The New Quantum Logic

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Abstract

It is shown how all the major conceptual difficulties of standard (textbook) quantum mechanics, including the two measurement problems and the (supposed) nonlocality that conflicts with special relativity, are resolved in the consistent or decoherent histories interpretation of quantum mechanics by using a modified form of quantum logic to discuss quantum properties (subspaces of the quantum Hilbert space), and treating quantum time development as a stochastic process. The histories approach in turn gives rise to some conceptual difficulties, in particular the correct choice of a framework (probabilistic sample space) or family of histories, and these are discussed. The central issue is that the principle of unicity, the idea that there is a unique single true description of the world, is incompatible with our current understanding of quantum mechanics.

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1 Introduction

The conceptual difficulties of standard quantum mechanics, defined as what one finds in standard textbooks, are encountered by students in their very first course in the subject. Part of the problem is unfamiliar mathematics, but even when the mathematics has been (more or less) mastered a serious problem remains in relating the mathematical formalism to some physical understanding of quantum systems. And it is no wonder that students are having difficulty if even textbook writers do not really understand the subject, and are sometimes bold enough to admit it \[1\]. Are there some basic principles which are missing from the textbooks, ideas which were they included therein would clear up quantum paradoxes?

The thesis of this paper is that there are such basic principles, which have been around in some form or another for nearly thirty years, and they deserve to be more widely known. Although ignored in much of the current literature, the (consistent or decoherent) histories approach appears capable of resolving every major conceptual difficulty of quantum mechanics, not least the infamous measurement problem. To be sure, the histories approach has not been completely ignored; a small but distinguished group of critics—who may possibly outnumber the advocates—have not hesitated to point out what they consider serious flaws; see the references given in \[2\]. One of the more generous of these critics, N. David Mermin, expressed what has probably troubled many others when he made the following comparison with special relativity, see p. 281 of \[3\] or p. 16 of \[4\]:

\[
\text{[But]} \ I \ am \ disconcerted \ by \ the \ reluctance \ of \ some \ consistent \ historians \ to \ acknowledge \ the \ utterly \ radical \ nature \ of \ what \ they \ are \ proposing. \ The \ relativity \ of \ time \ was \ a \ pretty \ big \ pill \ to \ swallow, \\
\text{but \ the \ relativity \ of \ reality \ itself \ is \ to \ the \ relativity \ of \ time \ as \ an \ elephant \ is \ to \ a \ gnat.}
\]

What the consistent historians are proposing is indeed radical, which should surprise no one familiar with Feynman’s famous remark that “no one understands quantum mechanics” (p. 129 of \[5\]). It occurs in a context where Feynman makes it abundantly clear that he considers quantum theory much more difficult than special relativity, though he does not quantify this by means of a zoological analogy. The historians’ proposal to revise some of the rules of reasoning which before the arrival of quantum mechanics were thought to apply universally, both in scientific reasoning and in everyday human affairs, is obviously more difficult to accept than the move from pre-relativistic to relativistic physics. It is much more like the transition our intellectual ancestors made when they abandoned the notion that the earth is motionless at the center of the universe in favor of the radical proposal that it moves around the sun as well as spinning around its axis. The question physicists should be asking is not whether the ideas in the consistent histories approach are radical, but rather whether they are internally consistent, and whether they genuinely resolve the serious conceptual issues which have beset quantum theory ever since its development in the mid 1920s.

However radical it may seem, the idea that quantum theory requires a new mode of reasoning is itself not at all new. In 1936, just four years after the appearance of von Neumann’s famous book, Birkhoff and von Neumann \[6\] published their proposal for a quantum logic as a replacement in the quantum domain for ordinary propositional logic. Through the years there has been a continuing, albeit modest, research effort attempting to develop quantum logic in hopes that it would lead to a solution of the quantum conceptual difficulties. Despite some early enthusiasm, e.g. \[7\], this program has not made a great deal of progress; for some discussion of the current situation see \[8,9\]. It may be that we physicists are simply not smart enough to reason in this fashion, and the quantum mysteries will have to rest until the day when superintelligent robots (with access to quantum computers?) can make sense of the quantum world. But will they be able (or even want to) explain it to us?

What is here called the new quantum logic has the same motivation and shares important ideas with the proposal of Birkhoff and von Neumann. It is in some respects a less radical break with conventional reasoning than the older quantum logic, and has turned out to be much more useful in terms of allowing human beings, including college seniors and beginning graduate students, to understand the quantum world in a consistent and coherent way. The present paper is devoted to explaining how this approach resolves the major conceptual problems of standard quantum mechanics listed in Table \[1\] and discussed in some detail in Sec. \[2\]. Following that, Sec. \[3\] summarizes the histories approach and how it addresses these difficulties. Next, Table \[2\] lists and Sec. \[4\] discusses various conceptual problems raised by the histories approach itself. A brief conclusion follows in Sec. \[5\].


2 Quantum Conceptual Difficulties

Table I is a list of major conceptual difficulties of standard quantum mechanics; that is, the treatment currently found in most textbooks. These are topics which have given rise to a lengthy and continuing discussion in the quantum foundations literature. While no such list can claim to be complete, the author believes that most of the significant interpretational problems fall in one or another of these categories.

Table 1: Major Conceptual Difficulties of Quantum Mechanics

| 1. Meaning of the wave function |
|-------------------------------|
| a. Ontological                |
| b. Time development           |
| c. Epistemological            |
| 2. Measurements               |
| a. Outcomes (pointer states)   |
| b. What was measured?         |
| c. Wave function collapse     |
| 3. Interference               |
| a. Particle vs. wave          |
| b. Delayed choice             |
| 4. Locality                   |
| a. Bell inequalities          |
| b. GHZ and Hardy              |

2.1 Meaning of a wave function

Students are taught that a wave function or wave packet or ket in the quantum Hilbert space is analogous to a point in a classical phase space; e.g., for a single particle it contains information about both the position and the momentum. Wave packets have a unitary time development governed by the Hamiltonian through Schrödinger’s equation, and under suitable circumstances the wave packet can be seen to “move” somewhat like a a classical particle—one thinks of the well-known Ehrenfest relations. Thus it is rather natural to conclude that the wave function represents whatever it is in the quantum world that is “really there,” a beable in Bell’s terminology [10]. Let us call this the ontological perspective.

There are other circumstances in which a wave function seems to play a different role. It can be used to calculate probabilities of the outcomes of a measurement. Students learn that the process of measurement makes a wave function collapse. This can, it seems, take place instantly, which for a wave function with significant extension in space might violate special relativity. It is certainly contrary to the unitary continuous time development induced by Schrödinger’s equation. Probabilities can be instantly updated according to new knowledge, and relativity theory need not be violated in such updating. So if a wave function is just a means of calculating the probability of something, there is no reason why it should not suddenly change. Such is the epistemic understanding of wave functions: rather than actually representing the physical state of affairs they only provide information about a quantum system. But what is this information about? Presumably quantum theory is able in principle, even if in practice the calculations may be very difficult, to tell us something about what is going on in systems which have been probed experimentally leading to the conclusion that classical mechanics is not an adequate representation of atomic systems. Is there something really there, the way experimentalists seem to think, and if so how is it related to the wave function? And how is discontinuous time development related to Schrödinger’s equation?

Reconciling the ontological and epistemic points of view is a serious conceptual problem, and it does not disappear when one replaces a wave function with a density operator, somewhat analogous to a classical
probability distribution. A classical distribution provides a probability of something definite, which either
occurs or does not occur. But what is the referent of a quantum probability distribution?

2.2 Measurements

Measurements play a central role in textbook expositions of quantum theory. The students are suspicious,
and rightly so. After all, the measuring apparatus is itself constructed from a large collection of atoms
whose behavior is governed by quantum laws, and therefore it should be possible, at least in principle, to
describe the entire measuring process, both the system being measured and the measuring apparatus, in fully
quantum mechanical terms. Providing an adequate and fully quantum description constitutes the infamous
measurement problem of quantum foundations.

There are actually two distinct measurement problems. The first, the one most often discussed in the
foundations literature, has to do with the fact that a measuring process amplifies microscopic differences
in such a way as to make them macroscopic. In the dated but picturesque terminology of this field, these
difference are ultimately revealed through different positions occupied by a macroscopic pointer that indicates
the measurement outcome. When unitary time evolution is applied to both the microscopic system being
measured and the apparatus the result will often be a quantum wave function which is a superposition of
different macroscopic pointer positions. (See Sec. 3.5 below for a particular measurement model.) How is
such a quantum state, nowadays often called a Schrödinger cat, to be understood? Is the pointer oscillating
back and forth between different positions unable, so-to-speak, to make up its mind? Superficial invocations
of decoherence do not really resolve the problem [11].

If one can somehow get the pointer to stop wiggling and settle down in a definite position, the second
measurement problem remains: how is this position related to the microscopic state of affairs that the
apparatus was designed to measure? Experimenters typically claim that the outcomes of their experiments
tell them something about a prior state of affairs. E.g., a gamma ray was detected coming form a decaying
nucleus, or a neutrino from a distant supernova was detected by the apparatus. This seems directly contrary
to the claim found in some textbooks that measurements tell one nothing about what was there before the
measurement took place.

The third item under the measurement heading in Table I wave function collapse, has already been
mentioned in Sec. 2.1. Obviously, an adequate and fully quantum mechanical description of the measurement
process should provide some insight into why collapse can be a useful epistemic perspective, or else replace
it with something else which will accomplish the same purpose.

2.3 Interference

Double-slit interference leads to a well-known paradox in which one must understand a quantum particle
as a wave that is sufficiently delocalized that it can in some sense pass through both of the slits in order to
produce an interference pattern. However, if detectors are placed immediately behind the slits only one, not
both, will be triggered, indicating that the particle passed through only one slit. And despite its wavelike
character the particle can arrive at a quite specific location in the interference region. Feynman’s superb
discussion in Ch. 1 of [12] can be recommended to any reader who has not yet encountered it. A very similar
paradox occurs in a Mach-Zehnder interferometer where a photon must in some sense be moving through
both arms in order to produce the expected interference at the second beam splitter, whereas a measurement
inside the interferometer will detect the photon in just one arm, not both. The paradox is even more striking
in Wheeler’s delayed choice version [13], where the final beam splitter in the Mach-Zehnder is either left in
place or else suddenly removed at a time when the photon has already passed through the first beam splitter.
In yet another version [14] the fact that one arm of the interferometer is blocked can seemingly be detected
by a photon which passes through the other arm a long distance away.

2.4 Locality

Bell inequalities [15] apply to a situation where two quantum particles, typically two photons, are
prepared in an entangled state and various measurements are used to determine the statistical correlations
of some of their properties. By making certain assumptions about the presence of physical properties in the
particles before measurement, and assuming locality, which is that influences only travel at a finite speed
from one point to another, Bell derived certain inequalities these correlations should satisfy. The inequalities
are violated by the predictions of quantum mechanics, and numerous experiments of increasing precision all agree with quantum theory and disagree with Bell’s inequalities. This has convinced most physicists that one or the other of Bell’s assumptions must be wrong. Hardy’s paradox [17], which also applies to correlations of properties of two separated particles, is to some degree more straightforward than Bell’s work as it is easier to see that quantum predictions for the correlations, again in accord with experiments, contradict what one might naively expect if measurements reveal prior properties. The paradox of Greenberger, Horne, and Zeilinger [18, 19] is similar to Hardy’s and preceded it in time, but refers to three particles rather than two. Once again, the predictions of quantum mechanics are supported by experiment, suggesting that something must be wrong with the reasoning that leads to these paradoxes.

The claim has often been made that the only reasonable conclusion to be drawn from the experimental violation of Bell inequalities and these other paradoxes is that the quantum world must be nonlocal, and allow for instantaneous interactions or influences between spatially separated systems, even in situations where this conflicts with special relativity. However, even those who believe in the existence of such influences agree that they cannot be used to transmit information, which conveniently makes them experimentally unobservable.

3 The New Quantum Logic

In resolving the conceptual difficulties listed in Table 1 the histories approach uses two main tools. The first is the new quantum logic, employed in Sec. 3.1 to discuss physical properties of a quantum system at a single instant of time, and extended to probabilities in Sec. 3.2. The second is stochastic time development, the subject of Sec. 3.3. Together these provide a way to understand the dynamics of macroscopic systems using quasiclassical frameworks, Sec. 3.4 and thus resolve both measurement problems, as discussed in Sec. 3.5. Interference and the locality of quantum mechanics are taken up in Secs. 3.6 and 3.7. Some brief remarks on approximations in Sec. 3.8 complete the discussion of the new logic and how it resolves quantum paradoxes.

3.1 Properties

The new quantum logic shares with its older counterpart a very fundamental idea, consistent with but seldom sufficiently emphasized in quantum textbooks. A quantum physical property, something which can be true or false—such as “the energy is between 2 and 3 J”—is represented in quantum mechanics by a (closed) subspace $P$ of the quantum Hilbert space or, equivalently, by the projector $P$ (orthogonal projection operator) onto this subspace. Subspaces (or their projectors) represent the quantum ontology, they are the mathematical counterparts of Bell’s “beables.” Note that a wave packet or any nonzero ket $|\psi\rangle$ in the Hilbert space corresponds to, or generates, a one-dimensional subspace consisting of all its multiples: kets of the form $c|\psi\rangle$, where $c$ is any complex number. When $|\psi\rangle$ is normalized we denote the projector onto this subspace by $[\psi] = |\psi\rangle\langle\psi|$. Used in this way a ket or wave function has an ontological meaning. However, kets can also play an epistemological role, as discussed below in Secs. 3.2 and 3.3. It is important to note that subspaces of dimension greater than one also represent quantum properties.

A classical phase space $\Gamma$ with a point in the phase space $\gamma$ representing the actual physical state, provides a useful analogy for the quantum Hilbert space. A collection of points $P$ in $\Gamma$ represents a classical property, and this property is true for a given system if the point $\gamma$ representing its physical state is in the set $P$. There is a one-to-one correspondence between the subset $P$ and the corresponding indicator function $P(\gamma)$, equal to 1 for $\gamma \in P$ and 0 otherwise. This classical indicator is thus analogous to a quantum projector, whose eigenvalues are 1 and 0.

In addition, classical mechanics employs various physical variables represented by real-valued functions on the phase space; e.g., energy, momentum, angular momentum. In quantum mechanics a physical variable, referred to as an observable, is represented by a Hermitian operator which can be written in the form

$$A = \sum a_j P_j, \quad P_j = P_j^1 = P_j^2, \quad \sum P_j = I,$$

where each eigenvalue $a_j$ of $A$ occurs but once in the sum: $j \neq k$ implies $a_j \neq a_k$. Here the $\{P_j\}$ are a collection of projectors that form a projective decomposition of the identity operator $I$, sometimes called a projector-valued measure or PVM. The property that the physical variable $A$ takes on or possesses the
value $a_j$, thus $A = a_j$, corresponds to the projector $P_j$ or, equivalently, the subspace $\mathcal{P}_j$ onto which $P_j$ projects. While measurement will be discussed in more detail below in Sec. 5.1, it is worth remarking that in quantum mechanics the measurement of an observable is the same thing as measuring the corresponding decomposition of the identity, determining which property $\mathcal{P}_j$, equivalently $P_j$, is, in fact true.

The negation of a classical property $\mathcal{P}$ corresponds to the complementary subset $\mathcal{P}^c$ in the phase space: the set of points in $\Gamma$ which are not in $\mathcal{P}$. Its indicator, $\mathcal{P}^c$ or $-P$, is $I - P$, where $I$ is the identity: $I(\gamma) = 1$ for every $\gamma \in \Gamma$. Von Neumann proposed that in quantum theory the negation $-P$ should be represented not by the set-theoretical complement of the corresponding Hilbert subspace, but instead by its orthogonal complement, the collection $\mathcal{P}^\perp$ of all kets which are orthogonal to every ket in $\mathcal{P}$. This is a subspace with projector $I - P$, where $I$ is the identity operator on the Hilbert space. This identification is consistent with textbook quantum mechanics, though often it is not properly discussed.

Let us consider two examples. For the two-dimensional Hilbert space representing a spin-half particle the one-dimensional subspaces corresponding to the orthogonal kets $|z^+\rangle$ and $|z^-\rangle$ (denoted by $|0\rangle$ and $|1\rangle$ in quantum information theory), “spin up” and “spin down”, are associated with the projectors $[z^+] = |z^+\rangle\langle z^+|$, $[z^-] = |z^-\rangle\langle z^-|$. (2) As they sum to $I$ they are negations of each other, and together form a projective decomposition of the identity. If the spin is not “down” it is “up”, consistent with what Stern and Gerlach observed in their famous experiment. If $S_z = +1/2$ (in units of $\hbar$) is true, then $S_z = -1/2$ is false, and vice versa. In the case of a quantum harmonic oscillator with energy eigenstates $|n\rangle$, eigenvalues $(n + \frac{1}{2})\hbar\omega$, and projectors $|n\rangle\langle n|$, “the energy is less than $2\hbar\omega$” is represented by the projector $P = |0\rangle + |1\rangle$, and its negation, “the energy is greater than $2\hbar\omega$”, by the projector $I - P = |3\rangle + |4\rangle + \cdots$. We shall return to these examples later.

With reference to quantum properties and their negations the old and the new quantum logic are identical. The difference begins to emerge when one considers the conjunction $P \land Q$, “$P$ AND $Q$”, of two properties. Birkhoff and von Neumann defined it using the set-theoretical intersection $\mathcal{P} \cap \mathcal{Q}$ of the two subspaces, itself a (closed) subspace. This has a precise analog in the classical phase space, where the conjunction of two properties is represented by the intersection of the two subsets $\mathcal{P}$ and $\mathcal{Q}$. However, if one follows the analogy of indicator functions and projectors there is an important difference. The indicator for a classical conjunction $P \land Q$ is the product $PQ$ of the indicators. But in the quantum case the product $PQ$ of two projectors is a projector if and only if $PQ = QP$, i.e., the two projectors commute, in which case the projectors, or the corresponding quantum properties, are said to be compatible. For compatible properties the product $PQ$ of the projectors projects on the subspace $\mathcal{P} \cap \mathcal{Q}$. However, when $P$ and $Q$ do not commute, that is, the quantum properties or their projectors are incompatible, neither $PQ$ nor $QP$ is a projector, and there is no simple relationship between either $PQ$ or $QP$ and the projector onto the subspace $\mathcal{P} \cap \mathcal{Q}$. The new quantum logic differs from the old in that it does not define $P \land Q$ when the projectors do not commute; the expression “$P \land Q$” in such a case is meaningless: quantum mechanics does not assign it a meaning.

The old quantum logic defines the disjunction “$P$ OR $Q$” (or both), $P \lor Q$, as the span of $\mathcal{P} \cup \mathcal{Q}$, the union of the two collections of kets. (Note that $\mathcal{P} \cup \mathcal{Q}$ is in general not a subspace.) The indicator for the classical property $\mathcal{P} \cup \mathcal{Q}$ is $P + Q - PQ$, and same expression works for the quantum projector when $PQ = QP$, but not otherwise. Again, the new quantum logic only defines the disjunction $P \lor Q$ when $P$ and $Q$ commute Otherwise the disjunction is undefined, thus meaningless.

The prohibition of conjunctions and disjunctions when $P$ and $Q$ do not commute is an application of the single framework rule, which plays a central role in the new quantum logic, and about which more will be said in Sec. 5.2. This prohibition is a syntactical rule governing the combination of meaningful expressions (propositions or properties) to form other meaningful expressions. Thus, for example, in ordinary logic the combination $P \land \neg Q$ has no meaning because it has not been constructed according to the rules for meaningful expressions. In a similar way the new logic forbids the combinations $P \land Q$ and $P \lor Q$ when $PQ \neq QP$; they are meaningless. Note the difference between a statement which is meaningful but false and one which is meaningless. The negation of a false statement is true, whereas the negation of a meaningless statement is equally meaningless.

The spin half example introduced above in (2) provides a useful illustration. If we define

$$|x^+\rangle = (1/\sqrt{2})(|z^+\rangle + |z^-\rangle), \quad |x^-\rangle = (1/\sqrt{2})(|z^+\rangle - |z^-\rangle),$$

with $|x^+\rangle$ and $|x^-\rangle$ the projectors for the properties $S_x = \pm 1/2$, it is easily checked that these do not commute with $|z^+\rangle$ and $|z^-\rangle$. In the old quantum logic the statement “$S_z = +1/2$ AND $S_x = +1/2$”
corresponds to the zero operator, which is always false, and consistent with this its negation \( S_z = -1/2 \) OR \( S_z = -1/2 \)" is always true. This last does not seem to make much physical sense, and will indeed lead to a contradiction if one follows the rules of ordinary reasoning—see Sec. 4.6 of [20]. Consequently, as Birkhoff and von Neumann [8] pointed out, it is necessary to modify the rules of ordinary reasoning, by removing the distributive laws, in order to construct a quantum logic free of contradictions. By contrast, in the new quantum logic, since \( S_z = +1/2 \) AND \( S_z = +1/2 \) is meaningless, its negation is equally meaningless, and thus no contradiction arises. The approach in textbook quantum theory is to say that \( S_z \) and \( S_z \) cannot be simultaneously measured. This is quite true, and one wishes that the textbooks would go on and state the reason for this: even the most skilled experimental physicists cannot measure that which does not exist!

The new quantum logic is in a sense a "subset" or restricted part of the old quantum logic, as the former accepts only a special collection of the formulas which are valid (constructed according to syntactical rules) for the latter. What is gained by adding this restriction is the ability to use the ordinary rules of reasoning—and, as discussed below, the ordinary rules of probability—to understand the quantum world without encountering contradictions and paradoxes.

### 3.2 Probabilities and the single framework rule

In the histories interpretation the time development of a quantum system is a stochastic process. Always, not just when measurements are being made. Furthermore, the probabilities in question obey the standard rules of probabilistic reasoning found in textbooks on probability theory (and which ought to be found in quantum textbooks). Three things are needed: a sample space \( S \) of mutually exclusive possibilities, one and only one of which is true, a Boolean event algebra \( \mathcal{E} \), and a probability measure \( \mathcal{M} \) that assigns probabilities to the elements of \( \mathcal{E} \). For present purposes we do not need sophisticated concepts. A finite, or at most countable, sample space will do very well, and \( \mathcal{E} \) can consist of all the subsets of \( S \), including \( S \) and the empty set.

A quantum sample space is always a projective decomposition of the identity, and any such projection can serve as a sample space. The event algebra consists of all projectors which are sums of some of the projectors in the sample space, including the zero projector \( 0 \) and the identity \( I \). Since the elements of a projective decomposition of the identity commute with each other, so do all the projectors in \( \mathcal{E} \). The term framework will be used either for \( S \) or \( \mathcal{E} \); given the close relationship between the two this ambiguity should not matter. When a distinction is important the elements of \( S \) will be called elementary events or projectors.

Two frameworks with sample spaces \( \{P_j\} \) and \( \{Q_k\} \) are said to be compatible if every \( P_j \) commutes with every \( Q_k \); otherwise they are incompatible. One arrives at exactly the same definition using projectors belonging to the two event algebras: either they all commute (compatible) or some do not (incompatible). In the compatible case there is always a smallest common refinement of the two frameworks with a sample space consisting of all nonzero products of the form \( P_j Q_k \), with duplicates eliminated. The event algebra of the refinement includes all the projectors in the two original event algebras. Two incompatible frameworks do not have a common refinement, and thus there is no way to combine the event algebras.

To assign probabilities we start with a collection of nonnegative numbers \( \{p_j\} \), one for each projector in \( S \), which sum to 1. Probabilities of events in \( \mathcal{E} \) are calculated in the obvious way; e.g., \( \Pr(P_2 + P_3 + P_5) = p_2 + p_3 + p_5 \). Where do the \( p_j \) come from? For ordinary (classical) probabilistic models they are simply parameters chosen by guesswork, or to agree with experiment: there are no hard and fast rules. The same is true in quantum theory except that for the time development of a closed system the Born rule and its extensions, Sec. 3.3, provide certain conditional probabilities which combined with appropriate assumptions (e.g., an initial state) yield a probability distribution. There is no reason quantum probabilities should not be applied to single systems as is done for the weather or to estimate the probability that the earth will collide with an asteroid of a given size during the next millennium.

The single framework rule is a central principle of the new quantum logic. What it says, in brief, is that a probabilistic calculation or a logical argument must be carried out using a single framework, a single event algebra generated by a single sample space, a single projective decomposition of the identity. Carrying out half of the reasoning or calculation using one framework and then transferring the result to a different framework for additional reasoning or calculations is prohibited. Since all projectors in a framework commute with each other, this immediately rules out combinations of noncommuting projectors using AND or OR as discussed in Sec. 3.3.

A useful illustration is provided by the harmonic oscillator, Sec. 3.1, where \( P = [0] + [1] \) is the property that the energy is not greater than \( 2\hbar \omega \). It might seem obvious that if \( P \) is true then the energy is either
can use ordinary logic to infer that if $P$ implies that it possesses the property $P$, neither of which has a well-defined energy. The frameworks violate that rule: “Let us suppose the oscillator is in the state $|+\rangle$. From this we infer (framework rule allows the use of either $F$ to the subspace $P$) that the oscillator has either the property $|+\rangle$ or the property $|-\rangle$, neither of which has a well-defined energy. The frameworks $F_2$ and $F_3$ are mutually incompatible because $|+\rangle$ and $|-\rangle$ do not commute with $|0\rangle$ and $|1\rangle$—the relationship is formally the same, if one restricts attention to the subspace $P$, as that between the $S_z$ and $S_x$ eigenstates of a spin-half particle. The single framework rule allows the use of either $F_2$ or $F_3$, but insists that they not be combined. Here is an argument that violates that rule: “Let us suppose the oscillator is in the state $|+\rangle$. From this we infer (framework $F_3$) that it possesses the property $P$. But it is obvious (framework $F_2$) that a system with the property $P$ has an energy of either $\hbar\omega/2$ or $3\hbar\omega/2$. Consequently the state $|+\rangle$ has one of these two energies.” It is this sort of reasoning, which in the quantum domain can lead to paradoxes, that is blocked by the single framework rule.

But why would one ever want to use a framework such as $F_3$? It is nowadays possible to prepare a harmonic oscillator, either a mechanical oscillator or the electromagnetic field inside a cavity, in a superposition of the ground and first excited state, and $F_3$ might be useful in describing such a situation. Thus in quantum mechanics it is quite possible to say that “the energy is less than $2\hbar\omega$ without implying that the energy is equal to either of the two possible energies that are less than this value, and this is precisely the significance of the projector $P$.

Four principles provide a compact summary of what the single framework rule does and does not mean. First, the physicist has perfect liberty to construct different, perhaps incompatible, frameworks when analyzing and describing a quantum system. No law of nature singles out a particular quantum framework as the “correct” one; from a fundamental point of view there is perfect Equality among different possibilities. However, the principle of Incompatibility prohibits combining incompatible frameworks into a single description, or in employing them for a single logical argument leading from premises to conclusions. The last principle is Utility: not every framework is useful for understanding a particular physical situation or addressing certain scientific questions. In addition it is important to avoid thinking that the physicist’s choice of framework somehow influences reality. Instead, quantum reality allows a variety of alternative descriptions, useful for different purposes, which when they are incompatible cannot be combined.

### 3.3 Time development

Von Neumann’s quantum mechanics had two distinct sorts of time evolution: unitary evolution, based on Schrödinger’s equation, and a separate stochastic evolution associated with measurements, Sec. V.1 of [21]. Few have found this satisfactory, but devising something better has proven difficult. In the Everett or many worlds interpretation [22,23] there is only unitary time development: a single unitarily evolving wave function of the universe, or univave in the terminology of [2]. Proponents of this approach then have to explain the probabilistic behavior of quantum systems observed in the laboratory, a not altogether easy task. The histories interpretation takes the opposite approach: all quantum time development is stochastic, and the deterministic Schrödinger equation is used to calculate probabilities. This is also what is done in textbooks, where physics is extracted from the formalism using absolute squares of transition amplitudes, though the whole matter is obscured through frequent (and unnecessary) references to measurements.

Stochastic time evolution requires a sample space with events at successive times, and in the histories approach each event is a quantum property. Thus for a sequence of times $t_0 < t_1 < \cdots t_f$ the sequence of properties

$$Y = F_0 \circ F_1 \circ \cdots F_f,$$

where each $F_j$ is a projector, is a history to which under appropriate conditions one can assign a probability. The $\circ$ in [4] indicates a tensor product. (While it would be perfectly correct to use the standard symbol
symbol \( \otimes \), it is often helpful when considering the time development of a quantum system possessing subsystems to employ a distinct symbol that separates situations at different times.) The operator \( Y \) is a projector on a subspace of the history Hilbert space
\[
\hat{\mathcal{H}} = \mathcal{H} \otimes \mathcal{H} \otimes \cdots \mathcal{H}
\]
formed from the tensor product of copies of the Hilbert space \( \mathcal{H} \) that describes the system at a single time. Its physical interpretation is that that the (quantum) event \( F_0 \) occurred or, equivalently, the property \( F_0 \) was true at the time \( t_0 \), \( F_1 \) at the time \( t_1 \), and so forth.

The sample space \( \mathcal{S} \) consists of a collection of orthogonal projectors of the kind shown in (4), the elementary histories, that sum to the history identity,
\[
\hat{I} = I \otimes I \otimes \cdots I,
\]
and thus constitute a projective decomposition of \( \hat{I} \). The corresponding event algebra \( \mathcal{E} \) consists of projectors which are sums of some of the projectors that make up \( \mathcal{S} \), and the probability of any history in \( \mathcal{E} \) is the sum of the probabilities of the elementary histories of which it is composed. The term “family of histories” is often employed in place “framework”, and depending on the context can refer to either \( \mathcal{S} \) or \( \mathcal{E} \). As in any probabilistic model, one and only one of the elementary histories, which are mutually exclusive, occurs in any given situation or “experimental run.”

A simple but fairly useful family employs a set of elementary histories
\[
Y^\alpha = [\psi_0] \otimes P_1^{\alpha_1} \otimes P_2^{\alpha_2} \otimes \cdots P_f^{\alpha_f},
\]
where \( [\psi_0] = |\psi_0\rangle \langle \psi_0| \) is a fixed initial state at \( t_0 \), and at the later time \( t_m \) the projector \( P_m^{\alpha_m} \), where \( \alpha_m \) is a label not an exponent, belongs to a fixed decomposition of the (single time) identity,
\[
\sum_{\alpha_m} P_m^{\alpha_m} = I,
\]
and \( \alpha = (\alpha_1, \alpha_2, \ldots, \alpha_f) \) is the label for the history \( Y^\alpha \). If one includes along with the \( Y^\alpha \) in (7) a special history \( Y^0 = (I - [\psi_0]) \otimes I \otimes I \otimes \cdots I \), the result is a family for which the projectors add to \( I \), as required for a sample space.

The rules for assigning probabilities to elementary histories of a closed quantum system whose unitary time development is generated by a Hermitian Hamiltonian are best explained using examples. The simplest situation is that in which \( f = 1 \), so only two times \( t_0 \) and \( t_1 \) are involved. Let \( T(t, t') \) be the unitary time development operator for the time interval from \( t' \) to \( t \); for a time-independent Hamiltonian \( H \) it is \( T(t_1, t_0) = \exp[-i(t_1 - t_0)H/\hbar] \). At time \( t_1 \) assume that the projective decomposition of the identity corresponds to an orthonormal basis \( \{|\phi_1^k\rangle\} \), and let \( P_1^k = |\phi_1^k\rangle \langle \phi_1^k| \). The Born rule assigns a conditional probability
\[
p_k = \text{Pr}(P_1^k | [\psi_0]) = |\langle \phi_1^k | T(t_1, t_0) | \psi_0 \rangle|^2
\]
to \( P_1^k \) at \( t_1 \) given the initial state \( [\psi_0] \) at \( t_0 \). If we assign a probability of 1 to \( [\psi_0] \) and 0 to \( I - [\psi_0] \), which is to say we assume the system starts at \( t_0 \) in the initial state \( [\psi_0] \), then \( p_k \) is the probability assigned to the elementary history \( Y^k = [\psi_0] \otimes |\phi_1^k\rangle \).

Textbooks use the same rule, but tend to word it differently. Solving Schrödinger’s equation with initial state \( [\psi_0] \) at \( t_0 \) yields
\[
|\psi(t)\rangle = T(t, t_0) |\psi_0\rangle,
\]
at time \( t \). Setting \( t = t_1 \) allows us to write \( p_k \) in (9) as
\[
p_k = |\langle \phi_1^k | \psi(t_1) \rangle|^2.
\]
That is, students are taught to first calculate the uniwave (10), and then use (11) to find probabilities. (The other difference is that the textbooks generally refer to \( p_k \) as the probability of a (macroscopic) measurement outcome, the pointer position, rather than as the probability of the (microscopic) property the measurement apparatus was designed to measure. The connection between the two will be discussed in Sec. 3.5 below.)

While this is an excellent calculational technique and gives the right answers, it has unfortunately given rise to the idea that the uniwave constitutes the fundamental quantum ontology, it represents quantum reality.
There are two ways to see that this is mistaken. First, if one regards \( |\psi(t_1)\rangle \) as a quantum property, then it will not commute with any \( |\phi_k^1\rangle \) for which \( 0 < p_k < 1 \). Thus except in special cases the single framework rule prevents \( |\psi(t_1)\rangle \) from being added to the set of properties \( \{|\phi_k^1\rangle\} \) under consideration. Second, note that \( p_k \) can be calculated using the formula

\[
p_k = |\langle \psi_0 | \hat{\phi}^k(t_0) \rangle|^2; \quad |\hat{\phi}^k(t)\rangle = T(t, t_1) |\phi_1^k\rangle.
\]  

(12)

That is, start with \( |\phi_k^1\rangle \) at time \( t_1 \) and integrate Schrödinger’s equation backwards in time to obtain \( |\hat{\phi}^k(t)\rangle \) at \( t = t_0 \). In this procedure (which is not particularly efficient, since to obtain probabilities for several different \( k \) requires integrating Schrödinger’s equation a comparable number of times) the uniwave never appears, which shows that it was merely a convenient calculational tool.

Referring to \( |\psi(t)\rangle \) as a “pre-probability” (something used to calculate a probability), as in Sec. 9.4 of [20], serves to emphasize that, at least in the situation under consideration, it is not to be regarded as a quantum property, a genuine “beable.” (Similarly, \( |\hat{\phi}^k(t)\rangle \) in (12) is a pre-probability.) Consequently, the appropriate use of “the wave function” obtained by solving Schrödinger’s equation is, at least in general, epistemic: it is employed to compute probabilities. The claim, as found for example in [24–27], that the wave function cannot be used in this way seems to be based on the use of classical hidden variables, which are inconsistent with Hilbert-space quantum mechanics [28].

When histories involve three or more times an extension of the simple Born rule is needed in order to generate a consistent set of probabilities for quantum histories. The essential features appear already in the case of three times, \( f = 2 \), but it will be convenient to consider the general case of a family of the type (7).

For each elementary history \( Y^\alpha \) define the chain ket

\[
|\alpha\rangle = (|\alpha_1, \alpha_2, \ldots, \alpha_f\rangle) = P_f^{\alpha_f} T(t_f, t_{f-1}) P_{f-1}^{\alpha_{f-1}} T(t_{f-1}, t_{f-2}) \cdots P_1^{\alpha_1} T(t_1, t_0) |\psi_0\rangle.
\]  

(13)

Provided these chain kets are mutually orthogonal, which is to say

\[
\langle \alpha | \alpha' \rangle = 0 \quad \text{whenever} \quad \alpha \neq \alpha'
\]  

(14)

where \( \alpha = \alpha' \) if and only if \( \alpha_j = \alpha'_j \) for every \( j \), then the history \( Y^\alpha \) is assigned the probability

\[
\Pr(\alpha | |\psi_0\rangle) = \langle \alpha | \alpha \rangle
\]  

(15)

conditioned on the initial state \( |\psi_0\rangle \). The consistency conditions (13) are automatically satisfied for histories only involving two times, the case \( f = 1 \), and then (15) gives the same Born probability as (9). However, for \( f = 2 \) or more the restriction (14), which depends both on the projectors making up the family and the unitary dynamics, is needed and is not trivial.

As noted previously, the single framework rule prohibits combining two sample spaces when the projectors do not commute. This applies also to combining two families of histories: the history projectors must commute with each other for the combination to be possible, and if this is not so we say the history families are incompatible. However, even if all the projectors in the two families commute, it may be the case that the common refinement fails to satisfy the consistency conditions, so there is no way to assign probabilities to this family using the extended Born rule for a closed quantum system. In that case we say the two families are incommensurate. It is then a natural extension of the single framework rule to prohibit incommensurate as well as incompatible families of histories. Or, to put it another way, the usual rules of probabilistic quantum dynamics can only be applied to a single consistent family, and results from two incommensurate, as well as from two incompatible, families cannot be combined. For an example of incommensurate families see the discussion of families \( \mathcal{A} \) and \( \mathcal{B} \) for the Aharonov and Vaidman three-box paradox given in Sec. 22.5 of [20].

### 3.4 Quasiclassical frameworks

It has been argued by Omnès [20,30], and Gell-Mann and Hartle [31,33] (also see Ch. 26 of [20]) that classical mechanics for macroscopic systems emerges as a good approximation to a more exact but unwieldy quantum description. The idea is to use a quasiclassical quantum framework employing coarse-grained projectors that project onto Hilbert subspaces of enormous, albeit finite, dimension, suitably chosen so as to be counterparts of classical properties such as those used in macroscopic hydrodynamics. The stochastic quantum dynamics associated with a family of histories constructed using these coarse-grained quasiclassical...
projectors gives rise, in suitable circumstances, to individual histories which occur with high probability and are quantum counterparts of the trajectories in phase space predicted by classical Hamiltonian mechanics. There are exceptions. For example, in a system whose classical dynamics is chaotic with sensitive dependence upon initial conditions one does not expect the quantum histories to be close to deterministic.

A quasiclassical family can hardly be unique given the enormous size of the corresponding Hilbert subspaces, but this is of no great concern provided classical mechanics is reproduced to a good approximation, in the sense just discussed, by any of them. Therefore all discussions which involve nothing but classical physics can, from the quantum perspective, be carried out using a single quasiclassical framework. As long as reasoning and descriptions are restricted to this one framework there is no need for the single framework rule, which explains why a central principle of quantum mechanics is absent from classical physics. And why ordinary propositional logic is adequate for the macroscopic world of everyday affairs.

3.5 Measurements

A simple measurement model based on the one proposed by von Neumann in Ch. V of [21] will illustrate how the histories approach addresses the measurement problems listed in Table II. Suppose properties of a system with Hilbert space $\mathcal{H}_s$, hereafter referred to as a “particle”, are to be measured by an apparatus, Hilbert space $\mathcal{H}_m$, with $\mathcal{H}_s \otimes \mathcal{H}_m$ the Hilbert space of the combined closed system. Let

$$|s_0\rangle = \sum_j c_j |s^j\rangle,$$

be the initial state of the particle, where $\{|s^j\rangle\}$ is an orthonormal basis of $\mathcal{H}_s$, and $|m_0\rangle$ the “ready” state of the apparatus at the initial time $t_0$. During the interval from $t_0$ to $t_1$ the particle and apparatus do not interact, so we set $T(t_1, t_0) = I = I_s \otimes I_m$ corresponding to a trivial dynamics. For the time interval from $t_1$ to $t_2$ during which they interact we assume that

$$T(t_2, t_1)(|s^j\rangle \otimes |m_0\rangle) = |s^j\rangle \otimes |m^j\rangle,$$

where the $|m^j\rangle$, associated with different pointer positions, are normalized and mutually orthogonal. Thus under unitary time evolution the initial state

$$|\Psi_0\rangle = |s_0\rangle \otimes |m_0\rangle$$

develops into

$$|\Psi_1\rangle = T(t_1, t_0)|\Psi_0\rangle = |\Psi_0\rangle, \quad |\Psi_2\rangle = T(t_2, t_1)|\Psi_1\rangle = \sum_j c_j |s^j\rangle \otimes |m^j\rangle$$

at the times $t_1$ and $t_2$.

Now consider various history families of the form (17), with the initial state $|\Psi_0\rangle = |\Psi_0\rangle/\langle\Psi_0|$ at time $t_0$ given by (15). One possibility is unitary time development:

$$\mathcal{F}_u: |\Psi_0\rangle \otimes \{[\Psi_1], I - [\Psi_1]\} \otimes \{[\Psi_2], I - [\Psi_2]\}$$

for times $t_0 < t_1 < t_2$, with different histories in the sample space constructed by choosing one of the projectors inside the curly brackets at each of the later times. Since the events $I - [\Psi_1]$ and $I - [\Psi_2]$ occur with zero probability they can be ignored, and the single history $|\Psi_0\rangle \otimes |\Psi_1\rangle \otimes |\Psi_2\rangle$ occurs with probability 1. While $\mathcal{F}_u$ is perfectly acceptable as a family of quantum histories, it cannot be used to discuss possible outcomes of the measurement because it does not include the projectors $\{[m^j]\}$ for the pointer positions at time $t_2$, nor can it be refined to include them, because $[\Psi_2]$ will not commute with some of the $[m^j]$, assuming at least two of the $c_j$ in (16) are nonzero. Thus the first measurement problem cannot be solved if all time development is unitary. This is a basic difficulty facing all quantum interpretations that make the uniwave fundamental to their ontology.

The histories approach can solve the first measurement problem by replacing $\mathcal{F}_u$ with the family

$$\mathcal{F}_1: |\Psi_0\rangle \otimes [\Psi_1] \otimes \{[m^j]\}.$$  

That is, the different histories agree at $t_0$ and $t_1$, but correspond to different pointer positions at $t_2$. The alternative $I - [\Psi_1]$, which occurs with zero probability, has been omitted at $t_1$, and at time $t_2$ we employ
the usual physicist’s convention that $|m^j| = |m^j⟩⟨m^j|$ means $I_s ⊗ |m^j⟩$ on the full Hilbert space $H_s ⊗ H_m$. An additional projector $R^j = I - \sum_j |m^j⟩⟨m^j|$ should be included at the final time in (12) so that the total sum is the $I$, but, again, it has probability zero. While $|Ψ_2⟩$ cannot be one of the properties at time $t_2$ in family $F_1$, see the discussion of $F_2$ above, it can be used as a pre-probability (see the discussion following (12)) to calculate probabilities of the different pointer positions at time $t_2$:

$$\Pr(|m^j⟩) = \text{Tr}(Ψ_2|m^j⟩⟨m^j|Ψ_2).$$

(22)

(Here and below we omit from $\Pr()$ the condition $|Ψ_0⟩$ at $t_0$, as it applies in all cases.)

In order to relate the measurement outcome, the pointer position, to a prior property of the measured particle and thus solve the second measurement problem yet another family is needed:

$$F_2: |Ψ_0⟩ ⊗ \{[s^j]\} ⊗ \{[m^k]\}.$$  

(23)

The decomposition $\{[s^j]\}$ at $t_1$ refers to properties of the particle, $[s^j]$ means $[s^j] ⊗ I_m$, without reference to the apparatus. It is straightforward to show that $F_2$ is consistent, leading to a joint probability distribution

$$\Pr([s^j], [m^k]) = |c_j|^2 δ_{jk},$$

(24)

where the subscripts on $[s^j]$ and $[m^k]$ indicate the time. The marginals are:

$$\Pr([s^j]_2) = \Pr([m^j]_2) = |c_j|^2.$$  

(25)

Thus if $|c_k|^2 > 0$ the conditional probability

$$\Pr([s^j]_1 | [m^k]_2) = δ_{jk}$$

(26)

implies that from the (macroscopic) measurement outcome or pointer position $[m^k]$ at time $t_2$ we can infer, using standard statistical inference, that the particle had the (microscopic) property $[s^k]$ at the earlier time $t_1$. This solves the second measurement problem. And because the probability of the $[s^j]$ at $t_1$ is the same as $[m^j]$ at $t_2$, textbooks in which students are taught to calculate $|c_j|^2$ for the particle alone and then ascribe the resulting probability to the outcome of a measurement, are not wrong. They would be less confusing if they provided a proper quantum analysis of the measurement process, such as given here.

Because it solves the second measurement problem, a measurement actually measures something, the histories approach is sometimes confused with the sorts of hidden variables approach studied by Bell and his followers. However, the basic hypothesis underlying most hidden variables schemes is that every property which could possibly be measured already “exists” in some sense in the particle before measurement. This leads to various difficulties such as the Bell-Kochen-Specker paradox, for which see the discussion in Sec. 22.1 of [20]. Suppose, for example, that a measurement is to be carried out on a spin half particle. Since this might be a measurement of $S_x$ or of $S_y$ or of $S_z$, it then seems natural to suppose that all three values are somehow “present” in the particle before it is measured. But this is contrary to Hilbert space quantum mechanics, since the different projectors do not commute; see the discussion in Sec. 3.1. The histories approach avoids the Bell-Kochen-Specker paradox by applying the single framework rule $F_2$. This point will come up again in the discussion of locality in Sec. 3.7 below. For the same reason the histories approach rejects the notion that quantum mechanics is contextual; see the detailed discussion in [35].

It is to be noted that all three history families or frameworks, $F_u$, $F_1$, and $F_2$ employed above satisfy the consistency conditions and thus provide legitimate quantum descriptions. There is no reason a priori to prefer one to another; at a fundamental level there is Equality. However, Utility plays a significant role. If measurement outcomes (pointer positions) at $t_2$ are under discussion, $F_u$ is unsatisfactory, as its event algebra does not contain them. Both $F_1$, and $F_2$ allow discussion of measurement outcomes, but if one is interested in how the outcomes are related to the properties the device was designed to measure, the $[s^j]$ at time $t_1$, $F_2$ makes this possible, whereas $F_1$ does not.

This does not mean that $F_2$ is always the “right” framework. Consider a situation in which Alice prepares a spin half particle with $S_x = +1/2$ and sends it through a field-free region to an apparatus that Bob has set up to measure $S_z$, and which yields the value $S_z = -1/2$. At the intermediate time the value of $S_x$ may be of interest to Alice—did the preparation device work as intended?—in which case the $F_1$ family is appropriate. However, if Bob is concerned with how his apparatus has functioned, $F_2$ is more relevant. Both frameworks are valid tools for quantum analysis, and they could both be used by the same person, e.g., someone who
sets up both the preparation and the measurement apparatus. The restriction which the new logic imposes is that they cannot be combined into a single description.

It is worth mentioning yet another family

\[ \mathcal{F}_3 : [\Psi_0] \otimes [\Psi_1] \otimes \{[s^k] \otimes [m^j]\}, \] (27)

which is similar to \( \mathcal{F}_1 \) except that at time \( t_2 \) we have added the final particle states to the description. It is easy to show, using \( |\Psi_2\rangle \) from (19) as a pre-probability, that the probability of any history with \( k \neq j \) is 0, and hence if \( c_j \) in (16) is nonzero,

\[ \Pr([s^k][m^j]) = \delta_{jk} \] (28)

That is to say, if at time \( t_2 \) the pointer is in the position \( [m^j] \) the particle is in the state \( [s^j] \). This is von Neumann’s (and the textbooks’) “wave function collapse”. But now it is simply an ordinary probabilistic inference using a conditional probability, so there is nothing at all mysterious about it, and it definitely does not have to be added to quantum theory as an independent principle. Other cases of wave function collapse (when it is being properly used) can also be replaced by conditional probabilities, thus eliminating another of the conceptual difficulties in Table 1. Note that \( \mathcal{F}_3 \) is a family useful for analyzing a preparation procedure for producing a particle in a well-defined initial state.

Our discussion has employed various simplifications not present in real measurements. In particular, pointer positions will always be associated with macroscopic properties, thus projectors onto enormous subspaces and not the pure states \( |m^k\rangle \) assumed above. Similarly, the initial state of a macroscopic apparatus should be described using a macroscopic projector, or an appropriate density operator. The discussion given above is extended in Ch. 17 of [20] to include these more realistic features, along with irreversible (in the thermodynamic sense) behavior of the apparatus. None of the conclusions discussed above is undermined by this extension.

### 3.6 Interference

The double slit and the similar Mach-Zehnder paradoxes were introduced in Sec. 2.3. The histories approach in the case of the Mach-Zehnder interferometer is discussed in considerable detail in Ch. 13 of [20], and the same principles apply to the double slit. Here are the end results of that analysis. A family of histories referring to the particle needs to include both its existence in a coherent state \( |\psi_0\rangle \) at a time \( t_0 \) before it encounters the slit system, and then its presence in a reasonably compact region in the interference zone at a later time \( t_2 \). One can then argue that a family of histories in which the particle passes through one or the other of the two slits at the intermediate time \( t_1 \) fails to satisfy the consistency conditions, and thus cannot be discussed in appropriate (probabilistic) quantum terms, in agreement with Feynman’s conclusion based on (excellent) physical intuition. There is, on the other hand, an alternative family in which the particle does pass through a definite slit and nothing is said about what happens later. Still another possibility is that the particle passes through a definite slit and the quantum detectors in the later interference region are left in a macroscopic quantum superposition (Schrödinger cat) state. Various possibilities are discussed in Ch. 13 of [20] using simplified models which are very useful for gaining a better intuitive understanding of quantum interference.

The consistency condition (14), the requirement that the inner product of chain kets for different elementary histories be zero, can be understood as requiring an “absence of interference” when calculating probabilities. This should not be misinterpreted to mean that the histories approach cannot be applied to physical situations, such as the double slit, where quantum interference is central to the phenomena under consideration. Instead, for any given physical situation, whether or not there is some form of interference, the histories approach and in particular the consistency condition singles out physically sensible and consistent ways of discussing what is going.

### 3.7 Locality

The following analogy shows how the histories approach counters the widespread claim that quantum mechanics is nonlocal because it violates Bell inequalities. Charlie in Chicago places a red slip of paper in an opaque envelope, a green slip in another, and shuffles the two before mailing one to Alice in Atlanta and the other to Bob in Boston. Knowing the protocol followed by Charlie, if Alice opens her envelope and sees a red slip of paper she can immediately conclude that Bob’s envelope contains (or contained) a green slip.
This inference has nothing to do with whether Bob opens the envelope earlier or later than Alice, or simply throws it away unopened. Furthermore, Alice’s “measurement” by opening and looking in the envelope has absolutely no influence on the color, or any other property, of the slip in Bob’s envelope.

How are things different if Charlie prepares two spin half particles $a$ and $b$ in a singlet state, and pushes a button that sends $a$ towards Alice’s apparatus set up to measure $S_{az}$ for this particle, and $b$ towards Bob’s apparatus set up to measure $S_{bz}$? If Alice’s measurement outcome, indicated by a suitable pointer, corresponds to $S_{az} = +1/2$ she is entitled, as a competent experimentalist who knows how her equipment functions, to infer that particle $a$ had this property before the measurement. By using a suitable framework that includes both $S_{az}$ and $S_{bz}$ values at this earlier time, and knowing the protocol followed by Charlie, she can infer the value $−1/2$ for $S_{az}$.

To be more specific, use the $F_2$ type of framework discussed in Sec. 3.5 with $|\Psi_0\rangle$ the initial state of Alice’s apparatus tensored with the spin singlet state of the two particles, and at the intermediate time $t_1$ use projectors onto the orthonormal basis (with a notation similar to (2))

$$|z_a^+, z_b^-\rangle, |z_a^+, z_b^+\rangle, |z_a^-, z_b^-\rangle, |z_a^-, z_b^+\rangle$$

of particle spin states. Their joint probability distribution at time $t_1$ given the singlet state at $t_0$ (and assuming no magnetic fields are present) is

$$\Pr(S_{az} = +\frac{1}{2}, S_{bz} = +\frac{1}{2}) = 0, \quad \Pr(S_{az} = +\frac{1}{2}, S_{bz} = −\frac{1}{2}) = \frac{1}{2},$$

$$\Pr(S_{az} = −\frac{1}{2}, S_{bz} = +\frac{1}{2}) = \frac{1}{2}, \quad \Pr(S_{az} = −\frac{1}{2}, S_{bz} = −\frac{1}{2}) = 0,$$

which gives conditional probabilities

$$\Pr(S_{bz} = −\frac{1}{2} | S_{az} = +\frac{1}{2}) = 1 = \Pr(S_{bz} = +\frac{1}{2} | S_{az} = −\frac{1}{2}).$$

Since Alice knows on the basis of her measurement outcome that $S_{az}$ had the value $+1/2$ at $t_1$, she can infer with certainty using the first equality in (31) that $S_{bz}$ had the value $−1/2$. Note that Bob, who also knows the protocol, can write down exactly the same probability formulas (30) and (31). The only difference is that Alice because she knows the outcome of her measurement can use (31) to infer the value of $S_{bz}$. Bob of course will get this result if he carries out a later measurement.

Wave function collapse never appears in the foregoing argument. It could be used as a calculational tool, as in $F_3$ in Sec. 3.5, to compute a conditional probability, but computational tools are not to be confused with physical processes, and wave functions serving as pre-probabilities should be carefully distinguished from quantum properties. Note also that the time at which Alice carries out a measurement, relative to when Bob does or does not carry out a measurement, is of no importance. (See [36] for a more extended discussion of this point.) There are no mysterious influences of Alice’s measurement on Bob’s particle, and thus no hint of any violation of the principles of special relativity.

And what if if Alice measures some other component of spin angular momentum, say $S_{zx}$ rather than $S_{az}$? In that case she can use an appropriate framework to infer the value of $S_{ax}$ before the measurement was made, and from it deduce the value of $S_{bx}$ for particle $b$. But that framework cannot be combined with the one which allows her to infer the earlier value of $S_{az}$ when that is measured, since the $x$ and $z$ components of angular momentum are incompatible quantum variables. Alice, a competent experimentalist, cannot measure both $S_{ax}$ and $S_{az}$ on the same particle for there is no combined property to be measured.

But even if Alice measures $S_{az}$ on this occasion, surely she could have instead measured $S_{ax}$ on the very same particle, and in that case the measurement would surely have revealed either $S_{ax} = +1/2$ or $−1/2$. And therefore before the measurement the particle must have had both a definite value of $S_{ax}$ as well as $S_{az}$.

What is wrong with this argument? The “if . . . would” construction betrays the presence of counterfactual reasoning: something actually happened, $S_{az}$ was measured, but one imagines a different world in which $S_{ax}$ was measured instead. As discussed in Sec. 19.4 of [20], it is important to subject counterfactual reasoning about quantum systems to the single framework rule. There is no difficulty imagining a counterfactual world with distinct macroscopic measurement setting, for these represent mutually exclusive alternatives represented in quantum theory by commuting projectors (their product is 0). But there is no room in the Hilbert space for simultaneous values of $S_{az}$ and $S_{ax}$, as noted earlier in Sec. 3.5.

Here, indeed, is the point where derivations of Bell inequalities are inconsistent with Hilbert space quantum mechanics: the inequalities are obtained using “hidden variables” not subject to the rules appropriate to a quantum Hilbert space. The fundamental issue has nothing to do with locality, for it already arises when considering measurements by just one party, in our case Alice, of two incompatible physical variables belonging to incompatible frameworks. For further discussion see [36][37].
3.8 Approximations

The condition for compatibility of two frameworks for a quantum system at a single time was stated in Sec. 3.1 in terms of commutation of the projectors from both collections. Would not approximate commutation suffice? The consistency condition for a family of histories at three or more times is rather stringent; would it suffice if it were approximately satisfied?

As physicists we prefer to have theories stated in precise terms, especially if the mathematical expressions are simple and “clean”, even if in practice it is almost always necessary to make approximations to what we believe are exact laws in order to have a theory which can be related to the real world of experience and experiments. Whether a particular approximation is adequate is an element of judgment, and it is often difficult if not impossible to provide precise error bounds. Quantum theory interpreted using the new logic is no different from other physical theories in this respect, and the material given above has deliberately been expressed in terms of exact rules. Nonetheless, since the new logic is intended to assist in providing a physical interpretation of quantum mechanics, let us add a couple of comments that may be helpful.

First, it is plausible that two rays $|\psi\rangle$ and $|\phi\rangle$ in the Hilbert space that are “near” each other in the sense that $|\langle\psi|\phi\rangle|$ is close to 1 should have a very similar physical interpretation. For example, the spin of a spin-half particle has a positive component in a direction which is close to the $z$ axis but not exactly aligned with it. Then we can, at least for certain purposes, think of it has having the property $S_z = +1/2$. For example, if the spin is measured in the $S_z$ basis the result will be $S_z = +1/2$ most of the time, with a probability of $1 - |\langle\psi|\phi\rangle|^2$ of obtaining $S_z = -1/2$. Nor will the difference between $|\psi\rangle$ and $|\phi\rangle$ increase in time under unitary time evolution. It is considerations of this sort that suggest that the condition that exact orthogonality of the projectors making up a quantum sample space can be relaxed in some cases without seriously distorting the physical interpretation.

A similar intuition applies to the consistency condition (14) for a family of histories of the form (7). If consistency is satisfied with small errors, then it can be argued that small alterations of the projectors in the sample space, with but small shifts in their physical interpretation, will yield a family that exactly satisfies the consistency conditions. In this case the matter is not as obvious as for a simple collection of almost orthogonal projectors, but at least it is plausible, see [38].

4 Conceptual Difficulties of the New Logic

The histories approach gives rise to various conceptual difficulties, and this section discusses those listed in Table 2. While it may not be complete, no important difficulty known to the author has been omitted from this list, which helps organize the material that follows. As the new logic is a scheme of reasoning, the issues it raises are conveniently divided into two categories. First, is it internally consistent, free from contradictions? This is addressed in Sec. 4.1. Second, assuming consistency, does it provide a good way to think about quantum mechanics? From the perspective of the physicist the second question is just as important as, and perhaps even more important than, the first: a scheme which is logically sound but does not help us understand the world will not resolve the problems plaguing quantum foundations. Sections 4.2 to 4.5 address issues of the second type, those numbered 2 to 5 in the table.

4.1 Internal consistency

Let us start by summarizing the new logic’s rules for probabilistic reasoning about the quantum world. First, choose a quantum framework. By definition this consists of a quantum sample space, a collection of projectors on the appropriate quantum Hilbert space that sum to the identity, together with the associated event algebra composed of all projectors made up of sums of sample space projectors. Within this framework the usual laws of classical probability theory and ordinary propositional logic apply without any change, as discussed in Secs. 3.1 and 3.2 because the projectors all commute with each other. And one can use the same intuition—e.g., the sample space is a collection of mutually exclusive possibilities, one and only of which is correct—employed in other uses of ordinary (Kolmogorov) probability theory. (The nontrivial issue of how to go about choosing a quantum framework is taken up in Sec. 4.3 below.)

The single framework rule, which is a central principle of the new logic, prohibits combining frameworks: any sort of probabilistic argument from premises to conclusions, including propositional logic as a special case when probabilities are 0 or 1, must employ just one framework. From this it follows that arguments that prove the consistency of ordinary probabilistic or propositional reasoning also demonstrate the consistency
Table 2: Conceptual Difficulties of the New Logic

1. Internal consistency

2. Stochastic time development
   a. Determinism abandoned
   b. The uniwave

3. Framework selection
   a. Numerous frameworks
   b. Incompatible frameworks
   c. Selection based on utility
   d. Single framework rule
   e. Choice influences reality?

4. Particular histories
   a. Which history occurs?
   b. Retrodiction from different measurements

5. Truth and reality
   a. Framework dependence of truth
   b. Unicity

of quantum reasoning based on the new logic. In particular, any contradiction that might arise within a fixed quantum framework will also have a counterpart in standard (classical) reasoning. Claims made in the literature that the histories approach is inconsistent, in the sense of leading to contradictions [39, 40], are flawed in that the authors have not taken the single framework rule seriously; see [34, 41].

It may help to supplement the preceding remarks with some comments about how probabilistic reasoning is actually carried out. Classical applications of probability theory also begin with a framework, which is to say a sample space and an event algebra, though this is sometimes done implicitly—e.g., random variables are introduced without bothering to say which space they are functions on, since the knowledgeable reader ought to know how to construct it. In quantum mechanics one needs to be a bit more careful, since many—perhaps most—quantum paradoxes are constructed by combining incompatible frameworks. The framework chosen must, of course, include all the events or histories one is interested in. Next some set of probabilities are assumed: the only strict rule for assigning them is that they must be additive and sum to 1. Then the typical argument proceeds from some initial data, assumed to be correct or perhaps assigned some initial probabilities, to final conclusions, also expressed using probabilities. If all the probabilities are 0 or 1, one has an ordinary logical argument. The “initial” in “initial data” refers to something assumed at the beginning of the argument, not necessarily properties of a physical system at the earliest time of interest, though these are often included in the initial data. Similarly, “final” refers to the end of the argument, not necessarily the latest time.

For a specific quantum example see the discussion of measurements in Sec. 3.3 using the family \( F_2 \). The state \( |\Psi_0\rangle \) at the earliest time \( t_0 \) together with the measurement outcome at time \( t_2 \) constitute the initial data needed to infer a property of the particle at an intermediate time \( t_1 \), which is the final conclusion. (In this instance the inference requires the use of probabilities obtained by applying the extended Born rule to a closed system in a situation involving three times, so an acceptable quantum framework must be a family of histories satisfying the consistency conditions.)

Often there is more than one framework which will contain the events of specific interest along with other events, and since two such frameworks could be incompatible, one might be concerned that they would lead to different results, i.e., different outcome probabilities. However, a basic consistency rule, discussed in more detail in Ch. 16 of [20], shows that the probabilities linking a particular set of conclusions to a specific collection of initial data are always the same for any framework that includes both. A useful heuristic in constructing arguments of this sort is to employ the coarsest possible framework, i.e., the smallest number
of projectors in the quantum sample space, that can accommodate all data and conclusions, since adding refinements is usually more work, and there is the danger that in constructing a complicated argument one may overlook something, such as a consistency condition, and arrive at incorrect conclusions.

4.2 Stochastic time development

A stochastic (probabilistic) time development in place of determinism should not represent a serious conceptual problem. One can without difficulty imagine a classical world in which the fundamental dynamical law has some stochastic element, and in the regime of classical chaos even deterministic equations can lead to behavior which is for all practical purposes indeterministic. Furthermore, the study of quasiclassical frameworks, Sec. 3.4, shows how fundamentally indeterministic quantum laws can, under suitable circumstances, give rise to what is for all practical purposes deterministic behavior for a macroscopic system. Even Einstein might have been willing to abandon determinism in order to achieve a theory which contains no mysterious nonlocal influences, no longer has measurement as a fundamental principle, and allows one to say the moon is there even when it is not being observed.

There is, however, another barrier, as much sociological as scientific. Students in their first quantum course are taught to reverence the deterministic Schrödinger equation as the central principle of quantum dynamics, whereas probabilities are treated as somewhat of an embarrassment, a necessary evil when measurements interfere with the “correct” time dependence, which however will resume again once the nasty measurement is over. Were students at the beginning of the course taught that the unitarily developing wave function, the uniwave, is simply a tool for calculating probabilities, not a representation of reality, and during the course supplied with a schematic but fully quantum description of measurements, this particular difficulty would likely disappear.

4.3 Framework selection

Quantum mechanics is similar to other branches of theoretical physics in that in order to apply it to a particular system one must construct a conceptual model following certain rules. The choice of which model to use depends on what one wants to discuss, but a variety of other considerations can enter that choice. All physical models are approximate in one way or another, and in this sense have varying degrees of “reality” associated with them. Simplifications are often introduced so as to allow an easier mathematical analysis, or in the hopes of gaining physical insight into the problem under discussion. Consequently the task of choosing a framework in which to carry out a discussion is not absent from classical physics, though it is generally simpler than in the quantum case. In particular, whenever probabilities are used, there must be either an explicit or implicit choice of a sample space and an event algebra. Difficulties arise in the quantum case both because there are a large number of possibilities, and also because one cannot combine incompatible alternatives into a single description. A useful way of exploring the quantum difficulties is to consider some classical systems, and ask which of the principles for quantum frameworks—Liberty, Equality, Incompatibility, Utility—introduced in Sec. 3.2 have classical analogs.

Let us begin with an everyday classical example. It is possible to view a coffee cup from below as well as from above, and the two perspectives give different types of information about, or describe different aspects of, a single object. One can choose either, and neither perspective is more fundamental than the other, so Liberty and Equality are represented in this mundane example. The Utility of each depends on what one is interested in learning: Is there coffee in the cup? Is there a crack on the bottom surface? Both perspectives are compatible: they can (in principle) be harmoniously combined into a single, more detailed, more refined description, containing all the information present in the views from above and below. And this compatibility is consistent with quantum theory: the relevant projectors, should one be so foolish as to attempt a quantum mechanical description of the coffee cup, will form a commuting set that is part of a quasiclassical framework.

Contrast this with the example in Sec. 3.5 where Alice prepares a spin-half particle in a state $S_x = +1/2$ and Bob later measures it and finds $S_z = -1/2$. To describe the particle at an intermediate time between preparation and measurement one can use either a consistent family of histories which contains the value of $S_x$ at this time, or one which contains the value of $S_z$. Either is perfectly acceptable from the perspective of quantum theory: there is no fundamental law that says that one should be used rather than the other. However, they are incompatible with each other and cannot be combined. Each family has its uses; e.g., in
addressing the question of whether the preparation was successful, or whether the measurement apparatus performed what it was designed to do.

It is Incompatibility that most clearly marks the border between classical and quantum physics, as can be seen by comparing the preceding example with a situation where the spin-half particle is replaced by a golf ball which Alice prepares with a positive \( x \) component of spin angular momentum, and Bob later measures the \( z \) component and finds it is negative. Again two valid descriptions at the intermediate time, but now they can be combined. Since the typical angular momentum of a spinning golf ball is on the order of \( 10^{30} \) in units of \( \hbar \) there is no problem constructing a quasi-classical framework in which both \( x \) and \( z \) components are represented approximately with a precision much more than adequate for all practical purposes.

Another partial analogy is provided by Lorentz transformations in classical special relativity. The physicist is at Liberty to choose different Lorentz frames, with none being more “fundamental” than another, and Utility may determine the choice; e.g., in scattering problems there is an advantage to using the center of mass. However, the situation is unlike quantum mechanics in that every Lorentz frame contains the same information as any other, since there is a well-defined means of transforming positions and momenta between different frames. Distinct quantum frameworks which are mutually incompatible obviously do not contain the same information. (As an aside we note that there is no particular problem in constructing a relativistic version of the histories approach; see, e.g., [42].)

Perhaps a closer analogy is provided by classical statistical mechanics where it is sometimes useful, for purposes of discussing irreversibility or the origin of hydrodynamic laws, to introduce a coarse graining of the classical phase space into nonoverlapping cells, with the coarse-grained description providing not the actual phase point of the system, but instead the label of the cell in which it is located. Here the choice of coarse graining is clearly one made by the physicist on the basis of its utility for the calculation he has in mind, and of course no coarse graining is more “fundamental” than any other. In addition, two coarse grainings do not in general contain the same information. However, given any two coarse grainings there is always a common refinement using the intersection of the cells, so with respect to Incompatibility the analogy with (incompatible) quantum frameworks breaks down.

To summarize the situation, classical physics provides analogies of many of the features which need to be taken into account when thinking about quantum frameworks. Frameworks are not automatic: they must be chosen. There are multiple possibilities, and alternative frameworks are not mutually exclusive in the sense that if one is right the others must be wrong. The physicist’s choice of framework has no influence on the reality being described, and is generally motivated by the desire to understand or describe a particular aspect of the system of interest. What classical physics does not provide is a good analogy for incompatible frameworks and the single framework rule that prohibits combining them. Computer languages provide a partial analogy: woe be to the programmer who mixes FORTRAN with C. But since a particular algorithm can be expressed using either, this analogy, while it may be helpful, is not exact.

### 4.4 Particular histories

A common objection to the histories approach is that there many consistent families of histories, and even if in each family only one elementary history can occur, this still leaves a large number of possibilities. How does one know which of these histories actually occurred? What is the one true history? What rule selects the correct family that contains it?

From a formal perspective the issue raised here is a particular instance of the framework selection problem discussed above. Quantum mechanics allows many different frameworks any one of which can be chosen by the physicist for constructing a description, as long as they are not combined in a way that violates the single framework rule. That prohibition includes, in the case of consistent families of histories, the combining of incommensurate families. Within a consistent family the elementary histories (those belonging to the sample space) are mutually exclusive, so one and only one of them occurs or is correct, even though quantum theory can in general only provide probabilities for different possibilities.

A formal statement is often insufficient to resolve some intuitive difficulty, and here is where a classical analogy may be helpful. History books written by professional historians are often quite different, but this by itself does not mean they are defective. The historian chooses material that provides a coherent narrative for the time period of interest while still remaining consistent with the facts insofar as they are known. No one would expect a history of the United States to cover the same territory as a history of Great Britain. The historian has Liberty to choose material that best serves his purpose, and there seems no reason to deny the quantum physicist a similar freedom.
To be sure we expect different histories of the world to be consistent to the extent that they deal with the same events, and it seems reasonable to expect quantum descriptions to satisfy similar conditions of consistency. And indeed they do. Given the same input data, without which one cannot assign probabilities, two consistent families of histories that include this data will always assign the same probabilities to other events that occur in both families; this is a consequence of the internal consistency of the histories approach discussed earlier in Sec. 4.1.

The example discussed earlier in which Alice prepares $S_x = +1/2$ and Bob measures $S_z = -1/2$ may help to illustrate this point. There are two incompatible consistent families, one containing $S_x$ and the other $S_z$ at the intermediate time $t_1$, and in each family one draws some conclusion about the spin angular momentum at $t_1$. The conclusions are indeed different, but they are not contradictory, for events involving $S_x$ cannot be combined with those involving $S_z$. A more striking example is provided by the three-box paradox of Aharonov and Vaidman [43], discussed in detail in Sec. 22.5 of [20], where again the single framework rule removes the apparent contradiction resulting from applying classical reasoning in a situation where it violates quantum principles.

To be sure, some may want to insist that “there just has to be a single history a single true story.” This is as much a philosophical position as a scientific objection, which does not mean it can simply be dismissed. The discussion in the following section is an attempt to get to the bottom of what is here at issue.

### 4.5 Truth and reality

The preceding examples and discussions help identify what is perhaps the central conceptual difficulty of the new logic. In quantum mechanics interpreted in this way any description of nature must be formulated using a framework of commuting Hilbert space projectors which can be assigned probabilities in a consistent fashion. Consistency is ensured by the single framework rule, which prohibits combining incompatible frameworks or incommensurate families of histories. Consequently, probability distributions are relative to frameworks; there is not a single probability distribution that can be used for every framework or every family of histories.

In a probabilistic model the limiting cases of probability 1 and 0 correspond to statements which in propositional logic are true and false, respectively. This same interpretation is employed in histories quantum mechanics. But then “true” and “false” must be understood relative to a framework. There is no single universally true state of affairs in histories quantum mechanics. This has undoubtedly been a major stumbling block standing in the way of its more general acceptance by the physics community, despite the fact that it provides a consistent resolution of all the usual quantum paradoxes, something which cannot be said of any other interpretation of quantum mechanics currently available. And it seems to be at the heart of Mermin’s objection, see Sec. 1.

Indeed, there is a deep-rooted faith or intuition, shared by scientists as well as the ordinary man on the street, that at any point in time there is a particular state of affairs which exists, which is “true”, and to which every true description of the world must conform. No one claims to know what this exact truth is, and indeed it must, if it exists, be beyond human knowledge. Let us call this belief the principle of unicity. Calling it into question seems heretical, contrary to both common sense and sound science. Nonetheless, in the quantum world it does not seem to be valid.

To see how and why it breaks down, recall that a classical space can be divided into finer and finer regions until one arrives at a single point representing the precise state of a physical system. All subsets of the phase space that contain this point represent properties that are simultaneously true; their intersection is the point itself, which is the ultimate truth. Hence classical mechanics provides a convenient mathematical picture for visualizing unicity, and the enormous success of classical mechanics lends support to its validity.

In the quantum Hilbert space the description of properties is “quantized”: subspaces have integer dimensions, and the smallest subspace representing a property which could possibly be true has dimension 1. So this ought to be the quantum analog of a single point in the classical phase space. But then one finds, as discussed in detail in Sec. 3.1, that at this level the structure of Hilbert space is significantly different from that of classical phase space. In particular one has properties that are incompatible—their projectors do not commute—in a manner which is completely foreign to classical physics. The obvious extrapolation of classical unicity, the notion of a single true property, runs into the logical difficulties understood by Birkhoff and von Neumann. Abandoning unicity, as in the new logic, may not be the only way to solve the problem, but simply ignoring it, which is what one finds in much modern work on quantum foundations, is not likely to result in progress.
To put the matter in a slightly different way, if we assume that the world is governed by classical principles we eventually run into disagreement with experiment. However, by assuming that it is governed by quantum principles, with quasiclassical frameworks explaining the success of classical physics at the macroscopic level, we can begin to understand the deep intuition that lies behind the notion of unicity, based on everyday human experience in the classical world. But at the same time we can understand why and in what circumstances this intuition breaks down.

To those who claim that classical notions of truth and reality are necessary truths, which are self evident, the appropriate response is to say that they are just as self evident as the fact, accepted by our intellectual ancestors, that the earth is at rest at the center of the universe. The history of science is marked by a set of important revolutions in thought in which things thought to be intuitively obvious and self-evident have been replaced by alternative explanations in better agreement with empirical observation. Why should quantum theory be different?

Abandoning unicity is not equivalent to abandoning logical thought, and it is worth stressing that the histories approach to quantum interpretation, the new quantum logic, is entirely consistent provided one pays attention to the rules, discussed with various examples in Sec. 3 and summarized in Sec. 4.1 for constructing quantum descriptions. Nor does abandoning unicity mean that one has to give up on physical intuition about what is “going on” in the quantum world. True, classical thinking is no longer satisfactory in the quantum domain, and the physicist has to develop an appropriate quantum intuition in its place. This takes effort, but it is not impossible. Nor does abandoning unicity require giving up the notion of a real world “out there”, one whose existence is independent of our thoughts, wishes, and beliefs. What the development of quantum mechanics and its consistent interpretation using the new logic indicates is that the certain features of this reality differ from what was thought to be the case before quantum theory was developed and successfully applied to understanding phenomena in the microscopic world.

5 Conclusion

The fundamental thesis of this paper is that the conceptual difficulties of quantum foundations listed in Table 1 and discussed in Sec. 2 can be, and in fact have been, successfully resolved using the new quantum logic embodied in the histories approach, as summarized in Sec. 3. The new logic, while not as radical as the older quantum logic, still represents an important break with ideas which have long seemed central to human thought in general and to the natural sciences in particular. The most significant changes, and the conceptual difficulties that they in turn give rise to, are indicated in Table 2 and discussed in Sec. 4. However novel it may seem, it is worth remembering that the new logic is consistent both with logical thought and with an independent reality that does not require human thought (or measurements) to bring it into existence.

There might be yet better ways of resolving quantum conceptual difficulties than those provided by the histories approach. What distinguishes it from alternative proposals at the present time is the combination of (i) the central role played by the noncommutation of quantum operators, in particular projectors, in the conceptual foundations of the subject; (ii) its insistence that all quantum time development is stochastic, not just when measurements take place; (iii) its success in resolving not just one or two, but all of the standard quantum paradoxes. In particular, the supposed superluminal influences that have infested quantum foundations for the last 50 years and make some other interpretations of quantum mechanics difficult to reconcile with relativity theory are absent from the histories approach; such influences are nothing but fudge factors needed to compensate for a lack of understanding of what quantum measurements measure, and the failure to use a fully consistent set of quantum principles when discussing entangled states.

In the end the acceptance or rejection of a set of ideas by individual scientists and by the scientific community is a matter of scientific judgment; there are no overwhelming arguments that establish the proof of any scientific theory. That scientists today believe that the earth moves, around its axis and around the sun, rather than lying fixed in place at the center of the universe, is a consequence not of rigorous logical proofs of the sort that appeal to some philosophers, but instead the fact that this way of looking at things clears up a number of conceptual difficulties in a way much simpler and seemingly more satisfactory than can be done by assuming the earth is fixed. The author believes that the same is true of the new quantum logic, and welcomes critical scrutiny by those who are willing to examine it in detail before publishing their conclusions. That the histories approach to interpreting quantum theory is radical must be acknowledged. This does not mean it is wrong.
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