Two-dimensional finite difference-time domain simulation of moving sound source and receiver

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Abstract: In this paper, a moving sound source and a receiver on an arbitrary trajectory are implemented in the two-dimensional finite difference-time domain (FDTD) method. Two methods are proposed to implement a moving point source and a receiver in the FDTD method in which physically valid analysis is possible including the Doppler effect. One is a direct method and the other is a convolution method. The formulations and numerical experiments are made for the two-dimensional sound field, and the accuracy of two proposed methods is compared. It is confirmed that both methods can be applied to the moving sound source and receiver including the Doppler effect, and that two methods have the almost same accuracy. It is found that the convolution method has an advantage that the source waveform and the moving speed can be freely changed at the time of convolution when either the source or receiver is stationary.

Keywords: Moving sound source, Moving receiver, Doppler effect, FDTD method, Convolution

1. INTRODUCTION

Finite difference-time domain (FDTD) method [1,2] is one of the most popular and powerful numerical tool for sound field analysis. In most cases in the FDTD calculation, sound source and receiver are usually fixed, then the response between them is mainly calculated. However, to analyze the noise of moving automobiles [3] or trains, it is necessary to implement moving sound sources in the FDTD method. In addition, moving receivers must be also implemented for analysis of drone audition [4,5] or bat sonar mechanism [6]. There are a few studies on the moving source in the electromagnetic analysis [7,8] using the FDTD method in which the Lorentz transformation is applied. There are few studies on the numerical analysis of the moving sound sources using the linearized Euler equations [9]. There are some methods that a fluid flow is analyzed instead of the sound source moving [10,11]. Other studies on the moving sound source are related to auralization in virtual reality [12–16], but physically valid analysis is difficult in their studies because the sound wave propagation cannot be calculated.

There are few studies on the moving receiver other than those related to the walk-through auralization [17].

In this paper, two methods are proposed to implement a moving sound source and a receiver in the FDTD method that provides physically valid analysis including the Doppler effect. One is the direct method, in which source waveform is radiated or sound pressure is received while switching grid points on the moving path of the sound source or receiver at every time step. In the direct method, when the source position is between the grid points, the adjacent grid points are driven by the source weighting function. The direct method can simulate physically valid sound source or receiver movement, but it has disadvantages that the calculation time takes for the sound source or receiver movement time, and the sound source waveform cannot be changed later.

The other is the convolution method, in which all impulse responses from the sound source or receiver position on the moving path at every time step are calculated in advance, then the source waveform is convoluted while switching the impulse response according to the sound source or receiver movement. In the convolution method, even if the number of impulse responses to be calculated increases, the calculation time is hardly increased by using the reciprocity. Moreover, there is an advantage that the sound source waveform and
the moving speed can be changed freely later in the convolution process.

In this paper, the compact explicit finite-difference time domain (CE-FDTD) method [18–22] is applied to the moving sound source and receiver, while proposed methods can also be applied to the standard FDTD method [1]. The CE-FDTD method is a high accuracy version of the standard FDTD method and is good at large-scale sound field analysis. We have been already implemented a moving sound source in the 2-D FDTD method [23], the same manner is applied to a moving receiver in this paper. Formulation and numerical experiments on the moving source and receiver with arbitrary trajectory are made for a two-dimensional sound field. In both methods, a point source which has no boundaries is assumed. Some numerical experiments are performed including the case where the sound source and the receiver move at the same time. Then, the numerical accuracy is compared between the direct method and the convolution method. Note that this paper is an English version of Japanese technical report [24].

2. THEORY

The two-dimensional wave equation in homogeneous media is given as

$$\frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2} = \frac{\partial^2 p}{\partial x^2} \frac{\partial^2 p}{\partial y^2},$$

(1)

where \( p \) is the sound pressure and \( c_0 \) is the sound speed. In the CE-FDTD method, the wave equation is directly discretized on the collocated grid on the basis of the central finite-difference. The grid intervals of the \( x \) and \( y \) directions are all set to the same \( \Delta \). Considering not only the axis direction but also the diagonal direction, Eq. (1) is discretized as [17, 22]

$$p_{i,j}^{n+1} = 2p_{i,j}^n - p_{i,j}^{n-1} + \chi^2(1 - 2a)p_{i+1,j}^{n} + p_{i-1,j}^{n} + p_{i,j+1}^{n} + p_{i,j-1}^{n}$$

$$+ 4\Delta^2(1 - a)p_{i,j}^n,$$

(2)

where \( p_{i,j}^n \) represents the sound pressure on the grid point \( (x, y) = (i\Delta, j\Delta) \) at time \( t = n\Delta t \) and \( \Delta t \) is the time step, \( \chi = c_0\Delta t / \Delta \) is the Courant number [25], and \( a \) (0 ≤ \( a \) ≤ 1/2) is the parameters for accuracy control. In this paper, \( a = 1/4 \) is used as the interpolated wide-band (IWB) scheme. The sound field analysis tool based on the CE-FDTD method has already been well verified in our previous researches [19–23].

2.1. Implementation of Moving Sound Source and Receiver in Direct Method

First, a moving sound source is described. In order to implement a moving sound source in the FDTD method directly, the grid points on the moving path of the sound source must be switched and driven according to the sound source movement. When the point source is just on the grid point, it is sufficient to drive the grid point. However, when it is between the grid points, it is necessary to distribute the sound source to the adjacent grid points and drive them.

Consider the two-dimensional case where the sound source is located at \((x^n_0, y^n_0) = (x^n_0 + d_{x}^n, y^n_0 + d_{y}^n)\) between the four grid points \( P_1 \sim P_4 \) at time \( t = n\Delta t \) as shown in Fig. 1(a) where \( 0 < d_{x,j}^n \leq \Delta \). When the local coordinate system \((\xi, \eta)\) is introduced, the sound source position is converted to \((\xi^n_0, \eta^n_0)\) as shown in Fig. 1(b) where \(-1 \leq \xi, \eta \leq 1\). The source weighting functions of four adjacent grid points \( P_1 \sim P_4 \) are given as follows [23]

\[
\begin{align*}
\eta_1^n &= (1 - \xi_0^n)(1 - \eta_0^n)/4, \\
\eta_2^n &= (1 + \xi_0^n)(1 - \eta_0^n)/4, \\
\eta_3^n &= (1 + \xi_0^n)(1 + \eta_0^n)/4, \\
\eta_4^n &= (1 - \xi_0^n)(1 + \eta_0^n)/4,
\end{align*}
\]

(3)

where \( \xi_0^n = 2d_{x,j}^n / \Delta - 1 \), \( \eta_0^n = 2d_{y,j}^n / \Delta - 1 \) are the local coordinates normalized by the grid interval \( \Delta \). A sound source between grids is expressed by simultaneously driving the grid points \( P_1 \sim P_4 \) with the amplitude of \( w_i^n s^n \) where \( i = 1 \sim 4 \), and \( s^n \) is the sound source signal at \( t = n\Delta t \). The direct method can be implemented by exactly the same manner not only in the IWB method but also in the standard FDTD method [1].

Next, a moving sound receiver is implemented. In the case of a moving sound receiver, the basic idea is the same as that of the moving sound source. As shown in Fig. 1(a), when the receiver is located between the grid points, the sound pressure at the receiver is interpolated from the sound pressures of the adjacent four grid points \( P_1 \sim P_4 \) using the weighting function given in Eq. (3).

\[
p^n = u_1^n p_{i1}^n + u_2^n p_{i2}^n + u_3^n p_{i3}^n + u_4^n p_{i4}^n,
\]

(4)

where \( p_{i1}^n \sim p_{i4}^n \) are sound pressures at grid points \( P_1 \sim P_4 \).
2.2. Implementation of Moving Sound Source and Receiver in Convolution Method

In order to implement a moving sound source in the FDTD method by the convolution method, we here consider a discrete-time system. When the sound source is fixed at the position \( \mathbf{r}_S \) and the receiver is fixed at the position \( \mathbf{r}_R \), the acoustic signal \( p \) received at the receiver is expressed as

\[
p(n) = \sum_{m=0}^{n} s(m)h(n - m, \mathbf{r}_R; \mathbf{r}_S),
\]

where \( s(m) \) is the sound source signal, and \( h(n) \) is the impulse response between the source and receiver calculated by the FDTD method. In the case of the moving sound source, the relative position between the sound source and receiver changes every time step, so that the impulse response \( h(n) \) also changes accordingly. When the sound source is located at the position \( \mathbf{r}_S(m) \) at a certain discrete time \( m \), the impulse response at the fixed receiving point \( \mathbf{r}_R \) is expressed as \( h(n, \mathbf{r}_R; \mathbf{r}_S(m)) \). Therefore the receiving signal for the moving sound source is expressed as

\[
p(n) = \sum_{m=0}^{n} s(m)h(n - m, \mathbf{r}_R; \mathbf{r}_S(m)).
\]

When the sound source position \( \mathbf{r}_S(m) \) is not on the grid point, the weighting functions expressed in Eq. (3) are also applied to interpolate the impulse response from the adjacent grid points.

Thus, in order to obtain the signal \( p(n) \) at the receiver, it is necessary to obtain \( h(n, \mathbf{r}_R; \mathbf{r}_S(m)) \) at all source positions \( \mathbf{r}_S(m) \) on the moving path in advance as shown in Fig. 2(a). In the FDTD calculation, it is generally necessary to repeat the calculation for the number of sound sources to calculate the impulse responses from multiple sound sources as shown in Fig. 2(a). On the other hand, it is easy to simultaneously obtain the impulse responses at multiple receivers if there is only one sound source as shown in Fig. 2(b). Therefore, by using the reciprocity \( h(n, \mathbf{r}_R; \mathbf{r}_S(m)) = h(n, \mathbf{r}_S(m); \mathbf{r}_R) \), it is easy to calculate all impulse responses on the sound source path when an impulse is radiated from the receiver position \( \mathbf{r}_R \) and calculate the responses at multiple sound source positions \( \mathbf{r}_S(m) \).

\[\text{Fig. 2} \quad \text{The reciprocity of the impulse responses between the moving sound source and receiver.}\]

In the case of moving receiver, the impulse response \( h(n - m, \mathbf{r}_R; \mathbf{r}_S(m)) \) in Eq. (6) should be replaced by \( h(n - m, \mathbf{r}_R(n); \mathbf{r}_S) \) which is one between the receiver position \( \mathbf{r}_R(n) \) and the fixed sound source position \( \mathbf{r}_S \). Therefore the receiving signal for the moving receiver is expressed as

\[
p(n) = \sum_{m=0}^{n} s(m)h(n - m, \mathbf{r}_R(n); \mathbf{r}_S).
\]

When both the sound source and receiving point are moving, the combination of the impulse responses becomes enormous number, so that the convolution method is not suitable in this case.

2.3. Doppler Shift

As shown in Fig. 3, it is assumed that a sound source moves with velocity \( \mathbf{v}_S \) and a receiver moves with velocity \( \mathbf{v}_R \). The Doppler shift is determined by the radial velocity, which is velocity along the line between the source and receiver. The position and its radial velocity of the sound source are evaluated at the time \( t_S \) when the sound wave is radiated, and that of the receiver are evaluated at the time \( t_R \) when the sound wave is received. This is because the Doppler effect caused by the movement of the sound source is determined when the sound wave is emitted from the source, and that of the receiver is determined when the sound wave is received at the receiver. Therefore it is necessary to consider the change in the position and the radial velocity of the source and receiver due to the time lag of sound propagation. The magnitude of radial velocity \( v_{SR} \) for the sound source and radial velocity \( v_{RS} \) for the receiver are respectively given as

\[
v_{SR} = \frac{(R_R(t_R) - R_S(t_S)) \cdot v_S(t_S)}{|R_R(t_R) - R_S(t_S)|},
\]

\[
v_{RS} = \frac{(R_S(t_R) - R_S(t_S)) \cdot v_R(t_R)}{|R_S(t_R) - R_S(t_S)|},
\]

where \( \mathbf{r}_S \) and \( \mathbf{r}_R \) denote the position vectors of the source and receiver, respectively. So the normalized Doppler shift \( D \) at the receiver \( R \) is given as

\[
D = \frac{f_R}{f_S} = \frac{c_0 - v_{RS}}{c_0 - v_{SR}},
\]

where \( f_R \) is the receiving frequency, and \( f_S \) is the radiated source frequency.

\[\text{Fig. 3} \quad \text{The positional relationship between the moving sound source S and receiving point R.}\]
3. NUMERICAL EXPERIMENTS

3.1. Direct Method

We first conduct numerical experiments on the direct method. Figure 4 shows the 2-D numerical model for the sound source or receiver moving on a linear trajectory for the direct method. The region size is $50\times50\text{m}^2$ and is divided into $5,000 \times 5,000$ FDTD cells. The grid size is $\Delta = 10\text{mm}$, the time step is $\Delta t = 29.4117\text{ms}$, and the sound speed is $c_0 = 340\text{m/s}$, so the Courant number $\chi$ is 1.

For the moving source, the moving point source $P$ is located at the center of the domain at $t = 0$ and moves linearly with a constant speed $v$ in the direction of $\theta$ with respect to the $x$-axis. The fixed receivers $Q_1$ and $Q_2$ are located $15\text{m}$ away from the sound source. For the moving receivers, the point $P$ is a moving receiver and the point $Q_1$ or $Q_2$ is a fixed source that radiates sound wave only from one or the other. The direction from the point $P$ to the point $Q_1$ is assumed to be positive. The boundary condition is the Higdon’s second order absorbing boundary [26]. The calculations were performed on the NVIDIA Quadro RTX 8000 graphics processing unit machine with single precision.

Figure 5 shows the sound pressure waveforms calculated at the receiver. The source or receiver moves with a constant speed of $v = 64\text{m/s}$ (the Mach number $M = 0.2$), $v = 128\text{m/s}$ ($M = 0.4$) in the direction of $\theta = 0$. A differential Gaussian pulse with a pulse width of $3\text{ms}$ is radiated from the source at the time $t = 0\text{s}$. The velocity at which the point $P$ moves toward the point $Q_1$ is assumed to be positive. Figure 5(a) shows the results for the moving source and Fig. 5(b) shows the moving receiver. It is shown that the waveform is compressed in the moving direction of the source or receiver and is expanded in the opposite direction due to the Doppler effect. Some artifacts can be slightly confirmed in the calculated waveform for the moving source, while there is no artifact for the moving receiver. The exact cause of the artifacts is unknown, but it may be related to the interpolation of the sound source position. Figure 6 shows the sound pressure waveforms calculated at the receiver for the moving source when the grid point closest to the sound source position is driven (direct method).

Figure 7 shows the normalized Doppler shift against the Mach number $M$ for $\theta = 0$. A continuous sine wave with a frequency of $500\text{Hz}$ is radiated from the source. Figure 7(a) shows the results for the moving source and Fig. 7(b) shows the moving receiver. The Doppler shift is...
calculated by FFT of the sound pressure waveform calculated at the receiver. The calculation is performed in the range of the Mach number \( M \) of \( \sqrt{C_0/2} \) to 2. When the source speed exceeds the sound speed (\( M > 1 \)), a shock wave front is formed in the direction of the Mach angle (\( \sin^{-1}(1/M) \)). Therefore the waveform cannot be calculated in the direction of the Mach angle due to the numerical dispersion error. However the waveform can be calculated in the traveling direction of the source, so the results for \( M > 1 \) is indicated in the figure as a reference. The calculation results agree well with the theoretical values even for \( M > 1 \).

The accuracy is evaluated by the root mean square (RMS) error between the calculated and the theoretical normalized Doppler shifts expressed as following equation:

\[
\varepsilon = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (d_i - D_i)^2},
\]

(11)

where \( N \) is the number of evaluation samples, \( d_i \) is the numerically calculated normalized Doppler shift, and \( D_i \) is the theoretical one calculated by Eq. (10). Figure 8 shows the root mean square error of the normalized Doppler shift against traveling direction \( \theta \) for various source frequencies in the range of \( M = -0.5 \) to 0.5. As a whole, the error for the moving receiver is smaller than that for the moving source because the artifacts shown in Fig. 5(a) cannot be ignored in the moving sound source. In the case of the moving sound source, the error tends to increase as the frequency increases while there is no clear tendency in the case of the moving receiver. In both cases, the RMS error shows a symmetrical characteristic around 45 degrees, which is a reasonable result on an equally spaced collaged grid in the FDTD calculation. The error is within the range of \( 2 \times 10^{-5} \) to \( 2.2 \times 10^{-3} \) in all directions and frequencies. Since the threshold of the human frequency discrimination is about 0.2\% [27], the RMS error almost clears this criteria. It is confirmed that the moving sound source and receiver can be calculated accurately by the direct method.

A moving sound source or receiver passing linearly in front of the receiver or source is next calculated. Figure 9 shows the numerical model. The sound source or receiver moves linearly with an initial speed of \( M = 0.1 \) and an acceleration of \( a = 0.30 \text{m/s}^2 \) at a position 2 m in front of the receiver or source. The source radiates a continuous
sine wave with a frequency of 500 Hz. Figure 10 shows the normalized Doppler shift calculated by the short-time Fourier transform of the sound pressure waveform at the receiver. In the figure, the center frequency of the power spectrum is calculated by the reassignment method [28] in MATLAB toolbox. The reassignment method is a technique for sharpening a time-frequency representation by mapping the data to time-frequency coordinates that are nearer to the true region of support of the analyzed signal. In both cases, the Doppler shift from high to low frequencies is well reproduced as the source or receiver passes. The calculated result and the theoretical result show good agreement even in the case of acceleration.

Figure 11 shows the root mean square error of the normalized Doppler shift against the traveling direction $\theta$ for the passing sound source or receiver (direct method). In the convolution method, it is necessary to calculate the impulse responses between all grid points located around the moving path of the sound source or receiver in advance. In the model, the sound source or receiver starts moving from the center position $P_0$ and moves a distance of 10 m to the end point $P_{1001}$. There are about 4,004 grid points around the path of the moving source or receiver, so it is necessary to calculate up to 4,004 impulse responses between $S_i \ (i = 0, 1, \cdots, 1,000)$ and $Q_1$, $Q_2$ in advance including adjacent nodes. In the general FDTD calculation, 4,004 times calculation of the impulse response are required. However, it is possible using the reciprocity that the impulse responses can be obtained simultaneously at 4,004 grid points by radiating an impulse from $Q_1$ and receiving at $S_i$. The same manner can be applied to $Q_2$, so a total of 8,008 responses can be obtained by only two
calculations. It is expected that the calculation can be significantly reduced.

Figure 13 shows the root mean square error of the normalized Doppler shift against the traveling direction $\theta$ for various source frequencies in the range of $M = 0.5$ to 0.5, corresponding to Fig. 8 by the direct method. In the convolution method, once the impulse responses are calculated, it is not necessary to recalculate the impulse responses when the trajectory is the same even if the speed or frequency is changed. The results of the convolution method show the same symmetrical characteristics as the direct method.

Figure 14 shows the root mean square error of the normalized Doppler shift against the propagation direction $\theta$ for the linearly passing sound source or receiver (convolution method).

3.3. The Case When Sound Source and Receiver Move at the Same Time

Finally, we examined the case when the sound source and receiver move at the same time. Numerical experiments are performed using only the direct method, as the number of combinations of the impulse response is usually huge when both the source and receiver are moving. Figure 15 shows the case when the sound source and receiver move on the linear trajectory facing each other. Figure 15(a) shows the numerical model and Fig. 15(b) shows the normalized Doppler shift. The initial velocity of both the sound source and receiver is $v_0 = 0.1$ and the acceleration is $a = 0, 30 \text{m/s}^2$. They move linearly over a distance of 40 m with an interval of 2 m. A continuous sine wave with a frequency of 500 Hz is radiated from the source. The calculated results and the theory are in good agreement.

The case when a reflective wall exists in the domain is next examined. Figure 16(a) shows the numerical model. A source-receiver pair moves linearly at the same speed
with the initial speed of \( M = 0.1 \) in the direction of the reflection wall. To extract only the reflected wave, the sound pressure waveform without the reflecting wall is also calculated, then the direct wave is removed by subtracting it from the waveform with the reflecting wall. Figure 16(b) shows the normalized Doppler shift. In the case of constant speed, the Doppler shift is constant. The calculated value is 1.2218 compared to the theoretical value of 1.2222, which is an error of \(-4.19 \times 10^{-4}\). In the case of acceleration, the Doppler shift increases almost linearly because the source and receiver speed increases with time.

The case when the source-receiver pair moves on a circular trajectory is lastly examined. It is assumed that the sound source and receiver are separated by 2 m, and their center of gravity moves in a circular trajectory with a radius of 15 m as shown in Fig. 17(a). So, the source and receiver move with the same angular velocity keeping the distance at 2 m. The initial velocity of the center of gravity is \( M = 0.1 \) or the initial angular velocity is 2.267 rad/s, and the acceleration is \( a = 0, 30 \text{ m}/\text{s}^2 \). As shown in Fig. 17(b), the Doppler shift changes sinusoidally in the case of uniform circular motion, while it changes significantly with time when accelerated. As described above, it is confirmed that the proposed direct method can be applied even when the sound source and receiver move in an arbitrary trajectory at the same time.

**Fig. 17** A moving sound source and a receiver on a circular trajectory when there is a reflecting wall (direct method).

4. CONCLUSIONS

A moving sound source and a receiver with an arbitrary trajectory were implemented in the two-dimensional FDTD method. The direct method and the convolution method were proposed in which physically valid analysis is possible including the Doppler effect. As the results of two-dimensional numerical experiments, it is confirmed that both methods can be applied to the moving sound source and receiver including the Doppler effect, and that two methods have the almost same accuracy. It is found that the convolution method has advantages that the source waveform and the moving speed can be freely changed at the time of convolution.

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