Saturation of product’s exoticity in compound nuclear reactions and its role in the production of new \( n \)-deficient nuclei with radioactive projectiles.

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Representation in terms of a new parameter, exoticity, a measure of \( n \)-deficiency or \( p \)-richness, clearly brings out the saturation tendency of the product’s maximum exoticity in a compound nuclear reaction as the compound nucleus is made more and more exotic using radioactive projectile (RIBs). The effect of this saturation on the production of new proton-rich species with RIBs over a wide \( Z \)-range has been discussed.

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The compound nuclear reaction has been used extensively in the last two decades for producing neutron-deficient (\( n \)-deficient) or proton-rich (\( p \)-rich) nuclei away from the \( \beta \)-stability. The fusion of two \( \beta \)-stable heavy-ions leads in most cases to a neutron-deficient compound system and the lightest (most \( n \)-deficient) compound nucleus of any atomic number \( Z \) can be reached through rather symmetric combination of target and projectile, e.g.
The evaporation of a neutron from such a compound nucleus (CN) takes the residue or the product towards the $p$-drip line while the evaporation of a proton or a $\alpha$-particle brings it closer to the $\beta$-stability line as compared to the said compound nucleus. The Coulomb barriers (CB) for proton and $\alpha$ usually make the evaporation of these particles energetically more costly compared to neutron evaporation and, therefore, neutron evaporation is usually more favoured. To the extent that the $n$-evaporation dominates one always gains, vis-a-vis production of exotic species, by choosing appropriate projectile-target combinations leading to the formation of lightest compound systems.

The possible availability of low energy (around Coulomb barrier) radioactive ion beams (RIBs) in near future will certainly allow formation of even lighter (as compared to the 'lightest' CN systems possible with stable projectile - stable target combinations) CN systems and it is important to assess to what extent these lighter CN systems can help in the production of new $n$-deficient nuclei in or around the $p$-drip line. In other words it is important to assess to what extent the naive expectation ”more exotic the compound nucleus, more exotic is the product” can be extrapolated as the compound nucleus becomes more and more lighter.

The above expectation is known to hold for compound systems upto a certain distance from the $\beta$-stability line but as one moves farther and farther away the binding energy of the last proton decreases very rapidly and that of the neutron increases sharply [1]. For any given $Z$ if the mass number $A$ of the compound system is less than a certain value, the energy cost of the last proton (binding energy + CB for proton) becomes actually
lower than that of a neutron making thereby $p$-evaporation a more likely process. The formation of compound systems beyond a certain extent of $n$-deficiency, thus, may not lead, given any limit of production cross-section, to the production of more $n$-deficient products. Intuitively, therefore, there is a possibility of saturation in the $n$-deficiency of the products. It is important to note that one needs to set a lower limit for the production cross-section because to the extent that no basic conservation laws (e.g. charge no., mass no. etc.) are violated, products of any exoticity can be obtained, in principle, from a given compound nucleus if one stops bothering whether the production cross section is 1 mb, or say a millionth of a millibarn.

The lightest possible compound nuclei with stable projectile - stable target combinations and obviously the even lighter compound nuclei which can be formed with $p$-rich RIBs fall mostly in the domain where effective separation energy of the last proton is either equal to or less than that of the neutron. The product pattern in such cases, therefore, are expected to show the effect of the possible saturation, that is the formation of more $n$-deficient compound systems may not lead, given any limit of production cross-section, to more $n$-deficient products.

To see whether there is indeed any saturation or not one needs, at first, to coin a definition of $n$-deficiency which is independent of $Z$. This is because the products of a given compound nucleus can have a range of $Z$ values starting from $Z_{CN}$ ($Z$ of the CN) to upto say, 7 or 8 units of atomic number less than $Z_{CN}$. To compare which one of any two products of different $Z$ is more $n$-deficient we need a $Z$-independent description of $n$-deficiency so that the products of different $Z$'s can be considered in the same footing. This can be achieved by defining a new parameter which we prefer to call "exoticity".
Keeping in mind that the absolute value of the $n$-deficiency, $A_s - A$ (where $A$ is the mass number of the nucleus of atomic number $Z$ and $A_s$ is the mass number of the most $\beta$-stable isotope for the same $Z$) alone can not be taken to be a measure of 'exoticity' of the compound nucleus or the product since the $p$-drip line is only a few neutrons away at lower $Z$ values while it is a few tens of neutrons away at higher $Z$'s, we choose to define the 'exoticity' as,

$$\zeta = 1 - (A - A_d)/(A_s - A)$$

where $A$ is the mass number of the nucleus of atomic number $Z$ and $A_d$ is the mass number of the isotope at the drip-line corresponding to the same $Z$. The exoticity is equal to 1 on the drip-line, is zero on the stability line and is greater than one beyond the drip line. The mass numbers $A_d$ were chosen from the compilation of Janecke and Masson [2] which offers compilation of the proton drip-line over a wide range.

The second hindrance in estimating the capability of a given compound nucleus to produce exotic products with production cross-section greater than any arbitrary chosen limit, is the excitation energy dependence of the production cross-section and product distribution which are typical to compound nuclear reactions. This makes any description involving the products themselves not suitable for the purpose (estimating the capacity of a CN to produce exotic products) since with increase in the excitation energy more and more new channels are opened up changing the products’ distribution and also the cross-section of a given product.

To see whether or not any representation independent of excitation energy is possible we have plotted in figure 1, as a typical example, the exoticity of the maximum exotic product
produced with cross-section greater than 1 mb as a function of the excitation energy for
different compound nuclei of Ce (Cerium) having different exoticities. The cross-section
values were computed using the code ALICE [3]. The plot reveals the interesting feature
that at lower CN exoticities an increase in the excitation energy leads to more exotic prod-
ucts but beyond a certain value of CN-exoticity the products’ exoticity becomes practically
independent of the excitation energy. It is important to note that the excitation energy
independence of the products’ exoticity does not mean that the ‘most exotic product’ satis-
ifying the minimum cross-section criterion ($\geq 1$ mb in this case) will remain the same at all
the excitation energies. At a given excitation energy there will be one product (given the
limit of production cross-section) which is most exotic. If one varies the excitation energy,
the most exotic product satisfying the minimum cross-section criterion may be a different
isotope but if one calculates its exoticity it will almost be the same as that of the most
exotic product at the earlier excitation energy. Further, the compound systems of Cerium
for which the excitation energy dependence practically vanish or becomes very weak are
those which are lighter than the CN for which the separation energy of a neutron equals to
the effective separation energy of a proton, that is $B_n = B_p^* \ (B_p^* = B_p + CB)$. For Cerium
$B_n$ equals to $B_p^*$ for $\zeta_{CN} = 0.54$ ( $A = 127$). Compound systems of other $Z$ values also
exhibit similar dependence of products’ exoticity on the excitation energy.

In this study we attempt to estimate the production of very $p$-rich exotic nuclei from
compound nuclei formed by use of $p$-rich RIBs and having $Z$ in the range $50 \leq Z_{CN} \leq 82$.
In this $Z$-range the Coulomb barrier for protons is quite high (favouring production of exotic
species) and also the compound nuclear formation cross-section and its subsequent decay
by light particle emission constitutes a major fraction of the total reaction cross-section for
projectile energies not much above the Coulomb barrier.

The compound nuclear systems of interest in the present study are those which are more exotic than the compound nuclei for which $B_n \sim B_p^*$. For example, for Ce the lightest CN that can be reached with stable target-stable projectile combination is $^{122}Ce$ for which $\zeta_{CN} = 0.75$. In the domain of our interest, therefore, an excitation energy independent description is possible if one chooses a representation in terms of exoticity of the compound nucleus and of the product rather than the usual representation in terms of $A$ and $Z$ of the products. It is important to mention here that an accurate estimation of cross-sections of very exotic products is not possible no matter which one of the presently available codes such as ALICE, CASCADE, PACE etc. is used for the purpose. Our intention is, therefore, not to predict the accurate cross-section values in a number of specific cases but to examine whether indeed there is any saturation of products’ exoticity and its implications on the production of new nuclei with RIBs.

The dependence of $\zeta_p^{max}$ on $\zeta_{CN}$ is shown in figure 2, where $\zeta$’s for odd-$Z$ products of maximum exoticity (that is $\zeta_p^{max}$) produced from compound nuclei of representative even $Z$s and with cross-sections greater than 1 mb are plotted against corresponding $\zeta_{CN}$’s. Exoticity of compound nucleus greater than one (i.e. beyond the drip-line) has also been considered. This is because the $p$-decay life time, due to the existence of CB is expected to be longer than CN decay time up to a certain distance beyond the $p$-drip line and one can attempt to form such compound systems so as to produce the maximum $p$-rich products. To decide how far beyond $\zeta_{CN} = 1$ one can go, the ground state $p$-decay life times, which are dependent on barrier (Coloumb and centrifugal) penetration probabilities, for the compound nuclei beyond
the $p$-drip line have been calculated for various angular momenta using WKB approximation. Only those compound nuclei for which the ground state $p$-decay life times have been found (for $l = 0$, to ensure a very conservative calculation) to be greater than $10^{-14}$ sec. (once again to ensure a conservative estimate) are considered. It can be seen from figure 2 that at each $Z_{CN}$, beyond a certain $\zeta_{CN}$, $\zeta_{p}^{max}$ shows a saturation tendency and the value of $\zeta_{p}^{max}$ at saturation increases with $Z$. The $\zeta_{p}^{max}$ at saturation, as expected, moves towards higher values as $Z$ increases. The gradient of the curves at various $Z_{CN}$’s are artefact of the relative binding energies or the evaporation probabilities of mainly protons and alphas at corresponding $Z_{CN}$’s.

The curves shown in figure 2 clearly bring out the limitation, as a result of the saturation, of the concept of forming more and more exotic compound systems for the production of more and more exotic $p$-rich species. The effect of the saturation is however not so serious vis-a-vis production of odd $Z$ nuclei on or around the drip-line. For odd $Z$ nuclei, the drip line can be reached for all nuclei having $Z \geq 51$ with cross-sections $\geq 1mb$. This is shown in fig.3 where odd $Z$ products of maximum exoticity that can be produced with RIBs which are 4-neutron away (deficient) from the lightest $\beta$-stable isotopes are shown in the $N-Z$ plane with two different cross-section limits of 1 mb and 0.01 mb. In the cross-section limit of 0.01 mb the drip-line can be reached for all nuclei with $Z \geq 45$ with only 4 neutron deficient RI projectiles.

For even $Z$ products, however, drip line can be reached with the same projectiles (4-neutron deficient), as the calculation reveals, only for $Z \geq 80$ with 1 mb cross-section limit. If the cross-section limit is relaxed to $\geq 0.01$ mb, the even $Z$ drip-line can be reached for
nuclei with $Z \geq 66$. The saturation thus affects seriously the prospect of reaching the $p$-drip line with reasonable cross-sections, say $\geq 10\mu b$ for all even $Z$ species with $Z \leq 66$.

One can, however, consider the use of more exotic projectiles to reach the even $Z$ drip line for $Z \leq 66$, although the beam intensity is likely to fall rather sharply with the exoticity. It is important to note however that various other factors e.g. the signal to noise ratio, detection efficiency, the type of measurement etc. together decide the lower limit of cross-section and the intensity of RIB that one needs in any given situation. The saturation, thus, in no way puts any absolute restriction and should rather be considered as a hindrance to be overcome by putting more efforts to increase beam intensity of RIBs, detection efficiency, background rejection etc.

It is important to note, in the context of discussions above, that production of new nuclei with RI projectiles usually offer a number of advantages as compared to the production of same nuclei with $\beta$-stable projectiles. An estimation of these advantages is necessary to decide the minimum usable beam intensity and the production cross-section (the product of these two represents a sort of quality factor) in a given situation. To illustrate the possible advantages of RIBs we have plotted in figure 4 the estimated cross-sections of isotopes of $Z = 70$ products from three different compound systems of $W$, i.e., $Z = 74$. The compound nucleus of minimum exoticity $\zeta_{CN} = 0.76$ is the lightest one that can be formed from the stable projectile - stable target combination ($^{64}Zn + ^{96}Ru$). The other two compound nuclei of $\zeta_{CN}$’s 0.92 and 1.07 are formed respectively from $^{60}Zn + ^{96}Ru$ and $^{56}Zn + ^{96}Ru$. These curves clearly bring out the advantage of RIBs in terms of enhanced production cross-section and in terms of enhancing the signal to background ratio. For example it can be seen from figure 4 that the production cross-section of $^{150}Yb$ (a new and very exotic nucleus) with RIB
$^{60}\text{Zn}, \zeta_{CN} = 0.93$ is about 300 times more than that with the stable projectile $^{64}\text{Zn}, \zeta_{CN} = 0.76$) and what is equally important for experimental measurements is the change in the relative production pattern. With stable projectile, the production cross-section of $^{150}\text{Yb}$ is almost four orders of magnitude less compared to the most favoured channel, whereas in the RIB case ($\zeta_{CN} = 0.93$), $^{150}\text{Yb}$ is produced with the maximum cross-section.

Such situations are very favourable in that they push down the lower limit of the needed RI beam intensity (for detection and other measurements) by several orders of magnitude or conversely the lower limit of production cross-section is pushed down allowing measurements on even more exotic species.

In this communication, we have attempted to address the question to what extent one can hope to produce more exotic products by realising more and more exotic compound systems using radioactive projectiles. It has been shown that the exoticity of the product saturates beyond a certain value of the exoticity of the CN. The value of the compound nucleus’ exoticity at which the saturation occurs depends on the atomic number $Z$ and moves, as expected, towards higher value of exoticity as $Z$ increases. The conclusions reached are practically independent of the excitation energy of the compound nuclei as long as it is a few tens of MeV above the Coulomb barrier.

This new revelation has important consequences in the production of new $p$-rich species using $p$-rich projectiles (RIBs). It tends to limit, to an extent, the utility of very $p$-rich projectiles vis-a-vis production of new $p$-rich species in or around the $p$-drip line. While the saturation does not affect that adversely the production of new odd $Z$ nuclei where the proton drip line can be reached with reasonable cross-sections ($\geq 10\mu b$) for all elements.
with $Z \geq 45$ with only moderately $p$-rich projectiles (4 neutron deficient), it does make production of $p$-drip line nuclei for even $Z$ (especially for $Z \leq 65$) difficult with moderate production cross-sections unless one uses quite exotic projectiles.

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[3] M. Blann Code ALICE/85/300
Figure Captions.

Figure-1. Dependence of maximum exoticity of products against the excitation energy (in MeV) of compound nuclei of different $\zeta_{CN}$.

Figure-2. The variation of maximum exoticity $\zeta_{p}^{max}$ for odd Z products with the exoticity of even Z compound nuclei $\zeta_{CN}$.

Figure-3. The $\beta$-stability line along with the proton drip line and lines showing the extent of production of exotic nuclei with RIBs which are only four neutron deficient compared to the lightest stable projectiles of corresponding Z (within the cross-section limits of 1mb and 0.01 mb).

Figure-4. The cross-section of $Z = 70$ isotopes against mass number $A$ at representative $\zeta_{CN}$ values.
Fig. 1

Max. Exoticity of Products, $\varepsilon_{\text{max}}$ vs Excitation Energy of CN (MeV)

Even Products
From $Z_{\text{cm}} = 58$
Fig. 2

Max. Exoticity of Product $\xi_p^{\text{max}}$

Exoticity of Compound Nucleus $\xi_{\text{CN}}$

Cross-Sections $\geq 1 \text{ mb}$
Fig. 3
Fig. 4