Proposed experiments to detect keV-range sterile neutrinos using energy-momentum reconstruction of beta decay or K-capture events

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Abstract
Sterile neutrinos in the keV mass range may constitute the galactic dark matter. Various proposed direct detection and laboratory searches are summarized. It is suggested that a promising method for searching for keV sterile neutrinos in the laboratory is complete energy-momentum reconstruction of individual beta-decay or K-capture events, by measuring the vector momentum of all decay products from atoms suspended in a magneto-optical trap. Reconstruction of the ‘missing mass’ would isolate any keV-range sterile neutrinos as a separated population. A survey of suitable nuclides is presented, together with the measurement precision required in a typical experimental configuration. Among the most promising are the K-capture nuclides $^{131}$Cs, which requires measurement of an x-ray and several Auger electrons in addition to the atomic recoil, and $^7$Be which has only a single decay product but needs development work to achieve a trapped source. A number of background effects are discussed. It is concluded that, with current time-of-flight precision, sterile neutrinos with masses down to the 10 keV region would be detectable with relative couplings $10^{-5}–10^{-6}$ in a 1–2 year running time, and with foreseeable future upgrades eventually able to reach coupling levels down to $10^{-10}–10^{-11}$ using high-population trapped sources.

1. Introduction

Weak interactions create neutrinos in one of three leptonic flavors: electron neutrinos, muon neutrinos, and tau neutrinos. These do not have individual masses but are described through a mixing matrix into three eigenstates of definite mass [1]. Neutrino oscillation experiments provide mass squared differences between these but do not fix the absolute mass scale. However various cosmological observations indicate that the sum of the three mass eigenstates is below 1 eV [1, 2]. (Note that particle physics ‘natural’ units $\hbar/2\pi = c = 1$ are used throughout this paper, with mass, momentum, and energy all in electron volts eV.)

For a number of theoretical reasons discussed in detail in [1, 2] it is believed that the neutrino family is incomplete, and that there exist one or more additional neutrinos that interact with the known neutrinos with a much lower coupling strength. These are known as ‘sterile neutrinos’. There are many proposed theoretical models of these, reviewed in detail by Abazajian [1], examples being the ‘3 + 1’ model, which contains the three standard neutrinos plus one sterile neutrino, and the ‘νMSM model’ (‘3 + 3’), which adds one sterile neutrino per lepton generation. Neutrino mixing is then represented in those examples by an extension of the mixing matrix from the standard $3 \times 3$ matrix to a $4 \times 4$ or $6 \times 6$ matrix with matrix elements $U(\nu_L, \nu_n)$ linking lepton neutrino flavor states $L$ to neutrino eigenstates $n$.

The large number of theoretical models predict, or are sufficiently flexible to allow for, a wide range of sterile neutrino masses, from the sub-eV level to the GeV level and higher [1, 2], but there has been specific interest in models allowing the production in the early universe of sterile neutrinos in the keV mass range as a possible candidate for the galactic dark matter, as discussed further in section 2 below.

It is important to note that the experimental technique investigated in this paper is independent of any theoretical model, and assumes only that the electron neutrino $\nu_e$ produced in beta decay is a mixture of the...
three standard neutrino mass eigenstates plus one (or more) additional sterile neutrino eigenstates \(\nu_s\) with relative amplitudes \(U(\nu_e, \nu_s)\), and observable probabilities \(|U(\nu_e, \nu_s)|^2\). This probability is also referred to as \(\sin^2 \theta_s\), where \(\theta_s\) is a mixing angle.

The present study is directed towards the experimental possibility of detecting keV-mass neutrinos through their emission as rare events in atomic beta decay or K-capture. By measurement of the momenta of all decay products, including the recoil nucleus, the mass of the missing neutrino could be reconstructed. This experimental principle was originally proposed in 1992 by Cook et al.\(^4\) and by Finocchiaro and Shrock \(^4\) following contemporary observations of an apparent distortion of the tritium beta spectrum suggestive of a neutrino of mass 17 keV. This idea was also foreshadowed 50 years earlier by measurements of atomic recoil energy to establish the form of the weak interaction \(^5\). Mass reconstruction was subsequently used by Hindi et al.\(^6\), using K-capture in \(^{37}\)Ar to set relative sterile neutrino coupling limits \(\sim 10^{-2}\) in the mass range 370–640 keV, and by Trinczek et al.\(^7\) using beta decay of \(^{38m}\)K to set coupling limits \(\sim 10^{-5}\) in the mass range 0.7–3.5 MeV. But for the reasons discussed below in section 2, there was increasing interest in the possible existence of sterile neutrinos at much lower masses \(\sim 5–10\) keV, and lower coupling levels \(< \sim 10^{-6}\), leading Bezrukov and Shaposhnikov \(^8\) to investigate the possibility of more precise reconstruction of tritium beta decay events, together with higher event rates, in order to reach these lower values. This was nevertheless seen as a challenging task and no experiments to reach keV-range neutrino masses have been attempted using the reconstruction technique. The object of the present paper is to investigate the potential capability of this method in greater practical detail.

2. KeV sterile neutrinos in astrophysics

In addition to the potential importance of sterile neutrinos in particle physics, their existence would have important consequences in astrophysics, reviewed in detail by Abazajian \(^9\). In particular, they could be created in the early universe and accumulate in sufficient numbers to form the dark matter that is inferred kinematically to dominate the mass of all galaxies, including our own. Direct searches for dark matter particles have been in progress for over three decades, the majority using underground detectors to search for hypothetical weakly interacting massive neutral particles in the mass range 1–1000 GeV, in particular the lightest hypothetical neutral particle predicted by supersymmetry theory. However, at the time of writing these searches continue to give null results, and this, along with the absence so far of evidence for supersymmetry at the Large Hadron Collider, has given rise to increasing interest in alternative candidates for the dark matter, including sterile neutrinos \(\nu_s\). Although these would be unstable (decaying into a standard neutrino and x-ray via an intermediate W and lepton), for a sufficiently small value of \(\sin^2 \theta_s < 10^{-4}\), the lifetime for this process, for a neutrino mass in the keV mass range, would exceed that of the Universe by many orders of magnitude \(12–14\). Thus sterile neutrino dark matter would be effectively stable on the scale of the Universe lifetime.

Specific estimates of the sterile neutrino mass range consistent with dark matter formation are 0.1–10 keV by Shi and Fuller \(^14\) and 7–36 keV by Cherry and Hortuchi \(^22\), and a lower mass limit of 0.4 keV follows from applying the standard Tremaine and Gunn \(^23\) phrase space limit to the dark matter density in dwarf galaxies. It is remarkable that these mass estimates overlap the mass region that could be verified in laboratory beta decay experiments, i.e. lie within the Q-value, or energy release, in these decays. The coupling \(\sin^2 \theta_s\) needs to be typically smaller than \(10^{-3}–10^{-4}\) to be consistent with the combined lower limits versus \(m_s\) profile shown in figure 1, from a number of individual beta decay experiments (referenced in \(^{24}\)), based on the analysis principle discussed in section 3 below.

However, on the specific assumption that sterile neutrinos constitute 100% of the dark matter, substantially lower limits to \(\sin^2 \theta_s\) can be obtained by searching for x-ray signals from sterile neutrino decay in galaxies and galaxy clusters known to contain a large amount of dark matter, since the decay photon would, by momentum conservation have an energy half that of the sterile neutrino mass.

In the absence of such signals the estimated dark matter density can be used to place limits on the coupling versus mass, as shown in figure 2 (in this case showing limits on \(\sin^2 (2\theta_s)\)). The limits are proportionally less stringent if sterile neutrinos contribute only some fraction of the dark matter, and do not provide any limits in the case that sterile neutrinos do not contribute at all to the dark matter.

Figure 2 also shows one apparent significant signal region, at \(M_{DM} = 7.1\) keV and \(\sin^2 (2\theta_s) \sim 10^{-10}\) which has been suggested as a possible dark matter signal, though at the time of writing its interpretation remains controversial \(^9\). If this signal is verified by further and more accurate observations, it is clear that this low coupling level will present a major challenge to confirm the existence of this particle in terrestrial experiments. Nevertheless, it is proposed in this paper that a sufficiently advanced version of the mass reconstruction method could in principle achieve this. If the x-ray signal proves to have an alternative explanation, then the neutrino
physics case for sterile neutrino searches remains, and we show in this paper that large improvements in the existing limits on keV-range sterile neutrinos would be experimentally possible.

### 3. Detection of keV-range sterile neutrinos

There are two possible types of detection method for sterile neutrinos in the keV mass range:

(a) if sterile neutrinos contribute to the galactic dark matter, then direct detection may be possible in addition to their decay to x-rays;

(b) whether or not sterile neutrinos form the dark matter, experiments involving weak decay could detect rare events with emission of sterile neutrinos.

This paper is concerned principally with laboratory experiments based on method (b), but we first summarize published work on method (a), noting first that no existing dark matter detectors would be sensitive to direct collisions from sterile neutrino dark matter, because the nuclear recoils from keV-mass sterile neutrinos would be too low in energy ($\sim 10^{-2}$ eV). Thus new methods are needed.

In principle, direct detection would be possible by coherent scattering from atomic electrons, by the two processes shown in figure 3. This has been investigated by Ando and Kusenko [25], showing that the scattering would transfer sufficient energy to produce ionization in a detector material. However, the rate would be only 1
event/year/kiloton for a sterile neutrino mass $m_s = 7$ keV and coupling $\sin^2 \theta = 10^{-9}$, with proportionality to $\sin^2 \theta$ and $m_s^2$. Thus this would be impractical in the foreseeable future.

A related possibility is that the decay processes producing x-rays from sterile neutrino dark matter (figure 2) could also in principle be seen in a terrestrial experiment consisting of a large evacuated volume with its surface covered with low background x-ray detectors. But the required detector volume and area would be impractically large. Assuming a dark matter density 0.3 GeV cm$^{-3}$, $\xi = 10^{-9}$ and $m_s = 7$ keV, in a km$^3$ volume with its 6 km$^2$ surface fully covered with zero background detector, less than ten 3.5 keV x-rays would be produced per year, the number in this case being proportional to $m_s^5$.

A further theoretical possibility, using macroscopic coherence, is that of coherent scattering from an array of sub-micron granules suspended in an ‘optical honeycomb’. This currently lies beyond existing technology but remains a future possibility with increasing proficiency in nanotechnology and optical suspension [26, 27].

An apparently more promising method of direct detection would be by inverse beta decay. Any beta decay process $A \rightarrow B + e^- + \nu$ can be inverted to $\nu + A \rightarrow B + e^-$, giving events beyond the normal beta spectrum end point, and fully separated from the latter by the neutrino mass. An example for Tritium decay is illustrated in figure 4.

This method was discussed over 30 years ago by Irvine and Humphries [28] as a possible means of detecting the relic background of standard neutrinos but requiring a target of at least 1 g tritium (subsequently corrected to 100 g tritium [30]), many orders of magnitude greater than the targets of a few $\mu$g typically in use at that time. Thus detecting the relic neutrinos in this way seemed out of reach in the 1980s. Now however, the PTOLEMY project [29] is planning to construct the required 100 g tritium target as a single atomic layer on the surface of an array of graphene layers of total area 10$^4$ m$^2$, a reminder that an apparently impractical idea can sometimes become feasible within a generation.

Application of inverse beta decay to sterile neutrino dark matter detection has been considered by a number of authors [31–38] estimating the target mass needed to obtain a significant peak of events beyond the beta spectrum end point. Li and Xing [36] find the optimum nuclides to be tritium $^3$H (lifetime $\sim$ 12 years, beta end point 18 keV) and $^{106}$Ru (lifetime $\sim$ 1 year, beta end point 39 keV). To obtain a significant number $\approx 10$ events yr$^{-1}$ would require target masses 10 kg $^3$H or 1000 kg $^{106}$Ru, the latter of course needing annual replacement to maintain the decay rate. The targets would need to be in the form of a gas or gaseous compound or thin surface layers to allow the electrons to escape to a (low background) detector. These formidable requirements are not feasible in the near future, but remain a future possibility for direct sterile neutrino dark matter detection. Subsequent to the above
studies it has been pointed out that stable \(^{163}\text{Dy}\) could also in principle be used as a direct detection target releasing an electron for incident sterile neutrino masses greater than the \(^{163}\text{Dy} \rightarrow ^{163}\text{Ho}\) Q-value, 2.8 keV, and this could avoid the problems of a radioactive target \[^{39}\].

For laboratory detection, the most familiar existing type of search for sterile neutrinos in the range \(1 \text{--} 1000\) keV is that of a possible distortion of the electron spectrum from beta decay of various nuclides. This could result from a small admixture of events involving the emission of a higher mass neutrino, giving a slight distortion of the electron spectrum (known as a ‘kink’) at a distance from the end point approximately given by the sterile neutrino mass, as illustrated in figure 5. This plot exaggerates the effect for illustration, and to change the spectrum by a measurably significant amount, the fraction of decays giving a higher mass neutrino needs typically to be \(>10^{-4}\) in most experiments, giving rise to the combined limits shown above in figure 1 for the experiments referenced in \[^{24}\].

To obtain sensitivity to smaller values of the mixing parameter using this method, a stronger beta source is needed, giving many orders of magnitude more events/year, along with a higher energy resolution to search for significant distortions in the spectrum. The largest and most sensitive such project, currently under construction, is the KATRIN spectrometer, 70 m total length, with a spectrometer section 24 m \(\times\) 10 m diameter \[^{2,40,42}\]. This is designed to achieve an energy resolution \(\sim 1\) eV, with the objective of measuring or placing limits on the largest of the standard neutrino mass eigenstates from the events at the end point of the electron spectrum. Extending this to integration over the whole spectrum could achieve sensitivity to any sterile neutrinos with mass \(<18\) keV \[^{41-43}\]. Figure 6, taken from \[^{42}\] summarizes the expected initial sensitivity to sterile neutrino masses and couplings, for two different analysis techniques, together with the future improved sensitivity possible with an upgraded detection system capable of a counting rate \(\sim 10^{10}\) counts per second to permit a full statistical analysis over the whole spectrum rather than just the end point. If this is achieved, along with a planned upgrade in tritium source luminosity to \(10^{11}\) decays s \(^{-1}\), a 90% confidence exclusion limit \(\sin^{2}\theta\) \(<10^{-7}-10^{-8}\), for masses \(1-17\) keV, could be achieved in a three-year active running time using two alternative spectrum analysis methods. This upgraded detection and readout system has the name TRISTAN \[^{44}\].
4. Isolation of a population of sterile neutrino events by energy-momentum reconstruction

It is clear that in preference to attempting to detect a small sterile neutrino event population within a much larger population of standard neutrino events, it would be preferable to isolate the individual sterile neutrino events as a separate population. This is the objective of the principle of full energy-momentum reconstruction already outlined in section 1 above and the subject of [3–8]. Two variations of this are possible, using either beta decay or K-capture nuclides, and are shown in figure 7. In figure 7 (a) an electron and anti-neutrino are emitted with continuous energy spectra. In figure 7 (b) an electron is captured from an atomic shell, usually the K-shell, to together with emission of a nearly mono-energetic neutrino. In each case the weak decay is accompanied, as indicated in figure 7 by a recoil of the nucleus, and several other particles.

The formulae accompanying the figure show how the square of the neutrino mass can be reconstructed from vector momentum measurements of the emitted particles, needing both position sensitive detectors and time-of-flight, the decay nuclide being chosen to provide an additional photon as time trigger. The precision of mass reconstruction will depend principally on the combined errors in the vector momenta. In addition to the width of $m_\nu^2$ due to measurement errors, there is a ‘natural’ line width for the emission of Auger and x-rays to be considered, but which is shown in section 6 to be a negligible contribution in this case.

In case (a) a nuclide needs to be chosen that provides a trigger by emitting a simultaneous nuclear gamma, of known energy-momentum but whose emission direction must still be measured. In case (b), the K-capture is followed within $<1$ ps by the filling of the atomic vacancy by emission of an x-ray, of known energy-momentum but also needing measurement of its emission direction. Although K-capture appears to have the considerable advantage of not requiring momentum measurement of the electron, the filling of the K vacancy leaves another unfilled vacancy in a higher shell, and the resulting rearrangement of atomic states results in the emission of several Auger electrons within a further $\sim 10^{-13}$–$10^{-15}$ s. These are of relatively low kinetic energy but have sufficient momenta to contribute significantly to the mass reconstruction and all need to be collected and measured. The two alternative methods will be discussed separately in sections 5 and 6 below.

In order to discuss the sterile neutrino mass range achievable by energy-momentum reconstruction, it is necessary first to discuss the spatial and temporal measurement precision attainable with available techniques. Since measurement of both momentum and direction are necessary, the former requires accurate timing over a
suitable path length, while the latter requires also position sensitivity to define the 3D emission directions of all the particles. The ability to do the latter results from the development over the past 30 years of the Cold Target Recoil Ion Momentum Spectroscopy (COLTRIMS) technique illustrated in figure 8.

Full details and examples of this technique are given in reviews [45] and [46]. In the example illustrated, the reaction products emitted from a small volume, defined by the interaction of two atomic beams in high vacuum, are directed on to position-sensitive detectors, using magnetic and electric fields to increase the solid angle captured. In the case of the recoil ion, the detector is an Micro-Channel Plate (MCP) preceded by a ~2 kV grid to accelerate the ion (since the latter will have a very low, ~eV, initial recoil kinetic energy) on to the MCP surface. An MCP can also be used to detect both the position and arrival time of one or more electrons, the principal alternative being a traditional multi-anode photomultiplier tube, or a multi-anode silicon photomultiplier (SiPM) or a Multi-Pixel Photon Counter. For the neutrino mass sensitivity estimates in this paper, the position and timing resolutions achievable with commercially available components and systems are assumed [47], giving a typical rms timing resolution ~0.2 ns (hence a combined time of flight resolution ~0.3 ns for the complete path), and a typical position resolution ~50 μm.

In the case of the decay products from a single atomic species, it would be possible in principle to define the source by exposing just a short length of an atomic beam, but a preferable solution has been to create a small stable volume of atoms in a Magneto-Optical Trap (MOT). An MOT confines and cools the atoms at the intersection of three retro-reflected laser beams and a quadrupole magnetic field provided by a pair of anti-Helmholtz coils (figure 9). Experiments with trapped atoms have been reviewed by Behr and Gwinner [48] and by Melconian [49]. If such a trap is used as the source in a configuration analogous to COLTRIMS, the technique is often referred to as MOTRIMS [50–53].

From the viewpoint of sterile neutrino searches, there are several important interrelated parameters for the cloud of trapped atoms:

(a) The effective atom temperature \( T \) should be sufficiently low to avoid the initial atom velocity contributing a significant time of flight fractional error (typically required to be < 10⁻⁴). Atom temperatures of 25–100 μK or below are possible [54] corresponding, for an atom with atomic number \( A \sim 100 \), to an rms velocity \( \sim 5–10 \text{ cm s}^{-1} \). This is small compared with typical recoil velocities \( \sim 10^2–106 \text{ cm s}^{-1} \), but can nevertheless have a significant effect on the rms spread of the reconstructed distribution of \( m_\nu^2 \).

(b) The rms radius \( \sigma \) of the atom cloud needs to be < ~10⁻³ \( L \), where \( L \) is the chosen recoil path length. Because the practical maximum peak trapped atom density \( \rho_0 \) is typically no higher than ~3 × 10⁸ atoms mm⁻³ [54], \( \sigma \) will increase with the total number \( N_T \) of trapped atoms, and also with the required trap lifetime \( \tau \) (s). Assuming a three-dimensional Gaussian distribution, the trap parameters are related approximately as:

\[
\sigma_3(\text{mm}) \sim (N_T / 10^9)^{0.33}
\]

showing that \( \sigma_3 \) can range typically from ~0.1 mm at \( N_T = 10^6 \) to ~2 mm at \( N_T = 10^{10} \).
for a two-dimensional projections \( \sigma_2(\text{mm}) \sim [N_T / 10^9]^{0.33} \times [2 / 3]^{0.5} \) (4.1b)

for a one-dimensional projections \( \sigma_1(\text{mm}) \sim [N_T / 10^9]^{0.33} \times [1 / 3]^{0.5} \). (4.1c)

Equation (4.1b) will be appropriate when considering source errors in measured emission angles.

To be sensitive to low values of the sterile neutrino coupling, the total number of trapped atoms needs to be as large as possible to maximize the number of decays per year. Camara et al [55] achieved an MOT loading of \( N_T = 1.3 \times 10^{11} \) Rb atoms, finding a somewhat larger exponent, \( \sim 0.35–0.39 \), in equation (4.1a).

(c) For a nuclide with a half life \( \tau_{1/2} \) (days) the decay rate per atom is \( \lambda = 0.69 / \tau_{1/2} \) per day and hence a source decay rate

\[
\frac{dN}{dt} = 0.69N_T / \tau_{1/2} \text{ per day}
= 250N_T / \tau_{1/2} \text{ decays per year (of active running time).}
\]

Thus nuclides with low values of \( \tau_{1/2} \) (e.g. between 3 and 100 d) are advantageous in providing faster accumulation of data, they would require frequent replenishment. For traps fed continuously from an accelerator beam, shorter half lives (< 1 d) would become usable.

(d) Although the recoil momentum could be calculated simply from the time-of-flight \( t_F \) between the trigger and the MCP signal, over a known path length \( L \), it is usual to add an axial electric field of a few volts cm\(^{-1}\) to extract the ions from the source region. This has four advantages:

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**Figure 9.** (From [53]). Schematic arrangement of magnetic fields and three retro-reflected laser beams, to produce a spherical trapped cloud of cold atoms in high vacuum.

**Figure 10.** Example of computed ion trajectories with axial electric field \( \sim 2 \) V cm\(^{-1}\), showing collection of recoils from both hemispheres. The MCP is located on the left hand side, with paths emitted towards the MCP in blue, paths emitted away from MCP shown in red.
(i) To increase the solid angle for capture of the recoil ions, through the focusing effect of the axial electric field.

(ii) To provide a controllable proportionality between recoil momentum differences $\Delta p_N$ and time of flight differences $\Delta t_F$. Since the recoil is non-relativistic, simple kinematics gives (to a sufficiently high approximation) the following linear relationship between momentum differences $\Delta p_N$ (keV), time of flight differences $\Delta t_F$ (s) and a (uniform) electric field $F$ (kV cm$^{-1}$) [45, 46]:

$$\Delta p_N = c_n . F . \Delta t_F, \quad (4.3)$$

where $c = 3 \times 10^{10}$ cm s$^{-1}$ and $F = V(kV)/L(cm)$, $n =$ ion charge multiplicity.

(iii) To provide compensation for the time of flight differences between decays from the front and back of the significant diameter source atom cloud. This is achieved by providing a uniform electric field over a distance $L_1$ from the source, followed by a field free region of length $L_2 = 2L_1$ [56]. The total flight time is then to a first approximation independent of small changes in $L_1$ arising from the finite source diameter. A more exact criterion, for field length $L_1$, field free length $L_2$, initial recoil velocity $u$ and final velocity $v$, is:

$$L_1/L_2 = (v^2 - u^2)/2v^2. \quad (4.4)$$

So that the factor 2 ratio applies if $u \ll v$, otherwise (4.4) can be used.

This longitudinal compensation is called 'McLaren focusing' or 'Time focusing'. Transverse focusing configurations have also been studied [45, 46, 57] in order to maximize overall momentum resolution.

(iv) To capture ions emitted in both the facing and back hemispheres (figure 10). For each there is a computable unique correspondence between the 2D detection co-ordinates and the polar and azimuthal emission angles, but the correspondence differs for front and back hemispheres. This gives a two-fold ambiguity in the emission angles, which can subsequently be resolved in the mathematical reconstruction of the event, one of the two alternatives giving an unrealistic value of reconstructed square mass.

5. Choice of nuclides with a beta spectrum

Figure 7 shows the two distinct types of nuclear beta decay available for a sterile neutrino search: (a) emitting a free electron, sometimes with a nuclear gamma, and (b) electron capture by the nucleus (usually from the atomic K-shell) followed by emission of an atomic x-ray and low energy Auger electrons. Of type (a), tritium has been adopted for measuring the mass of standard neutrinos and, as discussed in section 3 has also been suggested for sterile neutrino searches. However, for the full reconstruction technique, tritium does not provide a clear trigger to define $t = 0$ for time of flight measurements, and also would not detect a sterile neutrino with a mass larger than its 18 keV end point. This section considers the optimum choice of type (a) nuclides with a clear trigger and a wider mass range. Numerical details of nuclide decay schemes were obtained from the standard tables of Firestone and Shirley [58].

There are 147 beta decay nuclides with half-lives between ~3 d and 2 years. 65 of these decay by electron capture and are considered in section 6. The remaining 82 candidates were reduced by the requirement of a single prompt (~ps) gamma for use as a trigger, and rejecting the following.

(a) Positron decays.
(b) Decays to the nuclear ground state giving no gamma.
(c) Delayed gammas.
(d) Nuclear cascade giving multiple gammas.
(e) Complex sets of several beta decays and gammas.
(f) Gamma probability <0.1%.

This left 17 candidates, listed in table 1 in order of decay rate (per atom per year) obtained by multiplying the intrinsic decay rate equation (4.2) by the fraction of decays producing the trigger gamma.

Table 1 shows also the beta end point energy, gamma energy, and $Q$ value for the decay.

Not all of these are suitable for trapping in an MOT. Aside from hydrogen and the inert gases, the elements so far known to be feasible for trapping in an MOT are listed in table 2.
Some of these, Yb, Ag, Dy, Er, Tm, have been loaded into an MOT more efficiently via a buffer gas cell which has been used for many previously difficult compounds and radicals and may allow more elements to be considered for MOT sources. However, for the present purpose only those listed in table 2, already demonstrated to be feasible, will be considered. This in turn means that only five of the elements in table 1 are suitable for an MOT source, as indicated in the final column. Of those, three are in the lower part of the list, and thus less favorable from the viewpoint of event rate, while $^{225}$Ra has the disadvantage of a more difficult supply route via thorium. This leaves $^{203}$Hg, which has a 280 keV trigger gamma, a 212 keV beta endpoint, a $Q$ of 492 keV, and well-suited for loading into an MOT with high efficiency. Thus $^{203}$Hg provides a suitable example for assessing the feasibility of reaching keV-range sterile neutrino masses, through the decay process of figure 7(a). The latter shows the reconstruction of the neutrino mass $m_{\nu}^2$ via the relation $m_{\nu}^2 = (\text{total energy})^2 - (\text{total vector momentum})^2$, written in terms of measurable quantities (again using units $h/2\pi = c = 1$ with energy, momentum, and mass are all in keV):

$$m_{\nu}^2 = [Q - E_\gamma - E_{ek} - E_N]^2 - [\mathbf{p}_e + \mathbf{p}_k + \mathbf{p}_N]^2,$$

(5.1)

(where the first bracket consists of scalars and the second bracket vectors)

Since the emitted gamma energy/momentum is known, equation (5.1) requires the measurement of the other two momenta by time of flight, and the three momentum directions by position-sensitive detectors. Differentiation of equation (5.1) enables the rms spread of $m_{\nu}^2$ to be directly related to the rms spread of any of the measured parameters, and hence to the required measurement precision needed to achieve sensitivity to a given value of neutrino mass. Beginning with the time of flight measurements, and using the approximate equality of the two terms on the right hand side of equation (5.1):

$$d(m_{\nu}^2) \approx 2[Q - E_\gamma - E_{ek} - E_N] dp_N \approx 2[210 \text{ keV} - E_{ek}] dp_N.$$

(5.2a)

### Table 1. Beta decay nuclides with continuous electron spectrum, giving a single prompt gamma trigger, listed in order of triggered decay rate/year/atom (for number in MOT maintained constant).

| Z  | A  | Half life | Q | Trigger gamma | Fraction | Beta | Trigger rate |
|----|----|----------|---|--------------|----------|------|-------------|
| 79 | Au | 198      | 2.7 | 1372        | 412      | 0.98 | 960         |
| 53 | I  | 131      | 8.0 | 971         | 361      | 0.81 | 610         |
| 79 | Au | 199      | 3.1 | 453         | 208      | 0.21 | 245         |
| 88 | Ra | 225      | 14.9| 356         | 40       | 0.53 | 316         |<– |
| 71 | Lu | 177      | 6.7 | 498         | 113      | 0.20 | 385         |
| 41 | Nb | 95       | 35.0| 925         | 765      | 1.00 | 160         |
| 60 | Nd | 147      | 11.0| 896         | 92       | 0.28 | 804         |
| 80 | Hg | 203      | 46.6| 492         | 280      | 0.81 | 212         |<– |
| 40 | Zr | 95       | 64.0| 1124        | 758      | 0.98 | 366         |
| 58 | Ce | 141      | 32.5| 580         | 145      | 0.48 | 435         |
| 79 | Au | 196      | 6.2 | 586         | 426      | 0.08 | 260         |
| 26 | Fe | 59       | 44.5| 1565        | 1        | 0.43 | 476/274     |
| 70 | Yb | 175      | 4.2 | 470         | 114      | 0.02 | 356         |<– |
| 58 | Ce | 144      | 285 | 319         | 2        | 0.23 | 186/239     |
| 69 | Tm | 170      | 129 | 968         | 84       | 0.03 | 884         |<– |
| 68 | Er | 169      | 9.4 | 351         | 8        | 0.002| 343         |<– |
| 45 | Rh | 102      | 207 | 1150        | 556      | 0.02 | 594         |

### Table 2. Elements suitable for a Magneto-optical trap.

| From [48] | Additional elements added subsequently [54] |
|-----------|---------------------------------------------|
| Li, Na, Mg, K, Ca, Cr, Rh, Sr, Ag, Ca, Ba, Er, Yb, Hg, Ra | Dy, Tm, Cd |

$^{235}$U, $^{238}$U, $^{238}$Pu, $^{239}$Pu, $^{239}$U, $^{235}$Th, $^{233}$Th, $^{232}$Th, $^{232}$Th, $^{233}$U, $^{235}$U, $^{237}$Np, $^{239}$Pu, $^{240}$Pu, $^{241}$Pu, $^{242}$Pu, $^{244}$Pu, $^{248}$Cm, $^{251}$Cf, $^{252}$Cf, $^{254}$Cf, $^{255}$Cf, $^{259}$Fm, $^{264}$Fm, $^{266}$Fm, $^{267}$Fm, $^{268}$Fm, $^{269}$Fm, $^{270}$Fm, $^{271}$Fm, $^{272}$Fm, $^{273}$Fm, $^{274}$Fm, $^{275}$Fm, $^{276}$Fm, $^{277}$Fm, $^{278}$Fm, $^{279}$Fm, $^{280}$Fm, $^{281}$Fm, $^{282}$Fm, $^{283}$Fm, $^{284}$Fm, $^{285}$Fm, $^{286}$Fm, $^{287}$Fm, $^{288}$Fm, $^{289}$Fm, $^{290}$Fm, $^{291}$Fm, $^{292}$Fm, $^{293}$Fm, $^{294}$Fm, $^{295}$Fm, $^{296}$Fm, $^{297}$Fm, $^{298}$Fm, $^{299}$Fm, $^{300}$Fm, $^{301}$Fm, $^{302}$Fm, $^{303}$Fm, $^{304}$Fm, $^{305}$Fm, $^{306}$Fm, $^{307}$Fm, $^{308}$Fm, $^{309}$Fm, $^{310}$Fm, $^{311}$Fm, $^{312}$Fm, $^{313}$Fm, $^{314}$Fm, $^{315}$Fm, $^{316}$Fm, $^{317}$Fm, $^{318}$Fm, $^{319}$Fm, $^{320}$Fm, $^{321}$Fm, $^{322}$Fm, $^{323}$Fm, $^{324}$Fm, $^{325}$Fm, $^{326}$Fm, $^{327}$Fm, $^{328}$Fm, $^{329}$Fm, $^{330}$Fm, $^{331}$Fm, $^{332}$Fm, $^{333}$Fm, $^{334}$Fm, $^{335}$Fm, $^{336}$Fm, $^{337}$Fm, $^{338}$Fm, $^{339}$Fm, $^{340}$Fm, $^{341}$Fm, $^{342}$Fm, $^{343}$Fm, $^{344}$Fm, $^{345}$Fm, $^{346}$Fm, $^{347}$Fm, $^{348}$Fm, $^{349}$Fm, $^{350}$Fm, $^{351}$Fm, $^{352}$Fm, $^{353}$Fm, $^{354}$Fm, $^{355}$Fm, $^{356}$Fm, $^{357}$Fm, $^{358}$Fm, $^{359}$Fm, $^{360}$Fm, $^{361}$Fm, $^{362}$Fm, $^{363}$Fm, $^{364}$Fm, $^{365}$Fm, $^{366}$Fm, $^{367}$Fm, $^{368}$Fm, $^{369}
From figure 11, 80% of the electron spectrum lies in the range $\sim 20$–$140$ keV, averaging $\sim 80$ keV. Thus

$$d(m^2_e)(\text{keV}) \approx 260(\text{keV})d\nu_e(\text{keV}).$$  \hspace{2cm} (5.2b)$$

This can be converted to the time of flight precision needed to achieve a given $d$ (mass), by noting that the atomic recoil is non-relativistic so that $p_N = M_Nv_N/c$ with $v_N = L_N/t_N$, where $t_N(s)$ is the travel time for a path length $L_N$(cm). This gives:

$$d(m^2_e)(\text{keV}) \approx 260(\text{keV}) \times M_N(\text{keV}) \times (v_N/c)^2 \times (c\,dt_N/L_N).$$  \hspace{2cm} (5.3)$$

For the average (and most probable [61]) opening angle between electron and neutrino $\sim 90^\circ$ and isotropic gamma emission, the average recoil momentum is $320$ keV, with $v_N/c \sim 1.6 \times 10^{-6}$, giving:

$$d(m^2_e)(\text{keV}) \approx 4dt_N(\text{ns})/L_N(\text{cm}).$$  \hspace{2cm} (5.4)$$

This immediately shows that there would be no problem in obtaining the required precision for the momentum of the recoiling atom, since a time-of-flight measurement accuracy $\sim 10$ ns and a path length $\geq 50$ cm will give a spread $\pm 1$ keV$^2$ in the reconstructed (mass)$^2$. However, the electron presents a more severe problem because of its much higher velocity over most of its kinetic energy spectrum.

Differentiating equation (5.1) with respect to $p_e$, this time with both rhs brackets contributing, equations (5.3) and (5.4) become, for an average electron energy $E_eK \approx 80$ keV, velocity $v_e/c \approx 0.50$ and path length $L_e$:

$$d(m^2_e)(\text{keV}) \approx 2[210 \text{ keV} - E_{eK}]/(1 + v_e/c)d\nu_e(\text{keV})$$

$$d(m^2_e)(\text{keV}) \approx 1.5.10^6dt_e(\text{ns})/L_e(\text{cm}).$$  \hspace{2cm} (5.5)$$

In this case, even assuming (in a search for a sterile neutrino mass of 7 keV) a (mass)$^2$ precision of $(50 \text{ keV})^2 \pm 20 \text{ (keV)}^2$ and a minimum timing error of 0.3 ns, the required time-of-flight path would be $L_e > 200$ m. Assuming this to be impractical, at least for initial experiments, the path length can be reduced to a more practical value if only the lowest 0.5% of the electron spectrum is used, i.e. up to an energy $\sim 2.5$ keV, with an average $v_e/c \approx 0.05$. Equation (5.6) then becomes:

$$d(m^2_e)(\text{keV}) \approx 1.5.10^6dt_e(\text{ns})/L_e(\text{cm}).$$  \hspace{2cm} (5.7)$$

Thus using only the lowest 0.5% of the beta spectrum would reduce the required path length to $\sim 2$ m, at the expense of factor 200 loss in number of decays processed.

But even if this is acceptable, or the search confined to limits on larger sterile neutrino masses, the problem of measuring the three-momentum angular directions needs to be assessed.

The right hand side of equation (5.1) contains three vector products of the form $2p_xp_y\cos \theta_{x,y}$. As before, differentiation gives a relation between $d$(mass)$^2$ and an angular measurement error $d(\cos \theta)$.

$$d(m^2_e)(\text{keV}) \approx 2p_xp_y\sin \theta_{x,y}d\theta_{x,y}$$ (for each of the angles $\theta_{N,x},\theta_{e,y},\theta_{N,e}$).  \hspace{2cm} (5.8)$$
Using average values $p_{Nav} \sim 320$ keV, $p_{pav} \sim 80$ keV, $p_{av} = 280$ keV, $(\sin \theta_{av})_{av} \approx 0.64$:

$$d\theta_{N,av} \approx 2.10^{-3}d(m_{e}^{2})d\theta_{av} \approx 2.10^{-5}d(m_{e}^{2})d\theta_{N,av} \approx 5.10^{-7}d(m_{e}^{2}).$$ (5.9)

Expressing this in terms of detection position errors $ds$ at a distance $R$, we have $dR = ds/R$ (radians). For the example of $R = 100$ cm, a (mass)$^2$ error $\sim 10$ keV requires position errors $\sim 0.1$ mm for detection of both recoil atom and electron for a determination of $\theta_{N,av}$. This is achievable with the high resolution ($<40 \mu m$) structure of an MCP detector surface, but remains a significant problem for the 280 keV gamma for which the detection by an MCP has only 1%–2% efficiency.

A better solution for the latter would be to use scintillating plates followed by SiPM with 50 $\mu m$ pixels [62]. Since sub-ns timing is also required, the scintillator needs to be either LYSO, which has a 47 ns decay but a high light output so provides a few photoelectrons $<1$ ns, or BaF$_2$ which provides both a fast (0.6 ns) light output pulse for timing plus a larger but slower (600 ns) light output pulse for position measurement [63]. For BaF$_2$ the fast pulse is in the UV wavelength (220 nm) necessitating coating the SiPM with TPB wavelength shifter to convert to a wavelength suitable for the SiPM [64]. However, for a 280 keV gamma from $^{203}$Hg the BaF$_2$ scintillator needs to be 6 mm thick to obtain even a 50% detection efficiency, giving another order of magnitude loss, in addition to the two orders of magnitude loss in processed decay events arising from the above limitation to the lowest $<1\%$ of the beta energy spectrum. For these reasons LYSO scintillator appears to be the clear overall preference for an $^{203}$Hg trigger.

A larger fraction of the electron spectrum may be possible by applying a longitudinal magnetic field to give the electrons a spiral path to confine larger emission angles. From figure 11, 10% of the decays have energies $0\sim20$ keV, with average $(v/c) \sim 0.2$. This would increase the numerical coefficient in equation (5.7):

$$d(m_{e}^{2})(keV^2) \approx 3.10^{-2}d(N, cm)/L(c/m).$$ (5.10)
in turn increasing the required path length to $L_{c} \sim 18$ cm. However, with an axial magnetic field of 10 Gauss, an electron of momentum 10 keV, emitted at an angle $\alpha \sim 75^\circ$ to the axis $(\cos \alpha \sim 0.3)$, would have a spiral path of radius $\sim30$ cm (from figure 12) but an axial $v/c$ reduced to $\sim0.05$, allowing the axial path length to be restored to 2 m. However, for smaller emission angles the axial/transverse speed is greater, giving an increased mass$^2$ error for this reduced path length.

So the possibility of shortening the geometric length of the system with an axial magnetic field applies, in this example, to only $\sim10\%$ of the beta spectrum. The magnetic field enables a larger solid angle to be collected and focused onto a detector at a given distance, but does not allow sufficient time-of-flight precision for most of the events. Moreover, since the emission angle is calculated from the radial co-ordinate $r$ of the detection point [46], a significant fraction of events, for which the number of spiral turns is within $\pm0.1$ of any integer and hence returning to points close to $r \sim 0$, cannot be utilized.

A background effect arising from the use of a gamma trigger would be the interaction of the nuclear gamma with atomic electrons, ejecting an electron by internal conversion or Compton effects. This also leaves a vacancy giving rise to atomic rearrangement and Auger electron emission. From the formula in [65] this effect should be $<10^{-3}$ of the decays, with these events sufficiently distinctive to be rejected. Also rejected would be a minority of events with associated atomic x-ray and Auger emission, which constitute $\sim20\%$ of $^{203}$Hg decays [66].

Some of the preceding problems might be alleviated by replacing $^{203}$Hg with a nuclide with a lower energy gamma trigger, having a smaller kinematic contribution, in particular Tm, Yb, and Er in table 1. These are also suitable for trapping in an MOT, but each has a compensating disadvantage of a low fraction of decays giving the gamma trigger, and also a higher energy beta end point.
The conclusion is that all of the candidate beta-decay nuclides, although in principle able to reach reconstructed masses $<10$ keV, have a low overall efficiency of usable decays, typically $<10^{-3}$–$10^{-4}$ making it difficult to reach very low sterile neutrino coupling levels, e.g. $10^{-6}$ and possibly down to $10^{-10}$ (figure 2), even with an MOT filling $\sim 10^8$–$10^{10}$ atoms. As an alternative, nuclides decaying by K-capture have the advantage of being initially a simpler two-body decay, with the electron absorbed by the nucleus rather than emitted, but with the possible disadvantage of releasing a significant number of low energy Auger electrons. These nuclides are considered in the next section, and in the remainder of the paper.

### Table 3. K-capture nuclides with no accompanying nuclear gamma, showing possible x-ray trigger energies. Listed in order of triggered decay rate/year/atom (number in MOT maintained constant).

| Z | A | Half Life | Trigger X-rays | Fraction | Trigger Rate |
|---|---|-----------|----------------|----------|--------------|
| 55 | Cs | 131 | 9.6 | 352 | 29.8, 33.6, 34.4 | 1.00 | 26.23 | –< |
| 74 | W | 178 | 22 | 91 | 57.5, 65.2, 67.0 | 1.00 | 11.45 | –< |
| 38 | Sr | 82 | 26 | 180 | 13.4, 15.0, 15.2 | 1.00 | 9.69 | –< |
| 24 | Cr | 51 | 28 | 752 | 5.4, 5.9, 6.9 | 0.90 | 8.10 | –< |
| 77 | Ir | 189 | 13 | 532 | 63.0, 71.4, 73.4 | 0.39 | 7.56 | –< |
| 18 | Ar | 37 | 35 | 813 | 2.6, 2.8, 3.0 | 1.00 | 7.20 | –< |
| 4 | Be | 7 | 53 | 861 | 0.045 (Auger) | 0.90 | 4.28 | –< |
| 33 | As | 73 | 80 | 340 | 9.9, 11.0, 11.1 | 0.90 | 2.83 | –< |
| 34 | Se | 75 | 120 | 864 | 10.5, 11.7, 11.9 | 1.00 | 2.10 | –< |
| 66 | Dy | 159 | 144 | 366 | 44.5, 50.3, 51.7 | 0.73 | 1.28 | –< |
| 32 | Ge | 68 | 270 | 107 | 9.2, 10.3, 10.4 | 1.00 | 0.93 | –< |
| 23 | V | 49 | 338 | 602 | 4.5, 4.9, 5.6 | 1.00 | 0.75 | –< |
| 73 | Ta | 179 | 653 | 110 | 55.8, 63.1, 65.0 | 1.00 | 0.39 | –< |
| 26 | Fe | 55 | 986 | 232 | 5.9, 6.4, 6.8 | 1.00 | 0.26 | –< |
| 79 | Au | 195 | 186 | 227 | 66.8, 75.7, 77.9 | 0.10 | 0.14 | –< |

The conclusion is that all of the candidate beta-decay nuclides, although in principle able to reach reconstructed masses $<10$ keV, have a low overall efficiency of usable decays, typically $<10^{-3}$–$10^{-4}$ making it difficult to reach very low sterile neutrino coupling levels, e.g. $10^{-6}$ and possibly down to $10^{-10}$ (figure 2), even with an MOT filling $\sim 10^8$–$10^{10}$ atoms. As an alternative, nuclides decaying by K-capture have the advantage of being initially a simpler two-body decay, with the electron absorbed by the nucleus rather than emitted, but with the possible disadvantage of releasing a significant number of low energy Auger electrons. These nuclides are considered in the next section, and in the remainder of the paper.

### 6. Choice of nuclides decaying by electron capture

Electron capture nuclides have the basic advantage of an initial two-body decay releasing a nearly monoenergetic neutrino, and a recoil atom in an excited state, with equal and opposite momenta. Since the kinetic energy of the recoil is much smaller than the energy of the neutrino, emission of a sterile neutrino with a larger mass will have a lower momentum, resulting in a slightly lower momentum for the recoil atom, which is in principle measurable. The majority of electron captures are from the K shell, leaving a vacancy which is filled by a higher shell electron with the emission of an x-ray photon which provides a trigger for the event. Projecting back from the x-ray detection point to the source position provides the time origin for the subsequent recoil time-of-flight measurement.

However, an additional complication is that the filling of the K shell leaves another higher shell vacancy, and subsequent cascade of atomic electron transitions results in some being ejected from the atom as Auger electrons (typically within $10^{-13}$–$10^{-15}$ s [67]). The latter have significant momenta which has to be included in the mass reconstruction (figure 7(b)). Thus the K-capture decay mode, although initially giving only an atomic recoil followed by an x-ray, also needs measurement of one or more Auger electrons, but with the advantageous difference that, for appropriate choice of initial x-ray shell, the Auger electrons are much lower in energy/momentum compared with the beta decay spectra discussed in section 5, and hence in principle easier to measure.

There are 65 K-capture nuclides with half lives between a few days and two years, but many can be eliminated by including only those which decay directly to the nuclear ground state of the preceding element, to avoid additional nuclear gammas contributing to the kinematics. Those remaining are shown in table 3, listed as before in order of net event rate per year per atom.

As in the case of table 1, those elements which have been successfully trapped in an MOT, and listed in table 2, are indicated in the final column of table 3. Top of this list is $^{131}$Cs, which is also considered one of the
The x-ray case. BaF2 has both a fast decay gamma trigger. However, the limited light output and energy resolution requires more detailed consideration in the remainder of the table shown in section 9. The fast component is not increased at reduced temperature, and requires wavelength-shifting. However, the latter output can be doubled by operating at reduced temperature and requires wavelength-shifting. However, the total light output is only marginally sufficient for identifying the specific x-ray lines. An x-ray energy ~30 keV yields ~600 photons, producing, assuming a back reector and ~40% SiPM quantum efficiency, ~100 photoelectrons (pe) with a ~10% resolution, ~±10 pe or ±3 keV. This would be at first sight insufficient to distinguish the individual M and N x-rays. However, this can be done by a mathematical procedure, based on the fact that the x-ray lines have accurately known values. Starting from the K-capture mass reconstruction equation in figure 7(b) which we repeat as follows,

\[ m^{2}_{\alpha} = [Q - E_{\gamma} - E_{\alpha} - E_{N}]^2 - [p_{\alpha} + p_{\gamma} + p_{N}]^2, \] (6.1)

(where the subscript ‘\( \alpha \)’ denotes the total energy and net momentum of one or more Auger electrons) differentiation gives, by analogy with equation (5.2b), now with \( Q = 350 \text{ keV} \) and x-ray energy ~35 keV:

\[ \Delta(m^{2}_{\alpha})(\text{keV}^2) \approx 600(\text{keV})\times d(p_{\gamma})(\text{keV}) \] (6.2)

showing that an error ~1 keV will give a reconstructed value of \( m^{2}_{\alpha} \) deviating from zero by ± several hundred keV². Thus, for each x-ray from M and N shells, it is sufficient for the detector to identify the x-ray energy range 32–36 keV, and repeat the reconstruction (in real time) for each of the known M and N x-ray energies, to reject reconstruction as unphysical and identify the correct x-ray line. In the event of any ambiguity the event would be rejected. A similar procedure would apply in the case of LYSO scintillator.

The emission of an x-ray leaves a vacancy which is filled by an atomic electron usually leading to a cascade resulting in the emission of one or more Auger electrons. Figure 13 shows an illustrative example of the emission of two Auger electrons following the emission of a K–M x-ray. The energy of an emitted Auger electron is approximately the energy released by the internal atomic transition minus the binding energy of the shell from which the Auger electron is ejected.

Table 4. Electron-capture percentages for \(^{133}\text{Cs} \), with \(^{133}\text{Xe} \) x-ray energies and percentage transitions from higher shells.

| Electron captured from shell | K   | L   | M   |
|-----------------------------|-----|-----|-----|
| X-rays from shell           |     |     |     |
| O                           | 85% | 12.5% | 2.5% |
| N                           | 69.5% |     |     |
| M                           | 13.0% | 10.0% |     |
| X-ray energy range (keV)    |     |     |     |
| L                           | 29.4–29.8 |     |     |
| M                           | 33.6–33.7 | 3.6–4.5 |     |
| N                           | 34.4–34.5 | 4.6–5.3 | 0.7–0.9 |

best prospects for achieving an MOT source with \( 10^8–10^{10} \) atoms, and will therefore be assessed in more detail in the remainder of this section. The special case of \(^7\text{Be} \), which has particularly simple kinematics, with no x-ray but a single low energy Auger electron trigger, will be discussed in section 9.

Electron capture in \(^{133}\text{Cs} \) is predominantly from the K shell, but can also occur from higher shells. Using the theoretically calculated \( L/K \) ratio ~0.14 ± 0.01 [68] and the \( M/L \) ratio ~0.22 ± 0.01 [69], gives the relative capture percentages shown in the top row of table 4. These are consistent with the similar percentages reported for neighboring elements \(^{125}\text{I} \) and \(^{127}\text{Xe} \) [70]. The resulting capture vacancy is then filled by an electron from a higher shell with the emission of an x-ray. The x-ray energy ranges and percentages from \( L, M, \) and \( N \) shells are shown in the remainder of the table [58].

The x-rays in the 29–35 keV range would provide a suitable fast trigger, using either LYSO or BaF2 scintillator followed by multi-anode SiPM for position sensitivity, as discussed in section 5 for a higher energy gamma trigger. However, the limited light output and energy resolution requires more detailed consideration in the x-ray case. BaF2 has both a fast decay (0.6 ns, 220 nm, 1.4 photons keV⁻¹) and slow decay (600 ns, 310 nm, 9.5 photons keV⁻¹) light component. The latter output can be doubled by operating at reduced temperature [71]. The fast component is not increased at reduced temperature, and requires wavelength-shifting. However, the total light output is only marginally sufficient for identifying the specific x-ray lines. An x-ray energy ~30 keV yields ~600 photons, producing, assuming a back reflector and ~40% SiPM quantum efficiency, ~100 photoelectrons (pe) with a ~10% resolution, ~±10 pe or ±3 keV. This would be at first sight insufficient to distinguish the individual \( M \) and \( N \) x-rays. However, this can be done by a mathematical procedure, based on the fact that the x-ray lines have accurately known values. Starting from the K-capture mass reconstruction equation in figure 7(b) which we repeat as follows,

\[ m^{2}_{\alpha} = [Q - E_{\gamma} - E_{\alpha} - E_{N}]^2 - [p_{\alpha} + p_{\gamma} + p_{N}]^2, \] (6.1)

(where the subscript ‘\( \alpha \)’ denotes the total energy and net momentum of one or more Auger electrons) differentiation gives, by analogy with equation (5.2b), now with \( Q = 350 \text{ keV} \) and x-ray energy ~35 keV:

\[ \Delta(m^{2}_{\alpha})(\text{keV}^2) \approx 600(\text{keV})\times d(p_{\gamma})(\text{keV}) \] (6.2)

showing that an error ~1 keV will give a reconstructed value of \( m^{2}_{\alpha} \) deviating from zero by ± several hundred keV². Thus, for each x-ray from M and N shells, it is sufficient for the detector to identify the x-ray energy range 32–36 keV, and repeat the reconstruction (in real time) for each of the known M and N x-ray energies, to reject reconstruction as unphysical and identify the correct x-ray line. In the event of any ambiguity the event would be rejected. A similar procedure would apply in the case of LYSO scintillator.

The emission of an x-ray leaves a vacancy which is filled by an atomic electron usually leading to a cascade resulting in the emission of one or more Auger electrons. Figure 13 shows an illustrative example of the emission of two Auger electrons following the emission of a K–M x-ray. The energy of an emitted Auger electron is approximately the energy released by the internal atomic transition minus the binding energy of the shell from which the Auger electron is ejected.

The subshell binding energies of the recoil Xe atom are (in eV),

- L-shell: 5453, 5107, 4786
- M-shell: 1149, 1002, 941, 689, 676
- N-shell: 213, 147, 145, 69, 67
- O-shell: 23, 13, 12
From this it follows that there are, stemming from any vacancy left by the x-ray, many combinations of internal transitions, leading to a large range of numbers and energies of Auger electrons. However, the Auger electrons resulting from M- or N-shell vacancies are both fewer and lower in energy [72, 73], in particular the N-shell vacancy gives only two principal Auger sequences, type NNO and NOO (using standard Auger notation: shell of initial vacancy, shell of electron which fills vacancy, shell of ejected electron) with NOO energies \(\sim 100–120\ eV\) (single Auger) and NNO energies \(\sim 25–65\ eV\) (two Augers per decay). The ratio of NOO to NNO Augers is about 1:3 [72].

Auger electrons stemming from the M shell vacancy have a somewhat larger average energy, \(\sim 400\ eV\) for types MXY (X, Y = any higher shell) and \(\sim 100\ eV\) for types MMX [72]. From table 4, restriction to triggering on N-shell x-rays would allow the use of only 2.5% of decays, whereas using both N- and M-shell x-rays would increase this to 15% of decays. We therefore need to design (a) initially for N-shell x-ray triggers followed by time-of-flight measurement of single or double Augers from N and O-shells, and (b) the addition of M-shell x-ray triggers, extending the energy sensitivity to cover Augers up to 400 eV.

For a more detailed analysis of the decay process, the sequence of events is shown in figure 14.

(a) K-capture in \(^{131}\text{Cs}\) converts it to an excited \(^{131}\text{Xe}\) atom with emission of a neutrino. At this point the recoil and neutrino have equal and opposite momenta.

(b) The K-shell is filled from an outer shell, emitting one of several x-rays, as indicated in table 4. This causes a small change in nuclear recoil momentum, depending on the x-ray momentum and (random isotropic) emission angle \(\theta\).

(c) Further rearrangement of outer electrons results in the emission of one or more Auger electrons. This alters the nuclear recoil direction by an amount that depends on the random (isotropic) direction of the vector sum of the Auger electrons, denoted in figure 14 by \(\alpha\) relative to the final recoil direction. There is also a 30%–40% probability of an additional ‘shake-off’ electron of \(\sim 10\ eV\) [75, 76], which can be recognized by an additional charge on the recoil ion, and the event rejected.
Both x-ray and Auger electrons are emitted within $<1$ ps, so the x-ray detection (allowing for the transit time from source to detector) is therefore an adequate trigger for sub-ns time of flight measurement precision. The kinematics of the above sequence is given by the following formulae, in addition to equation (6.1):

$$p_{N1}^2 = (Q^2 - m_{N1}^2) / (1 + Q/M)$$

with sufficient approximation, from $p_{N1} = p_{N2}$, where $M = $ atomic mass in keV and $Q = 352$ keV (difference of $^{131}$Cs and $^{131}$Xe atomic ground states).

$$p_{N1}^2 = p_{N2}^2 + p_\gamma^2 + 2p_\gamma p_{N2} \cos \theta$$

It can be seen from figure 14 that these equations also cover the full three-dimensional kinematics, since the x-ray vector can be rotated $2\pi$ in azimuth around the $p_{N2}$ vector, and similarly the combined $p_{N2}$ vector can be rotated $2\pi$ in azimuth around the $p_{N2}$ vector, without affecting equations (6.3)–(6.5). Thus the two dimensional form of figure 14 covers all isotropic directions of emission of x-ray and Auger emission, provided the latter are combined into a single resultant $p_{N2}$.

It is also important to note that by expansion of equation (6.1) in the presence of measurement errors it follows that, zero mass, changes in the quantity $m_{N2}^2$ are linearly proportional to any individual error. Thus if the measurement errors are Gaussian, it is the distribution of $m_{N2}^2$ that will be Gaussian. This is the case for masses at least into the several hundred keV$^2$ region, and can be assumed in estimating the overlap of the tail of this distribution with any non-zero mass signal.

In addition to the width of $m_{N2}^2$ due to measurement errors, there is a small additional ’natural’ line width for the emission of Auger and x-rays that should be considered [74]. However, in the case of Auger electrons, the

![Figure 15.](image1)

**Figure 15.** Half-width of neutrino (mass)$^2$ distribution versus rms time-of-flight errors for $^{131}$Cs. The results are for an assumed time of flight path $L = 100$ cm, the effect of other path lengths being included by the factor 100 cm $L^{-1}$ in the axis parameter.

![Figure 16.](image2)

**Figure 16.** Half-width of neutrino (mass)$^2$ distribution versus emission angle errors for $^{131}$Cs, (with $d\cos(\theta, \alpha) = \sin(\theta, \alpha) d(\theta, \alpha)$ and the sin factor averaged for isotropic emission).
measurement of their momentum already includes this, and in the case of the M or N-shell x-rays the natural width is \( \sim 10 \text{ eV} \) giving, from (6.2) an additional \( \Delta (m^2) \approx 6 \text{ (keV)}^2 \), an order of magnitude smaller than the anticipated measurement errors. We can therefore neglect this.

To determine the effect of measurement errors, the above equations were set up as a numerical simulation, first calculating \( p_{\nu 3} \) with all errors set to zero, then reversing the process to calculate equation (6.1) in the presence of Gaussian timing errors for \( |p_{\nu 3}| \) and Gaussian errors in \( \theta \) or \( \alpha \) (using \( \Delta (\cos \theta) = \sin \theta \, d\theta \) and \( \Delta (\cos \alpha) = \sin \alpha \, d\alpha \), with the sin factors averaged for isotropic emission). The effect of individual measurement errors on the reconstructed \( \Delta (m^2) \) is summarized in figures 15 and 16, assuming an x-ray energy of 34 keV, and typical Auger energies of 60, 120, 360 eV. The time-of-flight path length \( L \) is included in the axis of figure 15. The dashed line in figure 16 for an x-ray of 4 keV corresponds to the possibility of using L-capture events, as discussed below in section 7. Errors in angle would be essentially detection position resolution divided by path length but require additional simulations if electric and/or magnetic fields are used to improve solid angle capture of the ion and electrons. An additional factor to be noted is the likelihood of several recoil ion charge states arising from emission of varying numbers of Auger electrons. This would be identifiable from the transit time and allowed for in the time-of-flight momentum calculation.

7. Illustrative experimental configuration

For an experimental sensitivity to sterile neutrino masses down to the 10 keV region (i.e. 50–100 keV\(^2\)), an rms reconstruction width \( \Delta (m^2) < \pm 30 \text{ keV}^2 \) is required, depending on the number of processed events per year needed to reach a given coupling level. Allowing for the fact that there are four vector measurement errors to be combined in quadrature, the individual errors in figures 15, 16, would need to be smaller, by factors dependent on the details of a particular design. For the relatively slowly moving recoil ion, a 50 cm path length is sufficient, but the faster Auger electrons need at least a 200 cm path. For the x-ray and Auger electrons, requiring an rms angular precision \( <0.0004 \) radians, a 0.2–0.4 mm position resolution at the detection point would require a path length typically of order 100 cm.

This leads to an illustrative configuration of the form and size shown schematically in figure 17. This shows a conventional MCP detector for the recoil ions, and a large solid angle of a proposed new type of ‘pho-switch’ detector. This would utilize LYSO or BaF\(_2\) scintillator for x-rays, and plastic scintillator for pre-accelerated (2 kV) electrons, the photons from both being registered by a multi-anode SiPM array. The latter would be coated with wavelength shifter to respond to the fast BaF\(_2\) component, but this would be unnecessary if LYSO scintillator is used. The incident surface of the plastic scintillator would be coated with a 300 Å reflective metallic layer, which pre-accelerated electrons can penetrate to produce light in the plastic scintillator isotropically, but 90% of the back-emitted light would be reflected into the forward direction by the metallic layer while still allowing the pre-accelerated electrons into the plastic scintillator.

An alternative detection scheme, giving a finer position resolution (\( <40 \mu\text{m} \)) would be obtainable for the Auger detection using an MCP array plus guiding electric and magnetic fields. However, using separate detectors for electrons and x-ray reduces the solid angle collection for each, which would reduce overall efficiency for both triggers and multiple Auger collection.

A system on this scale, using state-of-the-art sub-ns time and sub-mm position resolution, would be capable of separating reconstructed neutrinos with \( m^2 \) down to the region \( \leq 100 \text{ keV}^2 \), and hence sensitive to the sterile neutrino mass range predicted as a solution to the dark matter problem-in particular the claimed observation (figure 2) of an unpredicted x-ray signal that may correspond to the decay of a particle of mass 7 keV. The coupling sensitivity is largely independent of the measurement precision, and depends on the number of events that can be processed in, for example, a 1–2 y running time. Since the decay processes are isotropic, this depends, for a given nuclide, on a number of experimental factors: (a) the source atom population, (b) the fraction of decays with accepted triggers, (c) the solid angle capture efficiency of all decay products, and (d) the detection efficiency for each type of particle.

The estimated numbers for these processes are summarized in table 5, for an assumed four stages of development of the experiment. To minimize the Auger emission only the use of N-shell x-rays is assumed:

- Phase 1: K-capture with a source population of \( 10^8 \) atoms
- Phase 2: K-capture with a source population of \( 10^{10} \) atoms
- Phase 3: Increasing the source population to \( 10^{11} \) atoms, and adding events from L-capture, the latter doubling the x-ray triggers from the N-shell.
- Phase 4: If an increase in source population to \( 10^{12} \) atoms proves impractical, replicating ten identical experimental units would be more cost-effective than a large geometric scale-up.

Table 5 illustrates the possibility, with sufficient technical effort, of ultimately reaching the coupling levels suggested by the apparent x-ray signal shown in figure 2, should that be verified as a strong sterile neutrino dark matter candidate. A possible problem of using L-capture events, as seen from table 4, is that the M and N x-ray energies are in the lower range 4–5 keV. Using the BaF\(_2\) or LYSO resolution estimates for 35 keV x-rays in
section 6, the energy resolution becomes \(\sim 20\%\) at 4 keV, or \(\sim \pm 0.8\) keV, again exceeding the spacing of the L x-ray lines, but still with the possibility of using the comparative reconstruction procedure described in section 6 to choose the correct tabulated line. It thus seems possible in principle to utilize the L-capture events, but needs some additional numerical study (along with, eventually, practical confirmation). Otherwise the sensitivity in the third and fourth columns of table 5 would be reduced by a factor 2.

8. Event processing and identification

To achieve sensitivity to coupling levels below \(10^{-10}\), should this be necessary, an event processing rate \(\sim 10^7 - 10^8\) s\(^{-1}\) would be needed. An accepted event would consist of a 32–36 keV x-ray trigger, followed by a

| Table 5. Example of reconstructed \(^{136}\)Cs K-capture events per year assuming progressive improvement in source population and detection efficiency. |
|---|
| **Capture shell** | Phase 1 | Phase 2 | Phase 3 | Phase 4 |
| X-ray shell | K | K | K + L | K + L |
| Auger initial vacancy | N, N2 | N, N2 | N1, N2 | N1, N2 |
| Augers per trigger | 1 (25%) or 2 (75%) | 1 (25%) or 2 (75%) | 1 (25%) or 2 (75%) | 1 (25%) or 2 (75%) |
| Atoms in MOT | 1E9 | 1E10 | 1E11 | 1E11 |
| No of experimental units | 1 | 1 | 1 | 10 |
| Decays/atom | 26 | 26 | 26 | 26 |
| Fraction N shell decays | 0.025 | 0.025 | 0.05 | 0.05 |
| Total N-shell decays | 6.5E8 | 6.5E9 | 1.3E11 | 1.3E12 |
| X-ray capture efficiency | 0.3 | 0.6 | 0.9 | 0.9 |
| Triggers/atom | 2E8 | 4E9 | 1.2E11 | 1.2E12 |
| Ion capture hemispheres | 1 | 2 | 2 | 2 |
| Ion solid angle capture | 0.3 | 0.6 | 0.8 | 0.8 |
| Ion (MCP) detection efficiency | 0.6 | 0.6 | 0.6 | 0.6 |
| Total ion detection fraction | 0.18 | 0.36 | 0.48 | 0.48 |
| Electron capture hemispheres | 1 | 1 | 2 | 2 |
| Electron solid angle capture | 0.3 | 0.6 | 0.9 | 0.9 |
| Electron detection efficiency | 0.7 | 0.7 | 0.7 | 0.7 |
| Overall detection efficiency | 0.21 | 0.42 | 0.63 | 0.63 |
| Source decays/s | 8E2 | 8E3 | 8E4 | 8E5 |
| Events/s | 6 | 120 | 3800 | 38000 |
| Reconstructed events/s | 0.07 | 1 | 1100 | 11000 |
| Reconstructed events/y | 2E6 | 3E8 | 3E10 | 3E11 |
cluster of Auger events after $\sim 200$ ns, and completed by a recoil ion signal after $\sim 200 \mu s$ (for a singly charged ion). Applying time windows would largely reject the larger number of triggers not followed by both Auger and ion signals. Figure 18 illustrates (qualitatively) the possible time sequence of successive events, for the extreme case of an MOT with $10^{12}$ trapped $^{131}$Cs atoms. Figure 19 shows an alternative of overlapping events to allow for faster processing with many sequences rejected as missing some signals through detection efficiency.

9. The special case of nuclide $^7$Be

The K-capture nuclides all have the complication of emitting several Auger electrons, with the need to ensure that, for each accepted event, all are collected along with their momentum and direction, in order to achieve a complete reconstruction of the neutrino mass. The need to minimize their number, restriction to N-shell x-rays, or at most N + M shell x-rays, together with collection and detection efficiency factors for each Auger electron, considerably reduces the overall collection efficiency for completed events (table 5). The single exception to this in table 3 is the nuclide $^7$Be for which 90% of K-capture decays are direct to the $^7$Li ground state [75], with no x-ray emitted but the K-capture vacancy filled with the release of a single Auger electron of known kinetic energy 45 eV [76] (figure 20).

This has the following advantages compared with the sequence for other K-capture nuclides (figure 14).

(a) There is no x-ray to be detected, thus avoiding the need for a special detector for this.
(b) A single Auger electron is emitted, with fixed 45 eV energy, providing a time-of-flight trigger.

(c) Collection of the single electron can be achieved with a large solid angle using an electric field.

(d) MCP detection of the trigger electron provides 0.3 ns time resolution, and 40 μ position resolution.

(e) The Q value of 860 keV for $^7\text{Be}$ allows a wider range of sterile neutrino masses to be explored.

(f) The higher Q and lower atomic mass results in a recoil velocity 40 times higher, and kinetic energy 100 times higher, than for Cs, making the source temperature $\propto Mv^2$ correspondingly less critical.

The higher recoil speed necessitates a longer time-of-flight path than in the case of $^{131}\text{Cs}$, to achieve the same momentum and mass precision. For a combined timing error 0.3 ns (i.e. ±0.2 ns at start and finish), figure 21 shows the (mass)$^2$ error as (left) a function of recoil path length $L$, and (right) a function of the angle error (between electron and atom recoil) for a fixed $L = 200$ cm and timing error 0.3 ns.

The longer path length for the atom recoil would require a combination of axial magnetic and electric fields to capture a large range of emission angles. Typically, 400 gauss and 10 V cm$^{-1}$ are required to focus recoil ion emission angles up to $70^\circ$ in one hemisphere on to a 20 cm diameter MCP array at a distance 200 cm. Lower fields are required to capture the 45 eV Auger electrons, typically 5 G for the same voltage gradient 10 V cm$^{-1}$ at the source. This will focus electron emission angles up to $80^\circ$ (in the opposite hemisphere) on to a 20 cm diameter MCP array at a distance ~60 cm from the source. The two requirements could be matched by means of appropriate magnetic solenoids on the two sides, along with a pair of anti-Helmholtz coils required for the MOT. Note that although the field distribution will be non-uniform there remains a one-to-one correspondence between each detection point on the MCP and the polar and azimuthal emission angles from the source. This would be established by computer simulation, together with final calibration based on the majority of events necessarily reconstructing at zero neutrino mass.

Table 3 shows that for $^7\text{Be}$ the trigger rate per year is a factor ~6 lower than for $^{131}\text{Cs}$, but this is offset by the above advantages for the efficiency and precision of the experiment, and $^7\text{Be}$ would be the most favored choice for a sterile neutrino search were it not for the fact that at the time of writing it cannot be trapped efficiently in an MOT or a high density beam, and there are further problems arising from positive ion production [54].
Nevertheless, the above advantages provide a strong incentive for the development of a method for making this into a high density source or beam, perhaps via an extension of the ‘buffer cell’ technique\cite{59} which has been used to produce beams of some previously intractable atoms, compounds, and radicals, and thus suggests an optimistic prospect for the future use of trapped Be in this application.

10. Backgrounds

Figure 22 indicates the most likely background sources as follows (not in order of significance).

(a) Cosmic ray muons.
(b) Residual gas scatters by Auger electrons during flight to detectors.
(c) Radioactive impurities in walls and component materials.
(d) Atoms escaping from the trap, or lost during loading, adhering to walls and detector surfaces.
(e) Scattering of ions while emerging from the trapped source cloud.
(f) Radiative atomic transitions: radiative beta decay or radiative $K$.

A more detailed discussion of each is as follows.

(a) Cosmic ray muons (assuming the experiment to be located at the Earth’s surface) would give a rate ($100/\text{m}^2/\text{s}/\text{sr}$) of signals in individual components of the scintillator array, but these would be predominantly at the MeV-level (for example in 1 mm BaF$_2$ or LYSO scintillator plates), and can be rejected in real time by an energy cut, hence not contaminating the accepted events. The resulting dead time (from the $\sim$2 $\mu$s recovery time of the BaF$_2$) would be $<1\%$, but most of this would be concurrent with (and would not affect) the timing of the recoil ion to the MCP. Although a surrounding anti-coincidence muon detector could be designed, it is unlikely to be needed, since the scintillator array provides its own anti-coincidence.

(b) Scattering from residual gas molecules can be minimized with a cryo-pumped ultra-high vacuum. For the pressure $\sim 10^{-11}$ torr likely to be required to maximize the atom trap lifetime, the mean free path of ions and electrons will be at least four orders of magnitude longer than the apparatus dimension. Nevertheless, some rare scattering events may occur at the $10^{-4}$–$10^{-5}$ level, changing the direction of the decay products. These events will be rejected by the previously-mentioned fact that the (mass)$^2$ reconstruction equations (5.1) or (6.1) involves the difference between two large numbers $\sim 10^8$ keV$^2$. It is this that, on the one hand, gives the challenge of measuring these numbers with sufficient precision, but on the other hand also provides a significant advantage in rejecting spurious events. In particular, a residual gas scatter would result in an incorrect value for
the magnitude and direction of the electron momentum. The resulting large difference, sometimes negative, between the two terms of equations (5.1) or (6.1), would be sufficient to allow rejection of these events.

c) Radioactive impurities would be reduced to a minimum in the selection of cryostat, source and detector materials, but residual concentrations would emit electrons and gammas, giving random signals in detectors. These would largely be rejected by imposing time windows on the sequence of signals that constitute an acceptable event. After an x-ray trigger of the required M or N shell energy, a time window \( \sim 100-200 \) ns would be allowed for the arrival and detection of the Auger electrons, followed by a second time window \( \sim 0.5-1.5 \) ms later for the detection of the single recoil ion at the MCP (this range allowing for all sterile neutrino masses up to 300 keV). Only the detection of Auger and ion signals within these time intervals would constitute an acceptable event, leaving a large fraction, \( >99.9\% \) of the time (see figure 18), in which radioactive background signals would be automatically ignored. Moreover, any arrival within the correct time window would simulate an additional Auger electron for which the complete event would then not reconstruct consistently. In support of this, it should be noted that this background has not been reported as a problem in many published COLTRIMS experiments [45, 46].

d) Atoms are lost into the vacuum chamber from the MOT due to its finite trapping lifetime, which is of order \( (10^{-8}/p \text{ Torr}) \) seconds [48]. Thus with ultra-high cryo-pumped vacuum, with pressure \( p \sim 10^{-11} \) Torr, the lifetime could be \( 10^4-10^5 \) s. The lost atoms then move in straight lines to various surfaces, including the cryostat, the MCP and the large area x-ray/electron detector, where they decay with a 9.7d half-life. In equilibrium (loss rate from the MOT = decay rate) there would be a number of atoms, on surfaces in the system, exceeding those in the trap by a factor \( \sim 10-100 \), and hence, once this equilibrium is reached, decaying at a rate \( \sim 10-100 \) times those in the MOT. However, x-rays and Augers from these will either not both be detected, or not within the correct time windows. Specifically, for atoms either (i) on the detector surface or (ii) on its pre-acceleration grid, the Augers will be confined to that detector, and thus too close in time (i.e. within a few ns) to the x-ray signal (traveling at velocity \( c \)). Finally, (iii) for atoms on the interior surface of the chamber, although the emitted x-ray may reach a detector, the electrons can be confined to the surface by lining it with a grid or alternating potentials [77] and will not reach any detector. Thus decays of escaping atoms cannot be mistaken for real events, but nevertheless need to be kept to \(<10 \) times the source decay rate, to avoid pile-up or else too large a dead time from the rejection of these background signals at the highest planned event-processing rates. More detailed calculations, taking account of the restricted arrival time windows for ions and electrons, indicate that this background will be acceptable for source populations \( 10^9 \) and \( 10^{10} \) atoms (columns 1 and 2 of table 5), but for larger populations \( \sim 10^{13} \) atoms, the experimental runs may need to be interrupted for intermediate periods of bake-out to remove deposited atoms. Continuous desorption of deposited atoms with external UV light (LIAD process), fed in through optimally placed windows, may also prove possible.

e) Recoil ions can undergo scattering while passing out of the trapped cloud of atoms, due to the basic Langevin cross section \( (2\pi^2 \alpha/E_{cm})^{0.5} \) [78] where \( \alpha \) is the Cs polarizability in units (length)\(^2\) and \( E_{cm} \) is the center of mass energy in units (length\(^{-1}\)). For Xe\(^+\) + Cs this gives a collision cross section \( 3 \times 10^{-14} \) cm\(^2\). For a source with 3D Gaussian density \( 3 \times 10^8 \) atoms mm\(^{-3}\) and rms radius 0.8 mm. a numerical integral over all possible starting points and paths gives an average collision probability \( \sim 2 \times 10^{-4} \) for each ion recoiling out of the cloud. To estimate the resulting background in reconstructed (mass)\(^7\), we note (a) that the momentum loss in a collision with scattering angle \( \theta \) will be \( p_\theta(1-\cos \theta) \), giving a non-zero reconstructed mass which decreases at small angles, while (b) the differential scattering cross section increases at small angles [79, 80]. For a quantitative estimate we combine these and integrate over all angles to obtain a probability per atom versus reconstructed (mass)\(^7\), giving the result plotted in figure 23 for a \(^{133}\)Cs source, again for a likely peak MOT density of \( 3 \times 10^8 \) atoms mm\(^{-3}\). It is important to note that the vertical position of this curve could be lowered in proportion to the (controllable) peak MOT density, at the expense of increasing the source diameter. The Auger electrons, with a factor \( \sim 100 \) higher kinetic energy have a Langevin cross section a factor \( \sim 10 \) lower, and a correspondingly reduced contribution to the background.

f) There are several processes from which additional photons may be emitted:

(i) in the atomic transitions emitting Auger electrons, the energy can be released alternatively as low energy x-rays. Published calculations list a low relative probability for these, below \( 10^{-3} \) for the M shell and below \( 10^{-4} \) for the N shell [73]. These x-rays would have the same low energy as the corresponding Auger
transition, and hence not detected, giving unreconstructed events that would be rejected in real time by having incomplete energy collection;

(ii) for beta decay, a radiative diagram exists, in which the free electron emits a photon with a continuous spectrum up to the beta end point energy. This has been calculated for Tritium decay by Bezrukov [11]. An undetected photon of energy $E_{\gamma}$ in equation (5.1) would mimic a non-zero neutrino (mass) $\sigma m_{\nu}^{2}$, averaged over emission angles relative to the beta electron, with a probability $\sim 10^{-5} - 10^{-6}$ keV$^{-1}$ in the mass region 4–12 keV. This would appear to make it more difficult to detect a sterile neutrino coupling below this value unless these events can be rejected by detecting the coincident radiated photon;

(iii) in the case of electron-capture there is no free electron to radiate a photon, but radiative capture nevertheless occurs, and the resulting photon spectrum has been studied both theoretically and experimentally [81–84]. The differential spectrum depends strongly on the capture state (S or P). For K-capture (S-state) the spectrum has the form:

$$dN_{ph}/dy = (\alpha/\pi)(Q/m_{e})^{2}y(1 - y)^{2},$$

where $y = E_{ph}/E_{max}$, $E_{max} = Q$, $\alpha = 1/137$, $m_{e} = 511$ keV giving a total probability per K-capture

$$N_{ph} = (\alpha/12p)(Q/m_{e})^{2}.$$

The typical case of $^{119}$Sb (with $Q = 590$ keV) provides an example of agreement between theory and measurement [83] and similar agreement has been found for a number of other K-capture elements. For $^{131}$Cs, with $Q = 362$ keV, $dN_{ph}/dE_{ph}$ is shown as curve (a) in figure 24, confirmed by absolute measurements of its upper branch in [83]. Its potential effect as a background. In a sterile neutrino search is that an undetected photon of energy (and momentum) $E_{\gamma}$ in equation (6.1) will, after reconstruction, mimic a non-zero neutrino mass $m_{\nu}$, its value dependent on both $E_{ph}$ and the emission angle $\delta$ relative to the nuclear recoil. Using an average $\cos \delta \sim 0$ results in a reconstructed effective mass $m_{eff} \approx (2E_{\gamma}Q)^{0.5}$, which can be used to convert the $E_{ph}$ axis to an effective mass axis, at the same time using $dm_{eff}/dE_{ph} = [Q/2E_{ph}]^{0.5}$ to transform the dN/d$E_{ph}$ spectrum, shown in blue, to the required dN/dm$_{eff}$ spectrum, shown in red. For both spectra, $N$ is the number of events per keV per K-capture. Corresponding curves for $^{7}$Be ($Q = 860$ keV) are shown in figure 24.

Thus, for any value of reconstructed mass, curves (b) give the probability of a spurious event resulting from radiative capture. To magnify the low end of the spectra, they are shown also on log scales in figures 24 and 25. For $^{7}$Be, the higher Q value results in about a factor $\sim 2$ higher dN/d$E_{ph}$ background, but after conversion to an effective reconstructed mass the radiative background, at any specific mass, is about a factor 4 lower than that of $^{131}$Cs. In the mass region $m_{\nu} \sim 10$ keV, figures 24 and 25 show that this background would be sufficiently low to reach coupling levels $< 10^{-11}$. This rises to $10^{-8}$ at $\sim 100$ keV, in principle reducible by an order of magnitude if the radiated photon could be detected (in coincidence with the atomic x-ray) with 90% efficiency, enabling rejection of those events.

It is important to note that the continuous intrinsic backgrounds, (e) and (f), would not represent a lower limit on the observable neutrino coupling at a particular mass, but would provide a ‘known background’ on...
which the statistical significance of any apparent signal peak would be estimated using the tables of Feldman and Cousins [85] based on the frequentist Neyman construction.

11. Other possibilities using K-capture

Polarization of the atomic nucleus will create an angular asymmetry in the direction of the emitted neutrino relative to the initial spin direction, and hence in the recoil nucleus, whose recoil is initially opposite in direction to that of the neutrino. This has sometimes been taken to imply that for a 99% polarized nucleus the neutrino direction will be almost unidirectional, subject only to thermal fluctuations of the nuclear spin [86, 87]. Clearly this would be of value in the mass reconstruction, since one of the directions would be defined in advance, and also allow complete collection of the atomic recoils. However, although the atom and neutrino directions are initially equal and opposite, that direction can nevertheless lie at an angle $\theta$ to the initial polarization direction. The correct angular distribution was calculated by Treiman [88] who derived a dependence of the form $(1 - A \cos \theta)$ where, for a $5/2 \rightarrow 3/2$ transition, the constant $A$ is equal to the polarization $P$ and, for $P \sim 1$, the recoils will be predominantly in the same direction as the polarization [89]. While this has some forward/back asymmetry, it may not be sufficient to justify the additional complication of using a polarized source.

Polarization of $^{131}$Cs can be achieved by optical pumping but not while trapped in an MOT. However, if the MOT is turned off rapidly (less than a millisecond), the trapped ions will remain in place for long enough (1–2 ms) for polarization (<100 $\mu$s) and data-taking, after which the MOT can be re-established in ~2 ms [90]. There would thus be a repeated ~4 ms cycle of alternating trapped and polarized atoms (figure 26) Thus this loses a factor ~3 in data-taking time, and for this to be adopted in a sterile neutrino search, any advantage from polarization would need to offset this loss. A similar repetitive sequence without polarization, of simply turning off the MOT while data taking, could be adopted if there is too much perturbation of Auger electron paths by the anti-Helmholtz magnetic field required for the MOT.

Figure 24. (a) Radiative K-capture spectra for $^{131}$Cs and (b) converted to an apparent reconstructed mass.

Figure 25. (a) Radiative K-capture for $^7$Be and (b) converted to an apparent reconstructed mass.
12. Conclusions

Sterile neutrinos in the 10 keV mass region could account for all or part of the Galactic dark matter. Direct detection appears not to be feasible in the near future, making a strong case for the development of laboratory searches. The detailed study in this paper shows that such searches are feasible using full energy-momentum reconstruction of beta decay or K-capture events, from neutral atoms suspended in a MOT. This provides the only method so far proposed for isolating and observing individual sterile neutrino events as a keV-range mass population, separated from the standard neutrino distribution close to zero mass. A survey of suitable nuclides suggests that detection by precise reconstruction of K-capture events appears more feasible than those emitting a beta spectrum. For the most favorable cases, $^7$Be and $^{131}$Cs, the currently attainable resolution provides sufficient momentum precision to resolve sterile neutrino masses down to the 10 keV mass region in a laboratory-scale experiment. Of these two nuclides, $^{131}$Cs can be trapped in an MOT in large numbers, but presents a more difficult measurement problem because of the need to collect and measure, in addition to the nuclear recoil, an x-ray and several low energy Auger electrons. In contrast, $^7$Be has a simpler decay process which may make it the best choice from a measurement viewpoint, but cannot yet be efficiently loaded into an MOT or high density beam. Because of the latter, $^{131}$Cs remains the best practical choice for initial sterile neutrino searches using this scheme.

For any choice of nuclide, the more difficult prospect is the processing of a sufficient number of events per year to reach low coupling levels. To improve on existing coupling limits ($\sim 10^{-2}$) requires only $\sim 10^6$ decays per year, but to reach the coupling level $\sim 10^{-10}$, suggested by a recent unidentified astronomical x-ray signal (figure 2), may require processing of $\sim 10^{12}$ decays per year, depending on the efficiency of collection and detection. The latter efficiencies would be much higher (perhaps by 2–3 orders of magnitude) for the very simple $^7$Be decay, than in the case of $^{131}$Cs decay, giving a strong incentive for the future development of some technique to create a compact isolated source of $^7$Be atoms. A further possibility for increasing the event processing rate would be to design a more complex geometry containing multiple sources, or to make multiple replicas of a single-source apparatus, which would be justified given any hint of a sterile neutrino signal in a single system.

A number of background effects have been discussed. At least 99.9% of background signals can be rejected as not within the correct sequence of time windows for real events, or can be rejected by the reconstruction mathematics, which is sensitive to any perturbations arising, for example, from scattering in the vacuum chamber which would give signal sequences not obeying momentum conservation. Two background sources cannot be rejected in this way: (a) scattering within the trapped source, from which $\sim 10^{-4}$ of events will give a smooth and continuous falling background in reconstructed mass, on which significant signal peaks would still be observable, and (b) the photons from radiative K-capture, which on reconstruction would give a continuous background of non-zero neutrino masses, but which are calculated to be sufficiently small in the 10 keV mass region to allow couplings below $10^{-10}$ to be reached. There is thus good reason to believe that the principle of full four-momentum reconstruction in K-capture processes could become a leading method of searching for sterile neutrinos with masses down to the 10 keV level, and at the same time including higher masses up to the Q value of the chosen nuclide ($\sim 300$ keV for $^{137}$Cs or $\sim 800$ keV for $^7$Be).

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