ASYMPTOTIC FREEDOM FOR $\lambda \phi^4$ QFT IN SNYDER-DE SITTER SPACE

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Abstract. In this letter we analyze the model of a self-interacting $\phi^4$ scalar field theory in Snyder-de Sitter space. After analytically computing the one-loop beta functions, we solve the corresponding system of differential equations numerically. These results show that the model possesses at least one regime in which the theory is asymptotically free. Moreover, in a given region of the parameter’s space we also observe a peculiar running of the parameter associated to the curvature, which changes its sign and therefore can be interpreted as a transition from an IR de-Sitter space to an UV anti-de Sitter one.

1. Introduction

It is currently believed that noncommutative geometry [1, 2] may play an important role in the search for a quantum theory of gravity. The first example of noncommutative geometry was introduced by Snyder [3] with the hope of taming the divergences of quantum field theory (QFT) through the discretization of space. Of particular interest is therefore the investigation of QFT on noncommutative spaces, which has become an important area of research in recent times, especially in the context of the Moyal geometry [4, 5].

In [6], we have used the worldline formalism to investigate the QFT of a scalar model with quartic interaction on Snyder space, in an approximation of first order in the noncommutativeness parameter. The investigation has been performed using the standard formalism of noncommutative QFT. We have found that the model is nonrenormalizable, because divergences appear in the six-point function. This is due to the approximation used. A treatment at all orders in the noncommutativeness parameter gives strong clues of renormalizability and maybe even finiteness, at least for some choices of the interaction potential, although calculations have not been completed due to algebraic difficulties [7].

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However, the present status of these calculations cannot rule out the occurrence of the phenomenon of UV/IR mixing. UV/IR mixing is a property of some noncommutative QFTs, that manifests itself through the appearance of divergences in the UV-renormalized diagrams when the external momenta go to zero. This scenario however brings out several opportunities. The mixing could be for example used to generate small scales from the UV dynamics as pointed out in [8]. It served also as a motivation to find new well-behaved NC field theories. One of these is the Grosse-Wulkenhaar (GW) extension of the Moyal $\phi_4^4$ model [9]. The main peculiarity of this model is the presence in the action of an harmonic oscillator term that smooths the infrared behavior and in the end gives rise to an all-order perturbatively renormalizable theory.

It has been observed that the introduction of such term may be justified by the assumption of a de Sitter spacetime background [10]. For this reason, we extend our previous study of the Snyder scalar QFT to the case of a de Sitter background [11]. Some other approaches to noncommutative curved spaces can be found in [12, 13, 14], where the authors motivate their mathematical construction from Poisson-Lie algebras, and [15, 16], where a group-representation approach is used and some astrophysical consequences are discussed.

Coming back to our Snyder-de Sitter model, we shall show that it presents an effective action that at zeroth order in the noncommutative and curvature parameters coincides with that of the GW model (although the star-product is different), and might then share some of its properties.

Although our computations are limited to the linear approximation in the noncommutativenss and curvature coupling constants of the theory and therefore do not consent to drive conclusions about questions such as the UV/IR mixing, this is a starting point where the renormalization flow of the theory can be considered. Also, for simplicity, we shall use an action which is not de Sitter symmetric. We shall present a fully de Sitter-invariant treatment together with a more detailed exposition of the calculations in a forthcoming paper [17].

2. CURVED SNYDER SPACE

Snyder spaces are spaces in which noncommutativity is implemented in such a way that the Lorentz algebra and its action on the position ($\hat{x}_i$) and momentum ($\hat{p}_j$) operators is undeformed; in our Euclidean version, we can write the relevant commutation relations in terms of the generators $J_{ij}$ of the Lorentz algebra,

$$\begin{align*}
[J_{ij}, J_{kl}] &= i((\delta_{ik}J_{jl} - \delta_{il}J_{jk} - \delta_{jk}J_{il} + \delta_{jl}J_{ik}), \\
[J_{ij}, \hat{p}_k] &= i(\delta_{ik}\hat{p}_j - \delta_{jk}\hat{p}_i), \\
[J_{ij}, \hat{x}_k] &= i(\delta_{ik}\hat{x}_j - \delta_{jk}\hat{x}_i). \\
\end{align*}$$

These expressions do not fix the commutation relations among momentum and position operators, that constitute the (deformed) Heisenberg algebra. Indeed, there exist several representations for which the $\hat{p}_i$ commute among themselves, but the $\hat{x}_i$ do not [18]. The commutation relations of the $\hat{p}_i$ and $\hat{x}_i$ are then fixed almost uniquely by requiring the validity of the Jacobi identities.

In this letter, we consider instead an algebra in which the momenta do not commute: this can be interpreted as implying the presence of spacetime curvature. Explicitly, we choose the commutation relations\footnote{Be careful to the change $\beta \to \beta^2$ with respect to some previous works on the topic, as [19].} [11, 20, 21, 22] with respect to some previous works on the topic, as [19].

$$\begin{align*}
[\hat{x}_i, \hat{x}_j] &= i\beta^2 J_{ij}, \\
[\hat{p}_i, \hat{p}_j] &= i\alpha^2 J_{ij}, \\
[\hat{x}_i, \hat{p}_j] &= i(\delta_{ij} + \alpha^2 \hat{x}_i \hat{x}_j + \beta^2 \hat{p}_i \hat{p}_j + \alpha\beta(\hat{x}_j \hat{p}_i + \hat{p}_i \hat{x}_j)). \\
\end{align*}$$

The generators $J_{ij}$ can be realized in terms of the phase space variables as $J_{ij} = \frac{1}{2}(\hat{x}_i \hat{p}_j - \hat{x}_j \hat{p}_i + \hat{p}_j \hat{x}_i - \hat{p}_i \hat{x}_j)$. On the one hand, the position commutators correspond
to the simplest Snyder space realization. On the other hand, the momentum
commutators are the ones one would find for a de Sitter space in the usual commutative
ground. For $\beta \to 0$ one recovers the de Sitter algebra, while for $\alpha \to 0$ one gets the
Snyder one. As usual, the commutator that mixes positions and momenta is fixed
by the request that the Jacobi identities are satisfied. These observations justify the
name “Snyder-de Sitter space” (SdS) given to this geometry.

One interesting property of the SdS model is the fact that it can be transformed
with the aid of a nonunitary transformation into the usual Snyder space. In fact, we
can define the operators $\hat{x}_i$ and $\hat{p}_i$ in terms of new operators $X_i$ and $P_i$ that satisfy
the Snyder algebra,

$$[X_i, X_j] = i\beta^2 J_{ij}, \quad [P_i, P_j] = 0, \quad [X_i, P_j] = i(\delta_{ij} + \beta^2 P_i P_j),$$

employing the following definitions that involve an arbitrary parameter $t$ [20, 22] (in
the following it will be set to 0 for simplicity):

$$\hat{x}_i =: X_i + t \frac{\beta}{\alpha} P_i, \quad \hat{p}_i =: (1 - t)P_i - \frac{\alpha}{\beta} X_i,$$

Moreover, it is well known that the Snyder operators can be written in terms of
operators $x_i$ and $p_i$, obeying canonical commutation relations, as

$$P_i =: p_i = -i \partial_i, \quad X_i =: x_i + \beta^2 x_j p_j p_i = x_i - \beta^2 x_j \partial_j \partial_i.$$  

Although the $X_i$ operators so defined are non-hermitian, this problem can be overcome
by symmetrizing [19],

$$X_i \to X_i = \hat{x}_i = x_i + \frac{\beta^2}{2} (x_j p_j p_i + p_i p_j x_j).$$

After this sequence of transformations, the original momentum operator of SdS can be
finally written as

$$\hat{p}_i = p_i = \frac{\alpha}{\beta} x_i - \frac{\alpha \beta}{4} (x_j p_j p_i + p_i p_j x_j).$$

3. Self-interacting scalar quantum field theory on SdS

In the following, we shall define the free scalar field action in $D$ dimensions as

$$S_K = \int d^D x \phi (\hat{p}^2 + m^2) \phi,$$

where $\hat{p}^2$ is the kinetic operator, defined previously in (7).\footnote{Actually, in a de Sitter background, the correct kinetic operator would be $A_{dS} = p^2 + \frac{e^2}{4} J_{ij}$, and $S_K$ would have a nontrivial measure, but this does not modify our results in a substantial way. We shall consider this in a forthcoming publication [17].}

Using hermitian operators $\hat{x}$, $\hat{p}$ as given by eqs. (6) and (7), and after several
manipulations using the commutators of the algebra and integration by parts, we can
obtain a simpler expression, that up to first order in both $\alpha^2$ and $\beta^2$ reads

$$\hat{p}^2 = p^2 + \frac{\alpha^2}{\beta^2} x^2 + \alpha^2 (x_i x_j p_j p_i + x_j p_j x_i).$$

Hence, up to first order order in the noncommutative parameters one obtains the
Grosse-Wulkenhaar kinetic term, plus an extra contribution that mixes position and
momenta operators. It is important to notice that in this way the harmonic term,
added ad hoc in the Grosse-Wulkenhaar model, acquires a pure geometric meaning.
This possibility was proposed in a different context already in [10].

It is also interesting to remark that the form of the additional contribution $(x \cdot p)^2$ is not surprising, since in the usual commutative case on curved spaces there would also be similar contributions. Furthermore we expect that when considering the dS invariant kinetic term some further corrections of this type will arise.
Now that we have obtained an expression for the kinetic term, we need to introduce the interaction. In order to do this, let us briefly mention some properties of a powerful tool used to describe noncommutative theories, the star product. The noncommutative geometry can be implemented in terms of an algebra of functions with a deformed (noncommutative or star) product. In the case of the explicit realization we are using for the Snyder space, the star product $\star$ of plane waves has been found to be [19]

$$e^{ik \cdot x} \star e^{iq \cdot x} = \left[ 1 - \frac{\beta^2 k \cdot q}{1 + \sqrt{1 + \beta^2 k^2}} \right] k_\mu + \sqrt{1 + \beta^2 k^2} q_\mu,$$

where $D$ is the dimension of the space and

$$D_\mu(k, q) := \frac{1}{1 - \beta^2 k \cdot q} \left[ \left( 1 - \frac{\beta^2 k \cdot q}{1 + \sqrt{1 + \beta^2 k^2}} \right) k_\mu + \sqrt{1 + \beta^2 k^2} q_\mu \right].$$

Note that this star-product is both noncommutative and nonassociative.

At this point the addition of an interacting term is straightforward. If we propose a quartic term, we end up with the usual expression in Snyder space, where the noncommutativity among position operators can be traded for the star product [6]:

$$S_I = \frac{\lambda}{4!} \int d^D x \phi(\hat{x}) \phi^2(\hat{x}) \phi(\hat{x}) \phi(\hat{x})\Phi(\hat{x}) \Phi(\hat{x}) = \frac{\lambda}{4!} \int d^D x \phi(x) \star \left[ \phi(x) \star \left( \phi(x) \star \phi(x) \right) \right].$$

Needless to say, the action whose one-loop contributions will be considered in the next section is simply the sum of both the kinetic and interacting term, viz. eqs. (8) and (12):

$$S = S_K + S_I.$$

### 4. The One-Loop Effective Action

In order to perform the one-loop calculations we will use the Worldline Formalism in its noncommutative version [23]. This method has already proved its utility in the study of the exact nonperturbative propagator of the Grosse-Wulkenhaar model [24, 25].

Consider then the expression for the effective action $\Gamma$ up to one-loop corrections,

$$\Gamma[\phi] = S[\phi] - \frac{1}{2} \int_0^\infty \frac{dT}{T} \text{Tr} \left( e^{-T \delta^2 S} \right),$$

where the connection with the heat kernel of the second variation $\delta^2 S$ of the action is made explicit. The fact that the action involves the noncommutative and nonassociative product $\star$ implies that the computation of the variation should be performed with care. As an example of this type of calculations consider [6].

Instead of writing the second variation of the action, we report below its Weyl ordered expression $\delta^2 S_W$, since this is the relevant one for the Worldline Formalism:

$$\delta^2 S_W = p^2 + \omega^2 x^2 + \alpha^2 (x_i x_j p_j p_i) + m_{eff} + V_W,$$

where the subscript $S$ indicates a symmetrized expression and we have introduced the Weyl-ordered potential $V_W$, the frequency $\omega$ for the oscillator and the effective

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3As an example, consider for simplicity the one-dimensional case. If we have an expression $x^2 p^2$, its symmetrization gives $(x^2 p^2)_S = \frac{1}{4} \left( x^2 p^2 + 2xp^2 x + p^2 x^2 \right)$ [26].
mass $m_{\text{eff}}$, given by

$$V_W := \frac{\lambda}{4!} \int \frac{dq_1 dq_2}{(2\pi)^2} \left[ 4! e^{ix(q_1+q_2)} + \beta^2 (\alpha_{\mu\nu}(x)p^\mu p^{\nu} + \beta_\mu(x)p^\mu + \gamma(x)) \right] \tilde{\phi}_1 \tilde{\phi}_2,$$

$$\omega^2 := \frac{\alpha^2}{\beta^2},$$

$$m_{\text{eff}}^2 := m^2 - \frac{5\alpha^2}{2} D(D+1).$$

In the expression for $V_W$, the $x$-dependent coefficients $\alpha_{\mu\nu}$, $\beta_\mu$ and $\gamma$ are those given in Appendix A of [6].

It is interesting to notice that as a by-product of the symmetrization we get a negative contribution to the mass term. Even if this could seem awkward at a first sight, the appearance of such a term and the usually consequent infrared divergences are familiar in the commutative de Sitter case [27]. It is not the scope of this letter to inquire in this direction – instead, we point out that there have been some advances in this topic, for example regarding computations in the large-N limit of a O(N) model in usual commutative de Sitter[28].

Using expressions (15) and (16), the computation of the one-loop contribution to the effective action is lengthy but otherwise straightforward. In order to isolate the divergent contributions and proceed with the renormalization of the theory, we use dimensional regularization in $D = 4 - \varepsilon$ dimensions. In this way we find that the divergent contributions of the 2- and 4-point functions are given by

$$\Gamma_{\text{div}}^{(2)} = -\frac{\lambda}{96\pi^2 \varepsilon} \int d^D x \phi \left[ 2\beta^2 \omega^2 x_{\mu\nu}x_{\nu}(-i\partial^\mu)(-i\partial^\nu) - x^2 \beta^2 \omega^2 \partial^2 + x^2 \left( \frac{15\alpha^2 m_{\text{eff}}^2}{2} + 18\beta^2 m_{\text{eff}}^2 \omega^2 + 3\omega^2 \right) + x^4 \left( \frac{9\alpha^2 \omega^2}{2} + 12\beta^2 \omega^2 \right) - 163\alpha^2 + 6\beta^2 \omega^2 + \frac{3\alpha^2 m_{\text{eff}}^4}{\omega^2} + 6\beta^2 m_{\text{eff}}^4 + 3m_{\text{eff}}^2 \right] \phi,$$

$$\Gamma_{\text{div}}^{(4)} = \frac{1}{4!} \frac{3\lambda^2}{16\pi^2 \varepsilon} \int d^D x \phi^2 \left[ \frac{\alpha^2 \partial^2}{2\omega^2} - \frac{2m_{\text{eff}}^2 (\alpha^2 + 2\beta^2 \omega^2) + \omega^2}{\omega^2} - x^2 \frac{5\alpha^2 + 16\beta^2 \omega^2}{2} \right] \phi^2,$$

where we have defined

$$\phi_{*,(1)} := \frac{2}{3} \phi^3 \left( (D+2) + 2x^\mu \partial_\mu \right) \partial^2 \phi.$$
which upon expansion in the $\alpha$ parameter gives (among others) the desired contributions. So the introduction of this type of counterterms should not be seen as an inconsistency of our theory.

Another contribution related to the geometry of the curved space are the two terms combining $p$ and $x$. As explained in the previous section, they are of the form that usually arises when considering the Laplacian in curved commutative spaces. The fact that we should add a new term $x^2 p^2$ to our original action can be understood as a dynamical deformation of the original spacetime by the one-loop contributions of the scalar field. A similar effect could be seen in the commutative case, where the one-loop effects of the scalar field generate new terms in the gravitational sector [30, 31, 32, 33].

The remaining contributions, i.e. the $\phi_{\ast(1)}$ and the $\partial^2$ one in the 4-point function, can be explained if we look deeper into the noncommutative $\phi^4$ potential. A direct computation gives

$$\int dx \phi \ast (\phi \ast (\phi \ast \phi)) = \int dx \left[ \phi^4 + \beta^2 \phi_{\ast(1)} + O(\beta^4) \right],$$

so that these two divergent contributions were hidden in our original action. As a consequence, the noncommutative parameter $\beta$ should be renormalized, but in a different way for the two terms in the RHS of eq. (19) – in other words, we are in presence of a dynamical deformation of the noncommutative space. This is one of the key differences with the Grosse-Wulkenhaar model, in which the noncommutativeness has no running.

5. The beta functions

Upon the introduction of an energy scale $\mu$ to render the coupling $\lambda$ dimensionless in $D$ dimensions, we can compute the corresponding beta functions from (17) and (18), using the standard definition $\beta_x = \frac{\partial x}{\partial \log \mu}$ for the coupling $x$. For the sake of simplicity we limit ourselves to the terms that were present in the original action (13), whose coupling constants have the following beta functions:

$$\beta_\lambda = \frac{3\lambda^2 \left( 2m_{\text{eff}}^2 \left( \alpha^2 + 2\beta^2 \omega^2 \right) + \omega^2 \right)}{16\pi^2 \omega^2},$$

$$\beta_{\omega^2} = \frac{\lambda}{48\pi^2} \left( \frac{15\alpha^2 m_{\text{eff}}^2}{2} + 18\beta^2 m_{\text{eff}}^2 \omega^2 + 3\omega^2 \right),$$

$$\beta_{m_{\text{eff}}^2} = \frac{\lambda}{48\pi^2} \left( -163\alpha^2 + 6\beta^2 \omega^2 + \frac{3\alpha^2 m_{\text{eff}}^4}{\omega^2} + 6\beta^2 m_{\text{eff}}^4 + 3m_{\text{eff}}^2 \right),$$

$$\beta_{\alpha^2} = \frac{\lambda}{48\pi^2} (2\beta^2 \omega^2),$$

$$\beta_{\beta^2} = -\frac{3\lambda \beta^2 m_{\text{eff}}^2 \left( \alpha^2 + 2\beta^2 \omega^2 \right)}{8\pi^2 \omega^2}. $$

Contrary to the GW case, the field does not get renormalized at the one-loop level.

Recall that, in order to analyze the renormalization group equation, the relevant differential equations are obtained from the previous ones adding a term related to the dimension $d_x$ of the corresponding coupling $x$ in units of mass, i.e.

$$\frac{\partial x(\mu)}{\partial \log \mu} = \beta_x - d_x \bar{x}(\mu).$$

If one were to solve this system of differential equations, one would need to provide some initial conditions; the natural guess would be to fix them at an energy scale attainable experimentally, say $\mu_0 \sim \text{GeV}$, at which according to the available experimental data we could choose $\lambda_0 \sim 1$, a typical baryon mass $m_{\text{eff},0} \sim \text{GeV}$, $\alpha_0 \sim$
Figure 1. Numerical solutions for the running of all the parameters of the theory, choosing as initial conditions $v_1 = (1, 1, 10^{-5}, 10^{-4}, 10^{-1})$ at an energy $\mu_0 = 1$ (all the quantities are measured in the corresponding powers of eV).

$10^{-33}$eV$^{-1}$ (according to its relation to the cosmological constant) and $\beta_0 \sim 10^{-29}$eV, that is of the order of the Planck scale. These quantities fix the initial value of the frequency $\omega_0 \sim 10^{-4}$eV$^{-2}$, which turns out to be the only relevant consequence of noncommutativeness and curvature until one reaches large energies.

Since we are interested in the qualitative behaviour of the solutions, we will instead choose rather general initial conditions. In addition, we will use the vectorial notation $v = (\lambda, m_{\text{eff}}, \alpha^2, \beta^2, \omega)$ to simplify the following description.

Let us start by choosing $v_1 = (1, 1, 10^{-5}, 10^{-4}, 10^{-1})$ (all measured in the corresponding powers of eV) at $\mu_0$. Using the \texttt{odeint} function implemented in [34], we solve numerically the renormalization group equations, i.e. the system of differential equations (22), after the inclusion of the corresponding dimensional term (23). In this way we obtain the plots depicted in Figure 1. Notice that in some graphics the running of two parameters is shown. In order to avoid confusion, in the online version the curve and the ordinate scale of a given parameter are drawn with the same color, while for different parameters the colors differs. In the printed black and white version, the dashed curves correspond to $\beta^2$ and $m_{\text{eff}}^2$, the dashed-dotted to $\alpha^2$ and the continous to $\lambda$ and $\omega^2$.

Turning to the interpretation of the plots, we observe that the couplings behave as expected: the noncommutativeness becomes more relevant at high energies, contrary to what happens to the curvature (or, analogously, noncommutativeness of the momenta), while the mass and the frequency decay fast. On its side, the coupling constant increases slowly but faster than in the commutative case, since its beta function is always greater than the commutative one. As a consequence, there occurs a Landau pole for $\lambda$.

However, the situation can change drastically if we consider different initial conditions. Suppose that we had begun with an anti-Snyder ($\beta^2 < 0$) and anti-de Sitter ($\alpha^2 < 0$) space, given by $v_2 = (1, 10, -10^{-5}, -10^{-1}, 10^{-1})$ and depicted in Figure 2.
Being the curvature and noncommutative contribution to $\lambda$’s beta function negative, the asymptotic freedom of the theory is guaranteed for large enough masses. Also, as can be seen from Figure 2, the curvature tends to zero for large energies as in the previous case – analogously, the frequency, the mass and the noncommutative parameter show a behaviour analogous to the previous case (of course with an additional minus sign for $\beta^2$). All in all, in this situation the only relevant parameter in the UV turns out to be the noncommutativeness parameter.

Now notice that, after the running, the constraint for $\omega$ given in (16) is no longer valid. This suggests that we could also relax the constraint in the initial condition and consider more general ones. For example, let us choose an initial anti-Snyder space, say $m_{\text{eff}} = 1$ and $\beta^2 = -10^{-4}$ in the corresponding units. Then, $\lambda$’s beta function would have a negative contribution that would be nevertheless suppressed by the fast decay of the mass. If we choose instead the situation with a bigger mass, e.g. choosing $v_3 = (1, 10^2, 10^{-5}, 10^{-4}, 10^{-1})$, the negative term in the beta function of $\lambda$ is dominant and the theory becomes again asymptotically free. The plots in Figure 3 (left) depict the behaviour of $\lambda$ and $\beta^2$ in this case, while the remaining parameters show a behaviour qualitatively equal to the previous $v_1$ case.

Still another interesting situation shows up if we still assign initial anti-Snyder conditions, but with $v_4 = (1, 10, 10^{-5}, -10^{-1}, 10^{-1})$ at $\mu_0$ (all quantities given in the corresponding powers of eV). In this case $\alpha^2$ decreases so fast that becomes negative—our interpretation is thus that the geometry of the model becomes of anti-de Sitter type as a consequence of the one-loop dynamics of the theory. At the same time, the coupling constant stays in an asymptotic-free regime. The only parameter that changes its behaviour with respect to the previous case is $\alpha^2$, whose plot is shown in the right panel of Figure 3.

### 6. Conclusions

The formulation of a scalar field theory on noncommutative curved spacetime has many interesting features. In particular, we have shown that the Snyder-de Sitter model could provide a geometric motivation for the harmonic term introduced by Grosse and Wulkenhaar in their celebrated model [35]. In this respect, the emergence of a frequency given by the quotient of the noncommutativeness and curvature parameter turns out to be even more interesting, since it means that its effects could be several orders of magnitude bigger than expected. Of course it will be of interest to
analyze how the current experimental data from the standard model could constrain the presence of such an harmonic term.

Turning our attention to the beta functions, we have seen that according to the initial infrared initial conditions, it could be the case that the curvature and the noncommutativity render the theory asymptotically safe. For this effect to arise, it is a necessary condition to begin with an anti-Snyder-anti-de Sitter or an anti-Snyder-de Sitter scenario. In these cases it is clear that the theory does not suffer the illness of a Landau pole. However, it is important to emphasize that this is a situation different from the one set by the GW model, in which the existence of a vanishing of a beta function guarantees asymptotic safety.

Another interesting peculiarity is the fact that we are forced to consider the running of both the noncommutativeness and curvature parameters, very much akin to what happens when one takes into account the one-loop matter gravitational contributions in the usual QFT on curved spaces. Its consequence is that an initial symmetry, encoded in the constraint $\omega^2 = \alpha^2 / \beta^2$, is dynamically broken.

After a deeper look one realizes that this fact is even more interesting than expected: it gives the opportunity to analyze some proposals discussed in the literature of quantum gravity and asymptotic safety, such as [36]. Indeed, we have shown that a running from de Sitter space to a would-be anti-de Sitter is admitted in our model after generalizing the initial conditions, suggesting that, even if the renowned de Sitter-Swampland conjecture is true, a way out of it could be possible.

Finally, we would also like to make a point regarding gravitational corrections to the beta functions of coupling constants. In some articles (see for example [37] and references therein), it has been proposed that this type of corrections are not physical by analysing QFT in curved spaces. Our model could be a playground for considering this claim in a theory that can be seen as a step further towards Quantum Gravity.

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