An empirical strength criterion for deep rock incorporating the effect of fracture intensity using distinct element method

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Abstract. Deep rock strength is one of the most important research areas in rock mechanics and engineering which is closely related to bearing capacity and stability of underground structures. Due to high ground stress, the mechanical behaviours of deep rock are different from those of shallow rock. In order to characterize deep rock strength precisely, a great number of rock strength criterions have been proposed over the past few decades to quantitatively describe the relationship between complex stress state and rock strength parameters. Some microscopic characteristics like fracture intensity ($P_{32}$) have received some attention. They are thought to have significant influences on the strength of deep rock. Currently, most of the constitutive models and strength criterions focused on the shallow rock and macro mechanical parameters, and they can not directly explain the relationship between the microscopic fracture characteristics and the strength of deep rock. In this research, the effects of fracture characteristics and confining pressure on deep rock strength were investigated using distinct element method (DEM). Firstly, an improved bond contact model incorporating rolling and twisting resistances was implemented in the rock numerical specimens, and the Smooth-Joint contact model was used for simulating fractures of rocks. The fracture length distributions in specimens were characterised by an exponent parameter $a$. Then numerical simulation of triaxial compression tests were carried out on numerical rock specimens under different confining pressures and $P_{32}$. The results show an approximate linear relationship between the maximum principal stress and $P_{32}$. Rock strength decreases with the increase of $P_{32}$, and it decreases rapidly at a higher confining pressure. Based on the results, a new empirical strength criterion is proposed. The criterion considers the nonlinear relationship between maximum principal stress and confining pressure of deep rock, additionally reveals the correlation between microscopic fracture characteristics and rock strength. In principle stress space, the corresponding failure envelopes projected onto the $\pi$-Plane.
showed a good agreement with DEM simulation results, which indicates a good applicability for the prediction of deep rock strength.

1. Introduction

With the consumption of energy, mining engineering is going deeper as the existing shallow reserves are depleted\(^1\). Due to complicated geology environments like high ground stress, high earth temperature and high karst water pressure, the mechanical properties of deep rock are different from shallow rock. The present research on the nonlinear mechanical characters of deep rock showed the classic theories and methods for rock strength were partly not applicable\(^2\). In deep mining engineering, rock strength is closely related to the stability and bearing capacity in rock foundations and underground structures. In addition, joint distribution characteristics have great impact on the strength of rock\(^3\). Therefore, it is important to focus on the study of strength criterion and propose a criterion for deep jointed rock. Different stress state and mineral composition make the various failure characteristics of rock. In the past few decades, a lot of strength criterions were put forward based on laboratory tests or numerical simulations considering different failure mechanism. Hoek and Brown\(^4\) established an empirical strength criterion for intact rock based on Griffith failure criterion\(^5\) and a wide range of experiment data. The criterion used uniaxial compressive strength as a scaling parameter to describe the response of intact rock to the stress condition, and was widely applied to the preliminary design of slopes and mining excavation. In following studies, the geological strength index (GSI)\(^6\)-\(^8\) and rock mass classifications\(^9\) were adopted respectively to obtain the constants which were defined by Hoek-Brown criterion. Although Hoek-Brown criterion showed good applicability under low confining pressure, it is not applicable for deep rock in a state of relatively high confinement, and it assumes that the intermediate principle stress has a negligible influence on failure condition\(^10\). Considering nonlinear strength characteristics under high confining pressure, other criterions were proposed for deep rock. The empirical Bieniawski criterion\(^11\) with three parameters showed a small fitting deviations with experiment data and was considered good applicability for deep rock under high confining pressures\(^12\). Besides, You\(^13\) proposed a new strength criterion considering the impact of intermediate principle stress and construct an exponential form of equations for true-triaxial strength. This criterion also showed good agreement with experimental data for rocks under high confining pressures\(^12\). Furthermore, after a twin stress criterion proposed by Yu\(^14\), the criterion was then extended to a unified strength theory\(^15\) which considered the influences of all stress components on strength of materials and implemented cohesion parameters in terms of inner friction angle for the application in rock and soil materials. Based on the nonlinear twin shear strength criterion, Zan\(^16\) proposed a nonlinear unified strength criterion and generalized this criterion to jointed rock. In addition, considering the impact of microcracks, damage theory was implemented into rock strength criterion\(^17\). By adopting Mohr-Coulomb criterion, Cao\(^18\) proposed a damage constitutive model for strain-softening rock which contained conventional mechanical parameters and reflect the influences of confining pressure. After that, in order to describe residual strength for rock, another damage-softening statistical constitutive model based on Drucker–Prager and the Mohr–Coulomb criteria was proposed by Wang\(^19\) which took account of degradation of Young’s modulus due to damage behaviours. However, most current constitutive models and strength criterions were not suitable for deep rock due to nonlinear strength characteristics and high confining pressures. Moreover, previous studies concentrated more on the macro strength parameters, which had difficulties in describing microscopic fracture characteristics.

Distinct element method(DEM)\(^20\) proposed by Cundall has significantly showed advantages in microscopic simulations. Therefore, in this study, numerical simulations of triaxial compression tests were carried out using DEM to investigate the relationship between deep rock strength, fracture intensity\((P_{32})\) and high confining pressure. Furthermore, a new empirical strength criterion with parameter of \(P_{32}\) and confining pressure was proposed for deep jointed rock based on the numerical triaxial results.
2. Modelling of deep rock and joints

An improved three-dimensional (3D) bond model incorporating rolling and twisting resistances was used for numerical rock specimens\cite{21}. For DEM simulation of rock joints, fracture networks were build up using Smooth-Joint Contact Model(SJM). In order to generate homogeneous samples, the Multi-layer with Undercompaction Method(UCM) proposed by Jiang\cite{22} was employed in this research.

The numerical samples have a dimension of \( L=8.71\,\text{mm}, B=8.71\,\text{mm}, H=8.51\,\text{mm} \), and contain a total number of 160000 particles with the mean particle diameter \( d_{50} \) being \( 1.74\times 10^{-4}\,\text{m} \). The grain-size distribution adopted in this study is shown in figure 1. Table 1 list the parameter values of numerical rock specimens.

Fracture model parameters as shown in table 2 are calculated by:

\[
A_{50} = \pi \left( \frac{d_{50}}{2} \right)^2 = 2.378 \times 10^{-8}(m^2) \tag{1}
\]

\[
k_{n,j} = \frac{EA_{50}}{d_{50}} = 4.10 \times 10^9 (N/m) \tag{2}
\]

\[
k_{s,j} = \frac{k_{n,j}}{\kappa} = 2.73 \times 10^6 (N/m) \tag{3}
\]

where \( A_{50} \) is the contact area of the mean particle, \( E \) the particle effective modulus, \( \kappa \) normal-to-shear stiffness ratio, \( k_{n,j} \) normal and shear stiffness per unit area. Cementation and dilatancy are not considered in fracture model.

Dershowitz\cite{23} defined fracture intensity\( (P_{32}) \) as fracture area per unit volume and in this paper, the values of \( P_{32} \) varied from 100–600 \( \text{m}^2/\text{m}^3 \). To describe the spatial and length distributions of fracture networks, Bour and Davy presented a double power law to express the density fracture length distribution\cite{24,25}. The equation was simplified as follows:

\[
n(l) = l^{-a} \tag{4}
\]

where \( n(l) \) is the number of fractures with a length of \( l \), \( a \) the exponent of the frequency distribution of fracture lengths. The larger value of \( a \) leads to the larger proportion of small-size fractures. In this paper, the values of \( a \) are 2, 3 and 4, maximum principle stresses were obtained by the average amounts under different values of \( a \).

\[\text{Figure 1. Grain-size distribution curves and numerical specimen for DEM simulations.}\]

\[\text{Table 1. Rock model parameters used in numerical tests}\]

| Parameters                  | Value       |
|-----------------------------|-------------|
| Density \( \rho (\text{kg/m}^3) \) | \( 2.65 \times 10^4 \) |
| Particle effective modulus \( E (\text{Pa}) \)  | \( 3.0 \times 10^{10} \) |
| Normal-to-shear stiffness ratio \( \kappa \)      | 1.5         |
| Friction coefficient \( \mu \)                    | 0.3         |
| Local damping coefficient     | 0.7         |
| Bond effective modulus \( E_b (\text{Pa}) \)    | \( 1.35 \times 10^{10} \) |
| Bond normal-to-shear stiffness ratio \( \kappa_b \)| 1.5         |
| Bond tensile stress \( \sigma_t (\text{Pa}) \) | \( 3.5 \times 10^{7} \) |
Bond compression stress $\sigma_c$ (Pa)
Bond radius multiplier $\lambda$
Bond thickness $g_c$ (m)

1.4×10^9
1.0
1.0×10^{-5}

| Normal stiffness per unit area $k_{n,j}$ (N/m) | Shear stiffness per unit area $k_{s,j}$ (N/m) | Friction coefficient $\mu_j$ | Dilation angle (°) | Tensile strength (MPa) | Cohesion (MPa) |
|---------------------------------------------|---------------------------------------------|-----------------------------|-------------------|-----------------------|---------------|
| 4.10×10^6                                  | 2.73×10^6                                  | 0.3                         | 0                 | 0                     | 0             |

3. Empirical strength criterion based on numerical triaxial compression tests

Under different confining pressures, maximum principle stress decreases with the increase of $P_{32}$. After calculating the average value of maximum principle stress when $a=2$, 3 and 4, straight lines were used to fit the data points of stress as shown in figure 2. It is obvious that the two parameters have a good linear relationship. The maximum principle stress decreases against $P_{32}$, while it increases against confining pressure. Under different confining pressures, the fitting relationship can be expressed as a unified form:

$$\sigma_c = b_\sigma - k_\sigma P_{32} \quad (5)$$

where $b_\sigma$ and $k_\sigma$ are the intercept and slope of the fitting line respectively. Note that the value of constant term $b_\sigma$ in equation (5) is only related to confining pressure regardless of $P_{32}$ and it presents strength of whole rock without fractures.

Figure 2. Fitting relationship of maximum principal stress and fracture intensity under different confining pressures.

Figure 3 illustrates the values of slopes $k_\sigma$ in figure 2 against confining pressures. It is observed that, at higher confining pressures, the fitting lines of maximum principle stress exhibit higher slopes. And it shows a general linear relationship between slopes and confining pressures, as described in the following:

$$k_\sigma = 0.078 + 0.002\sigma_3 \quad (6)$$
Figure 3. The slopes of fitting line under different confining pressures. The intercepts $b_\sigma$ of the lines in figure 2 versus confining conditions are given in figure 4. Apparently, the intercepts increase nonlinearly against confining pressures. The value of intercept when the confining pressure is zero represents the uniaxial compressive strength of rock specimen without fractures. The data points can be well fitted by the following expression:

$$b_\sigma = 78 + 12\sigma_3^{0.623} = \sigma_c + 12\sigma_3^{0.623}$$  \hspace{1cm} (7)

where $\sigma_c$ is uniaxial compressive strength of rock.

Figure 4. The intercepts of fitting line under different confining pressures.

Finally, after substituting equation (6) and equation (7) into equation (5), the empirical strength criterion incorporating the effect of $P_{32}$ can be defined as:

$$\sigma_i = b_\sigma - k_\sigma P_{32} = (\sigma_c + 12\sigma_3^{0.623}) - (0.078 + 0.002\sigma_c)P_{32}$$ \hspace{1cm} (8)

4. Conclusions
In this paper, numerical triaxial compression tests of deep rock under high confining pressures were carried out by DEM. Based on the results, a nonlinear empirical strength criterion for deep rock was proposed. The main conclusions are summarized as follows:

(1) Maximum principal stress of deep rock shows a nonlinear relationship with confining pressure, in which it is higher under a larger confining pressure. While it decreases linearly with $P_{32}$. When $P_{32}$ is zero, the criterion can be used for intact rock specimens without fractures.

(2) On the basis of strength results from numerical triaxial compression tests, an empirical strength criterion was proposed. The criterion takes uniaxial compressive strength, $P_{32}$ and confining pressure as three scaling parameters which can express the relationship between macro strength and micro fracture characteristics under different confining conditions.

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