Z’ in the 3-3-1 model

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Abstract
Phenomenological implications of the Z’ in SU(3)×SU(3)×U(1) extension of the standard model are studied. The model improves the fit for R_b as well as the high E_T jet cross section observed by CDF.

1. Model: We consider the SU(3)_color×SU(3)×U(1) extension of the standard model, proposed earlier by Frampton [1] to determine its predictions for the observables that appear to show deviation from standard model. The SU(3)×U(1) interaction breaks down to SU(2)×U(1) at the scale of about 1 TeV. The model has several interesting features. Leptons in this model interact only with the weak SU(3), namely they do not directly couple to U(1). The U(1) gauge interaction with relatively strong coupling leads to a new leptophobic interaction. The U(1) coupling has to be relatively strong, else it leads to a light leptophobic Z’ which might already be ruled out by the CDF jet data. The anomaly cancellation occurs among the three generations and thus might provide a hint for their existence. The model also leads to doubly charged gauge bosons which provide lepton number violating interaction. The experimental limit on the masses of these gauge bosons is found to be 230 GeV [2].

In the present paper we show that a slight modification of the original model [1], leads to successful explanation of R_b as well as of the anomalous increase in the jet cross section seen in CDF [3]. The original large discrepancy in R_b [4] which generated considerable theoretical interest [5-10] has dissapeared and it now appears to be only a 2σ effect. The present model brings the theoretical prediction further close to experiments without significantly effecting the total Z width. It also provides a new leptophobic Z’ which might be necessary to explain the large deviation in jet cross section seen by CDF [3].

The lepton triplets,

\[
\begin{pmatrix}
  e \\
  \nu_e \\
  L_e
\end{pmatrix}_L,
\begin{pmatrix}
  \mu \\
  \nu_\mu \\
  L_\mu
\end{pmatrix}_L,
\begin{pmatrix}
  \tau \\
  \nu_\tau \\
  L_\tau
\end{pmatrix}_L
\]
are assigned to the 3 representation of the SU(3) group. The leptons $L_e$, $L_\mu$ and $L_\tau$ can be identified with $e_L^c$, $\mu_L^c$, $\tau_L^c$. The right handed neutrinos are singlets. The quark triplets

$$\begin{pmatrix} c \\ s \\ q_c \end{pmatrix}_L, \begin{pmatrix} t \\ b \\ q_t \end{pmatrix}_L,$$

are assigned to the 3 representation and have strong hypercharge $Y_S = 2/3$ and the triplet

$$\begin{pmatrix} d \\ u \\ q_u \end{pmatrix}_L$$

to the 3 representation and has strong hypercharge $Y_S = -4/3$. Note that this assignment is reversed in comparison to what was proposed in Ref. [1]. This is necessary so that the correction to $R_b$ is positive in this model.

It is not possible to identify the third members of the quark triplets with one of the known right handed quarks since the hypercharges do not agree. These have to be identified with new exotic heavy quarks. The right handed quarks are all singlets under SU(3) with strong hypercharge assignments given by,

$$Y_S(u_R) = Y_S(c_R) = Y_S(t_R) = -4/3$$
$$Y_S(d_R) = Y_S(s_R) = Y_S(b_R) = 2/3$$
$$Y_S(q_cR) = Y_S(q_tR) = 8/3$$
$$Y_S(q_uR) = -10/3$$

The anomalies cancel with this assignment.

The Higgs fields necessary for breakdown of this symmetry are discussed in section 3. We introduce three higgs fields $\phi$, $\phi_1$, and $\phi_2$ which respectively form (3, $-2$), (3, 0) and (3, 2) representations of the SU(3)$\times$U(1). The vacuum values of these fields have been assumed to be

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix}, \langle \phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \\ 0 \end{pmatrix}, \langle \phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_2 \\ 0 \\ 0 \end{pmatrix} \quad (1)$$

We discuss the Higgs potential that can yield such a vev and also the resulting physical Higgs masses in next section. We will assume that $v >> v_1, v_2$ and is responsible for the breakdown of SU(3)$\times$U(1) down to SU(2)$\times$U(1). The scale of this symmetry
breaking is determined by fitting the experimental value of $\sin^2 \theta_W$. In the present model $\sin^2 \theta_W$ at the symmetry breaking scale is found to be

$$\sin^2 \theta_W = \frac{1}{4[1 + g^2/(4g_S^2)]}$$

(2)

g and $g_S$ are the SU(3) and U(1) coupling constants respectively at the symmetry breaking scale. The SU(3) gauge bosons corresponding to the broken generators will be denoted as $Y^{++}$, $Y^{--}$, $Y^+$ and $Y^-$, consistent with the notation used in Ref. [1] and will acquire masses $M(Y) \approx g v/2$. We require that the $\sin^2 \theta_W$ measured at $M_Z$ in the MS scheme and evolved to energy scale $M(Y)$ should agree with the value given in Eq. [2]. The resulting values of $g_S$ and $v$ are displayed in Table 1. The corresponding values of $M(Y)$ and $M(Z')$, where $Z'$ is the vector boson corresponding to the superposition of the U(1) and $T_8$ generator of SU(3), are also shown. $Z'$ effectively acts as a leptophobic gauge boson and could lead to the enhancement in jet cross section at large $E_T$ needed to explain the CDF data. We discuss this later. The masses of $W^\pm$ in the present model turn out to be $M(W^\pm) = g \sqrt{v_1^2 + v_2^2}/2$. The electroweak hypercharges are given by

$$Y = \sqrt{3} \lambda_8 - Y_S$$

| $M(Z')$ (GeV) | $g_S$ | $M(Y)$ (GeV) | $v$ (GeV) |
|--------------|-------|-------------|--------|
| 1000         | 1.47  | 221         | 680    |
| 1200         | 1.54  | 254         | 779    |
| 1400         | 1.61  | 283         | 870    |
| 1600         | 1.66  | 314         | 964    |
| 1800         | 1.70  | 345         | 1059   |
| 2000         | 1.79  | 364         | 1117   |

Table 1: The masses of the gauge bosons $Z'$, $Y$ as a function of the U(1) gauge coupling $g_S$. The corresponding vacuum expectation values $v$ of the higgs field $\phi$ are also shown.

2. Results: We next determine the values of $R_b$ and $R_c$ within the present model. The model leads to corrections of the order $(v_1/v)^2$ and $(v_2/v)^2$ beyond the standard model. These corrections arise from the mixing of the electroweak $Z$ boson with the leptophobic $Z'$. The symmetry breakdown occurs in two stages, first SU(3)$\times$U(1) breaks to SU(2)$\times$U(1) at the scale $v \approx 1$ TeV, which then further breaks down to
U(1) at the electroweak scale. We identify the three neutral physical bosons Z, Z', and the photon approximately to order $v_1^2/v^2$ and $v_2^2/v^2$. The Z and Z' couplings to the fermions turn out to be,

$$\mathcal{L}_{Z,Z'} = \bar{\psi}_L \left[ \frac{g}{\cos \theta_W} (-Q \sin^2 \theta_W + T_3) + \frac{\delta}{2} g_S Y^L \right] \gamma_\mu Z^\mu \psi_L$$

$$+ \bar{\psi}_R \left[ -\frac{g}{\cos \theta_W} \sin^2 \theta_W Q + \frac{\delta}{2} g_S Y^R \right] \gamma_\mu \psi_R$$

$$+ \bar{\psi}_L g_S \frac{Y^L}{2} \gamma_\mu \psi_L + \bar{\psi}_R g_S \frac{Y^R}{2} \gamma_\mu \psi_R \tag{3}$$

Here we have neglected terms proportional to $g^2 \tan^2 \theta_W$ in comparison to $g_S^2$ since they give very small corrections. We have also set $\cos \theta = g_S/\sqrt{g^2 + g_S^2} \approx 1$, where $g' = g/\sqrt{3}$ at the scale of SU(3) breaking. We find that the mixing angle between Z and Z' is given by,

$$\delta \approx \frac{g}{2g_S v^2 \cos \theta_W} \left[ \frac{g \tan^2 \theta_W (v_2^2 - v_1^2)}{2g_S} - g_S v_2^2 \right]$$

The Z width is then found to be proportional to

$$(t_3 - 4Q \sin^2 \theta_W)^2 + (t_3)^2 + \frac{4 \cos \theta_W}{g} t_3 g_S Y^L \delta - \frac{8 \cos \theta_W}{g} Q \sin^2 \theta_W g_S (Y^L_S + Y^R_S) \delta \tag{3}$$

where $t_3$ is +1 for up type quarks and -1 for down type quarks and $Q$ is the electric charge. In this formula we have neglected terms of order $(g \tan \theta_W)^2$ which are an order of magnitude smaller in comparison to $g_S^2$. We find that the resulting correction to leptonic Z width is very small. All the hadronic decay modes, however, receive a correction. The total hadronic decay width receives a very small correction which is smaller than the current uncertainty in its measured value. The resulting corrections to $R_b$, $R_c$, and the total hadronic width are shown in table 2 for several different values of $M'_Z$ and $v_2$. We point out that $v_2$ is so far an adjustable parameter.
The shift in $R_b$, $R_c$ and hadronic width due to mixing with $Z'$. Several representative cases of $Z'$ mass for different values of $v_2$ are shown. $v_2$ can be treated as an adjustable parameter to obtain the best experimental fit. For comparison $R_{b\exp}^{\pm} = 0.2179 \pm 0.0012$ [11], $R_{b}^{SM} = 0.2158$, $R_{c}^{exp} = 0.1715 \pm 0.0056$ [11], $R_{c}^{SM} = 0.1723$, $\Gamma_{h\exp}^{\pm} = 1.7448 \pm 0.030$ GeV and $\Gamma_{h}^{SM} = 1.746$ GeV.

In Table 2, we have shown some typical results for several values of the mass of $Z'$ and the vacuum expectation value $v_2$. The model reduces the observed discrepancy in $R_b$. The experimental results are $R_b^{\exp} = 0.2179 \pm 0.0012$ [11], $R_c^{exp} = 0.1715 \pm 0.0056$ [11] and $\Gamma_h^{exp} = 1.7448 \pm 0.030$ GeV and the Standard model values are $R_b^{SM} = 0.2158$, $R_c^{SM} = 0.1723$ and $\Gamma_h^{SM} = 1.746$ GeV. The values of $R_b$ and $R_c$ in the present model are both within one sigma of the experimental results with the range of parameters shown in Table 2. The model produces very small change in the total hadronic width. The shift in other observables such as the asymetries is very small compared to the experimental error in these quantities. The resulting correction to the jet cross section is shown in Fig. 1 for three different values of $M'_{Z}$. It is clear from the figure that for certain range of parameters the model can explain the deviation observed in jet cross section by CDF. In all our discussion we have concentrated on somewhat large values of $g_S \geq 1.47$. For smaller values the model produces a large correction to the jet cross section and might already be ruled out as is clear from Fig. 1.
3. Higgs Potential: In the analysis so far we have introduced three higgs fields $\phi, \phi_1, \phi_2$ which are in the representation

$$
\phi : (3, -2), \phi_1 : (3, 0), \phi_2 : (3, 2)
$$

of $\text{SU}(3) \times \text{U}(1)$. The Higgs potential has to be chosen such that it breaks $\text{SU}(3) \times \text{U}(1)$ to $\text{SU}(2) \times \text{U}(1)$. In our analysis we have assumed that the symmetry breaking occurs such that the three higgs fields acquire vacuum expectation values given in equation 1. The Higgs potential that can accomplish this symmetry breaking can be chosen to have the form,

$$
V = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 - \mu_1^2 \phi_1^\dagger \phi_1 + \lambda_1 (\phi_1^\dagger \phi_1)^2 \\
- \mu_2^2 \phi_2^\dagger \phi_2 + \lambda_2 (\phi_2^\dagger \phi_2)^2 + \lambda_3 |\phi_1^\dagger \phi_1|^2 + \lambda_4 |\phi_2^\dagger \phi_2|^2 \\
+ \lambda_5 |\phi_1^\dagger \phi_2|^2 + (\lambda_6 \epsilon_{ijk} \phi_2^\dagger \phi_i \phi_j \phi_k + \lambda_7 (\phi_2^\dagger \phi_1)(\phi_1^\dagger \phi_1) + h.c) \tag{4}
$$

The $\lambda_6$ term assures that there are no more Goldstone particles in the spectrum other than the one’s needed to give masses to the gauge bosons. If $\lambda_6$ is set equal to
zero then we get one extra massless scalar particle in the physical spectrum. This is because in the absence of the this term, the U(1) transformation

\[ \phi \rightarrow e^{i\alpha}\phi, \quad \phi_1 \rightarrow e^{i\alpha}\phi_1, \quad \phi_2 \rightarrow e^{i\alpha}\phi_2 \]

is a symmetry of the action but not of the vacuum. The \( \lambda_6 \) term is needed to explicitly break this symmetry. The \( \lambda_7 \) term has been included to break the U(1) generator \( W \) corresponding to the transformation,

\[ \phi \rightarrow e^{i\alpha}\phi, \quad \phi_1 \rightarrow e^{-2i\alpha}\phi_1, \quad \phi_2 \rightarrow e^{i\alpha}\phi_2 \]

which is a symmetry of the action in the absence of this term. In this case, however, setting \( \lambda_7 = 0 \) does not lead to a Goldstone boson since the vacuum remains invariant under an additional U(1) whose generator \( Q' \) is given by

\[ Q' = -\frac{3}{2}Y_S + 2\sqrt{3}\lambda_8 + W \]

We need to show that for a certain acceptable range of parameters this potential does have the assumed global minima. We may parametrize the higgs minimum as

\[ <\phi^\dagger\phi> = v^2 \quad <\phi_1^\dagger\phi_1> = v_1^2 \quad <\phi_2^\dagger\phi_2> = v_2^2 \]

\[ <\phi_1^\dagger\phi_1> = vv_1 \sin \beta_1 e^{i\psi_1} \quad <\phi_2^\dagger\phi_2> = vv_2 \sin \beta_2 e^{i\psi_2} \quad <\phi_1^\dagger\phi_2> = vv_1v_2 \sin \beta_{12} e^{i\psi_{12}} \]

Minimizing the resulting potential with respect to \( \psi_1 - \psi_2 \) gives \( \lambda_7 \cos(\psi_1 - \psi_2) = -|\lambda_7| \). Henceforth we treat \( \lambda_7 > 0 \) and take \( \cos(\psi_1 - \psi_2) = -1 \). We first also set \( \lambda_6 = 0 \). The extremization with respect to \( \beta_1, \beta_2 \) and \( \beta_{12} \) leads to the following three types of minima, assuming \( v, v_1 \) and \( v_2 \) are all nonzero,

I. The three \(<\phi>s\) are orthogonal to one another.

II. \(<\phi>\) and \(<\phi_2>\) and parallel to one another but both of them are orthogonal to \(<\phi_1>\).

III. All of \(<\phi>s\) are parallel to one another.

The potential at these minima is,

\[ V_{\text{min}} = -\frac{1}{2}(\mu^2 v^2 + \mu_1^2 v_1^2 + \mu_2^2 v_2^2) \]

We want

\[ V_{II} - V_I = -\frac{1}{2}(\mu^2 \delta v^2 + \mu_1^2 \delta v_1^2 + \mu_2^2 \delta v_2^2) > 0 \]
and similarly $V_{II} - V_I > 0$. A sufficient, but not necessary, condition that will ensure this is for each $\delta v_i^2$ to be separately negative. The $\delta v_i^2$ for $V_{II} - V_I$ and $V_{III} - V_I$ are negative if,

$$\lambda_3 v_i^2 + \lambda_4 v_2^2 - \lambda_7 \frac{v_1^2 v_2}{v} > 0$$
$$\lambda_3 v^2 + \lambda_5 v_2^2 - 2 \lambda_7 v v_2 > 0$$
$$\lambda_4 v^2 + \lambda_5 v_1^2 - \lambda_7 \frac{v v_1^2}{v_2} > 0$$

Note that $v, v_1$ and $v_2$ are all positive numbers. The above conditions are complicated since $v_i$ are themselves determined in terms of $\lambda$’s. If we set $\lambda_7 = 0$ and take $\lambda_3, \lambda_4$ and $\lambda_5$ to be positive these conditions are clearly satisfied. It is clear that, if $\lambda_7$ is nonzero, there will be a range of $\lambda_7$ for which the above mentioned sufficient conditions are fulfilled. Now suppose we have a $\lambda_7$ such that $V_{II}$ and $V_{III}$ are higher than $V_I$ by atleast $\delta$. Then there is at least a range of $\lambda_6$ for which $V_I$ is a global minimum. Actually $\lambda_6$ term for $\lambda_6 < 0$, gives negative contribution to $V_I$ and zero contribution to $V_{II}$ and $V_{III}$. Hence for $\lambda_6$ negative $V_I$ will be a global minima.

There exist a total of ten massive scalar fields in the physical spectrum. These are identified by expanding about the minimum of the potential such that

$$\phi = <\phi> + \hat{\phi}$$

We give the results only for $\lambda_7 = 0$. We note that none of the massive scalar fields become massless when $\lambda_7$ is set equal to zero. Setting $\lambda_7 \neq 0$ therefore only changes the mass splittings and not the masses themselves as long as $\lambda_7$ is not too large. We define three complex scalar fields $H_1$, $H_2$ and $H_3$,

$$H_1 = \frac{v_1 \hat{\phi}_2^* + v_2 \hat{\phi}_{13}}{\sqrt{v_1^2 + v^2}}, \quad H_2 = \frac{v_2 \hat{\phi}_{23} + v_1 \hat{\phi}_1^*}{\sqrt{v_1^2 + v^2}}, \quad H_3 = \frac{v_1 \hat{\phi}_{22} + v_2 \hat{\phi}_{11}^*}{\sqrt{v_1^2 + v_2^2}}$$

which have masses

$$M_1^2 = \frac{(v^2 + v_1^2)}{2} \left[ \lambda_3 - \frac{\sqrt{2} \lambda_6 v_2}{v v_1} \right]$$
$$M_2^2 = \frac{(v^2 + v_2^2)}{2} \left[ \lambda_4 - \frac{\sqrt{2} \lambda_6 v_1}{v v_2} \right]$$

and

$$M_3^2 = \frac{(v^2 + v_2^2)}{2} \left[ \lambda_5 - \frac{\sqrt{2} \lambda_6 v}{v_1 v_2} \right]$$
respectively. We also have the real field

$$N \left[ \frac{\text{Im} \hat{\phi}_3}{v} + \frac{\text{Im} \hat{\phi}_{12}}{v_1} + \frac{\text{Im} \hat{\phi}_{21}}{v_2} \right]$$

with mass

$$-\frac{\lambda_6 v_1 v_2}{\sqrt{2}} \left[ \frac{1}{v^2} + \frac{1}{v_1^2} + \frac{1}{v_2^2} \right].$$

Finally we have the remaining three massive real fields $\text{Re} \hat{\phi}_3, \text{Re} \hat{\phi}_{12}$ and $\text{Re} \hat{\phi}_{21}$ with mass matrix,

$$
\begin{pmatrix}
2\lambda v^2 - \lambda_6 v_1 v_2 / \sqrt{2} v & \lambda_6 v_2 / \sqrt{2} & \lambda_6 v_1 / \sqrt{2} \\
\lambda_6 v_2 / \sqrt{2} & 2\lambda_1 v_1^2 - \lambda_6 vv_2 / \sqrt{2} v_1 & \lambda_6 v / \sqrt{2} \\
\lambda_6 v_1 / \sqrt{2} & \lambda_6 v / \sqrt{2} & 2\lambda_2 v_2^2 - \lambda_6 v_1 v / \sqrt{2} v_2
\end{pmatrix}
$$

**Acknowledgements:** We would like to thank John Ralston for making useful suggestions in this work.

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