Design method of compression-bending arches with web openings considering local buckling of web

Kuantang Xi, Yi Zhang

School of Civil Engineering, Xi'an University of Architecture and Technology, Xi'an, China
zy2840@chinahualueng.com
Zhangyi940726@163.com

Abstract—This paper studies the in-plane stability design method of two-hinged steel arches with web openings I-shaped cross sections when local web buckling is allowed. The paper compares two types of failure mechanisms, namely, global buckling and local buckling. Based on the analogy of calculation formula of beam bending, the author proposes a formula for calculating the bearing capacity when web buckling is taken into consideration.

1. INTRODUCTION

The actual stress state of the steel arch is compression and bending, such as vertical uniform load, concentrated force, etc. Its deformation and internal force changes are more complicated, with constantly changing distribution of bending moment, axial force and shear force [1]. Ultimately, the steel arch reaches the limit loading state.

Scholars have studied the stability in the arch plane. Timoshenko [2] gave an analytical solution based on the classical theory. Handbook of Structural Stability [3] prepared by Japan Pillar Research Council makes a summary of the theories related to buckling loads of circular arches and parabolic arches. Lin et al. [4] proposed an in-plane design formula for stable bearing capacity. Huang [5] et al. pointed out that the effects of slenderness ratio and vector span ratio must be considered at the same time. Lu [6] et al. analysed the failure morphology of the steel arch in the plane under different vector-span ratios based on experiments. Batemi [7] et al. studied the nonlinear theory of shallow arches under concentrated forces. Julia [8] et al. studied the local buckling of open-web beams by establishing explicit equations. Zhang [9] studied the stability performance of steel arch with I-shaped sections when the local web buckling is taken into consideration. Yuan [10] established the stability coefficient expression with local web buckling taken into consideration based on the regularized web height-thickness ratio and regularized slenderness ratio. Feng [11] et al. studied the failure pattern of open-web beams though experiments and proposed related design formulas.

The advanced design method of I-section components adopts high and thin webs. They will cause web buckling, but do not necessarily lead to lower bearing capacity. There is a considerable amount of post-buckling strength for use [12]. As shown in figure 1, this paper takes full-span and half-span vertical uniform loads as examples to give design suggestions under in-plane compression and bending.
2. FINITE ELEMENT MODEL AND VERIFICATION

The finite element software ABAQUS is used to establish a calculation model of a two-hinged arc steel arch structure with open-web I-shaped sections. Elastic modulus \( E = 2.06 \times 10^5 \) MPa; Poisson’s ratio \( \nu = 0.3 \); Yield strength \( f_y = 235 \) MPa.

With the bearing capacity of local web buckling in the plane taken into consideration, the author restricted the out-of-plane displacement at the interface between the web and the flange. A rigid cover plate is set at the arch foot, and the flanges at both ends and the web are fixed to the rigid cover plate, restricting the translational freedom of the central node of the rigid plate. The initial geometric defects include overall and local defects. The lowest-degree buckling deformation that causes overall instability under elastic buckling is selected as the overall defect with an amplitude of \( S/1000 \). The lowest-degree buckling deformation that causes web elastic buckling is selected as the local defect with an amplitude of \( h/500 \). In the actual situation, the situation of residual stress is more complicated. This paper doubles the overall geometric initial defect to \( S/500 \).

The test in literature [13] is used to verify the correctness of the finite element model in the circular arch plane under the action of compression and bending. The test was carried out on a two-hinged circular arch with four-structure welded I-shaped cross-section with a vector span ratio (m) of 0.2 and 0.3, under full-span vertical uniform load and half-span vertical uniform load, respectively. The arch span \( L \) is taken as 9 m. Details of the test pieces are shown in table 1:

| No. | Load conditions        | Span ratio | Section size (mm) |
|-----|------------------------|------------|-------------------|
| 1   | Full-span load         | 0.3        | Flange width \( b_f \) | 160 |
| 2   | Full-span load         | 0.2        | Flange thickness \( t_f \) | 12  |
| 3   | Half-span load         | 0.3        | Section height \( h \) | 240 |
| 4   | Half-span load         | 0.2        | Web thickness \( t_w \) | 10  |
Thus test stimulates the full-span vertical uniform load using 7 vertical concentrated forces, and stimulates the half-span vertical uniform load using 4 vertical concentrated forces. The corresponding stable bearing capacity results are shown in table 2:

| No. | Finite Element Value (kN) | Test Value (kN) | Error (%) |
|-----|---------------------------|----------------|-----------|
| 1   | 168.8×7                   | 145.9×7        | 15.7      |
| 2   | 198.2×7                   | 169.3×7        | 17.2      |
| 3   | 103.1×4                   | 109.3×4        | 5.71      |
| 4   | 106.8×4                   | 104.2×4        | 1.82      |

Based on the comparison between stable bearing capacity of the finite element and that of the experiment, it can be seen that the two values are basically similar. Under the half-span vertical uniform load, the error of the stable bearing capacity is about 5%.

In summary, based on the results of the stable bearing capacity and the load-displacement curve, the finite element ABAQUS calculation has highly accurate and reliable, which proves that the calculation model is correct.

3. FAILURE MECHANISM UNDER BENDING

Under the action of bending, higher height-to-thickness ratio of the web can reflect superior bearing performance. In order to study the similarities and differences of the failure mechanism of steel arches with web buckling and global buckling taken into consideration respectively, this paper sets \( \lambda = 70 \), vector span ratio \( m = 0.4 \), hole clear spacing \( g = 0.4h \), and hole radius \( r = 0.3h \), and then performs the elasto-plastic analysis. The size of the section is shown in table 3.

| No. | \( t_f \) (mm) | \( b_f \) (mm) | \( t_w \) (mm) | \( h \) (mm) | Web height-thickness ratio |
|-----|---------------|---------------|---------------|-------------|--------------------------|
| 1   | 24            | 20            | 20            | 600         | 30                       |
| 2   | 24            | 20            | 4             | 600         | 150                      |

Steel arch 1 and steel arch 2 differ in both the positions and the specific form of buckling and failure under the limit and failure states. This is mainly due to different internal force distributions. This paper takes the effect of uniformly distributed load as an example to analyse the two steel arches under the limit and failure states in detail.

Figure 4. Limit state under full-span load: \( h/tw = 30 \)

Figure 5. Limit state under full-span load: \( h/tw = 150 \)
As shown in figure 4 and 5, the lower flange at about one-eighth of the Steel Arch 1 reached the buckling stress first. Most of the lower flange and the web at the edge of the hole are fully buckled. The maximum in-plane horizontal displacement $w_m = 111.20\text{mm}$, the maximum relative displacement $w_m/L = 0.47\%$, the local maximum out-of-plane displacement of the web $v_m = 0.5\text{mm}$, and the maximum out-of-plane relative deformation $v_m/h = 0.08\%$. Buckling first appeared at the web near the arch foot of Steel Arch 2. The maximum in-plane horizontal displacement $w_m = 75.4\text{mm}$, the maximum relative displacement $w_m/L = 0.32\%$, the local maximum out-of-plane displacement of the web $v_m = 8.49\text{mm}$, and the maximum out-of-plane relative deformation $v_m/h = 1.41\%$. The overall initial stiffness of Steel Arch 1 and Steel Arch 2 remained basically the same. Under the action of compression and bending, the overall stiffness of Steel Arch 2 was degraded first because the web of this arch foot buckled first. Compared with Steel Arch 1, Steel Arch 2 has smaller overall displacement and a relative value smaller by 32.06\%. But it has larger local web deflection and a relative value larger by 3.4 times, which indicated that the failure mechanism of the steel arch has changed. Therefore, the larger web thickness-to-thickness ratio will also affect the bearing capacity and failure mode of the steel arch under the action of bending.

As shown in figure 6 and 7, the failure position is at the one-eighth of Steel Arch 1. It has more obvious overall antisymmetric deformation but less obvious local web deflection. The upper flange also buckled, and the lower flange was seriously deformed due to complete section rigidity loss. The maximum in-plane horizontal displacement ($w_m$) is $616.72\text{mm}$, the maximum horizontal relative displacement ($w_m/L$) is $2.63\%$, the local maximum local out-of-plane displacement ($v_m$) is $30.2\text{mm}$, and the maximum relative out-of-plane deformation ($v_m/h$) is $5.03\%$. The deformation position of the web is also limited to the plastic hinge, which is mainly due to the adaptive deformation of the connected webs resulting from the severe deformation of the lower flange. The Steel Arch 2 had severe local deflection at the web near the arch foot, and the upper and lower flanges are partially buckled. The maximum in-plane horizontal displacement $w_m = 132.36\text{mm}$, the maximum horizontal relative displacement $w_m/L = 0.564\%$, the maximum local out-of-plane web displacement $v_m = 77.25\text{mm}$, and the maximum out-of-plane relative deformation $v_m/h = 12.9\%$. The failure is mainly due to the loss of bearing capacity of the steel arch resulting from the loss of section stiffness, which is because of the web deflection.

The increase of the height-thickness ratio of the web leads to that the web buckles and enters the elasto-plastic stage before of the steel arch as a whole. Compared with pure compression, the bending moment of the second-order effect is larger under the action of bending and compression, so under this circumstance, $N/N_y$ is smaller [10].
4. DESIGN FORMULA FOR BEARING CAPACITY UNDER BENDING AND COMPRESSION

Under the combined effects of axial force, bending moment and shear force, the elasto-plastic load-bearing performance of steel arch is very different from that under the radial uniform load. Axial forces and bending moments are no longer uniformly distributed along the arch axis. Therefore, this paper proposes a bending and compression design formula similar as that for beam-column components.

\[
\frac{N}{\varphi Af_y} + \frac{M}{\alpha \gamma WF_y} \leq 1
\]  

(1)

\(\gamma_1\) is the comprehensive influence factor considering the shape development coefficient and different sections of the maximum axial force and the maximum bending moment. Its value is taken as 1.1. \(\alpha\) is the influence of lower section stiffness on the bending moment caused by the web opening, which is taken as 0.9. \(\varphi_1\) is the stability coefficient of the arch with web opening under uniform compression with web buckling taken into consideration.

\[
\varphi_1 = \varphi \times \eta
\]  

(2)

\(\varphi\) is the stability coefficient of the web-opening arch under uniform compression considering only global buckling. Refer to literature [5]. \(\eta\) is the reduction factor considering the influence of web buckling on the overall buckling. When \(t_f/t_w = 4\):

\[
\eta = \min\left(f(\lambda_{ng}, \lambda_{nw}), 1\right)
\]  

(3)

\[
f(\lambda_{ng}, \lambda_{nw}) = 1.13 - 0.22\lambda_{nw} + 0.04\lambda_{nw}^2 + 0.056\eta
\]  

(4)

\[
\gamma = -e + \delta + 1
\]  

(5)

\[
\delta = \frac{\lambda_{nw} - 1.47}{0.2}
\]  

(6)

\(\lambda_{ng}\) is the regularized slenderness ratio of steel arch. Refer to literature [5]. \(\lambda_{nw}\) is the regularized height-to-thickness ratio of the web.

\[
\lambda_{nw} = \sqrt{\frac{f_r}{\sigma_{cr, w}}}
\]  

(7)

\(\sigma_{cr, w}\) is the elastic buckling stress of the opened web:

\[
\sigma_{cr, w} = \frac{\beta k \pi^2 E}{12(1 - \mu^2)} \left(\frac{t_w}{h}\right)^2
\]  

(8)

\(k\) is the elastic buckling coefficient of the web, which is set as 6.97; \(\beta\) is the reduction coefficient of the web with hole opening taken into consideration.

\[
\beta = 0.99 - 1.76 \times \exp\left[-0.5 \times \left(\frac{g - 0.08}{0.15}\right)^2 - 0.5 \times \left(\frac{t - 3.47}{1.82}\right)^2\right]
\]  

(10)

When \(t_f/t_w \neq 4\), in order to simplify the design formula and facilitate utilization, the author assumed that the web was still fixed to the flange, and introduced the equivalent coefficient \(\chi\) for correction:

\[
\lambda_{nw} = \chi \lambda_{bw}
\]  

(11)

\[
\chi = 1.95 + 0.2 \frac{t_f}{t_w} - 0.1 \left(\frac{t_f}{t_w}\right)^2
\]  

(12)
Through the above formula, the height-to-thickness ratio of the web and flange is equivalent to $\lambda_{ew}$, which is used to replace $\lambda_{nw}$, the reduction factor $\eta$, in (3). In this way, the stability coefficient of steel arch in this case can also be calculated using this formula.

The calculation results of the formula and finite element are shown in figure 8.

![Figure 8. Calculation results of formula and finite element under bending and compression](image)

After correction, the range of application of bearing capacity design formula of the arch with web openings under bending and compression when local web buckling is taken into consideration is: vector span ratio $m = 0.1 \sim 0.4$; geometric slenderness ratio $\lambda = 40 \sim 150$; height-thickness ratio of web $h/tw = 80 \sim 160$; thickness ratio of flange web $tf/tw = 1.0 \sim 4.0$; width-to-thickness ratio of flange $bf/tf = 8 \sim 14$; hole radius $r = 0.25h \sim 0.35h$; net hole spacing $g = 0.35h \sim 0.5h$.

5. CONCLUSION

This paper conducts an elasto-plastic analysis on the web-opening arch with I-shaped sections when local web buckling is taken into consideration, and proposes the design formula for calculating the bearing capacity under bending and compression. The major conclusions are as follows:

(1) Based on the elasto-plastic analysis, this paper compares the failure mechanism of the steel arch when web buckling and global buckling is respectively taken into consideration. The result shows that when local web buckling occurs before global buckling, the bearing capacity of steel arches will be affected to some extent.

(2) Due to the different failure mechanism, this paper proposes a design formula for the in-plane arch with web openings under compression and bending when with web buckling is taken into consideration.

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