Research Article

Dissipative Filter Design for Nonlinear Time-Varying-Delay Singular Systems against Deception Attacks

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This paper applies a T-S fuzzy model to depict a class of nonlinear time-varying-delay singular systems and investigates the dissipative filtering problem for these systems under deception attacks. The measurement output is assumed to encounter random deception attacks during signal transmission, and a Bernoulli distribution is used to describe this random phenomena. In this case, the filtering error system modeled by a stochastic singular T-S fuzzy system is established and stochastic admissibility for this kind of system is defined firstly. Then, by combining some integral inequalities and using the Lyapunov–Krasovskii functional approach, sufficient delay-dependent conditions are presented based on linear matrix inequality techniques, where the system of filtering error can be stochastically admissible and strictly \((Q, S, R)\)-dissipative against randomly occurring deception attacks. Moreover, parameters of the desired filter can be obtained via the solutions of the established conditions. The validity of our work is illustrated through a mostly used example of the nonlinear system.

1. Introduction

In order to study various actual systems, such as large-scale systems, circuit systems, and biological systems, scholars have widely used the singular system model to describe these systems. In contrast with ordinary state space systems, singular systems can describe the performance characteristics of physical systems better [1]. On the other hand, time delay is a major factor that leads to system instability and performance degradation. Because of the coupling of the delay term and the functional equation, the study of singular time-delay systems is much more difficult than that of the standard time-delay systems [2]. Not only stability but also regularity and absence of impulse (or causality) are involved in the admissible analysis problem for singular time-delay systems. Up to now, various results for studying singular time-delay systems have been published. Reachable set estimation for continuous-time singular delay systems [3–5], sliding mode control for discrete-time singular delay systems [6–8], and \(H_{\infty}\) control for singular time-delay systems with Markovian jump parameters [9–11] are just a few examples of so many research works.

In the past decade, control researchers discovered that in practical or industrial applications, nonlinearities in system dynamics behavior can be fairly accurately described as a set of locally linear models mixed together with fuzzy membership functions. Thus, any complex nonlinear systems can be fuzzified and effectively approximated as a set of linear models by using the T-S fuzzy model approach [12]. In this case, a similar way to linear systems can be extended to analyze and synthesize nonlinear systems and lots of fruitful studies on the fields of stability theories, adaptive tracking control, \(H_{\infty}\) filter design, etc., have been achieved for T-S fuzzy systems [13–18]. Recently, using the T-S fuzzy model to approximate singular nonlinear systems has also been published in many literature studies. To mention a few, the problems of admissibility analysis and controller design for singular T-S fuzzy systems with mismatched membership functions were investigated in [19,20]; the problems of adaptive sliding mode controller design for T-S fuzzy singular systems were considered in [21,22]; and asynchronous filtering problems for T-S fuzzy singular
systems with Markovian jump parameters were investigated in [23,24].

On another active research frontier, cyber-attacks have become more and more important factors to threaten network security in the network control system since they could lead to the leakage of a large amount of confidential information. So far, denial-of-service attacks and deception attacks are two main kinds of cyber-attacks widely studied by scholars [25–29]. Especially, deception attacks are more seclusive and harder to detect which import mendacious data in the process of signal transmission and lead to performance damage of the target system. Therefore, it is very important to design a security filter for the system with deception attacks. For example, the topics include distributed recursive filtering for discrete time-delayed stochastic systems subject to both uniform quantization and deception attacks, recursive filtering for stochastic nonlinear time-varying complex networks with deception attacks, and event-triggered filter design for T-S fuzzy systems with deception attacks had been investigated in [30–32], respectively. As far as we know, control and filtering problems for T-S fuzzy singular systems with time-varying delays subject to deception attacks have not been fully investigated, not mention to dissipative filter design. Dissipativity, in simple terms, generally indicates that the increase in a system’s internal energy storage does not exceed the external energy supply of the system. The dissipativity has been analysed for large amounts of nonlinear systems and widely used in control theory and practice [33–35]. All the aforementioned facts motivate our research.

The main contribution of this paper is centered on dealing with the problem of dissipative filtering for T-S fuzzy singular systems with time-varying delays subject to deception attacks, where the measurement output is assumed to encounter random deception attacks based on a Bernoulli distribution during signal transmission. In this case, the filtering error system modeled by a stochastic singular T-S fuzzy system is established and the stochastic admissibility for this kind of system is defined. By using the Lyapunov–Krasovskii functional (LKF) approach and based on linear matrix inequality (LMI) techniques, sufficient delay-dependent conditions are established to guarantee the stochastic admissibility and strictly (Q, S, R)-dissipativity of the filtering error with randomly occurring deception attacks. Furthermore, based on these feasible conditions, parameters of the desired filter can be obtained. At last, we give an example of the nonlinear system to show the effectiveness of our result.

Notations. In this work, the dimensions of all the matrices are generally considered to be compatible. \( \mathbb{R}^n \) denotes the Euclidean space with \( n \) dimension; \( \mathbb{R}^{m \times n} \) represents the real matrices with \( n \times m \) dimension; \( I \) represents an identity matrix, and \( 0 \) denotes a zero matrix with appropriate dimension; \( \| \cdot \| \) denotes the Euclidean norm of a vector and its induced norm of a matrix; \( A + A^T \) is described by \( \text{sym}(A) \); \( \mathbb{Z}_2^+ \) is the space of integral vector over \( [0, \infty) \); for any real function \( x, y \in \mathbb{Z}_2^+ \) and real matrix \( M \), we define \( \langle x, M y \rangle_d = \int_0^d x(t) M y(t) dt \); the mathematical expectation operator is denoted as \( \mathbb{E}\{\cdot\} \).

2. Problem Formulation

For the description of an interval time-varying-delay nonlinear singular system, a delayed T-S fuzzy model is adopted in terms of \( r \) plant rules as follows.

Rule 1. If \( \theta_i(t) \) is \( \mu_{ij} \), and \( \ldots \) and \( \theta_r(t) \) is \( \mu_{rp} \), then
\[
\begin{align*}
E \dot{x}(t) & = A_i x(t) + A_{di} x(t - d(t)) + B_i \omega(t), \\
y(t) & = C_i x(t) + C_{di} x(t - d(t)) + D_i \omega(t), \\
z(t) & = L_i x(t), \\
x(t) & = \phi(t), \quad \forall t \in [-d_2, 0],
\end{align*}
\]
where \( x(t) \in \mathbb{R}^n \), \( y(t) \in \mathbb{R}^m \), and \( \omega(t) \in \mathbb{R}^l \) stand for the state vector, the measurement output, and the disturbance input, respectively; \( z(t) \in \mathbb{R}^q \) is the signal to be estimated; \( \phi(t) \) is the initial condition; the premise variable vector is described by \( \theta(t) = [\theta_1(t), \theta_2(t), \ldots, \theta_r(t)] \), and the fuzzy sets are \( \mu_{ij} (i = 1, \ldots, r; j = 1, \ldots, p) \); the time-varying delay \( d(t) \) satisfies the conditions of \( 0 \leq d_1 \leq d(t) \leq d_2, \; d(t) \leq \omega \), and \( d_1, d_2, \; 0 \leq \omega < 1 \) are constant scalars; \( E \in \mathbb{R}^{m \times m} \) may be a singular matrix with rank(\( E \)) = \( g \leq m \); and \( A_i, A_{di}, B_i, C_i, C_{di}, D_i, L_i \) are known real constant matrices with appropriate dimensions.

Then, we can generate the model of the T-S fuzzy singular systems when time-varying delay is considered:
\[
\begin{align*}
E \hat{x}(t) & = \sum_{i=1}^{r} h_i(\theta(t))[A_i x(t) + A_{di} x(t - d(t)) + B_i \omega(t)], \\
y(t) & = \sum_{i=1}^{r} h_i(\theta(t))[C_i x(t) + C_{di} x(t - d(t)) + D_i \omega(t)], \\
z(t) & = \sum_{i=1}^{r} h_i(\theta(t))L_i x(t),
\end{align*}
\]
where
\[
h_i(\theta(t)) = \frac{\prod_{j=1}^{p} \mu_{ij}(\theta_j(t))}{\sum_{i=1}^{r} \prod_{j=1}^{p} \mu_{ij}(\theta_j(t))},
\]
and \( \mu_{ij}(\theta_j(t)) \) stand for the grade of membership for \( \theta_j(t) \) in \( \mu_{ij} \). It is easy to verify that
\[
h_i(\theta(t)) \geq 0, \quad \sum_{i=1}^{r} h_i(\theta(t)) = 1.
\]

In this paper, for calculating the signal \( z(t) \), a fuzzy filter is derived as
\[
\begin{align*}
E \hat{x}_f (t) & = \sum_{i=1}^{r} h_i(\theta(t))[A_{fi} x_f(t) + B_{fi} y_a(t)], \\
z_f(t) & = \sum_{i=1}^{r} h_i(\theta(t))C_{fi} x_f(t),
\end{align*}
\]
where \( x_f(t) \in \mathbb{R}^n \) and \( z_f(t) \in \mathbb{R}^q \) represent the filter state and the filter output, respectively. \( y_a(t) \) is the sensor
measurement under randomly occurring deception attacks which can be given as
\[ y_a(t) = y(t) + \alpha(t)(-y(t) + \sigma(t)). \]  
(6)

\( \sigma(t) \) defines the deception signal imported into the output which is assumed to satisfy \( \| \sigma(t) \| \leq \| W y(t) \| \), and \( W \) is considered to be an appropriate dimension matrix; \( \alpha(t) \) is a Bernoulli-distributed variable and we have the following assumptions on \( \alpha(t) \):

\[
\begin{align*}
\text{Prob}[\alpha(t) = 1] &= E[\alpha(t)] = \alpha, \\
\text{Prob}[\alpha(t) = 0] &= 1 - \alpha.
\end{align*}
\]  
(7)

Remark 1. The model of deception attacks considered in this paper is established in (6). A Bernoulli distribution is applied to describe the random property of deception attacks. It is easy to say that \( \alpha(t) = 1 \) or \( \alpha(t) = 0 \) means sensor measurement is under deception attacks or not, respectively. Furthermore, the deception attacks are supposed to be norm bounded in (6), since there usually exists an upper bound for the attack signals to avoid detection.

We define the initial condition of system (5) as \( x_f(0) = x_{f0} \), and for any \( t \in [-d_2, 0] \), assume that \( x_f(t) = x_{f0} \), \( A_f, B_f, \) and \( C_f \) are filter gains to be determined.

Then, define \( e_f(t) = z(t) - z_f(t) \) and \( \eta(t) = \left[ x^T(t) \ x_f^T(t) \right]^T \). By combining systems (2) and (5), we can obtain the following filtering error system:

\[
\begin{align*}
\hat{E}\eta(t) &= \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\theta(t))h_j(\theta(t)) \left( \left[ \hat{A}_{ij} - \hat{\alpha}(t)\hat{B}_{fij} \right] \eta(t) + \left( \hat{A}_{dij} - \hat{\alpha}(t)\hat{B}_{f2ij} \right) \eta(t - d(t)) \\
+ \left( \hat{B}_{ij} - \hat{\alpha}(t)\hat{B}_{f3ij} \right) \omega(t) + aB_f \sigma(t) + \hat{\alpha}(t)B_{f4}\sigma(t) \right), \\
e_f(t) &= \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\theta(t))h_j(\theta(t))\hat{\alpha}(t)B_{f3ij} \eta(t), \\
\eta(t) &= \phi(t) = \left[ \phi^T(t) \ x_f^T(t) \right]^T, \quad \forall t \in [-d_2, 0],
\end{align*}
\]  
(8)

where

\[
\begin{align*}
\hat{A}_{ij} &= \left[ \begin{array}{cc} A_i & 0 \\
(1 - \alpha)B_f C_j & A_f \end{array} \right], \\
\hat{A}_{dij} &= \left[ \begin{array}{cc} A_i & 0 \\
(1 - \alpha)B_f C_d & 0 \end{array} \right], \\
\hat{B}_{ij} &= \left[ \begin{array}{cc} B_i \\
(1 - \alpha)B_f D_j \end{array} \right], \\
\hat{B}_{f2ij} &= \left[ \begin{array}{cc} 0 \\
B_f C_i \end{array} \right], \\
\hat{B}_{f3ij} &= \left[ \begin{array}{cc} 0 \\
B_f D_i \end{array} \right], \\
\hat{B}_{f4ij} &= \left[ \begin{array}{cc} 0 \\
B_f \end{array} \right], \\
\hat{L}_{ij} &= \left[ \begin{array}{cc} L_i & -C_f \end{array} \right], \\
\hat{\alpha}(t) &= \alpha(t) - \alpha.
\end{align*}
\]  
(9)

Denoting

system (8) can be given as

\[
\begin{align*}
\hat{E}\eta(t) &= \hat{A}(t)\eta(t) + \hat{A}_d(t)\eta(t - d(t)) + \hat{B}(t)\omega(t) + \hat{B}_1(t)\sigma(t), \\
e_f(t) &= \hat{L}(t)\eta(t).
\end{align*}
\]  
(11)

Remark 2. Due to a stochastic variable of Bernoulli distribution, filtering error system (11) is established as a stochastic T-S fuzzy singular system. In this condition, the definition of admissibility in [19] does not work for the stochastic T-S fuzzy singular system in (11) anymore. Motivated by the stochastic admissibility defined in [36] of
the discrete time case, we can generalize the definition of admissibility in [19] naturally and have the following definition of stochastic admissibility for system (11).

**Definition 1** (see [36]).

1. System (11) is said to be stochastically regular if \( \det(\mathcal{E} - A(t)) \) is not identically zero.
2. System (11) is said to be stochastically impulse-free if \( \deg(\det(\mathcal{E} - A(t))) = \operatorname{rank}(\mathcal{E}) \).
3. System (11) with \( \omega(t) = 0 \) is said to be stochastically stable if there exists a scalar \( \mathcal{M} > 0 \) such that
   \[
   \lim_{t \to \infty} \mathbb{E} \left\{ \int_0^t \eta^T(s, \phi(t))\eta(s, \phi(t))\,ds \right\} \leq \mathcal{M},
   \]
   where \( \eta(s, \phi(t)) \) represents the system solution.
4. System (11) with \( \omega(t) = 0 \) is said to be stochastically admissible if it is stochastically regular, impulse-free, and stable.

**Lemma 2** (see [33]). Give real symmetric matrices \( Q \) and \( R \) and matrix \( S \). For a scalar \( \gamma > 0 \), if the equation
   \[
   \mathbb{E} \left\{ \langle e_f, Qe_f \rangle_{t_i} \right\} + 2\mathbb{E} \left\{ \langle e_f, Se_f \rangle_{t_i} \right\} + \mathbb{E} \left\{ \langle \omega, R\omega \rangle_{t_i} \right\} \geq \gamma \mathbb{E} \left\{ \langle \omega, \omega \rangle_{t_i} \right\}
   \]
   holds under zero initial state with \( t_s > 0 \), system (11) is said to be strictly \((Q, S, R)\)-\( \gamma \)-dissipative. Furthermore, we suppose \( Q \leq 0 \) and \(-Q = Q^T Q_- \) for some \( Q_- \).

Before discussing the main results, we present a few lemmas that we need to use.

**Lemma 1** (see [37]). For a matrix \( Z > 0 \), the following inequality holds:

\[
-(k_2 - k_1)\int_{k_1}^{k_2} x^T(t)Zx(s)\,ds \leq -\Pi^T \text{diag}(Z, 3Z, 5Z)\Pi,
\]

where
\[
\Pi = \begin{bmatrix}
 x(k_2) - x(k_1) \\
 x(k_2) + x(k_1) - 2k_2 - k_1 \int_{k_1}^{k_2} x(s)\,ds \\
 x(k_2) - x(k_1) - 6k_2 - k_1 \int_{k_1}^{k_2} \zeta_{x,k_1}(s)x(s)\,ds
\end{bmatrix}
\]
and \( \zeta_{x,k_1}(s) = 2\left(s - k_1/k_2 - k_1\right) - 1 \).

**Lemma 2** (see [38]). Given a real scalar \( \beta \in (0,1) \), matrices \( Z_1 > 0 \) and \( Z_2 > 0 \), and any matrices \( \mathcal{W}_1 \) and \( \mathcal{W}_2 \), the inequality

\[
\begin{bmatrix}
 1 \\
 0 \\
 0
\end{bmatrix}
\begin{bmatrix}
 \frac{1}{\beta} & 0 \\
 0 & \frac{1}{1 - \beta}
\end{bmatrix}
\begin{bmatrix}
 Z_1 + (1 - \beta)\mathcal{F}_1 & (1 - \beta)\mathcal{W}_1 + \beta\mathcal{W}_2 \\
 (1 - \beta)\mathcal{W}_1 + \beta\mathcal{W}_2 & Z_2 + \beta\mathcal{F}_2
\end{bmatrix}
\begin{bmatrix}
 1 \\
 0 \\
 0
\end{bmatrix}
\]

holds, where
\[
\mathcal{F}_1 &= Z_1 - \mathcal{W}_2 Z_2^{-1}\mathcal{W}_1^T \\
\mathcal{F}_2 &= Z_2 - \mathcal{W}_1 Z_1^{-1}\mathcal{W}_2^T.
\]

**Lemma 3** (see [13]). If
\[
\Xi_{ii} < 0, \quad i = 1, 2, \ldots, r,
\]
then
\[
\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\theta(t))h_j(\theta(t))\Xi_{ij} < 0,
\]
where \( h_i(\theta(t)) \), \( i = 1, 2, \ldots, r \), satisfy (3) and (4).

### 3. Main Results

In this section, we will design a dissipative filter for system (1) with randomly occurring deception attacks. First, we give some notations in order to simplify the presentation:

\[
\zeta_1(s) = \frac{2s + d_1}{d_1} - 1,
\]
\[
\zeta_2(s) = \frac{2s + d(t)}{d(t) - d_1} - 1,
\]
\[
\zeta_3(s) = \frac{2s + d_2}{d_2 - d(t)} - 1,
\]
\[
\zeta_4(s) = \frac{2s + d_2}{d_2 - d_1} - 1.
\]
\[ \zeta(t) = \left[ \eta^T(t) E^{\wedge T} \right] \begin{bmatrix} t \\ t_{-d_1} \\ t_{-d_2} \\ t_{-d_1} \\ t_{-d_2} \end{bmatrix} \int_{t_{-d_1}}^t \eta^T(s) E^T ds \int_{t_{-d_2}}^t \zeta_1(s) \eta^T(s) E^T ds \int_{t_{-d_2}}^t \eta^T(s) E^T ds \right]^T, \]

\[ \xi_1(t) = \left[ \eta^T(t) \eta^T(t) E^{\wedge T} \eta^T(t-d_1) \eta^T(t-d_2) \right]^T, \]

\[ \xi_2(t) = \frac{1}{d(t)} \left[ \int_{t_{-d_1}}^t \eta^T(s) E^T ds \int_{t_{-d_2}}^t \zeta_1(s) \eta^T(s) E^T ds \right]^T, \]

\[ \xi_3(t) = \frac{1}{d(t)-d_1} \left[ \int_{t_{-d_1}}^t \eta^T(s) E^T ds \int_{t_{-d_2}}^t \zeta_2(s) \eta^T(s) E^T ds \right]^T, \]

\[ \xi_4(t) = \frac{1}{d(t)-d_2} \left[ \int_{t_{-d_2}}^t \eta^T(s) E^T ds \int_{t_{-d_1}}^t \zeta_3(s) \eta^T(s) E^T ds \right]^T, \]

\[ \xi_5(t) = (d(t)-d_1) \xi_1(t), \]

\[ \xi_6(t) = (d_2-d(t)) \xi_4(t), \]

\[ \xi(t) = [\xi_1(t) \xi_2(t) \xi_3(t) \xi_4(t) \xi_5(t) \xi_6(t) \sigma^T(t) \omega^T(t)]^T, \]

\[ \Pi_1 = \begin{bmatrix} \Psi_3^T 2R_2 \Psi_1 \\ \Psi_4 \\ \Psi_4 \end{bmatrix}, \]

\[ \Pi_2 = \Psi_5 \Psi_3^T, \]

\[ \Pi_3 = \begin{bmatrix} \Psi_3^T \Psi_2 \\ \Psi_4 \\ \Psi_4 \end{bmatrix}, \]

\[ \Pi_4 = \Psi_4^T R_2^{-1} \Psi_4. \]

\[ \Psi_1(d(t)) = \left[ \mathcal{J}_1^T \mathcal{J}_5 \mathcal{J}_1 \mathcal{J}_{12} \mathcal{J}_{14} (d_2-d(t)) \mathcal{J}_{12} + (d(t)-d_1) (\mathcal{J}_{13} - \mathcal{J}_{14}) \right]^T, \]

\[ \Psi_2 = \left[ \mathcal{J}_2^T \mathcal{J}_2 (\mathcal{J}_1 + \mathcal{J}_3)^T \mathcal{J}_2 \mathcal{J}_3 - \mathcal{J}_6 \mathcal{J}_{12} - \mathcal{J}_{14} \right]^T, \]

\[ \Psi_3 = \left[ \mathcal{J}_3 - \mathcal{J}_4 \right]^T \mathcal{J}_3 - \mathcal{J}_6 \mathcal{J}_{12} - \mathcal{J}_{14} \right]^T, \]

\[ \Psi_4 = \left[ \mathcal{J}_4 - \mathcal{J}_4 \right]^T \mathcal{J}_3 - \mathcal{J}_6 \mathcal{J}_{12} - \mathcal{J}_{14} \right]^T, \]

\[ f_1(d(t)) = (d(t)-d_1) \left[ \mathcal{J}_8 - \mathcal{J}_{12} \right], \]

\[ f_2(d(t)) = (d_2-d(t)) \left[ \mathcal{J}_{10} - \mathcal{J}_{14} \right], \]

\[ \mathcal{J}_i = \begin{bmatrix} 0_{n \times (i-1)n} I_{2n} 0_{n \times (15-2)n} 0_{n \times (m+1)} \end{bmatrix}, \quad i = 1, \ldots, 15, \]

\[ \mathcal{J}_6 = \begin{bmatrix} 0_{n \times 3n} I_n 0_{n \times 2n} \end{bmatrix}, \]

\[ \mathcal{J}_7 = \begin{bmatrix} 0_{n \times (3n+m)} I_1 \end{bmatrix}, \]

\[ \mathcal{J}_8 = \begin{bmatrix} I_n 0_{n \times (29+m+n)} \end{bmatrix}, \]

\[ \mathcal{J}_9 = \begin{bmatrix} 0_{n \times 6n} I_n 0_{n \times (23+m+n)} \end{bmatrix}, \]

\[ \mathcal{J}_1 = \text{diag} \{ R_2, 3R_2, 5R_2 \}. \]
Theorem 1. For given scalars $\alpha, \gamma > 0$, $0 \leq d_1 < d_2$, and $0 \leq \omega < 1$, matrices $W$ and $S$, symmetric matrices $Q$ and $R$ with $Q \leq 0$, and full column rank matrix $T$ with $E^T \Gamma = 0$, system (11) is said to be stochastically admissible and strictly $(Q, S, R)$-\(\gamma\)-dissipative, if there exist matrices $P = P_{11} P_{12} P_{13} P_{14} P_{15}$

\[
\begin{bmatrix}
    P_{11} & P_{12} & P_{13} & P_{14} & P_{15} \\
    \ast & P_{22} & P_{23} & P_{24} & P_{25} \\
    \ast & \ast & P_{33} & P_{34} & P_{35} \\
    \ast & \ast & \ast & P_{44} & P_{45} \\
    \ast & \ast & \ast & \ast & P_{55}
\end{bmatrix}
\]

such that $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $R_1 > 0$,

\[
\Omega (d(t)) = \text{sym} \left( \begin{array}{c}
    \Psi^T_1 (d(t)) P \Psi_2 + T F_1^T \Psi_2 + \mathcal{U}_1 f_1 (d(t)) + \mathcal{U}_2 f_2 (d(t)) + \left( \mathcal{F}_1^T \mathcal{F}_1 + \mathcal{F}_2^T \mathcal{F}_2 \right) \Phi - \left( L(t) \mathcal{F}_3 \right)^T S \mathcal{F}_1 \\
    + \alpha (C(t) \mathcal{F}_1 + C_d(t) \mathcal{F}_2 + D(t) \mathcal{F}_3)^T W^T W (C(t) \mathcal{F}_1 + C_d(t) \mathcal{F}_2 + D(t) \mathcal{F}_3) - \alpha \mathcal{F}_1^T \mathcal{F}_2 \\
    - \left( L(t) \mathcal{F}_1 \right)^T Q \left( \hat{L}(t) \mathcal{F}_1 \right) - \mathcal{F}_1^T \left( R - \gamma I \right) \mathcal{F}_1 + \mathcal{F}_1^T (Q_1 + Q_2 + Q_3) \mathcal{F}_1 - \mathcal{F}_1^T Q_1 \mathcal{F}_3 \\
    - (1 - \omega) \mathcal{F}_1^T Q_4 \mathcal{F}_2 - \mathcal{F}_2^T \mathcal{F}_3 Q_3 + \mathcal{F}_2^T (d_d^2 R_1 + (d_d - d_1)^2 R_2) \mathcal{F}_2 - \mathcal{F}_1^T (1 - \mathcal{F}_1^T \mathcal{F}_3)^T \hat{E} \mathcal{F}_1 - \hat{E} \mathcal{F}_3 \\
    - 3 \left( \hat{E} \mathcal{F}_1 + \hat{E} \mathcal{F}_3 - 6 \mathcal{F}_6 \right)^T R_1 \left( \hat{E} \mathcal{F}_1 + \hat{E} \mathcal{F}_3 - 6 \mathcal{F}_6 \right) - 5 \left( \hat{E} \mathcal{F}_1 + \hat{E} \mathcal{F}_3 - 6 \mathcal{F}_7 \right)^T R_1 \left( \hat{E} \mathcal{F}_1 + \hat{E} \mathcal{F}_3 - 6 \mathcal{F}_7 \right)
\end{array} \right)
\]

\[
\Phi = \begin{bmatrix} A(t) & -I_{2n} & 0_{2n} & A_d(t) & 0_{2n} & B(t) & \bar{B}(t) & \bar{B}(t) \end{bmatrix}.
\]

\[
\begin{align*}
A(t) &= \sum_{i=1}^{r} \sum_{j=1}^{r} h_i (\theta(t)) h_j (\theta(t)) \hat{A}_{ij}, \\
\bar{A}(t) &= \sum_{i=1}^{r} \sum_{j=1}^{r} h_i (\theta(t)) h_j (\theta(t)) \hat{A}_{ij}, \\
\bar{B}(t) &= \sum_{i=1}^{r} \sum_{j=1}^{r} h_i (\theta(t)) h_j (\theta(t)) \hat{B}_{ij}, \\
\bar{B}(t) &= \sum_{i=1}^{r} \sum_{j=1}^{r} h_i (\theta(t)) h_j (\theta(t)) \hat{B}_{ij}, \\
C(t) &= \sum_{i=1}^{r} h_i (\theta(t)) C_i, \\
C_d(t) &= \sum_{i=1}^{r} h_i (\theta(t)) C_d_i, D(t) = \sum_{i=1}^{r} h_i (\theta(t)) D_i.
\end{align*}
\]

Proof. First, we will show the stochastic admissibility for system (11) with $\omega(t) = 0$. It can be obtained from (18) and (19) that

\[
\begin{bmatrix}
    \Re_{11} - \hat{L}(t)^T \hat{Q} L(t) + Q_1 + Q_2 + Q_3 & \Re_{12} & \Re_{13} \\
    \hat{a} & \Re_{22} + d_d^2 R_1 + (d_d - d_1)^2 R_2 & \Re_{23} \\
    \hat{a} & \hat{a} & \Re_{33}
\end{bmatrix} < 0,
\]

where

\[
\Omega (d_1) - \Pi_1 + \Pi_2 < 0,
\]

\[
\Omega (d_2) - \Pi_3 + \Pi_4 < 0.
\]
where
\[
\mathcal{M}_{11} = \text{sym}\left(\mathcal{T}_1 \mathcal{A}(t) + E P_{11} \mathcal{A} + E P_{13} \mathcal{E}\right)
\]
\[
\mathcal{M}_{12} = E \mathcal{P}_{11} + Y T - \mathcal{T}_1 + \mathcal{A}(t) \mathcal{T}_2',
\]
\[
\mathcal{M}_{22} = \mathcal{T}_2' + \mathcal{T}_2.
\]

\(\mathcal{I}_1, \mathcal{I}_2, \) and \(\mathcal{I}_3\) represent matrices which have compatible dimensions and will not be used in the following discussion. Since \(Q_1 > 0, Q_2 > 0, Q_3 > 0, R_1 > 0, R_2 > 0,\) and \(-Q \geq 0,\) we have
\[
\begin{bmatrix}
\mathcal{M}_{11} & \mathcal{M}_{12} \\
\mathcal{M}_{12} & \mathcal{M}_{22}
\end{bmatrix} < 0.
\]

Premultiplying and postmultiplying (23) by \([I \mathcal{A} (t)^T] \) and \([I \mathcal{A} (t)^T] \)\(^T\), we can obtain
\[
V(\eta_1) = \xi^T(\eta_1) P\xi(t) + \int_{t - d_1}^{t} \eta^T(s) Q_1 \eta(s) ds + \int_{t - d_2}^{t} \eta^T(s) Q_2 \eta(s) ds + \int_{t - d_3}^{t} \eta^T(s) Q_3 \eta(s) ds
\]
\[
+ d_1 \int_{t - d_1}^{t} \eta^T(s) R_1 E \eta(s) ds d\theta + (d_2 - d_1) \int_{t - d_2}^{t} \eta^T(s) R_2 E \eta(s) ds d\theta.
\]

Set
\[
\begin{align*}
\xi(t) &= \begin{bmatrix} \xi_1^T(t) & \xi_2^T(t) & \xi_3^T(t) & \xi_4^T(t) & \xi_5^T(t) \end{bmatrix}^T, \\
\mathcal{F}_1 &= \begin{bmatrix} 0_{2n \times (i-1)2n} & I_{2n} & 0_{2n \times (15-i)2n} \end{bmatrix}, \quad i = 1, \ldots, 15, \\
\mathcal{F}_{16} &= \begin{bmatrix} 0_{n \times 30n} & I \times m \end{bmatrix}, \\
\mathcal{F}_{17} &= \begin{bmatrix} I_n & 0_{n \times (29n+m)} \end{bmatrix}, \\
\mathcal{F}_{19} &= \begin{bmatrix} 0_{n \times 6n} & I_n & 0_{n \times (23n+m)} \end{bmatrix}, \\
\Psi_1(d(t)) &= \mathcal{F}_1 \mathcal{E} \widehat{d}_1 \mathcal{F}_6 \mathcal{F}_7 \mathcal{F}_{12} \mathcal{F}_{14} \left( d_2 - d_1 \right) \left( \mathcal{F}_{12} + \mathcal{F}_{15} \right) + \left( d(t) - d_1 \right) \left( \mathcal{F}_{13} + \mathcal{F}_{14} \right), \\
\Psi_2 &= \begin{bmatrix} \mathcal{F}_2^T \left( \mathcal{F}_1 - \mathcal{F}_3 \right) \mathcal{E}^T \left( \mathcal{F}_1 + \mathcal{F}_3 \right) \mathcal{E} - 2 \mathcal{F}_6 \left( \mathcal{F}_3 - \mathcal{F}_5 \right) \mathcal{E}^T \left( \mathcal{F}_{12} + \mathcal{F}_{14} \right) \\
(d_2 - d_1) \left( \mathcal{F}_3 + \mathcal{F}_5 \right) \mathcal{E} - 2 \left( \mathcal{F}_{12} + \mathcal{F}_{14} \right) \end{bmatrix}^T, \\
\Psi_3 &= \begin{bmatrix} \mathcal{F}_3 - \mathcal{F}_5 \mathcal{E}^T \left( \mathcal{F}_3 + \mathcal{F}_4 \right) \mathcal{E} - 2 \mathcal{F}_8 \left( \mathcal{F}_3 - \mathcal{F}_4 \right) \mathcal{E} - 6 \mathcal{F}_9 \end{bmatrix}^T, \\
\Psi_4 &= \begin{bmatrix} \mathcal{F}_4 - \mathcal{F}_5 \mathcal{E}^T \left( \mathcal{F}_4 + \mathcal{F}_5 \right) \mathcal{E} - 2 \mathcal{F}_{10} \left( \mathcal{F}_4 - \mathcal{F}_5 \right) \mathcal{E} - 6 \mathcal{F}_{11} \end{bmatrix}^T, \\
f_1(d(t)) &= (d(t) - d_1) \begin{bmatrix} \mathcal{F}_8 \\
\mathcal{F}_9 \end{bmatrix}, \\
f_2(d(t)) &= (d_2 - d(t)) \begin{bmatrix} \mathcal{F}_{10} \\
\mathcal{F}_{11} \end{bmatrix}.
\[
\begin{aligned}
\hat{\Phi} &= \left[ \hat{A}(t) - I_{2n} \ 0_{2n} \ 
\hat{A}_d(t) \ 0_{2n \times 2n} \ 
\right], \\
\hat{\Pi}_1 &= \left[ \hat{\Psi}_3 \begin{bmatrix} 2\hat{R}_2 \ & \hat{\Psi}_3 \end{bmatrix} \right], \\
\hat{\Pi}_2 &= \hat{\Psi}_3 \hat{\Psi}_4 \hat{\Psi}_5, \\
\hat{\Pi}_3 &= \left[ \hat{\Psi}_3 \begin{bmatrix} \hat{R}_2 & \hat{\Psi}_4 \end{bmatrix} \right], \\
\hat{\Pi}_4 &= \hat{\Psi}_4 \hat{\Psi}_5 \hat{\Psi}_6.
\end{aligned}
\]

Define the infinitesimal $\mathcal{L}$ as $\mathcal{L}V(\eta) = \lim_{\Delta t \to 0} 1/\Delta \{ E[V(\eta) + \Delta \eta] - V(\eta) \}$. Along the trajectory of (11) with $\omega(t) = 0$, we have

\[
E[\mathcal{L}V(\eta)] \leq \xi'(t) \left( \text{sym} \left( \hat{\Psi}_1(d(t)) \hat{\Psi}_2 \right) + \hat{\Psi}_1(Q_1 + Q_2, \hat{\Psi}_3 - \hat{\Psi}_2 \hat{\Psi}_5) \right. \\
- (1 - \omega) \hat{\Psi}_1(Q_1 \hat{\Psi}_4 - \hat{\Psi}_3 Q_2 \hat{\Psi}_5) + \hat{\Psi}_2(d_2^2 R_1 + (d_2 - d_1)^2 R_2) \hat{\Psi}_3 \right) \xi(t) \\
- d_1 \int_{t-d_1}^{t} \hat{\eta}^T(s) \hat{E}_1 \hat{E}_2 \hat{\eta}(s) ds - (d_2 - d_1) \int_{t-d_2}^{t} \hat{\eta}^T(s) \hat{E}_1 \hat{E}_2 \hat{\eta}(s) ds.
\]

Using Lemma 1, it can be generated that

\[
- d_1 \int_{t-d_1}^{t} \hat{\eta}^T(s) \hat{E}_1 \hat{E}_2 \hat{\eta}(s) ds \leq - \xi'(t) \left( \hat{\Psi}_1 \hat{\Psi}_2 \right) \xi(t) \\
+ 3 \left( \hat{\Psi}_1 \hat{\Psi}_2 - 2 \hat{\Psi}_6 \right) R_1 (\hat{\Psi}_1 \hat{\Psi}_2 - 2 \hat{\Psi}_6) \\
+ 5 \left( \hat{\Psi}_1 \hat{\Psi}_2 - 2 \hat{\Psi}_6 \right) R_1 (\hat{\Psi}_1 \hat{\Psi}_2 - 2 \hat{\Psi}_6) \xi(t),
\]

\[
= - (d_2 - d_1) \int_{t-d_1}^{t} \hat{\eta}^T(s) \hat{E}_1 \hat{E}_2 \hat{\eta}(s) ds \\
\leq - \xi'(t) \left( \frac{d_2 - d_1}{d(t) - d_1} \hat{\Psi}_3 \hat{R}_2 \hat{\Psi}_5 + \frac{d_2 - d_1}{d(t) - d_1} \hat{\Psi}_4 \hat{R}_1 \hat{\Psi}_4 \right) \xi(t).
\]

Then, by Lemma 2, we can obtain

\[
-(d_2 - d_1) \int_{t-d_1}^{t} \hat{\eta}^T(s) \hat{E}_1 \hat{E}_2 \hat{\eta}(s) ds \leq - \frac{d_2 - d(t)}{d_2 - d_1} \xi'(t) \left( \hat{\Psi}_3 \hat{R}_2 \hat{\Psi}_5 + \hat{\Psi}_4 \hat{R}_1 \hat{\Psi}_4 \right) \xi(t). 
\]

(27)
Note that \( \hat{\xi}_5(t) = (d(t) - d_1) \hat{\xi}_5(t) \) and \( \hat{\xi}_6(t) = (d_2 - d(t)) \hat{\xi}_6(t) \). Thus, it is easy to see that for matrices \( \mathcal{U}_1 \) and \( \mathcal{U}_2 \),

\[
2\hat{\xi}^T(t) \left( \mathcal{U}_1 f_1(d(t)) + \mathcal{U}_2 f_2(d(t)) \right) \hat{\xi}(t) = 0. \tag{32}
\]

\[
2E \left[ \left[ \eta^T(t) \mathcal{F}_1 + (\hat{\xi}(t))^T \mathcal{F}_2 \right] \left[ -\hat{\xi}(t) + \hat{A}(t)\eta(t) + \hat{A}_d(t)\eta(t - d(t)) + \hat{B}_1(t)\sigma(t) \right] \right] = 0. \tag{33}
\]

It is easy to obtain from (33) that

\[
2\hat{\xi}^T(t) \left( \mathcal{F}_1^T \mathcal{F}_1 + \mathcal{F}_2^T \mathcal{F}_2 \right) \hat{\Phi}(t) = 0. \tag{34}
\]

Noticing \( \hat{E} = \Gamma = 0 \), we can obtain that for matrix \( \mathcal{Y} \) with appropriate dimension,

\[
\hat{\xi}^T(t) \left( a \left( C(t) \mathcal{F}_1 \hat{Q}_1 + C_d(t) \mathcal{F}_1 \right) \right) W^T W \left( C(t) \mathcal{F}_1 \hat{Q}_1 + C_d(t) \mathcal{F}_1 \right) - a \mathcal{F}_1^T \mathcal{F}_1 \hat{\Phi}(t) \geq 0. \tag{37}
\]

From (28)–(37), it can be verified that

\[
E[\mathcal{L}(\eta_0)] \leq \hat{\xi}^T(t) \left( \hat{\Omega}(d(t)) - \frac{d_2 - d(t)}{d_2 - d_1} (\hat{\Pi}_1 - \hat{\Pi}_3) - \frac{d(t) - d_1}{d_2 - d_1} (\hat{\Pi}_3 - \hat{\Pi}_4) \right) \hat{\xi}(t), \tag{38}
\]

where

\[
\hat{\Omega}(d(t)) = \text{sym} \left( \mathcal{F}_1^T \mathcal{F}_1 \mathcal{P}_1 \mathcal{F}_1 + \mathcal{F}_1^T \mathcal{F}_2 \mathcal{F}_1 + \mathcal{F}_2^T \mathcal{F}_2 \mathcal{P}_2 + \mathcal{F}_2^T \mathcal{F}_2 \mathcal{F}_2 + \mathcal{F}_1^T \mathcal{F}_2 \mathcal{F}_2 \right) \Phi - \mathcal{F}_1^T \mathcal{F}_1 \hat{Q}_1 \mathcal{F}_1
\]

\[
+ a \left( C(t) \mathcal{F}_1 \hat{Q}_1 + C_d(t) \mathcal{F}_1 \right) W^T W \left( C(t) \mathcal{F}_1 \hat{Q}_1 + C_d(t) \mathcal{F}_1 \right) - a \mathcal{F}_1^T \mathcal{F}_1 \hat{\Phi}(t) + \mathcal{F}_1^T \mathcal{F}_1 \hat{\Phi}(t)
\]

\[
- (1 - \omega) \hat{\mathcal{F}}_4 \hat{\mathcal{F}}_4 - \hat{\mathcal{F}}_5 \hat{\mathcal{F}}_5 + \mathcal{F}_2^T \mathcal{F}_3 \left( d_1^2 R_1 + (d_2 - d_1)^2 R_2 \right) \mathcal{F}_2 - \left( \mathcal{F}_1^T \mathcal{F}_1 \right)^T \mathcal{E} \mathcal{R}_1 \mathcal{E} \left( \mathcal{F}_1^T \mathcal{F}_1 \right)
\]

\[
- 3 \left( \hat{\mathcal{F}}_4^T + \hat{\mathcal{F}}_5^T - 2 \hat{\mathcal{F}}_6^T \right) R_1 \left( \hat{\mathcal{F}}_4^T + \hat{\mathcal{F}}_5^T - 2 \hat{\mathcal{F}}_6^T \right) R_1^T - 3 \left( \hat{\mathcal{F}}_4^T + \hat{\mathcal{F}}_5^T - 2 \hat{\mathcal{F}}_6^T \right) R_1 \left( \hat{\mathcal{F}}_4^T + \hat{\mathcal{F}}_5^T - 2 \hat{\mathcal{F}}_6^T \right) R_1^T \hat{\mathcal{F}}_4^T - \hat{\mathcal{F}}_3^T \hat{\mathcal{F}}_3
\]

We can obtain from (18) and (19) that \( E[\mathcal{L}(\eta_0)] < 0 \). Hence, we can find a scalar \( \beta > 0 \) so that \( E[\mathcal{L}(\eta_0)] \leq -\beta \| \eta(t) \|^2 \). Using Dynkin’s formula, it can be derived that

\[
E[V(\eta)] - E[V(\eta_0)] \leq -\beta E \left[ \int_0^t \| \eta(s) \|^2 \mathrm{d}s \right], \tag{40}
\]

which implies

\[
E \left[ \int_0^t \| \eta(s) \|^2 \mathrm{d}s \right] \leq \beta^{-1} E[V(\eta_0)]. \tag{41}
\]

From Definition 1, it can be concluded that system (11) in the condition of \( \omega(t) = 0 \) is considered to be stochastically stable.
To verify the dissipativity for system (11), by (18), (19), and (25), it can be derived that

\[ \mathcal{L} V (\eta_t) - e_j^T(t) Q e_j(t) - 2 e_j^T(t) \mathbf{\Theta} (t) - \omega^T(t) (\mathbf{R} - \gamma I) \omega(t) \]

\[ = \xi^T (t) \left( \Omega (d(t)) + \frac{d_2 - d(t)}{d_2 - d_1} (\Pi_1 - \Pi_2) - \frac{d(t) - d_1}{d_2 - d_1} (\Pi_3 - \Pi_4) \right) \xi(t) < 0. \]  

(42)

Thus, for any \( t_* \geq 0 \), it is easy to see that

\[ \mathbb{E} \left\{ V(\eta_{t_*}) - V(\eta_0) - \int_0^{t_*} e_j^T(t) Q e_j(t) dt - \int_0^{t_*} \omega^T(t) (\mathbf{R} - \gamma I) \omega(t) dt \right\} < 0. \]  

(43)

Considering \( \mathbb{E} \{ V(\eta_{t_*}) \} \geq 0 \) and \( V(\eta_0) = 0 \) under initial condition, by (43), we can obtain that (13) holds, which shows that system (11) is strictly \((Q, \mathcal{S}, \mathcal{R}) - \gamma\)-dissipative.

**Theorem 2.** For given scalars \( \alpha, \gamma > 0 \), \( 0 \leq d_1 < d_2 \), and \( 0 \leq \alpha < 1 \), matrices \( \mathbf{W} \) and \( \mathcal{S} \), symmetric matrices \( \mathbf{Q} \) and \( \mathbf{R} \) with \( \mathbf{Q} \leq 0 \) and \(-\mathbf{Q} = \mathbf{Q}^T \mathcal{Q}_-\), and full column rank matrix \( \mathbf{I} \) with \( \mathbf{E} \mathbf{I}^T \mathbf{I} = 0 \), if there are matrices \( \mathbf{P} = \begin{bmatrix} \mathbf{P}_1 & \ldots & \mathbf{P}_n \end{bmatrix} \)

\[ \begin{bmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} & \mathbf{P}_{13} & \mathbf{P}_{14} & \mathbf{P}_{15} \\ \mathbf{P}_{22} & \mathbf{P}_{23} & \mathbf{P}_{24} & \mathbf{P}_{25} \\ \mathbf{P}_3 & \mathbf{P}_{34} & \mathbf{P}_{35} \\ \mathbf{P}_{44} & \mathbf{P}_{45} \end{bmatrix} \]

\[ > 0, \quad Q_1 > 0, \quad Q_2 > 0, \quad Q_3 > 0, \quad R_1 > 0, \quad R_2 > 0, \quad \text{an invertible matrix } \mathbf{Y}, \quad \mathcal{M} = \mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3, \mathcal{M}_4 \]

\[ \mathcal{M}_i = \begin{bmatrix} Y_i, A_i - \mathbf{B}_f C_i, \mathbf{A}_f, \mathbf{B}_f, \mathbf{C}_f, i = 1, \ldots, r \end{bmatrix} \]

\[ \mathbf{i} = 1, 2, \ldots, r, \quad k = 1, 2, \]

(44)

\[ \mathbf{E} \mathbf{i}^T \mathbf{i} = 0, \quad \mathbf{E} \mathbf{f}^T \mathbf{f} = 0, \quad \mathbf{E} \mathbf{g}^T \mathbf{g} = 0, \quad \mathbf{E} \mathbf{h}^T \mathbf{h} = 0, \quad \mathbf{E} \mathbf{i}^T \mathbf{j} = 0, \quad i < j, \quad i = 1, 2, \ldots, r, \quad k = 1, 2, \]

(45)

where
Then, fuzzy filter (5) can guarantee system (11) to be stochastically admissible and strictly \((Q, S, R, \gamma)\)-\(\gamma\)-dissipative. Furthermore, from LMIs (44) and (45), the parameters of fuzzy filter (5) can be obtained by

\[
A_{fj} = Y^{-1} \bar{A}_{fj}, \\
B_{fj} = Y^{-1} \bar{B}_{fj}, \\
C_{fj} = \bar{C}_{fj}. 
\]

Proof. Using Lemma 3 and from (44) and (45), it can be verified that

\[
\begin{align*}
\sum_{i=1}^{r} \sum_{j=1}^{r} & h_i(\theta(t)) h_j(\theta(t)) \left( \Omega_{ij} (d_i) - \left( T_{ij} \Gamma_1 \right)^T Q \left( T_{ij} \Gamma_1 \right) - \Pi_1 + \Pi_2 \right) < 0, \\
\sum_{i=1}^{r} \sum_{j=1}^{r} & h_i(\theta(t)) h_j(\theta(t)) \left( \Omega_{ij} (d_i) - \left( T_{ij} \Gamma_1 \right)^T Q \left( T_{ij} \Gamma_1 \right) - \Pi_3 + \Pi_4 \right) < 0.
\end{align*}
\]

Remark 3. From Theorem 1 and Theorem 2, we can find that fuzzy filter (5) is designed successfully for a class of time-varying-delay T-S fuzzy singular systems subject to randomly occurring deception attacks based on the LKF approach and LMI techniques. According to Definitions 1 and 2, the filtering error system is guaranteed to be stochastically admissible and satisfy strictly \((Q, S, R, \gamma)\)-\(\gamma\)-dissipativity by conditions (44) and (45). Furthermore, Lemmas 1 and 2 are applied in (29)–(31) to lessen the estimation gaps of (47), which have been shown to reduce the conservatism in dealing with time delays in [37,38], respectively.

### 4. Numerical Example

Consider a time-delay nonlinear system which is borrowed from [39]:

\[
(1 + a \cos \varphi(t)) \ddot{\varphi}(t) = - b \dot{\varphi}(t)^3 + c \varphi(t) + c_d \varphi(t - d(t)) + f \omega(t),
\]
where $\dot{\psi}(t)$ is supposed to satisfy $|\dot{\psi}(t)| < \psi$. Choose $\psi = 2$, $a = -0.01$, $b = 0.5$, $c = -0.5$, $c_d = -0.05$, $f = -0.2$, and $d(t) = 0.3 + 0.2 \sin 0.5 \ t$. Set $x_1(t) = \varphi(t)$, $x_2(t) = \dot{\varphi}(t)$, and $x_3(t) = \ddot{\varphi}(t)$. Introduce a new variable $x(t) = [x_1^T(t) \ x_2^T(t) \ x_3^T(t)]^T$, and the nonlinear system in (31) can be exactly modeled as

$$
\begin{align*}
E \dot{x}(t) &= \sum_{i=1}^{3} h_i[A_i x(t) + A_{di} x(t - d(t)) + B_i \omega(t)], \\
y(t) &= \sum_{i=3}^{5} h_i[C_i x(t) + C_{di} x(t - d(t)) + D_i \omega(t)], \\
z(t) &= \sum_{i=3}^{5} h_i[L_i x(t)].
\end{align*}
$$

The parameters are given as

$$
\begin{align*}
h_1 &= \frac{x_1^2(t)}{\psi^2 + 2}, \\
h_2 &= \frac{1 + \cos x_1(t)}{\psi^2 + 2}, \\
h_3 &= \frac{\psi^2 - x_1^2(t) + 1 - \cos x_1(t)}{\psi^2 + 2}, \\
E &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \\
A_1 &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ c & -b(\psi^2 + 2) & a - 1 \end{bmatrix}, \\
A_2 &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ c & 0 & a - 1 - a \psi^2 \end{bmatrix}, \\
A_3 &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ c & 0 & a - 1 \end{bmatrix}, \\
A_{d1} &= A_{d2} = A_{d3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ c_d & 0 & 0 \end{bmatrix}, \\
B_1 &= B_2 = B_3 = \begin{bmatrix} 0 \\ 0 \\ f \end{bmatrix}
\end{align*}
$$

For $\gamma = 0.5$, we can obtain from conditions (44)–(47) in Theorem 2 that the desired filter parameters are

$$
\begin{align*}
C_1 &= \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}, \\
C_2 &= \begin{bmatrix} 0.6 & 0 & -0.4 \end{bmatrix}, \\
C_3 &= \begin{bmatrix} -0.2 & 0.1 & 0.55 \end{bmatrix}, \\
C_{d1} &= \begin{bmatrix} -0.25 & 0.3 & 0 \end{bmatrix}, \\
C_{d2} &= \begin{bmatrix} 0.12 & -0.6 & 0 \end{bmatrix}, \\
C_{d3} &= \begin{bmatrix} 0 & 0.8 & 0 \end{bmatrix}, \\
L_1 &= \begin{bmatrix} -0.36 & 2.5 & 1 \end{bmatrix}, \\
L_2 &= \begin{bmatrix} 0.09 \ \alpha &= 0.2, \\
Q &= -0.01, \\
S &= -1.1, \ \Re \\
F &= \begin{bmatrix} -0.2502 & 0.5137 & -0.0001 \\ 0.5429 & -1.8529 & 1.0002 \\ -0.2868 & 0.1419 & -0.9900 \end{bmatrix}, \\
A_{f1} &= \begin{bmatrix} 0.3353 & -0.2848 & 0.9769 \\ -0.3028 & -0.0022 & -1.0297 \\ -0.3269 & -0.1942 & 0.3735 \end{bmatrix}, \\
A_{f2} &= \begin{bmatrix} 0.3252 & -0.3790 & 0.9439 \\ -0.2717 & 0.1739 & -1.0781 \\ 0.1762 \end{bmatrix}, \\
A_{f3} &= \begin{bmatrix} 0.1641 \\ 0.3855 \\ -0.0724 \end{bmatrix}, \\
B_{f1} &= \begin{bmatrix} 0.0577 \\ 0.0994 \end{bmatrix}, \\
B_{f2} &= \begin{bmatrix} 0.6792 \\ -0.1021 \end{bmatrix}, \\
B_{f3} &= \begin{bmatrix} -0.2313 & 0.8816 & -1.0000 \end{bmatrix}, \\
C_{f1} &= \begin{bmatrix} -2.3880 & 2.7310 & 0.5015 \end{bmatrix}, \\
C_{f2} &= \begin{bmatrix} 0.2935 & 1.2410 & -2.0008 \end{bmatrix}.
\end{align*}
$$
Next, we will provide the simulation results to demonstrate the effectiveness of our filter design method against randomly occurring deception attacks. Let the initial condition be \([-0.1, 0.1]^T\) and the disturbance be \(\omega(t) = 0.1e^{-0.5t}\sin t\). Figure 1 shows the system state response \(x(t)\). The deception attack function is given as \(\sigma(t) = -\tanh(0.09y(t))\). The sensor measurement \(y_a(t)\) under randomly occurring deception attack is given in Figure 2. Figure 3 is the filtering error \(e_f(t)\). From Figures 1–3, we can find that our results are effective.
5. Conclusions

In this work, a method for dealing with the problem of dissipative filtering associated with T-S fuzzy singular systems with time-varying delays subject to deception attacks has been developed. Since the measurement output is supposed to encounter random deception attacks based on a Bernoulli distribution during signal transmission, the filtering error system is modeled by a stochastic singular T-S fuzzy system and the definition of stochastic admissibility for this kind of system has been presented. By using the LKF approach and LMI techniques, sufficient delay-dependent results have been generated, where the filtering error system can be stochastically admissible and strictly $(Q, S, R)$-dissipative against randomly occurring deception attacks. Besides, the desired filter parameters can be obtained by these solvable conditions. Finally, a frequently used example of the nonlinear system has been given to show the effectiveness of our work.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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