Expected sensitivity to test of standard solar models with future solar CNO neutrino flux measurement

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Abstract: Standard solar models built from spectroscopy and from the helioseismology predicted inconsistent solar metallicity. Solar neutrino fluxes can be used to test two models. Hypothesis test was performed using $^7\text{Be}$, $^8\text{B}$ CNO neutrino fluxes. Current experimental results already disfavored the low metallicity standard solar model at a significance of 2.1 $\sigma$. Considering new experiments that can improve the precision of the measured CNO neutrino flux are being planned, it is important to study their potentiality to discriminating two standard solar models. We evaluated the expected significance to reject the low metallicity solar model where the assumed central value and uncertainty of the measured CNO neutrino flux vary within the range of $3.5 \times 10^8 - 8 \times 10^8$ s$^{-1}$cm$^{-2}$ and 1%--20%, respectively. It was found that the potentiality of future experiments to reject the low metallicity standard solar model strongly depends on the central value of the CNO neutrino flux measurement. When the central value is the same as the value measured by Borexino in 2020, the required precision to reach 3 $\sigma$ and 5 $\sigma$ are 20% and 8%, respectively, which are both achievable.

1. Introduction
Solar metallicity is the fraction of total mass of the elements heavier than Helium in the Sun. The carbon-nitrogen-oxygen (CNO) cycle is one of two mechanisms how the Sun convert hydrogen to helium. It is expected to contribute about one percent of the total energy production [1,2]. Because the solar metallicity affects the efficiency of the CNO-cycle and the plasma opacity, and the latter indirectly changes the temperature of the core and modifies the evolution of the Sun and its density profile, it is important to determine the solar metallicity with high precision. However, current standard solar models built from two paths give inconsistent solar metallicity. Standard solar models inferred from helioseismology (Standard Solar Model of high metallicity, hereinafter as SSM-HZ [3]) predicted a higher solar metallicity compared with the prediction of those built from spectroscopy (Standard Solar Model of low metallicity, hereinafter as SSM-LZ [4]).

One way to settle down this controversy is to measure solar neutrino fluxes and compare results with the predictions of SSMs. Currently, the best measurements of pp, $^7\text{Be}$, pep [5] and CNO solar neutrinos [6] were achieved by the Borexino experiment, while the best measurement of $^8\text{B}$ solar neutrinos were achieved by the Super-Kamiokande [7] and SNO [8] experiments. Results from Borexino along were in tension with the SSM-LZ at a significance of 2.1 $\sigma$ [6].

Among all solar neutrinos, fluxes of neutrinos from CNO-cycle directly depend on the carbon and nitrogen abundances in the solar core. Part of next generation neutrino and dark matter experiments may improve the precision of the measurement of the flux of CNO solar neutrinos. Among them, experiments such as Jingping Neutrino Experiment and THEIA are developing technology to measure the momentum direction of the particle. If this measurement is possible, CNO neutrinos and natural radioactive decay...
background, especially those from $^{210}$Bi, can be well separated and thus would improve the precision of
the CNO neutrino flux measurement. Experiments using noble gas as targets, such as DarkSide and
Darwin, do not suffer from $^{11}$C backgrounds, so the high energy tail of CNO neutrino recoil electron
energy distribution would be visible given well controlled radon contamination and events in this energy
range can be used to discriminate between the CNO neutrinos and their major background, the $^{210}$Bi
events. In this work, we present the expected significance to reject SSM-LZ under various assumed
precision and central values of measured CNO solar neutrino fluxes.

2. Method
Hypothesis test is performed to test the compatibility between the experiment data and the SSM-LZ
model of solar neutrino fluxes. The null hypothesis is defined as that the measured pp, $^7$Be, $^8$B solar
neutrino fluxes [5-8], and the assumed CNO solar neutrino flux are compatible with the SSM-LZ model.
The pep neutrino flux is not used because the measured values depend on the assumption of the CNO
neutrino flux [5]. The test statistic is defined as the chi-square of neutrino fluxes:

$$
\chi^2 = \sum_{X=\text{pp, }^7\text{Be, }^8\text{B, }\text{CNO}} \frac{(R_{\text{exp.}}(X) - R_{\text{theo.}}(X))^2}{\sigma_{\text{exp.}}(X)^2 + \sigma_{\text{theo.}}(X)^2}
$$

where $R$ and $\sigma$ are central value and precision of the flux of neutrino $X$, respectively, and their values
are shown in Table 1. $R_{\text{theo.}}$ and $\sigma_{\text{theo.}}$ are predictions of SSM-LZ, and $R_{\text{exp.}}$ and $\sigma_{\text{exp.}}$ are the measured
values for pp, $^7$Be [5], $^8$B solar neutrinos [7, 8] or assumed values for CNO neutrinos. In our work, the
central values and uncertainties of CNO neutrino flux ranges from $3.5 \times 10^8$ s$^{-1}$cm$^{-2}$ to $8 \times 10^8$ s$^{-1}$
and from 1% to 20%, respectively.

The p-values of the test statistic are evaluated according to the distribution of chi-square of degrees
of freedom of 1 and are converted to the equivalent distance in a standard Gaussian distribution, namely
the number of sigmas. The number of sigmas is the square root of chi-square.

3. Results
The compatibility between measured and assumed neutrino fluxes and standard solar models were
evaluated using the chi-square test statistic. The evaluated chi-square can be expressed as

$$
\chi^2 = 3.34 + \frac{(\mu-3.51)^2}{(0.51)^2 + (\mu-\sigma)^2},
$$

where $\mu$ are the assumed CNO neutrino fluxes in $10^8$ cm$^{-2}$s$^{-1}$, and $\sigma$ is the relative uncertainty in
percentage. The first term 3.34 is the total chi-square contributed by pp, $^7$Be, and $^8$B neutrinos, see Table
1. The contribution of pp neutrinos is negligible because the experimental precision is not good enough.
The p-value and number of sigmas as a function of the central values and uncertainties of CNO neutrino
flux are shown in Figure 1.

Table 1. Used values of measured solar neutrino fluxes [5-8] and corresponding SSM-LZ predictions
[3, 4]. In the last column there is the chi-square of each neutrino. The unit of the second and third
columns from pp to CNO solar neutrinos are $10^{10}$ cm$^{-2}$s$^{-1}$, $10^9$ cm$^{-2}$s$^{-1}$, $10^8$ cm$^{-2}$s$^{-1}$, and $10^8$ cm$^{-2}$s$^{-1}$,
respectively. From the table we can see the contribution of pp solar neutrino is negligible due to large
experimental uncertainty.

| Neutrino types | Experiment | Theory (SSM-LZ) | chi-square |
|----------------|------------|-----------------|------------|
| pp             | 6.1(1±0.10) | 6.05(1±0.006)   | 0.01       |
| $^7$Be         | 4.99(1±0.027) | 4.50(1±0.06)   | 2.64       |
| $^8$B          | 5.35(1±0.003) | 4.50(1±0.12)   | 2.48       |
| CNO            | 7.0(1±0.29) | 3.51(1±0.14)   | 5.93       |
Figure 1. p-value using measured pp, $^7$Be, $^8$B and assumed central values of uncertainties of measurement of the CNO neutrino flux

Figure 2. Number of sigmas (right) of hypothesis test against SSM-LZ using measured pp, $^7$Be, $^8$B and assumed central values of uncertainties of measurement of the CNO neutrino flux

Figure 1 The p-value (left) and number of sigmas (right) of hypothesis test against SSM-LZ using measured pp, $^7$Be, $^8$B and assumed central values of uncertainties of measurement of the CNO neutrino flux. The 3 σ and 5 σ boundaries are shown as blue and red lines on the right figure, respectively. From the figure we can see that when the central value is less than $4.6 \times 10^8$ cm$^{-2}$s$^{-1}$ or the uncertainty is worse than 10%, it is impossible to reject SSM-LZ with more than 5 σ significance. Greater the value of central value and random error, greater the chi-square value and less possible to exclude SSM-LZ model.

From the figure we can see that when the central value of the measured CNO neutrino flux is $4.88 \times 10^8$ cm$^{-2}$s$^{-1}$ as predicted by SSM-HZ, even if the uncertainty is negligible, the maximum significance to reject SSM-LZ is only 3.3 σ due to predictions of central values of CNO neutrino fluxes are too close between two models compared with the theoretical precisions. When the central value is $7 \times 10^8$ cm$^{-2}$s$^{-1}$ as reported by Borexino [6], the required precision to reach 3 σ and 5 σ significance rejection of SSM-LZ are 20% and 8%, respectively. In both cases, the dominate contribution to total uncertainty are experimental, and the significance can be simplified to

$$\frac{n\sigma}{\sigma_r} \approx 100\%$$

4. Conclusions and Discussions
Fluxes of solar neutrinos can be used to discriminate between the SSM-HZ and SSM-LZ models and shed lights on the solar metallicity problems. The discrimination power mainly comes from $^7$Be, $^8$B, and CNO neutrinos, while the experimental uncertainty of pp and pep neutrinos is too large.

The significance to reject SSM-LZ models using measured $^7$Be, $^8$B neutrino flux and assumed CNO neutrino flux is evaluated. It is found that the power to reject SSM-LZ strongly depends on the central value of the measured CNO neutrino flux. When the measured central value is the same as the prediction of SSM-HZ, the maximum significance to reject SSM-LZ is 3.3 σ, while in the case of measured central value by Borexino, the required experimental precisions to reach 3 σ and 5 σ are 20% and 8%, respectively, which are both achievable.

In Borexino experiments, the major challenge to reach high precision is to stabilize the detector and suppress the convection motion. Similar technology can be applied to SNO+, which has larger volume. For THEIA and Jingping Neutrino Experiment, if directionality can be measured with a precision better than 25° in ~100 kT • year exposure, then the precision of measured CNO neutrino flux can reach 8% [9-11], and a 5 σ rejection of SSM-LZ can be demonstrated if the measured central value is the same as
the Borexino result. For liquid argon detectors, according to [12], the expected precision can reach 15% in 400 tonne • years exposure.

Besides providing hints on direction to check the source of discrepancy between two paths building SSMs, the measured CNO solar neutrino flux can also be used to directly determine the carbon and nitrogen abundances in the solar core region [13]. Currently the theoretical uncertainty is dominated by that of the nuclear reactions, with the largest coming from S_{114} (7.3%), S_{34} (3.4%), and S_{17} (3.5%). It is expected they can be further improved [14], and thus step further close to the solution to the long-standing solar metallicity problem.

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