DE-CROP: Data-efficient Certified Robustness for Pretrained Classifiers

Gaurav Kumar Nayak*  Ruchit Rawal*  Anirban Chakraborty

Department of Computational and Data Sciences
Indian Institute of Science, Bangalore, India
{gauravnayak, ruchitrawal, anirban}@iisc.ac.in

Abstract

Certified defense using randomized smoothing is a popular technique to provide robustness guarantees for deep neural networks against $l_2$ adversarial attacks. Existing works use this technique to provably secure a pretrained non-robust model by training a custom denoiser network on entire training data. However, access to the training set may be restricted to a handful of data samples due to constraints such as high transmission cost and the proprietary nature of the data. Thus, we formulate a novel problem of “how to certify the robustness of pretrained models using only a few training samples”. We observe that training the custom denoiser directly using the existing techniques on limited samples yields poor certification. To overcome this, our proposed approach (DE-CROP) generates class-boundary and interpolated samples corresponding to each training sample, ensuring high diversity in the feature space of the pretrained classifier. We train the denoiser by maximizing the similarity between the denoised output of the generated sample and the original training sample in the classifier’s logit space. We also perform distribution level matching using domain discriminator and maximum mean discrepancy that yields further benefit. In white box setup, we obtain significant improvements over the baseline on multiple benchmark datasets and also report similar performance under the challenging black box setup.

1. Introduction

Given large amounts of training data (such as ImageNet [8]) and compute power (powerful GPUs [29]), deep models are highly accurate on the respective tasks for which it is trained. However, these models can easily get fooled when they encounter adversarial images as input that are crafted using an adversarial attack [30]. A lot of efforts have been made in the literature to secure the models against adversarial attacks. Adversarial training [22, 13] is one of the most common ways to provide empirical defense where the models are trained to minimize the maximum training loss induced by adversarial samples. Such defenses are heuristic-based and are only robust to known or specific adversarial attacks. Powerful adversaries easily break them, hence are not truly robust against adversarial perturbations [1, 32, 7]. This motivated the researchers to develop methods where a trained model can be guaranteed to have a constant prediction in the input neighborhood. Such methods that provide formal guarantees are called certified defense methods [25, 34, 9, 17, 23].

Randomized smoothing [3, 21, 19, 20, 6] is a certified defense technique used to provide provable robustness against $l_2$ adversarial perturbations. It outperforms other certification methods and is also scalable to deep neural networks due to its architectural independence. Using randomized smoothing, any base classifier can be converted to a smoothed classifier, which is certifiably robust against $l_2$ attacks as it has the desirable property of constant predictions within the $l_2$ ball around the input. The prediction by the smoothed classifier on an input image is simply the most probable class predicted by the base classifier on random gaussian perturbations of the input. Note that the higher the probability with which the base classifier predicts the most probable class as the correct class, the higher the certified radius [6]. However, any vanilla trained / off-the-shelf classifier would not be robust to the input corrupted by gaussian noise. The predicted most probable class can be incorrect or may be predicted with very low confidence leading to poor certification. Thus, models are often trained from scratch using the gaussian perturbed samples [6] as augmentation, and also with adversarial training [26].

Training from scratch on gaussian-noise augmentation is not always a viable option, especially when the large pretrained models are shared as an API, either as a white box or a black box. Also, retraining these large cumbersome models adds a lot of computational overload. To avoid this, Salman et al. [27] proposed a “denoised smoothing” technique where a custom-trained denoiser is prepended before the pretrained classifier. Though their approach provides
certified robustness to pretrained models, they use the entire training data for training the denoiser. In fact, all the previous certification methods also assume the availability of entire training data. This assumption is unrealistic as the API provider may not share the whole large-scale training set. Due to heavy transmission costs associated with complete training data or proprietary reasons, they may provide access to only a few training samples. In such cases, when we perform denoised smoothing [27] directly, the certified accuracy decreases significantly as the number of training samples reduces from 100% to 1% as shown in Fig. 1.

In this work, we attempt to provide certified robustness to a pretrained non-robust model in the challenging setting of limited training data. We first explore whether the performance of denoised smoothing can be improved using different approaches such as weight decay as a regularization scheme or even generate extra data using augmentation and mix-up strategies [36] to avoid overfitting. However, these traditional approaches only show a marginal improvement (refer to Tables 1, 2). Hence, we propose our approach of data-efficient certified robustness (dubbed as ‘DE-CROP’) (refer to Sec. 4 and Fig. 2, 3) that generates better samples, yielding diverse features on the pretrained classifier similar to complete training data.

Since generative models are hard to train and perform poorly (on downstream tasks) in the presence of limited data, we adopt a simple and intuitive approach to generate additional data to minimize overfitting. We achieve this by generating adversarial samples (corresponding to our limited data) that we term ‘boundary’ samples. Adversarial samples act as an upper bound to the decision boundary [24] while maintaining the semantic content of the image, allowing the denoiser to learn from samples in its neighborhood. Moreover, we also generate interpolated samples (lying between original and boundary samples) that further increase the data diversity in the feature space. We train the denoiser by maximizing the similarity between the pretrained classifier’s features on the original data and denoised perturbed generated/original data. We achieve this by performing orthogonal modifications at two granularities: a) instance-level (using cosine-similarity) and b) distribution-level (using Maximum Mean Discrepancy [14] and negative gradients from a Domain-discriminator [11]).

Our contributions are summarized as follows:

- Given only limited training data, we provide robustness guarantees for a non-robust pretrained classifier against $l_2$ perturbations on both white-box and black-box setups. To the best of our knowledge, we are the first to provide certified adversarial defense using only the few training samples.

- To mitigate overfitting on limited training data, we propose a novel sample-generation strategy that synthesize ‘boundary’ and ‘interpolated’ samples (Sec. 4.1) to augment the limited training data, leading to improved feature-diversity on the pretrained classifier.

- The denoiser network trained with regular cross entropy loss provides limited benefit. To enhance the performance further, we proposed additional losses (Sec. 4.2) that align the feature representations of original and denoised gaussian perturbed generated/original samples at multiple granularities (both instance and distribution levels).

- We show benefit of our generated samples (Sec. 5.2) along with contributions from each of the proposed losses (Sec. 5.3), by reporting significant improvements observed across diverse sample budgets and noise levels in both white-box and black-box settings.

2. Related Works

We broadly categorize the relevant works that provide adversarial robustness and briefly discuss them below:

*Empirical Robustness:* Empirically motivated adversarial robustness defenses can be broadly classified into: a) adversarial training (AT) and b) non-adversarial training regularizations. AT [22, 13, 30] improves robustness by augmenting the training data with adversarial samples.
generated by a particular threat model. Although AT is widely regarded as the best empirical defense, it suffers from high-computational costs due to generation of adversarial samples at training time. Non-AT based approaches [5] attempt to reduce the computational burden (usually at the cost of drop in adversarial robustness) by mimicking properties observed in robust networks explicitly. AT is also highly dependent on the quantity of training data. Aditi et al. [4] demonstrated that using additional unlabeled data in a pseudo-labelling setup led to a significant increase in adversarial robustness. However, since performance of pseudo-labelling itself depends on the amount of labeled data, the performance of their technique drops considerably as labeled data is reduced. To alleviate this problem, Sehwag et al. [28] illustrated the benefit of using additional data generated from generative models for improving adversarial robustness.

Since the empirical defenses are designed based on heuristics, they can be easily fooled as stronger adversarial attacks are developed in the future. In contrast, we attempt to provably robustify pretrained classifier against $l_2$ adversarial attacks under the challenging constraint of limited training samples.

Certified Robustness: Unlike empirical defense, here the model predictions on the neighborhood region lying within ball of small radius around the input sample, is guaranteed to not vary and remain constant. The methods that provide certification are either ‘exact’ [10, 9, 17, 31] or ‘conservative’ [34, 37]. The former is not scalable to large architectures, is compute-intensive, and often uses less expressive networks but can verify with guarantees about the existence of any adversarial samples if lying within the radius of input. The latter is more scalable and requires less computation but can incorrectly decline the certification even if no adversarial sample is present. Both techniques require either customized or specific architectures, hence are not suitable for modern deep architectures.

Randomized smoothing is a popular technique that does not have any architectural dependency and was initially used as heuristic defense [3, 21]. It was first shown to provide certified guarantees by Lecuyer et al. [19] where techniques from ‘differential privacy’ were used for certification. It was later improved by Li et al. [20] who provided better guarantees using ideas from ‘information theory’. Both these methods have lower guarantees on the smoothed classifier. Cohen et al. [6] provided a tight certified guarantee against $l_2$ norm adversarial perturbations. After that, the certified accuracy was further improved by using adversarial training techniques in the randomized smoothing framework [26]. However, all these techniques trained the classifier from scratch while providing certified robustness. Recently, Salman et al. [27] provided provable robustness to pretrained models by appending a custom trained denoiser before it. We also train the custom denoiser but only using few training samples. Unlike [27], our limited data setup is more challenging and directly using their method yields poor certification results. Our generated samples with the added domain discriminator optimized using our proposed losses, handles the overfitting on the denoiser quite well and gives significant improvements on certified accuracy.

We now discuss the necessary preliminaries to give required background before explaining our approach.

3. Preliminaries

Notations: The complete original training dataset is denoted by $D_o = \{D_{\text{train}}, D_{\text{test}}\}$, where $D_{\text{train}}$ and $D_{\text{test}}$ are the training set and the test set respectively. The base classifier $B_c$ is pretrained on the entire $D_{\text{train}}$ which consists of $N$ training samples. The API provider has granted public access to the trained $B_c$ which can be used by clients to obtain predictions. However, only a limited amount of $D_{\text{train}}$ (denoted by $D_{\text{lim}}^{\text{train}}$) is shared to the clients. $D_{\text{lim}}^{\text{train}}$ is only $k\%$ of $D_{\text{train}}$ containing $N^k$ training samples such that $N^k \ll N$. The same relationship holds for each class of the classifier $B_c$, i.e. for any class $c$: $N^k_c \ll N_c$ and $N^k$ is $k\%$ of $N_c$. An $i^{th}$ sample of $D_{\text{lim}}^{\text{train}}$ (i.e. $x_o^i$) with label $y_o^i$ is perturbed by gaussian noise which is denoted by $x_o^i = x_o^i + \epsilon$ where $\epsilon \sim \mathcal{N}(0, \sigma^2 I)$, constituting the collection of perturbed samples as $D_{\text{lim}}^{\text{train}}$.

The penultimate features, logits and label predictions from the pretrained classifier $B_c$ on $x_o^i$ are denoted by $F_{B_c}(x_o^i)$, $\text{label}(B_c(x_o^i))$ and $B_c(x_o^i)$ respectively. Similarly, $\text{label}(S_c(x_o^i))$ is the label predicted by smoothed classifier $S_c$ on input $x_o^i$. The classifier $B_c$ is converted to smoothed classifier $S_c$, which is used for certified defense via randomized smoothing. The certified radius of an $i^{th}$ test sample is $R_i^c$. The evaluations are performed on $l_2$ perturbations at a radius $r$ denoted by $R_i^r$. The denoiser network $D_n$ and domain discriminator $D_d$ are parameterized by $\theta$ and $\phi$ respectively. The boundary sample and interpolated sample generated using an $i^{th}$ training sample of $D_{\text{lim}}^{\text{train}}$ (i.e. $x_o^i$) are represented by $x_b^i$ and $x_{int}^i$, respectively.

Randomized Smoothing (RS): This technique is used to build a new smoothed classifier $S_c$ from the given base classifier $B_c$. For any $i^{th}$ sample of $D_{\text{lim}}^{\text{train}}$ (i.e. $x_o^i$), the output of the classifier $S_c$ corresponding to input $x_o^i$ is the most likely class that has the highest probability to be get predicted by $B_c$ on $x_o^i$.

$$\text{label}(S_c(x_o^i)) = \arg\max_{c \in C} \text{Prob}(\text{label}(B_c(x_o^i + \epsilon)) = c)$$

where $\epsilon \sim \mathcal{N}(0, \sigma^2 I)$

(1)

Here, $\sigma$ is a hyperparameter which controls the noise level and $C$ is the set of unique target labels in $D_{\text{lim}}^{\text{train}}$. The process of RS does not assume $B_c$’s architecture and thus permits the $B_c$ to be any arbitrary large deep neural network.
Certified Robustness using RS: Lecuyer et al. [19] and Li et al. [20] gave robustness guarantees for the smoothed classifier $S_c$ using RS, but they were loose as $S_c$ was provably more robust than the obtained guarantees. Cohen et al. [6] gave a tight bound on $l_2$ robust guarantees using RS.

If the prediction on the base classifier $B_c$ for the gaussian perturbed copies of $x^i_o$ i.e. $\mathcal{N}(x^i_o, \sigma^2 I)$ is $c_1$ as the ‘most probable class’ with probability $p_1$ and $c_2$ as “runner-up” class with $p_2$, then the smoothed classifier $S_c$ is provably robust around the input $x^i_o$ within the radius $R_c = \sigma/2(\phi^{-1}(p_1) - \phi^{-1}(p_2))$. Here $R_c$ is the certified radius as the predictions are guaranteed to remain constant inside the radius and $\phi^{-1}$ denotes the inverse of standard gaussian CDF. It is impossible to calculate $p_1$ and $p_2$ exactly when $B_c$ is a deep neural network. Hence, Cohen et al., estimates lower bound on $p_1$ ($p_1$) and upper bound on $p_2$ ($\overline{p_2}$) using Monte Carlo technique.

**Theorem [Cohen et al., [6]]**: Let $B_c$ be any function that maps the input to one of the output class labels. Let $S_c$ be defined as in eq. 1. The noise $\epsilon$ is sampled from the normal distribution i.e. $\epsilon \sim \mathcal{N}(0, \sigma^2 I)$. If $c_1 \in C$ and $p_1 < \overline{p_2}$, (2) holds the below inequality:

$$
\text{Prob}(\text{label}(B_c(x^i_o + \epsilon)) = c_1) \geq p_1 \geq \overline{p_2} \implies \max_{c \neq c_1} \text{Prob}(\text{label}(B_c(x^i_o + \epsilon) = c))
$$

(2)

Then label $S_c(x^i_o + \epsilon) = c_1 \forall \|\epsilon\|_2 < R_c$, where

$$
R_c = \sigma/2(\phi^{-1}(p_1) - \phi^{-1}(\overline{p_2}))
$$

(3)

In practice, Cohen et al. used $R_c = \sigma\phi^{-1}(p_1)$ for $p_1 > 1/2$, assuming $\overline{p_2} = 1 - p_1$, otherwise abstained with $R_c = 0$. These above expressions can be derived using the “Neyman-Pearson” lemma for which we refer the reader to [6]. Now, we discuss our proposed approach in detail in the next section.

4. Proposed Approach

We aim to provide certified robustness to the given pretrained base classifier $B_c$. However, obtaining certification using randomized smoothing expects the model $B_c$ to be robust against the input perturbations with the random Gaussian noise, which may not be the case with the model $B_c$ supplied by the API provider. In order to make the base model $B_c$ appropriate for randomized smoothing based certification without modifying/retraining $B_c$, a denoiser network $D_n$ is prepended to $B_c$. Thus, $B_c \circ D_n$ is the new base classifier using which the prediction on smoothed classifier $S_c$ on an $i$th training sample is defined as follows:

$$
label(S_c(x^i_o)) = \arg \max_{c \in C} \text{Prob}(\text{label}(B_c(D_n(x^i_o + \epsilon))) = c)
$$

where $\epsilon \sim \mathcal{N}(0, \sigma^2 I)$

(4)

The above smoothed classifier $S_c$ is provably robust against the $l_2$ perturbations with certified radius $R_c$ (refer to Sec. 3). For high performance on the certified defense, high $R_c$ is required, which is directly proportional to $p_1$ (eq. 3). The probability ($p_1$) of predicting the most probable class ($c_1$) i.e. confidence depends on the performance of the denoiser network $D_n$. However, $D_n$ trained on the given limited training data $D_{lim}^{train}$ using the existing technique [27] yields poor certification (shown in Fig. 1). Even when we attempt to minimize the overfitting of $D_n$ on $D_{lim}^{train}$ (: $|D_{lim}^{train}| < |D_{train}|$) using different traditional approaches such as weight decay, augmentation, and mix-up strategies, we observe only a minor improvement (refer to Tables 1, 2) in certified accuracy. Hence, we propose our method (‘DE-CROP’) that overcomes this limitation by crafting boundary and interpolated samples using $D_{lim}^{train}$, followed by using them to train the denoiser with appropriate losses.

4.1. Generation of Boundary & Interpolated sample

In Fig. 2, we show the visualization of the logit layer features of the pretrained base classifier $B_c$ on a particular class via t-SNE plot. The class-features corresponding to available limited class samples of $D_{lim}^{train}$ is shown in green color (on left). We focus on improving the feature diversity of the limited training samples. For this, we first estimate class-boundary samples that would have respective features in the boundary region of the t-SNE.

Adversarial attacks [22] synthesize samples via an optimization procedure that carefully perturbs the input sample with a small human-imperceptible noise (i.e., adversarial noise). The model gets fooled on such samples (i.e., adversarial samples) as the prediction gets flipped to some other class. As these samples cross the decision boundary, which are constructed by adding a small noise to the input original sample, they often lie very close to the decision boundary. Moreover, they are human-imperceptible and preserve the class semantics of the input class sample. Hence, adversarial samples serve as a good candidate for a proxy of class-boundary samples. For any $i$th training sample of $D_{lim}^{train}$ (i.e., $x^i_o$), we obtain boundary sample ($x^i_b$) by computing

![Figure 2. t-SNE visualization of the pretrained base classifier’s features for training samples of a particular class of CIFAR-10. Our generated samples (interpolated and boundary) increases the feature diversity of limited-original training data.](image-url)
adversarial noise $\delta$ for which the following relation holds:

$$x_b^i = x_o^i + \delta,$$

$$\|\delta\|_\infty < \epsilon, \quad \epsilon > 0, \quad \text{label}(B_c(x_b^i)) \neq \text{label}(B_c(x_o^i))$$ \hspace{1cm} (5)

Next, we obtain interpolated features by performing a mixup between the features of generated boundary sample $x_b^i$ and original training sample $x_o^i$ as follows:

$$Logit_{int}^i = \alpha \times B_c(x_o^i) + (1 - \alpha) \times B_c(x_b^i)$$ \hspace{1cm} (6)

Here $\alpha$ is the mixing coefficient. The interpolated features, the class-boundary features, and the original limited class-sample features are highlighted in yellow, blue, and green colors respectively, in the t-SNE plot (Fig. 2). These interpolated features being label preserving help to improve the feature-diversity leading to improved certified accuracy (refer to Sec. 5.2). We craft the input sample corresponding to the interpolated feature $Logit_{int}^i$ (i.e., $x_{int}^i$) by perturbing the original sample $x_o^i$ to match its feature response to $Logit_{int}^i$ using mean square error loss ($L_{mse}$). Mathematically, we obtain interpolated sample $x_{int}^i$ as follows:

$$x_{int}^i \leftarrow \min_x \ L_{mse}(B_c(x), Logit_{int}^i)$$ \hspace{1cm} (7)

where $x$ is initialized with $x_o^i$ and kept trainable with ground truth as $Logit_{int}^i$. The model $B_c$ is non-trainable, but the gradients are allowed to backpropagate from the model to update the input $x$. Thus, we obtain the boundary and the interpolated samples corresponding to each training sample.

Refer supplementary for visualization of both the boundary and interpolated samples in the input space. All of them preserve the class semantics. Our generated boundary samples $x_b$ and interpolated samples $x_{int}$ are used along with limited training samples $x_o$ to train the denoiser network $D_n$, which we discuss in the subsequent subsection.

### 4.2. Training of the denoiser network ($D_n$)

The denoiser network $D_n$ is attached before the pretrained base classifier $B_c$ to make it suitable for randomized smoothing. Apart from this, we also add a domain discriminator network $D_d$ whose input is the normalized features of the penultimate layer of $B_c$. The discriminator $D_d$ learns to distinguish between distribution of clean samples and distribution of denoised outputs of gaussian perturbed input samples. Motivated from domain adaptation literature [12], we use gradient reversal layer (GRL) before feeding the normalized penultimate layer features to the discriminator that allows normal forward pass but reverses the direction of gradient in the backward pass. As a consequence, this negative gradients backpropagates to the denoiser network $D_n$ that helps it to produce denoised output which yields indistinguishable domain-invariant features on the pretrained $B_c$ classifier.

Refer to supplementary for architectural details of the networks $D_n$ and $D_d$. The overall steps involved in the proposed framework (‘DE-CROP’) are also shown in Fig. 3. The network is trained with our generated data and limited
training data \(D_{\text{lim}}^\text{train}\) by using different losses aimed at different objectives - label consistency to ensure correct predictions on \(B_c\) and matching of high-level feature similarity obtained on \(B_c\) for denoised output of gaussian perturbed input and clean original training samples both at the sample level and distribution level. The respective losses are described below:

**Ensuring label consistency**: Similar to [27], we use cross entropy loss \(L_{\text{ce}}\) to ensure that the label predicted by the pretrained network \(B_c\) on original clean data and denoised output of its gaussian perturbed counterpart are same.

\[
L_{\text{lc}} = \frac{1}{N^k} \sum_{i=1}^{N^k} L_{\text{ce}}(\text{softmax}(B_c(D_n(\tilde{x}^i_o))), \text{label}(B_c(x^i_o)))
\]  

(8)

**Enforcing feature similarity at sample level**: The pretrained base classifier \(B_c\) trained on complete training data \(D_{\text{train}}\) is highly discriminative. To leverage this on our crafted data, we use cosine similarity loss at the logits of the \(B_c\) network to encourage logit features on the denoised output of our generated data to be as discriminative as the features of the limited original training samples \(D_{\text{lim}}^\text{train}\) by maximizing this loss.

\[
L_{\text{cs}} = \frac{1}{N^k} \sum_{i=1}^{N^k} \left( \text{CS}(B_c(D_n(\tilde{x}^i_k)), B_c(x^i_o)) + \text{CS}(B_c(D_n(\bar{x}^i_{\text{int}})), B_c(x^i_o)) \right), \text{ s.t. CS}(w, z) = \frac{w^T z}{\|w\|\|z\|}
\]  

(9)

**Enforcing feature similarity at distribution level**: Unlike \(L_{\text{cs}}\) which is applied at sample level, here we enforce distribution level matching between the set of our denoised generated data and the set of limited original training data by using maximum mean discrepancy (MMD) [14] loss on the normalized pre-logit layer of \(B_c\) network.

\[
L_{\text{mmd}} = \text{MMD}(F_{B_c}(D_n(\tilde{x}_b)), F_{B_c}(x_o)) + \text{MMD}(F_{B_c}(D_n(\bar{x}_{\text{int}})), F_{B_c}(x_o))
\]  

(10)

Moreover, we also train the domain discriminator network \(D_d\) (parameterized by \(\phi\)) using binary cross entropy loss \(L_{\text{bce}}\) to distinguish the distribution of gaussian perturbed samples and clean samples.

\[
L_{\text{dd}} = \sum_{x^i \in D_{\text{lim}}^\text{train} \cup D_n(\tilde{D}_{\text{lim}}^\text{train})} L_{\text{bce}}(D_d(F_{B_c}(x^i)), d^i)
\]  

(11)

Here, if \(x^i \in D_{\text{lim}}^\text{train}\) then \(d^i = 1\) and if \(x^i \in D_n(\tilde{D}_{\text{lim}}^\text{train})\) then \(d^i = 0\). The negative gradients are backpropagated via GRL [11] (multiplies calculated gradient by \(-1\)), to update the parameters \(\theta\) of denoiser network \(D_n\) such that features of limited training data \(D_{\text{lim}}^\text{train}\) and its corresponding denoised output of gaussian corrupted data \(D_n(\tilde{D}_{\text{lim}}^\text{train})\) on network \(B_c\) are domain invariant.

Hence, the total loss can be written as follows:

\[
L(D_n^\phi, D_n^\phi) = \beta_1 L_{\text{lc}} - \beta_2 L_{\text{cs}} + \beta_3 L_{\text{mmd}} + \beta_4 L_{\text{dd}}
\]  

(12)

At test time, the trained denoiser network \(D_n\) with optimal parameters \(\theta^*\) prepended with base classifier \(B_c\) is used for evaluation.

**5. Experiments**

We demonstrate the effectiveness of our proposed approach (DE-CROP) on two widely-popular image-classification datasets, namely, CIFAR-10 [18] and Mini-ImageNet [33]. We limit the training set size of the datasets mentioned above by randomly selecting 1% and 10% samples respectively (ensuring class balance) from the \(D_{\text{train}}\). Our baseline corresponds to training the denoiser using the \(L_{\text{lc}}\) loss (similar to [27]). We fix the selected samples and use a ResNet-110 and ResNet-12 [16] network (for CIFAR-10 and Mini-ImageNet respectively) as our pre-trained classifiers \(B_c\) with a value of \(\sigma\) as 0.25 for all our ablations and state-of-the-art comparisons, unless otherwise specified. Refer to supplementary for additional ablations on different values of noise strength \(\sigma\) (0.12, 0.50, 1.00), quantity of limited training data \(D_{\text{train}}\) (5%, 10%, 20%, 100%) and choice of architecture for the pretrained classifier \(B_c\). We set the weights for the final loss-equation (refer eq. 12) i.e. \(\beta_1, \beta_2, \beta_3, \beta_4\) as 1, 4, 1, 1 respectively. In the subsequent subsections, we first show limited benefit of conventional techniques, followed by benefits of each component in DE-CROP and a comparison with state-of-the-art techniques.

**5.1. Improving Certification on Limited Training Data via Conventional Techniques**

\(D_n\) trained with the \(L_{\text{lc}}\) objective proposed by [27] severely overfits in the presence of limited data resulting in poor certification accuracy (refer Fig. 1). In this section, we explore whether conventional supervised-learning techniques such as explicit regularization and data-augmentation are equipped to meaningfully improve certification accuracy when dealing with limited data.

| Method | Standard Certified | Robust Certified |
|-------|------------------|-----------------|
|       | (\(r=0.00\)) | (\(r=0.25\)) | (\(r=0.50\)) | (\(r=0.75\)) |
| Without Reg. | 20.60 | 4.60 | 0.80 | 0.00 |
| \(L_1\) Reg. | 22.60 | 5.80 | 0.60 | 0.00 |
| \(L_2\) Reg. | 27.80 | 7.80 | 0.80 | 0.00 |

Table 1. Effect of adding weight decay regularizer (\(L_1\) and \(L_2\)) on limited data certification against adversarial attacks. \(L_2\) regularization obtains better certified standard and robust accuracies.

In Table 1, we regularize the \(D_n\) by applying \(L_1\) and \(L_2\) reg. (regularization) [15]. We observe that \(L_2\) reg. improves certified performance, whereas \(L_1\) reg. only results in a marginal improvement. Although \(L_2\) reg. limits the...
overfitting effect in the presence of limited data, it can’t help in improving the inherent data diversity. Thus, in Table 2, we explore whether traditional affine and specialized augmentation methods (such as mixup [36] and cutmix [35]), when combined with $L_2$ reg. further boost performance.

Rows 2-4 in Table 2 correspond to affine-transformations where the intensity of the augmentation is increased progressively (i.e. policy 1<policy 2<policy 3). We also experiment with widely-popular augmentation techniques: ‘mixup’ and ‘cutmix’. Surprisingly, we observe that policy 1(row-2; the lightest augmentation) performs the best, followed closely by ‘no-aug.’. We hypothesize that if the augmentation-policy is too aggressive, $B_c$ can make incorrect predictions leading to noisy gradients for the $D_n$ to learn from, resulting in poor certification accuracy.

| Method       | Standard Certified $(r=0.00)$ | Robust Certified $(r=0.25)$ | $(r=0.50)$ | $(r=0.75)$ |
|--------------|-------------------------------|-----------------------------|------------|------------|
| No Aug.      | 27.80                         | 7.80                        | 0.80       | 0.00       |
| Aug. (policy 1) | 29.80                      | 9.20                        | 1.40       | 0.00       |
| Aug. (policy 2) | 26.40                      | 7.40                        | 1.00       | 0.00       |
| Aug (policy 3)     | 21.00                      | 3.00                        | 0.20       | 0.00       |
| Mixup        | 24.20                         | 5.40                        | 0.60       | 0.00       |
| Cutmix       | 19.80                         | 3.20                        | 0.00       | 0.00       |

Table 2: Investigating the effect of augmentation at different intensity levels (policies), mixup and cutmix strategies in minimizing overfitting on limited training data. The augmentation with light intensity (policy 1) yields marginal improvement against $L_2$ perturbations of different radii compared to no-augmentation strategy.

Thus, a combination of policy 1 and $L_2$-reg. improves the certified standard and robust accuracies. We use this combination as a baseline for further experiments upon which we make orthogonal improvements.

### 5.2. Effectiveness of our Generated Data

One of the critical reasons for the drop in certified accuracy is the lack of diversity in the limited data. We address this problem by generating synthetic samples that provide diversity in the feature space. As elaborated in the proposed section (refer Sec. 4.1): adversarial samples (termed as boundary-samples i.e. $x_b$) serve as a good candidate for this task since they flip the classifier prediction with minimum possible perturbations, thus allowing the $D_n$ to train on samples from less-dense boundary regions. Moreover, we also generate samples whose features are an interpolation between the original and boundary samples, crafted by minimizing the $L_{mae}$ loss between the interpolated logits and the generated sample logits (as described by eq. 7).

We empirically validate our motivation in Table 3, where we observe that using $L_{cs}$ on both $x_b$ and corresponding $x_{int}$ leads to a massive improvement in performance over the baseline. Interestingly, the gain in performance when using only $x_b$ is comparatively modest. This observation further reinforces our intuition regarding the complementary nature of the information provided by the $x_b$ and $x_{int}$. Moreover, contrary to adversarial training that generates adversarial samples at every iteration during the training time, we only need to generate the boundary and interpolated samples once (amounting to negligible increase in training time), as pre-trained classifier $B_c$ is fixed and not trainable.

| Method                     | Standard Certified $(r=0.00)$ | Robust Certified $(r=0.25)$ | $(r=0.50)$ | $(r=0.75)$ |
|----------------------------|-------------------------------|-----------------------------|------------|------------|
| Baseline                   | 29.80                         | 9.20                        | 1.40       | 0.00       |
| Ours (with boundary samples) | 31.60                        | 7.80                        | 1.00       | 0.20       |
| Ours (with boundary + interpolated samples) | **48.80** | **22.00** | **6.00** | **0.80** |

Table 3: Benefit of our generated samples in improving certification. Using boundary and interpolated samples, we obtain significant boost in certified accuracy on original and $L_2$ perturbed data.

### 5.3. Distribution Alignment: Enhancing Certification in Limited Data Setup

In Table 4, we observe that using domain-discriminator $(D_d)$ along with the previously introduced $L_{cs}$ loss works quite well as the standard certified accuracy improves by 6% and certified robust accuracy improves by 4% (at $r=0.25$) compared to only using $L_{cs}$.

We also explore whether equipping the $D_n$ with explicit distribution discrepancy losses such as $L_{mmd}$ would also work well in combination with the $D_d$. Intuitively, applying $L_{mmd}$ along with $L_{cs}$ should make the job of $D_d$ harder, leading to a better $D_d$. Consequently, improving $B_c$’s feature representation via the negative gradients of domain-discriminator loss $(L_{dd})$.

We indeed observe this in Table 4, where using $L_{mmd}$ in combination with the $L_{cs}$ and the $L_{dd}$ setup leads to an improvement in performance across both standard and robust accuracies (across radii). Thus, using $L_{mmd}$ and $L_{dd}$ in conjunction with the previously discussed $L_{cs}$ and $L_{dc}$ constitutes our final approach: DE-CROP.

### 5.4. Comparison with state-of-the-art

In this section, we compare the effectiveness of our approach DE-CROP against state-of-the-art robustness certification techniques, namely, denoised-smoothing by Salman et al. [27] and gaussian-augmentation by Cohen et al. [6].

Since Salman et al. [27] don’t report performance on limited data scenarios in their paper, we use the code from their official implementation\(^2\) for evaluating their proposed method ($D_n$ with $L_{dc}$) in the presence of only 1% (for CIFAR-10) and 10% (for Mini-ImageNet) $D_{train}$. Similarly, for Cohen et al. [6], we re-train the classifier with

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\(^2\)https://github.com/microsoft/denoised-smoothing
gaussian augmentation only on the available limited training data. Although Cohen et al. [6]'s technique is unfeasible for our problem setup as the API provider may not prefer re-training and replacing the deployed model, we still compare our performance as gaussian-augmentation often outperforms previous denoiser-based approaches in presence of full training data.

As shown in Fig. 4, our proposed approach DE-CROP comfortably outperforms Salman et al. [27] on CIFAR-10, improving the certified standard accuracy by 27% and consistently improving robust accuracy across radii. The performance of Cohen et al. [6] drops to 10% across all radii, indicating that the network behaves like a random baseline (i.e. predicting each class with equal probability irrespective of the input). We observe similar trends for Mini-ImageNet, where we comfortably outperform both Salman et al. [27] and Cohen et al. [6] further demonstrating the wide applicability of our approach.

| Method | Standard Certified | Robust Certified |
|--------|--------------------|------------------|
|        | (r=0.00)           | (r=0.25)         | (r=0.50)         | (r=0.75) |
| Baseline | 29.80              | 9.20             | 1.40             | 0.00     |
| Ours (instance level) | 48.80              | 22.00            | 6.00             | 0.80     |
| Ours (instance + distribution level via discriminator) | 54.00              | 26.00            | 6.80             | 1.80     |
| Ours (instance + distribution level via discriminator and MMD) | 57.60 | 27.20 | 9.20 | 2.20 |

Table 4. Besides performance gains with instance-level feature matching, we observe further improvements in certified standard and robust accuracy when distributions of denoised and clean data are aligned in feature space via domain discriminator and MMD.

In previous sections we assumed white-box access to the pre-trained base classifier \( B_c \), i.e. we can backpropagate the gradients through \( B_c \) to optimize the denoiser \( D_m \). However, this may not always be the case as the API provider can limit access to only \( B_c \)'s predictions (i.e. black-box) due to proprietary reasons. Since the black-box setup restricts the gradient information of \( B_c \), we first use a black-box model stealing technique [2] to train a surrogate model: \( S_m \). We use \( S_m \) which allows gradient backpropagation, to train \( D_m \), using our proposed approach DE-CROP (refer Fig. 3). Finally, for evaluation we use the denoiser (trained via \( S_m \)) to certify robustness of the black-box classifier \( B_c \).

In Fig. 5, we compare the certification performance (of \( B_c \)) using denoisers trained via \( S_m \) (‘black-box access’) to the denoiser trained directly on \( B_c \) (‘white-box access’). We take \( B_c \) as Alexnet and two different choices for \( S_m \), namely ResNet-18 and Half-Alexnet. Our method DE-CROP yields very similar performance in the black box setting across different architectures of \( S_m \). Also, the performance drop compared to white box setting is marginal, highlighting the suitability of our technique even when the pretrained classifier weights are not shared.

6. Conclusion

We presented our approach (DE-CROP), which for the first time, solves the problem of providing provable robustness guarantees to a pretrained classifier in the challenging limited training data settings. Our method comprises a two-step process - a) generation of boundary and interpolated samples ensuring feature diversity and b) effectively utilizing the generated samples along with limited training samples for training the denoiser using the proposed losses to ensure feature similarity between the denoised output and clean data at two different granularities (instance level and distribution level). We validate the efficacy of the generated data as well as the contribution of individual losses by extensive ablations and experiments across CIFAR-10 and Mini-ImageNet datasets. Moreover, our method works quite well in the black-box setting as it provides similar certification performance compared to the white-box setup.

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