Universal critical behavior in single crystals and films of YBa$_2$Cu$_3$O$_{7-\delta}$

Hua Xu, Su Li, Steven M. Anlage, and C. J. Lobb

Center for Nanophysics and Advanced Materials, Department of Physics, University of Maryland, College Park, MD 20742-4111, USA

M. C. Sullivan

Department of Physics, Ithaca College, Ithaca, NY 14850, USA

Kouji Segawa and Yoichi Ando

Institute of Scientific and Industrial Research, Osaka University, Ibaraki, Osaka 567-0047, Japan

We have studied the normal-to-superconducting phase transition in optimally-doped YBa$_2$Cu$_3$O$_{7-\delta}$ in zero external magnetic field using a variety of different samples and techniques. Using DC transport measurements, we find that the dynamical critical exponent $z = 1.54 \pm 0.14$, and the static critical exponent $\nu = 0.66 \pm 0.10$ for both films (when finite-thickness effects are included in the data analysis) and single crystals (where finite-thickness effects are unimportant). We also measured thin films at different microwave frequencies and at different powers, which allowed us to systematically probe different length scales to avoid finite-thickness effects. DC transport measurements were also performed on the films used in the microwave experiments to provide a further consistency check. These microwave and DC measurements yielded a value of $z$ consistent with the other results, $z = 1.55 \pm 0.15$. The neglect of finite-thickness, finite-current, and finite-frequency effects may account for the wide ranges of values for $\nu$ and $z$ previously reported in the literature.

PACS numbers: 74.25.Fy, 74.25.Dw, 74.72.Bk

I. INTRODUCTION

The high critical temperatures, large penetration depths, and short coherence lengths of high-temperature superconductors make it possible to measure critical fluctuations in these materials, in contrast to conventional superconductors [1, 2]. In spite of nearly two decades of work, however, there is no experimental consensus on the critical exponents of the superconducting phase transition in zero magnetic field. By performing both DC and microwave measurements on thin films, and DC measurements on single crystals, and doing careful analysis of the data that properly accounts for finite size, current, and frequency effects, we are able to provide consistent values for the exponents.

There are two fundamental parameters which characterize a second-order phase transition such as the superconducting to normal transition [3]. The first is the temperature-dependent correlation length, $\xi(T)$, which close to the transition temperature $T_c$ varies as

$$\xi(T) \sim |T/T_c - 1|^{-\nu},$$

where $\nu$ is the static critical exponent. A second parameter is the relaxation time $\tau(T)$, which close to $T_c$ varies as

$$\tau \sim \xi^z \sim |T/T_c - 1|^{-z \nu},$$

where $z$ is the dynamic critical exponent.

It is generally accepted that, theoretically, $\nu \approx 0.67$ in a superconductor in zero magnetic field, since the phase transition belongs to the three dimensional (3D) XY universality class [3]. The theoretical situation for the dynamical exponent $z$ is less certain. Fisher, Fisher, and Huse [1] argue that the number of Cooper pairs is not conserved, so that model A dynamics [3], which give $z = 2$, should apply. Other theoretical considerations yield $z = 1.5$ [4], similar to model E dynamics. Lidmar [5] and Weber [6] present Monte Carlo simulations that suggest $z \approx 1.5$.

The exponent $\nu$ can be determined experimentally from a number of static experiments. In zero field, measurements of penetration depth, magnetic susceptibility, specific heat, and thermal expansivity largely agree that the static critical exponent $\nu \approx 0.67$, and indicate that the phase transition in zero field belongs to the 3D-XY universality class. (Note, however, that there are some measurements which yield different results, [5, 14].)

In principle DC conductivity measurements, which depend on both the statics and the dynamics of the order parameter near $T_c$, can determine both the static critical exponent $\nu$ and dynamical critical exponent $z$. The exponents $\nu$ and $z$ are expected to be universal, but values extracted from conductivity measurements are not consistent. For example, DC conductivity measurements yield a wide range of values for critical exponents: $\nu = 0.63$ to 1.2 and $z = 1.25$ to 9.3, [13, 16, 17, 18, 12, 20, 21].

AC measurements can determine both the real and imaginary parts of the fluctuation conductivity, providing another probe of critical dynamics. Measurements over a broad frequency range allow one to probe the dynamical behavior of the system and di-
rectly measure the fluctuation lifetime. These experiments are difficult and seldom done, and the available results are inconsistent, with values of \( z \) ranging from 2 to 5.6. Booth et al. investigated the frequency-dependent microwave conductivity of \( YBa_2Cu_3O_{6+\delta} \) (YBCO) films above \( T_c \) and obtained \( z = 2.3 \) to 3. Nakielksi et al. measured the conductivity of YBCO at low frequency (<2 GHz) and obtained \( z \approx 5.6 \). Osborn et al. did a similar experiment on \( Bi_2Sr_2CaCu_2O_{8+\delta} \) and obtained \( z \approx 2 \). For an optimally doped \( La_{2-x}Sr_xCuO_4 \) (LSCO) film, Kitano et al. found that their data were consistent with the 3D-XY model with diffusive dynamics, \( \nu \approx 0.67, z \approx 2 \) in a certain temperature range.

Although the critical exponents \( \nu \) and \( z \) should be universal, we see that there is at present no consensus in the literature as to their values for the zero-field transition in the cuprate superconductors.

In this paper we report the results of a variety of complimentary experiments which yield independent determinations of the critical exponents. In section II, we discuss DC transport measurements on both thin films and thick single crystals. We present different ways to infer each exponent from measurements, and show how finite-thickness effects in the films can confuse interpretation of the data in the limit of small currents. We also show that choice of the proper range of current can avoid the finite-thickness effects in films, and show how application of a small magnetic field allows the determination of \( \nu \) in both crystals and films.

In section III, we discuss microwave measurements on thin films. Just as with DC measurements, microwave measurements require a non-zero current density. How the applied microwave current density affects the measured response has not been systematically addressed. Recently Sullivan et al. argued that a finite thickness effects at low current density was the reason for previous inconsistent results in DC measurements. The question of whether a finite-thickness effect influences the AC measurement and the extracted critical exponents, as in DC measurements, inspired us to study the power dependence of the microwave fluctuation conductivity. We find that several length scales play a role in AC conductivity measurements, and only after their effects are properly accounted for can the underlying critical dynamics be understood. As a further check, after completing the microwave measurements, we re-patterned the same samples and performed DC measurements.

When finite-size effects are properly accounted for, we find that all of our results are consistent. As discussed in detail below, we find \( \nu \) is 0.66 ± 0.10 and \( z \) is 1.55 ± 0.15.

II. DC MEASUREMENTS ON SINGLE CRYSTALS AND FILMS

Our crystals are grown by a flux method using \( Y_2O_3 \) crucibles to ensure crystal purity. The films are prepared by the pulsed-laser deposition technique at 850 °C and 150 mbar oxygen pressure on \( SrTiO_3 \) substrates. Transport measurements were carried out using the standard four-probe method. The currents were applied along the ab plane for the films and along the a-axis for the crystals. All connections to the sample are made through double-T low pass filters to reduce noise. We measure the samples inside a cryostat covered with a \( \mu \)-metal shield so that the residual magnetic field is less than \( 2 \times 10^{-7} T \). At 96 K, the resistivity of the crystal is around 70 \( \mu \)-Omega, and the resistivity of the film is around 90 \( \mu \)-Omega cm.

In theory, log-log plots of electric field \( E \) vs. current density \( J \) isotherms above \( T_c \) have positive curvatures and display nonlinearities in the high currents, as shown schematically in Fig. 1(a). As \( T_c \) is approached from above, in the limit of \( J \to 0 \),

\[
\frac{E}{J} \sim \xi^{(D-2-z)}.
\]

Thus, in a log(\( E \)) vs. log(\( J \)) plot, isotherms above \( T_c \) exhibit ohmic behavior - a slope of one - at low currents.

![FIG. 1: Schematic plots of electric field E versus current density J. (a) E-J plot in log-log scale and (b) dlog(E)/dlog(J) vs. J in semi-log scale.](Image)

---

[24] [30]
The isotherms below $T_c$ have negative curvatures and display vanishing linear resistance ($R \to 0$ as $J \to 0$). At $T = T_c$, the critical isotherm is expected to show a power law behavior:

$$E \sim J^{(3+\nu)/2}$$

which is a line with a slope greater than one on a log($E$) vs. log($J$) plot.

In Fig. 2(a) we show selected $E$ vs. $J$ curves in a log-log plot for an untwinned YBCO single crystal. From the figure, all isotherms above the dashed line (triangles) have positive curvatures and have ohmic response in the limit of zero current, and all isotherms below the dashed line (squares) have negative curvatures and display vanishing linear resistivity. We can verify this curvature by fitting the $E-J$ curves to a second-order form of log($E$) = $a_0 + a_1 \log(J) + a_2[\log(J)]^2$, where we use the sign of $a_2$ to indicate the curvature of each isotherm. We find that $a_2$ is positive above 93.838 K and negative below 93.836 K. Thus, from Fig. 2(a) we find $T_c = 93.837 \pm 0.003$ K. The dashed line in Fig. 2(a) separates the superconducting and normal states of the sample and the high-current part can be fit to $E \sim J^{1.22 \pm 0.10}$. From Eq. (4) (using $D = 3$), we find

$$z = 1.44 \pm 0.2.$$  

Another way to evaluate the power law behavior is the derivative plot (of log($E$)) vs. $J$ plot. From Eq. (4), at the transition temperature $T_c$,

$$\left(\frac{\partial \log E}{\partial \log J}\right)_{T_c} = \frac{z + 1}{D - 1}$$

thus, the critical isotherm is a horizontal line parallel to the $J$ axis in a derivative plot. The critical isotherm separates the monotonically increasing isotherms above $T_c$ and the monotonically decreasing isotherms below $T_c$ in the schematic Fig. 1(b). The derivative plot generally displays the phase transition more clearly than a basic plot of $E$ vs. $J$.

In Fig. 2(b), we show a derivative plot of the data shown in Fig. 2(a). The dashed line in Fig. 2(b) separates the normal and superconducting phases. The intercept of the dashed line is $1.25 \pm 0.10$ and is expected to be $(z + 1)/2$ from Eq. (6). From the derivative plot, we find

$$z = 1.50 \pm 0.20,$$  

which is nearly identical to the result obtained from Fig. 2(a), using Eq. (1). Fig. 2(b) also qualitatively shows the change in sign of $a_2$ discussed above.

The exponent $\nu$ can be found from the low-current ohmic behavior $R_L$ above $T_c$ by combining Eqs. (1) and
FIG. 4: (a) $T_c - T_m$ vs. $\mu_0H$ of an untwinned YBCO single crystal. Here, as $T_c - T_m(H) \sim H^{1/2\nu}$, we can find $\nu$ from this line without assuming a value for $z$, and find $\nu = 0.68 \pm 0.10$. The inset is the melting line for the crystal up to 7 T. (b) A similar plot of $T_c - T_m$ vs. $\mu_0H$ for a YBCO thin film ($d \approx 150$ nm). From this curve we find $\nu = 0.63 \pm 0.10$. The inset is the glass transition line for the film up to 6.5 T.

\[ R_L \propto (T/T_c - 1)^{\nu(z-1)}. \]  

(8)

The slope of the log($R_L$) vs. log($T/T_c - 1$) plot in Fig. 3 combined with Eq. (8), determines $\nu$. We find $\nu = 0.71 \pm 0.30$  

(9)

from Fig. 3.

We can apply a perpendicular magnetic field and look at the transition in finite field. According to Ref. [1], the difference between the critical temperature $T_c$ and the melting temperature $T_m(g)(H)$ is

\[ T_c - T_m(g)(H) \sim H^{1/2\nu}, \]  

(10)

where $\nu$ is the zero-field static exponent. Eq. (10) is expected to be true for clean crystals, where the transition is a first order melting (m) transition, as well as for disordered films, where the transition is a glass (g) transition. We show $T_c - T_m(H)$ vs. $\mu_0H$ in Fig. 4(a) and find $\nu = 0.68 \pm 0.10$  

(11)

from a power law fit. This result is consistent with the 3D-XY model, and is also consistent with the result obtained from experiments using Eq. (8).

In Fig. 5(a), we show the $E - J$ curves for a 150 nm thick YBCO c-axis oriented optimally-doped film. The isotherms differ by 0.05 K from 92.075 K to 91.225 K. Unlike Fig. 2(a), we cannot find a single straight line in Fig. 5(a) that separates the isotherms into two groups which are either concave or convex exclusively. To help find the true critical isotherm, we again use the derivative plot [29], where the critical isotherm ideally will correspond to a horizontal straight line, as in Figs. 1(b) and 2(b). However, in Fig. 5(b), there is no horizontal isotherm, and there are isotherms monotonically decreasing above $2 \times 10^7$ A/m$^2$, and also monotonically increas-

FIG. 5: (a) $E - J$ isotherms for a 150 nm YBCO film in zero magnetic field. The spacing between isotherms is 50 mK. (b) The derivative plot of some selected isotherms from (a). At low current density regime, the phase transition is obscured by finite-size effects. The spacing between isotherms is 100 mK.
ing below $2 \times 10^7 \text{ A/m}^2$. So, if the experimental setup were to allow us to measure even smaller voltages, we would expect all of these isotherms would bend down toward 1 (ohmic behavior) in the derivative plot at smaller current densities.

The cause of this behavior is most likely finite-size effects [19, 32]. Below the transition temperature, thermal fluctuations take the form of vortex loops [34]. As discussed in the Appendix, vortex loops with length scales

\[ L_j \sim \sqrt{\frac{k_B T}{2\pi \Phi_0 J}} \tag{12} \]

are probed by current density $J$ [1, 33]. The loops with length scale smaller than $L_j$ will shrink and cause no dissipation. At high current density, such that $L_j$ is less than the thickness of the sample $d$, the vortex loops probed in the experiment are still 3D-like. However, at low current density, such that $L_j > d$, the size of the vortex loops probed will be limited by the thickness of the sample and vortex anti-vortex pairs will be probed. This will lead to a non-diverging energy barrier causing ohmic behavior even below the bulk transition temperature. When $L_j$ is equal to 150 nm, the thickness of the film used to produce the data in Fig. 5, the crossover current density is on the order of $5\times10^6$ A/m$^2$, which is close to where the isotherms bend towards ohmic behavior in Fig. 5(b).

Because of the finite-size effects, the conventional method picks an incorrect critical isotherm and exponent, contributing to the inconsistent results from previous transport experiments on high-$T_c$ films. However, high-current data are not affected by finite-size effects, and we can extract the $T_c$ and $z$ from the high-current regime [19]. If we only look at the high-current regime in Fig. 5(b), it looks very similar to the schematic derivative plot (Fig. 1(b)) and the actual derivative plot of crystal data (Fig. 2(b)). The dashed line in Fig. 5(b), which coincides in the high current regime with the isotherm of 91.825 K, separates the two phases of the film. The transition temperature determined from the high-current regime is $T_c = 91.825 \pm 0.025$ K, and the intercept of the dashed line is $1.27 \pm 0.07$. According to Eq. (6), from the high-current data,

\[ z = 1.54 \pm 0.14 \tag{13} \]

which agrees with the result from the crystal data. In addition, in our other $c$-axis oriented YBCO films with the thickness $d$ ranging from 100 nm to 300 nm, we get consistent values of $z$ ranging from 1.43 to 1.6 [35]. In passing, we note that one should be cautious about the adverse effect of joule heating when the measurement is to be done in the high-current regime to avoid the finite-size effect. In this regard, while the result reported in Ref. [36] is interesting in that it was the first I-V measurements of high-$T_c$ nano-strips, the extracted critical exponents were likely to be inaccurate because of the difficulty of avoiding Joule heating in nano-strips at high currents. In contrast, we have tested heating in our samples by using the low-frequency technique of Koch et al. [37], and we have found that heating does not affect the DC data in samples similar to those measured in this paper at current densities less than $\approx 10^9$ A/m$^2$. [38]

We are not aware of any way to remove finite-size effects that will allow us to use Eq. (8) to determine $\nu$ in films. Instead, we use Eq. (10) which is also applicable to the vortex-glass transition. By applying a magnetic field we introduce a magnetic length scale $l_B \propto \sqrt{\frac{\mu_0 B}{\Phi_0}}$, which is smaller than the film thickness for $B > 0.1$ T, effectively removing the finite-size limitation in the film. We show $T_c - T_g(H)$ vs. $H$ in Fig. 4(b) and find

\[ \nu = 0.63 \pm 0.10. \tag{14} \]

It is important to note that there is more disorder in the film than in the crystal. Besides the point-like oxygen vacancy disorder as in the untwinned crystals, there are other kinds of disorder existing in the film such as twin boundaries and lattice mismatch caused by the substrate. However, the similar values of $z$ and $\nu$ for the untwinned crystal and the film argue that the universality of the phase transition of high-$T_c$ materials is not affected by disorder.

Hence, taking into account results of DC transport measurements on both thin films and thick single crystals, we obtained the critical exponents

\[ z = 1.54 \pm 0.14 \tag{15} \]
\[ \nu = 0.66 \pm 0.10. \tag{16} \]

### III. MICROWAVE MEASUREMENTS ON THIN FILMS

The samples we used for microwave measurements are YBCO films ($d$ = 100 nm to 300 nm thickness) deposited via pulsed laser deposition on NdGaO$_3$ and SrTiO$_3$ substrates. AC susceptibility showed $T_c$ of the films around 90 K with transition widths about $\Delta T_c = 0.2$ K. The resistivity of the films is about 120 $\mu$Ω-cm at 2 K above $T_c$. Using a Corbino reflection technique, we measured the complex resistivity $\rho = \rho_1 + i\rho_2$ of the samples over a wide frequency range. The measured complex resistivity is converted to conductivity and the mean field contribution, as determined from the dc resistivity measured from room temperature down to the lowest temperature in the same experiment, is removed. [18, 39, 41]. The process is similar to the method described in [18] to obtain the fluctuation conductivity $\sigma_{fl}$ [44].

#### A. Frequency Dependent Fluctuation Conductivity and Power Dependence

According to Fisher-Fisher-Huse (FFH), in zero magnetic field when the current density is small the complex AC fluctuation conductivity should scale as

\[ \sigma_{fl}(T, \omega) \propto \xi^{z+2-D}S_\pm(\omega \tau). \tag{17} \]
In Eq. (17), \( \xi \) is the correlation length and \( \tau \) is the fluctuation lifetime. The function \( S_\pm \) is a universal scaling function above (below) \( T_c \), which should be the same for all members of a given universality class. As temperature approaches \( T_c \), both \( \xi \) and \( \tau \) will diverge according to Eqs. (11) and (12).

The scaling functions behave as \( S_+(y) \rightarrow \text{real constant} \) and \( S_-(y) \rightarrow 1/(-iy) \) for \( y \rightarrow 0 \), reflecting the low frequency behavior above and below \( T_c \) respectively. As \( y \rightarrow \infty \), representing \( T \rightarrow T_c \), \( S_+(y) \approx S_-(y) \approx c\hat{y}^{(D-2)(\pm 1)} \) where \( \hat{c} \) is a complex constant and \( D \) is the dimensionality of the system. [1,22]

The complex fluctuation conductivity can be written as \( \sigma_{fl} = |\sigma_{fl}|e^{i\phi} \), so both the magnitude and phase are predicted to scale

\[
|\sigma_{fl}| \approx \xi^{z+2-D}|S_\pm(\omega \xi^z)|, \tag{18}
\]

\[
\phi_\sigma = \Phi_{\pm}(\omega \xi^z) \tag{19}
\]

where \( \Phi_{\pm} \) is the phase of the scaling function \( S_\pm \). At \( T_c \), one expects [22]

\[
|\sigma_{fl}| \sim \omega^{-(z+2-D)/z}, \tag{20}
\]

and

\[
\phi_\sigma = \frac{\pi}{2}(z+2-D)/z. \tag{21}
\]

Fig. 5 sketches the expected Fisher-Fisher-Huse AC scaling behavior of the magnitude and phase of fluctuation conductivity near \( T_c \). [1,22]

Fig. 6 shows the measured complex fluctuation conductivity vs. frequency for various temperatures at two different microwave powers. These data display significant and systematic deviations from the expected FFH scaling sketched in Fig. 5. At high frequency, both the magnitude and phase of the fluctuation conductivity look similar to FFH theory. However, as frequency decreases, the measured magnitude of the fluctuation conductivity below \( T_c \) saturates, instead of bending up. All of the phase isotherms below \( T_c \) tend toward zero, indicating ohmic response, instead of approaching \( \pi/2 \) at low frequency. These deviations are qualitatively similar to the low current-density deviations of \( E \) vs. \( J \) in DC measurements seen in Fig. 5 [19].

Fig. 7 also shows that the applied microwave power affects the measured fluctuation conductivity, particularly at low frequencies. As frequency decreases, the higher applied microwave power decreases the magnitude of the fluctuation conductivity and depresses the phase. These phenomena cannot be explained by the AC scaling equation, Eq. (17), and we need to look at the full version of the FFH dynamic scaling function, which can be written in the following form with assumed dimensionality \( D = 3 \) [11]:

\[
\frac{E}{J} = \xi^{1-z} \chi_{\pm}(J \xi^2, \omega \xi^z, H \xi^2, \ldots). \tag{22}
\]

where \( E \) is the electric field.

Since the critical point is located in the limit of zero magnetic field \( H \), current density \( J \) and frequency \( \omega \), increased applied current should drive the system further away from the transition and thus into the ohmic regime. In our measurement, the magnetic field term \( H \xi^2 \) can be ignored. The two remaining terms are \( J \xi^2 \) and \( \omega \xi^z \). Qualitatively, at low frequency, \( \omega \xi^z \) is small so that the applied power term, \( J \xi^2 \), has more effect on the fluctuation conductivity.

To illustrate the effect of different powers, \( |\sigma_{fl}| \) vs. microwave power at different frequencies is plotted in Fig. 8. The power dependence of \( |\sigma_{fl}| \) clearly varies with frequency. At low frequencies (60 MHz, 80 MHz and 100 MHz), \( |\sigma_{fl}| \) vs. incident power increases first as power increases (Region I) and then saturates (Region II). At very high power, \( |\sigma_{fl}| \) decreases again (Region III). At high frequencies (> 0.5 GHz) the fluctuation conductivity is almost power independent.

The important features in Fig. 8 are that large applied power affects the fluctuation conductivity, and that even small power depresses the fluctuation conductivity at low frequency. While the high-frequency and high-power data in Fig. 8 are consistent with Eq. (22), and
FIG. 7: (Color online) (a) Magnitude $|\sigma_{fl}|$ and (b) phase $\phi_{r}$ vs. frequency at various temperatures for a typical YBCO film (xuh139). The black lines were measured with -22dBm power while the red lines were measured with -2dBm at the same temperature. (For clarity, only every other isotherm is shown.)

FIG. 8: (Color online) $|\sigma_{fl}|$ vs. incident microwave power at different frequencies. (T=89.140 K, sample xuh139 below $T_c$)

Thus can be explained by FFH scaling theory[1, 22, 23], the low-power low-frequency behavior is not consistent.

The similarity between this low power and low frequency deviation and the low current density deviation in DC conductivity measurement suggests the presence of a "probed length scale" for a finite frequency. As discussed in connection with Eq. (12) and in the appendix, when a current with density $J$ is applied, vortex loops (with large $r$) will "blow out" to infinite size (producing dissipation). Vortex loops with small $r$ shrink and annihilate (with no dissipation). A current density induced length scale $L_J$, given in Eq. (12) separates vortex loops into two categories, depending on their ultimate fate.

The shrinking of a loop takes time. This time depends on the size of the loop, thus relating the size of a vortex loop to a time scale. In AC measurements, small frequency means that large length scales are probed and vice versa. By generalizing the order parameter relaxation time scale in time-dependent Ginzburg Landau theory[40] one can construct a frequency-dependent length scale

$$L_{\omega} = \left(\frac{ck_BT_c}{h\omega}\right)^{1/z}\xi(0),$$  \hspace{1cm} (23)

where $c$ is a constant of order 1 and $\xi(0)/\xi(T) = |T/T_c - 1|^\nu$.

In AC conductivity measurements, the probed length scale should be determined by both frequency and current density. Since the smaller length scale dominates the measured fluctuation conductivity, we propose a plausible expression for the probed length scale for AC measurement $L_{AC}$,

$$\frac{1}{L_{AC}} = \frac{1}{L_J} + \frac{1}{L_{\omega}}.$$  \hspace{1cm} (24)

This formula has the correct limits as $J \rightarrow 0$ or $\omega \rightarrow 0$, which corresponds to frequency dependent $\omega \xi^2$ scaling or current density dependent $J \xi^2$ scaling, respectively. It is also qualitatively consistent with the two-term FFH scaling without the magnetic term $H \xi^2$ of Eq. (22) in the crossover range. Finite size effects come into play when $L_{AC}$ approaches the thickness of the film.

Fig. 9 summarizes the length scales in an AC measurement in terms of experimental quantities. In this figure, we use $\xi(0) = 5\AA$, $c = 1$ and $z = 1.5$. The dotted line in the figure gives the boundary $L_J = L_{\omega}$. To the right and below the dotted line, when $L_{\omega} \ll L_J$, the frequency induced length scale dominates, and one observes mainly frequency dependent scaling of the fluctuation conductivity. Above the dotted line, when $L_{\omega} \gg L_J$, current-induced nonlinear effects will dominate the behavior. This explains the features shown in Fig. 7 and Fig. 8, where the current density has less effect on the fluctuation conductivity at high frequency and a larger effect at low frequency.

At low frequency and small current density, $L_{AC}$ may approach the thickness of the sample ($d$) or some other length scale that interrupts the fluctuation vortex loops.
Hence deviations from the simple scaling theory are expected when $L_{AC} > d$.

In our AC measurements, we want to keep to the limit $L_{AC} < d$ to avoid finite-thickness effect. Hence we choose to stay at low $J$ but high $\omega$. In this region we can find the true critical behavior without getting into any finite-size effect or crossover difficulties. Our previous analysis strayed out of this region and this may account for the larger values of $z$ reported before [18] and elsewhere in the literature.

**B. Improved Data Analysis Method**

In this paper, with very small applied microwave power, -46dBm (corresponding to $J < 2.2 \times 10^5$ A/m²), and high frequency data, we investigated the frequency dependent fluctuation conductivity around $T_c$. Conventionally, examining experimental data with the scaling formulas one can search for the temperature at which the conductivity magnitude best fits to a power law and has a constant value of $\phi_\sigma$, to determine $T_c$ and the dynamic critical exponent $z$. In this analysis process, the determination of $T_c$ is crucial because it directly affects the value of $z$. Hence we improved the temperature stability and conductivity calibration techniques in the experiment, enabling the measurement of high quality data at small temperature intervals (50 mK).

Using this data, we improved the conventional data analysis method [18] to determine $T_c$. One expects a power-law behavior of $|\sigma_{fl}|$ on frequency at $T_c$, with a change of curvature on either side (a convex function below $T_c$ and a concave function above $T_c$), as sketched in Fig. 8(a). One also expects a plateau in the conductivity phase vs. frequency at $T_c$, with a change in the sign of the slope on either side, as sketched Fig. 8(b).

Unlike the DC I-V curve where Strachan et al. used an opposite concavity criterion to determine $T_c$ in a $dI/dV$ plot, [29] it is hard to take the frequency derivative of $|\sigma_{fl}(\omega)|$ because of noise. An alternative approach is to do a quadratic fit to the data on a log-log plot. Below $T_c$, the curve bends up with a positive coefficient of $[log(\omega)]^2$ and above $T_c$, the curve bends down with a negative coefficient of $[log(\omega)]^2$. Hence we did a quadratic fit and found that the coefficient of the $[log(\omega)]^2$ term changes sign between temperatures 89.192K and 89.245K, bracketing $T_c$.

The scaling theory also predicts a constant phase angle $\phi_\sigma(\omega)$ at $T_c$. $\phi_\sigma(\omega)$ vs. $log \omega$ is known to be a decreasing function below $T_c$ and an increasing function above $T_c$. A linear fit of $\phi_\sigma(\omega)$ vs. $log \omega$ also has been done and the result shows its has negative slope at 89.192K and positive slope at 89.245K, which is consistent with the quadratic fit result of $log[|\sigma_{fl}(\omega)|]$ vs. $log \omega$. The next step is to do a linear fit for $log[|\sigma_{fl}(\omega)|]$ versus $log(\omega)$ to get the slope of $log[|\sigma_{fl}(\omega)|]$, and take the average of the $\phi_\sigma(\omega)$ at $T_c$ to obtain the value of $z$. From this method, we get the critical temperature $T_c = 89.22 \pm 0.05K$ and the critical exponent $z = 1.62 \pm 0.20$.

In addition, we developed a new method to determine $T_c$ from the data. Consider the Wickham and Dorsey scaling function above $T_c$ [22]

$$S_+ (y) = \frac{2z^2[1 - \frac{D-2}{2} + \frac{iy - (1 - iy)(D-2+2z)/2}{y^2(D-2-z)(D-2)}]}$$

where $y = \omega \tau \propto \omega \xi^z$. We find at small $\omega$, corresponding to temperatures far above $T_c$, the function $S_+(y)$ is essentially independent of dimensionality $D$ and $z$ because the fluctuation contribution is small. According to Eq. (17), one can write $\sigma_{fl}(T, \omega) = \sigma_0(T)S(\omega/\omega_0)$ where $\sigma_0(T)$ and $\omega_0(T)$ are characteristic conductivity and frequency scales, respectively. Both the phase $\phi_\sigma(\equiv tan^{-1} [\sigma_{fl}^H/\sigma_{fl}^I])$ of $\sigma_{fl}$ and the magnitude $|\sigma_{fl}|/\sigma_0$ can be treated as scaled quantities with two temperature-dependent scaling parameters $\omega_0(T)$ and $\sigma_0(T)$. This is a new data collapse method, pioneered by Kitano et al. [20]. They pointed out that the advantage of this new collapse method is the independence of the two scaling parameters $\omega_0(T)$ and $\sigma_0(T)$. In this new data analysis method, the parameters $\omega_0(T)$ and $\sigma_0(T)$ are chosen at each temperature to collapse $\phi_\sigma(T)$ vs. $\omega/\omega_0$ and $|\sigma_{fl}|/\sigma_0(T)$ vs. $\omega/\omega_0$ to smooth and continuous curves, without a priori determination of $T_c$ or critical exponents.

First $\omega_0(T)$ is determined through a collapse plot of $\phi_\sigma$ vs. $\omega/\omega_0(T)$ from high temperature to low temperature (see Fig. 10(a)). Using the feature that $S_+(y)$ is not sensitive to dimensionality $D$ and $z$ far above $T_c$, the appropriate $\omega_0(T)$ for isotherms far above $T_c$ is chosen to make the measured $\phi_\sigma(\omega/\omega_0(T))$ overlap with the theoretical prediction from the known scaling function $\phi(S_+(y))$. Then at temperatures closer to $T_c$ where $S_+(y)$ starts to depend on $D$ and $z$, $\omega_0(T)$ for each temperature is chosen to connect smoothly to the existing curve of $\phi_\sigma(\omega/\omega_0(T))$ and to make all the temperature
From these two figures, Fig. 11(b) also shows that the blue line is straightest. These lines can be used to determine \( \omega_0(T) \) and \( \sigma_0(T) \) far above \( T_c \). In this figure, only the \( \omega_0(T) \) and \( \sigma_0(T) \) of the isotherm \( T = 89.763 \) K are shown to make the measured \( \phi_s(\omega/\omega_0(T)) \) and \( |\sigma_{fl}|/\sigma_0(T) \) vs. \( \omega/\omega_0(T) \) overlap with the theoretical prediction. For all the other isotherms, \( \omega_0(T) \) and \( \sigma_0(T) \) for each temperature are chosen to connect smoothly to the existing curve of \( \phi_s(\omega/\omega_0(T)) \) and \( |\sigma_{fl}|/\sigma_0(T) \) respectively, to make all the temperature curves collapse into one smooth and continuous curve.

curves collapse into one smooth and continuous curve. This process continues to lower and lower temperature until a temperature is reached where \( \phi_s(\omega/\omega_0(T)) \) can not be connected smoothly to the existing curve. In this way, \( \omega_0(T) \) for temperature points above \( T_c \) can be determined.

To scale the conductivity magnitude, we start with the determined \( \omega_0(T) \) for each temperature, then plot \( |\sigma_{fl}|/\sigma_0(T) \) vs. \( \omega/\omega_0(T) \), where \( \sigma_0(T) \), similarly to \( \omega_0(T) \), is determined for each temperature to make a smooth and continuous curve of \( |\sigma_{fl}|/\sigma_0(T) \) vs. \( \omega/\omega_0(T) \).(see Fig. 11(b))

Using the power-law assumption for \( \omega_0(T) \) and \( \sigma_0(T) \), \( T_c \) can be determined. Fig. 11 shows \( \omega_0(T) \) vs. \( t \) and \( \sigma_0(T) \) vs. \( t \) for different assumed values of \( T_c \). The correct \( T_c \) can be determined from the line showing a pure power-law. Fig. 11(a) shows that the blue line which corresponds to an assumed \( T_c = 89.25K \) is straightest. Fig. 11(b) also shows that the blue line is straightest. From these two figures, \( T_c \) is consistently determined to be \( T_c = 89.25 \pm 0.05K \). This result is also consistent with the \( T_c \) determined by the improved conventional method.

With the value of \( T_c \) determined here, we can do a linear fit for \( \log|\sigma_{fl}(\omega)| \) verses \( \log(\omega) \) to get the slope of \( \log|\sigma_{fl}(\omega)| \), and take the average of the \( \phi_s(\omega) \) at \( T_c \) to obtain the value of \( z \). Through this procedure, we obtained the critical exponent \( z = 1.55 \pm 0.20 \).

In the procedure outlined above, we take advantage of the broad microwave frequency range of the experiment, which includes frequencies of order 1/\( \tau \). High quality data at small temperature intervals are essential for the implementation of this method. Another advantage of this method is that many isotherms near \( T_c \) contribute to defining the scaling curve, not just the one closest to \( T_c \). The new method has the advantage of more precisely determining \( T_c \). So according to the two methods the critical temperature and exponent for sample xuh139 were determined to be \( T_c = 89.25 \pm 0.05K, z = 1.55 \pm 0.20 \).

The dynamic critical exponent should be sample independent. To check the results, we not only repeated measurements on the same sample, but also repeated the experiment on different samples. Films of different thickness (\( d = 100 \) nm to 300 nm) were examined, and \( z \) was found to be independent of the thickness, keeping in mind the constraints of Fig. 11. Experiments on 6 samples have been done giving

\[
z = 1.55 \pm 0.15.
\]

C. AC and DC Experiments on the Same Sample

We also performed DC current-voltage characteristic measurements on the same samples. Typical results are shown in Fig. 12 with no background subtraction. According to the negative curvature criterion, we determined the critical temperature to be 91.220±0.04 K and the critical exponent \( z = 1.75 \pm 0.2 \) from the derivative plot in Fig. 12(b). In Fig. 12 all the isotherms tend towards ohmic behavior at low current density, brought about by \( L_j > d \) finite-size effects, as discussed for the data shown previously in Fig. 6. From this data it is clear that when the current density is smaller than \( 1 \times 10^6 A/m^2 \), the sample will have only ohmic response around \( T_c \). The -46dBm applied power in the AC measurement corresponds to a maximum current density of \( 2.2 \times 10^8 A/m^2 < 1 \times 10^6 A/m^2 \). This means that for -46dBm incident power \( L_j > d \), verifying a feature of Fig. 4 and suggesting that one-parameter scaling should work when \( L_\omega < L_j, d \). Hence it is appropriate...
to determine $T_c$ and critical exponents with AC data at -46dBm applied power.

The difference of $T_c$ between DC and AC measurements is due to the different thermometer positions and temperature control techniques of the two experimental systems. The resistance vs. temperature plots from the AC and DC experiment have a temperature offset about 2.0 K, which is the difference of the determined $T_c$ from these two methods.

In DC measurements, disorder and heating lead one to systematically choose a lower temperature isotherm as $T_c$, resulting in an enhanced value of $z$.\[31] We find that films with lower normal-state resistivity and smaller $\Delta T_c$ have smaller values of $z$.\[30] The films used in AC conductivity measurements were grown on NdGaO$_3$ substrates, and these films have systematically higher resistivity and larger $\Delta T_c$ than films on SrTiO$_3$ substrates. In addition, performing DC measurements on the same film after AC measurements involves more processing steps than a DC measurement alone, and may result in additional disorder in the sample. This correspondingly gives larger values of $z$ (Fig. 12). We carefully repeated the DC measurements alone on different YBCO films grown on different substrates (SrTiO$_3$ and NdGaO$_3$) and found that the sample quality does affect the obtained value of $z$.\[41] However, for films with high $T_c$, sharp transition and small resistivity, the obtained value of $z \approx 1.50$, which is consistent with the AC result $z = 1.55 \pm 0.15$. In addition, DC measurements carried out the same way on high-quality crystals shown in the previous section also gave $z \approx 1.5$.

IV. SUMMARY AND CONCLUSIONS

In this paper, we performed a variety of complimentary experiments to determine the critical exponents in optimally-doped YBa$_2$Cu$_3$O$_{7-\delta}$.

The DC transport measurements on both thin films and thick single crystals show how finite-thickness effects in the films can confuse interpretation of the data in the limit of small currents. Only with the choice of the proper range of current, can one avoid the finite-thickness effects in films and obtain the correct exponent $z$, consistent with the value obtained from thick single crystals measurements (where finite-thickness effects are unimportant). We also show how application of a small magnetic field allows the determination of $\nu$ in both crystals and films. Using DC transport transport measurements, we find that the dynamical critical exponent $z = 1.54 \pm 0.14$, and the static critical exponent, $\nu = 0.66 \pm 0.10$ for both films and single crystals.

Microwave measurements on thin films at different frequencies and at different powers have also been performed, which allow us to systematically probe different length scales in the sample. After developing a comprehensive understanding of length scales in microwave measurements, we choose to stay at low $J$ but high $\omega$ to find the true critical behavior without getting into any finite-size or crossover effects. DC transport measurements were also performed on the films used in the microwave experiments to provide a further consistency check. These microwave and DC measurements yielded a value of $z$ consistent with the other results, $z = 1.55 \pm 0.15$.

To conclude, using two different measurement methods, we studied the dynamic fluctuation effects of YBa$_2$Cu$_3$O$_{7-\delta}$ single crystals and thin films around $T_c$. The results of both AC and DC measurements agree with the XY value for $\nu \approx 0.67$ and with model-E dynamics value for $z = 1.55 \pm 0.15$.\[5] The neglect of finite-thickness, finite-power, and finite-frequency effects may account for the wide ranges of values for $\nu$ and $z$ previously reported in the literature.

Acknowledgments

The authors thank A. T. Dorsey for insightful discussion. This work has been supported by NSF grant number DMR-0302596 and by the Maryland Center for...
Appendix: Currents and Length Scales in Superconductors

In this appendix, we consider a number of length scales in current-carrying superconductors to provide a clearer physical meaning for the length scale $L_J$ of Eq. (12). We first consider a simple model for fluctuations in superconductors, where we assume that the only fluctuations are circular vortex loops (or vortex “smoke rings”) of radius $r$. The energy of such a loop can be written as

$$U_{\text{loop}} = 2\pi \varepsilon(r)$$  \hspace{1cm} (27)

where $\varepsilon(r)$ is the energy per unit length of the vortex loop. For a straight vortex,

$$\varepsilon(r = \infty) = \frac{1}{4\pi \mu_0} \left(\frac{\Phi_0}{\lambda}\right)^2 K_0 \left(\frac{\lambda}{\xi}\right) \approx \frac{1}{4\pi \mu_0} \left(\frac{\Phi_0}{\lambda}\right)^2 \ln \left(\frac{\lambda}{\xi}\right)$$ \hspace{1cm} (28)

where $K_0$ is a modified Bessel function of the second kind and the approximate form holds in the limit of high $\kappa \equiv \lambda/\xi$. As a first approximation, we will assume the energy per unit length is constant, given by Eq. (28).

In an infinite superconductor with no applied current, vortex loops of different sizes occur with different probabilities as thermal fluctuations. The probability of finding a loop of size $r$ in a range $dr$ is given by

$$P(r) dr = \frac{e^{-\frac{2\pi \varepsilon(r)}{k_B T} r}}{\int_\xi^\infty e^{-\frac{2\pi \varepsilon(r)}{k_B T} r} dr}$$ \hspace{1cm} (29)

where interactions between the loops are neglected for simplicity.

We wish to find the size of a typical vortex loop, $r_{\text{med}}$. One way to do this is to find the fraction of loops $f$ with a radius greater than the median radius, or $r > r_{\text{med}}$. This fraction will be $f = \frac{1}{2}$, given by,

$$f = \frac{\int_{r_{\text{med}}}^\infty e^{-\frac{2\pi \varepsilon(r)}{k_B T} r} dr}{\int_\xi^\infty e^{-\frac{2\pi \varepsilon(r)}{k_B T} r} dr} \equiv \frac{1}{2}.$$ \hspace{1cm} (30)

If the energy per unit length of the loop is given by Eq. (28), then Eq. (30) leads to

$$r_{\text{med}} = \xi + \frac{k_B T}{2\pi \varepsilon} \ln 2.$$ \hspace{1cm} (31)

If the second term on the right hand side of Eq. (31) dominates, this gives

$$r_{\text{med}} \approx \frac{k_B T}{2\pi \varepsilon} \ln 2 \Rightarrow \varepsilon \approx \frac{k_B T}{2\pi r_{\text{med}}} \ln 2.$$ \hspace{1cm} (32)

Eq. (32) states that, within a factor of $\ln 2$, the total energy of a vortex loop of size $r_{\text{med}}$ is equal to $k_B T$, which is a plausible result.
To check whether the second term on the right side of Eq. (31) is the dominant one, we combine Eqs. (28) and (31). This leads to

$$r_{med} = \xi \left[ 1 + \frac{\xi}{\left( \frac{2\pi}{k_B T} \right)} \ln \frac{2 \xi^2}{\kappa^2} \right] = \xi \left[ 1 + \frac{\xi}{\Lambda T} \ln \frac{2 \xi^2}{\kappa^2} \right]$$

(33)

where $\Lambda T$ is defined in Eq. (1.1) of Fisher, Fisher, and Huse [1] (in cgs units with the Boltzmann constant $k_B$ defined to be 1). The second terms in Eqs. (31) and (33) dominate in the critical regime because $\xi$ diverges while $\Lambda T$ is fixed.

For simplicity, we drop the $\ln 2$ in Eq. (31), and use the physically plausible result

$$r_{med} = \frac{k_B T}{2\pi \varepsilon}$$

(34)

Next consider that a current per unit area $J$ is applied in a direction perpendicular to the plane of the loop. The total Lorentz force acting outward on the loop is

$$F_{ext} = 2\pi \varepsilon J \Phi_0.$$  

The energy defined in Eq. (27) gives rise to an inward force that the loop exerts on itself, $-2\pi \varepsilon$. Summing the forces and finding the point where the net force is equal to zero leads to a critical loop size

$$r_o = \frac{\varepsilon}{\Phi_0 J},$$

(36)

where, for simplicity, $\varepsilon(r)$ is again assumed to be independent of $r$. Note that Eq. (36) is not the equation for the current-dependent length scale $L_J$.

Physically, if a vortex loop has $r > r_o$, the external current “blows out” the loop to infinite size; this process leads to dissipation. If $r < r_o$, the vortex loop shrinks and annihilates.

One can interpret Eq. (36) in a different, but equivalent, way. The presence of a current density $J$ significantly alters the population of vortex loops with $r > r_o$, and has less effect on the vortex loops with $r < r_o$. In this sense, a current $J$ probes the physics on length scales of order $r_o$ and larger. This is the type of language that is sometimes used to describe $L_J$.

We next discuss the physical significance of comparing the lengths $r_o$ and $r_{med}$. Eqs. (34) and (36). If $r_{med} \ll r_o$, the current is probing a length scale where there are very few vortex loops. The current thus acts as a very small perturbation on the system. If $r_{med} \gg r_o$, the current is probing a very short length scale, and a large portion of the intrinsic vortex population is being disrupted by the current. The point where $r_{med} = r_o$ thus marks a crossover in the behavior from current acting as a small perturbation to current acting as a large perturbation.

How does a non-infinite film of thickness $d$ affect the physics? It is plausible to say $r_{med} \ll d$ is the three-dimensional limit, while $r_{med} \gg d$ is the two-dimensional limit, since in the second case most of the vortex loops are interrupted by the film thickness, while in the first case they are not. This is true as far as it goes, but misses the key point that an applied current probes physics at the scale of $r_o$ and larger. Thus, even in the limit $r_{med} \gg d$, if $r_o$ is small enough, current will probe physics on length scales smaller than $d$, and thus the measurement will not be affected by the finite thickness of the film.

In order for the thickness of the film to have a measurable effect, the current should probe a significant fraction of the loop population and should also probe lengths on the scale of the film thickness. For this to be true, we require

$$r_o = r_{med} \equiv L_J$$

(37)

Combining Eqs. (34), (36), and (37) gives

$$L_J = \left( \frac{k_B T}{2\pi \Phi_0 J} \right)^{\frac{1}{2}}.$$  

(38)

This argument leading to Eq. (38) motivates a physical description for $L_J$: For any $J$ there is a length scale $L_J$, given by Eq. (38), such that roughly half the equilibrium (zero current) vortex loop population is strongly affected by $J$. This is the length that one should compare to the film thickness for seeing whether or not measurements are in the two or three dimensional limit. The requirements are that there be a significant fraction of the loops that would exceed the film thickness, and, in addition, that the current is probing the same length scale.

[1] D. S. Fisher, M. P. A. Fisher, and D. A. Huse, Phys. Rev. B 43, 130 (1991); Nature 358, 553 (1992).
[2] C. J. Lobb, Phys. Rev. B 36, 3930 (1987).
[3] P. C. Hohenberg and B. I. Halperin, Rev. Mod. Phys. 49, 435 (1977).
[4] F. S. Nogueira and D. Manske, Phys. Rev. B 72, 014541 (2005).
[5] J. Lidmar, M. Wallin, C. Wengel, S.M. Girvin, and A.P. Young, Phys. Rev. B 58, 2827 (1998).
[6] H. Weber and H. J. Jensen, Phys. Rev. Lett. 78, 2620 (1997).
[7] S. Kamal, D.A. Bonn, N. Goldenfeld, P.J. Hirschfeld, R. Liang, and W.N. Hardy, Phys. Rev. Lett. 73, 1845 (1994).
[8] S.M. Anlage, J. Mao, J.C. Booth, D.H. Wu, and J.L. Peng, Phys. Rev. B 53, 2792 (1996).
[9] M. B. Salamon, S. E. Inderhees, J. P. Rice, B. G. Pazol, D. M. Ginsberg and N. Goldenfeld, Phys. Rev. B 38, 885 (1988); M.B. Salamon, J. Shi, N. Overend, and M.A. Howson, Phys. Rev. B 47, 5520 (1993); M.B. Salamon, W. Lee, K. Ghiron, J. Shi, N. Overend, and M.A. Howson, Physica A 200, 365 (1993).
[10] A. Pomar, A. Diaz, M.V. Ramallo, C. Torron, and J.A.
Veira, *Physica C* **218**, 257 (1993).

[11] R. Liang, D.A. Bonn, and W.N. Hardy, *Phys. Rev. Lett.* **76**, 835 (1996).

[12] N. Overend, M.A. Howson, and I.D. Lawrie, *Phys. Rev. Lett.* **72**, 3238 (1994).

[13] V. Pasler, P. Schweiss, C. Meingast, B. Obst, H. Wuhl, A.I. Rykov, and S. Tajima, *Phys. Rev. Lett.* **81**, 1094 (1998).

[14] S. E. Inderhees, *et al.*, *Phys. Rev. Lett.* **60**, 1178, (1988); S. E. Inderhees *et al.* *Phys. Rev. Lett.* **66**, 232 (1991).

[15] N. C. Yeh, W. Jiang, D. S. Reed, and U. Kriplani, *Phys. Rev. B* **47**, 6146 (1993).

[16] J. M. Roberts, Brandon Brown, B. A. Hermann, and J. Tate, *Phys. Rev. B* **49**, 6890 (1994).

[17] T. Nojima, T. Ishida, and Y. Kuwasawa, Czech. J. Phys. **46**, Suppl. S3, 1713 (1996).

[18] J. C. Booth, D. Wu, S. B. Qadri, E. F. Skelton, M. S. Ososky, A. Pique, and S. M. Anlage, *Phys. Rev. Lett.* **77**, 4438 (1996).

[19] M. C. Sullivan, D. R. Strachan, T. Frederiksen, R. A. Ott, M. Lilly, and C. J. Lobb, *Phys. Rev. B* **69**, 214524 (2004).

[20] K. Moloni, M. Friesen, S. Li, V. Souw, P. Metcalf, L. Hou, and M. McElfresh, *Phys. Rev. Lett.* **78**, 3173 (1997).

[21] P. Voss-de Haan, G. Jakob and H. Adrian, *Phys. Rev. B* **60**, 12443 (1999).

[22] R. A. Wickham and A. T. Dorsey, *Phys. Rev. B* **61**, 6945 (2000).

[23] D. N. Peligrad, M. Mehring and A. Dulčić, *Phys. Rev. B* **69**, 144516 (2004).

[24] G. Nakielski, D. Görlich, C. Stodte, M. Welters A. Krämer, and J. Kötzler, *Phys. Rev. B* **55**, 6077 (1997).

[25] K. D. Osborn, D. J. Harlingen, V. Aji, N. Goldenfeld, S. Oh, and J. N. Eckstein, *Phys. Rev. B* **68**, 144516 (2003).

[26] H. Kitano, T. Ohashi, A. Maeda, and I. Tsukada, *Phys. Rev. B* **73**, 092504 (2006).

[27] Kouji Segawa and Yoichi Ando, *Phys. Rev. Lett.* **86**, 4907 (2001).

[28] Kouji Segawa and Yoichi Ando, *Phys. Rev. B* **69** 104521 (2004).

[29] R. D. Strachan, M. C. Sullivan, P. Fournier, S. P. Pai, T. Venkatesan, and C. J. Lobb, *Phys. Rev. Lett.* **87**, 067007 (2001).

[30] S. Li, Ph. D. thesis, University of Maryland, 2007 (http://hdl.handle.net/1903/7276).

[31] M. C. Sullivan, T. Frederiksen, J. M. Repaci, D. R. Strachan, R. A. Ott, and C. J. Lobb, *Phys. Rev. B* **70**, 140503 (2004).

[32] The isotherms 93.838 K and 93.836 K are not shown in Fig. 2(a) for clarity.

[33] P. J. M. Wöltgens, C. Dekker, R. H. Koch, B. W. Husse, and A. Gupta, *Phys. Rev. B* **52**, 4536 (1995).

[34] A. K. Nguyen and A. Sudbo, *Phys. Rev. B* **60**, 15307 (1999), and references cited there in.

[35] Our previous work [12] reported $z \approx 2$. This included a normal-state background subtraction (following W. J. Skocpol and M. Tinkham, Rep. Prog. Phys. **38** 1049, (1975)). This subtraction is not valid in the critical regime, at or near $T_c$. Our previous work, when reanalyzed, yields $z \approx 1.5$.

[36] Y. Ando, H. Kubota and S. Tanaka, *Phys. Rev. Lett.* **69**, 2851 (1992).

[37] R. H. Koch, V. Foglietti, W. J. Gallagher, G. Koren, A. Gupta, and M. P. A. Fisher, *Phys. Rev. Lett.* **63**, 1511 (1989).

[38] D. R. Strachan, Ph.D. thesis, University of Maryland, 2002, Ch.2; M. C. Sullivan, Ph.D. thesis, University of Maryland, 2004, Ch. 5.

[39] J. C. Booth, D. H. Wu and S. M. Anlage, *Rev. Sci. Instr.* **65**, 2082 (1994).

[40] M. Tinkham, *Introduction to Superconductivity*, McGraw-Hill Book Co. New York, 1975.

[41] H. Xu, Ph. D. thesis, University of Maryland, 2007 (http://hdl.handle.net/1903/7587).

[42] M. C. Sullivan, D. R. Strachan, T. Frederiksen, R. A. Ott, and C. J. Lobb, *Phys. Rev. B* **72**, 092507 (2005).

[43] The sample presents a short circuit termination to the transmission line, to good approximation, resulting in a relation between applied microwave power and current density in the Corbino disk, $J(r) = \sqrt{2 P / Z_0 \pi r t}$, where $Z_0$ is the characteristic impedance of the coaxial cable, $t_o$ is the thickness of the measured sample, and $J(r)$ is the current density amplitude at the distance $r$ from the center of the Corbino disk. The maximum current density $J_{max}$ is at the inner radius of the Corbino disk.

[44] Because the AC mean-field conductivity remains finite and well-behaved through $T_c$, and because the critical regime is wide, a mean field subtraction can be performed with little effect on the determined critical behavior.

[45] T. P. Orlando and K. A. Delin, *Foundations of Applied Superconductivity*, Addison-Wesley, Reading, Massachusetts, 1991, p. 281.