The elimination of decoherence of two-state quantum systems interacting with a thermal reservoir through an external controllable driving field is discussed in the present paper. The restriction equation with which the external controllable driving field should agree will be derived. Based on this, we obtain the time-development equation of the off-diagonal elements of density operator in the supersymmetric multiphoton two-state quantum systems, which is helpful for studying the polarization evolution in this two-state quantum model.

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I. INTRODUCTION

Recently, an area called quantum computation, which involves computers that use the ability of quantum systems to be in a superposition of many states, attracts extensive attention of many researchers. However, it is not yet clear whether quantum computers are feasible to build [1]. One reason that quantum computers will be difficult, if not impossible, to build is decoherence. In the process of decoherence, some qubit or qubits of the computation become entangled with the environment, thus in effect “collapsing” the state of the quantum computer [1]. In literature, there may exist three schemes to reduce the decoherence: (i) error-avoiding codes [2]; (ii) error-correcting codes [3]; (iii) decoherence-avoiding scheme [4]. The third approach to the suppression of decoherence can be realized by eliminating the interaction between the (two-state) quantum system and environment (such as a noise field, bath, thermal reservoir and so on) in the presence of an external controllable driving field. In this paper, we will study the maintenance of coherence via the decoherence-avoiding scheme, and the time evolution of polarization in a supersymmetric multiphoton two-level model. On considering the latter problem, we assume that the environmental effect on the quantum system under consideration has been eliminated by using the decoherence-avoiding scheme, which enables us to treat the polarization evolution problem in the multiphoton model more conveniently (i.e., the polarization evolution problem will be investigated under the assumption that the decoherence of the quantum systems in the noise field has been reduced).

In section II, we introduce Zhang’s method for treating the decoherence problem [5]. In section III, by employing Zhang’s approach to the multiphoton two-state quantum system we obtain the time-development equation of the off-diagonal element of density operator in this two-state system.

II. MAINTENANCE OF COHERENCE

In this section, we review one of the formulation for dealing with the decoherence-avoiding scheme, which was suggested by Zhang [5] more recently. The reason for the detailed reappearance of Zhang [5] below is as follows: (i) first and foremost, Zhang’s formulation is helpful for treating the polarization evolution problem in the multiphoton model; (ii) in this paper, the elimination of decoherence is a prerequisite for simplifying the polarization evolution problem in the multiphoton model, namely, in studying the polarization evolution problem in the multiphoton two-state system, we assume that the entanglement of the quantum system with the environment (e.g., thermal reservoir)
has been eliminated by an external driving field. So, we need not consider the decoherence problem of the multiphoton system in section III.

Let us first consider the following model, the Hamiltonian of which takes the form (in the unit $\hbar = 1$)

$$H(t) = \frac{\omega_0}{2}\sigma_z + \sum_k \omega_k a_k^\dagger a_k + \sum_k g_k \sigma_z (a_k + a_k^\dagger) - \frac{d}{2}[E(t)\sigma_+ + E^*(t)\sigma_-],$$  \hspace{1cm} (2.1)

which can describe the interaction of a two-level atom with a noise field (thermal reservoir). In this Hamiltonian, $\omega_0$, $\omega_k$, $\sigma_z$, $g_k$ and $E(t)$ denote the atomic transition frequency, photon frequency with $k$-mode, the third-component Pauli matrix, the coupling coefficient (of atoms to the thermal reservoir) and the external driving field, respectively.

By using the unitary transformation

$$V(t) = \exp \left[ \frac{1}{i} \left( \frac{\omega_0}{2}\sigma_z + \sum_k \omega_k a_k^\dagger a_k \right) t \right],$$  \hspace{1cm} (2.2)

one can arrive at the Hamiltonian

$$H_1(t) = \sum_k g_k \sigma_z \left[ a_k \exp(-i\omega_k t) + a_k^\dagger \exp(i\omega_k t) \right] - \frac{d}{2}[E(t)\exp(i\omega_0 t)\sigma_+ + E^*(t)\exp(-i\omega_0 t)\sigma_-]$$  \hspace{1cm} (2.3)

in the interaction picture, where use is made of $H_1(t) = V(t)^\dagger [H(t) - i\frac{\partial}{\partial t}] V(t)$. The density operator of the two-level atomic system agrees with

$$i\frac{\partial \rho_1(t)}{\partial t} = [H_1(t), \rho_1(t)].$$  \hspace{1cm} (2.4)

Let $\rho_{ql}(t)$ denote the atomic reducible density operator. It follows that the reducible density operator equals

$$\dot{\rho}_{ql}(t) = \text{Tr}_r \rho_1(t) = -\int_0^t \text{Tr}_r[H_1(t), [H_1(t'), \rho_1(t')]]dt'.$$  \hspace{1cm} (2.5)

If we assume that the thermal reservoir is rather large, then it can be concluded that the reservoir may not change much during the time evolution process of the atom-reservoir system. Thus we have $\rho_1(t) \simeq \rho_{ql}(t)\rho_{rl}(0)$, where $\rho_{rl}(0) \simeq \exp(-\beta H_0)/\text{Tr}\exp(-\beta H_0)$ with $\beta = 1/k_B T$, $H_0 = \sum_k \omega_k a_k^\dagger a_k$. If we take $\rho_1(t') \simeq \rho_1(t)$ (Markoff approximation), then Eq.(2.5) can be rewritten as

$$\dot{\rho}_{ql}(t) = -\int_0^t \text{Tr}_r[H_1(t), [H_1(t'), \rho_{ql}(t)\rho_{rl}(0)]]dt',$$  \hspace{1cm} (2.6)

where the integrand can be rewritten as

$$\text{Tr}_r[H_1(t), [H_1(t'), \rho_{ql}(t)\rho_{rl}(0)]] = \text{Tr}_r[H_1(t)H_1(t')\rho_{ql}(t)\rho_{rl}(0) - H_1(t)\rho_{ql}(t)\rho_{rl}(0)H_1(t')$$

$$- H_1(t')\rho_{ql}(t)\rho_{rl}(0)H_1(t)] + \rho_{ql}(t)\rho_{rl}(0)H_1(t')H_1(t)].$$  \hspace{1cm} (2.7)

By the aid of the relations $< a_k^\dagger a_k >_R = \bar{n}_k$, $< a_k a_k^\dagger >_R = \bar{n}_k + 1$, $\bar{n}_k = 1/[\exp(\hbar \omega_k/k_B T) - 1]$, one can arrive at through lengthy calculation [5]

$$\dot{\rho}_{ql}(t) = -2 \left[ \frac{d}{2} \rho_{ql}(t) |E(t)|^2 \int_0^t \exp[i\omega_0(t - t')]|dt' - \rho_{ql}(t)A + \sigma_z \rho_{ql}(t)\sigma_z A \right]$$  \hspace{1cm} (2.8)

with $A = A_1 + A_2 + A_3 + A_4$, where

$$A_1 = \sum_k g_k^2 (\bar{n}_k + 1) \frac{1 - \exp(-i\omega_k t)}{i\omega_k}; \hspace{1cm} A_2 = \sum_k g_k^2 \bar{n}_k \frac{1 - \exp(i\omega_k t)}{-i\omega_k},$$

$$A_3 = \sum_k g_k^2 (\bar{n}_k + 1) \frac{-\exp(i\omega_k t)}{-i\omega_k}; \hspace{1cm} A_4 = \sum_k g_k^2 \bar{n}_k \frac{1 - \exp(-i\omega_k t)}{i\omega_k}.$$  \hspace{1cm} (2.9)
Ignoring the Lamb-shift term (i.e., taking the real parts of \( \dot{\rho}_{01} = \langle 0 | \dot{\rho}_{q1} | 1 \rangle \), \( \rho_{01} = \langle 0 | \rho_{q1} | 1 \rangle \), \( \dot{\rho}_{10} = \langle 1 | \dot{\rho}_{q1} | 0 \rangle \), \( \rho_{10} = \langle 1 | \rho_{q1} | 0 \rangle \)), we can obtain the time-development equation of the off-diagonal elements of density operator, i.e.,

\[
\dot{\rho}_{01} = -2 \left[ \frac{\sin(\omega_0 t)}{\omega_0} \frac{d^2}{4} |E(t)|^2 + \sum_k 2g_k^2(2\tilde{n}_k + 1) \frac{\sin(\omega_k t)}{\omega_k} \right] \rho_{01},
\]

\[
\dot{\rho}_{10} = -2 \left[ \frac{\sin(\omega_0 t)}{\omega_0} \frac{d^2}{4} |E(t)|^2 + \sum_k 2g_k^2(2\tilde{n}_k + 1) \frac{\sin(\omega_k t)}{\omega_k} \right] \rho_{10}.
\]

Thus it is readily verified that if the envelope of the external driving field satisfies the following condition

\[
\frac{\sin(\omega_0 t)}{\omega_0} \frac{d^2}{4} |E(t)|^2 + \sum_k 2g_k^2(2\tilde{n}_k + 1) \frac{\sin(\omega_k t)}{\omega_k} = 0,
\]

then one have \( \rho_{01}(t) = \rho_{01}(0) \) and \( \rho_{10}(t) = \rho_{10}(0) \), which means the suppression (elimination) of decoherence in this two-level quantum system interacting with the environment (thermal reservoir) through an external controllable field \( E(t) \).

It should be noted again that the above theory was proposed by Zhang [5]. In the next section we will investigate the polarization evolution problem in the two-level supersymmetric multiphoton Jaynes-Cummings model by making use of Zhang’s formulation.

### III. POLARIZATION EVOLUTION IN THE MULTIPHOTON TWO-LEVEL QUANTUM SYSTEM

The multiphoton two-level system that we will consider in this section is the supersymmetric multiphoton Jaynes-Cummings model [6,7], the Hamiltonian of which under the rotating wave approximation is given by

\[
H = \frac{\omega_0}{2} \sigma_z + \omega a^\dagger a + g(a^\dagger)^k \sigma_- + g^*a^k \sigma_+,
\]

where \( a^\dagger \) and \( a \) are the creation and annihilation operators for the electromagnetic field, and obey the commutation relation \([a, a^\dagger] = 1\); \( \sigma_\pm \) and \( \sigma_z \) denote the two-level atom operators which satisfy the commutation relation \([\sigma_z, \sigma_\pm] = \pm 2\sigma_\pm\); \( g(t) \) and \( g^*(t) \) are the coupling coefficients and \( k \) is the photon number in each atom transition process; \( \omega_0(t) \) and \( \omega(t) \) are respectively the transition frequency and the mode frequency.

The supersymmetric structure can be found in this multiphoton two-level quantum model by defining the following supersymmetric transformation generators [8,9]:

\[
N = a^\dagger a + \frac{k-1}{2} \sigma_z + \frac{1}{2} = \begin{pmatrix} a^\dagger a + \frac{k}{2} & 0 \\ 0 & a a^\dagger - \frac{k}{2} \end{pmatrix}, \quad N' = \begin{pmatrix} (a^\dagger)^k & 0 \\ 0 & (a^\dagger)^k a^k \end{pmatrix};
\]

\[
Q = (a^\dagger)^k \sigma_- = \begin{pmatrix} 0 \\ (a^\dagger)^k \end{pmatrix}, \quad Q^\dagger = a^k \sigma_+ = \begin{pmatrix} 0 \\ a^k \end{pmatrix}.
\]

It is easily verified that \((N, N', Q, Q^\dagger)\) form supersymmetric generators and have supersymmetric Lie algebra properties, i.e.,

\[
Q^2 = (Q^\dagger)^2 = 0, \quad [Q^\dagger, Q] = N' \sigma_z, \quad [N, N'] = 0, \quad [N, Q] = Q,
\]

\[
[N, Q^\dagger] = -Q^\dagger, \quad \{Q^\dagger, Q\} = N', \quad \{Q, \sigma_z\} = \{Q^\dagger, \sigma_z\} = 0,
\]

\[
[Q, \sigma_z] = 2Q, \quad [Q^\dagger, \sigma_z] = -2Q^\dagger, \quad (Q^\dagger - Q)^2 = -N',
\]

where \(\{\}\) denotes the anticommuting bracket.

Now let us obtain the Hamiltonian of the above multiphoton Jaynes-Cummings model in the interaction picture by using the following unitary transformation

\[
V(t) = \exp \left[ \frac{i}{\hbar} \left( \frac{\omega_0}{2} \sigma_z + \omega a^\dagger a \right) t \right],
\]

and the result is
\[ H(t) = g \exp(-i\delta t)Q + g^* \exp(i\delta t)Q^\dagger \]

(3.5)

with \( \delta = k\omega - \omega_0 \). Based on Eq. (2.7), by complicated calculation, one can arrive at

\[
\text{Tr}_r[H_1(t)H_1(t')\rho_{ql}(t)\rho_{ql}(0)] = \langle H_1(t)H_1(t')\rho_{ql}(t) \rangle_R
= gg^* \{ \exp[i\delta(t' - t)] < Q Q^\dagger \rho_{ql}(t) > R + \exp[-i\delta(t' - t)] < Q^\dagger Q \rho_{ql}(t) > R \},
\]

(3.6)

where \( Q^2 = (Q^\dagger)^2 = 0 \) is applied to the calculation, and

\[
\text{Tr}_r[H_1(t)\rho_{ql}(t)\rho_{ql}(0)H_1(t')] = \langle H_1(t)\rho_{ql}(t)H_1(t') \rangle_R
= g^2 \exp[-i\delta(t + t')] < Q \rho_{ql}(t)Q > R + (g^*)^2 \exp[i\delta(t + t')] < Q^\dagger \rho_{ql}(t)Q^\dagger > R
+ gg^* [\exp[i\delta(t' - t)] < Q \rho_{ql}(t)Q^\dagger > R + \exp[-i\delta(t' - t)] < Q^\dagger \rho_{ql}(t)Q > R],
\]

(3.7)

\[
\text{Tr}_r[H_1(t')\rho_{ql}(t)\rho_{ql}(0)H_1(t)] = g^2 \exp[-i\delta(t + t')] < Q \rho_{ql}(t)Q > R + (g^*)^2 \exp[i\delta(t + t')] < Q^\dagger \rho_{ql}(t)Q^\dagger > R
+ gg^* [\exp[i\delta(t' - t)] < Q \rho_{ql}(t)Q^\dagger > R + \exp[-i\delta(t' - t)] < Q^\dagger \rho_{ql}(t)Q > R],
\]

(3.8)

and

\[
\text{Tr}_r[\rho_{ql}(t)\rho_{ql}(0)H_1(t')] = gg^* \{ \exp[-i\delta(t' - t)] < \rho_{ql}(t)Q Q^\dagger > R + \exp[i\delta(t' - t)] < \rho_{ql}(t)Q^\dagger > R \}.
\]

(3.9)

Thus it follows from (2.7) and the above four expressions (3.6)-(3.9) that

\[
\text{Tr}_r[H_1(t), [H_1(t'), \rho_{ql}(t)\rho_{ql}(0)]] = \mathcal{T}_1 + \mathcal{T}_2 + \mathcal{T}_3,
\]

(3.10)

where

\[
\mathcal{T}_1 = gg^* \{ \exp[i\delta(t' - t)] < (a^\dagger)^k a^k > R \sigma_+ \sigma_+ + \exp[-i\delta(t' - t)] < a^k (a^\dagger)^k > R \sigma_- \sigma_- \} \rho_{ql}(t)
+ gg^* \rho_{ql}(t) \{ \exp[-i\delta(t' - t)] < (a^\dagger)^k a^k > R \sigma_- \sigma_+ + \exp[i\delta(t' - t)] < a^k (a^\dagger)^k > R \sigma_- \sigma_- \},
\]

(3.11)

\[
\mathcal{T}_2 = -2 \{ g^2 \exp[-i\delta(t + t')] < Q \rho_{ql}(t)Q > R + (g^*)^2 \exp[i\delta(t + t')] < Q^\dagger \rho_{ql}(t)Q^\dagger > R \}
\]

(3.12)

and

\[
\mathcal{T}_3 = -gg^* \{ \exp[i\delta(t' - t)] + \exp[-i\delta(t' - t)] \} \{ < Q \rho_{ql}(t)Q^\dagger > R + < Q^\dagger \rho_{ql}(t)Q > R \}.
\]

(3.13)

With the help of the following relations \( a^k (a^\dagger)^k |m\rangle = \frac{(m+k)!}{m!} |m\rangle \), \( < a^k (a^\dagger)^k > R = \frac{(m+k)!}{m!} \), \( < a^k a^k > R = \frac{m!}{(m-k)!} \), \( < a^k (a^\dagger)^k > R = 0 \), \( \sigma_+ \sigma_+ = 0 \), \( \sigma_- \sigma_- = 0 \), \( \langle + | \sigma_- = 0 \), \( \langle - | \sigma_+ = 0 \), one can arrive at

\[
\langle 0 | \mathcal{T}_1 | 1 \rangle = gg^* \exp[i\delta(t' - t)] \left[ \frac{(m+k)!}{m!} + \frac{m!}{(m-k)!} \right] \langle 0 | \rho_{ql}(t) | 1 \rangle,
\]

\[
\langle 0 | \mathcal{T}_2 | 1 \rangle = 0,
\]

\[
\langle 0 | \mathcal{T}_3 | 1 \rangle = -gg^* \{ \exp[i\delta(t' - t)] + \exp[-i\delta(t' - t)] \} \left[ \frac{(m+k)!}{m!} \right] \langle 0 \rangle \rho_{ql}(t) | 0 \rangle.
\]

(3.14)

If we set \( \rho_{01}(t) = \langle 0 | \rho_{ql}(t) | 1 \rangle \), \( \rho_{10}(t) = \langle 1 | \rho_{ql}(t) | 0 \rangle \), \( \rho_{01}(t) = \langle 0 | \rho_{ql}(t) | 1 \rangle \), and use the equation

\[
\dot{\rho}_{01}(t) = -\int_0^t \text{Tr}_r \{ [H_1(t), [H_1(t'), \rho_{ql}(t)\rho_{ql}(0)]] | 1 \rangle \langle t' \}
\]

(3.15)

in accordance with (2.6), then we will get

\[
\dot{\rho}_{01}(t) = c_1(t) \rho_{01}(t) - c_2(t) \rho_{10}(t)
\]

(3.16)

with

\[
c_1(t) = -gg^* \left[ \frac{(m+k)!}{m!} + \frac{m!}{(m-k)!} \right] \frac{1 - \exp(-i\delta t)}{-i\delta}, \quad c_2(t) = gg^* \left[ \frac{(m+k)!}{m!} \right] \frac{1 - \exp(-i\delta t)}{-i\delta} + \frac{1 - \exp(i\delta t)}{i\delta}.
\]

(3.17)
In the meanwhile, we can obtain the complex conjugation to Eq. (3.16), i.e.,
\[ \dot{\rho}_{10}(t) = c_1^*(t)\rho_{10}(t) - c_2(t)\rho_{01}(t). \] (3.18)

Thus we obtain the time-development equation of the off-diagonal elements of density operator in this supersymmetric multiphoton two-state quantum systems.

In order to indicate the physical meanings of Eqs. (3.16) and (3.18), we will set \( u = \rho_{01} + \rho_{10}, \) \( v = i(\rho_{01} - \rho_{10}). \) Eqs. (3.16) and (3.18) can therefore be rewritten as follows
\[
\begin{align*}
\frac{du}{dt} &= [\text{Re} c_1(t) - c_2(t)] u + \text{Im} c_1(t)v, \\
\frac{dv}{dt} &= [\text{Re} c_1(t) + c_2(t)] v - \text{Im} c_1(t)u.
\end{align*}
\] (3.19)

It is easily seen that here the physical meanings of \( u \) and \( v \) are as follows: \( u \) and \( v \) have close relation to the dispersion and the dissipation (absorption/amplification) of polarization, respectively.

**IV. BRIEF DISCUSSION**

To close this section, we briefly discuss the time-evolution equation (3.19) of \( u \) and \( v. \) Let us consider a simple case where the detuning frequency \( \delta \) from the atomic transition frequency is vanishing. It follows from the expressions (3.17) that \( c_1(t) = -gg^* \left[ \frac{m+k}{m!} - \frac{m+k}{m-k!} \right] t \) and \( c_2(t) = -2gg^* \frac{k}{m!} t. \) Note that here \( c_1(t) \) is a real function. So, according to Eq. (3.19), we obtain
\[
\begin{align*}
u(t) &= u_0 \exp \left[ \frac{1}{2} \lambda_u t^2 \right], \quad v(t) = v_0 \exp \left[ \frac{1}{2} \lambda_v t^2 \right],
\end{align*}
\] (4.1)

where \( \lambda_u = -gg^* \left[ \frac{m!}{(m-k)!} - \frac{m+k}{m!} \right] \) and \( \lambda_v = -gg^* \left[ \frac{m!}{(m-k)!} + \frac{k(m+k)}{m!} \right]. \) Note that since \( \lambda_u > 0 \) for \( k > 0, \) the function \( u(t) \) associated with the dispersion will exponentially increase in the process of evolution, while \( v(t) \) related to the absorption/amplification will decrease rapidly in this process. In a decoupling case where \( k = 0, \) \( \lambda_u = 0. \) In this case, the real part of polarization (i.e., \( u \)) will not vary with the development of the quantum system. However, the imaginary part \( (v) \) of polarization does not vanish.

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[1] P.W. Shor, Phys. Rev. A 52, R2493 (1995).
[2] P. Zanardi and M. Rasetti, Phys. Rev. Lett. 79, 3306 (1997) and references therein.
[3] D. Gottesman, Phys. Rev. A 54, 1862 (1996); D.P. Divincenzo and P.W. Shor, Phys. Rev. Lett. 77, 3260 (1996); B. Schumacher, Phys. Rev. A 54, 2614 (1996); B. Schumacher, Phys. Rev. A 54, 2629 (1996); A.R. Calderbank and P.W. Shor, Phys. Rev. A 54, 1098 (1996).
[4] L. Viola and S. Lloyd, Phys. Rev. A 58, 2733 (1998); C.P. Sun, H. Zhan, and X.F. Liu, Phys. Rev. A 58, 1810 (1998).
[5] D.Y. Zhang, Chin. J. Quant. Electronics 20, 454 (2003).
[6] C.V. Sukumar and B. Buck, Phys.Lett. A 83, 211 (1981).
[7] F.L. Kien, M. Kozierowski and T. Quang, Phys.Rev. A 38, 263 (1988).
[8] H.X. Lu, X.Q. Wang, and Y.D. Zhang, Chin. Phys. 9, 325 (2000).
[9] H.X. Lu and X.Q. Wang, Chin. Phys. 9, 568 (2000).