Interpolatory-Based Data-Driven Pulsed Fluidic Actuator Control Design and Experimental Validation

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Abstract—Pulsed fluidic actuators (PFAs) play a central role in the fluid flow experimental control strategy to achieve better performances of aeronautic devices. In this brief, we demonstrate, through an experimental test bench, how the interpolatory-based Loewner data-driven control (L-DDC) framework is an appropriate tool for accurately controlling the outflow velocity of this family of actuators. L-DDC combines the concept of ideal controller with the Loewner framework in a single data-driven rationale, appropriate to experimental users. The contributions of this brief are, first, to emphasize the simplicity and versatility of such a data-driven rationale in a constrained experimental setup and, second, to solve some practical fluid engineers concerns by detailing the complete workflow and key ingredients for successfully implementing a PFA controller from the data acquisition to the control implementation and validation stages.

Index Terms—Data-driven control, flow control, fluidic pulsed actuator, interpolatory methods, Loewner.

I. INTRODUCTION

The design of active closed-loop flow controllers constitutes an important field of research in fluid mechanics. Active flow control is considered in many applications among which flows over open cavities (see [1]) and backward-facing steps, boundary layer flows (see [2]), flows over airfoils, or in combustion processes (see [3], [4]). The possible objectives are to maintain laminarity or delay transition to turbulence, decrease turbulence level, reduce noise, increase lift and decrease drag, and enhance mixing and heat release. Without detailing the specificity and the control methodology employed in each case, in most of these contributions and the references therein, both the sensor(s) and the actuator(s) are supposed to be lumped and ideal. By this, one intends that the sensors are capable to deliver instantaneous and accurate measurements, while the actuators are able to deliver the exact control signals computed by the flow controller (e.g., with no delay, no noise, continuous control signal, and unbounded intervals). These developments are relevant for academic and methodological purposes. However, in order to move toward experimental applications and expect real-life validations, it is essential to consider a realistic setup instead of these idealized versions. Therefore, accurate consideration of the actuator-sensor combination is absolutely necessary. This constitutes the core of this brief.

Among the different flow actuator technologies available, which may be classified as mechanical (e.g., surfaces using electrical, hydraulic, and morphing actuation) or fluidic (e.g., pulsed or continuous blowing, and synthetic jet), the pulsed fluidic actuator (PFA) stands as a simple, ergonomic, economic and affordable solution. The latter is therefore widely used in fluid mechanics for control application. The most widely used PFA types are valves that control the output mass flow rate/pressure issued from a reservoir at rest with fixed pressure and temperature. A considerable amount of studies can be found in the literature on the attempts, with more or less success, to control flows in various applications. While their response time and operating frequency can be high enough compared to the main characteristic time of the flow to manipulate (at least one order of magnitude larger), it is noteworthy that this class of actuators is mainly used to excite the flow within the receptivity range of its most unstable modes, namely the large-scale structures. Their level of authority is, however, restricted since the PFA alone, acts on the flow on a limited spatial domain. In practical applications, these actuators are therefore usually installed in arrays to provide enough energy to gain authority over the considered flow region and mechanism to be controlled. Besides this, one major drawback is that they work either completely open or closed and are thus ON/OFF systems (see Fig. 1). To be integrated into a global flow control scheme, these actuators may be equipped with sensors measuring the current outflow velocity and be accurately controlled at high frequency with an inner controller so that the velocity of the blown air follows the reference control signal provided by the outer controller. Indeed, these actuators are not ideal as usually assumed, presenting asymmetric dynamics and noise, as shown in Fig. 1. Based on the above comments, there is a need for providing practitioners and applied fluid engineers a systematic and simple approach to design a controller tailored to PFA devices, allowing to track a given reference signal fed by an outer flow controller. As mentioned previously, in practice, multiple PFAs are generally employed in parallel. Due to manufacturing and installation versatility, some

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discrepancies in the characteristic responses are most of the time observed between the PFAJs installed. One additional need is, therefore, to provide a methodology that can be easily applied on multiple installed devices [5].

The main objective of the present contribution is, therefore, to provide practitioners and experimental researchers a simple but yet effective methodology and practical workflow to implement pulsed fluid actuators flow controllers directly from experiments. This is achieved through the Loewner data-driven control (L-DDC) algorithm. This technique relies on the definition of an ideal controller, derived from a reference model [6], [7], which allows to use the Loewner framework [8] to construct a reduced-order controller, simple to implement. While the contribution is more methodological rather than theoretical, authors believe that the interpolatory-based data-driven control design process presented in this brief solves practical problems faced by practitioners, namely, the complete PFA controller design using open-loop data collected directly on the experimental setup only. This approach has proved to be effective on infinite-dimensional systems [9] for numerical control [10] and relates to data-driven stability analysis [17].

The major benefit of this data-driven rationale is that the control design only requires one single set of open-loop data collected on the actuator, allowing to account for variations and discrepancies from an actuator to another and thus to design a control law tailored to each system. Instead of spending time and energy in an identification and control design process, the data-driven approach is less costly to deploy in practice and tailored to each actuator. In addition, as the proposed workflow does not require any optimization iteration, its application remains easy in an experimental context. Finally, given a set of open-loop data and an objective closed-loop specification, the proposed process automatically finds the controller structure and gains.

The control and experimental setup and closed-loop architecture are described in Section II. Section III recalls the interpolatory-driven L-DDC design approach and details the key steps to follow for proper implementation. Section IV is devoted to the illustration and analysis of the obtained experimental results. Conclusions and comments finally close this brief in Section V.

II. EXPERIMENTAL SETUP AND CLOSED-LOOP DESCRIPTION

A. Overall Control Setup

A schematic of the inner control loop is given in Fig. 2. From a control engineer perspective, the flow control may be referred to as the outer loop (out of this brief scope), while the PFA controller is referred to as the inner loop. Following Fig. 2, we are interested in the inner loop only and more specifically in the control design, implementation, and validation on a real PFA. With reference to Fig. 2, \( r(t_k) \) denotes the reference signal (typically the control signal provided by an outer controller) at time \( t_k \). \( \tilde{y}(t_k) \) denotes the measurement signal at time \( t_k \) (typically the instantaneous velocity measured at the output of the PFA), and \( e(t_k) = r(t_k) - \tilde{y}(t_k) \) denotes the error signal at time \( t_k \). Then, \( \tilde{u}(t_k) \) is the reference control signal computed at time \( t_k \) by the sampled-time controller \( K(z) \) to be designed. Then, \( u(t_k/N) \) is the ON/OFF control signal, at time \( t_k/N \), modulated in pulsewidth by the pulsed width modulation (PWM) block. The picture in Fig. 2 represents (left) the top view of the PFA and (right) the pressure sensor (PS), sensing \( y(t_k/N) \), here velocity, at time \( t_k/N \), which is then downsampled and averaged at time \( t_k \), leading to \( \tilde{y}(t_k) \). As shown in Fig. 2, two sampling frequencies denoted as \( f_{s1} \) and \( f_{s2} = Nf_{s2} (N \in \mathbb{N}) \), referring to sampling times \( t_k/N \) and \( t_k \), respectively, are required to implement the controller. Details of this multisampling setup are given later in this section.

B. Experimental Setup

The experimental test bench is composed of a PFA and a hot-wire probe (PS) as shown on the photograph in Fig. 2. The acquisition and generation of the command laws are carried out due to a real-time hardware system (National Instruments NI PXI-1042 mainframe).

The PFA used in the present study is an SX11F-BH microvalve from SMC, which can operate, according to the manufacturer, up to 1 kHz at a mean flow rate of 50 L/min. The asymmetric response times at opening and closing, again according to the manufacturer, are 0.55 and 0.4 ms, respectively. For the present purpose, the valve is alimented with compressed air at constant pressure (2 bar). A short tube of 5 mm diameter is used at the valve exit. The instantaneous velocity at 3-D downstream the tube exit is surveyed due to a hot-wire probe (Dantec 55P11) connected to a 55M10 DISA constant temperature anemometer (CTA). Note that since the purpose of this brief is to demonstrate the potential of the methodology introduced, no calibration of the hot-wire probe is effected. The voltage of the hot-wire probe is, therefore, directly considered as representative of the exit velocity and used as the output signal \( y(t_k/N) \). From an application point of view, PFA equipped with outflow velocity sensors needs to be...
developed (here, an external hot-wire probe system is used). This is currently being investigated in the Micro Opto Electro Mechanical Systems (MEMS) community.

C. PWM and Average Blocks

As the PFA works with only ON/OFF control, a PWM function has naturally been implemented on the software side of the setup to transform the sampled-time signal $\overline{u}(t_k)$ provided by the controller $K(z)$ into $u(t_k/N)$, a stair signal with varying length (or duty cycle $D$). In addition, in order to consider for noise measurement, since the probe sensor is not ideal, an average block is introduced, which operates on the measurement signal. The following presents the PWM and Average blocks in detail.

1) PWM Block: The PWM block uses a rectangular impulse signal taking values between $u_{\text{min}}$ and $u_{\text{max}}$, whose length is modulated. This modulation results in variation of the mean and a duty cycle $D \in [0, 1]$, the averaged value $u^{avg}(t_k)$ of the resulting signal is given by

$$u^{avg}(t_k) = \frac{1}{T_{s2}} \int_0^{T_{s2}} \overline{u}(t_k) dt_k$$

$$= \frac{1}{T_{s2}} \left( \int_0^{DT_{s2}} u_{\text{max}} dt_k + \int_{DT_{s1}}^{T_{s2}} u_{\text{min}} dt_k \right)$$

$$= Du_{\text{max}} + (1 - D)u_{\text{min}}$$

where $T_{s1} = 1/f_{s1}$ and $T_{s2} = 1/f_{s2}$. Obviously, the PWM should be sampled at rate $N$ times higher than that of the signal $u(t_k)$ to be modulated (with $N \in \mathbb{N}$). In practical applications, a simple way to generate the PWM is to use the intersection method that simply requires a saw-tooth carrier signal denoted $u_c(t)$, with frequency $f_{s2}$ and amplitude from $u_{\text{min}} = \min \overline{u}(t_k)$ to $u_{\text{max}} = \max \overline{u}(t_k)$, which should be compared to the incoming signal $\overline{u}(t_k)$. When $u_c(t_k) > \overline{u}(t_k)$, then $u(t_k/N) = u_{\text{max}}$, and $u(t_k/N) = u_{\text{min}}$ otherwise. Note that in the present case, the carrier signal has the same frequency as the control signal $\overline{u}(t_k)$.

2) Average Block: The average block consists in averaging the measurement value $y(t_k/N)$ over $N$ past samples. The resulting output is a downsampling signal $\overline{y}(t_k)$, which represents the averaged output to be controlled and should track $r(t_k)$. The main purpose of such block is to partially filter the measurement noise, making the averaged value $\overline{y}(t_k)$ more representative of the output state than the instantaneous value $y(t_k/N)$.

Remark 1 (About $N$): For some applications, the parameter $N$ is dictated by the hardware and setup. However, most of the time, it can be chosen by the user and is thus an additional tuning parameter. A rule of thumb is to choose $N \geq 10$ to ensure that the PWM block will be able to translate the control signal into an accurate binary output. Still, large $N$ leads to too strong filtering of the measured signal and may result in irrelevant behavior. On the other side, low $N$ may lead to noisy data and thus inaccurate data-driven controller design. Consequently, authors advise users to use this parameter accordingly to the setup limitations and use it as a tradeoff.

D. Signals Characteristics and Specifications

Now, the experimental setup and PWM/Average blocks have been presented, and let us summarize the characteristics of the signal as follows.

1) $r(t_k)$ (sampled at $f_{s2}$) is the reference signal to be tracked. This signal is continuous and is fed by the outer control. For the considered future application of turbulence control, its bandwidth is below $f_c = 5$ Hz\(^1\)

The knowledge of this signal characteristics is essential in the construction of the objective performance $M_k(s)$ (see Section III).

2) $y(t_k/N) \in [y_{\text{min}}, y_{\text{max}}]$ (sampled at $f_{s1}$) is the measurement signal obtained by the PS. It allows measuring the airflow velocity at the exit of the actuation. Note that the distance of the probe from the PFA exit has an impact on the control law by adding a delay in the time response of the device. This is an additional reason justifying for the data-driven approach: the design can be reproduced easily for different distances of/and different probe sensors.

3) $\overline{y}(t_k) \in [y_{\text{min}}, y_{\text{max}}]$ (sampled at $f_{s1}$) is the averaged and downsampled value of $y(t_k/N)$ on blocks of duration $1/f_{s1}$.

4) $\overline{u}(t_k) \in [0, u_{\text{max}}]$ (sampled at $f_{s2}$) is the control signal provided by the PFA controller to be modulated by

\(^1\)Note that this bandwidth may be considered as way too slow for fluid engineer. Still, this does not affect the method and higher bandwidth will be considered in future works.
the PWM block. The sampling frequency of this signal determines the carrier and frequency of the PWM.

5) \( u(t_{i,N}) \in [0, u_{\text{max}}] \) (sampled at \( f_{s1} \)) is the effective modulated (pulsed) control signal sent to the PFA.

In our configuration, the controller \( K(z) \) and the PWM modules, respectively, run at frequency \( f_{s2} = 100 \text{ Hz} \) and \( f_{s1} = 1000 \text{ Hz} \) (thus, \( N = 10 \) is the PWM multiplicity factor). Now, the setup and signals characteristics have been introduced, and let us describe the main result, namely the workflow adopted to obtain the controller \( K(z) \) structure and gains.

The method proposed in this brief, detailed in Section III, aims at designing a controller \( K(z) \) to improve the tracking performances of the couple PFA-PS. Indeed, as this system is stable and minimum phase, the system does not need to be stabilized. The emphasis will therefore be put on enhancing closed-loop performances while maintaining stability.

III. MAIN RESULT: DATA-DRIVEN FLUIDIC ACTUATOR CONTROL DESIGN

A. Overview of the Approach and Control Objective

Considering the closed-loop structure of Fig. 2, the objective is to track a reference signal \( r(t_k) \) up to a frequency of \( f_r = 5 \text{ Hz} \), being the maximal frequency of the outer loop (not detailed here). To this aim and for the simplicity and flexibility of the implementation, a frequency-domain interpolation-based data-driven approach has been chosen. The employed workflow is summed up in Algorithm 1 and the key steps are described in the remaining of this section.

B. Open-Loop Data Acquisition and Frequency Response Construction

The first three steps of Algorithm 1 are now detailed and commented. These steps consist in exciting the system to obtain a relevant open-loop frequency response.

1) Construction of a PRBS Exciting Signal: Open-loop data can be obtained using a pseudorandom binary sequence (PRBS) signal as input of the system. This signal consists in an ON/OFF sequence of varying length. A "random" sequence of these signals, whose duration should last enough to reach steady state output and short enough to excite frequencies above the cutoff one of the system, is defined. The Fourier transform of the sequence \( u_{\text{prbs}}(t_k/N) \) and the corresponding output \( y_{\text{prbs}}(t_k/N) \) signals, one computes the averaged signals \( \overline{u}_{\text{prbs}}(t_k) \) and \( \overline{y}_{\text{prbs}}(t_k) \) and their Fourier transform \( \tilde{u}_{\text{prbs}}(\omega) \) and \( \tilde{y}_{\text{prbs}}(\omega) \), respectively (\( i = (-1)^{1/2} \)). Then, the cross correlation transfer of these signals is used to construct \( \Phi_i \) as

\[
\Phi_i(\omega) = \frac{\tilde{y}_{\text{prbs}}(\omega_i)}{\tilde{u}_{\text{prbs}}(\omega_i)} \quad \text{(4)}
\]

In opposition to the spectral energy density, the cross (or intercorrelated) spectral density is a complex number which gain represents the interaction power and which arguments represents the phase between \( \overline{u}_{\text{prbs}}(t_k) \) and \( \overline{y}_{\text{prbs}}(t_k) \). Now, the open-loop data have been collected and treated, and the L-DDC process is deployed [9], [11].

2) Frequency Transfer Construction: Based on the input \( u_{\text{prbs}}(t_k/N) \) and corresponding output \( y_{\text{prbs}}(t_k/N) \) signals, one computes the averaged signals \( \overline{u}_{\text{prbs}}(t_k) \) and \( \overline{y}_{\text{prbs}}(t_k) \) and their Fourier transform \( \tilde{u}_{\text{prbs}}(i\omega) \) and \( \tilde{y}_{\text{prbs}}(i\omega) \), respectively (\( i = (-1)^{1/2} \)). Then, the cross correlation transfer of these signals is used to construct \( \Phi_i \) as

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\Phi_i(\omega) = \frac{\tilde{y}_{\text{prbs}}(i\omega_i)}{\tilde{u}_{\text{prbs}}(i\omega_i)} \quad \text{(4)}
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C. Loewner Data-Driven Control

The L-DDC algorithm allows to design a controller on the basis of frequency-domain data from the system to be controlled. As presented in [11] and [12], the L-DDC approach covers the choice of the reference model, the definition of the ideal controller (step 2, bullet 1), and its interpolation (step 2, bullet 2) and reduction based on the Loewner framework (step 2, bullet 3), but also a data-driven stability analysis of
the resulting closed-loop. These different aspects of the L-DDC control design are detailed hereafter for the proposed application.

**Remark 2 (Continuous Versus Sampled-Time L-DDC):** Note that, in the end, a continuous controller \( \hat{K}(s) \) is obtained and then discretized to obtain the control law \( K(z) \). A hybrid version of the L-DDC, allowing to obtain directly a discrete controller for a continuous system, also exists and can be found in [10], but has not been used here for practical reasons (remove work and work decoupling).

1) **Reference Model Specification:** As detailed in [9], the choice of the specifications is a key aspect of the L-DDC procedure and can be a difficult task for a practitioner.

2) **Ideal Controller Definition:** As detailed in [9], the choice of the specifications is a key aspect of the L-DDC procedure and can be a difficult task for a practitioner.

3) **Full and Reduced Controller Design via Loewner:** The Loewner framework allows to find a rational LTI model \( K_{\text{full}} \) achieving (7). Recent descriptions of the Loewner realization landmark by Mayo and Antoulas [8]. In short, let the interpolation points \( i \omega_k \) be divided in two equal subsets as follows (\( \lambda_i \in \mathbb{C} \) and \( \mu_j \in \mathbb{C} \)):

\[
\{z_k\}_{k=1}^{2m} = \{\mu_j\}_{j=1}^m \cup \{\lambda_i\}_{i=1}^m = \{i \omega_k\}_{k=1}^N.
\]

The Loewner realization framework in [8] consists in building the Loewner \( L \in \mathbb{C}^{m \times m} \) and shifted Loewner \( L_{\sigma} \in \mathbb{C}^{m \times m} \) matrices defined as follows (in the single input single output case), for \( i = 1, \ldots, m \) and \( j = 1, \ldots, m \):

\[
[L]_{j,i} = \frac{K^*(\lambda_i) - K^*(\lambda_j)}{\mu_j - \lambda_i}.
\]

Then, the model \( K_M \) given by the following descriptor realization:

\[
E^m \delta(x(t)) = A^m x(t) + B^m u(t) \quad \text{and} \quad y(t) = C^m x(t)
\]

(10)

where \( E^m = \alpha L, A^m = \alpha L_{\sigma}, [B^m]_k = K^*(\mu_k), \) and \([C^m]_k = K^*(\lambda_k)\) (for \( k = 1, \ldots, m \)) and whose related transfer function

\[
K^m(\xi) = C^m(\xi E^m - A^m)^{-1} B^m
\]

(11)

satisfies (7). In (10), \( \delta(\cdot) \) denotes the derivative operator in the continuous time and the forward one in the sampled time. Similarly, \( \zeta \) is the Laplace variable in the continuous case and the \( z \) operator in the sampled case. Assuming that the number \( 2m = N \) of available data is large enough, then it has been shown in [8] that a minimal model of dimension \( n \leq m \) that still interpolates the data can be built with a projection of (10) provided that, for \( k = 1, \ldots, 2m \), rank of \( (z_k L - L_{\sigma}) = \text{rank}((L, L_{\sigma}) = \text{rank}((H, L_{\sigma}^H) = n \) in that case, let us denote by \( Y \in \mathbb{C}^{m \times n} \) the matrix containing the first \( n \) left singular vectors of \( [L, L_{\sigma}] \) and \( X \in \mathbb{C}^{m \times m} \) the matrix containing the first \( n \) right singular vectors of \( \{L^H, L_{\sigma}^H \}^H \).

Then, \( K_{\text{full}} \) (\( Y^H E^m X, Y^H A^m X, Y^H B^m, C^m X \)) is the minimal McMillan degree rational function also satisfying the interpolation conditions (7) (see details in [15]).

However, \( K_{\text{full}} \) is often of very high order and such a controller would be too complex to be implemented. In the present case, the interpolation gives a high-order minimal realization, \( n = 290 \), principally due to noisy data.

Similar to the rank truncation performed above, the Loewner framework allows to control the complexity of the identified controller. It is possible to obtain a \( r \)th order reduced controller \( \hat{K} \) of the minimal realization \( K_{\text{full}} \) by applying the
singular value decomposition (SVD) on the Loewner pencil and truncating with a user-defined order $r < n$. The order of the controller becomes a parameter that can be tuned by the user, accordingly to the Loewner singular value decay. In the present case, the singular value decomposition SVD decay of the associated Loewner pencil is given in Fig. 3 (see [16] for practical details and [17, Ch. 2] for some applications).

Fig. 3 suggests an order $r = 3$ as it provides almost 80% singular value contribution (i.e., $\sigma_1 + \sigma_2 + \sigma_3 \approx 0.8 \sum_{i=1}^{n} \sigma_i$). However, selecting an order $r = 1$ still achieves to 50% of the information $\sigma_1 \approx 0.5 \sum_{i=1}^{n} \sigma_i$. Such choice leads to a pure integral control action, which discrete-time model reads

$$K(z) = \frac{k_j}{z - 1} \quad (12)$$

where $k_j \in \mathbb{R}_+$ is the integral gain, computed by the proposed L-DDC procedure. Fig. 4 shows the responses of the ideal controller, as well as the reduced integral controllers obtained with $r = 1$ (continuous and discrete), and compare it to the order $r = 3$ one. Note that the control order remains a parameter that the designer can choose according to its implementation limitations. Here, the simple integral choice has been considered as a good tradeoff between complexity and performances.
4) Preliminary Verification: The reduction of the controller model implies that the closed-loop, denoted $T_K$, obtained when inserting the reduced and sampled-time $K$ controller, will not be equal to the desired reference model $M$. Therefore, it is interesting to compute the frequency response of the expected closed loop $T_K$ on the basis of the available open-loop data $(t_{o_i},\Phi_i)_{i=1}^N$ from the system as follows, $\forall i = 1, \ldots, N$:

$$T_K(t_{o_i}) = \left(I + \Phi_i K(e^{i\omega_i T_s})\right)^{-1}\Phi_i K(e^{i\omega_i T_s}).$$

(13)

The result of the expected closed loops for three different controllers is shown later in Fig. 6. Note that for $r = 3$, authors considered that the gain was not important enough with respect to experimental benefits of such a simple integral structure and is therefore not illustrated.

5) About Stability: The choice of the reference model presented earlier ensures that the ideal controller stabilizes the system internally. However, there is no stability guarantee regarding the use of a reduced-order controller. In [9], the small-gain theorem is derived in a data-driven way; it is shown that limiting the controller modeling error during the reduction step ensures internal stability. This stability test is very conservative and another solution would be to use the projection-based technique mentioned in [13] or the one in [17] to conclude regarding the stability of the closed loop. Both approaches are data-driven one and may be used to guess the stability. Note also that in the single input-single output case, the Nyquist criteria can also be considered.

IV. EXPERIMENTAL RESULTS AND ANALYSIS

A. Considered Configurations

Based on the presented control design rationale and preliminary validation procedures presented in Section III, we are now ready to implement and validate the control setup on the real experimental test bench. The real-time acquisition and control functions, including the control law $K(z)$, the PWM, and Average blocks as shown in Fig. 2, are implemented within a LabView interface. For the considered flow system, three different controllers denoted $K_{fj}(z)$ with the same structure as in (12), sampled at $f_s = 100$ Hz, corresponding to three different performance objectives $M_{fj}(s)$ as defined in (5), for $j = \{1, 3, 5\}$, are constructed. These may be read as follows.

1) $K_{f1}$, where $k_1 = 0.0462$ ($M_{f1}$ with $f_1 = 1$ Hz).
2) $K_{f3}$, where $k_3 = 0.1385$ ($M_{f3}$ with $f_3 = 3$ Hz).
3) $K_{f5}$, where $k_5 = 0.2309$ ($M_{f5}$ with $f_5 = 5$ Hz).

For the considered application, following Fig. 2 notations, $N = 10$ and, thus, $f_{x1} = 1000$ Hz were chosen. The closed-loop performances are first validated with a sine sweep signal and second, with a more realistic reference trajectory.

B. Closed-Loop Sine Sweep Reference Signal

We first apply a reference signal $r(t_k)$ being a frequency sweep from 0.1 to 30 Hz, of duration 1000 s and amplitude ranging from the minimal to the maximal possible values of the system. The mean values $\bar{y}(t_k)^{3}$ for all three configurations and time-domain responses are first reported in Fig. 5.

The averaged output flow velocity well tracks the reference signal considered. In all three cases, the low frequencies are well tracked, which is already an important result for PFA systems and in view of fluid flow control. One can also observe that the tracking is more accurate in high frequencies with the more aggressive controller $K_{f5}$, synthesized to target a bandwidth of $f_5 = 5$ Hz, validating the reference model approach of the data-driven procedure. Fig. 6 compares the experimental frequency-domain responses from $r$ to $\bar{y}_j$, denoted $T_{r\bar{y}_j}$, with $T_{K_{fj}}$, the expected closed loop computed in (13). For each controllers, the resulting closed loop $T_{r\bar{y}_j}$ provides a

3Signal $\bar{y}(t_k)$ refers to $\bar{y}(t_k)$ in Fig. 2, for the $j$th configuration ($j = \{1, 2, 3\}$).
similar frequency response trends as the one expected and illustrated with $T_{K_{\text{h}}}$. Some differences persist but can fairly be attributed to the nonlinear and nonsymmetric nature of the pulsed actuator. Indeed, as shown in Fig. 1 and shown later in the section; when a reference amplitude close to zero is requested, some stick-like behaviors are observed, leading to a loss of accuracy. Still, the obtained control law designed solely through the lens of a pure data-driven approach provides the required performances.

C. Closed Loop Using Realistic Reference Signals

Let us demonstrate the efficiency of the proposed control with a more realistic reference signal $r(t_k)$. Fig. 7 shows the tracking performance of the mean output $y(t_k)$ and control $u(t_k)$ signals using the first controller $K_{\text{h}}$. Clearly, Fig. 7 exposes really satisfactory results in terms of tracking. As previously pointed, one can remark that some chattering artifact and difficulties appear when tracking in low amplitude references. This observation can be correlated with the asymmetric actuator characteristic, which produces a stick-like behavior in the valve opening. Even though not in the scope of this study, one way to limit this effect is to add high-frequency noise in the control signal to avoid the problem, but at the price of an actuator fatigue (note that the approach is similar to solutions used in friction control). The averaged control signal $\overline{u}(t_k)$ is also reported. Interestingly, a simple integral controller action, obtained from a single open-loop data collection, using only ON/OFF signals, allows tracking a complex reference signal with a good accuracy.

V. CONCLUSION AND DISCUSSION

One underlying objective of this brief was to bridge the gap between fluid experts and control engineers by providing practitioners a simple way to adjust a control law for a pulsed (ON/OFF) fluidic actuator, without spending too much energy in a time-consuming identification control process. To this aim, a frequency-domain data-driven approach (celebrated as L-DDC) is used. Such a procedure, which solely relies on a single experimental data set, allows to find the order and controller gains tailored to the system under consideration. The complete approach has been applied and validated through an experimental setup. As an interesting result, a simple integral action was shown to be enough for ensuring such a tracking task. We believe that the proposed workflow presents a valid alternative to the complex identify and control approach, especially in the experimental wind tunnel context where experiments are expensive and time is limited. In the coming steps of the project, 96 similar PFAs and their associated (integral) control will be installed and used over a complete 1-m wingspan. These 96 actuators may be lumped as a single one with a given bandwidth in order to control a flow phenomenon over a given geometry, through an outer loop control law. Connections with positive systems are also under investigation to integrate actuators’ limitations [18].

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4In Fig. 7, indices are removed and notations are the one of Fig. 2.

5Supplementary video material is also available on the first author’s webpage. https://sites.google.com/site/charlespoussotvassal/visual