Influence of Pressure Gradient on Streamwise Skewness Factor in Turbulent Boundary Layer

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Abstract. The paper shows an effect of favourable and adverse pressure gradients on turbulent boundary layer. The skewness factor of streamwise velocity component was chosen as a measure of the pressure gradient impact. It appears that skewness factor is an indicator of convection velocity of coherent structures, which is not always equal to the average flow velocity. The analysis has been performed based upon velocity profiles measured with hot-wire technique in turbulent boundary layer with pressure gradient corresponding to turbomachinery conditions. The results show that the skewness factor decreases in the flow region subjected to FPG and increases in the APG conditions. The changes of convection velocity and skewness factor are caused by influence of large-scale motion through the mechanism called amplitude modulation. The large-scale motion is less active in FPG and more active in APG, therefore in FPG the production of vortices is random (there are no high and low speed regions), while in the APG the large-scale motion drives the production of vortices. Namely, the vortices appear only in the high-speed regions, therefore have convection velocity higher than local mean velocity. The convection velocity affects directly the turbulent sweep and ejection events. The more flow is dominated by large-scale motion the higher values takes both the convection velocity of small-scale structures and sweep events induced by them.

1. Introduction
The effect of favourable (FPG) and adverse pressure gradients (APG) on turbulent boundary layer is not well understood. The most recent results of Harun et al. [1] show that the flow in FPG and APG conditions is driven by large-scale motion. Namely for FPG the decrease of large-scale energy occurs, while for APG the reaction is opposite as it was shown by Monty et al. [2] and Dróżdż & Elsner [3]. In the APG the energy of large-scale motion is enough high to create so-called outer peak of fluctuations. It is interesting how this high energy large-scale motion influence the production of turbulence near the wall. Mathis et al. [4] showed that in turbulent boundary layers there is amplitude modulation of the small-scale structures by large-scale motion. In this mechanism the amplitude of small-scales recorded by hot-wire probe increases in the large-scale high-speed regions. Latter on Mathis et al. [5] found that the wall-normal profile of the amplitude modulation between the large-scale and the envelope of the small-scales exhibits strong resemblance to the skewness profile of streamwise $u$ fluctuation component. The skewness factor, which is the moment of third order, is a measure of asymmetry of the probability distribution. Mathis et al. [5] decomposed streamwise velocity signal $u$ onto small-scale $u_s$ and large-scale $u_l$ and concluded that the skewness factor cross term $3u_s u_l^2$ is an alternative diagnostic measure to determined amplitude modulation rate. The
knowledge about the skewness factor distribution is important because it affects directly the small-scale turbulent quadrant events production associated with so-called sweep and ejection events. Sweep is an event, which has positive streamwise and negative wall-normal fluctuations, while ejection event has opposite relation. It was concluded that, for turbulent boundary layer in zero pressure gradient (ZPG), the sweep and ejection events are equally important for the turbulence production [6]. This is consistent with the equality of convection velocity to the mean local velocity in the outer region (at least for the small-scale motion). In case of pressure gradient flows the turbulence production is dominated by the ejection events (negative streamwise fluctuations) under FPG, while in APG by the sweep events (positive streamwise fluctuations). Therefore, with the enhancement of the sweep event or alternatively ejection event, the value of the streamwise skewness factor should increase or decrease respectively. The speculation presented above was proved by Simpson et al. [7] who showed that close to flow separation the skewness factor of \( u \) is positive, what indicates that there is enhanced sweep event. Nagano and Houra [8] suggested that “the rise in skewness is caused by structural changes in the near-wall region due to the pressure gradient”. Johansson and Alfredsson [9] showed that in the near-wall region of channel flow, “a positive skewness reflects the presence of high shear events imposed on a background of lower turbulence level”. The above literature review suggests that there is a direct relation between convection velocity and the increase of particular quadrant turbulent events. The aim of the paper is to confirm the direct relation between skewness factor and convection velocity of small-scale structures. The analysis has been performed based upon velocity profiles measured with hot-wire technique in turbulent boundary layer with pressure gradient corresponding to turbomachinery conditions. As a measure of pressure gradient influence on near-wall turbulence a skewness factor of streamwise velocity component was chosen.

2. Experimental apparatus and conditions

The experiment was performed in an open-circuit wind tunnel, where the turbulent boundary layer developed along the flat plate, which was 2807 mm long and 250 mm wide. The test section is located in the rear part of the wind tunnel. Its upper wall of test section was shaped according to the assumed distribution of pressure gradient corresponding to the conditions encountered in axial compressor blading. Additionally, to modify the flow circulation around the plate and to ensure negligible streamwise pressure gradient, the trailing edge flap and grid at the outlet from the test section was also applied. In order to obtain a fully developed turbulent boundary layer the tripping of boundary layer, after the leading edge of a flat plate was used. The facility is equipped with the computer-controlled, 2D traversing system (in streamwise and wall-normal direction).

Static pressure measurements were done using 70 pressure holes and the results of measurements is shown in figure 1. The pressure distribution is typical for turbomachinery case, where after short region of zero pressure gradient flow accelerates (from \( x_s = 197 \) mm) and then (from \( x_s = 427 \) mm) it decelerates. It is seen that pressure gradient values are within the range of \(-0.27 \div 0.28 \) Pa/mm. Velocity profiles were measured with single hot-wire anemometry probe of a diameter \( d = 3 \) \( \mu \)m and length \( l = 0.4 \) mm (Dantec Dynamics 55P31). The probe was combined with the DISA 55M hot-wire bridge connected to 14 bit PC card. Acquisition was maintained at frequency 50kHz with 10 seconds sampling records.

The position at the wall closest point of the hot-wire probe was determined using the mirrored image. Positions of 8 measuring traverses, dotted lines, are shown in figure 1. The distances of traverses from inlet plane, the corresponding dimensionless distances \( S_g = x_s / L \), where \( L \) is the length of the test section \( (L = 1067 \) mm) and the most important flow parameters are given in table 1. The conditions determined in inlet plane \( (S_g = 0) \), located in zero pressure gradient area are the mean velocity in core flow \( U_\infty = 15 \) m/s, turbulent intensity \( Tu = 0.4\% \).
Figure 1. The shape of upper wall and the corresponding static pressure and pressure gradient distributions along the flat plate.

Table 1. Location and parameters of the analysed profiles.

| PG | Traverse number | $x_s$ [mm] | $S_g$ | $U_\infty$ [ms$^{-1}$] | $u_\tau$ [ms$^{-1}$] |
|----|----------------|------------|-------|-----------------------|----------------------|
| FPG 1 | 1 | 197 | 0.185 | 15.14 | 0.632 |
| FPG 2 | 2 | 277 | 0.26 | 15.61 | 0.677 |
| FPG 3 | 3 | 367 | 0.344 | 16.97 | 0.744 |
| FPG 4 | 4 | 427 | 0.4 | 17.48 | 0.776 |
| APG 5 | 5 | 487 | 0.456 | 16.97 | 0.718 |
| APG 6 | 6 | 577 | 0.541 | 15.64 | 0.582 |
| APG 7 | 7 | 667 | 0.625 | 14.45 | 0.473 |
| APG 8 | 8 | 787 | 0.738 | 13.21 | 0.370 |

3. Results

3.1. Amplitude modulation mechanism

The physical significance of the amplitude modulation of small-scales by the large scales is not currently well understood [10]. It should be however, presumed that according to this mechanism, the large-scale motion induces low and high-speed regions near the wall and the amplitude of the small-scales is increased in the large-scale high-speed regions.

The qualitative relationship of amplitude modulation of the small-scale by the large-scale component where presented in figure 2 taken from Dróżdż and Elsner [3]. In order to have small- and large-scale signal components the carrier velocity signal was filtered by high- and low-pass filtering respectively. The cut-off frequency was set at timescale $\tau^+ \approx 160$, which is located in the middle between the inner and outer peaks. Figure 2 presents a carrier signal (upper time trace) recorded for
APG conditions in the buffer layer ($y^+ \approx 50$), large-scale signal component (middle time trace), while the lower time trace represents the small-scale signal component together with the envelope (dotted lines) corresponding to the large-scale signal. The envelopes were shifted in order to easily trace the variations of magnitude of the small-scale component. The coupling is evident, the small-scale component signal is clearly modulated by the large-scale motions in the APG. The small-scale amplitude to some extent varies with the envelope of the large-scales. It is clearly visible on the raw signal (upper trace) that the small scales in the high-speed large-scale regions are shifted towards positive values of scale, what suggest that they have convection velocity higher than the mean flow value.

![Figure 2](image.png)

**Figure 2.** Visual comparison of the scales coupling [3].

As it was shown by Harun et al. [1] the large-scale motion is less active in FPG and more active in APG. Therefore, in FPG the production of vortices is more or less random (there are no high and low speed regions). While in the APG the large-scale motion drives the production of vortices, which appear mainly in the high-speed regions. The more the flow is dominated by large-scale motion the higher values the skewness factor takes.

### 3.2. Skewness factor and convection velocity

In order to illustrate the relation between the skewness factor and convection velocity the skewness factor was calculated for streamwise velocity fluctuation, according to following formula:

$$ S_f = \frac{\overline{u^3}}{\overline{u^2}^{3/2}} $$

The results were shown in figure 3. The first group of the profiles (blue colour) were measured in FPG flow region, while the second group of profiles (red colour) were measured in APG flow region. As it can be observed the skewness factor is the highest close to the wall. Zero crossing is changing with the pressure gradient right above $y^+ \approx 15$, where well-known inner peak of fluctuation exist. It varies to the location $y^+ \approx 300$, where outer peak of fluctuation emerges in the APG region. Above this location the distributions are similar. The influence of pressure gradient on skewness factor is clearly visible. For the range of $y^+$ between 10 and 300 favourable pressure gradient causes the decrease of skewness factor, while adverse pressure gradient causes the increase of skewness factor. It is worth to
note that when the APG is strong enough, the skewness factor takes the positive values in the whole inner region of boundary layer.

Figure 3. Influence of FPG and APG on skewness factor $S_f$.  

The knowledge about the skewness factor distribution is important, because it seems that it is a good indicator of convection velocity of small-scale coherent structures, what was shown by Dróżdż & Elsner [11]. The paper confirmed the influence the pressure gradient on strength of turbulent quadrant events i.e. sweep and ejection. It was shown that in the FPG region the ejection event close the wall was enhanced while the sweep event was reduced. In APG region the reaction was opposite, namely the sweep event was enhanced, while ejection event was reduced. They conclude that the convection velocity was lower than the mean velocity in the FPG region and higher in the APG region. Particularly when convection velocity is higher the sweep event (with $u > 0$) is stronger, while when convection velocity is lower the ejection event (with $u < 0$) is stronger. In order to quantify the small-scale ejection and sweep changes the mean velocity $u_c$ between sweep event maximum $<u_{\text{sweep}}>$ and ejection event minimum $<u_{\text{ejection}}>$ for different pressure gradients were calculated according to following formula:

$$u_c = \frac{<u_{\text{sweep}}> + <u_{\text{ejection}}>}{2},$$

where $<_>_{\text{ave}}$ is phase-averaging of small-scale ejection or sweep events.

The results were shown in figure 4. First group of profiles (blue colour) were measured in FPG flow region, while the second group of profiles (red colour) were measured in APG flow region. As it can be observed the velocity $u_c$ distributions for different pressure gradient have similar shapes as distributions of skewness factor presented in figure 3. The positive values corresponds to convection velocity which is higher than mean local velocity, while negative values corresponds to convection velocity lower than mean local velocity. It is worth to note that below the inner peak of fluctuation ($y^+ = 15$) the skewness factor and $u_c$ is always positive and increase toward the wall. It coincidence with constant convection velocity below inner peak showed by Johansson [12]. In the same region the
mean velocity value decreases towards zero at the wall, therefore the difference between constant convection and increase of mean velocities is a result of what is shown in figure 4. This difference is also responsible for sweep events domination close the wall.

![Figure 4](image)

**Figure 4.** Influence of FPG and APG on phase-averaged velocity $u_C$.

The above results show that the convection velocity changes can be explained by amplitude modulation phenomenon, which is quantified by the skewness factor. With the increase of pressure gradient the energy of large-scale motion increases and it is the direct result of amplitude modulation. With the increase of large scale motion the small-scale structures are enhanced or created in the large-scale high-speed regions near the wall thus the small-scale structures convect at that velocity, which is above the local mean velocity and results with increase of sweep events strength.

The concept of this process has been clarified in figure 5, where was showed that strength of quadrant events is a function of their convection velocity. It occurs with the increase of large-scale motion energy not only with pressure gradient but also with Reynolds number. The upper schemes present the small-scale vortical structure for different convection velocity and how it influences the ejection and sweep events. Below the vortical structures schemes the areas of momentum transport changes is shown. For simplicity it was assumed that above the vortex center there is high momentum fluid (HMF), while below there is low momentum fluid (LMF). Middle case in figure 5 shows that when the difference between convection $U_C$ and mean $U$ velocities is zero, the energy of the sweep and ejection induced by the vortex is the same. The LMF occupy the same area as HMF, because the same amount of fluid is ejected as swept. It results in no change of total momentum in the area. Left case in figure 5 shows that transfer of the energy from sweep to ejection occurs. When the convection velocity is lower than mean velocity the difference between $U_C$ and $U$ is added to ejection event and subtracted from sweep event. The LMF area is greater than HMF area, because the higher amount of fluid is swept than ejected what results in increase of total momentum in the area. Right case in figure 5 shows that when the convection velocity is higher than mean velocity, the transfer of energy occurs from ejection to sweep. The difference between $U_C$ and $U$ is added to sweep event and subtracted from ejection event. The HMF area is greater than LMF area, because the higher amount of fluid is swept than ejected what results in increase of total momentum in the area.

Summing up in FPG and APG flows the large-scale motions cause the increase and decreases of small scales convection velocity respectively. Additionally, the momentum near the wall in FPG is lower.
(LMF area is greater than HMF area in figure 5), while in the APG is higher (HMF area is greater than LMF area in figure 5). The higher momentum near the wall is responsible for postpone of boundary layer separation in the APG region when the Reynolds number increases. Buckles et al. [13] showed that in the flow separation, the vortices send fluid toward the wall (sweep event) and entrain fluid from the reversed-flow region upward (increase of momentum). It is correlated with large, positive skewness values in the reversed-flow region caused by the passage of shear-layer vortices overhead. These observations led Buckles et al. [13] to suggest that the detached shear flow was driven by a mechanism other than just the external pressure gradient. Also Maciel et al. [14] pointed out that “pressure force and the turbulent transport no longer play an important dynamic role close to separation and flow become essentially as inertial flow”. The above literature proved that similar effect was observed, however the physical mechanism was not proposed.

Figure 5. Influence of large scale motions on quadrant events energy (LMF – low momentum fluid, HMF – high momentum fluid).

The observed phenomenon extends the knowledge about wall-normal momentum transfer in turbulent boundary layer. It was proved that the momentum transfer is additionally stimulated by large-scale motion, even though large-scale motion itself do not increase momentum close the wall, but only modulate small-scale structures production through the amplitude modulation mechanism. Especially, it is evident in APG region, where the production of small-scales vortices is increased in high-speed large-scale regions, the opposite reaction occurs in the low-speed regions. Therefore the structures have increased convection velocity, what in turn influence the quadrant events, which are responsible for wall-normal momentum transfer.

4. Conclusions
The results show that the skewness factor decreases in the flow region subjected to FPG and increases in the APG conditions. The change of both convection velocity and skewness factor is caused by influence of large-scale motion through the mechanism called amplitude modulation. It was proved that the momentum transfer is additionally stimulated by large-scale motion. The large-scale motion is less active in FPG and more active in APG. Therefore, in FPG the production of vortices is random (there are no high and low speed regions), while in the APG the large-scale motion drives the production of vortices, which appears mainly in the high-speed regions, therefore have convection velocity higher than local mean velocity. The more flow is dominated by large-scale motion the higher values takes both the convection velocity of small-scale structures and sweep events induced by them.
The observation could be useful in turbulent boundary layer separation control as well as in turbulence numerical modeling improvements.

5. References
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