Information Fusion of Discrete Variables and Continuous Variables Based on D-S Evidential Theory

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Abstract. In reliability analysis of a target, information from various aspects should be analyzed comprehensively. In this process, information fusion plays a key role in integrating and analyzing information from different channels. Aiming at the reliability analysis of discrete variables and continuous variables existing simultaneously, an information fusion method based on Dempster's rule of combination of D-S evidence theory was proposed to facilitate the reliability analysis by integrating information. Firstly, the continuous variables are discrete by using the continuous variables discretization algorithm based on the probability distribution of the job state with equal expectation scale, and the basic probability distribution of each discrete variable is carried out. Then Dempster's synthesis rule was used to synthesize the discrete variables. Deng entropy was used as the judgment standard in the synthesis process, so that the entropy of each step of fusion was reduced to ensure the credibility of fusion was increased. Finally, the feasibility and effectiveness of the method are verified by a case.

1. Introduction
With the acceleration of the process of intelligent society, multi-source information fusion plays an increasingly prominent role. Information fusion can also be regarded as data fusion, mainly because most fusion algorithms are used for data processing. Multi-source information fusion technology has been applied in environmental monitoring, fault analysis, artificial intelligence, image processing and other social hot industries. It can be said that information fusion technology has been widely applied in our daily life.

Literature [1] makes a comprehensive summary of information fusion, including various fusion methods and their advantages and disadvantages, as well as the application fields and applications of information fusion. Literature [2-3] mainly discusses the problem of information fusion of small
samples. We can learn information fusion methods under different conditions in many aspects. The importance of machine learning and its unique advantages in information fusion are also expounded. Literature [4] introduces the practical application of information fusion. Information fusion technology is used to analyze the reliability of natural gas pipelines by integrating various factors, and good results are achieved. [5-8] paradox discussed mainly for evidence theory, analyze the conflict evidence in the process of integration, respectively from the decision theory of distance, Angle cosine and long distance theory, a new method for enhanced belief measure differences (RB) evidence theory in the view of three deficiencies are optimized, effectively solve the insufficiency of evidence theory, give full play to the advantages of different sources of information for effective integration. Literature [9-11] mainly introduces the analysis and processing of interval variables, in which interval variables are discrete and weights are added to make the discretization process more reasonable. Literature [12-13] mainly introduces fusion methods, including information processing for fusion, and entropy changes in the fusion process to judge whether fusion is effective and reliable. Literature [14-16] mainly introduces the theoretical knowledge of entropy, indicating that it is feasible to use entropy to judge the effectiveness of information fusion when using evidence theory.

Evidence theory is a statistical reasoning method which focuses on feature level fusion and decision level fusion. The theory of Probability of Basic events to broaden the space for Basic event power set (also known as the recognition framework, the Frame of Discernment, FOD) and to identify framework to build the Basic Probability Assignment function (Basic aim-listed Probability the Assignment, BPA) to describe the uncertain information. Evidence theory is compatible with probability theory in simple environment, while it can effectively represent and process uncertain information in complex model. These characteristics make it widely applied in the field of multi-source information fusion. Under the framework of evidence theory, the research on information fusion mainly focuses on reasonable generation of BPA, conflict processing of evidence, and computational complexity. These studies lay a certain foundation for improving the reliability of information fusion results. However, in the field of information fusion, there is still a lack of research on the quality of information fusion. If the quality of information is low and there is no quality assessment and quality control mechanism for information, the fusion may lead to wrong results. For example, it is necessary to establish an effective information fusion quality discrimination scheme to improve the target recognition accuracy when multi-sensor cooperative detection. Therefore, this paper will take the evidence theory system as the framework to discuss the information processing screening method and the information fusion effectiveness measurement method.

2. Evidence theory

Dempster first proposed the evidence theory in 1967, and it was further developed by his student Shafer as an imprecise reasoning theory, so this theory is also known as Dempster/Shafer evidence theory (i.e. D-S evidence theory). Evidence theory is an uncertain reasoning method with the ability to process uncertain information and was first applied to expert system. The core of evidence theory is Dempster's synthesis rule, which has the main advantages of satisfying the weaker conditions than bayesian probability theory and the data needed in the theory can be obtained easily and intuitively. In addition, Dempster's synthesis rule makes evidence theory widely applied in information fusion, expert system, multi-attribute analysis and other fields. Its basic principles and concepts are as follows:

Identification framework D: D is a discrete set consisting of a series of statistically independent
and mutually exclusive finite discriminant hypotheses. D is a sample space of the variable x that contains all possible values of x. Basic probability distribution BPA: in D, BPA is a function m of $2^D \rightarrow [0,1]$, called mass function, satisfy the following conditions:

$$m(\Phi) = 0$$
$$\sum m(A) = 1$$

Where A that makes $m(A) > 0$ is called the focal element, and the value represents the degree of trust in the focal element. Reliability function: is a power set based on D, i.e. $2^D$. On D, the BPM based reliability function is defined as follow:

$$\text{Bel}(A) = \sum_{B \subseteq A} m(B)$$

Likelihood function: is a power set based on D, i.e. $2^D$. On D, the BPM based reliability function is defined as follow:

$$\text{Pl}(A) = \sum_{B \subseteq X \neq \Phi} m(B)$$

Dempster Synthesis rules (evidence synthesis formula):

$$m_1 \oplus m_2 = \frac{1}{K} \sum_{B \subseteq X \neq \Phi} m_1(B) m_2(C)$$

Where $K$ is normalized constant

$$K = \sum_{B \subseteq X \neq \Phi} m_1(B) m_2(C) = 1 - \sum_{B \subseteq X \neq \Phi} m_1(B) m_2(C)$$

3. Continuous interval variables are discrete

At present, the methods of interval variable discretization are frequency band discretization and interval discretization, which are widely used and representative. But it also has the inevitable deficiency, it is only a rough qualitative analysis of the data, the accuracy is poor. Besides, it will also cause the lack of data, so it is mostly used for the general analysis of the target when the accuracy is low. In this paper, the continuous interval discretization method based on expected value is adopted, which can make up for the lack of accuracy of interval discretization and frequency band discretization methods. The basic methods of this method are as follows:

First, the original continuous variable $x$ is analyzed to obtain its variance, mean and other information as well as the matrix composed of $m$ group data $X = (x_1, \cdots, x_m)^T$; Then combined with the analysis in the previous step, $n$ standard job states were selected to form the standard job state vector of $X$: $e = (e_{1}, \cdots, e_{m})^T$ Calculate the state distribution matrix $P_i$ based on the vector, $X = (x_1, \cdots, x_m)^T$ is decomposed into $P_i * e_i$, where $P_i$ is the state probability matrix of $m \times n_i$ dimension. In this case, the $i$th group data $x_i$ of $X$ matrix can be decomposed into $x_i = P_{i\omega} * e_i$, where $P_{i\omega}$ is the $i$th row of $P_i$. Using equation (1) to solve $P_{i\omega}$, $P_i$ can be derived by this method.

$$P_i (i, \cdot) = 1$$
$$P_i (i, \cdot) e_i = x_i$$

Equations of the two equations, so when the $e_i$ winning vertebral $n_i$ number more than 2, the
system of equations has infinitely many solutions, at this point the dimension reduction method, first of all to judgment, to find the variable data in the range and define when \( e_{ij} < x_i < e_{ij+1} \), will only \( x_i \) allocated \( e_{ij} \) and \( e_{ij+1} \) two standard job status, the equation (6) into (7):

\[
\sum P_x (i, :) = 1 \\
P_x (i, k) = 0, k \neq j, k \neq j + 1 \\
P_x (i, j) e_{ij} + P(i, j + 1) = X_i
\]

Based on expectations of the standard state probability distribution of continuous variables discretization algorithm will continuous variable values as expectations, in the selection of the status of all the standard that is all the discretization of the scalar value, the probability value in the form of probability distribution to each on the standard state, is a continuous variables is decomposed into continuous probability and the discrete state standards.

4. Dempster’s combination rule of evidence

Dempster's rule of combination, namely, the formula of evidence combination, is as follows:

For any \( A \) belonging to D, the combination rule of Dempster for the two mass functions \( m_1 \) and \( m_2 \) on D is follow:

\[
m_1 \oplus m_2 = \frac{1}{K} \sum_{B \cap C = A} m_1 (B) m_2 (C)
\]

(8)

Where \( K \) is normalized constant

\[
K = \sum_{B \cap C = \Phi} m_1 (B) m_2 (C) = 1 - \sum_{B \cap C = \Phi} m_1 (B) m_2 (C)
\]

(9)

For any \( A \) belonging to D, Dempster's synthesis rule of finite mass functions \( m_1, m_2, \ldots, m_n \) on D is follow:

\[
(m_1 \oplus m_2 \oplus \ldots \oplus m_n) (A) = \frac{1}{K} \sum_{A \cap A_1 \cap \ldots \cap A_n = A} m_1 (A_1) m_2 (A_2) \ldots m_n (A_n)
\]

(10)

Where \( K \) is normalized constant

\[
K = \sum_{A \cap A_1 \cap \ldots \cap A_n = \Phi} m_1 (A_1) m_2 (A_2) \ldots m_n (A_n) = 1 - \sum_{A \cap A_1 \cap \ldots \cap A_n = \Phi} m_1 (A_1) m_2 (A_2) \ldots m_n (A_n)
\]

(11)

Dempster's rule of composition is one of the core cornerstones of evidence theory. Its form is relatively simple, but the standardization process of composition may lead to paradoxes in reasoning results. Since Zadeh found this problem, conflict evidence has been one of the important issues concerned by D-S evidence theory. Domestic and foreign scholars have done a lot of research on this, and put forward a variety of evidence synthesis methods. Also apply evidence theory of information fusion, this paper is about the paradox of the entropy theory is optimized, such as the use of information entropy to express the characteristics of the information uncertainty degree, in the process of each evidence fusion for the determination of information entropy and making use of new information entropy is reduced after fusion, in order to determine each time the process of information fusion is effective, so that the fusion process is effective, the fusion method is feasible.
5. Isentropy

The definition of isentropic is expressed as follows:

\[ E(m) = -\sum_{A \in \mathcal{X}} m(A) \log_2 \left(\frac{m(A)}{2^{|A|-1}} \right) = \sum_{A \in \mathcal{X}} m(A) \log_2 \left(\frac{2^{|A|}-1}{\sum_{A \in \mathcal{X}} m(A)}\right) \]

(12)

Where, \( m \) is the BPA defined on the recognition frame \( \mathcal{X} \), \( A \) is the focal element in BPA, and \( |A| \) is the potential of \( A \), that is, the number of focal elements in \( A \). In information processing, entropy can measure the uncertainty of a random variable or the amount of information. For a piece of information (evidence under the evidence theory system, BPA), the higher the information entropy, the more uncertain the information is, and the more information is needed to resolve the uncertainty. In other words, in the process of information fusion, the increase or decrease of information entropy of evidence body is inevitable. If the information fusion algorithm reduces the entropy of the original evidence, it indicates that the fusion process reduces the uncertainty of the information, obtains more information from the fusion result than the original information, and lower information entropy means lower decision-making difficulty. On the other hand, there is a certain relationship between the change of information entropy and the conflict size of the evidence to be fused. The main reason for the conflict is that different evidence supports different primary focus elements. According to the characteristics of Dempster's combination rule, the reliability distribution after the conflicting evidence combination will become even, resulting in the increase of the reliability entropy. However, if the collision is small and suitable for fusion, the main focal element is focused after fusion, and the reliability entropy is generally reduced, which is more conducive to decision-making. To sum up, an effective information fusion process should be accompanied by the decrease of the information entropy of the evidence body.

6. Fusion process

Suppose that there are \( N \) pieces of evidence with fusion, choose any \( n_1 \) as the first piece of evidence for fusion, and then choose any \( n_2 \) and \( n_1 \) to be fused according to the principle of Demspert, get a new piece of evidence \( n_{(2)} \), and then judge the entropy of the new evidence. If the entropy of \( n_{(2)} \) is less than the entropy of the first two pieces of evidence, then the two pieces of evidence can be fused, and the new evidence after fusion is helpful for decision-making. If the principle of quotient subtraction is not met, \( n_2 \) is output. Then choose any evidence from the rest of the evidence \( n_3 \) and \( n_1 \) for fusion, the fusion principle and the judgment and treatment of fusion results are the same. After the first round of fusion, \( M \) pieces of evidence are output, then \( N-M \) pieces of evidence are fused and the new evidence is \( n_{(N-M)} \). Then, evidence \( n_{(N-M)} \) is used to fuse with \( M \) pieces of evidence output in the first round of fusion, and the specific method is modeled after the first step until the end of the second round of fusion. And so on until all the evidence that can be fused is fused. Suppose that \( J \) pieces of evidence are finally exported and cannot be fused to be exported, then \( J \) pieces of evidence are internally fused as a small whole, and then they are fused with evidence \( n_{(N-M)} \) after the fusion is completed. If \( K \) pieces of evidence are finally exported and cannot be fused, it means that \( K \) pieces of evidence are contrary to most of the evidence, so they are discarded. \( K \) should be much less than \( N \). If \( K \) is too large, it may be because the choice of \( n_1 \) and \( n_2 \) is not reasonable, and one of them is not credible, so the two evidences of the first fusion should be reconsidered.

7. Example

In this section, an example is given to illustrate the effectiveness of the proposed algorithm. Assumes
that the multisensor $a$, $b$, $c$, $d$, different aspects common to detect the same target, produced four detection information including $a$ detector for a range of continuous variable, $b$, $c$, $d$ detector as discrete variables, first of all to discretization of continuous variable interval, after and then use the discrete of discrete variables directly with detector to detect the discrete variables of fusion.

7.1. Continuous interval variables are discrete
Continuous variable x is uniformly distributed in the interval $[3, 5]$, $\bar{X} = 1.5$ its mean , variance $\sigma_x^2 = 1.25$. For the three groups of data $x = [3.4, 4.4, 4.7]^T$, three standard job states are selected. $e_{11} = 3$, $e_{12} = 4$, $e_{13} = 5$, that is, after the standard job state vector $e = [3, 4, 5]^T$, formula (1), $X = [3.4, 4.4, 4.7]^T$ can be decomposed according to formula (7):

$$X = [3.4, 4.4, 4.7]^T = \begin{bmatrix} 0.6 & 0.4 & 0 \\ 0 & 1 & 0 \\ 0 & 0.3 & 0.7 \end{bmatrix} [3, 4, 5]^T$$  \hspace{1cm} (13)

After dispersion, it was transformed into the evidence theoretical framework system. The BPA of the evidence in each state was as follows:

$$m(a_1) = 0.85 \quad m(a_2) = 0.87 \quad m(a_3) = 0.75$$

7.2. Discrete variable
Let sensors $b$, $c$ and $d$ respectively detect the target and generate three pieces of detection information, which are transformed into the identification framework of evidence theory. The BPA of the three pieces of evidence is as follows:

$$m(b) = 0.85 \quad m(c) = 0.8 \quad m(d) = 0.7$$

7.3. Fusion process
The various pieces of evidence are combined in accordance with the method described above. The specific process is as follows:

![Figure 1. Fusion process of each piece of evidence](image)

The entropy change of each piece of evidence in the fusion process is shown in the following table1:
Table 1. Entropy change during evidence fusion

| evidence | entropy | 1   | 2   | 3   | 4   | 5   |
|----------|---------|-----|-----|-----|-----|-----|
| a1       | 1.0464  |     |     |     |     |     |
| a2       | 0.9496  | 0.1834 |     |     |     |     |
| a3       | 1.5725  | 1.5725 | 0.0525 |     |     |     |
| b        | 1.1076  | 1.1076 | 1.1076 | 0.0228 |     |     |
| c        | 1.2623  | 1.2623 | 1.2623 | 1.2623 | 0.0145 |     |
| d        | 1.7545  | 1.7545 | 1.7545 | 1.7545 | 1.7545 | 0.0102 |

Figure 1 shows the process of gradual fusion with fusion information, and table 1 shows the entropy of each information before information fusion and the changes of each entropy along with the process of fusion. It is not difficult to see from the two tables that the process entropy of fusion is decreasing step by step, thus improving the reliability of information and the ease of decision making. This shows that the information processing method and fusion algorithm in this paper are effective, which proves the correctness of the theory proposed in this paper.

8. Conclusions
When there are two kinds of information: interval continuous variable and random discrete variable, this paper proposes an information fusion method based on equal expected value discretization of continuous interval and isentropic fusion criterion. This method has unique advantages in dealing with two different types of information fusion: interval continuous variable and random discrete variable. Compared with similar methods, the fusion idea is simple and clear, the method is easy to understand, and the entropy theory in the fusion process ensures the effectiveness of the fusion. Therefore, this method not only solves the problem of two kinds of information fusion, but also has simple, strong operability and effective results. In order to analyze the credibility of all kinds of information, comprehensively consider and analyze all kinds of evidence to increase the reliability of information and facilitate the final decision. The continuum variables discretization in the first place, then the recognition of discrete variables into evidence theory frame to give corresponding probability value, after the D-S evidence theory Demspert synthetic rules to each discrete variable line integration, process with discriminant is appropriate for business to the vertebral fusion, at last, through the actual case to verify the proposed fusion method in this paper. Through the above theory and case analysis, it is proved that the method described in this paper is effective, reduces the uncertainty of information, increases the credibility, and conducive to the final decision and subsequent information processing.

Acknowledgments
Foundation item: the National Natural Science Foundation of China (No.51965051), the Natural Science Foundation of Inner Mongolia Autonomous Region, China (No. 2020MS05065).

References
[1] Chen C, Zhang J L, Jia J Y and Wang Y 2019 Research progress of information fusion method. Science and Technology Vision. 17 32-3
[2] Feng J 2004 Research on method and application of reliability information fusion for complex system with small sample test. University of National Defense Science and Technology
[3] Meng T, Jing X Y and Yan Z 2020 A survey on machine learning for data fusion. Information Fusion 115-29
[4] Hu M L, Huang K and Wu X N 2011 The gas pipeline reliability analysis based on D-S evidence theory. Youqi Chuyun 7 490-3
[5] Yang Y X, Gao Z F, Zhu H and Zhao X 2018 A multi-source conflict evidence information fusion algorithm based on the combination of decision distance measurement and d-s evidence theory. Journal of Lanzhou University of Arts and Sciences (natural sciences) 6 62-7
[6] Dong Y, Cao Y L and Jiang K 2018 Research on improved method of conflict evidence in D-S evidence theory. Electronic Measurement Technology 23 29-33
[7] Li W L and Guo K H 2010 Combination rules of D-S evidence theory and conflicts problem. Systems Engineering Theory and Practice 8 1422-32
[8] Xiao F Y 2020 A new divergence measure for belief functions in D–S evidence theory for multisensor data fusion. Inf. Sci. (N, Y.) 462-83
[9] Fang H T 2017 The research on discretization of Continuous variables and weight assignment in multi-source information fusion. Shandong University 17-45
[10] Xu P F, Pan D B, Yang Y, Xi Y and Liu F 2014 Reliability research of asymmetric voting system based on d-s evidence theory. Dongnan Daxue Xuebao 12 193-200
[11] Ferson S, Ginzburg L, Kreinovich V, Nguyen H T and Starks S A 2002 Uncertainty in risk analysis: towards a general second-order approach combining interval, probabilistic, and fuzzy techniques. (Glasgow UK: Proceedings of the 2002 IEEE International Conference on)
[12] Meng X J, Jing S X, Liu J H, Zhang L X and Zhang H 2015 Reliability analysis method based on evidence theory under multi-source uncertainty Computer Integrated Manufacturing Systems 3 648-55
[13] Jiang W, Zhang Y and Xie C H 2019 Evidence theory classification information fusion method for multi-sensor collaborative detection. Navigation Positioning and Timing 5 32-7
[14] Sun J X, Shi H M and Wang H Q 2003 The theory relative to entropy in information fusion. Jisuanjixuebao 7 796-801
[15] Yong S W, Yu W X and Guo G R 1995 The information entropy theory of information fusion. Systems Engineering and Electronics 10 1-6
[16] Yu S R, Yin Y H, Xu B and Zhang D M 2006 Analysis of the random-fuzzy reliability based on the information entropy theory. JiXie Qiangdu 5 695-8
[17] University of Exeter 2020 Developing Methods for the Overarching Synthesis of Quantitative and Qualitative Evidence: the Interweave Synthesis Approach. Science Letter
[18] Simjanoska M, Kochev S, Tanevski J, Bogdanova A M, Papa G and Eftimov T 2020 Multi-level information fusion for learning a blood pressure predictive model using sensor data. Information Fusion 24-39