Giant-dipole Resonance and the Deformation of Hot, Rotating Nuclei

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The development of nuclear shapes under the extreme conditions of high spin and/or temperature is examined. Scaling properties are used to demonstrate universal properties of both thermal expectation values of nuclear shapes as well as the minima of the free energy, which can be used to understand the Jacobi transition. A universal correlation between the width of the giant dipole resonance and quadrupole deformation is found, providing a novel probe to measure the nuclear deformation in hot nuclei.

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The study of the properties of the giant-dipole resonance (GDR) at finite excitation energy (or temperature, $T$) and/or angular momentum, $J$, provides an insight into the behavior of nuclei under extreme conditions \cite{1}. A wealth of experimental data now exist for nuclei ranging from $44 \leq A \leq 208$ (see for example Refs. \cite{1–4}) detailing the properties of the GDR centroid $E_0$ and damping width $\Gamma$ with excitation energy and angular momentum of the compound nucleus. These experiments provide an important testing ground for theoretical models of the GDR; in particular, the role played by quantal and thermal fluctuations in the damping of the giant vibration. Based on both qualitative and quantitative agreement with experimental data, the thermal fluctuation model (TFM) \cite{5–8} provides a coherent picture of the GDR in nuclei at finite excitation energy and/or angular momentum. A systematic study of the thermal fluctuation model has recently revealed the existence of a universal scaling law for the width of the GDR for all $T$, $J$, and $A$ \cite{9}.

An important feature of the thermal fluctuation model is that the GDR strength function is directly related to the damping of the GDR and the mean quadrupole deformation. Our aim is to understand whether simple, universal behaviors exist for the evolution of nuclear shapes and their fluctuations. We also investigate the systematics of the free energy surfaces through the location of the minima $\beta_0$, $\gamma_0$, the deformation parameters $\beta$ and $\gamma$, including their mean values $\langle \beta \rangle$ and $\langle \gamma \rangle$ and their variances $\langle \beta^2 \rangle$ and $\langle \gamma^2 \rangle$, as a function of $T$, $J$, $A$, and $Z$. The universality we find for $\beta$ and $\gamma$ allows the tracing out of the Jacobi transition at high angular momentum. We further find a universal relationship between the full-width-at-half-maximum (FWHM) of the GDR strength function and the mean quadrupole deformation $\langle \beta \rangle$. Consequently, a single measurement of the FWHM can be simply related to the mean deformation of the system and readily extracted from experiment.

The thermal fluctuation model is based on the fact that large-amplitude thermal fluctuations of the nuclear shape play an important role in describing observed nuclear properties. If the time scale associated with thermal fluctuations is slow compared to the shift in the dipole frequency caused by the fluctuations (adiabatic motion), the observed GDR strength function can be obtained from a weighted average over all nuclear shapes and orientations. Projecting the $z$-component, $J_z$, of angular momentum, the GDR cross section is given by \cite{10}

$$\sigma(E) = Z_j^{-1} \int \frac{D[\alpha]}{I(\beta, \gamma, \theta, \psi)^{1/2}} \sigma(\vec{\alpha}, \omega ; J) e^{-F(\beta, \gamma, J, J_z)/T},$$

where $D[\alpha] = \beta^4 d\beta \sin(3\gamma) d\gamma \sin \theta d\theta d\omega d\psi$, $E$ is the photon energy, $Z_j = \int D[\alpha]/I^{1/2} e^{-F/T}$, $I$ is a function dependent on the principal moments of inertia and the Euler angles, and $F$ is the free energy at finite angular momentum. As with previous applications of the adiabatic picture, we model the GDR as a rotating, three-dimensional harmonic oscillator. Details describing the evaluation of $\sigma(\vec{\alpha}, \omega ; J)$ are given in Ref. \cite{8}.

The free energy is evaluated using the Nilsson-Strutinsky \cite{12} procedure and its extensions to finite temperature \cite{13}. It is composed of two components,

$$F = F_{LD} + F_{SH},$$

where $F_{LD}$ is the free energy of a liquid-drop and $F_{SH}$ is the Strutinsky shell correction. Shell corrections play an important role for well-deformed and doubly-magic nuclei at low temperature and exhibit no known global or universal behaviors. On the other hand, shell corrections weaken with increasing temperature, and most non-doubly magic nuclei have essentially \textit{melted} by $T = 1.25$ MeV. For doubly magic nuclei, such as $^{208}$Pb, the shell corrections at
\[ T = 1.5 \text{ MeV} \] have diminished by a factor of ten from their value at \( T = 0 \) \([7,8]\). Consequently, any derived universal scaling law using liquid-drop free energies should be applicable to non-doubly magic nuclei for \( T \geq 1.25 \text{ MeV} \) and essentially all nuclei for \( T \geq 1.75 \text{ MeV} \). In this work, we utilize a liquid-drop parameterization based on finite-temperature, extended Thomas-Fermi calculations \([14]\). This liquid-drop parameterization includes the curvature term and all coefficients are temperature-dependent.

In this work, we focus on the mass region \( A = 44 - 208 \). For the GDR parameters defining the central frequency for spherical shapes, \( E_0 \), and the intrinsic width \( \Gamma_0 \) defined in Ref. \([8]\) we use: \( (E_0 = 22, \Gamma_0 = 4) \) for \(^{44}\text{Ti}, (17, 4) \) for \(^{90}\text{Zr}, (15.5, 5) \) for \(^{120}\text{Sn}, (14, 4) \) for \(^{168}\text{Er} \) and \((13.6, 4) \) for \(^{208}\text{Pb} \), all in MeV.

We now examine in more detail the behavior of the nucleus shape under extreme conditions. Using Eq. (1), we define the mean value of any observable, and in particular moments of the quadrupole deformation parameter \( \beta \) (and similarly for \( \gamma \)) as

\[ \langle \beta^n \rangle = Z_J^{-1} \int \frac{D[\alpha]}{I(\beta, \gamma, \theta, \psi)} \beta^n e^{-F(T, \beta, J, \gamma)}/T. \]  

(3)

The spin dependence of the moments can be isolated by studying the normalized expectation values,

\[ \frac{\langle \beta^n(J, T, A) \rangle}{\langle \beta^n(0, T, A) \rangle}, \quad \frac{\langle \gamma^n(J, T, A) \rangle}{\langle \gamma^n(0, T, A) \rangle} \]  

(4)

at a fixed temperature \( T \). These are shown in Figs. 1(a-d) as a function of \( \xi \) for \( T = 1 \text{ MeV} \) and selected nuclei, displaying scaling. When we repeat this for other values of \( T \), we obtain a family of distinct curves. (We will focus now on \( \langle \beta \rangle \) and \( \langle \beta^2 \rangle \) to simplify the discussion.) Taking the same \( T \)-dependent power law as in \([9]\), we can remove the \( T \) dependence of the curves in Figs. 1(a-d) by considering \[ \left( \frac{\langle \beta^n(J, T, A) \rangle}{\langle \beta^n(0, T, A) \rangle} \right)^{(T+3)/4}, \]  

which now becomes both mass and temperature independent as seen in Figs. 2(a-b) (symbols). The results are the universal functions (solid) for each \( n \) and observable. We denote these \( w_n(\xi) \) for \( \beta \); shown as the solid lines in Figs. 2(a-b). The final quantity for these thermal fluctuation results is the \( J = 0 \) contribution to the average deformation. \( \langle \beta^n(J, T, A) \rangle \) at \( J = 0 \) is estimated from Eq. (3) to be approximately

\[ \langle \beta^n(J = 0, T, A) \rangle = \frac{4}{3\sqrt{\pi}} \Gamma \left( \frac{n+5}{2} \right) \left( \frac{T}{C_0(Z, A)} \right)^{n/2}, \]  

(5)

where \( C_0(Z, A) \simeq (1/2)a^2F/\partial a^2 \mid_{a=0} \). The \( Z \) and \( A \) dependence of \( C_0 \) can be estimated from the dominant contributions to the liquid drop energy \([11]\), giving

\[ C_0(Z, A) = a_1 A^{2/3} + a_2 \frac{(N - Z)^2}{A} + a_3 \frac{Z^2}{A} + a_4 \frac{Z^2}{A^{4/3}}. \]  

(6)

The calculated \( \langle \beta^n(J = 0, T, A) \rangle \) values are well reproduced with \( a_1 = 3.09, a_2 = -0.74, a_3 = 0.12, a_4 = -0.066 \), as seen in Fig. 3. These values compare quite well to the coefficients of the liquid-drop Free energy. Here we point out an interesting distinction between the \( J = 0 \) temperature dependence of \( \langle \beta \rangle \) and the GDR width \( \Gamma \): the former has \( T^{1/2} \) behavior while the latter has been found to behave as \( \log(1 + T) \) \([9]\).

Gathering the results, we have

\[ \langle \beta(J, T, A) \rangle = \frac{8}{3\sqrt{\pi}} \sqrt{\frac{T}{C_0}} w_1(\xi)^{4/(T+3)}, \]  

\[ \langle \beta^2(J, T, A) \rangle = \frac{5T}{2C_0} w_2(\xi)^{4/(T+3)}. \]  

(7)

(8)

While not unique, we follow \([9]\) and parameterize \( w_n \) as shifted Fermi-functions

\[ w_1(\xi) = 1 + \frac{a_1}{1 + \exp[(a_2 - \xi)/a_3]}, \quad w_2(\xi) = w_1(\xi)^2 \]  

where \( a_i = (4.3, 1.64, 0.31) \). Analogous functions can be found for \( \langle \gamma^n \rangle \) as is evident from Fig. 1. Eq. (7) describes the mean nuclear deformation of a hot nucleus of mass \( A \), temperature \( T \) and spin \( J \), demonstrating that the nucleus not only undergoes a smooth evolution under extreme conditions, but that the results are \( \textit{generic} \), with a simple function describing all the systematics.
While the same analysis is readily done for other observables, let us consider properties of the free energy surface itself, such as the location of the minimum of $F$ as a function of $J, T, A$, denoted by $\beta_{\text{min}}, \gamma_{\text{min}}$. The location of the minimum is shown in Fig. 2(right) for selected nuclei, at $T = 1 - 4$ MeV. We find that when plotted versus $\xi$, these display the same universality. The Jacobi transition now becomes evident at $\xi \sim 1.2$, where $\gamma_{\text{min}}$ displays an abrupt change from $\pi/3$ to 0. We can use this to define a critical spin for the transition: $J_c \sim 1.2A^{5/6}$. We note that although the curve plotted is for $T = 1$ MeV, little variation is exhibited for other values of $T$. In Fig. 4, we plot the generic evolution of the free energy surface minimum as a function of $\xi$ that arises in the TFM.

We have seen that simple scaling properties emerge for the average deformations and their powers. Taken together with the scaling previously seen in the GDR width $\Gamma$, it follows that there should be some functional relation between the average deformation $\langle\beta\rangle$ and $\Gamma$. This is likely since the primary mechanism for spreading the GDR modes is a vibrational dephasing caused by the shift in the centroids of the GDR modes at each deformation, which to first order is a linear function in $\beta$. Hence, we propose the ansatz

$$\langle\beta(J, T, A)\rangle = a' \frac{\Gamma(J, T, A) - \Gamma_0}{E_0} + c',$$  \hspace{1cm} (9)$$

and find remarkable agreement for all nuclei at all temperatures and spins below the Jacobi transition with $a' = 0.8$ and $c' = 0.12$ as illustrated in Fig. 5. Indeed, we see that for all nuclei, the average deformation depends linearly on the FWHM and is reproduced by Eq. (9) with an average fluctuation, or uncertainty, of $\delta(\beta) \approx \pm 0.05$. Hence, with the FWHM one can probe the evolution of the nuclear shape directly from experiment.

Eq. (9) provides a convenient and direct experimental measure of nuclear deformation. In Fig. 6 recent experimental data on Sn [1–4] is used to extract empirical deformations. The error bars in the figure were obtained by adding in quadrature the quoted experimental errors [1–4] and the uncertainty $\delta(\beta)$ from Eq. (9). The values $\langle\beta\rangle$ deduced from experimental data are also compared directly to TFM calculations (solid) as well as scaling law predictions (dashes) of Eq. (7). One can see that Eq. (9) provides a convenient measure of both the $J$ and $T$ dependence of the deformation, and is readily computed from the data.

It has recently been argued that the spin-dependence of the GDR width $\Gamma$ in $^{194}$Hg is not seen experimentally since the fluctuations $\sigma_\beta = \sqrt{\langle\beta^2\rangle - \langle\beta\rangle^2}$ are larger in magnitude than $\beta_{\text{min}}$, so that rotational effects due to the deformation are washed out [15]. Using (7)-(8), Fig. 2, and $C_0(A, Z)$ for $^{194}$Hg, we compute that $\sigma_\beta \sim \beta_{\text{min}}$ when $J \sim 37\sqrt{T}$. At $T = 1.3$ MeV, $J < 42$ agrees with observations and predictions in [15]. This condition complements the analysis of [9], providing a deformation-based, analytic approach to understanding the observed experimental features of the GDR.

In conclusion, within the framework of the thermal fluctuation model, we have studied new universal scaling properties. We find a simple scaling behavior for all nuclei at all temperatures and spins for the moments of parameters defining the quadrupole deformation, $\beta$ and $\gamma$. In addition, we find a remarkable linear relationship between the FWHM of the GDR and mean value of the quadrupole deformation, $\langle\beta\rangle$. Consequently, the FWHM may be viewed as an experimental probe of the the evolution of the nuclear shape.

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**FIG. 1.** Dependence of normalized shape parameters as a function of $\xi = J/A^{5/6}$ for $^{44}$Ti ($\times$), $^{90}$Zr (+), $^{120}$Sn ({$\ast$}), $^{168}$Er ({$\bigcirc$}), $^{208}$Pb (o) at $T = 1$ MeV, demonstrating scaling for fixed $T$.

**FIG. 2.** $\xi$ dependence of: (Left) normalized expectation values of deformation from $T = 1 – 4$ MeV for $^{44}$Ti ($\times$), $^{90}$Zr (+), $^{120}$Sn ({$\ast$}), $^{168}$Er ({$\bigcirc$}), $^{208}$Pb (o) indicating $T$ independence. Solid lines are Eqs. (4)-(9); (Right) Location of free energy minima ($\beta_{\text{min}}, \gamma_{\text{min}}$) for same nuclei as a function of $\xi$ for $T = 1 – 4$ MeV. Dashed lines are for guidance. The Jacobi transition is evident at $\xi \sim 1.2$. Below that, $\beta_{\text{min}} \sim \xi/5$ and $\gamma_{\text{min}} \sim 1$. 
FIG. 3. Temperature dependence of moments of deformation at $J = 0$ (symbols) compared to predictions from Eqs. (5)-(6). Symbols are: $^{44}$Ti (×), $^{90}$Zr (+), $^{120}$Sn (○), $^{168}$Er (□), $^{208}$Pb (o). Note that at a given $T$, the ordering of the symbols is not monotonic with $A$, and is well reproduced by Eq. (5).

FIG. 4. Location of the minimum of the free energy surface for hot, rotating nuclei. The circles are shown in increments of $\xi = 0.2$, starting with $\xi = 0$ at the origin.

FIG. 5. Linear relation of average nuclear deformation to the GDR width $\Gamma$, for $44 \leq A \leq 208$, $T \leq 4$ MeV and $J \leq 60\hbar$, illustrating that $\Gamma$ can be used as a direct experimental measure of nuclear deformation for any $J, T, A$ where shell corrections are unimportant. Symbols represent the same nuclei as in Figs. 1-3.
FIG. 6. Average nuclear deformation $\langle \beta \rangle$ extracted from experiment [1-4] using Eq. (9), as a function of $J$ and $T$. Comparison to exact TFM calculations (solid) and scaling predictions of Eq. (7) (dashes) are quite good.