Effects of finite-light-speed correction for the Coulomb interaction on nuclear binding energies and radii in spherical nuclei

Tomoya Naito (内藤智也)
Department of Physics, Graduate School of Science,
The University of Tokyo, Tokyo 113-0033, Japan and
RIKEN Nishina Center, Wako 351-0198, Japan
(Dated: June 29, 2021)

The finite-light-speed correction to the Coulomb interaction called the Breit correction is discussed with Skyrme Hartree-Fock calculations. It is found that the correction to the total energy is about 800 keV for $^{208}$Pb, while proton and neutron radii do not change significantly. Effects of the Breit correction are also compared to the correction due to the vacuum polarization. It is shown that the two contributions to the total energy are comparable in light nuclei, but the latter dominates in heavy nuclei. Since the covariant (relativistic) density functional theory often includes effects of magnetic interaction and retardation for electromagnetic interaction, the method presented in this work enables one to make a fair comparison between the nonrelativistic and the relativistic density functional calculations for electromagnetic energies.

I. INTRODUCTION

Atomic nuclei are composed of protons and neutrons, which interact via the nuclear and electromagnetic (EM) interactions. Since the nuclear interaction is much stronger than the EM one, the former determines most properties of the atomic nuclei. Nevertheless, the EM interaction plays an important role when one focuses on nuclear properties related to the isospin symmetry breaking. For more details, see reviews, e.g., Ref. 11.

The Hartree-Fock method or the density functional theory (DFT) [12–14] is a powerful method to calculate quantum many-fermion problems, including atomic nuclei. In DFT, an energy density functional (EDF), which contains information on interactions, governs the accuracy of a calculation. Although the Coulomb interaction is rather simple, first-principle EDFs, even for electronic systems, have been derived approximately using in the local density approximation (LDA) [15–17], and effects of the density gradient have been included empirically or phenomenologically [18–20]. In nuclear physics, the situation is more complicated than that in atomic physics. Hence, it is interesting to compare which contribution dominates to the nuclear total energy, the Breit correction or the vacuum polarization.

Another interesting point is a comparison between the Breit correction and the exact Coulomb exchange energy. The Breit correction had been taken into account for exchange [31–33] or exchange-correlation [44] EDF in the LDA level, and was already taken into account fully for atomic DFT [15]. It was found in Ref. [15] that the Breit correction contributes to the total energies of atoms in almost the same magnitude but the opposite to the density gradient effects. In the context of nuclear physics, it was shown in Refs. [31–37] that difference between the Coulomb exchange energy calculated by the exact-Fock term or GGA and that in LDA is non-negligible in nuclear binding energies. Thus, it should be tested to what extent the Breit correction contributes to nuclear binding energies.

The covariant density functional theory (CDFT), one of the nuclear DFT in the relativistic scheme, is based on the covariant Lagrangian with nucleon, meson, and photon fields. Hence, effects beyond the Coulomb in-
teraction, such as magnetic interaction and retardation, have often been considered \[34, 35, 158\]. Therefore, it is interesting to take into account the relativistic correction to the Coulomb interaction in SHF calculations to make a fair comparison with the CDFT calculations for electromagnetic energies.

The aim of this work is to take into account the Breit correction in SHF calculations using the Hartree-Fock-Slater approximation, i.e., LDA. I shall discuss contributions to proton and neutron radii, total energies, and single-particle energies. This paper is organized as follows: In Sec. II, the theoretical framework for the Breit correction will be shown. In Sec. III, the Breit correction to the total energy will be estimated analytically, and in Sec. IV numerical results will be shown. Finally, in Sec. V, conclusion and perspectives will be given.

II. THEORETICAL FRAMEWORK

In this section, I show the theoretical framework for the Breit correction and the DFT. Throughout this paper, an unit with \(4\pi\varepsilon_0 = 1\) is used and \(c\) denotes the speed of light.

A. Breit correction

Atomic nuclei can be described by using the Hamiltonian

\[
H = T + V_{\text{nucl}} + V_{\text{EM}},
\]

where \(T\), \(V_{\text{nucl}}\), and \(V_{\text{EM}}\) denote the kinetic energy operator, a nuclear interaction, and the EM interaction, respectively. In most works, the Coulomb interaction between protons

\[
V_{\text{Coul}}(r_j, r_k) = \frac{e^2}{r_{jk}}
\]

is used for \(V_{\text{EM}}\), where \(r_j\) and \(r_k\) are the spatial coordinates of the protons \(j\) and \(k\), \(r_{jk} = r_j - r_k\), and \(r_{jk} = |r_{jk}|\). Here, the point-particle approximation is used, i.e., a nucleon is assumed to be a pointwise particle \[19\]. In this paper, instead, I consider the finite-light-speed correction to the Coulomb interaction. According to the quantum electrodynamics, the leading-order instantaneous interaction, in which photons mediate between particles with \(c = \infty\), is the Coulomb interaction. The next-leading-order interaction \((O(1/c^2))\) in the Coulomb gauge is called the Breit correction, whose form is \[39\]

\[
V_{\text{Breit}}(r_j, r_k) = -\left[\frac{\mathbf{r}_{jk}}{2c^2r_{jk}} + \frac{\mathbf{r}_{jk}\cdot\mathbf{r}_{jk}}{2c^2r_{jk}^3}\right]
\]

with \(\mathbf{r}\) is the Pauli matrix

\[
\alpha = \left(\begin{array}{cc}
O_2 & \sigma_x \\
\sigma_x & O_2
\end{array}\right), \quad \left(\begin{array}{cc}
O_2 & \sigma_y \\
\sigma_y & O_2
\end{array}\right), \quad \left(\begin{array}{cc}
O_2 & \sigma_z \\
\sigma_z & O_2
\end{array}\right)
\]

After the Foldy-Wouthuysen-Tani transformation \[54\]

\[
\tilde{V}_{\text{Breit}}(r_j, s_j, r_k, s_k) = -\frac{\pi\hbar^2c^2}{M^2c^2}\delta(r_{jk}) - \frac{e^2}{2M^2c^2p_j} \left[\frac{1}{r_{jk}} + \frac{r_{jk}r_{jk}}{r_{jk}^3}\right] \cdot p_k
\]

\[
- 8\pi\hbar^2c^2\delta(r_{jk}) s_j \cdot s_k - \frac{\hbar^2c^2}{M^2c^2} \left[\frac{3r_{jk}r_{jk}}{r_{jk}^3} - \frac{1}{r_{jk}^3}\right] \cdot s_k + \frac{\hbar^2c^2}{M^2c^2} \frac{1}{r_{jk}^3} s_j \cdot [r_{jk} \times (2p_k - p_j)],
\]

where \(M\) is the mass of particles and \(p_j\) is the momentum of the particle \(j\). The first and second terms correspond to the Darwin term for the proton-proton Coulomb interaction, which is related to Zitterbewegung \[52\], and the retardation of the Coulomb interaction, respectively, and the remaining terms are related to the spin-orbit and the spin-magnetic interactions \[41, 58, 59\].

In this paper, I consider the Breit correction \(\tilde{V}_{\text{Breit}}\).
on top of the Coulomb interaction [Eq. (2)], i.e.,

$$V_{EM} = \frac{1}{2} \sum_{j, k \in \text{proton}} \left[ V_{Coul} (r_j, r_k) + \tilde{V}_{Breit} (r_j, s_j, r_k, s_k) \right],$$

(6)

which will be referred to as the Coulomb-Breit interaction. To isolate the Breit correction, I shall not consider any other EM contribution in this paper; that is, neither the nucleon finite-size effects nor the vacuum polarization is considered.

### B. Density functional theory

To calculate the ground-state density distribution and energy numerically, one needs to use a many-body calculation technique. In this paper, Skyrme Hartree-Fock is considered. Before performing numerical many-body calculations with the Breit correction, I estimate contribution of the Breit correction to the total energy with a simple model and discuss the systematic behavior. As in Ref. [32], I assume a hard-sphere distribution for proton distribution.

The Hartree and LDA exchange functionals for the Breit correction were proposed in Ref. [45] and Ref. [43], respectively, and their forms are

$$E_{H}^{\text{Breit}} [\rho_p] = -\frac{\pi h^2 c^2}{2 M^2 e^2} \int [\rho_p (r)]^2 dr,$$

(9a)

$$E_{\text{Breit}}^{\text{Breit}} [\rho_p] = \frac{3 \pi h^2 c^2}{2 M^2 e^2} \int [\rho_p (r)]^2 dr,$$

(9b)

respectively, and accordingly, the total correction is

$$E_{\text{tot}}^{\text{Breit}} [\rho_p] = E_{H}^{\text{Breit}} [\rho_p] + E_{\text{Breit}}^{\text{Breit}} [\rho_p]$$

(9c)

where $M \approx 938.919 \text{ MeV}/c^2$ is the nucleon mass. They should be considered on top of Eqs. (3). Only the first term of Eq. (3) contributes to $E_{\text{tot}}^{\text{Breit}} [\rho_p]$ [Eq. (9a)] in spin unpolarized systems. Equation (9c) corresponds to the exchange energy of the original Breit correction $V_{\text{Breit}}$ [Eq. (4)] for homogeneous density distribution [43, 45].

The energy densities $\mathcal{E}$, that is the integrands of the EDFs $E_{\text{Breit}}^{\text{tot}}, E_{\text{Breit}}^{\text{H}},$ and $E_{\text{Breit}}^{\text{LDA Exchange}},$ are plotted as functions of the proton density $\rho_p$, where $\rho_0 = 0.16 \text{ fm}^{-3}$ is the saturation density. For comparison, the Coulomb LDA exchange EDF $E_{\text{Coul}}^{\text{LDA Exchange}}$ is also plotted as dash-dotted line. (b) Ratio of $E_{\text{tot}}^{\text{Breit}}$ to $E_{\text{Coul}}^{\text{LDA Exchange}}$ as a function of $\rho_p$.

### III. ANALYTICAL ESTIMATION

The energy densities $\mathcal{E}$, that is the integrands of the EDFs $E_{\text{Breit}}^{\text{tot}}, E_{\text{Breit}}^{\text{H}},$ and $E_{\text{Breit}}^{\text{LDA Exchange}},$ are plotted as functions of the proton density $\rho_p$, where $\rho_0 = 0.16 \text{ fm}^{-3}$ is the saturation density. For comparison, the Coulomb LDA exchange EDF $E_{\text{Coul}}^{\text{LDA Exchange}}$ is also plotted. Obviously, with considering the Breit correction, the EM interaction becomes effectively weak, and the absolute value of the Breit correction to the Hartree term is three times smaller than those to the exchange one. The ratio of $E_{\text{tot}}^{\text{Breit}}$ to $E_{\text{Breit}}^{\text{Coul}}$ as a function of $\rho_p$ is shown in Fig. 1(b). This correction is about $-3\%$ at the half of the saturation density $\rho_p \approx 0.08 \text{ fm}^{-3}$. 

FIG. 1. (a) The integrands of the EDFs $E_{\text{tot}}^{\text{Breit}}$ (solid line), $E_{\text{H}}^{\text{Breit}}$ (long-dashed line), and $E_{\text{LDA Exchange}}^{\text{Breit}}$ (dashed line) as functions of the proton density $\rho_p$, where $\rho_0 = 0.16 \text{ fm}^{-3}$ is the saturation density. For comparison, the Coulomb LDA exchange EDF $E_{\text{Coul}}^{\text{LDA Exchange}}$ is also plotted as dash-dotted line. (b) Ratio of $E_{\text{tot}}^{\text{Breit}}$ to $E_{\text{Breit}}^{\text{Coul}}$ as a function of $\rho_p$. 

The Hartree and LDA exchange functionals for the Breit correction were proposed in Ref. [45] and Ref. [43], respectively, and their forms are

$$E_{\text{H}}^{\text{Breit}} [\rho_p] = \frac{\pi h^2 c^2}{2 M^2 e^2} \int [\rho_p (r)]^2 dr,$$

$$E_{\text{Breit}}^{\text{Breit}} [\rho_p] = \frac{3 \pi h^2 c^2}{2 M^2 e^2} \int [\rho_p (r)]^2 dr,$$

respectively, and accordingly, the total correction is

$$E_{\text{tot}}^{\text{Breit}} [\rho_p] = E_{\text{H}}^{\text{Breit}} [\rho_p] + E_{\text{Breit}}^{\text{Breit}} [\rho_p]$$

where $M \approx 938.919 \text{ MeV}/c^2$ is the nucleon mass. They should be considered on top of Eqs. (3). Only the first term of Eq. (3) contributes to $E_{\text{tot}}^{\text{Breit}} [\rho_p]$ [Eq. (9a)] in spin unpolarized systems. Equation (9c) corresponds to the exchange energy of the original Breit correction $V_{\text{Breit}}$ [Eq. (4)] for homogeneous density distribution [43, 45].

The energy densities $\mathcal{E}$, that is the integrands of the EDFs $E_{\text{Breit}}^{\text{tot}}, E_{\text{H}}^{\text{Breit}},$ and $E_{\text{LDA Exchange}}^{\text{Breit}},$ are plotted as functions of the proton density $\rho_p$, in Fig. 1(a), where $\rho_0 = 0.16 \text{ fm}^{-3}$ is the saturation density. For comparison, the Coulomb LDA exchange EDF $E_{\text{Coul}}^{\text{LDA Exchange}}$ is also plotted. Obviously, with considering the Breit correction, the EM interaction becomes effectively weak, and the absolute value of the Breit correction to the Hartree term is three times smaller than those to the exchange one. The ratio of $E_{\text{tot}}^{\text{Breit}}$ to $E_{\text{Breit}}^{\text{Coul}}$ as a function of $\rho_p$ is shown in Fig. 1(b). This correction is about $-3\%$ at the half of the saturation density $\rho_p \approx 0.08 \text{ fm}^{-3}$. 

### III. ANALYTICAL ESTIMATION

Before performing numerical many-body calculations with the Breit correction, I estimate contribution of the Breit correction to the total energy with a simple model and discuss the systematic behavior. As in Ref. [32], I assume a hard-sphere distribution for proton distribution.
\[ \rho_p(r) = \begin{cases} \rho_0^p & r \leq R_p, \\ 0 & r > R_p, \end{cases} \quad (10) \]

where \( R_p \) is the proton radius defined by

\[ R_p = \left( \frac{3Z}{4\pi\rho_0^p} \right)^{1/3} \quad (11) \]

and \( \rho_0^p \approx 0.08 \text{ fm}^{-3} \) is the half of the saturation density of atomic nuclei. With this density, one can estimate the Coulomb Hartree and exchange energies as \[ E \]

\[ E_{\text{Coul}}^H = \frac{3\pi^2}{5} \left( \frac{4\pi\rho_0^p}{3} \right)^{1/3} Z^{5/3} \approx 0.60Z^{5/3} \text{ MeV}, \quad (12a) \]

\[ E_{\text{Coul}}^x = -\frac{3e^2}{4} \left( \frac{3\rho_0^p}{\pi} \right)^{1/3} Z \approx -0.46Z \text{ MeV}, \quad (12b) \]

respectively.

With the relation

\[ \int |\rho_p(r)|^2 \, dr = \frac{4\pi R_p^3}{3} (\rho_0^p)^2 = \rho_0^p Z, \quad (13) \]

the Breit correction to the Hartree and exchange energies can be estimated as

\[ E_{\text{Breit}}^H = -\frac{\pi (hc)^2 e^2}{2(Mc^2)^2} \rho_0^p Z \approx -0.0080Z \text{ MeV}, \quad (14a) \]

\[ E_{\text{Breit}}^x = \frac{3\pi (hc)^2 e^2}{2(Mc^2)^2} \rho_0^p Z \approx 0.024Z \text{ MeV}. \quad (14b) \]

Accordingly, the total correction can be estimated as

\[ E_{\text{Breit}}^{\text{tot}} = E_{\text{Breit}}^H + E_{\text{Breit}}^x = \frac{\pi (hc)^2 e^2}{(Mc^2)^2} \rho_0^p Z \approx 0.016Z \text{ MeV}. \quad (14c) \]

On the other hand, in nuclear physics, both the vacuum polarization and Breit correction are corrections to the proton-proton Coulomb interaction. Thus, it is worthwhile to compare them. In Ref. [22], the vacuum polarization was taken into account in a nuclear structure calculation using the Uehling potential [63, 64]. The contribution of the vacuum polarization was estimated as

\[ E_{VP} \approx 0.0047Z^{5/3} \text{ MeV}. \quad (15) \]

Hence, the Breit correction and the vacuum polarization provide comparable contributions to the total energy in the light nuclei (e.g., for \( Z = 20 \), \( E_{\text{Breit}}^{\text{tot}} \approx 0.32 \text{ MeV} \) and \( E_{VP} \approx 0.69 \text{ MeV} \) but the latter dominates in heavy nuclei (e.g., for \( Z = 100 \), \( E_{\text{Breit}}^{\text{tot}} \approx 1.6 \text{ MeV} \) and \( E_{VP} \approx 10 \text{ MeV} \)). In the next section, I shall confirm this by performing numerical calculations.

## IV. SKYRME HARTREE-FOCK CALCULATION

The EDF with the Breit correction [Eq. (9)] is implemented to the self-consistent Skyrme Hartree-Fock calculation code SKYRME_. RPA [65]. In this work, the pairing interaction is not considered, and the spherical symmetry is assumed. The calculations are performed with a box of 15 fm with 0.1 fm mesh. The SAMi functional [65] is adopted for the nuclear EDF.

First of all, I shall mention effects to nuclear radii. Neither the proton nor the neutron radii is found to be changed more than 0.001 fm due to the Breit correction and the vacuum polarization. Therefore, the effect of the Breit correction on nuclear radii can be safely neglected.

Next, the ground-state energies for several nuclei are shown in Table I. The columns named “Coulomb” and “Coulomb-Breit” show results with the Coulomb LDA [Eq. (8)] and the Coulomb-Breit LDA [Eqs. (8) and (9)], respectively. For comparison, total energies calculated with the vacuum polarization on top of the Coulomb-Breit LDA are shown in a column named “C.-B. & Vacuum Pol.” Both the Breit correction and the vacuum polarization make the atomic nuclei less bound. The contribution of the former is around 190 keV in \(^{40}\text{Ca}\) and 810 keV in \(^{208}\text{Pb}\). It was shown in Refs. [31, 47] that the exact treatment or its simplified treatment called GGA of the Coulomb exchange energy makes \(^{40}\text{Ca}\) bound more by around 370 keV and \(^{208}\text{Pb}\) by around 870 keV. Hence, such difference and the Breit correction give almost comparable in magnitude but the opposite contribution to total energies, as in the case of atoms [45]. Thus, when both contributions are considered simultaneously, even it is not obvious whether the total energy decreases or increases. Indeed, this fact was pointed out in the context of atoms in Ref. [45], while no first-principles Coulomb-Breit GGA functional is available, which remains an open question.

I also compare the Breit correction and the vacuum polarization. The vacuum polarization also makes the atomic nuclei less bound, and contribution to the total
energy is approximately 400 keV in $^{40}$Ca and 3.7 MeV in $^{208}$Pb. Thus, as discussed in Sec. III, the vacuum polarization gives a comparable contribution to the total energy to the Breit correction in light nuclei, but it contributes more in heavy nuclei.

The systematic behavior of the Breit correction and the vacuum polarization shall be discussed next. The contributions of the Breit correction and the vacuum polarization can be seen as the difference of the second and the third columns and the third and the fourth columns in Table I, respectively. The proton number $Z$ dependence of these contributions are shown in Fig. 2. These energies are fitted with $E = aZ^b$, where $a$ and $b$ are adjustable parameters. The results are

$$ E_{\text{Breit}}^\text{tot} \simeq 0.0115Z^{0.972} \text{ MeV}, \hspace{1cm} (16a) $$
$$ E_{\text{VP}} \simeq 0.00398Z^{1.55} \text{ MeV}, \hspace{1cm} (16b) $$

which are consistent to Eqs. (14c) and (15).

The exchange contribution to the EM energies shall be further discussed. The ratio of the Coulomb or Breit exchange energy to the exact Coulomb exchange energy

$$ \Delta E_x = \frac{E_x^{\text{LDA}} - E_x^{\text{exact}}}{E_x^{\text{exact}}} \hspace{1cm} (17) $$

is shown in Fig. 3. Here, the exact Coulomb exchange energy is calculated by using the modified GGA-PBE functional with $\lambda = 1.25$, which reproduces the exact Coulomb exchange energy less than 500 keV error [51], and the Coulomb and the Coulomb-Breit LDA exchange energies are calculated by using $E_x^{\text{Coul}}$ [Eq. (8c)] and $E_x^{\text{Coul}} + E_x^{\text{Breit}}$ [Eqs. (8c) and (9b)], respectively. It is seen that the absolute value of the relative change $|\Delta E_x|$ gets smaller as $A$ increases in light nuclei and reaches almost constant $\Delta E_x \approx -6\%$ in heavy nuclei ($A \gtrsim 100$). The difference between $\Delta E_x$ for the Coulomb LDA and that for the Coulomb-Breit LDA is almost kept constant. This difference is consistent to relativistic Hartree-Fock calculations [48]. Here, the exact Coulomb exchange energy corresponds to Eq. (4) in Ref. [48], which was shown not to include the relativistic correction, and its difference is around 10\% in $^{208}$Pb. With considering such difference, $E_{\text{Coul}} + E_{\text{Breit}}$ calculated in this work is almost consistent to Fig. 1 of Ref. [48].

Lastly, a change of the single-particle energies is discussed, for which $^{208}$Pb is taken as an example. The single-particle energies calculated with the Coulomb and Coulomb-Breit LDA are shown in Table II. For comparison, the single-particle energies calculated with the vacuum polarization on top of the Coulomb-Breit LDA are shown. It is seen that the proton single-particle energies change by about 10–25 keV due to the Breit correction and further by about 100 keV due to the vacuum polarization. In contrast, both the Breit correction and the vacuum polarization change neutron single-particle

| Nuclei | Coulomb | Coulomb-Breit | C.-B. & Vacuum Pol. |
|--------|---------|----------------|---------------------|
| $^4\text{He}$ | -27.5263 | -27.5134 | -27.5050 |
| $^{16}\text{O}$ | -130.4800 | -130.4084 | -130.3188 |
| $^{48}\text{Ca}$ | -347.0848 | -346.8798 | -346.4702 |
| $^{48}\text{Ca}$ | -415.6148 | -415.4201 | -415.0190 |
| $^{48}\text{Ni}$ | -352.6388 | -352.3274 | -351.6077 |
| $^{100}\text{Sn}$ | -811.6641 | -811.0990 | -809.2893 |
| $^{132}\text{Sn}$ | -1103.0881 | -1102.6076 | -1100.9215 |
| $^{208}\text{Pb}$ | -1636.6149 | -1635.8023 | -1632.0911 |
| $^{310}\text{Cd}$ | -2131.4146 | -2130.1539 | -2122.8658 |

FIG. 2. Contribution of breit correction and vacuum polarization to the total energies, $E_{\text{tot}}^\text{Breit}$ and $E_{\text{VP}}$ shown as functions of proton number $Z$ in circle and square points, respectively. Empirical fits with Eqs. (16) are also shown by the solid and the dashed lines, respectively.

FIG. 3. Ratio of the Coulomb (square) or the Coulomb-Breit (circle) exchange energies in local density approximation to the exact Coulomb exchange energy, $\Delta E_x$ [Eq. (17)], shown as functions of mass number $A$. 

TABLE I. Ground-state energies calculated with the SAMi functional [22] shown in MeV. The columns named “Coulomb” and “Breit” shows results calculated with the Coulomb LDA [Eq. (8a)] and the Coulomb-Breit LDA [Eqs. (8c) and (9b)], respectively. For comparison, the total energies calculated with the vacuum polarization on top of the Coulomb-Breit LDA are shown in the column named “C.-B. & Vacuum Pol.”

$^{16}\text{O}$ is almost kept constant. This
considered, due to the short-range nature, the Breit correction itself makes the EM repulsive interaction weaker, the mean-field potential shallower. Note that the Breit correction was used instead of the original one, and, in addition, the local density approximation (LDA) was further introduced for the exchange term.

I found that the LDA Breit exchange energy is a times of the Breit Hartree one, and the total Breit correction is 2/3 times of the Breit LDA exchange energy. It was found that the Breit correction makes 40Ca less bound by 200 keV and 208Pb by 800 keV, which are comparable to the difference between the exact Coulomb exchange, or its simplified treatment called the generalized gradient approximation (GGA), and the LDA Coulomb exchange energies, but with opposite sign. Thus, once both the exact treatment and the Breit correction are considered at the same time, even the sign of the total correction is not obvious, as was pointed out also in the context of atomic physics [45].

The Breit correction was also compared with the vacuum polarization. Since the proton-number dependence of the vacuum polarization is stronger than that of the Breit energy, these two contributions to the total energy are comparable in light nuclei, but the former dominates in heavy nuclei. This behavior is in contrast to atoms, in which the vacuum polarization is sub-dominant to the Breit correction [42].

In relativistic Hartree-Fock calculation, the current-current Hartree interaction is considered, and the relativistic correction is also considered to the Fock term exactly or by using LDA. In this work, the Breit correction is consistently considered in the Hartree-Fock-Slater approximation. The magnetic (current-current) interaction and the retardation appear in the next-leading order of the electromagnetic interaction in the Lorentz gauge, which corresponds to the Breit correction in the LDA [50]. Hence, the difference between them is which gauge is introduced in quantum electrodynamics calculation [51 52 53]. Thus, it can be concluded that by introducing the Breit correction to nonrelativistic Hartree-Fock calculations as proposed in this work, one can directly compare electromagnetic energies obtained by nonrelativistic and relativistic calculations.

The Coulomb-Breit GGA functional is indispensable to achieve more accurate calculation in both atomic and nuclear structure calculations, which remains as a future perspective.

**V. CONCLUSION AND PERSPECTIVES**

In this paper, the Breit correction, that is the finite-light-speed correction to the Coulomb interaction, which includes magnetic contribution and retardation effects, was introduced in Skyrme Hartree-Fock calculations. To this end, the Foldy-Wouthuysen-Tani transformed Breit correction was used instead of the original one, and, in addition, the local density approximation (LDA) was further introduced for the exchange term.

**ACKNOWLEDGMENTS**

The author thanks Kouichi Hagino, Haozhao Liang, and Kenichi Yoshida for stimulating discussions. The author also thanks Kouichi Hagino for variable comments.

| States  | Coulomb | Coulomb-Breit | C.-B. & Vacuum Pol. |
|---------|---------|---------------|---------------------|
| α1s1/2  | −44.980 | −44.955       | −44.849             |
| α1p3/2  | −39.387 | −39.363       | −39.263             |
| α1p1/2  | −39.107 | −39.084       | −38.984             |
| α1d5/2  | −32.482 | −32.460       | −32.366             |
| α1d3/2  | −31.815 | −31.793       | −31.700             |
| α2s1/2  | −28.509 | −28.489       | −28.392             |
| α1f7/2  | −24.692 | −24.672       | −24.584             |
| α1f5/2  | −23.353 | −23.333       | −23.246             |
| α2p3/2  | −19.411 | −19.393       | −19.302             |
| α2p1/2  | −18.626 | −18.608       | −18.518             |
| α1g9/2  | −16.338 | −16.320       | −16.237             |
| α1g7/2  | −14.019 | −14.002       | −13.920             |
| α2d5/2  | −10.255 | −10.240       | −10.155             |
| α2d3/2  | −8.846  | −8.832        | −8.748              |
| α3s1/2  | −7.673  | −7.658        | −7.574              |
| α1h1/2  | −7.663  | −7.648        | −7.569              |

| States  | Coulomb | Coulomb-Breit | C.-B. & Vacuum Pol. |
|---------|---------|---------------|---------------------|
| ν1s1/2  | −59.291 | −59.285       | −59.270             |
| ν1p3/2  | −52.953 | −52.948       | −52.938             |
| ν1p1/2  | −52.656 | −52.651       | −52.641             |
| ν1d5/2  | −45.375 | −45.371       | −45.364             |
| ν2s1/2  | −41.962 | −41.959       | −41.952             |
| ν1f7/2  | −36.904 | −36.902       | −36.899             |
| ν1f5/2  | −32.094 | −32.093       | −32.089             |
| ν2p1/2  | −35.702 | −35.701       | −35.699             |
| ν2p3/2  | −31.344 | −31.343       | −31.341             |
| ν1g9/2  | −27.863 | −27.862       | −27.862             |
| ν1g7/2  | −25.791 | −25.791       | −25.793             |
| ν2d5/2  | −22.277 | −22.278       | −22.278             |
| ν3s1/2  | −19.959 | −19.960       | −19.961             |
| ν2d3/2  | −20.843 | −20.844       | −20.846             |
| ν1h1/2  | −18.533 | −18.534       | −18.536             |
| ν1h3/2  | −15.306 | −15.308       | −15.313             |
| ν2f7/2  | −12.672 | −12.673       | −12.677             |
| ν2f5/2  | −10.622 | −10.624       | −10.629             |
| ν3p3/2  | −9.859  | −9.861        | −9.864              |
| ν3p1/2  | −9.150  | −9.152        | −9.157              |
| ν3p1/2  | −9.063  | −9.065        | −9.069              |

Energies only by less than 10 keV. This is because the Coulomb interaction does not affect the neutrons directly, unless the finite-size effects are considered, and indirectly via the change of proton states. The protons get less bound due to both the Breit correction and the vacuum polarization, because both corrections make, in total, the mean-field potential shallower. Note that the Breit correction itself makes the EM repulsive interaction weaker, but once both the direct and exchange terms are considered, due to the short-range nature, the Breit correction forms repulsive-like mean-field potential as shown by Eqs. 9.
on the manuscript. The author acknowledges the RIKEN iTHEMS program and the JSPS Grant-in-Aid for JSPS Fellows under Grant No. 19J20543. The numerical calculations were performed on cluster computers at the RIKEN iTHEMS program.

[1] K. Okamoto, Coulomb energy of He³ and possible charge asymmetry of nuclear forces, Phys. Lett. 11, 150 (1964).
[2] E. Friedman, Coulomb energies and the anomaly in nuclear matter radii, Phys. Lett. B 35, 543 (1971).
[3] S. Shlomo and W. G. Love, Core Polarization and Coulomb Displacement Energies, Phys. Scr. 26, 280 (1982).
[4] T. Suzuki, H. Sagawa, and N. Van Giai, Charge independence and charge symmetry breaking interactions and the Coulomb energy anomaly in isobaric analog states, Phys. Rev. C 47, R1360 (1993).
[5] K. Kaneko, T. Mizusaki, Y. Sun, S. Tazaki, and G. de Angelis, Coulomb Energy Difference as a Probe of Isospin-Symmetry Breaking in the Upper fp-Shell Nuclei, Phys. Rev. Lett. 109, 092504 (2012).
[6] P. Bączyk, J. Dobaczewski, M. Konieczka, W. Satuła, T. Nakatsukasa, and K. Sato, Isospin-symmetry breaking in masses of N ≃ Z nuclei, Phys. Lett. B 778, 178 (2018).
[7] X. Roca-Maza, G. Colò, and H. Sagawa, Nuclear Symmetry Energy and the Breaking of the Isospin Symmetry: How Do They Reconcile with Each Other?, Phys. Rev. Lett. 120, 202501 (2018).
[8] P. Bączyk, W. Satuła, J. Dobaczewski, and M. Konieczka, Isobaric multiplet mass equation within nuclear density functional theory, J. Phys. G 46, 03LT01 (2019).
[9] B. M. Loc, N. Auerbach, and G. Colò, Isospin mixing and Coulomb mixing in ground states of even-even nuclei, Phys. Rev. C 99, 014311 (2019).
[10] J. M. Dong, J. Z. Gu, Y. H. Zhang, W. Zuo, L. J. Wang, Y. A. Litvinov, and Y. Sun, Beyond Wigener’s isobaric multiplet mass equation: Effect of charge-symmetry-breaking interaction and Coulomb polarization, Phys. Rev. C 99, 014319 (2019).
[11] N. Auerbach, Coulomb effects in nuclear structure, Rep. Prog. Phys. 98, 273 (1983).
[12] P. Hohenberg and W. Kohn, Inhomogeneous Electron Gas, Phys. Rev. 136, B864 (1964).
[13] W. Kohn and L. J. Sham, Self-Consistent Equations Including Exchange and Correlation Effects, Phys. Rev. 140, A1133 (1965).
[14] W. Kohn, Nobel Lecture: Electronic structure of matter—wave functions and density functionals, Rev. Mod. Phys. 71, 1253 (1999).
[15] S. H. Vosko, L. Wilk, and M. Nusair, Accurate spin-dependent electron liquid correlation energies for local spin density calculations: a critical analysis, Can. J. Phys. 58, 1200 (1980).
[16] J. P. Perdew and A. Zunger, Self-interaction correction to density-functional approximations for many-electron systems, Phys. Rev. B 23, 5048 (1981).
[17] T. Yokota and T. Naito, Ab initio construction of the energy density functional for electron systems with the functional-renormalization-group-aided density functional theory, Phys. Rev. Research 3, L012015 (2021).
[18] J. P. Perdew and Y. Wang, Accurate and simple density functional for the electronic exchange energy: Generalized gradient approximation, Phys. Rev. B 33, 8800 (1986).
[19] J. P. Perdew and Y. Wang, Accurate and simple analytic representation of the electron-gas correlation energy, Phys. Rev. B 45, 13244 (1992).
[20] J. P. Perdew, K. Burke, and M. Ernzerhof, Generalized Gradient Approximation Made Simple, Phys. Rev. Lett. 77, 3865 (1996).
[21] J. Drut, R. Furnstahl, and L. Platter, Toward ab initio density functional theory for nuclei, Prog. Part. Nucl. Phys. 64, 120 (2010).
[22] J. Dobaczewski, Ab initio derivation of model energy density functionals, J. Phys. G 43, 04LT01 (2016).
[23] T. Yokota, K. Yoshida, and T. Kunihiro, Functional renormalization-group calculation of the equation of state of one-dimensional uniform matter inspired by the Hohenberg-Kohn theorem, Phys. Rev. C 99, 024302 (2019).
[24] G. Salvioni, J. Dobaczewski, C. Barbieri, G. Carlsson, A. Idini, and A. Pastore, Model nuclear energy density functionals derived from ab initio calculations, J. Phys. G 47, 085107 (2020).
[25] G. Accorto, T. Naito, H. Liang, T. Nikšić, and D. Vretěriva, Nuclear energy density functionals from empirical ground-state densities, Phys. Rev. C 103, 044304 (2021).
[26] F. Marino, C. Barbieri, G. Colò, A. Lovato, F. Pedriva, X. Roca-Maza, and E. Vigezzi, Nuclear energy density functionals grounded in ab initio calculations (2021), arXiv:2103.14480 [nucl-th].
[27] B.-A. Li, L.-W. Chen, and C. M. Ko, Recent progress and new challenges in isospin physics with heavy-ion reactions, Phys. Rep. 464, 113 (2008).
[28] P. Danielewicz and J. Lee, Symmetry energy I: Semi-infinite matter, Nucl. Phys. A 818, 36 (2009).
[29] P. Danielewicz and J. Lee, Symmetry energy II: Isobaric analog states, Nucl. Phys. A 922, 1 (2014).
[30] T. Naito, R. Akashi, and H. Liang, Application of a Coulomb energy density functional for atomic nuclei: Case studies of local density approximation and generalized gradient approximation, Phys. Rev. C 97, 044319 (2018).
[31] T. Naito, X. Roca-Maza, G. Colò, and H. Liang, Coulomb exchange functional with generalized gradient approximation for self-consistent Skyrme Hartree-Fock calculations, Phys. Rev. C 99, 024309 (2019).
[32] T. Naito, X. Roca-Maza, G. Colò, and H. Liang, Effects of finite nucleon size, vacuum polarization, and electromagnetic spin-orbit interaction on nuclear binding energies and radii in spherical nuclei, Phys. Rev. C 101, 064311 (2020).
[33] D. Vautherin and D. M. Brink, Hartree-Fock Calculations with Skyrme’s Interaction. I. Spherical Nuclei, Phys. Rev. C 5, 626 (1972).
[34] J. Meng, H. Toki, S. Zhou, S. Zhang, W. Long, and L. Geng, Relativistic continuum Hartree Bogoliubov the-
T. Nakatsukasa, K. Matsuyanagi, M. Matsuo, and K. Yamagata, Time-dependent density-functional description of nuclear dynamics, Rev. Mod. Phys. 88, 045004 (2016)

G. Colò, Nuclear density functional theory, Adv. Phys.:X 5, 1740061 (2020)

T. Nikšić, D. Vretenar, and P. Ring, Relativistic nuclear dynamics, Rev. Mod. Phys. 73, 888 (2001)

G. Breit, Dirac's Equation and the Spin-Spin Interactions of Two Electrons, Phys. Rev. 34, 553 (1929)

G. Breit, The Fine Structure of HE as a Test of the Spin Interactions of Two Electrons, Phys. Rev. 36, 383 (1930)

G. Breit, The Effect of Retardation on the Interaction of Two Electrons, Phys. Rev. 34, 553 (1929)

M. I. Eides, H. Grotch, and V. A. Shelyuto, Theory of the Complete Breit Interaction, Phys. Rev. A 43, 63 (2001)

G. Breit, The Effect of Retardation on the Interaction of Two Electrons, Phys. Rev. 39, 616 (1932)

A. H. MacDonald and S. H. Vosko, A relativistic density functional formalism, J. Phys. C 12, 2977 (1979)

S. D. Kenny, G. Rajagopal, R. J. Needs, W.-K. Leung, M. J. Godfrey, A. J. Williamson, and W. M. C. Foulkes, Quantum Monte Carlo Calculations of the Energy of the Relativistic Homogeneous Electron Gas, Phys. Rev. Lett. 77, 1099 (1996)

T. Naito, R. Akashi, H. Liang, and S. Tsuneyuki, Relativistic density functional theory with finite-light-speed correction for the Coulomb interaction: a non-relativistic-reduction-based approach, J. Phys. B 53, 215002 (2020)

J. Le Bloas, M.-H. Koh, P. Quentin, L. Bonneau, and J. I. A. Ithnin, Exact Coulomb exchange calculations in the Skyrme-Hartree-Fock-BCS framework and tests of the Slater approximation, Phys. Rev. C 84, 014310 (2011)

X. Roca-Maza, L.-G. Cao, G. Colò, and H. Sagawa, Fully self-consistent study of charge-exchange resonances and the impact on the symmetry energy parameters, Phys. Rev. C 94, 044313 (2016)

H.-Q. Gu, H. Liang, W. H. Long, N. Van Giai, and J. Meng, Slater approximation for Coulomb exchange effects in nuclear covariant density functional theory, Phys. Rev. C 87, 041301 (2013)

E. Lindroth and A.-M. Mårtensson-Pendrill, Further analysis of the complete Breit interaction, Phys. Rev. A 39, 3794 (1989)

M. H. L. Pryce, The mass-centre in the restricted theory of relativity and its connexion with the quantum theory of elementary particles, Proc. Royal Soc. Lond. A 195, 62 (1948)

L. L. Foldy and S. A. Wouthuysen, On the Dirac Theory of Spin 1/2 Particles and Its Non-Relativistic Limit, Phys. Rev. 78, 29 (1950)

S. Tani, Connection between Particle Models and Field Theories, I —The Case Spin 1/2—, Prog. Theor. Phys. 6, 267 (1951)

L. L. Foldy, The Electromagnetic Properties of Dirac Particles, Phys. Rev. 87, 688 (1952)

T. Itoh, Derivation of Nonrelativistic Hamiltonian for Electrons from Quantum Electrodynamics, Rev. Mod. Phys. 37, 159 (1965)

S. D. Kenny, G. Rajagopal, and R. J. Needs, Relativistic corrections to atomic energies from quantum Monte Carlo calculations, Phys. Rev. A 51, 1898 (1995)

J. C. Slater, A Simplification of the Hartree-Fock Method, Phys. Rev. 81, 385 (1951)

P. A. Zyla, R. M. Barnett, J. Beringer, O. Dahl, D. A. Dwyer, D. E. Groom, C. J. Lin, K. S. Lugovsky, E. Pianori, D. J. Robinson, C. G. Wohl, W. M. Yao, K. Agashe, G. Aielli, B. C. Allanach, C. Amrister, M. Antonellini, E. C. Aschenauer, D. M. Asner, H. Baer, S. Banerjee, L. Baudis, C. W. Bauer, J. J. Beatty, V. I. Belousov, S. Bethke, A. Bettini, O. Bibel, K. M. Black, E. Blucher, O. Buchmüller, V. Burkert, M. A. Burchik, R. N. Calin, M. Carena, A. Cucco, A. Cerri, D. Chakraborty, R. S. Chivukula, G. Cowan, G. D’Ambrosio, T. Damour, D. de Florian, A. de Gouvea, T. DeGrand, P. de Jong, D. Dserti, B. A. Dobrescu, M. D’Onofrio, M. Doser, M. Drees, H. K. Dreiner, P. Erola, U. Egede, S. Eidelman, J. Ellis, J. Erler, V. V. Ezhela, W. Fetscher, B. D. Fields, B. Foster, A. Freitas, H. Gallagher, L. Garren, H. J. Gerber, G. Gierz, T. Gershon, Y. Gershtein, T. Gherghetta, A. A. Godz, M. C. Gonzalez-Garcia, M. Goodman, C. Grab, A. V. Gritsan, C. Grojean, M. Grünwald, A. Gurtu, T. Gutsche, H. E. Haber, C. Hanhart, S. Hashimoto, Y. Hayato, A. Hebecker, S. Heinemeyer, B. Heltlsley, J. J. Hernández-Rey, K. Hikasa, J. Hisano, A. Höcker, J. Holder, A. Holtkamp, J. Huston, T. Hyodo, K. F. Johnson, M. Kado, M. Karlener, U. F. Katz, M. Kenzie, V. A. Khoze, S. R. Klein, E. Klempt, R. V. Kowalewski, F. Krauss, M. Krebs, B. Kruse, Y. Kwon, O. Lahav, J. Laiho, L. P. Lellouch, J. Lesgourgues, A. R. Lidzle, Z. Ligeti, C. Lippmann, T. M. Liss, L. Littenberg, C. Lourenço, S. B. Lugovsky, A. Lusiani, Y. Makida, F. Maltoni, T. Mannel, A. V. Manohar, W. J. Marciano, A. Masoni, J. Matthews, U. G. Meißner, M. Mikkelsen, D. J. Miller, D. Milstead, R. F. Mitchell, K. Mönnig, P. Molaro, F. Moortgat, M. Moskovic, K. Nakamura, N. Narain, P. Nason, S. Navas, M. Neubert, P. Nevskii, Y. Nir, K. A. Olive, C. Patrignani, J. A. Peacock, S. T. Petcov, V. A. Petrov, A. Pich, A. Piepke, A. Pomarol, S. Proffumo, A. Quadl, K. Rabbertz, J. Rademacker, G. Raffelt, H. Ramani, M. Ramsay-Musolf, B. N. Ratcliff, P. Richardson, A. Ringwald, S. Roesler, S. Rolli, A. Romanikou, L. J. Rosenberg, J. L. Rosner, G. Rybka, M. Ryskin, R. A. Ryutin, Y. Salahi, G. P. Salam, S. Sarkar, F. Sauli, O. Schneider, K. Scholberg, A. J. Schwartz, J. Schwiening, D. Scott, V. Sharma, S. R. Sharpe, T. Shutt, M. Silar, T. Sjöström, P. Skands, T. Skwarnicki, G. F. Smoot, A. Soffer, M. S. Sozzi, S. Spanier, C. Spiering, A. Stahl, S. L. Stone, Y. Sumino, T. Sumiyoshi, M. J. Syphers, F. Takahashi, M. Tandabashi, J. Tanaka, M. Taševský, K. Terashi, J. Tennling, U. Thoma, R. S. Thorne, L. Tiator, M. Titov, N. P. Tkachenko, D. R. Tovey, K. Trabelsi, P. Urquijo,
The exchange-correlation energy of $\tilde{V}_{\text{Breit}}$ for homogeneous electron gas had been calculated using the diffusion Monte Carlo calculation in Ref. [44], and it was found in Ref. [45] that the exchange-correlation energy is numerically almost identical to the exchange energy derived in Ref. [43].

[62] The exchange-correlation energy of $\tilde{V}_{\text{Breit}}$ for homogeneous electron gas had been calculated using the diffusion Monte Carlo calculation in Ref. [44], and it was found in Ref. [45] that the exchange-correlation energy is numerically almost identical to the exchange energy derived in Ref. [43].

[63] E. A. Uehling, Polarization Effects in the Positron Theory, Phys. Rev. 48, 55 (1935).

[64] L. Wayne Fullerton and G. A. Rinker, Accurate and efficient methods for the evaluation of vacuum-polarization potentials of order $Z\alpha$ and $Z\alpha^2$, Phys. Rev. A 13, 1283 (1976).

[65] G. Colò, L. Cao, N. Van Giai, and L. Capelli, Self-consistent RPA calculations with Skyrme-type interactions: The skyrme_rpa program, Comput. Phys. Commun. 184, 142 (2013).

[66] X. Roca-Maza, G. Colò, and H. Sagawa, New Skyrme interaction with improved spin-isospin properties, Phys. Rev. C 86, 031306 (2012).