The Inspection of Power Quality Disturbances by Using Improved Extreme-value Lifting Morphological Wavelet Method

ZHANG Huaying, HU Ziheng, LI Yan and AI Jingwen
Shenzhen Power Supply Co.Ltd. Institute of Electric Power Science, Shenzhen 518020, Guangdong Province China
E-mail: 754797930@qq.com Tel: 15920443231

Abstract. For solving the difficulty of extracting the disturbing characteristics of the signals in power quality processing, and restraining the noise in the process of sampling, an improved extreme-value lifting morphological wavelet method is constructed. Firstly, morphological extreme-value operator is chosen to be the prediction operator and update operator, then the max-lifting and min-lifting morphological wavelets are constructed. Secondly, the improved extreme-value lifting morphological wavelet method is designed to further highlight local maxima and minima information of the disturbing signals based on the first step. Finally, the signals are dealt with this method, and the detail coefficients can be obtained, which have kept the disturbing characteristics of the signals. The result of simulation and analysis shows that this algorism owns the trait of simple, exactness, and de-noising.

Keywords. Power quality; Lifting scheme; Morphological wavelet; Signal processing

1. Introduction
Nowadays, the increasingly high requirements of power quality are requested in both power supply-side and consuming side, and more attention is paid to disturbances like voltage dip and voltage rise. Functions including accurately alarming, operation and protection in smart appliance are based on correct inspection of disturbance signal. How to accurately inspect the disturbance signal of the power system has become an urgent problem in the research of power quality.

The existing power quality disturbance inspection methods mainly include short-time Fourier transform, wavelet transform, and mathematical morphology methods. The analysis of short-time Fourier transform [1] is related to the selection of short-window function, with the only single resolution, which limits its application in power quality disturbance inspection; Wavelet transform [2-3] has a good performance to process mutated signals, but its analysis results are closely related to the selection of source wavelets, with a large amount of calculation; As a nonlinear time domain-based analysis method, mathematical morphology [4-5] has the characteristics of fast calculation speed and good denoising performance. However, the structural element size is often determined by experience in the signal processing process, which reduces the applicable performance of algorithm; Morphological wavelets is a research direction of nonlinear expansion of wavelet theory, both morphological characteristics of mathematical morphology and the multi-resolution characteristics of wavelets are taken into consideration, therefore, leads to a good detail retention and anti-noise performance, as well as smaller computational complexity, also applied well in image processing [6-
However, the traditional morphological wavelet transform algorithm has poor anti-interference performance and cannot highlight signal disturbance characteristics; The morphological wavelet algorithm proposed by the lifting lattice transform as the lifting lattice has the advantages of signal local extremum preservation and fast calculation as well as outstanding anti-interference performance. It has a good effect in the extraction of fault feature [10-11]. However, during the processing of the disturbance signal, the extracted disturbance characteristics are incomplete and not obvious enough.

Therefore, in order to overcome the shortcomings of the traditional morphological wavelet and the improved lattice morphological wavelet algorithm, the maximum and minimum Lifting morphological wavelet algorithms are established first in this paper by selecting the morphological extremum operator as the prediction and update operator; Then an improved extreme value Lifting morphological wavelet algorithm is obtained based on these two algorithms; Finally, the power quality disturbance signal is improved by the extreme value type Lifting morphological wavelet transform to suppress the noise interference and highlight its disturbance characteristics. The typical power quality disturbance is taken as an example to verify the simulation.

2. Morphological wavelets

2.1. Concept

Morphological wavelets unify most linear wavelets and nonlinear wavelets and then form a unified framework for multiresolution analysis. Among them, the concept of dual wavelet decomposition and non-dual wavelet decomposition is [6].

Assuming that set \( V_j \) and \( W_j \) for \( V_j \) is the \( j \)-th level signal space (\( j=0,1,2,\ldots,J-1 \), where \( J \) is the maximum number of decomposition layers), \( W_j \) indicates the \( j \)-th detail space. The analysis and synthesis of signal, as shown in Fig.1, can be performed separately: (1) Signal analysis refers to the decomposition of the signal by the signal analysis operator \( \psi_j : V_j \rightarrow V_{j+1} \) and the detail analysis operator \( \omega_j : V_j \rightarrow W_{j+1} \) along the direction of increasing \( j \); (2) The signal is synthesized by using the signal synthesis operator \( \psi_j^*: V_{j+1} \times W_{j+1} \rightarrow V_j \) along the direction of reducing \( j \). This decomposition scheme is called dual wavelet decomposition.

![Figure 1. One stage of the coupled wavelet decomposition scheme](image)

The above decomposition scheme must satisfy the complete representation of one signal, which means the signal analysis operator \( (\psi_j, \omega_j) : V_j \rightarrow V_{j+1} \times W_{j+1} \) and the signal synthesis operator \( \psi_j^*: V_{j+1} \times W_{j+1} \rightarrow V_j \) must be mutually inverse processes. Therefore, the following two conditions must be satisfied in the dual wavelet decomposition:

\[
\psi_j^*(\psi_j(x), \omega_j(x)) = x, x \in V_j
\]

\[
\left\{
\begin{aligned}
\psi_j^*(\Psi_j(x,y)) &= x, x \in V_{j+1}, y \in W_{j+1} \\
\omega_j^*(\Psi_j(x,y)) &= y, x \in V_{j+1}, y \in W_{j+1}
\end{aligned}
\right.
\]

Where equation (1) is the exact reconstruction condition, equation (2) is established for a non-redundant decomposition.
2.2. Improved lifting morphological wavelet

The lifting method proposed by Sweldens provides a practical and flexible method for designing nonlinear wavelets [12]. The promotion plan mainly includes two types of operators: forecasting promotion and updating promotion. Detail decomposition operator $\omega$ and the synthesis operator $\omega$ are used to improve the prediction lifting operator, while the signal analysis operator $\psi$ and the synthesis operator $\psi$ are intended to improve the update lifting operator. Assuming that the combination of operation "\( \land \)" and "\( \lor \)" can form a complete prediction - update lifting morphological wavelet, the specific implementation is shown in Fig.2.

![Figure 2 Analysis and synthesis of predicting and updating scheme](image)

Let the original signal written as $x(n)$, and the prediction and update operators be respectively $\pi(n)$ and $\lambda(n)$, then the signal is decomposed into:

$$x_1 = y_1 \land \pi(x_1) \quad \text{and} \quad x_1 = x_1 \lor \lambda(x_1).$$

After the signal is decomposed, it can be reconstructed and synthesized as long as the equations (1) and (2) are satisfied, which means the signal analysis and the signal synthesis are reciprocal processes.

3. Algorithm designment

The algorithm process in this paper is as follows:

1. Constructing the maximum and minimum lifting morphological wavelet, by using the morphological extremum operator as the prediction and update operator;
2. An improved extreme-value lifting morphological wavelet algorithm is established by comprehensively utilizing the extreme-value retention characteristics in the maximum and minimum lifting morphological wavelets;
3. An improved algorithm is used to process the power quality disturbance signal and extract the disturbance characteristics.

3.1. Maximum and minimum lifting morphological wavelet transform

By choosing the extreme-value operator as the prediction and update operator, we can construct the extreme-value lifting morphological wavelet:

Assuming that the original signal is $x(n)$, according to Lazy wavelet decomposition:

$$x_1(n) = x(2n), \quad y_1(n) = x(2n + 1) \quad (3)$$

The maximum operator is used as prediction and update operators to construct the lifting morphological wavelet, which is the maximum lifting morphological wavelet. And the prediction and update operators are:

$$\pi(x)(n) = x(n) \lor x(n + 1) \quad (4)$$
$$\lambda(y)(n) = -(0 \lor y(n - 1)) \lor y(n) \quad (5)$$

Where $\lor$ denotes morphological expansion operator, taking the maximum value.

Considering operations "\( \land \)" and "\( \lor \)" as subtraction, therefore, the raised signal and detail coefficients can be demonstrated as:
\[
\begin{align*}
\dot{x}_i(n) &= x_i(n) + (0 \lor y'_i(n-1) \lor y'_i(n)) \\
\dot{y}_i(n) &= y_i(n) - (x_i(n) \lor x_i(n+1))
\end{align*}
\tag{6}
\]

It can be proved that the equations (6)~(7) satisfy the condition of complete signal reconstruction and construct a dual wavelet.

Similarly, the morphological expansion operator \( \lor \) in equations (4) and (5) is turned into the morphological corrosion operator \( \land \), which means the minimum value is obtained. Thus, the improved signal and detail coefficients after finishing the construction of minimum lifting morphological wavelet are:

\[
\begin{align*}
\dot{x}_i(n) &= x_i(n) + (0 \land y'_i(n-1) \land y'_i(n)) \\
\dot{y}_i(n) &= y_i(n) - (x_i(n) \land x_i(n+1))
\end{align*}
\tag{8}
\]

\[
\begin{align*}
\dot{y}_i(n) &= y_i(n) - (x_i(n) \land x_i(n+1))
\end{align*}
\tag{9}
\]

The information of the local maximum value of the signal, in other words, the feature called convex is retained after the maximum lifting morphological wavelet transform; while the minimum lifting morphological wavelet transform preserves the information of the local minimum value of the signal, which is the feature of concave. This shows that the extreme-value morphological wavelet algorithm has good local extremum retention ability.

### 3.2. Improvement of algorithm

By using the excellent local extremum retention ability of the maximum and minimum lifting morphological wavelet transform, it is possible to construct a perturbation inspection algorithm with better performance than the traditional shape wavelet.

Presuming that the detail coefficient of the disturbance signal after the minimum lifting morphological wavelet transform is \( y'_i \), the detail coefficient of the signal after the maximum lifting morphological wavelet transform is \( y_i \), and subtract the above two to obtain:

\[
D_k = y_i - y'_i
\tag{10}
\]

Where \( k \) represents the number of layers of the morphological wavelet transform decomposition, \( D_k \) represents the co-directional superposition between \( y_i \) and \( y'_i \).

### 3.3. Improved algorithm performance analysis

Since the maximum and minimum lifting morphological wavelet transform retain the local maximum and minimum information of the signal respectively, the detail coefficients of the corresponding decomposition layer are subtracted, which makes the original characteristics of the signal are basically unchanged after the improvement. However, the disturbance characteristics become more prominent and therefore, easier to inspect disturbances. What’s more, since the minimum and maximum lifting morphological wavelet transform has its own noise suppression ability, the obtained coefficient \( D_k \) after the subtraction between two detail coefficients is rarely affected by noise, which means the algorithm has a good performance in noise suppression.

### 4. Case analysis

#### 4.1. Performance analysis without noise interference

This paper takes the voltage sag, voltage swell, and voltage interruption as examples to analyse. These three common power quality disturbances are simulated without noise, and then making comparisons with the maximum lifting morphological wavelet, the minimum lifting morphological wavelet and the db4 wavelet to verify the effectiveness of the improved extreme-value lifting morphological wavelet algorithm. Supposing that \( u \) is a noise-free voltage signal with a sampling frequency of \( f_s = 6.4 \text{ kHz} \). Fig3, Fig4, and Fig5 show the results of voltage sag, voltage swell, and voltage interrupt signal respectively. Where a) is the disturbance signal \( u \); b) is the detail coefficient of the maximum lifting morphological wavelet after the decomposition of one layer; c) is the detail coefficient of the minimum lifting morphological wavelet after the decomposition of one layer; d) is db4 wavelet after
the decomposition of one-layer high frequency sub-band; e) is the result of one-layer decomposition of the improved extreme-value morphological wavelet algorithm.

From the following three figures, we can find out that: 1) The maximum and minimum lifting morphological wavelets have good extreme-value retention ability, but the extracted disturbance characteristics are incomplete, which means the maximum lifting morphological wavelet can extract the initial disturbance characteristics to some extent. However, the feature extraction at the end of the disturbance is not obvious; the situation about the minimum lifting morphological wavelet is exactly the opposite; 2) the signal perturbation feature extracted by db4 wavelet are more comprehensively than the maximum and minimum lifting morphological wavelet, but the disturbance characteristics cannot be highlighted well. 3) The improved extreme-value morphological wavelet can effectively extract the disturbance characteristics. The extracted disturbance characteristics are more complete than the maximum and minimum lifting morphological wavelets, and more obvious than the db4 wavelet.

![Figure 3](image)

**Figure 3** The processing of voltage sag

![Figure 4](image)

**Figure 4** The processing of voltage swell
4.2. Performance analysis with noisy interference

In the case of noise interference, since the analysis process of voltage swell is similar to the voltage sag, only the analysis of voltage sag and voltage interruption is given. A white noise with an amplitude of 0.1 is added to the voltage signal, and the signal-to-noise ratio (SNR) is 20 dB to verify the performance of the algorithm. Suppose $u$ is a voltage signal containing noise with the sampling frequency $f_s = 6.4$ kHz, Fig. 6 and Fig. 7 are the processing results of the noise-containing voltage signal. where a) is the disturbance signal $u$; b) is the detail coefficient of the maximum lifting morphological wavelet after the decomposition of one layer; c) is the detail coefficient of the minimum lifting morphological wavelet after the decomposition of one layer; d) is db4 wavelet after the decomposition of one-layer high frequency sub-band; e) is the result of one-layer decomposition of the improved extreme-value morphological wavelet algorithm.

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**Figure 5** The processing of voltage interruption

**Figure 6** The processing of noisy voltage sag  
**Figure 7** The processing of noisy voltage interruption
It can be seen from the above figures that (1) the disturbance characteristics extracted by the maximum and minimum lifting morphological wavelets in the case of noise are almost the same as those in the case of no noise, but the extraction of disturbance feature is incomplete. (2) Compared with the case of no noise, the performance of db4 wavelet is significantly reduced, and the extraction of disturbance characteristics is suppressed while the noise disturbance is suppressed, which means in the voltage sag analysis, the characteristics at the beginning of the disturbance can be extracted but not prominent enough, it is almost impossible to extract the features at the end of the disturbance; in the voltage interruption analysis, the characteristics at the end of the disturbance can be extracted, and the characteristics at the beginning of the disturbance can hardly be extracted, which indicates that the anti-interference ability of the algorithm is worse in the presence of noise; 3) The improved extreme-value lifting morphological wavelet has the best performance against the noise interference of the above algorithm, and can also effectively retain and highlight the signal disturbance characteristics.

In summary, the improved extreme-value lifting morphological wavelet overcomes the shortcomings of traditional wavelet, for which the anti-interference and signal lifting is not prominent; By improving the lifting morphological wavelet, the extreme-value information of the signal can be well preserved after the signal decomposition, which highlight the disturbance characteristics of the signal and also have a certain ability to resist noise.

4.3. Anti-interference test for algorithm
In chapter 3.2, only the simulation results when SNR equals to 20db are given. In order to verify the anti-interference ability of the improved lifting morphological wavelet further, the simulation results under different SNR are given below. Since the simulation results of voltage sag, voltage swell and voltage interruption are similar, the simulation results of voltage sag only can be used to verify the anti-interference ability of the algorithm. The simulation results of voltage sag under SNR equal to 32db, 18db and 14db are given below.
It can be seen from Fig.8 to Fig.10 that the db4 wavelet analysis decreases with the improvement of the signal-to-noise ratio, and the anti-interference ability decrease gradually. When the signal-to-noise ratio is 14 db, the disturbance characteristics have been submerged by noise. The anti-interference ability of the improved lifting morphological wavelet is relatively stable. When the signal-to-noise ratio is reduced to 14db, the disturbance characteristics can still be extracted well. This proves that the algorithm has good anti-interference ability.

4.4. Case analysis

Entrusted by a power supply bureau, the author has finished power quality measurement and evaluation on the substation and stadiums that are relative to the 26th Universiade competition venue. Measurement point in one of venues was taken as field example to verify the performance of the algorithm. Fig.8 presents the current signal of measurement point and its processing result. Where u is the field sampling signal with the sampling frequency $f_s=6.4$kHz, which is shown in a); b) is the detail coefficient of the maximum lifting morphological wavelet after one-layer decomposing; c) is the detail coefficient of the minimum lifting morphological wavelet after the one-layer decomposition; d) is the high frequency sub-band of the db4 wavelet after the one-layer decomposition; e) is the processing result after the one-layer decomposition of the improved extreme-value morphological wavelet algorithm.

![Figure 11](image1.png)

**Figure 11** The processing of on-site sampled signal

It can be seen from the Fig.11 that under the condition of the filed signal, 1) the maximum and minimum lifting morphological wavelets can suppress the noise interference and extract the disturbance characteristics, but the extraction is not obvious enough. 2) the performance db4 wavelet of is the worst, basically submerged by noise and almost no disturbance characteristics can be extracted, which indicates that the algorithm has certain limitations in the actual measurement. 3) The improved extreme-value morphological wavelet algorithm presents superior performance in the actual measurement. It can be known from equation (10) that the extracted perturbation feature is the co-direction superposition of perturbed features extracted by the maximum and minimum lifting morphological wavelets. Not only the improved algorithm highlights the disturbance characteristics with no missed inspection, but also an excellent anti-interference ability is preserved.
It can also be seen from the case analysis that disturbance characteristics are not prominent enough for the maximum and minimum lifting morphological wavelets, which is difficult to meet the requirements of practical application; the db4 morphological wavelet is short of anti-interference ability, which does not meet the actual measurement conditions; while the improved extreme-value lifting morphological wavelet shows excellent performance in the actual measurement, indicating that it has a wide application space.

5. Conclusion

1) This paper analyses the principle of wavelet, the maximum and minimum lifting morphological wavelet, as well as improved extreme-value wavelet. What’s more, the advantages and disadvantages of each algorithm are also compared.

2) For the wavelet transform, the anti-interference ability is poor, and the extracted disturbance feature is not obvious enough. The improved lifting wavelet algorithm is proposed to enhance the anti-interference ability and highlight the disturbance characteristics.

3) In order to overcome the incompleteness of extracted disturbance feature of the maximum and minimum lifting morphological wavelet, and further highlight the disturbance characteristics, to improve these two problems, an improved extreme-value morphological wavelet algorithm is proposed.

4) Based on the morphological wavelets, the improved extreme-value lifting morphological wavelet combines the advantages of the maximum and minimum lifting morphological wavelet algorithm, and the co-direction superimposition of the two disturbance characteristics significantly highlights the disturbance characteristics and suppresses the noise interference. What’s more, a superior anti-interference ability can be a stable foundation for improving the accuracy of disturbance positioning.

Acknowledgement

This science project is funded by Shenzhen Power Supply Co., Ltd. Project number: SZKJXM20160169

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