Minimal rates for lepton flavour violation from supersymmetric leptogenesis

A Ibarra and C Simonetto

Physik-Department T30d, Technische Universität München, James-Franck-Straße, 85748 Garching, Germany
E-mail: alejandro.ibarra@ph.tum.de, cristoforo.simonetto@ph.tum.de

Abstract. The see-saw is a very attractive model for neutrino mass generation in particular in association with supersymmetry as a solution to the hierarchy problem. Under the plausible assumptions of hierarchical neutrino Yukawa eigenvalues and the absence of cancellations, we derive an upper bound on the lightest right-handed neutrino mass from the non-observation of $\mu \rightarrow e\gamma$ and $\mu - e$ conversion in nuclei. The ongoing experiment MEG as well as the planned experiments Mu2e, COMET and PRISM/PRIME will improve this bound if no evidence of lepton flavour violation is found. We lastly comment on the possibility of ruling out minimal leptogenesis if these experiments find no signal.

1. Introduction

In the see-saw the particle content of the Standard Model (SM) is augmented by three right-handed neutrinos. Being uncharged they have a Majorana mass term – unlike all other particles of the SM. Electroweak symmetry breaking generates also Dirac masses that lead to mixing of left- and right-handed neutrinos and finally to the observed small neutrino masses [1, 2, 3, 4]. Although very appealing theoretically, the see-saw mechanism faces the disadvantage of lacking predictability. To be specific the see-saw can accommodate any neutrino mass and the neutrino mass operator is the only mass-dimension 5 operator of the SM. Thus there is no low energy prediction of the see-saw.

In addition, the existence of such heavy neutrinos leads to quadratically divergent quantum corrections to the Higgs mass, claiming for a solution to the hierarchy problem. Hence the see-saw is naturally built into Supersymmetry (SUSY). In broken SUSY the slepton mass matrices can be affected by the see-saw mechanism [5]. Those flavour structures lead to lepton flavour violating processes, such as $\mu \rightarrow e\gamma$ or $\mu - e$ conversion in nuclei which can be searched for by experiment.

Unfortunately, the see-saw mechanism does not predict any model independent connection between lepton flavour violating processes and the neutrino mass matrix [6]. On the other hand, imposing any additional assumption will lead to constraints among the low energy parameters. Following this guideline the challenge is to find as plausible assumptions as possible having an impact on low energy observables.

In section 2 we will make use of the two plausible assumptions of hierarchical neutrino Yukawa eigenvalues and the absence of cancellations. This will allow to derive upper bounds on lightest right-handed neutrino mass from the non-observation of lepton flavour violation. Afterwards we
will show in section 3 how this might open the possibility of probing the minimal leptogenesis mechanism [7] where the observed baryon asymmetry is generated by the decay of the lightest right-handed neutrino.

2. Constraints on the see-saw parameters

2.1. The framework

Adding right-handed neutrinos \( \nu_R \) to the Minimal Supersymmetric SM (MSSM) the superpotential includes the following additional terms:

\[
W \supset \bar{\nu}_{Ri} Y_{\nu ij} L_j H_u - \frac{1}{2} \bar{\nu}_{Ri} D_{Mij} \nu_{Rj}.
\]  

(1)

We will work in a basis of diagonal heavy neutrino masses \( D_M = \text{diag}(M_1, M_2, M_3) \) and write the neutrino Yukawa according to the singular value decomposition as:

\[
Y_{\nu} = V_R D_Y V_L^\dagger, \quad D_Y = \text{diag}(y_1, y_2, y_3),
\]

(2)

where \( V_R \), \( V_L \) are unitary matrices. In this parametrisation the see-saw formula for the light neutrino mass matrix \( \mathcal{M} \) reads:

\[
\mathcal{M} = \langle H_u^0 \rangle^2 Y_{\nu}^T D_M^{-1} Y_{\nu} = \langle H_u^0 \rangle^2 V_L^T D_Y V_R^T D_M^{-1} V_R D_Y V_L^\dagger,
\]

(3)

being \( \langle H_u^0 \rangle \) the vacuum expectation value of the Higgs \( H_u \). We aim to solve for \( D_M \), \( V_R \). Therefore we rewrite the last equation

\[
V_R^\dagger D_M^{-2} V_R = \frac{1}{\langle H_u^0 \rangle^2} D_Y^2 \tilde{\mathcal{M}}^{-1} D_Y^2 \tilde{\mathcal{M}}^{-1},
\]

(4)

where we have defined for convenience \( \tilde{\mathcal{M}} = V_L^T \mathcal{M} V_L \).

Throughout this work we will assume hierarchical neutrino Yukawa eigenvalues \( y_1 \ll y_2 \ll y_3 \). This assumption is motivated by the fact that the three observed Yukawa matrices of quarks and leptons do follow this pattern. For the moment let us also consider the case of a less hierarchical matrix \( \tilde{\mathcal{M}} \). In particular if \( \tilde{\mathcal{M}}_{11} \) and \( \tilde{\mathcal{M}}_{12} \) are sizeable, one can identify the largest entry on the r.h.s. of eq. (4) and read off its largest mixing (see also [8, 9, 10, 11]):

\[
M_1 \simeq y_1^2 \langle H_u^0 \rangle^2 \frac{1}{|\tilde{\mathcal{M}}_{11}|}, \quad V_{R12} \simeq \frac{y_1 \tilde{\mathcal{M}}_{12}}{y_2 \tilde{\mathcal{M}}_{11}},
\]

(5)

As from experiment we know that \( \mathcal{M}_{11}, \mathcal{M}_{12} \) are sizeable, eq.(5) can only be circumvented for large mixing in \( V_L \). In particular the only possibly natural choice seems to be \( (V_L)_{k1} \sim U_{k1}^{MNS} \). This case has to be treated separately but will not modify the conclusions [12]. Thus we can assume eq. (5) to hold true independently of \( V_L \) unless there are unnatural cancellations.

2.2. Rare lepton decays

In the case of the MSSM the strongest bound on lepton flavour violation is presently set by the MEGA experiment \( \text{BR}(\mu \rightarrow e\gamma) \leq 1.2 \times 10^{-11} \) [13]. Very soon an improved bound can be expected from MEG at PSI where the final goal is to go down to \( \text{BR}(\mu \rightarrow e\gamma) \sim 10^{-13} \) [14]. This value seems to be the best one achievable with present techniques. On the other hand there are experiments in preparation for the search for \( \mu-e \) conversion in nuclei. Within the next few years Mu2e at Fermilab [15] and COMET at J-PARC [16] could be built which aim to a sensitivity of \( \text{R}(\mu \rightarrow e\gamma) \sim 10^{-16} \). Extending COMET with a FFAG muon storage ring could finally
provide a beam pure enough to lower this sensitivity again by two orders of magnitude in the PRISM/PRIME project [17].

These lepton flavour violating processes are in the MSSM mainly induced by slepton loops and can be approximated as [18]:

\[
R(\mu \text{Ti} \rightarrow e \text{Ti}) \simeq \alpha \text{BR}(\mu \rightarrow e\gamma) \simeq \frac{\alpha^4}{G_{F}^2} \frac{|m_{L12}^2|}{m_S^2} \tan^2 \beta . \tag{6}
\]

In a target medium different from Ti the conversion rate can differ by a factor of 2 [19]. We have denoted \(m_{L}^2\) the slepton soft mass matrix, \(m_S^2\) some average soft mass and \(\tan \beta\) the ratio of the Higgs vacuum expectation values.

Under the standard assumption of flavour blind SUSY breaking mediation – which consists in the most conservative assumption of lepton flavour violation – at some energy scale \(\Lambda\) where SUSY is broken it holds:

\[
m_{L}^2 \propto \frac{1}{\Lambda} \quad \text{at } \Lambda . \tag{7}
\]

Typically \(\Lambda\) is a very high scale like e.g. the Planck scale in Supergravity theories. As long as \(D_M < \Lambda\), RGE effects will transmit lepton flavour violation from the see-saw couplings to the slepton masses. This results into flavour off-diagonal mass entries at low energies that read roughly:

\[
m_{L12}^2 \simeq -\frac{3}{8\pi^2}m_S^2P_{12}, \quad \text{with} \quad P = Y_{\nu}^\dagger \log \frac{\Lambda}{D_M}Y_{\nu} . \tag{8}
\]

2.3. The minimal amount of flavour violation

The aim of this work is to set an upper bound on \(M_1\) from searches for the \(\mu-e\) transition as in \(\mu \rightarrow e\gamma\) or \(\mu-e\) conversion in nuclei. This corresponds to finding the minimal rate for rare lepton decays given fixed \(M_1\). According to eqs. (6), (8) the rate of the \(\mu-e\) flavour transition is minimised for minimal \(P_{12}\). Therefore let us express \(P_{12}\) in the parametrisation eq. (2):

\[
P_{12} = \left( V_L D_V V_R^\dagger \log \frac{\Lambda}{D_M} V_R D_Y V_L^\dagger \right)_{12} . \tag{9}
\]

If \(V_L\) has sizeable off-diagonal entries one can expect \(P_{12}\) to be dominated by \(V_{L13}Y_{3}^\dagger V_{L23} \log \frac{\Lambda}{M_2}\) as we assume \(D_Y\) to be hierarchical. But \(P_{12}\) could naturally be much smaller for \(V_L \sim I\). In this case

\[
P_{12}^\text{min} = y_1 V_{R1i}^\dagger \log \frac{\Lambda}{M_i} V_{R2y2} \simeq y_1 y_2 V_{R12} \log \frac{M_2}{M_1} , \tag{10}
\]

where insertions of two off-diagonal entries of \(V_R\) have been neglected – a good assumption as \(V_{R12}\) is the largest off-diagonal entry. Applying once again eq. (5) this reads:

\[
|P_{12}^\text{min}| = \frac{M_1|P_{12}|}{(H_u^0)^2} \log \frac{M_2}{M_1} . \tag{11}
\]

However, an even smaller value for \(P_{12}\) could possibly be achieved if small off-diagonal entries in \(V_L\) cancel this contribution. In order this to happen naturally, a model is necessary that relates \(V_L\) to \(V_R\), \(D_Y\) and the logarithm of the masses. Let us for example assume that the contribution from \(V_{L12}\) cancels the above \(P_{12}^\text{min}\), i.e. \(V_{L12} = - \frac{y_1}{y_2} V_{R12} \log \frac{\Lambda}{M_1} \log^{-1} \frac{\Lambda}{M_2}\). We find the existence of such a scenario very unplausible in particular because of the appearance of the logarithms and the SUSY breaking scale \(\Lambda\).
Figure 1. Allowed parameter space of thermal leptogenesis (in yellow, adopted from [20]), including the constraints on $M_1$ which stem from the assumed non-observation of the $\mu$-$e$ flavour transition in future experiments, under the assumption of hierarchical neutrino Yukawa eigenvalues and barring cancellations. The orange region corresponds to the range of $\tilde{m}_1$ for generic neutrino parameters. In this plot it is assumed $m_S \simeq 200$ GeV and $\tan \beta \simeq 10$.

2.4. The constraints
Now we have argued that eq. (11) indeed gives the minimal mixing of $\mu$-$e$, we can calculate the minimal conversion rate using eqs. (6), (8):

$$R(\mu \to e) \gtrsim \frac{\alpha^4}{G_F^2} \left( \frac{3}{8\pi^2} \frac{\tan \beta}{m_S^2} \right)^2 \left( \frac{M_1|\mathcal{M}_{12}|}{\langle H_u^0 \rangle^2} \log \frac{M_2}{M_1} \right)^2.$$  \hspace{1cm} (12)

This can of course be inverted to bound $M_1$ from above. For simplicity assume conservatively $\log \frac{M_2}{M_1} \sim \log 10$ and take $\sin \beta \sim 1$, giving:

$$M_1 \lesssim 5 \times 10^{12} \text{GeV} \left( \frac{R(\mu \to e)}{10^{-13}} \right)^{1/2} \left( \frac{m_S}{200 \text{ GeV}} \right)^2 \left( \frac{\tan \beta}{10} \right)^{-1}.$$  \hspace{1cm} (13)

Here we have normalised the conversion rate by the maximal possible rate in agreement with data from $\mu \to e\gamma$. We believe that it is interesting on its own to shrink the possible range of a fundamental parameter. But if future searches for SUSY and $\mu$-$e$ transitions bring down the constraint on $M_1$ to $\sim 10^9$ it could also serve as a probe of leptogenesis.

3. Probing leptogenesis
The baryon asymmetry generated through the leptogenesis mechanism depends, under the assumption of hierarchical neutrinos, essentially on two parameters: the lightest right-handed neutrino mass, $M_1$, and an effective neutrino mass $\tilde{m}_1$ [21, 22], defined as

$$\tilde{m}_1 = \frac{(Y_\nu Y_\nu^\dagger)_{11}}{M_1} \langle H_u^0 \rangle^2.$$  \hspace{1cm} (14)
which measures the strength of the coupling of the lightest right-handed neutrino to the thermal bath. Naively one expects $\tilde{m}_1$ to be of the order of the light neutrino mass splittings but strictly it is a free parameter. In the situation of hierarchical neutrino Yukawa couplings and minimal lepton flavour violation $\tilde{m}_1$ equals the solar mass splitting [12].

We show in Fig. 1 the impact of the bounds on the lightest right-handed neutrino mass stemming from the non-observation of the $\mu-e$ flavour transition on the parameter space of thermal leptogenesis. The yellow/orange region corresponds to the allowed region found by Blanchet and di Bari, and shown in Fig. 1 of [20]. The orange region marks the range of $\tilde{m}_1$ for generic neutrino parameters, i.e. the interval from the solar to the atmospheric mass splitting.

As thick dashed lines we show the most stringent upper bound on $M_1$, eq. (13), for the projected sensitivity of different experiments (cf. sec. 2.2). The solid lines bound the allowed region for certain leptogenesis scenarios: thermal (thin) vs. zero initial abundance (thick) and maximal (right) vs. no flavour effects (left). Further details can be inferred from [12].

We find that for generic $\tilde{m}_1$ the window for $M_1$ could be closed with PRISM/PRIME for $m_2^{\tan\beta} < \frac{(200 \, \text{GeV})^2}{50}$. Otherwise a better sensitivity is necessary.

4. Conclusions
The see-saw mechanism is a very elegant explanation for the smallness of neutrino masses, which in addition might account for the baryon asymmetry of the Universe, through the mechanism of leptogenesis. The supersymmetric version, which is probably the most natural arena to implement it, endows us with another clue to the see-saw parameters. Namely, the existence of the neutrino Yukawa couplings affects the flavour structure of the slepton mass matrices through quantum effects, leading to lepton flavour violation maybe visible in ongoing or future searches.

Working under very general and well motivated assumptions, namely the absence of cancellations and a hierarchical pattern in the neutrino Yukawa eigenvalues, we have identified the scenario yielding the minimal rate for the rare decay $\mu \to e\gamma$ and $\mu-e$ conversion in nuclei. In this scenario, the rate depends essentially on the lightest right-handed neutrino mass, $M_1$, and on supersymmetric parameters. Using the experimental constraint on $\text{BR}(\mu \to e\gamma)$ we have derived an upper bound $M_1 \lesssim 5 \times 10^{12}$ for typical soft SUSY breaking terms of 200 GeV and $\tan\beta = 10$. This should be compared with the lower bound required by the thermal leptogenesis scenario, $M_1 \gtrsim 10^9$ GeV. The upper bound derived in this paper scales as $\text{BR}(\mu \to e\gamma)^{1/2}$, therefore, future improvements in sensitivity to the process $\mu \to e\gamma$ and to $\mu-e$ conversion in nuclei can have important implications for the thermal leptogenesis scenario if no positive signal is found. But only in the case of large $\tan\beta$ a sensitivity of $R(\mu Ti \to e Ti) \sim 10^{-18}$, as expected from the PRISM/PRIME project, can rule out thermal leptogenesis models without fine-tuning and based on the decay of the lightest right-handed neutrino. On the other hand, our bound is very conservative and in general one can expect much larger lepton flavor violating rates in the case of supersymmetric leptogenesis.

Acknowledgments
CS wishes to thank the organizers for this very pleasant conference. This work was partially supported by the DFG cluster of excellence Origin and Structure of the Universe and by the Graduiertenkolleg “Particle Physics at the Energy Frontier of New Phenomena”.

References
[1] Minkowski P 1977 Phys. Lett. B67 421
[2] Gell-Mann M, Ramond P and Slansky R Print-80-0576 (CERN)
[3] Mohapatra R N and Senjanovic G 1980 Phys. Rev. Lett. 44 912
[4] Yanagida T In Proceedings of the Workshop on the Baryon Number of the Universe and Unified Theories, Tsukuba, Japan, 13-14 Feb 1979
[5] Borzumati F and Masiero A 1986 Phys. Rev. Lett. 57 961
[6] Davidson S and Ibarra A 2001 JHEP 09 013 (Preprint hep-ph/0104076)
[7] Fukugita M and Yanagida T 1986 Phys. Lett. B174 45
[8] Davidson S and Ibarra A 2003 Nucl. Phys. B648 345–375 (Preprint hep-ph/0206304)
[9] Davidson S 2003 JHEP 03 037 (Preprint hep-ph/0302075)
[10] Branco G C, Gonzalez Felipe R, Joaquim F R and Rebelo M N 2002 Nucl. Phys. B640 202–232 (Preprint hep-ph/0202030)
[11] Akhmedov E K, Frigerio M and Smirnov A Y 2003 JHEP 09 021 (Preprint hep-ph/0305322)
[12] Ibarra A and Simonetto C 2009 JHEP 08 113 (Preprint 0903.1776)
[13] Brooks M L et al. (MEGA) 1999 Phys. Rev. Lett. 83 1521–1524 (Preprint hep-ex/9905013)
[14] Mori, T et al 1999 Search for $\mu \to e \gamma$ down to $10^{-14}$ branching ratio, Research Proposal to Paul Scherrer Institut, see http://meg.web.psi.ch
[15] Bernstein, R H and Miller, J P et al 2008 Proposal to Search for $\mu^- N \to e^- N$ with a Single Event Sensitivity below $10^{-16}$, Mu2e Proposal, see http://mu2e.fnal.gov
[16] Kuno, Y et al 2007 An Experimental Search for Lepton Flavor Violating $\mu$–$e$ Conversion at Sensitivity of $10^{-16}$ with a Slow-Extracted Bunched Proton Beam, COMET Proposal, see http://comet.phys.sci.osaka-u.ac.jp
[17] Kuno Y 2005 Nucl. Phys. Proc. Suppl. 149 376–378
[18] Paradisi P 2005 JHEP 10 006 (Preprint hep-ph/0505046)
[19] Kitano R, Koike M and Okada Y 2002 Phys. Rev. D66 096002 (Preprint hep-ph/0203110)
[20] Blanchet S and Di Bari P 2007 JCAP 0703 018 (Preprint hep-ph/0607330)
[21] Buchmuller W, Peccei R D and Yanagida T 2005 Ann. Rev. Nucl. Part. Sci. 55 311–355 (Preprint hep-ph/0502169)
[22] Davidson S, Nardi E and Nir Y 2008 Phys. Rept. 466 105–177 (Preprint 0802.2962)