Mode Analysis Theory of Small Disturbance Operation Process of Power System

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Abstract. The power system currently has the problem that transient steady-state low-frequency oscillations after the power system suffers from small disturbances will affect the normal operation of the power grid. This paper proposes a model analysis of engineering mathematics for the power system’s small-disturbance transient low-frequency oscillations. First, this paper constructs the mathematical model and simplified model of the power system. Next, this paper calculates the initial steady state of the model and builds a linearized model. Then small disturbances are added to the model in order to simulate the transient steady-state process of a single-machine infinite-bus system under real conditions. Finally, the power system stabilizer (Power system stabilizer PSS) is designed through model analysis. Compared the transient steady-state process the PSS isn’t installed and the PSS is installed in the single-machine infinite power system, we can get experimental results, which show that the power system can obtain a new steady state after receiving small disturbances and eliminating small disturbances in time. At the same time, the installation of PSS can effectively increase the stability of the power system and the tolerance for the existence of interference.

Keywords: power system; single machine infinity; small disturbance; transient steady state; PSS; simulation

1. Introduction

There is a significant content that power system stability in the process of power system analysis. Especially in recent years, the rise of distributed power sources has brought us various forms of electric energy conversion, but it also puts forward more stringent requirements on the power system stability. Therefore, the model analysis of the power system and the construction of the power system model are the premise and basis for studying and observing the transient operation.

The current small-disturbance transient steady-state analysis is mainly to observe whether the power system can maintain isochronous operation after the power system suffers a certain degree of interference. Literature [1] built a single-machine infinite power system model based on MATLAB, and gave the power system a short-circuit fault within a period of time. Then this paper observed the influence of the fault existence time on the transient stability of the power system. Literature [2] used wavelet transform to extract and analyze the characteristics of system faults.

In summary, the current industry has achieved certain advantages in the model construction and failure analysis of a single-machine infinite power system. In the process of building a power system, Simulink and power system module libraries are basically used to simulate and analyze the power system. Although it is very intuitive, it relies too much on power devices and lacks more rigorous and mathematical models. For this reason, this paper proposes a model analysis of engineering
mathematics for the transient oscillations in the power system. The PSS is designed through model analysis and the transient steady-state process before and after the PSS is installed in the power system is compared. Under the mode analysis, the influence of the PSS designed by the residue method on the operation process of the power system will be observed. This paper lays a theoretical model foundation for the subsequent small disturbance transient steady-state mode analysis of multi-machine power systems.

2. Detailed Linearization Model

The detailed linearization model involved in this article is shown in Figure 1. In order to study the small disturbance transient steady-state process of a power system, the first model to be built is a synchronous generator model. Afterwards, for the sake of regulating and controlling the generator’s voltage, the system of excitation and an automatic voltage regulator (AVR) model must be built. Finally, we need to build a single-machine infinity system and its linearized model based on the synchronous generator model.

Figure 1. Detailed linearization model for mode analysis.

2.1. Synchronous Generator

When describing the characteristics of synchronous generators, this article uses the voltage, current and flux equations after Park transformation to express. After adding the two-phase damper windings D and Q of the rotor and the field winding f on the stator, the matrices are as shown in the equation (1)

\[
\begin{bmatrix}
\psi_d \\
\psi_f \\
\psi_d' \\
\psi_f'
\end{bmatrix} =
\begin{bmatrix}
X_{ad} & X_{af} & X_{df} & -I_f \\
X_{fd} & X_{ff} & X_{df} & I_d \\
X_{ad} & X_{af} & X_{df} & I_d \\
X_{fd} & X_{ff} & X_{df} & I_d
\end{bmatrix}
\begin{bmatrix}
\psi_d' \\
\psi_f' \\
-I_f \\
I_d
\end{bmatrix}
\]

Where, \(\psi, V, I, R, X\) are the flux linkage, voltage, current, resistance and self-reactance of each winding. Where \(\omega_0\) and \(\omega\) are synchronous speed and rotor speed. We can assume that the mutual reactance \(X_{ad}, X_{af}\) of the coaxial winding is equal.

The rotor motion equations of the synchronous motor are as shown in the equation (2)

\[
\begin{align*}
\delta &= \omega_0(\omega-1) \\
\dot{\omega} &= \frac{1}{M}[T_m - T_f - D(\omega-1)]
\end{align*}
\]

where, \(M\) is the rotor inertia constant; \(D\) is the damping coefficient of the rotor movement; \(\delta\) is the angular displacement of the synchronous motor rotor relative to the synchronous reference axis; \(T_m\) and \(T_f\) are the mechanical torque and electromagnetic torque received by the synchronous motor rotor movement.
2.2. Automatic Voltage Regulator

Excitation is the process of making the generator rotor form a rotating magnetic field based on the principle of electromagnetic induction. The excitation system plays a very important role in the stability of the power system and the safety of the power grid. It is not only a lever for reactive power and voltage regulation in the entire grid, but also a guarantee for the stable operation of the unit [3].

It can be known that the voltage is composed of two parts, one part is the voltage output by the AVR, and the other part is the constant excitation voltage. We set a terminal voltage reference value \( V_{t_{\text{ref}}} \) for the AVR. When a measured terminal voltage value \( V_t \) is input, if \( V_t < V_{t_{\text{ref}}} \), then the generator senses that the actual value of the terminal voltage is too small. The transfer function \( TE(s) \) output is positive. The generator excitation voltage, the excitation current and the magnetic field increase. Eventually, the terminal voltage increases and continues to approach the reference voltage. In this way, the tracking process from the terminal voltage to the reference voltage can be completed.

Therefore, combining the relationship equations between the generator excitation voltage and the deviation of the terminal voltage, we can get the mathematical model of the automatic voltage regulator as shown in the equation (3)

\[
\begin{align*}
    V_r &= V_{f0} + V'_{i} \\
    V'_{i} &= \frac{1}{T_A} V_{i} + \frac{K_A}{T_A} (V_{t_{\text{ref}}} - V_t + u_{\text{pss}})
\end{align*}
\]

where, \( V_{f0} \) is the constant excitation voltage; \( K_A \) is the gain of the automatic voltage regulator; \( u_{\text{pss}} \) is PSS’s stable control signal; \( T_A \) is the AVR’s time constant; \( V_{t_{\text{ref}}} \) and \( V_t \) are the given reference voltage and the generator terminal voltage.

2.3. Single Machine Infinity System

In fact, the power of the generator is usually much smaller than the total power of the system. We assume that the system is "infinite" to study the characteristics of generators or simple generator sets, thereby reducing variables and simplifying calculations. The transmission line and generator on the left can be seen as a single infinite power system. We can get the dynamic model of the power system. Next, we linearize equation (1) and equation (2). Then we can get the state space model of the single-machine infinite system as shown in the equation (4)

\[
\begin{align*}
    s[\Delta \delta & \Delta \omega \Delta V_r \Delta \psi_r \Delta \psi_i \Delta \psi_p \Delta \psi_q] \\
    &= A_m \Delta \delta + B_m \Delta \omega + C_m \Delta V_r + D_m \Delta \psi_r + E_m \Delta \psi_i + F_m \Delta \psi_p + G_m \Delta \psi_q + H_m u_m
\end{align*}
\]

(4)

3. Mode Analysis

After constructing the linearized model, we can get the state space model, where is the state matrix of the state equation. In the mode analysis, the mode decomposition is first required to obtain the state space model and mode decomposition form which are closely connected with the open loop system. Next, obtain the time domain responses of the state variables and the closed-loop system’s state matrix. Then, we can design the PSS according to the residue method and set the PSS parameters. Finally, the conclusion is obtained by hanging the designed PSS into the power system for simulation [4].

3.1. Mode Decomposition And Closed Loop System

We regard the power system as a linear system in the model analysis. First, get the state matrix, output vector and control vector of the open loop system. We define \( \psi_r \) and \( \psi_i \) satisfying the conditions of \( AV_r = \lambda_r \psi_r \) and \( w_r^T A = w_i^T \lambda_i \) as the right eigenvector and the left eigenvector respectively. After
introducing a new state variable, set \( X = VZ \) to obtain the state space expression containing the left and right eigenvectors after introducing the new state variable. In addition, the state space model and mode decomposition form of the open loop system can be obtained.

The inverse Laplace transform of the state-space model can obtain the time-domain response as

\[
x_k(t) = \sum_{i=1}^{M} v_{ki} z_i(0)e^{\lambda_i t},
\]

the time domain response of system state variables is closely related to the eigenvalues. If the real part of the eigenvalue is greater than or equal to 0, then the system is an unstable system; if the real part of the eigenvalue is less than 0, then the system is a stable system; if there is a pair of eigenvalues \( \lambda_{i,j} = \xi_i + j\omega_j \) that are conjugate each other, it means that the system corresponding to this part of the characteristic value oscillates over time. \( \omega_j \) determines the angular frequency of oscillation, and \( \xi_i \) determines whether the oscillation is attenuated or divergent. We call \( c_0^T v_i \) the observable index, and we call \( w_i^T b_0 \) the observable index. The residue is finally obtained, which is the product of the observable index and the controllable index, as shown in the equation (5).

\[
\mathcal{R}_e = w_i^T b_0 c_0^T v_i
\]  

If we use the residue to measure the degree of influence of the feedback controller parameter change on the closed-loop system mode, let \( \alpha \) be a parameter variable of the feedback controller, combined with equation (5), there is

\[
\frac{\partial \lambda_i}{\partial \alpha} = \mathcal{R}_e \frac{\partial H(\lambda_i, \alpha)}{\partial \alpha}. 
\]

This shows the sensitivity of the eigenvalues to changes in control parameters.

### 3.2. Application of Pattern Analysis in the Design of PSS

The automatic voltage regulator of the power system alone may not be able to greatly improve the stability of the power system, so when we add a power system stabilization device, set its related parameters, and input the stable signal of the power system stabilization device into the automatic voltage regulator Device, we can observe the influence of PSS on the stability of the power system.

In the above mode analysis, we have obtained the residue, which can measure the degree of influence of the controller parameter change on the closed-loop system mode, so the residue can be used to design the parameters of PSS. Generally speaking, the PSS can be composed of a lead-lag module. If \( T_{ps}(s) = K_{ps} \frac{(1+sT_2)(1+sT_3)}{(1+sT_2)(1+sT_3)} \), after installing the PSS, the closed-loop system oscillation mode is \( \lambda_c \), there is as shown in the equation (6)

\[
\lambda_c - \lambda_i = \mathcal{R}_e \frac{\partial H(\lambda_i, K_{ps})}{\partial K_{ps}} = \mathcal{R}_e \frac{(1+\lambda_i T_2)(1+\lambda_i T_3)}{(1+\lambda_i T_2)(1+\lambda_i T_3)} \Delta K_{ps}
\]

Here we hope that after installing the PSS, the frequency of the system oscillation mode will not change, but the stability and damping can be improved to make the system's low-frequency oscillation attenuation greater. So, we need to select the PSS parameters appropriately so that the phase angle of

\[
\frac{(1+\lambda_i T_2)(1+\lambda_i T_3)}{(1+\lambda_i T_2)(1+\lambda_i T_3)}
\]

and the phase angle of \( \angle K_{ps} = K_{ps} = -\frac{\xi_i - \xi_j}{\mathcal{R}_e H} \) can design PSS.

Finally, the control equation of PSS is incorporated into the state expression of the single-machine infinite system. The linearized model of the system with PSS can be obtained.
4. Power System Small Disturbance Transient Stability Calculation

There are many methods for calculating small-disturbance transient stability in power systems, but in general, two methods can be used, one is the direct method, and the other is the time-domain simulation method. This paper mainly uses the improved Euler method in time domain simulation [5] to survey the whole process of the power system being disturbed from the old steady state to the new steady state.

Therefore, the transient stability of the power system is judged by the curve of power angle change with time (generator rotor rocking curve). The correction value at the end of time period is as shown in the equation (7)

\[ x_{n+1} = x_{n+1} + \Delta x_{n+1} + \frac{1}{2} \left[ \frac{dx}{dt} \bigg|_{t_n} + \frac{dx}{dt} \bigg|_{t_n+1} \right] \Delta t \]

where, \( xx \) is any time period; \( x \) is the state variable; \( \Delta x \) is the change of the state variable.

5. Calculation Example-Single-Machine Infinite Power System

The parameters of the single-machine infinite power system (all standard unit values) are shown in table 1.

Table 1. Calculation example of single-machine infinite power system parameters.

| Generator parameters | \( X_d \) | \( X_q \) | \( X_{ad} \) | \( X_{aq} \) | \( X_d \) | \( X_q \) | \( X_f \) |
|----------------------|--------|--------|--------|--------|--------|--------|--------|
| \( R_d \) | 1.18   | 0.78   | 1.0    | 0.6    | 1.11   | 0.73   | 1.13   |
| \( R_f \) | 0.005  | 0.00075| 0.002  | 0.04   | 7      | 0      | 5      |
| \( M \) | 0.7065 | 8.4184 | 0.15   | 0.01   |
| \( T_{do} \) | 1.51   | 8.44   | -1.51  | 8.44   |

Automatic voltage adjustment parameters

| \( K_A \) | 100 |
| \( T_A \) | 0.01 |

Transmission line parameter

| \( X_j \) | 1.5 |

Steady state operating point parameters

| \( P_0 \) | \( V_0 \) | \( V_{do} \) |
| 0.5    | 1.05   | 1.0    |

From the comparison of the two sets of eigenvalues of the state matrix, the deviation from the set value before and after PSS is installed is shown in table 2.

Table 2. Deviation from the set value before and after PSS is installed.

| No PSS installed | Install PSS |
|------------------|-------------|
| \( \lambda_{6,7} \) | -0.7065±j8.4184 | -1.5203±j8.3842 |
| Deviation percentage | 53.212% | 0.682% |

It can be observed that the percentage of damping deviation from the designated position we set when the PSS is not installed is 53.212%; when the PSS is installed, the percentage of damping deviation from the designated position we set is 0.682%. Therefore, the PSS designed by the above-mentioned residue method successfully improved the damping.

6. Simulation Results

In this example, the simulation time is 10s. The short-circuit fault of the transmission line occurs at 0.5s, and the fault is cleared after 0.1s. The simulation result is seen in Figure 2(a); the simulation
result of the motor angular velocity varying with time is shown in Figure 2(b); The simulation results of the flux linkages on windings $d$, $D$ and $Q$ that change with time are shown in Figure 2(c), (d) and (e), and the simulation result of PSS control signal changes with time is shown in Figure 2(f).

![Figure 2](image)

**Figure 2.** Each state quantity angle of the single-machine infinite bus system without PSS and PSS installed with time.

It can be seen from the above figures that damping after the system is installed with a PSS with a residual number design affects the low-frequency oscillation successfully.

![Figure 3](image)

**Figure 3.** The power angle changes with time cuts off the interference at different times (a) No PSS installed (b) Install PSS.

Figure 3(a) compares the transient steady-state process of the single-machine infinite-bus system under the three conditions of adding small interference at 0.5s, and removing small interference at 0.6s, 0.6030s and 0.6060s, respectively. It can be known that with the increase of small disturbance time, the oscillation amplitude becomes larger. When it exceeds a critical time (0.6066s in the case of the PSS system without installation), the system will not be able to maintain a steady state, and will crash directly. Figure 3(b) compares the transient steady-state process under the four conditions of adding small interference at 0.5s, and removing small interference at 0.6s, 0.6050s, 0.6100s, and 0.6130s. It can be seen that as the existence time of small disturbances increases, the oscillation amplitude of the system becomes larger. When it exceeds a critical time (the case of installing PSS system is 0.6140s), the system will not be able to maintain a steady state and crash directly.
Comparing the Figure 3 (a) and (b), we can get: when the system is equipped with PSS, the critical point of system instability has changed from 0.6066s when no PSS is installed to 0.6140s, and the delay percentage is 112.121%. Therefore, the installation of PSS improves the tolerance of the power system to the existence time of small disturbances, and gains a certain time for manual or machine control to remove the fault.

7. Conclusion
In this paper, the mode analysis of the small disturbance transient steady-state low-frequency oscillation process occurring in the power system is carried out. The mathematical model is constructed, and the transient steady-state process under small disturbances is simulated for the models without PSS and PSS. The effect of PSS on the transient steady-state process of the power system after small disturbances is verified. Finally, the following conclusions are reached:

- If the system can be cut off in time after receiving small disturbances, the system can be restored to a new steady state; on the contrary, if the small disturbances are not cut off in time, the system will not be able to maintain the steady state and crash directly.
- The PSS designed by the residue method has successfully improved the damping, and has a restraining effect on the power system’s low-frequency oscillation.
- The installation of PSS improves the tolerance of the power system to the existence time of small disturbances, taking the single-machine infinity as an example, and gains a certain time for manual or machine control to remove the fault.

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