Analysis of stable operation region of the hybrid AC/DC distribution systems

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Abstract: Hybrid alternating current (AC)/direct current (DC) distribution is a promising technology emerging for future power systems owing to its advantages, such as high controllability and compatibility. The stability of grid-connected converters is essential for the operation of an AC/DC distribution system. This study analyses the stable operation requirements of converters under different control modes based on the short-circuit ratio (SCR). Then, a method to calculate the critical SCR (CSCR) of a specified converter is proposed. Based on the CSCR of each converter, the stability criterion to judge the stability of multi-terminal DC (MTDC) distribution networks is proposed. To verify the accuracy and effectiveness of the proposed method and criterion, detailed electromagnetic simulations using PSCAD are conducted. The simulations indicate that the analysis results obtained using the proposed method and criterion are accurate, and the presented stable operation requirements of the converters can help to enhance the controllability of hybrid AC/DC distribution systems.

Nomenclature

\begin{itemize}
\item $u_S$: fundamental voltage phasor of alternating current (AC) source
\item $u_C$: fundamental voltage phasor inverted by the converter
\item $i_S$: fundamental current phasor of AC system
\item $U_S$: amplitude of voltage phasor $u_S$
\item $U_C$: amplitude of voltage phasor $u_C$
\item $I_S$: amplitude of current phasor $i_S$
\item $U_N$: rated voltage of the AC system
\item $U_{\min}$: minimum voltage inverted by the converter
\item $U_{\max}$: maximum voltage inverted by the converter
\item $U_{dS}$: $d$-axis voltage of AC source in dq frame
\item $U_{qS}$: $q$-axis voltage of AC source in dq frame
\item $i_{dS}$: $d$-axis current of AC system in dq frame
\item $i_{qS}$: $q$-axis current of AC system in dq frame
\item $Z$: equivalent impedance of the AC system
\item $\delta$: angle between the voltages $u_S$ and $u_C$
\item $\vartheta$: impedance angle of $Z$
\item $P$: active power flowing through the converter
\item $Q$: reactive power flowing through the converter
\item $S_C$: short-circuit capacity
\item SCR: short-circuit ratio of a specified converter
\item CSCR: critical short-circuit ratio of a specified converter
\item SCR\textsubscript{sys}: short-circuit ratio of a specified multi-terminal DC system
\end{itemize}

1 Introduction

Hybrid alternating current (AC)/direct current (DC) distribution technologies are considered promising for future power systems owing to their high controllability and compatibility [1, 2]. To achieve the stable operation of hybrid AC/DC distribution systems, control and power management strategies to regulate the voltage and power flow are crucial [3–9]. However, the significant variations in the operation mode of DC distribution systems can lead to the instability of grid-connected converters. In addition, the factors affecting the steady-state operation of converters under different control modes are different. Therefore, it is necessary to study the stable operation requirements and regions of the converters under different control modes and propose a stability criterion for multi-terminal DC (MTDC) distribution networks.

Two-level voltage-source converters (VSCs) and modular-multiple-level converters (MMCs) have become the preferred topologies for grid-connected converters in hybrid AC/DC distribution systems, and the stability analysis for grid-connected VSCs and MMCs has been studied extensively [10–17]. Current research mainly focuses on the small-signal stability of grid-connected converters using frequency- and time-domain-based methods. In [10–13], the small-signal modelling of VSCs and MMCs in the frequency domain was reported, and the impedance-based stability criterion was proposed based on the Nyquist theorem. However, the internal factors affecting the dynamic characteristics of the subsystem cannot be identified with this method, so it is difficult to describe the dominant links affecting the small-signal stability of a complex distribution network. In [14–17], the small-signal modellings of VSCs and MMCs in the time domain were reported, and the small-signal stability of the complex systems was accurately analysed. In [14, 15], a comprehensive small-signal model for a two-level VSC was presented, and the impact of the system parameters on the small-signal stability was analysed. In [16], the small-signal stability of an MMC was assessed through a linear time-periodic framework considering the circulating current control loop. In [17], two reduced-order small-signal MMC models were proposed for different research objectives. The calculation of the initial power-flow is essential for small-signal stability analysis. In [18–22], a method to calculate the power flow in power systems under different control strategies and operating conditions was developed using radial basis function neural networks. This method is useful for ill-conditioned networks owing to its capability of calculating the partial derivatives of equations and taking the inverse of the Jacobian matrix [18]. Although the above-mentioned studies analysed the small-signal stability, they did not analyse the requirements of solvability and stability of the power flow and the stable operation region of the system under different control modes.

This study proposes the stable operation requirements of converters under two typical control modes based on the short-circuit ratio (SCR) and develops a method of calculating the critical SCR (CSCR) of a specified converter. Furthermore, this study proposes a stability criterion for MTDC systems based on the
The CSCR of each converter. The proposed method and stability criterion are validated via detailed simulations in PSCAD. The rest of this paper is organised as follows. Section 2 introduces the concept of SCR. Sections 3 and 4 present the stable operation range of converters under two classical modes. The stability criterion for an MTDC distribution network based on the CSCR of each converter is introduced in Section 5. Section 6 discusses the results of a simulation performed in PSCAD/EMTDC. The conclusions are presented in Section 7.

2 Concept and definition of SCR

In the traditional high-voltage DC transmission, the current source converter needs to use the reverse voltage of the AC network to turn off the thyristor in the receiving system. However, most receiving systems in the DC distribution network are typical weak AC systems, so they may not have enough AC voltage or moment of inertia to support the reverse voltage. Therefore, switchable VSCs are adopted in the DC distribution network. They can communicate without external voltage and supply power to passive or weak AC systems.

In traditional high-voltage DC transmission systems, the SCR is a critical factor affecting the DC distribution with VSCs. It helps to judge the stability of the converter in the DC distribution when connected to the grid. VSCs are increasingly used in the interconnection with weak AC systems in DC distribution projects. Related research focuses more on the weakest AC system to which converters can connect, which can be represented by the CSCR and the maximum power transferred from the converter to the AC system. The maximum output power of a converter is mainly affected by small-disturbance stability, transient stability, and steady-state operation characteristics. The former two are mainly related to specific control parameters and system-level macro-parameters and can be calculated and analysed with the time-domain small-signal model and electromagnetic transient simulations. The latter is mainly related to macro-parameters, such as the active power, reactive power, line impedance, and control mode, and is not affected by the specific control parameters [23]. Therefore, in a DC distribution network, the stability of a single converter and that of the whole distribution network can be judged from the SCR and the corresponding control mode. To obtain the maximum transmit power, the theoretical operating range of the system has to be determined by analysing the steady-state characteristics, so that the converter can maintain a stable operation state under critical conditions.

Independently from the topology, the external characteristics of the VSC are the same at the basic frequency. Fig. 1 shows the equivalent circuit of the converter and the AC system to which it is connected. The fundamental voltage phasor of the AC source is \( U_S \), and the fundamental voltage phasor inverted by the VSC is \( U_C \), which lags behind \( U_S \) by an angle \( \delta \). The total reactance on the line and bridge arm is \( Z \). With the power flow in the positive direction, as shown in Fig. 1, the transmission power between the converter and AC system can be expressed as (1) and (2)

\[
P = \frac{U_S U_C}{Z} \sin \delta \quad (1)
\]

\[
Q = \frac{U_S(U_S - U_C \cos \delta)}{Z} \quad (2)
\]

In the above equations, \( U_N \) is the rated voltage of the AC bus and \( P \) is the active power of the converter from the AC system. The short-circuit capacity \( S_c \) and SCR of the converter can be expressed as [24]

\[
S_c = \frac{U_N^2}{Z} \quad (3)
\]

\[
\text{SCR} = \frac{S_c}{|P|} = \frac{U_N^2}{|Z||P|} \quad (4)
\]

The CSCR of the converter is related to the stability constraints of the system. For the grid-connected converter, the characteristics of the steady-state operation are different under different control modes and thus have to be analysed first. Then, by reducing the SCR of the converter and observing its influence on the stability of the system, the SCR value when the system is critically stable, i.e. the CSCR value, can be obtained. Under the same voltage level, the rated capacities of the VSCs and MMCs are different. Therefore, to generalise the analysis, this study adopts the per-unit value method. The benchmark parameters of the VSCs and MMCs are shown in Table 1.

3 Stable operation region of constant power controlled converter

3.1 Analytical equations of control parameters and stability criterion

For the external-loop control, when the AC voltage is three-phase balanced and the \( d \)-axis is synchronised with the voltage of the AC source under steady-state operation conditions, we have \( U_{sd} = U_S \), \( U_{sq} = U_S \). According to \( d-q \) transformation, the expressions of the active and reactive powers injected from the AC system into the converter can be expressed as [25]

\[
\begin{align*}
P &= U_S i_{sd} \\
Q &= -U_S i_{sq}
\end{align*}
\]

Then, (1) and (2) can be written as follows:

\[
\begin{align*}
U_S U_C \sin \delta &= PD\sin \varphi - QZ\cos \varphi \\
U_S U_C \cos \delta &= PZ\cos \varphi + QZ\sin \varphi + U_S^2
\end{align*}
\]

For simplicity, in this section, the equivalent impedance angle of the AC system is set as \( \varphi = \pi/2 \), and thus \( Z = X \). In the constant power control mode, because \( P \) and \( Q \) can be regarded as constant, analytical expressions of parameters \( U_S \), \( \delta \), \( i_{sd} \) and \( i_{sq} \) can be obtained by combining (4) and (5)

\[
\begin{align*}
U_C &= \left( \frac{U_S^2 - 2XQ + \sqrt{U_S^4 - 4U_S^2XQ - 4X^2P^2}}{2} \right)^{1/2} \\
\delta &= \arcsin \left( \frac{PX}{U_S} \left( \frac{U_S^2 - 2XQ + \sqrt{U_S^4 - 4U_S^2XQ - 4X^2P^2}}{2} \right)^{1/2} \right) \\
i_{sd} &= \left( \frac{U_S^2 - 2XQ + \sqrt{U_S^4 - 4U_S^2XQ - 4X^2P^2}}{2} \right)^{1/2} P
\end{align*}
\]

Table 1 Reference parameters of converter

| Parameter | \( U_S \), kW | \( U_C \), kW | \( Z \), mH | \( P \), MW | \( Q \), MVar |
|-----------|----------------|----------------|---------------|-------------|--------------|
| VSC       | 10             | 10             | 2             | 5           | 1            |
| MMC       | 10             | 10             | 3             | 15          | 4            |

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At the same time, to ensure stable and safe operation of the converter when connected to the AC system, the following four constraints need to be satisfied:

\[
\begin{align*}
&dU_C/dQ < 0, \\
&d_{isq}/dP > 0, \\
&d_{isq}/dQ < 0, \\
&U_{\text{min}} \leq U_C \leq U_{\text{max}}.
\end{align*}
\]

(17) (18) (19) (20)

Equation (13) is the static voltage stability requirement of the AC system. Equations (14) and (15) are the stability requirements of the \(d\)- and \(q\)-axis control parameters of the converters. Considering (14) as an example, the \(d\)- and \(q\)-axis external-loops adopt proportional–integral (PI) regulation. When the converter increases the active power, the reference value of the active power \(P\) increases, leading to an increase in the output of the PI regulation. If requirement (14) is not satisfied, the increment in \(i_{sq}\) will lead to a reduction in \(P\), and the output of the \(d\)-axis PI regulator will be further increased, resulting in an unstable positive feedback process. Similarly, (15) is the stability requirement of the \(q\)-axis, and (16) is the requirement of the voltage stability range of the AC system, where \(U_{\text{min}} = 0.95\) p.u. and \(U_{\text{max}} = 1.05\) p.u.

3.2 Variation law of stable operation region of converter

From (7), (9), and (10), the following relationships can be obtained:

\[
\begin{align*}
&\frac{dU_C}{dQ} = \frac{-X - (U_C^2 X + U_C^2 X Q - 4 U_C^2 X_Q - 4 X^2 P^2)}{2k} < 0, \\
&\frac{d_{isq}}{dP} = \frac{1}{\sqrt{k}} + \frac{P X^2}{2 U_C^2 X_Q - 4 U_C^2 X_Q - 4 X^2 P^2} > 0, \\
&\frac{d_{isq}}{dQ} = -\frac{1}{\sqrt{k}} - \frac{Q(X + (U_C^2 X + U_C^2 X_Q - 4 U_C^2 X_Q - 4 X^2 P^2))}{2k}, \\
&k = \frac{\sqrt{U_C^2 - 4 U_C^2 X_Q - 4 X^2 P^2 + U_C^2} - X Q}{2}.
\end{align*}
\]

Under requirements (11) and (12), it is easy to prove that the stability requirements (13) and (14) can be satisfied by (17) and (18). However, the stability requirement (15) cannot be judged directly by (19). Therefore, it is necessary to calculate the variation law of \(i_{sq}\) with different SCRs and investigate the influence of the SCR with requirement (15). In the calculation of the CSCR, the SCR of the converter needs to be continuously reduced to reach the critical point. Using (3), in this section, the SCR is reduced by increasing the active power \(P\) and observing the relationship of \(U_C, i_{sq}\), and \(i_{sq}\) with \(Q\) under different SCRs. The results are shown in Fig. 2.

As shown in Fig. 2, as the active power \(P\) and reactive power \(Q\) increase and the SCR decreases, the stability requirements (11) and (12) are less likely to be satisfied, and the AC power flow equations tend to be unsolvable. When the power flow equation has a critical solution, the reactive power is assumed to be \(Q_1\). From Fig. 2a, it can be seen that the converter voltage \(U_C\) at the AC side decreases with the increase in the reactive power \(Q\) independently from the change in SCR, corresponding to (17). Fig. 2b shows that for any reactive power \(Q\), smaller SCRs lead to higher \(i_{sq}\), corresponding to (18). As shown in Fig. 2a, to meet the stability requirement (16), i.e. for the AC side voltage \(U_C\) to be within the safe operation limit, the adjustable range of reactive power \(Q\) reduces with the SCR. The upper and lower limits of the adjustable range of \(Q\) are denoted as \(Q_H\) and \(Q_L\). When SCR = 1, \(U_C\) cannot be within 0.95–1.05 p.u. for any value of \(Q\). When SCR = 2, for \(U_C\) to meet the requirement in (16), \(Q\) needs to be between −0.18 and −0.1 p.u. Hence, \([Q_L, Q_H] = [−0.18, −0.1]\) can be obtained.

As shown in Fig. 2c, when SCR = 1, \(i_{sq}\) has a minimum value, indicating that the stability requirement (15) can no longer be satisfied by increasing the reactive power \(Q\), and the converter cannot operate steadily. The critical value of the reactive power \(Q_2\) is −0.78 p.u., so the safe operating range of the reactive power should be \(Q < −0.78\) p.u. This has no intersection with the corresponding range \([Q_L, Q_H]\), so there is no stable operation range of the converter. Similarly, when SCR = 1.2, the critical reactive power value \(Q_2\) is −0.46 p.u.
Based on the above results, it is found that the reduction in SCR compromises the stable operation of the converter. In particular, when the SCR is reduced by a certain amount, the system cannot reach a stable state no matter how the external electrical quantities, such as the active and reactive powers and AC voltage, are adjusted. For reactive power $Q$, the security and stability of the converter are ensured by the following equation:

$$Q_i \leq Q \leq \min (Q_{i1}, Q_{i2}).$$  \hspace{1cm} (21)

3.3 Critical state of converter

The above analysis revealed that when the converter is in constant power control mode, the stability margin decreases with the decrease in the SCR. The CSCR is restricted by three stability requirements: the existence of a real solution of the power flow equation at the AC side, corresponding to (11) and (12); the upper and lower limits of the voltage $U_C$ at the AC side of the converter, corresponding to (16); the stability requirements for the $q$-axis reactive power control in the external loop, corresponding to (15). According to the analysis in Section 3.2, for any SCR, there exists $Q_1$, $Q_2$, $Q_1$, and $Q_H$ that satisfy the above requirements. The intersection of $Q_1$, $Q_2$, $Q_1$, and $Q_H$ ensures the steady operation of the converter.

However, as well as the SCR, other electrical parameters, such as $U_S$, $P$, and $X$, affect the stability requirements. Among those parameters, the equivalent impedance $X$ and active power $P$ can be considered in the SCR. In addition, once $U_S$ is determined, the stability requirements of the converter can be satisfied by adjusting $Q$ with different SCRs, ensuring the stable operation of the system. In this section, the CSCR is determined. At SCRs lower than the CSCR, the stability requirements cannot be satisfied no matter how the reactive power is adjusted.

The voltage $U_S$ of the AC system is assumed to fluctuate between 100 and 120% of the rated value, which is also the voltage range of the converter at the AC side, under various operation modes in the actual DC distribution network. The CSCR is calculated through the following steps:

(i) Given a certain SCR, the curves of $U_C$ and $i_{sq}$ as functions of the reactive power $Q$ are obtained by varying $Q$ over a certain range.

(ii) With the increase in the absolute value of $Q$, (11) and (12) tend to have no real solution. When the power flow equation has critical solutions, the critical reactive power, denoted as $Q_1$, is determined.

(iii) The critical reactive power $Q_2$ that satisfies the stability requirement of the $q$-axis reactive power control in the external loop is determined. From the curve of $i_{sq}$ as a function of $Q$, the value of reactive power $Q_2$ at the minimum point, satisfying $\frac{dq_{sq}}{dQ} = 0$, is determined.

(iv) Once the allowable voltage range of the AC system ($U_{min}$, $U_{max}$) is determined, $Q_1$ and $Q_H$ are determined as the upper and lower limits of the curve of $U_C$ as a function of $Q$, respectively.

(v) The curves of $Q_1$, $Q_2$, $Q_1$, and $Q_H$, as functions of SCR are obtained by repeating the above four steps for different values of the SCR. According to the curves, the critical reactive power $Q_{cri}$ is determined as the value satisfying $\min (Q_1, Q_2, Q_1, Q_H) = Q_{cri}$, and the CSCR is the corresponding SCR value.

Fig. 3 shows the trend of critical reactive power $Q$ ($Q_1$, $Q_2$, $Q_1$, $Q_H$) as a function of the SCR under different constraints and with different AC voltage $U_S$. The stable operation of the converter requires the relevant parameters to be below the curves of $Q_1$, $Q_2$, and $Q_{cri}$, and above the curve of $Q_H$, where the value of the SCR at the critical intersection point is CSCR. Table 2 shows the values of CSCRs obtained. From the table, it can be seen that a higher AC system voltage $U_S$ leads to a smaller CSCR of the converter, which is mainly constrained by the $q$-axis reactive power control and the upper voltage limit of the AC system.

### Table 2  CSCRs of the converter under constant power control

| Case | $U_S$, p.u. | $X$, p.u. | $P$, p.u. | $Q_{cri}$, p.u. | CSCR |
|------|-------------|-----------|----------|----------------|-------|
| 1    | 1.0         | 1         | 0.82     | −0.52          | 1.22  |
| 2    | 1.1         | 1         | 0.89     | −0.36          | 1.12  |
| 3    | 1.2         | 1         | 0.94     | −0.24          | 1.06  |

Fig. 3 Variation curves of the critical reactive power $Q$ with the SCR under different $U_S$

(a) $U_S = 1.0$ p.u., (b) $U_S = 1.1$ p.u., (c) $U_S = 1.2$ p.u.
4 Stable operation region of constant AC voltage amplitude controlled converter

When a DC distribution network participates in adjusting the AC voltage of the receiving AC system, it may also adopt a constant active power and constant AC-voltage-amplitude control mode. Therefore, this section studies the corresponding stability requirements and CSCR of the converter. The control loop of the converter is the classical current vector control.

4.1 Analytical equation of control parameters and stability criterion

For simplicity, in this section, the equivalent resistance in the AC lines is ignored, so the active power \( P \) and AC voltage amplitude of the converter \( U_C \) can be regarded as constants. According to (5) and (6), the analytical expressions parameters \( Q \), \( \delta \), \( i_{sd} \), and \( i_{sq} \) can be obtained as follows:

\[
Q = \frac{\sqrt{U_C^2 - P^2 X^2} - U_C^2}{X}.
\]

\[
\delta = \arcsin \left( \frac{P X}{U_C U_C} \right).
\]

\[i_{sd} = \frac{P}{U_C}.
\]

\[i_{sq} = \frac{U_C - \sqrt{U_C^2 - P^2 X^2}}{X U_C}.
\]

To ensure stable operation of the system and the existence of real solutions of power flow equations of the AC system, the following five stability requirements need to be satisfied:

\[U_C \geq \frac{PX}{U_S}.
\]

\[\frac{dQ}{dU_C} < 0,
\]

\[
\frac{di_{sd}}{dU_C} < 0,
\]

\[
\frac{di_{sq}}{dU_C} > 0,
\]

\[U_{\text{min}} \leq U_C \leq U_{\text{max}}.
\]

Equation (26) corresponds to the requirement of the power flow equation to have real solutions, and (29) is the stability requirement of the \( q \)-axis constant AC-voltage-amplitude control.

4.2 Variation law of stable operation region of converter

By putting (22)–(25) into the three stability requirements (27)–(29), the corresponding analytical expressions can be expressed as

\[
\frac{dQ}{dU_C} = \frac{(U_C^2 \delta^2 - U_C U_C - P^2 X^2) - 2U_C}{X},
\]

\[
\frac{di_{sd}}{dU_C} = \frac{1}{U_C} > 0,
\]

\[
\frac{di_{sq}}{dU_C} = \frac{2 - (U_C^2 \delta^2 - U_C U_C - P^2 X^2)}{X} + \frac{\sqrt{U_C^2 - P^2 X^2} - U_C^2}{X U_C}.
\]

According to (32), the \( d \)-axis active power control of the converter satisfies the stability requirement. However, the requirements in the other two equations cannot be directly verified. Therefore, the influence of the SCR on the stability judgment of the converter is investigated by obtaining the \( Q \) and \( i_{sd} \) curves as functions of \( U_C \) under different SCR values. From Fig. 4b, it can be observed that when \( U_C \) is constant, \( i_{sd} \) increases as the SCR decreases; the gradient of the curve is expressed as (32).

From Fig. 4a, it can be seen that the power flow equations of the AC system tend to have no real solution when \( U_C \) and SCR exceed the critical values. With SCR = 1, the power flow equation has no real solution when \( U_C < 1.05 \) p.u. Similarly, the AC voltage corresponding to the critical solution of the power flow equation is the critical voltage \( U_1 \).

Fig. 4a also shows that smaller \( U_C \) and SCR lead to worse static stability of the AC voltage. There exists a maximum of \( Q \) in all SCR curves. For example, when SCR = 1.2 and \( U_C = 0.95 \) p.u., the maximum point is \( Q = 0.45 \) p.u. The AC voltage corresponding to the maximum point is the critical voltage \( U_2 \). If the AC side voltage \( U_C \) of the converter is smaller than \( U_2 \) and \( dQ/dU_C > 0 \), the stability requirements (31) will not be satisfied. Therefore, \( U_C > U_2 \) is the equivalent stability requirement.

As shown in Fig. 4c, when \( U_C \) and SCR decrease below certain values, the \( q \)-axis AC voltage amplitude control becomes unstable. For example, when SCR = 1.5 and \( U_C = 0.9 \) p.u., there is a minimum point \( i_{sq} = 0.23 \) p.u. The critical AC voltage corresponding to the minimum point of the \( q \)-axis current is denoted as \( U_3 \). If the AC voltage \( U_C \) of the converter is smaller...
than \( U_3 \), the stability requirement (32) will not be satisfied, resulting in the unstable operation of the converter. Therefore, \( U_C > U_3 \) is required.

Based on the above analysis, to ensure the stable operation of the converter with the decrease in the SCR, the voltage at the AC side \( U_C \) must be higher than the three critical voltage values. The lower limit of \( U_C \) is denoted as \( U_{cri} \). To ensure the stability of the converter in the normal range of voltage operation, \( U_{cri} \) needs to be smaller than \( U_{min} \). Therefore, the following requirements have to be satisfied:

\[
U_{cri} = \max (U_1, U_2, U_3) \tag{34}
\]

\[
U_{cri} \leq U_{min}. \tag{35}
\]

### 4.3 Critical state of converter

As determined in the previous sections, when the converter is in constant AC voltage amplitude control mode, the stable operation region of the system is reduced with the decrease in the SCR, and the value of the CSCR is restricted by four stability requirements:

(i) The power flow equation at the AC side of the system must have real solutions, corresponding to (26).

(ii) The voltage at the AC side must be stable, corresponding to (27).

(iii) The \( q \)-axis constant AC-voltage-amplitude control must be stable, corresponding to (29).

(iv) The voltage at the AC side under normal operation of the converter must be below a critical value, corresponding to (35).

The four stability requirements are different when in reactive power control mode. The existence of critical voltages \( U_1, U_2, U_3, \) and \( U_{cri} \) makes the above requirements valid.

The values of the critical voltages \( U_1, U_2, \) and \( U_3 \) can be expressed as follows:

\[
U_1 = \frac{P_X}{U_5} = \frac{U_5}{U_5 \cdot SCR}. \tag{36}
\]

\[
U_2 = \sqrt{\frac{U_5^2 + P_X^2}{U_5^2}} = \sqrt{\frac{U_5^2 + U_5^2 \cdot SCR^2}{U_5^2 \cdot SCR^2}}. \tag{37}
\]

\[-U_5^2 U_1^2 + U_5^2 U_2^2 + U_5^2 = 0. \tag{38}\]

Equation (38) is a cubic equation of \( U_1^2 \) and can be solved by MATLAB. The three equations are related to \( U_S, SCR, \) and \( U_{SCR} \) and not to \( P \) or \( X \). Fig. 5 shows the curves of the critical voltages \( U_1, U_2, \) and \( U_3 \) as functions of the SCR with different \( U_5 \) values.

Table 3 shows the relevant data of each critical point in Fig. 5. It can be seen that the main constraints are requirements (3) and (4), and the following conclusions can be drawn. From the figure, it can be seen that with constant \( U_{cri} \), the CSCR decreases with increasing \( U_S \); with constant \( U_S \), the CSCR decreases with increasing \( U_{cri} \). The main restrictions to the safe and stable operation of the converters are the requirements of \( q \)-axis constant AC-voltage-amplitude control and the upper limit of the critical value of the AC voltage.

### 5 Stability criterion of MTDC distribution system based on the CSCR

In the MTDC distribution networks, the SCR of each terminal can be calculated separately, and the equivalent impedance of the AC receiving system of the converter can be regarded as the impedance of the Thevenin equivalent circuit from the connecting point to the AC system. Thus, the CSCR of all converters in the MTDC systems can be calculated using the proposed method. If the SCR of the converter is higher than the CSCR, the converter can be considered in a stable operation state. In contrast, if the SCR is lower than the CSCR, the converter may not be in the stable operation state. Considering the interaction between the converters, the stability conclusions of the MTDC system can be divided into three conditions.

**Conclusion 1**: The entire MTDC system operates normally, and both the AC and DC parts of all converters in the MTDC system are stable.

![Fig. 5 Curves of critical voltage as a function of the SCR with different \( U_5 \)](a) \( U_5 = 1.0 \) p.u., (b) \( U_5 = 1.1 \) p.u., (c) \( U_5 = 1.2 \) p.u.

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**Table 3** Values of the CSCR of the converter under constant AC voltage amplitude control modes

| Case | \( U_5 \), p.u. | \( X \), p.u. | \( P \), p.u. | \( U_{cri} \), p.u. | CSCR |
|------|----------------|-------------|-------------|----------------|------|
| 1    | 1.0            | 1           | 0.74        | 0.95           | 1.36 |
| 2    | 1.1            | 1           | 0.79        | 0.95           | 1.27 |
| 3    | 1.2            | 1           | 0.83        | 0.95           | 1.20 |
Conclusion 2: Both AC parts and DC parts of all converters in the MTDC system are unstable.

Conclusion 3: The DC parts of all converters in the MTDC system are unstable. The AC parts of the unstable converters cannot operate normally, while those of the stable converters are stable.

Fig. 6 shows the flow chart of the stability criterion of the MTDC system; the following steps are considered to judge the stability of the system:

**Step 1:** Calculate the SCRs and CSCRs of all converters in the MTDC system using the proposed method.

**Step 2:** Compare the SCR and CSCR of each converter and record the number of unstable converters (SCR < CSCR).

**Step 3:**
- If all converters are stable, conclusion 1 can be achieved.
- If there are unstable converters in the system and none of them is a constant-DC-voltage-controlled converter, conclusion 3 can be achieved.
- On the contrary, if the constant-DC-voltage-controlled converter is unstable, conclusion 2 can be achieved.

Based on the proposed CSCR calculation method and the steps to judge the stability of the system, the stability of the MTDC system can be obtained.

### 6 Case studies and simulation results

To verify the proposed CSCR calculation method and stability criterion of the MTDC system, electromagnetic simulations of single- and multi-terminal hybrid AC/DC distribution systems under different control modes were conducted using PSCAD.

#### 6.1 Case study 1: verification and analysis of CSCR calculation of constant-power-controlled converters

Fig. 7a shows the VSC- and MMC-based one-terminal AC/DC hybrid distribution systems built for simulation case 1. The constant power control mode is chosen for the grid-connected converters. Initially, the stable operation points of the systems are $U_S = 1 \text{ p.u.}$, $X = 1 \text{ p.u.}$, and $P = 0.5 \text{ p.u.}$ (SCR = 2). After 1 s, two changes are introduced, i.e. a 0.3 p.u. step-change in the reference active power $P_{ref}$ from 0.5 to 0.8 p.u., and a 0.35 p.u. step-change in $P_{ref}$ from 0.5 to 0.85 p.u. Figs. 8 and 9 show the curves of the DC voltage with $P_{ref}$ obtained via PSCAD electromagnetic simulations.

As shown in Fig. 8, when $P_{ref}$ changes from 0.5 to 0.8 p.u. (SCR = 1.25), the DC voltage of the VSC-based one-terminal AC/DC hybrid distribution system is stable. In contrast, when $P_{ref}$ changes from 0.5 to 0.85 p.u. (SCR = 1.18), the DC voltage is unstable. Fig. 9 shows the same stability conclusions for the MMC-based one-terminal system.

The simulation results confirm the accuracy of the proposed CSCR calculation method for constant-power-controlled converters (CSCR = 1.22).

#### 6.2 Case study 2: verification and analysis of the CSCR calculation of constant-amplitude-controlled converters

Fig. 7b shows the VSC- and MMC-based one-terminal AC/DC hybrid distribution systems built for simulation case 2. The constant AC-voltage-amplitude control mode is chosen for the grid-connected converters. Initially, the stable operation points of the systems are $U_S = 1 \text{ p.u.}$, $X = 1 \text{ p.u.}$, and $P = 0.5 \text{ p.u.}$ (SCR = 2). After 1 s, two changes are introduced, i.e. a 0.22 p.u. step-change in the reference active power $P_{ref}$ from 0.5 to 0.72 p.u., and a 0.27 p.u. step-change in $P_{ref}$ from 0.5 to 0.77 p.u. The curves of the AC voltage as a function of $P_{ref}$ obtained from PSCAD electromagnetic simulations are shown in Figs. 10 and 11.

As shown in Fig. 10, when $P_{ref}$ changes from 0.5 to 0.72 p.u. (SCR = 1.40), the AC voltage of the VSC-based one-terminal AC/DC hybrid distribution system becomes stable. In contrast, when $P_{ref}$ changes from 0.5 to 0.77 p.u. (SCR = 1.33), the AC voltage is unstable. Fig. 11 shows the same stability conclusion for the MMC-based one-terminal system. The simulation results confirm the accuracy of the proposed CSCR calculation method for constant-amplitude-controlled converters (CSCR = 1.36).
6.3 Case study 3: stability analysis of MTDC distribution system

Fig. 7 shows the structure of the MMC-based four-terminal star-topology DC distribution network built for simulation case 3. The master–slave control strategy is chosen for the system; the VSC1 is in constant DC voltage control mode, VSC2 and VSC3 are in constant power control mode, and VSC4 is in constant AC voltage amplitude control mode. Table 4 shows the impedance parameters, SCR, and CSCR of each MMC in the MTDC distribution system.

Based on the proposed stability criterion, the stability conclusions of the MTDC system can be obtained, as shown in Table 5. In case 1, only constant-AC-voltage-amplitude-controlled converter (VSC4) is unstable, so stability conclusion 3 can be achieved. In case 2, the constant-DC-voltage-controlled converter (VSC1) is unstable, so stability conclusion 2 can be achieved. In case 3, all the converters are stable, so stability conclusion 1 can be achieved.

Figs. 12 and 13 show the curves of the DC voltage and AC voltage in case 1 and case 2 obtained via PSCAD simulations. As shown in Fig. 12, the entire DC system is unstable, and the AC system of VSC4 is unstable, while those of VSC1, VSC2, and VSC3 are stable. As shown in Fig. 13, both the AC and DC systems of all converters in the MTDC system are unstable. Thus,
the stability of the MTDC system obtained by the proposed stability criterion agrees well with the results of the simulation using PSCAD, which is usually considered to be consistent with the response of the practical system. This confirms the accuracy of the proposed stability criterion.

7 Conclusion

This study analysed the stable operation requirements of converters based on the SCR. A method to calculate the CSCR under different control modes was proposed. The stability criterion for MTDC systems based on the CSCR of each converter was also proposed. The main conclusions are as follows:

(i) For constant-power-controlled converters, the CSCR is mainly constrained by solvable AC system power flow equations, stable constraints of q-axis reactive power control, and limitations of the steady-state AC voltage.

(ii) For constant-amplitude-controlled converters, the CSCR is mainly constrained by solvable AC system power flow equations, static stability constraints of the AC voltage, q-axis reactive power control stable constraints, and upper limit of the critical voltage.

(iii) The CSCR of the converter decreases with the increase in $U_s$, and the simulation results are consistent with the theoretical ones.

(iv) Considering the interaction between the converters, stability conclusions of the MTDC system can be divided into three conditions. The stability conclusion can be accurately achieved based on the proposed stability criterion.

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