Finite Element Study for Magnetohydrodynamic (MHD) Tangent Hyperbolic Nanofluid Flow over a Faster/Slower Stretching Wedge with Activation Energy

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Abstract: The below work comprises the unsteady flow and enhanced thermal transportation for Carreau nanofluids across a stretching wedge. In addition, heat source, magnetic field, thermal radiation, activation energy, and convective boundary conditions are considered. Suitable similarity functions use to transmuted partial differential formulation into the ordinary differential form, which is solved numerically by the finite element method and coded in Matlab script. Parametric computations are made for faster stretch and slowly stretch to the surface of the wedge. The progressing value of parameter A (unsteadiness), material law index \( \epsilon \), and wedge angle reduce the flow velocity. The temperature in the boundary layer region rises directly with exceeding values of thermophoresis parameter \( N_t \), Hartman number, Brownian motion parameter \( N_b \), Biot number \( B_i \) and radiation parameter \( R_d \). The volume fraction of nanoparticles rises with activation energy parameter \( E_E \), but it receded against chemical reaction parameter \( \Omega \), and Lewis number \( L_e \). The reliability and validity of the current numerical solution are ascertained by establishing convergence criteria and agreement with existing specific solutions.

Keywords: finite element method; tangent hyperbolic nanofluid; falkner-skan flow; wedge geometry; activation energy

1. Introduction

In the past few decades, the discovery of nanoparticles has to define another goal for researchers. A new roadmap has been launch to create a useful energy source. The foundation of modern technologies has been laid. The interesting noteworthy of these nanoparticles is identified with the advancements of solar energy systems, semiconductors, biomedical engineering, energy, pharmaceutical products, and materials manufacturing, etc. Generally, nanofluids are other types of fluid mixed with nanoparticles through conventional doping fluids (oil, water, gels, and polymers). These nanoparticles are mostly composed of metals, oxides, starches, nitrides, and non-metals, measuring somewhere between 1 and 100 nm. The fluid utilized in modern high technological areas with huge heat diffusivity capacity is called nanofluid. For the first time, such liquid is experimental studied by Choi, and Eastman [1]. Later, Buongiorno [2] has carried on an investigation regarding the heat transport phenomenon in nanofluids’ flow. Non-linear convection flow of Williamson nanofluid through a radially stretching surface presented by Ibrahim
et al. [3]. Khan et al. [4] examined the impacts of multi slip-on Jeffery fluid flow focus to a permeable stretched sheet. To transfer the nature of hybrid nanofluid convection inside a porous medium is scrutinized by [5]. Abbas et al. [6] investigated the different aspects of MHD cross nanofluid flow with the inspiration of thermal and joule heating. Zadeh et al. [7] numerically observe the flow, heat, and mass transfer of nano liquids over a vertical stretch sheet.

The stagnation point describes the liquid’s movement near the stagnant region in front of the blunt flow body for the solid bodies floating in a fluid. In 1911 Hiemenz [8] introduced simulations of similarity to mathematical models, as well as the Navier–Stokes model introduced the concept of stagnant flow. Awaludin et al. [9] discover the stability analysis about the stagnation point flow over the shrinking/stretching sheet. The stagnation point flow about temperature and concentration over the stretching/shrinking sheet’s dimensionless surface are investigated by Merkin and Pop [10]. Bhatti et al. [11] studied the effects of magnetizing on stagnation point flow over a shrinking sheet. To explore the impact of the variable thermal conductivity on the stagnation point flow reviewed by Shah et al. [12]. The effect of stagnation point flow on the micropolar based fluid over a permeable stretching plate examined by Fatumnbi and Adeniyan [13] along with the conclusion that an addition in the boundary parameter results in an increasing in the microrotation of the liquid constituents.

The current trend demonstrates that flow over wedge-shaped geometry has broad applications in the field of aerodynamics, heat exchangers, hydrodynamics, geothermal systems, groundwater pollution, oil recuperation, and so forth [14]. Falkner and Skan [15] acquired the flow over a static wedge brought about the advancement of the equation of Falkner–Skan. In the many previous years, numerous scientists have eventually contributed extraordinary eye-catching works on the Falkner–Skan flow in light of the impact of several thermophysical parameters [16]. Later, Watanabe [17] analyzed fluid flow behavior over a wedge with injection and suction, Ishak et al. [18] examined the MHD flow of past over the moving wedge. Ali et al. [19] obtained the numerical solution of the Falkner–Skan equation utilization the finite element numerical technique.

The investigation of chemical reaction finds enormous applications that incorporate food, contamination, the formation of fog, synthesis and oxidation materials, biochemical engineering, chemical processing types of equipment, plastic expulsion and metallurgy, and energy transfer in a drizzly cooling tower, and so on. The impact of the chemical reaction and the heat source/sink on the unsteady magnetohydrodynamic (MHD) flow of nanofluid over two equal radiating plates lowered in porous media briefly presented by Mohamed et al. [20]. The chemically reactive flow of magnetized Carreau nanofluid flow is a study by Ali et al. [21]. Muhammad et al. [22] studied the significance of nonlinear thermal radiation with a chemical reaction and Arrhenius activation energy in the 3D Eyring Powell nanofluids flow. Kalaivanan et al. [23] discussed the chemical reaction rate effects on second-grade nanofluid along with activation energy. The computational examination of chemically reactive fluid flow over a wedge shape geometry are investigated by Shahzad et al. [24]

Examination of non-Newtonian fluids attracts numerous scientist because of their significance in daily life and in mechanical and synthetic procedures. Taswar et al. [25] examined the MHD flow of tangent hyperbolic nanofluid along with the variable thickness. They found that the heat transfer rate was an increasing function of Prandtl number Pr. Examination of electro-magnetohydrodynamic (EMHD) non-Newtonian tangent hyperbolic nanofluid passed over a Riga plate considered by [26]. The main finding was that the modified Hartmann number maximize the skin friction coefficient and velocity of the fluid. Several numerical and analytical examinations have been reported to predict the characteristics of non-Newtonian fluids like, micropolar fluid [27], Casson fluid flow [28], Jeffrey nanofluid [29], non-Newtonian fluid flow [30], tangent hyperbolic fluid [31], micropolar nanofluid [32], and Oldroyd-B nanofluid [33].
In the previously mentioned investigation, for the most part, the wedge is either static or, on the other hand, moving. Less consideration is paid towards the tangent hyperbolic nanofluid flow across faster/slower stretching wedge. In this examination, five perspectives have been a focus. Firstly, to address the mass and heat transfer of tangent hyperbolic nanofluid. Secondly, to analyze the impact of thermal radiation. Thirdly, to examine the stagnation point flow. Fourthly, to study the effect of activation energy. Fifthly, the finite element approach for this elaborated problem. It solves boundary value problem adequately, rapidly and precisely [34–36]. The results have been computing on a finer mesh selected on the convergence criterion, where the solution’s accuracy is ascertaining adequately, rapidly and precisely [34–36]. The results have been computing on a finer mesh selected on the convergence criterion, where the solution’s accuracy is ascertaining adequately, rapidly and precisely [34–36].

2. Physical Model and Mathematical Formulation

2.1. Tangent Hyperbolic Constitutive Model

For the non-Newtonian fluids, there are numerous models in the literature which describe the different properties of a rheological fluid. The tangent hyperbolic fluid is one of the four constant non-Newtonian fluid models, describing the shear thinning behavior. The apparent viscosity gradually varies between zero and infinite shear rate. The constitutive equation for the tangent hyperbolic fluid is given by

\[
\tau = [\mu_\infty + (\mu_0 + \mu_\infty) \tanh(Γ\dot{\omega})] \dot{\omega},
\]

where, \(ε\), \(τ\), \(Γ\), \(μ_0\), and \(μ_\infty\) are the power law index, extra stress tensor, time constant, zero shear rate viscosity, and infinite shear rate viscosity, respectively, and \(\dot{\omega}\) is defined as:

\[
\dot{\omega} = \sqrt{\frac{1}{2} \sum_i \sum_j \dot{\omega}_{ij} \dot{\omega}_{ji}} = \sqrt{\frac{1}{2} \sqrt{\Pi}},
\]

where \(\Pi\) is the second invariant of the strain rate tensor and \(\Pi = \frac{1}{2} \text{tr}((\text{grad } V)^2)\). Consider the assumption \(u_\infty = 0\). The fact, we are focusing on the shear thinning behavior therefore \(Γ\dot{\omega} < 1\), the extra stress tensor \(τ\) is reduced to

\[
τ = μ_0[(Γ\dot{\omega})^2] \dot{\omega} = μ_0[(1 + Γ\dot{\omega} - 1)^2] \dot{\omega} = μ_0[1 + ε(Γ\dot{\omega} - 1)] \dot{\omega},
\]

2.2. Statement of the Problem

We assume a Falkner–Skan flow of an incompressible unsteady tangent hyperbolic nanofluid over a faster/slower stretching wedge in light of an applied magnetic field along with activation energy and magnetic field. The Reynolds number is considered very small, \(\text{Re} = \frac{\rho U H}{\mu}\), where \(H\) is the height of the wedge. We assume a Falkner–Skan flow of an incompressible unsteady tangent hyperbolic nanofluid over a faster/slower stretching wedge in light of an applied magnetic field along with activation energy and magnetic field. The Reynolds number is considered very small, \(\text{Re} = \frac{\rho U H}{\mu}\). We assume a Falkner–Skan flow of an incompressible unsteady tangent hyperbolic nanofluid over a faster/slower stretching wedge in light of an applied magnetic field along with activation energy and magnetic field. The Reynolds number is considered very small, \(\text{Re} = \frac{\rho U H}{\mu}\). We assume a Falkner–Skan flow of an incompressible unsteady tangent hyperbolic nanofluid over a faster/slower stretching wedge in light of an applied magnetic field along with activation energy and magnetic field. The Reynolds number is considered very small, \(\text{Re} = \frac{\rho U H}{\mu}\). We assume a Falkner–Skan flow of an incompressible unsteady tangent hyperbolic nanofluid over a faster/slower stretching wedge in light of an applied magnetic field along with activation energy and magnetic field. The Reynolds number is considered very small, \(\text{Re} = \frac{\rho U H}{\mu}\).
and impact of the induced magnetic field is neglected. Considering the above suppositions, the governing equations for the current modeled problem are as per the following \[41,42\]:

\[
\begin{align*}
\frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} &= \frac{\partial \bar{u}_c}{\partial t} + \bar{u}_c \frac{\partial \bar{u}_c}{\partial x} + \nu (1 - e) \left( \frac{\partial^2 \bar{u}}{\partial y^2} + \frac{\sqrt{2} \nu \Gamma e}{\partial y} \frac{\partial \bar{u}}{\partial y} \right) - \frac{\sigma B^2(t) \bar{u}}{\rho} (\bar{u} - \bar{u}_c), \\
\frac{\partial \bar{T}}{\partial t} + \bar{u} \frac{\partial \bar{T}}{\partial x} + \bar{v} \frac{\partial \bar{T}}{\partial y} &= k_f \frac{\partial^2 \bar{T}}{\partial y^2} + \nu D_B \frac{\partial \bar{C}}{\partial y} \frac{\partial \bar{T}}{\partial y} + \nu \frac{T}{T_{\infty}} \left( \frac{\partial \bar{T}}{\partial y} \right)^2 + \frac{Q_0}{\rho c_p} (T - T_{\infty}) - \frac{1}{\rho c_p} \frac{\partial q}{\partial y}, \\
\frac{\partial \bar{C}}{\partial t} + \bar{u} \frac{\partial \bar{C}}{\partial x} + \bar{v} \frac{\partial \bar{C}}{\partial y} &= D_B \frac{\partial^2 \bar{C}}{\partial y^2} + \frac{D_T}{T_{\infty}} \frac{\partial^2 \bar{T}}{\partial y^2} - k_f^2 (\bar{C} - \bar{C}_0) \left( \frac{\tilde{T}}{T_{\infty}} \right)^n \exp \left( \frac{-E_a}{k_B T} \right), \\
\bar{u} &= \bar{U}_w(x) = \lambda_s \bar{U}_c, \bar{v} = \bar{v}_w, \exp \left( \frac{\partial \bar{T}}{\partial y} \right) = \frac{h_f (T - T_{\infty})}{C_w(x) = 0, \quad \text{as} \quad y = 0,} \\
\bar{u} &\rightarrow \bar{U}_c, \quad T \rightarrow T_{\infty}, \quad \bar{C} \rightarrow \bar{C}_0, \quad \text{as} \quad y \rightarrow \infty. 
\end{align*}
\]  

(5)

\[
\bar{U}(x,t) = \frac{\bar{U}_w(x)}{1 - \sigma}
\]

Figure 1. Physical and schematic configuration with coordinate system.

Here, \((\bar{u}, \bar{v})\) are velocity components in \(x, y\) directions, respectively, \(\bar{T}\) and \(\bar{C}\) are the fluid temperature and nanoparticle volume concentration, \(D_B\) and \(D_T\) are the Brownian diffusion and thermophoretic diffusion coefficient respectively, \(\bar{U}_c\) is the free stream velocity, \(B_0, m, \rho, C_p, E_a, \epsilon, \) and \(\Gamma\) are the uniform magnetic field strength, Falkner–Skan power law parameter, fluid density, specific heat capacity, activation energy, the power law index, and Williamson parameter, respectively, \(Q_0\) denotes the temperature-dependent volumetric rate of heat source \((Q_0 > 0)\) and heat sink \((Q_0 < 0)\), \(q_f\) is given by \(q_f = \frac{4\alpha^2 D_T^2}{\pi K_1}\) (see\[34,43\]), here Stefan Boltzman constant is \(\alpha^2\) and \(K_1\) is the Roseland mean absorption coefficient. Further, the last term in Equation (4), \(k_f^2 (\bar{C} - \bar{C}_0) \left( \frac{\tilde{T}}{T_{\infty}} \right)^n \exp \left( \frac{-E_a}{k_B T} \right)\) shows the modified Arrhenius equation with a reaction rate of \(k_f^2\). Where \(\tilde{T}_w\) and \(\bar{C}_w\) are temperature and nanoparticle volume fraction at the surface. The corresponding ambient values are denoted by \(\tilde{T}_\infty, \bar{C}_\infty\) respectively.
Introducing following similarity transformations (see [41, 44]):

\[ \psi(x, y, t) = \sqrt{\frac{2vxU_e}{(m+1)}} f(\zeta), \quad \bar{u} = \frac{\partial \psi}{\partial x} = \tilde{U}_e f'(\zeta), \quad \bar{v} = -\frac{\partial \psi}{\partial y} = -\sqrt{\left(\frac{m+1}{2}\right)(vU_e/x)[f(\zeta) + (m-1)\zeta f'(\zeta)]}, \]

\[ \theta(\zeta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\zeta) = \frac{\tilde{C} - \tilde{C}_\infty}{\tilde{C}_w - \tilde{C}_\infty}, \quad \zeta = \sqrt{\left(\frac{m+1}{2}\right)\tilde{U}_e}. \]  

(6)

where \( \psi \) is the stream function and \( \zeta \) is the dimensionless coordinate.

In view of Equation (6), Equations (2)–(5) transform into the following nonlinear ODE’s:

\[ [1 - e + cWef'']f'' + ff'' - \beta(f'^2 - 1) - A(2 - \beta)(\frac{\zeta}{2} f'' + f' - 1) - Ha^2(2 - \beta)(f' - 1) = 0, \]  

\[ (1 + Rd)\theta'' + Pr(f'' - 2f'\theta) + Pr[Nb\theta'\phi' + Nt(\theta')^2 - \frac{A}{2}(2 - \beta)(3\theta + \zeta\theta') + Q\theta] = 0, \]  

\[ \phi'' - \frac{A}{2}Le(2 - \beta)(3\phi + \zeta\phi') + Le(f\phi' - 2f'\phi) + \frac{Nt}{Nb}\theta'' - 2\Omega Le\phi(1 + \gamma\theta)^n \exp\left(\frac{-EE}{1 + \gamma\theta}\right) = 0, \]  

\[ f(\zeta) = f_w, \quad f'(\zeta) = \lambda, \quad \theta'(\zeta) = -Bi(1 - \theta(\zeta)), \quad \phi(\zeta) = 1 \quad \text{at} \quad \zeta = 0, \]  

\[ f'(\zeta) \to 1, \quad \theta(\zeta) \to 0, \quad \phi(\zeta) \to 0 \quad \text{as} \quad \zeta \to \infty. \]  

(10)

The emerging parameters in Equations (07)—(10) are defined as:

\[ \lambda_s = \frac{U_w}{U_e}, \quad A = \frac{c}{ax^{m-1}}, \quad Pr = \frac{v}{\kappa}, \quad Le = \frac{v}{D_B}, \quad Rd = \frac{16\omega^* T_\infty^3}{3k^2 K}, \quad Nb = \frac{\tau D_B(v)^{-1}(\tilde{C}_w - \tilde{C}_\infty)}, \quad Nt = \frac{\tau D_T(T_w - T_\infty)}{vT_\infty}, \]

\[ EE = \frac{E_a}{k_B T_\infty}, \quad Q = \frac{2Qh x}{U_e (\rho C_p)}, \quad \Omega = \frac{x C_0(\kappa_K)^2}{U_w}, \quad \gamma = \frac{T_w - T_\infty}{T_\infty}, \quad \beta = \frac{2m}{m+1}, \quad Bi = -\frac{h_f}{\kappa_f} \left(\frac{(m+1)U_e}{2vU_w}\right)^{-\frac{1}{2}}, \]

\[ (Ha)^2 = \frac{\sigma B_0^2}{\kappa B x^{m-1}}, \quad We = \sqrt{\frac{T^2(m+1)(\tilde{U}_e)^3}{2v}}, \quad Re_s = \frac{\tilde{U}_w(x)}{v}, \quad f_w = -\frac{\tilde{v}_w}{\sqrt{(m+1)vU_w}}. \]

where \( \lambda_s \) is the velocity ratio parameter of wedge such that \( \lambda_s > 1 \) corresponds to faster stretching than that of free stream and \( \lambda_s < 1 \) corresponds to slower than that of free stream flow [45], \( A \) is the unsteadiness parameter \( Pr \) is the Prandtl number, \( Le \) is the Lewis number, \( Rd \) is the radiation parameter, \( Nb, Nt \) are the Brownian motion and thermophoresis respectively, \( EE \) is the dimensionless activation energy, \( Q \) is the heat generation/absorption parameter, \( \omega \) is the chemical reaction rate constant, \( \gamma \) is the temperature difference variable, \( \beta \) is the wedge angle parameter, \( Bi \) is the Biot number, \( Ha \) is the Hartmann number, \( We \) is the Weissenberg number, \( Re_s \) is the local Reynolds number, and \( f_w \) is the suction/injection parameter \((f_w > 0 \text{ for suction and } f_w < 0 \text{ for injection})\). Skin friction coefficient expressions, local Nusselt number, and Sherwood number are defined as:

\[ \tilde{C}_f = \frac{\tau_w}{\rho U_w^2}, \quad Nu = \frac{x q_w}{\kappa(U_w - T_\infty)}, \quad Shr = \frac{x q_m}{\tau_B(\tilde{C}_w - \tilde{C}_\infty)}. \]  

(11)

where the skin friction tensor at wall is \( \tau_w = \mu \left[(1 - e) \frac{\partial u}{\partial y} + \frac{\kappa}{\sqrt{2}} \left(\frac{\partial \theta}{\partial y}\right)^2\right]_{y=0} \), the wall heat transfer is \( q_w = -\kappa \left[(1 + 16a^3 T_\infty^3) \frac{\partial T}{\partial y}\right]_{y=0} \), and the mass flux from the sheet is \( q_m = -\left(\tilde{D}_B \frac{\partial \phi}{\partial y}\right)_{y=0} \). By the aid of similarity transformation Equation (7), we get:
\[ C_f \text{Re}_{\text{e}}^{1/2} = \sqrt{\frac{m+1}{2}} \left[ (1 - \epsilon)f''(0) + \frac{\epsilon}{2} We(f''(0))^2 \right], \]
\[ Nu_x \text{Re}_{\text{e}}^{-\frac{1}{2}} = -\sqrt{\frac{m+1}{2}} [(Rd + 1)\theta'(0)], \]
\[ Shr_x \text{Re}_{\text{e}}^{-\frac{1}{2}} = -\sqrt{\frac{m+1}{2}} [\phi'(0)]. \]

### 3. Finite Element Solutions

The FEM (finite element method) is well known to solve various types of differential equations. This technique’s basic idea is to comprise piecewise approximation of continuous polynomials functions that minimize the error size [46]. The fundamental steps and an outstanding description of this technique outlined by Jyothi [47], and Reddy [48]. It merits referencing that the finite element technique can solve the boundary value problem along with complex geometry precisely, rapidly, and accurately as compared to the finite difference method (FDM) [49,50] and solved many fluid-related engineering problems [51–54].

To solve the system of non-linear coupled partial differential Equations (7) to (9) together with boundary condition (10), firstly we consider:

\[ f' = p, \]

The set of Equations (07)–(10) thus reduces to

\[ [1 - \epsilon + e Wep']p'' + f p' - \beta(p^2 - 1) - A(2 - \beta)(\frac{\zeta}{2}p' + p - 1) - Ha^2(2 - \beta)(p - 1) = 0, \]
\[ (1 + Rd)\theta'' + Pr(f\theta' - 2p\theta) + Pr[Nb\theta'\phi' + Nt(\theta')^2 - \frac{A}{2}(2 - \beta)(3\theta + \zeta\theta') + Q\theta] = 0, \]
\[ \phi'' = \frac{A}{2} Le(2 - \beta)(3\phi + \zeta\phi') + Le(f\phi' - 2p\phi) + \frac{Nt}{Nb}\theta'' - 2\Omega Lef(1 + \gamma\theta)\exp(-\frac{EE}{1 + \gamma\theta}) = 0, \]
\[ f(\zeta) = f_w, \quad p(\zeta) = \lambda, \quad \theta'(\zeta) = -B, \quad \phi(\zeta) = 1 \text{ at } \zeta = 0, \]
\[ \phi(\zeta) \to 1, \quad \theta(\zeta) \to 0, \quad \phi(\zeta) \to 0 \text{ as } \zeta \to \infty. \]

#### 3.1. Variational-Formulations

The variational form connected with Equations (13)–(16) over a quadratic element \( \Omega \) is given by

\[ \int_{\zeta_1}^{\zeta_{i+1}} \bar{w}_1 \left\{ \frac{df}{d\zeta} - p \right\} d\zeta = 0, \]
\[ \int_{\zeta_1}^{\zeta_{i+1}} \bar{w}_2 \left\{ [1 - \epsilon + e Wep']p'' + f p' - \beta(p^2 - 1) - A(2 - \beta)(\frac{\zeta}{2}p' + p - 1) ight\} d\zeta = 0, \]
\[ \int_{\zeta_1}^{\zeta_{i+1}} \bar{w}_3 \left\{ (1 + Rd)\theta'' + Pr(f\theta' - 2p\theta) + Pr[Nb\theta'\phi' + Nt(\theta')^2 - \frac{A}{2}(2 - \beta)(3\theta + \zeta\theta') + Q\theta] \right\} d\zeta = 0, \]
\[ \int_{\zeta_1}^{\zeta_{i+1}} \bar{w}_4 \left\{ \phi'' = \frac{A}{2} Le(2 - \beta)(3\phi + \zeta\phi') + Le(f\phi' - 2p\phi) + \frac{Nt}{Nb}\theta'' ight\} d\zeta = 0. \]

Here \( \bar{w}_1, \bar{w}_2, \bar{w}_3, \) and \( \bar{w}_4 \) are trial functions.
3.2. Formulation of Finite-Element

The finite model of the element can be obtained from Equations (18)–(21) by replacing the following form:

\[ f = \frac{3}{\eta} f_n \psi_n, \quad \rho = \sum_{n=1}^{\eta} \rho_n \psi_n, \quad \vartheta = \sum_{n=1}^{\eta} \vartheta_n \psi_n, \quad \phi = \sum_{n=1}^{\eta} \phi_n \psi_n \]  

(22)

with \( \bar{w}_1 = \bar{w}_2 = \bar{w}_3 = \bar{w}_4 = \psi_n (n = 1, 2, 3) \), where the test functions \( \psi_n \) for a typical length element \( \Omega_c = (\zeta_a, \zeta_{a+1}) \) are given by.

\[ x_1, x_2, x_3, x_4, \ldots, x_p \]

In global coordinates

In local coordinates: For \( p = 2 \) (linear element)

\[ \psi_1 = \frac{\zeta - \zeta_{a+1}}{\zeta_a - \zeta_{a+1}}, \quad \psi_2 = \frac{\zeta_a - \zeta}{\zeta_a - \zeta_{a+1}}, \quad \zeta_a \leq \zeta \leq \zeta_{a+1}. \]

(23)

In local coordinates: For \( p = 3 \) (Quadratic element)

\[ \psi_1 = \frac{(\zeta_{a+1} - \zeta_a - 2\zeta)(\zeta_a - \zeta)}{(\zeta_{a+1} - \zeta_a)^2}, \quad \psi_2 = \frac{4(\zeta - \zeta_a)(\zeta_{a+1} - \zeta)}{(\zeta_{a+1} - \zeta_a)^2}, \]

\[ \psi_3 = -\frac{(\zeta_{a+1} - \zeta_a - 2\zeta)(\zeta - \zeta_a)}{(\zeta_{a+1} - \zeta_a)^2}, \quad \zeta_a \leq \zeta \leq \zeta_{a+1}. \]

(24)

The model of finite elements of the equations thus developed is given by:

\[
\begin{bmatrix}
W^{11} & W^{12} & W^{13} & W^{14} \\
W^{21} & W^{22} & W^{23} & W^{24} \\
W^{31} & W^{32} & W^{33} & W^{34} \\
W^{41} & W^{42} & W^{43} & W^{44}
\end{bmatrix}
\begin{bmatrix}
f \\ \\
\theta \\ \\
\phi
\end{bmatrix}
=
\begin{bmatrix}
b^1 \\ \\
b^2 \\ \\
b^3 \\
b^4
\end{bmatrix}
\]  

(25)

where \([W^{mn}]\) and \([b^m]\) (m,n=1,2,3,4) are defined as:

\[ W^{11}_{ij} = \int_{\xi_a}^{\xi_{a+1}} \psi_i \frac{d\psi_j}{d\xi} d\xi, \quad W^{12}_{ij} = -\int_{\xi_a}^{\xi_{a+1}} \psi_i \psi_j d\xi, \quad W^{13}_{ij} = W^{14}_{ij} = W^{21}_{ij} = W^{31}_{ij} = 0, \]

\[ W^{22}_{ij} = -(1 - \epsilon) \int_{\xi_a}^{\xi_{a+1}} \frac{d\psi_i}{d\xi} \frac{d\psi_j}{d\xi} d\xi + W e \int_{\xi_a}^{\xi_{a+1}} \bar{\eta} \psi_i \frac{d\psi_j}{d\xi} d\xi + \int_{\xi_a}^{\xi_{a+1}} \int_0^{\xi_{a+1}} \int_{\xi_a}^{\xi_{a+1}} \psi_i \frac{d\psi_j}{d\xi} d\xi d\zeta - Ha^2 (2 - \beta) \int_{\xi_a}^{\xi_{a+1}} \psi_i \psi_j d\xi \]

\[ - \beta \int_{\xi_a}^{\xi_{a+1}} \psi_i \psi_j d\xi - A (2 - \beta) \int_{\xi_a}^{\xi_{a+1}} \psi_i \psi_j d\zeta - A (2 - \beta) \frac{\bar{\eta}}{2} \int_{\xi_a}^{\xi_{a+1}} \psi_i \frac{d\psi_j}{d\xi} d\xi, \quad W^{23}_{ij} = W^{24}_{ij} = W^{32}_{ij} = W^{42}_{ij} = 0, \]
\[
W_{ij}^{33} = -(1 + Rd) \int_{\xi_0}^{\xi_0+1} \frac{d\psi}{d\xi} \frac{d\psi}{d\xi} d\zeta + Pr \int_{\xi_0}^{\xi_0+1} \frac{d\psi}{d\xi} \frac{d\psi}{d\xi} d\zeta - 2Pr \int_{\xi_0}^{\xi_0+1} \psi \frac{d\psi}{d\xi} d\zeta + PrNb \int_{\xi_0}^{\xi_0+1} \frac{d\psi}{d\xi} \frac{d\psi}{d\xi} d\zeta \\
+ PrNt \int_{\xi_0}^{\xi_0+1} \frac{d\psi}{d\xi} \frac{d\psi}{d\xi} d\zeta - \frac{3PrA}{2} (2 - \beta) \int_{\xi_0}^{\xi_0+1} \psi \frac{d\psi}{d\xi} d\zeta - APr(2 - \beta) \frac{\zeta}{2} \int_{\xi_0}^{\xi_0+1} \psi \frac{d\psi}{d\xi} d\zeta + PrQ \int_{\xi_0}^{\xi_0+1} \psi \frac{d\psi}{d\xi} d\zeta,
\]

\[
W_{ij}^{34} = W_{ij}^{41} = W_{ij}^{42} = 0, W_{ij}^{43} = \int_{\xi_0}^{\xi_0+1} \frac{d\psi}{d\xi} \frac{d\psi}{d\xi} d\zeta, W_{ij}^{44} = \int_{\xi_0}^{\xi_0+1} \frac{d\psi}{d\xi} \frac{d\psi}{d\xi} d\zeta + Le \int_{\xi_0}^{\xi_0+1} \psi \frac{d\psi}{d\xi} d\zeta \\
- 2Le \int_{\xi_0}^{\xi_0+1} \psi \frac{d\psi}{d\xi} d\zeta - \frac{3LeA}{2} (2 - \beta) \int_{\xi_0}^{\xi_0+1} \psi \frac{d\psi}{d\xi} d\zeta - ALe(2 - \beta) \frac{\zeta}{2} \int_{\xi_0}^{\xi_0+1} \psi \frac{d\psi}{d\xi} d\zeta \\
- 2\Omega Le \int_{\xi_0}^{\xi_0+1} (1 + \gamma \bar{\theta}) \exp(-\frac{EE}{1 + \gamma \bar{\theta}}) \psi \frac{d\psi}{d\xi} d\zeta.
\]

and

\[
b_i^3 = 0, b_i^4 = -(1 - \epsilon) \left( \frac{d\psi}{d\xi} \right)^{\xi_0+1}_{\xi_0} - \beta - A(2 - \beta) - Ha^2(2 - \beta),
\]

\[
b_i^3 = -(1 + Rd) \left( \frac{d\psi}{d\xi} \right)^{\xi_0+1}_{\xi_0}, b_i^4 = - \left( \frac{d\psi}{d\xi} \right)^{\xi_0+1}_{\xi_0} - \frac{Nt}{Nb} \left( \frac{d\psi}{d\xi} \right)^{\xi_0+1}_{\xi_0}
\]

with

\[
f = \sum_{j=1}^{3} f_j \psi_j, h = \sum_{j=1}^{3} h_j \psi_j, \theta' = \sum_{j=1}^{3} \theta_j \psi_j, \phi' = \sum_{j=1}^{3} \phi_j \psi_j.
\]

For computational purposes, the computational domain is divided into quadratic elements of equal size because Table 1 demonstrates no more variation against higher input of n (number of elements). Five functions are evaluated at each node, and 3005 \times 3005 order of stiffness matrix is acquired after the assembling of whole element equations. After applying the boundary condition (Equation (20)), the developed equations are non-linear, so an iterative scheme utilized to solve it with 0.000005 required precision.

Table 1. Finite element method (FEM) convergence results of f(\zeta), p(\zeta), \theta(\zeta), and \phi(\zeta) at the 1.5 of computational domain [0, 12] for different number of elements when Pr = 1, \lambda_s = 0.5, A = Q = We = 0.2, Ha = 0.5, Bi = EE = \gamma = 1, Nt = Nb = 0.3, \omega = 3, \epsilon = 0.3, Le = 3, Rd = 0.5, \beta = 0.5, n = 1, f_w = 0.5.

| Number of Elements | f(1.5)   | h(1.5)   | \theta(1.5) | \phi(1.5)  |
|-------------------|----------|----------|-------------|------------|
| 60                | 1.068133 | 0.914246 | 0.148457    | 0.054075   |
| 100               | 1.067776 | 0.914217 | 0.148492    | 0.054118   |
| 180               | 1.067636 | 0.914206 | 0.148507    | 0.054136   |
| 360               | 1.067585 | 0.914202 | 0.148512    | 0.054142   |
| 500               | 1.067582 | 0.914202 | 0.148513    | 0.054143   |
| 700               | 1.067578 | 0.914201 | 0.148513    | 0.054144   |
| 1000              | 1.067575 | 0.914201 | 0.148513    | 0.054144   |

4. Results And Discussion

Exploration for momentum, temperature and concentration fields are presented in pictorial form with consideration of three conditions at the boundary of geometric configuration viz wedge surface is stretched fast (\lambda_s > 1), stretched uniformly (\lambda_s = 1) and stretched slowly (\lambda_s < 1). Tables 2 and 3 show a comparison of f''(0) (skin contact coefficient) for certain values of the Hartmann number (Ha) and f_w (suction/injection). We observed from the tables, an excellent agreement is noticed which valid the acquired results. Further, to check the accuracy of finite element (FE) technique, a comparison
of Nusselt number \((-Re^{1/2}Nu)\) is performed with existing literature for higher input of Prandtl number \(Pr\) and wedge angle \(\beta\) in restricting cases. Here, an excellent agreement is also noticed (see Table 4). For numerical solutions, we have picked the non-dimensional parameter values \(Pr = Bi = 1, \beta = Ha = Rd = 0.5, A = We = 0.2, Le = 3, Nb = Nt = 0.3, EE = n = 1, \Omega = 3, f_w = 0.5, \epsilon = 0.3, Q = 0.2, \gamma = 1\) these parameters values are saved as common in whole investigation of present study apart from the variations in the corresponding figures. In this investigation, the graphs in blue dashed line show the slower stretching sheet \((\lambda_s = 0.5)\), the solid green colour indicate the stretched uniformly \((\lambda_s = 1)\), and the solid red line represent the faster stretching sheet \((\lambda_s = 1.7)\).

Table 2. Comparison of \(f''(0)\) obtained by FEM and that of Ariel [55] for \(\beta = 1\) and when all other parameters are fixed zero.

| \(Ha\) | Perturbation Solution | Approximate Solution | \(f''(0)\) (Current Results) | \(\%\ Error\) |
|---|---|---|---|---|
| 0.0 | 1.232588 | 1.224745 | 1.232589 | 0.000081 |
| 0.4 | 1.295290 | 1.288410 | 1.295369 | 0.000077 |
| 0.8 | 1.463725 | 1.462874 | 1.467977 | 0.000068 |
| 1.0 | 1.570687 | 1.581139 | 1.585331 | 0.000063 |
| 1.4 | 1.774774 | 1.840810 | 1.862849 | 0.000161 |
| 1.6 | 1.842391 | 2.005172 | 2.017155 | 0.000050 |
| 3.0 | - | 3.240355 | 3.240952 | 0.000062 |
| 5.0 | - | 5.147815 | 5.147968 | 0.000058 |
| 10.0 | - | 10.074740 | 10.074748 | 0.000069 |

Table 3. Comparison of \(f''(O)\) with \(f_w\) when \(\beta = 1\) and all other parameters are fixed zero.

| \(f_w\) | \(Ishak\) [56] | \(Ahmad and Khan\) [57] | \(Yin\) [58] | \(Imran Ullaha\) [59] | \(Postelnicu and Pop\) [60] | \(FEM\) Current Results |
|---|---|---|---|---|---|---|
| -1.0 | 0.7566 | 0.75655 | 0.7566 | 0.75658 | 0.75658 | 0.756576 |
| -0.5 | 0.9692 | 0.96922 | 0.9692 | 0.96923 | 0.96923 | 0.969232 |
| 0.0 | 1.2326 | 1.23258 | 1.2326 | 1.23259 | 1.23259 | 1.232591 |
| 0.5 | 1.5418 | 1.54175 | 1.5418 | 1.54175 | 1.54175 | 1.541756 |
| 1.0 | 1.8893 | 1.88931 | 1.8893 | 1.88931 | 1.88931 | 1.889321 |

Table 4. Numerical values of Nusselt number \((-Re^{1/2}Nu)\) for different values of Prandtl number \(Pr\) and wedge angle parameter \(\beta\) when \(\epsilon, We, Nt, Nb, f_w, Sc,\) and \(Ha\) are fixed zero.

| \(Pr\) | \(\beta = 0\) | \(\beta = 0.3\) | \(\beta = 0\) | \(\beta = 0.3\) |
|---|---|---|---|---|
| 0.1 | 0.1980 | 0.2090 | 0.198129 | 0.209153 |
| 0.3 | 0.3037 | 0.3278 | 0.303719 | 0.327831 |
| 0.6 | 0.3916 | 0.4289 | 0.391677 | 0.428928 |
| 0.7 | 0.4178 | 0.4592 | 0.418094 | 0.459555 |
| 1.0 | 0.4696 | 0.5195 | 0.469604 | 0.519524 |
| 2.0 | 0.5972 | 0.6690 | 0.597241 | 0.669056 |
| 6.0 | 0.8672 | 0.9872 | 0.867297 | 0.987299 |
| 10.0 | 1.0297 | 1.1791 | 1.029779 | 1.179182 |
Figure 2. Fluctuation of $f' (\zeta)$ (velocity profile) along with $f_w$ (suction/injection) (a), Hartmann number ($Ha$) (b), $\epsilon$ (material power law index) (c), $\beta$ (wedge angle) (d), Weissenberg number ($We$) (e), and unsteadiness parameter ($A$) (f).

Plots in Figure 2a–f in their respective order represent the varying pattern of rescaled fluid velocity $f' (\zeta)$ concerning the appropriate changing values of the parameters $f_w$, $Ha$, $\epsilon$, $\beta$, $We$ and $A$. The velocity trace in the form of the parabola for $\lambda_s > 1$, it is a straight line for $\lambda_s = 1$, and it sweeps out an inverted boundary when $\lambda_s < 1$. Figure 2a illustrates that high values of injection parameter ($f_w < 0$) makes the flow speedy in the boundary layer, whereas the suction ($f_w > 0$) causes to slow the speed of flow. This outcome is in agreement with the influence of mass transfer at the boundary. The Hartman number’s exceeding strength ($Ha$) decelerates the flow as depicted in Figure 2b. The very reason for this finding is the resistive force that calls into play due to the interaction of magnetic and electric fields. Figure 2c,d respectively exhibit the slowing speed of flow when material law index $\epsilon$ and wedge angle parameter ($\beta$) is an increment in case of $\lambda_s > 1$. An opposite phenomenon observes in the case of $\lambda_s < 1$. It is perceived that both these parameters impede the fluid flow. The rise in $\epsilon$ signifies the fluid’s shear-thickening more extensive and
an increase in $\alpha$ establishes the resistive force, which calls in to play due to the interaction of magnetic and electric fields. Weisenberg number ($We$), which corresponds to the increased relaxation time, Figure 2e discloses that with large values of $We$, the velocity recedes for ($\lambda_s < 1$), but it becomes fast for ($\lambda_s > 1$). Interestingly, the higher value of parameter $A$ (unsteadiness) characterizes the more lapse of time after the jerk to the stretching surface. Hence, the slowing of velocity ($f'(\zeta)$) results, as revealed in Figure 2f.

![Figure 3. Fluctuation of temperature profile ($\theta(\zeta)$) along with $f_w$ (suction/injection) (a), Hartmann number ($Ha$) (b), material power law index ($\epsilon$) (c), and Brownian motion ($Nb$) (d).](image)

A description for altering behavior of non-dimensional temperature $\theta(\zeta)$ is provided for $\lambda_s < 1$ (slow stretching) and $\lambda_s > 1$ (fast stretching). It observed that the temperature function for ($\lambda_s < 1$) is larger than the case of ($\lambda_s > 1$), as drawn in the following plots in Figure 3a–d to visualize respectively, the impacts of $f_w$, $Ha$, $\epsilon$, and Brownian motion parameter $Nb$. A first sight reveals that larger injection ($f_w < 0$) raises the curve of $\theta(\zeta)$ but higher suction ($f_w > 0$) declines $\theta(\zeta)$. The magnetic field’s intensified strength to yield greater value for $Ha$ has enlarged $\theta(\zeta)$, and the thermal boundary layer loses its thickness (see Figure 3b). The physical reason for this outcome is associated with the enhanced resistance to the flow of fluid. Figure 3c,d respectively exhibit the effect of incremented values of $\epsilon$ and $Nb$ resulted in higher temperature distribution because of the increasing $\epsilon$, the shear thickening of fluid is enhanced to capture more heat, and the high value of $Nb$ associated with the intensified random motion of the nanoparticles can efficiently diffuse heat in the fluid. $Nt$ measures thermophoresis, and it stands for transportation of nanoparticles from hot to cold regions.
Figure 4. Fluctuation of temperature profile ($\theta(\zeta)$) along with thermophoresis ($N_t$) (a), heat generation/absorption ($Q$) (b), Biot number ($Bi$) (c), wedge angle ($\beta$) (d), radiation ($R_d$) (e), and unsteadiness parameter ($A$) (f).

Figure 4a discloses the directly proportional behavior of $\theta(\zeta)$ in response to $N_t$. Figure 4b placed to delineate $\theta(\zeta)$ with variation in $Q$, the heat sink-source parameter. As expected, it is revealed that $\theta(\zeta)$ diminishes against $Q$ ($Q < 0$) but it rises with $Q$ ($Q > 0$). Biot number is a measure of ratio for convection at the surface to conduction; hence $\theta(\zeta)$ is incremented when $Bi$ made large as depicted in Figure 4c. The plot in Figure 4d presents the exposition that the incremented wedge parameter $\beta$ has reduced temperature $\theta(\zeta)$. The radiation parameter $R_d$ characterizes radiative heat transfer mode with a heat flux of greater strength at the surface. The temperature $\theta(\zeta)$ is raised directly with exceeding the value of $R_d$ as indicated from Figure 4e. The increasing parameter of unsteadiness ($A$) marked significant depreciation in the values of $\theta(\zeta)$ as disclosed in Figure 5a–d in respective order display the non-dimensional volume fraction of nanoparticles $\phi(\zeta)$ under the influences of activation energy parameter $EE$, chemical reaction parameter $\Omega$, $f_w$ and Lewis number $Le$. It is perceived from these graphs that $\phi(\zeta)$ upsurges with increment in $EE$ (see Figure 5a), but it diminishes against $\Omega$, $f_w$, and $Le$ (see Figure 5c,d).
Plots for the skin friction coefficient drawn under the variation of Ha and $f_w$. Figure 6a reveals that skin friction is intensified for larger values of Ha when there is slow stretching ($\lambda_s < 1$) but opposite pattern is observed for fast stretching ($\lambda_s > 1$). Moreover, the greater injection ($f_w < 0$) reduces the skin friction but larger suction ($f_w > 0$) enhances it when $\lambda_s < 1$. Mass transfer’s role at the surface (suction/injection) is reverse when $\lambda_s > 1$. It is also seen skin friction remains uniform at zero value for static wedge ($\lambda_s = 1$). The declining variation of Nusselt number against progressive values of thermophoresis and Brownian motion parameters $Nt$, $Nb$ is sketched in Figure 6b. This situation disclosed that the Nusselt number is stronger for slow stretching than the wedge surface’s fast stretching. Figure 6c implies that the Nusselt number enhances with the rising value of Prandtl number $Pr$ as well as that of radiation parameter $Rd$. Here the Nusselt number is higher for $\lambda_s > 1$ but lower for $\lambda_s < 1$. The delineation of Sherwood number against activation energy parameter $EE$ and chemical reaction parameter $\Omega$ is exposed in Figure 6d. It is perceived that Sherwood’s number is enhanced when $\Omega$ elevated, but it is reduced vividly against developing values of $EE$. Moreover, Sherwood number attains higher values for $\lambda_s > 1$ as compared to those for $\lambda_s < 1$.

![Figure 5](image.png)  
Figure 5. Fluctuation of nanoparticle concentration profile ($\phi(\zeta)$) along with activation energy ($EE$) (a), chemical reaction rate ($\Omega$) (b), suction/injection ($f_w$) (c), and Lewis number ($Le$) (d).
Figure 6. Fluctuation of $C_f R_e^{1/2}$ (Skin friction coefficient) along with Hartmann number ($Ha$) and suction/injection ($f_w$) (a), Nusselt number ($Nu_x R_e^{-1/2}$) with thermophoresis ($Nt$), Brownian motion ($Nb$) (b), Prandtl number ($Pr$), and radiation ($Rd$) (c), and Sherwood number ($Shr_x R_e^{-1/2}$) along with activation energy ($EE$) and chemical reaction rate ($\Omega$) (d).

5. Conclusions

The finite element solution for the unsteady motion of Carreau nanofluid over a fast or slow stretched wedge is explored in this work. The chemically reactive species of nanomaterial adheres to thermophoresis and Brownian movement slip conditions. Thermal transportation is based on a heat source, radiation mode, and convective boundary conditions. Some of the notable findings described briefly:

- Increased injection parameter ($f_w < 0$) makes the flow faster, whereas the suction ($f_w > 0$) causes the speed of flow to slow.
- The exceeding values of Hartman number $Ha$, suction/injection ($f_w$) material law index $\epsilon$, aligned magnetic field parameter $\alpha$ and unsteadiness parameter ($A$) recede the velocity $f'(\xi)$ when $\lambda_s > 1$ whereas enhance when $\lambda_s < 1$. An opposite trend is observed for Weissenberg number ($We$).
- The greater values of $Nt$, $Rd$, $Bi$, $Q$ ($Q > 0$) and $Nb$ results in increased temperature distribution whereas the $f_w$, $\beta$, and unsteadiness ($A$) causes it to decline in both cases ($\lambda_s > 1, \lambda_s < 1$).
- The greater values of $Ha$ and $\epsilon$ results in increased temperature distribution when $\lambda_s > 1$ but a decline is observed for $\lambda_s < 1$.
- The volume fraction of nanoparticles $\phi(\xi)$ is upsurged with increment in $EE$ but it diminishes against $\Omega$, $f_w$ and $Le$ in both cases ($\lambda_s > 1, \lambda_s < 1$).
- Skin friction grows larger with increment in values of $Ha$ when there is slow stretching ($\lambda_s < 1$), but the opposite pattern is observed for fast stretching ($\lambda_s > 1$).
- Nusselt number declines against progressive values of thermophoresis and Brownian motion parameters $Nt$, $Nb$. 
**Author Contributions:** B.A. and R.A.N. modeled the problem and wrote the manuscript. A.M. completed the formal analysis and revision. O.M.A. thoroughly checked the mathematical modeling, English corrections, formal analysis, and revision. B.A. solved the problem using MATLAB software. L.A. and R.A.N.: writing—review and editing. All authors finalized the manuscript after its internal evaluation. All authors have read and agreed to the published version of the manuscript.

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**Nomenclature**

| Symbol | Description                  |
|--------|------------------------------|
| \( \hat{T} \) | Non-dimensional temperature |
| \( \hat{T}_w \) | Temperature at surface       |
| \( \hat{C} \) | Non-dimensional nanoparticles concentration |
| \( \hat{C}_w \) | Concentration at surface     |
| \( \hat{T}_\infty \) | Temperature away from the surface |
| \( a, b, c \) | Positive constants           |
| \( \hat{C}_\infty \) | Concentration away from the surface |
| \( E \) | Activation energy            |
| \( \hat{n}_\infty \) | Motile organisms away from the surface |
| \( (\hat{u}, \hat{v}) \) | Velocity components         |
| \( \hat{U}_w(x, t) \) | Velocity of stretching/shrinking wedge |
| \( \hat{U}_e(x, t) \) | Free stream velocity        |
| \( \beta_T \) | Thermal expansion coefficient |
| \( \nu \) | Kinematic viscosity          |
| \( \rho_f \) | Density of fluid             |
| \( Pr \) | Prandtl number               |
| \( D_\hat{T} \) | Thermophoretic diffusion coefficient |
| \( D_B \) | Brownian diffusion coefficient |
| \( m \) | Falkner-Skan power law      |
| \( B_0 \) | Uniform magnetic field      |
| \( \sigma \) | Electrical conductivity      |
| \( Le \) | Lewis number                 |
| \( \beta \) | Wedge angle parameter       |
| \( We \) | Weissenberg number          |
| \( \rho C_p \) | Base fluid heat capacity    |
| \( Q_0 \) | Heat generation/absorption  |
| \( E \) | Activation energy            |
| \( \kappa_B \) | Boltzmann constant          |
| \( n \) | Fitted rate constant         |
| \( \sigma^* \) | Stefan-Boltzmann number     |
| \( K_1 \) | Mean assimilation coefficient |
| \( \psi \) | Stream function              |
| \( \epsilon \) | Power law index             |
| \( \Gamma \) | Williamson parameter        |
| \( Nb \) | Brownian motion             |
| \( Nt \) | Thermophoresis              |
| \( \omega \) | chemical reaction rate       |
| \( Bi \) | Biot number                 |
| \( Ha \) | Hartmann number             |
| \( Re_x \) | Local Reynolds number       |
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