Estimates of the subthreshold photoproduction of charm

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Abstract

Charm photoproduction rates off nuclei below the nucleon threshold are estimated using the phenomenologically known structure functions both for $x < 1$ and $x > 1$. The rates rapidly fall below the threshold from values $\sim 1$ nb for Pb at the threshold (7.8 GeV) to $\sim 1$ pb at 6 GeV.

1 Introduction

In view of the envisaged upgrade of the CEBAF facility up to 12 GeV it becomes important to have relatively secure predictions about the production rates of charm on nuclear targets below and immediately above the threshold for the nucleon target. This note aims at such predictions. From the start it has to be stressed that the dynamical picture of charm production at energies close to the threshold is much more complicated than at high energies (see e.g. [1]). Correspondingly the production rates cannot be described by the standard collinear factorization expression but involve gluon distributions both in $x$ and transverse momentum, which are unknown at small transverse momenta (in the confinement region). To obtain our predictions we shall recur to some crude assumptions about these distributions both at $x < 1$ and $x > 1$. In the latter case we shall exploit the (poorly) known behaviour of the nuclear structure functions, which fall exponentially with $x$, the slope being in the range $15 \div 16$. With all these simplifications, we hope to be able to predict the rates up to factor $2 \div 3$.

2 Kinematics

Consider the exclusive process shown in Fig. 1

$$\gamma + A \to C\bar{C} + A^*$$

(1)

where $A$ is the target nucleus of mass $m_A$ and $A^*$ is the recoil nuclear system of mass $m_A^*$. The notation for all the momenta involved can be read off the Fig. 1. We denote the total mass of the $C\bar{C}$ system as $M$. Obviously $M \geq 2m_c$ where $m_c$ is the mass of the C-quark, which for our modest purpose we take as 1.5 GeV. The inclusive cross-section for charm photoproduction is obtained after summing over all states of the recoil nuclear system.

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We choose a reference system in which the target nucleus is at rest and the incoming photon is moving along the z-axis in the opposite direction, so that \( q_+ = q_\perp = 0 \) The (part of) the photoproduction cross-section corresponding to (1) is then obtained via the imaginary part of the diagram in Fig. 1 as

\[
\sigma_A = A \int \frac{d^4k}{(2\pi)^3} \delta((Ap - k)^2 - m_A^2)x \left( \frac{\Gamma(k^2)}{k^2} \right)^2 \sigma_g(M^2, k^2)
\]  

(2)

Here \( \Gamma(k^2) \) is the vertex for gluon emission from the target; \( \sigma_g(M^2, k^2) \) is the photoproduction cross-section off the virtual gluon of momentum \( k \). We have also introduced the scaling variable for the gluon as \( x = k_+/p_+ \). Due to \( q_+ = 0 \) this is also the scaling variable for the observed charm. Note that this definition, which is standard at large energies and produced masses, is not at all standard at moderate scales. In particular this \( x \) does not go to unity at the threshold for the nucleon target. Rather the limits of its variation converge to a common value 0.76. For the nuclear targets with \( A \gg 1 \) its minimal value at the nucleon threshold is well below unity (\( \sim 0.64 \)). One should have this in mind when associating this \( x \) with the gluon distribution: it follows that for a nuclear target, for energies going noticeably below the nucleon threshold, the contributing values of \( x \) still remain below \( x = 1 \). This effect is essential for the charm cross-sections to remain not too small immediately below the threshold.

We use the \( \delta \)-function to integrate over \( k_\perp \) to obtain imaginary part of the diagram in Fig. 1 as

\[
\sigma_A = A \int \frac{dx d^2k_\perp}{2(A - x)(2\pi)^3} \left( \frac{\Gamma(k^2)}{k^2} \right)^2 \sigma_g(M^2, k^2)
\]  

(3)

In these variables we find

\[
M^2 = xs_1 + k^2, \quad s_1 = 2pq
\]  

(4)

and

\[
k^2 = xAm^2 - \frac{x}{A - x}m_A^2 - \frac{A}{A - x}k_\perp^2
\]  

(5)

where we have put \( p^2 = m^2 \), the nucleon mass squared, neglecting the binding.

The limits of integration in (3) are determined by the condition \( M^2 \geq 4m^2_c \), which leads to

\[
x(s_1 + Am^2) - \frac{x}{A - x}m_A^2 - \frac{A}{A - x}k_\perp^2 - 4m^2_c \geq 0
\]  

(6)

Since \( k_\perp^2 \geq 0 \) one gets

\[
x(s_1 + Am^2) - \frac{x}{A - x}m_A^2 - 4m^2_c \geq 0
\]  

(7)

from which one finds the limits of integration in \( x \):

\[
x_1 \leq x \leq x_2
\]  

(8)

where

\[
x_{1,2} = \frac{1}{2s}(As - m_A^2 + 4m^2_c) \pm \sqrt{[As - (m^* + 2m_c)^2][As - (m^* - 2m_c)^2]}
\]  

(9)

where \( s = s_1 + Am^2 \). The limits of integration in \( k_\perp \) at a given \( x \) are determined by (6).

Using (5) we may pass from the integration variable \( k_\perp^2 \) to \( |k^2| \):

\[
\sigma_A = \pi \int_{x_1}^{x_2} dx \int_{|k^2|_{\text{min}}}^{x_{1,2} - 4m^2_c} \frac{d|k^2|}{2(2\pi)^3} \left( \frac{\Gamma(k^2)}{k^2} \right)^2 \sigma_g(xs_1 - |k^2|, k^2)
\]  

(10)
where
\[ |k^2|_{\text{min}} = \frac{A}{A-x} x^2 m^2 \] (11)

The threshold energy corresponds to \( x_1 = x_2 \) or \( As \approx m_A + 2m_c \) with the minimal possible value of the recoiling system, which is just \( m_A \simeq Am \). In terms of the photon energy \( E \) we have \( s_1 = 2mE \) and the threshold energy is found to be
\[ E_A^{\text{th}} = 2m_c \left( 1 + \frac{m_c}{A m} \right) \] (12)
It steadily falls with \( A \) from the nucleon target threshold
\[ E_1^{\text{th}} = 2m_c \left( 1 + \frac{m_c}{m} \right) \simeq 7.8 \text{ GeV} \] (13)
down to the value \( 2m_c \) for infinitely heavy nucleus.

3 High-energy limit

To interpret the quantities entering Eq. (10) it is instructive to study its high-energy limit, which corresponds to taking \( s_1 \gg m_c^2 \) and both quantities much greater than the nucleon mass. Assuming that the effective values of the gluon virtuality are limited (and small) one then gets for the nucleon target (\( A = 1 \))
\[
\sigma_1 = \pi \int_{x_1}^{x_2} dx \int_0^{x_1} dx_1 \, \frac{d|k^2|}{2(2\pi)^3} \left( \frac{\Gamma(k^2)}{k^2} \right)^2 \sigma_g(x s_1) \] (14)
Here we also neglect the off-mass-shellness of the cross-section off the gluon, considering \(|k^2| \ll 4m_c^2\). The obtained formula is precisely the standard collinear factorization formula with the identification
\[
x g(x, M^2) = \pi \int_0^{M^2} \frac{d|k^2|}{2(2\pi)^3} \left( \frac{\Gamma(k^2)}{k^2} \right)^2 \equiv \int_0^{M^2} d|k^2| x \rho(x, |k^2|) \] (15)
Of course it is understood that this is only a part of the gluon distribution in \( x \) coming from a particular recoil state. The total gluon distribution is obtained after summing over all recoiling states. The quantity \( \rho(x, |k^2|) \) obviously has a meaning of the double distribution of gluons in \( x \) and \( |k^2| \). Eq.(15) allows to relate function \( \Gamma^2/k^4 \) entering our formulas of Section 2 to the double distribution \( \rho(x, |k^2|) \) and rewrite our formula for the cross-section at finite energies as
\[
\sigma_A = \int_{x_1}^{x_2} dx \int_{|k^2|_{\text{min}}}^{x_1 - 4m_c^2} d|k^2| \rho_A(x, |k^2|) \sigma_g(x s_1 - |k^2|, k^2) \] (16)

4 The gluon distribution \( \rho(x, |k^2|) \)

To make our estimates, we assume a simple factorizable form for the double density \( \rho(x, |k^2|) \) and choose the \( |k^2| \) dependence in accordance to the perturbation theory with an infrared cutoff in the infrared region:
\[
\rho(x, |k^2|) = \frac{a(x)}{|k^2| + \Lambda^2} \] (17)
Function $a(x)$ can be obtained matching (17) with the observed $xg(x, M^2)$ at a particular point $M_0^2$. Since we are interested in the threshold region, we take $M_0 = 2m_c$ to finally obtain

$$\rho(x, |k^2|) = \frac{g(x, 4m_c^2)}{\ln(4m_c^2/L^2 + 1)} \frac{1}{|k^2| + \Lambda^2}$$

(18)

Our calculations show that the results are practically independent of the infrared cutoff for energies above 6 GeV. However for lower energies they begin to rapidly fall with the growth of $\Lambda$.

For the proton at $x < 1$ the gluon distribution $g(x, 4m_c^2)$ can be taken from numerous existing fits to the experimental data. In our calculations we have used GRV95 LO [2]. For the nucleus at $x < 1$ we use the simplest assumption $g_A(x, Q^2) = Aq_1(x, Q^2)$ neglecting the EMC effect in the first approximation. Obviously this not a very satisfactory approximation at $x$ quite close to unity.

For the nuclei at $x > 1$ we the gluon distribution may be estimated using, first, the existing data for the structure function in this region and, second, the popular hypothesis that at sufficiently low $Q^2 = s_0$ the gluon distribution vanishes and the hadrons become constructed exclusively of quarks. Then one can, in principle, find the gluon distribution at a given $Q^2$ from the standard DGLAP evolution equation with the quark distributions determined from the experimental data on the structure functions at $x > 1$ evolved back to $Q^2 = s_0$. In the present estimates we recurred to a simpler approach, neglecting the $Q^2$ dependence of the quark distributions at $x > 1$ in accordance with the naive parton model and experimental evidence for the structure functions at $x > 1$ which depend on $Q^2$ very weakly. From [3] one concludes that at $x > 1$ the structure function of carbon falls with $x$ as approximately $\exp(-\Delta x)$ with $\Delta \simeq 16$, its value at $x = 1.05$ being $\sim 6 \times 10^{-5}$ and very weakly dependent on $Q^2$. From these data one can find the quark density at $x > 1$ and using the DGLAP equation find the corresponding gluon density at $x > 1$: 

$$xg_A(x, Q^2) = a \frac{6\alpha_s}{\pi \Delta} A^{1+0.3x} \frac{1}{x} e^{-\Delta x} \ln Q^2 s_0$$

(19)

with $\delta = 16., a = 100.$ and $s_0$ the scale at which the gluon contents of the hadrons vanishes. We take $s_0 = (0.4 \text{ GeV})^2$. The $A$-dependence has been chosen in accordance with the experimental data for hadron production at $x > 1$ [4].

5 Numerical results

To simplify calculations we take in (16) the photon-gluon fusion cross-section on the gluon mass shell where it is known to be [5]

$$\sigma_g(M^2) = \frac{\pi \alpha_em \alpha_s e_c^2}{M^4} \frac{1}{t_1^2} \int_{t_1}^{t_2} dt \left[ \frac{t}{u} + \frac{u}{t} + \frac{4m_c^2 M^2}{tu} \left( 1 - \frac{m_c^2 M^2}{tu} \right) \right]$$

(20)

where $e_c$ is the quark charge in units $e$, $u = -M^2 - t$ and the limits $t_{1,2}$ are given by

$$t_{1,2} = -\frac{M}{2} [M \pm \sqrt{M^2 - 4m_c^2}]$$

(21)

We take the strong coupling constant $\alpha_s = 0.3$, infrared cutoff in the gluon distribution $\Lambda = 0.3 \text{ GeV}$.

With these parameters we obtain the cross-sections for charm photoproduction shown in Fig. 2. To have the idea of the number of nucleons which have to interact together to produce charm at fixed energy below threshold we show the limits of integration $x_1$ and $x_2$ in Fig. 3.
As expected the cross-sections rapidly fall for energies below threshold. Their energy dependence cannot be fit with a simple exponential (in fact they fall faster than the exponential). As to the absolute values, for Pb at we have obtained values around 1 nb at the nucleon threshold (7.8 GeV) and around 1 pb at 6 GeV. The \( A \) dependence is close to linear.

6 Discussion

We have crudely estimated charm photoproduction rates for nuclear targets below and in the vicinity of the nucleon target threshold. The estimates require knowledge of the gluon distribution in both \( x \) and \( k^2_{\perp} \) in a wide region of the momenta including the confinement region.

Our estimates were based on a simple factorization assumption and introduction of an infrared cutoff. As mentioned at energies above 6 GeV the obtained cross-sections practically do not change at all when the cutoff is raised from 0.3 GeV to 0.4 GeV. However at 5 GeV they fall by nearly 30 times. Thus at energies much below the nucleon threshold and closer to the nuclear thresholds the results become strongly dependent on the cutoff. However at such energies the cross-sections becomes so small that do not present any interest from the practical point of view.

The behaviour in the nearest vicinity of the threshold depends crucially on the choice of the scaling variable \( x \) and the form of the nuclear gluon distribution at \( x \) smaller but very close to unity. Our choice of \( x \) as the part of the "+" component of the hadron momentum carried by the gluon is well founded from the theoretical point of view. However with this definition \( x \) remains below unity at the nucleon target threshold. This considerably enhances the cross-sections for nuclear targets in the vicinity and somewhat below the threshold. Our assumption that in this region the nuclear structure function is roughly proportional to \( A \) is quite crude without any doubt. To improve the estimates one has to specially investigate this region of \( x \) for a nuclear target following [1].

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8 References

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9 Figure captions

Fig. 1. The forward scattering amplitude corresponding to reaction (1).
Fig. 2. The charm photoproduction cross-sections for different photon energies \( E \) and targets. Curves from bottom to top correspond to \( A = 1, 12, 64 \) and 207.
Fig. 3. The limits of \( x \)-integration for different photon energies and nuclear targets. Curves from bottom to top correspond to \( A = 12, 64 \) and 207.
Fig. 1
