Determination of expansion pressure of heat exchange based on three material models

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Abstract. The hydraulic expansion of the heat exchange tube-to-tubesheet is to apply the liquid pressure to expand the inner wall of the tube uniformly through the device. It makes the tube wall deformed and contact the tube sheet. The expansion pressure which affects the quality of the expansion of the heat exchanger is one of the important factors for the hydraulic expansion of the heat exchanger. Therefore, in order to meet the expansion requirements, it is necessary to determine the expansion pressure of the heat exchange. The determination of hydraulic expansion pressure by some scholars are introduced in this paper.

1. Introduction

In hydraulic expansion[1], a uniform liquid pressure is applied on the inner surface of heat exchange tube initially. It makes the tube start to happen plastic deformation, until the outer wall of the heat exchange tube contacts the inner wall of the tube sheet. However, there is no deformation to tubesheet. This stage is called the deformation stage of the heat exchange tube. As the hydraulic expansion is increased, the tubesheet partly or all happen plastic deformation, this stage is called the loading to the tube sheet. Finally, liquid pressure is unloaded when the expansion pressure reaches a predetermined value. Because the tubesheet which happen elastic deformation or partly happen plastic deformation occur a elastic rebound and the tube which happen full plastic deformation can’t occur rebound, The outer wall of heat exchange tube and the inner wall of tubesheet are tightly attached together. Hydraulic expansion has the advantages of simple operation and high production efficiency. What’s more, There is not cold hardening which is produced by mechanical extrusion and less residual stress and corrosion for the hydraulic expansion. At the same time, the expansion pressure can be set precisely, the pull-out force can be predicted, and the quality of expansion can be guaranteed.

Since born to the present, the theory of expansion joint has been continuously developed and improved. Next, the determination of the expansion pressure of hydraulic expansion joints by some
scholars are introduced and the three model which these scholars apply to determine the hydraulic expansion are introduced simply. These three models are elastic-perfectly plastic model, hardening model and bilinear model.

2. determination expansion pressure using elastic-perfectly plastic model

Some scholars applied the elastic-perfectly plastic model for determining the expansion pressure of the heat exchanger. This model neglects the affect of the hardening of material in the expansion of the heat exchange. The assumption of the plane stress is used to determine the expansion pressure during the expansion process of the heat exchange tube. Fig.1 is the geometrical character of the heat exchange expansion model. Where the \( r_i \), \( r_o \) are respectively indicate the inner and outer tube radius, \( R_i \), \( R_o \) are respectively indicate the inner and outer tubesheet radius, \( c_0 \) is indicate the clearance of the tube and tubesheet.

\[
p_{\text{min}} = \frac{2\sigma_s}{3K_t^2 + 1} \cdot \left( K_t^2 - 1 \right) + \frac{\sigma_s (K_t^2 - 1)}{2}
\]

\[
p_{\text{max}} = \frac{\sigma_s}{\beta} \left( \frac{K_t^2}{K_i^2} + 1 \right) \sigma_s
\]

Where \( K_t, K_i \) are respectively indicate the ratio of radius, \( K_t = r_o/r_i, K_i = R_o/R_i \), \( \sigma_s \), \( \sigma_t \) are respectively the yield stress of tube and tubesheet, \( \mu \) is the Poisson’s ratio of the material.

In 2001, Based on the formula proposed by Krips and Podhorsky, Cheng gang et al.[3] Introduced a coefficient \( \beta \) when he considered the length of the expansion joint is generally smaller than the thickness of the tube sheet, and the influence of the material becoming rigid, etc. It make the results closer to reality. The coefficient \( \beta \) is generally selected between 0.8 and 0.9. After rewriting, the minimum expansion pressure and maximum expansion pressure \( p_{\text{min}} \) of the heat exchange tube are given:

\[
p_{\text{min}} = \frac{1}{\beta} \cdot \frac{2\sigma_s}{3K_t^2 + 1} \cdot \left( K_t^2 - 1 \right) + \frac{\sigma_s (K_t^2 - 1)}{2}
\]

\[
p_{\text{max}} = \frac{\sigma_s}{\beta} \left( \frac{K_t^2}{K_i^2} + 1 \right) \sigma_s
\]

In 1996, Yan Huigeng et al.[4] used the single tube model which is two concentric circles with a initial clearance to determine the expansion pressure. An assumption which The material is the elastic-perfectly plastic material and the ratio of the inner to outer diameters of the heat exchange tube is still \( K_t \) after deformation is made. Axial stress of tubesheet and heat exchange tube during expansion
is neglected and Von mises yield criteria are applied to determine expansion pressure at each deformation stage:

\[
p_{\text{max}} = \frac{2}{\sqrt{3}} \sigma_y \ln K \left(1 - 2 \epsilon \right)
\]

\[
p_{\text{max}} = \frac{2}{\sqrt{3}} \sigma_y \ln K + \frac{2}{\sqrt{3}} \sigma_m \ln K,
\]

Where \(c\) is a material constant associated with elastic modulus of the tube and tubesheet, \(K_t, K_s\) and the poisson's ratio of the tube and tubesheet.

In 2009, Laghzale and Bouzid [5] used the Tresca yield criteria to study the expansion pressure of the hydraulic expansion process of the heat exchange tubes. and the expansion pressure at each stage are given:

\[
p_{\text{ss}} = \sigma_y \ln K + \sigma_m \left(\frac{K^2 - 1}{2} K_t\right)
\]

\[
p_{\text{max}} = \sigma_y \ln K + \sigma_m \ln K,
\]

3. Determination expansion pressure using strain hardening model

Metal materials generally have power strain-hardening properties. The material properties have a significant effect on the expansion process, which leads to a larger error in the determination of the expansion pressure using the elastic-perfectly plastic model. Some scholars have considered the material hardening properties and use the material hardening model to determine the expansion pressure during the expansion process.

In 2007, Wang Haifeng et al.[6] comprehensively considered the material strain-hardening properties, heat exchange tube and tubesheet geometry and other factors. Power strain-hardening properties are used to describe the mechanical behavior of the heat exchange tubes and tube sheets during the expansion process. Constitutive relationship of the materials power strain-hardening is given:

\[
\sigma_i = \begin{cases} 
E \varepsilon_i \sigma_i \leq \sigma_s \\
A_1 \varepsilon_i \sigma_i > \sigma_s 
\end{cases}
\]

Where \(E\) is the elastic modulus of material, \(m\) is material exponential, \(A\) is the material constant, \(\varepsilon_i\) is the material strain, \(\sigma_i, \sigma_s\) are respectively indicate the material stress and yield stress.

Moreover, Von mises yield criteria and Hencky deformation theory are applied to determine the expansion pressure of each expansion joint in the expansion process of the heat exchange tube :

\[
p_{\text{max}} = k_2 (c_0 r_f)^m (r_0^{2m} - r_f^{2m})
\]

\[
p_{\text{max}} = \sigma_y \sqrt{\frac{2}{m_1}} (K^2 - 1) + k_1 r_f (c_0 \sigma_s + \frac{5 \sigma_0 R_0^2}{2E R})^m (r_0^{2m} - r_f^{2m})
\]

Where \(c_0\) is the initial clearance, \(m_1, m_2\) is material exponential of the tube and tubesheet, \(k_2\) is the constant.

In 2010, Xie T et al.[7] applied a general strain-stress model which is proposed by XP Huang et al.[8] in 2005 to describe the strain hardening during the expansion of the heat exchange tube.Constitutive relationship of the materials general strain-stress model is given:

\[
\sigma_i = \begin{cases} 
E \varepsilon_i \sigma_i \leq \sigma_s \\
A_1 + A_2 \varepsilon_i \sigma_i > \sigma_s 
\end{cases}
\]

Where \(A_1, A_2\) are the material constant, \(A_{12}\) are respectively indicate the tube material constant,
\( A_{i1}, A_{i2} \) are respectively indicate the tube-sheet material constant.

Xie applied Von mises yield criteria and Hencky deformation theory to determine the expansion pressure of heat exchange and the expansion pressure of each expansion joint in the expansion process of the heat exchange tube are given:

\[
P_{\text{min}} = \frac{2}{\sqrt{3}} A_{i1} \ln K_i + (\frac{2}{\sqrt{3}})^{m_{i+1}} A_{i2} \left( c_{0i} r_i \right)^m_i \left( \frac{1}{-2B_i} \right) (r_o - 2m_i - r_i^{-2m_i}) \]  
(13)

\[
P_{\text{max}} = \frac{2}{\sqrt{3}} A_{i1} \ln K_i + \frac{2}{\sqrt{3}} A_{i1} \ln K_i - \frac{1}{2B_i} (\frac{2}{\sqrt{3}})^{m_{i+1}} A_{i2} \left[ r_i \left( c_i + \frac{\sqrt{3} \sigma_o R_i^2}{2E R_i} \right) \right]^{m_i} (R_o - R_i^{-m_i}) \]  
(14)

In 2016, based on the general strain-stress model which is proposed by XP Huang, Bouzid and Mourad[9] determined the expansion pressure of each expansion joint in the expansion process of the heat exchange tube by using Von mises yield criteria, Hencky deformation theory and Tresca yield criteria:

\[
P_{\text{min}} = \frac{2A_{i1}}{\sqrt{3}} \ln K_i + \frac{2A_{i2}}{\sqrt{3}} (c_{0i} r_i)^m_i (r_o - r_i^{-2m_i}) \]  
(15)

\[
P_{\text{max}} = \frac{2A_{i1}}{\sqrt{3}} \ln K_i - \frac{A_{i2}}{\sqrt{3} B_i} \left( \frac{\sigma_{0i}}{E_i} \right)^m_i (1 - K_i^{-2m_i}) + \frac{2A_{i1}}{\sqrt{3}} \ln K_i - \frac{A_{i2} c_i^{m_i}}{\sqrt{3} m_i} \left( \frac{2c_i}{\sqrt{3}} + \frac{\sigma_{0i} R_i^2}{E_i R_i} \right)^m (R_o - R_i^{-m_i}) \]  
(16)

4. Determination expansion pressure using bilinear model

Strain-hardening model which consider the strain hardening of the material and the effect of the initial clearance is close to the real performance of the expanding material. The complexity of the model ,the cumbersome calculation and the relevant parameters which are difficult to determine in the formula make the strain-hardening model hard to apply to engineering practice. Based on this situation, it is necessary to simplify the strain-hardening model.

In order to simplify the material stress-strain hardening curve, a bilinear simplify model is proposed by Hong ying et al.[10] in 2018. The bilinear simplified model consists of two straight lines with different slopes. The slope of the first line is the elastic modulus of the material.The slope of the second line reflects the strength of the stress and strain hardening ability of the material. The stress-strain model of the model material is characterized by bilinearity as:

\[
\sigma_i = \begin{cases} 
E_i (\sigma_i \leq \sigma) \\
A_i + A_{i2} (\sigma_i > \sigma) 
\end{cases} 
\]  
(17)

After simplification, the expansion pressure of each expansion joint during the expansion process of the heat exchange tube is:

\[
P_{\text{min}} = \frac{2}{\sqrt{3}} A_{i1} \ln K_i - \frac{2}{3} A_{i2} c_i^{m_i} (c_i^{-2} - c_i^{-2}) \]  
(18)

\[
P_{\text{max}} = \frac{2}{\sqrt{3}} A_{i1} \ln K_i - \frac{2}{3} A_{i2} c_i^{m_i} (c_i + \frac{\sqrt{3} \sigma_o R_i^2}{2E R_i}) (R_o - R_i^{-m_i}) + \frac{2}{\sqrt{3}} A_{i1} \ln K_i - \frac{2}{3} A_{i2} c_i^{m_i} (c_i + \frac{\sqrt{3} \sigma_o R_i^2}{2E R_i}) (c_i^{-2} - c_i^{-2}) \]  
(19)

5. Conclusions

Elastic-perfectly plastic model is simple, but the calculation results of the expansion pressure based on the elastic-perfectly plastic model has a large error. The strain hardening model is more in line with the real properties of materials, the calculation results of the expansion pressure has a high accuracy, but
this model is complex and has a cumbersome calculation to determine the expansion pressure. The bilinear model had simplified the strain-hardening model. After simplifying the strain hardening model, the determination of the expansion pressure which is close to the determination based on the strain hardening model become simple. So, the determination of the expansion pressure based on the strain hardening model is convenient for practical application. The determination of the expansion pressure which is mentioned in the paper are based on the single tube model. In the practical production, it need to expand multi tubes at the same time. So, It’s necessary to determine the expansion pressure with multi tubes further. In the hydraulic expansion, the problem how to clean and dry the heat exchanger tube need to be solved.

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