Superwalkers

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A droplet of silicone oil can be made to bounce and walk on the surface of an oscillating bath of the same fluid\textsuperscript{1}. These walking droplets called walkers are propelled by the standing waves they generate on each bounce. A walker is a composite object comprising a wave and a particle. Such walkers have been shown to mimic several features intrinsic to quantum systems\textsuperscript{2}. Here we present observations of a new class of walking droplets coined ‘superwalkers’ due to their unusual dynamics. The superwalkers are characterised by their great size and speed and we have measured the largest superwalking droplets to have more than twice the size of the largest walker and travel at more than double the speed of the fastest walkers. Furthermore, superwalkers are fundamentally different from normal walkers in their interactions with other droplets; they may form tightly bound many-droplet crystals which undergo melting and evaporation on increasing the driving amplitude. The droplets which are evaporated from the crystals behave similar to an ideal gas of particles with nearly elastic collisions and ballistic dynamics. We have also observed several superwalkers to self-stabilise into a chasing mode forming droplet trains. The superwalkers open a pathway for observations of new phenomena in systems of walking droplets.

I. INTRODUCTION

In 2005, Couder et al. \textsuperscript{1} showed that if a bath of silicone oil is vibrated vertically with sinusoidal acceleration $\gamma \sin(2\pi f t)$, with $\gamma$ the peak acceleration and $f$ the frequency, then a droplet of the same liquid as the bath can be made to bounce indefinitely on the oscillating surface provided $\gamma > \gamma_B$, with $\gamma_B$ the bouncing threshold. Just above the bouncing threshold, the droplet bounces at frequency $f$. As $\gamma$ increases, the droplet undergoes a series of bifurcations, and for $\gamma > \gamma_W > \gamma_B$, with $\gamma_W$ the walking threshold, a robust walking state develops with the droplet bouncing at frequency $f/2$. This walking state emerges just below the Faraday instability threshold $\gamma_F$, above which the fluid-air interface becomes unstable to standing Faraday waves of frequency $f/2$. In this walking state, the droplet is a local exciter of Faraday waves whose decay time is a function of the ‘memory’ of the system, the proximity to the Faraday threshold. At high memory (near the threshold), waves generated by the droplet in the distant past continue to affect the motion of the droplet. This hydrodynamic system comprising a wave and a particle mimics several features of the quantum realm. These include particle diffraction through single and double slit arrangements\textsuperscript{3}, orbital quantization in rotating frames\textsuperscript{4} and harmonic potentials\textsuperscript{5,6}, wavelike statistics in confined geometries\textsuperscript{7,8}, tunneling across submerged barriers\textsuperscript{9}, and anomalous two-droplet correlations\textsuperscript{10}.

These walking droplets or ‘walkers’ have been shown to exist over the frequency range 35-125 Hz depending on the viscosity of the oil. For oil of viscosity 20 cSt that has been used in most of the experiments, the frequency range for walking is 60-80 Hz and the typical droplet radii are in the range 0.3-0.5 mm with walking speeds up to 15 mm/s\textsuperscript{11}. Interactions of a walker with barriers and other droplets are mediated through the wavefield they generate on each bounce. Interaction of a walker with submerged planar barriers results into non-specular reflections\textsuperscript{12}. Multiple walkers interact with each other through their wavefield and exhibit intricate dynamics. Interaction of two walkers can results into bound states of two droplets such as parallel walkers\textsuperscript{13}, promenading pairs that oscillate towards and away from one-another while parallel walking\textsuperscript{14,15}, and tightly bound orbiting states\textsuperscript{13,16,17}. Many bouncing droplets tend to self-organize into bound lattice structures\textsuperscript{13,18,19} and moving rafts\textsuperscript{20}.

Bouncing and walking dynamics of droplets at two frequency forcing at 80 and 64 Hz has been studied by Sampara and Gilet\textsuperscript{21} who found the vertical bouncing of the walker to become chaotic and the horizontal motion to become irregular when amplitudes of both frequencies are significant. Parametrically forcing a bath of fluid at two different frequencies $f$ and $f/2$ can result into Faraday instability with either harmonic or subharmonic waves depending on the amplitudes of the two frequencies and the phase difference between them\textsuperscript{22}. Here we present observations on a new class of walking droplets that emerge when the bath is driven at frequencies $f$ and $f/2$ with the acceleration $\gamma_f \sin(2\pi f t) + \gamma_{f/2} \sin(\pi f t + \phi)$ where $50 \lesssim f \lesssim 100$ Hz. We have found such super walking droplets to be up to double the radii ($\approx 1.2$ mm) of walkers and to walk up to more than double the speed ($\approx 40$ mm/s) and hence we call them superwalkers. Apart from their size and speed, superwalkers are also fundamentally different from walkers in their inter-droplet interactions. In Section\textsuperscript{11} we provide details of our experimental setup followed by the characterization of a single superwalker (Section\textsuperscript{11}) and exploring the interactions of multiple...
When a bath of silicone oil is vibrated at a single frequency of 80 Hz, droplets with radii $0.3 \leq R \leq 0.5$ mm can walk on the oscillating bath while droplet with radii $R > 0.5$ mm either just bounce or coalesce with the fluid in the bath\[1\]. We find that on adding a second frequency at 40 Hz with a phase difference $\phi$, these bigger droplets start bouncing and walking resulting into superwalkers. Observing the wavefield, we find that the wavefield of superwalkers is very similar to walker, and that they are also a local exciter of the 40 Hz Faraday standing waves. The motion of the superwalkers can be switched ‘on’ and ‘off’ by switching on and off the signal from the 40 Hz driving. Depending on the value of $\gamma_{80}$, medium sized superwalkers suddenly stop and become bouncers while large superwalkers coalesce with the bath when the 40 Hz signal is suddenly switched off. Below we characterize the walking dynamics of a single superwalker.

Figure 1 shows the different behaviours observed in the $\gamma_{80}$-$\gamma_{40}$ parameter space for a fixed phase difference of $\phi = 120^\circ$ and a droplet of radius $R \approx 0.6$ mm. For a single frequency forcing at 80 or at 40 Hz, the Faraday instability thresholds are $\approx 4.2g$ and $\approx 1.2g$ respectively with $g$ being the gravitational acceleration. Droplet coalesces with the bath at low $\gamma_{80}$ and $\gamma_{40}$. At large $\gamma_{80}$ the Faraday instability from 80 Hz forcing dominates while at large $\gamma_{40}$ the Faraday instability from 40 Hz forcing dominates. When both $\gamma_{80}$ and $\gamma_{40}$ are large, both Faraday patterns are excited simultaneously. For intermediate forcing, bouncing and superwalking region exist and are shown for a droplet of radius $R \approx 0.6$ mm.

We have studied the speed of a single superwalker walking along a (horizontally) straight line as a function of its radius by varying the two forcing acceleration amplitudes $\gamma_{80}$ and $\gamma_{40}$ at a fixed phase difference $\phi = 120^\circ$. The results of varying one of the forcing accelerations keeping the other fixed are shown in Figs. 2(a) and (b) showing the speed $(V)$ as a function of radius $(R)$. Figure 2(a) shows the results of increasing $\gamma_{40}$ at a fixed $\gamma_{80} = 3.4g$. This results into walking of larger droplets that previously bounced or coalesced at $\gamma_{40} = 0g$ and the speed of the larger droplets increases with increasing $\gamma_{40}$. For $\gamma_{40} = 0g$ (purple circles), the bath is driven at a single frequency and only walkers exist. Here we observe that the speed of the walker increases with its size until $R \approx 0.45$ mm and then suddenly drops to zero because the heavier, larger droplets are unable to walk. Increasing $\gamma_{40}$ to 0.25$g$ (yellow circles) and then to 0.6$g$ (red circles) powers the droplets of size $R \gtrsim 0.45$ mm and turns them into superwalkers. However, the speed increases with size until $R \approx 0.6$ mm and then decreases. On visualizing the bouncing motion with a high-speed camera, we find that the droplets on the left branch ($0.45 \lesssim R \lesssim 0.6$ mm) are bouncing in a mode similar to the (2,1) bouncing mode for walkers [11] but...
with larger contact time, while the droplet on the right branch \((R \gtrsim 0.6 \text{ mm})\) are bouncing in a mode similar to the \((2,2)\) bouncing mode of walkers\(^{11}\) with significant deformation during each bounce and the droplets hardly lift off. Droplets of radii \(0.3 \lesssim R \lesssim 0.45 \text{ mm}\) that were walkers at a single frequency forcing, are no longer resonant walkers in the \((2,1)\) bouncing mode at two frequency forcing and start walking irregularly. Further increasing \(\gamma_{40}\) to 

\(1 \text{ g} \) (blue circles) seems to only have a significant effect on the droplets of size \(R \gtrsim 0.6 \text{ mm}\) and boosts their velocity to \(V \approx 40 \text{ mm/s}\). For \(\gamma_{40} \gtrsim 1 \text{ g}\), Faraday waves instability from 40 Hz forcing kicks in and superwalkers become chaotic bouncers and eventually coalesce. Figure 2(b) shows speed \((V)\) as a function of radius \((R)\) of the droplet for a fixed \(\gamma_{40} = 1 \text{ g}\) and three different values of \(\gamma_{80}\). We find that increasing \(\gamma_{80}\) for a fixed \(\gamma_{40}\) results into a vertical dilation of the curve as \(\gamma_{80}\) increases. This suggests that increasing \(\gamma_{80}\) increases the speed of droplets of all size. At the onset of the Faraday threshold from 80 Hz forcing, we find that while normal walkers start bouncing chaotically, the superwalkers are still walking at the onset of this instability and their motion is only disturbed once the amplitude of the nonlinear standing Faraday waves becomes significantly large.

Figure 2(c) shows the speed as a function of \(\gamma_{40}\) for three different sized droplets for a fixed \(\gamma_{80} = 3 \text{ g}\) and phase difference \(\phi = 120^\circ\). A smaller droplet of radius \(R = 0.41 \text{ mm}\) (blue circles), which is a walker at \(\gamma_{40} = 0 \text{ g}\), starts bouncing and walking irregularly as \(\gamma_{40}\) is increased and its walking speed decreases. At \(\gamma_{40} \approx 1 \text{ g}\), the walker stops walking completely and is just a chaotic bouncer. A medium sized droplet of radius \(R = 0.68 \text{ mm}\) is a bouncer until \(\gamma_{40} \approx 0.2 \text{ g}\) and then starts walking with speed increasing with \(\gamma_{40}\) with the functional form of the curve being similar to that of walker at a single driving frequency \(^{11}\). A larger droplet of size \(R = 1.04 \text{ mm}\) cannot just bounce and can only walk. For \(\gamma_{40} \lesssim 0.45 \text{ g}\) the droplet coalesces with the bath while for \(\gamma_{40} \gtrsim 0.45 \text{ g}\), the droplet walks and its speed seems to increases linearly with \(\gamma_{40}\).

Figure 2(d) shows the speed of droplets of two different size \(R = 0.38 \text{ and } 0.63 \text{ mm}\) as a function of the phase different between the signals \(\phi\) at a fixed value of \(\gamma_{80} = 2.6 \text{ g}\) and \(\gamma_{40} = 0.8 \text{ g}\). A droplet of size \(R = 0.38 \text{ mm}\) is an irregular bouncer at these parameters and is not significantly affected by the phase difference. While a larger droplet of size \(R = 0.63 \text{ mm}\) suddenly starts walking for a narrow range of phase difference in the range \(70 \lesssim \phi \lesssim 140\) with the peak velocity near \(\phi \approx 110^\circ\). We find this to be independent of the droplet size of superwalkers. Hence superwalkers only exist for a range of phase differences \(70 \lesssim \phi \lesssim 140\) when driven at 80 and 40 Hz.

IV. HORIZONTAL DYNAMICS OF MULTIPLE SUPERWALKERS

A. Bound states of few superwalkers

Two superwalkers can form bound states like two walkers. At low memories above the superwalking threshold, two superwalkers can bind into a tight pair and walk together where the individual droplets are touching each other. If the droplets are of different size then they traverse a circular path (Fig. 3(a)) while same size droplets follow a straight line path (Fig. 3(b)). As the forcing acceleration is increased, the droplet form a state reminiscent of the promenading pair of walkers \(^{14,15}\) where the droplets walk parallel with sideways oscillations. Promenading pair of walkers are separated from each other by a wave barrier formed by their mutual wavefield at their minimum separation. However, promenading pair of superwalkers overshoot their equilibrium, and the droplets
collide and bounce off each other at their minimum separation in this oscillatory motion. The center-of-mass of identical superwalkers follows a straight line path while that of even slightly mismatched superwalkers tends to follow a circular trajectory (Fig. 3(c)–(e)). Such circular trajectories of promenading pairs of walkers were obtained in numerical simulations of identical in-phase droplets by Valani and Slim [23].

Two superwalkers also form orbiting pairs like two walkers. We find orbiting motion of mismatched superwalkers to be much more common than same sized superwalkers. For mismatched droplets, the bigger droplets orbit in a circle of larger radius while the smaller droplet orbits in a circle of smaller radius (Fig. 3(f)). If there is big difference between the size of the two droplets then we find that the orbiting pairs starts drifting as well (Fig. 3(g)). Trajectories have also been found where the orbiting pair reverses its direction intermittently.

We have found another type of bound pair, chasers, in experiments with both walkers and superwalkers that has been observed numerically for identical in-phase droplets by Valani et al. [10]. In this state, two droplets walk one behind the other at a constant speed (Fig. 3(h)). Here the droplets can be of the same size or different sizes. For droplets that are not the same size, the larger droplet is in the lead while the smaller droplet is dragged by the leading droplet. Chasing pair of superwalkers are more robust and form ubiquitously at high memory while chasing pair of walkers is rare to find. This bound state is different from the ratcheting pair motion that has previously been reported for normal walkers [20]. While ratcheting motion occurs below the walking threshold, we find that the chasers only dominate at high memory with their speeds an order of magnitude greater than reported for ratcheting pairs. We have also observed chasing motions with up to three droplets forming a droplet train but their occurrence is much more rare than chasing motion with two droplets. For chasing motion of three mismatched droplets, the leading droplets is the largest with size progressively decreasing for the trailing droplets. The chasing motion seems to become destroyed once the chasing pair hits the container wall, but if the size difference between the chasing pairs is large then they may remain in the chasing mode even after colliding with the wall. For two mismatched superwalkers, we also find transition between chasing motion and orbiting motion intermittently. In future, we aim to explore the chasers in more detail.

Other rich dynamical bound states of three droplets and interesting rafts dynamics of few droplet crystals are also observed. Two common occurring dynamical states for three droplets are shown in Fig. 3(i) and (j). In Fig. 3(i), the smaller droplet pulls the condensed pairs of droplets while oscillating back and forth towards and away from the condensed pair while in Fig. 3(j), three droplets form a molecule and walk in a straight line at constant speed.

B. Dynamics of many superwalkers

When many superwalkers are present, they tend to spontaneously bind into crystal structures at low memory below the superwalking threshold. This can be though of as a solid-like state of the superwalkers (Fig. 4(a)). As the forcing amplitude is increased, the crystal heats up and some droplets ‘melt’ apart forming two and three droplet molecules (Fig. 4(b)). Such crystal melting has been observed with oil droplets when driven at a single frequency [18]. As the forcing amplitude is increased
FIG. 4. Many-droplet dynamics: Intricate dynamics take place with many interacting superwalkers as the forcing acceleration ($\gamma_{40}$ or $\gamma_{80}$) is varied with the emergence of (a) a ‘solid’ state (at low memory) in which the droplets have condensed into crystal structure, (b) a ‘liquid’ state (at mid-memory) where the crystal begins to agitate and some droplet molecules break apart but such molten droplets still remain bound as two and three droplet composites, and (c) a ‘gaseous’ state (at high memory) where droplets are moving at high speed colliding with each other elastically like billiard balls moving ballistically between the collisions.

further, all the droplets ‘evaporate’ and start superwalking at high speeds and bouncing off each other like billiard balls (Fig. 4(c)). Intriguing dynamics take place in this high memory regime when many walkers and superwalkers are simultaneously created on the bath. In this regime, larger droplets (fast moving superwalkers) interact with smaller droplets (chaotically bouncing walkers) in two ways. First, they try to snatch the smaller droplets and drag them in a chasing pair, or, secondly, they destroy the smaller droplet when they approach them head on by either making the smaller droplet coalesce with the bath or the bigger droplet merging with the smaller droplet.

We also find another interesting dynamics when the forcing frequencies are slightly detuned from being exactly proportional by a factor of two. For example if the two forcing frequencies are 80 and 39.5 Hz, then the droplets perform a stop-and-go motion. The droplets walk for a second, then stop, then again walking for a second and so on. This stopping and starting is due to the beating behavior of the driving signal where the droplets cross the superwalking threshold for only a part of the cycle. It acts as though a discrete dynamical system motion arising in a continuous system.

V. CONCLUSION

We have uncovered a new type of walking droplet - superwalker, when a bath is driven at two frequencies with a relative phase shift. Superwalkers can be up to double the size and can walk up to more than twice the speed of normal walkers. Superwalkers exist only for a range of phase difference $\phi$ between the two frequencies. Increasing the forcing acceleration $\gamma_{80}$ increases the speed of all sized droplets while increasing $\gamma_{40}$ only powers the superwalkers resulting into their large speed. Superwalkers interact differently than walkers and form several novel bound states such as a tight pair where the droplets are touching, promenading pair with droplets bouncing off each other and chasing pairs of walkers and droplet trains at high memory. Interaction of many superwalkers is intriguing and can results into ‘evaporation’ of tightly bound droplet crystals where the evaporated droplets starting moving at high speeds and colliding with each other like billiard balls. Statistical mechanics of this apparent phase transition will be interesting to explore in future. Superwalkers will open up fascinating opportunities to explore new hydrodynamic quantum analogs and revisiting the previously explored phenomena of walking droplets.

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