Modeling of the Stiffness of Corrugated Cardboard Considering Material Non-linear Effect

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Abstract: This paper studies the application of corrugated cardboard in vibration isolators. Cardboard is known as a highly environment-friendly material. The focus of the study was on modeling of the stiffness of a cardboard or a group of cardboard (cardboard system) along the cardboard’s thickness direction which is also the vibration direction. Cardboard in this study is corrugated cardboard. There are two shortcomings in modeling the stiffness of such cardboard in literature: (1) neglect of the material non-linear effect of cardboard, and (2) neglect of the effect of cardboard width even for the situation that cardboard width is greater than cardboard length. A finite element method was applied to overcome these shortcomings - particularly by adopting an orthotropic material constitutive model and using a shell element. In addition, the study also examined the effect of inaccuracy in describing the core shape on prediction of the stiffness of cardboard. The experiment was performed, which shows significant improvements in the prediction accuracy with the new finite element model.

1. Introduction

The corrugated cardboard is widely used in packaging industry because it is cost-effective, easy for recycling, and environment-friendly [1]. A typical corrugated cardboard is illustrated in Figure 1, which is made up of paper and has a sandwich structure, in particular consisting of a corrugated core (flute) and two liners [2].

Figure 1. The outlook and structure of a corrugated cardboard

There are several studies on modeling and prediction of the stiffness of corrugated cardboard under compressive loading in its thickness direction and several finite element models were proposed. Lu et al. [3] proposed a finite element model to predict the compressive behavior of the corrugated cardboard under a uniform flat compressive loading. But the error between their analysis result and the experimental result is about 30%. They concluded that the error is caused by the use of inaccurate non-linear constitutive relation for the material used. Krusper et al. [4] refined the Lu’s model with a more accurate model. In their model, a linear elastic material property was however assumed. When the displacement over to 0.2mm in their paper, the model predicted result and experimental result can’t match..
On the other hand, the width of corrugated cardboard and peak load are two important parameters for the application of it for the vibration isolation. Difference in the width parameter could give rise to a significantly different stiffness of the cardboard. They can't overcome the challenge, because the beam element is used in their work. When the width of corrugated cardboard is longer than the length, the beam element is not suitable for modeling of the stiffness. In addition, inaccuracy is also contributed by the constitutive model which did not consider non-linear material property of the cardboard. Thus, it is necessary to present a new finite element model that includes the nonlinear material property and the width effect. That is the main motivation for the present study.

Considering the width effect and the peak load, the study in this paper developed a more accurate finite element model for the stiffness of the corrugated cardboard under compressive loading in its thickness direction. The experiment was conducted to verify the model. The study neglects the humidity and the thermal effect on the cardboard from the environment. The next section gives a detailed description of the proposed finite element model, followed by the experimental verification of the model. There is a conclusion at the end of the present paper.

2. Finite element modeling for stiffness of the cardboard

In this paper, an orthotropic material constitutive model was employed for the two liners and core of the corrugated cardboard. The orthotropic constitutive model consists of two parts: the linear elastic and the nonlinear plastic portions. The linear elastic portion is governed by orthotropic Hooke’s Law, while the plastic portion is governed by a quadratic Hill yield criterion.

2.1 Linear elastic material property

The linear elastic orthotropic constitutive model is represented by [5]:

\[
\begin{bmatrix}
\varepsilon_x & \varepsilon_y & \varepsilon_z \\
\gamma_{xy} & \gamma_{xz} & \gamma_{yz} \\
\varepsilon_{xy} & \varepsilon_{xz} & \varepsilon_{yz}
\end{bmatrix}
= \begin{bmatrix}
\frac{1}{E_x} & -\frac{v_{xy}}{E_x} & -\frac{v_{xz}}{E_x} & 0 & 0 & 0 \\
-\frac{v_{yx}}{E_y} & \frac{1}{E_y} & -\frac{v_{yz}}{E_y} & 0 & 0 & 0 \\
-\frac{v_{zx}}{E_z} & -\frac{v_{yz}}{E_z} & \frac{1}{E_z} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{G_{xy}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{G_{xz}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{G_{yz}}
\end{bmatrix}
\times
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\tau_{xy} \\
\tau_{xz} \\
\tau_{yz}
\end{bmatrix}
\]

(1)

Where \( \varepsilon_x, \varepsilon_y, \varepsilon_z \) is Strains in the x, y, z direction, \( \gamma_{xy}, \gamma_{xz}, \gamma_{yz} \) is Strains in the xy, xz, yz plane, \( E_x, E_y, E_z \) is Young’s modulus in the x, y, z direction, \( v_{xy}, v_{xz}, v_{yz} \) is Poisson ratio in the xy, xz, yz plane and \( G_{xy}, G_{xz}, G_{yz} \) is Shear modulus in the xy, xz, yz plane.

The symmetrical geometry of the cardboard leads to [5]:

\[
\frac{v_{xy}}{E_x} = \frac{v_{yx}}{E_y}, \quad \frac{v_{xz}}{E_x} = \frac{v_{zx}}{E_z}, \quad \frac{v_{yz}}{E_y} = \frac{v_{zy}}{E_y}
\]

(2)

There are nine unknown variable, and they are: \( E_x, E_y, E_z, v_{xy}, v_{xz}, v_{yz}, G_{xy}, G_{xz}, G_{yz} \). Generally, all these unknown variables are determined by measuring. However, the dimension of the liner and core in the thickness direction is too small to measure some variables. For the cardboard system as shown in Figure 1, the in-plane material parameters \( E_x \) was measured by the standard tensile test, while \( E_y, G_{xy}, G_{xz}, G_{yz} \) were derived empirically as follows. For the Young’s modulus in the thickness direction, it was approximated given by

\[
E_z = \frac{E_y}{200}
\]

(3)

The shear modulus are approximated by
For both liner and core, the value of $\nu_{xy}$, $\nu_{xz}$ and $\nu_{yz}$ were set according to Nordstand [6], which is 0.34, 0.01 and 0.01, respectively. In order to make the result as close as possible to the experimental result, we choose different material parameters corrugated cardboard for testing. Based on this trial and error procedure, we obtain the elastic material parameters for the finite model. Finally, Table 1 lists these elastic parameters.

### 2.2 Nonlinear plastic material property

In this study, Quadratic Hill yield criterion in ANSYS was used, as the model was simplified by assuming there is no difference in yield strength in tension and compression. The yield criterion was used with the isotropic hardening option, which is given by Equation (5) from [7].

$$f\{\sigma\} = \sqrt{\{\sigma\}^T [M] \{\sigma\} - \sigma_0 \varepsilon^p} = 0 \quad (5)$$

Where $\sigma_0$ is yield stress in the x direction, $\varepsilon^p$ is equivalent plastic strain, $\{\sigma\}$ is yield stress matrix and $[M]$ is plastic compliance matrix.

The plastic compliance matrix $[M]$ can be written as [7]

$$M = \begin{bmatrix} G + H & -H & -G & 0 & 0 & 0 \\ -H & F + H & -F & 0 & 0 & 0 \\ -G & -F & F + G & 0 & 0 & 0 \\ 0 & 0 & 0 & 2N & 0 & 0 \\ 0 & 0 & 0 & 0 & 2L & 0 \\ 0 & 0 & 0 & 0 & 0 & 2M \end{bmatrix} \quad (6)$$

Where $F, G, H, L, M$ and $N$ are material constants that can be determined experimentally. They were defined by [7]:

$$F = \frac{1}{2} \left( \frac{1}{R_{yy}} + \frac{1}{R_{zz}} - \frac{1}{R_{ss}} \right), G = \frac{1}{2} \left( \frac{1}{R_{xx}} + \frac{1}{R_{yy}} - \frac{1}{R_{ss}} \right), H = \frac{1}{2} \left( \frac{1}{R_{xx}} + \frac{1}{R_{yy}} - \frac{1}{R_{zz}} \right)$$

$$L = \frac{3}{2} \frac{1}{R_{yy}} , M = \frac{3}{2} \frac{1}{R_{zz}} , N = \frac{3}{2} \frac{1}{R_{yy}} \quad (7)$$

In the above equations, the yield stress ratios $R_{xx}, R_{yy}, R_{zz}, R_{xy}, R_{yz}$ and $R_{xz}$ can be found by [7]

$$R_{xx} = \frac{\sigma_{xy}^v}{\sigma_0}, R_{yy} = \frac{\sigma_{xy}^v}{\sigma_0}, R_{yy} = \frac{\sigma_{xy}^v}{\sigma_0}, R_{yy} = \sqrt{3} \frac{\sigma_{yz}^v}{\sigma_0}, R_{yz} = \sqrt{3} \frac{\sigma_{yz}^v}{\sigma_0}, R_{xz} = \sqrt{3} \frac{\sigma_{xz}^v}{\sigma_0} \quad (8)$$

Where $\sigma_0^v$ is the yield stress in the x, y, z, xy, yz and xz direction. Further, the plastic slope of the material after yield point is given by

$$E_p^p = \frac{E}{E_0}$$

Where $E_0$ is elastic modulus in the x direction and $E$ is tangent modulus after the yield point.

In the above equations, we need to determine $\sigma_0, E_0, R_{xx}, R_{yy}, R_{yy}, R_{yz}$ and $R_{xz}$. Specifically, the in-plane material parameters $\sigma_0, E_0, R_{xx}, R_{yy}, R_{yy}$ were derived from the results of tensile testing, while the values of $R_{xz}, R_{yz}, R_{xy}$ are related to the values of $\sigma_0^v, \sigma_0^v, \sigma_0^v, \sigma_0^v$, respectively, which are the yield stress in $z, xy, yz$ and $xz$ direction, respectively. According to [8], $\sigma_0^v$ was about 0.003-0.007 Gpa, while $\sigma_0^v, \sigma_0^v, \sigma_0^v$ are about 0.003 to 0.011 Gpa. In this work, for the liner and core, $\sigma_0^v$ was set to be 0.007Gpa and 0.004Gpa, and $\sigma_0^v, \sigma_0^v, \sigma_0^v$ were all set to be 0.011 and 0.004 for the first estimate, respectively. Further, $R_{xx}, R_{yy}, R_{yy}, R_{xz}$ were determined according to Equation (8). In order to get the result as close as possible to the experimental result, we also choose different material parameters corrugated cardboard for testing. Based on the trial and error procedure, we obtain the plastic material parameters for the finite model. Finally, these plastic parameters are listed in Table 1.
Table 1. Elastic and Plastic material parameters of the liner and core of the corrugated board

| Plastic material property | Liner | Core | Elastic material property | Liner | Core |
|---------------------------|-------|------|---------------------------|-------|------|
| $\sigma_0 (Gpa)$          | 0.030 | 0.011| $E_x (Gpa)$               | 3.200 | 5.000|
| $E_x (Gpa)$               | 2.500 | 0.100| $E_y (Gpa)$               | 0.016 | 0.025|
| $R_{xx}$                  | 0.350 | 0.300| $v_{xy}$                  | 0.340 | 0.340|
| $R_{yy}$                  | 0.230 | 0.300| $\nu_{yx}$                | 0.010 | 0.010|
| $R_{zz}$                  | 0.635 | 0.630| $\nu_{xz}$                | 0.010 | 0.010|
| $R_{zx}$                  | 0.635 | 0.630| $G_{xy}(Gpa)$             | 1.000 | 1.000|
| $R_{zy}$                  | 0.635 | 0.630| $G_{yx}(Gpa)$             | 0.058 | 0.050|
| $R_{yz}$                  | 0.010 | 0.010| $G_{zx}(Gpa)$             | 0.057 | 0.005|

2.3 Finite element model

Generally, the geometry of the core can be approximately as the sinusoidal shape. The comparison of the sinusoidal shape and actual shape is shown in Figure 2 [9]. However, the actual shape was considered in this study, due to the higher accuracy of the model. The dimensions of the cardboard for a single core were: the width is 7.6mm, the flute length is 7.6mm, the height is 3.6mm and the thickness of the liner and core is 0.23mm. And we use the share the same node to model the connected nodes between the liner and the tip of the core.

![Figure 2. The actual shape and sinusoidal shape](image)

Considering the width effect, we choose the shell element to represent the corrugated cardboard. By definition, a shell is a geometric form where the length and the width are of the same order of magnitude but the thickness of the element is considerably small in comparison with the length and width dimensions. Apparently, the element shell can be equivalent to the liner and core of the corrugated cardboard. We selected the shell 181 element in ANSYS. Because this type element includes reduced integration schemes, and can improve the computationally efficient. In order to simplify the model, we propose the following assumption of the boundary conditions and loads: the friction between the machine and the cardboard is so large that the upper and lower lines can't move in the horizontal direction. This assumption consists with the test result. Loads were applied on the finite element model as shown in Figure 3. The uniform vertical displacement load that applied on the top liner of the corrugated cardboard promises the compressive loads is uniform. As for the boundary condition, the top liner of the corrugated cardboard is constrained in all direction, and the bottom line is constrained in all direction. In addition, the symmetry boundary condition is performed so that a quarter finite element model can represent the full size finite element model. To be specific, the two edges of the corrugated cardboard are constrained only in the horizontal direction because of the symmetry.

![Figure 3. Compressive loads and boundary condition](image)
3. Results and discussions
The length of the corrugated cardboard that this paper studies is 60.8mm (8 flute length), and the width is 38mm. But the length of the finite element model is 30.4mm (4 flute length) and width is 19mm due to the symmetry, as shown in Figure 4. The finite element model result and measurement are shown in Figure 5. The finite element model result has excellent coherence to the measurement result until to 0.76mm. Because the plastic model property is not accurately available under the larger displacement. So that not all parameters in the model can be readily determined, which is a challenge and considered for future work.

![Figure 4. Geometry of FE model for specimen](image)

![Figure 5. Comparison of the measurement results and FEM results](image)

4. Conclusions
This paper presented a study of the improvement of the finite element model of the corrugated cardboard. An improved finite element model of the corrugated cardboard with a nonlinear orthotropic material constitutive model was presented in ANSYS environment. In particular, the peak load of the corrugated cardboard can be predicted. The experimental validation was conducted, which has shown that the improved finite element model has better accuracy for stiffness prediction than the models in literature.

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