We discuss the use of the $CP$ asymmetry parameter ($A_{CP}$) as a possible observable of $CP$ violation in the leptonic sector. In order to do this, we study for a wide range of values of $L/E$ the behavior of this asymmetry for the corresponding maximal value of the $CP$ violation factor allowed by all the present experimental limits on neutrino oscillations in vacuum and the recent Super-Kamiokande atmospheric neutrino result. We work in the three neutrino flavor framework.

PACS numbers: 11.30.Er; 14.60.Pq

I. INTRODUCTION

We are definitively living in very exciting times in neutrino physics. The recent results from Super-Kamiokande (SK) indicating evidence for neutrino oscillations in atmospheric showers [1], the reports of observed neutrino oscillations by the Los Alamos Liquid Scintillator Detector (LSND) [2] in the $\nu_{\mu} \rightarrow \nu_e$ and $\nu_\mu \rightarrow \nu_e$ channels in conjunction with other hints such as the results of solar neutrino experiments [3–6] makes it difficult to believe today that all of these facts are not related to neutrino properties beyond the standard model.

In the past neutrino physics has led to the discovery of neutral currents and provided the first indications in favor of the standard model of electroweak interactions. It may as well, if neutrino oscillations turn out to be confirmed by future experiments, reveal itself as an invaluable tool to cast some light on physics beyond the standard model, in particular, on the origin of $CP$ violation. One can hope this can be achieved in the study of the neutrino oscillation phenomena in some of the future experiments [7–9].

The recent KTeV result on $Re(\epsilon'/\epsilon)$ [10] finally establishes direct $CP$ violation in the Kaon system, rules out once and for all pure superweak theory and supports the notion of a nonzero phase in the CKM matrix. This makes it even more interesting to check if a similar effect also happens in the leptonic sector.

As it is well known $CP$ violation in neutrino oscillations can, in principle, be observed in neutrino experiments by looking at the differences of the transition probabilities between $CP$–conjugate channels, $\Delta P = P(\nu_\alpha \rightarrow \nu_\beta) - P(\nu_\alpha \rightarrow \nu_\beta)$. It has been pointed out by many authors that it may be, in practice, very difficult to get a reliable measurement of $\Delta P$ due to possible Earth matter effects in long baseline neutrino experiments [11,12]. However we believe it is worthwhile to try to evaluate the maximal size of $CP$ violation effect in vacuum since the vacuum oscillation experiments will be the ultimate proof of electroweakly induced neutrino flavor conversion. Both solar and atmospheric neutrino data can, in fact, be explained by alternative mechanisms invoking matter phenomena [17,19].

*Email: agago@charme.if.usp.br
*Email: vicente@ift.unesp.br
*Email: zukanov@charme.if.usp.br
Here we will use the asymmetry parameter, \( A_{CP} = \Delta P / [P(\nu_\alpha \rightarrow \nu_\beta) + P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)] \), suggested by Cabibbo \[1\], as an alternative to measure CP violation in the leptonic sector. We will investigate the possible values of this parameter and the corresponding maximal values of \( \Delta P \) allowed by present experimental data for different \( L/E \) situations, for a particular choice of neutrino mass squared differences. We will not make any assumption about the elements of the mixing matrix.

II. SCALE OF MASSES AND CP VIOLATION PARAMETERS

This work will be developed in the three flavor neutrino scheme and for this reason only two mass scale indications can be taken to be right \[20\]. We will fix them to be:

\[
\Delta m^2_{21} \approx 3.0 \times 10^{-3} \text{ eV}^2, \quad \Delta m^2_{32} \approx 0.4 \text{ eV}^2,
\]

which are taken within the allowed regions given by the atmospheric \[1\] and terrestrial neutrino experiments \[2,21,22\] respectively. We will also take into account the probability constraints coming from this two types of experiments. We admit here that the solar neutrino problem may be understood invoking other types of mechanisms \[17,23\].

We can get the analytical expressions for \( \Delta P \) and \( A_{CP} \) using the usual form of the CKM matrix parameterization:

\[
U = \begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
    -s_{13}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
    s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix},
\]

where \( c \) and \( s \) denote the cosine and the sine of the respective arguments. Thus, \( \Delta P \) in vacuum can be written as:

\[
\Delta P(\alpha, \beta) = P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) - P(\nu_\alpha \rightarrow \nu_\beta) = 4J_{CP}(\sin \Delta_{12} + \sin \Delta_{23} + \sin \Delta_{31}),
\]

with \( \alpha, \beta = e, \mu, \tau \) and

\[
\Delta_{ij} = 2.54 \left( \frac{\Delta m^2_{ij}}{1 \text{eV}^2} \right) \left( \frac{L}{\text{km}} \right) \left( \frac{1 \text{GeV}}{E} \right), \quad i, j = 1, 2, 3;
\]

where \( \Delta m^2_{ij} = m^2_j - m^2_i \) and the well known Jarlskog invariant \[24\]

\[
J_{CP} = c_{13}s_{12}c_{12}s_{23}s_{23} \sin \delta.
\]

We can see from Eqs. (3-5) that in vacuum \( \Delta P(\mu, \tau) = \Delta P(\mu, \bar{\tau}) = \Delta P(\tau, e) \) so they are all simply referred to as \( \Delta P \).

On the other hand \( A(\alpha, \beta)_{CP} \), which depends on the specific channel \( (\alpha, \beta) \), is given by:

\[
A(\alpha, \beta)_{CP} = \frac{P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) - P(\nu_\alpha \rightarrow \nu_\beta)}{P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) + P(\nu_\alpha \rightarrow \nu_\beta)}.
\]

In practice, for a real experiment, one has to average the probabilities in Eqs. (3) and (6) over the corresponding experimental conditions, i.e. neutrino energy spectrum, distance \( L \), efficiencies etc.

III. EXPERIMENTAL CONSTRAINTS

We have studied the experimental constraints imposed on the neutrino oscillation probabilities by several experimental results. In order to obtain realistic constraints we have, for each experiment, averaged
the corresponding probability over its distributions of $L$ and neutrino energy. This will be denoted by $\langle \cdots \rangle$ in what follows.

In the terrestrial case we have taken the results from two short baseline accelerator experiments LSND \cite{2} and E776 \cite{22} for the channels $\nu_\mu \to \nu_e$ and $\nu_\mu \to \nu_e$ respectively. To analyze LSND we have used the simple model quoted in Ref. \cite{25}. In the case of E776 the neutrino beam energy spectrum was taken from Ref. \cite{22}.

**LSND:**

$$\langle P(\nu_\mu \to \nu_e) \rangle = 0.31 \pm 0.10 \pm 0.05\%; \quad L \sim 30 \text{ m}, \quad E \sim 36 - 60 \text{ MeV}, \quad (7)$$

**E776:**

$$\langle P(\nu_\mu \to \nu_e) \rangle \leq 1.5 \times 10^{-3} \text{ at 90\% C.L.; } \quad L/E \sim 1. \quad (8)$$

For the oscillation channel $\nu_\mu \to \nu_\tau$ we have used the data from CHORUS \cite{26}. The CHORUS relevant information is available at their Web site \cite{27}. This short baseline accelerator experiment gives

$$\langle P(\nu_\mu \to \nu_\tau) \rangle \leq 6 \times 10^{-4} \text{ at 90\% C.L.; } \quad L/E \sim 0.02. \quad (9)$$

We have obtained from CHOOZ \cite{28}, a long baseline reactor experiment searching for the disappearance of the $\nu_e$, the limit :

$$1 - \langle P(\nu_e \to \nu_e) \rangle \leq 10^{-1} \text{ at 90\% C.L.; } \quad L/E \sim 300, \quad (10)$$

and from the Bugey \cite{21} reactor experiment

$$1 - \langle P(\nu_e \to \nu_e) \rangle \leq 10^{-2} \text{ at 90\% C.L.; } \quad L = 15, 40, 95 \text{ m, } \quad E \sim 1 - 6 \text{ MeV}. \quad (11)$$

We have read out the positron energy spectrum for CHOOZ from Ref. \cite{28} and for Bugey from Ref. \cite{21} and used the relation $E_\nu = E_{e^+} + 1.8 \text{ MeV}$ to extract the neutrino energy spectrum.

We show in Fig. 1 our own reproduction of the exclusion (allowed in the case of LSND) regions of these experiments. We observe a reasonable agreement with the original experimental plots that can be found in Refs. \cite{2,21,22,26,28}. This is a demonstration that Eqs. (7-11) correctly represent the constraints coming from these reactor and accelerator neutrino experiments.

In the case of atmospheric neutrinos, we only have used the latest SK \cite{1} result since it is the most precise one. We have performed our analysis of SK atmospheric data in the following way. We have separately used the muon neutrino events ($\mu$-like events) ratio

$$R_\mu(L/E) = \gamma \left( \langle P_\mu(L/E) \rangle + \frac{e_0}{\mu_0} \langle P_{e\mu}(L/E) \rangle \right), \quad (12)$$

and the electron neutrino events ($e$-like events) ratio

$$R_e(L/E) = \gamma \left( \langle P_{e\mu}(L/E) \rangle + \frac{\mu_0}{e_0} \langle P_{e\mu}(L/E) \rangle \right), \quad (13)$$

where $\mu_0 = \mu_0(L/E)$ and $e_0 = e_0(L/E)$ are the distributions of Monte Carlo $\mu$-like and $e$-like events taken from Ref. \cite{30} and $\gamma$ is a constant to take into account the overall normalization. We have taken the value of this constant to be 1.16 in accordance with Ref. \cite{29}. We have introduced in Eqs. (12) and (13) the shorthand $P_{\alpha\beta} = P(\nu_\alpha \to \nu_\beta), \alpha, \beta = e, \mu$. The probabilities $\langle P \rangle$ have been smeared by the resolution function given in Ref. \cite{31} at each $L/E$ value.

On the other hand, from Fig. 4 of Ref. \cite{1} we can read eight different values of $L/E$ and their corresponding $\mu$-like and $e$-like ratios. Using this we have calculated the simple average of each ratio obtaining:

$$\overline{R}_\mu = 0.76 \pm 0.08; \quad \overline{R}_e = 1.19 \pm 0.13, \quad (14)$$

In the case of atmospheric neutrinos, we only have used the latest SK \cite{1} result since it is the most precise one. We have performed our analysis of SK atmospheric data in the following way. We have separately used the muon neutrino events ($\mu$-like events) ratio

$$R_\mu(L/E) = \gamma \left( \langle P_\mu(L/E) \rangle + \frac{e_0}{\mu_0} \langle P_{e\mu}(L/E) \rangle \right), \quad (12)$$

and the electron neutrino events ($e$-like events) ratio

$$R_e(L/E) = \gamma \left( \langle P_{e\mu}(L/E) \rangle + \frac{\mu_0}{e_0} \langle P_{e\mu}(L/E) \rangle \right), \quad (13)$$

where $\mu_0 = \mu_0(L/E)$ and $e_0 = e_0(L/E)$ are the distributions of Monte Carlo $\mu$-like and $e$-like events taken from Ref. \cite{30} and $\gamma$ is a constant to take into account the overall normalization. We have taken the value of this constant to be 1.16 in accordance with Ref. \cite{29}. We have introduced in Eqs. (12) and (13) the shorthand $P_{\alpha\beta} = P(\nu_\alpha \to \nu_\beta), \alpha, \beta = e, \mu$. The probabilities $\langle P \rangle$ have been smeared by the resolution function given in Ref. \cite{31} at each $L/E$ value.

On the other hand, from Fig. 4 of Ref. \cite{1} we can read eight different values of $L/E$ and their corresponding $\mu$-like and $e$-like ratios. Using this we have calculated the simple average of each ratio obtaining:

$$\overline{R}_\mu = 0.76 \pm 0.08; \quad \overline{R}_e = 1.19 \pm 0.13, \quad (14)$$
defining two allowed bands, one for the $e$-like events and the other for the $\mu$-like events ratio. These are our selection criteria coming from SK data which were used in the following way: for a given set of values of the mass squared differences, the mixing angles and the phase we have computed the right hand side of Eq. (12) and Eq. (13) taking an average over the eight $L/E$ bins and verifying if they satisfy the conditions given by Eqs. (14). It is important to stress that working with $\mu$-like and $e$-like event ratios separately has the advantage that we are able to constraint independently $P(\nu_\mu \to \nu_e)$ and $P(\nu_e \to \nu_\mu)$. Indeed doing this we observe that these probabilities are always below a few $\%$, this is much more stringent that the SK analysis shown in Ref. [3].

It must be noted that this simple SK analysis is not totally rigorous, in fact by using the $L/E$ distribution from Ref. [1], we only work with a sub-sample of the SK data (fully-contained events). Moreover there are quite large uncertainties in the determination of this distribution due to the fact that one relies on the observed final lepton to infer the neutrino physical quantities, these uncertainties are discussed in Ref. [3]. Nevertheless we consider our approach good enough for the goals of this paper.

Since the Eqs. (1-3) are in fact independent of the number of neutrino generations we are now free to use these probability limits in the three neutrino flavor framework.

IV. LIMITS OF $J_{CP}$, $\Delta P$ AND $A_{CP}$

In order to determine the maximum permitted values of $\Delta P$, $A(\mu, e)_{CP}$ and $A(\mu, \tau)_{CP}$ we proceed in the following way. We choose randomly different values of the mixing parameters $s_{12}^2, s_{23}^2$ and $s_{13}^2$ taken in the interval $[0, 1]$. For each drawn set of these parameters we evaluate the values corresponding to Eqs. (6-13). We check if they simultaneously pass all the experimental constraints given in Eqs. (7-11) and Eq. (12). It really means that we look for a common allowed region among all experiments. In the positive case we compute the $CP$ violation factor $J_{CP}$ for fixed values of the phase angle $\delta$. We select among them the maximal value of $J_{CP}$, $J_{CP}^{max}$, for each $\delta$ and calculate the corresponding values of $\Delta P^{max}$, $A(\mu, e)_{CP}$ and $A(\mu, \tau)_{CP}$ for $L/E$ varying from 40 to 250.

In Tables I-II we show $J_{CP}^{max}$ for $\Delta m^2_{21} \approx 3 \times 10^{-3}$ eV$^2$ and $\Delta m^2_{32} \approx 0.27, 2.0$ eV$^2$. We have picked these values so one can have an idea of the order of the variations that $J_{CP}^{max}$ will have if one varies $\Delta m^2_{32}$ inside the LSND allowed region. In general we see that the values of $J_{CP}^{max}$ in the squared mass difference interval considered in this paper are more or less stable and of $O(10^{-3})$ in agreement with Ref. [3].

In Fig. 1 we show $\Delta P^{max}$ as a function of $L/E$ for sin $\delta = 1$. As one could guess the oscillating behavior of this curve is dictated by the sum of sines in Eq. (3). We are not showing here other curves for different values of sin $\delta$ since they have exactly the same form as these ones, only the oscillation amplitude will change in accordance to the corresponding $J_{CP}^{max}$. We observe that the maximal value of $\Delta P^{max}$, which increase with $L/E$, are of the $O(10^{-2})$. This is in accordance with the estimations given in Refs. [17, 23].

In Figs. 2 we show the behavior of $A(\mu, e)_{CP}$ and $A(\mu, \tau)_{CP}$ as a function of $L/E$ for sin $\delta = 1$. We can observe that $A(\mu, e)_{CP}$ grows and $A(\mu, \tau)_{CP}$ decreases as a function of $L/E$. Also the maximal values for $A(\mu, e)_{CP}$ are of the $O(1)$ while for $A(\mu, \tau)_{CP}$ they are of the $O(10^{-1})$. This result was expected since we know that the only difference between $A(\mu, e)_{CP}$ and $A(\mu, \tau)_{CP}$, in vacuum, comes from the denominator of Eq. (1) and $P(\nu_\mu \to \nu_e)$ is currently very much suppressed by data while SK data seems to support $P(\nu_\mu \to \nu_\tau)$ oscillations that can reach the order of $10^{-1}$ for neutrinos coming from bellow the horizon.

As we have already mentioned at the end of Sec. I, one has to be careful in interpreting our results in relation to future experiments and remember to take into account the average over the corresponding $L/E$ distribution. Because we did not want to make any ad-hoc hypothesis on $L/E$ distribution we have deliberately chosen not to do any smearing on $L/E$. We believe that presenting our results in this unfolded way make them more useful and ready to be applied to any real experimental situation.

To illustrate the implications of our results in future experiments we have put an arrow in Fig. 2 at the corresponding mean value of $L/E$ for MINOS [8], K2K [9] and one of the possible configurations of a long baseline neutrino experiment using neutrinos from a muon collider [13]. Based on our results we also have computed an estimation, averaging over the expected energy spectrum, of the maximal values that can be investigated at MINOS: $\langle A(\mu, e)_{CP} \rangle = 0.33$, $\langle A(\mu, \tau)_{CP} \rangle = 0.02$, $\langle \Delta P \rangle = 0.0022$; and K2K: $\langle A(\mu, e)_{CP} \rangle = 0.19$, $\langle A(\mu, \tau)_{CP} \rangle = 0.0017$, $\langle \Delta P \rangle = 0.0014$. All these values are, as expected, lower.
than the ones shown by the corresponding arrows in Fig. 3.4 since in the $L/E$ scope of MINOS/K2K the dominant scale is $\Delta m^2_{21}$, the contribution from $\Delta m^2_{32}$ being averaged out.

V. CONCLUSIONS

We have found the maximal allowed values for $\Delta P^{\text{max}}$, $A(\mu, e)_{CP}$ and $A(\mu, \tau)_{CP}$ as a function of $L/E$ and $\sin \delta$. This was done in the three neutrino flavor framework using the most stringent constraints from recent neutrino data and admitting the two mass squared differences to be $\Delta m^2_{21} \approx 3 \times 10^{-3}$ eV$^2$ and $\Delta m^2_{32} \approx 0.4$ eV$^2$. In fact this is in a way complementary to Ref. [36].

It is important to remark that we have adopted here a different approach from the authors of Refs. [16,34,36] since we do not make any assumptions about the mixing parameters other than the two squared mass difference scales and work with the probability expression without any approximation. Besides we have explicitly used the experimental resolution functions in our calculations.

We have seen that in general the values of $A_{CP}$ are much more sizable than the corresponding $\Delta P^{\text{max}}$. In fact, admitting the mass hierarchy considered in this paper, we have shown that the present neutrino data tremendously suppress the maximal values $\Delta P$ that can be investigated at the next generation of neutrino experiments. In addition since in the asymmetry the systematic errors cancel out, even if the absolute flux of the neutrino beam is determined with an accuracy of 10 % it may be possible to measure $CP$ violation at 1 % level. This is particularly interesting since in long baseline experiment there are expected matter effects that fake genuine $CP$ violation [17]. In a forthcoming paper [37] we will discuss the implications that the inclusion of matter effects will have on our present limits as well as how these limits change if one varies the squared mass difference scales.

ACKNOWLEDGMENTS

This work was supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), by Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP), and by Programa de Apoio a Núcleos de Excelência (PRONEX).

[1] Super-Kamiokande Collaboration, Y. Fukuda et al., Phys. Lett. B 433, 9 (1998); hep-ex/9807001.
[2] LSND Collaboration, C. Athanassopoulos et al., Phys. Rev. Lett. 75, 2650 (1995); ibid. 77, 3082 (1996); Phys. Rev. C 54, 2685 (1996); Phys. Rev. Lett. 81, 1774 (1998).
[3] B. T. Cleveland et al., Nucl. Phys. B (Proc. Suppl.) 38, 47 (1995); R. Davis, Prog. Part. Nucl. Phys. 32, 13 (1994).
[4] GALLEX Collaboration, W. Hampel et al., Phys. Lett. B 388, 384 (1996).
[5] D. N. Abdurashitov et al., Nucl. Phys. Proc. Suppl. 48, 299 (1996).
[6] Kamiokande Collaboration, Y. Fukuda et al., Phys. Rev. Lett. 77, 1683 (1996); Super-Kamiokande Collaboration, Y. Fukuda et al., Phys. Rev. Lett. 81, 1158 (1998).
[7] Y. Oyama, talk given at the YITP Workshop on Flavor Physics, Kyoto, Japan, 28-30 Jan. 1998 (hep-ex/9803011).
[8] MINOS Collaboration, Fermilab report NuMI-L-337 Technical Design Report, Aug. 1998.
[9] ICARUS Collaboration, P. Cennini et al., LNGS-94/99, May 1994.
[10] KTeV Collaboration, A. Alavi-harati et al., Phys. Rev. Lett. 83, 22 (1999).
[11] N. Cabibbo, Phys. Lett. B 72, 333 (1978).
[12] V. Barger, K. Whisnant, and R. J. N. Phillips, Phys. Rev. Lett. 45, 2084 (1980).
[13] S. M. Bilenky, J. Hosek, and S. T. Petcov, Phys. Lett. B 94, 495 (1980).
[14] S. Pakvasa, in: Proceedings of the XXth International Conference of High Energy Physics, Eds. L. Durand and L. G. Pondrom, AIP Conf. Proc. No. 68 (AIP, New York, 1981), Vol. 2, p. 1164.
[15] M. Tanimoto, Phys. Rev. D 55, 322 (1997); J. Arafune and J. Sato, Phys. Rev. D 55, 1653 (1997).
[16] H. Minakata and H. Nunokawa, Phys. Lett. B 413, 369 (1997); Phys. Rev. D 57, 4403 (1998).
[17] M. M. Guzzo, A. Masiero and S. T. Petcov, Phys. Lett. 260B, 154 (1991).
[18] P. I. Krastev and J. N. Bahcall, Talk given at Symposium on Flavor Changing Neutral Currents: Present and Future Studies (FCNC 97), Santa Monica, CA, 19-21 Feb 1997 (hep-ph/9703267).
[19] M. C. Gonzalez-Garcia et al., Phys. Rev. Lett. 82, 3202 (1999).
[20] A. Yu Smirnov, in: Proceeding of the XXVIIIth International Conference on High Energy Physics, Warsaw 1996, ICHEP’96, Vol. 1, p. 288 (hep-ph/961146).
[21] Bugey Collaboration, B. Achkar et al., Nucl. Phys. B 434, 503 (1995).
[22] E776 Collaboration, L. Borodovsky et al., Phys. Rev. Lett. 68, 274 (1992).
[23] S. W. Mansour and T. K. Kuo, hep-ph/9810510.
[24] C. Jarlskog, Phys. Rev. Lett. 55, 1039 (1985).
[25] Particle Data Group, C. Caso et al., Eur. Phys. J. C 3, 1 (1998).
[26] CHORUS Collaboration, E. Eskut et al., Nucl. Instrum. Meth. A 401, 7 (1997); CHORUS Collaboration, Talk given at 29th International Conference on High Energy Physics (ICHEP98), Vancouver, Canada, 23-29 July 1998 (hep-ex/9807024).
[27] See for instance the CHORUS proposal in http://choruswww.cern.ch/Publications/papers.html.
[28] CHOOZ Collaboration, M. Apollonio et al., Phys. Lett. B 420, 397 (1998).
[29] V. Barger, T. J. Weiler, and K. Whisnant, Phys. Lett. B 440, 1 (1998).
[30] M. D. Messier, Ph. D. Thesis (Boston University, 1999) available at http://budoe.bu.edu.
[31] See http://www.phys.hawaii.edu/~jgl/post/lores.gif.
[32] R. P. Thun and S. McKee, Phys. Lett. B 439, 123 (1998).
[33] T. Ohlsson and H. Snellman, hep-ph/9903252.
[34] S. M. Bilenky, C. Giunti, and W. Grimus, Phys. Rev. D 58, 33001 (1998).
[35] S. Geer, Phys. Rev. D 57, 6989 (1998), Erratum-ibid D 59, 039903 (1999). (1998).
[36] M. Tanimoto, hep-ph/990651.
[37] A. M. Gago, V. Pleitez, and R. Zukunovich Funchal, in preparation.

| $4 \times J_{CP}^{\text{max}}$ | $4 \times J_{CP}^{\text{max}} \sin \delta$ | $\sin \delta$ |
|-----------------------------|---------------------------------|----------------|
| 0.0011                      | 0.0044                          | 0.2588         |
| 0.0024                      | 0.0048                          | 0.5000         |
| 0.0032                      | 0.0045                          | 0.7071         |
| 0.0046                      | 0.0053                          | 0.8660         |
| 0.0052                      | 0.0054                          | 0.9659         |
| 0.0052                      | 0.0052                          | 1.0000         |

TABLE I. Maximal values of the Jarlskog factor obtained for different values of $\sin \delta$ with $\Delta m_{21}^2 \approx 3.0 \times 10^{-3}$ eV$^2$ and $\Delta m_{32}^2 \approx 0.27$ eV$^2$. 
TABLE II. Maximal values of the Jarlskog factor obtained for different values of sin δ with \( \Delta m_{21}^2 \approx 3.0 \times 10^{-3} \text{ eV}^2 \) and \( \Delta m_{32}^2 \approx 2.0 \text{ eV}^2 \).

| \( 4 \times J_{CP}^{\text{max}} \) | \( 4 \times J_{CP}^{\text{max}} / \sin \delta \) | \( \sin \delta \) |
|---|---|---|
| 0.0010 | 0.0041 | 0.2588 |
| 0.0021 | 0.0042 | 0.5000 |
| 0.0031 | 0.0044 | 0.7071 |
| 0.0038 | 0.0043 | 0.8660 |
| 0.0042 | 0.0044 | 0.9659 |
| 0.0043 | 0.0043 | 1.0000 |
FIG. 1. Our reproduction of the exclusion/allowed regions for different experiments in two generations: (a) E776, (b) LSND, (c) Bugey, (d) CHOOZ and (e) CHORUS. The exclusion regions at 90% C.L. are the ones to the right of the curves for all experiments except LSND. In the case of LSND the allowed region is the region between the two curves.

FIG. 2. $\Delta P^\text{max}$ as a function of $L/E$ for $\sin \delta = 1$. 
FIG. 3. Values of $A(\mu, e)_{CP}$ as a function of $L/E$ for $\sin \delta = 1$.

FIG. 4. Values $A(\mu, \tau)_{CP}$ as a function of $L/E$ for $\sin \delta = 1$. 