Logic Programming for Describing and Solving Planning Problems

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Abstract
A logic programming paradigm which expresses solutions to problems as stable models has recently been promoted as a declarative approach to solving various combinatorial and search problems, including planning problems. In this paradigm, all program rules are considered as constraints and solutions are stable models of the rule set. This is a rather radical departure from the standard paradigm of logic programming. In this paper we revisit abductive logic programming and argue that it allows a programming style which is as declarative as programming based on stable models. However, within abductive logic programming, one has two kinds of rules. On the one hand predicate definitions (which may depend on the abducibles) which are nothing else than standard logic programs (with their non-monotonic semantics when containing with negation); on the other hand rules which constrain the models for the abducibles. In this sense abductive logic programming is a smooth extension of the standard paradigm of logic programming, not a radical departure.

keywords: planning, abduction, non-monotonic reasoning.

Introduction
A number of recent papers argue in favour of a new logic programming paradigm based on stable model semantics (Gelfond & Lifschitz 1988). Marek & Truszczynski 1999 introduces Stable Logic Programming as a novel programming paradigm. The language is basically DATALOG extended with negation. Stable models of programs form a finite family of finite sets, hence the solutions to search problems can be represented as stable models of Stable Logic Programs. The programming style is to introduce constraints which restrict the stable models to the solutions. The paper refers to various related efforts: the closest and most prominent one is that of (Niemela 1999a; Niemela 1999b) which proposes function free logic programming with stable model semantics as a paradigm for constraint programming which “brings advantages of logic programming based knowledge representation techniques to constraint programming”. The Smodels system of (Niemelä & Simons 1996) has evolved in one of the leading implementation efforts. Lifschitz (Lifschitz 1999a; Lifschitz 1999b) introduces Answer Set Programming and explores its use in planning. The formalism is slightly more expressive: disjunctive heads are allowed and two forms of negation are used. However simple transformations can eliminate disjunctive heads and classical negation and Answer Set Programming basically relies on the same implementations as Stable Logic Programming for execution (for computing stable models). In the rest of this paper, we refer to this programming paradigm as Stable Logic Programming.

When the solution of a problem is some finite set, encoding it as a term which is the computed answer of a query of a logic program induces a level of indirection which increases the distance between the problem domain and the program. Indeed, in this “term-based” programming style (De Schreye & Denecker 1999), knowledge about the problem domain is not a “first class citizen” in the problem representation. One can rightfully say that such programs are less declarative and hence worse from knowledge representation point of view than their counterparts based on stable model semantics. As a concrete example, consider the n-queens problem. The traditional (constraint) logic programming approach represents the solution as a list of column-positions with the position in the list encoding the row. This is arguable less declarative than the solution in the above formalisms where the stable model can be a set of facts position(i,j).

However the stable model paradigm is a rather drastic departure from the standard paradigm of logic programming (Marek & Truszczynski 1999). Abduction (Kakas, Kowalski, & Toni 1992; Kakas, Kowalski, & Toni 1998) is another formalism which also computes models (for the so called abducibles). It can be interpreted as a smooth extension of the traditional logic programming paradigm. Indeed, part of an abductive logic program is a traditional logic program consisting of predicate definitions. The extension consists of additional rules that are constraining the possible models of the abducibles. Consequently, a solution is not only a substitution for the variables in the query but, more importantly, a model for the abducibles. At a rather informal level, this paper revisits logic programming, the treatment of negation, the extension to abduction and
discusses the suitability of abductive logic programming to solve the problems for which Stable Logic Programming is being promoted, in particular its use in solving planning problems.

The purpose of this paper is to argue that the abductive logic programming paradigm is comparable to the stable model logic programming paradigm in bringing advantages of better knowledge representation techniques to constraint programming in general and to planning problems in particular. Moreover, being an extension of the standard logic programming paradigm instead of a radical departure, abductive logic programming is perhaps easier to learn for experienced logic programmers than stable logic programming.

**Logic programming revisited**

The impact of Kowalski’s seminal paper (Kowalski 1979) on the use of definite Horn clauses for programming is in a large part due to the combination of extreme simplicity of the formalism (definite Horn clauses of the form \( h(t) \leftarrow b(t_1), \ldots, b(t_n) \) with \( h(t), b(t_1), \ldots, b(t_n) \text{ atoms} \) with high expressiveness (the power of the universal Turing machine) and with the ability to control the procedural behaviour. Being rooted in first order logic theorem proving, definite Horn clauses were initially introduced as a subset of first order logic. When it came to semantics, in particular semantics of negation, it became gradually clear that logic programs are not a subset of first order logic. For definite programs there is a consensus that the least Herbrand model provides the appropriate declarative meaning. It defines which atoms are true. Atoms which are not true are false, hence the Closed World Assumption (Reiter 1978) is the correct semantics for deriving negative information from definite programs. Although negation as finite failure and completion semantics (Clark 1978) characterises what SLD can proof about definite programs, it is not the appropriate semantics. The Closed World semantics has the practical drawback that the set of false atoms is in general not recursively enumerable, hence not effectively computable (Apt 1990). (The lack of decidability is a major motivation for the work on termination analysis of logic programs.) Circumscribing the unique least model requires higher order logic and definite programs are abbreviations for such higher order theories (Denecker 1998). Negation is classical negation with respect to the implicit higher order theory.

The situation is less clear once general clauses (with negation in the body) are considered. There is an overwhelming amount of literature about different semantics, different extensions, ... Having rejected completion semantics, one major direction is to give up the idea of a unique model. It leads to the stable model semantics and the programming paradigms mentioned in the introduction (weaker than the universal Turing machine, because giving up functions). The other direction sticks to the unique model property: stratification, local stratification, ... well-founded semantics (Van Gelder, Ross, & Schlipf 1991). Although one model is but a special case of several models, I have a preference for the unique model approach. My argument is that when performing a programming task, the programmer should define a unique model for the task at hand. Moreover, that unique model should be complete, in other words, the well-founded model should be two valued. If not, the program has an error. In this view, a logic program is a set of inductive definitions (in the mathematical sense) defining a unique model. Note that, once a concept\(^1\) is defined, its negation can be used to define another concept. This insight is at the basis of the various forms of stratification. As we argued for the case of CWA and definite programs, such use of negation is classical negation, not negation as failure to proof, with respect to the implicit higher order theory which is associated with the program. Denecker (Denecker 1998) has investigated in depth the mathematical notion of inductive definition and has shown that general programs with a two valued well-founded semantics are correct inductive definitions. Note that normal programs are non-monotonic. Extending a normal program with extra clauses can cause non-monotonic changes in the model of the implicit higher order theory.

A problem is that not all programming tasks can be achieved by writing an inductive definition. Coming back to the n-queens problem, it is impossible —apart from giving all solutions as facts—to give an inductive definition of the position/2 predicate that describes the positions of the n-queens on the board. The solution traditionally pursued in (constraint) logic programming is to encode the set of true position/2 atoms in a data structure and to write a correct definition of a program searching the space of possible configurations for a safe one. A typical program is as follows:

```prolog
queens(Q,N) :- generate(Q,N,N),
            safe(Q),
            instantiate(Q).

generate([],0,_,_).
generate([X|T],M,N) :- M > 0,
                  X in 1..N,
                  M1 is M - 1,
                  generate(T,M1,N).

safe([]).
safe([X|T]) :- noAttack(X,1,T),
             safe(T).

noAttack(_,_,[],_).
noAttack(X,N,[Y|Z]) :- X \= Y,  
                    X \= Y + N,  
                    Y \= X + N,  
                    S is N + 1,  
                    noAttack(X,S,Z).

instantiate([]).
instantiate([X|T]) :- enum(X),
                    instantiate(T).
```

This program is written in a rather procedural style

\(^1\)One can consider each ground instance of an atom as a separate concept.
with a double recursion inside the safe/2 predicate to set up the constraints. This is quite a bit away from the ideal of declarative programming. The mapping between the program representation and the problem domain is not as simple as it should be in truly declarative programming.

Here Abductive Logic Programming comes to the rescue. Our approach is to distinguish between on one hand predicates one does not know (abducible predicates) and on the other hand predicates one knows and is able to define correctly and completely (defined predicates) when assuming a correct and complete definition of the abducible ones. Although one is unable to define the abducible predicates, one typically has some partial knowledge about them. In the queens problem, not every collection of position/2 atoms makes up a valid definition. They should satisfy the constraint that they do not attack each other and that there is one on each row. Hence, besides a definition component, a program should also have a constraint component. Then, given a set of abducible predicates A, a definition D and a set of constraints C, the task of an abductive solver is to come up with a definition ∆ (as a set of facts) of the abducible predicates A such that the constraints are true for the definition D ∪ ∆, i.e. D ∪ ∆ |= C. Note that we impose that D ∪ ∆ is a correct inductive definition; hence its unique model includes ∆.

To make things concrete, let us look at a program for the n-queens problem.

**Example 1 n-queens.**

abducible position/2.

% DEFINITIONS
size(8).
row(C) :- size(N), C in 1..N.
column(C) :- size(N), C in 1..N.
row(R) :- size(N), R in 1..N.

% CONSTRAINTS
% the arguments of position/2 have the % correct types
row(R) <- position(R,C).
column(C) <- position(R,C).
% at least one queen on each row
row_has_queen(R) :- position(R,C).
% no more than one queen on each row
row_has_queen(R) <- row(R).
% a queen attacks a queen on a higher row
false <- position(R1,C1), position(R2,C2),
R1 R2,
(C1=C2 ; abs(R2-R1)=abs(C2-C1)).

The program has three components. The first component declares which predicates are the abducible ones. The second component consists of the definitions. We follow a PROLOG-like notation for it. It has a definition for the size of the board, for the concepts row and column, and for the concept of a row which has a queen. Some of the definitions make use of an assumed "built-in" X in Y..Z which returns the integers X in the interval Y to Z. row/1 and column/1 serve as "type definitions" for the arguments of the open predicate position/2. The third component contains the constraints. In principle, a constraint could be any first order logic formula. However, my preference is for rules, universally quantified implications. In my opinion, they offer the best compromise between conciseness and readability. A rule is a formula of the form head ← body with head a disjunction of literals (with "::" as separator) and body a conjunction of literals (with "::" as separator). Because it is common in PROLOG programs and to achieve more concise formulations, we also use a disjunction of bodies in the position of a body literal (as in the last constraint). Similarly one could allow a conjunction of heads in the position of a head literal; e.g. replacing the first two constraints by (row(R),column(C)) :: position(R,C). I write explicit true for an empty conjunction and false for an empty disjunction to remind these are constraints, not definitions or goals. The first two constraints express the types of the arguments of the open predicate. The next two state that there is exactly one queen on every row. Finally the last constraint states that a queen should not attack a queen on a later row i.e. should not be on the same column or diagonal. The symmetry of the attack relation is used to avoid redundant constraints (the presence of R1 < R2).

It is fair to say that this formulation is more declarative than the usual one in (Constraint) Logic Programming where a list of column positions is built which represents a safe configuration.

Expressing the same problem as a Smodels program, one obtains:

row(1..8) <- .
column(1..8) <- .
position(R,C) <- row(R), column(C),
    not negposition(R,C).
negposition(R,C) <- row(R), column(C),
    not position(R,C).
% at least one queen on each row
row_has_queen(R) :- position(R,C),
    row(R), column(C),
    not negposition(R,C).
% no more than one queen on each row
row_has_queen(R) <- row(R), column(C),
    position(R,C),
    not negposition(R,C).
% a queen attacks a queen on a higher row
false <- position(R1,C1), position(R2,C2),
    R1 < R2,
    (C1=C2 ; abs(R2-R1)=abs(C2-C1)).
% no two queens on same column
false <- position(R1,C), position(R2,C),
    R1 = R2,
    column(C1),
    abs(C2-C1).
% no two queens on same diagonal
false <- position(R1,C1), position(R2,C2),
    R1 = R2,
    C1 = C2,
    abs(C2-C1).

In this regard, I agree with the stable logic programming’s use of rules to represent a unit of knowledge.
This program is a minor variant of the program in [Niemelä 1999].

A first major difference with the abductive program is the pair of rules —typical for stable logic programming— with position/2 and negposition/2 in the head that defines the solution space of the problem: for each candidate pair of coordinates (i, j), either position(i, j) should be true —there is a queen with coordinates (i, j)— or negposition(i, j) should be true —there is no queen with coordinates (i, j)—. In the abductive program, the abduced position/2 facts are the definition of the position/2 predicate, hence there is no queen with coordinates (i, j) when no such fact is abduced. On the other hand, the abductive program has constraints to ensure that the abduced position/2 facts are well typed, i.e. have valid coordinates. A second major difference is in the way the constraint that each row must have a queen is expressed. In the abductive program the abduced position/2 facts are the definition of the position/2 predicate, hence there is no queen with coordinates (i, j) when no such fact is abduced. On the other hand, the abductive program has constraints to ensure that the abduced position/2 facts are well typed, i.e. have valid coordinates. A second major difference is in the way

This is a subtlety of stable logic programming which is non-trivial for beginners. Perhaps the difficulty for the beginning stable logic programmer lies in the problem of recognising that the same notation —rules— are being used for three purposes. In a rule with a head such as row_has_queen(R) <- row(R), not row_has_queen(R) expresses that a candidate stable model containing row(R) is extended with row_has_queen(n) which does not establish that there is a position(n, ... ) atom in the model. This is a subtlety of stable logic programming which is non-trivial for beginners. Perhaps the difficulty for the beginning stable logic programmer lies in the problem of recognising that the same notation —rules— are being used for three purposes. In a rule with a head such as row_has_queen(R) <- row(R), column(C), position(R, C), the rule defines the row_has_queen/1 predicate[]. A rule with an empty head such as <- row(R), not row_has_queen(R) is a constraint, eliminating a number of candidate stable models. Finally, rules with heads but which are not inductive definitions, due to the loop through negation (as in the rule pair with heads position(R, C) and negposition(R, C)) serve to generate a solution space for the predicate of interest (a negative predicate has to be introduced to make the negative information about the predicate of interest explicit).

A minor difference is that Smodels accepts only domain restricted programs, i.e. that each variable occurs in a positive “domain” predicate (row/1 or column/1). This ensures efficient handling of the so called grounding problem [Niemelä 1999]. Introducing some extra syntax for defining domains and for typing predicates, e.g.

| constant size == 8. |
| domain row == 1..size. |
| domain column == 1..size. |
| predicate position(row, column). |
| predicate negposition(row, column). |
| predicate row_has_queen(row). |

it is likely these extra calls could be automatically inserted. In fact, also the abductive program could be simplified when including domain declarations:

| constant size == 8. |
| domain row == 1..size. |
| domain column == 1..size. |
| abducible position(row, column). |

The definitions of row/1 and column/1 and the first two constraints enforcing the well-typedness of abduced position/2 facts can then be dropped.

### Planning

[Lifschitz 1999] describes the use of answer set programming for planning problems. He is building on the work of other authors and we refer to his paper for the broader context of this approach to planning and the contributions of others in its development. A plan is described by a history or evolution of a system over a fixed time interval. In the concrete case of the blocks world, a configuration of the world at a time point \( T \) is described by a number of facts on(B,L,T) expressing that block \( B \) is on \( L \) (another block or the table) at time \( T \). Evolution is caused by move(B,L,T) actions expressing that block \( B \) is moved to location \( L \) at time \( T \). The answer set programming approach consists of formulating constraints such that the stable models (describing the successive configurations and the move actions) only describe legal evolutions from some initial configuration to some final configuration. Our intention is to show that such problem can equally well be formulated in abductive logic programming. The main problem discussed in [Lifschitz 1999] is a blocks world problem in a blocks world with two grippers. The problem is to find a plan which converts an initial configuration into a final configuration. In developing our solution, we try to follow the answer set program (ASP) of Lifschitz as close as possible.

In what follows, the constant \( \text{maxt} \) is used to represent the time at which the final configuration must be reached. The ASP starts with defining the “solution space” by the rules

\[
\begin{align*}
on(B, L, 0); & \quad \neg \text{on}(B, L, 0) \leftarrow \text{(1)} \\
moves(B, L, T); & \quad \neg \text{moves}(B, L, T) \leftarrow (T \neq \text{maxt}) \text{(2)}
\end{align*}
\]

As there are other rules “defining” \text{on}/3 for non-zero time points, we isolate the initial configuration into an abducible \text{initially on}/2, hence we use two abducibles, \text{initially on}/2 and \text{move}/3. To
ensure the abduced facts are well typed, we introduce integrity constraints such as \[ \text{movetime}(T) \leftarrow \text{move}(B,L,T) \] where \( \text{movetime}/1 \) is defined to be any time point in the interval \( 0 \) to \( \text{maxt} - 1 \).

Next we define \( \text{on}/3 \).

\[
\begin{align*}
on(B,L,0) & : - \text{initially}_\text{on}(B,L) & (3) 
\text{on}(B,L,T+1) & : - \text{move}(B,L,T). & (4) 
\text{on}(B,L,T+1) & : - \text{on}(B,L,T), 
\end{align*}
\]

\[ \text{not}((\text{terminates}_\text{on}(B,T))) \] (5)

The first clause defines \( \text{on}/3 \) for time point 0. The second clause expresses that moving a block to a location causes that block to be at that location in the next time point. The third clause is the frame axiom, it expresses that blocks stay in place at time \( T + 1 \) if it was there at time \( T \) and its stay was not terminated at time \( T \). In addition we have to define \( \text{terminates}_\text{on}/2 \). It can be defined as

\[ \text{terminates}_\text{on}(B,T) : - \text{move}(B,L1,T), \]

\[ \text{on}(B,L0,T), L0 \neq L1 \] (6)

The ASP also has the rule for describing the effect of a move. For the frame axiom it uses:

\[ \text{on}(B,L,T+1) \leftarrow \text{on}(B,L,T), \]

\[ \text{not} \text{-on}(B,L,T+1)(t \neq \text{maxt})(7) \]

This rule expresses the default that \( B \) is still on \( L \) at time \( T+1 \) if it was there at time \( T \) and one fails to prove that it is not on \( L \) at time \( T+1 \). Because there is now a rule which has \( \text{not} \text{-on}(B,L,T+1) \) as a condition, i.e. the absence of a false fact, the ASP needs rules defining these false facts. It uses:

\[ \text{-on}(B,L,T) \leftarrow \text{on}(B,L1,T), L \neq L1. \] (8)

Our decision to define \( \text{on}/3 \) has forced us to deviate from the ASP. In my opinion it makes the program easier to understand. By the way, our approach is also feasible with stable models. Niemelä uses a rule similar to ours in his stable logic program for solving planning problems (Niemelä 1993).

Our definition of \( \text{on}/3 \) is but a variant of the well known acyclic PROLOG program solving the Yale Shooting Problem. It is a widespread belief that it relies on negation as failure. For acyclic programs (Apt & Hazem 1991), many semantics coincide, in particular the completion semantics coincides with the well-founded or inductive definition semantics. As already stated before, our view is that such programs make use of classical negation with respect to the implicit higher order theory circumscribing the unique model.

The rest of the program consists of constraints expressing the assumptions about the executability of \( \text{move}/3 \) steps. We have the following constraints (see the program below for the actual code) which also occur in the answer set program unless otherwise stated.

\[ * \text{It is one of the rules which I found hard to grasp when learning about answer set programming.} \]

- A block cannot be moved if another block is on top of it.
- A block cannot be moved to a moving target. (This subsumes the constraint that a block cannot be moved to itself).
- A block cannot be moved to two locations. In the answer set program, this constraint is subsumed by the constraint that a block cannot be at two locations.
- No two blocks can be on top of the same block.
- The robot can perform only two moves at a time which we express by the constraint that one cannot move three blocks at a time (because we already excluded to move the same block to two different locations). The answer set program uses (without real need?) the more complex constraint that there cannot be three moves at the same time.
- The answer set program also expresses the constraint that each block must be supported (directly or indirectly) by the table. It is easy to show that this property holds if it holds in the initial configuration (the target of a move is a block which is supported) or in the final configuration, hence we omit it. It would be needed if both initial and final configuration are incomplete. Likely the same argument holds for the answer set program.
- Constraints expressing the initial and the final configuration.

The complete abductive program, including an initial and final configuration, is as follows:

Example 2 A planning problem

abdicable(move/3).
abdicable(initially_on/2)

% DEFINITIONS

maxtime(3).
block(B) :- B in 1..6.
location(table).
location(L) :- block(L).

movetime(T) :- maxtime(T1), T0=T1-1,
T in 0..T0.

% definition of on/3
on(B,L,0) :- initially_on(B,L).
% a block is at L at t+1 if moved to L at t
on(B,L,T+1) :- move(B,L,T).
% frame axiom (uses negation)
on(B,L,T+1) :- on(B,L,T),
\text{not}((\text{terminates}_\text{on}(B,T))).
% auxiliary definition
terminates_on(B,T) :- move(B,L1,T),
on(B,L0,T), L0 \neq L1.

% CONSTRAINTS
% initial configuration
initially_on(1,2) <- true.
initially_on(2,table)<- true.
initially_on(3,4) <- true.
initially_on(4,table) <- true.
initially_on(5,6) <- true.
initially_on(6,table) <- true.

% initially_on/2 is correctly typed
block(B) <- initially_on(B,L).
location(L) <- initially_on(B,L).

% move/3 is correctly typed
block(B) <- move(B,L,T).
location(L) <- move(B,L,T).
movetime(T) <- move(B,L,T).

% it is illegal to move a block if no another block is on it
false <- move(B,L,T), on(B1,B,T).

% it is illegal to move a block to a moving target (implies it cannot be moved to itself)
false <- move(B1,B2,T), move(B2,L2,T).

% a block cannot be moved to two locations
false <- move(B,L1,T), move(B,L2,T), L1 \= L2.

% no two blocks can be on top of same block
false <- on(B1,B,T), on(B2,B,T),
        B1 \= B2, block(B).

% it is illegal to move three blocks at the same time
false <- move(B1,L1,T),move(B2,L2,T),
         move(B3,L3,T),
         B1 \= B2, B1 \= B3, B2 \= B3.

% the final configuration
on(1,table,3) <- true.
on(2,1,3) <- true.
on(3,2,3) <- true.
on(4,table,3) <- true.
on(5,4,3) <- true.
on(6,5,3) <- true.

I believe it is fair to claim that this program is from knowledge representation point of view as good as the program given by Lifschitz.

Conclusion
In this paper we have repeated the viewpoint of Marc Denecker [Denecker 1998] that good logic programs are inductive definitions and that the use of negation in normal logic programs is better not considered as negation as failure, but as classical negation with respect to the higher order logic theory which circumscribes the unique model of the program read as an inductive definition. A correct program divides its world (the Herbrand base of the program) in two halves, the true atoms and the false atoms. This logic is non-monotonic, adding new clauses to the program may cause non-monotonic changes to the unique model.

To cope with partial knowledge, we use a simple scheme for abduction, where a set of abducibles A identifies the predicates one is unable to define completely with an inductive definition and where integrity constraints I are expressing the partial knowledge about the abducibles. In this paradigm, the purpose is not to compute an answer to the variables in a query, but to find a model for the abducibles satisfying the integrity constraints. Representing the model as a set of ground facts Δ, the computational task is to find a Δ such that the integrity constraints I are true in the unique model of the logic program P ∪ Δ (which should be a correct inductive definition). It is currently unclear to me whether the requirement that P ∪ Δ is a correct inductive definition puts a limitation on the usability of this paradigm. It is also unclear whether there is need for the minimality [Kakas, Kowalski, & Toni 1998] of abductive solutions Δ.

With this form of negation, there is the problem that the negative information is not recursively enumerable. Hence, it is impossible to define a complete proof procedure. The answer to this is that there is a need to prove termination of the programs one write (for the proof strategy being used). In fact this is not different from the current situation as proof procedures are also incomplete in practice for other semantics. Perhaps the paradigm also creates a need for a methodology and for (automated) methods to verify that a program is a correct inductive definition. Partial solutions exist, for example acyclic programs [Apt & Bezem 1991] are correct inductive definitions. Note also that terminating proof procedures are feasible in the function free case which is addressed by the stable logic programming paradigm.

Through two examples, the n-queens problem and a planning problem, we have shown that our paradigm offers solutions to problems which are, from knowledge representation point of view, as good as their counterparts in the stable logic programming paradigm.

Personally, we believe that our approach even offers some advantages (though we realise this is a matter of taste and of familiarity with various formalisms). Our paradigm is a natural extension to logic programming. We only add abducibles and integrity constraints to it (and use a notation for the latter which distinguishes them from program clauses defining predicates). Stable logic programming is a rather radical departure from standard logic programming. Moreover we see a number of drawbacks in its programming paradigm.

5 A query ?- p(X) can be translated in an integrity constraint false <- p(X), x(X) with x/1 a new abducible, hence queries can still be supported
may well be due to our ignorance and lack of familiarity and which can possibly be solved by offering a more high-level language. We observed that the rule concept is used for different purposes: for defining what we call the abducibles and their solutions space (by rules which fail to be inductive definitions), for defining other predicates (by rules which are inductive definitions) and for expressing integrity constraints (by rules without head). Moreover there seems to be sometimes a need for explicitly defining what is false (e.g. the rule \( \neg P \)) in the planning problem; such rule is not required for the predicate \( \text{row has queen}/1 \) in the queens problem. My understanding is that such rules are needed when the negated predicate is used as a default (not \( \neg \ldots \)) in the condition of a rule. This seems to spoil somewhat the modularity of the represented knowledge.

We have not discussed implementation. With some minor transformations, our programs are executable under the abductive proof procedure SLDFNA (De Schreye & Denecker 1998). Recently this procedure has been integrated with a finite domain constraint solver (SLDNFAC) (Denecker & Van Nuffelen 1999). SLDNFAC is a Prolog meta interpreter which collects finite domain constraints and passes them on to the finite domain constraint solver. It is inspired by the work of Kukas on the ACLP system (Kukas & Mourlas 2000). Notwithstanding the overhead of meta interpretation it outperforms Smodels (Niemelä & Simons 1996; Niemelä 1999) on the n-queens problem (Pelov, De Mot, & Bruynooghe 2000). On other problems (graph colouring), the performance is comparable. In fact also other implementation paths are feasible, including translation to Smodels. E.g. (Sato & Iwayama 1991) describes how to transform an abductive problem in a stable logic program.

Technically speaking, this paper offers little that is new. Inductive definitions are being promoted by Marc Denecker for quite a while (Denecker 1998). Abduction has been widely studied and surveyed (Kakas, Kowalski, & Toni 1992; Kakas, Kowalski, & Toni 1998). If we added anything it was adding limitations: insistence on the simple view that \( P \cup \Delta \) should be an inductive definition in which the integrity constraints are true. While it is unclear whether this is expressive enough for all purposes, hopefully it offers the advantage that it can be understood with little effort by logic programmers unfamiliar with abduction and stable model programming. For example by practitioners doing finite domain CLP programming and which are now writing “procedural” programs for solving their problems using various versions of Prolog extended with a finite domain solver. The language we propose enhanced with some mechanisms to declare the finite domains over which the abducibles range could perhaps be a useful language for specifying finite domain CLP problems at a higher level than is currently the case.

Before us, other have proposed new languages incorporating abduction for solving the class of problems we addressed. The n-queens problem is also described in e.g. (Kowalski, Wetzel, & Toni 1998). The use of abduction for solving planning problems goes back to Eshghi 1988.

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