Soliton-impurity interaction in two coupled ferromagnetic chains

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Abstract. The propagation of soliton excitations in a system of two magnetic chains with impurities is studied. The inhomogeneous chains coupled through ferromagnetic interaction between the opposite spins are described by a Heisenberg model in the nearest-neighbor approximation. The presence of the impurities leads to linear perturbing terms in the evolutionary equations. We investigated numerically the influence of the soliton parameters, the interchain coupling and the defect strength on the soliton dynamics. The conditions of perfect soliton switching are obtained.

1. Introduction
The existence of solitons and their properties in magnetic systems have been subject of investigation for many years [1-7]. The most considered models are based on the one-dimensional spin Heisenberg chains. In the last years growing interest is devoted to the study of coupled spin chains (spin ladders) with different exchange interactions between them [8-11] as model to explain the magnetic and electronic structure of different experimentally discovered magnetic materials [12,13]. In a previous paper we have studied the dynamics of bright solitons in two coupled ferromagnetic chains [14]. We have obtained that the coupling between the two anisotropic ferromagnetic chains leads to linear and nonlinear terms in the dynamics equations. The condition for a perfect soliton switching in such a system was derived. We have observed that in contrast to the linear interchain coupling for the nonlinear coupling not only its value but also its sign plays a crucial role for the soliton dynamics.

Another interesting topic of research with practical importance is the dynamics of solitons in coupled parallel inhomogeneous chains [15-20]. In the present paper we study the interaction of bright solitons with impurities in two coupled anisotropic ferromagnetic chains.

2. Hamiltonian of the system
We consider two ferromagnetic Heisenberg chains of $N$ spins with magnitude $S$ described in the nearest-neighbor approximation by the following Hamiltonian:

$$
\hat{H} = -J \sum_{n=1}^{N} (\hat{S}_n \cdot \hat{S}_{n+1} + \hat{\sigma}_n \cdot \hat{\sigma}_{n+1}) - A \sum_{n=1}^{N} (|\hat{S}_n|^2 + |\hat{\sigma}_n|^2)
$$

$$
- \sum_{n=1}^{N} (\mu H_0 + \epsilon \delta_{n,n_0})(\hat{S}_n^z + \hat{\sigma}_n^z) - d \sum_{n=1}^{N} \hat{S}_n \cdot \hat{\sigma}_n,
$$

(1)
where $J > 0$ is the exchange integral and $A$ is the on-site anisotropy constant which can be positive (easy axis) or negative (easy plane). $H_0$ is the external magnetic field applied along the $z$-axis, so that in the ground state of the system all spins $\hat{S}_n$ and $\hat{\sigma}_n$ corresponding respectively to the first and the second chains are aligned in the $z$-direction, $\mu$ is the magnetic moment per spin. $d$ characterizes the coupling interaction between the two chains which can be ferromagnetic $(d > 0)$ or antiferromagnetic $(d < 0)$. The constant $\epsilon$ determines the strength of the impurity on site $n_0$, which can be attractive or repulsive. It corresponds to a change in the external magnetic field or in the magnetic moment.

We use for the scalar products in (1) the rule
\[
\hat{a} \cdot \hat{b} = \frac{\hat{a}^+ \hat{b}^- + \hat{a}^- \hat{b}^+}{2} + \hat{a}^z \hat{b}^z, \tag{2}
\]

\[
[\hat{S}_n^\pm, \hat{S}_j^\pm] = \mp \hat{S}_n^\pm \delta_{ij}, \quad [\hat{S}_n^\pm, \hat{S}_j^-] = 2\hat{S}_n^\pm \delta_{ij}, \quad [\hat{\sigma}_n^\pm, \hat{\sigma}_j^\pm] = \mp \hat{\sigma}_n^\pm \delta_{ij}, \quad [\hat{\sigma}_n^\pm, \hat{\sigma}_j^-] = 2\hat{\sigma}_n^\pm \delta_{ij}. \tag{3}
\]

All the other commutators are zero.

The equations of motion $i\dot{\hat{a}} = [\hat{a}, \hat{H}]$ for $\hat{S}_i^\pm$ and $\hat{\sigma}_i^\pm$ ($h = 1$) yield
\[
\pm i\dot{\hat{S}}_i^\pm = - J[\hat{S}_i^\pm (\hat{S}_{i-1}^\pm + \hat{S}_{i+1}^\pm) - \hat{S}_i^\pm (\hat{S}_{i-1}^\pm + \hat{S}_{i+1}^\pm)] + A(\hat{S}_i^\pm \hat{S}_i^\pm \hat{S}_i^\pm + d(\hat{S}_i^\pm \hat{\sigma}_i^\pm - \hat{S}_i^\pm \hat{\sigma}_i^\pm) + (\mu H_0 + \epsilon \delta_{nn_0})\hat{S}_i^\pm, \tag{4}
\]
\[
\pm i\dot{\sigma}_i^\pm = - J[\hat{\sigma}_i^\pm (\hat{\sigma}_{i-1}^\pm + \hat{\sigma}_{i+1}^\pm) - \hat{\sigma}_i^\pm (\hat{\sigma}_{i-1}^\pm + \hat{\sigma}_{i+1}^\pm)] + A(\hat{\sigma}_i^\pm \hat{\sigma}_i^\pm \hat{\sigma}_i^\pm + d(\hat{\sigma}_i^\pm \hat{\sigma}_i^\pm - \hat{\sigma}_i^\pm \hat{\sigma}_i^\pm) + (\mu H_0 + \epsilon \delta_{nn_0})\hat{\sigma}_i^\pm. \tag{4}
\]

In the semiclassical approximation valid for large values of $S$ the components of the spin operators are complex amplitudes, $\alpha_n = \hat{S}_n^+ / S$, $\alpha_n^* = \hat{S}_n^- / S$, $\beta_n = \hat{\sigma}_n^+ / S$, $\beta_n^* = \hat{\sigma}_n^- / S$ and $\hat{\sigma}_n^+ / S = \sqrt{1 - |\beta_n|^2}$. Then we have
\[
i\frac{\partial \alpha_n}{\partial t} = - JS \left( (\alpha_{n+1} + \alpha_{n-1}) \sqrt{1 - |\alpha_n|^2} - \alpha_n \left( \sqrt{1 - |\alpha_{n+1}|^2} + \sqrt{1 - |\alpha_{n-1}|^2} \right) \right) + 2AS \alpha_n \sqrt{1 - |\alpha_n|^2} - dS \left( \beta_n \sqrt{1 - |\alpha_n|^2} - \alpha_n \sqrt{1 - |\beta_n|^2} \right) + (\mu H_0 + \epsilon \delta_{nn_0}) \alpha_n, \tag{5}
\]
\[
i\frac{\partial \beta_n}{\partial t} = - JS \left( (\beta_{n+1} + \beta_{n-1}) \sqrt{1 - |\beta_n|^2} - \beta_n \left( \sqrt{1 - |\beta_{n+1}|^2} + \sqrt{1 - |\beta_{n-1}|^2} \right) \right) + 2AS \beta_n \sqrt{1 - |\beta_n|^2} - dS \left( \alpha_n \sqrt{1 - |\beta_n|^2} - \beta_n \sqrt{1 - |\alpha_n|^2} \right) + (\mu H_0 + \epsilon \delta_{nn_0}) \beta_n. \tag{5}
\]

The set of differential equations (5) describes our system.

3. Soliton solutions
We shall look for solutions in the form of amplitude-modulated waves
\[
\alpha_n(t) = \varphi_n(t)e^{i(kn - \omega t)}, \quad \beta_n(t) = \psi_n(t)e^{i(kn - \omega t)}, \tag{6}
\]
where $k$ and $\omega$ are the wave number and the frequency of the carrier waves (the lattice constant equals unity) and the envelopes $\varphi_n(t), \psi_n(t)$ are slowly varying functions of the position and time. In the continuum limit and for $\varphi^2, \psi^2 \ll 1$ equations (5) transform into the following
coupled modified nonlinear Schrödinger equations for the envelopes:

\[
i \left( \frac{1}{S} \frac{\partial \varphi}{\partial t} + 2J \sin k \frac{\partial \varphi}{\partial x} \right) = \left( \omega_0 + \epsilon \delta(x - x_0)S^{-1} - \omega S^{-1} \right) \varphi - J \cos k \frac{\partial^2 \varphi}{\partial x^2} + g |\varphi|^2 \varphi - \frac{d}{2} (2\varphi - |\varphi|^2 \varphi)
\]

\[
i \left( \frac{1}{S} \frac{\partial \psi}{\partial t} + 2J \sin k \frac{\partial \psi}{\partial x} \right) = \left( \omega_0 + \epsilon \delta(x - x_0)S^{-1} - \omega S^{-1} \right) \psi - J \cos k \frac{\partial^2 \psi}{\partial x^2} + g |\psi|^2 \psi - \frac{d}{2} (2\varphi - |\psi|^2 \varphi + |\psi|^2 \varphi),
\]

where

\[\omega_0 = \mu H_0 S^{-1} - 2g + d, \quad g = J(\cos k - 1 - A/J).\] (8)

\[|\alpha_n|^2 \quad |\beta_n|^2 \quad |\alpha_n|^2 \quad |\beta_n|^2 \quad (a) \quad (b) \quad (c) \quad (d) \quad (e) \]

Figure 1. Switching of the bound soliton-impurity solution (11) with \( J = S = 1, A = 1, L = 10 \) and \(|d| = 0.0314\) for \( \epsilon = -0.08 \) (a) and \( \epsilon = 0.08 \) (b). The time is in units of \( 10^3 / JS \).

Further we shall consider the case \( d = 0 \). For \( \epsilon = 0 \) the uncoupled homogeneous nonlinear equations (7) possess bright-soliton solutions of the form

\[\varphi(x, t) = \varphi_0 \text{sech} \frac{x - vt}{L}, \quad \psi(x, t) = \psi_0 \text{sech} \frac{x - vt}{L}\] (9)

with

\[\varphi_0^2 = \psi_0^2 = -\frac{2J \cos k}{gL^2}, \quad \omega = \omega_0 S - \frac{JS \cos k}{L^2}, \quad v = 2JS \sin k,\] (10)

which appear for \( gJ \cos k < 0 \). \( L \) and \( v \) are soliton’s width and velocity, respectively.
For the static inhomogeneous case \((k = v = 0 \text{ and } \epsilon \neq 0)\) the uncoupled nonlinear equations (7) possess bound soliton-impurity solutions of the form

\[
\varphi(x) = \psi(x) = \frac{2J}{AL^2} \text{sech} \left( \frac{|x - x_0|}{L} + \Delta \right), \quad \Delta = \text{Arth} \left( -\frac{\epsilon L}{2SJ} \right),
\]

which exists only for \(A > 0\).

For \(d \neq 0\) we shall investigate the evolution of a soliton which at the initial time \(t = 0\) is launched in one of the chains at the position \(n_0 = 500\). Figure 1 shows the evolution of a bound soliton-impurity state (11). We have obtained that the single peak solution \((\Delta > 0, \epsilon < 0, \text{attractive impurity})\) is stable and is transferred from one chain to the other and back [figure 1(a)]. The process is close to a perfect switch of the soliton in the linear case with a period \(t_0 = \pi/|d|\). The double peak solution which corresponds to a repulsive impurity \((\Delta < 0, \epsilon > 0)\) is not stable. It is split in two parts which can oscillate around the impurity or propagate with opposite velocities during the transfer process [figure 1(b)].

![Figure 1](image_url)

**Figure 1.** Scattering of a soliton \(\alpha_n\) launched at \(n_s = 450\) for \(k = 0.025\) from a repulsive defect with \(\epsilon = 0.01\) (a) and \(\epsilon = 0.03\) (b). All other parameters are the same as in figure 1.

### 4. Soliton scattering on the linear impurities

We shall investigate the evolution of a soliton with the form (9) which at the initial time \(t = 0\) is launched in one of the chains at the position \(n_s\) far enough from the impurity position \(n_0\)

\[
\alpha_n(0) = \frac{1}{L} \sqrt{\frac{2 \cos k}{A/J + 1 - \cos k} \text{sech} \frac{n - n_s}{L} e^{ikn}}, \quad \beta_n(0) = 0
\]

solving numerically the system (5). The simulations are carried out for large enough chains compared to the soliton width \(L\) and periodic boundary conditions. We consider anisotropy.
values for which $\varphi_0^2 \ll 1$. Then, the linear coupling between the chains is dominant in the system (7) and perfect soliton switching with the period $t_0$ can occur when the condition

$$\left| \frac{J \cos k}{2L^2d} \right| \ll 1$$

is fulfilled [14].

Now, when the soliton is perfectly transferred from one chain to the other, its scattering on the defects is like the soliton dynamics in one inhomogeneous chain. The evolution depends strongly on the initial soliton amplitude ($\varphi_0$), velocity ($v \sim k$) and the strength of the defect $\epsilon$.

First we consider slow solitons. In the case of repulsion $\epsilon > 0$ the soliton is either transmitted [figure 2(a)] or reflected [figure 2(b)].

The evolution is more complex in the case of attractive defects ($\epsilon < 0$, figure 3). For a fixed small initial velocities we observe between the regions of transmission and reflection regions where the soliton is trapped [figure 3(a)] or split in a trapped and a reflected part [figure 3(b)] depending on the defect strength.

Further we consider fast solitons. For large values of the velocity the soliton behaves like a wave and it is split in a transmitted and a reflected part. Their size depends on the defect strength and not on its sign (figure 4). We observe transmission [figure 4(a)], splitting [figure 4(b)] or reflection [figure 4(c)] of the initial soliton which in the same time is switched from one chain to the other. In all these cases the soliton-impurity interaction is not influenced by the coupling constant between the two chains.

5. Conclusion
Soliton dynamics in two inhomogeneous Heisenberg spin chains with ferromagnetic interaction between them is studied. We obtained that when the condition for perfect soliton switching is
Figure 4. Scattering of a soliton $\alpha_n$ launched at $n_s = 50$ for $k = 1.1$ from a defect with $|\epsilon| = 0.1$ (a), $|\epsilon| = 2$ (b) and $|\epsilon| = 10$ (c). The time is in units of $1/JS$. All other parameters are the same as in figure 1.

fulfilled the soliton-impurity interaction is not influenced by the coupling constant between the chains. The numerical investigation of the stability of bound soliton-defect solutions showed that the single-peak solution is stable, while the double-peak solution is unstable and easily destroyed.

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