B-quark mediated neutrinoless $\mu^- - \mu^-$ conversion in presence of $R$-parity violation

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Abstract

We found that in supersymmetric models with $R$-parity non-conservation ($R_p$/SUSY) the $b$-quarks may appreciably contribute to exotic neutrinoless $\mu^- - e^-$ conversion in nuclei via the triangle diagram with two external gluons. This allowed us to extract previously overlooked constraints on the third generation trilinear $R_p$ parameters significantly more stringent than those existing in the literature.

PACS: 12.60Jv, 11.30.Er, 11.30.Fs, 23.40.Bw

KEYWORDS: Lepton flavor violation, $\mu - e$ conversion in nuclei, supersymmetry, R-parity violation.

In the standard model (SM) the lepton flavors ($L_f$) are conserved quantum numbers as an accidental consequence of gauge invariance and field content. Thus, observation of $L_f$ non-conservation would imply the presence of physics beyond the standard model. Non-conservation of muon lepton flavor $L_\mu$ in neutrino oscillations has been recently established by the SuperKamiokande experiment in atmospheric neutrino measurements. In this case lepton flavor violation ($\mathcal{U}_f$) is generated by non-zero neutrino masses. Various sources of $\mathcal{U}_f$ can be probed by searching for certain exotic processes. Among them the neutrinoless muon-to-electron conversion in muonic atoms, $\mu^- + (A, Z) \rightarrow e^- + (A, Z)^*$, is known to be one of the most powerful tools to constrain $\mathcal{U}_f$ interactions $[1]-[3]$. In particular, it allows setting stringent constraints on the $\mathcal{U}_f$ interactions of the supersymmetric models with $R$-parity violation ($R_p$/SUSY). In the literature there have been obtained upper bounds on various products of the $\mathcal{U}_f$ trilinear $R$-parity violating ($R_p$) couplings $\lambda\lambda'$, $\lambda'\lambda'$ $[3]-[7]$. In the present letter we derive new constraints on the products $\lambda\lambda'$ with some other combinations...
of generation indexes. These constraints emerge from the previously overlooked contribution of b-quark to $\mu^- - e^-$ conversion via the triangle diagram shown in Fig. 1.

During the last decade $R_p$ SUSY models have been extensively studied in the literature. For the minimal field content the R-parity violating part of the super potential reads

$$W_{R_p} = \lambda_{ijk} L_i L_j E_k^c + \bar{\lambda}'_{ijk} L_i Q_j D_k^c + \mu_j L_j H_2 + \bar{\lambda}''_{ijk} U_i D_j^c D_k^c.$$  

(1)

The definition of the couplings $\bar{\lambda}'$, $\bar{\lambda}''$ corresponds to the gauge basis for the quark fields. We set $\bar{\lambda}'' = 0$, since these are irrelevant for our consideration. This “ad hoc” condition ensures proton stability and can be guaranteed by special discreet symmetries other than R-parity.

The leading quark-level tree diagrams with the trilinear $R_p$-couplings contributing to the $\mu^- - e^-$ conversion are listed in Ref. [5] and are of the following three types:

(i) $\mu_{L,R} \rightarrow e_{L,R}, d_L \rightarrow d_R$, with t-channel $\tilde{\nu}$ exchange,
(ii) $\mu_L \rightarrow d_R, d_R \rightarrow e_L$, with t-channel $\tilde{u}$ exchange,
(iii) $\mu_L u_L \rightarrow \tilde{d}_R \rightarrow e_L u_L$.

The photonic 1-loop diagrams can also significantly contribute to this process [4]. However they are irrelevant for the present case of heavy quark contribution and, therefore, are not included in our analysis. Integrating out the heavy intermediate SUSY-particles from the above mentioned diagrams and carrying out a Fierz rearrangement one obtains the following 4-fermion effective Lagrangian for $\mu^- - e^-$ conversion at the quark level [3]

$$L^q_{e\mu} = \frac{G_F}{\sqrt{2}} j_\mu \left[ \eta^{ui} J_{u(i)}^\mu + \eta^{di} J_{d(i)}^\mu \right] + \frac{G_F}{\sqrt{2}} \left( \eta_{L}^{di} j_L + \eta_{R}^{di} j_R \right) J_{d(i)}.$$  

(2)

The index $i$ denotes generation so that $u_i = u, c, t$ and $d_i = d, s, b$. Here $J_{q(i)}^\mu = \bar{q}_i \gamma^\mu q_i$, $J_{d(i)}^\mu = \bar{d}_i d_i$, $J_{d(i)}^\mu = \bar{d}_i d_i$, $J_{d(i)}^\mu = \bar{e} \gamma^\mu P_L \mu$, $J_{L,R} = \bar{e} P_{L,R} \mu$. In Eq. (2) we neglected the terms with axial-vector and pseudoscalar quark currents which do not contribute to the dominant coherent

\footnote{On leave of absence from the Joint Institute for Nuclear Research, Dubna, Russia}
The coefficients \( \eta \) in Eq. (3) accumulate the dependence on \( R_p \) SUSY parameters as

\[
\eta_{ui} = -\frac{1}{\sqrt{2}} \sum_{i,m,n} \frac{\lambda_{imi}^*}{G_F m_i^2} V_{im} V_{in}, \quad \eta_{di} = \frac{1}{\sqrt{2}} \sum_{i,m,n} \frac{\lambda_{imi}^*}{G_F m_i^2} V_{im} V_{in},
\]

\[
\eta^d_{R} = -\sqrt{2} \sum_{n} \frac{\lambda_{imi}^*}{G_F m_i^2}, \quad \eta^d_{L} = -\sqrt{2} \sum_{n} \frac{\lambda_{imi}^*}{G_F m_i^2}.
\]

(3)

Here \( m_{\tilde{q}(n)} \), \( m_{\nu(n)} \) are the squark and sneutrino masses. In Eq. (3) we introduced the couplings \( \lambda'_{ijk} = \lambda_{imm}^* \left( V^d_L \right)_{jm} \left( V^d_R \right)_{kn} \) corresponding to the \( R_p \) interactions in the quark mass eigenstate basis, related to the gauge basis \( q' \) through \( q_{L,R} = V^q_{L,R} q'_{L,R} \). The CKM matrix is defined in the standard way as \( V = V^u V^d \).

The contribution of the quark currents \( J_{u(i)}^\mu, J_{d(i)}^\mu, J_{d(i)}^\mu \) present in Eq. (2) to the corresponding nucleon currents can be parametrized in the form

\[
\langle N| \bar{q} \Gamma_K q| N \rangle = G^{(q,N)}_K \bar{\Psi}_N \Gamma_K \Psi_N,
\]

(4)

with \( q = \{u, d, s\} \), \( N = \{p, n\} \) and \( K = \{V, S\} \), \( \Gamma_K = \{\gamma_{\mu}, 1\} \). The maximum momentum transfer \( q \) in \( \mu^- - e^- \) conversion can be estimated as \( |q| \approx m_\mu/c \) with \( m_\mu = 105.6 \text{ MeV} \) being the muon mass. Since \( |q| \) is relatively small compared to the typical nucleon structure scales we can safely neglect in Eq. (4) the \( q^2 \)-dependence of the nucleon form factors \( G^{(q,N)}_K \) as well as the weak magnetism and the induced pseudoscalar terms which are proportional to the small momentum transfer.

Isospin symmetry requires that \( G^{(u,n)}_K = G^{(d,p)}_K \equiv G^{d}_K \), \( G^{(d,n)}_K = G^{(u,p)}_K \equiv G^{u}_K \), \( G^{(s,n)}_K \equiv G^{s} \), \( G^{(h,n)}_K = G^{(h,p)}_K \equiv G^{h}_K \), with \( K = V, S \) and \( h = c, b, t \). Furthermore, conservation of vector current requires the vector charge to be equal to the quark number of the nucleon. This allows fixing the vector nucleon constants as \( G^u_V = 2, G^d_V = 1, G^s_V = G^h_V = 0 \). Thus the strange and heavy quarks can contribute only to the b-quark nucleon current \( \bar{q}_b \). Since the scalar currents in Eq. (2) involve only down quarks \( d, s \), it follows that among the heavy \( c, b, t \)-quarks only the b-quark can contribute to the coherent \( \mu^- - e^- \) conversion. The heavy quarks contribution to the scalar current is realized via the triangle diagram in Fig. 1 with the two gluon lines. The heavy quark \( q_h \) scalar current induced by the diagrams of this type can be estimated using the heavy quark expansion \[11\]

\[
m_h \bar{q}_h q_h \approx -\frac{2}{3} \left( \frac{\alpha_s}{8\pi} \right) G G + O \left( \frac{\mu^2}{m_h^2} \right).
\]

(5)

Here \( \alpha_s \) and \( \mu \) are the QCD coupling constant and a typical hadronic scale respectively, and \( G G = G_{\mu\nu}^a G_{a\mu}^a \) where \( G_{\mu\nu}^a \) is the gluon field strength. The quark scalar currents and the gluon operator \( GG \) also contribute to the trace of the energy-momentum tensor

\[
\theta_{\mu}^\mu = m_u \bar{u} u + m_d \bar{d} d + m_s \bar{s} s + \sum_h m_h \bar{q}_h q_h - (b\alpha_s/8\pi) G G,
\]

(6)

\[2\text{They can also contribute to the axial-vector and pseudoscalar nucleon currents which are irrelevant for the coherent } \mu^- - e^- \text{ conversion considered in the present letter.} \]
where \( b = 11 - (2/3)n_f \) and \( n_f = 6 \) is the number of quark species.

The scalar form factors \( G_S^d \) can be extracted from the baryon octet \( B \) mass spectrum \( M_B \), expressed as \([12]\)

\[
\langle B|\theta^\mu_\mu|B\rangle = M_B\bar{B}B, \tag{7}
\]

and from the data on the pion-nucleon sigma term \( \sigma_{\pi N} = (1/2)\langle m_u + m_d \rangle \langle p|\bar{u}u + \bar{d}d|p \rangle \). Using Eqs. (5)-(7) in combination with \( SU(3) \) relations \([12]\) for the matrix elements in (6) we obtain

\[
G_S^u \approx 5.1, \quad G_S^d \approx 4.3, \quad G_S^s \approx 2.5, \quad G_S^b \approx 9 \times 10^{-3}. \tag{8}
\]

Here we used for \( \sigma_{\pi N} = 48 \text{ MeV} \) \([13]\), and \( m_u = 4.2 \text{ MeV}, m_d = 7.5 \text{ MeV}, m_s = 150 \text{ MeV}, m_b = 4.2 \text{ GeV} \).

Having the couplings \( G_V^d, G_S^d \) determined we rewrite the Lagrangian \([2]\) in terms of the nucleon currents

\[
\mathcal{L}_{\text{eff}}^N = \frac{G_F}{\sqrt{2}} \left[ \bar{e}\gamma_\mu (1 - \gamma_5) \mu \cdot J^\mu + \bar{e}\mu \cdot J^+ + \bar{e}\gamma_5 \mu \cdot J^- \right]. \tag{9}
\]

Here we defined \( J^\mu = \bar{N}\gamma^\mu (\alpha_V^{(0)} + \alpha_V^{(3)} \tau_3) N, J^+ = \bar{N} (\alpha_{S+}^{(0)} + \alpha_{S+}^{(3)} \tau_3) N \), where \( N^T = (p, n) \) is the nucleon isospin doublet. The isoscalar \( \alpha_0 \) and the isovector \( \alpha_3 \) coefficients are

\[
\alpha_V^{(0)} = \frac{1}{8} (G_V^u + G_V^d) (\eta_{u^1}^{d^1}), \quad \alpha_V^{(3)} = \frac{1}{8} (G_V^u - G_V^d) (\eta_{u^1}^{d^1}), \\
\alpha_{S+}^{(0)} = \frac{1}{16} (G_S^u + G_S^d) (\eta_{L}^{d^{1 L}} \pm \eta_{R}^{d^{1 R}}) + \frac{1}{8} G_S^d (\eta_{L}^{d^{2 L}} \pm \eta_{R}^{d^{2 R}}) + \frac{1}{8} G_S^b (\eta_{L}^{d^{3 L}} \pm \eta_{R}^{d^{3 R}}), \\
\alpha_{S+}^{(3)} = -\frac{1}{16} (G_S^u - G_S^d) (\eta_{L}^{d^{1 L}} \pm \eta_{R}^{d^{1 R}}). \tag{10}
\]

Following the approach of Refs. \([3, 9]\) we derive from the Lagrangian \([3]\) the coherent \( \mu - e \) conversion branching ratio in the form

\[
R_{\mu e^{-}} = \frac{G_F^2}{2\pi} Q \frac{p_e E_e (\mathcal{M}_\mu + \mathcal{M}_n)^2}{\Gamma(\mu^{-} \rightarrow \text{capture})} , \tag{11}
\]

where \( \Gamma(\mu^{-} \rightarrow \text{capture}) \) is the total rate of ordinary muon capture and

\[
Q = 2|\alpha_0^{(0)}|^2 + |\alpha_0^{(0)}|^2 + |\alpha_0^{(0)}|^2 + 2 \text{Re} \{\alpha_0^{(0)} [\alpha_0^{(0)} + \alpha_0^{(0)}]\}. \tag{12}
\]

In this formula we neglected the contribution of isovector currents which is small for most of the experimentally interesting nuclei \([3, 9]\). The numerical values of the nuclear matrix elements \( \mathcal{M}_{p,n} \) for the currently interesting have been calculated in Ref. \([9]\).

The most stringent experimental bounds on the branching ratio \( R_{\mu e} \) have been set by the SINDRUM2 experiment (PSI) with \( ^{197}\text{Au} \) and \( ^{48}\text{Ti} \) stopping targets:

\[
R_{\mu e}^{^{197}\text{Au}} = \frac{\Gamma(\mu^{-} \rightarrow ^{197}\text{Au} \rightarrow e^{-} + ^{197}\text{Au})}{\Gamma(\mu^{-} \rightarrow ^{197}\text{Au} \rightarrow \nu_\mu + ^{197}\text{Pt})} \leq 5.0 \times 10^{-13}, \quad (90\% \text{ C.L.}) \quad [14], \tag{13}
\]

\[
R_{\mu e}^{^{48}\text{Ti}} = \frac{\Gamma(\mu^{-} \rightarrow ^{48}\text{Ti} \rightarrow e^{-} + ^{48}\text{Ti})}{\Gamma(\mu^{-} \rightarrow ^{48}\text{Ti} \rightarrow \nu_\mu + ^{48}\text{Sc})} \leq 6.1 \times 10^{-13}, \quad (90\% \text{ C.L.}) \quad [15]. \tag{14}
\]
Note that a $^{48}\text{Ti}$ target will be used in the future experiment planned at the muon factory at KEK (Japan) \cite{16}. This experiment is going to increase the sensitivity up to $R_{\mu e} \leq 10^{-18}$.

In the near future new bounds are expected from the MECO (Brookhaven) experiment with an $^{27}\text{Al}$ target

$$R_{\mu e}^{^{27}\text{Al}} = \frac{\Gamma(\mu^- + ^{27}\text{Al} \rightarrow e^- + ^{27}\text{Al})}{\Gamma(\mu^- + ^{27}\text{Al} \rightarrow \nu_\mu + ^{27}\text{Mg})} \leq 2 \times 10^{-17} \quad [17]$$

From these experimental limits it is straightforward to extract upper bounds on various products of the type $\lambda\lambda'$, $\lambda'\lambda'$. Many of them have been previously derived in Refs. \cite{3}-\cite{5}. In Table 1 we present the new upper bounds that are associated with the b-quark contribution. We show the three cases, corresponding to the experimental limits in Eqs. (13)-(15).

| Parameters | Previous limits | Present limits (Au) | Present limits (Ti) | Expected limits (Al) |
|------------|----------------|---------------------|--------------------|---------------------|
| $|\lambda'_{133}\lambda_{121}|$ | $6.9 \cdot 10^{-5}$ | $8.5 \cdot 10^{-7}$ | $2.0 \cdot 10^{-6}$ | $1.2 \cdot 10^{-8}$ |
| $|\lambda'_{233}\lambda_{212}|$ | $7.4 \cdot 10^{-3}$ | $8.5 \cdot 10^{-7}$ | $2.0 \cdot 10^{-6}$ | $1.2 \cdot 10^{-8}$ |
| $|\lambda'_{333}\lambda_{312}|$ | $2.8 \cdot 10^{-2}$ | $8.5 \cdot 10^{-7}$ | $2.0 \cdot 10^{-6}$ | $1.2 \cdot 10^{-8}$ |
| $|\lambda'_{333}\lambda_{321}|$ | $3.2 \cdot 10^{-2}$ | $8.5 \cdot 10^{-7}$ | $2.0 \cdot 10^{-6}$ | $1.2 \cdot 10^{-8}$ |

Table 1: The new upper bounds from the b-quark contribution to $\mu^- - e^-$ conversion. The previous bounds were taken from \cite{7}. The scaling factors $B_{1,2,3}$ are defined in the text.

In the derivation of these bounds we assumed, as usual, the dominance of only one of these products with the specific combination of generation indices. We also assumed that all the scalar masses in Eq. (3) are equal $\tilde{m}_{uL(n)} \approx \tilde{m}_{dL,L(n)} \approx \tilde{m}_{u(n)} \approx \tilde{m}$.

As can be seen from Table 1, our new limits(columns 3-5) are significantly more stringent than those previously known in the literature \cite{7} (column 2). In Table 1 the quantities $B_{1,2,3}$ denote scaling factors defined as

$$B_1 = (R_{\mu e}^{\exp}/5.0 \cdot 10^{-13})^{1/2}, \quad B_2 = (R_{\mu e}^{\exp}/6.1 \cdot 10^{-13})^{1/2}, \quad B_3 = (R_{\mu e}^{\exp}/2.0 \cdot 10^{-17})^{1/2}.$$  \hspace{1cm} (16)

They allow recalculating limits given in Table 1 to the case corresponding to the experimental upper bounds on the branching ratio $R_{\mu e}^{\exp}$ other than in Eqs. (13)-(15).

In summary, we found new important contribution to $\mu^- - e^-$ conversion originating from the b-quark sea of the nucleon. We have shown that among the heavy $c, b, t$-quarks only the b-quark can contribute to the coherent $\mu^- - e^-$ conversion via the scalar interactions involving down quarks $d, s, b$. The heavy quarks contribution to the scalar current is materialized with the gluon exchange shown in the triangle diagram of Fig. 1. From the existing data and expected experimental constraints on the branching ratio $R_{\mu e}^{\exp}$ we obtained new upper limits on the products of the trilinear $R_p$ parameters of the type $\lambda_{n12}\lambda'_{n33}, \lambda_{n21}\lambda'_{n33}$ which are significantly more stringent than those existing in the literature.
This work was supported in part by Fondecyt (Chile) under grant 8000017, by a Cátedra Presidencial (Chile) and by RFBR (Russia) under grant 00-02-17587.

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