Crack propagation modeling of strengthening reinforced concrete deep beams with CFRP plates

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Abstract
Fracture analysis of reinforced concrete deep beam strengthened with carbon fiber-reinforced polymer (CFRP) plates was carried out. The present research aimed to discover whether crack propagation in a strengthened deep beam follows linear elastic fracture mechanics (LEFM) theory or nonlinear fracture mechanics theory. To do so, a new energy release rate based on nonlinear fracture mechanics theory was formulated on the finite element method and the discrete cohesive zone model (DCZM) was developed in deep beams. To validate and compare with numerical models, three deep beams with rectangular cross-sections were tested. The code results based on nonlinear fracture mechanics models were compared with the experimental results and the ABAQUS results carried out based on LEFM. The predicted values of initial stiffness, yielding point and failure load, energy absorption, and compressive strain in the concrete obtained by the proposed model were very close to the experimental results. However, the ABAQUS software results displayed greater differences compared to the experimental result. For instance, the predicted failure load for the shear-strengthened deep beam using the proposed model only had a 6.3% difference from the experimental result. However, the predicted failure load using ABAQUS software based on LEFM indicated greater differences (25.1%) compared to the experimental result.

1. Introduction
Reinforced concrete deep beams play an essential role in bridges, buildings, offshore structures, foundations, and military structures [1–3]. Pile caps in foundations, coupling beams in buildings, transfer beams, load-bearing walls, and bunker walls conduct as deep beams [4]. In the deep beam, the span to effective depth ratio is less than or equal to 2.0 [5] and the transverse plane sections before bending do not remain plane after bending [6]. In general, cracks in a concrete structure, such as deep beams, start in the tension zone due to increasing stresses or the presence of initial cracks [7]. Therefore, these cracks must be studied correctly. Two approaches are available to study crack propagation in concrete structures [8]. They can be divided into two general categories as Linear Elastic Fracture Mechanics (LEFM) and nonlinear fracture mechanics theories. LEFM theory was applied, for the first time, to analyze fracture mechanics on ships used in World War II [9]. LEFM considers the stress ahead of the crack to be infinite. The stress intensity factor (SIF) is applied in LEFM to calculate the stress condition near the tip of the crack. A fracture occurs if the SIF reaches the fracture toughness value. The SIF changes with the size of the crack, load, geometry of the structure, and material properties. One of the characteristics of LEFM theory is stress singularity at the crack tip. Kaplan’s research [10] showed that LEFM theory is not acceptable to study crack propagation of normal-sized concrete structures but can be used to
investigate the propagation of cracks in mass structures such as concrete dams. Later, researchers discovered a zone in front of cracks in a normal-sized concrete structure and called it the fracture process zone (FPZ). The behavior of this zone is non-linear and a great deal of energy is accumulated in this area which the presence of the FPZ explains the softening behavior in the stress-crack opening curve after maximum load [11–14]. Modeling of this area which has been studied a great deal in recent years is important for concrete structural members such as beams, joints, and deep beams. The FPZ is not used in LEFM theory [15, 16]. One of the most successful and accurate methods in modeling this area is the discrete cohesive zone model (DCZM) that was developed in the previous studies [17–19].

By using different kinds of strengthening, the crack propagation of concrete changes. In recent years, Carbon Fiber Reinforced Polymer (CFRP) plates have been applied in concrete structures [20, 21]. Advantages of this type of retrofitting are low weight, corrosion-resistant, easy installation, and high tensile strength [22–24]. Nowadays, strengthening using CFRP is of interest to highly versatile materials; available as sheets, plats, bars, and tubes [25]. The use of CFRP composites is now identified as a successful, suitable, and efficient technique to strengthen structures [26]. Because CFRP plates affect the initiation and propagation of cracks in concrete members, including deep beams, it is necessary to model, test, and study these members. Due to shear crack, the strengthening of deep beams in the shear region is significant. Shear cracks propagate more in deep beams compared with flexural cracks. However, it was expected in advance that flexural strengthening of a deep beam in the flexural zone (soil strength of the deep beam) cannot enhance their load-displacement behavior. To verify this issue, such strengthening was performed in this study. Indeed, the capacity of deep beams is affected by shear-strengthening (side face of the deep beam) rather than flexural-strengthening. Many experimental research studies on deep beams and CFRP shear-strengthened deep beam exist [27, 28]. However, there is limited knowledge on crack propagation in a concrete deep beam strengthened with CFRP plates in flexural and shear regions. It is vital to carry out experimental tests to better understand the crack propagation pattern of concrete deep beams strengthened with CFRP plates.

The present research aims to discover whether the crack propagation in a strengthened deep beam follows LEFM theory or nonlinear fracture mechanics theory. The objective of this paper is to develop a new crack propagation criterion based on nonlinear fracture mechanics theory in deep beams strengthened with CFRP plates. Therefore, the DCZM was developed in deep beams and the crack propagation criteria were proposed. As a result, a new energy release rate based on nonlinear fracture mechanics theory was formulated using the finite element method. In this study, a numerical model was proposed to simulate the FPZ. To validate and compare with the numerical models, three deep beam specimens with rectangular cross-sections were tested. One of the deep beams was considered a control specimen. Another deep beam was strengthened in flexural with CFRP plates at the bottom and the last one was strengthened in shear with CFRP plates at both sides of the shear span of the beam. The three beams were tested by a four-point bending test until failure. The experimental results were compared with the results obtained by a LEFM simulation and a simulation based on the nonlinear fracture mechanics. For the LEFM simulation ABAQUS software was applied, the nonlinear simulation was implemented in FEAPpv.

2. Materials and methods

2.1. Numerical model

2.1.1. Proposed numerical model based on nonlinear fracture mechanics

The Finite Element Analysis Programs personal version (FEAPpv) is open-source software to analyze complex structures with user-defined functions. FEAPpv has only a command language and was designed for research by Taylor [29]. In this program, the solution algorithm is written by the operator. Therefore, each operator can describe a solution plan that meets specific needs. The system includes enough commands that can be applied for use in mechanics, structural, heat transfer, fluid, and other areas by differential equations. Several numerical models have been accomplished by FEAPpv [30–35].

To estimate shear crack propagation for concrete deep beam, a new formulation of energy release rate based on the finite element method was introduced in this study. To compute the strain energy release, the virtual crack closure technique (VCCT), which is the most popular and powerful tool in DCZM, was used [17]. A small part of concrete deep beam in the shear span is shown in figure 1. A truss element as the interface is set between interfacial node pairs, nodes ‘1’ and ‘2’. Initially, nodes ‘1’ and ‘2’ have the same coordinates. In figure 1, the gap between nodes was exaggerated.

The stiffness of the interface element selected as truss element in the elastic zone (no crack propagates) based on VCCT is given by [36]:

\[ k_x = \frac{EB\Delta}{h}, \]  

(1)
where $E$, $G$, $B$, $\Delta$, and $h$ are Young’s modulus of concrete, shear modulus, the width of the deep beam, mesh size, and height of the deep beam, respectively. The cracking stress proposed from experimental by Thomas et al.\cite{37} was adopted to model the initial shear crack as:

$$\sigma_{\text{start}} = 0.24\sqrt{1.25f'_c},$$

where $\sigma_{\text{start}}$ and $f'_c$ are cracking stress and concrete compressive strength, MPa, respectively. If the principal stress in Nodes ‘1’ or ‘2’ was equal to equation (3), the stiffnesses in equations (1) and (2) were zero and cracks were created. The normal stress of crack propagation from experimental by Walraven\cite{38} was adopted to find strain energy release rates in the deep beam.

$$\sigma_N = CK(100f'_c)^{1/3},$$

where $\sigma_N$, $\rho$, $K$, and $C$ are normal stress, longitudinal reinforcement ratio, size factor, and a coefficient (approximately 0.12), respectively. The $K$ is given by:

$$K = 1 + \sqrt{200/d},$$

where $d$ is the effective depth in mm. Equation (3), and (4) were used and formulated in the proposed approach to predict crack propagation modeling of RC deep beams. The new proposed approach was validated with the experimental results of three deep beams. It was assumed that the cross-section of the truss element is equal to $A = B \times \Delta$. The strain energy release rates of Mode I, $G_I$ is expressed as follows, according to\cite{18}

$$G_I = \frac{E_N(x_1 - x_2)}{2B\Delta} = \frac{\sigma_N(x_1 - x_2)}{2},$$

where $E_N$, $x_1$ and $x_2$ are nodal force, displacement of node ‘1’ and ‘2’ in $x$ direction, respectively. The shear stress of crack in the deep beam was adopted by Zhang et al.\cite{39} and represented as follows.

$$\tau_N = A + B \sigma_N,$$

where $A$ and $B$ are the shear cohesive and frictional that are:

$$A = 0.347f_{\tau}^{0.665},$$

$$B = \frac{0.4f^'_c - 0.37 - 0.347f_{\tau}^{0.665}}{0.25f^'_c}. $$

Therefore, the strain energy release rates for shear, $G_{II}$, was based on a study by Xie et al.\cite{36} as follows:

$$G_{II} = \frac{\tau_N(y_1 - y_2)}{2},$$

On the other hand, the $G_F$ is the critical fracture energy of concrete deep beam which is equal to equation (11) according to\cite{40}

$$G_F = 0.073(f_{\tau}^{0.18}),$$

Figure 1. A part of a concrete deep beam.
where $f_t$ is the tensile strength of concrete. Therefore, when $G_I + G_{II} < G_{fs}$, the stiffnesses are equal to equations (1) and (2). When $G_I + G_{II} > G_{fs}$, the crack is propagated, the interface element is removed, and equations (1) and (2) become zero. In addition, to find failure stress, ultimate shear stress by Thomas et al [34] was used. Equations (5)–(10) were implemented as a User-Element subroutine to model interface element, and crack propagation criterion was applied with the Material library. The flowchart of the present numerical model, experimental model, and ABAQUS for analysis of control deep beam are shown in figure 2. It is worth mentioning that a correct estimation of energy release rate is important to predict nonlinear crack propagation in concrete structures such as deep beams [14]. The nonlinear fracture mechanics theory is different from the nonlinearity of structure. It is now well known that in order to accurately model cracking behavior of structures such as deep beams, the fracture process zone (FPZ) must be properly modeled to consider the gradual energy dissipation during cracking.

A numerical method was developed to model crack propagation based on the mentioned new strain energy release rates in the reinforced concrete deep beam which is flexural–strengthened with CFRP [41, 42]. Furthermore, in this study, a computer code based on work by Shahbazpanahi et al [43] was developed specifically to model shear-strengthening of the reinforced concrete deep beam by modifying the crack propagation criteria.

A 2D plane stress FEAPpv was used to study nonlinear fracture mechanics where four-node isoparametric elements were applied to model bulk concrete with isotropic behavior. Truss elements with elastic behavior were used to model the soffit CFRP. The side face CFRP had linear elastic behavior and was simulated by four-node isoparametric elements. Longitudinal steels were modeled by truss elements with elastic–perfect plastic behavior. The bond-slip between longitudinal steels and concrete were assumed completely. To model slip between CFRP and concrete, the constitutive model obtained by Nakaba et al [44] was used. To find the load-displacement curve of the deep beams obtained by FEAPpv, load was assumed to be incremental rather than displaced. Therefore, load-controlled condition was used.
2.1.2. Numerical model based on LEFM

To compare with the experimental results and FEAPpv based on nonlinear fracture mechanics, the ABAQUS software was used to model crack propagation by conventional cohesive elements. ABAQUS software can only model the crack propagation based on linear fracture mechanics and is not capable of simulating nonlinear fracture mechanics. The concrete parameters used for deep beam based on LEFM in ABAQUS are given in Table 1.

Three elements were applied to simulate deep beams by ABAQUS: truss, shell, and solid. Truss elements with elastic-perfect plastic behavior were applied for steel bars because this element can hold just axial while shell elements with linear elastic behavior were used for modeling CFRP plates. Solid elements with plastic behavior were used to model concrete. As mentioned, the simplified flowchart for analysis of control deep beam by ABAQUS of the tests conducted are shown in Figure 2.

2.2. Experimental test of control and CFRP deep beams

Experimental tests were conducted under static monotonic load to validate the simulation results. Results from both models were compared with the experimental results of the three deep beams.

2.2.1. Specimens

Figure 3 illustrates the deep beam dimensions which were 500 mm in depth, 150 mm in width, and 900 mm in length. The geometry supports and details of the deep beams are also shown in Figure 3. The deep beams were reinforced longitudinally with two 16 mm diameter steel rebars at the bottom and two 12 mm diameter steel rebars at the top with 30 mm clear concrete cover. The deep beams were not reinforced transversally to guarantee the shear failures. Only two stirrups were used out of the shear zone to hold the longitudinal reinforcement in place as shown in Figure 3. The yield strength of the bars was 400 MPa based on the manufacturer’s report.

The average concrete compressive strength was 28 MPa (at 28 days) based on testing three cylinders with a diameter of 150 mm and height of 300 mm. Table 2 shows a mix design for the concrete used to cast the deep beams. The mechanical properties of the CFRP plates were found according to ASTM D7565 [43]. The properties of the CFRP plates and steel reinforcement are given in Table 3. B-0, B-1, and B-2 were used to designate the control deep beam, the flexural strengthened and the shear strengthened deep beams, respectively. All the deep beams had the same steel reinforcement as shown in Figure 2.

Three plywood formworks were built to cast the deep beams. Following casting, the beams were tested after 28 days. The deep beams were simply supported and tested under two equal concentrated loads. For the B-1 deep beam, the CFRP plate was externally bonded at the bottom of the beam. For the B-2 deep beam, the CFRP plate was bonded into both sides of the deep beam in the shear span. Although the CFRP shear-strengthened plates were not fully wrapped, they were interrupted at 100 mm from the top of the beam.

After demolding, all beams were cured and covered with wet burlap at 20°C until 28 days, and then the CFRP plates were installed with resin. Before strengthening the specimens with CFRP plates, the concrete surface was cleaned to remove any surface grease. The surfaces of the specimens were further cleaned by grinding and brushing. An epoxy adhesive was uniformly applied in a thin layer on the bonding surfaces. The CFRP was placed over it and pressed firmly by plastic rollers. External strengthening with single-layer CFRP plates had a thickness of 0.8 mm and an elastic modulus of 250 GPa. The ultimate tensile stress of the CFRP plates was 210 MPa.

2.2.2. Instrumentation

A hydraulic jack with 800 kN capacity was used on the deep beam. The mid-span displacements (Figure 4) were recorded by Linear Variable Displacement Transformers (LVDTs). Two strain gauges were attached directly under the CFRP plates and concrete below the load to observe strain. The supports were placed at the bottom of the concrete beams. The applied loads were measured by a load cell. All the measurements were automatically recorded using a data logger.

| Parameter | Description | Value |
|-----------|-------------|-------|
| ε         | Eccentricity | 0.1   |
| ψ         | Dilation angle (degree) | 56    |
| μ         | Viscosity Parameter | 0.0001 |
| K         | The second stress invariant/tensile meridian | 0.66  |

Table 1. ABAQUS Parameters of concrete deep beam based on LEFM.
2.2.3. Test set-up and technique

Figure 5 illustrates the test setup details for the control beam (B-0), the beam with flexure strengthened by CFRP (B-1), and the beam with shear strengthened by CFRP plates (B-2). The test set-up of the control deep beam included a load distribution frame (with steel I-beam) with a clear span of 900 mm, a 50 mm overhang at each side, pin support at one end, and roller support at the other end.

Table 2. Mix proportions for the concrete mix.

| Mix design | W/C | Cement kg/m³ | Water kg/m³ | Coarse aggregate kg/m³ | Fine aggregate kg/m³ |
|------------|-----|---------------|-------------|------------------------|----------------------|
| Concrete   | 0.41| 415.00        | 170         | 1000                   | 746                  |

Table 3. Properties of the CFRP plates and steel reinforcement.

| Item                                | Value     |
|-------------------------------------|-----------|
| Type of CFRP                        | bidirectional |
| Elastic modulus (GPa)               | 250       |
| Strain (%)                          | 1.09      |
| Tensile strength (MPa)              | 210       |
| Thickness (mm)                      | 0.8       |
| Epoxy resin Density (kg/l)          | 1.7       |
| Epoxy resin tensile strength (MPa)  | 14        |
| Modulus of Elasticity of epoxy resin (GPa) | 4        |
| Yield strength (MPa) of steel       | 400       |
| Ultimate strength (MPa) of steel    | 580       |
| Modulus of elasticity (GPa) of steel| 200       |

2.2.3. Test set-up and technique

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A strain gauge was mounted on the surface of the concrete to record strain in the concrete. A four-point bending test setup was used to test the deep beams. One LVDT was set to monitor the displacements under the deep beam. A load cell was utilized to measure the applied load. The test was conducted under a gradually increasing monotonic load (at a loading rate of 0.4 mm min$^{-1}$) until beam failure was reached.

### 3. Results

This section presents the validation and comparison of crack propagation obtained by the proposed model based on the nonlinear fracture mechanics theory, the experimental results, and ABAQUS software results based on LEFM.

#### 3.1. Crack propagation in the control beam (B-0)

The control beam was studied first to compare the proposed model with the experimental results. A comparison of the proposed model based on nonlinear fracture mechanics theory with the experimental results is shown in figure 6. A good agreement between the experimental results and the proposed model results based on nonlinear fracture mechanics theory was found for the load-deflection curves in the mid-span as shown in figure 6. The load-deflection curves indicated nonlinear behavior. The initial stiffness obtained by the nonlinear fracture mechanics model coincides with the stiffness of the beam at the elastic zone, as predicted by the experimental results. The initial stiffness obtained by the linear elastic fracture mechanics model by ABAQUS was overestimated and had significantly higher stiffness in the mid-span. For the deep beams simulated based on LEFM by ABAQUS, at the same load level, displacements were lower compared to the experimental results.

The results of the proposed model based on nonlinear fracture mechanics theory provided an accurate steel yield load point compared with the experimental results. Thus, the nonlinear fracture mechanics model can detect yield stress in the steel with reasonable accuracy. The most obvious effect of the nonlinear fracture mechanics model on deep beam response can be seen in the plastic zone. The proposed model corresponds closely to the experimental results in the plastic zone. Based on the nonlinear fracture mechanics model, the failure load was 350.7 kN, whereas in the experimental result, this figure was 331.2 kN. Failure load based on the LEFM model by ABAQUS was 401.3. Thus, the nonlinear fracture mechanics model can predict the failure load more accurately. Figure 7 displays the crack paths of the control beam, which were modeled using the nonlinear fracture mechanics model. Only half of the beam was modeled by considering symmetry. At mid-span, one flexural crack was predicted with 350 mm length perpendicular to the axis of the control beam. Furthermore, it can be observed from figure 10, there is only one flexural crack under the load. The nonlinear fracture mechanics model was predicted one flexural-shear crack at shear span and two shear cracks near the support.

Figure 8 illustrates the crack path through the experimental test. This crack pattern could be compared with the predicted result shown in figure 7. The nonlinear fracture mechanics model predicted three cracks within the shear span compared with the three cracks observed in the experimental test. Shear span is the distance between

![Figure 4. Details of the loading system and measurement schemes.](image)
a support and the nearest load point. The nonlinear fracture mechanics model observed one big and one small flexural crack within the flexural zone whereas only one crack was observed in the experimental results. Flexural span is the distance between the point loads. The agreement between the crack paths obtained in the nonlinear fracture mechanics model and the experimental test is sufficient to justify the validity of the nonlinear fracture mechanics model. The crack patterns obtained by the nonlinear fracture mechanics model and experimental results were propagated to the last quarter of the section height. Figure 9 shows the crack paths of the control beam simulated using the ABAQUS software based on LEFM. The shear crack near the support cannot be modeled by conventional ABAQUS software. Therefore, the nonlinear fracture mechanics model predicts crack propagation more objectively compared with the ABAQUS software. As shown in figures 7, 8, and 9, the model of control deep beam works better with FEAPpv based on nonlinear fracture mechanics than ABAQUS based on LEFM.

Figure 5. Deep beams set up before test: (a) control, (b) flexural, and (c) shear–strengthened.
Figure 6. Load-deflection curves of control deep beam (B-0).

Figure 7. Crack path obtained by nonlinear fracture mechanics model (B-0).

Figure 8. Crack path observed by experimental test (B-0).

Figure 9. Crack path by ABAQUS software (B-0).
Figure 10 compares the load versus the mid-span deflection of the deep beam obtained by the proposed model based on nonlinear fracture mechanics theory, the results of the experimental test, and the ABAQUS software. The results of the proposed model are consistent with the results of the nonlinear fracture mechanics. The stiffness of the deep beam predicted based on the LEFM had smaller differences compared with the experimental results. However, the stiffness of the deep beam predicted based on ABAQUS is greater than the experimental results' stiffness. The steel yield load predicted by LEFM had a difference of approximately 4% compared to the experimental result.

However, the steel yield load by ABAQUS software had greater error (31%) compared to the experimental result, which indicates that it is less capable of estimating the steel yield load. Load failure in the proposed model was predicted within a different range of 5.6% to 8% compared to that in the experimental results. The load-deflection curve of the nonlinear fracture mechanics model was similar to the experimental results. However, the load-deflection curve by ABAQUS software was higher than the experimental curve. The ABAQUS software results had a greater amount of difference (29.9%–39.4%) compared with the experimental results. The proposed model based on the nonlinear fracture mechanics model could be used to perform analysis of reinforced deep beam flexural–strengthened by CFRP plates. Figure 11 shows the comparison of crack patterns between the proposed model, the experiment, and the ABAQUS software in the deep beam with flexure strengthened by CFRP plates.

The predicted cracking pattern of the proposed model was generally consistent with the experimental observations. In both cases, the flexural cracks were closely spaced near the beam mid-span as shown in figures 11(a) and (b). Within the shear span, the model predicted two shear–flexural cracks, whereas only one crack was observed in the experiment (left shear span) in half of the beam. The model predicted one shear crack near the support, but no crack was observed in the test. The shear cracks could have been too small to be noticed in the experimental test. As shown in figure 11(c), the ABAQUS software could not simulate this crack pattern because neither the accurate stiffness of the FPZ nor the crack propagation criterion was considered in the software. A flexural crack was observed by the nonlinear fracture mechanics model in the control beam near the mid-span at a load level of 280 kN. However, the crack of the beam strengthened in flexure occurred at a relatively higher load level (300 kN) than that for the control beam. The observed flexural crack load for the flexural strengthened deep beam was approximately 300 kN based on the nonlinear fracture mechanics and the experimental results. The deep beam strengthened in flexure showed approximately 2% shorter flexural crack length compared to the control beam. As expected, the behavior of the flexural strengthened deep beam with CFRP plates was slightly changed compared to the control deep beam. Finally, the beam failed because of shear failure with a large shear crack.

3.2. Crack propagation in the shear–strengthened deep beam (B-2)

The load-deflection curves obtained from the experimental test, the nonlinear fracture mechanics model, and the ABAQUS software for the deep beam with shear strengthened by CFRP plates are shown in figure 12. The results of the proposed model are close to that of the experimental one. This finding indicates that the nonlinear fracture mechanics model is validated by the test results. The yield point of the load-deflection curve in the nonlinear fracture mechanics model is similar to that in the experimental test result (approximately a 3% difference). However, this point obtained in the LEFM simulations by ABAQUS was higher than that in the experimental test result (approximately a 21% difference). The accuracy of the proposed model was also
confirmed by the close value of the failure load obtained from the proposed model and the test (approximately 2\% difference at 4 mm deflection). Furthermore, the stiffness of the beam with shear strengthened by CFRP as analyzed by the ABAQUS software was over-estimated.

Figure 13 shows the crack paths obtained by the proposed model, the experimental results, and the cracks predicted in the FEA by ABAQUS. A good agreement was observed for the crack paths predicted by the proposed model compared with the experimental results as shown in figures 13(a) and (b). Figures 13(a) and (b) demonstrate that only one flexural crack initiated and propagated towards the loading point. Only one
shear–flexural crack was observed near the CFRP plate. The shear cracks stopped initiation and propagation in the shear span because the shear strengthened with CFRP plates as reported by other works [2, 5, 13]. The flexural crack began to appear at a load of approximately 240 kN. The flexural–shear cracks propagated from the mid-depth of the beam toward the point of the applied load. As the load increased, the flexural–shear crack propagated to the final failure.

A comparison of the shear cracks in the beam strengthened in shear [see figure 13(a)] and the control beam [see figure 8] shows that there are more shear cracks in the control beam. The use of CFRP plates affected and delayed the propagation of the shear crack. Figure 13(c) indicates the crack paths obtained by ABAQUS based on the LEFM. By comparing figures 13(a), (b), and (c), the nonlinear fractures mechanics model for the shear–strengthened deep beam with CFRP is shown to be comparatively better than the LEFM by ABAQUS.

Mesh sensitivity study of the nonlinear fracture mechanics model was presented in terms of load–deflection in figure 14. Mesh (a) is a coarse mesh with 260 elements and 440 interface elements. Mesh (b) had 780 elements and 1132 interface elements as a medium mesh. Fine mesh (Mesh (c)) had 1278 elements and 1896 interface elements. The prediction of load versus deflection at midspan in the three meshes was close to each other. Therefore, the dependency of mesh size does not significantly influence the overall load–deflection curves.

Figure 15 shows the effect of shear strengthened by CFRP on crack propagation. Only one small crack formed under the CFRP plate as shown in figure 15. CFRP plates delay and control cracking in the beam and confine the crack propagation of concrete and this behavior has been reported by other researchers [20, 45, 46]. Shear strengthened by CFRP plate reduces the width of cracks in concrete. Figure 15 shows that crack was not observed in the shear span because of the presence of CFRP plates. The crack length was restrained as well.

In table 4, some factors such as accuracy of predicted initial stiffness, yielding point (based on the same load), plastic zone, and failure point for each model were compared with the experimental results. The predicted values of initial stiffness, yielding point, and failure point obtained by non-linear fracture mechanics were very close to the experimental results. However, the ABAQUS software results based on LEFM had a greater discrepancy. For instance, the steel yielding point in the control deep beam was predicted by non-linear fracture mechanics within a difference of 1.2%. Furthermore, the steel yield load by ABAQUS software based on LEFM displayed a higher difference (13%) compared to the control beam, which indicates that it was less capable of estimating the steel yield load. The failure loads based on the non-linear fracture mechanics and the experimental results for the shear strengthened deep beam were approximately 485 and 498 kN corresponding respectively to mid-span displacements of 4.1 and 4.5 mm. However, these values for the model based on LEFM were equal to 510 kN with mid-span deflections of 3.7 mm. For the shear strengthened deep beam (B-2), delamination of the CFRP plates was evident based on the non-linear fracture mechanics model and the experimental results. However, the model based on LEFM showed that failure occurs because of the rupture of the CFRP plates on the shear span.

In table 5, the energy absorption (the area under the load–deflection curve) is listed for each deep beam at failure load. The calculated energy absorptions for the control deep beam were 987, 1029, and 1154 kN mm based on the experimental test, the proposed model, and the LEFM results, respectively. The energy absorption for the control deep beam obtained from the experimental results was slightly lower than that of the proposed model. Moreover, for the strengthened deep beams in shear, the energy absorption obtained by LEFM was much higher than that of the experimental results and the proposed model. The shear strengthened deep beam showed an increase in the energy absorption up to 67.1% compared to the control deep beam. The experimental
compressive strain for the shear-strengthened deep beam was greater than the control deep beam compressive strain due to shear strengthening. The compressive strain reported for the shear-strengthened deep beam was approximately 0.00318. In both models and the experimental results, the control deep beam had the lowest concrete compressive strain compared to the other two deep beams that were also reported in other works [2, 5]. The shear strengthened deep beam simulated by the proposed model showed an increase in the compressive strain of 1.2% compared to the experimental results, while this increase was 10.4% for the deep beam modeled by LEFM. Therefore, the nonlinear fracture mechanics model demonstrated better performance for estimating concrete compressive strain compared to the LEFM model.

Here, it is worth mentioning that the concept of FPZ in the nonlinear fracture mechanics theory is broadened to one of interface process zone, which includes the nonlinear behavior of the fracture process.
Based on [1, 6, 14, 22], there is not any numerical model to simulate nonlinear crack propagation in deep beams. Therefore, nonlinear fracture mechanics models should be used to model crack propagation of deep beams to achieve greater accuracy. A new computational approach to implement finite element method to study crack initiation and growth in the deep beam was presented and validated. The novelties of the present study are that strain energy release rates of cracks in the deep beams can be calculated simultaneously as FEA is performed and nonlinear crack propagation can also be directly analyzed. To do so, three reinforced concrete deep beams strengthened by Carbon Fiber-reinforced PolymeCFRP plates was analyzed as an example of the use of the new finite element implementation of the nonlinear crack model propagation. As a result, positive contributions were found for the finite element simulations and the predicted values of initial stiffness, yielding point and failure load, energy absorption, and compressive strain in the concrete obtained by the proposed model were very close to the experimental results.

4. Conclusions

In the present study, two models for reinforced concrete deep beam strengthened with CFRP plates to predict the fracture behavior were discussed. One of the models was based on nonlinear fracture mechanics by FEAPpv. Furthermore, a new energy release rate was formulated. The DCZM was developed and the crack propagation criteria was proposed. The second model was based on linear elastic fracture mechanics by ABAQUS software. Experimental testing on reinforced concrete deep beams strengthened with CFRP plates was carried out to compare with the models. The proposed model can reasonably predict the experimental results in terms of stiffness, steel yielding load, plastic zone, and failure load. However, the ABAQUS results for the aforementioned parts were in great contrast to the experimental results. Furthermore, the nonlinear fracture mechanics by FEAPpv accurately predicted compared to the linear elastic fracture mechanics model by ABAQUS. For
Table 4. Comparison of both models with experimental results (at the same load).

| Deep beam          | Model            | stiffness (%) | Yielding (%) | Plastic zone (%) | Failure load (%) | Failure mode by models | Failure mode by experimental |
|--------------------|------------------|---------------|--------------|------------------|------------------|------------------------|-----------------------------|
| Control (B-0)      | Proposed model   | 2–6           | 1.2          | 9–11             | 6.3              | Shear                  | Shear                       |
|                    | LEFM             | 12–19         | 40           | 16–24            | 23.6             | Shear                  |                             |
| Flexural Strengthening (B-1) | Proposed model | 1–4           | 4            | 5–7              | 5.6–8.0          | Shear                  | Shear                       |
|                    | LEFM             | 10–16         | 31.1         | 14–20            | 29.9–39.4        | Shear                  |                             |
| Shear Strengthening (B-2) | Proposed model | 6–10          | 3            | 1–3              | 2.6              | Delamination           | CFRP Delamination           |
|                    | LEFM             | 22–35         | 21           | 34–53            | 25.1             | CFRP rupture           |                             |
example, in the control deep beam, the most obvious effect of the nonlinear fracture mechanics model on deep beam response can be seen in the plastic zone. The proposed model corresponds closely to the experimental results in the plastic zone.

Furthermore, the failure load of the shear-strengthened deep beam was predicted by the proposed model with a 6.3% difference compared to the experimental results whereas the failure load predicted by the ABAQUS software based on LEFM indicated a greater difference of 25.1% with the experimental results. The proposed model captured the crack patterns of the strengthened reinforced concrete deep beam with CFRP plates quite satisfactorily compared to the experimental observations. For the shear strengthened deep beams, the energy absorption obtained by LEFM was much greater than those of the experimental results and the proposed model. Furthermore, the shear strengthened deep beam simulated by the proposed model showed an increase in the compressive strain of 1.2% compared to the experimental results, while this increase was 10.4% for the deep beam modeled by LEFM. Therefore, the nonlinear fracture mechanics model is a reasonable choice for simulating the fracture mechanics of the reinforced concrete deep beam strengthened with CFRP plates. Lack of an internal stirrup in the desired shear failure region of the deep beams and the neglect of the behavior of concrete in the compression zone and crushing constitute the limitations of the proposed model.

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Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

Author Contributions

Conceptualization, S.S., H.K., W. A. and A. M.; methodology, S.S., H. K., S.S. and W. A.; software, S.S., H. K., S.S. and W. A.; validation, A.M.; formal analysis, S.S., H. K., S.S. and W. A.; investigation, S.S., H. K., S.S. and W. A.; resources, U.R., and A.M.; data curation, S.S., H. K., S.S. and W. A.; writing—original draft preparation, S.S., H. K., S.S. and W. A.; writing—review and editing, S.S., H. K., S.S., W. A., U.R., and A.M.; visualization, S.S., H. K., S.S. and W. A.; supervision, U.R. and A.M.; project administration, A.M.; funding acquisition, A.M. All authors have read and agreed to the published version of the manuscript.

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