Calculation of reinforced concrete plates with hole at long-term loading

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Abstract. The article is devoted to improving the calculation of reinforced concrete slabs, taking into account the deformation under prolonged load and spatial design work. As a reference for the calculation of the impact study is taken shape, size and location of the holes in the circuit boards of fracture, as well as the proposed expression for calculation of deflections for one of the cases. Use of the deformed scheme allows to develop a settlement unit, approximating to the actual calculation of the plates, as well as to achieve a significant reduction in materials. We consider the continuous slab, supported on a loop and a plate having a square hole in the center.

For practical basis for calculating the deflection plates, it was created in the design scheme of the complex system Ing + Gen_3dim 2014.

Key words: reinforced concrete slab, long loading, bending, hole, break the scheme.

1. Introduction

Overlapping in large-panel residential buildings in most cases is made of solid reinforced concrete slabs the size of a room or half of a structural cell supported along a contour or three sides and working in two directions.

The developed new method of calculating these plates takes into account their spatial work and allows to reduce the consumption of materials. In contrast to the existing one, the new method takes into account the change in the geometry of the plate during deformation and the increase in this connection of the arms and the moments of internal forces in the design sections due to deflections.

The positive influence of pinching plates in the platform joints is also taken into account, which is taken into account in calculations for the limiting states of the second group and allows in many cases to fully exploit the effect of spatial work in calculating the strength.

After the determination of the moments, all other calculations are carried out in full accordance with the operating [1]. With the new method, the strength of the calculation increases from 5 to 20%, depending on the ratio of spans and the relative flexibility of the plates. Accordingly, the consumption of steel can be reduced.

The developed method of calculation is based on the theory of limiting equilibrium.

2. Formulation of problem

The increase in spans of modern structures, the transition to high-strength materials and thin-walled sections with a limited height, the acceleration of the commissioning of structures leads to the fact that it is not uncommon for the structural possibilities and sections of reinforced concrete elements to be dictated by the rigidity conditions, i.e. calculation of the deformations makes it necessary to make corrections to the dimensions of the cross sections that satisfy the strength calculation requirements.

Deflections in reinforced concrete structures are of interest to us not only when calculating the second limiting state. The estimation of displacements is necessary for calculation on the first limiting condition...
(calculation of strength) at definition of internal efforts in statically indeterminate systems. This particularly applies to the calculation of reinforced concrete slabs weakened by holes.

As is known, the presence of holes does not introduce any fundamental features into the methods of calculating plates that are supported along the contour. To calculate plates loosen by holes, a number of formulas have already been proposed which, however, did not allow for a change in the geometry of the plates during their deformation under load. This approach leads to an underestimation of the bearing capacity and rigidity of the plates.

Thus, the adaptation of the deformed scheme calculation method to the plates weakened by the holes will allow to reduce the cross-section of the working re in comparison with the results of the techniques currently used by a wide range of specialists.

One of the directions of improving the calculation, developed recently, is the registration of plate deformations under load and spatial design work. The kinematic principle of the theory of limiting equilibrium is adopted as the initial basis for the new calculation method. Using a deformed scheme, it is possible to develop a calculating apparatus that approximates the results of calculations to the actual work of plates, and also to achieve a significant reduction in materials and money. Theoretical and physical prerequisites, effectiveness and reliability of the method are confirmed by the analysis of domestic and foreign experimental studies.

The effect of the shape, size and location of the hole on the fracture pattern of slabs was investigated, and expressions for the calculation of the bearing capacity for such cases were also proposed.

3. Method of solving the problem

In rectangular, freely supported plates, under the action of a uniformly distributed load, two points of intersection of the plastic hinges are formed [2]. Since such a plate has two symmetry axes, its destruction is also symmetrical and both centers of the fracture are located on an axis parallel to the large sides of the plate. Due to symmetry, the fracture scheme under consideration consists of two trapezoidal and two triangular elements. The diagram of the break of such a plate is shown in Fig. 1.

![Fig.1. Calculation scheme of a rectangular plate with a hole in the center at uniform load](image)

Figure 1 gives a diagram of the rates of mutual rotation of hard disks A, B, C, D around the reference and transit plastic hinges at the speed of translational displacement of the horizontal fracture line equal to 1.

The fracture line is assumed, but more likely when the hole is located in the center of the plate. Then the intensity of the destructive load for such a case of hole location:
\[ p = \frac{24[m_1(\gamma - \xi) + m_2]}{l_1^2[(3\gamma - 1) - 6\xi^2 + 3\xi^3]} \]  

(1)

where \( \gamma = \frac{l_1}{l_2} \) - the ratio of the sides of the slab, \( \xi \) - the coefficient characterizing the size of the slab hole; \( m_i \) - is the moment perceived by the reinforcement in the corresponding plastic hinge.

In the limiting state, the plate, depending on the ratio of the sides and the limiting moments in the spans, can have two types of kink circuits. The scheme of the first type has two centers, and the fracture line connecting them is parallel to the horizontal (Fig. 2). Triangular and trapezoidal elements of this scheme are the same, that is, the diagram of fracture is adopted unchanged [3].

![Fig.2. Calculation scheme of a rectangular plate with a hole displaced from the center of the plate (diagram of fracture 1).](image)

The proposed fracture scheme of the second type also has two centers connected by a horizontal fracture line. However, in contrast to the first type of fracture scheme, only triangular elements are identical here, and the trapezoid elements have different heights (Fig. 3) [4]. The real diagram of fracture is considered to be the one that corresponds to the smallest load-bearing capacity.

![Fig.3. The design scheme of a rectangular plate with a hole displaced from the center of the plate (diagram of fracture 2).](image)

With a given hole arrangement, the plate can have two types of diagram of fracture. The fracture scheme of the first type has two fracture centers, and the fracture line connecting them is parallel to the horizontal (Fig. 4).
Fig. 4. Calculation scheme of the plate with a hole located at the point of intersection of plastic hinges (diagram of fracture 1)

The diagram of the break of the second type (Fig. 5) is similar to the scheme of the first type with the only difference that the position of the line joining the centers of the fracture is given by a $V$ variable value.

Fig. 5. The design scheme of a plate with a hole whose center coincides with the point of intersection of the plastic hinges (diagram of fracture 2)

We determine the load-bearing capacity of the plate under consideration under uniform load for the diagram of fracture 1. For this case, the volume formed by the surface of the deformed plate with a horizontal plane in the support level will be:

$$V = \frac{1}{6} \left( l_2 (3l_2 - l_1) - (\varepsilon_1) (1 - \varepsilon) \frac{1}{3} \varepsilon_1 \frac{\varepsilon_1}{2} - \frac{1}{2} \varepsilon_2 \frac{\varepsilon_2}{2} \right) = \frac{l_2^2}{6} (3\gamma - 1) - 6\varepsilon^2 + 3.5\varepsilon^3.$$

The work of external forces, carried out by the load on the corresponding possible displacements:

$$A_{ewn} = \frac{q l_2^2}{6} \left( 3\gamma - 1 \right) - 6\varepsilon^2 + 3.5\varepsilon^3. \quad (2)$$

Define the work of internal forces on the same possible displacement. The reinforcement group, parallel to the short side, within the linear plastic hinge AC creates the work:
\[
\bar{m}_{11} \frac{4}{l_1} \left( t_2 - t_1 \right) - 0,5 \bar{h}_1 = 4m_{11}(y - 1 - 0,5 \xi). \tag{3}
\]

The same group of reinforcements on the plastic hinges AD, CD, AB, CB creates the work:

\[
\bar{m}_{12} \frac{4}{l_1} \left[ t_1 - 0,5 \bar{h}_1 \right] = 4m_{12}(1 - 0,5 \xi). \tag{4}
\]

A group of reinforcement parallel to the long side of the plate within the plastic hinges AD, CD, AB, CB creates the work:

\[
\bar{m}_{22} \frac{4}{l_1} \left[ t_1 - 0,5 \bar{h}_1 \right] = 4m_{22}(1 - 0,5 \xi). \tag{5}
\]

Equating the work of external and internal forces, we determine the intensity of the breaking load:

\[
q_{ul}^w = \frac{24 \cdot \left( m_{11}(y - 1 - 0,5 \xi) + m_{12}(1 - 0,5 \xi) + m_{22}(1 - 0,5 \xi) \right)}{l_1 \left( 3y - 1 - 6\xi^2 + 3,5\xi^3 \right)}. \tag{6}
\]

With increasing hole size, the intensity of the breaking load per unit area of the plate increases somewhat, and the total load on the whole plate decreases [5]. This directly affects the area of the reinforcement that fell into the hole section and the deflections of the plate. The main criterion limiting the opening of the hole is the observance of structural requirements, such as permissible deflections, and the conditions supported by the contour of a rectangular plate [6,7].

Let us consider a special case of calculation of deflections in a reinforced concrete slab with a hole under long-term loading.

The deflection in the limiting state is determined by the formula:

\[
f_{ul} = S \cdot \frac{1}{r_{ul}} l_1^2; \tag{7}
\]

where \( S \) - coefficient, depending on the design scheme and type of load:

\[
S = \{ 0,141 \text{ - for reinforcements of A-240, A-300, A-400 }, \text{ and as well, } A_{s2} 0,1 \text{-for reinforcement B500.} \}
\]

The curvature of the cross sections in the limiting state by strength is determined by the formula:

\[
\frac{1}{r_{ul}} = \frac{2 \cdot R_{s1}}{h_{01} \cdot E_{s1}} \left( 1 + \frac{1,2 \mu \alpha}{\tilde{e}_{ul}} \right); \tag{8}
\]

where \( E_{b1} \) - is the modulus of deformation of compressed concrete, is assumed to be:

long-term loading

\[
E_{b1} = \frac{E_b}{1 + \varphi_{b,cr}}; \tag{9}
\]

where \( \varphi_{b,cr} \) is the creep coefficient of concrete, taken as a function of the relative humidity of the air and the class of concrete [8];

\( \alpha \) - is the ratio of the modulus of elasticity of reinforcement and concrete: \( \alpha = \frac{E_{s1}}{E_b} \).

The reinforcement factor \( \mu \) is determined by the formula:

\[
\mu = \frac{1}{2} \left( \frac{A_{s1}}{h_{01} \cdot l_2} + \frac{A_{s2}}{h_{02} \cdot l_1} \right). \tag{10}
\]
The relative depth of the compressed zone $\xi_{ul}$, in the ultimate state of strength, is determined by the formula:

$$\xi_{ul} = 0.1 + 0.5 \cdot \mu \cdot \frac{R_{s1}}{R_b}$$  \hspace{1cm} (11)$$

From the reinforcement required for the calculation for a solid plate without a hole, exclude the bars, intersected by the contours of the hole (see Fig. 6):

Fig.6. Plate scheme with a square hole in the center

$$A_{s1}^0 = A_{s1} - 1.101 \cdot l_{1hoi}$$; \hspace{1cm} (12)

$$A_{s2}^0 = A_{s2} - 1.102 \cdot l_{2hoi}$$; \hspace{1cm} (13)

In the future, the strength of the plate with the hole is checked.

4. Calculation and results

Initial data[9]. The plate is made in a horizontal position. Concrete - heavy class B25 with design characteristics (MPa): $R_b=14.5$; $R_{bt}=1.05$; $R_{b,ser}$ ($R_{n}$) =18.5; $R_{bt,ser}$ ($R_{bt,n}$) = 1.55; $E_{b1}=30 \cdot 10^3$. Working reinforcement in the direction $L_1$ 19 - Ø 8 mm class A400 ($A_{s1} = 9.44$), in the direction $L_2$ 10 - Ø 5 mm class B-I ($A_{s2} = 1.78$) with design resistances (MPa): $R_{s1} = 355$; $R_{s2} = 360$; $R_{s,ser_1}$ ($R_{s,n}$) = 390; $R_{s,ser_2}$ ($R_{s,n}$)= 395; $E_{s1}=2.0 \cdot 10^5$; $E_{s2}=1.8 \times 10^5$. Protective layer of concrete to the bottom of the reinforcement 20mm. The reinforcement of both directions is uniformly distributed, $\varphi_c = 1$.

Checking the strength of the plate according to the deformed scheme is made by the formulas (7 - 11):

$$\mu = \left( \frac{9.44}{13.6 \cdot 642} + \frac{1.78}{12.9 \cdot 292} \right) = 0.00078;$$

$$\alpha = \frac{2 \cdot 10^5}{8572} = 23.33;$$

$$E_{b1} = \frac{3 \cdot 10^4}{1+2.5} = 8572;$$

$$\xi_{ul} = 0.1 + 0.5 \cdot 0.00078 \cdot \frac{355}{14.5} \cdot 0.11;$$

$$\frac{1}{r_{ul}} = 2 \cdot 355 \cdot 13.6 \cdot 2 \cdot 10^5 \left( 1 + \frac{1.2 \cdot 0.00078 \cdot 23.33}{0.11} \right) = 31.3 \cdot 10^{-5} \frac{1}{cm};$$

$$f_{ul} = 0.141 \cdot 31.33 \cdot 10^{-5} \cdot 292^2 = 3.76 \text{cm.}$$

If the short-term loading $E_{b1} = 0.85 \cdot E_b$, the value of the strain modulus is greater than $E_{b1} = 25.5 \cdot 10^3$ than with long-term loading.

$$\alpha = \frac{2 \cdot 10^5}{25.5 \cdot 10^3} = 7.84.$$
We calculate the value of deflections for short-term loading:
\[ f_{ul} = 0.141 \cdot 18.87 \cdot 10^{-5} \cdot 292^2 = 3.25 \text{ cm}. \]
Consequently, under long-term loading, the deflection value is greater than for short-term loading:
\[ f_{ul} = 3.25 \text{ cm} < f_{ul} = 3.76 \text{ cm}. \]
Selection of operating reinforcements in a plate with a hole shown on Fig. 7.

![Fig. 7. Reinforcement of the plate with a hole](image)

By the formulas (12 - 13):
\[ A_{o1} = 9.44 - \frac{9.44}{292} \cdot 1 = 6.2 \text{ cm}^2; \]
\[ A_{o2} = 1.78 - \frac{1.78}{642} \cdot 1 = 1.5 \text{ cm}^2; \]
We check the strength of the plate with the hole in the deformed scheme:
\[ \mu = \frac{1}{2} \left( \frac{6.2}{13.6} + \frac{1.50}{292} \right) = 0.00111; \]
\[ \alpha = \frac{2 \cdot 10^5}{8572} = 23.33; \]
\[ E_b = 3 \cdot 10^4; \]
\[ \xi_{ul} = 0.1 + 0.5 \cdot \frac{355}{14.5} = 0.1136; \]
\[ \frac{1}{r_{ul}} = \frac{2 \cdot 355}{141.2 \cdot 10^5} \cdot \left( 1 + \frac{1}{\frac{0.1136}{12.35}} \right) = 32.08 \cdot 10^{-5} \frac{1}{\text{ cm}}. \]
\[ f_{ul} = 0.141 \cdot 32.08 \cdot 10^{-5} \cdot 292^2 = 3.86 \text{ cm}. \]
In the plate with a hole, relatively solid plate, we observe an increase in deflection:
\[ f_{ul} = 3.76 \text{ cm} < f_{ul} = 3.86 \text{ cm}. \]
As the hole is enlarged, the reinforcement will grow, following which the deflections will increase.
For the comparative analysis, models of fixed plates and a plate with a hole were created in the program Ing + Gen_3dim, with a long-term loading (see Fig. 8) Concrete class B25. The deformation modulus was adopted for long-term loading \( E_b = 8572 \text{ MPa}. \) The plates are evenly distributed load \( q = 10.38 \text{ kPa}. \)
\[ \text{Min } Mr = -0.0285019 \text{ kNm/m}, \text{ Min } Mr = -0.0319575 \text{ kNm/m}, \]
\[ \text{Max } Mr = 10.0979 \text{ kNm/m} \text{ Max } Mr = 14.455 \text{ kNm/m} \]
Fig. 8. Schemes of isometric of moments for plates

In Fig. 9 is a diagram of isometric by deflections

![Diagram of isometric by deflections](image)

a) Solid plate
   \[ U_z = 3.23246 \text{cm} \]

b) Plate with hole
   \[ U_z = 3.53855 \text{cm} \]

Fig. 9. Diagram of isometric by deflections

Comparison of the results of the analytical method of calculation, according to the method described, with the data of the finite element method shows satisfactory convergence, the discrepancies do not exceed 8-12%.

5. Conclusions

1. Determination of the breaking load of reinforced concrete slabs is effected by the influence of the dimensions, shapes, locations of the holes and schemes of the fracture.
2. Comparative analysis of the bearing capacity and deflections according to the design schemes is shown taking into account the different arrangement of the holes in the plate.
3. The results of schemes of the fracture and deflections of reinforced concrete slabs at different locations of the hole are given.

References

[1] SP 52-01-2003. Concrete and reinforced concrete structures. Basic provisions. - M.: GUP TSPP, 2004. - 24 p.
[2] Korobko V.I., Chernyaev A.A. About calculation of the average thickness of the plates stringency conditions. / Regional architecture and engineering. 2012. №1. P.83-89.
[3] Zinoviev R.V., Zinoviev N.F., Frakter A.M. Concrete slabs with holes. -Moscow: Stroyizdat 1975.
[4] Nowak, A. S. Reliability of Structures, McGraw-Hill / A.S.Nowak, K.R.Collins.—New York, 2000. – 397 p.
[5] Karpenko, N.I. *General mechanics model of reinforced concrete* / Karpenko N.I.-M.: Stroyizdat, 1996. - 416p.

[6] Tamrazyan, A. *Searching method of optimization of bending reinforced concrete slabs with simultaneous assessment of criterion function and the boundary conditions* / A. Tamrazyan, E. Filimonova // Applied Mechanics and Materials / International Conference on Materials Science and Mechanical Engineering: conference proceedings. - Kuala Lumpur, Malaysia, 2013.

[7] Tamrazyan, A.G., Filimonova E.A. *Evaluation of the bearing capacity and rigidity of the monolithic concrete slab with defects*. XV Scientific and Methodical Conference "Defects buildings. Strengthening of building structures. "VITI. March 24, 2011. St. Petersburg, Russia.

[8] Tamrazyan, A.G. *About the impact of reducing the hardness of concrete slabs on the bearing capacity under long-term stress* / A.G. Tamrazyan, E.A. Filimonova // PGS.- 2012.- №7.- P.30-32.

[9] Zyryanov, V.S. *Guidelines for calculation and design of solid slabs of large-panel buildings*. Moscow (TSNIIEPzhilishcha), 1989.