Pion transverse charge density and the edge of hadrons

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We use the world data on the pion form factor for space-like kinematics and a technique used to extract the proton transverse densities, to extract the transverse pion charge density and its uncertainty due to experimental uncertainties and incomplete knowledge of the pion form factor at large values of $Q^2$. The pion charge density at small values of $b < 0.1$ fm is dominated by this incompleteness error while the range between 0.1-0.3 fm is relatively well constrained. A comparison of pion and proton charge densities shows that the pion is denser than the proton for values of $b < 0.2$ fm. The pion and proton distributions seem to be the same for values of $b$=0.2-0.6 fm. Future data from JLab 12 GeV and the EIC will increase the dynamic extent of the data to higher values of $Q^2$ and thus reduce the uncertainties in the extracted pion charge density.

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I. INTRODUCTION

Measurements of form factors play an important role in our understanding of the structure and interactions of hadrons based on the principles of QCD. One of the simplest hadronic systems available for study is the pion, whose valence structure is a bound state of a quark and an antiquark. Its elastic electromagnetic structure is parameterized by a single form factor, $F_\pi(Q^2)$. Calculations of the pion charge form factor have been used to investigate the transition from the low-momentum transfer confinement region to the regime where perturbative methods are applicable. There is a long history of experimental measurements. In particular, $F_\pi(Q^2)$ has been measured at space-like momentum transfers through pion-electron scattering and pion electroproduction on the nucleon with high precision up to $Q^2=2.5$ GeV$^2$, and new measurements are planned with the 12 GeV era at the Thomas Jefferson National Accelerator Facility (JLab) and envisioned for a future Electron-Ion Collider (EIC).

The concept of transverse charge densities has emerged recently as a framework providing an interpretation of electromagnetic form factors in terms of the physical charge and magnetization densities. It has been explored in a number of recent works. These transverse densities are obtained as two-dimensional Fourier transforms of the elastic form factors and describe the distribution of charge and magnetization in the plane transverse to the propagation direction of a fast moving nucleon. They are related to the Generalized Parton distributions (GPDs), which are expected to provide a universal (process-independent) description of the nucleon, and simultaneously encode information on parton distributions and correlations in both momentum (in the longitudinal direction) and coordinate (in the transverse direction) spaces.

There have been two previous analyses of the pion transverse charge density. In the former a wide range of models was used. No estimate of the uncertainty caused by incomplete kinematic knowledge of the form factor was made. The latter was based on data taken in the time-like region and extended to the space-like region through the use of dispersion relations and models needed to obtain the separate real and imaginary parts of the observable quantity $|F_\pi|^2$. The present paper is aimed at avoiding models and determining the impact of potential new experiments.

In particular, the goal of the present analysis is to evaluate the world’s data on the space-like pion form factor, to extract the corresponding transverse pion charge distribution within current uncertainties, and estimate the influence of the planned experiments on the pion transverse charge density. Examining the current data requires forming a superset with a single global uncertainty, taking into account the individual uncertainties and the differences in the form factor extraction method. This is done in Sec. II. We use the infinite radius approximation technique applied to analyze proton form factor data described in Ref. to estimate the uncertainty due to the limited kinematic coverage of the currently available data in Sec. III. Results for the pion transverse charge distribution are presented in Sec. IV. An interesting application of transverse charge densities is the analysis of the spatial structure of the nucleon’s pion cloud. Recent work found that the non-chiral core of the charge density dominated up to rather large distances $\sim 2$ fm implying a large proton core. The proton and pion transverse charge density are compared in Sec. V, and the impact of future experiments is assessed in Sec. VI.
II. EXTRACTION OF THE PION FORM FACTOR FROM WORLD DATA

The pion’s elastic electromagnetic structure is parameterized by a single form factor, \( F_\pi(Q^2) \), which depends on \( Q^2 = -q^2 \), where \( q^2 \) is the four-momentum squared of the virtual photon. \( F_\pi \) is well determined up to values of \( Q^2 \) of 0.28 GeV\(^2\) by elastic \( \pi - e \) scattering \( \pi \cdot e \), from which the charge mean radius has been extracted. Determining \( F_\pi(Q^2) \) at larger values of \( Q^2 \) requires the use of pion electroproduction from a nucleon target. The longitudinal part of the cross section for pion electroproduction, \( \sigma_L \), contains the pion exchange process, in which the virtual photon couples to a virtual pion inside the nucleon. This process is expected to dominate at small values of the Mandelstam variable \(-t\), thus allowing for the determination of \( F_\pi \). A comprehensive review on the extraction of \( F_\pi \) from pion electroproduction data can be found in Refs. [7, 8].

Pion electroproduction data have previously been obtained for values of \( Q^2 \) of 0.18 to 9.8 GeV\(^2\) at the Cambridge Electron Electron Accelerator (CEA), at Cornell [11, 12] and at the Deutsches Elektronen-Synchrotron (DESY) [13, 15]. Most of the high \( Q^2 \) data have come from experiments at Cornell. In these experiments, \( F_\pi \) was extracted from the longitudinal cross sections, which were isolated by subtracting a model of the transverse contribution from the unseparated cross sections. Pion electroproduction data were also obtained at DESY [13, 15] for values of \( Q^2 \) of 0.35 and 0.7 GeV\(^2\), and longitudinal (L) and transverse (T) cross sections were extracted using the Rosenbluth L/T separation method. With the availability of the high-intensity, continuous electron beams and well-understood magnetic spectrometers at JLab it became possible to determine L/T separated cross sections with high precision, and thus to study the pion form factor in the regime of \( Q^2 = 0.5-3.0 \) GeV\(^2\) [16, 19, 34].

The pion form factor has been compared with different empirical fits and model calculations based on pQCD, lattice QCD, dispersion relations with QCD constraint, QCD sum rules, Bethe-Salpeter Equation, local quark-hadron duality, constituent quark model, holographic QCD, etc. in Ref. [8]. A new method has recently been developed to calculate \( F_\pi \) on the entire region of \( Q^2 \) using the Dyson-Schwinger equation framework [35]. The results are in very good agreement with the world \( F_\pi \) data. Many models in the literature approach the monopole form \( F_\pi^{\text{monopole}}(Q^2) = 1/(1+Q^2r^2/6) \) for large values of \( Q^2 \).

Our analysis of the uncertainty due to lack of \( F_\pi(Q^2) \) data at values of \( Q^2 > 9.8 \) GeV\(^2\) requires the use of an upper bound and a lower bound on \( F_\pi \) in the region where it is not measured. An upper bound [3, 8] for the pion form factor is given by the monopole form, so we use this form to provide an upper bound in our analysis. Our lower bound is chosen to be a light front constituent quark model that does not converge to the monopole asymptotically yet still describes the data well. There are many models available in this category, which typically differ in the treatment of the quark wave functions of relativistic effects. The model of Ref. [36] provides a relativistic treatment of quarks spins and center of mass motion. It uses a power-law wave function with parameters determined from experimental data on the charged pion decay constant, the neutral pion two-photon decay width, and the charged pion electromagnetic radius. This model is in very good agreement with the world \( F_\pi \) data.

Fig. 1 shows the world data on the pion form factor together with the results of the empirical monopole form and a Light Front model (LF) calculation based on Ref. [36]. Both, the monopole and the Light-Front models are in very good agreement with the data up to values of \( Q^2 \approx 2.5 \) GeV\(^2\). Above that, \( Q^2F_\pi^{\text{monopole}} \) and \( Q^2F_\pi^{\text{LF}} \) deviate from each other. \( Q^2F_\pi^{\text{monopole}} \) tends to a constant value while \( Q^2F_\pi^{\text{LF}} \) decreases as \( Q^2 \to \infty \). All other models of the pion form factor fall between these two curves. No distinction can be made between the models based on the current data due to their large uncertainties in particular at values of \( Q^2 \) between 3 GeV\(^2\) and 10 GeV\(^2\).

The transverse pion charge distribution is given in terms of the pion charge form factor. Elastic pion-electron scattering has been measured up to 0.28 GeV\(^2\) and the form factor has been extracted with high precision up to \( Q^2 = 2.5 \) GeV\(^2\) from pion electroproduction.
data. The two main sources of uncertainty in the extraction of the transverse densities are the published experimental uncertainties from the measurements of the pion form factor and uncertainties due to the lack of form factor data at values of $Q^2 > 9.8$ GeV$^2$.

The pion form factor was extracted from a global analysis of the world $F_\pi$ data. The analysis follows that of Ref. [3]. The slope of $F_\pi$ at $Q^2 \to 0$ GeV$^2$ is constrained by an analysis of the low $Q^2$ elastic pion-electron scattering data. The value of the radius was found to be $0.672 \pm 0.008$ fm [4]. The current $F_\pi$ data show a systematic departure from the monopole curve above $Q^2 \sim 1.5$ GeV$^2$. In our analysis we thus use an empirical fit form including a dipole component.

$$F_\pi(Q^2) = A \cdot \frac{1}{(1 + B \cdot Q^2)^2} + (1 - A) \cdot \frac{1}{(1 + C \cdot Q^2)^2}$$  \hspace{1cm} (1)$$

Here, parameters $B$ and $C$ are related to the monopole and dipole pion charge radius, and $A$ denotes the fractional contribution of the monopole and dipole components to the overall fit. To evaluate effect of the fitted normalization factors, the normalization factor from a single data set was varied around its best fit value while all other parameters were allowed to vary without constraints. The resulting curves with the form of Equation 1 were fitted to the data. For each fit, the experimental points were randomly recreated following a Gaussian distribution around their central values. The results of these fits are shown in the black/hatched band in Fig. 2. We found the best coefficients for these fits to be $A = 0.384 \pm 0.071$, $B = 1.203 \pm 0.101$ GeV$^{-2}$ and $C = 1.054 \pm 0.080$ GeV$^{-2}$. Using these coefficients we extracted an RMS radius of the pion of $0.641 \pm 0.025$ fm from the slope of $F_\pi$ at $Q^2 \to 0$, which is consistent with the result of Ref. [4]. The present fit is obtained by varying all of the values of the parameters $A$, $B$ and $C$ simultaneously.

The dominance of the dipole term over the monopole term on the fit differs from the result of Ref. [3] which found the monopole dominant. The constraints on the fit in Ref. [3] were different from our present fit in that the values of $B$ and $C$ were kept fixed to reproduce the squared radii of 0.431 fm$^2$ for the monopole term and 0.411 fm$^2$ for the dipole term. Furthermore, our present fit included additional data points up to $Q^2 = 9.8$ GeV$^2$. The impact of the additional higher $Q^2$ data points on the present fit was small due to their large experimental uncertainties; the fit parameters change by less than 0.5%. However, including these points here despite their large uncertainties is important for the truncation of the series expansion in Eq. 1 and thus the incompleteness error, which results from the region in $Q^2$ where no measurements exist at all. As Fig. 3 shows the incompleteness error dominates over the experimental error.

![Figure 2](image.png)

**FIG. 2**: (Color online) Empirical fit to the experimental $F_\pi$ data (black/hatched band) used to evaluate the pion charge distribution. The band represents the systematic uncertainties due to combining the different measurements. The error bars represent the statistical and systematic uncertainties of the individual measurements. The curves are the same as shown in Fig. 1.

### III. EXTRACTION OF THE PION TRANVERSE DENSITY DISTRIBUTION

The pion transverse charge distribution $\rho_\pi(b)$ is the matrix element of the light-front density operator integrated over longitudinal distance [27] and is given by the two dimensional Fourier transform of the space-like pion form factor $F_\pi(Q^2)$:

$$\rho_\pi(b) = \frac{1}{(2\pi)^2} \int d^2 q e^{-i\vec{q} \cdot \vec{b}} F_\pi(Q^2). \hspace{1cm} (2)$$

The distribution function $\rho_\pi(b)$ denotes the probability that a charge is located at a transverse distance $b$ from the transverse center of momentum with normalization condition, $\int d^2 b \rho_\pi(b) = 1$. If we consider an azimuthal symmetry of the charge distribution $\rho_\pi$, Equation 2 reduces to an one-dimensional integral:

$$\rho_\pi(b) = \frac{1}{2\pi} \int_0^\infty Q dQ J_0(Qb) F_\pi(Q^2). \hspace{1cm} (3)$$

Intuitively we expect the charge of the pion to be localized within a volume of radius $R$. This assumption is called finite radius approximation and we use it to simplify Eq. 2. For values of $b$ less than the chosen distance parameter $R$, the function $\rho_\pi(b)$ can be expanded in a series of the Bessel function $J_0$ as
\[ \rho_\pi(b) = \sum_{n=1}^{\infty} c_n J_0 \left( X_n \frac{b}{R} \right), \quad (4) \]

where \( X_n \) is the \( n \)-th zero of \( J_0 \) and \( c_n \) is given by the expression

\[ c_n = \frac{1}{\pi R^2} \frac{2}{\left( J_1(X_n) \right)^2} F_\pi \left( Q_n^2 \right). \quad (5) \]

Here, \( Q_n \) is defined as

\[ Q_n = \frac{X_n}{R}. \quad (6) \]

Equations (4) and (5) combined yield the following expression for \( \rho_\pi(b) \)

\[ \rho_\pi(b) = \frac{1}{\pi R^2} \sum_{n=1}^{\infty} F_\pi \left( Q_n^2 \right) \frac{J_0 \left( X_n \frac{b}{R} \right)}{\left( J_1(X_n) \right)^2}. \quad (7) \]

This expansion provides the transverse density for values of \( b < R \) for measurements of the pion form factor up to momentum transfers of \( Q_n^2 \). In this finite radius approximation, the orthogonality of the Bessel function was considered for the evaluation of the coefficients from Equation (5), requiring that the charge distribution vanish for \( b > R \).

The extraction of the pion transverse density requires as input the experimental value of \( F_\pi \) obtained from the fits shown in Fig. 2. The uncertainty on the extraction thus also depends on the experimental uncertainties. The total uncertainty on \( \rho_\pi(b) \) has two main sources: 1) experimental uncertainties on the individual measurements and combining data from different experiments in the region where data exist and 2) uncertainties due to the lack of data in the region beyond \( Q^2 = Q_{\text{max}}^2 \), where no measurements exist. The experimental uncertainties are taken into account directly in the coefficients \( c_n \) through equation (5). However, uncertainty due to lack of data for values of \( Q^2 > Q_{\text{max}}^2 \) must also be estimated. Both sources of uncertainty are discussed next.

### A. Experimental Uncertainty

The series expansion in equation (7) is truncated to values of \( Q_n^2 \) for which the pion form factor has been extracted from data. Uncertainties from these data were used to estimate the uncertainty in \( \rho_\pi \) due to uncertainties in the experimental measurements. The form factor \( F_\pi \) has been measured with high precision up to \( Q^2 = 2.5 \text{ GeV}^2 \). We also use lower precision data with large systematic uncertainties 50-70\% for values \( Q^2 = 3.3-9.8 \text{ GeV}^2 \). Thus we take the form factor as a measured quantity for \( Q^2 < Q_{\text{max}}^2 = 10 \text{ GeV}^2 \). This corresponds to an upper limit of \( n_{\text{max}} = 10 \), with a value of \( R = 2 \text{ fm} \), in Eq. (6). This value of \( R \) is chosen to minimize the effect of oscillations due to a sharp cutoff of the charge density.

The uncertainty on \( F_\pi \) directly results in an uncertainty on the coefficients \( c_n \) and thus directly contributes to \( \rho_\pi(b) \). The contribution of the experimental uncertainty due to the experimental uncertainty is 3.5\% of the transverse density at \( b=0 \text{ fm} \) and decreases with increasing \( b \). The fractional uncertainty is 3.0\% up to \( b=1 \text{ fm} \) as illustrated by the black/hatched area shown in Fig. 3.

![Figure 3](image-url)

**FIG. 3:** (Color online) Uncertainties of the pion charge distribution due to \( F_\pi \) experimental data uncertainties (black/hatched band) and the incompleteness error, when considering the monopole model (blue band) or the Light-Front model (gray band).

### B. Incompleteness Error

Uncertainties due to lack of knowledge of the pion form factor for values of \( Q^2 > Q_{\text{max}}^2 \), where no measurements exist, must be estimated. The first step is understanding the truncation that have to be made.

Equation (7) uses the finite radius approximation requiring knowledge of the form factor in the full range of \( Q^2 \). Since \( F_\pi \) measurements are limited to a region \( Q^2 = Q_{\text{max}}^2 \), the series expansion has to be truncated to \( Q_{\text{max}}^2 \). The effects of this truncation in the calculation of \( \rho_\pi(b) \) is estimated in the incompleteness error.

The basic transverse pion densities are obtained using equation (7) for values of \( Q^2 < 10 \text{ GeV}^2 \) corresponding to values of \( n_{\text{max}} = 10 \). A maximum error was estimated using representative theoretical models with very different asymptotic behavior, which describe the existing \( F_\pi \) data well. Out of the available models we chose the monopole and the Light-Front model from Ref. 28 as an upper and lower bound respectively. These models encompass all other models of \( F_\pi \) that describe the data and thus include the true value of \( F_\pi \) in the region where no data...
exist. The incompleteness error for our two chosen models is estimated using

$$\Delta_{\text{model}}(b) = \left| \frac{1}{\pi R^2} \sum_{n=n_{\text{max}}+1}^{\infty} F_{\text{model}}(Q_n^2) \frac{J_0(X_n b_R)}{(J_1(X_n))^2} \right|,$$

as a function of $b$, where $n_{\text{max}} = 10$ is the last term of the charge distribution series where $F_\pi(Q_{n_{\text{max}}})$ had been measured. The results are summarized in Fig. 3. The incompleteness error likely overestimates the error.

### IV. PION TRANSVERSE CHARGE DISTRIBUTION

We turn to our stated goal of using the world data on the space-like pion form factor to extract the pion charge density. Fig. 4 shows the pion charge density evaluated using the series expansion of Equation (7) with the experimental uncertainty based on our fits of $F_\pi$ (from Fig. 2) and with the incompleteness error estimated using the monopole and LF models as described above.

![Image of Fig. 4](image_url)

**FIG. 4:** (Color online) TOP: The pion charge distribution (red curve) calculated from the 2D Fourier transform of the pion form factor. BOTTOM: The pion charge distribution multiplied by the Jacobian $b$. Uncertainties from experimental $F_\pi$ data are represented by the black/hatched band, while the incompleteness error was estimated using the monopole model (blue band) and the Light-Front model (gray band).

As we are working in polar coordinates, the spatial transverse element of area is $d^2b = 2\pi db^2$, for a given impact parameter $b$. Thus the bottom panel of Fig. 4 shows the pion transverse charge distribution multiplied by the Jacobian $b$.

For $\rho(b)$ the uncertainties due to the incompleteness error for $b > 0.1$ fm are relatively small compared to the ones for the region $b < 0.1$ fm. The oscillatory behavior can be attributed to the truncation of the Bessel function in equation (8). On the other hand, in the region $b < 0.1$ fm the incompleteness error is very large, which is due to the lack of the pion form factor data at very large values of $Q^2$.

### V. NUCLEON PION CLOUD AND PION CHARGE DENSITIES

A recent work explored the proton transverse charge density finding that the non-chiral core is dominant up to relatively large distances of $\sim 2$ fm. This suggests that there is a non-pionic core of the proton, as one would obtain in the constituent quark or vector meson dominance models. One does not usually think of the pion having a meson cloud since, e.g., $\rho\pi$ component would involve a high excitation energy. Therefore it is interesting to compare the proton and pion transverse charge densities. This is done in Fig. 5 and Fig. 6 for different ranges of $b$.

![Image of Fig. 5](image_url)

**FIG. 5:** (Color online) TOP: Comparison of the pion charge distribution (red curve) to the proton charge distribution (from [30]) shown in the green band. BOTTOM: ratio of pion to proton charge densities. The uncertainties for the pion charge distribution are as in Fig. 4. Those for the proton charge distribution are included in the band. The charge distribution curves coalesce in the region $b > 0.2$ fm while the pion charge density appears denser than that of the proton in the region $b < 0.2$ fm.
For values of $b$ less than about 0.2 fm the transverse charge density of the pion is larger than that of the proton. This higher density might be expected because the pion’s radius $0.672$ fm is smaller than the proton’s $0.84$ fm. As previously noted [26, 29], it is possible that the pion’s transverse density is singular for small values of $b$. An interesting feature is that the curves seem to coalesce in the region $b > 0.2$ fm (at least within current uncertainties). This is not expected. A possible explanation could be obtained by regarding the pion to be a $q\bar{q}$ pair bound by a color octet exchange mechanism (proportional to $\lambda_i \cdot \lambda_j$ (where the 8 components of $\lambda_i$ are generators of SU(3) in color space) and regarding the proton as a quark-diquark [37, 38] system that interacts also bound via a color octet exchange mechanism. Similarity in binding forces could lead to a similarity in transverse densities. The result that the pion and proton transverse densities are similar in their core may be a first experimental glimpse at the transition between proton core and meson cloud, e.g., the “edge” of the nucleon.

VI. IMPACT OF FUTURE EXPERIMENTS

Future data would improve the extraction of the pion charge density, which in the region of $b > 0.2$ fm would be of great interest for further studies of the “edge” of hadrons. Experiments at the 12 GeV JLab [20, 21] will extend the $Q^2$ range of high precision pion form factor data to $Q^2 = 6$ GeV$^2$ and 9 GeV$^2$. The envisioned EIC will further extend this reach to $Q^2$ of about 25 GeV$^2$. This $Q^2$ region would add data into the region of interest for studying the hadron edge improving the precision of the extraction of the pion charge distribution. The measurements would also add data into thus far unmeasured regions of small $b$. The projected uncertainties of these future experiment are shown in Fig. 6 together with existing data and the monopole and LF calculations.

![Graphical representation of experimental data and projected uncertainties](image)

**FIG. 6**: (Color online) TOP: Comparison of the pion charge distribution to the proton charge distribution as in Fig. 5, showing a range of higher impact parameter $b$. BOTTOM: ratio of pion to proton charge densities. Oscillations of $\rho_\pi$ come from the truncation of the series expansion of Eq. 7.

**FIG. 7**: (Color online) $F_\pi$ world data (red circles) and projected uncertainties for experiments at JLab at 12 GeV (blue diamonds) and those for measurements of the form factor with an Electron-Ion Collider. The projected uncertainties at the EIC are divided into three groups depending on the energy $E_p$ of the ion beam (magenta triangles with $E_p=5$GeV, green stars with $E_p=10$GeV, brown squares with $E_p=15$GeV). The error bars shown include both projected statistical and systematic uncertainties. The black/hatched band represents an empirical fit using equation 1 taking into account present data.

The projected uncertainties of the new data will add sufficient precision to distinguish between theoretical models at values of $Q^2$ greater than 3 GeV$^2$, and thus narrow down our selection of models for estimating the incompleteness error. Assuming that all data from both 12 GeV JLab and the EIC are measured, we analyze the impact of the new data on the precision of the extraction of the pion charge distribution. If the data were better described by a different model, the resulting precision on the pion charge distribution would not be greater than the estimate for the monopole model, since the monopole form gives the largest possible values the pion form factor can attain. The estimated uncertainties considering future data are shown in Fig. 8.
Including the projected uncertainties of the future data the precision of the pion charge distribution would be better than 20\% for $b > 0.1$ fm. This would greatly constrain the transverse charge density and determine the proton and pion transverse charge densities really are the same for moderate values of $b$.

VII. SUMMARY

In this paper we used the world data on the space-like pion form factor to extract the transverse pion charge density. The extraction method is based on that used for the proton in Ref. \cite{30} and includes the use of Bessel series expansion and finite radius approximation to determine the impact of experimental uncertainties and the incompleteness error due to the lack of data for $Q^2 > Q^2_{\text{max}}$. Two theoretical models, monopole and LF, are used to estimate the incompleteness error. Those models provide a very conservative upper and lower bound for describing $F_\pi(Q^2 > Q^2_{\text{max}})$. The resulting uncertainty on the extracted pion charge density is dominated by the incompleteness error at values of $b < 0.1$ fm. The relative uncertainty in the region $0.1$ fm $< b < 0.3$ fm is smallest and the region above $b > 0.3$ fm is dominated by oscillations from the truncation of the Bessel series.

A comparison of the pion to the proton charge densities shows a larger density of the pion in the region $b < 0.2$ fm. The two curves coalesce for values $0.3$ fm $< b < 0.6$ fm, which may be interpreted in terms of the spatial structure of the nucleon consisting of a core and a meson cloud. The coming together of the two curves at the edge of their density suggest a common confinement mechanism for pions and nucleons. Future experiments at 12 GeV JLab and EIC will add high precision data at higher $Q^2$ and would reduce the uncertainty, which in the region of $b > 0.2$ fm could be of great interest for studies of a common transverse charge density.

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