Diffractive $J/\psi$ production through Color-Octet mechanism at hadron colliders

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Abstract

We propose the color-octet mechanism combined with the two gluon exchange model for the diffractive $J/\psi$ production in hadron collisions. In the leading logarithmic approximation (LLA) in QCD, we find that the diffractive $J/\psi$ production rate is related to the off-diagonal gluon density in the proton and to the nonperturbative color-octet matrix element of $J/\psi$. The rate is found to be very sensitive to the gluon density at very small values of $x$ (down to $x = O(10^{-6})$). As a result, this process may provide a wide window for testing the two-gluon exchange model, and may be particularly useful in studying the small $x$ physics. And it may also be a golden place to test the color-octet mechanism proposed by solving the $\psi' (J/\psi)$ surplus problem at the Tevatron.

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In recent years, there has been a renaissance of interest in diffractive scattering. These diffractive processes are described by the Regge theory in terms of the Pomeron ($P$) exchange \[1\]. The Pomeron carries quantum numbers of the vacuum, so it is a colorless entity in QCD language, which may lead to the “rapidity gap” events in experiments. However, the nature of the Pomeron and its interaction with hadrons remain a mystery. For a long time it had been understood that the dynamics of the “soft Pomeron” was deeply tied to confinement. However, it has been realized now that much can be learnt about QCD from the wide variety of small-$x$ and hard diffractive processes, which are now under study experimentally. Of all these processes, the diffractive heavy quarkonium production has drawn specially attention, because their large masses provide a natural scale to guarantee the application of perturbative QCD. In Refs. \[2,3\], the diffractive $J/\psi$ and $\Upsilon$ production cross sections have been formulated in photoproduction processes and in DIS processes in perturbative QCD. In the framework of perturbative QCD the Pomeron is assumed to be represented by a pair of gluons in the color-singlet state. This two-gluon exchange model can successfully describe the experimental results from HERA \[4\]. An important feature of this perturbative QCD model prediction is that the cross section for the diffractive $J/\psi$ production is expressed in terms of the square of the gluon density.

So far the previous studies are focused on the diffractive processes at $ep$ collider (photoproduction and DIS processes), we should expect that the two-gluon exchange model can also be used to describe the diffractive processes at hadron colliders. In this paper, we extend the idea of perturbative QCD description of diffractive processes from $ep$ colliders to hadron colliders. Unlike in the case of the photoproduction process, at hadron colliders (shown in Fig.1), in the diffractive process the final state $c\bar{c}$ pair, which carry the quantum number of the incident gluon, is in a color-octet configuration. This color-octet $c\bar{c}$ pair must evolve into the physical color-singlet bound state of $J/\psi$ for the experimental observation. Here we use the color-octet mechanism based on the NRQCD factorization formalism \[5\] to describe this evolving process. In the last few years, the application of the color-octet mechanism has gained some important success. For example, by including this color-octet mechanism one might explain the $\psi'$ ($J/\psi$) surplus measured by the CDF Collaboration at the Tevatron \[6\].

The diffractive $J/\psi$ production process in Fig.1 can be realized via the color-octet $^3S_1$ channel, in which the $c\bar{c}$ pair in a configuration of $^3S_1^{(8)}$ is produced in hard process as the incident gluon interacts with the proton by $t$ channel color-singlet exchange (the two-gluon ladder parametrized Pomeron), and then evolve into the physical state $J/\psi$ through the long distance process (nonperturbative). In this long distance process some soft gluons will be emitted to change the color of the $c\bar{c}$ pair to coincide with the quantum number of the final state $J/\psi$. The soft gluons carry little momentum and will cause little change to the final state $J/\psi$ spectrum in the diffractive processes. In the old color-singlet model \[7\] the emitted gluons are hard, which will lead to a quite different momentum distribution in experiment. In NRQCD, the long distance evolving process is described by the nonperturbative matrix elements of four-fermion operators. In the process we considered in this paper, the associated matrix element is $\langle O_8^{\psi}(^3S_1^{(8)}) \rangle$, which is also used to describe the large $p_t$ prompt $J/\psi$ production at the Tevatron and has been determined by several authors \[8\].

Based on the NRQCD factorization formalism, as a general factorization ansatz, the rate
for the diffractive process \( gp \to \psi p \) can be factorized into the short distance part and the long distance part as the following form,

\[
|A(gp \to J/\psi p)|^2 = |A(gp \to (c\bar{c})[^3S_1^{(8)}]p)|^2 \times |A((c\bar{c})[^3S_1^{(8)}] \to J/\psi)|^2. \tag{1}
\]

Here \( A(gp \to (c\bar{c})[^3S_1^{(8)}]p) \) is the short distance amplitude which can be calculated in perturbative QCD. The amplitude \( A((c\bar{c})[^3S_1^{(8)}] \to J/\psi) \) describes the long distance evolving process which is related to the nonperturbative color-octet matrix element as \( |A((c\bar{c})[^3S_1^{(8)}] \to J/\psi)|^2 = \langle \mathcal{O}_8^{\psi}(3S_1) \rangle / 24m_c \).

For the diffractive subprocesses, \( gp \to J/\psi p \), the leading contribution comes from the diagrams shown in Fig.2. Due to the positive signature of these diagrams (color-singlet state), we know that the real part of the amplitude cancels out in the leading logarithmic approximation. The first two diagrams are similar to those calculated in photoproduction process, and the rest diagrams are new due to the existence of the gluon-gluon interaction vertex. From the following calculations, we can see that the diagrams other than the first two are needed to guarantee the gauge invariance.

For convenience, we perform our calculations in terms of the Sudakov variables. That is, for the involved particles the four momenta are decomposed as, \( k_i = \alpha_i q + \beta_i p + \vec{k}_iT \), where \( q \) and \( p \) are the momenta of the incident gluon and the proton, \( q^2 = 0, p^2 = 0 \), and \( 2p \cdot q = W^2 = s \). Here \( s \) is the total c.m. energy of the gluon-proton system, \( \alpha_i \) and \( \beta_i \) are the momentum fraction of \( q \) and \( p \) carried by the momentum \( k_i \). \( k_T \) is the transverse momentum, which satisfies \( k_T \cdot q = 0, k_T \cdot p = 0 \). In our calculations, we set the momentum transfer \( t \) equal to zero, i.e., \( t = (k - k')^2 = 0 \). All of these Sudakov variables are fixed by considering the mass shell condition on the crossed lines shown in Fig.2. Take Fig.2(a) as an example, the Sudakov variables are,

\[
\alpha_k = \alpha_{k'} = -\frac{k_T^2}{s}, \quad \beta_k = -\frac{M_\psi^2 + 2k_T^2}{s}, \quad \beta_{k'} = -\frac{2k_T^2}{s},
\]

where \( M_\psi \) is the mass of \( J/\psi \) which is equal to \( 2m_c \).

Following Ref. [4], the calculations are straightforward. When we perform the integral over the loop momentum \( k \), the main large logarithmic contribution comes from the region \( \frac{1}{R_N} \ll k_T^2 \ll M_\psi^2 \) (\( R_N \) is the nucleon radius) [2]. So, we calculate the amplitude as an expansion of \( k_T^2 \).

For Fig.2(a), the imaginary part of the short distance amplitude \( A(gap \to (c\bar{c})b[^3S_1^{(8)}]p) \) is, to leading order contribution,

\[
\text{Im} A^{(a)} = F \times \frac{1}{9} \delta^{ab} \int \frac{dk_T^2}{k_T^2} \frac{1}{m_c^2} G(k), \tag{2}
\]

1 Exactly, the final state of this diffractive process should include some soft gluons. However, because the soft gluons only carry little momenta, they do not change the final state kinematic distribution much. In this paper, we neglect the soft gluon effects in the long distance evolving process.
where \( F = \frac{3\pi}{32\sqrt{2}} g^3 m s, \) \( a \) and \( b \) are the color indexes of the incident gluon and the \( c\bar{c} \) pair in color-octet \( 3S_1 \) state. Factor \( \frac{1}{9} \) is the color factor. The function \( G(k) \) specifies the probability of finding the gluon in the proton. In the simplest three valence quark model, \( G = \frac{4\alpha_s}{3\pi} \times 3. \) For Fig.2(b), the result is,

\[
\text{Im}\mathcal{A}^{(b)} = -F \times \left( -\frac{1}{72} \delta^{ab} \right) \int \frac{dk_T^2}{k_T^4} \frac{1}{4m_c^2 + k_T^2} G(k),
\]

(3)

where the color factor is \( -\frac{1}{72} \). Unlike in the case of the diffractive photoproduction processes, the color factors of these two diagrams (Fig.2(a) and Fig.2(b)) are not the same. The leading part of the contributions from these two diagrams (which is proportional to \( \frac{1}{k_T^4} \)) can not cancel out each other. After integrating the loop momentum \( k \), for small \( k_T \) this will lead to a linear singularity, not a logarithmic singularity (proper in QCD) as that in diffractive photoproduction process \[2\]. So, here in the case of diffractive process at hadron colliders, there must be some other diagrams to cancel out the leading part of Fig.2(a) and Fig.2(b) to obtain the correct result. This is also due to the gauge invariance requirement. As mentioned above, in QCD due to the nonabelian \( SU(3) \) gauge theory there are additional diagrams shown in Fig.2(c)-(e) as compared with that in photoproduction at \( ep \) colliders. By summing up all these diagrams together, we expect that in the final result the leading part singularity which is proportional to \( \frac{1}{k_T^4} \) will be canceled out, and only the terms proportional to \( \frac{1}{k_T^2} \) will be retained.

The contribution from Fig.2(c) is,

\[
\text{Im}\mathcal{A}^{(c)} = F \times \left( -\frac{1}{2} \delta^{ab} \right) \int \frac{dk_T^2}{k_T^2} \frac{1}{4m_c^2 + k_T^2} G(k),
\]

(4)

where \( -\frac{1}{2} \) is color factor. From Eqs. (2), (3), (4), we can see that the leading part singularity from Fig.2(a)-(c) are canceled out as expected.

By the same reason, for Fig.2(d) and Fig.2(e), the leading part of each diagram is proportional to \( \frac{1}{k_T^2} \). However, their sum is only proportional to \( \frac{1}{k_T^4} \) because the leading part is canceled out. Their final results is,

\[
\text{Im}\mathcal{A}^{(de)} = F \times \left( -\frac{1}{2} \delta^{ab} \right) \int \frac{dk_T^2}{k_T^2} \frac{2}{16m_c^4} G(k).
\]

(5)

Adding all the contributions from Fig.2(a)-(e), we get the imaginary part of the short distance amplitude,

\[
\text{Im}\mathcal{A}(gp \rightarrow (c\bar{c})[3S_1]p) = F \times \left( -\frac{13}{18} \delta^{ab} \right) \int \frac{dk_T^2}{k_T^2} \frac{1}{M_4^2} f(x', x'\prime; k_T^2),
\]

(6)

Here, we rewrite the function \( G(k) \) as \( G(k) = f(x', x'\prime; k_T^2), \)

\[
f(x', x'\prime; k_T^2) = \frac{\partial G(x', x'\prime; k_T^2)}{\partial \ln k_T^2}
\]

(7)

where the function \( G(x', x'\prime; k_T^2) \) is the so-called off-diagonal gluon distribution function \[8\]. Here, \( x' \) and \( x'\prime \) are the momentum fraction of the proton carried by the two gluons. It is
expected that for small $x$, there is no big difference between the off-diagonal and the usual diagonal gluon densities $\bar{Q}^2 = M_{\psi}^2$. So, in the following calculations, we estimate the production rate by approximating the off-diagonal gluon density by the usual diagonal gluon density, $G(x', x^g; Q^2) \approx xg(x, Q^2)$, where $x = M_{\psi}^2/s$.

Finally, in the leading logarithmic approximation (LLA), we obtain the imaginary part of the short distance amplitude,

$$\text{Im} A (gp \rightarrow (c\bar{c})[^3S_1^{(8)}]p) = \left( -\frac{13}{18} \delta_{ab} \right) \frac{F}{M_{\psi}^4} xg(x, Q^2),$$

where we set $\bar{Q}^2 = M_{\psi}^2$. The factorization scale in the gluon density is very important because we know that parton distributions at small $x$ change rapidly with this scale $\bar{Q}$. In the literature there are various choices $\bar{Q}$. Here, we choose the scale to be $M_{\psi} = 2m_c$ which is typically used in NRQCD factorization approach $\bar{Q}$.

Using Eqs. (10), we get the cross section for the diffractive subprocess $gp \rightarrow J/\psi p$,

$$\frac{d\hat{\sigma}(gp \rightarrow J/\psi p)}{dt}|_{t=0} = \frac{|A|^2}{16\pi s^2} = \frac{169\pi^4 m_c}{32 \times 27} \frac{\alpha_s(\bar{Q}^2)\langle\mathcal{O}_8^\psi[^3S_1]\rangle}{M_{\psi}^8} [xg(x, \bar{Q}^2)]^2. \quad (9)$$

We have also calculated the color-octet $^1S_0$ and $^3P_J$ subprocesses. As expected they do not contribute to the diffractive $J/\psi$ production at hadron colliders. That is, only the color-octet $^3S_1$ contributes in this process. So the diffractive $J/\psi$ production considered in this paper is sensitive to the matrix element $\langle\mathcal{O}_8^\psi[^3S_1]\rangle$, which is very important to describe the prompt $J/\psi$ production at the Tevatron $\bar{Q}$. Therefore, the diffractive $J/\psi$ production at hadron colliders would provide a golden place to test the color-octet mechanism in the heavy quarkonium production.

Provided the partonic cross section (10) above, we can get the cross section of diffractive $J/\psi$ production at hadron level. However, as we know, there exist nonfactorization effects in the hard diffractive processes at hadron collisions $\bar{Q}$. As argued by D.E. Soper, these effects may be accounted by a suppression factor $F_S$. At the Tevatron, the value of $F_S$ may be as small as $F_S \approx 0.1 \bar{Q}$. That is to say, the total cross section of the diffractive processes at the Tevatron may be reduced down by an order of magnitude due to the nonfactorization effects. So, the cross section for the diffractive process $pp(\bar{p}) \rightarrow J/\psi p$ can be formulated as

$$\frac{d\sigma(pp(\bar{p}) \rightarrow J/\psi p)}{dt}|_{t=0} = F_S \frac{169\pi^4 m_c}{32 \times 27} \frac{\alpha_s(\bar{Q}^2)\langle\mathcal{O}_8^\psi[^3S_1]\rangle}{M_{\psi}^8} \int_{x_{1\text{min}}} dx_1 g(x_1, \bar{Q}^2)[xg(x, \bar{Q}^2)]^2,$$

where $x_1$ is the longitude momentum fraction of the proton (or antiproton) carried by the incident gluon. So, the c.m. energy of the gluon-proton system is $s = x_1 S$, where $S$ is the total c.m. energy of the proton and proton (antiproton) system (e.g., $S = (1800 GeV)^2$ at the Tevatron). Then, $x = M_{\psi}^2/s = M_{\psi}^2/x_1 S$. We can see that at the Tevatron, $x$ may lower down to the $10^{-6}$ level. The lower bound of the integral variable $x_1$ is set to satisfy the relation $M_{\psi}^2 \ll s$ which guarantee the validity of our calculations for this high energy process.
For the numerical calculations, we choose the input parameters as, $M_\psi = 3.10\text{GeV}$, $\alpha_s(2m_c) = 0.27$, $\langle O^\psi_S(\bar{3}S_1) \rangle = 0.0106\text{GeV}^3$. The value of the color-octet matrix element is taken from [14]. For the parton distribution function, we use the GRV NLO set [17]. The region for $x$ is limited in GRV set to $10^{-5} < x < 1$, but in our case, the value of $x$ can reach the level down to $10^{-6}$, so we extrapolate the GRV set to these values according to their fitting parameters.

In Fig.3, we plot the differential cross section $d\sigma/dt (t = 0)$ as a function of $x_{1\text{min}}$ for $p\bar{p}$ collider at the Fermilab Tevatron. (The value of the suppression factor of $\mathcal{F}_S$ is taken as 0.1). From this figure, we see that the main contribution to the total cross section comes from the region of $x_1 > 10^{-3}$, which contributes 87% of the total cross section. $x_1 > 10^{-3}$ corresponds to the region of $x < 2.9 \times 10^{-3}$. For $x_1 > 10^{-2}$, the contribution is over 58%, which corresponds to the region of $x < 2.9 \times 10^{-4}$. For $x_1 > 10^{-1}$, the contribution is above 16%, which corresponds to $x < 2.9 \times 10^{-5}$. We can see that the gluon density at small $x$ is very important to the diffractive $J/\psi$ production at hadron colliders. Therefore, we may measure the gluon density at small $x$ by observe the diffractive $J/\psi$ production. At the Tevatron, the gluon density can be probed down to the level of $10^{-6}$ through this process. At the LHC, the situation is even more optimistic.

After integrating over $x_1$, we get the total cross section for the diffractive production $J/\psi$ at the Tevatron, $\sigma(p\bar{p} \rightarrow J/\psi p) = 66\text{nb}$. Here, we assume the $J/\psi$ diffractive slope $b$ to take the same value as in the photoproduction process, i.e., $b = 4.5\text{GeV}^{-2}$ [4]. We have also estimated the gluon $k_T$ effects following [2], which will increase the total cross section by a factor within 5%. Our result shows that the rate of this process is more than three order of magnitude larger than that of large $p_T$ $J/\psi$ production in diffractive process by soft Pomeron exchange [18], where the cross section at large $p_T$ ($\geq 8\text{GeV}$) is of order of 0.01nb.

However, the above results must be viewed as a rough estimate because in our calculations there are large theoretical uncertainties. The parton distribution function (PDF) is unknown to such low $x$ region, and different sets of PDF will result in different result for the cross section. And the main uncertainty may come from the charm quark mass because the cross section scales like the eighth power of $m_c$. Even a modest change in the charm mass $m_c$ will result in huge changes in the overall normalization of the cross section. Another uncertainty may come from the color-octet matrix element $\langle O^\psi_S(\bar{3}S_1) \rangle$. In our estimate, we follow the determination of Ref. [16]. So far our formula is only to the LLA level, higher order corrections are neglected. Nevertheless, as a first order approximation, our result may provide much sense about the diffractive $J/\psi$ production rate at hadron colliders.

Another important issue is about the nonfactorization effects of this process. In the above, we use the general suppression factor $\mathcal{F}_S$ to describe these effects. Further investigation about the nonfactorization effects is needed but is beyond the scope of this work. Nevertheless, we believe that the nonfactorization effects may not destroy the main results obtained in our paper, and the process in this paper may also be viewed as a test of the factorization of the hard diffractive processes in hadron collisions.

To sum up, in this paper, we give the formula for the diffractive $J/\psi$ production at hadron colliders in LLA QCD by using the two-gluon exchange model. We introduce the color-octet mechanism to realize the color-octet $c\bar{c}$ pair evolving into $J/\psi$ meson. Though there may exist large theoretical uncertainties in our calculations, our results show the importance of diffractive $J/\psi$ production at hadron colliders to the study of small $x$ physics, the property of
diffractive process, the nature of the Pomeron and even for the test of color-octet production mechanism of heavy quarkonium. We hope the experimental measurement will be carried on in the near future.

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Figure Captions

Fig.1. Sketch diagram for the diffractive $J/\psi$ production at hadron colliders in perturbative QCD. The black box represents the long distance process for color-octet $c\bar{c}$ pair in $^3S_1$ state evolving into $J/\psi$.

Fig.2. The lowest order perturbative QCD diagrams for $J/\psi$ production at hadron colliders.

Fig.3. The differential cross section $d\sigma/dt\ (t = 0)$ at the Fermilab Tevatron as a function of the lower bound of $x_1$ in the integral of Eq. (11).
FIGURES
Fig. 1
Fig. 2
Fig. 3

\[ \frac{d\sigma}{dt} \bigg|_{t=0} (10^{-4} \text{ GeV}) \times 1 \text{ (min)} \]