Research progress on the medium frequency expansion method based on statistics energy

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Abstract. The SEA medium frequency extension methods are described. Then, medium frequency extension methods (FE-SEA (finite element-statistical energy analysis), CI-FE/SEA (CHEBYSHEV interval finite-element/statistical energy analysis), ES-FE-SEA (Edge smoothing -finite element- statistical energy analysis)) are compared. In FE-SEA method, neutron structure assembly and modeling method, interval parameter uncertainty and hybrid interval parameter analysis, interval variables and the hybrid interval variable response method are emphatically expounded. And on this basis, the existing methods are compared and analyzed. Based on the differences among the CHEBYSHEV expansion CI-FE/SEA method, the edge smooth domain ES-FE-SEA method and the Taylor series expansion FE-SEA method in solving parameter interval range and solution precision, the hybrid and wide range interval algorithm combined existing methods is expected to develop.

1. Introduction

statistical energy analysis, SEA is a statistical method to study and analyze vibration and acoustics from an energy point of view. It divides the whole system into weakly coupled substructures, describes the States of each subsystem with power and energy, and obtains the acoustic-vibration characteristics of the system through energy flow and transfer feedback to the equilibrium equation. It is currently the most widely used method to solve the problem of high-frequency acoustic vibration [1]. Compared with the deterministic method, SEA model reduces the scale and greatly improves the computational efficiency. Generally, due to the limitation of theory and calculation period, Monte Carlo simulation of structures under high frequency excitation using finite element method (FEM) and boundary element method (BEM) is inefficient [2],[3], while SEA method can perfectly describe the average vibration characteristics of the system and is a powerful tool to solve the high frequency acoustic-vibration problem. SEA method is applied to solve the high frequency acoustic-vibration problem. The calculation accuracy depends on the accuracy of its parameters, i.e. modal density, internal loss factor, ILF), coupling loss factor, CLF ) input power, system energy, etc. A typical SEA subsystem model is shown in Figures 1 and 2. In the past few decades, the classical SEA analysis theory has been extended to a large extent to deal with non-conservative coupling systems, strong coupling systems and indirect coupling systems [4],[5]. Up to now, SEA method has been widely used in vehicle, ship, aviation and other transportation engineering fields [6],[7]. However, due to
many assumptions in SEA method, it is not suitable for acoustic-vibration coupling in intermediate frequency range. In order to solve the problem of mid-frequency band, many scholars have carried out relevant research. GRICE et al [8] have combined finite element method with analytical impedance method to carry out the vibration research of beam element. LANGLEY et al [9]–[11] proposed an intermediate frequency solution, which combines FEM and SEA methods to construct deterministic components of the system in a hybrid finite element analysis framework for structural performance prediction. In addition, in recent years, in some research fields, such as non-uniform modal energy statistical energy analysis, energy distribution [12], low modal density [13] and non-resonant response [14], SEA model variance prediction [15], SEA expansion technology [16], Finite element -statistical energy analysis (FE-SEA) method [17], transient statistical energy analysis [18], the above research work has made further development and expanded the application of sea method from high frequency to intermediate frequency. At present, the following methods have been gradually formed to solve the "mid" - [band] acoustic-vibration coupling problem. The first one is deterministic method. By improving the computational efficiency of deterministic method, the application range of analysis frequency is expanded to intermediate frequency. The research methods include: FEM method with modified integral formula, modified form function formula, multi-scale FEM method, high-order integral method, etc. Although the first method can improve the calculation frequency, it ignores the influence of uncertain factors in the intermediate frequency problem and the calculation accuracy is difficult to be guaranteed. The second is the generalized SEA method, which expands the statistical energy analysis frequency to the middle frequency band by relaxing the restriction of the basic hypothesis of the statistical energy theory. The research methods include: Test Statistical Energy Method (ESEA), Statistical Modal Energy Analysis Method (SMEDA), Waveguide Method (WGA ), etc. The third is the hybrid method of finite element technology and statistical energy, including FE-SEA method, CI-FE/SEA method, ES-FE-SEA method, etc. CI-FE/SEA is the combination of Chebyshev polynomial and FE-SEA method, ES-FE-SEA is the combination of edge smooth gradient algorithm and FE-SEA method. The latter two methods are improved in accuracy and efficiency compared with FE-SEA method, and are also the extension and extension of FE-SEA method. The common characteristic of this kind of methods is to solve the problem of intermediate frequency band by fusing the finite element deterministic method and the statistical energy nondeterministic method [19,20].

Figure 1. Two subsystems SEA power flow transmission model
2. ESEA method

2.1. Application of ESEA Method

Classical SEA analysis needs to input power load into the structure by means of PIM Power input method (PIM) to obtain system loss factor [21]-[23]. However, due to physical constraints, it is difficult to carry out such measurement. Literature [4] points out that the main error of SEA model comes from the measurement error of input power. The essence of Experimental statistical energy analysis (ESEA) is to test the assembled structure to verify CLF or make theoretical estimation. When accurate theoretical estimation is not possible, experimental data for SEA model are obtained [25]-[27]. The testing process is shown in figs. 3 and 4 [28]. The method does not need to input measurement power, only needs to measure the square of the average speed of the excitation point and the response point, or the square of the speed transmission modulus, and realizes spatial averaging, thus overcoming the system error caused by the measurement uncertainty of the input power, and expanding the application range of the classical SEA method to a large extent, thereby realizing the expansion from high frequency to intermediate frequency. ESEA theory usually uses the propagation energy ratio, propagation path, random energy method and other methods to evaluate the overall mean value of the system. The research in this area includes: GUASCH et al [29] proposed an ESEA analysis method, and established a statistical energy model composed of three subsystems by using the theories of direct and indirect transmission energy ratio and transmission rate. BOUHAIJ [30] proposed a random energy method. Monte Carlo simulation program was used to study the ESEA population mean value problem. The confidence interval of normalized energy was estimated and the superiority of the method was verified by comparing the energy matrix with the obtained results. Document [31]-[33] proposes a new ESEA method in the framework of transmission path analysis.
3. FE-SEA Method

3.1. Substructure Assembly and Modeling

Because ESEA method has large errors in the testing process, hybrid FE-SEA and its extension methods have been developed. As shown in Figures 5 and 6, FE-SEA method combines finite element method and statistical energy method through the principle of direct-hybrid field reciprocity [9],[34] - [37], and decomposes complex structures into dynamic substructures that are assembled. The assembly principle is to define FE components using deterministic analysis method as the main system and SEA components with uncertain parameters as subsystems. The main system and subsystem are connected together through deterministic connection or deterministic components. The response of the assembled structure consists of two parts, one part is provided by the degree of freedom of nodes from FE components, and the other part is provided by the vibration energy of SEA subsystem. Under normal circumstances, the uncertainty modeling method in mixed FE-SEA theory is nonparametric, which satisfies the assumptions of highly random SEA subsystem and statistical distribution of characteristic frequency [20], [37]. The nonparametric uncertainty modeling method does not need to identify specific random physical parameters, and generates an uncertainty set by introducing parameter uncertainty into FE components. The set consists of two parts, one is a nonparametric SEA subsystem and the other is a parameterized FE component [38]. Document [39] proposes a combination method to solve the problem of uncertainty modeling. The method combines FE-SEA method with Laplace method to evaluate the probability that the system response variables exceed the limit value, and verifies the effectiveness of the method through two combination boards with uncertainty characteristics.

![Figure 5. Straight-Mixed Field Mutual Benefit Relation](image)

![Figure 6. DC-Mixed Field Power Transfer](image)
3.2. Parametric Uncertainty

3.2.1. Classification of Parameter Uncertainty. In mixed FE-SEA uncertainties, parameter uncertainties are generally divided into two categories: probabilistic uncertainties and non-probabilistic uncertainties. Probabilistic uncertainty includes methods such as fuzzy set theory and stochastic theory, and non-probabilistic uncertainty includes interval analysis method and mixed parameter method. The advantage of probabilistic uncertainty method is that it can provide mean and variance, and also can provide the probability distribution of uncertain output. Therefore, this method is designated as the optimal solution strategy for uncertain problems with stochastic uncertainties. However, in the early stage of design, it is not always sufficient to construct precise probability distribution parameters of random distribution with available information. In this case, it is necessary to assume the probability distribution of uncertain parameters. The calculation accuracy of the probability distribution has great errors, and the probability method needs to solve the probability density function of uncertain parameters [40], [41].

3.2.2. Interval Analysis. Interval analysis, also known as interval technique, is an effective method for non-probabilistic uncertain parameter analysis. Its advantage is that only less data information is needed to obtain interval solutions including real values when solving uncertain problems, thus reducing sample space of data and improving calculation efficiency. The commonly used methods of interval analysis include interval gauss elimination, interval iteration method, non-probability interval method, interval correspondence method, interval perturbation finite element method, Legendre orthogonal polynomial method, etc. [42] - [48]. In recent years, with more in-depth research on interval analysis, some new and effective analysis methods have emerged. WU [49], [50] uses Chebyshev polynomial interval method to study the dynamic response. The advantage of this method is that it does not need to determine the probability distribution of parameters, only needs to determine the range of parameter intervals, and obtains the response interval of the system by solving the parameter equation. However, it is only applicable to the case where the range of interval parameters is small and the solution error is large for large range of interval parameters. WANG et al [51] propose an improved algorithm of interval parameter perturbation method, and apply this method to prediction of external sound field noise. The improved interval parameter perturbation method retains higher-order terms and is used to calculate the inverse of interval matrix. Taylor series correction algorithm is applied to approximate the interval matrix vector. The difference between Monte Carlo simulation method and interval parameter perturbation method is compared through an example. ZHANG et al [52] proposed a method to solve the interval linear equation of the system by using finite element method and distribution method, which separates the interval values of uncertain parameters and determines each interval boundary by calculating the extreme value problem of the equation solution. SU et al [53] proposed an interval inverse analysis method for unknown parameters. The effects of different test accuracy and different target parameters on the inverse analysis results were compared and studied, and the convergence conditions of the solution were deduced. XIA et al [54] proposed a method to solve the dynamic response of the system by using interval random model.

3.2.3. Mixed parameter analysis. In FE-SEA method, interval parameter uncertainty analysis is widely used due to its small sample size and simple calculation. Its basic theory is based on non-probabilistic uncertainty method, while probability distribution and other methods are mainly based on probabilistic uncertainty method. However, there are often mixed types of parameter uncertainty problems in engineering practice. Therefore, mixed parameter uncertainty analysis has received more and more attention in recent years. Related researches include: GAO et al [55] proposed a stochastic interval perturbation method for mixed parameter analysis of uncertain structures, and used this method to determine the stochastic distribution and interval parameters. Yin et al [56] introduced the mixed non-probabilistic fuzzy theory and interval uncertainty into the mixed FE construction. Based on the combination of non-parametric mixed fuzzy and interval parameter uncertainty, a mixed model
of non-parametric combined mode was established, and a Fuzzy interval finite element/Statistical energy analysis (FIFE/SEA) framework was proposed to obtain the uncertain response of the combined system. In order to deal with the uncertainty of mixed parameters effectively, document [57] introduces the first-order fuzzy interval perturbation method into the mixed FE-SEA framework and proposes the first-order fuzzy interval perturbation method (FFI PFE / SEA) and verifies the effectiveness of the method. Yin and YU [58] established non-parametric equations and non-parametric hybrid models based on the introduction of interval parameter uncertainty and dynamic interval into the structural acoustic system by hybrid FE-SEA framework. The interval parameter uncertainty parameters of the structural acoustic system were obtained. In order to further improve the calculation accuracy, a second-order perturbation finite element / statistical energy analysis (SIPFM / SEA) method is proposed. In the SIPFM / SEA method, the expectation of the second-order response is derived from the standard values of the second-order Taylor series expansion interval parameters. In order to calculate efficiently, the non-diagonal elements of Hessian matrix are ignored. The cross-power spectrum of vibration energy boundary and response is obtained by searching for the objective function of maximizing or minimizing the objective interval parameters. However, this method is only suitable for narrow parameter interval analysis because the high-order terms of Taylor series are ignored. XIA et al [59] proposed a mixed perturbation vertex method for uncertain structures, and conducted acoustic analysis on mixed random interval parameters. CHEN [60] developed a mixed perturbation method to analyze the external sound field with uncertain random interval parameters. Literature [61],[62] uses hybrid FE-SEA method and Modal component synthesis (MCS) to deal with mixed uncertainties of fuzzy random parameters, and verifies the effectiveness of the method. Literature [57], [63], [64] proposes a hybrid fuzzy random reliability analysis method based on transformation mode by studying the equivalent transformation between fuzzy variables and random transformation.

4. CI-FE/SEA Method

4.1. CHEBYSHEV Interval Method

Among many FE-SEA methods, the mixed interval perturbation method based on Taylor series expansion term is widely used in predictive uncertainty analysis because of its simple calculation. For most applications of Taylor series expansion interval perturbation method, the dynamic response solution is based on low-order Taylor interval parameter series. This method restricts the interval perturbation method to inter-cell uncertainty analysis. The accuracy of uncertainty analysis can be improved by using Taylor series high-order perturbation technique. However, the solution of high-order derivatives is very complicated, especially in the case of FE-SEA mixed model. The system matrix is the product of several matrices, and the computational efficiency is low. In order to solve this problem, in recent years, CHEBYSHEV based on orthogonal Chebyshev polynomials has been used. The interval method has been developed and applied to the response analysis of dynamic time-domain nonlinear systems [65]. In the CHEBY-SHEV interval method, the CHEBYSHEV series is based on the interval mathematical theory, and the truncated CHEBYSHEV series and boundary are used to approximate the response of uncertain parameters.

4.2. CI-FE/SEA Method Application

In the application of CI-FE/SEA method, a CHEBYSHEV series interval method is proposed in reference [47] to analyze the dynamic response of nonlinear systems with uncertain bounded parameters. It is verified that the truncated CHEBYSHEV series expansion is closer to the general solution than the truncated Taylor series expansion. It is concluded that the CHEBYSHEV inclusion function is more anti-jamming than the traditional Taylor inclusion function. In reference [50], an uncertain analysis method is proposed. The dynamic response of mechanical system is studied by using CHEBYSHEV sequence. It is verified that CHEBYSHEV inclusion function can obtain more precise interval boundaries than traditional Taylor inclusion function. Yin [66] et al. proposed an
improved mixed CHEBYSHEV interval finite element/statistical energy analysis method (CI-FE/SEA). The mixed FE-SEA equation was treated analytically. The response boundary of truncated CHEBYSHEV series relative to interval parameters was calculated by Monte Carlo simulation. The differences of the second-order interval perturbation finite element/statistical energy analysis (SIPFEM/SEA), CI-FE/SEA, MSC-FE/SEA (Monte Carlo simulation of the hybrid FE/SEA model, MSC-FE/SEA) methods are compared. ES-FE-SEA Method

4.3. S-FEM and ES-FEM methods
In the mixed FE-SEA theory, the finite element FE component is an important component of the mixed FE-SEA method. As is known to all, there are interpolation errors and dissipative errors in solving intermediate frequency problems, and the two kinds of errors will increase sharply with the expansion of analysis frequency. At the same time, FE components have some inherent defects in medium and high frequency analysis. These defects are closely related to the "excessive rigidity" characteristic of FEM method and its sensitivity to numerical errors [67,68]. In order to solve this problem, LIU et al. [69] - [71] combined FEM method with gradient smoothing theory to propose Smooth finite element method (S-FEM), and its smooth domain processing process is shown in fig. 7[72]. The principle is to discretize the tetrahedral element, connecting the two end points of the edge k with the centroid of the tetrahedron and the centroid of the two face triangles in the tetrahedron containing the edge k respectively. This kind of method breaks through the weak form constraint of the minimum displacement principle, well regulates the continuous system stiffness, and makes the model closer to the real system stiffness. Then, based on the smooth finite element method (S-FEM), WU[73] and others proposed a new edge-based smooth finite element and statistical energy analysis (ES-FE-SEA) method. This method applies gradient smoothing technology based on element edges to embedded models to soften discrete systems. According to the smoothing domain type S-FEM method used, there are various models, including smoothing domains based on nodes and nodes, nodes and edges, the construction process is as shown in fig. 8[73]. Different S-FEM models have different properties. The smoothing domain based on nodes adopts the Node smooth finite element method (NS-FEM) [69, 74], and the soft-hard mode is adjusted to generate the upper bound solution. Edge smoothing finite element method (ES-FEM) [75, 76] is used to solve stable and ultra-accurate numerical solutions of dynamic problems.

Figure 7. Is based on the tridimensional integral smoothing domain with tetrahedron as the center.
Table 1. Typical existing types of smoothing domains (SD’s)

| Type               | Method for creation and number of SD’s                                                                 | S-FEM models     | Dimension of problem |
|--------------------|--------------------------------------------------------------------------------------------------------|------------------|----------------------|
| Cell-based SD (CSD)| SD’s or smoothing cells (SC’s) are divided from and located within the elements                       | CS-FEM, nCS-FEM  | 1D, 2D, 3D           |
| Edge-based SD (ESD)| SD’s are created based on edges by connecting portions of the surrounding elements sharing the associated edge | ES-FEM           | 2D, 3D               |
| Node-based SD (NSD)| SD’s are created based on nodes by connecting portions of the surrounding elements sharing the associated node | NS-FEM           | 1D, 2D, 3D           |
| Face-based SD (FSD)| SD’s are created based on faces by connecting portions of the surrounding elements sharing the associated face | FS-FEM           | 3D                   |

5. Conclusion

Based on the above review, this paper summarizes the technical difficulties that may be encountered in the research of SEA IF expansion methods, including:

5.1. ESEA Method
(1) The problems of ESEA method lie in the influence of uncertainties on the accuracy of system solution, such as the uncertainty of subsystem partition, the lack of effective excitation and response sampling, the difficulty of obtaining the quality of non-uniform subsystem, and the digital uncertainty of matrix equation inversion, etc. These uncertainties will produce systematic errors in the process of solution.

5.2. FE-SEA Method
(1) The deterministic part of FE-SEA method is composed of FE components, which need to be modeled by finite element method. There is a risk of low interpolation accuracy and numerical error [107]. At the same time, there is a need to use finer finite element meshes to obtain accurate results, so the cost of calculation will increase.

5.3. CI-FE/SEA Method
In the CI-FE/SEA method, in order to obtain more accurate results, it is necessary to increase the truncation order of CHEBYSHEV polynomials, and the increase of the truncation order will lead to an
exponential increase in the number of iterations. Therefore, the CI-FE/SEA method is only applicable to the medium frequency problem with moderate interval number and cannot satisfy the large interval parameters.

5.4. ES-FE-SEA Method
ES-FE-SEA method converts "appropriate softening effect" into discrete model, effectively solves the problem of too "rigid" finite element model, and makes the calculation results closer to the actual situation, but there is a risk of low interpolation accuracy. Like FE-SEA method, it is only suitable for dynamic response analysis with small interval range.

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