Analytical Method for Prediction of Suction Performance of Ejector-Jet*

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To reduce the cost of space transportation, air-breathing engines are considered to be candidates for propulsion. However, to cover a wide range of flight speeds, the propulsion system has to operate in various modes to be efficient under incoming atmospheric-air conditions. The Japan Aerospace Exploration Agency is proposing a rocket-based combined cycle engine for operation under various condition, an ejector-jet mode being adopted for the low-speed regime. The suction performance ejector-jets has long been studied experimentally and numerically at JAXA, and little success has been achieved in explaining the deterioration of suction performance with high-temperature gas or light gas such as helium. In the present study, based on former models, a simple one-dimensional model was introduced incorporating the mixing effects of the primary flow (rocket flow) and secondary flow (induced air flow). The results were compared using several experimental and numerical data to check the plausibility of the model. It was found that if greater mixing occurs, suction performance is degraded, explaining the actual phenomena of the experiments.

Key Words: Air-breathing Engine, Combined Cycle, Ejector-jet

Nomenclature

\[ A: \text{area} \]
\[ C_p: \text{specific heat at constant pressure} \]
\[ E: \text{energy} \]
\[ G_c: \text{incompressible growth rate of mixing layer} \]
\[ G_f: \text{compressible growth rate of mixing layer} \]
\[ H: \text{height} \]
\[ K: \text{momentum} \]
\[ M: \text{Mach number} \]
\[ P: \text{total pressure} \]
\[ R: \text{gas constant} \]
\[ T: \text{total temperature} \]
\[ W: \text{width} \]
\[ \dot{m}: \text{mass flow} \]
\[ \rho: \text{static pressure} \]
\[ r: \text{velocity ratio} \]
\[ s: \text{density ratio} \]
\[ t: \text{static temperature} \]
\[ u: \text{velocity} \]
\[ w: \text{molecular weight} \]
\[ \alpha = \left( \frac{T_1 w_2}{T_2 w_1} \right)^\frac{\gamma - 1}{\gamma} \]
\[ \beta: \text{mixing parameter} \]
\[ \gamma: \text{specific heat ratio} \]
\[ \mu: \text{suction rate} \]
\[ \rho: \text{density} \]

Subscripts

1: primary flow (rocket flow)
2: secondary flow (induced air flow)

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1. Introduction

To reduce the cost of space transportation, engine efficiency is an important aspect. However, conventional rocket engines have come close to optimal performance, and the next step in the evolution of space propulsion has to incorporate a fundamentally more efficient cycle to enhance the effective specific impulse (\(e_{isp}\)). The combined cycle engine, which is a hybrid rocket-jet engine, has been proposed for this purpose and extensively studied. At the JAXA Kakuda Space Propulsion Center, rocket-base combined cycle engines (RBCC) have been studied since the turn of the century.1) Since all space transporters have to accelerate from zero to orbital speed, the engine has to operate in a wide range of flight speeds. The rocket can inherently achieve this requirement but with limited \(e_{isp}\). On the other hand, air-breathing engines can produce thrust with a much higher \(e_{isp}\), but their range of speed in which high performance can be achieved is rather limited, depending on the operation cycle. JAXA’s RBCC uses both a rocket and several types of air-breathing cycles to maintain higher performance throughout the entire ascent flight (Fig. 1). In the supersonic or hypersonic region, the RBCC operates a ram/scramjet combustion cycle, while in the subsonic or transonic region, where ram pressure is not sufficient, it uses a rocket as the main thrust generator. To enhance the specific impulse even in such a low-speed region, the air induced into the RBCC flow duct by the rocket exhaust is used as an oxidizer for extra
fuel. The operation cycle is designated as the "ejector-jet" mode. Ram and scramjet engines had been studied at Kakuda for several years\(^2,3\) prior to the RBCC research. However, the ejector-jet had not been well explored at Kakuda.

To study the ejector-jet and ejector effect, several tests using small-scale models with room temperature gas as rocket exhaust have been carried out\(^4,5\). Also large-scale engine tests with hydrogen/oxygen rockets have been conducted\(^6\). The results showed that the choked air flow (secondary flow) can be established with nitrogen gas at room temperature. On the other hand, with the very same geometrical configuration, the choked condition is hard to establish when using lighter molecular weight gas (light gas), such as helium or high-temperature gas. Since the amount of the intake-air is directly related to the performance of the ejector-jet, it is imperative to study the reason for the degradation of suction performance in order to design an RBCC engine with high performance. In 2009, to study the suction performance under low subsonic conditions, experiments with a hybrid sounding rocket, CAMUI\(^7\), were carried out in cooperation with Hokkaido University and the Hokkaido Aerospace Science and Technology Incubation Center. A cylindrical duct was attached around the nozzle section of the CAMUI to form an ejector duct. Comparing the small-scale tests with nitrogen gas, no degradation of suction performance was observed in the CAMUI experiments. The results further confused the understanding of the ejector mechanism.

Along with the experiments, several models of ejector flow field have been examined. Aoki et al.\(^9\) proposed a simple momentum transfer model based on the Fabri model\(^10,11\). However, that method does not explain the difference of suction performance caused by the difference in the nature of the primary gas. The present study aims to enhance this method by considering the effect of mixing rocket gas (primary flow) and air flow (secondary gas). In short, Aoki’s method assumes the pressure of the choked secondary flow at the terminal point where the momentum exchange of two flows is completed. The present method assumes higher pressure due to the mixing of two flows at the terminal point. Analysis with the present method was carried out to compare the experimental results and to see if mixing is the reason for the degradation of suction performance.

2. Analytical Model

Aoki et al.\(^9\) proposed a momentum transfer model which considers the exchange of momentum between the primary and secondary flow at the contact surface. It is assumed that, through the contact surface, the momentum which equals the product of the pressure and contact area is exchanged, whereas no other properties (mass flow or energy) remain constant for each flow. At the end of the interaction, the pressure of two flows matches, and the secondary flow reaches its choked condition (i.e., sonic condition). The suction performance (initial condition of the secondary flow) is iteratively determined, so that pressure matching and choked condition are simultaneously achieved. The model is applicable if the secondary flow is in the subsonic condition, and so is true for the present model described below.

In the present study, it is assumed that primary and secondary flows form a mixing layer. At some distance from the initial contact point, primary, secondary and tertiary (mixed) flows complete an exchange of mass, momentum and energy, and reach a uniform pressure condition. To calculate the flow properties, momentum exchange and mass transfer to the mixing layer should be simultaneously considered, but this would require an intricate calculation process.

To avoid complexity, in the present method, two processes (momentum transfer and mixing) are separately modeled and are assumed to occur in two steps. In the first step, only momentum transfer is considered. In the second step, starting from the momentum-transferred conditions, the mixing process of two flows is considered. The present procedure is outlined in Figs. 2 and 3.

The primary aim of the present analysis was to study the degradation of suction performance due to the pressure rise caused by mixing. Neither physical mechanism of mixing nor the mixing rate were sufficiently modeled in this procedure. Instead, a single mixing parameter was introduced, and the relation between the parameter and the suction performance was calculated in the analysis. Here, the mixing parameter \( \beta \) is defined as the ratio of half of the mixing layer thickness \( (H_m) \) to the smaller duct height \( (H_1 \text{ or } H_2) \) of the primary or secondary flow.

\[
\beta = \frac{H_m}{MIN(H_1, H_2)},
\]

where \( MIN( ) \) is a function that gives the smaller number from arguments. At the beginning of the procedure, the incoming secondary flow condition (i.e., the Mach number, \( M_2 \)) is assumed. The momentum exchange is then evaluated to obtain the intermediate state of the two flows. Unlike Aoki’s method, the secondary flow does not necessarily reach the choked condition and only pressure matching of the two flows is imposed. After the momentum exchange, the mixing process is calculated from the intermediate state. In this process, an arbitrary \( H_m(\beta) \) is given and the three layers of flow field are equalized in pressure at the terminal point. If a match is established, it is considered that the assumed incoming secondary flow condition is the solution.
for the given $\beta$. If the mixing procedure does not yield an equalized pressure state, the secondary flow condition is iteratively changed, and the solution is sought.

For the detailed mathematical procedure of each model, the unknowns and equations are summarized in the following subsections. In general, mass, momentum and energy of the flow ($\dot{m}, K$ and $E$, respectively) are evaluated by the following equations in the subsections.

\[ \dot{m} = \rho u A \]  
\[ K = p A + \dot{m} u \]  
\[ E = \dot{m} \left( C_p \cdot t + \frac{u^2}{2} \right), \]

where $C_p$ is specific heat at constant pressure.

### 2.1. Momentum transfer process

Assuming that the initial condition of the secondary flow is known, five parameters for each primary and secondary flow ($\rho, p, t, u$ and $A$) at the intermediate condition are obtained in the process. Each flow satisfies the mass, momentum and energy conservation from the initial state to the intermediate state. Additionally, the ideal gas law, $p = \rho R_t$, is adopted (four equations for each flow). For momentum conservation, it is assumed that the pressure force on the contact surface transfers momentum from one flow to the other:

\[ A_i(\rho_i u_i + \rho_i u_i^2) - p_i \Delta A = A_i(\rho_i u_i + \rho_i u_i^2), \]

where subscripts 1 and 2 denote the primary and secondary flow, respectively. Also note that the subscript $i$ indicates an initial (incoming) condition, and $x$ indicates an intermediate state. $p_i$ is the pressure along the boundary of the two layers and $\Delta A$ is the variance of area between $A_{1i}$ and $A_{2i}$. In this process, it is assumed that a flow with higher pressure expands to the lower pressure stream. Thus, if $p_{1i} > p_{2i}$, primary flow expands, and $p_i \Delta A$ is positive. If otherwise, secondary flow expands and $p_i \Delta A$ becomes negative. In all, eight equations for 10 parameters are given. However, since the pressure matches at the intermediate state and overall area at the intermediate state is also known, two more relations can be incorporated. Thus, all quantities can be obtained.

### 2.2. Mixing process

The mixing model assumes that the mixing layer develops with the same height $H_m$ into both the primary and secondary flows. The mass of the primary flow and that of the secondary flow that go into the mixing layer, $\dot{m}_{1m}$ and $\dot{m}_{2m}$, are obtained from the initial state of the flow (i.e., $\rho_i u_i H_m W$ and $\rho_i u_i H_m W$ where $W$ is the width of flow field), which is constant for both primary and secondary flow passage for the sake of simplicity. The mass conservation of the mixing layer is written as follows:

\[ \dot{m}_1 + \dot{m}_2 = \dot{m}_m \]

Here, subscript $m$ denotes the mixing layer condition. As for the properties in the mixing layer, the unknown parameters are five: namely, $\rho_m$, $p_m$, $t_m$, $u_m$ and $A_m$. The flows out of the mixing layer (i.e., primary and secondary flows that are left unmixed) are assumed to change isentropically. These two flows satisfy mass conservation and ideal gas law (two equations for each flow).

\[ \rho_{1i} u_{1i} A_1 = \dot{m}_{1m} = \rho_{1i} u_{1i} A_{1i} \]  
\[ p_{1i} = \rho_{1i} R_1 t_{1i}, \]

where subscript $t$ denotes the terminal condition. If flow is assumed to be in an isentropic condition, static pressure and temperature can be written in isentropic relations (two equations):
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\[ t_{1t} = t_{1s} \left\{ \frac{2 + (\gamma - 1)M^2\text{fl}}{2 + (\gamma - 1)M^2\text{mc}} \right\} \]

By adding the Mach number relation, \( M = u/\sqrt{(\gamma R T)} \), five equations can be obtained for the flow out of the mixing layer, which has six unknowns \((p_1, \rho_1, \rho_2, u_1, A_m, \text{and} \ M)\). Furthermore, the sum of momentum and energy of primary, secondary and tertiary flows are conserved (two equations):

\[ K_{1i} + K_{2i} = K_{1v} + K_m + K_{2v} \]

\[ E_{1i} + E_{2i} = E_{1v} + E_m + E_{2v} \]

For the flow inside the mixing layer, the ideal gas relation is imposed. Thus, 14 equations for 17 unknowns are obtained. However, since the pressure matches at the terminal point, two pressure values can be neglected and unknowns are reduced to 15.

\[ p_m = p_{1i} = p_{2i} \]

In this analysis, for the sake of simplicity, the entire flow area is constant from initial to terminal points, giving a 15th equation:

\[ (H_1 + H_2)W = A_{1i} + A_{2i} = A_{1v} + A_m + A_{2v} \]

Thus, there are 15 equations for 15 unknowns. If the intermediate state is given, terminal values can be solved.

3. Experiments and CFD Analysis

Figure 4 shows a typical diagram of the measurement system. Wall pressures were measured by various pressure transducers (300 kPa, 700 kPa FS) which were electrically scanned. The signal was then A/D converted and stored in a computer. The total pressure of the primary gas was obtained with a sensor of 5 MPa FS, separately from measurement of the wall pressures. To synchronize the time of the two systems, the starting signal was recorded in both systems and matching of the time lines was manually established by signal pattern comparison. The suction performance was estimated based on the wall pressure at the throat of the model assuming the isentropic process in the inlet section. Prior to the experiments, the measurement system was calibrated, and its end-to-end uncertainty was confirmed to be less than 0.5% of full scale.

To investigate the growth of a mixing layer, a gas sampling was conducted. The gas was sampled at several locations downstream of the nozzle exit, with nitrogen as the primary flow. A gas chromatograph, with an accuracy of 0.2 vol %, was utilized for analyzing the flow composition. The mixing layer thickness was evaluated based on the distribution of the nitrogen. The uncertainty of the location of sampling was ± 0.5 mm for each probe.

Computational fluid dynamics (CFD) analysis of the ejector jet was also carried out to validate the present method. The Reynolds averaged Navier-Stokes equation was solved on a structured grid. The in-house code is based on the control volume method, with the Spalart-Allmaras turbulence model. The computational domain was 2-dimensional and had a simple rectangular shape like the one shown in Fig. 2. The length of the ejector duct was 19 times the duct height of the flow passage \((H_1 + H_2)\), four times that upstream and 15 times that downstream from the point where the mixing initiated. The total amount of nodes inside the duct was about 57,000.

4. Results

In some experiments, the suction performance varied by changing the chemical or thermal condition of the primary flow. In general, the higher the temperature of the primary gas, or the lighter the molecular weight of the species for primary flow, the more the suction performance was degraded. This phenomenon cannot be explained solely by the momentum transfer model. In this section, the present analysis method is applied to actual experimental conditions and it is confirmed whether degradation can be explained by considering mixing.

Hereafter, suction rate, \( \mu \), is defined by the ratio of the mass flow:

\[ \mu = \frac{\dot{m}}{\dot{m}_c} \]

where \( \dot{m} \) is the mass flow of the secondary flow, and \( \dot{m}_c \) is also the mass of secondary flow under choked condition at the entrance \((i.e M_2 = 1)\). By definition, \( \mu \) varies from 0 to 1.

4.1. Effect of mixing

Before applying actual experimental conditions, the present procedure was applied to a virtual simple geometry ejector jet model in order to check the general tendency of the mixing effect.

The duct had a constant rectangular section, and the ratio of the primary and secondary flows was set to 2 to 1. The Mach number of the primary flow was 3.0 and the property of pure nitrogen gas was assumed. The total pressure of the primary flow was 2.0 MPa. As for the secondary flow, the total pressure and temperature were 101.3 kPa and 300 K (atmospheric condition), respectively. Figure 5 shows the effect of the temperature of the primary flow on suction performance. The vertical axis denotes the mixing parameter, \( \beta \). As the temperature increased, the suction performance of
the flow was drastically reduced. At 1,000 K or less, the drop in suction performance was virtually non-existent. The broken line shows the results of the CFD. The CFD also showed no loss in suction performance at temperatures less than 1,000 K, whereas at 2,000 or 3,000 K, deterioration in performance occurred. By comparing the CFD and the present analysis, β of 20 to 45% gives a match.

The effect of molecular weight was also surveyed with the present procedure using the same geometry. The total temperature and pressure were 2,000 K and 1.2 MPa, respectively. The Mach number was 3.0. Atmospheric conditions were assumed for the secondary flow. As can be seen in Fig. 6, the suction performance decreased with the increase of β. The defect was greater with the gas of lighter molecular weight. Helium showed significant sensitivities to β, but nitrogen and argon did not show such different tendencies, though argon is much heavier than nitrogen. The additional comparison with total temperature at 300 K showed that nitrogen and argon do not have any sensitivities to β, while helium showed the deterioration of performance with higher β. The higher sensitivity of helium will be reexamined in section 4.3.

By these evaluations, it was concluded that suction performance is affected by the difference in natures of the two flows. When the natures of the two flows widely differ, the suction performance becomes quite sensitive to β, resulting in large degradation of performance with a small amount of mixing.

4.2. The comparison with experiments of scale models

The present procedure was also examined by comparison with the experimental results. In all of the experiments with small engine models conducted at JAXA, nitrogen gas at room temperature was used for the primary flow. Also, helium was used to check the effect of gas species.

Figure 7 shows a simple schematic of the models tested. A rocket nozzle is located at the mid-point of the model. The primary flow is accelerated through the conical nozzle. In the actual configuration, however, there was a rearward face (base area) around the nozzle exit, which the present model cannot explicitly incorporate. In reality, Ad, the downstream duct area, should be

\[
A_d = A_1 + A_2 + A_b,
\]

(16)

where \(A_b\) is the base area, and \(A_1\) and \(A_2\) are the upstream primary and secondary flow areas, respectively. In the present analysis, the base area was neglected and the geometry was modified as follows:

\[
A_d = A_1 + A_2
\]

(17)

From the results of the previous section, mixing is the key phenomena that affects suction performance. The base area (i.e., both its size and shape) contributes to form a secondary flow that alters the mixing process. A larger base area delays the formation of the mixing layer (small β) and a complex base geometry enhances mixing (large β). In the present analysis, the value of β is the effect produced by the base area.

Figure 8 shows the model geometry and suction performance using a light gas for the rocket. The total pressure of the primary gas was 1.6 MPa and the Mach number at the exit of the conical nozzle was estimated to be 2.3 from the area ratio. Broken lines denote the experimental data. In the experiments, the secondary flow achieved a choked condition using nitrogen gas as the primary flow. However, a 5% loss in suction performance was observed with helium as the primary flow. If 15% of β is assumed, the present procedure can reproduce the loss.

At JAXA, a 3-m-long rocket-based combined cycle engine (RBCC, designated as E36) has been tested in the ramjet engine test facility (RJTF). A configuration of the model is provided in Fig. 9.
The model consists of a double-staged ramp inlet, an isolator that enfolds the rockets and an expanding combustor. It has several ram fuel injection holes in the combustor section, as well as a pair of rockets with a gas (oxygen and hydrogen) propellant. By changing the rockets and ram fuel injection method, the model can be operated in the ejector-jet, ram and scramjet modes. As for the ejector-jet mode, the chamber pressure of the rocket was 3 MPa. To compare the results, a one-fifth-scale model with nitrogen gas rocket flow was also tested under static condition at sea level.\(^1\) The scale model easily achieved choked condition with nitrogen gas injected at 2 or 3 MPa. However, E3 could not achieve the choked condition at the same pressure level. With the present procedure, there was relatively little degradation on suction performance with nitrogen gas, whereas the loss of suction was obtained with a hydrogen and oxygen mixture, as is indicated in Fig. 9. It should be noted that the physical quantities of the combustion gas were estimated by chemical equilibrium with applications (CEA).\(^1\) The experimental suction performance is indicated by broken lines in the figure for 2 and 3 MPa cases. The loss in suction performance coincides with the condition in which 20% of \(\beta\) is assumed.

To check the suction performance with low-speed incoming flow, an ejector-jet experiment was conducted with a modified CAMUI sounding rocket in 2009.\(^2\) CAMUI\(^3\) is a hybrid propellant rocket using polyethylene as a fuel and liquid oxygen as an oxidizer. In that experiment, a shroud duct was mounted around the rocket nozzle, thus forming an ejector flow passage (see Fig. 10). In this case, the shape of the ejector jet was cylindrical. Since the present analysis assumes one-dimensional preservation of the total amount of mass, momentum and energy, inherently, the shape of the flow geometry should not affect the results of the analysis. For pre-checking the actual flight test, a 1/3-scale model was prepared and tested in a low subsonic wind tunnel. Again, the rocket gas was simulated using nitrogen gas. Additionally, a ground test having an actual rocket motor with a shrouded duct was carried out. For comparison with these

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**Fig. 8.** Comparison to small-scale model with He injection.

**Fig. 9.** Comparison to E3 models.

**Fig. 10.** Comparison to CAMUI models.
tests, physical quantities of gas were estimated by CEA. Unlike the E3 cases, the difference of rocket gas had no effect on suction performance. Even a hot gas produced maximum suction (i.e., choked flow in the CAMUI configuration). On the other hand, the present procedure resulted in a loss of suction as \( \beta \) becomes greater with hot gas. This result suggests that mixing in the actual model was poor.

### 4.3 Effects of Total Temperature and Molecular Weight on the Mixing Process

As was observed in the previous sections, the differences in nature of the primary and secondary gases affected suction performance. In this section, two properties of the gas, total temperature and molecular weight, were investigated analytically for their effect on mixing parameter, \( \beta \), and the pressure rise due to mixing.

As for \( \beta \), it was simply assumed that \( \beta \) is related to the mixing layer growth rate because a larger growth rate results in larger \( \beta \). The growth rate, \( G_r \), is evaluated as follows:

\[
G_r = G_r(M_1),
\]

where \( G_r \) is an incompressible growth rate of the mixing layer described as:

\[
G_r = \frac{2H_m}{x} = 0.17 \frac{(1 - r)(1 + s^2)}{1 + rs^2} \tag{19}
\]

\( r \) denotes the ratio of flow velocity (\( u_2/u_1 \)) and \( s \) is the ratio of density (\( \rho_2/\rho_1 \)). \( H_m \) is half of the mixing layer thickness, as defined in the former section.

\( f(M_1) \) is the function of a convective Mach number, \( M_{c1} \), to adjust the compressible effect on \( G_r \):

\[
f(M_1) = 0.2 + 0.8e^{-3M_1^2}, \tag{20}
\]

where \( M_{c1} \) is defined as follows:

\[
M_{c1} = \frac{u_1 - u_c}{a_1} \tag{21}
\]

\[
u_c = \frac{u_1 + \sqrt{s}u_2}{1 + s^2} \tag{22}
\]

Since the pressure of two flows is assumed to be identical, \( s \) can be described as the product of the molecular weight ratio and the total temperature ratio.

\[
s = \frac{\rho_2}{\rho_1} = \frac{w_2I_1}{w_1I_2} \tag{23}
\]

\[
= \frac{w_2T_1}{w_1T_2} + \frac{(\gamma_2 - 1)M_1^2}{w_1T_2} + \frac{(\gamma_1 - 1)M_1^2}{w_1T_2} \tag{24}
\]

By introducing \( \alpha \), which is defined as:

\[
\alpha = \frac{\frac{T_1w_2}{T_2w_1}}{\gamma_1} \tag{25}
\]

Eqs. (19) and (20) can be described as follows:

\[
G_r = 0.17 \left\{ \frac{1 - \frac{M_2}{M_1} \left( \frac{\gamma_2T_1w_1}{\gamma_1I_2w_2} \right)^{\frac{1}{\gamma_1}}}{1 + \left( \frac{w_2}{w_1} \right)^{\frac{1}{\gamma_1}}} \right\} \left\{ 1 + \left( \frac{w_2}{w_1} \right)^{\frac{1}{\gamma_1}} \right\} \tag{26}
\]

\[
M_{c1} = M_1 \frac{\alpha}{1 + \frac{\alpha}{\gamma_1}} \left\{ \frac{\gamma_1}{\gamma_2} \right\} \frac{1}{\gamma_1} \frac{1}{M_1^2} \right\}, \tag{27}
\]

As can be seen, both \( G_r \) and \( M_{c1} \) (and thus \( f(M_{c1}) \)) are functions of \( \alpha \). From the definition, when the total temperature of the primary gas becomes greater, or the molecular weight of the primary gas becomes lighter, \( \alpha \) becomes greater and the effect results in greater \( G_f \) as depicted in Fig. 11. With the fixed secondary flow condition (atmospheric air at \( M_2 = 0.5 \), \( T_2 = 300 \text{ K} \) and \( w_2 = 28.8 \)) and the fixed total pressure of the primary flow (\( P_1 = 2 \text{ MPa} \)), three cases were evaluated and the results are shown in this figure. The solid line with circle symbols shows the case with nitrogen as the primary gas, varying its total temperature. The other two lines correspond to the cases where the total temperature of the primary gas was fixed to be 300 K, varying the molecular weight with a different specific heat ratio. For clarity, additional axes show the equivalent total temperature or the molecular weight of the primary gas. In any case, the minimum \( G_f \) appeared around \( \alpha = 1 \) and \( G_f \) became greater roughly proportionally to \( \alpha \). Thus, it is expected qualitatively that \( \beta \) becomes larger as the value of \( \alpha \) increases. It should be noted that the local maximum was observed at \( \alpha = 0.6 \), but that \( \alpha \) corresponds to an unpractical condition (primary gas of extremely low temperature or heavy molecular weight) for an ejector-jet application.

As for the pressure rise due to mixing, one-dimensional analysis for two flows forming a mixed flow was carried out. Here, for the sake of simplicity, it is assumed two flows have the same sectional area, \( A \), and the mixed flow has an area twice the size, \( 2A \). The conservation equations and ideal gas relation were assumed for the mixing process, and the supersonic solution for the mixing flow was evaluated.

\[
\hat{m}_1 + \hat{m}_2 = \hat{m} = \rho_mu_m2A \tag{28}
\]
The properties of the mixed flow, $R_m$ and $Cp_m$, were obtained as a mole weighted average of the two flows. To further simplify the relation, the following relation is also adopted from Mayer’s relation:

$$\frac{R_m}{Cp_m} = \frac{\gamma_m - 1}{\gamma_m} \tag{32}$$

As a result, $p_m$ can be described as follows:

$$p_m = \frac{K}{2A(y_m + 1)} \tag{33}$$

The sum of the momentum and the product of mass and energy of two flows are described as follows:

$$K = K_1 + K_2$$
$$= pA + \rho_1 u_1^2 A + pA + \rho_2 u_2^2 A \tag{34}$$
$$= pA(2 + \gamma_1 M_1^2 + \gamma_2 M_2^2)$$

$$\dot{m}E = (\dot{m}_1 + \dot{m}_2)(\dot{m}_1 Cp_1 T_1 + \dot{m}_2 Cp_2 T_2)$$
$$= \dot{m}_1^2 Cp_1 T_1 + \dot{m}_2^2 Cp_2 T_2 + \dot{m}_1 m_2 (Cp_1 T_1 + Cp_2 T_2) \tag{35}$$

By substituting these values in Eq. (32), the normalized pressure, $p_m/p$, is derived as follows:

$$2(y_m + 1)\frac{p_m}{p} = (2 + \gamma_1 M_1^2 + \gamma_2 M_2^2)$$

$$- \left(\frac{\gamma_1^2}{2} + \frac{\gamma_1 M_1^2 + \gamma_2 M_2^2}{2} - 2(y_m - 1) \left[\frac{\gamma_1 M_1^2}{\gamma_1 - 1} \left(1 + \frac{\gamma_1 - 1}{2} M_1^2 \right) + \frac{\gamma_2 M_2^2}{\gamma_2 - 1} \left(1 + \frac{\gamma_2 - 1}{2} M_2^2 \right) \right] \right)$$

$$+ M_1 M_2 \left[\gamma_1 \gamma_2 \left(1 + \frac{\gamma_1 - 1}{2} M_1^2 \right) \left(1 + \frac{\gamma_2 - 1}{2} M_2^2 \right) \right]^{\frac{1}{2}} \left(\frac{\gamma_1}{\gamma_1 - 1} - \frac{\gamma_2}{\gamma_2 - 1} \right) \right]^{\frac{1}{2}} \beta. \text{ This phenomena can be explained as resulting from a larger rise in pressure.}$$

4.4. Mixing layer observation in CAMUI geometry

To check the actual formation of a mixing layer, an experiment was conducted using the scale model of CAMUI described in section 4.2. The gas sampling was carried out at the end of the constant area section, downstream of the nozzle exit with a distance of roughly seven times the size of the height of the secondary flow passage.

Pure nitrogen was used for the primary gas and its total pressure was 2.0 MPa, which is high enough to establish the choked condition of the secondary flow. It should also be noted that the theoretical pressure at the exit of the nozzle roughly coincided with the pressure of the choked secondary flow. Figure 13 shows the distribution of nitrogen along the radial direction. Pure nitrogen was observed in the region exactly downstream of the nozzle exit. The dissipation of nitrogen gas was rather restricted, roughly comparable with the error of the probe location. From the distribution of nitrogen, the mixing layer thickness was estimated to be roughly 1 mm, which can be translated into 0.13 of $\beta$. From Fig. 10, $\mu$ can be as high as 0.9, which corresponds to the

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and secondary loss of suction, thus indicating that mixing of the primary in the momentum transfer model. This model resulted in a performance in an ejector-jet, a mixing model was incorporated

5. Conclusion

good suction performance observed in the experiments.

5. Conclusion

To explore the reason for the degradation of suction performance in an ejector-jet, a mixing model was incorporated in the momentum transfer model. This model resulted in a loss of suction, thus indicating that mixing of the primary and secondary flows could affect suction performance. The following results were obtained by the present analysis.

1. As the temperature of the primary flow becomes higher, loss of suction performance becomes greater. This is also true with light molecular gases. The present analysis showed that, as the difference in the physical nature of two flows becomes greater, the loss of suction performance is keenly affected by the quantity of the mixing flow.

2. In the case of the CAMUI experiment, it was suggested that mixing the two flows was minimal since the analysis showed that a loss in performance would occur when mixing the two flows.

3. Analytical investigations showed that the ratios of the total temperature and molecular weight of two flows are the major parameter for determining the mixing layer thickness and pressure rise. The degradation of suction performance is also affected by this parameter.

4. Analytical investigation also indicated that the molecular weight non-linearly affects pressure rise due to mixing. The result explains the extreme degradation of suction performance when helium is used for the primary flow.

5. Nitrogen distribution at the exit of the CAMUI model showed limited growth of the mixing layer. Good suction performance was achieved due to poor mixing, as predicted by the present method.

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