A dynamical approach to link low energy phases with leptogenesis

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If lepton masses and mixings are explained by a flavour symmetry in seesaw model which leads to $U_{e3} = 0$ at leading order, we find that, under reasonable assumptions, a future observation of low energy lepton CP violation implies, barring accidental cancellations, a lepton asymmetry both in flavoured leptogenesis and in its one-flavour approximation. We explicitly implement this approach with a predictive seesaw model for Tri-Bimaximal Mixing (TBM) and show how cosmological baryon asymmetry can be directly trigged by low energy phases appearing in $U_{e3}$. Thanks to this direct correlation we can derive a lower bound on the reactor angle $\theta_{13}$: $\sin^2 \theta_{13} \gtrsim 0.005$.

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A. Introduction. All experimental data widely confirm the existence of neutrino masses, significantly smaller than those of charged fermions. Lepton mixing pattern is also very different from $V_{\text{CKM}}$ because it contains a nearly maximal atmospheric angle $\theta_{23}$ and a very precise tri-maximally mixed solar angle, i.e. $\sin^2 \theta_{12} = 1/3$ [1,2]. The appealing feature of neutrino mass structure can be nicely explained by (type I) seesaw mechanism implemented with an appropriate flavour symmetry. The super-heavy right-handed Majorana neutrinos, added to the standard model (SM), violate the lepton number by two units. Then their out-of-equilibrium decay can play a role in the generation of the observed baryon asymmetry of the universe (BAU) through leptogenesis [3]. CP violation in the leptonic sector is a necessary ingredient in order to implement leptogenesis. However, the seesaw lagrangian generically contains a larger number of free parameters than its effective light neutrino sector and consequently has a poor predictive power. In particular, in addition to the CP violating phases present in the neutrino mixing matrix $U_{\text{PMNS}}$, there are high energy phases not directly observable in low energy experiments. Establishing a direct connection between leptogenesis and low energy phases is a very important issue and might offer a possible test of seesaw mechanism. In one-flavour limit, only high energy phases explicitly appear in the lepton asymmetry and such a connection is generically believed to arise only from minimal [3] /texture zeros [6] seesaw approaches. Otherwise, one can take into account flavour effects and in this case the lepton asymmetries depend on $U_{\text{PMNS}}$. An explicit connection between leptogenesis and low energy phases can then arise when high energy phases are simply assumed to be absent [3].

In the present letter, we propose a dynamical approach to link leptogenesis with low energy CP violating phases. We exploit the possibility that an underlying flavour symmetry naturally leads to both $U_{e3} = 0$ and vanishing lepton asymmetries at leading order. We show that, under reasonable assumptions, leptogenesis is always related with the low energy CP violating phases both in “flavoured” and “unflavoured” regimes. As an explicit example, we will also present a predictive seesaw model for TBM based on $A_4$ flavour symmetry in which low energy phases are responsible for the BAU.

B. General Consideration. The smallness of the reactor angle $\theta_{13}$ is a popular ingredient to characterize a flavour symmetry. We will begin our analysis by recalling the general mass structure which automatically leads to a vanishing $\theta_{13}$ [8]. Consider for a moment the effective lepton sector in the flavour basis $-\mathcal{L} = e^\dagger m_{\ell} e + \nu m_{\nu}^\dagger \nu$ where $m_{\ell} = \text{diag}(m_e, m_\mu, m_\tau)$ and

$$m_{\nu} = U_{\text{PMNS}}^\dagger D_m U_{\text{PMNS}}$$

(1)

where $D_m \equiv \text{diag}(m_1, m_2, m_3)$ with $m_i \geq 0$. Using the standard parametrization for the neutrino mixing matrix $U_{\text{PMNS}}$ and imposing $U_{e3} = 0$, one can immediately see that $m_{\nu}^e$ takes the form:

$$
\begin{pmatrix}
 a - b & d\sqrt{2} \cos \gamma & d\sqrt{2} \sin \gamma \\
 d\sqrt{2} \cos \gamma & a + (c - b) \cos 2\gamma & (c - b) \sin 2\gamma \\
 d\sqrt{2} \sin \gamma & (c - b) \sin 2\gamma & a + (c - b) \cos 2\gamma
\end{pmatrix}
$$

(2)

where the parameters depend on the low energy observable quantities and in particular $\gamma = \theta_{23}$. This matrix is invariant under a parity operator $G_{23} \simeq Z_2$:

$$G_{23} = \begin{pmatrix}
 1 & 0 & 0 \\
 0 & \cos \gamma & \sin \gamma \\
 0 & \sin \gamma & -\cos \gamma
\end{pmatrix}
$$

(3)
which exactly corresponds to the $\nu_{\mu} - \nu_{\tau}$ exchange symmetry when $\gamma = \pi/4$ \cite{4}. $L_e - L_\mu - L_\tau$ symmetry \cite{10} is the most simple example corresponding to the case $a = b = c = 0$. Bi-Maximal mixing pattern \cite{11} can be realized by choosing simultaneously $\gamma = \pi/4$ and $c = 0$ while TBM \cite{12} requires $\gamma = \pi/4$ and $c = d$. Then our following analysis can in principle work within all these models. A realistic flavour symmetry group $G_f$ should be usually larger than $G_{23}$ in order to enforce also diagonal and hierarchical charged lepton masses. Indeed, the peculiar feature of the flavour basis for this class of models can be achieved dynamically by vacuum misalignment in the spontaneous breaking of $G_f$. Recently, models in which charged fermion hierarchies are directly generated by vacuum alignment that maximally breaks the $\nu_{\mu} - \nu_{\tau}$ symmetry \cite{13,14} become of great interest.

Now we consider the seesaw lagrangian

$$-\mathcal{L} = \nu^c Y_{\nu} h^u + \nu^c M_{\nu} v^c + h.c.$$  

which gives rise to light neutrino masses $m_\nu = -v^2 Y_{\nu}^T M^{-1} Y_{\nu}$ after the electroweak (EW) symmetry breaking. The theory can be further implemented by a flavour symmetry $G_f$ broken spontaneously by a set of flavon fields $\Phi$. We require that the scalar potential for $\Phi$ allows two different alignments in such a way that the effective neutrino sector preserves a symmetry containing $G_{23}$ and the charged leptons are diagonal. For the model building, it is quite natural to ensure that the leading order lepton mixing matrix is already encoded in the right-handed neutrino mass matrix $M$ which should be then diagonalized by the transformation:

$$U_0^\dagger M U_0^* = \text{diag}(M_1, M_2, M_3) \equiv D_M \ , \quad (4)$$

where $(U_0)_{13} = 0, M_i > 0$ and $U_0$ differs from $U_{PMNS}$ by subleading order contributions $\sim \langle \Phi \rangle / \Lambda \ll 1$. Observe that any symmetry spontaneously broken by $\Phi$ which can lead to a mass pattern of the form \cite{2} at effective level can be realized also for right-handed neutrino mass matrix $M$ which should be then diagonalized by the transformation:

$$m_{\text{diag}} = v^2 U_0^T Y_{\nu}^T U_0^+ M_{\text{diag}}^{-1} Y_{\nu} U_0^T$$

Defining $\tilde{Y}_{\nu} = U_0^\dagger Y_{\nu} U_0$, the solution to the previous equation is given by

$$\tilde{Y}_{\nu} = \text{diag}(\pm \sqrt{m_1 M_1}, \pm \sqrt{m_2 M_2}, \pm \sqrt{m_3 M_3}) / v \quad (5)$$

and this is equivalent to require that $\tilde{Y}_{\nu}$ has the same structure of $M$ displayed in \cite{2}.

We suppose that the flavour symmetry is broken at a very high scale $10^{12} \text{GeV} < \langle \Phi \rangle \lesssim M_1$. In this regime we can study the leptogenesis in the so-called one-flavour approximation. The CP-violating asymmetry, summed over all flavours, can be expressed in the form (for hierarchical heavy neutrinos):

$$\epsilon_j \cong -\frac{3 M_j}{16 \pi v^2} \sum_i \text{Im} (m_i^2 R_{ij}^2) \frac{1}{\sum_i m_i |R_{ij}|^2} \quad (6)$$

where the orthogonal complex matrix $R$ contains all the information on high energy phases of the seesaw model and is given by \cite{16}

$$R = v M_{\text{diag}}^{-1/2} \tilde{Y}_{\nu} U_{PMNS} M_{\text{diag}}^{-1/2} \quad (7)$$

where $\tilde{Y}_{\nu}$ is neutrino yukawa coupling in the basis of diagonal right-handed neutrinos. Apparently the low energy phases present in $U_{PMNS}$ do not play any role in generating lepton asymmetry. However, in our context, the neutrino yukawa coupling $Y_{\nu}$ is subject to the condition \cite{15} and we obtain a trivial $R$:

$$R = \begin{pmatrix} \pm 1 & 0 & 0 \\ 0 & \pm 1 & 0 \\ 0 & 0 & \pm 1 \end{pmatrix} \quad (8)$$

As an immediate consequence the lepton asymmetry $\epsilon_j$ vanishes (even including flavour effects). It is important to remind that the vanishing asymmetry is not a consequence of the preserved $G_{23}$ symmetry. Our viewpoint is quite different from that pointed out by authors of \cite{17}, indeed, in their models the vanishing $\epsilon_j$ are tightly due to the $\nu_{\mu} - \nu_{\tau}$ exchange symmetry.

The previous analysis, however, is expected to be modified by higher order corrections suppressed by $\langle \Phi \rangle / \Lambda$. When these corrections are accounted for, $U_{3} \equiv U_{PMNS}$ is no longer vanishing and, at the same time, the matrix $R$ will slightly differ from identity. Then, barring accidental cancellations, a non-vanishing $\epsilon_j$ is related to non-vanishing Dirac or/and Majorana phases appearing in $U_{3} \equiv U_{PMNS}$. This result is quite general and the dependence of generated lepton asymmetry from low energy phases is dictated by flavor symmetries. However, since the next-to-leading order (NLO) corrections are not determined by flavour symmetries, a direct bridge between the BAU and the element $U_{3}$ cannot be generally established. In the following we will illustrate a case of leptogenesis in the $A_4$ seesaw model proposed in \cite{14} in which a relationship between the low energy phases and the generated BAU arises naturally.

C. A constrained seesaw model for TBM. So far the discussion was completely general. Now we will focus on a special case of \cite{2} with $\gamma = \pi/4$, $c = d$ and $a = -2d$ which corresponds to a possible realization of TBM pattern. Here we consider the seesaw model for TBM based on the flavour symmetry $A_4 \times Z_3 \times Z_4$ \cite{14}. We recall that the discrete group $A_4$ is the group of even
TABLE I: The transformation properties of leptons and EW Higgs doublets under $A_4 \times Z_3 \times Z_4$.

| Field | $\ell$ | $e^c$ | $\mu^c$ | $\tau^c$ | $\nu_e$ | $\nu_\mu$ | $\nu_\tau$ |
|-------|--------|-------|--------|--------|--------|--------|--------|
| $A_4$ | 3      | 1     | 1      | 1      | 3      | 1      | 1      |
| $Z_3$ | 1      | 1     | 1      | $\omega$ | 1     | 1      | 1      |
| $Z_4$ | 1      | $-i$  | 1      | 1      | $1-i$  | 1      | $-i$   |

permutations of 4 objects and has one triplet and three singlet $(1, 1', 1'')$ representations.

The lepton and EW Higgs content, together with their transformation properties under the flavour group, is displayed in Table I. The flavour symmetry breaking sector consists of the scalar fields neutral under the SM gauge group: $(\varphi_T, \xi')$ for charged leptons, $(\varphi_S, \xi, \zeta)$ for neutrinos. The additional $Z_3 \times Z_4$ discrete factor is needed in order to reproduce the desired alignment of scalar fields and at the same time allows a hierarchy between VEVs of scalars in different sectors as we will see in a moment.

For the charged lepton sector we choose $(\varphi_T, \xi') \sim (3, 1')$ of $A_4$. They are all invariant under $Z_3$ and carry a charge $i$ under $Z_4$. It is not difficult to obtain a stable alignment [14] of $(\varphi_T, \xi')$ as follows:

$$\langle \varphi_T \rangle = (0, v_T, 0), \quad \langle \xi' \rangle = u'. \quad (9)$$

This structure of vacua automatically reproduces diagonal and hierarchical charged lepton masses through the following lagrangian:

$$\mathcal{L}_c = \alpha_1 e^c (l\varphi_T) h_d / \Lambda + \beta_1 \mu^c (l\varphi_T)' h_d / \Lambda^2 + \beta_3 \mu^c (l\varphi_T^2) h_d / \Lambda^2 + \gamma_1 e^c (\xi')^2 (l\varphi_T)' h_d / \Lambda^3 + \gamma_2 \mu^c (\xi')^2 (l\varphi_T^2)' h_d / \Lambda^3 + \gamma_3 e^c (l\varphi_T^3) h_d / \Lambda^3 + h.c. + \cdots$$

After EW and flavour symmetry breakings one obtains:

$$m_l = \begin{pmatrix} \sim \varphi_T^2 / \Lambda^3 & 0 & 0 \\ 0 & \sim \varphi_T^2 / \Lambda^2 & 0 \\ 0 & 0 & \sim \varphi_T / \Lambda \end{pmatrix} v_d. \quad (10)$$

The required hierarchy among $m_\ell$, $m_\mu$, and $m_\tau$ is approximately described providing

$$\lambda_\ell \ll v_T / \Lambda \sim u' / \Lambda \ll \lambda_c^2,$$

being $\lambda_c$ the Cabibbo angle.

The neutrino sector is given by the seesaw lagrangian with 3 heavy right-handed neutrinos $\nu_i^c$:

$$\mathcal{L}_\nu = y_\nu (\nu^c l) \overline{c} h^c / \Lambda + x_\nu \xi (\nu^c v^c) + x_\nu (\varphi_S \nu^c v^c) + h.c.$$

where $(\varphi_S, \xi, \zeta) \sim (3, 1, 1)$ under $A_4$ and have only $Z_3$ charge: $(\omega, \omega, \omega^2)$. We find that the minimization of the scalar potential leads to the following VEVs:

$$\langle \varphi_S \rangle = (v_S, v_S, v_S), \quad \langle \xi \rangle = u, \quad \langle \zeta \rangle = v. \quad (11)$$

The combinations $\zeta$ and $\zeta\varphi_S$ are invariant under the abelian part of the flavour group and can affect the charged lepton sector as corrections at the next-to-next-to-leading order (NNLO). These corrections are however suppressed by $1/\Lambda^2$ and we have a certain freedom to choose flavour symmetry breaking scale in the neutrino sector without destroying the hierarchical structure obtained for charged leptons [10]. We will assume

$$u / \Lambda \sim v_S / \Lambda \sim v / \Lambda \sim \lambda_c \div \lambda_\ell^2. \quad (12)$$

Differently from conventional models [12] aimed to explain TBM, this possibility can also describe a relatively large value of $\theta_{13}$ [13]. The leading contributions to the Dirac and Majorana masses are

$$m_D^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \frac{y \nu v_u}{\Lambda}, \quad m_M = \begin{pmatrix} b + 2d & -d & -d \\ -d & 2d & b - d \\ -d & b - d & 2d \end{pmatrix} u,$$

where $b \equiv x_a$ and $d \equiv x_b v_S / u$. As the general situation analyzed in the beginning, the leading order mixing matrix, corresponding to TB mixing in this case, diagonalizes the right-handed neutrino mass matrix by [14].

More precisely, $U_0 = U_{TB}\Omega$ where

$$U_{TB} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & +1/\sqrt{2} \end{pmatrix}, \quad (13)$$

$$\Omega = \text{diag}\{e^{i\phi_1 / 2}, e^{i\phi_2 / 2}, e^{i\phi_3 / 2}\} \quad \text{and} \quad \phi_1, \phi_2, \phi_3 \text{ are respectively phases of } b + 3d, b - 3d. \quad \text{Moreover, the flavour symmetry automatically leads to a Dirac mass of the form } (2), \text{ then the condition } (13) \text{ is fulfilled. The physical masses of } \nu_i \text{ are given by } M_1 = |b + 3d|, M_2 = |b| \text{ and } M_3 = |b - 3d| \text{ and those of light neutrinos are } m_i = |y v_u|^2 / (\Lambda^2 M_i) \sim y^2 \lambda_\ell^2 v_u / M_i \div y^2 \lambda_\ell^2 v_u^2 / M_i.$$

In this model the only relevant NLO corrections to the lagrangian $\mathcal{L}_c + \mathcal{L}_\nu$ appear in the Dirac mass and they are of type $(\nu^c l \varphi_S) h^c / \Lambda^2$ with $\varphi \in \{\varphi_S, \xi\}$. The correction to the Dirac mass $\delta m_D^0$ that breaks the condition imposed by [15] has the following form:

$$\delta m_D^0 = \begin{pmatrix} 0 & y_1 + y_3 & y_2 - y_3 \\ y_1 - y_3 & y_2 & y_3 \\ y_2 + y_3 & -y_3 & y_1 \end{pmatrix} v_u \frac{v_S^2}{\Lambda^2},$$

where $y_i$ are generally complex numbers of order 1. Including $\delta m_D^0$, the TB mixing should receive a small correction according to $U_{PMNS} = U_0 \delta U$. The correction to leading mixing matrix $\delta U$ can be calculated perturbatively and we find that, due to the special structure of $\delta m_D^0$, only the $(13)$ and $(31)$ elements of $\delta U$ survive. Moreover we should expect that $(\delta U)_{13} = -(\delta U)_{31} \sim \lambda_c \div \lambda_\ell^2$ depending on the scale of VEVs in [12]. As
a consequence, the tri-bimaximal prediction for $\theta_{12}$ remains unchanged at the first order in $(\delta U)_{13}$ and $(\delta U)_{13}$ simultaneously induces a departure of $\theta_{13}$ and of $\theta_{23} - \pi/4$ from zero. Then we can derive the following sum-rule:

$$\sin^2 \theta_{23} = 1/2(1 + \sqrt{2} \cos \delta \sin \theta_{13}) + O(\theta_{13}^2) \tag{14}$$

where $\delta$ is the CP-violating Dirac phase in the standard parametrization of $U_{PMNS}$.

D. Unflavoured Leptogenesis from low energy phases.

As an illustration we will discuss a case of unflavoured leptogenesis [18] which mainly depends on low energy phases and its implication on reactor angle $\theta_{13}$. We only consider the case of normal hierarchy for light neutrinos. In this case, the right-handed neutrinos are naturally hierarchical according to $M_3 \ll M_2 \approx 1/2M_1$. We can simply estimate the natural mass range of the lightest right-handed neutrino $\nu_3^c$ by taking neutrino mass scale as $\sqrt{\Delta m_{atm}^2} \approx 0.05 \text{ eV}$ and the scale of $m^D$ as $v_u \lambda_1^2 + v_e \lambda_2^2$ with $v_u = 174 \text{ GeV}$ obtaining $M_3 \sim 10^{12} \text{ GeV} \div 3 \times 10^{13} \text{ GeV}$.

As the general analysis made in the beginning, leptogenesis does work in this class of models only taking into account NLO corrections. In the model presented in the previous chapter, although these corrections are quite simple, their contribution to CP-violating asymmetries does not depend only on $(\delta U)_{13}$ but also on $\delta m^D$ itself. Indeed the NLO off-diagonal corrections to $R$ are perturbatively given by:

$$R = m_{\text{diag}}^{1/2} U^0_0 m^D U_0 \delta U m_{\text{diag}}^{-1/2} + m_{\text{diag}}^{1/2} U^0_0 \delta m^D U_0 m_{\text{diag}}^{-1/2}. \tag{15}$$

Since $U^0_0 m^D U_0 = \text{diag}(1, 1, -1)$ the first row in (15) is directly related with $(\delta U)_{13}$. However, the presence of $\delta m^D$ generally destroys a hopeful alignment between $R$ and $\delta U$. This means that, without further considerations, a direct bridge between the BAU and the element $U_{e3}$ cannot be established.

Now we observe that for normally ordered hierarchical light neutrinos, it is suitable to consider the limit $m_1 \ll m_3$. In this case it is not difficult to see that $(U^0_0 \delta m^D U_0)_{31} \approx (\delta U)_{13}$. With this approximation, from [18] one immediately finds that

$$R_{31} = 2\sqrt{m_3/m_1}(\delta U)_{13}.$$ 

This is a very nice feature of this model with hierarchical neutrino spectrum because a same small rotation matrix $\delta U$ is responsible both for corrections to TBM and in generating off-diagonal elements of $R$ responsible for leptogenesis.

Now, it is convenient to parametrize $(\delta U)_{13}$ in terms of low energy physical quantities $U_{e3} = e^{-i\delta} \sin \theta_{13}$ and $\phi_{13} = (\phi_1 - \phi_3)/2$:

$$(\delta U)_{13} = \sqrt{3/2} e^{-i\phi_{13}} U_{e3}. \tag{16}$$

From (6) we obtain a lepton asymmetry for the decay of $\nu_3^c$ which explicitly depends on the low energy Dirac phase $\delta$ and the physical Majorana phase $\phi_{13}$:

$$\epsilon_3 = \frac{3M_3 m_1}{4\pi v_u} \text{Im} [(\delta U)_{13}^2]$$

$$= \frac{9M_3 m_1}{8\pi v_u^2} \sin(2\delta + 2\phi_{13}) \sin^2 \theta_{13}.$$ 

We are in the strong wash-out regime since

$$\hat{m}_3 = \sum_a m_a |R_{3a}|^2 \approx m_3 \approx (\Delta m_{atm}^2)^{1/2}.$$ 

In this regime, the final lepton asymmetry can be approximately reproduced by [2]:

$$Y_L \approx 0.3 \frac{\epsilon_3}{g_*} \left( \frac{0.55 \times 10^{-3} \text{eV}}{\hat{m}_3} \right)^{1.16}.$$ 

Using the observed central value of $Y_{\nu}^{\text{obs}} = 8.6 \times 10^{-11}$ [3] we obtain the following lower bound on $\sin^2 \theta_{13}$:

$$\sin^2 \theta_{13} \gtrsim 0.005 \times 3 \times 10^{13} \text{GeV} / M_3 \tag{18}$$

where we have used $g_* = 217/2$ and $m_1 = (\Delta m_{atm}^2)^{1/2} \approx 0.01 \text{ eV}$. Since $M_3$ cannot be too much larger than $10^{13} \text{ GeV}$, a neutrino spectrum of normal hierarchy in this model favors a value of $\sin^2 \theta_{13}$ larger than 0.005.

In summary, we have considered a general class of seesaw models in which leptogenesis can be related with low energy CP violating phases. The flavour symmetry which leads to a vanishing $U_{e3}$ at leading order plays a central role. We gave an explicit model realization of this correlation which also explains the TBM. The same approach can also be applied to many other flavour models [8, 9, 10, 11, 12, 13] and it demonstrates that, even in flavoured leptogenesis, a connection between low and high energy CP violation can be established naturally.

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