The effect of non-uniform magnetic field on the energy spread of a low energy electron beam

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Abstract. This work aimed to enhance a resolution of the electron diffraction pattern by improving the longitudinal and lateral coherence of electron beam. Using a uniform magnetic field and slits is a cheap and straightforward method. However, such magnet requires a larger space. Smaller solenoids which create a non-uniform magnetic field are more applicable, especially when space was limited. Therefore, the effect of a non-uniform magnetic field on the energy spread of the electron beam was investigated numerically and experimentally. The simulation results agreed well (within 10%) with the measurements. It has been found that the ferrite core solenoids together with a narrow slit can be used to improve the beam coherence. However, beam shape observed from the real measurement significantly deviates from the simulation result. In addition the non-uniform magnetic field, the beam trajectory was affected by the electric field from electron gun which causes the change of beam shape and spatial distribution.

1. Introduction
Electron beam diffraction has various applications in quantum sensors [1]. Improving coherence of an electron beam leads to higher contrast and resolution of the diffraction pattern [2]. A simple method to increase beam coherence is using a bending magnet and a slit. This method improves energy spread but reduces beam intensity [3]. Since electron diffraction experiments require an ultra-high vacuum, using a small size chamber with limited space is a good economical choice. Hence, all components need to be redesign to make our setup fits into the chamber. The biggest components considered here is the bending magnet which consists of a pair of solenoids with magnetic material cores. Ferrite cylinder cores is inserted inside the coils to enhance the magnetic field and to minimize the coil current resulting in the reduction of the coil size [4]. The calculation of the magnetic field generated by solenoids with magnetic material core can be done analytically or numerically [5] by using different techniques [6]. Although there are a plenty of magnetostatics software or codes such as COMSOL Multiphysics, GEANT4, ANSYS and the Monte Carlo simulation as found in ref.[7], there are only a few achievable budget softwares which are able to accurately compute the magnetic field from a complex coil profile with a magnetic material core. In this work, the magnetic field was calculated based on Biot-Savart law and the known core material permeability. The calculation method used here was verified by the real measurement. When a good agreement was obtained between the calculation and the measurement within 10%, we then further investigated the effect of a non-uniform magnetic field on the electron beam motion by using a cathode ray tube (CRT) demonstration apparatus. Even though the trajectories of electrons in a non-uniform magnetic field have been calculated
analytically [8], the system and parameters used in each study were different. Additionally, there are numerous factors that affects electron beam motion [9]. We, therefore, performed simulations and measurements to reveal the effect of the non-uniform magnetic field on the velocity distribution and a spatial distribution of the electrons in our system.

2. Magnetic field of solenoids with ferrite core

2.1. Calculation

First, the real coils, which are different from ideal current loops, were design and modeled in a cylindrical coordinate as a function of angle $\theta$ in radian,

$$
r_{n+1} = \begin{cases} 
  r_0 + nD & \text{when } 2n\pi \leq \theta \leq \frac{2n+1}{2} \pi \\
  r_n + \frac{\pi}{2}\theta & \text{when } (2n + 1)\pi \leq \theta \leq (2n + 2)\pi \text{ and } n = m \\
  z_0 & \text{when } (n - \frac{1}{2})\pi \leq \theta \leq 2n\pi \text{ and } n = m, 2m, 3m, ... \\
  z_0 + (m - 1)D & \text{when } (m - \frac{1}{2})\pi \leq \theta \leq m\pi \\
  \frac{\theta D}{2\pi} - D & \text{otherwise}
\end{cases}
$$

where $n \geq 0$ is an ordinal number of turns, $D$ is the wire diameter and $m$ is the number of turns per layer.

After wire path was created, the magnetic field strength $\vec{H}$ along the path can be simply calculated by using Bio-Sawart law,

$$
\vec{H}(\vec{r}) = \frac{I}{4\pi} \int_C \frac{d\vec{l} \times \vec{r}}{|\vec{r}|^3}
$$

where $\vec{r}$ is the displacement vector from the wire of length $d\vec{l}$ carrying a current $I$. In case that the field strength is weak and the magnetic material core is not saturated, the magnetic field $\vec{B}(\vec{r})$ can be calculated by using linear relation between $\vec{B}(\vec{r})$ and $\vec{H}(\vec{r})$,

$$
\vec{B}(\vec{r}) = \mu \vec{H}(\vec{r})
$$

Here $\mu$ is the magnetic permeability of the core. The following boundary conditions between magnetic material core and air were also taken into account in determination of the magnetic field,

$$
\vec{B}_{\perp,\text{air}}(\vec{r}) = \vec{B}_{\perp,\text{m}}(\vec{r}) \quad \mu_{\text{air}} = \frac{\mu_{\text{m}}}{\mu_{\text{m}}}
$$

where $\vec{B}_{\perp}$ stands for the magnetic field perpendicular to the surface of the media or material and $\vec{B}_{||}$ is the magnetic field parallel to the surface of media or material. $\mu_{\text{air}}$ and $\mu_{\text{m}}$ are permeability of air and magnetic material, respectively.

2.2. Measurement

A magnetic field scanner was built by attaching 49E linear hall sensor to a 3D printer. The 3D printer with Malin’s firmware [10] was connected to a PC via USB port serial interface in python. The python script generated G-code for controlling the hall probe sensor position. The highest scanning resolutions were $\pm 0.1$ mm in each axis. An actual magnetic field at different positions was measured by the hall sensor. The sensor output voltage was directly proportional to the magnetic field strength. The analog voltage signal was converted to a digital signal by using 16 bits AD1115 analog to digital module. Then the module were interfaced with Arduino Nano [11] which sent the digital data to PC as shown in figure 1. Finally, the sensor were linearly calibrated with Lakeshore 425 Gaussmeter with error of less than 0.01 mT.
2.3. Comparison between calculated and measured magnetic fields

One of the most important parameters in our study was the permeability of the magnetic material core. Here ferrite was chosen and employed as the solenoid core. A ferrite core was removed from Toshiba flyback transformer TFB4099AD and cut into a size by using a diamond grinder. The solenoid core was 15 mm long and had diameter of 7.05 mm. In order to determine the permeability of the ferrite core, the air core solenoid was fixed on the print bed of the 3D printer and then the hall sensor was placed at a fixed distance $z_m$ in front of the coil. The magnetic field strength ($\vec{H}$) was varied by changing the coil supplied current. The magnetic field ($\vec{B}$) generated at different current was measured. Then the ferrite core was inserted into the solenoid and the same procedure was followed. According to the relation between $\vec{B}$ and $\vec{H}$ given in equation 4, the permeability of the ferrite core of $2.05 \pm 0.03 \times 10^{-6}$ T with linear regression coefficient of 0.999 was obtained. The magnetic field at any points in space was calculated from the numerical integration and the measured permeability. The calculation results shown in figure 2 indicates that the calculation matches well with measurement. The noticeable error probably originated from coils misalignment. Another important factor is a dimensional shrinkage of coil supporter which made from polylactic acid (PLA) thermoplastic. The dimensional shrinkage always occurs after cooling down of 3D heat printed PLA material. Hence, there will be dimension mismatched between originally designed coils and 3D printed coil supporter parts leading to the deviation of the measured magnetic field from the expected value.

Figure 2. (a) Calculated magnetic field at different points in space, (b) measured and calculated magnetic field along x axis at the coil current of 2 A and (c) magnetic field along y axis at the coil current of 1 A, 2 A and 3 A.
3. Effect of magnetic field on electron beam motion
The effect of non-uniform magnetic field on electron beam motion was investigated numerically and experimentally. The particle in cell (PIC) method [12] was employed to calculate the external fields, e.g., an electric field, a magnetic field and a gravitational field on the grid points. The field at any points could be achieved by interpolation from the data on the grid. In this work, the magnetic field on equal spaced grid was calculated and the electron acceleration at the grid points was obtained from Lorentz force equation,
\[ \vec{a} = \frac{q}{m} \vec{E} + \frac{q}{m} \vec{v} \times \vec{B}. \] (7)
Integrating the acceleration gives the velocity. Likewise, the electron position can be obtained by the integration of the velocity. The initial velocity and the position of each electron were randomly selected by using a normal distribution. The average electron velocity was approximated from the applied accelerating voltage of 300 V. The full width at half maximum (FWHM) of the initial velocity distribution was \(4.85 \times 10^2\) m/s and the spatial distribution on x-y plane has the same FWHM of \(1.10 \times 10^{-3}\) m. For simplicity, we neglected the relativistic effect and restricted calculation only in classical region. The FWHM of the beam profile was set to 1 mm as approximated from the CRT demonstration apparatus. Because the CRT was filled with inert gas, we could easily observe the electron beam from the luminescence emitted when electrons collide with the inert gas atoms. All input parameters used in our calculation were obtained from the CRT specifications.

3.1. Electron bending radius
The first parameter we focused on was the path of the bent electron beam at different current. However, measuring exact dimension of the beam in vacuum glass tube was not an easy task (see figure 3 (a)). We, therefore, compared only the distance along z-axis between the point where the electron beam hits the tube wall and the center point of acceleration plate (\(h\)). The distance \(h\) obtained from both simulation and experiment decreases with increasing coil supplied current, i.e., the radius of curvature of the bent electron beam becomes smaller when the supplied current rises. The linear relation between the distance \(h\) and an inverse of the supplied current similar to a motion of an electron in a uniform magnetic field, i.e., \(h \approx r = \frac{mv}{qB(I)}\) is expected. Here \(m\) is the electron mass, \(v\) is the electron velocity which depends on the accelerating voltage, \(q\) is the electron charge and \(B(I)\) is the magnetic field which depends on the supplied current. However, \(B(I)\) need to be rescaled according to an effective value of the non-uniform magnetic field. However, non-linear curves are clearly observed in figure 3 (c). This would result from the measurement offset.

![Figure 3](image-url)

Figure 3. (a) Calculated magnetic field at different points in space, (b) measured and calculated magnetic field along x axis at the coil current of 2 A and (c) the distance along z-axis where the beam hit the CRT wall (\(h \approx r\)) versus 1/I at coil current of 1 A, 1.5 A, 2 A, 2.5 A and 3 A.
3.2. Electron velocity distribution after bending

To investigate the effect of a non-uniform magnetic field coupled with a narrow slit on the electron velocity distribution, the computer simulation was used instead of measurement because it was not possible to put a slit inside the CRT. A narrow adjustable collimator slit was placed along the direction parallel to ferrite cylinders in the simulation setup as shown in figure 4. The center of the slit was placed where the electron beam bending angle reached 90 degree. Figure 5 shows the comparison between velocity distribution of the incident beam and the bent beam after passing through a 1-mm slit and a 2-mm slit. The electron beam detected behind the 1-mm slit had narrower velocity distribution and lower intensity as expected (see table 1).

![Figure 4. Slit position in simulation setup.](image)

![Figure 5. Electron beam velocity distribution after beam pass through slit at 90 degree bending angle obtained from simulation.](image)

| The coils current (A) | The beam FWHM (m/s) | The beam intensity (%) |
|-----------------------|---------------------|------------------------|
| 1.50                  | $1.86 \times 10^2$  | 58.23                  |
| 2.00                  | $2.05 \times 10^2$  | 71.61                  |
| 2.50                  | $2.44 \times 10^2$  | 66.60                  |
| 3.00                  | $2.54 \times 10^2$  | 90.04                  |

3.3. Electron spatial distribution after bending

The electron beam cross sections shown in figure 6 (a) and (b) were calculated from the electron beam cross section at the beginning and after 90 degree bending, respectively. The spatial distributions in x and z planes of the beam after passing through the bending magnet indicate that the electron beam tends to be elongated along z axis which is parallel to the magnetic material core (see figure 6 (c) and (d)). However, the simulation result significantly deviated from the real measurement presented in figure 7 (a). Hence, the effect of the electric field was included in the simulation in order to achieve the better result.
Figure 6. Cross section of (a) the initial electron beam and (b) the bent electron beam at 90 degree bending angle and the electron distributions calculated from beam cross section at 90 degree bending angle (c) along x axis and (d) along z axis obtained from simulations. The color bars represent the distance of each electron from the initial beam center.

Since the suppressor, the accelerator ring and the filament were located inside the CRT, we were unable to measure their dimensions directly. The only measurable parameter is the potential on all metal plates. Knowing the potential on all metal piece allowed us to solve Laplace’s equation. However, the boundary conditions were still missing. Therefore, we used symmetry to approximate the solution. The finite-difference were used to solve the Laplace’s equation for the potential in 3D space [13]. The electric field was determined by the potential gradient [14]. The simulation was also done by numerical integration in equation 7. When the effect of the electric field was taken into account, the deviation of the simulation from the real measurement decreased. Furthermore, the upward shift of the electrons around the beam center and the downward shift of the electrons at the edge were observed (see figure 7 (b)).

Figure 7. (a) Experimental result and (b) simulation result obtained by adding electric field. The color bar represents the distance of each electron from the initial beam center before bending.
4. Conclusion
In this work, we have investigated the effect of a non-uniform magnetic field on the electron beam to improve the energy spread for further electron diffraction applications. The calculation method of the magnetic field generated by the linear magnetic material core solenoid was successfully verified by using measured data. It has been proved that using the non-uniform magnetic field and a narrow slit helps to improve the electron beam coherence. However, the drawback of this method is that the beam intensity is reduced. After passing through a non-uniform magnetic field, the beam elongation has been observed in both simulation and measurement. Beside the coil current, the electric field from electron gun also affects the shape of the beam cross section and spatial distribution of electron trajectory. Further investigation is needed to gain a better understanding of electron path modification observed under the electric field coupled with non-uniform magnetic field.

Acknowledgments
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