THE EARLY YEARS OF STRING THEORY:
A PERSONAL PERSPECTIVE

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Abstract

This article surveys some of the highlights in the development of string theory through the first superstring revolution in 1984. The emphasis is on topics in which the author was involved, especially the observation that critical string theories provide consistent quantum theories of gravity and the proposal to use string theory to construct a unified theory of all fundamental particles and forces.

Based on a lecture presented on June 20, 2007 at the Galileo Galilei Institute
1 Introduction

I am happy to have this opportunity to reminisce about the origins and development of string theory from 1962 (when I entered graduate school) through the first superstring revolution in 1984. Some of the topics were discussed previously in three papers that were written for various special events in 2000 [1, 2, 3]. Also, some of this material was reviewed in the 1985 reprint volumes [4], as well as the string theory textbooks [5, 6]. In presenting my experiences and impressions of this period, it is inevitable that my own contributions are emphasized.

Some of the other early contributors to string theory have presented their recollections at the Galileo Galilei Institute meeting on “The Birth of String Theory” in May 2007. Since I was unable to attend that meeting, my talk was given at the GGI one month later. Taken together, the papers in this collection should convey a fairly accurate account of the origins of this remarkable subject.¹

The remainder of this paper is divided into the following sections:

• 1960 – 68: The analytic S matrix (Ademollo, Veneziano)
• 1968 – 70: The dual resonance model (Veneziano, Di Vecchia, Fairlie, Neveu)
• 1971 – 73: The RNS model (Ramond, Neveu)
• 1974 – 75: Gravity and unification
• 1975 – 79: Supersymmetry and supergravity (Gliozzi)
• 1979 – 84: Superstrings and anomalies (Green)

For each section, the relevant speakers at the May meeting are listed above. Since their talks were more focussed than mine, they were able to provide more detail. In one section (gravity and unification) my presentation provided more detail than the others.

2 1960 – 68: The analytic S matrix

In the early 1960s there existed a successful quantum theory of the electromagnetic force (QED), which was completed in the late 1940s, but the theories of the weak and strong nuclear forces were not yet known. In UC Berkeley, where I was a graduate student during the period 1962 – 66, the emphasis was on developing a theory of the strong nuclear force.

¹Since the history of science community has shown little interest in string theory, it is important to get this material on the record. There have been popular books about string theory and related topics, which serve a useful purpose, but there remains a need for a more scholarly study of the origins and history of string theory.
I felt that UC Berkeley was the center of the Universe for high energy theory at the time. Geoffrey Chew (my thesis advisor) and Stanley Mandelstam were highly influential leaders. Also, Steve Weinberg and Shelly Glashow were impressive younger faculty members. David Gross was a contemporaneous Chew student with whom I shared an office.\(^2\)

Geoffrey Chew’s approach to understanding the strong interactions was based on several general principles \(^8, 9\). He was very persuasive in advocating them, and I was strongly influenced by him. The first principle was that quantum field theory, which was so successful in describing QED, was inappropriate for describing a strongly interacting theory, where a weak-coupling perturbation expansion would not be useful. A compelling reason for holding this view was that none of the hadrons (particles that have strong interactions) seemed to be more fundamental than any of the others. Therefore a field theory that singled out some subset of the hadrons did not seem sensible. Also, it was clearly not possible to formulate a quantum field theory with a fundamental field for every hadron. One spoke of nuclear democracy to describe this situation.\(^3\)

For these reasons, Chew argued that field theory was inappropriate for describing strong nuclear forces. Instead, he advocated focussing attention on physical quantities, especially the S Matrix, which describes on-mass-shell scattering amplitudes. The goal was therefore to develop a theory that would determine the S matrix. Some of the ingredients that went into this were properties deduced from quantum field theory, such as unitarity and maximal analyticity of the S matrix. These basically encode the requirements of causality and nonnegative probabilities.

Another important proposal, due to Chew and Frautschi, whose necessity was less obvious, was maximal analyticity in angular momentum \(^10, 11\). The idea is that partial wave amplitudes \(a_l(s)\), which are defined in the first instance for angular momenta \(l = 0, 1, \ldots\), can be uniquely extended to an analytic function of \(l, a(l, s)\), with isolated poles called Regge poles. The Mandelstam invariant \(s\) is the square of the invariant energy of the scattering reaction. The position of a Regge pole is given by a Regge trajectory \(l = \alpha(s)\). The values of \(s\) for which \(l\) takes a physical value, correspond to physical hadron states. The necessity of branch points in the \(l\) plane, with associated Regge cuts, was established by Mandelstam.

\(^2\)It was a particularly nice office, which was being reserved for Murray Gell-Mann, whom Berkeley was trying to hire. It was felt that students would be easier to dislodge than a faculty member. Gross and I wrote one joint paper in 1965 \(^7\), which I felt was rather clever.

\(^3\)The quark concept arose during this period, but the prevailing opinion was that quarks are just mathematical constructs. The SLAC deep inelastic scattering experiments in the late 1960s made it clear that quarks and gluons are physical (confined) particles. It was then natural to try to base a quantum field theory on them, and QCD was developed a few years later with the discovery of asymptotic freedom.
Their role in phenomenology was less clear.

The theoretical work in this period was strongly influenced by experimental results. Many new hadrons were discovered in experiments at the Bevatron in Berkeley, the AGS in Brookhaven, and the PS at CERN. Plotting masses squared versus angular momentum (for fixed values of other quantum numbers), it was noticed that the Regge trajectories are approximately linear with a common slope

$$\alpha(s) = \alpha(0) + \alpha' s \quad \alpha' \sim 1.0 \text{ (GeV)}^{-2}.$$ 

Using the crossing-symmetry properties of analytically continued scattering amplitudes, one argued that exchange of Regge poles (in the $t$ channel) controlled the high-energy, fixed momentum transfer, asymptotic behavior of physical amplitudes:

$$A(s, t) \sim \beta(t)(s/s_0)^{\alpha(t)} \quad s \to \infty, \; t < 0.$$ 

In this way one deduced from data that the intercept of the $\rho$ trajectory, for example, was $\alpha_\rho(0) \sim .5$. This is consistent with the measured mass $m_\rho = .76 \text{ GeV}$ and the Regge slope $\alpha' \sim 1.0 \text{ (GeV)}^{-2}$.

The ingredients discussed above are not sufficient to determine the $S$ matrix, so one needed more. Therefore, Chew advocated another principle called the bootstrap. The idea was that the exchange of hadrons in crossed channels provide forces that are responsible for causing hadrons to form bound states. Thus, one has a self-consistent structure in which the entire collection of hadrons provides the forces that makes their own existence possible. It was unclear for some time how to formulate this intriguing property in a mathematically precise way. As an outgrowth of studies of finite-energy sum rules in 1967 [12, 13, 14, 15, 16] this was achieved in a certain limit in 1968 [17, 18, 19]. The limit, called the narrow resonance approximation was one in which resonance lifetimes are negligible compared to their masses. The observed linearity of Regge trajectories suggested this approximation, since otherwise pole positions would have significant imaginary parts. In this approximation branch cuts in scattering amplitudes, whose branch points correspond to multiparticle thresholds, are approximated by a sequence of resonance poles.

The bootstrap idea had a precise formulation in the narrow resonance approximation, which was called duality. This is the statement that a scattering amplitude can be expanded in an infinite series of $s$-channel poles, and this gives the same result as its expansion in an infinite series of $t$-channel poles.\(^4\) To include both sets of poles, as usual Feynman diagram techniques might suggest, would amount to double counting.

\(^4\)One defines divergent series by analytic continuation.
3 1968 – 70: The dual resonance model

I began my first postdoctoral position (entitled *instructor*) at Princeton University in 1966. For my first two and a half years there, I continued to do work along the lines described in the previous section (Regge pole theory, duality, etc.). Then Veneziano dropped a bombshell – an exact analytic formula that exhibited duality with linear Regge trajectories [20]. Veneziano’s formula was designed to give a good phenomenological description of the reaction $\pi + \pi \to \pi + \omega$ or the decay $\omega \to \pi^+ + \pi^0 + \pi^-$. Its structure was the sum of three Euler beta functions:

$$T = A(s, t) + A(s, u) + A(t, u)$$

$$A(s, t) = \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s) - \alpha(t))}$$

where $\alpha$ is a linear Regge trajectory

$$\alpha(s) = \alpha(0) + \alpha'.$$

An analogous formula appropriate to the reaction $\pi + \pi \to \pi + \pi$ was quickly proposed by Lovelace and Shapiro [21, 22]. A rule for building in adjoint $SU(N)$ quantum numbers was formulated by Chan and Paton [23]. This symmetry was initially envisaged to be a global (flavor) symmetry, but it later turned out to be a local gauge symmetry.

The Veneziano formula gives an explicit realization of duality and Regge behavior in the narrow resonance approximation. The function $A(s, t)$ can be expanded in terms of the $s$-channel poles or the $t$-channel poles. The motivation for writing down this formula was mostly phenomenological, but it turned out that formulas of this type describe tree amplitudes in a perturbatively consistent quantum theory!

Very soon after the appearance of the Veneziano amplitude, Virasoro proposed an alternative formula [24]

$$T = \frac{\Gamma(-\frac{1}{2}\alpha(s))\Gamma(-\frac{1}{2}\alpha(t))\Gamma(-\frac{1}{2}\alpha(u))}{\Gamma(-\frac{1}{2}\alpha(t) - \frac{1}{2}\alpha(u))\Gamma(-\frac{1}{2}\alpha(s) - \frac{1}{2}\alpha(u))\Gamma(-\frac{1}{2}\alpha(s) - \frac{1}{2}\alpha(t))}.$$ 

which has similar virtues. Since this formula has total $stu$ symmetry, it is only applicable to particles that are singlets of the Chan–Paton group.

Over the course of the next year or so, string theory (or *dual models*, as the subject was then called) underwent a sudden surge of popularity, marked by several remarkable discoveries. One was the discovery of an $N$-particle generalization of the Veneziano formula.
\[ A_N(k) = g_{\text{open}}^{N-2} \int d\mu_N(y) \prod_{i<j} (y_i - y_j)^{\alpha k_i k_j}, \]

where \( y_1, y_2, \ldots, y_N \) are real coordinates, any three of which are \( y_A, y_B, y_C \), and

\[ d\mu_N(y) = |(y_A - y_B)(y_B - y_C)(y_C - y_A)|^{N-1} \prod_{i=1}^{N-1} \theta(y_{i+1} - y_i) \]

\[ \times \delta(y_A - y_A^0)\delta(y_B - y_B^0)\delta(y_C - y_C^0) \prod_{i=1}^{N} dy_i. \]

The formula is independent of \( y_A^0, y_B^0, y_C^0 \) due to its \( SL(2,\mathbb{R}) \) symmetry, which allows them to be mapped to arbitrary real values. This formula has cyclic symmetry in the \( N \) external lines.

Soon thereafter Shapiro formulated an \( N \)-particle generalization of the Virasoro formula [30]:

\[ A_N(k_1, k_2, \ldots, k_N) = g_{\text{closed}}^{N-2} \int d\mu_N(z) \prod_{i<j} |z_i - z_j|^{\alpha k_i k_j}, \]

where \( z_1, z_2, \ldots, z_N \) are complex coordinates, any three of which are \( z_A, z_B, z_C \), and

\[ d\mu_N(z) = |(z_A - z_B)(z_B - z_C)(z_C - z_A)|^2 \]

\[ \times \delta^2(z_A - z_A^0)\delta^2(z_B - z_B^0)\delta^2(z_C - z_C^0) \prod_{i=1}^{N} d^2 z_i. \]

The formula is independent of \( z_A^0, z_B^0, z_C^0 \) due to its \( SL(2,\mathbb{C}) \) symmetry, which allows them to be mapped to arbitrary complex values. This amplitude has total symmetry in the \( N \) external lines.

Both of these formulas were shown to have a consistent factorization on a spectrum of single-particle states described by an infinite number of harmonic oscillators [31, 32, 33, 34, 35]

\[ \{a_{\mu m}^\dagger \} \quad \mu = 0, 1, \ldots, d-1 \quad m = 1, 2, \ldots \]

with one set of such oscillators in the Veneziano case and two sets in the Virasoro case. These results were interpreted as describing the scattering of modes of a relativistic string [35, 36, 37, 38, 39, 40]: open strings in the first case and closed strings in the second case.

Amazingly, the formulas preceded the interpretation. Although, we did not propose a string interpretation, Gross, Neveu, Scherk, and I did realize that the relevant diagrams of the
loop expansion were classified by the possible topologies of two-dimensional manifolds with boundaries [41].

Having found the factorization, it became possible to compute radiative corrections (loop amplitudes). This was initiated by Kikkawa, Sakita, and Virasoro [42] and followed up by many others. Let me describe my role in this. I was at Princeton, where I collaborated with Gross, Neveu, and Scherk in computing one-loop amplitudes. In particular, we discovered unanticipated singularities in the “nonplanar” open-string loop diagram [43]. The world sheet is a cylinder with two external particles attached to each boundary. Our computations showed that this diagram gives branch points that violate unitarity. This was a very disturbing conclusion, since it seemed to imply that the classical theory does not have a consistent quantum extension. This was also discovered by Frye and Susskind [44]. (The issue of quantum consistency turned out to be a recurring theme, which reappeared many years later, as discussed in Section 7.)

Soon thereafter Claude Lovelace pointed out [45] that these branch points become poles provided that
\[ \alpha(0) = 1 \quad \text{and} \quad d = 26. \]

Until Lovelace’s work, everyone assumed that the spacetime dimension was \( d = 4 \).
\(^{5}\) As we were not yet talking about gravity, there was no reason to consider anything else. Later, these poles were interpreted as closed-string modes in a one-loop open-string amplitude. Nowadays this is referred to as open string–closed string duality.

Lovelace’s analysis also required there to be an infinite number of decoupling conditions. These turned out to be precisely the Virasoro constraints, which were discovered at about the same time [46, 47]. A couple of years later Brink and Olive constructed a physical-state projection operator [48], which they used to verify Lovelace’s conjecture that the nonplanar loop amplitude actually contains closed-string poles when the decoupling conditions in the critical dimension are imposed [49].

Thus, quantum consistency was restored, but the price was high: a spectrum with a tachyon and 22 extra dimensions of space. In 1973, the origin of the critical dimension and the intercept condition were explained in terms of the light-cone gauge quantization of a fundamental string by Goddard, Goldstone, Rebbi, and Thorn [50]. Prior to this paper the string interpretation of dual models was only a curiosity. The GGRT approach was extended to interacting strings by Mandelstam [51].

\(^{5}\)The idea of considering a higher dimension was suggested to Lovelace by David Olive.
4 1971 – 73: The RNS model

In January 1971 Pierre Ramond constructed a dual-resonance model generalization of the Dirac equation [52]. He reasoned as follows: just as the total momentum of a string, $p^\mu$, is the zero mode of a momentum density $P^\mu(\sigma)$, so should the Dirac matrices $\gamma^\mu$ be the zero modes of densities $\Gamma^\mu(\sigma)$. Then he defined the modes of $\Gamma \cdot P$:

$$F_n = \int_0^{2\pi} e^{-i n \sigma} \Gamma \cdot P d\sigma \quad n \in \mathbb{Z}.$$ 

In particular,

$$F_0 = \gamma \cdot p + \text{oscillator terms}.$$

He proposed the wave equation

$$(F_0 + m)|\psi\rangle = 0,$$

which is now known as the Dirac–Ramond Equation. Its solutions give the spectrum of a noninteracting fermionic string.

Ramond also observed that the Virasoro algebra generalizes to

$$\{F_m, F_n\} = 2L_{m+n} + \frac{c}{3}m^2 \delta_{m,-n}$$

$$[L_m, F_n] = (\frac{m}{2} - n)F_{m+n}$$

$$[L_m, L_n] = (m - n)L_{m+n} + \frac{c}{12}m^3 \delta_{m,-n}.$$ 

The free fermion spectrum should be restricted by the super-Virasoro constraints $F_n|\psi\rangle = L_n|\psi\rangle = 0$ for $n > 0$.

André Neveu and I proposed a new bosonic dual model, which we called the dual pion model, in March 1971 [53]. It has a similar structure to Ramond’s free fermion theory, with the periodic density $\Gamma^\mu(\sigma)$ replaced by an antiperiodic one $H^\mu(\sigma)$. Then the modes

$$G_r = \int_0^{2\pi} e^{-i r \sigma} H \cdot P d\sigma \quad r \in \mathbb{Z} + 1/2$$

satisfy a similar super-Virasoro algebra. The free particle spectrum is given by the wave equation $(L_0 - 1/2)|\psi\rangle = 0$ supplemented by the constraints $G_r|\psi\rangle = 0$ for $r > 0$. (These

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6His paper does not include the central terms.

7We submitted another publication [54] one month earlier that contained some, but not all, of the right ingredients.
formulas are appropriate in the $F_2$ picture discussed below.) We also constructed $N$-particle amplitudes analogous to those of the Veneziano model.

The $\pi + \pi \rightarrow \pi + \pi$ amplitude computed in the dual pion model turned out to have exactly the form that had been proposed earlier by Lovelace and Shapiro. However, the intercepts of the $\pi$ and $\rho$ Regge trajectories were $\alpha_{\pi}(0) = 1/2$ and $\alpha_{\rho}(0) = 1$. These were half a unit higher than was desired in each case. This implied that the pion was tachyonic and the rho was massless.

Soon after our paper appeared, Neveu traveled to Berkeley, where there was considerable interest in our results. This led to Charles Thorn (a student of Stanley Mandelstam at the time) joining us in a follow-up project in which we proved that the super-Virasoro constraints were fully implemented [55]. This required recasting the original description of the string spectrum (called the $F_1$ picture) in a new form, which we called the $F_2$ picture. The three of us then assembled these bosons together with Ramond’s fermions into a unified interacting theory of bosons and fermions [56, 57], thereby obtaining an early version of what later came to be known as superstring theory.

The string world-sheet theory that gives this spectrum of bosons and fermions is

$$S = \int d\sigma d\tau \left( \partial_\alpha X^\mu \partial^\alpha X_\mu - i \bar{\psi}_\mu \rho^\alpha \partial_\alpha \psi_\mu \right),$$

where $\psi_\mu$ are two-dimensional Majorana spinors and $\rho^\alpha$ are two-dimensional Dirac matrices. Later in 1971 Gervais and Sakita observed [58] that this action has two-dimensional global supersymmetry described by the infinitesimal fermionic transformations

$$\delta X^\mu = \bar{\epsilon} \psi^\mu,$$

$$\delta \psi^\mu = -i \rho^\alpha \epsilon \partial_\alpha X^\mu.$$

This is actually part of a much larger superconformal symmetry. There are two possible choices of boundary conditions for the fermi fields $\psi^\mu$, one of which gives the boson spectrum (Neveu–Schwarz sector) and the other of which gives the fermion spectrum (Ramond sector). Five years later, a more fundamental world-sheet action with local supersymmetry was discovered [60, 61]. It has the virtue of accounting for the super-Virasoro constraints as arising from covariant gauge fixing.

Bruno Zumino explored the RNS string’s gauge conditions associated to the two-dimensional superconformal algebra [62]. Following that, he and Julius Wess began to consider the possibility of constructing four-dimensional field theories with analogous features. This resulted
in their famous work [63] on globally supersymmetric field theories in four dimensions. As a consequence of their paper,\(^8\) supersymmetry quickly became an active research topic.

The dual pion model has a manifest \( \mathbb{Z}_2 \) symmetry. Since the pion is odd and the rho is even, this symmetry was identified with \( G \) parity.\(^9\) It was obvious that one could make a consistent truncation (at least at tree level) to the even \( G \)-parity sector and that then the model would be tachyon free. Because of the desired identification with physical hadrons, there was no motivation (at the time) to do that. Rather, considerable effort was expended in the following years attempting to modify the model so as to lower the intercepts by half a unit. As discussed in the next section, none of these constructions was entirely satisfactory.

One of the important questions in this period was whether all the physical string excitations have a positive norm. States of negative norm (called \textit{ghosts}) would represent a breakdown of unitarity and causality, so it was essential that they not be present in the string spectrum. The first proof of the \textit{no-ghost theorem} for the original bosonic string theory was achieved by Brower [66], building on earlier work by Del Giudice, Di Vecchia, and Fubini [67]. This work showed that a necessary condition for the absence of ghosts is \( d \leq 26 \), and that the critical value \( d = 26 \) has especially attractive features, as we already suspected based on the earlier observations of Lovelace.

I generalized Brower’s proof of the no-ghost theorem to the RNS string theory and showed that \( d = 10 \) is the critical dimension and that the ground state fermion should be massless [68]. This was also done by Brower and Friedman a bit later [69]. An alternative, somewhat simpler, proof of the no-ghost theorem for both of the string theories was given by Goddard and Thorn at about the same time [70]. Other related work included [71, 72, 73].

Later in 1972, thanks to the fact that Murray Gell-Mann had become intrigued by my work with Neveu, I was offered a senior research appointment at Caltech. I think that the reason Gell-Mann became aware of our work was because he spent the academic year 1971-72 on a sabbatical at CERN, where there was an active dual models group. I felt very fortunate to receive such an offer, especially in view of the fact that the job market for theoretical physicists was extremely bad at the time. Throughout the subsequent years at Caltech, when my work was far from the mainstream, and therefore not widely appreciated, Gell-Mann was always very supportive. For example, he put funds at my disposal to invite visitors. This facilitated various collaborations with Lars Brink, Joël Scherk, and Michael

\(^8\)The work of Golfand and Likhtman [64], which was the first to introduce the four-dimensional super-Poincaré group, was not known in the West at that time.

\(^9\)\( G \) parity is a hadronic symmetry that is a consequence of charge conjugation invariance and isotopic spin symmetry.
Green among others.

One of the first things I did at Caltech was to study the fermion-fermion scattering amplitude. Using the physical-state projection operator $[48]$, Olive and Scherk had derived a formula that involved the determinant of an infinite matrix $[74]$. C.C. Wu and I $[75]$ discovered that this determinant is a simple function. We derived the result analytically in a certain limit and then verified numerically that it is exact everywhere. (The result was subsequently verified analytically $[76]$.) To our surprise, the fermion-fermion scattering amplitude ended up looking very similar to the bosonic amplitudes. This might have been interpreted as a hint of spacetime supersymmetry, but this was before the Wess–Zumino paper, and that was not yet on my mind.

String theory is formulated as an on-shell S-matrix theory in keeping with its origins discussed earlier. However, the SLAC deep inelastic scattering experiments in the late 1960s made it clear that the hadronic component of the electromagnetic current is a physical off-shell quantity, and that its asymptotic properties imply that hadrons have hard pointlike constituents. With this motivation, I tried for the next year or so to construct off-shell amplitudes. Although some intriguing results were obtained $[77, 78, 79]$, this was ultimately unsuccessful. Moreover, all indications were that strings were too soft to describe hadrons with their pointlike constituents.

At this point there were many good reasons to stop working on string theory: a successful and convincing theory of hadrons (QCD) was discovered, and string theory had many severe problems as a hadron theory. These included an unrealistic spacetime dimension, an unrealistic spectrum, and the absence of pointlike constituents. Also, convincing theoretical and experimental evidence for the standard model was rapidly falling into place. Understandably, given these successes and string theory’s shortcomings, string theory rapidly fell out of favor. What had been a booming enterprise involving several hundred theorists rapidly came to a grinding halt.

Given that the world-sheet descriptions of the two known string theories have conformal invariance and superconformal invariance, it was a natural question whether one could obtain new string theories described by world-sheet theories with extended superconformal symmetry. The $N = 2$ case was worked out in $[80]$. The critical dimension is four, but the signature has to be $(2, 2)$. For a long time it was believed that the critical dimension of the $N = 4$ string is negative, but in 1992 Siegel argued that (due to the reducibility of the constraints) the $N = 4$ string is the same as the $N = 2$ string $[81]$. 

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5 1974 – 75: Gravity and unification

The string theories that were known in the 1970s (the bosonic string and the RNS string) had many shortcomings as a theory of hadrons. The most obvious of these was the necessity of an unrealistic spacetime dimension (26 or 10). Another is the occurrence of tachyons in the spectrum, which implies that the vacuum is unstable. However, the one that bothered us the most was the presence of massless particles in the spectrum, which do not occur in the hadron spectrum.

In both string theories the spectrum of open strings contains massless spin 1 particles, and the spectrum of closed strings contains a massless spin 2 particle as well as other massless particles (a dilaton and an antisymmetric tensor in the case of oriented bosonic strings). These particles lie on the leading Regge trajectories in their respective sectors. Thus, the leading open-string Regge trajectory has intercept \( \alpha(0) = 1 \), and the leading closed-string Regge trajectory has intercept \( \alpha(0) = 2 \). Is was tempting (in the RNS model) to identify the leading open-string trajectory as the one for the \( \rho \) meson. In fact, it had most of the properties expected for that case except that the empirical intercept is about \( \alpha_{\rho}(0) = 1/2 \).

The leading closed-string trajectory carries vacuum quantum numbers, as expected for the Pomeron trajectory. Moreover, the factor of two between the open-string and closed-string Regge slopes is approximately what is required by the data. However, the Pomeron intercept was also double of the desired value \( \alpha(0) = 1 \), which is the choice that could account for the near constancy (up to logarithmic corrections) of hadronic total cross sections at high energy.

For these reasons we put considerable effort in the years 1972–74 into modifying the RNS theory in such a way as to lower all open-string Regge trajectories by half a unit and all closed-string Regge trajectories by one unit. Some successes along these lines actually were achieved, accounting for some aspects of chiral symmetry and current algebra [82, 83]. However, none of the schemes was entirely consistent. The main problem is that changing the intercepts and the spacetime dimensions meant that the Virasoro constraints were not satisfied, and so the spectrum was not ghost-free. Also, the successes for nonstrange mesons did not extend to mesons made from heavy quarks, and the Ramond fermions didn’t really look like baryons.

The alternative to modifying string theory to get what we wanted was to understand better what the theory was giving without modification. String theories in the critical dimension clearly were beautiful theories, with a remarkably subtle and intricate structure,
and they ought to be good for something. The fact that they were developed in an attempt to understand hadron physics did not guarantee that this was necessarily their appropriate physical application. Furthermore, the success of QCD made the effort to formulate a string theory of hadrons less pressing.

The first indication that such an agnostic attitude could prove worthwhile was a pioneering work by Neveu and Scherk [85], which studied the interactions of the massless spin 1 open-string particles at low energies (or, equivalently, in the *zero-slope limit*) and proved that their interactions agreed with those of Yang–Mills gauge particles in the adjoint representation of the Chan–Paton group. In other words, open-string theory was Yang–Mills gauge theory modified by higher dimension interactions at the string scale. This implies that the Chan–Paton group is actually a Yang–Mills gauge group. Prior to this work by Neveu and Scherk, it was always assumed that the Chan–Paton symmetry is a global symmetry.

I am struck by the fact that Yang and Mills in their original paper on SU(2) gauge theory [84], tried to identify the gauge symmetry with isotopic spin symmetry and the gauge fields with \( \rho \) mesons. In our failed efforts to describe hadrons, we had been making essentially the same mistake.

I arranged for Joël Scherk, with whom I had collaborated in Princeton, to visit Caltech in the winter and spring of 1974. Our interests and attitudes in physics were very similar, and so we were anxious to start a new collaboration. Each of us felt that string theory was too beautiful to be just a mathematical curiosity. It ought to have some physical relevance. We had frequently been struck by the fact that string theories exhibit unanticipated miraculous properties. What this means is that they have a very deep mathematical structure that is not fully understood. By digging deeper one could reasonably expect to find more surprises and then learn new lessons. Therefore, despite the fact that the rest of the theoretical high energy physics community was drawn to the important project of exploring the standard model, we wanted to explore string theory.

Since my training was as an elementary particle physicist, gravity was far from my mind in early 1974. Traditionally, elementary particle physicists had ignored the gravitational force, which is entirely negligible under ordinary circumstances. For these reasons, we were not predisposed to interpret string theory as a physical theory of gravity. General relativists, the people who did study gravity, formed a completely different community. They attended different meetings, read different journals, and had no need for serious communication with

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10This work relating string theory and Yang–Mills theory followed an earlier study by Scherk describing how to obtain \( \phi^3 \) field theory in the zero-slope limit [86].
particle physicists, just as particle physicists felt they had no need for relativists who studied topics such as black holes or the early universe.

Despite all this, we decided to do what could have been done two years earlier: we explored whether it is possible to interpret the massless spin 2 state in the closed-string spectrum as a graviton. This required carrying out an analysis analogous to the earlier one of Neveu and Scherk. This time one needed to decide whether the interactions of the massless spin 2 particle in string theory agree at low energy with those of the graviton in general relativity (GR). Success was inevitable, because GR is the only consistent possibility at low energies (i.e., neglecting corrections due to higher-dimension operators), and critical string theory certainly is consistent. At least, it contains the requisite gauge invariances to decouple all but the transverse polarizations. Therefore, the harder part of this work was forcing oneself to ask the right question. Finding the right answer was easy. In fact, by invoking certain general theorems, due to Weinberg [87], we were able to argue that string theory agrees with general relativity at low energies [88]. Although we were not aware of it at the time, Tamiaki Yoneya had obtained the same result somewhat earlier [89, 90].

In [88], Scherk and I proposed to interpret string theory as a quantum theory of gravity, unified with the other forces. This meant taking the whole theory seriously, not just viewing it as a framework for deriving GR and Yang–Mills theory as limits. Our paper was entitled *Dual Models for Non-Hadrons*, which I think was a poor choice. It emphasized the fact that we were no longer trying to describe hadrons and their interactions, but it failed to emphasize what we were proposing to do instead. A better choice would have been *String Theory as a Quantum Theory of Gravity Unified with the Other Forces*.

This proposal had several advantages: First, gravity was required by both of the known critical string theories. A forceful way of expressing this is the assumption that the fundamental physical entities are strings predicts the existence of gravity. In fact, even today, this is really the only direct experimental evidence that exists in support of string theory, though there are many other reasons to take string theory seriously.

Second, string theories are free from the UV divergences that typically appear in point-particle theories of gravity. The reason can be traced to the extended structure of strings. Specifically, string world sheets are smooth, even when they describe interactions. So they do not have the short-distance singularities that are responsible for UV divergences. These divergences always occur when one attempts to interpret general relativity (with or without matter) as a quantum field theory, since ordinary Feynman diagrams do have short-distance
This can also be expressed forcefully by saying the assumption that the fundamental physical entities are point particles predicts that gravity does not exist! I found this line of reasoning very compelling.\(^{12}\)

Third, extra dimensions could be a very good thing, rather than a problem, since in a gravity theory the geometry of spacetime is determined by the dynamics. (Prior to our proposal, their appearance as critical dimensions of string theories was viewed as a shortcoming of the theories rather than as a reason to study their possible implications.) In the gravitational setting one could imagine that the equations of motion would require (or at least allow) the extra dimensions to form a very small compact manifold. In 1974 there had been essentially no work on Kaluza–Klein theory for many years (and certainly none in the particle physics community) at the time of this work, so the notion of extra dimensions seemed very bizarre to most particle theorists. It is discussed so much nowadays that it is easy to forget this fact. Since the only length scale in string theory is the string scale, determined by the string tension, that would be the natural first guess for the size of the compact space. Given this assumption, the value of Newton’s constant in four dimensions can be deduced. The observed strength of gravity requires a Regge slope \(\alpha' \sim 10^{-38} \text{GeV}^{-2}\) instead of \(\alpha' \sim 1 \text{GeV}^{-2}\), which is the hadronic value. Thus, the change in interpretation meant that the tension of the strings, which is proportional to the reciprocal of \(\alpha'\), needed to be increased by 38 orders of magnitude. Equivalently, the size of the strings decreased by 19 orders of magnitude. This was a big conceptual leap, though the mathematics was unchanged.

Fourth, unification of gravity with other forces described by Yang–Mills theories was automatic when open strings are included. Of course, it was immediately clear that the construction of a realistic vacuum would be a great challenge. Indeed, that is where much of the effort these days is focussed. Other ways of incorporating gauge interactions in string theory were discovered many years later. These include heterotic string theory, where gauge fields appear as closed-string modes in ten dimensions, coincident D-brane world-volume theories, and certain types of singularities in M theory or F theory.

Scherk and I were very excited by the possibility that string theory could be the Holy

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\(^{11}\)There has been recent speculation about the possible finiteness of \(\mathcal{N} = 8\) supergravity. If true, this would be a counterexample to my assertion. Whether or not this is the case, the nonperturbative completion of \(\mathcal{N} = 8\) supergravity is likely to lead one back to ten-dimensional superstring theory [91].

\(^{12}\)In light of subsequent developments, the distinction made here no longer seems quite so sharp. A single theory can have dual descriptions based on different fundamental entities. One description is weakly coupled when the other one is strongly coupled. For example, certain AdS/CFT duality relates string theory in a particular background geometry to a more conventional quantum field theory.
Grail of unified field theory, overcoming the problems that had stymied other approaches. In addition to publishing our work in scholarly journals, we gave numerous lectures at conferences and physics departments all over the world. We even submitted a paper entitled Dual Model Approach to a Renormalizable Theory of Gravitation to the 1975 essay competition of the Gravity Research Foundation [92]. The first paragraph of that paper reads as follows:

A serious shortcoming of Einstein’s theory of gravitation is the nonrenormalizability of its quantum version when considered in interaction with other quantum fields. In our opinion this is a genuine problem requiring a modification of the theory. It is suggested in this essay that dual resonance models may provide a suitable framework for such a modification, while at the same time achieving a unification with other basic interactions.

For the most part our work was received politely — as far as I know, no one accused us of being crackpots. Yet, for a decade, very few experts showed much interest. Part of the problem may have been that some key people were unaware of our proposal. Unfortunately, Scherk passed away midway through this 10-year period, though not before making some other important contributions that are discussed in the following sections. In the decade following its publication, our paper [88] only received about 20 citations in papers written by people other than Scherk or myself. The authors of these papers include the following distinguished physicists: Lars Brink, Peter Freund, Michael Green, Bernard Julia, David Olive, Tamiaki Yoneya, and Bruno Zumino. After the subject took off in the autumn of 1984, [88] became much better known.

6 1975 – 79: Supersymmetry and supergravity

Following the pioneering work of Wess and Zumino, discussed earlier, the study of supersymmetric quantum field theories became a major endeavor. One major step forward was the realization that supersymmetry can be realized as a local symmetry. This requires including a gauge field, called the gravitino field, which is vector-spinor. In four dimensions it describes a massless particle with spin 3/2, which is the supersymmetry partner of the graviton. Thus, local supersymmetry only appears in gravitational theories, which are called supergravity theories.

The first example of a supergravity theory was $\mathcal{N} = 1$, $d = 4$ supergravity. It was formulated in a second-order formalism by Ferrara, Freedman, and Van Nieuwenhuizen [93] and subsequently in a first-order formalism by Deser and Zumino [94]. The first-order formalism

\footnote{It would have been better to say ultraviolet finite instead of renormalizable.}
simplifies the analysis of terms that are quartic in fermi fields.

The two-dimensional locally supersymmetric and reparametrization invariant formulation of the RNS world-sheet action was constructed very soon thereafter [60, 61].\textsuperscript{14} This construction was generalized to the $N = 2$ string of [80] by Brink and me [95]. Reparametrization-invariant world-sheet actions of this type are frequently associated with the name Polyakov, because he used them very skillfully five years later in constructing the path-integral formulation of string theory [96, 97]. Since neither Polyakov nor the authors of [60, 61] are happy with this usage, the new textbook [6] refers to this type of world-sheet action as a \textit{string sigma-model action}.

The RNS closed-string spectrum contains a massless gravitino (in ten dimensions) in addition to the graviton discussed in the previous section.\textsuperscript{15} Since this is a gauge field, the only way the theory could be consistent is if the theory has local supersymmetry. This requires, in particular, that the spectrum should contain an equal number of bosonic and fermionic degrees of freedom at each mass level. However, as it stood, this was not the case. In particular, the bosonic sector contained a tachyon (the “pion”), which had no fermionic partner.

In 1976 Gliozzi, Scherk, Olive [98, 99] proposed a projection of the RNS spectrum – \textit{the GSO Projection} – that removes roughly half of the states (including the tachyon). Specifically, in the bosonic (NS) sector they projected away the odd G-parity states, a possibility that was discussed earlier, and in the fermionic (R) sector they projected away half the states, keeping only certain definite chiralities. Then they counted the remaining physical degrees of freedom at each mass level. After the GSO projection the masses of open-string states, for both bosons and fermions, are given by $\alpha' M^2 = n$, where $n = 0, 1, \ldots$ Denoting the open-string degeneracies of states in the GSO-projected theory by $d_{\text{NS}}(n)$ and $d_{\text{R}}(n)$, they showed that these are encoded in the generating functions

$$f_{\text{NS}}(w) = \sum_{n=0}^{\infty} d_{\text{NS}}(n) w^n$$

$$= \frac{1}{2\sqrt{w}} \left[ \prod_{m=1}^{\infty} \left( \frac{1 + w^{m-1/2}}{1 - w^m} \right)^8 \right. - \left. \prod_{m=1}^{\infty} \left( \frac{1 - w^{m-1/2}}{1 - w^m} \right)^8 \right].$$

\textsuperscript{14}This generalized the one-dimensional result obtained a bit earlier for a spinning point particle [59].

\textsuperscript{15}More precisely, as was understood later, there are one or two gravitinos depending on whether one is describing a type I or type II superstring.
and

\[ f_R(w) = \sum_{n=0}^{\infty} d_R(n) w^n = 8 \prod_{m=1}^{\infty} \left( \frac{1 + w^m}{1 - w^m} \right)^8. \]

In 1829, Jacobi proved the remarkable identity [100]

\[ f_{NS}(w) = f_R(w), \]

though he used a different notation. Thus, there are an equal number of bosons and fermions at every mass level, as required. This was compelling evidence (though not a proof) for ten-dimensional \textit{spacetime supersymmetry} of the GSO-projected theory. Prior to this work, one knew that the RNS theory has world-sheet supersymmetry, but the realization that the theory should have spacetime supersymmetry was a major advance.

Since a Majorana–Weyl spinor in ten dimensions has 16 real components, the minimal number of supercharges is 16. In particular, the massless modes of open superstrings at low energies are approximated by an \( \mathcal{N} = 1, d = 10 \) super Yang–Mills theory with 16 supersymmetries. This theory was constructed in [99, 101]. When this work was done, Brink and I were at Caltech and Scherk was in Paris. Brink and I wrote to Scherk informing him of our results and inviting him to join our collaboration, which he gladly accepted. Brink and I were unaware of the GSO collaboration, which was underway at that time, until their work appeared. Both papers pointed out that maximally supersymmetric Yang–Mills theories in less than ten dimensions could be deduced by dimensional reduction, and both of them constructed the \( \mathcal{N} = 4, d = 4 \) super Yang–Mills theory explicitly.

Having found the maximally supersymmetric Yang–Mills theories, it was an obvious problem to construct the maximally supersymmetric supergravity theories. Nahm showed [102] that the highest possible spacetime dimension for such a theory is \( d = 11 \). Soon thereafter, in a very impressive work, the Lagrangian for \( \mathcal{N} = 1, d = 11 \) supergravity was constructed by Cremmer, Julia, and Scherk [103]. It was immediately clear that 11-dimensional supergravity is very beautiful, and it aroused a lot of interest. However, it was puzzling for a long time how it fits into the greater scheme of things and whether it has any connection to string theory. Clearly, supergravity in 11 dimensions is not a consistent quantum theory by itself, since it is very singular in the ultraviolet. Moreover, since superstring theory only has ten dimensions, it did not seem possible that it could serve as a regulator. It took more than fifteen years to find the answer to this conundrum [107, 108]: At strong coupling Type IIA superstring theory develops a circular 11th dimension whose radius grows with the string coupling constant. In the limit of infinite coupling one obtains
M theory, which is presumably a well-defined quantum theory that has 11 noncompact dimensions. Eleven-dimensional supergravity is the leading low-energy approximation to M theory. In other words, M theory is the UV completion of 11-dimensional supergravity.

In 1978–79, I spent the academic year at the École Normale Supérieure in Paris supported by a Guggenheim Fellowship. I was eager to work with Joël Scherk on supergravity, supersymmetrical strings, and related matters. After various wide-ranging discussions we decided to focus on the problem of supersymmetry breaking. We wondered how, starting from a supersymmetric string theory in ten dimensions, one could end up with a nonsupersymmetric world in four dimensions. The specific supersymmetry breaking mechanism that we discovered can be explained classically and does not really require strings, so we explored it in a field theoretic setting [104, 105]. The idea is that in a theory with extra dimensions and global symmetries that do not commute with supersymmetry ($R$ symmetries and $(-1)^F$ are examples), one could arrange for a twisted compactification, and that this would break supersymmetry. For example, if one extra dimension forms a circle, the fields when continued around the circle could come could back transformed by an $R$-symmetry group element. If the gravitino, in particular, is transformed then it acquires mass in a consistent manner.

An interesting example of our supersymmetry breaking mechanism was worked out in a paper we wrote together with Eugène Cremmer [106]. We started with maximal supergravity in five dimensions. This theory contains eight gravitinos that transform in the fundamental representation of a $\text{USp}(8)$ $R$-symmetry group. We took one dimension to form a circle and examined the resulting four-dimensional theory keeping the lowest Kaluza–Klein modes. The supersymmetry-breaking $R$-symmetry element is a $\text{USp}(8)$ element that is characterized by four real mass parameters, since this group has rank four. These four masses give the masses of the four complex gravitinos of the resulting four-dimensional theory. In this way we were able to find a consistent four-parameter deformation of $\mathcal{N} = 8$ supergravity.

Even though the work that Joël Scherk and I did on supersymmetry breaking was motivated by string theory, we only discussed field theory applications in our articles. The reason I never wrote about string theory applications was that in the string theory setting it did not seem possible to decouple the supersymmetry breaking mass parameters from the compactification scales. This was viewed as a serious problem, because the two scales are supposed to be hierarchically different. In recent times, people have been considering string theory brane-world scenarios in which much larger compactification scales are considered. In such a context our supersymmetry breaking mechanism might have a role to play. Indeed, quite a few authors have explored various such possibilities.
7 1979 – 84: Superstrings and Anomalies

Following Paris, I spent a month (July 1979) at CERN. There, Michael Green and I unexpectedly crossed paths. We had become acquainted in Princeton around 1970, when Green was at the IAS and I was at the University, but we had not collaborated before. In any case, following some discussions in the CERN cafeteria, we began a long and exciting collaboration. Our first goal was to understand better why the GSO-projected RNS string theory has spacetime supersymmetry.

Green, who worked at Queen Mary College London at the time, had several extended visits to Caltech in the 1980–85 period, and I had one to London in the fall of 1983. We also worked together several summers in Aspen. On several of these occasions we also collaborated with Lars Brink, who had visited Caltech and collaborated with me a few times previously.

After a year or so of unsuccessful efforts, Green and I discovered a new light-cone gauge formalism for the GSO-projected theory in which spacetime supersymmetry of the spectrum and interactions was easily proved. This was presented in three papers [109, 110, 111]. The first developed the formalism, while the next two used this light-cone gauge formalism to compute various tree and one-loop amplitudes and elucidate their properties. At this stage only open-string amplitudes were under consideration.

Our next project was to identify more precisely the possibilities for superstring theories. The GSO work had identified the proper projection for open strings, but it left unclear what one should do with the closed strings. Green and I realized that there are three distinct types of supersymmetry possible in ten dimensions and that all three of them could be realized by superstring theories. In [112] we formulated the type I, type IIA, and type IIB superstring theories. (We introduced these names a little later.) The type I theory is a theory of unoriented open and closed strings, whereas the type II theories are theories of oriented closed strings only.

Brink, Green, and I formulated $d$-dimensional maximally supersymmetric Yang–Mills theories and supergravity theories as limits of superstring theory with $10 - d$ of the ten dimensions forming a torus. By computing one-loop string-theory amplitudes for massless gauge particles in the type I theory and gravitons in the type II theory and taking the appropriate limits, we showed that both the Yang–Mills and supergravity theories are ultraviolet finite at one loop for $d < 8$ [113]. The toroidally compactified string-loop formulas exhibited T-duality symmetry, though this was not pointed out explicitly in the article.
We also spent considerable effort formulating superstring field theory in the light-cone gauge [114, 115, 116]. This work became relevant about 20 years later, when the construction was generalized to the case of type IIB superstrings in a plane-wave background spacetime geometry.

The fact that our spacetime supersymmetric formalism was only defined in the light-cone gauge was a source of frustration. Brink and I had found a covariant world-line action for a massless superparticle in ten dimensions [117], so it was just a matter of finding the suitable superstring generalization. After a number of attempts, Green and I eventually found a covariant world-sheet action with manifest spacetime supersymmetry (and non-manifest kappa symmetry) [119, 120]. This covariant action reduces to our previous one in the light cone gauge, of course. It was natural to try to use it to define covariant quantization. However, due to a subtle combination of first-class and second-class constraints, it was immediately apparent that this action is extremely difficult to quantize covariantly. Numerous unsuccessful attempts over the years bear testimony to the truth of this assertion. More recently, Berkovits seems to have found a successful scheme. However, as far as I can tell, its logical foundations are not yet entirely clear.

Another problem of concern during this period was the formulation of ten-dimensional type IIB supergravity, which is the leading low-energy approximation to type IIB superstring theory. Some partial results were obtained in separate collaborations with Green [121] and with Peter West [122]. A challenging aspect of the problem is the presence of a self-dual five-form field strength, which obstructs a straightforward construction of a manifestly covariant action. Therefore, I decided to focus on the equations of motion, instead, which I presented in [123]. Equivalent results were obtained in a superfield formalism by Howe and West [124].

Let me now turn to the issue of anomalies. Type I superstring theory is a well-defined ten-dimensional theory at tree level for any $SO(n)$ or $Sp(n)$ gauge group [125, 126]. However, in every case it is chiral (i.e., parity violating) and the $d = 10$ super Yang–Mills sector is anomalous. Evaluation of a one-loop hexagon diagram exhibits explicit nonconservation of gauge currents of the schematic form

$$\partial_\mu J^\mu \sim \varepsilon^{\mu_1 \cdots \mu_{10}} F_{\mu_1 \mu_2} \cdots F_{\mu_9 \mu_{10}},$$

which is a fatal inconsistency.

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16Casalbuoni considered similar superparticle systems in four dimensions several years earlier [118].

17Type IIA supergravity can be obtained by dimensional reduction of 11-dimensional supergravity, but type IIB supergravity cannot be obtained in this way.
Alvarez-Gaumé and Witten derived general formulas for gauge, gravitational, and mixed anomalies in an arbitrary spacetime dimension [127], and they discovered that the gravitational anomalies (nonconservation of the stress tensor) cancel in type IIB supergravity. This result was not really a surprise, since the one-loop type IIB superstring amplitudes are ultraviolet finite. It appeared likely that type I superstring theory is anomalous for any choice of the gauge group, but an explicit computation was required to decide for sure. In this case there are divergences that need to be regulated, so anomalies are definitely possible.

Green and I explored the anomaly problem for type I superstring theory off and on for almost two years until the crucial breakthroughs were made in August 1984 at the Aspen Center for Physics. That summer I was the organizer of a workshop entitled “Physics in Higher Dimensions” at the Aspen Center for Physics. This attracted many participants, even though string theory was not yet fashionable, because by that time there was considerable interest in supergravity theories in higher dimensions and Kaluza-Klein compactification. We benefitted from the presence of many leading experts including Bruno Zumino, Bill Bardeen, Dan Friedan, Steve Shenker, and others.

Green and I had tried unsuccessfully to compute the one-loop hexagon diagram in type I superstring theory using our supersymmetric light-cone gauge formalism, but this led to an impenetrable morass. In discussions with Friedan and Shenker the idea arose to carry out the computation using the covariant RNS formalism instead. At that point, Friedan and Shenker left Aspen, so Green and I continued on our own.

It soon became clear that both the cylinder and Möbius-strip diagrams contributed to the anomaly. Before a workshop seminar by one of the other workshop participants (I don’t remember which one), I remarked to Green that there might be a gauge group for which the two contributions cancel. At the end of the seminar Green said to me “SO(32),” which was the correct result. Since this computation only showed the cancellation of the pure gauge part of the anomaly, we decided to explore the low-energy effective field theory to see whether the gravitational and mixed anomalies could also cancel. Before long, with the help of the results of Alvarez-Gaumé and Witten and useful comments by Bardeen and others, we were able to explain how this works. The effective field theory analysis was written up first [128], and the string loop analysis was written up somewhat later [129]. We also showed that the UV divergences of the cylinder and Möbius-strip diagrams cancel for SO(32) [130]. Nowadays such cancellations are usually understood in terms of tadpole cancellations in a dual closed-string channel.

The effective field theory analysis showed that \( E_8 \times E_8 \) is a second gauge group for which
the anomalies could cancel for a theory with $\mathcal{N} = 1$ supersymmetry in ten dimensions. In both cases, it is crucial for the result that the coupling to supergravity is included. The $SO(32)$ case could be accommodated by type I superstring theory, but we didn’t know a superstring theory with gauge group $E_8 \times E_8$. We were aware of the article by Goddard and Olive that pointed out (among other things) that there are just two even-self-dual Euclidean lattices in 16 dimensions, and these are associated with precisely these two gauge groups [131]. However, we did not figure out how to exploit this fact before the problem was solved by others.

Before the end of 1984 there were two other major developments. The first one was the construction of the heterotic string by Gross, Harvey, Martinec, and Rohm [132, 133, 134]. Their construction actually accommodated both of the gauge groups. The second one was the demonstration by Candelas, Horowitz, Strominger, and Witten that Calabi–Yau compactifications of the $E_8 \times E_8$ heterotic string give supersymmetric four-dimensional effective theories with many realistic features [135].

By the beginning of 1985, superstring theory – with the goal of unification – had become a mainstream activity. In fact, there was a very sudden transition from benign neglect to unbounded euphoria, both of which seemed to me to be unwarranted. After a while, most string theorists developed a more realistic assessment of the problems and challenges that remained.

8 Postscript

The construction of a dual string theory description of QCD is still an actively pursued goal. It now appears likely that every well-defined (finite or asymptotically free) four-dimensional gauge theory has a string theory dual in a curved background geometry with five noncompact dimensions. The extra dimension corresponds to the energy scale of the gauge theory. The cleanest and best understood example of such a duality is the correspondence between $\mathcal{N} = 4$ supersymmetric Yang-Mills theory with an $SU(N)$ gauge group and type IIB superstring theory in an $AdS_5 \times S^5$ spacetime with $N$ units of five-form flux threading the sphere [136]. In particular, the (off-shell) energy–momentum tensor of the four-dimensional gauge theory corresponds to the (on-shell) graviton in five dimensions.

Such possibilities were not contemplated in the early years, so it understandable that success was not achieved. Moreover, the dual description of QCD is likely to be considerably more complicated than the example described above. For one thing, for realistic numbers of
colors and flavors, the five-dimensional geometry is expected to have string-scale curvature, so that a supergravity approximation will not be helpful. However, it might still be possible to treat the inverse of the number of colors as small, so that a semiclassical string theory approximation (corresponding to the planar approximation to the gauge theory) can be used. If one is willing to sacrifice quantitative precision, one can already give constructions that have the correct qualitative features of QCD. One of their typical unrealistic features is that the Kaluza–Klein scale is comparable to the QCD scale. I remain optimistic that a correct construction of a string theory configuration that is dual to QCD exists. However, finding it and analyzing it might take a long time.

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