Superluminal Phenomena as Instantaneous Transferring of Excitations in Correspondence with the Wigner Principle of Causality

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It is shown that the transferring of light signal faster-than-c can take place exclusively as the instantaneous quantum tunneling. Its extent (space size of instanton) must be inversely relative to lack of energy till nearest stable or resonant state. "Superluminality" leads to nonlocality "in small", in a near field zone, and can be described by 4-vector $A_\mu$, the $E$ and $H$ fields remain local.

These assertions are proven, independently, in the frameworks of general field theory and by the theory of time duration of scattering, with taking into account the entropy growing. They are correlated with peculiarities of tunneling and frustrated total reflection. The results correspond to the Wigner formulation of causality: "The scattered wave cannot escape a scatterer before the initial wave reaches it" and are conformed to experimental data on superluminal transferring.

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1. INTRODUCTION

In the experiments [1-10] has been observed that the light signal could be transferred on definite distances via specially chosen media with speed greater vacuum light velocity ("superluminal" or "light faster-than-c"). Almost in all theoretical researches it has been adopted, from the outset, that such exotic speed is unreal, primary impossible, and therefore these observations would be explainable by some reformulation of interference conditions, by account of reshaping of pulses form and so on (e.g. [11-13]).

However, these experimental data can be considered as some manifestation of reality of the superluminal phenomena. Such point of view requires search of the exclusive conditions and opportunities at which superluminal phenomena are not absolutely impossible. Therewith such possibilities must be revealed on the most general basis without introduction of any additional hypotheses, and just such approach is the main purpose of the paper.

Let us begin with overview of the experimental data. Superluminal transmission of light pulses (or of its forward front only?) through some media has been registered in rarefied gas [1] (the most impressive observations of this effect have been made nearby the resonant frequencies [2, 3]), as well as under propagation of microwaves through the air (several experiments of two independent groups [4]). The superluminal speed of light pulses has been registered also at their propagation through solid-state films [5] and interfering filters [6], at frustrated total internal reflection (FTIR [7]). It results in some features of
diffraction pictures [8] and even the speed of front of ionization can be superluminal [9]. Moreover, in electronic systems such phenomena can be observed also [10]. Reviews of many experimental data and some theoretical models are given in [11] and in the proceedings of conferences [12], but we do not consider these models here [13].

These observations are indicative for the reality of the superluminal propagation or of signal transferring, at least along comparatively small, characteristic distances, and require a reasonable theoretical interpretation. Notice that to these observations must be added the long time noted discrepancy with results of tunneling calculations that show negative duration of such transitions (e.g. [14]).

How the strict restriction of signal speed over the large distances can be reconciled with these observations?

The regular measurements of signal speed were carried out in the far field, i.e. at distances, which are by far larger than characteristic scales, determined by the wavelengths and/or the uncertainty relations. Hence the superluminal phenomena could be only attributed to such scales that are negligible in the far field measurements, i.e. they should be limited by a near field zone. The energy propagation (group) speed determined by the Poynting’s vector in far field could remain subluminal.

The question under consideration arises from the beginning for some specification of the causality principle itself. Its conventional statement: “No outgoing signal can leave a point before it is reached by the initial one” is essentially classical. It is not applicable literally to the quantum theory of measurements, does not admitting so exact localization of the area of absorption and/or emission. Thereby, one has to adopt the weaken formulation given by Wigner [15]: “The scattered wave can not leave a scatterer before the initial wave reaches it”, which can be viewed as the general (or quantum) causality principle. It must be noted that the Wigner principle is reconcileable, in particular, with occurrence of some items distinct from zero outside of the light cone in the causal propagator - just they are suggestive for the appearance of superluminal phenomena [16].

The singularity and apparent exotic character of a problem requires its research from different standpoints. Therefore we shall begin our consideration from the rigorous mathematical, but sufficiently general investigation of an opportunity of existence and properties of the superluminal phenomena in the scope of general field theory, without concretization of media, in which can be observed such phenomena (Section 2, the Appendix A).

Then we consider an opportunity of description of superluminal phenomena within the framework of general theory of time delay with taking into account restrictions, which are imposed on parameters of processes by the requirements of entropy growth (Section 3-1, the Appendix B).

The opportunities of confirmation of general results are considered within the quantum electrodynamics (QED, Subsection 3-2), in the theory of tunneling (Subsection 3-3) and, in particular, in the phenomenon of the frustrated total internal reflection (FTIR, Subsection 3-4) as the most known and the most old example of tunneling. Then in the Section 4 we carry out comparison of our
results with experimental data. Some possible models and interpretations of results are discussed in the Section 5 and thereby are outlined some possibilities of further investigations.

As the method used in the Section 2 is not traditional one, let’s forewarning it by such remarks. We examine passage of a signal throw linear, homogeneous, stationary medium (without its further concretization). This process is described by the integral relationship:

\[ O(t, r) = \int dt' dr' R(t - t', r - r') f(t', r'). \]  

(1.1)

The set of response functions \( \{R(t, r)\} \) of this system may be divided formally on two classes: the set of local functions \( \{R_L(t, r)\} \), for which \( ct \geq r \), and the set of nonlocal functions \( \{R_{NL}(t, r)\} \), which are characterized by nonequality \( ct < r \) and thereby described the superluminal signal (energy) transferring only. The analysis of such rigorous division of functions of response onto two classes shows the exact limitation of peculiarities of the class of nonlocal functions.

The further analysis of this class of functions will showed, that the instant transfer of signal can occur only onto the certain distances \( a \), which must be determined by the difference between the energy of scattered particle and the energy of nearest stable or resonant state. Moreover, such transition can take place exclusively by the tunneling, i.e. they present a pure quantum phenomenon and corresponding response function does not depend on magnitudes of vectors \( \mathbf{E} \) and \( \mathbf{H} \). Therefore, just as in the effects of Aharonov-Bohm [17], Casimir (e.g. [18]) and so on, superluminal phenomena should be described via potential \( A_\mu \) only.

All subsequent items demonstrate concretization and illustrations of these principal results. Notice that the outstripping of particles on completely determined distances can be described as the ”gain duration” in its kinematics and is not reflected on properties of far field.

2. PERMISSIBLE FORMS OF RESPONSE FUNCTIONS

2-1. LOCAL AND NONLOCAL INTERACTIONS

The method of division of response functions on various classes is based on usage of projectors of properties of physical systems introduced by von Neumann [19] for description of qualitative (alternative) properties of systems, which are completely present or are completely absent, such as causality, locality, mass-spectrality, definite symmetries, etc.

In the quantum mechanics projectors of type \( P_\psi = |\psi\rangle \langle \psi| \) are used for separation of certain states of systems in the Hilbert space, on the base of which are constructed matrices of density [20], etc. Unlike it, we shall consider projectors of allowable supports of response functions (areas of change of variables, outside of which functions equal zero) in the usual time-space, the corresponding method was proposed and developed in [21, 22].

Projectors of locality (of light cone) \( P_L \) and of nonlocality (the areas outside of light cone) \( P_{NL} \) are, accordingly, represented as

\[ P_L(x) = \theta(x^2) \equiv \theta(t^2 - r^2), \quad P_{NL}(x) = 1 - \theta(x^2) \equiv \theta(-x^2). \]  

(2.1)
Any function can be decomposed on local and nonlocal parts: \( R(x) = R_L(x) + R_{NL}(x) \). These parts can be separated from any function by the appropriate projector:

\[
R(x) = P_L(x) R(x), \quad R_{NL}(x) = P_{NL}(x) R(x) .
\] (2.2)

The condition of completeness, \( P_L + P_{NL} = 1 \) (see the Appendix A), allows to deduce from (2.2) the equations of orthogonality for these functions:

\[
P_{NL}(x) R_L(x) = 0; \quad P_L(x) R_{NL}(x) = 0,
\] (2.3)

i.e. the projector of light cone \( P_L \) is orthogonal to functions with support out of the cone, etc.

From the law of energy conservation for classical systems or from the principle of unitarity for quantum systems follows that functions, which describe input and output signals, are square integrable. Therefore for them and for the function of medium response \( R(t, r) \) exist the complete and partial Fourier-transforms (in the rigged Hilbert space [21] at least):

\[
O(\omega, k) = R(\omega, k) I(\omega, k) \quad (1.1')
\]

The Fourier transformation of (2.3) to \( (\omega, k) \)-representation with simultaneous account of causality (the projector \( P_C(x) = \theta(-t) \), i.e. at allocation only the bottom part of light cone) leads to relativistic generalization of Kramers-Kröning dispersion relations of complex structure [22], though admitting the approximated forms [23].

Let's consider these equations of restriction in other variables. In variables \((t, k)\) the projector gets a kind:

\[
P_L(t|k) \equiv F_k[P_L(t, r)] = (\frac{1}{2\pi})^3 \int d\rho e^{-ik\rho} \theta(t^2 - \rho^2) = \frac{1}{2\pi^2 k^3} \sin(kt) - kt \cos(kt) .
\] (2.5)

Hence, \( P_L(t|k) \to \delta(k) \) at \( t \to \infty \), and at \( t \to 0 \) it is presented as

\[
P_L(t|k) \to \frac{1}{2\pi^2 k^3} \sum_{n=1}^{\infty} (-1)^n n \frac{t^n}{(kt)^{2n+1}} . \quad (2.6)
\]

The Fourier transformation of (2.3) leads to the integral equation for local functions in these variables:

\[
R_L(t|k) = \int d\rho P_L(t|\rho) R_L(t|k - \rho) , \quad (2.7)
\]

from which, via (2.6) and so on, some asymptotic estimations can be received.

The research of features of completely nonlocal transfer functions on the basis of (2.4) is the most interesting. The Fourier transform of this equation results in the equation in the equation:

\[
\int d\rho P_L(t|\rho) \ R_{NL}(t|k - \rho) = 0 . \quad (2.8)
\]

As its feasibility does not depend on magnitudes of \( t \) and \( k \), it is reduced to the algebraic equation:

\[
P_L(t|\rho) \ R_{NL}(t|k - \rho) = 0 . \quad (2.9)
\]

and its equality to zero is determined by features of \( P_L(t|\rho) \) as function of variable \( t \).

On the basis of fundamental definition \( \xi \times \delta(\xi) = 0 \) can be shown that if function \( P(x) \) is nonsingular with only isolated zeros in the points \( x_m \) (not higher \( n \)-th order), the general solution of the algebraic equation \( P(x) \times f(x) = 0 \) is of the such form:
\[ f(x) = A \sum_n \sum_m a_{n,m} \delta^{(n)}(x-x_m) \] (2.10)

with arbitrary factors \(a_{n,m}\), which other reasons or models may determine.

Hence, the solution of (2.9) has the form:

\[ R_{NL}(t|k, q) = f(t|k, q) \delta(P_L(t|k)), \] (2.11)

and it is necessary to determine in the argument of \(\delta\)-function its zeros as function of temporary variable. As in the expression (2.5) the values \(kt = 2\pi n\) are no its roots, from the argument of \(\delta\)-function can be factored out \(\cos(kt)/2\pi^2 k^3\), and thus in (2.11) remains \(\delta(\tan(kt) - kt)\). The first numerical roots of the transcendent equation \(\tan x = x\) are equal to 0; 4.493; 7.725; (approximately). As it will be shown below, the minimal extent of path at non-local interaction is equal to the wavelength \(\lambda\), therefore if \(kt = b\), the maximal speed of pulse transfer must be \(u \approx \lambda/t = 2\pi c/b\). Hence, at decomposition of the argument of \(\delta\)-function all roots, except the first two, can be omitted.

In accordance with (2.6) the zero in point \(t = 0\) is of the third order and consequently the general solution of (2.11) must be written down as

\[ R_{NL}(t|k, q) = A\delta(P_L(t|k, q)) = \varphi_0(0) + \varphi_1(0) + \varphi_2(0) + \varphi_3(0) + ... \] (2.12)

with arbitrary functions \(\varphi_n = \varphi_n(k, q)\). Last term of (2.12) results to \(\lambda/t \sim 1.4c\) and apparently is of type of harbinger or aftershock and therefore further is not examined.

Other terms of (2.12) show that if there is the nonlocal transfer of interaction, it should occur instantaneously, without dependence on time and consequently must be described by the stationary Laplace equation. The substitution of \(R_{NL}(t, r)\) in (1.1) leads to the expression

\[ O_{NL}(t, r) = \int d^3r' f_0(r-r') + f_1(r-r')\left(\partial/\partial t\right) + f_2(r-r')\left(\partial/\partial t\right)^2 \] \(I(t, r'), \)

(2.13)

where functions \(f_n(r)\), which describe medium, are not concretized. The expression (2.13) show that the nonlocal causal response of system depends on spatial properties of medium and on velocity and acceleration of the source changing, i.e. on forces operating in it.

The instantaneous transferring of all three characteristics of process is necessary and is sufficient for complete restoration of the initial form and dynamics of process. Three spatial transfer functions included in (2.13) can be considered as independents, concerning, accordingly, to static, kinematical and dynamic parts of nonlocal field.

2-2. DISTANCES OF INSTANTANEOUS JUMPS OF PHOTONS

Let’s consider the projector describing instant jump of particle into the \(x\) direction on distances not smaller \(a\). Projectors of such property should include the bottom part and the top part of light cone remote at the moment \(t = 0\) on distance \(a\):

\[ P_a(t, x) = \theta(-t)\theta(t^2 - x^2) + \theta(t)\theta(t^2 - (x+a)^2). \] (2.14)

This projector separates within the whole set of response functions those functions, which correspond to jump on distances not smaller \(a\):
\( R_\alpha(t, r) = P_\alpha(t, x) R(t, r). \) (2.15)

Thereby they should satisfy the equation of orthogonality:

\( (1 - P_\alpha(t, x)) R_\alpha(t, r) = 0. \) (2.15')

Fourier-transform of the projector in (2.15') as function of temporary variable has the form:

\[
F_\omega[1 - P_\alpha(t, x)] = \left( \frac{1}{2\pi} \right) \omega \left( e^{-i\omega|x|} - e^{-i\omega|x+a|} \right), \quad (2.16)
\]

and the response function of system describing instant jumps on distances \( l \geq a \) is of type

\[
R_\alpha(\omega|\omega' - a) = f(\omega|\omega') \delta(\omega - \omega'), \quad (2.17)
\]

The zeros of argument of \( \delta \)-function correspond to the equality \( \omega a = 2\pi n \).

Thus the spatial extent of instant jumping can be equal to one or several wavelengths:

\[
a = n\lambda, \quad n = 1, 2, 3, ..., \quad (2.18)
\]

i.e. exceeds, already with \( n = 1 \), the meaning of uncertainty \( \Delta x \Delta k \geq \frac{1}{2} \), \( \Delta x \approx a, \Delta k \approx 2\pi/\lambda \) and can be measurable. (Such possibility was announced in [24].)

If the considered process is determined by the difference between energies of scattered particle and of stable (or resonant) state \( \Delta \omega = \omega - \omega_0 \), it is necessary to carry out, already in (2.15), the appropriate subtraction:

\[
R_\alpha(\omega|\omega - a) \rightarrow R_\alpha(\Delta \omega|\omega'). \quad (2.19)
\]

The replacement \( \omega \rightarrow \Delta \omega \) in (2.16) leads to the length of instant tunnel jump:

\[
a = \left( \frac{2\pi c}{\Delta \omega} \right) n, \quad n = 1, 2, 3, ..., \quad (2.18')
\]

that corresponds, at \( n = 1 \), to the distance or to the "gain time", measured in the experiments [2].

### 2-3. ENERGY-MOMENT RELATION

The interaction functions (matrix elements of transition) in variables \((\omega, k)\) can be naturally divided, up to an exit on the mass surface, onto two classes: \( R_E(\omega, k) \), at which \(|\omega| > |k|\), i.e. there is a surplus of energy relative to moment, and \( R_{NE}(\omega, k) \), at which \(|\omega| < |k|\), i.e. the energy is lesser than value appropriate to moment.

By introducing the projectors of 4-cone \((\omega, k)\),

\[
P_E(\omega, k) = \theta(\omega^2 - k^2), \quad P_{NE}(\omega, k) = \theta(k^2 - \omega^2) = 1 - P_E(\omega, k), \quad (2.20)
\]

we receive, by the exact analogy with (2.3-4), the equations of orthogonality:

\[
P_{NE}(\omega, k) R_E(\omega, k) = 0, \quad (2.21)
P_E(\omega, k) R_{NE}(\omega, k) = 0. \quad (2.22)
\]

The Fourier transforms of these projectors to variables \((t, k)\) are

\[
P_{NE}(t|k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \omega e^{-i\omega t} \frac{\sin(kt)}{\pi t}, \quad (2.23)
P_E(t|k) = \delta(t) - P_{NE}(t|k). \quad (2.23')
\]

The Equation (2.21) results, in view of the condition of causality \((f_E(t|k) = 0\) at \( t > 0 \), in the integral relation:

\[\int_{-\infty}^{t} dt R_E(t - t|k) \frac{\sin(kt)}{\pi t} = 0. \quad (2.24)\]
As integrand does not depend on magnitudes of $t$ and $k$, it should be equal to zero, and its solution, as well as above, can be expressed through $\delta$-function depending on $t$ and $k$:

$$R_E(t|k) = f(t|k) \delta(\sin(kt)/t) = f(t|k) t \delta(\sin(kt)). \quad (2.25)$$

Delta-function of periodic argument is represented by series:

$$\delta(\sin x) = \sum_{0}^{\infty} \delta(x - \pi n),$$

and consequently the solution of (2.21) can be written as

$$R_E(t|k) = \pi t^2 f(t|k) \sum_{1}^{\infty} \delta(k - \pi n/t), \quad (2.26)$$

in which, by virtue of the extra factor $t$ in (2.25), there is no term with $n = 0$.

From here follows that if the energy of system exceeds the value appropriate to moment, the instant transfer of interaction is impossible, since it requires the infinite large moment!

The expression (2.26) corresponds, as though, to natural condition of resonance transfer of the ideal retarded signal in any system with superfluous energy.

Let’s consider $(\omega, r)$-representation of the Equation (2.22). As it is identical to (2.9) with replacements $(t, k) \rightarrow (\omega, r)$ only, then by analogy with (2.12) it must be written that

$$R_{NE}(\omega|r) = \varphi_0(r)\delta(\omega) + \varphi_1(r)\delta'(\omega) + \varphi_2(r)\delta''(\omega) + \varphi_3(r)\delta(\omega - 4.5/r) + ... \quad (2.27)$$

or, in the temporary representation,

$$R_{NE}(t|r) = f_0(r) + t f_1(r) + t^2 f_2(r) + ... \quad (2.28)$$

Hence, at the lacking of energy concerning value of moment (it is just the case of tunneling) the opportunity of instant transition at $t = 0$ is not excluded and may be described by the function $f_0(r)$.

Here should be specially underlined, that this nonlocality does not lead to nonlocality of $E$ and $H$ fields. Such conclusion follows from consideration of the commutators of fields, as response functions must be expressed through Green functions:

$$[E_i(x), E_j(y)] = [H_i(x), H_j(y)] = \frac{1}{4\pi^2} \{ \partial_j \partial_j - \delta_{ij} \partial_j^2 \} D(x - y);$$

$$[E_i(x), H_j(y)] = \frac{1}{4\pi^2} \partial_j \partial_j D(x - y), \quad (2.29)$$

and differentiation of response function removes the unique nonlocal term $f_0(r)$ in (2.28).

Thus, nonlocality should be observable basically in such phenomena as the Aharonov-Bohm effect, the Casimir effect, effects of near field optics, etc., caused by the peculiarities of field $A_\mu$. Moreover, the possibility of movement with superluminal speed is the pure quantum phenomenon, and therefore, in the complete consent with the theory of relativity, cannot be described in the scope of classical theory.

3. DURATION OF INTERACTION
3-1. DURATION OF STATE FORMATION AND PRINCIPLE OF PASSIVITY

The Wigner’s causality admitted a negative “durations of delay” $\tau > -a$ and it was assumed that $a$ can be the Compton wavelength. It seems natural to
suggest that for photons such estimation must be conformed to the uncertainty principle and/or can be associated with an extent of near field.

Let us consider this problem in detail. The complex function of duration of interaction can be determined by expansion of $R(\omega)$ or its logarithm in the vicinity of some characteristic frequency of medium $\omega_0$ as:

$$\ln R(\omega, \mathbf{r}) = \ln R(\omega_0, \mathbf{r}) + i(\omega - \omega_0) \tau(\omega_0, \mathbf{r}) + \frac{1}{2}(\omega - \omega_0)^2 \tau'(\omega_0, \mathbf{r}) + \ldots \quad (3.1)$$

with

$$\tau(\omega, \mathbf{r}) \equiv \tau_1 + i\tau_2 = (\partial_i/\partial\omega) \ln R(\omega, \mathbf{r}). \quad (3.2)$$

In this representation $\tau_1(\omega)$ is the time-delay during elastic scattering (e.g. [25]) and $\tau_2(\omega)$ is the duration of final state formation, some of their peculiarities are outlined in the Appendix.

The physical significance of $\tau_2$ becomes more transparent in its formulation in the S-matrix theory as $\tau_2 = \partial \ln |S|/\partial \omega$. Hence, $\tau_2$ can be considered as a measure of temporary non-completion of the final (free photon) state: by definition, S-matrix should be unitary while describing transition between real physical states (cf. [26]).

If we shall restrict our consideration to first two terms of (3.1), then since for passive linear media $|R(\omega)|^2 = |R(\omega_0)|^2 \exp[-2(\omega - \omega_0)\tau_2] \leq 1$, the inequality

$$|\omega - \omega_0|\tau_2(\omega_0, \mathbf{r}) \geq 0 \quad (3.3)$$

should be valid, and, therefore, the formation time $\tau_2 < 0$ under some values of $\omega < \omega_0$.

Thus, the advanced or instantaneous, in the sense of expression (1.1), phenomena could be observed within a frequency range below the resonance one, just in the correspondence with the Subsection 2-3.

In macroscopic theories the role of medium response function plays the Fourier-transform of dielectric susceptibility, $\varepsilon(\omega, \mathbf{r})$. Possibility of the negative sign of formation time in these theories is emerged from such general precept: in passive media the principle of entropy growing results in the strict inequality:

$$\partial(\omega \varepsilon(\omega))/\partial \omega \geq 0 \quad [27 \cdot$$

At substitution $R \rightarrow \varepsilon(\omega) - \varepsilon(\infty) = \varepsilon_1 + i\varepsilon_2$ in (3.1) this general inequality can be rewritten as

$$\tau_2 \leq \frac{1}{\omega} - \tau_1 \frac{i\varepsilon_2}{\varepsilon_1^2}. \quad (3.4)$$

Hence in the region of abnormal dispersion, where must be expected discordance between maxima of $\tau_1$ and $\tau_2$, the value of $\tau_2$ can be negative.

Temporal functions of the simplest form can be obtained in a model for dielectric susceptibility of media with single isolated resonance:

$$\varepsilon(\omega; \mathbf{r}_1, \mathbf{r}_2) = \rho(\mathbf{r}_1, \mathbf{r}_2)/[\omega_0^2 - (\omega + \frac{1}{2}\Gamma)^2]. \quad (3.5)$$

After substitution in the definition (3.2), this gives at $|\omega| \ll \omega_0 >> \Gamma$:

$$\tau_1 = \frac{\Gamma}{2\omega^2 + (\Gamma/2)^2} + \frac{\Gamma}{2\omega^2 + (\Gamma/2)^2} \approx \frac{\Gamma}{2\omega^2 + (\Gamma/2)^2}, \quad (3.6)$$

$$\tau_2 = \frac{\delta\omega}{2\omega^2 + (\Gamma/2)^2} - \frac{\delta\omega}{2\omega^2 + (\Gamma/2)^2} \approx \frac{\delta\omega}{2\omega^2 + (\Gamma/2)^2}, \quad (3.7)$$

where $\delta\omega = (\omega_0 - \omega)$, $\omega = (\omega_0 + \omega)/2$.

Due to its conformity with the Breit-Wigner formula the physical sense of expression (3.6) is obvious. The expression for $\tau_2$ is close to the uncertainty value, evidently admits negative $\tau_2$ and could be interpreted as advanced emission of photons or as their instantaneous jumps onto the distance $c|\tau_2|$.

Making use of these expressions, the condition (3.4) transforms in
\[ \tau_2 \leq \frac{1}{\omega} - \frac{\omega^2 + (\Gamma/2)^2}{\omega^2 - \omega^2 + \Gamma^2/4}. \quad (3.8) \]

At the margin of resonant line, \(|\delta \omega| \geq \Gamma/2\), one can accept that
\[ \tau_{\text{eff}} \equiv \tau_2 - \frac{1}{\omega} \leq -\frac{1}{4 \delta \omega} \left( \frac{\Gamma^2}{\delta \omega^2 + (\Gamma/2)^2} \right). \quad (3.9) \]

Thus, a negative duration of formation (the "gain of time" at superluminal jump) in gaseous media is possible only and only at \( \omega < \omega_0 \), and it completely confirms results of Subsection 2-3 above (just such frequencies were used in the quoted experiments). Moreover, the length of superluminal “jump” exceeds the atomic sizes, at \( c(\tau_2 - \frac{1}{\omega}) \sim 10^{-8} \), only if \(|\delta \omega| \leq 10^{11} \), i.e. only within the specially prepared media of a type described in [1-3].

3-2. QUANTUM ELECTRODYNAMICS

The causal, Stueckelberg-Feynman, propagator \( D_c \) includes the space-like part, nonvanishing beyond of light cone, i.e. contains the superluminal terms. This peculiarity is usually intuitively understood as a result of vacuum fluctuations, descriptively interpreting through uncertainty principles.

Really, the photon propagator of the lowest order is written as
\[ D_c(t, r) = \bar{D} + \frac{1}{4} D^{(1)} = \frac{1}{4\pi} \delta(c^2t^2 - r^2) + \frac{1}{2\pi} \frac{ic}{c^2t^2 - r^2}, \quad (3.10) \]
where the first term (the Pauli - Jordan function) is supported in the light cone, but the second term (the Hadamard function) partially goes out the cone, though with a fast attenuation in the outside region. Classical electrodynamics is based on the retarded interaction or on the propagator \( \bar{D} \) only [28] and does not include superluminal terms.

In the general case, the first term of the causal propagator is responsible for the restriction of support of space-time parameters into the light cone, \( c^2t^2 - r^2 \geq 0 \), and the second one is responsible for the restriction of support of energy-moments parameters: \( E^2 - p^2c^2 \geq m^2c^4 \). These requirements were considered in the Section 2 from another point of view. Fulfillment of both requirements by any single function is impossible; these two functions have different physical sense, which can conform to different space-time effects [22]. Therefore the durations, resulting from these two parts, may have different physical sense and different magnitudes.

In the \((\omega, k)\) representation at \( \eta \to +0 \) the causal propagator of the lowest order has in the Feynman gauge such form:
\[ D_c(\omega, k) = \frac{4\pi}{\omega^2 - k^2 + i\eta} = \frac{4\pi}{\omega^2 - k^2} \exp \left( \frac{-i\eta}{\omega^2 - k^2} \right). \quad (3.11) \]

Thereby the delay time and duration of formation are:
\[ \tau_1(\omega, k) = -2\pi \delta(\omega^2 - k^2), \quad (3.12) \]
\[ \tau_2(\omega, k) = \frac{2\omega}{\omega - k^2} \sim \frac{1}{\omega^2 - |k|^2}. \quad (3.12') \]

The function \( \tau_1 \) obviously describes the reemission of dressed photon. The function \( \tau_2 \) shows that an outstripping is possible only at the condition of certain mismatch between photon frequency and moment. Just such mismatch takes place under conditions of the abnormal dispersion and frustrated total internal reflection, when the absolute value of moment grows up.

In our case, the response function (transfer amplitude) consists of the photon propagators and the vertex parts. It can be assumed that in the lowest order
the delay and duration times at each elementary step of photon transfer will be determined by the propagators only.

Let’s continue our examinations in the \((\omega, r)\) representation:

\[
D_c(\omega, r) = -\frac{1}{4\pi r} e^{i|\omega| r}; \quad D^{(1)}(\omega, r) = -\frac{1}{2\pi r} \sin(\omega r),
\]

and we receive by consideration the equation (B.1) such temporal functions:

\[
\tau_c(\omega, r) = r \text{sgn} \omega; \quad (3.14)
\]

\[
\tau^{(1)}(\omega, r) = -ir \cot(\omega r). \quad (3.15)
\]

The expression (3.14) shows that causal propagators describe free photons propagations in wave zone and it means that the usual QED calculations practically exclude phenomena connected with near fields.

The expression (3.15) reveals that response function tends to peaks as \(\omega r \to 2\pi n\) from below, i.e. at \(\tau^{(1)}(\omega, r) \to -\infty\). Subsequent terms of the expansion (3.1), as they are expressed through derivatives of \(\tau^{(1)}\), only sharpen these peaks and therefore it can be concluded that the results of QED completely confirm our main condition (2.18).

3-3. TUNNELING

Tunnel transitions, as is known, are intimately related with the imaginary values of momenta. As the demonstrative example of such phenomena the tunnel transition of particle with energy \(E\) through the rectangular barrier \(U(x)\) in the 1-D space range \((-a, a)\), where \(E < U_0\), can be considered.

Energy of particle moving in the potential \(U(x)\) is equal to \(E = p^2/2m + U(x)\). For reaching the classically forbidden region, \(E < U\), kinetic energy should be negative, corresponding to imaginary momenta \(p\). In the WKB approximation the wave function \(\psi(x) \sim \exp(i\Phi(x))\) with \(\Phi(x) = \pm \int_{x'}^{x} dx'p(x')/\hbar + O(\hbar),\) where \(p(x) = [2m (U(x) - E)]^{1/2}\) is the local momentum. In the classical region, the wave function is oscillatory one, while in the classically forbidden region (corresponding to imaginary momenta) the wave function is exponentially suppressed.

The matrix element of this transition is of order

\[
M \sim \exp(-\int_{-a}^{a} [2m (U(x) - E)/\hbar^2]^{1/2} dx), \quad (3.16)
\]

where \(a\) is determined from the equality \(U(\pm a) = E\). This representation leads to the following expression for time durations:

\[
\tau(E) = \hbar \frac{\partial}{\partial E} \ln M = -\text{Im} \int_{-a}^{a} [2m (U(x) - E)/\hbar^2]^{-1/2} dx, \quad (3.17)
\]

which can be rewritten via imaginary moment of tunneling particle. Here can be noted that at the considering of tunnel processes by the Landau method the transition to an ”imaginary” time is often employed, but as the formal procedure only [29].

For a sufficiently common case of the oscillator (parabolic) barrier \(U(x) = U_0 - \frac{1}{2} \kappa x^2\) the expression (3.17) leads to such results:

\[
\tau_1(E) = 0; \quad \tau_2(E) = -\frac{2\pi}{\kappa U_0} = -\frac{\lambda_0}{4c} \quad (3.18)
\]

with an oscillator frequency \(\omega_0 = (k/m)^{1/2}\), i.e. behind the barrier the complete energy of particle (wave) must appear instantaneously and regardless.
of its value, as $\tau_2$ in (3.18) corresponds to $\lambda/4c$. This property is retained even under energy absorption at the tunneling state, e.g. at substitution $U_0 \rightarrow U_0 + U_1 \cos \omega t$, if $U_0 + U_1 > E$. Note that $|\omega_0 \tau_2|$ is bigger than uncertainty magnitude and hence can be measurable.

Independence from the energy does not represent the common property of tunnel transitions. So for the rectangular barrier $\tau_2(E) = -[2m a^2/(U - E)]^{1/2}$, and here the advancing duration (extent of instant jump) varies with $U(t)$.

The expression (3.17) really shows that the tunnel transition proceed without delays as $\tau_1 = 0$, but $\tau_2$ is negative and in general qualitatively corresponds to the uncertainty value. For $E > U_0$, evidently, $\tau_1 \neq 0$ and both its signs are, in principle, possible.

Let’s consider, as a more specific example, the variation of wave function in this process by direct calculations. Let the process begins with the Gaussian wave packet, centered around $k_1$, on the left side of barrier

$$\psi_i(t, x) = \int_{-\infty}^{\infty} dk f(k - k_1) \exp[i(kx - E(k)t)], \quad (3.19)$$

The usual calculation of function on the right side of barrier leads to

$$\psi_f(t, x) \approx \psi_i(t, x - 2a), \quad (3.21)$$

which shows that the transmission through barrier is instantaneous (unessential phase factors are omitted).

The strange, contradictory and curious, as seems, effect of instantaneous packet transfer is usually explained as erroneous, explicable by physical non-completeness of nonrelativistic Schrödinger equation (e.g. [14]). But as was shown by different methods, just similar instantaneous jumps are characteristic for completely relativistic QED expressions. Hence may be concluded that the possibilities of superluminal or even instant transitions are the characteristic peculiarity of quantum tunneling.

3-4. FRUSTRATED TOTAL INTERNAL REFLECTION (FTIR)

For analysis of experiments [7] the basic peculiarities of phenomenon of FTIR must be considered. As FTIR can be described classically, we can begin the consideration with the Fresnel formulas. In accordance with them amplitudes of evanescent waves, tunneling into optically lesser dense medium under the angle $\vartheta \geq \sin^{-1}(1/n)$, and amplitudes of appropriate usual waves in $(x, z)$ plane are connected on the borderline $z = 0$ by the response function [30]:

$$S_{e}(\omega|t, x, z) = A(n, \vartheta) \exp[-i\omega(t - x n \sin \vartheta) - \omega s z], \quad (3.22)$$

where $s = (n^2 \sin^2 \vartheta - 1)^{1/2}$, $n > 1$, factor $A(n, \vartheta)$ depends on polarization, etc.

The condition of completeness of (3.22) should be expressed as

$$\int_{0}^{\infty} |S_e|^2 dz = 1, \quad (3.23)$$

which shows that in the description of FTIR must be included all trajectories of evanescent “rays” with $z \in [0, \infty)$. But if durations of their passage depend on $z$, the short initial wave pulse should be effectively extended after each act of FTIR. However, already the waveguides practice shows absence of appreciable expansion of short pulses [31] (just absence of an expansion is the basic one for
single mode light guides). Therefore it should be assumed, that the durations of "geometrically differing trajectories of rays" do not depend on \( z \) and these durations should be (approximately, at least) equal, i.e. the transverse shifts of "trajectories" of evanescent rays must be instant: the duration of signal propagation in \( x \) direction in the area of FTIR should been determined by the speed component \( v_x = (c/n) \sin \theta \) only.

The direct analogy to the consideration carried out above permits to rewrite (3.22) as \( S_e(\omega; r) = |S_e| \exp(i\Phi) \) with the real phase \( \Phi(\omega, r) \) and then, according to (3.2),

\[
\tau_1 = \frac{\partial \Phi}{\partial \omega} = -(t - n \sin \theta), \quad (3.24)
\]

\[
\tau_2 = \frac{n}{c} \ln |S_e| = -z[s + \frac{x}{2} n \sin^2 \theta \frac{\partial n}{\partial \omega}], \quad (3.24')
\]

Hence the response function for transparent linear medium and weak light flux can be written down at \( |\omega| \sim \omega_0 \) as

\[
S(\omega, r) = S(\omega_0, r) \exp\{i \tau_1(\omega_0, r) - \tau_2(\omega_0, r)\}. \quad (3.25)
\]

On the other hand, we can begin with (3.25) as with the independent determination of response function. Then according to [32] the changes of moment of particle during quantum transition is determined as \( \Delta p = \rho \langle E \rangle V T_p \), where \( \rho \) is the average density of charges in volume of interaction \( V \), \( \langle E \rangle \) is the average intensity of an internal field, \( T_p \) is the duration of process. At FTIR the magnitude and direction of additional moment, received by photon in the area of FTIR, should be determined by parameters of field \( E \) in immediate proximity to media interface layer. And actually this formula conducts to the condition, described above: the performance of (3.22) corresponds to the transformation of moment, \( k_z \to ik_z \), and (3.25) corresponds to the representation of complex temporal function \( T_p \to \tau_1 + i\tau_2 \), which at \( \tau_1 = 0 \) leads to concordance of both approaches.

Note that on the same basis some other phenomena, analogs of FTIR, can be considered (their description is given e.g. in [33]).

4. COMPARISON WITH EXPERIMENTS

The cited experiments, except FTIR, can be divided into two groups: 1). The resonant ones, in which "gain time" can be, in principle, measurable after single scattering act, and 2). Nonresonant (relative to atomic frequencies) processes of light flux passage through media, where tiny "gain" durations can be accumulated via sequential interactions.

1). The best demonstration of existence of superluminal propagation is presented in the articles [2]: the weak almost resonant laser pulse of \( \lambda = 0.8521 \) mcm was passed through tube of 6 cm length with atomic Cesium at temperature of 30\(^{\circ}\)C. At the deviation of frequency from resonance into the abnormal dispersion side on \( \Delta \nu = 1.9 \) MHz the light pulse, propagated through tube, outstrips light pulse, transferred through vacuum, on \( (62 \pm 1) \) nsec, which can be named "the gain time".

The estimation of free path length and almost complete similarity of the forms of input and output pulses show, that the pulse has undergone only one act of outstripping scattering in tube. In support of this assumption can be noted that the output signal is of order 40% of an entrance signal that corresponds
to division of an entrance pulse in each scattering act on approximately equal retarding and advancing parts. As the width of top level $\Gamma = 0.37 \cdot 10^8 \text{ sec}^{-1}$ [20], the estimation of advancing by the formula (3.12) conducts to the "gain duration" $\tau_{e,f} \sim \tau_2 - 1/\omega \sim -59.4 \text{ nsec}$ in the consent with measured value of advancing or outstripping.

The experiments [3] are of special interest to suggested theory. In them the superluminal features of pulses propagated through microcavity containing a few cold atoms of $^{85}\text{Rb}$ were observed. Number of atoms was varied and, in principle, the results could be extrapolated till single atom in the resonator. The observations of superluminal propagation in such "substance" evidently prove that this phenomenon is not caused by rearrangement of wave front [11-13] and so on, but it must be related to processes of scattering on individual atoms.

The numerical treatment of these experiments show that with detuning of initial laser frequency on $53 \text{ MHz}$ the advancing of signal on $170 \text{ nsec}$ is observed, but with detuning on $45 \text{ MHz}$ it gave way to delay on the $440 \text{ nsec}$. It means that the strict resonance is above the frequency of superluminal signal not more than on $8 \text{ MHz}$. On the other hand the observed outstrip on $170 \text{ nsec}$ corresponds, in accordance with (2.12), to difference of frequencies of order of $5.88 \text{ MHz}$ from resonance into the region of abnormal dispersion.

Hence we should conclude the conformity of these experimental data with suggested theory.

Now let consider the first, as far as we know, observation of the superluminal phenomenon [1]. In this experiment it was established that the pulse of He-Ne laser ($\lambda = 0.6328 \text{ mcm}$) passes through a tube with $^{20}\text{Ne}$ ($p = 2.6 \text{ Torr}$, $L = 16 \text{ cm}$) with the group velocity $u = 1.0003c$.

Let's try to be limited by consideration of this experiment to account of the closest resonance on $\lambda_{21} = 0.6334 \text{ mcm}$ with $A_{21} = 1.36 \cdot 10^7 \text{ sec}^{-1}$ only. Density of atoms in the pipe with such pressure of gas is of order $N_a = 9.2 \cdot 10^{16} \text{ atoms/cm}^3$ (i.e. $N_e \sim 8N_a$ electrons of upper atomic shell considered as scatterers). The cross-section of unbiased elastic scattering $\sigma = 16\pi \left[ \frac{2i+1}{2i+1} \right] \cdot \delta \omega \left( \frac{2}{i} \right)^2 |r_{e,f}| \approx 3 \cdot 10^{-20} \text{ cm}^2$. Hence the free path length $l = 1/N_e \sigma = 45 \text{ cm}$ and each photon cannot suffer more than one scattering act. Thereby the "gain duration" should be calculated for single atoms: as the difference of frequencies is of order of $453 \text{ GHz}$, it leads, in accordance with (3.12), to the advancing on $0.35 \text{ psec}$. These values gives $u/c \sim 1.0006$, qualitatively corresponding to the results of [1].

2). Let's consider the experiments, in which the superluminal phenomena in nonresonant (relative to atomic levels) conditions were observed. For such cases all photons can be divided on "superluminal" and usual ones. For "superluminal" photons, which have experienced consecutive outstripping (advanced) interactions only, the average number of such interactions on distance $L$ will be of order $N = L/(l + \Delta l)$, where $l = 1/\rho \sigma$ is the free path length, $\rho$ is the density of scatterers, $\sigma$ is the complete cross-section of $e-\gamma$ scattering and $\Delta l = c|\tau_2|$ for $\tau_2 < 0$. Therefore, the durations of light flux flight through vacuum and superluminal photons flight and jumps through medium are equal, accordingly,
to
\[ T = \frac{L}{c}; \quad T_{\text{adv}} = \frac{(L - N\Delta l)}{c} \to T - \Delta T. \] (4.1)

It results in an obvious expression for average speed of “superluminal” photons:
\[ \frac{u}{c} = \frac{T}{T_{\text{adv}}} = \frac{1}{1 - c\rho\sigma|\tau_2|}. \] (4.2)

In the experiments [5] superluminal propagation of pulse of intensity $3 \div 100$ W/cm$^2$ through the film of GaP:N with changes of thickness of film in an interval $9.5 \div 76$ mcm was investigated with variation of the laser radiation frequency around the isolated A-line (534 nm). The received diagram of dependence of duration of delay (positive and negative) as function of frequency of light qualitatively corresponds to the expression (3.4) with the dramatic transition from subluminal to superluminal speeds.

In the series of experiments [6] passage of light pulse through multilayer dielectric mirrors was investigated. The mirrors contained alternating layers of thickness $\lambda/4$ with high ($H$) and low ($L$) indices of refraction, so they were of structure $(HL)^m H$, in whole were investigated mirrors with $m = 3 \div 11$.

From the classical point of view each pair of layers should completely reflect falling resonant wave. But as some photons are passed, due tiny superluminal jumps, for those times on larger distances, they slip through interferential reflecting planes. Here can be accepted that as at nonresonant scattering two Feynman graphs (retarded and advanced) lead to approximately equal contributions, so each pair of layers approximately halves passed photon flux into reflected and transmitted parts. Therefore, if the photon free path length $l < \lambda/4$, intensity of light, missed through such mirror, should contain an outstripping flow of intensity $J_{\text{adv}}(m) \sim J_0/2^m$ that corresponds to measurements.

As the examined process is nonresonant, then according to principle of uncertainty or (3.4), where far from all self frequencies the second term can be omitted, we accept that $\tau_2 = -1/2\omega$, and it means, that the process of scattering occurs on almost free electrons. In the same approximation, according to the optical theorem of scattering theory, complete cross-section of interaction $\sigma_{\text{tot}} = (4\pi c/\omega) r_0$, $r_0 = e^2/mc^2$. Thus at density of external electrons $\rho = 1.3 \cdot 10^{21}$ cm$^{-3}$, the relation (4.2) conducts to $u/c = 1.56$, that corresponds to the experimental data.

The experiments in the microwave range [4] are more difficult for analyzing, as in them the role of boundary conditions can be essential. Thereby we shall limit ourselves by qualitative consideration of superluminal propagation noticed at pass of GHz waves by air between two aerials. If such process can be described by instantaneous jump of wave on $\Delta l$ (outstripping radiation) at distance $x$ between megaphones or aerials, it becomes necessary to expect change of relative speed as $u/c = 1/[1 - \Delta l/x]$ and analogical dependence was observed in these experiments.

It is necessary to notice, that the independence of all these effects on polarization is caused, obviously, by absence of conductors on the way of light fluxes. The interesting qualitative results are submitted in [8], where was investigated the diffraction of THz waves on thin wires and plates and was received, that the superluminal phenomena are appreciable at polarization parallel to conduc-
tors and are absent or are not appreciable at perpendicular polarization. These results seem explainable, as waves of parallel polarization should generate an alternating current in a long conductor, which thereby becomes the radiating aerial with the extended near field zone. In case of perpendicular polarization the induction is much weaker and diffracted beams of far field are imposed on weak radiation of near field.

Let discuss the experiments on FTIR.

In the experiments [7 a)] "gain duration" (3.24') in the area of FTIR was measured in THz region: the distribution of wave packages with central wave length of 1 mm and pulse duration of 0.8 psec was investigated at depths of FTIR from 0 up to 8 mm. The researches showed outstripping character of evanescent waves, the numerical estimation of this advancing, obviously, linearly depends on the size of an interval, in which these waves are observed. According to (3.24') and if omit the term $\partial n/\partial \omega$, the observed data conducted to $\tau_2 = -0.41$ psec on 1 mm of depth of evanescent waves penetration.

In the experiments [7 b)] dependences of outstripping of evanescent waves on depth $d$ was measured at various light polarizations ($\lambda = 3.39$ mc, $d = 0 \div 25$ mcm) and it has been shown, that the duration of an advancing does not depend on polarization. Up to distance $d = 8$ mcm it grows and then remains approximately constant, of order of 0.2 psec. These supervisions do not contradict our approach, but do not give opportunities of quantitative comparison.

5. DISCUSSION

As the opportunity of instantaneous propagation of signal (or excitation) within a near field zone, its transferring, is established and as it is demonstrated that by this phenomenon can be explained a set of experimental facts, connected with "gain times", then now the attempts to consider the problem with more general positions become possible.

Let’s examine some theoretical prerequisites and possibilities for interpretation of the deduced results.

1. It is necessary to emphasize, at the beginning, that the principle of locality experimentally was checked up only in the far field, only for fields $\mathbf{E}$ and $\mathbf{H}$. Hence, the a priori excluding of possibilities of nonlocality of those parts of electromagnetic field, which are not included into the (transverse) far electrical and magnetic fields, represents not obvious hypothesis.

The hidden opportunity of nonlocality "in small" can be contained in conditions of gauge invariance: the classical Lorentz condition, $\partial A_\mu/\partial x_\mu = 0$, is replaced in QED by the Lorentz - Fermi condition [34]:

$$\frac{\partial A_\mu}{\partial x_\mu} |_0 = 0, \quad (5.1)$$

which requires a mutual indemnification or disappearance of the "superfluous" components of $A_\mu$, "pseudophotons", only in the average. Therefore it does not exclude possible nonlocality of interaction in a near zone of these fields.

All attempts of fields’ quantization without introduction of 4-vector $A_\mu$ were unsuccessful. Then the Aharonov-Bohm effect brightly showed, that these "needless" field components have definite physical sense and the impossibil-
ity of their omitting has, apparently, deep roots. This conclusion could be strengthened by existence and features of Casimir forces and, probably, by the phenomena of near field optics (e.g. the review [35]).

It can be added also, that consideration of some concrete phenomena connected to longitudinal part of electromagnetic field (cf. [36]) leads to phenomenological introduction of nonlocality, of some effective "smearing" of charge, depending on its 4-moment.

So, the problem of locality of these components of field is not resolved yet.

2. The consideration of all these phenomena invites further investigations of the basic equations of theory. So, if to examine the Klein-Gordon equation
\[ \partial_t^2 - \Delta - m^2 f(x) = 0, \quad (5.2) \]
as the wave equation, i.e. to search its solution as
\[ f(x) = f(\nu t - r), \quad (5.3) \]
then non-fading decision exists only at \( v \leq c \). Just this circumstance forbids movement with superluminal speed as effective nonlocality on any distances. However there is not formal restriction on the existence of non-wave solutions in a scope of near field with arbitrary value of \( v \). So it is necessary to consider such solutions, which can transfer interaction via field \( A_\mu \) with speed greater \( c \) on small distances, i.e. effectively nonlocal.

3. Let's point out such formal opportunity of interpretation of the equations describing processes faster-than-\( c \). Projectors in (3.6-7) can be expressed through the Green functions for Klein-Gordon equation with imaginary mass: for example,
\[ P_{NE}(\omega, k) = \theta(-k^2) = \int_0^\infty dm^2 \delta(-\omega^2 + k^2 - m^2) = 2 \int_0^\infty dm^2 \Delta_1(\omega, k, \text{im}), \quad (5.4) \]
i.e. it formally describes interaction transferred by tachyons, hypothetical particles with velocity always-bigger \( c \). From here for (3.7) in the complete \( x \)-representation follows such dispersion relation:
\[ f_{NE}(x) = 2\pi \int_0^\infty dm^2 \int d^4y \Delta_1(x - y, \text{im}) f_{NE}(y). \quad (5.5) \]
For functions \( f_E(x) \) is received the same representation, but with real mass.

Thus, it is not excluded a formal possibility for description the superluminal phenomena via tachyons (cf. [37]).

4. At transition to imaginary time, e.g. at transformation of variables \( (t, r) \rightarrow (i\tau, r) \) at \( \tau_1 = 0 \), the Equation (5.2) transforms from hyperbolic into elliptic one, in the 4-D Poisson equation:
\[ [\partial_t^2 + \nabla^2] D_c(i\tau, r) = -\delta(x), \quad (5.6) \]
and hence, all signals in such field are transferred instantly: the field becomes instanton ones (cf. [38]). It means that the photon, absorbed on one side of such pseudoparticle of radius \( \Delta l = c|\tau_2| \), instantly appears on the opposite side. Thus instead discussion of the advanced character of emission, instantaneous jumps and so on, we can take into account the existence of virtual instanton-type states (of zero mass in considered case) with parameters specified by the usual propagators via time duration terms.

Such description seems sufficiently simple and physically reasonable.
But a somewhat more complicated will be the structure of near field in the FTIR phenomenon. Its description by the substitution \( k_z \rightarrow ik_z \) into the Fresnel formulae leads to replacement of the usual ansatz \( Q_n(k) = k_0^2 - n^2k^2 \) for the far field on ansatz \( Q_x(k) = k_0^2 + n^2(k_x^2 - k_y^2 - k_z^2) \) for the FTIR zone. It means transition from the wave (hyperbolic) equation into the ultrahyperbolic equation [39] for near-surface evanescent waves:

\[
L(ix)S_e \equiv (\partial^2_t + \partial^2_z - \partial^2_x - \partial^2_y)S_e = \pm \delta(x). \quad (5.7)
\]

This equation can be considered as the difference of two 2-D Laplace equations \((\Delta_{t,z} - \Delta_{x,y})f(x) = \delta(x)\) with Green functions (propagators)

\[
G^\pm(x) = 1/(L(x) \pm i0) = 1/(t^2 + z^2 - x^2 - y^2 \pm i0). \quad (5.8)
\]

Their difference should been the Green function of corresponding homogeneous equation. However, as the full Fourier-transforms \( F_k[G^\pm(x)] = 4\pi/(-G(k) \pm i0) \) of both functions (5.8) coincide, the homogeneous form of equation (5.7) has only the zero general solution. It means the absence of free waves, which would propagate in \( z \)-direction. On the other hand, it means that the \( x \)-component of light flux speed at FTIR depends on \( k_x \) component of moment only, and for this reason, as was noted above, light pulses at FTIR are not widening.

However, for complete reliance in their existence the further researches are necessary.

**CONCLUSIONS**

The received results demonstrate, in accordance with the experimental data, the reality of superluminal signal transferring. Hence they prove the validity of Wigner’s general principle of causality with its admitting of instantaneous transferring. It seems that these reasonings can be considered as the first proof of existence of instantons as virtual particles or, more precisely, as pseudoparticles.

The principle of causality is refined, and it is shown that ”sizes of scatterers” or ”sizes of scattering processes”, often introduced in different investigations, can be identified, at least in definite cases, with the instanton sizes. It is not excluded that just such instantaneous transferring can take place at transfer of excitations or even binding energy between some constituents of condensed media (the Förster law and so on). Hence some of their parameters can be defined by the extent of area of tunneling (probably, in the scope of near field).

All it shows the necessity of (formal) refinement of the first postulate of relativity: *the speed of signal propagation in far field cannot exceed the vacuum velocity of light, but the transferability of excitation can be instantaneous on the length of tunneling (within the scope of near field)*. From the kinematical point of view such transitions can be described via magnitudes of ”gain time” at processes of particle (wave) propagation. Note that the physical sense, as was discussed above, reveals that the expression ”superluminal transferring” is more exact and preferable than the usual now ”superluminal propagation” and so on.

It can be pointed also, that the physical sense of instantaneous character of transition consists, in particular, in the answer on very old naive question: where resides particle at the time of tunnel transition? And also: how can be
imaged a process of gradually exit of emitting wave (on its length, at least) from
the source?

On the other hand, it must be underlined that in the chain of these investiga-
tions had been demonstrated the significance and possibilities of theory of
temporal functions, at least in their specific forms. Moreover, it must be noted
the suggesting and developing of the specific mathematical method of equations
of orthogonality, which will be more comprehensively considered elsewhere.

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APPENDIX

A. CONDITIONS OF COMPLETENESS AND EQUATIONS OF
ORTHOGONALITY
The most serious assumption at deduction of conditions of orthogonality
(2.3-4) and others consists in essentially used classical condition of completeness:
\( P_L + P_{NL} = 1, \) (A.1)

Really it means an implicit introduction of the assumption about absence of
features connected to intersection of areas, \( P_L \cap P_{NL} = 0, \) and requires special
researches. Notice that the Heaviside unit functions \( \theta(\xi), \) from which projectors
are constructing, are not definite in the point \( \xi = 0; \) the considered problem,
just as many others in the theory, is intimately connected with this uncertainty.

The decomposition (A.1) can be generalized as
\( P_L + P_{NL} + P_{\delta L} = 1, \) (A.2)

into which are introduced the "quasilocal terms" (cf. [40]):
\( P_{\delta L} = a_0 \delta(x^2) + a_1 \partial_\mu^2 \delta(x^2) + \ldots \) (A.3)

At limiting by the first term of (A.3) and as in the \((t,k))-representation the
Fourier transform \( F_k[\delta(x^2)] = \frac{1}{2\pi^2} \sin(k|t|), \) we receive that the argument of
\( \delta \)-function in (2.11) is replaced on
\[
\cos(kt) \left[ (1 + a_0 k^2) \tan(kt) - k t \right] \quad (A.4)
\]

and consequently the transcendental equation, which roots determine prop-
erties of nonlocal functions, is slightly varied.

However, insofar our main result, the conclusions about instantaneous ex-
citation transferring by tunneling, does not change, we can do no more than
mention a possibility of consideration of quasilocal terms in decomposition of
projectors and accordingly in decomposition of response functions.

B. TO INTRODUCTION AND DETERMINATION OF TEM-
PORAL FUNCTIONS
The designation (3.2) can be rewritten as the
equation for \( S \)-matrix:
\[
(\partial/\partial \omega) S(\omega, \mathbf{r}) = \tau(\omega, \mathbf{r}) S(\omega, \mathbf{r}). \quad (B.1)
\]
Logically another way seems more reasonable: the transition to such equation by the complete Legendre transformation \((t, r) \rightarrow (\omega, k)\) of the Schrödinger equation for \(S\)-matrix:

\[
\left(\frac{i}{\partial t}\right) S(t, r) = H(t, r) S(t, r), \quad (B.2)
\]

This way leads, instead of (B.1), to somewhat different form:

\[
\left(\frac{\partial}{i\partial \omega}\right) S(\omega, k) = T(\omega, k) S(\omega, k) \quad (B.3)
\]

with temporal function instead of Hamiltonian.

It should be noticed that just these functions arise into theory in the quite reasonable way: they are naturally revealed in the standard QED calculations [41]. That is the reason that we did not discuss here many other suggested forms of time-delays, e.g. [42].

Fourier transformation of (B.1),

\[
\left(\frac{\partial}{i\partial \omega}\right) S(\omega, k) = \int dq \tau(\omega, q) S(\omega, k - q), \quad (B.4)
\]

shows that both definitions are rather close if it can be suggested that the main role in discussed processes play closely related magnitudes of wave numbers. (These items will be discussed more comprehensively elsewhere.)

It must be noted that the first approaches and views on the problem require, in accordance with a common intuition, consideration of their relation to the uncertainties principles. Therefore must be considered peculiarities of uncertainty principle, connected with temporal functions [43].

Let's use the general method of deduction of uncertainties relations given by Schrödinger [44]. It starts with decomposition of the product of operators on Hermitian and anti-Hermitian parts as \(AB = \binom{1}{2}(AB + BA) + \binom{1}{2}(AB - BA)\) with subsequent quadrates of this expression, its averaging over the complete system of \(\psi\)-functions and replacement of operators by differences of operators and their averaged values: \(A \rightarrow A - \langle A \rangle\). This leads, due the Schwartz inequality, to such final expression:

\[
(\Delta A)^2(\Delta B)^2 \geq \binom{1}{4}(\langle AB - BA \rangle)^2 + \binom{1}{4}(\langle AB + BA \rangle - 2 \langle A \rangle \langle B \rangle)^2, \quad (B.5)
\]

where to the common form became added the last term, which is an Hermitian function and its magnitude must be real. Note that the Heisenberg limit of (B.5) shows a minimal value of uncertainties, which can be achieved at the determined conditions.

In considered case the canonically conjugated operators must be substituted by differences \(A \rightarrow E - \langle E \rangle\) and \(B \rightarrow T - \langle T \rangle\) and there is needed their averaging, instead of \(\psi\)-functions, over the complete system of \(S(E)\) functions, nonunitary in general, as

\[
\langle A \rangle = \int_{-\infty}^{\infty} dE S^* S \cdot \int_{-\infty}^{\infty} dE |S|^2. \quad (B.6)
\]

The evident calculations in \(E\)-representation with operators \(T\) and \(E\) give such result:

\[
(\Delta E)^2(\Delta T)^2 \geq \binom{1}{4} h^2 + \binom{1}{4}((\langle E \rangle - 2 \langle E \rangle \langle T \rangle)^2, \quad (B.7)
\]

i.e. the general form of uncertainty principle does not depend on the duration of state formation or on the "gain time" at jumps, if \(\tau_2\) is negative. It can means that peculiarity of \(\tau_2\) must be considered as the internal property of forming particle, which does not depend on measurement procedures.
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