RADIAL ARC STATISTICS: A NEW POWERFUL PROBE OF THE CENTRAL DENSITY PROFILE OF GALAXY CLUSTERS

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ABSTRACT

We show that an expected number ratio of radial arcs (gravitationally lensed images whose major axes lie in the radial direction of a cluster-lens potential) to tangential arcs (gravitationally lensed images whose major axes lie in the tangential direction) has strong dependence on the central density profile of galaxy clusters and has little dependence on other parameters, e.g. cluster temperature, background source galaxy redshift etc.. A comparison of the expected number ratios with observed ratios provides a robust test to constrain the central density profile of galaxy clusters. A tentative comparison with the observational data shows that the central density profile of galaxy clusters is $\rho(r) \propto r^{-1.5}$. This result indicates that the dark matter is collisionless at least on cluster scale. Our result gives an upper limit on a collision cross-section of self-interaction of dark matter ($\sigma_{\text{coll}}$) as $\sigma_{\text{coll}}/m < 0.1 \text{ cm}^2/\text{g}$ where $m$ is a dark matter particle mass.

Subject headings: galaxies: clusters: general — dark matter — gravitational lensing

1. Introduction

The central density profile of dark matter halos around galaxies and galaxy clusters is now highlighted in the dark matter cosmology. Navarro et al. (1997) have performed cosmological N-body simulations based on cold dark matter models and show that the equilibrium density profiles of dark matter halos have self-similar profiles. The profile (hereafter, NFW profile) is thought to be the final relaxed state of self-gravitating collisionless particles and thereby called a universal profile. The NFW profile has a central cusp diverging as $\rho(r) \propto r^{-1}$. Recent simulations with higher resolution have confirmed
existence of a universal profile but show that the central density profile gets steeper and is better represented by $\rho(r) \propto r^{-1.5}$ (Fukushige & Makino 1997; Moore et al. 2000) although slight variation of the central density profiles from halo to halo is reported (Jing & Suto 2000).

Rotation curve measurements for dwarf galaxies (e.g. Moore 1994; Dalcanton & Bernstein 2000) have shown that central density profiles are flat and a density profile with a central cusp is inconsistent with the mass distribution of dwarf galaxies. To solve the discrepancy between the theoretical prediction and the observations for dwarf galaxies, Spergel & Steinhardt (2000) have proposed that the dark matter has a finite cross-section to allow strong self-interactions between the dark matter particles themselves. This type of dark matter is called self-interacting dark matter (hereafter, SIDM). The introduction of the self-interaction may reduce the central mass concentration found in the universal profile obtained by pure collisionless dark matter simulations. Therefore, the precise measurement of central density profiles of dark matter halos is one of the key questions to solve the nature of the dark matter.

Both observational and theoretical studies have been done to constrain the SIDM cross-section. Higher resolution measurements of rotation curves of dwarf galaxies (e.g. van den Bosch & Swaters 2000) have rejected the central profile of $r^{-1.5}$ although the data cannot discriminate whether the central density profiles have constant density cores or $r^{-1}$ cuspy profiles. Miralda-Escude (2000) has shown that ellipticity of the dark halo core of the lensing cluster MS 2137.3$-$2353 is rather high using the strong lensing data. He has proposed to use the measured ellipticity to give an upper limit on the SIDM cross-section. Yoshida et al. (2000) have performed N-body simulations of cluster formation with various values of SIDM cross-sections. They show that a few collisions per particle per Hubble time at the halo center can substantially affect the central density profile. Therefore, precise
measurements of central density profiles can provide a strong constraint on the SIDM cross-section.

We examine how a number ratio of radial arcs to tangential arcs depends on the central density profile of galaxy clusters. A radial arc is a gravitationally lensed image whose major axis lies in the radial direction of a cluster-lens potential. A tangential arc is a gravitationally lensed image whose major axis lies in the tangential direction of a cluster-lens potential. We define ‘radial arc statistics’ as a statistical average of number ratios of radial arcs to tangential arcs found in a certain cluster sample. Recently, Wyithe et al. (2000) have proposed an arc statistics for probing the central density profile of galaxy clusters. In their arc statistics, the absolute frequency of finding arcs appeared in a cluster must be used. However, the absolute frequency of finding arcs in a certain cluster sample is known to be sensitive to yet uncertain parameters e.g. evolution of background galaxies and details of mass distribution of galaxy clusters (Hattori et al. 1997; Hamana & Futamase 1997; Molikawa et al. 1999). No model has succeeded to reproduce the observed high frequency of finding arcs yet. As shown in this paper, the proposed radial arc statistics is free from the systematic errors coming from above uncertainties. Throughout this paper, a cosmological model of \( \Omega_{m0}, \Omega_{\Lambda0}, H_0 \) = (0.3, 0.7, 100h km/s) is assumed.

2. Radial arc statistics

The density profile we use is

\[
\rho(r) = \frac{\tilde{\rho}_s}{\tilde{r}^a(1+\tilde{r})^{3-a}}; \quad \tilde{r} \equiv \frac{r}{r_s},
\]

where \( r_s \) is the scale radius, \( \rho_s \) is the critical density times the (dimensionless) characteristic over-density, and adopted values of \( a \) are 0.5, 1.0, 1.5, and 2.0. A projected mass \( m_a(x) \) can
be written as

\[ m_a(x) = 4\pi r_s^2 \rho_s \int_0^\infty I_a(x, z) dz, \tag{2} \]

where \( \tilde{r}^2 = x^2 + z^2 \) (\( z \) is the line-of-sight direction.) and

\[ I_a(x, z) \equiv \int_0^x \frac{x'dx'}{\left(\sqrt{x'^2 + z^2}\right)^a \left(1 + \sqrt{x'^2 + z^2}\right)^{3-a}}. \tag{3} \]

The lens equation reads

\[ y = x - b \hat{\alpha}_a(x), \tag{4} \]

where \( y \equiv \beta/\theta_s \) is a source position, \( x = \theta/\theta_s \) is an image position, \( b \equiv 16\pi G \rho_s r_s D_{OL} D_{LS}/c^2 D_{OS} = 4\rho_s r_s/\Sigma_c r \) is called lens parameter and \( \theta_s \equiv r_s/D_{OL} \).

\( D_{OL} \) is the angular distance from the observer to the lens, \( D_{LS} \) is that from the lens to the source galaxy, \( D_{OS} \) is that from the observer to the source galaxy, and \( \Sigma_c \) is the critical surface mass density. For example, \( b = 1.1 \) corresponds to a cluster at redshift of 0.38 with a virial temperature \( k_B T_{\text{vir}} = 8 \text{ keV} \) and a source redshift of 1.0 (e.g. Eke et al. 1998).

An expected number ratio of radial arcs to tangential arcs is calculated by calculating a ratio of a cross-section forming radial arcs to that forming tangential arcs. It is assumed that source galaxies are circular and their size is infinitesimal, i.e. lensing magnification matrix is same everywhere in the source and is represented by that at the source center. A quantity \( \lambda_r \equiv (dy/dx)^{-1} \) represents stretching-contracting factor in the radial direction of a lens potential. A quantity \( \lambda_t \equiv (y/x)^{-1} \) represents stretching-contracting factor in the tangential direction of a lens potential. Hereafter, we use the word ‘radial arc’ for an image which satisfies \(|\lambda_t| > |\lambda_r|\) and ‘tangential arc’ for an image which satisfies \(|\lambda_t| > |\lambda_t|\). The axis ratios of a radial arc \( R(x) \) and a tangential arc \( T(x) \) are given by

\[ R(x) = \frac{|\lambda_t(x)|}{|\lambda_r(x)|}, \quad T(x) = \frac{|\lambda_t(x)|}{|\lambda_t(x)|}, \tag{5} \]

respectively. The radial and tangential cross-sections are calculated by solving the following
inequalities:
\[ \epsilon_{\text{th}} < R(x) \leq \infty, \quad \epsilon_{\text{th}} < T(x) \leq \infty, \]
\[ (6) \]
where \( \epsilon_{\text{th}} \) is the threshold axis ratio. We calculate ratios of radial cross-sections to tangential cross-sections for \( a = 0.5, 1.0, 1.5, 2.0 \). The NFW profile corresponds to \( a = 1 \).

In Figure 1, we show the expected number ratios of radial arcs whose axis ratios are larger than the threshold value \( \epsilon_{\text{th}} \), to tangential arcs which have the same threshold axis ratio, against the threshold axis ratio \( \epsilon_{\text{th}} \). Calculations were done for \( b = 0.5, 1.0, 1.5, 2.0 \), and 2.5 which cover almost all practical combinations of cluster redshifts and their masses, and source redshifts.

Figure 1 shows that the number ratios drastically vary with \( a \) values and variation due to variation of \( b \) values is much smaller. This indicates that the number ratios of radial arcs to tangential arcs in a certain cluster sample is a powerful probe for the central density profile of galaxy clusters. Biases in selection of a cluster sample may be reflected in biases of cluster masses and their redshifts, and the deepness of the observations which is equivalent to source redshifts. Changing values of these parameters only results in change of \( b \) values. Therefore, insensitivity of the radial arc statistics to \( b \) values shows that the selection biases of a cluster sample less affects the constraint from the radial arc statistics on the central density profile. A dramatic increase of the radial arc number with increasing of \( a \) from 1.5 to 2 found in Figure 1 can be understood as follows. When the central density profile is gentler than \( r^{-2} \), a lens could have radial caustics where the stretching factor in the radial direction become infinite, and radial arcs are produced in two different ways (Hattori et al. 1999). When the central density profile is gentler than \( r^{-1.5} \), radial arcs are produced only when source galaxies touch the radial caustics. On the other hand, when the central density profile is steeper than \( r^{-1.5} \), the width of the source image along the tangential direction is contracted. Therefore, radial arcs can be produced without significant stretching in the
radial direction. As a result, the number of radial arcs are dramatically increased when the central density profile is steeper than $r^{-1.5}$. Since intrinsic source size of $L_*$ galaxies at a redshift of $\sim 1$ which are the dominant sources of observed arcs, is $\sim 1$ arcsec, high resolution instruments with the Full-Width-at-Half-Maximum of better than $0.2 - 0.1$ arcsec., like Hubble Space Telescope (HST), Subaru etc., is required to make a reliable sample for the radial arc statistics.

3. Comparison with observations

In TABLE 1, we list clusters for which existence of radial arcs were reported. In most of the cases, the radial arcs are thin and small. Special attention is necessary to find a radial arc even on the HST images. The listed clusters in Table 1 could be only a set for which existence or non-existence of radial arcs have been securely checked. This sample is, of course, not uniform and is highly biased by the interests of the observer for each cluster. However, as mentioned in previous section, our radial arc statistics is little affected by the selection biases. We could safely use this sample for the comparison with theoretical results. The hyphenated arcs in Table 1 are folded or separated but nearly folded images and, therefore, each ‘hyphenated arcs’ is counted as a single arc because those (nearly) folded images are produced from a single source. The length of these arcs is calculated as a half (in the case of merging of two images) or one third (in the case of merging of three images) of the total length. Since all of the reported radial arcs have axis ratio larger than 4, the threshold value can be set to 4. Tangential arcs with axis ratios larger than 4 in the same clusters are also listed in TABLE 1.

TABLE 1 shows that the observational number ratio of radial arcs to tangential arcs is 0.7 on average. This indicates $a = 1.0 \sim 1.5$. The central density profile with $a > 1.5$ over-produces radial arcs and can be rejected. On the other hand, the central density profile
with $a < 1.0$ predicts too few radial arcs compared with the observed results and also can be rejected. Therefore, the radial arc statistics supports that the central density profile of cluster is in the range of $r^{-1.0} \sim r^{-1.5}$ and dark matter is collisionless at least on the cluster scale.

### 4. Discussion

We have shown that the radial arc statistics which is defined as a number ratio of radial arcs to tangential arcs appeared in a certain sample of clusters, strongly depends on the central density profile, and is almost independent of other cluster properties and source redshifts. Therefore, our radial arc statistics which uses the number ratio can provide a robust test to constrain the central density profile of galaxy clusters. A tentative comparison with observations shows that the cluster central density profile is as steep as $r^{-1.0} \sim r^{-1.5}$. This result indicates that the dark matter should be collisionless at least in cluster scale.

Therefore, even if the dark matter has a finite collision cross-section of the self-interaction $\sigma_{\text{coll}}$, the mean free time $t_{\text{free}}$ of a dark matter particle (mass $m$) should be longer than a cluster age $t_{\text{age}}$. The mean free time $t_{\text{free}}$ can be estimated as $t_{\text{free}} \simeq (\rho_s/m\sigma_{\text{coll}}V_{\text{cl}})^{-1}$ where $V_{\text{cl}} \simeq 2000(T_{\text{vir}}/8 \text{ keV})^{1/2} \text{ km/s}$ is a cluster 3D velocity dispersion, assuming that a characteristic density of the dark matter in clusters is represented by $\rho_s$. Assuming a cluster redshift of 0.38, its virial temperature of 8 keV, and a cluster age of $6.6h^{-1}$ Gyr, the upper limit is obtained as $\sigma_{\text{coll}}/m < 0.11h^{-1} \text{ cm}^2/\text{g}$. According to the numerical simulations (Yoshida et al. 2000), this upper limit allows weak self-interaction of the dark matter which produces a finite core in cluster mass distribution. Therefore, the above mentioned upper limit on $\sigma_{\text{coll}}/m$ should be taken as conservative limit. As shown by Yoshida et al. (2000), the collision rate expected from this small cross-section in the dwarf galaxies is too small to reproduce core radii of dwarf galaxies inferred from rotation curve measurements.
Therefore, the existence of cores in dwarf galaxies is still mystery.

Although we have assumed infinitesimal source size, this assumption is valid only if a variation of the lensing amplification within a source is negligible. In Figure 2, we show the width of the region on the source plane which satisfies the condition of Eq.(6) for a infinitesimal source. The width is scaled by $D_{OS}$, for each threshold axis ratio using a typical value of $b = 1.0$ and $r_s = 0.24 \, h^{-1} \, \text{Mpc}$. Since size of $L_\ast$ galaxy is $\sim 1$ arcsec at a redshift of $\sim 1$ which is the dominant source for arcs, Figure 2 shows that the infinitesimal source approximation could provide reasonable estimations for the frequency of the tangential arcs but this assumption could not be good approximation for estimating the frequency of radial arcs for $a < 2.0$. The quantitative studies of the effect of the finite source extents as well as the lens-clusters ellipticity and irregularity in mass distribution, will be done in the forthcoming paper.

The cluster sample used for comparison with theoretical prediction should be improved. It should be mentioned, for example, that all the clusters listed in TABLE 1 are XD clusters. An XD cluster is a cluster which has a bright elliptical galaxy at the center of the X-ray emission. Since radial arcs are expected to appear in the very central region of a galaxy cluster and all the radial arcs listed in TABLE 1 are found near cDs, it must be checked whether a local central density increment by the mass of the cD galaxy plays an important role on forming radial arcs. Therefore much more number of clusters, including non-XD clusters, should be included in the radial arc statistics sample by future high resolution observation.

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Fig. 1.— Ratios of radial arcs to tangential arcs against threshold axis ratios. The uppermost five solid lines are for $a = 2.0$. The following five dashed lines are for $a = 1.5$. The following five dotted lines are for $a = 1.0$. The rest two dot-dashed lines are for $a = 0.5$. Symbols are square: $b = 2.5$, circle: $b = 2.0$, triangle: $b = 1.5$, cross: $b = 1.0$, and asterisk: $b = 0.5$. If and only if $a < 1$, the central surface density is finite. If $a < 1$, $b$ must satisfy the condition $b/2(1 - a)(2 - a) > 1$ to have radial arcs (e.g. Subramanian & Cowling 1986). The radial and tangential cross-sections are partly merge where plot points are lost.

Fig. 2.— Angular lengths between the lower and upper limits of equation (6) on the source plane against threshold axis ratios for $b = 1.0$. The solid lines with boxes are for $a = 2.0$, the dashed lines with circles are for $a = 1.5$, and the dotted line with triangles are for $a = 1.0$. The non-filled symbols represent angular lengths between the limits for $R(x)$ in equation (6), and the filled symbols are for $T(x)$ in the same equation. Typical source size is $\sim 1$ arcsec at a redshift of $\sim 1$. 
Table 1. The cluster sample and observational number ratios

| Cluster† | Radial arc | Tangential arc | Number ratio |
|----------|------------|----------------|--------------|
| A 370    | R1-2       | A0,A1-2,B2-3   | 0.3          |
| AC 114   | A4-5       | B,D            | 0.5          |
| MS 0440  | A16, A17   | A2-3, A8-9     | 1.0          |
| MS 2137  | AR         | A0             | 1.0          |

†Reference: A370: Bézecourt et al. (1999), AC114: Natarayan et al. (1998), MS0440: Gioia et al. (1998), MS2137: Hammer et al. (1997)
Length between the two limits [arcsec]

Threshold axis ratio