zones

A two-emitting-region model has been proposed for several blazars in order to describe the broadband SED in flaring and quiescent states. Moreover, the superposition of two polarized...
Figure 8. The $R$-band differential photometry data from 2013 April 13 to 19. Each of the seven panels shows the changes in the differential $R$-band magnitudes. The duration of the monitoring in hours from the beginning to the end for each night is shown on the $X$-axis.
Figure 9. Intranight polarimetric data from 2013 April 13 to 19. Each of the seven panels shows the polarization degree (top) and position angle (bottom) variations. The duration of the monitoring in hours from the beginning to the end for each night is shown on the X-axis.
components (one constant and the other variable) coming from different zones has been widely suggested to describe the variability behavior (Holmes et al. 1984; Brindle 1996; Qian 1993). Following Holmes et al. (1984), the resulting polarization degree and the position angle of the two optically thin synchrotron-emitting regions can be written as

\[ p^2 = \frac{P_{\text{con}}^2 + P_{\text{var}}^2 I_{c/j}^2 + 2P_{\text{con}} P_{\text{var}} I_{c/j} \cos 2\xi}{(1 + I_{c/j})^2} \]

and

\[ \tan 2\theta = \frac{P_{\text{con}} \sin 2\theta_{\text{con}} + P_{\text{var}} I_{c/j} \sin 2\theta_{\text{var}}}{P_{\text{con}} \cos 2\theta_{\text{con}} + P_{\text{var}} I_{c/j} \cos 2\theta_{\text{var}}} \]

respectively, where \( \xi = \theta_{\text{con}} - \theta_{\text{var}} \) and \( I_{c/j} = F_{\text{var}}/F_{\text{con}} \) is the flux ratio of different zones, one variable and one constant. Taking into account the obtained average Stokes parameters \( U_c = (1.65 \pm 0.01) \) and \( Q_c = (0.35 \pm 0.01) \) mJy and following Jones et al. (1985), the constant values found are \( P_{\text{con}} = 3.25\% \pm 0.70\% \) and \( \theta_{\text{con}} = 219^\circ 01 \pm 0^\circ 02 \). Considering the limit cases \( F_{\text{var}} > F_{\text{con}} \) and \( F_{\text{var}} < < F_{\text{con}} \), the resulting polarizations are \( P \approx P_{\text{var}} \) and \( P \approx P_{\text{con}} \), respectively.

The previous results suggest that the optical emission observed during the 2013 flares could be produced in two different regions through the injection of additional high-energy electrons.

The first region is related to the flare observed in 2013 April, and the second is related to the flare detected in 2013 May. The optical flux associated with the emitting region that originates the TeV \( \gamma \)-rays and X-rays has a polarization degree of \( P_{\text{con}} = 3.25\% \pm 0.70\% \) with small variations, and the region associated with the emission observed in optical and radio wavebands has a variable polarization degree. Zhang & Böttcher (2013) presented a full three-dimensional radiation transfer code, considering a helical magnetic field and geometry effects through the jet. Synchrotron radiation in this code is assumed to come from an ordered magnetic field, and SSC is handled with the two-dimensional Monte Carlo/Fokker–Planck code. Using several scenarios for leptonic models in flaring events, they showed that depending on the energy range analyzed, the blazar Mrk 421 had different behaviors in the polarization degree and position angle. In particular, in the scenario for the injection of additional high-energy electrons, the polarization properties at lower energies had small variations (quasi-constant). At higher energies, the polarization degree was expected to vary significantly. This scenario would show that two different electron populations could describe the flares observed in 2013 April and later in May. The additional high-energy electrons had cooled sufficiently to make the former electron population dominant again.

It is common to suggest that the low polarization degree observed in Mrk 421 might be associated with the emission originated from the accretion disk or emitting regions outside of the jet (Jermak et al. 2016). However, it is shown that both emitting optical fluxes are originated by synchrotron radiation where one flux is much more polarized than the other.
Figure 10. Normalized Stokes parameter correlations found in the $u$–$q$ plane from 2013 April 13 to 19.
Note. Columns (2) and (3) show the slope and Pearson’s correlation coefficients. Columns (4)–(6) are the average values of the electron power indexes.

### Table 8

| Parameter | Max (1) | Min (2) | Average (3) | Y (%) (4) | μ (%) (5) | \( \mathcal{F} \) (6) |
|-----------|---------|---------|-------------|-----------|----------|----------|
| \( m_{\nu, w} (\chi^2) \) | | | | | | |
| \( \tau_{\nu, w} (p \text{ value}) \) | | | | | | |
| \( \alpha_x \) | | | | | | |
| \( \beta = 0.1 (\chi^2) \) | | | | | | |
| \( \beta = 0.15 (\chi^2) \) | | | | | | |
| \( \beta = 0.20 (\chi^2) \) | | | | | | |

First period: From 2013 April 10 (MJD 56392) to 13

| Parameter | Max | Min | Average | Y (%) | μ (%) | \( \mathcal{F} \) |
|-----------|-----|-----|---------|-------|-------|-------|
| \( F_R \) (mJy) | 93.60 ± 3.53 | 53.82 ± 2.11 | 69.99 ± 0.69 | 56.72 | 0.99 | 0.27 |
| \( P \) (%) | 3.77 ± 0.17 | 1.86 ± 0.18 | 2.59 ± 0.10 | 72.59 | 3.87 | 0.34 |
| \( \theta (\text{deg}) \) | 154.9 ± 1.1 | 137.7 ± 1.8 | 144.93 ± 0.86 | 11.69 | 0.59 | 0.06 |

Second period: From 2013 April 13 (MJD 56395) to 15

| Parameter | Max | Min | Average | Y (%) | μ (%) | \( \mathcal{F} \) |
|-----------|-----|-----|---------|-------|-------|-------|
| \( F_R \) (mJy) | 57.34 ± 2.23 | 48.90 ± 1.98 | 51.64 ± 0.19 | 15.95 | 0.37 | 0.08 |
| \( P \) (%) | 2.98 ± 2.4 | 2.19 ± 0.18 | 1.97 ± 0.04 | 73.42 | 2.02 | 0.40 |
| \( \theta (\text{deg}) \) | 190.1 ± 2.0 | 94.2 ± 0.16 | 119.48 ± 0.50 | 55.40 | 0.42 | 0.26 |

Third period: From April 15 (MJD 56397) to 19

| Parameter | Max | Min | Average | Y (%) | μ (%) | \( \mathcal{F} \) |
|-----------|-----|-----|---------|-------|-------|-------|
| \( F_R \) (mJy) | 55.92 ± 2.31 | 47.00 ± 2.11 | 51.74 ± 0.06 | 16.89 | 0.12 | 0.09 |
| \( P \) (%) | 7.41 ± 0.17 | 1.81 ± 0.16 | 3.84 ± 0.01 | 171.08 | 0.33 | 0.68 |
| \( \theta (\text{deg}) \) | 194.7 ± 1.7 | 185.9 ± 1.4 | 185.63 ± 0.11 | 13.48 | 0.06 | 0.07 |

From 2013 May 10 (MJD 56422) to 17 (MJD 56429)

| Parameter | Max | Min | Average | Y (%) | μ (%) | \( \mathcal{F} \) |
|-----------|-----|-----|---------|-------|-------|-------|
| \( F_R \) (mJy) | 64.69 ± 2.49 | 47.77 ± 1.91 | 53.02 ± 0.35 | 31.33 | 0.65 | 0.15 |
| \( P \) (%) | 11.00 ± 0.18 | 7.16 ± 0.19 | 8.92 ± 0.07 | 42.85 | 0.80 | 0.21 |
| \( \theta (\text{deg}) \) | 217.8 ± 0.8 | 190.9 ± 0.8 | 200.59 ± 0.33 | 13.39 | 0.16 | 0.07 |

Note. The parameters \( Y \) (%), \( \mu \) (%), and \( \mathcal{F} \) are the amplitude of the variations, the fluctuation index, and the fractional variability index.

### 6.2. Modeling the SED

The most widely adopted scenario to explain the broadband SED of HBL objects with the fewest parameters is the one-zone SSC model. In this context, the electrons within the emitting region are moving at ultra-relativistic velocities in a collimated jet and are confined in this region by the magnetic field. Then, photons are radiated via synchrotron emission and upscattered to higher energies by the same electron population via inverse Compton scattering (Fraija et al. 2012; Fraija 2014a). Photons from radio to X-ray bands are interpreted as synchrotron radiation, and the \( \gamma \)-rays are described by inverse Compton emission. After the Fermi-LAT era, the best way to fit the SED of Mrk 421 was using a broken power law (two power-law functions). During the Fermi-LAT multiband campaign in 2008–2010, it was found that a simple broken power law did not adequately fit the broadband SED (Abdo et al. 2011). In this work, three power-law functions (i.e., two breaks) are used to describe the electron population. The double-break power laws can be written as

\[
\frac{dn_e}{d\gamma_e} = N_{0,e} \begin{cases} 
\gamma^{-\alpha_1} & \gamma_e < \gamma_e \leq \gamma_{c1} \\
\gamma^{-\alpha_2} \gamma_0^{\alpha_1-\alpha_2} & \gamma_{c1} < \gamma_e \leq \gamma_{c2} \\
\gamma^{-\alpha_3} \gamma_0^{\alpha_1-\alpha_2-\alpha_3} & \gamma_{c2} < \gamma_e \leq \gamma_{\text{max}}
\end{cases}
\]

(14)

where \( N_{0,e} \) is the number density of electrons; \( \alpha_1, \alpha_2, \text{and} \alpha_3 \) are the spectral indexes, and \( \gamma_{\text{min}}, \gamma_{c1}, \gamma_{c2}, \text{and} \gamma_{\text{max}} \) are the electron Lorentz factors for minimum, break (1 and 2), and maximum, respectively. It is worth noting that the two broken power laws have three more parameters than a single power law. In order to describe the SED, the one-zone SSC model used in this work follows the methodology presented by Fraija & Marinelli (2016) with some modifications related to the double-break.
power laws. The observed synchrotron spectrum (with an additional power law to that shown in Fraija & Marinelli 2016) is obtained through Equation (14) and emissivity \( \epsilon(E, \epsilon_c) d\epsilon_c = (-dE_c/\mu) N(\epsilon_c) dE_c \) (Rybicki & Lightman 1986; Longair 1994). For the VHE \( \gamma \)-ray fluxes, the effect of the extragalactic background light absorption modeled by Franceschini et al. (2008) is introduced. We did the \( \chi^2 \) minimization using the ROOT software package (Brun & Rademakers 1997; Fraija 2014b) to fit the data and get the values of the Doppler factor, the size of the emitting region, the electron number density, and the strength of the magnetic field. Additionally, the Bezier function was used to smooth the SEDs.\(^{21}\) With the previous values obtained from the best fit, we estimated several physical properties of Mrk 421 in distinct

\(^{21}\) gnuplot.sourceforge.net

Figure 11. Intrnight variations of the electron power index as a function of time (\( \Delta t \sim \) minutes) and different fractions of ordered and chaotic magnetic fields.
states: the total jet power $L_{\text{jet}}$ and energy densities carried by electrons $U_e$, magnetic field $U_B$, and protons $U_p$. The total jet power is defined by (Celotti & Ghisellini 2008)

$$L_{\text{jet}} = \sum_{i=e,p,B} L_i,$$

where $L_i = \pi r_i^2 \Gamma^2 U_i$, with $U_i = m_i N_i \langle \gamma_i \rangle = m_i \int_{\gamma_{\text{min}}}^{\gamma_{\text{max}}} \frac{d\gamma}{d\gamma} \Gamma \gamma_i$. $U_p = N_p m_p$, and $U_B = \frac{B^2}{8\pi}$. Here $m_p$ is the proton mass and $\Gamma \approx \delta_D$.

Figure 12 shows the fit of the observed SED of Mrk 421 during the flaring events in 2008 May (top panels) and 2010 March (middle panels) and the quiescent states (bottom panels) from 2008 August 5 (MJD 54683) to 2009 June 18 (MJD 55000) and from 2010 November 16 (MJD 55516) to 2012 June 28 (MJD 56106). This figure shows that the one-zone SSC model generated by the electron population described through the double-break power laws is successful in describing the SED during the flaring and quiescent states. The homogeneous one-zone SSC model was used in each panel, and the best-fit parameters are reported in Table 9. This table displays all the parameter values obtained and derived in and from the fit for the values of spectral indexes $\alpha_1 = 2.2$, $\alpha_2 = 2.7$, and $\alpha_3 = 4.7$ and for $\gamma_{\text{min}} = 800$ (Abdo et al. 2011). The values reported in Table 9 after fitting are in the range of those obtained by Abdo et al. (2011) and Aleksic et al. (2015b). From the values of Doppler factors and emitting radii reported in Table 9, the variability timescales (Equation (3)) are $\sim 9.6$ and 18.2 hr for the flaring events of 2008 May and 2010 March, respectively, and $\sim 24$ hr for the quiescent states. The large values of $\gamma_{\text{min}}$ used in this work to describe the SEDs imply that electrons are efficiently accelerated by the Fermi mechanism only above this energy, and below this energy they are accelerated by a different mechanism that produces the hard electron distribution. The proton luminosity is computed using the charge neutrality condition to justify a comparable number of electrons and protons ($N_e = N_p$; Sikora et al. 2009; Abdo et al. 2011; Böttcher et al. 2013; Petropoulou et al. 2014; Fraija et al. 2017). The value of the proton luminosity found corresponds to a small fraction ($\sim 10^{-2}$) of the Eddington luminosity $L_{\text{Edd}} \sim 2.5 \times 10^{46}$ erg s$^{-1}$ that is estimated by considering the value of the supermassive black hole ($2 \times 10^8 M_\odot$; Barth et al. 2003). The cooling Lorentz factor for radiative electrons $\gamma_e = 6\pi m_e \langle \sigma_T (1 + Y) B^2 \rangle / m_e$ is obtained by equaling the dynamical and synchrotron cooling times. The maximum electron Lorentz factor $\gamma_{\text{max}} = (3q_e/\langle \sigma_T \rangle)^{1/2} B^{-1/2}$ is calculated by equaling the acceleration and cooling timescales. Here, $Y$ is the Compton parameter as defined in Fraija & Marinelli (2016), and $\rho_d$ is defined in Equation (3). Comparing the values of $\gamma_e$ with $\gamma_{\text{cl}}$ and $\gamma_{\text{c2}}$ shows that the second break in the electron population described by the double-break power law is similar to the second-break Lorentz factor $\gamma_{\text{cl}}$, probably due to synchrotron cooling. As suggested by Abdo et al. (2011), the first break $\gamma_{\text{cl}}$ is associated with the acceleration mechanism, and energetic electrons above this break are accelerated less efficiently. Taking into account the values of the magnetic field, electron and proton luminosities, and their ratios $\lambda_j = L_j / L_i$, a principle of equipartition could be present in the jet of Mrk 421 in order to relate these luminosities. The emitting radius found is of the order of $\sim 10^{18}$ cm, which is three orders of magnitude larger than the gravitational radius $r_g = 5.9 \times 10^{13}$ cm. It is worth noting that the electron energy density is one order of magnitude larger than the magnetic energy density, as reported previously by Abdo et al. (2011) and Aleksic et al. (2012, 2015b).

6.3. Optical–Radio Comparison

Figure 6 shows the optical flares of 2013 April, followed by the increase of the polarization degree and radio flux in 2013 May. The polarization degree increased from $\sim 2\%$ (in April, first period) to its largest value observed in the $R$ band of $11\%$ (in 2013 May, during the flare) and also brightened at radio wavelengths. A similar variability behavior was observed by Tosti et al. (1998) during the monitoring study done from 1994 October to 1997 June. A large optical flare occurred from 1996 November 13 to 1997 March 15 and was followed by a radio flare with a delay of 30–60 days. In the same observational period, the polarization level increased to the largest value observed: $\sim 12\%$ in the $V$ band.

In Figure 13, the superposition of the optical ($V$ and $R$ bands) and radio ($22$ and $95$ GHz) flares observed in 1996–1997 (black points) and 2013 April (magenta points) are shown. The figure clearly shows that the polarization degree (from both epochs) increased to one of the highest values ever observed.

In order to compare with the data obtained by Tosti et al. (1998), $R$-band average flux values per run were obtained and are shown as blue points in the figure. The variability behavior of the radio flux (bottom panel) and the polarization degree (middle panel) present a similar structure. For instance, the radio fluxes at 95 and 22 GHz (bottom panel) present, first, variations around average values of $0.512 \pm 0.003$ and $0.514 \pm 0.017$, respectively, and second, a rise to a peak followed by a decay. The slopes of rise and decay are $(4.26 \pm 0.01) \times 10^{-3}$ and $(-3.62 \pm 0.01) \times 10^{-3}$ for the radio flux at 95 GHz and $(2.54 \pm 0.68) \times 10^{-3}$ and $(-3.19 \pm 0.65) \times 10^{-3}$ for the radio flux at 22 GHz. Although the optical $R$-band flux (top panel) is much higher than that of the $V$ band, they vary in a similar pattern. In this figure, it is observed that both flares present a temporal difference of $\sim 16.34$ yr.

6.4. Periodicity Analysis of Long-term Light Curves

In this section, the historical light curve of Mrk 421 obtained from 1900 January 29 to 1991 January 10, which was built by Liu et al. (1997), will be used along with our optical data in order to look for periodical variations (see Figure 14). Historical data were obtained in the optical $B$ band. It is worth noting that optical fluxes from different bands (in particular, $R$ and $B$) are found to be clearly correlated (e.g., Tosti et al. 1998; Horan et al. 2009; Acciari et al. 2011; Aleksic et al. 2015b).

Periodograms of the long-term Mrk 421 light curves were obtained by using two different methods: Lomb–Scargle (Scargle 1982) and RobPer (Benítez et al. 2015; Thieler et al. 2016).

Using the historical optical data, an analysis with and without new data was done. The periods found without the new data are $23.8 \pm 2.1$ and $15.9 \pm 1.3$ yr. These periods are similar to those found by Liu et al. (1997), which are $23.1 \pm 1.1$ and $15.3 \pm 0.7$ yr; however, for these authors, the $\sim 23$ yr period is more statistically significant. Adding our data, the $\sim 16$ yr period result is more statistically significant. The periods found with the new data are displayed in Figure 15. This figure shows the analysis done with the Lomb–Scargle (left panel) periodogram and the $R$ package RobPer (right panel). The analysis with the Lomb–Scargle and RobPer methods provides the same periodicities of 1365 and 5935 days. The periods of 1 day and 365 days are due to the window function (Dawson & Fabrycky 2010).
Therefore, only the period of $5935 \pm 650$ days is real, and it is in agreement with one of the periods found by Liu et al. (1997). This period naturally explains the similar flares observed in the radio and optical bands of 1996–1997 and 2013, i.e., $\approx 16$ yr later.

6.5. $V$ and $R$ Spectral Index Variability Analysis

Figure 16 shows the intranight spectral index variations as a function of time for different values of $\beta$. In this figure, the $V$ and $R$ optical bands are considered when Mrk 421 exhibited unprecedented flaring activity from April 9 (MJD 56391) to 19. The $V$-band optical data used in this section are reported in Sinha et al. (2016). Electrons in a magnetized plasma are usually cooled by synchrotron radiation, then cooling timescales of electrons in a few hours could be related to variations in the $V$ and $R$ optical bands. In fact, the changes in the $V$ and $R$ optical bands, together with the observed intranight flux variability, suggest that the acceleration timescale is less than

Figure 12. The one-zone SSC model was used to fit the SEDs of Mrk 421 during the flares observed in 2008 May, 2010 March, and the quiescent states from 2008 August 5 (MJD 54683) to 2009 June 18 (MJD 55000) and from 2010 November 16 (MJD 55516) to 2012 June 28 (MJD 56106). The best-fit parameters are reported in Table 9.
Table 9

Leptonic Model Parameters

| Obtained Quantities | MJD 54590 (2008 May 4) | MJD 54591 (2008 May 5) | MJD 54594 (2008 May 8) | MJD 55268 (2010 March 12) | MJD 55270 (2010 March 14) | Quiescent 1 | Quiescent 2 |
|---------------------|-------------------------|------------------------|------------------------|--------------------------|--------------------------|-------------|-------------|
| $\delta_D$          | 24                      | 24                     | 24                     | 22                       | 22                       | 20          | 20          |
| $B$ ($\times 10^{-3}$ G) | 55.3                    | 54.6                   | 54.9                   | 43.3                     | 43.1                     | 34.3        | 34.1        |
| $r_d$ ($\times 10^{16}$ cm) | 2.5                     | 2.5                    | 2.5                    | 4.3                      | 4.3                      | 5.0         | 5.0         |
| $N_e$ ($\times 10^{-1}$ cm$^{-3}$) | 1.9                     | 1.7                    | 1.6                    | 2.6                      | 2.3                      | 1.2         | 1.0         |
| Derived Quantities  |                         |                        |                        |                          |                          |             |             |
| $\gamma_c$ ($\times 10^2$) | 2.7                     | 2.8                    | 2.8                    | 2.6                      | 2.6                      | 3.6         | 3.7         |
| $\gamma_{c1}$ ($\times 10^2$) | 5.8                     | 5.8                    | 5.8                    | 2.7                      | 2.7                      | 7.8         | 7.8         |
| $\gamma_{c2}$ ($\times 10^2$) | 3.1                     | 3.1                    | 3.1                    | 2.8                      | 2.8                      | 4.0         | 4.0         |
| $\gamma_{c,max}$ ($\times 10^2$) | 1.2                     | 1.2                    | 1.2                    | 1.4                      | 1.4                      | 1.6         | 1.6         |
| $U_e/U_B$ ($\times 10^3$) | 1.1                     | 1.0                    | 0.9                    | 2.2                      | 2.0                      | 1.7         | 1.4         |
| $L_e$ ($\times 10^{43}$ erg s$^{-1}$) | 1.7                     | 1.5                    | 1.4                    | 5.7                      | 5.1                      | 3.0         | 2.5         |
| $L_B$ ($\times 10^{42}$ erg s$^{-1}$) | 4.1                     | 4.0                    | 4.1                    | 6.3                      | 6.2                      | 4.4         | 4.4         |
| $L_p$ ($\times 10^{43}$ erg s$^{-1}$) | 1.0                     | 0.9                    | 0.8                    | 3.3                      | 2.9                      | 1.7         | 1.4         |
| $L_{jet}$ ($\times 10^{43}$ erg s$^{-1}$) | 3.1                     | 2.8                    | 2.6                    | 9.6                      | 8.6                      | 5.1         | 4.4         |

Note. Here $\delta_D$ is the Doppler factor, $B$ is the magnetic field, $r_d$ is the size of the emitting radius, $N_e$ is the electron number density, $\gamma_c$ is the electron-cooling Lorentz factor, and $\gamma_{c1}$, $\gamma_{c2}$, and $\gamma_{c,max}$ are the electron Lorentz factors for the breaks (1 and 2) and maximum. In addition, $U_e$ and $U_B$ are the densities carried by electrons and the magnetic field; $L_e$, $L_B$, and $L_p$ are the electron, magnetic field, and proton luminosities; and $L_{jet}$ is the total jet power. Quiescent state 1: from 2008 August 5 (MJD 54683) to 2009 June 18 (MJD 55000). Quiescent state 2: from 2010 November 16 (MJD 55516) to 2012 June 28 (MJD 56106).
the cooling timescale in the emitting region, and the particle acceleration occurs at the shock front with a magnetic field produced by the plasma compression (Marscher & Gear 1985). Considering a variability timescale of 5 hr ($\tau_{\text{var}}$ $\sim$ 5 hr; see Table 6) and a typical value of Doppler factor $\delta_D$ $\sim$ 20 (see Table 9), the magnetic field and emitting region become $B \lesssim 1.2$ G and $r_d = 9.8 \times 10^{15}$ cm, respectively. These values are in agreement with those reported by Abdo et al. (2011).

Considering the scattering of the polarization degree $\delta P$ and the emitting region $r_d$, the coherence length of the large-scale field can be estimated as (Jones et al. 1985)

$$l_B = \left( \frac{k \Pi_0 \delta P}{r_d^{3/2}} \right)^{-2/3}. \quad (15)$$

Taking into account the values of $\delta P \sim 2\%$ (see Figure 16 for $\alpha = 2.2$), $k = 0.5$ (Sorcia et al. 2013), $\Pi_0 = 0.705$, and $r_d \approx 9.8 \times 10^{15}$ cm, the coherence length is $l_B = 0.26$ pc. Taking into account the intranight variation associated with the distance traveled by the relativistic shocks, we can compare the

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**Figure 14.** Historical light curve of Mrk 421 from 1900 to 2016.

**Figure 15.** Periodograms obtained with the historical light-curve data of Mrk 421 (see Figure 14) obtained using Lomb–Scargle (left panel) and RobPer (right panel).
linear scale (Equation (15)) with this distance. Following Rees (1967) and Qian et al. (1991), we estimate the distance traveled by the relativistic shocks using

\[ D = \beta_s \delta_z \Gamma_t \Delta t \left(1 + z \right) \]

(16)

where \( \beta_s \) is the shock speed, \( \Delta t \) is the observed timescale (Qian et al. 1991), and \( \Gamma_t \) is the Lorentz factor. Using the values of shock speed given in Piner & Edwards (2004) and Piner et al. (2008), the distance traveled by the shocks is 0.29 pc. The good agreement found between the values of the distance traveled by shocks along the jet and the field turbulence scale encourages us to think that the intranight variations observed in the polarization degree and spatial changes in the magnetic field could be strongly correlated with inhomogeneities. These inhomogeneities could have their origin in the jet or plasma by compressing and reordering the complex structure of the magnetic field (Laing 1980).

7. Conclusions

Long-term R-band photopolarimetric observations of the HBL Mrk 421 carried out from 2008 February to 2016 May have been presented. The highest brightness state of 11.29 ± 0.03 mag (93.60 ± 1.53 mJy) and the polarization degree value of 11.00% ± 0.44% have never been detected before in the R-band light curves. Extensive multiwavelength observations covering radio to TeV \( \gamma \)-rays around the flares observed in 2008 May, 2010 March, and 2013 April have been studied. From the photopolarimetric analysis presented in this work in the optical R band, the following results are found.

From the flaring activity in 2008 May, we have shown that the optical flux varies on timescales of 15.4 days and is anticorrelated with TeV \( \gamma \)-rays, hard/soft X-rays, the polarization degree, and the position angle. In addition, a strong correlation between the polarization degree and position angle was found. These results suggest that Mrk 421 could have different emitting regions that might play a role in different situations. In 2010 March, a correlation among TeV \( \gamma \)-ray, X-ray, and optical fluxes was found during the high-activity state, in agreement with the one-zone SSC model.

The most important results were found in 2013, where this object displayed the highest activity levels. In order to do a more detailed analysis, the flare observed in 2013 April was divided into three periods. During the first period, Mrk 421 presented very high activity in the TeV \( \gamma \)-ray, soft X-ray, and optical bands. The optical and TeV \( \gamma \)-ray fluxes were found to be anticorrelated, and the polarization degree and position angle were found to be correlated. During the second period, a large rotation of the position angle of \( \sim 100^\circ \) was detected when the degree of polarization was \( \sim 2\% \). This result could be due to shocks traveling along helical magnetic field lines (Marscher et al. 2008). The TeV \( \gamma \)-ray and hard/soft X-ray flares were accompanied by a modest increase in optical activity. The optical flux was found to be strongly correlated with the position angle and anticorrelated with the polarization degree. Additionally, a strong anticorrelation between the polarization degree and position angle was found. In the third period, a maximum value of the polarization degree of \( \sim 8.33\% \) was detected when the TeV \( \gamma \)-ray, X-ray, and optical fluxes were decreasing. No strong correlations were found among the optical flux with the rest of the higher-energy fluxes. The normalized Stokes parameters \( q \) and \( u \) were found to be
strongly correlated, thus suggesting that the observed variability was due to a single variable component with constant polarization properties (Hagen-Thorn et al. 2008).

The entire data set obtained in 2013 was used to analyze the Stokes parameters. The superposition of two polarized components coming from distinct emitting regions was found: one was a constant polarized component, and the other was a variable one. The constant component had a polarization degree of $P = 3.25\% \pm 0.70\%$ and a position angle of $\theta_c = 219^\circ 01 \pm 0^\circ 02$. The component that accounts for the variability behavior can be naturally explained due to the propagation of the shocks (Holmes et al. 1984; Brindle 1996).

The first intranight photopolarimetric variability study in the $R$ band of Mrk 421 from 2013 April 13 to 19 is presented in this work. For the OAN-SPM data, the following results were found.

1. The minimum variability timescale of the flux was found to be $\sim 2.3\text{ hr}$, whereas the polarization degree and spectral index vary on timescales of $\Delta \tau \sim \text{ minutes}$. Variations of the polarization degree could be related to the propagation of the shocks by means of an inhomogeneous plasma compressing and reordering the magnetic field lines (Laing 1980).

2. From night to night, the polarization degree variability presented variations of $\sim 2\% - 3\%$. In addition, for the spectral index in the range of $2 \leq \alpha_v \leq 3$ (see Abdo et al. 2011), the ratio of ordered to chaotic magnetic field intensity was found to be in the range of $0.1 \leq \beta \leq 0.3$. These variations might be explained when the shocks travel through the jet and find different regions with distinct magnetic field properties, leading a changing ratio of ordered to chaotic magnetic field intensities (Barres de Almeida et al. 2010).

3. The correlation found between the normalized Stokes parameters ($q$ and $u$) analyzed per night showed a temporal evolution. It could suggest that the total polarized flux is produced by more than one polarized component (Hagen-Thorn et al. 2008).

Using the 100 yr historical light curve of Mrk 421 and our SPM data, a periodicity of $5935 \pm 650\text{ days} (16.26 \pm 1.78\text{ yr})$ was found. One possible explanation for the periodicity is given through the dynamics of an orbiting binary supermassive black hole system and its accretion disks.

In addition, the analysis of the radio and photopolarimetric light curves observed during the flares of 1996–1997 and $\approx 16.34\text{ yr}$ later in 2013 indicate that both flares could have been produced in the same emitting region. This result has important implications for theoretical models that intend to explain the optical (with polarimetry) and radio variability of this source. These results will be analyzed in a forthcoming paper.

The broadband SEDs were fitted with the one-zone SSC model. An electron population with three power-law functions was required to describe the SEDs during the flaring and quiescent states. The full set of parameters derived by using the SED are: Doppler factor $\delta_D = 20 – 24$, magnetic field $B = (34.3 – 43.1) \times 10^{-3}\text{ G}$, emitting radius $r_e = (2.5 – 5.0) \times 10^{16}\text{ cm}$, and electron density $N_e = (0.1 – 0.26) \text{ cm}^{-3}$. These values are in agreement with those reported by Abdo et al. (2011) and Aleksić et al. (2015b, 2012). The values of the magnetic field and electron density found are higher in flaring than in quiescent states. This indicates that the radiative efficiency of electrons is higher in flaring events. The fit applied to the broadband SEDs yields the variability timescales that range from 9 hr to 1 day. The largest value of 1 day corresponds to the description of the quiescent states, and the smaller value (9 hr) is related to the flaring event. This shows that the variability timescale could be related to the state of activity and with the acceleration processes in the jet and the cooling mechanisms of electrons.

A detailed analysis of the electron spectral index with the $V$ and $R$ optical bands was performed. The results indicate that the coherence length of the large-scale magnetic field and the distance traveled by the relativistic shocks are of the same order, thus suggesting a connection between the intranight variations observed in the polarization degree and the spatial changes in the magnetic field.

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**References**

Abdo, A., Abeysekara, A., Allen, B. T., et al. 2014, ApJ, 782, 110
Abdo, A. A., Ackermann, M., Ajello, M., et al. 2009, ApJ, 700, 597
Abdo, A. A., Ackermann, M., Ajello, M., et al. 2010, Natur, 463, 919
Ahnen, M. L., Ansoldi, S., Antonelli, L. A., et al. 2016, A&A, 593, A91
Albert, J., Anderhub, H., Antoranz, P., et al. 2007, ApJ, 663, 125
Aleksić, J., Alvarez, E. A., Antonelli, L. A., et al. 2012, A&A, 542, A100
Aleksić, J., Ansoldi, S., Antonelli, L. A., et al. 2015a, A&A, 578, A22
Aleksić, J., Ansoldi, S., Antonelli, L. A., et al. 2015b, A&A, 576, A126
Aller, H. D., Aller, M. F., Latimer, G. E., & Hodge, P. E. 1985, ApJS, 59, 513
Angel, J. R. P., & Stockman, H. S. 1980, ARA&A, 18, 321
Baloković, M., Purniss, A., Madejski, G., & Harrison, F. 2013, ATel, 4974, 1
Baloković, M., Paneque, D., Madejski, G., et al. 2016, ApJ, 819, 156
