Finding neutral Higgs bosons in a two-Higgs-doublet model with spontaneous CP violation

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Abstract

We consider a particular two-Higgs-doublet model with spontaneous CP violation, where two neutral Higgses have no definite CP properties and the third one is CP-even. In this model, as the parameter $\theta$ of CP violation increases, the masses of the neutral Higgs bosons are drawn towards the mass of the lightest one and the chance of detection of neutral Higgses at an $e^+e^-$ collider in $Z+Higgs, b\bar{b}+Higgs$ and $t\bar{t}+Higgs$ production increases with respect to the CP conserving case.
1 Introduction

The origin of CP violation is among the most important physics issues. An elegant way of introducing CP violation is based on the enlargement of the Higgs sector of the Standard Model [1][2][3][4].

Already the simplest extension of the SM, with two scalar Higgs-doublets, predicts the existence of five physical Higgs bosons. Among them, two are a charged pair and three are electrically neutral.

In the CP conserving version of the two-Higgs-doublet model (2HDM), two neutral Higgs bosons are CP-even and one is CP-odd. But, for example in the minimal supersymmetric model, with soft supersymmetric CP violating phases, the neutral Higgs states will mix beyond the Born approximation, leading to CP-impure mass eigenstates [5].

However, in a general 2HDM the Higgs sector itself may also generate spontaneous and/or explicit CP violation. Then, in a CP violating 2HDM the neutral Higgs states mix already at the tree level; the mass eigenstates have indefinite CP parity, i.e., the neutral Higgses have both scalar and pseudoscalar Yukawa couplings to quarks and leptons [6].

In this note, we consider the neutral Higgs production in the higher energy $e^+e^-$ colliders, in a particular 2HDM with spontaneous CP violation. In this model, let $h_1, h_2, h_3$ to be the three neutral Higgs bosons in the order of increasing mass, $h_3$ is CP-even and $h_1, h_2$ are CP-impure mass states, depending on the relative phase $\theta$ of the vacuum expectation values of the Higgs doublets. In the limit of CP conservation, $\theta \to 0$, $h_1$ becomes CP-odd. Meanwhile $h_2$ becomes CP-even and degenerates in mass with $h_3$; but their mass (for $m(h_1) \neq 0$) is divergent. In the opposite limit, $\theta \to \frac{\pi}{2}$, the CP-impure $h_1$ and $h_2$ degenerate in mass and $m(h_3) \to \sqrt{2}m(h_1)$.

In $e^+e^-$ collisions the processes of production of neutral Higgs bosons are (1) the Higgs-strahlung $e^+e^- \to Zh_k$, (2) the Higgs pair production $e^+e^- \to h_jh_k$ and (3) the Yukawa processes with Higgs radiation off (heavy) fermions $e^+e^- \to f\bar{f} \to f\bar{f}h_k$. In order to treat these processes on the same footing, we consider the $f\bar{f}h_k$ final state at future $e^+e^-$ colliders. The process (1) contributes to this final state when $Z \to f\bar{f}$ and the process (2) when $h_j \to f\bar{f}$. Now, the unitarity of the model implies in general a number of sum rules for the Higgs-gauge boson [7][8] and Higgs-fermion [9][10] couplings. (In our particular 2HDM they have a very simple form.) As well discussed in ref.(9), these sum rules guarantee the detection in $e^+e^-$ collisions of any neutral Higgs boson that is sufficiently light to be cinematically accessible in (a) Higgs-strahlung and Higgs pair production or (b) Higgs-strahlung and $b\bar{b}$+Higgs and $t\bar{t}$+Higgs.

In our particular model, as the parameter $\theta$ of CP violation, increases, the masses of the neutral Higgs bosons are drawn towards the mass of the lightest one. We can have two or even all three neutral Higgses ”relatively” light. This reduces their production thresholds and increases the chance of their finding in the above-
mentioned processes. In this sense, we are more interested on the cumulative 
\[ \sum_{k=1}^{3} \sigma(e^+e^- \rightarrow f \bar{f} h_k) \] rather than the single cross section \( \sigma(e^+e^- \rightarrow f \bar{f} h_1) \).

The paper is organized as follows. In section 2, we present our two-Higgs-
doublet model with spontaneous CP violation and the corresponding ZZ-Higgs,
Z-Higgs-Higgs and Higgs Yukawa couplings. In section 3, we present the cross
section formula for \( e^+e^- \rightarrow f \bar{f} h_k \) processes. In section 4 we comment on some
numerical examples of \( \sum_{k} \sigma(e^+e^- \rightarrow f \bar{f} h_k) \), for \( f = t, b \).

2 The two-Higgs-doublet model with spontaneous CP violation

The 2HDM of the electroweak interaction is the extension of the Standard Model
by an extra SU(2) Higgs doublet. The two Higgs doublets \( \Phi_1 \) and \( \Phi_2 \) are assumed
to couple to quarks and leptons in such a way that there are no flavour-changing
neutral coupling at the tree level. This natural flavour conservation constraint is
enforced (see, for instance [11]) customarily by imposing a discrete \( Z_2 \) symmetry
\[ \Phi_2 \rightarrow -\Phi_2 \quad u_{iR} \rightarrow -u_{iR} \]

The requirement that the potential breaks this symmetry only softly, excludes
terms as \( (\Phi_1^\dagger \Phi_2)[\lambda_6(\Phi_1^\dagger \Phi_1) + \lambda_7(\Phi_2\Phi_2)] + h.c. \) with operator dimension four, be-
cause they breaks \( Z_2 \) symmetry in a hard way. The renormalizable scalar potential
has the form
\[
V(\Phi_1, \Phi_2) = m_1^2\Phi_1^\dagger \Phi_1 + m_2^2\Phi_2^\dagger \Phi_2 - (m_3^2\Phi_1^\dagger \Phi_2 + h.c.) + \lambda_1(\Phi_1^\dagger \Phi_1)^2 + \lambda_2(\Phi_2^\dagger \Phi_2)^2 + \]
\[ + \lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{\lambda_5}{2}[(\Phi_1^\dagger \Phi_2)^2 + h.c.] \] (1)

We have explicit CP breaking if \( \xi = \text{Im}(m_3^4\lambda_5^*) \neq 0 \). We assume \( \xi = 0 \).

The potential is CP invariant and all parameters are real. Minimization of
the potential yields the vacuum expectation values of the two Higgs fields (the
phase of \( \Phi_1 \) can be adjusted such that the v.e.v. of \( \Phi_1 \) is real and positive, and
the phase of \( \Phi_2 \) such that \( \lambda_5 \) is real and positive)
\[
< \Phi_1 >= \frac{v_1}{\sqrt{2}} \quad < \Phi_2 >= \frac{v_2}{\sqrt{2}} e^{i\theta} \] (2)

where \( v_1, v_2 \) are real and positive parameters satisfying the experimental con-
straint \( v = \sqrt{v_1^2 + v_2^2} = 246 \) Gev. The requirement that our vacuum be at
least a stationary point of the potential results in the following constraints (\( \bar{\lambda} \equiv \lambda_3 + \lambda_4; s_\theta = \sin \theta; c_\theta = \cos \theta \))
\[ m_1^2 = -\lambda_1v_1^2 + \frac{1}{2}(\lambda_5 - \bar{\lambda})v_2^2 \] (3)
\[ m_2^2 = -\lambda_2 v_2^2 + \frac{1}{2}(\lambda_5 - \bar{\lambda}) v_1^2 \]  

\[ s_\theta(m_2^2 - \lambda_5 v_1 v_2 c_\theta) = 0 \]  

The last condition entails the interesting case of a non-zero CP violating phase \( \theta \), provided \(|m_3^2/\lambda_5 v_1 v_2| < 1 \).

After SU(2) \( \times \) U(1) gauge symmetry breaking, in the neutral Higgs sector the would-be Goldstone boson which is eaten in giving mass to the Z boson, is the combination \( \sqrt{2}(c_\beta \text{Im}\phi_1^0 + s_\beta \text{Im}\phi_2^0) \) where the mixing angle \( \beta \) is \( \tan \beta = v_2/v_1 \). The physical neutral Higgs bosons are the eigenstates of the mass squared matrix \( M_{ij} = \frac{\lambda}{2} \mu_{ij} \) in the remaining three neutral degrees of freedom. The elements of the reduced matrix \( \mu_{ij} \), in the basis \((\sqrt{2}\text{Re}\phi_1^0, \sqrt{2}\text{Re}\phi_2^0, \sqrt{2}(s_\beta \text{Im}\phi_1^0 - c_\beta \text{Im}\phi_2^0)) \equiv (\varphi_1^0, \varphi_2^0, \chi_c^0) \), read [12]

\[
\begin{align*}
\mu_{11} &= 2\lambda_1 c_\beta^2 + \lambda_5 s_\beta^2 c_\theta^2 \\
\mu_{12} &= (\bar{\lambda} - \lambda_5 s_\theta^2) s_\beta c_\beta \\
\mu_{13} &= \lambda_5 s_\beta s_\theta c_\theta \\
\mu_{22} &= 2\lambda_2 s_\beta^2 + \lambda_5 c_\beta^2 c_\theta^2 \\
\mu_{23} &= \lambda_5 c_\beta s_\theta c_\theta \\
\mu_{33} &= \lambda_5 s_\theta^2
\end{align*}
\]

where we have used (3),(4) and (5) to exclude \( m_1, m_2, m_3 \) from the equations.

Going to the basis \((c_x \varphi_1^0 - s_x \varphi_2^0, s_x \varphi_1^0 + c_x \varphi_2^0, \chi_c^0)\), the new matrix elements

\[
\begin{align*}
\mu'_{13} &= -\lambda_5 s_\theta c_\theta s_{x-\beta} \\
\mu'_{12} &= (c_x^2 - s_x^2)(\bar{\lambda} - \lambda_5) s_\beta c_\beta + s_x c_x(2\lambda_1 c_\beta^2 - 2\lambda_2 s_\beta^2) - \lambda_5 s_\theta^2 s_{x-\beta} c_{x-\beta}
\end{align*}
\]

can vanish if \( x = \beta \)

\[ (2\lambda_1 + \bar{\lambda} - \lambda_5)c_\beta^2 = (2\lambda_2 + \bar{\lambda} - \lambda_5)s_\theta^2 (\equiv \zeta) \]

In this case

\[
\begin{align*}
\mu'_{11} &= \zeta + \lambda_5 - \bar{\lambda} \\
\mu'_{22} &= \zeta + \lambda_5 c_\theta^2 \\
\mu'_{33} &= \lambda_5 s_\theta^2 \\
\mu'_{23} &= \lambda_5 s_\theta c_\theta
\end{align*}
\]

Therefore

\[ c_\beta \varphi_1^0 - s_\beta \varphi_2^0 \]

is a mass eigenstate with reduced mass \( \zeta + \lambda_5 - \bar{\lambda} \). The other two eigenstates are

\[-s_\psi(s_\beta \varphi_1^0 + c_\beta \varphi_2^0) + c_\psi \chi_c^0 \\
c_\psi(s_\beta \varphi_1^0 + c_\beta \varphi_2^0) + s_\psi \chi_c^0
\]
and have the reduced masses\(^2\) \(\frac{1}{2}(\zeta + \lambda_5 - \sqrt{\zeta^2 + \lambda_5^2 + 2\lambda_5\zeta \cos 2\theta})\) and \(\frac{1}{2}(\zeta + \lambda_5 + \sqrt{\zeta^2 + \lambda_5^2 + 2\lambda_5\zeta \cos 2\theta})\) respectively. In eq.(12) the mixing angle \(\psi\) is

\[
\sin 2\psi = \frac{\lambda_5 \sin 2\theta}{\sqrt{\zeta^2 + \lambda_5^2 + 2\lambda_5\zeta \cos 2\theta}}; \cos 2\psi = \frac{\zeta + \lambda_5 \cos 2\theta}{\sqrt{\zeta^2 + \lambda_5^2 + 2\lambda_5\zeta \cos 2\theta}}
\]

(13)

This is the model we assume, with a further condition which makes it very simple and attractive from the point of view of the mass spectrum: with

\[
\zeta = \lambda_5 \quad \tilde{\lambda} = 0
\]

(14)

we have

\[
\psi = \frac{\theta}{2}
\]

(15)

and the masses\(^2\) become \(\lambda_5v^2, \lambda_5 v^2 \sin^2 \psi, \lambda_5 v^2 \cos^2 \psi\). Therefore, in this model we identify (without loss of generality we choose \(-\frac{\pi}{2} \leq \theta \leq +\frac{\pi}{2}\), that is \(-\frac{\pi}{4} \leq \psi \leq +\frac{\pi}{4}\))

\[
\begin{align*}
    h_1 &= \sqrt{2}[-s_{\psi}(s_{\beta} \text{Re}\phi_1^0 + c_{\beta} \text{Re}\phi_2^0) + c_{\psi}(s_{\beta} \text{Im}\phi_1^0 - c_{\beta} \text{Im}\phi_2^0)] \\
    h_2 &= \sqrt{2}[c_{\psi}(s_{\beta} \text{Re}\phi_1^0 + c_{\beta} \text{Re}\phi_2^0) + s_{\psi}(s_{\beta} \text{Im}\phi_1^0 - c_{\beta} \text{Im}\phi_2^0)] \\
    h_3 &= \sqrt{2}(c_{\beta} \text{Re}\phi_1^0 - s_{\beta} \text{Re}\phi_2^0)
\end{align*}
\]

(16)

Clearly \(h_3\) is CP-even, \(h_1\) and \(h_2\) generally are CP-impure. The masses satisfy a Pythagorical sum rule

\[
m(h_1)^2 + m(h_2)^2 = m(h_3)^2
\]

(17)

For given \(m(h_1) \neq 0\), we have \(m(h_2) = m(h_1)/|\tan \psi|\) and \(m(h_3) = m(h_1)/|\sin \psi|\). If \(\theta \rightarrow \pm \frac{\pi}{2}\) (\(\psi \rightarrow \pm \frac{\pi}{4}\)) then \(m(h_1) = m(h_2) = m(h_3)/\sqrt{2}\); if \(\theta \rightarrow 0\) (\(\psi \rightarrow 0\)) then \(h_1\) becomes CP-odd and \(h_2\) becomes CP even but \(m(h_2) \rightarrow m(h_3) \rightarrow \infty\).

Once the model is defined, we can give the explicit forms of the couplings. The Yukawa interactions of the \(h_i\) \((i = 1, 2, 3)\) mass eigenstates are given by

\[
\mathcal{L} = \frac{m_v}{v} \bar{u}[s_{\psi} - i c_{\psi} \gamma_5]h_1 - \frac{c_{\psi} + i s_{\psi} \gamma_5}{t_{\beta}} h_2 + h_3]u + \\
+ \frac{m_d}{v} \bar{d}[(s_{\psi} - i c_{\psi} \gamma_5)t_{\beta} h_1 - (c_{\psi} + i s_{\psi} \gamma_5)t_{\beta} h_2 - h_3]d
\]

(18)

for up-type and down-type quarks and similarly for charged leptons.

For large \(t_{\beta}\), eq.(18) shows the typical enhancement of the couplings to down-type quarks over the coupling to up-type quarks as regards \(h_1\) and \(h_2\), but this is not the case of \(h_3\).
Let \( \mathcal{L} = \bar{f}(S^f_i + iP^f_i \gamma_5)f h_i \) and \( \hat{S}^f_i = \frac{v_{hf}}{m_f} S^f_i, \hat{P}^f_i = \frac{v_{hf}}{m_f} P^f_i \), the check of the sum rule\[9\]

\[
s^\beta_2[(\hat{S}^t_i)^2 + (\hat{P}^t_i)^2] + c^\beta_2[(\hat{S}^b_i)^2 + (\hat{P}^b_i)^2]
\]

is immediate.

The couplings ZZ-Higgs are \( g_{ZZh} = \frac{gm_Z}{c_w} C_i \) with

\[
C_1 = -s_{\psi}s_{2\beta} \quad C_2 = c_{\psi}s_{2\beta} \quad C_3 = c_{2\beta}
\]

For the couplings Z-Higgs-Higgs we have \( g_{Zh_h} = \frac{g}{2c_w} C_{ij} \) with

\[
C_{12} = c_{2\beta} \quad C_{23} = -s_{\psi}s_{2\beta} \quad C_{31} = c_{\psi}s_{2\beta}
\]

antisymmetric in the indices. Clearly

\[
C_i = C_{jk}
\]

for \((i, j, k)\) being any permutation of \((1, 2, 3)\). The sum rule \( \sum_k C^2_k = 1 \) becomes

\[
C_i^2 + C_j^2 + C_{ij}^2 = 1
\]

\((i \neq j; i, j = 1, 2, 3)\) which requires at least one of the \( ZZ_{hi}, ZZ_{hj} \) and \( Zh_ih_j \) couplings to be substantial in size. At last, one can very easily check also the following sum rules

\[
2\hat{S}^t_is_{2\beta}^2 + C_i = 2\hat{S}^b_i c_{2\beta}^2 + C_i = 0 \quad (i = 1, 2)
\]

\[
2\hat{S}^t_is_{2\beta}^2 + C_3 = -(2\hat{S}^b_i C_3 + C_3) = 1
\]

in agreement with the general sum rules\[9\]

\[
(\hat{S}^t_i)^2 + (\hat{P}^t_i)^2 = \frac{1}{t_{2\beta}^2}[1 + C_i(2\hat{S}^b_i + \frac{C_i}{c^2_\beta})]
\]

\[
(\hat{S}^b_i)^2 + (\hat{P}^b_i)^2 = t_{2\beta}^2[1 + C_i(2\hat{S}^t_i + \frac{C_i}{s^2_\beta})]
\]

\((i=1,2\text{and }3)\).

In ref.[9], it is well shown that the set of these sum rules makes it apparent that the Higgs finding strategy at \( e^+e^- \) colliders should include the Yukawa processes with Higgs radiation off top and bottom quarks (the resort to heavy flavour sector is obvious) together with Higgs-strahlung and Higgs pair production.
3 Higgs boson production in $e^+e^-$ colliders

We consider now the production of the neutral Higgs boson $h_i$ in association with a fermion pair in $e^+e^-$ collisions

$$e^+e^- \rightarrow f\bar{f}h_i$$

It can proceed via (1) bremsstrahlung off the Z boson $e^+e^- \rightarrow Z^* h_i \rightarrow f\bar{f}h_i$, (2) Higgs pair production $e^+e^- \rightarrow h_i^* h_i \rightarrow f\bar{f}h_i$, $e^+e^- \rightarrow h_i^* h_i \rightarrow f\bar{f}h_i$ (where $i, j, k$ are any permutations $(1,2,3)$) and (3) Yukawa processes with Higgs radiation off a fermion line $e^+e^- \rightarrow f\bar{f} \rightarrow f\bar{f}h_i$, $e^+e^- \rightarrow f^* \bar{f} \rightarrow f\bar{f}h_i$. (On the importance of the Yukawa processes in a CP conserving 2HDM see [13], [14].)

The total cross section $\sigma(e^+e^- \rightarrow f\bar{f}h_i) \equiv \sigma(f\bar{f}h_i)$ of our process can be written as follows. Let $N_c$ denote the number of colors, $\sigma_0 = 4\pi \alpha^2/s$ the standard normalization cross section, where $\sqrt{s}$ is the total c.m. energy, $q_f$ the electric charge, $v_f$ and $a_f$ the vector and axial Z charges of the fermion $f$

$$v_f = \frac{2I_{L,f} - 4q_f s_W^2}{4s_W c_W}, \quad a_f = \frac{2I_{L,f}}{4s_W c_W} \quad (25)$$

with $I_{L,f} = \pm \frac{1}{2}$ being the weak isospin of the left-handed fermions. We have

$$\sigma(f\bar{f}h_i) = N_c \frac{\sigma_0}{(4\pi)^2} \int_{4f}^{(1-\sqrt{t})^2} dy \{ [q_f^2 q_j^2 + 2q_f v_f I(z) + (v_f^2 + a_f^2) Q(z)] [H_0^{(1)}(y) + Q(z) [a_f^2 H_0^{(2)}(y) + H_1^{(1)}(y) + H_2^{(1)}(y)] + (v_f^2 + a_f^2) H_1^{(2)}(y) + H_3(y)] + [q_f v_f I(z) + (v_f^2 + a_f^2) Q(z)] H_2^{(2)}(y) \} \quad (26)$$

In eq.(26)

$$I(z) = \frac{q_e v_e (1 - z)}{(1 - z)^2 + \gamma_z z}, \quad Q(z) = \frac{v_e^2 + a_e^2}{(1 - z)^2 + \gamma_z z}$$

where $z = \frac{m_f^2}{s}, \quad \gamma_z = \frac{\Gamma_Z}{s}$ are the reduced mass and width of the Z boson. Analogously, $f = \frac{m_f^2}{s}, \quad h_i = \frac{m(h_i)}{s}$ are the reduced masses of the fermion and the $h_i$ neutral Higgs boson, and in the following we shall use $\gamma_i = \frac{\Gamma_{h_i}}{m_{h_i}}$ for the reduced width of $h_i$.

In order to specify the $H_m^{(n)}(y)$ functions, we introduce the triangular function

$$\lambda_i \equiv \lambda(1, h_i, y) = (1 + h_i - y)^2 - 4h_i \quad (27)$$

and

$$\Delta_i \equiv \left[ \frac{y - 4f}{y - \lambda_i} \right]^{1/2}, \quad B_i \equiv \ln \frac{1 + h_i - y + \Delta_i}{1 + h_i - y - \Delta_i} \quad (28)$$
and the combinations of the scalar \( S^f_i \) and pseudoscalar \( P^f_i \) couplings
\[
\alpha_i^{f(\pm)} = (S^f_i)^2 \pm (P^f_i)^2
\]  
(29)

We have
\[
H_0^{(1)}(y) = 2\alpha_i^{f(+) \Delta_i} + \frac{(2f-h_i)(1+2f)y}{f\lambda_i+h_iy} \Delta_i + \frac{(1+2f-y)^2+(2f-h_i)^2+4y}{1+h_i-y} B_i
\]  
(30a)
\[
+4f\alpha_i^{f(-)} \left[ \frac{(1+2f)y}{f\lambda_i+h_iy} \Delta_i + \frac{2+2f-y}{1+h_i-y} B_i \right]
\]

\[
H_0^{(2)}(y) = 2\alpha_i^{f(+) \Delta_i} - \frac{y-2}{f\lambda_i+h_iy} \Delta_i - \frac{2f\lambda_i+2(3f-h_i)(1+2f-y)+6fh_i}{1+h_i-y} B_i
\]  
(30b)
\[
-24f\alpha_i^{f(-)} \left[ \frac{y}{f\lambda_i+h_iy} \Delta_i + \frac{1+2f-y}{1+h_i-y} B_i \right]
\]

\[
H_1^{(1)}(y) = \frac{g_{ZH}^2}{(y-z)^2+\gamma z^2} 2f[-12z+(\frac{y}{z}-2)\lambda_i] \Delta_i
\]  
(30c)
\[
H_1^{(2)}(y) = \frac{g_{ZH}^2}{(y-z)^2+\gamma z^2} 4z(y+2f)(1+\frac{\lambda_i}{12y}) \Delta_i
\]  
(30d)

\[
H_2^{(1)}(y) = S^f_i g_{ZH} \frac{y-z}{(y-z)^2+\gamma z^2} \sqrt{\frac{f}{z}} \left\{ [(1+h_i-y)\gamma z-6z] \Delta_i + 2[3z(4f-h_i)+f\lambda_i+h_iy] B_i \right\}
\]  
(30e)
\[
H_2^{(2)}(y) = S^f_i g_{ZH} \frac{y-z}{(y-z)^2+\gamma z^2} \sqrt{\frac{f}{z}} \left\{ [2\gamma z(4f-h_i)+f\lambda_i+h_iy] B_i \right\}
\]  
(30f)

\[
H_3(y) = \left\{ \frac{[S^f_j C_{ij}(y-h_k)+(j \leftrightarrow k)]^2(y-4f)+[P^f_j C_{ij}(y-h_k)+(j \leftrightarrow k)]^2 y}{8s^2 w \lambda \gamma z^2 [(y-h_j)^2+\gamma_j h_j][(y-h_k)^2+\gamma_k h_k]}ight\}
\]
\[
+\frac{a f g_{ZH} \sqrt{\frac{f}{z}}}{s w c w} \frac{P^f_j C_{ij}(y-h_j)}{(y-h_j)^2+\gamma_j h_j} \lambda_i \Delta_i
\]  
(30g)
\[
+\frac{a f}{s w c w} \frac{(S^f_j P^f_i+S^f_i P^f_j) C_{ij}(y-h_j)}{(y-h_j)^2+\gamma_j h_j} \lambda_i \Delta_i
\]

\[
\lambda_i \Delta_i
\]

\[y(1+h_i-y)\Delta_i-2(f\lambda_i+h_iy) B_i
\]

\((i, j, k)\) being any permutation of \((1, 2, 3)\).

The functions \( H_0^{(1)}(y), H_0^{(2)}(y) \) describe the Yukawa processes with \( h_i \) neutral Higgs radiation off the fermions. The functions \( H_1^{(1)}(y), H_1^{(2)}(y) \) describe the Higgs-strahlung off the Z boson. The functions \( H_2^{(1)}(y), H_2^{(2)}(y) \) describe the interference between the above mentioned processes. The function \( H_3(y) \) describe the Higgs pair production (2) and its interference with the other two mechanisms of \( h_i \) production.

This result is in right agreement with ref.[9]. A part a 1/4\pi common factor, already taken into account, the \( H_m^{(n)} \) functions are integrated forms of the functions \( F_n, G_n \) or their combinations.
We can now proceed to comment our results with some examples.

As we argued above, the violation of CP in the 2HDM taken into account, plays a rôle not only in determining the couplings of the neutral Higgs bosons but also in adjusting their mass spectrum. For growing $\psi = \frac{\theta}{2}$, the tendency of the masses towards the lightest one, makes more interesting the $\sum_{k=1}^{3} \sigma (f\bar{f}h_k)$ (with $f = t, b$) rather than the single $\sigma (f\bar{f}h_k)$ cross sections.

In the numerical examples, we have taken $m_b = 5$Gev, $m_t = 175$Gev, $\gamma_k = 0.02m(h_k)$. Moreover, whenever $m(h_1)$ is given, then $m(h_1) = 100$Gev.

The fractional contributions of the single $\sigma (t\bar{t}h_k)$ cross sections to their sum, is shown in Fig.1. Their dependence on $\psi$, for given $\tan \beta$, is obvious owing to the influence of $\psi$ both on the production thresholds and on the coupling constants. In this respect, one can compare the panels (a) and (b) and the panels (c) and (d) of Fig.1.

In the same figure, the comparison of the panels (a) and (c) and of the panels (b) and (d) exhibits the dependence on $\tan \beta$ of the fractional contributions $\sigma (t\bar{t}h_k)$, for given $\psi$. Clearly, the increase of $\tan \beta$ grows the fractional contribution of $h_3$ (whose coupling to quarks do not depend on $\tan \beta$ in our model) with respect to those of the other two neutral Higgs, owing to the break down of their coupling to up-type quarks for large $\tan \beta$.

As regards $\sum_k \sigma (b\bar{b}h_k)$, an opposite remark holds, as it is apparent by comparison of the panels (a) and (c) and of the panels (b) and (d) of Fig.2. Now, for given $\psi$, the rise of $\tan \beta$ reduces the fractional contribution of $h_3$, owing to the independence on $\tan \beta$ of its coupling to quarks, while the couplings of $h_1$ and $h_2$ to down-type quarks are enhanced for large $\tan \beta$.

The dependence on $\psi$ of the fractional contributions $\sigma (b\bar{b}h_k)$ to their sum, for given $\tan \beta$, is particularly evident by comparison of the panels (a) and (b) and of the panels (c) and (d) of Fig.2. For rising $\psi$, the production thresholds of $b\bar{b}h_2$ and $b\bar{b}h_3$ move backwards and strikingly modifie the profile of $\sum_k \sigma (b\bar{b}h_k)$ versus the total c.m. energy.

As an example of these effects, we note that the hunting of the neutral Higgs bosons at $\sqrt{s} = 500$Gev, $\tan \beta = 10$, and $\psi = 27^\circ$, is practically the hunting of $h_3$, the most massive(!) of the neutral Higgs bosons in our model.

The Fig.3 shows the behaviour of $\sum_k \sigma (t\bar{t}h_k)$ versus $\sqrt{s}$ for different choices of $\psi$, for given $\tan \beta$. Its growing with $\psi$ is clearly shown.

The Fig.4 shows the behaviour of $\sum_k \sigma (b\bar{b}h_k)$ versus $\sqrt{s}$ for given $\tan \beta$ and various values of $\psi = \theta/2$, which parametrizes in some way the CP breaking. In this figure the curve for the CP conserving case ($\psi = 0$) is invisible in the chosen scale. For growing $\psi$, the lowering of the production thresholds and the growth of $\sum_k \sigma (b\bar{b}h_k)$ is also well evident.

In Fig.5 one can see the dependence of $\sum_k \sigma (t\bar{t}h_k)$ on the parameter $\psi$ of CP breaking, at given $\tan \beta$, for different choices of the c.m. total energy $\sqrt{s}$. The
common feature of these curves (a part that they are symmetric in $\psi$) is their remarkable increase with $\psi(>0)$, at least if $\psi$ is not too small. Similar remarks hold in Fig.6 where is shown the behaviour of $\sum_k \sigma(b\bar{b}h_k)$ versus $\psi$, at given $\tan\beta$ and for various $\sqrt{s}$. In this case, it is well evident that the width of the plateau around $\psi = 0$ (where the gain of $\sum_k \sigma(b\bar{b}h_k)$ with respect to the CP conserving case $\psi = 0$ is unsizable) decreases as $\sqrt{s}$ increases.

In the Figs.7 and 8, are reported the behaviours of $\sum_k \sigma(t\bar{t}h_k)$ and $\sum_k \sigma(b\bar{b}h_k)$ versus $m(h_1)$, at given $\sqrt{s}$ and at some values of $\tan\beta$ and $\psi$. At given $\tan\beta$, the minimal values of $\sum_k \sigma(t\bar{t}h_k)(\sum_k \sigma(b\bar{b}h_k))$ are realized in the CP conserving case $\psi = 0$. In effect, in this case, only $\sigma(t\bar{t}h_1)(\sigma(b\bar{b}h_1))$ contributes, because the other two neutral Higgs are enormously massive. In any case, the maximal values are achieved for ”maximal” CP violation, that is for $\psi = \frac{\pi}{4}(\theta = \frac{\pi}{4})$. Independently on the mass of the lightest Higgs, this confirms what we have already seen for $m(h_1) = 100$ Gev. Also the occurrence that the smaller is $\tan\beta$ (at given $\psi$) and the larger is the $\sum_k \sigma(t\bar{t}h_k)$ (the smaller is the $\sum_k \sigma(b\bar{b}h_k)$), is consistent with the previous considerations concerning the dependence on $\tan\beta$ of the couplings of the $h_1$ and $h_2$ bosons to up-type and down-type quarks.

But we can well note in Fig.8 that the maximal $\sum_k \sigma(b\bar{b}h_k)$ at $\tan\beta = 10$ can be larger than the minimal one at $\tan\beta = 100$. In other terms, a small $\tan\beta$ and a large $\psi$ can yield a cumulative cross section $\sum_k \sigma(b\bar{b}h_k)$ larger than the one with a larger $\tan\beta$ but a smaller $\psi$. This can be observed also for $\Sigma_t = \sum_k \sigma(t\bar{t}h_k)$ in the panel (a) of Fig.9, where the contour lines for $\Sigma_t$ are reported in the plane $(\tan\beta, \psi)$, at given $m(h_1) = 100$ Gev and $\sqrt{s} = 600$ Gev. E.g., for $\tan\beta = 0.1$ and $\psi = 30^0$ one has a $\Sigma_t$ larger than for $\tan\beta = 0.07$ and $\psi = 10^0$: the growing of $\tan\beta$ yields effects opposite to those due to the growing of $\psi$.

The panels (b) and (c) of Fig.9 show the contour lines for the cumulative cross section $\Sigma_t$ for two different choices of $\tan\beta$, at given $\sqrt{s} = 600$ Gev, in the plane $(m(h_1), \psi)$. Also here, a small $m(h_1)$ is not a guarantee of a great $\Sigma_t$: that depends on $\psi$. E.g., $m(h_1) = 60$ Gev, $\psi = 5^0$ has a $\Sigma_t$ smaller than the one for $m(h_1) = 150$ Gev, $\psi = 45^0$ (at $\tan\beta = 0.1, \sqrt{s} = 600$ Gev). Analogous considerations can be made concerning $\Sigma_b = \sum_k \sigma(b\bar{b}h_k)$, as it is shown in the panel (d) of Fig.9.

In our 2HDM, at given $m(h_1)$, to change $\psi$ is equivalent to change $m(h_2)$ (and $m(h_3)$). Therefore the former figures can be turned in the plane $(m(h_1), m(h_2))$ (or $(m(h_1), m(h_3))$). They are shown in Fig.10. In the panel (a) are reported the contour lines for $\Sigma_t$, at $\tan\beta = 0.1$ and c.m. energy $\sqrt{s} = 600$ Gev, in the plane $(m(h_1), m(h_2))$ (solid lines) and $(m(h_1), m(h_3))$ (dashed lines). Analogously in the panel (b) one can see the contour lines for $\Sigma_b$ (with $\tan\beta = 10$ and $\sqrt{s} = 400$ Gev).

We can conclude that the spontaneous CP violation in our 2HDM not only do not jeopardize but, on the contrary, increases our ability to find light neutral
Higgs bosons. Furthermore, to find a neutral Higgs boson do not necessarily mean to have found the lightest one: it can happen that the largest contribution to this hunting is due to the heaviest neutral Higgs boson.

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Figure captions

Fig.1 The cumulative \( \sum_{i=1}^{3} \sigma(e^+e^- \rightarrow \bar{t}h_{k}) \equiv \sum_{k} \sigma(\bar{t}h_{k}) \) cross section and its contributions \( \sigma(\bar{t}h_{i}) \), \( i = 1,2,3 \), versus the c.m. energy \( \sqrt{s} \), for \( m(h_{1}) = 100 Gev \) and (a) \( \tan \beta = 0.1, \psi = 9^o \); (b) \( \tan \beta = 0.1, \psi = 27^o \); (c) \( \tan \beta = 1.0, \psi = 9^o \); (d) \( \tan \beta = 1.0, \psi = 27^o \).

Fig.2 The cumulative \( \sum_{i=1}^{3} \sigma(e^+e^- \rightarrow \bar{b}h_{k}) \equiv \sum_{k} \sigma(\bar{b}h_{k}) \) cross section and its contributions \( \sigma(\bar{b}h_{i}) \), \( i = 1,2,3 \), versus the c.m. energy \( \sqrt{s} \), for \( m(h_{1}) = 100 Gev \) and (a) \( \tan \beta = 10, \psi = 9^o \); (b) \( \tan \beta = 10, \psi = 27^o \); (c) \( \tan \beta = 100, \psi = 9^o \); (d) \( \tan \beta = 100, \psi = 27^o \).

Fig.3 The \( \sum_{k} \sigma(\bar{t}h_{k}) \) cross section versus \( \sqrt{s} \), for \( m(h_{1}) = 100 Gev, \tan \beta = 0.1 \) and for various values of \( \psi \), from \( \psi = 0^o \) (CP conserving case) to \( \psi = 45^o \) ("maximal" CP breaking).

Fig.4 The \( \sum_{k} \sigma(\bar{b}h_{k}) \) cross section versus \( \sqrt{s} \), for \( m(h_{1}) = 100 Gev, \tan \beta = 10 \) and for various values of \( \psi \). The curve corresponding to the CP conserving case \( \psi = 0^o \) is undrawn because it is too small in the chosen scale.

Fig.5 The dependence on \( \psi \) of the \( \sum_{k} \sigma(\bar{t}h_{k}) \) cross section, for \( m(h_{1}) = 100 Gev, \tan \beta = 0.1 \) and for various values of the c.m. energy \( \sqrt{s} \).

Fig.6 The \( \sum_{k} \sigma(\bar{b}h_{k}) \) cross section versus \( \psi \), for \( m(h_{1}) = 100 Gev, \tan \beta = 10 \) and for various values of \( \sqrt{s} \).

Fig.7 The cumulative \( \sum_{k} \sigma(\bar{t}h_{k}) \) cross section versus the mass of the lightest neutral Higgs boson \( m(h_{1}) \), at given \( \sqrt{s} = 800 Gev \), for \( \tan \beta = 0.1 \) and various values of \( \psi \), and for \( \tan \beta = 1.0 \) and various values of \( \psi \).

Fig.8 the \( \sum_{k} \sigma(\bar{b}h_{k}) \) cross section versus \( m(h_{1}) \), at \( \sqrt{s} = 400 Gev \). There are two sets of lines: for \( \tan \beta = 10 \) and various values of \( \psi \), and for \( \tan \beta = 100 \) and various values of \( \psi \).

Fig.9 (a) Contour lines for \( \sum_{k} \sigma(\bar{t}h_{k}) \equiv \Sigma_{t} \) in the plane \( (\tan \beta, \psi) \) with \( m(h_{1}) = 100 Gev \) and \( \sqrt{s} = 600 Gev \); (b) contour lines for \( \Sigma_{t} \) in the plane \( (m(h_{1}), \psi) \), for \( \tan \beta = 0.1 \) and \( \sqrt{s} = 600 Gev \); (c) same of (b) but for \( \tan \beta = 1.0 \); (d) contour lines for \( \sum_{k} \sigma(\bar{b}h_{k}) \equiv \Sigma_{b} \) in the plane \( (m(h_{1}), \psi) \), for \( \tan \beta = 10 \) and \( \sqrt{s} = 400 Gev \).

Fig.10 (a) Contour lines for \( \Sigma_{t} = \sum_{k}(\bar{t}h_{k}) \) in the plane \( (m(h_{1}), m(h_{2})) \) (solid lines) or in the plane \( (m(h_{1}), m(h_{3})) \) (dashed lines). Here \( \tan \beta = 0.1 \) and \( \sqrt{s} = 600 Gev \); (b) same of (a) but for \( \Sigma_{b} = \sum_{k}(\bar{b}h_{k}) \), with \( \tan \beta = 10 \) and \( \sqrt{s} = 400 Gev \).
Fig. 9
Fig. 3

\sum_k \sigma(t\bar{t}h_k)(fb)

\psi = 45°

\psi = 36°

\psi = 27°

\psi = 18°

\psi = 9°

\psi = 0°

\tan \beta = 0.1