DARK ENERGY, DARK MATTER AND FERMION FAMILIES IN THE TWO MEASURES THEORY

E. I. GUENDELMAN AND A. B. KAGANOVICH

Physics Department,
Ben Gurion University of the Negev,
Beer Sheva 84105, Israel
E-mail: guendel@bgumail.bgu.ac.il and alexk@bgumail.bgu.ac.il

A field theory is proposed where the regular fermionic matter and the dark fermionic matter are different states of the same "primordial" fermion fields. In regime of the fermion densities typical for normal particle physics, each of the primordial fermions splits into three generations identified with regular fermions. In a simple model, this fermion families birth effect is accompanied with the right lepton numbers conservation laws. It is possible to fit the muon to electron mass ratio without fine tuning of the Yukawa coupling constants. When fermion energy density becomes comparable with dark energy density, the theory allows new type of states - Cosmo-Low Energy Physics (CLEP) states. Neutrinos in CLEP state can be both a good candidate for dark matter and responsible for a new type of dark energy. In the latter case the total energy density of the universe is less than it would be in the universe free of fermionic matter at all. The (quintessence) scalar field is coupled to dark matter but its coupling to regular fermionic matter appears to be extremely suppressed.

1. Introduction

The existence of the lepton and quark generations is one of the greatest puzzles of the particle physics. Resolution of this puzzle implies an explanation of the origin of the mass spectrum of the elementary fermions and observable flavor properties of their electroweak interaction. In this talk side by side with the fermion families problem we will study the dark matter and dark energy puzzle. It turns out that these, for the first glance, absolutely different problems are very much connected in the framework of the so called Two Measures Theory (TMT).

Understanding of the nature of the dark matter and dark energy should be the basis for explanation of the cosmic coincidence. A promising approach towards solving the cosmic coincidence problem in the scenario of an accelerating expansion for the present day universe was developed in
the variable mass particles (VAMP) models\textsuperscript{1}. However such a modification of the particle physics theory underlying the quintessence scenarios has a fundamental problem: although there are some justifications for choices of certain types of dark matter-dark energy coupling in the Lagrangian, there is a necessity to assume the absence or extremely strong suppression of the barion matter-dark energy coupling. Actually this problem was known from the very beginning in the quintessence models since generically there are no reasons for the absence of a direct coupling of the quintessence scalar field $\phi$ to the barion matter. Such coupling would be the origin of a long range scalar force because of the very small mass of the quintessence field $\phi$. This "fifth-force" problem might be solved\textsuperscript{2} if there would be a shift symmetry $\phi \rightarrow \phi + \text{const}$ of the action. However the quintessence potential itself does not possess this symmetry. The situation with the fifth-force problem becomes still more critical in the discussed above models since one should explain now why the direct quintessence-dark matter coupling is permissible in the Lagrangian while the same is forbidden for the barion matter.

Modifications of the particle physics models in Refs.\textsuperscript{1} are based on the assumption that all the fields of the fundamental particle theory should be divided into two large groups: one describing detectable particles (ordinary matter) and the other including dark matter particles. The main purpose of this talk is to point out that TMT is able to propose a resolution for the above problems by an absolutely new way: \textit{the dark matter is not introduced as a special type of matter but rather it appears as the solution of equations of motion describing a new type of states of the (primordial) neutrino field}; in other words, \textit{the dark neutrino matter and the regular neutrino generations (electron, muon and $\tau$ neutrinos) are different states of the same primordial fermion field}.\textsuperscript{3}

TMT has been originally constructed with the aim to solve the "old" cosmological constant problem\textsuperscript{3}. TMT is a generally coordinate invariant theory with the action of the following general form

$$S = \int L_1 \Phi d^4x + \int L_2 \sqrt{-g} d^4x$$

(1)

where $\Phi$ is a scalar density built of four scalar fields $\varphi_a$ ($a = 1, 2, 3, 4$)

$$\Phi = \varepsilon^{\mu\nu\alpha\beta} \varepsilon_{abcd} \partial_\mu \varphi_a \partial_\nu \varphi_b \partial_\alpha \varphi_c \partial_\beta \varphi_d$$

(2)

and $L_1$, $L_2$ are independent of the measure fields $\varphi_a$.

We proceed in the first order formalism where all fields, including also vierbeins $e_{a\mu}$, spin-connection $\omega_{a\mu}^{\nu}$ and the measure fields $\varphi_a$ are indepen-
dent dynamical variables. All the relations between them are results of equations of motion. It turns out that the measure fields $\varphi_a$ affect the theory only via the scalar field $\zeta \equiv \Phi/\sqrt{-g}$ which is determined by a constraint in the form of an algebraic equation. The latter is exactly a consistency condition of equations of motion and determines $\zeta$ in terms of matter fields.

After transformation to new variables (conformal Einstein frame), all equations of motion take canonical GR form of equations for gravity and matter fields. All the novelty consists in the structure of the dilaton $\phi$ and the Higgs field effective potentials, masses and interactions of fermions as well as the structure of their contributions to the energy-momentum tensor: all these now depend on the fermion densities via $\zeta$.

2. $SU(2) \times U(1)$ gauge invariant and scale-invariant model

The TMT model we have studied in Ref. 4 possesses spontaneously broken global scale symmetry which allows to suggest a simultaneous resolution of both the fermion families problem and the fifth-force problem. The theory starts from one primordial fermion field for each type of leptons and quarks. For example, in $SU(2) \times U(1)$ gauge theory the fermion content is the left doublets and right singlets constructed from the primordial neutrino $N$ and electron $E$ fields and primordial $U$ and $D$ quark fields.

We allow in both $L_1$ and $L_2$ all the usual contributions considered in standard field theory models in curved space-time. Keeping the general structure (1), it is convenient to represent the integrand of the action in the following form:

$$\Phi L_1 + \sqrt{-g}L_2 = e^{\alpha\phi/M_p}(\Phi + b\sqrt{-g}) + \frac{1}{2}g^{\mu\nu}\phi,\mu\phi,\nu + \frac{1}{2}g^{\mu\nu}(D_\mu H)\text{D}_\nu H$$

$$- e^{2\alpha\phi/M_p} [\Phi V_1(H) + \sqrt{-g}V_2(H)] + e^{\alpha\phi/M_p}(\Phi + k\sqrt{-g})L_{fk}$$

$$- e^{\frac{\gamma}{2\alpha\phi/M_p}} [(\Phi + h_E\sqrt{-g})f_E\text{L}_L H E_R + h.c.]$$

$$- e^{\frac{\gamma}{2\alpha\phi/M_p}} [(\Phi + h_N\sqrt{-g})f_N\text{L}_L H^c N_R + h.c.]$$

$$- \frac{1}{4}g^{\alpha\mu}g^{\beta\nu} (B_{\alpha\beta}B_{\mu\nu} + W^a_{\alpha\beta}W^a_{\mu\nu})$$

Here $R(\omega, e)$ is the scalar curvature in the vierbein-spin-connection formalism; $L_{fk}$ is the fermion kinetic term including the standard interaction to the gauge fields $\bar{W}_\mu$ and $B_\mu$; $H$ is the Higgs doublet. Constants $b, k, h_N, h_E$ are non specified dimensionless real parameters of the model and we will only assume that orders of their magnitudes are not too much different.
The real positive parameter $\alpha$ is assumed to be of the order of one. For short we omit here and in what follows the primordial quarks since they appear in a form similar to the primordial leptons. The action incorporates also the dilaton field $\phi$ which provides global scale symmetry ($\Psi$ denotes all the primordial fermions):

$$e^\alpha \to e^{\theta/2}e^\alpha, \quad \omega^\mu_{ab} \to \omega^\mu_{ab}, \quad \phi_a \to \lambda_a \phi_a \quad \text{where} \quad \Pi \lambda_a = e^{2\theta},$$

$$\phi \to \phi - \frac{M_p}{\alpha} \theta, \quad \Psi \to e^{-\theta/4} \Psi, \quad \bar{\Psi} \to e^{-\theta/4} \bar{\Psi}; \quad \theta = \text{const},$$

$$H \to H, \quad \tilde{W}_\mu \to \tilde{W}_\mu, \quad B_\mu \to B_\mu.$$ (4)

In the context of cosmology, $\phi$ plays the role of a quintessence field.

In the new variables ($\phi, H, B_\mu$ and $\tilde{W}_\mu$ remain unchanged) which we call the Einstein frame,

$$\tilde{g}_{\mu\nu} = e^{2\phi/M_p}(\zeta + b)g_{\mu\nu}, \quad \tilde{e}_{a\mu} = e^{2/3 \alpha \phi/M_p}(\zeta + b)^{1/2} e_{a\mu},$$

$$\Psi_i' = e^{-2 \alpha \phi/M_p}(\zeta + k)^{1/2}(\zeta + b)^{3/4} \Psi_i, \quad i = N, E,$$ (5)

the gravitational equations take the form $G_{\mu\nu}(\tilde{g}_{\alpha\beta}) = \frac{8\pi}{3} T_{\mu\nu}^{eff}$ where $G_{\mu\nu}(\tilde{g}_{\alpha\beta})$ is the Einstein tensor in the Riemannian space-time with the metric $\tilde{g}_{\mu\nu}$ and

$$T_{\mu\nu}^{eff} = \phi_{,\mu}\phi_{,\nu} - \frac{1}{2}\tilde{g}_{\mu\nu}\tilde{g}^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta} + (D_\mu H)^{\dagger}D_\nu H - \frac{1}{2}\tilde{g}_{\mu\nu}\tilde{g}_{ab}(D_a H)^{\dagger}D_b H$$

$$+ \tilde{g}_{\mu\nu}V_{eff}(\phi, v; \zeta) + T_{\mu\nu}^{(gauge)} + T_{\mu\nu}^{(ferm,can)} + T_{\mu\nu}^{(ferm,noncan)},$$ (6)

$$V_{eff}(\phi, v; \zeta) = b \left[ M^4 e^{-2\alpha \phi/M_p} + V_1(v) \right] - V_2(v);$$ (7)

$T_{\mu\nu}^{(gauge)}$ is the canonical energy momentum tensor for the $SU(2) \times U(1)$ gauge fields sector; $T_{\mu\nu}^{(ferm,can)}$ is the canonical energy momentum tensor for (primordial) fermions $N'$ and $E'$ in curved space-time including also their standard $SU(2) \times U(1)$ gauge interactions. $T_{\mu\nu}^{(ferm,noncan)}$ is the non-canonical contribution of the fermions into the energy momentum tensor $T_{\mu\nu}^{(ferm,noncan)} = -\tilde{g}_{\mu\nu}(F_N(\zeta, v)N^iN_i + F_E(\zeta, v)E^iE_i) \equiv -\tilde{g}_{\mu\nu}\Lambda^{(ferm)}_{\text{dyn}}$.

The scalar field $\zeta$ is determined by the constraint

$$\frac{1}{(\zeta + b)^2} \left\{ (b - \zeta) \left[ M^4 e^{-2\alpha \phi/M_p} + V_1(v) \right] - 2V_2(v) \right\} = \Lambda^{(ferm)}_{\text{dyn}}$$ (8)

where

$$F_i(\zeta, v) = \frac{v f_i}{2 \sqrt{2(\zeta + k)(\zeta + b)^{3/2}}}(\zeta - \zeta_1^i)(\zeta - \zeta_2^i), \quad i = N, E.$$ (9)
\[ \zeta^{(i)} = \frac{1}{2} \left[ k - 3h_i \pm \sqrt{(k - 3h_i)^2 + 8b(k - h_i) - 4kh_i} \right], \quad i = N, E. \quad (10) \]

The \( \zeta \) depending "masses" of the primordial fermions are of the form

\[
m_N(\zeta, \nu) = \frac{v f_N(\zeta^{(N)} + h_N)}{\sqrt{2}(\zeta^{(N)} + k)(\zeta^{(N)} + b)^{1/2}}, \quad m_E(\zeta, \nu) = \frac{v f_E(\zeta^{(E)} + h_E)}{\sqrt{2}(\zeta^{(E)} + k)(\zeta^{(E)} + b)^{1/2}},
\]

where \( v \) is the VEV of the Higgs field in the unitary gauge.

### 3. Fermions in normal particle physics conditions

The simple analyze shows that at fermion energy densities corresponding to normal laboratory particle physics conditions ("high fermion density") the balance imposed by the constraint can be realized only through one of the following two ways: (I) \( F_i(\zeta, \nu) \approx 0 \Rightarrow \zeta = \zeta^{(i)}_1 \) or \( \zeta = \zeta^{(i)}_2, i = N, E; \) (II) \( \zeta = \zeta_3 \approx -b. \)

Two constant solutions \( \zeta^{(i)}_{1,2} \) correspond to two different states of the primordial leptons with different constant masses determined by Eq.(11) where we have to substitute \( \zeta^{(i)}_{1,2} \) instead of \( \zeta \). We identify these two states of the primordial leptons with the mass eigenstates of the first two generations of the regular leptons. If the free primordial electron is in the state with \( \zeta = \zeta_1^{(E)} \) (or \( \zeta = \zeta_2^{(E)} \)) it is detected as the regular electron \( e \) (or muon \( \mu \)) and similar for the electron and muon neutrinos with masses respectively:

\[
m_{\nu_e}(\nu_e) = \frac{v_0 f_N(\zeta^{(N)}_{12} + h_N)}{\sqrt{2}(\zeta^{(N)}_{12} + k)(\zeta^{(N)}_{12} + b)^{1/2}}; \quad m_{\nu_\mu}(\nu_\mu) = \frac{v_0 f_E(\zeta^{(E)}_{12} + h_E)}{\sqrt{2}(\zeta^{(E)}_{12} + k)(\zeta^{(E)}_{12} + b)^{1/2}}.
\]

One can show that the model provides right flavour properties of the electroweak interactions, at least for the first two lepton generations.

Turning now to the old problem of the ratio \( m_\mu/m_e \approx 207 \) one can notice at once that the fitting of the Yukawa couplings has no relation to this problem because electron and muon emerge as different states of the same primordial field \( E' \). However, our TMT model opens a possibility to get the desirable ratio \( m_\mu/m_e \) by means of a soft enough restrictions on the dimensionless parameters \( b, k \) and \( h_E \). We will assume in what follows that

\[ b > 0, \quad k < 0, \quad h_E < 0, \quad \frac{|k|}{b} = n > 2 \quad \text{and} \quad |h_E| \gg |k|. \quad (13) \]

Then it follows from Eq.(10) that \( \zeta_1 \approx 3|h_E| - 3n(4n - 2)/|k|, \quad \zeta_2 \approx \)
\[ 3^{-1}|k|(1 - 2/3n) \] and Eq.(12) gives \( m_\mu/m_e \approx 207 \) if

\[ \frac{|h_E|}{|k|} = 9.8 \frac{n + 1}{n} \tag{14} \]

One can show that our TMT model provides the universality, (i.e. \( \zeta \) independence) of the Lagrangian of the electro-weak interaction. Therefore with the same condition (14) we obtain the right ratio of the physical (renormalized) muon and electron masses.

The effective interaction of the dilaton \( \phi \) with the regular leptons and quarks of the first two generations appears to be extremely suppressed because the appropriate Yukawa coupling is proportional to \( F_i(\zeta, \nu) \). In other words, the interaction of the dilaton with matter observable in gravitational experiments is practically switched off, and that solves the fifth-force problem.

The solution of the type II, \( \zeta(i) = \zeta_3^{(i)} \approx -b \) we associate with the third fermion generations. These states should be realized via fermion condensate.

4. Vacuum and/or very low fermion density?

One can show that in the fermion vacuum, the effective potential of the scalar sector including the quintessence-like field \( \phi \) and the Higgs field \( \nu \) is

\[
V_{eff}^{(0)}(\phi) = \frac{[V_1(\nu_0) + M_4 e^{-2\alpha\phi/M_p}]^2}{4[b(V_1(\nu_0) + M_4 e^{-2\alpha\phi/M_p}) - V_2(\nu_0)]} \tag{15}
\]

where \( \nu_0 \) is determined by the equation

\[
\left[ b - 2V_2(\nu_0) \left(V_1(\nu_0) + M_4 e^{-2\alpha\phi/M_p}\right)^{-1} \right] V_1'(\nu_0) + V_2'(\nu_0) = 0 \tag{16}
\]

The structure of the potential (15) allows to construct a model, where zero vacuum energy is achieved without fine tuning\(^3\) when \( V_1(\nu_0) + M_4 e^{-2\alpha\phi/M_p} = 0 \). In fact one may get such situation multiple times, therefore naturally obtaining a multiple degenerate vacuum as advocated by Bennett, Froggatt and Nielsen\(^7\).

In alternative models where the potential \( V_{eff}(\phi) \) has no zeros, it can monotonically decrease to the cosmological constant

\[
\Lambda^{(0)} = \frac{V_1^2(\nu_0)}{4[bV_1(\nu_0) - V_2(\nu_0)]}.
\]
This takes place if \( bV_1(\nu_0) > 2V_2(\nu_0) \). Then in the context of cosmology of the late time universe, \( \phi \) plays the role of the quintessence-like field and (15) is the dark energy potential.

Physics of fermions at very low densities, as it is governed by the constraint (8), turns out to be very different from what we know in normal particle physics. The term very low fermion density means here that the fermion energy density is comparable with the dark energy density. In this case, the noncanonical contribution (proportional to \( F_i(\zeta, \nu) \bar{\Psi}_i \Psi_i \)) to the energy-momentum tensor of the primordial fermion fields can be larger and even much larger than the canonical one. The theory predicts that in this regime the primordial fermion may not split into generations. Instead of this, for instance, in the FRW universe, the primordial fermion can participate in the expansion of the universe by means of changing its own parameters. We call this effect "Cosmo-Particle Phenomenon" and refer to such states as Cosmo-Low Energy Physics (CLEP) states.

As the first step in studying Cosmo-Particle Phenomena, we restrict ourselves to the consideration of a simplified cosmological model where universe is filled with a homogeneous scalar field \( \phi \) and uniformly distributed non-relativistic (primordial) neutrinos in CLEP states. A possible way to get up such a CLEP state might be spreading of the wave packet during its free motion lasting a very long (of the cosmological scale) time. The constraint shows that decreasing of the neutrino probability density may be compensated by approaching \( \zeta \to -k \) (recall that \( F_i(\zeta, \nu) \propto (\zeta + k)^{-2} \)). This regime is accompanied by increasing of the neutrino mass \( \nu_{CLEP} \sim (\zeta + k)^{-1} \). For a particular value \( \alpha = \sqrt{3}/8 \), the cosmological equations allow the following analytic solution for the late time universe: \( \phi(t) = \frac{M_P}{2^{5/2}} \zeta_0 + \frac{M_P}{\sqrt{3}} \ln(M_plt), \quad a(t) \propto t^{1/3} e^{\lambda t}, \) where \( a = a(t) \) is the scale factor, \( \lambda = \frac{1}{2} \sqrt{\Lambda/3} \) and

\[
\Lambda = \frac{V_2(\nu_{CLEP}) + |k|V_1(\nu_{CLEP})}{(b - k)^2}
\]

is the cosmological constant of the universe in the CLEP state. Such CLEP-neutrino matter is detectable practically only through gravitational interaction and this is why it can be regarded as a model of a dark matter. The mass of CLEP-neutrino increases as \( a^{3/2} \propto e^{3\lambda t} \). This dark matter is also cold one in the sense that kinetic energy of neutrinos is negligible as compared to their mass. However due to the dynamical fermionic term \( \Lambda \) generated by neutrinos in CLEP state, this cold dark matter has negative pressure and its equation of state approaches \( p_{d.m.} = -\rho_{d.m.} \) as \( a(t) \to \infty \).
Besides, the energy density of this dark matter scales in a way very similar to the dark energy which includes both a cosmological constant and an exponential potential. Thus, in this toy model, the CLEP dark matter displays itself as a sort of a dark energy.

The remarkable feature of such a Cosmo-Particle solution is that the total energy density of the universe in this case is less than it would be in the universe free of fermionic matter at all. In particular, one can check in a simple model for the Higgs prepotentials $V_1$, $V_2$ that $\Lambda < \Lambda(0)$. This means that there are two different vacua: one is usual vacuum free of the particles, which is actually a false vacuum, and another one, a true vacuum, which could be called ”Cosmo-Particle Vacuum” since it is a state containing neutrinos in CLEP state. Therefore one should expect the possibility of soft domain walls connecting these two vacua, that may be similar to soft domain walls studied in Ref.8.

One can expect that spherically symmetric solutions in the regime of the CLEP states may play an important role in the resolution of the halos dark matter puzzle.

References
1. See for example: G.W. Anderson and S.M. Carroll, astro-ph/9711288; L. Amendola, Phys. Rev. D62, 043511 (2000); D.J. Holden and D. Wands, ibid D61, 043506 (2000); A.P. Bilyard and A.A. Coley, ibid. D61, 083503 (2000); N. Bartolo and M. Pietroni, ibid. D61, 023518 (2000); L.P. Chimento, A.S. Jakubi and D. Pavon, ibid D62, 063508 (2000); D. Tocchini-Valentini and L Amendola, ibid., D65, 063508 (2002); D. Comelli, M. Pietroni and A. Riotto, Phys. Lett. B 571, 115 (2003); G.R. Farrar and P.J.E. Peebles, astro-ph/0307316; M.B. Hoffman, astro-ph/0307350; U. Franca and R. Rosenfeld, astro-ph/0308149; G.Huey and B.D.Wandelt, astro-ph/0407196.
2. S.M. Carroll, Phys. Rev. Lett. 81, 3067 (1998).
3. E.I. Guendelman and A.B. Kaganovich, Phys. Rev. D53, 7020 (1996); ibid. D55, 5970 (1997); ibid. D56, 3548 (1997); ibid. D57, 7200 (1998); ibid. D60, 065004 (1999).
4. E.I. Guendelman and A.B. Kaganovich, gr-qc/0312006.
5. E.I. Guendelman, Mod. Phys. Lett. A14, 1043 (1999); ibid. A14, 1397 (1999); Class. Quant. Grav. 17, 361 (2000); Found. Phys. 31, 1019 (2001).
6. E.I. Guendelman and A.B. Kaganovich, Int. J. Mod. Phys. A17, 417 (2002); Mod. Phys. Lett. A17, 1227 (2002).
7. D.L. Bennett, C.D. Froggatt and H.B. Nielsen, hep-ph/9504294.
8. C.T. Hill, D.N. Schramm and J.N. Fry, Comments Nucl.Part.Phys. 19, 25 (1989).