$k^+ = 0$ Modes in Light-Cone Quantization

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ABSTRACT

We investigate the light-cone quantization of $\phi^3$ theory in 1+1 dimensions with a regularization of discretized light-cone momentum $k^+$. Solving a second-class constraint associated with the $k^+ = 0$ mode, we show that the $k^+ = 0$ mode propagates along the internal lines of Feynman diagrams in any order of perturbation, hence our theory recovers the Lorentz invariance.

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1. Introduction

The light-cone quantization is a useful tool in particle physics. However, it is a long-standing problem how to treat the mode with the vanishing light-cone momentum $k^+$.\cite{1-3} In conventional calculation, one simply ignores such modes. However, the absence of the $k^+ = 0$ modes causes some problems, \textit{e.g.}, the breakdown of Lorentz invariance,\cite{4} uneasiness about the vacuum definition, etc. The subtlety comes from constraints over the zero momentum mode. In the light-cone frame, the conjugate momentum of bosonic field does not contain $x^+$- derivative (where $x^+ = \frac{1}{\sqrt{2}}(x^0 + x^1)$), hence the definition of momentum must be considered as a second-class constraint. The theory must be quantized by the Dirac bracket, in which the $k^+ = 0$ mode generates secondary constraint. The secondary constraint includes the information of the interaction.

Recently, some researchers surveyed the problem from the point of view of vacuum (non-)triviality,\cite{5} and the spontaneous break-down of symmetry.\cite{6} McCartor and Robertson investigated light-cone quantization of discretized 3+1 dimensional Yukawa theory taking the modes into account.\cite{7} They solved the zero mode equation, and showed that the modes propagate in internal lines of Feynman diagrams, and the inclusion of the modes improves bad behaviors of loop amplitudes. In their model, it is rather easy to solve the constraint, because the equation is linear with respect to the zero mode.

In this letter, we study the light cone problem of $\phi^3$-theory in a 1+1 dimensions. Although the constraint equation is not linear, it exhibits some interesting feature. In the following, we solve the equation perturbatively, and show that McCartor and Robertson’s result is able to be extended in any order of perturbation for this case.

This letter is organized as follows: In section 2, we quantize the 1+1 dimensional $\phi^3$-theory imposing a periodic boundary condition for $x^- = \frac{1}{\sqrt{2}}(x^0 - x^1)$. In section 3, we solve the zero mode constraint in each order of coupling constant. Substituting the solution into the interaction hamiltonian of the model, we show
that the \( k^+ = 0 \) mode propagates along all internal lines of Feynman diagrams.

In section 4, a problem of our normal ordering is discussed. The last section is devoted to conclusion.

2. \( \phi^3 \)-model in 1+1 dimension

To make the problem tractable, we consider a \( \phi^3 \)-model in 1+1 dimensions. The lagrangian density of the model is

\[
L = \partial_+ \phi \partial_- \phi - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{3!} \phi^3. \tag{2.1}
\]

We impose a periodic boundary condition \( \phi(x^- = -\frac{V}{2}) = \phi(x^- = \frac{V}{2}) \), from which the light-cone momentum is discretized as \( p^+ = \frac{2\pi}{V} n \) (\( n \) is an integer). Splitting the field \( \phi \) as \( \phi = \tilde{\phi} + \phi_0 \) (where \( \phi_0 = \frac{1}{V} \int dx^- \phi(x^-) \)), the lagrangian can be written as

\[
L = \int dx^- \partial_+ \tilde{\phi} \partial_- \tilde{\phi} - \int dx^- \frac{1}{2} m^2 \tilde{\phi}^2 - \frac{1}{2} m^2 V \phi_0^2 \\
- \int dx^- \frac{\lambda}{3!} (\tilde{\phi}^3 + 3 \phi_0 \tilde{\phi}^2) - \frac{\lambda}{3!} V \phi_0^3. \tag{2.2}
\]

Here, we do not consider the ordering of operators. It will be discussed later.

From (2.2), the conjugate momenta are defined as \( \tilde{\Pi}(x) \equiv \frac{\delta L}{\delta \partial_+ \phi(x)} = \partial_- \tilde{\phi}(x) \) and \( \Pi_0 \equiv \frac{\delta L}{\delta \phi_0} = 0 \). The momentum \( \Pi_0 \) generates a secondary constraint:

\[
-m^2 V \phi_0 - \frac{\lambda}{2} V \phi_0^2 - \frac{\lambda}{2} \int dx^- \tilde{\phi}^2 = 0. \tag{2.3}
\]

The canonical momenta have no fields with time \((x^+)\)- derivative. Using the Dirac
bracket quantization, we obtain canonical commutation relations as follows.

\[
[\phi_p, \phi_q] = \frac{1}{2p^+}\delta_{p+q,0}, \quad [\phi_p, \phi_0] = -\frac{1}{2p^+m^2 - \lambda\phi_0},
\]

(2.4)

where \(\phi_p\) is an expansion coefficient of \(\phi\) defined by

\[
\tilde{\phi}(x^+, x^-) = \frac{1}{\sqrt{V}} \sum_{p \neq 0} \phi_p e^{-ip^+x^-}.
\]

(2.5)

In the following, we proceed the calculation in the interaction picture. The Hamiltonian associated with lagrangian (2.2) is divided as

\[
H = H_{\text{free}} + H_{\text{int}},
\]

where

\[
H_{\text{free}} = \frac{1}{2}m^2 \int dx^- \tilde{\phi}^2,
\]

\[
H_{\text{int}} = \frac{1}{2}m^2V\phi_0^2 + \frac{\lambda}{3!}V\phi_0^3 + \frac{\lambda}{3!} \int dx^- (\tilde{\phi}^3 + 3\phi_0\tilde{\phi}^2).
\]

(2.6)

We include \(\frac{1}{2}m^2V\phi_0^2\) term (zero-mode mass term) not in \(H_{\text{free}}\) but in \(H_{\text{int}}\).

Free Heisenberg equations

\[
i\dot{\phi}_p = [H_{\text{free}}, \phi_p] = \frac{m^2}{2p^+}\phi_p, \quad i\dot{\phi}_0 = [H_{\text{free}}, \phi_0] = 0
\]

(2.7)

are consistent with the free equation of motion \(\partial_+\partial_-\phi - m^2\phi = 0\). They decide \(x^+\)-dependence of \(\phi_p\) in interaction picture.

From the commutation relation (2.4) and the definition of vacuum

\[
\phi_p|0\rangle = 0 \quad \text{for } p > 0,
\]

(2.8)
the free propagator of $\phi$ can be calculated as follows:

$$\tilde{\Delta}(x, y) = \langle T \left( \tilde{\phi}(x^+, x^-)\tilde{\phi}(y^+, y^-) \right) \rangle$$

$$= \theta(x^+ - y^+) \frac{1}{V} \sum_{p^+ > 0} \frac{1}{2p^+} e^{-i(p^+(x^- - y^-) + \frac{m^2}{2p^+}(x^+ - y^+))}$$

$$+ \theta(y^+ - x^+) \frac{1}{V} \sum_{p^+ > 0} \frac{1}{2p^+} e^{i(p^+(x^- - y^-) + \frac{m^2}{2p^+}(x^+ - y^+))}$$

(2.9)

In the conventional equal-time quantization where $x^0$ is chosen as the time, the discretized expression of propagator is expressed as

$$\Delta(x, y) = \Delta_0(x, y) + \tilde{\Delta}(x, y)$$

$$= \frac{i}{2\pi V} \int_{-\infty}^{\infty} dp^- \sum_{p^+ > -\infty} \frac{1}{2p^+p^- - m^2 + i\epsilon} e^{-i(p^+(x^- - y^-) + p^-(x^+ - y^+))}$$

(2.10)

where $\Delta_0$ is $p^+ = 0$ part of $\Delta$, i.e.,

$$\Delta_0(x, y) = \frac{i}{2\pi V} \int_{-\infty}^{\infty} dp^- \frac{1}{-m^2} e^{-ip^-(x^+ - y^+)} = \frac{-i}{m^2 V} \delta(x^+ - y^+).$$

(2.11)

The propagator (2.10) is covariant in continuum limit ($V \to \infty$).

If we simply ignore $\phi_0$, the propagator $\langle T(\phi\phi) \rangle$ does not provide $\Delta_0$ part. However, taking the zero-mode constraint (2.3) into account, $\Delta_0$ is recovered at least in all internal lines of Feynman diagrams. The purpose of our discussion below is to demonstrate the recovery of $\Delta_0$. 

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3. Solving $\phi_0$-constraint

From the constraint (2.3), $\phi_0$ is given in terms of $\tilde{\phi}$ as

$$
\phi_0 = -\frac{m^2}{\lambda} \left( -1 \pm \sqrt{1 - \frac{\lambda^2}{m^4V} \int dx \tilde{\phi}^2} \right)
$$

(3.1)

Since the negative sign in the parenthesis corresponds to a classically unstable vacuum $\phi_0 = \frac{2m^2}{\lambda}$, we ignore it and choose the positive sign solution, which is expressed in a power expansion form as

$$
\phi_0 = -\frac{m^2}{\lambda} \sum_{n=1}^{\infty} \frac{(2n-3)!!}{n!} \left( \frac{\lambda^2}{2m^4V} \int dx \tilde{\phi}^2 \right)^n.
$$

(3.2)

Substituting (3.2) into (2.6), the interaction hamiltonian $H_{\text{int}}$ is expressed as

$$
H_{\text{int}} = \sum_{n>0} H_{\text{int}}^{(n)} + \frac{\lambda}{3!} \int dx - \tilde{\phi}^3
$$

(3.3)

where

$$
H_{\text{int}}^{(n)} = -\frac{m^6V}{\lambda^2} \frac{(2n-3)!!}{(n+1)!} \bigg( \frac{\lambda^2}{2m^4V} \int dx \tilde{\phi}^2 \bigg)^{n+1}.
$$

(3.4)

The symbol $\hat{} \hat{} \hat{}$ denotes a certain normal ordering. Note however that it is not an usual normal ordering, but it is defined in the next section.

From (3.3), the exponent of $e^{-i \int H_{\text{int}} dt}$ has terms as follows:

$$
-i \int H_{\text{int}}^{(n)} dt = (-i\lambda)^{2n} \frac{(2n-3)!!}{(n+1)!} \left( \frac{-i}{m^2V} \right)^{2n-1} V^{n-1} \prod_{i=1}^{n+1} dx_i \frac{1}{2} \tilde{\phi}^2(x_i) \hat{} \hat{} \hat{}.
$$

(3.5)

The above expression describes $2n$ cubic interaction vertices which are connected one another by $2n - 1$ $\Delta_0$ propagators. The factor $(-i\lambda)^{2n}$ implies that $H_{\text{int}}^{(n)}$
describes 2nth-order interactions where the factor \( \left( \frac{-i}{m^2 V} \right)^{2n-1} \) comes from \( 2n - 1 \) \( \Delta_0 \) propagators which combine 2n vertices. The operators \( \left( \frac{1}{2} \tilde{\phi}^2 \right)^{n+1} \) remain uncontracted. The rest of vertices, of which number is \( n - 1 = 2n - (n + 1) \), are fully contracted. Since \( \Delta_0 \) is independent of \( x^- \), \( x^- \)-integrations of fully contracted vertices are trivially worked out, and generate the factor \( V^{n-1} \).

The factor \( \frac{1}{(n+1)!} \) will be canceled by a number of combinations when we contract all \( \left( \frac{1}{2} \int \tilde{\phi}^2 \right)^{n+1} \). The factor \( (2n - 3)!! \) is a number of patterns to connect \( n + 1 \) points with \( n - 1 \) lines using cubic interactions without making loop. It can be proved by induction as follows.

First, for \( H^{(1)}_{\text{int}} \), the statement is true because there is only one way to connect two points by one line \( \left( (2 \times 1 - 3)!! = 1 \right) \). Second, we will show that if the number of contractions in \( H^{(k)}_{\text{int}} \) is \( (2k - 3)!! \), the number in \( H^{(k+1)}_{\text{int}} \) is \( (2k - 1)!! \). A Feynman diagram in \( H^{(k+1)}_{\text{int}} \) is obtained by adding one \( \frac{1}{2} \tilde{\phi}^2 \) to \( H^{(k)}_{\text{int}} \). The \( \Delta_0 \) propagator should be connected from the new \( \frac{1}{2} \tilde{\phi}^2 \) to somewhere in \( \Delta_0 \) propagators in \( H^{(k)}_{\text{int}} \).

As already explained, each diagrams for \( H^{(k)}_{\text{int}} \) has \( (2k - 1) \) \( \Delta_0 \) propagators. Hence we have \( 2k - 1 \) alternatives to make \( H^{(k+1)}_{\text{int}} \)-diagram from one \( H^{(k)}_{\text{int}} \)-diagram. As the result, the number of possible diagrams for \( H^{(k+1)}_{\text{int}} \) is \( (2k - 3)!! \times (2k - 1) = (2k - 1)!! \). Q.E.D.

Eventually, (3.3) is rewritten as

\[
-i \int H^{(n)}_{\text{int}} dt = \sum_{\text{possible diagrams}} \phi \int_{\Delta_0} \prod_{i=1}^{n+1} dx_i^2 \prod_{j=1}^{n-1} dy_j^2 \frac{(-i \lambda)^{2n}}{(n+1)!} \Delta_0^{2n-1} \frac{1}{2} \tilde{\phi}^2(x_i) \phi.
\]

where \( \Delta_0 \)'s connect \( \{x_i, y_j\} \) with cubic interactions. Each \( y_j \) integration is trivial and its result is \( V \). The summation runs over all combinations of \( \Delta_0 \). In fact, each combination of \( \Delta_0 \) gives same result. As stated above, \( H^{(n)}_{\text{int}} \) contains \( 2n \) vertices, \( n + 1 \) \( \tilde{\phi}^2 \) terms, \( n - 1 \) internal interaction points \( (y_j) \) and \( 2n - 1 \) \( \Delta_0 \) propagators.

For \( n = 1 \),

\[
-i \int dt H^{(1)}_{\text{int}} = \frac{(-i \lambda)^2}{2} \phi \int dx^2 dy^2 \left( \frac{1}{2} \tilde{\phi}^2(x) \right) \Delta_0(x, y) \left( \frac{1}{2} \tilde{\phi}^2(y) \right) \phi.
\]
describes nothing but the interaction in which the $k^+ = 0$ mode propagates between two $\frac{1}{2}\tilde{\phi}^2$s (See Fig. 1). For $n = 2$, $-i \int dt H_{\text{int}}^{(2)}$ describes the interaction in which three $\frac{1}{2}\tilde{\phi}^2$s are connected with three $\Delta_0$s (Fig. 2). Similarly, for $n = 3$, $-i \int dt H_{\text{int}}^{(3)}$ describes three patterns of the interaction in which four $\frac{1}{2}\tilde{\phi}^2$s are connected with five $\Delta_0$'s (Fig. 3).

As already explained, if we ignore $\phi_0$, there is no $\Delta_0$ propagator in internal lines of Feynman diagrams. However, the lack of $\Delta_0$ is recovered by $H_{\text{int}}^{(n)}$. Namely, $H_{\text{int}}^{(n)}$ supplies any diagram which is obtained by replacing some $\tilde{\Delta}$ propagators to $\Delta_0$ propagators in a diagram which is constructed from $\tilde{\Delta}$ only.

There is no ‘zero mode loop’ (See Fig. 4 for example) contribution in $H_{\text{int}}^{(n)}$. The amplitudes of Fig. 4 diverge because of multiplication of $\delta$-functions. Our normal ordering prescription is defined in such a way that automatically subtracts such a divergent diagram.

4. How to Define Normal Ordering

For loop-level, we must consider self-contraction in $H_{\text{int}}^{(n)} (n \geq 1)$. The simplest example is two-point function with one-loop (See Fig. 5a):

$$
\int dx' \int dy' \left\langle T \left( \phi(x) \frac{1}{2!} (-i \lambda \tilde{\phi}^3(x'))(-i \lambda \tilde{\phi}^3(y')) \phi(y) \right) \right\rangle
$$

$$
= \frac{(-i\lambda)^2}{2} \int dx' dy' \Delta(x, x') \Delta(x', y') (\Delta(x', y'))^2 \Delta(y', y)
$$

(4.1)

where $\frac{1}{2}$ is a symmetric factor. There should be an amplitude which is obtained by replacing one of the internal lines of (4.1) to $\Delta_0$ (See Fig. 5b), namely,

$$
(-i\lambda)^2 \int dx' dy' \Delta(x, x') \Delta(x', y') \Delta_0(x', y') \Delta(y', y).
$$

(4.2)
Substituting the definition of $\tilde{\Delta}$ (2.9) to (4.2), we obtain

$$\frac{1}{2} \frac{i \lambda^2}{m^2 V^2} (x^+ - y^+) \sum_{p^+ > 0} \left( \frac{1}{2p^+} \right)^3 e^{-i(p^+(x^- - y^-) + \frac{m^2}{2p^+}(x^+ - y^+))}. \quad (4.3)$$

Note that internal propagator $\tilde{\Delta}(x', y')$ provides the factor $\frac{1}{2p^+}$ in momentum space (external propagators provide the factor $\frac{1}{2p^+}$). The factor $\frac{1}{2}$ comes from

$$\lim_{x^+ \to y^+} \Theta(x^+ - y^+) = \frac{1}{2}. \quad (4.4)$$

The amplitude (4.2) is obtained from self-contraction of $H^{(1)}_{\text{int}}$ as

$$\left\langle T \left( \phi(x) (-i \int dt H^{(1)}_{\text{int}}(y)) \phi(y) \right) \right\rangle$$

$$= \frac{(-i \lambda)^2}{2} \int dx' dy' \left\langle T \left( \phi(x) \phi_2 (x') \Delta_0(x', y) \frac{1}{2} \phi_2 (y) \phi(y) \right) \right\rangle. \quad (4.5)$$

In order to get correct factor (4.2), $H^{(1)}_{\text{int}}$ should be normal-ordered as

$$- i \int dt H^{(1)}_{\text{int}} = \frac{(-i \lambda)^2}{2} \int dt \sum_{p^+ > 0} \tilde{\phi}_{-p^+} \tilde{\phi}_{p^+} \frac{-i}{m^2 V} \sum_{q^+ > 0} \tilde{\phi}_{-q^+} \tilde{\phi}_{q^+} :$$

$$= \frac{(-i \lambda)^2}{2} \int dt \left[ \sum_{p^+ > 0} \tilde{\phi}_{-p^+} \tilde{\phi}_{p^+} \sum_{q^+ > 0} \tilde{\phi}_{-q^+} \tilde{\phi}_{q^+} : \right. + \left. \sum_{p^+ > 0} \tilde{\phi}_{-p^+} \frac{1}{2p^+} \tilde{\phi}_{p^+} : \right]\quad (4.6)$$

where $: :$ is the usual normal ordering, in which $\phi_p(p > 0)$ should be put on the right of $\phi_p(p < 0)$. The second term in the parenthesis of the last expression of (4.6)
contributes to (4.2). Note that, since $H_{\text{int}}^{(1)}$ is instantaneous in $x^+$, an ordering of $\tilde{\phi}^2(x')$ and $\tilde{\phi}^2(y')$ is fixed. There is no freedom to choose $\tilde{\phi}^2(x')$ or $\tilde{\phi}^2(y')$ when we contract with $\phi(x)$ or $\phi(y)$. Hence we get $2 \times 2$ as a multiplication factor of the contraction (these two 2 arise due to bi-linearity of two $\tilde{\phi}^2$s). In this case, naive ordering $(: \tilde{\phi}^2 : \tilde{\phi}^2 :)$ gives correct result. Unfortunately, it is not true in higher order. The normal-ordering $\tilde{\phi}^2 \tilde{\phi}^2$ of higher order interaction hamiltonians should be decided according to the requirement of such a consistency.

Let us consider the diagram obtained from $H_{\text{int}}^{(n)}$ by self-contractions. When we contract $n_1$ $\tilde{\phi}^2$s in $H_{\text{int}}^{(n)}$, the factor $n_{+1}P_{n_1} = \frac{n_{+1}!}{(n_{+1} - n_1)!}$ arises if the ordering of $\tilde{\phi}^2$ was not fixed. It is a desired factor, but the ordering is fixed in fact. The factor $n_{+1}C_{n_1} = \frac{n_{+1}!n_1!}{(n_{+1} - n_1)!n_1!}$ arises if we set $\tilde{\phi}^2 \tilde{\phi}^2$ in $H_{\text{int}}^{(n)}$. Adding to it, we have to consider the fact that the propagator $\tilde{\Delta}(x, y)$ gives the factor $\frac{1}{2}$ in the instantaneous limit ($x^+ = y^+$). We must attach the factor $\left(\frac{1}{2}\right)^n n_{+1}P_{n_1}$ to the contracted operators by hand. It means that $\tilde{\phi}^2 \tilde{\phi}^2$ in $H_{\text{int}}^{(n)}$ must include the term

$$\left(\frac{1}{2}\right)^n n_{+1}P_{n_1} : \left(\sum_{p^+ > 0} \tilde{\phi}_{-p^+} \tilde{\phi}_{p^+}\right)^{n_{+1} - n_1} \left(\sum_{p_1^+ > 0} \tilde{\phi}_{-p_1^+} \left(\frac{1}{2p_1^+}\right)^{n_1 - 1} \tilde{\phi}_{p_1^+}\right) : \tag{4.7}$$

For attaching these factors to each all contracted operators, $\tilde{\phi}^2 \tilde{\phi}^2$ is decided as follows:
\[ \varnothing \left( \sum_{p^+ > 0} \tilde{\phi}_{-p^+} \tilde{\phi}_{p^+} \right)^{n+1} = \varnothing : \left( \sum_{p^+ > 0} \tilde{\phi}_{-p^+} \tilde{\phi}_{p^+} \right)^{n+1} : + \sum_{\text{possible } n_1, n_2, \ldots, n_I > 0} \sum_{p^+ > 0} \left( \sum_{p^+_1 > 0} \tilde{\phi}_{-p^+_1} \left( \frac{1}{2p^+_1} \right)^{n_1} \tilde{\phi}_{p^+_1} \right) \left( \sum_{p^+_2 > 0} \tilde{\phi}_{-p^+_2} \left( \frac{1}{2p^+_2} \right)^{n_2} \tilde{\phi}_{p^+_2} \right) \cdots \left( \sum_{p^+_I > 0} \tilde{\phi}_{-p^+_I} \left( \frac{1}{2p^+_I} \right)^{n_I} \tilde{\phi}_{p^+_I} \right) : \]  

\begin{equation}
(4.8)
\end{equation}

The factor \( \left( \frac{1}{2} \right)^{n_1} \cdot \cdots \cdot \left( \frac{1}{2} \right)^{n_I} \cdot \left( \sum_{p^+_i > 0} \tilde{\phi}_{-p^+_i} \tilde{\phi}_{p^+_i} \right)^{n_1+1} \) is obtained from a self-contraction of \( \left( \sum_{p^+ > 0} \tilde{\phi}_{-p^+} \tilde{\phi}_{p^+} \right)^{n+1} \).

5. Conclusion

In this letter, we have considered \( \phi^3 \)-theory as a simple example. We show that, if we quantize the theory in a proper way, \( k^+ = 0 \) mode propagates along internal lines of Feynman diagrams in any order of perturbation. The inclusion of the zero mode is expected to recover breakdown of Lorentz invariance of a naively light-cone quantized theory after limiting procedure \( V \to \infty \). In fact, in 1+1 dimension, the Lorentz transformation is the scale transformation of \( p^+ \) and \( p^- \) and it does not mix zero modes and non-zero modes. The Lorentz invariance becomes important in higher dimensions.

The extension of the result of this paper to other theories is straightforward.
For example, in the case of $\phi^4$, we must solve the equation

$$-m^2 V\phi_0 - \frac{\lambda}{3!} V\phi_0^3 - \frac{\lambda}{2} \int dx \bar{\phi}^2 \phi_0 - \frac{\lambda}{3!} \int dx \bar{\phi}^3 = 0. \diamond OB \quad (5.1)$$

The constraint equation becomes more complicated than $\phi^3$-theory. In such a case, redefinition of mass $m$ is needed (See ref 8).

Finally we comment on the problem of vacuum (non-)triviality. A remarkable simplicity of $\phi^3$-theory is the fact that $\phi_0$ commutes with $H_{\text{free}}$. In the case of $\phi^3$-theory, the time-development of $\phi$ by $H_{\text{free}}$ obeys the free equation of motion. In $\phi^4$ (or more complicate interaction)-theory, it is not the case. In such cases, $H_{\text{free}}$ does not reproduce correct free equation of motion. Hence, in general, $H_{\text{free}}$ in this letter is no more a ‘free’ hamiltonian. This discrepancy might make some clue for the problem of vacuum (non-)triviality. These subjects remain for future investigation.

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FIGURE CAPTIONS

Figure 1. Two $\frac{1}{2}\phi^2$ are connected by a $\Delta_0$ propagator (broken line).

Figure 2. There $\frac{1}{2}\tilde{\phi}^2$ are connected by three $\Delta_0$ propagators.

Figure 3. Four $\frac{1}{4}\tilde{\phi}^2$ are connected by five $\Delta_0$ propagators. Three patterns exist.

Figure 4. Examples in which $\Delta_0$ propagators make a loop by themselves.

Figure 5a. One-loop diagram with two external lines and two internal lines.

Figure 5b. One of internal lines of Figure 5a is replaced to $\Delta_0$. 

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