Inflation models, spectral index and observational constraints.

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We have evaluated the observational constraints on the spectral index $n$, in the context of a $\Lambda$CDM model. For $n$ scale-independent, as predicted by most models of inflation, present data require $n \simeq 1.0 \pm 0.1$ at the 2-$\sigma$ level. We have also studied the two-parameter scale-dependent spectral index, predicted by running-mass inflation models. Present data allow significant variation of $n$ in this case, within the theoretically preferred region of parameter space.

1 Introduction

It is generally supposed that structure in the Universe originates from a primordial gaussian curvature perturbation, generated by the quantum fluctuations of the inflaton field during slow-roll inflation. Then the spectrum of the curvature perturbation $\delta_H(k)$ is determined by the inflaton potential $V(\phi)$. In this paper we will consider the scale–dependence of the primordial spectrum, defined by the spectral index $n$:

$$n(k) - 1 = 2 \frac{\partial \ln \delta_H}{\partial \ln k} = 2M_{Pl}^2(V''/V) - 3M_{Pl}^2(V'/V)^2,$$

where the potential and its derivatives are evaluated at the epoch of horizon exit $k = aH$. The value of $\phi$ at this epoch is given by $\ln(k_{end}/k) = N(\phi) = M_{Pl}^{-2} \int_{\phi_{end}}^{\phi} (V/V')d\phi$, where $k_{end}$ is the scale leaving the horizon at the end of slow roll inflation and $N(\phi)$ is the number of e-folds. In the majority of the inflation models, $n$ is practically scale-independent so that $\delta_H^2 \propto k^{n-1}$, but we shall also discuss an interesting class of models giving significant scale dependence.

2 The observational constraints on the $\Lambda$CDM model

In the interest of simplicity and due to present observations\(^\dagger\), we adopt the $\Lambda$CDM model, with $\Omega_{tot} = 1$ and cold non-baryonic dark matter with negligible interaction. We shall constrain this model, including the spectral index, by performing a least-squares fit to the key observational quantities.

The parameters of the $\Lambda$CDM model are the primordial spectrum $\delta_H(k)$, the Hubble constant $h$ (in units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$), the total matter density $\Omega_0$, the baryon density $\Omega_b$, and the reionization redshift $z_R$ (we consider complete and sudden reionization). $z_R$ can be estimated in terms of the other parameters because it can be related to the density perturbation and the fraction of collapsed matter $f$ at the epoch of reionization, so we exclude it from the least-squares fit. In the case of the constant $n$ models we fix it at a reasonable value ($z_R = 20$), while in the case of the running mass models we compute it assuming that reionization occurs when a fixed fraction of the matter $f \simeq 1$ collapses. The spectrum is conveniently specified by its value at the COBE scale $k_{COBE} = 6.6H_0$, and the spectral index $n(k)$. Excluding $z_R$, the $\Lambda$CDM model is therefore specified by five parameters in the case of a constant spectral index, or by six parameters in the case of running mass inflation models.

\(^\dagger\) $M_{Pl} = 2.4 \times 10^{18} \text{ GeV}$ is the Planck mass, $a$ is the scale factor and $H = \dot{a}/a$ is the Hubble parameter, and $k/a$ is the wavenumber.
Table 1: Fit of the ΛCDM model to presently available data. The spectral index $n$ is a parameter of the model, as are the next four quantities. Every quantity except $n$ is a data point, with the value and uncertainty listed in the first two rows taken from the references in superscript. The result of the least-squares fit is in the lines three to five for $z_R = 20$. All uncertainties are at the nominal 1-$\sigma$ level. The total $\chi^2$ is 2.4 for 2 degrees of freedom.

|        | $n$  | $\Omega_b h^2$ | $\Omega_0$ | $h$  | $10^2 \delta H$ | $\Gamma$ | $\sigma_8$ | $\sqrt{C}_{\text{peak}}$ |
|--------|------|----------------|------------|------|-----------------|----------|------------|-------------------|
| data   | —    | 0.019          | 0.35       | 0.62 | 1.94            | 0.23     | 0.56       | 80 $\mu$K         |
| error  | —    | 0.002          | 0.075      | 0.075| 0.075           | 0.035    | 0.055      | 10 $\mu$K         |
| fit    | 1.01 | 0.019          | 0.36       | 0.63 | 1.95            | 0.19     | 0.58       | 72 $\mu$K         |
| error  | 0.05 | 0.002          | 0.06       | 0.06 | 0.075           | —        | —          | —                 |
| $\chi^2$ | —    | $4 \times 10^{-5}$ | $1 \times 10^{-2}$ | 0.1  | $5 \times 10^{-3}$ | 1.3      | 0.2        | 0.8               |

Taking as our starting point a study performed three years ago, we consider seven observational quantities: the cosmological quantities $h$, $\Omega_0$, $\Omega_B$, which we are also taking as free parameters, and moreover the shape parameter $\Gamma$, $\sigma_8$, the COBE normalization and the first peak height in the cmb anisotropy. The adopted values and errors are given in the second and third line of Table 1. For a discussion of the data, see. In common with earlier investigations, we assume the errors to be uncorrelated and random errors to dominate over systematic ones.

3 Results

We perform the least–squares fit with the CERN Minuit package; the quoted error bars use the parabolic approximation, while the exact errors computed by Minuit agree with the approximated ones to better than 10%.

In order to simplify the numerical procedure, we follow and parameterize the predicted value of $\sqrt{C}_{\text{peak}}$ with the analytical formula 

$$\sqrt{C}_{\text{peak}} = 77.5 \, \mu K \left( \frac{2h(k_{\text{COBE}})}{1.94 \times 10^{-3}} \right)^{\nu/2}$$

where

$$\nu = 0.88(n_{\text{COBE}} - 1) - 0.37 \ln(h/0.65) - 0.16 \ln(\Omega_0/0.35) + 5.4h^2(\Omega_b - 0.019) - 0.65g(\tau)\tau$$

and $\tau = 0.035 \frac{\Omega_b}{\Omega_0} h \left( \sqrt{\Omega_0(1 + z_R)^3} + 1 - \Omega_b - 1 \right)$. The formula is fitted to the CMBfast results and agrees within 10% for a 1-$\sigma$ variation of the cosmological parameters, $h$, $\Omega_0$ and $\Omega_b$, and $n = 1.0 \pm 0.05$. With the function $g(\tau)$ set equal to 1, the formula contains the usual factor exp($-\tau$). By fitting the output of CMBfast, we introduce also $g(\tau) = 1 - 0.165\tau/(0.4 + \tau)$, so that our formula is accurate to a few percent over the interesting range of $\tau$.

Constant spectral index. For the case of a constant spectral index our result is given in Table 1 for $z_R = 20$. In the case of no reionization ($z_R = 0$) we obtain a slightly smaller spectral index, $n = 0.98 \pm .05$, and cosmological parameters within the observational error bar, in agreement with previous analysis. This result is not enough yet to exclude completely proposed inflationary models, but a better determination of the peak height could strengthen the bound sufficiently to discriminate between them, especially in the case of new inflation models, which give low values of $n$.

Running mass models. We have also considered the scale-dependent spectral index, predicted in inflation models with a running inflaton mass. In these models, one–loop corrections to the potential are taken into account by evaluating the scale dependent inflaton mass $m^2(Q)$ at $Q \approx \phi$. Then the spectral index can be parameterized by just two quantities:

$$\frac{n(k) - 1}{2} = \sigma e^{-cN(\phi)} - c$$

(3)
where $\sigma$ is an integration constant and $c$ is related to the inflaton coupling responsible of the mass running. The different signs of $\sigma$ and $c$ give raise to four different models of inflation. In general, without fine tuning, we expect

$$|c| \lesssim |\sigma| \lesssim 1 \quad |c| \simeq g^2 \tilde{m}^2 M_{Pl} / V_0$$

with $g$ denoting the gauge or Yukawa coupling of the inflaton, $\tilde{m}^2$ the soft supersymmetry breaking mass of the particles in the loop and $V_0$ the value of the potential energy during inflation. With gravity-mediated susy breaking, typical values of the masses are $\tilde{m}^2 \sim V_0 / M_{Pl}^2$, which makes $c$ of order of the coupling strength. For a gauge coupling, or an unsuppressed Yukawa coupling, we expect $|c| \sim 10^{-1}$ to $10^{-2}$.

Extremizing with respect to all other parameters, we have computed the $\chi^2$ in the $\sigma$ vs. $c$ plane and obtained contour levels for $\chi^2$ corresponding to the 70\% and 95\% confidence level in two variables. The results are shown in Figure 1.

In the case of Models (ii) and (iv), the allowed region corresponds to $|c|$ and $|\sigma|$ both small, giving a practically scale-independent spectral index, with a red and blue spectrum respectively.

In contrast, the allowed region for Models (i) and (iii) allows strong scale-dependence. In
Model (i), a large departure from a constant spectral index is allowed for large $\sigma$; for the theoretically favored value $\sigma \sim 1$ the variation between $k_{COBE}$ and $8^{-1}h\text{Mpc}^{-1}$ can be as large as 0.05, while the maximal change allowed by the data is 0.2. For Model (iii), a much larger departure from a constant spectral index is allowed, but in the theoretically favored regime $|\sigma| \geq c$ one again finds a variation of at most 0.05.

4 Conclusion

We have evaluated the observational constraints on the spectral index $n$, using a range of data, and we find, for constant $n$ at 2-$\sigma$ level, $0.88 \leq n \leq 1.11$ for $0 \leq z_R \leq 20$.

We have also investigated the running mass models, parameterized by $c$ and $\sigma$. For $c$ and $\sigma$ with the same sign, we have found that indeed $n$ can vary by about 0.05 between the COBE scale and $8h^{-1}\text{Mpc}$. Moreover, if $c$ is positive as it would be for a gauge coupling, $n - 1$ can change sign between the COBE and $8h^{-1}\text{Mpc}$ scales. It will be very interesting to see how the present situation changes with the advent of better data.

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