Constraints of Dark Energy at High Redshift

Qiping Su\textsuperscript{a,1}, Rong-Gen Cai\textsuperscript{2}

\textsuperscript{1}Department of Physics, Hangzhou Normal University, Hangzhou, 310036, China
\textsuperscript{2}State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, P.O. Box 2735, Beijing 100190, China

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Abstract

Constraints of dark energy (DE) at high redshift from current and mock future observational data are obtained. It is found that present data give poor constraints of DE even beyond redshift \( z=0.4 \), and mock future 2298 type Ia supernovae data only give a little improvement of the constraints. We analyze in detail why constraints of DE decrease rapidly with the increasing of redshift. Then we try to improve the constraints of DE at high redshift. It is shown that the most efficient way is to improve the error of observations.

1 Introduction

The current expansion of the universe is found to be in accelerating \cite{1,2} and it is believed that the dark energy (DE), whose equation of state \( w_{de} = p_{de}/\rho_{de} \) is less than \(-1/3\), plays the role to drive the accelerated expansion of the universe. Plenty of DE models have been proposed \cite{3,4,5,6,7,8,9} but the origin of DE is still unknown. We have only known a little about DE from observations.

Several parametrization methods of \( w_{de} \) have been proposed to fit with observations to get constraints of \( w_{de} \). Usually in parametrization models \( w_{de} \) is assumed to be a simple function of redshift \( z \), e.g., the CPL parametrization \cite{10,11}: \( w_{de}(z) = w_0 + w_a z/(1 + z) \), and the parametrization of redshift expansion \cite{12,13}: \( w_{de}(z) = w_0 + w_z z \). It has been found that \( w_{de} \) is very close to -1 and is varying very slowly (if it is dynamical). But these fitting results depend on the parametrization forms of \( w_{de}(z) \) used. Moreover, one parametrization form of \( w_{de}(z) \) could only approximate the real \( w_{de} \) well in a very limited region of redshift. So fitting results from a single model should not be used to analyzing behaviors of DE at both low and high redshift. There are also “model-independent” methods \cite{14,15,16,17}, in which the redshift of data is usually divided into several bins and in each redshift bin \( w_{de} \) is assumed to be a simple function of \( z \). All “model-independent” methods show that constraints of \( w_{de} \) at higher redshift are much weaker than that at low redshift \cite{15,19,20,21,22,23}. It is important and necessary to get good constraints of \( w_{de} \) at high redshift to reveal the nature of DE, such as the dynamical behaviors of DE.

In this paper, we’d like to analyze constraints of \( w_{de} \) at high redshift. Since present constraints of DE are from observational data, we will mainly analyze effects of observational data on constraining DE at high redshift. We will start with type Ia supernovae (SNIa) data. Almost all present data for DE are related to the comoving distance and the most data of this distance type are from SNIa. Commonly used DE data other than the distance type are CMB data and Hubble parameter data only. And main information in CMB data can be extracted to distance priors (such as the shift parameter \( R \) date): there is only a little difference between constraints of DE from full CMB data and from the distance priors \cite{24}. The distance priors are related to distances, i.e., one can convert the CMB data to data of distance type. So our analyses will be mainly based on the distance type of data, and study why distance type of data give poor constraints of DE at high redshift.

The paper is organized as follows. In section II, constraints of \( w_{de} \) at low redshift and high redshift from present and mock future data are obtained. In section III, we analyze the properties of \( w_{de} \) at high redshift in detail and try to find out reasons of the poor constraints. In section IV, we try to improve constraints of \( w_{de} \) at high redshift by adding number of date points,


2 Constraints from distance type of data

At present, almost all observational data for DE are the distance type, such as data of SnIa, BAO parameter A and CMB shift parameter R, which are related to a comoving distance: 

\[ r_c = \int \frac{dt}{a} = \int \frac{dz}{H(z)} \]

At first, we’d like to estimate the effect of SnIa data (since it gives the most date points for DE) on constraints of DE, especially constraints at high redshift. Since a single SnIa date set has poor constraints on DE, the BAO parameter A will also be adopted to alleviate the degeneracy between equation of state \( w \) and dimensionless density energy \( \Omega_{de} \) of DE.

We will use the UBE method to get constraints of \( w_{de} \) from present and future mock data. We divide the redshift of data into two bins and set

\[
    w_{de}(z) = \begin{cases} 
    w_1, & 0 \leq z \leq z_1 \\
    w_2, & z_1 < z
    \end{cases}
\]

where \( w_1 \) and \( w_2 \) are just constants and \( z_1 \) is the divided position of low and high redshift. In the numerical calculations we will set the prior \( w_2 > -20 \), or \( w_2 \) will run to large minus value in MCMC procedure and the lower error of \( w_2 \) will be extremely large. This will be discussed in detail in the next section. For each date set the figure of merit (FoM) \([25,26]\) is also calculated, which is defined as:

\[
    \text{FoM} = \left| \det(C(w_1, w_2)) \right|^{-1/2},
\]

where \( C(w_1, w_2) \) is the covariance matrix of \( w_1 \) and \( w_2 \) after marginalizing out all other parameters. In general, FoM is used to estimate the the goodness of the data in constraining \( w_{de} \).

Our calculations show that the correlation between \( w_1 \) and \( w_2 \) is very small, i.e., the errors of \( w_1 \) and \( w_2 \) obtained can be treated as independent with each other. So errors of \( w_2 \) (\( w_1 \)) just represents constraints of \( w_{de} \) at high (low) redshift.

2.1 Constraints from present date set

Here we get constraints of \( w_{de} \) from Union2.1 SnIa data \([26]\) and BAO Parameter A date from \([28]\). To estimate constraints of \( w_{de} \) at different high redshift bins, the divided position \( z_1 \) will be set as 0.4, 0.5, 0.6 and 0.7, respectively. The best-fitted parameters and their 68.3%, 95.4% confidence level (C.L.) errors are obtained by using the Markov chain Monte Carlo (MCMC) method, which are shown in Table 1. We have following conclusions:

1. \( w_{de} \) at high redshift \((z > 0.4)\) are highly unconstrained by present data, especially compared with \( w_{de} \) at low redshift. With \( z_1 = 0.4 \), the average 95.4% C.L. error for \( w_2 \) (noted as \( 2\sigma(w_2) \)) is

\[
    2\sigma(w_2) = (0.67 + 1.05)/2 = 0.86
\]

and \( 2\sigma(w_1) \) is only about 0.15, as shown in Table 1.

| \( z_1 \)  | \( w_1 \)         | \( w_2 \)         | FoM    |
|---------|----------------|----------------|--------|
| 0.4     | \(-1.04^{+0.03}_{-0.14}\) | \(-1.04^{+0.16}_{-0.67}\) | 28.47  |
| 0.5     | \(-1.04^{+0.07}_{-0.14}\) | \(-1.19^{+0.68}_{-1.15}\) | 12.68  |
| 0.6     | \(-1.04^{+0.06}_{-0.13}\) | \(-1.34^{+1.30}_{-2.13}\) | 5.52   |
| 0.7     | \(-1.04^{+0.06}_{-0.12}\) | \(-1.50^{+1.24}_{-1.44}\) | 2.36   |

Table 1 The best-fitted values and their 68.3% and 95.4% C.L. errors of \( w_1 \) and \( w_2 \) from present observational data, the divided positions are \( z_1 = 0.4, 0.5, 0.6, 0.7, \) respectively.

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Fig. 1 The likelihoods of \( w_2 \) from present data, with divided position \( z_1 = 0.4, 0.5, 0.6, 0.7, \) respectively.

Fig. 2 68% C.L. contour plot in \( w_2 \) ~ \( w_2 \) plane from present data, with divided position \( z_1 = 0.4, 0.5, 0.6, 0.7, \) respectively.
2. Errors of $w_{de}$ at high redshift (i.e., errors of $w_2$) increase rapidly with the divided point $z_1$, especially the lower errors of $w_2$. As shown in Fig. 1, $z_1$ is larger, the likelihood of $w_2$ decreases more slowly with the decreasing of $w_2$. We will analyze this phenomenon in detail in the next section.

3. Effect of the divided position $z_1$ on errors of $w_{de}$ at low redshift (i.e., $w_1$) is very weak. As shown in Table 1 with the increasing of $z_1$ variations of errors of $w_1$ are negligible. It implies that present date set at low redshift ($z < 0.4$) is sufficient and constraints on $w_{de}$ in this region is strong.

4. Here, values of FoM can be used to estimate goodness of constraints of $w_{de}$ at high redshift. FoM from Eq. 2 is proportional to the inverse area of the 1σ error ellipse in the $w_1 \sim w_2$ plane. As shown from the contour plots in Fig. 2 and errors of $w_1$ in Table 1, the decreasing of FoM (with respect to the increasing of $z_1$) is due to the increasing of errors of $w_2$. So value of FoM is larger, errors of $w_2$ are smaller, and vice versa. Indeed, FoM increases rapidly with the decreasing of $z_1$, as shown in Table 1 and Fig. 5.

2.2 Constraints from future data

To estimate the constraints of $w_{de}$ at high redshift from future data, we fit the 2-binned UBE model with 2298 simulated SNe Ia data (denoted as elementary data set in this paper), which contain 1998 SNe Ia data with redshift $0.1 < z < 1.7$ from a SNAP-like JDEM survey and 300 SNe Ia data with $z < 0.1$ from the NSNF [30,31]. To alleviate the degeneracy between $\Omega_{m0}$ and $w_{de}$ [32], the date of BAO Distance Parameter $A$ [28] will also be included. To simulate the mock SNe Ia data, we assume the fiducial model as $w_{de}(z) = -1$. The error of the distance modulus ($\delta$) for each supernova is set as 0.13 [17]. To estimate the effect of the divided position $z_1$, we have set $z_1$ as 0.4, 0.6, 0.8 and 1.0, respectively.

As shown in Table 2 with much more date points, the errors of $w_{de}$ from future data are smaller than that from the present data. E.g., here $2\delta(w_2) = 0.43$ for $z > 0.4$, which is just a half of that from the present data (i.e., 0.86). Errors of $w_{de}$ at high redshift (i.e., $w_2$) also increase rapidly with $z_1$, e.g., here $2\delta(w_2)$ for $z > 0.6$ is 0.88. It also shows that constraints of $w_{de}$ beyond $z = 1$ are extremely weak. On the other hand, constraints of $w_{de}$ at low redshift (i.e., constraints of $w_1$) are also improved, but not as much as that of $w_2$.

In all, present data gives poor constraints of $w_{de}$ at high redshift (e.g., $z > 0.4$), and the mock future 2298 SNe Ia data do not give sufficient improvement on constraints of $w_{de}$ at high redshift.

3 Analyzing constraints

Now we try to answer two questions:

3.1 Why data of distance type give poor constraints on $w_{de}$ at high redshift?

Reasons are:

1. Date points in a redshift bin will constrain $w_{de}$ in this bin and in lower redshift bins, but can not constrain $w_{de}$ in higher redshift bins, e.g., $w_{de}$ in the lowest redshift bin can be constrained by all date points. Moreover, the redshift is larger the corresponding distance is farther, and it will be harder to measure the distance. And the farther a supernova is, the harder it can be detected. While the most date points for DE are from SNe Ia. In all, there are much less efficient date points in higher redshift bins.

2. In higher redshift, the effect of dark energy on the luminosity distance

$$D_l(z) = (1 + z) \int_0^z \frac{dz'}{H(z')}$$

is much less. Here $H^2(z') \sim \rho_{de}(z') + \rho_{dm}(z')$. With respect to the increase of redshift $z$, energy density of DE ($\rho_{de}$) evolves very slowly, while energy density of DM ($\rho_{dm}$) increases very rapidly since $\rho_{dm} \sim (1 + z)^3$. In higher redshift bin, $\rho_{dm}$ is much less than $\rho_{de}$. So constraints of DE at high redshift from distance type of data will be weaker.

3.2 Why lower errors of $w_{de}$ at high redshift bins are extremely large

This can be shown by simple calculations as follows. In a flat FRW universe the Friedmann equation reads:

$$H^2(x) = \frac{1}{3} [\rho_{m0}(1 + x)^3 + \rho_{de}(x)]$$

where $\rho_{de}(x) = \rho_{de0}e^{3H_0\int_0^x \frac{1}{1 + y} dy}$. 

| $z_1$ | $w_1$ | $w_2$ | FoM |
|------|------|------|-----|
| 0.4  | -0.96+0.03+0.07 | -1.12+0.17+0.30 | 119.65 |
| 0.6  | -0.97+0.03+0.08 | -1.14+0.27+0.45 | 33.23 |
| 0.8  | -0.96+0.04+0.08 | -1.11+0.43+0.73 | 11.68 |
| 1.0  | -0.95+0.02+0.06 | -1.11+0.89+1.28 | 5.89 |

Table 2 The best-fitted values and their 68.3% and 95.4% C.L. errors of $w_1$ and $w_2$ from mock future data, the divided positions are $z_1 = 0.4, 0.6, 0.8, 1.0$, respectively.
The redshift is divided into n bins and in each bin \( w_{de} \) is set as a constant. Supposing there is a tiny variation of \( w_{de} \) in the \( i \)th bin (\( \delta w_i \)) and no variations of \( w_{de} \) in all other bins, from Eq. (3) and (4) one gets

\[
\delta D_i(z) = \left\{ \begin{array}{ll}
0, & 0 \leq x \leq z_i \\
-\frac{3}{2}(1+z) \int_{z_i}^{z+1} \left[ 1 + \frac{\Omega_{de} \ln \left( \frac{1+z}{1-z} \right)}{1+z} \right] dx, & z_i < x \leq z+i
\end{array} \right.
\]

and Eq. (5) for \( \delta D_i(z) \), where \( \Omega_{de} = \rho_{de}/(\rho_{de} + \rho_{dm}) \).

One can see that \( \delta w_i \) always appears together with \( \Omega_{de} \) in Eq. (6). At higher redshift (i.e., larger \( i \)), with the same \( \delta w_i \), \( \Omega_{de} \) is much smaller and \( \delta D_i(z) \) is also smaller. It indicates that the redshift is higher, to lead to the same variation of \( D_i \) the variation of \( w_i \) must be larger, i.e., \( D_i(z) \) is less sensitive to \( w_i \) at higher redshift. In all, constraints on \( w_{de} \) from the luminosity distance are weaker in higher redshift bins.

As shown in Eq. (6), \( \delta D_i \) in the \( i \)th bin is related to an integration, which is mainly determined by \( \Omega_{de} \) and the width of this redshift bin. The width of a redshift bin is larger, the integration will be larger and so will be \( \delta D_i \). In higher redshift bins \( \Omega_{de} \) is smaller since

\[
\Omega_{de}(z_i < z < z+i) \sim (1+z)^{\delta w_i},
\]

so \( \delta D_i \) will be smaller. Note that \( \Omega_{de} \) is also dependent on \( w_i \).

Now let’s answer the question. In a high redshift bin if \( w_i \) is much smaller than \(-1\), \( \Omega_{de} \) in this bin will be so small that \( \delta D_i \) keeps small even the bin is wide and \( \delta w_i \) is large, i.e. with different small values of \( w_i \) one gets almost the same \( D_i \). In this case, the likelihood of \( w_i \) will be very flat for \( w_i < -1 \) as shown in Fig. 1. Thus the lower errors of \( w_i \) at high redshift bins are always extremely large. In general, the constraint of \( w_{de} \) in the last redshift bin is the weakest.

| multiple | \( w_1 \) | \( w_2 \) | FoM |
|----------|---------|---------|-----|
| \( \times 2 \) | \(-0.96^{+0.03+0.06}_{-0.08-0.13} \) | \(-1.33^{+0.82+1.13}_{-1.45-1.86} \) | 6.64 |
| \( \times 3 \) | \(-1.00^{+0.02+0.06}_{-0.05-0.11} \) | \(-1.01^{+0.54+0.93}_{-0.86-3.48} \) | 15.41 |
| \( \times 4 \) | \(-1.00^{+0.03+0.09}_{-0.04-0.08} \) | \(-1.20^{+0.49+0.89}_{-0.72-2.49} \) | 22.03 |
| \( \times 5 \) | \(-0.99^{+0.02+0.06}_{-0.04-0.10} \) | \(-1.00^{+0.37+0.63}_{-0.47-1.27} \) | 92.48 |

**Table 3** The best-fitted values and their 68.3% and 95.4% C.L. errors of \( w_1 \) and \( w_2 \) from simulated SnIa data. The numbers of SnIa data are 2, 3, 4, and 5 times of 2298 elementary data, respectively. The divided position \( z_1 \) is set to 1.0.

### 4 Improving constraints

According to previous results and analyses, we try to improve the constraints of \( w_{de} \) at high redshift. Three methods will be implemented and their efficient will be compared.

#### 4.1 Adding the number of SnIa data

In the section 2.2 it is shown that \( \sim 2300 \) mock future SnIa data still give poor constraints of \( w_{de} \) beyond \( z \sim 0.6 \). Here we try to use more mock SnIa data, with 2, 3, 4 and 5 times of the 2298 elementary data. The proportional distribution of redshift in these multiple data sets is the same as the elementary data set. The divided position \( z_1 \) of the two bins is now set to 1.0.

The results are shown in Table 3. It shows that FoM increases with the increasing of number of data points, but the increase is not very efficient. With 5 times number of the elementary SnIa data set (about 11500 supernovae), constraints of \( w_{de} \) beyond \( z = 1 \) are still weak: \( 2\sigma(w_2) = 0.96 \).

#### 4.2 Improving accuracy of the data

Increasing the number of SnIa data seems not efficient enough in improving constraints of \( w_{de} \) at high redshift. It is expected that with the increase of number of data the statistical errors of the data will be decreased. Moreover, the systematic errors of the data will be improved by future observations. To estimate effects of the error \( \delta \) of distance modulus of supernovae, here we set \( \delta = 0.13, 0.1, 0.05, 0.02 \), respectively. The number

| \( \delta \) | \( w_1 \) | \( w_2 \) | FoM |
|----------|---------|---------|-----|
| 0.13 | \(-1.00^{+0.02+0.06}_{-0.05-0.11} \) | \(-1.01^{+0.54+0.93}_{-0.86-3.48} \) | 15.41 |
| 0.10 | \(-1.00^{+0.01+0.05}_{-0.04-0.07} \) | \(-1.01^{+0.41+0.72}_{-0.55-1.52} \) | 30.57 |
| 0.05 | \(-1.00^{+0.01+0.03}_{-0.01-0.04} \) | \(-1.01^{+0.22+0.39}_{-0.24-0.55} \) | 386.48 |
| 0.02 | \(-1.00^{+0.00+0.00}_{-0.00-0.00} \) | \(-1.00^{+0.00+0.00}_{-0.00-0.00} \) | 2478.16 |

**Table 4** The best-fitted values and their 68.3% and 95.4% C.L. errors of \( w_1 \) and \( w_2 \) with errors in distance modulus \( \delta = 0.13, 0.10, 0.05, 0.02 \), respectively. The number of SnIa used is 3 times of 2298 elementary SnIa data. The divided positions \( z_1 \) is set to 1.0.
The error of R data (0.016) is very small, compared to the redshift, main reasons are:

- This parameter is defined as:

\[ R = \sqrt{D_{\text{m}0}} \int_0^{z_s} \frac{dz}{E(z)}, \quad E = H/H_0 \]

which is related to the distance of decoupling epoch (\( z_s \sim 1091 \) is the redshift of decoupling).

Now We combine shift parameter R from WMAP9 with Union2.1 and BAO data to get constraints of \( w_{de} \).

The addition of a single date point R gives a big improvement of FoM and constraints of \( w_{de} \) at high redshift, main reasons are:

1) The error of R date (0.016) is very small, compared with the present errors of Sna. As shown in previous results, the error of date is smaller, the constraints of \( w_{de} \) will be better. Moreover, the corresponding redshift for the parameter R is \( z \sim 1091 \), it gives good constraints of \( w_{de} \) in all redshift bins.

2) To involve the date of parameter R, the second redshift bin should be \((z_1, 1091)\). As analyzed in section 3.2, the bigger a redshift bin is, the effect of \( w_{de} \) on \( D_l \) will be bigger. So the constraint of \( w_{de} \) from parameter R is good.

3) The shift parameter R date can further alleviate the degeneracy between \( w_{de} \) and \( \Omega_{de} \).

| \( z_1 \) | \( w_1 \) | \( w_2 \) | FoM |
|--------|--------|--------|------|
| 0.4    | -1.03^{+0.08+0.15}_{-0.07-0.16} | -0.86^{+0.20+0.37}_{-0.25-0.54} | 72.22 |
| 0.5    | -1.03^{+0.08+0.15}_{-0.05-0.14} | -0.82^{+0.25+0.45}_{-0.35-0.84} | 50.45 |
| 0.6    | -1.03^{+0.08+0.15}_{-0.05-0.14} | -0.74^{+0.30+0.48}_{-0.50-1.39} | 35.56 |
| 0.7    | -1.04^{+0.10+0.15}_{-0.03-0.11} | -0.67^{+0.34+0.53}_{-0.76-2.78} | 13.13 |

Table 5 The best-fitted values and their 68.3% and 95.4% C.L. errors of \( w_1 \) and \( w_2 \) from data of SN+A+R, the divided positions are \( z_1 = 0.4, 0.5, 0.6, 0.7 \), respectively.

### 4.3 Combined with other type of data

There is few type of date other than distance type, the typical one is the Hubble parameter data which constrain \( w_{de} \) directly; there is no integration between the Hubble parameter and \( w_{de} \).

Here we combine 22 Hubble parameter data\(^{[50]}\) with the Union2.1 and BAO data to show effects of the Hubble parameter data. The results are shown in Table 6 and Fig. 3 With the addition of the 22 Hubble date points, FoM is improved, but not as much as that with shift parameter R. Though one needs not to have integration in fitting Hubble data, the errors of Hubble data is much bigger than that of shift parameter R date and the width of second bin related to the Hubble data is much smaller than that of R date.

The addition of a single date point R gives a big improvement of FoM and constraints of \( w_{de} \) at high redshift, main reasons are:

1) The error of R date (0.016) is very small, compared with the present errors of Sna. As shown in previous results, the error of date is smaller, the constraints of \( w_{de} \) will be better. Moreover, the corresponding redshift for the parameter R is \( z \sim 1091 \), it gives good constraints of \( w_{de} \) in all redshift bins.

2) To involve the date of parameter R, the second redshift bin should be \((z_1, 1091)\). As analyzed in section 3.2, the bigger a redshift bin is, the effect of \( w_{de} \) on \( D_l \) will be bigger. So the constraint of \( w_{de} \) from parameter R is good.

3) The shift parameter R date can further alleviate the degeneracy between \( w_{de} \) and \( \Omega_{de} \).

### 5 Summary

We have analyzed constraints of \( w_{de} \) at high redshift from current and future observations. It was shown that...
at higher redshift, constraints of $w_{de}$ from all observational date sets are much weaker. The present data give poor constraints on $w_{de}$ beyond $z \sim 0.4$, whose average 95.4% C.L. error is 0.86. With the future 2298 mock data, average 95.4% C.L. error of $w_{de}$ beyond $z \sim 0.4$ is about 0.43. We have carefully analyzed why constraints of DE at high redshift from observational data are so poor. Since almost all DE data is of distance type, our analyses are mainly based on distance type of data. The analyses show that it is hard to get good constraints of $w_{de}$ at high redshift from distance type of data. Then we tried to improve constraints of DE at high redshift by adding more numbers of future mock SNeIa data, improving the errors of observational data, and combining other type of data (Hubble parameter data) with distance types of data. It was shown that improving the error of observations is the most efficient way. About 6900 SNeIa data with observational errors $\sigma = 0.02$ can constrain $w_{de}$ beyond $z = 1$ within 0.1 at 68.3% C.L. and within 0.2 at 95.4% C.L.

At present, many projects for DE are in progress or in planning. It is expected that a great deal of date points will be released. The distance types of date will still be the most important ones for revealing the nature of DE. To reveal the nature of DE, good constraints of $w_{de}$ at high redshift is required. Our results show that much more date points will be needed, but improvements for accuracy of the observations are much more efficient and necessary. While the most present observational data for DE are distance type or can be converted into distance type, another way to improve the constraints of $w_{de}$ is developing more other types of data.

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