Ramond-Ramond Flux Stabilization of D-Branes

Jacek Pawelczyk\textsuperscript{a,c,d} and Soo-Jong Rey\textsuperscript{b,d}

Institute of Theoretical Physics, Warsaw University
Hoza 69, PL-00-681, Warsaw, Poland \textsuperscript{a}

School of Physics \& Center for Theoretical Physics
Seoul National University, Seoul 151-742 Korea \textsuperscript{b}

Institut für Theoretische Physik, Universität München
Theresienstr. 37, München, Germany \textsuperscript{c}

Erwin Schrödinger Institute for Mathematical Physics
Boltzmanngasse 9, Wien A-1090 Austria \textsuperscript{d}

jacek.pawelczyk@physik.uni-muenchen.de sjrey@gravity.snu.ac.kr

abstract

In $\text{AdS}_n \times S^m$ threaded by $N$-units of Ramond-Ramond flux, the dynamics of a $D_m$-brane wrapped partially on $S_{m-1} \subset S_m$ is investigated. Under the condition that flux of the dual gauge field on the D-brane worldvolume is quantized integrally, it is found that the wrapped D-brane is stable both locally and globally and, on AdS$_n$, behaves effectively as a fundamental string. It is also claimed that the semi-infinite, partially wrapped $D_m$-brane can ‘lasso’ around the $S^m$.

\textsuperscript{1} The work of S.-J.R. was supported in part by BK-21 Initiative in Physics (SNU Project-2), KRF International Collaboration Grant, KOSEF Interdisciplinary Research Grant 98-07-02-07-01-5, and KOSEF Leading Scientist Program 2000-1-11200-001-1. The work of J.P. was supported in part by the EC Contract HPRN-CT-2000-00152 and the Alexander-von-Humboldt Foundation.
1 Introduction

Recently, it has been shown that a D2-brane wrapped on \( S_2 \subset S_3 \) is a stable configuration \([1]\), despite that \( H_2(S_3) = 0 \). The stability is provided by a nontrivial NS-NS background potential, threaded through \( S_2 \). The argument of \([1]\) relied heavily on the CFT description of open strings in NS-NS background in terms of SU(2) WZW model \([2]\).

In this paper, we would like to explore further D-brane dynamics in a nontrivial background, in particular, in the R-R background. While some of the examples are related to the NS-NS background by S-duality, we will also uncover several new features as well as certain puzzling aspects, e.g. fate of the charges of the effective branes extending in \( AdS \) part of the background. We shall analyze the dynamics of branes in terms of the DBI action. Thus, most of the calculations would be similar to those of \([1]\). However, there is one important difference — for instance, in \( AdS_3 \times S^3 \) space, one could have disregarded the \( AdS \) part of the dynamics for NS-NS background, but not for R-R background.

The set-up of D-brane configurations considered in this paper will be as follows. We shall be looking for a classical configuration of \( D_m \)-brane embedded in \( AdS_n \times S^m \), in which flux of R-R tensor field is threaded through \( S^m \). In particular, we shall consider \( D_m \)-branes wrapped on \( S^{m-1} \subset S^m \) \((m = 3, 5)\) and show that they transmute into effective F-strings extended into the \( AdS \) space. In section 2 and 3, we will study D3-brane in \( AdS_3 \times S^3 \) space and D5-brane in \( AdS_5 \times S^5 \) space, respectively. In section 4, we will discuss charge nonconservation and relation to baryon vertex. We conclude, in section 5, with posing several unresolved, open problems.

2 D3-brane in \( AdS_3 \times S^3 \) (R-R)

2.1 Brane Configuration

We begin with classical configuration of a D3-brane in \( AdS_3 \times S^3 \). String theory in this background has been studied extensively in numerous recent works \([3,4]\). The relevant supergravity background in the near horizon limit is

\[
ds^2 = \ell^2 \alpha' \left[ u^2 (-dt^2 + (dx)^2) + \frac{du^2}{u^2} + d\Omega_3^2 \right]
\]

\[
H_{RR} = 2Q_5 \alpha' (\epsilon_3 + *_6 \epsilon_3)
\]

\[
e^{-2\phi} = \frac{1}{g_6^2 Q_1 Q_5}
\]

where \(*_6\) denotes Hodge duality in \( AdS_3 \times S^3 \) space, \( \epsilon_3 \) is the volume element of the unit 3-sphere, and \( \ell^2 = g_6 \sqrt{Q_1 Q_5} \). The background is characterized by the constant dilaton and
the non-trivial R-R 3-form fields. We will find it convenient to represent the metric on $S^3$ as $d\Omega_3^2 = d\vartheta^2 + \sin^2 \vartheta d\Omega_2^2$: a 2-sphere $S^2$ of radius $\sin \vartheta$ is located at latitude angle $\vartheta$.

At leading order in $\alpha'$, D3-brane world-volume effective action in the above background is given by

$$
S_{\text{DBI}} = T_3 \left( - \int_{\text{Vol}} d^4 \sigma e^{-\phi} \sqrt{-\det[(X^*G + 2\pi \alpha' F)_{ab}]} \pm 2\pi \alpha' F \wedge X^*C_2^{\text{RR}} \right). \tag{1}
$$

If we rescale as $F \rightarrow \ell^2 F$ and factor out all powers of $\alpha'$, $\ell^2$, $Q_5$ from the metric and $C_2^{\text{RR}}$, then the above action becomes

$$
S_{\text{DBI}} = T_3 Q_5 \ell^2 \alpha'^2 \left( - \int_{\text{Vol}} d^4 \sigma \sqrt{-\det[(X^*G + 2\pi \alpha' F)_{ab}]} \pm 2\pi F \wedge X^*C_2^{\text{RR}} \right). \tag{2}
$$

Locally, we may integrate $H^{\text{RR}} = dC_2^{\text{RR}}$ and obtain the 2-form potential $C_2^{\text{RR}}$ on $S^3$: $C_2^{\text{RR}} = (\vartheta - \nu - \frac{1}{2} \sin(2\vartheta)) \epsilon_2$, where $\nu$ is an integration constant and $\epsilon_2$ denotes the volume-form of the unit $S^2$. It is not possible to define $C_2^{\text{RR}}$ globally over the whole $S^3$, as $H^{\text{RR}}$ is a non-trivial element of $H^3(S^3, \mathbb{R}) = \mathbb{Z}$.

Consider now a D3-brane embedded in $AdS_3 \times S^3$, whose world-volume is extended along $\sigma^\mu = (t, u, S^2)$ and is located at fixed $\vartheta = \vartheta_0$ and $x$. The embedding implies that the pull-back of the two-form R-R potential $X^*C_2^{\text{RR}}$ has components only along $S^2$ in $S^3$. Hence, among the world-volume gauge field strength, only the $F_{0u}$ component couples linearly to the background R-R potential $C_2^{\text{RR}}$ through the Chern-Simons term. The coupling in turn leads to $F_{0u} \neq 0$, which is needed for a non-trivial extremum to exist. As the partially wrapped D3-brane is extended along $u$-direction, the coupling also implies that the D3-brane behaves effectively as a fundamental string (F-string). We have found two classes of the extremum of (2):

$$
2\pi F = \text{constant}, \quad \vartheta_0 = 0, \pi \tag{3}
$$

$$
2\pi F = \mp \cos \vartheta_0, \quad \vartheta_0 = \text{constant}. \tag{4}
$$

The first one corresponds to the collapsed cycle and will not be discussed here. The second family of solutions depends on $\vartheta_0$ as a moduli space parameter. In the S-dual situation, where nontrivial background involves NS-NS tensor field [1], the $\vartheta_0$ moduli space has been discrete as a result of the requirement that the effective D-brane carries an appropriate R-R charge. In the present case, the effective fundamental string in $AdS_3$ carries NS-NS charge, which can be derived from terms in (2) linear in the NS-NS 2-form $B_2$. Taking into account of the non-trivial background of $E$, we find:

$$
T_3 \int_{S^2} \frac{\delta S_{\text{DBI}}}{\delta (2\pi F_{ab})} X^*B_{ab} = \mp T_3 Q_5 \alpha' \text{Vol}(S^2) (\vartheta_0 - \nu) X^*B_{0u}, \tag{5}
$$

where $T_{Dp} = 1/((2\pi)^p \alpha' (p+1)/2 g_{st})$ and the last expression has been derived at the classical extremum. Recall that $\delta S_{\text{DBI}}/\delta (2\pi F_{ab})$ is proportional to the dual gauge field strength.
\[ \epsilon_{ab}^{\text{cd}} \tilde{F}_{\text{cd}} \] Thus, in a simplified notation, the NS-NS electric charge density in (\[ \textbf{5} \]) has the form \( T_3[X^*B \wedge (2\pi \alpha' \tilde{F})] \), viz. the string charge density agrees precisely with an expression which is electric-magnetic dual to the one obtained for the case \( AdS_3 \times S^3 \) with NS-NS 3-form flux \([4]\). Indeed, this is what one would have expected from the S-duality transformation of the Type IIB string background and the self-duality of the D3-brane. Thus, at strong coupling, \([\textbf{3}]\) should be the correct expression, as it is precisely S-dual to the known situation with NS-NS background \([4]\). In particular, precisely as in \([4]\), the integral flux quantization in \([\textbf{3}]\) ought to refer to the dual gauge field \( (2\pi \alpha' \tilde{F}) \) itself, not to the one involving the R-R two-form: \( (2\pi \alpha' \tilde{F} - X^* C_{\text{RR}}^2) \). At weak coupling regime, we do not expect corrections to the definition of the charge \([\textbf{3}]\), as the wrapped D-brane is a BPS state.

We can also fix the integration constant \( \nu \) by demanding that the NS-NS charge should be zero for \( \vartheta = 0 \). This sets \( \nu = 0 \). The flux quantization condition implies that the last expression of \([\textbf{3}]\) equals the integer multiple of the fundamental string charge. It yields

\[ \vartheta_\alpha = \frac{n}{Q_5} \pi, \quad 0 \leq n \leq Q_5. \]  

(6)

This is precisely the same result as for the \( AdS_3 \times S^3 \) with nontrivial NS-NS background. However, in contrast to the NS-NS background case, we do not have yet any direct argument for the above quantisation from boundary conformal field theory or other string theoretic framework. One can also derive the tension of the effective string as:

\[ \mathcal{E} = \left( F_{0u} \frac{\delta \mathcal{L}}{\delta F_{0u}} - \mathcal{L} \right) = T_3 Q_5 l^2 \alpha'^2 \text{Vol}(S^2) \sin \vartheta. \]  

(7)

The results \([\textbf{3}]\) and \([\textbf{7}]\), which are main results of this section, may be summarized by posing the following puzzle. The NS-NS charge density \([\textbf{3}]\) does not exhibit \( \vartheta \to \pi - \vartheta \) symmetry, whose origin can be traced back to the fact that the R-R 2-form potential \( C_{\text{RR}}^2 \) cannot be defined globally, as \( H_3(S^3) = \mathbb{Z} \). On the other hand, the tension is a periodic function of \( \vartheta \), being proportional to \( \sin \vartheta \). Thus, it appears that the charge density and the tension are not proportional to each other, except near the north pole, \( \vartheta \sim 0 \). Moreover, comparison of \([\textbf{3}]\) and \([\textbf{7}]\) suggests that the charge of the effective string is defined only modulo a multiple of \( Q_5 \) (at large \( Q_5 \) approximation). These are apparently the same phenomena as for the NS-NS background and calls for a deeper understanding.

### 2.2 Stability

The D3-brane is wrapped partially on \( S^2 \subset S^3 \), hence, is apt to shrink down to a singular configuration. Let us begin with local analysis of the brane system. Denote the spacetime coordinates as \((012)(345)(6789)\), where \((012)\) is along the \( AdS_3 \) directions, \((345)\) is along the \( S^3 \) directions, and \((6789)\) are the four-dimensional spectator directions. The \( AdS_3 \times S^3 \) is
provided by stack of $D1$ and $D5$ branes whose world-volumes are oriented along $(01)$ and $(016789)$ directions, respectively. The partially wrapped $D3$-brane world-volume is oriented along $(0234)$ directions. As such, the $D3$-brane is a supersymmetric configuration with respect to both the $D1$-branes with relative co-dimension 4 and the $D5$-branes, with relative co-dimension 8. Hence, the partially wrapped $D3$-brane ought to be a stable configuration, at least locally for each point on $S^2 \subset S^3$.

We next examine global stability of the classical solution explicitly by expanding $D3$-brane world-volume fields in harmonic fluctuations around the classical configuration. Denote small fluctuations of the $\vartheta$-field by $\xi$ and those of $F$-field by $f$. From (2), we get the following effective string Lagrangian density for $\xi, A_u$ up to quadratic terms (after rescaling $A_u \rightarrow \sin \vartheta A_u$):

$$L = \frac{T_3}{2} \int_{S^2} \left[ \frac{1}{u^2} (\partial_0 \xi)^2 - u^2 (\partial_u \xi)^2 - (\partial_i \xi)^2 + 2f^2 + f^2 \mp 4f \xi - u^2 F^2_{tu} \right].$$  (8)

Here, the indices $i$ refers to the coordinates on $S^2$. Choosing the gauge $A_0 = 0$ so that $f = \partial_0 A_u$, the Gauss' law constraint reads

$$0 = \frac{\delta L}{\delta A_0} = \partial_u (\partial_0 A_u \mp 2\xi).$$  (9)

Let us expand the fluctuating fields into spherical harmonics on $S^2$:

$$\xi(t, u, y) = \sum_{l \geq 0} Y_{(l,m)}(y) \xi_{(l,m)}(t, u), \quad A_u(t, u, y) = \sum_{l > 0} Y_{(l,m)}(y) \alpha_{(l,m)}(t, u).$$  (10)

Then, the equations of motion for $\alpha, \xi$ read

$$u^{-2} \partial_0^2 \xi - \partial_u \left( u^2 \partial_u \xi \right) + l(l + 1) \xi - 2\xi \mp 2\partial_0 \alpha = 0$$

$$\partial_0^2 \alpha \mp 2\partial_0 \xi + l(l + 1)u^2 \alpha = 0,$$  (11)

where we have suppressed the $(l, m)$ indices, as they are common to all the fields at the linearized level. We change the field variables by introducing a new field $\eta = \partial_0 \alpha \mp 2\xi$. For $l = 0$ mode, the second equation in (11) gives $\partial_0 \eta = 0$ i.e. $\eta$ is not a dynamical field. The first equation in (11) can be solved substituting $\eta = 0$. It then leads to

$$\frac{1}{u^2} \partial_0^2 \xi - \partial_u (u^2 (\partial_u \xi)) + 2\xi = 0$$

viz. the $l = 0$ mode is massive. Next, let $l > 0$, then differentiating the second equation with respect to time variable we get

$$u^{-2} \partial_0^2 \xi - \partial_u \left( u^2 \partial_u \xi \right) + [l(l + 1) + 2] \xi \pm 2\eta = 0$$

$$\partial_0^2 \eta + l(l + 1)u^2 (\eta \pm 2\xi) = 0.$$
The resulting mass matrix takes the form:

$$
\begin{pmatrix}
    l(l+1) + 2 & \pm 2 \\
    \pm 2l(l+1) & l(l+1)
\end{pmatrix}.
$$

(12)

It is exactly the same mass matrix as obtained in [1]. Thus, the fluctuation spectrum is

$$
m_l^2 = \begin{cases} 
(l+1)(l+2) & \text{for } l = 0, 1, \ldots \\
(l-1)l & \text{for } l = 1, 2, \ldots
\end{cases}
$$

(13)

It is clear that there is a triplet of massless modes for $l = 1$, corresponding to the Goldstone modes of the broken rotational symmetry $SO(4) \rightarrow SO(3)$. All higher modes are massive and hence proving the harmonic stability of the partially wrapped D3-brane.

This stability analysis shows that the partially wrapped D3-brane has the same properties as S-dual counterpart viz. the case with NS-NS background [1]. The agreement is quite suggestive because even the massive spectra of quadratic fluctuations are the same. This might indicate that even the D3-brane wrapped on $S^2$-cycle in R-R background can be described in terms of fuzzy geometry [7].

3 D5-brane in $AdS_5 \times S^5$ (R-R)

3.1 Brane Configuration

We now study dynamics of partially-wrapped D5-brane in $AdS_5 \times S^5$ threaded by R-R flux. The situation is parallel to that of the D3-branes in $AdS_3 \times S^3$, so we will be brief and limit foregoing discussion only to essential steps. The ten-dimensional Type IIB supergravity background is given by [8]

$$
\begin{align*}
    ds^2 &= R^2 \alpha' \left[ u^2(-dt^2 + dx_i^2) + \frac{du^2}{u^2} + d\Omega_5^2 \right] \\
    G_5 &= 4R^4 \alpha'^2 (\epsilon_5 + *\epsilon_5) \\
    e^{-\phi} &= 1, \quad R^2 = \sqrt{4\pi g_{st}N}. \quad (14)
\end{align*}
$$

Here, $\epsilon_5$ denotes the volume element of the $S^5$. Again, consider a D5-brane, wrapped partially on $S^4 \subset S^5$. The world-volume effective action of the D5-brane is given by

$$
S_{DBI} = T_5 \left( - \int_{\text{Vol}} d^4\sigma e^{-\phi} \sqrt{-\det[(X^*G + 2\pi \alpha' F)_{ab}] + 2\pi \alpha' F \wedge X^*C_4} \right) \quad (16)
$$

As before, decompose the metric on $S^5$ in polar coordinates so that $d\Omega^2_5 = d\vartheta^2 + \sin^2 \vartheta d\Omega^2_4$, where $\Omega_4$ denotes the metric on $S^4$, and consider the D5-brane whose world-volume wraps partially on $S^4$ at a fixed $\vartheta$ and also extends along $u$-direction in $AdS_5$. Due again to
the Chern-Simons coupling, the $F_{0u}$ component of the world-volume gauge field acquires a non-vanishing expectation value, $F_{0u} = \mp \cos \vartheta$.

In $AdS_5$, extended along $u$-direction, the partially wrapped D5-brane behaves as an effective string. Again, the effective string is nothing but a multiple of fundamental string. To see this, we calculate the string charge density from the minimal coupling of the D5-brane to the NS-NS 2-form $B_2$. Taking into consideration of the non-trivial $F$-field background, from (2), we find the following effective coupling with $B_2$:

$$T_5 \int_{S^4} \frac{\delta S_{DBI}}{\delta (2\pi \alpha' F_{ab})} X^* B_{ab} = \mp T_5 R^4 \text{Vol}(S^4) \frac{3}{2} (\vartheta - \frac{1}{2} \sin 2\vartheta) X^* B_{0u}$$

$$= \mp \frac{N}{2\pi^2 \alpha'} (\vartheta - \frac{1}{2} \sin 2\vartheta) X^* B_{0u}. \quad (17)$$

Here, the last equality has been calculated at the classical extremum. As in the case of D3-brane in $AdS_3 \times S^3$ (R-R) discussed in section 2, we did not include in (17) the field corrections proposed in [6]. The charge formula (17) shows that, for a given D5-brane, the effective string charge can not exceed $N$.

On the other hand, as in $AdS_3 \times S^3$ (R-R), the tension of the effective string can be estimated from the mass density and turns out to be proportional to $\sin^3 \vartheta$. Once again, the charge density and the tension of the effective string are not proportional to each other. Only around $\vartheta = 0$, they both grow as $\vartheta^3$ and hence become proportional. Thus, we conclude that the properties of the effective strings in this case are analogous to the $AdS_3 \times S^3$ case.

### 3.2 Stability

Stability analysis of the partially wrapped D5-brane in $AdS_5 \times S^5$ proceeds essentially as in the $AdS_3 \times S^3$ case. Local analysis shows that the partially wrapped brane should be a BPS state. Denoting by (0123) the directions along D3-brane (whose near horizon geometry is $AdS_5 \times S^5$), we embed the D5-brane in (045678). As the relative co-dimension is 8, the brane system preserve half of the supersymmetries of the background.

We next proceed with the global analysis of harmonic fluctuations. Expanding the fluctuations of the $\vartheta$ and $A_u$ fields in spherical harmonics on $S^4$, we find:

$$u^{-2} \partial_\vartheta^2 \xi - \partial_u \left(u^2 \partial_u \xi \right) + l(l+3)\xi - 4\xi \pm 4\partial_\vartheta \alpha = 0$$

$$\partial_\vartheta^2 \alpha \mp 4\partial_\vartheta \xi + l(l+3)u^2 \alpha = 0.$$

As in the previous section, we introduce a new field $\eta = \partial_\vartheta \alpha \mp 4\xi$. Then, the mass matrix for $\xi, \eta$ ($l > 0$) is given by

$$\begin{pmatrix}
    l(l+3) + 4 & \pm 4 \\
    \pm 4l(l+3) & l(l+3)
\end{pmatrix}, \quad (18)$$
yielding the fluctuation spectrum as

\[ m^2_l = \begin{cases} 
(l + 3)(l + 4) & \text{for } l = 0, 1, \ldots \\
(l - 1)l & \text{for } l = 1, 2, \ldots 
\end{cases} \]

For \( l = 1 \), we get 5 Goldstone modes in fundamental representation of \( SO(5) \) arising from spontaneously broken rotational symmetry \( SO(6) \to SO(5) \). As anticipated from vector spherical harmonics, the above spectrum bears a similar structure as that in the \( AdS_3 \times S^3 \) case.

## 4 Charge Nonconservation and Baryon Vertex

In the previous sections, we have studied partially wrapped D3- or D5-brane on \( S^2 \subset S^3 \), respectively, \( S^4 \subset S^5 \). On \( AdS_3 \) or \( AdS_5 \) space, the brane was assumed to extend along the \( u \)-direction infinitely \( u = [0, \infty] \), hence, connecting two boundary points of the anti-de Sitter space.

Now, let us consider a variant situation: the partially wrapped D-brane forms a semi-infinite string running between \( u = [u_0, \infty] \) with \( N \) units of NS-NS charge\(^2\). The configuration is clearly unstable. For one thing, due to nonzero tension, the effective string will shrink to a zero size. In this case, we will need to understand one more piece of physics: fate of the string charge. We now would like to argue that, instead of shrinking to the boundary of anti-de Sitter space, the semi-infinite effective string, whose configuration on \( S^3 \) or \( S^5 \) is a co-dimension one hypersphere, will ‘lasso’ around \( S^3 \) or \( S^5 \) completely, say, from the North-to the South-pole. If one assumes the mod-\( N \) charge conservation rule, then we find that the same mechanism will stabilize a single string. \( \ddagger \)From the charge density formula, we note that the charge deficiency when ‘lassoing’ around the sphere is precisely \( N \) units, which is the same amount as the induced effective string charge.

The final configuration would then be ( for \( AdS_3 \times S^3 \) ) the superposition of the D7-branes: a D7-brane wrapped partially on \( S^2 \subset S^3 \) and its ‘lasso’ image wrapped entirely on \( S^3 \). The superposed D7-brane composite is a configuration of relative co-dimension two, hence, corresponds to a non-threshold bound-state. Indeed, the bound-state is formed precisely by the world-volume gauge field, which is directly coupled to the NS-NS two form. Thus, the above argument provides a complementary picture of the baryon vertex from the string creation phenomenon. Namely, the partially wrapped D-brane will try to ‘lasso’ around the entire \( S^m \) and absorbs the induced NS-NS fluxes on \( S^m \).

In order to see whether this is the case, let us reconsider the baryon vertex in \( AdS_5 \times S^5 \). \( \ddagger \)From the \( \mathcal{N} = 4 \) super Yang-Mills theory point of view, the baryon is constructed by

\(^2\)We set \( N = Q_5 \) in this section.
wrapping a D5-brane on $S^5$ at $u = u_0$ and connecting $N$ fundamental string between the wrapped D5-brane and $N$ D3-branes at the boundary of AdS$_5$. World-volume dynamics of the D5-brane wrapped on $S^5$ exhibits two novel features. First, the D5-brane carries $N$ units of fundamental string charge. This is because world-volume action of the D5-brane wrapped on $S^5$ includes the following Wess-Zumino term [9]:

$$S_{WZ} = \int_{S^5 \times T} (B + 2\pi \alpha' F) \wedge C_4 = -\int_T (\Lambda_1 + 2\pi \alpha' A) \wedge \int_{S^5} G_5.$$  \hspace{1cm} (19)

Utilizing that $\oint_{S^5} G_5/2\pi = N$, we conclude that $N$ units of electric flux or, gauge equivalently, $N$ units of fundamental string charge is induced on the D5-brane. As the world-volume of the D5-brane wrapped on $S^5$ is compact, Gauss’ law dictates that the flux has to leak out to ‘infinity’. This is achieved by attaching the $N$ fundamental strings connecting the D5-brane wrapped on $S^5$ and the D3-branes – the flux on D5-brane then leaks out to the D3-branes (whose world-volume is noncompact) via the $N$ fundamental strings. Now, the requisite $N$ fundamental strings are provided precisely by the D5-brane wrapped on $S^4 \subset S^5$. In other words, the entire configuration of the baryon vertex and the fundamental string is made solely out of two intersecting D5-branes – one wrapped on $S^5$ and another wrapped on $S^4 \subset S^5$! Second, the D5- and D3-branes have relative co-dimension 8, so form a supersymmetric configuration. We have seen that the D5-brane wrapped partially on $S^4 \subset S^5$, which behaves effectively as $N$ fundamental strings stretched along $u$-direction, also have relative co-dimension 8, so is a supersymmetric configuration. In doing so, the aforementioned two D5-branes wrapped on $S^4$ and $S^5$, respectively, form a non-threshold bound-state. This is exactly the same situation as the system of D7- and D3-branes exhibits in AdS$_3 \times S^3$, as already mentioned.

5 Open problems

Our discussion is far from conclusive and calls for deeper understanding of the physics of branes on $AdS_m \times S^n$ spaces. Let us list here several open problems. It is clear that our results are subject to curvature corrections [10], as the D-branes are wrapped on $S^{n-1} \subset S^n$. What is more important, we lack a proper definition of the D-brane charges when various background fields are present. Our calculations suggest that the charge might be conserved modulo $N$ ($Q_5$ for AdS$_3 \times S^3$) (see also [11]), but we do not know any possible mechanism underlying such a behaviour directly from the D-brane physics. We also lack a proper understanding of corrections to effective string charges of the type described in [4] in the R-R field background. The S-duality enforces lack of such contributions for the case of the
R-R background, while they were necessary for the NS-NS background. Lastly, the partially wrapped D-branes have unusual charge-tension relations, which certainly calls for further studies.

Acknowledgement

We thank A. Recknagel, S.Ramgoolam and S. Theisen for interesting conversations. We are grateful to organizers, H. Grosse, M. Kreuzer and S. Theisen, for inviting us to ”Duality, String Theory and M-Theory” workshop at Erwin Schrödinger Institute, where part of this work has been accomplished.

References

[1] C. Bachas, M. Douglas, C. Schweigert, J. High-Energy Phys. 0005 (2000) 048, hep-th/0003037; J. Pawelczyk, hep-th/0003057.

[2] A. Yu. Alekseev, V. Schomerus, Phys. Rev. D60 (1999) 061901, hep-th/9812193; A. Yu. Alekseev, A. Recknagel, V. Schomerus, J. High-Energy Phys. 9909 (1999) 023, hep-th/9908040.

[3] J. Maldacena, A. Strominger, J. High-Energy Phys. 9812 (1998) 005, hep-th/9804088.

[4] J. de Boer, Nucl.Phys. B548 (1999) 139, hep-th/9806104; S. Deger, A. Kaya, E. Sezgin, P. Sundell, Nucl.Phys. B536 (1998) 110, hep-th/9804166.

[5] C. Schmidhuber, Nucl. Phys. B 467 (1996) 146, hep-th/9601003; A.A. Tseytlin, Nucl. Phys. B 469 (1996) 51, hep-th/9602064.

[6] W. Taylor, hep-th/0004141.

[7] R. C. Myers, J. High-Energy Phys. 9912 (1999) 022, hep-th/9910053; D. Kabat and W. Taylor, Adv. Theo. Math. Phys. 2 (1998) 181, hep-th/9711078; S.-J. Rey, hep-th/9711081; A. Yu. Alekseev, A. Recknagel, V. Schomerus, J. High-Energy Phys. 0005 (2000) 010, hep-th/0003187.

[8] G. Horowitz and A. Strominger, Nucl. Phys. B360 (1991) 197.

[9] E. Witten, J. High-Energy Phys. 9807 (1998) 006, hep-th/9805112.
[10] C. P. Bachas, P. Bain and M. B. Green, J. High-Energy Phys. 9905 (1999) 011, hep-th/9903210.

[11] S. Stanciu, hep-th/0006145;
A. Alekseev, V. Schomerus, hep-th/0007096.