Spatiotemporal isotropic-to-anisotropic meta-atoms

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Abstract
Metamaterials and metasurfaces are designed by spatially arranging (periodically or non-periodically) subwavelength geometries, allowing a tailored manipulation of the electromagnetic response of matter. Here, we exploit temporal variations of permittivity inside subwavelength geometries to propose the concept of spatiotemporal meta-atoms having time-dependent properties. We exploit isotropic-to-anisotropic temporal boundaries within spatially subwavelength regions where their permittivity is rapidly changed in time. In so doing, it is shown how resulting scattered waves travel in directions that are different from the direction of the impinging wave, and depend on the values of the chosen anisotropic permittivity tensor. To provide a full physical insight of their performance, multiple scenarios are studied numerically such as the effect of using different values of permittivity tensor, different geometries of the spatiotemporal meta-atom and time duration of the induced isotropic-to-anisotropic temporal boundary. The intrinsic asymmetric response of the proposed spatiotemporal meta-atoms is also studied demonstrating, both theoretically and numerically, its potential for an at-will manipulation of scattered waves in real time. These results may open new paradigms for controlling wave–matter interactions and may pave the way for the next generation of metamaterials and metasurfaces by unleashing their potential using four-dimensional unit cells.

1. Introduction
Metamaterials and metasurfaces (as their 2D version) have opened new ways to control and manipulate fields and waves at-will, and have been a hot research topic given their ability to produce artificially engineered media with electromagnetic (EM) properties not easily available in nature [1–4]. They have been proposed and experimentally demonstrated in multiple frequency ranges including radio frequencies, microwave and millimeter up to the optical regime [5–10] and they have been used in groundbreaking applications such as sensors [11–13], quantum devices and technologies [14], antennas and lenses [15–24], beam steers [25–27], tunable metamaterials and surfaces [28–31], optical circuits [32, 33], analogue computing [34–36] and parity-time-symmetric systems [37, 38], to name a few.

To design such artificial media, one can exploit subwavelength unit cells (known as meta-atoms) spatially placed in often periodic (and sometimes randomly located) arrangements. In so doing (and considering the materials, geometries and orientation of such subwavelength meta-atoms) it is possible to design artificial EM media having an at-will tailored physical responses (of effective permittivity, ε, and effective permeability, μ) enabling extreme values such as negative or near-zero values of refractive index [39–44].

Since their conception, metamaterials and metasurfaces have been mostly investigated in the frequency domain (time-harmonic scenario) where EM wave propagation can be manipulated by using spatial inhomogeneities (spatial modulation). Recently, the manipulation of EM waves using temporal and
spatiotemporal metamaterials has gained a prominent attention in the scientific community, opening new ways to fully manipulate EM radiation both in space and time \((x, y, z, t)\). Temporal modulation of media was first studied by Morgenthaler \cite{45} and Fante \cite{46} several decades ago by considering a monochromatic wave traveling within a spatially unbounded medium. It was shown theoretically that, if the relative permittivity of the whole medium is rapidly changed (with a time duration smaller than the period \(T\) of the incident wave) from \(\varepsilon_{r1}\) to \(\varepsilon_{r2}\) at \(t = t_1\) (both values isotropic and larger than one) a set of forward (FW) and backward (BW) waves can be produced and, interestingly, vector \(k\) is preserved while frequency is changed. Such step-like temporal function of EM parameters of media in time are known as *temporal boundaries* with the FW and BW waves being the temporal analogue of the transmitted and reflected waves, respectively, produced at the spatial interface between two spatially semi-infinite media (spatial boundary) \cite{47}.

Temporal and spatiotemporally modulated metamaterials and metasurfaces have become a cutting-edge research field enabling interesting potential applications \cite{48–53} ranging from nonreciprocity \cite{54, 55}, Fresnel drag \cite{56}, temporal gratings \cite{57}, effective medium theory \cite{58, 59}, antireflection temporal coatings \cite{60}, frequency conversion \cite{61–63}, inverse prism \cite{64}, holography \cite{65}, temporal aiming \cite{66}, temporal metamaterials with loss and gain \cite{67, 68} and transmission without reflection in time modulated metamaterials using techniques such as our recently proposed temporal Brewster angle \cite{69}. In most of these examples, EM waves are manipulated by considering that the whole medium where the waves travel is modulated in time to induce a temporal boundary. Recently, the interaction of EM waves using externally time-modulated meta-atoms has also been explored showing how metamaterials and metasurfaces can benefit from sinusoidally temporal modulations of subwavelength geometries \cite{70}. However, one may ask: what would happen if isotropic-to-anisotropic temporal boundaries are applied to spatially subwavelength geometries? What interesting physical phenomena can be produced when rapidly changing in time the relative permittivity of a meta-atom from an isotropic value to an anisotropic tensor?

Motivated by the interesting opportunities that temporal and spatiotemporal metamaterials can provide, in this work we propose the effects of temporal boundaries in spatially subwavelength material regions to enable what we call *spatiotemporal isotropic-to-anisotropic meta-atoms*. In a recent work \cite{66} we have demonstrated theoretically and numerically how the direction of energy propagation \((S)\) of an obliquely incident \(p\)-polarized wave can be modified in real time by rapidly changing in time the permittivity of the medium where the wave travels from isotropic \(\varepsilon_{r1}\) to an anisotropic permittivity tensor \(\varepsilon_{r2} = \{\varepsilon_{r2x}, \varepsilon_{r2y}, \varepsilon_{r2z}\}\) (subscript ‘\(r\)’ denotes the relative values with respect to that of free space. All values are assumed to be larger than unity. The material resonance frequencies are assumed to be much higher than the operating frequency, i.e. no dispersion is considered). We have also exploited this temporal aiming technique \cite{66} to achieve FW wave propagation and frequency conversion without exciting BW waves; i.e. a technique that we called temporal Brewster angle \cite{69}. Here, we explore such isotropic-to-anisotropic temporal boundaries in two-dimensional (2D) subwavelength geometries rather than in spatially unbounded media. We provide an in-depth study of such spatiotemporal meta-atoms demonstrating both numerically and theoretically how the direction of the resulting ‘temporal’ scattering produced by the isotropic-to-anisotropic temporal boundaries induced in subwavelength geometries can be manipulated at-will by properly engineering the values of the relative permittivity tensor \(\varepsilon_{r2} = \{\varepsilon_{r2x}, \varepsilon_{r2y}, \varepsilon_{r2z}\}\). All designs are validated using the time-domain solver of the commercial full-wave simulation software COMSOL Multiphysics®.

## 2. Methods

### 2.1. Concept: isotropic to anisotropic temporal boundaries

To begin with, a schematic representation of the proposed spatiotemporal isotropic-to-anisotropic meta-atom is shown in figure 1. In this work, we consider all media to be non-magnetic, i.e. with \(\mu_r = 1\). The background medium is isotropic with a constant (time-invariant) relative permittivity defined as \(\varepsilon_{\text{background}} = \varepsilon_{r1}\). We consider a single 2D subwavelength meta-atom being immersed in such spatially unbounded background medium and being illuminated by an obliquely incidence \((\text{incidence angle } \theta_1)\) \(p\)-polarized wave \((\text{with the magnetic field polarized along the out-of-plane } y\text{-axis})\). All quantities are independent of \(y\) variable.

As shown in figure 1(a), for times \(t < t_1\), \(\varepsilon_r\) of the meta-atom is \(\varepsilon_{r1}\); i.e. the same as the background medium, so no scatterer is present before \(t < t_1\). Note that we have chosen this value as the same as the background medium to remove any spatial scattering for times before inducing the temporal boundaries \((t < t_1)\) in the meta-atoms. Our concept would still be applicable if the scatterer is different from the background medium for \(t < t_1\), but here we consider this assumption for the sake of simplicity. At \(t = t_1\), the \(\varepsilon_r\) of the meta-atoms is changed to an anisotropic permittivity tensor \(\varepsilon_{r2} = \{\varepsilon_{r2x}, \varepsilon_{r2y}, \varepsilon_{r2z}\}\). In so doing, the
meta-atom will act as a new source creating a cylindrical wave (see figures 1(b) and (c) for a schematic representation) which will travel in a direction determined by the induced tensor $\varepsilon_{\text{r}} = \varepsilon_{\text{r}1} = \varepsilon_{\text{background}}$. Importantly, differently to our previous temporal aiming work [66] where the $\varepsilon_{\text{r}}$ of the whole spatially unbounded medium was changed from isotropic-to-anisotropic tensor, here the rapid change of $\varepsilon_{\text{r}}$ is only applied to the subwavelength 2D meta-atom, meaning that the new radiated wave from the meta-atom travel within the time-invariant background medium, enabling the possibility of an arbitrary manipulation of the direction of such ‘temporal’ scattering. Finally, to avoid ‘spatial’ scattering, the $\varepsilon_{\text{r}}$ of the meta-atom is then quickly returned to the initial isotropic value $\varepsilon_{\text{r}1}$ at $t = t_2$.

To calculate the angles ($\theta_2$) of the emitted wave produced by the proposed spatiotemporal meta-atom, we can exploit our recent approach for isotropic-to-anisotropic temporal boundaries in spatially unbounded media [66, 69]. As we have shown in [66, 69], the wave that experiences such temporal boundary will preserve the wavenumber $k$ while direction of the energy propagation defined by the Poynting vector ($S$) is modified $[\{\theta_1 = \theta_{1xS} = \theta_{2S}\} \neq \{\theta_2 = \theta_{2S}\}]$. This new direction of the energy propagation $S (\theta_2)$ can be theoretically calculated by considering that the permittivity tensor $\varepsilon_{\text{r}2} = \{\varepsilon_{r2x}, \varepsilon_{r2z}\}$ will modify the amplitude of the $E_z$ and $E_x$ components of the electric field. Hence, $\theta_2$ can be mathematically expressed via the following closed-form equation [66]:

$$\theta_2 = \theta_{2S} = \tan^{-1} \left[ \tan (\theta_1) \left( \frac{\varepsilon_{r2z}}{\varepsilon_{r2x}} \right) \right].$$

(1)

To evaluate the implications of the expression above, the analytical values of $\theta_2$ are shown in figure 2 considering different incident angles of the monochromatic $p$-polarized wave (namely $\theta_1 = 5^\circ$, $\theta_1 = 45^\circ$, and $\theta_1 = 65^\circ$). We consider that $\varepsilon_{\text{r}}$ is changed from $\varepsilon_{\text{r}1} = 10$ to a tensor $\varepsilon_{\text{r}2} = \{\varepsilon_{r2x}, \varepsilon_{r2z}\}$ at $t = t_1$. As it is shown in figure 2, $\theta_2$ is strongly dependent on $\theta_1$, $\varepsilon_{\text{r}1}$ and $\varepsilon_{\text{r}2}$, as expected from equation (1). For instance, it can be observed how larger values of $\theta_1$ can be achieved when increasing the angle of the monochromatic wave before inducing the temporal boundary ($\theta_1$). To guide the eye and for the sake of completeness, we extracted $\theta_2$ from figures 2(a)–(c) considering two different fixed values of $\varepsilon_{r2x} = 12.5$ and $\varepsilon_{r2z} = 5$ while varying $\varepsilon_{r2x}$. The results are presented in figures 2(d)–(f), respectively. From figures 2(d)–(f) one can clearly note the influence of both $\theta_1$ and $\varepsilon_{\text{r}2}$ on $\theta_2$. For example, if $\varepsilon_{r2z} = 12.5$, $\theta_2$ can be modified within the range of $\{\sim-0.4^\circ \text{ to } \sim8^\circ\}$, $\{\sim4.5^\circ \text{ to } \sim58^\circ\}$ or $\{\sim9.7^\circ \text{ to } \sim74^\circ\}$ when the $\varepsilon_{r2z}$ is varied from $\varepsilon_{r2z} = 1$ to $\varepsilon_{r2z} = 20$, respectively considering incident angles of $\theta_1 = 5^\circ$, $\theta_1 = 45^\circ$ and $\theta_1 = 65^\circ$, respectively (see black lines in figures 2(d)–(f)). The values of $\theta_2$ can then be further increased to be within the range of $\{\sim1^\circ \text{ to } \sim19.3^\circ\}$.
Figure 2. Analytically evaluated values of the angle of the energy propagation for times $t > t_1$ ($\theta_2$, calculated using equation (1)) for a monochromatic $p$-polarized wave traveling in a spatially unbounded medium with an incidence angle $\theta_1$. The values of $\theta_2$ are calculated after $\varepsilon_r$ of the spatially unbounded medium is changed from isotropic $\varepsilon_r^1 = 10$ to a tensor $[\varepsilon_r^2, \varepsilon_r^2]$ at $t = t_1$. (a)–(c) $\theta_2$ as a function of $\varepsilon_r^{2z}$ and $\varepsilon_r^{2x}$ for incidence angles $\theta_1 = 5^\circ$, $\theta_1 = 45^\circ$ and $\theta_1 = 65^\circ$, respectively. (d)–(f) $\theta_2$ extracted from the horizontal dotted lines in (a)–(c), respectively, considering fixed values of $\varepsilon_r^{2x} = 5$ (blue lines) and $\varepsilon_r^{2x} = 12.5$ (black lines) while varying $\varepsilon_r^{2z}$.

$[\sim 11.3^\circ$ to $\sim 76^\circ]$ or $[\sim 23.2^\circ$ to $\sim 83.4^\circ]$ when using the same range of $\varepsilon_r^{2z}$ and values of $\theta_1$, respectively, by simply reducing $\varepsilon_r^{2x}$ to $\varepsilon_r^{2x} = 5$ (see blue lines in figures 2(d)–(f)).

Note that this approach, which we have proposed and demonstrated numerically in spatially unbounded temporal metamaterials, is also valid for the spatiotemporal meta-atom shown in figure 1(a) where, as it will be shown in the following sections, the newly emitted wave, resulted from applying the first isotropic-to-anisotropic temporal change of $\varepsilon_r^{2z}$, will have the same angle defined by equation (1). Hence, given the subwavelength size of the meta-atom, the new radiated wave will exit the meta-atom and preserve its direction while traveling within the time-independent background medium, as explained above.

2.2. Numerical simulations

All the numerical calculations were carried out using the time-domain solver of the commercial software COMSOL Multiphysics® following a similar setup as in [66]. In all the simulations, a rectangular box of dimensions $20\lambda \times 12\lambda$ was implemented as the background medium and the subwavelength particles (2D circle/square) were immersed and placed at the center of the box ($x = z = 0$). A scattering boundary condition was implemented on the top boundary of the simulation box and used to apply the incident wave with the magnetic field polarized perpendicular to the plane of incidence. The incident wave was modelled as a Gaussian beam with a beamwaist of diameter $9\lambda$ in order to account for an approximately planar phase of the illumination upon the subwavelength particles. The Gaussian beam was calculated by implementing the full non-paraxial Gaussian beam expression via the angular spectrum technique for plane waves, as in [66], where the incident wave is defined as the integral of multiple planewaves propagating with the same magnitude of wavenumber $k$ but in different directions [71]. (Since this Gaussian beam has a wide waist, it approximately resembles a plane wave with a given direction of propagation when illuminates the particles.) Scattering boundary conditions were also implemented on the left, bottom and right boundaries of the simulation box to avoid unwanted reflections. The rapid changes in $\varepsilon$ in all the studies were modelled by implementing rectangular analytical functions with smooth transitions having rise/fall times smaller than the period of the incident wave ($\sim 0.003T$). Moreover, two continuous derivatives were applied to these transitions to ensure convergence in the calculations. Finally, a triangular mesh was implemented with minimum and maximum sizes of $1.5 \times 10^{-4}\lambda$ and $0.15\lambda$, respectively, to ensure accurate results.
3. Results and discussion

In this section we evaluate the performance of the proposed 2D spatiotemporal meta-atom considering different 2D subwavelength geometries, namely cylinders and squares, of diameter/lateral size of $l \ll \lambda$ ($l = 0.1\lambda$ with $\lambda$ being the wavelength of the incident wave inside the background medium). The relative permittivity of the background medium is again considered to be time invariant with a value of $\varepsilon_{r1} = 10$. Without loss of generality, we make use of an obliquely incident monochromatic $p$-polarized wave (modelled as a Gaussian beam with a beam waist of diameter $9\lambda$, see section 2.2) with an incidence angle $\theta_1 = 25^\circ$ to illuminate the meta-atoms. As in the previous section, all media in this work is considered to be non-magnetic ($\mu_r = 1$). The numerical calculations are carried out using the time-domain solver of the commercial software COMSOL Multiphysics® with the same setup as in [66].

3.1. Tailoring the temporally scattered waves using spatiotemporal meta-atoms

The analytical results of the angles of the energy propagation after inducing an isotropic-to-anisotropic temporal boundary ($\theta_2$), calculated from equation (1), are shown in figure 3(a) as a function of the relative permittivity tensor $\varepsilon_{r2} = \{\varepsilon_{r2x}, \varepsilon_{r2z}\}$. For completeness, the values of $\theta_2$ as a function of $\varepsilon_{r2x}$ when fixing $\varepsilon_{r2z}$ to $\varepsilon_{r2z} = 1, 3, 5, \text{and } 8$ are shown in figure 3(b). As observed, similar results as those shown in figure 2 are obtained where $\theta_2$ is strongly dependent on $\theta_1$ and $\varepsilon_{r2z}$, as expected.
Let us know numerically evaluate the scattered wave being emitted by the proposed spatiotemporal meta-atom, with circular-cylindrical cross section of radius 0.05λ, using different values of the tensor $\mathbb{e}_{22}$. For the numerical simulations, we consider that $\varepsilon_r$ of the meta-atoms is rapidly changed in time from isotropic $\varepsilon_{r1} = 10$ to a tensor $\mathbb{e}_{22} = \{\varepsilon_{22x}, \varepsilon_{22y}\}$ at a time $t = t_1 = 12.01T$ and it is returned to $\varepsilon_{r1} = 10$ at $t = t_2 = 12.05T$. As described in the previous section, note that here $\tau = t_2 - t_1 \ll T$ in order to avoid/lessen spatial scattering (i.e. the scattering that would be produced if the incident monochromatic $p$ polarized wave could interact with an anisotropic subwavelength particle). A discussion about such spatial scattering will be presented later in this manuscript. With this configuration, the numerical results of the scattered magnetic field distribution (calculated as $H_{\text{scat}} = H_{\text{total}} - H_{\text{inc}}$, with $H_{\text{total}}$ and $H_{\text{inc}}$ being the magnetic field with and without the presence of the spatiotemporal meta-atom, respectively) at different times after inducing the temporal boundaries ($t > t_2$) are shown in figures 3(c) and (d) considering a tensor of $\mathbb{e}_{22} = \{\varepsilon_{22x} = 8, \varepsilon_{22y} = 1\}$ and $\mathbb{e}_{22} = \{\varepsilon_{22x} = 1, \varepsilon_{22y} = 5\}$, respectively. Note that the scattered field distributions shown in figures 3(c) and (d) are not completely smooth, which is more evident for longer times after applying the temporal boundary (see right panels of the same figure). This is due to the finite size of the mesh used which, despite being extremely small (see section 2.2) does not smoothly capture all the details of the radiated field for longer times. This is also partially due to the subwavelength nature of the 2D circular/square particles which, given the induced temporal boundary, produces a temporal scattering related to the size of the particle, as expected. However, as it is shown in figure 3 and the following studies, the numerical results allow us to accurately verify the change of direction of the scattered field when inducing a temporal boundary within the subwavelength 2D geometries.

From these results, one can clearly note how the scattered field (emitted wave) traveling within the (time-independent) background medium has an angle of $\theta_2 \sim 80^\circ$ and $\theta_2 \sim 5^\circ$ for each case of $\mathbb{e}_{22}$ in figures 2(c) and (d), respectively, in line with the analytical values from figure 2(a) which predicts $\theta_2 = 75^\circ$ and $\theta_2 = 5.32^\circ$, respectively. Importantly, note that the analytically predicted values consider the case when the $\varepsilon_r$ is changed from isotropic-to-anisotropic and kept to this value while the numerical simulations consider that $\varepsilon_r$ is returned to isotropic at $t = t_2$. As discussed before, when applying the first temporal boundary at $t = t_1$, the spatiotemporal meta-atom will emit a cylindrical wave with angle $\theta_2$. Given the subwavelength size of the meta-atom, this scattered wave almost completely leaves the meta-atom and propagates in the background medium with an angle being approximately equals to $\theta_2$. This has interesting implications as the proposed spatiotemporal meta-atom could potentially be engineered to emit such scattered waves at any desired output angle by simply modifying the values of the tensor $\mathbb{e}_{22}$, a mechanism that can open new directions for EM wave manipulation using 4D metamaterials. Finally, note how the amplitude of the scattered field is different when using different values of $\mathbb{e}_{22}$, as shown in figures 3(c) and (d). These results are in agreement with our previous findings [66] where it was shown how the amplitude of the FW and BW waves produced by an isotropic-to-anisotropic temporal change of $\varepsilon_r$ applied within a spatially unbounded medium is different depending on $\mathbb{e}_{22}$ and incident angle $\theta_1$. For instance, if one considers the values of $\theta_1$ and $\varepsilon_{22}$ used in figures 3(c) and (d), the predicted FW based on [66] would be the indicated figure.
3.2 Changing the geometry of the spatiotemporal meta-atom

In section 3.1 we discussed the scattered wave produced by 2D circular-cylindrical spatiotemporal meta-atoms demonstrating how its direction can be tailored by using isotropic-to-anisotropic temporal boundaries with properly chosen values for the tensor $\varepsilon_{r2}$. However, what would happen if the shape of the meta-atom is different? Intuitively, one can foresee that the shape of the meta-atom would not be an important parameter for the resulting angle of the scattered wave ($\theta_2$) given its subwavelength size ($l \ll \lambda$).

In the following study, this question will be addressed by considering 2D square (with a lateral size of $0.1\lambda$) and circular (with a diameter of $0.1\lambda$) shapes for the spatiotemporal meta-atom. Here the spatiotemporal meta-atoms are illuminated with a monochromatic obliquely incident $p$-polarized wave with an incidence angle of $\theta_1 = 45^\circ$ or $\theta_1 = 25^\circ$ (see figures 4(a) and (b), respectively).

The numerical results of the scattered magnetic field distribution for the squared and cylindrical meta-atoms after inducing the temporal boundaries ($t > t_2$) are shown in figures 4(a) and (b) considering values of the tensor $\varepsilon_{r2}$ of $\varepsilon_{r2} = \{\varepsilon_{r2z} = 6, \varepsilon_{r2x} = 1\}$ and $\varepsilon_{r2} = \{\varepsilon_{r2z} = 1, \varepsilon_{r2x} = 15\}$, respectively. By comparing the results for both 2D shapes of the spatiotemporal meta-atoms, one can clearly notice that the direction of the scattered wave $\theta_2$ is effectively the same, as expected due to the subwavelength size of the meta-atoms. Finally, note that the resulting values of $\theta_2$ are $\theta_2 \sim 77^\circ$ and $\theta_2 \sim 2^\circ$ when using approximately two times for the case with $\varepsilon_{r2} = \{\varepsilon_{r2z} = 1, \varepsilon_{r2x} = 5\}$ compared to the scenario with $\varepsilon_{r2} = \{\varepsilon_{r2z} = 8, \varepsilon_{r2x} = 1\}$, in agreement with the results shown in figures 3(c) and (d).
Figure 6. Effect of temporal duration $\tau = t_2 - t_1$: (a)–(f) temporal variation of $\varepsilon_r$ of the 2D cylindrical meta-atom considering a value of $\tau = 0.1T$ and $\tau = T$ (b)–(e) and (g)–(j) scattered magnetic field at different times for the temporal variation of $\tau = 0.1T$ and $\tau = T$, respectively. In all these results an incident angle of $\theta_1 = 25^\circ$ (from top-to-bottom) is considered. In all the simulations, the incident magnetic field $H_{inc}$ has an amplitude of $3 \times 10^{-3}$ A m$^{-1}$. All the color scales are in $10^{-3}$ A m$^{-1}$.

$\varepsilon_r = \{\varepsilon_{r2}, 6, \varepsilon_{r2} = 1\}$ and $\varepsilon_r = \{\varepsilon_{r2}, 1, \varepsilon_{r2} = 15\}$, respectively, in agreement with equation (1) which theoretically predict values of $\theta_2 = 80.54^\circ$ and $\theta_2 = 1.78^\circ$, respectively. Note that in this section, and in all the studies in the main part of this manuscript, we consider $l \ll \lambda$ for all the 2D particles to approximate the radiated scattered field as a single 2D dipole radiating a cylindrical wave. If larger geometries are used, then the scattering produced by such larger particles will be different. This is due to the fact that larger objects should be modeled by more than a single radiating 2D dipole, hence the problem becomes different from the case proposed here using small dielectrics modeled as single dipoles. We provide an example in appendix A where it is shown how the results look like when we use the larger 2D circles/squares.

### 3.3. Asymmetric response of the spatiotemporal meta-atoms

Based on the results shown in figures 2–4, one may ask the following question: does this spatiotemporal meta-atom has an intrinsically nonreciprocal response? We have discussed how the direction of the scattered wave produced by the spatiotemporal meta-atom can be tailored by changing the tensor $\varepsilon_{r2}$ and incident angle $\theta_1$. Hence, one may foresee a nonreciprocal behavior of the meta-atom.

To study this, let us consider the case shown in figure 5(a) using a 2D cylindrical shape (diameter of $0.1\lambda$) of the meta-atom. As in the previous section, an incident monochromatic $p$-polarized wave with $\theta_1 = 25^\circ$ is again traveling from top-to-bottom on the $zx$ plane. The $\varepsilon_r$ of the meta-atom is isotropic with $\varepsilon_{st} = 10$ for $t < t_1$, it is changed to an anisotropic tensor $\varepsilon_{r2} = \{\varepsilon_{r2}, 5, \varepsilon_{r2} = 2\}$ at $t = t_1$ and returned to $\varepsilon_{st} = 10$ at $t = t_2$ (with $\tau = 0.04T$). With this setup, the numerical results of the incident magnetic field distribution ($H_{inc}$) at a time $t < t_1$ is shown in figure 5(d) along with the scattered magnetic field distribution ($H_{scat}$) calculated at different times $t > t_1$ (second and third panels). From these results, the direction of the scattered wave produced by the spatiotemporal meta-atom is $\theta_2 = 49^\circ$, in agreement with the theoretically predicted value from equation (1) of $\theta_2 = 49.38^\circ$. To evaluate the possibility of using the proposed spatiotemporal meta-atom as a nonreciprocal subwavelength particle, we can then use the angle...
\[ \theta_2 \sim 49^\circ \] as the new incidence angle of the incident monochromatic wave and illuminate the meta-atom from bottom-to-top with an angle of \( \theta_1 = 49^\circ \). With this configuration, the instantaneous incident \( H_{\text{inc}} \) and scattered \( H_{\text{scat}} \) magnetic field distribution calculated at the same time as the result discussed in figure 5(d) are shown in figure 5(e), respectively. From these results, once can clearly see how the resulting scattered wave produced by the spatiotemporal meta-atom is directed to an angle \( \theta_2' \sim 71^\circ \) (again in agreement with equation (1) which predicts \( \theta_2' \sim 70.8^\circ \)), i.e. with \( \theta_2' \neq \theta_1 \), the scattered field is not directed towards 25\(^\circ\) (incidence angle used for figure 5(d)), a value that would account for a reciprocal behavior of the meta-atom.

These results demonstrate how spatiotemporal meta-atoms that are modulated in time with isotropic-to-anisotropic functions of \( \varepsilon_2 \) can expectedly exhibit an intrinsic nonreciprocal behavior and can be used to steer the generated scattered wave in real time, features that could open the way towards new paradigms for spatiotemporal control of EM wave propagation using 4D metamaterials.

3.4. Effect of changing \( \tau = t_2 - t_1 \)

All the results presented in the previous sections have been obtained considering \( \tau = t_2 - t_1 \sim 0.04T \). Here we will discuss the effect of this parameter on the scattered wave produced by the spatiotemporal meta-atom with induced temporal boundaries. Here we will use two different values of \( \tau \), namely \( \tau = 0.1T \) and \( \tau = T \) (both larger than those used in the previous sections). The temporal function of \( \varepsilon_2(t) \) for the spatio-temporal meta-atoms is schematically shown in figures 6(a) and (f), respectively. Let us first examine the case with \( \tau = 0.1T \). The numerical results of the scattered magnetic field distribution at different times are shown in figures 6(b)–(e). As observed, we have chosen times at \( t = t_1^+ \), \( t_1 < t < t_2 \), \( t = t_2^− \) and \( t > t_2 \), respectively. From these results, one can notice how the resulting scattered wave resembles that shown in figure 5(d). However, note that here the effect of the second temporal boundary at \( t = t_2 \) starts to become more evident (see figure 6(e)) with the scattered wave having more oscillations that that shown in figure 5(d). However, note that spatial scattering in this case is almost negligible as the duration of \( t \ll T \), as expected.

What would happen if we increase the duration of \( \tau \)? The numerical results of the magnetic field distribution for the case with \( \tau = T \) are shown in figures 6(g)–(j) calculated at the same times as in figures 6(b)–(e), respectively. Interestingly, by comparing the results shown in figures 6(d) and (i), one can note how the scattered wave produced by the first temporal boundary is present in both cases along with a small spatial scattering (see also figures 6(c) and (h)), as expected. Moreover, for a time \( t = 12.46T \), two main differences can be noticed by comparing the results with \( \tau = 0.1T \) (figure 6(e)) and \( \tau = T \) (figure 6(j)). One is that the number of oscillations of the scattered wave is smaller for the case with \( \tau = T \), which is a direct consequence of only one temporal boundary being induced at that time while two temporal boundaries have been induced for the case with \( \tau = 0.1T \). The second difference is that spatial scattering is observable for the case with \( \tau = T \) which is not noticeably present for \( \tau = 0.1T \). Again, this is an expected result given that at a time \( t = 12.46T \), the \( \varepsilon_2 \) of the meta-atom is still anisotropic for the case...
with \( \tau = T \) while it is isotropic and equals to the background medium for the case with \( \tau = 0.1T \). These results demonstrate how mainly temporal or both spatial and temporal scattering can be produced depending on the values of \( \tau \) in spatiotemporal meta-atoms with induced isotropic-to-anisotropic temporal boundaries.

4. Conclusions

In conclusion, we have shown and studied, both numerically and theoretically, the use of isotropic-to-anisotropic temporal changes of permittivity within spatially subwavelength regions rather than in spatially unbounded media. In this context, we have proposed what we called spatiotemporal isotropic-to-anisotropic meta-atoms. Several configurations have been presented such as different values of permittivity tensor and shape of the subwavelength particle. We have shown how the direction of the scattered wave produced by the meta-atom when inducing such temporal changes of permittivity can be manipulated at will by carefully crafting the values of the permittivity tensor of the meta-atom. A discussion about the intrinsically non-reciprocal response of the spatiotemporal meta-atom was presented along with the effect of the time duration \( (\tau = t_2 - t_1) \) of the temporal boundary, showing how either only temporal or both spatial and temporal scattering can be achieved by using different values of \( \tau \). These results may open new avenues for exploration and exploitation of the next generation of metamaterials and metasurfaces by using 4D spatiotemporally modulated meta-atoms.

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Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

Appendix A. Effect of larger particles

In the results presented in the main part of this manuscript, we have considered that the region where \( \varepsilon_r \) is changed from an isotropic to an anisotropic tensor consist of a subwavelength 2D circle/square with a size \( l \ll \lambda \). In doing so, the radiated scattered field produced by the particles (due to the induced temporal boundaries) is a cylindrical wave that is comparable to the radiation from a single dipole. Here we explore the effect of larger particle sizes with different shapes. The numerical results of the scattered magnetic field distributions at times after the \( \varepsilon_r \) is changed from isotropic to anisotropic and then returned to the initial isotropic value are shown in figures 7(a) and (b) considering 2D circles/squares with sizes of \( l = 0.25\lambda \) and \( l = 0.5\lambda \), respectively. In these examples, the temporal function of \( \varepsilon_r \) for the particles is the same as the one used in the results shown in figure 4(b). However, as observed by this simulation, the scattering produced by different shapes become different when increasing their size. This is an expected result given that now the scattering is due to radiation from many dipoles, instead of a single dipole.

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